The effective complex heavy-quark potential in an anisotropic quark-gluon plasma

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Abstract. We introduce a method for reducing anisotropic heavy-quark potentials to isotropic potentials by using an effective screening mass that depends on the quantum numbers $l$ and $m$ of a given state. We demonstrate that, using the resulting 1D effective potential model, one can solve a 1D Schrödinger equation and reproduce the full 3D results for the energies and binding energies of low-lying heavy-quarkonium bound states to relatively high accuracy. This includes the splitting of different p-wave polarizations. The resulting 1D effective model provides a way to include momentum anisotropy effects in open quantum system simulations of heavy-quarkonium dynamics in the quark-gluon plasma.

1 Introduction

The survival probability of the heavy quarkonium states, such as $J/\Psi$ and $\Upsilon$ has been widely used as a sensitive probe to study the quark-gluon plasma (QGP) formed in relativistic heavy-ion experiments at RHIC and LHC \cite{1, 2}. Due to non-relativistic nature of heavy quarkonium states, one can obtain their in-medium properties, such as masses and decay rates by solving a Schrödinger equation with a complex heavy-quark (HQ) potential. The real part of the HQ potential provides the binding energy, whereas the imaginary part provides information about the decay of a quarkonium state via wave function decoherence \cite{3}. One can obtain the HQ potential at short distances by making use of hard-thermal-loop (HTL) resummed perturbation theory in the weak-coupling limit. Recently, several attempts have been made to develop complex-valued potential models to understand the in-medium properties of quarkonia quantitatively \cite{10, 19}. During the last decade, many prior works have treated the QGP as an anisotropic medium by incorporating momentum-space anisotropies generated by longitudinal expansion into the underlying parton distribution functions. To make a phenomenological study of this effect in heavy-ion collisions, we consider the following spheroidal distribution function ansatz in the local rest frame (LRF) of the QGP \cite{20}

\begin{equation}
    f_{\text{iso}}^{\text{aniso}}(k) \equiv f_{\text{iso}}\left(\frac{1}{\lambda} \sqrt{k^2 + \xi(k \cdot n)^2}\right).
\end{equation}
This form takes into account the rapid longitudinal expansion of the QGP at early times and allows for explicit pressure anisotropies in the LRF [21, 22]. Here, \( f_{\text{iso}} \) is an arbitrary isotropic distribution function, \( \lambda \) is a temperature-like scale, which becomes the temperature \( T \) of the system in the thermal equilibrium limit. The degree of momentum-space anisotropy (\( \xi \)) in the range \(-1 < \xi < \infty \) is given by

\[
\xi = \frac{1}{2} \left( \frac{k_z^2}{k_\perp^2} \right) - 1 \tag{2}
\]

where \( k_z \equiv \mathbf{k} \cdot \mathbf{n} \) and \( k_\perp \equiv \mathbf{k} - \mathbf{n}(\mathbf{k} \cdot \mathbf{n}) \) correspond to the particle momenta along and perpendicular to the direction of anisotropy (\( \mathbf{n} \)), respectively. Many prior works have studied heavy quarkonium physics by considering the momentum-space anisotropy inside the QGP [10, 13, 16–18, 20, 21, 23]. Here we focus on how to efficiently take momentum-space anisotropy into account in a one-dimensional effective theory and compare the one- and three-dimensional results for static and dynamical quantities numerically.

In this proceedings contribution, we summarize our previous works where the real part of a 3D anisotropic HQ potential has been reduced to 1D effective potential [24, 25]. This work is organized as follow: In sec. 2 we describe the isotropic complex HQ potential model, in sec. 3 we obtain the anisotropic complex HQ potential model, in sec. 4 we obtain our effective complex HQ potential model, in sec. 5 we present our static results, and in sec. 6 we present our dynamic results.

## 2 Isotropic Potential Model

The Fourier transform of the real time gluon propagator in the static limit gives the complex HQ potential in an isotropic QGP [26].

\[
V(\lambda, r) = -g^2 C_F \int\frac{d^3p}{(2\pi)^3}(e^{ip\cdot r} - 1)D^{00}(p_0 = 0, \mathbf{p}, \lambda) \tag{3}
\]

### 2.1 Perturbative Contribution

The perturbative contribution to the complex HQ potential can be obtained from HTL resummed perturbation theory. The real and imaginary parts of this perturbative contribution are given by

\[
\begin{align*}
\Re V_p(\lambda, r) &= -g^2 C_F \int\frac{d^3p}{(2\pi)^3}(e^{ip\cdot r} - 1)\left(\frac{1}{p^2 + m_D^2} - \frac{1}{p^2}\right) \equiv \alpha m_D(\mathcal{I}_1(\hat{\mathbf{r}}) - 1), \\
\Im V_p(\lambda, r) &= -g^2 C_F \int\frac{d^3p}{(2\pi)^3}(e^{ip\cdot r} - 1)\frac{-\pi\lambda m_D^2}{p(p^2 + m_D^2)} \equiv \alpha\lambda(\mathcal{I}_2(\hat{\mathbf{r}}) - 1),
\end{align*}
\]

where the integrals \( \mathcal{I}_1(\hat{\mathbf{r}}) \) and \( \mathcal{I}_2(\hat{\mathbf{r}}) \) are

\[
\begin{align*}
\mathcal{I}_1(\hat{\mathbf{r}}) &= 4\pi \int\frac{d^3\hat{\mathbf{p}}}{(2\pi)^3}e^{i\phi_{\hat{\mathbf{r}}}}\frac{1}{\hat{\mathbf{p}}^2(\hat{\mathbf{p}}^2 + 1)} = 1 - e^{-\hat{\mathbf{r}}}, \\
\mathcal{I}_2(\hat{\mathbf{r}}) &= 4\pi^2 \int\frac{d^3\hat{\mathbf{p}}}{(2\pi)^3}e^{i\phi_{\hat{\mathbf{r}}}}\frac{1}{\hat{\mathbf{p}}(\hat{\mathbf{p}}^2 + 1)^2} = \phi_2(\hat{\mathbf{r}}),
\end{align*}
\]

with

\[
\phi_{\hat{\mathbf{r}}}(\hat{\mathbf{r}}) = 2 \int_0^\infty dz \sin(z\hat{\mathbf{r}}) \frac{z}{z^2 + 1}. \tag{7}
\]

Here, \( \hat{\mathbf{p}} \equiv \mathbf{p}/m_D, \hat{\mathbf{r}} \equiv \mathbf{r}/m_D \), and the strong coupling constant \( \alpha = g^2 C_F/(4\pi) \). We also subtracted a term \( 1/p^2 \) in eq. (4) to make the \( r \)-independent part finite.
2.2 Non-perturbative Contribution

The gluon propagator also contains a non-perturbative string contribution which arises from a dimension two gluon condensate. Its Fourier transform gives us the non-perturbative contributions [25]

\[
\text{Re } V_{\text{np}}(\lambda, r) = -g^2 C_F m_G^2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left( e^{i \Phi \cdot \mathbf{r}} - 1 \right) \frac{p^2 + 5 m_D^2}{(p^2 + m_D^2)^2} \equiv -\frac{2\sigma}{m_D} (I_3(\hat{r}) - 1), \quad (8)
\]

\[
\text{Im } V_{\text{np}}(\lambda, r) = -g^2 C_F m_G^2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left( e^{i \Phi \cdot \mathbf{r}} - 1 \right) \frac{4\pi \lambda m_D^2 (p^2 - 2 m_D^2)}{p(p^2 + m_D^2)^4} \equiv \frac{4\sigma \lambda}{m_D^2} (I_4(\hat{r}) - 1), \quad (9)
\]

where \(\sigma = \alpha m_G^2/2\) and \(m_G^2\) is a dimensionful constant. The integrals appearing above are

\[
I_3(\hat{r}) = 4\pi \int \frac{d^3 \mathbf{p}}{(2\pi)^3} e^{i \Phi \cdot \mathbf{r}} \hat{p}^2 + 5 \frac{(\hat{p}^2 + 1)^3}{(\hat{p}^2 + 1)^3} = (1 + \hat{r}/2)e^{-\hat{r}},
\]

\[
I_4(\hat{r}) = 8\pi^2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} e^{i \Phi \cdot \mathbf{r}} \frac{2 - \hat{p}^2}{\hat{p}^6 (\hat{p}^2 + 1)^4} = -2\phi_3(\hat{r}) + 6\phi_4(\hat{r}). \quad (10)
\]

2.3 Total Isotropic potential

The sum of the perturbative and non-perturbative contributions give us the total complex isotropic HQ potential

\[
\text{Re } V_{\text{Is}}(r) = \text{Re } V_{\text{pt}}(\lambda, r) + \text{Re } V_{\text{np}}(\lambda, r) = \alpha m_D \left( 1 - \frac{e^{-r m_D}}{r m_D} \right) - \alpha m_D - \frac{\sigma}{m_D} (2 + r m_D) e^{-r m_D} + \frac{2\sigma}{m_D} \frac{\alpha}{r}, \quad (11)
\]

\[
\text{Im } V_{\text{Is}}(r) = \text{Im } V_{\text{pt}}(\lambda, r) + \text{Im } V_{\text{np}}(\lambda, r) = \alpha \lambda \phi_2(r m_D) - \alpha \lambda = \frac{8\sigma \lambda}{m_D^2} \phi_3(r m_D) + \frac{24\sigma \lambda}{m_D^2} \phi_4(r m_D) - \frac{4\sigma \lambda}{m_D^2}. \quad (12)
\]

We include a relativistic correction, \(-0.8\sigma/(m_{b/c}^2 r)\), in the potential model while solving the Schrödinger equation for charmonia and bottomonia [10], where the masses of the charm and bottom quarks are taken to be \(m_c = 1.3\) GeV and \(m_b = 4.7\) GeV, respectively.

3 3D Anisotropic Potential Model

The real and imaginary part of the 3D anisotropic potential model as derived in our previous work [25] are

\[
\text{Re } V_{\text{Aniso}}(r, \theta, \xi) = \alpha m_D^\lambda \left( 1 - \frac{e^{-r m_D^\lambda}}{r m_D^\lambda} \right) - \alpha m_D^\lambda - \frac{\sigma}{m_D^\lambda} (2 + r m_D^\lambda) e^{-r m_D^\lambda} + \frac{2\sigma}{m_D^\lambda} \frac{\alpha}{r}, \quad (13)
\]

\[
\text{Im } V_{\text{Aniso}}(r, \theta, \xi) = \alpha \lambda^\lambda \phi_2 \left( r m_D^\lambda \right) - \alpha \lambda^\lambda = \frac{8\sigma \lambda}{m_D^\frac{1}{2} \lambda} \phi_3 \left( r m_D^\frac{1}{2} \lambda \right) + \frac{24\sigma \lambda}{m_D^\frac{1}{2} \lambda} \phi_4 \left( r m_D^\frac{1}{2} \lambda \right) - \frac{4\sigma \lambda}{m_D^\frac{1}{2} \lambda}, \quad (14)
\]

where,

\[
m_D^\lambda = m_D \left( 1 - \frac{\xi}{6} \right), \quad \lambda^\lambda = \lambda \left( 1 - \frac{\xi}{6} \right). \quad (15)
\]
and

\[ m_A^D = m_D \left[ 1 + \xi (0.108 \cos 2\theta - 0.131) \right], \quad m_i^D = m_D \left[ 1 + \xi (0.026 \cos 2\theta - 0.158) \right]. \quad (16) \]

Eq.(15) assures a correct asymptotic behavior of the potential. Eq. (16) was obtained by matching effective and exact result at \( \hat{r} = 1 \) as described in [25].

4 1D Effective Potential Model

Due to the angular dependence in the 3D anisotropic potential model, solving a 3D Schrödinger equation to find various in-medium properties of the quarkonium states is rather time consuming and much more complicated. One possible solution to this problem is to introduce an angle-averaged effective screening mass \( M_{lm}(\lambda, \xi) \) [24]

\[
M_{lm}^{K,l}(\lambda, \xi) = \langle Y_{lm}(\theta, \phi)|m_D^{K,l}(\lambda, \xi, \theta)|Y_{lm}(\theta, \phi)\rangle,
\]

\[
= \int_{-1}^{1} d\cos \theta \int_{0}^{2\pi} d\phi Y_{lm}(\theta, \phi)m_D^{K,l}(\lambda, \xi, \theta)Y_{lm}^{*}(\theta, \phi),
\]

and where \( Y_{lm}(\theta, \phi) \) refers to the spherical harmonics with azimuthal quantum number \( l \) and magnetic quantum number \( m \).

The real and imaginary part of the 1D effective potential model as derived in our previous work [25] are

\[
\text{Re} \ V_{\text{Eff}}(r, \xi) = \alpha m_{D}^{A} \left( 1 - e^{-r M_{lm}^{\text{eff}}} \right) - \alpha m_{D}^{A} - \frac{\sigma}{m_{D}^{A}} \left( 2 + r M_{lm}^{\text{eff}} \right) e^{-r M_{lm}^{\text{eff}}} + \frac{2\sigma}{m_{D}^{A}} - \frac{\alpha}{r},
\]

\[
\text{Im} \ V_{\text{Eff}}(r, \xi) = \alpha \lambda^{A} \phi_{2} \left( r M_{lm}^{\text{eff}} \right) - \alpha \lambda^{A} - \frac{8\sigma \lambda^{A}}{(m_{D}^{A})^{2}} \phi_{3} \left( r M_{lm}^{\text{eff}} \right) + \frac{24\sigma \lambda^{A}}{(m_{D}^{A})^{3}} \phi_{4} \left( r M_{lm}^{\text{eff}} \right) - \frac{4\sigma \lambda^{A}}{m_{D}^{A}},
\]

where,

\[
K_{lm} = \frac{2l(l + 1) - 2m^2 - 1}{4l(l + 1) - 3}
\]

and

\[
M_{lm}^{\text{eff}} = m_{D} \left[ 1 + \xi (0.216 K_{lm} - 0.239) \right], \quad M_{lm}^{I} = m_{D} \left[ 1 + \xi (0.052 K_{lm} - 0.184) \right].
\]

The \( l \) and \( m \) values of various quarkonium states are given in Table 1.

5 Static Results

For the static solutions, we used a previously developed 3D eigensolver called quantumFDTD [27, 28]. Using this code, we compared results obtained with the 1D effective potential and the full 3D anisotropic potential. In Table 2 we list the exact results of the eigenenergies (Re \( E \)), decay widths (Im \( E \)) and the binding energies (\( E_{\text{bind}} \)) with the anisotropy parameter \( \xi = 1 \) for \( \Upsilon(1S) \) and \( J/\Psi \).

In the numerical evaluations, we took \( \alpha = 0.272 \) and \( \sigma = 0.215 \text{ GeV}^2 \). For the \( \Upsilon(1S) \) state, we used a lattice size of \( N^3 = 512^3 \) with a lattice spacing of \( a = 0.020 \text{ GeV}^{-1} \approx 0.004 \text{ fm} \) giving a lattice size of \( L = Na \approx 2.05 \text{ fm} \). For the \( J/\Psi \), we used a lattice size of \( N^3 = 256^3 \) with a lattice spacing of \( a = 0.085 \text{ GeV}^{-1} \approx 0.017 \text{ fm} \) giving a lattice size of \( L = Na \approx 4.35 \text{ fm} \).
Table 1. $l$ and $m$ values of various quarkonium states.

| Bottomonium | $l$, $m$ values | states           |
|-------------|-----------------|-----------------|
| s-wave      | $l = 0$, $m = 0$| $\Upsilon(1S), \Upsilon(2S), \Upsilon(3S)$ |
| p-wave      | $l = 1$, $m = -1$| $\chi_{b1}(1P)$ |
|             | $l = 1$, $m = 0$| $\chi_{b0}(1P)$ |
|             | $l = 1$, $m = +1$| $\chi_{b+1}(1P)$ |

| Charmonium | $l$, $m$ values | states           |
|-------------|-----------------|-----------------|
| s-wave      | $l = 0$, $m = 0$| $J/\Psi(1S), \psi(2S), \psi(3S)$ |
| p-wave      | $l = 1$, $m = -1$| $\chi_{c-1}(1P)$ |
|             | $l = 1$, $m = 0$| $\chi_{c0}(1P)$ |
|             | $l = 1$, $m = +1$| $\chi_{c+1}(1P)$ |

Table 2. The exact 3D results of the complex eigenenergies ($E$) and binding energies ($E_{\text{bind}}$) for different quarkonium states at various temperatures with $\xi = 1$. $\delta E$ are the differences in results obtained using 1D effective and 3D anisotropic potentials. Here $T_o$ is 192 MeV and all results are in MeV [25].

| $T_o$ | $E$ | $\delta E$ | $E_{\text{bind}}$ | $\delta E_{\text{bind}}$ |
|-------|-----|------------|-------------------|--------------------------|
| $T_o$ | 128.869 | 0.611 | -662.669 | 11.838 | 0.027 |
| $1.1T_o$ | 174.957 | 0.593 | -570.612 | 14.830 | 0.031 |
| $1.2T_o$ | 166.556 | 0.573 | -493.689 | 18.190 | 0.034 |
| $1.4T_o$ | 148.439 | 0.531 | -372.540 | 26.004 | 0.039 |

| $T_o$ | $E$ | $\delta E$ | $E_{\text{bind}}$ | $\delta E_{\text{bind}}$ |
|-------|-----|------------|-------------------|--------------------------|
| $T_o$ | 439.336 | 1.230 | -406.202 | 41.980 | 0.107 |
| $1.1T_o$ | 422.207 | 1.163 | -323.362 | 51.467 | 0.105 |
| $1.2T_o$ | 404.597 | 1.095 | -255.648 | 61.698 | 0.098 |
| $1.3T_o$ | 386.604 | 1.028 | -199.583 | 72.564 | 0.086 |
| $1.4T_o$ | 368.301 | 0.963 | -152.678 | 83.958 | 0.070 |

6 Dynamical Results

In order to solve the 3D Schrödinger equation in real time, we used a split-step pseudospectral method [29] with temporal step size $\Delta t = 0.001$ fm/c. Once again we compare results obtained with the full 3D anisotropic potential to those obtained with the 1D effective potential. We evolve the wave function from $\tau = 0$ fm/c to $\tau = 0.25$ fm/c in the vacuum ($T = 0$). Starting at $\tau = \tau_0 = 0.25$ fm/c, we consider a fixed anisotropy parameter $\xi = 1$ and boost-invariant Bjorken evolution for the hard scale

$$\lambda(\tau) = \lambda_0 \left(\frac{T_0}{\tau}\right)^{1/3}.$$  (22)

Here we take the initial hard scale to be $\lambda_0 = 630$ MeV. Further details of the numerical method can be found in [25].
Figure 1. The top row shows the overlaps of $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ resulting from real-time solution of the Schrödinger equation. Here we initialized the wave function as pure $\Upsilon(1S)$ eigenstate. The bottom row shows the time evolution of the bottomonium p-wave overlaps resulting from initialization with different p-wave polarizations [25].

6.1 Bottomonium

For bottomonium states we take the box size to be $L = 2.56$ fm, $m_b = 4.7$ GeV, and use $N = 128$ lattice points in each direction. The top row of fig. 1 shows the time evolution of overlaps of the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ using a pure $\Upsilon(1S)$ eigenstate as the initial condition. Whereas the bottom row shows the time evolution of the bottomonium p-wave overlaps resulting from initialization with different p-wave polarizations. Results with pure $\Upsilon(2S)$ and $\Upsilon(3S)$ eigenstate and a Gaussian as the initial condition can be found in Ref. [25].

6.2 Charmonium

For charmonium states we take $L = 5.12$ fm, $m_c = 1.3$ GeV, and use $N = 128$ lattice points in each direction. The top row of the fig. 2 shows the time evolution of overlaps of the $J/\psi$, $\psi(2S)$, and $\psi(3S)$ by using pure $J/\psi$ eigenstate as the initial condition. Whereas the bottom row shows the time evolution of the charmonium p-wave overlaps resulting from initialization with different p-wave polarizations. The results with pure $\psi(2S)$ and $\psi(3S)$ eigenstate and Gaussian as the initial condition can be found in Ref. [25].
Figure 2. The top row shows the overlaps of $J/\psi$, $\psi(2S)$, and $\psi(3S)$ resulting from real-time solution of the Schrödinger equation. Here we initialized the wave function as pure $J/\psi$ eigenstate. The bottom row shows the time evolution of the charmonium p-wave overlaps resulting from initialization with different p-wave polarizations [25].

7 Conclusions

We have reduced anisotropic heavy-quark potentials to isotropic ones by introducing an effective screening mass that depends on the quantum numbers $l$ and $m$ of a given state. We demonstrated that, using the resulting 1D effective potential model, one can reproduce the full 3D results for the energies and binding energies of low-lying heavy-quarkonium bound states to relatively high accuracy. This finding is important because it can be used to incorporate anisotropy effects into one-dimensional real-time Schrödinger equations which underpin phenomenological calculations of bottomonium suppression in open quantum systems approaches.

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