A cellular automaton for the factor of safety field in landslides modeling

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Landslide inventories show that the statistical distribution of the area of recorded events is well described by a power law over a range of decades. To understand these distributions, we consider a cellular automaton to model a time and position dependent factor of safety. The model is able to reproduce the complex structure of landslide distribution, as experimentally reported. In particular, we investigate the role of the rate of change of the system dynamical variables, induced by an external drive, on landslide modeling and its implications on hazard assessment. As the rate is increased, the model has a crossover from a critical regime with power-laws to non power-law behaviors. We suggest that the detection of patterns of correlated domains in monitored regions can be crucial to identify the response of the system to perturbations, i.e., for hazard assessment.

Introduction As for earthquakes and forest fires, there is a compelling evidence that the landslide frequency-size distributions are power-law functions of the area [Turcotte et al., 2002]. The presence of these broad distributions has crucial consequences on both the basic understanding of these phenomena and the practical and relevant purposes, such as the evaluation of natural hazards. Here, we introduce a cellular automaton that is aimed at modeling the general features of landslides, and is focused on the dynamical evolution of a space and time dependent factor of safety field. This model is very simple, but it is able to give a comprehensive picture of the avalanching phenomena and to reproduce some well-known properties of landslide distributions.

Several authors invoked the paradigm of self-organized criticality (SOC) [Bak et al., 1987; Jensen, 1998; Turcotte, 1999] to explain landslide distributions [Turcotte et al., 2002; Pelletier et al., 1997; Hergarten and Neugebauer, 2000]. Although the “critical” nature of the present phenomenon is not yet assessed and many authors believe that deviations from power-law appear to be systematic for small landslides data [Stark and Hovius, 2001; Pelletier et al., 1997; Brardinoni and Church, 2004; Malamud et al., 2004], several regional landslide inventories records show robust power-law distributions of large events with an exponent around \( \alpha \approx 2.5 \) [Turcotte et al., 2002], ranging approximately from \( \alpha \approx 1.75 \) for rockfalls to \( \alpha \approx 2.8 \) for mixed landslides (see [Dussauge et al., 2003; Faillettaz et al., 2004] and references therein). The universality of such an exponent is still debated (see [Turcotte et al., 2002; Dussauge et al., 2003; Faillettaz et al., 2004; Malamud et al., 2004] and references above), and its reported values are far from the one in the original “sandpile model” [Bak et al., 1987], where \( \alpha \approx 1.0 \). Recently, the reported values of \( \alpha \) have been obtained by introducing two-thresholds mechanisms in models that relate landslide dynamics both to SOC [Pelletier et al., 1997; Hergarten and Neugebauer, 2000] and to non SOC cellular automata [Faillettaz et al., 2004]. Furthermore, models based on \( \Gamma \) [Malamud et al., 2004] or Pareto [Stark and Hovius, 2001] distributions have been proposed.

Here, we consider a model inspired to an anisotropic version of the Olami-Feder-Christensen (OFC) [Olami et al., 1992] cellular automaton and subject to a finite driving rate [Hamon et al., 2002]. The model describes the evolution of a space and time dependent factor of safety, which is investigated for the first time in the present framework. In particular, we outline the essential role played by the rate of change of the system dynamical variables (variation of pore water pressure, lithostatic stress, cohesion coefficients, etc., [Helley et al., 2004; Iverson et al., 2000]) induced by external triggers. We find the model to be at the edge of the SOC limit. Actually, such a limit, which is achieved only in the asymptotic condition of vanishing driving rate, is hardly attainable in a real landslide process. The model is able to reproduce power-law distributions with exponents very close to the observed values. Power-laws are robust even though their exponent smoothly depends on system parameters (e.g., time derivative of the factor of safety and its dissipation level, see below). In this sense, although the SOC paradigm to some extent may be applied to landslides [Turcotte et al., 2002], the idea of universality, within this model, must be restricted to the shape of the frequency-size distribution rather than to its exponent, as can be deduced from some catalogues [Dussauge et al., 2003; Faillettaz et al., 2004]. Finally, in presence of strong driving rates we find that the model has Gaussian behaviors. We examine below the implications of our results on hazard assessment.

The Model The empirical Mohr-Coulomb failure criterion establishes that landslides occur when the shear stress exceeds a maximum value, which is given by \( \tau_{\text{max}} = c + (\sigma - u) \tan \phi \), with \( \sigma \) the total normal stress, \( u \) the pore-fluid pressure, \( \phi \) the angle of internal friction of the soil and \( c \) the cohesional (non-frictional) component of the soil strength [Terzaghi, 1962]. In literature, the factor of safety, \( FS \), against slip is defined by the ratio of the maximum shear strength \( \tau_{\text{max}} \) to the disturbing shear stress \( \tau \)

\[
FS = \frac{\tau_{\text{max}}}{\tau}.
\]

(1)

If \( FS > 1 \), resisting forces exceed driving forces and the slope remains stable. Slope failure starts when \( FS = 1 \). Although the practical determination of \( FS \) is difficult, simple one-dimensional infinite-slope models can quantify how \( c, u \) and \( \phi \) influence the Coulomb failure and show that the groundwater term has the most widely ranging influence [Iverson et al., 1997]. Traditional models generally treat soils and rocks as continuous porous media that obey to the Darcy’s law. Actually, field evidence indicates that the hydrology of some natural slopes is strongly influenced by discontinuities such as fractures and macropores. In practice, observations of large spatial and temporal fluctuations of water flow, within slopes at different sites, support the assertion that water-flow paths and permeability continually change...
within the slopes and also during the failure, providing different local values of the pore pressures and of the cohesion [Iverson et al., 1997; Helley et al., 2004; Iverson et al., 2000].

In order to take into account the complex non-homogeneous structure of a slope in the above failure condition, we consider a site and time dependent factor of safety, $F_S$. In particular, we approximate a natural slope by a two-dimensional (square) grid and define on each cell, $i$, of the lattice a local variable $e_i = 1/F_S$. Such a local inverse factor of safety is the fundamental dynamical variable of our model. The presence of diffusion, dissipative and driving mechanisms acting in the soil, such as those on the water content, inspires the dynamics of our model, which is defined by the following operations. Starting from a random and "stable" initial configuration ($e_i < 1 \forall i$), the system is subject to changes caused by some external trigger, for example a uniform rainfall, and the values of $e_i$ on each of our grid change at a given rate $\nu$, $e_i \to e_i + \nu$. For the sake of simplicity, we consider here only a uniform driving rate, $\nu$, but different choices can be made to simulate the effect of different hydrologic and external triggering mechanisms. The model is driven as long as $e_i < 1$ on all sites $i$. Then, when the generic site $i$ becomes unstable (i.e., overpasses the threshold, $e_i \geq 1$), it relaxes with its neighbors according to the rule:

$$e_i \to 0; \quad e_{nn} \to e_{nn} + f_{nn} e_i,$$

where the index $nn$ denotes the nearest neighbors of site $i$ and $f_{nn}$ is the fraction of $e_i$ toppling on $nn$ (after failure we set $1/F_S = 0$ for simplicity, as any other finite level would work [Jensen, 1998]). This kind of chain relaxations ("avalanches") is considered to be instantaneous compared to the time scale of the overall drive and it lasts until all sites are below threshold. The model is said to be conservative if $C = \sum_{nn} f_{nn}$ is finite. Since many complex dissipative phenomena (such as evaporation, mechanism, infiltration, volume concentrations, etc. [Fredlund and Rahardjo, 1993]) contribute to a dissipative stress transfer, we consider the non-conservative case $C < 1$, which is different from previous landslide models [Pelletier et al., 1997; Hergarten and Neugebauer, 2000].

Since gravity individuates a privileged direction, we consider an anisotropic model where the fraction of $e_i$ moving from the site $i$ to its "downward" (resp. "upward") neighbor on the square lattice is $f_i$ (resp. $f_i > 0$). The fraction to each of its "left" and "right" neighbor, in particular, we assume $f_a < f_a$ and $f_i = f_i < f_a$, respectively. This choice of parameters is made in the attempt to sketch the complex relaxation processes occurring in a slope failure. The conservation level, $C$, and the anisotropy factors, $f_s$, which we assume to be uniform, are actually related to the local soil properties (e.g., lithostatic, frictional and cohesive properties), as well as to the local geometry of the slope (e.g., its morphology). The rate of change of the inverse factor of safety, $\nu$, which is induced by the external drive (e.g., rainfall) and is related to soil and slope properties, quantifies how the triggering mechanisms affect the time derivative of the FS field.

**Numerical Results**

We consider a $64 \times 64$ square lattice, implementing both cylindrical (open along the vertical axis and periodical along the horizontal axis) and open boundary conditions, which do not give appreciable differences. Once the system has attained a stationary state in its dynamics, we study the probability distribution, $P(s)$, of avalanches of size $s$. During a run (we treat statistics of over $10^6$ events per run) the conservation level $C$ and the rate, $\nu$, are kept fixed. Examples of the frequency-size distribution of avalanches, $P(s)$, are shown in figures 1 and 2. In figure 1, the different curves correspond to different values of the rate $\nu$, for $C = 0.4$, and in figure 2 to different values of $C$, for $\nu = 5 \cdot 10^{-3}$. In the limit of very small driving rate, i.e., $\nu \to 0$, the distribution of events, $P(s)$, exhibits the typical SOC structure (see figure 1): a power law characterized by a critical exponent $\alpha$, $P(s) \sim s^{-\alpha}$, in agreement with the experimental evidence on medium and large landslides [Turcotte et al., 2002; Pelletier et al., 1997; Hergarten and Neugebauer, 2000; Faillettaz et al., 2004; Brizardoni and Church, 2004; Malamud et al., 2004; Dussauge et al., 2003], followed by a size dependent exponential cutoff [Jensen, 1998]. By increasing the rate $\nu$, the power-law regime shifts towards larger sizes and at some point the probability distribution apparently shows a maximum for a value $s^*$. There are two regimes: for large landslides ($s > s^*$) the above structure $P(s) \sim s^{-\alpha}$ is found, while for small events ($s < s^*$) an increasing function of $s$ is observed. Such a complex structure is absent in SOC models, instead a maximum is found in landslide inventory maps for small landslide data, although there is no consensus about the nature of such a feature [Stark and Hovius, 2001; Brizardoni and Church, 2004; Malamud et al., 2004]. The values of the power-law exponent, $\alpha$, by varying the rate, $\nu$, and $C$ are very close to those experimentally found [Turcotte et al., 2002; Dussauge et al., 2003]. As in the original isotropic OFC model [Olami et al., 1992], the critical exponent decreases with the level of conservation, $C$ (see inset in figure 2). The value of $\alpha$ slightly changes with the anisotropic ratios $f_{na}/f_{aa}$ and $f_{la}/f_{aa}$, except when they get too small [Piegari et al., 2005] where, as found also in other models [Amitrano et al., 1999; Amitrano, 2003; Faillettaz et al., 2004], the event size distribution is considerably modified. The power-law regime is crucially robust to changes in system parameters. For instance, in the case of figure 1 it can be found for $\nu$ up to approximately $10^{-2}$, all over the range $C \in [0.4, 0.8]$. It is worth noticing that the $\alpha$ values here obtained are comparable to those found in models of failure in fiber bundle [Henner et al., 1992; Hansen et al., 1994; Hidalgo et al., 2002].

As it can be seen in figure 1, a further increase of the driving rate (above $\nu \sim 10^{-2}$) causes a crossover to a markedly different regime where power-laws are no longer apparent and a bell shaped (Gaussian) distribution emerges, whose peak shifts towards larger sizes and shrinks up. Such a behavior is to be expected since for strong driving rates all internal correlations are washed out.

Summarizing, the conservation level, $C$, and the time derivative of $1/F_S$, $\nu$, turn out to be important to determine landslide probability distributions: in the limit $\nu \to 0$, the model is indeed in the SOC class; for small but finite $\nu$ the system is at the edge of SOC and the critical behaviors are still largely observed; finally, as $\nu$ gets large enough, Gaussian properties are found. Thus, in the small $\nu$ regime, our model reproduces the general properties of existing catalogs and can help interpreting them, while in the large $\nu$ regime it foresees a different class of behavior. Nevertheless, for the sake of clarity, we have considered the simple case where the rates $\nu$ and $C$ are uniform. Thus, the distributions of figure 1 may be not directly comparable to landslide inventories, which gather events with non-uniform driving rates.

Pictures of a typical "avalanche" in the different regimes discussed above are plotted in figure 3 (upper panels) with the corresponding values of the factor of safety, $F_S$, on the modelgrid after the avalanche (lower panels). The snapshots in the upper panels, taken in two systems driven at different rates (left $\nu = 5 \cdot 10^{-3}$, right $\nu = 5 \cdot 10^{-2}$), show a typical event with size $s = 230$ (such a value is chosen because it has approximately the same probability in the two cases, see figure 1): the system on the left is in the power-law regime; the one on the right is in the non power-law regime. The difference of the avalanche geometry in the two cases is impressive. Domino effects are crucial to determine a "catastrophic" event when the system is governed.
by a power-law statistics, where a huge compact landslide is present [Pietronero and Schneider, 1991] with a typical size of the order of the system size (left-upper panel). Conversely, large events are expected at higher $\nu$ (where indeed the average size $s$ is much larger than 230), but in such a regime a typical event with $s = 230$ is made of many tiny unconnected avalanches (summing up to $s = 230$).

Interestingly, even though the $P(s)$ is very different in the two cases, the probability distribution, $P(F_S)$, of the spatial values of $F_S$ on the grid has a similar Gaussian shape, with comparable averages ($\langle F_S \rangle$) and fluctuations ($\Delta F_S^2$): ($\langle F_S \rangle = 2.20$ and $\Delta F_S^2 = 0.16$ for $\nu = 5 \cdot 10^{-3}$, $\langle F_S \rangle = 2.53$ and $\Delta F_S^2 = 0.06$ for $\nu = 5 \cdot 10^{-2}$) laying far above the instability threshold $F_S = 1$. Thus, a measure of just an average safety factor on the investigated area could provide only very partial information about the statistics governing landslide events.

The origin of the striking difference of the $P(s)$ in the two considered cases traces back to the relative extension of spatial correlations of the factor of safety, $F_S$, which is derived from the correlation function:

$$C(\vec{z}) = \frac{\langle F_S(\vec{r})F_S(\vec{r} + \vec{z}) \rangle - \langle F_S(\vec{r}) \rangle^2}{\langle F_S(\vec{r})^2 \rangle - \langle F_S(\vec{r}) \rangle^2}$$

where $F_S(\vec{z})$ is $F_S$ at position $\vec{z}$ (here we take $\vec{z}$ along the direction of the slope) and the average is over the system sites. As it is well-known [Jensen, 1998], we find that $C(\vec{z}) \propto \exp(-z/\xi)$, where $\xi$ is the spatial correlation length of $F_S$. The value of $F_S$, on site $s$ is shown in the lower panels of figure 3, in gray scale, for the same cases pictured in the upper panels: patterns of large correlated areas (regions with similar values of $F_S$ site, i.e., the same color) are apparent in the left bottom panel and, in practice, absent in the right one. In the power-law regime (e.g., $\nu = 5 \cdot 10^{-3}$), the correlation length, $\xi$, is of the order of the system size; thus, even a very small perturbation (say, a drop of water) at one single point can trigger huge system responses. Instead, in the non power-law regime (e.g., $\nu = 5 \cdot 10^{-2}$) large-scale correlations are absent; here, large events trivially occur just because the strong external driving rate makes likely that many cells simultaneously approach the instability threshold. The detection of patterns of correlated domains (i.e., the size of $\xi$) in investigated areas results, thus, to be a crucial tool to identify the response of the system to perturbations, i.e., for hazard assessment.

**Conclusions** To summarize, we have investigated a continuously driven anisotropic cellular automaton model for the characterization of landslide size distributions. The model may help in interpreting the general behaviors observed in real systems. In particular, we have found that different values of the driving rate give rise to different statistical distributions of events. The determination of correlated domains in the factor of safety becomes crucial for landslide classification and, consequently, for hazard assessment.

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Figure 1. The probability density distribution, $P(s)$, of avalanches of size $s$ is plotted for the shown values of the time derivative of the inverse factor of safety, $\nu$ (model size $L^2 = 64 \times 64$, conservation level $C = 0.4$, anisotropy coefficients $f_a/f_d = 2/3$ and $f_d/f_a = 5/6$). The power law $P(s) \sim s^{-10}$ found in the limit $\nu \rightarrow 0$ is partially preserved by increasing $\nu$ up to a point where a bell shaped behavior is clearly observed.
**Figure 2.** The probability distribution, $P(s)$, of avalanches of size $s$ is plotted for the shown values of level of conservation, $C$ ($\nu = 5 \cdot 10^{-3}$, other params as in figure 1). The inset shows the exponent $\alpha$ (confidence interval 95%) of the power-law fit as a function of $C$.

**Figure 3.** Top panels: Two snapshots of a typical landslide event of size $s = 230$ on our $64 \times 64$ grid, in two cases with a driving rate $\nu = 5 \cdot 10^{-3}$ (left figure) and $\nu = 5 \cdot 10^{-2}$ (right figure). The cells marked in black are those which reached the instability threshold. Bottom panels: The pictures plot the local value of the factor of safety, FS, corresponding to the stable configurations reached after the avalanches shown in the upper panels. The FS values have been associated to ten levels of color from white to black, in order to measure the distance of a cell from its instability condition: the darker the color, the farther is the cell from the instability threshold. In the panels it is possible to recognize as dark areas the avalanches shown in the corresponding upper grids. In particular, dark areas are related to previous landslide events, as the lighter areas indicate regions of future events. In the left figure large correlated regions (compact areas with same color) are observed, whereas their size is small in the right figure.
