Multipath Wave Particle Duality with a Spooky Path Detector

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According to Bohr’s principle of complementarity, a quanton can behave either as a wave or a particle, depending on the choice of the experimental setup. Some recent two-path interference experiments have devised methods where one can have a quantum superposition of the two choices, thus indicating that a quanton may be in a superposition of wave and particle nature. These experiments have been of interest from the point of view of Wheeler’s delayed-choice experiment. However, it has also been claimed that this experiment can violate complementarity. Here we theoretically analyze a multipath interference experiment that has a which-path detector in a quantum superposition of being present and absent. We show that a tight multipath wave-particle duality relation is respected in all such situations, and complementarity holds good. The apparent violation of complementarity may be due to incorrect evaluation of path distinguishability in such scenarios.

I. INTRODUCTION

The discourse of wave-particle duality has always attracted attention from the early days of quantum mechanics. It is believed that it lies at the heart of quantum mechanics [1]. It was understood from the beginning that the object exhibits both wave and particle nature. Objects showing both wave and particle nature are often called quantons [2]. It was Bohr who first pointed out that both properties are mutually exclusive and formulated it as a principle of complementarity [3]. Wootters and Zurek [4] revisited Bohr’s complementarity principle from the information-theoretic approach, looking at two-slit interference in the presence of a path detector, and found that simultaneous observation of both nature is possible with the proviso that the more you observe one, more it will obscure the other. Later, Greenberger and Yasin [5] formulated a quantitative bound in terms of the predictability and fringe visibility. The predictability was defined as a priori information i.e., it tells one the difference between probabilities of going through different paths. Englert [6] proposed a stronger path quantifier which was based on a posteriori path information acquired using a path detector, and derived a bound on the path distinguishability and fringe visibility, \( D^2 + V^2 \leq 1 \). This relation, generally called the wave particle duality relation, is understood to be a quantitative statement of Bohr’s principle. Of late the concept of wave particle duality has been generalized to multipath interference [7–11].

In a Mach-Zehnder interferometer, it is understood that in the balanced mode, only one of the detectors registers all the photons, and no photons arrive at the other detector due to destructive interference. In this situation, it is logical to believe that the photon follows both the paths, which later interfere. If the second beam-splitter is removed, photons from one path can only reach a particular detector. So it is logical to assume that each photon detected by any detector, came from only one path, and not both. So the presence of the second beam-splitter makes the photons behave as a wave, following both paths, and in its absence they behave like particles, following only one path at a time. John Wheeler had introduced an idea that if the choice of removing or retaining the beam-splitter is made after the photon has traversed most of its path, one can affect the past of the particle in the sense of making sure, even after a delay, that the photons behave like a wave or like a particle [12]. This “delayed choice” idea has been a subject of debate for long. Some years back, a proposal was made by Ionicioiu and Terno [13] suggesting that the second beam-splitter could be put in a quantum superposition...
of being present and absent (see FIG. 1). The idea was that this would force the photon to be in a superposition of wave and particle natures. This “quantum delayed choice” experiment, with a spooky beam-splitter immediately became a subject of attention, and many experimental and theoretical studies were carried out [14–19].

Apart from the obvious relevance of this new class of experiments to Wheeler’s delayed choice idea, there have been speculations that the superposition of wave and particle natures might violate complementarity. In particular, some claims of exceeding the bound set by the two-path duality relation of the kind $D^2 + V^2 \leq 1$ have been made [15]. In this work, we investigate the issue of wave particle duality in the more general scenario of $n$-path interference, where the path detector is spooky, i.e., it can be in a quantum superposition of being present and absent.

II. WAVE-PARTICLE DUALITY IN MULTIPATH INTERFERENCE

A. Duality relation for pure quanton and QPD

Consider an $n$-path interference experiment with pure initial quanton state

$$|\psi_{in}\rangle = \sum_{i=1}^{n} \sqrt{p_i} |\psi_i\rangle,$$  \hspace{1cm} (1)

where $p_i$ is the probability of acquiring $i^{th}$ path and $|\psi_i\rangle$ forms an orthonormal basis.

We use a quantum path detector (QPD) to detect the path acquired by quanton. There are two degrees of freedom associated with it. One is its location, which is assumed to have two states, $|Y\rangle_L$ corresponding to it being present in the paths of the quantum and $|N\rangle_L$ corresponding to being absent from the path. The other degree of freedom is its internal state denoted by $|d_i\rangle$, which corresponds to it detecting the path of the quanton.

Initially, the QPD is assumed to be in the state $|d_0\rangle$, and if the quanton goes through the $k^{th}$ path, the QPD state changes to $|d_k\rangle$. So the full initial detector state is given by

$$|\phi_0\rangle = |d_0\rangle (c_1 |Y\rangle_L + c_2 |N\rangle_L),$$ \hspace{1cm} (2)

where $c_1$ is the amplitude of QPD presence and $c_2$ the amplitude of its absence; $c_1^2 + c_2^2 = 1$. The state represents the QPD being in a superposition of the two locations.

Initially, the joint state of quanton and QPD is given by

$$\rho_{sd}^{(in)} = \sum_{i,j=1}^{n} \sqrt{p_i p_j} |\psi_i\rangle \langle \psi_j| \otimes |\phi_0\rangle \langle \phi_0|,$$ \hspace{1cm} (3)

which denotes a pure state of the quanton with amplitude $\sqrt{p_k}$ to go through the $k^{th}$ path, being in the state $|\psi_k\rangle$. The interaction can be represented by a controlled unitary operation, $U$. The combined state of quanton and QPD can be written as

$$\rho_{sd} = \sum_{i,j=1}^{n} \sqrt{p_i p_j} |\psi_i\rangle \langle \psi_j| \otimes U_i |\phi_0\rangle \langle \phi_0| U^d_j.$$ \hspace{1cm} (4)

where $U_i |\phi_0\rangle = c_1 |d_i\rangle |Y\rangle_L + c_2 |d_0\rangle |N\rangle_L$.

The above interaction creates entanglement between the quanton and path detector. Thus, for gaining knowledge of the path of the quanton, it is sufficient to do a measurement on the states $|d_i\rangle$ of the QPD. Here we will use the unambiguous quantum state discrimination (UQSD) method for gaining the path information [7, 8].

For wave information we will use $l_1$ norm measure of coherence [8, 20, 21]. Let us now look at the path distinguishability and the measure of coherence.

Path distinguishability: Based on UQSD, the path-distinguishability for $n$-path interference [7, 8], is given by

$$D_Q := 1 - \frac{1}{n-1} \sum_{i \neq j} \sqrt{p_i p_j} \left| \langle \phi_0 | U^d_j U_i | \phi_0 \rangle \right|^2,$$

$$= 1 - \frac{1}{n-1} \sum_{i \neq j} \sqrt{p_i p_j} \left( c_1^2 |\langle d_j | d_i \rangle| + c_2^2 \right).$$ \hspace{1cm} (5)

It is essentially the maximum probability with which the states $U_i |\phi_0\rangle$ can be unambiguously distinguished from each other.

Quantum coherence: Quantum coherence [8, 20, 21] gives the wave nature of a quanton, given by

$$C(\rho) := \frac{1}{n-1} \sum_{i \neq j} |\rho_{ij}|,$$ \hspace{1cm} (6)

where $n$ is the dimensionality of the Hilbert space. The reduced density matrix of the quanton can be obtained by tracing out all the states of the QPD

$$\rho_s = \sum_{i,j=1}^{n} \sqrt{p_i p_j} \text{Tr} \left( U_i |\phi_0\rangle \langle \phi_0| U^d_j \right) |\psi_i\rangle \langle \psi_j|,$$ \hspace{1cm} (7)
The set \( \{|\psi_i\rangle\} \) forms a complete basis for the \( n \) path setup. Thus, the coherence can be obtained using the reduced density matrix

\[
C = \frac{1}{n-1} \sum_{i \neq j} \left| \langle \psi_i | \rho_s | \psi_j \rangle \right|
= \frac{1}{n-1} \sum_{i \neq j} \sqrt{p_i p_j} \left| \text{Tr} \left( U_i |\phi_0 \rangle \langle \phi_0 | U_j^\dagger \right) \right|, \tag{8}
\]

Using Eq. (2), we get the following form:

\[
C = \frac{1}{n-1} \sum_{i \neq j} \sqrt{p_i p_j} \left( c_1^2 |\langle d_j | d_i \rangle \rangle + c_2^2 \right). \tag{9}
\]

Combining Eq. (5) and Eq. (9), we get

\[
D_Q + C = 1. \tag{10}
\]

This is a tight wave-particle duality relation which had been derived earlier for \( n \)-path interference [8]. So, the relation continues to hold even in the case of a QPD.

**Two-path experiment:** For \( n = 2 \) and \( p_1 = p_2 = \frac{1}{2} \), the path distinguishability (5) and coherence (9) becomes

\[
D_Q = c_1^2 \left( 1 - |\langle d_1 | d_2 \rangle \rangle \right) \tag{11}
\]

\[
C = 1 - c_1^2 + c_2^2 |\langle d_1 | d_2 \rangle \rangle. \tag{12}
\]

Our result reproduces the earlier result [22] for a two path experiment in the presence of a QPD, while recognizing that for two paths, the coherence \( C \) is identical to the traditional visibility \( V \) [21]. It also satisfies Eq. (10) in the same way.

**B. Superposition of wave and particle natures**

The preceding analysis is for the behavior of the quanton irrespective of the location state of the QPD. One might argue that one would get the same result if QPD were not in the superposition state (2), but in a mixed state of being present and absent. To really see the effect of the QPD being in a superposition, one should look at the behavior of the quanton conditioned on obtaining a superposition location state of the QPD. To this end, let us assume the QPD state is measured in a certain basis and collapses to

\[
|\phi_0 \rangle = \cos \alpha |Y \rangle_L + \sin \alpha |N \rangle_L \tag{13}
\]

which is state just for the location degree of the QPD.

The interaction can be represented by a controlled unitary operation, \( U \). The combined state of quanton and QPD can be written as

\[
\rho_{sd} = \sum_{i,j=1}^{n} \sqrt{p_i p_j} |\psi_i \rangle \langle \psi_j | \otimes |d'_i \rangle \langle d'_j |. \tag{14}
\]

where \( |d'_i \rangle \equiv \langle \phi_0 | U_i | \phi_0 \rangle = c_1 \cos \alpha |d_1 \rangle + c_2 \sin \alpha |d_0 \rangle \); with normalization condition \( c_1^2 \cos^2 \alpha + c_2^2 \sin^2 \alpha = 1 \).

The above interaction creates the entanglement between the quanton and path detector, with the QPD out of the picture now. Following the earlier procedure, we will use the UQSD method for gaining the path information and coherence for wave information. Based on UQSD, the path-distinguishability for \( n \)-path interference is given by

\[
D_Q = 1 - \frac{1}{n-1} \sum_{i \neq j} \sqrt{p_i p_j} \left| \langle c_1^2 \cos^2 \alpha \langle d_j | d_i \rangle + c_2^2 \sin^2 \alpha \right| + \frac{c_2^2}{2} \sin 2\alpha \langle d_j | d_0 \rangle + \langle d_0 | d_i \rangle \rangle \right| \tag{15}
\]

The reduced density matrix of the quanton can be obtained by tracing out the detector states

\[
\rho_s = \sum_{i,j=1}^{n} \sqrt{p_i p_j} \text{Tr} \left( |d'_i \rangle \langle d'_j | \right) |\psi_i \rangle \langle \psi_j |. \tag{16}
\]

The set \( \{|\psi_i \rangle\} \) forms a complete incoherent basis for \( n \) path setup. Thus, the coherence can be obtained using the reduced density matrix

\[
C = \frac{1}{n-1} \sum_{i \neq j} \sqrt{p_i p_j} \langle d'_i | \langle d'_j | \rangle. \tag{17}
\]

Using Eq. (2), we get the following form:

\[
C = \frac{1}{n-1} \sum_{i \neq j} \sqrt{p_i p_j} \left| \langle c_1^2 \cos^2 \alpha \langle d_j | d_i \rangle + c_2^2 \sin^2 \alpha \right| + \frac{c_2^2}{2} \sin 2\alpha \langle d_j | d_0 \rangle + \langle d_0 | d_i \rangle \rangle \right| \tag{18}
\]

Combining Eq. (15) and Eq. (18), we get

\[
D_Q + C = 1. \tag{19}
\]

Thus, even when quanton is forced to be in a superposition of wave and particle nature, the usual wave-particle duality relation continues to hold. This is at variance with earlier claims suggesting that wave-particle duality relations are violated in such a situation.

**C. Perspectives**

At this stage, it may be useful to analyze these results in the light of various earlier works. It is widely believed that the superposition of wave and particle nature may lead to a violation of the complementarity. However, most experiments that have been carried out, do not involve a path-detecting device. Rather the beam-splitter BS1 (see FIG. 1) is in a superposition of being present and absent. So, in the situation where BS1 is in a superposition, there is no way of knowing if a particular photon received at (say) D1, followed one path or both the paths. In such a situation, one can only talk of the probability of taking one path or the other, the duality relation that is meaningful is the one derived by Greenberger and Yasin [5]. The duality relation pertaining to
detecting which path the quanton followed, derived by 
Englert [6], is not applicable in such scenarios.

The analysis carried out in the previous subsections shows that complementarity is always respected in the 
multipath interference experiment which has a path-
detecting device in the superposition of being present 
and absent. Eq. (5) has a nice interpretation that the 
path-detecting states \( |d_i\rangle \) are present with a 
probability \( c_i^2 \) and absent with probability \( c_i^2 \). And it leads to 
the perfect duality relation (10). However, if one naively 
uses the same definition, which appears reasonable, for 
the case where is the quanton is really forced to be in a 
superposition of wave and particle behavior, one will run 
into a problem. With that reasoning one would imagine 
that the path-detecting states \( |d_i\rangle \) are present with a 
probability \( c_i^2 \cos^2\alpha \) and absent with probability prob-
bility \( c_i^2 \sin^2\alpha \). The distinguishability will then come 
out to be \( D_Q = 1 - \frac{1}{n-1} \sum_{i \neq j} \sqrt{\frac{n}{p_{ij}}} \left| \left( c_i^2 \cos^2\alpha \langle d_j | d_i \rangle + c_i^2 \sin^2\alpha \right) \right| \). But the coherence in this situation will 
be \( C = \frac{1}{n-1} \sum_{i \neq j} \sqrt{\frac{n}{p_{ij}}} \left| \left( c_i^2 \cos^2\alpha \langle d_j | d_i \rangle + c_i^2 \sin^2\alpha + \frac{c_i c_j}{n} \sin 2\alpha \left( \langle d_j | d_0 \rangle + \langle d_0 | d_i \rangle \right) \right) \right| \). It is easy to see that the 
sum \( D_Q + C \) may exceed 1 because of the term 
\( \frac{c_i c_j}{n} \sin 2\alpha \left( \langle d_j | d_0 \rangle + \langle d_0 | d_i \rangle \right) \), which is a signature of inter-
ference between the wave and particle natures. One 
may naively interpret it as a violation of complementar-
ity. However, recognizing that the paths of the quanton 
are correlated with \( |d_i\rangle \equiv \langle \phi_0 | U_i | \phi_0 \rangle = c_1 \cos \alpha |d_i\rangle + c_2 \sin \alpha |d_0\rangle \), and not just with \( |d_i\rangle \), one can see that 
the unambiguous discrimination of \( |d_i\rangle \) is what will yield 
the correct distinguishability (15). This distinguishability 
leads to the correct duality relation (19).

So we see that even in the scenario where there is an 
interference between the wave and particle natures, quan-
tum complementarity is fully respected, governed by the 
wave particle duality relation (19). In the experiments 
where there is no real path-detector in place, it is all the 
more likely to come to an erroneous conclusion regarding 
the violation of complementarity.

D. Generalized Duality relation

We extend our analysis for noisy scenario. The mixed 
quantum state can be taken as \( \rho_{\text{in}} = \sum_{i,j} \rho_{ij} |\psi_i\rangle \langle \psi_j| \). The initial 
joint state of a quanton and a detector system can 
be written as \( \rho_{\text{in}}^{(0)} = \rho_{\text{in}} \otimes \rho_{\text{in}}^{(0)} \). The effect of noise on 
the QPD can be represented as

\[
\rho_{\phi}^{(0)} \rightarrow \rho_{\phi}^{(0)} = \sum_i K_i \rho_{\phi}^{(0)} K_i^\dagger,
\]

with completeness relation \( \sum_i K_i^\dagger K_i = I \). The spectral 
decomposition of the transformed QPD can then be written as

\[
\rho_{\phi}^{(0)} = \sum_k r_k |\phi_k\rangle \langle \phi_k|,
\]

where \( \sum_k r_k = 1 \), \( r_k \geq 0 \), and \( \langle \phi_k | \phi_l \rangle = \delta_{kl} \).

The combined quanton-QPD state, when QPD is con-
sidered in state Eq. (13), can be written as

\[
\rho_{sd} = \sum_{i,j=1}^n \rho_{ij} |\psi_j\rangle \langle \psi_j| \otimes \sum_k r_k |d_{ki}\rangle \langle d_{kj}| \tag{22}
\]

where \( |d_{ki}\rangle \equiv \langle \phi_i | U_i | \phi_i \rangle = c_1 \cos \alpha |d_{ki}\rangle + c_2 \sin \alpha |d_{kj}\rangle \). The path 
distinguishability for mixed QPD (21) can be calculated using

\[
D_Q' = 1 - \frac{1}{n-1} \sum_{k \neq j} r_k \sqrt{\rho_{ij} \rho_{jj} \langle \langle d_{kj} | d_{ki} \rangle \rangle} \tag{23}
\]

To find the measure of coherence, let us first calculate the 
reduced density matrix of the quanton, given by

\[
\rho_s' = \sum_{i,j=1}^n \rho_{ij} \text{Tr} \left( |d_{ki}\rangle \langle d_{kj}| \right) |\psi_i\rangle \langle \psi_j| \tag{24}
\]

The coherence comes out to be

\[
C' = \frac{1}{n-1} \sum_{i \neq j} |\rho_{ij}| \sqrt{\sum_k r_k \langle d_{ki} | d_{kj} \rangle} \tag{25}
\]

Combining Eq. (23) and Eq. (25), we get

\[
D_Q' + C' + \frac{1}{n-1} \sum_{k \neq j} r_k \sum_{i \neq j} \sqrt{\rho_{ij} \rho_{jj} \langle \langle d_{kj} | d_{ki} \rangle \rangle} = 1 \tag{26}
\]

Every principal 2x2 sub matrix of (22) is positive semi-
definite [23], thus we have

\[
\sqrt{\rho_{ii} \rho_{jj} \langle \langle d_{kj} | d_{ki} \rangle \rangle} \geq 0 \tag{27}
\]

Therefore, we find that Eq. (26) reduces to

\[
D_Q' + C' \leq 1 \tag{28}
\]

III. ARE EXPERIMENTS WITH A QUANTUM DEVICE REALLY UNIQUE?

Two-path interference experiments with a quantum de-
vice have attracted lot of attention. But are these experi-
ments really unique? In this section we try to answer this 
question.

Let us consider the setup shown in FIG. 1. For sim-
plicity let us consider the beam-splitter to be in an equal 
superposition state \( |\psi\rangle = \frac{1}{\sqrt{2}} (|Y\rangle + |N\rangle) \). \( |Y\rangle \) represents the 
situation when BS1 is in the path, and \( |N\rangle \) when it is not. 
Let the quanton in the two paths be also in an equal su-
pereposition state \( |\psi\rangle = \frac{1}{\sqrt{2}} (|\psi_1\rangle + |\psi_2\rangle) \). The effect of BS1 
is to take \( |\psi_1\rangle \) \( , |\psi_2\rangle \) to \( |D_1\rangle \), \( |D_2\rangle \), the detector states of 
the two detectors \( D_1 \) and \( D_2 \), respectively. The transfor-
mation can be written as \( U_{\gamma} |\psi_1\rangle = \frac{1}{\sqrt{2}} (|D_1\rangle - |D_2\rangle) \) and
$U_Y|\psi_2\rangle = \frac{1}{\sqrt{2}}(|D_1\rangle + |D_2\rangle)$. If BS2 is absent, the transformation is as follows: $U_Y|\psi_1\rangle = |D_1\rangle$ and $U_Y|\psi_2\rangle = |D_2\rangle$. Using this, the effect of the superposed beam-splitter on the quanton can be written as follows:

$$U|\psi\rangle = \frac{1}{2} \left( (U_Y(|\psi_1\rangle + |\psi_2\rangle)|Y\rangle + U_N(|\psi_1\rangle + |\psi_2\rangle)|N\rangle \right) = (\frac{1}{\sqrt{2}}|Y\rangle + \frac{1}{2}|N\rangle)|D_1\rangle + \frac{1}{2}|N\rangle)|D_2\rangle. \quad (29)$$

The above relation implies that detectors $D_1$ and $D_2$ click with probability $3/4$ and $1/4$, respectively. Experimentally, this cannot be distinguished from the situation if BS1 were an ordinary 75:25 beam-splitter. The original proposal claimed that one can correlate the detected quanta with the $|Y\rangle$ and $|N\rangle$ states, and get wave or particle natures [13]. But even in doing that, at a time one can see either wave nature or particle nature. The same thing can be achieved by randomly removing BS2 from the quanton path.

Recognizing the fact that correlating with $|Y\rangle$ and $|N\rangle$ states was like a statistical effect, some authors suggested that the quanton be observed conditioned on detection of the state $|\phi_\alpha\rangle = \cos \alpha |Y\rangle + \sin \alpha |N\rangle$ [15, 17, 19]. The (unnormalized) state of the quanton in that situation will be

$$\langle \phi_\alpha |U|\psi\rangle = (\frac{1}{\sqrt{2}} \cos \alpha + \frac{1}{2} \sin \alpha)|D_1\rangle + \frac{1}{2} \sin \alpha |D_2\rangle. \quad (30)$$

There is nothing special about even this state. An appropriately biased BS2 will produce such a quanton state. There is no particular reason to call it a superposition of wave and particle nature.

What about a two-path interference experiment with a real two-state path-detecting device, which is in a superposition of being present and absent, one may ask. In the following, we will show even this experiment is completely equivalent to a two-path interference experiment with a real two-state path-detecting device, which is always present, and is not spooky in the sense that is being discussed here. Let us consider a two-path interference experiment with a which-way detector whose two states that correlate with the two paths of the quanton are not orthogonal to each other. The state of the quanton and path-detectator may be written as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle|d_1\rangle + |\psi_2\rangle|d_2\rangle), \quad (31)$$

where $|d_1\rangle|d_2\rangle \neq 0$. Now it can be shown that the states $|d_1\rangle, |d_2\rangle$ can be represented in terms of an expanded Hilbert space as follows [24, 25]

$$|d_1\rangle = \gamma |q_1\rangle + \beta |q_3\rangle$$
$$|d_2\rangle = \gamma |q_2\rangle + \beta |q_3\rangle, \quad (32)$$

where $|q_1\rangle, |q_2\rangle, |q_3\rangle$ are orthonormal states, and $\gamma, \beta$ are certain constants which we need not specify for the present purpose. In this basis, the entangled state (31) has the following form

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \gamma (|\psi_1\rangle|q_1\rangle + |\psi_2\rangle|q_2\rangle)$$
$$+ \frac{1}{\sqrt{2}} \beta (|\psi_1\rangle + |\psi_2\rangle)|q_3\rangle. \quad (33)$$

This state can be interpreted as a representation of a superposition of wave and particle nature. The quanton state correlated with $|q_3\rangle$ represents a quanton showing wave nature, and the rest showing particle nature. If one were to measure an observable $Q$ which has $|q_1\rangle, |q_2\rangle, |q_3\rangle$ as three eigenstates with distinct eigenvalues, the quantons detected in coincidence with $|q_3\rangle$ will show full interference, and those detected in coincidence with $|q_1\rangle, |q_2\rangle$ will show full particle nature. This state will show all the features that the state (4) can show, although it involves only a conventional path detector and not a quantum device. Such a state can also be produced without expanding the Hilbert space, but by introducing a two-state ancilla system interacting with the path-detector [26].

From this analysis, we conclude that although a lot of research interest was generated by the interference experiments with a quantum device, the effect they show can also be seen in conventional interference experiments.

IV. CONCLUSIONS

In conclusion, we have theoretically analyzed an $n$-path interference experiment where the path-detector is assumed to be spooky, meaning it can exist in a superposition of being present and absent from the interference path. We have shown that the $n$-path wave particle duality relation derived earlier [8] continues to hold even in this case. The duality relation remains tight even in the situation where there is expected to be interference between the wave and particle nature of the quanton. So, the various interference experiments, with a quantum device, may be of interest for various reasons but are completely within the realm of complementarity. We have also shown that the effects seen due to a spooky path detector, can also be seen in interference experiments with a conventional which-way detector. The effects seen by the quantum delayed choice experiment, i.e., without a real path detector, but with a spooky beam-splitter, can also be seen in a conventional Mach-Zehnder setup with a biased beam-splitter.

V. ACKNOWLEDGEMENTS

MAS acknowledges the National Key R&D Program of China, Grant No. 2018YFA0306703.
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