Numbering method for the kinematic chain isomorphism recognition of a planar mechanism

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Abstract. The kinematic chain isomorphism identification of a planar mechanism is an important part of the innovative design of kinematic chain regeneration. Based on the structural characteristics of the kinematic chain of a planar mechanism, a new method, i.e. the numbering method, is proposed for isomorphism identification. Firstly, the links of a kinematic chain are numbered according to certain rules, and they are normalized. Then, the normalized contracted link adjacency matrix is used to represent the connection of the kinematic chain graph. Based on whether the contracted link adjacency matrix of two kinematic chains is the same, we can judge whether the two links are similar.

1. Introduction
With the development of science and technology, the identification of Graph isomorphism is widely applied to many fields, such as pattern recognition [1], artificial visual sense [2], data mining [3], bioinformatics [4] and mechanisms [5-8]. Isomorphism identification of mechanism kinematic chain is a difficult task in the innovative design of kinematic chain regeneration [5], but is also an essential step in kinematic mechanism synthesis. So far, many methods for isomorphic recognition have been proposed to address this problem [9], such as characteristic polynomial method, code-based approaches, Hamming number method and distance-based approach, etc.

In the field of mechanism topology, isomorphism of two kinematic chains means that the links between the two kinematic chains and their connections have one-to-one correspondence. In this paper, a novel numbering method is proposed to judge the isomorphism of kinematic chains according to the characteristics of the kinematic chains of a planar mechanism. The advantage of this method is simple and reliable. The rest structure of this paper is organized as follows: the basic knowledge of graph method for kinematic chain isomorphism identification are introduced in Section 1. In Section 2, the algorithm of numbering method is described. Section 3 presents the isomorphic recognition algorithm in detail. An example is given to verify the effectiveness of our method in Section 4. Finally, the content of this paper is summarized in Section 5.

2. Fundamentals
Basic link: A link that contains no less than 3 motion pairs is defined as a basic link. As shown in Figure 1, the links 1, 4, 5, and 7 in the randomly-numbered contracted link kinematic chain are basic links.
Contracted link: The two pairs of links connected in series in a kinematic chain are defined as a contracted. For example, the links 2, 3, 6, 8 and 9 are shown in Figure 1.

Link degree: $d_i$ represents the link degree of the $i$-th link. The number of movement pairs of each basic link in a kinematic chain is defined as the link degree of the basic link, e.g. $d_2=d_6=4$, $d_3=d_8=3$ shown in Figure 1. The link degree of a contracted link is defined as the inverse of the number of binary links included in the contracted link. For example, in Figure 1, the link degree of the contracted links 2, 3, 6, 8 with a binary link is $d_2=d_3=d_6=d_8=-1$, and the link degree of the contracted link 9 with two binary links $d_9=-2$.

Contracted link adjacency matrix: Contracted link adjacency matrix is a form that uses numerical values to represent the topological graph of a kinematic chain.

A contracted link adjacency matrix $M_c$ is defined as:

$$M_c = [e_{ij}]_{ij=1,2,\ldots,n}$$

where $N$ is the number of links in a kinematic chain; diagonal element $e_{ii}$ is called the link degree element, and $e_{ii} = d_i$; non-diagonal element $e_{ij}$ is called the relevancy degree element, which is used to indicate the connection between link $i$ and link $j$. When link $i$ and link $j$ are connected, relevancy degree element is 1; otherwise, it is 0. The $M_c$ of the contracted link adjacency matrix shown in Table 1:

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 2 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 3 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 4 | 0 | 1 | 4 | 1 | 0 | 0 | 0 | 1 |

Table 1. Link numbering.

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | 3 | 1 | 0 | 1 |
| 2 | 1 | 0 | 0 | 1 | -1 | 0 | 0 | 0 |
| 3 | 1 | 0 | 0 | 0 | 0 | 3 | 1 | 1 |
| 4 | 0 | 0 | 0 | 1 | 0 | 1 | -1 | 0 |
| 5 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | -2 |

Link degree sequence of a kinematic chain: The row vectors composed in the descending order of link degree are called link degree sequence $D_L$. The link degree sequence of the contracted link kinematic chain shown in Figure 1 is $D_L = [4, 4, 3, 3, -1, -1, -1, -1, -2]$.

2.1. Principle for link numbering

The structure of a kinematic chain can be represented by a contracted link adjacency matrix which is determined by the number of links. So there is a unique mapping relationship between the number of links and the contracted link adjacency matrix. The mapping function exactly represents the structure of a kinematic chain. For the isomorphic judgment of different kinematic chains, we should number...
the links using the unified principle, perform normalized processing, and then judge whether the elements in the normalized contracted link adjacency matrix have correspondence.

In this paper, the numbers of the links in a contracted link kinematic chain are determined according to the following principles: ① All links are numbered from small value to large value starting with the natural number 1. ② All links are numbered in order of link degree. ③ The numbers of links with equal link degrees must be continuous, and the order must follow the following principle: use the relation code to determine the link similarity (5), arbitrarily number the similar links with continuous numbers, and preferentially number the unsimilar links with large relation code sum (the concept of relation code sum will be explained below).

2.2. Similarity recognition

The similarity of two links means that in the process of logical reasoning, the obtained divergence results are equivalent when these two links are selected as functional links.

Relation code: A represents a link, and the weights of links adjacent to A are arranged in order of magnitude, and the numbers obtained are called the relation code of link A, and it is represented by the symbol \( I_mD_n \), where \( m \) represents the level of the relation code, and \( n \) represents the link number. In a contracted link kinematic chain graph, the relation code obtained by assigning the link degree of each link as a weight to each link is called the I-level relation code of the link. The I-level relation code of each link is assigned to each link as a weight, and the obtained relation code is called the II-level relation code of the link, and the III-level relation code can be obtained by analogy.

Relation code sum: Two links with different "level" relation codes must be different. The relation code sum of the current "level" when two links with different "level" relation codes are selected, and it is represented by the symbol \( S_n \). The numbers obtained are called the relation code of link A, and it is represented by the symbol \( I_mD_n \), where \( m \) represents the level of the relation code, and \( n \) represents the link number.

The relation code and relation code sum of a contracted link kinematic chain is shown in Table 2.

| Component | \( d_n \) | \( I_1D_n \) | \( I_2D_n \) | \( I_3D_n \) | \( S_n \) |
|-----------|---------|---------|---------|---------|---------|
| 1         | 4       | 3-1-1-1 | 4444434-2-1 | DFEE    | 0       |
| 4         | 4       | 3-1-1-2 | 4444434-1-1 | CHEE    | -1      |
| 5         | 3       | 4-1-1   | 43333-1-1-1-2 | BGF    | 2       |
| 7         | 3       | 4-1-2   | 43333-1-1-1 | AGH     | 1       |
| 2         | -1      | 44      | 3-1-1-13-1-1-2 | BA     | 8       |
| 3         | -1      | 44      | 3-1-1-13-1-1-2 | BA     | 8       |
| 6         | -1      | 43      | 4-1-13-1-1   | AC      | 7       |
| 8         | -1      | 33      | 4-1-14-1-2   | DC      | 6       |
| 9         | -2      | 43      | 4-1-23-1-1-2 | BD      | 7       |

Note: \( n \) represents the vertex number, where \( A=4444434-2-1, B=4444434-1-1, C=43333-1-1-1-2, D=43333-1-1-1, E=3-1-1-13-1-1-2, F=4-1-13-1-1, G=4-1-14-1-2, H=4-1-23-1-1-2. \)

According to relation code method, for a 6-link kinematic chain, the links with the same I-level relation code are similar; for a 8-link kinematic chain, the links with the same II-level relation code are similar, and the II-level relation code of similar links are the same; for a 10-link (or more) kinematic chain, the links with the same III-level relation code are similar, and the III-level relation code of similar links are the same.

Based on the principle of link numbering, the principle of similarity recognition, and the relation code and relation code sum shown in Table 1, the links in Figure 1 are re-numbered to obtain the normalized contracted link kinematic chain graph, as shown in Figure 2.
2.3. General steps for link numbering

(1) Links are numbered from natural number \( l \) arbitrarily. After reading the number, we can obtain the topological information of the kinematic chain, the number of links \( N \), and the adjacency matrix \( M_c \). Let \( L \) be a sequence of \( 1 \sim N \). Then the kinematic chain is renumbered.

(2) If \( N \leq 6 \), let \( k = 1 \); if \( 6 < N \leq 8 \), let \( k = 2 \); if \( 8 < N \), let \( k = 3 \).

(3) Read the two links \( i \) and \( j \) in sequence. If \( d_i < d_j \), swap the positions of \( i \) and \( j \) in \( L \) and go to step (6); if \( d_i > d_j \), go directly to step (6); otherwise let \( m = 1 \), go to the next step.

(4) Calculate \( I_m D_i \) and \( I_m D_j \) according to \( M_c \). If \( I_m D_i \neq I_m D_j \) and \( S_i < S_j \), swap the positions of \( i \) and \( j \) in \( L \), and go to step (6). If \( I_m D_i \neq I_m D_j \) and \( S_i > S_j \), go directly to step (6); if \( I_m D_i \neq I_m D_j \), then \( m = m + 1 \), go to the next step.

(5) If \( m > k \), go to step (6); otherwise go to step (4).

(6) If all the two links are read, go to the next step, otherwise go to step (3).

(7) The links corresponding to the \( L \) elements in the sequence are re-numbered with \( l \sim N \) to obtain the normalized contracted link kinematic chain graph and return to the adjacency matrix \( M_{cs} \).

(8) End.

3. Algorithm for isomorphic recognition

A normalized contracted link adjacency matrix can uniquely reflect the structure of a contracted link kinematic chain, so it is concluded that if the elements in a normalized contracted link adjacency matrix are correspondingly the same, the kinematic chains have isomorphism; and if the elements in a normalized contracted link adjacency matrix are not the same, the kinematic chains do not have isomorphism. To improve the recognition efficiency, the two kinematic chains with different link degree sequences are directly judged as non-isomorphism. The main steps for isomorphic recognition are as follows:

(1) Calculate the link degree sequence \( D^1_L \) and \( D^2_L \) according to the kinematic chain graph with arbitrary numbers. If \( D^1_L \neq D^2_L \), then the two kinematic chains are not isomorphism, go to step (4); otherwise go to the next step.

(2) Call the numbering function NO () to renumber the kinematic chain to obtain the normalized kinematic chain graph and the normalized contracted link adjacency matrices \( M_{cs}^1 \) and \( M_{cs}^2 \).

(3) Determine whether the elements in \( M_{cs}^1 \) and \( M_{cs}^2 \) have the same one-to-one correspondence. If yes, the two kinematic chains are isomorphic; otherwise, the two kinematic chains are not isomorphic.

(4) End.

4. Example research

Please judge whether the two single-degree-of-freedom kinematic chains shown in Figure 3 are isomorphic.
Figure 3. Two chains of one degree of freedom.

(1) Number the two kinematic chains arbitrarily, as shown in Figure 3, the degree sequence of the two kinematic chain members are $D_1 = D_2 = [4, 3, 3, 3, 3, 1, -1, -1, -2]$.

(2) Call the numbering function NO() to re-number the two kinematic chains. The specific numbering steps are not repeated here, and a normalized contracted link kinematic chain is obtained as shown in Figure 4.

(3) According to the normalized kinematic chain graph and Equation (1), we can obtain the normalized contracted link adjacency matrix $M_{cs}^1$ and $M_{cs}^2$, so the two kinematic chains in the figure are isomorphic.

Figure 4. Normalized contracted link kinematic chain.

5. Conclusions
A normalized contracted link adjacency matrix can uniquely reflect the structure of a contracted link kinematic chain, so that the similarity of links can be reliably recognized. When judging the isomorphism of kinematic chains, it is feasible to check whether the normalized contracted link adjacency matrices are the same. This method is simple to operate, does not involve complex theories, and is convenient for computer programming, thereby ensuring the efficiency of isomorphism recognition of kinematic chains. Verified by a large number of examples, the numbering method is suitable for the isomorphic recognition of multi-link and multi-degree-of-freedom kinematic chains.

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