Unification Scale in String Theory

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Abstract

We study the unification scale and gauge coupling constant in 4D string theory. We show that the fine structure constant is determined by the dimension of the hidden gauge group and only $SU(6)$ and $SO(9)$ are consistent with minimal string unification while the unification scale can be of order of $10^{16}$ GeV.
String theory provides with the only possibility, up till now, of unifying all interactions as formulated in the critical dimensions it has only one free parameter which is taken as the Planck mass. But once it is compactified to four dimensions much of its uniqueness is lost because there are a great number of consistent string vacua. Nevertheless it is possible to study general 4D string models by concentrating on properties shared by all models and consider the model dependent quantities as free parameters. These models contain, after compactification, two separate gauge groups $E_6 \times E'_8$ or subgroups thereof. The group $E_6$, called the visible sector, is supposed to have the standard model as a subgroup while the other group $E'_8$ (or a subgroup) is referred to as the hidden gauge group. In many cases these two sectors interact through gravity only. In this context supersymmetry (SUSY) will be broken in the hidden sector via a gaugino condensate and it will be transmitted to the observable sector through gravity. The scale at which the condensate is formed is related to the condensation scale, defined as the scale where the gauge coupling constant becomes strong. Clearly it will vary for different gauge groups. Since in string theory all gauge coupling constants are unified at the string scale and are mainly given by the v.e.v. of the dilaton field $S$, the condensation scale for each gauge group will be determined by its one-loop beta function coefficient $\beta_0$. The larger $\beta_0$ is, the larger the condensation scale will be, and we will then expect that SUSY is broken by the gauge group with largest $\beta_0$. Clearly, some relevant model dependent quantities are then the hidden gauge group and its spectrum which determine the scale of SUSY breaking. The standard model can then be viewed as a globally supersymmetric model with explicit soft supersymmetric breaking terms and the study of these terms is relevant in determining the viability of the models.

Another generic property is the invariance of the effective Lagrangian under duality symmetry. Although it has only been proved to be exact to all orders in perturbation theory one expects that non-perturbative effects will respect duality. This symmetry is intimately related to the contribution of the infinite number of Kaluza-Klein modes, always present in string theory. These modes become relevant in the low energy effective theory for small values of the compactified radius given by the real part of the (1,1) moduli. In the simplest case, the moduli fields transform under duality as an element of the SL(2,Z) group and in this paper we will only consider those moduli.

In string theory the gauge coupling constant and the unification scale are determined dynamically and are given in terms of the v.e.v.s of the dilaton and moduli fields. Assuming that the low energy model is that of the minimal supersymmetric standard model (MSSM) unification of the gauge coupling constants of the standard model restricts the values of the gauge coupling constant at the unification scale and the unification scale to a narrow band. In particular the unification scale should be of order $10^{16}$GeV but in string theory it is usually much larger. The reason is that the unification scale is given in terms of a modular invariant function of the moduli fields. If the v.e.v.s of the moduli is of order one, which is their v.e.v. at "tree" level, the unification scale is of order of the string scale. However, it was recently shown that once loop corrections of the strong binding effects, which lead to a gaugino condensate, are taking into account, the v.e.v. of the moduli can be much larger. Furthermore, their v.e.v.s may differ allowing for squeezed orbifolds which are better candidates for minimal string unification. Here we will study whether it is possible to obtain the desired values for the unification scale and gauge coupling constant.

We study the possibility of having unification of the gauge coupling constants assuming the minimal string unification scheme (MSU). This scheme consists of having the chiral matter content in the visible sector of the theory to be just that of the minimal supersymmetric model (MSSM) while the hidden sector remains unspecified. Recent precision measurements for the weak coupling and strong coupling constant at LEP have permitted to refine the analysis of the unification of the gauge coupling constants. The $SU(5)$ prediction is now ruled out but the

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1) By tree level we understand that it does not include loop corrections of the strongly binding effects.
supersymmetric extension is in good agreement with the experimental values [14].

The gauge coupling constants in the standard model (SM) are measured at the electroweak energy scale. Using the well-known renormalization group equation these couplings can be determined for any other energy scale. The gauge coupling constants have a logarithmic evolution with respect to energy and at the one-loop level they are given by

\[ g_a^{-2}(\Lambda) = k_ag_0^{-2}(\Lambda_0) + 2b_a ln(\frac{\Lambda}{\Lambda_0}). \]  

(1)

\( \Lambda_0 \) is the energy scale at which the gauge coupling constant \( g_0 \) is measured, \( g_a \) is the gauge coupling constant at the arbitrary scale \( \Lambda \), and \( b_a \) is the one-loop \( \beta \) function coefficient for the \( G_a \) gauge group while \( k_a \) is its corresponding Kac-Moody level. In the SM the beta function coefficients are given by 16\( \pi^2b_3 = 7 \), 16\( \pi^2b_2 = -19/6 \) and 16\( \pi^2b_1 = -41/6 \) with \( k_3 = k_2 = k_1 = 3/5 = 1 \) for \( SU(3), SU(2) \) and \( U(1) \) respectively.

As mentioned above, it has recently been shown that the evolution of the gauge coupling constants in the SM do not become unified, i.e., they do not meet for any given energy. However, interestingly enough they do meet if one includes supersymmetry (SUSY) and assumes the minimal supersymmetric standard model (MSSM). The main reason is that in the latter case the one-loop beta function coefficients change due to the inclusion of the extra states and they are very much restricted by phenomenological constraints. For these values of the one-loop beta function coefficient and assuming that the supersymmetric breaking scale \( M_{ss} \) is of order 10\(^{3\pm1} \) GeV, the gauge coupling constants are unified at a value for the fine structure constant given by (in the minimal subtraction scheme \( \overline{MS} \)) [1]

\[ a_{gut}^{-1} = \left( \frac{g_{gut}^2}{4\pi} \right)^{-1} = 25.7 \pm 1.7 \]  

(2)

and unification scale

\[ \Lambda_{gut} = 10^{16\pm0.3} \text{ GeV}. \]  

(3)

Though in the MSSM one has three free parameters \( M_{ss}, g_{gut} \) and \( \Lambda_{gut} \) to predict three coupling constants, meaning that there will always be a solution to the unification of the gauge coupling constant, this fact does not mean that the unification scheme is empty because the values of the parameters \( M_{ss}, g_{gut} \) and \( \Lambda_{gut} \) are very much restricted by phenomenological constraints. In fact, one requires \( M_{Planck} > \Lambda_{gut} > 10^{15} \text{ GeV} \) to avoid a fast decaying proton, \( M_{ss} \) must be at the most of order 1 TeV to explain the mass hierarchy problem and larger then 100 GeV to prevent light SUSY states that would have been detected already. Finally \( g_{gut}^2 \) must be positive and small so that we stay in the perturbative regime.

The running of the couplings in string theory is given by [27-29]

\[ \frac{1}{g_a^2(\Lambda)} = \frac{k_a}{g_s^2} + b_a ln(\frac{\Lambda^2}{M_s^2}) + \Delta_a \]  

(4)

where \( b_a = \frac{1}{16\pi^2}(3C(G_a) - \Sigma_{R_a} h_{R_a} T(R_a)) \) is the N=1 \( \beta \)-function coefficient, \( C(G_a) \) the quadratic Casimir operator and \( h_{R_a} \) the number of chiral fields in a representation \( R_a \) (the Latin indices of the beginning of the alphabet \((a,b,..)\) represent gauge indices while those in the middle of the alphabet \((i,j,...)\) refer to the type of moduli \( T_i \)). \( M_a \), as defined below, is the renormalization scale below which the coupling constant begins to run. The \( o \)-index refers to the hidden sector gauge group only while the indices \( a \) and \( b \) correspond to generic gauge groups (visible or hidden sector).

The existence of an infinite number of massive states (Kaluza and winding states), above the string scale, give rise to string threshold contributions \( \Delta_a \) which are relevant to the determination of the coupling constant at the string scale. These threshold effects can be directly
The scale $M_a$, below which the coupling constant starts to run, is in general a moduli dependent quantity

$$M_a^2 = \Sigma_i (T_{ri})^{\alpha_i} M_s^2$$

where $T_{ri} = (T + \bar{T})_i$ and the constant $\alpha_i$ is model and gauge dependent and $M_s$ is the string scale \[^2\]. In the case of a single overall (1,1) moduli $T_i = T$, $i = 1, 2, 3$, with $\alpha = \Sigma \alpha_i = -1$ one obtains the “naive” field theoretical expression

$$M_a = (Re S Re T)^{-1/2}.$$ 

The contribution to the gauge coupling constant from the moduli fields in eq.(3) can be calculated by computing world-sheet string amplitudes involving external gauge fields and moduli \[^28, 29\], and has been done for (2,2) symmetric orbifold compactification. Another possible way to calculate the threshold corrections is by imposing target space modular invariance and the cancellation of target space modular anomalies.

The scale $M_a$, below which the coupling constant starts to run, is in general a moduli dependent quantity

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$$M_a = (Re S Re T)^{-1/2}.$$ 

The threshold term in eq.(4) is given by

$$\Delta_a = \Sigma_i (b_a^i - k_a \delta_{GS}) \ln|\eta(T_i)|^4,$$

$$b_a^i = \frac{1}{16\pi^2} (C(G_a) - \Sigma R_a h R_a T(R_a)(1 + 2n R_a))$$

where $n_i$ is the modular weight for a chiral matter superfield with respect to the $i$-moduli and $\eta(T)$ the Dedekind-eta function. In the case of an overall moduli $b_a^i = \Sigma b_a^i = \frac{1}{16\pi^2} (3C(G_a) - \Sigma R_a h R_a T(R_a)(3 + 2n R_a)) = b_a - 2\Sigma R_a h R_a T(R_a)(1 + n R_a)$ with $n R_a = \Sigma n_i R_a$.

The universal Green-Schwarz coefficient $\delta_{GS}$ in eq.(3) is needed to cancel, using the Green-Schwarz mechanism, the gauge independent part of the target space modular anomaly. The threshold contribution of the massive fields, $b_a - k_a \delta_{GS}$, is in general non-vanishing if at least one of the orbifold twists leaves the $i$-plane unrotated. In this sector, the massive spectrum is N=2 space-time supersymmetric and $b_a - k_a \delta_{GS}$ is proportional to the N=2 $\beta$-function coefficient which is in general non zero. On the other hand if all orbifold twists rotate a specific plane, than the spectrum is N=4 supersymmetric and $b_a - k_a \delta_{GS} = 0$, giving no threshold contribution. In this case the gauge coupling constant is independent of the $T_i$ moduli and one has that $b_a^i/k_a$ must be equal for all gauge groups, i.e. $b_a^i/k_a = b_b^i/k_b$.

Eq.(4) can be rewritten in a similar form as eq.(3)

$$g_a^{-2}(\Lambda) = k_a g_{gut}^{-2}(\Lambda_{gut}) + 2b_a \ln(\frac{\Lambda}{\Lambda_{gut}})$$

with the unification scale defined by

$$\Lambda_{gut} = M_s(\Pi_i T_{ri})^{i/4}|\eta(T_{ri})|^4 \alpha_i^{\alpha_i/2}$$

where

$$\alpha_i = \frac{\delta_{GS} k_a - b_a^i}{b_a}$$

and the gauge coupling constant at the unification scale given by

$$g_{gut}^{-2} = \frac{Y}{2}$$

\[^2\] Numerically the string scale is $0.7 g_{gut} 10^{15} GeV$ \[^2\].
eliminate the Green-Schwarz term in eq.(8) and the unification scale becomes \[ b \] part of the target modular anomaly and terms of the gaugino bilinear of the hidden sector \[ \eta \]

where at tree level \( Y \) is given in terms of the v.e.v. of the dilaton field \( S \). For two gauge coupling constants to become equal, i.e. \( \frac{g_i^2(\Lambda_{\text{gut}})}{b_i} = \frac{g_j^2(\Lambda_{\text{gut}})}{b_j} = \frac{g^2_{\text{gut}}(\Lambda_{\text{gut}})}{b} \), at the unification scale \( \Lambda_{\text{gut}} \) the coefficients defined in eq.(9) for different gauge groups must be the same, i.e. \( \alpha_{i}^a = \alpha_{j}^b \). From eq.(8) one obtains the condensations scale, defined as the scale where the gauge coupling constant becomes strong, and it is given by

\[
\Lambda_{\text{gut}}^a = \Lambda_{\text{gut}}^a e^{-k_a Y/4b_a}.
\] (11)

From eq.(8) we note that a unification scale smaller than the string scale necessarily requires the exponent to be positive, \( \alpha > 0 \) since the modular invariant function \( T_{rs} \eta_{s}^{4} \) is given. The Kahler potential, superpotential and gauge kinetic functions are \[ 15 \]-\[18 \] respectively. The Kahler potential, superpotential and gauge kinetic function at the string scale is given by \[ 27 \]-\[29 \],\[30 \] and it is completely specified once the v.e.v. of the moduli and the \( b \) coupling constant becomes strong, and it is given by

\[
\Lambda_{\text{gut}} = M_s (\Pi_i T_{ri} |\eta(T_{ri})|^4)^{\frac{k'_i - b'_i}{2(\eta_{0} - b_0)}},
\] (12)

and it is completely specified once the v.e.v. of the moduli and the \( b_0 \) and \( b'_i \) coefficients are determined. A positive \( \alpha \) or equivalently a positive exponent in eq.(12) forces the compactified space to have chiral matter fields with modular weights different than \(-1/3\) (untwisted fields have modular weights \(-1/3\)) otherwise \( b'_i = b_i/3 \).

In string theory the different parameters of the low energy theory like unification scale, supersymmetry breaking scale and gauge coupling constant at the unification scale are dynamically determined once supersymmetry is broken. The most common and probably the best way for breaking SUSY is via gaugino condensate \[13 \]. In order to study the breaking of SUSY one determines the effective potential and the vacuum. The effective interaction involving the gaugino bilinear can be obtained by demanding the complete Lagrangian to be anomaly free under the R-symmetry under which the gauginos transform non-trivially. The low energy degrees of freedom are then the dilaton field \( S \), moduli fields \( T_i \), chiral matter fields \( \varphi_i \), gauge fields and the Goldstone mode \( \Phi \) associated with the spontaneously broken R-symmetry plus their supersymmetric partners. In orbifold compactification there are always three diagonal moduli whose real parts represent the size of the compactified complex plane and here we will only consider these moduli. The 4D string model is given by an N=1 supergravity theory and it is specified once the Kahler potential \( G = K + \ln \frac{1}{2} |W|^2 \) and the gauge kinetic function \( f \) are given. The Kahler potential, superpotential and gauge kinetic functions are \[21 \]-\[26 \]

\[
K = -\ln \left( S + \bar{S} + 2\Sigma_i (k_0 \delta_{GS}^i - b_0^i) \ln T_{ri} \right) - \Sigma_i \ln(T_{ri}) + K_i |\varphi_i|^2,
\] (13)

\[
W_0 = \Pi_i \eta^{-2}(T_i) \Phi + W_m
\] (14)

and \[18 \]

\[
f = f_0 + \frac{2}{3} b_0 \ln(\Phi)
\] (15)

respectively. \( W_m \) is the superpotential for the chiral matter superfields and the gauge kinetic function at the string scale is given by \[27 \]-\[29 \],\[31 \]

\[
f_0 = S + 2\Sigma_i (b_0^i - k_0 \delta_{GS}^i) \ln [\eta(T_i)^2]
\] (16)

where \( \eta \) is the Dedekind-eta function and \( b_0 \) the one-loop beta function for the hidden gauge group. The coefficient \( \delta_{GS} \) is the Green-Schwarz term needed to cancel the gauge independent part of the target modular anomaly and \( b_0^i \) define in eq.(8).

Through the equation of motion of the auxiliary field of \( \Phi \), the scalar component is given in terms of the gaugino bilinear of the hidden sector \[18 \]

\[
\phi = \frac{e^{-K/2\xi}}{2\Pi_i \eta^{-2}(T_i)} \bar{\lambda}_R \lambda_L
\]
with $\xi = 2b_0/3$. The model described in eqs. [13,15] is anomaly free and duality invariant. The duality transformation for the fields read

\[
S \rightarrow S + 2\Sigma_i(k_a\delta_G - b_a^i)\ln(ieT_i + d), \\
T_i \rightarrow \frac{aT_i - ib}{ieT_i + d}, \\
\phi \rightarrow \phi,
\]

with $a, b, c, d \in \mathbb{Z}$ and $ad - bc = 1$. This model generates a four-Gauginon interaction and reproduces the tree level scalar potential used by other parameterizations of gaugino condensate\cite{14,21,22}. It also permits the determination of the radiative corrections and use of NJL technique\cite{33} to extract non-perturbative information in the regime of strong coupling. After minimizing the complete scalar potential (tree level plus one-loop potential), the vacuum structure is quite different than the tree level one, thus permitting us to find a stable solution for the dilaton with the inclusion of a single gaugino condensate. We will show that the value of the v.e.v.'s of the dilaton and moduli fields can give a good prediction of the fine structure constant at the unification scale and unification scale allowing for minimal string unification to work.

In supergravity theory the tree level scalar potential is given by\cite{31}

\[
V_0 = h_i(G^{-1})^i_jh^j - 3m^2_{3/2} 
\]

where the auxiliary fields are $h_i = -e^{G/2}G_i + \frac{1}{4}f_i\lambda_R\lambda_L$. For the choice of Kahler potential and gauge kinetic function given in eqs. [13,15] one has

\[
V_0 = e^G B_0 = \frac{1}{4}e^{K}\Pi_i|\eta(T_i)|^{-4} |\phi|^2 B_0 
\]

with

\[
B_0 = (1 + \frac{Y}{\xi})^2 + \Sigma_i \frac{Y}{Y + a_i}(1 - \frac{a_i}{\xi})^2 \frac{T_i^2}{4\pi^2}|\hat{G}_2(T_i)|^2 - 3, 
\]

\[
a_i = 2(k_a\delta_G - b_a^i) \quad \text{and} \quad Y = S + \bar{S} + 2\Sigma_i(k_0\delta_G - b_0^i)\ln T_i. 
\]

The gravitino mass is given by

\[
m^2_{3/2} = \frac{1}{4}e^{K}\Pi_i|\eta(T_i)|^{-4} |\phi|^2. 
\]

For a fixed value of $S$ the extremum solution to eq. [19] for the moduli fields gives a v.e.v. of $<T_i> \simeq 1.2$\cite{16,17}. This value is independent of $S$ and yields an unification scale of order of the string scale much larger than the required value (cf. eq. [8]). Furthermore, there is no stable solution in the dilaton direction, it is a runaway potential for $S \to \infty$ and it is unbounded from below for $S \to 0$. This is not surprising because we haven’t included the contribution from loop corrections of the strong coupling constant responsible for the gaugino binding. These contributions can be calculated using the Coleman-Weinberg one-loop potential $V_1$ and it is given by\cite{33,34}

\[
V_1 = \frac{1}{32\pi^2}Str \int d^2p \ln|p^2 + M^2| 
\]

where $M^2$ represents the square mass matrices and $Str$ the supertrace. By solving the mass gap equation $\frac{d}{d\phi}(V_0 + V_1) = 0$ one is effectively summing an infinite number of gaugino bubbles. After integrating $\phi$ out ($\phi = \Lambda^i$) one can minimize $V_0 + V_1$ with respect to the moduli fields and one obtains that the extremum equations are satisfied if they either take the dual invariant values ($<T_i> = 1, e^{\pi/6}$) or they take a common “large” value (i.e. $<T_i> = <T_j>$). For those moduli $T_i$ that take a “large” v.e.v. the corresponding $\alpha_0$ parameter defined in eq. [3] must be the same, i.e. $\alpha_0 = \Lambda^i$, and from now on we will drop the $i$-index in $\alpha_0$. 

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The extremum equations for the dilaton and moduli fields yield the constraint \[ B_0 = \frac{9}{2\gamma b_0^2} \left( 1 + \frac{2\alpha_0 - 1}{3\alpha_0 - 1} \epsilon \right)^{-1} \] (23)
and
\[ \epsilon \equiv x_g \ln(x_g)|_{\text{min}} = \frac{4b_0}{Y} (3\alpha_0 - 1) \] (24)
where \( B_0 \) is given in eq. (20), \( x_g = \frac{m_g^2}{\Lambda_c^2} \) with \( m_g^2 = \frac{3\xi^2 B_0^2}{2n_g^2} \) the gaugino mass square and \( \Lambda_c \) the condensation scale. Since at the minimum \( x_g < 1 \) eq. (24) requires \( \alpha_0 < 1/3 \). After eliminating \( \phi \) the gravitino mass becomes
\[ m_{3/2}^2 = \frac{1}{4Y} M_\phi^2 \Pi(T_i|\eta(T_i)|^4)^{3\alpha_0 - 1} e^{-3Y/2b_0} \] (25)
and
\[ x_g = \left( \frac{b_0}{6} \right)^2 \frac{B_0^2}{Y} \Pi(T_i|\eta(T_i)|^4)^{\alpha_0 - 1} e^{-Y/2b_0}. \] (26)

From eqs. (23) and (24) it follows that for reasonable solutions the dominant term in \( B_0 \) is given by the contribution from v.e.v. of the auxiliary field of the dilaton \( h_s \) and one can approximate \( B_0 \simeq \left( \frac{3Y}{2b_0} \right)^2 \). The v.e.v. of \( Y \) and \( T_i \) are then given in terms of the dimension of the hidden gauge group, its one-loop N=1 \( \beta \)-function coefficient and \( \alpha \) by
\[ Y \simeq 8\pi \sqrt{\frac{1}{n_g} \left( 1 + \frac{2\alpha - 1}{3\alpha - 1} \epsilon \right)^{-1}}, \] (27)
\[ \Pi(T_i|\eta(T_i)|^4) = \left( \frac{x_g^3 B_0^3 m_{3/2}}{32 Y x_g^{3/2}} \right). \] (28)

Eq. (28) is obtained from eqs. (25) and (26) and the sum is over all moduli that acquire a v.e.v. different from the dual points. To leading order the v.e.v. of the moduli is
\[ \Sigma(1 - \alpha_0)T_i = \frac{3Y}{\pi b_0.} \] (29)

If the gauge group is broken down from \( E_8 \) to a lower rank group such as \( SU(N) \) with \( 5 \leq N \leq 9 \), as can be easily done by compactifying on an orbifold with Wilson a large hierarchy can be obtained with only one gaugino condensate.

In any given string vacuum the gauge groups and particle spectrum are entirely determined and therefore the gauge coupling constant, unification scale and gravitino mass are not free parameters. In practice, there is a large number of consistent vacua having the standard model gauge group and three generations of particles and until now there is has been no procedure to choose one. To avoid this problem, we work in a model independent form by allowing the coefficients \( n_g, b_0 \) and \( \alpha_0 \) of the hidden sector to be free parameters from which the the gauge coupling constant, unification scale and gravitino mass are obtained. Again, it would seem that there is not any predictive power, since we are replacing the three parameters of the MSSM with three other ones. Yet, this point of view is not entirely fair since the new parameters can take only a limited number of discrete values and in principle they are not free at all, as argued above. One could further restrict the minimal string unification by demanding that the gauge coupling constants of the visible sector and of the hidden sector become unified at the unification scale. In what follows we will consider both possibilities; (i) that all gauge coupling constants are unified at the same scale and (ii) that only the gauge coupling constants of the SM are unified. Let us consider the (i) case first. From eq. (8), we notice that if all gauge coupling constants are to be unified, then the \( \alpha \) parameters of the hidden and visible sector must be the
same, i.e. $\alpha_0^i = \alpha_a^i = \alpha_b^i$. From the minimization condition we obtained that $\alpha_0^i = \alpha_0^j$ for those moduli with "large" v.e.v.'s and thus all $\alpha$'s must have the same value.\(^{4}\)

Assuming that the gauge group in the hidden sector is of rank less or equal eight (i.e. a subgroup of $E_8$) then $n_g \leq 248$ (the dimension of $E_8$), $0 < 16\pi^2 b_0 < 90$ and $0 < \alpha_0 < 1/3$. If the hidden sector is broken down to an $SU(N)$ subgroup (with $N \leq 9$) then the coefficients are restricted to take values in the range

$$0 < 16\pi^2 b_0 \leq 3N,$$

$$0 < \alpha_0 < \frac{1}{3}$$

and

$$n_g = N^2 - 1.$$  \hspace{1cm} (32)

Thus we see that a very small set of values for $n_g$, $b_0$ and $\alpha_0$ are allowed and that the values of the MSSM parameters (cf. eq.(2-3)) that can be obtained is far from trivial. Furthermore, the structure constant at the unification scale is given by

$$\hat{\alpha}_{\text{gut}}^{-1} = \frac{16\pi^2}{\sqrt{n_g}}.$$  \hspace{1cm} (33)

From eqs.(2) and (33), a coupling constant consistent with the range of MSSM requires a gauge group with $33 < n_g < 43.4$, which fixes the hidden gauge group to be $SU(6)$ or $SO(9)$. For these gauge groups, the fine structure constant would be $\hat{\alpha}_{\text{gut}} \approx 26.7$ and $26.3$ respectively.

To gain a better insight into the solution obtained in eqs.(26,24) we will trade the one-loop beta function coefficient of the hidden gauge group for the gravitino mass and we will rewrite the unification scale in terms of $m_{3/2}$, $n_g$ and $\alpha_0$. From eqs.(8) and (25) we have

$$\Lambda_{\text{gut}} = M_s \Pi_i (T_{ri} |\eta_i|^4)^{\alpha_0/2}$$

$$\Lambda_{\text{gut}} = M_s \left( 4Y m_{3/2}^2 / M_s^6 \right)^{\alpha_0/2} e^{-Y / 2 \eta_0 (1-\alpha_0)}$$

and the $b_0$ dependence is in the exponent only. Using eq.(26), one can eliminate this dependence and we get

$$\Lambda_{\text{gut}} = M_s \left[ \frac{\xi^3 B_0^3 m_{3/2}}{32Y x_g^{3/2}} \right]^{\alpha_0/2}$$

with $b_0$ given by

$$b_0 = -\frac{3Y}{2} \left[ \ln \left( 4Y m_{3/2}^2 / M_s^6 \left[ \frac{\xi^3 B_0^3 m_{3/2}}{32Y x_g^{3/2}} \right]^{1-3\alpha_0} \right) \right]^{-1}.$$  \hspace{1cm} (36)

The r.h.s. of eqs.(25) and (26) still depend on $b_0$ through $x_g$, $\xi$ and $B_0$ but the unification scale depends on $b_0$ now only linearly. It is not possible to solve eq.(36) analytically for $b_0$, although its dependence on the r.h.s. is only logarithmic. By setting $16\pi^2 b_0 = 3N$ on the r.h.s., one obtains a good approximation.

We can now plot $\Lambda_{\text{gut}}$ vs. $\alpha_0$ for a fixed value of the gravitino mass. In fig.6.1 we show the graph for an $SU(6)$ gauge group with $m_{3/2} = 82\text{ GeV}$. The unification scale has a minimum at around $\alpha_{\min} = 0.3$ with a value of

$$\Lambda_{\text{gut}} \approx 2.8 \times 10^{16}\text{ GeV}.$$  \hspace{1cm} (37)

\(^{3}\)For moduli with dual invariant v.e.v. the modular invariant quantity $(T_{rk} |\eta(T_k)|^4)^{\alpha_0^k} \approx 1$ and we will neglect it in the unification scale.
It is remarkable that the minimum value for the unification scale is just about the value required by the minimal unification of couplings in the MSSM. The value of $\alpha_{min}$ does not depend on the gravitino mass. For decreasing value of the gravitino mass, the unification scale is also reduced as it is clear from eq. (35). If we require that $m_{3/2} > 35 \text{ GeV}$, then $16\pi^2 b_0 > 14.8$. Since $b_0$ can only take discrete values, we have set $16\pi^2 b_0 = 15$ which gives $m_{3/2} \simeq 82 \text{ GeV}$. The value of the moduli for this specific example are $T_r/2 = 22.3, 12.1, 8.7$ for one, two or three moduli with "large" v.e.v., respectively. It is precisely the fact that the v.e.v. of the moduli get a "large" v.e.v. that permits the unification scale to be much smaller then the string scale (note that for $T_r \simeq 1.2$ as obtained at tree level the unification scale will always be of the same order of magnitude as the string scale). Furthermore, these solutions allow for squeezed orbifolds which where found better candidates for minimal string unification.

We will now consider the case in which the hidden sector gauge coupling constant does not necessarily become unified with the gauge coupling constants of the standard model. This would be the case when the threshold corrections to the gauge coupling constant differ for the hidden and visible sector gauge groups. In this case, there is no connection between the $\alpha$ parameters of the hidden sector and the ones in the visible sector. Thus, the unification scale is

$$\Lambda_{\text{gut}} = M_s \left[ \frac{\xi^3 B_3 m_{3/2}}{32 Y x_g} \right]^{\tilde{\alpha}/2}$$

(38)

The term in square brackets is independent of $\alpha_a$ and $\tilde{\alpha}$ is the average of the $\alpha_a^i$ whose corresponding moduli $T_i$ get a “large” v.e.v., i.e. $\tilde{\alpha} = \frac{1}{n_l} \sum_i \alpha_a^i = \frac{1}{n_l} \sum_i \alpha_b^i$, where $n_l$ is the number of moduli with “large” v.e.v. It is easy to see that in this case a unification scale consistent with MSSM is possible. The value of $\alpha_0$ is restricted only to be smaller than 1/3 and it can take negative values. As an example, we could consider a hidden gauge group with matter field in the untwisted sector only and taking $\delta_{GS} = 0$ one has $\alpha_0 = -1/3$. For this value of $\alpha_0$ and choosing $\tilde{\alpha} = 0.32$, one obtains a gravitino mass and a unification scale of

$$m_{3/2} \simeq 632 \text{ GeV}$$

(39)

$$\Lambda_{\text{gut}} \simeq 2 \times 10^{16} \text{ GeV}.$$ 

(40)

In this example the v.e.v. of the moduli are $T_r/2 = \text{Re} T = 22, 12.3, 9$ for one, two or three with "large" v.e.v. and $16\pi^2 b_0 = 11$. The same value of $\Lambda_{\text{gut}}$ can be obtained for larger gravitino mass and $\tilde{\alpha}$.

To conclude, we have shown that after finding a stable solution to the scalar potential including the contribution from gaugino binding effects, the v.e.v. obtained for the moduli and dilaton fields give the required values of MSSM for the gauge coupling constant and supersymmetry breaking scale (i.e. gravitino mass). The unification scale can also be consistent with MSSM for specific values of $\alpha$. Furthermore, since the value of the fine structure constant at the unification scale is fixed by the dimension of the hidden sector gauge group this group must be $SU(6)$ or $SO(9)$.

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Figure 1: Unification scale as a function of $\alpha$, keeping $m_{3/2}$ fixed.
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