Top Decay in
Topcolor-Assisted Technicolor

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Abstract

Conventional technicolor models with light charged technipions ($\pi_T^\pm$) lead to an unacceptably large contribution to $t \to \pi_T^+ b$ decay rate. Topcolor-assisted technicolor models also have additional PGBs called top-pions ($\pi_t^\pm$) which may contribute to this decay. We study the potentially dangerous mixing of charged top-pion and technipions in toy models of ‘natural’ topcolor-assisted technicolor. We find that the $t \to \pi_t^+ T b$ decay rate in such models can be within experimental limits due to a combination of heavy top-pion and small $\pi_t - \pi_T$ mixing.
A natural, dynamical explanation for electroweak and flavor symmetry breaking is a desirable alternative to the Higgs sector in the standard model of electroweak interactions. In technicolor (TC) theories [1], electroweak symmetry breaking is accomplished by the chiral symmetry breaking of technifermions which transform nontrivially under a new strong and unbroken gauge interaction called technicolor. This yields the right masses of the weak gauge bosons when the characteristic energy scale of technicolor interactions is about a TeV. In order to give masses to the fermions without using fundamental scalars, one invokes an additional, spontaneously broken, gauge interaction called extended technicolor (ETC) [2], [3].

Experimental constraints from flavor changing neutral currents [3] and the value of the S parameter [4] seem to suggest that technicolor is a walking [5] gauge theory. In addition, ETC interactions seem to be inadequate to account for the extremely large top quark mass [6], [7].

Topcolor-assisted technicolor (TC2) [8] is a recent attempt to address the unsatisfactory features of the technicolor scenario. The basic idea is that of generation sensitive gauge group replication. In the simplest version of TC2, the third generation is assumed to transform with the usual quantum numbers under strong $SU(3)_1 \times U(1)_1$ while the lighter generations transform identically under a different (and weaker) group $SU(3)_2 \times U(1)_2$. At scales of about 1 TeV, $SU(3)_1 \times SU(3)_2$ and $U(1)_1 \times U(1)_2$ are spontaneously broken to ordinary color ($SU(3)$) and weak hypercharge, respectively. Electroweak symmetry breaking is still driven primarily by technicolor interactions. In addition, the topcolor interactions (felt only by the third generation quarks) with a scale near 1 TeV generate $\langle \bar{t}t \rangle$ and the very large top-quark mass. The ETC interactions are still required to generate the light fermion masses and a small but important contribution to the mass of the top quark ($m^*_t$). The reason for a nonzero $m^*_t$ is to give mass to the Goldstone bosons of t, b chiral symmetry breaking (top-pions).

As was pointed out by Chivukula, Dobrescu and Terning [9], generic TC2 models suffer from a ‘$(\rho - 1)/naturalness$’ problem, where $\rho = M^2_W / M^2_Z \cos^2 \theta_W$. In other words, it may be unnatural to have $\rho - 1$ to be within experimental limits. This is because $U(1)_1$ (like topcolor $SU(3)_1$) is expected to be strong so that the top interactions are critical while the bottom interactions are sub-critical. Then, the technifermion doublet responsible for the top and bottom ETC masses has custodial isospin violating $U(1)_1$ couplings (and unacceptably large $\rho - 1$) even when the technifermions are degenerate. A small $U(1)_1$
coupling requires the topcolor couplings to be unnaturally fine tuned for top (but not bottom) condensation.

Natural TC2 was introduced by Lane and Eichten [10] to address the $\frac{\rho - 1}{\text{natural}}$ problem. They employ two different technidoublets for bottom and top ETC masses, an additional doublet for the lighter generations, and thereby transfer the isospin violating interactions to the weak $U(1)_2$. The model has no gauge anomalies. The ETC gauge group is unspecified; instead nonrenormalizable operators allowed after imposing constraints (such as $U(1)_{1,2}$ symmetry, gauge anomaly cancellation and desired intragenerational mixing pattern) are listed. Subsequently, a more ambitious model (with colored technifermions) was developed by Lane [11] to explain topcolor breaking and the observed magnitude of generational mixing in TC2 models.

In this note, we shall discuss top decay in TC2 models. A light charged technipion in conventional ETC models can be ruled out because of the large $t \to \pi^+_T b$ decay rate. This is due to the large coupling $m_t/F_{\pi_T}$, where $F_{\pi_T} = v/\sqrt{N_D}$, $v = 246$ GeV and $N_D$ is the number of techni-doublets. The decay rate $\Gamma(t \to \pi^+_T b)$ is then given by

$$\Gamma(t \to \pi^+_T b) = \frac{1}{16\pi} \left( \frac{m_t^2 - m_{\pi_T}^2}{F_{\pi_T}^2 m_T} \right)^2,$$

where $m_{\pi_T}$ is the technipion mass. The branching ratio of top to bottom quark and W is measured to be $B(t \to W^+ b) = 0.87^{+0.13}_{-0.30}(\text{stat})^{+0.13}_{-0.11}(\text{syst})$ [12]. The standard model value for $\Gamma(t \to W^+ b)$ is 1.6 GeV with an essentially 100% branching ratio. The bounds on $F_{\pi_T}$ and $m_{\pi}$ are plotted in Figure 1 for different values of $B(t \to W^+ b)$. We see that small values of $F_{\pi_T}$ and $m_{\pi_T}$ are excluded.

However, in TC2 models, the contribution of a light technipion contribution to top decay is small since the $t - \pi_T - b$ coupling is only $m_t^{\text{ETC}}/F_{\pi_T}$ and $m_t^{\text{ETC}}/m_t$ is $0.01 - 0.1$ [8]. The top-pion, on the other hand, couples with strength $m_t^{\text{dyn}}/F_t$, which is large since $m_t^{\text{dyn}} \approx m_t$ and $F_t = 70$ GeV is the top-pion decay constant. The ETC interactions responsible for ordinary top quark mass induce mixing between top-pions and technipions. The resulting pseudo-Goldstone bosons (PGBs) can lead to an unacceptably large $t \to \pi b$ decay rate if allowed by phase space and if the top-pion component in the mixed PGB is large (see, e.g., Ref. [13]). Hence, this note will discuss the effect on top quark decay of the mixing of the charged PGBs in the top doublet sector (top-pions) and the techniflavor sector (technipions) in natural TC2 models. We carried out a detailed analysis in the toy
models of [10]. The study of top decay in these can give us an idea about what can happen in more general TC2 models.

In [10], the gauge group is $G_{TC} \times SU(3)_1 \times U(1)_1 \times SU(3)_2 \times U(1)_2 \times SU(2)_W$. There are three doublets of techniquarks: $T^l$, $T^t$ and $T^b$, where $T = (U, D)$. The three technidoublets are assumed to transform under the same complex irreducible representation of the technicolor gauge group, $G_{TC}$. They are $SU(3)_{1,2}$ singlets; for details on hypercharge assignments see [10]. Hence, the flavor symmetry group (ignoring for the moment broken $U(1)_1$ and ETC interactions) in the techniflavor sector is $SU(6)_L \times SU(6)_R$. When TC interactions become strong, this is spontaneously broken to a $SU(6)$ subgroup. The flavour symmetry in the top sector is $SU(2)_L \times U(1)$ which breaks spontaneously to $U(1)_Y$. It is the charged Goldstone bosons that are of relevance here. They obtain mass from ETC and $U(1)_1$ interactions.

The ETC gauge group is unspecified. Instead, the model is assumed to have certain ETC–generated four–fermion operators consistent with all gauge symmetries. Firstly, there are the ETC–generated two–technifermion (2T) interactions required for quark hard mass generation. These are of the following form:

\[
\mathcal{H}_{uu} = \frac{g_{ETC}^2}{M_{ETC}^2} q^l_i \gamma^\mu T^l_i \bar{U}^j R \gamma^\mu u^j_R + \text{h.c.}
\]
\[
\mathcal{H}_{dd} = \frac{g_{ETC}^2}{M_{ETC}^2} q^l_i \gamma^\mu T^l_i \bar{D}^j_R \gamma^\mu d^j_R + \text{h.c.}
\]
\[
\mathcal{H}_{tt} = \frac{g_{ETC}^2}{M_{ETC}^2} q^h_i \gamma^\mu T^l_i \bar{U}^j_R \gamma^\mu t^j_R + \text{h.c.}
\]
\[
\mathcal{H}_{bb} = \frac{g_{ETC}^2}{M_{ETC}^2} q^h_i \gamma^\mu T^l_i \bar{D}^j_R \gamma^\mu b^j_R + \text{h.c.}
\]

Here, $q^l_i = (u_i, d_i)_L$ for $i = 1, 2$ stands for the two light doublets and $q^h_i = (t, b)_L$. Also, $g_{ETC}$ and $M_{ETC}$ are generic ETC couplings and gauge boson masses. The model also has two sets of ETC–generated four–technifermion (4T) interactions corresponding to the two allowed choices of technifermion $U(1)_1$ charges called cases A and B in [10]. The 4T interactions in set A are $\mathcal{H}_{tt\bar{t}b}$, $\mathcal{H}_{tb\bar{t}b}$, $\mathcal{H}_{bb\bar{t}t}$, $\mathcal{H}_{t\bar{t}b\bar{t}}$, $\mathcal{H}_{t\bar{b}b\bar{b}}$ and $\mathcal{H}_{\text{diag}}$, where, for example,

\[
\mathcal{H}_{tt\bar{t}b} = \frac{g_{ETC}^2}{M_{ETC}^2} T^l_i \gamma^\mu T^l_i (a_u \bar{U}^j_R \gamma^\mu U^j_R + a_d \bar{D}^j_R \gamma^\mu D^j_R) + \text{h.c.}
\]
\[
\mathcal{H}_{\text{diag}} = \frac{g_{ETC}^2}{M_{ETC}^2} T^l_i \gamma^\mu T^l_i (b_u \bar{U}^j_R \gamma^\mu U^j_R + b_d \bar{D}^j_R \gamma^\mu D^j_R).
\]
The constants $a_{U,D},\ldots$ stand for unknown ETC–model–dependent factors and, in the diagonal interaction, $i,j = l,t,b$. The allowed operators in Case B are $H_{lltb}, H_{lbbt}$ and $H_{diag}$. The operators in Eqs. (1) and (2) are renormalized at scale $M_{ETC}$. In addition, there are also $4T U(1)_{1}$ operators generated by $Z'$ exchange (expected to be comparable to $4T$ ETC operators because the $U(1)_{1}$ coupling is large) which are determined by the $U(1)_{1}$ charges in the two sets (see [10]).

In the presence of the broken ETC and $U(1)_{1}$ interactions, $H'$, the spontaneously broken chiral symmetries are also explicitly broken and the Goldstone bosons (except those responsible for W and Z masses) become massive. For weak perturbations, the masses can be estimated using chiral perturbation theory [14].

We now state the values of the parameters used in our analysis. Since there are three techni-doublets in this model, $F_{\pi T} = 246 GeV/\sqrt{3}$. The value of $F_{t}$ follows from the Pagels-Stokar formula in the fermion loop approximation [13]:

$$F_{t}^{2} = \frac{16\pi^{2}m_{t,dyn}^{2}}{N_{c}} \left( \ln \frac{\Lambda_{t}}{m_{t,dyn}} + k \right)$$

Comparing with the naive dimensional analysis [16] estimate $\langle \bar{t}t \rangle = 4\pi \kappa_{t} F_{t}^{3}$, we obtain $\kappa_{t} \approx 3.5$. The technifermion condensate $\langle \bar{U}_{L}^{i}U_{R}^{j}\rangle_{\Lambda T}$ (no sum over techniflavor index $i$) is similarly estimated to be $2\pi \kappa_{T} F_{T}^{3}$, where $\kappa_{T}$ is $O(1)$. The ETC-generated top quark mass is given by

$$m_{t}^{ETC} (M_{ETC}) = \frac{g_{2}^{2}}{M_{ETC}^{2}} \langle \bar{U}^{t}U^{t}\rangle_{METC} \approx \frac{g_{2}^{2}}{M_{ETC}^{2}} \left( \frac{M_{ETC}}{\Lambda_{T}} \right) \gamma_{m} \langle \bar{U}^{t}U^{t}\rangle_{\Lambda T}$$

where $\gamma_{m}$ is the anomalous dimension of $\bar{U}^{t}U^{t}$ and expected to be close to 1 in a walking gauge theory [3]. The ETC-generated top mass renormalised at scale $\Lambda_{T}$ is given by

$$m_{t}^{ETC}(\Lambda_{T}) \approx \frac{12}{29} \left( \frac{g_{3}^{2}(\Lambda_{T})}{g_{3}^{2}(M_{ETC})} \right) \left( \frac{g_{1}^{2}(\Lambda_{T})}{g_{1}^{2}(M_{ETC})} \right)$$

For $M_{ETC} = 30$ TeV (appropriate for the third generation), $g_{2}^{2}(M_{ETC}) = 4\pi$, $g_{3}^{2}(\Lambda_{T}) = g_{3}^{2}(\Lambda_{T}) \approx 10$, $\gamma_{m} = 0.8$, and $\Lambda_{T} = 4\pi F_{\pi T} \approx 1.8$ TeV, one obtains $m_{t}^{ETC}(\Lambda_{T}) = [1]$ The ETC coupling is expected to be large in a walking gauge theory [17].
4.9\kappa T \text{ GeV. To complete this sample analysis, } \kappa T \approx 1.7 \text{ yields } m_t = m_t^{\text{dyn}} + m_t^{\text{ETC}} = 175 \text{ GeV} \text{[7].}

There are several qualitative comments we should make about the vacuum alignment studied here. First, the 4T interactions and \( \mathcal{H}_{tt} \) are the only operators of importance when studying vacuum alignment. In the extreme walking limit (\( \gamma_m = 1 \)), the vacuum energy contribution of the 4T piece is independent of the \( \LambdaTC \). Hence, if the coefficients of the 2T and 4T interactions are of the same order, the 2T contribution to the vacuum energy is smaller than the 4T piece by a factor of \( \langle \bar{t}t \rangle^{\text{ETC}}/\langle \bar{T}T \rangle^{\text{ETC}} \approx \kappa_t F_t^3 \Lambda_t / \kappa_T F_{\pi T}^3 \LambdaTC \approx 0.01 \). The mixing between top-pion and technipion is given by this ratio and so is small. In contrast, in QCD-like technicolor, the two pieces are comparable, and consequently there will be significant mixing. A crude estimate of the top-pion mass is \( M_{\pi T}^2 \approx m_t^{\text{ETC}} \langle \bar{t}t \rangle/F_t^2 \approx \kappa_t 4\pi m_t^{\text{ETC}} F_t \), i.e., \( M_{\pi T} \approx 150 \text{ GeV} \). The mass of the technipions are typically \( M_{\pi T}^2 \approx (4\pi g_{\text{ETC}} \kappa_T F_{\pi T}^2 / \LambdaTC)^2 \approx (250 \kappa_T g_{\text{ETC}})^2 \text{ GeV}^2 \). In a model with more technifermion doublets (such as in [11]), the technipions are lighter and there is likely to be more mixing between the top-pion and the technipions.

The \( t - \pi^+_t - b \) coupling is [8]

\[
\epsilon m_t^{\text{dyn}} \sqrt{2} F_t \left[ \bar{t} (1 - \gamma^5) b \pi^+_t + h.c. \right], \tag{8}
\]

where \( m_t^{\text{dyn}} \) is the dynamical top quark mass (167 GeV here) and \( \epsilon \) is the top-pion component in the normalized technipion mass-eigenstate (for instance, \( \epsilon = 1 \) in the absence of 2T ETC interactions). This modifies the decay rate of \( t \to \pi^+_t b \) to

\[
\Gamma(t \to \pi^+_t b) = \frac{|\epsilon|^2}{16\pi} \left( \frac{m_t^{\text{dyn}}}{m_t} \right)^2 \left( m_t^2 - m_{\pi T}^2 \right)^2 \frac{1}{F_t^2 m_t} \tag{9}
\]

which depends sensitively on \( m_{\pi T} \).

We now carry out a numerical analysis. The condensate matrix \( \langle T^i_L T^j_R \rangle \propto W_{ij} \), where \( W_{ij} \) is an \( SU(6) \) matrix, depends on the choice of the interactions and is determined numerically. The PGB mass-squared matrix is thus determined (using Dashen’s formula [14]) and the masses and the mixings are then obtained on diagonalization. The conservation of electric charge implies that \( W_{ij} \) is block-diagonal, i.e., \( W = W_u \oplus W_d \), where \( W_u \) and \( W_d \) are \( 3 \times 3 \) condensate matrices in the up and down sectors respectively. In the isospin symmetric case, either \( W_u = W_d \) or \( W_u = W_d^* \) (see [14]). The masses of the PGBs and their mixings are different for the two cases, but that does not affect our general conclusions.
The allowed set of 4T ETC interactions for case A are $\mathcal{H}_{ltb}$, $\mathcal{H}_{ibdt}$, $\mathcal{H}_{blit}$, $\mathcal{H}_{lilb}$, $\mathcal{H}_{iblb}$ and $\mathcal{H}_{diag}$. Isospin symmetry implies $a_U = a_D$, $b_U = b_D$ etc. We studied the patterns of vacuum alignment in the isospin limit for various values of $g_{ETC}^2/4\pi$ (chosen to be between 0.4 and 1.0) for the 4T ETC operators in case A and including $\mathcal{H}_{lt}$. The scale of the ETC interactions is taken to be 30 TeV, which is appropriate for the third generation.

The coefficients were chosen so that the vacuum was aligned non-trivially and not close to a symmetry limit. Also, there are no massless Goldstone bosons, other than the ones corresponding to the longitudinal components of $W^\pm$ and $Z^0$. We find generically that there is only one PGB with mass less than $m_t$ with a typical coupling to the top-pion of about $(0.4) m_t^{dyn}/F_t$. The branching ratio $B(t \to Wb)$ is then found to be about 0.6 (or more), which is consistent with current experimental results. Note that $\Gamma(t \to \pi^+_T b)$ is a sensitive function of $m_{\pi_t}$ and $\epsilon$. The top-pion is found to have a mass in excess of 200 GeV. Walking has been assumed (approximated here by assuming a constant $\gamma_m = 0.8$), which raises the values of the PGB masses.

The isospin symmetric interactions chosen for case B included 4T interaction $\mathcal{H}_{iblb}$, suggested by $U(1)_1$ interactions (apart from the 4T ETC interactions $\mathcal{H}_{lilb}$, $\mathcal{H}_{ibdt}$ and $\mathcal{H}_{diag}$ and $\mathcal{H}_{lt}$) so as to make vacuum alignment non-trivial and break all chiral symmetries. In this case, we find that the generic situation is that there are two light PGBs (since some $g_{U(1)}^2$ are negative) with masses less than $m_t$ and with the magnitude of $\epsilon$ typically of $\mathcal{O}(0.10)$. The top-pion is found to be heavier than the top quark. Here we find that $B(t \to Wb) \geq 0.7$, which is also consistent with the experimental result stated above. The contribution to $\Gamma(t \to \pi^+_T b)$ due to the technipion component is small as $m_t^{ETC}/m_t^{dyn} \sim 0.05$ and can hence be ignored.

In conclusion, we find that the potential problems associated with top decay into the light PGBs in the toy models of natural TC2 studied here can be resolved if technicolor is a walking gauge theory. This is because the top-pion is then generally heavier than the top quark and the top-pion does not mix significantly with lighter technipion(s) to cause an unacceptably large top decay rate. In a more elaborate model (such as in [3]), the PGBs are expected to be lighter than in the models studied here and the technipions are expected to mix significantly with the top-pion, because of the smaller value of $F_{\pi_T}$. A significantly better experimental determination of the $Br(t \to Wb)$ would severely restrict the allowed parameter space in such models.
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Figure Caption

[1] The limits on the charged technipion mass as a function of $F_\pi$ from $B(t \to W^+b)$ [12]. The curves (from left to right) correspond to $B(t \to W^+b) = 0.25, 0.5$ and $0.87$ respectively. The excluded regions lie below the curves.
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Figure 1