Atom chips with two-dimensional electron gases: theory of near surface trapping and ultracold-atom microscopy of quantum electronic systems

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We show that current in a two-dimensional electron gas (2DEG) can trap ultracold atoms < 1 µm away with orders of magnitude less spatial noise than a metal trapping wire. This enables the creation of hybrid systems, which integrate ultracold atoms with quantum electronic devices to give extreme sensitivity and control: for example, activating a single quantized conductance channel in the 2DEG can split a Bose-Einstein condensate (BEC) for atom interferometry. In turn, the BEC offers unique structural and functional imaging of quantum devices and transport in heterostructures and graphene.

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Figure 1: (Color) Schematic of a heterojunction-based atom chip comprising GaAs and (AlGa)As layers (labelled) in the $x-y$ plane (axes inset). Ionized Si donors (red layer) supply electrons to the 2DEG (blue), which lies in the $z = 0$ plane. GaAs and (AlGa)As layers below the 2DEG are shown semi-transparent for clarity. Insulating regions (gray) in the 2DEG enclose distinct conducting channels, 1 and 2, which are typically a few µm or nm wide respectively. A BEC (red) is confined in a harmonic trap, centered in the $z = z_0$ plane, produced by applied field, B, (green arrow), and current through either the metal surface wire (yellow Z shape) or 2DEG Channel 1. Rectangular yellow regions are metal surface gates, which can be biased negatively to produce a QPC (between light green depletion regions) in the lower arm of Channel 2.
This, combined with low Johnson noise and weak CP attraction to membranes, makes 2DEGs – in heterojunctions or in graphene – ideal for producing the smooth stable near-surface traps required for creating hybrid cold-atom/quantum electronic systems. To illustrate the potential of such systems, we show that the BEC is so sensitive to the quantum conductor that it can detect the opening or closure of a single quantized conductance channel in a quantum point contact (QPC) [11, 30] – enabling functional imaging of quantum electron transport and devices over hundreds of μm. Opening and closing the channel splits and remerges the BEC, demonstrating that delicate quantum electronic transport processes offer robust control of atomic matter waves.

We consider an atom chip built on a GaAs/(AlGa)As heterojunction, which traps a BEC near a 2DEG (Fig. 1). In the first part of the paper, the trapping potential is produced by current, \( I_m \), through a \( Z \)-shaped metal surface wire (yellow in Fig. 1), combined with a uniform magnetic field \( \mathbf{B} = (B_x, B_y, B_z) \), which positions the trap center at distance \( z_0 \) from conducting Channel 1 in the 2DEG [8]. The 2DEG is formed by electrons from ionized donors in a narrow \( (\approx 15 \text{ nm}) \) thick sheet (blue in Fig. 1). Insulating regions in the 2DEG (gray in Fig. 1), made by implanting Ga ions [32], enclose two distinct conduction channels, labelled 1 and 2 in Fig. 1.

Atom chips are usually built on a bulk substrate, which generates strong CP attraction, thus preventing atoms from being trapped closer than a few μm from the surface [13, 35]. Recently, suspended trapping wires were used to reduce the CP potential [26]. We consider a heterojunction membrane of width \( w = 130 \text{ nm} \), as in recent experiments [25], and calculate the CP potential energy \( V_{CP}(z) \approx -wC_5/z^5 (z \gg w) \), where \( C_5 = 2 \times 10^{-34} \text{ Jm}^4 \) via Eqs. (25-29) of [34]. As shown below, \( V_{CP} \) is weak enough to allow submicron trapping.

We take typical heterojunction parameters with \( n_d = 3.3 \times 10^{15} \text{ m}^{-2} \) ionized donors at distance \( d = 50 \text{ nm} \) from the 2DEG (Fig. 1) [11, 13, 35]. The 2DEG is 65 nm below the surface and of mean electron density equal to \( n_d \). Since the 2DEG is so thin, surface fluctuations are negligible: a major advantage over metal wires for near-surface trapping. We take the heterojunction temperature to be 4.2 K (as in superconducting atom chips [10]) to ensure high 2DEG conductivity, \( \sigma = 7.2 \times 10^{-2} \Omega^{-1} \), and that inhomogeneity in the current originates only from non-uniformity of the ionized donors [26, 28].

These donors create a spatially-varying attractive potential, which is partially screened by the 2DEG [11]. In the Thomas-Fermi screening model [35], the potential energy of a 2DEG electron at position \( r = (x, y) \) is

\[
\Phi(r) = \frac{e^2}{4\pi\varepsilon_0} \int e^{-k_d p_d(k)} e^{ik \cdot r} d^3k,
\]

where \( k_d = (k_x, k_y) \), \( p_d(k) \) is the 2D Fourier transform of the spatial ionized donor density, \( p_d(r) = n_d + \Delta p_d(r) \), \( \epsilon \) is the relative permittivity of GaAs, and the screening wave vector, \( k_s = e^2 m^*/(2\epsilon\epsilon_0\pi\hbar^2) \), depends on the electron charge, \( -e \), and effective mass, \( m^* \) [35].

We now consider the electrostatic properties and current profile of Channel 1, whose width is henceforth taken to be \( z_0 \). Figure 2 shows (a) a typical uncorrelated ionized donor distribution, \( p_d(r) \), and (b,c) the screened potential energy in the 2DEG, \( \Phi(r) \) [27].

When an applied voltage creates an electric field, \( \mathbf{E} \), along Channel 1, the local current density is \( j(r) = \sigma \mathbf{E} + \Delta j(r) \) [36], where the inhomogenous component \( \Delta j(r) = \sigma \nabla \Phi(r)/e \) originates from fluctuations in \( p_d(r) \) and \( \Phi(r) \).

The black curves in Fig. 2(b,c) show current streamlines calculated for \( E = 2.5 \text{ kV m}^{-1} \). Inhomogeneity of \( \Phi(r) \) makes these streamlines deviate from the \( x \)-direction. Consequently, the resulting magnetic field at position \( r = (x, y) \) in the \( z = z_0 \) plane has a non-zero \( x \)-component given, from the Biot-Savart law [7, 36], by

\[
B_x(r, z_0) = \frac{\mu_0 e \sigma}{4\pi\epsilon_0} \int \frac{k_y \Delta p_d(k) e^{-k \cdot (d+z_0)} e^{ik \cdot r}}{k + k_s} d^3k,
\]

where \( \Delta p_d(k) \) is the 2D Fourier transform of the donor density fluctuations, \( \Delta p_d(r) \). Equation (2) reveals two key results. First, the fluctuations of \( B_x(r, z_0) \) depend directly on the donor distribution via \( \Delta p_d(k) \). Second, due to the \( e^{-k \cdot (d+z_0)} \) term, the field fluctuations become weaker and smoother as \( z_0 \) increases. This can be seen by comparing the color maps of \( B_x(r, z_0) \) in Fig. 2(d,e).
We now consider how the field fluctuations affect a BEC, henceforth called BEC A, comprising $10^4 \ ^{87}\text{Rb}$ atoms in state $|F = 2, m_F = 2\rangle$, whose position in the $z = z_0$ plane is shown by the gray regions in Fig. [2] (d,e).

Since the BEC is strongly confined along $y$ and $z$, its density profile is sensitive only to fluctuations in $B_z(x, z_0)$. Along the $x-$axis [$r = (x, 0)$], the atom density fluctuations are $\Delta n(x) = -\langle m_F g_{F\mu B}/2\omega_0 a_s B_z(x, z_0) \rangle$, where the $g$-factor $g_F = 1/2$, $\mu_B$ is the Bohr magneton, $\omega_r$ is the trap frequency in the $y-z$ plane, and $a_s$ is the $s$-wave scattering length.

The variation of the oscillatory amplitudes of $B_z(x, z_0)$ with $z_0$ is crucial for understanding, and exploiting, the effect of 2DEG current on a BEC. To demonstrate this, we first consider the rms average, $B_{\text{rms}}^z(z_0)$, of $B_z(x, z_0)$ along $z$ at given $z_0$. If the spatial donor correlation function, $\langle \Delta p_d(r)\Delta p_d(r') \rangle = C(r, r') = C(|x-x'|, y, y')$, is homogeneous along $x$ and $\to 0$ as $|x-x'| \to \infty$, the rms spatial average is equivalent to an ensemble average, denoted (...), at any given $x$. Choosing $x = 0$ gives

$$B_{\text{rms}}^z(z_0) = \left(\frac{\mu_0 e\sigma}{4\epsilon_0}\right) \int \int \frac{k_y k_y' S(k, k')}{(k + k_s)(k' + k_s)} e^{-(k+k')(d+z_0)} d^2k d^2k' \right)^{1/2}. \tag{3}$$

Equation (3) reveals that $B_{\text{rms}}^z(z_0)$ depends on, and can hence probe, the correlation function of the ionized donors, $S(k, k') = \langle \Delta p_d(k)\Delta p_d(k') \rangle$, in $k$-space.

For a random donor distribution (Fig. [3] right inset), $S(k, k') \propto \delta(k + k')$. Using this in Eq. (3) gives $B_{\text{rms}}^z(z_0) \sim 1/z_0^2$ (solid curve in Fig. [3]), meaning that for $z_0 \gtrsim 1.5 \mu m$, field fluctuations decay more slowly above a 2DEG than above a metal wire (dotted curve in Fig. [3]). At this point, $S(k, k') \propto \delta(k_0 + k_0)\delta(k_y' - k_0) + \delta(k_y - k_0)\delta(k_y' + k_0)$, where $k_0 = 2\pi/\lambda_y$. Using this in Eq. (3) gives $B_{\text{rms}}^z(z_0) \sim \exp(-z_0/\lambda_y)$, shown for $\lambda = 660 \text{ nm}$ by the dotted curve in Fig. [3]. This rapid decay occurs because, as in a magnetic mirror, the current streamlines align with the striped donor pattern (Fig. [3] left inset), which reduces their meander and, hence, $B_{\text{rms}}^z(z_0)$.

The ability to tailor the potential landscape of the 2DEG, and the resulting field fluctuations, is a unique feature of heterojunctions, which can be exploited for trapping, manipulating, and imaging with, ultracold Bose gases. Crucially, exponential decay makes the $B_{\text{rms}}^z(z_0)$ curve for the periodically-modulated donor distribution (dotted in Fig. [3]) fall rapidly below that for a metal wire (dot-dashed curve in Fig. [3]). At $z_0 \gtrsim 0.8 \mu m$, the field fluctuations above the 2DEG are more than 3 orders of magnitude smaller than for the metal wire.

Consequently, 2DEGs have great potential for creating near-surface microtraps. To demonstrate this, we consider the trap produced by a 355 $\mu \text{A current through 2DEG Channel 1 (of width 3 $\mu m$ and central arm length 60 $\mu m$) only}$, setting $I_w = 0$. The solid curve in Fig. [4] (a) shows the total potential energy $V_{\text{tot}}(z) = V_{Iw}(z) + V_{\text{CP}}(z)$ calculated for an $\ ^{87}\text{Rb}$ atom, where $V_{Iw}(z)$ originates from the magnetic field produced by Channel 1 and an applied field $B = (40, 536, 0) \text{ mG}$. Since the CP attrac-
one-dimensional plane wave states up to the Fermi level, \( G \) to motion across it. Along the channel, whose quantized energy levels, \( E \) are only \( \approx 2 \hbar \nu \) is only \( \approx 20 \) \( \mu \) from the 2DEG \((\text{solid curve in Fig.} \, 4\text{[b]})\) because \( B^x_{\text{res}}(z_0) \) is only \( \approx 20 \) \( \text{nT} \) \((\text{Fig.} \, 3\text{[b]}\text{)}\). Even at 4.2 K, the resistance of Channel 1 is \( \approx 10 \) times higher than a similarly-sized gold conductor at room temperature \([11]\). Consequently, 2DEGs offer lower Johnson noise and spin-flip loss rates than metal wires.

Near-surface trapping makes the BEC highly sensitive to magnetic field variations arising from the geometry of the conduction channels, such as local narrowing. As an example, suppose that \( B \) is adjusted to hold BEC \( B \) across the middle arm of U-shaped Channel 2 \([\text{Figs.} \, 1\text{[a]} \text{and} \, 4\text{[b]}\text{]}\). Channel 2 is narrow enough \((20 \text{ nm})\) for the electrons to populate a small number, \( N \), of discrete energy levels, \( E_i \) \((i = 0, 1, 2, ..., N)\), corresponding to motion across it. Along the channel, whose quantized conductance is \( G = 2N\hbar^2/e \) \([11, 24]\), electrons occupy one-dimensional plane wave states up to the Fermi level, \( E_F \). Applying a negative voltage, \(-|V_g|\), to metal surface gates \((\text{yellow rectangles in Fig.} \, 4\text{)}\), which are on either side of the lower arm of Channel 2 and sufficiently far from the BEC to have negligible electrostatic effect on it, locally depletes the 2DEG. This narrows the conduction channel, forming a QPC and raising \( E_i \). As \( V_g \) increases, the energy levels successively exceed \( E_F \) and depopulate \([11, 29, 30]\). Depopulation of each level decreases \( N \) and, hence, the current through Channel 2 by \( \Delta J = 2e^2V/h \), where \( V \) is the voltage dropped across the QPC.

The dashed curve in Fig. \( 4\text{[b]} \) shows the density profile, \( n(x) \), of BEC \( B \) when the QPC in Channel 2 is fully depleted \((N = 0)\). Since the QPC carries no current in this case, \( n(x) \) is the unperturbed ground state of the trapping potential shown by the solid curves in Fig. \( 4\text{[a,b]}\). Opening a single conduction channel in the QPC \((N = 1)\) changes the trap profile sufficiently to almost completely split the BEC \([\text{dotted curve in Fig.} \, 4\text{[b]}\text{]}. Consequently, the BEC can detect quantized changes in the QPC’s conductance, which, conversely, can manipulate the BEC, for example splitting and recombining it in atom interferometry. More complex shaping of the atom cloud is also possible: an array of QPCs could imprint and control a wide range of sub-\( \mu \)m patterns in the BEC.

In summary, quantum electron transport in heterojunctions can create smooth, low-noise, magnetic traps, which provide the sub-\( \mu \)m control required to integrate ultracold atoms with quantum electronic systems. By tailoring the donor distribution, magnetic field fluctuations can be suppressed exponentially. Quantum transport processes in a 2DEG can imprint strong density modulations in a BEC, which, conversely, provide a non-invasive probe of those processes. The ability to measure the potential landscape and mobility of a 2DEG independently may yield new insights for understanding how the two relate and, hence, for increasing 2DEG mobility \([38, 39]\). Suspended graphene membranes, which combine low CP attraction with high room-temperature mobility \([12]\), could be the ultimate material for sub-\( \mu \)m atom trapping and BEC microscopy of quantum transport.

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Assuming $E$ is large enough for $\Phi(\mathbf{r})$ to weakly perturb the current streamlines. Atom density fluctuations then result only from $\Delta j_y(\mathbf{r})$ and are thus independent of $E$. 

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