We investigate the implications of a seesaw type mass matrix, i.e., $M_f \simeq m_L M_F^{-1} m_R$, for quarks and leptons $f$ under the assumption that the matrices $m_L$ and $m_R$ are common to all flavors (up-/down- and quark-/lepton- sectors) and the matrices $M_F$ characterizing the heavy fermion sectors have the form $[(\text{unit matrix}) + b_f (\text{a democratic matrix})]$ where $b_f$ is a flavor parameter. We find that by adjusting the complex parameter $b_f$, the model can provide that $m_t \gg m_b$ while at the same time keeping $m_u \sim m_d$ without assuming any parameter with hierarchically different values between $M_U$ and $M_D$. The model with three adjustable parameters under the “maximal” top quark mass enhancement can give reasonable values of five quark mass ratios and four KM matrix parameters.
1. Introduction

One of the most mysterious facts in the quark mass spectrum is why top quark mass $m_t$ is so much larger than the bottom quark mass $m_b$, while $u$ quark mass $m_u$ is of the order of $d$ quark mass $m_d$. In the usual discussion of fermion masses, this drastic generation dependence of the mass splitting between members of each isomultiplet of quarks is attributed to an arbitrary hierarchy among the input parameters which is not completely satisfactory. It is therefore important to seek alternative ways to understand this feature. In this paper we argue that within the see-saw\[1\] type mass formula for quark masses discussed in the context of gauge models \[2\], a very simple explanation of this feature is obtained by imposing a specific universality ansatz for various flavor matrices. We then find that a slight generalization of this ansatz provides an extremely good fit to all the quark mass ratios and mixings.

Our starting point is the following specific see-saw type ansatz proposed by one of the authors \[3\] for quark and lepton mass matrices:

$$M_f = M_e^{1/2}O_fM_e^{1/2},$$

(1.1)

where $M_e^{1/2} = \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$. Here, for the up-quark mass matrix $M_u$, the matrix $O_f$ ($f = u$) is given by

$$O_f = 1 + 3a_fX,$$

(1.2)

where $1$ is a unit matrix and $X$ is a democratic-type matrix \[4\]

$$X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

(1.3)

which satisfies the relation $X^2 = X$. The up-quark mass matrix can then successfully give a quark mass ratio \[3,5\]

$$\frac{m_u}{m_c} \simeq \frac{3m_e}{4m_\mu},$$

(1.4)

for a large value of the parameter $a_u$. The value of $a_u$ is adjusted from the mass ratio $m_c/m_t$. 

2
Stimulated by the phenomenological success of the mass matrix form (1.1) – (1.3), the authors [6] have applied the mass matrix form to down-quark mass matrix, by considering that the parameter $a_d$ is complex. They have found that the value of a complex parameter $a_d$ which fits the mass ratios $m_d/m_s$ and $m_s/m_b$ gives reasonable values of not only Kobayashi-Maskawa (KM) [7] matrix elements $V_{ij}$ ($i,j$ denote family indices) but also up-to-down-quark mass ratios $m_u/m_d$, $m_c/m_s$ and $m_t/m_b$.

Suggested from the form (1.1), it may be expected that such phenomenological success will also be obtained in the context of a seesaw-type mass matrix

$$M_f \simeq m_L M^{-1}_F m_R,$$

(1.5)

with $m_L \propto m_R \propto M^{1/2}$ and $M_F \propto O_f^{-1}$. Here, the expression (1.5) is derived from the $6 \times 6$ mass matrix for fermions $(f,F)$

$$(f_L \ T_L) \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} \begin{pmatrix} f_R \\ F_R \end{pmatrix} + h.c.,$$

(1.6)

for the case of $O(M_F) \gg O(m_R),O(m_L)$, where $f = (f_1,f_2,f_3)$ are three family quarks and leptons, and $F = (F_1,F_2,F_3)$ are vector-like heavy fermions corresponding to $f$.

The re-interpretation of the model (1.1) based on the seesaw model (1.5) seems to be plausible because of the following reasons. The inverse of the matrix $O_f$ with a simple form $[(\text{unit matrix})+(\text{democratic-type matrix})]$ has also a simple form $[(\text{unit matrix})+(\text{democratic-type matrix})]$, i.e.,

$$O_F \equiv O_f^{-1} = 1 + 3b_f X,$$

(1.7)

where the complex coefficients $a_f$ and $b_f$ are related by

$$a_f = -b_f/(1 + 3b_f).$$

(1.8)

In the mass matrix model (1.1), we need hierarchically different values [6] of the parameters $a_f$, i.e., $a_u = 28.65$ and $|a_d| = 0.4682$, in order to provide reasonable quark masses and KM mixings, while, as seen from (1.8), the values $|a_u| \gg 1$ and $a_d \simeq -1/2$ correspond to $b_u \simeq -1/3$ and $b_d \simeq -1$ in the inverse matrix (1.7), respectively. In the present paper, we are interested in such a model that $M_u$ and
M_d are “almost” symmetric, i.e., they have almost the same structure and they take parameter values which are not so hierarchically different between M_u and M_d. The parameter ratio \( |a_u/a_d| \simeq 60 \) in the model (1.1) can be reduced to the ratio \( |b_u/b_d| \simeq 3 \) in (1.7).

However, when we consider a model (1.6) (not (1.5)) with \( M_F \propto O_F \), one problem arises: Recently the CDF collaboration [8] has reported \( m_t = 174 \pm 10^{+13}_{-12} \) GeV as top quark mass from \( pp \) collision data at \( \sqrt{s} = 1.8 \) TeV. On the other hand, the universal mass matrix \( m_L \) which breaks the SU(2)_L gauge symmetry should be of the order \( \Lambda_W = (\sqrt{2} G_F)^{-1/2}/\sqrt{2} = 174 \) GeV~\( m_t \), or less. Then, the approximate expression (1.4) for up-quarks is not valid any longer, because if (1.5) is valid, \( O(m_L) \sim m_t \) means \( M_U^{-1} m_R \sim O(1) \), so that it does not satisfy the condition \( O(M_F) \gg O(m_R) \) for the validity of the seesaw expression (1.5). This is also understood from the fact that the limit \( |a_u| \to \infty \) means the limit \( b_u \to -1/3 \) and the determinant of \( M_U \) becomes zero in the limit, so that the expansion of \( M_f \) in \( M_F^{-1} \) can not be a good approximation.

In this paper, we do not use the approximate relation (1.5), but calculate directly the \( 6 \times 6 \) mass matrix (1.6). In Sect. 2, we will give the outline of our mass matrix model. In Sect. 3, we will give an expression of \( M_f \) which is valid in the limit of \( b_f \to -1/3 \), i.e., \( \det M_F = 0 \), instead of the well-known seesaw expression (1.5), and discuss the up-quark mass ratios which are expressed in terms of lepton mass ratios and our adjustable parameters (see the next section). In Sec. 4, we discuss the fermion mass spectra by numerically evaluating the \( 6 \times 6 \) mass matrix. In Sect. 5, KM matrix parameters are discussed numerically. In the present model, under some basic assumptions (see Sects. 2 and 5), the parameter fitting for quark mass ratios and KM matrix parameters (5+4=9 observables) is done by three adjustable parameters \( k/K, b_d \) and \( \beta_d \) (see the next section for the definitions). We will find that the value of \( m_t \) takes the largest enhancement at \( b_u = -1/3 \), while the relations \( m_u \sim m_d \) and (1.4) are kept. We can obtain reasonable values of quark mass ratios (not only \( m_u/m_c, m_c/m_t, m_d/m_s \) and \( m_s/m_b \), but also \( m_u/m_d, m_c/m_s \) and \( m_t/m_b \)) and the KM matrix parameters, by taking \( b_u = -1/3 \) and \( b_d \simeq -1 \).

2. Outline of the model

In addition to the conventional quarks and leptons \( f_i \), where \( f \) is the flavor index \( (f = u, d, \nu \text{ and } e \text{ denote up-quarks, down-quarks, neutrinos and charged leptons}), \) and \( i \) is the family index \( (i = 1, 2, 3) \), We consider vector-like fermions \( F_i \)
correspondingly to \( f_i \). These fermions belong to \( f_L = (2, 1), f_R = (1, 2), F_L = (1, 1) \) and \( F_R = (1, 1) \) of \( SU(2)_L \times SU(2)_R \). A “would-be” seesaw mass matrix for the fermions \((f, F)\) is given by (1.6). Gauge models which realize the mass matrix form (1.6) have been proposed by many authors [2]. Although the interest of most authors is how to embed the model (1.6) into a unification model in the framework of gauge theory, our interest is how to give realistic quark mass spectra and family mixing from the phenomenological point of view.

Suggested by the phenomenological success of the model (1.1), we assume the following mass matrix [9]

\[
M = \begin{pmatrix}
0 & m_L \\
 m_R & M_F
\end{pmatrix} = m_0 \begin{pmatrix}
0 & Z \\
 kZ & KO_F
\end{pmatrix},
\]

where the matrices \( m_L \) and \( m_R \) (i.e., \( m_0, h \) and the matrix \( Z \)) are common to all of \( f = u, d, \nu, e \), and only \( M_F \) depends on flavors \( f \) through the complex parameter \( b_f \). Hereafter, we denote the complex parameter \( b_f \) in (1.7) as \( b_f e^{i\beta_f} \) (\( b_f \) is real and \( |\beta_f| \leq \pi/2 \)) in (2.2) below. The vector-like fermions \( F \) acquire large masses \( M_F \) at an energy scale \( \mu = m_0 K \). We consider that the energy scale \( m_0 K \) is not as large as the ground unification scale, but an intermediate energy scale. At the present stage, the origin of the democratic form

\[
O_F = 1 + 3b_f e^{i\beta_f} X = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} + b_f e^{i\beta_f} \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix},
\]

is an open question. We may attribute the origin of the democratic term \( X \) to a permutation symmetry \( S_3 \) [10], a BCS-like mechanism [11], a composite model based on the analogy of hadronic \( \pi^0-\eta-\eta' \) mixing [12], and so on. In the present phenomenological analysis, we do not discuss its origin moreover.

The present model is left-right symmetric except for \( k \neq 1 \). At an energy scale \( \mu = m_0 k \) (\( \mu = m_0 \)) at which \( SU(2)_R \) (\( SU(2)_L \)) is broken, the mass term \( F_L m_R f_R (\bar{f}_L m_L F_R) \) appears, so that we consider \( k \sim m(W_R)/m(W_L) \). The relation \( m_L = m_R/k = m_0 Z \) is merely a phenomenological working hypothesis. The matrix \( Z \) takes a diagonal form

\[
Z = \text{diag}(z_1, z_2, z_3),
\]

(2.3)
with the normalization condition $z_1^2 + z_2^2 + z_3^2 = 1$. (In other words, in the family basis in which $Z$ is diagonal, we have assumed that the matrix $O_F$ is given by (2.2)). For the charged leptons, since $m_\tau \ll m_0 \sim m_W$, it is clear that the seesaw expression $M_e = m_0(k/K)ZO_F^{-1}Z$ is well satisfied, so that we can fix the parameter $z_i$ as

$$
\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau + m_\mu + m_\tau}}.
$$

Here, we have assumed $b_e = 0$ according to the phenomenological success [3] of the model (1.1). In the present paper, we do not discuss why $z_i$ are given by the relation (2.4), because the purpose of the present paper is to study quark mass ratios and KM matrix parameters phenomenologically, so that charged lepton masses are regarded as inputs in the numerical estimates. Since the evolution effects of fermion mass ratios (not the absolute values) from $\mu = m_0K$ to $\mu = m_0$ are, at most, several percent, for simplicity, we use the values of $z_i$ which are fixed by using the formula (2.4) with the observed charged lepton masses [13].

For the case of $K \gg k \gg 1$, the quark mass ratios and the KM matrix parameters (nine observables) are described by five real parameters $k/K$ (not $k$ and $K$ separately), $b_u$, $\beta_u$, $b_d$ and $\beta_d$. As we will discuss in Sections 3 and 4, the maximal top-quark-mass enhancement occurs at $b_u = -1/3$ and $\beta_u = 0$. We will put an ansatz of “maximal top-quark-mass enhancement”, so that we will fix the parameters $b_u$ and $\beta_u$ to $b_u = -1/3$ and $\beta_u = 0$. The numerical fitting for the nine observables is then tried by adjusting only three parameters $k/K$, $b_d$ and $\beta_d$. However, as will be discussed in Sect. 5, a straightforward application of the mass-matrix model (2.1) cannot lead to reasonable predictions of the KM matrix parameters. We will therefore introduce a sign factor by replacing $m_L = m_0Z$ in (2.1) by $m_L^f = m_0P_fZ$, where $P_u = \text{diag}(1,1,1)$, while $P_d = \text{diag}(1,1,-1)$. The adjustable parameters are still three, i.e., $k/K$, $b_d$ and $\beta_d$. The phase matrices $P_f$ do not affect the discussion of the mass spectrum. For a time being in Sects. 3 and 4, we will neglect the phase matrices $P_f$.

### 3. Expression of $M_f$ in the case of $b_f \simeq -1/3$

One of the purposes in the present paper is to obtain a reliable expression of $M_f$ in the case of $b_f \simeq -1/3$, because the case leads to $\det M_F \simeq 0$, so that the seesaw expression (1.5) which is obtained by expanding it in $M_F^{-1}$ is not valid any
As shown in Appendix, in general, the transformation of the $6 \times 6$ mass matrix $M$ into

$$U_L M U_R^\dagger \equiv U_L \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} U_R^\dagger = M' \equiv \begin{pmatrix} M'_{11} & 0 \\ 0 & M'_{22} \end{pmatrix},$$

is done by the following two $6 \times 6$ unitary matrices,

$$U_L = \begin{pmatrix} (1 + \rho_L \rho_L^\dagger)^{-1/2} & (1 + \rho_L \rho_L^\dagger)^{-1/2} \rho_L \\ -(1 + \rho_L^\dagger \rho_L)^{-1/2} \rho_L^\dagger & (1 + \rho_L^\dagger \rho_L)^{-1/2} \end{pmatrix}$$

(3.1)

and $U_R$ with $L \leftrightarrow R$ in (3.2). The so-called seesaw expression $M'_{11} \equiv M_f \simeq m_L \mathcal{M}_F^{-1} m_R$ is obtained by expanding $M'_{11}$ in $\mathcal{M}_F^{-1}$. Since our mass matrix (2.1) is not Hermitian, for evaluating the KM matrix (family mixing of left-handed fermions), it is useful to define the $3 \times 3$ Hermitian matrix $H_f$:

$$H_f \equiv M'_{11} M'^{\dagger}_{11} = (1 + \rho_L \rho_L^\dagger)^{-1/2} \mathcal{H}_f (1 + \rho_L \rho_L^\dagger)^{+1/2} .$$

(3.3)

As seen in (A.22), (A.24) and (A.27), the matrix $\mathcal{H}_f$ is given by

$$\mathcal{H}_f \equiv \rho_L m_R \rho_L m_L^\dagger = (m_L + \rho_L M_F) m_L^\dagger ,$$

(3.4)

and it satisfies the following equation:

$$\mathcal{H}_f^2 m_L^{\dagger -1} - \mathcal{H}_f m_L^{\dagger -1} \left( M_F \mathcal{M}_F + m_L^\dagger m_L + \mathcal{M}_F^{-1} m_R m_R^\dagger M_F \right) + m_L \mathcal{M}_F^{-1} m_R m_R^\dagger M_F = 0 .$$

(3.5)

Our interest is in the expression of $\mathcal{H}_f$ in the case of $\det M_F \simeq 0$. However, since it is hard to obtain the general formulation in such the case, we confine ourselves to investigating the special form (2.1) with (2.2).

For the investigation of the case of $b_u \simeq -1/3$, it is convenient to define the parameter

$$3 \varepsilon \equiv \Delta b = b + \frac{1}{3} .$$

(3.6)

Then, the matrix $O_F$ is represented by

$$O_F = Y + \varepsilon X ,$$

(3.7)
where

\[ Y = 1 - X , \quad (3.8) \]

and the matrices \( X \) and \( Y \) satisfy the relations \( X^2 = X, \ Y^2 = Y, \) and \( XY = YX = 0 \) from the definitions (1.3) and (3.8), so that the inverse of \( O_F, \) (3.7), is given by

\[ O_F^{-1} = Y + X/\varepsilon . \quad (3.9) \]

For the case of \( (k/K)^2 \ll \varepsilon^2 \ll 1, \) from the equation (3.5), we obtain

\[ \tilde{H}_f \simeq m_0^2 \left( \frac{k}{K} \right)^2 Z \left( Y + \frac{1}{\varepsilon} X \right) Z^2 \left( Y + \frac{1}{\varepsilon} X \right) Z , \quad (3.10) \]

which corresponds to the well-known seesaw expression \( M_f \simeq m_0(k/K)ZO_F^{-1}Z. \)

For a general case, we assume an approximate form

\[ \tilde{H}_u \simeq m_0^2 Z \left( \frac{k}{K} Y + x X \right) Z^2 \left( \frac{k}{K} Y + x X \right) Z , \quad (3.11) \]

from an analogy to the form (3.10). By substituting (3.11) into (3.5), we find

\[ x \simeq \left[ \frac{\varepsilon}{2k/K} + \frac{1}{3} + \left( \frac{\varepsilon}{2k/K} \right)^2 \right]^{-1} . \quad (3.12) \]

For \( \varepsilon^2 \gg (k/K)^2, \) (3.12) reproduces (3.10). For \( \varepsilon^2 \ll (k/K)^2, \) we obtain

\[ \tilde{H}_u \simeq 3m_0^2 Z \left( X + \frac{1}{\sqrt{3} K} Y \right) Z^2 \left( X + \frac{1}{\sqrt{3} K} Y \right) Z . \quad (3.13) \]

This expression (3.13) is the expression which should be used in the case of \( \det M_F \simeq 0 \) as a substitute for the well-known seesaw expression (3.10).

The mass eigenvalues are calculated from \( \text{Tr}H_u = \text{Tr}\tilde{H}_u, \ ((\text{Tr}H_u)^2 - \text{Tr}H_u^2)/2 = ((\text{Tr}\tilde{H}_u)^2 - \text{Tr}\tilde{H}_u^2)/2 \) and \( \det H_u = \det\tilde{H}_u. \) We obtain up-quark masses

\[ m_u \simeq \frac{3}{2} z_1^2 \frac{k}{K} m_0 , \quad m_c \simeq 2z_2^2 \frac{k}{K} m_0 , \quad m_t \simeq \frac{1}{\sqrt{3}} \frac{1}{\sqrt{1 + 27(\Delta b)^2(k/K)^2}} m_0 , \quad (3.14) \]
from (3.11) and (3.12), where \( \varepsilon = \Delta b/3 \) (3.6). We find that the relation (1.4) is also valid in the case \( (\Delta b)^2 \ll (k/K)^2 \ll 1 \), even in the limit of \( b_u = -1/3 \).

4. Numerical study of quark mass ratios

Numerical evaluation of the eigenvalues of the \( 6 \times 6 \) mass matrix (2.1) can easily be done with the help of a computer. Numerical study is helpful for checking analytical calculations based on the formalism of the previous section. In Fig. 1, in order to give an overview of the mass spectrum in our mass matrix model, we illustrate the light fermion mass spectrum \( m_f^i \) \( (i = 1, 2, 3) \) versus the parameter \( b_f e^{i\beta_f} \). Here, we have taken \( k = 10 \) and \( K/k = 50 \) as a trial. (The choices of \( k \) and \( K/k \) are discussed later.) In order to fix the values of the parameters \( z_i \) at \( b_e = 0 \), we have used the observed charged leptons masses [13] as inputs.

The spectrum for the case of \( \beta_f = 0 \) (solid lines) shows the following characteristics:

1. The third fermion mass is sharply enhanced at \( b_f = -1/3 \).
2. Level crossing (mass degeneration) occurs at \( b_f = -1/2 \) and \( b_f = -1 \).

These characteristics become mild when \( \beta_f \) takes a sizable value (dashed lines).

For comparison, we list the observed running quark mass values (in unit of GeV) [14] at \( \mu = \Lambda_W \equiv (\sqrt{2} G_F)^{-1/2} = 174 \) GeV:

\[
\begin{align*}
m_u &= 0.00230 \pm 0.00045, & m_c &= 0.612^{+0.010}_{-0.023}, & m_t &= 166^{+21}_{-26}, \\
m_d &= 0.00406 \pm 0.00045, & m_s &= 0.082 \pm 0.014, & m_b &= 2.874^{+0.012}_{-0.023}.
\end{align*}
\]

In the previous section, we have showed that the up-quark mass ratio \( m_u/m_c \) is given by (1.4) in the limit \( \varepsilon \ll (k/K)^2 \ll 1 \), see (3.14). The relation can be checked by a numerical study. We find that the ratio \( m_c/m_u \) at a fixed \( K/k \) is insensitive to the choice of \( k \), for \( k \geq 10 \). Also, the ratio is insensitive to the parameters \( K/k \) and \( \Delta b_u \) for large \( K/k \); for example, \( m_c/m_u = 260.8, 260.8, 259.2, \) and \( 259.2 \), for \( (K, k, \Delta b_u) = (10^3, 10, 0), (10^5, 10, 0), (10^3, 10, 0.003) \) and \( (10^5, 10, 0.003) \), respectively, while \( m_c/m_u \) \( \exp \) \( = 266^{+70}_{-49} \). Thus, we conclude that the relation (1.4) is valid almost independently of the values of \( k \) and \( K/k \) for the case of \( K \gg k \gg 1 \).

Next, we study the up-quark mass ratio \( m_t/m_c \). We find that the ratio is also insensitive to the value of \( k \) for \( k \geq 10 \). Therefore, we illustrate the behavior of \( m_t/m_c \) versus \( K/k \) for the case of \( k = 10 \) in Fig. 2. It is noticeable that, for \( \Delta b_u \simeq +0.00388 \) and \( \Delta b_u \simeq -0.00362 \), the ratio \( m_t/m_c \) comes near the experimental value.
Further numerical estimates. Of quark mass values, hereafter, we simply adopt the integral solution \( \beta \)

\[ \text{Hereafter, we adopt the ansatz of the "maximal top-quark-mass enhancement",} \]

\[ \text{may be unfavorable from the point of view of the perturbative electroweak theory.} \]

\[ \text{Therefore, a small value of} \]

\[ \text{as we have shown in (3.14). On the other hand, as seen in Fig. 3, the case (c) \( \Delta b_u \simeq \pm 0.004 \) gives reasonable predictions not only for} \]

\[ \text{gives us} \]

\[ \exp 271 \text{and} \]

\[ 50 \text{from the observed ratio of} \]

\[ \text{The ratios} \]

\[ \text{The value} \]

\[ \text{may be large} \]

\[ \text{may be unfavorable from the point of view of the perturbative electroweak theory.} \]

\[ \text{Hereafter, we adopt the ansatz of the \"maximal top-quark-mass enhancement\",} \]

\[ \text{i.e.,} \]

\[ \text{such a large value} \]

\[ \text{be unfavorable from the point of view of the perturbative electroweak theory.} \]

\[ \text{On the other hand, the down-quark masses are given by adjusting two parameters} \]

\[ \text{As seen in Fig. 1, the case of} \]

\[ \text{The ratios} \]

\[ \text{As far as we see in Fig. 4, the cases} \]

\[ \text{Considering the present experimental uncertainty of quark mass values, hereafter, we simply adopt the integral solution} \]

\[ \text{For the case of} \]

\[ \text{down-quark masses are given by} \]

\[ m_d \simeq z_1^2 \dfrac{1}{\beta_d} \dfrac{k}{K} m_0 \]

\[ m_s \simeq z_2^2 z_3^2 \beta_d \dfrac{k}{K} m_0 \]

\[ m_b \simeq \dfrac{1}{2} \dfrac{k}{K} m_0 \]
In the present model, the up-to-down quark mass ratio $m_u/m_d$ is given by

$$\frac{m_u}{m_d} \simeq 3 \frac{m_s}{m_c} \simeq \frac{3}{2} \beta_d ,$$

so that the ratios $m_u/m_d$ and $m_s/m_c$ can be fitted independently of $m_t/m_c$ (i.e., $K/k$) by adjusting the parameter $\beta_d$.

When we take $b_d = -1.0$ and $\beta_d = -18^\circ$ (and $k = 10$ and $K/k = 50$), we can obtain reasonable quark mass values:

$$m_u(\Lambda_W) = 0.00234 \text{ GeV} , \quad m_c(\Lambda_W) = 0.610 \text{ GeV} , \quad m_t(\Lambda_W) = 166 \text{ GeV} ,$$

$$m_d(\Lambda_W) = 0.00475 \text{ GeV} , \quad m_s(\Lambda_W) = 0.0923 \text{ GeV} , \quad m_b(\Lambda_W) = 3.450 \text{ GeV} ,$$

where we have taken $m_0(\Lambda_W) = 288 \text{ GeV}$ to have $m_t(\Lambda_W) = 166 \text{ GeV}$.  

So far, except for (4.4), we have discussed only quark mass ratios and not the absolute values, because the ratios are comparatively insensitive to the evolution from $\mu = m_0 K$ to $\mu = m_0$. The common value $m_0(\Lambda_W) = 288 \text{ GeV}$ does not give the absolute magnitudes of the charged lepton masses, $(k/K)m_0 = m_\tau + m_\mu + m_e$.  

We find

$$\left. \frac{(m_0 k/K)_q}{(m_0 k/K)_\ell} \right|_{\mu = \Lambda_W} = 3.05 ,$$

where $(m_0 k/K)_q(\ell)$ denotes the value of $m_0 k/K$ in the quark (lepton) sector. It is not likely that the factor 3.1 comes only from the evolution from $\mu = m_0 K$ to the present scale $\mu = \Lambda_W$. Since we consider the case where the parameters $m_0$ and $k$ (i.e., $m_L$ and $m_R$) are universal for all flavors $f = u, d, \nu, e$, the discrepancy (4.5) should come from the difference in $K$ between the quark- and lepton-sectors, i.e., $K_q \neq K_\ell$. Although it is possible that the coupling constants of the colored heavy fermions with Higgs bosons which generate the democratic-type matrix (2.2) are smaller than that of the colorless heavy fermions by a factor 1/3, i.e., $K_\ell/K_q = 3$, we do not discuss the origin of $K_\ell/K_q = 3$ in the present paper. In the present model, we practically consider that $m_L$ and $m_R$ are universal for quarks and leptons, while $M_F$ are not so, and $K_u = K_d \equiv K_q \neq K_\nu = K_\ell \equiv K_\ell$. Hereafter, we denote $K_q$ simply as $K$.  

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Similarly, with the same parameter values as in (4.4), the heavy quark masses are given as follows:

\[ m_u^4(\Lambda_W) = 1.66 \text{ TeV} \, , \quad m_u^5(\Lambda_W) = 144 \text{ TeV} \, , \quad m_u^6(\Lambda_W) = 144 \text{ TeV} \, , \]
\[ m_d^4(\Lambda_W) = 144 \text{ TeV} \, , \quad m_d^5(\Lambda_W) = 144 \text{ TeV} \, , \quad m_d^6(\Lambda_W) = 298 \text{ TeV} \, . \tag{4.6} \]

These numerical results are also obtained from the approximate relations for \( b_u = -1/3 \) and \( b_d = -1 \):

\[ m_u^4 \simeq (k/\sqrt{3})m_0 \, , \quad m_u^5 \simeq m_u^6 \simeq K m_0 \, , \tag{4.7} \]
\[ m_d^4 \simeq m_d^5 \simeq K m_0 \, , \quad m_d^6 \simeq 2\sqrt{1 + 3\beta_d^2/4Km_0} \, . \tag{4.8} \]

Note that the fourth up-quark \( u_4 \) becomes considerably lighter than the other heavy quarks, at the cost of the enhancing the top-quark mass. The absolute magnitudes the heavy quark masses in (4.6) should not be taken solidly, because they depend on both \( k \) and \( K \). We have chosen \( K/k = 50 \) in order to fit \( m_t/m_c \), but the choice \( k = 10 \) was only a trial choice, because the predictions for light fermions (quarks and leptons) are insensitive to the value of \( k \). Only constraint on the value \( k \) comes from the relation \( k \sim m(W_R^+) / m(W_L^+) \). The present lower bound of the right-handed weak boson mass \( m(W_R) \) is given in Ref. [15], so that we cannot choose too small value of \( k \). Since \( m_u^4 \) is of the order of \( km_0 \), as seen in (4.7), we can expect to observe the fourth up-quark at the energy scale where the right-handed weak bosons \( W_R \) are observed.

5. KM matrix parameters

In the present model, the parameter fitting for five quark-mass ratios and four KM matrix parameters is done by five parameters, \( k/K \) (not \( k \) and \( K \)), \( b_u, \beta_u, b_d \) and \( \beta_d \). When we adopt the ansatz of “maximal top-quark-mass enhancement”, we have fixed the parameters \( b_u \) and \( \beta_u \) to \( b_u = -1/3 \) and \( \beta_u = 0 \), and the remaining adjustable parameters are \( k/K \), \( b_d \) and \( \beta_d \). We have pointed out that the relation between up-quark mass ratio \( m_u/m_c \) and \( m_c/m_\mu \), (1.4), is satisfied independently of these parameters for the case \( b_u \simeq -1/3 \). The parameter \( K/k \) was fixed to \( K/k = 50 \) from the observed up-quark mass ratio \( m_t/m_c \), see Fig. 2. In the previous section, we have shown that the remaining two parameter \( b_d \) and \( \beta_d \) can be fitted to three observed quark mass ratios \( m_d/m_s \), \( m_s/m_b \) and \( m_u/m_d \) reasonably (see Fig. 4). Then, our final task in the present phenomenological study is to check
whether these parameter values can also give reasonable predictions for the four KM matrix parameters.

The KM matrix $V$ is given by $V = U_u U_d^\dagger$, where $U_q$ ($q = u, d$) are the unitary matrices to diagonalize the light fermion mass matrices $M_f M_f^\dagger$, where $M_f \equiv M'_{11}$ ($f = u, d$) defined by (3.2). Unfortunately, our parameter values $K/k \simeq 50$, $b_d \simeq -1$ and $\beta_d \simeq -18^\circ$ give rise to the KM matrix parameters far away from the observed values [13]. Therefore, we must slightly modify our model.

So far, we have assumed that the matrices $m_L$ and $m_R$ are universal for up- and down-sectors. However, in the present section, let us distinguish the matrix $m_L$ in the up-quark sector, $m^u_L = m_0 Z_u$, from that in down-quark sector, $m^d_L = m_0 Z_d$. We assume that $Z_u$ and $Z_d$ are given by $Z_q = P_q Z$ ($q = u, d$), where $Z$ is given by (2.3) and (2.4), and $P_q$ are phase matrices. (It is not essential whether we also assume a similar modification on $m_R$ or not, because the KM matrix is related only to the family mixing among the left-handed fields.) Such a modification does not change our predictions on the fermion masses in Sects. 3 and 4, while the KM matrix $V$ is changed into the following expression:

$$V = U_u P U_d^\dagger,$$

(5.1)

where $U_q$ ($q = u, d$) are unitary matrices to diagonalize the unchanged matrices $M_f M_f^\dagger$ (i.e., in the case of $P_u = P_d = 1$), and $P = P_u P_d^\dagger$. In general, the phase matrix $P$ can have two independent phase parameters such as $P = \text{diag}(1, e^{i\delta_2}, e^{i\delta_3})$. However, since we do not want more adjustable parameters, we examine a simpler ansatz that the phase matrix $P$ is real, i.e., $\delta_i = 0$ or $\pi$. Thus, we keep three adjustable parameters, $k/K$, $b_d$ and $\beta_d$, at the cost of putting the additional ansatz on $P$.

As a result, we find that only for the case

$$P = \text{diag}(1, 1, -1),$$

(5.2)

we can obtain reasonable values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$. We show $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ versus $\beta_d$ in Fig. 5. The same parameter values as in (4.4), $K/k = 50$, $b_d = -1$ and $\beta_d = -18^\circ$, give reasonable predictions

$$|V_{us}| = 0.220, \quad |V_{cb}| = 0.0598, \quad |V_{ub}| = 0.00330, \quad |V_{td}| = 0.0155,$$

$$J = -3.18 \times 10^{-5},$$

(5.3)
where $J$ is the rephasing invariant $[16] J = \text{Im}(V_{cb} V_{us}^* V_{cs}^* V_{ub}^*)$. Although the origin of the phase inversion $P = \text{diag}(1, 1, -1)$ is not clear and the predicted value of $V_{cb}$ is somewhat large, it is a noticeable feature of the present model that the parameters which were fixed by the observed quark-mass ratios can roughly give reasonable predictions for all the KM matrix parameters.

6. Conclusions

In conclusion, we have demonstrated that the seesaw-type mass matrix (2.1) with $M_F$ given by (2.2) can give top-quark-mass enhancement without assuming any parameters with hierarchically different values between $M_U$ and $M_D$, i.e., with $b_u \simeq -1/3$ and $b_d \simeq -1$. The enhancement $m_t/m_b \gg 1$ comes from the fact that the democratic part $X$ in the inverse matrix $M_F^{-1}$ in (1.2), is enhanced as to $b_f \to -1/3$ because $|a_f| \to \infty$ in the limit as seen in (1.8). On the other hand, the result $m_u \sim m_d$ comes from the feature that the democratic-type mass matrix can give rise to a large mass only to the third family, i.e., the effect of $|a_u| \to \infty$ contributes mainly to $m_t$.

In the present model, the parameter fitting for the five quark mass ratios and the four KM matrix parameters has been done by five parameters $k/K$ (not $k$ and $K$ separately), $b_u$, $\beta_u$, $b_d$ and $\beta_d$. (The parameters $z_i$ were fixed by charged lepton masses.) When we adopt the ansatz of “maximal top-quark-mass enhancement”, the parameters $b_u$ and $\beta_u$ are fixed to $b_u = -1/3$ and $\beta_u = 0$, and the remaining adjustable parameters are $k/K$, $b_d$ and $\beta_d$. The parameter $K/k$ is then fixed by the observed up-quark-mass ratio $m_t/m_c$ to be $K/k = 50$. The remaining two parameters $b_d$ and $\beta_d$ are then free parameters by which four quark mass ratios $m_u/m_c, m_d/m_s, m_s/m_b$ and $m_u/m_d$, and four KM parameters are fitted. As shown in Sects. 4 and 5, by choosing $b_d \simeq -1$ and $\beta_d \simeq -18^\circ$, we have obtained reasonable fitting for the quark-mass ratios, and also for the KM matrix parameters with the ansatz (5.2).

A few remarks are in order.

In the present model, flavor changing neutral currents (FCNC) can, in principle, appear. However, the FCNC due to the SU(2)$_L$ (SU(2)$_R$) doublet Higgs boson exchange through $f-F$ mixing are highly suppressed by a GIM-like mechanism [17]. The FCNC due to the Z-boson exchange through $f-F$ mixing are also suppressed because the effective coupling constants are order of $1/K$ (we can find that those are of the order of $10^{-8}$ in the case of $k = 10$), so that the FCNC rare decay modes
are suppressed by $10^{-16}$.

The $CP$ violating phases come only from the heavy fermion mass matrix $M_F$, i.e., from the parameter $\beta_f$. In the up-quark sector, the parameter $\beta_u$ must be $\beta_u = 0$, because the top quark mass enhancement becomes mild when $\beta_u \neq 0$. On the other hand, if $\beta_d = 0$, we cannot fit down-quark mass ratios $m_d/m_s$ and $m_s/m_b$ for any values of $k/K$ and $b_d$. We must choose a sizable value of $\beta_d$. Thus, in our model, the $CP$ violating phase in quarks comes only from the down-quark sector $M_D$.

In the present paper, we have discussed a seesaw mass matrix model with the form of $M_F = m_0KO_F$ given by (2.2). As far as the phenomenological predictions are concerned, we can choose other family-basis, for example, a rather simple form of $O_F$

$$O_F = 1 + 3b_fe^{i\beta_f} \text{diag}(0, 0, 1), \quad (6.1)$$

instead of the democratic form (2.2). However, in order to obtain reasonable predictions of quark mass ratios and KM matrix parameters, the matrix $Z$ cannot be a diagonal form such as in (2.3), and it must be given by

$$Z = \frac{1}{6} \begin{pmatrix}
3(z_2 + z_1) & -\sqrt{3}(z_2 - z_1) & -\sqrt{6}(z_2 - z_1) \\
-\sqrt{3}(z_2 - z_1) & 4z_3 + z_2 + z_1 & -\sqrt{2}(2z_3 - z_2 - z_1) \\
-\sqrt{6}(z_2 - z_1) & -\sqrt{2}(2z_3 - z_2 - z_1) & 2(z_3 + z_2 + z_1)
\end{pmatrix}, \quad (6.2)$$

where $z_i$ are given by (2.4). Which family basis is reasonable is not essential as far as we discuss only the fermion masses and KM mixing parameters, but it will become important for model-building.

We believe that our phenomenological mass-matrix model is worth serious attention, not only because it has fewer adjustable parameters than conventional models do, but also because it gives $m_t \gg m_b$ and $m_u \sim m_d$ simultaneously despite its “almost” up-down symmetric mass matrices (i.e., $b_u/b_d$ is not so large as $m_t/m_b$).
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Appendix: Diagonalization of $2n \times 2n$ matrix

The transformation of $2n \times 2n$ matrix

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \quad (A.1)$$

into

$$M' \equiv \begin{pmatrix} M'_{11} & 0 \\ 0 & M'_{22} \end{pmatrix} \quad (A.2)$$

is done by two $2n \times 2n$ unitary matrices,

$$U_L = \begin{pmatrix} (1 + \rho_L \rho_L^\dagger)^{-1/2} & (1 + \rho_L \rho_L^\dagger)^{-1/2} \rho_L \\ -(1 + \rho_L^\dagger \rho_L)^{-1/2} \rho_L^\dagger & (1 + \rho_L^\dagger \rho_L)^{-1/2} \rho_L \end{pmatrix} \quad (A.3)$$

and $U_R$ with $L \leftrightarrow R$ in (A.3) as

$$M' = U_L M U_R^\dagger \quad (A.4)$$

where $M_{ij}$, $M'_{ij}$, $\rho_L$, $\rho_R$ are $n \times n$ matrices.

The conditions $M'_{12} = 0$ and $M'_{21} = 0$ lead to the relations

$$M_{12} - M_{11} \rho_R + \rho_L M_{22} - \rho_L M_{21} \rho_R = 0 \quad (A.5)$$

and

$$M_{21} + M_{22} \rho_R^\dagger - \rho_L^\dagger M_{11} - \rho_L^\dagger M_{12} \rho_R^\dagger = 0 \quad (A.6)$$

respectively, which lead to

$$\rho_R = (M_{11} + \rho_L M_{21})^{-1} (M_{12} + \rho_L M_{22}) \quad (A.7)$$
\[ \rho_R = (M_{21}^\dagger - M_{11}^\dagger \rho_L)(M_{12}^\dagger \rho_L - M_{22}^\dagger)^{-1} . \]  

(A.8)

By eliminating \( \rho_R \) from (A.7) and (A.8), we obtain

\[ (M_{12} + \rho_L M_{22})(M_{12}^\dagger \rho_L - M_{22}^\dagger) = (M_{11} + \rho_L M_{21})(M_{21}^\dagger - M_{11}^\dagger \rho_L) , \]

(A.9)

or

\[ M_{11} M_{21}^\dagger + M_{12} M_{22}^\dagger - (M_{11} M_{11}^\dagger + M_{12} M_{12}^\dagger) \rho_L \]

\[ + \rho_L (M_{21} M_{21}^\dagger + M_{22} M_{22}^\dagger) - \rho_L (M_{21} M_{11}^\dagger + M_{22} M_{12}^\dagger) \rho_L = 0 . \]

(A.10)

Similarly, we obtain the relation

\[ M_{11}^\dagger M_{12} + M_{21}^\dagger M_{22} - (M_{11}^\dagger M_{11} + M_{21}^\dagger M_{21}) \rho_R \]

\[ + \rho_R (M_{12}^\dagger M_{12} + M_{22}^\dagger M_{22}) - \rho_R (M_{12}^\dagger M_{11} + M_{22}^\dagger M_{21}) \rho_R = 0 . \]

(A.11)

Eliminating \( M_{22} \) from (A.5) and (A.6), we obtain

\[ \rho_L M_{22} \rho_R^\dagger = (M_{11} \rho_R + \rho_L M_{21} \rho_R - M_{12}) \rho_R^\dagger \]

\[ = \rho_L (\rho_L^\dagger M_{11} + \rho_L^\dagger M_{12} \rho_R^\dagger - M_{21}) , \]

(A.12)

so that

\[ (1 + \rho_L \rho_L^\dagger)(M_{11} + M_{12} \rho_R^\dagger) = (M_{11} + \rho_L M_{21})(1 + \rho_R \rho_R^\dagger) . \]

(A.13)

Similarly, eliminating \( M_{11} \) from (A.5) and (A.6), we obtain

\[ \rho_L^\dagger M_{11} \rho_R = \rho_L^\dagger (M_{12} + \rho_L M_{22} - \rho_L M_{21} \rho_R) \]

\[ = (M_{21} + M_{22} \rho_R^\dagger - \rho_L^\dagger M_{12} \rho_R^\dagger) \rho_R , \]

(A.14)

so that

\[ (1 + \rho_L^\dagger \rho_L)(M_{22} - M_{21} \rho_R) = (M_{22} - \rho_L^\dagger M_{12})(1 + \rho_R^\dagger \rho_R) . \]

(A.15)

By using the relations (A.13) and (A.15), we obtain

\[ M_{11}' = (1 + \rho_L \rho_L^\dagger)^{-1/2}(M_{11} + M_{12} \rho_R^\dagger + \rho_L M_{21} + \rho_L M_{22} \rho_R^\dagger)(1 + \rho_R \rho_R^\dagger)^{-1/2} \]

\[ = (1 + \rho_L \rho_L^\dagger)^{-1/2}(M_{11} + \rho_L M_{21})(1 + \rho_R \rho_R^\dagger)^{+1/2} \]

(A.16)

\[ = (1 + \rho_L \rho_L^\dagger)^{+1/2}(M_{11} + M_{12} \rho_R^\dagger)(1 + \rho_R \rho_R^\dagger)^{-1/2} , \]

(A.17)

\[ M_{22}' = (1 + \rho_L \rho_L^\dagger)^{-1/2}(\rho_L^\dagger M_{11} \rho_R - \rho_L^\dagger M_{12} - M_{21} \rho_R + M_{22})(1 + \rho_R^\dagger \rho_R)^{-1/2} \]

\[ = (1 + \rho_L^\dagger \rho_L)^{+1/2}(M_{22} - M_{21} \rho_R)(1 + \rho_R^\dagger \rho_R)^{-1/2} \]

(A.18)
\[ = (1 + \rho_L^\dagger \rho_L)^{-1/2} (M_{22} - \rho_L^\dagger M_{12})(1 + \rho_R^\dagger \rho_R)^{+1/2}. \]  

(A.19)

The matrices \(\rho_L\) and \(\rho_R\) are obtained as solutions of the equations (A.10) and (A.11), respectively. When the \(2n \times 2n\) mass matrix \(M\) (A.1) is Hermitian, we can set \(\rho_L = \rho_R \equiv \rho\), so that the calculation becomes easier.

When \(M\) is not Hermitian, instead of the \(n \times n\) mass matrices \(M'_{11}\) and \(M'_{22}\), the diagonalization is done for the following Hermitian matrices

\[
H_1 \equiv M'_{11} M_{11}^{\dagger} = (1 + \rho_L \rho_L^\dagger)^{-1/2} \tilde{H}_1 (1 + \rho_L \rho_L^\dagger)^{+1/2},
\]

(A.20)

\[
H_2 \equiv M'_{22} M_{22}^{\dagger} = (1 + \rho_L^\dagger \rho_L)^{+1/2} \tilde{H}_2 (1 + \rho_L^\dagger \rho_L)^{-1/2},
\]

(A.21)

where

\[
\tilde{H}_1 = (M_{11} + \rho_L M_{21})(M_{11}^\dagger + \rho_R M_{12}^\dagger),
\]

(A.22)

\[
\tilde{H}_2 = (M_{22} - M_{21} \rho_R)(M_{22}^\dagger - M_{12} \rho_L).
\]

(A.23)

We are interested in the diagonalization of (A.22). By using (A.5), we can rewrite (A.22) into

\[
\tilde{H}_1 = A + \rho_L B,
\]

(A.24)

where

\[
A = M_{11} M_{11}^\dagger + M_{12} M_{12}^\dagger,
\]

(A.25)

\[
B = M_{21} M_{11}^\dagger + M_{22} M_{12}^\dagger.
\]

(A.26)

By eliminating \(\rho_L\) from (A.10) and (A.24), we find that the matrix \(\tilde{H}_1\) satisfies the following equations

\[
\tilde{H}_1^2 - \tilde{H}_1 (A + B^{-1} DB) + AB^{-1} DB - CB = 0,
\]

(A.27)

where

\[
C = M_{11} M_{21}^\dagger + M_{12} M_{22}^\dagger,
\]

(A.28)

\[
D = M_{11} M_{21}^\dagger + M_{22} M_{22}^\dagger.
\]

(A.29)
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Figure Captions

Fig. 1. Masses $m_f^i$ ($i = 1, 2, 3$) versus $b_f$ for the case of $k = 10$ and $K/k = 50$. The solid and broken lines denote for the cases of $\beta_f = 0$ and $\beta_f = -20^\circ$, respectively. The parameters $k$ and $K$ are defined by (2.1). The figure should be taken as that for the quark mass ratios. For the absolute value of quark masses, see a comment on (4.5) in the text.

Fig. 2. Mass ratio $m_t/m_c$ versus $K/k$ for $k = 10$. The curves (a) - (d) denote the cases (a) $\Delta b_u = 0$, (b) $\Delta b_u = +1.00 \times 10^{-3}$ and $\Delta b_u = -0.980 \times 10^{-3}$, (c) $\Delta b_u = +3.88 \times 10^{-3}$ and $\Delta b_u = -3.62 \times 10^{-3}$, (d) $\Delta b_u = +10.0 \times 10^{-3}$ and $\Delta b_u = -8.53 \times 10^{-3}$. The horizontal lines denote the experimental values $(m_t/m_c)_{exp} = 271 \pm 46$.

Fig. 3. Top quark mass $m_t$ in unit of $m_0$ versus $K/k$ for $k = 10$. The curves (a) - (d) denote the cases (a) $\Delta b_u = 0$, (b) $\Delta b_u = +1.00 \times 10^{-3}$ and $\Delta b_u = -0.980 \times 10^{-3}$, (c) $\Delta b_u = +3.88 \times 10^{-3}$ and $\Delta b_u = -3.62 \times 10^{-3}$, (d) $\Delta b_u = +10.0 \times 10^{-3}$, and (d) $\Delta b_u = -8.53 \times 10^{-3}$.

Fig. 4. Mass ratios $m_s/m_d$ and $m_b/m_s$ versus $\beta_d$ for $b_d = -0.90$ (a dotted line), $b_d = -1.0$ (a solid line) and $b_d = -1.1$ (a broken line) in the case of $k = 10$ and $K/k = 50$.

Fig. 5. Kobayashi-Maskawa matrix elements $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ versus $\beta_d$ in the case of $k = 10$, $K/k = 50$, $b_u = -1/3$ and $\beta_u = 0$. 

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