Interaction between dielectric particles enhances the Q factor.

E. N. Bulgakov, K.N. Pichugin, and A. F. Sadreev
Kirensky Institute of Physics, Federal Research Center KSC SB RAS, 660036 Krasnoyarsk, Russia

Abstract—We consider resonant modes of two dielectric identical particles which can be classified as symmetric and anti symmetric combinations of the resonant modes of individual particles. We show that an approaching of two particles gives rise to an avoided crossing of resonant poles because of interaction between the disks. That in turn results in strong enhancement of the quality factor of two disks compared to isolated disks.

0.1. INTRODUCTION

It is rather challenging for optical resonators to support resonances of simultaneous sub wavelength mode volumes and high quality factor (Q factors). The traditional way for increasing of the Q factor of optical cavities is a suppression of leakage of resonance mode into the radiation continua either of free space or open channels of photonic crystal waveguides. That is achieved usually by decreasing of coupling of the resonant mode with the continua. The most known example of such a strategy is exploitation of Fabry-Perot resonators or whispering gallery modes [1] for which the Q factor reaches extremal values. The decisive breakthrough came with paper by Friedrich and Wintgen (FW) [2] which put forward idea of destructive interference of two neighboring resonant modes leaking into the continuum. Based on a simple generic two-level model they formulated the condition for the bound state in the continuum (BIC) as the state with zero resonant width for crossing of eigenlevels of the cavity. After this principle was explored in open plane wave resonator when the BIC occurs at points or vicinity of degeneracy of the closed integrable resonator [3]. However these BICs exist provided that they embedded into single continuum of propagating modes of directional waveguide. In photonics the optical BICs embedded into the radiation continuum can be realized by two ways. The first way is realized in optical cavity coupled with the continuum of 2d photonic crystal (PhC) waveguide [4] that is an optical variant of microwave system [3]. More perspective way for the BICs is achieved in periodic PhC systems or arrays of dielectric particles in which resonant modes are leaked into restricted number of diffraction continua [5, 6, 7, 8, 9]. Although the exact BICs can exist only in the infinite periodical arrays [10, 11], the finite arrays demonstrate resonant modes with the very high Q factor which grow quadratically [12] or even cubically with the number of particles [13]. However the price of that is a large space extension of the array.

Recently ITMO group has demonstrated a way to achieve high Q factor (super cavity modes) in isolated high-index dielectric disk [14, 15]. Such super-cavity modes originate from avoided crossing of nearest resonant modes, specifically the Mie-type resonant mode and the Fabry-Pérot resonant mode for variation of aspect ratio of dielectric disk. That resulted in a significant enhancement of the Q factor similar to that as the avoided crossing of nearest resonant modes gives rise to the BICs in the framework of two-mode non Hermitian Hamiltonian [2, 3, 16, 17]. It is worthy also to notify the formation of long-lived, scar like modes near avoided resonance crossings in optical deformed microcavities [18]. The dramatic Q factor enhancement was predicted by Boriskina [19] for avoided crossing of very highly excited whispering gallery modes in symmetrical photonic molecules of dielectric cylinders. Recently Taghizadeh and Chung demonstrated very high Q factor for few unit cell structures [20]. In the present paper we consider similar way of the avoided crossing to enhance the Q factor by variation of distance between two identical dielectric particles. We consider the avoided crossing of basic low excited resonant modes (monopole and dipole) for approaching of two dielectric particles of different simple shapes like coaxial disks and parallel cylinders. When the particles are separated rather long distance we have a family of degenerate resonant modes such as monopole, dipole etc modes. For approaching of the particles they are coupled via leaky resonant modes of each particle that gives rise to the avoided resonance crossing. We show that is complementing by spiral behavior of poles when interaction between the particle is weak. With further
approaching of particles interaction is increasing to give rise to strong repulsion of the resonant poles. One of the poles can approach close to real axis distinguishing the high $Q$ factor compared to the $Q$ factor of isolated particle.

### 0.2. TWO COAXIAL DISKS

Fig. 1(a) illustrates pole behavior for the case of two dielectric coaxial disks with variation of the distance between disks. In spite of visible complexity of the behavior of poles in Fig. 1 (a) zoomed pictures in 1 (b) and (c) reveal remarkably simple behavior of the resonant poles in the form of spiral convergence of pole pairs into the degenerate resonant pole of isolated disk shown by closed circle for enlarging of the distance between the disks when it exceeds the wavelength of light. When the distance becomes sufficiently small the interaction becomes stronger to give rise to the repulsion of the poles as shown in insets in Fig. 1.

The mechanism of interaction is the following. Assume for a moment we pump up resonant mode of the first disk that results in radiation which scatters by the second disk resulting in an interaction between disks. In order to quantitatively evaluate this interaction we write the effective non Hermitian Hamiltonian in the two-level approximation

$$
H_{eff} = H^{(0)}_{eff} + V = \begin{pmatrix} \omega_r & 0 \\ 0 & \omega_r \end{pmatrix} + \begin{pmatrix} a & b \\ b & a \end{pmatrix},
$$

where $\omega_r$ is the complex eigenfrequency of the isolated disk marked by closed red circles in Fig. 1. The complex matrix element $b$ is the result of interaction between the disks owing to the scattering of leaking resonant mode from the first disk by the second one. The complex matrix element $a$ is the result of two scattering processes, at first by the second disk and then by the first disk. Therefore one can expect that $\arg(b) = kL$, $\arg(a) = 2kL$, $|b| \sim 1/L^\lambda$, $|a| \sim 1/L^{2\lambda}$ where $\lambda$ depends on the effective dimension of the system. Fig. 2 shows the behavior of the matrix elements by the absolute values and phase. The matrix elements $a$ and $b$ can be easily found through the poles versus the distance $L$ shown in Fig. 1 (d)

$$
\omega_1,2 = \omega_r + a \pm b,
$$

to be equal $b = \frac{\omega_1 - \omega_2}{2}$, $a = \frac{\omega_1 + \omega_2}{2} - \omega_r$. As the result we evaluate that $\lambda \approx 2$ for the case of coaxial disks. Respectively hybridized resonant eigenmode is given by symmetric and antisymmetric combinations of the resonant modes of the isolated disk

$$
\psi_{s,a}(r, \theta, z) = \psi_r(r, \theta, z - L/2) \pm \psi_r(r, \theta, z + L/2)
$$

where $\psi_r(r, \theta, z)$ is the corresponding resonant mode of the isolated disk.
0.3. TWO PARALLEL CYLINDERS

The problem of scattering of electromagnetic waves from two parallel infinitely long dielectric cylinders was solved long time ago in papers [21, 22] that allows us to easily find resonant modes. In contrast to couple of disks the effective dimension of this problem is reduced to two. Therefore role of interaction is increased. Indeed interaction of the monopole or dipole resonances is so strong that there is no convergence to the case of degenerate resonances of isolated cylinders. Results of computation for two types of interaction of dipole resonant modes are presented in Fig. 2. When the dipole resonant modes of isolated cylinder have nodal lines perpendicular to line connected two cylinders (Fig. 2 (a)) we observe stronger interaction compared to the case of the dipole resonant modes with the nodal lines parallel to this line (Fig. 2 (a)). Respectively in the first case the resonant pole of the system after one circling around the Mie dipole resonance go away and undergo avoided crossing around other Mie resonances shown in Fig. 2 (a) by crosses. While in the second case of dipole resonant mode the resonances of the system demonstrate spiral behavior around the Mie resonances similar to the case of coaxial disks. However with further increasing of distance between the cylinders the resonance also go away to the next Mie resonance as shown in Fig. 2 (c). The reason is related to exponential growth of the resonant modes (the Gamov states) at far distances from the cylinder. The behavior of the Q factors is shown in Fig. 2 (b) and (d) at right.

0.4. CONCLUSION

We considered the behavior of low lying resonant frequencies of the system two coaxial disks and two parallel infinitely long dielectric cylinders versus a distance between them. Similar to papers [14, 15, 18, 19, 20] we observe multiple events of the avoided crossing due to interaction between the objects that gives rise to multiple enhancement of the Q factor.
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