Modeling of a Temperature Distribution in Optomechanical Thermal Microsystems

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Abstract: An approach for modeling the temperature distribution in optomechanical thermal microsystems is presented. Three types of the microsystems are considered: microsystem with two supporting microbeams in adjacent corners of the microplate; microsystem with two supporting microbeams in opposite corners of the microplate; microsystem with four supporting microbeams. In the structure of each type of the microsystems, the modeling domains are marked out, which are divided into zones with homogeneous equivalent parameters. Analytical expressions for the temperature distribution in the zones of optomechanical thermal microsystems are obtained. The method of modeling was used to determine the temperature distribution in bi-material sections in cantilever microbeams in the optomechanical thermal microsystems under consideration. The dependence of the temperature difference on the boundaries of the bi-material section on the thickness of the aluminum layer in this section was determined.

Keywords: optomechanical thermal microsystem, temperature distribution, method of eigenfunctions, microplate, cantilever microbeam, bi-material

1. Introduction
The modern stage of the mechanical engineering development is characterized by the extensive use of microsystems. Optomechanical thermal microsystems represent a new type of microsystems, in which the advantages of the three physical effects are combined: optical, mechanical and thermal. The development and investigation of these microsystems began about twenty years ago and are intensively continuing at the present time. The feature of these microsystems is that in them the electrical part is excluded from the transformation processes and, accordingly, the influence of electromagnetic effects on the characteristics of the microsystem is not significant.

The operation of the optomechanical thermal microsystems is based on the following principles:
– formation of an elevated temperature field in the microsystem structure by converting a measured or control action into a thermal signal;
– usage of a bimetallic effect in the cantilever microbeams of the structure to move the microplate;
– usage of optical reading to determine the amount of a microplate movement;
– usage of structures made by MEMS technology to increase a temperature sensitivity of bimetallic elements.

For the first time, optomechanical thermal microsystems in the form of bimetallic cantilever microbeams were used as chemical sensors for measuring the amount of heat released in a chemical thermocatalytic reaction by the substance being determined [1]. At present, an important and promising application of optomechanical thermal microsystems is their use as sensors for measuring temperature [2], uncooled IR detectors [3-6] and elements of scanning IR images [7-12].
The characteristics of optomechanical thermal microsystems depend on three subsystems: optical, mechanical and thermal. Among these subsystems, the main role is played by the thermal subsystem, which to a large extent affects the dynamic characteristics of the optomechanical thermal microsystem [13, 14] and its sensitivity. To take into account this influence of the thermal subsystem, it is necessary to know the temperature distribution in the structure of the optomechanical thermal microsystem. Currently, this issue is being intensively developed.

2. Formation of the Problem
Optomechanical thermal microsystems are manufactured using surface micromachining. They consist of the following elements:
1) the microplate with a functional layer in which the energy of input action is converted into thermal energy;
2) the cantilever microbeams which on the one side are fixed on the base and on the other side support the microplate over the base;
3) bi-material section that are part of the microbeams and induce the deflection of the microplate a result of the bimetallic effect.

The basic structures of optomechanical thermal microsystems are shown in Fig. 1. In these microsystems, an important role is assigned to bi-material sections that are part of cantilever microbeams. From the structure and geometric dimensions of these sections and the distribution of temperature in them depends the displacement of the microplate. In its turn, the temperature distribution in the bi-material sections is determined by the structure of an optomechanical thermal microsystem. Thus, the determination of the temperature distribution in the structure of the optomechanical thermal microsystem is an important task, the solution of which makes it possible to determine the displacement of the microplate and, accordingly, the output signal of the optical subsystem.

In this connection, the aim of this paper is to simulate the temperature distribution in the structure of an optomechanical thermal microsystem to determine the temperature distribution in the bi-materials sections of, and to assess the influence of the design parameters of the microsystem on this distribution.

3. Theory. Method of Modeling
To find the temperature distribution in the structure of the optomechanical thermal microsystem, an analytical method, proposed specifically for these purposes and taking into account the features of thermal microsystems [15], will be used.

The algorithm of the method has the following sequence of steps.
1. Taking into account the symmetry of the structure of the optomechanical thermal microsystem (a microplate with a functional layer and supporting cantilever microbeams), the domain of modeling is highlighted in it (Fig. 1). The conditions for highlighting this domain of modeling are, firstly, the formation of the entire structure from the domain of modeling by means of the corresponding symmetry transformation, and, secondly, accounting the peculiarities of the temperature modeling method presented in [15]. This method allows us to reduce the size of the domain of modeling due to symmetric boundary conditions. As can be seen from the data presented in Fig. 1, the domains of modeling for the microsystems under consideration have the same form.
2. The domain of modeling is divided into a series of rectangular zones (Fig. 2). The dimensions of these zones are determined by the composition of the layers. Each zone is replaced by an equivalent zone with homogeneous parameters: equivalent thickness, \(d_j\), and an equivalent thermal conductivity, \(\lambda_j\), whose values are determined as follows

\[
d_j = \sum_{i=1}^{n} k_i^{(j)} d_i^{(j)},
\]   

(1)
Figure 1. Optomechanical thermal microsystems on the basis of rectangular microplate supported by (a) two microbeams in adjacent corners, (b) two microbeams in opposite corners, (c) four microbeams: 1 – microplate with a functional layer; 2 – supporting microbeam; 3 – bi-material section; 4 – domain of modeling.

\[ \lambda_j = \left( \sum_{i=1}^{n} k_i^{(j)} d_i^{(j)} \lambda_i^{(j)} \right) / d_j, \]

where \( d_i^{(j)} \) is the thickness of layer \( i \) in zone \( j \); \( n \) is the number of layers in zone \( j \); \( k_i^{(j)} \) is the coefficient equal to the relation of the area of layer \( i \) in zone \( j \) to the total area of this zone; \( \lambda_i^{(j)} \) is the thermal conductivity of layer \( i \) in zone \( j \). The division of the domains of modeling into zones for the microsystems under consideration is different due to the features of their structures.

3. For each zone of the domain of modeling, the conditions of its heat exchange with the environment are determined. Through the boundary with the neighboring zones and the substrate, heat transfer is carried out by thermal conductance. From the lower and upper surfaces of the zone heat exchange is carried out by means of convective and conductive heat transfer through the gaseous medium and by radiative heat exchange. The total heat loss from the upper and lower surface of the zone \( j \), normalized per unit area (\( N_i^{(j)} \)), can be represented as follows...
Figure 2. Division of the domain of modeling into zones for microsystems based on rectangular microplate supported by (a) two microbeams in adjacent corners, (b) two microbeams in opposite corners, (c) four microbeams; the division of the microbeam into zones in the variants (b) and (c) is analogous to the variant (a).

$$N_{1}^{(j)} = A_j (T_j - T_{en}), \quad (3)$$

where

$$A_j = h_c + \frac{\lambda_a}{d_1^{(j)}} + 4\sigma (\varepsilon_{\text{l}}^{(j)} + \varepsilon_{\text{u}}^{(j)}) T_{en}^3; \quad (4)$$

$T_j$ and $T_{en}$ are the temperature of zone $j$ and the environment temperature, respectively; $A_j$ is the total coefficient of the heat transfer from the lower and upper surfaces of zone $j$; $h_c$ is the convective coefficient of the heat transfer for the given structure of the microsystem; $\lambda_a$ is the thermal conductivity of the air medium around the microsystem; $d_1^{(j)}$ is the distance between the lower surface of zone $j$ and the substrate surface; $\sigma$ is the Stefan-Boltzmann constant; $\varepsilon_{\text{l}}^{(j)}$ and $\varepsilon_{\text{u}}^{(j)}$ are the emissivities of the lower and upper surfaces of zone $j$, respectively.

In the equation (4), the total heat transfer coefficient takes into account the following ways of heat transfer from the lower and upper surfaces of the zone. The first term determines the heat transfer from the upper surface of the zone by convection. The second term characterizes the heat removal from the lower surface of the zone by means of thermal conductance through the air gap between this surface
and the substrate surface. The third term determines the dissipation of heat from the upper and lower surfaces of the zone by radiation heat exchange.

4. For each zone, a stationary differential heat equation and boundary conditions are determined. The solution of the heat equation is carried out by the method of eigenfunctions and is written in an analytical form. The heat fluxes densities between the zones included in the solution are represented as a sum of orthogonal functions with weight coefficients whose values are unknown. The stationary differential heat equation for all zones can be represented in the following general form

\[
\frac{\partial^2 T_j'}{\partial x_j^2} + \frac{\partial^2 T_j'}{\partial y_j^2} - p_j^2 T_j' = \varphi_j, \tag{5}
\]

where

\[
T_j' = T_j - T_{en};
\]

\[
p_j = \frac{A_j}{\lambda_j d_j};
\]

\[
\varphi_j = -\frac{q_j}{\lambda_j d_j};
\]

\(q_j\) is the heat power density in zone \(j\).

The solution of the differential heat equation (5) for all zones has the following form

\[
T_j = D^{(j)}_{0,0} + 2 \sum_{k=1}^{\infty} D^{(j)}_{k,0} \cos(k \pi x_j / l_j) + 2 \sum_{m=1}^{\infty} D^{(j)}_{0,m} \cos(m \pi y_j / b_j) +
\]

\[
+ \frac{4}{l_j b_j \lambda_j} \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} D^{(j)}_{k,m} \cos(k \pi x_j / l_j) \cos(m \pi y_j / b_j), \tag{6}
\]

where

\[
D^{(j)}_{k,m} = \frac{(-1)^k \delta^{(js)}_m - (-1)^m \delta^{(jt)}_k + \delta^{(ju)}_m + \delta^{(jv)}_k}{l_j b_j \lambda_j [(k \pi / l_j)^2 + (m \pi / b_j)^2]}; \tag{7}
\]

\(l_j, b_j\) are the length and the width of zone \(j\), respectively; \(\delta^{(js)}_m, \delta^{(jt)}_k, \delta^{(ju)}_m,\) and \(\delta^{(jv)}_k\) are the weight coefficients determining heat flux densities on boundaries of the zone \(j\) and the zones \(s, t, u,\) and \(v\), respectively; \(k, m\) are the summation indexes on \(x\) and \(y\) coordinates, respectively.

The zones \(s, t, u,\) and \(v\) are localized, respectively, on the right, above, left, below the zone \(j\). The expressions for the temperature distribution in the microsystem zones contain the weight coefficients \(\delta^{(js)}_m, \delta^{(jt)}_k, \delta^{(ju)}_m,\) and \(\delta^{(jv)}_k\), which characterize the heat flux densities on the boundaries of the zones and whose values are unknown.

5. To determine the unknown values of the weight coefficients entering into equation (6), we use the equations for the temperature equality from the conjugate boundary conditions between neighboring zones. In the general case, the problem of determining the values of the weight coefficients reduces to solving a system of linear differential equations. After finding the values of the weight coefficients, the temperature distributions in all zones of the optomechanical thermal microsystem are determined.
Knowing the temperature distribution in the zone with bi-material (zone 2), one can find the deflection of microbeams and, accordingly, the deviation of the microplate. For this, the differential equation of the bending of a cantilever beam with a bi-material is used:

\[
d^2z\over dy^2 = 6\left(\alpha_2 - \alpha_1\right)\left[\frac{d^2}{d_1^2} + \frac{d^2}{d_2^2}\right] \left[T_2(y) - T_2(y)\right]_y = 0,
\]

where \(\alpha_1\) and \(\alpha_2\) are the thermal expansion coefficients of layers 1 and 2 in zone 2, respectively;

\[
K = 4 + 6\left(\frac{d_2^2}{d_2^2}\right) + 4\left(\frac{d_2^2}{d_2^2}\right)^2 + \frac{E_1}{E_2}\left(\frac{d_2^2}{d_2^2}\right)^3 + \frac{E_2}{E_1}\left(\frac{d_2^2}{d_2^2}\right);
\]

\(E_1\) and \(E_2\) are the Young's modulus of materials of layers 1 and 2 in zone 2, respectively. The solution of the differential equation (8) allows one to find the microplate deflection.

4. Results of Modeling

The proposed model was applied to determine the temperature distribution in the bi-material sections in three types of optomechanical thermal microsystems with an absorbing layer (Fig. 1). In the simulation, the following values of parameters were used:

- size of microplate: length \(-50\ \mu\text{m}\); width \(-50\ \mu\text{m}\); thickness \(-0.5\ \mu\text{m}\);
- the dimensions of the supporting L-shaped microbeams: the length of the long section is \(50\ \mu\text{m}\); the length of the short section attached to the microplate is \(4\ \mu\text{m}\); width \(-4\ \mu\text{m}\); thickness \(-0.5\ \mu\text{m}\);
- material of the microplate and the microbeams – silicon dioxide with the following properties: thermal conductivity \(-1.2\ \text{W/(m-K)}\); emissivity \(-0.5\); coefficient of thermal expansion \(-6.1 \times 10^{-6}\ \text{K}^{-1}\); Young's modulus is \(0.65\ \text{GPa}\);
- material of the absorbing layer - platinum with the following properties: thermal conductivity \(-72\ \text{W/(m-K)}\); emissivity \(-0.9\);
- the thickness of the absorbing layer is \(0.4\ \mu\text{m}\);
- dimensions of the section with bi-material in supporting microbeams: length \(-38\ \mu\text{m}\); width \(-4\ \mu\text{m}\);
- the thickness of the layer 2 of the section with bi-material is \(0.05 \ldots 0.5\ \mu\text{m}\);
- material of layer 2 of the section with bi-material - aluminum with the following properties: thermal conductivity \(-236\ \text{W/(m-K)}\); emissivity \(-0.55\); the coefficient of thermal expansion \(-23.3 \times 10^{-6}\ \text{K}^{-1}\); Young's modulus is \(70\ \text{GPa}\).

The absorbing layer occupies the entire area of the microplate on which it is located. The microsystem is located in a housing whose dimensions are larger than the dimensions of the microsystem itself. The distance between the lower surface of the microplate and the substrate was assumed to be \(20\ \mu\text{m}\). The values of the other parameters were chosen as follows: the thermal conductivity of the ambient air was \(0.026\ \text{W/(m-K)}\); coefficient of convective heat transfer \(-5.6\ \text{W/(m}^2\cdot\text{K)}\); ambient temperature \(-300\ \text{K}\); the power density of the input radiation is \(10,000\ \text{W/m}^2\).

In Fig. 3, the overheating temperature distribution on the surface of the the bi-material section of the cantilever microbeam in the optomechanical thermal microsystems of the types under consideration is shown for a thickness of layer 2 (aluminum) of the bi-material section equal to \(0.1\ \mu\text{m}\). The overheating temperature distribution along the bi-material section of the cantilever microbeam is shown in Fig. 4. The dependence of the temperature difference on the boundaries of the bi-material section of the cantilever microbeam in the optomechanical thermal microsystems under consideration on the thickness of layer 2 (aluminum) of the bi-material section is shown in Fig. 5.

5. Results Discussion

The analysis of the overheating temperature distribution on the surface of the bi-material section of the cantilever microbeam in the optomechanical thermal microsystems presented in Fig. 3, shows that for identical designs of the microplates and the microbeams and the same heating conditions in microsystem designs with two supporting microbeams, the temperature distribution on the surface of the bi-material section is practically identical. The same is confirmed by the data for the overheating...
temperature distribution along the bi-material section of the cantilever microbeam (Fig. 4). In comparison with the Microsystems in which the microplate is supported by two microbeams, in the microsystem with four microbeams, the temperature of the bi-material section of the cantilever microbeam has the less value (Fig. 3).

![Temperature distribution table]

**Figure 3.** The overheating temperature distribution in the bi-material section of the cantilever microbeam (zone 2) in the optomechanical thermal Microsystems on the basis of rectangular plate supported by (a) two microbeams in adjacent corners, (b) two microbeams at opposite corners, (c) four microbeams.

In all types of Microsystems, the overheating temperatures along the bi-material section of the cantilever microbeam in a direction from the attachment point of the microbeam to the place of its attachment to the microplate increase linearly (Fig. 4). The temperature difference at the ends of the bi-material section of the cantilever microbeam for the thickness of the aluminum layer of 0.1 μm is:

- a) for Microsystems with two support microbeams (regardless of the attachment point) – 0.36 K;
- b) for microsystem with four supporting microbeams – 0.33 K.

As can be seen from the presented data, the replacement of four supporting microbeams with two support microbeams in the considered structures of the optomechanical thermal Microsystems makes it possible to increase the difference in temperature at the ends of the bi-material section of the cantilever microbeam by no more than 10 percent.

The importance in designing optomechanical thermal Microsystems has the choice of the thickness of layer 2 of the bi-material section. This layer is applied to the microbeam and together with the microbeam material forms a bi-material section. Its thickness, mechanical and thermophysical properties determine the displacement of the microbeam end and influence on the sensitivity of the microsystem. From the data shown in Fig. 5, it is seen that with increasing the thickness of the aluminum layer (layer 2 of the bi-material section in the Microsystems under consideration), the temperature difference at the ends of the bi-material section decreases nonlinearly. At small layer
thicknesses (0.05 ... 0.2 μm), a significant decrease in the temperature difference (by 3.5 times) is observed, and for large layer thicknesses (0.2 ... 0.5 μm), the temperature difference decreases by 2.4 times.

![Graph showing temperature distribution](image)

**Figure 4.** The distribution of the overheating temperature along the bi-material section of the cantilever microbeam in optomechanical thermal microsystems on the basis of rectangular plate supported by (1) two microbeams in adjacent corners, (2) two microbeam beams at opposite corners, (3) four microbeams.

6. Conclusion
The approach for modeling the stationary temperature distribution in the structure of the optomechanical thermal microsystems consisting of microplate with the functional layer and the supporting cantilever microbeams with the bi-material section is proposed. Based on this approach, the temperature distribution in the structure of microsystems and, in particular, in the bi-material sections of the cantilever microbeams is determined. It is established that the distinction in the temperature difference at the ends of the bi-material section of the cantilever microbeam in the microsystems with two and four supporting microbeams does not exceed 10 percent. With increasing thickness of layer 2 of the bi-material section, the temperature difference decreases according to a nonlinear law.

The proposed approach for modeling the stationary temperature distribution in the structure of optomechanical thermal microsystems can be used to develop the design of these microsystems, the choice of materials and the design dimensions of the microsystem elements. Using the proposed approach, it is possible to establish the influence of the properties of the used materials and the geometric dimensions of the elements on the important characteristic of microsystems - the temperature difference at the ends of the bi-material section of the cantilever microbeams.

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Figure 5. Dependence of the temperature difference on the boundaries of the bi-material section of the cantilever microbeam in optomechanical thermal microsystems on the basis of rectangular plate supported by (1) two microbeams in adjacent corners, (2) two microbeams at opposite corners, (3) four microbeams on the thickness of the aluminum layer.