Tunable and dynamic polarizability tensor for asymmetric metal-dielectric super-cylinders

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We propose a novel type of bi-anisotropic hybrid metal-dielectric structure comprising dielectric and metallic cylindrical wedges wherein the composite super-cylinder enables advanced control of electric, magnetic and magneto-electric resonances. We establish a theoretical framework in which the electromagnetic response of this meta-atom is described through the electric and magnetic multipole moments (based on multiple expansion theory) that are excited by arbitrary incident plane waves. The complete dynamic polarizability tensor, expressed in a compact form, is derived as a function of the Mie scattering coefficients. Further, the constitutive parameters -determined analytically- illustrate the tunability of the structure’s frequency and strength of resonances in light of its high degree of geometric freedom. In addition, we show that the highly versatile bi-anisotropic meta-atom is amenable to being designed for the desired electromagnetic response, such as electric dipole-free and zero/near-zero (backward and forward) scattering. The results reported herein contribute toward improving the physical understanding of wave interaction with artificial materials composed of asymmetric elongated metal-dielectric inclusions and open the potential of its application in antenna (Huygens scatterer) and metamaterial designs.

I. INTRODUCTION

Modeling of two-dimensional structures illuminated by electromagnetic waves has been of leading research interest for a long time. The problem of deriving the polarizability of elongated structures has been of central interest in the study of antenna theory over the last century [1]. Due to their simplicity and practicality, long metallic and dielectric cylinders and wire media structures have received much attention among all the various canonical 2D structures. These include structures realizing artificial plasmas [2], hyperbolic media [3,6], creating exotic material properties [4,12], including those for antenna applications [13,14], tailoring the phase of reflection waves in metasurface applications [15,22], imaging and endoscopy [23,25], manipulating Casimir forces [26], enhancing the coupling to quantum sources [27,29] and enabling single-molecule biosensors [30].

The first fundamental step to describing the EM response of a metamaterial is to model the response of an individual particle. In this method each inclusion serves as a polarizable particle which is modeled with a pair of electromagnetic polarizable dipoles, which in turn becomes a new source of electromagnetic fields that lead to corresponding local fields. Ultimately, these effects form the macroscopic constitutive parameters. Unusual properties of this material are observed near resonance and its dependence on the geometry and EM properties of the individual inclusions are self-evident [31].

A number of recent studies retrieve the EM response for elongated cylinders under specific conditions of incidence, such as normal incidence [31]. For example, static expressions for the transverse polarizability components of circular cylinders are well established, and recently have been extended to the dynamic case based on far-field scattering considerations [32]. However, because of symmetry restrictions of the infinitely long cylinder, this particle does not have any magnetoelectric coupling response. In general, no magnetoelectric coupling may exist in the microscopic polarizability of a scatterer having both temporal and inversion symmetry [33,34]. Considering the inability of these symmetric meta-atoms, even with flexibility in design, to yield the desired EM response, exploring an asymmetric meta-atom with different geometrical parameters is of high research importance.

Metal-dielectric ‘super-cylinder’ as an asymmetric meta-atom with a tunable wedge angle presents high degrees of design freedom. However, the scattering problem of a finite metal circular wedge has not been extensively studied in the literature [35]. Accordingly, we proceed to first extend the theory to the case of a metal-dielectric super-cylinder with large degrees of freedom, and thereafter we derive the full dynamic polarizability tensor for this meta-atom.

It is shown in the current paper that a metal-dielectric super-cylinder could be an adjustable 2D meta-atom with a unique advantage that the frequency and strength of electric, magnetic and magneto-electric resonance can be tuned. Accordingly, it has significant potential for the engineering and design of desired and exotic electromagnetic responses. Explicit analytical description to characterize their dynamic polarizability tensor is presented. More precisely, despite the increasing geometrical complexity of the meta-atom, we show that it is possible to describe analytically the scattering due to a metal-dielectric super-cylinder illuminated by a plane wave; that is, the Mie scattering coefficients in terms of a unique...
set of dipole moments. In fact, based on multiple expansion theory there is a significant link between the multipole moments and its well-known Mie scattering coefficients that encompasses all the information of the scatterer \cite{36}. Based on the standing wave method \cite{37,38}, the full dynamic polarizability tensor is extracted and the magnetoelastic coupling response to a normally incident plane wave is surprisingly observed. Lastly, through a practical example, we demonstrate the enormous potential of the proposed meta-atom and in particular the ability to achieve a transparent meta-atom.

The article herein is organized as follows. First, we theoretically discuss the problem of scattering from a metal-dielectric super-cylinder (section II). Then, the multipole moments and polarizability tensor of the meta-atom are derived using the multipole expansion method and standing wave approach (section III). Thereafter, we show that by changing the geometry and constitutive parameters of the meta-atoms it is possible to design a manifold of the structure at different angles. This is achieved by fixing the incident wave angle. In order to solve the problem in the most general manner, we consider the incident wave to impinge upon the structure at different angles. This is achieved by fixing the incident wave while rotating the structure to the desired angle (\(\alpha\)) (see Fig. 1). It is noteworthy that in this study we assume the time dependence to be \(e^{-j\omega t}\).

Considering that the structure is of infinite extent along the longitudinal axis it is clear that \(\frac{\partial}{\partial z} = 0\) for all the parameters. Given that the problem of EM scattering from a finite PEC wedge has been solved previously for TE and TM polarizations \cite{35}, here we extend the structure by filling-in the remaining part of the cylinder with a dielectric and thus enabling the analysis to simply avail the same scattering wave forms.

II. SCATTERING ANALYSIS OF A METAL-DIELECTRIC SUPER-CYLINDER

The configuration of the structure and illumination is shown in Fig. 1. The presented meta-atom consists of two finite wedges which together form a complete cylinder. The material properties for these wedges are different, one is a metal and the other is a dielectric where \(\varepsilon_c\) and \(\mu_c\) are its permittivity and permeability, respectively. \(r_0\) is the radii of the structure and \(\beta\) is the metal wedge angle. In order to solve the problem in the most general manner, we consider the incident wave to impinge upon the structure at different angles. This is achieved by fixing the incident wave while rotating the structure to the desired angle (\(\alpha\)) (see Fig. 1). It is noteworthy that in this study we assume the time dependence to be \(e^{-j\omega t}\).

The preceding equations are derived using the EM boundary conditions, where the \(a_n\) and \(b_n\) are the Mie scattering coefficients. These are derived using the EM boundary conditions (a detailed procedure for obtaining Mie scattering coefficient is outlined in the Appendix). Further, these can be written in the following way where 'e' and 'o' superscripts allude to the even and odd modes in the Mie scattering coefficient.

\[
E_{z,sc}^{TM} = \sum_{n=0}^{\infty} \left( a_n^{TM} \cos(n(\varphi + \alpha)) + b_n^{TM} \sin(n(\varphi + \alpha)) \right) H_n^{(1)}(k_0r)
\]  

\[
H_{TE,\text{far}}^{TM} = \sqrt{\frac{2}{\pi k_0 r}} \left( a_0^{TM} + a_1^{TM} \cos(\alpha) + b_1^{TM} \sin(\alpha) \right) \sin(\varphi) \right) \exp \left( j(k_0r - \pi/4) \right)
\]  

\[
H_{TE, \text{far}}^{TM} = \sqrt{\frac{2}{\pi k_0 r}} \left( a_0^{TM} + a_1^{TM} \cos(\alpha) + b_1^{TM} \sin(\alpha) \right) \sin(\varphi) \right) \exp \left( j(k_0r - \pi/4) \right)
\]  

\[
\begin{bmatrix}
    a_n^{TM/TE} \\
    b_n^{TM/TE}
\end{bmatrix}_{n \times 1} = \begin{bmatrix}
    I_n \\
    \chi_{nm}(e),TM/TE
\end{bmatrix}_{1 \times n}^{-1} \begin{bmatrix}
    \chi(e),TM/TE
\end{bmatrix}_{m \times 1}
\]
The relations for \( \chi_n^{(o),TM} \) and \( \chi_n^{(r),TM} \) can be written similar to \( \chi_n^{(e),TM} \) and \( \chi_n^{(r),TM} \) using the following substitutions: \( \nu_i \rightarrow \tau_i, \varepsilon_n \rightarrow 1, a_m^{inc} \rightarrow b_m^{inc}, \) \( \sin(\theta_1) + \sin(\theta_2) \rightarrow \sin(\theta_1) - \sin(\theta_2), \) and \( i = n = 0 \) \( \rightarrow \) \( i = n = 1 \) (indexes of the summation). Also the dual relations for \( \chi_n^{(o),TE} \) and \( \chi_n^{(r),TE} \) can be written with the following substitutions in \( \chi_n^{(o),TM} \) and \( \chi_n^{(o),TM} \): \( \nu_i \rightarrow \tau_i, \tau_i \rightarrow \nu_i, J_{n}(\chi), H_{n}(\chi) \rightarrow \frac{dJ_{n}(\chi)}{d\chi}, \frac{dH_{n}(\chi)}{d\chi} \) and \( Z_2(\pi-\beta/2)Z_0^{1/\varepsilon} \rightarrow Z_2(\pi-\beta/2)Z_0, \) where \( Z_0 \) and \( Z_c \) are wave impedance of background medium and dielectric cylinder, respectively. Also, \( i, \nu_i, \varepsilon_n, \) and \( \delta_{nm} \) are defined in Appendix. For the thin metal-dielectric super-cylinder which are of interest here, scattering is dominated by the \( n = 0 \) and 1 in TM and TE harmonics, that obviously depending on the incident polarization, the geometric (\( \alpha \) and \( \beta \) angles and \( r_0 \)) and constitutive parameters \( (\varepsilon_c = \varepsilon_r \varepsilon_0 \) and \( \mu_c = \mu_r \mu_0 \) of dielectric wedge).

III. DYNAMIC POLARIZABILITY TENSOR EXTRACTION

Modelling of the electromagnetic fields using multipole moments has several advantages. The electromagnetic fields are linearly dependent on the moments and as a result do not involve complicated integrals [31]. The resulting multiple moments are vital elements in the description of homogeneous effective medium theories such as Maxwell-Garnet and other nonlocal methods [25,39].
In the following sections, we first derive the analytical expression for the induced electric and magnetic dipoles based on the multiple moments theory. Next, we extract the polarizability tensors using the standing wave approach which describe the EM interaction of the proposed structure.

A. Extraction of multipole moments

The radiated fields due to a set of electric and magnetic dipoles can be approximately expressed as presented by [10]

\[ \vec{E} \simeq Z_0 \left\{ \left[ k_0^2 \vec{p} c_0 G + (\vec{p} c_0 \nabla) \nabla G \right] + j k_0 [\nabla G \times \vec{m}] \right\} \]

\[ \vec{H} \simeq -j k_0 [\nabla G \times \vec{p} c_0] + \left[ k_0^2 \vec{m} G + (\vec{m} \nabla) \nabla G \right], \]

where \( k_0, c_0 \) are the wave vector and the speed of light, respectively. \( \nabla = \hat{r} \frac{\partial}{\partial r} \) (\( \hat{r} \) is a cylindrical unit vector along the radial direction). Here, we are going to derive the induced electric and magnetic dipole moments in a 2D structure. Consequently, we can use the 2D cylindrical Green function \( G = j \frac{1}{4} H_0^{(1)}(k_0r) \) as an alternative.

As a result, we utilize the analytical expression for the electromagnetic radiation of electric and magnetic dipole moments wherein the radiated fields are expressed as TE and TM modes [31] using far field approximation.

\[ \vec{E}_{TM}^{far} = \frac{j Z_0 k_0^2}{4} \sqrt{\frac{2}{\pi k_0 r}} (p_z c_0 - 2m_z) \exp \left(j(k_0 r - \pi/4)\right), \]  

(12)

\[ \vec{H}_{TM}^{far} = -\frac{j k_0^2}{4} \hat{\varphi}(p_z c_0 - 2m_z) \exp \left(j(k_0 r - \pi/4)\right), \]  

(13)

\[ \vec{E}_{TE}^{far} = \varphi \frac{j Z_0 k_0^2}{4} \sqrt{\frac{2}{\pi k_0 r}} (p_\varphi c_0 + m_\varphi) \exp \left(j(k_0 r - \pi/4)\right), \]  

(14)

\[ \vec{H}_{TE}^{far} = \frac{j k_0^2}{4} \hat{\varphi}(p_\varphi c_0 + m_\varphi) \exp \left(j(k_0 r - \pi/4)\right), \]  

(15)

It is noteworthy, that \( p_\varphi \) and \( m_\varphi \) describe projections of the moments along the azimuthal direction \((p_\varphi = p y \cos \phi - p_z \sin \phi, m_\varphi = m_y \cos \phi - m_z \sin \phi)\) and should not be confused with the azimuthal components since the dipoles \( P \) and \( M \) are located at the origin of the axes. Observing the above equations, it is clear that the radiating fields correspond to typical cylindrical TEM waves, satisfying \( \vec{H} = \hat{\varphi} \times \vec{E} / Z_0 \).

The dipole moments excited in the metal-dielectric super-cylinder can now be derived using the approach presented in [31]. First, we analytically extract the far-field scattering TE and TM fields for the metal-dielectric super-cylinder (see section II), and then we derive the
FIG. 4: Normalized transverse magnetic and longitudinal electric polarizability which exited by a TM polarized incident wave for dielectric cylinder ($\varepsilon_r = 18$).

The static magnetic and electric polarizability are illustrated to verify the validity of the method [31, 32].

far-field radiation due to the set of electric and magnetic dipoles (Eqs. 12 to 15). Thereafter, we extract the induced dipole moments by comparing the Eqs. 3 and 4 and Eqs. 12 to 15.

The relation between the Mie coefficients and the induced dipole moments is derived:

$$P = \hat{x} \left( \frac{4 (b_1^{TE} \cos(\alpha) - a_1^{TE} \sin(\alpha))}{j k_0^2 c_0} \right) + \hat{y} \left( \frac{4 (a_1^{TE} \cos(\alpha) + b_1^{TE} \sin(\alpha))}{j k_0^2 c_0} \right) + \hat{z} \left( \frac{4 a_0^{TM}}{j k_0^2 Z_0 c_0} \right)$$

$$M = \hat{x} \left( \frac{2 (b_1^{TM} \cos(\alpha) - a_1^{TM} \sin(\alpha))}{j k_0^2 Z_0} \right)$$

$$- \hat{y} \left( \frac{2 (a_1^{TM} \cos(\alpha) + b_1^{TM} \sin(\alpha))}{j k_0^2 Z_0} \right) + \hat{z} \left( \frac{4 a_0^{TE}}{j k_0^2} \right)$$

Upon observation of the equations, it is clear that the extracted dipole moments can be easily tuned by varying the angular rotation $\alpha$ and the scattering Mie-coefficients by changing the wedge angle $\beta$. In order to verify the validity of our approach, the far-field scattering pattern of a metal-dielectric super-cylinder ($\varepsilon_r = 18$) with wedge angle of $\pi$ was derived using the full-wave simulation (CST Microwave Studio), the radiation of the induced dipole moments, and also the closed-form Mie scattering of the metal-dielectric super-cylinder (section II). The incident wave is assumed to be a TM plane wave with the operating wave length of $\lambda_0 = 6 \mu m$. Figure. 2 presents the mentioned results for three metal-dielectric cylinders with different radii 600 nm, 800 nm, and 1 $\mu$m, respectively. It is clear from Fig. 2 that the radiation of the derived polarization is in good agreement with the full wave simulation. It is noteworthy that the small difference between the scattering patterns is due to the truncation of the scattering series and the dipole approximation.

B. Extraction of polarizability tensor

Due to the asymmetry of the proposed meta-atom, in contrast to the symmetric cases of the sphere and the cylinder meta-atoms, extraction of the polarizability tensor is complicated. There are several methods to extract the polarizability tensor for a desired bianisotropic meta-atom [37, 38, 11, 44]. In this paper, we use the standing wave approach to derive the polarizability tensor as presented in recent studies [37, 38]. We can simply write a standing wave as a superposition of two plane waves traveling in opposite directions (see Fig. 3). Using the extracted EM multiple moments from the previous section, the polarizability tensors excited by the standing wave can be derived using the superposition theorem. As it is clear from Fig. 3 the magnetic fields are out of phase in the center of coordinates, therefore by adding or subtracting the two plane waves we can derive the pure electric and magnetic components of the polarizability tensors, respectively. As an example 12 polarizability tensor components can be derived using $E_x = E_0 e^{\pm j k y}$ and $H_z = \pm H_0 e^{\pm j k y}$ plane waves.

$$\alpha_{qq}^{ex} = \frac{P_q^{3\pi/2}}{2 E_0} + \frac{P_q^{3\pi/2}}{2 E_0}, \alpha_{qq}^{me} = \frac{m_q^{3\pi/2} + m_q^{3\pi/2}}{2 E_0}$$

$$\alpha_{qz}^{ex} = \frac{P_q^{3\pi/2}}{2 E_0} - \frac{P_q^{3\pi/2}}{2 E_0} \eta, \alpha_{qz}^{em} = \frac{m_q^{3\pi/2} - m_q^{3\pi/2}}{2 E_0} \eta$$

where $q = \{x, y, z\}$, and moments superscripts (i.e., $\pi/2$ and $3\pi/2$) indicate the wave vector angle with respect to the direction $x$. All the other components can be derived by simply using different configurations for the standing wave (see Fig. 3) similar to [37, 38]. After complete consideration of all the derived polarizability tensor components, the complete polarizability tensor can be presented as follows:
In order to present a compact expression for the extracted polarizability tensor components, we define the following arbitrary parameters which depend on the Mie coefficients:

\[ \psi_{N,M}^{R,\pm} = N_{M}^{R,2\pi} \pm N_{M}^{R,\pi} \]  

(21)

\[ \psi_{N,M}^{\dagger,R,\pm} = N_{M}^{R,3\pi/2} \pm N_{M}^{R,\pi/2} \]  

(22)

Polarizability tensor components

\[
\begin{bmatrix}
\alpha_{xx}^{ee} & \alpha_{xy}^{ee} & 0 & 0 & 0 & \alpha_{xz}^{em} \\
\alpha_{yx}^{ee} & \alpha_{yy}^{ee} & 0 & 0 & 0 & \alpha_{yz}^{em} \\
0 & 0 & \alpha_{zz}^{ee} & \alpha_{zx}^{em} & \alpha_{zy}^{em} & 0 \\
0 & 0 & \alpha_{zy}^{me} & \alpha_{yy}^{mm} & \alpha_{yz}^{mm} & 0 \\
\alpha_{zx}^{me} & \alpha_{zy}^{me} & 0 & 0 & 0 & \alpha_{zz}^{mm}
\end{bmatrix}
\]

\[ P \]

\[
\begin{bmatrix}
E_{\text{loc}} \\
H_{\text{loc}}
\end{bmatrix}
\]

(20)

where R=TM or TE, N= a or b and M=0 or 1. We normalized all the presented polarizabilities to provide fair comparison: \( \alpha_{ee}, \alpha_{em}, \alpha_{me} \) and \( \alpha_{mm} \) are normalized respectively to \( \epsilon_0, (\epsilon_0 Z_0)^{-1}, Z_0 \) and 1. Using the above expressions, the non-zero polarizability components are obtained as follows:

It is self-evident from the above relations that the induced polarizability tensor components depend on the Mie coefficients (wedge angle and dielectric constitutive parameters) and the meta-atom orientation, which make this meta-atom an excellent candidate for engineering and tuning of its EM response. The present analysis has allowed us to define a fully dynamic expression for the entire polarizability tensor of the metal-dielectric super-
FIG. 5: The real (solid lines) and imaginary parts (dashed lines) of a) electric, b) and c) magnetoelectric, and d) magnetic normalized dipole polarizability of asymmetric metal-dielectric super-cylinder ($\varepsilon_r = 18$) which exited by TM polarized incident wave for variant values of $\beta$ with respect to radius.

cylinder in closed-form. In the following section, we discuss the properties of this tensor and present some practical examples.

IV. PROPERTIES OF THE POLARIZABILITY TENSOR

The polarizability tensor extracted in the previous section provides a fully dynamic, compact description of the EM response of a metal-dielectric super-cylinder excited by an arbitrary electromagnetic wave. As noted, the validity of the tensor is limited to the case where the meta-atom is smaller than the wavelength. Also, its scattering response can be approximated by using only the electric and magnetic dipole terms. Although our derivation considered an infinitely long metal-dielectric super-cylinder, the Mie scattering results are well-known to characterize a finite meta-atom with a length to diameter ratio of greater than 5 [31].

We verify our results by simply comparing the polarizability components for the special case of an infinitely long dielectric cylinder ($\beta = 0$) in both static [31] and dynamic regimes [32]. Fig. 4 shows that the normalized transverse magnetic ($\alpha_{xx,yy}^m/(\pi r_0^2)$) and longitudinal electric polarizabilities ($\alpha_{zz}^e/(\pi r_0^2)$) are in good agreement with results of [31, 32].

The overarching objective of developing a tunable meta-atom is to enable the design of any desired EM response, that is, to engineer a meta-atom for any given polarizability tensor. To-date, there are only a handful of meta-atoms represented by a closed-form EM response [32, 45]. Further, it is noteworthy that the EM response of these meta-atoms is limited owing to the simplicity and symmetry of their structures. However, the metal-dielectric super-cylinder presented here is a novel meta-atom capable of realizing complex EM responses through the tunability of its polarizability tensor. The geometry of the presented meta-atom clearly shows that we have several degrees of freedom. We can simply tune the EM response and polarizability tensor by changing the constitutive and geometric parameters such as dielectric properties, wedge angle, and angle of rotation. These parameters affect the resonant frequency and the strength of the induced magnetic, electric and magnetoelectric responses. Since the tunability of the dielectric permittivity is limited to existing materials, the wedge angle plays an important role in the design of the meta-atom. Figs. 5 and 6 illustrate the effect of wedge angle on the strength and position of the resonance with respect to the super-cylinder radius. Here, we choose to illustrate a set of selected polarizability components for conciseness. Examining Figs. 5 and 6, we see that the resonant frequency of the polarizability components ex-
FIG. 6: The real (solid lines) and imaginary parts (dashed lines) of a) electric, b) and c) magnetoelectric, and d) magnetic normalized dipole polarizability of asymmetric metal-dielectric super-cylinder ($\varepsilon_r = 18$) which is exited by TE polarized incident wave for variant values of $\beta$ with respect to radius.

cited by the TM wave can be tuned by adjusting the $\beta$ angle. However, adjusting the wedge angle only affects the strength of the resonance in the TE excitation. This result is due to the use of a non-magnetic dielectric in the meta-atom. Nevertheless, it is possible to simultaneously adjust both the strength and position of resonance with wedge angle when using a magnetic dielectric in the meta-atom.

The presence of magnetoelectric effects in such a simple structure might be surprising at first sight. In fact, the geometrical asymmetry and inhomogeneity of this meta-atom breaks the symmetry in EM response and allows for the magnetoelectric effects to arise. One can also deduce this result from the standing wave approach. The EM response of an asymmetric structure is dissimilar for incident waves with opposite propagation directions, that is, the superposition of induced magnetic dipoles (see Eqs. 18 and 19) give rise to magnetoelectric polarizability components. In the following sections, we show the flexibility of the proposed meta-atom through two practical examples.

V. ILLUSTRATIVE EXAMPLES

Here, we demonstrate the ability of the proposed bianisotropic meta-atom in achieving exotic EM responses.

In the following examples the meta-atom is tuned to be electromagnetically transparent. There are several ways to attain transparency, for instance, one can achieve a transparent meta-atom by vanishing the induced electrical polarizability [46, 52] or the forward and backward scattering [47–50]. Such meta-atoms could be used in various applications demanding minimal total scattering.
such as miniature sensors and ultra-fast nano-lasers.

### A. Zero electric polarization

First, we look for a condition where the electric polarization \( P \) in the meta-atom can be made negligible (electric dipole-free meta-atom). We assume that the meta-atom is illuminated by a TM-polarized plane wave propagating in the x direction. According to Eq. 16 the electric polarization \( P_z \) directly depends on the first harmonic of Mie scattering \( a_0 \) of the metal-dielectric super-cylinder. Targeting \( a_0 = 0 \) as a goal of the optimization, we obtain the conditions under which the induced electric dipole is nearly zero \( (\varepsilon_r = 11.16, r = 0.09\lambda, \alpha = 62.96^\circ, \beta = 125.27^\circ) \). Fig. 7 shows the magnitude of electric polarization \( P_z \) and transverse magnetic polarization \( M_t \). Under this condition electric quadrupoles and magnetic dipoles entirely determine the wave-matter interaction which is illustrated through the 3D scattering pattern of the optimized meta-atom. It is worth noting that it is not possible to achieve a precisely zero value for the electric dipole in a passive structure. However, a recent study shows that this condition can be achieved when using a meta-atom structure which on the whole is lossless, that is, a structure comprising a loss-compensated dimer [52].

### B. Near zero forward and backward scattering

Another way of achieving a transparent meta-atom is to determine the condition under which forward and backward scattering of the meta-atom vanishes. Based on the optical theorem [51] the total extinction cross section of an object \( \sigma_{ext} \) (sum of absorption and total scattering cross sections) is related to the normalized scattering amplitude in the forward direction \( \sigma_\phi(0) \) in the following way

\[
\sigma_{ext} = \frac{\lambda_0^2}{\pi} \text{Im}[\sigma_\phi(0)]
\]

where \( \lambda_0 \) is the operating wavelength. Optical theorem applies to the EM response of any object illuminated by a linearly polarized plane wave. Eq. 23 implies that a near-zero forward scattering amplitude results in zero total scattering, which means that a meta-atom with near zero forward scattering will be transparent. Recent studies have demonstrated that in order for forward scattering to be zero, part of the meta-atom must be active. Therefore, in the passive case it is not possible to achieve exactly zero forward scattering [51, 52].

Assuming that the meta-atom does not consist of absorptive material, the extinction cross section and scattering cross section would be equal. Hence, the total cross section can be calculated using the scattering fields.
in the following way.

\[ \sigma_T = \frac{1}{2\pi} \int_0^{2\pi} \sigma(\varphi) \, d\varphi \]  

\[ \sigma_T = \frac{1}{4\pi r_0} \left\{ \sum_{n=0}^{\infty} |a_{n}^{\text{sc}}|^2 \varepsilon_n + \sum_{n=1}^{\infty} |b_{n}^{\text{sc}}|^2 \right\} \]  

Now that we have the analytical expression for EM scattering from the proposed meta-atom, we can easily find the condition under which the forward and backward scattering are zero. Fig. 8 presents the zero forward and backward scattering pattern under these specific conditions.

Here, the specific properties of the structure which possess zero forward and backward scattering are presented. The electrical permittivity of the dielectric, radius of the cylinder, rotation angle, and wedge angle of a meta-atom with near-zero forward scattering are as follows: \((\varepsilon_r = 17.5, r = 0.0507\lambda, \alpha = 90.01^\circ, \beta = 0.02^\circ)\).

Fig. 8 a illustrates the 2D scattering of the meta-atom with the aforementioned properties and the total scattering with respect to wavelength. A similar condition for near-zero back scattering is found \((\varepsilon_r = 14.807, r = 0.0721\lambda, \alpha = 279.87^\circ, \beta = 51.36^\circ)\) and the corresponding scattering pattern is shown in Fig. 8 b. Figs 8 a and 8 b show that the forward scattering and the total cross section are highly correlated and that the minimum total scattering and the forward scattering occur at the same wavelength. Consequently, near-zero forward scattering results in a transparent meta-atom.

VI. CONCLUSION

An ideal meta-atom is one which contains all possible bianisotropic electromagnetic responses wherein the structure of the meta-atom permits independent tunability of all the elements comprising the constitutive parameters. Nevertheless, there will be some physical constraints to the design of any meta-atom. In this paper, we propose a novel metal-dielectric super-cylinder as a tunable meta-atom which consists of metallic and dielectric finite wedges. Numerous design parameters provide a good degree of freedom which enable the proposed meta-atom to be an excellent candidate for engineering the EM response. We first derive the analytical expression for the EM scattering due to the structure. Next, we extract the polarizability tensor and explaining the response of such a meta-atom in the dipole region. Finally, we show the flexibility of the proposed inclusion through practical examples that give rise to exotic EM responses. The examples verify the validity of the analytical formulation and illustrate the capability of the proposed meta-atom to independently tune the electromagnetic polarizability components.

Appendix A: Boundary Conditions

In this Appendix, we detail the analytical steps for deriving EM scattering from an asymmetric metal-dielectric super-cylinder of infinite extent along its axis.

In the dielectric region, we construct an expansion for the total electric field \(E^d = E^d_\varphi \) the TM case, which follows from the solution of Maxwell’s equations in cylindrical coordinates.

\[ E^d_z(r, \phi) = \sum_{i=0}^{\infty} \left( U^1_i J_\nu_i(\kappa r) \cos(\nu_i(\phi + \alpha)) \right) + \]  

\[ \sum_{i=1}^{\infty} \left( U^II_i J_\tau_i(\kappa r) \sin(\tau_i(\phi + \alpha)) \right) \]

where \( \kappa = \omega \sqrt{\mu_r \varepsilon_r} \) denotes the wavenumber. The \( \zeta^I \) and the \( \zeta^I \) are the (unknown) coefficients. Also, the incident plane wave has a following form

\[ E^{inc}_z(r, \phi) = \sum_{i=0}^{\infty} \left( U^1_i J_\nu_i(\kappa_0 r) \cos(\nu_i(\phi + \alpha)) \right) + \]  

\[ \sum_{i=1}^{\infty} \left( U^II_i J_\tau_i(\kappa_0 r) \sin(\tau_i(\phi + \alpha)) \right) \]

where

\[ U^1_n = \frac{2E_0 j^n}{\varepsilon_n} \cos(n(\gamma + \alpha)) \]

\[ U^II_n = \frac{2E_0 j^n}{\varepsilon_n} \sin(n(\gamma + \alpha)) \]

and also \( E^{sc}_z \) is presented in the equation (1). On the metal part of dielectric-metal super-cylinder \((r = r_0)\) the tangential part of the total electric field must vanish, and at the dielectric-air interface \((r = r_0)\), the tangential parts of the total electric and total magnetic field have to be continuous.

\[ E^sc_z|_{r=r_0} + E^{inc}_z|_{r=r_0} = \begin{cases} 0 & \text{if } S_1 < \phi < S_2 \\ E^d_z|_{r=r_0} & \text{Otherwise} \end{cases} \]

\[ H^{sc}_\varphi|_{r=r_0} + H^{inc}_\varphi|_{r=r_0} = H^d_{\varphi}|_{r=r_0} \]

where \( S_1 \) and \( S_2 \) are \(-\pi + (\beta/2 + \alpha)\) and \(\pi - (\beta/2 + \alpha)\).

Using the orthogonality relations of the trigonometric functions, we can compute the unknown scattering coefficient \(a_n\) and \(b_n\) in equations (1) and (2) which are presented in equations (5) and (6).

Also, \( \tau, \nu, \varepsilon_n \), and \( \delta_{nm} \) are defined in the following
\[ \tau_i = \frac{i\pi}{\pi - \beta} \quad i = 1, 2, 3, \ldots \quad (A7) \]

\[ \nu_i = \frac{\pi(2i + 1)}{2(\pi - \beta)} \quad i = 0, 1, 2, \ldots \quad (A8) \]

\[ \varepsilon_n = \begin{cases} 2 & n = 0 \\ 1 & n \neq 0 \end{cases} \quad (A9) \]

\[ \delta_{nm} = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases} \quad (A10) \]
[45] B. B. Dasgupta and R. Fuchs, Phys. Rev. B 24, 2 (1981).
[46] P. Grahn, A. Shevchenko, and M. Kaivola, Phys. Rev. B 86, 3 (2012).
[47] M. Kerker, D.-S. Wang, and C. L. Giles, J. Opt. Soc. Am. 73 (1983).
[48] R. V. Metha, R. Patel, R. Desai, R. V. Upadhyay, and K. Parekh, Phys. Rev. Lett. 96, 127402 (2006).
[49] B. Garca-Cmara, F. Moreno, F. Gonzalez, and J. M. Saiz, Phys. Rev. Lett. 98, 179701 (2007).
[50] Y. Li, M. Wan, W. Wu, Z. Chen, P. Zhan, and Z. Wang, Sci. Rep. 5, 12491, (2015).
[51] A. Alu and N. Engheta, J. Nanophotonics, 4, 1 (2010).
[52] M. Safari, M. Albooyeh, C. R. Simovski, and S. A. Tretyakov, Phys. Rev. B 97, 085412 (2018).