Chiral Symmetry Restoration and the $\pi^-$ meson

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The $\pi^-$ meson, which is at best a broad resonance in the vacuum, may suffer a substantial red-shift and appear as a soft and narrow collective mode at finite temperature and baryon densities. The physics behind this softening phenomenon and its relation to the partial restoration of chiral symmetry are discussed. Possible experiments in laboratory to test the idea are also summarized.

1. Introduction

One of the most intriguing phenomena in QCD is the dynamical breaking of chiral symmetry (DB). This explains the existence of the pion and dictates most of the low energy phenomena in hadron physics. DB is associated with the condensation of quark-antiquark pairs in the vacuum, $qq\bar{q}$, which is analogous to the condensation of Cooper pairs in the theory of superconductivity$^3)$. As the temperature ($T$) and/or the baryon density increase, the QCD vacuum undergoes a phase transition to the chirally symmetric phase where $qq\bar{q}$ vanishes$^4)$. For massless 2-avors at finite $T$ with vanishing baryon density, the chiral transition is likely to be of second order from the renormalization group analysis with the universality hypothesis$^2) and from the direct lattice QCD simulations$^3)$. The chiral transition at zero $T$ with vanishing baryon density is not well understood, but some of the effective theories suggest the first order transition at several times the nuclear matter density.

The general wisdom ofm any-body physics$^5) tells us that the fluctuation of the order parameter becomes large as the system approaches to the critical point of a second order or weak first order phase transition. In QCD, the fluctuations of the phase and the amplitude of the chiral order parameter $qq\bar{q}$ correspond to the pion ($\pi$) and the sigma-$\pi$ meson ($\pi$), respectively. Their role in the dynamical phenomenon near the critical point of chiral phase transition at finite $T$ was rst studied by the present authors in ref.$^6) it was shown in$^6) that the chiral restoration gives rise to a softening (the red-shift) of the $\pi$, which in turn leads to a small width due to the suppression of the phase-space of the decay $\pi \rightarrow \pi \pi$. Therefore, the $\pi^- \pi^+$ may appear as a narrow resonance at finite $T$ although it is at best a very broad resonance in the free space with a width comparable to its mass$^7)$. For the phenomenological applications of the above idea (the softening and narrowing of
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Fig. 1. Spectral functions in the channel (A) and in the channel (B) for $T=50, 120,$ and $145$ MeV.\cite{11} in (B) shows only a broad bump at low $T$ (a), while the spectral concentration is developed as $T$ increases as (a)! (b)! (c).

at finite $T$ in relation to the relativistic heavy ion collisions, see\cite{10}.

Further theoretical analysis\cite{11} by taking account the coupling § 2 shows that (i) the spectral function is the most relevant quantity for studying the nature of which was already discussed in\cite{6}, and (ii) the spectral function of has a characteristic enhancement just above the two-pion threshold near and below the critical temperature $T_c$ of chiral transition. This enhancement may be measured by the diphoton spectrum from the hot plasma created in the relativistic heavy ion collisions\cite{11,12}. In Fig. 1, shown are the spectral functions in the $-\pi$- and the $-\pi$-channels at finite $T$ calculated in the O(4) linear model. The apparent two characteristic features are the broadening of the pion peak (Fig. 1(A)) and the spectral concentration just above the 2-threshold (! 2m ) in the $-\pi$-channel (Fig. 1(B)).

The notion of the chiral soft mode has recently been applied to finite baryon density in\cite{13}. It was shown that the near-threshold enhancement could be seen as long as the partial restoration of chiral symmetry occurs, irrespective of the order of the phase transition. Since $h_{q\bar{q}}$ is expected to decrease almost by 35% in nuclear matter as shown below, one may observe the near-threshold enhancement in the dipion and diphoton spectra in hadron-nucleus and photon-nucleus reactions as was pointed out in\cite{14}.

In the following, we will focus on the recent development at finite baryon density and illustrate the essential ideas behind the physics of in-medium .

2. In-medium spectral function of

2.1. Quark condensate in nuclear matter

$h_{q\bar{q}}$ at finite baryon density ( ) obeys an exact theorem in QCD\cite{15}:

$$\frac{h_{q\bar{q}}}{h_{q\bar{q}}_0} = 1 - \frac{E^2 m^2}{N + m} \frac{d}{dm} \frac{E(\cdot)}{A}; \quad (2.1)$$
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where \( N = 45 \) 10 MeV is the pion-nucleon sigma term and \( E_{\pi N} \approx A \) is the nuclear binding energy per particle with \( m \) being the current quark mass. \( h_{\pi q_i} \), \( (225 \text{ MeV})^3 \) denotes the chiral condensate in the vacuum. The density-expansion of the right hand side of (2.1) gives a reduction of almost 35% of \( h_{\pi q_i} \) already at the nuclear matter density \( \rho = 0.17 \text{fm}^{-3} \); notice that the rest density modification comes linearly in \( \rho \).

2.2. Basic idea of the spectral enhancement

A direct evidence of partial chiral restoration could be an enhancement of the spectral strength in the scalar-isoscalar channel near the 2 GeV threshold as shown in Fig.1(B). The physics behind this phenomena is the following: Suppose we have Landau free-energy of a double-well type written in terms of the order parameter \( h_{\pi q_i} \),

\[
V(\rho) = \frac{a}{2} \rho^2 + \frac{b}{4} \rho^4; \tag{2.2}
\]

where \( a \) is positive in the vacuum but changes the sign at the critical point, while \( b \) remains positive. The minimum of the effective potential \( \rho_0 \) and the curvature at the minimum read

\[
\rho_0 = \frac{a}{b}; \quad \frac{1}{2} \frac{d^2 V(\rho)}{d \rho^2} = a; \tag{2.3}
\]

Therefore, as \( \rho \) becomes small, not only the order parameter \( \rho_0 \) but also the curvature decreases. This is nothing but the softening of the oscillational mode of the order parameter associated with the second order phase transition \( T \).

In the real world, the situation is not that simple, since the \( \rho \) has a large width from the decay \( \pi \to 2 \). Nevertheless, there is an interesting possibility that the spectral function just above the 2 GeV threshold could be enhanced due to the change of \( \rho_0 \). To describe the general features of this spectral enhancement, let us consider the propagator of the scalar-isoscalar \( \pi^- \) meson at rest in the medium: \( D^{-1}(\pi^-) = \frac{1}{\rho_0^2} \frac{d^2 V(\rho)}{d \rho^2} \); here \( m \) is the mass of the \( \pi^- \) in the tree-level, and \( (\pi^-) \) is the loop corrections in the vacuum as well as in the medium. The corresponding spectral function is given by \( (\pi^-) = \frac{1}{\rho_0^2} \text{Im} D(\pi^-) \). Near the 2 GeV threshold, the imaginary part in the one-loop order reads

\[
\text{Im} / (\pi^-, 2m) = \frac{4m^2}{\rho_0^2}; \tag{2.4}
\]

When chiral symmetry is being restored, \( m \) (effective mass) of the pion as a zero of the real part of the propagator \( \text{Re} D^{-1}(\pi^-) = 0 \) approaches to \( m \). Therefore, there exists a density \( \rho_0 \) at which \( \text{Re} D^{-1}(\pi^-) = 0 \) vanishes even before the complete degeneracy is realized; \( \text{Re} D^{-1}(\pi^-) = (\pi^-, 2m) \approx m^2 \). \( \text{Re } D^{-1}(\pi^-) = 0 \) At this point, the spectral function can be solely represented by the imaginary part of the self-energy:

\[
(\pi^- 2m) = \frac{1}{\text{Im}} / (\pi^- 2m) = \frac{(\pi^- 2m)}{4m^2/\rho_0^2}; \tag{2.5}
\]
which shows an enhancement of the spectral function at the \(2^\circ\) threshold. One should note that this enhancement is owing to the partial restoration of chiral symmetry and hence generic.

2.3. A calculation based on the \(O(4)\) \(-\) model

To make the argument more quantitative, let us evaluate \(\langle \rangle\) in the \(O(4)\) linear \(-\) model:

\[
L = \frac{1}{4} \text{tr}\left[ \sigma \mathbb{M} \, \mathbb{M} \,^\gamma \, 2 \mathbb{M} \,^\mu \, \gamma \, \frac{2}{4!} \left( \mathbb{M} \,^\mu \, ^\gamma \, \mathbb{M} \,^\nu \, \mathbb{M} \,^\nu \right) \right]; \quad (2.6)
\]

where \(\text{tr}\) is for the flavor index and \(\mathbb{M} = + i \, \sim\). Although the model has only a limited number of parameters and is not a precise low energy representation of QCD, it can describe the pion dynamics qualitatively well up to \(1\text{GeV}\). The coupling constants \(\gamma\); and \(h\) have been determined in the vacuum to reproduce \(f = 93\) \(\text{MeV}\), \(m = 140\) \(\text{MeV}\) as well as the \(s\)-wave scattering phase shift in the one-loop order. We parameterize the chiral condensate in nuclear matter \(h, i\) as

\[
h_i = \left( \begin{array}{c} 0 \\ 0 \end{array} \right) \quad (2.7)
\]

Depending on how one relates \(h, i\) with \(h, i\), \(= 0\) may take a value in the range \(0.6 \to 0.9\).

The in-medium spectral function in the \(-\) channel calculated in eq.(2.6) in the one-loop level are shown in Fig.2: The characteristic enhancement of the spectral function is seen just above the \(2^\circ\) threshold as expected. To confirm this threshold enhancement experimentally, measuring \(2^0\) and \(2^\circ\) in experiments with hadron/photon beam so the heavy nuclear targets are useful\(^{14}\). Measuring \(2^0\) is experimentally feasible\(^{17}\), which is free from the meson background inherent in the \(+\) measurement. Measuring \(2^\circ\) is also interesting because of the small \(\pi\) state interactions, although the branching ratio is small. (One

\[
\Phi(\rho) = 0.75
\]

\[
\Phi(\gamma) = 0.85
\]

\[
\Phi(\tau) = 0.6
\]

\[
\Phi(\omega) = 0.9
\]

\[
\Phi(\sigma) = 1.0
\]

\[
\Phi(\pi) = 0.13
\]

Fig. 2. Spectral function for \(\pi\) and the real part of the inverse propagator for several values of \(\omega\) as well as the baryon density increases.

\(\Phi(\rho) = 0.75\) Similar spectral concentration with Fig.1(B) can be seen as the baryon density increases.
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needs also to ght with the large background of photons mainly coming from \(^0\)s.\) Nevertheless, if the enhancement is prominent, there is a chance to nd the signal. There is also a possibility that one can detect dilepton through the scalar-vector mixing in matter: \(\phi \rightarrow e^+ e^-\). In this case, the dileptons are produced only when \(I = J = 0\) channel. A possible experiment is the formation of the mesonic nuclei through the nuclear reactions such as the \((d, t)\) reaction. The incident kinetic energies of the deuteron in the laboratory system \(E\) can be estimated to be \(1 \sim 5 \text{ GeV} < E < 10 \text{ GeV}\), to cover the spectral function in the range \(2m < \sqrt{s} < 750 \text{ MeV}\).

Recently CHAO S collaboration\(^{19}\) reported the data on the \(^+\) invariant mass distribution \(M^A\) in the reaction \(A (^+; ^+ )A^0\) with the mass number \(A\) ranging from 2 to 208. They observed that the yield for \(M^A\) near the 2 threshold is close to zero for \(A = 2\), but increases dramatically with increasing \(A\). They identi ed that the \(^+\) pairs in this range of \(M^A\) is in the \(I = J = 0\) state. This experiment was originally motivated by a possibility of strong correlations in nuclear matter\(^{20}\). However, the state-of-the-art calculations using the nonlinear chiral lagrangian together with \(N\) many-body dynamics do not reproduce the cross sections in the \(I = 0\) and \(I = 2\) channels simultaneously\(^{21}\); the nal state interactions of the emitted two pions in nuclei give rise to a slight enhancement of the cross section in the \(I = 0\) channel, but is not su cient to reproduce the experimental data. This indicates that some additional mechanism such as the partial restoration of chiral symmetry first proposed in\(^{13}\) may be relevant for explaining the data (see also the later studies\(^{22}\)).

In a recent paper, to make a close connection between the idea of the spectral enhancement and the CHAO S data, the in-medium \(-\) cross section has been calculated in the linear and non-linear models\(^{23}\). It was shown that, in both cases, substantial enhancement of the pion-pion correlation in the \(I = J = 0\) channel near the threshold can be seen due to the partial restoration of chiral symmetry, and an eective \(4\) \(N-N\) vertex responsible for the enhancement is identi ed. In Fig. 3, the in-medium \(-\) cross section in the \(I = 0\) channel calculated in the \(O(4)\) linear model

![In-medium \(\pi \pi\) cross section in \(I=J=0\) channel](image)

Fig. 3. In-medium \(\pi \pi\) cross section in the \(I = J = 0\) channel for different values of \(\Phi\). The cross section is shown in the arbitrary unit (A.U.)\(^{23}\).
is shown, where the same parameter sets are used with Fig.1 and Fig.2. It is of great importance to make an extensive analysis of the interaction in nuclear matter with the new vertex which has not been considered before in the analysis of the CHAOS data.

3. Conclusion

The light -meson does not show up clearly in the free space because of its large width due to the strong coupling with two pions. However, it may appear as a soft and narrow collective mode in the hadronic medium when the chiral symmetry is (partially) restored. A characteristic signal is the enhancement of the spectral function and the cross section near the 2 threshold. They could be observed in the hadronic reactions with heavy nuclear targets as well as in the heavy ion collisions. Detecting such signal provides us with a better understanding of the non-perturbative structure of the QCD vacuum and its quantum fluctuations.

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