Oscillation power as a diagnostic tool for stellar turbulent spectra

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ABSTRACT

Recent observations and theoretical studies support the theory that solar-type oscillations are intrinsically stable but excited stochastically by the turbulent convection in the outer layers of the star. The acoustic noise generated by the convective motion depends on the details of the turbulent energy spectrum. In this paper we present a general formulation for the acoustic noise generation rate based on previous works by other authors. In this general formulation any model for the spatial turbulent energy spectrum and for the turbulent time spectrum can be assumed. We compute acoustic power spectra and oscillation amplitudes of radial oscillations for models of the Sun and Procyon A. The results are compared with recent observations.

1. Introduction

Instruments aboard the SOHO spacecraft have provided high-quality data of solar oscillations. Comparing theoretical estimates of acoustic power spectra with such high-quality data leads to a better understanding of the excitation processes of p-mode oscillations and provides more details of the characteristics of stellar turbulence. The acoustic power injected into the p modes by turbulent convection has been modelled by several authors (e.g., Goldreich & Keeley 1977; Christensen-Dalsgaard & Frandsen 1983; Balmforth 1992b; Goldreich, Murray & Kumar 1994; Musielak et al. 1994). Balmforth (1992b) and Goldreich, Murray & Kumar (1994) investigated the contributions of the fluctuating entropy to the noise generation rate additional to the contributions of the fluctuating Reynolds stresses but reported different results. Balmforth found the entropy fluctuations to be less important relative to the Reynolds stress contributions. Goldreich, Murray & Kumar (1994, hereafter GMK), however, concluded that the fluctuating entropy contribution is about 3–4 times larger than the Reynolds stress contribution, a result which was also found by the hydrodynamical simulations of Stein & Nordlund (1991).

Here, we adopt the formulation of Samadi et al. (2000), which takes into account contributions both from the Reynolds stresses and from the fluctuations of the entropy. Moreover, the new formulation allows a consistent investigation of the effects of using different forms of the turbulent time spectrum and turbulent energy spectrum. In particular, we study the effects of using various forms of the spatial turbulent energy spectrum and turbulent time spectrum on the acoustic power of radial p-mode oscillations for a model of the Sun and for a model of Procyon A and compare the results with recent observations. Better agreement
with the observations is found for calculations in which the turbulent energy spectrum includes contributions from convective elements with spatial scales larger than one mixing length.

2. Excitation of stellar p modes by turbulent convection

2.1. Acoustic noise generation rate

The acoustic power injected into the oscillations is defined (e.g., GMK) in terms of the damping rate $\eta$, the mean-square amplitude $\langle A^2 \rangle$, the mode inertia $I$ and oscillation frequency $\omega_0$:

$$P = 2 \eta^{1/2} \langle A^2 \rangle I \omega_0^2. \quad (1)$$

The mean-square amplitude is determined by the balance between the energy gain from the turbulent flow and the energy drain by thermal and mechanical damping processes. It can be derived as (e.g., Balmforth 1992b)

$$\langle A^2 \rangle \propto \eta^{-1} \int_0^M dm \; \rho_0 \; w^4 \left( \frac{\partial \xi_r}{\partial r} \right)^2 S(\omega_0, m) \quad \text{with} \quad S(\omega_0, m) = \int_0^\infty dk \; \phi(k) \; \chi_k(\omega_0), \quad (2)$$

where $\xi_r$ is the radial displacement eigenfunction, $\rho_0$ the density, $k$ the wavenumber of an eddy, $w$ is the vertical rms velocity of the convective elements and $\delta$ is the turbulent source term, describing contributions to the noise generation rate from eddies with different sizes. The integration is performed over the stellar mass $M$. The mean-square amplitude is indirect proportional to the damping rate, $\eta$, (e.g. Balmforth 1992b) and consequently the noise generation rate (or acoustic power) $P$ becomes independent of $\eta$. Thus comparing the estimated noise generation rates with observations avoids the additional uncertainties in the modelling of the damping rates (GMK). Both Reynolds stresses and entropy fluctuations contribute to the excitation of p modes, and proper models are needed for their computations. The turbulent source $S$, describing the turbulent spectrum, can be separated into a spatial turbulent energy spectrum $\phi(k)$ and in a turbulent time spectrum $\chi_k(\omega)$ (Stein 1967). The present formulation differs from the formulation of Balmforth (1992b) and GMK mainly in the way the entropy fluctuations are modelled (for more details see Samadi et al. 2000).

2.2. Turbulent ingredients

The correlation time of the turbulent eddies is modelled by the turbulent time spectrum $\chi_k(\omega)$; several forms of $\chi_k(\omega)$ have been proposed: the commonly used Gaussian spectrum (e.g., Goldreich & Keeley 1977, Balmforth 1992b), the exponential spectrum (Stein 1967; Musielak et al. 1994) and the “Modified Gaussian” spectrum as suggested by Musielak et al. (1994). Similarly, various forms for the spatial energy spectrum $\phi(k)$ have been proposed: the Kolmogorov spectrum (KS), the spectrum suggested by Spiegel (1962) which includes a representation for the nonlinear interaction between turbulent modes (SS), and the “Raised Kolmogorov” spectrum (RKS) proposed by Musielak et al. (1994), which was derived empirically from the solar observations of Muller (1989). Only the RKS spectrum includes a description that takes into account contributions from eddies having smaller wavenumbers than those considered in the KS and SS spectra. These eddies correspond to meso-granulation.
3. Solar model and calibration

We considered a solar model computed with the CESAM code (Morel 1997) assuming the model parameters displayed in Fig. 1. The oscillation properties were obtained from the adiabatic FILOU pulsation code of Tran Minh & Leon (1995). We computed models with various timespectra and concluded that the Gaussian time spectrum provides the best agreement between the computed and observed (Libbrecht 1988) acoustic power spectrum. In particular the shape of the acoustic spectrum and the frequency of the maximum value of the acoustic power are closest to the observations for models computed with the Gaussian time spectrum.

Using the observed linewidths of Libbrecht (1988) we computed the mean surface velocities from the power estimates. The results are plotted in Fig. 1 as functions of oscillation frequencies for model computations using the RKS (continuous curve), the KS (dashed curve) and the SS (dot-dashed curve) spectrum. For all three model computations the Gaussian time spectrum was assumed. Amplitudes obtained with the RKS spectrum are closest to the observed values. Moreover, use of the RKS spectrum leads to the smallest frequency shift between the computed and observed maximum value of the velocity amplitudes.

In order to estimate acoustic power spectra for other stars we need to calibrate our formulation, i.e., to scale the free parameters, which are inherent in the formulation of the noise generation rate (for details see Samadi et al. 2000). The noise generation rate is calibrated for all three turbulent spectra in such a way as to predict the same maximum value for the velocity amplitude of $18 \text{ cm s}^{-1}$ as suggested by the BBSO observations of Libbrecht (1988).

4. What can we learn from other stars?

Using the same programmes with the same input physics as used for the solar model described above, we computed a model for Procyon A. For the model parameters we assumed a mass $M = 1.46 M_\odot$, an effective temperature $T_{\text{eff}} = 6395 \text{ K}$, $\alpha = 1.785$ and the same chemical composition as for the solar model (see Fig. 1). The left panel of Fig. 2 shows the normalized power versus the oscillation frequencies.
computed with the RKS (continuous curve), the KS (dotted curve) and the SS (dot-dashed curve) spectrum. The results suggest large differences at high frequencies between models computed with the RKS and the KS spectrum. At low frequencies the shape of the noise generation rate (power) is predominantly determined by the modal inertia, whereas at high frequencies the shape of the eigenfunctions becomes more important. This dependence on the eigenfunctions at high frequencies is more pronounced in the model of Procyon than in the solar model. However, only small differences are found between power estimates obtained from computations including only the Reynolds stress contribution and for computations including both Reynolds stress and entropy contributions, assuming the same turbulent spectrum.

In order to compute velocity amplitudes we need estimates of the pulsation damping rates, $\eta$. The damping rates for radial oscillations were obtained from a non-adiabatic pulsation programme introduced by Balmforth (1992a). In this programme convection is treated with a time-dependent, nonlocal generalization of the mixing-length formulation (Gough 1976, 1977). Computation details can be found in Balmforth (1992b) and in Houdek et al. (1999). The right panel of Fig. 2 shows the estimated surface velocities computed with the RKS and the KS spectra assuming the computed damping rates $\eta$ and the same parameters as suggested by a solar model which has been calibrated to the observations. We observe a large frequency shift of the maximum values of the estimated velocities between models computed with the RKS and the KS spectrum.

In the calculations for the amplitudes we assumed that the effect of using different eigenfunctions for computing the power with the programme of Tran Minh & Leon and for the damping rates with the
programme of Balmforth are small compared to the uncertainties inherent in the formulation of estimating the acoustic power. This inconsistency may affect the absolute value of the surface velocity but has no effect on the comparison between velocity estimates obtained with different turbulent spectra.

Observations of Procyon have been carried out by Martic et al. (1999) and Barban et al. (1999). These authors concluded that the maximum velocity is observed around $\nu \simeq 1 \text{ mHz}$ with an upper limit of $V_{\text{max}} \lesssim 60 \text{ cm s}^{-1}$. The velocity estimates using the RKS spectrum (continuous curve in the right panel of Fig. 2) exhibit a maximum at $\nu \simeq 1 \text{ mHz}$, in fair agreement with the observations. However, the predicted surface velocities are too large relative to the observations. It should be noted that some of the uncertainties in the computed amplitudes do stem from the uncertainties in the damping rate estimations, $\eta$.

5. Conclusion and perspectives

We have presented results of a more general and consistent formulation for estimating the acoustic noise generation rate in solar-type stars. In accordance with GMK, the entropy contribution has been found to be roughly three times larger than the contributions from the Reynolds stresses. However, preliminary results suggest only small differences in the maximum amplitude values in other stars between models computed with the Reynolds stress contribution alone and models computed with both Reynolds stress and entropy contributions, provided the amplitudes in both cases are calibrated first to solar observations for a solar model.

For the solar case we conclude that the “Raised Kolmogorov” spectrum (RKS) and the Gaussian time spectra provide the best agreement with the observations. Computations for a model of Procyon A support this conclusion. For the Procyon model the differences between the RKS and the KS spectra are larger than in the solar case.

Further investigations of the proposed formulation are necessary such as testing it against the results of hydrodynamical simulations. Extending the investigation of the noise generation rate to other stars improves our understanding of stellar turbulence. Moreover, the results of modelling solar-type oscillation properties are of great importance for the selection process of target stars in future space projects such as COROT (COnvection and ROTation), MONS (Measuring Oscillations in Nearby Stars) or MOST (Microvariability & Oscillations of STars).

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DISCUSSION

DOUGLAS GOUGH: Why does the ‘‘Raised Kolmogorov’’ spectrum lead to larger power estimates at high frequencies?

REZA SAMADI: The ‘‘Raised Kolmogorov’’ spectrum (RKS) leads to a smaller depth of excitation. The RKS takes into account eddies of smaller wavenumbers than the Kolmogorov spectrum (KS). There is also an excess of power at low wavenumbers. Consequently, for a given wavenumber k the correlation time of an eddy ($\tau_k$) is larger with the RKS than with the KS spectrum. The major contribution to mode excitation comes from eddies with $\tau_k \omega_0 < 1$. Thus the region where $\tau_k \omega_0 < 1$ should also be smaller with the RKS than with the KS. Therefore the RKS induces a smaller depth of the excitation region.