Updated neutrino mass constraints from galaxy clustering and CMB lensing-galaxy cross-correlation measurements

Isabelle Tanseri\textsuperscript{a}, Steffen Hagstotz\textsuperscript{b,c,a}, Sunny Vagnozzi\textsuperscript{d,*}, Elena Giusarma\textsuperscript{e}, Katherine Freese\textsuperscript{a,f,g}

\textsuperscript{a}The Oskar Klein Centre for Cosmoparticle Physics, Department of Physics, Stockholm University, Roslagstullsbacken 21A, SE-106 91 Stockholm, Sweden
\textsuperscript{b}Universität-Sternwarte, Fakultät für Physik, Ludwig-Maximilians Universität München, Scheinerstraße 1, D-81679 München, Germany
\textsuperscript{c}Excellence Cluster ORIGINS, Boltzmannstraße 2, D-85748 Garching, Germany
\textsuperscript{d}Kavli Institute for Cosmology, University of Cambridge, Cambridge CB3 0HA, UK
\textsuperscript{e}Department of Physics, Michigan Technological University, Fisher Hall 118, 1400 Townsend Drive, Houghton, MI 49931, USA
\textsuperscript{f}Theory Group, Department of Physics, The University of Texas at Austin, 2515 Speedway, C1600, Austin, TX 78712-0264, USA
\textsuperscript{g}Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden

Abstract

We revisit cosmological constraints on the sum of the neutrino masses $\Sigma m_\nu$ from a combination of full-shape BOSS galaxy clustering $[P(k)]$ data and measurements of the cross-correlation between Planck Cosmic Microwave Background (CMB) lensing convergence and BOSS galaxy overdensity maps $[C_\kappa g^\ell]$, using a simple but theoretically motivated model for the scale-dependent galaxy bias in auto- and cross-correlation measurements. We improve upon earlier related work in several respects, particularly through a more accurate treatment of the correlation and covariance between $P(k)$ and $C_\kappa g^\ell$ measurements. When combining these measurements with Planck CMB data, we find a 95\% confidence level upper limit of $\Sigma m_\nu < 0.14\text{ eV}$, while slightly weaker limits are obtained when including small-scale ACTPol CMB data, in agreement with our expectations. We confirm earlier findings that (once combined with CMB data) the full-shape information content is comparable to the geometrical information content in the reconstructed BAO peaks given the precision of current galaxy clustering data, discuss the physical significance of our inferred bias and shot noise parameters, and perform a number of robustness tests on our underlying model. While the inclusion of $C_\kappa g^\ell$ measurements does not currently appear to lead to substantial improvements in the resulting $\Sigma m_\nu$ constraints, we expect the converse to be true for near-future galaxy clustering measurements, whose shape information content will eventually supersede the geometrical one.

Keywords: Neutrinos, Cosmic Microwave Background, Large-Scale Structure

*Corresponding author
Email addresses: isabelle.tanseri@gmail.com (Isabelle Tanseri)

Preprint submitted to Journal of High Energy Astrophysics July 25, 2022
1. Introduction

Neutrinos, while being among the most abundant particle species in the Universe, remain also one of the most elusive [1]. The observation of solar and atmospheric neutrino oscillations indicates that at least two out of three neutrino mass eigenstates are massive [2–6], a fact which remains the only direct evidence for new physics beyond the Standard Model of Particle Physics. It should therefore not come as a surprise that the value of the sum of the neutrino masses $\Sigma m_\nu$ is an extremely important experimental target. Oscillation experiments are insensitive to the absolute neutrino mass scale, and therefore to $\Sigma m_\nu$, which instead can be constrained by others types of probes: the kinematics of $\beta$-decay [14, 15], neutrino-less double-$\beta$ decay searches [16, 17] and, last but not least, cosmological observations [18–20]. Moreover, oscillation experiments are currently insensitive to the sign of the largest (atmospheric) mass-squared splitting, $|\Delta m_{31}^2|$, leaving two possibilities open for the so-called neutrino mass ordering (or hierarchy): the normal ordering with $\Delta m_{31}^2 > 0$ and $m_1 < m_2 < m_3$, and the inverted ordering with $\Delta m_{31}^2 < 0$ and $m_3 < m_1 < m_2$, where $m_1$, $m_2$, and $m_3$ are the masses of the three neutrino mass eigenstates, and the mass-squared splittings are defined as $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$.

As of today, cosmological probes provide the tightest constraints on $\Sigma m_\nu$, although such constraints are inevitably associated to a certain degree of model-dependence (see e.g. [19] for an up-to-date review). In particular, measurements of anisotropies in the thermal radiation from recombination, the Cosmic Microwave Background (CMB) [21–23], in combination with measurements of Baryon Acoustic Oscillations (BAO) in galaxy clustering data [24–26], have been able to provide extremely strong bounds on $\Sigma m_\nu$. Currently, one of the tightest upper bounds on $\Sigma m_\nu$ is $\Sigma m_\nu < 0.12 \text{ eV}$ at 95% confidence level (C.L.), inferred from a combination of CMB data from the Planck 2018 legacy data release and BAO data from the MGS, 6dFPGS, and BOSS DR12 galaxy surveys [23]. Cosmology is in principle also able to constrain the mass ordering, and recent cosmological observations have been argued to slightly favor the normal mass ordering (see e.g. [69–83]).

Neutrinos decouple from the primordial plasma at temperatures of $\mathcal{O}(\text{MeV})$, thus while highly relativistic, therefore behaving as radiation early on, including at the time of recombination (given current constraints on $\Sigma m_\nu$ which exclude the possibility that neutrinos were heavy enough to already behave as matter then). After they decouple, neutrinos start free-streaming with high thermal velocities. At late times, at least two out of three neutrino mass eigenstates become non-relativistic, and contribute to the matter component of the Universe. This aspect, in combination with their free-streaming nature, leads to massive neutrinos behaving as a hot dark matter component and suppressing power on small-scales. This small-scale power suppression is in principle one of the

1. See e.g. [7–13] for recent global fits to active and sterile neutrino parameters (which include both cosmological and non-cosmological probes), and discussions thereof.
2. See for instance [27–61] for examples of other recent works investigating constraints on neutrino masses and properties within various cosmological scenarios. See [62–66] for examples of mass-varying or decaying neutrino scenarios. See also [67, 68] for discussions on the implications of a detection or non-detection of the cosmological neutrino background.
tell-tale cosmological signatures of massive neutrinos [84–86], and can be searched for
instance through measurements of the clustering of tracers of the large-scale structure
(LSS), such as galaxies and quasars.

The effect of massive neutrinos on the CMB is instead more subtle. In general,
discussions as to what the effects on the CMB of changing a given cosmological parameter
are require specifying what other quantities are being kept fixed while the parameter in
question is varied, and $\Sigma m_{\nu}$ is no exception. In the CMB, the effect of non-zero $\Sigma m_{\nu}$ is
best discussed while fixing both the acoustic scale $\theta_s$ and the redshift of matter-radiation
equality $z_{\text{eq}}$. This choice a) ensures that the position and height of the first acoustic
peak in the CMB, both tightly constrained by observations [23], are left unchanged
while varying $\Sigma m_{\nu}$, and b) helps disentangling “direct” neutrino perturbation effects
from “background” effects which may instead be re-absorbed by suitably shifting other
cosmological parameters. Increasing $\Sigma m_{\nu}$ while keeping $\theta_s$ and $z_{\text{eq}}$ fixed leads to a small
reduction of power on large angular scales (due to a reduced late integrated Sachs-Wolfe
effect, but swamped by cosmic variance) alongside tiny shifts in the damping scale. 4

The most important effect on the CMB is instead related to the reduction of the lensing
potential, a direct consequence of the neutrino-induced small-scale structure suppression.
As the effect of lensing is to smooth the higher CMB acoustic peaks, increasing $\Sigma m_{\nu}$
slightly sharpens these peaks.

The above discussion makes it clear that LSS clustering measurements are a promising
probe of neutrino masses, and advances in the field lead us to expect that the strongest
bounds on $\Sigma m_{\nu}$ will soon come from datasets probing the imprint on $\Sigma m_{\nu}$ on structure
growth, rather than on the background evolution [93–95]. The neutrino-induced sup-
pressed structure growth is most cleanly imprinted on the small-scale amplitude of the
matter power spectrum, $P_{\text{mm}}(k)$: the latter can be measured indirectly through LSS
tracers such as galaxies [96, 97], or through the gravitational lensing of the CMB [96, 98]
or of background galaxies [99, 100]. Here, we shall mainly focus on galaxies as LSS
tracers, and consider full-shape (FS) measurements of the galaxy power spectrum. FS
galaxy power spectrum analyses do not come without significant challenges. One im-
portant challenge is related to our limited ability to model the underlying matter power
spectrum in the mildly non-linear regime. Another related challenge pertains to the
fact that galaxies are biased tracers of the underlying matter distribution, and therefore
do not faithfully trace the latter. The statistical relation between the galaxy and matter
overdensity fields is encapsulated in the galaxy bias parameter(s), which ultimately
capture complexities associated to galaxy formation and evolution.

While galaxy power spectrum analyses are undoubtedly challenging, and have often
been performed in the context of large collaborations (see e.g. [101–110]), various works
in recent years have attempted to extract information on cosmological parameters from
these types of measurements, adopting different theoretical models. Among these we
mention the Effective Field Theory of LSS (EFTofLSS), a formalism allowing to predict

---

3The acoustic scale is given by the ratio between the comoving sound horizon at recombination and
the angular diameter distance to recombination.

4See e.g. Fig. 4.10 in [87] and the associated discussion for an example of this exercise. Note, in
addition, that fixing $\theta_s$ and $z_{\text{eq}}$ ensures that massive neutrinos leave no visible imprint on the early
integrated Sachs-Wolfe effect, whose amplitude is very tightly constrained by CMB observations (see for example [88–92]).
the clustering of the LSS in the mildly non-linear regime in a robust symmetry-driven way (see e.g. [111–115]). Recent advances in the EFTofLSS have allowed for applications on real data from the BOSS survey, with extremely promising results (see e.g. [116–130] for examples in these directions). 5

Other works have applied related perturbation theory-based models (see e.g. [142, 143]) for the redshift-space galaxy power spectrum to real data [144, 145] or forecasts [146–151], tested these models on simulations [152–155], or investigated other ways of extracting (possibly compressed) information from the redshift-space FS galaxy power spectrum and higher order multipoles [156–167]. Finally, various works have used simulations to investigate the imprint of neutrino properties on the clustering of the LSS (see for e.g. [168–177]), as well as related cosmological observables (see e.g. [178–190]).

The strength and main advantage of several of these well-motivated first-principles theoretical models is that they are able to take into account and separate at the modeling level the effects of various different aspects pertaining to galaxy formation and biasing, with contributions to the redshift-space galaxy power spectrum captured by different operators and/or different galaxy bias parameters. At the same time, this strength comes at the cost of several extra nuisance parameters to be marginalized over, which can be problematic if the level of precision of the data is not sufficient as to be able to constrain them in a meaningful way, effectively leading to saturation of cosmological constraints. One may in principle attempt to exploit theory- or simulations-based relations to enforce prior relations between the various nuisance parameters, or set informative priors on some of them, as is routinely done in order to speed up analyses. However, following this route requires (more or less explicitly) making the assumption that one can indeed reliably model galaxy formation in the mildly non-linear regime, and any incorrect assumption in these assumed relations will propagate (as a modeling systematic) to the inferred cosmological parameters.

One important example in this sense is the EFTofLSS, the most general, symmetry-driven model for the mildly non-linear clustering of biased LSS tracers, which integrates out the complex and poorly-known details of short-scale physics by parametrizing these through a series of counterterms with functional form fixed by symmetry considerations, and amplitudes which are effectively treated as nuisance parameters to be fit to the data (see e.g. [191] for a recent review). The state-of-the-art implementation of the EFTofLSS to model the multipoles of the full-shape redshift-space galaxy power spectrum in the CLASS-PT Boltzmann solver [123] introduces up to 11 nuisance parameters per galaxy sample. 6 In practice, however, (physically motivated) Gaussian priors need to be imposed on most of these nuisance parameters in order to aid the convergence of analyses: see e.g. Eqs. (6.4,6.5) in [123], Eqs. (D.1,D.2) in [133], and Eqs. (11,12) in [134]. This effectively suggests that current galaxy clustering data may not yet be sufficiently precise to meaningfully constrain a large number of nuisance parameters. This highlights an

5See instead [131–141] for applications of the EFTofLSS and related models with an eye to analyses of the redshift-space bispectrum as well as higher-order correlators.

6These parameters are: linear bias \( b_1 \), quadratic bias \( b_2 \), (quadratic) tidal bias \( b_{G2} \), third-order tidal bias \( b_{G3} \), \( k^2 \) counterterms for the monopole, quadrupole, and hexadecapole \( c_0, c_2, \) and \( c_4 \), \( k^4 \) Fingers-of-God counterterm \( \tilde{c}_G k^4 \), shot noise parameter \( P_{\text{shot}} \), and scale-dependent shot noise parameters \( a_0 \) and \( a_2 \). The theoretical modeling of higher-order correlators such as the bispectrum requires the introduction of up to an order of magnitude more nuisance parameters.
interesting trade-off between model complexity and data precision, already appreciated in earlier works (see also [192]).

An alternative possibility, explored by some of us in recent years (see e.g. [193–195]), is instead to adopt a minimal, phenomenological, yet still physically motivated theoretical (bias) model, at least as long as the (limited) precision of current data allows. This should be sufficiently precise for the purposes of current data, but not overly complex to the point that the associated nuisance parameters cannot be meaningfully constrained by data. Of course, as future data becomes more precise, such a model should be refined with the introduction of additional ingredients and nuisance parameters, and at a certain point adopting well-motivated first-principles approaches such as the EFTofLSS becomes inevitable.

For the underlying matter power spectrum, we adopt a linear model with non-linear corrections on top following the HALOFIT prescription [196], while also including effects due to survey geometry, linear redshift-space distortions, the Alcock-Paczynski effect, and a phenomenological modeling of non-linear redshift-space distortions (Fingers-of-God). As done by some of us in [195], we go beyond the simple large-scale linear bias model by considering the leading scale-dependent correction, scaling as $k^2$. This scale-dependent bias model is simple but strongly motivated from various independent theoretical approaches, including but not limited to peaks theory [197, 198], the excursion-set approach [199], and standard perturbation theory [97, 200]. We do not make prior assumptions on the relations between the bias parameters, but leave these to be constrained by the data. Similar simplified models, in some cases including an extra $k^4$ bias term, have been used in other recent works (see e.g. [201–203]).

Moreover, in addition to galaxy clustering data, we use measurements of the cross-correlation between the CMB lensing convergence and galaxy overdensity fields (CMB lensing-galaxy cross-correlation in short). Joint analyses of galaxy clustering and CMB lensing-galaxy cross-correlation measurements can help constrain the linear galaxy bias parameter (as it enters differently in the two measurements), which in turn is beneficial for improving constraints on $\Sigma m_\nu$, as demonstrated in [195]. We also note that cross-correlations between CMB lensing and tracers of the LSS (including galaxies, but also other tracers such as galaxy clusters, quasars, filaments, and galaxy groups) have recently been considered in a huge number of works, including but not limited to [204–229].

Our aim in the present work is to revisit and improve the analysis performed by some of us in [195], which found that $C^{\kappa g}_\ell$ had a small but not insignificant impact on the bound on the neutrino mass, and highlighted the importance of moving beyond the simplified scale-independent bias model adopted previously. We go beyond the earlier work of [195] in various respect, which include but are not limited to the following:

- accounting for the reduction of power in the matter-galaxy cross-power spectrum caused by the decorrelation between galaxy and matter fields (see Sec. 2.2);
- accounting for the non-negligible cross-covariance between galaxy-galaxy and galaxy-matter power spectra, and therefore between galaxy clustering and CMB lensing-galaxy cross-correlation measurements, which had previously been neglected, although we find a posteriori that the effect of including the cross-covariance is very small (see Sec. 3.2).
- updating CMB data from Planck 2015 to the Planck 2018 legacy data release [23];
• investigating and testing in more detail the robustness of the underlying theoretical model and whether the resulting bounds of $\Sigma m_{\nu}$ are competitive with the bounds gained obtained from a combination of CMB and BAO (non-FS) measurements.

With these improvements, we find that the addition of $C_{\kappa g}^{\ell}$ has a negligible impact with current data, while it will become important in future datasets. The reason why it does not appear to be important in current datasets (contrary to the findings in [195]) is the use of the updated Planck observations (including importantly small-scale polarization data), which by themselves significantly reduce the error on $\Sigma m_{\nu}$ (implicitly putting much stronger requirements on other datasets, or equivalently reducing the benefits of including additional datasets), and of the relatively low signal-to-noise of current $C_{\kappa g}^{\ell}$ measurements. Nonetheless, we expect that the inclusion of $C_{\kappa g}^{\ell}$ (as well as the adoption of a flexible, simple, but well-motivated galaxy bias model) will be very important when considering upcoming galaxy clustering data, e.g. from Euclid [230] or DESI [231]. We also stress that the scale-dependent bias we are considering here is relevant on small scales (large $k$), and is totally distinct from the spurious scale-dependence studied by some of us in [232], appearing on large scales (small $k$) in the presence of massive neutrinos if the bias itself is not correctly defined (see also [201, 203, 233, 234]).

The rest of this work is then structured as follows. In Sec. 2 we review signatures of $\Sigma m_{\nu}$ in galaxy clustering measurements, and issues pertaining to modeling the scale-dependent galaxy bias in auto- and cross-correlation measurements. In Sec. 3 we discuss the adopted observational datasets, theoretical modeling thereof, and analysis methodology. In Sec. 4 we discuss the resulting constraints on $\Sigma m_{\nu}$ and investigate the robustness of the underlying theoretical model. Finally, in Sec. 5 we provide concluding remarks.

2. Massive neutrinos and large-scale structure: theory and modeling

In this section, we first review the physical imprints of massive neutrinos on the clustering of the large-scale structure (LSS). We follow this up by a description of the scale-dependent galaxy bias model we adopt, before discussing in more detail our modeling of the theoretical (redshift-space) galaxy power spectrum.

2.1. Signatures of massive neutrinos in LSS data

Neutrinos decouple from the primordial plasma at a temperature of $\mathcal{O}$ (MeV), when they were highly relativistic. While ultra-relativistic, neutrinos are unable to cluster on scales smaller than their free-streaming wavenumber, $k \gg k_{fs}$, as their large thermal velocities prevent them from falling within gravitational potential wells. This leads to a well-known small-scale suppression of structure growth, which is one of the most distinctive signatures of neutrino masses in cosmological observations [84].

In our work, as discussed in more detail later in Sec. 3, we shall adopt the so-called degenerate approximation wherein the total neutrino mass $\Sigma m_{\nu}$ is equally distributed among the three mass eigenstates. This approximation is extremely robust given the sensitivity of current cosmological observations, which are only sensitive to the neutrino mass sum rather than the masses of the individual eigenstates (see e.g. [235–237]). At the time when neutrinos are ultra-relativistic, the free-streaming wavenumber $k_{fs}$ is roughly equal to the inverse of the Hubble horizon scale. This implies that during this regime neutrinos do not contribute to clustering on any physical scale. However, at late times
massive neutrinos transition to becoming non-relativistic. At this point, $k_{fs}$ starts growing slower than the horizon scale. Therefore, on scales $k \ll k_{fs}$, the massive neutrino eigenstates behave as a cold dark matter component, and are able to cluster. On the other hand, for smaller scales, $k \gg k_{fs}$, the massive neutrino eigenstates still cannot cluster, and structure formation is suppressed. As a result, matter perturbation modes entering the horizon after the non-relativistic transition evolve without ever experiencing free-streaming due to the massive neutrino eigenstates. These effects lead to a characteristic step-like suppression in power on scales below the free-streaming wavenumber of neutrinos at their non-relativistic transition ($k_{nr}$), which is given by (see e.g. [18]):

$$k_{nr} = 0.0178\Omega_m^{1/2} \left( \frac{\Sigma m_\nu}{\text{eV}} \right)^{1/2} h \text{Mpc}^{-1},$$

where $\Omega_m$ is the present matter density parameter. Besides suppressing power on small scales, massive neutrinos also slow down the growth of matter perturbations $\delta(a)$, with $a$ the scale factor. In the presence of massive neutrinos, matter perturbations grow at a rate $\propto a^{-3f_\nu/5}$ rather than $\propto a^1$, where $f_\nu \equiv \Omega_\nu/\Omega_m$ is the fractional neutrino contribution to the matter density, and similarly $\Omega_\nu$ is the density parameter of (massive) neutrinos.

In the linear regime and at redshift $z = 0$, the relative suppression of the matter power spectrum $P_{mm}(k)$ due to the presence of massive neutrinos has a characteristic step-like/"kink" feature, and saturates at [86]:

$$\frac{P_{mm}(k)}{P_{mm}(k)_{z=0}} \approx 1 - 8f_\nu \quad (k \gg k_{nr}, z = 0).$$

In the non-linear regime, the suppression actually saturates at $\simeq -10f_\nu$, a result which has been independently confirmed by means of N-body simulations (see e.g. [238–247]), as well as higher-order perturbative calculations (see e.g. [248–252]). This step-like suppression is a special feature of massive neutrinos not easily mimicked by other cosmological parameters or systematic effects. Therefore, a possible way of constraining $\Sigma m_\nu$ from LSS clustering data is to measure the full-shape power spectrum of LSS tracers (such as galaxies) for wavenumbers around $k_{nr}$.

However, besides its amplitude and shape, LSS full-shape power spectrum measurements also contain precious geometrical information which helps pinning down $\Sigma m_\nu$. In particular, the position of the Baryon Acoustic Oscillation (BAO) wiggles in $k$-space is directly related to the ratio $r_s/D_V$, with $r_s$ the sound horizon at baryon drag, and $D_V$ the volume-averaged distance to the effective redshift of the sample of LSS tracers whose power spectrum is being measured (see e.g. recent discussions on this point, and more generally on the information content of full-shape power spectrum measurements, in [117, 128, 256]). This geometrical information is crucial in breaking the “geometrical degeneracy” [257–259], which in the context of a spatially flat Universe involves the matter density parameter $\Omega_m$ and the Hubble constant $H_0$, various combinations of which

\[\text{Note that light but massive relics (such as certain eV-scale relics) which become non-relativistic during radiation domination (unlike neutrinos which become non-relativistic during matter domination) lead to a similar feature, which however saturates at $-14f_X$, with $f_X$ the fractional light relic contribution to the matter density (see for instance [253–255]).}\]
lead to a virtually identical CMB power spectrum. The geometrical information contained in the full-shape power spectrum is helpful in breaking the geometrical degeneracy as it helps pinning down both $H_0$ and $\Omega_m$ (by excluding extremely low/high values of $H_0$ or $\Omega_m$ respectively, which would otherwise be tolerated by CMB data alone), and thereby improving constraints $\Sigma m_\nu$ which, we recall, contributes to $\Omega_m$ at late times: see for instance [117, 256] for related discussions on these aspects.

2.2. Galaxy bias

Although the LSS neutrino mass signature is cleanly imprinted in the matter power spectrum $P_{mm}(k)$, this is not a directly observable quantity, as we cannot directly observe the clustering of the matter field, but only that of its luminous tracers, such as galaxies. Galaxies (and other tracers) are biased tracers of the underlying matter density field: their clustering properties are related, but not identical, to those of the matter field. On large scales, the relation between matter overdensity $\delta_m$ and galaxy overdensity $\delta_g$ is fully deterministic and can be captured via a linear relation [271]:

$$\delta_g = b\delta_m, \quad (3)$$

where the proportionality factor $b$ is a constant referred to as (linear) galaxy bias. We stress that Eq. (3) is valid only on sufficiently large, linear scales, where gravitational interactions are dominant. The exact value of the galaxy bias varies depending on the properties of the galaxy sample of interest and is therefore generally treated as a nuisance parameter which is subsequently marginalized over: see [97] for a recent very complete review on galaxy bias. We stress that, in the presence of massive neutrinos, the linear galaxy bias is scale-independent only if the bias is defined with respect to the dark matter-plus-baryons field rather than the total matter field (the latter including massive neutrinos). Nonetheless, this distinction is not important given the sensitivity of current galaxy clustering data, and will only become relevant with upcoming data (e.g. from Euclid or DESI), as discussed for instance in [201, 203, 232–234].

The validity of the linear bias model breaks down as we enter mildly non-linear scales ($k \gtrsim O(0.1) \, \text{hMpc}^{-1}$ at $z = 0$), where complications associated to galaxy formation become increasingly relevant over gravitational interactions. As a result, the biasing relation between the galaxy and matter fields, while still deterministic, is expected to become scale-dependent (see e.g. [272–274] for early seminal works in this direction). This can be understood in terms of an expansion of the galaxy overdensity field in higher-order spatial derivatives of the matter overdensity field $\nabla^{(n)}\delta_m$, which in Fourier space is simplified to an expansion in factors of $k^n$:

$$\delta_g = \left(b_{\text{lin}} + b_{k^2}k^2\right)\delta_m + \mathcal{O}(k^4). \quad (4)$$

Note that odd powers of $k$ are excluded on the basis of statistical isotropy and the equivalence principle [97, 275].

\*\*For more recent detailed discussions on the geometrical degeneracy and implications for parameter estimation, as well as different ways of breaking it with additional late-time datasets, see for instance [260–270].
We note that the linear-plus-$k^2$ galaxy bias parametrization in the mildly non-linear regime we will adopt is minimal, yet highly motivated from different independent theoretical frameworks, including but not limited to peaks theory [197, 198], excursion-set approach [199], and standard perturbation theory [97, 200]. Within the EFTofLSS, the $k^2$ correction can be viewed as the leading order counterterm to the redshift-space monopole, with the associated coefficient being a generalization of the real-space dark matter sound speed (more concretely, see for instance Eqs. (2.7,2.15,2.23) of [123]).

Importantly, we also note that other works have adopted similar simplified models, without the inclusion of a $k^4$ term, which could further improve the fit (see e.g. [201–203]). A more physically motivated model has also been proposed by [276], and includes non-local bias terms up to third-order in the density field (see e.g. Eq. (3.3) of [203]). Recent work by [277] instead computed the complete expression for the redshift-space galaxy power spectrum up to 1-loop order, which includes 28 independent loop integrals and 5 additional free parameters, and the same has been done within the context of the EFTofLSS. In general, on sufficiently small scales, various bias contributions enter in such a way that the relation between the redshift-space galaxy power spectrum and the underlying linear matter power spectrum is no longer a direct (albeit scale-dependent) proportionality such as in Eq. (4), and introduce several additional nuisance parameters. In this work, our aim is instead to adopt a minimal yet physically motivated bias model going beyond linear bias which, while simplified, is still sufficiently useful given the precision of current data, provided the analysis is limited to sufficiently large scales.

We note that the amplitude of the large-scale full-shape galaxy power spectrum scales as $b_{\text{lin}}^2$, and more precisely depends on the combination $b_{\text{lin}}^2 \sigma_8^2$, where $\sigma_8$ is the present day linear theory amplitude of matter fluctuations averaged in spheres of radius $8 h^{-1}\text{Mpc}$. It is therefore clear that jointly fitting another observable which carries a different functional dependence on $b_{\text{lin}}$ (and $\sigma_8$) can significantly help improving cosmological parameter constraints obtained from galaxy clustering measurements (and possibly help break the $b_{\text{lin}} - \sigma_8$ degeneracy). To this end, we shall include an observable which is sensitive to the matter-galaxy cross-spectrum $P_{\text{mg}}(k,z)$: as $P_{\text{mg}}(k,z)$ only correlates one power of the galaxy density field, its large-scale amplitude scales as $b_{\text{lin}}^2 \sigma_8^2$. As we will discuss later in this Section, we use measurements of the angular cross-spectrum between the CMB lensing convergence and the galaxy overdensity field $C_{\kappa g}^{\gamma}$, see Eq. (12), connected to the matter-galaxy cross-spectrum.

We adopt the following linear-plus-$k^2$ scale-dependent galaxy bias model for measurements of the galaxy-galaxy (auto) and galaxy-matter (cross) power spectra (where, for notational simplicity, all redshift dependencies will be omitted henceforth):

$$P_{\text{gg}}(k) = b_{\text{auto}}(k)^2 P_{\text{mm}}(k) \approx \left(b_{\text{lin}} + b_{k^2,\text{auto}} k^2\right)^2 P_{\text{mm}}(k),$$

$$P_{\text{mg}}(k) = b_{\text{cross}}(k) P_{\text{mm}}(k) \approx \left(b_{\text{lin}} + b_{k^2,\text{cross}} k^2\right) P_{\text{mm}}(k),$$

where we have assumed that the scale-dependent bias parameters in the auto-power ($b_{k^2,\text{auto}}$) and cross-power ($b_{k^2,\text{cross}}$) spectra are not necessarily identical. This assumption is supported by theoretical predictions from [277], and evidence from N-body simulations carried out in [278] and [279]. The theoretical predictions from [277] foresee that potential velocity bias contributions – e.g. those arising from galaxy formation effects or baryonic pressure perturbations – would affect the mapping between redshift- and rest-frame. As
such, contributions from a velocity bias would not enter into the cross-power spectra, but only into the auto-power spectra (scaled as $\propto k^2$).\footnote{An expression of the cross-power spectra using the framework of [277] is missing therein but can be retrieved from e.g. [149].}

As for the evidence from simulations, [278] and [279] clearly show that, as $k$ is increased, $db_{\text{cross}}(k)/dk > 0$ and $db_{\text{auto}}(k)/dk < 0$. Physically speaking this different behavior can be explained as follows. The small-scale matter-galaxy cross-correlation function in real space traces the density profile of host halos [280], and therefore increases on small scales, which in Fourier space translates into $b_{\text{cross}}$ increasing with increasing $k$. On the other hand, halos are extended objects, which cannot overlap in the initial Lagrangian space [281]. This halo exclusion principle places strong constraints on the small-scale behavior of the real-space galaxy 2-point correlation function, which has to approach $\xi(r) \to -1$ on sufficiently small scales. In Fourier space, this requirement translates into $b_{\text{auto}}$ decreasing with increasing $k$. We refer the reader to [195] for further discussions on these two different behaviors.

We have so far ignored stochastic contributions to the relation between $\delta_g$ and $\delta_m$. We refer to stochastic contributions as being those which are independent of the matter density field [96]. The largest stochastic contribution relevant to our work arises from the fact that the tracers we are using to sample the underlying matter density field, namely galaxies, are discrete rather than continuous. In the simplest scenario, this leads to the appearance of a Poisson noise term, $1/\bar{n}$, where $\bar{n}$ is the mean number density of galaxies in our sample. The second stochastic contribution that we consider emerges from the fact that processes associated to galaxy formation eventually lead to decorrelations between galaxy and matter density fields. This results in the presence of small-scale fluctuations which are decorrelated from (and thus largely independent of) the large-scale fluctuations [155]. This noise and the Poisson noise enter the matter power spectrum as additive terms (to leading order), and therefore are mutually degenerate. We therefore include an effective stochastic parameter, labeled $P_{\text{shot}}$, in the auto-power spectrum, as follows:

$$P(gg)(k) \approx (b_{\text{lin}} + b_{\text{auto}} k^2)^2 P_{\text{mm}}(k) + P_{\text{shot}}.$$\footnote{Note that $P_{\text{shot}}$ leaves the matter-galaxy cross-power spectrum (directly) unchanged. The underlying reason is that any stochastic contributions are completely independent.}

In our later discussion, we normalize $P_{\text{shot}}$ to the fiducial Poisson noise of our galaxy sample $1/\bar{n}$, where the average galaxy number density for the sample we are considering is $\bar{n} \approx 3 \times 10^{-4} h^3 \text{Mpc}^{-3}$ [282]. Therefore, if $P_{\text{shot}}$ deviates from unity, it should be interpreted as the presence of non-Poissonian noise. We have only considered scale-independent components, although in principle the stochastic contributions may be expanded similarly to the deterministic components of the galaxy bias in Eq. (4). We are assuming that scale-dependent stochastic contributions are negligible on the scales of interest (i.e. $k < 0.13 h \text{Mpc}^{-1}$), an assumption which is supported by earlier findings (see for instance [154, 155, 197, 283, 284]). As a consistency check, we verified that this scale-dependent model improves the residual fit to the observed galaxy power spectrum as compared to a scale-independent model (only involving the linear galaxy bias), as we see in Fig. 1.
Figure 1: Upper panel: measured monopole of the BOSS DR12 CMASS power spectrum (black), along-side the best-fit theoretical prediction from the model used in this work, with a scale-independent (blue) or scale-dependent (red) bias. The white k-band range represents the wavenumber range to which we limit our fit (0.03 < k/(h Mpc\(^{-1}\)) < 0.13 at z = 0.57): we exclude the remaining range of wavenumbers (grey) due to observational systematics (large scales, small k) or to avoid non-linearities (small scales, large k), as clarified in Sec. 3.1. Lower panel: residuals with respect to a fit adopting a scale-independent (blue) or scale-dependent (red) bias. The residuals clearly show the improvement in fit which follows from considering a scale-dependent bias. The effect of linear redshift-space distortions (Kaiser effect) is included, together with a phenomenological model for non-linear redshift-space-distortions (Fingers-of-God) which is modeled by following Eq. (11).
of the matter field by definition, and thus can only show up in the auto-power spectrum. Nevertheless, the matter-galaxy cross-power spectrum can still be indirectly affected by stochastic contributions, in a way which can instead be captured by the cross-correlation coefficient:

$$r(k, z) = \frac{P_{mg}(k, z)}{\sqrt{P_{gg}(k, z)P_{mm}(k, z)}}$$ \hspace{1cm} (8)

which quantifies the loss of information caused by scatter in the $\delta_g - \delta_m$ relation [285]. This scatter may originate from other effects besides those associated to the stochastic components, such as differing values of the scale-dependent bias parameters $b_{k2\text{auto}}$ and $b_{k2\text{cross}}$, as well as redshift-space distortions (RSD), which are discussed later on in this section.

To include the cross-correlation coefficient, we rescale the matter-galaxy cross-power spectrum as follows:

$$P_{mg}(k, z) \rightarrow r(k, z)P_{mg}(k, z),$$ \hspace{1cm} (9)

By including $r(k, z)$, the cross-power spectrum is damped even on linear scales (where $b_{k2}k^2 \approx 0$) as a result of $r(k, z)$ not converging to unity but to a value that is proportional to the relative size between the linear galaxy auto-power spectrum and the stochastic component $\approx 1 - P_{\text{shot}}/(2b_{lin}^2 P_{mm}(k))$. On smaller scales, the cross-power spectrum experiences a larger amount of damping as the contribution of the scale-dependent galaxy bias parameter grows. We show the behaviour of $r(k, z)$ in Fig. B.5 of Appendix B, which displays all the aforementioned effects.

Having now defined our theoretical galaxy bias model in auto- and cross-correlation measurements, we discuss our modeling of the observed redshift-space galaxy power spectrum and the angular cross-spectrum between the CMB lensing convergence and the galaxy overdensity field. The galaxy auto-power spectrum is observed from three-dimensional galaxy clustering data, and therefore in so-called redshift space, since distances along the third dimension are computed from the observed redshift assuming a fiducial cosmology. It is thus sensitive to peculiar velocities along the line-of-sight. On linear scales, peculiar velocities are dominated by the coherent motion of galaxies falling into the gravitational wells of overdense regions, an effect commonly referred to as the “Kaiser effect”, or linear RSD. The Kaiser effect induces a dependency on the line-of-sight angle for the otherwise isotropic galaxy power spectrum, the strength of this line-of-sight angle dependency being connected to the infall rate of the galaxies. To model this effect, we assume that on large scales the coherent motion of galaxies is described by linear perturbation theory, which makes the distortion proportional to the linear growth rate of structures [286]. In the following, we shall only be interested in the monopole of the full-shape power spectrum, i.e. the angle-averaged (spherically averaged) power spectrum. This average results in the dependency on the line-of-sight angle being integrated out, so that the effect of linear RSD is captured by a growth rate-dependent global enhancement of the power spectrum. The linear RSD-corrected galaxy power spectrum monopole is given by:

$$P_{th}^{gg}(k) = b_{auto}^2(k)\left(1 + \frac{2}{3} \beta(k) + \frac{1}{5} \beta^2(k)\right)P_{cb}(k) + P_{\text{shot}},$$ \hspace{1cm} (10)
where $b_{\text{auto}}^2(k) = b_{\text{lin}} + b_{\text{auto}}k^2$ and $\beta = f/b_{\text{auto}}(k)$, and $f$ is the linear growth rate of structure, which we approximate as $\Omega_m(z)^{0.545}$, an approximation which is valid to very high accuracy under the assumption of general relativity and a cosmological constant as the dark energy component (see for instance [287–289]). Finally, $P^{\text{cb}}$ is the cold dark matter-plus-baryons power spectrum. It is this quantity which appears in Eq. (10) rather than $P^{\text{mm}}$ as this choice has been shown to result in the linear galaxy bias being scale-independent and universal (independent of $\Sigma m_\nu$) on large scales, reflecting the fact that neutrinos do not participate in clustering on the scales relevant for galaxy formation [201, 203, 232–234]. At any rate, given the precision of current observational data, the distinction between $P^{\text{cb}}$ and $P^{\text{mm}}$ in Eq. (10) is irrelevant, as shown in [201, 232], but this difference will become important for upcoming LSS surveys. We append the subscript “th” to $P_{\text{gg}}(k)$ to distinguish the theoretical and observed galaxy auto-power spectra.

On top of the large-scale infall described by the Kaiser effect, and captured by the $\beta$-dependent terms in Eq. (10), one of the key non-linear RSD contributions arises from random peculiar velocities of galaxies which further distort small-scale clustering in redshift space, an effect known as Fingers-of-God (FoG). The simplest modeling of FoG exponentially suppresses Eq. (10) on small scales (see e.g. [295, 296]):

$$P_{\text{th}}(k) - P_{\text{shot}} \rightarrow (P_{\text{th}}(k) - P_{\text{shot}}) \exp \left[ (-k\sigma_{\text{FoG}})^2 \right],$$

where $\sigma_{\text{FoG}}$ is the typical scale above which the power spectrum is suppressed (or equivalently, the suppression occurs above a typical wavenumber $k_{\text{FoG}} \sim \sigma_{\text{FoG}}^{-1}$). For the BOSS CMASS galaxy sample we are interested in and given the scale cuts we will apply (all of which will be discussed in Sec. 3), we expect the FoG contribution to be dominated by the virialized motions of satellite galaxies inside host halos and to be small, as shown for instance in [283]. Nonetheless, for completeness we test this expectation in our analysis, finding that it is met (see Sec. 4.1.1).

For the cross-power spectrum, we use measurements of the cross-correlation between Planck 2015 CMB lensing (convergence) maps and BOSS DR12 galaxy overdensity maps, $C_{\kappa g}^{\ell}$ [300]. The CMB lensing convergence field is related to the integrated effect of the intervening matter between the last-scattering surface and us [98]. The cross-correlation between CMB lensing convergence and galaxy overdensity reads [301]:

$$C_{\kappa g}^{\ell, \text{th}} = \int dz \frac{H(z)}{\chi^2(z)} W^2(z) f^2(z) P_{m\kappa} \left( k = \frac{\ell + 1/2}{\chi(z)}, z \right),$$

where $H(z)$ is the Hubble parameter, $\chi$ is the comoving scale, and we apply the Limber approximation.
approximation [302]. $W^\kappa(z)$ is the lensing convergence kernel:

$$W^\kappa(z) = \frac{3\Omega_m}{2} \frac{H_0^2}{H(z)} (1 + z) \chi(z) \frac{\chi(z_{\text{CMB}}) - \chi(z)}{\chi(z_{\text{CMB}})},$$

(13)

where $H_0$ is the Hubble parameter today, and $z_{\text{CMB}}$ is the redshift of recombination. Lastly, in Eq. (12), $f^g(z)$ is the normalized redshift distribution of the galaxy overdensity maps:

$$f^g(z) = \frac{dN/dz}{\int d\bar{z}' dN/d\bar{z}'},$$

(14)

where $dN/dz$ is the redshift distribution of the galaxy sample. We neglect the effect of lensing magnification given that this effect is dependent on redshift and our redshift bin is fairly small [303, 304]. Finally, we do not include relativistic effects, as these are only relevant on very large scales, beyond those probed here (see e.g. [305–309]). Finally, to model non-linear corrections to the underlying matter power spectrum in the presence of massive neutrinos (which nonetheless are very small compared to current observational uncertainties on the scales we are interested in, see Fig. 1 of [194]) we make use of the \textsc{Halofit} prescription discussed in [196].

3. Datasets and methodology

We consider a 7-parameter model, which extends the 6-parameter $\Lambda$CDM model by allowing the sum of the neutrino masses $\Sigma m_\nu$ to vary. The 7 free parameters we consider are then: the physical baryon and cold dark matter densities $\omega_{b}h^2$ and $\omega_{cdm}h^2$, the acoustic scale $\theta_s$, the optical depth to reionization $\tau$, the amplitude and tilt of the primordial scalar power spectrum $A_s$ and $n_s$, and finally the sum of the neutrino masses $\Sigma m_\nu$. Concerning the neutrino mass spectrum, we adopt the so-called degenerate approximation, wherein the total neutrino mass $\Sigma m_\nu$ is equally distributed among the three mass eigenstates, each carrying an individual mass $m_{\nu_i} = \Sigma m_\nu / 3$. Various works have argued that this approximation is extremely robust given the sensitivity of current cosmological observations, which are only sensitive to the neutrino mass sum rather than the masses of the individual eigenstates (see e.g. [235–237]). Prospects for distinguishing the normal and inverted orderings based on physical effects associated to the individual mass eigenstates (as opposed to overall parameter space volume effects) do not appear promising in near-future cosmological data (see [237]).

3.1. Datasets

We now discuss the datasets adopted, starting from galaxy clustering and CMB lensing-galaxy cross-correlation data:

- Angle-averaged (monopole moment) full-shape power spectrum of the BOSS DR12 CMASS galaxies, measured at an effective redshift $z_{\text{eff}} = 0.57$, as measured in [315]. We only use measurements within the wavenumber range $0.03 < k[h\text{Mpc}^{-1}] < 0.13$.

\footnote{See however [310–314] for other works reaching slightly different conclusions.}
The choice of large-scale cutoff $k_{\text{min}} = 0.03 \, h\text{Mpc}^{-1}$ is dictated by the fact that larger scales have significantly lower signal-to-noise ratio and are dominated by observational systematics (e.g. related to stellar density, seeing requirements, missing close-pairs, fiber collisions, and redshift failures, see [316]). The choice of small-scale cutoff $k_{\text{max}} = 0.13 \, h\text{Mpc}^{-1}$ is instead limited by the ability to reliably model non-linear effects (discussed further below). This dataset is referred to as $P_{\text{gg}, \text{obs}}(k)$. For simplicity and especially for consistency and ease of comparison to our earlier related work [193–195], here we have chosen not to include measurements of the quadrupole moment of the BOSS DR12 CMASS full-shape power spectrum.  

- Measurements of the cross-correlation between CMB lensing convergence maps from the Planck 2015 data release and galaxy overdensity maps from the BOSS DR12 CMASS sample [300]. This dataset is referred to as $C_{\kappa g, \text{obs}}$. There is some degree of overlap between the $C_{\kappa g}$ and $P_{\text{gg}, \text{obs}}(k)$ measurements as the part of sky covered by respective galaxy samples overlap with the Planck lensing maps. In [195] this overlap had not been taken into account and the two measurements had been treated as independent. Here, we go beyond this simplification, and self-consistently take into account the overlap between the $P_{\text{gg}, \text{obs}}(k)$ and $C_{\kappa g, \text{obs}}$ measurements by including the cross-covariance between the two (see Sec. 3.2 for further discussions), although we find a posteriori that the effect of neglecting the cross-covariance is small given the precision of current CMB lensing and full-shape galaxy clustering data.

As discussed earlier, $P_{\text{gg}}(k)$ measurements are particularly useful when complemented with CMB data, as they help breaking the geometrical degeneracy, through the geometrical information contained in the reconstructed BAO peak(s): see for instance [117, 256]. In particular, $P_{\text{gg}}(k)$ can help excluding low/high values of $H_0$ or $\Omega_m$, respectively, which would otherwise be tolerated by CMB data alone. This in turn improves constraints $\Sigma m_\nu$, which contributes to $\Omega_m$ at late times. We therefore complement the above datasets by the latest CMB measurements, along with BAO distance and expansion rate measurements. CMB data is particularly helpful in constraining the 6 $\Lambda$CDM parameters. BAO data, on the other hand, provide a late-time standard ruler calibrating the matter density parameter $\Omega_m$, and absolute scale of the expansion rate $H_0$. More specifically, we consider the following datasets:

- Measurements of the CMB temperature ($TT$), E-mode polarization ($EE$), and temperature-polarization cross-correlation ($TE$) anisotropy spectra from the Planck 2018 data release [317]. We include the full $TT$ ($2 \leq \ell \leq 2508$) and $EE$ ($2 \leq \ell \leq 1996$) ranges, as well as the high-$\ell$ $TE$ ($30 \leq \ell \leq 1996$) range. For the high-$\ell$ ($\ell \geq 30$) $TT$, $TE$, and $EE$ measurements, we adopt the P14k likelihood [317]. We refer to this dataset as Planck.

- Small-scale CMB $TT, TE, EE$ anisotropy measurements from the Atacama Cosmology Telescope Polarimeter (ACTPol) Data Release 4 (DR4) [318, 319]. We use

\[14\] However, we note that the peculiar velocity information contained within the quadrupole through redshift-space distortions would help tightening parameter constraints compared to the monopole-only ones, as this information helps breaking the $b_{\text{lin}}-\sigma_8$ degeneracy. Nonetheless, the inclusion of CMB lensing-galaxy cross-correlation measurements also helps breaking this degeneracy.
measurements in the multipole range $\ell_{\text{min}} \leq \ell \leq 4000$. In particular, we truncate at the large-scale cut-off $\ell_{\text{min}} = 1800$ for $TT$ and $\ell_{\text{min}} = 350$ for $TE,EE$, as suggested in [318] in order to ensure that the errors arising from neglecting the cross-covariance between ACTPol and Planck datasets are negligible and in any case at most 5%. We refer to this dataset as ACTPol$^{15}$. 

- Baryon Acoustic Oscillation (BAO) measurements from the: SDSS Main Galaxy Sample (MGS, $z_{\text{eff}} = 0.15$) [320]; Six-degree-Field Galaxy Survey (6dFGS, $z_{\text{eff}} = 0.106$) [321]; and lastly, the post-reconstructed (consensus) results constructed from the BOSS DR12 galaxy samples ($z_1 = 0.38, z_2 = 0.51, z_3 = 0.61$) [322]. Note that our galaxy probes $P_{\text{obs}}(k)$ and $C_{\ell,\text{obs}}$ are also constructed from the BOSS DR12 galaxy samples, however they are only dependent on the CMASS sample that is situated at an effective redshift $z_{\text{eff}} = 0.57$, which overlaps with the two upper $z$-bins of the BAO BOSS DR12 consensus dataset. Hence, we exclude the redshift bins $z_2$ and $z_3$ from the BAO BOSS DR12 consensus dataset whenever it is used simultaneously with our $P_{\text{obs}}(k)$ and $C_{\ell,\text{obs}}$ measurements. We denote this reduced BAO dataset, together with MGS and 6dFGS, BAO$^{15}$. Therefore, BAO$^{15}$ includes BAO measurements from the MGS, 6dFGS, and BOSS DR12 $z_1$ galaxy samples, whereas BAOcons includes BAO measurements from the MGS, 6dFGS, and complete BOSS DR12 ($z_1, z_2$, and $z_3$) galaxy samples.

For conciseness, hereafter we shall refer to the combination Planck + BAO$^{15}$ + $P_{\text{obs}}(k)$ as base: this combination defines our reference baseline dataset against which we will compare all our results later on.

We make a few final amendments to our theoretical galaxy power spectrum, $P_{\text{th}}(k)$, in order to correct for survey-specific effects impacting the observed galaxy power spectrum. Firstly, the finite survey geometry introduces mode-coupling between otherwise independent $k$-modes. In practice, we model this effect through the window function $W(k_i, k_j)$, which we convolve with the theoretical galaxy power spectrum as follows:

$$P_{\text{th}}(k_i, z_{\text{eff}}) \rightarrow \sum_k \frac{P_{\text{th}}(k = k_j a_{\text{scl}}^3, z_{\text{eff}})}{a_{\text{scl}}^3},$$

where $z_{\text{eff}} = 0.57$ is the effective redshift of the BOSS DR12 CMASS sample ($z_{\text{eff}}=0.57$), and the parameter $a_{\text{scl}}$ is defined in Eq. (16) below. In addition, we also model the Alcock-Paczynski (AP) effect, a well-known effect resulting from the need to assume a fiducial cosmology in order to convert redshifts into comoving coordinates to estimate the power spectrum [323], where the assumption of a wrong fiducial cosmology will lead to geometrical distortions in the observed clustering pattern. To model the AP effect, we follow [324–327], and adopt the scaling factor $a_{\text{scl}}$:

$$a_{\text{scl}} = \frac{D_A(z_{\text{eff}})^2 / H(z_{\text{eff}})}{D_A^\text{fid}(z_{\text{eff}})^2 / H^\text{fid}(z_{\text{eff}})},$$

The actpollite_dr4-software is available at https://lambda.gsfc.nasa.gov/product/act/act_dr4_likelihood_get.cfm.
where $D_A$ is the angular diameter distance, and the superscript "fid" denotes quantities evaluated in the fiducial cosmology assumed by the BOSS collaboration to compute $P^\text{obs}_g(k)$. The AP effect is implemented by evaluating the theoretical power spectrum at re-scaled wavenumbers $\hat{k} = k(a_{\text{eff}})^{-1/3}$, and re-scaling the power spectrum by a factor of $a_{\text{eff}}$. We note, however, that the effect on parameter estimation of not including the AP effect is negligible given the precision of current galaxy clustering data.

3.2. Parameter estimation

Our $P^g(k)$ and $C^g_{\ell}$ datasets probe large, linear scales, where density perturbations are approximately Gaussian. Hence, they are approximately described by normal random variables. This enables us to express the joint $P^g(k)-C^g_{\ell}$, $\mathcal{C}$, as a multivariate normal probability density function:

$$\ln \mathcal{L} \sim (t(\theta) - d)^T C^{-1}(t(\theta) - d),$$

where $t(\theta)$ is the theoretical prediction for the observational data vector $d$ given a set of model parameters $\theta$, and $C_{ij}$ is the covariance matrix quantifying the amount of covariance between two elements of model parameters $\theta$. In our case, in order to properly model the fact that our $P^g(k)$ and $C^g_{\ell}$ measurements are not independent (as they are obtained from datasets which overlap on the sky), we are considering a joint $P^g(k)-C^g_{\ell}$ likelihood, which means that the data vector $d$ holds the measurements of both $P^\text{obs}_g(k)$ and $C^g_{\ell,\text{obs}}$, and similarly for $t$ with the corresponding theoretical predictions:

$$d = \left[ P^g_{\text{obs}}(k_1), \ldots, P^g_{\text{obs}}(k_n), C^g_{\ell_1,\text{obs}}, \ldots, C^g_{\ell_m,\text{obs}} \right],$$

$$t(\theta) = \left[ P^g_{\text{th}}(k_1), \ldots, P^g_{\text{th}}(k_n), C^g_{\ell_1,\text{th}}, \ldots, C^g_{\ell_m,\text{th}} \right],$$

where we are considering $n$ bins with $P^g_{\text{obs}}(k)$ measurements in the wavenumber range $k_1 \leq k \leq k_n$ and $m$ bins with $C^g_{\ell,\text{obs}}$ measurements within the multipole range $\ell_1 \leq \ell \leq \ell_m$.

The fact that $P^g_{\text{obs}}(k)$ and $C^g_{\ell,\text{obs}}$ are not statistically independent is reflected in the full covariance matrix. More specifically, it is useful to think of the covariance matrix $C$ as a $(n+m) \times (n+m)$ block matrix, partitioned into 2 row groups and 2 column groups:

$$C = \begin{bmatrix}
\text{Cov} [\hat{P}^g(k), \hat{P}^g(k')] & \text{Cov} [\hat{P}^g(k), \hat{C}^g_{\ell,\text{th}}] \\
\text{Cov} [\hat{P}^g(k), \hat{C}^g_{\ell,\text{th}}]^T & \text{Cov} [\hat{C}^g_{\ell,\text{th}}, \hat{C}^g_{\ell',\text{th}}]
\end{bmatrix},$$

where $\text{Cov} [\hat{P}^g(k), \hat{P}^g(k')]$ and $\text{Cov} [\hat{C}^g_{\ell,\text{th}}, \hat{C}^g_{\ell',\text{th}}]$ are the covariance matrices of the individual $\hat{P}^g(k)$ and $\hat{C}^g_{\ell,\text{th}}$ measurements, themselves estimators of the observational datasets $P^g_{\text{obs}}(k)$ and $C^g_{\ell,\text{obs}}$ respectively. Then, the statistical correlation between $P^g_{\text{obs}}(k)$ and $C^g_{\ell,\text{obs}}$ is captured by the off-diagonal block $\text{Cov} [\hat{P}^g(k), \hat{C}^g_{\ell,\text{th}}]$ and its transpose: we shall refer to this block as the cross-covariance between $\hat{P}^g(k)$ and $\hat{C}^g_{\ell,\text{th}}$, occasionally denoting it by $C^{\text{cross}}$.

The covariance matrix for $P^g_{\text{obs}}(k)$ has been measured by the BOSS collaboration using dedicated mocks [26]. On the other hand, we have assumed a Gaussian likelihood for $C^g_{\ell,\text{obs}}$, with covariance matrix estimated by jackknife resampling of 37 equal-weight
regions of the CMASS survey area. We refer the reader to [300] for further details. In order to write down the full joint \( P_{\text{obs}}(k)\)-\( C_{\text{obs}}^\text{ng} \) likelihood, we therefore require an estimate for \( C_{\text{cross}} \). We write down an analytical estimator for \( C_{\text{cross}} \) based on two assumptions: Gaussian density perturbations, implying that the cross-covariance is independent of the matter trispectra; and flat-sky approximation for \( C_{\text{obs}}^\text{ng} \), valid as long as we are not observing perturbations on ultra-large scales. The full derivation of \( C_{\text{cross}} \) is reported in Appendix Appendix C, and we simply cite the result of this calculation below:

\[
\text{Cov} \left[ \hat{P}_{\text{obs}}(k, z_{\text{eff}}), \hat{C}_{\text{obs}}^\text{ng} \right] = \int dz \frac{H(z)}{\chi^2(z)} W^\kappa(z) f^3(z) \times \frac{V_1}{V_s(k_\ell)} D^2_s(z) \left. 2 P_{\text{obs}}(k, z_{\text{eff}}) P_{\text{obs}}^\text{ng} \right|_{|k_\ell - \delta k_\ell| \leq \frac{\delta k_\ell}{2}}.
\]

(20)

In the above, we have denoted the shot noise-less galaxy power spectrum by \( P_{\text{gg}} = P_{\text{th}} = P_{\text{shot}} \) and the linear growth function by \( D_s \). Furthermore, \( V_s(k_\ell) \) is the volume of a spherical shell centred upon \( k_\ell \) and \( \delta k_\ell \) is the size of the bin associated to \( k_\ell \); \( \delta k_\ell = (k_{\ell+1} - k_{\ell-1})/2 \). Finally, \( V_1 = (2\pi)^3 / V_{\text{surv}} \) is the volume of the fundamental cell that depends on the galaxy survey volume \( V_{\text{surv}} \). Note that this result ignores non-Gaussian corrections to the covariance, an approach which was also adopted in the earlier related work of [328], see also [329, 330].

As can clearly be seen in Eq. (20), the cross-covariance is by definition a function of the scale-dependent bias parameters and the \( \Lambda \)CDM parameters. Therefore, it would in principle require a new evaluation for each sample in our Markov Chain Monte Carlo (MCMC) analysis. However, as often done in these contexts with current data, we fix all parameters to given fiducial values, evaluate the covariance matrix for these sets of parameters, and assume that the covariance matrix is then fixed and does not vary with parameters.\(^{16}\) We fix the bias parameters to the following values: \( b_{\text{lin}} = 2 \), \( b_{\text{cross}} = b_{\text{auto}} = 0 \) \( h^{-2}\text{Mpc}^2 \). Note that while \( b_{\text{lin}} \) is dimensionless, \( b_{\text{cross}} \) and \( b_{\text{auto}} \) carry dimensions of \( h^{-2}\text{Mpc}^2 \), reflecting the fact that the quantities \( b_{\text{cross}} k^2 \) and \( b_{\text{auto}} k^2 \) need to be dimensionless (as they carry the same units as \( b_{\text{lin}} \)). Moreover, we normalize the stochastic shot noise component \( P_{\text{shot}} \) in units of the fiducial Poisson shot noise 1/\( n \), where \( n \) is the average number density of the galaxy survey in question, which for the BOSS DR12 CMASS sample is \( n \simeq 3 \times 10^{-4} h^3\text{Mpc}^{-3} \) [282]. Therefore, our fiducial Poisson shot noise is \( 1/n \simeq 0.33 \times 10^4 h^{-3}\text{Mpc}^3 \), and we implicitly normalize \( P_{\text{shot}} \) in these units (e.g. \( P_{\text{shot}} = 1 \) really means \( P_{\text{shot}} = 0.33 \times 10^4 h^{-3}\text{Mpc}^3 \)). Finally, we fix the \( \Lambda \)CDM parameters to their respective best-fit values as inferred from the Planck 2018 TT, TE, EE (across the full \( \ell \)-range) legacy measurements alone [23], i.e. \( \Omega_b h^2 = 0.0224 \), \( \Omega_c h^2 = 0.120 \), \( \theta_s = 0.0104 \), \( \tau = 0.054 \), \( \ln(10^{10} A_s) = 3.045 \), and \( n_s = 0.966 \).

We compute theoretical predictions for the cosmological observables we consider through the Boltzmann solver CAMB [333]. To sample the joint posterior distribution for the cosmological and nuisance parameters (including the scale-dependent bias parameters), we employ MCMC methods, with samples generated through a suitably modified version of the cosmological MCMC sampler CosmoMC [334]. The convergence of the generated chains is evaluated by computing the Gelman-Rubin parameter \( R - 1 \) [335], a measure of the ratio between the intra-chain and inter-chain variances, adopting \( R - 1 < 0.01 \)

\(^{16}\)Note that with future more precise galaxy clustering data, this simplification may no longer be adequate (see for instance [331, 332]).
Table 1: Ranges for the (flat) priors on the galaxy bias and shot noise parameters. We normalize the stochastic shot noise component $P_{\text{shot}}$ in units of the fiducial Poisson shot noise $1/n$, where $\bar{n}$ is the average number density of the galaxy survey in question, which for the BOSS DR12 CMASS sample is $\bar{n} \approx 3 \times 10^{-4} \text{h}^{-3} \text{Mpc}^{-3}$ [282], so our fiducial Poisson shot noise ($P_{\text{shot}} = 1$) actually corresponds to $1/\bar{n} \approx 0.33 \times 10^{4} \text{h}^{-3} \text{Mpc}^{-3}$.

| Datasets | $\theta_{\text{lin}}$ | $b_{k^2\text{cross}}$ | $b_{k^2\text{auto}}$ | $P_{\text{shot}}$ | $\Sigma m_{\nu}$ (95% C.L.) |
|----------|-------------------|-------------------|-------------------|-------------------|-------------------|
| base | $[0.0, 5.0]$ | $[-70.0, 30.0]$ | $[-70.0, 30.0]$ | $[0.03, 5.00]$ | |
| base + Planck + BAO/2$h$ + $P_{\text{obs}}(k)$ | 1.97 ± 0.05 | -27.2 ± 10.0 | 1.30 ± 0.44 | < 0.14 | |
| base + $C_{l}^{\ell}$ | 1.97 ± 0.05 | 9.1 ± 3.1 | -29.7 ± 10.1 | 1.51 ± 0.44 | < 0.14 |
| base + $C_{\text{cross}}$ (including $C_{\text{cross}}$) | 1.97 ± 0.05 | 8.8 ± 3.0 | -28.7 ± 10.0 | 1.47 ± 0.43 | < 0.14 |
| base + $C_{\text{obs}}$ ($P_{\text{shot}} = 0$) | 2.00 ± 0.05 | 5.1 ± 0.7 | -16.1 ± 1.0 | < 0.14 | |
| base + $C_{l}^{\ell}$ (max = 0.14h$/\text{Mpc}$) | 1.98 ± 0.06 | 7.0 ± 2.4 | -22.2 ± 7.8 | 1.18 ± 0.36 | < 0.14 |
| base + $C_{l}^{\ell}$ (max = 0.16h$/\text{Mpc}$) | 2.01 ± 0.05 | 4.8 ± 1.8 | -15.8 ± 5.8 | 0.86 ± 0.39 | < 0.14 |
| base + $C_{l}^{\ell}$ (max = 0.18h$/\text{Mpc}$) | 2.00 ± 0.05 | 5.3 ± 1.9 | -17.0 ± 6.5 | 0.92 ± 0.31 | < 0.14 |
| base + $C_{l}^{\ell}$ (max = 0.20h$/\text{Mpc}$) | 2.04 ± 0.05 | 4.3 ± 1.6 | -13.5 ± 4.7 | 0.75 ± 0.24 | < 0.13 |
| base + ACT | 1.98 ± 0.06 | -27.2 ± 10.0 | 1.30 ± 0.45 | < 0.17 | |

Table 2: Constraints on the scale-dependent bias parameters and $\Sigma m_{\nu}$. For the scale-dependent bias parameters we report 68% C.L. intervals, whereas for $\Sigma m_{\nu}$ we report the 95% C.L. upper limit. If the value of $k_{\text{max}}$ is not mentioned, it is set to 0.13h$/\text{Mpc}^{-1}$. We normalize the stochastic shot noise component $P_{\text{shot}}$ in units of the fiducial Poisson shot noise $1/n$, where $\bar{n}$ is the average number density of the galaxy survey in question, which for the BOSS DR12 CMASS sample is $\bar{n} \approx 3 \times 10^{-4} \text{h}^{-3} \text{Mpc}^{-3}$ [282], so our fiducial Poisson shot noise ($P_{\text{shot}} = 1$) actually corresponds to $1/\bar{n} \approx 0.33 \times 10^{4} \text{h}^{-3} \text{Mpc}^{-3}$. For the dataset combination including $C_{\text{cross}}$, the cross-covariance between $P_{\text{obs}}(k)$ and $C_{l,\text{obs}}$ has been properly included, although we find a posteriori that the impact thereof is negligible.

as stopping criterion. We set uniform priors on all cosmological parameters. We allow $\Sigma m_{\nu}$ to be as small as 0eV, ignoring prior information from oscillation experiments, which set a lower limit of 0.06eV (see e.g. [34] for further discussions on advantages associated to using this prior). Table 1 summarizes the priors on the galaxy bias and shot noise parameters. Finally, to compute parameter constraints and produce plots of the respective posterior distributions, we make use of the GetDist Python analysis package [336].

4. Results & Discussion

The obtained marginalized constraints on $\Sigma m_{\nu}$ and the galaxy bias parameters are summarized in Tab. 2. We always report 68% C.L. intervals except for cases where only an upper/lower limit is available (as with $\Sigma m_{\nu}$), in which case we quote a 95% C.L. upper/lower limit. We begin by discussing the 95% C.L. upper limits on $\Sigma m_{\nu}$. Posterior distributions for $\Sigma m_{\nu}$ obtained from different dataset combinations are presented in Fig. 2, whereas the corresponding 95% C.L. upper limits are given in the last column of Tab. 2.
4.1. Baseline constraints on neutrino masses

We begin by discussing the constraints on $\Sigma m_\nu$ we obtain from our base dataset combination, which we recall is given by the combination $\text{Planck} + BAO_z + P_{\text{gg}}^{\text{obs}}(k)$. By comparing the 95% C.L. upper limit obtained from the base combination against the same limit obtained from $\text{Planck}$ alone, we see that the inclusion of LSS data has significantly improved constraints on $\Sigma m_\nu$, bringing the upper limit from 0.26 eV to 0.14 eV. This is not unexpected, as the inclusion of LSS data helps easing the geometrical degeneracy affecting $H_0$ and $\Omega_m$, by cutting out the part of parameter space associated to low/high values of $H_0/\Omega_m$ respectively, which would otherwise be tolerated by CMB data alone. The tighter constraints on $\Omega_m$ naturally results in tighter constraints on $\Sigma m_\nu$. We have checked that neutrino masses below 0.14 eV would result in an induced suppression of power in the galaxy power spectrum which is of the order or less than the suppression obtained by propagating the uncertainty on $b_{\text{auto}}^2(k)$.

We find that the upper limit on $\Sigma m_\nu$ obtained from the base dataset combination is comparable to the one we would obtain if we were to use purely geometrical information from the reconstructed BAO peak(s) using the $\text{Planck} + BAO_{\text{com}}$ dataset combination, i.e. removing the full-shape $P_{\text{gg}}^{	ext{obs}}(k)$ measurement and replacing it by the BOSS DR12 $z_2$ and $z_3$ BAO measurements. In the latter case, we find an upper limit of 0.12 eV, consistent with the bound reported by the Planck collaboration from the same dataset combination [23]. We recall once more that $BAO_z$ includes BAO measurements from the MGS, 6dFGS, and BOSS DR12 $z_1$ galaxy samples, whereas $BAO_{\text{com}}$ includes BAO measurements from the MGS, 6dFGS, and complete BOSS DR12 ($z_1$, $z_2$, and $z_3$) galaxy samples. We also recall that the reason why we only use the BOSS DR12 $z_1$ BAO measurements (in addition to the MGS and 6dFGS BAO measurements, which are always included) when including the $P_{\text{gg}}^{	ext{obs}}(k)$ dataset is that the $z_2$ and $z_3$ samples partially overlap with the BOSS DR12 CMASS sample. Therefore, the power spectrum of the BOSS DR12 CMASS sample (i.e. the $P_{\text{gg}}^{	ext{obs}}(k)$ dataset) cannot be used simultaneously with the BOSS DR12 $z_2$ and $z_3$ BAO measurements to avoid double-counting data.

These findings suggests that current BOSS full-shape information and purely geometrical information from the reconstructed BAO peak(s) [17] carry comparable constraining power once combined with $\text{Planck}$ CMB data. This somewhat surprising conclusion agrees with the same conclusion reached in [117, 119], where it was argued that this fact is merely a coincidence given the current volume and redshift coverage of the BOSS survey as well as the efficiency of current BAO reconstruction algorithms [26, 337, 338]. With future spectroscopic galaxy surveys covering a much larger volume and redshift range, together with expected substantial improvements in the efficiency of BAO reconstruction algorithms (see e.g. [339, 340]), this trend is expected to be reversed, with the full-shape information eventually superseding the purely geometrical information (see [148, 231, 341]). [18]

Thus, for what concerns $\Sigma m_\nu$ bounds, we conclude that $P_{\text{gg}}^\text{obs}(k)$-only shape information is approximately as informative as geometrical information from reconstructed

---

17We use the plural for “peaks” as the peak in the real-space correlation function translates to a series of (damped) peaks in the power spectrum.

18See also [194, 342] for similar conclusions reached using earlier data. Moreover, these same works argued that this result may be reversed in extensions to $\Lambda$CDM where shape information can play a crucial role. A recent explicit example of this has been provided in [343].
BAO peak(s). Alongside the reasons outlined in [119] and discussed above, another possibility previously raised in [194] and [195] is that this may be at least partially due to the introduction of extra nuisance parameters when analyzing full-shape $P^{gg}(k)$ data, such as the scale-dependent bias and shot noise parameters. To more thoroughly harness the shape information, it is therefore desirable to add other measurements which help nailing down or at least breaking degeneracies related to the bias parameters. To this end, we include measurements of the CMB lensing-galaxy angular cross-power spectrum $C_{\ell}^{\kappa g}$: this dataset is sensitive to $b_{\text{in}}\sigma_8^2$, while $P^{gg}(k)$ is sensitive to $b_{\text{in}}^2\sigma_8^2$: therefore, the base + $C_{\ell}^{\kappa g}$ combination can help disentangle $b_{\text{in}}$ and $\sigma_8$. Moreover, $C_{\ell}^{\kappa g}$ suffers from a different set of observational systematics compared to $P^{gg}(k)$, as discussed for instance in [328, 344–347].

4.1.1. Impact of Fingers-of-God

Earlier in Sec. 2.2, we argued that the impact of FoG is expected to be negligible given our galaxy sample and scale cuts. We test this expectation explicitly, by including our FoG modeling given in Eq. (11). More specifically, we include an extra parameter $\sigma_{\text{FoG}}$, for which we set a prior linear in the range $[1; 100] h^{-1}$ Mpc. Considering the base dataset combination, we then test whether the inclusion of $\sigma_{\text{FoG}}$ significantly improves the fit and/or alters the inferred values of the other parameters.

We find that including FoG does not lead to meaningful changes in the inferred cosmological or bias parameters. To within the precision at which we report constraints, the upper limit on $\Sigma m_\nu$ is unchanged, and so are the inferred values of all the bias parameters. The only exception is $b_{k2,\text{auto}}$, which shifts very slightly to less negative values, to compensate the extra FoG-induced suppression. As expected, we only infer upper limit on $\sigma_{\text{FoG}}$, with $\sigma_{\text{FoG}} < 3.2 h^{-1}$ Mpc at 68% C.L. and $< 4.7 h^{-1}$ Mpc at 95% C.L.: these limits can be roughly translated to lower limits on the wavenumber $k_{\text{FoG}}$ at which FoG become non-negligible, $k_{\text{FoG}} \gtrsim 0.33 h$ Mpc$^{-1}$ (68% C.L.) and $\gtrsim 0.21 h$ Mpc$^{-1}$ (95% C.L.), limits within which our scale cuts are safely inside. We thus conclude that for the purposes of our analysis FoG can be safely neglected, although we stress that all our subsequent results include FoG modeling.

4.2. Including the CMB lensing-galaxy cross-correlation

We now complement the previously discussed base dataset combination with measurements of $C_{\ell}^{\kappa g}$. Doing so, we find that the upper limit on $\Sigma m_\nu$ is essentially unchanged. In order to investigate whether this is due to a “poor” fit to the data or to the data uncertainties we perform a goodness-of-fit analysis that is detailed in Appendix Appendix A. We find that there is an underfit between the data and the model, as the significance is determined to $p$-value$\approx0.01$. This may either imply that the dataset errors are too optimistic or that our model is insufficient to represent the data.

Another possibility is that there is some tension between the $C_{\ell}^{\kappa g}$ and $P^{gg}(k)$ measurements. In fact, as already pointed out in [195], $C_{\ell}^{\kappa g}$ measurements (including the one we adopted) systematically appear show a lack of power on large angular scales [300, 348–350], which can be interpreted as a preference for a lower value of the linear galaxy bias compared to that inferred from galaxy clustering. 19 The most plausible explana-

---

19 See for instance [208, 301, 351] for other measurements of CMB lensing-galaxy cross-correlations which do not find this deficit of power.
tions for this lack of power attribute it to systematics in CMB lensing mapp (see for instance [349, 352–357]), such as thermal Sunyaev-Zel’dovich contamination, for which a novel cleaning procedure was recently proposed in [358] and applied to ACT data in [359]. Overall, we therefore find that including shape information from \( C_{\kappa g}^{\ell} \) has not improved our constraints on \( \Sigma m_\nu \). It is however expected that this conclusion should change with expected improvements in the quality of future CMB lensing maps and overlapping galaxy redshift surveys, where CMB lensing-galaxy cross-correlations will be a major science driver (see for instance [328, 360–363]).

The previous work of [195] found that \( C_{\kappa g}^{\ell} \) had a small but not insignificant impact on the bound on the neutrino mass, while here we find that the impact of \( C_{\kappa g}^{\ell} \) is essentially negligible. This can be attributed to the use of the updated Planck dataset (from Planck 2015 to Planck 2018), as this dataset by itself leads to a large reduction of the uncertainties on the neutrino mass. Implicitly, this puts much stronger requirements on other datasets for them to make an impact. The main improvement in going from Planck 2015 to 2018 is that for the latter we have also included small-scale polarization data: the use of high-\( \ell \) polarization data in Planck 2015 was earlier cautioned against due to possible residual systematics in the dataset, which is no longer the case for the Planck legacy data release.

4.3. Including ACTPol data

Finally, we further include the latest ACT small-scale CMB temperature and polarization anisotropy measurements [318]. Unlike Planck, ACT does not display a preference for extra lensing (as captured by the lensing amplitude \( A_{\text{lens}} > 1 \) in Planck). Therefore, we expect the Planck + ACT dataset combination to prefer slightly higher values of \( \Sigma m_\nu \), or in any case for the \( \Sigma m_\nu \) constraints resulting from such a dataset combination to be slightly degraded compared to the same dataset not including ACT. The reason is that increasing \( \Sigma m_\nu \) decreases the amplitude of lensing in the CMB, in the direction required by ACT. This expectation is confirmed by our analysis, as reported in the last rows of Tab. 2, where we find that the 95% C.L. upper limit of \(< 0.14 \text{ eV} \) from the base dataset combination is degraded to \( 0.17 \text{ eV} \) within the base + ACT dataset combination.

The extent to which the bound degrades is not very drastic, since the \( BAO_z \) and \( P_{g g}(k) \) datasets (included in the base dataset) are still the main drivers for the improvement in the constraints on \( \Sigma m_\nu \) compared to the CMB-only constraints, through the improved determination of \( \Omega_m \). Finally, we further include the \( C_{\kappa g}^{\ell} \) dataset, thus considering the base + ACT + \( C_{\kappa g}^{\ell} \) dataset combination, finding no significant shift in the upper bound on \( \Sigma m_\nu \), which remains compatible with \( 0.17 \text{ eV} \). Marginalized posterior distributions for \( \Sigma m_\nu \) obtained from the dataset combinations discussed so far are shown in Fig. 2, whereas the corner plot in Fig. 3 shows 2D joint and 1D marginalized posteriors for \( \Sigma m_\nu \) and the scale-dependent bias parameters obtained from the base + \( C_{\kappa g}^{\ell} \) dataset combination.

Our interpretation of these results is that the difference between the bounds on \( \Sigma m_\nu \) when including ACT vs Planck is partially a reflection of the mild \( \approx 2.5 \sigma \) tension existing between these CMB measurements, which has been well documented in the literature both by the ACT collaboration [318] and by [364]. Ultimately, part of this tension can be brought down to the fact that Planck primary CMB measurements appear at face value to prefer extra lensing in the small-scale temperature data (as the higher acoustic peaks are more smoothed than expected), as indicated by \( A_{\text{lens}} > 1 \). This naturally
Figure 2: 1D marginalized posterior distributions for $\Sigma m_\nu$ obtained from various dataset combinations discussed in the main text. The base dataset combination significantly improves the bound on $\Sigma m_\nu$ compared to Planck data alone (from 0.26 eV to 0.14 eV). The bound resulting from the base dataset combination is also comparable to the Planck + BAO cons bound (0.12 eV). The y axis is in arbitrary units, as we are plotting normalizable probability distributions.
disfavors heavier neutrinos. as these would suppress structure and reduce the lensing signal, whereas ACT sees no preference for extra lensing, and therefore can accommodate heavier neutrinos. This explains why unsurprisingly the upper limit on $\Sigma m_\nu$ degrades when including ACT data.

We note that, while is some disagreement between ACT and Planck as to the height of the first acoustic peak, this is not relevant to our discussion, as we are only using small-scale (high-$f$) data from ACT. Our conclusion is that until the reason underlying the preference for $A_{\text{lens}} > 1$ in Planck data is well understood, within a $\Lambda$CDM + $\Sigma m_\nu$ model the CMB side of the data is in principle still able to tolerate neutrino mass limits $\gtrsim 40\%$ weaker than those obtained when making use of Planck data.  

4.4. Galaxy bias parameters and shot noise: detection significance and degeneracies

The small-scale galaxy bias parameters ($b_{k^2\text{auto}}$ and $b_{k^2\text{cross}}$), as well as the shot noise parameter $P_{\text{shot}}$, are all detected at moderate significance: 2.9$\sigma$, 2.9$\sigma$, and 3.4$\sigma$ respectively, see Tab. 2 for the inferred mean values and uncertainties. We note that $b_{k^2\text{auto}}$ and $b_{k^2\text{cross}}$ are anti-correlated, which further justifies our choice of treating $b_{k^2\text{auto}}$ and $b_{k^2\text{cross}}$ as separate parameters modeling the galaxy bias behavior in the the galaxy-galaxy auto-spectrum and galaxy-matter cross-spectrum, respectively.

Another important degeneracy we find is that between the shot noise parameter $P_{\text{shot}}$ and the scale-dependent term of the auto-spectrum bias ($b_{k^2\text{auto}}$). In fact, we find that fixing $P_{\text{shot}}$ decreases the uncertainty in $b_{k^2\text{auto}}$ by an order of magnitude (see the fourth row in Tab. 2). This increases the detection significance from 2.9$\sigma$ to 16.5$\sigma$, confirming that it is of vital importance to include (and have precise measurement of) the shot noise term in order to better constrain the scale-dependent bias, and viceversa. Moreover, the negative correlation between the two parameters, already noted earlier in [195], is not surprising. Decreasing $P_{\text{shot}}$ decreases power on all scales, but the effect is particularly noticeable on small scales, as power is naturally larger on larger scales, for wavenumbers beyond the matter-radiation equality turn-around in $P_\text{gg}(k)$. This can be compensated by increasing $b_{k^2\text{auto}}$, as it enhances clustering and hence power on small scales.

We also comments on the signs and values of the bias parameters $b_{k^2\text{cross}}$ and $b_{k^2\text{auto}}$. In general, the bias parameters are considered nuisance parameters which are marginalized over. Nevertheless, the inferred values of the bias parameters can to some extent provide information on the scales at which physical processes connected to the galaxy bias parameters themselves start to play an important role. In the case of the phenomenological $b_{k^2\text{cross}}$ and $b_{k^2\text{auto}}$ parameters, these are loosely speaking associated to complexities inherent to galaxy formation. In particular, galaxy formation gives rise to non-local interactions associated to a characteristic scale, $k_{gf}$, which we can loosely speaking identify as $\approx 1/\sqrt{|b_{k^2\text{cross}}|}$ or $\approx 1/\sqrt{|b_{k^2\text{auto}}|}$ (for details, see Sec. 2.6 of [97]). Given our values of $b_{k^2\text{cross}}$ and $b_{k^2\text{auto}}$, the estimated scale is approximately $k_{gf} \sim 0.3 h\text{Mpc}^{-1}$, which agrees with our expectation concerning the scale at which effects due to galaxy formation start becoming important.

Moreover, the signs of the inferred constraints on $b_{k^2\text{cross}}$ and $b_{k^2\text{auto}}$ also agree with our expectation that $b_{k^2\text{cross}} > 0$ and $b_{k^2\text{auto}} < 0$. We recall, as discussed in Sec. 2.2, that

\[^{20}\text{See also the recent work of [365], where a stronger version of this point was made, and the related results of [366, 367] obtained from other datasets.}\]
the expectation that $b_{k^2\text{cross}} > 0$ comes from the fact that the small-scale matter-galaxy cross-correlation function in real space traces the density profile of host halos, whereas we expect $b_{k^2\text{auto}} < 0$ on the basis of the halo exclusion principle. We stress that we had not provided any information on the expected signs of $b_{k^2\text{cross}}$ and $b_{k^2\text{auto}}$ at the level of priors. Therefore, the fact that the inferred signs of these two parameters are consistent with theoretical expectations is completely data-driven. Finally, we note that similar observations had been made earlier in [195].

4.5. Robustness tests

We perform a set of robustness tests on our model, investigating the impact of including versus excluding the cross-covariance as well as the cross-correlation coefficient, and examining potential dependencies of $\Sigma m_\nu$ and the bias model parameters on the $k_{\text{max}}$ cut-off of the galaxy power spectrum.

4.5.1. Cross-covariance

One of the aims of this work was to investigate the impact of including versus excluding the usually neglected cross-covariance between $P^\delta\delta(k)$ and $C^\kappa g_\ell$, see Eq. (20). We used a Gaussian analytical approximation to estimate of the effects of including the cross-covariance. We ran the combination base + $C^\kappa g_\ell$ with and without cross-covariance, with results given in Tab. 2. We found no significant impact of including the cross-covariance. We interpret this as indicating that the effect of the cross-covariance is negligible at least with the current datasets, which justifies a posteriori the approximation adopted in [195]. With future datasets, however, the cross-covariance might become an important contributor, in which case the non-Gaussian contributions could also be considered, e.g. those related to mode-couplings or dependent on the binning-scheme [368–370].

4.5.2. Cross-correlation coefficient

As another robustness test, we examined the difference caused by excluding versus including the cross-correlation coefficient, $r(k, z)$. The inclusion of the cross-correlation coefficient is detailed in the paragraphs following Eq. (8). The exclusion of the cross-correlation coefficient was achieved by setting $r(k, z) = 1$. Note that in doing so one is in a sense assuming that $b_{k^2\text{cross}} = b_{k^2\text{auto}}$, as the cross-correlation coefficient accounts for decorrelation effects caused by $b_{k^2\text{cross}} \neq b_{k^2\text{auto}}$. However, we know that $b_{k^2\text{cross}} = b_{k^2\text{auto}}$ does not agree with predictions from simulations nor from theory, as discussed in Sec. 2.2. Indeed, we find that the data strongly prefers to treat $b_{k^2\text{cross}}$ and $b_{k^2\text{auto}}$ as separate parameters.

To test if including the cross-correlation coefficient is preferred by the data, we treated $b_{k^2\text{cross}}$ and $b_{k^2\text{auto}}$ as separate parameters while still setting $r(k, z)$ to unity. We found that effectively including $r(k, z)$ in this way gave significantly tighter constraints on $b_{k^2\text{cross}}$, as its detection significance increased approximately by a factor of 2. We did not find any significant change to the other bias parameters ($b_{\text{lin}}$, $b_{k^2\text{auto}}$, $P_{\text{shot}}$). The increased significance of our $b_{k^2\text{cross}}$ inference can be understood from the fact that the

---

21 The cross-correlation coefficient also accounts for decorrelations caused by stochastic contributions, as well as redshift-space distortions. However, compared to $b_{k^2\text{cross}}$ and $b_{k^2\text{auto}}$ these are relatively small and can therefore be ignored in the following discussion.
term in $P_{\text{mg}}(k)$ associated to $b_2\tilde{z}_{\text{cross}}$ is increased by a factor of $2b_{\text{lin}}$: without $r(k,z)$, $b_2\tilde{z}_{\text{cross}}$ enters into $P_{\text{mg}}(k)$ as a factor of $b_2\tilde{z}_{\text{cross}}k^2$, whereas with $r(k,z)$ it enters as a factor of $2b_{\text{lin}}b_2\tilde{z}_{\text{cross}}k^2$. Despite achieving a tighter constraint on $b_2\tilde{z}_{\text{cross}}$, we did not find strong evidence that a better fit was achieved when including the effects of $r(k,z)$. In fact, the $\chi^2$ for $C_\ell^{\g\g}$ remained relatively unchanged. However, we stress that there are no drawbacks of adding $r(k,z)$ into the model on mildly non-linear scales: $r(k,z)$ only depends on existing bias parameters and thus does not add any degrees of freedom that could lead to an overfitting of the data.

4.5.3. Small-scale wavenumber cut

We now investigate to what extent the inferred bounds on $\Sigma m_\nu$ are affected by the choice $k_{\text{max}}$ while remaining within the mildly non-linear regime. The full BOSS DR12 galaxy power spectrum monopole measurements cover wavenumbers within the range $0.002 \leq k/(h \text{Mpc}^{-1}) \leq 0.317$ [315]. Previously, we chose to restrict our analysis to the wavenumber range between $k_{\text{min}} = 0.030 h \text{Mpc}^{-1}$ and $k_{\text{max}} = 0.13 h \text{Mpc}^{-1}$. The choice of large-scale cut-off $k_{\text{min}}$ was dictated by the fact that $P_{\text{gg}}(k)$ measurements on larger scales become increasingly contaminated by observational systematics related to stellar density, seeing requirements, missing close-pairs, fiber collisions, and redshift failures, as discussed in detail in [316]. While these effects are modelled through systematic weights at the map level, we have conservatively chosen to exclude scales where these systematics play an important role, following earlier analyses [193–195]. Similarly, the choice of small-scale cut-off $k_{\text{max}}$ is dictated by the fact that non-linearities and complexities associated to galaxy formation start playing a very important role [371], and are not adequately captured by our simplified theoretical (bias) model.

With these caveats in mind, we investigate how the inferred constraints on $\Sigma m_\nu$ and other parameters (including the scale-dependent bias parameters) change if a higher $k_{\text{max}}$ is adopted. To do so, we adopt the base $+G_{\ell}^\nu$ dataset and increase $k_{\text{max}}$ starting from our baseline $k_{\text{max}}$ of $0.13 \text{hMpc}^{-1}$ up to $0.205 \text{hMpc}^{-1}$. The results of this test are reported in Tab. 2, from the fifth to the eighth row. We find that there are no significant changes neither in the inferred limits on $\Sigma m_\nu$ (see the last column of the Table), nor in the inferred values of the 6 $\Lambda$CDM parameters (not reported in the Table).

One possible interpretation of these results is related to our earlier observation, and similar earlier findings in [117, 119, 194], that current BOSS full-shape information and purely geometrical information coming from the reconstructed BAO peak(s) carry comparable constraining power once combined with CMB data, particularly given that the latter carries significant statistical power. If this is the case, most of the improvement gained by adding $P_{\text{gg}}(k)$ measurements to CMB data comes from breaking the geometrical degeneracy and better constraining $\Omega_m$ (and thus indirectly $\Sigma m_\nu$). The first two BAO peaks in $P_{\text{gg}}(k)$ lie between $0.05 \text{hMpc}^{-1}$ and $0.1 \text{hMpc}^{-1}$, and between $0.1 \text{hMpc}^{-1}$ and $0.15 \text{hMpc}^{-1}$ respectively. Once the first BAO peak(s) in $P_{\text{gg}}(k)$ have been measured, the geometrical information in $P_{\text{gg}}(k)$ has mostly been “exhausted” (also given that FoG damp peaks at higher $k$), and there is not much gain in moving to smaller scales.

Another possible interpretation of these results is that the bias nuisance parameters (almost) completely absorb the additional shape information when moving into the more non-linear regime. If so this would imply that our simplified theoretical model is able to reliably obtain the shape information from $P_{\text{gg}}(k)$ while extensively covering the mildly non-linear regime ($\leq 0.205 h \text{Mpc}^{-1}$), at least at the current level of precision of BOSS
full-shape data. Nonetheless, while this is an instructive test, we caution against over-interpretating its results since our theoretical $P_{g g}(k)$ model cannot safely be extended down to the scales to which we pushed the test. The fact that nuisance parameters absorb at least part of the small-scale information, even when moving across different cosmological models, has been noted also in the context of the EFTofLSS (see e.g. [124, 125, 343]), consistently with our findings.

We have discussed above that the inferred constraints on $\Sigma m_\nu$ are hardly affected by the choice of $k_{\text{max}}$ beyond 0.13 $h$Mpc$^{-1}$, as most of the geometrical information has been “exhausted” by then, and the galaxy bias parameters absorb the additional shape information which would be gained from moving within the more non-linear regime. Here we perform a similar analysis focused on the galaxy bias and shot noise parameters. The inferred values thereof for different choices of $k_{\text{max}}$ are given in Tab. 2. We find that as $k_{\text{max}}$ increases, the detection significance for all three parameters ($b_k^{\text{cross}}$, $b_k^{\text{auto}}$, and $P_{\text{shot}}$) remains roughly constant. What changes are the inferred central values of the parameters, which decrease by $\approx 0.6-\sigma$ for each added $k$-bin. These changes may be understood in terms of different bias contributions (e.g. tidal bias and other higher-order bias terms) entering and playing a role at different scales (see for instance [97, 149]). On the other hand, we are instead capturing these contributions through an “effective” $k^2$ scale-dependent term, which is the leading-order correction to a constant linear bias. Note that the constancy of the relative uncertainties implies that the errors propagated from $P_{g g}(k)$ into the galaxy bias parameters is not improved by including additional scales. Thus, additional shape information beyond $k_{\text{max}} = 0.13 h$Mpc$^{-1}$ does not appear to improve the precision at which the galaxy bias parameters and $\Sigma m_\nu$ are inferred, at least when considering current data, and within our simplified model.

It is also interesting to look at deviations in the inferred value of the shot noise parameter $P_{\text{shot}}$ from the fiducial Poissonian shot noise of the BOSS DR12 CMASS galaxy sample (i.e. deviations from $P_{\text{shot}} = 1$, given our choice of normalization) while varying $k_{\text{max}}$. We find that $P_{\text{shot}}$ goes from being super-Poissonian for $k_{\text{max}} = 0.13 h$Mpc$^{-1}$ to sub-Poissonian for $k_{\text{max}} = 0.12 h$Mpc$^{-1}$. We have assumed that any higher-order stochastic contributions to $P_{g g}(k)$ can be neglected. However, the differences in $P_{\text{shot}}$ for variations in $k_{\text{max}}$ might suggest potential benefits in including next-to-leading-order scale-dependent contributions, i.e. $k^4$ terms besides the $k^2$ term we considered, as a $k^4$ scale-dependent term is consistent with the bias upturn observed on small scales from both theoretical considerations and simulations (see for instance [372, 373] for considerations of this sort). For instance [201–203] recently adopted a phenomenological underlying theoretical model which is very close to ours, while including both $k^2$ and $k^4$ terms, and finding a very good agreement with simulations.

In the EFTofLSS context, recent work has found improvements with the inclusion of additional parameters capturing the scale-dependence of the shot noise (referred to as $a_0$ and $a_2$ in a large number of recent papers, see e.g. [133, 134]), originally not included in the fit. These phenomenological parameters may capture effects such as scale-dependent stochasticity, halo exclusion, and so on. It may be beneficial to include similar parameters in our model as well.

\footnote{A super-Poissonian shot noise contribution may be understood as an enhancement due to a high fraction of satellite galaxies (see for instance [283]).}
Figure 3: Triangular plot showing 2D joint and 1D marginalized posterior probability distributions for the galaxy bias and shot noise parameters resulting from the base + $C_l^{\gamma g}$ dataset combination.
5. Conclusions

In this work, we have revisited cosmological neutrino mass constraints from current full-shape galaxy power spectrum data (BOSS DR12 CMASS), in combination with measurements of the cross-correlation between CMB lensing convergence and galaxy overdensity maps. We adopt an underlying model which is minimal yet theoretically motivated, particularly in light of the precision of current data. We improve on the earlier work carried out by some of us in [195] in several respects, most notably through a more careful treatment of the correlations and covariance between galaxy clustering and CMB lensing-galaxy cross-correlation measurements, for which we construct a tractable model, and by performing a number of additional robustness tests.

When combining galaxy clustering data with current CMB data from Planck, we find a 95% C.L. upper limit on the sum of the neutrino masses \( \Sigma m_\nu \) of 0.14 eV, compatible with the bound of 0.12 eV one would obtain when replacing the full-shape information with purely geometrical information from the reconstructed BAO peak(s). This conclusion, already reached independently with a similar theoretical model in [194] and in the EFTofLSS-based analysis in [119] and related works, indicates that full-shape and purely geometrical information carry the same level of constraining power given the level of precision, volume, and reconstruction efficiency of current BOSS data. This interpretation is confirmed by our robustness tests which show that including clustering information from smaller scales does not improve our parameter constraints, suggesting that beyond a certain wavenumber all the geometrical information has been “exhausted”.

We find that the inclusion of CMB lensing-galaxy cross-correlation measurements does not have a significant impact on our results, which slightly disagrees with the earlier findings of [195]. This is partially due to the use of the updated Planck dataset (and in particular to the use of small-scale polarization data), as this dataset by itself leads to tight constraints on \( \Sigma m_\nu \): this implicitly sets much stronger requirements on other datasets or, equivalently, reduces the benefits of including additional datasets. In addition, the fact that including CMB lensing-galaxy cross-correlation measurements appears to not have a significant impact is a direct consequence of the relatively low signal-to-noise level of the current measurements. Furthermore, we have explored the role of CMB data by including small-scale temperature and polarization data from ACT. We have found that including the latter degrades the previous constraints by \( \approx 40\% \). This is related to the fact that unlike Planck data, ACT data does not appear to show any indication for extra lensing (as captured by the phenomenological \( A_{\text{lens}} \) parameter).

We expect that the full-shape information content of near-future galaxy clustering measurements at much higher signal-to-noise (for instance from Euclid or DESI) will supersede the geometrical one. In turn, this will significantly increase the importance of CMB lensing-galaxy cross-correlation measurements, which appear to not (yet) play a significant role in current data. Therefore, improvements in the precision and robustness of neutrino mass constraints from future galaxy surveys will require a more robust theoretical modeling, ultimately requiring the introduction of several extra nuisance parameters beyond the ones considered here (including possibly scale-dependent stochastic terms, as suggested by our robustness tests on the inferred shot noise parameter, and similar results in the context of the EFTofLSS finding improvements with the addition of scale-dependent shot noise terms). This will require a study weighing the systematic biases introduced by including too few nuisance parameters against the parameter
degeneracies introduced by including many parameters: in other words, whether the information gain from the decrease in observational uncertainties overcomes the increased complexity of the required theoretical model, or if there is a sweet spot compromising between the two, an issue which we plan to return to.

Finally, it is worth noting that cross-correlations between future CMB lensing [93, 360, 361] and galaxy clustering [231, 374] measurements will be detected at much higher statistical significance, particularly in light of the expected substantial overlap in sky fraction between future surveys. This will considerably increase the information content gain from the proposed joint analysis of galaxy clustering and CMB lensing-galaxy cross-correlations [362]. Therefore, future work along these lines is very timely and warranted.

Acknowledgements

We thank Rishi Babu and Shirley Ho for collaboration in the initial stages of the project. We acknowledge the use of computing facilities at NERSC and at the Texas Advanced Computing Center. S.V. thanks Misha Ivanov, Shubham Kejriwal, and Oliver Philcox for useful discussions. I.T., S.H., and K.F. acknowledge support by the Vetenskapsrådet (Swedish Research Council) through contract No. 638-2013-8993 and the Oskar Klein Centre for Cosmoparticle Physics. S.V. is supported by the Isaac Newton Trust and the Kavli Foundation through a Newton-Kavli Fellowship, and by a grant from the Foundation Blanceflor Boncompagni Ludovisi, née Bildt. S.V. acknowledges a College Research Associateship at Homerton College, University of Cambridge. E.G. acknowledges support from the Physics Department at Michigan Technological University. K.F. is grateful for support from the Jeff and Gail Kodosky Chair of Physics at the University of Texas, Austin, and from the U.S. Department of Energy, Office of Science, Office of High Energy Physics program under Award Number DE-SC0022021.

Appendix A. Goodness-of-Fit Test

We performed a goodness-of-fit analysis in order to evaluate how closely our theoretical model fits the observed data. This analysis can indicate if there are any under-/overestimation of errors, as well as general discrepancies between theory and data, including under-/overfits. To perform this analysis, we estimated the effective degrees of freedom \( \nu_{\text{eff}} \) for \( C_{\ell}^{\kappa g} \) and \( P_{\text{gg}}(k) \), which are later used to calculate the significance \( (p\text{-value}) \) of our theoretical models.

We wish to estimate \( \nu_{\text{eff}} \) because of the general rule that the minimized \( \chi^2 \)-values \( \chi^2_{\text{bestfit}} \) should approximately be equal to \( \nu_{\text{eff}} \). For example, if \( \chi^2_{\text{bestfit}} \) is much smaller than \( \nu_{\text{eff}} \) (i.e. large \( p\)-value) we may have an overfit to the data (potentially due to degenerate parameters), and/or an overestimation of errors. On the contrary, if \( \chi^2_{\text{bestfit}} \) is much larger than the \( \nu_{\text{eff}} \) (i.e. small \( p\)-value) we may have an underfit to the data (due to disagreements between model or data), and/or underestimation of errors.

The quantity \( \nu_{\text{eff}} \) may be estimated in several different ways: a rudimentary method is to calculate \( \nu_{\text{eff}} = D - M \) where \( D \) is the number of data points and \( M \) is the number of model parameters, both of which we know a priori. However, we can only use this estimate with confidence if we know how many of the model parameters are actually effective. For example, we have seen that some of the model parameters, such as \( P_{\text{shot}} \).
are tightly constrained by $P_{gg}^{}(k)$ but not by $C_{\ell}^{\kappa g}$, Therefore, if we were to assume that all model parameters are important for $C_{\ell}^{\kappa g}$, we would be subsequently underestimating $\nu_{\text{eff}}$ for $C_{\ell}^{\kappa g}$.

To have a more robust estimate of the effective number of degrees of freedom for $C_{\ell}^{\kappa g}$ and $P_{gg}^{}(k)$, we sampled their $\chi^2$-distributions from mock data, generated according to the following steps:

1. Assume that $P_{gg}^{}(k)$ ($C_{\ell}^{\kappa g}$) can be described by multivariate Gaussian distributions. Let the variance be represented by the covariance matrices we specified in Sec. 3, and the means by the theoretical bestfits of $P_{gg}^{}(k)$ ($C_{\ell}^{\kappa g}$) that we obtained in Sec. 2 for the base + $C_{\ell}^{\kappa g}$ dataset combination.
2. From these distributions, we draw a number of $n$ random samples, and thereby generate a number of $n$ mock datasets. In our case, we set $n = 100$.
3. For each generated mock dataset, calculate the minimized $\chi^2$-value.
4. With this $\chi^2$-distribution, estimate $\nu_{\text{eff}}$.

The results from the above steps are presented in Fig. A.4: the left panel contains the results for $P_{gg}^{}(k)$, whereas the right panel shows the equivalent for $C_{\ell}^{\kappa g}$.

We start by commenting on the results for $P_{gg}^{}(k)$. Based on the obtained significance value ($p=0.8$), we find that $P_{gg}^{}(k)$ is a good fit to the data. We also find a good agreement between the $\nu_{\text{eff}}$ shown in the Figure with the $\nu_{\text{eff}}$ that we would estimated by using the equation mentioned earlier: $\nu_{\text{eff}} = D - M = 19 - 4 = 15$, where the value of $M$ has been taken directly from the number of bias model parameters in our model: $b_{\text{lin}}$, $b_{k^2\text{cross}}$, $b_{k^2\text{auto}}$, and $P_{\text{shot}}$, as these were indicated to be effective for $P_{gg}^{}(k)$ in Tab. 2.

For $C_{\ell}^{\kappa g}$, we find that the significance is relatively low ($p=0.01$, i.e. within a 3-$\sigma$ confidence level). This low significance indicates the possibility that the errors for $C_{\ell}^{\kappa g}$ are too optimistic, and/or that there are disagreements between the model and data. As for the latter, there have been consistent findings for discrepancies between theory and modeling of $C_{\ell}^{\kappa g}$, possibly related to observational systematics: we discuss this in detail.

Figure A.4: $\chi^2$-distributions for $P_{gg}^{}(k)$ (left) and $C_{\ell}^{\kappa g}$ (right). In each figure, the vertical dotted line represents the $\chi^2$ value of the bestfit. The green histogram contains mock datasets that were generated using the method described in detail in the text. The green histogram was least-square fitted to a theoretical $\chi^2$-distribution, illustrated by the dotted red curve.
Figure B.5: Behaviour of the cross-correlation coefficient [see Eq. (8)] for a scale-independent (blue) versus scale-dependent (red) bias model. As discussed in the main text, the reduction in power is clearly stronger for the scale-dependent model.

in Sec. 4.2. Lastly, we find a good agreement between the $\nu_{\text{eff}}$ from the Figure with the $\nu_{\text{eff}}$ that we would estimated from: $\nu_{\text{eff}} = D - M = 13 - 1 = 12$. Indeed, the results in Table 2 indicates that it is mostly $b_{k^2,\text{cross}}$ which is constrained by $C_{\kappa g}^\ell (M = 1)$, whereas the other bias parameters are constrained by $P_{gg}(k)$.

Appendix B. Cross-correlation coefficient

As our formulation of the cross-correlation coefficient depends on non-linear quantities, we performed a cross-check that the cross-correlation coefficient behaves as intended. We expect the cross-correlation coefficient to capture the gradual reduction in power while moving to smaller scales, coming from decorrelations between the galaxy and matter field. As we have mentioned in Sec. 2, one such type of decorrelation is the stochastic component, $P_{\text{shot}}$, which we have demonstrated in Figure B.5 for a linear (scale-independent) galaxy bias model. Another source for decorrelation is the differing scale-dependent components between the matter and galaxy field. This is also portrayed in Figure B.5: in particular, the scale-dependent model predicts a larger suppression than the linear galaxy bias model. Both the scale-dependent component and the stochastic component lead to a gradual reduction in $r(k, z)$ on smaller scales. This demonstrates
that the cross-correlation quantity behave as intended within our \( k \)-range of interest. Lastly, note that the decorrelation effect coming from RSD has been included into both functions in Figure B.5 and also has been confirmed to contribute in the same way as the two aforementioned effects, although with a smaller contribution.

Appendix C. Cross-covariance calculation

The cross-covariance we use is based on the formulation of the estimators for 3D and 2D Gaussian fields in [368]. First, we incorporate our galaxy bias model into the aforementioned estimators, and second, we use these estimators to calculate an analytical form of the cross-covariance between the galaxy clustering data, \( P^{gg}_{\text{obs}}(k) \), and the CMB lensing convergence-galaxy cross-correlation, \( C^{\kappa g}_{\ell,\text{obs}} \). The assumptions that go into this calculation are primarily that non-Gaussian modes can be considered negligible for the purposes of this work (given current observational uncertainties), and that spherical harmonics defining the estimator for \( C^{\kappa g}_{\ell,\text{obs}} \) can be approximated by 2D Fourier modes.

We demonstrate the steps that are required to perform this calculation by first deriving the estimator for the galaxy clustering data, \( \hat{P}^{gg}(k_i) \), and the estimator for the CMB lensing convergence-galaxy cross-correlation, \( \hat{C}^{\kappa g}_{\ell} \).

Using the convention in [368], we define the unbiased estimator of the matter power spectrum used to compute the estimators for \( P^{gg}_{\text{obs}}(k) \) and \( C^{\kappa g}_{\ell,\text{obs}} \) as follows:

\[
\hat{P}^{mm}(k_i, z) = V_i \int_{k_i^{\text{bin}}} \frac{d^3k}{V_s(k_i)} \delta_m(k, z) \delta_m(-k, z),
\]

where \( \hat{P}^{mm} \) is the bin-averaged matter power spectrum taken over a thin spherical shell, \( V_s \), that radially extends within the interval: \( k_i^{\text{bin}} = [k_i - \delta k_i/2, k_i + \delta k_i/2] \), where \( \delta k_i \) is the estimated bin-size associated to \( k_i \): \( \delta k_i = (k_{i+1} - k_{i-1})/2 \). Lastly, \( V_i = (2\pi)^3/V_{\text{surv}} \) is the volume of the fundamental cell, which takes into account the finite volume of the survey, \( V_{\text{surv}} \).

The bin-averaged galaxy power spectrum depends on the bin-averaged matter power spectrum in Eq. (C.1) analogously to how the theoretical galaxy power spectrum depends on the theoretical matter power spectrum in Eq. (10):

\[
\hat{P}^{gg}(k_i, z) = V_i \int_{k_i^{\text{bin}}} \frac{d^3k}{V_s(k_i)} \delta_g(k, z) \delta_g(-k, z)
\]

\[
= b^{\text{auto}}(k_i) \left[ 1 + \frac{2}{3} \beta(k_i, z) + \frac{1}{3} \beta^2(k_i, z) \right] \hat{P}^{mm}(k_i, z) + P_{\text{shot}},
\]

which works well as long as the scale-dependent galaxy bias, \( b^{\text{auto}}(k) \), as well as the redshift-space distortion corrections (in this case, the Kaiser effect) are approximately constant within \( k_i^{\text{bin}} \).

Having now defined our estimator for the galaxy power spectrum from the formulation of the matter power spectrum in Eq. (C.1), we will perform a similar procedure for \( C^{\kappa g}_{\ell,\text{obs}} \). \( C^{\kappa g}_{\ell} \) depends on the projection of the mass overdensities:

\[
A(\hat{n}) = \int dz W^A(z) \delta_m(\chi(z)\hat{n}),
\]

33
Figure C.6: Gaussian cross-covariance matrix from Eq. (C.11) alongside the covariance matrices for the individual probes. The upper left block shows the covariance matrix for $P_{gg}^{obs}(k)$ (blue), and the lower right block shows the covariance matrix for $C^{\kappa g}_{\ell, obs}$ (orange). For the cross-covariance, the grey entries are essentially zero, whereas the non-zero entries are in green and capture the Gaussian modes.
where $A$ is either $\kappa$ or $g$ and therefore represents the convergence field or the 2D galaxy field, respectively. The fields are weighted along the line-of-sight with the help of their respective window functions. As we have projected mass densities, we could expand in terms of spherical harmonic modes over the unit sphere: $A(\hat{n}) = \sum_{\ell,m} A_{\ell m} Y_{\ell m}(\hat{n})$. However, the use of spherical harmonic modes would involve integration over spherical Bessel functions for the computation of the cross-covariance. To circumvent this issue, we will follow the approach in [368] and employ the flat-sky approximation. For $C_{\ell, \text{obs}}^{\kappa g}$, this relies on expanding the spherical harmonic modes into Cartesian Fourier modes:

$$A(\hat{n}) = \int \frac{d^2 \ell}{(2\pi)^2} e^{i\ell \cdot \hat{n}} A(\ell), \quad \text{(C.4)}$$

Using this formulation, the cross-covariance between the convergence mass overdensity and the projected galaxy overdensity is:

$$\langle \kappa(\ell) g(\ell') \rangle = (2\pi)^2 \delta^{(2)}(\ell - \ell') P^{\kappa g}(\ell), \quad \text{(C.5)}$$

where $\delta^{(2)}$ is the Dirac delta.

We insert the expressions of $\kappa(\ell)$ and $g(\ell)$ – which can be straightforwardly derived from Eq. (C.3) and (C.4) – into the above equation and obtain that the cross-power spectrum evaluates to:

$$P^{\kappa g}(\ell) = \int dz \frac{H(z)}{\chi^2(z)} W^\kappa(z) f^g(z) P^{\kappa g} \left( k = \frac{\ell}{\chi(z)}, z \right), \quad \text{(C.6)}$$

which is nothing but the Limber approximation to $C_{\ell, \text{th}}^{\kappa g}$ given in Eq. (12). Thus, $P^{\kappa g}(\ell)$ and our $C_{\ell, \text{th}}^{\kappa g}$ are equivalent, implying that the statistical estimator for $C_{\ell, \text{obs}}^{\kappa g}$ using $\hat{P}^{\kappa g}(\ell)$ is sufficient for the evaluation of the cross-covariance.

To calculate $\hat{P}^{\kappa g}(\ell)$, we only need to replace $P^{\text{mm}}(k)$ (contained within $P^{\kappa g}(k)$) with its estimator that we defined in Eq. (C.1): we assume that the other $k$-dependent components in $P^{\kappa g}(k)$ (i.e. $b_{k,\text{cross}}$ and $r(k, z)$) are approximately constant within a $k$-bin. This yields:

$$\hat{P}^{\kappa g}(\ell) = \int dz \frac{H(z)}{\chi^2(z)} W^\kappa(z) f^g(z) \times b_{\text{cross}}(k_i) r(k_i, z) P^{\text{mm}}(k_i, z) \bigg|_{k_i = \frac{\ell}{\chi(z)}}, \quad \text{(C.7)}$$

where we have made the substitution $P^{\kappa g} \rightarrow b_{\text{cross}} r P^{\text{mm}}$ for clarity.

With the estimators for $P^{\kappa g}_{\text{obs}}(k)$ and $C_{\ell, \text{obs}}^{\kappa g}$ at hand, we calculate the cross-covariance:

\begin{align*}
\text{Cov} \left[ \hat{P}^{\kappa g}(k_i, z_{\text{eff}}), \hat{P}^{\kappa g}(\ell_j) \right] \\
= b_{\text{auto}}^2(k_i) \left[ 1 + \frac{2}{3} \beta(k_i, z_{\text{eff}}) + \frac{1}{5} \beta^2(k_i, z_{\text{eff}}) \right] \\
\times \int dz \frac{H(z)}{\chi^2(z)} W^\kappa(z) f^g(z) b_{\text{cross}}(k_j) r(k_j, z) \\
\times \text{Cov} \left[ \hat{P}^{\text{mm}}(k_i, z_{\text{eff}}), \hat{P}^{\text{mm}}(k_j, z) \right] \bigg|_{k_j = \frac{\ell_j}{\chi(z)}}, \quad \text{(C.8)}
\end{align*}
The expression can easily be checked by identifying the pre-factors belonging to \( P_{\text{shot}}^{gg}(k) \) through comparison with Eq. (10) and similarly, the integrand belonging to \( C_{\ell \thicksim 1}^{mm} \) by comparison with Eq. (12). Note that \( P_{\text{shot}} \) does not directly appear in the expression as it is a constant and is statistically decorrelated from the density perturbations. However, \( P_{\text{shot}} \) still affects the cross-covariance through its indirect impact on the cross-correlation coefficient, \( r(k_j, z) \).

To continue evaluating Eq. (C.8), we expand the covariance of the matter power spectrum in Eq. (C.1), the covariance of the matter power spectrum evaluates to:

\[
\text{Cov}\left[ \hat{P}_{mm}(k_i, z_{\text{eff}}), \hat{P}_{mm}(k_j, z) \right] = V_i^2 \int_{k_{\text{bin}}^{i}} \frac{d^3 k}{V_i(k_i)} \int_{k_{\text{bin}}^{j}} \frac{d^3 k'}{V_j(k_j)} \times \left[ \langle \delta_m(k, z_{\text{eff}}) \delta_m(k', z) \rangle \langle \delta_m(-k, z_{\text{eff}}) \delta_m(-k', z) \rangle - \langle \delta_m(k, z_{\text{eff}}) \delta_m(-k', z) \rangle \langle \delta_m(-k, z_{\text{eff}}) \delta_m(k', z) \rangle \right],
\]

where we have neglected higher-order correlators beyond the power spectra. If the matter perturbations are evaluated at the same redshifts (i.e. \( z = z_{\text{eff}} \)), the above equation simply gives the Gaussian contributions to the covariance matrix for the matter power spectrum. However, in our case, the matter perturbations are evaluated at different redshifts as the value of \( z \) varies within the integration limits set by the redshift distribution \([f^g(z), \text{see Eq. (14)}]\). Given that this redshift distribution spans over a relatively small redshift interval \((0.43 \leq z \leq 0.69)\), which encompasses \( z_{\text{eff}} = 0.57 \), we normalize the matter perturbations evaluated at \( z \) to redshift \( z_{\text{eff}} \) by assuming linear growth of matter perturbations \( \delta_m(k, z) = D^+(z) \delta_m(k, 0) = [D^+(z)/D^+(z_{\text{eff}})] \delta_m(k, z_{\text{eff}}) \), yielding:

\[
\text{Cov}\left[ \hat{P}_{mm}(k_i, z_{\text{eff}}), \hat{P}_{mm}(k_j, z) \right] = \frac{V_i}{V_i(k_i)} \frac{D^2(z)}{D^2(z_{\text{eff}})} \left[ P_{mm}(k_i, z_{\text{eff}}) \right]^2 \left| \frac{k_i - k_j}{k_i} \right|. \quad (C.9)
\]

Therefore, we retrieve a Gaussian covariance that has been normalized by the amplitude of the growing mode, \( D^+ \). The Gaussian covariance has been proven to work very well for modes \( k \leq 0.1h\text{Mpc}^{-1} \). For \( k > 0.1h\text{Mpc}^{-1} \), however, the covariance is dependent on the mode-couplings to a larger extent [368–370]. If the cross-covariance is significant, ignoring these mode-couplings could impact the bounds on the bias parameters. The Gaussian contribution, however, is likely to be sufficient for our purposes: we aim to investigate if there is an indication that including the cross-covariance is of significant importance, and have found the answer to be no, at least given the precision of current galaxy clustering data. Note that with the increased precision of future datasets as well as the with use of more non-linear models, non-Gaussian contributions to the cross-covariance need to be considered. Therefore, in future work, it will be extremely important to extend the covariance to include non-Gaussian contributions, including e.g. trispectrum contributions and binning-scheme dependent contributions.

By inserting the Gaussian covariance in Eq. (C.10) into Eq. (C.8), we arrive at the
final expression of the cross-covariance:

\[
\text{Cov}\left[\hat{P}^{\text{obs}}(k_i, z_{\text{eff}}), \hat{P}^{\text{obs}}(k_j)\right] = \int dz \frac{H(z)}{\chi^2(z)} W^\ell(z) f^\ell(z) \times \frac{V_\ell}{V_\ell(k_i)} \frac{D^2(z)}{D_\ell^2(z_{\text{eff}})} 2P^{\text{obs}}(k_i, z_{\text{eff}})P^{\text{obs}}(k_i, z_{\text{eff}})\left|\chi_{k_i} - \chi_{k_j}\right| \leq \frac{1}{2},
\]

where we have absorbed the following factors: the auto-galaxy bias and the redshift-space distortions into \(P^{\text{sys}} = P^{\text{th}} - P^{\text{shot}}\) and the cross-galaxy bias and cross-correlation coefficient into \(P^{\text{mg}} = b_{\text{cross}}P^{\text{mm}}\). For illustrative purposes, we plot the full cross-covariance matrix in Fig. C.6 together with the individual covariance matrices for \(P^{\text{obs}}(k)\) and \(C_{LL}^{\text{obs}}\), and find that the results fully confirm our qualitative expectations.

References

[1] R. N. Mohapatra et al., Theory of neutrinos: A White paper, Rept. Prog. Phys. 70 (2007) 1757–1867, [hep-ph/0510213].
[2] Super-Kamiokande collaboration, Y. Fukuda et al., Evidence for oscillation of atmospheric neutrinos, Phys. Rev. Lett. 81 (1998) 1562–1567, [hep-ex/9807003].
[3] SNO collaboration, Q. R. Ahmad et al., Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory, Phys. Rev. Lett. 89 (2002) 011301, [nucl-ex/0204008].
[4] M. C. Gonzalez-Garcia and M. Maltoni, Phenomenology with Massive Neutrinos, Phys. Rept. 460 (2008) 1–129, [0704.1800].
[5] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, Global fit to three neutrino mixing: critical look at present precision, JHEP 12 (2012), [1209.3023].
[6] P. F. de Salas, S. Gariazzo, O. Mena, C. A. Ternes and M. Tortola, Neutrino Mass Ordering from Oscillations and Beyond: 2018 Status and Future Prospects, Front. Astron. Space Sci. 5 (2018) 36, [1806.11051].
[7] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tortola and J. W. F. Valle, Status of neutrino oscillations 2018: 3\(\nu\) hint for normal mass ordering and improved CP sensitivity, Phys. Lett. B 782 (2018) 633–640, [1708.01186].
[8] F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri and A. Palazzo, Global constraints on absolute neutrino masses and their ordering, Phys. Rev. D 95 (2017) 096014, [1609.08511].
[9] I. Esteban, M. C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni and T. Schwetz, Global analysis of three-flavour neutrino oscillations: synergies and tensions in the determination of \(\theta_{23}, \Delta C_{\ell}^\nu,\) and the mass ordering, JHEP 01 (2019) 106, [1811.06487].
[10] S. Hagedorn, P. F. de Salas, S. Gariazzo, M. Gerbino, M. Lattanzi, S. Vagnozzi et al., Bounds on light sterile neutrino mass and mixing from cosmology and laboratory searches, Phys. Rev. D 104 (2021) 123524, [2003.02289].
[11] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou, The fate of hints: updated global analysis of three-flavor neutrino oscillations, JHEP 09 (2020) 178, [2007.14792].
[12] F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri and A. Palazzo, Unfinished fabric of the three neutrino paradigm, Phys. Rev. D 104 (2021) 083031, [2107.00532].
[13] M. Archidiacono and S. Gariazzo, Two Sides of the Same Coin: Sterile Neutrinos and Dark Radiation, Status and Perspectives, Universe 8 (2022) 175, [2201.10319].
[14] E. W. Otten and C. Weinheimer, Neutrino mass limit from tritium beta decay, Rept. Prog. Phys. 71 (2008) 086201, [0909.2104].
[15] M. Aker, A. Beglarian, J. Behrens, A. Berlev, U. Besserer, B. Bieringer et al., First direct neutrino-mass measurement with sub-eV sensitivity, arXiv e-prints (May, 2021) arXiv:2105.08533, [2105.08533].
[16] S. Dell’Oro, S. Marcocci, M. Viel and F. Vissani, Neutrinoless double beta decay: 2015 review, Adv. High Energy Phys. 2016 (2016) 2162659, [1601.07512].
[17] M. J. Dolinski, A. W. P. Poon and W. Rodejohann, Neutrinoless Double-Beta Decay: Status and Prospects, Ann. Rev. Nucl. Part. Sci. 69 (2019) 219–251, [1902.04097].
A Short Review on the Latest Neutrinos Mass and Number Constraints from Cosmological Observables, Universe 8 (2022) 284.

A. J. Cuesta et al., First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Preliminary maps and basic results, Astrophys. J. Suppl. 148 (2003) 1–27, [astro-ph/0302207].

PLANK collaboration, P. A. R. Ade et al., Planck 2015 results. XIII. Cosmological parameters, Astron. Astrophys. 594 (2016) A13, [1502.01589].

PLANK collaboration, N. Aghanim et al., Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. 641 (2020) A6, [1807.06299].

2dFGRS collaboration, S. Cole et al., The 2dF Galaxy Redshift Survey: Power-spectrum analysis of the final dataset and cosmological implications, Mon. Not. Roy. Astron. Soc. 362 (2005) 505–534, [astro-ph/0501174].

SDSS collaboration, D. J. Eisenstein et al., Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies, Astrophys. J. 653 (2005) 560–574, [astro-ph/0501171].

A. J. Cuesta et al., The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: Baryon Acoustic Oscillations in the correlation function of LOWZ and CMASS galaxies in Data Release 12, Mon. Not. Roy. Astron. Soc. 457 (2016) 1770–1785, [1509.06371].

A. J. Cuesta, V. Niro and L. Verde, Neutrino mass limits: robust information from the power spectrum of galaxy surveys, Phys. Dark Univ. 13 (2016) 77–86, [1511.05953].

S. Wang, Y.-F. Wang, D.-M. Xia and X. Zhang, Impacts of dark energy on weighing neutrinos: mass hierarchies considered, Phys. Rev. D 94 (2016) 083519, [1608.00672].

C. S. Lorenz, E. Calabrese and D. Alonso, Distinguishing between Neutrinos and time-varying Dark Energy through Cosmic Time, Phys. Rev. D 96 (2017) 043510, [1706.00730].

S. Wang, Y.-F. Wang and D.-M. Xia, Constraints on the sum of neutrino masses using cosmological data including the latest extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample, Mon. Not. Roy. Astron. Soc. 468 (2017) 4404–4409, [1710.10264].

L. Chen, Q.-G. Huang and K. Wang, New cosmological constraints with extended-Baryon Oscillation Spectroscopic Survey DR14 quasar sample, Eur. Phys. J. C 77 (2017) 762, [1707.02742].

M.-M. Zhao, J.-F. Zhang and X. Zhang, Measuring growth index in a universe with massive neutrinos: A revisit of the general relativity test with the latest observations, Phys. Lett. B 779 (2018) 473–478, [1710.02391].

R. C. Nunes and A. Bonilla, Probing the properties of relic neutrinos using the cosmic microwave background, the Hubble Space Telescope and galaxy clusters, Mon. Not. Roy. Astron. Soc. 473 (2018) 4404–4409, [1710.10264].

S. Vagnozzi, S. Dhawan, M. Gerbino, K. Freese, A. Goobar and O. Mena, Constraints on the sum of the neutrino masses in dynamical dark energy models with $w(z) \geq -1$ are tighter than those obtained in $\Lambda$CDM, Phys. Rev. D 98 (2018) 083501, [1801.06553].

R.-Y. Guo, J.-F. Zhang and X. Zhang, Exploring neutrino mass and mass hierarchy in the scenario of vacuum energy interacting with cold dark matter, Chin. Phys. C 42 (2018) 095103, [1803.06910].

S. Roy Choudhury and S. Choubey, Updated Bounds on Sum of Neutrino Masses in Various Cosmological Scenarios, JCAP 09 (2018) 017, [1806.10832].

S. Roy Choudhury and A. Naskar, Strong Bounds on Sum of Neutrino Masses in a 12 Parameter Extended Scenario with Non-Phantom Dynamical Dark Energy $w(z) \geq -1$ with CPL parameterization, Eur. Phys. J. C 79 (2019) 262, [1807.02860].

A. Bonilla, R. C. Nunes and E. M. C. Abreu, Forecast on lepton asymmetry from future CMB experiments, Mon. Not. Roy. Astron. Soc. 485 (2019) 2486–2491, [1810.06356].

C. S. Lorenz, E. Calabrese and S. Hannestad, Time-varying neutrino mass from a supercooled phase transition: current cosmological constraints and impact on the $\Omega_m-\sigma_8$ plane, Phys. Rev. D 99 (2019) 023501, [1811.01991].

B. Bolliet, T. Brinckmann, J. Chluba and J. Lesgourgues, Including massive neutrinos in thermal Sunyaev Zeldovich power spectrum and cluster counts analyses, Mon. Not. Roy. Astron. Soc. 497 (2020) 1332–1347, [1906.10359].
S. Vagnozzi, New physics in light of the $H_0$ tension: An alternative view, Phys. Rev. D 102 (2020) 023518, [1907.07569].

W. Yang, S. Pan, R. C. Nunes and D. F. Mota, Dark calling Dark: Interaction in the dark sector in presence of neutrino properties after Planck CMB final release, JCAP 04 (2020) 008, [1910.08821].

N. Palanque-Delabrouille, C. Yèche, N. Schöneberg, J. Lesgourgues, M. Walther, S. Chabanier et al.,Hints, neutrino bounds and WDM constraints from SDSS DR14 Lyman$\alpha$ and Planck full-survey data, JCAP 04 (2020) 038, [1911.00973].

R. C. Nunes, S. K. Yadav, J. F. Jesus and A. Bernui, Cosmological parameter analyses using transversal BAO data, Mon. Not. Roy. Astron. Soc. 497 (2020) 2133–2141, [2002.09293].

W. Yang, E. Di Valentino, O. Mena and S. Pan, Dynamical Dark sectors and Neutrino masses and abundances, Phys. Rev. D 102 (2020) 023535, [2003.12552].

M. Zhang, J.-F. Zhang and X. Zhang, Impacts of dark energy on constraining neutrino mass after Planck 2018, Commun. Theor. Phys. 72 (2020) 125402, [2005.04647].

H.-L. Li, J.-F. Zhang and X. Zhang, Constraints on neutrino mass in the scenario of vacuum energy interacting with cold dark matter after Planck 2018, Commun. Theor. Phys. 72 (2020) 125401, [2005.12041].

W. Yang, E. Di Valentino, S. Pan and O. Mena, Emergent Dark Energy, neutrinos and cosmological tensions, Phys. Dark Univ. 31 (2021) 100762, [2007.03927].

W. Giare, E. Di Valentino, A. Melchiorri and O. Mena, New cosmological bounds on hot relics: axions and neutrinos, Mon. Not. Roy. Astron. Soc. 505 (2021) 2703–2711, [2011.14704].

S. Roy Choudhury, S. Hannestad and T. Tram, Updated constraints on massive neutrino self-interactions from cosmology in light of the $H_0$ tension, JCAP 03 (2021) 084, [2012.07519].

T. Brinckmann, J.-H. Chang and M. LoVerde, Self-interacting neutrinos, the Hubble parameter tension, and the cosmic microwave background, Phys. Rev. D 104 (2021) 063523, [2012.11830].

E. Di Valentino, S. Pan, W. Yang and L. A. Anchordoqui, Touch of neutrinos on the vacuum metamorphosis: Is the $H_0$ solution back?, Phys. Rev. D 103 (2021) 123527, [2012.05641].

E. Di Valentino, S. Gariazzo and O. Mena, Most constraining cosmological neutrino mass bounds, Phys. Rev. D 104 (2021) 083504, [2012.15267].

L. A. Anchordoqui, E. Di Valentino, S. Pan and W. Yang, Dissecting the $H_0$ and S8 tensions with Planck + BAO + supernova type Ia in multi-parameter cosmologies, JHEAP 32 (2021) 28–64, [2017.13932].

L. Feng, R.-Y. Guo, J.-F. Zhang and X. Zhang, Cosmological search for sterile neutrinos after Planck 2018, Phys. Lett. B 827 (2022) 136940, [2019.06111].

E. Di Valentino, S. Gariazzo, C. Giunti, O. Mena, S. Pan and W. Yang, Minimal dark energy: Key to sterile neutrino and Hubble constant tensions?, Phys. Rev. D 105 (2022) 103511, [2110.03990].

F. Renzi, N. B. Hogg and W. Giare, The resilience of the Ethertonn–Hubble relation, Mon. Not. Roy. Astron. Soc. 513 (2022) 4004–4014, [2112.05701].

S.-J. Jin, R.-Q. Zhu, L.-F. Wang, H.-L. Li, J.-F. Zhang and X. Zhang, Impacts of gravitational-wave standard siren observations from Einstein Telescope and Cosmic Explorer on weighing neutrinos in interacting dark energy models, arXiv e-prints [Apr., 2022] arXiv:2204.04089, [2204.04089].

S. Kumar, R. C. Nunes and P. Yadav, Updating non-standard neutrinos properties with Planck-CMB data and full-shape analysis of BOSS and eBOSS galaxies, arXiv e-prints (May, 2022) arXiv:2205.04292, [2025.04292].

A. Reeves, L. Herold, S. Vagnozzi, B. D. Sherwin and E. G. M. Ferreira, Restoring cosmological concordance with early dark energy and massive neutrinos?, arXiv e-prints (July, 2022) arXiv:2207.01501, [2207.01501].

E. Di Valentino, S. Gariazzo and O. Mena, Model marginalized constraints on neutrino properties from cosmology, arXiv e-prints (July, 2022) arXiv:2207.05167, [2207.05167].

C. Wetterich, Growing neutrinos and cosmological selection, Phys. Lett. B 655 (2007) 201–208, [0706.4427].

M. Wali Hossain, R. Myrzakulov, M. Sami and E. N. Saridakis, Unification of inflation and dark energy à la quintessential inflation, Int. J. Mod. Phys. D 24 (2015) 1530014, [1410.6105].

C.-Q. Geng, C.-C. Lee, R. Myrzakulov, M. Sami and E. N. Saridakis, Observational constraints on varying neutrino-mass cosmology, JCAP 01 (2016) 049, [1504.08141].

Z. Chacko, A. Dev, P. Du, V. Poulin and Y. Tsai, Determining the Neutrino Lifetime from Cosmology, Phys. Rev. D 103 (2021) 043519, [2002.08401].
[66] C. S. Lorenz, L. Funcke, M. Löffler and E. Calabrese, Reconstruction of the neutrino mass as a function of redshift, Phys. Rev. D 104 (2021) 123518, [2102.13618].

[67] D. Green and J. Meyers, Cosmological Implications of a Neutrino Mass Detection, arXiv e-prints (Nov., 2021) arXiv:2111.01096, [2111.01096].

[68] J. Alvey, M. Escudero, N. Sabti and T. Schwetz, Cosmic neutrino background detection in large-neutrino-mass cosmologies, Phys. Rev. D 105 (2022) 063501, [2111.14870].

[69] Q.-G. Huang, K. Wang and S. Wang, Constraints on the neutrino mass and mass hierarchy from cosmological observations, Eur. Phys. J. C 76 (2016) 489, [1512.05899].

[70] S. Hannestad and T. Schwetz, Cosmology and the neutrino mass ordering, JCAP 11 (2016) 035, [1606.04691].

[71] L. Xu and Q.-G. Huang, Detecting the Neutrino Mass Hierarchy from Cosmological Data, Sci. China Phys. Mech. Astron. 61 (2018) 09521, [1611.05178].

[72] M. Gerbino, M. Lattanzi, O. Mena and K. Freese, A novel approach to quantifying the sensitivity of current and future cosmological datasets to the neutrino mass ordering through Bayesian hierarchical modeling, Phys. Lett. B 775 (2017) 239–250, [1611.07847].

[73] W. Yang, R. C. Nunes, S. Pan and D. F. Mota, Effects of neutrino mass hierarchies on dynamical dark energy models, Phys. Rev. D 95 (2017) 103522, [1703.02556].

[74] F. Simpson, R. Jimenez, C. Pena-Garay and L. Verde, Strong Bayesian Evidence for the Normal Neutrino Hierarchy, JCAP 06 (2017) 029, [1703.03425].

[75] T. Schwetz, K. Freese, M. Gerbino, E. Giussarma, S. Hannestad, M. Lattanzi et al., Comment on “Strong Evidence for the Normal Neutrino Hierarchy”, arXiv e-prints (Mar., 2017) arXiv:1703.04585, [1703.04585].

[76] A. J. Long, M. Raveri, W. Hu and S. Dodelson, Neutrino Mass Priors for Cosmology from Random Matrices, Phys. Rev. D 97 (2018) 043510, [1711.08434].

[77] S. Gariazzo, M. Archidiacono, P. F. de Salas, O. Mena, C. A. Torres and M. Tortola, Neutrino masses and their ordering: Global Data, Priors and Models, JCAP 03 (2018) 011, [1801.04946].

[78] A. F. Heavens and E. Sellentin, Objective Bayesian analysis of neutrino masses and hierarchy, JCAP 04 (2018) 047, [1802.09450].

[79] C. Mahony, B. Leistedt, H. V. Peiris, J. Braden, B. Joachimi, A. Korn et al., Target Neutrino Mass Precision for Determining the Neutrino Hierarchy, Phys. Rev. D 101 (2020) 083513, [1907.04331].

[80] S. Roy Choudhury and S. Hannestad, Updated results on neutrino mass and mass hierarchy from cosmology with Planck 2018 likelihoods, JCAP 07 (2020) 037, [1907.12598].

[81] L. T. Hergt, W. J. Handley, M. P. Hobson and A. N. Lasenby, Bayesian evidence for the tensor-to-scalar ratio r and neutrino masses m.sub.Nu: Effects of uniform vs logarithmic priors, Phys. Rev. D 103 (2021) 123511, [2102.11511].

[82] R. Jimenez, C. Pena-Garay, K. Short, F. Simpson and L. Verde, Neutrino Masses and Mass Hierarchy: Evidence for the Normal Hierarchy, arXiv e-prints (Mar., 2022) arXiv:2203.14247, [2203.14247].

[83] S. Gariazzo, M. Gerbino, T. Brinckmann, M. Lattanzi, O. Mena, T. Schwetz et al., Neutrino mass and mass ordering: No conclusive evidence for normal ordering, arXiv e-prints (May, 2022) arXiv:2205.02195, [2205.02195].

[84] J. R. Bond, G. Efstathiou and J. Silk, Massive Neutrinos and the Large Scale Structure of the Universe, Phys. Rev. Lett. 45 (1980) 1980–1984.

[85] D. J. Eisenstein and W. H. Hu, Power spectra for cold dark matter and its variants, Astrophys. J. 511 (1997) 5, [astro-ph/9710252].

[86] W. Hu, D. J. Eisenstein and M. Tegmark, Weighing neutrinos with galaxy surveys, Phys. Rev. Lett. 80 (1998) 5255–5258, [astro-ph/9712057].

[87] S. Vagnozzi, Cosmological searches for the neutrino mass scale and mass ordering, arXiv e-prints (July, 2019) arXiv:1907.08010, [1907.08010].

[88] Z. Hou, R. Keisler, L. Knox, M. Millea and C. Reichardt, How Massless Neutrinos Affect the Cosmic Microwave Background Damping Tail, Phys. Rev. D 87 (2013) 083008, [1104.2333].

[89] G. Cabass, M. Gerbino, E. Giussarma, A. Melchiorri, L. Pagano and L. Salvati, Constraints on the early and late integrated Sachs-Wolfe effects from the Planck 2015 cosmic microwave background anisotropies in the angular power spectra, Phys. Rev. D 92 (2015) 063534, [1507.07586].

[90] J. A. Kable, G. E. Addison and C. L. Bennett, Deconstructing the Planck TT Power Spectrum to Constrain Deviations from ΛCDM, Astrophys. J. 905 (2020) 164, [2008.01785].

[91] S. Vagnozzi, Consistency tests of ΛCDM from the early integrated Sachs-Wolfe effect: Implications for early-time new physics and the Hubble tension, Phys. Rev. D 104 (2021) 40.
BOSS R. Neveux, E. Burtin, A. de Mattia, A. Semenaite, K. S. Dawson, A. de la Macorra et al.,
J. J. M. Carrasco, M. P. Hertzberg and L. Senatore,

DES H. Gil-Marín, L. Verde, J. Noreña, A. J. Cuesta, L. Samushia, W. J. Percival et al.,
D. Baumann, A. Nicolis, L. Senatore and M. Zaldarriaga,
A. Semenaite et al.,
H. Gil-Marín et al.,
E. Pajer and M. Zaldarriaga,

BOSS

F. Bernardeau, S. Colombi, E. Gaztanaga and R. Scoccimarro,
T. Sprenger, M. Archidiacono, T. Brinckmann, S. Clesse and J. Lesgourgues,
M. Ruiz-Granda and P. Vielva,
V. Desjacques, D. Jeong and F. Schmidt,
K. N. Abazajian, P. Adshead, Z. Ahmed, S. W. Allen, D. Alonso, K. S. Arnold et al.,
Y. Oyama, K. Kohri and M. Hazumi,

H. Hildebrandt et al.,

Effective Fluid

KiDS-450: Cosmological parameter constraints from tomographic weak gravitational lensing,
Mon. Not. Roy. Astron. Soc. 465 (2017) 1454, [1606.05338].

DES collaboration, T. M. C. Abbott et al.,
Dark Energy Survey Year 3 results: Cosmological constraints from galaxy clustering and weak lensing,
Phys. Rev. D 105 (2022) 023520, [2105.13549].

H. Gil-Marín, L. Verde, J. Noreña, A. J. Cuesta, L. Samushia, W. J. Percival et al.,
The power spectrum and bispectrum of SDSS DR11 BOSS galaxies – II. Cosmological interpretation,
Mon. Not. Roy. Astron. Soc. 452 (2015) 1914–1921, [1408.0027].

BOSS collaboration, J. N. Grieb et al.,
The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey:
Cosmological implications of the Fourier space wedges of the final sample,
Mon. Not. Roy. Astron. Soc. 467 (2017) 2085–2112, [1607.03143].

BOSS collaboration, A. G. Sanchez et al.,
The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey:
combining correlated Gaussian posterior distributions,
Mon. Not. Roy. Astron. Soc. 464 (2017) 1493–1501, [1607.03146].

BOSS collaboration, A. G. Sanchez et al.,
The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey:
cosmological implications of the configuration-space clustering wedges,
Mon. Not. Roy. Astron. Soc. 464 (2017) 1640–1658, [1607.03147].

BOSS collaboration, F. Beutler et al.,
The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey:
baryon acoustic oscillations in the Fourier space,
Mon. Not. Roy. Astron. Soc. 464 (2017) 3409–3430, [1607.03149].

BOSS collaboration, F. Beutler et al.,
The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey:
Anisotropic galaxy clustering in Fourier-space,
Mon. Not. Roy. Astron. Soc. 466 (2017) 2242–2260, [1607.03150].

H. Gil-Marín et al.,
The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample:
structure growth rate measurement from the anisotropic quasar power spectrum in the redshift range 0.8 < z < 2.2,
Mon. Not. Roy. Astron. Soc. 477 (2018) 1604–1638, [1801.02689].

H. Gil-Marín et al.,
The Completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey:
measurement of the BAO and growth rate of structure of the luminous red galaxy sample from the anisotropic power spectrum between redshifts 0.6 and 1.0,
Mon. Not. Roy. Astron. Soc. 498 (2020) 2492–2531, [2007.08994].

A. Semenaite et al.,
Cosmological implications of the full shape of anisotropic clustering measurements in BOSS and eBOSS,
Mon. Not. Roy. Astron. Soc. 512 (2022) 5657–5670, [2111.03156].

R. Neveux, E. Burtin, A. de Mattia, A. Semenaite, K. S. Dawson, A. de la Macorra et al.,
Combined full shape analysis of BOSS galaxies and eBOSS quasars using an iterative emulator,
arXiv e-prints (Jan., 2022) arXiv:2201.04679, [2201.04679].

D. Baumann, A. Nicolas, L. Senatore and M. Zaldarriaga,
Cosmological Non-Linearities as an Effective Fluid,
JCAP 07 (2012) 051, [1004.2488].

J. J. M. Carrasco, M. P. Hertzberg and L. Senatore,
The Effective Field Theory of Cosmological Large Scale Structures,
JHEP 09 (2012) 082, [1206.2926].

E. Pajer and M. Zaldarriaga,
On the Renormalization of the Effective Field Theory of Large
Bias in the Effective Field Theory of Large Scale Structures, *JCAP* 08 (2013) 037, [1301.7182].

[114] L. Senatore and M. Zaldarriaga, *The IR-resummed Effective Field Theory of Large Scale Structures*, *JCAP* 02 (2015) 013, [1404.5954].

[115] L. Senatore, *Bias in the Effective Field Theory of Large Scale Structures*, *JCAP* 11 (2015) 007, [1406.7843].

[116] G. D’Amico, J. Gleyzes, N. Kokron, K. Markovic, L. Senatore, P. Zhang et al., *The Cosmological Analysis of the SDSS/BOSS data from the Effective Field Theory of Large-Scale Structure*, *JCAP* 05 (2020) 005, [1909.06271].

[117] M. M. Ivanov, M. Simonović and M. Zaldarriaga, *Cosmological Parameters from the BOSS Galaxy Power Spectrum*, *JCAP* 05 (2020) 042, [1909.05277].

[118] T. Colas, G. D’Amico, L. Senatore, P. Zhang and F. Beutler, *Efficient Cosmological Analysis of the SDSS/BOSS data from the Effective Field Theory of Large-Scale Structure*, *JCAP* 06 (2020) 001, [1909.07951].

[119] M. M. Ivanov, M. Simonović and M. Zaldarriaga, *Cosmological Parameters and Neutrino Masses from the Final Planck and Full-Shape BOSS Data*, *Phys. Rev. D* 101 (2020) 083504, [1912.08208].

[120] O. H. E. Philcox, M. M. Ivanov, M. Simonović and M. Zaldarriaga, *Combining Full-Shape and BAO Analyses of Galaxy Power Spectra: A 1.6\% CMB-independent constraint on H0*, *JCAP* 05 (2020) 032, [2002.04335].

[121] G. D’Amico, L. Senatore and P. Zhang, *Limits on wCDM from the EFTofLSS with the PyBird code*, *JCAP* 01 (2021) 006, [2003.07956].

[122] T. Nishimichi, G. D’Amico, M. M. Ivanov, L. Senatore, M. Simonović, M. Takada et al., *Blind challenge for precision cosmology with large-scale structure: results from effective field theory for the redshift-space galaxy power spectrum*, *Phys. Rev. D* 102 (2020) 123541, [2003.08277].

[123] A. Chudaykin, M. M. Ivanov, O. H. E. Philcox and M. Simonović, *Nonlinear perturbation theory extension of the Boltzmann code CLASS*, *Phys. Rev. D* 102 (2020) 063533, [2004.10607].

[124] M. M. Ivanov, E. McDonough, J. C. Hill, M. Simonović, M. W. Toomey, S. Alexander et al., *Constraining Early Dark Energy with Large-Scale Structure*, *Phys. Rev. D* 102 (2020) 103502, [2006.11235].

[125] G. D’Amico, L. Senatore, P. Zhang and H. Zheng, *The Hubble Tension in Light of the Full-Shape Analysis of Large-Scale Structure Data*, *JCAP* 05 (2021) 072, [2006.12420].

[126] O. H. E. Philcox, B. D. Sherwin, G. S. Farren and E. J. Baxter, *Determining the Hubble Constant without the Sound Horizon: Measurements from Galaxy Surveys*, *Phys. Rev. D* 103 (2021) 023538, [2008.08084].

[127] D. Wadekar, M. M. Ivanov and R. Scoccimarro, *Cosmological constraints from BOSS with analytic covariance matrices*, *Phys. Rev. D* 102 (2020) 123521, [2009.00622].

[128] A. Chudaykin, K. Dolgikh and M. M. Ivanov, *Constraints on the curvature of the Universe and dynamical dark energy from the Full-shape and BAO data*, *Phys. Rev. D* 103 (2021) 023507, [2009.10106].

[129] G. D’Amico, Y. Donath, L. Senatore and P. Zhang, *Limits on Clustering and Smooth Quintessence from the EFTofLSS*, arXiv e-prints (Dec., 2020) arXiv:2012.07554, [2012.07554].

[130] M. M. Ivanov, *Cosmological constraints from the power spectrum of eBOSS emission line galaxies*, *Phys. Rev. D* 104 (2021) 103514, [2106.12580].

[131] O. H. E. Philcox, *Cosmology without window functions. II. Cubic estimators for the galaxy bispectrum*, *Phys. Rev. D* 104 (2021) 123529, [2107.06287].

[132] O. H. E. Philcox, J. Hou and Z. Slepian, *A First Detection of the Connected 4-Point Correlation Function of Galaxies Using the BOSS CMASS Sample*, arXiv e-prints (Aug., 2021) arXiv:2108.01670, [2108.01670].

[133] M. M. Ivanov, O. H. E. Philcox, T. Nishimichi, M. Simonović, M. Takada and M. Zaldarriaga, *Precision analysis of the redshift-space galaxy bispectrum*, *Phys. Rev. D* 105 (2022) 063512, [2110.10161].

[134] O. H. E. Philcox and M. M. Ivanov, *BOSS DR12 full-shape cosmology: \Lambda CDM constraints from the large-scale galaxy power spectrum and bispectrum monopole*, *Phys. Rev. D* 105 (2022) 043517, [2112.04515].

[135] G. Cabas, M. M. Ivanov, O. H. E. Philcox, M. Simonović and M. Zaldarriaga, *Constraints on Single-Field Inflation from the BOSS Galaxy Survey*, arXiv e-prints (Jan., 2022) arXiv:2201.07238, [2201.07238].

[136] G. D’Amico, M. Lewandowski, L. Senatore and P. Zhang, *Limits on primordial non-Gaussianities from BOSS galaxy-clustering data*, arXiv e-prints (Jan., 2022)
A. Aviles, A. Banerjee, G. Niz and Z. Slepian,
S. Cheng and B. Ménard,
F. Kamalinejad and Z. Slepian,
A. Moradinezhad Dizgah, G. K. Keating, K. S. Karkare, A. Crites and S. R. Choudhury,
S. Zhou et al.,
S. Bird, M. Viel and M. G. Haehnelt,
G. Cabass, M. M. Ivanov, M. Lewandowski, M. Mirbabayi and M. Simonović,
E. J. Baxter et al.,
C.-T. Chiang, M. LoVerde and F. Villaescusa-Navarro,
D. Valcin, F. Villaescusa-Navarro, L. Verde and A. Raccanelli,
B. D. Sherwin et al.,
A. Wu, A. Jiang and W. Fang,
J. Lee, S. Ryu and M. Baldi,
W. Liu, A. Jiang and W. Fang,
D. Valcin, F. Villaescusa-Navarro, L. Verde and A. Raccanelli,
E. Giusarma, S. Vagnozzi, S. Ho, S. Ferraro, K. Freese, R. Kamen-Rubio et al.,
S. Ryu and J. Lee,
M. Musso, A. Paranjape and R. K. Sheth,
S. Vagnozzi, E. Giusarma, O. Mena, K. Freese, M. Gerbino, S. Ho et al.,
S. Vagnozzi, E. Giusarma, O. Mena, S. Vagnozzi, S. Ho and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena, S. Vagnozzi, S. Ho and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
S. Vagnozzi, E. Giusarma, O. Mena and K. Freese,
[208] F. Bianchini et al., Toward a tomographic analysis of the cross-correlation between Planck CMB lensing and H-ATLAS galaxies, *Astrophys. J.* **825** (2016) 24, [1511.05116].

[209] DES, SPT collaboration, E. J. Baxter et al., A measurement of CMB cluster lensing with SPT and DES year 1 data, *Mon. Not. Roy. Astron. Soc.* **476** (2018) 2674–2688, [1708.01360].

[210] S. He, S. Alam, S. Ferraro, Y.-C. Chen and S. Ho, The detection of the imprint of filaments on cosmic microwave background lensing, *Nature Astron.* **2** (2018) 401–406, [1709.02543].

[211] S. Raghunathan, F. Bianchini and C. L. Reichardt, Imprints of gravitational lensing in the Planck cosmic microwave background data at the location of WISE+SCOS galaxies, *Phys. Rev. D* **98** (2018) 043506, [1710.09770].

[212] J. Han, S. Ferraro, E. Giusarma and S. Ho, Probing Gravitational Lensing of the CMB with SDSS-IV Quasars, *Mon. Not. Roy. Astron. Soc.* **485** (2019) 1720–1726, [1809.04196].

[213] DES, SPT collaboration, Y. Omori et al., Dark Energy Survey Year 1 Results: Tomographic cross-correlations between Dark Energy Survey galaxies and CMB lensing from South Pole Telescope+Planck, *Phys. Rev. D* **100** (2019) 043501, [1810.02342].

[214] S. Singh, R. Mandelbaum, U. Seljak, S. Rodríguez-Torres and A. Slosar, Cosmological constraints from galaxy-lensing cross-correlations using BOSS galaxies with SDSS and CMB lensing, *Mon. Not. Roy. Astron. Soc.* **491** (2020) 51–68, [1811.06499].

[215] G. A. Marques and A. Bernui, Tomographic analyses of the CMB lensing and galaxy clustering to probe the linear structure growth, *JCAP* **05** (2020) 052, [1908.04654].

[216] A. Krolikowski, S. Ferraro, E. F. Schlady and M. White, unWISE tomography of Planck CMB lensing, *JCAP* **05** (2020) 047, [1909.07412].

[217] D. Alonso, E. Bellini, C. Hale, M. J. Jarvis and D. J. Schwarz, Cross-correlating radio continuum surveys and CMB lensing: constraining redshift distributions, galaxy bias and cosmology, *Mon. Not. Roy. Astron. Soc.* **502** (2021) 876–887, [2009.01817].

[218] ACT collaboration, M. S. Madhavacheril et al., The Atacama Cosmology Telescope: Weighing Distant Clusters with the Most Ancient Light, *Astrophys. J. Lett.* **903** (2020) L13, [2009.07772].

[219] Q. Hang, S. Alam, J. A. Peacock and Y.-C. Cai, Galaxy clustering in the DESI Legacy Survey and its imprint on the CMB, *Mon. Not. Roy. Astron. Soc.* **501** (2021) 1481–1498, [2010.00466].

[220] E. Kitanidis and M. White, Cross-Correlation of Planck CMB Lensing with DESI-Like LRGs, *Mon. Not. Roy. Astron. Soc.* **501** (2021) 6181–6198, [2010.04698].

[221] X. Lin, Z. Cai, Y. Li, A. Krolewski and S. Ferraro, Constraining the Halo Mass of Damped Lyman Absorption Systems (DLAs) at \( z = 2.53 \) Using the Quasar-CMB Lensing Cross-correlation, *Astrophys. J.* **905** (2020) 176, [2011.01234].

[222] A. Krolikowski, S. Ferraro and M. White, Cosmological constraints from unWISE and Planck CMB lensing tomography, *JCAP* **12** (2021) 028, [2105.03421].

[223] F. Dong, P. Zhang, L. Zhang, J. Yao, Z. Sun, C. Park et al., Detection of a Cross-correlation between Cosmic Microwave Background Lensing and Low-density Points, *Astrophys. J.* **923** (2021) 153, [2107.09694].

[224] Z. Sun, J. Yao, F. Dong, X. Yang, L. Zhang and P. Zhang, Cross-correlation of Planck CMB lensing with DESI galaxy groups, *Mon. Not. Roy. Astron. Soc.* **511** (2022) 3548–3560, [2109.07387].

[225] M. White et al., Cosmological constraints from the tomographic cross-correlation of DESI Luminous Red Galaxies and Planck CMB lensing, *JCAP* **02** (2022) 007, [2111.09888].

[226] T. Chen and M. Remazeilles, Impact of thermal SZ effect on cross-correlations between Planck CMB lensing and SDSS galaxy density fields, *Mon. Not. Roy. Astron. Soc.* **514** (2022) 596–606, [2203.04809].

[227] A. Kusiaik, B. Bolliet, A. Krolikowski and J. C. Hill, Constraining the galaxy-halo connection of infrared-selected unWISE galaxies with galaxy clustering and galaxy-CMB lensing power spectra, arXiv e-prints (Mar., 2022) arXiv:2203.12583, [2203.12583].

[228] C. Chang, Y. Omori, E. J. Baxter, C. Doux, A. Choi, S. Pandey et al., Joint analysis of DES Year 3 data and CMB lensing from SPT and Planck II: Cross-correlation measurements and cosmological constraints, arXiv e-prints (Mar., 2022) arXiv:2203.12440, [2203.12440].

[229] S.-F. Chen, M. White, J. DeRose and N. Kolkon, Cosmological Analysis of Three-Dimensional BOSS Galaxy Clustering and Planck CMB Lensing Cross Correlations via Lagrangian Perturbation Theory, arXiv e-prints (Apr., 2022) arXiv:2204.10392, [2204.10392].

[230] L. Amendola et al., Cosmology and fundamental physics with the Euclid satellite, *Living Rev. Rel.* **21** (2018) 2, [1606.00180].

[231] DESI Collaboration, The DESI Experiment Part I: Science, Targeting, and Survey Design, arXiv e-prints (Oct., 2016) arXiv:1611.00036, [1611.00036].
due to neutrinos must not uncorrect'd go measurements of neutrino masses with future cosmological data observations neutrino properties on the estimation of inflationary parameters from current and future (2019) 035, 049, massive neutrinos II: on the universality of the halo mass function and bias, JCAP 02 (2014) 049, [1311.1212].

C. Fidler, N. Sujata and M. Archidiacono, Relativistic bias in neutrino cosmologies, JCAP 06 (2019) 035, [1812.09266].

M. Archidiacono, T. Brinckmann, M. Gerbino, J. Lesgourgues and V. Poulin, Physical effects involved in the measurements of neutrino masses with future cosmological data, JCAP 02 (2017) 052, [1610.09852].

M. Archidiacono, S. Hannestad and J. Lesgourgues, What will it take to measure individual neutrino mass states using cosmology?, JCAP 09 (2020) 021, [2003.03354].

J. Brandbyge, S. Hannestad, T. Haugbølle and B. Thomsen, The Effect of Thermal Neutrino Motion on the Non-linear Cosmological Matter Power Spectrum, JCAP 08 (2008) 020, [0802.3700].

J. Brandbyge and S. Hannestad, Grid Based Linear Neutrino Perturbations in Cosmological N-body Simulations, JCAP 05 (2009) 002, [0812.3149].

J. Brandbyge and S. Hannestad, Resolving Cosmic Neutrino Structure: A Hybrid Neutrino N-body Scheme, JCAP 01 (2010) 021, [0908.1969].

M. Viel, M. G. Haehnelt and V. Springel, The effect of neutrinos on the matter distribution as probed by the Intergalactic Medium, JCAP 06 (2010) 015, [1003.2422].

Y. Ali-Haimoud and S. Bird, An efficient implementation of massive neutrinos in non-linear structure formation simulations, Mon. Not. Roy. Astron. Soc. 428 (2012) 3375–3389, [1209.0461].

E. Castorina, C. Carbone, J. Bel, E. Sefusatti and K. Dolag, DEMNUni: The clustering of large-scale structures in the presence of massive neutrinos, JCAP 07 (2015) 043, [1505.07148].

A. Banerjee and N. Dalal, Simulating nonlinear cosmological structure formation with massive neutrinos, JCAP 11 (2016) 015, [1606.06167].

J. Liu, S. Bird, J. M. Z. Matilla, J. C. Hill, Z. Haiman, M. S. Madhavacheril et al., MassiveNuS: Cosmological Massive Neutrino Simulations, JCAP 03 (2018) 049, [1711.10524].

A. Banerjee, D. Powell, T. Abel and F. Villaescusa-Narvarro, Reducing Noise in Cosmological N-body Simulations with Neutrinos, JCAP 09 (2018) 028, [1801.03906].

C. Partmann, C. Fidler, C. Rampf and O. Hahn, Fast simulations of cosmic large-scale structure with massive neutrinos, JCAP 09 (2020) 018, [2003.07387].

Y. Y. Y. Wong, Higher order corrections to the large scale matter power spectrum in the presence of massive neutrinos, JCAP 10 (2008) 035, [0809.0693].

J. Lesgourgues, S. Matarrese, M. Pietroni and A. Riotto, Non-linear Power Spectrum including Massive Neutrinos: The Time-RG Flow Approach, JCAP 06 (2009) 017, [0901.4550].

D. Blas, M. Garny, T. Konstandin and J. Lesgourgues, Structure formation with massive neutrinos: going beyond linear theory, JCAP 11 (2014) 039, [1408.2995].

F. Führer and Y. Y. Y. Wong, Higher-order massive neutrino perturbations in large-scale structure, JCAP 03 (2015) 046, [1412.2764].

S. Hannestad, A. Upadhye and Y. Y. Y. Wong, Spoon or slide? The non-linear matter power spectrum in the presence of massive neutrinos, JCAP 11 (2020) 002, [2006.04995].

M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese and A. Riotto, Constraining warm dark matter candidates including sterile neutrinos and light gravitinos with WMAP and the Lyman-alpha forest, Phys. Rev. D 71 (2005) 063534, [astro-ph/0501522].

N. DePorzio, W. L. Xu, J. B. Muñoz and C. Dvorkin, Finding eV-scale light relics with cosmological observables, Phys. Rev. D 103 (2021) 023504, [2006.09380].

W. L. Xu, J. B. Muñoz and C. Dvorkin, Cosmological constraints on light but massive relics, Phys. Rev. D 105 (2022) 095029, [2107.09664].

S. Vagnozzi, E. Di Valentino, S. Gariazzo, A. Melchiorri, O. Mena and J. Silk, The galaxy power spectrum take on spatial curvature and cosmic concordance, Phys. Dark Univ. 33 (2021) 100851, [2010.02230].

J. R. Bond, G. Efstathiou and M. Tegmark, Forecasting cosmic parameter errors from microwave background anisotropy experiments, Mon. Not. Roy. Astron. Soc. 291 (1997)
[335] A. Gelman and D. B. Rubin, *Inference from Iterative Simulation Using Multiple Sequences*, Statist. Sci. 7 (1992) 457–472.

[336] A. Lewis, GetDist: a Python package for analysing Monte Carlo samples, arXiv e-prints (Oct., 2019) arXiv:1910.13970, [1910.13970].

[337] D. J. Eisenstein, H.-j. Seo, E. Sirko and D. Spergel, *Improving Cosmological Distance Measurements by Reconstruction of the Baryon Acoustic Peak*, Astrophys. J. 664 (2007) 675–679, [astro-ph/0604362].

[338] N. Padmanabhan, X. Xu, D. J. Eisenstein, R. Scalzo, A. J. Cuesta, K. T. Mehta et al., A 2 per cent distance to \( z = 0.35 \) by reconstructing baryon acoustic oscillations - I. Methods and application to the Sloan Digital Sky Survey, Mon. Not. Roy. Astron. Soc. 427 (2012) 2132–2145, [1202.0090].

[339] B. Lévy, R. Mohayaee and S. von Hausegger, *A fast semidiscrete optimal transport algorithm for a unique reconstruction of the early Universe*, Mon. Not. Roy. Astron. Soc. 500 (2021) 1165–1185, [2012.09574].

[340] S. von Hausegger, B. Lévy and R. Mohayaee, *Accurate Baryon Acoustic Oscillations Reconstruction via Semidiscrete Optimal Transport*, Phys. Rev. Lett. 128 (2022) 201302, [2110.08688].

[341] T. Brinckmann, D. C. Hooper, M. Archidiacono, J. Lesgourgues and T. Sprenger, *The promising future of a robust cosmological neutrino mass measurement*, JCAP 01 (2019) 059, [1808.05955].

[342] J. Hamann, S. Hannestad, J. Lesgourgues, C. Rampf and Y. Y. Y. Wong, *Cosmological parameters from large scale structure - geometric versus shape information*, JCAP 07 (2010) 022, [1003.3999].

[343] R. C. Nunes, S. Vagnozzi, S. Kumar, E. Di Valentino and O. Mena, *New tests of dark sector interactions from the full-shape galaxy power spectrum*, Phys. Rev. D 105 (2022) 123506, [2203.08093].

[344] S. Singh, R. Mandelbaum and J. R. Brownstein, *Cross-correlating Planck CMB lensing with SDSS: Lensing-lensing and galaxy-lensing cross-correlations*, Mon. Not. Roy. Astron. Soc. 464 (2017) 2120–2138, [1606.08841].

[345] C. Doux, M. Penna-Lima, S. D. P. Vitenti, J. Tréguer, E. Aubourg and K. Ganga, *Cosmological constraints from a joint analysis of cosmic microwave background and spectroscopic tracers of the large-scale structure*, Mon. Not. Roy. Astron. Soc. 480 (2018) 5386–5411, [1706.04583].

[346] J. R. Bermejo-Climent, M. Ballardini, F. Pinelli, D. Paoletti, R. Maartens, J. A. Rubiño Martín et al., *Cosmological parameter forecasts by a joint 2D tomographic approach to CMB and galaxy clustering*, Phys. Rev. D 103 (2021) 103502, [2106.05267].

[347] M. Ballardini and R. Maartens, *Constraining the neutrino mass using a multitracer combination of two galaxy surveys and cosmic microwave background lensing*, Mon. Not. Roy. Astron. Soc. 510 (2022) 4295–4301, [2109.03763].

[348] DES collaboration, T. Giannantonio et al., *CMB lensing tomography with the DES Science Verification galaxies*, Mon. Not. Roy. Astron. Soc. 456 (2016) 3213–3244, [1507.05551].

[349] A. Kuntz, *Cross-correlation of CFHTLenS galaxy catalogue and Planck CMB lensing using the halo model prescription*, Astron. Astrophys. 584 (2015) A53, [1510.00398].

[350] C. Shekhar Saraf, P. Bielawicz and M. Chodorowski, *Cross-correlation between Planck CMB lensing potential and galaxy catalogues from HELP, arXiv e-prints* (June, 2021) arXiv:2106.02551, [2106.02251].

[351] F. Bianchini and C. L. Reichardt, *Constraining gravity at large scales with the 2MASS Photometric Redshift catalogue and Planck lensing*, Astrophys. J. 862 (2018) 81, [1801.03736].

[352] C. M. Hirata, N. Padmanabhan, U. Seljak, D. Schlegel and J. Brinckmann, *Cross-correlation of CMB with large-scale structure: Weak gravitational lensing*, Phys. Rev. D 70 (2004) 103501, [astro-ph/0406004].

[353] K. M. Smith, O. Zahn and O. Dore, *Detection of Gravitational Lensing in the Cosmic Microwave Background*, Phys. Rev. D 76 (2007) 043510, [0705.3980].

[354] S. Das et al., *Detection of the Power Spectrum of Cosmic Microwave Background Lensing by the Atacama Cosmology Telescope*, Phys. Rev. Lett. 107 (2011) 021301, [1103.2124].

[355] A. van Engelen, S. Bhattacharya, N. Sehgal, G. P. Holder, O. Zahn and D. Nagai, *CMB Lensing Power Spectrum Biases from Galaxies and Clusters using High-angular Resolution Temperature Maps*, Astrophys. J. 786 (2014) 13, [1310.7023].

[356] J. Liu and J. C. Hill, *Cross-correlation of Planck CMB Lensing and CFHTLenS Galaxy Weak Lensing Maps*, Phys. Rev. D 92 (2015) 063517, [1504.05598].

[357] S. Ferraro and J. C. Hill, *Bias to CMB Lensing Reconstruction from Temperature Anisotropies
due to Large-Scale Galaxy Motions, Phys. Rev. D 97 (2018) 023512, [1705.06761].

[358] M. S. Madhavacheril and J. C. Hill, Mitigating Foreground Biases in CMB Lensing Reconstruction Using Cleaned Gradients, Phys. Rev. D 98 (2018) 023534, [1802.08230].

[359] O. Darwish, M. S. Madhavacheril, B. D. Sherwin, S. Aiola, N. Battaglia, J. A. Beall et al., The atacama cosmology telescope: a cmb lensing mass map over 2100 square degrees of sky and its cross-correlation with boss-cmass galaxies, Monthly Notices of the Royal Astronomical Society 500 (Nov, 2020) 2250–2263.

[360] Simons Observatory collaboration, P. Ade et al., The Simons Observatory: Science goals and forecasts, JCAP 02 (2019) 056, [1808.07445].

[361] Simons Observatory collaboration, M. H. Abitbol et al., The Simons Observatory: Astro2020 Decadal Project Whitepaper, Bull. Am. Astron. Soc. 51 (2019) 147, [1907.08284].

[362] X. Fang, T. Eifler, E. Schaan, H.-J. Huang, E. Krause and S. Ferraro, Cosmology from clustering, cosmic shear, CMB lensing, and cross correlations: combining Rubin observatory and Simons Observatory, Mon. Not. Roy. Astron. Soc. 509 (2021) 5721–5736, [2108.00628].

[363] B. Yu, S. Ferraro, Z. R. Knight, L. Knox and B. D. Sherwin, The physical origin of dark energy constraints from Rubin observatory and CMB-S4 lensing tomography, Mon. Not. Roy. Astron. Soc. 513 (2022) 1887–1894, [2108.02801].

[364] W. Handley and P. Lemos, Quantifying the global parameter tensions between ACT, SPT and Planck, Phys. Rev. D 103 (2021) 063529, [2007.08496].

[365] E. Di Valentino and A. Melchiorri, Neutrino Mass Bounds in the Era of Tension Cosmology, Astrophys. J. Lett. 931 (2022) L18, [2112.02993].

[366] R. K. Sharma, K. Lal Pandey and S. Das, Multi-parameter Dynamical Dark Energy Equation of State and Present Cosmological Tensions, arXiv e-prints (Feb., 2022) arXiv:2202.01749, [2202.01749].

[367] A. Chudaykin, D. Gorbunov and N. Nedelko, Exploring ΛCDM extensions with SPT-3G and Planck data: 4σ evidence for neutrino masses, full resolution of the Hubble crisis by dark energy with phantom crossing, and all that, arXiv e-prints (Mar., 2022) arXiv:2203.03666, [2203.03666].

[368] R. Scoccimarro, M. Zaldarriaga and L. Hui, Power spectrum correlations induced by nonlinear clustering, Astrophys. J. 527 (1999) 1, [astro-ph/9901099].

[369] M. Takada and W. Hu, Power spectrum super-sample covariance, Physical Review D 87 (Jun, 2013).

[370] I. Mohammed, U. Seljak and Z. Vlah, Perturbative approach to covariance matrix of the matter power spectrum, Monthly Notices of the Royal Astronomical Society 466 (Dec, 2016) 780–797.

[371] A. R. Cooray, Non-linear galaxy power spectrum and cosmological parameters, Mon. Not. Roy. Astron. Soc. 348 (2004) 250–260, [astro-ph/0311515].

[372] V. Assassi, D. Baumann, D. Green and M. Zaldarriaga, Renormalized Halo Bias, JCAP 08 (2014) 056, [1402.5916].

[373] M. Biagetti, V. Desjacques, A. Kehagias and A. Riotto, Nonlocal halo bias with and without massive neutrinos, Phys. Rev. D 90 (2014) 043502, [1405.1435].

[374] LSST collaboration, v. Ivezić et al., LSST: from Science Drivers to Reference Design and Anticipated Data Products, Astrophys. J. 873 (2019) 111, [0805.2366].