Threshold Dynamics of a Semiconductor Single Atom Maser

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We demonstrate a single-atom maser consisting of a semiconductor double quantum dot (DQD) that is embedded in a high quality factor microwave cavity. A finite bias drives the DQD out of equilibrium, resulting in sequential single electron tunneling and masing. We develop a dynamic tuning protocol that allows us to controllably increase the time-averaged repumping rate of the DQD at a fixed level detuning, and quantitatively study the transition through the masing threshold.

The SeSAM consists of a DQD that is embedded in a microwave cavity [24]. This cavity quantum electrodynamics (QED) approach provides a template for a variety of single emitter lasing experiments involving optical cavities that are coupled to either natural or artificial atoms [25–29]. Of particular interest is the observation of non-classical optical phenomena, such as Fock state generation and thresholdless lasing [26]. In the microwave domain, circuit-QED has enabled dramatic improvements in the coupling between solid-state devices and microwaves, with the demonstration of single photon sources, tomography of itinerant photon states, and even the stabilization of cat states of light [7–11]. An on-chip single atom amplifier and maser have also been demonstrated using voltage-biased superconducting junctions [12, 13].

Examining photon emission from single quantum emitters provides a window into the interaction between light and matter. The first single atom maser was realized by passing single Rydberg atoms – which provided a transient gain medium – through a superconducting cavity [1]. This cavity quantum electrodynamics (QED) approach provides a template for a variety of single emitter lasing experiments involving optical cavities that are coupled to either natural or artificial atoms [2, 3]. Of particular interest is the observation of non-classical optical phenomena, such as Fock state generation and thresholdless lasing [20]. In the microwave domain, circuit-QED has enabled dramatic improvements in the coupling between solid-state devices and microwaves, with the demonstration of single photon sources, tomography of itinerant photon states, and even the stabilization of cat states of light [7–11]. An on-chip single atom amplifier and maser have also been demonstrated using voltage-biased superconducting junctions [12, 13].

Semiconductor double quantum dots (DQDs) have been placed in microwave cavities with charge-cavity coupling rates $g_c/2\pi = 10 – 100$ MHz [14–18], and the strong-coupling regime has recently been achieved in three separate experiments [19–21]. DQDs allow a great level of experimental control, as their energy level structure is electrically tunable [22, 23]. Furthermore, non-equilibrium physics can be explored by applying a source-drain bias across the DQD or by periodically driving the energy level detuning $\epsilon$ [17, 24, 25]. These characteristics have enabled a wide range of quantum optics experiments with DQDs, such as photon emission between hybridized single-electron states [26–28]. Masing can be observed in these systems when the gain exceeds the loss, as recently demonstrated by placing two voltage biased InAs DQDs in a microwave cavity [24].

In this Letter, we examine the threshold dynamics of a semiconductor single atom maser (SeSAM) consisting of a single DQD that is embedded in a microwave cavity and driven by single electron tunneling events. In contrast with previous experiments that required multiple emitters to exceed the masing threshold [23], we demonstrate masing with a single DQD emitter through improvements in the cavity quality factor $Q_c$ and $g_c$. We introduce a dynamic tuning protocol that changes the effective repumping rate of the DQD, allowing us to directly observe the transition from below threshold, where the system is dominated by incoherent emission, to above threshold, where the emission is coherent. The threshold behavior is in qualitative agreement with a single atom maser theory from atomic physics [4]. We obtain quantitative agreement with the data by including photomission events that originate from tunneling between the DQD and the source-drain electrodes [27, 29–32].

The SeSAM consists of a DQD that is embedded in a half-wavelength $\lambda/2$ superconducting cavity [Fig. 1(a)]. The cavity has a resonance frequency $f_c = 7.5$ GHz and total decay rate $\kappa_{tot}/2\pi = 1.5$ MHz, corresponding to $Q_c = 5000$. The inset of Fig. 1(a) shows a scanning electron microscope image of a single InAs nanowire that is placed on top of 5 metallic gates that are electrically biased to form a double well confinement potential along the length of the nanowire [33]. Charges trapped in the double well have a dipole moment that directly interacts with the cavity field, resulting in a large $g_c/2\pi \approx 70$ MHz [15, 24, 25]. In comparison with previous work, we can achieve masing with a single DQD emitter [15, 26, 33]. The experiment has been improved by reducing microwave leakage through dc gate biasing lines and increasing $g_c$ by fabricating devices with a smaller gap between the source and drain electrodes [24].

The SeSAM is powered by a source-drain bias $V_{SD} = 2$ mV that is applied to the DQD via the $LC$ filtered bias tee connected to the cavity voltage node [34]. Figure 1(b) shows the resulting current $I_e$ as a function of the gate voltages $V_L$ and $V_R$. Charge states are labeled $(N_L, N_R)$, where $N_L$($N_R$) indicates the electron number in the left...
Evidence for photon emission due to electron tunneling is shown in Fig. 1(c), where $P_{\text{out}}$ and $I_e$ are plotted as a function of DQD energy level detuning $\epsilon$. The source-drain bias results in a peak current of 6 nA at $\epsilon = 0$, where the left and right dot energy levels are resonant. In contrast, the output power peaks at $\epsilon = 0.3$ meV due to a large phonon sideband, where the current $I_{\text{on}} = 4$ nA and $P_{\text{out}} = 160$ fW. For the conditions that result in the maximum output power, the level detuning is approximately 9 times the cavity photon energy (30 $\mu$eV for a 7.5 GHz cavity). Energy is conserved by a process where a phonon and photon are simultaneously emitted [27]. The source-drain bias repumps the DQD to the excited state at a rate $|I_e/\epsilon|$ and generates the population inversion required for stimulated emission.

The strong emission that is observed is suggestive of above-threshold masing that is triggered by current flow through a single DQD. To investigate the threshold behavior we now measure the statistics of the emitted microwave field as a function of repump rate. In conventional solid state lasers, such as a diode laser, threshold behavior is studied by measuring the emitted power as a function of dc biasing conditions. Such an approach is not directly applicable to DQD devices since the resonant current (and therefore the repump rate) is independent of $V_{SD}$ once the two dots levels are within the transport window [22]. Moreover, tuning the tunnel rates also changes the DQD energy level structure and $q_c$. We therefore develop a dynamic tuning process that changes the time-averaged current through the DQD.

As schematically illustrated in Fig. 2(a), a square wave with a period $\tau = 50$ ns and duty cycle $D$ is applied to both the left and right gates to toggle the electron current on and off while keeping $\epsilon$ fixed. The locations of the “on” and “off” states in the stability diagram are indicated by the red and black dots in Fig. 1(b) and the corresponding energy level configurations are shown in the lower insets of Fig. 1(b). The time-averaged current through the DQD, $I_\text{ave}$, is plotted as a function of $D$ in Fig. 2(b). As expected, the measured current scales linearly with $D$.

The dynamic tuning process is effective at setting the threshold behavior because $\kappa_{\text{out}}/2\pi = f_\text{c}/Q_c = 1.5$ MHz is much less than the tunneling rate through the DQD, $I_{\text{on}}/|\epsilon| \approx 25$ GHz. Due to the fact that photon emission is driven by single electron tunneling events, the effective repump rate is proportional to the average current when $|\epsilon|/I_{\text{on}}$ $\ll$ $\tau < 1/\kappa_{\text{out}}$ [25].

The time-series of the demodulated quadrature-phase component of the cavity field $Q(t)$ qualitatively illustrates the crossover from below threshold to above threshold behavior as $I_e$ is increased [Fig. 3(a)]. When $I_e < 2.5$ nA the output is thermal noise, which is mainly

![Figure 1: (a) Optical micrograph of the DQD SeSAM. The source-drain bias $V_{SD}$ is applied through a $LC$ filtered bias tee. The cavity is connected to input and output ports with coupling rates $\kappa_{\text{in}}$ and $\kappa_{\text{out}}$. Inset: Scanning electron microscope image of an InAs nanowire DQD. (b) $I_e$ as a function of $V_L$ and $V_R$ with $V_{SD} = 2$ mV. (c) $I_e$ (black) and $P_{\text{out}}$ (red) plotted as a function of $\epsilon$. The maximum output power for this device tuning configuration is $P_{\text{out}} = 160$ fW.](image1)

![Figure 2: (a) Upper panel: Pulse sequence used to tune the time-averaged current flowing through the device. A square wave with a period $\tau = 50$ ns and duty cycle $D$ is applied to toggle the current on and off. The resulting time-averaged current is $I_e = DI_{\text{on}}$. Lower panel: DQD level configuration with the current on (left panel) and off (right panel). (b) The measured $I_e$ (red circles) as a function of $D$. The solid line is the prediction $I_e = DI_{\text{on}}$.](image2)
attributed to background noise in the amplification chain. When \( I_e > 2.5 \text{ nA} \) periodic voltage oscillations become visible, with an amplitude that increases with \( I_e \). The oscillating field is indicative of coherent emission. We note that the maser occasionally blinks off even for large \( I_e \) (see for example \( t = 0.2 \text{ ms} \) at \( I_e = 3.8 \text{ nA} \)). We attribute the blinking to large charge fluctuations that shift the DQD level detuning \[24\]. Similar behavior has been observed in other solid state lasers \[36\]. These data show that the dynamical detuning method effectively changes the DQD repump rate.

The maser emission statistics can be quantitatively studied by measuring histograms of the output field as a function of increasing \( I_e \). For each value of \( I_e \), we sample the in-phase and quadrature-phase components of the cavity field \((I, Q)\) at a rate of 12.3 MHz and then plot \( 1.7 \times 10^7 \) samples in a two-dimensional histogram \[24\][25]. Histograms with \( I_e = 1.7, 2.9 \) and \( 4 \text{ nA} \) are plotted in Figs. 3(b–d). With a small current of \( I_e = 1.7 \text{ nA} \), the histogram is centered within a narrow range of the origin in the \( IQ \) plane, as the detected field is dominated by the detection background noise. With increasing current, \( I_e = 2.9 \text{ nA} \), the histogram broadens out into a larger range as shown in Fig. 3(c). For \( I_e = 4 \text{ nA} \) \((D = 100\%)\), the \( IQ \) histogram has a ring shape that is indicative of above-threshold maser emission [Fig. 3(d)] and a small thermal component around \((I, Q) = (0,0)\). As noted in previous work, the events around \((I, Q) = (0,0)\) in Fig. 3(d) are attributed to blinking events (circled in black) that are visible in the time series data of Fig. 3(a) \[24\].

Measurements of \( P_{\text{out}} \) also provide insight into the threshold behavior and can be compared with existing single atom maser theories \[6\]. Figure 4 plots \( P_{\text{out}} \) as a function of \( I_e \) for two different devices. Focusing on Fig. 4(a), for small \( I_e = 1.5 \text{ nA} \) we measure \( P_{\text{out}} \approx 10^{-5} \text{ pW} \). \( P_{\text{out}} \) gradually increases with increasing \( I_e \) until \( I_e \approx 2.5 \text{ nA} \). There is a dramatic factor of \( \approx 50 \) increase in \( P_{\text{out}} \) in the range \( 2.5 < I_e < 2.8 \text{ nA} \). For \( I_e > 2.8 \text{ nA} \) we find that \( P_{\text{out}} \) increases slowly again with \( I_e \). These data indicate that the maser crosses threshold when \( I_e \approx 2.5 \sim 2.8 \text{ nA} \) and is well above threshold for \( I_e > 2.8 \text{ nA} \).

We can understand the threshold behavior with an expanded form of the semiclassical theory of a single-atom laser \[9\]. In particular, we model the DQD SeSAM using semiclassical laser equations that account for photon emission events during interdot tunneling, as well as during tunneling onto and off of the leads

\[
\dot{u} = -[\gamma + \Gamma_p(n_c) + R]u + \Gamma_p(n_c) - Rn_c u, \tag{1}
\]

\[
\dot{n}_c = -\kappa_{\text{tot}} n_c + Rn_c u + Ru, \tag{2}
\]

\[
\Gamma_p(n_c) = [1 + \alpha(n_c + 1)]\Gamma_p^0, \tag{3}
\]

where \( u \) is the population of the state with one electron in the right dot (see Fig. 2), \( \Gamma_p(n_c) \) is the tunneling rate between the source-drain electrodes and the DQD states, \( \Gamma_p^0 \) is the bare tunneling rate, \( \alpha \) is the fraction of lead tunneling events that result in photon emission, \( \gamma \) is the decay rate of the upper state of the DQD into modes (e.g.,...
phonons) other than the cavity, $R$ is the photon emission rate from the upper state of the DQD, and $\kappa_{\text{out}}$ is the total cavity decay rate. The dependence of the lead tunneling on cavity photon number arises from photon-assisted tunneling events between the leads and the DQD. When $\alpha = 0$, Eqs. (1–2) reduce to the conventional semiclassical theory of the single-atom laser [3]. Spontaneous emission is accounted for by the terms where $R$ and $\alpha$ appear independently of $n_e$.

We model the tuning cycle by coarse graining Eqs. (1–2) over one period with a duty cycle $D$, which effectively replaces $u$ in Eq. (2) by $Du$. Similarly, we model the average current through the DQD as $I_e = eD(\gamma + R + R\kappa_c)u$. The crucial parameter that relates $I_e$ to $P_{\text{out}}$ is the fraction of electron tunneling events through the DQD that result in photon emission, which we define as $\beta$. We can estimate $\beta$ by noting that the threshold current in our model is given by $I_{\text{th}} = |e|\kappa_{\text{tot}}/\beta$ with $\beta = R/\gamma + 2\alpha$. The full expression for $P_{\text{out}}$ is derived in the supplemental material [35], with the final expression given here:

$$P_{\text{out}} = \frac{G_{\text{out}}^T h f_e \kappa_{\text{out}}}{2\xi R/\gamma} \left[ \frac{I_e}{I_{\text{th}}} - 1 + \sqrt{\left( \frac{I_e}{I_{\text{th}}} - 1 \right)^2 + \frac{4\xi I_e}{I_{\text{th}}}} \right].$$

(4)

$\kappa_{\text{out}}$ is the coupling rate to the output port of the cavity and is designed to be $\kappa_{\text{out}}/2\pi = 0.8$ MHz for both devices. Here we have also defined a correction parameter associated with the lead emission $\xi = 1 - 2\alpha I_e/|e|I_{\text{th}}$. The prefactor $G_{\text{out}}^T/G_{\text{out}}^E$ accounts for the systematic error in the total gain of the detection chain, $G_{\text{out}}^T$, from the output port of the cavity to the top of the fridge.

The theoretical prediction is in good agreement with data from two devices, as shown in Fig. 4. Fit parameters are listed in Table I. From calibrations of the amplifier gain, and room temperature measurements of the losses in the coax lines, we estimate the total gain of the amplification chain to be $G_{\text{out}}^E = 74.5$ dB for device 1, and $G_{\text{out}}^E = 72.8$ dB for device 2 in another cool down. Given experimental uncertainties in $\kappa_{\text{out}}$, losses in the device packaging, and the temperature dependent losses in the coax lines, these values are in overall agreement with the best fit value $G_{\text{out}}^T = 76.3$ dB for device 1 and $G_{\text{out}}^T = 76.2$ dB for device 2 [25, 37]. The quantitative agreement of this model with the data suggests that lead emission events play an important role in the charge-cavity dynamics of our device [27, 29, 32]. The reader is referred to the supplemental material for a comparison to the standard single atom laser theory, which does not account for lead emission [35].

In conclusion, we have measured the threshold dynamics of a semiconductor single atom maser, which allows for investigations of maser physics in the simplest case of a single emitter in a cavity. Photon emission in the SeSAM is generated by single electron tunneling events. By implementing a dynamic tuning protocol, we quantitatively analyze the behavior of the SeSAM as it crosses threshold. The data are in agreement with a modified single atom maser theory that includes a correction due to lead emission.

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Table I: Fit parameters for the threshold behavior in Fig. 4

|                     | Device 1       | Device 2       |
|---------------------|----------------|----------------|
| Measured $f_e$      | 7.5 GHz        | 7.6 GHz        |
| $\kappa_{\text{tot}}/2\pi$ | 1.5 MHz       | 1.8 MHz       |
| Calibrated $G_{\text{out}}^T$ | 74.5 dB       | 72.8 dB       |
| Free parameters $I_{\text{th}}$ | 2.63 nA       | 6.14 nA       |
| $\alpha$            | $1.2 \times 10^{-4}$ | $0.3 \times 10^{-4}$ |
| Calculated $\beta = \kappa_{\text{tot}}/|e|I_{\text{th}}$ | $5.7 \times 10^{-4}$ | $2.9 \times 10^{-4}$ |
| $R/\gamma - 2\alpha$ | $3.3 \times 10^{-4}$ | $2.3 \times 10^{-4}$ |

Figure 4: $P_{\text{out}}$ (red circles) as a function of $I_e$ and fits to theory (solid lines) for two different devices. (a) Device 1 crosses threshold around $I_e \approx 2.6$ nA. (b) Device 2 crosses threshold around $I_e \approx 6.1$ nA. Both devices exhibit qualitatively similar threshold behavior.
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