Parity of Anti-Decuplet Baryons Revisited from Chiral Soliton Models

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We recalculate masses and widths of anti-decuplet baryons in the case of positive parity from chiral soliton models, provided that the member $\Xi_{3/2}$ of the anti-decuplet has a mass 1.86 GeV as reported recently. Calculations show that there are no convincing candidates for the nonexotic members of the anti-decuplet available in the baryon listings. Up to the leading order of $m_s$ and $1/N_c$, the width formula for the decay of the anti-decuplet baryons to the octet depends only on $SU(3)$ symmetry model-independently, except the coupling constant. Similarly we give a width formula for the decay of negative parity baryons belong to certain $SU(3)$ baryon multiplet by pure symmetry consideration. By this formula, we find that if we have an anti-decuplet with negative parity and that the masses are the same as those given by chiral soliton models, the identification of $N(1650)$ as $N_{\pi\pi}$ are inconsistent with experiments for $N(1650) \to N\pi$ while the widths agree with other two decay channels involving strangeness. And $\Sigma(1750)$ seems to be a reasonable candidate for $\Sigma_{\pi\pi}$.

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Due to recent reports\textsuperscript{1,2,3,4,5} for the existence of pentaquark state $\Theta^+$, Skyrme’s old idea\textsuperscript{6} of identifying baryons as solitons has aroused unprecedented interest. In the large $N_c$ limit, the soliton picture of baryons can be proved consistent with QCD. The quantization of the $SU(3)$ Skyrmion in collective coordinates\textsuperscript{6} not only gives the correct baryon octet and decuplet, but also predicts the next lighter baryon multiplet, the anti-decuplet \textsuperscript{7}, in which $\Theta^+$ is the lightest member. In the quark language, $\Theta^+$ is of the minimal five-quark configuration $uudd\pi$, and thus has the exotic strangeness number $S=+1$. Predictions about the mass and width of $\Theta^+$ from chiral soliton models\textsuperscript{12,13,14,15} have played an important role in the searches of $\Theta^+$. However, there are controversies about the parity of $\Theta^+$. Naively, the lowest $qqqq$ state should have negative parity in quark models\textsuperscript{14,15}. Analysis from QCD sum rule\textsuperscript{17} as well as earlier lattice QCD calculation\textsuperscript{18} show that $\Theta^+$ can be consistently identified as a pentaquark state provided its $J^P=\frac{1}{2}^+$, however, analysis of the stability of pentaquarks $uudd\pi$ shows that $\Theta^+$ $p$-shell with positive parity is lower than $s$-shell with negative parity for pentaquarks. The quark cluster models\textsuperscript{20} also predict a positive parity for $\Theta^+$. In literatures\textsuperscript{14,15}, the anti-decuplet baryons are predicted to have positive parity from chiral soliton models. However, there are also opinions\textsuperscript{21} that the success predictions from chiral soliton models could be fortuitous. Up to now, the parity of $\Theta^+$ has not been experimentally determined. Combining the observed pentaquark $\Theta^+$ mass from available experiments\textsuperscript{1,2,3,4,5} and the pentaquark $\Xi_{3/2}$ mass from a recent experiment\textsuperscript{22}, Diakonov and Petrov\textsuperscript{23} re-identified $N_{\pi\pi}$ and $\Sigma_{\pi\pi}$. However, there are no such two corresponding baryons with $J^P=\frac{1}{2}^+$ in available baryon listings\textsuperscript{24}. Thus, they suggested that there must be two new baryons as the missing members of the anti-decuplet. And in\textsuperscript{25}, the modified PWA analysis also indicates that the $N(1710)$ is not the appropriate candidate to be a member of the anti-decuplet, instead, $N(1680)$ or $N(1730)$ with positive parity was suggested.

The main purpose of this note is to discuss the probability of picturing $\Theta^+$, $\Xi(1862)$, $N(1650)$ and $\Sigma(1750)$ as the members of an anti-decuplet with negative parity model-independently by pure symmetry consideration. In soliton picture, there also exist pentaquark states with one heavy anti-quark $(\bar{Q}qqqq)$ and with $J^P=\frac{1}{2}^-$ in the bound state approach\textsuperscript{26,27}. However, in the collective quantization approach, how to describe both positive and negative parity baryons is still not solved. In the $SU(3)$ chiral soliton model, the fundamental object is the chiral field $U(x)$

$$U(x) = \exp \left[i \lambda_6 \phi_6(x)/f_\pi \right],$$

where $f_\pi \approx$93 MeV is the observed pion decay constant, $\lambda_6$ are the eight Gell-Mann $SU(3)$ matrices and $\phi_6(x)$ are the eight pseudoscalar meson fields. Under the space inversion transformation, $\phi_6(x)$ transforms as

$$\hat{P} \phi_6(x, t) = -\phi_6(-x, t),$$

where $\hat{P}$ is the parity operator. Accordingly, $U(x)$ transforms as

$$\hat{P}U(x) \hat{P}^\dagger = \exp \left[-i \lambda_6 \phi_6(-x, t)/f_\pi \right] = U^\dagger(-x, t).$$

In chiral limit, the action of Skyrme model is of the form

$$I = \frac{\alpha^2}{2} \int d^4x \text{Tr} \left( \partial_\mu U \partial^\mu U^\dagger \right) + \frac{1}{32\pi^2} \int d^4x \left[ \text{Tr} \left( \partial_\mu U U^\dagger, \partial_\nu U U^\dagger \right) \right]^2 + N_c \Gamma,$$
which has a hedgehog soliton solution under the assumption of maximal symmetry

$$U_1(x) = \begin{pmatrix} \exp \left[ i(\vec{r} \cdot \tau) F(r) \right] & 0 \\ 0 & 1 \end{pmatrix},$$  

where $e$ is introduced to stabilize the solitons by Skyrme; $\Gamma$ is the Wess-Zumino term; $F(r)$ is the spherical-symmetric profile of the soliton, the solution of the nonlinear equation of motion; $\tau$ are the three Pauli matrices; and $\vec{r}$ is the unit vector in space. The action is invariant under $SU(3)_L \times SU(3)_R$ transformation. However, we are only interested in those $U(x)$ with the same vacuum at $r \to \infty$

$$U(x) = A(t)U_1(x)A(t)^{-1}, \quad A \in SU(3).$$  

Inserting (5) into (6) gives

$$\hat{P}U(x)\hat{P}^\dagger = U(x),$$  

and

$$\hat{P}A(t)\hat{P}^\dagger = A(t),$$

$$\hat{P}F(r)\hat{P}^\dagger = -F(r).$$

Then quantize the system about this solitonic solution for collective coordinates $A$. To leading order only symmetry modes (collective coordinates) are important, thus we only treat collective coordinates quantum mechanically, and the SU(3) symmetric effective action in the large $N_c$ limit leads to the collective Hamiltonian:

$$\hat{H} = M_d + \frac{1}{2I_2} \left[ \hat{C}^{(2)} - \frac{1}{12} (N_c B)^2 \right] + \left( \frac{1}{2I_1} - \frac{1}{2I_2} \right) \hat{J}^2,$$

where $M_d$, $I_1$ and $I_2$ are given by the 3-dimensional space coordinate integrals of even functions of $F(r)$ and $e$, and are treated model-independently and fixed by experimental data in our work. $M_d$ is the classical soliton mass; $I_1$ and $I_2$ are moments of inertia; $\hat{C}^{(2)} = \sum_{a=1}^{8} \hat{G}_a^2$ is the quadratic (Casimir) operator of the vectorial group $SU(3)_c$, and in the representation $(p, q)$, its eigenvalue $C^{(2)} = \frac{1}{4}(p^2 + q^2 + pq + 3(p + q))$; $\hat{G}_a(A)$ $(a = 1-8)$ are the generators of SU(3)$_c$; and $\hat{J}_i(A)$ $(i = 1-3)$ are the generators of the spin group SU(2)$_c$. Using Eq. (8), we have

$$\hat{P}\hat{H}(A)\hat{P}^\dagger = \hat{H}(A),$$

$$\hat{P}\hat{G}_a(A)\hat{P}^\dagger = \hat{G}_a(A),$$

$$\hat{P}\hat{F}(r)\hat{P}^\dagger = -F(r).$$

The wave function $\Psi^{(\mu)}_{\nu\nu'}(A)$ of baryon $B$ in the collective coordinates is of the form

$$\Psi^{(\mu)}_{\nu\nu'}(A) = \sqrt{\dim(\mu)}D^{(\mu)}_{\nu\nu'}(A),$$

where $(\mu)$ denotes an irreducible representation of the SU(3) group; $\nu$ and $\nu'$ denote $(Y, I, I_1)$ and $(1, J, -J_3)$ quantum numbers collectively; $Y$ is the hypercharge of $B$; $I$ and $I_3$ are the isospin and its third component of $B$ respectively; $J_3$ is the third component of spin $J$; and $D^{(\mu)}_{\nu\nu'}(A)$ are representation matrices. Eq. (5) means that the wave function has positive parity. However, in the procedure of collective coordinate quantization, only collective parts are quantized, and modes orthogonal to the symmetry modes are treated classically. In our case, these modes should be related with the coordinate $r$. However, by Eq. (9), we know that a function of $r$ may still possess negative parity, this may be suggestive to extend soliton picture to negative parity baryons because the part we treat classically may still contribute negative parity. The symmetry breaking Hamiltonian is

$$H' = \alpha D^{(8)}_{88} + \beta Y + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D^{(8)}_{8i}J^i,$$

where the coefficients $\alpha$, $\beta$, $\gamma$ are proportional to the strange quark mass and model dependent, and are treated model-independently and fixed by experiments; and $D^{(8)}_{ma}(A)$ is the adjoint representation of the SU(3) group and defined as

$$D^{(8)}_{ma}(A) = \frac{1}{2} \text{Tr}(A^\dagger \lambda^m A \lambda^a).$$

We can use perturbation theory to calculate the baryon states in collective coordinates on the basis of flavor symmetry states:

$$|N\rangle = |N; 8\rangle + C^{10}_{10} |N; 10\rangle + C_{27} |N; 27\rangle,$$

$$|\Sigma\rangle = |\Sigma; 8\rangle + C^{10}_{10} |\Sigma; 10\rangle + \frac{\sqrt{6}}{3} C_{27} |\Sigma; 27\rangle,$$

$$|\Xi\rangle = |\Xi; 8\rangle + C_{27} |\Xi; 27\rangle,$$

$$|\Theta^+\rangle = |\Theta^+; 10\rangle,$$

$$|N_{10}\rangle = |N; 10\rangle - C_{10} |N; 8\rangle + \frac{\sqrt{30}}{80} C^{10}_{27} |N; 27\rangle,$$

$$|\Sigma_{10}\rangle = |\Sigma; 10\rangle - C_{10} |\Sigma; 8\rangle + \frac{\sqrt{6}}{4\sqrt{5}} C^{10}_{27} |\Sigma; 27\rangle,$$

$$|\Xi_{3/2}\rangle = |\Xi_{3/2}; 10\rangle + \frac{\sqrt{6}}{16} C^{10}_{27} |\Xi_{3/2}; 27\rangle,$$

and the coefficients for other multiplets are:

$$C_{10} = -\frac{1}{3\sqrt{5}} (\alpha + \frac{7}{3} \beta) I_2,$$

$$C_{27} = -\frac{\sqrt{6}}{3\sqrt{5}} (\alpha - \frac{7}{3} \beta) I_2,$$

$$C^{(10)}_{27} = - (\alpha - \frac{7}{3} \beta) I_2,$$

where $|B; \mu\rangle$ denotes a flavor symmetry state $\Psi^{(\mu)}_{\nu\nu'}(A)$ with $\nu\nu'$ denoting the quantum numbers of baryon $B$. From the analysis above, we see that this mixing is due to $H'$, and is irrelevant to parity. Thus this mixing is
still valid if the anti-decuplet baryons have negative parity. We know that in a baryon multiplet, different baryon functions are related by \( \hat{T}_+ = \hat{G}_1 \pm i \hat{G}_2 \), \( \hat{V}_+ = \hat{G}_3 \pm i \hat{G}_5 \) and \( \hat{U}_+ = \hat{G}_6 \pm i \hat{G}_7 \), which commute with \( \hat{P} \) by (12). Therefore, to fix the parity, we can only find a candidate with definite parity from experiments, which has the same mass and width to the corresponding member of the anti-decuplet, and then fix the parity of other baryons in the multiplet accordingly.

Ref. 22 first reported evidence for the existence of a narrow \( \Xi^- \pi^- \) baryon resonance with mass of 1.862 ± 0.003 GeV and width below the detector resolution of about 0.018 GeV, and this state is considered as a candidate for the pentaquark \( \Xi^- \) in the anti-decuplet predicted from chiral soliton models 14. Provided \( \Theta^+ \) with mass of 1.54 GeV and \( \Xi_+^0 \) with mass of 1.86 GeV, we recalculate the coefficients in (10) and (14) as well as the masses of the other two members of the anti-decuplet. The results are as follows

\[
1/I_1 = 154 \text{ MeV}; \ 1/I_2 = 399 \text{ MeV}; \ \Sigma \approx 78 \text{ MeV}; \ \alpha = -663 \text{ MeV}; \ \beta = -12 \text{ MeV}; \ \gamma = 185 \text{ MeV}; \ m_{N\Xi'} = 1.65 \text{ GeV}; \ m_{\Sigma\Xi'} = 1.75 \text{ GeV},
\]

(27)

where \( \varphi_m \) is any pseudoscalar meson, \( \bar{\psi} \) is a baryons belonging to baryon multiplet \( (\mu) \) with negative parity, and \( \psi \) is a baryons belonging to baryon multiplet \( (\mu') \) with positive parity. In this paper, we will only deal with an anti-decuplet with negative parity. Thus we get the widths of anti-decuplet baryons in Table 1 by using this formula and identifying \( \Theta^+ \), \( N(1650) \), \( \Sigma(1750) \), and \( \Xi(1862) \) as the members of the anti-decuplet, and the results calculated in the case of positive parity from chiral soliton models are still list in Table 1.

Table 1. The widths of baryons in the anti-decuplet

| Mode                     | estimation(MeV) | \( J^P = \frac{1}{2}^- \) (MeV) | \( J^P = \frac{1}{2}^+ \) (MeV) |
|--------------------------|-----------------|---------------------------------|---------------------------------|
| \( \Theta^+ \to K\pi \) | < 25            | 25(input)                       | 25(input)                       |
| \( N(1650) \to N\pi \)   | 80 ~ 171        | 12                              | 25                              |
| \( N(1650) \to N\eta \)  | 4 ~ 19          | 8                               | 8                               |
| \( N(1650) \to \Lambda K \) | 4 ~ 21        | 4                               | 1.5                             |
| \( \Sigma(1750) \to \bar{N}\bar{K} \) | 6 ~ 64        | 7                               | 3                               |
| \( \Sigma(1750) \to \Sigma\pi \) | < 12.8        | 8                               | 14                              |
| \( \Sigma(1750) \to \Sigma\eta \) | 9 ~ 88        | 2                               | 0.2                             |
| \( \Sigma(1750) \to \Lambda\pi \)  | seen          | 12                              | 28                              |
| \( \Xi_{3/2} \to \Xi\pi \)   | <18(?)         | 24                              | 46                              |
| \( \Xi_{3/2} \to \Sigma K \)  |                | 18                              | 15                              |
In summary, we recalculated the masses and widths of the baryons in the anti-decuplet. The calculated results show that if we accept $\Theta^+$ and the reported particle $\Xi(1862)$ in $\Xi^0$ as members of the anti-decuplet, we will get the mass of $N_{\Xi^0}$ around 1.65 GeV and the mass of $\Sigma_{\Xi^0}$ around 1.75 GeV. From available baryon listings [24], we find that there are only $N(1650)$ and $\Sigma(1750)$ with negative parity. And inspired by this, we give a width formula for the negative parity baryon decay on symmetry consideration and give all the widths of baryons of an anti-decuplet with negative parity but the same masses as calculated from chiral soliton models (Table 1). From Table 1, The width of $\Sigma(1750)$ in the case of $J^P = \frac{1}{2}^-$ fits experimental data better, but for $N(1650)$, there is an obvious deviation in the process $N(1650) \rightarrow N\pi$ while the widths agree with other two decay channels $N(1650) \rightarrow N\eta$ and $N(1650) \rightarrow \Lambda K$.

There could be a possibility for a missing negative parity $N_{\Xi^0}$ with mass around 1650 MeV and a narrow width or that the SU(3) breaking plays a role. The width of $\Xi_{\Sigma^0}$ with negative parity also agrees better with experimental observation. In the quark picture of the anti-decuplet, $N_{\Xi^0}$ and $\Sigma_{\Xi^0}$ contain hidden quark-antiquark pairs, while $N(1650)$ and $\Sigma(1750)$ are the orbital angular excitations in the quark model. Thus, to determine the parity of the anti-decuplet, we need further experiments to find missing non-exotic members and also to determine their parities.

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