Notes on teleparallel cosmology with nonminimally coupled scalar field

H. M. Sadjadi
Department of Physics, University of Tehran,
P. O. B. 14395-547, Tehran 14399-55961, Iran

March 4, 2013

Abstract
We consider the spatially flat Friedmann-Lemaitre-Robertson-Walker space time in the teleparallel model of gravity and assume that the universe is filled nearly by cold dark matter and a nonminimally coupled scalar field with a power-law potential as dark energy. We investigate the possibility that the universe undergoes a transition from quintessence to phantom phase. An analytical solution for the scalar field is obtained and necessary conditions required for such a transition are discussed.

1 Introduction
Teleparallel model of gravity, is a description of classical gravity where instead of the torsion-less Levi-Civita connection, the curvature-less Weitzenböck connection is employed [1]. In this model, the action reads

\[ S = \int \left( \frac{1}{16 \pi} T + \mathcal{L} \right) \det(e^i_\mu) d^4 x. \]  

(1)

In terms of dynamical vierbeins fields, \( e^i_\mu \), the metric is \( g_{\mu \nu} = \eta_{mn} e^n_\mu e^m_\nu \) and \( \det(e^i_\mu) = \sqrt{-g} \). \( \mathcal{L} \) is matter Lagrangian density. The scalar torsion \( T \) (this notation is borrowed from [2]) is given by

\[ T = \frac{1}{4} T^{\alpha \nu \rho} T_{\alpha \nu \rho} + \frac{1}{2} T^{\mu \nu \rho} T_{\mu \nu \rho} + T^{\nu \rho} T^\alpha_\rho, \]  

(2)

where

\[ T^\alpha_\nu = e^\alpha_i (\partial_\nu e^i_\mu - \partial_\mu e^i_\nu). \]  

\[ \text{mohsenisad@ut.ac.ir} \]

\[ ^1\text{We use units} \ h = c = G = 1 \text{ throughout the paper.} \]
Recently, some attempts have been performed to study the present positive acceleration of the universe expansion [3] in the framework of the teleparallel and the modified teleparallel models of gravity [4]. In the general theory of relativity, the universe acceleration may be realized by introducing an exotic scalar field, $\phi$, dubbed as quintessence dark energy [5]. The equation of state parameter (EoS) of the quintessence cannot be less than $-1$, $w_{\phi} \geq -1$, therefore this simple model cannot explain the data which favor an evolving dark energy whose EoS is less than $-1$ in the present epoch [6]. To remedy this problem, one can introduce nonminimal couplings between the scalar field and gravity [7]. In the teleparallel framework, and in the minimal coupling case, the cosmological consequences of the quintessence model are the same as those in the general relativity [8].

In the quintessence model in general relativity, adding a term proportional to $R\phi^2$, where $R$ is the Ricci scalar, to the Lagrangian density is required for renormalizability of the theory [9]. The effects of this nonminimal coupling term, in particle physics and inflationary cosmology such as super-acceleration were explained in [10]. Inspired by this model and to allow the phantom divide line crossing in teleparallel cosmology, a dark energy scalar field which is coupled to the scalar torsion via a term proportional to $T\phi^2$ (instead of the aforementioned term $R\phi^2$), was considered in [8]. Note that, in the study of cosmological models in the teleparallel context, it is customary to consider the scalar torsion as a substitute for the Ricci scalar [2] [4]. In [11], similar results as [8] for more general potentials were numerically derived. The cosmological phase space analysis of the same model in the presence of an interaction between dark sectors was performed in [12], and it was shown that although $w_{\phi}$ can cross the phantom divide line at late time, but the coincidence problem is not alleviated. The similarity of the model to the ELKO (Eigenspinoren des LadungsKonjugationsOperators) spinor dark energy model [13] was also briefly investigated. This similarity may have root in the relation between the torsion and the spinor fields [14]. In [15], in the absence of the scalar field potential, some analytical solutions were proposed, confirming the possible occurrence of the phantom phase.

In this manuscript, we consider a universe which is nearly composed of a scalar field dark energy with a power law potential, and cold dark matter in the framework of the teleparallel model of gravity. Like [8], the scalar field is assumed to be coupled to the scalar torsion via a term proportional to $T\phi^2$ in the Lagrangian density. We investigate the possibility of a transition from quintessence to phantom phase for the universe and try to obtain an analytical solution for the scalar field near the transition time. Based on this solution, required conditions for such a transition are obtained. We also confirm our results via numerical methods.
2 Super acceleration in the teleparallel gravity

The universe is assumed to be nearly filled with a homogeneous scalar field, $\phi$, and cold dark matter. Following [8, 11] the action is taken as

$$S = \int \left( \frac{1}{16\pi} T + \frac{1}{2} \left( \partial_\nu \phi \partial^\nu \phi + \xi T \phi^2 \right) - V(\phi) + \mathcal{L}_m \right) \det(e^i_\mu) d^4x,$$

where $\mathcal{L}_m$ is the cold dark matter energy density. The scalar field is non-minimally coupled to the scalar torsion by the term $\xi T \phi^2$, where $\xi$ is a real number.

By taking the dynamical vierbeins as

$$e^i_\mu = \text{diag}(1, a(t), a(t), a(t)),$$ (5)

where $a(t)$ is the scale factor, we obtain the metric of the flat Friedmann-Lemaître-Robertson-Walker (FLRW) space-time

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2).$$ (6)

We have chosen the metric signature $(1, -1, -1, -1)$. From (2), the torsion scalar is derived in terms of the Hubble parameter, $H = \frac{\dot{a}(t)}{a(t)}$, as $T = -6H^2$. By variation of the action (4) with respect to the vierbeins one obtains the Friedmann equations:

$$H^2 = \frac{8\pi}{3} \left( \rho_\phi + \rho_m \right),$$ (7)

and

$$\dot{H} = -4\pi (\rho_\phi + P_\phi + \rho_m).$$ (8)

$\rho_m$ is the cold dark matter energy density and $\rho_\phi$ and $P_\phi$ are the effective energy density and pressure of the scalar field respectively, given by

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) - 3\xi H^2 \phi^2,$$

$$P_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) + 4\xi H \dot{\phi} \phi + \xi \left( 3H^2 + 2\dot{H} \right) \phi^2.$$ (9)

Using (9), we can rewrite (7) and (8) as

$$H^2 = \frac{8\pi}{3} \left( \frac{\dot{\phi}^2 + V(\phi) + \rho_m}{1 + 8\pi \xi \phi^2} \right),$$ (10)

and

$$\dot{H} = -4\pi \frac{\dot{\phi}^2 + 4\xi H \dot{\phi} \phi + \rho_m}{1 + 8\pi \xi \phi^2}.$$ (11)

The scalar field equation

$$\ddot{\phi} + 3H \dot{\phi} + 6\xi H^2 \phi + V'(\phi) = 0,$$ (12)
may be derived from the continuity equation

\[ \dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0. \]  

(13)

The matter energy density also satisfies its own continuity equation

\[ \dot{\rho}_m + 3H\rho_m = 0. \]  

(14)

Note that using the continuity equations and one of the Friedmann equations, one can derive the other Friedmann equation.

In the following we assume that the Hubble parameter is a differentiable function of time in the epoch where the possible transition from quintessence to phantom phase occurs and try to obtain consistent analytic solutions to the scalar field and the Friedmann equations realizing such a transition. The Taylor expansion of the Hubble parameter about the transition time which is taken to be \( t = 0 \), is given by \([16]\)

\[ H = h_0 + h_1 t^k + \mathcal{O}(t^{k+1}), \quad k \geq 2, \quad h_1 > 0. \]  

(15)

\( h_0 \) is the value of the Hubble parameter at the transition time, \( k \) is the order of the first non zero derivative of \( H \) at \( t = 0 \), and

\[ h_1 = \frac{1}{k!} \left. \frac{d^k H}{dt^k} \right|_{t=0} > 0. \]  

(16)

If one of the Friedmann equations, and also the continuity equations are satisfied by (15), then the model is capable of describing the phantom divide line crossing. This can be simply seen from \( w = -1 - \frac{2}{3} \frac{\dot{H}}{H^2} \), where \( w \) is the EoS of the universe. As \( w \) is given by

\[ w = \frac{P_\phi}{\rho_m + \rho_\phi} = \left( \frac{\rho_\phi}{\rho_m + \rho_\phi} \right) w_\phi, \quad w \leq -1 \]

implies \( w_\phi \leq -1 \), although the reverse is not true.

In our study, we assume that

\[ \ddot{\phi} \ll 3H\dot{\phi}, \]  

(17)

which has been vastly employed in the literature as one of the slow roll conditions. With this assumption, (12) reduces to

\[ \dot{\phi} = -\frac{6\xi H^2 \dot{\phi} + V'(\phi)}{3H}. \]  

(18)

By substituting (18) into (11) we get

\[ \dot{H} = -\frac{4\pi}{1 + 8\pi\xi\dot{\phi}^2} \left( -4\xi^2 H^2 \dot{\phi}^2 + \rho_m + \frac{V'(\phi)}{9H^2} \right). \]  

(19)

In minimal models (i.e. \( \xi = 0 \)) the universe is always in the quintessence phase. For \( \xi \neq 0 \) but in the absence of the potential and matter, \( \dot{H} \) is still
negative (this can be seen from (19) and (11)). So for the transition, besides \( \xi \neq 0 \), we also need to have the presence of the matter and the potential.

To go further we choose the quadratic potential

\[
V(\phi) = \frac{1}{2} m^2 \phi^2,
\]

where \( m \) is the mass of the scalar field. In terms of dimensionless parameters

\[
\tau = h_0 t, \quad \tilde{h}_1 = \frac{h_1}{h_0}, \quad \tilde{m} = \frac{m}{h_0},
\]

the solution of the field equation (12), in the limit (17), is

\[
\phi(\tau) = \phi(0) \exp \left[ \frac{-\tau}{3k} \left( 12k\xi - 6\xi \Phi \left( -\tau^k \tilde{h}_1, 1, \frac{1}{k} \right) + \tilde{m}^2 \Phi \left( -\tau^k \tilde{h}_1, 1, \frac{1}{k} \right) \right) \right],
\]

where the Lerchphi function, \( \Phi \), has the series representation

\[
\Phi(z, s, \alpha) = \sum_{n=0}^{\infty} \frac{z^n}{(n + \alpha)^s}.
\]

To find conditions of validity of our approximation (17), by inserting (21) back into (17), and after some computations we arrive at

\[
|6\xi + \tilde{m}^2| \ll 1,
\]

leading to \( m^2 = \frac{d^2V(\phi)}{d\phi^2} \ll h_0^2 \), and \(|6\xi| \ll 1\).

By substituting (15) into (14), the cold matter energy density is obtained as

\[
\tilde{\rho}_m = \tilde{\rho}_m(0) \exp \left( -3 \left( \tau + \frac{\tilde{h}_1}{k+1} \tau^{k+1} \right) \right),
\]

where as before \( \tilde{\rho}_m = \frac{\rho_m}{h_0^2} \), and \( \tilde{\rho}_m(0) = \tilde{\rho}_m(t = 0) \). We can now consider the equation (11), and examine the validity of the solution (15). Inserting (21), (24), and (15) into (11) gives

\[
kh_1 \tau^{k+1} + O(\tau^k) = H_1 + H_2 \tau + O(\tau^2)
\]

where

\[
H_1 = \frac{4\pi}{9} \frac{36\xi^2 \phi^2(0) - 9\rho_m(0) - \tilde{m}^4 \phi^2(0)}{1 + 8\pi \xi \phi^2(0)},
\]

and

\[
H_2 = \frac{-8\pi \phi^2(0)}{27(1 + 8\pi \xi \phi^2(0))} \left[ -\tilde{m}^6 + 216\xi^3 + 72\pi \xi \tilde{m}^2 \tilde{\rho}_m(0) \right] + 36\xi^2 \tilde{m}^2 - 6\xi \tilde{m}^4 + 432\pi \xi^2 \tilde{\rho}_m(0) - 324\pi \xi \tilde{\rho}_m(0) - 40.5\tilde{\rho}_m(0).
\]
The equation (25) requires \( k = 2, \) \( H_1 = 0, \) and \( \tilde{h}_1 = \frac{H_2}{2}. \) So at transition time the cold dark matter energy density must be

\[
\tilde{\rho}_m(0) = \left( 4\xi^2 - \frac{\tilde{m}^4}{9} \right) \phi^2(0).
\]

(28)

Positivity of energy density yields

\[
|6\xi| > \tilde{m}^2,
\]

(29)

which implies that for \( \xi = 0 \) no transition occurs as expected. By applying the approximation (23) in (27) and by considering \( H_1 = 0, \) after some computations, \( \tilde{h}_1 \) becomes

\[
\tilde{h}_1 \simeq \frac{4\pi}{3} \frac{\phi^2(0)}{1 + 8\pi\xi\phi^2(0)} (36\xi^2 - \tilde{m}^4)
\]

\[
= \frac{12\pi\tilde{\rho}_m(0)}{1 + 8\pi\xi\phi^2(0)},
\]

(30)

which is positive provided that

\[
1 + 8\pi\xi\phi^2(0) > 0.
\]

(31)

by expressing the transition rate of the Hubble parameter in terms of \( \tilde{\rho}_m(0), \) reveals the key rôle of matter density in the transition.

Collecting all together, we can conclude that the Friedmann equations have the solution

\[
H = h_0(1 + \frac{12\pi\tilde{\rho}_m(0)}{1 + 8\pi\xi\phi^2(0)} t^2) + \mathcal{O}(t^3)
\]

(32)

near \( t = 0, \) allowing a transition from quintessence to phantom phase at \( t = 0, \) provided that (23) and (29) and (31) hold. To get an estimation about the energy density of the scalar field (9), we use (21) to obtain

\[
\frac{d\phi}{dt}(0) = -\frac{1}{3} (6\xi + \tilde{m}^2) \phi(0).
\]

(33)

This equation together with (23), give

\[
\rho_\phi(0) \simeq \frac{1}{2} (\tilde{m}^2 - 6\xi) \phi^2(0)h_0^2.
\]

(34)

Note that for \( \xi < 0, \) (29) results in \( \rho_\phi > 0. \) In contrast to minimal model, i.e. when \( \xi = 0, \) (34) does not imply that the main part of the energy density is coming from the scalar field potential. From (28), we have also

\[
\rho_m(0) \simeq \left( 4\xi^2 - \frac{\tilde{m}^4}{9} \right) \phi^2(0)h_0^2.
\]

(35)
Therefore from (23) we conclude that \( \rho_m(0) \ll \rho_\phi(0) \) and the coincidence problem is note alleviated in our non-interacting model.

We can confirm our results by using numerical method. Let us take (10), (12), and (14) as our independent equations. Inspired by the aforementioned discussions, we choose the initial conditions as \( \{ \phi(0) = 18.47, \rho_m(0) = 1.33 \times 10^{-5}, \frac{d\phi}{d\tau}(0) = 3.1 \times 10^{-3}, \xi = -10^{-4}, \tilde{m} = 10^{-2} \} \), and depict \( \frac{H^2}{H_0^2} \) numerically in terms of dimensionless time \( \tau \) in fig. (1) using Maple13 plots package.

\[ \text{Figure 1: } \frac{H^2}{H_0^2} \text{ depicted in terms of dimensionless time } \tau \text{ with initial conditions and parameters } \{ \phi(0) = 18.47, \rho_m(0) = 1.33 \times 10^{-5}, \frac{d\phi}{d\tau}(0) = 3.1 \times 10^{-3}, \xi = -10^{-4}, \tilde{m} = 10^{-2} \}. \]

This figure shows that \( H \) has a minimum at \( \tau = 0 \), where the transition from quintessence to phantom phase occurs. In the same way in fig (2), \( \frac{dH}{d\tau} \) is depicted with the same initial conditions taken in the previous figure.

\[ \text{Figure 2: } \frac{dH}{d\tau} \text{ depicted in terms of dimensionless time } \tau \text{ with initial conditions and parameters } \{ \phi(0) = 18.47, \rho_m(0) = 1.33 \times 10^{-5}, \frac{d\phi}{d\tau}(0) = 3.1 \times 10^{-3}, \xi = -10^{-4}, \tilde{m} = 10^{-2} \}. \]

When one generalizes the potential to embrace higher powers of the
scalar field, mathematical computations are not straightforward and obtaining scalar field compact solutions like (21), even if possible, is very complicated. But, in principle, one can use numerical analysis to see whether the phantom divide can be crossed in a specific model by specified parameters. As an example, for the cubic potential

\[ V(\phi) = \frac{\lambda}{3}\phi^3, \]  

and for the parameters \( \{\xi = -10^{-2}, \bar{\lambda} = \frac{\lambda}{h_0} = 10^{-5}\} \) and initial conditions \( \{\phi(0) = -1.97, \dot{\rho}_m(0) = 1.6 \times 10^{-3}, \frac{d\phi}{d\tau}(0) = -3.95 \times 10^{-2}\} \), the behaviors of \( H \) and \( \frac{dH}{d\tau} \) are depicted in fig.3 and fig.4 respectively, using Maple13 plots(odeplots) package, illustrating the occurrence of the phantom divide line crossing at \( \tau = 0 \).

![Figure 3: \( \frac{\dot{H}}{h_0^2} \) depicted in terms of dimensionless time \( \tau \) with initial conditions and parameters \( \{\phi(0) = -1.97, \dot{\rho}_m(0) = 1.6 \times 10^{-3}, \frac{d\phi}{d\tau}(0) = -3.95 \times 10^{-2}\}, \{\xi = -10^{-2}, \lambda = 10^{-5}\} \), for the cubic potential.](image)
Figure 4: \( \frac{dH}{d\tau} \) depicted in terms of dimensionless time \( \tau \) with initial conditions and parameters \( \{\phi(0) = -1.97, \tilde{\rho}_m(0) = 1.6 \times 10^{-3}, \frac{d\phi}{d\tau}(0) = -3.95 \times 10^{-2}\} \), \( \{\xi = -10^{-2}, \tilde{\lambda} = 10^{-5}\} \), for the cubic potential.

3 Conclusion

After a brief introduction to the teleparallel cosmology, we considered a spatially flat FLRW space-time filled nearly with cold dark matter and a single scalar field with quadratic potential and nonminimally coupled to gravity. Focusing on the phantom divide line crossing, we find an analytical solution for the scalar field. Based on this solution, conditions required for such a transition in terms of the parameters of the model, i.e. the mass of the scalar field and the coupling coefficient of the scalar field to gravity were obtained. We obtained the Hubble parameter and the transition rate and showed that the transition is not allowed in the absence of matter. It was explained that, although our model is capable of describing the phantom divide line crossing, but the coincidence problem is not alleviated in this context. Finally we confirmed our results via numerical methods and showed numerically that the same phase transition may occur for higher order power law potentials.

References

[1] A. Unzicker and T. Case, arXiv:physics/0503046 [physics.hist-ph]; K. Hayashi and T. Nakano, Prog. Theor. Phys. 38, 491 (1967); C. Pellegrini and J. Plebanski, K. Dan. Vidensk. Selsk. Mat. Fys. Skr. 2, No. 2 (1962); G. R. Bengochea and R. Ferraro, Phys. Rev. D 79, 124019, (2009).

[2] E. V. Linder, Phys. Rev. D 81, 127301 (2010).

[3] A. G. Riess et al., Astron. J. 116, 1009 (1998); S. Perlmutter et al., Nature (London) 391, 51 (1998); P. M. Garnavich et al., Astrophys. J.
509, 74 (1998); M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006); S. Tsujikawa, arXiv:1004.1493; M. Li, X. D. Li, S. Wang, and Y. Wang, Commun. Theor. Phys. 56, 525 (2011).

[4] K. Bamba, R. Myrzakulov, S. Nojiri, and S. D. Odintsov, Phys. Rev. D 85, 104036 (2012); B. Li, T. P. Sotiriou, and J. D. Barrow, Phys. Rev. D 83, 104017 (2011), arXiv:1103.2786 [astro-ph.CO]; K. Bamba, M. Jamil, D. Momeni, and R. Myrzakulov, arXiv:1202.6114 [physics.gen-ph]; H. M. Sadjadi, Phys. Lett. B 718, 270 (2012), arXiv:1210.0037v2; K. Bamba, J. de Haro, S. D. Odintsov, arXiv:1211.2968v2 [gr-qc]; A. V. Astashenok, E. Elizalde, J. de Haro, S. D. Odintsov, and A. V. Yurov, arXiv:1301.6344v1 [gr-qc]; Z. Haghani, T. Harko, H. R. Sepangi, and S. Shahidi, arXiv:1202.1879 [gr-qc]; A. Banijamali, and B. Fazlpour, arXiv:1206.3580 [physics.gen-ph]; K. Karami and A. Abdolmaleki, JCAP 04 (2012) 007; V. F. Cardone, N. Radicella, and S. Camera, arXiv:1201.5294 [astro-ph.CO].

[5] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998); P. J. E. Peebles and A. Vilenkin, Phys. Rev. D 59, 063505 (1999); V. Faraoni and C. S. Protheroe, arXiv:1209.3726 [gr-qc].

[6] B. Feng, X. L. Wang, and X. M. Zhang, Phys. Lett. B 607, 35 (2005); E. Komatsu et al., Astrophys. J. Suppl. 192, 18 (2011), arXiv:1001.4538 [astro-ph.CO].

[7] V. Faraoni, Phys. Rev. D 62, 023504 (2000), arXiv:gr-qc/0002091; K. Nozari, and S. D. Sadatian, Eur. Phys. J. C 58, 499 (2008), arXiv:0809.4741 [gr-qc]; K. Nozari and S. Shafizadeh, arXiv:1006.1027 [gr-qc]; S. Sushkov, Phys. Rev. D 85, 123520 (2012), arXiv:1204.6372 [gr-qc]; H. M. Sadjadi and P. Goodarzi, arXiv:1203.1580 [gr-qc]; H. M. Sadjadi, Phys. Rev. D 83, 107301 (2011), arXiv:1012.5719 [gr-qc]; H. Farajollahi and A. Salehi, arXiv:1207.1642 [gr-qc]; A. D. Felice and S. Tsujikawa, Phys. Rev. D 84, 124029 (2011); M. Jamil, D. Momeni, and R. Myrzakulov, Eur. Phys. J. C 72, 2075 (2012), arXiv:1208.0025 [gr-qc].

[8] C. Q. Geng, C. C. Lee, E. N. Saridakis, and Y. P. Wu, Phys. Lett. B 704, 384 (2011) 384, arXiv:1109.1092 [hep-th].

[9] D. Freedman and E. Weinberg, Ann. Phys. (NY) 87, 354 (1974); D. Freedman, I. Muzinich, and E. Weinberg, Ann. Phys. (N.Y.) 87, 95 (1974); C. G. Callan, S. Coleman, and R. Jackiw, Ann. Phys. (NY) 59, 42 (1970).

[10] V. Sahni and S. Habib, Phys. Rev. Lett. 81, 1766 (1998); V. Faraoni, Phys. Rev. D 62, 023504 (2000); E. Elizalde, S. Nojiri, and S. Odintsov, Phys. Rev. D 70, 043539 (2004); P. Wang, P. Wu, and H. Yu, Eur. Phys. J. C 72, 2245 (2012), arXiv:1301.5832 [gr-qc].
[11] C. Xu, E. N. Saridakis, and G. Leon, arXiv:1202.3781 [gr-qc]; C. Q. Geng, C. C. Lee, and E. N. Saridakis, arXiv:1110.0913 [astro-ph.CO].

[12] H. Wei, Phys. Lett. B 712, 430 (2012), arXiv:1109.6107 [gr-qc].

[13] D. V. Ahluwalia, D. Grumiller, JCAP 07, 012 (2005); S. Kouwn, J. Lee, T. H. Lee, and P. Oh, arXiv:1211.2981 [gr-qc]; H. M. Sadjadi, Gen. Rel. Grav. 44, 2329 (2012), arXiv:1109.1961 [gr-qc]; A. Basak, J. R. Bhatt, S. Shankaranarayanan, and K. V. P. Varma, arXiv:1212.3445 [astro-ph.CO].

[14] L. Fabbri, S. Vignolo, and C. Stornaiolo, Annalen Phys. 524, 826 (2012), arXiv:1201.0286 [gr-qc]; L. Fabbri, S. Vignolo, Int. J. Theor. Phys. 51, 3186 (2012), arXiv:1201.5498 [gr-qc].

[15] J. A. Gu, C. C. Lee, and C. Q. Geng, Phys. Lett. B 718, 722 (2013).

[16] H. M. Sadjadi and M. Alimohammad, Phys. Rev. D 74, 043506 (2006), gr-qc/0605143. H. M. Sadjadi and M. Honardoost, Phys. Lett. B 647, 231 (2007), arXiv:gr-qc/0609076