Free-piston reciprocating cryogenic expander utilizing phase controller

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Abstract. In a free-piston expander which eliminates mechanical linkages, a prescribed behaviour of the free-piston movement is the key to an expander performance. In this paper, we have proposed an idea of reducing complexity of the free-piston expander. It is to replace both multiple solenoid valves and reservoirs that are indispensable in a previous machine with a combination of a single orifice-reservoir assembly. It functions as a phase controller like that of a pulse tube refrigerator so that it generates time-delay of pressure variation between the warm-end and the reservoir resulting in the intended expansion of the cold-end volume down to the pre-set reservoir pressure. The modeling of this unique free-piston reciprocating expander utilizing phase controller is developed to understand and predict the performance of the new-type expander. Additionally, the operating parameters are analysed at the specified conditions to enable one to develop a more efficient free-piston type cryogenic expander.

1. Introduction

Expanders can be generally classified into two types: One is the velocity type, such as axial turbine expanders; the other is the volume type, such as screw expanders, scroll expanders and reciprocating piston expanders [1]. The velocity type expanders which can be also categorized as either centrifugal or axial flow turbines are commonly used in systems of cryogenic refrigerators and gas liquefaction plants in order to utilize the produced work during an expansion process of a high pressure gas to drive a compressor in most cases [2-4]. On the other hand, the volume type expanders which are operated by the increase of closed volume containing a constant amount of fluid mass are usually selected in preference to turbines in micro-scale systems because of their low flow rates, high pressure ratios and good performance such as micro-Rankine cycle [5-6]. Another expander which is not regarded as each of the two types is an ejector that has no mechanically moving part, which are mainly applied in jet refrigerators for the requirement of a high reliability.

The free-piston (floating piston) expander, first studied by Jones and developed by Smith [7], is an expander which eliminates a mechanical linkage to a reciprocating piston. By employing a free-piston, the expander could provide a high reliability because of less mechanical friction. Utilization of both solenoid valves and reservoirs at the warm-end of the expander to manipulate the movement of the piston was one of the proposed ideas and developed by AMTI.

1.1. Multi-valve expander (AMTI) [8]
AMTI specifically developed a cryogenic reciprocating expander by introducing the free-piston concept for the application of small-scale Collins cycle. The key to the operation of the expander is the active control of the free-piston movement. AMTI implemented four on/off valves for the purpose of active control and reservoirs at the warm-end to manipulate the piston movement as shown in (a) of Figure 1. However, this way of expander inevitably needs some complicated control system to activate solenoid valves and large space for the multiple reservoirs. Thus, as shown in Figure 1 (b), a new idea is proposed in this paper to replace the multiple valves and reservoirs with a phase controller which does not require any active control.

1.2. Mode of operation
A phase controller which is a combination of a single orifice-reservoir assembly can function as a phase (pressure) control device between the warm-end and the reservoir volume. It generates time-delay of pressure variation between the warm-end and the reservoir resulting in the intended expansion of the cold-end volume down to the pre-set reservoir pressure accompanying a pressure phase difference between the two volumes. The new expander cycle presented in this paper is composed of three processes as the follows with Figure 2.

(1) Intake process (a)-(b)
When the intake valve is opened, high pressure fluid flows into the cold-end volume pushing the piston up. Then, the pressure of the warm-end volume is increased because of compression caused by the ascending piston. The pressure of the reservoir, however, is not immediately increased so much as that of the warm-end volume because of the orifice valve that imposes flow resistance. Thus, there must be a pressure difference across the orifice valve which is a potential for the cold-end volume to exert work by expansion.

(2) Expansion process (b)-(c)
After the intake valve is closed, the cold-end volume continuously expands due to the fact that the mass of the warm-end flows into the reservoir due to the previously described pressure difference. By the time all the pressures in the cylinder are equal, the expansion process ends and the exhaust process begins.

(3) Exhaust process (c)-(d)
When the exhaust valve is opened, the mass of the cold-end volume flows out of the expander at low temperature. When the exhaust valve is closed, a cycle is completed and the same processes are repeated.
With this intended operation of the cycle, the PV work can be generated and dissipated at the warm-end of the expander.

2. Numerical model
The detailed expander cycle is simulated to investigate its thermodynamic behaviour and predict a performance of the new-type expander. The operating parameters affecting the expander performance are examined. In this model, several assumptions are introduced as the follows.

(i) Operating fluid behaves as an ideal gas.
(ii) Both cylinder and piston are thermally insulated (adiabatic).
(iii) Reservoir temperature is constant (isothermal).

The three individual control volumes are separately considered for analysis; (1) cold-end volume, (2) warm-end volume and (3) reservoir volume. All the thermodynamic states are determined by the energy conservation law (the first law of thermodynamics) as well as the mass balance.

2.1. Intake process

\[
\frac{dE_{\text{cold}}}{dt} = \dot{Q} - \dot{W} + (mh)_{\text{in}} - (mh)_{\text{out}}
\]  

(1)

Neglecting kinetic and potential energy and applying the aforementioned assumptions, we can modify equation (1) to equation (2) during the time step \( \Delta t \). Solving equation (2) for \( P_f \), we can obtain equation (3).

\[
\frac{c_v((PV)_f - (PV)_\text{cold})}{R} = (-P_f(V_f - V_i) + ((m_f - m_i)c_p T)_\text{in} - ((m_f - m_i)c_p T)_\text{out})_{\text{cold}}
\]  

(2)
\[ P_{f,cold} = \frac{1}{V_{f,cold}}(P_{i,cold}V_{i,cold} + (k-1)\times(-P_{i,cold}(V_{f,cold} - V_{i,cold}) + \Delta m_i c_p T_{in} - \Delta m_{out} c_p T_{cold})) \] (3)

Mass change in the cold-end volume can be calculated by considering the pressure difference across each valve as equation (4) and leakage through the clearance gap between the moving piston and the cylinder wall as equation (5) [9-10]. \( P_{ratio} \) is the pressure ratio of pressure across each valve, \( k \) is the specific heat ratio, \( c \) is the radial clearance gap, \( \mu_{visc} \) is the viscosity of the working fluid and \( f_{correction} \) is the correction factor of each valve which will be explained in the section 3.

\[ \frac{dm_{in}}{dt} = f_{correction} \times A_{in} \times P_{in} \times \left( \frac{2k}{k-1} \times P_{ratio}^{2/k} \times (1-P_{ratio})^{(k-1)/k} \right)^{1/2} \] (4)

\[ \frac{dm_{out}}{dt} = \frac{\pi D_{cylinder}^3 (P_{cold}^2 - P_{warm}^2)}{12 \mu_{visc} L_{piston} R T_{cold}} \] (5)

Next, the displacement of the piston from the bottom of the cylinder can be determined by using the dynamic equation of motion (6) during the infinitesimal step time. \( M_{piston} \) is a mass of the piston, \( \Delta P \) is a pressure difference across the piston, \( A_{cylinder} \) is a cross sectional area of the cylinder, \( f_{friction} \) is a friction force caused by the piston and cylinder, \( g \) is a gravitational acceleration, \( a \) is an acceleration of the piston, \( v \) is a velocity of the piston and \( x \) is a displacement of the piston from the bottom of the cylinder.

\[ a = \frac{1}{M_{piston}}(\Delta P A_{cylinder} - M_{piston} g - f_{friction}) \]

\[ \Delta v = a \times \Delta t \]

\[ \Delta x = v \times \Delta t + \frac{1}{2} a \times \Delta t^2 \] (6)

Once the physical states in the cold-end volume are determined at every time steps with the above equations, the states in the warm-end volume and the reservoir volume are subsequently determined step by step.

2.2. Expansion process

Expansion process occurs at the event of closing the intake valve. The difference of this process from the intake process is that there is no mass flow into the cold-end volume. The mass in the cold-end volume is decreased due to the leakage to the warm-end volume. Other procedures are same as the intake process.

\[ P_{f,cold} = \frac{1}{V_{f,cold}}(P_{i,cold}V_{i,cold} + (k-1)\times(-P_{i,cold}(V_{f,cold} - V_{i,cold}) - \Delta m_{out} c_p T_{cold})) \] (7)

2.3. Exhaust process

Exhaust process is the reverse of the intake process. It is triggered by the event of opening the exhaust valve. Mass in the cold-end volume is decreased due to the relatively higher pressure at the cold-end volume than that of the exit. Equation (8) is the same form with the equation (3) except the temperature.
After the intake, expansion and exhaust processes finished, a cycle is then completed and the next cycle with the same procedure should begin. Number of cycles and each valve timings are pre-set before the simulation processes. Table 1 shows the various parameters to be examined in this model which are separated into two categories: The geometric parameters that are fixed in this model and the operating parameters that are varied to be analysed. In this paper, the values of the geometric constraints are chosen by the actual experimental apparatus to be experimented in the future.

3. Correction factor of the mass flow equation

Three valves are to be implemented in the expander: two on/off valves (intake and exhaust) at the cold-end and an orifice valve at the warm-end. Mass flow equation of each valve is important and to be adjusted due to the unusual transient characteristic of the actual situation in order to closely predict the expander performance. By introducing the correction factor \( f_{correction} \) in the equation (4) and adjusting the values by trial and error method despite of the flow coefficient \( (C_v) \) provided by manufacturer, the mass flow equation of each valve applied in the simulation can be fitted to the experiment. Figure 3 demonstrates a pressure-time curve of a certain volume with each valve in the case of being pressurized or depressurized. All the cases show different pressure-time curves between the experiment result and the one calculated by \( C_v \), which implies that it needs not to use just \( C_v \) to obtain the pressure-time variation. By comparing the simulation results with the actual experiments under the same condition, adequate \( f_{correction} \) of the intake, exhaust and orifice valve can be found as 0.6, 1.2 and 0.22 (100%

\[
P_{f,cold} = \frac{1}{V_{f,cold}}(P_{i,cold}V_{i,cold} + (k-1)\times(-P_{e,cold}(V_{f,cold} - V_{i,cold}) - \Delta m_{w}c_{p}T_{cold} + \Delta m_{w}c_{p}T_{warm})) \quad (8)
\]

\( P_{i,cold} \) is the pressure at the intake valve, \( V_{i,cold} \) is the volume at the intake valve, \( k \) is the polytropic coefficient, \( P_{e,cold} \) is the pressure at the exchage valve, \( V_{f,cold} \) is the volume at the exchange valve, \( \Delta m_{w} \) is the mass of the working fluid, \( c_{p} \) is the heat capacity of the working fluid, \( T_{cold} \) is the temperature at the intake valve, and \( T_{warm} \) is the temperature at the warm-end.
Table 2. Simulated results by the numerical model during 100 cycles ($P_{\text{ratio}} = 10, V_{\text{res}} = 0.5 \text{ L}$)

| Intake period (ms) | Expansion period (ms) | Exhaust period (ms) | Opening of orifice valve (%) | PV work rate (W) At first cycle | PV work rate (W) At 100th cycle (cyclic steady state) |
|--------------------|----------------------|---------------------|-----------------------------|---------------------------------|--------------------------------------------------------|
| 100                | 200                  | 100                 | 100% opening                 | 13.69                           | 3.31                                                   |
|                    |                      |                     | 80% opening                  | 11.15                           | 2.64                                                   |
|                    |                      |                     | 50% opening                  | 7.58                            | 1.77                                                   |
|                    |                      |                     | 20% opening                  | 1.74                            | 0.27                                                   |
|                    |                      |                     | 0% opening                   | 0.30                            | 0.40                                                   |
| 200                | 400                  | 200                 | 100% opening                 | 11.92                           | 7.75                                                   |
|                    |                      |                     | 80% opening                  | 11.29                           | 6.57                                                   |
|                    |                      |                     | 50% opening                  | 9.06                            | 4.78                                                   |
|                    |                      |                     | 20% opening                  | 2.07                            | 0.91                                                   |
|                    |                      |                     | 0% opening                   | 0.13                            | 0.25                                                   |

Specifically, $f_{\text{correction}}$ of the orifice valve linearly varies as to the degree of opening based on the experiments.

4. Simulation results

Figure 4 shows the simulation results of pressure-volume and pressure-time curves in the case of $t_{\text{in}} = 200 \text{ ms}$, $t_{\text{exp}} = 400 \text{ ms}$, $t_{\text{exh}} = 200 \text{ ms}$, $P_{\text{ratio}} = 10$ and 80% opening of the orifice valve at 100th cycle (cyclic steady state). Pressures of the cold-end, warm-end and reservoir volume are indicated during the cycle operation. During the intake process, both the pressures in the cold expansion volume and the warm end volume are increasing rapidly, but the reservoir pressure is not following up because of the flow resistance of the orifice valve. In the expansion process after the intake valve is closed, there exists a pressure difference between the warm-end volume and the reservoir volume which results in both an expansion of the cold-end volume and pressure increase of the reservoir. Pressure of the cold expansion volume is slightly higher than that of the warm-end volume, which acts as a driving force of moving up the piston. After the expansion process ends, the exhaust valve is opened and the gas in the cold-end volume is released.
volume flows out to the environment because its pressure is lower than that of the cold expansion volume. Similarly, the pressures of the warm-end and the cold expansion volume are immediately decreasing but the reservoir’s does not. Once the exhaust valve is closed, a cycle is completed. While a number of cycle is running at pre-set valve timing and orifice valve opening, cyclic steady state can be obtained as shown in Figure 4.

5. Analysis on the operating parameters
The operating parameters which significantly affect the performance of the expander are to be examined with the developed numerical model. Criterion of the expander performance is defined as the PV work rate which is the work generation of the cold-end volume divided by the cycle period. The specified valve timing conditions and the results are shown in Table 2.

Depending on the opening duration of each valve and a degree of the orifice valve opening, the pressure of the reservoir is converged to a certain value. If this pressure becomes close to the intake pressure, there will be no expansion process because of no pressure difference between the warm-end volume and the reservoir volume. It results in no expansion work due to the fact that the expansion process progresses until both the pressure of the warm-end volume and the pressure of the reservoir volume become an equilibrium pressure. Figure 5 shows an increasing reservoir pressure resulting the decrease of the PV work rate. On the contrary, if the converged pressure approaches to the exhaust pressure, the pressure difference across the orifice valve will be increased, and consequently generate large expansion work. Thus, the adequately converged pressure of the reservoir which is subject to both the valve timing and the opening of the orifice valve is to be adjusted for the maximum PV work rate. With respect to the value of the PV work rate, there is no tendency about the ratio of each period time. Even though two

**Figure 5.** Pressure-time and PV work rate-time curves in the case of $t_{in} = 200$ ms, $t_{exp} = 400$ ms, $t_{exh} = 200$ ms and 80% opening of the orifice valve during 100 cycles ($P_{ratio} = 10$, $V_{res} = 0.5$ L)

**Figure 6.** Effects of the reservoir volume and pressure ratio ($P_{in}/P_{out}$) in the case of $t_{in} = 200$ ms, $t_{exp} = 400$ ms, $t_{exh} = 200$ ms and 80% opening of the orifice valve at 100th cycle (cyclic steady state)
cases have the same ratio of the intake, expansion and exhaust time as illustrated in Table 2, the PV work rates are quite different for two cases as 1.77 W and 4.78 W at the 50% opening.

Figure 6 shows the effects of the reservoir volume and pressure ratio \( \frac{P_{in}}{P_{out}} \) on the PV work rate in the case of \( t_{in} = 200 \text{ ms} \), \( t_{exp} = 400 \text{ ms} \), \( t_{exh} = 200 \text{ ms} \) and 80% opening of the orifice valve at 100\textsuperscript{th} cycle which can be considered as cyclic steady state. If the reservoir volume increases, the PV work rate also increases until the volume of the reservoir is around 0.5 L. After that value, however, the PV work rate is almost same even though the reservoir volume increases. It implies that it is not always beneficial to set the reservoir volume large to maintain the low converged pressure of the reservoir. In respect to the pressure ratio, the higher the pressure ratio is, the larger PV work rate can be obtained. It is natural that the higher pressure ratio can have more potential to generate the PV work. However, it may cause a collision problem between the piston and the cylinder in real experiment because of the high velocity of the piston. Adequate pressure ratio should be considered in actual experiment.

6. Conclusions
In this research paper, the modeling of free-piston reciprocating expander utilizing a phase controller was carried out for cryogenic application. Effect of the orifice valve opening was mainly examined to see how the expander performance changes. The converged pressure of the reservoir after the repeated cycles is the key parameter to determine the magnitude of the PV expansion work and ultimately defines the expander performance. Parameters such as the reservoir volume and the pressure ratio also affect the performance of the expander. The other parameters studies and experimental validation as the proof of the concept should be conducted in the future.

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