Optical-Cavity Limits on Higher-Order Lorentz Violation

Yuta Michimura,1 Matthew Mewes,2 Nobuyuki Matsumoto,1 Yoichi Aso,1 and Masaki Ando1,3

1Department of Physics, University of Tokyo, Bunkyo, Tokyo 113-0033, Japan
2Department of Physics and Astronomy, Swarthmore College, Swarthmore, Pennsylvania 19081, USA
3National Astronomical Observatory of Japan, Mitaka, Tokyo 181-8588, Japan

(Dated: December 30, 2013)

An optical ring cavity is used to place the first laboratory constraints on parity-odd nonrenormalizable Lorentz violation. Variations in resonant frequencies are limited to parts in \(10^{15}\). Absolute sensitivity to Lorentz-violating operators of mass dimension 6 is improved by a factor of a million over existing parity-even microwave-cavity bounds. Sensitivity to dimension-8 violations is improved by fourteen orders of magnitude.

PACS numbers: 03.30.+p, 06.30.Ft, 11.30.Cp, 42.60.Da

Lorentz invariance lies at the foundations of both the Standard Model of particle physics and General Relativity. However, attempts to reconcile these two theories have led to the idea that Lorentz invariance may only be approximate at attainable energies [1]. The prospect of measuring quantum-gravity-induced Lorentz violation has spurred a large number of experimental and theoretical studies in recent decades [2]. These studies include modern versions of the classic Michelson-Morley experiment [3] that are based on electromagnetic resonant cavities [4–9]. Here we present a search for Lorentz violation using an optical ring cavity. We obtain the first bounds on parity-odd nonbirefringent nondispersive Lorentz-violating operators of mass dimensions \(d = 6\) and 8. Our results also represent the first optical-frequency laboratory test of Lorentz violation associated with operators of dimensions \(d = 6\) and 8, yielding sensitivities that are many orders of magnitude beyond existing microwave constraints.

Lorentz violation introduces a number of exotic phenomena in resonant cavities, such as an unconventional relationship between frequency and wavelength [10, 11]. Since rotations are a part of Lorentz symmetry, violations also typically introduce orientation dependence, a key signature in resonant-cavity tests. Our experiment searches for Lorentz violation by comparing the resonant frequencies \(\nu_{\pm}\) of two counter-propagating directions in a ring cavity as it is rotated in the laboratory. Significant Lorentz violation would lead to modulations in the fractional difference \(\Delta \nu/\nu = (\nu_+ - \nu_-)/\nu\) at harmonics of the rotation rate \(\omega_{\text{rot}}\). We would also expect slow modulations at the sidereal frequency \(\omega_{\text{sid}} \approx 2\pi/(23\text{ hr 56 min})\) due to the Earth’s rotation. So variations in \(\Delta \nu/\nu\) at frequencies \(\omega_{\text{sid}} = m\omega_{\text{rot}} + m'\omega_{\text{sid}}\) are a sign of Lorentz violation.

Ring-cavity experiments have recently emerged as promising parity-odd Lorentz tests [4, 5] that complement existing parity-symmetric experiments based on microwave cavities [6, 7] and Fabry-Pérot resonators [8, 9]. The counter-propagating waves in a ring are parity mirrors of each other. As a result, the signal \(\Delta \nu/\nu\) changes sign under a parity transformation and can only depend on parity-odd changes to the resonant frequencies \(\nu_{\pm}\). Therefore, our experiment is only sensitive to Lorentz violations that are odd under parity.

While modulations in \(\Delta \nu/\nu\) are a generic signature of Lorentz violation, a complete theoretical model of the underlying physics is required to predict the precise form of the modulations and how they relate to other experiments. The general description of Lorentz violation is provided by the Standard-Model Extension (SME), which encompasses all known physics and all realistic violations of Lorentz invariance [11, 12].

A Lorentz-violating term in the Lagrange density of the SME is constructed from a conventional operator contracted with a constant tensor coefficient to form an observer-independent scalar. In natural units with \(\hbar = c = 1\), an unconventional term is partially characterized by the mass dimension \(d\) of the operator. For example, a dimension-\(d\) operator in the pure photon sector contributes through a term of the generic form \(\mathcal{L} \supset K_{\alpha_1 \alpha_2 \ldots \alpha_d} A_{\alpha_1} \partial_{\alpha_2} \ldots \partial_{\alpha_d} A_{\alpha_d}\), where \(A_{\alpha}\) is the photon field and \(K_{\alpha_1 \alpha_2 \ldots \alpha_d}\) are constant coefficients that govern the size of the Lorentz-violating effects. The operator, constructed from fields and derivatives of fields, has mass dimension \(d\), and dimensional analysis shows that the dimension of the coefficient is \(4 - d\). By convention, measurements of coefficients are reported in units of GeV\(^{4-d}\).

Restriction to renormalizable dimensions \(d = 3\) and \(d = 4\) yields the so-called minimal SME (mSME). The mSME has been studied extensively, but nonminimal terms with \(d > 4\) have received comparatively less attention due to the large variety and complexity of the higher-order violations [2]. However, recent theoretical work has established phenomenology that opens up the nonminimal sector to experimentation [11]. The push to consider nonminimal terms in the SME is motivated in part by the apparent nonrenormalizability of gravity and by the possibility that higher-order violations with \(d > 4\) might dominate. Theories based on noncommutative spacetime coordinates provide an example where Lorentz violation emerges in the form of operators of non-renormalizable dimension only [13].

Within the photon sector of the SME, a subset of violations cause vacuum birefringence or dispersion, which
can be tightly constrained through astrophysical observations. However, a relatively large class of nonbirefringent nondispersive operators exist that have no leading-order effect on light propagating in vacuum. These are the so-called camouflage terms, whose coefficients are denoted \((\varphi_F^{(d)})_{n,m}\) for even dimensions \(d \geq 6\). While the camouflage coefficients cannot be probed directly in astrophysical tests, they can be tested in cavity-based experiments. The goal of this work is to search for unconstrained parity-odd camouflage coefficients for dimensions \(d = 6\) and \(d = 8\).

Many features of Lorentz violation in electromagnetism can be understood by examining the structure of the Lagrange density. Terms with \(d = 4\) have the same number of derivatives as the usual case, so the effects of \(d = 4\) Lorentz violation generally scale with photon energy or frequency in the same way as conventional physics. However, for \(d > 4\), the additional derivatives imply that the effects of Lorentz violation typically grow with frequency by a factor of \(\nu^{d-4}\) relative to Lorentz-invariant physics. This scaling gives optical cavities an inherent advantage over microwave cavities. We naively expect an increase in sensitivity by a factor of \(\sim 10^{(d-4)}\).

The couplings of fields and derivatives to the coefficient tensors are also the source of anisotropies, which we detect as modulations in \(\Delta \nu / \nu\). The effects are similar to those in anisotropic dielectrics, where the coefficients for Lorentz violation act as a dielectric tensor filling space. The additional derivatives in the higher-\(d\) terms imply added complexity and higher harmonics in the modulations of \(\Delta \nu / \nu\). For example, the \(d = 6\) camouflage coefficients can be shown to introduce monopole, dipole, and quadrupole behavior, while the \(d = 8\) coefficients also give hexapole and octupole structure. Our experiment is only sensitive to parity-odd dipole and hexapole anisotropies.

The experimental setup is illustrated in Fig. 1. It is based on a triangular optical ring constructed from three mirrors rigidly fixed on a spacer. A silicon piece is placed along one side of the triangle. The measured refractive index of the silicon piece was \(n = 3.69\) at the operational wavelength \(\lambda = 1550\) nm. All of the optics are placed in a vacuum enclosure and rotated by a turntable. A detailed description of the apparatus can be found in Ref. [5]. The time is chosen so that \(t = 0\) when the beam inside the silicon is aligned with the \(x\) axis (south).

The transmitted light of the counterclockwise beam is then reflected back into the cavity in clockwise direction. This creates the signal that is proportional to the frequency difference between the counterclockwise beam and the clockwise resonance. The double-pass configuration ensures that most of the effects from environmentally induced cavity-length fluctuations are canceled in the differential frequency measurement.

The data used in this analysis were taken at the University of Tokyo for 393 days between August 2012 and September 2013. Positive and reverse rotations of 420° were repeated alternately at the rotation speed \(\omega_{\text{rot}} = 30\)°/sec. Our analysis only uses the middle 360° of the rotation where the rotation rate is constant and fluctuations from the Sagnac effect can be neglected. The ring cavity was rotated approximately 1.7 million times during the data acquisition.

To test for Lorentz violation, one could perform a direct search for variations at the frequencies \(\omega_{m\nu'} = m\omega_{\text{rot}} + \nu'\omega_\phi\). However, a demodulation method is appropriate since \(\omega_{\text{rot}} \gg \omega_\phi\). We first consider a decomposition of the frequency difference into harmonics of \(\omega_{\text{rot}}\),

\[
\frac{\Delta \nu}{\nu} = \sum_{m > 0} \left[ C_m \cos (m\omega_{\text{rot}}t) + S_m \sin (m\omega_{\text{rot}}t) \right].
\]

A turntable rotation of 180° effectively interchanges the two counterpropagating solutions, reversing the sign of \(\Delta \nu / \nu\). This implies that we can restrict attention to odd values of \(m\). The dipole anisotropies introduced by the \(d = 6\) SME coefficients can only give rise to \(m = 1\). The dipole and hexapole structure for \(d = 8\) can give \(m = 1, 3\) harmonics. Note, however, that Lorentz-violating operators of higher dimension can lead to harmonics with \(m \geq 5\).

FIG. 1. Diagram showing the optical ring cavity and its orientation with respect to the standard laboratory frame defined in Ref. [10]. The time is chosen so that \(t = 0\) when the beam inside the silicon is aligned with the \(x\) axis (south).
The amplitudes $C_m$ and $S_m$ vary at harmonics of the sidereal frequency $\omega_{\oplus}$ and can be expanded as

$$
C_m = \sum_{m' \geq 0} \left[ C^{C}_{m m'} \cos (m' \alpha) + C^{S}_{m m'} \sin (m' \alpha) \right],
$$
$$
S_m = \sum_{m' \geq 0} \left[ S^{C}_{m m'} \cos (m' \alpha) + S^{S}_{m m'} \sin (m' \alpha) \right],
$$

where $\alpha = \omega_{\oplus} T_{\oplus}$ is the right ascension of the local zenith [11]. Any nonnegative $m'$ can contribute, but the multipole structure predicted by the SME gives $m' \leq 1$ for $d = 6$ and $m' = 0, 1, 2, 3$ for $d = 8$, so we limit our focus to $0 \leq m' \leq 3$.

The analysis starts by demodulating the data at frequencies $\omega_{\text{rot}}$ and $3\omega_{\text{rot}}$ to extract the amplitudes $C_m$ and $S_m$ in Eq. (1) for each rotation. Time series data for $C_m$ and $S_m$ are split into one-day intervals and fit to Eq. (2) by the least-squares method to extract the modulation amplitudes $C^{C}_{m m'}$, $C^{S}_{m m'}$, $S^{C}_{m m'}$, and $S^{S}_{m m'}$ for each day. Data sets for some of the amplitudes are shown in Fig. 2, as illustrations. Taking the weighted average over the 393 days gives our measured values for the modulation amplitudes, which are listed in Table I.

The error bars for $m = 1$ amplitudes are approximately two times larger than the errors for $m = 3$ amplitudes. The measured noise level in relative frequency was $5 \times 10^{-13}/\sqrt{\text{Hz}}$ at the rotation frequency $\omega_{\text{rot}}$ and $4 \times 10^{-13}/\sqrt{\text{Hz}}$ at $3\omega_{\text{rot}}$ when the turntable is not rotated. However, error bars in Table I are more than one order of magnitude larger than the expected value from these noise levels. This suggests the effect of turntable vibration, which is likely to be larger at the rotation frequency. The exact vibration level was not measurable in the current setup since the rotation is reciprocated during the data acquisition.

A number of systematic effects was studied. A major source of the systematic offset was the tilt of the turntable, which creates a slight change in the alignment of the incident beam. This effect was estimated to be less than 10% of the statistical error. The Sagnac effect also gives systematic offset to the modulation amplitudes if the rotational speed of the turntable fluctuates in $1/m$ of a rotational period. The measured fluctuation was less than 1 mrad/sec, and this effect is more than 4 orders of magnitude below our statistical error. Also, there was 3% uncertainty in the calibration of the signal for Lorentz violation. This uncertainty originated mainly from a slight drift of a nonzero detuning in the laser frequency servo.

To get constraints on SME coefficients, we consider each dimension $d = 6$ and $d = 8$ separately and place constraints under the assumption that only one of the two sets of coefficients is nonzero. The calculation outlined in Ref. [16] yields the relationship between the SME camouflage coefficients $(\beta_d^{(d)}|^{(0E)})$ and the modulation amplitudes $C^{C}_{m m'}$, $C^{S}_{m m'}$, $S^{C}_{m m'}$, and $S^{S}_{m m'}$. Each of the amplitudes is a linear combination of SME coefficients that depends on the orientation, length, and index of refraction of each arm of the cavity and the colatitude ($\chi = 54.3^\circ$) of the laboratory.

For $d = 6$, variations in the frequency difference with $m = 1$ and $m' = 0, 1$ arise through linear combinations of three camouflage coefficients. The measurements of the modulation amplitudes yield individual constraints.
TABLE II. Measurements of SME $d = 6, 8$ camouflage coefficients with $1\sigma$ errors.

| Coefficient                                                                 | Measurement            |
|----------------------------------------------------------------------------|------------------------|
| $(\tilde{c}_F^{(d) (OE)})_{110}$                                           | $(-0.1 \pm 1.5) \times 10^3$ GeV$^{-2}$ |
| $\text{Re}[(\tilde{c}_F^{(d) (OE)})_{111}]$                                | $(0.8 \pm 1.1) \times 10^2$ GeV$^{-2}$ |
| $\text{Im}[(\tilde{c}_F^{(d) (OE)})_{111}]$                                | $(-0.6 \pm 1.0) \times 10^2$ GeV$^{-2}$ |
| $(\tilde{c}_F^{(8) (OE)})_{310} - 0.020(\tilde{c}_F^{(8) (OE)})_{110}$    | $(-0.2 \pm 1.9) \times 10^9$ GeV$^{-4}$ |
| $\text{Re}[(\tilde{c}_F^{(8) (OE)})_{111} - 0.020(\tilde{c}_F^{(8) (OE)})_{111}]$ | $(1.4 \pm 1.3) \times 10^9$ GeV$^{-4}$ |
| $\text{Im}[(\tilde{c}_F^{(8) (OE)})_{111} - 0.020(\tilde{c}_F^{(8) (OE)})_{111}]$ | $(0.1 \pm 1.3) \times 10^9$ GeV$^{-4}$ |
| $(\tilde{c}_F^{(8) (OE)})_{330}$                                           | $(-0.8 \pm 3.3) \times 10^9$ GeV$^{-4}$ |
| $\text{Re}[(\tilde{c}_F^{(8) (OE)})_{331}]$                                | $(-0.3 \pm 1.9) \times 10^9$ GeV$^{-4}$ |
| $\text{Im}[(\tilde{c}_F^{(8) (OE)})_{331}]$                                | $(-0.2 \pm 1.9) \times 10^9$ GeV$^{-4}$ |
| $\text{Re}[(\tilde{c}_F^{(8) (OE)})_{332}]$                                | $(2.2 \pm 1.3) \times 10^9$ GeV$^{-4}$ |
| $\text{Im}[(\tilde{c}_F^{(8) (OE)})_{332}]$                                | $(0.2 \pm 1.3) \times 10^9$ GeV$^{-4}$ |
| $\text{Re}[(\tilde{c}_F^{(8) (OE)})_{333}]$                                | $(-0.1 \pm 1.6) \times 10^9$ GeV$^{-4}$ |
| $\text{Im}[(\tilde{c}_F^{(8) (OE)})_{333}]$                                | $(-0.1 \pm 1.6) \times 10^9$ GeV$^{-4}$ |

on these three coefficients. The results are summarized in Table II. Note that SME coefficients $(\tilde{c}_F^{(d) (OE)})_{njm}$ with $m \neq 0$ are complex, so we bound both the real and imaginary pieces. There are a total of three parity-odd camouflage coefficients for $d = 6$, so our experiment constrains the entire coefficient space accessible to parity-odd cavity experiments.

Considering only $d = 8$ coefficients for Lorentz violation, we find that ten combinations of coefficients contribute to modulations of the frequency difference $\Delta \nu/\nu$. Consequently, the bounds on the twenty-eight amplitudes in Table I reduce to ten bounds on combinations of $d = 8$ SME coefficients. These bounds are also summarized in Table II. There are a total of thirteen parity-odd camouflage coefficients for $d = 8$, so three linear combinations of coefficients remain untested. These may be accessed by future ring-cavity experiments with different configurations, yielding sensitivities to different combinations of coefficients.

The results in Table II are the first bounds on parity-odd camouflage coefficients for Lorentz violation. The current best bounds on the parity-even coefficients come from the microwave-cavity experiment in Ref. [7]. While this experiment and our experiment probe two independent sets of Lorentz violations, a comparison of the two illustrates the improvement in sensitivity that results from higher frequencies. Naive estimates suggest improvements of roughly eight orders of magnitude for $d = 6$ and sixteen orders of magnitude for $d = 8$ may be possible. However, due to noise differences, our experiment achieves an increase in sensitivity that is closer to a factor of a million for $d = 6$ and a factor of $10^{14}$ for $d = 8$.

Our bounds on SME coefficients for nonrenormalizable dimensions $d = 6$ and $d = 8$ are consistent with zero at $2\sigma$, showing no significant evidence for Lorentz violation. We achieved raw sensitivity on the order of $10^{-15}$ to orientation dependence in the frequency difference $\Delta \nu/\nu$. Parity-even optical cavities have achieved sensitivities at the $10^{-17}$ level in tests of the mSME [9], suggesting the potential for a hundredfold improvement in future cavity tests of higher-order Lorentz violation.

**ACKNOWLEDGMENTS**

We thank Alan Kostelecký and Shigemi Otsuka for useful discussions. This work was supported by Grant-in-Aid for JSPS Fellows No. 2510386.

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