Worldline Supersymmetry and Dimensional Reduction

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Abstract

For any worldline reformulation of a quantum field theory for Dirac fermions, this paper shows that worldline supersymmetry may generally be enforced by the vanishing of the commutator of the Dirac operator with the worldline Hamiltonian. The action of supersymmetry on the worldline Lagrangian may not, however, be written in terms of the variations on the fields in the usual way, except when the spinning particle couples just to a one-form. By reduction from six to four dimensions of the worldline reformulation for a spinning particle coupled to a three-form, corrections to the superworldline Lagrangian are presented which are needed in order to reproduce correct field theory results from worldline perturbation theory in an unambiguous way.
1. Introduction

As the point particle limit of superstring theory, the worldline formalism for spinning particles is expected to have a $D = 1$, $N = 1$ supersymmetry or worldline supersymmetry. This has been shown to be true for a spinning particle coupled to an Abelian [1] and a non-Abelian [2] vector gauge field. For a review of supersymmetric quantum mechanics, see ref. [3]. The worldline Lagrangians for a spinning particle coupled to a scalar and pseudoscalar [4] as well as to a vector and axial vector [5] (all four fields Abelian) have been constructed in a manifestly supersymmetric way as the description of a spinning particle on a superworldline [1], although the actual supersymmetry transformations have not been discussed.

Recently [6], the worldline reformulation for a multiplet of Dirac fermions coupled to the most general set of non-Abelian background scalar, pseudoscalar, vector, axial vector and antisymmetric tensor fields has been derived from field theory. For the Abelian scalar, pseudoscalar and vector, the derived Lagrangian agrees with the result of [5], while for the Abelian axial vector there is agreement provided the auxiliary fields coming from the superworldline Lagrangian are treated in a somewhat ambiguous way. However, when including an antisymmetric tensor or a non-Abelian axial vector, the derived Lagrangian curiously contains terms which are not expected to appear in a manifest superworldline construction but are needed in order to produce correct field theory results. In other words, the full worldline Lagrangian derived from field theory is apparently not supersymmetric. Why should some worldline Lagrangians be supersymmetric and others turn out not to be?

By defining supersymmetry transformations on the worldline in a more general way than the superworldline construction, this paper shows in the next section that the spinning particle Lagrangian is actually always supersymmetric for any background coupling. Specifically, identification of the Dirac operator with the supercharge is made and the action of supersymmetry on the worldline Hamiltonian is given simply by its commutator evaluated at $\hbar = 0$ with the supercharge. It is shown that the action of supersymmetry on the Lagrangian can not be given by the usual functional variation, except when the spinning particle couples just to a one-form.

The last section investigates why the superworldline Lagrangian becomes problematic for the coupling to an axial vector or to an antisymmetric tensor. It is shown that the superworldline construction for a spinning particle coupled in any dimension to higher p-forms generally does not produce a number of terms present in the corresponding Lagrangian derived from field theory, except for the special case $p = 1$. From the dimensional
reduction of the worldline reformulation for a spinning particle coupled to a three-form in six dimensions, the superworldline Lagrangian for the coupling to an Abelian axial vector and an antisymmetric tensor, expressed in terms of the auxiliary fields introduced in [4 and 5], is recovered plus the correction terms which render the whole expression both correct and invariant under the new supersymmetry transformations. Finally, these insights make it possible to give simple unambiguous rules for all worldline perturbation theory, including expressions involving auxiliary fields.

2. Worldline Supersymmetry

Consider the worldline reformulation of a quantum field theory for a Dirac fermion with general couplings in $D$-dimensions, analytically continued to Euclidean space. The Dirac operator, $\hat{\Sigma}$, should appropriately be taken without loss of generality to be hermitian and to be of fermionic grading $\dagger$. From refs. [6 and 7], $\hat{\Sigma}$ may generally be cast as

$$\hat{\Sigma} = \Gamma_A \hat{p}_A - \sum_{r=0}^{2r+1} \hat{K}^{(2r+1)}_{A_1 \ldots A_{2r+1}} , \quad (2.1)$$

where $\hat{K}^{2r+1} = K^{2r+1}(\hat{x})$ is an hermitian Abelian $(2r+1)$-form, $\Gamma_A$, $A = 1, \ldots, D$ are the hermitian generators of the Euclidean Clifford algebra in $D$ dimensions, $\Gamma_{A_1 \ldots A_{2r+1}}$ is the totally antisymmetric product of $\Gamma_{A_1}, \ldots, \Gamma_{A_{2r+1}}$ and $\hat{p}_A$ is the hermitian momentum operator. The spinning particle worldline Lagrangian is then

$$L = L_K + H , \quad (2.2a)$$

where the kinetic part of the Lagrangian, $L_K$, is given by

$$L_K = -i \hat{x}_A \hat{p}_A + \frac{1}{4} \psi_A \dot{\psi}_A $$

and the worldline Hamiltonian, $H$, is obtained from the Hamiltonian operator

$$\hat{H} = \hat{\Sigma}^2 $$

in the limit $\hbar \to 0$. “Observable” classical quantities are obtained from the quantum mechanical operators in the $\hbar \to 0$ limit by the rules

$$\Gamma_{A_1 \ldots A_{2r+1}} \to \psi_{A_1} \cdots \psi_{A_{2r+1}} \quad (2.3a)$$

$$\{\hat{p}, F(\hat{x})\} \to 2pF(x) . \quad (2.3b)$$

$\dagger$ Such a Dirac operator is required in a worldline reformulation and may always be obtained from a general one by doubling the fermionic degrees of freedom [6 and 7].
Now, since $\hat{\Sigma}$ is hermitian and fermionic, it may be identified as the supercharge generating the $D = 1$ supersymmetry algebra of eqn.(2.2c). Use will now be made of the fact that the Hamiltonian commutes with the supercharge,

$$[\hat{H}, \hat{\Sigma}] = 0 ,$$

in order to define worldline supersymmetry in a general way. First, supersymmetry transformations on the fields are defined as

$$\delta_\alpha x_A = [\hat{x}_A, \alpha \hat{\Sigma}]_{(\hbar \to 0)} = i \alpha \psi_A$$

$$\delta_\alpha p_A = [\hat{p}_A, \alpha \hat{\Sigma}]_{(\hbar \to 0)} = i \alpha \sum_{r=0}^{\infty} i^r \psi_{A_1} \cdots \psi_{A_{2r+1}} \partial_A K_{A_1 \cdots A_{2r+1} A_2 \cdots A_{2r+1}}^{(2r+1)}$$

$$\delta_\alpha \psi_A = [\Gamma_A, \alpha \hat{\Sigma}]_{(\hbar \to 0)} = -2 \alpha p_A + 2 \alpha \sum_{r=0}^{\infty} i^r \psi_{A_2} \cdots \psi_{A_{2r+1}} K_{A_1 \cdots A_{2r+1} A_{2r+2} \cdots A_{2r+1}}^{(2r+1)} ,$$

where $\alpha$ is the Grassmann parameter for the supersymmetry transformations. These transformations may equivalently be defined in terms of the Poisson bracket \(^\dagger\)

$$\delta_\alpha f = [f, \alpha \hat{\Sigma}]_{[P,B]} , \quad f = x, p \text{ or } \psi ,$$

defined as

$$[A, B]_{[P,B]} = i \frac{\partial A}{\partial x_A} \frac{\partial B}{\partial p_A} - i \frac{\partial A}{\partial p_A} \frac{\partial B}{\partial x_A} - 2 \left( \delta_{0B} (-)^{A+B} + \delta_{1B} \right) \frac{\partial A}{\partial \psi_A} \frac{\partial B}{\partial \psi_A} .$$

These brackets may be shown to define a graded Lie algebra \([8]\). We have also introduced the observable supercharge

$$\Sigma = \hat{\Sigma}_{\hbar \to 0} = \psi_A p_A - \sum_{r=0}^{\infty} i^r \psi_{A_1} \cdots \psi_{A_{2r+1}} K_{A_1 \cdots A_{2r+1}}^{(2r+1)} ,$$

where use has been made of the rules in (2.3).

Now, under the variations of (2.5 or 6), the kinetic part of the Lagrangian is generally invariant (up to a total derivative in the propertime) with its variation defined as

$$\delta_\alpha L_K \equiv L_K(x + \delta_\alpha x, p + \delta_\alpha p, \psi + \delta_\alpha \psi) - L_K(x, p, \psi) = \partial_\tau \left( \frac{1}{4} \psi_A \delta_\alpha \psi_A - ip_A \delta_\alpha x_A - \alpha \Sigma \right) .$$

The action of supersymmetry on the Hamiltonian is defined to be

$$\delta_\alpha H \equiv [\hat{H}, \alpha \hat{\Sigma}]_{(\hbar \to 0)} ,$$

\(^\dagger\) This bracket may more correctly be referred to as a Dirac bracket \([8]\).
which must vanish by (2.4). Thus quite generally, the worldline Lagrangian, $L$, given by (2.2a), is invariant up to a total derivative under the supersymmetry transformations (2.8, and 9). However, it will now be demonstrated that the variation of the Lagrangian, $L$, is not given by the usual functional variation, i.e.,

$$\delta_\alpha L \neq L(x + \delta_\alpha x, p + \delta_\alpha p, \psi + \delta_\alpha \psi) - L(x, p, \psi) ,$$

except when the fermion couples just to a one-form.

In order to demonstrate (2.10), it suffices to show that the variation of the Hamiltonian, $H$, is not given by the usual functional variation in terms of the variations on the fields, $\delta_\alpha x$, $\delta_\alpha p$ and $\delta_\alpha \psi$. So, to see what the transformation (2.9) on the Hamiltonian looks like in terms of the fields, it is instructive to first consider the coupling just to the one-form ($r = 0$) in (2.1), which gives

$$H = (p - K)^2 + i\psi_A \psi_B \partial_A K_B .$$

It is easy now to check by (2.5 or 6) that

$$\delta_\alpha H = H(x + \delta_\alpha x, p + \delta_\alpha p, \psi + \delta_\alpha \psi) - H(x, p, \psi) = [H, \alpha \Sigma]_{P.B.} \left( = 0 \right) .$$

In particular, the commutator $[\hat{H}, \alpha \hat{\Sigma}]_{(\hbar \to 0)}$ and the Poisson bracket $[H, \alpha \Sigma]_{P.B.}$ are identical term by term (and of course these terms add to zero). For example,

$$[i\Gamma_{AB} \partial_A \hat{K}_B, \alpha \Gamma_{C\hat{P}C}]_{(\hbar \to 0)} = [i\psi_A \psi_B \partial_A K_B, \alpha \psi_{C\hat{P}C}]_{P.B.} = 2i\alpha (p_A \psi_B \partial_B K_A - p_A \psi_B \partial_A K_B) .$$

The one-form coupling may be used for example to obtain the usual gauge, scalar and pseudoscalar couplings in four dimensions by reduction from dimension $D \geq 6$ [4].

Next, consider the coupling just to the three-form ($r = 1$) in (2.1), which gives

$$H = p^2 - 6i\psi_A \psi_B p_C K_{ABC} - \psi_A \psi_B \psi_C \psi_D \partial_A K_{BCD} + 6K_{ABC} K_{ABC} - 9\psi_A \psi_B \psi_C \psi_D K_{EAB} K_{CDE} .$$

Now notice that

$$H(x + \delta_\alpha x, p + \delta_\alpha p, \psi + \delta_\alpha \psi) - H(x, p, \psi) = [H, \alpha \Sigma]_{P.B.} = 12i\alpha \psi_A \partial_A K_{BCD} K_{BCD} \neq 0 .$$

Thus, curiously

$$[\hat{H}, \alpha \hat{\Sigma}]_{(\hbar \to 0)} \neq [H, \alpha \Sigma]_{P.B.} .$$
in general. Thus the supersymmetry transformation (2.9) for the Hamiltonian is not given simply by the usual functional variation (i.e. Poisson bracket) as in (2.12) except for the special case when a fermion couples just to a one-form. This surprise can be understood in the following way. The commutator and the Poisson bracket both agree on the term that generates (2.15):

\[
\hat{K}_{ABC} \hat{K}_{ABC}, \alpha \Gamma_{D} \hat{p}_{D}|_{\hbar \to 0} = \left[ K_{ABC} K_{ABC}, \alpha \psi_{D} p_{D} \right]_{\text{P.B.}} = 12 i \alpha \psi_{A} \partial_{A} K_{BCD} K_{BCD} .
\]

(2.17)

For the commutator, the cancellation of (2.15 or 17) comes from a piece of the term

\[
[- \Gamma_{ABCD} \partial_{A} \hat{K}_{BCD}, - i \alpha \Gamma_{EFG} \hat{K}_{EFG}]|_{\hbar \to 0} \sim - 12 i \alpha \psi_{A} \partial_{A} K_{BCD} K_{BCD} + 36 i \alpha \psi_{A} \partial_{B} K_{ACD} K_{BCD} + \cdots .
\]

(2.18)

The commutator and Poisson bracket agree perfectly on the terms denoted as \( \cdots \). These terms arise due to a single contraction of \( \Gamma \)-matrices under the commutator, which is equivalent to the single derivative by worldline fermions on each argument under the Poisson bracket (2.6). However, the first two terms in (2.18) arise due to three contractions of \( \Gamma \)-matrices under the commutator, which at the observable level is equivalent to three derivatives by worldline fermions on each argument. Such higher derivative terms are obviously not accommodated by the Poisson bracket (2.6). Moreover, the second term in (2.18) is cancelled at the commutator level by a piece of the term

\[
[- 3 i \Gamma_{AB} \{ \hat{p}_{C}, \hat{K}_{ABC} \}, - i \Gamma_{DEFG} \hat{K}_{DEFG}]|_{\hbar \to 0} \sim - 36 i \alpha \psi_{A} \partial_{B} K_{ACD} K_{BCD} + \cdots .
\]

(2.19)

This leading term can again only be obtained at the observable level by higher derivatives lacking in the definition of the Poisson bracket in (2.6).

It is possible, however, to define a generalized Poisson bracket (G.P.B.) as a formula for \( \delta_{\alpha} H \) in terms of observables by analyzing how the higher contractions between \( \Gamma \)-matrices go in the commutator of (2.9). For the case of a three-form, the higher contractions are given specifically by (2.18) and (2.19) and so the (vanishing) variation of the Hamiltonian may be cast as

\[
\delta_{\alpha} H = [H, \alpha \Sigma]_{\text{P.B.}} + \frac{i}{2} \frac{\partial^{3} H}{\partial \psi_{A} \partial \psi_{B} \partial p_{C}} \frac{\partial^{3} (\alpha \Sigma)}{\partial \psi_{A} \partial \psi_{B} \partial q_{C}} + \frac{1}{3} \frac{\partial^{3} H}{\partial \psi_{A} \partial \psi_{B} \partial \psi_{C}} \frac{\partial^{3} (\alpha \Sigma)}{\partial \psi_{A} \partial \psi_{B} \partial \psi_{C}}
\]

\[\equiv [H, \alpha \Sigma]_{\text{G.P.B.}} .\]

(2.20)

This generalized Poisson bracket generates the same equations of motion and supersymmetry transformations for the fields \( x, p \) and \( \psi \) as the Poisson bracket (2.6). However, the generalized Poisson bracket does not obey the super Jacobi identity.
This situation becomes increasingly more severe as the fermion couples to higher forms. Variation of the Hamiltonian by the Poisson bracket leaves numerous terms uncancelled. These terms can only be cancelled using the correct variation formula (2.9) or equivalently variation by a generalized Poisson bracket with a suitably high number of derivatives.

3. Superworldline and Dimensional Reduction

The fact that $\delta_\alpha H \neq [H, \alpha \Sigma]_{PB}$, except when the fermion couples to a one-form ($r = 0$) will now be shown to mean equivalently that the worldline Lagrangian $L$ of (2.2a) derived from field theory differs from the Lagrangian obtained from the superworldline construction, except when $r = 0$. Firstly, using the equations of motion for the momentum, the supersymmetry transformations (2.5) on the fields become

$$\delta_\alpha x = i\alpha \psi_A$$  \hspace{1cm} (3.1a)
$$\delta_\alpha \psi_A = -i\alpha \dot{x}_A .$$  \hspace{1cm} (3.1b)

The same variations on the fields can be obtained by considering a superworldline ($\tau, \theta$), where $\theta$ is a Grassmann number. By defining the superfield $X$, supercharge $Q$ and superderivative $D$ as

$$X_A = x_A + \theta \psi_A , \quad Q = \frac{1}{i} (\partial_\theta + \theta \partial \tau) \quad \text{and} \quad D = \frac{1}{i} (\partial_\theta - \theta \partial \tau) ,$$  \hspace{1cm} (3.2)

supersymmetry variations (3.1) are reproduced by

$$\delta_\alpha x_A = \int d\theta \left[ X_A, \alpha Q \right] \quad \text{and} \quad \delta_\alpha \psi_A = \int d\theta \left[ X_A, \alpha Q \right] .$$  \hspace{1cm} (3.3)

Now, the Lagrangian (2.2a) for the coupling to just a one-form ($r = 0$) with the momentum integrated out becomes

$$L = \frac{\dot{x}^2}{4} + \frac{1}{4} \psi_A \dot{\psi}_A - i\dot{x}_A K_A + i\psi_A \psi_B \partial_A K_B .$$  \hspace{1cm} (3.4)

This Lagrangian can be formulated as a superworldline Lagrangian, $L_s$,

$$L_s \equiv \int d\theta \left( \frac{1}{4i} DX_A D^2 X_A - DX_A K_A \right) = L .$$  \hspace{1cm} (3.5)

This Lagrangian is invariant under supersymmetry transformations (3.1) with simply a functional variation:

$$\delta_\alpha L \equiv L(x + \delta_\alpha x, \psi + \delta_\alpha \psi) - L(x, \psi) = \partial_\tau \left[ \alpha \psi_A \left( K_A - \frac{i}{4} \dot{x}_A \right) \right] .$$  \hspace{1cm} (3.6)
This is equivalent to the fact that the variation of the Hamiltonian was given simply by a functional variation when $r = 0$.

Next, the Lagrangian (2.2a) for the coupling to just a three-form ($r = 1$) with the momentum integrated out becomes

$$L = \frac{i^2}{4} + \frac{1}{4} \psi_A \dot{\psi}_A + 3\psi_A \psi_B \dot{x}_C K_{ABC} - \psi_A \psi_B \psi_C \psi_D \partial_A K_{BCD} - 12 K_{ABC} K_{ABC} . \quad (3.7)$$

The analogue of (3.5) for the superworldline Lagrangian is

$$L_s \equiv \int d\theta \left( \frac{1}{4i} DX_A D^2 X_A + i DX_A DX_B DX_C K_{ABC} \right) . \quad (3.8)$$

However, although $L_s$ has a simple supersymmetry invariance

$$\delta_\alpha L_s \equiv L_s(x + \delta_\alpha x, \psi + \delta_\alpha \psi) - L_s(x, \psi) = \partial_\tau \left[ i \alpha \psi_A (\psi_B \psi_C K_{ABC} - \frac{1}{4} \dot{x}_A) \right] , \quad (3.9)$$

it does not generate the last term in (3.7), $-12 K_{ABC} K_{ABC}$. This means that $L_s$ will not reproduce correct field theory results and it means that $L$ is not invariant under a simple functional variation like that in (3.9). Nonetheless, $L$ was shown to be supersymmetric in the previous section when it was expressed in terms of the Hamiltonian. Notice also that the term $K_{ABC} K_{ABC}$ is exactly the same term that similarly forbade the Hamiltonian from being supersymmetric simply with the Poisson bracket variation. Finally, as the degree of the form coupled to the fermion increases, the superworldline Lagrangian, $L_s$, will fail to generate more and more terms that will be present in the correct worldline Lagrangian, $L$.

As a consequence of this analysis, consider the Lagrangian for the three-form coupling given by (3.7) in dimension $D = 6$, which comes from the six dimensional Dirac operator, $\Sigma = \Gamma_A \hat{p}_A - i \Gamma_{ABC} \hat{K}^{(3)}_{ABC}$. By defining the usual axial vector and antisymmetric tensor in four dimensions to be $B_\mu \equiv 6 K^{(3)}_{\mu56}$ and $K_{\mu\nu} \equiv 3 K^{(3)}_{\mu\nu6}$, respectively, the reduction to $D = 4$ of (3.7) gives

$$L = \frac{\dot{x}_5^2}{4} + \frac{\dot{x}_6^2}{4} + \frac{\dot{x}_6^2}{4} + \frac{1}{4} \psi_\mu \dot{\psi}_\mu + \frac{1}{4} \psi_5 \dot{\psi}_5 + \frac{1}{4} \psi_6 \dot{\psi}_6 + \psi_\mu (\bar{x}_5 \psi_6 - \bar{x}_6 \psi_5) B_\mu - \bar{x}_6 \psi_\mu \psi_\nu K_{\mu\nu}$$

$$+ \psi_5 \psi_6 \bar{x}_\mu B_\mu - 2 \psi_\mu \psi_5 \bar{x}_6 K_{\mu\nu} - \psi_\mu \psi_6 \psi_5 \bar{x}_6 \partial_\mu B_\nu - \psi_\mu \psi_6 \psi_\nu \psi_5 \partial_\mu K_{\nu\rho} - 2 B^2 - 4 K_{\mu\nu} K_{\mu\nu} , \quad (3.10)$$

\[\text{† It may be possible to generate this term by adding a term to (3.8) where } K_{ABC} \text{ is contracted with a three-form auxiliary superfield, } X_{ABC}. \text{ However, the auxiliary fermionic component of } X_{ABC} \text{ would have to be constrained so as not to introduce new degrees of freedom.}\]
where the auxiliary fields are defined as $\bar{x}_{5,6} \equiv -\dot{x}_{5,6}$. The last two terms in (3.10), which are quadratic in the background fields, are of course the terms which are not predicted by the superworldline Lagrangian, $L_s$, of (3.8). This is why Lagrangian (3.10) is presented in the superworldline approach of [5] for the axial coupling without the $-2B^2$ term. It is the verdict of this paper that using the worldline Lagrangian (3.10) derived from field theory, worldline perturbation theory reproduces Feynman diagrams, free of any of the ambiguities discussed in [5] on the treatment of $G_F^2$. In particular, it is now well understood from [6,7 and 9] that worldline fermions simply represent $\Gamma$-matrices. This is the only fact needed to determine that $G_F^2$ should always be taken to be unity even if it multiplies a $\delta$-function. The point is that by understanding worldline fermions as representing $\Gamma$-matrices, there is no mixing in the bosonic and fermionic sectors in worldline perturbation theory. This assertion has been checked on various Feynman diagrams. Similarly, the auxiliary fields may be integrated out of (3.10), but only at the $\Gamma$-level (just like the momentum [6]):

\[
L = \frac{\dot{x}^2}{4} + \frac{1}{4} \psi_\mu \dot{\psi}_\mu + \frac{1}{4} \psi_5 \dot{\psi}_5 + \frac{1}{4} \psi_6 \dot{\psi}_6 + \psi_5 \psi_6 \dot{x}_\mu B_\mu - 2\psi_\mu \psi_6 \dot{x}_\nu K_{\mu\nu} - 2K_{\mu\nu} K_{\mu\nu} \\
- \psi_\mu \psi_\nu \psi_5 \dot{\psi}_6 \partial_\mu B_\nu - \psi_\mu \psi_\nu \psi_\rho \psi_6 \partial_\mu K_{\nu\rho} - 2\psi_\mu \psi_\nu \psi_5 \dot{\psi}_6 \partial_\mu \bar{K}_\nu - \psi_\mu \psi_\nu \psi_\rho \psi_6 \partial_\mu \bar{K}_\rho
\]

(3.11)

This is the exact result obtain in the worldline reformulation of the four dimensional Dirac operator $\hat{\Sigma} = \Gamma_\mu \partial_\mu - i\Gamma_\mu \Gamma_5 \hat{B}_\mu - i\Gamma_{\mu\nu} \Gamma_6 \hat{K}_{\mu\nu}$.

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† $G_F(u)$ is the fermion propagator $\langle \psi(u)\psi(0) \rangle$, which is a step function.
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