CONIFOLD DEGENERATIONS OF FANO 3-FOLDS AS HYPERSURFACES IN TORIC VARIETIES

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ABSTRACT. There exist exactly 166 4-dimensional reflexive polytopes $\Delta$ such that the corresponding 4-dimensional Gorenstein toric Fano variety $\mathbb{P}_\Delta$ has at worst terminal singularities in codimension 3 and the anticanonical divisor of $\mathbb{P}_\Delta$ is divisible by 2 in its Picard group. For every such a polytope $\Delta$, one naturally obtains a family $\mathcal{F}(\Delta)$ of Fano hypersurfaces $X \subset \mathbb{P}_\Delta$ with at worst conifold singularities. A generic 3-dimensional Fano hypersurface $X \in \mathcal{F}(\Delta)$ can be interpreted as a flat conifold degeneration of some smooth Fano 3-folds $Y$ whose classification up to deformation was obtained by Iskovskikh, Mori and Mukai. In this case, both Fano varieties $X$ and $Y$ have the same Picard number $r$. Using toric mirror symmetry, we define a $r$-dimensional generalized hypergeometric power series $\Phi$ associated to the dual reflexive polytope $\Delta^*$. We show that if $r = 1$ then $\Phi$ is a normalized regular solution of a modular $D_3$-equation that appears in the Golyshhev correspondence. We expect that the power series $\Phi$ can be used to compute the small quantum cohomology ring of all Fano 3-folds $Y$ with the Picard number $r \geq 2$ if $Y$ admit a conifold degeneration $X \in \mathcal{F}(\Delta)$.

INTRODUCTION

A smooth projective $d$-dimensional algebraic variety $V$ is called Fano $d$-fold if the anticanonical class $-K_V$ is an ample Cartier divisor. The maximal integer $m$ such that $-K_V = mL$ for some Cartier divisor on $V$ is called the index of $V$.

It has been proved by Kollar, Mori and Myaoka that there exist only finitely many smooth Fano $d$-folds of fixed dimension $d$ up to deformation \cite{KMM92}. The classification of smooth Fano $d$-folds is known only for $d \leq 3$. There exist 17 types of Fano 3-folds with the Picard number 1 (they were classified by Iskovskikh \cite{Isk77,Isk78}) and 105 types of Fano 3-folds with the Picard number $\geq 2$ (see the classification due to Mori and Mukai \cite{MoMu81,MoMu03}). Among all 122 types of Fano 3-folds there are exactly 18 Fano 3-folds which are toric varieties (these varieties were classified by the first author \cite{Ba81} and Watanabe-Watanabe \cite{WaWa82}).

It is known that for any fixed dimension $d$ there exist only finitely many $d$-dimensional toric Fano varieties with at worst Gorenstein singularities \cite{Ba82}. These varieties are of particular interest because of their relation to mirror symmetry \cite{Ba94}. We also remark that there is a bijection between $d$-dimensional Gorenstein toric Fano varieties ans so called reflexive $d$-dimensional lattice polytopes. The second author together with Skarke has classified all reflexive polytopes of dimension 3 and 4 \cite{KS98,KS00}. This classification has been obtained by a computer and it contains of
4319 3-dimensional reflexive polytopes and 473,800,776 4-dimensional reflexive polytopes.

In this paper we connect the Iskovskikh-Mori-Mukai classification of Fano 3-folds to the classification of Fano hypersurfaces with conifold singularities in 4-dimensional Gorenstein toric Fano varieties $\mathbb{P}_\Delta$ of index 2 corresponding to some special 4-dimensional reflexive polytopes $\Delta$. Our approach is motivated by a theorem of Namikawa [Na97] that claims that every 3-dimensional Gorenstein Fano variety $X$ with at worst terminal singularities admits a flat smoothing to a Fano 3-fold $Y$. In this paper we construct many examples of such singular 3-dimensional Gorenstein Fano varieties as generic hypersurfaces in 4-dimensional Gorenstein toric Fano varieties $\mathbb{P}_\Delta$ corresponding to 4-dimensional reflexive polytopes $\Delta$ that satisfy the following two conditions:

1. $\Delta$ is isomorphic to $2\Delta'$ for some 4-dimensional lattice polytope $\Delta'$ (i.e., $\Delta$ is "divisible by 2");
2. all 2-dimensional faces of the dual reflexive polytope $\Delta^*$ are either basic triangles, or parallelograms consisting of two basic triangles.

Among 473,800,776 4-dimensional reflexive polytopes we find 5363 polytopes satisfying the first condition and 198,849 polytopes satisfying the second one. By intersecting these two sets, we obtain 166 4-dimensional reflexive polytopes $\Delta$ satisfying both conditions (1) and (2). Among 166 polytopes there are exactly 100 reflexive polytopes $\Delta \cong 2\Delta'$ such that $\Delta'$ that are pyramids over 3-dimensional reflexive polytopes $\Theta$ that define 3-dimensional toric Fano varieties $\mathbb{P}_\Theta$ with at worst conifold singularities (i.e. $\mathbb{P}_\Theta$ is a conifold toric degeneration of a smooth Fano 3-fold). We remark that these toric degenerations were classified independently by Nill [Ni05] and Galkin [Ga08, Ga12]. The interest to toric degenerations is motivated by the mirror symmetry for complete intersections in Fano varieties [BCKS98, BCKS00, Ba04]. On the other hand, toric degenerations are useful for constructing mirrors of Fano varieties via Landau-Ginzburg models [Pr07c].

1. Fano hypersurfaces

Let $M \cong \mathbb{Z}^d$ be a free abelian group of rank $d$ and $N := \text{Hom}(M, \mathbb{Z})$ be the dual group with the natural pairing $\langle *, * \rangle : M \times N \to \mathbb{Z}$. We consider also $\mathbb{R}$-vector spaces $M_\mathbb{R} := M \otimes \mathbb{R}$ and $N_\mathbb{R} := N \otimes \mathbb{R}$ together with the pairing $\langle *, * \rangle : M_\mathbb{R} \times N_\mathbb{R} \to \mathbb{R}$. Recall that a $d$-dimensional polytope $\Delta \subset M_\mathbb{R}$ is called reflexive if it contains $0 \in M$ in its interior, all vertices of $\Delta$ belong to the lattice $M \subset M_\mathbb{R}$, and all vertices of the dual polytope

$$\Delta^* := \{ y \in N_\mathbb{R} \mid \langle x, y \rangle \geq -1 \ \forall x \in \Delta \}$$

belong to the dual lattice $N \subset N_\mathbb{R}$. If $\Delta$ is reflexive then $\Delta^*$ is also reflexive and $(\Delta^*)^* = \Delta$. If $\Theta \subset \Delta$ is a proper face of a reflexive pooytope $\Delta$ then we set

$$\Theta^* := \{ y \in \Delta^* \mid \langle x, y \rangle = -1 \ \forall x \in \Theta \}$$
and call $\Theta^*$ the dual face of $\Delta$. One has $\dim \Theta + \dim \Theta^* = d - 1$. If $\Delta \subset M_\mathbb{R}$ is a $d$-dimensional reflexive polytope then we denote by $\Sigma(\Delta^*)$ (or simply by $\Sigma$) the complete fan consisting of all cones $\sigma(\Theta^*):= \mathbb{R}_{\geq 0} \Theta^* \subset N_\mathbb{R}$ where $\Theta^*$ runs over all proper faces $\Theta^*$ of the dual reflexive polytope $\Delta^*$. The fan $\Sigma(\Delta^*)$ defines a $d$-dimensional projective variety over $\mathbb{C}$ that we denote by $\mathbb{P}_\Delta$. One can define the projective toric variety $\mathbb{P}_\Delta$ also as Proj of the graded semigroup algebra over $\mathbb{C}$ corresponding to the graded monoid of all lattice points $(k, m) \in \mathbb{Z}_{\geq 0} \times M$ such that $m \in k\Delta$. There exists a natural stratification of $\mathbb{P}_\Delta$ by torus orbits $T_\Theta$ where $\Theta$ runs over all faces of $\Delta$. Then $T_\Theta = T_\Delta$ is the open dense orbit and $\dim T_\Theta = \dim \Theta$ $\forall \Theta \subset \Delta$. A generic ample divisor $X \subset \mathbb{P}_\Delta$ admit a stratification

$$X = \bigcup_{\dim \Theta > 0} X_\Theta$$

where $X_\Theta := X \cap T_\Theta$. The toroidal singularities of $X$ along $X_\Theta$ are determined by the cone $\sigma(\Theta^*)$. By the combinatorial characterisation of terminal singularities, one obtains that the Gorenstein singularities of $X$ along $X_\Theta$ are terminal if and only if all lattice points in the dual face $\Theta^* \subset \Delta^*$ are its vertices. It is no difficult to show that there exist exactly two 2-dimensional polygons $P \subset \mathbb{R}^2$ with vertices in $\mathbb{Z}^2$ (up to an $\mathbb{Z}$-isomorphism) such that $P \cap \mathbb{Z}^2$ is exactly the set of vertices of $P$:

- the triangle with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$;
- the unit square with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$.

This implies that Gorenstein singularities in codimension 3 are locally conifold singularities defined by the equation:

$$xy = zt \sim_c x^2 + y^2 + z^2 + t^2 = 0.$$

We remark that the anticanonical class $-K_{\mathbb{P}_\Delta}$ is divisible in $\text{Pic}(\mathbb{P}_\Delta)$ by some positive integer $k$ (i.e. $-K_{\mathbb{P}_\Delta} = kH$ for some $H \in \text{Pic}(\mathbb{P}_\Delta)$) if and only if the reflexive polytope $\Delta$ is isomorphic to $k\Delta'$ for some lattice polytope $\Delta' \subset M_\mathbb{R}$.

From now on we consider the case $d = 4$.

**Definition 1.1.** Let $\Delta$ be a 4-dimensional reflexive polytope such that $\Delta \cong 2\Delta'$ for some lattice polytope $\Delta' \subset M_\mathbb{R}$. We denote by $\mathcal{F}(\Delta)$ the family of Fano hypersurfaces in $\mathbb{P}_\Delta$ such that the Newton polytope of their equations equals $\Delta'$.

**Proposition 1.2.** A generic Fano hypersurface $X \in \mathcal{F}(\Delta)$ hast at worst Gorenstein terminal singularities if and only if every 2-dimensional face $\Theta^* \subset \Delta^*$ is a lattice polygon that is isomorphic to the standard triangle, or to the unit square.

Using the classification of all 4-dimensional reflexive polytopes and [12], we obtain

**Theorem 1.3.** There exist exactly 166 4-dimensional reflexive polytopes $\Delta \cong 2\Delta'$ such that a generic Fano hypersurface $X \in \mathcal{F}(\Delta)$ in 4-dimensional Gorenstein toric Fano variety $\mathbb{P}_\Delta$ has at worst terminal Gorenstein singularities.

On the other hand, there is a following statement due to Namikawa [Na97, Theorem 11]:
Theorem 1.4. Let $X$ be 3-dimensional Fano variety with Gorenstein terminal singularities. Then there is a flat deformation of $X$ to a smooth Fano 3-fold $Y$.

Corollary 1.5. Every 4-dimensional reflexive polytope $\Delta$ satisfying the conditions (1) and (2) gives rise to a deformation type of smooth Fano 3-folds $Y$ in the Iskovskikh-Mori-Mukai classification.

Our next purpose will be to explain how to compute the the topological invariants of the smoothing $Y$ via combinatorics of reflexive polytopes $\Delta$ and $\Delta^*$.

Since the degree of projective varieties remains unchanged under flat deformations we get:

**Proposition 1.6.** The anticanonical degree $(-K_Y)^3$ of the smoothing $Y$ of Fano hypersurfaces $X \in \mathcal{F}(\Delta)$ is equal to

$$4!\text{vol}(\Delta') = \frac{4!\text{vol}(\Delta)}{16}.$$ 

Our next interest is the Picard number of $Y$. It is important observation that in our situation also the Picard group remains unchanged under the flat deformation [JR06]. Therefore, it remains to compute the Picard number of the Fano hypersurface $X \subset \mathbb{P}_\Delta$. By Lefschetz-type arguments, the latter is equal to the Picard number of the toric variety $\mathbb{P}_\Delta$.

Let $\Delta \subset M_\mathbb{R}$ be a 4-dimensional reflexive polytope satisfying (1) and (2) and let $\{v_1, \ldots, v_n\}$ be the set of vertices of the dual reflexive polytope $\Delta^*$. Denote by $PL(\Delta^*)$ the sublattice in $\mathbb{Z}^n$ consisting of all $n$-tuples $(l_1, \ldots, l_n) \in \mathbb{Z}^n$ such that $l_{i_1} + l_{i_3} = l_{i_2} + l_{i_4}$ holds whenever $v_{i_1}, v_{i_2}, v_{i_3}, v_{i_4}$ are vertices of a 2-dimensional face $\Theta^*$ of $\Delta^*$ satisfying the equation $v_{i_1} + v_{i_3} = v_{i_2} + v_{i_4}$. We set

$$rk(\Delta^*) := n - \text{rank } PL(\Delta^*).$$

**Proposition 1.7.** Consider the monomorphism

$$\varphi : M \to \mathbb{Z}^n$$

$$m \mapsto (\langle m, v_1 \rangle, \ldots, \langle m, v_n \rangle)$$

Then the image of $\varphi$ is contained in $PL(\Delta^*) \subset \mathbb{Z}^n$ and the Picard group of the toric variety $\mathbb{P}_\Delta$ is isomorphic to the group

$$PL(\Delta)/\varphi(M).$$

In particular, the Picard number of the toric variety $\mathbb{P}_\Delta$ is equal to

$$n - 4 - rk(\Delta^*),$$

where $n = \text{vert}(\Delta^*)$ is the number of vertices of $\Delta^*$.

**Proof.** The statement follows from the standard computation of the Picard group of a toric variety [Ei92].
Corollary 1.8. The second Betti number of a Fano 3-fold $Y$ that admits a flat conifold degeneration to a Fano hypersurface $X \in \mathcal{F}(\Delta)$ is at most 5. Its distribution for 166 reflexive polytopes $\Delta$ is given by the following table:

| $B_2(Y)$ | 1  | 2  | 3  | 4  | 5  |
|----------|----|----|----|----|----|
| the number of polytopes $\Delta$ | 23 | 69 | 54 | 18 | 2  |

The last topological invariant that we want to compute is the Hodge number $h^{2,1}(Y) = 1/2B_3(Y)$. This number depends on the number of conifold singularities on the Fano hypersurface $X \in \mathcal{F}(\Delta)$.

Proposition 1.9. Let $sq(\Delta^*) := \sum_{\Theta^* \subset \Delta, \dim \Theta^* = 2} (|\Theta^* \cap N| - 3)$ be the number of 2-dimensional faces $\Theta^* \subset \Delta^*$ containing 4 vertices (i.e. the number of “squares”). Then the number of conifold points in $X$ is equal to

$$dp(\Delta^*) := \sum_{\Theta^* \subset \Delta, \dim \Theta^* = 2} (|\Theta^* \cap N| - 3)(|\Theta \cap M| - 1).$$

There exists a small resolution $\hat{X} \to X$ of these singularities such that $\rho(\hat{X}) - \rho(X) = \text{rk}(\Delta^*)$ where $\rho(V)$ denotes the Picard number of $V$.

Definition 1.10. Let us put $py(\Delta) := 1$ if $\Delta = 2\Delta'$ and $\Delta'$ is a pyramid over a 3-dimensional reflexive polytope and $py(\Delta) := 0$ otherwise.

Proposition 1.11.

$$h^{2,1}(Y) = \frac{1}{2}B_3(Y) = 1 + dp(\Delta^*) - \text{rk}(\Delta^*) - py(\Delta) = 1 - py(\Delta) + dp(\Delta^*) + \rho(X) - \rho(\hat{X}).$$

Proof. The smooth Fano 3-fold $Y$ is obtained from the smooth almost Fano 3-fold $\hat{X}$ by so called conifold transition. First we compute the Hodge number $h^{2,1}(X)$ of the Fano hypersurface $X \subset \mathbb{P}_\Delta$ using the formulas of Danilov-Khovanskii [DKh86]. We note that the Newton polytope of the equation of $X$ is $\Delta'$. There is no interior points in $\Delta'$ and there is exactly one interior point in $2\Delta' = \Delta$. A codimension 1 face of $\Delta'$ contains an interior lattice point if and only if $\Delta'$ is a pyramid over a 3-dimensional reflexive polytope. In this case only one facet has an interior lattice point. This shows that $h^{2,1}(X) = 1 - py(\Delta)$. Now we apply standard Clemens arguments to get

$$h^{2,1}(Y) = h^{2,1}(X) + dp(\Delta^*) + \rho(X) - \rho(\hat{X}).$$

2. The power series $\Phi$

Let $\Delta$ be a 4-dimensional reflexive polytopes satisfying the conditions (1) and (2) as above. Denote by $n := \text{vert}(\Delta^*)$ the number of vertices of the dual polytope $\Delta^*$. 
Consider the lattice of rank $n - 4$:

$$\Lambda(\Delta^*) := \{ \mathbf{k} = (k_1, \ldots, k_n) \in \mathbb{Z}^n \mid \sum_{i=1}^{n} k_i v_i = 0 \}.$$ 

One has the natural pairing

$$\text{PL}(\Delta^*) \times \Lambda(\Delta^*) \to \mathbb{Z}$$

$$(l, \mathbf{k}) \mapsto \sum_{i=1}^{n} l_i k_i$$

that vanish for all $l = (l_1, \ldots, l_n) \in \varphi(M)$, because for any $m \in M$ and any $\mathbf{k} = (k_1, \ldots, k_n) \in \Lambda(\Delta^*)$ one has

$$\sum_{i=1}^{n} k_i \langle m, v_i \rangle = \langle m, \sum_{i=1}^{n} k_i v_i \rangle = \langle m, 0 \rangle = 0.$$

So one obtains the pairing between $\text{Pic}(\mathbb{P}_\Delta) \cong \text{PL}(\Delta^*)/\varphi(M)$ and $\Lambda(\Delta^*)$. Let $\lambda_1, \ldots, \lambda_r \in \text{PL}(\Delta^*)/\varphi(M)$ be a $\mathbb{Z}$-basis. We denote by $\Lambda^+(\Delta^*)$ the semigroup $\Lambda(\Delta^*) \cap \mathbb{Z}_{\geq 0}^n$ and by $\kappa$ the element of $\text{PL}(\Delta^*)/\varphi(M) \cong \text{Pic}(\mathbb{P}_\Delta)$ such that $2\kappa$ ist the anticanonical class of $\mathbb{P}_\Delta$, i. e., $k_1 + \cdots + k_n = 2(\kappa, \mathbf{k})$. Define the multidimensional series $\Phi$ by the formula

$$\Phi(t_1, \ldots, t_r) = \sum_{\mathbf{k} \in \Lambda^+(\Delta^*)} \frac{((\kappa, \mathbf{k})!)^2}{k_1! \cdots k_n!} t_1^{(\lambda_1, \mathbf{k})} \cdots t_r^{(\lambda_r, \mathbf{k})}.$$ 

These series were first suggested in [BaSt95] in connection to the mirror symmetry for complete intersections in toric varieties. In our situation, it corresponds to mirrors of $K3$-surfaces that are complete intersections of two divisors in the 4-dimensional Gorenstein toric Fano variety $\mathbb{P}_\Delta$.

There is a specialization $\Phi_0(t)$ of $\Phi$ to a 1-parameter series corresponding to the class $\kappa \in \text{Pic}(\mathbb{P}_\Delta)$ that restricts to the anticanonical class in $\text{Pic}(X)$:

$$\Phi_0(t) := \sum_{\mathbf{k} \in \Lambda^+(\Delta^*)} \frac{((\kappa, \mathbf{k})!)^2}{k_1! \cdots k_n!} t^{(\kappa, \mathbf{k})}.$$ 

**Example 2.1.** There exists only one simplex $\Delta$ among 166 reflexive polytopes satisfying the conditions (1) and (2). The vertices of the dual reflexive simplex $\Delta^*$ satisfy the single relation

$$4v_0 + v_1 + v_2 + v_3 + v_4 = 0.$$ 

The corresponding hypersurface $X$ is isomorphic to $\mathbb{P}^3$ that considered as a hypersurface of degree 4 in the 4-dimensional weighted projective space $\mathbb{P}(4, 1, 1, 1, 1)$. So we have

$$\Phi_0(t) = \sum_{k \geq 0} \frac{((4k)!)^2}{(4k)! k!^4} t^{4k} = \sum_{k \geq 0} \frac{(4k)!}{(k!)^4} t^{4k}.$$
Example 2.2. There exist exactly 2 reflexive 4-dimensional polytopes from the list of 166 ones such that $\Delta^*$ has 6 vertices satisfying two independent relations

$$3v_0 + v_1 + v_3 + v_4 = 0, \quad 3v_0 + v_1 + v_2 + v_5 = 0$$

with the corresponding power series

$$\Phi_0(t) = \sum_{k \geq 0} \sum_{k_1 + k_2 = k} \frac{(3k)!}{(k!)(k_1!)^2(k_2!)^2} t^{3k} = \sum_{k \geq 0} \frac{(3k)!}{(k!)^5} t^{3k},$$

or the relations

$$v_0 + v_1 + v_3 + v_4 = 0, \quad v_0 + v_1 + v_2 + v_5 = 0$$

with the corresponding power series

$$\Phi_0(t) = \sum_{k \geq 0} \sum_{k_1 + k_2 = k} \frac{((2k)!)^2}{(k!)(k_1!)(k_2!)^2} t^{2k} = \sum_{k \geq 0} \frac{(2k)!}{(k!)^6} t^{2k}.$$

In the above calculations we used the equality

$$\sum_{k_1 + k_2 = k} \frac{(k!)^2}{(k_1!)(k_2!)^2} = \frac{(2k)!}{(k!)^2}.$$

3. Hypersurfaces with the Picard number 1

There exists exactly 23 4-dimensional reflexive polytopes $\Delta$ satisfying the conditions (1) and (2) such that a generic Fano hypersurface $X \in F(\Delta)$ has the Picard number 1.

We will use the following notations: $\deg := (-K_X)^3$, $h^{2,1} := h^{2,1}(Y)$, $rk := rk(\Delta^*)$, $dp := dp(\Delta^*)$, $sq := sq(\Delta^*)$, $py := py(\Delta)$, vert$(\Delta^*)$ denotes the number of vertices of $\Delta^*$. The deformation type of a Fano 3-fold $Y$ with the Picard number 1 is completely determined by the index $m$ and the integer $\deg / (2m^2)$. The pair $(m, \deg / (2m^2))$ is called the type of the Fano 3-fold $Y$. There exist exactly 17 types of smooth Fano 3-folds with the Picard number 1:

$$(4, 2), (3, 3)$$

$$(2, k), \quad 1 \leq k \leq 5$$

$$(1, k), \quad 1 \leq k \leq 9, \quad k = 11$$

Among 17 types there are 13 ones that admit conifold degenerations $X \in F(\Delta)$. There remaining 4 types are hypersurfaces or complete intersections in weighted projective spaces:

$$(2, 1) : V_6 \subset P(3, 2, 1, 1, 1)$$

$$(1, 1) : V_6 \subset P(3, 1, 1, 1, 1)$$

$$(1, 2) : V_4 \subset P^4$$

$$(1, 3) : V_{2,3} \subset P^5$$

The table below describes vertices of 23 dual polytopes $\Delta^*$ together with its properties and topological invariants of $Y$. 
Type (4, 2)

$V(1)$: $\deg = 64$, $h^{2,1} = 0$, $rk = 0$, $dp = 0$, $sq = 0$, $py = 1$, $\vert \Delta^* \vert = 5$

\[
\begin{array}{cccc}
1 & 0 & 0 & -4 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 1 & -1 \\
\end{array}
\]

Type (3, 3)

$V(2)$: $\deg = 54$, $h^{2,1} = 0$, $rk = 1$, $dp = 1$, $py = 1$, $\vert \Delta^* \vert = 6$

\[
\begin{array}{cccc}
1 & 0 & 0 & -3 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 1 & -1 \\
\end{array}
\]

Type (2, 2)

$V(3)$: $\deg = 16$, $h^{2,1} = 10$, $rk = 3$, $sq = 6$, $dp = 12$, $py = 0$, $\vert \Delta^* \vert = 8$

\[
\begin{array}{cccc}
1 & -1 & -1 & -1 \\
1 & 0 & 2 & 0 \\
-1 & 0 & -1 & 1 \\
-1 & 1 & -1 & 1 \\
\end{array}
\]

Type (2, 3)

$V(4)$: $\deg = 24$, $h^{2,1} = 5$, $rk = 2$, $sq = 3$, $dp = 6$, $py = 0$, $\vert \Delta^* \vert = 7$

\[
\begin{array}{cccc}
1 & -1 & 1 & -1 \\
0 & 2 & -1 & 0 \\
0 & -1 & 0 & 1 \\
1 & 1 & 1 & 0 \\
\end{array}
\]

Type (2, 4)

$V(5)$: $\deg = 32$, $h^{2,1} = 2$, $rk = 1$, $sq = 1$, $dp = 2$, $py = 0$, $\vert \Delta^* \vert = 6$

\[
\begin{array}{cccc}
1 & 0 & 0 & -1 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{array}
\]

Type (2, 5)

$V(6)$: $\deg = 32$, $h^{2,1} = 2$, $rk = 4$, $sq = 6$, $dp = 6$, $py = 1$, $\vert \Delta^* \vert = 9$

\[
\begin{array}{cccc}
1 & -1 & 1 & -1 \\
0 & 2 & -1 & 0 \\
1 & -1 & 0 & 1 \\
1 & 1 & 0 & 0 \\
\end{array}
\]

Type (2, 5)

$V(7)$: $\deg = 40$, $h^{2,1} = 0$, $rk = 3$, $sq = 3$, $dp = 3$, $py = 1$, $\vert \Delta^* \vert = 8$

\[
\begin{array}{cccc}
0 & -4 & -2 & 1 \\
0 & -1 & -1 & 0 \\
0 & -1 & 0 & 1 \\
1 & -1 & 0 & 0 \\
\end{array}
\]
| Type (1, 4): |
| V(8): $\deg = 8, h^{2,1} = 14, rk = 11, sq = 24, dp = 24, py = 0, \vert(\Delta^*) = 16$: |
| $\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \\
| \hline \\
| 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
| -1 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 \\
| -1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 \\
| -1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 \\
| \hline \\
| \end{array}$ |

| Type (1, 5): |
| V(9): $\deg = 10, h^{2,1} = 12, rk = 9, sq = 18, dp = 18, py = 0, \vert(\Delta^*) = 14$: |
| $\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \\
| \hline \\
| 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 \\
| -1 & 1 & 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\
| -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 0 & -1 & 0 \\
| -1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 \\
| \hline \\
| \end{array}$ |

| Type (1, 6): |
| V(10): $\deg = 12, h^{2,1} = 7, rk = 7, sq = 13, dp = 13, py = 0, \vert(\Delta^*) = 12$: |
| $\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \\
| \hline \\
| 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 \\
| -1 & 0 & 1 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & -1 \\
| 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & -1 \\
| -1 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
| \hline \\
| \end{array}$ |

| Type (1, 7): |
| V(11): $\deg = 12, h^{2,1} = 7, rk = 8, sq = 14, dp = 14, py = 0, \vert(\Delta^*) = 13$: |
| $\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \\
| \hline \\
| 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 \\
| -1 & 1 & 0 & -1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\
| -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \\
| -1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
| \hline \\
| \end{array}$ |

| Type (1, 8): |
| V(12): $\deg = 14, h^{2,1} = 5, rk = 6, sq = 9, dp = 10, py = 0, \vert(\Delta^*) = 11$: |
| $\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \\
| \hline \\
| 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
| -1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\
| 0 & 1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 \\
| -1 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
| \hline \\
| \end{array}$ |

| Type (1, 9): |
| V(13): $\deg = 14, h^{2,1} = 5, rk = 6, sq = 10, dp = 10, py = 0, \vert(\Delta^*) = 11$: |
| $\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \\
| \hline \\
| 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 & -1 & -1 \\
| 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 \\
| 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 1 \\
| -1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & -1 \\
| \hline \\
| \end{array}$ |

| Type (1, 10): |
| V(14): $\deg = 14, h^{2,1} = 5, rk = 7, sq = 11, dp = 11, py = 0, \vert(\Delta^*) = 12$: |
| $\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \\
| \hline \\
| 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 \\
| -1 & 1 & 0 & -1 & 1 & 0 & -1 & 0 & 1 & 0 & 1 \\
| -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \\
| -1 & 1 & 1 & 0 & 1 & 1 & 0 & -1 & 0 & 0 & 0 \\
| \hline \\
| \end{array}$ |

| Type (1, 11): |
| V(15): $\deg = 16, h^{2,1} = 3, rk = 5, sq = 7, dp = 7, py = 0, \vert(\Delta^*) = 10$: |
| $\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|} \\
| \hline \\
| -1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
| -1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
| -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \\
| -1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
| \hline \\
| \end{array}$ |
There are 8 types of smooth Fano 3-folds with the Picard number 1 that do not admit a toric conifold degeneration, but admit a conifold degeneration to a hypersurface in a toric variety. These types are the following: (2, 2), (2, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9).
4. The Golyshев correspondence

In [Go04] Golyshев has discovered a wonderful bijective correspondence between 17 types of Fano 3-folds with the Picard number 1 and 17 types of differential equations of D3-type satisfying some modularity conditions.

Let \( Y \) be a Fano 3-fold with Picard number 1. We put \( H := -K_Y \) and define the \( 4 \times 4 \)-matrix

\[
A := \begin{pmatrix}
a_{00} & a_{01} & a_{02} & a_{03} \\
1 & a_{11} & a_{12} & a_{13} \\
0 & 1 & a_{22} & a_{23} \\
0 & 0 & 1 & a_{33}
\end{pmatrix}
\]

where

\[
a_{ij} = a_{3-j,3-i} = \frac{j - i + 1}{(-K_Y)^3} \langle H^{3-i}, H^j \rangle_{j-i+1}.
\]

and \( \langle H^{3-i}, H^j \rangle_{j-i+1} \) is the number of maps \( f : \mathbb{P}^1 \to Y \) of degree \( (j - i + 1) \) such that \( f(0) \in L_{3-i}, f(\infty) \in L_j \) (here \( L_k \) is the subvariety of codimension \( k \) representing the cohomology class \( H^k \)). The matrix \( A \) is called counting matrix of \( Y \). One considers the equivalence class of such matrices up to adding a scalar matrix: \( A \sim \lambda E_4 + A \). The matrix \( A \) defines a D3-differential operator \( D_A \) expressed as polynomial in \( D = t \frac{\partial}{\partial t} \):

\[
D_A := D^3 - t(2D + 1) \left( (a_{00} + a_{11})D^2 + (a_{00} + a_{11})D + a_{00} \right) + t^2(D + 1) \left( (a_{11}^2 + a_{00}^2 + 4a_{11}a_{00} - a_{12} - 2a_{01})D^2 + (8a_{11}a_{00} - 2a_{12}D - 4a_{01} + 2a_{11}^2)D + 6a_{11}a_{00} - 4a_{01} \right) - t^3(D + 3)(D + 2)(D + 1)(a_{00}^2a_{11} + a_{11}^2a_{00} - a_{12}a_{00} + a_{02} - a_{11}a_{01} - a_{01}a_{00}) + t^4(D + 3)(D + 2)(D + 1)(-a_{00}^2a_{12} + 2a_{00}a_{12} + a_{00}^2a_{11}^2 - a_{03} + a_{01}^2 - 2a_{01}a_{11}a_{00})
\]

The D3-operators corresponding to equivalent matrices are called equivalent to each other. The following result is due to Golyshев [Go04]:

**Theorem 4.1.** There exists exactly 17 types of D3-operators \( D_A \) such that the corresponding Picard-Fuchs equation \( D_A F(t) = 0 \) comes from the degree d cyclic covering of a modular family over the universal elliptic curve with level \( N \), so called \( (d, N) \)-modular family. The set of pairs of integers \( (d, N) \) is exactly the set of all types smooth Fano 3-folds \( Y \) with the Picard number 1.

It was proved case by case that the counting matrix for 17 \( (d, N) \)-modular families is equivalent to the counting matrix of the Fano 3-fold \( Y \) (see [Pr07a, Pr07b]).

Our purpose was to find D3-operators from conifold degenerations \( X \in \mathcal{F}(\Delta) \) as operators that annihilate the hypergeometric series \( \Phi_0(t) \) defined in the previous section.

Our result is the following:
Theorem 4.2. For all 23 4-dimensional reflexive polytopes $\Delta$ such that the Fano hypersurface $X \in F(\Delta)$ has the Picard number 1, the power series $\Phi_0(t)$ satisfies a $D_3$-equation. The matrices of these $D_3$-equations are equivalent to the matrices in the Golyshev's list (see [Go07]) and they coincide with them except for the case $(1, 11)$ where the operator has the form

$$D^3 - 2tD(1 + D)(1 + 2D) - 8t^2(1 + D)(12 + 22D + 11D^2) - 150t^3(1 + D)(2 + 2D)(3 + 2D) - 304t^4(1 + D)(2 + D)(3 + D)$$

and the corresponding matrix is

$$
\begin{pmatrix}
0 & 24 & 198 & 880 \\
1 & 2 & 44 & 198 \\
0 & 1 & 2 & 24 \\
0 & 0 & 1 & 0
\end{pmatrix}
$$

We expect that the multidimensional power series $\Phi(t_1, \ldots, t_r)$ can be used in a similar way to get Gromov-Witten invariant of Fano 3-folds $Y$ with the Picard number $r \geq 2$ that admit a degeneration to a Fano hypersurface $X \in F(\Delta)$. For this purpose we present the list of all $143 = 166 - 23$ polytopes in the case $r \geq 2$ and their invariants in the remaining sections.

5. $B_2 = 2$

| V(24) | $\deg = 12$, $h^{1,2} = 9$, $rk = 6$, $sq = 13$, $dp = 14$, $py = 0$, $\vert(\Delta^*)\vert = 12$: |
|-------|---------------------------------|
| -1    | -1                             |
| 0     | 0                              |
| 0     | 0                              |
| 0     | 1                              |

| V(25) | $\deg = 12$, $h^{1,2} = 9$, $rk = 6$, $sq = 14$, $dp = 14$, $py = 0$, $\vert(\Delta^*)\vert = 12$: |
|-------|---------------------------------|
| -1    | 1                              |
| 0     | 1                              |
| 0     | 1                              |
| -1    | -1                             |

| V(26) | $\deg = 14$, $h^{1,2} = 5$, $rk = 5$, $sq = 9$, $dp = 9$, $py = 0$, $\vert(\Delta^*)\vert = 11$: |
|-------|---------------------------------|
| 0     | 0                              |
| 0     | 1                              |
| 0     | 0                              |
| 0     | 1                              |

| V(27) | $\deg = 14$, $h^{1,2} = 9$, $rk = 4$, $sq = 9$, $dp = 12$, $py = 0$, $\vert(\Delta^*)\vert = 10$: |
|-------|---------------------------------|
| 0     | 0                              |
| 0     | 0                              |
| 0     | 0                              |
| 0     | 1                              |

| V(28) | $\deg = 16$, $h^{1,2} = 3$, $rk = 4$, $sq = 5$, $dp = 6$, $py = 0$, $\vert(\Delta^*)\vert = 10$: |
|-------|---------------------------------|
| 0     | 0                              |
| 0     | 0                              |
| 0     | 0                              |
| 0     | 0                              |
| 0     | 0                              |
| 0     | 0                              |
| 0     | 0                              |
| 0     | 0                              |
| 0     | 0                              |
| 0     | 0                              |


\[ V(29): \text{deg} = 16, h^{1,2} = 3, \, rk = 4, \, sq = 6, \, dp = 6, \, py = 0, \, \text{vert}(\Delta^*) = 10: \]
\[
\begin{array}{cccccccc}
0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[ V(30): \text{deg} = 16, h^{1,2} = 5, \, rk = 4, \, sq = 8, \, dp = 8, \, py = 0, \, \text{vert}(\Delta^*) = 10: \]
\[
\begin{array}{cccccccc}
-1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\
-1 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\
-1 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 \\
\end{array}
\]

\[ V(31): \text{deg} = 16, h^{1,2} = 5, \, rk = 4, \, sq = 7, \, dp = 8, \, py = 0, \, \text{vert}(\Delta^*) = 10: \]
\[
\begin{array}{cccccccc}
0 & 0 & -1 & -1 & 1 & 0 & 0 & -1 & 0 & 1 \\
1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\
\end{array}
\]

\[ V(32): \text{deg} = 16, h^{1,2} = 5, \, rk = 5, \, sq = 9, \, dp = 9, \, py = 0, \, \text{vert}(\Delta^*) = 11: \]
\[
\begin{array}{cccccccc}
0 & 0 & 0 & -1 & 0 & -1 & -1 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 & -1 & 1 & 0 \\
-1 & 1 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\
\end{array}
\]

\[ V(33): \text{deg} = 16, h^{1,2} = 5, \, rk = 5, \, sq = 9, \, dp = 9, \, py = 0, \, \text{vert}(\Delta^*) = 11: \]
\[
\begin{array}{cccccccc}
0 & -1 & -1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & -1 & 0 & -1 & -1 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 \\
\end{array}
\]

\[ V(34): \text{deg} = 18, h^{1,2} = 5, \, rk = 3, \, sq = 5, \, dp = 7, \, py = 0, \, \text{vert}(\Delta^*) = 9: \]
\[
\begin{array}{cccccccc}
-1 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & -1 & 1 & 0 & 1 & -1 & 0 & 0 \\
\end{array}
\]

\[ V(35): \text{deg} = 18, h^{1,2} = 5, \, rk = 4, \, sq = 7, \, dp = 8, \, py = 0, \, \text{vert}(\Delta^*) = 10: \]
\[
\begin{array}{cccccccc}
-1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 \\
\end{array}
\]

\[ V(36): \text{deg} = 20, h^{1,2} = 1, \, rk = 3, \, sq = 3, \, dp = 3, \, py = 0, \, \text{vert}(\Delta^*) = 9: \]
\[
\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[ V(37): \text{deg} = 20, h^{1,2} = 2, \, rk = 3, \, sq = 4, \, dp = 4, \, py = 0, \, \text{vert}(\Delta^*) = 9: \]
\[
\begin{array}{cccccccc}
-1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \\
0 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
\end{array}
\]

\[ V(38): \text{deg} = 20, h^{1,2} = 2, \, rk = 3, \, sq = 3, \, dp = 4, \, py = 0, \, \text{vert}(\Delta^*) = 9: \]
\[
\begin{array}{cccccccc}
-1 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\
-1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
\section*{V(49): deg = 20, $h^{1,2} = 2$, $rk = 4$, $sq = 5$, $dp = 5$, $py = 0$, $\text{vert}(\Delta^*) = 10$:}

\begin{verbatim}
-1 0 0 -1 0 -1 -2 0 1 0
0 1 0 -1 -1 0 0 0 0 0
-1 0 0 -1 0 0 -1 -1 0 1
-1 0 1 0 0 0 -1 0 0 0
\end{verbatim}

\section*{V(50): deg = 20, $h^{1,2} = 3$, $rk = 2$, $sq = 2$, $dp = 4$, $py = 0$, $\text{vert}(\Delta^*) = 8$:}

\begin{verbatim}
0 0 1 0 0 -1 -1 1
-1 0 0 0 1 0 -1 0
0 0 0 1 0 0 0 -1
0 1 0 0 0 0 0 1
\end{verbatim}

\section*{V(51): deg = 20, $h^{1,2} = 3$, $rk = 3$, $sq = 5$, $dp = 5$, $py = 0$, $\text{vert}(\Delta^*) = 9$:}

\begin{verbatim}
-1 0 -1 -1 1 0 0 1 0
0 1 0 -1 1 0 0 -1
-1 0 0 0 0 1 0 -1
-1 0 0 -1 0 0 1 0 0
\end{verbatim}

\section*{V(52): deg = 20, $h^{1,2} = 3$, $rk = 3$, $sq = 4$, $dp = 5$, $py = 0$, $\text{vert}(\Delta^*) = 9$:}

\begin{verbatim}
-1 0 -1 -1 1 0 0 1 0
0 0 0 -1 1 0 0 -1
-1 0 0 -1 0 1 0 -1
-1 1 0 0 0 0 0 -1
\end{verbatim}

\section*{V(53): deg = 20, $h^{1,2} = 3$, $rk = 4$, $sq = 6$, $dp = 6$, $py = 0$, $\text{vert}(\Delta^*) = 10$:}

\begin{verbatim}
-1 0 0 -1 1 0 1 -1 0 0
-1 -1 0 0 1 0 0 -1 -1
0 1 1 -1 0 0 0 -1 -1
-1 0 0 -1 0 0 0 1 0
\end{verbatim}

\section*{V(54): deg = 20, $h^{1,2} = 3$, $rk = 4$, $sq = 6$, $dp = 6$, $py = 0$, $\text{vert}(\Delta^*) = 10$:}

\begin{verbatim}
-1 0 -1 0 -1 -2 -1 0 0 1
-1 1 -1 0 0 -1 0 -1 0 0
0 0 -1 0 0 -1 -1 0 1 0
-1 0 0 1 0 -1 -1 0 0 0
\end{verbatim}

\section*{V(55): deg = 20, $h^{1,2} = 3$, $rk = 9$, $sq = 12$, $dp = 12$, $py = 1$, $\text{vert}(\Delta^*) = 15$:}

\begin{verbatim}
-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 1
1 1 0 0 1 2 2 2 2 2 1 0 0 1 1 -1
1 0 1 0 0 0 1 2 1 1 2 2 2 -1
1 0 0 1 0 0 0 1 1 2 2 2 -1
\end{verbatim}

\section*{V(56): deg = 22, $h^{1,2} = 2$, $rk = 2$, $sq = 2$, $dp = 3$, $py = 0$, $\text{vert}(\Delta^*) = 8$:}

\begin{verbatim}
1 0 1 0 0 0 0 -1 0
0 0 0 1 0 0 0 -1
-1 0 0 0 -1 1 0 0
-1 1 0 0 -1 0 0 0
\end{verbatim}

\section*{V(57): deg = 22, $h^{1,2} = 2$, $rk = 3$, $sq = 4$, $dp = 4$, $py = 0$, $\text{vert}(\Delta^*) = 9$:}

\begin{verbatim}
-1 0 1 0 0 0 -1 -1 -2
0 1 0 0 0 -1 0 -1 -1
-1 0 0 1 0 0 0 -1 -1
-1 0 0 0 1 0 0 0 -1
\end{verbatim}

\section*{V(58): deg = 22, $h^{1,2} = 2$, $rk = 4$, $sq = 5$, $dp = 5$, $py = 0$, $\text{vert}(\Delta^*) = 10$:}

\begin{verbatim}
-1 0 0 1 0 0 -1 -2 0 -1
-1 0 1 0 0 0 -1 -1 -2
0 1 0 0 0 -1 0 -1 -1
-1 0 0 0 1 0 0 -1 0 -1
\end{verbatim}
V(49): deg = 22, \( h^{1,2} = 4 \), \( rk = 2 \), \( sq = 3 \), \( dp = 5 \), \( py = 0 \), \( vert(\Delta^*) = 8 \):
\[
\begin{pmatrix}
-1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 & -1 & -1 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\
\end{pmatrix}
\]

V(50): deg = 22, \( h^{1,2} = 4 \), \( rk = 3 \), \( sq = 6 \), \( dp = 6 \), \( py = 0 \), \( vert(\Delta^*) = 9 \):
\[
\begin{pmatrix}
-2 & -1 & 0 & -1 & -1 & -1 & 0 & 0 & 1 \\
-1 & 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 \\
-1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 \\
-1 & 0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 \\
\end{pmatrix}
\]

V(51): deg = 24, \( h^{1,2} = 1 \), \( rk = 2 \), \( sq = 2 \), \( dp = 2 \), \( py = 0 \), \( vert(\Delta^*) = 8 \):
\[
\begin{pmatrix}
-1 & 0 & -1 & -1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & -1 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

V(52): deg = 24, \( h^{1,2} = 1 \), \( rk = 2 \), \( sq = 2 \), \( dp = 2 \), \( py = 0 \), \( vert(\Delta^*) = 8 \):
\[
\begin{pmatrix}
0 & 0 & 0 & -1 & -1 & -1 & 1 & 0 \\
-1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
\end{pmatrix}
\]

V(53): deg = 24, \( h^{1,2} = 1 \), \( rk = 3 \), \( sq = 3 \), \( dp = 3 \), \( py = 0 \), \( vert(\Delta^*) = 9 \):
\[
\begin{pmatrix}
-1 & 0 & 0 & -1 & -1 & -1 & -1 & -2 & 0 & 0 & 1 \\
-1 & 0 & 0 & -1 & 0 & -1 & -1 & -1 & 1 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

V(54): deg = 24, \( h^{1,2} = 1 \), \( rk = 7 \), \( sq = 8 \), \( dp = 8 \), \( py = 1 \), \( vert(\Delta^*) = 13 \):
\[
\begin{pmatrix}
-1 & 0 & 1 & 0 & -1 & -3 & -2 & -1 & -2 & 0 & -1 & -2 & 2 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & -1 \\
\end{pmatrix}
\]

V(55): deg = 24, \( h^{1,2} = 2 \), \( rk = 2 \), \( sq = 2 \), \( dp = 3 \), \( py = 0 \), \( vert(\Delta^*) = 8 \):
\[
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 \\
-1 & 0 & 0 & 1 & 0 & 0 & -1 & -2 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

V(56): deg = 24, \( h^{1,2} = 2 \), \( rk = 3 \), \( sq = 3 \), \( dp = 4 \), \( py = 0 \), \( vert(\Delta^*) = 9 \):
\[
\begin{pmatrix}
0 & -1 & 0 & -1 & -2 & -1 & 0 & 0 & 1 \\
0 & -1 & -1 & 0 & -1 & -2 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & -1 & -1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\
\end{pmatrix}
\]

V(57): deg = 26, \( h^{1,2} = 0 \), \( rk = 6 \), \( sq = 6 \), \( dp = 6 \), \( py = 1 \), \( vert(\Delta^*) = 12 \):
\[
\begin{pmatrix}
1 & 0 & -1 & 0 & -2 & -2 & 0 & 1 & -1 & -2 & -2 & 0 \\
0 & -1 & -1 & 0 & 0 & 1 & 1 & 0 & -1 & -1 & 0 & 0 \\
0 & 1 & 0 & 1 & -1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & -1 & 1 \\
\end{pmatrix}
\]

V(58): deg = 26, \( h^{1,2} = 0 \), \( rk = 6 \), \( sq = 6 \), \( dp = 6 \), \( py = 1 \), \( vert(\Delta^*) = 12 \):
\[
\begin{pmatrix}
-1 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & -3 & -2 & -2 & 2 \\
1 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\
1 & -1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 \\
\end{pmatrix}
\]
\[
\begin{align*}
V(59): \ deg = 26, \ h^{1,2} = 2, \ rk = 1, \ sq = 1, \ dp = 2, \ py = 0, \ \text{vert}(\Delta^*) = 7: \\
-1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & -1 \\
-1 & 0 & 0 & 1 & 0 & -1 & -1 \\
-1 & 0 & 0 & 0 & 0 & 0 & -1 \\
\end{align*}
\]

\[
\begin{align*}
V(60): \ deg = 26, \ h^{1,2} = 2, \ rk = 2, \ sq = 3, \ dp = 3, \ py = 0, \ \text{vert}(\Delta^*) = 8: \\
-1 & -1 & 0 & 0 & 1 & 0 & -1 & -1 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & -1 \\
-1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
\end{align*}
\]

\[
\begin{align*}
V(61): \ deg = 26, \ h^{1,2} = 2, \ rk = 6, \ sq = 8, \ dp = 8, \ py = 1, \ \text{vert}(\Delta^*) = 12: \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 0 & 0 & 2 & 1 & 1 & 0 & 2 & 1 & -1 \\
0 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 1 & 0 & -1 \\
1 & 2 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 2 & 1 & -1 \\
\end{align*}
\]

\[
\begin{align*}
V(62): \ deg = 28, \ h^{1,2} = 0, \ rk = 5, \ sq = 5, \ dp = 5, \ py = 1, \ \text{vert}(\Delta^*) = 11: \\
1 & -1 & 0 & 0 & -2 & -2 & 0 & 1 & 0 & -2 & 0 & -2 \\
0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 1 & -1 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{align*}
\]

\[
\begin{align*}
V(63): \ deg = 28, \ h^{1,2} = 0, \ rk = 5, \ sq = 5, \ dp = 5, \ py = 1, \ \text{vert}(\Delta^*) = 11: \\
-1 & 0 & 1 & 0 & -2 & -1 & -3 & -1 & -2 & -2 \\
1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & -1 & -1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
\end{align*}
\]

\[
\begin{align*}
V(64): \ deg = 28, \ h^{1,2} = 0, \ rk = 6, \ sq = 6, \ dp = 6, \ py = 1, \ \text{vert}(\Delta^*) = 12: \\
-1 & 0 & -1 & 0 & 0 & -2 & -1 & 0 & 0 & -1 & 1 & -1 \\
0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{align*}
\]

\[
\begin{align*}
V(65): \ deg = 30, \ h^{1,2} = 0, \ rk = 4, \ sq = 4, \ dp = 4, \ py = 1, \ \text{vert}(\Delta^*) = 10: \\
1 & 1 & 0 & 0 & -2 & 0 & -2 & -1 & -2 & -2 \\
0 & 1 & 0 & 1 & -1 & 0 & -1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & -1 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{align*}
\]

\[
\begin{align*}
V(66): \ deg = 30, \ h^{1,2} = 0, \ rk = 4, \ sq = 4, \ dp = 4, \ py = 1, \ \text{vert}(\Delta^*) = 10: \\
-1 & 0 & -2 & 0 & 0 & -1 & 0 & 0 & -1 & -2 \\
1 & 1 & -1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & -1 & -1 \\
-1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 \\
\end{align*}
\]

\[
\begin{align*}
V(67): \ deg = 30, \ h^{1,2} = 0, \ rk = 5, \ sq = 5, \ dp = 5, \ py = 1, \ \text{vert}(\Delta^*) = 11: \\
-2 & 0 & -1 & -1 & 0 & 0 & 0 & -1 & -2 \\
1 & 0 & -1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \\
\end{align*}
\]

\[
\begin{align*}
V(68): \ deg = 30, \ h^{1,2} = 1, \ rk = 1, \ sq = 1, \ dp = 1, \ py = 0, \ \text{ vert}(\Delta^*) = 7: \\
1 & 0 & 1 & 0 & 0 & 1 & 1 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 \\
\end{align*}
\]
| V(69): deg = 30, $h^{1,2} = 1$, $rk = 4$, sq = 5, $dp = 5$, $py = 1$, $\vert\Delta^*\vert = 10$: |
|---|---|---|---|---|
| 1 0 0 -2 -2 1 0 0 -2 -2 |
| 0 0 0 -1 0 0 -1 1 0 1 |
| 0 1 0 0 -1 1 1 0 0 -1 |
| 0 0 1 0 0 1 1 0 -1 -1 |

| V(70): deg = 32, $h^{1,2} = 1$, $rk = 0$, sq = 0, $dp = 0$, $py = 0$, $\vert\Delta^*\vert = 6$: |
|---|---|---|---|---|
| -1 0 0 0 1 0 |
| 0 1 0 0 0 -1 |
| 0 0 1 0 0 -1 |
| 0 1 0 0 0 -1 |

| V(71): deg = 32, $h^{1,2} = 1$, $rk = 1$, sq = 1, $dp = 1$, $py = 0$, $\vert\Delta^*\vert = 7$: |
|---|---|---|---|---|
| -2 0 0 -1 0 0 1 |
| -1 0 0 -1 1 0 0 |
| -1 0 0 -1 0 1 0 |
| -1 1 0 0 0 0 0 |

| V(72): deg = 32, $h^{1,2} = 1$, $rk = 3$, sq = 4, $dp = 4$, $py = 1$, $\vert\Delta^*\vert = 9$: |
|---|---|---|---|---|
| -3 0 -1 -2 0 0 -2 0 -3 1 |
| -1 -1 -1 -1 0 1 0 0 0 0 |
| 0 1 1 0 1 0 1 0 -1 0 |
| -1 1 0 0 0 0 0 1 -1 0 |

| V(73): deg = 32, $h^{1,2} = 0$, $rk = 3$, sq = 3, $dp = 3$, $py = 1$, $\vert\Delta^*\vert = 9$: |
|---|---|---|---|---|
| 1 0 1 -2 0 0 -2 -4 -2 |
| 0 0 1 0 1 0 -1 -1 0 |
| 0 0 1 -1 0 1 0 -1 0 |
| 0 1 0 0 0 0 0 -1 -1 |

| V(74): deg = 34, $h^{1,2} = 0$, $rk = 4$, sq = 4, $dp = 4$, $py = 1$, $\vert\Delta^*\vert = 10$: |
|---|---|---|---|---|
| -3 -2 -4 0 -1 0 1 -2 -2 0 |
| -1 0 -1 0 -1 1 0 0 -1 0 |
| 0 0 -1 1 1 0 0 -1 0 0 |
| -1 1 0 0 0 0 0 -1 0 0 |

| V(75): deg = 34, $h^{1,2} = 0$, $rk = 5$, sq = 5, $dp = 5$, $py = 1$, $\vert\Delta^*\vert = 11$: |
|---|---|---|---|---|
| -5 0 -2 -3 -2 -1 0 0 -3 -2 1 |
| -1 0 -1 0 -1 1 0 0 -1 0 |
| 0 0 -1 1 1 0 0 -1 0 0 |
| 0 1 0 0 0 0 0 -1 -1 1 |

| V(76): deg = 38, $h^{1,2} = 0$, $rk = 2$, sq = 2, $dp = 2$, $py = 1$, $\vert\Delta^*\vert = 8$: |
|---|---|---|---|---|
| -3 0 1 0 -1 -2 0 -3 |
| 0 1 0 1 1 -1 0 -1 |
| -1 0 0 1 0 0 0 -1 |
| -1 0 0 0 -1 0 1 0 |

| V(77): deg = 38, $h^{1,2} = 0$, $rk = 3$, sq = 3, $dp = 3$, $py = 1$, $\vert\Delta^*\vert = 9$: |
|---|---|---|---|---|
| -2 -1 -1 -3 0 0 1 0 -2 |
| 1 3 1 0 1 0 0 -1 |
| -1 -1 0 1 0 0 1 0 |
| -1 0 -1 -1 0 1 0 0 |

| V(78): deg = 38, $h^{1,2} = 0$, $rk = 3$, sq = 3, $dp = 3$, $py = 1$, $\vert\Delta^*\vert = 9$: |
|---|---|---|---|---|
| -2 -1 -1 -3 0 0 1 0 -2 |
| 3 3 1 0 1 0 0 -1 |
| -1 -1 0 1 0 0 1 0 |
| -1 0 -1 -1 0 1 0 0 |
V(79): $\deg = 38$, $h^1,2 = 0$, $rk = 3$, $sq = 3$, $dp = 3$, $py = 1$, $\text{vert}(\Delta^*) = 9$:

\[
\begin{bmatrix}
1 & 0 & 1 \\
0 & 0 & -1 \\
0 & 1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
-1 & 0 & 1 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 1 & 1 \\
0 & 0 & 1 \\
0 & 1 & 1
\end{bmatrix}
\]

V(80): $\deg = 40$, $h^1,2 = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{bmatrix}
2 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -2 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

V(81): $\deg = 40$, $h^1,2 = 0$, $rk = 3$, $sq = 3$, $dp = 3$, $py = 1$, $\text{vert}(\Delta^*) = 9$:

\[
\begin{bmatrix}
-3 & 0 & 0 \\
-1 & 0 & 0 \\
-1 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-2 & -2 & -1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-3 & 0 & 0 \\
1 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}
\]

V(82): $\deg = 40$, $h^1,2 = 1$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 0$, $\text{vert}(\Delta^*) = 6$:

\[
\begin{bmatrix}
2 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

V(83): $\deg = 40$, $h^1,2 = 1$, $rk = 2$, $sq = 3$, $dp = 3$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{bmatrix}
-3 & 0 & 0 \\
-1 & 0 & 0 \\
-1 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-2 & -1 & -1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

V(84): $\deg = 46$, $h^1,2 = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 7$:

\[
\begin{bmatrix}
-3 & 0 & 0 \\
-1 & 0 & 0 \\
-1 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-2 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

V(85): $\deg = 46$, $h^1,2 = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 7$:

\[
\begin{bmatrix}
-3 & 0 & 0 \\
-1 & 0 & 0 \\
-1 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-3 & 0 & 0 \\
1 & 1 & 0 \\
1 & 0 & 0
\end{bmatrix}
\]

V(86): $\deg = 46$, $h^1,2 = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{bmatrix}
-3 & 0 & 0 \\
-1 & 0 & 0 \\
-1 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-2 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

V(87): $\deg = 48$, $h^1,2 = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 7$:

\[
\begin{bmatrix}
-3 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-2 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

V(88): $\deg = 54$, $h^1,2 = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 6$:

\[
\begin{bmatrix}
-3 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]
### CONIFOLD DEGENERATIONS OF FANO 3-FOLDS

V(89): deg = 54, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\vert \Delta^{*} \vert = 6$:

$$
\begin{pmatrix}
1 & 0 & 1 & 0 & 0 & -3 \\
1 & 0 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & 1 & 0 & -1 \\
-1 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
$$

V(90): deg = 54, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\vert \Delta^{*} \vert = 7$:

$$
\begin{pmatrix}
-5 & 0 & -2 & 0 & 0 & 1 \\
-2 & -1 & 0 & -1 & 1 & 0 \\
-1 & -1 & 0 & 0 & 0 & 1 \\
-1 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
$$

V(91): deg = 56, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\vert \Delta^{*} \vert = 6$:

$$
\begin{pmatrix}
-1 & 0 & 0 & 1 & 0 & -2 \\
2 & 1 & 0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 & 1 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
$$

V(92): deg = 62, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\vert \Delta^{*} \vert = 6$:

$$
\begin{pmatrix}
-1 & 0 & 0 & 1 & 0 & -2 \\
2 & 1 & 0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 & 1 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
$$

6. $B_{2} = 3$

V(93): deg = 12, $h^{1,2} = 8$, $rk = 5$, $sq = 12$, $dp = 12$, $py = 0$, $\vert \Delta^{*} \vert = 12$:

$$
\begin{pmatrix}
-1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\
0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & -1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & -1 & 0
\end{pmatrix}
$$

V(94): deg = 18, $h^{1,2} = 3$, $rk = 3$, $sq = 5$, $dp = 5$, $py = 0$, $\vert \Delta^{*} \vert = 10$:

$$
\begin{pmatrix}
0 & 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1
\end{pmatrix}
$$

V(95): deg = 22, $h^{1,2} = 1$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 0$, $\vert \Delta^{*} \vert = 9$:

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 1 \\
0 & 0 & -1 & 0 & -1 & 0 & -1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & -1 & 0 & 0
\end{pmatrix}
$$

V(96): deg = 24, $h^{1,2} = 1$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 0$, $\vert \Delta^{*} \vert = 8$:

$$
\begin{pmatrix}
0 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0
\end{pmatrix}
$$

V(97): deg = 24, $h^{1,2} = 1$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 0$, $\vert \Delta^{*} \vert = 9$:

$$
\begin{pmatrix}
-2 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\
-1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\
-1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$
V(98): deg = 24, $h^{1,2} = 1$, $rk = 6$, $sq = 7$, $dp = 7$, $py = 1$, $vert(\Delta^*) = 13$:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & -1 & 0 & 1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

V(99): deg = 26, $h^{1,2} = 0$, $rk = 5$, $sq = 5$, $dp = 5$, $py = 1$, $vert(\Delta^*) = 12$:

$$
\begin{bmatrix}
1 & -1 & 0 & -1 & 0 & -1 & 0 & 1 & 1 & 2 & 0 & -2 & 0 & -2 & 0 & -2 \\
0 & 1 & -1 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & -1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

V(100): deg = 26, $h^{1,2} = 3$, $rk = 1$, $sq = 1$, $dp = 3$, $py = 0$, $vert(\Delta^*) = 8$:

$$
\begin{bmatrix}
0 & -1 & -2 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & -1 & 2 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

V(101): deg = 28, $h^{1,2} = 0$, $rk = 4$, $sq = 4$, $dp = 4$, $py = 1$, $vert(\Delta^*) = 11$:

$$
\begin{bmatrix}
1 & -1 & -1 & -2 & 0 & 0 & 1 & -2 & 0 & -2 & 0 & -2 & 0 & -2 & 0 & -2 \\
0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 1 & -1 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & -1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 & -1 & 1 \\
\end{bmatrix}
$$

V(102): deg = 28, $h^{1,2} = 0$, $rk = 5$, $sq = 5$, $dp = 5$, $py = 1$, $vert(\Delta^*) = 12$:

$$
\begin{bmatrix}
-2 & 0 & 0 & -2 & -1 & -1 & -2 & 0 & -1 & -2 & 0 & -1 & -2 & 0 & -1 & -2 & 0 & -1 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

V(103): deg = 28, $h^{1,2} = 1$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 0$, $vert(\Delta^*) = 7$:

$$
\begin{bmatrix}
1 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

V(104): deg = 28, $h^{1,2} = 1$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 0$, $vert(\Delta^*) = 8$:

$$
\begin{bmatrix}
-2 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

V(105): deg = 28, $h^{1,2} = 1$, $rk = 4$, $sq = 5$, $dp = 5$, $py = 1$, $vert(\Delta^*) = 11$:

$$
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & -2 & -2 & 0 & -1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 1 & 1 \\
\end{bmatrix}
$$

V(106): deg = 28, $h^{1,2} = 1$, $rk = 4$, $sq = 5$, $dp = 5$, $py = 1$, $vert(\Delta^*) = 11$:

$$
\begin{bmatrix}
-2 & -1 & -1 & -3 & 0 & -1 & -2 & -2 & 0 & 1 & 0 \\
-1 & -1 & -1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
$$

V(107): deg = 30, $h^{1,2} = 0$, $rk = 3$, $sq = 3$, $dp = 3$, $py = 1$, $vert(\Delta^*) = 10$:

$$
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & -3 & -2 & -1 & -2 & -2 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
\end{bmatrix}
$$
| V(108): deg = 30, $h^{1,2} = 0$, rk = 4, sq = 4, dp = 4, py = 1, vert($\Delta^*$) = 11: |
|---|---|---|---|---|---|---|
| -1 | 0 | 0 | 1 | 0 | -3 | -2 | -2 | -1 | -2 | 0 |
| -1 | -1 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | -1 | 0 |
| 1 | 1 | 1 | 0 | 0 | -1 | 0 | -1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | -1 | -1 | 0 | 1 | 0 | 1 |

| V(109): deg = 30, $h^{1,2} = 0$, rk = 4, sq = 4, dp = 4, py = 1, vert($\Delta^*$) = 11: |
|---|---|---|---|---|---|---|
| -3 | -2 | -1 | -3 | -2 | -4 | 0 | 0 | -1 | 0 | 1 |
| -1 | -1 | 1 | 0 | 0 | -1 | 1 | 0 | -1 | 0 | 0 |
| 0 | 0 | -1 | -1 | -1 | -1 | 0 | 0 | 1 | 1 | 0 |
| -1 | 0 | 0 | -1 | 0 | -1 | 0 | 1 | 0 | 0 | 0 |

| V(110): deg = 32, $h^{1,2} = 0$, rk = 3, sq = 3, dp = 3, py = 1, vert($\Delta^*$) = 10: |
|---|---|---|---|---|---|---|
| -1 | -2 | -2 | 0 | 0 | 1 | 0 | -2 | -1 | -2 |
| 0 | -1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | -1 |
| -1 | -1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | -1 | -1 | 0 |

| V(111): deg = 32, $h^{1,2} = 0$, rk = 3, sq = 3, dp = 3, py = 1, vert($\Delta^*$) = 10: |
|---|---|---|---|---|---|---|
| -2 | 0 | -1 | -2 | 0 | -3 | -2 | -1 | 0 | 1 |
| 0 | 0 | -1 | -1 | 0 | -1 | 0 | 0 | 1 | 0 |
| -1 | 0 | 1 | 0 | 1 | 0 | 0 | -1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | -1 | -1 | 1 | 0 | 0 |

| V(112): deg = 32, $h^{1,2} = 0$, rk = 3, sq = 3, dp = 3, py = 1, vert($\Delta^*$) = 10: |
|---|---|---|---|---|---|---|
| -3 | 0 | -2 | -1 | -2 | 0 | -4 | -1 | 1 | 0 |
| 0 | 1 | 0 | -1 | -1 | 0 | -1 | 1 | 0 | 0 |
| -1 | 0 | -1 | 1 | 0 | 0 | -1 | -1 | 0 | 1 |
| -1 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 |

| V(113): deg = 32, $h^{1,2} = 0$, rk = 4, sq = 4, dp = 4, py = 1, vert($\Delta^*$) = 11: |
|---|---|---|---|---|---|---|
| -2 | 0 | -2 | 0 | -3 | -2 | -3 | -2 | -1 | 0 | 1 |
| 0 | 0 | -1 | 0 | -1 | -1 | 0 | 0 | 1 | 0 |
| -1 | 0 | 0 | 1 | 0 | -1 | -1 | 0 | -1 | 0 | 0 |
| 0 | 1 | 0 | 0 | -1 | 1 | 0 | -1 | 1 | 0 | 0 |

| V(114): deg = 32, $h^{1,2} = 1$, rk = 0, sq = 0, dp = 0, py = 0, vert($\Delta^*$) = 7: |
|---|---|---|---|---|---|
| -1 | 0 | -1 | 0 | 0 | 0 | 0 | 1 |
| -2 | 0 | 0 | -1 | 1 | 0 | 0 |
| -1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| V(115): deg = 32, $h^{1,2} = 1$, rk = 2, sq = 3, dp = 3, py = 1, vert($\Delta^*$) = 9: |
|---|---|---|---|---|---|---|---|---|---|
| 1 | -2 | -3 | -2 | -1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | -1 | -1 | 0 | 1 | 0 | 1 |
| 0 | -1 | -1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | -1 | -1 | 0 | 1 | 0 | 0 | 1 | 0 |

| V(116): deg = 34, $h^{1,2} = 0$, rk = 2, sq = 2, dp = 2, py = 1, vert($\Delta^*$) = 9: |
|---|---|---|---|---|---|---|---|---|---|
| -1 | 0 | 0 | 1 | 0 | -2 | -1 | -2 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | -1 | -1 |
| 1 | 1 | 0 | 0 | 0 | -1 | -1 | 1 |
| -1 | 0 | 0 | 0 | 1 | 0 | 0 | -1 |

| V(117): deg = 34, $h^{1,2} = 0$, rk = 3, sq = 3, dp = 3, py = 1, vert($\Delta^*$) = 10: |
|---|---|---|---|---|---|---|---|---|---|
| 0 | -1 | -2 | -2 | -1 | 0 | -2 | -3 | 0 | 1 |
| 0 | -1 | -1 | 0 | 0 | -1 | -1 | 1 | 0 | 0 |
| 0 | 1 | 0 | -1 | -1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | -1 | -1 | 0 | 0 |
V(118): $deg = 34$, $h^{1,2} = 0$, $rk = 3$, $sq = 3$, $dp = 3$, $py = 1$, $vert(\Delta^*) = 10$:

$$
\begin{array}{cccccccc}
-2 & -1 & -3 & 0 & -2 & 0 & 0 & -3 & -2 \\
-1 & 0 & -1 & 0 & 0 & 1 & 0 & -1 & -1 \\
1 & 1 & 0 & 1 & -1 & 0 & 0 & -1 & 0 \\
-1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\
\end{array}
$$

V(119): $deg = 36$, $h^{1,2} = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $vert(\Delta^*) = 9$:

$$
\begin{array}{cccccccc}
-1 & 0 & 0 & 0 & 1 & 0 & -3 & -2 & -2 \\
0 & 0 & 1 & 0 & 0 & -1 & -1 & -1 & 0 \\
1 & 1 & 0 & 0 & -1 & 0 & 1 & 1 & -1 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
\end{array}
$$

V(120): $deg = 36$, $h^{1,2} = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $vert(\Delta^*) = 9$:

$$
\begin{array}{cccccccc}
-2 & 0 & -2 & -1 & 0 & -2 & -1 & 0 & 1 \\
-1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\
0 & 1 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 \\
\end{array}
$$

V(121): $deg = 36$, $h^{1,2} = 0$, $rk = 3$, $sq = 3$, $dp = 3$, $py = 1$, $vert(\Delta^*) = 10$:

$$
\begin{array}{cccccccc}
1 & 0 & -2 & 0 & 0 & 1 & -2 & -1 & -1 \\
0 & 1 & -1 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\
0 & 1 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \\
\end{array}
$$

V(122): $deg = 36$, $h^{1,2} = 0$, $rk = 3$, $sq = 3$, $dp = 3$, $py = 1$, $vert(\Delta^*) = 10$:

$$
\begin{array}{cccccccc}
-3 & 0 & -1 & 0 & -3 & -2 & -4 & -2 & 0 \\
-1 & 0 & 0 & 0 & -1 & 0 & -1 & -1 & 1 \\
0 & 1 & -1 & 0 & -1 & 0 & -1 & 0 & 0 \\
-1 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 \\
\end{array}
$$

V(123): $deg = 36$, $h^{1,2} = 0$, $rk = 3$, $sq = 3$, $dp = 3$, $py = 1$, $vert(\Delta^*) = 10$:

$$
\begin{array}{cccccccc}
1 & 0 & -2 & 0 & 0 & 0 & 1 & -2 & -1 \\
0 & 1 & -1 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\
0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\
\end{array}
$$

V(124): $deg = 38$, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $vert(\Delta^*) = 8$:

$$
\begin{array}{cccccccc}
-1 & -2 & -3 & 0 & 1 & 0 & -2 & 0 \\
-1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & -1 & 0 & 0 & 1 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & -1 \\
\end{array}
$$

V(125): $deg = 38$, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $vert(\Delta^*) = 8$:

$$
\begin{array}{cccccccc}
1 & -2 & -2 & 0 & 0 & 0 & 1 & -1 \\
0 & -1 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & -1 & 0 & 0 & 1 & -1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
\end{array}
$$

V(126): $deg = 38$, $h^{1,2} = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $vert(\Delta^*) = 9$:

$$
\begin{array}{cccccccc}
1 & 1 & 0 & 0 & 0 & -2 & 0 & -2 \\
0 & 0 & 0 & 1 & 0 & -1 & -1 & 0 \\
0 & 1 & 0 & 0 & 1 & -1 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\
\end{array}
$$

V(127): $deg = 38$, $h^{1,2} = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $vert(\Delta^*) = 9$:

$$
\begin{array}{cccccccc}
-1 & 0 & 0 & 1 & 0 & -2 & -2 & -1 \\
1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 \\
0 & 1 & 0 & 0 & 1 & -1 & -1 & 1 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
\end{array}
$$
V(128): deg = 38, $h^{1,2} = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $\text{vert}(\Delta^*) = 9$:

\[
\begin{array}{cccccccc}
1 & -2 & -1 & -3 & 0 & -2 & 1 & 0 \\
0 & 1 & 1 & 0 & -1 & 1 & 1 & 0 \\
0 & -1 & -1 & -1 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \\
\end{array}
\]

V(129): deg = 38, $h^{1,2} = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $\text{vert}(\Delta^*) = 9$:

\[
\begin{array}{cccccccc}
-2 & -1 & 0 & -2 & 0 & -3 & -1 & 0 & 1 \\
0 & -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\
-1 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\
\end{array}
\]

V(130): deg = 38, $h^{1,2} = 0$, $rk = 3$, $sq = 3$, $dp = 3$, $py = 1$, $\text{vert}(\Delta^*) = 10$:

\[
\begin{array}{cccccccc}
1 & 1 & 0 & -2 & -1 & 0 & 0 & 0 & -2 & -3 \\
0 & 0 & 0 & -1 & -1 & -1 & 0 & 1 & 0 & -1 \\
0 & 1 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & -1 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
\end{array}
\]

V(131): deg = 40, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{array}{cccccccc}
1 & -2 & -2 & 0 & 1 & 0 & -3 & 0 \\
0 & 1 & -1 & 0 & 1 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & -1 & 0 & 1 & 1 & 0 & -1 & 0 \\
\end{array}
\]

V(132): deg = 42, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{array}{cccccccc}
1 & 1 & 0 & -1 & 0 & 2 & 0 & -3 \\
0 & 1 & 1 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 \\
\end{array}
\]

V(133): deg = 42, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{array}{cccccccc}
-2 & -1 & 0 & -2 & 0 & -3 & -1 & 1 & 0 \\
-1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\
\end{array}
\]

V(134): deg = 42, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{array}{cccccccc}
-4 & -2 & -2 & 0 & 0 & -1 & 0 & 1 \\
-1 & -1 & 0 & 1 & 0 & -1 & 0 & 0 \\
-1 & 0 & -1 & 0 & 1 & 1 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{array}
\]

V(135): deg = 42, $h^{1,2} = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $\text{vert}(\Delta^*) = 9$:

\[
\begin{array}{cccccccc}
1 & 2 & 1 & -2 & 0 & 0 & 0 & -3 & -3 \\
0 & 1 & 0 & -1 & 0 & 0 & 1 & -1 & -1 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{array}
\]

V(136): deg = 42, $h^{1,2} = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $\text{vert}(\Delta^*) = 9$:

\[
\begin{array}{cccccccc}
-4 & -1 & 0 & -2 & -3 & -2 & 0 & 0 & 1 \\
-1 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\
-1 & -1 & 0 & 0 & -1 & -1 & 0 & 1 & 0 \\
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

V(137): deg = 44, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 7$:

\[
\begin{array}{cccccccc}
-1 & 0 & 0 & 1 & 0 & 3 & -2 \\
1 & 1 & 0 & 0 & 0 & -1 & -1 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 \\
\end{array}
\]
V(138): deg = 44, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{pmatrix}
-3 & 0 & 0 & -3 & -4 & -2 & 0 & 1 \\
-1 & 0 & 0 & -1 & -1 & -1 & 1 & 0 \\
0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & -1 & 0 & 0 & 0
\end{pmatrix}
\]

V(139): deg = 46, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 7$:

\[
\begin{pmatrix}
1 & 0 & -2 & 0 & 1 & 0 & -3 \\
0 & 1 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
\]

V(140): deg = 46, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{pmatrix}
-4 & 0 & -3 & -1 & -2 & 0 & 0 & 1 \\
-1 & 0 & -1 & -1 & -1 & 1 & 0 & 0 \\
-1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
-1 & 1 & 0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

V(141): deg = 48, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 7$:

\[
\begin{pmatrix}
-2 & 0 & 0 & 1 & 0 & -2 & -2 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
\]

V(142): deg = 48, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 7$:

\[
\begin{pmatrix}
-1 & 0 & 0 & 1 & 0 & -2 & -2 \\
1 & 0 & 1 & 0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
\]

V(143): deg = 48, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{pmatrix}
-3 & 0 & 0 & -2 & -2 & -1 & 0 & 1 \\
-2 & 0 & 0 & -1 & 0 & -1 & 1 & 0 \\
1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

V(144): deg = 50, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 7$:

\[
\begin{pmatrix}
-1 & 0 & 0 & 1 & 0 & -2 & -1 \\
0 & 0 & 1 & 0 & 0 & -1 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
\]

V(145): deg = 50, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 7$:

\[
\begin{pmatrix}
1 & 0 & -2 & 0 & 1 & 0 & -2 \\
0 & 1 & -1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 \\
0 & 0 & 0 & 1 & 0 & 0 & -1
\end{pmatrix}
\]

V(146): deg = 52, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 7$:

\[
\begin{pmatrix}
-1 & 0 & 0 & 1 & 0 & -2 & -1 \\
1 & 0 & 1 & 0 & 0 & -1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
\]
7. $B_2 = 4$

V(147): $\deg = 24, h^{1,2} = 1, r_k = 0, sq = 0, dp = 0, py = 0, \vert \Delta^* \vert = 8$:
\[
\begin{pmatrix}
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

V(148): $\deg = 24, h^{1,2} = 1, r_k = 5, sq = 6, dp = 6, py = 1, \vert \Delta^* \vert = 13$:
\[
\begin{pmatrix}
-1 & -2 & -2 & 0 & 0 & 1 & -1 & 0 & -2 & -1 & -2 & 0 & -1 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & -1 \\
-1 & -1 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 1 & 1
\end{pmatrix}
\]

V(149): $\deg = 28, h^{1,2} = 1, r_k = 0, sq = 0, dp = 0, py = 0, \vert \Delta^* \vert = 8$:
\[
\begin{pmatrix}
-1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

V(150): $\deg = 28, h^{1,2} = 1, r_k = 3, sq = 4, dp = 4, py = 1, \vert \Delta^* \vert = 11$:
\[
\begin{pmatrix}
-1 & -2 & -2 & 0 & 0 & 1 & 0 & -2 & -1 & 0 & -2 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & -1 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0
\end{pmatrix}
\]

V(151): $\deg = 30, h^{1,2} = 0, r_k = 3, sq = 3, dp = 3, py = 1, \vert \Delta^* \vert = 11$:
\[
\begin{pmatrix}
-2 & 0 & -1 & -1 & 0 & 1 & 0 & -1 & -1 & -2 & 0 \\
-1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & -1 & -1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & -1
\end{pmatrix}
\]

V(152): $\deg = 32, h^{1,2} = 0, r_k = 2, sq = 2, dp = 2, py = 1, \vert \Delta^* \vert = 10$:
\[
\begin{pmatrix}
-2 & 0 & 0 & -1 & -1 & 0 & -1 & 1 & -2 & -1 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & 0
\end{pmatrix}
\]

V(153): $\deg = 32, h^{1,2} = 0, r_k = 2, sq = 2, dp = 2, py = 1, \vert \Delta^* \vert = 10$:
\[
\begin{pmatrix}
-1 & 0 & 0 & -1 & 2 & 1 & -2 & -1 & 0 & -1 \\
1 & 1 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 & 1 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{pmatrix}
\]

V(154): $\deg = 34, h^{1,2} = 0, r_k = 2, sq = 2, dp = 2, py = 1, \vert \Delta^* \vert = 10$:
\[
\begin{pmatrix}
-2 & 0 & 0 & 0 & -2 & -3 & 2 & -1 & -3 & 0 & 1 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0
\end{pmatrix}
\]

V(155): $\deg = 34, h^{1,2} = 0, r_k = 3, sq = 3, dp = 3, py = 1, \vert \Delta^* \vert = 11$:
\[
\begin{pmatrix}
-2 & -3 & 0 & 1 & 0 & -2 & 0 & -3 & -3 & -2 & -4 \\
0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & -1 & -1 \\
-1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1
\end{pmatrix}
\]

V(156): $\deg = 36, h^{1,2} = 0, r_k = 1, sq = 1, dp = 1, py = 1, \vert \Delta^* \vert = 9$:
\[
\begin{pmatrix}
-2 & 0 & 0 & -2 & -3 & 2 & -1 & 0 & 1 \\
-1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0
\end{pmatrix}
\]
V(157): $\text{deg} = 36$, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 9$:

\[
\begin{array}{cccccccc}
-4 & -2 & -1 & 0 & 0 & -1 & 0 & 1 \\
-1 & -1 & 1 & 0 & 1 & 0 & -1 & 0 \\
-1 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \\
-1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\end{array}
\]

V(158): $\text{deg} = 36$, $h^{1,2} = 0$, $rk = 2$, $sq = 2$, $dp = 2$, $py = 1$, $\text{vert}(\Delta^*) = 10$:

\[
\begin{array}{cccccccc}
-2 & 0 & -2 & 0 & -1 & -2 & -1 & 0 \\
0 & 0 & -1 & 0 & -1 & 0 & 0 & -1 \\
-1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & -1 & 1 \\
\end{array}
\]

V(159): $\text{deg} = 38$, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 9$:

\[
\begin{array}{cccccccc}
-2 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
-1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\
\end{array}
\]

V(160): $\text{deg} = 40$, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{array}{cccccccc}
1 & 0 & -2 & -2 & 0 & 1 & 0 & -3 \\
0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 1 & 1 & -1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
\end{array}
\]

V(161): $\text{deg} = 40$, $h^{1,2} = 0$, $rk = 1$, $sq = 1$, $dp = 1$, $py = 1$, $\text{vert}(\Delta^*) = 9$:

\[
\begin{array}{cccccccc}
0 & -1 & -2 & -2 & 0 & -2 & -3 & 0 \\
0 & -1 & -1 & 0 & 0 & -1 & -1 & 1 \\
0 & 1 & 0 & -1 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \\
\end{array}
\]

V(162): $\text{deg} = 42$, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 0 & -2 & -2 & -2 \\
0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\
\end{array}
\]

V(163): $\text{deg} = 44$, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 1 & -2 & -2 & -1 \\
0 & 0 & 0 & 1 & 1 & -1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\
\end{array}
\]

V(164): $\text{deg} = 46$, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, $\text{vert}(\Delta^*) = 8$:

\[
\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 0 & -2 & -2 & 0 \\
0 & 0 & 1 & 1 & 0 & -1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & -1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\
\end{array}
\]
8. $B_2 = 5$

\begin{verbatim}
V(165): deg = 36, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, vert($\Delta^*\ast$) = 9:
-2 0 0 0 -1 -2 1 -2 -1
0 1 0 0 1 0 0 -1 -1
0 0 1 0 -1 -1 0 0 1
-1 0 0 1 0 0 0 0 0
\end{verbatim}

\begin{verbatim}
V(166): deg = 36, $h^{1,2} = 0$, $rk = 0$, $sq = 0$, $dp = 0$, $py = 1$, vert($\Delta^*\ast$) = 9:
-1 0 0 0 -2 -1 -1 -2 1
1 1 0 0 -1 -1 1 0 0
0 0 1 0 0 1 -1 -1 0
-1 0 0 1 0 0 0 0 0
\end{verbatim}

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