Determination of the coefficient of friction using spinning motion

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Abstract. The paper presents a method for accurate determination of the coefficient of dynamic friction between two different materials. The principle of the method consists in evaluation of angular deceleration of a body of revolution that rotates positioned on another revolution body. The main advantages of the approach consist in high precision and reduced dimensions of the involved bodies.

1. Introduction

Friction is a phenomenon frequently met in everyday life. The effects of friction are varied, friction being one of the main factors contributing to the energy consumption and wear production. Eldrege, [1], shows that „6% din gross domestic product is spent on friction and wear issues including the replacement of worn parts and reduced efficiency”.

Describing the friction phenomenon is not a simple task. The attempt of a global approach is from start expected to be unsuccessful. To sustain this statement, the example of the completely different behaviour manner of two dynamical systems where fluid friction and dry friction respectively, occur can be given. While the fluid friction leads to linear differential equations, [2], the dry friction performs according to differential equations with strong nonlinear degree, [3]. Moreover, the fact that the behaviour of a system where dry friction occurs is described using inequalities makes more complicate the study of these systems and the notion of unilateral constraint is required. Thus, the presence of dry friction in a dynamical system is responsible for the occurrence of bifurcation phenomenon, [4]. The parameter essential to describe the friction force is the coefficient of dry friction. But the distinction has to be made between the cases when relative motion exists or not between the surfaces, therefore two coefficients of friction must be defined, a static one and a dynamic (or kinetic) one. In the absence of relative motion the friction force can take any value within a range dependant on the coefficient of static friction and on the normal force. Once the relative motion between the contacting surfaces happens, according to Amonton-Coulomb law, the friction force is completely determined by the coefficient of dynamic friction, normal force and relative velocity, [5].

Concerning the three mentioned parameters, the most difficult to assess is the coefficient of dynamic friction. The attempt to go beyond this stage by studying the literature is discouraged by the fact that the values of this coefficient must be chosen between two limiting values, hardly ever with supplementary indications. Additionally, the stunning technological progress came with new materials that have not yet complete characterisation in literature. Therefore, it is recommended that whenever it possible, the coefficient of friction should be evaluated for the actual conditions.
There are a number of methods for finding the coefficient of dynamic friction. A most convenient method is the inclined plane methodology, when the coefficient of dynamic friction results as the tangent of the angle for which a body placed on the inclined plane starts moving. The technique presents two main inconveniences: a) after the start of motion, due to the fact that the limit value of the coefficient of static friction is greater than the coefficient of dynamic friction, the body begins an accelerated motion, not a uniform one - as the hypothesis for definition of friction angle requires; b) the necessity to provide relatively large regions where the tribological characteristics of the two surfaces must remain constant. It results that for finding the coefficient of dynamic friction there are demanded methods implying surfaces of reduced dimensions and between which a permanent relative motion must exist in order to avoid the contribution of static friction, [6].

2. Calculation of the spinning torque
It is considered the Hertzian contact between two axi-symmetric bodies and therefore, the pressure distribution on the contact surface is a semi-ellipsoidal one, as shown in figure 1.

![Figure 1. Pressure and relative motion on the contact area.](img)

The elementary spinning torque is:

\[ dM_f = \mu \rho p(\rho, \theta)dA, \]  

(1)

where \( \mu \) is the dynamic coefficient of sliding friction, \( \rho \) is the current radius of point \( M \), \( p(\rho, \phi) \) is the pressure on contact area in the considered point and \( dA \) is the elementary area. Considering the axial symmetry of the problem, the contact area is a circular one, having the radius, [7]:

\[ r_{\text{max}} = \frac{3}{2} \kappa Q. \]  

(2)

The pressure distribution on the contact area has a semi-ellipsoidal form:

\[ p(\rho, \phi) = p_0 \sqrt{1 - \frac{\rho^2}{r_{\text{max}}^2}}, \]  

(3)

where:

\[ p_0 = \frac{1}{\pi} \frac{3}{2} \kappa^2 Q. \]  

(4)
is the maximum contact pressure, \( Q \) being the normal force. In relations (2) and (4):

\[
\eta = \frac{1}{E_1} \left( 1 - v_1^2 \right) + \frac{1}{E_2} \left( 1 - v_2^2 \right)
\]

(5)

is the contact stiffness, where \( E_{1,2} \) represent the Young modulus of the materials and \( v_{1,2} \) are the Poisson coefficients respectively. The parameter \( \kappa \) represents the reduced curvature of contact which, considering the axial symmetry of the two bodies, takes the form:

\[
\kappa = \frac{1}{2R_1} + \frac{1}{2R_2},
\]

(6)

\( R_{1,2} \) being the principal curvature radii of the bodies in the theoretical contact point. The total moment of friction is found by integrating equation (1) on the entire contact area:

\[
M_f = \int_0^{2\pi} \int_0^{r_{\text{max}}} \rho \mu p_0 \left( 1 - \frac{\rho^2}{r_{\text{max}}^2} \right) \rho \, d\rho \, d\phi.
\]

(7)

For given body geometries and constant value of normal force \( Q \), \( r_{\text{max}} \) and \( p_0 \) are constant and consequently, the friction torque is constant. Subsequent to this observation, the methodology is proposed. The contact between two axi-symmetric bodies, a mobile one and a fixed one, is considered.

The normal force \( Q \) is given by the weight of mobile body. By writing the moment of momentum theorem with respect to the rotation axis, for the mobile body:

\[
J_z \ddot{\epsilon} = M_f,
\]

(8)

where \( J_z \) is the axial moment of inertia with respect to rotation axis, \( \epsilon \) is the angular acceleration, one concludes that the angular acceleration is invariable and as a consequence, the angular velocity will have a linear variation. By performing the double integration in equation (7) and considering equation (2) and equation (4), the following expression of coefficient of friction is found:

\[
\mu = -\frac{3\pi}{16} \sqrt{\frac{3 \eta}{2 \kappa Q^4}}.
\]

(9)

3. Dynamic simulation of mobile body motion

In experimental tests, the authors considered the contact between a ball from a bearing and a glass concave lens. The concave glass lens was the option due to the fact that by fixing the lens, the contacting ball has five degrees of freedom, namely three rotations and two translations. The concave shape of the lens ensures an oscillatory motion for the ball’s centre. Initially, the ball was launched by simply rotating it by hand. It was noticed that motion of the ball after motion initiation differs substantially from one launch to another. Beside the rotation motion, the ball took a precession motion that sometimes asymptotically diminished but other time amplified itself and making the ball leave the surface of the lens. The contact comes apart when the initial velocity of the ball exceeds a certain value. This aspect was revealed by the dynamic simulation accomplished with the MSC-ADAMS software, shown in figure 2. In figure 3 there are presented the projections on a horizontal plane of the trajectories of the ball’s centre for several values of the coefficient of friction.
Figure 2. Modelling of the ball-lens contact using MSC-ADAMS.

Figure 3. Projection on a horizontal plane of ball’s centre trajectory.

In figure 4 and figure 5 there are presented the variations with time of angular velocity and velocity of mass centre for different values of coefficient of friction. The simulations were made considering both static friction and dynamic friction incidence, the values of the friction coefficients being presented on plots, \( \mu_s \) and \( \mu_d \) are the static and dynamic sliding friction coefficients, respectively. One can notice that there are different variations for angular velocity and centre of mass velocity. Another aspect to be emphasised is that for small values of coefficients of friction, the angular velocity presents a quasi-linear decrease (arrow (a) in figure 4). The intriguing fact is that the values of motion amplitude increase towards the end of the motion. The final oscillations would lead to the impossibility of a motion with a single degree of freedom.

Figure 4. Angular velocity of the ball variation. Figure 5. Velocity of the centre of mass.

The graph of angular velocity dependence upon the velocity of mass centre is extremely suggestive, as seen from figure 6. All the plots from figure 6 start from the same point (1) corresponding to the launching instant. At the final, all the graphs take the shape of a line passing through origin, (2). On the straight line region of the plot, there is proportionality between the velocity of mass centre and the angular velocity of the ball, condition characteristic to pure rolling.

Figure 7 presents the values of velocities components with respect to a trihedral fixed on the ball, for the case with minimum friction considered in figure 6, \( \mu_s=0.2 \) and \( \mu_d=0.1 \). One can notice
that only the $\omega_y$ and $\omega_z$ components are not zero, but while the spin angular velocity $\omega_y$ decreases asymptotically to zero, the precession velocity $\omega_z$ remains actually constant beginning from a certain instant. This remark shows that the pure rolling condition is ensured by the precession velocity while the sliding friction influences only the spinning component.

4. Test rig and experimental results

Figure 8 presents a first option of the device, where it can be noticed the non-contact tachometer 1 used in evaluation of angular velocity of the ball 3 that is in contact with the concave lens 2. To drive into motion the ball, a drilling machine 4 is used, with the axis oriented on the vertical in the centre of the ball. In the mandrel of the machine is placed an element of soft copper 5 that presents an inner conical cavity used in ball driving when the part is brought into contact to the ball.

Even after using the drilling machine in driving the ball, one could notice that the time required for stabilizing the motion of the ball is relatively long. The motion of the ball was substantially improved after replacing it with a smaller ball with a collar attached as fly-wheel, as presented in figure 9.

In figure 9 and figure 10 there are presented the angular velocities variations for the ball and the ball with fly-wheel. In figure 10 one can observe the presence of a rectilinear section (1) of the graph which will be further used in estimation of angular velocity. The experimental results registered using the tachometer are represented in figure 11 together to the interpolation line.

The angular acceleration can be found as the slope of the interpolation line from figure 12. Next, the experimental data for two cases will be presented:

1) the mobile body is a steel ball, 40mm in diameter, placed on a lens of 200mm radius. The value of angular acceleration is equal to the gradient of the interpolation graph for the angular velocity variation, figure 12;

2) the mobile body is a bearing ball, with the diameter 30mm, introduced in a steel disc of diameter 98mm and thickness of 8mm, with a related hole.

It must be remarked that unlike the dynamic simulation presented above, during the experimental tests it was noticed that regardless the shape of mobile body, after a certain period, the body motion became a rotation around the vertical passing through the lower point of the lens. As expected, this motion appears more rapid in the case of ball with attached collar. The results are presented in table 1.

The values obtained for the coefficient of friction are very close together and in concordance to the values from literature. In [8] it is given for the glass-metal contact a kinetic coefficient of friction in the range $\{0.2 \div 0.3\}$. 

Figure 6. Variation of angular velocity of the ball versus the velocity of mass centre.

Figure 7. Variation of components of angular velocity in time.
Figure 8. Experimental test rig.

Figure 9. Substitution of the ball with a ball with fly-wheel.

Figure 10. Variation of angular velocity of the ball.

Figure 11. Variation of the angular velocity of the ball with collar.

Figure 12. Experimental results and the interpolation curve.
Table 1. Experimental data

| Body/parameter | $M$ [kg] | $J_z$ [kg·m²] | $r_{ball}$ [m] | $r_{lens}$ [mm] | $\varepsilon$ [rad / sec²] | $\mu$ |
|----------------|---------|---------------|----------------|----------------|-----------------------------|------|
| Ball           | 0.261   | 4.4.18$\cdot$10$^{-5}$ | 20$\cdot$10$^{-3}$ | 200$\cdot$10$^{-3}$ | $-0.401$ | 0.158 |
| Ball with collar | 0.537   | 4.74$\cdot$10$^{-4}$ | 15$\cdot$10$^{-3}$ | 50$\cdot$10$^{-3}$ | $-0.085$ | 0.162 |

5. Conclusions
The paper presents a method and corresponding test rig for evaluation of dynamic coefficient of friction. The principle of the method consists in finding the angular velocity variation of a mobile axi-symmetric body. From the law of angular velocity variation and for the case of dry friction with constant coefficient of friction, the angular acceleration of the body can be found by approximating the graph with a line. The most difficult task is to drive the mobile body into steady motion. It was noticed that for the general case the motion of the body is very complex. To obtain a steady motion, the mobile body was placed on a concave lens, thus ensuring an asymptotic motion of the centre of mass. A dynamical simulation made with specialised software proved that the angular velocity has two components, rotation and precession. It was noticed that due to friction, the precession component diminishes and therefore the motion of the body is a rotation around the vertical axis. Estimation of the coefficient of friction must be made using the law motion corresponding to this phase of motion. Initially, a bearing ball was used as mobile body. In a while it was remarked that the time required to attain steady motion around fix axis reduces considerably when a collar playing the fly-wheel is attached to the ball. The experimental results of coefficient of friction obtained for the two rolling bodies are very close together and in concordance to the values from literature.

6. References
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