A Precision Closed-loop Driving Scheme of Silicon Micromachined Vibratory Gyroscope

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Abstract. This paper describes a precision closed-loop driving scheme for Silicon Micromachined Vibratory Gyroscope (SMVG). It decouples the angle and gain of the self-oscillation-driven, optimizes the angle to reduce the relative difference between drive frequency and resonant frequency of the drive mode and achieves the closed-loop self-oscillation-driven by nonlinear relation between DC voltage using for control and drive force. The experiments show that the standard deviation of drive frequency is 0.009Hz , with relative drift 2.2ppm and the standard deviation of the amplitude is 0.0025mV, with relative drift 15ppm in one hour respectively. The closed-loop drive scheme improves the precision and stability of drive frequency and the amplitude of the gyroscope well. The paper analyses and tests the noise of the self-oscillation-driven. The result shows that the self-oscillation-driven has a rms noise below -100dB.

Keywords: Silicon Micromachined Vibratory Gyroscope (SMVG); closed-loop; self-oscillation-driven; decouple.

1. Introduction

Silicon Micromachined Vibratory Gyroscopes (SMVG) are slight in volume, light in weight, low in cost, compatible with micro-electron processing technology and easy to achieve the mass production. It is one of popular developing directions of the gyroscope, with a widespread application prospect. Presently, most Silicon Micromachined Vibratory Gyroscopes utilize capacitive excitation, which requests the driving frequency approach the natural frequency of drive mode as far as possible, as well as a high stability of the driving frequency and the amplitude. To satisfy those requests, the actuation of SMVG must achieve the closed-loop control. Nowadays the closed-loop actuation of SMVG commonly adopts AGC (automatic gain control)\(^2\)\(^3\)\(^4\), which achieves the closed-loop drive by means of a non-linearity control, reducing the closed-loop gains of the entire loop to 1. However, influenced by floatation and boundary affect, the differential of actuate capacity to driving displacement(\(9C/9x\)) is not as linear as expected, resulting in an unpleasant control effect. In order to achieve the self-oscillation-driven of SMVG, the closed-loop control must satisfy two conditions: 1. The Phase angle of the whole loop \( \theta = 2\pi n \) (n is an integer); 2. The gain of the whole loop \( A > 1 \). But the AGC used
commonly, with a coupled angle and gain conditions, fails to optimize the parameter of the closed-loop control.

This article suggests an improved scheme of self-oscillation-driven of silicon micromachined vibratory gyroscope, which achieves the closed-loop control utilizing a non-linear relation between DC voltage using for control and driving force. The scheme succeed in decoupling the Phase angle and gain of the self-oscillation-driven, where the angle and the gain can be optimized and adjusted separately, thereby increasing the stability of drive frequency and the amplitude.

![Fig. 1. Driving principle diagram](image)

1. The driving requirement of SMVG
The driving scheme of a Z-Axis SMVG is shown in Fig.1. The drive mode of SMVG is equivalent to a second-order system of spring-mass-damp and its motion equation is:

\[ x'' + \frac{\omega_1}{Q_1} x' + \omega_1^2 x = \frac{f_d}{M_1}, \quad f_d = F_d \sin \omega_d t. \quad (1) \]

where \( x \) refers to the displacement in drive direction, \( \omega_1, Q_1 \) and \( M_1 \) refer respectively to the natural frequency, the quality factor and the driving mass in drive direction, \( f_d \) refers to the driving force, \( F_d \) refers to the amplitude of driving force, \( \omega_d \) is the driving frequency. When \( \omega_d = \omega_1 \), the steady-state output of SMVG is

\[ x = A \sin(\omega_d t + \phi). \quad (2) \]

in the eq.(2)

\[ A = F_d / (k_1 \sqrt{4(\Delta \omega)^2 + \frac{1}{Q_1^2}}) \quad \text{(2a)} \]

\[ \phi = -\tan^{-1}(1/(2Q_1 \Delta \omega / \omega_1)). \quad \text{(2b)} \]

\( k_1 \) refers to the stiffness coefficient of drive mode, \( \Delta \omega = \omega_1 - \omega_d \).

It can be concluded from (2a) that the amplitude \( A \) is related not only to \( Q_1 \) and \( F_d \), but also to the relative frequency difference \( \Delta \omega / \omega_1 \). When \( \omega_d = \omega_1 \), \( A \) is proportional to \( Q_1 \) and \( F_d \), which means that in condition of the same amplitude \( A \), if \( Q_1 \) is bigger, \( F_d \) has to be smaller, so reduces the needed drive power. In fact, \( \omega_d \neq \omega_1 \), which means that the drive frequency always has a slim difference to
natural frequency and with the increase of the relative frequency difference, the amplitude of driving displacement $A$ will drop rapidly, shown in Fig.2. When $-4.5<\lg(\Delta \omega / \omega_1)<-3$, $Q_1$ is bigger, $A$ will drop rapidly with the increase of relative frequency difference $\Delta \omega / \omega_1$. To fully implement the high drive efficiency brought by high quality factor, the relative frequency difference must be controlled in a certain value, that is $\Delta \omega / \omega_1<3.2\times10^{-5}$ ($1000<Q_1<10000$).

From Eq(2b), the relative frequency difference $\Delta \omega / \omega_1$ is:

$$|\Delta \omega / \omega_1| = 1/(2Q_1tg\phi) \quad (3)$$

The relative frequency difference $|\Delta \omega / \omega_1|$ is proportional inversely to $Q_1$ and $tg\phi$. In the condition of a certain $\phi$, when $Q_1$ is bigger, the relative frequency difference will be smaller while the frequency stability and the drive is better with efficiency higher. If $\phi$ is more closed to $90^\circ$, the relative frequency difference will be smaller, as shown in Fig 3. When $\phi=90^\circ$, $|\Delta \omega / \omega_1|$ is close to 0. For that reason, in closed-loop control, a slim relative frequency difference can be achieved through driving $\phi$ to $90^\circ$ closely.

2. Scheme of closed-loop self-oscillation-driven
From the analysis above, the role of closed-loop control of driving circuit is to reduce the $|\Delta \omega / \omega_1|$ as far as possible and increase the stability of driving frequency and amplitude. This paper provides a closed-loop control scheme to decouple the closed-loop gain and phase angle. Fig. 4 is the Matlab simulation frame of the closed-loop self-oscillation-driven. In figure, $W$ and $Q$ refer to the natural frequency and quality factor of drive mode respectively, $s$ is complex variable, $K$ is the coefficient of drive displacement to capacitance, $DC$ is the direct current bias applied to capacitance, $R$ is a large resistance, $B$ is a constant, $G$ is the gain of integrator, and $L$ and $J$ are the zero and the poles of the integrator separately. The module of “limited boundary condition” is mainly designed to control the boundary of integrator output, while the module of “force function” transforms the voltage to the driving force. The module of “phase angle” picks up the input information of phase angle only, with an invariable output amplitude all along. The phase angle of the whole closed-loop system is controlled by phase-angle-control-loop I, and gain is controlled by gain-control-loop II. These two loops achieve two conditions of closed-loop control apart, adjusting and optimizing the closed loop parameter separately. In order to prove whether the whole circuit achieves decoupling, we cut off circuit before the “phase angle module” in Fig 4 and add to an AC signal, whose frequency is the same as the natural frequency of drive mode. As a result, only the gain-control-loop is in effect in the whole circuit. Fig. 5 compares the “direct current control z” (in Fig 4) in this condition with full closed-loop control. The result shows the direct current control $z$, immunizing from the driving frequency, is the same in the two conditions, so the angle and gain of the self-oscillation-driven is decoupled. Fig.5 provides a partly magnified picture of the stabilized direct current $z$. It is evidence that the direct current $z$ in the full closed loop reaches a higher precision and a more stability, because the whole system can adjust the frequency and amplitude in a small range automatically.
2.1. Phase angle control

Phase angle control aims to meet the request of closed-loop control. In order to get minimums of \( |\Delta \omega / \omega_0| \) and drift of drive frequency \( \omega_d \), the phase angle \( \Phi \) of transfer function of SMVG should be close to \( 90^\circ \) as far as possible. To satisfy the closed-loop control, it has to be that \( \Phi + \theta + 90^\circ = 360^\circ \), with \( \theta \) the phase shifting of “phase angle module” in Fig.4, \( 90^\circ \) the phase shifting of differential module and the ideal situation that other modules have no phase shifting. In this article, the phase shifting \( \theta \) is \( 180^\circ \), which is implemented through the opposite phase function of operational amplifier. The phase shifting \( \theta \) is slight in distortion and high in precision, resulting in a precise control that the phase angle \( \Phi \) approaches \( 90^\circ \). Fig. 6 is the output driving frequency waveform of “phase angle module”. Fig 7 is the trace of relative frequency difference. Since the relative frequency difference is very slim, it is implementable to diminish the relative frequency difference through the phase angle control, a rather high control precision in addition.

2.2. Gain Control

The gain control tends to make the gain of the whole circuit 1 and achieve the closed-loop self-oscillation-driven, getting the whole closed-loop circuit to work at a suitable state and improving the control precision. The SMVG in this text is manufactured using bulk silicon micromachining technology, with a thicker structure, a larger drive mass, a small influence by the floatation and boundary affect and a high linearity. From above, the gain and phase angle are decoupled. So we can make SMVG work in a better linear interval by gain control to diminish the nonlinear of the structure. This scheme utilizes the nonlinear relation between the direct current \( z \) (in Fig 4) and driving force to manage the closed-loop control. The force function is shown as the following equation:

\[
f(z) = f_d / M_s = \frac{\partial C}{\partial x} (V + \frac{z}{2}) (V - \frac{z}{2}) \text{sq}(\omega_d t)^2 / M_s,
\]

where \( f_d \) refers to driving force; the differential of driving capacitance to displacement \( 9C/9x \) is a constant relating to the structure; \( M_s \) refers to the driving mass; \( \text{sq}(\omega_d t) \) refers to square wave, with amplitude 1 and driving frequency \( \omega_d \), \( z \) direct current controller, \( z \in [0, 2V] \); \( V \) is a direct current voltage. Thus the useful driving force \( f'(z) \) should be:

\[
f'(z) = \frac{\partial C}{\partial x} (V + \frac{z}{2}) (V - \frac{z}{2}) \text{sq}(\omega_d t) / M_s.
\]

It is obvious from equation (2a) and Fig.7 that the phase angle control has hold the relative frequency difference in a certain scale, with influence of relative frequency difference neglected, only basic frequency signal of square wave in driving action. Thereby, the amplitude gain \( A \) of transfer function of drive equation is:

\[
A = \frac{\partial C}{\partial x} (V + \frac{z}{2}) (V - \frac{z}{2}) \text{sq}(\omega_d t) / k_1.
\]

So the whole closed loop control equation is:

\[
-G \int \left( B + AK \omega_d \right) \cdot DC \cdot R \cdot \frac{1}{2} dt = z.
\]
Here the integrator module of Fig.4 is simplified into the basic integral function indeed, and 1/2 is the gain of commutator. Simplified eq (7):

\[
\frac{\partial C}{\partial x} G Q_{k} \omega_{d} \cdot DC \cdot R = \frac{\partial C}{\partial x} G Q_{k} \omega_{d} \cdot DC \cdot RV^2 \quad \frac{z}{8k_i} \quad z^2 = \frac{2k_i}{2k_i} \quad GB. \tag{8}
\]

Eq (8) is a first order nonlinear differential equation, which is hard to get analytic solution. Fig 8 is numerical solution of z according to parameters shown in Table 1. It can conclude from Fig.8 that controller z, when stabilized at 0.835V after 300ms, plays an active controlling role. The difference between Fig.8 and Fig.5(b) lies in the change of controller z in rising moment, since Fig.5(b) contains the balance course of second order system of SMVG. In addition, the achievement of closed-loop self-oscillation-driven in advantage of nonlinear between controller z and driving force mainly illustrates that the relation between controller z and driving force is nonlinear before the balance of closed-loop. However, after balanced, with fixed z, the applied voltage is linear with drive force. The choice of parameter B is very important. If value B is changed in a certain scale, at that time z works at a different point, the whole closed-loop system control well, through which can optimize the gain control. Suppose B ∈ [B_1, B_2], B_1 is defined by minimum signal sensed by the circuits and minimums linear displacement of SMVG, while B_2 is defined by the supply voltage in circuit and the max linear displacement of SMVG.

| Table 1. The value of simulation parameter |
|-------------------------------------------|
| Parameter(unit) | value |
| Q1             | 1000  |
| G              | 2000  |
| K(F/m)         | 3.2×10^{-6} |
| DC(V)          | 12    |
| R(MΩ)          | 30    |
| V(V)           | 12    |
| B(V)           | 0.3   |
| ηc/αx(F/m)     | 1.46×10^{-8} |
| ω_1(rad/s)     | 25120 |
| k1(N/m)        | 80    |

B_1 ≤ B ≤ B_2 : The gain of the whole loop is 1, z works at a definite voltage, and the closed-loop circuit works well.

B > B_2 : The gain of the whole loop is more than 1, z is 0. At this moment the voltage V can be limited boundary condition of the circuit, compelling SMVG to work in, or inside a max linear scale,
with phase angle control loop working normally and gain control loop out of action. The original state is fixed in this working scale. It can adjust $B \in [B_1, B_2]$, to optimize the gain control.

$B < B_1$: The gain of the whole loop is less than 1, $z$ is 2V, the closed-loop circuit works in no effect.

3. Experiment result and discussion

The whole experiment circuit is constructed on the thought of Fig.4, shown in Fig.9. The experiment result shows that the SMVG achieves closed-loop self-oscillation-driven, and the drive frequency works near the natural frequency of SMVG all along. It is practical to find a better working point for SMVG by adjusting $B$ in Fig.4. This paper tests the noise of the self-oscillation-driven, shown in Fig.10. The result shows that the self-oscillation-driven has a rms noise below -100dB. Fig.10 is the frequency drift of driving signal for 1 h. Fig.11 is the amplitude drift of driving signal for 1 h. The experiments show that the standard deviation of drive frequency is 0.009Hz, with relative drift 2.2ppm and the standard deviation of the amplitude is 0.0025mV, with relative drift 15ppm respectively. As a result, both stabilities of driving frequency and amplitude are rather high and the closed loop control is very successful.

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