Formation of the Lunar Dust Ejecta Cloud

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Abstract

The Lunar Dust Experiment (LDEX) on board the Lunar Atmosphere and Dust Environment Explorer (LADEE) mission orbited the Moon from 2014 September to 2015 April and observed a dynamic, permanently present dust cloud produced by continual meteoroid bombardment. For the latitudes observed by LDEX, the sporadic background contribution to the impacting dust flux is dominated by helion (HE), apex (AP), and antihelion (AH) sources oscillating with lunar phase. Using improved impact ejecta distributions, a three-dimensional model was implemented to estimate the inner and outer ejecta cone angles from LDEX plume measurements. Expanding upon this single-plume model and the derived ejecta cone angles, we implemented a global lunar model fitted to LDEX measurements of the sporadic background to constrain the product of meteoroid impactor fluxes and ejecta mass yield per source. We use the observed asymmetry between sunward and antisunward impactor fluxes to discuss the possible contributions of β-meteoroids, in addition to the HE and AH sporadic meteoroid sources. We find that if β-meteoroids are responsible for the day/night asymmetry, they must have an impact ejecta yield of at least 107.

Unified Astronomy Thesaurus concepts: Meteoroids (1040); The Moon (1692)

1. Introduction

Impact bombardment is a primary source of space weathering for all airless bodies in the solar system, affecting both the topology and chemical distributions of their surfaces over time (Pieters & Noble 2016). Characterizing these meteoroid impactor sources is essential for understanding the origin and evolution of airless bodies within our solar system (DeMeo et al. 2009; Brunetto et al. 2015). The Moon, as an airless body, is of particular interest, as measurements of impact ejecta from the lunar surface serve to help characterize the meteoroid environment near Earth and guide model predictions for the dust environment and regolith evolution of asteroids.

Observations of the lunar ejecta environment by the Lunar Dust Experiment (LDEX) on board the Lunar Atmosphere and Dust Environment Explorer (LADEE) mission discovered a permanently present dust cloud produced by continual meteoroid bombardment (Horányi et al. 2015). This dust cloud is highly sensitive to meteoroid showers and changes in impactor flux rates (Szalay et al. 2016; Szalay & Horányi 2016a). Of the sporadic meteoroid sources, impact ejecta was found to be produced by the helion (HE), apex (AP), and antihelion (AH) sources with a small contribution from the antiapex (AA) source and possible contributions from northern (NT) and southern (ST) toroidal sources, each oscillating with lunar phase (Szalay & Horányi 2015, 2016a; Szalay et al. 2019). The fluxes of these four sporadic sources can be constrained by LDEX observations through a forward-modeling approach. The β-meteoroids, dust grains on hyperbolic orbits moving away from the Sun, have been suggested to be an additional source of impact ejecta at the Moon due to the persistent dust ejecta density enhancement on the Moon’s sunward side (Szalay et al. 2020a).

Constraints on mass yields, the ratio of the total mass of the produced dust ejecta over the mass of the initial impacting particle, are necessary for deriving impactor fluxes from modeled ejecta clouds. While the relationship between impactor mass, speed, and mass yield has been measured in a variety of experiments (e.g., Koschny & Grün 2001a, 2001b), these do not cover the relevant impactor compositions, sizes, speeds, or characteristics of the surface regolith for the lunar environment. It is the goal of this paper to constrain the product of impactor flux and mass yield per source via a forward-modeling approach expanding a single-plume model (Bernardoni et al. 2019) to a global one. We also explore the possibility of β-meteoroids producing observable impact ejecta (Szalay et al. 2020a).

2. Particle Discretization

Ejecta for each impact plume were modeled discretely (Bernardoni et al. 2019), describing the initial ejecta velocity (ν), mass (m), and angle from surface normal (ϕ) distributions as (Horányi et al. 2015; Szalay et al. 2016; Szalay & Horányi 2016a; Bernardoni et al. 2019)

\[ f(ν) = \frac{2R_m ν}{λν_r^2(1 - (ν/ν_r)^2)^2} e^{-R_m/(ν/ν_0)^α}, \]

\[ f(m) = \frac{α}{m_{min} - m_{max}} m^{-(1+α)}, \]

\[ f(ϕ) = \frac{sin ϕ}{cos ϕ_1 - cos ϕ_0}, \]

where \( ν_r = 2.4 \text{ km s}^{-1} \) is the lunar escape speed, \( R_m = 1737 \text{ km} \) is the Moon’s radius, and \( λ = 200 \text{ km} \) is the characteristic scale height derived from the dust density distribution measurements (Szalay & Horányi 2016a). The mass exponent (\( α = 0.91 \pm 0.003 \)) was measured by LDEX consistent across all local times (Horányi et al. 2015), and the mass bounds (\( m_{min} = 10^{-17} \text{ kg} \) and \( m_{max} = 10^{-8} \text{ kg} \) were...
determined by the detection limits of LDEX at the spacecraft’s apex velocity. The outer and inner ejecta cone angles, \( \phi_0 \) and \( \phi_1 \) (see Figure 1), were fixed at 8° and 0°, respectively (Bernardoni et al. 2019). While Sachse et al. (2015) suggested coupled speed and size ejecta distributions, the ejecta distributions used here are treated as separable, as there is a lack of altitude dependence in the detected impact charge distribution of LDEX (Bernardoni et al. 2019; Horányi et al. 2015). For simulating the contribution of a single plume’s ejecta to LDEX’s measured ejecta impact rate, we sampled 200,000 simulated ejecta particle trajectories based on \( f(\phi) \) and \( f(v) \). Positions and velocities for each simulated particle trajectory were calculated at fixed time steps of \( dt = 1 \text{s} \) using elliptical orbit equations. As LDEX’s lowest detectable mass threshold is velocity-dependent, \( f(m) \) was implemented as a weight for the simulated impact rate from ejecta into LDEX, \( I_i \), for impact \( i \) (Szalay et al. 2016),

\[
I_i = \frac{R_i}{dV} = \frac{A(\omega)}{dV} \cos(\omega_i) \frac{(C_m v_{\text{rel}}/A)^{4/7} - m_{\text{min}}^{\alpha}}{m_{\text{min}}^{\alpha} - m_{\text{max}}^{\alpha}} v_{\text{rel}},
\]

\[
\cos \omega_i = -\frac{v_{\text{rel}} \cdot \hat{x}}{v_{\text{rel}}},
\]

\[
v_{\text{rel}} = (v_i \cos \phi_i - v_{\text{imp}})\hat{x} + v_i \sin \phi_i \hat{y} + v_{\text{imp}} \hat{z},
\]

where \( v_{\text{imp}} \) is the spacecraft speed with \( \hat{x} \) chosen in the direction of the spacecraft velocity vector, \( \omega \) is the angle between the detector’s boresight and the ejecta’s relative velocity vector \( v_{\text{rel}} \). \( dV \) is the simulated volume element, and \( A \) is the detector’s effective area, which is maximal at \( \omega = 0 \) and zero for \( \omega \geq 68^\circ \) (Horányi et al. 2015). Each plume in this model is considered to have these same initial ejecta distributions. The effects of impactor mass, speed, and obliquity of the impact are contained entirely within the mass yield (number of ejecta per plume) and the impactor flux (number of plumes per spatial-temporal bin). This is done for simplicity, as such effects are not sufficiently constrained over the relevant impact parameters. Formulating these additional effects, particularly asymmetric plumes produced from oblique impacts, could lead to a possible improvement of this model.

3. Multiple-plume Model

3.1. One Bin

For simulating the contribution of multiple plumes to the detected impact rate, we first consider the simplified case of a single volume element, \( dV \), with constant impactor flux, \( \Phi_{\text{imp}} \). For a single simulated particle at time \( t_i \) and azimuthal angle \( \phi_i \) with respect to the plume’s origin and spacecraft velocity (Figure 1), the contribution to the simulated impact rate, \( I_i \), is given by Equation (2). Thus, the measured average impact rate of LDEX, \( I_{\text{LDEX}} \), is

\[
I_{\text{LDEX}} = \frac{N_p M_p}{M_i} \sum_i I_i,
\]

where \( N_p \) is the total number of plumes produced per simulated time step within the lunar surface element \( dA \), \( M_p \) is the average total mass of one plume, and \( M_i \) is the simulated mass of one plume. The simulated mass of one plume consists of the sum of all particles in the simulation multiplied by the average mass, based on the distribution from Equation (1), while we treat the total mass of an actual plume as an unknown quantity. Note that there are some additional complications to the total simulated mass, as the azimuthal component is treated as continuous while the initial ejecta distributions are sampled discretely. Since the impactor mass flux for characteristic impactor mass \( m_{\text{imp}} \) can be written as \( \Phi_{\text{imp}} = m_{\text{imp}} N_p / dA \), we...
rewrite Equation (3) as

$$ I_{LD} = \frac{N_p}{dV M_i} \sum_i R_i $$

$$ = \frac{N_p}{dA} \int_{R_m}^{R_m+h+dz} \frac{R_m^2}{r^2 dr} M_p \sum_i R_i $$

$$ = Y \Phi_{imp} \left( \frac{3R_m^2}{(R_m + h + dz)^3 - (R_m + h)^3} \right) \frac{1}{M_i} \sum_i R_i, \quad (4) $$

where $h$ is the altitude of the spacecraft, $Y = M_p/m_{imp}$ is the mass yield of a plume, and $dz$ is the altitude bin size (Figure 1). Matching the simulation to LDEX data is then a matter of scaling $Y\Phi_{imp}$.

3.2. Multiple Bins

The previous case holds true for a constant impactor flux within the lunar surface patch considered. As the lunar impactor flux varies with local time, we expand this approach by considering local time bins, assuming that each bin encompasses a constant impactor flux. To standardize the notation of the following sections, $i$ denotes a simulated impact sampled from ejecta particle trajectories as described in Section 2. A subscript of $k$ denotes the observing surface-altitude bin (i.e., the bin that contains the measured impact rate), while $j$ denotes the surface being observed (i.e., surface patch possibly containing the plume origin, corresponding to simulated impact $i$). Thus, the subscript of $dA_{ijk}$ is read as “for simulated ejecta measurement $i$ originating from surface bin $j$ as measured within surface-altitude bin $k$.”

One subtlety that was glossed over was the location of $dA$ relative to $dV$, as $dA$ is only located directly beneath $dV$ in the case where the angular distance of the simulated impact from the center of the plume $\theta_i = 0$. For a single simulated impact at time $t_i$ and position $(\theta_i, \phi_i)$ relative to the center of the plume, $dA$ is shifted from surface patch corresponding to $dV$. Thus, for a particular $(\theta_i, \phi_i)$, the corresponding $dA$ will overlap up to four local time bins, as shown in Figure 2.

For the illustrated case in Figure 2, $dA$ projected from bin $k$ overlaps the fixed surface bins $j_1, j_2, j_3, j_4$ with surface areas $dA_{j_1k}, dA_{j_2k}, dA_{j_3k}, dA_{j_4k}$, respectively. This simulated impact contribution to the measured impact rate is

$$ I_k = \frac{M_p}{M_i} \left( \frac{N_{j_1k}}{dV} + \frac{N_{j_2k}}{dV} + \frac{N_{j_3k}}{dV} + \frac{N_{j_4k}}{dV} \right) $$

$$ = \frac{M_p}{M_i} \left( \frac{3R_m^2}{(R_m + h + dz)^3 - (R_m + h)^3} \right) \times \left( \frac{N_{j_1k}}{dA} + \frac{N_{j_2k}}{dA} + \frac{N_{j_3k}}{dA} + \frac{N_{j_4k}}{dA} \right) $$

$$ = \frac{1}{M_i} \left( \frac{3R_m^2}{(R_m + h + dz)^3 - (R_m + h)^3} \right) \times Y \left( \frac{dA_{j_1k}}{dA} + \frac{dA_{j_2k}}{dA} + \frac{dA_{j_3k}}{dA} + \frac{dA_{j_4k}}{dA} \right), \quad (5) $$

Figure 2. Illustration of the possible plume impact location for a given simulated ejecta measurement $i$ relative to the surface patch $dA$ directly below the simulated volume element $dV$. This possible plume location is characterized by the projection of surface element $dA$ given that the simulated measurement traveled a great circle distance of $\theta_i$ at an angle $\phi_i$ with respect to the spacecraft velocity. Dashed lines represent local time-latitude bins, each with a constant flux. For the illustrated case, the simulated impact $i$ has contributions from bins $j_1, j_2, j_3,$ and $j_4$ weighted by the area ratio in Equation (5). Note that $k$ is also binned by altitude with the corresponding bin determined by the height of simulated ejecta impact $i$. 
where $\Phi_j$, $\Phi_k$, $\Phi_j$, and $\Phi_k$ are the impactor fluxes for local time
bins $j_1$, $j_2$, $j_3$, and $j_4$, respectively. Note that it is with the
assumed constant flux that $\Phi_j = m_{nm}N_{jk}/dA_{jk}$ for each $i$ and $k$.
Expanding this to all $j$ bins and summing over all simulated
impacts $i$ gives the generalized form of Equation (4),

$$I_{\text{LDEX},k} = \frac{3R_m^2}{(R_m + h + dz)^3} - \frac{(R_m + h)^2}{R_m}$$
\[
\times \frac{1}{M_j} \sum_{q} Y_{\Phi_j} R_i dA_{jk}/dA
\]
\[
= \sum_{j} Y_{\Phi_j} S_{jk}.
\] (6)

Matching all surface and altitude bins of data becomes a
minimization with $Y_{\Phi_j}$ as parameters. In this formulation, the
values of $S_{jk}$ contain all of the geometric information from the
numerical simulation separated from the desired parameters to
be estimated. Specifically, $R_i$ contains the geometry of the
instrument, while the rest of $S_{jk}$ contains the geometry of the
system.

3.3. Latitude Variation

3.3.1. Grid Motivation

While the latitude coverage of the LDEX data set is limited
(Figure 4), expanding this model to a full three-dimensional
sphere may prove useful for future data sets, as well as to
minimize errors introduced by assuming a purely equatorial
orbit. In the case of a grid covering the entire surface of a
sphere, Equation (6) still holds true with different values for
d$A_{jk}$. However, complications arise with the increasing number
of parameters and computation time. For example, surface grid
cells defined by equal latitude and longitude steps are no longer
of equal area, as was the case in Figure 2, and whose overlap
with arbitrary $\theta_i$ and $\phi_i$ proves computationally expensive to
perform for each simulated impact.

Polygonal grids also introduce an additional parameter in the
relative orientation of the projected cell to the considered cell.
For example, in the case of Figure 2, $dA_{jk}$ would have a
different value from the one illustrated if the projected square
was tilted by an arbitrary angle.

Here we use overlapping spherical caps as surface grid cells
with grid points defined as the center of these spherical caps
(Figure 3). As this will introduce error in the form of surface
patches not covered or double-counted, we construct the grid
cell size such that the sum of all surface grid cell areas equals
the surface area of the sphere. This represents our best estimate,
as the double-counted regions should come close to accounting
for the regions not covered by the cells. Additionally, surface
grid cells are positioned to meet the following criteria:

1. maximize the smallest distance between any two grid
   points and
2. minimize the number of unique grid point–to–grid point
   distances within an effective range.

The first criterion is imposed to achieve a close-to-uniform
covering of the sphere, while the second criterion is imposed to
reduce the computation time of the simulation. As the distance
between grid points will become a parameter in determining
$S_{jk}$ (Appendix A, Equations (A2)–(A3)), reducing the number
of unique grid point–to–grid point distances significantly
increases the performance of the simulation. The “effective
range” mentioned in the criteria (and Appendix B, Figure 9)
refers to surface grid cells within the maximum $\theta_i$ of all
simulated impacts plus twice the radius of the surface grid cells,
as any cell beyond this range does not contribute to the
simulated impact rate. We used the HEALPix framework
(Gorski et al. 2005) to generate the locations of these grid
cells, as it has a reasonable agreement with our criteria and the
flexibility to scale the resolution as needed. To balance
accuracy with computation time, we chose a resolution of
$N_{\text{side}} = 2^5$, resulting in 12,288 surface grid cells.

3.3.2. Minimization

With the $S_{jk}$ values simulated for Equation (6) over all
relevant surface-altitude bins, we can fit the measured LDEX
data $I_{\text{LDEX},k}$ using the parameters $Y_{\Phi_j}$. Starting with a least-
squares approach, we minimize

$$\epsilon_{\text{tot}}(Y_{\Phi_j}) = \sum_{k} \left( I_{\text{LDEX},k} - \sum_{j=1}^{n} Y_{\Phi_j} S_{jk} \right)^2$$
\[
= \sum_{k} (I_{\text{LDEX},k} - I_{\text{fit},k})^2.
\] (7)

With no additional constraints, $n$ is the number of surface grid
cells. To directly connect to physical quantities, we will use the
four dominant impactor sources: HE, AH, AP, and AA. These
are equatorial sources at local times of 10.3, 1.7, 6, and 18 LT,
respectively (Szalay et al. 2019). Each bin $Y_{\Phi_j}$ is broken up by
source $\alpha$,

$$Y_{\Phi_j} = \sum_{\alpha=1}^{4} Y_{\Phi_{\alpha}} \cos^2(s_{\alpha}) \Theta(s_{\alpha} - \pi/2),$$
\] (8)
where \( \cos^2(s_{\alpha}) \) comes from how \( Y \) scales with impact angle and \( \cos(s_{\alpha})\Theta(s_{\alpha} - \pi/2) \) comes from the surface area projection of \( \Phi_k \) using the Heaviside function \( \Theta \) (Szalay & Horányi 2015). Equation (A4) is used for \( s_{\alpha} \) with the \((\gamma, \lambda)\) of the impactor source used for one of the entries. With this substitution, \( I_{\text{fit}, k} \) takes the form of a linear sum,

\[
I_{\text{fit}, k} = \sum_{\alpha=1}^{4} Y_{\Phi, \alpha} \sum_{j=1}^{n} S_{jk} \cos^3(s_{\alpha})\Theta(s_{\alpha} - \pi/2) \\
= \sum_{\alpha=1}^{4} Y_{\Phi, \alpha} x_{\alpha,k}. \tag{9}
\]

In this form, the minimization is now a linear regression without a constant bias, \( Y_{\Phi, \alpha} \) are the regression coefficients, and \( x_{\alpha,k} \) are the independent variables. Since the dependence of \( Y \) on the obliqueness of an impact is described by \( x_{\alpha,k} \), \( Y \) now represents the mass yield for impacts with normal incidence to the surface.

4. Results

Using the LDEX impact rate measurements with the coverage shown in Figure 4, fitted values for \( Y\Phi \) were calculated for the three equatorial source radiants 10.3, 1.7, and 6 LT, corresponding to HE, AH, and AP, respectively. These sources are labeled by their radiants in Figure 5, as these values represent all possible sources originating from that direction, not just HE, AH, and AP. We excluded data within 5 days of the Geminid and Quadrantid meteor showers to isolate the contribution from sporadic background sources (Szalay & Horányi 2016b). The segment of data coverage in Figure 4 from 12 to 22 LT was excluded from the fit, as the Sun was within LDEX’s field of view generating elevated noise levels in the instrument. As this excluded range covers the majority of the 18 LT (AA) source’s contribution to the lunar ejecta environment, the AA source was not included in this fit. Similarly, nonequatorial sources, such as the NT/ST, that may also contribute to ejecta production (Szalay et al. 2019) were not included, as the latitude coverage of Figure 4 is not sufficient to constrain them. Using the ejecta mass production weights from this latitude coverage for AP and NT/ST sources, \( w_{\text{AP}} = 0.303 \) and \( w_{\text{NT/ST}} = 0.303 \), respectively (Szalay et al. 2019), we estimate the possible contribution to 6 LT from these sources. Assuming latitudes of 60° for the NT source, the relative expected contribution is derived as

\[
\frac{w_{\text{NT}}\cos^3(60° - 18°)}{w_{\text{AP}}\cos^3(18° - 0°)} = 0.22, \tag{10}
\]

where 18° is the average latitude of all LDEX measurements taken near 6 LT. While we assume that the entirety of the 6 LT source is from AP meteoroids for the following discussion, note that up to 22% may be from toroidal sources. Note that the fitted values for \( Y\Phi \) are highly dependent upon \( m_{\text{max}} \), partially from the weight in Equation (2) but primarily from \( M_i \) in Equations (3)–(6). For this reason, we include estimates for \( m_{\text{max}} = 10^{-8} \) kg (opaque points in Figure 5) corresponding to 100 \( \mu \)m ejecta taken as the peak in the sporadic background mass distribution and \( m_{\text{max}} = 10^{-11} \) kg (transparent points in Figure 5) corresponding to 10 \( \mu \)m ejecta and the largest ejecta particle detected by LDEX. The error bars for Figure 5 represent the standard error of that parameter for the linear regression. As shown in Figures 5 and 6, LT (AP) is the dominant contributing source to the lunar dust environment for both cases with a mission-averaged impact ejecta mass production (mass yield times impactor mass flux) of \((1.62 \pm 0.02) \times 10^{-14} \) kg m\(^{-2}\) s\(^{-1}\) for \( m_{\text{max}} = 10^{-8} \) kg and \((7.29 \pm 0.07) \times 10^{-15} \) kg m\(^{-2}\) s\(^{-1}\) for \( m_{\text{max}} = 10^{-8} \) kg at...
normal incidence, or 14.6 ± 0.1 and 6.6 ± 0.1 t day⁻¹, respectively, averaged over the entire lunar surface. However, the asymmetry between HE and AH sources observed in prior studies (Szalay & Horányi 2015; Janches et al. 2018) remains despite considerations for the spacecraft’s trajectory with mission-averaged values of 10.5 ± 0.1 and 4.7 ± 0.1 t day⁻¹ for the HE source and 4.9 ± 0.1 and 2.2 ± 0.1 t day⁻¹ for the AH source. This asymmetry runs counter to expectation, as models of the sporadic background do not indicate a significant asymmetry between the HE and AH impactor fluxes, and the impactors for the two sources should be of similar size and speed and thus mass yield (Pokorný et al. 2019). The ground-based radar observations are somewhat less clear, as there are observations suggesting symmetry (Janches et al. 2018) and others suggesting an asymmetry between the influx from the HE and AH sources throughout the year (Campbell-Brown & Jones 2006). As the total meteoroid flux measured at Earth is 43 t day⁻¹, we expect a total impactor flux of a few tons per day for the lunar environment (Szalay et al. 2019; Carrillo-Sanchez et al. 2016). Thus these fitted values for the three primary impactor sources suggest yields on the order of 10 consistent with Szalay et al. (2019).

Whether the HE/AH asymmetry is an enhancement on the dayside of the moon or a deficit on the nightside is unclear. However, as we only have possible physical explanations for an enhancement on the dayside, we will proceed under that assumption. For comparison, the following section introduces the empirical formula fit from Pokorný et al. (2019) into our model. It should be noted that while a thermal dependence on the soil’s mass yield has been suggested as a potential cause for this enhancement, no physical mechanism has been proposed to link the temperature of the lunar surface to its impact ejecta yield, nor has there been any experimental evidence to suggest that this is the case. As a possible physical explanation, we discuss the potential contribution from β-meteoroids as an additional impactor source in Section 4.2 (Szalay et al. 2020a).

4.1. Yield Variance

Our understanding of how the lunar regolith responds to meteoroid bombardment is limited. Effects such as surface temperature, UV radiation, or solar wind have been suggested as possible dependencies that may enhance ejecta mass production on the lunar dayside (Janches et al. 2018;
Pokorný et al. 2019). An empirical relation for the “excess”
dayside lunar ejecta was found by comparing to expectations
from dynamical models (Pokorný et al. 2019). To incorporate
this effect within our model, we introduce the following term
into $S_{j0}$ of Equation (9) as a function of local time per surface
bin (Pokorný et al. 2019),

$$Y_j(LT) = \begin{cases} 
1 & |LT_j - 12| \geq \delta, \\
1 + 0.8(1 - |12 - LT_j|/\delta) & |LT_j - 12| \leq \delta.
\end{cases}$$

(11)

With this additional local time dependence on the mass
yield, we reproduce Figure 5 for $m_{max} = 10^{-8}$ kg as Figure 6.
Note that the fitted values plotted are of the $Y$ on the lunar
nightside, as the enhancement on the dayside is absorbed into
$S_{j0}$. Under this empirical formula, HE and AH sources have
a much closer agreement with mission-averaged values of
5.9 ± 0.1 t day$^{-1}$ for the HE source and 5.0 ± 0.1 t day$^{-1}$ for
the AH source. It should be noted that this formula is derived
from the observed HE/AH asymmetry in lunar ejecta and not
from surface response relations.

4.2. β-meteoroids

As a possible physical explanation for the excess dayside
impact ejecta, we consider the contribution of an additional
impactor source, β-meteoroids (Szalay et al. 2020a). Since
β-meteoroids are of a similar radiant to the HE source at 11 LT
(Szalay et al. 2020a), we would expect an enhancement of the
total lunar ejecta mass flux on that side. As such, we label the
HE/AH contribution as the lower of the two in Figure 7. We
cannot take the difference between the total ejecta mass flux of
10.3 and 1.7 LT fitted values as the β-meteoroid contribution,
as the mass distribution of ejecta may not be the same as that of
Equation (1). However, when considering the ejecta size
distributions per local time bin, the values for $\alpha$ remain
consistent, with no significant deviations from the fit. For
this reason, we use the same ejecta mass distribution for the
β-meteoroid source as the sporadic background sources, with
only the value for $m_{max}$ changing, as we do not expect an
impactor to produce ejecta larger than itself. To provide an
estimate range, we consider the cases of $m_{max} = 10^{-14}$ kg
corresponding to 1 μm ejecta and $m_{max} = 10^{-15}$ kg corresponding
to 0.5 μm ejecta. Figure 7 plots the fitted values for $Y_{\Phi}$ under these
stipulations, indicating that β-meteoroids may have a mission-
averaged impact ejecta mass production of 0.75 ± 0.15 t day$^{-1}$ for
$m_{max} = 10^{-14}$ kg and 0.51 ± 0.5 t day$^{-1}$ for $m_{max} = 10^{-15}$ kg.
Note that while fitted values for the sporadic background sources
use $m_{max} = 10^{-11}$ kg in Figure 7, values for the β-meteoroid total
lunar ejecta mass flux are similar for the $m_{max} = 10^{-8}$ kg,
case (~3 × 10$^{-6}$ difference).

Based on a variety of spacecraft measurements, the total number flux of β-meteoroids is expected to range from 10 to
600 km$^{-2}$ s$^{-1}$ (Berg & Grün 1973; Wehry & Mann 1999;
Zaslavsky et al. 2012; Szalay et al. 2020b). For the size range
of 0.5 – 0.2 μm with a bulk density of 2 g cm$^{-3}$, the expected
impact mass flux for the β-meteoroids is $\Phi_{\beta} = 2$–300 g day$^{-1}$.
Using the mission-averaged values from Figure 7, this corre-
sponds to mass yields at a normal incident of $Y_\beta = 2.3 \times 10^4 - 3.4 \times 10^3$ using $m_{max} = 10^{-14}$ kg for 0.5 and 0.2 μm
β-meteoroids and $Y_\beta = 1.6 \times 10^4 - 2.3 \times 10^4$ using $m_{max} = 10^{-15}$ kg. Extrapolating from experimentally measured yields to
impactor speeds of 100 km s$^{-1}$, we expect high yields for
β-meteoroids of $\gtrsim 10^3 - 10^4$, consistent with our results
(Koschny & Grün 2001a; Szalay et al. 2020a). Note that while β-meteoroids appear as a possible solution to the HE/AH ejecta
production asymmetry, it is by no means certain under this
cursory comparison. Further work is necessary to narrow down
what effects are contributing to the observed asymmetry.

5. Conclusion

Expanding upon the description of a single plume (Bernardoni
et al. 2019), we constructed a global multiplume model and
constrained the product of the mass yield and impactor flux for the
three dominant equatorial sources of the sporadic meteoroids
bombarding the Moon: AP, HE, and AH (Figure 5). Further
latitude coverage or constraints relative to these equatorial sources
are necessary to introduce additional sources into the model, such
as NT/ST. Asymmetry between HE and AH sources persists
despite considerations for spacecraft trajectory. Considering
the cases of surface variation in ejecta mass yield (Pokorný et al.
2019) and a β-meteoroid source (Szalay et al. 2020a), we
constrained the values for the mass yield and impactor mass flux

\[ Y_j(LT) = \begin{cases} 
1 & |LT_j - 12| \geq \delta, \\
1 + 0.8(1 - |12 - LT_j|/\delta) & |LT_j - 12| \leq \delta.
\end{cases} \]
required (Figures 6 and 7). Restrictions on the required mass yield extrapolated from the experiment for \( \beta \)-meteoroid ejecta are consistent with our results for the expected impactor mass flux with ejecta mass yields on the order of \( 10^3 \) – \( 10^5 \). This represents a significant increase in mass yield compared with values on the order of 1–10 for the sporadic background sources, which have average impactor sizes much larger than \( \beta \)-meteoroids. The larger inferred yields for \( \beta \)-meteoroids may be a combination of their higher impact speeds and the fact that, for smaller impactors, the surface will appear more solid and less regolith in nature. A similar mission to LADEE carrying an LDEX-type dust instrument on a near-polar orbit could provide the missing observations to precisely describe the ejecta production on the lunar surface. This could be critical to gauge the loss and accumulation of volatiles in the polar permanently shadowed regions, as well as to assess the effects of interplanetary dust bombardment on the evolution of the surface regolith of other airless bodies near 1 au.

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Appendix A

Grid Derivation

To achieve the area described in Section 3.3.1, the angular radius \( r_c \) of the spherical caps is given as

\[
\cos r_c = 1 - \frac{2}{N_{\text{grid}}},
\]

where \( N_{\text{grid}} = 12,288 \) is the number of grid cells on the surface. To calculate \( dA_{ijk} \) in Equation (6), we use the following equation for the overlap between two spherical caps of radius \( r_c \) separated by a great circle distance between their centers of \( d_{ijk} \) (Tovchigrechko & Vakser 2001),

\[
\frac{dA_{ijk}}{dA} = \frac{\pi - \arccos(-\cot^2 r_c + \cos d_{ijk} \csc^2 r_c) - 2\arccos\left(\cot r_c \tan \frac{d_{ijk}}{2}\right) \cos r_c}{\pi (1 - \cos r_c)}.
\]

Note that for this section, \( k \) denotes the surface bin directly below the surface-altitude bin under consideration. Additionally, the great circle distance between the projected area of a simulated impact \( (\theta_i, \phi_i) \) and surface grid cell \( j \) is given by the

Figure 8. Illustration of the area overlap \( dA_{ijk} \) of impact \( i \) projected from cell \( k \) with cell \( j \). Spherical cap \( k \) represents the surface bin below the surface-altitude bin where simulated ejecta measurement \( i \) is collected. The dotted circle indicates where on the surface ejecta could have come from, given that it traveled a great circle distance \( d_{ijk} \) at an angle \( \phi_i \) with respect to the spacecraft’s velocity \( v_{sc} \). To determine the contribution of surface grid cell \( j \) on the simulated ejecta measurement \( i \), \( dA_{ijk} \) must be calculated from \( d_{ijk} \) using Equation (A2). Here \( d_{ijk} \) is calculated from \( s_{jk} \), \( b_{jk} \), and \( \phi_i \) using Equation (A3). The blue shaded region indicates the change in the overlapping area as \( \phi_i \) is integrated.
spherical law of cosines,

\[ \cos d_{jk} = \cos \theta_i \cos s_{jk} + \sin \theta_i \sin s_{jk} \cos (\phi_i - b_{jk}), \quad (A3) \]

where \( s_{jk} \) is the great circle distance between surface grid cells \( j \) and \( k \), and \( b_{jk} \) is the bearing of surface grid cell \( j \) with respect to the latitude line of cell \( k \) (Figure 8). Here \( s_{jk} \) and \( b_{jk} \) are given in terms of latitude, \( \gamma \), and longitude, \( \lambda \), as

\[
\begin{align*}
\cos d_{jk} &= \cos \theta_i \cos s_{jk} + \sin \theta_i \sin s_{jk} \cos (\phi_i - b_{jk}), \\
\sin s_{jk} &= 2 \arctan \left( \frac{\sin \frac{\gamma_j - \gamma_k}{2} \cos \gamma_j \cos \gamma_k \sin^2 \frac{\lambda_j - \lambda_k}{2}}{1 - \sin^2 \frac{\gamma_j - \gamma_k}{2} - \cos \gamma_j \cos \gamma_k \sin^2 \frac{\lambda_j - \lambda_k}{2}} \right), \\
b_{jk} &= \arctan \left( \frac{\cos \gamma_j \sin \gamma_j - \sin \gamma_k \cos \gamma_j \cos (\lambda_j - \lambda_k)}{\sin (\lambda_j - \lambda_k) \cos \gamma_j} \right).
\end{align*}
\]

(A4)

Using Equations (A2)–(A4), \( dA_{ijk} \) becomes a function of \( (\theta_i, \phi_i, s_{jk}, b_{jk}) \), where \( s_{jk} \) and \( b_{jk} \) need only be calculated once when the grid is generated. Combining this with Equations (2) and (6) results in the equation for the simulated impact rate \( S_{jk} \) as a function of \( (v_i, v_{ij}, \theta_i, \phi_i, s_{jk}, b_{jk}) \) summed over simulated impacts \( i \).

**Appendix B: Optimization**

With the addition of latitude variation in the form of Equations (A2)–(A4), the production of \( S_{jk} \) in Equation (6) becomes computationally expensive. To increase efficiency, the structure of the simulation follows that of Figure 9. We note that steps G1–G3 rely only on the number and radius of the surface grid cells, with G4 containing the additional information on the geometry of the instrument, while S1 and S2 contain all of the ejecta particle dynamics of a plume. Thus, G1–G4 do not need to be recalculated with additional ejecta dynamical information, such as collisions or different initial ejecta distributions, to be possibly evaluated in the future. As the dependence of \( S_{jk} \) on \( b_{jk} \) closely follows that of \( A \cos (b_{jk}) + B \), we chose to only evaluate at \( b_{jk} \in \{0, \frac{\pi}{2}, \pi\} \) and use this fitted function to calculate all other \( b_{jk} \). This results in increased performance with the additional benefit of allowing for changes in spacecraft bearing. Any such variation in bearing need only require repeating the last part of S3, which can be done in a matter of seconds. This is one consequence of the circular symmetry of the spherical caps used for the surface grid cells.
**Figure 9.** Schematic of the simulation’s order of operations for the multiple-plume model with latitude variation. Here G1–G3 depends only on the number and size of the surface grid cells, while G4 contains information on the geometry of the instrument. Separate from these factors, S1 and S2 contain all of the dynamics of the plume. Thus, G1–G4 need not be recalculated with additional ejecta dynamics, such as introducing collisionality or changes in the initial ejecta distributions. The fit for the second part of S3 means that changes in the bearing of the instrument need only require recalculation of the last part of S3.

**References**

Berg, O. E., & Grün, E. 1973, Space Research XIII, 2, 1047
Bernardoni, E. A., Szalay, J. R., & Horányi, M. 2019, GeoRL, 46, 534
Brunetto, R., Loeffler, M. J., Nesvorný, D., Sasaki, S., & Strazzulla, G. 2015, in Asteroids IV, ed. P. Michel, F. E. DeMeo, & W. F. Bottke (Tucson, AZ: Univ. Arizona Press), 597
Campbell-Brown, M., & Jones, J. 2006, MNRAS, 367, 709
Carrillo-Sanchez, J. D., Nesvorny, D., Pokorný, P., Janches, D., & Plane, J. M. C. 2016, GeoRL, 43, 11,979
DeMeo, F. E., Binzel, R. P., Slivan, S. M., & Bus, S. J. 2009, Icar, 202, 160
Gorski, K. M., Hivon, E., Banday, A. J., et al. 2005, ApJ, 622, 759
Horányi, M., Szalay, J. R., Kempf, S., et al. 2015, Natur, 522, 324
Horányi, M., Sternovsky, Z., Lankton, M., et al. 2014, SSRv, 185, 93
Janches, D., Pokorný, P., Sarantos, M., et al. 2018, GeoRL, 45, 1713

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Koschny, D., & Grün, E. 2001a, *Icar*, 154, 402
Koschny, D., & Grün, E. 2001b, *Icar*, 154, 391
Pieters, C. M., & Noble, S. K. 2016, *JGRE*, 121, 1865
Pokorný, P., Janches, D., Sarantos, M., et al. 2019, *JGRE*, 124, 752
Sachse, M., Schmidt, J., Kempf, S., & Spahn, F. 2015, *JGRE*, 120, 1847
Szalay, J. R., & Horányi, M. 2015, *GeoRL*, 42, 10.580
Szalay, J. R., & Horányi, M. 2016a, *GeoRL*, 43, 4893
Szalay, J. R., & Horányi, M. 2016b, *Icar*, 275, 221

Szalay, J. R., Horányi, M., Colaprete, A., & Sarantos, M. 2016, *GeoRL*, 43, 6096
Szalay, J. R., Pokorný, P., & Horányi, M. 2020a, *ApJ*, 890, L11
Szalay, J. R., Pokorný, P., Sternovsky, Z., et al. 2019, *JGRE*, 124, 143
Szalay, J. R., Pokorny, P., Bale, S. D., et al. 2020b, *ApJS*, 246, 27
Tovchigrechko, A., & Vakser, I. A. 2001, *Protein Science*, 10, 1572
Wehry, A., & Mann, I. 1999, *A&A*, 341, 296
Zaslavsky, A., Meyer-Vernet, N., Mann, I., et al. 2012, *JGRA*, 117, 5102