Localized states at zigzag edges of multilayer graphene and graphite steps

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Abstract – We report the existence of zero-energy surface states localized at zigzag edges of \(N\)-layer graphene. Working within the tight-binding approximation, and using the simplest nearest-neighbor model, we derive the analytic solution for the wave functions of these peculiar surface states. It is shown that zero-energy edge states in multilayer graphene can be divided into three families: i) states living only on a single plane, equivalent to surface states in monolayer graphene; ii) states with finite amplitude over the two last, or the two first layers of the stack, equivalent to surface states in bilayer graphene; iii) states with finite amplitude over three consecutive layers. Multilayer graphene edge states are shown to be robust to the inclusion of the next-nearest-neighbor interlayer hopping. We generalize the edge state solution to the case of graphite steps with zigzag edges, and show that edge states measured through scanning tunneling microscopy and spectroscopy of graphite steps belong to family i) or ii) mentioned above, depending on the way the top layer is cut.

Introduction. – In the past few years carbon physics presented new challenges to the scientific community, increasing the list of rather unusual phenomena occurring in this life-support element. On the one hand, the discovery of metal-free carbon-based magnetism opened a new research field in fundamental physics, with possible applications in spin electronics [1–3]. On the other, the isolation of a single graphite layer —graphene— revealed an ultra-relativistic system full of unconventional electronic properties, and regarded with great expectation from the point of view of applications [4–6].

The origin of the observed magnetism in carbon-based materials is still under debate, but the presence of open edges seems to be an ubiquitous feature [3]. In proton-bombarded graphite, which shows room temperature ferromagnetism, proton irradiation induces hydrogen-terminated edges [7,8]. In activated carbon fibers and graphitized nanodiamond particles —known as nanographite— Curie-Weiss behavior and an enhanced paramagnetic susceptibility have been reported [2]. In these nanographites edges play a predominant role due to the built-in nanodimension. Edges are assumed to induce \(\pi\)-localized electrons due to surface (edge) states, which has been seen as a key ingredient to understand carbon’s magnetic behavior [1,3]. Indeed, the existence of edge states localized at zigzag edges of single-layer graphene, induced either by extended defects or vacancies, is now well documented and their magnetic behavior has been extensively reported [3–12].

Despite the positive correlation between edge state magnetism in graphene single layer and magnetic phenomena in graphite and nanographite, strictly speaking, neither of them are a single layer of graphene. Although the interlayer coupling is known to be very small, its effect is not negligible. To give an example, massless Dirac fermions in single-layer graphene turn out to be massive in bilayer graphene [6]. This brings about the question whether edge states are robust to stacking, or in other words, whether multilayer graphene can support edge states localized on zigzag edges. Moreover, with the advent of graphene physics, also graphene multilayers (bilayer, trilayer, . . .) were isolated. These graphene multilayers show interesting properties on their own [4,6], dissimilar from their single-layer constituent, and can be

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Fig. 1: (Colour on-line) (a) Side and top views of bilayer graphene, and its two families of edge states: monolayer (M) and bilayer (B); the vertical axes represent the associated charge densities (squared amplitudes). (b) Trilayer family of edge states (T) occurring in trilayer graphene (vertical axes represent charge densities), and all other possible edge states (in schematic view). (c) Edge states in N-layer graphene (in schematic view).

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even more suitable for some device applications [13–15]. Therefore, the question whether multilayer graphene possesses edge state physics is of paramount importance.

In this letter we show that zero-energy states localized at zigzag edges do exist in multilayer graphene. Using the simplest first-nearest-neighbor tight-binding model, we derive the analytical expression for multilayer-graphene edge states and show that their number is always equal to the number of layers occurring at the edge. The effect of second-nearest-neighbor interlayer hopping is considered, and the robustness of multilayer-graphene edge states is shown. Finally, we generalize the edge state solution to graphite steps, where experimental evidence for edge states has been widely reported [3]. The theoretical solution given in this letter agrees well with experimental findings. Also, we predict that edge states in graphite steps should be seen in scanning tunneling microscopy (STM) even when the step occurs underneath the first graphite layer.

Model. – We model AB-stacked multilayer graphene as shown in fig. 1(a) for the simplest case of a bilayer), where non-interacting π-electrons are allowed to hop only between A and B sublattices. In what follows we use the terminology balcony layers for layers represented with dashed (red) lines, and non-balcony layers for those represented with full (black) lines. Without loss of generality we assume all edge atoms belong to the A sublattice. The zigzag edge breaks translational invariance along its perpendicular direction, enabling us to write an effective one-dimensional Hamiltonian for a given momentum $k \in [0, 2\pi]$ along the edge (in units of $a^{-1}$). The first-nearest-neighbor tight-binding Hamiltonian can be written as

$$H_k = -t \sum_{i} \sum_{n} a_i^\dagger(k,n)(-e^{ik/2}D_k b_{i,k,n} + b_{i,k,n-1})$$

$$-t_{\perp} \sum_{i} \sum_{n} a_i^\dagger(k,n)b_{i+1,k,n} + \text{h.c.},$$

(1)

where $a_{i,k,n}$ ($b_{i,k,n}$) is the annihilation operator at momentum $k$ and position $n$ in sublattice $Ai$ (Bi), $i$ is the layer index and $D_k = -2 \cos(k/2)$. The first term in eq. (1) describes in-plane hopping while the second term parameterizes the inter-layer coupling ($t_{\perp} \ll t$). The symbol $\bullet$ indicates a sum over non-balcony layers. Afterwards we consider the second-nearest-neighbor interlayer hopping between A and B sublattices, which implies an extra term in eq. (1) given by $-\gamma_{3} \sum_i \sum_n b_i^\dagger(k,n)(e^{-ik}a_{i+1,k,n} - e^{-ik/2}D_k a_{i+1; k,n+1}) + \text{h.c.}$, where $\gamma_{3} \sim t_{\perp} \ll t$.

Edge states in N-layer graphene. – Multilayer-graphene edge states are investigated by solving the Schrödinger equation, $H_k|\psi_k\rangle = E_k|\psi_k\rangle$. The wave function $|\psi_k\rangle$ is written as a linear combination of the site amplitudes along the edge’s perpendicular direction, $|\psi_k\rangle = \sum_n \alpha_k(a,n)|a_i,k,n\rangle + \beta_k(b,n)|b_i,k,n\rangle$, where we have introduced the one-particle states $|a_i,k,n\rangle = c_i^\dagger e^{-ik\cdot n}|0\rangle$, with $c_i = a_i,b_i$. In addition we require the boundary conditions $\alpha_k(k,n \to \infty) = \beta_k(k,n \to \infty)$.$\alpha_k(k,n \to 0) = \beta_k(k,n \to 0) = 0$, accounting for the existence of the edge at $n = 0$. Within our model, the Fermi energy of multilayer graphene always occurs at zero energy. Therefore, we expect zero-energy edge states to have interesting physical consequences, and we set $E_k = 0$. As a result, the two sublattices become completely decoupled, and only the sublattice to which edge atoms belong can support edge states\textsuperscript{1}. This means that we always have $\beta_k(k,n) = 0$.

It was recently shown [17] that bilayer graphene supports two types of zero-energy edge states localized at zigzag edges for $2\pi/3 < k < 4\pi/3$: one type restricted to the balcony layer and coined monolayer family, with amplitudes equivalent to edge states in single-layer graphene,

$$\alpha_2(k,n) = \alpha_2(k,0)D_k^0 e^{-i\frac{\pi}{3}n};$$

(2)

and a new type coined bilayer family, with finite amplitudes over the two layers,

$$\alpha_1(k,n) = \alpha_1(k,0)D_k^0 e^{-i\frac{\pi}{3}n},$$

$$\alpha_2(k,n) = -\alpha_1(k,0)D_k^{-1} t e^{-i\frac{\pi}{3}(n-1)}(n - \frac{D_k^2}{1-D_k^2}),$$

(3)

where the normalization constants are given by $|\alpha_2(k,0)|^2 = 1 - D_k^2$ and $|\alpha_1(k,0)|^2 = (1 - D_k^2)^3/[3(1 - D_k^2)^2 + t_{\perp}^2/2]$. The charge densities (squared amplitudes)\textsuperscript{1}In the ribbon geometry the two sublattices are equivalent, supporting edge states localized in opposite ribbon edges. In the semi-infinite system, only those localized in the edge sublattice survive [16,17].

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associated with the two families of edge states are represented in fig. 1(a). Let us now consider a trilayer as shown in fig. 1(b), where a non-balcony layer is sandwiched between two balcony layers. Clearly, the bilayer family is not an edge state solution for this trilayer, as any finite amplitude at a non-balcony layer implies, through eq. (1), a finite amplitude over adjacent layers. We note, however, that our model ignores the coupling between next-nearest layers. Thus, if we construct a trilayer wave function whose amplitudes over balcony/non-balcony layers mimic those for the bilayer family, it is guaranteed, apart from a normalization factor, that we have an edge state solution. More precisely, we arrive at a new type of edge state with finite amplitudes over three consecutive layers —trilayer family— whose analytic form can be written as

$$\alpha_1(k, n) = -\alpha_2(k, 0)D_k^{n-1}\frac{t_\perp}{t}e^{-i\frac{\pi}{3}(n-1)} - \frac{D_k^2}{1 - D_k^2},$$

$$\alpha_2(k, n) = -\alpha_2(k, 0)D_k^n e^{-i\frac{\pi}{3}n},$$

$$\alpha_3(k, n) = -\alpha_2(k, 0)D_k^{n-1}\frac{t_\perp}{t}e^{-i2(n-1)} - \frac{D_k^2}{1 - D_k^2},$$

where the normalization constant is given by $$|\alpha_2(k, 0)|^2 = (1 - D_k^2)^2/[|1 - D_k^2|^2 + 2D_k^2/t^2].$$ The charge density (squared amplitude) associated with the trilayer family of edge states is represented in fig. 1(b).

Additionally, the trilayer we have been discussing also supports edge states of the monolayer family localized at balcony layers, as schematically shown in fig. 1(b). In fact, this is a general result. Balcony layers have an edge sublattice which is not connected through $$t_\perp$$ to adjacent layers. Thus, the monolayer family is always an edge state solution in N-layer graphene. Even more generally, we can look at a balcony layer as a buffer layer. As can be seen from eq. (1), a finite amplitude over a balcony layer does not imply finite amplitudes over adjacent layers. For the trilayer shown at the bottom of fig. 1(b), where a balcony layer is sandwiched between two non-balcony layers, monolayer edge states certainly exist at the middle balcony layer. But because of the buffer layer character, also bilayer edge states are present, localized either at the two top or the two bottom layers. An immediate consequence of the buffer layer concept is the fact that the trilayer family is the most general edge state family we can have, and exists localized at any non-balcony layer and its two adjacent layers, with all other site amplitudes equal to zero. Therefore, we have three families of edge states occurring in multilayer graphene: i) monolayer family for each balcony layer, eq. (2); ii) bilayer family for each non-balcony layer that starts and/or ends the multilayer, eq. (3); iii) trilayer family for each non-balcony layer sandwiched between two balcony ones, eq. (4). This is schematically represented in fig. 1(c). Note that the number of edge state families is always equal to the number of edge layers.

Effect of $\gamma_3$. — In multilayer graphene the effect of $\gamma_3$ is of the order of $$\frac{t_\perp}{t},$$ and should be included in a consistent edge state solution. The buffer layer concept introduced previously, however, does not survive at a finite $\gamma_3$. In order to generalize the edge state solution to the present case we use the transfer matrix technique, following ref. [17]. For bilayer graphene the transfer matrix, defined as $|\alpha_1(k, n), \alpha_2(k, n)|^T = e^{-ikn/2}T(2)|\alpha_1(k, 0), \alpha_2(k, 0)|^T,$ is given by

$$T(2) = \begin{bmatrix} u & v \\ x & D_k \end{bmatrix},$$

where $u = D_k(1 - \xi),$ $v = -\frac{\gamma_3}{4}e^{-ik/2}(1 - D_k^2),$ and $x = -\frac{\gamma_3}{4}e^{ik/2},$ with $\xi = t_\perp \gamma_3/t^2.$ The edge states are completely determined by the eigenvalues $\lambda_\pm$ and eigenvectors $\psi_\pm$ of the transfer matrix. If $|\lambda_\pm| < 1,$ then edge states exist and are given by $|\alpha_1(k, n), \alpha_2(k, n)|^T \propto e^{-ikn/2}\lambda_\pm^n \psi_\pm,$ apart from a normalization constant. Diagonalizing eq. (5), we obtain $\lambda_\pm = D_k(1 - \xi) \pm \sqrt{\lambda_\pm^2 + \xi^2}$ with $\lambda_\pm = \frac{1}{2}(1 + 1/t^2).$ Simple algebra shows that for $\lambda_\pm,$ the convergence condition implies $2\cos^{-1}((\sqrt{1 + \xi/2}))/<k < 2\cos^{-1}(-1 - \xi/2)$ or $4\pi/3 < \xi < 2\cos^{-1}(-\sqrt{1 + \xi/2}),$ while for $\lambda_-,$ it implies $2\cos^{-1}((1 + \xi/2))/<k < 2\pi/3$ or $2\cos^{-1}(-1 - \xi/2) < \xi < 2\cos^{-1}(-\sqrt{1 + \xi/2}).$ We conclude that bilayer graphene still has two families of edge states for $\gamma_3 \neq 0,$ though the $k$ range is slightly changed when compared with the $\gamma_3 = 0$ case. In particular, we have only one family for $k \in [2\pi/3, 2\cos^{-1}(-1 - \xi/2)]$ and $k \in [2\cos^{-1}(-1 - \xi/2), 4\pi/3],$ although the existence of edge states for $k < 2\pi/3$ and $k > 4\pi/3$ compensates this reduction, and we still have edge states for $1/3$ of the possible $k$s, as in the $\gamma_3 = 0$ case. As a test to what has just been said, we have numerically computed the energy spectrum for a bilayer ribbon with zigzag edges ($t_\perp = \gamma_3 = 0.2t$ and width 400 unit cells). The result is shown in fig. 2(a). Four flat bands at zero energy are clearly seen, and can be identified with the above-mentioned two families of edge states, two per edge. The insets reveal the $k$ restrictions mentioned before.
Fig. 3: (Colour on-line) Number of edge states per layer in N-layer graphene as a function of k: (a) \( \gamma_3 = 0 \); (b) \( \gamma_3 = t_\perp \); (c) \( \gamma_3 = 2t_\perp \). We set \( t_\perp = 0.2t \).

In fact, the values of \( k \) that limit the existence or number of edge states coincide with the Dirac points and satellite Fermi points that are \( \gamma_3 \neq 0 \) [18], as indicated by the thin red lines (the mismatch is due to the finite width of the ribbon, and consequent edge state overlap). The transfer matrix eigenvectors can be written as \( \chi^\pm = [\lambda_{\pm} - D, -\xi D] e^{\pm \xi k^2/2 T} \), from which we can write two families of wave functions, \( |\psi_{\pm}\rangle = C_{\pm} \sum_{n=0}^{\infty} e^{-\xi kn/2} \lambda_{\pm}^n |\chi_{\pm}\rangle \), where the normalization constant is given by \( C_{\pm} = \left| 1 - |\lambda_{\pm}|^2 \right| / (|\lambda_{\pm} - D|^2 + t_\perp^2/\mu^2)^{1/2} \). Note, however, that \( \chi^+ \) and \( \chi^- \) are not orthogonal, implying the non-orthogonality of the two solutions \( |\psi_{\pm}\rangle \). It is convenient to orthogonalize \( |\psi_{-}\rangle \) with respect to \( |\psi_{+}\rangle \), whose result can be written as \( |\psi_{-}\rangle = (|\psi_{-}\rangle - \langle \psi_{+}|\psi_{-}\rangle |\psi_{+}\rangle) / (1 - \langle \psi_{+}|\psi_{-}\rangle^2) \), where \( \langle \psi_{+}|\psi_{-}\rangle = C_{+} C_{-} (t_\perp^2/\mu + \xi D^2_\perp - \xi) / (1 - D^2_\perp + \xi) \). In fig. 2(b) we show the squared amplitudes associated with \( |\psi_{+}\rangle \) (left) and \( |\psi_{-}\rangle \) (right). The thin lines represent the edge states in bilayer graphene for \( \gamma_3 = 0 \), as given by eqs. (2) and (3). Clearly, as long as \( \gamma_3 \ll t_\perp \), we can identify \( |\psi_{+}\rangle \) with the monolayer family and \( |\psi_{-}\rangle \) with the bilayer family discussed previously. For \( \gamma_3 \sim t_\perp \), the edge state \( |\psi_{+}\rangle \), former monolayer family, already has an appreciable weight on both layers.

The analysis made for bilayer graphene with \( \gamma_3 \neq 0 \) can be extended to N-layer graphene. Defining the transfer matrix as \( [\alpha_{1}(k,n), \ldots, \alpha_{N}(k,n)]^T = e^{-\xi kn/2 T} \mathbf{T}_N^{(N)} [\alpha_{1}(k,0) , \ldots, \alpha_{N}(k,0)]^T \), we can derive its general pattern for a given number of layers \( N \),

\[
\mathbf{T}_N = \begin{bmatrix}
 u & v & -\xi D_k \\
 -\xi D_k & x & D_k \\
 & -\xi D_k & v & u \\
 & \ddots & \ddots & \ddots \\
 & & \ddots & \ddots & \ddots \\
 & & & \ddots & \ddots & \ddots \\
\end{bmatrix}
\]

(6)

The periodic structure of \( \mathbf{T}_N \) is readily identified, and the transfer matrix for any multilayer graphene is easily constructed. By diagonalizing eq. (6), and checking whether the eigenvalues \( \lambda \) satisfy \( |\lambda| < 1 \), we can conclude about the existence of edge states in N-layer graphene for \( \gamma_3 \neq 0 \). In fig. 3 the number of edge states per layer is shown in the plane number of layers \( N \) vs. \( k \). Panel 3(a) confirms what has been said for \( \gamma_3 = 0 \): same number of edge states as the number of layers for \( 2\pi/3 < k < 4\pi/3 \). As shown in panels 3(b) and 3(c), around the Dirac points the number of edge states for \( \gamma_3 \neq 0 \) may be smaller than the number of layers, as we have seen for \( N = 2 \). However, there is always a broad region in between the Dirac points where the number of edge states equals the number of layers. Thus, we conclude that multilayer graphene edge states are robust to second nearest-neighbor interlayer hopping. This result agrees with first principles calculations, where the presence of edge states and edge magnetism have been reported in bilayer [19,20] and graphite [21] zigzag nanoribbons.

**Graphite steps.**— Finally, we generalize the edge state solution to graphite steps, where experimental evidence for edge states has been widely reported [3,22–28]. The local density of states (LDOS) peak seen in STM of graphite steps has been interpreted as the experimental confirmation of the theoretically predicted single layer edge states [29]. However, we can easily convince ourselves that monolayer edge states do not always provide an eigenstate for a zigzag step-edge. To see why, we consider fig. 4(a), where the two possible zigzag step-edges on the surface of graphite are shown. These two terminations are denoted \( \alpha \)-type and \( \beta \)-type. For an \( \alpha \)-type termination the edge carbon atoms occur exactly on top of carbon atoms of the underlying layer, while for a \( \beta \)-type termination they occur at the center of the hexagons. Obviously, monolayer edge states as given by eq. (2) cannot be eigenstates for \( \alpha \)-steps, as some finite amplitude must be induced on the second layer through eq. (1). For \( \beta \)-steps, however, single layer edge states are indeed eigenstates.

In order to have a step-edge we need at least two graphene layers. Indeed, step-edges are easily obtained from bilayer graphene just by growing one of the layers beyond the edge. So, localized states at graphite steps can be understood by studying generalized bilayers where bottom and top graphene layers have different widths. The two families of edge states we have found to exist at zigzag edges of bilayer graphene remain eigenstates even when one of the layers is wider than the other. In particular, the solution given by eq. (2) and the non-orthogonalized solution for edge states of the bilayer family given by

\[
\begin{align*}
\alpha_1(k,n) &= \alpha_1(k,0) D_k e^{-i\xi n}, \\
\alpha_2(k,n) &= -\alpha_1(k,0) n D_k^{-1} t_\perp e^{-i\xi (n-1)},
\end{align*}
\]

(a simple linear combination of eqs. (2) and (3)), can be adapted to the generalized bilayer just by adjusting the unit cell index \( n \). Then, Gram-Schmidt orthogonalization gives the final solution. We should note, however, that the overlap between the two types of edge states is exponentially suppressed as the difference in layer width gets larger. When the width of one layer becomes infinite —the case of a perfect step— only one of the two possible solutions exists. Therefore, the possible localized solutions...
for zigzag step-edges (occurring at $n = 0$) are
\[ \alpha_{\text{top}}(k, n) = C_k D_k^n e^{-i\frac{k}{2}n}, \]
\[ \alpha_{\text{bottom}}(k, n) = -C_k n D_k^{n-1} \frac{t_{\perp}}{\epsilon} e^{-i\frac{k}{2}(n-1)}, \] (8)

for an $\alpha$-type step, and
\[ \alpha_{\text{top}}(k, n) = (1 - D_k^2) D_k^n e^{-i\frac{k}{2}n}, \] (9)

for a $\beta$-type step, where $n \geq 0$ and the normalization constant in eq. (8) is given by $|C_k|^2 = (1 - D_k^2)^2/(1 - D_k^2 + (1 + D_k^2)t_{\perp}^2/t^2)$. As in edge states discussed previously, the amplitudes in eqs. (8) and (9) refer to sites belonging to the same sublattice as the edge carbon atoms, while the amplitudes at the other sublattice are zero. Example charge densities for the given edge states are shown in fig. 4(b) for both the $\alpha$-type and the $\beta$-type step-edges.

As can be seen from eqs. (8) and (9), or by inspection of fig. 4(b), there is an apparent asymmetry between the two families of edge states: edge states at $\beta$-type steps live only on the top layer, while at $\alpha$-type steps both the top layer and the underlying layer have a finite edge state amplitude. Consequently, we expect a similar asymmetry to be present in the LDOS peak induced by edge states at the Fermi level, which, ultimately, should be seen with STM. To better appreciate this effect, we have computed the LDOS of a generalized bilayer — bottom layer wider than the top layer — using the recursive Green’s function method [30]. The calculated LDOS, which was accumulated in the range 0.01$\epsilon$ near the Fermi energy, should be proportional, in the simplest approximation, to the local tunnel currents in the experimental STM images [25,31]. In fig. 4(c) we show, for the top layer (edge sublattice), the LDOS difference between $\beta$-type and $\alpha$-type terminations as we move away from the step at $n = 0$. As expected, the LDOS at $\beta$-steps is higher and extends further into the bulk, a trend that is still present for realistic values of $\gamma_\alpha$, as shown in fig. 4(c). This behavior agrees with STM results, where two types of edge states with different penetration depths have been seen [26]. Edge states with reduced penetration depth have been observed at $\alpha$-type steps, whereas at $\beta$-type steps the edge states extend further into the bulk, as we have obtained here for the top layer component. However, right at the edge, the STM intensity has been found to be higher at $\alpha$-steps than $\beta$-steps [26]. According to our analytical result the opposite should be seen. This discrepancy is most probably due to edge state admixture, as experimentally both $\alpha$-type and $\beta$-type steps coexist on the same step-edge.

In fig. 4(d) and 4(e) we show the top layer LDOS map for $\alpha$- and $\beta$-steps, respectively. The former presents not only a reduced penetration depth, as previously discussed, but also higher intensity at sites connected to the underlying layer through $t_{\perp}$, as opposed to standard LDOS maps on the surface of bilayer graphene and graphite. This behavior is characteristic of edge states at $\alpha$-type steps, as given by eq. (8). Figures 4(f) and 4(g) show the underlying layer LDOS map for $\alpha$- and $\beta$-steps, respectively. As edge states at $\alpha$-steps (eq. (8)) have a finite amplitude over the underlying layer, the LDOS map for this layer shows an increased intensity at and near the step (fig. 4(f)), although the lattice discontinuity only exists at the other layer. As a consequence, we expect $\alpha$-steps to be detected in STM experiments even when they occur underneath the top layer. This feature is not seen in $\beta$-step maps (fig. 4(g)).

**Conclusions.** — We have demonstrated the existence of zero-energy states localized at zigzag edges of multilayer graphene and graphite steps. Stability to the presence of interlayer hopping $\gamma_3$ has been shown. The electron-hole symmetry breaking terms $\gamma_4$ (interlayer) and $t_{\perp}$ (inplane) are expected to induce edge state dispersion, but not to qualitatively modify the present results [32,33]. It should be noted that only perfect zigzag edges have been discussed here. However, we expect edge state properties to be present in multilayer graphene and graphite steps even for irregular edges, as long as some zigzag units are present, as recently demonstrated for single-layer...
graphene [34,35]. On the other hand, zigzag edges have been recently observed in epitaxial graphene monolayer [36], providing the first indication that the edge shape can be a controllable parameter in the future. Our findings are relevant in the context of carbon-based magnetism, where edge states seem to play an important role [1,3], and also in the context of graphene physics, where the reported self-doping in monolayer graphene [36] and suppression of conductance fluctuations near the neutrality point in bilayer and trilayer graphene [37] can be seen as edge-states–driven effects.

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