Coherent Control of Cooperative Photon Emission from Indistinguishable Quantum Emitters

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Photon-mediated interactions between atomic systems are the cornerstone of quantum information transfer. They can arise via coupling to a common electromagnetic mode or by quantum interference. This can manifold in cooperative light-matter coupling, yielding collective rate enhancements such as those at the heart of superradiance, or remote entanglement via measurement-induced path erasure. Here, we report coherent control of cooperative emission arising from two distant but indistinguishable solid-state emitters due to path erasure. The primary signature of cooperative emission, the emergence of “bunching” at zero-delay in an intensity correlation experiment, is used to characterise the indistinguishability of the emitters, their dephasing, and the degree of correlation in the joint system which can be coherently controlled. In a stark departure from a pair of uncorrelated emitters, we observe photon statistics resembling that of a weak coherent state in Hong-Ou-Mandel type interference measurements. Our experiments establish new techniques to control and characterize cooperative behaviour between matter qubits using the full quantum optics toolbox, a key stepping stone on the route to realising large-scale quantum photonic networks.

Cooperative photon emission can arise between quantum emitters due photon-mediated interaction via a shared electromagnetic mode. This can occur with indistinguishable atoms, or artificial atoms, which emit identical photon wavepackets and cannot be spatially distinguished. Atomic indistinguishability leads to entangled multi-particle states referred to as Dicke states [1]. In ensembles of densely packed atomic or solid-state emitters, Dicke states can yield sub- and super-radiant emission with modified temporal, spectral, and directional properties [2–6]. At the few emitter level, cooperative emission has been observed with emitters positioned closely (interatomic separation Δ < λ, the photon wavelength) in free space [7, 8] or coupled to one-dimensional waveguides [9–14]. Dicke states offer intriguing potential to engineer quantum states for applications in quantum information processing [15–19] and precision metrology [20, 21].

Atomic correlations can also occur from quantum interference between distant quantum emitters. This is commonly achieved by using a beam-splitter to erase the which-path information from two indistinguishable emitters, providing a route to realize scalable quantum networks [22–27]. Similarly, interference in the far-field of spatially separated sources of indistinguishable single photons also gives rise to entanglement and Dicke states [28–30]. Compared to the spontaneous emission from a single atom or a group of distinguishable atoms, a signature of the atomic entanglement that underlies cooperative emission is a change in the second-order intensity correlations; for instance, “bunching”, rather than “anti-bunching”, arises at zero delay in a Hanbury Brown and Twiss interferometer (HBT) [10, 12–14, 31–33]. While cooperative emission, and in particular sub- and super-radiance, have been extensively explored in both theory and experiment, coherent control of the correlations and a clear understanding of the effects of indistinguishability and dephasing have yet to be investigated. Further, higher-order intensity-intensity correlations that characterise the coherence [34] of cooperative emission remain unexplored.

Here we report cooperative emission from two proximate semiconductor quantum dots (QDs) which are electrically tuned into resonance. Although Δ > λ for the two QDs, collection of the emission by a diffraction-limited focus at Δ/2 erases the spatial distinguishability of the photons, creating emitter entanglement. By probing the second-order intensity correlations, we characterise the indistinguishability of the QDs under different excitation conditions and detuning. Via pulsed resonance fluorescence, we achieve coherent control of the Dicke state population and observe increased correlations, a signature of entanglement of the two emitters. Compared to strict resonance fluorescence, we observe decreased correlations using quasi-resonant excitation (into the phonon sideband, blue-detuned from the zero-phonon line) due to increased emitter dephasing and time jitter of the exciton population. Time-resolved resonance fluorescence of the independent (spectrally detuned) QDs and the correlated indistinguishable QD system reveals identical lifetimes, indicating an absence of temporal modification in the emission as typically associated with superradiance [2]. This indicates that the zero delay peak in the HBT experiment is not a sufficient witness of superradiant emission with collective rate enhancements, but is a sensitive probe of cooperative

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emission. Finally, we perform higher-order intensity-intensity correlations with a Hong-Ou-Mandel type interferometer [35] and observe photon statistics equivalent to that of a weak coherent state from an attenuated laser [36]. Our work establishes new techniques from the quantum optics toolbox to control and characterize collective light-matter interactions.

Figure 1(a) shows a schematic of our experiment: a confocal microscope with a diffraction-limited focus is used for both optical excitation and photon collection from two QDs. Crucially, the focal spot is optimized to ensure equal excitation and collection from both emitters, erasing the spatial distinguishability of each emitter. This is illustrated graphically in Figure 1(b), in which two identical QDs, each with spontaneous emission rate $\gamma$, emit photons with wave vector $k$ orthogonal to the separation between the emitters $r$. The indistinguishability of detected photons ensures that, upon measurement of a single photon from a two-emitter system initially in the doubly-excited state $|e_1, e_2\rangle$, the wave function collapses to the maximally entangled Dicke state $1/\sqrt{2}(|e_1, g_2\rangle + |g_1, e_2\rangle)$ (cf. Methods section). The entanglement enables both emitters to cooperatively take part in the second photon emission process, which enhances photon coincidences compared to the emission from uncorrelated emitters [37]. Note that, here, the entanglement is induced by the measurement process. In contrast, entanglement between emitters in the superradiant regime arises from free radiative decay alone [1, 2]. Figure 1(d) shows examples of the second-order correlation function $g^{(2)}(\tau)$ near zero time delay (cf. supplementary material S1) for continuous wave (CW) driving of: (i) a single quantum emitter, which exhibits perfect anti-bunching $g^{(2)}(0) = 0$ (blue curve); (ii) two distinguishable quantum emitters, which exhibit $g^{(2)}(0) = 0.5$ (grey curve); and (iii) two indistinguishable quantum emitters with different dephasing rates $\gamma_d = \gamma$ and $10\gamma$ (purple and red curves, respectively). An instructive example for which an analytic solution is available is the special case of incoherent pumping with equal pump and decay rates $\gamma_p = \gamma$, where one obtains a delay-time dependence of cooperative emission of the form [37]

$$g^{(2)}(\tau) = 1 - (e^{-2\gamma \tau} - e^{-(2\gamma + \gamma_d)\tau})/2.$$  

This yields the “anti-dip” at $\tau = 0$ with a width determined by the dephasing $\gamma_d$. As the main effect of dephasing here is to reduce the coherence between states with a single excitation in either emitter and therefore the correlations, the strong dependence of the $g^{(2)}(\tau)$ signal on the dephasing further highlights the importance of inter-emitter entanglement for the enhancement of photon coincidences.

**Tuning two near degenerate quantum dots into resonance** The sample consists of self-assembled InGaAs QDs embedded in a Schottky diode to control the QD charge state via Coulomb blockade [38] and allow a small range of energy tuning via the DC Stark effect [39]. Details on the device design are found in the Methods section. The self-assembly process leads to random spatial positions and an inhomogeneous distribution of QD energies and spontaneous emission lifetimes ($\gamma$). We therefore search the sample for the unlikely situation in which two QDs: (i) are close enough to each other to optically couple to the same focal point, (ii)
Figure 2. **Tuning two near degenerate quantum dots into resonance.** a PL spectra (non-resonant excitation) of negatively charged exciton transitions from QD1 and QD2 as a function of applied gate voltage ($V_G$). b Individual PL spectra at $V_G = -0.46, -0.54$ and $-0.58$ V. The QDs are tuned into degeneracy at $V_G = -0.54$ V. c Spatial profile of both QDs at $V_G = -0.46$ V. The Gaussian fits (solid lines) to the experimental data (circles) give the separation of two QDs of $\Delta = 512$ nm. d (Top) Second-order intensity correlation for emissions from both QDs (QD1 + QD2) reveals a anti-dip around zero time delay for the case at zero detuning ($\delta = 0$) and a dip showing a $g^{(2)}(0) \approx 0.5$ for non-zero detuning ($\delta = 3.9 \mu$eV). (Bottom) Second order intensity correlation ($g^{(2)}$) for emission from QD1 reveals dip at the zero-delay, showing a $g^{(2)}(0) \approx 0.06$, signifying single photon emission. The slight bunching in coincidences away from the zero-delay for the single emitter case, due to spectral diffusion, disappears for the case of two emitters.

have very similar transition energies and $\gamma$, and (iii) have different permanent dipole moments such that the QDs can be tuned into resonance with a vertical electric field. We choose to work with negatively-charged exciton ($X^\text{\textsc{l}}^-$) transitions, which unlike neutral excitons lack fine-structure splitting. Figure 2(a) shows the photoluminescence (PL) (using non-resonant excitation) for our chosen QD pair ($X^\text{\textsc{l}}^-\text{transitions}$) as a function of gate bias. The three line cuts at different applied biases in Figure 2(b) demonstrate the ability to tune the two QDs to the same emission wavelength using only the applied gate voltage. The resonance is found at $V_G = -0.540$ V. Here, $\lambda \approx 971$ nm in free space while inside the GaAs (with index of refraction $n \approx 3.67$), $\lambda_{\text{GaAs}} \approx 265$ nm. Fixing $V_G = -0.460$ V (where both QDs are spectrally distinguishable, detuned by $\delta \approx 70$ meV), we spatially map the precise locations of the two QDs by scanning the sample position while recording the emission spectra. Assigning the peak at lower (higher) wavelength to be QD1 (QD2), we obtain the spatial profile of each QD, as shown in Figure 2(c). Gaussian fits to the intensity vs scanner position gives the spatial separation of the two QDs: $\Delta = 513.8 \pm 2$ nm [40]. We note that for the two QDs in GaAs, $\Delta \approx 2\lambda_{\text{GaAs}}$, beyond the expected range for a significant superradiant emission enhancement. Here, we define the origin $(X,Y)=(0,0)$ as the optimal spatial position to ensure equal collection from both QDs. For all subsequent experiments, we perform all measurements at this position and utilize the Stark shifts to render the emitters degenerate or detuned.

To confirm the indistinguishability and cooperative emission of the two QDs when degenerate, we obtain resonance fluorescence under CW driving and measure the second-order correlation function $g^{(2)}(\tau)$ with the HBT interferometer. Figure 2(d) shows $g^{(2)}(\tau)$ for three scenarios at an excitation power ($P$) of $P/P_{\text{sat}} \approx 0.1$, where $P_{\text{sat}}$ is the excitation power at saturation [41]. When the QDs are detuned and only QD1 is addressed resonantly, $g^{(2)}(0) \to 0$, indicating single photon emission. To equally drive the QDs when they are detuned ($\delta \approx 3.9 \mu$eV), we excite both emitters slightly off resonance with a laser detuned by $\pm \delta/2$, and obtain $g^{(2)}(0) \to 0.5$, as expected for two distinguishable emitters emitting uncorrelated single photons. Finally, $g^{(2)}(0)$ for resonance fluorescence from the two degenerate QDs ($\delta = 0$) reveals the emergence of a zero-delay anti-dip, in agreement with previous reports for superradiant emission [10, 12, 13]. This signature confirms the coupling between both emitters. We fit the experimental data to analytical equation (c.f. Eq. 1), convolved with the Gaussian instrumental response function (FWHM=0.240 ns) and obtain a decay time of $(2\gamma)^{-1} = 0.880 \pm 0.022$ ns, dephasing time of $\gamma_d^{-1} = 0.199 \pm 0.010$ ns and hence a coherence time of $\tau_{\text{coh}} = 0.162 \pm 0.009$ ns, indicating the $1/e$ width of the anti-dip. While we observe $g^{(2)}(0) \approx 0.87$ from the data/fit, upon deconvolution we obtain $g^{(2)}(0) \approx 1$, as expected from collective emission from two indistinguishable emitters. Please refer to Figure S3.2a in Section S3 in the Supplementary Material for the fits to the data.

Finally, as shown in Section S4 of the Supplementary Material, we observe Rabi oscillations in the $g^{(2)}(\tau)$ coincidence histogram as we increase the driving strength, in excellent agreement with simulated data produced by our model in Section S1 of the Supplementary Material.

**Coherent Control of Cooperative Emission** To manipulate the Dicke ladder populations and probe the transient emission behavior, we perform resonance flu-
Figure 3. **Coherent control of two quantum dots.**

- **a** Emission intensity as a function of excitation power and pulse area from QD1 (blue), QD2 (green) and Both Dots when degenerate (purple) under resonant excitation.
- **b** Time-resolved emission profiles of the emission from the degenerate QDs (top) and individual emitters (bottom, QD1 and QD2) exhibit similar lifetimes of $T_1 = 0.643\, (1)$ and $0.668\, (1)$, $0.635\, (1)$ ns, respectively. The pulse area is $\pi/2$ for each measurement.
- **c** Second-order intensity correlation of the emission under resonant excitation (QD1 (blue), Both QDs at nonzero detuning ($\delta \neq 0$, grey) and zero detuning ($\delta = 0$, RF, purple)) and non-resonant excitation for the degenerate QDs (NR, orange). A zoom-in of the zero-delay peaks is shown in the dashed box. The data for “Reference” is the side peak at 12.44 ns for the RF case. Measurements for RF, $\delta \neq 0$ and QD1 are taken at the pulse area of $\pi/2$, while measurement for NR is taken at the saturation power.
- **d** Normalized zero-delay coincidences from the HBT measurement, $g^{(2)}(\tau)$ under resonant excitation, as a function of excitation power, within 0.3 ns (red) and 10 ns (purple) integration windows.
- **e** Coincidence histogram at a pulse area of $\pi$, $2\pi$ and $3\pi$. The data for $\pi$ and $3\pi$ are shifted temporally by -2 and 2 ns, respectively.

Fluorescence and time-resolved measurements using a pulsed laser. Figure 3(a) shows coherent Rabi oscillations in the detected emission count rate as a function of pulse area for each individual emitter (when detuned) and for both QDs when degenerate. Each individual QD exhibits the same excitation power to Rabi frequency conversion and similar count rates, while the degenerate case yields the identical power to Rabi frequency conversion and the measured count rates are a sum of the individual QD count rates. Fixing the excitation power corresponding to a pulse area of $\pi/2$, we measure the temporal emission profile for each case with a time-resolved resonance fluorescence measurement, as illustrated in Figure 3(b). Here we observe nearly identical temporal profiles and lifetimes in each case: $T_1 = 0.668\, (1)$, $0.635\, (1)$, and $0.643\, (1)$ ns for emission from QD1, QD2, and both QDs when degenerate, respectively. These results confirm that the temporal properties of the cooperative emission process are unchanged from the independent QDs; superradiance with a modified temporal profile is not observed, as expected for QDs with $\Delta > \lambda_{\text{GaAs}}$.

Figure 3(c) presents $g^{(2)}(\tau)$ for resonance fluorescence at a pulse area of $\pi/2$ for four representative cases, with each data set offset by a few ns on the x-axis for clarity. A zoom-in around zero-delay (without x-axis offset) is shown in the bottom panel of Fig. 3(c). Overall, the pulsed resonance fluorescence behavior is analogous to that obtained under CW driving. At zero-delay, we observe $g^{(2)}(0) \rightarrow 0$, $g^{(2)}(0) \approx 0.5$, and $g^{(2)}(0) \rightarrow 1$ for an individual emitter (QD1), distinguishable emitters ($\delta \neq 0$), and indistinguishable QDs (RF), respectively. For the individual emitter (QD1), the two QDs are detuned by $\sim 100\, \mu$eV. Integrating the coincidences within a 10 ns window around the zero-delay, we obtain $\Delta\tau = 10\, \text{ns}$ for emission from QD1 and distinguishable QDs (RF), respectively. For the individual emitter (QD1), the two QDs are detuned by $\sim 100\, \mu$eV. Integrating the coincidences within a 10 ns window around the zero-delay, we obtain $\Delta\tau = 10\, \text{ns}$ for emission from QD1 and distinguishable QDs (RF), respectively.
Instead obtain a much higher $g_{\Delta T=0.3\, \text{ns}}^{(2)}(0) = 0.904$ for two indistinguishable QDs, approaching the theoretical maximum of 1. Comparison of these two integration windows of $g_{\Delta T}^{(2)}(0)$ for the degenerate QDs suggests that the cooperative emission is sensitive to the degree of indistinguishability or dephasing between the two emitters. This effect is easily visualized by comparing the $g_{\Delta T}^{(2)}(0)$ peak for the indistinguishable QD (RF) to a reference peak (e.g. $\tau = 12.44\, \text{ns}$, black plot), as shown in the bottom panel. Here, we observe narrowing of the zero-delay peak compared to the side peaks at 12.44 ns. We then fit the experimental data (zero-delay peak) with Eq. 2 (convolved with the Gaussian instrument response function with FHWM=0.240 ns) to extract the dephasing parameter, $\gamma_d$.

$$g_{\text{Pulsed}}^{(2)}(\tau) = \frac{e^{-\gamma \tau} + e^{-(\gamma + \gamma_d) \tau}}{2}. \tag{2}$$

Using the measured lifetime, $\gamma^{-1} = 0.643\, (1)\, \text{ns}$, we extract a dephasing time of $\gamma_d^{-1} = 0.280\, (7)\, \text{ns}$ and hence a coherence time of $(\gamma + \gamma_d)^{-1} = 0.195\, (3)\, \text{ns}$ from the fit.

To experimentally demonstrate that the $g_{\Delta T}^{(2)}(0)$ value for cooperative emission is sensitive to dephasing, we excite the degenerate QDs using pulsed non-resonant, phonon-assisted excitation (NR, orange plot in Fig. 3(c)) and observe reduced ‘bunching’ compared to strict resonance fluorescence: $g_{\Delta T=0.3\, \text{ns}}^{(2)}(0) = 0.624\, (7)$ and $g_{\Delta T=10\, \text{ns}}^{(2)}(0) = 0.556\, (4)$ within 0.3 and 10 ns integration windows, respectively. Additionally, fitting the data to Eq. 2 results in a coherence time of 0.043 (2) ns, significantly shorter than that observed with coherent driving. We ascribe this reduction to reduced indistinguishability between the two QDs when using incoherent excitation, likely due to decoherence from the phonon-assisted transitions as well as increased charge noise and time jitter. Additional detailed spectroscopy results (power dependence and detuning dependence) under resonant and non-resonant excitation are included in Section S4 of the Supplementary Material.

To demonstrate coherent control and the ability to manipulate the populations on the Dicke ladder, we vary the excitation power of the resonant pulse and observe that the $g_{\Delta T=0.3\, \text{ns}}^{(2)}(0)$ ($g_{\Delta T=10\, \text{ns}}^{(2)}(0)$) value oscillates with pulse area: a maximum of 0.9 (0.73) at $2\pi$ pulse area and a value of 0.84 (0.63) at $\pi$ pulse area. The power dependence and the coincidence histograms that correspond to the three example pulse areas are depicted in Figure. 3(d) and 3(e), respectively. This result suggests that the measured $g_{\Delta T}^{(2)}(0)$ depends on the population of the bright, symmetric Dicke state: a maximal $g_{\Delta T}^{(2)}(0)$ value at pulse areas of even multiples of $\pi$, where the two-emitter system remains approximately in the ground state following the excitation pulse and a minimal $g_{\Delta T}^{(2)}(0)$ value at pulse areas of odd multiples of $\pi$, where the system is close to the doubly-excited state which then collapses onto the symmetric Dicke state following the first photon detection event. Full interpretation and modeling of this trend require the sufficient knowledge of multiple key parameters, including, but not limited to re-excitation rate [42, 43], excitation-induced dephasing rate [44], and signal-to-background ratio due to laser leakage as a function of driving power. This complete model is beyond the scope of this manuscript [37].

Finally, we note the bunching around zero delay demonstrates entanglement between the two QDs. In the absence of detector jitter, the limit $g_{\Delta T}^{(2)}(\tau \to 0)$ indicates the degree of the instantaneous entanglement immediately after the first photon detection event as it is related to the occupation of the Dicke state (cf. Methods section or Section S2 in the Supplementary Material). However, in a realistic experimental setting, this limit is largely determined by finite detection jitter (with $g_{\Delta T}^{(2)}(0) = 1$ for perfect detectors) and thus is not a reliable emitter entanglement metric. Therefore, we consider the integral of the entire zero-delay peak, relative to the adjacent uncorrelated side-peaks, as a more representative indicator of the degree of entanglement. This quantity $S_{\Delta T}^{(2)}$ has the further advantage of being resilient to details of emitter dephasing that determine $g_{\Delta T}^{(2)}(\tau \to 0)$. The degradation in the the degree of entanglement due to the increase in the dephasing under non-resonant excitation (at saturation) results in a much lower value of $S_{\Delta T=10\, \text{ns}}^{(2)} \approx 0.56$ compared to that under resonant excitation $S_{\Delta T=10\, \text{ns}}^{(2)} \approx 0.67$ (at a pulse area of $\pi/2$). Please refer to Section S3 in the Supplementary Material for the discussion on this entanglement measure in the presence of noise and detection jitter.

Higher-Order Intensity Correlations of Cooperative Emission

Next, we probe the coherence of the cooperative emission by interfering subsequently emitted photons in a conventional Hong-Ou-Mandel-type interferometer setup. The measurement setup consists of an unbalanced Mach Zehnder interferometer with a delay to match the temporal separation of the excitation pulses, i.e. $\Delta T = 12.44\, \text{ns}$. To quantify the interference visibility $V_{\text{HOM}}(\tau)$, we define

$$V_{\text{HOM}}(\tau) = 1 - g_{\parallel}^{(2)}(\tau)/g_{\perp}^{(2)}(\tau), \tag{3}$$

where in the case of single photon input, $V_{\text{HOM}}(0)$ is the ratio of coincidences at zero delay when photons in the two paths of the interferometer are rendered indistinguishable ($g_{\parallel}^{(2)}(0)$) and distinguishable ($g_{\perp}^{(2)}(0)$) in polarization. While $V_{\text{HOM}}(\tau)$ gives the single photon indistinguishability for a single photon input, it provides a measure of the degree of coherence [34] of entangled photons from both emitters.

Figure 4(a) and (b) show the results of the experiment for the emission from only QD1 and from both
QDs when degenerate, respectively, under CW resonant driving. The temporal post-selected indistinguishability for QD1 yields \( V_{\text{HOM}}(0) \rightarrow 1 \), as expected for an individual emitter. However, for cooperative emission from both QDs, we observe a maximum visibility of \( V_{\text{HOM}}(0) \approx 0.5 \). This result signals a significant change in emitted light, from being anti-bunched (sub-Poissonian) for a single QD to coherent (Poissonian) due to cooperative emission. Notably, the \( 1/e \) width of \( V_{\text{HOM}} \), which gives the coherence time of the emission, for the cooperative case (\( \approx 1.34 \) ns) is comparable to the single emitter case (\( \approx 1.15 \) ns). Using the coherence time window (CTW) [45–47], i.e. the integrated area of \( V_{\text{HOM}} \), as a figure of merit, we find the CTW for the single emitter case (1.37 (1) ns) is more than twice that of the cooperative emission (0.53 (1) ns), reflecting the difference for each case in \( V_{\text{HOM}}(0) \) and \( 1/e \) widths. Beyond the time averaged picture, Figure 4(c) and (d) show the result of Hong-Ou-Mandel type interference for the emission from QD1 and Both QDs under pulsed resonant driving, respectively. For QD1, we observe \( V_{\text{HOM}} = 0.40 (1) \) for the non-post-selected (10 ns integration window) visibility and a maximum post-selected (0.1 ns integration window) visibility of \( V_{\text{HOM}} = 0.79 (10) \). These results indicate partial distinguishability between successively emitted photons from QD1. Similarly, compared to \( V_{\text{HOM}} = 0.5 \) obtained from the CW measurement and \( V_{\text{HOM}} = 0.37 (2) \) in the 0.1 ns integration window, degradation in the non-post-selected visibility for the cooperative emission is observed: \( V_{\text{HOM}} = 0.20 (1) \) within a 10 ns integration window.

**DISCUSSION**

In summary, we coherently control and probe two quantum dots coherently coupled by a photon-mediated interaction enabled by the indistinguishability of their photon wavepackets and path erasure of their spatial positions. We find that both \( g^{(2)}(\tau) \) and lifetime measurements are necessary to fully describe the nature of the coherent coupling; \( g^{(2)}(\tau) \) on its own is not a sufficient witness of superradiance. Compared to incoherent excitation, we demonstrate that coherent driving enables both increased emitter indistinguishability and control of the populations on the Dicke ladder. We find that after emission of the first photon, both emitters are in an entangled state, as signified by \( g^{(2)}(0) \rightarrow 1 \). Further, we show that \( g^{(2)}(\tau) \) is a sensitive probe of the dephasing of the entangled emitters. For coherent driving, the dephasing likely originates from emitter coupling to the solid-state environment (phonon-induced dephasing [47, 48] or charge and spin noise [49]), which can be at least partially mitigated with improved device quality, choice of charge state, and Purcell enhancement. Finally, we show that a Hong-Ou-Mandel type interferometer can characterize the coherent nature and Poissonian statistics of cooperative emission. The photon correla-
tions observed in this experiment are equivalent to that of a weak coherent state from an attenuated laser [36]. This result demonstrates the coherence of the cooperative emission and evokes the analogy between cooperative emission, in which indistinguishable atomic dipoles are locked in-phase, and lasing [2]. Exciting prospects would be to increase the number of cooperative emitters using scalable approaches and include spin control of the coherently coupled emitters. This would create more intermediate states on the Dicke ladder, allow better understanding of the collective behavior of interacting many-body systems, and provide a potential route to tailor the cooperative behavior and harness collective light-matter interaction effects for photon-mediated applications.

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METHODS

Sample Structure The experiments are performed on self-assembled InGaAs quantum dots embedded in a GaAs Schottky diode for deterministic charge control via the applied gate voltage. The gate voltage induces D.C. Stark shift and shifts the emission wavelength of the emitter, with a typical linear response of 1 – 2 µeV/mV. A broadband planar cavity antenna and solid immersion lens (not shown in Figure 1(a)) is used to enhance the photon extraction efficiency [50]. Specific details of the sample heterostructure are described in Ref. [42].

Spectroscopy Setup The sample is kept at a temperature of 4 K in a closed-cycle helium flow cryostat. A a polarization-based dark-field confocal microscope is used to excite and collect the resonance fluorescence from the QDs while suppressing the scattered laser light (typical extinction ratio ≈ 10^7) [51]. The photons are sent to either a spectrometer (resolution of 40 µeV) equipped with a liquid nitrogen cooled CCD or a pair of superconducting nanowire single photon detectors with a nominal detection efficiency of 90 % at 950 nm and time jitter of ≈ 90 ps. For Hong-Ou-Mandel measurements, we spectrally filter the phonon sideband using a grating filter with a FWHM of 120(1) µeV and 45 – 50 % fibre-to-fibre transmission efficiency. The CW experiments are done with a narrow-band (sub-MHz linewidth) diode laser while the pulsed driving is achieved with a mode-locked femtosecond laser, stretched to ~ 40 ps pulse width, with an 80 MHz repetition rate. Emitter lifetime and photon correlations (HBT and HOM) are made with a time-correlated single photon counting system.

Coincidence Measurements under Cooperative Emission Photon coincidences provide a useful tool to characterise cooperative emission. To derive the signatures of cooperative emission, we consider the light-matter interaction between two QDs described as two-level systems, where |g⟩ and |e⟩ are ground and excited states of the i-th QD. Introducing the QD operators σ^−_i = |e⟩⟨g| and σ^+_i = |g⟩⟨e| as well as photon creation and annihilation operators a^+_k and a_k, the light-matter interacting Hamiltonian is

\[ H_1 = \sum_k \hbar g \left[ e^{i k \cdot r / 2} \sigma^+_1 + e^{-i k \cdot r / 2} \sigma^+_2 \right] a^+_k + h.c. \]  \hspace{1cm} (4)

If the two QDs are fully distinguishable, the emission of photons can be described by non-degenerate perturbation theory. The signal registered at the detectors is then simply a sum of the individual detection events

\[
S^{(2)}_{\text{indep.}}(t, \tau) = \frac{2}{\sum_{i,j=1}^2 \langle \sigma^+_i(t) \sigma^-_j(t + \tau) \rangle \langle \sigma^+_j(t + \tau) \sigma^-_i(t) \rangle}.
\]  \hspace{1cm} (5)

Following the emission of a photon, one QD will be in its ground state and can thus no longer contribute to the immediate emission of a second photon, i.e. \( \sigma^-_i \sigma^-_j = 0 \). Hence, only emission processes from different emitters \( i \neq j \) contributes to the numerator, whereas all emission processes contribute to the denominator. For initially uncorrelated and distinguishable QDs with exciton populations \( n_1 \) and \( n_2 \) at time \( t \), the zero-delay coincidences are therefore

\[
S^{(2)}_{\text{indep.}}(t, 0) = \frac{2n_1 n_2}{n_1 + n_2} \leq \frac{1}{2},
\]  \hspace{1cm} (6)

where the limit \( 1/2 \) is reached for \( n_1 = n_2 \).

However, when the QDs are indistinguishable, both QDs feed into the same electromagnetic field modes and a signal measured at the detector can no longer resolve from which QD a detected photon originated. To describe the measurement process, it is instructive to rewrite the light-matter interaction

\[ H_1 = \sum_k \hbar g \sqrt{2} \left( \sigma^+_k a^+_k + \sigma^-_k a_k \right) \]  \hspace{1cm} (7)

in terms of operators \( \sigma^\pm_k = \frac{1}{\sqrt{2}} (e^{i k \cdot r / 2} \sigma^+_1 + e^{-i k \cdot r / 2} \sigma^+_2) \), which describe the dipole operator responsible for driving the photon mode with wave vector \( k \).

In our setup, we predominantly detect photons emitted perpendicular to the plane containing the QDs, i.e. photons with wave vectors \( k \approx k_0 \), where \( k_0 \) is a reference wave vector with \( k_0 \perp r \) and modulus \( |k_0| \) matching the common transition frequency of the QDs. In this situation, \( e^{\pm i k_0 \cdot r} \approx 1 \) so that the photon modes picked up by the detector are driven by the dipole operators \( \sigma^\pm_3 = \frac{1}{\sqrt{2}} (\sigma^+_1 + \sigma^+_2) \), describing transitions through the symmetric Dicke state \( |\psi_S\rangle = \frac{1}{\sqrt{2}} (|e_1, g_2\rangle + |g_1, e_2\rangle) \) [cf. Figure 1(c)]. In particular, the second-order coherence takes the form

\[
S^{(2)}_{\text{coop.}}(t, \tau) = \frac{\langle \sigma^+_3(t) \sigma^+_3(t + \tau) \sigma^-_3(t + \tau) \sigma^-_3(t) \rangle}{\langle \sigma^+_3(t) \sigma^-_3(t) \rangle \langle \sigma^-_3(t + \tau) \sigma^+_3(t + \tau) \rangle}. \]  \hspace{1cm} (8)

With occupation \( n_{e_1, e_2} \) of the doubly excited state \( |e_1, e_2\rangle \) and occupations \( n_S \) of the symmetric state \( |\psi_S\rangle \), the zero-delay coincidences are

\[
S^{(2)}_{\text{coop.}}(t, 0) = \frac{n_{e_1, e_2}}{(n_{e_1, e_2} + n_S)^2}. \]  \hspace{1cm} (9)
which for initially uncorrelated states with equal exciton populations results in $g_{\text{coop}}^{(2)}(t, 0) = 1$.

The decisive difference to the situation of independent emitters is that the emission of the first photon does not impede the emission of a second photon. Instead, both emitters cooperatively contribute to both emission processes, leading to zero-delay coincidences that exceed the limit of $1/2$ for independent emitters.

A minimal model for calculating the time dependence of $g^{(2)}(t, \tau)$ is given in the supplementary material S1.
Supplementary material

S1. MINIMAL MODEL FOR $g^{(2)}(\tau)$

| Emission model | $\frac{\partial \rho}{\partial t} =$ | $G^{(2)}(\tau) =$ |
|----------------|---------------------------------|------------------|
| single         | $\gamma L[\sigma_{1}^{-}] (\rho) + \gamma_p L[\sigma_{1}^{+}] (\rho) + \gamma_d L[\sigma_{1}^{+} \sigma_{1}^{-}] (\rho)$ | $\langle \sigma_{1}^{+} \sigma_{1}^{+} (\tau) \sigma_{1}^{-} (\tau) \sigma_{1}^{-} \rangle$ |
| independent    | $\gamma L[\sigma_{1}^{-}] (\rho) + \gamma_p L[\sigma_{1}^{+}] (\rho) + \gamma_d L[\sigma_{1}^{+} \sigma_{1}^{-}] (\rho)$ | $\sum_{ij=1,2} \langle \sigma_{j}^{+} \sigma_{i}^{+} (\tau) \sigma_{j}^{-} (\tau) \sigma_{i}^{-} \rangle$ |
| cooperative    | $\gamma L[\sigma_{1}^{-}] (\rho) + \gamma_p L[\sigma_{1}^{+}] (\rho) + \gamma_d L[\sigma_{1}^{+} \sigma_{1}^{-}] (\rho)$ | $\langle \sigma_{2}^{+} \sigma_{2}^{+} (\tau) \sigma_{2}^{-} (\tau) \sigma_{2}^{-} \rangle$ |
| superradiant   | $\Gamma L[\sigma_{2}^{-}] (\rho) + \gamma_p L[\sigma_{1}^{+}] (\rho) + \gamma_d L[\sigma_{1}^{+} \sigma_{1}^{-}] (\rho)$ | $\langle \sigma_{2}^{+} \sigma_{2}^{+} (\tau) \sigma_{2}^{-} (\tau) \sigma_{2}^{-} \rangle$ |

Table I. Equations of motion and expressions for the second order (intensity) correlation function $G^{(2)}$ for the cases of a single emitter, independent emitters, selectively measured emitters, and super-radiantly coupled emitters.

Here, we formulate a minimal model to describe the time delay dependence of $g^{(2)}(\tau)$ for single emitters, independent emitters, selectively measured coherently coupled emitters, and super-radiantly coupled emitters. To this end, we need to calculate two-time correlation functions of the form

$$
\langle a_{k_1}^{\dagger}(t_1) a_{k_2}^{\dagger}(t_1 + \tau) a_{k_2}(t_1 + \tau) a_{k_1}(t_1) \rangle
$$

where the first term describes radiative decay with rate $\gamma$, the second term accounts for incoherent pumping with rate $\gamma_p$, and the third term phenomenologically describes pure-dephasing with a dephasing rate $\gamma_d$. The latter has the effect of reducing coherence between the ground $|g_1\rangle$ and excited state $|e_1\rangle$ and originates microscopically, e.g., from interactions with the phonon environment.

The equations of motion for two independent emitters are obtained by adding to the equations of motion for a single emitter the same set of equations but with operators $\sigma_{1}^{\pm}$ replaced by $\sigma_{2}^{\pm}$. In the case of superradiance,
we assume the pumping of both emitters to be independent, but radiative decay involves transitions through the symmetric state

$$\frac{d}{dt} \rho = \Gamma [\sigma^-] \rho + \gamma_p \mathcal{L} [\sigma^+ \sigma^-] \rho + \gamma_p \mathcal{L} [\sigma^+ \sigma^+] \rho, \quad (S4)$$

where for ideal (no pure dephasing) super-radiance $\Gamma = 2\gamma$. The respective equations of motion for the different cases together with the corresponding expressions for the second order coherences are listed in Table I. With these results, the numerical calculation of the second order correlation function is straightforward. If not stated otherwise, we use parameters $\gamma_p = \gamma$ and $\gamma_d = 0$, where $\gamma = 1$ defines the unit of time.

### S2. RELATION TO SUPERRADIANCE

To further clarify the nature of cooperative emission in absence of superradiance, we now describe the link to superradiance on the one hand and to independent emission on the other hand in terms of wave vector selectivity of coincidence measurements.

Consider two detectors that predominantly register photons at wave vectors $k^{(1)}_0$ and $k^{(2)}_0$, respectively. Then, a first photon detection event described by the operators $\sigma^\pm_{k_0^{(1)}}$ from an initially doubly occupied two-emitter system leads to a collapse of the emitter state to $|\psi_{k_0^{(1)}}\rangle$. The conditional probability of detecting a second photon by detector 2 as described by operators $\sigma^\pm_{k_0^{(2)}}$ is given by

$$G^{(2)}_{k_0^{(1)}k_0^{(2)}} (t, 0) = \langle \sigma^+_{k_0^{(1)}} \sigma^+_{k_0^{(2)}} \sigma^-_{k_0^{(1)}} \sigma^-_{k_0^{(2)}} \rangle = \langle \psi_{k_0^{(1)}} | \sigma^+_{k_0^{(1)}} \sigma^-_{k_0^{(1)}} | \psi_{k_0^{(1)}} \rangle = \langle \psi_{k_0^{(2)}} | \psi_{k_0^{(2)}} \rangle = \frac{1}{2} \left[ 1 + \cos \left( \langle (k_0^{(2)} - k_0^{(1)}) \cdot r \rangle \right) \right], \quad (S5)$$

which shows a strong dependence of photon coincidences on the relative emission directions of the first and the second photon. Eq. (S5) can be interpreted as follows: The first photon measurement process prepares the system in a state whose oscillator strength for the emission of a second photon strongly depends on the emission direction $k_0^{(2)}$, encoded in the relative phase $e^{i k_0^{(1)} \cdot r}$ between states $|e_1, e_2\rangle$ and $|g_1, e_2\rangle$ in the correlated intermediate state $|\psi_{k_0^{(1)}}\rangle$. 

![Figure S1.1. Second-order correlation function ($g^{(2)}(\tau)$) for selective measurement and super-radiant case. $g^{(2)}(\tau)$ as a function of detection time delay, $\tau$ shows similar anti-dip feature around $\tau = 0$ as a result of emission from two indistinguishable emitters, under the selective measurement ($\gamma_d \neq 0$, solid lines) and super-radiance (dashed-dotted line). In the absence of pure dephasing ($\gamma_d = 0$, solid line), $g^{(2)}(\tau) = 1$ at all $\tau$. For the case of two distinguishable emitters (dashed line), this feature disappears and $g^{(2)}(0) = 0.5$ instead.](image-url)
In the superradiant regime \( \mathbf{k}_0^{(1)} \cdot \mathbf{r} \approx 0 \), so that the states involved in the transitions have identical phases \( |\psi_{\mathbf{k}_0^{(2)}}\rangle = |\psi_{\mathbf{k}_0^{(1)}}\rangle = |\psi_S\rangle \), leading to a perfect overlap \( |\langle \psi_{\mathbf{k}_0^{(2)}} | \psi_{\mathbf{k}_0^{(1)}} \rangle|^2 = 1 \) and hence a maximal value of \( G_{\mathbf{k}_0^{(1)} \mathbf{k}_0^{(2)}}^2(t, 0) = 1 \), irrespective of the emission directions defined by \( \mathbf{k}_0^{(1)} \) and \( \mathbf{k}_0^{(2)} \).

By contrast, if the distance \( r \) is larger than the wavelength of the emitted light and all photons are collected, i.e. no particular wave vectors are selected, then the relative phases \( e^{i \mathbf{k}_0^{(1)} \cdot \mathbf{r}} \) are random, resulting in destructive interference of the cosine term in Eq. (S5). This leads to exactly half as many photon coincidences as in the superradiant regime, analogous to what we obtain from simply considering independent emitters.

Finally, in our confocal microscope, we predominantly detect photons emitted perpendicular to the sample plane containing the QDs, which implies that all detected photons have similar wave vectors \( \mathbf{k}_0^{(2)} = \mathbf{k}_0^{(1)} \). Therefore, with \( \cos ((\mathbf{k}_0^{(2)} - \mathbf{k}_0^{(1)}) \cdot \mathbf{r}) = 1 \), the same enhancement of \( G^2(t, 0) = 1 \) is found as in the superradiant regime. This remains true even though the distance between the emitters may be significantly larger than the wavelength of the emitted light. In both of the ‘cooperative’ cases, i.e. superradiance as well as the present case of cooperative emission without superradiance, the enhancement of photon coincidences with respect to independent emitters can be attributed to the perfect matching between intermediate states involved in the emission processes of the first and the second photon.

### S3. CORRECTION TO THE ENTANGLEMENT MEASURE DUE TO NOISE AND DETECTION JITTER

In this section, we provide an analysis of the entanglement measure for simultaneous detection of the emission from two indistinguishable emitters in the presence of the noise and detection jitter. As described in Sec. 8 of the Supplementary Material in Ref. [10], one can extract the ratio of noisy photons compared to single photons \( p_n \) extracted from the \( g_{\text{QD1}}^2(0) \) for individual QD emission, i.e.

\[
\frac{g_{\text{QD1}}^2(0)}{g_{\text{QD1}}^2(0, 0)} = \frac{p_n(2 + p_n)}{(1 + p_n)^2}. \tag{S6}
\]

Using this information, we can obtain the entanglement measure \( g_{\text{Corr.}}^2(0) \) using

\[
g_{\text{Corr.}}^2(0) \geq \frac{(g_{\text{Corr.}}^2(0) (1 + p_n)^2 - p_n(2 + p_n))^2}{g_{\text{Corr.}}^2(0) (1 + p_n)^2}. \tag{S7}
\]

In the noiseless limit (i.e. \( p_n = 0 \)), this is given solely by the measured (normalized) coincidences at the zero-delay in the second-order intensity correlation function \( g_{\text{Corr.}}^2(0) \). The presence of surplus ‘noisy’ photons in the signal (which may originate from further emitters or the excitation laser) and the timing jitter in the detection setup result in a decrease in the entanglement measure. To account for both effects, we extract the ratio of noisy photons in the single photon stream from an intensity correlation measurement with only a single emitter. We fit the experimental data in Figure 2d and 3c by convolving the theoretical expressions in Eq. 1 and 2 with the independently measured detection timing jitter (full-width-at-half-maximum of 0.240 ns) and compare them with the corresponding deconvolved fits. The fits, along with the experimental data are depicted in Figure S3.2. We summarize the results, i.e. the change in the \( g^2(0) \) value as a result of each contribution in Table II.

It is important to note that after deconvolution, the zero-delay peak (pulsed excitation) and the anti-dip (continuous wave excitation) have a maximum close to one, as expected from the ideal noise-less case for the emission from two indistinguishable emitters. This shows that the maximum of the zero-delay peak is largely determined by detector jitter and hence is inadequate to quantify the entanglement of the emitters. This is analogous to the interpretation of the zero-delay anti-bunching dip in the Hong-Ou-Mandel interference measurement of subsequently emitted single photons; please refer to Refs. [45–47] for more details. Instead, the integration over the full zero-delay peak within a large integration window \( g_{\text{A}}^2(0) \) gives information about the dephasing of the emitted photons, independent of the detector jitter, and hence is more suitable and useful to characterize the entanglement of the emitters.
Figure S3.2. **Comparison between experimental data, the convolved and deconvolved fit for the second order intensity correlation measurement with two indistinguishable emitters.** Normalized coincidence (to the raw coincidences at longer delays of \( \approx 10 \mu s \)) under continuous wave resonant (CW RF, a), pulsed resonant (RF, b) and non-resonant excitation (NR, c). Convolved and deconvolved fits are given by the solid and dashed lines, respectively. Experimental data and the convolved fits are the same as in Figure 2d (CW) and 3c (Pulsed). The uncorrelated side peak at 12.44 ns and the correlated zero-delay peak are shifted by -1 and 1 ns, respectively.

Table II. The entanglement measure, indicated by the normalized zero-delay coincidences \( g^{(2)}(0) \) from two degenerate QDs under CW and pulsed resonant (RF) and non-resonant (NR) excitation in the presence of noise, given by the ratio of noisy photons in the QD signal \( p_n \) and the detection timing jitter (modelled as a Gaussian with a full-width-at-half-maximum of 240 ps). The correction to this quantity, \( g^{(2)}_{corr}(0) \), as a result of the presence of noisy photons is estimated using the expression in Sec. 8 of the Supplementary Material in Ref. [10] using the extracted ratio of noisy photons to QD signal, \( p_n \) in the single emitter case. For the case with pulsed measurements, the values reported here are extracted from the coincidence histogram with integration windows of 10 ns \((g^{(2)}_{\Delta x=10\text{ns}}(0))\). The values with temporal post-selection of \( \leq 0.1 \text{ ns} \) \((g^{(2)}_{\Delta x\leq0.1\text{ns}}(0))\) are listed in parenthesis and printed in red.

|                      | CW                     | Pulsed         |                      |
|----------------------|------------------------|----------------|----------------------|
|                      | RF\(^a\)               | RF\(^b\)       | NR\(^c\)            |
| Measured QD1 \( g^{(2)}_{\text{QD1}}(0) \) | 0.06                   | 0.05 (0.05)    | - \(_{\text{NR}}\) |
| Ratio of noisy photons \( p_n \) | 0.031                  | 0.026 (0.026)  | - \(_{\text{NR}}\) |
| Measured QD1+QD2 \( g^{(2)}(0) \) | 0.87                   | 0.67 (0.93)    | 0.56 (0.65)         |
| Deconvolved \( g^{(2)}_{\text{Deconv.}}(0) \) | 1                      | 0.68 (1.04)    | 0.54 (0.98)         |
| Corrected \( g^{(2)}_{\text{Corr.}}(0) \) \(^d\) | 0.80                   | 0.60 (0.88)    | 0.56 (0.65)         |
| Corrected \( g^{(2)}_{\text{Deconv.\ Corr.}}(0) \) \(^e\) | 0.94                   | 0.61 (0.99)    | 0.54 (0.98)         |

\(^a\) Data taken at \( P \approx 0.3 P_{\text{sat}} \), \(^b\) Data taken at \( \pi/2\)-pulse, \(^c\) Data taken at saturation, \(^d\) Correction to the \( g^{(2)}(0) \), \(^e\) Correction to the \( g^{(2)}_{\text{Deconv.}}(0) \).

**S4. ADDITIONAL DATA ON COLLECTIVE EMISSION UNDER RESONANT AND NON-RESONANT EXCITATION.**

See captions of Figure S4.3 and S4.4 for additional information on the theory-experiment comparison of the power dependence \( g^{(2)} \) data under CW resonant excitation and spectroscopy of the emission from two QDs under non-resonant pulsed excitation, respectively.
Figure S4.3. **Power-dependent continuous wave resonant excitation of Both QDs tuned into resonance.** A comparison of the experimental (a) and simulated data (b) for $g^{(2)}$ as a function the ratio of the excitation power, $P$. The excitation power in the experimental data is normalized by the saturation power, $P_{\text{sat}}$.

Figure S4.4. **Pulsed non-resonant excitation of Both QDs.** (a) Emission count rates from Both QDs, when degenerate, as a function of excitation power at bias of -0.45 V. (b) Time-resolved emission profile of the cooperative emission at power of 0.58 $\mu$W. (c) Normalized zero-delay coincidences from the HBT measurement, $g^{(2)}(0)$ as a function of applied bias (excitation power = 0.58 $\mu$W). Inset shows the normalized coincidences from the HBT measurement at -0.52 V ($\delta \approx 95 \mu$eV, grey) and -0.45 V ($\delta \approx 0 \mu$eV, purple). (d) Two-photon interference between subsequently emitted photons, separated by 12.44 ns, for the case where input photons, from both QDs tuned into the resonance ($\delta = 0$) are prepared in parallel ($g^{(2)}$, blue) and perpendicular ($g^{(2)}_\perp$, orange) polarization. The non-post-selected (post-select within 100 ps window) interference visibility for this measurement is 0.041 (0.212). All measurements are done with 14 ps excitation pulses, frequency detuned by $\approx 0.4$ meV from the two-dot-degeneracy at $\lambda = 971.169$ nm.