Influence of Opinion Dynamics on the Evolution of Games

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Abstract

Under certain circumstances such as lack of information or bounded rationality, human players can take decisions on which strategy to choose in a game on the basis of simple opinions. These opinions can be modified after each round by observing one’s own or others’ payoff results but can be also modified after interchanging impressions with other players. In this way, the update of the strategies can become a question that goes beyond simple evolutionary rules based on fitness and becomes a social issue. In this work, we explore this scenario by coupling a game with an opinion dynamics model. The opinion is represented by a continuous variable that corresponds to the certainty of the agents with respect to which strategy is best. The opinions transform into actions by making the selection of an strategy a stochastic event with a probability regulated by the opinion. A certain regard for the previous round payoff is included but the main update rules of the opinion are given by a model inspired in social interchanges. We find that the fixed points of the dynamics of the coupled model are different from those of the evolutionary game or the opinion models alone. Furthermore, new features emerge such as the independence of the fraction of cooperators with respect to the topology of the social interaction network or the presence of a small fraction of extremist players.

Introduction

Evolutionary game theory has been introduced as a framework to study the processes of selection of genes or behaviors in biological and social systems [1–3]. Its aim is to characterize the choices in terms of strategies of individuals of a population playing a game. A particular strategy generates a payoff to the individual playing it that depends on the selection of the rest of individuals. The key assumption of the evolutionary theory is that the fitness of an individual to reproduce directly relates to the payoff obtained [1]. Consequently, most successful strategies in terms of payoff are also those that multiply faster and can eventually become dominant after some generations.

These ideas find an analytical expression in the form of the so-called replicator equation [4–6]. If \( x_i \) stands for the fraction of individuals in the population playing strategy \( i \), \( f_i(\bar{x}) \) for their payoff and \( \bar{f}(\bar{x}) \) for the average payoff over all the population, the replicator equation reads:

\[
\frac{dx_i}{dt} = x_i (f_i(\bar{x}) - \bar{f}(\bar{x})).
\]  

(1)

The fixed points and limit cycles of the equation define the final state of the system regarding the distribution of strategies in the population [3–5,7]. Moreover, the study of the stability of the solutions, particularly if they are formed by single strategies, to invasion by other strategies motivates the definition of evolutionary stable strategies (ESS) [7]. To illustrate the predictions of
of social networks enhances collaboration via the heterogeneity of individual roles that the different positions in the network produce [15–20]. Also random mutations or the individuals’ free exploration to search for a best response to the strategies of their counterparts are another element that can promote collaboration [13,21–24]. Finally, the fixed points of the system dynamics, including the level of cooperation, are affected too by the way in which the system updates either by taking into account discrete versus continuous dynamics [25,26] or by altering the update rules [27–30].

In this work, we explore a mechanism that can also play an important role to raise collaboration levels in social systems. The basic idea goes back to the fact that humans not always take the most rational option when presented with a dilemma [31–33]. This has been observed in experiments in controlled environments in which participants, in general students, were playing Prisoner’s dilemma [34–39]. Also, in other level, it is a well known behavior in the world of finances where decisions on buying and selling can be taken based on rumors or on a general state of opinion about the possibilities of an investment [40]. Our proposal is to increase the dimensionality of the system by noting that the opinion on which is the best strategy is an important variable to incorporate, even though in some cases such belief can be wrong or baseless with respect to actual performance in the game. The evolution of the system includes thus a purely social ingredient related to opinion formation [41] followed by a process of decision taking that relies on the formed opinion. In the abstract representation of Equation (1), the addition of a variable of opinion can be modeled as

\[
\begin{align*}
\frac{dx_i}{dt} &= g(\phi_i, x_i), \\
\frac{d\phi_j}{dt} &= h(\phi, x),
\end{align*}
\]

where the index describing the opinion j can be continuous or, as in this example, discrete, \( \phi \), represents the fractions of individuals holding opinion j, \( g() \) is a function that relates the opinion j with the probability of playing strategy i and the function \( h() \) describes the evolution of the opinions given the state of the system and the outcome of the game. The addition of the new field \( \phi \) corresponding to the opinions of the individuals and the new rules of update given by the interchange of opinions between individuals can lead to extremely different fixed points and solutions for this system. In the following, we provide an example with a simple model that shows how these ideas can be implemented in practice and how the dynamic and stationary predictions of evolutionary game theory can dramatically change due to the coupling between opinion and games.

**Model**

We take as basis a well-known model for opinion dynamics, the Deffuant model [42], and a game inspired by the dilemma of the tragedy of commons [43,44]. The opinion in the Deffuant model is described by a continuous variable \( \phi \) between -1 and 1. Considering a population of N agents, each one placed on a node of a network, the update of opinions is carried out by randomly choosing an agent i and one of her neighbors j and comparing their opinions at time t, \( \phi_i \) and \( \phi_j \). If \( |\phi_i - \phi_j| < \epsilon \), the interaction occurs and the new opinions are given by

\[
\begin{align*}
\phi_i(t+1) &= \phi_i + \mu(\phi_j - \phi_i), \\
\phi_j(t+1) &= \phi_j + \mu(\phi_i - \phi_j).
\end{align*}
\]

Otherwise, if the difference between \( \phi_i \) and \( \phi_j \) is larger than \( \epsilon \), there is no interaction. The parameter \( \mu \) is the so-called convergence parameter since it regulates to which new value the opinions converge after interaction. In this work, we set it at \( \mu = 1/2 \) which implies that the final opinion is the average over both agents opinion. The Deffuant’s model shows bounded confidence in the sense that interactions between agents whose opinions are further apart than \( \epsilon \) are forbidden. The value of \( \epsilon \) is thus a key parameter to take into account in the following study.

For the game, we consider a simple set of rules that permit the exploration of a dilemma and a harmony scenario by tuning a single parameter. This allows us to show the validity of our findings regardless of the game’s ESS. In the rules every time that an agent i plays, she does so with all her k_i neighbors. An unit of wealth is then distributed among all of them. If everybody cooperates then the payoff is \( 1/k_i \) for each agent. Otherwise, each defector is given priority and takes a portion \( p \) as payoff. If the total amount requested by the defectors, \( p n_D \), is larger than 1 nobody takes anything. On the contrary, if \( p n_D \leq 1 \), the cooperators evenly divide the remaining \( 1 - p n_D \). Note that for low values of \( p \), \( p < 1/k_i \), collaboration is the strategy with the largest payoff and in a pure evolutionary framework becomes the only survival. The same occurs on the other extreme for high values of \( p \), strictly speaking for \( p > 1 \) defection has a zero payoff. In the area of intermediate \( p \) values, the equilibrium of our system is equivalent to that of the public goods game and show the effects of the tragedy of commons dilemma because defection is the most advantageous strategy but if every agent opts for it none of them get any payoff [43,44].

After describing the opinion dynamics and the game rules, it is important to explain how both are coupled. As illustrated in Figure 1, the two extremes of the opinion variable \( \phi \) are identified with the strategies D and C. \( \phi \) represents thus the opinion of the agents about which is the best strategy to win the game. The pass from an agent’s opinion to real action is taken by assuming a probability \( p_C = (1+\phi)/2 \) of playing C and \( p_D = 1-p_C = (1-\phi)/2 \) of choosing D. It is important to stress that the game is actually played in a mixed strategy framework and that this way of implementing opinion and action is assuming incomplete information, actions based on impressions and a social component in the way the players move towards the selection of a strategy. In practice, the model is updated by choosing a random agent i in each time step, then she plays the game with her neighbors and after this her opinion is updated depending on the earned payoff. For updating the opinions, a neighbor of i, j, is randomly selected and the new opinions are calculated using Deffuant’s model of Eq. (3) only if j’s payoff is equal or higher than i’s. Note that only i’s opinion is updated, which introduces an asymmetry in Deffuant’s rules. This asymmetry prevents players that are doing better from changing opinion due to interactions with others performing worse, and it also breaks the strong conservation of the average opinion that is a feature of the original Deffuant’s model.

**Results**

Let us start by considering a mean-field situation in which in each time step a randomly selected agent interacts with a group formed by four other agents chosen at random. The first results
Figure 1. Sketch showing the coupling between the opinion variable $\phi$ and the probability of opting for one of the two strategies in the game collaboration (C) or defection (D).

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Figure 1 demonstrates the relationship between the opinion variable $\phi$ and the probability of choosing between the collaboration (C) or defection (D) strategies in a repeated Prisoner's Dilemma game. The curves in Figure 1 show the probability distribution for different values of $\phi$, ranging from -1 to 1, with the probability of playing C being $(1+\phi)/2$ and the probability of playing D being $(1-\phi)/2$. The curves are color-coded to represent different strategies: blue for $\phi = 0.1$, green for $\phi = 0.4$, and red for $\phi = 0.8$. The curves visually illustrate how the probability of choosing C or D changes as $\phi$ varies, reflecting the dynamic interplay between cooperation and defection in the game.
number connections of the nodes (degree, \( k \)). For ER, the distribution of degrees is Poissonian centered around the average \( \langle k \rangle \), while for the BA the distribution of degree is a power-law decaying function with exponent \(-3\), \( P(k) \propto k^{-3} \). In each case, an agent plays each round of the game with her nearest neighbors alone. In Fig. 4A, the fraction of cooperators \( f_C \) is displayed as a function of the parameter \( p \) for different network topologies and \( \epsilon = 1/2 \). The fraction of cooperators is not very sensitive to the topology. One can find a stronger difference in the distribution of opinions as can be seen in Figure 4B and C, where it can be seen that a model with random interactions or scale-free networks have more marked peaks. We have also explored the spatial distribution of opinions and strategies when the game is played in a 2D square lattice with 4 neighbors per node (Fig. 4D and E). As occurs with the Prisoner’s dilemma in replicator dynamics [11], the reduced dimensionality allows for the formation of clusters of agents with close opinions playing similar strategies. The local character of the interactions makes that clusters of collaborators can survive. In Figure 4, we explore also the effect that the heterogeneity in the
degree of the agents in the social networks can have on the opinion. The agents' opinion in an instance and the average opinion over many realizations is displayed as a function of the agents' degree (plots F and G). The average opinion tends to be more negative, closer to defection, for better connected agents regardless of the particular characteristics of the network. Even though all the results shown in Figs. 2–5 are for systems of approximately 1000 agents, we have explored larger systems and networks. For instance, for systems with 10000 agents the dynamics becomes slower but the main features such as opinion distributions, fraction of cooperators and formation of domains in lattices are maintained in the stationary regime.

A final aspect of the model that we analyze is the effect that a small fraction of radical agents can have on the opinion and strategies played by the rest of the population. There are two precedents that justify the concern with the role that the extremists can play. One is the existence of such radical individuals playing always the same strategies either cooperation or defection in the experiments [38,39]. The second is that the effect of extremists, who go under the name of contrarians or zealots in the literature, is well known in the opinion dynamic models [47–49] or even in the evolution of games [50,51]. A small fraction of extremists can drive the system out of consensus. The fraction of cooperators obtained with the model as a function of $p$ and the opinion distribution for $p = 0.8$ are depicted in Figure 5. The curves for the model with a fraction of extremists of 5% either of players $C$ or $D$ are over-imposed to the baseline without extremists. As can be seen, the average fraction of collaborators $f_C$ is weakly dependent on the presence of extremists or zealots. Apart from a slight shift due to the additional 5% players of pure strategies, no major change is observed. However, the same cannot be said regarding the opinion distributions. Both models with extremists show different distributions even though the effect is more dramatic if the zealots are playing “defect”.

**Discussion**

In summary, we have introduced a model that couples opinion dynamics and strategies selection in a game. Our main assumptions are that the agents have not certainty on which strategy is optimal and that they form an opinion on this issue which can be...
updated by social pressure. In particular, for the game we have selected a model based on the rules proposed in the Tragedy of Commons by G. Hardin that allows us to explore two possible final equilibria by tuning a single parameter $p$. For $p$ below 0.2, the rules of our system produce a scenario that reminds a harmony game, while for $p > 0.2$ a social dilemma equivalent to the public goods game is found. For the opinion dynamics, we use the Deffuant’s model that characterizes by having a continuous opinion variable $\phi$ and a bounded confidence mechanism embodied by the parameter $\epsilon$. If the opinions of two agents are further away than $\epsilon$, no interaction is possible. We take advantage of the continuous nature of $\phi$ to couple opinions and actions via a mixed strategy scenario. The two available strategies $C$ and $D$ become thus an action that is taken with certainty only in the limits of opinion $\phi = 1$ and $-1$, respectively. Any intermediate value of the opinion can be translated into a probability of choosing $C$ or $D$ with a bias towards the closest extreme in $\phi$.

Once the coupling of opinion and game dynamics is on, the outcome of the game changes. Of course, the model is stochastic and so a certain amount of dispersion in the main descriptive variables is expected due to the inherent randomness. However, variables such as the average fraction of collaborators or the distribution of opinion reach fixed points in the dynamics different from the de-coupled systems that reflect the constraints that opinion and game payoff put on each other. This effect is enhanced when the parameter is decreased imposing a more strict bounded confidence regime. Cooperation can thus be increased with a more social dynamics for the evolution of the strategies but this is not the only feature that calls for attention in our results.

The presence of the variable of opinion allows the system to adapt to different interaction topologies or to the existence of extremist players in a very particular way. In correspondence to the empirical observations, in the coupled model the fraction of cooperators is not altered by the consideration of different topologies or by the introduction of extremists. It is the opinion distribution instead which is modified to absorb the impact of the new conditions. In the experiments, this phenomenon was explained by the presence of moody players that have into account previous strategies when a new strategic decision was taken. In our model this role is played by the memory effect that the opinion variable provides. In this work, we have selected particularly simple rules for the game and the opinion dynamics. In order to gain further insights in the decision process of real players more theoretical and experimental work is needed. Nevertheless, the interplay between opinion and actions and the fact that the opinion gets updated by social pressure can significantly modify the scenario in evolutionary games.

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Author Contributions
Conceived and designed the experiments: FG, JJR. Performed the experiments: FG, JJR. Analyzed the data: FG, JJR. Contributed reagents/materials/analysis tools: FG, JJR. Wrote the paper: FG, JJR.

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