Translocality and a Duality Principle in Generally Covariant Quantum Field Theory

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Abstract

It is argued that the formal rules of correspondence between local observation procedures and observables do not exhaust the entire physical content of generally covariant quantum field theory. This result is obtained by expressing the distinguishing features of the local kinematical structure of quantum field theory in the generally covariant context in terms of a translocal structure which carries the totality of the nonlocal kinematical informations in a local region. This gives rise to a duality principle at the dynamical level which emphasizes the significance of the underlying translocal structure for modelling a minimal algebra around a given point. We discuss the emergence of classical properties from this point of view.

1 Introduction

Quantum field theory studies the properties of algebras which are expected to give accurate mathematical descriptions of physical systems. In general, the manner in which one can extract informations of direct physical relevance from the algebraic description is very subtle because, for a given abstract algebra, there may exist in general many (unitarily inequivalent) representations in terms of operator algebras acting on a Hilbert space. Therefore, the basic problem of quantum field theory concerns the characterization of physically admissible representations.

This problem is considerably simplified in the presence of space-time symmetries. For example, in quantum field theory in Minkowski space, because of the Lorentz symmetry, it is always possible to refer to a representation containing the physical vacuum. A similar simplification could, in principle, arise in any theory admitting at least a group of

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space-time symmetry with a global time-like generator.

It turns out that in a generally covariant quantum field theory, because of the dynamical role played by the space-time metric, no \textit{a priori} notion of space-time symmetry exists. Consequently, considerable difficulties arise if one wants to characterize the physically admissible representations.

Because of the lack of \textit{a priori} space-time symmetries in the generally covariant context, it is useful for the general treatment of the basic problem of quantum field theory in that context to isolate those features of the problem which can be discussed without reference to any pre-assigned space-time symmetries. It is perfectly possible that this may not resolve the problem completely, nevertheless attempts in this direction may provide important indications for understanding the physical content of generally covariant quantum field theory. The present paper contemplates a consideration of this issue within the scope of the algebraic approach to quantum field theory [1].

We first briefly discuss the question of how general covariance can be incorporated into the conventional framework of quantum field theory [2]. The basic idea is to start with free algebras, i.e. algebras which are free from \textit{a priori} relations. The need for this is obvious, since otherwise we have \textit{a priori} no principle at hand ensuring that the algebraic relations are kept unchanged under the action of an arbitrary space-time diffeomorphism.

The general scheme we shall now describe is a generalization of the scheme used in [2-5]. We consider a differentiable manifold \( M \) and assume the existence of a net of free algebras over \( M \) generated by what we call kinematical procedures. In specific terms we require an intrinsic correspondence between open sets \( O \in M \) and a free involutive algebra \( A(O) \) such that the additivity

\[
A(O) \subset A(O'), \text{ if } O \subset O'
\]

holds. The attribute 'intrinsic' means that the principle of general covariance is implemented by considering the group \( Diff(M) \) of all diffeomorphisms of the manifold as acting by automorphisms on the net of the algebras \( A(O) \), i.e. each diffeomorphism \( \chi \in Diff(M) \) is represented by an automorphism \( \alpha_\chi \) such that

\[
\alpha_\chi(A(O)) = A(\chi(O))
\]

holds. Given such an intrinsic correspondence between open sets and algebras, we call a self adjoint element of \( A(O) \) a kinematical procedure in \( O \).

We should emphasize that, because there is no diffeomorphism invariant notion of locality, it is by no means clear whether there is an \textit{a priori} correspondence between kinematical procedures and local properties in the underlying manifold. For example we may find a coordinate system in which the kinematical procedures carry the global properties of the entire manifold in a "local domain", i.e., in a finite range of that coordinate system. Typical examples of such coordinate systems in general relativity are coordinate systems which compactify the structure of infinity. Indeed, the exploration of the question concerning the characterization of local kinematical procedures is one of the basic tasks of the present analysis. It will be dealt with in the next section.

There could be many kinematical procedures which are equivalent with respect to the action of a physical system on them which is, in general, expected to connect the kinematical
procedures with dynamical procedures (traditionally identified with observables). Thus, the essential question is how to identify the dynamical procedures of the net $O \to A(\mathcal{O})$ as suitable equivalence classes of kinematical procedures?

For this aim, we first note that the precise mathematical description of a physical system is given in terms of a state which is taken to be a positive linear functional over the total algebra of kinematical procedures $\mathcal{A} := \bigcup \mathcal{A}(O)$. Given a state $\omega$, one gets via the GNS-construction a representation $\pi_\omega$ of $\mathcal{A}$ by an operator algebra acting on a Hilbert space $\mathcal{H}_\omega$ with a cyclic vector $\Omega_\omega \in \mathcal{H}_\omega$. In the representation $(\pi_\omega, \mathcal{H}_\omega, \Omega_\omega)$ one can select a family of related states on $\mathcal{A}$, namely those represented by vectors and density matrices in $\mathcal{H}_\omega$. It corresponds to the set of normal states of the representation $\pi_\omega$, the so called folium of $\omega$.

Once a physical state $\omega$ has been specified, one can consider in each subalgebra $\mathcal{A}(O)$ the equivalence relation

$$A \sim B \iff \omega'(A - B) = 0, \quad \forall \omega' \in \mathcal{F}_\omega.$$  

(3)

Here $\mathcal{F}_\omega$ denotes the folium of the state $\omega$. The set of such equivalence relations generates a two-sided ideal $\mathcal{I}_\omega(O)$ in $\mathcal{A}(O)$. One can construct the algebra of dynamical procedures $\mathcal{A}_\omega(O)$ from the algebra of kinematical procedures $\mathcal{A}(O)$ by taking the quotient

$$\mathcal{A}_\omega(O) = \mathcal{A}(O)/\mathcal{I}_\omega(O).$$  

(4)

By this annihilation all the relevant relations between the dynamical procedures can be characterized by the totality of elements in the kernel of the representation $\pi_\omega$, namely the total ideal $\mathcal{I}_\omega$

$$\mathcal{I}_\omega = \bigcup \mathcal{I}_\omega(O).$$  

(5)

This construction implies that the mapping from kinematical procedures to dynamical procedures becomes fundamentally state-dependent. This aspect reflects one of the characteristic features of generally covariant quantum field theory.

Crucial for further investigations is the realization that a diffeomorphism $\chi \in Diff(M)$ can act as an automorphism $\alpha_\chi$ on the net $O \to \mathcal{A}_\omega(O)$ provided

$$\alpha_\chi(\mathcal{I}_\omega(O)) = \mathcal{I}_\omega(\chi(O))$$  

(6)

holds. Any diffeomorphism satisfying this condition is called dynamical (or proper). Otherwise it is called nondynamical (or improper). Nondynamical diffeomorphisms can not be represented as automorphisms on the algebra of the dynamical procedures. For dynamical diffeomorphisms such a representation is possible. They generate a group $G_\omega$ which is called in the following the dynamical group of $\omega$ and will be denoted by $G_\omega$. The elements of $G_\omega$ correspond to state-dependent automorphisms of the algebra of dynamical procedures with a pure geometric action.
2 Local inertial sector

One of the basic difficulties of the above scheme is that, in general, the GNS-representation of a physical state cannot be unitarily fixed in an intrinsic manner, because the structure of the total ideal $I^ω$ depends crucially on the particular coordinates one uses. For example, starting from the GNS-representation of a physical state one can obtain another representation if the kinematical procedures are transformed by a nondynamical diffeomorphism. The representation obtained in this way may not be in the equivalence class of the former because it may have a different kernel. Thus a physical state will, in general, provide us with a variety of unitarily inequivalent representations depending on the nature of the coordinates that happened to have been chosen for a given problem, and *a priori* it is not known which representation is physical.

The problem can be addressed on various levels. One possibility is to take a global point of view and select the equivalence class of representations for a physical state $ω$ as that for which the dynamical group $G_ω$ is nontrivial and acts globally on the manifold $M$. The geometric action of this group would then determine the nature of the equivalence class of coordinates to which the representation refers. These are coordinates which are related by the geometric action of the dynamical group $G_ω$. Such coordinates may be considered as typical examples of global inertial coordinates.

A criterion of this type may be useful to analyze the particular type of a physical theory resulting from the transition from the generally covariant description of a physical state to the special relativistic one.

For the description of a physical state in the generally covariant context we shall formulate a local variant of the above criterion. Specifically, we assume that, given a physical state $ω$, we can assign to any point $x \in M$ a neighbourhood $O_x^ω$ so that by the restriction of the GNS-representation $π^ω$ to $O_x^ω$ a nontrivial dynamical group $G_ω$ is established which acts on $O_x^ω$. To emphasize the individuality of the point $x$, we shall assume that the geometric action of $G_ω$ on $O_x^ω$ leaves the point $x$ invariant. In an alternative formulation we shall require the invariance of the local ideal $I^ω(O_x^ω)$ under the (nontrivial) action of the dynamical group $G_ω$, namely

$$α(I^ω(O_x^ω)) = I^ω(O_x^ω), \quad ∀α ∈ G_ω \quad (7)$$

with $x$ being invariant under the geometric action of $G_ω$. For any physical state $ω$ this acts as a criterion to select a characteristic local equivalence class of representations. In symbols we shall write for this local equivalence class $\{π^ω|O_x^ω\}$ and refer to it as a local inertial sector of a physical state $ω$. Correspondingly the equivalence class of local coordinate systems to which $\{π^ω|O_x^ω\}$ refers are called the equivalence class of local inertial coordinates with the origin at $x$. The neighbourhood $O_x^ω$ is called a normal neighbourhood.

3 Local and translocal properties

One consequence of a local inertial sector of a physical state would be the distinction it would draw between the two different ideal sets of kinematical procedures. Given
a physical state $\omega$ and a point $x \in \mathcal{M}$, consider a local inertial sector $\{\pi^{\omega} | \mathcal{O}^{\omega}_x\}$. A kinematical procedure in $A \in \mathcal{A}(\mathcal{O}^{\omega}_x)$ is called translocal (or absolute) if it escapes the local action of the dynamical group $G_{\omega}$ in $\mathcal{O}^{\omega}_x$. In mathematical terms, this is taken to mean that for an arbitrary element $\alpha \in G_{\omega}$ we have $\alpha(A) - A \in \mathcal{I}(\mathcal{O}^{\omega}_x)$. A kinematical procedure $A \in \mathcal{A}(\mathcal{O}^{\omega}_x)$ for which this condition can not be satisfied is called local$^2$.

It can be shown that this distinction between local and translocal kinematical procedures is preserved at the dynamical level of the theory. In fact we prove the following

**Statement:** For a local (respectively translocal) kinematical procedure $A \in \mathcal{A}(\mathcal{O}^{\omega}_x)$ the corresponding equivalence class in the sense of (3) contains local (respectively translocal) kinematical procedures only.

Consider first the case of a translocal kinematical procedure $A \in \mathcal{A}(\mathcal{O}^{\omega}_x)$. We show that any kinematical procedure $B \in \mathcal{A}(\mathcal{O}^{\omega}_x)$ which is equivalent to $A$ is translocal. For an arbitrary element $\alpha$ of the dynamical group $G_{\omega}$ we have $\alpha(A) = A + I$ with $I \in \mathcal{I}(\mathcal{O}^{\omega}_x)$. Since $B \sim A$ we also have $B = A + I'$ with $I' \in \mathcal{I}(\mathcal{O}^{\omega}_x)$. It then follows for all $\alpha \in G_{\omega}$ that

$$\alpha(B) = \alpha(A) + \alpha(I') = A + I + \alpha(I') = B - I' + I + \alpha(I).$$

This together with the invariance of the ideal, relation (4), implies $\alpha(B) - B \in \mathcal{I}(\mathcal{O}^{\omega}_x)$. Thus $B$ is translocal. Now consider the case of a local kinematical procedure $A \in \mathcal{A}(\mathcal{O}^{\omega}_x)$. We show that any kinematical procedure $B \in \mathcal{A}(\mathcal{O}^{\omega}_x)$ which is equivalent to $A$ is local. We have $B = A + I$ with $I \in \mathcal{I}(\mathcal{O}^{\omega}_x)$. Since $A$ is local there exist an element $\alpha$ of the dynamical group so that the difference $\Delta = \alpha(A) - A$ does not lie in $\mathcal{I}(\mathcal{O}^{\omega}_x)$. It follows that

$$\alpha(B) = \alpha(A) + \alpha(I) = A + \Delta + \alpha(I) = B - I + \Delta + \alpha(I)$$

from which one infers that $\alpha(B) - B$ can not be in $\mathcal{I}(\mathcal{O}^{\omega}_x)$. Thus $B$ is local.

From this consideration it follows that the dynamical procedures of a local inertial sector decompose into two distinct sets, namely the sets containing all equivalence classes of local and translocal kinematical procedures respectively. A member of the first set (respectively the second set) is called a local (respectively translocal) dynamical procedure. We emphasize that this distinction between dynamical procedures takes the concept of dynamical activity in a local inertial sector as basic. A translocal dynamical procedure in a local inertial sector is taken to be a dynamical procedure that continually transforms into itself by the local action of the dynamical group. They correspond to absolute properties of a local inertial sector.

It should be emphasized that the appearance of the translocal kinematical procedures in $\mathcal{A}(\mathcal{O}^{\omega}_x)$ illustrates a novel effect of the principle of general covariance. In fact, any restriction to local kinematical procedures inside a local inertial sector $\{\pi^{\omega} | \mathcal{O}^{\omega}_x\}$ would be fundamental only to the extent to which the diffeomorphism group refers only to the properties inside the normal neighbourhood $\mathcal{O}^{\omega}_x$. That this is not the case is seen by the following consideration which furnishes the necessary prerequisite for our subsequent presentations.

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$^2$In reality there are certain limitations on the applicability of this definition, because of the limited accuracy of actual experiments which makes it impossible to determine the ideal $\mathcal{I}^{\omega}(\mathcal{O}^{\omega}_x)$ exactly. We shall ignore problems of this type.
Consider the identification of points inside $O^\omega_x$, made in a system of local inertial coordinates, and consider a kinematical procedure parametrized in a local coordinate system outside $O^\omega_x$. In general a kinematical procedure of this type characterizes a nonlocal kinematical property outside $O^\omega_x$ which does not transform under the change of the system of local inertial coordinates inside $O^\omega_x$, so it escapes the local action of the dynamical group in $O^\omega_x$. Now consider a diffeomorphism acting entirely outside $O^\omega_x$. We shall call a diffeomorphism of this type a gauge transformation. The essential point is that it needs only to apply an appropriate gauge transformation, namely a gauge transformation which has its image inside $O^\omega_x$, to convert a nonlocal kinematical property outside $O^\omega_x$ into a translocal kinematical procedure inside $O^\omega_x$. This argument demonstrates that a translocal kinematical procedure is the image of a non-local kinematical procedure outside $O^\omega_x$ under an appropriate gauge transformation. Thus, gauge transformations can be applied to generate the totality of all translocal kinematical procedures inside $O^\omega_x$ as the local codifications of the totality of all nonlocal kinematical procedures outside $O^\omega_x$. This connection between a local inertial sector and the associated appearance of translocal (absolute) properties is the distinctly marked conclusion of the present analysis.

At this point a clarifying remark concerning the status of translocal kinematical procedures with respect to the conventional quantum field theory appears to be in place. From our presentation one can immediately observe that, in any theory in which one finds a dynamical group globally acting on the underlying (space-time) manifold, there would be no obvious way to introduce (quasi) invariant kinematical procedures with respect to that group, so a translocal kinematical procedure would not be obvious in the fundamental description of the theory. This is specially so in Minkowski-space quantum field theory with the Lorentz-group playing the role of a global dynamical group. In particular, in the latter theory the proven statement at the begin of this section trivializes because all kinematical procedures becomes essentially local, because they can not escape the global action of a nontrivial Lorentz-transformation.

4 The axioms of translocality

From the scheme presented so far one can immediately infer that the set of all translocal dynamical procedures in a local inertial sector $\{\pi^\omega_x|O^\omega_x\}$ is closed under the algebraic operations. This statement may not have in general an analog with respect to the local dynamical procedures. Actually, there is a principal possibility that a translocal dynamical procedure can be approximated by finite algebraic operations on local dynamical procedures. In such a situation the dynamical informations monitored by an actual measurement on a physical system would algebraically connect both the local and the translocal properties. It is not the objective of this paper to develop the particular mathematical formalism needed to describe physics of this sort which is indeed a very

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3 The situation would change if one considers the embedding of the manifold with a global dynamical group into a larger manifold without extending the action of the dynamical group. In this case any kinematical procedure which lies outside the initial manifold can obviously be interpreted as a (quasi)invariant object with respect to the action of dynamical group.
The kind of behavior, that we may expect to occur for a large class of physical systems in the generally covariant context, is that it should categorically not possible to connect the local and translocal dynamical procedures by a finite (or infinite in an admissible sense) algebraic process in a local inertial sector. Mathematically, this requirement may be converted into the first axiom of translocality formulated as the statement:

The set of all local dynamical procedures in a local inertial sector \( \{ \pi^\omega | O^\omega_x \} \) generates a weakly closed subalgebra of \( \mathcal{A}^\omega(O^\omega_x) \) which has a trivial intersection\(^5\) with the algebra of translocal dynamical procedures inside \( O^\omega_x \).

This axiom emphasizes the feasibility of a substantial distinction between the local and translocal properties inside a local inertial sector.

We shall exclusively deal with theories satisfying this axiom. For such theories the local kinematical procedures in \( \mathcal{A}(O^\omega_x) \) can be identified with ordinary local observation procedures (pure description of possible laboratory measurements) and their corresponding equivalence classes in \( \mathcal{A}^\omega(O^\omega_x) \) with local observables. The equivalence classes of translocal kinematical procedures in \( \mathcal{A}^\omega(O^\omega_x) \) correspond to the properties which do not respond to a local measurement process inside \( O^\omega_x \). We denote the algebra generated by local observables of a normal neighbourhood \( O^\omega_x \) by \( \mathcal{A}^\omega_{\text{obs}}(O^\omega_x) \). It is considered as a weakly closed subalgebra of \( \mathcal{A}^\omega(O^\omega_x) \).

Particular attention should be directed to the transformation properties of a local inertial sector \( \{ \pi^\omega | O^\omega_x \} \) under various automorphisms of \( \mathcal{A}^\omega(O^\omega_x) \). Consider first the case of an inner-automorphism \( \alpha \) of \( \mathcal{A}^\omega(O^\omega_x) \) generated by a translocal dynamical procedure \( \mathcal{U} \), namely

\[
\alpha(A) = \mathcal{U} A \mathcal{U}^{-1}, \quad \forall A \in \mathcal{A}(O^\omega_x). \tag{8}
\]

An inner-automorphism of this kind is called a translocal morphism. The properties of a physical system in the generally covariant context depends very crucially on the particular way in which a translocal morphism acts geometrically. The second axiom of translocality assumes a one to one correspondence between the action of a translocal morphism and the action of a gauge transformation. More precisely, this axiom emphasizes that a given translocal morphisms has a geometric action corresponding to the action of a gauge transformation and conversely a given gauge transformation has an algebraic action corresponding to a translocal morphism.

Since gauge transformations are diffeomorphisms acting entirely outside \( O^\omega_x \), it follows that a translocal morphism should not affect the local observables inside the normal neighbourhood \( O^\omega_x \). This would require an arbitrary element \( \mathcal{U} \) of the algebra of the translocal dynamical procedures to commute with all local observables of a local inertial sector \( \{ \pi^\omega | O^\omega_x \} \), namely

\[
[\mathcal{A}, \mathcal{U}] = 0, \quad \forall A \in \mathcal{A}^\omega_{\text{obs}}(O^\omega_x). \tag{9}
\]

\(^4\)It may be expected that nonunitary evolution would be the dominating feature of physics of this type.

\(^5\)The difference between a trivial intersection and an empty intersection is that the former is allowed to contain multiples of the identity element.
Thus the second axiom of translocality implies that the total activity of translocal dynamical procedures inside a local inertial sector can be reduced to the presence of a (nontrivial) commutant of the algebra of local observables in that sector. We call it the translocal commutant of a local inertial sector. It will be denoted by $[\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x)]'$. 

Using the first axiom of translocality we can establish a general property of the translocal commutant $[\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x)]'$. We prove, namely, that $[\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x)]'$ should have a trivial center: Let us assume the opposite case. Then, by applying the bicommutant property $[\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x)]'' = \mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x)$, we would get a nontrivial intersection of the local elements of $\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x)$ and the translocal elements of $[\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x)]'$. This is a contradiction to the first axiom of translocality. Thus, the triviality of the center of $[\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x)]'$ becomes imperative.

We may note that the triviality of the center of $[\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x)]'$ may be illustrated as a statement about the global definiteness of the totality of all (non-local) complementary properties of a local inertial sector $\{\pi^\omega|\mathcal{O}_x^\omega\}$. In the generally covariant context, this definiteness seems to be important in determining the long range dynamical coupling of a physical state with distant sources. In particular this global definiteness proves to be very crucial in forming the algebraic action of dynamical group $G_\omega$ inside a local inertial sector $\{\pi^\omega|\mathcal{O}_x^\omega\}$. To illustrate this point, we note first that, by assumption, this action leaves the translocal dynamical procedures in $\{\pi^\omega|\mathcal{O}_x^\omega\}$ unchanged. The most immediate way to manifestly express this property is to approximate an element $\alpha \in G_\omega$ inside $\mathcal{O}_x^\omega$ by an inner-automorphism of $\mathcal{A}^\omega(\mathcal{O}_x^\omega)$ generated by a corresponding element $L_\alpha \in \mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x^\omega)$, namely

$$
\alpha(A) = L_\alpha A L_\alpha^{-1}, \quad \forall A \in \mathcal{A}^\omega(\mathcal{O}_x^\omega). \quad (10)
$$

This relation can be used to study the nature of the group-operator $L_\alpha$. We are particularly interested in a situation in which the group-operator $L_\alpha$ is uniquely determined by this relation. In general, this relation leaves us an ambiguity concerning the choice of the group-operator $L_\alpha$. In fact, with $(10)$ we get the freedom to replace the group-operator $L_\alpha$ by $L_\alpha C$, where $C$ is an arbitrary element in the center of the translocal commutant $[\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x^\omega)]'$. We infer that the triviality of the center of $[\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x^\omega)]'$, which was implied by the first axiom of translocality, appears to be a powerful restriction in order to characterize the group-operator $L_\alpha$.

Putting the totality of the translocal dynamical procedures into the translocal commutant $[\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x^\omega)]'$ by no means implies that correlations can not occur between the local observables and the translocal dynamical procedures inside $\mathcal{O}_x^\omega$. Indeed, an essential input is to make an assumption of general nature to characterize the form of the correlations implied by the activity of the translocal dynamical procedures. This issue is addressed by formulating the third axiom of translocality which reflects the impossibility of isolating the algebra generated by local observables with respect to the dynamical activity of the translocal commutant. To arrive at its mathematical formulation we shall require that for a physical state $\omega$, the corresponding vector $\Omega^\omega$ in a local inertial sector $\{\pi^\omega|\mathcal{O}_x^\omega\}$ be a separating vector for the algebra of local observable $\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x^\omega)$. This means that it should not be possible to annihilate the vector $\Omega^\omega$ by elements of $\mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x^\omega)$, namely

$$
A \Omega^\omega = 0 \rightarrow A = o, \quad \forall A \in \mathcal{A}_{\text{obs}}^\omega(\mathcal{O}_x^\omega). \quad (11)
$$
By the standard theorems of the theory of operator algebras [6] the above requirement can alternatively be replaced by the requirement of the cyclicity of the vector \( \Omega^\omega \) with respect to the translocal commutant \( \mathcal{A}_{\text{obs}}(\mathcal{O}^\omega_x)' \). In this formulation the third axiom of translocality emphasizes the distinguishing role played by translocal dynamical procedures inside a local inertial sector in singling out a dense subset of the corresponding Hilbert space.

## 5 Commutant duality

In reality, the more informations which should, in principle, be available in the form of correlations between local observables and translocal dynamical procedures has a significant effect on the effective description of the short-distance behavior of the underlying theory. To understand this effect, one has to extrapolate the physical informations carried by the members of the translocal commutant to the short-distance characteristics of a local inertial sector. This issue can be addressed by formulating a duality principle which, in essence, connects the long distance properties of states with their corresponding short-distance counterparts using a gauge transformation:

Given a local inertial sector \( \{ \pi^\omega | \mathcal{O}^\omega_x \} \), we call any neighbourhood \( \mathcal{O}_x \subset \mathcal{O}^\omega_x \) of the point \( x \) which is invariant under the geometric action of the dynamical group \( G^\omega \) an invariant neighbourhood of the origin.

We argue that to any local inertial sector \( \{ \pi^\omega | \mathcal{O}^\omega_x \} \) one can assign a characteristic invariant neighbourhood of the origin. By definition, the origin \( x \) is invariant under the geometric action of the dynamical group. Thus, one needs only to pass from the origin to one of its neighborhoods \( \mathcal{O}_x \) on which the action of the dynamical group remains still arbitrarily close to the identity such that no local observable can properly be affiliated to \( \mathcal{O}_x \). The operational way to achieve this is as follows: One may start with a contracting sequence of neighborhoods \( \mathcal{O}_x^\lambda \subset \mathcal{O}^\omega_x \) of the point \( x \)

\[
\mathcal{O}_x^{\lambda+1} \subset \mathcal{O}_x^\lambda, \tag{12}
\]

which is ideally taken to shrink to the point \( x \) as \( \lambda \to \infty \), and continue to truncate the sequence at some sufficiently large \( \lambda \). The prefix ‘sufficiently large’ characterizes an index \( \lambda \) for which the set of numbers

\[
|\omega'(\alpha(A)) - \omega'(A)|, \quad A \in \mathcal{A}^\omega(\mathcal{O}_x^\lambda)
\]

taken for all states \( \omega' \) in the folium \( \mathcal{F}^\omega \) of \( \omega \) and for all elements \( \alpha \) of the dynamical group \( G^\omega \), remains smaller than a characteristic nonvanishing small number \( \epsilon \) characterizing the limited accuracy of the local measurements. In this way the correspondence between a local inertial sector and a characteristic invariant neighbourhood of the origin may be established. For this neighbourhood we use the name the continuous image of the origin.

Our objective is now to apply the second axiom of translocality to derive a duality principle which emphasizes the significance of translocal dynamical procedures for modelling the algebra corresponding to the continuous image of the origin in a local inertial sector. Consider a gauge-transformation \( \sigma \) in a local inertial sector, which according to the second
axiom of translocality, has the algebraic action corresponding to a translocal morphism. Given a local inertial sector \( \{ \pi^\omega|O^\omega_x \} \) it is always possible to obtain a representation in the equivalence class of \( \pi^\omega \) by applying a gauge transformation \( \sigma \) to \( A^\omega(O^\omega_x) \). Let us now consider a gauge transformation \( \sigma \) which sends the totality of points outside \( O^\omega_x \) into the continuous image of the origin inside a local inertial sector \( \{ \pi^\omega|O^\omega_x \} \). It follows that for the image of the translocal commutant under \( \sigma \) we can establish the inclusion property

\[
\sigma[A^\omega_{obs}(O^\omega_x)]' \subset A^\omega(O_x)
\]

which holds for any neighbourhood \( O_x \subset O^\omega_x \), which contains the continuous image of the origin as a proper subset. The inclusion property (13) tells us that the gauge-transformation \( \sigma \) can be used to affiliate the translocal commutant into the continuous image of the origin. Since gauge transformations are symmetry operations inside a local inertial sector we infer that by restricting a state to the continuous image of the origin the folium of \( \omega \) becomes indistinguishable from the set of normal states over the translocal commutant. This is the expression of what we call the commutant duality.

We should emphasize that the geometric gauge transformations underlying the formulation of commutant duality is a novel feature of the principle of general covariance and can not be exemplified in conventional models of quantum field theory with no geometric gauge group. Since the geometric gauge transformations can, in principle, be used to affiliate the translocal commutant to any open region inside the normal neighbourhood \( O^\omega_x \), one can generally say that the theory deals profoundly with two different phases inside a local inertial sector, depending on whether the local or the translocal properties are considered as primary properties. Once this has been recognized, then the investigation of a possible symmetry between these two distinct phases appears to be a problem of direct physical relevance. This symmetry, which can generally be termed under the name of ‘duality’, needs the study of those coordinate transformations exchanging the local and translocal dynamical procedures which are related, in a specific model, to different sets of dynamical variables. One can generally expect that the formulation of this symmetry would reflect new geometric gauge invariance which is not visible inside a local inertial sector. It is needless to say that such a development would also shed new light on the symmetry behind the currently discussed duality of supersymmetric gauge theory and string theory.

Our last remark in this section concerns the notion of quantum equivalence principle. There exists a formulation of this principle in the framework of quantum field theory in curved space which takes the correspondence between the leading short-distance singularity of states and the corresponding singularity of the vacuum in Minkowski space [8][2] as basic. In the present context, the commutant duality requires a profoundly smooth short-distance behavior, so there is the need to reformulate the quantum equivalence principle in a different way. This formulation is implied by the commutant duality itself. In fact, combining it with the third axiom of translocality it follows that the state-vector \( \Omega^\omega \) can be considered as a cyclic vector for any algebra \( A^\omega(O_x) \subset A^\omega(O^\omega_x) \), for which the neighbourhood \( O_x \) contains the continuous image of the origin as a proper subset. This cyclicity

\[6\] See [7] and references therein.
property establishes an exact correspondence between the structure of correlations of the state \( \omega \) in a local inertial sector and that of the vacuum state in Minkowski-space\[7\]. We may, therefore, take this correspondence, which is implied by the commutant duality, as a coded form of a quantum equivalence principle.

6 Classical properties

We analyze now the consequence of commutant duality in an idealized limit which destroys the algebraic informations of the translocal commutant in a local inertial sector. At this level of description a state is unable to monitor the exact form of all conceivable correlations between the local observables and the individual members of the translocal commutant and the description of a state is transferred to a positive linear functional over the algebra of local observables in a local inertial sector. This corresponds to the conventional description of states in quantum field theory. However, the essential point is that, at such a level of description, the ignorance concerning the accurate form of algebraic informations contained in the translocal commutant implies a structural dependence of the short-distance behavior of the underlying theory on classical properties. We have to clarify this.

Given a local inertial sector \( \{ \pi^{\omega}\mathcal{O}_{x}\} \), we may ideally transfer \(^\dagger\) the description of the state \( \omega \) to a positive linear functional over the algebra of local observables in that sector. The question we shall address is how this change of the level of description will alter the nature of the translocal commutant. The resolution is quite immediate. Indeed, the inclusion relation (13) implies that the translocal commutant can then be approximated by a commutative algebra lying in the center of any subalgebra \( \mathcal{A}_{\text{obs}}^{\omega}(\mathcal{O}_{x}) \subset \mathcal{A}_{\text{obs}}^{\omega}(\mathcal{O}_{x}) \) for which the neighbourhood \( \mathcal{O}_{x} \subset \mathcal{O}_{x}^{\omega} \) contains the continuous image of the origin as a proper subset. In this way the emergence of classical properties in a local inertial sector may be an irreducible feature of the theory if we transfer to the conventional level of the description of a state in quantum field theory.

7 Concluding remarks

In this paper we have discussed the impact of the principle of general covariance on the algebraic framework of quantum field theory. At first sight the implementation of this principle seems to create confusion concerning a substantial identification of local properties. We have proposed a tentative resolution of the problem which takes the dynamical activity in a local inertial sector as basic. However, the principle of general covariance implies that the set of all local properties in a local inertial sector may not be considered as a completed totality. The notion of translocality was introduced to address this issue. In our approach there is an effective crossover from local properties to the translocal properties, once the short-distance scaling is performed inside a local inertial sector. This

\(^7\)Actually, in Minkowski-space there is a general result, obtained by Reeh and Schlieder, which states that the vacuum is cyclic not only for the whole algebra but also for the algebra of any open region [1]
interrelation of short-distance scaling with the translocal properties which is implied by the commutant duality may be of particular importance for expressing Mach’s principle [9] within the framework of quantum field theory [5]. In particular it emphasizes that the short-distance property of quantum field theory in the generally covariant context is profoundly different from ordinary quantum field theory. Remarkably, this is especially so for an important class of currently discussed theories generally termed by string theory. We can not at the present understand how an exemplification of the general principles of generally covariant quantum field theory in a model can be related to string theory. But, nevertheless it can be expected that for the unification of quantum field theory with certain features of string theory the commutant duality may have a vital role to play.

The next point implied by commutant duality concerns the transition to the conventional description of states in quantum field theory. On this level of description the dominant structure of a generally covariant quantum field theory has been recognized to be the occurrence of classical properties. It is an interesting subject to analyze the interrelation of such classical properties with the classical space-time metric of general relativity. We hope to address the issue elsewhere.

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