Local variance asymmetries in Planck temperature anisotropy maps

Saroj Adhikari\textsuperscript{1*}
\textsuperscript{1}Institute for Gravitation and the Cosmos, The Pennsylvania State University, University Park, PA 16802, USA

25 August 2014

\textbf{ABSTRACT}

Recently, it was shown that local variance maps of temperature anisotropy are simple and useful tools for the study of large scale hemispherical power asymmetry. This was done by studying the distribution of dipoles of the local variance maps. In this work, we extend the study of the dipolar asymmetry in local variance maps using foreground cleaned Planck 143 GHz and 217 GHz data to smaller scales. In doing so, we include the effect of the CMB doppler dipole. Further, we show that it is possible to use local variance maps to measure the doppler dipole in these Planck channel maps after removing large scale features (up to $l = 600$), at a significance of about $3\sigma$. At these small scales, we do not find any power asymmetry in the direction of the anomalous large scale power asymmetry beyond that expected from cosmic variance. At large scales, we verify previous results i.e. the presence of hemispherical power asymmetry at a significance of at least $3.3\sigma$.

\textbf{Key words:} cosmology: observations, methods: statistical

1 INTRODUCTION

A number of studies have been performed that show approximately $3.5\sigma$ hemispherical power asymmetry at large scales in WMAP and Planck CMB temperature fluctuations (Eriksen et al. 2004, 2007; Planck Collaboration et al. 2013a; Flender & Hotchkiss 2013; Hoftuft et al. 2009). Recently, Akrami et al. (2014) used a conceptually simple pixel space local variance method to demonstrate the presence of asymmetry at large scales in WMAP and Planck data at a significance of at least $3.3\sigma$; they find that none of the 1000 isotropic Planck FFP6 simulations had a local variance dipole amplitude equal to or greater than that found in data for disk radii $6^\circ \leq r_{\text{disk}} \leq 12^\circ$. In this method, a local variance map ($m_r$) is generated at a smaller HEALPIX (Görski et al. 2005) resolution $N_{\text{side}}$ from a higher resolution CMB temperature fluctuations map by computing temperature variance inside disks of certain radius $r_{\text{disk}}$ centered at the center of each pixel of the HEALPIX map with resolution $N_{\text{side}}$. Isotropic simulations are used to get the expected mean map ($\bar{m}_r$), and each map is normalized:

$$m_r^n = \frac{m_r - \bar{m}_r}{\bar{m}_r}.$$  \hspace{1cm}(1)

Then, we obtain local variance dipoles by fitting for dipoles in each of these normalized maps (both simulations and Planck data). The local variance dipole obtained from data can then be compared to the distribution obtained from simulations.

In this work, we make use of the local variance method and extend the results obtained in Akrami et al. (2014) to include smaller disk radii. The authors in Akrami et al. (2014) focused on large scale power asymmetry, and therefore only looked at large values of $r_{\text{disk}}$. After confirming their results at large disk radii ($r_{\text{disk}} \geq 4^\circ$), we perform the same local variance dipole analysis using smaller disk sizes. We find that, for smaller disk radii, the contribution of \textit{doppler dipole} becomes increasingly significant. The doppler dipole in the local variance map is an expected signal because of our velocity with respect to the CMB rest frame. While the direction and magnitude of CMB dipole has been known from previous CMB experiments (Jarosik et al. 2011), the doppler dipole signal in the temperature fluctuations is rather weak and reported only recently by the Planck Collaboration (Planck Collaboration et al. 2013) using harmonic space estimators (Amendola et al. 2011; Kosowsky & Kahnishvili 2011). We work towards detecting the expected doppler dipole in the local variance maps after removing large scale features from the maps. Our goal therefore is two fold: first, extend the local variance dipole study of the hemispherical power asymmetry to smaller disk radii and, second use the method of local variance to detect the doppler dipole whose amplitude is much smaller but is expected to contribute at all angular scales.

Before presenting the details of the analysis and results, we would like to point out and clarify a difference between our analysis and that of Akrami et al. They used 3072 disks ($N_{\text{side}} = 16$ healpix map) for all sizes of disks they considered ($r_{\text{disk}} \geq 1^\circ$). However, we find that for $r_{\text{disk}} = 1^\circ$ and $2^\circ$, 3072 disks are not enough to cover the whole sky. Therefore, we use $N_{\text{side}} = 32$ (12288 disks) for $r_{\text{disk}} = 2^\circ$ and $N_{\text{side}} = 64$ (49152 disks) for $r_{\text{disk}} = 1^\circ$. Once we do this, we find that, unlike the results in Akrami et al. (see Fig 2(a) and Table 1 in Akrami et al. 2014), none of our 1000 isotropic simulations produce local variance dipole amplitude larger than that of our foreground cleaned channel maps, for $r_{\text{disk}} = 1^\circ, 2^\circ$. In fact, the effect of anomalous dipole (with respect to the isotropic case) can be observed for even smaller angular disk radii (see Figure 3). This result, however, is not surprising because...
2 SIMULATIONS AND DATA

For our analysis, we use 1000 realizations of FFP6 Planck simulations that include lensing and instrumental effects. The FFP6 simulations (CMB and noise realizations separately) are provided at each Planck frequency. We generate local variance maps from which we obtain local variance dipole distributions following the method briefly introduced in the previous section (described in more detail in Akrami et al. (2014)). Since the FFP6 simulations for CMB and noise processed through the component separation procedure are not yet publicly available, we will generate foreground cleaned CMB channel maps at two frequencies 143 GHz and 217 GHz using the SEVEM method (Planck Collaboration et al. 2013b; Kim & Komatsu 2013). We use the same four template maps as the Planck Collaboration; these are difference maps of two near frequency channels: (30-44) GHz, (44-70) GHz, (545-353) GHz and (857-545) GHz. Before generating the four templates, we smooth the larger frequency channel map to the resolution of the smaller frequency channel map: $d_m^{\text{large}} \rightarrow d_m^{\text{large}} \times B_l^{\text{small}} / B_l^{\text{large}}$, where $B_l$ is the beam transfer function for the given frequency channel map. See Appendix C of Planck Collaboration et al. (2013b) for details of this method.

Once the linear coefficients for the four template maps are obtained, we use the same coefficients to process combinations of noise FFP6 simulations in order to generate noise maps for our foreground cleaned simulated maps. We process the Planck maps and the simulated maps identically in each step. In addition to the isotropic local variance maps (obtained from isotropic realizations of CMB plus noise), we obtain two new sets of simulated local variance maps from two other models which are obtained from the isotropic realizations as explained below:

Doppler model: The expected temperature modulation from the doppler effect, with an amplitude $0.00123b_v$ along the direction $(l, b) = (264^\circ, 48^\circ)$ (Jarosik et al. 2011). We take $b_v = 1.96$ for the 143 GHz map and $b_v = 3.07$ for the 217 GHz map (Planck Collaboration et al. 2013). The doppler modulation is generated in pixel space simply as a dipole modulation of the form:

$$\frac{\Delta T}{T}|_{\text{dop}}(\hat{n}) = (1 + 1.23 \times 10^{-3}b_v \hat{n} \cdot \hat{p}) \frac{\Delta T}{T}|_{\text{iso}}(\hat{n}) \quad (2)$$

Modulation model: The doppler modulation plus a large angle modulation, corresponding to a modulation at smoothing $f_{\text{whm}} = 1.25^\circ$, along the direction $(l, b) = (218^\circ, -20^\circ)$, with an amplitude $A_T = 0.073$ (Planck Collaboration et al. 2013a). The modulation:

$$\frac{\Delta T}{T}|_{\text{mod}}(\hat{n}) = (1 + A_T \hat{n} \cdot \hat{p}) \frac{\Delta T}{T}|_{\text{iso}}(\hat{n}) \quad (3)$$

is only applied to a doppler modulated map after smoothing at $f_{\text{whm}} = 1.25^\circ$. Here we have used the notation $A_T$ for the amplitude of modulation for temperature fluctuations in the above equation to distinguish it from the dipole modulation amplitude in local variance maps $A_{\text{LV}}$. Our choice of the smoothing scale $f_{\text{whm}} = 1.25^\circ$ is guided by our attempt to fit the obtained local variance asymmetry in data for different $r_{\text{disk}}$ that we have considered. For example, we find that a larger $f_{\text{whm}} = 5^\circ$ modulation does not reproduce the local variance asymmetry in data for $r_{\text{disk}} = 1^\circ$ or smaller. Similarly, using a smaller fwhm produces local variance asymmetry distribution with amplitudes too large to be consistent with $2\sigma$ that with seen in data for some disk radii ($r_{\text{disk}} = 1^\circ$ and $4^\circ$, for example). Note that when we apply modulation at large scales in this manner, we are using a filter that will suppress modulation at scales much smaller than the smoothing fwhm specified. Therefore, by construction, the modulation is only generated at large scales. The asymmetry generated by our modulation model in harmonic space is plotted (up to $l = 600$) in Figure [1].

3 RESULTS

First, let us comment that we get results consistent with Akrami et al. (2014) for $r_{\text{disk}} = (4^\circ \text{ to } 16^\circ)$, in terms of the significance of the magnitude, and the direction of the anomalous dipole. Also, our modified set of simulations that includes temperature modulation at large angular scales produces dipole distributions consistent with data. All our results presented in this paper were analyzed using the foreground cleaned 217 GHz maps unless otherwise stated.
Local variance asymmetries in Planck temperature anisotropy maps

Figure 2. Foreground cleaned maps at channels 143 GHz and 217 GHz. Planck U73 mask is used in both maps.

Figure 3. Local variance dipole analysis for $r_{\text{disk}} = 1, 4$ degrees. The vertical line and the text describe the measurements from Planck maps. The histograms are obtained from simulations: (i) light red: isotropic FFP6 simulations, (ii) blue: doppler effect added, and (iii) green: doppler and large scale modulation added. For $r_{\text{disk}} = 2^\circ$ (not shown), we get local variance dipole of amplitude $A_{LV} = 0.026 (2.8\sigma)$ in the direction $(l, b) = (203^\circ, -1^\circ)$. The significance values in the above figures are computed using the doppler modulated distributions. If instead we use the isotropic distribution, we obtain values of $3.5\sigma, 3.4\sigma$ and $4.0\sigma$ for $1^\circ, 2^\circ$ and $4^\circ$ disk radii respectively.

Figure 4. Local variance dipole analysis for $r_{\text{disk}} = 0.25, 0.18$ degrees. The vertical line and the text describe the measurements from Planck maps. The histograms are obtained from simulations: (i) red: plain FFP6 simulations, (ii) blue: doppler effect added, and (iii) green: doppler and large scale modulation added. The significance in the figures are calculated using the doppler modulated distributions. If instead we use the isotropic distribution, the corresponding significance becomes $3.78\sigma$ and $3.82\sigma$ respectively.
For large scale results i.e. relating to the anomalous dipole, we get similar results using the 143 GHz maps.

3.1 Local variance dipole for \( r_{\text{disk}} = 1^\circ, 2^\circ \)

For \( r_{\text{disk}} = 1^\circ, 2^\circ \), our results in Figure 3 indicate that the effect of the doppler dipole is increasing compared to larger disk radii. Further, the results also show that the modulation model that we have chosen (modulated at large scales corresponding to a smoothing of \( f_{\text{whm}} = 1.25^\circ \), with an amplitude \( A_T = 0.073 \)) is consistent with data, while the no modulation (isotropic) model is disfavored at approximately 3.5\( \sigma \). Note that the distribution of \( A_{\text{LV}} \) is not exactly Gaussian, but we assume the distribution to be Gaussian to obtain standard deviation significance values as rough guides.

Next, we extend the analysis to apply to even smaller disk radii.

3.2 Local variance dipole for sub-degree disk radii

We will now consider disks of radii \( r_{\text{disk}} = 0.25^\circ \) and 0.18\( ^\circ \). This requires that we increase the number of disks to cover the whole sky. We therefore use disks centered on pixels of a healpix map of \( N_{\text{side}} = 256 \) (786432 disks). Figure 4 shows our results for these two disk sizes. From the figures, it is clear that for these disk radii, the effect of doppler dipole cannot be neglected when computing the significance of the dipole amplitude obtained in the data. If we do not include the doppler effect, then the local variance dipole in the Planck map is anomalous at approximately 3.8\( \sigma \) for both \( r_{\text{disk}} = 0.25^\circ \) and 0.18\( ^\circ \), whereas with respect to the doppler modulated distribution the significance drops to about 2\( \sigma \). Also, the direction of the dipole detected from Planck maps gets closer to the doppler dipole as the disk radius is decreased. This is illustrated in Figure 5. However, as seen in Figure 4, the effect is still small and the distributions of the isotropic simulations and doppler model simulations overlap quite a bit. We will now investigate if, by removing large scale features from the temperature fluctuation maps, it is possible to separate the doppler modulated local variance dipole distribution from the isotropic one, and therefore measure the doppler dipole in the Planck maps.

3.3 The doppler dipole in local variance maps

We remove large scale features by simply filtering a map using a high-\( l \) filter i.e. set the low-\( l \) \( a_{l,m} \) values to zero in multipole space. Since we are looking for an expected signal that is a vector (i.e. to test the significance of the measurement we need to look at both the direction and magnitude of the dipole signals), we will determine the distribution of the component of the dipoles obtained in the direction of the known CMB dipole. This was also the approach taken by the Planck Collaboration [Planck Collaboration et al. 2013], the quantity whose distribution they plot, called \( \beta_m \), is the component of their estimator in the CMB dipole direction.

In Figure 5 we show results for our local variance analysis using disk radius \( r_{\text{disk}} = 0.18^\circ \) but using CMB temperature maps with \( a_{l,0} \) = 0 for \( l \leq \ell_{\text{min}} \) = 600 i.e. large scale features removed up to \( l = 600 \). The signal in the direction parallel to the CMB dipole is consistent with the doppler modulated model distribution while the isotropic model is disfavored at 3.2\( \sigma \). When using a larger disk radius \( r_{\text{disk}} = 8^\circ \) (also shown in Figure 5), we measure the doppler dipole signal at approximately 2.7\( \sigma \).

We have repeated this analysis using other disk radii, different \( \ell_{\text{min}} \) and the 143 GHz channel map. The results are summarized in Table 1. Unexpectedly, we obtain higher values of amplitude for the 143 GHz channel but in most cases the dipole amplitude in the direction of CMB dipole is within 2\( \sigma \) of the expected distribution obtained from the doppler modulated model. The only exception being the case of \( \ell_{\text{min}} = 900 \), 143 GHz, \( r_{\text{disk}} = 0.18^\circ \) in which the signal in data is at 3.1\( \sigma \) from the doppler distribution. In all cases, no amplitude in excess of 2\( \sigma \) is obtained in two directions orthogonal to the CMB dipole and with each other (see Figure 2 of Planck Collaboration et al. [2013]).

The local variance dipole directions obtained for various cases are shown in Figure 6.

3.4 Small scale power asymmetry

We can also investigate the presence of power asymmetry at small scales in the direction \( (l, b) = (218^\circ, -20^\circ) \), in our small scale maps that have \( a_{l,m} \)'s up to \( l = 600 \) set to zero. A similar study of the power asymmetry at small scales (but in multipole space) was performed in Flender & Hotchkiss [2013] using foreground cleaned SMICA maps; they report no such small scale asymmetry after accounting for a number of effects, including estimates of power asymmetry due to the doppler effect. They report an upper bound on the modulation amplitude of 0.0045 (95\%) at these scales. Another important previous work on constraining hemispherical power asymmetry at smaller scales is Hirata [2009] using quasars that report \(< A_T < 0.012 \approx 0.5\)hMpc\(^{-1} \). Thus, we subtract this shift to get the value of intrinsic local variance power asymmetry. Using \( r_{\text{disk}} = 0.18^\circ \), we obtain (at 1\( \sigma \)):

\[
A_{\text{LV}} = (7.05 \pm 13.3) \times 10^{-3}
\]  

(4)

However, we can see from our results for doppler modulated distributions that the relation between \( A_T \) of a dipole modulated model and the most likely value of \( A_{\text{LV}} \) obtained from the corresponding local variance maps is not quite simple. This can be directly seen in Figure 5 in which the doppler model had \( A_T = 3.07 \times 0.00123 \approx 0.0038 \) whereas the obtained distributions for \( A_{\text{LV}} \) are different for the two cases with different \( r_{\text{disk}} \). To estimate the \( A_T \) constraint from the \( A_{\text{LV}} \) constraint in eqn (4) let us assume that the relationship between them is linear for small values of \( A_T \) up to approx-

| \( \ell_{\text{min}} \) | \( r_{\text{disk}} \) | 217 GHz | 143 GHz |
|---|---|---|---|
| 600 | 0.18 | 0.0042 (2.7\( \sigma \)) | 0.0075 (3.4\( \sigma \)) |
| 900 | 0.18 | 0.0027 (2.9\( \sigma \)) | 0.0047 (4.1\( \sigma \)) |
| 600 | 8.0 | 0.0048 (2.7\( \sigma \)) | 0.006 (2.4\( \sigma \)) |
| 900 | 8.0 | 0.0042 (2.6\( \sigma \)) | 0.0071 (2.9\( \sigma \)) |

Table 1. Summary of doppler dipole detection results. The significance of detection is computed with respect to the corresponding isotropic distribution for local variance amplitudes (1000 simulations) in the CMB dipole direction.

© 0000 RAS, MNRAS 000, 000–000
Local variance asymmetries in Planck temperature anisotropy maps

Figure 5. The component of local variance dipole amplitudes in the direction \((l, b) = (264^\circ, 48^\circ)\) using disk of radius \(r_{\text{disk}} = 0.18^\circ, 8^\circ\) after removing large scales features up to \(l_{\min} = 600\). In both cases, one obtains the doppler signal at nearly 3\(\sigma\), while no effect is seen in the orthogonal directions (not shown in figures). As in the previous figures, the light red distribution is obtained from the isotropic case while the blue is obtained from the doppler modulated model. The results for our large scale modulation model (green) are also plotted above but they mostly coincide, as expected since we have removed the large scale features, with the doppler models.

Figure 6. Here we plot some of the directions for the local variance dipoles in our analysis in addition to the CMB dipole direction (black square) and the large scale power asymmetry direction (black cross). The blue dots are directions obtained for the local variance dipole using foreground cleaned 217 GHz Planck maps with the labeled value of \(r_{\text{disk}}\). For smaller \(r_{\text{disk}}\), the dipole direction moves towards the CMB dipole direction. We also plot directions obtained using maps that have large scale features removed (red triangles), with \(l_{\min} = 600\) (triangle up) and \(l_{\min} = 900\) (triangle down).

imately \(A_T = 0.0038\) (for a given frequency channel, \(r_{\text{disk}}\) and \(l_{\min}\)). Then, for the case of 217 GHz channel maps, \(r_{\text{disk}} = 0.18^\circ\) and \(l_{\min} = 600\), we can use the correspondence between \(A_T\) and \(A_{LV}\) in Figure 5 and translate the constraint in eqn 4 to:

\[
A_T = (0.9 \pm 1.6) \times 10^{-3}
\]

To further check our translation between \(A_{LV}\) and \(A_T\), we repeated this small scale analysis with a \(A_T,\text{input} = 0.0009\) modulation model and recovered the intrinsic local variance dipole amplitude obtained in eqn 4. Using other larger values of \(r_{\text{disk}}\) or the 143 GHz channel map, we obtained slightly weaker but consistent constraints for small scale power asymmetry \(A_T\) in the direction of hemispherical power asymmetry.

4 DISCUSSION AND SUMMARY

In this short work, we have used local variance maps to study the power asymmetry in Planck temperature anisotropy maps, extending the work done in Akrami et al. (2014) that used the same method to smaller scales. We have shown that the effect of doppler dipole is small for local variance dipole measurements at large disk
asymmetry as statistical assumptions that we make about the universe at large scales. Any direction consists of learning more about the fundamental statistical possibilities are important since the implications of progress in 2014. The statistical fluke hypothesis still remains a possibility too. 

Existing proposals exist (Dai et al. 2013; Schmid & Hui 2013; Lyth et al. 2014). While a satisfying theoretical explanation for the power asymmetry anomaly still lacks in the literature, several interesting papers exist (Dai et al. 2013; Schmid & Hui 2013; Lyth et al. 2014). The statistical fluke hypothesis still remains a possibility too.

For our large scale asymmetry analysis, we find similar results using both 217 GHz channel maps (reported in figures in this paper) and 143 GHz channel maps. This is in agreement with previous works (Hofuift et al. 2009; Hansen et al. 2004) that have investigated channel dependence of hemispherical power asymmetry in WMAP maps.

Throughout our analysis, we included a large scale modulation model in which temperature anisotropies were modulated only for scales smooth (Gaussian smoothing) at fwhm=1.25 degrees and found that the anomalous power asymmetry seen at all r_disk analyzed in this paper using the local variance method is consistent with this simple phenomenological model. This tells us about the scale dependence of the power asymmetry seen in data as our modulation model generates a scale dependent hemispherical asymmetry in C_\ell (see Figure 1).

We summarize important aspects of our work and results in the following points:

- We have verified the hemispherical power asymmetry results obtained in Akrami et al. (2014), with the exception of r_disk = 1, 2 degrees for which we have identified the need to use more number of disks in order to cover the whole sky.
- Once we use smaller disk radii in our local variance analysis, the effect of doppler modulation becomes increasingly important which can be directly observed from our dipole amplitude distributions.
- After removing large scale features up to l = l_{min} = \{600, 900\}, we could detect the expected doppler modulation in Planck temperature anisotropy maps at a significance of approximately 3\sigma.
- We have obtained constraint on dipolar modulation amplitude at small scales (l > 600) in the direction of hemispherical power asymmetry as A_T = 0.0009 \pm 0.0016(\sigma).

We expect this work to be useful for a better understanding of the power asymmetries that are known to exist in the CMB data. In particular, we hope that our work will shed more light on local variance statistics in CMB maps which is already being used to compare theoretical models of power asymmetry with data (Jazayeri et al. 2014). While a satisfying theoretical explanation for the power asymmetry anomaly still lacks in the literature, several interesting proposals exist (Dai et al. 2013; Schmid & Hui 2013; Lyth et al. 2014). The statistical fluke hypothesis still remains a possibility too (Bennett et al. 2011).

ACKNOWLEDGMENTS

This work is supported by the National Aeronautics and Space Administration under Grant No. NNX12AC99G issued through the Astrophysics Theory Program. The computations for the project were performed using computing resources at the Penn State Research Computing and Cyberinfrastructure (RCC). The author would like to thank Yashar Akrami and Sarah Shandera for discussions and many useful suggestions, Donghua Jeong for useful suggestions on an earlier draft of this work, and Julian Borrill for help with getting the Planck FFP6 simulation data.

REFERENCES

Akrami Y., Fantaye Y., Shafieloo A., Eriksen H. K., Hansen F. K., Banday A. J., G¨orski K. M., 2014, ApJ, 784, L42, [1402.0870] ADS

Amendola L., Catena R., Masina I., Notari A., Quartz M., Quercellini C., 2011, JCAP, 7, 27, [1008.1183] ADS

Bennett, C. L., Hill, R. S., Hinshaw, G., et al. 2011, ApJS, 192, 17, [1001.4758] ADS

Chluba J., Dai L., Jeong D., Kamionkowski M., Yoho A., 2014, MNRAS, 442, 670, [1404.2798] ADS

Dai L., Jeong D., Kamionkowski M., Chluba J., 2013, Phys. Rev. D, 87, 123005, [1303.6949] ADS

Erickcek A. L., Hirata C. M., Kamionkowski M., 2009, Phys. Rev. D, 80, 083507, [0907.0705] ADS

Erickcek A. L., Kamionkowski M., Carroll S. M., 2008, Phys. Rev. D, 78, 123520, [0806.0377] ADS

Eriksen H. K., Banday A. J., G¨orski K. M., Hansen F. K., Lilje P. B., 2007, ApJ, 660, L81, [astro-ph/0701089] ADS

Eriksen H. K., Hansen F. K., Banday A. J., G¨orski K. M., Lilje P. B., 2004, ApJ, 605, 14, [astro-ph/0307057] ADS

Flender S., Hotchkiss S., 2013, JCAP, 9, 33, [1307.6069] ADS

G¨orski K. M., Hivon E., Banday A. J., Wandelt B. D., Hansen F. K., Reinecke M., Bartelmann M., 2005, ApJ, 622, 759, [astro-ph/0409513] ADS

Hansen F. K., Banday A. J., G¨orski K. M., 2004, MNRAS, 354, 641, [astro-ph/0404206] ADS

Hirata C. M., 2009, JCAP, 9, 11, [0907.0703] ADS

Hofuift J., Eriksen H. K., Banday A. J., G¨orski K. M., Hansen F. K., Lilje P. B., 2009, ApJ, 699, 985, [0903.1229] ADS

Jarosik, N., Bennett, C. L., Dunkley, J., et al., 2011, ApJS, 192, 14, [1001.4744] ADS

Jazayeri S., Akrami Y., Firouzjahi H., Solomon A. R., Wang Y., 2014, ArXiv e-prints, [1408.3057] ADS

Kim J., Komatsu E., 2013, Physical Review D, 88, 101301

Kosowsky A., Kahniashvili T., 2011, Physical Review Letters, 106, 191301, [1007.4539] ADS

Lyth D. H., 2013, JCAP, 8, 7, [1304.1270] ADS

Namjoo M. H., Baghram S., Firouzjahi H., 2013, Phys. Rev. D, 88, 083527, [1305.0813] ADS

Planck Collaboration et al. 2013b, ArXiv e-prints, [1303.5072] ADS

Planck Collaboration et al. 2013a, ArXiv e-prints, [1303.5083] ADS

Planck Collaboration et al. 2013, [1303.5087] ADS

Schmid F., Hui L., 2013, Physical Review Letters, 110, 011301, [1210.2965] ADS

© 0000 RAS, MNRAS 000, 000–000