QCD sum rules at finite density in the large-$N_c$ limit:
The coupling of the $\rho$-nucleon system to the $D_{13}(1520)$

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QCD sum rules are studied for the vector-isovector current at finite baryon density in the limit of large number of colors $N_c$. For the condensate side it is shown that in this limit the four-quark condensate factorizes also for the finite density case. At the hadronic side the medium dependence is expressed in terms of the current-nucleon forward scattering amplitude. Generalizing vector meson dominance we allow for a direct coupling of the current to the nucleon as well as a coupling via the $\rho$ meson. We discuss the $N_c$ dependence of (a) modifications of the pion cloud of the $\rho$ meson, (b) mixing with other mesons (in particular $a_1$ and $\omega$) and (c) resonance-hole excitations $R N^{-1}$. We show that only the last effect survives in the large-$N_c$ limit. Saturating the sum rules with a simple hadronic ansatz which allows for the excitation of the $D_{13}(1520)$ we determine the coupling of the latter to the $\rho$-nucleon and the photon-nucleon system. These couplings are hard to determine from vacuum physics alone.

Keywords: QCD sum rules, large-$N_c$ expansion, meson properties, nuclear matter

I. INTRODUCTION

The question how hadrons once put in a strongly interacting medium change their properties provides a very active field of research. In the language of non-perturbative QCD, in-medium modifications are indicated by the change of condensates like the quark condensate which provides an order parameter of chiral symmetry breaking. On the other hand, changes like the melting of the condensates do not automatically tell what this means for the properties of a particular hadron like its mass or lifetime. The situation is such that the condensates are closer to QCD as the underlying theory of the strong interaction, whereas the hadron properties are closer to experimental observation. The QCD sum rule method is supposed to bridge that gap by connecting integrals over hadronic spectral functions with an expansion in terms of quark and gluon condensates. Originally they were introduced for the vacuum but later on generalized to in-medium situations.

A particularly interesting probe to study in-medium modifications are neutral vector mesons. The reason is that such mesons can decay into dileptons. If such a decay happens in the medium the dileptons leave the system untouched by strong interactions. In that way in-medium information is carried to the detectors. (For an overview see e.g. [4].) The present paper deals with $\rho$-mesons placed in an infinitely extended system with finite baryonic density. Such a scenario is an idealization of a finite nucleus or a heavy-ion collision. For simplicity we study $\rho$-mesons which are at rest with respect to the nuclear medium.

Originally it was expected that the use of nuclear medium QCD sum rules would yield model independent predictions for in-medium changes of hadronic properties — just like the vacuum sum rules yield in an impressive way parameter-free predictions of vacuum hadronic properties. However, further studies have revealed that for the case of a nuclear medium there is instead a rather large model dependence in the possible parameterizations which enter the hadronic side of the sum rules. In [2,3] it has been “predicted” that the mass of the $\rho$-meson should drop in a nuclear medium. The type of parameterization for the spectral function of the $\rho$-meson was adopted from the vacuum case: a state with practically negligible width (in the vacuum this approach is very successful [2]). On the other hand, in the last years hadronic model builders have accumulated evidence that in a nuclear medium the $\rho$-meson spectral function looks much different from a small-width resonance [4,5,6,7,8,9,10,11,12,13,14,15,16,17]. Clearly, model dependences are inherent in these more complicated in-medium spectral functions. In any case it is important to realize that the choice for the parameterization of the spectral information which enters the hadronic side of the sum rules is not an output of the sum rule method but an input. The predictions or lessons one deduces from the sum rules depend on the chosen input. In [18] a specific hadronic model has been fed into the sum rules. It has been shown that the sum rules require an additional downward mass shift on top of the chosen hadronic model. This result was basically in agreement with the original stable-$\rho$-meson approximation used in [2,3]. On the other hand, the hadronic model used in [10] did not yield a sizable mass shift of the $\rho$-meson but a significant peak broadening. It has been demonstrated

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that this specific model saturates the sum rules without further modifications on top of the hadronic input. In [19] a one-peak parameterization with arbitrary width and arbitrary peak position was plugged into the sum rules. This already involved too many free input parameters (in total four) to deduce predictions. Still a correlation between width and mass was found which agreed with the previously discussed limiting cases of small width and downward mass shift [3, 6] on the one hand side and large width and no mass shift [17] on the other. A parameterization with additional peaks caused by vector–axial-vector mixing has been studied in [20]. On top of these model dependences for the hadronic input there are also problems on the condensate side. Already for the vacuum case the size of the four-quark condensate provides a source of uncertainty (cf. e.g. [19]). The influence of the in-medium behavior of the four–quark condensate has been studied in detail in [21].

In [22] it was concluded that the nuclear medium sum rule method might still be useful to constrain a given hadronic model or judge its quality, but the originally expected predictive power seemed to be lost to a large degree. It is one purpose of the present work to regain part of this predictive power by involving a new QCD based argument.\(^1\) In [23] it has been suggested to treat the number of colors \(N_c\) as a hidden parameter of QCD. In that spirit we will study both sides of the sum rules as a function of \(N_c\). We will demand that both sides match not only for \(N_c = 3\), but for arbitrary \(N_c\). In particular, we will be interested in the large-\(N_c\) limit where several important scaling relations are known [23, 24]. In the large-\(N_c\) limit it will be possible to reduce the uncertainties on both sides of the sum rules to a large degree. Several hadronic in-medium effects will drop out in that limit. In addition, the in-medium four-quark condensate can now be related to the two-quark condensate.

It will turn out that baryonic resonances play an important role on the hadronic side of the QCD sum rules. Such resonances are formed in collisions of the \(\rho\)-meson with a nucleon from the medium. Such a process is also called excitation of a resonance-hole state. Especially the \(D_{13}(1520)\) resonance might play an important role for the in-medium properties of the \(\rho\)-meson at rest [12, 14, 15, 16, 25]. It is important to realize, however, that such an analysis relies on extractions of the resonance parameters from two-pion production data. Unfortunately such extractions are not completely model-independent, mainly because the \(N^\ast(1520)\) is nominally subthreshold with respect to the \(\rho\)-nucleon system. Therefore the present situation is such that we have to face a rather broad range of possible sizes for the coupling of the \(\rho\)-nucleon-\(D_{13}\) system. As we will see the QCD sum rule method constitutes a completely independent source of information about this coupling constant.

Concerning the importance of resonances, the present work is close in spirit to [26] where the three-momentum dependence of the sum rules is related to the excitation of resonances which couple to the \(\rho\)-nucleon system with a \(p\)-wave. Here, however, we study \(\rho\)-mesons at rest and consequently their coupling with nucleons to \(s\)-wave resonances. We also note that \(N_c\)-scaling arguments were not considered in [26].

The paper is organized in the following way: In the next section we study in-medium QCD sum rules for the \(\rho\)-meson channel and in particular their condensate side in the large-\(N_c\) limit. In Sec. III we analyze the \(N_c\) scaling of various hadronic in-medium changes of the hadronic side of the sum rules. In Sec. IV we become more quantitative and saturate the sum rules by allowing the excitation of the \(D_{13}(1520)\). We determine from the sum rules its couplings to the \(\rho\)-nucleon and photon-nucleon system. Finally we summarize our results in Sec. V.

II. QCD SUM RULES AT LARGE \(N_c\)

In this work we study the properties of a vector-isovector current

\[
j_{\mu} := \frac{1}{2} \left( \bar{u} \gamma_{\mu} u - \bar{d} \gamma_{\mu} d \right)
\]

(1)

which is at rest with respect to the nuclear medium. As outlined e.g. in [27] in-medium QCD sum rules can be obtained from an off-shell dispersion relation which integrates over the energy at fixed (here vanishing) three-momentum of the current. We also restrict ourselves to small densities \(\rho_N\) by using the linear density approximation. Effectively this means that the current is at rest with respect to the nucleon on which it scatters. The Borel sum rule is given by

\(^1\) Of course, it will turn out that things are now somewhat more complicated than they were originally thought to be.
leading terms. If all in-medium contributions scaled in this way, we would not find in-medium changes in leading order in $\rho_o N$. As we shall see, however, at finite baryonic density there appear in-medium modifications in leading order in $N_c$. The next term on the right hand side of (2) is the Landau damping contribution. The nucleon mass $m_N$ is of order $o(N_c)$

$$m_N \approx 1 \text{ GeV}$$

Concerning the external parameter, the density $\rho_o N$ is correctly reproduced. As a first step we will analyze the non-relativistic quark model for the description of a nucleon with momentum $p_c$ and momentum $\rho_o N$. We do not expect that this approximation is over-simplified. The nucleon mass in the chiral limit is $m_o N_c$ (cf. [19] and references therein) (1)

$$m_o N_c = 24 \pi^2 \left( 1 - \frac{2}{\pi} \frac{\alpha_s}{N_c} \right) \left( 1 - e^{-s_0/M^2} \right) - \frac{1}{4 M^2} \frac{\rho_o N}{m_N}$$

$$+ \frac{m_q(q\bar{q})_{med}}{M^4} + \frac{\alpha_s G^2}{4 M^4} \left( \frac{\alpha_s}{\pi} \right)_{med} + \frac{m_N a_2 \rho_o N}{M^4}$$

$$- \frac{1}{9 M^6} \frac{N_c^2 - 1}{N_c^2} \{2\} \pi \alpha_s \langle O^V_1 \rangle_{med} - \frac{5}{4 M^6} m_N^4 a_4 \rho_o N.$$ (2)

As a first step we will analyze the $N_c$ dependence of the various terms appearing on the right hand side of the Borel sum rule. The $N_c$ dependence of the perturbative contribution (first term on the right hand side) is easily obtained from perturbative QCD. Note that in the spirit of large-$N_c$ QCD the strong coupling $\alpha_s$ is $o(1/N_c)$. In total, the perturbative contribution to the sum rule is $o(N_c)$.

The continuum threshold $s_0$ is supposed to lie between the lowest hadronic state in the considered channel — here the $\rho$-meson — and the higher lying states. As the meson masses are $o(N_c^0)$ it is natural to assume that $s_0$ scales in the same way.

Concerning the external parameter, the density $\rho_o N$ we treat it formally as $o(N_c^0)$. However, this does not matter at all: Below we will compare the terms linear in the density from both sides of the sum rule with each other only (and not with the vacuum terms). Hence, in the end the "size" of $\rho_o N$ in powers of $N_c$ is actually irrelevant.

The next term on the right hand side of (2) is the Landau damping contribution. The nucleon mass $m_N$ is of order $o(N_c)$

$$m_N \approx 1 \text{ GeV}$$

The nucleon mass in the chiral limit is $m_o N_c$. The finite density part, however, is only $o(N_c^0)$. In a non-relativistic quark model we can replace $\bar{q}q$ by $q^\dagger q$. The latter operator counts the number of constituent quarks. Therefore we obtain

$$\frac{\sigma_N}{m_q} = 2 \langle N|\bar{q}q|N \rangle = \langle N|\bar{u}u + \bar{d}d|N \rangle \approx \langle N|u^\dagger u + d^\dagger d|N \rangle = N_c.$$ (4)

We want to stress here that we do not claim that the non-relativistic quark model reproduces the right value for $\sigma_N$, but we expect that the scaling with $N_c$ is correctly reproduced.

The in-medium value of the gluon condensate can be deduced from the trace anomaly (28):

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_{med} = \langle \frac{\alpha_s}{\pi} G^2 \rangle_{vac} - \frac{8}{N_c} m_N^{(0)} \rho_o N.$$ (5)

Like the quark condensate the vacuum gluon condensate is $o(N_c)$. The finite density part, however, is only $o(N_c^0)$, i.e. suppressed by one power in the number of colors. Note that the nucleon mass in the chiral limit is $m_o N_c$. We note in passing that even for $N_c = 3$ the in-medium change of the gluon condensate is rather small.

Both, the next term and the last one on the right hand side of (2) come from the nucleon structure functions. $a_2$ denotes the fraction of the nucleon momentum carried by the constituent quarks, i.e. the first moment of the corresponding structure function. $a_4$ denotes the corresponding third moment. In the low energy regime which we consider ($\mu^2 \approx 1 \text{ GeV}^2$) most of the momentum is carried by the constituent quarks. We do not expect that this changes in the large-$N_c$ limit, i.e. $a_2 \approx 1 = o(N_c^0)$. To explore the scaling of $a_4$ we adopt the following — obviously over-simplified — quark model for the description of a nucleon with momentum $p$: Take $N_c$ constituent quarks each with a mass $m_N/N_c$ and momentum $p/N_c$. In this case we get $a_2 = 1$ and $a_4 = 1/N_c^2$. Again we do not regard that

2 Note that we neglect effects of isospin violation.
as a realistic model for the structure function but only use it to determine the $N_c$ scaling behavior. Nonetheless, it is amusing to see that such a model reproduces the physical values for $a_2$ and $a_4$ within 10% (cf. Tab. 1). We conclude that both the $a_2$- and the $a_4$-term yield in-medium changes of order $N_c$ in (2), i.e. changes which are as important as the vacuum contributions from quark and gluon condensates.

Finally we turn to the four-quark condensate $\langle O^{V}_4 \rangle_{\text{med}}$. The explicit expression is given e.g. in [21] for three colors. In the following we do not need this explicit expression. Instead, we start with two arbitrary color-neutral two-quark operators $A$ and $B$. Note that using Fierz transformations any color-neutral four-quark operator can be written as a sum of products of type $AB$. In the linear density approximation the in-medium expectation value of $AB$ is given by

$$\langle AB \rangle_{\text{med}} = \langle AB \rangle_{\text{vac}} + \rho_N \langle N|AB|N \rangle. \quad (6)$$

According to [21] the vacuum part factorizes in the large-$N_c$ limit:

$$\langle AB \rangle_{\text{vac}} = \langle A \rangle_{\text{vac}} \langle B \rangle_{\text{vac}} + o(N_c). \quad (7)$$

Note that $\langle A \rangle_{\text{vac}}$, $\langle B \rangle_{\text{vac}} = o(N_c)$. The nucleon expectation value can be decomposed in the following way:

$$\langle N|AB|N \rangle = \langle A \rangle_{\text{vac}} \langle N|B|N \rangle + \langle N|A|N \rangle \langle B \rangle_{\text{vac}} + \langle N|AB|N \rangle_{\text{connected}}. \quad (8)$$

The last term on the right hand side describes the scattering of $A$ with a nucleon into $B$ and a nucleon. According to the large-$N_c$ rules developed in [21] this contribution is $o(N_c)$. In contrast, the other two terms on the right hand side of (8) are $o(N^2_c)$ (cf. (11)). Hence we find (in linear density approximation)

$$\langle AB \rangle_{\text{med}} = \langle A \rangle_{\text{vac}} \langle B \rangle_{\text{vac}} + \rho_N \left(\langle A \rangle_{\text{vac}} \langle N|B|N \rangle + \langle N|A|N \rangle \langle B \rangle_{\text{vac}} \right) + o(N_c), \quad (9)$$

i.e. "factorization" of any in-medium four-quark condensate in the large-$N_c$ limit. The quotation marks are meant to indicate that there is no $\rho^2_N$ term which would appear if factorization was taken literally. In particular we find:

$$\langle O^{V}_4 \rangle_{\text{med}} = \langle \bar{q}q \rangle_{\text{vac}}^2 + 2\rho_N \langle N|\bar{q}q|N \rangle \langle \bar{q}q \rangle_{\text{vac}} + o(N_c) = \langle \bar{q}q \rangle_{\text{vac}}^2 + \rho_N \frac{\sigma_N}{m_q} \langle \bar{q}q \rangle_{\text{vac}} + o(N_c). \quad (10)$$

Finally the four-quark condensate is multiplied by $\alpha_s = o(1/N_c)$ (cf. [21]). Hence, in total the "factorized" part of the four-quark condensate enters the sum rule in leading (=linear) order in $N_c$.

The outcome of our analysis is summarized in Tab. II together with the actual values we take for the quantitative analysis outlined below. Of course, strictly speaking we do not know the values of all required quantities for $N_c \rightarrow \infty$. Therefore for the quantitative analysis we use the physical ($N_c = 3$) values and assume that the modifications are not too large. This is in the same spirit as all other large-$N_c$ approaches. To be formally more precise we assume the following: for an arbitrary quantity $Q$ which scales like $N^\gamma_c$ we assume

$$\lim_{N_c \rightarrow \infty} \left( \frac{3^\gamma}{N^\gamma_c} Q \right) \approx Q \bigg|_{N_c = 3}. \quad (11)$$

Numerically we find that most of the in-medium changes of the right hand side of (2) found for $N_c = 3$ remain present in the large-$N_c$ limit.

To be more sensitive to the in-medium modifications we differentiate the Borel sum rule (2) with respect to the density:

$$\frac{1}{\pi M^2} \int_0^{s_0} ds e^{-s/M^2} \frac{\partial}{\partial \rho_N} \text{Im} R_{\text{HAD}}(s, \rho_N) \bigg|_{\rho_N = 0} = \frac{1}{M^2} \frac{N_c}{24\pi^2} \left(1 + \frac{N_c^2 - 1}{8\pi} \frac{\alpha_s}{N_c} \right) e^{-s_0/M^2} s'_0 + \frac{1}{M^2} c_2 + \frac{1}{M^6} c_3 \quad (12)$$

with

$$c_2 = \frac{m_N a_2}{4} + \frac{\sigma_N}{2}, \quad (13a)$$

$$c_3 = -\frac{7}{9} \frac{N_c^2 - 1}{N_c^2} \pi \alpha_s \langle \bar{q}q \rangle_{\text{vac}} \frac{\sigma_N}{m_q} - \frac{5}{24} m_N^3 a_4, \quad (13b)$$

$s_0 = s_0(\rho_N = 0)$ and

$$s'_0 = \frac{ds_0}{d\rho_N} \bigg|_{\rho_N = 0}. \quad (14)$$
Note that we have only kept the terms which remain present in the large-$N_c$ limit, i.e. we have neglected the Landau damping contribution and the in-medium change of the gluon condensate. Next we rewrite (12):

\[
\frac{1}{\pi} \int_0^{s_0} ds e^{(s_0-s)/M^2} \frac{\partial}{\partial \rho_N} \text{Im} \mathcal{R}_{\text{HAD}}(s, \rho_N) \bigg|_{\rho_N=0} = \frac{N_c}{24\pi^2} \left( 1 + \frac{N_c^2 - 1}{N_c} \frac{3}{8} \alpha_s \right) s_0' + \frac{1}{M^2} c_2 e^{s_0/M^2} + \frac{1}{M^2} c_3 e^{s_0/M^2}, \quad (15)
\]

expand both sides in powers of $1/M^2$ and compare the corresponding coefficients on right and left hand side:

\[
\frac{1}{\pi} \int_0^{s_0} ds \frac{\partial}{\partial \rho_N} \text{Im} \mathcal{R}_{\text{HAD}}(s, \rho_N) \bigg|_{\rho_N=0} = \frac{N_c}{24\pi^2} \left( 1 + \frac{N_c^2 - 1}{N_c} \frac{3}{8} \alpha_s \right) s_0', \quad (16a)
\]

\[
\frac{1}{\pi} \int_0^{s_0} ds (s_0 - s) \frac{\partial}{\partial \rho_N} \text{Im} \mathcal{R}_{\text{HAD}}(s, \rho_N) \bigg|_{\rho_N=0} = c_2, \quad (16b)
\]

\[
\frac{1}{\pi} \int_0^{s_0} ds (s_0 - s)^2 \frac{\partial}{\partial \rho_N} \text{Im} \mathcal{R}_{\text{HAD}}(s, \rho_N) \bigg|_{\rho_N=0} = 2 (c_2 s_0 + c_3). \quad (16c)
\]

In this way we have obtained weighted finite energy sum rules. The advantage of finite energy type sum rules as compared to Borel sum rules lies in the fact that with the former we have got rid off the Borel mass and the problem how to determine a reliable Borel window etc. (see e.g. [19] and references therein). On the other hand, the standard finite energy sum rules are rather sensitive to the modeling of the transition region from the hadronic part \text{Im} \mathcal{R}_{\text{HAD}} to the continuum (see e.g. [20] and references therein). Indeed, the first equation (16a) is plagued by that problem. The latter two equations, however, are not since the transition region is suppressed by powers of $(s_0 - s)$ [35]. Therefore (16b) and (16c) are more reliable as they are insensitive to details of the threshold modeling at $s_0$. Hence these weighted finite energy sum rules combine the advantages of Borel and standard finite energy sum rules. In general, the disadvantage is that there are only two properly weighted finite energy sum rules as compared to three standard finite energy sum rules. In our case, however, this does not reduce the available information: The in-medium change of the threshold parameter encoded in $s_0'$ is anyway unknown \textit{a priori}. Fortunately, it only appears in the first (anyway less reliable) sum rule (16a). The two preferable sum rules (16b) and (16c) are independent of $s_0'$. We shall use them for the subsequent analysis. Note that the vacuum threshold $s_0$ appears in (16b) and (16c). This, however, can be fixed by an independent vacuum sum rule analysis which is free of all in-medium uncertainties. For the actual calculation we adopt the point of view of [31, 52, 33] and use $s_0 \approx (3/N_c) (4\pi f_\pi)^2 \approx 1.3\text{GeV}^2$ with the pion decay constant $f_\pi$. Note that the latter scales with $\sqrt{N_c}$ as can be deduced e.g. from the Gell-Mann–Oakes–Renner relation [34].

| quantity | size | scaling | ref. |
|----------|------|---------|------|
| $g_V$    | 6.05 | $1/\sqrt{N_c}$ | [10] |
| $m_V$    | 770 MeV | 1 | [30] |
| $s_0$    | $1.3\text{GeV}^2$ | 1 | [31, 32, 33] |
| $m_N$    | 940 MeV | $N_c$ | [30] |
| $\sigma_N$ | 45 MeV | $N_c$ | [5] |
| $a_2$    | 0.9  | 1 | [5] |
| $a_4$    | 0.12 | $1/N_c^2$ | [5] |
| $\alpha_s$ | 0.36 | $1/N_c$ | [5] |
| $\langle \bar{q}q \rangle_{\text{vac}}$ | ($-240\text{MeV})^3$ | $N_c$ | [33, 34] |
| $m_q$    | 6 MeV | 1 | [30] |
| $m_{D13}$ | 1520 MeV | $N_c$ | [30] |
| $m_{D13} - m_N$ | 580 MeV | 1 | [30] |
| $f_{\pi}$ | 92 MeV | $\sqrt{N_c}$ | [30] |

**TABLE I:** Sizes and $N_c$-scaling of all relevant quantities.
III. HADRONS IN-MEDIUM CHANGES AT LARGE $N_c$

We now turn to the left hand side of the sum rules (23) or (16). It is well-known that the vector-isovector current $j_{\mu}$ strongly couples to the $\rho$-meson. In the vector meson dominance (VMD) picture which is phenomenologically rather successful it is even assumed that all the interaction of $j_{\mu}$ with hadrons is mediated by the $\rho$-meson. In the following we will not adopt the strict VMD picture. Nonetheless, the in-medium modifications of the $\rho$-meson will significantly influence the current $j_{\mu}$. Suppose for simplicity that in the medium there is one additional channel besides the $\rho$-meson which couples directly to the current. Later on this channel will be specified to be a resonance-hole excitation. For the moment, however, we keep the formalism a little bit more general. The vertex which gives rise to this channel also couples directly to the current $j_{\mu}$ with strength $f_{\rho}$. A generalization to several channels is straightforward. Following (37) we get

$$\text{Im} R_{\text{had}}(s, \rho_N) = -\frac{1}{g_V^2} \left[ \text{Im} \Pi_V(s) |d_V(s) - 1|^2 + \text{Im} \Pi_B(s) |d_V(s) - r|^2 \right]$$

with

$$d_V(s) = \frac{s - r \Pi_B(s) - \Pi_V(s)}{s - m_V^2 - \Pi_B(s) - \Pi_V(s)}$$

$$r = \frac{f_\gamma}{f_\rho}$$

and in the linear density approximation

$$\Pi_B(s) = \rho_N T(s)$$

with the $\rho$-$N$ forward scattering amplitude $T$. The vacuum self energy of the $\rho$-meson is given by $\Pi_V$, $m_V$ denotes the vacuum $\rho$-meson mass and $g_V$ the $\rho$-pion-pion coupling.

In a nuclear medium there are various ways how the nucleons which form the medium can interact with the $\rho$-meson or directly with the current $j_{\mu}$. We classify the important in-medium effects in the following way:

(a) Modifications of the pion cloud (cf. Fig. Ia). In the vacuum the $\rho$-meson can decay into pions. The latter get strongly modified in the medium, e.g. by their coupling to $\Delta(1232)$-hole states. This in turn changes the $\rho$-meson properties.

(b) Mixing with other mesons (cf. Fig. Ib): The nucleons which form the medium carry a pion cloud. These pions can interact with the $\rho$-mesons (or directly with the current) and cause a mixing with other mesons, e.g. with the $\omega$-meson or with the $a_1$-meson (chiral mixing).

(c) Excitation of resonance-hole pairs (cf. Fig. Ic): Instead of these indirect interactions via pions the $\rho$-meson can also couple directly to a nucleon from the medium and form a nucleon or a baryonic resonance. For $\rho$-mesons at rest which we study here, one can only form baryons which couple to the $\rho$-nucleon system in an $s$-wave. This excludes e.g. the nucleon and the $\Delta(1232)$ state. Concerning moving $\rho$-mesons and their $p$-wave interaction cf. also e.g. (11, 20, 36).

(d) Changes of the vacuum structure at the underlying quark-gluon level (described e.g. by Brown-Rho scaling): If the density and/or temperature of the medium is high enough one expects a transition to a state where the single quarks (and gluons) are the relevant degrees of freedom. As a precursor of these changes the non-perturbative QCD vacuum structure already changes in the hadronic medium (expressed e.g. in the melting of the condensates). The in-medium change of the underlying vacuum structure influences the quark properties and in turn the properties of the hadrons which consist out of quarks. From the hadronic point of view it is not clear whether such effects are distinct from the hadronic effects discussed above or whether it is just a different language for the same physics. It is also not clear whether these “exotic” effects show up at all in linear order in the density. After all, the linear density approximation describes the interaction of the probe (the current) with one nucleon at a time. The key ingredient is the current-nucleon forward scattering amplitude, i.e. a vacuum quantity. In the following we will disregard such “exotic” effects and figure out whether the sum rule can be saturated by the standard hadronic interactions discussed above.
Above we have described in-medium modifications of the $\rho$-meson. Note, however, that all these effects (a-d) can also directly influence the current instead of the $\rho$-meson.

As a next step we will analyze the $N_c$ scaling behavior of the hadronic effects (a-c) discussed above. We recall the large-$N_c$ counting rules developed in [23, 24]:

- Meson mass: $o(N^0)$.  
- Baryon mass: $o(N_c)$.
- Mesonic interactions are suppressed in the large-$N_c$ limit. The decay width of a meson into two other mesons is e.g. $o(1/N_c)$. Therefore, in vacuum mesons are stable in the large-$N_c$ limit.
- In contrast, meson-baryon interactions are not suppressed. Meson-baryon scattering amplitudes are $o(N^0)$. A meson-baryon-baryon vertex might even be enhanced, $o(\sqrt{N_c})$. Therefore, in general, baryonic contributions e.g. to the meson self energy are as important as the meson mass. This actually opens the possibility that a baryonic medium influences the meson properties in leading order in $N_c$.

There are remarkable exceptions to the last rule: The pseudovector interaction of a pion with a nucleon is proportional to the inverse of the pion decay constant $a$. Recall that the latter scales with $\sqrt{N_c}$. Therefore the pseudovector-pion-nucleon vertex is suppressed in the large-$N_c$ limit. Also the vector interaction of the $\rho$-meson with the nucleon is suppressed (whereas the tensor interaction is not) [46, 47, 48, 49]. This fits well with the approach to introduce the coupling of the $\rho$-meson to other hadrons via minimal substitution: The $\rho$-pion-pion coupling constant $g_{\rho}$ is $o(1/\sqrt{N_c})$. Minimal substitution introduces the same coupling for the $\rho$-nucleon-nucleon vertex and yields the suppression mentioned above. In addition, the principle of minimal substitution creates an elementary four-point coupling (Kroll-Ruderman type) $\rho$-nucleon-nucleon-pion from the pseudovector pion-nucleon interaction. Consequently the corresponding inelastic scattering amplitude $\rho$-nucleon to pion-nucleon is $o(1/N_c)$ and not $o(N^0)$.

It is also interesting to realize that the large-$N_c$ limit yields a justification for two commonly used approximations in the vacuum QCD sum rule approach, namely factoring the four-quark condensate and approximating the $\rho$-meson peak by a delta function [2]. Hence the standard treatment of the $\rho$-meson in the QCD sum rule method is practically identical to its large-$N_c$ treatment. In turn one can therefore conclude that one gets reasonable results for the vacuum properties of the $\rho$-meson by combining QCD sum rules with large-$N_c$ considerations. In the present work we apply that combination of techniques to the in-medium situation.

Let us now come back to the hadronic in-medium effects (a-c) discussed above. Both effects (a) and (b) are mediated by pions. The nucleon interaction to the pions already generates a suppression due to the $1/f_\pi$ factors. In addition, the coupling of the pions to the $\rho$-meson is suppressed. In total, the effects (a) and (b) are suppressed in the large-$N_c$ approximation as compared to the vacuum properties. This can also be deduced in detail by inspecting the appropriate expressions given e.g. in [10]. In contrast, the direct coupling of the $\rho$-meson to nucleons discussed in (c) in general is not suppressed. Therefore, the large-$N_c$ approximation allows to disentangle different hadronic in-medium effects. Studying both sides of the sum rule as functions of $N_c$ suggests that the sizable in-medium modification of the condensate side does not reflect modifications of the pion cloud or mixing with other mesons but the excitation of resonance-hole states. This is a simple consequence of the fact that the effects (a) and (b) show a scaling with $N_c$ which does not comply with the condensate side. In turn, we can now use the sum rule in the large-$N_c$ approximation to quantify the effect of resonance-hole states.

So far we have tried to be as model-independent as possible concerning the modeling of hadronic interactions. To make further progress, however, we need some phenomenological input. In [12, 14, 15] the influence of baryonic resonances on the in-medium properties of the $\rho$-meson were thoroughly discussed. It is shown there that the most important in-medium change for $\rho$-mesons at rest is caused by the excitation of the $D_{13}(1520)$ resonance (see also...
We should add, however, that such an analysis relies on extractions of the resonance parameters from two-pion production data. Unfortunately such extractions are not completely model-independent, mainly because the $N^*(1520)$ is nominally subthreshold with respect to the $\rho$-nucleon system. Therefore different analyses yield a rather broad range of possible sizes for the coupling of the $\rho$-nucleon-$D_{13}$ system. For a detailed discussion we refer to [13]. It is one purpose of the present work to determine that coupling constant by a completely independent approach, the QCD sum rule method. In the following we shall neglect all other baryonic resonances besides the $D_{13}(1520)$. Note again that several other possibly important channels like the nucleon or the $\Delta(1232)$ are $p$-waves. Consequently they do not couple to a $\rho$-meson which is at rest relative to the nucleon from the medium. We will try to saturate the sum rules [19] by including only the $D_{13}(1520)$ in the $\rho$-nucleon scattering amplitude $T$ introduced in [20].

IV. THE $\rho$-NUCLEON-$D_{13}$ SYSTEM

To utilize the finite energy sum rules [13] we need the density derivative of [17]. In the large-$N_c$ limit, $\Pi_V \to 0$ and

$$\frac{\partial}{\partial \rho_N} \text{Im} R_{\text{HAD}}(s, \rho_N)\bigg|_{\rho_N=0} = -\frac{1}{g_V^2} s \left\{ \text{Im} T(s) \left[ (r-1)^2 - m_V^4 \frac{d}{ds} \text{Re} D_V(s) - 2m_V^2 (r-1) \text{Re} D_V(s) \right] \right. $$

$$ + \left. \text{Re} T(s) \left[ -m_V^4 \frac{d}{ds} \text{Im} D_V(s) - 2m_V^2 (r-1) \text{Im} D_V(s) \right] \right\} \tag{21}$$

with the free $\rho$ meson propagator

$$D_V(s) = \frac{1}{s - m_V^2 + i\epsilon}. \tag{22}$$

Allowing for the excitation of a $D_{13}$ state the imaginary part of the $\rho$-$N$ forward scattering amplitude for vanishing three-momentum is given by

$$\text{Im} T(s) = \left( \frac{f_\rho}{m_N} \right)^2 \frac{2}{3} s \text{Im} D_{13}(q_0 = \sqrt{s} + m_N, \bar{q} = 0) \tag{23}$$

with the propagator $D_{13}$ to be specified below. The amplitude is deduced from the non-relativistic lagrangian [13]

$$\mathcal{L} = \frac{f_\rho}{m_N} \psi_R^\dagger S_i^+ \tau_\alpha \psi_N \left( \partial_i \rho_0^\alpha - \partial_0 \rho_i^\alpha \right) + \text{h.c.} \tag{24}$$

where $\psi_R$ denotes the $D_{13}$-resonance isospinor-spinor, $\psi_N$ the nucleon isospinor-spinor, $\rho_0^\alpha$ the $\rho$-meson isovector-vector, $S_i^+$ the spin-$\frac{1}{2}$ to $\frac{3}{2}$ transition operator and $\tau_\alpha$ the (Pauli) isospin matrix. Finally h.c. stands for hermitian conjugate.

The simplest version of a $D_{13}$ propagator is a non-relativistic one with vanishing width

$$D_{13}(q_0, \bar{q}) = \frac{1}{q_0 - m_{D_{13}} - \frac{\bar{q}^2}{2m_{D_{13}}} + i\epsilon}. \tag{25}$$

Here $m_{D_{13}}$ is the resonance mass. Following [50] we assume that the mass difference between the nucleon and its excited state is $o(N_c^0)$. Whether neglecting the width of the baryon resonance is a reasonable approximation will be discussed later. For the moment we are aiming at an expression with a minimal number of free parameters. For the imaginary part of the forward scattering amplitude we obtain

$$\text{Im} T(s) = -\left( \frac{f_\rho}{m_N} \right)^2 \frac{2}{3} \frac{2\pi (m_{D_{13}} - m_N)^3}{3} \delta \left( s - (m_{D_{13}} - m_N)^2 \right). \tag{26}$$

We determine the corresponding real part from a dispersion relation such that the amplitude does not diverge for large $s$:

$$\text{Re} T(s) = \frac{1}{\pi} \int_0^s ds' \frac{\text{Im} T(s')}{s - s'} + \text{const.} \tag{27}$$
and determine the constant such that the real part vanishes at the photon point (gauge invariance)

\[ \text{Re} T(s) = -\frac{1}{\pi} \int_{0}^{s_0} ds' \, \text{Im} T(s') \left( \frac{1}{s - s'} + \frac{1}{s'} \right). \]  

(28)

Thus

\[ \text{Re} T(s) = \left( \frac{f_\rho}{m_V} \right)^2 \frac{2}{3} 2 \left( m_{D13} - m_N \right) \frac{s}{s - \left( m_{D13} - m_N \right)^2} 
\quad \times \left[ \begin{array}{c} \frac{1}{\sqrt{s - \left( m_{D13} - m_N \right)}} - \frac{1}{\sqrt{s + \left( m_{D13} - m_N \right)}} \end{array} \right]. \]  

(29)

The last expression shows that we have achieved the inclusion of an s- and a u-channel process.

Now we are in the position to determine the free parameters \( f_\rho \) and \( r \) from the sum rules (16b) and (16c) using (21), (26) and (29). Both of the two sum rules relate \( f_\rho^2 \) to \( r \). These relations are depicted in Fig. 2. The two relations intersect for

\[ f_\rho^2 \approx 28.7, \quad r \approx 1.17 \]  

(30)

which finally yields

\[ |f_\gamma| = |r f_\rho| \approx 6.26. \]  

(31)

Before we compare these results with other approaches we take these values (30) as an input for (16a) and determine the in-medium change of the threshold parameter expressed by \( s_0' \). We find

\[ s_0' \approx 0.002 \frac{s_0}{\rho_{\text{n.m.}}} \]  

(32)

with normal nuclear matter density

\[ \rho_{\text{n.m.}} = 0.17 \text{ fm}^{-3}. \]  

(33)
Recall that the sum rule (16a) is less reliable as already discussed above. Still we can take (32) as an indication that the change of the threshold parameter is extremely small. This finding is in contrast to the traditional result that the threshold sizably drops in a nuclear medium (e.g. [5, 10, 19]). Both results are easy to interpret: The ρ-meson is the lowest state in the vector-isovector channel. The higher states are effectively taken into account by the continuum contribution which starts at $s_0$. Traditionally a dropping mass of the ρ-meson is deduced from the QCD sum rule approach (cf. discussion in the introduction). It is natural to assume in such a scenario that also the masses of the higher-lying states drop which effectively leads to a lowering of the threshold $s_0$. On the other hand, the present picture deduced from the large-$N_c$ considerations suggests that there is no change of the ρ-peak (besides level repulsion, see below), but the appearance of an additional peak, the collective resonance-hole excitation. If the ρ-peak is unchanged there is no reason why the higher-lying states should move. Consequently the threshold remains more or less the same.

It is also interesting to discuss the pole structure of (17) or (18), respectively. Besides the ρ-meson pole which is already present in the vacuum, there appears a second pole caused by the collective resonance-hole excitation. At normal nuclear matter density the two poles are at $m_1 \approx (1480 - 940) \text{MeV} = 540 \text{MeV}$ and $m_2 \approx 830 \text{MeV}$. As compared to the “free” poles ($1520 - 940) \text{MeV} = 580 \text{MeV}$ and $770 \text{MeV}$, level repulsion shifts the collective pole downwards and the ρ-pole upwards. Note that we went beyond the linear density approximation by putting the self energy (20) in the denominator of (18). Only by this iteration the ρ-peak shifts.

Our final numerical result is given by (30). Nonetheless it is illuminating to obtain an analytic result which provides a rough estimate. For that purpose we observe that our final result for $r$ is rather close to 1. Therefore we evaluate the weighted finite energy sum rules for $r = 1$ (which is strict VMD [37]). The left hand side of (16a) vanishes — if the coupling does not diverge. (This can also be observed in the strong rise of the dotted curve in Fig. 2) Therefore it is of no use for our present purpose. In contrast, (16a) does not vanish. Inspecting the right hand side, we observe that the numerically strongest contribution comes from the $a_2$ term (cf. [13d], [18a] and Tab. I). This is actually an interesting observation since the size of this term is rather save as compared to the size of $\sigma_N$. The latter enters the in-medium parts of the two- and four-quark condensate. Therefore we feel rather comfortable with our approach where the strongest contribution comes from a well-known term. Using for simplicity $a_2 \approx 1$ and $g_{\gamma}^2 s_0/m_V^2 \approx 24\pi^2/N_c$ [37], neglecting all other terms we obtain after some calculation:

$$ f_\rho \approx \frac{9\pi^2 m_N}{(m_{D_{13}} - m_N) N_c} \approx 48, \quad \text{rough estimate!} \tag{34} $$

This indicates that parametrically $f_\rho$ and therefore also the ρ-nucleon scattering amplitude is $o(N_c^0)$ as it should be.

Clearly the value for the ρ-nucleon-$D_{13}$ coupling constant crucially depends on the input lagrangian used for the calculation. Therefore for a comparison of our results to other approaches it is more illuminating to calculate quantities which are measurable or at least closer to measurable ones. For that purpose we use $f_\rho$ as an input for eq. (9) in [12] to calculate the partial decay width $D_{13} \to \rho N$. We obtain $\approx 10 \text{MeV}$. This value compares favorably with 12MeV obtained by [51]. It is smaller than 26MeV deduced in [52, 53] but much bigger than 2MeV from [17]. Note that the $N^*(1520)$ is nominally subthreshold with respect to the decay products ρ-meson plus nucleon. The decay is only possible due to the finite width of the ρ-meson. This has two interrelated aspects: Technically, for a reasonable width calculation for such a baryonic resonance it is crucial to respect a proper width (self-energy) parameterization for the ρ-meson. Therefore we use the momentum-dependent width caused by the two-pion decay of the ρ-meson [12]. The second aspect is the principal one already discussed above: Extracting a coupling constant for such a decay situation from the two-pion data introduces a rather high model dependence into the analysis. It is appealing that we have extracted here additional information from a completely different source.

We can also calculate the partial decay width of the $D_{13}$ into a nucleon and an isovector photon:

$$ \Gamma_{N\gamma} = \frac{\mu^2 m_N q_{cm}^3}{3\pi m_{D_{13}}} \tag{35} $$

with the center-of-mass momentum

$$ q_{cm} = \frac{m_{D_{13}}^2 - m_N^2}{2m_{D_{13}}} \tag{36} $$

and [37]

$$ \mu = \frac{e f_\gamma}{g_{\gamma} m_V} \tag{37} $$

where $e$ denotes the electric charge. We obtain $\approx 1 \text{MeV}$ which can be compared to the analysis of Arndt et al. [54] where $\approx 0.51 \text{MeV}$ is deduced.
At first sight, one might feel disappointed about this factor of two mismatch. However, we do not share that point of view for the following reasons: First, it is important to stress that we have presented a completely parameter-free determination of the coupling constants. It is by far non-trivial to obtain agreement within a factor of two. Second, one has to recall that the uncertainty in the value for the decay width into $\rho$-nucleon is much larger than a factor of two, as discussed above. Therefore we could actually narrow the range of possible values in that case. Third, several approximations entered the calculation: We worked in the large-$N_c$ limit (otherwise we could not sort out the various hadronic in-medium effects). We neglected all other resonances besides the $D_{13}$ (otherwise we would not have enough equations to determine the various coupling constants). We used a non-relativistic model for $D_{13}$ and neglected the width of the $D_{13}$. It is interesting to note that this last point is actually not crucial: We have found numerically that the obtained values for the coupling constants $f_\rho$ and $f_\pi$ practically do not depend on the value for the $D_{13}$ mass. Therefore a finite width cannot change the results. In view of all these considerations we regard our results for the coupling constants as rather satisfying.

Before summarizing we would like to comment on the possible nature of the $D_{13}(1520)$-resonance. It has been suggested recently \(^{17, 55}\) that this resonance (among others) could be a consequence of coupled-channel dynamics and not a three-quark resonance. In the present work we have treated the resonance as an elementary field (cf. (24)). This, however, merely tells about the way how to effectively describe the resonance and not about its nature. The more relevant question is whether the resonance survives in the large-$N_c$ limit. Clearly a three-quark resonance does. Whether this is also true for a dynamically created resonance actually depends on the details of the underlying model. In particular, it depends on the question whether the four-point meson-baryon interactions studied in \(^{17, 55}\) survive or become suppressed in the large-$N_c$ limit. In principle, such vertices can be $O(N_c^0)$ as discussed above. In this case our analysis still applies. In \(^{17}\) no statement about the $N_c$ scaling of the coupling constants was made and could be made, as they were introduced on a phenomenological level. In contrast, in \(^{55}\) it was claimed that the main mechanism for dynamical resonance formation is the Weinberg-Tomozawa interaction. The latter is completely fixed by the pion decay constant and produces four-point vertices of $O(1/N_c)$. For such a scenario our analysis does not fit, since no resonance would be formed in the large-$N_c$ limit. Nonetheless the analysis of \(^{55}\) is not and was not meant to be a complete analysis of the negative parity, spin-3/2 resonances. Only channels with Goldstone boson and baryon decuplet states where considered there while e.g. channels with vector mesons and baryon octet states where missing. It is not clear whether the interaction in the latter channels (and its mixing with the former ones) also vanishes in the large-$N_c$ limit. \textit{A priori} there is no reason why it should vanish. If this interaction was sufficient to form the $D_{13}(1520)$ — maybe with a different mass — our analysis might still be applicable.

V. SUMMARY

The present work relies on two basic assumptions. First, that the QCD sum rule approach provides a connection between hadronic and quark-gluon properties not only for $N_c = 3$ but for arbitrary number of colors, in particular for $N_c \rightarrow \infty$. Second, that for $N_c \rightarrow \infty$ the input parameters (Tab. 1) do not differ much from their $N_c = 3$ values (besides the appropriate rescaling factors $3/N_c$ to some power). The condensate side of the QCD sum rules for vector mesons changes sizably in a nuclear medium. Under the basic assumptions given above most of this change survives in the $N_c \rightarrow \infty$ limit. We have shown that the in-medium four-quark condensate can be related to the two-quark condensates in the large-$N_c$ limit. For $N_c = 3$ the unknown size of the four-quark condensate provides the major source of uncertainty on the condensate side. On the hadronic side we have discussed various in-medium modifications. For $N_c = 3$ none of them can be neglected \textit{a priori}. It was a long-standing problem which of these hadronic in-medium modifications (or which combination of them) corresponds to the large in-medium change observed on the condensate side. We have found here that the different hadronic effects show different scaling in powers of $1/N_c$. In particular, in the large-$N_c$ limit only the coupling to resonance-hole loops survives. Therefore this effect provides the hadronic counterpart of the large in-medium change of the condensate side. It is interesting to note that the hadronic effects which vanish in the large-$N_c$ limit mainly cause a broadening of the spectrum and a (small) mass shift. On the other hand, the coupling to resonance-hole excitations generates new peak structures. Hence the large-$N_c$ analysis shows that the QCD sum rules do not point towards an in-medium mass shift or a peak broadening but indicate the appearance of one or several new peaks. This is the qualitative result of our analysis.

To become more quantitative we had to further specify the hadronic input. Here we were guided by our phenomenological experience \(^{12, 14, 15}\) that among the baryonic resonances it is the $D_{13}(1520)$ which provides most of the in-medium change of $\rho$-mesons (at rest). In that spirit we have developed an ansatz where the collective excitation of a $D_{13}$-resonance and a nucleon-hole couples to the $\rho$-meson as well as directly to the corresponding current. The coupling constants were then determined from the QCD sum rules. In view of the approximations made (large-$N_c$ limit, neglecting all other resonances, non-relativistic model for $D_{13}$, neglecting the width of the $D_{13}$) we regard it as satisfying to obtain reasonable values for the partial decay widths as discussed above. In that context it is interesting
to note that we have found that the obtained values for the coupling constants $f_\rho$ and $f_\gamma$ practically do not depend on the value for the $D_{13}$ mass. Therefore we do not expect that an analysis with a finite $D_{13}$ decay width changes our results. Also if the mass of the $D_{13}$ in the large-$N_c$ limit should significantly deviate from the physical one, we would not find a noticeable change — provided that the scaling behavior given in Tab. I remains untouched. We have also found that the in-medium change of the threshold parameter is extremely small. In other words, the only in-medium change that is seen in the QCD sum rule approach (at least in the large-$N_c$ limit) is the collective excitation of the $D_{13}(1520)$ and a nucleon-hole.

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