Numerical modeling of Vickers indentation on polycrystalline alumina

Zhonglei Liu¹, Chao Wang¹, Zhanying Chen¹, Yiming Rong⁴ and Xuekun Li¹,²,³,⁵

¹ Department of Mechanical Engineering, Tsinghua University, Beijing 100084, China
² Beijing Key Lab of Precision/Ultra-Precision Manufacturing Equipment and Control, Beijing 100084, China
³ State Key Lab of Tribology, Beijing 100084, China
⁴ Department of Mechanical and Energy Engineering, Southern University of Science and Technology, Shenzhen 518055, China
⁵ E-mail: xli@mail.tsinghua.edu.cn

Abstract. Indentation techniques are usually simple and effective methods to characterize ceramics properties. This article establishes a numerical model of Vickers indentation by using the general exponent Drucker-Prager model, which is suitable for materials whose compressive yield strength is much higher than tensile strength. In addition, a standard method for calculating the diagonal length of indentation is also proposed. Using this FE model, several numerical simulations of Vickers indentation process are conducted. Then, through measuring the indentation morphology with the proposed method, the Vickers hardness is calculated, which has excellent agreement with the experiment results. The comparison results of simulation and experiment show that the proposed numerical model is robust and effective. It is very promising in modeling the indentation process.

1. Introduction

High performance ceramics, represented by Al₂O₃, SiC, Si₃N₄, etc., have been widely used in some special occasions due to their excellent mechanical, physical and chemical properties [1, 2]. These properties will directly determine their service performance in practice. Due to the advantages of nondestructive (micro-loss) and convenience, the indentation method has been one of the main methods of testing the mechanical properties of materials for a long history. Compared with other testing methods, indentation method is simple to operate, convenient for sample preparation, and has low requirements for the test environment. Therefore, indentation techniques have been widely used on ceramics to characterize various mechanical properties. By analyzing the final geometry size of the indentation morphology and the applied load on the indenter, the hardness index of the material can be obtained. According to different shapes of the indenters, hardness test methods can be divided into Brignell, Vickers, Rockwell, Richter, Shore hardness, etc. [3]. During these measurement methods, the Vickers hardness test, due to its good testing repeatability and wide measurement range, has become one of the mostly used methods.

In these indentation test methods, it is of great importance to obtain accurate indentation morphology and size, which is the basic parameter to calculate the material properties [4, 5]. However, for some amorphous and polycrystalline materials, the geometry boundaries of the indentation are not clear, leading to the difficulty in measuring the sizes [6]. Then, the calculated hardness value based on
the geometry size will have a large deviation. Apart from this, for materials with high hardness like engineered ceramics, hardness testing would cause more or less damage to the instrument. Therefore, the numerical modeling of indentation is very important, and it is also the focus of indentation test research [7, 8].

In this paper, taking polycrystalline alumina as the test material, a numerical model of the Vickers indentation is established. Through using the simulated indentation morphology and the applied force, the Vickers hardness is calculated and then compared with the measurement result of the indentation experiment. The results prove the effectiveness and accuracy of the established model in simulating Vickers indentation morphology. Therefore, this model can be used as an effective supplement to identify the Vickers hardness of ceramic materials and provides a technical basis for testing other mechanical parameters.

2. Principle of Vickers hardness test
Vickers hardness measurement is usually performed on the special Vickers hardness instrument. The standard Vickers indenter is a regular quadrangular pyramid diamond indenter with the face angle of 136°. By loading the indenter at a certain speed and holding it for a period of time on the sample surface, a tetrahedral indentation will be left on the surface of the sample (Figure 1). Based on the applied force and the diagonal length of the tetrahedral indentation, Vickers hardness HV can be calculated by this formula:

\[
HV = \frac{P}{A} = \frac{2P \sin(\alpha/2)}{d^2}
\]

(1)

\(P\) is loading force, \(A\) is indentation area after unloading, \(\alpha = 136°\) is face angle, \(d\) is the diagonal length of the tetrahedral indentation.

![Figure 1. Schematic of Vickers indentation.](image)

3. Material modeling
In this paper, Drucker-Prager model [9] is employed to characterize plastic flow behaviour of polycrystalline alumina during indentation. The Drucker-Prager constitutive model, which originates from the Tresca and Von Mises criteria, is mainly used to describe materials such as soil and rocks whose yield strength is dependent on the pressure. It is also very suitable for materials whose compressive yield strength is much higher than tensile strength. It is for this reason that we selected it as the material model for ceramics. More specifically, the Exponent Drucker-Prager model (EDP) [10] is selected and calibrated for the 95% alumina.

3.1. The yield surface
The EDP model has a convex yield surface, and its function form is:

\[
F = aq^b - p - p_s = 0
\]

(2)
and \(b\) are material constants, \(q\) is the Mises equivalent stress, \(p\) is hydrostatic pressure, \(p_i\) is hardening parameter.

3.2. The plastic potential surface
The EDP model’s plastic potential function can be expressed as:
\[
G = \sqrt{(\varepsilon \bar{\sigma}_0 \tan \psi + q^2 - p \tan \psi)}
\]
(3)
\(\varepsilon\) is eccentricity, \(\bar{\sigma}_0\) is the initial yield stress, \(\psi\) is the dilation angle.

3.3. The hardening law
The hardening law is determined by the functional relation of the equivalent stress \(\bar{\sigma}\) and equivalent plastic strain \(\varepsilon_p\). The equivalent stress \(\bar{\sigma}\) can be defined as compressive strength \(c\), tensile strength \(t\), or the cohesive force \(d\). The equivalent plastic strain \(\varepsilon_p\) is defined as:
\[
d\varepsilon_p = \frac{\sigma \cdot d\varepsilon_p}{\bar{\sigma}}
\]
(4) \(\sigma\) is the Cauchy stress.

3.4. Identification of the material model constants
To the authors’ knowledge, there are no existing material constants for the EDP model of 95% Al\(_2\)O\(_3\) in the literatures. And it is time-consuming and high-cost to obtaining these constitutive parameters through mechanical experiments. Therefore, to save time and cost, we calibrated Drucker-Prager plasticity model through the Johnson-Holmquist-II (JH-2) model [11, 12], which is often used to characterize the dynamic response of ceramics.

JH-2 material model consists of the following three models [11]:

(1) State model. The equation of state (EOS) is defined by a polynomial of volume strain \(\mu\):
\[
P = K_1 \mu + K_2 \mu^2 + K_3 \mu^3 + \Delta P, \quad \text{if } \mu \geq 0
\]
\[
P = K_i \mu, \quad \text{if } \mu < 0
\]
(5) \(P\) is pressure, \(\Delta P\) is the incremental pressure caused by damage, \(K_1, K_2, K_3\) are material state parameters.

(2) Strength model. The normalized strength can be described by normalized intact strength \(\sigma_i^*\), normalized strength \(\sigma_j^*\) at complete destruction state and the damage variables \(D\)
\[
\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_j^*)
\]
(6) where, \(\sigma^* = \sigma/\sigma_{HEL}\), \(\sigma_i^* = \sigma_i/\sigma_{HEL}\), \(\sigma_j^* = \sigma_j/\sigma_{HEL}\), \(\sigma_{HEL}\) is the equivalent stress at the Hugoniot elastic limit (HEL).

The normalized stress strength \(\sigma_i^*\) and \(\sigma_j^*\) are:
\[
\sigma_i^* = A \left( P^* + T^* \right)^N \left( 1 + C \ln \varepsilon^* \right), \quad \sigma_j^* = B \left( P^* \right)^M \left( 1 + C \ln \varepsilon^* \right)
\]
(7) where, \(A, B, C, M, N\) are material parameters, \(P^*\) and \(T^*\) are the normalized hydrostatic pressure and the maximum tensile strength by the hydrostatic pressure when the material is at HEL: \(P^* = P/P_{HEL}\), \(T^* = T/P_{HEL}\).

(3) Damage model. Damage variable \(D\) is determined by equivalent strain \(\varepsilon_p^*\):
\[ D = \sum \frac{\Delta \varepsilon^p}{\varepsilon^p(P)} \]  

\( \Delta \varepsilon^p \) is equivalent plasticity strain increment, \( \varepsilon^p \) is equivalent plastic strain when the material is fully fractured.

The EDP model constants for 95% Al_2O_3 can be calculated from the JH-2 model constants of 95% Al_2O_3 in Literature [12] by using the method in Literature [11]. The yield surface of the general exponent Formula (2) can be recast into in the following form:

\[ q = \frac{1}{a^b} (p + p_i)^{1/b} \]  

(9)

The intact strength of JH-2 model can be recast into in the following form:

\[ \sigma_i = \frac{A\sigma_{HEL}^N}{P_{HEL}^N} (P + T)^N \]  

(10)

The two formulas have the same form, so the coefficients are correspondingly equal:

\[ a = \frac{P_{HEL}}{(A\sigma_{HEL})^{1/N}}, \quad b = \frac{1}{N}, \quad p_i = T \]  

(11)

The hydrodynamic pressure of alumina can be described as [11]:

\[ p = \frac{\rho_0 c_0^2}{(1 - \eta)^2} \]  

(12)

where, \( \rho_0 \) is reference density, \( c_0 \) and \( s \) are material state constants, \( \eta = \mu/(1 + \mu) \). Expanding this formula with respect to \( \mu \), the linear and quadratic coefficients should be respectively equal to \( K_1 \) and \( K_2 \) in JH-2 model, which gives:

\[ K_1 = \rho_0 c_0^2, \quad K_2 = \rho_0 c_0^2 (2s - 1) \]  

(13)

Using the Formulas (11) and (13), and according to the JH-2 material constants of 95% Al_2O_3, the EDP plasticity model parameters can be obtained, as shown in Table 1.

**Table 1.** Exponent Drucker-Prager material parameters of 95% Al2O3.

| Constants                      | Values         | Units      |
|--------------------------------|----------------|------------|
| Reference density \( \rho_0 \) | 3.741×10^{-9}  | Ton/mm^3   |
| Material yield constant \( a \)| 7.082×10^{-2}  | Mpa^{-1,0} |
| Material yield constant \( b \)| 1.3089         |            |
| Uniaxial compressive strength \( \sigma_c \) | 2076.48        | MPa        |
| Material state constant \( c_0 \) | 7.024×10^{-6}  | mm/s       |
| Material state constant \( s \) | 1.004          |            |

**4. Finite element modeling**

**4.1. Geometric model**

The Vickers indentation process is a nonlinear process with large deformation. Therefore, Dassault’s Abaqus software is adopted to build the numerical model of the Vickers indentation, for it is ideal for nonlinear calculations. Firstly, a three-dimensional model is built in Abaqus/CAE. According to the symmetry of the model, a quarter model is adopted in order to reduce the calculation cost.
constraint is imposed on the bottom of the model and symmetry constraint is imposed on the split face. In order to improve the analysis efficiency without reducing the accuracy, the part near the tip of the indenter is meshed finely with meshes of 0.001mm, while the meshes of other part away from the tip are relatively coarse, about 0.02mm. Furthermore, the specimen is discretized by the structured meshes and C3D8 element type is used and the indenter is discretized by the unstructured meshes and C3D4 element type is used. Finally, the total number of elements is 216053. Figure 2 shows the whole model and partitions, while Figure 3 shows one-quarter model and its meshes. In order to characterize the behavior of the material correctly, it is necessary to edit the input file to work with this geometric model, as shown in Table 2, the relevant data of which correspond to the data in Table 1.

![Figure 2. The whole model and partitions.](image)

![Figure 3. One-quarter model and meshes.](image)

| Table 2. EDP model for 95% Al₂O₃ in Abaqus input file. |
|---------------------------------------------------------|
| *Material, name=alumina_drucker_prager                   |
| *DENSITY                                                |
| 3.741e-9                                               |
| *EOS, TYPE=USUP                                        |
| 7023845, 1.0035, 0.                                       |
| *ELASTIC, TYPE=SHEAR                                    |
| 120340,                                                 |
| *DRUCKER PRAGER, shear criterion=exponent              |
| 7.082325e-2,1.3089, 0.0                                  |
| *DRUCKER PRAGER HARDENING, type=COMPRESSION            |
| 2076.48                                                 |

4.2. Diagonal measurement method

The key to calculate the Vickers hardness is to obtain accurate diagonal length of indentation. However, up to now, there is no uniform method and standard in the finite element software for it. In addition, the Abaqus software has no special function to generate this value automatically. Therefore, we propose a specific method, which is very easy to operate, to measure the diagonal length in Abaqus. The method is as follows:

1. After the indenter is completely unloaded, create a “path” (Figure 4) along the intersecting line of the diagonal plane and the deformed surface of the specimen;
2. Create the “XY date” from the “path” created in step 1 (Figure 5). The X value is chosen to be “True distance”, and the value of Y is the node displacement along the indentation depth;
3. The starting point of the half diagonal is the centre point, while the end point should be the first point on the path outward from the centre point where the displacement is 0. Then the true distance between the starting and end point is half of the diagonal length.
4.3. Results under different friction coefficients

Friction coefficient is an important physical parameter that may affect the calculation result. We conduct numerical experiments with different friction coefficients varying from 0 to 0.5 and compare the calculation results, which are shown in Table 3. It can be concluded from the results in the table that the friction coefficient has little influence on the diagonal lengths. Its influence is mainly reflected on the maximum applied load, though very small. In spite of this, the difference between the maximum value (11260.22MPa) and minimum value (10919.45MPa) is only 3.12%. This proves that the established model is robust and insensitive to the friction coefficient. The average simulation Vickers hardness is 11193.36MPa. For further validation of the simulation’s correctness, the actual indentation tests are carried out in next section.

Table 3. Vickers hardness under different friction coefficients.

| Number | Friction coefficient | Maximum indentation depth (µm) | Maximum load (N) | Diagonal d/2 (µm) | Vickers Hardness (HV/MPa) |
|--------|----------------------|--------------------------------|-----------------|-----------------|-------------------------|
| 1      | 0                    | 5                              | 6.0826          | 16.07           | 10919.45                |
| 2      | 0.1                  | 5                              | 6.1852          | 15.97           | 11243.13                |
| 3      | 0.2                  | 5                              | 6.2016          | 15.99           | 11244.76                |
| 4      | 0.3                  | 5                              | 6.2109          | 16.00           | 11247.55                |
| 5      | 0.4                  | 5                              | 6.1940          | 15.98           | 11245.04                |
| 6      | 0.5                  | 5                              | 6.1946          | 15.97           | 11260.22                |

AVERAGE VICKER HARDNESS 11193.36

5. Experiment validation

In order to verify the simulation results, a series of Vickers hardness experiments are conducted on the HV-1000 Microhardness Tester (Figure 6). The sample material is 95% alumina ceramic. The sample is fabricated by slip casting and sintered at 1500°C. To facilitate the experiment and subsequent measurement, the sample is machined into a size of 15×15×10mm (Figure 7). Furthermore, in order to improve the measurement accuracy, the sample surface is polished. The sample is firstly roughly and finely ground with 3000# and 5000# sandpapers, and then polished in turn by polishing cloth with 1.5 and 0.05µm diamond powders until it is the same as the mirror surface, and finally cleaned by ultrasonic cleaning. After these preparations are completed, the experiment is ready to begin.
Firstly, the applied load on the indenter is gradually improved to a predetermined value (1Kgf) at a speed of 0.5N/s. During this process, the indenter is gradually penetrated into the sample. After reaching the preset load, the indenter is hold on for 10s, then returns to the origin at the same speed. As a result, an impression will form on the sample surface. In order to observe and measure the indentation more clearly, the sample’s morphology is observed with Zeiss Merlin Compact Scanning Electron Microscope (SEM, shown in Figure 8). The sample surface is not uniform, with many pores and crystals of various sizes (shown in Figure 9). This brings considerable troubles to the experiment and diagonal dimension measurement. The diagonal boundary is not obvious, which will cause measurement error to some extent. What’s more, the vertices of the diagonal are still determined by the eye, which will further add to the error. After the vertices of the diagonals are determined, the lengths of the two diagonals are calculated from the diagram using the scale of the picture. Using the two diagonal lengths and applied load, the hardness of 95% Al₂O₃ can be calculated according to Formula (1). In order to eliminate the measurement errors as much as possible, 6 tests are conducted at different locations under the same applied load. The hardness values obtained from the tests are shown in Table 4.

![The polished surface](image1)

**Figure 6.** HV-1000 microhardness tester.

![The polished surface](image2)

**Figure 7.** The sample with polished surface.

![Zeiss Merlin compact SEM](image3)

**Figure 8.** Zeiss Merlin compact SEM.

![Vickers indentation on the surface](image4)

**Figure 9.** Vickers indentation on the surface.

It can be seen from the Table 4 that the average hardness of the six measurement results is 10400.62Mpa. The error between the simulation result (11193.36MPa) and the experimental result (10400.62) is 7.62%, which provides strong evidence for the effectiveness of the proposed model. Apart from this, due to the inhomogeneity of the polycrystalline alumina surface, the difference between the maximum value (11242.54MPa) and minimum value (9649.744MPa) is up to 16.51%,
which is much higher than 3.12% for the simulation results. This further demonstrates the stability of the numerical model in identifying mechanical properties.

Table 4. Measurement results of 95% Al₂O₃ Vickers hardness.

| Number | Load (Kgf) | Diagonal \( d_1 \) (µm) | Diagonal \( d_2 \) (µm) | Vickers Hardness (HV/MPa) |
|--------|------------|-----------------|-----------------|-----------------|
| 1      | 1          | 41.470          | 45.352          | 9649.744        |
| 2      | 1          | 42.352          | 46.411          | 9232.373        |
| 3      | 1          | 42.882          | 42.352          | 10012.69        |
| 4      | 1          | 42.529          | 37.941          | 11233.32        |
| 5      | 1          | 40.86           | 40.34           | 11033.07        |
| 6      | 1          | 38.62           | 41.82           | 11242.54        |

AVERAGE VICKER HARDNESS 10400.62

6. Conclusions
In the present work, FE model of the Vickers indentation for 95% Al₂O₃ is established by using the general exponent form of extend Drucker-Prager model, whose material model constants are calibrated from the JH-2 model of 95% Al₂O₃. In order to calculate the Vickers hardness accurately, a standard method for measuring the indentation diagonal length in Abaqus is proposed. This method overcomes the problem of unclear indentation boundary of polycrystalline ceramics. The simulation results of Vickers hardness based on the above work are proved by actual indentation tests. What’s more, compared with the experimental results, the simulation results have better consistency. The proposed model in this paper could be treated as an effective candidate for the Vickers hardness measurement of ceramics and it also provided a technical basis for modeling other indentation process and testing other mechanical parameters.

Acknowledgments
The research is supported by The Science and Technology Major Project of China 2017ZX04007001. And the support from the Project SKLT2020B02-Research on precision grinding technology of high surface consistency curve roll is also appreciated.

References
[1] Jin Z H, Gao J Q and Qiao G J 2000 Engineering Ceramic Materials (Xi'an: Xi'an Jiaotong University Press)
[2] Guo J K, Kou H M and Li J 2011 Research on High Temperature Structural Ceramics (Beijing: Beijing Science Press)
[3] Tabor D 1948 A simple theory of static and dynamic hardness Proc R Soc Lond A Math Phys Sci 192 247-274
[4] Evans A G and Charles E A 1976 Fracture toughness determination by indentation J Am Ceram Soc 59 371-376
[5] Anstis G R, Chantikul P, Lawn B R and Marshall D B 1981 A critical evaluation of indentation techniques for measuring fracture toughness I: direct crack measurements J Am Ceram Soc 64 533-538
[6] Wang J L, Ma D J, Chen W, Huang Y and Bai M L 2015 Simulation and Experimental Analysis on Vickers Indentation Morphology of Ceramic Materials J Mater Eng 43 71-76
[7] Liu M, Lin J Y, Lu C, Tieu K A, Zhou K and Koseki T 2017 Progress in indentation study of materials via both experimental and numerical methods Crystals 7 258
[8] Lee J H, Gao Y F, Johanns K E and Pharr G M 2012 Cohesive interface simulations of
indentation cracking as a fracture toughness measurement method for brittle materials \textit{Acta Mater} \textbf{60} 5448-5467

[9] Drucker D C and Prager W 1952 Soil mechanics and plastic analysis of limit design \textit{Q Appl Math} \textbf{10} 157-165

[10] Fei K and Zhang J W 2010 \textit{Application of ABAQUS in geotechnical engineering} (Beijing: China WaterPower Press)

[11] ABAQUS 2016 \textit{ABAQUS Documentation} (Dassault Systemes, Providence, RI, USA)

[12] Yang Z Q, Pang B J, Wang L W and Chi R Q 2010 JH-2 model and its application in numerical simulation of low speed impact of Al$_2$O$_3$ ceramic \textit{Explos Shock Waves} \textbf{30} 18-26