Generalized uncertainty principle and quantum non-locality

S. Aghababaei1 · H. Moradpour2

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Abstract
The emergence of the generalized uncertainty principle and the existence of a non-zero minimal length are intertwined. On the other hand, the Heisenberg uncertainty principle forms the core of the EPR paradox. Subsequently, here, the implications of resorting to the generalized uncertainty principle (or equally, the minimal length) instead of the Heisenberg uncertainty principle on the quantum non-locality are investigated by focusing on the Franson experiment in which energy-time entanglement is the backbone of understanding and explaining the results. The survey also unveils the power of such an experiment in verifying the generalized uncertainty principle.

Keywords Quantum gravity · Generalized uncertainty principle · Franson experiment · Quantum non-locality

1 Introduction

Certainly, quantum non-locality (QNL) is one of the most intriguing subjects in physics rooted in the famous paper by Einstein, Podolsky, and Rosen (EPR) [1]. Basically, there is a deep connection between QNL and the Heisenberg uncertainty principle (HUP) [1–4]. Indeed, this property has been obtained since people could reach quantum energy levels. The validity of the Schrödinger equation (or equally, the quantum mechanics) decreases as the system energy increases, and at high energy levels, quantum mechanics should be replaced by quantum field theory. Therefore, it is a significant
challenge to study the quality of non-locality in high energy physics to answer the question of whether there is a change in QNL with increasing energy or not. In this regard, the effects of special relativity and curved spacetime on the behavior of QNL have extensively been studied [5–13].

Despite the success of general relativity, the relation between quantum mechanics and gravity is still mysterious [14], and attempts to find a quantum gravity scenario continue [15, 16]. One common feature of quantum gravity (QG) scenarios is the generalization of HUP called the Generalized Uncertainty Principle (GUP). It leads to the existence of a nonzero minimum length [17–19]. Such deformation is also expected in deformed field theories [20, 21]. Indeed, minimum observable (effective) length [16] means that the quantum mechanical commutators of operators should change, a change that is expected to become very noticeable near the Planck energy [22]. This correspondence is a great motivation to replace GUP with HUP, which stimulates us to study the implications of this replacement on QNL.

In general, when the quantum features of gravity are considered, canonical operators $x$ and $p$ are replaced with their generalized counterparts $X$ and $P$, respectively, and up to the first order of the GUP parameter $\beta$, we can write $P_i = p_i (1 + \beta f (p))$ in the position representation, where $X = x$. Applications of this representation appeared in low-energy effective field theories [20, 21] are also studied in many works such as [22, 23]. Additionally, in order to get an insight into the implications of QG (GUP) on the current physics, one may estimate the effects of QG as the perturbations to quantum mechanics, classical mechanics, and quantum field theory [16, 23–31].

As it has been argued, QNL is the result of HUP (the result of the non-commutative of operators) [1–4]. The dependency of the square of the Bell operators to the commutators is a bright signal from it [32, 33]. This point is addressed in Ref. [34] where, considering the effects of GUP on angular momentum algebra, it is shown that the Bell operators square of two partite systems changes. Based on this paper, the Bell operator and thus its expectation value do not change as this operator consists of operators with eigenvalues $\pm 1$, and hence, it remains unchanged. Therefore, there is an incompatibility, i.e., while the Bell operator does not change, its square changes. Indeed, mathematically and compared to the square of the Bell operator, the commutator’s role (or equally, HUP) is not obvious in the Bell operator due to the fact that the inequality is not a function of HUP. Hence, the fundamental question of whether GUP affects a Bell-type experiment (and thus QNL) or not still needs to be studied further.

In quantum mechanics, HUP is expressed as

$$\Delta x \Delta p \geq \frac{\hbar}{2},$$

(1)

for the operators $x$ and $p$, a result of the non-commutativity of the operators, i.e., $[x, p] = i\hbar$. There is also an uncertainty relation between energy and time as $\Delta t \Delta E \geq \frac{\hbar}{2}$ which is not a principle. However, for photons, this relation and HUP are equal as we have $x = ct$ and $E = pc$. For this massless particle, uncertainty in the transit time is equal to its position uncertainty ($\Delta x = c\Delta t$) [3]. The same is true for its energy and momentum [29]. Therefore, considering photon, due to the equivalence between its time and position from one side and its energy and momentum from the other side, one may conclude that the effects of GUP on QNL are studiable by employing
Fig. 1 Franson experiment: A source produces a pair of photons $\gamma_1$ and $\gamma_2$, sent through two Mach–Zehnder interferometers. At the end of the path, each of the photons is registered by either one of two detectors ($D_1$, $D'_1$ and $D_2$, $D'_2$, respectively). $\phi_1$ and $\phi_2$ are the phase shifts from different paths (S and L) of each photon. The dash lines are the half-silvered mirrors to split and recombine the beams to the shorter and longer paths, respectively.

an experiment that verifies QNL using energy-time entanglement of photons. This is essential because the existence of minimal length affects momentum (energy) and correspondingly, GUP proposes a generalization to (1).

In the Franson experiment [3], based on the energy-time entanglement, uncertainty principle has a crucial role in the results expressed using the coincidence rate [3, 35, 36]. Consequently, in order to study the effects of GUP on a Bell-type experiment, we focus on this test. The paper is structured as follows. After providing general remarks on the Franson experiment in section (II), the implications of the existence of a minimal length on its outcomes are addressed in Sec. (III). A summary has also been presented in the last section.

2 Franson experiment

In the Franson experiment, there is a source emitting a pair of entangled photons that are propagated in opposite directions and move toward two Mach–Zehnder interferometers. Figure 1 illustrates the setup in which two distant interferometers located at the end of the long (L) and short (S) paths. This setup produces a phase shift ($\phi$) and a time delay ($\Delta T$). The entangled photons are generated using an atomic cascade including three quantum levels, while the highest energy level has energy $E_1$ with a relatively long lifetime $\tau_1$. The intermediate and ground states have energy $E_2$ (with lifetime $\tau_2 \ll \tau_1$) and $E_3$ with a very long lifetime $\tau_3$ ($\tau_1 < \tau_3$), respectively. $\Delta T$ is the difference between transit times corresponding to the shorter and longer paths. It stores the uncertainty in position through the $c\Delta T = \Delta x$ relation, is assumed the same for both photons, and meets the $\tau_2 \ll \Delta T \ll \tau_1$ condition (see Ref. [3] for more info). The superposition of the photon states arriving at the two detectors $D_1$ and $D'_1$ and
\( D_2(D'_2) \) (see Fig. 1) is described as

\[
\varphi_k(x_1, t) = \frac{1}{2} \varphi_{k,0}(x_1, t) + \frac{1}{2} e^{i\phi_1} \varphi_{k,0}(x_1, t - \Delta T),
\]

and

\[
\varphi_k(x_2, t) = \frac{1}{2} \varphi_{k,0}(x_2, t) + \frac{1}{2} e^{i\phi_2} \varphi_{k,0}(x_2, t - \Delta T),
\]

respectively. Here, \( \varphi_{k,0}(x_i, t) \) denote the photon state with momentum \( k \) which removes the half-silver mirrors, and \( x_i \) are the locations of detectors. \( \phi_1 \) and \( \phi_2 \) are phases that store the information related to the shifts due to the half-silvered mirrors in the short path and long path of two interferometers, and they can be adjusted as desired. \( \Delta T \) is the difference between the transit times via the longer and shorter paths, assumed to be the same for both photons. \( R_c \) as the coincidence rate for detecting photons at the same time in the horizontal detectors is calculated as

\[
R_c = \eta_1 \eta_2 \left\langle 0 \left| \varphi_k^\dagger(x_1, t) \varphi_k^\dagger(x_2, t) \varphi_k(x_2, t) \varphi_k(x_1, t) \right| 0 \right\rangle,
\]

where \( \eta_i \) denotes the efficiency of the corresponding detector, and we can briefly write

\[
R_c = \frac{1}{16} \eta_1 \eta_2 \langle 0 | A \dagger A | 0 \rangle,
\]

in which

\[
A = \varphi_{k,0}(x_1, t) \varphi_{k,0}(x_2, t) + e^{i\phi_1} e^{i\phi_2} var \varphi_{k,0}(x_1, t - \Delta T) \varphi_{k,0}(x_2, t - \Delta T).
\]

Whenever \( \Delta T \ll \tau_1 \), the amplitude of detecting a pair of photons at time \( t - \Delta T \) will be approximately equal to the amplitude of detecting a pair of photons at time \( t \), and they have only a constant phase difference causing

\[
\varphi_{k,0}(x_1, t) \varphi_{k,0}(x_2, t) = \sum_{k_1, k_2} c_{k_1} c_{k_2} e^{i(k_1 \cdot x_1 - \omega_1 t)} e^{i(k_2 \cdot x_2 - \omega_2 t)},
\]

\[
\varphi_{k,0}(x_1, t - \Delta T) \varphi_{k,0}(x_2, t - \Delta T) = \sum_{k_1, k_2} c_{k_1} c_{k_2} e^{i(\omega_1 + \omega_2)\Delta T} \times e^{i(k_1 \cdot x_1 - \omega_1 t)} e^{i(k_2 \cdot x_2 - \omega_2 t)},
\]

where \( c_{k_1}, c_{k_2} \) are the expansion coefficients in the Fourier transformation and can be determined by the system evaluation. Energy conservation yields \( \omega \equiv \omega_1 + \omega_2 = (E_1 - E_3) / \hbar + \Delta \omega \), where \( E_1 \) and \( E_3 \) are the unperturbed energies of initial and final states, respectively, and therefore, \( \Delta E = E_1 - E_3 \gg \Delta \omega \) [3]. \( \Delta \omega \sim \frac{1}{\tau_1} + \frac{1}{\tau_3} \) and
satisfies $\Delta \omega \Delta T \gg 1$ which finally leads us to $\Delta E \Delta T \gg 1$. In fact, $\Delta \omega$ is much less than the individual uncertainty of $\omega_i$ since $\tau_2$ is relatively short [3], and thus

$$\varphi_{k,0}(x_1, t - \Delta T) \varphi_{k,0}(x_2, t - \Delta T) = e^{i(E_1 - E_3) \Delta T / \hbar} \varphi_{k,0}(x_1, t) \varphi_{k,0}(x_2, t), \quad (8)$$

leading to

$$R_c = \frac{1}{16} R_0 \left[ 1 + e^{-i(\Delta E \Delta T / \hbar + \phi_1 + \phi_2)} \right] \times \left[ 1 + e^{i(\Delta E \Delta T / \hbar + \phi_1 + \phi_2)} \right], \quad (9)$$

in which

$$R_0 = \left\langle 0 \left| \varphi_{k,0}(x_1, t) \right|^2 \varphi_{k,0}(x_2, t) \right| \left. \right|^2 0 \right\rangle = \eta_1 \eta_2 \left\langle 0 \left| \sum_{k_1, k_2} c_{k_1}^\dagger c_{k_2}^\dagger c_{k_2} c_{k_1} \right| 0 \right\rangle, \quad (10)$$

is the coincidence rate with the half-silvered mirrors removed (shorter lengths). Finally, one finds [3]

$$R_c = \frac{1}{4} R_0 \cos^2 \left( \frac{\Delta E \Delta T / \hbar + \phi_1 + \phi_2}{2} \right)$$

$$= \frac{1}{4} R_0 \cos^2 \left( \phi'_1 - \phi'_2 \right), \quad (11)$$

where

$$\phi'_1 = \phi_1 / 2, \quad \phi'_2 = - (\phi_2 + \Delta E \Delta T / \hbar) / 2, \quad (12)$$

in which $R_c$ denotes the amplitude probability of detecting entangled photons in detectors and hence $0 \leq R_c \leq 1$. Indeed, the Franson experiment can be described by a part including a single phase $\phi_1 + \phi_2$ instead of two phases [37] depending on the optical path difference, and a part $R_0$ which depends on the distance of detectors [38]. We have $\Delta L = c \Delta T$ for the length differences between the two paths of each interferometer, and $\Delta E = c p$ for the photon energy. Therefore, as energy (momentum) is changed by the existence of minimum length, this experiment may open a window to determine the effects of GUP on entanglement. In the next section, we investigate the changes in the coincidence rate due to replacing GUP with HUP.

### 3 Franson experiment in the presence of minimum length

In this section, we intend to study the implications of GUP on Eq. (11) and thus QNL, by employing the perturbation theory.
3.1 Minimal length framework

A well-known GUP is written as [16–19, 22, 23]

\[ \Delta X \Delta P \geq \frac{\hbar}{2} \left( 1 + \beta (\Delta P)^2 \right), \tag{13} \]

where \( \beta = \frac{\beta_0}{M_p c^2} = \frac{\beta_0 l_p^2}{\hbar^2} \), in which a fundamental minimal length is found as \( \Delta X_{\text{min}} = \hbar \sqrt{\beta} = \sqrt{\beta_0 l_p} \), in order of Planck’s length. The HUP algebra can be modified in this frame as

\[ [X, P] = i \hbar (1 + \beta P^2), \tag{14} \]

in which \( X \) and \( P \) refer to the position and momentum operators, and \( \beta \) is well-known as the GUP parameter. There are two general representations, namely the position and momentum representations to describe the GUP. Here, affected by the simplicity of applying the perturbation theory in the position representation, we adopt this representation [22, 23, 26, 27]

\[ X = x, \quad P = p(1 + \beta p^2), \tag{15} \]

where \( x \) and \( p \) denote the position and momentum in the quantum mechanic space, respectively. Now, suppose that two emitted photons in the Franson experiment are affected by the quantum gravity modifications. It means that the Hamiltonian of atoms and thus their energy levels are also perturbed by the modifications of the QG scenarios, and thus, we have \( \hat{H}_{\text{GUP}} = \hat{H} + \beta \hat{H}_p \), where \( \hat{H}_p \) refers to the perturbed Hamiltonian in the GUP frame, and finally, we have \( E_{\text{GUP}} = E + \beta E_p \) for the energy levels. \( E_p \) can be determined using the perturbation theory (up to the desired level) [26]. Therefore, due to the existence of minimal length, the state of each photon is modified as \( \varphi_{k}^{\text{GUP}}(x) = \varphi_k(x) + \beta \varphi_k^p(x) \), where the index \( p \) denotes the correction terms in the GUP framework [16, 29]. The time evolution of \( \varphi_{k}^{\text{GUP}}(x) \) is obtained by

\[ \varphi_{k}^{\text{GUP}}(x, t) = e^{i \hat{H}_{\text{GUP}}t/\hbar} \varphi_{k}^{\text{GUP}}(x) e^{-i \hat{H}_{\text{GUP}}t/\hbar} = \varphi_k(x, t) - i \beta \Gamma_k(x, t) + \beta \varphi_k^p(x, t) + \mathcal{O}(\beta^2), \tag{16} \]

where \( \Gamma_k(x, t) = [\varphi_k(x, t), \hat{H}_p] t / \hbar \).

Indeed, there have been many works to detect low-energy signatures of quantum gravity effects [26, 27, 39–42], which included accurate spectroscopic measurements, gravitational bar detectors, and optomechanical experiments. Thus, new phenomena in several precision experiments at low-energy systems can be a window to follow the GUP effects. To this end, we will focus on the GUP effects in the Franson experiments using the perturbation method.
3.2 Coincidence rate with minimal length

For the counterparts of Eqs. (2) and (3), similar to the above argument, and by following the Franson approach, the state corresponding to the \( i \)th photon, at the detector \( D_i \) can be written as

\[
\psi_{GUP}^k (x_i, t) = \frac{1}{2} \psi_{k,0}^{GUP} (x_i, t) + \frac{1}{2} e^{i \phi_1} \psi_{k,0}^{GUP} (x_i, t - \Delta T).
\]

Therefore, the corresponding coincidence rate \( R_c^{GUP} \) is achieved by

\[
R_c^{GUP} = \eta_1 \eta_2 \langle 0 | \psi_{k}^{GUP,\dagger} (x_1, t) \psi_{k}^{GUP,\dagger} (x_2, t) \\
\times \psi_{k}^{GUP} (x_2, t) \psi_{k}^{GUP} (x_1, t) | 0 \rangle,
\]

summarized into

\[
R_c^{GUP} = \eta_1 \eta_2 \langle 0 | B \dagger B | 0 \rangle,
\]

in which

\[
B = \psi_{k}^{GUP} (x_2, t) \psi_{k}^{GUP} (x_1, t) = \frac{1}{4} \left\{ \psi_{k,0} (x_2, t) \psi_{k,0} (x_1, t) \right. \\
+ e^{i \phi_1} e^{i \phi_2} \psi_{k,0} (x_2, t - \Delta T) \psi_{k,0} (x_1, t - \Delta T) \\
+ \beta \left[ -i \psi_{k,0} (x_2, t) \Gamma_{k,0} (x_1, t) \\
+ \psi_{k,0} (x_2, t) \psi_{k,0}^p (x_1, t) - i \Gamma_{k,0} (x_2, t) \psi_{k,0} (x_1, t) \\
+ \psi_{k,0} (x_2, t) \psi_{k,0} (x_1, t) - i \psi_{k,0} (x_2, t - \Delta T) \Gamma_{k,0} (x_1, t - \Delta T) \\
- i e^{i \phi_1} e^{i \phi_2} \psi_{k,0} (x_2, t - \Delta T) \Gamma_{k,0} (x_1, t - \Delta T) \\
+ e^{i \phi_1} e^{i \phi_2} \psi_{k,0}^p (x_2, t - \Delta T) \psi_{k,0} (x_1, t - \Delta T) \\
+ e^{i \phi_1} e^{i \phi_2} \psi_{k,0}^p (x_2, t - \Delta T) \psi_{k,0} (x_1, t - \Delta T) \right\} + O(\beta^2),
\]

where we employed Eq. (17) and considered the condition \( \Delta T \ll \tau_1 \) that vanishes some terms. Using the Fourier expansion, one can write

\[
\psi_{k,0}^p (x_i, t) = \sum_{k_i} c'_{k_i} e^{i (k_i \cdot x_i - \omega_i t)},
\]
\[ \Gamma_{k,0}(x_i, t) = \sum_{k_i} c_{k_i}' e^{i(k_i \cdot x_i - \omega_i t)} , \quad (22) \]

in which \( c_{k_i}' \) and \( c_{k_i}'' \) are the corresponding dimensional coefficients in the Fourier expansion of the modifications of photon state in the GUP frame.

By applying the Fourier expansions and substituting Eq. (20) into Eq. (18), one can find

\[
R_{GUP}^c = \frac{1}{16} \eta_1 \eta_2 \sum_{k_1, k_2} \left\{ \langle 0 | c_{k_1}^\dagger c_{k_2}^\dagger c_{k_2} c_{k_1} | 0 \rangle + \beta \left[ \langle 0 | c_{k_1}^\dagger c_{k_2}^\dagger c_{k_2} c_{k_1} | 0 \rangle + \langle 0 | c_{k_1}^\dagger c_{k_2} c_{k_2} c_{k_1} | 0 \rangle + \langle 0 | c_{k_1}^\dagger c_{k_2}^\dagger c_{k_2} c_{k_1} | 0 \rangle \right] \right\}
\times \left( 1 + e^{-i \left( \frac{(E_{GUP}^3 - E_{GUP}^1) \Delta T}{\hbar} + \phi_1 + \phi_2 \right)} \right)
\times \left( 1 + e^{i \left( \frac{(E_{GUP}^3 - E_{GUP}^1) \Delta T}{\hbar} + \phi_1 + \phi_2 \right)} \right). \quad (23) \]

In addition, the corresponding energy conservation in the GUP frame leads to

\[
\omega_1 + \omega_2 = \frac{(E_{GUP}^3 - E_{GUP}^1)}{\hbar} = \frac{\Delta E}{\hbar} + \beta \Delta E_p / \hbar, \quad (24) \]

where \( \Delta E = E_3 - E_1 \), and \( \Delta E_p = (E_{3,p} - E_{1,p}) \) is the different energy of two levels coming from the quantum mechanic and the GUP effects, respectively. For simplicity, one can consider the coefficients of Fourier expansions to be real, so the coincidence rate in the GUP frame yields

\[
R_{GUP}^c = \frac{1}{16} \left( R_0 + 2\beta (R_1' + R_2') \right)
\times \left[ 1 + e^{-i \left( \frac{\Delta E \Delta T}{\hbar} + \beta \Delta E_p \Delta T}{\hbar} + \phi_1 + \phi_2 \right)} \right]
\times \left[ 1 + e^{i \left( \frac{\Delta E \Delta T}{\hbar} + \beta \Delta E_p \Delta T}{\hbar} + \phi_1 + \phi_2 \right)} \right], \quad (25) \]

and thus

\[
R_{GUP}^c = \frac{1}{4} R_{0}^{GUP} \cos^2 \left( \frac{\Delta E \Delta T}{\hbar} + \beta \Delta E_p \Delta T}{\hbar} + \phi_1 + \phi_2 \right) \right) \]
\[
= \frac{1}{4} R_{0}^{GUP} \cos^2 \left( \Phi_1 - \Phi_2 \right) , \quad (26) \]
where \( R_{0}^{\text{GUP}} = R_{0} + 2\beta(B_{1} + B_{2}) \) denotes the coincidence rate of the shorter length near the Planck scale, and

\[
R_{GUP} = R_{0} + 2\beta(B_{1} + B_{2})
\]

denotes the coincidence rate of the shorter length near the Planck scale, and

\[
B_{1} = \left< 0 \mid \varphi_{k,0}(x_{1}, t) \mid^{2} \varphi_{k,0}(x_{2}, t) \varphi_{k,0}^{p}(x_{2}, t) \mid 0 \right>
= \eta_{1}\eta_{2} \left< 0 \mid \sum_{k_1, k_2} \overline{c}_{k_1} \overline{c}_{k_2} \overline{c}_{k_1}^{'} \overline{c}_{k_2}^{'} \mid 0 \right>,
\]

\[
B_{2} = \left< 0 \mid \varphi_{k,0}(x_{2}, t) \mid^{2} \varphi_{k,0}(x_{1}, t) \varphi_{k,0}^{p}(x_{1}, t) \mid 0 \right>
= \eta_{1}\eta_{2} \left< 0 \mid \sum_{k_1, k_2} \overline{c}_{k_1} \overline{c}_{k_2} \overline{c}_{k_1}^{'} \overline{c}_{k_2}^{'} \mid 0 \right>. \tag{27}
\]

These are the crossing probabilities of the usual state with the GUP corrected states (see Eq. 16). Moreover, we have

\[
\Phi_{1}' = \phi_{1}/2, \quad \Phi_{2}' = -\left( \phi_{2} + \Delta E\Delta T / \hbar + \beta \Delta E_{p}\Delta T / \hbar \right) / 2. \tag{28}
\]

for the new phases \( \Phi_{i}' \). From Eq. (26), one can see that quantum gravity can modify the coincidence rate of photons in the Franson experiment, where the non-locality of the theory is confirmed by modification of adjustable new phases. It is obvious that, at the limit of \( \beta \rightarrow 0 \) the desired results, obtained in quantum mechanics, are recovered.

Relying on the accuracy of the setup employed in Franson experiment, one can find the rate of difference between the coincidence rate in the GUP frame and that of quantum mechanics as

\[
\frac{\Delta R_{c}}{R_{c}} = \frac{R_{c}^{\text{GUP}} - R_{c}}{R_{c}} = 4\beta \frac{B'}{R_{0}}, \tag{29}
\]

where we suppose that the crossing probabilities coming from GUP are the same for two photons, \( B_{1} = B_{2} = B' \). Moreover, since Eq. (26) implies the equality of dimensions of \( \beta B' \) and \( R_{0} \), one deduces that GeV\(^{2}\) is the dimension of \( B' \). By defining the dimensionless parameter \( B_{0}' = \frac{B'}{M_{p}c^{2}} \) and using the accuracies (the relative error) of the setups introduced in Refs. \([43, 44]\) to measure \( R_{c} \) (0.991 ± 0.013 and 0.853 ± 0.004, respectively), one can find \( \beta_{0}B_{0}' \sim 3.3 \times 10^{-3} R_{0} \) and \( \beta_{0}B_{0}' \sim 1.1 \times 10^{-3} R_{0} \), respectively. \( \beta_{0}B_{0}' \) and \( R_{0} \) refer to the coincidence rate through the shorter path in the GUP and QM frames, respectively. Now, accepting \( \beta \sim 10^{-2} \) GeV\(^{-2}\) (\( \beta_{0} \sim 10^{36} \))\([45]\), one finds that \( \frac{B_{0}'}{R_{0}} \) should be of the order of \( 10^{-39} \). It is so small and shows that it is probably impossible to observe the GUP effects using the current Franson experiments. In Table 1, some estimations on the GUP parameter by assuming some values for \( R_{0} \), and \( B_{0}' \) are provided with the experimental uncertainty \( \frac{\delta R_{c}}{R_{c}} \sim 10^{-3} \), where it should be required \( \frac{\delta R_{c}}{R_{c}} < \frac{\Delta R_{c}}{R_{c}} \). The parameters \( R_{0} \) and \( B_{0}' \) in Eq. (29) are independent of
Table 1  Bounds on a Franson experiment’s ability to detect GUP

| $R_0$  | $B_0'$ | $\beta_0$ | $\beta (\text{GeV})^{-2}$ |
|-------|--------|-----------|--------------------------|
| 1     | 1      | $25 \times 10^{-5}$ | $25 \times 10^{-43}$ |
| 1     | $10^{-3}$ | $25 \times 10^{-2}$ | $25 \times 10^{-40}$ |
| $10^{-1}$ | $10^{-5}$ | $25 \times 10^{-1}$ | $25 \times 10^{-39}$ |
| $10^{-2}$ | $10^{-7}$ | $25 \times 10^{0}$  | $25 \times 10^{-38}$ |
| $10^{-3}$ | $10^{-9}$ | $25 \times 10^{+1}$ | $25 \times 10^{-37}$ |

If a Franson experiment has accuracy $\delta R_c / R_c \sim 10^{-3}$ and parameters $R_0$ and $B_0'$ shown on the two left columns, then it can detect GUP effects when the GUP parameter $\beta = \beta_0 M_{p,c}^2$ is greater than the values in the two right columns.

The GUP parameter. Thus, by assigning a variety of values to these two parameters, the order values of $\beta$ ($\beta_0$) become foreseeable, gathered in Table 1. Therefore, if an experiment with sufficient accuracy is designed, then one may expect to improve the chance of detecting the effects of GUP. It is obtained that $\Delta E$ and $\Delta T$ have a clear role in the results. Indeed, a setup in which the $\Delta E_p \Delta T$ term is comparable with other terms in Eq. (26) helps us increase the possibility of observing the GUP effects. In this line, establishing a Franson experiment with big $\Delta T$ (or equally, $\Delta L$), or using high energy photons whose energies are near the Planck energy may become useful. At these energy states, the effects of GUP are supposed to become dominant, an expectation that enhances the weight of the $\Delta E_p \Delta T$ term versus the term $\Delta E \Delta T$.

4 Summary

It seems that GUP is unavoidable leading to a minimum length [16, 17]. Consequently, motivated by the deep connection between HUP and the EPR paradox leading to the emergence of QNL, and also Ref. [34], showing that the square of the Bell operators formed by angular momentum operators, changes when HUP is replaced by GUP, we tried to clarify the relationship between QNL and GUP. To achieve this goal, and also motivated by the equivalency of energy (time) and momentum (position) of photons, we resorted to the Franson experiment including energy-time entanglement, and obtained that GUP affects the coincidence rate spectrum. It has been found out that although using the Franson experiment, the study of the effects of GUP on QNL is theoretically possible, more accurate versions of this experiment are needed to detect the effects. It is beyond the scope of this paper to provide a version of this experiment for this purpose, but in this line, we think that the use of (i) photons with energy states comparable with the Planck energy state and (ii) setups including big arms (large values of $\Delta L$) [3] increases the possibility of verifying the effects of GUP on QNL. Indeed, the results show a window that may be opened in future by the precise Franson
experiments to investigate the relationship between QG (the existence of minimal length) and QNL by studying the implications of QG on Bell-type experiments.

Data availability Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Conflict of interest The authors declare no conflict of interest.

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