$\alpha_s$ and the $\tau$ hadronic width

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Different choices exist for the renormalisation group resummation in the determination of $\alpha_s$ from hadronic $\tau$ decays: namely fixed-order (FOPT) and contour-improved perturbation theory (CIPT). The two approaches lead to systematic differences in the resulting $\alpha_s$. On the basis of a model for higher-order terms in the perturbative series, which incorporates well-known structure from renormalons, it is found that while CIPT is unable to account for the fully resummed series, FOPT smoothly approaches the Borel sum. Employing the model to determine $\alpha_s$ yields $\alpha_s(M_\tau) = 0.316 \pm 0.006$, which after evolution leads to $\alpha_s(M_Z) = 0.1180 \pm 0.0008$.

1. INTRODUCTION

Due to its particular mass of $M_\tau \approx 1.8$ GeV, the $\tau$ lepton constitutes an excellent system to study QCD at low energies, as in about 65% of all cases it decays into hadrons, while the QCD description remains largely perturbative. In the seminal work [1], the framework for a precise determination of the QCD coupling $\alpha_s$ from the total $\tau$ hadronic width

$$R_\tau = \frac{\Gamma[\tau^- \to \text{hadrons} \nu_\tau(\gamma)]}{\Gamma[\tau^- \to e^- \tau_\nu(\gamma)]} = 3.640(10),$$

was developed, while afterwards invariant mass distributions were incorporated into the analysis as well [2,3]. The most recent study of the ALEPH spectral function data on the basis of the final full LEP data set yielded $\alpha_s(M_\tau) = 0.344 \pm 0.005_{\exp} \pm 0.007_{\text{th}}$, which after evolution to the $Z$-boson mass scale results in $\alpha_s(M_Z) = 0.1212(11)$. The dominant quantifiable theory uncertainty resides in as yet uncalculated higher-order QCD corrections and improvements of the perturbative series through renormalisation group (RG) methods.

Most suitable for the $\alpha_s$ determination is the $\tau$ decay rate into light $u$ and $d$ quarks $R_{\tau,V/A}$ via a vector or axialvector current, since in this case power corrections are especially suppressed.

Theoretically, $R_{\tau,V/A}$ takes the form [1]

$$R_{\tau,V/A} = \frac{N_c}{2} S_{\text{EW}} |V_{ud}|^2 \left[ 1 + \delta^{(0)} + \delta'_{\text{EW}} + \sum_{D \geq 2} \delta^{(D)}_{ud,V/A} \right],$$

where $S_{\text{EW}} = 1.0198(6)$ [5] and $\delta'_{\text{EW}} = 0.0010(10)$ [6] are electroweak corrections, $\delta^{(0)}$ comprises the perturbative QCD correction, and the $\delta^{(D)}_{ud,V/A}$ denote quark mass and higher $D$-dimensional operator corrections which arise in the framework of the operator product expansion (OPE).

When computing the total $\tau$ hadronic width a phase-space integral over the physical spectrum has to be calculated, which by analyticity can be related to a contour integral over QCD correlators in the complex $s$-plane, where $s$ is the invariant mass of the final state hadronic system. As we also intend to RG improve the perturbative series, the questions arises if the RG resummation should be performed before or after evaluating the contour integral? If the “true” all-order result were available, both treatments should agree, but to any finite order in perturbation theory, significant differences may arise.

The approach of first RG-improving the correlators and then performing the contour integral was introduced in [7,8] and termed contour-improved perturbation theory (CIPT), as unarguably large running effects of $\alpha_s(\sqrt{s})$ along the contour are resummed. However, it is known that
the QCD series is divergent, being asymptotic at best. Hence, also the explicit expansion coefficients of the perturbative series are bound to become large. If cancellations between explicit coefficients and running effects occur, just resumming the running effects may not lead to a good approximation, but it could be better to perform a consistent expansion in powers of $\alpha_s(M_\tau)$, being called fixed-order perturbation theory (FOPT).

While it can be demonstrated that, as expected, CIPT is a good approximation, and CIPT as well as FOPT are compatible, when the running effects dominate [9], in the large-$\beta_0$ approximation, in which $\delta^{(0)}$ is exactly calculable, FOPT gives a better approximation to the true result related to the minimal terms in the large-$\beta_0$ approximation effects dominate [9], in the large-

\[ \delta^{(0)}_{\text{FO}} = \sum_{n=1}^{\infty} a(M^{2})^{n} \sum_{k=1}^{n} k c_{n,k} J_{k-1}, \]

(3)

where $a(\mu^2) \equiv a_\mu \equiv \alpha_s(\mu)/\pi$, and $c_{n,k}$ are the coefficients which appear in the perturbative expansion of the vector correlation function,

\[ \Pi_V(s) = -\frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} a_\mu^n \sum_{k=0}^{n+1} c_{n,k} \ln^k \left( \frac{-s}{\mu^2} \right). \]

(4)

At each perturbative order, the coefficients $c_{n,1}$ can be considered independent, while all other $c_{n,k}$ with $k \geq 2$ are calculable from the RG equation. Further details can for example be found in ref. [13]. Finally, the $J_l$ are contour integrals which are defined by

\[ J_l \equiv \frac{1}{2\pi i} \oint \frac{dx}{x} (1-x)^3 (1+x) \ln^l (-x). \]

(5)

The first three which are required up to $O(\alpha_s^3)$ take the numerical values

\[ J_0 = 1, \quad J_1 = -\frac{19}{12}, \quad J_2 = \frac{285}{72} - \frac{\pi^2}{3}. \]

(6)

At order $\alpha_s^3$ FOPT contains unsummed logarithms of order $\ln^l (-x) \sim \pi^l$ with $l < n$ related to the contour integrals $J_l$. CIPT sums these logarithms, which yields

\[ \delta^{(0)}_{\text{CI}} = \sum_{n=1}^{\infty} c_{n,1} J_n^a(M_{\tau}^2) \]

(7)

in terms of the contour integrals $J_n^a(M_{\tau}^2)$ over the running coupling, defined as:

\[ J_n^a(M_{\tau}^2) \equiv \frac{1}{2\pi i} \oint \frac{dx}{x} (1-x)^3 (1+x) a^n(-M_{\tau}^2 x). \]

(8)

In contrast to FOPT, for CIPT each order $n$ just depends on the corresponding coefficient $c_{n,1}$. 2. PERTURBATIVE CORRECTION $\delta^{(0)}$

In the following, we shall only be concerned with the purely perturbative correction $\delta^{(0)}$ which gives the dominant contribution to $R_{\tau,V/A}$. In FOPT it takes the general form

\[ \delta^{(0)}_{\text{FO}} = \sum_{n=1}^{\infty} a(M^{2})^{n} \sum_{k=1}^{n} k c_{n,k} J_{k-1}, \]

(3)
Thus, all contributions proportional to the coefficient $c_{n,1}$, which in FOPT appear at all perturbative orders equal or greater than $n$ are ressumed into a single term.

Numerically, the two approaches lead to significant differences. Using $\alpha_s(M_\tau) = 0.34$ in eqs. (9) and (10), one finds

$$\delta^{(0)}_{\text{FO}} = 0.2200 \ (0.2288),$$
$$\delta^{(0)}_{\text{CI}} = 0.1984 \ (0.2021),$$

where the first number in both cases employs the known coefficients up to $\mathcal{O}(\alpha_s^3)$ and the numbers in brackets include an estimate of the $\mathcal{O}(\alpha_s^5)$ term with $c_{5,1} \approx 283$. Inspecting the individual contributions from each order, up to $\mathcal{O}(\alpha_s^5)$ the CIPT series appears to be better convergent.

In the following discussion it is slightly more convenient to utilize the related function $\hat{D}(s)$. Its Borel transform $B[\hat{D}](t)$ is defined by the relation

$$\hat{D}(\alpha) = \int_0^\infty dt \ e^{-t/\alpha} B[\hat{D}](t).$$

The integral $\hat{D}(\alpha)$, if it exists, gives the Borel sum of the original divergent series. It was found that the Borel-transformed Adler function $B[\hat{D}](t)$ obtains infrared (IR) and ultraviolet (UV) renormalon poles at positive and negative integer values of the variable $u \equiv 9t/(4\pi)$, respectively $\delta^n_{\text{IR}}$ and $\delta^n_{\text{UV}}$. (With the exception of $u = 1$.)

Apart from very low orders, where a dominance of renormalon poles close to $u = 0$ has not yet set in, intermediate orders should be dominated by the leading IR renormalon poles, while the leading UV renormalon, being closest to $u = 0$, dictates the large-order behaviour of the perturbative expansion. Assuming that only the first two orders are not yet dominated by the lowest IR renormalons, one is led to the ansatz

$$B[\hat{D}](u) = B[\hat{D}^{\text{UV}}_1](u) + B[\hat{D}^{\text{IR}}_2](u) + B[\hat{D}^{\text{IR}}_3](u) + d_0^{\text{PO}} + d_1^{\text{PO}} u,$$

which includes one UV renormalon at $u = -1$, the two leading IR renormalons at $u = 2$ and $u = 3$, as well as polynomial terms for the two lowest perturbative orders. Explicit expressions for the UV and IR renormalon pole terms $B[\hat{D}^{\text{UV}}_p](u)$ and $B[\hat{D}^{\text{IR}}_p](u)$ can be found in section 5 of ref. [13].

Apart from the residues $d_p^{\text{UV}}$ and $d_p^{\text{IR}}$, the full structure of the renormalon pole terms is dictated by the OPE and the RG. Therefore, the model [13] depends on five parameters, the three residues $d_1^{\text{UV}}$, $d_2^{\text{IR}}$ and $d_3^{\text{IR}}$, as well as the two polynomial parameters $d_0^{\text{PO}}$ and $d_1^{\text{PO}}$. These parameters can be fixed by matching to the perturbative expansion of $\hat{D}(s)$ up to $\mathcal{O}(\alpha_s^5)$. Thereby one also makes use of the estimate for $c_{5,1}$. The parameters of the model [13] are then found to be:

$$d_1^{\text{UV}} = -1.56 \cdot 10^{-2}, \ d_2^{\text{IR}} = 3.16, \ d_3^{\text{IR}} = -13.5,$$
$$d_0^{\text{PO}} = 0.781, \ d_1^{\text{PO}} = 7.66 \cdot 10^{-3}.$$

The fact that the parameter $d_1^{\text{PO}}$ turns out to be small implies that the coefficient $c_{2,1}$ is already reasonably well described by the renormalon pole contribution, although it was not used to fix the residue. Therefore, one could set $d_0^{\text{PO}} = 0$ and actually work with a model which only has four parameters. The predicted value $c_{5,1} = 280$ in this model turns out very close to the estimate, which can be viewed as one test of the stability of the model.

A graphical account of the model [13] for the (reduced) Adler function $\hat{D}(M_\tau^2)$ is displayed in figure 1. The full circles denote the partial sums of the perturbative series up to order $n$. The

3. A PHYSICAL MODEL

To clarify whether FOPT or CIPT results in a better approximation to $\delta^{(0)}$, one needs to construct a physically motivated model for its series. The corresponding model will be based on the Borel transform of the Adler function $D_V(s)$:

$$D_V(s) \equiv -s \frac{d}{ds} \Pi_V(s) = \frac{N_c}{12\pi^2} [1 + \hat{D}(s)].$$

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minimal term of the series at the 5th order is marked by a grey diamond. The perturbative results are compared with the Borel sum of the series at the 5th order.

The implications of the model (13) for $\delta^{(0)}_{\text{FO}}$ in FOPT and CIPT is graphically represented in figure 2. The full circles denote the result for $\delta^{(0)}_{\text{FO}}$ in FOPT, and the grey circles the one for $\delta^{(0)}_{\text{CI}}$, as a function of the order $n$ up to which the terms in the perturbative series have been summed. The straight line corresponds to the principal value Borel sum of the series, $\delta^{(0)}_{\text{BS}} = 0.2371$, and the shaded band provides an error estimate based on the imaginary part divided by $\pi$. The order at which the series have their smallest terms is indicated by the grey diamonds.

As is obvious from figure 2, like for the Adler function itself, FOPT displays the behaviour expected from an asymptotic series: the terms decrease up to a certain order around which the closest approach to the resummed result is found, and for even higher orders, the divergent large-order behaviour of the series sets in. For CIPT, on the other hand, the asymptotic behaviour sets in earlier, and the series is never able to come close to the Borel sum.

The finding that in the model (13) CIPT misses the full Borel sum can be traced back to the fact that in CIPT the running effects along the complex contour are resummed to all orders, while explicit contributions of the $c_{n,1}$ at a certain order are dropped. However, being an asymptotic series, also the Adler function coefficients $c_{n,1}$ become large, and cancellations between the explicit contributions and the running effects take place. As was shown in section 4 of [13], in the large-$\beta_0$ approximation, for $R_{\tau}$ the leading IR renormalon cancels completely, and also the large-order divergence of the series is softened. Even though in real QCD the leading IR does not anymore cancel completely, for $R_{\tau}$ it is still suppressed by a factor $1/n^2$, and furthermore a sign-alternating UV renormalon component does not yet show up in the known coefficients. Thus, the cancellations between running effects and explicit coefficients are also expected to prevail in full QCD.

The deficiency of CIPT to approach the Borel sum of the series, which leads to the marked differences of $\delta^{(0)}_{\text{FO}}$ and $\delta^{(0)}_{\text{CI}}$, can also be observed when the Adler function is inspected along the

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1 Also shown as the shaded band is an estimate of the uncertainty inferred from the complex ambiguity which arises while defining the Borel integral over the IR renormalon poles. For details see appendix A of ref. [13].
complex circle. This is discussed in detail in appendix B of ref. 13. While FOPT converges to the Borel sum on the full circle (though rather badly close to the Minkowskian axis), in some regions of the circle CIPT largely differs from the resummed result, again due to the missed cancellations.

As the behaviour of CIPT versus FOPT hinges on the contribution of the leading IR renormalon at \( u = 2 \), in principal also models can be constructed for which CIPT provides a good account of the Borel sum. These would generally be models where \( d_2^{(n)} \) is much smaller than the value quoted in eq. (13). While such models can at present not be excluded, the pattern of the individual contributions appears more unnatural than in the model (13): the known \( c_{n,1} \) can only be reproduced when one allows for large cancellations between the individual terms. Thus, the behaviour generally expected from the presence of renormalon poles, namely dominance of leading IR poles at intermediate orders, would be lost.

4. Determination of \( \alpha_s \)

The starting point for a determination of \( \alpha_s \) from hadronic \( \tau \) decays is eq. (2) for the decay rate of the \( \tau \) lepton into light \( u \) and \( d \) quarks. The analysis will be based on FOPT together with the ansatz (13) for higher-order terms discussed in the last section. Due to the observation that CIPT is not able to approach the resummed series, it will not be employed below.

The first step of the \( \alpha_s \) analysis consists in estimating the values of the power corrections \( \delta_{ud,V+A}^{(D)} \), which arise from higher-dimensional operators in the framework of the OPE. Given these estimates and experimental data, a phenomenological value of \( \delta^{(0)} \) can be calculated using eq. (2). This allows to determine the value of \( \alpha_s(M_\tau) \) by requiring that the theoretical value \( \delta_{FO}^{(0)} \) matches the phenomenological value \( \delta_{phen}^{(0)} \). Errors are estimated by varying all parameters within their uncertainties.

In view of the smallness of the light quark masses \( m_u \) and \( m_d \), as well as the suppression of the dimension-4 contribution in \( R_\tau \), the dominant power correction arises from the six dimensional 4-quark condensates. As the number of contributing operators is too large to treat the 4-quark condensates individually, conventionally the so-called vacuum-saturation approximation (VSA) is employed. Then the corresponding contribution takes the form

\[
\delta_{ud,V+A}^{(6)} = - \frac{512}{27} \pi^3 \alpha_s \rho(\bar{q}q)^2 \frac{M_\rho^2}{M_\pi^2} \quad \text{with} \quad \rho = 2 \pm 1 \quad \text{assumed}
\]

where for the numerical estimate the required quark condensate is taken from the GMOR relation. Because the perturbative series for this correlator does not converge very well, the approach of refs. [19,20] will be followed. The main idea is to replace the QCD expression for the pseudoscalar correlator by a phenomenological representation. The dominant contribution to the pseudoscalar spectral function stems from the well-known pion pole, giving

\[
\delta^{\pi\rho}_{ud,S+P} = -16\pi^2 \frac{f_\pi^2 M_\rho^2}{M_\pi^2} \left( 1 - \frac{M_\pi^2}{M_\rho^2} \right)^2
\]

plus small corrections from higher-excited pionic resonances. Repeating the analysis of ref. [19] and updating the input parameters, one finds

\[
\delta_{ud,S+P} = (-2.64 \pm 0.05) \cdot 10^{-3}
\]

Collecting all contributions, and adding the errors in quadrature, one arrives at the total estimate of all power corrections:

\[
\delta_{PC} = (-7.1 \pm 3.1) \cdot 10^{-3}
\]

The value (18) is consistent with the most recent fit to the \( \tau \) spectral functions performed in ref. [4].

As a matter of principle, the OPE of correlation functions in the complex s-plane could be

\[
\sum_{i=1}^{\infty} \delta_i = \sum_{i=1}^{\infty} \frac{\alpha_i}{s_i}
\]

The scalar correlator, being proportional to \((m_u - m_d)^2\), is completely negligible.
inflicted with so-called “duality violations” [21]. These arise from the contour integral close to the physical region which even though suppressed in $R_r$ could be sizeable [22]. Nevertheless, before a possible additional duality violating contribution can be extracted consistently from a combined fit to spectral moments, it shall be omitted.

Employing the value $R_{r,V+A} = 3.479 \pm 0.011$, which results from eq. [1] in conjunction with $R_{r,S} = 0.1615 \pm 0.0040$ [4], as well as $|V_{ud}| = 0.97418 \pm 0.00026$ [23], from eqs. [2] and [15] the phenomenological value for $\delta^{(0)}$ can be derived:

$$\delta^{(0)}_{\text{phen}} = 0.2042 \pm 0.0050 . \quad (19)$$

The dominant experimental uncertainty in [19] is due to $R_{r,V+A}$ and the theoretical one to the dimension-6 condensate. The final step in the extraction of $\alpha_s(M_r)$ now consists in finding the values of $\alpha_s$ for which $\delta^{(0)}$ matches the theoretical prediction, which yields [13]

$$\alpha_s(M_r) = 0.3156 \pm 0.0030 \text{exp} \pm 0.0051 \text{th} . \quad (20)$$

Evolving this result to the $Z$-boson mass scale finally leads to

$$\alpha_s(M_Z) = 0.11795 \pm 0.00076 , \quad (21)$$

in perfect agreement with the world average [21].

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