Compton tensor with heavy photon in the case of longitudinally polarized fermion.

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Abstract

The matrix element squared for Compton scattering of a heavy photon on a longitudinally polarized electron is calculated. The gauge–invariant tensor structure of the heavy photon polarization is considered, assuming that all kinematical invariants and the invariant mass of the heavy photon are large compared with the electron mass. The expressions for the Born and first order radiative corrections are obtained. Applications are discussed.

In the case of unpolarized fermions the Compton tensor with heavy photon was calculated in papers [1, 2] years ago. It accumulates a considerable part of radiative corrections and can be used as a building block in calculations of various processes. The tensor was used for precision calculations of radiative corrections to Bhabha scattering at LEP [3], cross–section of deep inelastic scattering with tagged photons [4], and other.

We will restrict ourselves here in considering only that part of the Compton tensor, which contains the degree of polarization of the initial electron. The absence in literature of a closed expression for this quantity and the importance of it for many applications is the motivation for this investigation.

Let us consider the process (see Fig. 1)

\[
\gamma^*(q) + e(p_1) \rightarrow \gamma(k_1) + e(p_2), \quad q^2 < 0, \quad k_1^2 = 0, \quad p_1^2 = p_2^2 = m^2, \quad p_1 + q = p_2 + k_1,
\]

where \(m\) is the electron mass.

The Compton tensor is defined as

\[
K_{\rho\sigma} = (8\pi\alpha)^{-2}\Sigma M_{\rho}^{\gamma^*\rightarrow e\gamma}(M_{\sigma}^{\gamma^*\rightarrow e\gamma})^*,
\]

where the matrix element \(M\) describes the Compton scattering process [1]. It is read

\[
M_{\rho} = M_{0\rho} + M_{1\rho} = \bar{u}(p_2)O_{\rho\mu}u(p_1)\gamma_{\rho}^\mu(k_1),
\]

\[
O_{\rho\mu} = \frac{\alpha}{4\pi}O_{\rho\mu}^{(0)}, \quad O_{\rho\mu}^{(0)} = \gamma_{\rho}^\mu\frac{(p_2 - \hat{q} + m)}{t}\gamma_{\mu} + \gamma_{\mu}\frac{(p_1 + \hat{q} + m)}{s}\gamma_{\rho},
\]

The quantities \(O_{\rho\mu}^{(0)}\) and \(O_{\rho\mu}^{(1)}\) take into account the lowest and the first orders of perturbation theory respectively.
We will separate the contributions, associated with the electron polarization:

\[ s \sim -t \sim -u \sim -q^2 \gg m^2, \]
\[ s = 2p_2k_1, \quad t = -2p_1k_1, \quad u = -2p_1p_2, \quad q^2 = s + t + u. \]  

(4)

So, we will neglect the electron mass in all places, where possible. Note that for the unpolarized case in [1] the mass was taken into account.

\[ K_{\rho\sigma} = K^*_{\sigma\rho}, \]  

(5)

We will separate the contributions, associated with the electron polarization:

\[ K_{\rho\sigma} = K^0_{\rho\sigma} + \frac{\alpha}{4\pi} \left( K^1_{\rho\sigma} + K^1_{\rho\sigma}^* \right), \]  

(6)

where \( \xi \) is the degree of the initial electron polarization. Quantities \( B_{\rho\sigma} \) and \( T_{\rho\sigma} \) correspond to the case of unpolarized electron:

\[ B_{\rho\sigma} = B_9 \tilde{g}_{\rho\sigma} + B_{11} \tilde{p}_{1\rho} \tilde{p}_{1\sigma} + B_{22} \tilde{p}_{2\rho} \tilde{p}_{2\sigma}, \]  

(7)

\[ B_9 = \frac{1}{st} \left[ (s + u)^2 + (t + u)^2 \right] - 2m^2q^2 \left( \frac{1}{s^2} + \frac{1}{t^2} \right), \]  

\[ B_{11} = \frac{4q^2}{st} - \frac{8m^2}{s^2}, \quad B_{22} = \frac{4q^2}{st} - \frac{8m^2}{t^2}. \]
where the new variables
\[
\tilde{g}_{\rho \sigma} = g_{\rho \sigma} - \frac{q_{\rho} q_{\sigma}}{q^2}, \quad \tilde{p}_{1 \rho} = p_{1 \rho}^0 - \frac{p_{1 \rho} q_{\rho}}{q^2} q^0 \tag{8}
\]
provide an explicit fulfillment of gauge conditions: \(q_{\rho} K_{\rho \sigma} = 0, q_{\alpha} K_{\rho \sigma} = 0\). Quantity \(T_{\rho \sigma}\) has a rather cumbersome form, it is given in [1].

For the case of the most general form for the electron polarization vector
\[
\Sigma_{u}(p) \bar{u}(p) = (\hat{p}_1 + m)(1 - \xi \gamma_5 \hat{a}) \tag{9}
\]
one obtains (see also [5, 6])

\[
P_{0 \rho \sigma} = 4m \left\{ (p_1 q_{\rho \sigma}) \left[ \frac{qa - 2p_2 a}{st} + (p_2 q_{\rho \sigma}) \left[ \frac{qa}{t^2} + \frac{p_2 a}{t} \left( \frac{1}{s} - \frac{1}{t} \right) \right] \right. \\
+ (qaq_{\rho \sigma}) \left[ \frac{q^2}{st} - \frac{1}{s} - \frac{1}{t} - m^2 \left( \frac{1}{s^2} + \frac{1}{t^2} \right) \right] \right\}, \tag{10}
\]
where we used the notation
\[
(abcde) \equiv i \epsilon_{\alpha \beta \gamma \delta} a^\alpha b^\beta c^\gamma d^\delta. \tag{11}
\]
This object obeys the Shouten identity:

\[
(abcde)ef = (fbcde)ae + (afced)be + (abfde)ce + (abcfde). \tag{12}
\]

In this paper we restrict ourselves by considering only the case of longitudinally polarized fermion:

\[
\Sigma_{u}(p_1) \bar{u}(p_1) = \hat{p}_1 (1 - \xi \gamma_5). \tag{13}
\]
This is the most interesting case for physical applications. In the Born approximation we obtain

\[
P_{\rho \sigma} = \xi \left[ P_{0 \rho \sigma} + \frac{\alpha}{4\pi} P_{1 \rho \sigma} \right], \tag{14}
\]

\[
P_{0 \rho \sigma} = P_{0t \rho \sigma} + P_{0s \rho \sigma} = \frac{2}{st} \left[ (u + t)(p_1 q_{\rho \sigma}) + (u + s)(p_2 q_{\rho \sigma}) \right].
\]

Here and below the upper indexes \(t\) and \(s\) mean the contributions of Feynman diagrams. It is useful to present the explicit expressions for \(P_{0t,s}^{\rho \sigma}\):

\[
P_{0t \rho \sigma} = \frac{1}{st} \left[ 4(p_1 p_2 q_{\sigma})(p_{1 \rho} + p_{2 \rho}) + 2(t - s)(p_{1 \rho} p_{2 \rho} q_{\sigma}) + 2(s + u)(p_{2 \rho} q_{\rho \sigma}) \right],
\]

\[
P_{0s \rho \sigma} = \frac{1}{st} \left[ -4(p_1 p_2 q_{\sigma})(p_{1 \rho} + p_{2 \rho}) + 2(s - t)(p_{1 \rho} p_{2 \rho} q_{\sigma}) + 2(s + t)(p_{1 \rho} q_{\rho \sigma}) \right]. \tag{15}
\]

It is easy to check the following relations:

\[
q_{\rho} P_{0 \rho \sigma} = q_{\sigma} P_{0 \rho \sigma} = 0, \quad (P_{0t,s}^{\rho \sigma})* = P_{0s,t}^{\rho \sigma}, \quad P_{0t,s}^{\rho \sigma} q_{\rho} = 0, \quad P_{0t,s}^{\rho \sigma} q_{\sigma} \neq 0. \tag{16}
\]

Note now that we may consider in calculations only half of the full set of 8 Feynman diagrams in 1-loop level drawn in Fig.2, namely, the diagrams (a),(b),(c),(d). Really the whole contribution
may be obtained knowing the values of the contributions arising from Feynman diagrams Fig.2 using the rearrangement operator:

\[ P_{\rho\sigma}^1 = (1 + \hat{H})(1 - \hat{P})(P_{\rho\sigma}^{a,b} + P_{\rho\sigma}^{1c} + P_{\rho\sigma}^{1d}) + P_{\rho\sigma}^{\text{soft}}, \]  

(17)

where the operator \( \hat{P} \) is defined as

\[ \hat{P} F(\rho, \sigma, p_1, p_2, q, s, t) = F(\rho, \sigma, p_2, p_1, -q, t, s), \]  

(18)

and the hermitization operator \( \hat{H} \) acts as:

\[ \hat{H} a_{\rho\sigma} = a_{\sigma\rho}^*. \]  

(19)

Note that \( \hat{P} P_{\rho\sigma}^{0s,t} = -P_{\rho\sigma}^{0t,s} \). The last term in Eq.(17) describes the contribution due to the emission of additional soft photon \[ \square \]:

\[ P_{\rho\sigma}^{\text{soft}} = P_{\rho\sigma}^0 \delta_{\text{soft}}, \]  

(20)

\[ \delta_{\text{soft}} = -\frac{4\pi\alpha}{16\pi^3} \int \frac{d^3 k}{\omega} \left( \frac{p_1}{p_1 k} - \frac{p_2}{p_2 k} \right)^2 = \frac{\alpha}{\pi} \left[ (L_u - 1) \ln \frac{m^2(\Delta\varepsilon)^2}{\alpha^2 \varepsilon_1 \varepsilon_2} + \frac{1}{2} L_u^2 \right. \]

\[ - \left. \frac{1}{2} \ln^2 \frac{\varepsilon_1}{\varepsilon_2} - \frac{\pi^2}{3} + Li_2 \left( 1 + \frac{u}{4 \varepsilon_1 \varepsilon_2} \right) \right], \quad L_u = \ln \frac{-u}{m^2}. \]

Here \( \Delta\varepsilon \) is the maximal energy of additional soft photon escaping the detector; quantities \( \varepsilon_{1,2} = p_{1,2}^0 \) are the energies of the initial and the final electron in the laboratory reference frame (rest reference frame of the target).

Considering the contribution of Feynman diagrams Fig.2,a,b we may use the result, given in the preprint of paper \( \square \), namely

\[ (M^a_\sigma + M^b_\sigma)(-i(4\alpha\pi)^2)^{-1} = \frac{\alpha}{2\pi} \bar{u}(p_2)\gamma_\sigma [m A_1(\hat{e} - \hat{k}_1 \frac{p_1 e}{p_1 k_1}) + A_2 \hat{k}_1 \bar{e}] u(p_1). \] 

(21)

Note that this result may be reproduced using the loop integrals list given in Appendix and the standard renormalization procedure. We see that only structure in front of coefficient \( A_2 \) survives in the limit \( m \to 0 \). After simple algebra we obtain:

\[ P_{\rho\sigma}^{a,b} = 2L_u - 1 \frac{2}{st_2} [2(p_1 p_2 q\sigma)p_{2\rho} + (u + s)((p_2 q\rho\sigma) - (p_1 p_2 \rho\sigma))]. \]  

(22)

The remaining Fig.2c,d contributions have a form:

\[ P_{\rho\sigma}^{1c} = \frac{1}{t} \int \frac{d^4 k}{i\pi^2} \frac{1}{a_0 a_2 a_q} \left[ \frac{1}{4} \text{Tr} \hat{p}_2 \gamma_\lambda (\hat{p}_2 - \hat{k}) \gamma_\sigma (\hat{p}_2 - \hat{q} - \hat{k}) \gamma_\lambda (\hat{p}_2 - \hat{q}) \gamma_\mu \hat{p}_1 \gamma_5 \hat{O}_{\rho\mu}^0 \right] \]  

(23)

and

\[ P_{\rho\sigma}^{1d} = \int \frac{d^4 k}{i\pi^2} \frac{1}{a_0 a_1 a_2 a_q} \left[ \frac{1}{4} \text{Tr} \hat{p}_2 \gamma_\lambda (\hat{p}_2 - \hat{k}) \gamma_\sigma (\hat{p}_2 - \hat{q} - \hat{k}) \gamma_\mu (\hat{p}_1 - \hat{k}) \gamma_\lambda \hat{p}_1 \gamma_5 \hat{O}_{\rho\mu}^0 \right] \]  

(24)

where

\[ a_0 = k^2 - \lambda^2, \quad a_1 = k^2 - 2p_1 k, \quad a_2 = k^2 - 2p_2 k, \quad a_q = (p_2 - q - k)^2 - m^2. \]  

(25)
and the matrix $\hat{O}_{\rho\mu}^0$ differs from $O_{\rho\mu}^0$ (see Eq.(3)) by reversal order of gamma matrices. Using the integrals given in Appendix one may perform the loop momenta integration in right hand parts of expressions for $P^{1c}, P^{1d}$ and obtain the total expression for the Compton tensor. Its explicit form will be given below.

Now we will concentrate our attention on the terms containing the infrared singularities. There are three sources of them. The first one is the renormalization constant
\begin{equation}
Z_1 = 1 - \frac{\alpha}{2\pi} \left( \frac{1}{2} L_\Lambda + 2 \ln \frac{\Lambda}{m} + \frac{9}{4} \right), \quad L_\Lambda = \ln \frac{\Lambda^2}{m^2},
\end{equation}
which is needed to remove the ultraviolet divergence of the vertex function, entering into $P_{1c}$. The next source is a part of the box contribution $P_{1d}$, which comes from the terms from the numerator which does not contain loop momenta. Really for the Feynman diagram Fig.2d they are associated with the scalar integral,
\begin{equation}
I = \int \frac{d^4 k}{i\pi^2} \frac{1}{a_0 a_1 a_2 a_q} = \frac{1}{tu} \left[ 2L_u \ln \frac{m}{\Lambda} - L_q^2 + 2L_t L_u - \frac{\pi^2}{6} - 2Li_2(1 - \frac{q^2}{u}) \right],
\end{equation}
\begin{equation}
L_q = \ln \frac{-q^2}{m^2}, \quad L_t = \ln \frac{-t}{m^2}, \quad Li_2(z) = -\int_0^1 \frac{dx}{x} \ln(1 - zx).
\end{equation}
The third source is the emission of additional soft photons, which was given above. The infrared singularities are cancelled in the total sum.

Let us consider the contribution from one–loop corrections (see Fig.2a,b,c,d)
\begin{equation}
P^{t}_{\rho\sigma} = (P^{a,b} + P^{1c} + P^{1d})_{\rho\sigma}.
\end{equation}

Extracting the leading logarithmic terms and infrared singularities, we may present it as follows:
\begin{equation}
P^{t}_{\rho\sigma} = P^{0\rho\sigma} \left[ -L_u^2 - 4(L_u - 1) \ln \frac{m}{\Lambda} + 3L_u \right] + R^{t}_{\rho\sigma}.
\end{equation}

After hermitization and rearrangement operations and adding of the soft photon contribution we arrive to the result
\begin{equation}
P_{\rho\sigma} = P^{0}_{\rho\sigma} \left\{ 1 + \frac{\alpha}{\pi} \left[ (L_u - 1) \ln \frac{(\Delta \varepsilon)^2}{\varepsilon_1 \varepsilon_2} + \frac{3}{2} L_u - \frac{1}{2} \ln^2 \frac{\varepsilon_2}{\varepsilon_1} + \ln^2 \frac{\varepsilon_1}{\varepsilon_2} - \frac{\pi^2}{3} + Li_2(\cos^2 \theta) \right] \right\} + \frac{\alpha}{4\pi} R_{\rho\sigma}.
\end{equation}
Quantities $R^{t}_{\rho\sigma}$ and $R_{\rho\sigma}$ collect non–leading terms. They are free from infrared singularities. Tensor $R^{t}_{\rho\sigma}$ can be presented in the form
\begin{equation}
R^{t}_{\rho\sigma} = A(2q_\sigma p_\rho) + B(1q_\sigma p_\rho) + C(12q_\sigma p_1 + D(12q_\sigma p_2) + E(12q_\sigma q_\rho) + F(12q_\sigma) + G(12q_\sigma pq_\rho) + H(12q_\sigma pq_\rho). (31)
\end{equation}
The coefficients $A - F$ have a rather cumbersome form, we are not going to present them here. Note only that they obey the condition
\begin{equation}
C p_1 q + D p_2 q + E q^2 - F = 0,
\end{equation}
because of gauge invariance in respect to index $\rho$. 

5
The rearrangement operation gives
\[
(1 - \hat{p}) R^t_{\rho\sigma} = (A + \tilde{B})(2q\sigma\rho) + (B + \tilde{A})(1q\sigma\rho) + (C - \tilde{D})(12q\sigma)p_{1\rho} + (D - \tilde{C})(12q\sigma)p_{2\rho} + (E + \tilde{E})(12q\sigma)q_{\rho} + (F + \tilde{F})(12q\sigma) \equiv A_1(1q\sigma\rho) + A_2(2q\sigma\rho) + B_1(12q\sigma)p_{1\rho} + B_2(12q\sigma)p_{2\rho} + C_1(12q\sigma)q_{\rho} + F_1(12q\sigma). \tag{33}
\]

Tests of gauge invariance gives an important check of our calculations:
\[
q^\rho(1 - \hat{p}) R^t_{\rho\sigma} = B_1(12q\sigma)p_{1\rho} + B_2(12q\sigma)p_{2\rho} + C_1(12q\sigma)q_{\rho} + F_1(12q\sigma) = 0. \tag{34}
\]

The above conditions yield
\[
F_1 = 0, \quad C_1 = -B_1 \frac{p_{1\rho}}{q^2} - B_2 \frac{p_{2\rho}}{q^2}, \quad B_1 p_{1\rho} + B_2 p_{2\rho} + C_1 q_{\rho} = B_1 \tilde{p}_{1\rho} + B_2 \tilde{p}_{2\rho}, \tag{35}
\]
\[
\tilde{p}_{1\rho} = p_{1\rho} - \frac{p_{1\rho}}{q^2} q_{\rho}, \quad \tilde{p}_{2\rho} = p_{2\rho} - \frac{p_{2\rho}}{q^2} q_{\rho}.
\]

By straightforward calculations we checked these relations.

The hermitization gives
\[
R_{\rho\sigma} = (1 + H)(1 - \hat{p}) R^t_{\rho\sigma} = (A_1 + A_1^*)(1q\sigma\rho) + (A_2 + A_2^*)(2q\sigma\rho) + (12q\sigma)[B_1 \tilde{p}_{1\rho} + B_2 \tilde{p}_{2\rho}] - (12q\rho)[B_1^* \tilde{p}_{1\sigma} + B_2^* \tilde{p}_{2\sigma}], \tag{36}
\]
where
\[
A_1 = \frac{2}{st} \left[ \frac{2u(2s - u)}{a} L_{qu} + \frac{4us}{a} \left( \frac{u}{a} L_{qu} - 1 \right) \right] + \frac{2}{c} \left[ \frac{ub}{c} - \frac{u}{c} \frac{2u^2 + us - s^2}{L_{sq} + \frac{usb}{c^2} L_{sq}} \right] (37)
\]
\[
B_1 = \frac{2}{st} \left[ \frac{8u}{a} \left( \frac{u}{a} - 1 \right) L_{qu} \right] + \frac{6t}{b} L_{qt} + \frac{2(u^2 - 2s^2 - su)}{L_{sq}} L_{sq} \right] + \frac{2b}{c} \left( \frac{1}{c} L_{sq} \right) + \frac{2}{s} (2c - s) L_{tu} + \left( -2 - \frac{4c^2}{st} - \frac{12b}{t} - \frac{4s^2}{ut} \right) L_{qu} + \left( \frac{2b}{t} + \frac{2b^2}{t^2} \right) \tilde{G} + 6 \right], \tag{37}
\]
\[
G = (L_q - L_u)(L_q + L_u - 2L_t) - \frac{\pi^2}{3} - 2Li_2 \left( 1 - \frac{q^2}{u} \right) + 2Li_2 \left( 1 - \frac{t}{q^2} \right), \quad A_2 = (s \leftrightarrow t) A_1, \quad B_2 = -(s \leftrightarrow t) B_1, \quad \tilde{G} = (s \leftrightarrow t) G.
\]

Note that the above expressions are free from kinematical singularities. Really, in the limits
\[ a \to 0, b \to 0 \text{ and } c \to 0 \] the quantities are finite. The symmetry between \( A_1, B_1 \) and \( A_2, B_2 \) is because of the initial symmetry between \( p_1 \) and \( p_2 \) in the traces.

Thus we calculated the part of the leptonic tensor, proportional to the initial longitudinal polarization. This tensor describes Compton scattering with one off-shell photon, which is connected with a certain target.
The calculation allows to obtain the correction coming from one–loop effects to quantities observable in different polarization experiments. Let us consider for definiteness the task of calculation of $\alpha^2$ order radiative correction in polarized deep inelastic scattering. The results for the lowest order QED correction for nucleon and nuclear targets can be found in refs. [5, 6]. Both the Born cross-section ($\sigma_{\text{Born}}$) and the cross-section at the level of radiative corrections ($\sigma_{\text{RC}}$) can be split into unpolarized and polarized parts

$$\sigma_{\text{Born,RC}} = \sigma_{\text{Born,RC}}^{\text{unp}} + \xi_b \xi_t \sigma_{\text{Born,RC}}^{\text{pol}},$$

where $\xi_b$ and $\xi_t$ are polarization degrees of beam and target. The correction to asymmetry ($A = \sigma^{\text{pol}} / \sigma^{\text{unp}}$):

$$\Delta A = \frac{\sigma_{\text{RC}}^{\text{pol,unp}} \sigma_{\text{Born}}^{\text{unp}} - \sigma_{\text{Born}}^{\text{unp}} \sigma_{\text{RC}}^{\text{pol,unp}}}{\sigma_{\text{Born}}^{\text{unp}}(\sigma_{\text{Born}}^{\text{unp}} + \sigma_{\text{RC}}^{\text{unp}})} (39)$$

is usually not large because of mutual cancellation of large factorizing terms in eqn. (39). It is clear that in such cases when relatively small correction is obtained as a difference of two large terms the radiative correction cross-section has to be calculated with the most possible accuracy, and special attention has to be paid to non–factorizing terms like (37).

Now the new methods of experimental data processing, where experimental information about spin observables is extracted directly from polarized part of cross-section (difference of observed cross sections with opposite spin configurations) [8] is actively developed. It makes new requirements for accuracy of radiative correction calculation. We note that there is no any cancellation of leading contributions in this case, and factorizing terms in (29) give the basic contribution.

The kinematical regions with very high $y$ ($y \sim 0.9$) can be reachable in the current polarization experiments on DIS [9, 10]. In this region radiative correction to cross–section is comparable or larger of Born cross–section. Basically it is originated by contributions of radiative tails from elastic and quasielastic peaks. This calculation firstly allows to obtain the contribution of these tails with taking into account loop effects in the nest–to–leading approximation.

There is one particular interesting phenomenon. Note, that $P^{(1)}_{\rho\sigma}$ contains not only the imaginary part, but also a certain real part, which comes from the imaginary parts of $A_1$ and $B_1$. The conversion of this real part of $P^{(1)}_{\rho\sigma}$ with the ordinary symmetrical part of the hadronic tensor will give rise to one–spin azimuthal asymmetry for the final electron [7]. The asymmetry is proportional to the degree of polarization of the initial electron. It is small because of the extra power of $\alpha_{\text{QED}}$ and the absence of large logarithms.

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Appendix

Here we put the list of relevant one-loop integrals calculated in the approximation $s \sim -t \sim -u \sim -q^2 \gg m^2$. We use the notation:

\[
(I, I_\mu, I_{\mu\nu}) = \int \frac{d^4 k}{i\pi^2} \frac{(1, k, k, k)}{a_0 a_1 a_2 a_q}, \quad (i, i_\mu, i_{\mu\nu}) = \int \frac{d^4 k}{i\pi^2} \frac{(1, k, k, k)}{a_1 a_2 a_q}, \\
(j, j_\mu, j_{\mu\nu}) = \int \frac{d^4 k}{i\pi^2} \frac{(1, k, k, k)}{a_0 a_2 a_q}, \quad (n, n_\mu, n_{\mu\nu}) = \int \frac{d^4 k}{i\pi^2} \frac{(1, k, k, k)}{a_0 a_1 a_q}, \\
(z, z_\mu, z_{\mu\nu}) = \int \frac{d^4 k}{i\pi^2} \frac{(1, k, k, k)}{a_0 a_1 a_2}.
\] (A.1)

Using the explicit expression for the denominators (23), we obtain a decomposition for vector-type integrals:

\[
I_\mu = I_1 p_1 + I_2 p_2 + I_3 q_\mu, \quad i_\mu = i_1 p_1 + i_2 p_2 + i_3 q_\mu, \quad n_\mu = n_1 p_1 + n_2 (p_2 - q_\mu), \\
\dot{j}_\mu = j_2 p_2 + j_3 q_\mu, \quad z_\mu = z_1 (p_1 + p_2),
\] (A.2)

In the same way we get for tensor-type integrals:

\[
I_{\mu\nu} = I_{1\mu} g_{\mu\nu} + I_{11} p_{1\mu} p_{1\nu} + I_{22} p_{2\mu} p_{2\nu} + I_{23} q_\mu q_\nu + I_{12} (p_1 p_2 + p_2 p_1) \\
+ I_{13} (p_1 q_\nu + q_\mu p_1) + I_{23} (p_2 q_\nu + q_\mu p_2), \\
i_{\mu\nu} = i_{1\mu} g_{\mu\nu} + i_{11} p_{1\mu} p_{1\nu} + i_{22} p_{2\mu} p_{2\nu} + i_{23} q_\mu q_\nu + i_{12} (p_1 p_2 + p_2 p_1) \\
+ i_{13} (p_1 q_\nu + q_\mu p_1) + i_{23} (p_2 q_\nu + q_\mu p_2), \\
\dot{j}_{\mu\nu} = j_{1\mu} g_{\mu\nu} + j_{11} p_{1\mu} p_{1\nu} + j_{22} p_{2\mu} p_{2\nu} + j_{23} q_\mu q_\nu + j_{12} (p_2 q_\nu + q_\mu p_2), \\
n_{\mu\nu} = n_{1\mu} g_{\mu\nu} + n_{11} p_{1\mu} p_{1\nu} + n_{22} (p_2 p_2 - q_\mu q_\nu) + n_{13} (p_1 q_\nu + q_\mu p_1) \\
- n_{13} (p_1 q_\nu + q_\mu p_1) + n_{23} (p_2 q_\nu + q_\mu p_2), \\
z_{\mu\nu} = z_{1\mu} g_{\mu\nu} + z_{11} (p_1 p_1 + p_2 p_2) + z_{12} (p_2 p_1 + p_1 p_2),
\] (A.3)

The quantities, entering into vector and tensor integrals, are:

\[
I_1 = \frac{1}{d} \left[ (ut - sq^2)i + b^2 j + t(s - u)n - ub Y \right], \\
I_2 = \frac{1}{d} \left[ (us - tq^2)i + (ts - uq^2)j + tcn + uc Y \right], \\
I_q = \frac{1}{d} \left[ u(t - s)i + ubj - utn - u^2 Y \right], \\
Y = z - tI, \quad d = 2stu, \quad b = u + s, \quad a = s + t, \quad c = u + t.
\] (A.4)

For coefficients in tensor structures of $I_{\mu\nu}$ we have

\[
I_g = \frac{1}{2}(i + tI_q), \quad I_{11} = \frac{1}{d} \left[ b^2 (i + tI_q) + (ut - sq^2)i_1 + t(s - u)n_1 - ub (z_1 - tI_1) \right], \\
I_{22} = \frac{1}{d} \left[ c^2 (i + tI_q) + (us - tq^2)i_2 + (ts - uq^2)j_2 + tcn_2 + uc (z_1 - tI_2) \right], \\
I_{12} = \frac{1}{d} \left[ (st - uq^2)(i + tI_q) + (us - tq^2)i_1 + tcn_1 + uc (z_1 - tI_1) \right].
\]
\[ I_{1q} = \frac{1}{d} \left[ bu(i + 2tI_q) + (ut - sq^2)i_q + b^2 j_q + t(u - s)n_2 \right], \]
\[ I_{2q} = \frac{1}{d} \left[ -uc(i + 2tI_q) + (us - tq^2)i_q + (ts - uq^2)j_q - tcn_2 \right], \]
\[ I_{qq} = \frac{1}{d} \left[ u^2(i + 2tI_q) + u(t - s)i_q + tun_2 + ubj_q \right]. \quad (A.5) \]

The \(i\)-vector–type integrals are read
\[
i_1 = \frac{1}{a^2} \left[ q^2 ai + (q^2 + u)L_u - 2q^2 L_q + 2a \right], \quad i_2 = i - i_1, \]
\[
i_q = \frac{1}{a^2} \left[ uai + 2uL_u - (q^2 + u)L_q + 2a \right]. \quad (A.6) \]

The tensor–type integrals are
\[
i_g = \frac{1}{4} L_\Lambda + \frac{3}{8} + \frac{1}{4a} (uL_u - q^2 L_q), \]
\[
i_{11} = \frac{1}{a^3} \left[ (q^2)^2 ai + \frac{1}{2} (3(q^2)^2 + 4q^2 u - u^2)L_u - 3(q^2)^2 L_q + a(4q^2 - u) \right], \]
\[
i_{22} = \frac{1}{a^3} \left[ u^2 ai + \frac{1}{2} (-q^2)^2 + 4q^2 u + 3u^2)L_u + q^2(q^2 - 4u)L_q + 3ua \right], \]
\[
i_{qq} = \frac{1}{a^3} \left[ u^2 ai + 3u^2 L_u + \frac{1}{2} ((q^2)^2 - 4q^2 u - 3u^2)L_q + a(4u - q^2) \right], \]
\[
i_{12} = \frac{1}{a^3} \left[ -uq^2 ai - \frac{1}{2} ((q^2)^2 + 4q^2 u + u^2)L_u + q^2(q^2 + 2u)L_q - a(2q^2 + u) \right], \]
\[
i_{1q} = \frac{1}{a^3} \left[ uq^2 ai + \frac{u}{2} (5q^2 + u)L_u - \frac{q^2}{2} (q^2 + 5u)L_q + \frac{3a}{2} (q^2 + u) \right], \]
\[
i_{2q} = \frac{1}{a^3} \left[ -u^2 ai - \frac{u}{2} (q^2 + 5u)L_u + \frac{1}{2} (-q^2)^2 + 5uq^2 + 2u^2)L_q + \frac{a}{2} (q^2 - 7u) \right]. \quad (A.7) \]

The remaining vectors coefficients are:
\[
j_2 = \frac{2q^2}{b^2} L_q - \frac{q^2}{b^2} L_t - \frac{t}{b} j, \quad j_q = \frac{1}{b} (L_t - L_q), \]
\[n_1 = \frac{1}{t} (tn - 2L_t + 2), \quad n_2 = \frac{1}{t} (L_t - 2), \quad z_1 = \frac{1}{u} L_u. \quad (A.8) \]

Remaining tensor coefficients are:
\[
j_g = \frac{1}{4} L_\Lambda + \frac{3}{8} + \frac{1}{4b} (tL_t - q^2 L_q), \quad j_{qq} = \frac{1}{2b} (L_q - L_t), \]
\[
j_{2q} = \frac{1}{2b^2} \left[ (q^2 - 2t)(L_t - L_q) - b \right], \]
\[
j_{12} = \frac{1}{b^3} \left[ \frac{1}{2} (4q^2 t + 3t^2 - (q^2)^2)L_t + q^2(q^2 - 4t)L_q + \frac{b}{2} (q^2 + t) + t^2 b j \right], \]
\[
n_g = \frac{1}{4} L_\Lambda - \frac{1}{4} L_t + \frac{3}{8}, \quad n_{11} = n + \frac{1}{t} (-3L_t + \frac{9}{2}), \]
\[
n_{22} = \frac{1}{2t} (L_t - 2), \quad n_{1q} = \frac{1}{2t} (-L_t + 3), \]
\[
z_g = \frac{1}{4} (L_\Lambda - L_u + \frac{3}{2}), \quad z_{11} = \frac{1}{2u}, \quad z_{12} = \frac{1}{2u} (L_u - 1). \quad (A.9) \]

9
We use the notation

\[ L_A = \ln \frac{\Lambda^2}{m^2}, \quad L_t = \ln \frac{-t}{m^2}, \quad L_u = \ln \frac{-u}{m^2}. \tag{A.10} \]

The ultraviolet cut–off momentum parameter \( \Lambda \) will be eliminated from the final answer after the renormalization procedure. Really, accounting the renormalization constant

\[ Z_1 = -\frac{\alpha}{2\pi} \left( \frac{1}{2} L_A + 2 \ln \frac{\lambda}{m} + \frac{9}{4} \right) \tag{A.11} \]

for lepton Green functions, we get

\[ L_A \rightarrow -4 \ln \frac{\lambda}{m} - \frac{9}{2}, \tag{A.12} \]

where \( \lambda \) is a fictitious photon mass, \( \lambda \ll m \). We put below the scalar integrals:

\[ i = \frac{1}{2a} (L_q^2 - L_u^2), \quad z = \frac{1}{2u} \left[ 4 \ln \frac{m}{\lambda} L_u + L_u^2 - \frac{\pi^2}{3} \right], \]

\[ n = \frac{1}{2t} \left[ L_q^2 + \frac{2\pi^2}{3} \right], \quad j = \frac{1}{b} \left[ L_q (L_q - L_t) + \frac{1}{2} (L_q - L_t)^2 + 2 Li_2 \left( 1 - \frac{t}{q^2} \right) \right]. \tag{A.13} \]

Note that the quantity \( Y = z - tI \) (the scalar integral \( I \) is given in Eq.(27)) is free from infrared singularities:

\[ Y = \frac{1}{u} \left[ \frac{1}{2} L_u^2 + L_q^2 - 2 L_u L_t + 2 Li_2 \left( 1 - \frac{q^2}{u} \right) \right]. \tag{A.14} \]

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