SPHERICAL COLLAPSE IN MODIFIED NEWTONIAN DYNAMICS

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ABSTRACT

Modeling the structure formation in the universe, we extend the spherical-collapse model in the context of modified Newtonian dynamics (MOND) starting with the linear Newtonian structure formation followed by the MONDian evolution. In MOND, the formation of structures speeds up without a need for dark matter. Starting with the top-hat overdense distribution of the matter, the structures virialize with a power-law profile of the distribution of matter. We show that the virialization process takes place gradually from the center of the structure to the outer layers. In this scenario, the smaller structures enter the MONDian regime earlier and evolve faster, hence they are older than larger structures. We also show that the virialization of the structures occurs in the MONDian regime, in which the smaller structures have stronger gravitational acceleration than the larger ones. This feature of the dynamical behavior of the structures is in agreement with the fact that the smaller structures, as the globular clusters or galactic bulges, have been formed earlier and need less dark matter in cold dark matter scenario.

Key words: cosmology: theory – dark matter – galaxies: formation – gravitation – large-scale structure of universe

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1. INTRODUCTION

The conjecture for the existence of dark matter dates back to Zwicky (1933), who failed to explain the dynamics of Coma Cluster by the virial theorem through the distribution of visible matter (Zwicky 1933). Since then to interpret the Coma Cluster by the virial theorem through the distribution of matter, the structures virialize with a power-law profile of the distribution of matter. We show that the virialization process takes place gradually from the center of the structure to the outer layers. In this scenario, the smaller structures enter the MONDian regime earlier and evolve faster, hence they are older than larger structures. We also show that the virialization of the structures occurs in the MONDian regime, in which the smaller structures have stronger gravitational acceleration than the larger ones. This feature of the dynamical behavior of the structures is in agreement with the fact that the smaller structures, as the globular clusters or galactic bulges, have been formed earlier and need less dark matter in cold dark matter scenario.

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where the pressure can bend spacetime sufficiently to replace the roles of dark energy, cold dark matter, and heavy neutrinos in explaining anomalous accelerations at all scales (Zhao 2007).

Finally the third approach, with which we are concerned in this work, is the modification of the conventional Newtonian law, that is the so-called modified Newtonian dynamics (MOND). The dynamics of a structure in MOND under the gravitational field is given by Milgrom (1983):

\[ \mu(g/a_0)g = g_N, \]  

where \( g_N \) is the Newtonian gravitational acceleration, \( a_0 = 1.2 \times 10^{-10} \text{ms}^{-2} \) is a fundamental acceleration parameter, and \( \mu(x) \) is a function for the transition from the Newtonian to the MONDian regime (e.g., \( \mu(x) = x/\sqrt{1+x^2} \)). The dynamics of the structure for \( g < a_0 \) deviates from the Newtonian law by \( \mu(x) \) and for \( a_0 < g \) we recover the Newtonian dynamics (i.e., \( \mu = 1 \)). On the other hand for \( g \ll a_0 \), the so-called deep MOND regime, \( \mu(x) = x \) and the effective acceleration is given by \( g = \sqrt{g_N a_0} \).

Due to confusion in the definition of the dynamical concepts in MOND, this model can be interpreted as a modification to the gravity law instead of dynamics. Bekenstein & Milgrom (1984) used a nonconventional Newtonian action for the gravity to extract the modified Poisson equation as follows:

\[ \nabla \cdot (\mu(g/a_0)\nabla \phi) = 4\pi G \rho, \]  

where in the spherical symmetric systems this equation reduces to Equation (1). One of the problems with MOND is that it is not a covariant gravity model. Bekenstein (2004) proposed a covariant formulation of this model. This theory in addition to the metric has a scalar field as well as a four-vector field called TeVeS, where in the limit of nonrelativistic, small accelerations, the field equation reduces to that in MOND. The total potential
in this theory is given by the sum of the Newtonian potential, \( \phi_N \), and the potential due to the scalar field, \( \phi_s \):

\[
\Phi = \Phi_N + \phi_s, \quad (3)
\]

where the added scalar field plays the role of the dark matter. TeVeS theory also has a Newtonian limit for the nonrelativistic dynamics with significant acceleration. In the nonrelativistic limit, we can simplify the gravity inside the spherical system as

\[
g(r) = -\nabla \Phi \simeq \begin{cases} \sqrt{a_0 g_N}, & \phi_s \lesssim a_0; \\ 0, & \text{otherwise}. \end{cases} \quad (4)
\]

The advantage of MOND is that it could provide a successful fit to the rotation curve of spiral galaxies and dispersion velocity of elliptical galaxies (Milgrom & Sanderes 2003; Sanders 1996). It has been tested against the cosmic background radiation (Skordis et al. 2006), gravitational lensing (Chen & Zhao 2006; Zhao et al. 2006; Angus et al. 2007), stellar systems and galactic dynamics (Haghi et al. 2006; Hipoti et al. 2007; Tiet & Combes 2007), solar system (Bekenstein & Magueijo 2006; Sereno & Jetzer 2006), and Tully–Fisher and Freeman laws (McGaugh & de Blok 1998; McGaugh et al. 2000). MOND also decreases the mass discrepancy in the cluster of galaxies (Pointecouteau & Silk 2005) but yet in clusters it remains necessary to invoke the undetected matter, possibly in the form of a massive neutrino (Sanders 1999; Sanders 2003; Aguirre et al. 2002; Sanders & McGaugh 2002).

Studying the cosmology and formation of the large structures in the universe is another tool to examine MOND. Studying the MONDian scenario of the structure formation has been started by Feltan (1984) and Sanders (1998). In the paper by Sanders (1998), it is shown that a patch of universe in the MONDian regime smaller than the horizon size can evolve with a different rate than the background, hence the structures can be naturally formed through this scale-dependent dynamics. In this scenario, small structures form before the larger ones and this provides a bottom-up hierarchical procedure for the formation of the structures in the universe. The problem with this model is that the center of collapse is not identified and every point in the space depending on the choice of the coordinate system can be considered as the center of the structure. For solving this problem, one can consider MOND formula applied to a peculiar acceleration developing from density fluctuation rather than Hubble expansion (Sanders 2001; Nusser 2002). Nusser (2002) used the amplitude of the cosmic microwave background (CMB) anisotropies as the initial condition in the N-body simulation to simulate the large-scale structures in the universe. However, to have a compatible result, one has to either decrease \( a_0 \) by 1 order of magnitude or reduce the amplitude of the fluctuations at the initial condition. Recent studies in the formation of the galaxies by Sanders (2008) shows that massive elliptical galaxies may be formed at \( z \geq 10 \), as a consequence of the monolithic dissipationless collapse. Applying MOND with cooling mechanism puts an upper limit to the stellar clustering in the form of the galaxy. Extending the structure formation in TeVeS theory has been done by linear perturbation of metric, vector, and scalar fields. The predictions are compatible with the observations of the structures (Dodelson & Liguori 2006). Skordis (2008) also used a generalized TeVeS theory to construct the primordial adiabatic perturbations of a general family of scalar field kinetic functions.

In this work, we extend the spherical collapse model in MOND for studying the general behavior of the structures during their formation. The initial baryonic density contrast for the structures is taken from the CMB anisotropy. We obtain the dynamics of the baryonic structures in the early epochs with the linear Newtonian analysis until the entrance of the structure in the MONDian regime and follow the evolution with MOND. The dynamical evolution with MOND shows that the structures virialize with power-law distribution of matter. We show that while all of the structures recollapse and virialize in the MONDian regime, the gravitational acceleration of the structures in this stage inversely depends on the size of the structure. This dynamical behavior of the structure formation is compatible with the lesser existence of the dark matter in the globular clusters and central parts of the galaxies in CDM scenario.

The organization of the paper is as follows: in Section 2, we give a brief review of the spherical collapse model in MOND and extend it by looking at the dynamics of each layer in the onion model. In Section 3, we discuss the results of the calculation, showing that the density profile inside the structure is a time-varying function during the evolution and the structure at the final stage virializes from the center to the outer areas. In Section 4, we summarize and discuss the results.

2. STRUCTURE FORMATION IN MOND

In this section, we introduce the MONDian cosmology and apply MOND in the spherical collapse model for studying the formation of the structures. Here, we take the onion model for the spherical structure, dividing the sphere into concentric shells and studying the evolution of each layer separately and the structure as a whole.

2.1. MONDian Cosmology

In the standard cosmology, the dynamics of the universe is determined by the Hubble parameter and the evolution of the density contrast. Sanders (1998) used this approach to obtain the dynamics of the universe in the MONDian scenario. In MONDian cosmology, a patch of universe can evolve with a different rate than the background as soon as the acceleration fulfills the condition \( g < a_0 \). The result is dynamical decoupling of the smaller scales from the background, which causes the production of overdense regions in the universe. The reason for this feature of cosmological dynamics in MOND is that unlike the Newtonian mechanics, the acceleration in the comoving frame depends on the length scale.

Let us take a spherical region with radius, \( r \), from the background in which \( a_0 \ll g(r) \). Using the Newtonian dynamics, the acceleration is given by

\[
\ddot{r} = -\frac{GM}{r^2}, \quad (5)
\]

where \( M \) is the active gravitational mass and to have a compatible relation with the relativistic results, we define it to be composed of the relativistic and nonrelativistic matter as

\[
M = \frac{4\pi r^3}{3}(\rho + 3p), \quad (6)
\]

where \( \rho \) and \( p \) are the density and pressure of the cosmic fluid. Substituting Equation (6) in Equation (5), the acceleration is given by

\[
\ddot{r} = -\frac{4\pi G}{3}(\rho + 3p)r. \quad (7)
\]
In Equation (7), the gravitational acceleration increases linearly with \( r \). This implies that there should be a critical radius \( r_c \), where inside it the acceleration is smaller than the MOND threshold \( a_0 \) and the dynamics is given by the MOND, and where outside that radius, it is Newtonian. This critical length scale is obtained by equating the left-hand side of Equation (7) with \( \dot{r} = -a_0 \) as

\[
 r_c = \frac{3a_0}{4\pi G(\rho + 3p)}.
\]

This length scale separates the Newtonian (\( r > r_c \)) and the MONDian (\( r < r_c \)) domains. As the density of universe changes with the expansion of the universe, the critical radius also changes with time. From the continuity equation, the matter and the radiation densities vary as \( \rho = \rho_0 a^{-3} \) and \( p = p_0 a^{-4} \). Using the definition of the critical density, \( \rho_c = 3H_0^2/8\pi G \).

Equation (8) can be written in terms of the density parameters \( \Omega \), and the scale factor

\[
 r_c = \frac{2a_0}{H_0^2(\Omega_b^{(0)} a^{-3} + 2\Omega_r^{(0)} a^{-4} - 2\Omega_m^{(0)})},
\]

where we adapt \( H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \Omega_b^{(0)} = 0.02 \), \( \Omega_r^{(0)} = 5 \times 10^{-5} \), and \( \Omega_m^{(0)} = 0 \).

Now we do a comparison of the size of a structure with \( r_c \). From Equation (9), the critical radius, \( r_c \), changes with the scale factor as \( r_c \propto a^3 \) in the radiation and \( r_c \propto a^2 \) in the matter dominant epochs. On the other hand, the size of the structure is proportional to the scale factor (i.e., \( \lambda \propto a \)). So, we expect that the Newtonian structures eventually will enter the MONDian regime as \( r_c \) grows faster than \( \lambda \). Using the adapted cosmological parameters, the critical radius at the present time is obtained, \( r_c \approx 10H_0^{-1} \), which means that the whole observable universe resides in a MONDian domain. Comparing these two length scales at the last scattering surface results in \( r_c / H^{-1} \approx 10^{-4} \). The mass corresponding to this critical radius at last scattering surface is about \( M_c \approx 10^{11} M_\odot \), and for \( M < M_c \), the dynamics is given by MOND. While we expect to have density contrast growth for the scales \( r < r_c \) at the decoupling, comparing the critical mass \( M_c \approx 10^{11} M_\odot \) with the Jeans mass of \( M_J \approx 10^8 M_\odot \) at this time indicates that the structures at these scales should be washed out by the pressure (Sanders 1998).

The other feature of MONDian cosmology is that unlike standard cosmology where decoupling redshift is smaller than the equality redshift (\( z_{de} < z_{eq} \)), in MOND the equality epoch is much later than the decoupling time. This feature results from the fact that decoupling is related to the baryonic density of the universe and the temperature, and both parameters depend only on the scale factor, independent of the dynamics of the universe. So, we expect to have the same decoupling redshift in the MOND as standard cosmology. However, since dark matter does not exist in MONDian cosmology, the equality will be shifted to the lower redshifts. For our adopted cosmological parameters, the equality redshift is obtained as \( z_{eq} = 400 \).

### 2.2. Structure Formation: Spherical Collapse Model

In this section, we model the evolution of a structure with an overdense spherical region in MOND. To calculate the evolution of this spherical patch, for simplicity we take this overdense region with a top-hat distribution of matter. As the acceleration of this structure depends on the distance from the center, we expect to have different dynamics for each radius.

![Figure 1. Comparison of the accelerations in the Newtonian (dashed line) and MONDian (solid line) regimes for a region with galaxy mass scale (\( M = 10^{11} M_\odot \)) as a function of the redshift. For the early universe, the difference between the two dynamics is small but increases at later times. The corresponding redshift for the entrance of the structure into the critical radius is \( z_{\text{ent}} = 146 \).](image)

(Hence, we divide the structure into cocentric spherical shells like an onion model in cosmology (Lemaître 1933; Tolman 1934; Bondi 1947) and calculate the dynamics of each shell separately. The initial density contrast for this overdense region is taken from the fluctuations of the last scattering surface. In the Newtonian treatment of the structure formation (\( r_c < \lambda \)), the density contrast grows linearly (\( \delta \propto a \)) from the decoupling epoch up to the entrance of the structure to \( r_c \). For the MONDian regime (\( \lambda < r_c \)), we switch the dynamics to MOND to calculate the evolution of the structure. As an example let us take a sphere with a mass of \( M = 10^{11} M_\odot \) and find its acceleration with respect to the center. In Figure 1, we compare the acceleration of this spherical structure in MONDian and Newtonian dynamics as a function of the redshift. We note that the redshift is defined according to the dynamics of the scale factor in the background. At the early epochs, the difference between these two dynamics is small as \( r_c < \lambda \), but after entering the structure into the MONDian regime \( \lambda \lesssim r_c \), the evolution of the structure by Newtonian dynamics and MOND starts to diverge. This deviation of the dynamics from that of Newtonian plays the role of dark matter in the standard scenario of structure formation.

For calculating the dynamics of each shell into the onion model, we take the following notation: \( r^i(t) \) is the radius of the \( i \)th shell as a function of time and \( t_{\text{ent}}^i \) and \( r_{\text{ent}}^i \) are the entering time and radius of the \( i \)th shell into the MONDian domain, respectively. The velocity of the \( i \)th shell in terms of the Hubble parameter at entrance time is given by \( v_{\text{ent}}^i = H_0 t_{\text{ent}}^i (1 - \delta_{\text{ent}}^i) \), where \( H_0 \) is the Hubble parameter of the Newtonian background and \( \delta_{\text{ent}}^i \) is the density contrast of the sphere enveloped by \( r^i \). As we discussed in the previous section, all the shells will eventually enter the MONDian regime and we take this time as the initial condition for each shell in the MONDian evolution of the structures. Table 1 shows the initial density contrast, radius, and the corresponding redshift of the entrance of each shell into the MONDian regime. In the MONDian regime, the acceleration...
The first column represents the index of shells. The shells are taken equidistant when all the structures are in the Newtonian regime. The second column indicates the mass that is enveloped by the corresponding shell. The third column shows the radius of each shell at the time of crossing the MOND domain and the fourth column is the corresponding redshift. The fifth column is the density contrast at the entrance time and the sixth column shows the ratio of the kinetic energy per mass to \( W = rdV/dr \) of each shell at the entrance time.

### Table 1

| \( i \) | \( M_i (10^{11} M_\odot) \) | \( r_i (\text{ent}) \) (kpc) | \( z_{\text{ent}} \) | \( \delta_{\text{ent}} \times 10^{-5} \) | \( 2T_{\text{ent}}^{i}/W_{\text{ent}}^{i} \) |
|---|---|---|---|---|---|
| 1 | 0.001 | 0.45 | 400 | 2.71 | 2.24 |
| 2 | 0.008 | 1.25 | 297 | 3.70 | 2.28 |
| 3 | 0.027 | 2.40 | 242 | 4.52 | 2.32 |
| 4 | 0.064 | 3.55 | 218 | 5.03 | 2.36 |
| 5 | 0.124 | 5.03 | 195 | 5.61 | 2.41 |
| 6 | 0.215 | 6.35 | 184 | 5.97 | 2.48 |
| 7 | 0.341 | 8.17 | 170 | 6.47 | 2.51 |
| 8 | 0.510 | 9.77 | 161 | 6.81 | 2.53 |
| 9 | 0.725 | 11.44 | 154 | 7.12 | 2.55 |
| 10 | 1.000 | 13.57 | 146 | 7.51 | 2.57 |

Note.

* The first column represents the index of shells. The shells are taken equidistant when all the structures are in the Newtonian regime. The second column indicates the mass that is enveloped by the corresponding shell. The third column shows the radius of each shell at the time of crossing the MOND domain and the fourth column is the corresponding redshift. The fifth column is the density contrast at the entrance time and the sixth column shows the ratio of the kinetic energy per mass to \( W = rdV/dr \) of each shell at the entrance time.

(A color version of this figure is available in the online journal.)

approximately is \( \sqrt{g_N a_0} \) and the evolution of each shell is as follows:

\[
\dot{r}_i = -\sqrt{\frac{G M_i a_0}{r_i}},
\]

where \( M_i \) is the mass of the structure enveloped by the \( i \)th shell. Using the initial conditions given by Table 1, we obtain the evolution of each shell as shown in Figure 2. To visualize the evolution of the shells, we divide the sphere into 10 equidistant shells when all the structures are in the Newtonian regime and obtain their evolution as a function of the background redshift. The dynamics of shells shows that the inner shells evolve faster, reaching a maximum radius, and then collapse earlier than the outer ones. Here, the initial radius of the outermost shell is about 14 kpc at the entrance time of \( z_{\text{ent}} \sim 146 \) into the MOND regime. This shell expands up to a maximum radius of \( \sim 49 \) kpc at \( z_{\text{max}} \sim 28 \). Eventually, the shell starts to collapse and virialize at \( z \sim 18 \) with radius of \( \sim 28.5 \) kpc. The maximum radius of each shell, \( r_{\text{max}} \), is obtained by integrating Equation (10), letting \( \dot{r}(t) = 0 \), as follows:

\[
r_{\text{max}}^i = r_{\text{ent}}^i e^{\alpha}, \quad \alpha = \frac{\dot{v}_{\text{ent}}^2}{2\sqrt{GM_i a_0}}.
\]

The next phase of the evolution of shells after reaching a maximum radius is recollapsing. Similar to the standard scenario of the spherical-collapse models we expect that the global radial velocity of the structure during the free-fall collapse converts to the dispersion velocity and prevents the structure from a catastrophic collapse. This steady stage of the structure is given by the virial theorem. The corresponding radius that fulfills the virial condition is called the virial radius and is calculated from

\[
\frac{1}{2} \int_0^{r_i} \frac{dV(r)}{dr} + V(r_i) = E_i.
\]

where \( V(r_i) \) is the gravitational potential at the \( i \)th shell. The potential in the Newtonian or MONDian regimes is given by

\[
V(r_i) = \begin{cases} \sqrt{GM_i a_0} \ln(r_i), & \text{MOND;} \\ \frac{GM_i}{r_i} + \sqrt{GM_i a_0} \ln(r_i), & \text{Newt.} \end{cases}
\]

For the nondissipative evolution of the structure, the total energy is conserved and we substitute the right-hand side of Equation (12) by the energy of the system at the entrance time into the MONDian regime, \( E = \frac{1}{2} \dot{H}_{\text{ent}}^2 r_{\text{ent}}^2 + \sqrt{GM_i a_0} \ln(r_{\text{ent}}^i) \). Using the potentials given by Equation (13) into the left-hand side of the Equation (12), the virial radius for the MONDian and Newtonian regimes is obtained as

\[
r_{\text{vir}}^i = \begin{cases} \left( r_{\text{ent}}^i \frac{a_0}{G M_i} \right)^{-1/2}, & \text{MOND;} \\ \left( r_{\text{ent}}^i - \frac{2 \dot{H}_{\text{ent}}^2 r_{\text{ent}}^i}{G M_i} \right)^{-1}, & \text{Newt.} \end{cases}
\]

If the virialization of the structure takes place in the MONDian regime, \( r_{\text{vir}}^i > r_{\text{ent}}^i \) implies \( \alpha > 1/2 \) which results in \( 2T_{\text{ent}}^i/W_{\text{ent}}^i > 1 \), where \( T_{\text{ent}}^i \) is the kinetic energy for a unit mass and \( W_{\text{ent}}^i = \dot{r}^i dV^i/dr^i \). On the other hand, if the structure virializes in the Newtonian regime, \( r_{\text{vir}}^i < r_{\text{ent}}^i \) condition from Equation (14) implies \( 2T_{\text{ent}}^i/W_{\text{ent}}^i < 1 \). We calculate the expression for \( 2T_{\text{ent}}^i/W_{\text{ent}}^i \) for each shell (see Table 1) and show that all the shells of the structure virialize in the MONDian regime (i.e., \( r_{\text{vir}}^i > r_{\text{ent}}^i \)).

Table 2 shows the parameters of shells for the moment of the maximum radius and the virialization stage, and Figure 2 visualizes quantitative behavior of the shells during their evolution. In this figure, the spots on the evolution curves show the two critical stages of the maximum and the virialization radii, as reported in Table 2. The evolution lines of the shells show that the inner shells evolve faster and virialize at higher redshifts, while the outer shells evolve slower.

### 3. Predictions of the Model

In this section, we discuss the evolution of the density contrast and the profile of matter distribution inside the structure. We use...
the definition of the density contrast of the shells in our model:

$$\delta_i(t) = \frac{\rho_i(t) - \bar{\rho}(t)}{\bar{\rho}(t)}, \quad (15)$$

where $\rho_i$ is the density of the $i$th shell and $\bar{\rho}$ is the density of the background. As the collapsing of the shells starts from the inner to the outer parts of the structure, we will not have a shell-crossing during the evolution of the structure and from the conservation of mass, we can perform the Jacobian transformation from the initial distribution of matter inside the sphere to the evolved distribution as follows:

$$\rho_i = \rho_{ent}(\frac{r_{ent}}{r})^2 \frac{\delta_i}{\delta i}, \quad (16)$$

where $\delta r^i$ represents the thickness of the $i$th shell. Substituting the dynamics of each shell from the previous section into Equation (16), we obtain the evolution of the density contrast for the shells up to the virialization stage as shown in Figure 3. The inner shells evolve faster and reach the nonlinear regime ($\delta > 1$) at the higher redshifts while the outer ones evolve slower. Figure 4 shows the dependence of the corresponding nonlinear redshift on the mass of the structure. Comparing a small-scale structure with a mass of $\sim 10^5 M_{\odot}$ with a galaxy having a mass of $\sim 10^{11} M_{\odot}$ shows that the former structure enters the nonlinear regime at $z_{nl} \approx 106$ while the latter one becomes nonlinear at $z_{nl} \approx 33$. More details on the characteristic redshifts of the structures in terms of their masses are reported in Table 2. From this table, we extrapolate the dependence of the nonlinear redshift, maximum radius redshift, and virialization redshift on the mass of the structure with the following functions (see Figure 4):

$$\log(z_{nl}) = -0.167 \log \left( \frac{M}{M_{\odot}} \right) + 3.38,$$

$$\log(z_{max}) = -0.192 \log \left( \frac{M}{M_{\odot}} \right) + 3.56,$$

$$\log(z_{vir}) = -0.203 \log \left( \frac{M}{M_{\odot}} \right) + 3.48. \quad (17)$$

In what follows, we describe the qualitative predictions of this simple model.

### 3.1. Density Profile

In this section, we compare the evolution of the density profile of the structures in the spherical-collapse model for the Newtonian and MONDian regimes. In the Newtonian regime ($g > a_0$), we have seen that the dynamics of the shells in the structure depends only on time, and transforming to a comoving frame, the dynamics is scale independent (see Equation (7)). This means that the dynamics is invariant under the scale transformation and the result preserves the initial profile of the structure. In MOND ($g < a_0$), since the dynamics depends on the scale unlike the Newtonian case the density profile will change with time. In Figure 5, we plot the spatial variation of density from Equation (16) for a galaxy mass structure in four different stages of $z = 400, 87, 23,$ and 18. The initial stage at the redshift of $z = 400$ is taken as a top-hat distribution for the density when the innermost shell enters into the MONDian...
of each shell. While the structure evolves, the distance of shells changes with time as shown with a point on the profile representing the position of each shell (Figure 5). As the inner shells enter the MONDian regime earlier then we expect, the density profile deviates from the homogeneous distribution. Figure 5 shows a small deviation of the density profile from the homogeneous one at $z = 87$. At this moment all the shells are in the MONDian domain, however as the inner shells have evolved faster, they are more dense than the outer shells. Each shell after virialization freezes and preserves its density. We fit the density profile of the structure when the outermost shell, the latest part of the structure, virializes and the result is a power-law function of $\rho \propto r^\beta$ with an index of $\beta \simeq -1.22$.

### 3.2. Age of a Structure

A question that may be answered in this simple model is the dependence of the age of the structure on the mass. Observations show that small isolated structures, as such as globular clusters or center of galaxies, are older than the spiral arms (Chanoyer et al. 1998). In this model, we have seen that the smaller structures enter the MONDian regime earlier, evolve faster, and virialize at higher redshifts compared to the larger structures. As an example, in Table 2 it is shown that a structure with a mass of $10^8 M_\odot$ virializes at $z \simeq 71$ while whole of the galaxy virializes at $z \simeq 18$. The age of a structure from the virialization up to now is given by $t_{age} = \int_{t_0}^{t_{vir}} dt$, where $t_0$ is the present age of the universe. Changing the variable to the redshift results in

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**Figure 4.** Dependence of the nonlinear redshift (solid line), maximum radius redshift (dashed line), and virialized redshift (dotted-dashed line) as a function of the mass of the structure in logarithmic scale. Smaller structures enter the nonlinear regime at higher redshifts and the larger ones enter at lower redshifts. This feature of the dependence of the mass on the characteristic redshifts reveals that the smaller structure in this model forms before the larger ones.

(A color version of this figure is available in the online journal.)

**Figure 5.** Evolution of density profile in logarithmic scale normalized to the initial density, $y = \rho / \rho_{initial}$, as a function of the distance from the center of the structure in four different redshifts. The initial density profile is taken as a top-hat distribution with 10 equidistant shells for representing the evolution of the density. The innermost shell in this figure enters into the MOND domain at $z = 400$ (upper left panel). Each point on the curves notifies the position of the shell. For $z = 17$ (lower right panel), all the shells virialize and the density profile reaches a power-low function of $\rho \propto r^\beta$ with $\beta \simeq -1.22$.

(A color version of this figure is available in the online journal.)
Figure 6. Newtonian gravitational acceleration of each shell normalized to $a_0$ as a function of the shell size at the virialization time. Spots on the curve represent the acceleration and the position of each shell. The gravitational acceleration in the inner shells is larger than in the outer shells. (A color version of this figure is available in the online journal.)

$$t_{\text{age}} = H_0^{-1} \int_{z=0}^{z_{\text{vir}}} \frac{dz}{(1+z)^2 \sqrt{\Omega_b^0(1+z)^3 + \Omega_{c}^0(1+z)^4 + (1 - \Omega_b^0)(1+z)^2}},$$

(18)

where we have adopted the cosmological parameters of Section 2.1. The age of structures depending on their masses is given in Table 2.

3.3. Contribution of Dark Matter in the Smaller Structures

The other question in the formation of the structures is that why the smaller structures are mainly made of the baryonic matter and have less dark matter compared to the larger ones. This feature of the structures can be explained if we can show that in addition to the fast evolution of the smaller structures, the acceleration of the structure in terms of $a_0$, $(g_N/a_0)$ gets larger. To show this property of the structures in MOND, we calculate the gravitational acceleration at the virialization time of each shell and compare it with $a_0$. As the density of a structure at this stage changes with $\rho \propto r^\beta$, the gravitational acceleration will depend on $r$ as $g_N(r) \propto r^{\beta+1}$. For $\beta = -1.22$, the smaller radii should have larger acceleration compared to the larger ones.

In Figure 6, we plot the gravitational acceleration for each shell at the time of virialization in terms of the virialization radius. For the outer shells, the gravitational acceleration, $g_N/a_0$, is smaller than the inner shells. This means that the smaller structures after virialization need less dark matter compared to the larger ones in the standard CDM scenario. Finally, we plot the mass of the structure after virialization in terms of its size in Figure 7. This feature reveals the general property of structures in MOND, and an accurate result to compare with the observation may be obtained by $N$-body simulation of the structure formation.

4. CONCLUSION

Summarizing this work, we extend the spherical collapse model to study the generic properties of the structure formation in MOND. We showed that in the MONDian scenario, structures can evolve without a need for dark matter and have the following three main features: (1) MONDian spherical collapse unlike that of CDM does not preserve the initial density profile of the structure. In MOND starting with an initial homogeneous density profile the structure virializes with a power-law distribution of the matter, having singularity at the center. (2) We showed that the small-scale structures enter the MONDian regime earlier than the larger ones, evolve faster, and virialize at higher redshifts. This picture from the spherical-collapse model in MOND provides a bottom-top scenario for the structure formation in which the smaller structures formed before the larger ones. This result is compatible with the observations where the old stars are found to be located in the smaller structures, such as the globular clusters or center of galaxies. (3) Finally, we showed that while the smaller structures enter the MONDian regime before the larger ones, they have larger gravitational acceleration at the virialization time and hence need less dark matter in the CDM scenario.

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REFERENCES

Aguirre, A., Schaye, J., & Quataert, E. 2002, AJ, 561, 550
Angus, G.W., et al. 2007, ApJ, 654, L13
Bekenstein, J. D. 2004, Phys. Rev. D, 70, 083509
Bekenstein, J., & Magueijo, J. 2006, Phys. Rev. D, 73, 103513
Bekenstein, J. D., & Milgrom, M. 1984, ApJ, 286, 7
Bondi, H. 1947, MNRAS, 107, 410
Bosch, V. D., & Dalcanton, J. J. 2000, ApJ, 534, 146
Bosma, A. 1981, AJ, 86, 1825
Carroll, S. M., et al. 2004, Phys. Rev. D, 70, 043528
Chanover, B., et al. 1998, ApJ, 494, 98
Chen, D. M., & Zhao, H. S. 2006, ApJ, 650, L9
Dodelson, S., & Liguori, M. 2006, Phys. Rev. Lett., 97, 231301
Felten, J.E. 1984, ApJ, 286, 3
Haghi, H., Rahvar, S., & Hasani-Zonooz, A. 2006, ApJ, 652, 354
Halle, A., Zhao, H. S., & Li, B. 2008, ApJS, 177, 1
Klypin, A., et al. 1999, ApJ, 522, 82
Kroupa, P., Theis, C., & Boily, C. 2004, Astron. Nachr. Suppl., 325, 55
Lemaître, G. 1933, Ann. Soc. Sci. Brux. Ser. I, Sci. Math. Astron. Phys. A 53, 51 (for an English translation, see Lemaître, G. 1997, Gen. Rel. Grav., 29, 641)
McGaugh, S. S., & de Blok, W. J. G. 1998, ApJ, 499, 66
McGaugh, S. S., et al. 2000, ApJ, 533, L99
Milgrom, M. 1983, ApJ, 270, 365
Milgrom, M., & Sanders, R. H. 2003, ApJ, 599, L25
Moore, B., et al. 1999, ApJ, 524, L19
Nipoti, C., Londrillo, P., & Ciotti, L. 2007, ApJ, 660, 256
Nusser, A. 2002, MNRAS, 331, 909
Pointecouteau, E., & Silk, J. 2005, MNRAS, 364, 654
Saffari, R., & Rahvar, S. 2008, Phys. Rev. D, 77, 104028
Sanders, R. H. 1996, ApJ, 473, 117
Sanders, R. H. 1998, MNRAS, 296, 1009
Sanders, R. H. 1999, ApJ, 512, L23
Sanders, R. H. 2001, ApJ, 560, 1
Sanders, R. H. 2003, MNRAS, 342, 90
Sanders, R. H. 2008, MNRAS, 386, 1588
Sanders, R. H., & McGaugh, S. 2002, ARA&A, 40, 263
Sereno, M., & Jetzer, P. 2006, MNRAS, 371, 626
Skordis, C., et al. 2006, Phys. Rev. Lett., 96, 011301
Skordis, C. 2008, Phys. Rev. D, 77, 123502
Sobouti, Y. 2007, A&A, 464, 921
Spergel, D. N., et al. 2003, ApJS, 148, 175
Tiret, O., & Combes, F. 2007, A&A, 464, 517
Tolman, R. C. 1934, PANS, 20, 410
Zhao, H. S. 2007, ApJ, 671, L1
Zhao, H. S., et al. 2006, MNRAS, 368, 171
Zwicky, F. 1933, Helv. Phys. Acta, 6, 110