Computing RPC using robust selection of GCPs

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Abstract. The Rational Function Model (RFM) has gained widespread acceptance in photogrammetric processing, e.g. georeferencing, Digital Elevation Model (DEM) generation, 3D reconstruction, ortho-rectification etc. Most satellite image vendors provide the Rational Polynomial Coefficients (RPCs) instead of the rigorous sensor model. In this paper we present a novel approach to estimate RPCs using only Ground Control Points (GCPs). We ensured the RPC computing and validation with the aid satellite image and some number GCPs for which image coordinates and 3D ground coordinates are known. We obtained the system of the nonlinear equations on these well-established GCPs. Each equation describes the relationship between the image space coordinates and the object space coordinates. Then, we examined the RFM accuracy using well-established ground check points. This accuracy heavily depends on the distribution of GCPs. We propose a new method of computing RPCs using the robust selection of well-established GCPs. Our method is based on the analysis of spectral properties of the system of equations generated from these points.

1. Introduction
The RPCs of the RFM can be solved with or without knowing the rigorous sensor models [1]. One of the standard approaches to solving the RPCs is based on the use of a known 3D physical sensor model of the sensor. This sensor model allows us to obtain an accurate 3D description based on the ephemeris. A cubic grid of normalised geodetic coordinates can be established with a known rigorous model. Those points are commonly described in the virtual GCPs. For each virtual GCP point of the grid, the pixel coordinates in the image are calculated by projecting the rays with some definite step. The new RPCs can be estimated using the virtual GCPs by the least squares estimation [2].

If the rigorous sensor model is not known, then the RPCs can be solved using well-established GCPs. The RFM is essentially a generalised 3D physical sensor model [3]. This model establishes the relationship between the geodetic coordinates object and its coordinates in the image using the RPCs. The RPCs can be estimated using a test image that contains georeferencing information. This information contains a certain number of georeference points: the GCPs with known geographic and pixel coordinates in a given coordinate system. The georeferenced points are formed using the Global Positioning System (GPS) and landmarks on the surface of the Earth. The correspondence between
ground coordinates and corresponding image coordinates can be established more easily after specific preprocessing procedures, e.g. geometric distortion correction [4] and atmospheric correction [5]. The origin georeferenced points can be randomly distributed over the image.

To solve this problem, a set of the georeferenced points is divided into subsets of training and test points. Through well-established coordinates of the training points, a system of nonlinear (but linear in the required parameters) equations is obtained. Each equation describes the relationship between the geodetic coordinates and the pixel coordinates on the satellite image. Thus, in solving the system the parameters of the RFM model can be estimated.

Usually remote sensing satellite vendors provide the meta-information containing the RPCs instead of the rigorous model. However, often the attached information is not accurate enough, or completely absent. To improve the accuracy of the image georeferencing, it is sufficient to only refine the RPC coefficients if the RFM model is not accurate enough. In this way, it is enough to refine the initial file RPC using a small number (1-5) of GCPs for operational processing Earth remote sensing data. The usual example would be when it is required to process such data (DEM generation, orthophoto) on a fragment that is several times smaller than the original image.

Now consider a situation in which there is no meta-information, i.e. RPCs are not known. In this case it is necessary to calculate them using a sufficiently large number of the georeferenced points. At formation of the systems of nonlinear equations, at least 7 points are required for determining the RPCs of the first-order RFM model, while 19 are needed for the second order and 39 for the third. In such a manner, the RPC accuracy is highly dependent on the method of RFM model formation, the order of the model, and also the number of given georeferenced points.

Increasing the number of georeferenced points used improves the accuracy of determining RPCs. However, at the same time, both the requirements to computational resources and the user time for manual measurement of the corresponding pixel and geodetic coordinates are increasing. In addition, as studies show, the accuracy of estimating RPC coefficients depends not only on the number of georeferenced points, but also on the distribution of the training points. Therefore, many researchers are trying to develop methods for the formation of a training set of georeferenced points.

The technology proposed by Zhang [6] suggests splitting the image by a square grid into equal fragments. In each resulting fragment, two points with a minimum and a maximum height are selected. The problem with this approach is that the size of the grid on the image usually has to be selected manually. In the image there may be non-informative areas where there are a few georeferenced points, or they are completely absent, e.g. plains, lakes etc. In this case, the number of georeferenced points used is small and they are located on the same vertical (or horizontal) grid; as a result, the design matrix becomes ill conditioned [2]. This leads to high sensitivity in solutions regarding small errors from a given set.

We propose a new method of computing the parameters of RFM model using a robust selection of the georeferenced points. Our method is based on the analysis of the spectral properties of the system of equations generated from these points. This method is easy to implement and allows for automatic selection of georeferenced points. The advantage of the proposed method is sensitivity not only to image space distribution points in the satellite image, but also to information about 3D ground coordinates. The proposed approach is described in the third section. We present the results of experiments, demonstrating the usefulness of our method. The fourth section provides the results of experiments, demonstrating the usefulness of our method.

2. Problem statement

We address issues related to the DEM generation. An important step in this technology is the 3D reconstruction. The computing RPC for each stereo image as a stage of the DEM generation in detail was considered in the previous author’s paper [7]. After computing RPCs, the 3D reconstruction can be performed by using the RFM and the conjugate points of the stereo images. The proposed method for estimating RPCs [7] is insensitive to outliers or to inaccurate source values (georeferencing points).

In this paper we continue research in this direction. We consider the computing RPCs without knowing the rigorous sensor models or other satellite orbit ephemeris information. The RPCs were
estimated via the few GCPs based on the GPS. Therefore, the problem is to select a group of known control points in such a way that the RPCs can be assessed with fitting accuracy [2].

The RFM model provides accurate normalised 3D points \( \{P, L, H\} \) to normalised 2D image points \( \{X, Y\} \). In this paper the proposed method was conducted for the first-order RFM model. As discussed in Introduction, the minimum number of GCPs for the first-order RFM model is seven. The defined ratio of polynomials for image coordinate \( Y \) in the forward projective form is expressed as equation (1).

\[
Y = \frac{a^T u}{b^T u} = \frac{a_0 + a_1 P + a_2 L + a_3 H}{b_0 + b_1 P + b_2 L + b_3 H},
\]

where \( a = [a_0, a_1, a_2, a_3]^T \), \( b = [b_0, b_1, b_2, b_3]^T \) – required RPCs and vector \( u = [L, P, H]^T \) consist of the normalised coordinate values \( P, L, H \), which are obtained by normalisation of the ground coordinates \( \phi, \lambda, h \). The problem consists in determining the estimates \( \hat{a}, \hat{b} \) of the parameters \( a, b \) from the given \( N \) GCPs and their corresponding points. We note that the method is considered for the coordinate \( Y \). For given one \((i)\) GCP, and considering that we always have \( b_0 = 1 \), an equation (1) becomes:

\[
Y(i) + Y(i)\left[\frac{b^T u}{b^T u}\right] = a^T u,
\]

where \( b' = [b_1, b_2, b_3]^T \), \( u' = [L, P, H]^T \). In view of the above notation, \( b', u', a, u \), equation (2) can then be formed as:

\[
Y_i = a_0 + a_1 \cdot L + a_2 \cdot P + a_3 \cdot H - b_1 L Y_i - b_2 P Y_i - b_3 H Y_i.
\]

For the given \( N \) well-known GCPs, for which \( Y_i, L_i, P_i, H_i \) are known, the matrix form of equation (2) can be formed into equation (4):

\[
Y = MJ + \xi,
\]

where

\[
Y = [Y_1, Y_2, \ldots, Y_N]^T, \quad M = \begin{bmatrix}
1 & L_1 & P_1 & H_1 & -Y_1 L_1 & -Y_1 P_1 & -Y_1 H_1 \\
1 & L_2 & P_2 & H_2 & -Y_2 L_2 & -Y_2 P_2 & -Y_2 H_2 \\
M & M & M & M & M & M & M \\
1 & L_N & P_N & H_N & -Y_N L_N & -Y_N P_N & -Y_N H_N
\end{bmatrix}, \quad J = [a_0, a_1, a_2, a_3, b_1, b_2, b_3]^T,
\]

and \( \xi = [\xi_1, \xi_2, \ldots, \xi_N]^T \) – random error vector, caused by inaccurate values in coordinates \( L, P, H \) or \( Y \) in satellite images. The least squares estimate is obtained from equation (4), as shown in equation (5):

\[
\hat{J} = (M^T M)^{-1} M^T Y.
\]

Note, that equations (3), (4) and (5) can also be formed for the image coordinate \( X_i \). It is clear from (5) that the estimated parameters \( \hat{J} \) of the RFM in equation (1) depends on the properties of the matrix \( M^T M \). In particular, even with small errors in the original data, large errors in the \( \hat{J} \) are possible when this matrix is ill conditioned. The matrix elements are composed of the given points of the coordinates involved with \( N \). Obviously the accuracy assessment depends on their distribution across the scene. We propose a method for selecting the most informative subset of given points. The method is based on computing a sufficient condition estimate of the information matrix \( M^T M \).

### 3. Informativeness estimation method

As stated above, we set the task of selecting the most informative learning set by analysing the condition of the information matrix \( M^T M \). Let’s introduce the description for the information matrix to reduce further equations:
Matrix $\mathbf{A}$ is a Gramian matrix. For this class of matrices, the following conditioning characteristics are known. The minimal eigenvalue of the matrix $\mathbf{A}$:

$$\lambda_{\text{min}}(\mathbf{A}),$$

condition number

$$k(\mathbf{A}) = \frac{\lambda_{\text{max}}(\mathbf{A})}{\lambda_{\text{min}}(\mathbf{A})},$$

where $\lambda_{\text{max}}(\mathbf{A})$ and $\lambda_{\text{min}}(\mathbf{A})$ are maximal and minimal eigenvalues of $\mathbf{A}$ respectively.

These conditioning measures are often used to assess the quality of solutions. In particular, for errors in solutions the estimates in equations (9) and (10), based on the use of the minimum eigenvalue as a condition measure, are known as:

$$\|\Delta \mathbf{j}\|_2 \leq \lambda_{\text{min}}^{-1}(\mathbf{A}) \cdot \|\mathbf{\xi}\|_2,$$

or

$$\|\Delta \mathbf{j}\|_2 \leq \lambda_{\text{min}}^{-1}(\mathbf{A}) \cdot \|\mathbf{\zeta}\|_2,$$

where $\Delta \mathbf{j} -$ the solution error, and $\mathbf{\zeta} = \mathbf{M^T} \mathbf{\xi}.$

It is also possible to consider estimates for relative errors in solutions:

$$\delta_j \leq \lambda_{\text{min}}^{-1}(\mathbf{A}) \cdot \delta_\|\mathbf{\xi}\|_2,$$

$$\delta_j \leq \lambda_{\text{min}}^{-1}(\mathbf{A}) \cdot \delta_\|\mathbf{\zeta}\|_2,$$

where $\delta_j = \frac{\|\Delta \mathbf{j}\|_2}{\|\mathbf{J}\|_2},$ $\delta_\|\mathbf{\xi}\|_2 = \frac{\|\mathbf{\xi}\|_2}{\|\mathbf{\xi}\|_2},$ $\delta_\|\mathbf{\zeta}\|_2 = \frac{\|\mathbf{\zeta}\|_2}{\|\mathbf{\zeta}\|_2}.$

If the errors of the original data are contained only in the vector $\mathbf{Y},$ the following estimates for the relative errors are also valid. The estimates are obtained by using the condition number (equation (8)):

$$\delta_j = k(\mathbf{A}) \cdot \delta_\|\mathbf{\xi}\|_2,$$

where $\delta_j$ is shown above,

$$\delta_\|\mathbf{\xi}\|_2 = \frac{\|\mathbf{\xi}\|_2}{\|\mathbf{\xi}\|_2},$$

and $\mathbf{b} = \mathbf{M^TY}.$

The above estimates of the accuracy of solutions which depend on the minimum eigenvalue and the condition number are the strongest criteria for estimating the sensitivity of the least-squares problem to errors in the original data. However, the computation of these estimates represents computational difficulties, especially in a situation when the matrix $\mathbf{A}$ is ill conditioned. Therefore, along with these conditioning measures, we will also use sufficient estimates based on the calculation of the diagonal dominant index $\Phi(\mathbf{A})$ of the matrix $\mathbf{A}$:

$$\Phi(\mathbf{A}) = \left(\sum_{i=1}^{M} a_{ii}\right)^{2} / \sum_{i,j=1}^{M} a_{ij}^{2},$$

where $a_{ij}$ – the elements of an information matrix $\mathbf{A}.$

The diagonal dominant index $\Phi(\mathbf{A})$ is based on the dependence with eigenvalues. Hence, the index $\Phi(\mathbf{A})$ can be used in informativeness estimation method. In particular, according to the properties of the Gramian matrix, the following equations are obtained:
\[
\sum_{i=1}^{M} \lambda_i = tr(A), \quad (16)
\]

\[
\sum_{i=1}^{M} \lambda_i^2 = tr(A^2), \quad (17)
\]

It can be shown that if the \( \Phi(A) \) satisfies the inequations
\[
M - 1 < \Phi(A) \leq M, \quad (18)
\]
we have the following lower bound for the eigenvalues:
\[
\lambda_{\min}(A) \geq M^{-1}tr(A)[1 - \sqrt{(M / \phi - 1)(M - 1)}], \quad (19)
\]

Studies have shown that the index \( \Phi(A) \), equation (15), defines the degree of condition of the information matrix \( A \) over a wider range of values than those indicated in inequation in equation (18).

Let us introduce the following criterions \( Q_1, Q_2, Q_3 \) for the above measures of condition. In addition, we bring them to the same range for convenience of comparison.
\[
Q_1 = \lambda_{\min}(\hat{A}). \quad (20)
\]

Instead of the condition number in equation (8), we will use the inverse quantity as equation (21).
\[
Q_2 = k^{-1}(A) = \frac{\lambda_{\min}(A)}{\lambda_{\max}(A)}. \quad (21)
\]

The criterion, equation (21), is the inverse condition number. It is clear that the tending of this index to zero indicates that \( A \) is ill conditioned, and if its maximum value is 1 then all eigenvalues are the same.

Hence, in consequence of the properties, shown in equations (18) and (19), instead of the index \( \Phi(A) \) we will use the value reduced to the interval \([0,1]\):
\[
Q_3 = \Phi(A) - M + 1. \quad (22)
\]

Now we can formulate the optimisation problem. Let we have \( N \) well-established GCPs on the image. The goal of the proposed method is selection \( K \) points for which the information matrix \( \hat{A} \) composed of the coordinates of these points by the equations (3), (4) and (5), the maximum of the criterion \( Q_j, j \in \{1,2,3\} \), is reached:
\[
Q_j(\hat{A}) \rightarrow \max_{i \in \mathbb{N}^k_{x,\hat{A}}} Q_j(\hat{A}), \quad j \in \{1,2,3\} \quad (23)
\]

where \( Q_j \) – one of the criterions (equations (20), (21) and (22)), \( \hat{A} \) – information matrix composed from training subset of \( K \) points.

The solution of equation (23) can be obtained by using a full search of \( C_k^N \) variants of a training subset. If the number of given points \( N \) is small, this approach is applicable.

4. Experimental results

The initial image is shown in figure 1. Thirty white squares mark the source of the GCPs on the satellite image.

In this experiment, 12 GCPs and their corresponding image points were used to solve the RPCs. The given 30 control points were employed for accuracy checking. The parameters \( \hat{a}, \hat{b} \) of the RFM, equation (1), were estimated for the normalised image coordinates \( Y \) and \( X \) respectively. Then, error values (residuals) on each axis (OX and OY) of the image using estimation coordinates \( \hat{y}_i \) and \( \hat{x}_i \) were calculated. The RFM accuracy was checked using the Root Mean Square Error (RMSE) for each axis and total RMSE, as shown in equations (24), (25) and (26) respectively.
Figure 1. Distribution of the well-established GCPs for which image coordinates and 3D ground coordinates are known.

\[
RMSE_{ox} = \left( \frac{1}{N} \sum_{i=1}^{N} (\hat{x}_i - x_i)^2 \right)^{1/2},
\]

(24)

\[
RMSE_{oy} = \left( \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2 \right)^{1/2},
\]

(25)

\[
RMSE_{Total} = \left( \frac{1}{N} \sum_{i=1}^{N} (\hat{x}_i - x_i)^2 + \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2 \right)^{1/2},
\]

(26)

where \( \hat{x} \) and \( \hat{y} \) are estimates of the image coordinates, which can be computed from the RFM model for the given ground point coordinates \( \varphi, \lambda, h \), as shown in equation (1). Table 1 presents the RPC residuals, equations (24), (25) and (26), in pixels for three cases of distribution points: evenly, diagonally and vertically. Also, table 1 shows condition numbers \( k(A) \) and a diagonal dominant index \( \Phi(A) \).

Table 1. RPC residuals, condition numbers and diagonal dominant indexes for three cases of distribution points.

| Distribution of 12 points | \( k(A) \) | \( \Phi(A) \) | \( RMSE_x \) | \( RMSE_y \) | \( RMSE_{Total} \) |
|---------------------------|---------|--------|------------|------------|----------------|
| Evenly                    | 180     | 2.92   | 1.67       | 1.49       | 2.2           |
| Diagonally                | 2732.6  | 2.44   | 3.31       | 0.89       | 3.4           |
| Vertically                | 6869.1  | 1.68   | 8.2        | 0.81       | 8.2           |

The results show a small error \( RMSE_{Total} \) of 2.2 pixels for uniformly distributed points. As expected, low-accuracy is achieved due to vertical and diagonal distributions. The proposed criterions could successfully select informative training subsets.

5. Conclusion

In the course of the experiments, the condition number of the informative matrix and the diagonal prevalence index depend on the distribution of the control points. Therefore, it was possible to select an informative subset of given control points by which the RPCs can be determined with acceptable accuracy. High accuracy of RPCs can be provided using the diagonal dominant index as well as condition number.

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