Testing Kerr Black Hole Mimickers with Quasi-Periodic Oscillations from GRO J1655-40

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The measurements of quasi-periodic oscillations (QPOs) provide a quite powerful tool to test the nature of astrophysical black hole candidates in the strong gravitational field regime. In this paper, we use QPOs within the relativistic precession model to test a recently proposed family of rotating black hole mimickers, which reduce to the Kerr metric in a limiting case, and can represent traversable wormholes or regular black holes with one or two horizons, depending on the values of the parameters. In particular, assuming that the compact object of GRO J1655-40 is described by a rotating black hole mimicker, we perform a $\chi^2$-square analysis to fit the parameters of the mimicker with two sets of observed QPO frequencies from GRO J1655-40. Our results indicate that although the metric around the compact object of GRO J1655-40 is consistent with the Kerr metric, a regular black hole with one horizon is favored by the observation data of GRO J1655-40.

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I. INTRODUCTION

The first observations of gravitational waves by LIGO [1] and the first image of a black hole in the galaxy M87 [2] have ushered us into a new era of testing general relativity (GR) in the strong gravity regime. On the other hand, quasi-periodic oscillations (QPOs) are observed in the X-ray flux from black hole and neutron star X-ray binary systems, and detected as narrow peaks in the power density spectrum [3]. QPOs are believed to be associated with motion and accretion-related timescales in a region of order the Schwarzschild radius around the compact object, which makes QPOs excellent probes of the strong gravitational field regime [4–8]. In black hole systems, the observed frequencies of QPOs range from mHz to hundreds of Hz. While low frequency QPOs ($\lesssim$ 30 Hz) are commonly observed from black hole X-ray binaries [9], high frequency QPOs ($\gtrsim$ 60 Hz) are very rare. In fact, the Rossi X-ray Timing Explorer (RXTE), operational between 1996 and 2012, first detected high frequency QPOs in black hole systems [10, 11]. Interestingly, a pair of simultaneous high-frequency QPOs was first discovered in the X-ray flux from GRO J1655-40 by RXTE [12]. It was noted that the frequencies of the two high-frequency QPOs are in a 3:2 ratio, suggesting a resonance between orbital and epicyclic motion of accreting matter near the innermost stable circular orbit (ISCO) of black holes [13]. Later, three simultaneous QPO frequencies, consisting of two higher frequencies and one lower frequency, were also observed from the X-ray data of GRJ1655-40 [14].

On the theoretical side, various models have been proposed to explain QPOs by relating them to the orbital and epicyclic frequencies of geodesics, such as the relativistic precession model (RPM) [15], the tidal disruption model...
[16], the parametric resonance model [17], the resonance model [13, 18, 19], the warped disk oscillation model [20] and the non-axisymmetric disk oscillation model [21, 22]. With these models, the observation data of QPO frequencies have been used to constrain the parameters of X-ray binary systems and test the nature of various gravity theories [23–37].

Particularly, it showed that the X-ray data of GRO J1655-40, especially the QPO triplet, fit nicely in the RPM, and the mass and spin of the compact object of GRO J1655-40 can be precisely determined [14]. Remarkably, the inferred mass is in great agreement with the dynamical mass measurement [23]. The RPM was originally proposed to explain QPOs in low-mass X-ray binaries with a neutron star [38], and later extended to systems with stellar-mass BH candidates [15]. In the RPM, QPO frequencies are assumed to be related to fundamental frequencies of a test particle orbiting a central object. The twin higher frequencies are regarded as the azimuthal frequency \( \nu_\phi \) and the periapsis precession frequency \( \nu_{\text{per}} \) of quasi-circular orbits in the innermost disk region, respectively. The low-frequency QPO is identified as the nodal precession frequency \( \nu_{\text{nod}} \), which is emitted at the same radius where the twin higher frequencies are generated.

On the other hand, curvature singularities can be formed during a gravitational collapse. It is commonly believed that singularities can be avoided through quantum gravitational effects. Consequently, since Bardeen proposed the first regular black hole [39], constructing and studying classical black holes without singularities have been a topic of considerable interest in GR and astrophysics communities due to their non-singular property [40–47]. Recently, Simpson and Visser proposed a static and spherically symmetric regular spacetime described by the line element (dubbed the SV metric henceforth),

\[
ds^2 = - \left( 1 - \frac{2M}{\sqrt{r^2 + \ell^2}} \right) dt^2 + \left( 1 - \frac{2M}{\sqrt{r^2 + \ell^2}} \right)^{-1} dr^2 + (r^2 + \ell^2) \left( d\theta^2 + \sin^2 \theta d\phi^2 \right),
\]

where \( M \geq 0 \) represents the ADM mass, and \( \ell > 0 \) is a parameter responsible for regularizing the center singularity [48–50].

In this paper, we test gravity with QPOs frequencies observed from GRO J1655-40 within the RPM for the rotating SV metric. The content of this paper is as follows. After we briefly review the RPM and the rotating SV metric in sections II and III, respectively, the epicyclic frequencies of the rotating SV are computed in section III. In section IV, we use the data of GRO J1655-40 to put constraints on the parameters of the rotating SV metric. Section V is devoted to our conclusions. Throughout the paper, we use units in which \( G = c = 1 \).

II. EPICYCLIC FREQUENCIES

In this section, we consider the timelike geodesic equations in a stationary and axially symmetric spacetime, and then derive the expressions of the epicyclic frequencies. The metric of a stationary and axially symmetric spacetime which satisfies the circularity condition is given by [61]

\[
ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + 2g_{t\phi} dtd\phi + g_{\theta\theta} d\theta^2 + g_{\phi\phi} d\phi^2,
\]

where the metric \( g_{\mu\nu} \) is a function of \( r \) and \( \theta \), and we drop the coordinate dependence of the metric functions to simplify the notation. For a massive particle travelling along a time-like world line \( z^\mu (\tau) \) with \( \tau \) being the proper time, the four-velocity \( U^\mu \) is defined by \( U^\mu = \dot{z}^\mu = dx^\mu / d\tau \), which satisfies \( U^\mu U_\mu = -1 \). Due to the stationarity and axisymmetry, the metric (2) admits two Killing vectors,

\[
K^\mu = (\partial_\tau)^\mu = (1, 0, 0, 0) \quad \text{and} \quad R^\mu = (\partial_\phi)^\mu = (0, 0, 0, 1).
\]

The two Killing vectors correspond to two conserved quantities of geodesic motion,

\[
E = -K_\mu \frac{dx^\mu}{d\tau} = -g_{tt} \dot{t} - g_{t\phi} \dot{\phi},
\]

\[
L_z = R_\mu \frac{dx^\mu}{d\tau} = g_{t\phi} \dot{t} + g_{\phi\phi} \dot{\phi},
\]

(4)
which can be interpreted as the energy per unit mass and the angular momentum per unit mass along the axis of symmetry, respectively. In terms of $E$ and $L_z$, one can express $\dot{t}$ and $\dot{\phi}$ as

$$\dot{t} = \frac{g_{\phi\phi}E + g_{t\phi}L_z}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad \dot{\phi} = \frac{g_{tt}E + g_{t\phi}L_z}{g_{tt}g_{\phi\phi} - g_{t\phi}^2}. \quad (5)$$

Using the above equations, we can rewrite $U^\mu U_\mu = -1$ as

$$g_{rr}\dot{r}^2 + 2g_{r\theta}\dot{r}\dot{\theta} + g_{\theta\theta}\dot{\theta}^2 = V_{\text{eff}}(\theta, \phi), \quad (6)$$

where the effective potential $V_{\text{eff}}(r, \theta)$ is defined as

$$V_{\text{eff}}(r, \theta) = \frac{E^2 g_{\phi\phi} + 2EL_zg_{t\phi} + L_z^2g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}} - 1. \quad (7)$$

We consider a circular geodesic at $r = \bar{r}$ on the equatorial plane with $\theta = \pi/2$, which means that the effective potential may develop a double root at $r = \bar{r}$ on the equatorial plane, i.e., $V_{\text{eff}}(\bar{r}, \pi/2) = \partial_r V_{\text{eff}}(\bar{r}, \pi/2) = 0$. Along the circular orbit, the angular velocity of the particle measured by an observer at infinity is defined by

$$\Omega_\phi = \frac{d\phi}{dt} = \frac{\dot{\phi}}{\dot{t}} = \frac{-g_{tt}E + g_{t\phi}L_z}{g_{t\phi}E + g_{t\phi}L_z} \bigg|_{r=\bar{r}, \theta=\pi/2}. \quad (8)$$

Solving $V_{\text{eff}}(\bar{r}, \pi/2) = 0$ with eqn. (8) gives the specific energy and angular momentum of the particle,

$$E = -\frac{g_{tt} + g_{t\phi}\Omega_\phi}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega_\phi - g_{\phi\phi}\Omega_\phi^2}} \bigg|_{r=\bar{r}, \theta=\pi/2}, \quad (9)$$

$$L_z = \frac{g_{t\phi} + g_{\phi\phi}\Omega_\phi}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega_\phi - g_{\phi\phi}\Omega_\phi^2}} \bigg|_{r=\bar{r}, \theta=\pi/2}. \quad (9)$$

Solving $\partial_r V_{\text{eff}}(\bar{r}, \pi/2) = 0$ for $\Omega_\phi$, one obtains the angular velocity of the particle,

$$\Omega_\phi = -\frac{\partial_r g_{t\phi} \pm \sqrt{(\partial_r g_{t\phi})^2 - (\partial_r g_{t\phi})(\partial_r g_{\phi\phi})}}{\partial_r g_{\phi\phi}} \bigg|_{r=\bar{r}, \theta=\pi/2}. \quad (10)$$

where the sign $+/-$ corresponds to a prograde/retrograde orbit. The stability of the circular orbit is determined by the sign of $\partial^2_r V_{\text{eff}}(\bar{r}, \pi/2)$, i.e., $\partial^2_r V_{\text{eff}}(\bar{r}, \pi/2) > 0 \leftrightarrow$ unstable and $\partial^2_r V_{\text{eff}}(\bar{r}, \pi/2) < 0 \leftrightarrow$ stable. The transition between stable and unstable circular orbits, which is determined by $\partial^2_r V_{\text{eff}}(\bar{r}_{\text{ISCO}}, \pi/2) = 0$, is the ISCO, which is located at $r = r_{\text{ISCO}}$ on the equatorial plane.

To derive the epicyclic frequencies associated with the circular orbit, we consider small perturbations of the orbit in both the radial and the vertical directions,

$$r(t) = \bar{r} + \delta r(t), \quad \theta(t) = \frac{\pi}{2} + \delta \theta(t). \quad (11)$$

Inserting eqn. (11) into eqn. (6) yields the differential equations for the perturbations $\delta r(t)$ and $\delta \theta(t)$,

$$\frac{d^2\delta r(t)}{dt^2} + \Omega_r^2 \delta r(t) = 0, \quad \frac{d^2\delta \theta(t)}{dt^2} + \Omega_\theta^2 \delta \theta(t) = 0, \quad (12)$$

where the frequencies of the oscillations are

$$\Omega_r^2 = -\frac{1}{2g_{rr}t^2} \frac{\partial^2 V_{\text{eff}}(\theta, \phi)}{\partial t^2} \bigg|_{r=\bar{r}, \theta=\pi/2}, \quad \Omega_\theta^2 = -\frac{1}{2g_{\theta\theta}t^2} \frac{\partial^2 V_{\text{eff}}(\theta, \phi)}{\partial \theta^2} \bigg|_{r=\bar{r}, \theta=\pi/2}. \quad (13)$$

We then define $\nu_\phi = \Omega_\phi/2\pi$, $\nu_r = \Omega_r/2\pi$ and $\nu_\theta = \Omega_\theta/2\pi$ as the azimuthal, radial and vertical epicyclic frequencies, respectively. The periastron precession frequency $\nu_{\text{per}}$ and the nodal precession frequency $\nu_{\text{nod}}$ are defined by $\nu_{\text{per}} = \nu_\phi - \nu_r$ and $\nu_{\text{nod}} = \nu_\phi - \nu_\theta$, respectively.
III. ROTATING SIMPSON-VISSER METRIC

In [58], a rotating generalization of the static and spherically symmetric metric (1) has been constructed by employing the Newman–Janis procedure [62]. This stationary and axially symmetric metric can describe a rotating traversable wormhole and a rotating regular black hole with one or two horizons. In particular, the rotating SV metric reads

$$ds^2 = -\left(1 - \frac{2M\sqrt{r^2 + \ell^2}}{\Sigma}\right)dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 - \frac{4Ma\sin^2\theta\sqrt{r^2 + \ell^2}}{\Sigma}dtd\phi + \frac{A\sin^2\theta}{\Sigma}d\phi^2,$$

(14)

with

$$\Sigma = r^2 + \ell^2 + a^2 \cos^2\theta,$$

$$\Delta = r^2 + \ell^2 + a^2 - 2M\sqrt{r^2 + \ell^2},$$

$$A = (r^2 + \ell^2 + a^2)^2 - \Delta a^2 \sin^2\theta,$$

(15)

where $a$ is the spin parameter. The rotating SV metric will reduce to the SV metric (1) if $a = 0$ and to the Kerr metric if $\ell = 0$. Interestingly, the rotating SV metric is everywhere regular when $\ell > 0$ [58].

The horizons of the rotating SV metric are determined by $\Delta = 0$, whose solutions are

$$r_{\pm} = \sqrt{(M \pm \sqrt{M^2 - a^2})^2 - \ell^2}.$$

(16)

As shown in [58], the phases of the rotating SV metric are determined by the existence of $r_{\pm}$. Specifically, the rotating SV metric represents

- a traversable wormhole: $M < a$ or $l > M + \sqrt{M^2 - a^2}$;
- a regular black hole with one horizon (RBH-I): $M - \sqrt{M^2 - a^2} < l < M + \sqrt{M^2 - a^2}$ and $M > a$;
- a regular black hole with two horizons (RBH-II): $l < M - \sqrt{M^2 - a^2}$ and $M > a$;
- there limiting cases: a one-way wormhole with a null throat when $l = M + \sqrt{M^2 - a^2}$, a regular black hole with one horizon and a null throat when $l = M - \sqrt{M^2 - a^2}$ and an extremal regular black hole when $M = a$. 

FIG. 1. Plots of the azimuthal epicyclic frequency $\nu_\phi$ (left panel), the periastron precession frequency $\nu_{\text{per}}$ (middle panel) and the nodal precession frequency $\nu_{\text{nod}}$ (right panel) of prograde (solid lines) and retrograde (dashed lines) orbits at $\bar{r} = 6$ as a function of $\ell$ for $a = 0$ (red lines), $a = 0.4$ (green lines) and $a = 0.8$ (blue lines) in the rotating SV metric with $M = 1$. The $a = 0$ case corresponds to the SV metric, which has $\nu_{\text{nod}} = 0$ due to spherical symmetry. Except $\nu_{\text{nod}}$ with $a = 0$, the magnitudes of $\nu_\phi$, $\nu_{\text{per}}$ and $\nu_{\text{nod}}$ decrease as $\ell$ grows with a fixed $a$. For the retrograde orbits in the rotating SV metric with $a > 0$, the values of $\nu_{\text{nod}}$ are shown to be negative.
For a circular orbit at \( r = \bar{r} \) on the equatorial plane, substituting the rotating SV metric (14) into eqns. (8) and (13) gives epicyclic frequencies in the rotating SV metric,

\[
\nu_\phi = \frac{1}{2\pi} \frac{M^{1/2}}{(\bar{r}^2 + \ell^2)^{3/4} + a M^{1/2}},
\]

\[
\nu_r = \frac{r \nu_\phi}{(\bar{r}^2 + \ell^2)^{1/2}} \sqrt{1 - \frac{6M}{(\bar{r}^2 + \ell^2)^{1/2}} - \frac{3a^2}{(\bar{r}^2 + \ell^2)^{1/2}} \pm \frac{8a M^{1/2}}{(\bar{r}^2 + \ell^2)^{3/4}}},
\]

\[
\nu_\theta = \nu_\phi \sqrt{1 \mp \frac{4a M^{1/2}}{(\bar{r}^2 + \ell^2)^{3/4}} + \frac{3a^2}{(\bar{r}^2 + \ell^2)}},
\]

which can be used to constrain the four parameters \( \ell, r, M, \) and \( a \) of the rotating SV metric. Here, the top/bottom row of the ± and \( \mp \) signs corresponds to the orbit co-rotating/counter-rotating with the spacetime. To illustrate the dependence of the epicyclic frequencies on \( \ell \), we plot \( \nu_\phi, \nu_{\text{per}} \) and \( \nu_{\text{nod}} \) as a function of \( \ell \) for various values of \( a \) in FIG. 1, where \( M = 1 \) and \( \bar{r} = 6 \). The prograde and retrograde cases are represented by solid and dashed lines, respectively. The left panel shows that, for a fixed \( a \), the azimuthal epicyclic frequency \( \nu_\phi \) decreases with \( \ell \) increasing in both prograde and retrograde cases, whereas \( \nu_\phi \) of the prograde orbit is smaller than that of the retrograde orbit. The periastron precession frequency \( \nu_{\text{per}} \) is displayed in the middle panel, and also decreases as \( \ell \) increases for both prograde and retrograde orbits. Like \( \nu_\phi \), the retrograde orbits have larger \( \nu_{\text{per}} \). Note that retrograde circular orbits of radius \( \bar{r} = 6 \) do not exist when \( \ell \) is small enough. However, as shown in the right panel, the nodal precession frequency \( \nu_{\text{nod}} \) of the prograde/retrograde orbits decreases/increases as \( \ell \) increases for a given \( a \). More interestingly, when \( a > 0 \), \( \nu_{\text{nod}} \) of the retrograde orbits is negative while that of the prograde orbits is positive. Finally, it is noteworthy that the ISCO radius \( r_{\text{ISCO}} \) is determined by [58],

\[
r^2_{\text{ISCO}} + \ell^2 - 6M \sqrt{r^2_{\text{ISCO}} + \ell^2} \pm 8a \sqrt{M \sqrt{r^2_{\text{ISCO}} + \ell^2} = 3a^2},
\]

where \( +/\sim \) is associated with the prograde/retrograde ISCO.

IV. CONSTRAINING ROTATING SIMPSON-VISSER METRIC BY QUASI-PERIODIC OSCILLATIONS

In this section, we use the RPM along with the QPO frequencies from GRO J1655-40 to put constraints on the parameters of the rotating SV metric. GRO J1655-40 is an X-ray binary, consisting of a primary star and a compact companion [63]. The measurement of the X-ray spectrum was found to exhibit type-C low-frequency QPOs and simultaneous high-frequency QPOs, which are observed in pairs and therefore dubbed lower and upper high-frequency QPOs [12, 14]. In particular, we consider two sets of QPOs with the observed frequencies based on the RXTE observations [14],

\[
\begin{align*}
\nu_{1U} &= 441 \text{ Hz}, \quad \sigma_{1U} = 2 \text{ Hz}, \\
\nu_{1L} &= 298 \text{ Hz}, \quad \sigma_{1L} = 4 \text{ Hz}, \\
\nu_{1C} &= 17.3 \text{ Hz}, \quad \sigma_{1C} = 0.1 \text{ Hz}
\end{align*}
\]

and

\[
\begin{align*}
\nu_{2U} &= 451 \text{ Hz}, \quad \sigma_{2U} = 5 \text{ Hz}, \\
\nu_{2L} &= - \text{ Hz}, \\
\nu_{2C} &= 18.3 \text{ Hz}, \quad \sigma_{2C} = 0.1 \text{ Hz}.
\end{align*}
\]

In the RPM, three simultaneous QPO frequencies are generated at the same radial coordinate in the accretion disk. The upper high-frequency QPOs correspond to the azimuthal epicyclic frequency \( \nu_\phi \), the lower high-frequency QPOs to the periastron precession frequency \( \nu_{\text{per}} \), and the low-frequency QPOs to the nodal precession frequency \( \nu_{\text{nod}} \). Moreover, it is reasonable to assume that the above two sets of QPOs result from two circular orbits of different radii, i.e., \( r_1 \) and \( r_2 \). In short, we have five free parameters: the mass \( M \), the spin parameter \( a \), the \( \ell \) parameter, and the radii \( r_1 \) and \( r_2 \) corresponding to the QPOs with three frequencies and two frequencies, respectively. To obtain
the observed frequencies of the QPOs in GRO J1655-40 are consistent with a Kerr black hole (the rotating SV metric with \( \ell = 0 \)), but they also allow for large deviations from the Kerr black hole solution. In particular, a regular black hole with one horizon is favored over the other phases of the rotating SV metric, e.g., a regular black hole with two horizons and a traversable wormhole.

We present the best estimate and the 1\( \sigma \), 2\( \sigma \) and 3\( \sigma \) confidence levels (C.L.) of the spin parameter \( a/M \), the mass \( M/\odot \), the ratio of angular momentum \( \ell/M \) and the radius \( r/\odot \) of the ISCO, occurring at the best estimate of \( M, a, \ell \), and \( r \) and \( r_2 \). The range of the parameters at a confidence level (C.L.) is determined by the interval \( \chi^2_{\text{min}} + \Delta \chi^2 \). In the case of five degrees of freedom, the intervals with \( \Delta \chi^2 = 5.89, 11.29 \) and 17.96 correspond to 68.3\%, 95.4\% and 99.7\% C.L., respectively, which are the probability intervals designated as 1, 2, and 3 standard deviation limits, respectively.

Computing \( \chi^2 \), we find \( \chi^2_{\text{min}} = 0.195 \) and obtain the best fits of the parameters of the rotating SV metric within 68.3\% credibility,

\[
\begin{align*}
M/\odot &= 5.305^{+0.041}_{-0.028},
\quad a/M = 0.286^{+0.003}_{-0.002},
\quad \ell/M = 0.347^{+0.01}_{-0.034}.
\end{align*}
\]

Note that our result is consistent with the measurement of the mass \( M \) by optical and infrared observations, which give \( M = 5.4 \pm 0.3 \odot \) [23]. We present the best estimate and the 1\( \sigma \), 2\( \sigma \) and 3\( \sigma \) contour levels of \( M/\odot, a/M \) and \( \ell/M \) in FIG. 2, where the yellow/blue regions represent the RBH-I/RBH-II phases of the rotating SV metric. As shown in the left and middle panels, while the hypothesis that the compact object of GRO J1655-40 is described by a Kerr black hole is consistent with the interpretation of the QPOs’ data in the RPM, significant deviations from the Kerr metric are allowed. In fact, the best-fit values are in the parametric region of the RBH-I phase, and the regions within 1-, 2- and 3-standard deviation limits are almost in the RBH-I region. Therefore, the observation of GRO J1655-40 favors a regular black hole with one horizon if the compact object of GRO J1655-40 is described by the rotating SV metric. The best-fit values of the radii of the circular orbits associated with the two sets of QPOs are found to be \( r_1 = 5.669M = 1.1304\text{r}_{\text{ISCO}} \) and \( r_2 = 5.563M = 1.1094\text{r}_{\text{ISCO}} \), respectively, where \( \text{r}_{\text{ISCO}} = 5.105M \) is the innermost stable circular orbit evaluated for the rotating SV metric with the best-fit values (22). Consequently, the two circular orbits responsible for generating the two sets of QPOs lie in the close vicinity of the ISCO, and hence are in the strong-field region of the rotating SV metric.

### V. CONCLUSIONS

In this paper, we explored potential deviations from the GR predictions of astrophysical black holes using QPOs observed in the power density spectrum of GRO J1655-40. Specially, we modelled the spacetime around the compact object of GRO J1655-40 by the rotating SV metric, and interpreted the observed QPOs within the RPM, which relates...
the QPO frequencies to epicyclic frequencies of geodesics. The rotating SV metric reduces to a Kerr black hole in the limit of $\ell = 0$, and possesses multiple phases, e.g., a regular black hole with one or two horizons and a traversable wormhole. To test the nature of the the compact object of GRO J1655-40, we performed a $\chi^2$ analysis by fitting the QPO frequencies computed in the RPM with the observations of two sets of QPOs from GRO J1655-40. Our results show a preference towards a regular black hole with one horizon compared to the Kerr black hole predicted in GR.

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[1] B.P. Abbott et al. Observation of Gravitational Waves from a Binary Black Hole Merger. Phys. Rev. Lett., 116(6):061102, 2016. arXiv:1602.03837. doi:10.1103/PhysRevLett.116.061102.
[2] Kazunori Akiyama et al. First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole. Astrophys. J. Lett., 875:L1, 2019. arXiv:1906.11238. doi:10.3847/2041-8213/ab0ec7.
[3] M. van der Klis. Millisecond oscillations in x-ray binaries. Ann. Rev. Astron. Astrophys., 38:717–760, 2000. arXiv:astro-ph/0001167. doi:10.1146/annurev.astro.38.1.717.
[4] Emanuele Berti et al. Testing General Relativity with Present and Future Astrophysical Observations. Class. Quant. Grav., 32:243001, 2015. arXiv:1501.07274. doi:10.1088/0264-9381/32/24/243001.
[5] Dimitrios Psaltis. Probes and Tests of Strong-Field Gravity with Observations in the Electromagnetic Spectrum. Living Rev. Rel., 11:9, 2008. arXiv:0806.1531. doi:10.12942/lrr-2008-9.
[6] Dimitrios Psaltis. Two approaches to testing general relativity in the strong-field regime. J. Phys. Conf. Ser., 189:012033, 2009. arXiv:0907.2746. doi:10.1088/1742-6596/189/1/012033.
[7] Kinet Jusufi, Mustapha Abramowicz, Mubasher Jamil, Shao-Wen Wei, Qiang Wu, and Anzhong Wang. Quasinormal modes, quasiperiodic oscillations, and the shadow of rotating regular black holes in nonminimally coupled Einstein-Yang-Mills theory. Phys. Rev. D, 103(2):024013, 2021. arXiv:2008.08450. doi:10.1103/PhysRevD.103.024013.
[8] Mustapha Azreg-Aïnou, Zihang Chen, Bojun Deng, Mubasher Jamil, Tao Zhu, Qiang Wu, and Yen-Kheng Lim. Orbital mechanics and quasiperiodic oscillation resonances of black holes in Einstein-Æther theory. Phys. Rev. D, 102(4):044028, 2020. arXiv:2004.02602. doi:10.1103/PhysRevD.102.044028.
[9] Adam Ingram and Sara Motta. A review of quasi-periodic-oscillations from black hole X-ray binaries: observation and theory. New Astron. Rev., 85:101524, 2019. arXiv:2001.08758. doi:10.1016/j.newar.2020.101524.
[10] E. H. Morgan, R. A. Remillard, and J. Greiner. RXTE Observations of QPOs in the Black Hole Candidate GRS 1915+105. Astrophys. J., 482:993–1010, 1997. doi:10.1086/304191.
[11] Ronald A Remillard, Edward H Morgan, Jeffrey E McClintock, Charles D Bailyn, and Jerome A Orosz. Rxte observations of 0.1-300 Hz quasi-periodic oscillations in the microquasar gRO J1655-40. The Astrophysical Journal, 552(1):397, 1999.
[12] Tod E Strohmayer. Discovery of a 450 Hz quasi-periodic oscillation from the microquasar gRO J1655-40 with the rossi x-ray timing explorer. The Astrophysical Journal Letters, 552(1):L49, 2001. I, IV
[13] Marek Artur Abramowicz and Włodek Kluzniak. A Precise determination of angular momentum in the black hole candidate GRO J1655-40. Astron. Astrophys., 374:L19, 2001. arXiv:astro-ph/0105077. doi:10.1051/0004-6361:20010791.
[14] S. E. Motta, T. M. Belloni, L. Stella, T. Muñoz Darias, and R. Fender. Precise mass and spin measurements for a stellar-mass black hole through X-ray timing: the case of GRO J1655–40. Mon. Not. Roy. Astron. Soc., 437(3):2554–2565, 2014. arXiv:1309.3652. doi:10.1093/mnras/stt2068. I, IV
[15] Luigi Stella, Mario Vietri, and Sharon Morsink. Correlations in the qpo frequencies of low mass x-ray binaries and the relativistic precession model. Astrophys. J. Lett., 524:L63–L66, 1999. arXiv:astro-ph/9907346. doi:10.1086/312291.
[16] A. Cadez, M. Calvani, and U. Kostic. On the tidal evolution of the orbits of low-mass satellites around black holes. Astron. Astrophys., 487:527–532, 2008. arXiv:0809.1783. doi:10.1051/0004-6361:200809483.
[17] Paola Rebusco. Twin peaks kHz QPOs: Mathematics of the 3:2 orbital resonance. Publ. Astron. Soc. Jpn., 56:553, 2004. arXiv:astro-ph/0403341. doi:10.1093/pasj/56.3.553.
[18] W. Kluzniak and M. A. Abramowicz. Parametric epicyclic resonance in black hole disks: qpos in micro-quasars. 3 2002. arXiv:astro-ph/0203314.
[19] Marek A. Abramowicz, Włodek Kluzniak, Zdenek Stuchlik, and Gabriel Torok. The Orbital resonance model for twin peak kHz QPOs: Measuring the black hole spins in microquasars. 1 2004. arXiv:astro-ph/0401464.
[20] Shoji Kato. Frequency Correlations of QPOs Based on a Disk Oscillation Model in Warped Disks. Publ. Astron. Soc. Jap., 59:451, 2007. arXiv:astro-ph/0701085. doi:10.1093/pasj/59.2.451.
[21] M. Bursa, M. A. Abramowicz, V. Karas, and Włodek Kluzniak. The Upper kHz QPO: A Gravitational lensing vertical oscillation. Astrophys. J. Lett., 617:L45–L48, 2004. arXiv:astro-ph/0406586. doi:10.1086/427167.
[22] Gabriel Torok, Pavel Bakala, Eva Sramkova, Zdenek Stuchlik, and Martin Urbanec. On mass-constraints implied by the relativistic precession model of twin-peak quasi-periodic oscillations in Circinus X-1. Astrophys. J., 714:748–757, 2010.
Alex Simpson. Traversable Wormholes, Regular Black Holes, and Black-Bounces. Master’s thesis, Victoria U., Wellington, 2011. arXiv:1102.0010, doi:10.1111/j.1365-2966.2012.18466.x.

George Pappas. What can quasi-periodic oscillations tell us about the structure of the corresponding compact objects? Mon. Not. Roy. Astron. Soc., 322:2581–2589, 2012. arXiv:1201.6071, doi:10.1111/j.1365-2966.2012.20817.x.

Cosimo Bambi. Probing the space-time geometry around black hole candidates with the resonance models for high-frequency QPOs and comparison with the continuum-fitting method. JCAP, 09:04, 2012. arXiv:1205.6348, doi:10.1088/1475-7516/2012/09/014.

Cosimo Bambi. Testing the nature of the black hole candidate in GRO J1655-40 with the relativistic precession model. Eur. Phys. J. C, 75(4):162, 2015. arXiv:1312.2228, doi:10.1140/epjc/s10052-015-3396-7.

Andrea Maselli, Leonardo Guaitieri, Paolo Pani, Luigi Stella, and Valeria Ferrari. Testing Gravity with Quasi Periodic Oscillations from accreting Black Holes: the Case of the Einstein-Dilaton-Gauss-Bonnet Theory. Astrophys. J., 801(2):115, 2015. arXiv:1412.3473, doi:10.1088/0004-637X/801/2/115.

Arthur George Suvorov and Andrew Melatos. Testing modified gravity and no-hair relations for the Kerr-Newman metric through quasi-periodic oscillations of galactic microquasars. Phys. Rev. D, 93:024004, 2016. arXiv:1512.02291, doi:10.1103/PhysRevD.93.024004.

Cosimo Bambi and Sourabh Nampalliwar. Quasi-periodic oscillations as a tool for testing the Kerr metric: A comparison with gravitational waves and iron line. EPL, 116(3):30006, 2016. arXiv:1604.02643, doi:10.1209/0295-5075/116/30006.

Songbai Chen, Mei Wang, and Jiliang Jing. Testing gravity of a regular and slowly rotating phantom black hole by quasi-periodic oscillations. Class. Quant. Grav., 33(19):195002, 2016. arXiv:1604.07106, doi:10.1088/0264-9381/33/19/195002.

Ali Reza Allahyari and Lijing Shao. Testing No-Hair Theorem by Quasi-Periodic Oscillations: the quadrupole of GRO J1655–40. JCAP, 06:043, 2021. arXiv:2102.02232.

Songbai Chen, Zeyun Wang, and Jiliang Jing. Testing gravity of a disformal Kerr black hole in quadratic degenerate higher-order scalar-tensor theories by quasi-periodic oscillations. JCAP, 06:043, 2021. arXiv:2103.11788, doi:10.1088/1475-7516/2021/06/043.

Indrani Banerjee, Sumanta Chakraborty, and Soumitra SenGupta. Looking for extra dimensions in the observed quasi-periodic oscillations of black holes. JCAP, 2021. arXiv:2105.06636.

Vittorio De Falco, Marialucia De Laurentis, and Salvatore Capozziello. Epicyclic frequencies in static and spherically symmetric wormhole geometries. Mon. Not. Roy. Astron. Soc., 422:2581–2589, 2012. arXiv:1201.6071, doi:10.1088/0004-637X/726/1/11.

Katherine Rink, Ilaria Caiazzo, and Jeremy Heyl. Testing General Relativity using Quasi-Periodic Oscillations from X-ray binaries. Astrophys. J. Lett., 492:L9, 1998. arXiv:astro-ph/9709085, doi:10.1086/311075.

James M. Bardeen. Non-singular general-relativistic gravitational collapse, in proceedings of the International Conference GR5. Tbilisi, USSR, page 174, 1968. I

Thomas A. Roman and Peter G. Bergmann. Stellar collapse without singularities? Phys. Rev. D, 28:1265–1277, 1983. doi:10.1103/PhysRevD.28.1265. I

Sean A. Hayward. Formation and evaporation of regular black holes. Phys. Rev. Lett., 96:031103, 2006. arXiv:gr-qc/0506126, doi:10.1103/PhysRevLett.96.031103.

James M. Bardeen. Black hole evaporation without an event horizon. 6 2014. arXiv:1406.4098.

Valeri P. Frolov. Notes on nonsingular models of black holes. Phys. Rev. D, 94(10):104056, 2016. arXiv:1609.01758, doi:10.1103/PhysRevD.94.104056.

Pablo A. Cano, Samuele Chimento, Tomás Ortín, and Alejandro Ruipérez. Regular Stringy Black Holes? Phys. Rev. D, 99(4):046014, 2019. arXiv:1806.08377, doi:10.1103/PhysRevD.99.046014.

James M. Bardeen. Models for the nonsingular transition of an evaporating black hole into a white hole. 11 2018. arXiv:1811.06683.

Raul Carballo-Rubio, Francesco Di Filippo, Stefano Liberati, Costantino Pacilio, and Matt Visser. On the viability of regular black holes. JHEP, 07:023, 2018. arXiv:1805.02675, doi:10.1007/JHEP07(2018)023.

Raul Carballo-Rubio, Francesco Di Filippo, Stefano Liberati, and Matt Visser. Phenomenological aspects of black holes beyond general relativity. Phys. Rev. D, 98(12):124009, 2018. arXiv:1809.08238, doi:10.1103/PhysRevD.98.124009.

Alex Simpson and Matt Visser. Black-bounce to traversable wormhole. JCAP, 02:042, 2019. arXiv:1812.07114, doi:10.1088/1475-7516/2019/02/042.

Alex Simpson, Prado Martin-Moruno, and Matt Visser. Vaidya spacetimes, black-bounces, and traversable wormholes. Class. Quant. Grav., 36(14):145007, 2019. arXiv:1902.04232, doi:10.1088/1361-6382/ab28a5.

Alex Simpson. Traversable Wormholes, Regular Black Holes, and Black-Bounces. Master’s thesis, Victoria U., Wellington, 2019. arXiv:2104.14065.
[51] M. S. Churilova and Z. Stuchlik. Ringing of the regular black-hole/wormhole transition. *Class. Quant. Grav.*, 37(7):075014, 2020. [arXiv:1911.11823, doi:10.1088/1361-6382/ab7717].

[52] Tian-Yi Zhou and Yi Xie. Precessing and periodic motions around a black-bounce/traversable wormhole. *Eur. Phys. J. C*, 80(11):1070, 2020. doi:10.1140/epjc/s10052-020-08661-w.

[53] J. R. Nascimento, A. Yu. Petrov, P. J. Porfirio, and A. R. Soares. Gravitational lensing in black-bounce spacetimes. *Phys. Rev. D*, 102(4):044021, 2020. [arXiv:2005.13096, doi:10.1103/PhysRevD.102.044021].

[54] Xiao-Tong Cheng and Yi Xie. Probing a black-bounce, traversable wormhole with weak deflection gravitational lensing. *Phys. Rev. D*, 103(6):064040, 2021. doi:10.1103/PhysRevD.103.064040.

[55] Naoki Tsukamoto. Gravitational lensing by two photon spheres in a black-bounce spacetime in strong deflection limits. 5 2021. [arXiv:2105.14336].

[56] Kirill A. Bronnikov, Roman A. Konoplya, and Thomas D. Pappas. General parametrization of wormhole spacetimes and its application to shadows and quasinormal modes. *Phys. Rev. D*, 103(12):124062, 2021. [arXiv:2102.10679, doi:10.1103/PhysRevD.103.124062].

[57] Merce Guerrero, Gonzalo J. Olmo, Diego Rubiera-Garcia, and Diego Sáez-Chillón Gómez. Shadows and optical appearance of black bounces illuminated by a thin accretion disk. 5 2021. [arXiv:2105.15073].

[58] Jacopo Mazza, Edgardo Franzin, and Stefano Liberati. A novel family of rotating black hole mimickers. *JCAP*, 04:082, 2021. [arXiv:2102.01105, doi:10.1088/1475-7516/2021/04/082].

[59] Shafqat Ul Islam, Jitendra Kumar, and Sushant G. Ghosh. Strong gravitational lensing by rotating Simpson–Visser black holes. 4 2021. [arXiv:2104.00696].

[60] Rajibul Shaikh, Kunal Pal, Kuntal Pal, and Tapobrata Sarkar. Constraining alternatives to the Kerr black hole. 2 2021. [arXiv:2102.04299, doi:10.1093/mnras/stab1779].

[61] Subrahmanyan Chandrasekhar. *The mathematical theory of black holes*. 1985.

[62] E. T. Newman and A. I. Janis. Note on the Kerr spinning particle metric. *J. Math. Phys.*, 6:915–917, 1965. doi:10.1063/1.1704350.

[63] Jerome A. Orosz and Charles D. Bailyn. Optical observations of GRO J1655-40 in quiescence I: A Precise mass for the black hole primary. *Astrophys. J.*, 477:876, 1997. [arXiv:astro-ph/9610211, doi:10.1086/303741].