Nearly Autoparallel Maps, Tensor Integral and Conservation Laws on Locally Anisotropic Spaces

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Abstract

We formulate the theory of nearly autoparallel maps (generalizing conformal transforms) of locally anisotropic spaces and define the nearly autoparallel integration as the inverse operation to both covariant derivation and deformation of connections by nearly autoparallel maps. By using this geometric formalism we consider a variant of solution of the problem of formulation of conservation laws for locally anisotropic gravity. We note that locally anisotropic spaces contain as particular cases various extensions of Kaluza–Klein, generalized Lagrange and Finsler spaces.
I. INTRODUCTION

Theories of field interactions on locally anisotropic curved spaces form a new branch of modern theoretical and mathematical physics. They are used for modelling in a self-consistent manner physical processes in locally anisotropic, stochastic and turbulent media with beak radiational reaction and diffusion [1-8]. The first model of locally anisotropic space was proposed by P. Finsler [9] as a generalization of Riemannian geometry; here we also cite the fundamental contribution made by E. Cartan [10] and mention that in monographs [5-7] detailed bibliographies are contained. In this work we follow conventions of R. Miron and M. Anastasiei [1,2] and base our investigations on their general model of locally anisotropic (la) gravity (in brief we shall write la-gravity) on vector bundles, v–bundles, provided with nonlinear and distinguished connection and metric structures (we call a such type of v–bundle as a la-space if connections and metric are compatible).

The study of models of classical and quantum field interactions on la-spaces is in order of the day. For instance, in papers [11,12] the problem of definition of spinors on la-spaces is solved and some models of locally anisotropic Yang–Mills and gauge like gravitational interactions are analyzed (see alternative approaches in [5,8]). The development of this direction entails great difficulties because of problematical character of the possibility and manner of definition of conservation laws on la-spaces. It will be recalled that, for instance, conservation laws of energy–momentum type are a consequence of existence of a global group of automorphisms of the fundamental Mikowski spaces (for (pseudo)Riemannian spaces one considers automorphisms on tangent bundle and particular cases when there are symmetries generated by existence of Killing vectors). No global or local automorphisms exist on generic la-spaces and in result of this fact the formulation of la-conservation laws is sophisticated and full of ambiguities. R. Miron and M. Anastasiei firstly pointed out the nonzero divergence of the matter energy-momentum tensor, the source in Einstein equations on la-spaces, and considered an original approach to the geometry of time–dependent Lagrangians [1,2,13]. Nevertheless, the rigorous definition of energy-momentum values for la-gravitational and
matter fields and the form of conservation laws for such values have not been considered in present–day studies of the mentioned problem.

Our aim is to develop a necessary geometric background (the theory of nearly autoparallel maps and tensor integral formalism on la-multispaces) for formulation and a detailed investigation of conservation laws on la-spaces.

The question of definition of tensor integration as the inverse operation of covariant derivation was posed and studied by A.Moór [14]. Tensor–integral and bitensor formalisms turned out to be very useful in solving certain problems connected with conservation laws in general relativity [15,16]. In order to extend tensor–integral constructions we have proposed [17,18] to take into consideration nearly autoparallel [19-21] and nearly geodesic [22] maps, na– and ng–maps, which forms a subclass of local 1–1 maps of curved spaces with deformation of the connection and metric structures. A generalization of the Sinyukov’s ng–theory for spaces with local anisotropy was proposed by considering maps with deformation of connection for Lagrange spaces (on Lagrange spaces see [23,1,2]) and generalized Lagrange spaces [24-27]. Tensor integration formalism for generalized Lagrange spaces was developed in [28,29]. One of the main purposes of this paper is to synthesize the results obtained in the mentioned works and to formulate them for a very general class of la–spaces. As the next step the la–gravity and analysis of la–conservation laws are considered.

We note that proofs of our theorems are mechanical, but, in most cases, they are rather tedious calculations similar to those presented in [22,20,30]. We sketch some of them in Appendixes A and B.

In Sec. II we present some basic results, necessary for our further considerations, on nonlinear connections in bundle spaces [1,2]. Sec. III is devoted to the formulation of the theory of nearly autoparallel maps of la–spaces. Classification of na–maps and formulation of their invariant conditions are given in Sec. IV. In Sec. V we define the nearly autoparallel tensor–integral on locally anisotropic multispaces. The problem of formulation of conservation laws on spaces with local anisotropy is studied in Sec. VI. We present a definition of conservation laws for la–gravitational fields on na–images of la–spaces in Sec. VII. Outlook
and conclusions are contained in Sec. VIII.

II. GEOMETRY OF LOCALLY ANISOTROPIC SPACES

As a general model of locally anisotropic space (generalizing the concept of Finsler and Lagrange spaces, see R.Miron and M.Anastasiei monographs [1,2]) we consider a (for simplicity, trivial) v–bundle $\xi = (E, F, \pi, M)$, where $M$ is a differentiable manifold of dimension $n, \dim M = n$, the typical fibre $F$ is a real vector space of dimension $m$ and $\pi : E \to M$ is a differentiable surjection. Local coordinates on the base space $M$ are denoted by $x = (x^i)$ and on the total space $E$ by $u = (x, y) = u^a = (x^i, y^a)$ (we shall use indices $i, j, k, ... = 1, 2, ... n$ for $M$-components, $a, b, c, ... = 1, 2, ... m$ for $F$-components, $y$-coordinates on $F$ can be interpreted as parameters of local anisotropy, and Greek indices $\alpha, \beta, ...$ as general ones on v-bundle $\xi$).

**Definition 1** A nonlinear connection, briefly an N-connection, in tangent bundle $\xi$ is a global decomposition of the tangent bundle $T\xi$ into horizontal $H\xi$ and vertical $V\xi$ subbundles:

$$T\xi = H\xi \oplus V\xi. \quad (1)$$

To an N-connection on $\xi$ one can associate a covariant derivation on $M$:

$$\nabla_Y A = Y^i \{ \frac{\partial A^a}{\partial x^i} + N^a_i(x, A) \} s_a, \quad (2)$$

where $s_a$ are local linearly independent sections of $\xi$, $A = A^a s_a$ and $Y = Y^i s_i$ are vector fields. The differentiable functions $N^a_i$, as functions of $x^i$ and $y^i$, i.e. $N^a_i(x, y)$, are called the coefficients of the N-connection.

In v–bundle $\xi$ we can define a local basis (frame) adapted to a given N–connection:

$$X_\alpha = (X_i = \frac{\delta}{\delta x^i} = \partial_i - N^a_i(x, y) \frac{\partial}{\partial y^a}, X_a = \frac{\delta}{\delta y^a} = \frac{\partial}{\partial y^a}) \quad (3)$$
The dual basis to $X_\alpha$ is written as

$$X^\alpha = (X^i = \delta x^i = dx^i, \ , X^a = \delta y^a = dy^a + N^\alpha_i(x, y)dx^i).$$

By using adapted bases one can introduce the algebra of distinguished tensor fields (d-fields or d-tensors) $T = \bigoplus T^{pr}_{qs}$ on $\xi$, which is the tensor algebra on the vector bundle (v–bundle) $\xi_d$ defined by $\pi_d : H\xi \bigoplus V\xi \rightarrow \xi$. An element $t \in T^{pr}_{qs}$, d–tensor field of type $\begin{pmatrix} p & r \\ q & s \end{pmatrix}$, is written in local form as

$$t = t^{i_1 \cdots i_p \ a_1 \cdots a_r}_{j_1 \cdots j_q \ b_1 \cdots b_s}(x, y) \frac{\delta}{\delta x^{i_1}} \otimes \cdots \otimes \frac{\delta}{\delta x^{i_q}} \otimes dx^{j_1} \otimes \cdots \otimes dx^{j_q} \otimes \frac{\partial}{\partial y^{a_1}} \otimes \cdots \otimes \frac{\partial}{\partial y^{a_r}} \otimes \delta y^{b_1} \otimes \cdots \otimes \delta y^{b_s}.$$

In addition to d–tensors we can consider d–objects with various group and coordinate transforms adapted to the global splitting (1).

**Definition 2** A linear d–connection is a linear connection $^N D$ on $\xi$ conserving at a parallel translation the Whitney sum $H\xi \bigoplus V\xi$ associated to a fixed $N$–connection on $\xi$.

The components $^N \Gamma^\alpha_{\beta\gamma}$ of d–connection $^N D$ are defined in the form :

$$^N D_\gamma X_\beta = ^N D_\gamma X_\beta = ^N \Gamma^\alpha_{\beta\gamma} X_\alpha$$

The torsion $^N T^\alpha_{\beta\gamma}$ and curvature $^N R^\alpha_{\beta\gamma}$ of the connection $^N \Gamma^\alpha_{\beta\gamma}$ can be introduced in a standard manner :

$$^N T(X_\gamma, X_\beta) = ^N T^\alpha_{\beta\gamma} X_\alpha,$$

where

$$^N T^\alpha_{\beta\gamma} = ^N \Gamma^\alpha_{\beta\gamma} - ^N \Gamma^\alpha_{\gamma\beta} + w^\alpha_{\beta\gamma},$$

and

$$^N R(X_\delta, X_\gamma, X_\beta) = ^N R^\alpha_{\beta\gamma} X_\alpha,$$

where
\[ NR_{\beta,\gamma}^{\alpha} = X_\delta N_{\beta,\gamma}^{\alpha} - X_\gamma N_{\beta,\delta}^{\alpha} + N_{\beta,\gamma}^{\varphi} N_{\gamma,\delta}^{\alpha} - N_{\beta,\delta}^{\varphi} N_{\gamma,\varphi}^{\alpha} + N_{\beta,\gamma}^{\alpha} w_{\delta}^{\varphi}. \] (5)

In formulas (4) and (5) we have used the nonholonomical coefficients \( w_{\beta,\gamma}^{\alpha} \) of the adapted frame (3) defined as

\[ [X_\alpha, X_\beta] = X_\alpha X_\beta - X_\beta X_\alpha = w_{\alpha,\beta}^{\gamma} X_\gamma. \] (6)

Now we introduce the metric (fundamental) tensor \( g_{ij}(x, y) \). It is a second order, covariant and nondegenerate d–tensor field on \( M \) (in general \( g_{ij}(x, y) \) is nonhomogeneous in the variables \( y^i \) and it does not need to be generated by a fundamental function and by a Lagrangian, as in Finsler and, respectively, Lagrange geometry). For our purposes we consider v–bundles provided with a metric structure

\[ G(u) = G_{\alpha,\beta} du^\alpha du^\beta \] (7)

being compatible to given N–connection \( N_i^a(u) \) and d–connection \( D \) structures. In this case we can split the metric (7) into horizontal and vertical parts with respect to the locally adapted frame (3):

\[ G = G_{\alpha,\beta} du^\alpha \otimes du^\beta = \]

\[ g_{ij}(x, y) dx^i \otimes dx^j + h_{ab}(x, y) \delta y^a \otimes \delta y^b \] (8)

with components of d–metric \( h_{ab}(x, y) \) defined from relations

\[ G_{ia} - N_i^k h_{ka} = 0. \] (9)

The compatibility conditions with d–connection \( D \) are written as

\[ D_\alpha G_{\beta,\gamma} = 0. \] (10)

The components of a d–connection \( D \) satisfying conditions (10) are denoted as \( \Gamma_{\beta,\gamma}^{\alpha} \). The corresponding formulas for torsion \( T_{\beta,\gamma}^{\alpha} \) and curvature \( R_{\beta,\gamma}^{\alpha} \) are written in a similar manner as (4) and (5) by omitting the left upper index N.
We note that considering a similar to (10) metric structure on the tangent bundle $TM$ being compatible with both $N$–connection and almost Hermitian structure we obtain the so–called almost Hermitian model of generalized Lagrange geometry [1,2], which contains as particular cases the Lagrange and Finsler geometry. Finally, we remark that la–geometric constructions with respect to locally adapted frames are very similar to those for Riemannian spaces with corresponding generalizations for torsion and nonmetricity.

III. NEARLY AUTOPARALLEL MAPS OF LOCALLY ANISOTROPIC SPACES

In this section we shall extend the ng– [9] and na–map [16,4-8,12,13] theory by introducing into consideration maps of vector bundles provided with compatible $N$–connection, $d$–connection and metric structures.

Our geometric arena consists from pairs of open regions $(U, \bar{U})$ of la–spaces, $U \subset \xi, \bar{U} \subset \bar{\xi}$, and 1–1 local maps $f : U \rightarrow \bar{U}$ given by functions $f^\nu(u)$ of smoothly class $C^r(U)$ ($r > 2$, or $r = \omega$ for analytic functions) and their inverse functions $f^\nu(u)$ with corresponding non–zero Jacobians in every point $u \in U$ and $\bar{u} \in \bar{U}$.

We consider that two open regions $U$ and $\bar{U}$ are attributed to a common for $f$–map coordinate system if this map is realized on the principle of coordinate equality $q(u^\alpha) \rightarrow q(u^\alpha)$ for every point $q \in U$ and its $f$–image $\bar{q} \in \bar{U}$. We note that all calculations included in this work will be local in nature and taken to refer to open subsets of mappings of type $\xi \supset U \xrightarrow{f} \bar{U} \subset \bar{\xi}$. For simplicity, we suppose that in a fixed common coordinate system for $U$ and $\bar{U}$ spaces $\xi$ and $\bar{\xi}$ are characterized by a common $N$–connection structure (in consequence of (8) by a corresponding concordance of $d$–metric structure), i.e.

$$N^\alpha_j(u) = \bar{N}^\alpha_j(u) = \overline{\bar{N}}^\alpha_j(u),$$

which leads to the possibility to establish common local bases, adapted to a given $N$–connection, on both regions $U$ and $\bar{U}$. We consider that on $\xi$ it is defined the linear $d$–connection structure with components $\Gamma^{\nu}_{\alpha \beta}$. On the space $\bar{\xi}$ the linear $d$–connection is con-
sidered to be a general one with torsion

\[ T^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} - \Gamma^\alpha_{\gamma\beta} + u^\alpha_{,\beta\gamma}, \]

and nonmetricity

\[ K_{\alpha\beta\gamma} = D_{\alpha} G_{\beta\gamma}. \quad (11) \]

Geometrical objects on \( \xi \) are specified by underlined symbols (for example, \( A^\alpha, B^{\alpha\beta} \)) or underlined indices (for example, \( A_\alpha, B_{\alpha\beta} \)).

For our purposes it is convenient to introduce auxiliary symmetric d–connections, \( \gamma^\alpha_{,\beta\gamma} = \gamma^\alpha_{,\gamma\beta} \) on \( \xi \) and \( \gamma^\alpha_{\beta\gamma} = \gamma^\alpha_{\gamma\beta} \) on \( \xi \) defined, correspondingly, as

\[ \Gamma^\alpha_{,\beta\gamma} = \gamma^\alpha_{,\beta\gamma} + T^\alpha_{,\beta\gamma} \quad \text{and} \quad \Gamma^\alpha_{\beta\gamma} = \gamma^\alpha_{\beta\gamma} + T^\alpha_{\beta\gamma}. \]

We are interested in definition of local 1–1 maps from \( U \) to \( U \) characterized by symmetric, \( P^\alpha_{,\beta\gamma} \), and antisymmetric, \( Q^\alpha_{\beta\gamma} \), deformations:

\[ \gamma^\alpha_{,\beta\gamma} = \gamma^\alpha_{,\beta\gamma} + P^\alpha_{,\beta\gamma}, \quad (12) \]

\[ \gamma^\alpha_{\beta\gamma} = \gamma^\alpha_{\beta\gamma} + T^\alpha_{\beta\gamma}. \quad (13) \]

The auxiliary linear covariant derivations induced by \( \gamma^\alpha_{,\beta\gamma} \) and \( \gamma^\alpha_{\beta\gamma} \) are denoted respectively as \( (\gamma) D \) and \( (\gamma) D \).

Let introduce this local coordinate parametrization of curves on \( U \):

\[ u^\alpha = u^\alpha(\eta) = (x^i(\eta), y^i(\eta)), \ \eta_1 < \eta < \eta_2, \]

where corresponding tangent vector field is defined as

\[ v^\alpha = \frac{du^\alpha}{d\eta} = \left( \frac{dx^i(\eta)}{d\eta}, \frac{dy^i(\eta)}{d\eta} \right). \]
Definition 3: A curve \( l \) is called auto parallel, or a–parallel, on \( \xi \) if its tangent vector field \( v^\alpha \) satisfies a–parallel equations:

\[
v D v^\alpha = v^\beta(\gamma) D_\beta v^\alpha = \rho(\eta)v^\alpha, \quad (14)
\]

where \( \rho(\eta) \) is a scalar function on \( \xi \).

Let curve \( l \subset \xi \) is given in parametric form as \( u^\alpha = u^\alpha(\eta), \eta_1 < \eta < \eta_2 \) with tangent vector field \( v^\alpha = \frac{du^\alpha}{d\eta} \neq 0 \). We suppose that a 2–dimensional distribution \( E_2(l) \) is defined along \( l \), i.e. in every point \( u \in l \) is fixed a 2-dimensional vector space \( E_2(l) \subset \xi \). The introduced distribution \( E_2(l) \) is coplanar along \( l \) if every vector \( p^\alpha(u^\beta_0) \subset E_2(l), u^\beta_0 \subset l \) rests contained in the same distribution after parallel transports along \( l \), i.e. \( p^\alpha(u^\beta(\eta)) \subset E_2(l) \).

Definition 4: A curve \( l \) is called nearly autoparallel, or in brief an na–parallel, on space \( \xi \) if a coplanar along \( l \) distribution \( E_2(l) \) containing tangent to \( l \) vector field \( v^\alpha(\eta) \), i.e. \( v^\alpha(\eta) \subset E_2(l) \), is defined.

We can define nearly autoparallel maps of la–spaces as an anisotropic generalization (see also [24, 25]) of ng–[22] and na–maps [17–21]:

Definition 5: Nearly autoparallel maps, na–maps, of la–spaces are defined as local 1–1 mappings of \( v \)-bundles, \( \xi \rightarrow \xi \), changing every a–parallel on \( \xi \) into a na–parallel on \( \xi \).

Now we formulate the general conditions when deformations (12) and (13) characterize na–maps: Let a–parallel \( l \subset U \) is given by functions \( u^\alpha = u^{(\alpha)}(\eta), v^\alpha = \frac{du^\alpha}{d\eta}, \eta_1 < \eta < \eta_2 \), satisfying equations (14). We suppose that to this a–parallel corresponds a na–parallel \( l \subset U \) given by the same parameterization in a common for a chosen na–map coordinate system on \( U \) and \( U \). This condition holds for vectors \( v^\alpha(2) = vDv^\alpha(1) \) and \( v^\alpha(2) = vDv^\alpha(1) \) satisfying equality

\[
v^\alpha(2) = a(\eta)v^\alpha + b(\eta)v^\alpha(1), \quad (15)
\]

for some scalar functions \( a(\eta) \) and \( b(\eta) \) (see Definitions 4 and 5). Putting splittings (12) and (13) into expressions for \( v^\alpha(2) \) and \( v^\alpha(1) \) in (15) we obtain:
\[
v^\beta v^\gamma v^\delta (D_\beta P^\alpha_{\gamma\delta} + P^\alpha_{\beta\gamma} P^\gamma_{\beta\delta} + Q^\gamma_{\beta\gamma} P^\beta_{\gamma\delta}) = b v^\gamma v^\delta P^\alpha_{\gamma\delta} + a v^\alpha,
\]

where

\[
b(\eta, v) = \frac{b}{2} - 3\rho, \quad \text{and} \quad a(\eta, v) = a + b\rho - v^b \partial_b \rho - \rho^2
\]

are called the deformation parameters of na–maps.

The algebraic equations for the deformation of torsion \(Q^\alpha_{\beta\gamma}\) should be written as the compatibility conditions for a given nonmetricity tensor \(K^\alpha_{\beta\gamma}\) on \(\xi\) (or as the metricity conditions if d–connection \(D_\alpha\) on \(\xi\) is required to be metric):

\[
D_\alpha G^\beta_{\gamma\delta} - P^\delta_{\alpha(\beta} G_{\gamma)\delta} - K^\delta_{\alpha\beta\gamma} = Q^\delta_{\alpha(\beta} G_{\gamma)\delta},
\]

where ( ) denotes the symmetric alternation.

So, we have proved this

**Theorem 1** The na–maps from la–space \(\xi\) to la–space \(\xi\) with a fixed common nonlinear connection \(N^\alpha_j(u) = N^\alpha_j(u)\) and given d–connections, \(\Gamma^\alpha_{\beta\gamma}\) on \(\xi\) and \(\Gamma^\alpha_{\beta\gamma}\) on \(\xi\) are locally parametrized by the solutions of equations (16) and (18) for every point \(u^\alpha\) and direction \(v^\alpha\) on \(U \subset \xi\).

We call (16) and (18) the basic equations for na–maps of la–spaces. They generalize the corresponding Sinyukov’s equations [22] for isotropic spaces provided with symmetric affine connection structure.

**IV. CLASSIFICATION OF NA–MAPS OF LA–SPACES**

Na–maps are classed on possible polynomial parametrizations on variables \(v^\alpha\) of deformations parameters \(a\) and \(b\) (see (16) and (17)).

**Theorem 2** There are four classes of na–maps characterized by corresponding deformation parameters and tensors and basic equations:
1. for $\mathcal{na}_0$–maps, $\pi_0$–maps,
\[ P^\alpha_{\beta\gamma}(u) = \psi_{(\beta\delta)} \]
($\delta^\alpha_\beta$ is Kronecker symbol and $\psi_\beta = \psi_\beta(u)$ is a covariant vector d–field) ;

2. for $\mathcal{na}_1$–maps

\[ a(u, v) = a_{\alpha\beta}(u)v^\alpha v^\beta, \quad b(u, v) = b_{\alpha}(u)v^\alpha \]

and $P^\alpha_{\beta\gamma}(u)$ is the solution of equations

\[ D(\alpha P^\delta_{\beta\gamma}) + P^\tau_{(\alpha\beta\gamma)} - P^\tau_{(\alpha\beta\gamma)} = b_{(\alpha} P^\delta_{\beta\gamma)} + a_{(\alpha\beta\delta\gamma)}, \quad (19) \]

3. for $\mathcal{na}_2$–maps

\[ a(u, v) = a_{\beta}(u)v^\beta, \quad b(u, v) = \frac{b_{\alpha\beta}v^\alpha v^\beta}{\sigma_\alpha(u)v^\alpha}, \quad \sigma_\alpha v^\alpha \neq 0, \]

\[ P^\tau_{\alpha\beta}(u) = \psi_{(\alpha\delta)} + \sigma_{(\alpha F^\tau_{\beta})} \]

and $F^\alpha_\beta(u)$ is the solution of equations

\[ D(\gamma F^\alpha_\beta) + F^\alpha_\delta F^\delta_{\gamma\sigma} - Q^\alpha_\tau(\beta F^\gamma_\tau) = \mu_{(\beta F^\alpha_\gamma)} + \nu_{(\beta\delta\gamma)} \]

$(\mu_\beta(u), \nu_\beta(u), \psi_\alpha(u), \sigma_\alpha(u)$ are covariant d–vectors) ;

4. for $\mathcal{na}_3$–maps

\[ b(u, v) = \frac{\alpha_{\beta\delta}v^\beta v^\gamma v^\delta}{\sigma_\alpha v^\alpha v^\gamma}, \]

\[ P^\alpha_{\beta\gamma}(u) = \psi_{(\beta\delta)} + \sigma_{\beta\gamma}\varphi^\alpha, \]

where $\varphi^\alpha$ is the solution of equations

\[ D_\beta\varphi^\alpha = \nu\delta^\alpha_\beta + \mu_\beta\varphi^\alpha + \varphi^\gamma Q^\alpha_{\gamma\delta}, \quad (21) \]

$\alpha_{\beta\gamma\delta}(u), \sigma_{\alpha\beta}(u), \psi_\beta(u), \nu(u) \quad \text{and} \quad \mu_\beta(u)$ are d–tensors.
The proof of the theorem is sketched in the Appendix A.

We point out that for \( \pi_{(0)} \)-maps we have not differential equations on \( P_{\beta \gamma} \) (in the isotropic case one considers a first order system of differential equations on metric [22]; we omit constructions with deformation of metric in this work).

To formulate invariant conditions for reciprocal \( n \alpha \)-maps (when every \( a \)-parallel on \( \xi \) is also transformed into \( n \alpha \)-parallel on \( \xi \)) it is convenient to introduce into consideration the curvature and Ricci tensors defined for auxiliary connection \( \gamma_{\beta \gamma}^\alpha \):

\[
 r_{\alpha \beta \tau}^\delta = \partial_{[\beta \gamma] \alpha} + \gamma_{\rho \beta \gamma\tau}^\rho + \gamma_{\alpha \beta \gamma}^\beta \psi_{\beta \tau}
\]

and, respectively, \( r_{\alpha \tau} = r_{\alpha \tau}^\gamma \), where \([ \ ]\) denotes antisymmetric alternation of indices, and to define values:

\[
 (0) \Gamma_{\alpha \beta}^\mu - T_{\alpha \beta}^\mu - \left( \frac{1}{n + m + 1} \right) \delta_{(\alpha}^{\mu} \Gamma_{\beta \gamma \delta)}^\delta - \delta_{(\alpha}^{\mu} T_{\beta \gamma \delta)}^\delta,
\]

\[
 (0) W_{\alpha \beta \gamma}^\tau = r_{\alpha \beta \gamma}^\tau + \left( \frac{1}{n + m + 1} \right) \left[ \gamma_{\phi \tau}^\tau \delta_{(\alpha}^{\tau \beta \gamma \delta)}^\beta - \delta_{(\alpha}^{\tau \beta \gamma \delta)}^\beta \right] - \delta_{(\alpha}^{\tau \beta \gamma \delta)}^\beta \left( 2 \gamma_{\phi \tau}^\beta \phi_{\alpha \beta} - 2 \gamma_{\phi \tau}^\alpha \phi_{\beta \gamma} \right) - \delta_{(\alpha}^{\tau \beta \gamma \delta)}^\beta \left( 2 \gamma_{\phi \tau}^\beta \phi_{\alpha \gamma} - \gamma_{\phi \tau}^\gamma \phi_{\alpha \gamma} \right)
\]

\[
 (3) \Gamma_{\alpha \beta}^\delta = \gamma_{\alpha \beta}^\delta + \epsilon_{\phi}^{\gamma \delta} D_{\beta \gamma} + \frac{1}{n + m} (\delta_{\alpha}^\gamma - \epsilon_{\phi}^{\gamma \delta} \phi_{\alpha}) \left[ \gamma_{\beta \gamma}^\tau + \epsilon_{\phi}^{\gamma \delta} D_{\beta \gamma} + \frac{1}{n + m} (\delta_{\beta}^\gamma - \epsilon_{\phi}^{\gamma \delta} \phi_{\beta}) \left[ \gamma_{\alpha \tau}^\beta + \epsilon_{\phi}^{\gamma \delta} D_{\alpha \tau} + \frac{1}{n + m - 1} (\epsilon_{\phi}^{\gamma \delta} \gamma_{\alpha \tau}^\beta + \phi_{\gamma} \phi_{\tau}^\beta D_{\alpha \tau}) \right] + \frac{1}{n + m} (\epsilon_{\phi}^{\gamma \delta} \phi_{\alpha \tau} + \phi_{\gamma} \phi_{\tau}^\beta D_{\alpha \tau}) \right] \]

\[
 (3) W_{\alpha \beta \gamma}^\delta = \rho_{\beta \gamma}^{\alpha \delta} + \epsilon_{\phi}^{\alpha \delta} \phi_{\alpha \gamma} + \left( \delta_{\alpha}^\beta \gamma - \epsilon_{\phi}^{\alpha \beta} \phi_{\alpha} \right) p_{\beta \gamma} - \left( \delta_{\gamma}^\alpha \gamma - \epsilon_{\phi}^{\alpha \beta} \phi_{\alpha} \right) p_{\beta \gamma} - \left( \delta_{\beta}^\alpha \gamma - \epsilon_{\phi}^{\alpha \beta} \phi_{\alpha} \right) p_{\gamma \delta},
\]

\[
 (n + m - 2) p_{\alpha \beta} = -\rho_{\beta \gamma}^{\alpha \delta} - \epsilon_{\phi}^{\alpha \delta} \phi_{\alpha \gamma} + \frac{1}{n + m} \left[ \epsilon_{\phi}^{\alpha \beta} \rho_{\beta \gamma}^\gamma - \epsilon_{\phi}^{\alpha \beta} \rho_{\gamma \beta}^\gamma + \epsilon_{\phi}^{\alpha \beta} \rho_{\gamma \beta}^\gamma + \epsilon_{\phi}^{\alpha \beta} \rho_{\beta \gamma}^\gamma \right] \epsilon_{\phi}^{\alpha \beta} \rho_{\gamma \beta}^\gamma \]

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where \( q_\alpha \varphi^\alpha = \epsilon = \pm 1 \),

\[
\rho^\alpha_{\beta\gamma\delta} = r^\alpha_{\beta\gamma\delta} + \frac{1}{2} (\psi_{(\beta\delta\varphi)} + \sigma_{\beta\varphi}\varphi^\tau) w^\tau_{\beta\gamma\delta}
\]

( for a similar value on \( \xi \) we write \( \rho^\alpha_{\beta\gamma\delta} = r^\alpha_{\beta\gamma\delta} - \frac{1}{2} (\psi_{(\beta\delta\varphi)} - \sigma_{\beta\varphi}\varphi^\tau) w^\tau_{\beta\gamma\delta} \) ) and \( \rho_{\alpha\beta} = \rho^\alpha_{\alpha\beta} \).

Similar values,

\[
(0)^T_{\beta\gamma\lambda} W_{\alpha\beta\gamma} = (0)^T_{\beta\gamma\lambda} \, \hat{W}_{\alpha\beta\gamma}, \quad \tilde{T}_{\alpha\beta\gamma}, \quad \tilde{W}_{\alpha\beta\gamma}, \quad (3) \tilde{T}_{\alpha\beta\gamma}, \quad (3) \tilde{W}_{\alpha\beta\gamma},
\]

and \( (3) W_{\beta\gamma\delta} \) are given, correspondingly, by auxiliary connections \( \Gamma^\mu_{\alpha\beta} \),

\[
\star \gamma_{\beta\lambda} = \gamma^\beta_{\lambda \alpha} + \epsilon F^\gamma_{\alpha} D_{(\beta} F_{\lambda)}^{\tau}, \quad \tilde{\gamma}^\beta_{\lambda \alpha} = \tilde{\gamma}^\beta_{\lambda \alpha} + \epsilon F^\lambda_{\alpha} \tilde{D}_{(\beta} F_{\lambda)}^{\tau},
\]

\[
\tilde{\gamma}^\beta_{\lambda \alpha} = \gamma^\beta_{\lambda \alpha} + \sigma_{\beta\varphi}\varphi^\tau, \quad \hat{\gamma}^\beta_{\lambda \alpha} = \tilde{\gamma}^\beta_{\lambda \alpha} + \tilde{\sigma}_{\beta\varphi}\varphi^\tau,
\]

where \( \tilde{\sigma}_{\beta} = \sigma_{\alpha} F_{\beta}^{\alpha} \).

**Theorem 3** Four classes of reciprocal na–maps of la–spaces are characterized by corresponding invariant criterions:

1. for a–maps \( (0)^T_{\alpha\beta} = (0)^T_{\alpha\beta} \),

\[
(0)^W_{\alpha\beta\gamma} = (0)^W_{\alpha\beta\gamma},
\]

(22)

2. for na(1)–maps

\[
3(\gamma) D_{\lambda} P_{\alpha\beta}^\delta + P_{\tau\lambda}^\delta P_{\alpha\beta}^\tau = r^\delta_{(\alpha\beta)\alpha} - \sum^\delta_{(\alpha\beta)\alpha} + \]

\[
[T_{\alpha\beta\gamma} P_{\alpha\beta\gamma}^\tau + Q_{\alpha\beta\gamma}^\tau + b_{(\alpha\beta)\alpha} + \delta_{(\alpha\beta)\alpha}];
\]

(23)

3. for na(2)–maps \( T_{\alpha\beta\gamma} = \star T_{\alpha\beta\gamma} \),

\[
\hat{W}_{\alpha\beta\gamma} = \star W_{\alpha\beta\gamma};
\]

(24)

4. for na(3)–maps \( (3) T_{\alpha\beta\gamma} = (3) T_{\alpha\beta\gamma} \),

\[
(3) W_{\alpha\beta\gamma} = (3) W_{\alpha\beta\gamma};
\]

(25)
Proof of this theorem is sketched in the Appendix B.

For the particular case of $na_{(3)}$–maps when $\psi_\alpha = 0, \varphi_\alpha = g_{\alpha\beta}\varphi^\beta = \frac{\delta}{\delta u}(\ln \Omega), \Omega(u) > 0$ and $\sigma_{\alpha\beta} = g_{\alpha\beta}$ we define a subclass of conformal transforms $\underline{g}_{\alpha\beta}(u) = \Omega^2(u)g_{\alpha\beta}$ which, in consequence of the fact that d–vector $\varphi_\alpha$ must satisfy equations (21), generalizes the class of concircular transforms (see [22] for references and details on concircular mappings of Riemannian spaces).

We emphasize that basic $na$–equations (19)–(21) are systems of first order partial differential equations. The study of their geometrical properties and definition of integral varieties, general and particular solutions are possible by using the formalism of Pfaff systems [17]. Here we point out that by using algebraic methods we can always verify if systems of $na$–equations of type (19)–(21) are, or not, involute, even to find their explicit solutions it is a difficult task (see more detailed considerations for isotropic $ng$–maps in [22] and, on language of Pfaff systems for $na$–maps, in [20]). We can also formulate the Cauchy problem for $na$–equations on $\xi$ and choose deformation parameters (17) as to make involute mentioned equations for the case of maps to a given background space $\underline{\xi}$. If a solution, for example, of $na_{(1)}$–map equations exists, we say that space $\xi$ is $na_{(1)}$–projective to space $\underline{\xi}$. In general, we have to introduce chains of $na$–maps in order to obtain involute systems of equations for maps (superpositions of $na$–maps) from $\xi$ to $\underline{\xi}$:

$$U \xrightarrow{ng_{<i_1>}} U_1 \xrightarrow{ng_{<i_2>}} \cdots \xrightarrow{ng_{<i_{k-1}>}} U_{k-1} \xrightarrow{ng_{<i_k>}} U$$

where $U \subset \xi, U_1 \subset \xi_\bot, \ldots, U_{k-1} \subset \xi_{k-1}, \underline{U} \subset \xi_k$ with corresponding splittings of auxiliary symmetric connections

$$\underline{\Gamma}_{\beta\gamma}^{\alpha} = <i_1> P_{,\beta\gamma}^{\alpha} + <i_2> P_{,\beta\gamma}^{\alpha} + \cdots + <i_k> P_{,\beta\gamma}^{\alpha}$$

and torsion

$$\underline{T}_{\beta\gamma}^{\alpha} = T_{,\beta\gamma}^{\alpha} + <i_1> Q_{,\beta\gamma}^{\alpha} + <i_2> Q_{,\beta\gamma}^{\alpha} + \cdots + <i_k> Q_{,\beta\gamma}^{\alpha}$$

where cumulative indices $<i_1> = 0, 1, 2, 3$, denote possible types of $na$–maps.
Definition 6 Space $\xi$ is nearly conformally projective to space $\xi$, $nc : \xi \rightarrow \xi$, if there is a finite chain of na–maps from $\xi$ to $\xi$.

For nearly conformal maps we formulate:

Theorem 4 For every fixed triples $(N^a_j, \Gamma^\alpha_{\beta\gamma}, U \subset \xi)$ and $(N^a_j, \Sigma^\alpha_{\beta\gamma}, U \subset \xi)$, components of nonlinear connection, d–connection and d–metric being of class $C^r(U), C^r(U)$, $r > 3$, there is a finite chain of na–maps $nc : U \rightarrow U$.

Proof is similar to that for isotropic maps [18,20,24]. For analytic functions it is a direct consequence from the Cauchy–Kowalewski theorem [31].

Now we introduce the concept of the Category of la–spaces, $\mathcal{C}(\xi)$. The elements of $\mathcal{C}(\xi)$ consist from $\text{Ob}_\mathcal{C}(\xi) = \{\xi, \xi_{<i_1>, \xi_{<i_2>, \ldots, \}}\}$ being la–spaces, for simplicity in this work, having common N–connection structures, and $\text{Mor}_\mathcal{C}(\xi) = \{nc(\xi_{<i_1>}, \xi_{<i_2>})\}$ being chains of na–maps interrelating la–spaces. We point out that we can consider equivalent models of physical theories on every object of $\mathcal{C}(\xi)$ (see details for isotropic gravitational models in [17–21] and anisotropic gravity in [24,25]). One of the main purposes of this paper is to develop a d–tensor and variational formalism on $\mathcal{C}(\xi)$, i.e. on la–multispaces, interrelated with nc–maps. Taking into account the distinguished character of geometrical objects on la–spaces we call tensors on $\mathcal{C}(\xi)$ as distinguished tensors on la–space Category, or dc–tensors.

Finally, we emphasize that presented in that section definitions and theorems can be generalized for v–bundles with arbitrary given structures of nonlinear connection, linear d–connection and metric structures. Proofs are similar to those presented in [21,22] but rather cumbersome.

V. NEARLY AUTOPARALLEL TENSOR–INTEGRAL ON LA–SPACES

The aim of this section is to define tensor integration not only for bitensors, objects defined on the same curved space, but for dc–tensors, defined on two spaces, $\xi$ and $\xi$, even it is necessary on la–multispaces. A. Moór tensor–integral formalism [14] having a lot of
applications in classical and quantum gravity [32-34,15] was extended for locally isotropic
multispaces in [17,18]. The unispacial locally anisotropic version is given in [28,29].

Let \( T_u^\xi \) and \( T_{u_\xi}^\xi \) be tangent spaces in corresponding points \( u \in U \subset \xi \) and \( u_\xi \in U \subset \xi \) and, respectively, \( T_u^*\xi \) and \( T_{u_\xi}^*\xi \) be their duals (in general, in this section we shall not consider that a common coordinatization is introduced for open regions \( U \) and \( U_\xi \)). We call as the dc–tensors on the pair of spaces \((\xi,\xi)\) the elements of distinguished tensor algebra

\[
(\otimes_\alpha T_u^\xi) \otimes (\otimes_\beta T_{u_\xi}^\xi) \otimes (\otimes_\gamma T_u^*\xi) \otimes (\otimes_\delta T_{u_\xi}^*\xi)
\]
defined over the space \( \xi \otimes \xi \), for a given \( nc : \xi \to \xi \).

We admit the convention that underlined and non–underlined indices refer, respectively, to the points \( u_\xi \) and \( u \). Thus \( Q^\beta_\alpha \), for instance, are the components of dc–tensor \( Q \in T_u^\xi \otimes T_{u_\xi}^\xi \).

Now, we define the transport dc–tensors. Let open regions \( U \) and \( U_\xi \) be homeomorphic to sphere \( \mathcal{R}^{2n} \) and introduce isomorphism \( \mu_{u,u_\xi} \) between \( T_u^\xi \) and \( T_{u_\xi}^\xi \) (given by map \( nc : U \to U_\xi \)). We consider that for every d–vector \( v^\alpha \in T_u^\xi \) corresponds the vector \( \mu_{u,u_\xi}(v^\alpha) = v^\alpha_\xi \in T_{u_\xi}^\xi \), with components \( v^\alpha_\xi \) being linear functions of \( v^\alpha \):

\[
v^\alpha_\xi = h^\alpha_\xi(u_\xi, u_\xi)v^\alpha, \quad v^\alpha = h^\alpha_\xi(u, u_\xi)v^\alpha,
\]
where \( h^\alpha_\xi(u, u_\xi) \) are the components of dc–tensor associated with \( \mu_{u,u_\xi}^{-1} \). In a similar manner we have

\[
v^\alpha = h^\alpha_{u_\xi}(u_\xi, u)v^\alpha_\xi, \quad v^\alpha_\xi = h^\alpha_{u_\xi}(u, u_\xi)v^\alpha.
\]

In order to reconcile just presented definitions and to assure the identity for trivial maps \( \xi \to \xi, u_\xi = u \), the transport dc-tensors must satisfy conditions :

\[
h^\alpha_{\xi}(u, u)h^\beta_\xi(u_\xi, u) = \delta^\beta_\alpha, \quad h^\alpha_{\xi}(u, u)h^\alpha_\xi(u_\xi, u_\xi) = \delta^\alpha_{\xi}
\]
and

\[
\lim_{(u \to u_\xi)} h^\alpha_{\xi}(u, u) = \delta^\alpha_\xi, \quad \lim_{(u_\xi \to u)} h^\alpha_\xi(u_\xi, u_\xi) = \delta^\alpha_{\xi}.
\]

Let \( \mathcal{S}_p \subset U \subset \xi \) is a homeomorphic to \( p \)-dimensional sphere and suggest that chains of na–maps are used to connect regions :

\[
U \xrightarrow{nc(1)} \mathcal{S}_p \xrightarrow{nc(2)} U.
\]
Definition 7 The tensor integral in \( \overline{u} \in S_p \) of a dc–tensor \( N_{\varphi, \gamma, \alpha_1, \ldots, \alpha_p} (\overline{u}, u) \), completely antisymmetric on the indices \( \overline{\alpha}_1, \ldots, \overline{\alpha}_p \), over domain \( S_p \), is defined as

\[
N_{\varphi, \gamma, \alpha_1, \ldots, \alpha_p} (u, u) = \int_{(S_p)} N_{\varphi, \mu, \alpha_1, \ldots, \alpha_p} (\overline{u}, u) dS_{\overline{\alpha}_1 \ldots \overline{\alpha}_p},
\]

where \( dS_{\overline{\alpha}_1 \ldots \overline{\alpha}_p} = \delta u_{\alpha_1} \wedge \cdots \wedge \delta u_{\alpha_p} \).

Let suppose that transport dc–tensors \( h^\alpha_\alpha \) and \( h^\alpha_\mu \) admit covariant derivations of order two and postulate existence of deformation dc–tensor \( B_{\alpha, \beta} (u, u) \) satisfying relations

\[
D_\alpha h^\beta_\beta (u, u) = B_{\alpha, \beta} (u, u) h^\beta_\gamma (u, u)
\]

and, taking into account that \( D_\alpha \delta_\gamma^\beta = 0 \),

\[
D_\alpha h^\beta_\mu (u, u) = -B_{\alpha, \beta} (u, u) h^\gamma_\mu (u, u).
\]

By using formulas (4) and (5) for torsion and, respectively, curvature of connection \( \Gamma^\alpha_{\beta \gamma} \) we can calculate next commutators:

\[
D_\alpha D_\beta h^\gamma_\lambda = -(R^\lambda_\gamma, \alpha, \beta ) + T^\tau_{\alpha, \beta} B^\lambda_{\tau, \gamma}) h^\gamma_\lambda.
\]

On the other hand from (27) one follows that

\[
D_\alpha D_\beta h^\gamma_\lambda = (D_\alpha B^\lambda_{\beta, \gamma} + B^\lambda_{\alpha, \gamma} B^\lambda_\beta B^\lambda_{\gamma, \beta} ) h^\gamma_\lambda,
\]

where \( |\tau| \) denotes that index \( \tau \) is excluded from the action of antisymmetrization \([ \ ] \).

From (28) and (29) we obtain

\[
D_\alpha B^\lambda_{\beta, \gamma} + B_{|\beta|, \gamma} B^\lambda_{\alpha} = (R^\lambda_\gamma, \alpha, \beta + T^\tau_{\alpha, \beta} B^\lambda_{\tau, \gamma} )
\]

Let \( S_p \) be the boundary of \( S_{p-1} \). The Stoke’s type formula for tensor–integral (26) is defined as

\[
I_{S_p} N_{\varphi, \gamma, \alpha_1, \ldots, \alpha_p} (\overline{u}, u) dS_{\overline{\alpha}_1 \ldots \overline{\alpha}_p} = I_{S_{p+1}} \ast (\partial \overline{\mu}) N_{\varphi, \gamma, \mu, \alpha_1, \ldots, \alpha_p} dS_{\overline{\alpha}_1 \ldots \overline{\alpha}_p},
\]
where

\[ *_{(p)} D_{[\gamma [\varphi, \tau, [x_1, \ldots, x_p]]} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} = \]

\[ D_{[\gamma [\varphi, \tau, [x_1, \ldots, x_p]]} + p T_{[\varphi, \tau, [x_1, \ldots, x_p]]} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} - B_{[\varphi, \tau, [x_1, \ldots, x_p]]} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} + B_{[\varphi, \tau, [x_1, \ldots, x_p]]} N^{\varphi, \tau}_{[x_1, \ldots, x_p]}, \]  

(32)

We define the dual element of the hypersurfaces element \( dS_{\beta_1, \ldots, \beta_q} \) as

\[ dS_{\beta_1, \ldots, \beta_q} = \frac{1}{p!} \epsilon_{\beta_1, \ldots, \beta_p \alpha_1, \ldots, \alpha_p} dS^{\alpha_1, \ldots, \alpha_p}, \]

(33)

where \( \epsilon_{\gamma_{1}, \ldots, \gamma_q} \) is completely antisymmetric on its indices and

\[ \epsilon_{12 \ldots (n+m)} = \sqrt{|G|}, \quad G = \text{det}|G_{\alpha \beta}|, \]

\( G_{\alpha \beta} \) is taken from (7). The dual of dc–tensor \( N^{\varphi, \tau}_{[x_1, \ldots, x_p]} \) is defined as the dc–tensor \( N^{\varphi, \tau}_{[x_1, \ldots, x_p]} \) satisfying

\[ N^{\varphi, \tau}_{[x_1, \ldots, x_p]} = \frac{1}{p!} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} \epsilon_{\beta_1, \ldots, \beta_p \alpha_1, \ldots, \alpha_p}. \]

(34)

Using (26), (33) and (34) we can write

\[ I_{\beta_1, \ldots, \beta_q} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} dS^{\varphi_{1}, \ldots, \varphi_p} = \int_{S_{p+1}} \mp D_{\gamma} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} dS_{\beta_1, \ldots, \beta_{n+m-p+1}}, \]

(35)

where

\[ \mp D_{\gamma} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} = \]

\[ D_{\gamma} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} - (-1)^{(n+m-p)}(n + m - p + 1) T_{[\varphi, \tau, [x_1, \ldots, x_p]]} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} - \]

\[ B_{[\varphi, \tau, [x_1, \ldots, x_p]]} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} + B_{[\varphi, \tau, [x_1, \ldots, x_p]]} N^{\varphi, \tau}_{[x_1, \ldots, x_p]} - \]

To verify the equivalence of (34) and (35) we must take in consideration that

\[ D_{\gamma} \epsilon_{\alpha_1, \ldots, \alpha_k} = 0 \quad \text{and} \quad \epsilon_{\beta_1, \ldots, \beta_{n+m-p} \alpha_1, \ldots, \alpha_p} \epsilon_{\beta_{n+m-p} \gamma_1, \ldots, \gamma_p} = p! (n + m - p)! \delta_{[\gamma_1, \ldots, \gamma_p]} \]

The developed in this section tensor integration formalism will be used in the next section for definition of conservation laws on spaces with local anisotropy.
VI. CONSERVATION LAWS ON LOCALLY ANISOTROPIC SPACES

To define conservation laws on locally anisotropic spaces is a challenging task because of absence of global and local groups of automorphisms of such spaces. Our main idea is to use chains of na–maps from a given, called hereafter as the fundamental la–space to an auxiliary one with trivial curvatures and torsions admitting a global group of automorphisms. The aim of this section is to present a brief introduction into the la–gravity (see [1,2] as basic references and [11,12] for further developments for gauge like and spinor la–gravity) and formulate classes of conservation laws by using dc–objects and tensor–integral values, na–maps and variational calculus on the Category of la–spaces.

A. Locally Anisotropic Gravity in Vector Bundles

The Einstein equations on a v–bundle $\xi$, associated to a metric d–connection $D$, with locally adapted coefficients

\[
\Gamma^\alpha_{\beta\gamma} =
\]

\[
(\Gamma^i_{jk} = L^i_{jk}, \Gamma^i_{ja} = C^i_{ja}, \Gamma^i_{aj} = 0, \Gamma^a_{ab} = 0, \Gamma^a_{jb} = 0, \Gamma^a_{bk} = L^a_{bk}, \Gamma^a_{bc} = C^a_{bc}),
\] (36)

where coefficients

\[
L^i_{jk} = \frac{1}{2} g^{ih} \left( \frac{\delta g_{jh}}{\delta x^k} + \frac{\delta g_{kh}}{\delta x^j} - \frac{\delta g_{jk}}{\delta x^h} + g_{kl} T^l_{hj} + g_{lj} T^l_{hk} - g_{hi} T^l_{jk} \right),
\] (37)

\[
L^a_{bi} = \frac{\partial N^a_i}{y^b} + \frac{1}{2} h^{ac} \left( \frac{\delta h_{bc}}{\delta x^i} - \frac{\partial N^d_i}{\partial y^b} h_{de} - \frac{\partial N^d_i}{\partial y^c} h_{db} \right),
\]

\[
C^i_{jc} = \frac{1}{2} g^{ik} \frac{\partial g_{jk}}{\partial y^c},
\]

\[
C^a_{bc} = \frac{1}{2} h^{ad} \left( \frac{\delta h_{db}}{\delta y^c} + \frac{\delta h_{dc}}{\delta y^b} - \frac{\delta h_{bc}}{\delta y^d} + h_{be} S^e_{bc} + h_{ce} S^e_{db} - h_{de} S^e_{bc} \right)
\]

are constructed with respect to locally adapted coefficients of metric (8), N–connection (1),(2) and presribed torsions $T^i_{jk}$ and $S^a_{bc}$, which in turn, hand in hand with (36), define the torsion (see formulas (4))
\[ T^\alpha{}_{\beta\gamma} = \Gamma^\alpha{}_{\beta\gamma} - \Gamma^\alpha{}_{\gamma\beta} + w^\alpha{}_{\beta\gamma} \quad (38) \]

having following components

\[ T^i_{jk} = T^i_{jk}, \quad T^i_{ja} = C^i_{ja}, \quad T^i_{aj} = -C^i_{ja}, \quad T^i_{ja} = 0, \quad T^a_{bc} = S^a_{bc}, \]

\[ T^a_{ij} = \frac{\delta N^a_i}{\delta x^j} - \frac{\delta N^a_j}{\delta x^i}, \quad T^a_{bi} = P^a_{bi} = \frac{\partial N^a_j}{\partial y^b} - L^a_{bj}, \quad T^a_{ib} = -P^a_{bi}, \]

written by Miron and Anastasiei [1,2] for a generalization of general relativity to the case of la–spaces. They introduced the Ricci tensor \( R_{\beta\gamma} = R^\alpha{}_{\beta\gamma\alpha} \) constructed in a usual manner by using curvature of connection (36) (see similar formulas (5)) having nonvanishing components:

\[ R^i_{jk} = \frac{\partial L^i_{hj}}{\partial x^k} - \frac{\partial L^i_{hk}}{\partial x^j} + L^m_{hj} L^i_{mk} - L^m_{hk} L^i_{mj} + C^i_{ha} R^a_{jk}, \quad (39) \]

\[ R^a_{jk} = \frac{\partial L^a_{bj}}{\partial y^a} - \frac{\partial L^a_{bk}}{\partial x^j} + L^c_{bj} L^a_{ck} - L^c_{bk} L^a_{cj} + C^a_{bc} R^c_{jk}, \]

\[ P^i_{ja} = \frac{\partial L^i_{ja}}{\partial y^a} - \frac{\partial C^i_{ja}}{\partial x^k} + L^i_{jk} C^a_{ja} - L^i_{jk} C^a_{ta} - L^c_{ak} C^a_{jc} + C^i_{jb} P^b_{ka}, \]

\[ P^c_{ka} = \frac{\partial L^c_{bk}}{\partial y^a} - \frac{\partial C^c_{ba}}{\partial x^k} + L^c_{ak} C^d_{ba} - L^d_{bk} C^c_{da} - L^d_{ak} C^c_{bd} + C^c_{bd} P^d_{ks}, \]

\[ S^i_{jb} = \frac{\partial C^i_{jb}}{\partial y^c} - \frac{\partial C^i_{jc}}{\partial y^b} + C^h_{jb} C^i_{hc} - C^h_{jc} C^i_{hb}, \]

\[ S^a_{cd} = \frac{\partial C^a_{bd}}{\partial y^c} - \frac{\partial C^a_{cd}}{\partial y^b} + C^e_{bc} C^a_{ed} - C^e_{bd} C^a_{ec}. \]

The components of the Ricci d–tensor \( R_{\alpha\beta} \) with respect to locally adapted frame (3) are as follows:

\[ R_{ij} = R^k_{ij}, \quad R_{ia} = -2P_{ia} = -P^k_{ia}, \quad (40) \]

\[ R_{ai} = P_{ai} = P^b_{ib}, \quad R_{ab} = S_{ab} = S^c_{ab}. \]

We point out that because, in general, \( 1^P_{ai} \neq 2P_{ai} \) the Ricci d–tensor is nonsymmetric.

The scalar curvature \( R = G^{\alpha\beta} R_{\alpha\beta} \) is given by

\[ R = R + S, \quad (41) \]
where $R = g^{ij} R_{ij}$ and $S = h^{ab} S_{ab}$.

Now we can write the Einstein–Cartan equations for la–gravity

$$R_{\alpha\beta} - \frac{1}{2} G_{\alpha\beta} R + \lambda G_{\alpha\beta} = \kappa_1 T_{\alpha\beta}, \quad (42)$$

and

$$T^\alpha_{\beta\gamma} + G^\alpha_{\beta\gamma} T^\gamma_{\alpha\tau} - G^\alpha_{\gamma\tau} T^\gamma_{\beta\tau} = \kappa_2 Q^\alpha_{\beta\gamma}, \quad (43)$$

where $T_{\alpha\beta}$ and $Q^\alpha_{\beta\gamma}$ are respectively d–tensors of energy–momentum and spin–density of matter on la–space $\xi$, $\kappa_1$ and $\kappa_2$ are corresponding interaction constants and $\lambda$ is the cosmological constant. We have added the algebraic system of equations for torsion (43) in order to close the system of field equations (here we also point to the constraints (9)).

We have proposed a gauge like version of la–gravity for which a set of conservation laws similar to those for Yang–Mills fields on la–spaces hold [12]. In this work we restrict our considerations only to the Einstein–Cartan la–gravity.

B. Nonzero Divergence of the Energy–Momentum D–Tensor on LA–Spaces

R.Miron and M.Anastasiei [1,2] pointed to this specific form of conservation laws of matter on la–spaces: They calculated the divergence of the energy–momentum d–tensor on la–space $\xi$,

$$D_{\alpha} T^\alpha_{\beta} = \frac{1}{\kappa_1} U_{\alpha}, \quad (44)$$

and concluded that d–vector

$$U_{\alpha} = \frac{1}{2} (G^{\beta\delta} R^\gamma_{\phi\beta} T^\phi_{\alpha\gamma} - G^{\beta\delta} R^\gamma_{\phi\alpha} T^\phi_{\beta\gamma} + R^\phi_{\phi\alpha} T^\phi_{\beta\gamma})$$

vanishes if and only if d–connection $D$ is without torsion.

No wonder that conservation laws, in usual physical theories being a consequence of global (for usual gravity of local) automorphisms of the fundamental space–time, are more
sophisticated on the spaces with local anisotropy. Here it is important to emphasize the multi-connection character of la–spaces. For example, for a d–metric (8) on ξ we can equivalently introduce another (see (36)) metric linear connection \( \tilde{D} \) with coefficients

\[
\tilde{\Gamma}_{\alpha \beta \gamma} = \frac{1}{2} G^{\alpha \phi} \left( \frac{\delta G_{\beta \phi}}{\delta u^\gamma} + \frac{\delta G_{\gamma \phi}}{\delta u^\alpha} - \frac{\delta G_{\beta \gamma}}{\delta u^\phi} \right).
\]

The Einstein equations

\[
\tilde{R}_{\alpha \beta} - \frac{1}{2} G_{\alpha \beta} \tilde{R} = \kappa_1 \tilde{T}_{\alpha \beta}
\]

constructed by using connection (43) have vanishing divergences \( \tilde{D}^\alpha (\tilde{R}_{\alpha \beta} - \frac{1}{2} G_{\alpha \beta} \tilde{R}) = 0 \) and \( \tilde{D}^\alpha \tilde{T}_{\alpha \beta} = 0 \), similarly as those on (pseudo)Riemannian spaces. We conclude that by using the connection (43) we construct a model of la–gravity which looks like locally isotropic on the total bundle \( E \). More general gravitational models with local anisotropy can be obtained by using deformations of connection \( \Gamma_{\alpha \beta \gamma} \),

\[
\Gamma_{\alpha \beta \gamma} = \tilde{\Gamma}_{\alpha \beta \gamma} + P_{\alpha \beta \gamma} + Q_{\alpha \beta \gamma},
\]

were, for simplicity, \( \Gamma_{\alpha \beta \gamma} \) is chosen to be also metric and satisfy Einstein equations (43). We can consider deformation d–tensors \( P_{\alpha \beta \gamma} \) generated (or not) by deformations of type (12) and (13) for na–maps. In this case d–vector \( U_\alpha \) can be interpreted as a generic source of local anisotropy on \( \xi \) satisfying generalized conservation laws (46).

C. Deformation d–tensors and tensor–integral conservation laws

From (26) we obtain a tensor integral on \( C(\xi) \) of a d–tensor:

\[
N_{\xi}^2(u) = I_{\bar{\pi}} N_{\bar{\gamma}} \bar{\pi}_{1 \ldots p} (\bar{\pi}) h_{\bar{\gamma}}(\bar{u}, \bar{\pi}) h_{\bar{\gamma}}(\bar{u}, \bar{\pi}) dS_{\bar{\gamma}_{1 \ldots p}}.
\]

We point out that tensor–integral can be defined not only for dc–tensors but and for d–tensors on \( \xi \). Really, suppressing indices \( \varphi \) and \( \gamma \) in (34) and (35), considering instead of a deformation dc–tensor a deformation tensor
\[ B_{\alpha\beta}(u, u) = B_{\alpha\beta}^\gamma(u) = P_{\alpha\beta}^\gamma(u) \]  

(48)

(see deformations (12) induced by a nc–transform) and integration \( I_{s_p} \ldots dS^{\alpha_1 \ldots \alpha_p} \) in la–space \( \xi \) we obtain from (26) a tensor–integral on \( C(\xi) \) of a d–tensor:

\[ N_{\beta}^\gamma(u, u) = I_{s_p} N_{\tau, \alpha_1 \ldots \alpha_p}^\kappa(u, u) h_{\kappa}^\gamma(u, u) dS^{\alpha_1 \ldots \alpha_p}. \]

Taking into account (30) and using formulas (5) and (6) we can calculate that curvature

\[ R_{\gamma, \alpha\beta} = D_{[\beta}B_{\alpha]\gamma} + B_{[\alpha|\gamma]}B_{\beta]\tau} + T_{\tau, \alpha\beta}^\gamma \]

of connection \( \Gamma_{\gamma, \alpha\beta}(u) = \Gamma_{\gamma, \alpha\beta}(u) + B_{\alpha\beta}^\gamma(u) \) with \( B_{\alpha\beta}^\gamma(u) \) taken from (48), vanishes, \( R_{\gamma, \alpha\beta} = 0 \).

So, we can conclude that la–space \( \xi \) admits a tensor integral structure on \( C(\xi) \) for d–tensors associated to deformation tensor \( B_{\alpha\beta}^\gamma(u) \) if the nc–image \( \underline{u} \) is locally parallelizable. That way we generalize the one space tensor integral constructions in \([15,28,29]\), were the possibility to introduce tensor integral structure on a curved space was restricted by the condition that this space is locally parallelizable. For \( q = n + m \) relations (35), written for d–tensor \( N_{\underline{\alpha}}^{\beta\gamma} \) (we change indices \( \underline{\alpha}, \underline{\beta}, \ldots \) into \( \alpha, \beta, \ldots \)) extend the Gauss formula on \( C(\xi) \):

\[ I_{s_{q-1}} N_{\underline{\alpha}}^{\beta\gamma} dS_{\underline{\alpha}} = I_{s_q} q^{-1} D_{\underline{\alpha}} N_{\underline{\alpha}}^{\beta\gamma} dV, \]

where \( dV = \sqrt{|G_{\alpha\beta}| du^1 \ldots du^q} \) and

\[ q^{-1} D_{\underline{\alpha}} N_{\underline{\alpha}}^{\beta\gamma} = D_{\underline{\alpha}} N_{\underline{\alpha}}^{\beta\gamma} - T_{\underline{\alpha}}^\gamma N_{\underline{\alpha}}^{\beta\gamma} - B_{\underline{\alpha}}^{\gamma\tau} N_{\underline{\alpha}}^{\beta\tau} + B_{\underline{\alpha}}^{\beta\gamma} N_{\underline{\alpha}}^{\tau\tau}. \]  

(50)

Let consider physical values \( N_{\underline{\alpha}}^{\beta\gamma} \) on \( \underline{u} \) defined on its density \( N_{\underline{\alpha}}^{\beta\gamma} \), i. e.

\[ N_{\underline{\alpha}}^{\beta\gamma} = I_{s_{q-1}} N_{\underline{\alpha}}^{\beta\gamma} dS_{\underline{\alpha}} \]

with this conservation law (due to (49)):

\[ q^{-1} D_{\underline{\alpha}} N_{\underline{\alpha}}^{\beta\gamma} = 0. \]  

(52)

We note that these conservation laws differ from covariant conservation laws for well known physical values such as density of electric current or of energy–momentum tensor. For example, taking density \( T_{\beta}^\gamma \), with corresponding to (50) and (52) conservation law,
\[
\mathcal{I}^{-1} \mathcal{D}_{\gamma} \mathcal{T}_{\delta}^{\gamma} = \mathcal{D}_{\gamma} \mathcal{T}_{\delta}^{\gamma} - \mathcal{T}_{\Gamma_{\delta}^\lambda} \mathcal{T}_{\delta}^{\gamma} - \mathcal{B}_{\Gamma_{\delta}^\lambda} \mathcal{T}_{\delta}^{\gamma} = 0,
\]

we can define values (see (49) and (51))

\[
\mathcal{P}_{\alpha} = I_{\mathcal{S}_{q-1}} \mathcal{T}_{\delta}^{\gamma} d\mathcal{S}_{\gamma}.
\]

Defined conservation laws (53) for \(\mathcal{T}_{\delta}^{\gamma}\) have nothing to do with those for energy–momentum tensor \(T_{\alpha}^{\gamma}\) from Einstein equations for the almost Hermitian gravity [1,2] or with \(\mathcal{T}_{\alpha}^{\gamma}\) from (46) with vanishing divergence \(D_{\gamma} \mathcal{T}_{\alpha}^{\gamma} = 0\). So \(\mathcal{T}_{\alpha}^{\gamma} \neq T_{\alpha}^{\gamma}\). A similar conclusion was made in [15] for unispacial locally isotropic tensor integral. In the case of multispacial tensor integration we have another possibility (firstly pointed in [17,18] for Einstein-Cartan spaces), namely, to identify \(\mathcal{T}_{\delta}^{\gamma}\) from (53) with the na-image of \(\mathcal{T}_{\alpha}^{\gamma}\) on la–space \(\xi\). We shall consider this construction in the next section.

VII. THE EINSTEIN EQUATIONS ON NA–IMAGES OF LA–SPACES AND CONSERVATION LAWS FOR LA–GRAVITATIONAL FIELDS

It is well known that the standard pseudo–tensor description of the energy–momentum values for the Einstein gravitational fields is full of ambiguities. Some light was shed by introducing additional geometrical structures on curved space–time (bimetrics [35,36], bi–connections [37], by taking into account background spaces [38,34], or formulating variants of general relativity theory on flat space [39,40]). We emphasize here that rigorous mathematical investigations based on two (fundamental and background) locally anisotropic, or isotropic, spaces should use well–defined, motivated from physical point of view, mappings of these spaces. Our na–model largely contains both attractive features of the mentioned approaches; na–maps establish a local 1–1 correspondence between the fundamental la–space and auxiliary la–spaces on which biconnection (or even multiconnection) structures are induced. But these structures are not a priory postulated as in a lot of gravitational theories, we tend to specify them to be locally reductible to the locally isotropic Einstein theory [38,30].
Let us consider a fixed background la–space $\xi$ with given metric $G_{\alpha\beta} = (g_{ij}, h_{ab})$ and d–connection $\tilde{\Gamma}_{\beta\gamma}^{\alpha}$, for simplicity in this subsection we consider compatible metric and connections being torsionless and with vanishing curvatures. Supposing that there is an nc–transform from the fundamental la–space $\xi$ to the auxiliary $\xi$, we are interested in the equivalents of the Einstein equations (46) on $\xi$.

We consider that a part of gravitational degrees of freedom is "pumped out" into the dynamics of deformation d–tensors for d–connection, $P_{\alpha\beta\gamma}$, and metric, $B_{\alpha\beta} = (b^{ij}, b^{ab})$. The remained part of degrees of freedom is coded into the metric $G_{\alpha\beta}$ and d–connection $\tilde{\Gamma}_{\beta\gamma}^{\alpha}$.

Following [38,30] we apply the first order formalism and consider $B_{\alpha\beta}$ and $P_{\alpha\beta\gamma}$ as independent variables on $\xi$. Using notations

\begin{align*}
P_{\alpha} &= P_{\beta\alpha}, \quad \Gamma_{\alpha} = \Gamma_{\beta\alpha}, \\
\dot{B}_{\alpha\beta} &= \sqrt{|G|} B_{\alpha\beta}, \quad \dot{G}_{\alpha\beta} = \sqrt{|G|} G_{\alpha\beta}, \quad \dot{G}_{\alpha\beta} = \sqrt{|G|} G_{\alpha\beta}
\end{align*}

and making identifications

\begin{align*}
\dot{B}_{\alpha\beta} + \dot{G}_{\alpha\beta} &= \dot{G}_{\alpha\beta}, \quad \Gamma_{\beta\gamma}^{\alpha} - P_{\alpha\beta\gamma} = \Gamma_{\beta\gamma}^{\alpha},
\end{align*}

we take the action of la–gravitational field on $\xi$ in this form:

\begin{equation}
\mathcal{S}^{(g)} = -(2c\kappa_1)^{-1} \int \delta^{q} u \mathcal{L}^{(g)}, \quad (54)
\end{equation}

where

\begin{align*}
\mathcal{L}^{(g)} &= \dot{B}_{\alpha\beta}(D_{\beta}P_{\alpha} - D_{\tau}P^{\tau}_{\alpha\beta}) + \\
&\quad \dot{G}_{\alpha\beta}(P_{\tau}P^{\tau}_{\alpha\beta} - P^{\alpha}_{\alpha\kappa}P^{\kappa}_{\beta\tau})
\end{align*}

and the interaction constant is taken $\kappa_1 = \frac{4\pi}{c^4} k$, \quad ($c$ is the light constant and $k$ is Newton constant) in order to obtain concordance with the Einstein theory in the locally isotropic limit.

We construct on $\xi$ a la–gravitational theory with matter fields (denoted as $\varphi_A$ with $A$ being a general index) interactions by postulating this Lagrangian density for matter fields.
\[
\mathcal{L}^{(m)} = \mathcal{L}^{(m)}(\hat{\mathcal{G}}^{\alpha\beta} + \hat{B}^{\alpha\beta}; \frac{\delta}{\delta u^\gamma}(\hat{\mathcal{G}}^{\alpha\beta} + \hat{B}^{\alpha\beta}); \frac{\delta}{\delta u^\gamma}, \phi_A; \frac{\delta}{\delta u^\gamma})].
\]

Starting from (54) and (55) the total action of la–gravity on $\xi$ is written as

\[
\mathcal{S} = (2c\kappa_1)^{-1} \int \delta^q u \mathcal{L}^{(g)} + c^{-1} \int \delta^{(m)} \mathcal{L}^{(m)}.
\]

Applying variational procedure on $\xi$, similar to that presented in [38] but in our case adapted to N–connection by using derivations (3) instead of partial derivations, we derive from (56) the la–gravitational field equations

\[
\Theta_{\alpha\beta} = \kappa_1 (\mathfrak{L}_{\alpha\beta} + \mathfrak{T}_{\alpha\beta})
\]

and matter field equations

\[
\frac{\Delta \mathcal{L}^{(m)}}{\Delta \phi_A} = 0,
\]

where $\frac{\Delta}{\Delta \phi_A}$ denotes the variational derivation.

In (57) we have introduced these values: the energy–momentum d–tensor for la–gravitational field

\[
k_1 \mathfrak{L}_{\alpha\beta} = (\sqrt{|G|})^{-1} \frac{\Delta \mathcal{L}^{(g)}}{\Delta G_{\alpha\beta}} = K_{\alpha\beta} + P^\gamma_{\alpha\beta} P_\gamma - P^\gamma_{\alpha\tau} P^\tau_{\beta\gamma} + \frac{1}{2} G_{\alpha\beta} G^\gamma_{\gamma\tau}(P^\phi_{\gamma\tau} P_\phi - P^\phi_{\gamma\tau} P^\phi_{\phi\tau}),
\]

(59)

(where

\[
K_{\alpha\beta} = D_\gamma K_{\alpha\beta}^\gamma,
\]

\[
2K_{\alpha\beta}^\gamma = -B^{\tau\gamma} P^\tau_{(\alpha G_{\beta})\epsilon} - B^{\tau\epsilon} P^\epsilon_{(\alpha G_{\beta})\tau} + \frac{G_{\alpha\beta}}{2} G^{\tau\gamma}(G^\epsilon_{\epsilon\phi} P^\phi_{\phi\tau} G_{\phi\tau} - G^\epsilon_{\epsilon\phi} P^\phi_{\phi\tau} - G^{\tau\epsilon} D_{\tau\epsilon} D_{\epsilon(\alpha B_{\beta})\tau}
\]

and the energy–momentum d–tensor of matter

\[
\mathfrak{T}_{\alpha\beta} = 2 \frac{\Delta \mathcal{L}^{(m)}}{\Delta \mathcal{G}_{\alpha\beta}} - G_{\alpha\beta} G^{\gamma\delta} \frac{\Delta \mathcal{L}^{(m)}}{\Delta \hat{G}^{\gamma\delta}}.
\]

(60)
As a consequence of (58)–(60) we obtain the $d$–covariant on $\xi$ conservation laws

$$D_\alpha (t^{\alpha\beta} + T^{\alpha\beta}) = 0. \quad (61)$$

We have postulated the Lagrangian density of matter fields (55) in a form as to treat $t^{\alpha\beta} + T^{\alpha\beta}$ as the source in (57).

Now we formulate the main results of this section:

**Proposition 1** The dynamics of the Einstein $la$–gravitational fields, modeled as solutions of equations (46) and matter fields on $la$–space $\xi$, can be equivalently locally modeled on a background $la$–space $\xi$ provided with a trivial $d$-connection and metric structures having zero $d$–tensors of torsion and curvature by field equations (57) and (58) on condition that deformation tensor $P^{\alpha\beta\gamma}$ is a solution of the Cauchy problem posed for basic equations for a chain of $na$–maps from $\xi$ to $\xi$.

**Proposition 2** Local, $d$–tensor, conservation laws for Einstein $la$–gravitational fields can be written in form (61) for $la$–gravitational (59) and matter (60) energy–momentum $d$–tensors. These laws are $d$–covariant on the background space $\xi$ and must be completed with invariant conditions of type (22)-(25) for every deformation parameters of a chain of $na$–maps from $\xi$ to $\xi$.

The above presented considerations represent the proofs of both propositions.

We emphasize that nonlocalization of both locally anisotropic and isotropic gravitational energy–momentum values on the fundamental (locally anisotropic or isotropic) space $\xi$ is a consequence of the absence of global group automorphisms for generic curved spaces. Considering gravitational theories from view of multispaces and their mutual maps (directed by the basic geometric structures on $\xi$ such as $N$–connection, $d$–connection, $d$–torsion and $d$–curvature components, see coefficients for basic $na$–equations (19)-(21)), we can formulate local $d$–tensor conservation laws on auxiliary globally automorphic spaces being related with space $\xi$ by means of chains of $na$–maps. Finally, we remark that as a
matter of principle we can use d–connection deformations of type (47) in order to modelate
the la–gravitational interactions with nonvanishing torsion and nonmetry. In this case
we must introduce a corresponding source in (61) and define generalized conservation laws
as in (44) (see similar details for locally isotropic generalizations of the Einstein gravity
in Refs [16,17,21]).

VIII. OUTLOOK AND CONCLUSIONS

In this paper we have presented a detailed study of the problem of formulation of con-
ervation laws on spaces with local anisotropy. The need for such an investigation was
often expressed in order to develop a number of geometrical models of interactions in locally
anisotropic media or to extend in a self–consistent manner some gravitational and matter
field theories on tangent and vector bundles. As a geometric background of our consider-
ations we have chosen the R.Miron and M.Anastasiei [1,2] general approach of modelling
la–spaces on vector bundles provided with compatible nonlinear and distinguished connec-
tions and metric structures. This allowed us to obtain a similarity, within certain limits, with
the Einstein–Cartan spaces and to generalize some our results on both locally anisotropic
and isotropic gravitational theories [11,12,16-21,24-30].

We have formulated the theory of nearly autoparallel maps of la–spaces. Such maps
generalize the conformal transforms of curved spaces, are characterized by similar invariants
as Weyl tensor and Thomas parameters and appear to have a lot of applications in modern
classical and quantum gravity. We have shown that, as a matter of principle, we can modelate
locally equivalent physical theories on every la–spaces interrelated with the fundamental
space–time by mean of na–maps. We can introduce a new classification of la-spaces with
respect to nearly conformal transforms to background one with trivial connections and
vanishing torsions and curvatures. On such backgrounds admitting global on corresponding
vector bundles, but locally anisotropic on base space, groups of automorphisms we can
define in a usual manner, for a chosen model of field la–interactions, the conservation laws.
We must complete these laws by a set of invariant conditions being associated to chains of na–maps.

The main advantage of our geometric constructions is theirs compatibility with similar ones introduced in the framework of the tensor integral formalism. We have introduced bi– and multitensors and defined nearly autoparallel tensor integral on bi– and multispaces. We have formulated corresponding conservation laws on la–multispaces.

Our results points the possibility of formulation of physical models of locally anisotropic field interactions and definition of conservation laws on la–spaces in spite of the fact that, at a glance, the generic local anisotropy of such spaces cast doubt on the general possibility of formulation of such problems.

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APPENDIX A: PROOF OF THE THEOREM 2

We sketch the proof respectively for every point in the theorem:

1. It is easy to verify that a–parallel equations (14) on $\xi$ transform into similar ones on $\xi$ if and only if deformations (12) with deformation d–tensors of type $P^\alpha_{\beta\gamma}(u) = \psi(\beta^\delta_{\gamma})$ are considered.

2. Using corresponding to $na_{(1)}$–maps parametrizations of $a(u,v)$ and $b(u,v)$ (see conditions of the theorem) for arbitrary $v^\alpha \neq 0$ on $U \in \xi$ and after a redefinition of deformation parameters we obtain that equations (16) hold if and only if $P^\alpha_{\beta\gamma}$ satisfies (13).

3. In a similar manner we obtain basic $na_{(2)}$–map equations (20) from (16) by considering $na_{(2)}$–parametrizations of deformation parameters and d–tensor.
4. For $na_{(3)}$–maps we must take into consideration deformations of torsion (13) and introduce $na_{(3)}$–parametrizations for $b(u, v)$ and $P^\alpha_{\beta\gamma}$ into the basic na–equations (16). The last ones for $na_{(3)}$–maps are equivalent to equations (21) (with a corresponding redefinition of deformation parameters).

**APPENDIX B: PROOF OF THE THEOREM 3**

1. Let us prove that a–invariant conditions (22) hold. Deformations of d–connections of type

$$^{(0)}\gamma_\alpha^\mu = \gamma_\alpha^\mu + \phi_\alpha \delta_\beta^\mu$$  \hspace{1cm} (B1)

define a–applications. Contracting indices $\mu$ and $\beta$ we can write

$$\phi_\alpha = \frac{1}{m + n + 1} \left( \gamma_\beta^\beta - \gamma_\beta^\beta \right).$$  \hspace{1cm} (B2)

Introducing d–vector $\phi_\alpha$ into previous relation and expressing

$$\gamma_\alpha^\beta = -T_\alpha^\beta + \Gamma_\alpha^\beta$$

and similarly for underlined values we obtain the first invariant conditions from (22).

Putting deformation (B1) into the formula for

$$^{\tau}_{\alpha, \beta\gamma} \quad \text{and} \quad ^{\tau}_{\alpha\beta} = ^{\tau}_{\alpha\tau\beta\tau}$$

we obtain respectively relations

$$^{\tau}_{\alpha, \beta\gamma} - ^{\tau}_{\alpha, \beta\gamma} = \delta_\alpha^\gamma \phi_\gamma + \phi_\alpha \delta_\beta^\gamma + \delta_\gamma^\alpha \phi_\beta w_\alpha^\beta \gamma$$  \hspace{1cm} (B3)

and

$$^{\tau}_{\alpha\beta} = \phi_\alpha \phi_\beta + (n + m - 1)(\phi_\alpha \phi_\beta + \phi_\gamma w_\gamma^\alpha \beta + \phi_\alpha w_\alpha^\beta \phi_\gamma),$$  \hspace{1cm} (B4)

where

$$\phi_\alpha = \phi_\alpha.$$
Putting (B1) into (B3) we can express $\psi_{[\alpha\beta]}$ as

$$\psi_{[\alpha\beta]} = \frac{1}{n+m+1} \left[ L_{[\alpha\beta]} + \frac{2}{n+m+1} \gamma^\tau_{\varphi\tau} w^\varphi_{[\alpha\beta]} - \frac{1}{n+m+1} \gamma^\tau_{\tau[\alpha} w^\varphi_{\beta]} - \right. $$

$$\left. \frac{1}{n+m+1} \left[ r_{[\alpha\beta]} + \frac{2}{n+m+1} \gamma^\tau_{\varphi\tau} w^\varphi_{[\alpha\beta]} - \frac{1}{n+m+1} \gamma^\tau_{\tau[\alpha} w^\varphi_{\beta]} \right] \right]. \quad (B5)$$

To simplify our consideration we can choose an $a$–transform, parametrized by corresponding $\psi$–vector from (B1), (or fix a local coordinate cart) the antisymmetrized relations (B5) to be satisfied by $d$–tensor

$$\psi_{\alpha\beta} = \frac{1}{n+m+1} \left[ L_{\alpha\beta} + \frac{2}{n+m+1} \gamma^\tau_{\varphi\tau} w^\varphi_{\alpha\beta} - \frac{1}{n+m+1} \gamma^\tau_{\alpha\tau} w^\varphi_{\beta}\varphi - r_{\alpha\beta} - \right. $$

$$\left. \frac{2}{n+m+1} \gamma^\tau_{\varphi\tau} w^\varphi_{\alpha\beta} + \frac{1}{n+m+1} \gamma^\tau_{\alpha\tau} w^\varphi_{\beta}\varphi \right]. \quad (B6)$$

Introducing expressions (B1),(B5) and (B6) into deformation of curvature (B2) we obtain the second conditions (22) of $a$-map invariance:

$$(0)W_{\alpha\beta\gamma}^\delta = (0)W_{\alpha\beta\gamma}^\delta,$$

where the Weyl $d$–tensor on $\xi$ (the extension of the usual one for geodesic maps on (pseudo)–Riemannian spaces to the case of $v$–bundles provided with $N$–connection structure) is defined as

$$(0)W_{\alpha\beta\gamma}^\delta = \frac{1}{n+m+1} \left[ L_{\alpha\beta\gamma} + \frac{1}{n+m+1} \left[ \gamma^\tau_{\varphi\tau} w^\varphi_{[\alpha\beta\gamma]} - \delta^\gamma_{\alpha\beta\gamma} \right] \right] - $$

$$\frac{1}{(n+m+1)^2} \left[ \delta^\gamma_{\alpha} \left( 2\gamma^\tau_{\varphi\tau} w^\varphi_{[\alpha\beta\gamma]} - \gamma^\tau_{\tau[\gamma} w^\varphi_{\beta]\varphi} \right) + \delta^\gamma_{\beta} \left( 2\gamma^\tau_{\varphi\tau} w^\varphi_{[\alpha\beta\gamma]} - \gamma^\tau_{\alpha\gamma} w^\varphi_{\beta}\varphi \right) - $$

$$\delta^\gamma_{\gamma} \left( 2\gamma^\tau_{\varphi\tau} w^\varphi_{[\alpha\beta\gamma]} - \gamma^\tau_{\alpha\tau} w^\varphi_{\beta}\varphi \right) \right].$$

The formula for $(0)W_{\alpha\beta\gamma}^\tau$ written similarly with respect to non–underlined values is presented in Section IV.

2. To obtain $na_{(1)}$–invariant conditions we rewrite $na_{(1)}$–equations (19) as to consider in explicit form covariant derivation $(^\gamma D)$ and deformations (12) and (13):

$$2(^\gamma D_\alpha P^\delta_{\beta\gamma} + (^\gamma D_\beta P^\delta_{\alpha\gamma} + (^\gamma D_\gamma P^\delta_{\alpha\beta} + P^\delta_{\alpha\gamma} P^\tau_{\beta\gamma} + P^\delta_{\tau\beta} P^\tau_{\alpha\gamma} + P^\delta_{\tau\gamma} P^\tau_{\alpha\beta}) =$$

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\[ T^\delta_{\tau(\alpha P^r_{\beta \gamma})} + H^\delta_{\tau(\alpha P^r_{\beta \gamma})} + b_{(\alpha P^\delta_{\beta \gamma})} + a_{(\alpha \beta \delta \gamma)}. \] (B7)

Alternating the first two indices in (B7) we have

\[ 2(r^\delta_{(\alpha \beta)\gamma} - r^\delta_{(\alpha \beta)\gamma}) = 2^{(\gamma)} D^\gamma_{\alpha \beta} P^\delta_{\alpha \gamma} - 2^{(\gamma)} D^{(\gamma)}_{\gamma} P^\delta_{\alpha \gamma} + P^\delta_{\tau \alpha} P^r_{\beta \gamma} + P^\delta_{\tau \beta} P^r_{\alpha \gamma} - 2P^\delta_{\tau \gamma} P^r_{\alpha \beta}. \]

Substituting the last expression from (B7) and rescaling the deformation parameters and d–tensors we obtain the conditions (19).

3. Now we prove the invariant conditions for \( na_{(0)} \)–maps satisfying conditions

\[ \epsilon \neq 0 \quad \text{and} \quad \epsilon - F^\alpha_{\beta} F^\beta_{\alpha} \neq 0 \]

Let define the auxiliary d–connection

\[ \tilde{\gamma}^\alpha_{\beta \gamma} = 2^{(\gamma)}_{\beta \gamma} - \psi_{(\beta \delta \gamma)} = \gamma^\alpha_{\beta \gamma} + \sigma_{(\beta F^\alpha_{\gamma})} \] (B8)

and write

\[ \tilde{D}^\gamma_{\alpha} = \gamma^\alpha_{\gamma} + \sigma_{(\beta F^\alpha_{\gamma})} - \epsilon \sigma_{\beta \delta}^\gamma, \]

where \( \sigma_{\beta} = \sigma_{\alpha F^\alpha_{\beta}} \), or, as a consequence from the last equality,

\[ \sigma_{(\alpha F^\gamma_{\beta})} = \epsilon F^\gamma_{\beta} (\gamma^\alpha_{\gamma}) D^\alpha_{(\alpha F^\alpha_{\beta})} - \tilde{D}^\alpha_{(\alpha F^\alpha_{\beta})} + \tilde{\sigma}_{(\alpha \delta \beta)}^\gamma. \]

Introducing auxiliary connections

\[ \gamma^\alpha_{\beta \lambda} = \gamma^\alpha_{\beta \lambda} + \epsilon F^\alpha_{\gamma} D^\gamma_{(\beta F^\gamma_{\lambda})} \]

and

\[ \tilde{\gamma}^\alpha_{\beta \lambda} = \tilde{\gamma}^\alpha_{\beta \lambda} + \epsilon F^\alpha_{\gamma} \tilde{D}^\gamma_{(\beta F^\gamma_{\lambda})} \]

we can express deformation (B8) in a form characteristic for a–maps:

\[ \tilde{\gamma}^\alpha_{\beta \gamma} = \tilde{\gamma}^\alpha_{\beta \gamma} + \tilde{\sigma}_{(\beta \delta \beta)}^\gamma. \] (B9)
Now it’s obvious that $na_{(2)}$–invariant conditions (24) are equivalent with $a$–invariant conditions (22) written for $d$–connection (B9). As a matter of principle we can write formulas for such $na_{(2)}$–invariants in terms of ”underlined” and ”non–underlined” values by expressing consequently all used auxiliary connections as deformations of ”prime” connections on $\xi$ and ”final” connections on $\xi$. We omit such tedious calculations in this work.

4. Finally, we prove the last statement, for $na_{(3)}$–maps, of the theorem 3. Let

$$q_\alpha \varphi^\alpha = e = \pm 1,$$

where $\varphi^\alpha$ is contained in

$$\gamma^\alpha_{\beta \gamma} = \gamma^\alpha_{\beta \gamma} + \psi_{(\beta \delta_\gamma)} + \sigma_{\beta \gamma} \varphi^\alpha.$$ (B11)

Acting with operator $\gamma D_\beta$ on (B10) we write

$$\gamma D_\beta q_\alpha = (\gamma) D_\beta q_\alpha - \psi_{(\alpha \varphi _\beta)} - e \sigma_{\alpha \beta}.$$ (B12)

Contracting (B12) with $\varphi^\alpha$ we can express

$$e \varphi^\alpha \sigma_{\alpha \beta} = \varphi^\alpha ((\gamma) D_\beta q_\alpha - (\gamma) D_\beta q_\alpha) - \varphi^\alpha q_\alpha q_\beta - e \psi_\beta.$$ (B13)

Putting the last formula in (B11) contracted on indices $\alpha$ and $\gamma$ we obtain

$$(n + m) \psi_\beta = \gamma^\alpha_{\alpha \beta} - \gamma^\alpha_{\alpha \beta} + e \psi_\alpha \varphi^\alpha q_\beta + e \varphi^\alpha \varphi^\beta ((\gamma) D_\beta - (\gamma) D_\beta).$$ (B13)

\(\quad\)From these relations, taking into consideration (B10), we have

$$(n + m - 1) \psi_\alpha \varphi^\alpha =$$

$$\varphi^\alpha (\gamma^\alpha_{\alpha \beta} - \gamma^\alpha_{\alpha \beta}) + e \varphi^\alpha \varphi^\beta ((\gamma) D_\beta - (\gamma) D_\beta).$$

Using the equalities and identities (B12) and (B13) we can express deformations (B11) as the first $na_{(3)}$–invariant conditions from (25).
To prove the second class of $na_{(3)}$-invariant conditions we introduce two additional d–tensors:

\[ \rho^\alpha_{\beta\gamma\delta} = r^\alpha_{\beta\gamma\delta} + \frac{1}{2}(\psi(\beta\delta\varphi) + \sigma_{\beta\varphi}\varphi^\tau)w^\varphi_{\gamma\delta} \quad \text{and} \quad \rho^\alpha_{\beta\gamma\delta} = L^\alpha_{\beta\gamma\delta} - \frac{1}{2}(\psi(\beta\delta\varphi) - \sigma_{\beta\varphi}\varphi^\tau)w^\varphi_{\gamma\delta}. \]  

(B14)

Using deformation (B11) and (B14) we write relation

\[ \tilde{\psi}^\alpha_{\beta\gamma\delta} = \rho^\alpha_{\beta\gamma\delta} - \rho^\alpha_{\beta\gamma\delta} = \psi_{\beta}[\delta unknown \delta^\alpha] - \psi_{[\gamma\delta]}\delta^\beta_{\beta\gamma} - \sigma_{\beta\gamma\delta}\varphi^\alpha, \]  

(B15)

where

\[ \psi_{\alpha\beta} = (\gamma)D_{\beta}\psi_{\alpha} + \psi_{\alpha}\psi_{\beta} - (\nu + \varphi^\tau\varphi_{\tau})\sigma_{\alpha\beta}, \]

and

\[ \sigma_{\alpha\beta\gamma} = (\gamma)D_{[\gamma}\sigma_{\beta\alpha] - \mu_{[\gamma}\sigma_{\beta\alpha] - \sigma_{\alpha[\gamma}\sigma_{\beta]}}\varphi^\tau. \]

Let multiply (B15) on $q_{\alpha}$ and write (taking into account relations (B10)) the relation

\[ e\sigma_{\alpha\beta\gamma} = -q_{\tau}\tilde{\psi}^\alpha_{\alpha\beta\delta} + \psi_{\alpha[\beta}q_{\gamma]} - \psi_{[\beta\gamma]}q_{\alpha}. \]  

(B16)

The next step is to express $\psi_{\alpha\beta}$ trough d–objects on $\xi$. To do this we contract indices $\alpha$ and $\beta$ in (B15) and obtain

\[ (n + m)\psi_{[\alpha\beta]} = -\sigma^\tau_{\tau\alpha\beta} + eq_{\tau}\varphi^\lambda\sigma^\tau_{\lambda\alpha\beta} - e\tilde{\psi}_{[\alpha}\tilde{\psi}_{\beta]. \]

Then contracting indices $\alpha$ and $\delta$ in (B15) and using (B16) we write

\[ (n + m - 2)\psi_{\alpha\beta} = \tilde{\psi}^\tau_{\alpha\beta\tau} - eq_{\tau}\varphi^\lambda\tilde{\psi}_{\alpha\beta\lambda} + \psi_{[\beta\alpha]} + e(\tilde{\psi}_{[\beta}q_{\alpha} - \tilde{\psi}_{[\alpha}q_{\beta}), \]  

(B17)

where $\tilde{\psi}_{\alpha} = \varphi^\tau\varphi_{\alpha\tau}$. If the both parts of (B17) are contracted with $\varphi^\alpha$, it results that

\[ (n + m - 2)\tilde{\psi}_{\alpha} = \varphi^\tau\sigma^\lambda_{\tau\alpha\lambda} - eq_{\tau}\varphi^\lambda\varphi^\delta\sigma^\tau_{\lambda\alpha\delta} - eq_{\alpha}, \]

and, in consequence of $\sigma^\alpha_{\beta(\gamma\delta)} = 0$, we have

\[ (n + m - 1)\varphi = \varphi^\beta\varphi^\gamma\varphi^\alpha_{\beta\gamma\alpha}. \]
By using the last expressions we can write

$$
(n + m - 2)\psi_\alpha = \varphi^\tau \sigma_{\tau\alpha\lambda} - eq_\tau \varphi^\lambda \rho^\delta \sigma_{\lambda\alpha\delta} - e(n + m - 1)^{-1} q_\alpha \varphi^\tau \rho^\delta \sigma_{\tau\lambda\delta}. \quad (B18)
$$

Contracting (B17) with $\varphi^\beta$ we have

$$(n + m)\hat{\psi}_\alpha = \varphi^\tau \sigma_{\alpha\tau\lambda} + \ddot{\psi}_\alpha$$

and taking into consideration (B18) we can express $\hat{\psi}_\alpha$ through $\sigma_{\alpha\beta\gamma\delta}$.

As a consequence of (B16)–(B18) we obtain this formulas for d–tensor $\psi_{\alpha\beta}$:

$$
(n + m - 2)\psi_{\alpha\beta} = \sigma^\tau_{\alpha\beta\tau} - eq_\tau \varphi^\lambda \rho^\delta \sigma_{\tau\alpha\lambda} +
\frac{1}{n + m} \left\{-\sigma^\tau_{\alpha\beta\alpha} + eq_\tau \varphi^\lambda \rho^\delta \sigma_{\lambda\beta\lambda} - q_\beta (e\varphi^\tau \sigma_{\alpha\tau\lambda} - q_\tau \varphi^\lambda \rho^\delta \sigma_{\alpha\lambda\delta}) +
\right.
\left.
eq_a [\varphi^\lambda \sigma^\tau_{\alpha\beta\lambda} - eq_\tau \varphi^\lambda \rho^\delta \sigma^\tau_{\lambda\beta\delta} - \frac{e}{n + m - 1} q_\beta (e\varphi^\tau \sigma_{\tau\gamma\delta} - eq_\tau \varphi^\lambda \rho^\delta \sigma_{\tau\beta\lambda\delta}\right].
$$

Finally, putting the last formula and (B16) into (B15) and after a rearrangement of terms we obtain the second group of $na_{(3)}$-invariant conditions (25). If necessary we can rewrite these conditions in terms of geometrical objects on $\xi$ and $\xi$. To do this we must introduce splittings (B14) into (25).
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