B(s) to D(s) semileptonic decays with NRQCD-HISQ valence quarks

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Semileptonic B to D decays
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\[ \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu_\ell)} \]

HFAG 2015 (exp.)

1606.08030
HPQCD (2015)
FNAL/MILC (2015)
FNAL/MILC (2012)
1205.5442 (HQET)
Motivation

$B_s \rightarrow D(s)\ell\nu$ semileptonic decays provide determination of $V_{cb}$

Two lattice calculations of $B_s \rightarrow D_s\ell\nu$ at or near zero recoil

- FNAL/MILC
  - Bailey et al [FNAL/MILC], PRD 85 (2012) 114502

- Twisted mass fermions
  - Atoui et al, EPJ C 74 (2014) 2861

First unquenched analysis at nonzero recoil

Combined analysis of $B_s \rightarrow D_s\ell\nu$ and $B \rightarrow D\ell\nu$ to reduce theoretical uncertainties in determination of $B(B_s \rightarrow \mu^+\mu^-)$

Forthcoming LHCb analysis of $B_s \rightarrow D_s\ell\nu$ and $B_s \rightarrow K\ell\nu$ for $|V_{ub}/V_{cb}|$
Form factors

\[ q^\mu = p^\mu_{B_s} - p^\mu_{D_s} \]

\[ \langle D_s(p_{D_s})| V^\mu | B_s(p_{B_s}) \rangle = f_+(q^2) \left[ p^\mu_{D_s} + p^\mu_{B_s} - \frac{M^2_{B_s} - M^2_{D_s}}{q^2} q^\mu \right] + f_0(q^2) \frac{M^2_{B_s} - M^2_{D_s}}{q^2} q^\mu \]

Convenient to determine

\[ \langle D_s(p_{D_s})| V^\mu | B_s(p_{B_s}) \rangle = \sqrt{2M_{B_s}} \left[ f_\parallel(q^2) \frac{p^\mu_{B_s}}{M_{B_s}} + f_\perp(q^2) \left( p^\mu_{D_s} - \frac{p_{B_s} \cdot p_{D_s}}{M^2_{B_s}} p^\mu_{B_s} \right) \right] \]

In \( B_s \) rest frame

\[ \langle D_s(p_{D_s})| V^0 | B_s(p_{B_s}) \rangle = \sqrt{2M_{B_s}} f_\parallel(q^2), \quad \langle D_s(p_{D_s})| V^k | B_s(p_{B_s}) \rangle = \sqrt{2M_{B_s}} p^k_{D_s} f_\perp(q^2) \]

Reconstruct form factors

\[ f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}} \left[ f_\parallel(q^2) (M_{B_s} - E_{D_s}) f_\perp(q^2) \right] \]

\[ f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M^2_{B_s} - M^2_{D_s}} \left[ (M_{B_s} - E_{D_s}) f_\parallel(q^2) \left( E^2_{B_s} - M^2_{D_s} \right) f_\perp(q^2) \right] \]
Heavy quark currents

NRQCD bottom and HISQ charm valence quarks

\[ J^{(0)}_\mu = \bar{\psi}_c \gamma_\mu \psi_b, \quad J^{(1)}_\mu = -\frac{1}{M_b} \bar{\psi}_c \gamma_\mu \gamma \cdot \nabla \psi_b \]

Match currents via lattice perturbation theory

\[ \langle V_\mu \rangle_{\text{QCD}} = (1 + \alpha_s \rho_\mu) \langle J^{(0)}_\mu \rangle + \langle J^{(\text{sub})}_\mu \rangle \]

\[ J^{(\text{sub})}_\mu = J^{(1)}_\mu - \alpha_s \zeta_\mu J^{(0)}_\mu \]
Ensembles

MILC 2+1 asqtad ensembles

Cont. phys. point

area $\propto N_{tsrc}$

$m_l^{sea}/m_s^{sea}$

0.4
0.2
0.1
Correlators

$B_s$ meson

$$ \Phi^{\alpha} \Phi^{\alpha^\dagger} = a^3 \sum_{x'} \overline{\Psi}_b(x', t_0) \phi^\alpha(x' - x) \gamma_5 \psi_s(x, t_0) $$

Construct 2x2 correlator matrix

$$ C_{B_s}^{\beta,\alpha}(t, t_0) = \frac{1}{L^3} \sum_{x,y} \langle \Phi_{B_s}^\beta(y, t) \Phi_{B_s}^{\alpha^\dagger}(x, t_0) \rangle $$

$D_s$ meson

$$ \Phi^{\dagger} = a^3 \overline{\psi}_c(x, t_0) \gamma_5 \psi_s(x, t_0) $$

Two-point correlator function

$$ C_{D_s}(t, t_0; p) = \frac{1}{16 L^3} \sum_{x,y} e^{ip \cdot (x-y)} \langle \Phi_{D_s}(y, t) \Phi^{\dagger}_{D_s}(x, t_0) \rangle $$
Correlators

\[ C^\alpha_J(t, t_0, T; \mathbf{p}) = \frac{1}{L^3} \sum_{x, y, z} e^{i \mathbf{p} \cdot (z-x)} \left\langle \Phi_{D_s}(x, t_0 + T) J_\mu(z, t) \Phi_{B_s}^{\alpha \dagger}(y, t_0) \right\rangle \]

Four momenta \([(0,0,0), (1,0,0), (1,1,0), (1,1,1)]\)

Four values of \(T\): \([12,13,14,15]\) and \([21,22,23,24]\)
Correlator fits

Bayesian multi-exponential fitting strategy \([\text{corrfitter, lsqfit}]\)

\[
C_\alpha^j(t, T; p) = \sum_{j=0}^{N_{Bs}-1} \sum_{k=0}^{N_{Ds}-1} A_{jk}^\alpha e^{-E_j^{Ds}t} e^{-E_k^{Ds}(T-t)} + \ldots
\]

Extract three-point amplitudes

\[
A_{jk}^\alpha = \frac{\langle 0 | \Phi_{Ds} | E_j^{Ds} \rangle \langle E_j^{Ds} | J_\mu | E_k^{Bs} \rangle \langle E_k^{Bs} | \Phi_{Bs}^\dagger | 0 \rangle}{(2a^3 E_j^{Ds})(2a^3 E_k^{Bs})}
\]

Obtain required matrix element

\[
\langle D_s | J_\mu | B_s \rangle = A_{00}^\alpha \frac{(2a^3 E_0^{Ds})(2a^3 M_{Bs})}{\langle 0 | \Phi_{Ds} | E_0^{Ds} \rangle \langle M_{Bs} | \Phi_{Bs}^\dagger | 0 \rangle}
\]
Two-point fits

Stability plots: multi-exponential fits

| $aM_{D_s}$ | $aM_{D_s}$ |
|------------|------------|
| \[1.193\]  | \[0.851\]  |
| \[1.192\]  | \[0.850\]  |
| \[1.191\]  | \[0.849\]  |
| \[1.190\]  | \[0.848\]  |
| \[1.189\]  | \[0.847\]  |
| \[1.188\]  | \[0.847\]  |
| \[1.187\]  | \[0.847\]  |

$N_{\exp}$ | $N_{\exp}$
---|---
2 | 2
3 | 3
4 | 4
5 | 5
6 | 6
7 | 7
8 | 8

Ensemble C1 | Ensemble F1

Preliminary
Two-point fits

$D_s$ dispersion relation

$\frac{m^2+p^2}{E^2} = 1 + \frac{\alpha_s(ap)^2}{10}$
Three-point fits

Stability plots: T-combinations

$A_{00} \times 10^2$

Preliminary

Ensemble C2

Ensemble F1
Three–point fits

Correlations between momenta

Ensemble C2

Ensemble F1
Chiral–continuum fits

z-expansion

\[ z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - q_{\text{max}}^2}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - q_{\text{max}}^2}} \]

\[ t_+ = (M_{B_s} + M_{D_s})^2 \]

\[ q_{\text{max}}^2 = (M_{B_s} - M_{D_s})^2 \]

Fit to BCL parameterisation

\[ f_0 = \frac{1}{1 - q^2/M_0} \sum_{k=0}^{K-1} a_0^{(k)} z^k \]

\[ f_+ = \frac{1}{1 - q^2/M_{B^*_c}} \sum_{k=0}^{K-1} a_+^{(k)} \left[ z^k - (-1)^{k-K} \frac{k}{K} z^K \right] \]

Modified expansion coefficients

\[ a_{0,+}^{(k)} = \tilde{a}_{0,+}^{(k)} D_{0,+}^{(k)} (m^{\text{val}}, m^{\text{sea}}, a) \]

chiral logs and discretisation effects

Bourrely et al, PRD 79 (2009) 013008

Na et al [HPQCD], PRD 82 (2010) 114506

Bouchard et al [HPQCD], PRL 111 (2013) 162002
Modified z-expansion

Results from coarse ensembles
Modified z-expansion

$B \rightarrow D\ell\nu$ form factors

Na et al [HPQCD], PRD 92 (2015) 054510
B to D form factors

With BaBar data

\[ |V_{cb}| = 0.0402(17)(13) \quad R(D) = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu_\ell)} = 0.300(8) \]

Aubert et al [BaBar], PRL 104 (2010) 011802
Existing results

Experimental data not yet available

Form factor ratio at (almost) zero recoil

\[ \frac{f_0^{(s)}(M^2_\pi)}{f_0^{(d)}(M^2_\pi)} = 1.054(50) \]

[Atoui et al, EPJ C 74 (2014) 2861]

Form factor normalisation at zero recoil with twisted mass fermions

[Bailey et al [FNAL/MILC], PRD 85 (2012) 114502]
Conclusion...

$B_s \rightarrow D_s \ell \nu$ semileptonic decays provide determination of $V_{cb}$

Preliminary results for $B_s \rightarrow D_s \ell \nu$ form factors at non-zero recoil

... and outlook

Finalise chiral/continuum extrapolation

Simultaneous fit to determine form factor ratio at zero recoil

Combine data to extract $|V_{ub}/V_{cb}|$

previous HPQCD $B_s \rightarrow K \ell \nu$ analysis

forthcoming LHCb analysis
Thank you

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EXTRA SLIDES
## ENSEMBLES

MILC 2+1 asqtad ensembles

| Set | $r_1/a$ | $m_l^{\text{sea}}/m_s^{\text{sea}}$ | $N_{\text{conf}}$ | $N_{\text{tsrc}}$ | $L^3 \times N_t$ | $aM_b$ | $am_l^{\text{val}}$ | $am_s^{\text{val}}$ | $am_c^{\text{val}}$ |
|-----|--------|-----------------------------------|------------------|------------------|-----------------|-------|-----------------|-----------------|-----------------|
| C1  | 2.647  | 0.005/0.050                      | 2096             | 4                | $24^3 \times 64$ | 2.650 | 0.0070          | 0.0489          | 0.6207          |
| C2  | 2.618  | 0.010/0.050                      | 2256             | 2                | $20^3 \times 64$ | 2.688 | 0.0123          | 0.0492          | 0.6300          |
| C3  | 2.644  | 0.020/0.050                      | 1200             | 2                | $20^3 \times 64$ | 2.650 | 0.0246          | 0.0491          | 0.6235          |
| F1  | 3.699  | 0.0062/0.031                     | 1896             | 4                | $28^3 \times 96$ | 1.832 | 0.00674         | 0.0337          | 0.4130          |
| F2  | 3.712  | 0.0124/0.031                     | 1200             | 4                | $28^3 \times 96$ | 1.826 | 0.01350         | 0.0336          | 0.4120          |
CORRELATOR FITS

Meson correlators

\[ C_{B_s}^{\beta,\alpha}(t) = \sum_{j=0}^{N_{B_s}-1} b^\beta_k b^{\alpha*}_k e^{-E_j^{B_s}\text{sim} t} + \sum_{j=0}^{N'_{B_s}-1} b'^\beta_k b'^{\alpha\dagger}_k (-1)^t e^{-E'_j^{B_s}\text{sim} t} \]

\[ C_{D_s}(t, p) = \sum_{j=0}^{N_{D_s}-1} |d_k|^2 \left( e^{-E_j^{D_s} t} + e^{-E_j^{D_s} (N_t-t)} \right) + \sum_{j=0}^{N'_{D_s}-1} |d'_k|^2 (-1)^t \left( e^{-E'_j^{D_s} t} + e^{-E'_j^{D_s} (N_t-t)} \right) \]
CORRELATOR FITS

Three-point correlator

\[
C^\alpha_j(t, T; \mathbf{p}) = \sum_{j=0}^{N_{B_s}-1} \sum_{k=0}^{N_{D_s}-1} A_{jk}^\alpha e^{-E^D_j t} e^{-E^s_k (T-t)} \\
+ \sum_{j=0}^{N'_{B_s}-1} \sum_{k=0}^{N_{D_s}-1} B_{jk}^\alpha e^{-E^D_j t} e^{-E^s_k (T-t)} (-1)^{(T-t)} \\
+ \sum_{j=0}^{N_{B_s}-1} \sum_{k=0}^{N'_{D_s}-1} C_{jk}^\alpha e^{-E'^D_j t} e^{-E^s_k (T-t)} (-1)^t \\
+ \sum_{j=0}^{N'_{B_s}-1} \sum_{k=0}^{N'_{D_s}-1} D_{jk}^\alpha e^{-E'^D_j t} e^{-E'^s_k (T-t)} (-1)^T
\]
Chiral–continuum fits

“Modified z-expansion”

\[
f_0 = \frac{1}{1 - q^2/M_0} \sum_{k=0}^{K-1} a_0^{(k)} z^k
\]

\[
f_+ = \frac{1}{1 - q^2/M_{B_c^*}} \sum_{k=0}^{K-1} a_+^{(k)} \left[ z^k - (-1)^{(k-K)} \frac{k}{K} z^K \right]
\]

Where

\[
a_{0,+}^{(k)} = \tilde{a}_{0,+}^{(k)} D_{0,+}^{(k)} (m_{\text{val}}, m_{\text{sea}}, a)
\]

\[
D^{(k)} = 1 + \frac{M_\pi^2}{(4\pi f_\pi)^2} \left\{ c_1^{(k)} + c_2^{(k)} \log \left[ \frac{M_\pi^2}{(4\pi f_\pi)^2} \right] \right\}
\]

\[
+ c_3^{(k)} \left\{ \frac{(M_{\text{asqtad}}^2 - (M_{\pi}^{\text{HISQ}})^2)^2}{2(4\pi f_\pi)^2} + \frac{(M_K^{\text{asqtad}})^2 - (M_K^{\text{HISQ}})^2}{(4\pi f_K)^2} \right\}
\]

\[
+ d_1^{(k)} (a m_c^2) + d_2^{(k)} (a m_c)^4 + e_1^{(k)} \left( \frac{a E_{D_s}}{\pi} \right)^2 + e_2^{(k)} \left( \frac{a E_{D_s}}{\pi} \right)^4
\]