Relativistic Bullets Ejection from Supernovae and Generation of Gamma Ray Bursts

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ABSTRACT

It is generally believed that cosmological Gamma Ray Bursts (GRBs) are produced by the deceleration of relativistic objects with $\Gamma \gtrsim 100$. We study the possibility that some GRBs are produced along with relativistic matter ejection from supernovae. In this model, it is quite likely that the matter has to travel through the progenitor’s thick envelope before generating GRBs. Under the assumption that the ejected matter is described as a single collective matter, we obtain constraints on the matter to have $\Gamma \gtrsim 100$ at the breakout of the progenitor. One advantage of considering this type of model is that the expected GRB energy is sufficiently large, in contrast to the GRB generation model by the shock breakout in the energetic supernova explosion. We find that in general the cross section of the matter has to be very small compared with the progenitor’s radius and thus the matter has to be bullet (or jet)-like rather than shell-like.

Subject headings: gamma rays: bursts — relativistic shock — stars: supernovae: general

1. Introduction

It is generally believed that cosmological Gamma Ray Bursts (GRBs) are produced by relativistic matter with Lorentz factor $\Gamma \gtrsim 100$ when the kinetic energy of the matter is converted into radiation by inelastic collision with other relativistic matter — internal shock model —, or by collision with interstellar matter (ISM) — external shock model (e.g. Piran 1999).

Although it is still controversial, there are indications that some GRBs might be associated with type Ib/c supernovae (SNe) (e.g., Galama et al. 1998, Bloom et al. 1999, Castro-Tirado & Gorosabel 1999, Reichart 1999). Especially a type Ic SN, SN1998bw, which was discovered
in the error box of GRB980425 has been considered to be the most promising candidate for the GRB-SN association (Galama et al. 1998, Iwamoto et al. 1998, Nakamura 1999). In Iwamoto et al. (1998) we considered the following scenario for the GRB formation: at the shock breakout in the energetic explosion, the surface layer becomes highly relativistic, and the interaction with ISM may generate a GRB. One difficulty with this kind of model was that the total energy stored in the relativistic region was very small, and hence inconsistent with the observed GRB energy (see also Woosley, Eastman & Schmidt 1999). Therefore, something more than the energetic explosion is necessary to produce a GRB from a SN explosion. This led to the investigation of jet-like explosion (Khokhlov et al. 1999) or the effects of disk accretion around a black hole (BH) (MacFadyen & Woosley 1999).

In this Letter, we assume that during or after core collapse, a small fraction of the matter acquires a large energy, becomes relativistic deep inside the SN progenitor, and is ejected. This type of matter ejection may be related to the “mystery spot” of SN1987A (e.g., Rees 1987, Piran & Nakamura 1987, Cen 1999). We show that such relativistic matter can generate sufficiently energetic GRBs if the cross section of the matter is sufficiently small.

The collision of the relativistic matter with the surrounding matter produces an external shock, which decelerates the matter. If the relativistic matter is in an optically thin region, prompt radiation from it may be observed as a GRB. This kind of situation may be realized, if the collapsed star is rapidly rotating and forms an accreting BH, and if matter is sufficiently evacuated along the rotational axis as in the model of Fryer & Woosley (1998). However, numerical simulations of such system (MacFadyen & Woosley 1999) suggest that although the density is significantly reduced near the central BH along the polar direction, it is still high. Also the central BH and accretion disk are still surrounded by the envelope of the progenitor. Furthermore, light curve modeling of SN1998bw suggests that the SN explosion was globally well described by the usual SN model, i.e., the explosive SN shock should propagate through the normal stellar envelope (Iwamoto et al. 1998, Woosley et al. 1999). Therefore, it is likely that the relativistic object produced near the center has to travel through the optically thick envelope of the progenitor. The key question is, thus, if the object can still have $\Gamma \sim 100$, when it breaks out of the envelope. By considering a simple model, we obtain the constraints for a relativistic object produced in the deep interior of a SN progenitor to have $\Gamma \simeq 100$ when it breaks out of the envelope.

2. Basic equations

Here, the basic equations and some solutions are shown to describe the slowing down of a relativistic object via collision with the surrounding matter.

We assume that the slow-down of the relativistic object occurs by a series of infinitesimal inelastic collisions between the object and infinitesimal masses. We denote the rest-frame energy (rest-mass and thermal energy) of the object located at radius $r$ by $M(r)$, its Lorentz factor by
\(\Gamma(r)\), the mass of the envelope that has already collided with the object by \(m(r)\), and the thermal energy produced in the collision by \(E\). Then the energy and momentum conservation yield (e.g., Piran 1999),

\[
\frac{d\Gamma}{\Gamma^2 - 1} = -\frac{dm}{M}, \quad \text{and} \quad dE = (\Gamma - 1)c^2 dm.
\] (1)

The variation of \(M\) is given by

\[
dM = (1 - \epsilon)\frac{dE}{c^2} + dm,
\]

where \(\epsilon\) is the fraction of the shock-generated thermal energy that is radiated. When the object is in the envelope, we assume \(\epsilon \sim 0\). If \(\Gamma_i \gg 1\) and \(\Gamma \gg 1\) these equations can be easily integrated to give:

\[
M(r) \simeq M_0 \left(\frac{\Gamma_i}{\Gamma(r)}\right)^{1-2\epsilon},
\] (2)

where \(M_0\) and \(\Gamma_i\) are initial rest-mass and Lorentz factor, and

\[
m(r) \simeq \frac{M_0\Gamma_i}{2 - \epsilon}(\Gamma(r)^{-2+\epsilon} - \Gamma_i^{-2+\epsilon}).
\] (3)

After breaking out from the envelope, the object emits radiation as it is slowed down. If the collisions between relativistic objects are not efficient, then slow-down occurs mostly via collision with ISM (external shock model). Eq. (3) shows that, by the time the object lose half of its kinetic energy (\(\Gamma \simeq \Gamma_i/2\)), the mass of \(m_s \sim m(R)/\Gamma\) has been plowed. The total thermal energy produced is (Meszaros & Rees 1992),

\[
E \simeq \int_0^{m_s} \Gamma e^2 dm \sim m(R)e^2,
\] (4)

which is assumed to be roughly equal to the total GRB energy.

In the external shock model, the characteristic synchrotron energy is given by

\[
h\nu = 160 \text{ keV} \epsilon_B^{1/2} \epsilon_e^2 (\Gamma/100)^4 n^{1/2},
\]

where \(\epsilon_B\) is the ratio of the magnetic field energy density to the total thermal energy, \(\epsilon_e\) is the fraction of the total thermal energy which goes into random motions of the electrons, and \(n\) is the ISM number density per cm\(^3\) (e.g. Piran 1999). Therefore, \(\Gamma \gtrsim 100\) is required for efficient gamma ray emission.

### 3. Conditions for GRB generation

We assume that the objects become relativistic near the collapsed core, and travel through the envelope of the SN progenitor. In the following, we consider two cases for the deceleration of the objects. One is when the relativistic objects do not lose their energy significantly in the envelope of the progenitor, i.e., \(\Gamma_i \simeq \Gamma_f\), where \(\Gamma_f\) is the Lorentz factor at breakout. The second case is when the objects are initially highly relativistic and slow down to \(\Gamma_f \ll \Gamma_i\) through the envelope. In this paper, for simplicity, we only consider the case with \(\Gamma_f \simeq 100\). In both cases we find that the cross sections of the relativistic objects must be very small compared with progenitor’s radius. Hence the object should be bullet (or beam or jet)-like, rather than shell-like. Note that Eq. (1) applies only when the object can be treated as collective matter, and hence it does not apply to long beams or jets.
3.1. Case I: $\Gamma_i \simeq \Gamma_f \simeq 100$

The energy $E$ required to accelerate $N'$ pieces of bullets with mass $M_0$ up to $\Gamma_i$ is $E = N' M_0 c^2 \Gamma_i$. Therefore,

$$N' M_0 = 5.5 \times 10^{-4} E_{51} \Gamma_i^{-1} (M_\odot),$$

where $E_{51} \equiv E/(10^{51} \text{ erg})$. As shown in the previous section, when each bullet plows a mass of $\sim M_0/\Gamma$, $\Gamma$ becomes roughly half. Therefore, if the bullet is not to be significantly decelerated after traversing a mass $M_{\text{env}}$ and radius $R$, the diameter of the bullets $d_b$ (here the diameter is assumed to be constant) should satisfy the condition:

$$d_b < 10^{-4} \sqrt{E_{51}/N'(100/\Gamma)} R.$$  \hspace{1cm} (6)

The situation considered here is similar to the model considered in Heinz & Begelman (1999). They assume that the central engine of the burst releases $N'$ bullets distributed over an opening angle of $\theta \sim 10^\circ$ with $\Gamma_i \sim 1000$. Each bullet is assumed to be expanding sideways with a velocity $v_s = \alpha c/\Gamma \ll c \Gamma$ measured in the observer’s frame ($\alpha c$ is the expansion velocity in the comoving frame). Since the viewing angle $1/\Gamma \ll \theta$, the observed number of bullets is $N \simeq N'/(\theta^2 \Gamma^2)$. They considered this model to explain the short-time variability of the canonical GRBs, and claimed that the external shock model can explain that variability if $N \sim 100$, $\alpha \sim 0.01$, $\Gamma \sim 10^3$, and the ambient gas density $n \sim 10^8 \text{cm}^{-3}$. In this paper we do not attempt to produce short time variability, since it is not certain that a GRB associated with a SN should show it.

Each bullet may expand if it has sufficiently large internal energy, though the expansion rate depends on several assumptions. If we assume that the bullets expand sideways with comoving velocity $\alpha c$, as in Heinz & Begelman (1999), then condition (6) is somewhat relaxed as shown in the following. The cross section of the bullet in the observer’s frame $S(r)$ is given by

$$S(r) \simeq (2v_s t \Gamma^2)^2 = (\alpha r/\Gamma)^2,$$

where $v_s = \alpha c/\Gamma$ is the sideways velocity in the observed frame, and $t \simeq r/(2c \Gamma^2)$ is the observer’s time when the bullet is located at radius $r$. The total mass plowed by the bullet $m_s$ is given by,

$$m_s = \int_{r_0}^R \rho(r) S(r) dr,$$  \hspace{1cm} (7)

where $R$ is the radius of the progenitor, $r_0 (\ll R)$ is the radius where the bullet is accelerated to $\Gamma = \Gamma_i$, and $\rho(r)$ is the density of the progenitor.

If $\rho \propto r^{-b} (b < 3)$, and $r_0 \ll R$, Eq.(7) yields $m_s \simeq (4\pi)^{-1} M_{\text{env}} \alpha^2 \Gamma^{-2}$, where $M_{\text{env}} \equiv \int_{r_0}^R 4\pi r^2 \rho(r) dr$ is the progenitor mass above radius $r_0$. Note that $m_s$ is independent of $b$. A constraint on $\alpha$ is given by $m_s < M_0/\Gamma$:

$$\alpha < (4\pi \Gamma M_0/M_{\text{env}})^{1/2} = 0.08 \sqrt{E_{51}}/N' M_{\text{env}} (M_\odot).$$  \hspace{1cm} (8)
The diameter of the bullet at breakout \( r = R \), \( d(R) \) is

\[
d(R) = \alpha R / \Gamma < 10^{-3} \sqrt{\frac{E_{51}}{N' M_{\text{env}}(M_{\odot})}} R. \tag{9}
\]

After the bullet breaks out of the envelope, a GRB is produced. If it is produced by the external shock, total energy of the order of \( E_{\text{GRB}} \sim M c^2 \) is released as the bullet plows through a mass of \( \sim M / \Gamma_f \). Here \( M \) is the rest-energy of the bullet at \( r = R \), which is estimated by Eq.\( (3) \), \( M \approx M_0 \Gamma_i / \Gamma_f \approx M_0 \). Using Eq.\( (3) \), we obtain the total GRB energy \( E_{\text{GRB}} \sim 10^{49} E_{51} (100 / \Gamma_f) (N / N') \) erg. Note that if many bullets \( (N' \gg 1) \) are ejected within the opening angle \( \theta \), the total GRB energy seen by an observer is reduced by the factor of \( N / N' \approx 1 / (\theta \Gamma)^2 \).

### 3.2. Case II: \( \Gamma_i \gg \Gamma_f \approx 100 \)

Suppose again that the bullets expand sideways with constant velocity \( \alpha \) measured in the comoving frame. Then the diameter of the bullet at radius \( r \) in the observed frame is given by,

\[
d(r) = \int_{r_0}^{r} \frac{\alpha / \Gamma(r')}{dr'} dr'. \tag{10}
\]

In general \( \Gamma(r) \) is a function of \( \alpha \) and \( \rho(r) \). However, here we simply assume that \( \Gamma(r) = \Gamma_i (r / r_0)^{-a} \) and \( \rho(r) = \rho_0 (r / r_0)^{-b} \) in order to get a rough constraint without obtaining an exact solution for \( \Gamma(r, \alpha, \rho(r)) \). Here the subscript 0 represents the quantity at \( r = r_0 \) in the observer’s frame. Strictly speaking, assuming that both \( a \) and \( b \) are constants is not self-consistent, however the result given in Eq.\( (12) \) is not so sensitive to the value of \( a \) and \( b \). Therefore, this assumption is sufficient for a qualitative analysis. Now Eq. \( (10) \) gives \( d \propto r^{1+a} \). Hence we denote \( d(r) = d_0 (r / r_0)^{1+a} \), where \( d_0 = \alpha r_0 / [(a + 1) \Gamma_i] \). If constants \( a \) and \( b \) satisfy \( 0 < b < 3 \), and \( 2a + 3 - b > 0 \), then Eqs.\( (3) \) and \( (7) \) give

\[
m_a \simeq \frac{d_0^2 r_0^3 \rho_0}{2a + 3 - b} \left( \frac{R}{r_0} \right)^{2a + 3 - b} \simeq \frac{\Gamma_i}{2 \Gamma_f^2}. \tag{11}
\]

We define the envelope mass by \( M_{\text{env}} \equiv \int_{r_0}^{R} 4 \pi r^2 \rho(r') dr' \simeq (4 \pi r_0^3 \rho_0 / 3 - b) (R / r_0)^{3-b} \). Now, the diameter of the bullet \( d \) in the observed frame at the progenitor surface \( (r = R) \) is given by

\[
d(R) = d_0 \left( \frac{R}{r_0} \right)^{1+a} = \frac{R}{\Gamma_f} \sqrt{\frac{2a + 3 - b}{3 - b}} \frac{2 \pi M_0 \Gamma_i}{M_{\text{env}}} \simeq \frac{0.06 R}{\Gamma_f} \sqrt{\frac{2a + 3 - b}{3 - b}} \frac{E_{51}}{N' M_{\text{env}}(M_{\odot})}, \tag{12}
\]

or

\[
\alpha = 0.06(a + 1) \sqrt{\frac{2a + 3 - b}{3 - b}} \frac{E_{51}}{N' M_{\text{env}}(M_{\odot})}. \tag{13}
\]

Note that if \( \alpha \) is significantly smaller than the right hand side of this equation, Case I applies. This result is not sensitive to the values of \( a \) and \( b \) as long as \( 2a + 3 - b / 3 - b \sim O(1) \). This justifies our
approximate analysis assuming that \( a \) and \( b \) are constants. The total GRB energy (in the external shock model) can be estimated as \( E_{\text{GRB}} \sim 10^{49} E_{51}(100/\Gamma_f)(N/N') \) erg, which is same as Case I.

4. Discussion

We have investigated relativistic matter ejection from SN progenitors in order to study the possibility that SNe generate GRBs. If we consider this type of model, it is likely that the relativistic matter has to travel through the optically thick envelope of the SN progenitor before generating the GRB. We have obtained conditions for such relativistic matter to have \( \Gamma \gtrsim 100 \) after passing through the thick envelope of the progenitor. In general, in order to satisfy these conditions, the cross section of the objects must be small. Typically the diameter of the objects must be at least factor of \( 10^{3} E_{51}^{-1/2} \) smaller than the radius of the envelope when they break out of the envelope. Therefore, the objects should be bullet (or beam or jet)-like rather than shell-like. Here we should note that the opening angle of GRB can be significantly larger than the size of each bullet at breakout, because the size of the bullet may increase significantly by the time of GRB emission owing to its expansion or to merging with other bullets.

We make no attempt to explain how such bullets may be produced during core collapse since the explosion mechanism of massive stars are not well understood. Indeed, currently no mechanism is known to make such bullets. It is only speculation but such bullets may be produced if the exploding core is inhomogeneous (e.g. Burrows & Hayes 1996) and the explosion energy is concentrated in some small regions, or if the collapsing core forms an accreting black hole and relativistic bullets or jets are produced around the black hole (a jet of short duration may look like a bullet).

Next let us estimate the initial rest-frame density of the bullets. Suppose the bullet has a volume of roughly \( d_0^3 \) in the observed frame, and has \( \Gamma = \Gamma_i \) at mass coordinate \( M_r = 2M_\odot \) in a SN progenitor. If we take a progenitor model of SN1998bw, at \( M_r = 2M_\odot \) the radial coordinate is \( r_0 = 2 \times 10^8 \) cm. From Eqs. (9) and (12), we find that \( d_0 \sim 10^{-1} \Gamma_a^{-1} (E_{51}/N' M_{\text{env}}(M_\odot))^{1/2} r_0 \) if we require \( \Gamma_f \gtrsim 100 \). Here \( \Gamma_a > 100 \) for Case I and \( \Gamma_a = \Gamma_i > 100 \) for Case II. The bullet mass is \( M_0 \sim \Gamma_0 d_0^3 \rho_0 \). Thus, using Eq. (5), one obtains a constraint \( \rho_i \gtrsim 10^8 \Gamma_a (N'/E_{51})^{1/2} M_{\text{env}}^{3/2}(M_\odot)(\text{g cm}^{-3}) \). This density is much larger than the progenitor’s density at this radius \( \rho(r_0) \sim 10^8 \text{g cm}^{-3} \), which means that the bullets must be produced deeper inside.

The GRB energy produced by each bullet is \( E_{\text{GRB}} \sim 10^{49} E_{51} N^{-1}(100/\Gamma_f) \) erg (in the external shock model). If \( E_{51} \lesssim 1 \), this is sufficiently large and not too large to explain the energy of GRB980425 (~\( 10^{48} f_b^2 \) erg, if its distance is ~40Mpc, where \( f_b \leq 1 \) is the beaming factor) which might be associated with SN1998bw.

The GRB980425 lasted about 30 seconds and this must be explained in our model, if it is produced by bullets ejected from the progenitor. The time scale of the GRB in this model is
determined by the time scale of the deceleration of the bullets, or by the life time of the central engine. If it is determined by the latter, we can assume anything because almost nothing is known about the central engine. In the following, assuming the external shock model, we discuss how the time scale is determined in the former case.

In our model, the bullet mass at breakout is \( M \simeq M_0 \Gamma_i / \Gamma_f \). Each bullet emits a GRB as it plows ISM mass of \( m_s \sim M / \Gamma_f \). The mass plowed depends on ISM density and on the bullet’s cross section. If the ISM is produced by wind mass loss from the progenitor, the number density can be estimated by \( n_{\text{ISM}}(r) = 1.5 \times 10^{36} \dot{M}_{-6} v_{20}^{-1} r^{-2} \text{ cm}^{-3} \), where \( \dot{M}_{-6} \) is the mass loss rate in units of \( 10^{-6} M_\odot \text{ yr}^{-1} \) and \( v_{20} \) is the wind velocity in units of \( 20 \text{ km s}^{-1} \). \( \dot{M}_{-6} / v_{20} \) is roughly \( \sim 0.1 \) for O stars and \( \sim 1 - 100 \) for red giant stars. The time evolution of the bullet size is uncertain. The radius may increase due to internal expansion or by merging with other bullets. Here we assume that the diameter of a bullet increases as \( \propto r^{1+k} \) with \( k \sim O(1) \). Suppose the diameter of the bullet is \( \sim 10^{-2} (E_{51}/N')^{1/2} \Gamma_f^{-1} R \) at \( r = R \), where \( R \) is the radius of the order of the progenitor radius. For the model of SN1998bw progenitor, \( R \sim 10^{11} \text{ cm} \). Then the time scale of the GRB duration, \( \tau \), is estimated by

\[
\tau = \frac{\Delta R}{2 \Gamma_f^2 c} \sim \frac{R_{11}}{\Gamma_f^2} \left( \frac{10^{11}}{\dot{M}_{-6} v_{20}^{-1} R_{11}} \right)^{1/1+2k} \text{ sec},
\]

where Eqs. (5) and (7) are used, \( \Delta R \) is the distance the bullet travels during GRB emission, and \( R_{11} \equiv R / 10^{11} \text{ cm} \). This result shows that if \( k = 0 \), \( \tau \) is independent of \( R \) and typically too long: \( \tau > 10^5 (100 / \Gamma_f)^2 \text{ sec} \). On the other hand, if \( k \sim 0.5 \), \( \tau \sim 30 \text{ sec} \) is possible. If \( k \) is larger, even a milli-second time scale may be possible. Therefore, we may conclude that GRB durations of milli-seconds to months may be realized with this model, depending on the parameters and situations. Especially, the expansion rate of the bullet radius is very important to determine the time scale. In this Letter the expansion rate was treated as a free parameter, and constrained it for successful GRB generation. However, the rate may be estimated if the pressure inside and outside the bullets is given. The evolution of bullets including their expansion rate will be further explored by performing hydrodynamical simulations in future work.

Finally we comment on the internal shock model for the GRB generation. In the last paragraph, we mentioned that the cross section of the bullets may be increased via merging with other bullets. If this is the case, the GRB may be produced by the internal shocks produced during merging. If the merging bullets grow larger than the angular diameter of \( \Gamma^{-1} \) before losing their kinetic energy, their subsequent evolution will be similar to the model by Kumar & Piran (1999). They considered collision of relativistic blobs with total kinetic energy \( 10^{52} \text{ erg} \) as the source of GRBs, and showed that the observed diversity of the GRB energy can be explained by the inhomogeneity of the blobs and the differences of lines of sight. In general, the efficiency of the kinetic to thermal energy conversion is much worse in the internal shock model than in the external shock model. However, the time scale of energy loss can be several orders of magnitudes smaller than the external shock model and this may be an advantage (e.g. Piran 1999).
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