THRESHOLD PION ELECTROPRODUCTION IN CHIRAL PERTURBATION THEORY *

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ABSTRACT

Electroproduction of pions on the nucleon near the threshold is analyzed within the framework of baryon chiral perturbation theory. We give a thorough discussion of the low–energy theorems related to charged and neutral electropionproduction. It is shown how the axial radius of the nucleon can be related to the S–wave multipoles $E_{0+}^{(-)}$ and $L_{0+}^{(-)}$. The chiral perturbation theory calculations of the $\gamma^*p \rightarrow \pi^0p$ reaction are found to be in good agreement with the recent near threshold data. We also discuss the influence of some isospin–breaking effects in this channel. For future experimental tests of the underlying chiral dynamics, extensive predictions of differential cross sections and multipole amplitudes are presented.

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I. INTRODUCTION AND SUMMARY

Threshold pion photo- and electroproduction has received renewed interest over the past few years from both the experimental and the theoretical side. Extensive data were obtained for the process $\gamma + p \rightarrow \pi^0 + p$. These data spurred numerous theoretical investigations concerning the low-energy theorem for the electric dipole amplitude $E_{0+}$ and also a critical reexamination of the data was performed. In the framework of QCD these topics were addressed in big detail in Ref.[1] (which from now on will be referred to as I). Using virtual photons as probes one can get more detailed information about the structure of the nucleon in the non-perturbative regime of QCD due to the longitudinal coupling of the virtual photon to the nucleon spin. For example, charged pion electroproduction encodes information about fundamental quantities like e.g. the axial form factor of the nucleon. Many of these topics are addressed in the monograph by De Alfar et al. [2]. Also, new experimental information has recently become available for the process $\gamma^* + p \rightarrow \pi^0 + p$, where $\gamma^*$ denotes the virtual photon [3]. This experiment is a major step beyond previous measurements which were characterized by poor energy resolution and did not come close enough to the production threshold. Therefore, the results of the old measurements were dominated by the P-wave $M_{1+}$ multipole. In contrast, the data presented in [3] were obtained in the energy range of 0 to 2.5 MeV above threshold and at photon four-momenta squared of $k^2 = -0.04$ to $-0.1$ GeV$^2$ ($k^2 < 0$ in the physical region of the electroproduction reaction). More accurate data also for the production of charged pions are expected to come from NIKHEF, MAMI at Mainz and MIT Bates in the near future.

In this paper, we will be concerned with a systematic analysis of the processes $\gamma^* + p \rightarrow \pi^+ + n$, $\gamma^* + n \rightarrow \pi^- + p$ and $\gamma^* + p \rightarrow \pi^0 + p$ in the threshold region making use of baryon chiral perturbation theory (CHPT). In general, CHPT allows one to systematically investigate the strictures of the spontaneously broken chiral symmetry of QCD. It is based on the observation that in the three flavor sector of QCD, the quark masses are small and that the theory in the limit of vanishing quark masses admits an exact chiral symmetry. The latter is dynamically broken which leads to the appearance of massless pseudoscalar excitations, the Goldstone bosons. In the real world, the quark masses are not exactly zero and thus the Goldstone bosons acquire a small mass. The interaction of these particles with each other and matter fields like e.g. the nucleons are weak at low energies as mandated by Goldstone’s theorem. This fact is at the heart of CHPT which amounts to a systematic and
simultaneous expansion of the QCD Green functions in small momenta and quark masses. To perturbatively restore unitarity it is mandatory to consider pion loop diagrams. In what follows, we will work in the one–loop approximation which has been shown to be of sufficient accuracy for many threshold phenomena. For a general review see Ref.[4]. At next–to–leading order, one has not only pion loop graphs but also local contact terms. The latter are accompanied by a priori unknown coefficients, the so–called low energy constants. These can either be determined from phenomenology [5] or estimated from resonance exchange [6]. For the case under consideration, ∆(1232) and vector meson exchanges, the knowledge of some nucleon electromagnetic radii as well as the pion charge radius allow us to pin down all low energy constants appearing here.

The starting point of our investigation will be flavour SU(2) with equal masses and we work to first order in the electromagnetic coupling constant. As a consequence the one–loop approximation does not include isospin–breaking effects as they are revealed in the difference of the proton and neutron or the charged and neutral pion masses. To minimally account for these effects we will also present calculations in which the respective particles have their physical masses. For the neutral pion photoproduction it was argued in [7] that indeed the pion mass difference is the most important effect of the isospin breaking. In that reference it was also shown that this procedure leads to a much improved description of the data for $E_{\pi^0p}$ in the threshold region. We should stress, however, that ultimately a complete calculation including effects of higher loops and of higher order in the fine structure constant should be performed. This is beyond the scope of the present paper.

The pertinent results of our investigation can be summarized as follows:

(i) To one–loop order and to first order in the electromagnetic coupling, there are 66 topologically different Feynman diagrams including pion loops. These diagrams can be divided into three separately gauge invariant subsets (the first class was already discussed in ref.[14]). After mass and coupling constant renormalization, the corresponding invariant amplitudes $A_i^{(a)}$ ($i = 1, 2, 3, 4, 5, 6; a = +, 0, −$) contain further UV divergences which can be absorbed by two low–energy constants related to the isovector charge radius of the nucleon and the electromagnetic radius of the pion. In addition, there are six finite contact terms of order $q^3$. Four of the unknown coefficients were already determined in our study of threshold pion photoproduction [1] and the others can be fixed from the axial and the isoscalar charge radii of the nucleon.
(ii) We have discussed in some detail the low–energy theorems (LETs) for the electric dipole amplitude $E_{0+}^{(a)}$ and the longitudinal amplitude $L_{0+}^{(a)}$ ($a = +, 0, -$). For neutral pion electroproduction, virtual pions generate infrared singularities which modify the familiar form of the energy expansion [2,17]. In QCD, the expansion for $E_{0+}^{(a)}$ and $L_{0+}^{(a)}$ in powers of $\mu = M_\pi/m$ (the ratio of the pion to the nucleon mass) and $\nu = k^2/m^2$ (with $k^2$ the four–momentum of the virtual photon) is given in eq.(5.1). For $\gamma^* p \to \pi^0 p$ and $\gamma^* n \to \pi^0 n$, these agree with the result given in ref.[14]. For the (–) amplitudes which are probed in charged pion electroproduction, we have found a modification of the LET due to Nambu et al. [18]. This was already discussed in ref.[19] where it was shown that an additional pion loop effect modifies the relation between $dE_{0+}(k^2)/dk^2$ at $k^2 = 0$ and the nucleon axial radius $<r_A^2>^{1/2}$ (already in the chiral limit). In fact, this correction leads to a better agreement between the axial radius determined from (anti)neutrino scattering [13] and from pion electroproduction [31]. For the planned high precision charged electroproduction experiments it is mandatory to include this effect in the analysis of the data. We have also worked out the terms of order $\mu^3$ and $\mu^3 \ln \mu^3$ from the one–loop graphs for $L_\pi^0 p(\mu, 0)$ and shown that the expansion in $\mu$ converges slowly. This agrees with our findings for photoproduction reported in ref.[1].

(iii) We have confronted the chiral prediction with the recent very accurate NIKHEF data for $\gamma^* p \to \pi^0 p$ [3]. As already shown in ref.[30], one–loop effects are necessary to understand the $k^2$–dependence of the S–wave cross section for photon four–momenta $|k^2| \leq 0.1$ GeV$^2$. The theoretical prediction for $L_{0+}$ at the photon point, $|L_{0+}|^2 = 0.2 \mu b$ is in fair agreement with the result of ref.[3], $|L_{0+}|^2 = 0.13 \pm 0.05 \mu b$.

(iv) The calculated multipole amplitudes are presented. For future experimental tests of our predictions, extensive results are also given for the transverse ($T$), longitudinal ($L$), interference ($I$) and polarization ($P$) differential cross sections $d\sigma_{T,L,I,P}/d\Omega$ and are shown for all three channels, i.e. $\gamma^* p \to \pi^+ n$, $\gamma^* n \to \pi^- p$ and $\gamma^* p \to \pi^0 p$. To see the dynamical content of our CHPT calculations, these results are compared with the traditional calculations using pseudo-vector pion coupling with the nucleon. Our results strongly suggest that to test the dynamical content of the chiral perturbation theory prediction, it is mandatory to perform a transverse/longitudinal separation of the data. We have also shown that for photon four–momenta $|k^2| > 0.05$ GeV$^2$, the
loop corrections become large and thus it would be preferable to have experiments at lower photon four–momenta.

(v) For neutral electropionproduction of the proton, we have investigated the dominant effect of isospin breaking, namely the charged to neutral pion mass difference in the various loop functions. We have given a gauge invariant prescription to implement this effect. The resulting S–wave cross section is marginally different from the one presented in ref.[30]. Only at \( k^2 = 0 \), isospin breaking seems to be of importance (due to the dominance of the class I diagrams). We get an improved prediction for \( L_{0+} \) at the photon point, \( \left| L_{0+} \right|^2 = 0.17 \mu b \). At finite \( k^2 \), we could only find a substantial effect on \( d\sigma_P/d\Omega \) (at \( k^2 = -0.04 \text{ GeV}^2 \)). The largest uncertainty therefore resides in the knowledge of the low–energy constant \( d_1 \) (see ref.[1]).

The paper is organized as follows. In section 2 we discuss some formal aspects of pion electroproduction to fix our notation. The effective Lagrangian which will be used is given in section 3. We only exhibit the new terms related to the electroproduction reaction and refer the reader to I for more detailed discussions. Section 4 contains the invariant amplitudes for the reaction \( \gamma^* p \rightarrow \pi^0 p \). Pion electroproduction low energy theorems are derived and discussed in section 5. We also review critically previous attempts to formulate these low energy theorems. The numerical results are presented in section 6 together with some proposals for future experiments. Various technicalities are relegated to the appendices.

**II. PION ELECTROPRODUCTION: SOME FORMAL ASPECTS**

In this section, we define the current matrix elements for the \( \gamma^* N \rightarrow \pi N \) process in terms of invariant amplitudes which will then be subject to the chiral expansion in section IV. The formulas used in our calculations of the \( N(e,e'\pi)N \) cross sections from the current matrix elements can be found in Refs.[8,9] and are therefore omitted here. To see the dynamics at threshold, the multipole decomposition of the current matrix elements at threshold is presented explicitly in this section. The formulae needed for calculating the current matrix elements from the multipole amplitudes tabulated in tables 1a,b,c are given in Appendix C. This will allow the readers to calculate the CHPT predictions of \( (e,e'\pi) \) spin observables accessible to possible future experiments by using the well known
formulæ such as given in Ref.[9]. The reader familiar with these topics is invited to skip this section.

II.1. BASIC CONSIDERATIONS

Consider the process \( \gamma^*(k) + N(p_1) \rightarrow \pi^a(q) + N(p_2) \), with \( N \) denoting a nucleon (proton or neutron), \( \pi^a \) a pion with an isospin index \( a \) and \( \gamma^* \) the virtual photon with \( k^2 < 0 \). The case of photoproduction, \( k^2 = 0 \), was discussed in detail in I. The pertinent Mandelstam variables are \( s = (p_1 + k)^2 \), \( t = (p_1 - p_2)^2 \) and \( u = (p_1 - q)^2 \) subject to the constraint \( s + t + u = 2m^2 + M_\pi^2 + k^2 \). Here, \( m \) and \( M_\pi \) denote the nucleon and pion mass, in order. To first order in the electromagnetic coupling, \( e \), to which we will work, the transition current matrix element is given as

\[
J^\mu(p_2, s_2; q, a|p_1, s_1; k) = i \bar{u}_2 \gamma_5 \{ \gamma^\mu k B_1 + 2P^\mu B_2 + 2q^\mu B_3 + 2k^\mu B_4 + \gamma^\mu B_5 + P^\mu k B_6 + k^\mu k B_7 + q^\mu k B_8 \} u_1
\]  

(2.1)

with \( s_j \) the spin index of nucleon \( j \) and \( P = (p_1 + p_2)/2 \). The amplitudes \( B_i(s, u) \) have the conventional isospin decomposition (in the isospin symmetric limit),

\[
B_i(s, u) = B_i^{(+)}(s, u) \delta_{a3} + B_i^{(-)}(s, u) \frac{1}{2} [\tau_a, \tau_3] + B_i^{(0)}(s, u) \tau_a.
\]  

(2.2)

Not all of the eight \( B_i(s, u) \) are independent. Indeed, gauge invariance \( k_\mu J^\mu = 0 \) requires

\[
2k^2 [B_1(s, u) + 2B_4(s, u)] + (s - u) B_2(s, u) + 2(s + u - 2m^2) B_3(s, u) = 0
\]

\[
4 B_5(s, u) + (s - u) B_6(s, u) + 4k^2 B_7(s, u) + 2(s + u - 2m^2) B_8(s, u) = 0
\]  

(2.3)

so that one can express the transition current matrix element in terms of six independent invariant functions, conventionally denoted by \( A_i(s, u) \), \((i = 1, ..., 6)\),

\[
J^\mu = i \bar{u}_2 \gamma_5 \sum_{i=1}^{6} \mathcal{M}_i^\mu A_i(s, u) u_1
\]  

(2.4)

with

\[
\mathcal{M}_i^\mu = \frac{1}{2} (\gamma^\mu k^\gamma - k^\gamma \gamma^\mu), \quad \mathcal{M}_2^\mu = P^\mu (2q \cdot k - k^2) - P \cdot k (2q^\mu - k^\mu),
\]

\[
\mathcal{M}_3^\mu = \gamma^\mu q \cdot k - k^\mu q^\mu, \quad \mathcal{M}_4^\mu = 2\gamma^\mu P \cdot k - 2k^\mu P^\mu - m \gamma^\mu k^\gamma + m k^\gamma \gamma^\mu,
\]

\[
\mathcal{M}_5^\mu = k^\mu q \cdot k - q^\mu k^2, \quad \mathcal{M}_6^\mu = k^\mu k^\gamma - \gamma^\mu k^2
\]  

(2.5)
The $A_i(s, u)$ are related to the $B_i(s, u)$ via

\begin{align*}
A_1 &= B_1 - mB_6, & A_2 &= \frac{2B_2}{M_\pi^2 - t}, \\
A_3 &= -B_8, & A_4 &= -\frac{1}{2}B_6, & A_6 &= B_7, \\
A_5 &= \frac{2}{s + u - 2m^2} \left\{ B_1 + 2B_4 + \frac{(s - u)B_2}{2(t - M_\pi^2)} \right\} = \frac{1}{k^2} \left\{ \frac{s - u}{t - M_\pi^2} B_2 - 2B_3 \right\}.
\end{align*}

(2.6)

For $A_5$ we have given two equivalent forms to make clear that the possible zeros of $s + u - 2m^2$ in the physical region do not lead to a pole (infinity) of the current transition matrix element in the physical region. In what follows, we will use both sets of invariant functions (see section IV). Under $(s \leftrightarrow u)$ crossing the amplitudes $A_{1, 2, 4}^{(+, 0)}, A_{3, 5, 6}^{(-)}, B_{1, 2, 6}^{(+, 0)}, B_1^{(-)} + 2B_4^{(-)}, B_{3, 5, 7, 8}^{(-)}$ are even, while $A_{3, 5, 6}^{(+, 0)}, A_{1, 2, 4}^{(-)}, B_1^{(+, 0)} + 2B_4^{(+, 0)}, B_{3, 5, 7, 8}^{(+, 0)}, B_1^{(-)}$ are odd.

In terms of the isospin components, the physical channels under consideration are

\begin{align*}
J_\mu(\gamma^*p \to \pi^0 p) &= J_\mu^{(0)} + J_\mu^{(+)} \\
J_\mu(\gamma^*n \to \pi^0 n) &= J_\mu^{(+)} - J_\mu^{(0)} \\
J_\mu(\gamma^*p \to \pi^+ n) &= \sqrt{2}[J_\mu^{(0)} + J_\mu^{(-)}] \\
J_\mu(\gamma^*n \to \pi^- p) &= \sqrt{2}[J_\mu^{(0)} - J_\mu^{(-)}].
\end{align*}

(2.7)

Having constructed the current transition matrix element $J_\mu$ it is then straightforward to calculate observables. The pertinent kinematics and definitions are outlined in refs.[8,9] to which the reader is referred for details (see also Berends et al. [10]).

**II.2. MULTIPOLe DECOMPOSITION AT THRESHOLD**

For the discussion of the low energy theorems in section V, we have to spell out the corresponding multipole decomposition of the transition current matrix element at threshold. In the $\gamma^*N$ center of mass system at threshold i.e. $q_\mu = (M_\pi, 0, 0, 0)$ one can express the current matrix element in terms of the two $S$-wave multipole amplitudes, called $E_0^+$ and $L_0^+$,

\begin{equation}
\vec{J} = 4\pi i (1 + \mu) \chi_i^f \left\{ E_{0+}(\mu, \nu) \hat{\sigma} + [L_{0+}(\mu, \nu) - E_{0+}(\mu, \nu)] \hat{k} \cdot \hat{k} \right\} \chi_i
\end{equation}

(2.8)
with $\chi_{i,f}$ two component Pauli-spinors for the nucleon. For the later discussion we have introduced the dimensionless quantities

$$\mu = \frac{M_\pi}{m}, \quad \nu = \frac{k^2}{m^2}. \quad (2.9)$$

The multipole $E_{0+}$ characterizes the transverse and $L_{0+}$ the longitudinal coupling of the virtual photon to the nucleon spin. Alternatively to $L_{0+}$, we will also use the scalar multipole $S_{0+}$ defined via:

$$S_{0+}(s, k^2) = \frac{1}{|\vec{k}|} L_{0+}(s, k^2) \quad (2.10)$$

At threshold, we can express $E_{0+}$ and $L_{0+}$ through the invariant amplitudes $A_i(s, u)$ via (suppressing the isospin indices)

$$E_{0+} = \frac{m\sqrt{(2 + \mu)^2 - \nu}}{8\pi(1 + \mu)^{3/2}} \left\{ \mu A_1 + \mu m \frac{\mu(2 + \mu) + \nu}{2(1 + \mu)} A_3 + m \frac{\mu(\mu^2 - \nu)}{2(1 + \mu)} A_4 - \nu mA_6 \right\},$$

$$L_{0+} = E_{0+} + \frac{m\sqrt{(2 + \mu)^2 - \nu}}{16\pi(1 + \mu)^{5/2}} (\mu^2 - \nu) \left\{ -A_1 - B_2 + B_1 + 2B_4 - \mu mA_4 - m(2 + \mu)A_6 \right\} \quad (2.11)$$

with the $A_i(s, u)$ and $B_i(s, u)$ evaluated at threshold $s_{th} = m^2(1 + \mu)^2$ and $u_{th} = m^2(1 - \mu - \mu^2 + \mu\nu)/(1 + \mu)$. This completes the necessary formalism.

### III. EFFECTIVE LAGRANGIAN

In this section, we will briefly discuss the effective lagrangian on which our one-loop calculation in CHPT is based. We heavily borrow from our previous work on threshold pion photoproduction with $k^2 = 0$. Here, we will only discuss the terms which have to be added because of $k^2 < 0$ in the electroproduction case. For the details we refer the reader to I.

To systematically work out the consequences of the spontaneously broken chiral symmetry at low energies, one makes use of an effective lagrangian of the asymptotically observed fields, in our case the Goldstone bosons (pions) and the nucleons (i.e. proton and neutron). We work in flavor SU(2) and mostly in the isospin limit $m_u = m_d = \hat{m}$. The proton and neutron are collected in the isodoublet Dirac-spinor,

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix} \quad (3.1)$$
which transforms non-linearly under the chiral group $SU(2)_L \times SU(2)_R$,

$$\Psi(x) \rightarrow \mathcal{K}[g_L, g_R, U(x)]\Psi(x)$$

(3.2)

where $g_{L,R}$ are group elements of $SU(2)_{L,R}$ and the unitary unimodular matrix $U(x)$ embodies the pion fields. For our calculation, it is most convenient to work in the so-called $\sigma$-model gauge,

$$U(x) = [\sigma(x) + i\vec{\tau} \cdot \vec{\pi}(x)]/F, \quad \sigma^2 + \vec{\pi}^2 = F^2$$

(3.3)

with $F$ the pion decay constant in the chiral limit. To leading order, one calculates tree diagrams from the effective Lagrangian with the least number of derivatives and quark mass insertions. It is given by

$$\mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi \pi}$$

(3.4)

where the upper index indicates the so-called chiral power according to the counting rules spelled out in I. The explicit expressions for the lowest order pion-nucleon, $\mathcal{L}^{(1)}_{\pi N}$, as well as the pion lagrangian, $\mathcal{L}^{(2)}_{\pi \pi}$, are also given therein. At next-to-leading order, one has two types of contributions. These are the one pion loop diagrams which only involve the few parameters occurring at lowest order. However, there are also local contact terms which in general are necessary to renormalize the divergences of the one-loop graphs. To one loop order, i.e. chiral power $q^3$, their generic form is

$$\mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \Delta \mathcal{L}^{(0)}_{\pi N} + \Delta \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(4)}_{\pi \pi}.$$  

(3.5)

The terms $\Delta \mathcal{L}^{(0,1)}_{\pi N}$ relevant to renormalize the nucleon mass and pion-nucleon coupling constant are discussed by GSS [11] and in I. In the meson sector, $\mathcal{L}^{(4)}_{\pi \pi}$ renormalizes the pion decay constant $F_\pi$ and the pion mass and there is one term which will give extra contributions due to $k^2 < 0$. It enters the pion charge form factor [5],

$$\mathcal{L}^{(4)}_{\pi \pi} = \frac{i l_6}{2} \text{Tr}([u^\mu, u^\nu]f_{\mu\nu}^\pm)$$

(3.6)

with

$$f_{\mu\nu}^\pm = e(\partial_\mu A_\nu - \partial_\nu A_\mu)(uQu^\dagger \pm u^\dagger Qu),$$

$$u^\mu = iu^\dagger \nabla^\mu Uu^\dagger, \quad u = \sqrt{U}.$$  

(3.7)
Here, $Q = \text{diag}(1, 0)$ is the charge matrix and $A_\mu$ denotes the (external) photon field. The constant $l_6$ has a divergent piece (pole at $d=4$), its finite part can be fixed from the empirical value of the pion mean square charge radius,

$$< r^2 >_\pi = -\frac{1}{8 \pi^2 F_\pi^2} \left( \ln \frac{M_\pi}{\lambda} + \frac{1}{2} \right) - \frac{12}{F_\pi^2} l_6^\pi(\lambda)$$

(3.8)

where the following renormalization prescription was used to cancel the infinities from the loops

$$l_6 = -\frac{L}{6} + l_6^\pi(\lambda), \quad L = \frac{\lambda^{d-4}}{16 \pi^2} \left\{ \frac{1}{d-4} + \frac{1}{2} (\gamma_E - 1 - \ln 4\pi) \right\}$$

(3.9)

with $\lambda$ a scale introduced in dimensional regularization. Using the empirical value $< r^2 >_\pi = 0.439 \text{ fm}^2$ [12], we have $l_6^\pi(1 \text{ GeV}) = 6.6 \cdot 10^{-3}$. The other low-energy constants which occurred already in photoproduction are $l_3$ and $l_4$. These are related to the chiral corrections of the pion decay constant and pion mass at order $m$ and $m^2$, respectively. In the $\pi N$ sector, there are three new terms contributing to pion electroproduction at order $q^3$. These read

$$\mathcal{L}^{(3)}_{\pi N} = \frac{b_9}{F_\pi^2} \overline{\psi} \gamma^{\mu} D^{\nu} f^\mu_{\nu} \psi + \frac{\bar{b}_9}{F_\pi^2} \overline{\psi} \gamma^{\mu} \psi \text{Tr} (D^{\nu} f^\mu_{\nu})$$

$$+ \frac{g_A}{12} b_{13} \overline{\psi} \gamma^5 \gamma^{\mu} ([D^{\nu}, f^\mu_{\nu}] + \frac{i}{2} [u^{\nu}, f^\mu_{\nu}]) \psi.$$  

(3.10)

The first two terms in eq.(3.10) are related to the electric mean square charge radii of the proton and the neutron. It is well-known that these radii develop a logarithmic singularity in the chiral limit. The diagrams responsible for this behaviour are in fact divergent and the constants $b_9$, $\bar{b}_9$ absorb these infinities according to the renormalization prescription

$$b_9 = \frac{L}{6} (1 - g^2_A) + b_6^\pi(\lambda), \quad \bar{b}_9 = \frac{L}{12} (g^2_A - 1) + \bar{b}_6^\pi(\lambda).$$

(3.11)

Notice that the combination $b_9 + 2 \bar{b}_9$ is finite. The pertinent Dirac form factors of the proton and neutron to one loop order read,

$$F_1^p(k^2) = 1 + \frac{2k^2}{F_\pi^2} [b_6^\pi(\lambda) + \bar{b}_6^\pi(\lambda)] + \frac{k^2 (g^2_A - 1)}{96 \pi^2 F_\pi^2} \left( \ln \frac{M_\pi}{\lambda} + \frac{1}{2} \right) + \left( \frac{g^2_{\pi N}}{m^2} - \frac{1}{F_\pi^2} \right) \bar{J}^{\pi\pi}_{22}(k^2)$$

$$+ g^2_{\pi N} \left\{ -4 \bar{\Gamma}_3(k^2) - 16 m^2 \bar{\Gamma}_4(k^2) - 2 \bar{\Gamma}_3(k^2) + 4 m^2 \bar{\Gamma}_4(k^2) + k^2 [\bar{\Gamma}_4(k^2) - \bar{\Gamma}_5(k^2)] \right\};$$

$$F_1^n(k^2) = \frac{2k^2}{F_\pi^2} \bar{b}_9(\lambda) + \frac{k^2 (1 - g^2_A)}{96 \pi^2 F_\pi^2} \left( \ln \frac{M_\pi}{\lambda} + \frac{1}{2} \right) + \left( \frac{1}{F_\pi^2} - \frac{g^2_{\pi N}}{m^2} \right) \bar{J}^{\pi\pi}_{22}(k^2)$$

$$+ g^2_{\pi N} \left\{ 4 \bar{\Gamma}_3(k^2) + 16 m^2 \bar{\Gamma}_4(k^2) - 4 \bar{\Gamma}_3(k^2) + 8 m^2 \bar{\Gamma}_4(k^2) + 2 k^2 [\bar{\Gamma}_4(k^2) - \bar{\Gamma}_5(k^2)] \right\}.$$

(3.12)
where the loop functions $\bar{\gamma}_i(t)$ and $\bar{\Gamma}_i(t)$ are defined in appendix B. From the empirical charge mean square radii, $< r_E^2 >_p = 0.74 \text{ fm}^2$ and $< r_E^2 >_n = -0.12 \text{ fm}^2$ we can fix the coefficients at $\lambda = 1 \text{ GeV}$ as $b_{13}(1\text{GeV}) = 3.85 \times 10^{-3}$ and $\tilde{b}_{13}(1\text{GeV}) = 1.45 \times 10^{-3}$. The last term in eq.(3.10) is related to the slope of the axial form factor of the nucleon, $G_A(k^2)$. To one-loop order it can be written as

$$G_A(t) = g_A \left\{ 1 + \frac{t}{6} b_{13} + g_{\pi N}^2 \left[ -2 \bar{\Gamma}_3(t) - 4 m^2 \bar{\Gamma}_4(t) + t (\bar{\Gamma}_4(t) - \bar{\Gamma}_5(t)) \right] \right\} \quad (3.13)$$

The one loop contribution to the axial mean square radius is finite and very small, therefore the low-energy constant $b_{13}$ amounts for most of its value. Using the average empirical value from (anti)-neutrino scattering experiments, $< r_A^2 > = (0.416 \pm 0.02) \text{ fm}^2$ [13] we find $b_{13} = (10.05 \pm 0.62) \text{ GeV}^{-2}$. All other terms in $L_{\pi N}^{(2,3)}$ which are relevant and already appeared in the photoproduction process are discussed in I.

**IV. INVARIANT AMPLITUDES TO ONE-LOOP**

In this section, we will be concerned with the chiral expansion of the invariant amplitudes of pion electroproduction as introduced in section II. To keep this section short we will give only the expressions for the reaction $\gamma^* p \rightarrow \pi^0 p$ which was already discussed in the letter [14]. In appendix A, the amplitudes for $\gamma^* n \rightarrow \pi^0 n$ as well as the minus amplitudes $A_i^{(-)}(s,u), B_i^{(-)}(s,u)$ can be found. Together with the ones displayed here, these allow to calculate the charged electroproduction channels $\gamma^* p \rightarrow \pi^+ n$ and $\gamma^* n \rightarrow \pi^- p$ by using eqs.(2.7). To keep the notation compact, we will give $A_{1,3,4,6}(s,u), B_2(s,u)$ and $B_1(s,u) + 2B_4(s,u)$ or an independent subset of the $B_i(s,u)$, $(i \neq 3, 5)$. These are then converted into the $A_i(s,u)$ via eqs.(2.6).

**IV.1. GENERAL REMARKS**

We are seeking the chiral expansion of the invariant amplitudes $A_i(s,u)$, i.e. the expansion in small external momenta and quark masses,

$$A_i = A_i^{\text{tree}} + A_i^{1\text{-loop}} + A_i^{\text{ct}}. \quad (4.1)$$

Here, $A_i^{\text{tree}}$ is the contribution calculated from the lowest order effective lagrangian eq.(3.4). $A_i^{1\text{-loop}}$ stems from the one pion loop diagrams generated by the lowest order vertices.
and involves only the parameters $g_A, F$ and $m$ and finally $A_i^{ct}$ is calculated from tree diagrams with exactly one insertion from $\mathcal{L}_{\pi \pi}^{(4)}$ or $\mathcal{L}_{\pi N}^{(2,3)}$. They carry the information from the local counter terms whose coefficients are not fixed by chiral symmetry requirements.

Before giving the explicit formulae, it is instructive to first discuss the general structure of the $A_i(s, u)$ which emerges at one loop. As in the case of photoproduction there are 66 topologically inequivalent one-loop diagrams. These can be grouped into three separately gauge invariant classes, labelled class I, II and III, respectively. While class I scales like $g_A \sim g_{\pi N}$ (if one uses the Goldberger-Treiman relation), the diagrams in class II and III scale as $g_{\pi N}^3$, cf. figs. 2, 3, 4 in I, with $g_{\pi N}$ the strong pion nucleon coupling constant. In essence, any of the $A_i(s, u)$ has a contribution from electric and magnetic Born terms, the non-pole loop contributions from classes I, II and III plus the additional counter term contributions.

The electric Born term is the tree amplitude multiplied with the corresponding hadronic electromagnetic form factor consistently calculated to one-loop and it involves the physical values of the pion-nucleon coupling $g_{\pi N}$ and the nucleon mass $m$. For the channel $\gamma^* p \rightarrow \pi^0 p$, the relevant form factor is $F_p^1(k^2)$, the Dirac form factor of the proton. Similarly, the magnetic Born term involves the proton Pauli form factor $F_p^2(k^2)$ as generated by some graphs in class II and a counter term which allows to adjust the proton anomalous magnetic moment. Notice that the latter is not present at tree level but rather generated by loops and higher order counterterms. As in the photoproduction case many of the one loop diagrams simply lead to a mass and coupling constant renormalization (as discussed in I). Another important topic concerns the isovector nucleon electric form factor $F_1^v(k^2)$ and the pion electric form factor $F_\pi^v(k^2)$. These appear naturally in some of the amplitudes exhibited below. Since we are starting from a gauge invariant theory and because these form factors are build up by loop and counterterm contributions in a fashion that conserves the pertinent symmetries, there is no need of setting $F_1^v(k^2) = F_\pi^v(k^2)$ as it is done in many model calculations to preserve gauge invariance [2]. This feature is particularly clearly shown in the discussion of the low-energy theorems in section 5. After these general remarks, let us now discuss the various contributions to $A_i(s, u)$ for the process $\gamma^* p \rightarrow \pi^0 p$ at one loop order.

**IV.2. ELECTRIC AND MAGNETIC BORN TERMS**
The electric Born term modified by the proton Dirac form factor $F_1^p(k^2)$ (see eq.(3.12)) leads to the following amplitudes

$$A_1(s, u) = -B_2(s, u) = e g_{\pi N} \left( \frac{1}{s - m^2} + \frac{1}{u - m^2} \right) F_1^p(k^2),$$

$$B_1(s, u) + 2B_4(s, u) = A_{3,4,6}(s, u) = 0.$$  \hspace{1cm} (4.2)

It is important to notice that $F_1^p(k^2)$ is not the full physical form factor but rather its one loop expansion. This is a point frequently overlooked in the literature. We will come back to this point in section V in connection with the discussion of the low energy theorems. A brief discussion of this point has already been given in ref.[14]. Part of the magnetic Born term comes from the counter term which corrects the proton anomalous magnetic moment,

$$A_1(s, u) = \frac{e g_{\pi N}}{2m^2} \delta \kappa_p,$$

$$A_3(s, u) = -\frac{e g_{\pi N}}{2m} \delta \kappa_p \left( \frac{1}{s - m^2} - \frac{1}{u - m^2} \right),$$

$$A_4(s, u) = -\frac{e g_{\pi N}}{2m} \delta \kappa_p \left( \frac{1}{s - m^2} + \frac{1}{u - m^2} \right).$$

where

$$\delta \kappa_p = \kappa_p + 8g_{\pi N}^2 m^2 \{\Gamma_4(m^2, 0) - 2\gamma_4(m^2, 0)\} = 1.288$$ \hspace{1cm} (4.4)

is adjusted to give the empirical value of the proton anomalous magnetic moment, $\kappa_p = 1.793$. In contrast to what was done in I, we have not subtracted the loop contribution to $F_2^p(k^2)$ from class II and therefore only $\delta \kappa_p$ appears in eq.(4.3).

VI.3. CONTRIBUTIONS FROM CLASSES I, II, III AND COUNTER TERMS

For class I and II the following representation is most economic

$$A_1(s, u) = a_1(s, k^2) + a_1(u, k^2) - B_2(s, u),$$

$$B_2(s, u) = \frac{k^2}{s - m^2} \left[ b_{14}(m^2, k^2) - b_{14}(s, k^2) \right] + \frac{k^2}{u - m^2} \left[ b_{14}(m^2, k^2) - b_{14}(u, k^2) \right],$$

$$B_1(s, u) + 2B_4(s, u) = b_{14}(s, k^2) - b_{14}(u, k^2), \quad A_3(s, u) = a_3(s, k^2) - a_3(u, k^2),$$

$$A_4(s, u) = a_3(s, k^2) + a_3(u, k^2), \quad A_6(s, u) = a_6(s, k^2) - a_6(u, k^2).$$

(4.5)
Class I, which scales as $g_{\pi N}$, leads to the following contributions,

$$
\begin{align*}
    a_1(s, k^2) &= \frac{2e g_{\pi N}}{F^2_\pi} (s - m^2)[\gamma_6(s, k^2) - \gamma_4(s, k^2)], \\
    b_{14}(s, k^2) &= \frac{e g_{\pi N}}{F^2_\pi} \{(s + 3m^2)[2\gamma_6(s, k^2) - \gamma_2(s, k^2)] + (s - m^2)[\gamma_1(s, k^2) - 2\gamma_5(s, k^2)]\}, \\
    a_3(s, k^2) &= \frac{4e g_{\pi N}}{F^2_\pi} m\gamma_4(s, k^2), \\
    a_6(s, k^2) &= \frac{2e g_{\pi N}}{F^2_\pi} m[2\gamma_6(s, k^2) - \gamma_2(s, k^2)]. \\
\end{align*}
$$

The various loop functions $\gamma_i(s, k^2)$ are defined in appendix B.

The class II diagrams give the following expressions proportional to $g_{\pi N}^3$,

$$
\begin{align*}
    a_1(s, k^2) &= \frac{e g_{\pi N}^2}{4m^2} \{(s - m^2)[4\gamma_4(s, k^2) - 4\gamma_6(s, k^2) + \Gamma_1(s, k^2) - \Gamma_2(s, k^2) \\
    &\quad - 2\Gamma_4(s, k^2) + 2\Gamma_6(s, k^2)] + 8m^2[2\gamma_4(s, k^2) - \Gamma_4(s, k^2)]\}, \\
    b_{14}(s, k^2) &= \frac{e g_{\pi N}^2}{4m^2} \{(s + 3m^2)[4\gamma_5(s, k^2) - 2\gamma_1(s, k^2) + \Gamma_1(s, k^2) - \Gamma_2(s, k^2) - 2\Gamma_5(s, k^2)] \\
    &\quad + \frac{s^2 + 10sm^2 + 5m^4}{s - m^2}[2\gamma_2(s, k^2) - 4\gamma_6(s, k^2) + 2\Gamma_6(s, k^2)]\}, \\
    a_3(s, k^2) &= \frac{e g_{\pi N}^2}{2m} \{4\gamma_6(s, k^2) - 8\gamma_4(s, k^2) + 4\Gamma_4(s, k^2) - \Gamma_1(s, k^2) + \Gamma_2(s, k^2) \\
    &\quad - 2\Gamma_6(s, k^2) + \frac{8m^2}{s - m^2}[\Gamma_4(s, k^2) - 2\gamma_4(s, k^2)]\}, \\
    a_6(s, k^2) &= \frac{e g_{\pi N}^2}{2m} \{4\gamma_5(s, k^2) - 2\gamma_1(s, k^2) + \Gamma_1(s, k^2) - \Gamma_2(s, k^2) - 2\Gamma_5(s, k^2) \\
    &\quad + \frac{4(s + m^2)}{s - m^2}[\gamma_2(s, k^2) - 2\gamma_6(s, k^2) + \Gamma_6(s, k^2)]\}. \\
\end{align*}
$$

Notice that the one loop representation contains the magnetic (Pauli) form factor of the proton through the $k^2$ dependence of the loop functions. It is proportional to the residue of $a_3(s, k^2)$ at $s = m^2$. Furthermore, one observes that $a_6(s, k^2)$ and $b_{14}(s, k^2)$ do not have such poles since $2\gamma_6(m^2, k^2) - \gamma_2(m^2, k^2) = 0 = \Gamma_6(m^2, k^2)$. This can be easily seen from the Feynman representation eq.(B.2,3) by changing the integration variable $x \rightarrow 1 - x$. Finally, we have to give the amplitudes for class III, which are most tedious to work out.
A compact representation is obtained by giving the $B_i(s, u)$ $(i \neq 3, 5)$,

$$B_1(s, u) = eg_\pi^3 \left\{ \frac{\Gamma_0^N}{s - m^2} - \frac{2G_4(s, t, k^2)}{s - m^2} - \frac{2G_6(s, t, k^2)}{s - m^2} + (s - m^2)G_1(s, t, k^2) - 2G_4(s, t, k^2) - k^2G_5(s, t, k^2) + (u - s)G_6(s, t, k^2) + (t + k^2 - M^2)G_7(s, t, k^2) - tG_9(s, t, k^2) + (t - 4m^2)G_8(s, t, k^2) + 2\Omega_4(s, u, k^2) + \frac{s - m^2}{4m^2} \left[ 4\gamma_4(s, k^2) - 4\gamma_6(s, k^2) - \Gamma_1(s, k^2) + 4\gamma_5(s, k^2) + 2\Gamma_4(s, k^2) - 2\Gamma_6(s, k^2) \right] \frac{k^2}{4m^2} \left[ -2\gamma_1(s, k^2) + 2\gamma_2(s, k^2) + 4\gamma_5(s, k^2) - 4\gamma_6(s, k^2) - \Gamma_1(s, k^2) + \Gamma_2(s, k^2) + 2\Gamma_5(s, k^2) - 2\Gamma_6(s, k^2) \right] \right\} + (s \leftrightarrow u),$$

$$B_2(s, u) = eg_\pi^3 \left\{ -\frac{\Gamma_0^N(t)}{s - m^2} + \frac{2G_4(s, t, k^2)}{s - m^2} - \frac{2G_6(s, t, k^2)}{s - m^2} + (s - m^2)G_2(s, t, k^2) - G_3(s, t, k^2) + \Omega_2(s, u, k^2) - 2G_6(s, u, k^2) + 2G_4(s, t, k^2) + k^2G_5(s, t, k^2) + (2m^2 - s - u)G_6(s, t, k^2) + (M^2 - t - k^2)G_7(s, t, k^2) + tG_9(s, t, k^2) + (u - s - 4m^2 + M^2 - k^2)G_8(s, t, k^2) + 2(s - m^2 - M^2)G_10(s, t, k^2) + \Omega_{10}(s, u, k^2) - 2\Omega_4(s, u, k^2) + 2(M^2 - 3m^2 - s)\Omega_8(s, u, k^2) + 2\gamma_2(s, k^2) + \frac{k^2}{4m^2} \left[ 2\gamma_1(s, k^2) - 2\gamma_2(s, k^2) - 4\gamma_5(s, k^2) + 4\gamma_6(s, k^2) + \Gamma_1(s, k^2) - \Gamma_2(s, k^2) - 2\Gamma_5(s, k^2) + 2\Gamma_6(s, k^2) \right] \right\} + (s \leftrightarrow u),$$

$$B_1(s, u) + 2B_4(s, u) = eg_\pi^3 \left\{ (s - m^2) \left[ G_1(s, t, k^2) + G_2(s, t, k^2) - G_3(s, t, k^2) + 2G_5(s, t, k^2) + \Omega_1(s, u, k^2) - \Omega_3(s, u, k^2) - 2\Omega_5(s, u, k^2) \right] + 2(M^2 - 3m^2 - s)G_6(s, t, k^2) + G_{10}(s, t, k^2) + \Omega_6(s, u, k^2) - \Omega_{10}(s, u, k^2) + 2(2s - 2m^2 - M^2)G_7(s, t, k^2) + \Omega_7(s, u, k^2) + 2(m^2 + M^2 - s)G_9(s, t, k^2) + \Omega_9(s, u, k^2) + \gamma_0(s, k^2) - \frac{s + 3m^2}{2m^2} \gamma_1(s, k^2) + \frac{s - m^2}{4m^2} \left[ -\Gamma_1(s, k^2) + \Gamma_2(s, k^2) + 2\Gamma_5(s, k^2) - 2\Gamma_6(s, k^2) + 2\gamma_2(s, k^2) + 4\gamma_5(s, k^2) - 4\gamma_6(s, k^2) \right] \right\} - (s \leftrightarrow u),$$

$$B_6(s, u) = \frac{e}{m} g_\pi^3 \left\{ \Gamma_1(s, k^2) - \Gamma_2(s, k^2) + 2\Gamma_6(s, k^2) + 4\gamma_6(s, k^2) - 8m^2G_8(s, t, k^2) - 8m^2\Omega_8(s, u, k^2) \right\} + (s \leftrightarrow u),$$

14
\[ B_7(s, u) = \frac{e}{2m} g_{\pi N}^3 \{ \Gamma_1(s, k^2) - \Gamma_2(s, k^2) - 2\Gamma_5(s, k^2) + 2\gamma_1(s, k^2) \\
- 4\gamma_5(s, k^2) - 8m^2 G_6(s, t, k^2) + 8m^2 G_{10}(s, t, k^2) \\
- 8m^2 \Omega_6(s, u, k^2) + 8m^2 \Omega_{10}(s, u, k^2) \} - (s \leftrightarrow u), \]

\[ B_8(s, u) = \frac{e}{2m} g_{\pi N}^3 \{ \Gamma_1(s, k^2) - \Gamma_2(s, k^2) + 2\Gamma_6(s, k^2) + 4\gamma_6(s, k^2) \\
- 8m^2 G_{10}(s, t, k^2) - 8m^2 \Omega_{10}(s, u, k^2) \} - (s \leftrightarrow u). \] (4.8)

An excellent numerical check on these rather involved expressions is given by the gauge invariance relations eq.(2.3) (for that, one also calculates \( B_{3,5}(s, u) \)), with the exception of \( B_1(s, u) \) which is not constrained by the gauge invariance condition \( k_\mu J^\mu = 0 \). As a further check, all the above expressions of course match to the formulae given in I for the case \( k^2 = 0 \). Finally, for the channel \( \gamma^* p \to \pi^0 p \) there is one counter term contribution to \( A_4(s, u) = a_{\pi^0 p}^4 \) from \( \mathcal{L}_{\pi N}^{(3)} \). The same term was already present in the photoproduction calculation and its coefficient was estimated from resonance saturation. Equivalently, one can say that the determination of the constant \( a_{\pi^0 p}^4 \) in I amounted to an overall best fit to the total photoproduction cross section for \( \gamma p \to \pi^0 p \) in the threshold region based on the Mainz data [15] (with \( E_\gamma \leq 160 \text{ MeV} \)).

With the invariant amplitudes given here and in appendix A, we are now in the position of discussing the pertinent physics issues related to threshold pion electroproduction.

V. LOW ENERGY THEOREMS

V.1. GENERAL REMARKS

Pion electroproduction low energy theorems (LETs) have been discussed in many articles [16]. One of the most recent discussions is due to Scherer and Koch [17]. The underlying idea is to expand the threshold \( S \)-wave multipoles \( E_{0+} \) and \( L_{0+} \) in terms of the dimensionless and small parameters \( \mu = M_\pi/m \simeq 1/7 \) and \( \nu = k^2/m^2 \). Evidently, \( \nu \) can be made arbitrary small as \( k^2 \) approaches zero, the so-called photon point. One of the main differences to the photoproduction case was first pointed out by Nambu, Lurié and

* Here \( E_{0+} \) and \( L_{0+} \) stand as generic symbol for any isospin channel.
Shrauner (NLS) [18], namely that charged electroproduction involves the axial form factor of the nucleon $G_A(t)$. It therefore gives an other possibility of determining this fundamental quantity. However, the LET of NLS refers strictly to the chiral limit and one therefore has to add corrections for the physical case of a finite pion mass. Such corrections are discussed for example in the monograph [2]. As we will show in some detail later, CHPT allows to systematically calculate the next-to-leading order corrections to the LET of NLS, as already reported in ref.[19]. This is only one particular example of the chiral expansion done consistently. In fact, the one loop calculation performed here allows us to calculate all terms of the multipole amplitudes $E_{0+}(\mu, \nu)$ and $L_{0+}(\mu, \nu)$ up to and including order $O(\mu^2, \nu)$. The results of this calculation will be reported below.

First, however, some clarifying words about the meaning of LETs and their relation to the CHPT calculation are in order since in the present literature one finds many erroneous statements. In QCD, the quark masses are a priori free parameters. For the two flavor sector, one can set $m_u = m_d = 0$ to a good approximation. In this limit, the so-called chiral limit, the pions are exactly massless, as mandated by the Goldstone theorem. CHPT now allows to systematically work out the consequences of the spontaneous and the explicit chiral symmetry breaking in QCD, as an expansion in small external momenta and quark masses. For the case of pion electroproduction at threshold, these small expansion parameters are the pion mass $M_\pi^2 \sim \hat{m}$ and the photon four-momentum $k^2$. As it is well-known and most clearly spelled out by Weinberg [20], CHPT embodies the very general principles of gauge invariance, analyticity, crossing and PCAC (pion pole dominance). Therefore, to lowest order, one recovers the venerable current algebra LETs, which are based on these principles. The effective chiral lagrangian is simply a tool to calculate these but also all next-to-leading order corrections in a systematic and controlled fashion. The question of interest here is now: What are the corrections to the lowest order statements for $E_{0+}(\mu, \nu)$ and $L_{0+}(\mu, \nu)$ at threshold? This question can be unambiguosly answered by making use of the CHPT machinery. For that it is mandatory to consider pion loop diagrams. It was already shown in the photoproduction case that the conventional LET due to Vainshtein and Zakharov [21] and de Baenst [22] was incomplete and has to be modified at next-to-leading order, $O(\mu^2)$, due to a logarithmic singularity of certain loop diagrams in the chiral limit of vanishing pion mass. Such an effect can never be found without explicit calculation of all relevant diagrams. In the light of this result we also expect similar modifications to
show up in pion electroproduction. This expectation was already borne out by an explicit calculation of the class I diagrams for the reaction $\gamma^* p \rightarrow \pi^0 p$ [14]. In the following section, we will give the complete list of LETs for electropion production (in the isospin basis).

In view of the above arguments, it should be clear that simply calculating Born (tree) diagrams supplemented by form factors (in a gauge invariant fashion) can not give all corrections at next-to-leading order [17,23,24]. To be more specific, consider first charged pion electroproduction. The leading term in $E_{0+}(\mu, \nu)$ and $L_{0+}(\mu, \nu)$ are of order one, which is nothing but the generalization of the famous Kroll-Ruderman theorem [25]. In ref.[17] it was claimed that all corrections up to and including $O(\mu^2, \nu)$ can be calculated from PCAC and current conservation without considering loop diagrams. From the above remarks on a consistent expansion in CHPT, it should be obvious that this expectation is too naive. It is based on the incorrect assumption that while PCAC is a very general principle, CHPT is derived from a particular lagrangian and would be more restrictive. Quite contrary, any theory or model which claims to embody PCAC should lead to the same result as CHPT if one goes to the same level of sophistication. Similarly, the recent discussion of Ohta [24] is misleading. When calculating the LETs in QCD, one never has to worry about half off–shell nucleon form factors and thus the findings reported in refs.[14,19] are model–independent and do not rely on any assumptions about these form factors. We hope that with these remarks we have been able to shed some light on the seemingly controversial theoretical interpretation of the LETs. In fact, the meaning of the LETs is unique (experimental quantities are expanded in the chiral symmetry breaking parameter $M_\pi$ and small external momenta) and their accuracy at a given order can be tested experimentally. Let us now present the pertinent complete results at next-to-leading order.

V.2. FORMULATION AND DISCUSSION OF THE LOW ENERGY THEOREMS

The derivation of the LETs for $E_{0+}^{(+,0,-)}$ and $L_{0+}^{(+,0,-)}$ in electropion production is a straightforward generalization of the photoproduction case investigated in I. Here, we only wish to point out that for this calculation it is most convenient to make use of the heavy mass formulation of baryon CHPT [26]. It was shown in ref.[27] that a one loop calculation in this framework reproduces all relevant terms. We therefore skip the computational
Some of the kinematical prefactors in eqs.(5.1) have not been expanded to keep the notation compact.

\[
E_{0+}^{(0)}(\mu, \nu) = \frac{e g_{\pi N}}{32 \pi m} \{ -2 \mu + \mu^2 (3 + \kappa_v) - \nu (1 + \kappa_v) + \frac{\mu^2 m^2}{4 \pi F^2} \Xi_1(-\nu \mu^{-2}) \} + O(q^3),
\]
\[
L_{0+}^{(+)}(\mu, \nu) = E_{0+}^{(+)}(\mu, \nu) + \frac{e g_{\pi N}}{32 \pi m} (\mu^2 - \nu) \left\{ -\kappa_v + \frac{m^2}{4 \pi F^2} \Xi_2(-\nu \mu^{-2}) \right\} + O(q^3),
\]
\[
E_{0+}^{(0)}(\mu, \nu) = \frac{e g_{\pi N}}{32 \pi m} \{ -2 \mu + \mu^2 (3 + \kappa_s) - \nu (1 + \kappa_s) \} + O(q^3),
\]
\[
L_{0+}^{(0)}(\mu, \nu) = E_{0+}^{(0)}(\mu, \nu) + \frac{e g_{\pi N}}{32 \pi m} (\nu - \mu^2) \kappa_s + O(q^3),
\]
\[
E_{0+}^{(-)}(\mu, \nu) = \frac{e g_{\pi N}}{8 \pi m} \left\{ 1 - \mu + C \mu^2 + \nu \left( \frac{\kappa_v}{4} + \frac{1}{8} + \frac{m^2}{6} < r_A^2 > \right) \right\} + \frac{\mu^2 m^2}{8 \pi^2 F^2} \Xi_3(-\nu \mu^{-2}) + O(q^3),
\]
\[
L_{0+}^{(-)}(\mu, \nu) = E_{0+}^{(-)}(\mu, \nu) + \frac{e g_{\pi N}}{8 \pi m} (\mu^2 - \nu) \left\{ \frac{\kappa_v}{4} + \frac{m^2}{6} < r_A^2 > + \frac{\sqrt{(2 + \mu)^2 - \nu}}{2(1 + \mu)^{3/2}(\nu - \mu^2 - \mu^3)} \right\}
\]
\[
+ \left( \frac{1}{\nu - 2 \mu^2} - \frac{1}{\nu} \right) (F^N_{\pi}(m^2 \nu) - 1) + \frac{m^2}{8 \pi^2 F^2} \Xi_4(-\nu \mu^{-2}) \right\} + O(q^3).
\]

(5.1)

where the functions \( \Xi_j(-\nu/\mu^2) \), \( j = 1, 2, 3, 4 \) can not be further expanded since the argument \(-\nu/\mu^2\) counts as order one. They are given by

\[
\Xi_1(\rho) = \frac{\rho}{1 + \rho} + \frac{(2 + \rho)^2}{2(1 + \rho)^{3/2}} \arccos \left( \frac{-\rho}{2 + \rho} \right),
\]
\[
\Xi_2(\rho) = \frac{2 - \rho}{(1 + \rho)^2} - \frac{\rho^2 + 2 \rho + 4}{2(1 + \rho)^{5/2}} \arccos \left( \frac{-\rho}{2 + \rho} \right),
\]
\[
\Xi_3(\rho) = \sqrt{1 + \frac{4}{\rho}} \ln \left( \sqrt{1 + \frac{\rho}{4}} + \frac{\sqrt{\rho}}{2} \right) + 2 \int_0^1 dx \sqrt{(1 - x)}[1 + x(1 + \rho)] \arctan \frac{x}{\sqrt{(1 - x)[1 + x(1 + \rho)]}},
\]
\[
\Xi_4(\rho) = \int_0^1 dx \frac{x(1 - 2x)}{\sqrt{(1 - x)[1 + x(1 + \rho)]}} \arctan \frac{x}{\sqrt{(1 - x)[1 + x(1 + \rho)]}}.
\]

(5.2)

These functions are shown in fig.1 for \( 0 \leq \rho \leq 10 \). They exhibit a very smooth behaviour.

\( \kappa_v = \kappa_p - \kappa_n = 3.71 \) and \( \kappa_s = \kappa_p + \kappa_n = -0.12 \) are the isovector and isoscalar anomalous magnetic moment of the nucleon, respectively. We have to make various remarks about

\[\text{notation compact.}\]
the LETs exhibited in eqs.(5.1). First, adding the (+) and (0) multipoles, one recovers
the LETs for $\gamma^* p \rightarrow \pi^0 p$ derived in refs.[14,27]. One sees that class II and III do not at
all contribute at order $O(\mu^2, \nu)$. This feature is most easily understood in the heavy mass
formulation of CHPT, where simple selection rules determine those few diagrams which are
non-vanishing at threshold (see ref.[27]). Second, notice that the LETs do not contain the
full electromagnetic form factors of the nucleon. This is due to the fact that in the consistent
chiral power counting on which CHPT is based these form factor effects are already of order
$O(\mu \nu) = O(q^3)$ and therefore consistently have to be dropped. Furthermore, one sees only
the normalization of the magnetic form factors, i.e. the respective anomalous magnetic
moments. Of particular interest are the LETs for the $(-)$ amplitudes. In the chiral limit,
the LET for $E_{0+}^{(-)}$ agrees with the one of NLS [18] at order $O(\nu)$. The constant $C$ which
appears also in photoproduction is of order one and it is given by
\[ C = \frac{9}{8} - \frac{m^2}{8\pi^2 F_\pi^2} (1 + \ln \mu) + \frac{2m^3}{eg_{\pi N}} \left\{ 4m a_1^{(-)} + a_3^{(-)} \right\} \] (5.3)
where the coefficients $a_{1,3}^{(-)}$ have been estimated in ref.[1] using resonance saturation. For the
central values given there, $a_1^{(-)} = 1.4$ GeV$^{-4}$ and $a_3^{(-)} = -9.9$ GeV$^{-3}$, we find $C = 0.40$. In
ref.[19], the consequences of the LET for $E_{0+}^{(-)}$ were discussed. If one expands $\Xi_3(-\nu \mu^{-2})$
in powers of $k^2 = \nu m^2$ and picks up all terms proportional to $\nu$ (i.e. all terms which
contribute to the slope of $E_{0+}^{(-)}$ at $k^2 = 0$), one finds
\[ \frac{\partial E_{0+}^{(-)}}{\partial k^2} \bigg|_{k^2=0} = \frac{2\kappa}{8m^2} + \frac{1}{6} < r_A^2 > + \frac{1}{128 F_\pi^2} \left( 1 - \frac{12}{\pi^2} \right) \right) + O(\mu) \] (5.4)
where the first two terms on the right hand side of eq.(5.4) have first been given by NLS.
As discussed in ref.[19], the new term at order $k^2$ (not vanishing in the chiral limit) due
to loop effects has a numerical value of $-0.0456 \text{ fm}^2$ and this closes the gap between the experimental determination of the nucleon axial mean square radius $< r_A^2 >$ from neutrino
experiments, $< r_A^2 > = 0.42 \text{ fm}^2$, and pion electroproduction analysis, $< r_A^2 > = 0.37 \text{ fm}^2$ [31]. To get an idea about the higher orders in $k^2$, we show in fig.[2] the two functions
\[ \Phi_1(k^2) = 1 + \frac{k^2}{6} < r_A^2 >, \]
\[ \Phi_2(k^2) = 1 + \frac{k^2}{6} < r_A^2 > + \frac{M_\pi^2}{8\pi^2 F_\pi^2} \left\{ \Xi_3(-\frac{k^2}{M_\pi^2}) - \frac{\pi^2}{8} - \frac{1}{2} \right\} \] (5.5)
for $< r_A^2 >= 0.42 \text{ fm}^2$. For $|k^2| \leq 0.5 \text{ GeV}^2$, the corrections of order $k^4$ (and higher) from one loop are quite small. Most important for the analysis of new precise electroproduction experiments is that one takes into account the novel term at order $k^2$ appearing in eq.(5.4).

Let us now turn to the LET for $L_{0+}^{(-)} - E_{0+}^{(-)}$ in eq.(5.1). In contrast to the electroweak nucleon form factors, the full one loop expression of the pion charge form factor $F_\pi^v(k^2)$ enters. This can be traced back to the following fact. The one loop representation of $F_\pi^v(k^2)$ takes the form

$$F_\pi^v(k^2) - 1 = \frac{1}{F_\pi^2} \left\{ c_1 k^2 \ln M_\pi + c_2 \frac{k^4}{M_\pi^2} + c_3 \frac{k^6}{M_\pi^4} + \ldots \right\} \quad (5.6)$$

and all terms in the curly bracket are of the same order $O(q^2)$. This feature is similar to the functions $\Xi_j(-\nu\mu^{-2})$ whose expansion cannot be truncated at any finite order. To the order we are working, one can even identify the one loop pion charge form factor with the empirical one, since the differences are order $q^4$. To get an idea of the higher order corrections, we can use the two loop representation of ref.[28]. At $k^2 = -0.2 \text{ GeV}^2$, i.e. $\nu = -0.23$, one finds that the one loop and the two loop representation of $F_\pi^v(k^2)$ differ by 22%. Finally, let us stress again the two salient features of the LETs presented in eqs.(5.1). First, in some cases virtual pion loops generate contributions with chiral singularities which modify the form of the LETs based on an incomplete calculation of tree diagrams including electroweak form factors. These unfamiliar terms are given by the $\Xi$-functions. Second, in the consistent expansion only the first moment of the nucleon electromagnetic form factors and the first and second moment of the nucleon axial form factor survive. The pion charge form factor, however, enters with its full one loop expression. The LETs presented in eqs.(5.1) are the ones which follow from the spontaneous (and explicit) beaking of chiral symmetry in QCD.

It might be instructive to investigate how fast the convergence of the LETs is. In the photoproduction case it was shown that the $O(\mu^3)$ contributions of the one loop approximation to $E_{0+}^{\pi^0 p}(\mu)$ substantially reduce the $O(\mu^2)$ terms. Such a trend can also be seen here. To be more specific, consider $L_{0+}^{\pi^0 p}(\mu, 0)$ at the photon point. For that we have evaluated all the class I contributions up-to-and-including $O(\mu^3, \mu^3 \ln \mu)$. The result reads

$$L_{0+}^{\pi^0 p}(\mu, 0) = \frac{e g_{\pi N}}{8\pi m} \left\{ -\mu + \frac{3}{2} \mu^2 - \frac{15}{8} \mu^3 + \frac{m^2 \mu^2}{8\pi F_\pi^2} \left[ 1 + \frac{4}{\pi} \mu \ln \mu + \mu \left( \frac{1}{\pi} - \frac{3}{2} - \frac{\pi}{8} \right) \right] \right\} \quad (5.7)$$

$$= -0.0247 \cdot (1 - 0.797 + 0.377) \text{ GeV}^{-1} = -0.0143 \text{ GeV}^{-1}$$
where we have exhibited the resulting contributions at order $\mu, \mu^2$ and $\mu^3$, in order. One sees that the series actually converges slowly for $\mu \simeq 0.14$. Notice, however, that these results should be considered indicative since higher loop corrections will modify the coefficients of the $\mu^3$ and $\mu^3 \ln \mu$ terms. The result of the full one loop calculation will be given in section VI and compared to the recent experimental determination \[3\]. In the charged channels the corrections are increasing with $|k^2|$ due to form factor effects (see section 6). This is different from the photoproduction case, where the loop corrections to $E^\gamma p \rightarrow \pi^+ p(\mu)$ and $E^\gamma n \rightarrow \pi^- p(\mu)$ are small and move the chiral prediction closer to the empirical values.

V.3. INCLUSION OF SOME ISOSPIN BREAKING EFFECTS

As already stated, the LETs presented so far refer strictly to the isospin limit and do not account for isospin breaking either through quark mass differences ($\sim m_d - m_u$) or (higher order) electromagnetism ($\sim e^2$). At present, we are not able to implement in a consistent way all possible effects of isospin breaking. However, to get an idea about the size of such effects, we will present the results of a simplified approach. It is well-known that the mass of neutral and charged pion differ slightly,

$$\frac{M_{\pi^+} - M_{\pi^0}}{M_{\pi^+}} = 0.033$$ \hspace{1cm} (5.8)

and this difference is almost entirely of electromagnetic origin \[29\]. One therefore expects some effect to come from this mass difference of pions in the loop and the asymptotic state. Indeed, taking this mass difference into account in neutral pion photoproduction leads to a substantial correction and $E^\gamma p \rightarrow \pi^0 p(\mu) = -1.97 \cdot 10^{-3}/M_{\pi^+}$ in good agreement with the commonly accepted empirical value $E^\gamma p \rightarrow \pi^0 p = -(2.0 \pm 0.1) \cdot 10^{-3}/M_{\pi^+}$ \[7\].

A similar mechanism is also operative in the function $\Xi_{1,2}(\rho)$ which enter neutral pion electroproduction. Expanding in $\mu_0 = M_{\pi^0}/m$ and using $\rho_0 = -\nu/\mu_0^2$ we find

$$\hat{\Xi}_1(\rho_0) = \frac{4}{M_{\pi^0}} \left\{ -\sqrt{M_{\pi^+}^2 - M_{\pi^0}^2} + \int_0^1 dx \sqrt{M_{\pi^+}^2 - x^2 M_{\pi^0}^2 + k^2 x(x - 1)} \right\},$$

$$\hat{\Xi}_2(\rho_0) = 2 \int_0^1 dx \frac{x(1 - 2x)M_{\pi^0}}{\sqrt{M_{\pi^+}^2 - x^2 M_{\pi^0}^2 + k^2 x(x - 1)}}. \hspace{1cm} (5.9)$$

* In ref.\[7\], isospin breaking was only considered in the class I diagrams which give the dominant loop–effect at $k^2 = 0$.  

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In fig. [3], we show the functions $\tilde{\Xi}_{1,2}(\rho_0)$. Their $k^2$ dependence is similar to the isospin symmetric ones $\Xi_{1,2}(\rho)$, but the value at the photon point is substantially reduced. While when neglecting the $\pi^+ - \pi^0$ mass difference, $\Xi_1(0) = \pi \simeq 3.14$ and $\Xi_2(0) = 2 - \pi \simeq -1.14$, we have now $\tilde{\Xi}_1(0) = 2.28$ and $\tilde{\Xi}_2(0) = -0.74$, which amount to a sizable reduction of 27% and 35%, respectively. The main point to stress here is that in neutral pion electroproduction it is mandatory to go beyond $O(q^3)$ and to include at least in a simplified manner isospin breaking effects (like the charged and neutral pion mass difference). A systematic study of these effects is beyond the scope of this paper.

Differentiating between the neutral and charged pion masses in the loops amounts to a redefinition of some loop functions as detailed in appendix D. In essence, one has to disentangle the two thresholds, $s^0_{\text{thr}} = m + M_{\pi^0}$ and $s^+_{\text{thr}} = m + M_{\pi^+}$ (the neutron-proton mass difference is not accounted for). In particular, some of the low-energy constants which enter the proton charge radius and anomalous magnetic moment have to be readjusted. We find $b^0_9(1\text{GeV}) + \tilde{b}^0_9(1\text{GeV}) = 5.17 \cdot 10^{-3}$ as compared to $5.30 \cdot 10^{-3}$ in the isosymmetric case (see eq.(3.12)). Similarly, $\delta \kappa_p$ defined in eq.(4.4) changes from 1.288 to 1.291. That this prescription conserves gauge invariance can be checked after constructing the amplitudes $B_i(s, u)$ and using eqs.(2.3). We should also stress that in this case of pion electroproduction one has to include the pion mass difference in the diagrams of all classes (the procedure used in ref.[7] for photoproduction is, however, legitimate).

VI. RESULTS AND DISCUSSION

VI.1. ISOSYMMETRIC CASE

First, we consider the isosymmetric case, i.e. we do not differentiate between the proton and the neutron masses and also not between the neutral and charged pion masses. Throughout, we set $m_p = m_n = m = 938.27$ MeV, $F_\pi = 93.1$ MeV, $g_{\pi N}^2/4\pi = 14.28$ and $e^2/4\pi = 1/137.036$. When considering charged pion production we set $M_\pi = M_{\pi^\pm} = 139.57$ MeV and for neutral pion production $M_\pi = M_{\pi^0} = 134.97$ MeV. This is a consistent procedure within the one-loop approximation we are using.

In Fig.4 we compare the CHPT prediction with the recent NIKHEF data [3]. In the same figure, we also show the results from the tree diagrams (open circles) and from the pseudovector (PV) Born terms with form factors (open squares) calculated in Ref.[8]. Note
that all of the PV results presented here do not include vector meson–exchange and final state interaction terms introduced in [8]. We see that the one–loop contribution drastically reduces the calculated cross sections and brings the CHPT results within the experimental uncertainties. The results from the PV terms are also not too different from the data. To have a more rigorous test of CHPT more precise measurements at small \( k^2 \) are needed.

Let us now turn to analyzing the dynamical content of the CHPT calculations. We consider a kinematical situation with the final pion–nucleon invariant mass \( W = 1080/1074 \) MeV (for charged/neutral pion production) which is perhaps most realistic for future experiments. At this near threshold energy (\( W_{\text{thr}} = 1077.84/1073.24 \) MeV for charged/neutral production) the cross sections are dominated by the S–wave amplitudes \( E_{0+} \) and \( L_{0+} \) (\( S_{0+} \)). In Figs.5 and 6 we display the real parts of these two amplitudes for \( p(\gamma^*, \pi^+)n \), \( n(\gamma^*, \pi^-)p \) and \( p(\gamma^*, \pi^0)p \), in order. In all cases the tree and PV results are rather similar whereas the one–loop prediction is significantly different from these. This comes in part from the irreducible class I,II,III diagrams and to a large extent from the difference in the \( k^2 \) dependence of the pion form factor and the isovector nucleon charge form factor which is a natural ingredient in our approach.\(^*\) In Ref.[30] we had already shown that the S–wave cross section for neutral pion production off the proton

\[
a_0 = |E_{0+}|^2 - \epsilon \frac{k^2}{k_0^2} |L_{0+}|^2
\]

where \( \epsilon \) and \( k_0 = (s - m^2 + k^2)/2\sqrt{s} \) represent, respectively, a measure of the transverse linear polarization and the energy of the virtual photon in the \( \pi N \) rest frame, follows nicely the data of Welch \textit{et al.} [3] whereas PV and tree are at variance with the data with increasing \( |k^2| \). In this channel we find at the photon point \( (k^2 = 0) \) \( |L_{0+}|^2 = 0.2 \mu b \) in fair agreement with the result of [3], \( |L_{0+}|^2 = 0.13 \pm 0.05 \mu b \). Clearly, a precise measurement of the \( k^2 \) dependence is the most effective way to test the CHPT predictions. We should stress that one should consider \( |k^2| \leq 0.05 \text{ GeV}^2 \) since otherwise the loop corrections are too large. To further illustrate this point, we show in Fig.7 the transverse and longitudinal cross sections \( \sigma_T \) and \( \sigma_L \), respectively, for \( p(\gamma^*, \pi^+)n \), \( n(\gamma^*, \pi^-)p \) and \( p(\gamma^*, \pi^0)p \), in order. In all cases the one–loop predictions are rather different from the PV and tree results. We find an enhancement of \( \sigma_L \) and a depletion of \( \sigma_T \). Note, however, that the sums

\(^*\) The role of the various form factors has also been stressed by Olsson \textit{et al.} in ref.[31].
\[ \sigma = \sigma_T + \sigma_L \] are very similar for tree, PV and one–loop. To test the dynamical content of the chiral symmetry prediction, it is therefore mandatory to perform a transverse/longitudinal separation. To further exhibit these differences, we plot in Figs.8 to 11 the differential cross sections \( d\sigma_T/d\Omega, d\sigma_L/d\Omega, d\sigma_I/d\Omega \) and \( d\sigma_P/d\Omega \) for three values of \( k^2 = -0.001, -0.04 \) and \(-0.08 \) GeV\(^2\). The most striking difference appears in \( d\sigma_T/d\Omega \) for neutral pion production. While the CHPT prediction becomes forward peaked for \(|k^2| > 0.04 \) GeV\(^2\), the PV and tree results remain backward peaked until \(|k^2| \simeq 0.07 \) GeV\(^2\). For \(|k^2| \leq 0.05 \) GeV\(^2\) the shapes of \( d\sigma_I/d\Omega \) and \( d\sigma_P/d\Omega \) are not too different between the tree, loop and PV predictions in case of charged pion electroproduction. For neutral electropionproduction, the angular distribution \( d\sigma_P/d\Omega \) shows a significant difference between the tree, PV and one loop results even at low photon four–momenta. We will come back to this in section 6.2. It is worth to stress that as \(|k^2| \) increases, the contributions from the diagrams of classes II and III become more and more important. To illustrate this, consider the S–wave cross section (6.1) at \( k^2 = -0.06 \) GeV\(^2\). The reduction of the one loop result compared to the tree (or PV) prediction [30] is almost entirely due to the class II and III diagrams (with equal share). The class I diagrams contribute insignificantly to \( a_0 \) at this \( k^2 \). This is completely different from the photoproduction case discussed in refs.[1,7].

It is important to note that these results at \( W = 1074/1080 \) MeV shown in Figs.5 to 11 contain contributions from P–wave multipoles. In the threshold region these are small but they can nevertheless contribute significantly to the various cross sections through the interference with the S–wave amplitudes. In table 1a,b,c we give the real and imaginary parts for \( E_{0+}, S_{0+}, E_{1+}, S_{1+}, S_{1-}, M_{1+} \) and \( M_{1-} \) for \( k^2 = -0.001, -0.04 \) and \(-0.06 \) GeV\(^2\) for \( p(\gamma^*, \pi^+)n, n(\gamma^*, \pi^-)p \) and \( p(\gamma^*, \pi^0)p \), in order. The knowledge of these multipoles allows one to construct any observable one wishes to measure, in particular the many which appear in the case of polarized electrons. Some pertinent formulae are summarized in appendix C.

To explore the sensitivity of the charged pion production amplitudes to the axial form factor, we have performed a set of calculations at \( k^2 = -0.04 \) GeV\(^2\) varying the dipole mass between 0.96 and 1.16 GeV, corresponding to squared axial radii between 0.35 and 0.51 fm\(^2\) (remember that our central value is \( M_A = 1.06 \) GeV). From the LETs discussed before, eq.(5.1), one expects that a smaller axial radius leads to an increase in transverse strength and a decrease of the longitudinal strength. This expectation is borne out by the actual
calculations. In fig.12 we show the differential cross sections \(d\sigma /d\Omega\), \(d\sigma_L/d\Omega\), \(d\sigma_I/d\Omega\) and \(d\sigma_P/d\Omega\) for this range of axial radii for \(p(\gamma^{\star}, \pi^+)n\) (the results for \(n(\gamma^{\star}, \pi^-)p\) are similar and not shown). Particularly sensitive to \(M_A\) is the ratio \(\sigma_T/\sigma_L\) which decreases/increases by approximately 20 per cent for \(M_A\) lowered/enhanced by 0.1 GeV. In contrast, the sum \(\sigma_L+\sigma_T\) is almost independent of the axial radius which again points towards the importance of a transverse/longitudinal separation. A summary of these results is given in table 2.

Furthermore, we have investigated the influence of the counterterm \(d_1\) which already appears in the photoproduction reaction \(p(\gamma, \pi^0)p\). Reducing its strength by a factor 2, the transverse cross section is diminished since the electric dipole amplitude is smaller (at the photon point). For finite \(k^2\), however, this trend reverses and eventually leads to enhancement of \(\sigma_T\) (by 50 per cent at \(k^2 = -0.04\) GeV\(^2\)). The longitudinal cross section is essentially unaffected by this counter term. The charged electroproduction amplitudes are insensitive to this finite contact term. We should stress again that its central value could be considered as coming from a best fit of the precise total photoproduction cross section data [15]. We will come back to this when discussing the inclusion of some isospin–breaking effects. We did not vary the two counter terms appearing in the charged photoproduction channels since their influence on the pertinent observables is fairly small.

**VI.2. INCLUDING ISOSPIN–BREAKING**

As discussed in section 5.3, for the case of neutral pion electroproduction it is mandatory to consider the dominant isospin breaking effect which stems from the pion mass difference. In what follows, we consider \(\gamma^* p \rightarrow \pi^0 p\) using \(M_{\pi^+} = 139.57\) MeV and \(M_{\pi^0} = 134.97\) MeV for the pertinent loop functions (see appendix D). In all kinematical factors the neutral pion mass enters. Before presenting the results, it is worth to stress that the gauge invariance conditions (2.3) are fulfilled within machine accuracy.

In fig.13, the S–wave cross section \(a_0\) defined in eq.(6.1) is shown in comparison to the isosymmetric CHPT [30] and the PV result (for \(\epsilon = 0.58\)). Although the pion mass difference has a pronounced effect on \(E_{0+}(\mu, 0)\) and \(L_{0+}(\mu, 0)\) (e.g. \(|L_{0+}|^2 = 0.17 \mu b\)), at finite \(k^2\) the two CHPT curves do not differ significantly. The conclusion of ref.[30] that loop effects are needed to explain the trend of the NIKHEF data is not invalidated by the inclusion of isospin-breaking. This is due to the dominating effect of the class II and class III diagrams at finite \(k^2\).
To get a better handle on the possible isospin-breaking effects, we show in fig.14 the angular distributions $d\sigma_{L,T,P,I}/d\Omega$ at $k^2 = -0.04$ GeV$^2$. Only in the case of $d\sigma_P/d\Omega$ one finds a significant change from the isosymmetric case. The height in the peak differs by almost a factor of three. In all other cases, the differences are marginal. This can also be seen from the comparison of $\sigma_T$ and $\sigma_L$ shown in fig.15 for $-0.1 \leq k^2 \leq 0$ GeV$^2$. The largest sensitivity of these results stems from the low–energy constant $d_1$. Reducing its strength by a factor of two, one finds an enhancement of $d\sigma_T/d\Omega$ in forward direction (at $\theta = 0$, the enhancement factor is about 1.7) and of $d\sigma_P/d\Omega$ (the peak height is increased by about 1.4). The other two angular distributions remain almost unaltered. In the S–wave cross section $a_0$ this amounts to an increase of about 21 percent. We conclude that the largest uncertainty of our predictions stems from the knowledge of the low–energy constant $d_1$.

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APPENDIX A: INVARIANT AMPLITUDES OF PION ELECTROPRODUCTION

Here, we will complete the list of invariant amplitudes for pion electroproduction by giving those for the reaction \( \gamma^* n \rightarrow \pi^0 n \) as well as the (-) amplitudes.

**A.1. THE CHANNEL \( \gamma^* n \rightarrow \pi^0 n \)**

Electric and magnetic Born terms:

\[
A_1(s, u) = -B_2(s, u) = -e g_{\pi N} \left( \frac{1}{s-m^2} + \frac{1}{u-m^2} \right) F_1^n(k^2), \quad \text{(A.1)}
\]

\[
B_1(s, u) + 2B_4(s, u) = A_{3,4,6}(s, u) = 0.
\]

with the neutron Dirac form factor \( F_1^n(k^2) \) given in eq.(3.12).

\[
A_1(s, u) = -\frac{e g_{\pi N}}{2m^2} \delta \kappa_n, \quad A_{2,5,6}(s, u) = 0, \]

\[
A_3(s, u) = \frac{e g_{\pi N}}{2m} \delta \kappa_n \left( \frac{1}{s-m^2} - \frac{1}{u-m^2} \right), \quad \text{(A.2)}
\]

\[
A_4(s, u) = \frac{e g_{\pi N}}{2m} \delta \kappa_n \left( \frac{1}{s-m^2} + \frac{1}{u-m^2} \right).
\]

where

\[
\delta \kappa_n = \kappa_n + 16 g_{\pi N}^2 m^2 \{ \Gamma_4(m^2, 0) + \gamma_4(m^2, 0) \} = 1.821 \quad \text{(A.3)}
\]

is adjusted to reproduce the empirical value of the neutron anomalous magnetic moment, \( \kappa_n = -1.913 \).

For class I and II the following representation is again most economic

\[
A_1(s, u) = a_1(s, k^2) + a_1(u, k^2) - B_2(s, u),
\]

\[
B_2(s, u) = \frac{k^2}{s-m^2} [b_{14}(m^2, k^2) - b_{14}(s, k^2)] + \frac{k^2}{u-m^2} [b_{14}(m^2, k^2) - b_{14}(u, k^2)],
\]

\[
B_1(s, u) + 2B_4(s, u) = b_{14}(s, k^2) - b_{14}(u, k^2), \quad A_3(s, u) = a_3(s, k^2) - a_3(u, k^2),
\]

\[
A_4(s, u) = a_3(s, k^2) + a_3(u, k^2), \quad A_6(s, u) = a_6(s, k^2) - a_6(u, k^2). \quad \text{(A.4)}
\]

\[27\]
Class I:

\[ a_1(s, k^2) = \frac{eg\pi N}{F_\pi^2}(s - m^2)[2\gamma_6(s, k^2) - 2\gamma_4(s, k^2) + \Gamma_1(s, k^2) - \Gamma_2(s, k^2) - 2\Gamma_4(s, k^2) + 2\Gamma_6(s, k^2)], \]

\[ b_{14}(s, k^2) = \frac{eg\pi N}{F_\pi^2}\{(s + 3m^2)[2\gamma_6(s, k^2) - \gamma_2(s, k^2) + 2\Gamma_6(s, k^2)] \\
+ (s - m^2)[\gamma_1(s, k^2) - 2\gamma_5(s, k^2) + \Gamma_1(s, k^2) - \Gamma_2(s, k^2) - 2\Gamma_5(s, k^2)]\}, \]

\[ a_3(s, k^2) = \frac{4eg\pi N}{F_\pi^2}m[\gamma_4(s, k^2) + \Gamma_4(s, k^2)], \]

\[ a_6(s, k^2) = \frac{2eg\pi N}{F_\pi^2}m[2\gamma_6(s, k^2) - \gamma_2(s, k^2) + 2\Gamma_6(s, k^2)]. \]

\[(A.5)\]

Class II:

\[ a_1(s, k^2) = \frac{eg^3}{2m^2}\{(s - m^2)[2\gamma_4(s, k^2) - 2\gamma_6(s, k^2) - \Gamma_4(s, k^2)] + \Gamma_2(s, k^2) + 2\Gamma_4(s, k^2) - 2\Gamma_6(s, k^2)] \\
+ 2\Gamma_4(s, k^2) - 2\Gamma_6(s, k^2)] + 8m^2[\gamma_4(s, k^2) + \Gamma_4(s, k^2)]\}, \]

\[ b_{14}(s, k^2) = \frac{eg^3}{2m^2}\{(s + 3m^2)[2\gamma_5(s, k^2) - \gamma_1(s, k^2) - \Gamma_1(s, k^2) + \Gamma_2(s, k^2) + 2\Gamma_5(s, k^2)] \\
+ \gamma_2(s, k^2) - 2\gamma_6(s, k^2) - 2\Gamma_6(s, k^2)]\}, \]

\[ a_3(s, k^2) = \frac{eg^3}{m}\{2\gamma_6(s, k^2) - 4\gamma_4(s, k^2) - 4\Gamma_4(s, k^2) + \Gamma_1(s, k^2) - \Gamma_2(s, k^2) + 2\Gamma_6(s, k^2) - \frac{8m^2}{s - m^2}[\gamma_4(s, k^2) + \gamma_4(s, k^2)]\}, \]

\[ a_6(s, k^2) = \frac{eg^3}{m}\{2\gamma_5(s, k^2) - \gamma_1(s, k^2) - \Gamma_1(s, k^2) + \Gamma_2(s, k^2) + 2\Gamma_5(s, k^2) + 2(\frac{s + m^2}{s - m^2})[\gamma_2(s, k^2) - 2\gamma_6(s, k^2) - 2\Gamma_6(s, k^2)]\}. \]

\[(A.6)\]
Class III:

We give the $B_i(s, u)$ with $i \neq 3, 5$ which allow uniquely to construct the $A_i(s, u)$.

\[
B_1(s, u) = e g_{\pi N}^3 \left\{ 2(s - m^2)G_1(s, t, k^2) - 4G_4(s, t, k^2) - 2k^2G_5(s, t, k^2) \\
+ 2(u - s)G_6(s, t, k^2) + 2(t + k^2 - M_\pi^2)G_7(s, t, k^2) + 2(t - 4m^2)G_8(s, t, k^2) \\
- 2tG_9(s, t, k^2) + 2\Omega_4(s, u, k^2) + \frac{s - m^2}{2m^2} \left[ 2\gamma_4(s, k^2) - 2\gamma_6(s, k^2) - \Gamma_1(s, k^2) \right] \\
+ \Gamma_2(s, k^2) + 2\Gamma_4(s, k^2) - 2\Gamma_6(s, k^2) \right\} + (s \leftrightarrow u),
\]

\[
B_2(s, u) = e g_{\pi N}^3 \left\{ 2\Gamma_1(t) + (s - m^2) \left[ 2G_2(s, t, k^2) - 2G_3(s, t, k^2) + \Omega_2(s, u, k^2) \right] \\
- 2\Omega_6(s, u, k^2) \right\} + 4G_4(s, t, k^2) + 2k^2G_5(s, t, k^2) + 2(2m^2 - s - u)G_6(s, t, k^2) \\
+ 2(M_\pi^2 - t - k^2)G_7(s, t, k^2) + 2tG_9(s, t, k^2) + 2(u - s - 4m^2 + M_\pi^2 - k^2)G_8(s, t, k^2) \\
+ 2(s - m^2 - M_\pi^2) \left[ 2G_{10}(s, t, k^2) + \Omega_{10}(s, u, k^2) \right] + 2(M_\pi^2 - 3m^2 - s)\Omega_8(s, u, k^2) \\
- 2\Omega_4(s, u, k^2) + 2\gamma_2(s, k^2) + \frac{k^2}{2m^2} \left[ \gamma_1(s, k^2) - \gamma_2(s, k^2) - 2\gamma_5(s, k^2) \right] \\
+ \gamma_6(s, k^2) + \Gamma_1(s, k^2) - \Gamma_2(s, k^2) - 2\Gamma_5(s, k^2) + 2\Gamma_6(s, k^2) \right\} + (s \leftrightarrow u),
\]

\[
B_1(s, u) + 2B_4(s, u) = e g_{\pi N}^3 \left\{ (s - m^2) \left[ 2G_1(s, t, k^2) + 2G_2(s, t, k^2) - 2G_3(s, t, k^2) \right] \\
- 4G_5(s, t, k^2) + \Omega_1(s, u, k^2) - \Omega_3(s, u, k^2) - 2\Omega_5(s, u, k^2) + 2(M_\pi^2 - 3m^2 - s) \right\} \\
+ \left[ 2G_6(s, t, k^2) - 2G_{10}(s, t, k^2) + \Omega_6(s, u, k^2) - \Omega_{10}(s, u, k^2) \right] + 2(2s - 2m^2 - M_\pi^2) \\
+ \left[ 2G_7(s, t, k^2) + \Omega_7(s, u, k^2) \right] + 2(m^2 + M_\pi^2 - s) \left[ 2G_9(s, t, k^2) + \Omega_9(s, u, k^2) \right] \\
+ \frac{s + 3m^2}{2m^2} \gamma_1(s, k^2) + \frac{s - m^2}{2m^2} \left[ -\Gamma_1(s, k^2) + \Gamma_2(s, k^2) + 2\Gamma_5(s, k^2) \right] \\
- 2\Gamma_6(s, k^2) + \gamma_2(s, k^2) + 2\gamma_5(s, k^2) - 2\gamma_6(s, k^2) \right\} \right\} - (s \leftrightarrow u),
\]

\[
B_6(s, u) = \frac{2e}{m} g_{\pi N}^3 \left\{ \Gamma_1(s, k^2) - \Gamma_2(s, k^2) + 2\Gamma_6(s, k^2) + 2\gamma_6(s, k^2) \\
- 8m^2G_8(s, t, k^2) - 4m^2\Omega_8(s, u, k^2) \right\} + (s \leftrightarrow u),
\]
\[ B_7(s, u) = \frac{e}{m} g_{\pi N}^3 \{ \Gamma_1(s, k^2) - \Gamma_2(s, k^2) - 2\Gamma_5(s, k^2) + \gamma_1(s, k^2) - 2\gamma_5(s, k^2) \\
+ 4m^2[-2G_6(s, t, k^2) + 2G_{10}(s, t, k^2) - \Omega_6(s, u, k^2) + \Omega_{10}(s, u, k^2)] \} - (s \leftrightarrow u), \]

\[ B_8(s, u) = \frac{e}{m} g_{\pi N}^3 \{ \Gamma_1(s, k^2) - \Gamma_2(s, k^2) + 2\Gamma_6(s, k^2) + 2\gamma_6(s, k^2) \\
- 8m^2G_{10}(s, t, k^2) - 4m^2\Omega_{10}(s, u, k^2) \} - (s \leftrightarrow u). \]  

(A.7)

### A.2. THE (-) AMPLITUDES

**Electric and magnetic Born terms:**

\[ A_1^{(-)}(s, u) = -B_2^{(-)}(s, u) = \frac{e}{2} g_{\pi N} \left( \frac{1}{s - m^2} - \frac{1}{u - m^2} \right) F_1^v(k^2), \quad A_{3,4}^{(-)}(s, u) = 0, \]

\[ B_1^{(-)}(s, u) + 2B_4^{(-)}(s, u) = \frac{e g_{\pi N}}{t - M_\pi^2} F_\pi^v(k^2) + \frac{e g_{\pi N}}{k^2} [F_1^v(k^2) - F_\pi^v(k^2)], \]  

\[ A_6^{(-)}(s, u) = \frac{eg_{\pi N}}{2mk^2} [F_1^v(k^2) - 1] \]

Here, \( F_1^v(k^2) = F_1^p(k^2) - F_1^n(k^2) \) denotes the nucleon isovector Dirac form factor and

\[ F_\pi^v(k^2) = 1 + \frac{k^2}{6} < r^2 > _\pi - \frac{2}{F_\pi^2} \frac{\bar{m}_\pi}{2} \]  

\[ (A.9) \]

is the one loop representation of the pion charge form factor. It is important to note that the Born amplitude is more than just the tree amplitude (with form factors equal to unity) multiplied by the appropriate form factors generated by the loops. It is known that such a simple multiplication prescription violates gauge invariance, unless the (unphysical) constraint \( F_1^v(k^2) = F_\pi^v(k^2) \) is imposed (or one adds some additional seagull terms [2,16]).

Within the framework of CHPT the form factors generated by loops and counterterms are automatically included in a gauge invariant fashion.

\[ A_{1,2,5,6}^{(-)}(s, u) = 0, \]

\[ A_3^{(-)}(s, u) = -\frac{eg_{\pi N}}{4m} \delta \kappa_v \left( \frac{1}{s - m^2} + \frac{1}{u - m^2} \right), \]  

\[ A_4^{(-)}(s, u) = -\frac{eg_{\pi N}}{4m} \delta \kappa_v \left( \frac{1}{s - m^2} - \frac{1}{u - m^2} \right). \]  

\[ (A.10) \]

where \( \delta \kappa_v = \delta \kappa_p - \delta \kappa_n \) is adjusted to reproduce the empirical value of the nucleon isovector anomalous magnetic moment, \( \kappa_v = 3.71 \).
For class I and II the following representation of the (−) amplitudes is most economic,

\[
A_1^-(s, u) = a_1(s, k^2) - a_1(u, k^2) - B_2^-(s, u),
\]

\[
B_2^-(s, u) = -\frac{k^2}{s - m^2} b_{14}(s, k^2) + \frac{k^2}{u - m^2} b_{14}(u, k^2),
\]

\[
B_1^-(s, u) + 2B_4^-(s, u) = b_{14}(s, k^2) + b_{14}(u, k^2),
\]

\[
A_3^-(s, u) = a_3(s, k^2) + a_3(u, k^2),
\]

\[
a_4^-(s, u) = a_3(s, k^2) - a_3(u, k^2),
\]

\[
A_6^-(s, u) = a_6(s, k^2) + a_6(u, k^2).
\]

(A.11)

Class I:

\[
a_1(s, k^2) = \frac{e g_{\pi N}}{F_\pi^2} (s - m^2)[\gamma_6(s, k^2) - \gamma_4(s, k^2)],
\]

\[
b_{14}(s, k^2) = \frac{e g_{\pi N}}{2F_\pi} \{(s + 3m^2)[2\gamma_6(s, k^2) - \gamma_2(s, k^2)] + (s - m^2)[\gamma_1(s, k^2) - 2\gamma_5(s, k^2)]\},
\]

\[
a_3(s, k^2) = \frac{2e g_{\pi N}}{F_\pi^2} m\gamma_4(s, k^2),
\]

\[
a_6(s, k^2) = \frac{e g_{\pi N}}{F_\pi^2} m[2\gamma_6(s, k^2) - \gamma_2(s, k^2)].
\]

(A.12)

Class II:

\[
a_1(s, k^2) = \frac{e g_{\pi N}}{8m^2} \{(s - m^2)[8\gamma_4(s, k^2) - 8\gamma_6(s, k^2) - \Gamma_1(s, k^2) + \Gamma_2(s, k^2)
\]

\[+ 2\Gamma_4(s, k^2) - 2\Gamma_6(s, k^2)] + 8m^2[4\gamma_4(s, k^2) + \Gamma_4(s, k^2)]\},
\]

\[
b_{14}(s, k^2) = \frac{e g_{\pi N}}{8m^2} \{(s + 3m^2)[8\gamma_5(s, k^2) - 4\gamma_1(s, k^2) - \Gamma_1(s, k^2) + \Gamma_2(s, k^2) + 2\Gamma_5(s, k^2)]
\]

\[+ \frac{s^2 + 10sm^2 + 5m^4}{s - m^2}[4\gamma_2(s, k^2) - 8\gamma_6(s, k^2) - 2\Gamma_6(s, k^2)] + 4m^2[-8\gamma_5(k^2)
\]

\[+ 4\tilde{\gamma}_1(k^2) + \tilde{\Gamma}_1(k^2) - 2\tilde{\Gamma}_5(k^2) + 8m^2(-2\gamma_4(k^2) + 4\gamma_6(k^2) + \Gamma_6(k^2))\}],
\]

\[
a_3(s, k^2) = \frac{e g_{\pi N}}{4m} \{[8\gamma_6(s, k^2) - 16\gamma_4(s, k^2) - 4\Gamma_4(s, k^2) + \Gamma_1(s, k^2)
\]

\[- \Gamma_2(s, k^2) + 2\Gamma_6(s, k^2) - \frac{8m^2}{s - m^2}[\Gamma_4(s, k^2) + 4\gamma_4(s, k^2)]\},
\]

\[
a_6(s, k^2) = \frac{e g_{\pi N}}{4m} \{8\gamma_5(s, k^2) - 4\gamma_1(s, k^2) - \Gamma_1(s, k^2) + \Gamma_2(s, k^2) + 2\Gamma_5(s, k^2)
\]

\[+ \frac{4(s + m^2)}{s - m^2}[2\gamma_2(s, k^2) - 4\gamma_6(s, k^2) - \Gamma_6(s, k^2)] - 8\gamma_5(k^2) + 4\tilde{\gamma}_1(k^2)
\]

\[+ \tilde{\Gamma}_1(k^2) - 2\tilde{\Gamma}_5(k^2) + 8m^2[-2\gamma_4(k^2) + 4\gamma_6(k^2) + \Gamma_6(k^2)]\}.
\]

(A.13)
Here, the prime on the loop functions denotes the partial derivative with respect to $s$ evaluated at $s = m^2$. Note, that we have $b_{14}(m^2, k^2) = a_6(m^2, k^2) = 0$ such that $B^{(-)}_2(s, u)$ has no more pole at $s = m^2$ or $u = m^2$. The residue at these poles is proportional to the isovector Dirac form factor, already contained in the electric Born term. Similarly, $A^{(-)}_6(m^2, m^2) = 0$.

**Class III:**

\[
B^{(-)}_1(s, u) = \frac{e^3}{2g_{\pi N}} \left\{ \Gamma_0^N(s) - \frac{M_\pi^2}{s - m^2} \Gamma_0^N(s) - \Gamma_2^N(s) + (m^2 - s)G_1(s, t, k^2) + 2G_4(s, t, k^2) + k^2G_5(s, t, k^2) + (s - u)G_6(s, t, k^2) + (M_\pi^2 - k^2 - t)G_7(s, t, k^2) + tG_9(s, t, k^2) + (4m^2 - t)G_8(s, t, k^2) + \frac{s - m^2 + k^2}{4m^2} \left[ \Gamma_1(s, k^2) - \Gamma_2(s, k^2) + 2\Gamma_6(s, k^2) \right] \right. \\
+ \frac{m^2 - s}{2m^2} \left[ \Gamma_4(s, k^2) - \frac{k^2}{2m^2} \Gamma_5(s, k^2) \right] \left\} - (s \leftrightarrow u),
\]

\[
B^{(-)}_2(s, u) = \frac{e^3}{2g_{\pi N}} \left\{ -\Gamma_0^N(s) + \frac{M_\pi^2}{s - m^2} \Gamma_0^N(s) + \Gamma_2^N(s) + (m^2 - s) \left[ G_2(s, t, k^2) - G_3(s, t, k^2) \right] - 2G_4(s, t, k^2) - k^2G_5(s, t, k^2) + (s + u - 2m^2)G_6(s, t, k^2) + (t + k^2 - M_\pi^2)G_7(s, t, k^2) + (s - u + 4m^2 - M_\pi^2 + k^2)G_8(s, t, k^2) - tG_9(s, t, k^2) + 2(m^2 + M_\pi^2 - s)G_{10}(s, t, k^2) + \frac{k^2}{4m^2} \left[ -\Gamma_1(s, k^2) + \Gamma_2(s, k^2) + 2\Gamma_5(s, k^2) - 2\Gamma_6(s, k^2) \right] \right\} - (s \leftrightarrow u),
\]

\[
B^{(-)}_1(s, u) + 2B^{(-)}_4(s, u) = \frac{e^3}{2g_{\pi N}} \left\{ (s - m^2) \left[ -G_1(s, t, k^2) - G_2(s, t, k^2) + G_3(s, t, k^2) + 2G_5(s, t, k^2) \right] + 2(s - M_\pi^2 + 3m^2) \left[ G_6(s, t, k^2) - G_{10}(s, t, k^2) \right] + 2(2m^2 + M_\pi^2 - 2s)G_7(s, t, k^2) + 2(s - m^2 - M_\pi^2)G_9(s, t, k^2) + \frac{s - m^2}{4m^2} \left[ \Gamma_1(s, k^2) - \Gamma_2(s, k^2) - 2\Gamma_5(s, k^2) + 2\Gamma_6(s, k^2) \right] - \tilde{\Gamma}_1(t) + \frac{1}{t - M_\pi^2} \left[ (t - 4m^2)\tilde{\Gamma}_1(t) + (4m^2 - M_\pi^2)\tilde{\Gamma}_1(M_\pi^2) \right] \right\} - (s \leftrightarrow u),
\]

\[
B^{(-)}_6(s, u) = \frac{e^3}{2m g_{\pi N}} \left\{ -\Gamma_1(s, k^2) + \Gamma_2(s, k^2) - 2\Gamma_6(s, k^2) + 8m^2G_8(s, t, k^2) \right\} - (s \leftrightarrow u),
\]

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\[
B_7^{(-)}(s, u) = \frac{e}{4m}g_{\pi N}^3 \left\{ -\Gamma_1(s, k^2) + \Gamma_2(s, k^2) + 2\Gamma_5(s, k^2) + 8m^2 G_6(s, t, k^2) - 8m^2 G_{10}(s, t, k^2) \right\} + (s \leftrightarrow u),
\]

\[
B_8^{(-)}(s, u) = \frac{e}{4m}g_{\pi N}^3 \left\{ -\Gamma_1(s, k^2) + \Gamma_2(s, k^2) - 2\Gamma_6(s, k^2) + 8m^2 G_{10}(s, t, k^2) \right\} + (s \leftrightarrow u).
\]

It is important to note that in the chiral limit \( M_\pi = 0 \) one has the relation

\[
B_7^{(-)}(s = u = m^2, t = k^2) = e g_{\pi N} \frac{\sqrt{-32 \pi}^2}{2m} \left[ 1 - \frac{1}{g_A} G_A(t) \right] \tag{A.14}
\]

with \( G_A(t) \) the one loop expression for the nucleon axial form factor. This relation contains the basic current algebra statement, that the nucleon axial form factor enters the pion electroproduction amplitudes. Since in general current algebra statements are exact only in the chiral limit we can not expect eq.(A.15) to hold also for finite pion mass. In that case corrections beyond current algebra show up which are not constrained a priori.

**APPENDIX B: LOOP FUNCTIONS AND THEIR IMAGINARY PARTS**

Here, we will heavily borrow from appendix B of ref.[1] where all the loop function occurring in photoproduction have been defined and given in terms of their Feynman parameter representations. We will write down here only the relevant extensions to \( k^2 \neq 0 \). The one loop expressions of the form factors involve the loop function

\[
\frac{\pi \pi}{22}(k^2) = -\frac{1}{32 \pi^2} \left\{ \frac{k^2}{6} + \int_0^1 dx [M_\pi^2 + k^2 x(x - 1)] \ln \left[ 1 + \frac{k^2}{M_\pi^2} x(x - 1) \right] \right\} \tag{B.1}
\]

Furthermore, we have in our electroproduction amplitudes

\[
\gamma_i(s, k^2) = \frac{1}{16 \pi^2} \int_0^1 dx \int_0^1 dy \frac{(1 - y)p_i(x, y)}{h_\gamma(x, y; s, k^2)} \quad (i \neq 3), \tag{B.2}
\]

with \( h_\gamma(x, y; s, k^2) = M_\pi^2(1 - y) + m^2 y^2 + (s - m^2) x y (y - 1) + k^2 (1 - y)^2 x (x - 1) \) and the \( p_i(x, y) \) are the same as in ref.[1]. An other set of new functions is:

\[
\Gamma_i(s, k^2) = \frac{1}{16 \pi^2} \int_0^1 dx \int_0^1 dy \frac{y q_i(x, y)}{h_\Gamma(x, y; s, k^2)} \quad (i \neq 3), \tag{B.3}
\]
with \( h_\Gamma(x, y; s, k^2) = M^2_\pi(1 - y) + m^2 y^2 + (s - m^2)xy(y - 1) + k^2 y^2 x(x - 1) \) and the \( q_i(x, y) \) are the same as in ref.[1]. For the nucleon form factors their values at \( s = m^2 \) occur which we denote by a tilde,

\[
\tilde{\gamma}_i(k^2) = \gamma_i(m^2, k^2), \quad \tilde{\Gamma}_i(k^2) = \Gamma_i(m^2, k^2), \quad (i \neq 3),
\]

\[
\tilde{\gamma}_3(k^2) = \frac{1}{32\pi^2} \int_0^1 dx \int_0^1 dy (1 - y) \ln \frac{h_\gamma(x, y; m^2, k^2)}{h_\gamma(x, y; m^2, 0)},
\]

\[
\tilde{\Gamma}_3(k^2) = \frac{1}{32\pi^2} \int_0^1 dx \int_0^1 dy y \ln \frac{h_\Gamma(x, y; m^2, k^2)}{h_\Gamma(x, y; m^2, 0)},
\]

and an overbar means subtraction at \( k^2 = 0 \).

The box graphs give rise to the \( G \)-functions,

\[
G_i(s, t, k^2) = \frac{1}{16\pi^2} \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{(1 - x)y^2 b_i(x, y, z)}{h_G^2(x, y, z; s, t, k^2)}, \quad (i \neq 4),
\]

\[
G_4(s, t, k^2) = \frac{1}{32\pi^2} \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{(x - 1)y^2}{h_G(x, y, z; s, t, k^2)},
\]

with

\[
h_G(x, y, z; s, t, k^2) = M^2_\pi[1 - y + x(x - 1)y^2z] + m^2 y^2 + (s - m^2)xy(y - 1) + t(1 - x)^2y^2z(z - 1) + k^2x(x - 1)y^2(1 - z)
\]

and the \( \Omega \)-functions,

\[
\Omega_i(s, u, k^2) = \frac{1}{16\pi^2} \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{(1 - y)x r_i(x, y, z)}{h_\Omega^2(x, y, z; s, u, k^2)}, \quad (i \neq 4),
\]

\[
\Omega_4(s, u, k^2) = \frac{1}{32\pi^2} \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{(y - 1)x}{h_\Omega(x, y, z; s, u, k^2)},
\]

with

\[
h_\Omega(x, y, z; s, u, k^2) = M^2_\pi[1 - y + x(x - 1)y^2] + m^2 y^2 + (s - m^2)(1 - x)y(y - 1)z + (u - m^2)xy(1 - y)(z - 1) + k^2(1 - y)^2z(z - 1).
\]

The polynomials \( b_i(x, y, z) \) and \( r_i(x, y, z) \) are the same as in ref.[1].

Finally, we give the expressions for the imaginary parts of the new \( k^2 \)-dependent scalar loop functions (those with \( i = 0 \)). The imaginary part of the vector and tensor functions are obtained in the standard way. One has to write down appropriate linear relations among these functions. The solution of a subset of these linear equations with maximal rank

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gives rise to explicit formulae for the imaginary parts of the vector and tensor functions expressed through those of the scalar ones.

The quadratic polynomial \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \) is called the Källén or triangle function. Let us abbreviate \( \lambda_\pi = \lambda(s, m^2, M_\pi^2) \) and \( \lambda_k = \lambda(s, m^2, k^2) \). Using the Cutkosky cutting rules we obtain the following imaginary parts:

\[
\text{Im}\gamma_0(s, k^2) = \frac{1}{16\pi\sqrt{\lambda_\pi}} \ln \frac{(s - m^2 + M_\pi^2)(s - m^2 + k^2) - 2sk^2 + \sqrt{\lambda_\pi\lambda_k}}{(s - m^2 + M_\pi^2)(s - m^2 + k^2) - 2sk^2 - \sqrt{\lambda_\pi\lambda_k}}
\]

\[
\text{Im}\Gamma_0(s, k^2) = \frac{1}{16\pi\sqrt{\lambda_\pi}} \ln \frac{(s + m^2 - M_\pi^2)(s - m^2 + k^2) - 2sk^2 + \sqrt{\lambda_\pi\lambda_k}}{(s + m^2 - M_\pi^2)(s - m^2 + k^2) - 2sk^2 - \sqrt{\lambda_\pi\lambda_k}}
\]

\[
\text{Im}G_0(s, t, k^2) = \frac{\sqrt{\lambda_\pi}}{32\pi\sqrt{B_G^2 - A_GC_G}} \ln \frac{A_G + B_G + \sqrt{B_G^2 - A_GC_G}}{A_G + B_G - \sqrt{B_G^2 - A_GC_G}}
\]

\[
A_G(s, t, k^2) = m^2s^2 - s(2m^4 + k^2M_\pi^2) + m^6 + k^2M_\pi^2(k^2 + M_\pi^2 - 3m^2),
\]

\[
B_G(s, t, k^2) = -\frac{t}{2}\lambda_\pi + \frac{1}{2}M_\pi^2(M_\pi^2 - k^2)(M_\pi^2 + 2k^2 - 3m^2 - s),
\]

\[
C_G(s, t, k^2) = t\lambda_\pi + M_\pi^2(M_\pi^2 - k^2)^2,
\]

\[
\text{Im}\Omega_0(s, u, k^2) = \frac{\sqrt{\lambda_\pi}}{32\pi\sqrt{B_\Omega^2 - A_\Omega C_\Omega}} \ln \frac{A_\Omega + B_\Omega + \sqrt{B_\Omega^2 - A_\Omega C_\Omega}}{A_\Omega + B_\Omega - \sqrt{B_\Omega^2 - A_\Omega C_\Omega}}
\]

\[
A_\Omega(s, u, k^2) = M_\pi^2(s - m^2)^2 + k^2M_\pi^2(M_\pi^2 - 3m^2 - s) + k^4m^2,
\]

\[
B_\Omega(s, u, k^2) = -\frac{u}{2}\lambda_\pi + \frac{s^2}{2}(m^2 - M_\pi^2) + s(-m^4 + 2m^2M_\pi^2 - \frac{3}{2}M_\pi^4 - m^2k^2 + \frac{3}{2}M_\pi^2k^2)
\]

\[
+ \frac{1}{2}m^6 - \frac{7}{2}m^4M_\pi^2 + 2m^2M_\pi^4 + k^2(m^4 + \frac{5}{2}m^2M_\pi^2 - \frac{3}{2}M_\pi^4) - k^4m^2,
\]

\[
C_\Omega(s, u, k^2) = u\lambda_\pi + 2s(M_\pi^2 - m^2)(M_\pi^2 - k^2) + (2m^2 - M_\pi^2 - k^2)(3m^2M_\pi^2 - 2M_\pi^4 - k^2m^2).
\]

The formulae given above hold for \( k^2 \leq 0, \ t \leq 0, \ u \leq (m - M_\pi)^2 \) and \( s \geq (m + M_\pi)^2 \). The physical region is completely contained within this domain of the Mandelstam plane.

**APPENDIX C: CALCULATION OF CURRENT MATRIX ELEMENTS FROM MULTIPOLAR AMPLITUDES**

To calculate polarization observables using the formulas of Ref.9, we need to construct current matrix elements in the center-of-mass frame of the final \( \pi N \) system. The photon
momentum defines the z-axis and hence we have \( k^\mu = (k_0, 0, 0, k) \). The amplitude of the \( \gamma^* N \to \pi N \) process can then be written as

\[
<m_{s'} | \epsilon^\mu(\lambda) J_\mu | m_s> = \sum_{i=1,6} <m_{s'} | O_i | m_s> f_i(k^2, W, x) \tag{C.1}
\]

with

\[
O_1 = i\sigma \cdot \bar{\epsilon}_\lambda \\
O_2 = \sigma \cdot \hat{q} \epsilon \cdot (\hat{k} \times \bar{\epsilon}_\lambda) \\
O_3 = i\sigma \cdot \hat{k} \epsilon \cdot \bar{\epsilon}_\lambda \\
O_4 = i\sigma \cdot \hat{q} \epsilon \cdot \bar{\epsilon}_\lambda \\
O_5 = i\sigma \cdot \hat{k} \epsilon \cdot \bar{\epsilon}_\lambda \\
O_6 = i\sigma \cdot \hat{k} \epsilon \cdot \bar{\epsilon}_\lambda \tag{C.2}
\]

Here we have defined \( \hat{q} \) as the pion momentum in \( \pi N \) center-of-mass frame, \( \hat{q} = \hat{q}/|\hat{q}| \) and \( \hat{k} = \hat{k}/|\hat{k}| \). The spherical unit vectors are defined as \( \bar{\epsilon}_{\pm1} = \mp1/\sqrt{2}(\hat{x} \pm i\hat{y}) \) and \( \bar{\epsilon}_0 = \hat{z} \). The photon polarization vectors for \( \lambda = \pm1,0 \) in Eq.(C.1) are defined as

\[
e^\mu(\pm1) = (0, \bar{\epsilon}_{\pm1}) \\
e^\mu(0) = \frac{1}{\sqrt{-k^2}}(|\vec{k}|, k_0 \bar{\epsilon}_0) \tag{C.3}
\]

where \( k^2 = k_0^2 - \vec{k}^2 < 0 \). The coefficients \( f_i(k^2, W, x) \) are only functions of \( \pi N \) invariant mass \( W \), four-momentum transfer square \( k^2 \) and \( x = \hat{k} \cdot \hat{q} \). In terms of the multipole amplitudes, we have (all multipoles are functions of \( k^2 \) and \( W \))

\[
f_1(k^2, W, x) = \sum_l (E_{l+} P'_{l+1}(x) + E_{l-} P'_{l-1}(x) + M_{l+} P'_{l+1}(x) + (l+1)M_{l-} P'_{l-1}(x)) \\
f_2(k^2, W, x) = \sum_l ((l+1)M_{l+} P'_l(x) + lM_{l-} P'_l(x)) \\
f_3(k^2, W, x) = \sum_l (E_{l+} P''_{l+1}(x) + E_{l-} P''_{l-1}(x) - M_{l+} P''_{l+1}(x) - M_{l-} P''_{l-1}(x)) \tag{C.4} \\
f_4(k^2, W, x) = \sum_l (-E_{l+} P''_{l+1}(x) - E_{l-} P''_{l-1}(x) + M_{l+} P''_{l+1}(x) - M_{l-} P''_{l-1}(x)) \\
f_5(k^2, W, x) = \sum_l ((l+1)S_{l+} P'_l(x) + lS_{l-} P'_l(x)) \\
f_6(k^2, W, x) = \sum_l ((l+1)S_{l+} P'_{l+1}(x) - lS_{l-} P'_{l-1}(x))
\]
By using Eq.(C.3), it is easy to see that

\[
< m_s' | J_x | m_s > = -1/\sqrt{2} ( < m_s' | e^{\mu}(+1)J_{\mu} | m_s > - < m_s' | e^{\mu}(-1)J_{\mu} | m_s > )
\]

\[
< m_s' | J_y | m_s > = i/\sqrt{2} ( < m_s' | e^{\mu}(+1)J_{\mu} | m_s > + < m_s' | e^{\mu}(-1)J_{\mu} | m_s > )
\]

(C.5a)

By using the current conservation condition \( k_0 J_0 = \vec{k} \cdot \vec{J} = k J_z \), Eq.(C.1) for \( \lambda = 0 \) leads to

\[
< m_s' | J_z | m_s > = \frac{\sqrt{1-k^2}}{k_0} < m_s' | e^{\mu}(0)J_{\mu} | m_s >
\]

(C.5b)

The matrix elements defined in Eq.(C.5) are needed to calculated various polarization observables defined in Ref.9.

**APPENDIX D: MODIFICATION OF INVARIANT AMPLITUDES IN THE CASE OF UNEQUAL CHARGED AND NEUTRAL PION MASSES.**

In ref.[7] is was observed that isospin breaking effects due to the difference of the neutral and charged pion masses turn out to be quite sizeable in the case of neutral pion photoproduction close to threshold. From this experience one expects that this isospin breaking effect will also be relevant for \( \pi^0 \) electroproduction from protons. Therefore we will display here the relevant modifications of the invariant amplitudes \( A_i(s,u) \) for the reaction \( \gamma^*p \rightarrow \pi^0p \) which arise if we do not set equal the neutral and charged pion mass as it was done in the chapter IV. Notice that in contrast to the photoproduction case, one has to consider diagrams from all three gauge invariant classes.

**Class I and II:**

The expressions given in eqs.(4.5,4.6,4.7) hold still for the invariant amplitudes. There is only some change in the loop functions \( \gamma_i(s,k^2) \) and \( \Gamma_i(s,k^2) \) of the following form. \( \gamma_i(s,k^2) \) has to be evaluated with \( M_\pi = M_{\pi^0} \) and is denoted \( \gamma_i(s,k^2) \) whereas \( \Gamma_i(s,k^2) \) has to be calculated with \( M_\pi = M_{\pi^0} \) denoted by \( \Gamma_i(s,k^2) \).

**Electric and magnetic Born terms:**

Here, the proton Dirac form factor \( F_1^p(k^2) \) in eq.(3.12) gets modified. The \( \tilde{\gamma}_i(k^2) \) and \( \tilde{\Gamma}_i(k^2) \) are replaced by \( \tilde{\gamma}_{i+}(k^2) \) and \( \tilde{\Gamma}_{i0}(k^2) \), respectively. Furthermore, \( \bar{\gamma}^2 \pi \pi (k^2) \) of eq.(B.1) has to be evaluated with \( M_\pi = M_{\pi^+} \). Of course, now the counter term contribution proportional to \( b_y^2(\lambda) + b_y^2(\lambda) \) has to be readjusted in order to reproduce the empirical proton mean square charge radius. In a similar fashion the value of the magnetic moment counter term \( \delta \kappa_p \) is obtained from

\[
\delta \kappa_p = \kappa_p + 8 g_{\pi NN}^2 m^2 \{ \Gamma_{40}(m^2,0) - 2 \gamma_{4+}(m^2,0) \} = 1.291
\]

(D.1)

**Class III:**

Most of the terms in the \( B_i(s,u) \) can be taken over from eq.(4.8) by simply replacing the respective loop functions by \( \gamma_{i+}(s,k^2) \), \( \Gamma_{i0}(s,k^2) \), \( \tilde{\Gamma}_{10}(t) \), \( G_{i0}(s,t,k^2) \) and
$\Omega_{i+}(s, u, k^2)$. In all kinematical prefactors like $M_\pi^2 - t - k^2$, $M_\pi$ reads $M_{\pi^0}$. The modified loop functions $\Omega_{i+}$ need some explanation since they involve both the neutral and charged pion. The proper denominator for their Feynman parameter representation is now

$$h_{\Omega+}(x, y, z; s, u, k^2) = M_\pi^2 (1 - y) + M_{\pi^0}^2 x (x - 1) y^2 + m^2 y^2 + (s - m^2)(1 - x) y (y - 1) z + (u - m^2) x y (1 - y) (z - 1) + k^2 (1 - y)^2 z (z - 1).$$

(D.2)

The amplitudes $B_{1, 2, 3}(s, u)$ involve the combination

$$\Gamma_{0}^{\pi N}(s) - \frac{M_\pi^2}{s - m^2} \Gamma_{00}^{\pi N}(s) - \Gamma_{2}^{\pi N}(s).$$

(D.3)

It gets replaced by

$$2 \Gamma_{0+}^{\pi N}(s) - \frac{2M_\pi^2}{s - m^2} \Gamma_{00}^{\pi N}(s) - 2 \Gamma_{2+}^{\pi N}(s) - \Gamma_{00}^{\pi N}(s) + \frac{M_{\pi^0}^2}{s - m^2} \Gamma_{00}^{\pi N}(s) + \Gamma_{20}^{\pi N}(s)$$

(D.4)

where the Feynman parameter representation of the loop functions reads

$$\Gamma_{i+}^{\pi N}(s) = \frac{1}{16\pi^2} \int_0^1 dx \int_0^1 dy \frac{yg_i(x, y)}{M_\pi^2 (1 - y) + M_{\pi^0}^2 x y^2 (x - 1) + m^2 y^2 + (s - m^2) x y (y - 1)}$$

(D.5)

with $g_0 = 1$, $g_1 = 1 - y$, $g_2 = 1 - xy$ and $\Gamma_{i0}^{\pi N}(s)$ is obtained if $M_{\pi^+}$ is set equal $M_{\pi^0}$.

**REFERENCES**

1. V. Bernard, N. Kaiser and Ulf-G. Meißner, *Nucl. Phys.* B383 (1992) 442.
2. E. Amaldi, S. Fubini and G. Furlan, *Pion–Electroproduction*, Springer Verlag, Berlin 1979.
3. T. P. Welch et al., *Phys. Rev. Lett.* 69 (1992) 2761.
4. Ulf-G. Meißner, *Rep. Prog. Phys.* 56 (1993) 903.
5. J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* 158 (1984) 142; J. Gasser and H. Leutwyler, *Nucl. Phys.* B250 (1985) 465.
6. G. Ecker, J. Gasser, A. Pich and E. de Rafael, *Nucl. Phys.* B321 (1989) 311; J. F. Donoghue, C. Ramirez and G. Valencia, *Phys. Rev.* D39 (1989) 1947.
7. V. Bernard, N. Kaiser and Ulf-G. Meißner, “Testing nuclear QCD: $\gamma p \to \pi^0 p$ at threshold”, in the $\pi N$ Newsletter No. 7 (1992), 62.
8. S. Nozawa and T.-S. H. Lee, *Nucl. Phys.* A513 (1990) 511.
9. S. Nozawa and T.-S. H. Lee, *Nucl. Phys.* A513 (1990) 543.
10. F.A. Berends, A. Donnachie and D.L. Weaver, *Nucl. Phys.* B4 (1967) 1.
11. J. Gasser, M.E. Sainio and A. Švarc, *Nucl. Phys.* B 307 (1988) 779.
12. S. R. Amendolia et al., *Nucl. Phys.* B277 (1986) 168.
13. T. Kitagaki *et al.*, *Phys. Rev.* D28 (1983) 436;
   L.A. Ahrens *et al.*, *Phys. Rev.* D35 (1987) 785;
   L.A. Ahrens *et al.*, *Phys. Lett.* B202 (1988) 284.
14. V. Bernard, N. Kaiser and Ulf-G. Meißen, *Phys. Lett.* B282 (1992) 448.
15. R. Beck *et al.*, *Phys. Rev. Lett.* 65 (1990) 1841.
16. S. Fubini, G. Furlan and C. Rossetti, *Nuovo Cim.* 40 (1965) 1171;
   Riazuddin and B.W. Lee, *Phys. Rev.* 146 (1966) B1202;
   S.L. Adler and F.J. Gilman, *Phys. Rev.* 152 (1966) B1460;
   S.L. Adler, *Ann. Phys.* (N.Y.) 50 (1968) 189;
   N. Dombey and R.J. Read, *Nucl. Phys.* B60 (1973) 65.
17. S. Scherer and J. H. Koch, *Nucl. Phys.* A534 (1991) 461.
18. Y. Nambu and D. Lurie, *Phys. Rev.* 125 (1962) 1429;
   Y. Nambu and E. Shrauner, *Phys. Rev.* 128 (1962) 862.
19. V. Bernard, N. Kaiser and Ulf-G. Meißen, *Phys. Rev. Lett.* 69 (1992) 1877.
20. S. Weinberg, *Physica* 96A (1979) 327.
21. I.A. Vainshtein and V.I. Zakharov, *Nucl. Phys.* B36 (1972) 589.
22. P. de Baenst, *Nucl. Phys.* B24 (1970) 633.
23. S. Scherer, J. H. Koch and J. L. Friar, *Nucl. Phys.* A552 (1993) 515.
24. K. Ohta, *Phys. Rev.* C47 (1993) 2344.
25. N.M. Kroll and M.A. Ruderman, *Phys. Rev.* 93 (1954) 233.
26. E. Jenkins and A.V. Manohar, *Phys. Lett.* B255 (1991) 558.
27. V. Bernard, N. Kaiser, J. Kambor and Ulf-G. Meißen, *Nucl. Phys.* B388 (1992) 315.
28. J. Gasser and Ulf-G. Meißen, *Nucl. Phys.* B357 (1991) 90.
29. J. Gasser and H. Leutwyler, *Phys. Reports* C87 (1982) 77.
30. V. Bernard, N. Kaiser, T.–S. H. Lee and Ulf-G. Meißen, *Phys. Rev. Lett.* 70 (1993) 367.
31. A. del Guerra *et al.*, *Nucl. Phys.* B107 (1976) 65;
   M.G. Olsson, E.T. Osypowski and E.H. Monsay, *Phys. Rev.* D17 (1978) 2938.
FIGURE CAPTIONS

Fig.1. The functions $\Xi_{1,2,3,4}(\rho)$ defined in eq.(5.2) with $\rho = -k^2/M^2$. The solid, dotted, dashed and dash–dotted lines refer to $\Xi_1$, $\Xi_2$, $\Xi_3$ and $\Xi_4$, respectively.

Fig.2. The functions $\Phi_{1,2}(k^2)$ defined in eq.(5.5). The solid and dotted lines refer to $\Phi_1$ and $\Phi_2$, in order.

Fig.3. The functions $\tilde{\Xi}_{1,2}(\rho_0)$ defined in eq.(5.9) with $\rho_0 = -k^2/M^2_\pi$. The solid and dotted lines refer to $\tilde{\Xi}_1$ and $\tilde{\Xi}_2$, respectively.

Fig.4. Comparison of the theoretical predictions with the recent NIKHEF data for $p(\gamma^*, \pi^0)p$ [3]. The solid circles, open circles and open squares refer to the one–loop CHPT, tree and PV Born term calculation, in order.

Fig.5. The longitudinal $S_{0+}$ multipole amplitudes for $p(\gamma^*, \pi^+)n$, $n(\gamma^*, \pi^-)p$ and $p(\gamma^*, \pi^0)p$. The solid, dotted and dashed lines refer to the one–loop CHPT, the tree and the PV Born calculation, in order.

Fig.6. The transversal $E_{0+}$ multipole amplitudes for $p(\gamma^*, \pi^+)n$, $n(\gamma^*, \pi^-)p$ and $p(\gamma^*, \pi^0)p$.

Fig.7. $k^2$–dependence of $\sigma_T$ and $\sigma_L$ for $p(\gamma^*, \pi^+)n$, $n(\gamma^*, \pi^-)p$ and $p(\gamma^*, \pi^0)p$.

Fig.8. Angular distributions of the transverse cross sections for $p(\gamma^*, \pi^+)n$, $n(\gamma^*, \pi^-)p$ and $p(\gamma^*, \pi^0)p$. For notations see Fig.5.

Fig.9. Same as Fig.8 except for the longitudinal cross sections.

Fig.10. Same as Fig.8 except for the interference cross sections.

Fig.11. Same as Fig.8 except for the polarization cross sections.

Fig.12. Dependence of the axial cut–off mass for $p(\gamma^*, \pi^+)n$. We show the angular distributions $d\sigma_T/d\Omega$, $d\sigma_L/d\Omega$, $d\sigma_I/d\Omega$ and $d\sigma_{III}/d\Omega$ at $k^2 = -0.04$ GeV$^2$ for $M_A = 0.96, 1.06, 1.16$ GeV corresponding to the dotted, solid and dashed lines, respectively.

Fig.13. The S–wave cross section $a_0$ for $p(\gamma^*, \pi^0)p$ including isospin–breaking for $W = 1074$ MeV and $\epsilon = 0.58$. The solid, dotted and dashed lines represent the isosymmetric CHPT, CHPT with isospin–breaking and PV predictions, in order.

Fig.14. Angular distributions for $p(\gamma^*, \pi^0)p$ including isospin–breaking. For notations see Fig.13.

Fig.15. $k^2$–dependence of $\sigma_T$ and $\sigma_L$ for $p(\gamma^*, \pi^0)p$ including isospin–breaking. For notations see Fig.13.
### TABLES

| $k^2$ | $-0.001$ GeV$^2$ | $-0.04$ GeV$^2$ | $-0.06$ GeV$^2$ |
|-------|-----------------|-----------------|-----------------|
| $E_{0^+}$ | $(25.77, 0.2663)$ | $(15.00, 0.5830)$ | $(9.899, 0.6602)$ |
| $S_{0^+}$ | $(-18.44, 0.1890)$ | $(-21.32, 0.2296)$ | $(-22.58, 0.2425)$ |
| $E_{1^+}$ | $(0.7625, 2.022 \times 10^{-6})$ | $(0.6366, -1.169 \times 10^{-6})$ | $(0.5707, -7.883 \times 10^{-8})$ |
| $S_{1^+}$ | $(-0.4098, 2.631 \times 10^{-6})$ | $(-0.3335, 7.441 \times 10^{-6})$ | $(-0.2849, 9.871 \times 10^{-6})$ |
| $S_{1^-}$ | $(-3.373, 9.221 \times 10^{-3})$ | $(-4.831, 0.01405)$ | $(-5.152, 0.01523)$ |
| $M_{1^+}$ | $(-1.037, -5.792 \times 10^{-5})$ | $(-1.135, -9.365 \times 10^{-5})$ | $(-1.168, -1.058 \times 10^{-4})$ |
| $M_{1^-}$ | $(0.2250, -6.891 \times 10^{-5})$ | $(-0.8534, 2.085 \times 10^{-3})$ | $(-1.271, 2.989 \times 10^{-3})$ |

Table 1a: Various multipoles for the reaction $\gamma^* p \rightarrow \pi^+ n$ in units of $10^{-3}/M_{\pi^+}$.

| $k^2$ | $-0.001$ GeV$^2$ | $-0.04$ GeV$^2$ | $-0.06$ GeV$^2$ |
|-------|-----------------|-----------------|-----------------|
| $E_{0^+}$ | $(-29.99, -0.3543)$ | $(-16.96, -0.6121)$ | $(-10.86, -0.6657)$ |
| $S_{0^+}$ | $(22.98, -0.08447)$ | $(26.55, -0.08553)$ | $(27.24, -0.08384)$ |
| $E_{1^+}$ | $(-0.7534, -4.067 \times 10^{-11})$ | $(-0.6263, 3.178 \times 10^{-9})$ | $(-0.5611, 5.027 \times 10^{-9})$ |
| $S_{1^+}$ | $(0.4010, -1.952 \times 10^{-10})$ | $(0.3174, -7.329 \times 10^{-10})$ | $(0.2691, -1.377 \times 10^{-9})$ |
| $S_{1^-}$ | $(3.336, -8.639 \times 10^{-3})$ | $(4.778, -0.01237)$ | $(5.137, -0.01300)$ |
| $M_{1^+}$ | $(1.242, 3.158 \times 10^{-11})$ | $(1.350, -3.199 \times 10^{-9})$ | $(1.341, -5.050 \times 10^{-9})$ |
| $M_{1^-}$ | $(-0.5396, 1.667 \times 10^{-3})$ | $(0.5531, 3.483 \times 10^{-4})$ | $(1.059, -1.967 \times 10^{-4})$ |

Table 1b: Various multipoles for the reaction $\gamma^* n \rightarrow \pi^- p$ in units of $10^{-3}/M_{\pi^+}$.
### Table 1c: Various multipoles for the reaction $\gamma^\ast p \to \pi^0 p$ in units of $10^{-3}/M_{\pi^+}$.

| $k^2$    | $-0.001 \text{ GeV}^2$ | $-0.04 \text{ GeV}^2$ | $-0.06 \text{ GeV}^2$ |
|----------|--------------------------|------------------------|------------------------|
| $E_{0^+}$| (-1.304, 0.5103)         | (1.625, 0.9461)        | (2.668, 1.045)         |
| $S_{0^+}$| (3.199, 0.1805)          | (3.758, 0.2102)        | (3.386, 0.2176)        |
| $E_{1^+}$| $(6.989 \times 10^{-3}, -2.634 \times 10^{-4})$ | $(1.609 \times 10^{-3}, -4.267 \times 10^{-4})$ | $(-2.648 \times 10^{-3}, -4.385 \times 10^{-4})$ |
| $S_{1^+}$| $(-0.01311, -7.260 \times 10^{-5})$ | $(-0.03610, -7.342 \times 10^{-5})$ | $(-0.04627, -7.053 \times 10^{-5})$ |
| $S_{1^-}$| $(0.09471, 0.01126)$      | $(0.2800, 0.01760)$     | $(0.3872, 0.01912)$     |
| $M_{1^+}$| $(0.1162, 2.884 \times 10^{-4})$ | $(0.04723, 9.844 \times 10^{-5})$ | $(-0.02483, 6.400 \times 10^{-5})$ |
| $M_{1^-}$| $(-1.521, -1.233 \times 10^{-4})$ | $(-2.303, 2.877 \times 10^{-3})$ | $(-2.520, 4.016 \times 10^{-3})$ |

### Table 2a: Dependence on the axial cut-off mass $M_A$ of various observables for the reaction $\gamma^\ast p \to \pi^+ n$.

| $M_A[\text{GeV}]$ | 1.06 | 1.16 | 0.96 |
|-------------------|------|------|------|
| $\text{Re}E_{0^+}[10^{-3}/M_{\pi^+}]$ | 15.00 | 15.98 | 13.71 |
| $\text{Re}S_{0^+}[10^{-3}/M_{\pi^+}]$ | -21.32 | -20.42 | -22.51 |
| $\sigma_T[\mu b]$ | 10.81 | 12.17 | 9.04 |
| $\sigma_L[\mu b]$ | 16.87 | 15.51 | 18.70 |
| $(\sigma_T + \sigma_L)[\mu b]$ | 27.68 | 27.68 | 27.68 |
| $\sigma_T/\sigma_L$ | 0.64 | 0.79 | 0.48 |
| $M_A$[GeV] | 1.06 | 1.16 | 0.96 |
|------------|-----|------|------|
| Re$E_{0+}[10^{-3}/M_{π^+}]$ | -16.96 | -17.94 | -15.67 |
| Re$S_{0+}[10^{-3}/M_{π^+}]$ | 26.55 | 25.65 | 27.74 |
| $σ_T[μb]$ | 13.70 | 15.27 | 11.74 |
| $σ_L[μb]$ | 25.58 | 23.95 | 27.88 |
| $(σ_T + σ_L)[μb]$ | 39.28 | 39.21 | 39.62 |
| $σ_T/σ_L$ | 0.54 | 0.64 | 0.42 |

Table 2b: Dependence on the axial cut–off mass $M_A$ of various observables for the reaction $γ^*n → π^-p$. 

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