Error Analysis of Braking-type Plate Shearing Machine

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Abstract. In this paper, geometric errors of braking-type plate shearing machine were analyzed based on Multi-body system theory and sensitivity coefficient. The general expression of pose, movement and errors were expressed based on a 6×3200 braking-type plate shearing machine. The mathematic model was built up and the geometric error was identified based on sensitivity coefficient which calculated by first-order sensitivity function and data analysis. The improvement measures were listed to increase machining accuracy of braking-type plate shearing machine according to above theoretical analysis and practical production.

1. Introduction
In the processing of machine manufacturing, the accuracy of product is up to precision of CNC machine. There are two measures to improve the manufacturing precision [1]: increasing the precision of CNC machines and compensating errors during the manufacturing. So far more and more researches are about machine precise modeling and errors analyzing. The geometry of complex multi-body systems was linearized by Andreas Pott et al. [2]; prediction models was established and analysed to prove the accuracy characteristics and robustness properties of mathematical model by E Miao [3]; the method for geometrical errors analysis in MMTs was proposed and the viability of using an adaptive control system to improve the operational characteristics of an MMT was proved by A Caballero-Ruiz et. al. [4]; the development of kinematic error models in the Vertical Machining Center was presented and the volumetric errors in the multi-axis machine tool by homogeneous coordinate transformation was calculated by A.C. Okafor et al. [5].

Braking-type plate shearing machine was widely used for forging and forming. However, so far most researches are about optimization of structure and hydraulic system of shearing machine, there is no literature about its mathematic models and theoretical error analysis. This paper calculated, modeled and analyzed to define and decrease errors of shearing machine for increasing product precision.

2. Braking–type plate shearing machine and its error analysis

2.1 Braking-type plate shearing machine
This paper was researched under a domestic 6×3200 braking-type plate shearing machine showed as Figure 1 below. It is made up of top blade carrier, stander, pressing settings, back gauge, hydraulic cylinder, CNC system and shearing adjustment device. During the shearing process, plates were pushed under the control of CNC system and pressed under the cylinder. Then top blade carriers were pushed by the main cylinder and moved down against below blade. Due to the existence of shear angle, top blade gradually shears plate from left to right until finished.
2.2 Factors affecting working accuracy

During the process of shearing, the major factors affecting working accuracy of braking-type plate shearing machine were shown as the following:

a) Positioning accuracy of back gauge

The dimensional precision of plate was directly influenced by the positioning accuracy and reliability of back gauge. During the process of shearing, especially shearing thick plate, back gauge is impacted. Therefore it is need to ensure parallelism between back gauge and tool to avoid out of tolerance.

b) Dimensional precision of plate

There is inevitably some defect in the plates sheared, such as section bulging, inclination and deformation at the edge. Therefore, it is need to repeatedly measure to reflect the size of plate correctly and avoid measurement error caused by defect locations, such as convex plate, burr and ends.

c) Shearing movement and deformation

During the process of shearing, plate will move and deform due to large shearing stress and the settings lacking press. The machining accuracy will be influenced greatly by these movement.

2.3 Errors of braking-type plate shearing machine

During CNC machining, the machining accuracy is up to relative positions of tool and works. In the braking-type plate shearing machine, geometric errors in X axis and Z axis were considered due to location, shearing only happened in these two direction. In the same time, shearing belongs to cold modeling, so we don’t research error linking to thermal error respectively. The errors of braking-type plate shearing machine are shown as Table 1.

| Name               | Errors                        | Code  |
|--------------------|-------------------------------|-------|
| Translation in X axis | Linear displacement error | $\delta_{xx}$ |
|                    | Straightness errors in Y axis | $\delta_{yy}$ |
|                    | Straightness errors in Z axis | $\delta_{zz}$ |
|                    | Rolling error               | $\epsilon_{xx}$ |
|                    | Deflecting error            | $\epsilon_{yx}$ |
|                    | Pitching error              | $\epsilon_{zx}$ |
|                    | Linear displacement error    | $\delta_{zz}$ |
|                    | Straightness errors in X axis | $\delta_{xz}$ |
|                    | Straightness errors in Y axis | $\delta_{yz}$ |
| Translation in Z axis | Rolling error               | $\epsilon_{zz}$ |
|                    | Deflecting error            | $\epsilon_{xz}$ |
|                    | Pitching error              | $\epsilon_{yz}$ |

3. Modeling based on Multi-body system theory

3.1 Multi-body system theory
Firstly, the relatively static, relatively moving structure were described and the topology was divided. Secondly, a chain was made up of the components, interrelating with each other, of CNC machine. Then, the errors were analyzed, the feature matrix of CNC machine was calculated based on ideal and practical motion feature matrix. Lastly, the comprehensive error modelling was calculated and the sensitivity was analyzed based on sensitivity coefficient. The specific steps were shown as Figure 2.

![Figure 2. Steps of modeling based on the Multi-body system theory](image)

### 3.2 The topology structure and low order body of braking-type plate shearing machine

The topology structure of braking-type plate shearing machine was described as: workpiece (3)-below blade (2)-working table (1)-wallboard (4)-clearance adjustment device (5)-blade carrier (6)-top blade (7). And it includes two chains, workpiece chain: working table (1)-below blade (2)-workpiece (3) and tool chain: working table (1)-wallboard (4)-clearance adjustment device (5)-blade carrier (6)-top blade (7).

![Figure 3. Structure of braking-type plate shearing machine](image)

![Figure 4. Topology of braking-type plate shearing machine](image)

A typical body $B_j$ of braking-type plate shearing machine was chose to define the nth-order low order body of it: $L^n(j) = i$, and $L$ was defined as the symbol operator of lower body. In the same time, the calculation of low order body was required to obey to following rules.

\[
L^n(j) = L\left(L^{n-1}(j)\right) \tag{3-1}
\]
\[
L^0(j) = j \tag{3-2}
\]
\[
L^n(0) = 0 \tag{3-3}
\]

$B_j$ was defined as nth low order body of $B_i$. If $B_j$ was adjacent to $B_i$, $L(j) = 1$.

Thus, the low order body array was derived. The low order body arrays and the freedom of adjacent arrays were listed following tables.
Owing to \( R \), the F-type plate shearing machine of braking was expressed as the relative position change of \( R_k \) and \( R_j \), and rotation transformation of \( n_{k1}, n_{k2}, n_{k3} \) and \( n_{j1}, n_{j2}, n_{j3} \) were expressed as the relative movement transformation of \( R_k \) and \( R_j \).

As the Figure 5 shows,

\[
O_jO_k = q_k + s_k \\
[n_{k1}, n_{k2}, n_{k3}] = [n_{j1}, n_{j2}, n_{j3}][TJK]_{mn} \\
[TJK]_{mn} \text{ is a } 3 \times 3 \text{ order transformation matrix, and transformation matrix } [TJK] \text{ was expressed as location transformation of adjacent low order body in the multi-body system. In order to simplify the error model research, the motion transformation of braking-type plate shearing machine was transformed to mathematical model.}
\]

### 4. Motion model of braking-type plate shearing machine

Errors of braking-type plate shearing machine exist in the parts manufacturing and motion, such as \( q_{ke} \) and \( s_{ke} \). \( q_{ke} \) was expressed as relative position error, such as errors caused by thermal deformation and forced deformation; and \( s_{ke} \) was expressed as relative displacement error, such as geometric errors and motion errors.

\[
O_jO_k = q_k + q_{ke} + s_k + s_{ke}
\]
\[
\{O_j D_k \} = [T O J](q_k + q_{ke})_{r_j} + [T O K_p](q_k + q_{ke})_{r_j}
\]

Based on Homogeneous Coordinate Transformation,
\[
[T J K] = [T J K]_p[T J K]_s
\]

If \( R_k \) was expressed as combination of six basic movement (revolving and translation along coordinate system), \([T J K]_p\) is the product of basic motion transformations (\(x, y, z\) was expressed as the displacement of \(B_k\) moving in coordinate axis of \(B_j\); \(\alpha, \beta, \gamma\) is expressed as the Euler angle of \(B_k\) against \(B_j\)).

\[
[T J K]_p = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \alpha & -\sin \alpha & 0 \\
0 & \sin \alpha & \cos \alpha & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \beta & 0 & \sin \beta & 0 \\
0 & 1 & 0 & 0 \\
-\sin \beta & 0 & \cos \beta & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \gamma & -\sin \gamma & 0 & 0 \\
0 & 1 & 0 & 0 \\
\sin \gamma & \cos \gamma & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos \beta \cos \gamma & -\cos \beta \sin \gamma & \sin \beta & x \\
\cos \alpha \sin \gamma + \sin \alpha \sin \beta \cos \gamma & \cos \alpha \cos \gamma - \sin \alpha \sin \beta \sin \gamma & -\sin \alpha \cos \beta & \gamma \\
\sin \alpha \sin \gamma - \cos \alpha \sin \beta \cos \gamma & \cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma & \cos \alpha \cos \beta & \zeta \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

The relative position errors of plate shearing machine, such as \(\Delta \alpha, \Delta \beta, \Delta \gamma\), were caused by thermal deformation and forced deformation. As the value is small, the equation can be seen as: \(\sin \Delta \alpha = \Delta \alpha, \cos \Delta \alpha = 1\). Therefore,

\[
[T J K]_{\Delta s} = \begin{bmatrix}
1 & -\Delta \gamma & \Delta \beta & \Delta x \\
\Delta \gamma & 1 & -\Delta \alpha & \Delta y \\
-\Delta \beta & \Delta \alpha & 1 & \Delta z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

If \(B_k\) moved along the X axis against \(B_j\), there were six errors in the \(B_k\).

\[
[T J K]_{\Delta x} = [T J K]_x = \begin{bmatrix}
1 & -\varepsilon_{xx} & \varepsilon_{yx} & \delta_{xx} \\
\varepsilon_{xx} & 1 & -\varepsilon_{xx} & \delta_{yy} \\
-\varepsilon_{yx} & \varepsilon_{yy} & 1 & \delta_{xz} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -\varepsilon_{xx} & \varepsilon_{yx} & 0 \\
\varepsilon_{xx} & 1 & -\varepsilon_{xx} & 0 \\
-\varepsilon_{yx} & \varepsilon_{yy} & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

is the angular error matrix, 
\[
\begin{bmatrix}
1 & 0 & 0 & \delta_{xx} \\
0 & 1 & 0 & \delta_{yy} \\
0 & 0 & 1 & \delta_{xz} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

is the linear error matrix.

If \([T J K]\) was a combination of six basic movement (revolving and translation along coordinate system), \([T J K]_s\) and \([T J K]_{\Delta s}\) also can be calculated as the above ways.

According to the principle of error transformation, actual motion feature matrix of \(B_k\) against \(B_j\) was as following.

\[
[T J K] = [T J K]_p[T J K]_{\Delta p}[T J K]_{\Delta s}[T J K]_s
\]

The transformation matrix of adjacent low order body was obtained by shearing process and low order body array, and small errors were ignored to simplify calculation. The error feature matrix of adjacent low order body were listed as following,

\[
[T J K]_{12p} = E, [T J K]_{12\Delta p} = E, [T J K]_{12s} = E, [T J K]_{12\Delta s} = E;
[T J K]_{23p} = E, [T J K]_{23\Delta p} = E,
[T J K]_{23s} = \begin{bmatrix}
1 & 0 & 0 & x \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, [T J K]_{23\Delta s} = \begin{bmatrix}
1 & -\varepsilon_{xx} & \varepsilon_{yx} & \delta_{xx} \\
\varepsilon_{xx} & 1 & -\varepsilon_{xx} & \delta_{yy} \\
-\varepsilon_{yx} & \varepsilon_{yy} & 1 & \delta_{xz} \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]
\[ [TJ]_{14p} = E, [TJ]_{14\Delta p} = \begin{bmatrix} 1 & 0 & 0 & \eta_{xz} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, [TJ]_{14s} = E, [TJ]_{14\Delta s} = E; \]

\[ [TJ]_{45p} = E, [TJ]_{45\Delta p} = E, \]

\[ [TJ]_{45s} = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, [TJ]_{45\Delta s} = \begin{bmatrix} 1 & -\varepsilon_{zx} & \varepsilon_{yx} & \delta_{xx} \\ \varepsilon_{zx} & 1 & -\varepsilon_{xx} & \delta_{yx} \\ -\varepsilon_{yx} & \varepsilon_{xx} & 1 & \delta_{zx} \\ 0 & 0 & 0 & 1 \end{bmatrix}; \]

\[ [TJ]_{56p} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, [TJ]_{56\Delta p} = \begin{bmatrix} 1 & -\varepsilon_{zx} & \varepsilon_{yz} & \delta_{zx} \\ \varepsilon_{zx} & 1 & -\varepsilon_{xz} & \delta_{yx} \\ -\varepsilon_{yz} & \varepsilon_{xz} & 1 & \delta_{zx} \\ 0 & 0 & 0 & 1 \end{bmatrix}; \]

\[ [TJ]_{56s} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, [TJ]_{56\Delta s} = \begin{bmatrix} 1 & -\varepsilon_{zx} & \varepsilon_{yy} & \delta_{xx} \\ \varepsilon_{zx} & 1 & -\varepsilon_{xx} & \delta_{yx} \\ -\varepsilon_{yy} & \varepsilon_{xx} & 1 & \delta_{zx} \\ 0 & 0 & 0 & 1 \end{bmatrix}; \]

\[ [TJ]_{67p} = E, [TJ]_{67\Delta p} = E; [TJ]_{67s} = E, [TJ]_{67\Delta s} = E. \]

The processing error of shearing machine can be obtained as following if machining position of top blade and workpiece in own coordinate system were known:

\[ E = [TJK]_{03}[TJK]_{14}[TJK]_{45}[TJK]_{56}[TJK]_{67}[Tx, Ty, Tz, \Delta x, \Delta y, \Delta z] - [TJK]_{01}[TJK]_{12}[Wx, Wy, Wz, \Delta x, \Delta y, \Delta z]; \]

\[ E_x, E_y, \text{ and } E_z \text{ are errors along the coordinate axis. } [Tx, Ty, Tz, \Delta x, \Delta y, \Delta z] \text{ and } [Wx, Wy, Wz, \Delta x, \Delta y, \Delta z] \text{ are position coordinates of top blade and workpiece.} \]

5. Error analysis of braking-type plate shearing machine

5.1 First-order sensitivity function

At present, there are three ways to analyze sensitivity of CNC machine: direct method, first-order sensitivity function and higher-order sensitivity function. First-order sensitivity function was widely used to analyze error models based on minor calculation and tractable result.

Vector \( y = [y_1, y_2, ..., y_n]^T \) was used to describe function parameters of CNC machine. The relation of \( y \) and \( a \) was expressed as:

\[ y_i(t, a) = y_i(t, a_1, a_2, ..., a_r), i = 1, 2, ..., n \]

If \( a_0 \) was the parameters of result, \( y_0 \) was the value of parameter \( y \); if \( a_i = a_{o_i} + \Delta a, i = 1, 2, ..., r \) was the parameters of result, the corresponding parameter value was expressed as:

\[ y_i(t, a) = y_i(t, a_{o_i} + \Delta a_1, ..., a_{o_j} + \Delta a_j), i = 1, 2, ..., n \]

Therefore, the system error caused by parameter transformation was expressed as:

\[ \Delta y_i = y_i(t, a_0 + \Delta a) - y_i(t, a_0), i = 1, 2, ..., n \]

According to the Taylor Formula, neglecting second and higher orders of above result,

\[ \Delta y_i(t, a) \approx \sum_{j=1}^{r} \frac{1}{j!} d^j y_i(t, a), a = a_0, i = 1, 2, ..., n \]

If \( F(x) \) is derivable, first-order sensitivity function was expressed as:

\[ S = \frac{\partial F(x)}{\partial x_j} \]

The influencing degree of total error in CNC machine was different from each error. The degree which every error influenced processing is obtained based on the relationship of each error and overall error. The function relationship of overall error \( E \) and each geometric error was expressed as:

\[ E = S \frac{\partial E(x)}{\partial x_j} \frac{\partial E(x)}{\partial x_j} \]

\[ \varepsilon_j \]

was expressed as each part of error model and \( \varepsilon_j \) was expressed as the j-th error element. The comprehensive error model CNC machine was described as:

\[ \varepsilon_j \]

\[ \varepsilon_j \]

\[ \varepsilon_j \]
\[ E = F(L, P_W, U, U_W, U_t) \]

In the model, \( L \) was expressed as error vector composed by geometric errors of CNC machine; \( P_W \) was expressed as vector of parts position in workpiece coordinate; \( U \) was expressed as vector of position in each part coordinate; \( U_W \) was expressed as vector of position in workpiece coordinate; \( U_t \) was expressed as vector of position in tool coordinate. During the analyzing, \( P_W, U, U_W \) and \( U_t \) can be given constantly. Based on first-order sensitivity function, the matrix of CNC machine sensitivity was expressed as:

\[
S = \frac{\partial E}{\partial L} = \begin{bmatrix}
\frac{\partial f_1}{\partial l_1} & \cdots & \frac{\partial f_1}{\partial l_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial l_1} & \cdots & \frac{\partial f_n}{\partial l_n}
\end{bmatrix} \tag{5-7}
\]

5.2 Sensitive analysis of braking-type plate shearing machine

The research method of CNC machine can be applied to braking-type plate shearing machine. The installation position of tool, parts and each error parameter were put into error model to calculate error of workpiece in this position. The universal volumetric error was accumulated by each part error and the influence of each error was different. The influencing degree of error was calculated by sensitivity coefficient in the comprehensive error model.

The error feature matrix of adjacent low order body in plate shearing machine has been given in fourth chapter, and sensitivity formula has been given in formula 5-6. The sensitivity matrix \( S_i \) was the absolute value of partial derivative of each error.

\[
S_i = \left| \frac{\partial E}{\partial \Delta e_i} \right|, i = 1, 2, \ldots \tag{5-8}
\]

There were 24 errors in the simplified module and sensitivity matrix was expressed as:

\[
E = S_{4 \times 24} \cdot E_{24 \times 1} \tag{5-9}
\]

In the above matrix, \( S_{4 \times 24} \) was expressed as the sensitivity matrix of 24 errors and \( E_{26 \times 1} \) was expressed as the error matrix of 24 error elements. The sensitivity expression of each error was calculated by taking partial derivatives to 24 errors \( E_{24 \times 1} \) of error model \( E \).

During the research about the shearing machine, the result of angular error and linear error are approximate 0.01. Therefore, the value of angular error \( \delta \) was set to 0.01°, the value of linear error was set to 0.01mm and the value of initial position was set to 1mm.

The sensitivity analysis of error modelling in the x, y and z direction was listed as Table 4.

| Partial derivative | \( E_x \) | \( E_y \) | \( E_z \) | Partial derivative | \( E_x \) | \( E_y \) | \( E_z \) | Partial derivative | \( E_x \) | \( E_y \) | \( E_z \) |
|-------------------|--------|--------|--------|-------------------|--------|--------|--------|-------------------|--------|--------|--------|
| \( \frac{\partial}{\partial \delta} \) | 0 | 0 | 0 | \( \frac{\partial}{\partial \delta} \) | -1 | 0 | 0 | \( \frac{\partial}{\partial \delta} \) | 0.01 | 0.01 | 1 |
| \( \frac{\partial}{\partial \delta} \) | 1 | 0 | 0 | \( \frac{\partial}{\partial \delta} \) | 0 | -1 | 0 | \( \frac{\partial}{\partial \delta} \) | -0.01 | 0 | 0 |
| \( \frac{\partial}{\partial \delta} \) | -0.01 | -0.01 | 0 | \( \frac{\partial}{\partial \delta} \) | 0 | 0 | 0 | \( \frac{\partial}{\partial \delta} \) | 0.01 | 0 | 0 |
| \( \frac{\partial}{\partial \delta} \) | 0 | -1 | 0.01 | \( \frac{\partial}{\partial \delta} \) | 0 | -0.01 | -0.01 | \( \frac{\partial}{\partial \delta} \) | -1.01 | 0.02 | 0.02 |
| \( \frac{\partial}{\partial \delta} \) | 0.01 | 0.99 | -0.01 | \( \frac{\partial}{\partial \delta} \) | 0 | 0.01 | -0.01 | \( \frac{\partial}{\partial \delta} \) | 0 | 0.01 | 0.01 |
| \( \frac{\partial}{\partial \delta} \) | -0.01 | 0.01 | 0.01 | \( \frac{\partial}{\partial \delta} \) | 0 | -0.01 | -0.0001 | \( \frac{\partial}{\partial \delta} \) | -1 | -0.02 | 0.02 |
| \( \frac{\partial}{\partial \delta} \) | 0.01 | 0 | 0 | \( \frac{\partial}{\partial \delta} \) | 0 | -0.02 | 0.01 | \( \frac{\partial}{\partial \delta} \) | 0 | -0.01 | -0.01 |
| \( \frac{\partial}{\partial \delta} \) | -0.02 | 0 | 0 | \( \frac{\partial}{\partial \delta} \) | 0 | 0 | 0 | \( \frac{\partial}{\partial \delta} \) | 0 | 0.01 | -0.01 |
| \( \frac{\partial}{\partial \delta} \) | 0 | 0.02 | -0.01 | \( \frac{\partial}{\partial \delta} \) | -1 | -0.01 | 0.01 |
It is known according Figure 6:

(1) The relative position errors of braking-type plate shearing machine only exist between clearance adjustment device and blade carrier, and errors with the biggest sensitive coefficient are relative angular position error around X-axis and Y-axis. Therefore, it is need to improve positioning accuracy between clearance adjustment device and blade carrier to reduce above errors during the installation.

(2) The relative motion errors of braking-type plate shearing machine exist among below blade, workpiece, wallboard, clearance adjustment device and blade carrier. Errors with the biggest sensitive coefficient are relative motion position error around X-axis and Y-axis. Therefore, it is need to improve tolerance of below blade, wallboard and clearance adjustment device during the designing. In the same time, it is need to strengthen monitoring and compensating above errors during the producing to improve precision and quality of products.

6. Conclusions

The structure of braking-type plate shearing machine is different from precision multi-axis CNC machine, but the research methods also can be applied in shearing machine, such as Multi-body system, building error model, sensitivity analyzing formula, etc. The analysis of geometric error, calculation of sensitivity coefficient and comparison about all errors in braking-type plate shearing machine offer important evidence to structural designing, manufacturing, error monitoring and error compensating.

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