Non-collinear ground state and stable bimerons from four-spin chiral interactions in $D_{3h}$ magnet

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Abstract. We demonstrate that four-spin interactions in crystals with $D_{3h}$ point group of symmetry can cause a phase transition from a collinear state to a non-collinear magnetic ground state (such as magnetic vortices or magnetic skyrmions), while all anti-symmetric chiral terms are forbidden by symmetry in such crystals. Moreover, $D_{3h}$ point group rather common among two dimensional magnets. Taking into account possible four-spin chiral exchange interactions is important for understanding noncollinear magnetic order in these systems. We also address a possible stabilization of bimerons by the same contribution.

1. Introduction

Electron-mediated indirect asymmetric exchange between a pair of localized spins in lattices with broken inversion symmetry or with external symmetry breaking is commonly referred to as the Dzyaloshinsky-Moriya interaction (DMI) [1]. The DMI leads to a variety of magnetic structures from the collinear state (ferromagnetic or anti-ferromagnetic) to complex, non-collinear spin textures [2]. Indeed, the presence of linear-in-gradient terms in micromagnetic energy, that DMI may induce, can make a collinear order unstable and create helix, cone, or skyrmion crystals. Those are important structures that are promising candidates for the role of information signal carrier in spintronics and also may be used to store and manipulate information. Such structures were observed in many conducting magnets or magnetic multilayers with broken inversion symmetry such as MnSi, FeGe, Ir/Co/Pt or Pt/CoFeB/MgO [3]. However, a little attention has been paid to the study of two-dimensional (2D) magnets. A breakthrough occurred in 2017, when long-range magnetic order was discovered in 2D van der Waals materials Cr$_2$Ge$_2$Te$_6$ [4] and CrI$_3$ [5]. Soon they were accompanied by a metallic ferromagnet Fe$_2$GeTe$_2$ [6, 7], where spin helical phase also has been detected [8]. These materials are characterized by $D_{3h}$ point group of symmetry with honeycomb magnetic lattice, for which all Lifshitz invariant terms in micromagnetic functional are forbidden, while multi-spin chiral exchange interactions are allowed by symmetry [9]. That is why the origin of non collinear magnetic order in this material cannot be correctly explained by DMI [10]. It can affect magnetic order only at the sample boundaries. In this Letter, we discuss another possible origin of noncolliner magnetic textures in ferromagnets with the $D_{3h}$ point group symmetry. As in $D_{3h}$, all antisymmetric contributions vanish, it is thus worthwhile to consider terms of the fourth order with respect to the vector of local magnetization direction.

2. Helical phase from four-spin chiral interactions

Free energy functional of $D_{3h}$ ferromagnet depends on a dimensionless magnetization vector $m(r)$ of the unit length and reads:
\[ F[m] = \int d^2r [A \Sigma_a (\nabla m_a)^2 + 8BI + Km^2 - Hm], \]

where \( H \) stands for external magnetic field measured in energy units, \( A > 0, K, B \) are exchange, anisotropy and four-spin interaction constants, respectively. The group \( D_{3h} \) is characterized by two independent fourth order invariants with respect to the field \( m(r) \). They can be written as

\[ I_1 = m_x(m_x^2 - 3m_y^2)\nabla m_y, \quad I_2 = m_x(m_x^2 - 3m_y^2)\nabla m_x. \]

Obviously, for a 2D system, the second one can be disregarded. Hence, we are left with a single fourth order term \( I_1 \).

A non-collinear ansatz for local magnetization field \( m(r) \), that has the biggest chance to become a ground state, can be written in a form:

\[ m(r) = n \cos \alpha + [m_1 \cos(kr) + m_2 \sin(kr)] \sin \alpha, \]

where \( n, m_1 \) and \( m_2 \) are mutually perpendicular unit vectors: \( m_1 = (\cos \phi, -\cos \phi, 0)^T \), \( m_2 = (\cos \phi \cos \theta, \sin \phi \cos \theta, -\sin \theta)^T \), \( n = m_1 \times m_2 \). The angle \( \alpha = 0 \) corresponds to a collinear state, \( \alpha = \pi/2 \) to a pure helix state. We assume that the ferromagnet is kept well below Curie temperature, hence \( |m| = 1 \). The angle between the vectors \( k \) and \( m \) is an important characteristic of the state that can be measured in experiment.

Substituting (3) into (1) we minimize equation (1) with respect to \( k_x, k_y \) and get for \( I_1 \) the free energy as a function of three variables \( \theta, \phi, \alpha \):

\[ \frac{F[m]}{VB^2} = -\frac{9}{64} (\sin \alpha + 5 \sin 3\alpha)^2 \sin^4 \theta \cos^2 \theta + \frac{KA}{B^2} \cos^2 \theta \cos^2 \alpha 
\]

\[ + \frac{KA}{2B^2} \sin^2 \theta \sin^2 \alpha - \frac{A}{B^2} \sin \alpha Hm, \]

where the four-spin interaction sets the energy scale \( A/B^2 \) that defines the non-collinear order.

It is interesting to note that states described by equation (4) are degenerate with respect to \( \phi \). It means the free energy does not depend on the azimuthal angle of the average magnetization vector \( m \). At the same time, for helical state with a finite wave vector, the angles between \( k \) and \( m \) are different for different values of \( \phi \):

\[ k \cdot m \propto \sin 3\phi. \]

It is seen in figure 1. Thus, by controlling the direction of the average magnetization it is also possible to control the propagation direction of the non-collinear state.

**Figure 1.** Geometry of the structure of the \( D_{3h} \) magnet. The direction of the helix propagation is defined by the wavevector \( k \). Panels show three directions of the wavevector and the in-plane component of \( m \).

Further we minimize equation (4) with respect to \( \theta, \phi, \alpha \) numerically and the result of the energy minimization is illustrated in figure 2 by plotting the dependence \( \alpha \) (an angle describing a transfer from collinear to non-collinear state) on both \( KA/B^2 \) and \( HA/B^2 \) at the absolute energy minimum for the cases of out-of-plane and in-plane magnetic field. The graph shows a phase transition from a collinear state (\( \alpha = 0 \)) to non-collinear state (\( \alpha \neq 0 \)) and back. In addition, the angle \( \alpha \) smoothly decreases from
$\pi/6$ to almost 0 with $H$ increase. It can be seen that the span of magnetic cone may, at best, only slightly exceed the value $\pi/6$, while the pure helix, $\pi/2$, is never reached. The corresponding jumps are also seen in the left panels.

![Figure 2. Phase diagram of the collinear and non-collinear phases as a function of anisotropy $K$ and external magnetic field in lateral (upper panel) and perpendicular (lower panel) directions. The right panel shows the cross-sections of the phase diagrams along the lines marked with dashed lines.]

3. Stabilization of bimerons

It is natural to explore the effect of four-spin interactions on skyrmions. It however appears that due to the antisymmetric nature of the term in equation 2 its contribution to the radially symmetric spin distributions such as skyrmion is zero. At the same time, topological excitations lacking the radial symmetry should be affected by the four-spin interactions. As an example, we consider a bimeron, an in-plane counterpart of skyrmion with the magnetization profile given by $m = \vec{R}_z(\phi_0)m_0$, where $\vec{R}_z$ is the matrix of rotation with respect to $z$ axis, and $m_0$ reads:

$$m_0 = [\cos \Theta(r), \sin \Theta(r) \cos \Phi(\phi), \sin \Theta(r) \sin \Phi(\phi)],$$

where $\Phi = Q\phi + \delta$ with integer $Q$ being the topological charge, and an unknown $\Theta(r)$ satisfies the boundary conditions $\Theta(0) = \pi$, $\theta|_{r \to \infty} = 0$. In what follows we assume the positive exchange constant $A>0$. If we substitute the ansatz (6) for $Q = 1$ to the expression for the free energy we get the following Lagrange equation:

$$\Theta''(r) + \frac{\Theta'(r)}{r} - \frac{\sin 2\Theta(r)}{2r^2} - \frac{KA}{4B^2} \sin 2\Theta(r) + \frac{3B}{2|\rho|} \sin (2\phi_0 + \delta) [5 \cos^2 \Theta(r) - 1] \sin^2 \Theta(r) = 0.$$  (7)

The specific profile for $\Theta$ can be obtained by numerically solving the corresponding Euler-Lagrange equations. In figure 3(a) we show profiles of $\Theta$ for different values of the dimensionless parameter $KA/B^2$ and $2\phi_0 + \delta = \pi/2$.

The role of four-spin interaction can be revealed by a simple analysis. If we suppose that the specific radial bimeron profile $\Theta_{BM}(r) = \varphi_{BM}(r/r_0)$ is characterized by a single characteristic bimeron radius $r_0$ we can rewrite the free energy as

$$F = AF_{\text{exch}} - B\rho_0 F_{\text{spin}} + \frac{KA}{2B^2} \rho_0^2 F_{\text{anisotropy}},$$

where $\rho_0 = (B/|B|) \sin (2\phi_0 + \delta)$ and $F_{\text{exch}}, F_{\text{spin}}, F_{\text{anisotropy}}$ are scale-invariant contributions from exchange, four-spin interactions and anisotropy respectively. We can immediately note, that since
$F_{\text{anisotropy}} = \int \rho d\rho \sin^2 \varphi_{BM}(\rho) > 0$, the free energy has a minimum at finite $\rho_0$ only for $K>0$, i.e. for the case of easy-plane anisotropy. Further, let us employ a standard ansatz for the radial profile $\varphi_{BM} = \pi - 2 \arcsin(\tanh(\rho/\rho_0))$. Substituting this ansatz to the free energy functional and minimizing it with respect to $\rho_0$ we finally obtain the following equation for the $\rho_0$ corresponding to the minimal energy

$$\rho_0 = \frac{4KB^2}{K\chi} \sin(2\varphi_0 + \delta).$$

Thus, whenever $B = (B/|B|) \sin(2\varphi_0 + \delta) > 0$ there exist a bimeron solution. The minimal energy corresponds to the condition $2\varphi_0 + \delta = \pm \pi/2$ for positive (negative) $B$. In figure 3(a) we plot the ansatz solution with the proper radius $\rho_0$ with the dashed lines. It is seen that ansatz solution accurately describes the exact numerical solution at small radius but substantially discrepancies are seen at larger $\rho$. The scaling of the mean bimeron radius is also well described by the simple linear relation as can be seen in figure 3(b).

**Figure 3.** (a) Radial profile of the bimeron state for different values of $KA/B^2$ obtained via numerical solution of the Euler-Lagrange equation (solid lines) and with an ansatz. (b) Dependence of the mean bimeron radius on $B^2/KA$ obtained numerically (blue line) and using the ansatz (green line).

### 4. Conclusion

In conclusion, we demonstrate that the DMI in a two-dimensional magnet with D$_{3h}$ point group of symmetry does not contribute to the energy functional and cannot cause an instability of the collinear order, while the existence of the four-spin indirect magnetic interaction may be responsible for the appearance of a non-collinear magnetic order. We also address a possible stabilization of bimerons by the same contribution.

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