Strongly-interacting massive particle and dark photon in the era of intensity frontier

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(Dated: April 30, 2020)

A strongly interacting massive particle (SIMP) is an interesting candidate for dark matter (DM) because its self-interaction cross section can be naturally strong enough to address the astrophysical problem of small-scale structure formation. A simple model was proposed by assuming a monopole condensation, where composite SIMP comes from a “strongly interacting” U(1)d gauge theory. In the original model, the DM relic abundance is determined by the 3→2 annihilation process via the Wess-Zumino-Witten term. In this letter, we discuss that the DM relic abundance is naturally determined also by a semi-annihilation process via a kinetic mixing between the hypercharge gauge boson and the dark U(1)d gauge boson (dark photon). The dark photon can be discovered by LDMX-style missing momentum experiments in the near future.

Introduction.— The intensity frontier is one of the broad approaches to new physics in collider experiments and recently became more important as the Large Hadron Collider has not yet found a clear signal for new physics. We should also note the null results in direct-detection experiments of dark matter (DM), which may indicate that the mass of DM is not of order the electron or TeV scale. We therefore focus on the case in which the DM mass is in a sub-GeV region, which can be tested via rare events rather than by a direct production from high-energy particles. Among proposed high-intensity accelerators, the Light Dark Matter eXperiment (LDMX) [1] is designed to measure missing momentum in high-rate electron fixed-target reactions and can be a powerful discovery tool for such a light DM particle.

From the perspective of cosmology, the strongly-interacting massive particle (SIMP) proposed in Refs. [2, 3] naturally fits sub-GeV DM. They pointed out that the relic abundance of sub-GeV DM is consistent with the observed value if the 3→2 annihilation process dominates at the time of freeze-out of DM and its cross section is determined by the mass scale of DM with an O(1) coupling. SIMPs can be naturally realized by composite particles like pions. The 3→2 annihilation process is actually realized by the Wess-Zumino-Witten term in the low-energy dark sector. Interestingly, the model predicts a self-interaction cross section of DM which is potentially favored by the observations of small-scale structure in cosmology [4–8] (see Ref. [9] for a review). This is dubbed as the SIMP miracle. However, there is a difficulty in maintaining thermal equilibrium between the dark and visible sectors during the freeze-out of the 3→2 annihilation process, which is required for the SIMP miracle to work. This can be realized in rather complicated models like the ones proposed in Refs. [10–12] (see Refs. [13–19] for recent works).

In Ref. [20], we have proposed a simple model of the SIMP, where the composite DM “pions” consist of dark-sector “electrons” and “positrons” connected by a U(1)d gauge interaction rather than a strong non-Abelian gauge interaction. We introduce a fundamental “monopole” for U(1)d at a high-energy scale and assume a “monopole” condensation at the sub-GeV scale. One cannot write down the Lagrangian of this kind of theory including both a “monopole” and an “electron”. However, this does not mean that the theory does not exist. In fact, theories with “monopoles” and “electrons” have been extensively studied in N = 2 [21–24] and N = 1 supersymmetry [25–28] without specifying the Lagrangian. In this letter, we revisit our SIMP model and propose a scenario in which the DM relic abundance is determined by a 2→2 semi-annihilation process via the kinetic mixing between the U(1)d gauge boson and U(1)v gauge boson rather than the 3→2 annihilation process. The model is quite economical [29]: we do not need to introduce any other particles but just introduce dark-sector “electrons”, a “monopole”, and the U(1)d gauge boson (dark photon), the latter of which plays the roles of confinement and mediator to the visible sector. Although the SIMP miracle does not work in this scenario, the model is simple and all small dimensionless parameters are expected to be naturally small due to non-trivial anomalous dimensions.

The detectability and testability of our model is quite
different from other DM models. Since there is no “pion”-“pion”-photon interaction and the semi-annihilation process is p-wave suppressed, it is very difficult to directly or indirectly detect the DM “pions”. However, the kinetic mixing allows us to discover the dark photon by LDMX-like experiments. Our model is unique in the sense that it can be tested only by experiments designed to measuring missing momentum in high-rate electron fixed-target reactions.

Hidden “pions” from a “monopole” condensation.— We introduce a scalar “monopole” φ and N\textsubscript{F} pairs of dark-sector “electrons” ψ\textsubscript{e} and “positrons” ψ\textsubscript{μ} with U(1)\textsubscript{d} gauge field [20]. To ensure the stability of “pions” in the low-energy dark sector, we assume SU(N\textsubscript{F}) flavor symmetry under which the “electrons” and “positrons” transform in the fundamental and anti-fundamental representations, respectively. We call the U(1)\textsubscript{d} gauge boson as a dark photon.

We consider the case where the U(1)\textsubscript{d} gauge symmetry is spontaneously broken by the “monopole” condensation in the low-energy dark sector, just like the Higgs mechanism [30]. Each pair of “electrons” and “positrons” is then confined and connected by a string formed by the “monopole” condensation [30] and composes mesons while there is no baryon state in the low-energy dark sector [31]. The string tension is determined by the energy scale of the “monopole” condensation, Λ, and sets the dynamical scale of the system. We assume the condensation of “electrons” and “positrons” that dynamically breaks the chiral symmetry and the “pions” are the lightest composite states in the low-energy dark sector. We also assume that the chiral symmetry for the “electrons” and “positrons” is only an approximate symmetry so that the mass of the “pions” is as large as (but smaller than) the condensation scale Λ [3, 32].

After the “monopole” condensation, there are N\textsubscript{π} ≈ N\textsubscript{F} – 1 “pions”, the radial component of “monopole”, and a massive U(1)\textsubscript{d} gauge boson in the effective field theory. The “monopole” and the gauge boson are assumed to be heavier than the “pions”, which we identify as DM.

There is only one energy scale in the dark sector Λ, which is of order the masses of “pions”, “monopole”, and dark photon denoted by m\textsubscript{π}, m\textsubscript{φ}, and m\textsubscript{γ}, respectively. We introduce \mathcal{O}(1) constants c\textsubscript{1} that represents our ignorance of an \mathcal{O}(1) uncertainty in the low-energy effective field theory [32]. For example, we define m\textsubscript{π} = c\textsubscript{1}Λ = c\textsubscript{m}\textsubscript{π} = c\textsubscript{m}\textsubscript{γ} = c\textsubscript{m}\textsubscript{γ}. We also introduce other \mathcal{O}(1) parameters associated with interactions in the dark sector specified below. To calculate the conservative bounds, we take c\textsubscript{1} ∈ (0.1, 1) throughout this letter.

Self-interactions.— The “pions” have self-interactions whose cross sections are determined by the size of “pions”, which is of order Λ\textsuperscript{-1}. Representing an \mathcal{O}(1) factor by c\textsubscript{1}, we write the cross section as

\[
\frac{\sigma_{\text{el}}}{m_{\pi}} = \frac{(4\pi)^4 c^2_{1} m_{\pi}}{4\pi \Lambda^4} \approx 2.7 \text{ cm}^2/\text{g} \left( \frac{c\textsubscript{1} \Lambda^2}{4\pi} \right)^2 \left( \frac{m_{\pi}}{100 \text{ MeV}} \right)^{-3}.
\]

from the dimensional analysis.\textsuperscript{1} This is of order the upper bound on the interaction cross section of DM from the observations of cluster collisions, including the bullet cluster, and ellipticity on Milky way and cluster scales [34–38]. These constraints and discussions have \mathcal{O}(1) uncertainties due to, say, the difficulties of numerical simulations, and hence we consider that they are marginally consistent with \frac{\sigma_{\text{el}}}{m_{\pi}} = 0.1 – 1 \text{ cm}^2/\text{g}. The recent observations of small-scale structure potentially favors the self-interacting DM with a cross section of the same order [4–8, 39]. We note that m\textsubscript{π} can be as small as about 10 MeV if c\textsubscript{Λ} = c\textsubscript{1} = 0.1.

Kinetic mixings and 2 → 2 semi-annihilation process.— There must be a nonzero kinetic mixing ε between the U(1)\textsubscript{d} gauge boson and the U(1)\textsubscript{γ} gauge boson because it is allowed by any symmetry [31]. There are two types of kinetic mixing terms in theories consisting simultaneously of both a “monopole” and an “electron”: εB\textsubscript{μν}F\textsuperscript{μν} and εB\textsuperscript{μν}F\textsuperscript{μν}, where B\textsubscript{μν} and F\textsubscript{μν} are the field strengths of U(1)\textsubscript{γ} and U(1)\textsubscript{d} gauge bosons, respectively, and \tilde{F}\textsuperscript{μν} \equiv \left(1/2\right)ε\tilde{B}\textsuperscript{μνρσ}F\textsubscript{ρσ}. If the CP symmetry is conserved, either of these mixing terms is allowed.\textsuperscript{2} However, one may expect that the CP symmetry is violated in the dark sector and both mixing terms are present in general.

The U(1)\textsubscript{d} gauge theory may be conformal in the presence of “monopole” as well as “electrons” [23, 24], which implies that the gauge field strength F\textsubscript{μν} has an scaling dimension larger than 2 as is guaranteed by the unitarity bound [40]. As a result, the kinetic mixing terms are irrelevant operators and are suppressed at low energy [20], if present. This naturally results in small ε and ε in our model. Hereafter we represent B\textsubscript{μν} as the photon field strength and absorbs the Weinberg angle into ε and ε for notational simplicity.

In this letter, we mainly consider the case with εB\textsubscript{μν}F\textsuperscript{μν} and without εB\textsuperscript{μν}F\textsuperscript{μν} for simplicity unless otherwise stated. In the dual basis, our model looks similar

\textsuperscript{1} We assume c\textsubscript{1}m\textsubscript{π}^2 ≤ (4\pi)^{-1} throughout this letter so that the scattering cross section is less than the geometrical cross section, 4\pi/m\textsubscript{γ}^2, that is below the Unitarity bound for v < ε [33];

\textsuperscript{2} One may think that εB\textsubscript{μν}F\textsuperscript{μν} itself violates the CP symmetry. In general, either of F\textsubscript{μν} and B\textsubscript{μν} can be chosen to be a tensor and the other one is a pseudo-tensor. If we choose the definition in which B\textsubscript{μν} and F\textsubscript{μν} are tensors and B\textsubscript{μν} and F\textsubscript{μν} are pseudo-tensors, the kinetic mixing term εB\textsubscript{μν}F\textsuperscript{μν} conserves the CP symmetry. In this case, dark “pions” transform as ε → −ε (rather than π → −π) under the CP, so that (Tr [\tilde{B}\textsubscript{μν}π\partial\textsubscript{μ}π] → (μ ↔ ν)) is also a tensor and can be mixed with F\textsubscript{μν}.
to the standard spontaneously broken U(1)$_d$ gauge theory, where the U(1)$_d$ symmetry is (spontaneously) broken by the condensation of the “Higgs” field (i.e., the scalar “monopole” in the original basis) and the kinetic mixing term looks the same as the usual one, $\epsilon B_{\mu \nu} F^{\mu \nu}$. Then we can quote constraints on the kinetic mixing parameter to compare our result with the present and future constraints. We will explain that our result does not change much even if $\epsilon'$ is nonzero and is as large as $\epsilon$.

Here we note that $F_{\mu \nu}$ does not satisfy the Bianchi identity, $\varepsilon^{\mu \nu \rho \sigma} \partial_{\rho} F_{\sigma \rho} = 0$, in theories consisting simultaneously of both a “monopole” and an “electron” (see, e.g., Ref. [41]). Then an operator mixing between $F_{\mu \nu}$ and $\mathrm{Tr} \left[ \tau_{i} \partial_{\mu} \pi \partial_{\nu} \pi \right]$ is allowed in those theories. Therefore, once we allow the nonzero kinetic mixing, $\epsilon B_{\mu \nu} F_{\mu \nu}$, we can have a term like

$$\mathcal{L} \supset c_{\epsilon} \frac{(4\pi)^2}{\Lambda^2} \epsilon B_{\mu \nu} \mathrm{Tr} \left[ \pi \partial_{\mu} \pi \partial_{\nu} \pi \right].$$

where $c_{\epsilon}$ is an $O(1)$ constant. This operator leads to a semi-annihilation process of $\pi \pi \rightarrow \pi \gamma$ only in the presence of a “monopole” and “electrons”. If $F_{\mu \nu}$ satisfied the Bianchi identity, one could write $F_{\mu \nu} = \partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu}$ with $V_{\mu}$ being a (magnetic) gauge field of U(1)$_d$. Then the kinetic mixing operator $B_{\mu \nu} F_{\mu \nu}$ could be written as $-2 \partial_{\mu} B^{\mu \nu} V_{\nu} = 0$ after the integration by parts for on-shell photon. However, $F_{\mu \nu}$ does not satisfy the Bianchi identity in the presence of a “monopole” as well as “electrons”. There is no reason that we prohibit the operator of Eq. (2) and the on-shell photon is produced by the annihilation process, $\pi \pi \rightarrow \pi \gamma$.

The operator of Eq. (2) vanishes for $N_{\pi} < 3$ since it is antisymmetric in the flavor SU($N_{\pi}$), so that we assume $N_{\pi} \geq 2$ in our model. We note that the “pions” transform as an adjoint representation of the flavor SU($N_{\pi}$). The two “pions” in the initial state must be antisymmetric in the flavor SU($N_{\pi}$) to contact with the one “pion” in the final state. On the other hand, the initial state of the semi-annihilation process must be symmetric in terms of the “pion” exchange because “pions” are bosons. These observations imply that the initial angular momentum must be antisymmetric and the semi-annihilation process is $p$-wave suppressed. We thus expect that its cross section can be estimated as

$$(\sigma v)_{\pi \pi \rightarrow \pi \gamma} \sim c_{\epsilon}^2 c_\lambda^2 \frac{(4\pi)^{4} m_{\pi}^{4}}{4 \Lambda^{6}} \left( \frac{T}{m_{\pi}} \right),$$

from the dimensional analysis, where we absorb an $O(1)$ uncertainty into $c_{\epsilon}$. This interaction is in thermal equilibrium at a temperature higher than $m_{\pi}$ for $c_{\epsilon} c_\lambda \gtrsim 4 \times 10^{-12} c_{\lambda}^{-3} (m_{\pi}/100 \text{ MeV})^{1.2}$. The temperature of the “pions” is the same as that of the SM sector until the semi-annihilation process freezes out at $T/m_{\pi} \sim 1/20$.

**Relic abundance of “pions”.**—As the temperature becomes lower than the “pion” mass, the number density of “pions” is suppressed by the Boltzmann factor and eventually the $\pi \pi \rightarrow \pi \gamma$ semi-annihilation process freezes out. We note that the $\pi \pi \rightarrow \pi \gamma$ semi-annihilation process is similar to but is slightly different from the standard annihilation process in the weakly-interacting massive particle (WIMP) scenario. The important difference is that the “pion” in the final state can be relativistic and may heat the dark sector [14, 42, 43]. From the Boltzmann equation of the “pions”, the evolution equations of the yield $Y_{\pi} \equiv n_{\pi}/s$ and the inverse temperature $x_{\pi} (\equiv m_{\pi}/T_{\pi})$ are approximated as

$$\frac{d}{dx} Y_{\pi} \approx - \frac{\lambda}{x^2} Y_{\pi}^{1/2},$$

$$\frac{d}{dx} \left( \frac{x_{\pi}}{x} \right) \approx \frac{2}{3} \lambda Y_{\pi} \left( \frac{x_{\pi}}{x} \right)^2,$$

for $x (\equiv m_{\pi}/T) > x_{\text{FO}} (\equiv m_{\pi}/T_{\text{FO}})$, where $s = (2\pi^{2}/45) g_{*} T^{3}$, $T$ is the temperature of the SM particles, and $T_{\text{FO}}$ is the freeze-out temperature (see Ref. [42] for the original equations without using approximations). The effective number of relativistic degrees of freedom, $g_{*}$, is taken to be about 10. The dimensionless reaction rates are given by

$$\lambda = \frac{x s (\sigma v_{\text{rel}})}{2 H}, \quad \bar{\lambda} \approx - (\gamma - 1) \lambda,$$

where $\gamma = (5/4)$ is the Lorentz factor that DM achieves through semi-annihilation.

Assuming $x_{\text{FO}} \sim 20$, we numerically solve Eqs. (4) and (5) and obtain the asymptotic value of the yield and the temperature of “pions” as

$$Y_{\pi,\infty} \sim c_{\gamma} \frac{2 x_{\text{FO}}}{\lambda(x_{\text{FO}})} \quad x_{\pi} \sim c_{x} \frac{x^2}{x_{\text{FO}}},$$

for $x_{\pi} \gg x_{\text{FO}}$, where $c_{\gamma} = O(0.1)$ and $c_{x} = O(0.1)$ are numerical constants. We note that there are $O(1)$ uncertainties in these results, though they are accurate enough for our purpose. These results are different from the ones for the WIMP scenario by a factor of order 0.1. This is because the relativistic “pion” in the final state of the semi-annihilation process heats the dark sector, which results in the relative increase for the $p$-wave semi-annihilation rate. The energy density of the “pions” is present is consistent with the observed value of the DM relic density when

$$\epsilon \sim 5 \times 10^{-7} c_{\gamma}^{1/2} c_{\epsilon}^{-1} c_{\lambda}^{-3} \left( \frac{m_{\pi}}{100 \text{ MeV}} \right).$$

The kinetic mixing can be as large as, e.g., $O(10^{-3})$ for $m_{\pi} = 100 \text{ MeV}$ if $c_{\lambda} = c_{\epsilon} = 0.1$.

The second term in the right-hand side of Eq. (5) becomes negligible after the freeze-out if the semi-annihilation process is $p$-wave suppressed and $\bar{\lambda} \propto 1/x_{\pi}$.

3 The initial condition is taken to be $Y_{\pi} = c_{\text{ini}} x_{\text{FO}}^{2}/\lambda(x_{\text{FO}})$ and $x_{\pi} = x$ at $x = x_{\text{FO}}$ with $c_{\text{ini}}$ being an $O(1)$ constant. The numerical coefficients $c_{\gamma}$ and $c_{x}$ depend on $c_{\text{ini}}$ only logarithmically while they linearly depend on $x_{\text{FO}}$. 


Then the temperature of the “pions” scales as $T_\pi \propto 1/a^2$ just like the non-relativistic matter and the DM “pions” is cold, where $a$ is the scale factor. This is in contrast to the case of a $s$-wave semi-annihilation process discussed in Ref. [42], where $T_\pi \propto 1/a$ and the DM is warm.

We show the allowed region of the kinetic mixing parameter $\epsilon^2$ in Fig. 1. We assume that $c_1, c_\Lambda, c_\epsilon \in (0.1, 1)$ with a condition of $c_1 c_\Lambda^2 < (4\pi)^{-1}$ (see footnote 1) for a conservative analysis while we take $c_\epsilon = 0.1$ and $c_{m_\nu} = 1/4$ for simplicity. The shaded regions are parameters in which the DM relic abundance can be consistent with the observed DM abundance and the self-interaction cross section can be $\sigma_{\text{els}}/m_\pi \in (0.1, 1) \text{ cm}^2$/g.

In the darkly shaded region, $\sigma_{\text{els}}/m_\pi$ can be as large as 1 cm$^2$/g while in the lightly shaded region it is smaller than 1 cm$^2$/g but can be larger than 0.1 cm$^2$/g. The upper-left corner of the shaded region is bounded by the condition that $c_\Lambda$ should not be smaller than about 0.1 in Eq. (8). In the upper-right (lower-left) corner of the figure, the self-interaction cross section of “pions” becomes too small (large) to be consistent with the observations of the small-scale structure.

For the case of $N_F \geq 3$, there may be a term similar to the Wess-Zumino-Witten term in strong SU(N) gauge theories, which leads to the $3 \rightarrow 2$ annihilation process [3, 20]. We find that the $3 \rightarrow 2$ annihilation process is negligible during the freeze-out process when

$$\epsilon \gtrsim 2 \times 10^{-6} c_\epsilon^{-1} \left(\frac{WZW}{0.1}\right)^{3/5} \left(\frac{m_\pi}{100 \text{ MeV}}\right)^{1/10}. \tag{9}$$

**Experimental constraints.**—Since there is no $\pi - \pi - \gamma$ (or dark photon) interaction due to the flavor SU($N_F$), the “pions” cannot be detected by the direct-detection experiments of DM. On the other hand, the dark photon can be produced via the kinetic mixing and can be discovered by some experiments employing missing momentum and/or energy techniques. In the figure, we plot the constraints on the kinetic mixing parameter by BaBar [44–46] and NA64 [47, 48] in the magenta and green lines, respectively. We can see that most of the parameter space is consistent with the present upper bound. We also find that (Extended) LDMX experiment, whose sensitivity curve is shown as the blue (red) dashed line [1], can cover a large parameter space.

Note that the dark photon cannot decay into two “pions” in our model. This implies that the dark photon cannot decay solely into the dark sector for the case of $m_\nu < 3m_\pi$. On the other hand, the dark photon dominantly decays into the dark “pions” for the case of $m_\nu > 3m_\pi$. The LDMX experiment is designed to measure missing momentum in this kind of process. As we hope to indirectly detect the DM particle by LDMX-like experiments, we assume $m_\nu = 4m_\pi$ $(> 3m_\pi)$, i.e., $c_{m_\nu} = 1/4$, to plot the figure. We predict that $m_\nu$ is larger than about 30 MeV because we require $m_\nu > 3m_\pi$ and $m_\pi \gtrsim 10$ MeV.

Here we comment on the case in which there is only the other kinetic mixing term $\epsilon B_{\mu\nu} F_{\mu\nu}$ rather than $\epsilon B_{\mu\nu} F_{\mu\nu}$. In this case, Eq. (2) should be replaced by a term like $c_\epsilon (4\pi)^2/\Lambda^3 \epsilon B_{\mu\nu} \text{Tr}[\pi \partial_\mu \pi \partial_\nu \pi]$ though our analysis of the semi-annihilation process does not change qualitatively. The standard model particles cannot emit on-shell dark photons while the dark-sector particles can be produced via the off-shell (dark) photons via the kinetic mixing. We expect that the cross section of such a process with missing particles is then given by the replacements of $m_\nu$ by $\Lambda$ and $\epsilon^2$ by $\epsilon^2$ with an additional factor of $N_F a_\nu/(2\pi) \ln(E/\Lambda)$ ($\sim O(1)$) for $E \gtrsim \Lambda$, where $E$ ($= O(1) \text{ GeV}$) is the energy of the scattering process [45]. We note that the additional factor is just an $O(1)$ factor and the difference between $m_\nu$ and $\Lambda$ is also an $O(1)$ factor. We may absorb these factors into $c_\epsilon$ and $c_{m_\nu}$, respectively. Then the result is similar to the one shown in Fig. 1 with $\epsilon^2 \rightarrow \epsilon^4$. Even in the presence of both kinetic terms, the result does not change qualitatively because their effects are additive for the production process in the experimental setups as well as for the semi-annihilation process.

We also comment on the region near the lower bound on the “pion” mass ($\sim 10$ MeV). As the “pions” are non-relativistic and are suppressed by the Boltzmann factor during the freeze-out process of neutrinos, the effect of “pion” decoupling is almost negligible for observables such as the effective number of neutrinos. However, it is argued that its effect can be detected in the near future by the Simons Observatory [49] and CMB-S4 [50, 51] if the “pion” mass is as small as about 10-15 MeV [52].

Finally, we note that the constraint from the indi-
rect detection experiments of DM is not relevant in our model because the semi-annihilation process is $p$-wave suppressed and is not efficient in the galactic scale (see, e.g., Ref. [53]).

**Discussion and conclusions.**—We proposed a SIMP model with dark-sector “electrons” and a “monopole” in U(1)$_d$ gauge theory, motivated by the small-scale crisis in cosmology. The model is quite economical: the U(1)$_d$ gauge boson plays the roles of confinement and the mediator for the annihilation of “pions”. The number of flavor $N_F$ can be as small as two to introduce an operator for the semi-annihilation process. We assume SU($N_F$) flavor symmetry to ensure the stability of “pions”. One can promote this flavor symmetry to a gauge symmetry without changing our scenario qualitatively if the gauge coupling constant is small enough.

Finally, we comment on a mixing between the “monopole” $\phi$ and the SM Higgs field $H$ below the Higgs and “monopole” condensation scales via a quartic interaction of $|\phi|^2 |H|^2$. There is a strong collider constraint on the mixing parameter from the Higgs-decay channel into two “monopoles” [54]. The “monopoles” can decay into muons after they are produced from the Higgs decay [55]. In this case, the branching ratio of the Higgs decay into the “monopoles” must be smaller than about 1% [56], which requires that the above quartic coupling must be smaller than of order $10^{-3}$. Such a small coupling may be naturally realized in our model because our model may be conformal above the “monopole” and “electron” mass scale and the “monopole” has a relatively large anomalous dimension [23, 24]. The search for the Higgs decay into muons may also be an interesting direction to test our model in the near future.

**Acknowledgments.**—T. T. Y. deeply thanks the experimental groups at TDLI for the discussions on the search for the dark photon. Without the discussion, we could not have reached the conclusion in this letter. We thank K. Yonekura for useful discussion. A. K. was supported by Institute for Basic Science under the project code, IBS-R018-D1. M. Y. was supported by Leading Initiative for Excellent Young Researchers, MEXT, Japan. T. T. Y. was supported in part by the China Grant for Talent Scientific Start-Up Project and the JSPS Grant-in-Aid for Scientific Research No. 16H02176, No. 17H02878, and No. 19H05810 and by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. M. Y. thanks the hospitality during his stay at DESY. T. T. Y. thanks Kavli IPMU for their hospitality during the corona virus outbreak.

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