Optical properties of biased bilayer graphene
due to gap parameter effects

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Abstract

We address the optical conductivity of undoped bilayer graphene in the presence of a finite bias voltage at finite temperature. The effects of gap parameter and stacking type on optical conductivity are discussed in the context of tight binding model Hamiltonian. Green’s function approach has been implemented to find the behavior of optical conductivity of bilayer graphene within linear response theory. We have found the frequency dependence of optical conductivity for different values of gap parameter and bias voltage. Also the dependence of optical conductivity on the temperature has been investigated in details. A peak appears in the plot of optical conductivity versus frequency for different values of temperatures and bias voltage. Furthermore we find the frequency position of broad peak in optical conductivity goes to higher values with increase of gap parameter for both bernal and simple stacked bilayer graphenes.

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1 Introduction

The isolation of monolayer graphene has generated a lot of interesting topics for the physics community. On the one hand, the electronic spectrum of graphene can be described by the two dimensional Dirac equation.\[\text{[1, 2, 3]}\]. However, the gapless electron spectrum of monolayer graphene makes it difficult to turn off the electrical current due to tunneling. Bilayer graphene, on the other hand, can provide a finite band gap up to hundreds of meV, when the inversion symmetry between top and bottom layers is broken by an applied perpendicular electric field\[\text{[4, 5]}\]. This system is composed of two layers, each of which has carbon atoms arranged in a honeycomb lattice with two sublattices. Each layer can be considered by strong interlayer hopping integral between two different sublattices sites in the two layers. One can consider a simple model for bilayer graphene spectrum is a pair of chiral parabolic electron and hole bands touching each other at the Dirac point.

In contrast to the case of single-layer graphene (SLG) low energy excitations of the bilayer graphene have parabolic spectrum, although, the chiral form of the effective 2-band hamiltonian persists because the sublattice pseudospin is still a relevant degree of freedom. The pristine or undoped bilayer graphene has attracted a lot of interest since the presence of a single Fermi point and quadratic dispersion can lead to a host of exotic phenomena \[\text{[6, 7]}\].

The low energy approximation in bilayer graphene is valid only for small doping \(n < 10^{12} \text{cm}^{-2}\), while experimentally doping can obtain 10 times larger densities. For such a large doping, the 4-band model\[\text{[8]}\] should be used instead of the low energy effective 2-band model.
Furthermore, an electronic band gap can be introduced in a dual gate bilayer graphene \cite{8,9}, and it makes BLG very appealing from the point of view of applications. It was shown theoretically \cite{4,8} and demonstrated experimentally \cite{10,9} that a graphene bilayer is the only material with semiconducting properties that can be controlled by electric field effect \cite{5}.

The optical properties of graphene are of considerable importance for technological applications and all variants of graphene are also of potential interest and should be examined. The dynamical conductivity of graphene has been extensively studied theoretically \cite{11} and experiments have largely verified the expected behavior \cite{12}. The conductivity for Bernal-stacked bilayer graphene has been studied theoretically \cite{13} and observed \cite{14}. Some preliminary work on the absorption coefficient of undoped AA-stacked graphene in zero magnetic field has been reported \cite{15} however most materials naturally occur with charge doping where the Fermi level is away from charge neutrality. Furthermore, the interesting feature for practical applications is the variation of optical properties with doping, usually achieved through a field effect transistor structure. The optical conductivity of a few-layer epitaxial graphite \cite{16} and oriented pyrolytic graphite \cite{17} in finite external magnetic field has been reported recently, as well as for graphene \cite{18}. There have also been theoretical studies \cite{19,20} of the conductivity, including discussions of optical sum rules \cite{21} which continue to provide useful information on the electron dynamics. In the other theoretical work, frequency dependence of dynamical conductivity of AA-stacked bilayer graphene in the presence of electron doping has been calculated within Dirac approximation \cite{22}. For in-plane response, AA-stacked graphene shows a Drude weight at charge neutrality along with Pauli blocking at low frequencies below the onset of a flat interband absorption.
In an experimental work the effects of both bias voltage and impurity doping on Raman spectrum and electrical resistivity have been studied\[23\]. In this experimental study the transport results presents a variable range hopping conduction near the charge neutrality point at low temperatures. Such results provide evidence for the impurity level inside the gap. Also magnetoresistance of biased bilayer graphene as a function of magnetic field has been studied. The electron-impurity scattering rate as a function of quasi particle energy for doped bilayer graphene has been theoretically calculated\[24\]. The results show that scattering rate for bilayer graphene enhance on increasing impurity concentrations. Also electron impurity scattering rate of bilayer graphene goes to a constant value for zero limit of quasi particle energy. The other theoretical works study the impurity levels and the effects of impurity doping on the local density of states in the gapped doped bilayer graphene\[25, 26, 27, 28\].

In this paper, we study the effects of bias voltage and energy gap on the optical conductivity of both simple and bernal stacked undoped graphene bilayer within full band tight binding approximation. Full band calculation beyond Dirac approximation has been implemented to derive in-plane optical conductivity spectra. We have exploited Green’s function approach to calculate the optical conductivity, i.e. the time ordered electrical current correlation. The effects of energy gap, bias voltage and stacking types on the frequency dependence of optical conductivity of bilayer graphene have been studied. Also we discuss and analyze to show how bias voltage value affects the frequency dependence of the optical conductivity. We study the optical spectra of bilayer graphene along zigzag direction for both types of bilayer graphene versus photon frequency. Finally we have compared the behavior of optical conductivity of bilayer graphene for two different stacking types in details.
2 Theoretical Model and Method

To calculate the optical conductivity of bilayer graphene we consider bilayer graphene composed of two graphene single layers arranged in both cases of the simple (AA) and Bernal (AB) stacking [29]. This lattice structure of each graphene layer has been shown in Fig.(1). The unit cell vectors of bilayer graphene is similar to those of single layer one. The primitive unit cell vectors are given by

\[ a_1 = \frac{\sqrt{3}}{2}i + \frac{j}{2}, \quad a_2 = \frac{\sqrt{3}}{2}i - \frac{j}{2}, \]  

(1)

where \( i \) and \( j \) are unit cell vectors along zigzag and armchair directions, respectively. Also the length of unit cell vectors is considered to be one. In order to obtain optical properties of bilayer graphene we must first examine the band structure and provide an expression for the electronic Green’s function. We start from a tight binding model incorporating nearest neighboring intralayer and interlayer hopping terms. An on-site potential energy difference between two layers is included to model the effect of an external voltage. Also a finite difference between on-site energies of two different sublattice atoms of honeycomb structure has been applied in the model Hamiltonian. For the case of AA-stacking, an A (B) atom in the upper layer is stacked directly above A(B) atom in the lower layer. The spin independent tight binding model hamiltonians in the nearest neighbor approximation for AA-stacked bilayer graphene \((H_{AA})\) and AB one \((H_{AB})\) are given by

\[
H_{AA} = -t \sum_{l=1,2} \sum_{i,\delta} (b_{l,i+\delta}^\dagger a_{l,i} + h.c.) - \gamma \sum_i a_{1,i}^\dagger a_{2,i} + h.c. - \gamma \sum_i b_{1,i}^\dagger b_{2,i} + h.c. \\
+ \frac{V}{2} \sum_i (a_{1,i}^\dagger a_{1,i} + b_{1,i}^\dagger b_{1,i}) - \frac{V}{2} \sum_i (a_{2,i}^\dagger a_{2,i} + b_{2,i}^\dagger b_{2,i}) + \Delta \sum_i (a_{1,i}^\dagger a_{1,i} + a_{2,i}^\dagger a_{2,i}) \\
- \Delta \sum_i (b_{1,i}^\dagger b_{1,i} + b_{2,i}^\dagger b_{2,i}),
\]
\[
H_{AB} = -t \sum_{l=1,2} \sum_{i,\delta} (b_{l+\delta,i}^\dagger a_{l,i} + \text{h.c.}) - \gamma \sum_i a_{1,i}^\dagger b_{2,i} + \text{h.c.} \\
+ \frac{V}{2} \sum_i (a_{1,i}^\dagger a_{1,i} + b_{1,i}^\dagger b_{1,i}) - \frac{V}{2} \sum_i (a_{2,i}^\dagger a_{2,i} + b_{2,i}^\dagger b_{2,i}) + \Delta \sum_i (a_{1,i}^\dagger a_{1,i} + a_{2,i}^\dagger a_{2,i}) \\
- \Delta \sum_i (b_{1,i}^\dagger b_{1,i} + b_{2,i}^\dagger b_{2,i}).
\]  

(2)

In Eq.(2), \(a_{l,i}(b_{l,i})\) denotes the annihilation operator for an electron which is on an A(B)-atom site with unit cell label \(i\) in the graphene layer indexed by \(l\). \(\delta\) is one of three vectors that connects the unit cells of nearest-neighbor lattice sites and given by \(\delta = 0, a_1, a_2\) according to Fig.(1). \(t \approx 2.6\text{eV}\) is the nearest neighbour intralayer hopping terms for electrons to move within a given plane. The third term in Eq.(2) corresponds to the interlayer hopping between graphene sheets with amplitude \(\gamma\). In bilayer graphene, we have \(\gamma \approx 0.2\text{eV}\) \([15, 30]\). \(V\) refers to the potential energy difference between the first and second layers induced by a bias voltage.

The following Fourier transformations for fermionic operators \(a_{l,i}^\dagger, b_{l,i}^\dagger\) have been given by

\[
a_{l,i}^\dagger = \frac{1}{\sqrt{N}} \sum_k e^{-i\mathbf{k} \cdot \mathbf{R}_i} a_{l,i}^\dagger, \\
b_{l,i}^\dagger = \frac{1}{\sqrt{N}} \sum_k e^{-i\mathbf{k} \cdot \mathbf{R}_i} b_{l,i}^\dagger,
\]

(3)

where \(N\) is the number of unit cells and \(\mathbf{k}\) is wave vector belonging to the first Brillouin zone of honeycomb structure. \(\mathbf{R}_i\) introduces the position vector of \(i\) th unit cell in graphene layer.

In terms of Fourier transformation of operators, one can rewrite the clean tight binding part of the Hamiltonians in Eq.(2) as

\[
H_{AA} = \sum_k \phi_k^\dagger H_{AA}(k) \phi_k, \\
H_{AB} = \sum_k \phi_k^\dagger H_{AB}(k) \phi_k,
\]

(4)
in which the vector of fermion creation operators is defined as \( \phi_k^\dagger = (a_{1,k}^\dagger, b_{2,k}^\dagger, a_{2,k}^\dagger, b_{1,k}^\dagger) \). The nearest neighbor approximation gives us the following matrix forms for \( H_{AA}(k) \) and \( H_{AB}(k) \) as

\[
H_{AA}(k) = \begin{pmatrix}
\Delta + V/2 & 0 & \gamma & f(k) \\
0 & -\Delta - V/2 & f^*(k) & \gamma \\
\gamma & f(k) & \Delta - V/2 & 0 \\
f^*(k) & \gamma & 0 & -\Delta + V/2
\end{pmatrix},
\]

\[
H_{AB}(k) = \begin{pmatrix}
\Delta + V/2 + g(k) & 0 & 0 & f(k) \\
0 & -\Delta - V/2 & f^*(k) & 0 \\
0 & f(k) & \Delta - V/2 & \gamma \\
f^*(k) & 0 & \gamma & -\Delta + V/2
\end{pmatrix}.
\] (5)

Using the definition for \( \delta \), the function \( f(k) \) are expressed based on the following relations

\[
f(k) = -t \left( 1 + 2 \cos \left( \frac{k_y}{2} \right) \exp \left( -i \frac{\sqrt{3}}{2} k_x \right) \right).
\] (6)

After diagonalizing of the Hamiltonians in Eq.(5), the band structures for simple stacked bilayer graphene are given by following eigenvalues

\[
E_1(k) = \sqrt{\Delta^2 + \frac{V^2}{4} + \gamma^2 + |f(k)|^2 + 2 \sqrt{\left( \frac{V^2}{4} + \gamma^2 \right) \left( |f(k)|^2 + \Delta^2 \right)}},
\]

\[
E_2(k) = \sqrt{\Delta^2 + \frac{V^2}{4} + \gamma^2 + |f(k)|^2 - 2 \sqrt{\left( \frac{V^2}{4} + \gamma^2 \right) \left( |f(k)|^2 + \Delta^2 \right)}},
\]

\[
E_3(k) = -\sqrt{\Delta^2 + \frac{V^2}{4} + \gamma^2 + |f(k)|^2 + 2 \sqrt{\left( \frac{V^2}{4} + \gamma^2 \right) \left( |f(k)|^2 + \Delta^2 \right)}},
\]

\[
E_4(k) = -\sqrt{\Delta^2 + \frac{V^2}{4} + \gamma^2 + |f(k)|^2 - 2 \sqrt{\left( \frac{V^2}{4} + \gamma^2 \right) \left( |f(k)|^2 + \Delta^2 \right)}}.
\] (7)
Also the band structure for AB stacked bilayer graphene in the presence of bias voltage, next nearest neighbor hopping and gap parameter is readily obtained as following expression

\[
E_1(k) = \sqrt{\Delta^2 + \frac{V^2}{4} + \frac{\gamma^2}{2} + |f(k)|^2 + \sqrt{\frac{\gamma^4}{4} + \gamma^2 \left(|f(k)|^2 - V\Delta\right)}} + V^2(|f(k)|^2 + \Delta^2),
\]

\[
E_2(k) = \sqrt{\Delta^2 + \frac{V^2}{4} + \frac{\gamma^2}{2} + |f(k)|^2 - \sqrt{\frac{\gamma^4}{4} + \gamma^2 \left(|f(k)|^2 - V\Delta\right)}} + V^2(|f(k)|^2 + \Delta^2),
\]

\[
E_3(k) = -\sqrt{\Delta^2 + \frac{V^2}{4} + \frac{\gamma^2}{2} + |f(k)|^2 + \sqrt{\frac{\gamma^4}{4} + \gamma^2 \left(|f(k)|^2 - V\Delta\right)}} + V^2(|f(k)|^2 + \Delta^2),
\]

\[
E_4(k) = -\sqrt{\Delta^2 + \frac{V^2}{4} + \frac{\gamma^2}{2} + |f(k)|^2 - \sqrt{\frac{\gamma^4}{4} + \gamma^2 \left(|f(k)|^2 - V\Delta\right)}} + V^2(|f(k)|^2 + \Delta^2) \quad (8)
\]

Using band energy spectrum in Eq.(7), the Hamiltonian in Eq.(4) can be rewritten by

\[
H = \sum_{k_x,p,\eta=1,2,3,4} E_\eta(k)c_{\eta,k}^\dagger c_{\eta,k}. \quad (9)
\]

The electronic Green’s function can be defined using the Hamiltonian in Eq.(9) as following expression

\[
G_\eta(k, \tau) = -\langle T_\tau c_{\eta,k}(\tau)c_{\eta,k}^\dagger(0)\rangle, \quad (10)
\]

where \(\tau\) is imaginary time. Using the model Hamiltonian in Eq.(9), the Fourier transformations of Green’s function is given by

\[
G_\eta(k, i\omega_n) = \int_0^{1/k_BT} d\tau e^{i\omega_n \tau} G_\eta(k, \tau) = \frac{1}{i\omega_n - E_\eta(k)}. \quad (11)
\]

Here \(\omega_n = (2n + 1)\pi k_BT\) denotes the fermionic Matsubara frequency in which \(T\) is equilibrium temperature. In the following, the relation of the real part of in-plane optical conductivity of AA stacked bilayer graphene is presented using Green’s function method[31]. When bilayer graphene is excited by the electromagnetic field with polarization of electric field along \(x\) direction presented in Fig.(1), an Hamiltonian term as \(\frac{e}{mc} A_p\) is added to original Hamiltonian.
\[ H = H_0 + H_{\text{imp}}. \]

The optical conductivity, as dynamical correlation function between electrical current operators, for single band model Hamiltonian \cite{32} have been obtained in terms of Green’s function using Wick’s theorem. One can generalize this result for multiband model Hamiltonian and the optical conductivity of bilayer graphene along \( x \) zigzag direction are related Green’s function as

\[
\sigma(\omega) = \frac{e^2 k_B T}{4N} \sum_{k, \eta} \left( \frac{\partial E_\eta(k)}{\partial k_x} \right)^2 \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \frac{n_F(\epsilon + \omega) - n_F(\epsilon)}{\omega} \left( 2 \text{Im}G_{\eta}(k, i\omega_n \rightarrow \epsilon + i0^+) \right)^2, \tag{12}
\]

where \( n_F(x) = \frac{1}{e^{x/k_BT} + 1} \) is the Fermi-Dirac distribution function and \( T \) denotes the equilibrium temperature. Substituting electronic Green’s function into Eq.(12) and performing the numerical integration over wave vector through first Brillouin zone, the results of optical absorption have been obtained. Here, the contribution of both inter and intra band transitions on the optical conductivity in Eq.(12) has been considered.

3 Results and conclusions

We have obtained the optical conductivity of the gapped biased undoped bilayer graphene for both stacking cases for polarization of electromagnetic wave along \( x \) direction as shown in Fig.(1). We have implemented a tight binding model Hamiltonian including gap parameter and bias voltage. We have obtained the optical spectrum of this structure by means of Green’s function approach so that the optical conductivity has been obtained by calculating the electrical current correlation function. Using the band structures in Eqs.(7,8), the electronic Green’s function is found based on Eq.(11) and afterwards the photon frequency dependence of optical absorption is obtained by Eq.(12). In the obtaining the following numerical results, the
intralayer nearest neighbor hopping parameter \( t \) is set to 1. Therefore the other parameters in the model Hamiltonian are expressed as \( \gamma/t, V/t \).

The optical absorption of gapped simple stacked bilayer graphene structure as a function of normalized frequency \( \omega/t \) for different values of gap parameters, namely \( \Delta/t = 0.3, 0.6, 0.9, 1.2 \) for half filling case \( \mu/t = 0 \) are shown in Fig.(2). For gap parameter cases \( \Delta/t = 0.3, 0.6 \), optical absorption \( \sigma(\omega) \) displays a Drude response or ballistic transport [33] at zero frequency limit which arises from intraband transitions. Such infinite conductivity originates from classical motion of electrons under long wave length of photons. This Drude response has been predicted under Dirac approximation for gapless graphene [34]. Upon more increasing gap parameter, i.e. \( \Delta/t = 0.9, 1.2 \), the band gap width in density of states increases so that Drude weight coefficient in optical conductivity goes to zero as shown in Fig.(2). Insulating behavior in bilayer graphene becomes more remarkable with increasing gap parameter and consequently the Drude weight vanishes for normalized gap parameters 0.9,1.2. Interband transition contributes to the optical conductivity at higher frequencies with increasing gap parameter. It can be understood from this fact that the interband transition requires photons with more frequencies when the band gap in density of states increases. In other words, the characteristic frequency position of peak in optical absorption tends to higher frequencies with gap parameter \( \Delta \) according to Fig.(2). Upon increasing frequency above the peak position, the optical conductivity reduces so that it vanishes at \( \omega/t \approx 7.0 \) for all values of gap parameter.

In Fig.(3), we have plotted optical absorption of undoped AA stacked bilayer graphene as a function of frequency for the various normalized temperature, namely \( k_B T/t = 0.04, 0.05, 0.063, 0.07, 0.075 \) for bias voltage \( V/t = 0.0 \) at \( \Delta/t = 0.0 \). Due to increase of transition rate of electrons with tem-
perature, an increasing behavior for $\sigma(\omega)$ is clearly observed on the whole range of frequency. Both inter and intraband transition rates raise with temperature so that the Drude weight and height of peak increase in optical spectrum of bilayer graphene in Fig.(3). The frequency position of peaks in optical absorption is independent of temperature and is around $\omega/t \approx 2.5$.

Also the frequency dependence of optical spectrum goes to zero in the frequency region above normalized value 7.0 for all values of temperature.

In Fig.(4), we present in-plane optical conductivity of the undoped AA stacked bilayer graphene versus normalized frequency ($\omega/t$), for different values of bias voltage, namely $V/t = 0.0, 0.5, 1.0, 1.3$ for fixed temperature $k_B T/t = 0.06$ and $\Delta/t = 0.0$. For high values of bias voltages $V/t = 1.0, 1.3$, the dynamical conductivity displays a Drude weight at low frequency due to intraband transitions. However the optical conductivity has no Drude weight for lower bias voltages $V/t = 0.0, 0.5$. The emergence of Drude weight has been predicted by using Dirac cone approximation [22]. The optical conductivity curves in Fig.(4) have a broad peak at finite non zero frequency for all values of bias voltage. Such peak arises from interband electronic transition so that the frequency position of peaks is around $\omega/t \approx 2.75$. However the height of peaks reduces with bias voltage as shown in Fig.(4). At fixed frequencies below normalized value 2.0, the optical conductivity decreases with bias voltage. For bias voltage values $V/t = 0.0, 0.5$, the optical absorption gets the zero values in frequency region $0 < \omega/t < 0.75$. In fact electromagnetic wave is not absorbed by bilayer structure in this frequency region for $V/t = 0.0, 0.5$.

We have also studied the optical conductivity of AB stacked bilayer graphene. In Fig.(5), we have plotted the frequency dependence of optical conductivity of gapped bernal stacked
bilayer graphene structure as a function of normalized frequency $\omega/t$ for different values of gap parameters for half filling case in the absence of bias voltage. For gap parameter cases $\Delta/t = 0.3$, optical absorption $\sigma(\omega)$ displays a Drude response or ballistic transport \[33\] at zero frequency limit which arises from intraband transitions. Upon more increasing gap parameter, i.e. $\Delta/t = 0.6, 0.9, 1.2$, the band gap width in density of states increases so that Drude weight coefficient in optical conductivity goes to zero as shown in Fig.(5). Insulating behavior in bilayer graphene becomes more remarkable with increasing gap parameter and consequently the Drude weight vanishes for normalized gap parameters 0.6, 0.9 and 1.2. Interband transition contributes to the optical conductivity at higher frequencies with increasing gap parameter. It can be understood from this fact that the interband transition requires photons with more frequencies when the band gap in density of states increases. In other words, the characteristic frequency position of optical absorption tends to higher frequencies with gap parameter $\Delta$ according to Fig.(5). Upon increasing frequency above the peak position, the optical conductivity reduces so that it vanishes at $\omega/t \approx 7.0$ for all values of gap parameter. The comparison between Figs.(2,5) shows the Drude weight for optical conductivities of simple stacked bilayer graphene is higher than that of bernal bilayer graphene at gap parameter $\Delta/t = 0.3$

The optical absorption of undoped unbiased AB stacked bilayer graphene as a function of frequency for the various normalized temperature has been plotted in Fig.(6). The gap parameter $\Delta$ has been assumed to be zero. Due to increase of transition rate of electrons with temperature, an increasing behavior for $\sigma(\omega)$ is clearly observed on the whole range of frequency. Both inter and intraband transition rates raises with temperature so that the Drude weight and height of peak increase in optical spectrum of bilayer graphene in Fig.(6). Also the
frequency dependence of optical spectrum goes to zero in the frequency region above normalized value 7.0 for all values of temperature.

In Fig.(7), we present in-plane optical conductivity of the undoped AB stacked bilayer graphene versus normalized frequency \((\omega/t)\), for different values of bias voltage for fixed temperature \(k_B T/t = 0.06\) and \(\Delta/t = 0.0\). For high values of bias voltages \(V/t = 1.0, 1.3\), the dynamical conductivity displays a Drude weight at low frequency due to intraband transitions. However the optical conductivity has no Drude weight for lower bias voltages \(V/t = 0.0, 0.5\). The optical conductivity curves in Fig.(7) have a broad peak at finite non zero frequency for all values of bias voltage. Such peak arises from interband electronic transition so that the frequency position of peaks moves to higher frequencies with bias voltage. Moreover the height of peaks reduces with bias voltage as shown in Fig.(7). At fixed frequencies below normalized value 1.5, the optical conductivity decreases with bias voltage. For bias voltage values \(V/t = 0.0, 0.5\), the optical absorption gets the zero values in frequency region \(0 < \omega/t < 0.5\). In fact electromagnetic wave is not absorbed by bilayer structure in this frequency region for \(V/t = 0.0, 0.5\).

Finally we have compared the behaviors of optical conductivities of simple and bernal stacked bilayer graphenes at fixed gap parameter \(\Delta/t = 0.9\) for \(V/t = 1.0\) at \(k_B T/t = 0.07\). The frequency position of broad peak in optical conductivity of bernal stacking locates at higher frequency rather than that in optical spectrum of simple stacked bilayer graphene. Moreover the height of peak in optical spectrum of bernal stacked bilayer graphene is more than that of simple stacking case. Also the Drude weight in optical spectrum of simple stacked case gets higher value in comparison with bernal stacked bilayer graphene case.
The optical conductivity of bilayer graphene is in agreement with the theoretical study of optical conductivity of bilayer graphene under Dirac cone approximation \cite{22}. The Drude weight in optical conductivity of bilayer graphene has been obtained in our results in agreement with theoretical work \cite{22}. The effects of inter and intrasite interactions between electrons on the physical properties of the system such as excitonic corrections of optical properties can be studied using GW method. However the study of electronic interactions in bilayer graphene is out of scope of the present manuscript.

4 Conclusions

In summary, we have studied the optical properties of bilayer graphene in the presence of bias voltage and gap parameter. The effects of stacking type on the frequency dependence of dynamical conductivity have been studied. Also we have investigated the effects of temperature on the conductivity. The Drude weight due to intraband transition vanishes upon increasing gap parameter. However the increase of temperature increases the Drude weight for stacking types of bilayer graphene. The comparison between simple and bernal bilayer graphene shows the Drude weight in AA stacked bilayer graphene is higher than the Drude weight in AB one.

References

[1] K. S. Novoselov, A. K. Geim, S. V. Morosov, D. Jiang, Y. Zhang, S. V. Dubonos, I. V. Grigorieva and A. A. Firsov, Electric Field Effect in Atomically Thin Carbon Films, Science 306, 666 (2004)
Figure 1: Crystal structure of honeycomb lattice with two different sublattices. $\mathbf{a}_1$ and $\mathbf{a}_2$ are the primitive unit cell vectors.
Figure 2: Optical conductivity ($\sigma(\omega)$) of unbiased AA stacked bilayer graphene as a function of normalized frequency $\omega/t$ for various amounts of gap parameter $\Delta/t_\parallel$ for fixed temperature $k_B T/t_\parallel = 0.06$ at zero value for chemical potential.
Figure 3: Optical conductivity ($\sigma(\omega)$) of unbiased AA stacked bilayer graphene as a function of normalized frequency $\omega/t$ for various amounts of normalized temperature $k_B T/t_\parallel$ at fixed zero gap parameter. The chemical potential has been assumed to be zero.
Figure 4: Optical conductivity ($\sigma(\omega)$) of biased undoped AA stacked bilayer graphene as a function of normalized frequency $\omega/t$ for various amounts of normalized bias voltage $V/t_\parallel$ at fixed zero gap parameter. The normalized temperature has been assumed to be $k_B T/t = 0.06$. 
Figure 5: Optical conductivity ($\sigma(\omega)$) of unbiased AB stacked bilayer graphene as a function of normalized frequency $\omega/t$ for various amounts of gap parameter $\Delta/t_{||}$ for fixed temperature $k_B T/t_{||} = 0.06$ at zero value for chemical potential.
Figure 6: Optical conductivity ($\sigma(\omega)$) of unbiased AB stacked bilayer graphene as a function of normalized frequency $\omega/t$ for various amounts of normalized temperature $k_B T/t$ at fixed zero gap parameter. The chemical potential has been assumed to be zero.
Figure 7: Optical conductivity ($\sigma(\omega)$) of biased undoped AB stacked bilayer graphene as a function of normalized frequency $\omega/t$ for various amounts of normalized bias voltage $V/t_\parallel$ at fixed zero gap parameter. The normalized temperature has been assumed to be $k_B T/t = 0.06$. 
Figure 8: The comparison between optical conductivities ($\sigma(\omega)$) of simple (AA) and bernal (AB) stacked bilayer graphene as a function of normalized frequency $\omega/t$ for $k_B T/t = 0.07$. The normalized bias voltage has been fixed at $V/t_\parallel = 1$. The normalized gap parameter has been assumed to be $\Delta/t = 0.9$. 
[2] K. S. Novoselov, A. K. Geim, S. V. Morosov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, A. A. Firsov, Two-dimensional gas of massless Dirac fermions in graphene, Nature 438, 197 (2005)

[3] A. H. Castro Neto, F. Guinea, n. M. R. Peres, K. S. Novoselov and A. K. Geim, The electronic properties of graphene, Rev. Mod. Phys 81, 109 (2009)

[4] E. McCann, Asymmetry gap in the electronic band structure of bilayer graphene, Phys. Rev. B 74, 161403 (2006)

[5] H. Min, B. Sahu, S. K. Banerjee, and A. H. MacDonald, Ab initio theory of gate induced gaps in graphene bilayers, Phys. Rev. B 75, 155115 (2007)

[6] Y. Barlas and K. Yang, Many-body instability of Coulomb interacting bilayer graphene: Renormalization group approach, Phys. Rev. B 80, 161408(R) (2009)

[7] R. Nandkishore and L. Levitiv, Dynamical Screening and Excitonic Instability in Bilayer Graphene, Phys. Rev. Lett 104, 156803 (2010)

[8] Ed. McCann and V. I. Falko, Landau-Level Degeneracy and Quantum Hall Effect in a Graphite Bilayer, Phys. Rev. Lett 96, 086805 (2006)

[9] E. V. Castro, K. S. Novoselov, S. V. Morosov, N. M. R. Peres, J. M. B. Lopes dos Santos, J. Nilsson , F. Guinea, A. K. Geim, and A. H. Castro Neto, Biased Bilayer Graphene: Semiconductor with a Gap Tunable by the Electric Field Effect, Phys. Rev. Lett 99, 216802 (2007)
[10] J. B. Oostinga, H. B. Heersche, X. Liu, A. F. Morpurgo and L. M. K. Vandersypen, 
Dynamical Thermal Conductivity of Doped AA-Stacked Bilayer Graphene in the Presence 
of Bias Voltage, Nature 38, 2082 (2007)

[11] T. Ando, Y. Zhang, and H. Suzuura, Dynamical Conductivity and Zero-Mode Anomaly 
in Honeycomb Lattices, J. Phys. Soc. Jpn 71, 1318 (2002)

[12] R. R. Nair, B. Blake, A. N. Grigorenko, K. S. Novoselov, T. J. Booth, T. Stauber, N. 
M. R. Peres, and A. K. Geim, Fine Structure Constant Defines Visual Transparency of 
Graphene, Science 320, 1308 (2008)

[13] D. S. L. Abergal and V. I. Falko, Optical conductivity of bilayer graphene with and without 
an asymmetry gap, Phys. Rev. B 77 155409 (2008)

[14] L. M. Zhang, Z. Q. Li, D. N. Basov, M. M. Fogler, Z. Hao, and M. C. Martin, Determination 
of the electronic structure of bilayer graphene from infrared spectroscopy, Phys. Rev. B 
78, 235408 (2008)

[15] Y. Xu, X. Li, and J. Dong, Infrared and Raman spectra of AA-stacking bilayer graphene, 
Nanotechnology 21, 065711 (2010)

[16] M. L. Sadowski, G. Martinez, M. Potemski, C. Berger, and W. Ade Heer, Landau Level 
Spectroscopy of Ultrathin Graphite Layers, Phys. Rev. Lett 97, 266405(2006)

[17] Z. Q. Li, S. W. Tasi, W. J. Padilla, S. V. Dordvic, K. S. Burch, Y. J. Wang, and D. 
N. Basov, Infrared probe of the anomalous magnetotransport of highly oriented pyrolytic 
graphite in the extreme quantum limit, Phys. Rev. B 74, 195404 (2006)
[18] Z. Jiang, E. A. Henriksen, L. C. Tung, Y.-J. Wang, M. E. Schwarts, M. Y. han, P. Kim, and H. L. Stormer, Infrared Spectroscopy of Landau Levels of Graphene, Phys. Rev. Lett 98, 197403 (2007)

[19] L. A. Falkovaky and A. A. Varlamov, Space-time dispersion of graphene conductivity, Euro. Phys. J. B 56, 281 (2007)

[20] V. P. Gusynin, S. G. Sharapov and J. P. Carbotte, Magneto-optical conductivity in graphene, J. Phys: Condense. Mtter 19, 026222(2007)

[21] V. P. Gusynin, S. G. Sharapov and J. P. Carbotte, Sum rules for the optical and Hall conductivity in graphene, Phys. Rev. B 75, 165407 (2007)

[22] C. J. Tabert and E. J. Nicol, Dynamical conductivity of AA-stacked bilayer graphene, Phys. Rev. B 86, 075439 (2012)

[23] Q. Han, B. Yan, Z. Jia, J. Niu, D. Yu and X. Wu, Effect of impurity doping in gapped bilayer graphene, Applied Physics Letters 107, 163104 (2015)

[24] D. K. Patel and A. C. Sharma, Electron-impurity scattering rate in doped bilayer graphene, AIP Conf. Proc 1447, 973 (2012)

[25] J. Nilsson, A. H. Castro Neto, F. Guinea, and N. M. R. Peres, Electronic Properties of Graphene Multilayers, Phys. Rev. Lett 97, 266801 (2006)

[26] M. Koshino, Electronic transport in bilayer graphene, New. J. Phys 11, 095010 (2009)
[27] A. Ferreira, J. Viana-Gomes, J. Nilsson, E. R. Mucciolo, N. M. R. Peres, and A. H. Castro Neto, Unified description of the dc conductivity of monolayer and bilayer graphene at finite densities based on resonant scatterers, Phys. Rev. B 83, 165402 (2011)

[28] V. V. Mikhitaryan and E. G. Mishchenko, Localized States due to Expulsion of Resonant Impurity Levels from the Continuum in Bilayer Graphene, Phys. Rev. Lett 110, 086805 (2013)

[29] J. Nilsson, A. H. Castro Neto, F. Guinea, and N. M. R. Peres, Transmission through a biased graphene bilayer barrier, Phys. Rev. B 76, 165416 (2007)

[30] I. Lobato and B. Partoens, Multiple Dirac particles in AA-stacked graphite and multilayers of graphene, Phys. Rev. B 83, 165429 (2011)

[31] G. D. Mahan, Many Particle Physics, Plenum Press, New York, 1993

[32] I. Paul and G. Kotliar, Thermal transport for many-body tight-binding models, Phys. Rev. B 67, 115131 (2003)

[33] G. Grosso and G. P. Parravicini, Solid state Physics, Academic Press, Singapour, 2000

[34] T. Stauber and N. M. R. Peres, Effect of Holstein phonons on the electronic properties of graphene, J. Phys.: Condens. Matter 20, 055002 (2008)