Liquid-gas Phase Transition in Strange Hadronic Matter with Weak Y-Y Interaction

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Abstract

The liquid-gas phase transition in strange hadronic matter is reexamined by using the new parameters about the Λ – Λ interaction deduced from recent observation of $^6_{\Lambda\Lambda}He$ double hypernucleus. The extended Furnstahl-Serot-Tang model with nucleons and hyperons is utilized. The binodal surface, the limit pressure, the entropy, the specific heat capacity and the Caloric curves are addressed. We find that the liquid-gas phase transition can occur more easily in strange hadronic matter with weak Y-Y interaction than that of the strong Y-Y interaction.

PACS numbers: 21.65.+f; 11.30.Rd; 21.80.+a

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I. INTRODUCTION

Strangeness opens a new dimension to nuclear physics. Nuclear system with strangeness has many implications in astrophysics and cosmology and thus arouses much research interest. Nuclear systems with strangeness can be categorized into two species: strange quark matter (or strangelet) and strange hadronic matter (SHM). Quantum chromodynamics predicts that the strangeness-rich system could be strange quark matter or strangelet, consisting of $u$, $d$ and $s$ quarks. Strangelet has been a hot research topic for both theoretical physics and experimental physics, but there is still no convincing experimental result to confirm its existence. The other possible existence of strangeness is SHM, in which quarks are localized within nucleons and hyperons. The research of SHM is attractive because of the complexity of its structure and ingredients, especially, the phase transition including the quark deconfinement phase transition and liquid-gas (L-G) phase transition.

This paper evolves from an attempt to study the L-G phase transition in SHM at finite temperature. In a previous paper [1], we have studied the possibility of an L-G phase transition in SHM utilizing the extended Furnstahl-Serot-Tang (FST) model. After investigating the mechanical and chemical stabilities of SHM, we found a critical pressure on the binodal surface for $T = 13 MeV$ and a limit pressure on the binodal surface for lower temperature. After examining the continuities of entropy and specific heat capacity, we found the L-G phase transition in SHM is of second order. Employing the chiral $SU(3)$ quark mean field model, the same conclusion has also been drawn in ref [2].

Recently, Takahashi et al. [3] observed the $\Lambda - \Lambda$ energy in a $^6_{\Lambda\Lambda}He$ double hypernucleus. Their observation deduced the $\Lambda - \Lambda$ energy to be $\Delta B_{\Lambda\Lambda} = 1.01 \pm 0.20^{+0.15}_{-0.11} MeV$, which is much smaller than the previous estimation $\Delta B_{\Lambda\Lambda} \simeq 4 \sim 5 MeV$ from earlier experiments [4, 5, 6, 7, 8]. This leads to a significant decrease of potential $V^{(\Lambda)}_{\Lambda}$ from about $20 MeV$ to $5 MeV$ [9, 10]. The new data imply that the hyperon-hyperon interaction ($Y-Y$ interaction) can be weaker than known before [9]. We employed two of many candidates of phenomenological SHM models [10, 11, 12, 13, 14, 15, 16, 17], namely, extended FST model [10, 11, 12] and modified quark meson coupling (MQMC) model [9], and the weak $Y-Y$ interaction to reexamine the stability of SHM. We came to a conclusion that, while the system with the strong $Y-Y$ interaction and in a quite large strangeness fraction region is more deeply bound than the ordinary nuclear matter due to the opening of new degrees of
freedom, the system with weak Y-Y interaction is rather loosely bound compared to the latter. This conclusion is not model-dependent on the extended FST model or MQMC model and true for both zero and finite temperature.

Noting that the stability affects on the L-G phase transition directly, the stability of SHM with weak Y-Y interaction is quite different from that with strong Y-Y interaction. It is of interest to investigate the influence of this difference on the L-G phase transition in SHM. To employ the extended FST model and reexamine the L-G phase transition in SHM with weak Y-Y interaction is the main purpose of this paper. We will show that there is still a second order L-G phase transition in SHM with weak Y-Y interaction, but the binodal surface, limit pressure and the equation of state are very different from those of strong Y-Y interaction.

The paper is organized as follows. In Sec. II, the extended FST model is briefly introduced and the parameters for strong and weak Y-Y interactions are given. In Sec. III we reexamine the L-G phase transition in SHM with weak Y-Y interaction. The last section is the summary of the main results.

II. THE EXTENDED FST MODEL

In refs. [10, 11, 12], we extended the original FST model to include not only nucleons and $\sigma$, $\omega$ mesons, but also $\Lambda$, $\Xi$ hyperons and strange mesons $\sigma^*$ and $\phi$. In ref. [1], we used this model to study the L-G phase transition in SHM. The details of this model can be found in refs. [1, 10, 11, 12]. Hereafter, we only give brief formulae, which are necessary for discussing the L-G phase transition.

In mean-field approximation, the Lagrangian of the extended FST model is presented as follows:

$$\mathcal{L}_{MF} = \bar{\psi}_N (i\gamma^\mu \partial_\mu - g_{\omega N} \gamma^0 V_0 - M_N + g_{\sigma N} \sigma_0) \psi_N$$

$$+ \bar{\psi}_\Lambda (i\gamma^\mu \partial_\mu - g_{\omega \Lambda} \gamma^0 V_0 - g_{\phi \Lambda} \gamma^0 \phi_0 - M_\Lambda + g_{\sigma \Lambda} \sigma_0 + g_{\sigma^* \Lambda} \sigma^*_0) \psi_\Lambda$$

$$+ \bar{\psi}_\Xi (i\gamma^\mu \partial_\mu - g_{\omega \Xi} \gamma^0 V_0 - g_{\phi \Xi} \gamma^0 \phi_0 - M_\Xi + g_{\sigma \Xi} \sigma_0 + g_{\sigma^* \Xi} \sigma^*_0) \psi_\Xi$$

$$+ \frac{1}{2} \left( 1 + \eta \frac{\sigma_0}{S_0} \right) m_\omega^2 V_0^2 + \frac{1}{4!} \zeta (g_{\omega N} V_0)^4 + \frac{1}{2} m_\phi^2 \phi_0^2 - \frac{1}{2} m_{\sigma^*}^2 \sigma^*^2$$
\[-H_q \left( 1 - \frac{\sigma_0}{S_0} \right)^{4/d} \left[ \frac{1}{d} \ln \left( 1 - \frac{\sigma_0}{S_0} \right) - \frac{1}{4} \right] \]  

(1)

\( g_{ij} \) are the coupling constants of baryon \( j \) to meson \( i \) field.

By using the standard technique of statistical mechanics, the thermodynamic potential \( \Omega \) is obtained

\[
\Omega = V \left\{ \frac{1}{4} \left( \frac{1}{d} \ln \left( 1 - \frac{\sigma_0}{S_0} \right) - \frac{1}{4} \right) \right. \\
- \frac{1}{2} \left( 1 + \eta \frac{\sigma_0}{S_0} \right) m^2 V_0^2 - \frac{1}{4!} \zeta (g_{\omega N} V_0)^4 - \frac{1}{2} m_\phi^2 \phi_0^2 + \frac{1}{2} m^2 s \sigma_0^2 \right\} \\
- 2k_B T \left\{ \sum_{i,k} \ln \left[ 1 + e^{-\beta(E_i^*(k) - \nu_i)} \right] + \sum_{i,k} \ln \left[ 1 + e^{-\beta(E_i^*(k) + \nu_i)} \right] \right\} 
\]

(2)

where \( \beta \) is the inverse temperature and \( V \) is the volume of the system.

The effective masses of the hyperons and nucleons are

\[
M_i^* = M_i - g_{si} \sigma_0 - g_{s* i} \sigma_0^* \quad (i = \Lambda, \Xi), \\
M_i^* = M_i - g_{si} \sigma_0 \quad (i = N) 
\]

(4)

(5)

The mean-field values \( \phi_0, V_0, \sigma_0 \) and \( \sigma_0^* \) are determined by the corresponding extreme conditions of the thermodynamic potential.

The baryon densities \( \rho_{Bi} \) is given by

\[
\rho_{Bi} = \langle \psi_i^+ \psi_i \rangle = \frac{g_i}{\pi^2} \int dkk^2 \left[ n_i(k) - \bar{n}_i(k) \right] 
\]

(6)

where \( g_i = 2 \) for \( i = N \) or \( \Xi \), \( g_i = 1 \) for \( i = \Lambda \). The baryon and anti-baryon distributions are, respectively, expressed as

\[
n_i(k) = \left\{ \exp[\beta(E_i^*(k) - \nu_i)] + 1 \right\}^{-1} 
\]

(7)

and

\[
\pi_i(k) = \left\{ \exp[\beta(E_i^*(k) + \nu_i)] + 1 \right\}^{-1} 
\]

(8)

\( \nu_i \) are related to chemical potential \( \mu_i \) by

\[
\mu_N = \nu_N + g_{\omega N} V_0, \\
\mu_\Lambda = \nu_\Lambda + g_{\omega \Lambda} V_0 + g_{\phi \Lambda} \phi_0, \\
\mu_\Xi = \nu_\Xi + g_{\omega \Xi} V_0 + g_{\phi \Xi} \phi_0. 
\]

(9)
Since the system has equal number of protons and neutrons, and equal number of \( \Xi^0 \) and \( \Xi^- \), the chemical equilibrium conditions for the reactions \( \Lambda + \Lambda \rightleftharpoons n + \Xi^0 \) and \( \Lambda + \Lambda \rightleftharpoons p + \Xi^- \) are unified as

\[
2\mu_\Lambda = \mu_N + \mu_\Xi. \tag{10}
\]

Eq. (10) implies that only two components of \( N, \Lambda \) and \( \Xi \) are independent. The SHM is regarded as a two-component system, for instance, the nucleon component and the \( \Xi \) component. The strangeness fraction \( f_S \) is introduced as

\[
f_S \equiv \frac{\rho_{B\Lambda} + 2\rho_{B\Xi}}{\rho_B}, \tag{11}
\]

which plays the similar role as that of the asymmetric parameter \( \alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p) \) in the asymmetric nuclear matter. We can use the same method as that of refs. [1, 18, 19, 20, 21, 22] to address the L-G phase transition.

Following the usual procedure of statistical physics, we can calculate the other thermodynamic quantities from thermodynamic potential \( \Omega \). For example, the pressure and entropy density are calculated by formulas

\[
p = \sum_i \frac{g_i}{6\pi^2} \int \frac{k^4}{E_i^*(k)} \left[ n_i(k) + \pi_i(k) \right] - H_q \left\{ \left( 1 - \frac{\sigma_0}{S_0} \right)^{\frac{3}{4}} \left[ \frac{1}{4} \ln \left( 1 - \frac{\sigma_0}{S_0} \right) - \frac{1}{4} \right] + \frac{1}{4} \right\} + \frac{1}{2} \left( 1 + \eta \frac{\sigma_0}{S_0} \right) m_\sigma^2 V_0^2 \frac{\beta g_\omega N V_0^4 + \frac{1}{2} m_\sigma^2 \phi_0^2 - \frac{1}{2} m_\sigma^2 \phi_0^*}{m_\sigma^2 \phi_0^*}, \tag{12}
\]

\[
s \equiv \frac{S}{V} = \sum_i \frac{g_i}{6\pi^2} \int \frac{k^4}{E_i^*(k)} \left\{ \frac{\beta^2 \left( E_i^*(k) - \nu_i \right) \exp \left[ \beta \left( E_i^*(k) - \nu_i \right) \right]}{\left[ \exp \left[ \beta \left( E_i^*(k) - \nu_i \right) + 1 \right] \right]^2} + \frac{\beta^2 \left( E_i^*(k) + \nu_i \right) \exp \left[ \beta \left( E_i^*(k) + \nu_i \right) \right]}{\left[ \exp \left[ \beta \left( E_i^*(k) + \nu_i \right) + 1 \right] \right]^2} \right\}. \tag{13}
\]

In the previous calculation [1], we employed the parameter set T1 given by ref. [11], and the earlier data of strong Y-Y interaction. In refs. [9, 10], we have the new data of weak Y-Y interaction and adjusted the parameters from \( g_{\sigma^*\Lambda}^2 = 48.31 \) and \( g_{\sigma^*\Xi}^2 = 154.62 \) for strong Y-Y interaction, to \( g_{\sigma^*\Lambda}^2 = 28.73 \) and \( g_{\sigma^*\Xi}^2 = 129.06 \) for weak Y-Y interaction. The details of the parameters are summarized in Table 1. We will use the parameters for weak Y-Y interaction to address the L-G phase transition in the next section.
| $m_s$ | $S_0$ | $\xi$ | $\eta$ | $d$ | $g^2_{sN}$ | $g^2_{\omega N}$ | $g^2_{s\Lambda}$ | $g^2_{s\Xi}$ |
|------|------|------|------|-----|-------------|----------------|----------------|------------|
| 509  | 90.6 | 0.0402 | −0.496 | 2.70 | 99.3        | 154.5          | 37.32         | 9.99       |

$g^2_{\sigma \Lambda}$ (Strong) $g^2_{\sigma \Xi}$ (Strong) $g^2_{\sigma \Lambda}$ (Weak) $g^2_{\sigma \Xi}$ (Weak)

48.31 154.62 28.73 129.06

TABLE I: The parameters used in calculation

III. L-G PHASE TRANSITION IN SHM WITH WEAK Y-Y INTERACTION

In this section, we employ the extended FST model to investigate the L-G phase transition in the SHM with weak Y-Y interaction.

Using the formulae in Sec.II and the parameters of Table 1, we calculate the pressure and study the equation of state first. The results for pressure vs. baryon density $\rho_B$ curves at $T = 10\, MeV$ are shown in Fig.1, with weak and strong Y-Y interactions, respectively. We can see that the equation of state for two cases are quite different in two aspects. Firstly, with weak Y-Y interaction, when $f_S > 0.7$, the pressure increases with density monotonically and there are no mechanical unstable regions. While with strong Y-Y interaction, there are always mechanical unstable regions for each $f_S$. Secondly, with weak Y-Y interaction, the minimum of the pressure-density curve rises monotonically with $f_S$, till $f_S > 0.7$, where there is no minimum. While with strong Y-Y interaction, the minimum drops with $f_S$ and reaches the lowest value on the curve of $f_S = 1.3$. When $f_S > 1.3$, the pressure-density curves ascend with the increase of $f_S$. These features are consistent with the behavior of energy-density curves and the stability of SHM in ref.[12] and [10].

To discuss the chemical instability of SHM with weak Y-Y interaction, we show the chemical potential isobars for nucleons and $\Xi$ against $f_S$ at temperature $T = 10\, MeV$ in Fig.2 for $p = 0.05, 0.10, 0.20, 0.40, 0.50$ and $0.60\, MeVfm^{-3}$ respectively. An inflection point could be spotted on the curve with the pressure $p^i = 0.50\, MeVfm^{-3}$. There are chemical unstable regions on curves with $p < p^i$. This behavior is similar to that of strong Y-Y interaction but the inflection points are different for these two cases.

The chemical, thermal and mechanical Gibbs equilibrium conditions for two separate phases require

$$T^L = T^G = T$$ (14)
\[
\mu_q^L(T, \rho^L, f_s^L) = \mu_q^G(T, \rho^G, f_s^G), (q = N, \Xi),
\]

\[
p^L(T, \rho^L, f_s^L) = p^G(T, \rho^G, f_s^G),
\]

where the superscripts \( L, G \) denote the liquid and gas phases, respectively. We follow the procedures used in refs. [1, 18, 19, 20, 21] to solve out Eqs.(14),(15),(16). Collecting up all the solutions and we get the phase boundary, i.e., binodal surface. The binodal surface at \( T = 10 \text{ MeV} \) are shown in Fig.3, where the dotted line refers to weak Y-Y interaction and the solid line to strong Y-Y interaction. We see from Fig.3 that the differences between two binodal surfaces are very remarkable. Though the limit pressures, above which the L-G phase transition cannot take place, exist for both two cases, the values are very different. The limit pressure for weak Y-Y interaction \( p_{lim}^W = 0.50 \text{ MeV fm}^{-3} \) is much higher than that of the strong Y-Y interaction as \( p_{lim}^S = 0.095 \text{ MeV fm}^{-3} \). The enclosed region of binodal surface with weak Y-Y interaction is much larger than that of the strong Y-Y interaction. A higher limit pressure and a larger region of phase separation imply that the system is likely to be less stable and the L-G phase transition can take place more easily. This result is of course very reasonable if we notice the conclusion given by refs. [9, 10] that the SHM with weak Y-Y interaction is less stable.

To determine the order of the L-G phase transition, we examine the entropy density and the specific heat capacity of SHM with weak Y-Y interaction. By Ehrenfest’s definition, the first order phase transition is characterized by the discontinuities of the first order derivatives of the chemical potential, such as the discontinuities of entropy and volume, while the second order phase transition unfolds the discontinuous behavior for the second order derivatives of the chemical potential, such as the specific heat capacity. The entropy per baryon is

\[
s(T, p, f_s) = \frac{S(T, p, f_s)}{\rho_B}. \quad (17)
\]

where \( S(T, p, f_s) \) can be calculated from Eq.(13).

The specific heat capacity is

\[
C_p = T \left( \frac{\partial S}{\partial T} \right)_{p, f_s}
= T \sum_i \frac{g_i}{6\pi^2} \int \frac{dk}{k} \frac{k^4}{E_i^*(k)} \left[ \frac{d^2n_i}{dT^2} + \frac{d^2\pi_i}{dT^2} \right],
\]

where

\[
d^2n_i \over dT^2 = \frac{e^{\beta(E_i^*(k)-\nu)}}{T^4(e^{\beta(E_i^*(k)-\nu_i)}+1)^3} \left[ (E_i^*(k)-\nu_i)^2(e^{\beta(E_i^*(k)-\nu_i)}-1) - 2T(E_i^*(k)-\nu_i)(e^{\beta(E_i^*(k)-\nu_i)})+1 \right].
\]

\[
(19)
\]
\[
\frac{d^2 \pi_i}{dT^2} = \frac{e^{\beta(E_i^*(k) + \nu_i)} \left[ (E_i^*(k) + \nu_i)^2 (e^{\beta(E_i^*(k) + \nu_i)} - 1) - 2T(E_i^*(k) + \nu_i)(e^{\beta(E_i^*(k) + \nu_i)} + 1) \right]}{T^4(e^{\beta(E_i^*(k) + \nu_i)} + 1)^3}
\] (20)

The entropy vs. temperature curves for \( f_S = 0.0, 0.1 \) and \( 0.4 \) at \( p = 0.11MeV fm^{-3} \) are shown in Fig.4. We see from Fig.4 that the entropy evolves continuously through the phase transition when \( f_S > 0.0 \). But when \( f_S = 0.0 \), our model reduces to the one-component symmetric nuclear matter. The entropy becomes discontinuous. The L-G phase transition becomes the first order. This result is consistent with theoretical and experimental results about L-G phase transition in symmetric nuclear matter.

The specific heat capacity vs. temperature curves for \( f_S = 0.1 \) and \( 0.4 \) at \( p = 0.11MeV fm^{-3} \) are shown in Fig.5. The specific heat capacity curves are discontinuous for \( f_S = 0.1 \) and \( 0.4 \). The continuity of entropy together with the discontinuity of specific heat capacity demonstrates that the L-G phase transition in SHM with weak Y-Y interaction is still of second order, as expected.

One of the important signals for L-G phase transition is the investigation of Caloric curves\[23, 24, 25, 26\]. The Caloric curve reflects the relationship between excited energy of hadronic matter and the temperature. The energy of SHM is

\[
\varepsilon = \sum_i \frac{g_i}{\pi^2} \int dk k^2 E_i^*(k)[n_i(k) + \pi_i(k)] + H_q \left\{ \left( 1 - \frac{\sigma_0}{S_0} \right)^4 \left[ \frac{1}{4} \ln \left( 1 - \frac{\sigma_0}{S_0} \right) - \frac{1}{4} \right] + \frac{1}{4} \right\}
\]

\[
-\frac{1}{2} \left( 1 + \eta \frac{\sigma_0}{S_0} \right) m_\omega^2 V_0^2 - \frac{1}{4} \zeta g_{\omega N} V_0^4 - \frac{1}{2} m_\phi^2 \phi_0^2 + \frac{1}{2} m_\sigma^2 \sigma_0^2
\]

\[
+ g_{\omega N} V_0 \rho_B N + (g_{\omega A} V_0 + g_{\phi A} \phi_0) \rho_B A + (g_{\omega \Xi} V_0 + g_{\phi \Xi} \phi_0) \rho_B \Xi
\] (21)

and the excited energy \( E^*/\rho_B \) is

\[
E^*/\rho_B = \frac{\lambda \varepsilon^L(\rho^L, f s^L, T) + (1 - \lambda) \varepsilon^G(\rho^G, f s^G, T)}{\lambda \rho^L + (1 - \lambda) \rho^G} - \left( \frac{\varepsilon^L(\rho^L, f s^L, T)}{\rho^L} \right)_{T=0}
\] (22)

We present the Caloric curves at pressure \( p = 0.11MeV fm^{-3} \) for \( f_S = 0.0, 0.1 \) and \( 0.4 \) in Fig.6. There are slopes during the L-G phase transition for \( f_S = 0.1 \) and \( 0.4 \), while for \( f_S = 0.0 \) there is a platform. This platform-shaped Caloric curve has been got in experiments of the L-G phase transition in normal symmetric nuclear matter\[24, 25\]. For \( f_S > 0.0 \), the slopes mean that temperature does not remain constant during phase transition. This is the very feature of second order phase transition as was first pointed out by Müller and Serot\[18\].
To exhibit the behavior of Caloric curves of SHM with strong and weak Y-Y interactions clearly, we plot the temperature $T$ vs. $E^*/\rho_B$ curves at $p = 0.11MeVfm^{-3}$, $f_S = 0.4$ for strong and weak Y-Y interactions in Fig.7. We find the temperature region of phase transition for weak Y-Y interaction is lower than that of strong Y-Y interaction and the slope for weak Y-Y interaction is more acclivitous. Perhaps these distinctions can provide us a possibility to check the Y-Y interaction be strong or weak, and then the $\Lambda - \Lambda$ energy.

IV. SUMMARY

We have used the new parameters about the weak Y-Y interaction derived from recent experiment data to reexamine the L-G phase transition in SHM. We have employed the extended FST model with nucleons and $\Lambda$, $\Xi$ hyperons and simplified the SHM into a two-component system to discuss its L-G phase transition. We come to the following conclusions:

1. No matter the Y-Y interaction is strong or weak, the L-G phase transition can take place in SHM. The L-G phase transition is of second order for $f_S \neq 0$, the entropy is continuous and the heat capacity is discontinuous at the phase transition point. But for $f_S = 0$ the L-G phase transition becomes the first order. The entropy is discontinuous.

2. Due to the difference between the stability of SHM with strong and weak Y-Y interactions, the physical features of L-G phase transition, including the limit pressure, the regions enclosed by the binodal surface, the Caloric curves, the slope of phase transition curves are very different for weak and strong Y-Y interactions.

3. After reexamining the features of SHM, we come to a conclusion that the L-G phase transition can occur in SHM with weak Y-Y interaction more easily than that with strong Y-Y interaction.

V. ACKNOWLEDGEMENTS

This work is supported in part by National Natural Science Foundation of China under Nos. 10235030, 10247001, 10375013, 10347107 10047005, 10075071, the National Basic
Research Programme 2003CB716300, the Foundation of Education Ministry of China under contract 2003246005 and CAS Knowledge Innovation Project No. KJCX2-N11.

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FIG. 1: Pressure as a function of baryon density at temperature $T = 10 \text{MeV}$ for various strangeness fractions $f_S$ with weak and strong Y-Y interactions. Note that the scales of the two figures are not the same.

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FIG. 2: Chemical potential isobars as functions of $f_S$ at $T = 10 MeV$. The curves have pressures $p = 0.05, 0.10, 0.20, 0.40, 0.50$ and $0.60 MeV fm^{-3}$ respectively. The inflection point (A) is at $p^i = 0.50 MeV fm^{-3}$. 
FIG. 3: Binodal surface at $T = 10 MeV$. The dotted line is the binodal surface for weak Y-Y interaction, while the solid line is that for strong Y-Y interaction. The binodal surfaces are cut off at limit pressure $p_{lim} = 0.50 MeV fm^{-3}$ and $p_{lim} = 0.095 MeV fm^{-3}$ respectively.
FIG. 4: Entropy as a function of temperature at constant pressure $p = 0.11\text{MeV fm}^{-3}$ for strangeness fraction $f_S = 0.0, 0.1$ and 0.4. For $f_S = 0.1$ and 0.4, the entropy evolves continuously through the phase transition, while for $f_S = 0.0$ the entropy is discontinuous.
FIG. 5: Specific heat capacity as a function of temperature at constant pressure $p = 0.11\text{MeVfm}^{-3}$ for strangeness fraction $f_S = 0.1$ and 0.4. For $f_S = 0.1$ and 0.4, the entropy is discontinuous through the phase transition.
FIG. 6: Temperature as a function of excited energy at constant pressure $p = 0.11 MeV fm^{-3}$ for strangeness fraction $f_S = 0.0, 0.1$ and $0.4$. For $f_S = 0.1$ and $0.4$, there are slopes during the phase transition, while for $f_S = 0.0$ there is a platform.
FIG. 7: Temperature as a function of excited energy at constant pressure $p = 0.11 MeV fm^{-3}$ for strangeness fraction $f_S = 0.4$. 