About the mechanisms of fibre reinforced plastic behaviour under cycling loading

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Abstract. In order to study the peculiarities of the behavior of fiber reinforced plastics under cyclic loading, tests of specimens made of ELUR-P carbon tape and cold curing binder XT-118 with a stacking sequence $[+45^\circ / -45^\circ]_s$ ($s$ - the number of laminas) for cyclic tension were carried out. It is established that stabilization of the secant modulus of elasticity is observed as the number of cycles increases. The experimental data were compared with the results of numerical modeling of the deformation processes under study. Based on numerical calculations and analytic solutions found, for a number of types of loads and creep kernels, the value of the secant modulus of elasticity does not depend on the load amplitudes, when using relations with the Abel creep kernel, the ratio of viscous parts of strain shifted by values, does not depend on the period of cyclic load, the amplitude of the load and the parameter that determines the degree of viscosity of the material, but depends only on the parameter that determines the degree of attenuation of the creeping process. By using the method of decomposing the load in a series of trigonometric functions with the same cyclic periods, it is shown that the effect is manifested in the general case of cyclic loading using relations with the Abel creep kernel. There is established the independence of secant and tangential moduli of elasticity from the absolute values of maximum and minimum stresses for any kinds of loads and creep kernels using the linear theory of hereditarily elastic body. A method has been developed for identifying creep kernel parameters, relations for viscoplastic and elastic strain based on the analysis of experimental data on cyclic loading. For the separation of hereditarily elastic and viscoplastic strain, the hypothesis was introduced that their rates at large times are very different. This made it possible, on the basis of theoretical results obtained for the Abel kernel, to obtain ratios from which, independently of other mechanical characteristics, one can find the attenuation parameter from the strain values, measured in the experiment in times shifted by an integer number of periods. The method is applied to processing the data of real experiments conducted by the authors.

1. The results of experimental studies
Currently, in the scientific literature there are a large number of works devoted to the study of the mechanical properties of fiber reinforced plastics (FRP) under cyclic loading. Most of them are devoted to studying the residual strength and stiffness of FRP under cycling loading by comparing the elastic modulus and tensile strength after a certain number of loading cycles with their values obtained in the first loading cycle. On the basis of experimental data, as a rule, a relationship is built between the current

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modulus of elasticity of the composite and the number of cycles. For example, the results of such experiments for a cantilever specimen of a FRP can be found in [1]. A review of various models characterizing the residual stiffness of composites can be found, in particular, in [2]. The work [3] is devoted to the problems of building models describing the decreasing of the elastic modulus in the process of multiple loading for a FRP based on carbon fiber and an epoxy binder. It assumes that the stiffness at failure is proportional to the actual stress. In [4], a study was carried out for a unidirectional composite based on glass fibers under shear conditions for repeated cyclic loading of the specimens with a stacking sequence \([+45^\circ/-45^\circ]_s\). From the experimental stress-strain diagrams, the secant shear modulus \(G_{12}\) was determined and it was concluded that it decreases after loading the specimen with ten cycles.

The authors of this work carried out a series of experimental studies of a unidirectional FRP based on carbon tape ELUR-P and a binder HT-118 under multiple loading by testing specimens with a stacking sequence \([+45^\circ/-45^\circ]_s\) for tension and compression. The tests were carried out according to the method described in the article [5], on specimens aged eight months after their manufacture. Measurement of axial strain was carried out using a contact extensometer with a base of measurement 50 mm. In Figure 1 they are presented for the case of \(\sigma_{x}^{\text{max}} = 55\) MPa for the purpose of illustration.

It was revealed that the loading and unloading curves form a hysteresis loop, and the axial strain \(\varepsilon_x\) on each subsequent cycle increases by the value \(\Delta\varepsilon_x^{(i)}\) (Fig. 1), where \(i\) is the cycle number. With the increase in the number of cycles, the value \(\Delta\varepsilon_x^{(i)}\) decreases. Figure 2 shows the dependences of the secant moduli of elasticity under loading and unloading at three stress levels (hereinafter, \(E_x^{(i)}\) - the secant modulus under loading, \(E_x^{(u)}\) - during unloading).

\[\text{Figure 1. Stress-strain curve (a) and dependence of residual strain increment on number of cycles (b)}\]
Figure 2. Dependence of stress on time (a) and moduli of elasticity on cycle number for displacement rate 0.5 mm/min

Analysis of the results of the above and other experiments, as well as a review of the scientific literature indicates the presence of a number of features of the behavior of specimens from materials such as carbon fiber reinforced plastic under cyclic loading [6, 7]. They consist in the fact that, in the general case, on each cycle, the mechanical characteristics change, which determine elastic strain, creep strain, and also strain caused by microrearrengement of the composite structure. In this connection, the task of a theoretical description of the stress-strain state being formed in a specimen is rather complicated, since it is necessary to construct the dependences of the rheological and elastic characteristics of the material not only on the level of average stresses, but also on the number of cycles, asymmetry coefficients.

To clarify the structure of strain, the authors carried out a series of experimental studies not only on cyclic loading, but also on long-term tension with long-continued exposure under stress of specimens from cross-ply FRP [8]. Based on the analysis of the obtained results, it was found that the total axial strain can be represented as a sum of the following components:

\[ \varepsilon = \frac{\sigma}{E_0} + \varepsilon' (\sigma, t) + \varepsilon^r (\sigma) + \varepsilon^\text{nel} (\sigma) + \varepsilon^\text{R} (\sigma), \]

(1)

\[ \varepsilon' = \int_0^t f(\sigma) H(\sigma, t - \tau) d\tau, \]

(2)

\[ \frac{d\varepsilon'}{dt} = F(\sigma, \varepsilon'). \]

(3)

Here \( \varepsilon^\text{nel} = \varphi(\sigma) \) – is a nonlinearly reversible part, \( \varepsilon^\text{R} (\sigma) \) – is an irreversible instantaneous part, \( \varepsilon^r (\sigma, t) \) – is an irreversible creep strain, \( \varepsilon' \) – is a hereditarily elastic (viscoelastic) reversible component, \( H \) – is the creep kernel.

In this paper, we will consider in what follows a fair generalization of Kachanov’s hypothesis [9] that the strain \( \varepsilon' \), \( \varepsilon^\text{R} \), \( \varepsilon^\text{nel} \), \( \varepsilon' \) develop independently of each other.

The choice of the physical ratio for \( \varepsilon' \) must be determined in accordance with the results of experiments. As such a relation, you can use the view representation (the model with the Abel creep kernel)

\[ \varepsilon' = B \int_0^t \frac{\sigma(\tau)}{(t - \tau)^\alpha} d\tau, \quad \alpha < 1, \quad B > 0 \]

(4)

which, as shown by an analysis of the literature and the processing results of the author's experiments, is well confirmed for FRP. Equation (4) also has the advantage of having the least amount of mechanical characteristics.
Next, let us accept the experimentally confirmed hypothesis that the damping of the creep strain rate is much faster than the damping of the strain rate of hereditary elasticity. This suggests that the increment of instantaneous residual strain after a sufficiently large number of cycles represents only an increment of instantly reversible and hereditarily elastic strain.

2. Theoretical studies of hereditarily elastic strain

Assuming that the stresses in the relation (4) vary according to an arbitrary cyclic law with a period $T$

$$\sigma = \sigma(t) = \sigma(t + T)$$

we will conduct a theoretical study of the problem of changes in hereditarily elastic strain. Since $\sigma(\tau)$ a periodic function, it can be decomposed into the following trigonometric series

$$\sigma(\tau) = \sigma_0 \left( 1 + \sum_{m=1}^{\infty} \left[ s_m \sin \left( \frac{2\pi m \tau}{T} \right) + c_m \cos \left( \frac{2\pi m \tau}{T} \right) \right] \right).$$

When introducing a change of variables $\tau^* = \tau / T$, $t^* = t / T$,

relation (6) will be the following

$$\sigma(\tau) = \sigma_0 \left( 1 + \sum_{m=1}^{\infty} \left[ s_m \sin(2\pi m \tau^*) + c_m \cos(2\pi m \tau^*) \right] \right) = \sigma_0 \cdot f(\tau^*),$$

which allow relation (4) represent in the form of

$$\varepsilon^v(t) = BT^{1/\alpha} \sigma_0 \int_0^{\tau^*} \frac{f(\tau^*)}{(t^* - \tau^*)^\alpha} d\tau^*$$

We now write the ratio of strain $\varepsilon^v$ calculated with a shift, when the time in question differs from some time on a loading cycle by an amount multiple of the cycle period. Then we get

$$\varepsilon^v(t + Tn) / \varepsilon^v(t + Tk) = \int_0^{\tau^*} \frac{f(\tau^*)}{(t^* + n - \tau^*)^\alpha} d\tau^*/\int_0^{\tau^*} \frac{f(\tau^*)}{(t^* + k - \tau^*)^\alpha} d\tau^*$$

It follows that the ratio of viscous parts of the strain at such times will not depend on $T$, $B$, $\sigma_0$.

3. Inverse problems of identification of mechanical properties

We now consider the inverse problem of cyclic loading performed, for example, by a multilinear law (Fig. 2). The issue in the problem of identification of rheological characteristics is the issue of separation in the experiment of viscoelastic strain $\varepsilon^v$ and creep strain $\varepsilon^r$. For such a separation, we accept the experimentally confirmed hypothesis that the decay of the rate of irreversible creep strain occurs much earlier than the decay of the rate of strain of hereditary elasticity. Therefore, provided that $t_i, t_j \in t_i$ (the time of the first cycle), it is possible with some error (which we will assume to be small) to write the relation

$$\Delta_j \varepsilon^{exp} = \varepsilon_j^{exp} - \varepsilon_j^{\alpha} = \varepsilon_j^r - \varepsilon_j^v$$

Writing them for different times $t_i, t_j$, but with the same $k$, we obtain from (5) a system of equations for the parameter $\alpha$. By minimizing the quadratic residual of this system, a constant $\alpha$ can be found that is included in the approximating expressions (4) for the creep kernel $H$. Then, from (11) we again obtain the system of equations, but already containing the parameter $B$. The minimization of the quadratic residual of this second system can be found $B$.

Next, it is necessary to identify the parameters of relation (3) for the theory of aging. In the future, it was chosen in its simplest form with respect to stress

$$d\varepsilon^r / dt = \chi_0 \sigma (l + \chi_1 (\varepsilon^m)^m),$$

(12)
where \( m, \gamma_0, \gamma_1 \) — unknown constants.

At the last stage, one can find the secant modulus of elasticity \( E \), considering the strain-stress curve at the initial part of the first loading cycle, but taking into account the laws of viscoelastic and viscoplastic strain already found.

As a result of processing the data of the experiments, the following values were obtained

\[
B = 4.01 \times 10^{-6} \text{ sec}^{1-1}/\text{MPa}, \quad \alpha = 0.7101, \quad E = 12400 \text{ MPa},
\]

\[
\gamma_0 = 4.015 \times 10^{-8} \text{ (sec-MPa)}^{-1}, \quad \gamma_1 = 1.913 \times 10^3, \quad m = 22.1
\]

Figure 3 shows for clarity only the strain values minus the elastic parts at maximum stresses. Strictly speaking, maximum strain are achieved with some delays compared to stresses. However, this delay for the considered case is almost indistinctly graphically.

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