Shear viscosity of nucleons and pions in heavy-ion collisions at energies of NICA

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Abstract. The shear viscosity is calculated microscopically via the Green-Kubo relation for the series of snapshots in the central region in an ongoing relativistic collision simulated via the UrQMD framework for various bombarding energies in the anticipated NICA experiments. In previous works the shear viscosity was calculated as function of temperature, while the chemical potential of baryon charge was kept constant. In present work we extract, in various time windows, the average energy density, the net baryon density and the small though nonzero net strangeness density. By fitting these parameters to statistical model, one can get temperature and both chemical potentials of baryon charge and strangeness. Simultaneously, these parameters are used as input to simulations in a box, again within the UrQMD transport model. The autocorrelations in time of the energy stress tensor are extracted, and subsequently via the Green-Kubo identities the shear viscosity coefficient of that equilibrium hadronic system is obtained. Then we calculate partial viscosity both for nucleons and pions for five collision energies from $E_{\text{lab}} = 5$ to 40 AGeV. It appears that substantial part of the contribution to total shear viscosity of the system comes out from pion-nucleon and other correlators.

1. Introduction

The main aim of experiments on heavy-ion collisions at ultrarelativistic energies is the study of properties of a new state of matter called quark-gluon plasma (QGP). For present status of the field see, e.g., [1] and references therein. Nowadays the QGP is considered as an almost perfect strongly interacting fluid rather than ideal gas of non-interacting partons. When the hot fireball with QGP expands it experiences phase transition to hadronic matter at a certain transition temperature. The order of this transition depends on the energy of the collision. Analysis of experimental data on Au+Au and Pb+Pb collisions at energies of $\sqrt{s_{NN}} = 200$ GeV at RHIC and $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV at LHC indicates that the matter experiences a smooth crossover. In contrast, at intermediate energies, say, above few GeV, the deconfinement phase transition should be of first order. The curve of the first order phase transition on the QCD phase diagram ends up at the tricritical point, where the phase transition becomes of the second order. Exact position of the tricritical point should be determined experimentally. Its search for is among the primary goals of experiments at beam energy scan (BES) at RHIC, SPS at CERN, and at coming in a nearest future FAIR and NICA facilities.

Shear viscosity, $\eta$, is a promising signal to probe the tricritical point on the phase diagram because its ratio to entropy density, $\eta/s$, reaches there minimum for all known substances [2].
It is interesting to study also how close is this ratio for hot and dense nuclear matter to the absolute limit, estimated within the AdS/CFT correspondence as $1/(4\pi)$ [3]. These issues were studied by means of microscopic models in, e.g., [4–15]. Usually, the temperature dependence of $\eta/s$ was investigated at fixed baryochemical potential and zero chemical potential. In our analysis we will use technique developed in [16–18] which permits us to study the evolution of the $\eta/s$ ratio during the course of heavy-ion collision as a function of temperature $T$, baryon chemical potential $\mu_B$, and strangeness chemical potential $\mu_S$ simultaneously.

2. Technique and method

The Green-Kubo formalism [19,20] is employed to determine the shear viscosity in the system. Its important assumption is that the closed system, which is initially out of equilibrium, should evolve towards the equilibrated state. Then, in system of natural units $c = \hbar = k_B$ the shear viscosity reads

$$\eta(t_0) = V \frac{T}{\tau} \int_{t_0}^{\infty} \langle \pi(t) \pi(t_0) \rangle_t dt$$

Here $V$ and $T$ is the volume and the temperature of the system, respectively, $t_0$ is initial time and $t$ is the final time. The correlator in equation (1) can be calculated as

$$\langle \pi(t) \pi(t_0) \rangle_t = \lim_{t_{\text{max}} \to \infty} \frac{1}{t_{\text{max}} - t_0} \int_{t_0}^{t_{\text{max}}} \pi^{ij}(t + t') \pi^{ij}(t') dt'$$

$$\approx \langle \pi(t_0) \pi(t_0) \rangle \exp \left(-\frac{t - t_0}{\tau} \right),$$

with $\tau$ being the effective relaxation time. Finally, the tensor $\pi^{ij}(t)$ is the non-diagonal part of the stress energy tensor $T^{ij}$

$$\pi^{ij}(t) = \frac{1}{V} \sum_{k=1}^{\text{particles}} \frac{p_k^i(t) p_k^j(t)}{E_k(t)},$$

where $E_k$ and $p_k^{i(j)}$ is the energy and $i(j)$ components of momentum of particle $k$. Inserting equation (3) into equation (1) we get

$$\eta(t_0) = \frac{V_T}{T} \langle \pi(t_0) \pi(t_0) \rangle,$$

indicating that one needs to determine the correlator $\langle \pi(t_0) \pi(t_0) \rangle$, the relaxation time, and the temperature to find the shear viscosity.

We employ the well-known UrQMD model [21,22] for our calculations. The model successfully describes various features of hadronic and nuclear interactions in a broad energy range. The bombarding energy of central Au+Au collisions studied in our paper varies from $E_{\text{lab}} = 5$ to 40 GeV, accessible for NICA. Because even in most central heavy-ion collisions the net baryon charge is not distributed uniformly within the whole volume, we opted for the central cubic cell with volume $V = 5 \times 5 \times 5 = 125 \text{ fm}^3$. Previous studies [23–28] revealed that such cell is well suited for the investigation of the relaxation process in hot and dense nuclear matter. - Recall, that we have to determine temperature of the (sub)system, and there is no rigorous definition of the temperature for the out-of-equilibrium systems. - To do this, we employ the formalism developed in [23,25]. Namely, one has to extract values of the energy density, the net baryon density, and the net strangeness out of the microscopic calculations and insert it then to system
of nonlinear equations provided by the statistical model (SM) of ideal hadron gas

\[ \varepsilon^{\text{mic}} = \sum_i \varepsilon_i^{SM}(T, \mu_B, \mu_S) \]

\[ \rho_B^{\text{mic}} = \sum_i B_i n_i^{SM}(T, \mu_B, \mu_S) \]

\[ \rho_S^{\text{mic}} = \sum_i S_i n_i^{SM}(T, \mu_B, \mu_S) \],

where both partial number density \( n_i^{SM} \) and energy density \( \varepsilon_i^{SM} \) of hadron specie "i" are just functions of temperature \( T \) and both chemical potentials, \( \mu_B \) and \( \mu_S \). Total chemical potential of specie "i" depends on its baryon \( B_i \) and strangeness \( S_i \) content

\[ \mu_i = B_i \mu_B + S_i \mu_S \].

In the statistical model of ideal hadron gas the values of \( \varepsilon_i^{SM}, n_i^{SM}, \) and partial pressure \( P_i^{SM} \) are derived via the distribution function \( f(p, m_i) \) as

\[ n_i^{SM} = \frac{g_i}{(2\pi)^3} \int_0^\infty f(p, m_i) d^3 p \]

\[ \varepsilon_i^{SM} = \frac{g_i}{(2\pi)^3} \int_0^\infty \epsilon_i f(p, m_i) d^3 p \]

\[ P_i^{SM} = \frac{g_i}{(2\pi)^3} \int_0^\infty \frac{p^2}{3\epsilon_i} f(p, m_i) d^3 p \]

\[ f(p, m_i) = \left[ \exp \left( \frac{\epsilon_i - \mu_i}{T} \right) \pm 1 \right]^{-1} \].

Here \( g_i \) is the spin-isospin degeneracy factor, \( p \) is the hadron momentum, \( \epsilon_i = \sqrt{p^2 + m^2} \) is the hadron energy, and \( m \) is its mass. Sign + in equation (13) stands for fermions, and sign – is for bosons. Comparing the particle yields and energy spectra, obtained in microscopic model calculations, to those given by the SM we can find the beginning of equilibrium and determine, therefore, values of \( T, \mu_B, \) and \( \mu_S \). Then, we have to determine (i) the effective relaxation time \( \tau \) and (ii) the correlator \( \langle \pi(t_0)\pi(t_0) \rangle \). This can be done by studying the relaxation process in the UrQMD box with periodic boundary conditions \[29, 30\], which preserve the energy density and the net quark content in the box. To initialize the box with volume \( 10 \times 10 \times 10 = 1000 \text{ fm}^3 \) we use again the values of \( \varepsilon, \rho_B, \rho_S \) extracted from the cell. Net baryon density is provided by neutrons and protons, taken in equal proportion, whereas the nonzero strangeness density can be generated by admixture of Lambdas or kaons. Relaxation process in the box takes much longer times compared to the cell calculations \[29, 30\], therefore we run the calculations until \( t_{\text{box}} = 1000 \text{ fm/c} \).

### 3. Results

Results of our calculations of the shear viscosity and the ratio \( \eta/s \) in the central cell of central gold-gold collisions at \( E_{\text{lab}} = 5, 10, 20, 30 \) and 40 AGeV can be found elsewhere \[16–18\]. In present paper we would like to study partial contributions of nucleons and pions to the total shear viscosity. To do this we replace the summation over all particles in equation (4) to the
summation over either all nucleons or all pions in the system. For each of 5 bombarding energies, the central cell parameters were extracted at times from \( t = 1 \text{ fm/c} \) up to \( t = 20 \text{ fm/c} \) with the time step \( \Delta t = 1 \text{ fm/c} \).

Figure 1 displays the partial shear viscosity of nucleons \( \eta_N \) as a function of initial time \( t_0 \). For small values of \( t_0 \) and for early times in the cell corresponding to the overlap of colliding nuclei, the values of \( \eta_N \) are large. They quickly drop to an approximately constant value for \( 200 \leq t_0 \leq 800 \text{ fm/c} \). In order to reduce statistical errors, therefore, we averaged the values of \( \eta_N \) over the plateau \( t_0 \in [200, 800] \text{ fm/c} \). Another interesting feature is the decrease of \( \eta_N \) for all times with increasing bombarding energy, whereas pions demonstrate the opposite trend.

The total shear viscosity of nuclear matter calculated within the aforementioned initial time interval is shown in figure 2. For early cell conditions the total \( \eta \) seems to increase with rising energy. - Note, however, that the matter is out-of-equilibrium here. Therefore, results for the very beginning of nuclear collisions should be treated with great care. - At \( t \geq 6 \text{ fm/c} \), when the matter approaches equilibrium, all curves \( \eta(t_{cell}) \) sit on the top of each other.

Figure 3 depicts the partial contributions of nucleons and pions to the total shear viscosity. Results concerning the non-equilibrium stages are shown by dashed lines. The ratios of \( \eta_N/\eta \) and \( \eta_\pi/\eta \) are quite peculiar. At lower bombarding energies, \( E_{lab} = 5 \) and 10 AGeV, the contribution of nucleons, which dominate the particle spectrum, is more than 50% for early times of the collision. It drops to 15-25% at the late stages. For higher energies, the nucleon contribution to \( \eta \) does not exceed 10-15%. Pions, as seen in figure 3, make rather modest though stable contribution at any times for all considered energies. Their part does not exceed 10-15% of total \( \eta \). Where is the missing 60%? It would be interesting, therefore, to study the role of \( \pi - N \) correlators, as well as resonances, in the formation of shear viscosity of hadrons.

![Figure 1](image-url)  
**Figure 1.** Shear viscosity of nucleons \( \eta_N(t_0) \) for five collision energies from 5 AGeV to 40 AGeV and for the cell times \( t \in [1, 20] \text{ fm/c} \).
Figure 2. Shear viscosity of hadrons as function of time in the central cell in gold-gold collisions at energies from 10 AGeV to 40 AGeV.

Figure 3. Time evolution of the ratio of shear viscosity of nucleons (a) and pions (b) to total shear viscosity in the cell for Au+Au collisions at energies from 5 AGeV to 40 AGeV.

4. Conclusions

The UrQMD model is employed to study the partial shear viscosity of nucleons and pions in the central zone of central Au+Au collisions at energies between $E_{lab} = 5$ and 40 AGeV accessible for NICA. The self-consistent procedure to determine $\eta$ at temperatures and chemical potentials corresponding to those in heavy-ion collisions is developed. It is based on (i) application of the statistical model to determine temperature $T$, baryon chemical potential $\mu_B$, and strangeness chemical potential $\mu_S$, and (ii) UrQMD box calculations to determine the relaxation rates and correlators, employed further within the Green-Kubo formalism to calculate $\eta$.

The developed procedure was used to calculate partial shear viscosity of nucleons and pions based on nucleon-nucleon and pion-pion correlators. At lower energies the shear viscosity of nucleons is about 50-70% of total $\eta$ right after beginning of the equilibrium. At late stages of the matter evolution it drops to 15-25%. For higher bombarding energies $\eta_N$ is below 20% regardless of time. Pion contribution does not practically depend on collision energy or evolution time. However, pions contribute just 10-15% to the total shear viscosity. It means that substantial part of $\eta$ comes out from other correlators. This question deserves further investigations.
Acknowledgments

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