Increasing Lifetime of Recurrent Sunspot Groups within the Greenwich Photoheliographic Results

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Abstract

Long-lived (>20 days) sunspot groups extracted from the Greenwich Photoheliographic Results (GPR) are examined for evidence of decadal change. The problem of identifying sunspot groups which are observed on consecutive solar rotations (recurrent sunspot groups) is tackled by first constructing manually an example dataset of recurrent sunspot groups and then using machine learning to generalise this subset to the whole GPR. The resulting dataset of recurrent sunspot groups is verified against previous work by A. Maunder and other Royal Greenwich Observatory (RGO) compilers. Recurrent groups are found to exhibit a slightly larger value for the Gnevyshev-Waldmeier Relationship than the value found by Petrovay and van Driel-Gesztelyi (\textit{Solar Phys.} \textbf{51}, 25, 1997), who used recurrence data from the Debrecen Photoheliographic Results. Evidence for sunspot group lifetime change over the previous century is observed within recurrent groups. A lifetime increase of 1.4 between 1915 and 1940 is found, which closely agrees with results from Blanter \textit{et al.} (\textit{Solar Phys.} \textbf{237}, 329, 2006). Furthermore, this increase is found to exist over a longer period (1915 to 1950) than previously thought and provisional evidence is found for a decline between 1950 and 1965. Possible applications of machine-learning procedures to the analysis of historical sunspot observations, the determination of the magnetic topology of the solar corona and the incidence of severe space-weather events are outlined briefly.

Keywords: Sunspots, neural networks, long-term change, non-linear, lifetime, Greenwich, sunspot nests, sunspot nestlet,

1. Introduction

The influence of the Sun on the Earth has attracted renewed attention in the context of climate change (Friis-Christensen and Svensmark, 1997). Sunspots

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are a good measure of solar activity and have been observed systematically for hundreds of years. The Royal Greenwich Observatory (RGO) began an effort to record the position and size of sunspot groups in 1874 and maintained this programme of solar observations until the end of 1976. The resulting dataset is unrivalled in its longevity and homogeneity.

Several attempts to model the total quantity of solar radiation arriving at the Earth (the total solar irradiance) have been undertaken using various indices (Lean, Beer, and Bradley, 1995; Fligge and Solanki, 1997; Balmaceda, Krivova, and Solanki, 2007). Some of these attempts have relied on the measurements of sunspot number, since this index extends back for a few centuries. Modern high resolution imaging and measurement of sunspot properties are of limited use because the characteristic times of solar change, on top of the 22-year solar cycle, are expected to take place on centennial time scales (Blanter et al., 2006).

Studies of the temporal properties of sunspot groups (lifetime, maximum size, heliographic position, etc) are hampered by two factors: short-lived sunspot groups may be missed due to nightfall (Solanki, 2003) and the rotation of the Sun carries groups out of view from an Earth-bound observer. In addition, the effects of fore-shortening and limb darkening will hamper reliable observation away from the central meridian (Pierce and Slaughter, 1977).

In order to quantify the limits of reliable observation of sunspot groups within the GPR, the longitude distribution of the apparent maximum size is presented in Figure 1. From this illustration one can conservatively conclude that observations at distances greater than 60° from the central meridian are difficult. This result is consistent with theoretical findings (Kopecký, 1985).

Sunspot groups that are recorded at any stage with a central meridian distance < -60° or > +60° are classified as ‘unreliably observed’. This subset is named GPR(unreliable) herein.

The considerable asymmetry of the distribution in Figure 1 may be explained by a combination of sunspot group decay rate and recurrence. This matter is briefly treated by Henwood (2008). However, a detailed investigation is outside the intended scope of this paper.

Blanter et al. (2006), performed a non-linear study of short-term correlation properties of solar activity in order to reveal long-lifetime variations. This method was applied to the GPR and an increase of lifetime by a factor of 1.4 was observed from 1915 to 1940. A dataset of sunspot group lifetimes that are not truncated by solar rotation would allow direct measurement of lifetime and hence verify the observation of Blanter et al.

In this study a training dataset of recurrent sunspot groups is constructed by hand from longitude-time plots of GPR data. This subset of GPR data is balanced to ensure an equal number of linked and unlinked examples, and presented as training input to a number of feed-forward neural network architectures. Each network is trained using 10-fold cross validation and the network that displays the lowest over or under fitting is selected. The chosen network is then trained and applied to the entire GPR dataset. Finally, a simple filter is applied to the linked dataset and validation is performed.
Figure 1. The longitudinal distance from the central meridian of the Sun at which a sunspot group achieves maximum wholespot size. Error limits are the width of the bins (5°) in the x-direction and the square-root of the number of elements in the y-direction. GPR data are filtered to remove sunspot groups with only one observation, since these do not make meaningful maximum-size contributions. The resulting GPR data contain a maximum at around -75°. This is attributed to long-lived spots which are growing as they reach the west limb of the Sun or declining as they appear over the east limb of the Sun. The χ² value for this distribution gives a high statistical significance (>99%) to the departure from uniformity. The declining measurements within the grey regions indicate that reliable observations towards the solar limbs are difficult.

2. Method

A meticulous observer can follow a long-lived sunspot group for successive days until it disappears from view over the west limb of the Sun. By taking note of the latitude of the last reliable observation, and with knowledge of the solar rotation period, the observer could wait to see if a sunspot group appeared over the east limb at roughly the same latitude and at the appropriate time. If such a prediction is fulfilled, one may conclude that the same sunspot group has been observed on consecutive rotations and that it should not be recorded as two separate sunspot groups. Ideally, a recurrent sunspot group should be identified as such, possibly by using the same sunspot group number for such recurrent observations. Within the digital version of the GPR, no recurrent information is recorded. Sunspot observations are grouped together and allocated a unique Greenwich group number if the group is observed on consecutive days.

Becker (1955) studied Sonnenfleckenherde, which may be translated as ‘focus of sunspots’. The method employed to identify recurrence in that study was a “statistical method”. Sonnenfleckenherde were defined as “an area on the Sun in which, during a longer period of time, spots appear or are being built”. Size,
heliographic latitude and longitude are all taken into consideration during the search for recurrent groups. This method was applied to GPR data from 1879 to 1941 and 46 Sonnenfleckenherde were catalogued.

Subsequently, Castenmiller, Zwaan, and van der Zalm (1986) introduced the term sunspot nest to describe the same phenomenon on the Sun. They defined nests as evolving within one month, lasting for 6 to 15 rotations and not expanding in latitude or longitude. In their study, the primary means of investigating sunspot nests was by visualising the GPR in heliographic longitude-time diagrams (Figure 2) and recording recurrent spots manually. Practically, Castenmiller, Zwaan, and van der Zalm (1986) required the following data for their analysis: Carrington longitude, latitude and number of “visible days” a group was observed during one solar rotation. This method was applied to the period August 1959 – December 1964 of GPR data, and 41 probable sunspot nests were found.

Within this paper a less stringent concept of a sunspot nest is employed, namely a sunspot nestlet. A nestlet is defined as two or more ‘unreliably’ observed sunspot groups which are linked together because they are likely to be the same group but have different group numbers in the GPR. Hence a nestlet physically corresponds to a white-light sunspot group observed on consecutive solar rota-
tions. It is possible Nestlets are found preferentially in “active longitudes” or “hot spots” zones, but such a study is outside the intended scope of this paper.

The method employed to find all the nestlets within the GPR is as follows:

1. Construct a longitude-time diagram for a chosen solar cycle.
2. By inspection, create a list of recurrent sunspot groups.
3. Use this list to train a neural network.
4. Apply the trained neural network to the whole GPR dataset.
5. Post-process to remove outliers.

2.1. Solar Differential Rotation

Sunspot groups move on the photosphere with different speeds depending on latitude because of differential solar rotation (Schröter, 1985; Thompson et al., 1996). According to the annals published by the Royal Greenwich Observatory, “it should be noted that longitudes are based on the ephemeris given in the Astronomical Ephemeris, assuming a solar rotation period constant at all latitudes” (Royal Greenwich Observatory, 1980). Hence, after an interval of one rotation, recurring groups would be expected to show a drift in longitude due to differential solar rotation. Using the analysis of solar rotation from GPR data, performed by Balthasar, Vázquez, and Wöhl (1986), one should expect a group at an extreme latitude of 35° to exhibit a backwards drift of at most 1°day⁻¹ with respect to the equator. This would correspond to a drift of not more than 18° during the whole unreliable passage (18 days). The figure of 18 days is derived using a synodic solar rotation period of 27 days and an unseen passage of 180° (the far-side of the Sun) + the unreliable regions of the near side of the Sun (2 × 30°). This gives: \( \frac{2}{3} \times 27 \text{ days} = 18 \text{ days} \).

Figure 4 demonstrates that sunspot groups rarely move more than 5° in latitude and 15° in longitude over the duration of their observed life. Such movement appears to be independent of solar cycle development. Hence the worst case of 18° longitude drift due to differential rotation is uncommon.

2.2. Presenting GPR Data to the Neural Network

The purpose of the proceeding approximate analysis of sunspot movement is to provide boundary conditions when converting GPR(unreliable) data into a form suitable for the neural network. This process consists of two stages. First, selecting groups that are possibly linked. Second, reducing each possible linkage into a suitable network input.

For the first stage, the group movement metrics calculated above provide approximate boundary conditions when considering if any two sunspot groups could be linked. It is asserted here that for such a link to exist from an initial group, the linked group (post group) must appear within latitude and longitude bounds of ±15° and ±50° respectively. These bounds are at the extreme end of observed group movement against an average solar rotation (Section 2.1).

In addition, a post group must appear in the future (with respect to the initial group) and within a time window that starts when the first longitude bound appears at the east limb of the Sun, and ends when the last longitude
bound appears at the east limb of the Sun. These bounds are calculated using an ephemeris (Meeus, 1991). A deliberately generous window is chosen because decision making regarding recurrence is performed by the neural network, not during pre-processing of the data.

Every group which is ever observed near the unreliable west limb of the Sun has this first stage applied to it and a set of potential linked candidates are created. Once a pair of potential linked candidates have been identified, they are encoded for presentation to the neural network. In the first step, the two groups which make up the linking candidates are classified as an “initial group” and a “post group”. The “initial group” is the one that is observed first in time and disappears over the west limb of the Sun. The “post group” is the one that is observed later in time, and emerges over the east limb of the Sun.

The encoding of initial and post groups takes place as follows. The observation closest to the west limb is found for the initial group. For each observation in time, sunspot groups are quantised into a given number of bins for both latitude and longitude. Longitude and latitude are encoded separately. For the initial group, five latitude bins and seven longitude bins are used. The first observation of the group is placed in the middle bin (1) with the other bins at this time step set to zero (Figure 3, line 11 for longitude and line 27 for latitude). Tracking back from the west limb of the Sun, time steps are assumed to be $\frac{180}{15}^\circ$ /day (Figure 3, lines 11 – 17 and lines 27 – 33). This time step corresponds to the typical observed movement of a group due to solar rotation. If no group is observed during a particular time step, all the bins are set to zero (0). Subsequent group observations are placed in bins relative to the previous observation of the group. Bin widths are $1/2^\circ$ and $1/7^\circ$ for longitude and latitude respectively. This process is repeated for seven time steps and is illustrated for group numbers 8929 and 8965 in Figure 3. Uncoded groups are visualised on the left, with the corresponding network encoding on the right.

The process for treatment of the post group is similar. For the post group, 9 latitude bins and 21 longitude bins are used (Figure 3, lines 19 – 25 and lines 3 – 9). Again, seven observations in time are made. In the case of the post group, however, all binning is performed relative to the last observed longitude of the initial group. Extreme deviations are limited at the bin furthest from the middle bin.

2.3. Constructing Training Data

In this study, the judgment of the first author (Henwood, 2008) was initially used to decide if any two sunspot groups were close enough to represent a recurrent group. The dataset was balanced and a feed-forward neural network system was then trained with these judgments. In this context, a balanced dataset is one with an equal number of positive and negative examples. The system generalises from the examples presented to it and captures the uncertainty of the task. When a previously unseen case of possible recurrence is introduced, the trained neural network provides a probability that this is indeed a case of recurrence.

Solar cycle 15 (August 1913 – August 1923) has been selected as the period to identify recurrence manually, since this cycle is neither the longest nor the
Figure 3. Schematic example of encoding a link between two groups a (initial) and b (post). “Zeros” and “ones” signify the absence and presence (respectively) of a sunspot group in a longitude or latitude bin. The moderate variation in longitude is visible in the encoded data (lines 11–17). The post link group is encoded with observations nearest the limb ranked highest. Thus the ninth line of the data representation encodes the longitude of group b observed closest to the east limb. The line of “ones” to the right of centre indicates that this group appears at a greater longitude than the initial group. Similarly, latitude indicated by colour is encoded in lines 27–33 for group a and lines 19–25 for group b. Line 37 encodes the judgement on whether or not the two groups shown are the same recurrent group; accordingly, ‘zero’ is false, ‘one’ is true. Line 35 includes the Greenwich group numbers of the two groups. This is not used in decision making, it is used to identify groups after the neural network decision making process is complete.

shortest. It can also be described as a period when the overall sunspot group number is neither particularly large nor small. In short, this cycle is chosen because it is “typical”.

Only sunspot groups in the GPR(unreliable) dataset are considered for recurrence. Pairs of groups which appear to be linked by recurrence are selected for the entire ten years of GPR(training). The training data, GPR(training), consists of 4073 examples, 621 of which are “linked” or probability = 1, the remaining are “unlinked” or probability = 0. Such a dataset, which contains unequal numbers of true and false examples, is called “unbalanced”. Provost (2000) highlights the problems associated with using unbalanced data with standard machine learning algorithms. Fawcett (2004) suggests the use of receiver operating characteristic
Figure 4. Extreme deviation is measured for all sunspot groups within the GPR that are observed to exist for more than ten days. The extreme deviation is measured as the difference between the absolute maximum and minimum recorded heliographic positions (“maximum deviation”, \(x\)-direction). Deviation counts are binned into solar cycle bins. Each solar cycle is divided into nine bins (“solar cycle bin”, \(y\)-direction). Deviation counts are normalised (“normalised occurrence”, \(z\)-direction). The extreme latitude deviation (top) illustrates that groups commonly move a few degrees but rarely more than five degrees. Extreme longitude (bottom) deviations indicate a greater spread but with groups rarely moving more than 15°.

In this study, the training dataset is balanced using two techniques: firstly, the addition of random noise (switching a single random adjacent bit in the network representation) and secondly a technique based on a priori knowledge of the problem domain. This second technique uses the following assumption: If one accepts that a given pair of groups are linked, any pair which have a smaller unseen latitude and/or longitude deviation must also be linked. Hence, new linked groups can be created from existing groups by re-encoding a linked pair of groups with a reduced sensitivity than described in Section 2.2. Using these two techniques, the training data is balanced to produce 3756 positive examples and 3827 negative examples.

Ten-fold cross validation (Kohavi, 1995) has been used during training to provide the learning and error rates. These are measured to identify network designs that exhibit overfitting or underfitting (Moore, 2001). Overfitting is detected by observing that the error rate does not increase as learning is taking place. Under-fitting is detected by observing that learning has approached a stable (ROC) curves to evaluate a classifier trained on unbalanced data, while Lawrence et al. (1998) provide a number of suggestions in order to balance unbalanced data.
minimum. Six network designs were constructed with varying numbers of hidden layers and interconnects. These designs are documented in the Master’s Thesis of Henwood (2008), which also provides an assessment of the performance of each network. The trained network that exhibits the least overfitting or underfitting is presented with the GPR(unreliable) dataset and, after classification, returns a new dataset with recurrent sunspots grouped and labelled with the group number of the first observed sunspot group. This dataset is called GPR(linked) and contains 5374 group linkages.

A comparison of GPR(training) with GPR(linked) was then performed by hand. This revealed 20 or so linked groups identified by the neural network that exhibited a large deviation of latitude or longitude during unseen passage. We judged these to be physically unlikely but since there is no absolute criterion for this classification these could in principle be identified as linked groups by another observer. We have chosen a criterion with which to filter the data.

A filter was constructed which removed linked groups that exhibited a large latitude deviation (> 8.5°) or the post link was not observed near to the East limb, which corresponds to an large (> 19.5°) longitude deviation. This filter removed 17 linkages (3%) from GPR(training) and 244 (5%) linkages from GPR(linked). This suggests that the majority of the “physically unlikely” groups were not a result of overfitting or underfitting but are the result of subjectivity during selection of linked groups for the training dataset GPR(training).

A final dataset, GPR(linked) was produced by applying the filter described above to GPR(linked). The result is a dataset with contentious linkages removed. This dataset is used throughout the remainder of this study and is available at the UK Solar System Data Centre (http://www.ukssdc.ac.uk/wdcc1/.greenwich/recurrence).

3. Comparison with Manual Datasets

The UK Solar System Data Centre (http://www.ukssdc.ac.uk) maintains a complete set of annals that were published by the Royal Greenwich Observatory; these provided the source for the GPR. As an appendix to the Greenwich observations, a “Catalogue of Recurrent Groups of Sun Spots, 1874 – 1906” was compiled by A.S.D. Maunder and published in 1909. Between 1916 and 1955, the “Ledgers of Groups of Sunspots” included two sections: “Recurrent” and “Non-Recurrent”. Recurrent groups were also identified between 1907 and 1915 but tabulation of the information during that time was different.

While a digital version of these records is not known to exist, the method used to compile recurrent spots is documented (in the Greenwich Photographic Results, 1956):

Recurrent groups were selected upon the following plan, reference being made to the General Catalogue:- If any spot when first seen was 60° or more to the east of the central meridian, the catalogue and, if necessary, the Daily Results also, were searched some fifteen to sixteen days earlier to ascertain whether a spot group of similar heliographic longitude and latitude was then near the west limb of the Sun. Similarly, if any spot group when last seen was 60° or
more to the west of the central meridian, a search was made fifteen to sixteen
days later. When there appeared to be a case of probable continuity between
groups in consecutive rotations of the Sun, the character of the groups, their
areas and their longitude and latitude have been carefully compared before
accepting them as recurrent groups.

Between 1874 and 1906, Maunder catalogued 624 recurring groups: 468 were
seen only in two rotations, 118 appeared in three rotations, 25 in four rotations,
12 in five rotations, and 1 (somewhat doubtfully) in six rotations.

The process of verification addresses the question of whether or not the work
described in this paper has been completed correctly. One approach is to compare
GPR(linked) with the recurrence observations that were published by the Royal
Greenwich Observatory. Recurrence data for 1896 and 1958 were typed in and
compared with GPR(linked). For interest, the performance of the neural network
against the training data, GPR(training), was also evaluated.

Table 1 compares these three different linked sunspot group datasets with
GPR(linked) using metrics for a confusion matrix (Kohavi and Provost, 1998).
True positive counts are links which are in both the manual and GPR(linked)
datasets. False positives are links which are only in GPR(linked). False negatives
are links which are in the manual dataset but not in GPR(linked). True negatives
are defined as: \( TN = \text{Total} - TP - FP - FN \).

By rapidly constructing longitude-time diagrams that correspond to the pe-
riod of interest the instances of false classification have been investigated individ-
ually. As may be expected, GPR(training) data performs the best overall. It does
not achieve 100% success compared to the human classification. On inspection
of the “False Positive” classifications made by the neural network, all but one of
these linkages were discovered to be physically likely linkages which the human
classifier had overlooked. During inspection by longitude–time diagrams it was
observed that these “False Positive” results occurred in dense complexes of many
groups. The neural network out-performs the human under these conditions.

The Royal Greenwich Observatory 1958 recurrence records show the highest
number of false positives. On investigating the set of examples classified as false
positives, it was observed that these were again a result of a complication arising
during observation of longitude–time diagrams. It should be noted that 1958 was a year during which the sunspot number reached a record high. This level of activity corresponds to a greater number of observations to display on a longitude–time diagram, which increases the challenge to a human of identifying all potentially linked groups.

In addition, the method used during 1958 was restricted to providing only one link between two given groups. However, the neural network classifier could find multiple linkages between groups. Finally, the neural network classifier could also make a link between groups even if one of the groups was only visible for a single day. Such a linkage is apparently not permitted within the 1958 recurrence dataset.

The RGO 1896 recurrence records showed the greatest discrepancy from the neural network linked dataset. This is probably because the criteria used to identify a recurrent group are different from both the 1958 and the GPR(training) datasets. Maunder used heliographic position, allowing for a sunspot group to remain unseen for some days and marking it as recurrent if another group was observed at the same heliographic position. The method employed here is to require a sunspot group to meet the “unreliable observed” criteria. Some of the groups marked as recurrent by Maunder failed to be seen within 30° of the relevant limb. It should also be noted that 1958 was a year of considerable activity on the Sun, whereas 1896 was relatively quiet, reducing the opportunity to observe recurrence.

4. Gnevyshev–Waldmeier Rule within Recurrent Groups

After classifying recurrence, one might expect the GPR(linked, reliable) dataset to exhibit some of the well established physical characteristics of non-recurrent sunspot groups. The Gnevyshev–Waldmeier rule (Gnevyshev, 1938; Waldmeier, 1955) states that sunspot group maximum area ($A_0$) and lifetime ($T$) are proportional:

$$A_0 = \bar{D}_{GW} T \approx 10 \text{ MSH day}^{-1}.$$  \hspace{1cm} (1)

where $\bar{D}_{GW}$ is the constant of proportionality measured in millionths of the solar hemisphere (MSH) per day.

In their paper on sunspot decay, Petrovay and van Driel-Gesztelyi (1997) used the Debrecen recurrence dataset (Dezső, Gerlei, and Kovács, 1987; Dezső, Gerlei, and Kovács, 1997). They choose to use groups which “were born on the visible hemisphere and also died there”, so that their lifetimes could be determined accurately. Since only one group satisfied this criterion, they applied the Gnevyshev–Ringnes correction to the remaining recurrent groups to make a total dataset of 128 groups. The Gnevyshev–Ringnes correction provides the probability that birth and death will occur in the visible hemisphere. After binning, they found a least squares linear fit of $\bar{D}_{GW} = 10.89 \pm 0.48 \text{ MSH day}^{-1}$.

There are 841 groups in GPR(linked, reliable) that are reliably observed according to the criteria that both the birth and death of a group must take
Figure 5. Recurrent sunspot group lifetime (measured in days) plotted against maximum wholespot size measured in millionths of the solar hemisphere (MSH). Lifetime is measured as the duration between first and last observation of a sunspot group. The GPR(linked, reliable) dataset contains 841 observations and a linear fit of $D_{GW} = 11.73 \pm 0.26$ is found. The data are divided into three age categories (grp 1, grp 2 and grp 3) and the centre of mass of each is indicated with a cross. The error bounds are marked by dashed lines and $D_{GW} = 10$ is included for comparison.

place within $\pm 60^\circ$ of the solar central meridian. Using reliably observed groups means the Gnevyshev–Ringnes correction is not required. Figure 5 shows a linear fit through GPR(linked, reliable) of $D_{GW} = 11.73 \pm 0.26$.

All of the lifetimes within GPR(linked, reliable) are accurate to within one day. The corresponding measurement of group maximum size is subject to some uncertainty because rotation of the Sun carries each recurrent group out of sight for a portion of its lifetime. The effect is that the value observed as the maximum is either the true maximum or a smaller value.

The data show a large scatter around the linear fit (Figure 5). Regions between the bands of points (where no reliable observation is possible because either the birth or death of the group is unseen) make it somewhat difficult to determine the linear relationship. Three age categories are defined as follows: grp 1, ages 17 to 45 days; grp 2, 46 to 72 days; grp 3, greater than 72 days. The centre of mass of each of these groups is indicated by a cross. In addition, since only recurrent sunspot groups are examined, there are no data points between the origin and $\approx 18$ days.

Compared to the data analysed by Petrovay and van Driel-Gesztelyi (1997), the GPR(linked, reliable) data are more numerous. In addition, the GPR(linked, reliable) data do not have a limit on the maximum size measured and only
5. Temporal Variations of Sunspot Group Lifetime

The GPR(linked) dataset presents a unique opportunity to investigate changes of sunspot lifetime with time. Previous studies of this property were complicated for the reasons already outlined; namely, incomplete recurrence data compiled by different individuals using different criteria.

Blanter et al. (2006) considered the topic of sunspot lifetime. They performed a nonlinear study of the short-term correlation properties of solar activity in order to reveal their long-lifetime variations. Their method was applied to GPR(unreliable) and allows for the problems associated with recurrence within the dataset and short-lived sunspots. These authors mitigate such factors by examining the population of sunspot lifetimes that are between 1 and 15 days. These observations were used to create a 22-year running averaged series.

One of the conclusions of the study by Blanter et al. (2006) was that they found sunspot lifetime of GPR(unreliable) had increased over the duration of the dataset. In addition, they were able to quantify the change in lifetime, which increased by a factor of 1.4 over the interval from 1915 to 1940. During this period, GPR(linked) indicates a change from below to above average lifetimes, as shown in Figure 6.

Blanter et al. (2006) discuss the problems associated with both small sunspot groups, whose lifetime cannot be perfectly measured, and the lack of observations from the invisible side of the Sun. Because of this, they developed a technique that used sunspot group size and the Gnevyshev–Waldmeier relationship to infer sunspot group lifetimes. Figure 6 presents GPR data (grey pattern region) and GPR(linked) (grey solid region) from our study.

Large errors are observed in the calculation of lifetime from GPR data containing recurrent groups. GPR(linked) alone contains much reduced error and a trend can be observed. While GPR(linked) only contains groups that have a sufficiently long lifetime to be observed on more than one solar rotation, there are proportionally fewer such groups, which reduces the lifetime average during the sample window.

GPR(linked) data shown in Figure 6 indicate a marked increase between 1910 and 1950. Blanter et al. (2006) found that sunspot lifetime had increased by a factor of 1.4 between 1915 and 1940. The results presented here largely agree with that value. In addition, Figure 6 suggests that the increase may extend over a longer interval and also augments the work of Blanter et al. (2006) by introducing an uncertainty measure.
Figure 6. Lifetime of sunspot groups versus time, calculated for the longest-lived sunspot on a given day for 22-year (8035 days) moving averages. The GPR age estimate takes no account of recurrent sunspot groups and is plotted as a grey pattern contained within the bounds of error. GPR(linked) is plotted as a dark grey solid region containing the bounds of uncertainty. Error is calculated as the longest and shortest lifetime estimates of a sunspot group lifetime. Longest lifetime assumes that group birth was at the earliest possible unseen time and death was at the latest possible unseen time. Shortest lifetime is calculated assuming the earliest observation in GPR is the birth and the latest observation in GPR is the death. Variations in sunspot number are shown to indicate individual solar cycles.

6. Conclusions

Neural networks have been applied previously to various problems in space science. In particular, they have been used in the prediction of geomagnetic phenomena (Lundstedt, 1992; Lundstedt and Wintoft, 1994; Calvo, Ceccato, and Piacentini, 1995; Gleisner, Lundstedt, and Wintoft, 1996; Williscroft and Poole, 1996; Wu and Lundstedt, 1996; Weigel et al., 1999), the classification of asteroid spectra (Howell, Merényi, and Lebofsky, 1994), and ionogram processing (Galkin et al., 1996). In addition, they have also been applied to the important problem of automatically classifying sunspots from data obtained by processing SOHO/MDI satellite images (Nguyen et al., 2006). However, the authors are not aware of the previous use of neural networks in studies of recurrent sunspot groups.

It has been shown in this study that it is possible to train a neural network to identify recurrent sunspot groups within the Greenwich Photoheliographic Results (GPR), which extend over the long interval 1874 – 1976 (Section 2). Once trained, the neural network can often outperform a human classifier, particularly when a large number of sunspot groups are present on the solar disc at the same time/years.
time. Since the neural network performs deterministically when classifying a pair of linked sunspot groups, it operates with a consistency of judgement that exceeds the sterling endeavours of various individuals over more than a century.

Nevertheless, the neural network method has some limitations. These are most clearly revealed when considering false-positive linkages (Section 3). In such cases, the black-box nature of the decision process becomes problematic and confidence in the neural network is partially undermined. In the method discussed in this paper, post-processing of the data is used to reduce false-positive linkages but in any future study it might be valuable to investigate alternative machine learning systems.

Despite these limitations, the particular choice of network and data reduction procedure employed in this study could also be applied to Rome Daily Sunspot Reports (1958 – 2000), USSR Station Data (1968 – 1991), Mount Wilson Individual Sunspot Data (1917 – 1985), Kodaikanal Individual Sunspot Data (1906 – 1987) and Greenwich/Debrecen Observations (1874 – 2007), without re-training the existing neural network. All of these datasets are available through either the National Geophysical Data Centre (http://www.ngdc.noaa.gov) or the UK Solar System Data Centre (http://www.ukssdc.ac.uk).

The constants of proportionality in the Gnevyshev–Waldmeier rule \((A_0 = \bar{D}_{GW}T)\) derived in this study (Section 4) have been found to be greater than previous estimates. Petrovay and van Driel-Gesztelyi (1997), who used information on recurrent sunspot groups extracted from the Debrecen Photoheliographic Results, also found a value of \(\bar{D}_{GW}\) that was larger \((10.89 \text{ MSH day}^{-1})\) than previous estimates. In a study of sunspot group lifetimes, Zuccarello (1993) presented results which show a change in the rotation rate between short-lived and long-lived (11-day old) sunspot groups, pointing to a difference in “aggregation capability” of the group within the convection zone. The present results suggest that there may be some additional physics present within longer lived groups in the GPR(linked, reliable) dataset and thereby imply that this matter warrants further investigation.

Evidence has been found for an increase in the lifetime of recurrent sunspot groups by a factor of about 1.4 between 1915 and 1940 (Section 5), which is in excellent agreement with the result obtained by Blanter et al. (2006). Indeed, this increase in lifetime actually occurs over a longer period \((1915 – 1950)\) than previously thought and there is also provisional evidence for a slight decrease in lifetime between 1950 and 1965 (see Figure 6).

Solar changes over periods longer than a few decades are currently of considerable interest as the solar output is a significant input to climate models (Haigh et al., 2005). The Gleissberg cycle is detected in sunspot number and has a measured period of approximately 80-120 years (Gleissberg, 1967; Hoyt, Schatten, and Nesmes-Ribes, 1994; Garcia and Mouradian, 1998). Garcia and Mouradian (1998) estimated the most recent Gleissberg cycle minima to be around 1900 and the maxima around 1965. The results presented here suggest that, if the lifetimes of recurrent sunspot groups are a good proxy for the Gleissberg cycle, the maxima occurred some years before 1965 during the 1950s. Further study of this topic would be facilitated by applying the trained neural network to sunspot data in the interval 1977 – 2009.
The analysis of historical sunspot observations using automated methods, such as the neural network method presented in this paper, is required to estimate the (true) total number of sunspots emerging on the solar surface during the solar cycle. Numerical simulations of the centennial evolution of magnetic flux on the solar surface scale the sunspot emergence rate to the total sunspot number, measured without considering recurrent sunspots and their variable lifetimes in great detail (Wang, Lean, and Sheeley, 2002). A sunspot number that corrects for the recurrence of long-lived sunspots may improve the numerical predictions by providing a better description of emergence. The numerical simulations over centennial scales could be compared to the most recent estimates of long-term variation of the open magnetic flux on the solar surface (Rouillard, Lockwood, and Finch, 2007). Neural networks could also be optimised to detect the location of the umbra and penumbra of sunspots and to estimate the total magnetic flux inside these large-scale active regions. Such a rough estimate of the magnetic flux on the photosphere could then be used to estimate the magnetic topology of the corona using simplified potential field source surface models.

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Increasing Lifetime of Recurrent Sunspot Groups within the Greenwich Photoheliographic Results

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Abstract

Long-lived (>20 days) sunspot groups extracted from the Greenwich Photoheliographic Results (GPR) are examined for evidence of decadal change. The problem of identifying sunspot groups which are observed on consecutive solar rotations (recurrent sunspot groups) is tackled by first constructing manually an example dataset of recurrent sunspot groups and then using machine learning to generalise this subset to the whole GPR. The resulting dataset of recurrent sunspot groups is verified against previous work by A. Maunder and other Royal Greenwich Observatory (RGO) compilers. Recurrent groups are found to exhibit a slightly larger value for the Gnevyshev-Waldmeier Relationship than the value found by Petrovay and van Driel-Gesztelyi (Solar Phys. 51, 25, 1997), who used recurrence data from the Debrecen Photoheliographic Results. Evidence for sunspot group lifetime change over the previous century is observed within recurrent groups. A lifetime increase of 1.4 between 1915 and 1940 is found, which closely agrees with results from Blanter et al. (Solar Phys. 237, 329, 2006). Furthermore, this increase is found to exist over a longer period (1915 to 1950) than previously thought and provisional evidence is found for a decline between 1950 and 1965. Possible applications of machine-learning procedures to the analysis of historical sunspot observations, the determination of the magnetic topology of the solar corona and the incidence of severe space-weather events are outlined briefly.

Keywords: Sunspots, neural networks, long-term change, non-linear, lifetime, Greenwich, sunspot nests, sunspot nestlet,

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1. Introduction

The influence of the Sun on the Earth has attracted renewed attention in the context of climate change (Friis-Christensen and Svensmark, 1997). Sunspots are a good measure of solar activity and have been observed systematically for hundreds of years. The Royal Greenwich Observatory (RGO) began an effort to record the position and size of sunspot groups in 1874 and maintained this programme of solar observations until the end of 1976. The resulting dataset is unrivalled in its longevity and homogeneity.

Several attempts to model the total quantity of solar radiation arriving at the Earth (the total solar irradiance) have been undertaken using various indices (Lean, Beer, and Bradley, 1995; Fligge and Solanki, 1997; Balmaceda, Krivova, and Solanki, 2007). Some of these attempts have relied on the measurements of sunspot number, since this index extends back for a few centuries. Modern high resolution imaging and measurement of sunspot properties are of limited use because the characteristic times of solar change, on top of the 22-year solar cycle, are expected to take place on centennial time scales (Blanter et al., 2006).

Studies of the temporal properties of sunspot groups (lifetime, maximum size, heliographic position, etc) are hampered by two factors: short-lived sunspot groups may be missed due to nightfall (Solanki, 2003) and the rotation of the Sun carries groups out of view from an Earth-bound observer. In addition, the effects of fore-shortening and limb darkening will hamper reliable observation away from the central meridian (Pierce and Slaughter, 1977).

In order to quantify the limits of reliable observation of sunspot groups within the GPR, the longitude distribution of the apparent maximum size is presented in Figure 1. From this illustration one can conservatively conclude that observations at distances greater than 60° from the central meridian are difficult. This result is consistent with theoretical findings (Kopecký, 1985).

Sunspot groups that are recorded at any stage with a central meridian distance < -60° or > +60° are classified as ‘unreliably observed’. This subset is named GPR(unreliable) herein.

The considerable asymmetry of the distribution in Figure 1 may be explained by a combination of sunspot group decay rate and recurrence. This matter is briefly treated by Henwood (2008). However, a detailed investigation is outside the intended scope of this paper.

Blanter et al. (2006), performed a non-linear study of short-term correlation properties of solar activity in order to reveal long-lifetime variations. This method was applied to the GPR and an increase of lifetime by a factor of 1.4 was observed from 1915 to 1940. A dataset of sunspot group lifetimes that are not truncated by solar rotation would allow direct measurement of lifetime and hence verify the observation of Blanter et al..

In this study a training dataset of recurrent sunspot groups is constructed by hand from longitude-time plots of GPR data. This subset of GPR data is balanced to ensure an equal number of linked and unlinked examples, and presented as training input to a number of feed-forward neural network architectures. Each network is trained using 10-fold cross validation and the network that displays the lowest over or under fitting is selected. The chosen network is then trained
Figure 1. The longitudinal distance from the central meridian of the Sun at which a sunspot group achieves maximum wholespot size. Error limits are the width of the bins (5°) in the x-direction and the square-root of the number of elements in the y-direction. GPR data are filtered to remove sunspot groups with only one observation, since these do not make meaningful maximum-size contributions. The resulting GPR data contain a maximum at around -75°. This is attributed to long-lived spots which are growing as they reach the west limb of the Sun or declining as they appear over the east limb of the Sun. The χ² value for this distribution gives a high statistical significance (>99%) to the departure from uniformity. The declining measurements within the grey regions indicate that reliable observations towards the solar limbs are difficult.

and applied to the entire GPR dataset. Finally, a simple filter is applied to the linked dataset and validation is performed.

2. Method

A meticulous observer can follow a long-lived sunspot group for successive days until it disappears from view over the west limb of the Sun. By taking note of the latitude of the last reliable observation, and with knowledge of the solar rotation period, the observer could wait to see if a sunspot group appeared over the east limb at roughly the same latitude and at the appropriate time. If such a prediction is fulfilled, one may conclude that the same sunspot group has been observed on consecutive rotations and that it should not be recorded as two separate sunspot groups. Ideally, a recurrent sunspot group should be identified as such, possibly by using the same sunspot group number for such recurrent observations. Within the digital version of the GPR, no recurrent information is recorded. Sunspot observations are grouped together and allocated a unique Greenwich group number if the group is observed on consecutive days.
Figure 2. A longitude-time diagram of GPR (unreliable) groups for the last six months of 1935. Time is in the \( x \)-direction, longitude in the \( y \)-direction. GPR (unreliable) groups are displayed as a connected series of coloured circles. Colour corresponds to latitude (right-hand colour bar); radius indicates size, corrected for foreshortening and measured in millionths of the solar hemisphere. Example radii are provided for sunspot sizes of 1000, 2000, 3000 and 4000 millionths of the solar hemisphere (left of colour bar). Slanting grey dots mark the Carrington longitude of the east and west limb of the Sun.

Becker (1955) studied Sonnenfleckenherde, which may be translated as ‘focus of sunspots’. The method employed to identify recurrence in that study was a “statistical method”. Sonnenfleckenherde were defined as “an area on the Sun in which, during a longer period of time, spots appear or are being built”. Size, heliographic latitude and longitude are all taken into consideration during the search for recurrent groups. This method was applied to GPR data from 1879 to 1941 and 46 Sonnenfleckenherde were catalogued.

Subsequently, Castenmiller, Zwaan, and van der Zalm (1986) introduced the term sunspot nest to describe the same phenomenon on the Sun. They defined nests as evolving within one month, lasting for 6 to 15 rotations and not expanding in latitude or longitude. In their study, the primary means of investigating sunspot nests was by visualising the GPR in heliographic longitude-time diagrams (Figure 2) and recording recurrent spots manually. Practically, Castenmiller, Zwaan, and van der Zalm (1986) required the following data for their analysis: Carrington longitude, latitude and number of “visible days” a group was observed during one solar rotation. This method was applied to the period August 1959 – December 1964 of GPR data, and 41 probable sunspot nests were found.

Within this paper a less stringent concept of a sunspot nest is employed, namely a sunspot nestlet. A nestlet is defined as two or more ‘unreliably’ observed
sunspot groups which are linked together because they are likely to be the same
group but have different group numbers in the GPR. Hence a nestlet physically
corresponds to a white-light sunspot group observed on consecutive solar rota-
tions. It is possible Nestlets are found preferentially in “active longitudes” or
“hot spots” zones, but such a study is outside the intended scope of this paper.

The method employed to find all the nestlets within the GPR is as follows:

1. Construct a longitude-time diagram for a chosen solar cycle.
2. By inspection, create a list of recurrent sunspot groups.
3. Use this list to train a neural network.
4. Apply the trained neural network to the whole GPR dataset.
5. Post-process to remove outliers.

2.1. Solar Differential Rotation

Sunspot groups move on the photosphere with different speeds depending on
latitude because of differential solar rotation (Schröter, 1985; Thompson et al.,
1996). According to the annals published by the Royal Greenwich Observatory,
“it should be noted that longitudes are based on the ephemeris given in the
Astronomical Ephemeris, assuming a solar rotation period constant at all lat-
itudes” (Royal Greenwich Observatory, 1980). Hence, after an interval of one
rotation, recurring groups would be expected to show a drift in longitude due to
differential solar rotation. Using the analysis of solar rotation from GPR data,
performed by Balthasar, Vázquez, and Wöhl (1986), one should expect a group
at an extreme latitude of 35° to exhibit a backwards drift of at most 1°day⁻¹
with respect to the equator. This would correspond to a drift of not more than
18° during the whole unreliable passage (18 days). The figure of 18 days is
derived using a synodic solar rotation period of 27 days and an unseen passage
of 180° (the far-side of the Sun) + the unreliable regions of the near side of the
Sun (2 × 30°). This gives: \( \frac{2}{7} \times 27 \text{ days} = 18 \text{ days}. \)

Figure 4 demonstrates that sunspot groups rarely move more than 5° in lati-
tude and 15° in longitude over the duration of their observed life. Such movement
appears to be independent of solar cycle development. Hence the worst case of
18° longitude drift due to differential rotation is uncommon.

2.2. Presenting GPR Data to the Neural Network

The purpose of the proceeding approximate analysis of sunspot movement is
to provide boundary conditions when converting GPR(unreliable) data into a
form suitable for the neural network. This process consists of two stages. First,
selecting groups that are possibly linked. Second, reducing each possible linkage
into a suitable network input.

For the first stage, the group movement metrics calculated above provide
approximate boundary conditions when considering if any two sunspot groups
could be linked. It is asserted here that for such a link to exist from an initial
group, the linked group (post group) must appear within latitude and longitude
bounds of ±15° and ±50° respectively. These bounds are at the extreme end of
observed group movement against an average solar rotation (Section 2.1).
In addition, a post group must appear in the future (with respect to the initial group) and within a time window that starts when the first longitude bound appears at the east limb of the Sun, and ends when the last longitude bound appears at the east limb of the Sun. These bounds are calculated using an ephemeris (Meeus, 1991). A deliberately generous window is chosen because decision making regarding recurrence is performed by the neural network, not during pre-processing of the data.

Every group which is ever observed near the unreliable west limb of the Sun has this first stage applied to it and a set of potential linked candidates are created. Once a pair of potential linked candidates have been identified, they are encoded for presentation to the neural network. In the first step, the two groups which make up the linking candidates are classified as an “initial group” and a “post group”. The “initial group” is the one that is observed first in time and disappears over the west limb of the Sun. The “post group” is the one that is observed later in time, and emerges over the east limb of the Sun.

The encoding of initial and post groups takes place as follows. The observation closest to the west limb is found for the initial group. For each observation in time, sunspot groups are quantised into a given number of bins for both latitude and longitude. Longitude and latitude are encoded separately. For the initial group, five latitude bins and seven longitude bins are used. The first observation of the group is placed in the middle bin (1) with the other bins at this time step set to zero (Figure 3, line 11 for longitude and line 27 for latitude). Tracking back from the west limb of the Sun, time steps are assumed to be 180/15 °/day (Figure 3, lines 11–17 and lines 27–33). This time step corresponds to the typical observed movement of a group due to solar rotation. If no group is observed during a particular time step, all the bins are set to zero (0). Subsequent group observations are placed in bins relative to the previous observation of the group. Bin widths are 1/2° and 1/7° for longitude and latitude respectively. This process is repeated for seven time steps and is illustrated for group numbers 8929 and 8965 in Figure 3. Uncoded groups are visualised on the left, with the corresponding network encoding on the right.

The process for treatment of the post group is similar. For the post group, 9 latitude bins and 21 longitude bins are used (Figure 3, lines 19–25 and lines 3–9). Again, seven observations in time are made. In the case of the post group, however, all binning is performed relative to the last observed longitude of the initial group. Extreme deviations are limited at the bin furthest from the middle bin.

2.3. Constructing Training Data

In this study, the judgment of the first author (Henwood, 2008) was initially used to decide if any two sunspot groups were close enough to represent a recurrent group. The dataset was balanced and a feed-forward neural network system was then trained with these judgments. In this context, a balanced dataset is one with an equal number of positive and negative examples. The system generalises from the examples presented to it and captures the uncertainty of the task. When a previously unseen case of possible recurrence is introduced, the trained neural network provides a probability that this is indeed a case of recurrence.
Figure 3. Schematic example of encoding a link between two groups $a$ (initial) and $b$ (post). “Zeros” and “ones” signify the absence and presence (respectively) of a sunspot group in a longitude or latitude bin. The moderate variation in longitude is visible in the encoded data (lines 11–17). The post link group is encoded with observations nearest the limb ranked highest. Thus the ninth line of the data representation encodes the longitude of group $b$ observed closest to the east limb. The line of “ones” to the right of centre indicates that this group appears at a greater longitude than the initial group. Similarly, latitude indicated by colour is encoded in lines 27–33 for group $a$ and lines 19–25 for group $b$. Line 37 encodes the judgement on whether or not the two groups shown are the same recurrent group; accordingly, ‘zero’ is false, ‘one’ is true. Line 35 includes the Greenwich group numbers of the two groups. This is not used in decision making, it is used to identify groups after the neural network decision making process is complete.

Solar cycle 15 (August 1913 – August 1923) has been selected as the period to identify recurrence manually, since this cycle is neither the longest nor the shortest. It can also be described as a period when the overall sunspot group number is neither particularly large nor small. In short, this cycle is chosen because it is “typical”.

Only sunspot groups in the GPR(unreliable) dataset are considered for recurrence. Pairs of groups which appear to be linked by recurrence are selected for the entire ten years of GPR(training). The training data, GPR(training), consists of 4073 examples, 621 of which are “linked” or probability = 1, the remaining are “unlinked” or probability = 0. Such a dataset, which contains unequal numbers of true and false examples, is called “unbalanced”. Provost (2000) highlights the
Figure 4. Extreme deviation is measured for all sunspot groups within the GPR that are observed to exist for more than ten days. The extreme deviation is measured as the difference between the absolute maximum and minimum recorded heliographic positions ("maximum deviation", x-direction). Deviation counts are binned into solar cycle bins. Each solar cycle is divided into nine bins ("solar cycle bin", y-direction). Deviation counts are normalised ("normalised occurrence", z-direction). The extreme latitude deviation (top) illustrates that groups commonly move a few degrees but rarely more than five degrees. Extreme longitude (bottom) deviations indicate a greater spread but with groups rarely moving more than 15°.

problems associated with using unbalanced data with standard machine learning algorithms. Fawcett (2004) suggests the use of receiver operating characteristic (ROC) curves to evaluate a classifier trained on unbalanced data, while Lawrence et al. (1998) provide a number of suggestions in order to balance unbalanced data.

In this study, the training dataset is balanced using two techniques: firstly, the addition of random noise (switching a single random adjacent bit in the network representation) and secondly a technique based on a priori knowledge of the problem domain. This second technique uses the following assumption: If one accepts that a given pair of groups are linked, any pair which have a smaller unseen latitude and/or longitude deviation must also be linked. Hence, new linked groups can be created from existing groups by re-encoding a linked pair of groups with a reduced sensitivity than described in Section 2.2. Using these two techniques, the training data is balanced to produce 3756 positive examples and 3827 negative examples.

Ten-fold cross validation (Kohavi, 1995) has been used during training to provide the learning and error rates. These are measured to identify network designs
that exhibit overfitting or underfitting (Moore, 2001). Overfitting is detected by observing that the error rate does not increase as learning is taking place. Underfitting is detected by observing that learning has approached a stable minimum. Six network designs were constructed with varying numbers of hidden layers and interconnects. These designs are documented in the Master’s Thesis of Henwood (2008), which also provides an assessment of the performance of each network. The trained network that exhibits the least overfitting or underfitting is presented with the GPR(unreliable) dataset and, after classification, returns a new dataset with recurrent sunspots grouped and labelled with the group number of the first observed sunspot group. This dataset is called GPR(linked\') and contains 5374 group linkages.

A comparison of GPR(training) with GPR(linked\') was then performed by hand. This revealed 20 or so linked groups identified by the neural network that exhibited a large deviation of latitude or longitude during unseen passage. We judged these to be physically unlikely but since there is no absolute criterion for this classification these could in principle be identified as linked groups by another observer. We have chosen a criterion with which to filter the data.

A filter was constructed which removed linked groups that exhibited a large latitude deviation (> 8.5°) or the post link was not observed near to the East limb, which corresponds to an large (> 19.5°) longitude deviation. This filter removed 17 linkages (3%) from GPR(training) and 244 (5%) linkages from GPR(linked\'). This suggests that the majority of the “physically unlikely” groups were not a result of overfitting or underfitting but are the result of subjectivity during selection of linked groups for the training dataset GPR(training).

A final dataset, GPR(linked) was produced by applying the filter described above to GPR(linked\'). The result is a dataset with contentious linkages removed. This dataset is used throughout the remainder of this study and is available at the UK Solar System Data Centre (http://www.ukssdc.ac.uk/wdcc1/greenwich/recurrence)

3. Comparison with Manual Datasets

The UK Solar System Data Centre (http://www.ukssdc.ac.uk) maintains a complete set of annals that were published by the Royal Greenwich Observatory; these provided the source for the GPR. As an appendix to the Greenwich observations, a “Catalogue of Recurrent Groups of Sun Spots, 1874 – 1906” was compiled by A.S.D. Maunder and published in 1909. Between 1916 and 1955, the “Ledgers of Groups of Sunspots” included two sections: “Recurrent” and “Non-Recurrent”. Recurrent groups were also identified between 1907 and 1915 but tabulation of the information during that time was different.

While a digital version of these records is not known to exist, the method used to compile recurrent spots is documented (in the Greenwich Photographic Results, 1956):

Recurrent groups were selected upon the following plan, reference being made to the General Catalogue:- If any spot when first seen was 60° or more to the east of the central meridian, the catalogue and, if necessary, the Daily Results
Table 1. Recurrent sunspot groups found by Henwood (2008), termed GPR(training), and two other recurrent group datasets tabulated within the Royal Greenwich Observatory publications are compared with GPR(linked). The columns of values, from left to right are: the number of linked groups which the human and neural network classifier agree on (True Positive count); the number of linkages which both the human and neural network agree are not linked (True Negative count); the number of links which the neural network classifies as true but the human does not (False Positive count); the number of linkages the human thinks are true but the neural network does not (False Negative count).

| Dataset    | True Positives | True Negatives | False Positives | False Negatives |
|------------|----------------|---------------|----------------|----------------|
| GPR(training) | 450            | 3799          | 34             | 170            |
| RGO 1958   | 50             | 2032          | 106            | 11             |
| RGO 1896   | 15             | 180           | 13             | 11             |

Also, were searched some fifteen to sixteen days earlier to ascertain whether a spot group of similar heliographic longitude and latitude was then near the west limb of the Sun. Similarly, if any spot group when last seen was 60° or more to the west of the central meridian, a search was made fifteen to sixteen days later. When there appeared to be a case of probable continuity between groups in consecutive rotations of the Sun, the character of the groups, their areas and their longitude and latitude have been carefully compared before accepting them as recurrent groups.

Between 1874 and 1906, Maunder catalogued 624 recurring groups: 468 were seen only in two rotations, 118 appeared in three rotations, 25 in four rotations, 12 in five rotations, and 1 (somewhat doubtfully) in six rotations.

The process of verification addresses the question of whether or not the work described in this paper has been completed correctly. One approach is to compare GPR(linked) with the recurrence observations that were published by the Royal Greenwich Observatory. Recurrence data for 1896 and 1958 were typed in and compared with GPR(linked). For interest, the performance of the neural network against the training data, GPR(training), was also evaluated.

Table 1 compares these three different linked sunspot group datasets with GPR(linked) using metrics for a confusion matrix (Kohavi and Provost, 1998). True positive counts are links which are in both the manual and GPR(linked) datasets. False positives are links which are only in GPR(linked). False negatives are links which are in the manual dataset but not in GPR(linked). True negatives are defined as: $TN = Total - TP - FP - FN$.

By rapidly constructing longitude-time diagrams that correspond to the period of interest the instances of false classification have been investigated individually. As may be expected, GPR(training) data performs the best overall. It does not achieve 100% success compared to the human classification. On inspection of the “False Positive” classifications made by the neural network, all but one of these linkages were discovered to be physically likely linkages which the human classifier had overlooked. During inspection by longitude–time diagrams it was observed that these “False Positive” results occurred in dense complexes of many groups. The neural network out-performs the human under these conditions.
The Royal Greenwich Observatory 1958 recurrence records show the highest number of false positives. On investigating the set of examples classified as false positives, it was observed that these were again a result of a complication arising during observation of longitude–time diagrams. It should be noted that 1958 was a year during which the sunspot number reached a record high. This level of activity corresponds to a greater number of observations to display on a longitude–time diagram, which increases the challenge to a human of identifying all potentially linked groups.

In addition, the method used during 1958 was restricted to providing only one link between two given groups. However, the neural network classifier could find multiple linkages between groups. Finally, the neural network classifier could also make a link between groups even if one of the groups was only visible for a single day. Such a linkage is apparently not permitted within the 1958 recurrence dataset.

The RGO 1896 recurrence records showed the greatest discrepancy from the neural network linked dataset. This is probably because the criteria used to identify a recurrent group are different from both the 1958 and the GPR(training) datasets. Maunder used heliographic position, allowing for a sunspot group to remain unseen for some days and marking it as recurrent if another group was observed at the same heliographic position. The method employed here is to require a sunspot group to meet the “unreliable observed” criteria. Some of the groups marked as recurrent by Maunder failed to be seen within 30° of the relevant limb. It should also be noted that 1958 was a year of considerable activity on the Sun, whereas 1896 was relatively quiet, reducing the opportunity to observe recurrence.

4. Gnevyshev–Waldmeier Rule within Recurrent Groups

After classifying recurrence, one might expect the GPR(linked, reliable) dataset to exhibit some of the well established physical characteristics of non-recurrent sunspot groups. The Gnevyshev–Waldmeier rule (Gnevyshev, 1938; Waldmeier, 1955) states that sunspot group maximum area ($A_0$) and lifetime ($T$) are proportional:

$$A_0 = \bar{D}_{GW} T \quad \bar{D}_{GW} \approx 10 \text{MSHday}^{-1}. \quad (1)$$

where $\bar{D}_{GW}$ is the constant of proportionality measured in millionths of the solar hemisphere (MSH) per day.

In their paper on sunspot decay, Petrovay and van Driel-Gesztelyi (1997) used the Debrecen recurrence dataset (Dezsö, Gerlei, and Kovács, 1987; Dezsö, Gerle, and Kovács, 1997). They choose to use groups which “were born on the visible hemisphere and also died there”, so that their lifetimes could be determined accurately. Since only one group satisfied this criterion, they applied the Gnevyshev–Ringnes correction to the remaining recurrent groups to make a total dataset of 128 groups. The Gnevyshev–Ringnes correction provides the
Figure 5. Recurrent sunspot group lifetime (measured in days) plotted against maximum wholespot size measured in millionths of the solar hemisphere (MSH). Lifetime is measured as the duration between first and last observation of a sunspot group. The GPR(linked, reliable) dataset contains 841 observations and a linear fit of $\bar{D}_{GW} = 10.89 \pm 0.48$ MSH day$^{-1}$ is found. The data are divided into three age categories (grp 1, grp 2 and grp 3) and the centre of mass of each is indicated with a cross. The error bounds are marked by dashed lines and $\bar{D}_{GW} = 10$ is included for comparison.

probability that birth and death will occur in the visible hemisphere. After binning, they found a least squares linear fit of $\bar{D}_{GW} = 10.89 \pm 0.48$ MSH day$^{-1}$.

There are 841 groups in GPR(linked, reliable) that are reliably observed according to the criteria that both the birth and death of a group must take place within $\pm 60^\circ$ of the solar central meridian. Using reliably observed groups means the Gnevyshev–Ringnes correction is not required. Figure 5 shows a linear fit through GPR(linked, reliable) of $\bar{D}_{GW} = 11.73 \pm 0.26$.

All of the lifetimes within GPR(linked, reliable) are accurate to within one day. The corresponding measurement of group maximum size is subject to some uncertainty because rotation of the Sun carries each recurrent group out of sight for a portion of its lifetime. The effect is that the value observed as the maximum is either the true maximum or a smaller value.

The data show a large scatter around the linear fit (Figure 5). Regions between the bands of points (where no reliable observation is possible because either the birth or death of the group is unseen) make it somewhat difficult to determine the linear relationship. Three age categories are defined as follows: grp 1, ages 17 to 45 days; grp 2, 46 to 72 days; grp 3, greater than 72 days. The centre of mass of each of these groups is indicated by a cross. In addition, since only recurrent
sunspot groups are examined, there are no data points between the origin and \approx 18 \text{ days}.

Compared to the data analysed by Petrovay and van Driel-Gesztelyi (1997), the GPR(linked, reliable) data are more numerous. In addition, the GPR(linked, reliable) data do not have a limit on the maximum size measured and only recurrent groups are included in the fitting. For these reasons, one should not expect exact agreement between $\bar{D}_{GW}$ in Figure 5 and values found from previous investigations. In this study, $\bar{D}_{GW}$ has a smaller uncertainty but is larger than previous estimates. The discrepancy between the value obtained in this paper and the one found by Petrovay and van Driel-Gesztelyi (1997) is small, if allowance is made for the appropriate error bounds, which suggests that for recurrent groups $\bar{D}_{GW}$ is probably closer to 11 MSH day$^{-1}$ than 10 MSH day$^{-1}$.

5. Temporal Variations of Sunspot Group Lifetime

The GPR(linked) dataset presents a unique opportunity to investigate changes of sunspot lifetime with time. Previous studies of this property were complicated for the reasons already outlined; namely, incomplete recurrence data compiled by different individuals using different criteria.

Blanter et al. (2006) considered the topic of sunspot lifetime. They performed a nonlinear study of the short-term correlation properties of solar activity in order to reveal their long-lifetime variations. Their method was applied to GPR(unreliable) and allows for the problems associated with recurrence within the dataset and short-lived sunspots. These authors mitigate such factors by examining the population of sunspot lifetimes that are between 1 and 15 days. These observations were used to create a 22-year running averaged series.

One of the conclusions of the study by Blanter et al. (2006) was that they found sunspot lifetime of GPR(unreliable) had increased over the duration of the dataset. In addition, they were able to quantify the change in lifetime, which increased by a factor of 1.4 over the interval from 1915 to 1940. During this period, GPR(linked) indicates a change from below to above average lifetimes, as shown in Figure 6.

Blanter et al. (2006) discuss the problems associated with both small sunspot groups, whose lifetime cannot be perfectly measured, and the lack of observations from the invisible side of the Sun. Because of this, they developed a technique that used sunspot group size and the Gnevyshev–Waldmeier relationship to infer sunspot group lifetimes. Figure 6 presents GPR data (grey pattern region) and GPR(linked) (grey solid region) from our study.

Large errors are observed in the calculation of lifetime from GPR data containing recurrent groups. GPR(linked) alone contains much reduced error and a trend can be observed. While GPR(linked) only contains groups that have a sufficiently long lifetime to be observed on more than one solar rotation, there are proportionally fewer such groups, which reduces the lifetime average during the sample window.

GPR(linked) data shown in Figure 6 indicate a marked increase between 1910 and 1950. Blanter et al. (2006) found that sunspot lifetime had increased by a
Figure 6. Lifetime of sunspot groups versus time, calculated for the longest-lived sunspot on a given day for 22-year (8035 days) moving averages. The GPR age estimate takes no account of recurrent sunspot groups and is plotted as a grey pattern contained within the bounds of error. GPR(linked) is plotted as a dark grey solid region containing the bounds of uncertainty. Error is calculated as the longest and shortest lifetime estimates of a sunspot group lifetime. Longest lifetime assumes that group birth was at the earliest possible unseen time and death was at the latest possible unseen time. Shortest lifetime is calculated assuming the earliest observation in GPR is the birth and the latest observation in GPR is the death. Variations in sunspot number are shown to indicate individual solar cycles.

factor of 1.4 between 1915 and 1940. The results presented here largely agree with that value. In addition, Figure 6 suggests that the increase may extend over a longer interval and also augments the work of Blanter et al. (2006) by introducing an uncertainty measure.

6. Conclusions

Neural networks have been applied previously to various problems in space science. In particular, they have been used in the prediction of geomagnetic phenomena (Lundstedt, 1992; Lundstedt and Wintoft, 1994; Calvo, Ceccato, and Piacentini, 1995; Gleisner, Lundstedt, and Wintoft, 1996; Williscroft and Poole, 1996; Wu and Lundstedt, 1996; Weigel et al., 1999), the classification of asteroid spectra (Howell, Merényi, and Lebofsky, 1994), and ionogram processing (Galkin et al., 1996). In addition, they have also been applied to the important problem of automatically classifying sunspots from data obtained by processing SOHO/MDI satellite images (Nguyen et al., 2006). However, the authors are
not aware of the previous use of neural networks in studies of recurrent sunspot
groups.

It has been shown in this study that it is possible to train a neural network
to identify recurrent sunspot groups within the Greenwich Photoheliographic
Results (GPR), which extend over the long interval 1874 – 1976 (Section 2). Once
trained, the neural network can often outperform a human classifier, particularly
when a large number of sunspot groups are present on the solar disc at the same
time. Since the neural network performs deterministically when classifying a
pair of linked sunspot groups, it operates with a consistency of judgement that
exceeds the sterling endeavours of various individuals over more than a century.

Nevertheless, the neural network method has some limitations. These are
most clearly revealed when considering false-positive linkages (Section 3). In
such cases, the black-box nature of the decision process becomes problematic
and confidence in the neural network is partially undermined. In the method
discussed in this paper, post-processing of the data is used to reduce false-positive
linkages but in any future study it might be valuable to investigate alternative
machine learning systems.

Despite these limitations, the particular choice of network and data reduc-
tion procedure employed in this study could also be applied to Rome Daily
Sunspot Reports (1958 – 2000), USSR Station Data (1968 – 1991), Mount Wil-
son Individual Sunspot Data (1917 – 1985), Kodaikanal Individual Sunspot Data
(1906 – 1987) and Greenwich/Debrecen Observations (1874 – 2007), without re-
training the existing neural network. All of these datasets are available through
either the National Geophysical Data Centre (http://www.ngdc.noaa.gov) or the
UK Solar System Data Centre (http://www.ukssdc.ac.uk).

The constants of proportionality in the Gnevyshev–Waldmeier rule ($A_0 = \bar{D}_{GW} T$) derived in this study (Section 4) have been found to be greater than
previous estimates. Petrovay and van Driel-Gesztelyi (1997), who used information
on recurrent sunspot groups extracted from the Debrecen Photoheliographic
Results, also found a value of $\bar{D}_{GW}$ that was larger (10.89 MSH day$^{-1}$) than
previous estimates. In a study of sunspot group lifetimes, Zuccarello (1993) pre-
sented results which show a change in the rotation rate between short-lived and
long-lived (11-day old) sunspot groups, pointing to a difference in “aggregation
capability” of the group within the convection zone. The present results suggest
that there may be some additional physics present within longer lived groups in
the GPR (linked, reliable) dataset and thereby imply that this matter warrants
further investigation.

Evidence has been found for an increase in the lifetime of recurrent sunspot
groups by a factor of about 1.4 between 1915 and 1940 (Section 5), which is in
excellent agreement with the result obtained by Blanter et al. (2006). Indeed,
this increase in lifetime actually occurs over a longer period (1915 – 1950) than
previously thought and there is also provisional evidence for a slight decrease in
lifetime between 1950 and 1965 (see Figure 6).

Solar changes over periods longer than a few decades are currently of consid-
erable interest as the solar output is a significant input to climate models (Haigh
et al., 2005). The Gleissberg cycle is detected in sunspot number and has a mea-
sured period of approximately 80-120 years (Gleissberg, 1967; Hoyt, Schatten,
and Nesmes-Ribes, 1994; Garcia and Mouradian, 1998). Garcia and Mouradian (1998) estimated the most recent Gleissberg cycle minima to be around 1900 and the maxima around 1965. The results presented here suggest that, if the lifetimes of recurrent sunspot groups are a good proxy for the Gleissberg cycle, the maxima occurred some years before 1965 during the 1950s. Further study of this topic would be facilitated by applying the trained neural network to sunspot data in the interval 1977 – 2009.

The analysis of historical sunspot observations using automated methods, such as the neural network method presented in this paper, is required to estimate the (true) total number of sunspots emerging on the solar surface during the solar cycle. Numerical simulations of the centennial evolution of magnetic flux on the solar surface scale the sunspot emergence rate to the total sunspot number, measured without considering recurrent sunspots and their variable lifetimes in great detail (Wang, Lean, and Sheeley, 2002). A sunspot number that corrects for the recurrence of long-lived sunspots may improve the numerical predictions by providing a better description of emergence. The numerical simulations over centennial scales could be compared to the most recent estimates of long-term variation of the open magnetic flux on the solar surface (Rouillard, Lockwood, and Finch, 2007). Neural networks could also be optimised to detect the location of the umbra and penumbra of sunspots and to estimate the total magnetic flux inside these large-scale active regions. Such a rough estimate of the magnetic flux on the photosphere could then be used to estimate the magnetic topology of the corona using simplified potential field source surface models.

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