DO PHANTOM CUNTZ-KRIEGER ALGEBRAS EXIST?

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Abstract. If phantom Cuntz-Krieger algebras do not exist, then real rank zero Cuntz-Krieger algebras can be characterized by outer properties. In this survey paper, a summary of the known results on non-existence of phantom Cuntz-Krieger algebras is given.

1. Introduction

The Cuntz-Krieger algebras were introduced by J. Cuntz and W. Krieger in 1980, cf. [8], and are a generalization of the Cuntz algebras. Given an \( n \times n \) matrix \( A \) with entries in \( \{0, 1\} \), its associated Cuntz-Krieger algebra \( O_A \) is defined as the universal \( C^* \)-algebra generated by \( n \) partial isometries \( s_1, \ldots, s_n \) satisfying the relations

\[
1 = s_1 s_1^* + \cdots + s_n s_n^*,
\]

\[
s_i^* s_i = \sum_{j=1}^{n} A_{ij} s_j s_j^* \text{ for all } i = 1, \ldots, n.
\]

The Cuntz-Krieger algebras arise from shifts of finite type, and it has been shown that the Cuntz-Krieger algebras are exactly the graph algebras \( C^*(E) \) arising from finite directed graphs \( E \) with no sinks or sources.

Neither of the two equivalent definitions of Cuntz-Krieger algebras give an outer characterization of Cuntz-Krieger algebras; i.e., neither give a way of determining whether a \( C^* \)-algebra is a Cuntz-Krieger algebra, unless it is constructed from a graph or a shift of finite type.

A Cuntz-Krieger algebra is purely infinite if and only if it has real rank zero, and in the following we will mainly restrict to real rank zero Cuntz-Krieger algebras since we will rely on classification results that only hold in the purely infinite case. The Cuntz-Krieger algebra \( O_A \) is purely infinite if and only if \( A \) satisfies Cuntz’s condition (II), and equivalently the Cuntz-Krieger algebra \( C^*(E) \) is purely infinite if and only if the graph \( E \) satisfies Krieger’s condition (K).

The notion of \( C^* \)-algebras over a topological space is useful for defining phantom Cuntz-Krieger algebras and for defining filtered \( K \)-theory, and in [10], Eberhard Kirchberg proved some very powerful classification results for \( O_\infty \)-absorbing \( C^* \)-algebras over a space \( X \) using \( KK(X) \)-theory. A \( C^* \)-algebra \( A \) over the finite \( T_0 \)-space \( X \) is a \( C^* \)-algebra equipped with a lattice-preserving map from the open
sets of $X$ to the ideals in $A$, denoted $U \mapsto A(U)$ and extended to locally closed subsets as $U \setminus V \mapsto A(U)/A(V)$. In particular a $C^*$-algebra with finitely many ideals is a $C^*$-algebra over its primitive ideal space.

**Definition 1.1.** A $C^*$-algebra $A$ with primitive ideal space $X$ looks like a Cuntz-Krieger algebra if

1. $A$ is unital, purely infinite, nuclear, separable, and of real rank zero,
2. $X$ is finite
3. for all $x \in X$, the group $K_*(A(x))$ is finitely generated, the group $K_1(A(x))$ is free, and $\text{rank} K_0(A(x)) = \text{rank} K_1(A(x))$,
4. for all $x \in X$, $A(x)$ is in the bootstrap class of Rosenberg and Schochet.

A $C^*$-algebra that looks like a Cuntz-Krieger algebra but is not isomorphic to a Cuntz-Krieger algebra, is called a phantom Cuntz-Krieger algebra.

All real rank zero Cuntz-Krieger algebras look like Cuntz-Krieger algebras. It is not known whether all $C^*$-algebras that look like Cuntz-Krieger algebras (and quack like Cuntz-Krieger algebras) are Cuntz-Krieger algebras. If it is established that they are, i.e., that phantom Cuntz-Krieger algebras do not exist, then the above definition gives a characterization of the real rank zero Cuntz-Krieger algebras.

An example to point out the relevance of such a characterization is given by Proposition 1.2. If phantom Cuntz-Krieger algebras do not exist, the proposition determines exactly when an extension of real rank zero Cuntz-Krieger algebras is a real rank zero Cuntz-Krieger algebra.

By a result of Lawrence G. Brown and Gert K. Pedersen, Theorem 3.14 of [7], an extension of real rank zero $C^*$-algebras has real rank zero if and only if projections in the quotient lift to projections in the extension. Hence, if a $C^*$-algebra $A$ with primitive ideal space $X$ has real rank zero, then $K_0(A(Y \setminus U)) \to K_1(A(U))$ vanishes for all $Y$ and $U$ where $Y$ is a locally closed subsets of $X$ and $U$ is an open subsets of $Y$. Using this, an induction argument shows that for a $C^*$-algebra that looks like a Cuntz-Krieger algebra, (3) and (4) of Definition 1.1 hold for all locally closed subsets $Y$ of $X$.

**Proposition 1.2.** Consider a unital extension $0 \to I \to A \to A/I \to 0$ and assume that $A/I$ is a real rank zero Cuntz-Krieger algebra and that $I$ is stably isomorphic to a real rank zero Cuntz-Krieger algebra. Then $A$ looks like a Cuntz-Krieger algebra if and only if the induced map $K_0(A/I) \to K_1(I)$ vanishes.

**Proof.** By Theorem 3.14 of [7], the $C^*$-algebra $A$ is of real rank zero if and only if the induced map $K_0(A/I) \to K_1(I)$ vanishes. It is well-known or easy to check that the other properties stated in Definition 1.1 are closed under extensions. □

### 2. Special cases

One of the first places one would look for phantom Cuntz-Krieger algebras are among the matrix algebras over real rank zero Cuntz-Krieger algebras. Clearly, if $O_A$ is a Cuntz-Krieger algebra of real rank zero, then $M_n(O_A)$ looks like a Cuntz-Krieger algebra for all $n$. Since $M_n(O_A)$ is a graph algebra, one then immediately asks if a graph algebra can be a phantom Cuntz-Krieger algebra. It turns out that it cannot.
Theorem 2.1 ([2]). Let $E$ be a directed graph and assume that its graph algebra $C^*(E)$ is unital and satisfies $\text{rank}K_0(C^*(E)) = \text{rank}K_1(C^*(E))$. Then $C^*(E)$ is isomorphic to a Cuntz-Krieger algebra.

Theorem 2.2 ([2]). Let $A$ be a unital $C^*$-algebra and assume that $A$ is stably isomorphic to a Cuntz-Krieger algebra. Then $A$ is isomorphic to a Cuntz-Krieger algebra.

As a small corollary to the work of Eberhard Kirchberg on $KK(X)$-theory, phantom Cuntz-Krieger algebras cannot have vanishing $K$-theory.

Theorem 2.3 ([10]). Let $A$ and $B$ be unital, nuclear, separable $C^*$-algebras with primitive ideal space $X$. Then $A \otimes \mathcal{O}_2$ and $B \otimes \mathcal{O}_2$ are isomorphic.

Corollary 2.4. Let $A$ be a $C^*$-algebra that looks like a Cuntz-Krieger algebra, and assume that $K_*(A) = 0$. Then $A$ is a Cuntz-Krieger algebra.

Proof. Let $X$ denote the finite primitive ideal space of $A$. Since $K_*(A) = 0$ and $A$ looks like a Cuntz-Krieger algebra, $K_*(A(x)) = 0$ for all $x \in X$. So for all $x \in X$, $A(x)$ is $\mathcal{O}_2$-absorbing since it is a UCT Kirchberg algebra with vanishing $K$-theory. By applying Theorem 4.3 of [14] finitely many times, we see that $A$ itself is $\mathcal{O}_2$-absorbing. Let $O_B$ be a Cuntz-Krieger algebra with primitive ideal space $X$ and with $O_B(x)$ (stably) isomorphic to $\mathcal{O}_2$ for all $x \in X$. Then by Theorem 2.3, $A$ is isomorphic to $O_B$.

3. Using filtered $K$-theory

Via $K$-theoretic classification results it can be established that a phantom Cuntz-Krieger algebra cannot have a so-called accordion space as its primitive ideal space. We will first restrict to the cases where the primitive ideal space has at most 2 points in order to describe the historical development and due to the importance and powerlessness of the results needed. The most crucial result is by Eberhard Kirchberg who showed in [10] that for stable, purely infinite, nuclear, separable $C^*$-algebras $A$ and $B$ with finite primitive ideal space $X$, any $KK(X)$-equivalence between $A$ and $B$ lift to a $*$-isomorphism.

Simple $C^*$-algebras that look like Cuntz-Krieger algebras are UCT Kirchberg algebras, hence the classification result by Eberhard Kirchberg and N. Christofer Phillips applies. For a unital $C^*$-algebra $A$ with unit $1_A$, denote by $[1_A]$ the class of $1_A$ in $K_0(A)$. For unital $C^*$-algebras $A$ and $B$ an isomorphism from $(K_*(A), [1_A])$ to $(K_*(B), [1_B])$ is defined as a pair $(\phi_0, \phi_1)$ of group isomorphisms $\phi_i : K_i(A) \to K_i(B)$, $i = 0, 1$, for which $\phi_0([1_A]) = \phi_0([1_B])$.

Theorem 3.1 ([11]). Let $A$ and $B$ be unital, simple, purely infinite, nuclear, separable $C^*$-algebras in the bootstrap class. If $(K_*(A), [1_A])$ and $(K_*(B), [1_B])$ are isomorphic, then $A$ and $B$ are isomorphic.

The range of $K_*$ for graph algebras has been determined by Wojciech Szymański, and his result has been extended by Søren Eilers, Takeshi Katsura, Mark Tomforde, and James West to include the class of the unit.

Theorem 3.2 ([9]). Let $G$ and $F$ be finitely generated groups, let $g \in G$, and assume that $F$ is free and that $\text{rank}G = \text{rank}F$. Then there exists a simple Cuntz-Krieger algebra $O_A$ of real rank zero realising $(G \oplus F, g)$ as $(K_*(O_A), [1_{O_A}])$. 
Corollary 3.3. Simple phantom Cuntz-Krieger algebras do not exist.

Proof. Let $A$ be a simple $C^*$-algebra that looks like a Cuntz-Krieger algebra. By Theorem 3.2 there exists a Cuntz-Krieger algebra $O_B$ of real rank zero for which $(K_*(A), [1_A]) \cong (K_*(O_B), [1_{O_B}])$. Since $A$ and $O_B$ are UCT Kirchberg algebras, it follows from Theorem 3.1 that $A$ and $O_B$ are isomorphic.

For $C^*$-algebras with exactly one nontrivial ideal, the suitable invariant seems to be the induced six-term exact sequence in $K$-theory.

Definition 3.4. Let $X_{six}$ denote the space $\{1, 2\}$ with $\{2\}$ open and $\{1\}$ not open. For a $C^*$-algebra $A$ with primitive ideal space $X_{six}$, $K_{six}(A)$ is defined as the groups and maps

$$
\begin{array}{c c c}
K_0(A(2)) & \overset{i}{\longrightarrow} & K_0(A) \\
\delta & & \delta \\
K_1(A(1)) & \overset{r}{\longrightarrow} & K_1(A) \\
\end{array}
$$

induced by the extension $0 \to A(2) \to A \to A(1)$. For unital $C^*$-algebras $A$ and $B$ with primitive ideal space $X_{six}$, an isomorphism from $(K_{six}(A), [1_A])$ to $(K_{six}(B), [1_B])$ is defined as a triple $(\phi^Y, \varphi_{six}, \chi^Y_i)$ of graded isomorphisms $\phi^Y : K_*(A(Y)) \to K_*(B(Y)), Y \in \{\{2\}, X_{six}, \{1\}\}$, that commute with the maps $i$, $r$, and $\delta$ and satisfies $\phi^0_{six}([1_A]) = [1_B]$.

This invariant was originally introduced by Mikael Rørdam to classify Cuntz-Krieger algebras with exactly one nontrivial ideal up to stable isomorphism. Alexander Bonkat established a UCT for $K_{six}$ (that was later generalized by Ralf Meyer and Ryszard Nest), and by combining his UCT with the result of Eberhard Kirchberg (and a result by Gunnar Restorff and Efren Ruiz in [13] to achieve unital and not stable isomorphism) one obtains the following theorem.

Theorem 3.5 ([6] [10]). Let $A$ and $B$ be unital, purely infinite, nuclear, separable $C^*$-algebras with primitive ideal space $X_{six}$, and assume that $A(x)$ and $B(x)$ are in the bootstrap class for all $x \in \{1, 2\}$. Then $(K_{six}(A), [1_A]) \cong (K_{six}(B), [1_B])$ implies $A \cong B$.

The range of $K_{six}$ for graph algebras has been determined by Søren Eilers, Takeshi Katsura, Mark Tomforde, and James West.

Theorem 3.6 ([9]). Let a six-term exact sequence

$$
\begin{array}{c c c c c c}
\mathcal{E} : & G_1 & \overset{ }{\longrightarrow} & G_2 & \overset{ }{\longrightarrow} & G_3 \\
& F_3 & \overset{ }{\longleftarrow} & F_2 & \overset{ }{\longleftarrow} & F_1 \\
\end{array}
$$

be given with $G_1, G_2, G_3$ and $F_1, F_2, F_3$ finitely generated groups, and let $g \in G_2$. Assume that the groups $F_1, F_2, F_3$ are free, and that $\text{rank} G_i = \text{rank} F_i$ for all $i = 1, 2, 3$. Then there exists a Cuntz-Krieger algebra $O_A$ of real rank zero with primitive ideal space $X_{six}$ realising $(\mathcal{E}, g)$ as $(K_{six}(O_A), [1_{O_A}])$.

Corollary 3.7. Phantom Cuntz-Krieger algebras with exactly one nontrivial ideal do not exist.
The generalization of the invariant $K_{\text{six}}$ to larger primitive ideal spaces is called filtered $K$-theory or filtrated $K$-theory and was introduced by Gunnar Restorff and by Ralf Meyer and Ryszard Nest. Filtered $K$-theory consists of the six-term exact sequences induced by all extensions of subquotients. A smaller invariant, the reduced filtered $K$-theory $\text{FK}_R$ originally defined by Gunnar Restorff to classify Cuntz-Krieger algebras, has so far proven suitable for classifying $C^*$-algebras that look like Cuntz-Krieger algebras.

Let $X$ be a finite $T_0$-space. For $x \in X$, we denote by $\{x\}$ the smallest open subset of $X$ containing $x$, and we define $\partial(x)$ as $\{x\} \setminus \{x\}$. For $x, y \in X$ we write $y \to x$ when $y \in \partial(x)$ and there is no $z \in \partial(x)$ for which $y \in \partial(z)$.

**Definition 3.8.** For a $C^*$-algebra $A$ with primitive ideal space $X$, its reduced filtered $K$-theory $\text{FK}_R(A)$ consists of the groups and maps

$$K_1(A(x)) \xrightarrow{\delta} K_0(A(\partial(x))) \xrightarrow{i} K_0(A(\{x\}))$$

induced by the extension $0 \to A(\partial(x)) \to A(\{x\}) \to A(x) \to 0$, for all $x \in X$, together with the groups and maps

$$K_0(A(\{y\})) \xrightarrow{i} K_0(A(\partial(x)))$$

induced by the extension $0 \to A(\{y\}) \to A(\partial(x)) \to A(\partial(x) \setminus \{y\}) \to 0$, for all $x, y \in X$ with $y \to x$.

**Example 3.9.** Let $X = \{1, 2, 3\}$ be given the topology $\{\emptyset, \{1\}, \{3, 2\}, \{3, 1\}, X\}$. Then for a $C^*$-algebra $A$ with primitive ideal space $X$, its reduced filtered $K$-theory $\text{FK}_R(A)$ consists of the groups and maps

$$\begin{array}{ccc}
K_1(A(2)) & \xrightarrow{\delta} & K_0(A(\{3, 1\})) \\
& & \xrightarrow{i} \\
K_0(A(3)) \xrightarrow{i} K_0(A(\{3, 2\}))
\end{array}$$

Together with the group $K_1(A(3))$.

It is shown in [1] that if $A$ is a $C^*$-algebra of real rank zero with primitive ideal space $X$, then the sequence

$$\bigoplus_{y \to x, y \to x'} K_0(A(\{y\})) \xrightarrow{(i^2 - i^2)} \bigoplus_{x \in X} K_0(A(\{x\})) \xrightarrow{(i)} K_0(A) \to 0$$

is exact.

**Definition 3.10.** For a unital $C^*$-algebra $A$ of real rank zero with primitive ideal space $X$, $1(A)$ is defined as the unique element in

$$\bigoplus_{x \in X} K_0(A(\{x\})) / \bigoplus_{y \to x, y \to x'} K_0(A(\{y\}))$$

that is mapped to $[1_A]$. For $A$ and $B$ unital $C^*$-algebras of real rank zero with primitive ideal space $X$, an isomorphism from $(\text{FK}_R(A), 1(A))$ to $(\text{FK}_R(B), 1(B))$
is defined as a family of isomorphisms

\[ \phi_{\{x\}} : K_1(A(x)) \to K_1(B(x)) \]

\[ \phi_{\partial(x)} : K_0(A(\partial(x)) \to K_0(B(\partial(x))) \]

\[ \phi_{\{x\}} : K_0(A(\{x\})) \to K_0(B(\{x\})) \]

for all \( x \in X \) that commute with the maps \( i \) and \( \delta \) and maps \( 1(A) \) to \( 1(B) \).

Using Theorem 3.6, Rasmus Bentmann, Takeshi Katsura, and the author have established the range of reduced filtered \( K \)-theory \( FK_R \) for graph algebras.

**Theorem 3.11 ([1]).** Let \( B \) be a \( C^\ast \)-algebra that looks like a Cuntz-Krieger algebra. Then there exists a Cuntz-Krieger algebra \( O_A \) of real rank zero with \( \text{Prim}(O_A) \cong \text{Prim}(B) \) for which \( (FK_R(O_A), [1_{O_A}]) \) is isomorphic to \( (FK_R(B), [1_B]) \).

**Definition 3.12.** A finite, connected \( T_0 \)-space \( X \) is called an accordion space if for all \( x \in X \) there are at most two elements \( y \in X \) for which \( y \to x \), and if there is at least two elements \( x \in X \) for which there is exactly one element \( y \in X \) for which \( y \to x \).

The notion of accordion spaces was introduced by Rasmus Bentmann in [4]. Intuitively, a space is an accordion space if and only if the Hasse diagram of the ordering defined by \( y \leq x \) when \( y \in \{x\} \), looks like an accordion. All finite, linear spaces are accordion spaces, and the following five spaces are examples of connected spaces that are not accordion spaces.

**Definition 3.13.** Define a topology on the space \( X = \{1, 2, 3, 4\} \) by defining \( U \subseteq X \) to be open if \( U \) is empty or \( 4 \in U \). Define \( X^{\text{op}} \) as having the opposite topology. Then \( X \) and \( X^{\text{op}} \) have Hasse diagrams

\[
\begin{array}{c}
1 \searrow 2 \swarrow 3 \\
4
\end{array}
\quad
\begin{array}{c}
3 \searrow 4 \swarrow 2 \\
1
\end{array}
\]

respectively. Define a topology on the space \( Y = \{1, 2, 3, 4\} \) by defining \( U \subseteq X \) to be open if \( U \in \{\emptyset, \{4\}\} \) or if \( \{3, 4\} \subseteq U \). Define \( Y^{\text{op}} \) as having the opposite topology. Then \( Y \) and \( Y^{\text{op}} \) have Hasse diagrams

\[
\begin{array}{c}
1 \searrow 3 \\
2 \downarrow 4
\end{array}
\quad
\begin{array}{c}
2 \searrow 3 \\
1 \downarrow 4
\end{array}
\]

respectively. Finally, define a topology on the space \( D = \{1, 2, 3, 4\} \) as the open sets being \( \{\emptyset, \{4\}, \{3, 4\}, \{2, 4\}, \{2, 3, 4\}, D\} \). Then \( D \) has Hasse diagram

\[
\begin{array}{c}
1 \searrow 2 \\
3 \downarrow 4
\end{array}
\]
Ralf Meyer and Ryszard Nest showed in [12] that if $X$ is a finite, linear space, then filtered $K$-theory is a complete invariant for all stable, purely infinite, nuclear, separable $C^*$-algebras $A$ with primitive ideal space $X$ that satisfy that $A(x)$ are in the bootstrap class for all $x \in X$. They also gave a counter-example to completeness of filtered $K$-theory for the space $X$. Using their methods, Rasmus Bentmann and Manuel Köhler showed in [5] that filtered $K$-theory is a complete invariant for such $C^*$-algebras exactly when their primitive ideal space $X$ is an accordion space.

However, Gunnar Restorff, Efren Ruiz, and the author showed in [3] that for the spaces $X, X^\text{op}, Y$, and $Y^\text{op}$, filtered $K$-theory is a complete invariant for such $C^*$-algebras if one adds the assumption of real rank zero. And in [1], Rasmus Bentmann, Takeshi Katsura, and the author showed that for the space $D$, reduced filtered $K$-theory is a complete invariant for $C^*$-algebras that look like Cuntz-Krieger algebras. It is also shown in [1] that for $C^*$-algebras that look like Cuntz-Krieger algebras and have either an accordion space or one of the spaces defined in Definition 3.13 as primitive ideal space, any isomorphism on reduced filtered $K$-theory can be lifted to an isomorphism on filtered $K$-theory.

The five spaces of Definition 3.13 are so far the only non-accordion spaces for which such results have been achieved. However, combining these results with Theorem 2.1 of [13] gives the following theorem, cf. [1].

**Theorem 3.14** ([12], [4], [3], [1]). Let $X$ be either an accordion space or one of the spaces defined in Definition 3.13. Let $A$ and $B$ be $C^*$-algebras that look like Cuntz-Krieger algebras and have $X$ as primitive ideal space. Then $(FK_R(A), 1(A)) \cong (FK_R(B), 1(B))$ implies $A \cong B$.

**Corollary 3.15.** Let $X$ be either an accordion space or one of the spaces defined in Definition 3.13. Then phantom Cuntz-Krieger algebras with primitive ideal space $X$ do not exist.

4. **Summary**

The results stated in this article, are recaptured in the following theorem.

**Theorem 4.1.** Let $A$ be a $C^*$-algebra that looks like a Cuntz-Krieger algebra. If $A$ satisfies either of the following conditions,

- $A$ is a graph algebra,
- $K_*(A) = 0$,
- $\text{Prim}(A)$ is an accordion space,
- $\text{Prim}(A)$ is one of the five four-point spaces of Definition 3.13

then $A$ is isomorphic to a Cuntz-Krieger algebra.

It is unknown whether phantom Cuntz-Krieger algebras exist in general.

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