Mathematical model of the flow of polymer melt in the extruder forming die

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Abstract. Steady creeping flow of viscoelastic fluid with the free surface which is realized when fluid polymer is entering the forming channel and leaving it is investigated. Fluid motion is described by equations conservation of mass and momentum supplemented by Metzner’s determinative rheological constitutive equation of the fluid. On the basis of finite element method a stable numerical algorithm for solving the problem was developed.

1. Introduction
The continuous growth of the production of chemical fibers and yarns requires the solution for a number of issues, among which should be noted such as improving the continuity of technological processes and their intensification, creation of a new high-efficiency equipment, reducing labor intensity by mechanization and automation of production processes. One of the main directions of development of mechanical engineering for the production of the finished products from polymer melts is an increase of spinning rate and combination of technological operations, decrease of energy and metal intensity of the equipment, mechanization and automation of processes using microprocessor technology. When solving all the above problems the development of mathematical models, methods of computation and optimization of molding process by extrusion with subsequent verification and implementation of the obtained results into production is becoming particularly urgent [1,2].

2. Governing equations
Motion of an incompressible viscoelastic fluid is described by the equations of conservation of mass and momentum.

In a cylindrical coordinate system creeping flow of a viscoelastic fluid in the absence of gravity is described by the system equations conservation of mass and momentum which are closed by the Giesekus determinative rheological constitutive equation [6, 7]:

$$\nabla \cdot \mathbf{v} = 0,$$

(1)
Here $\rho$ – density of polymer melt, $P$ – pressure, $\mathbf{v}$ – velocity vector, $\mathbf{\tau}$ – stress deviator. The system of equations (1) - (2) is supplemented by the rheological constitutive relation connecting the stress deviator $\mathbf{\tau}$ with the strain velocity tensor $D$ in the Metzner form

$$\tau = \tau_v + 2\eta_N \mathbf{D}, \quad \tau_v = \bar{\tau} = 2\eta_v \mathbf{D}$$

(3)

where $\lambda$ – fluid relaxation time, $\eta_v$, $\eta_N$ – efficient dynamic viscosities of solvent and polymer melt, respectively. Dependence of viscosity on shear rate has the form

$$\eta_v = (\eta_0 - \eta_\infty) \left[ 1 + (k I_2)^{1/2} \right] + \eta_\infty,$$

where $\eta_0$, $\eta_\infty$ – the greatest and the smallest effective shear viscosities of the polymer, $I_2$ – second invariant of the strain velocity tensor, $k$, $n$ – model parameters. Operator $\tau_v$ defines the upper convective derivative of a tensor as

$$\tau_v = \frac{\partial \tau}{\partial t} + \mathbf{v} \cdot \nabla \tau = -\mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v}' . \tau.$$

Polymer flow in the computational region is described by the equations (1) – (3) velocity profiles and complete stress distributions are specified at the entrance boundary of the computational region $\mathbf{v} = \mathbf{v}_0(x_2)$, $\mathbf{\tau} = \mathbf{\tau}_0(x_2)$, steady uniform profile of velocity and stresses is specified at the region outlet $\mathbf{v} = 0$, $\partial \mathbf{u} / \partial x = 0$, $\partial \mathbf{\tau} / \partial x = 0$, kinematic and dynamic boundary conditions in the following form should be satisfied at the free boundary

$$\mathbf{v} \cdot \mathbf{n} = 0, \quad \mathbf{n} \cdot \tau = 2H / \chi,$$

where $\mathbf{n}$, $\mathbf{t}$ – unit normal vector and unit vector tangent to the free surface, $2H$ – radius of curvature of the free surface, $\chi$ – surface tension coefficient.

For the free surface described by the equation $F(\chi, t) = 0$ the following relationship is satisfied

$$\frac{\partial F}{\partial t} + \mathbf{v} \cdot \nabla F = 0$$

(4)

Approximation of equations (1) – (3) and calculations are carried out by the finite element method of the second order on irregular meshes condensing to the zone of outflow of the polymer from the nozzle. Since the desired functions vary significantly only in the vicinity of outflow of the polymer, the use of fine meshes in this zone and sparse meshes outside it plays a significant role in saving computational costs. The sequence of condensing meshes of 9-node quadrilateral elements (the number of nodes 2000, 8400) was created for the calculations. Linear quadrilateral elements were used for the calculation of the stresses. Location of the deformable free surface is found from the approximation of the kinematic condition (4) by the finite element method then mesh of finite elements near it is rearranged to obtain solutions of matrix equations (1) – (3) using which the field of velocities, pressures and stresses at the new time step is found. Stationary solution of the problem is found by establishing the evolutionary problem using traditional algorithms for this class of equations.

3. Results

The problem of outflow of Newtonian fluid from the pipe at different Reynolds numbers was solved for approbation of the numerical algorithm. Increase of diameter of the output stream is characterized by parameter $h_f$ which is equal to the ratio of stream diameter to the channel diameter and is called the coefficient of die swell. The figure shows the forms of the free surface of a Newtonian fluid at various Re numbers: 1 – 4.09; 2 – 12.5; 3 – 17.2; 4 – 27.3; 5 – 47.4 as compared to the experimental data [3] (solid symbols).
It should be noted that there is satisfactory agreement between the calculated data and the experimental results which have an error of approximately ±0.01.

The obtained results show that effective finite element algorithm for calculating flows of viscoelastic fluids in regions with moving boundaries was created.

References
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