Analysis and Reduction of Slot Frequency Vibration of PMSMs with the Force-Harmonic Coupling Model Considering Phase Angle Characteristic

ZHIYAN GUO¹, JIAKUAN XIA¹, ZEXING LI¹ AND CHEN ZHAO²
¹School of Electrical Engineering, Shenyang University of Technology, No. 111, Shenliao West Road, Shenyang, China
²Assets Equipment Management Office, Shenyang Institute of Engineering, No. 18, Puchang Road, Shenyang, China

Corresponding author: ZHIYAN GUO (e-mail: zhiyan_guo@163.com).

This work is supported by the National Natural Science Foundation of China under Grant 52077142, the University Innovation Team Project of Liaoning Province under Grant LT2013006, and the Natural Science Foundation of Liaoning Province Education Department under Grant LJC201914.

ABSTRACT This paper establishes the force-harmonic coupling model considering phase angle characteristic for the analysis and reduction of the slot frequency force harmonic of PMSMs. Compared with the traditional analytical model, the significant influence of the force phase angle on the coupling relationship among force components is considered in the model. Firstly, the force-harmonic coupling model considering phase angle characteristic is deduced based on the Maxwell stress tensor, and considering the influence of the force phase angle on the coupling relationship among force components contributing to the slot frequency force harmonic, these force components are divided into a series of combinations of positive and negative components. Combined the finite element model, the coupling model is verified. Based on the coupling relationship, the principle of the rotor-step skewing is further explained as the mutual compensation effect of the positive and negative force components, and the slot frequency vibration is effectively offset through choosing the appropriate segment number and skewed angle. Finally, the optimized prototype is tested, and the measured results are in good agreement with the FEM results.

INDEX TERMS Slot frequency force harmonic, Force-harmonic Coupling model, PMSMs, Rotor-step skewing.

I. INTRODUCTION
The permanent-magnet synchronous machines (PMSMs) are the most attractive candidates for the use as the power sources for underwater vehicles and contribute to the performance of the power system due to its ease of fabrication, inherent high efficiency, and high efficiency [1]-[2]. However, as an essential performance for the underwater vehicles, concealment is vulnerable to low-frequency vibration and noise caused by PMSMs [3]-[4]. In the low frequency domain, the slot frequency vibration plays a significant vibroacoustic role for PMSMs, the frequency of which is equal to the multiplication of slot number and mechanical rotation frequency [5]-[6].

Some works have been done to analyze and reduce the slot frequency vibration characteristics of PMSMs [7]-[9]. In [10], the slot frequency component of the slot number order radial force was considered as the main source of the zeroth mode slot frequency vibration for integer slot PMSMs, the contribution of which was significant in the low frequency domain. In [11], based on the analytical model of radial force density considering the rotor-step skewing, the principle of the rotor-step skewing to reduce the slot frequency was explained. In [12], effect comparison of the zigzag skew PM pole and straight skew slot for slot frequency vibration was carried out by 3-D FEM and vibration experiment, which shows that the zigzag skew PM pole is better. In [13], the structure of the piecewise staggering poles with continuous skew edge was proposed, and the slot frequency vibration and pole frequency vibration were reduced. In [14], the copper ring was applied to reduce the slot frequency vibration based on the damping effect by caused the eddy current in the copper ring. In
summary, the slot frequency component of the slot number order radial force is the main source of the zeroth mode slot frequency vibration, so the slot frequency vibration is common and significant for integer slot PMSMs. In addition, some methods have been used for reducing corresponding vibration peaks.

It is well known that the electromagnetic vibration is caused by electromagnetic exciting force, and the slot frequency force harmonic is the main exciting source of the slot frequency vibration [15]. In the above literatures, the analysis methods of the slot frequency force harmonic mainly include FEM and analytical method. The FEM, with high accuracy, is used in some complex models, but the physical concept is not clear [16]-[17]. In the process of traditional analytical method, the analytical model of the radial force wave considering key parameters is established based on the Maxwell stress tensor method, which has clear physical relation [11]. However, these analytical models often only focus on the spatial-temporal distribution of the radial force wave, and the phase angle characteristic is ignored, which is closely related to the relationship among force components.

In fact, for the specific force harmonic, it is usually coupled by a series of force components, and the coupling relationship among force components plays an important role for the analysis and reduction of the force harmonic. Especially for the harmonic injection method to reduce the vibration, the accurate coupling relationship is vital [18]. However, the coupling relationship of force components is still ambiguous due to the neglect of the phase angle characteristic in above literatures, which is difficult to provide valuable reference for further research. In other words, when only analyzing the spatial-temporal distribution, the traditional model is feasible and effective, but when performing the amplitude superposition and modulation among the force waves, the phase relationship among the force wave components must be considered.

Herein, this paper establishes the force-harmonic coupling model considering the phase angle characteristic for the analysis and reduction of the slot frequency force harmonic of PMSMs. Based on the proposed coupling model, there is not only superposition effect among force components, but also compensation effect among these force components, which is different from the traditional analytical model of the radial force wave. In Section 2, the model of the prototype with 6-poles 36-slots is shown, and its corresponding basic parameters are given. In section 3, the force-harmonic coupling model considering the phase angle characteristic is deduced, and the influence of the phase angle on the relationship among force components is discussed in detail. In section 4, taking a 6-poles 36-slots PMSM as an example, the slot frequency force harmonic is analyzed with the proposed force-harmonic coupling model, and the coupling model is verified by FEM. Based on the coupling relationship, the principle of the rotor-step skewing is further explained as the mutual compensation effect of the positive components and the negative components of force waves. Finally, the prototype is tested, and the measured results are in good agreement with the FEM results. Considering the slot frequency force harmonic is caused by the magnetic field excited by PMs, the simulations and experiments are carried out on no-load. Compared the traditional analysis, the proposed force-harmonic coupling model considering the phase angle characteristic can more clearly explain the relationship between specific electromagnetic force harmonics and magnetic field harmonics, thus providing a valuable reference for the analysis and reduction of the corresponding force harmonic.

II. PROPOSED MODELING OF PMSM

The 3D structure view of the prototype investigated in this paper is shown Fig. 1, which consists of a front end cap (1 in Fig. 1), a shift (2 in Fig. 1), Poles (3 in Fig. 1), a rotor (4 in Fig. 1), a stator core (5 in Fig. 1), stator windings (6 in Fig. 1), a back end cap (7 in Fig. 1), a motor shell (8 in Fig. 1).

The simplified analysis model is shown in Fig. 2, where $\theta$ is the rotor position, and $\theta=0^\circ$ when the center line of a specified tooth coincides with the center line between the two magnetic poles. $\alpha$ is the angle of the rotation of the rotor relative to the referent position. The basic parameters of the prototype are listed in Table 1.
TABLE 1. Basic parameters of the 1.9 kW PMSM

| Specifications          | Units | Values |
|-------------------------|-------|--------|
| Rated power             | kW    | 1.9    |
| Rated speed             | r/min | 1500   |
| Rated current           | A     | 22.8   |
| Number of poles and slots |       | 6/36   |

III. THE FORCE-HARMONIC COUPLING MODEL CONSIDERING FORCE PHASE ANGLE CHARACTERISTIC

A. ANALYSIS OF AIR-GAP FLUX DENSITY

Considering the initial phase angle of each harmonic, the magneto motive force (MMF) in the air gap generated by PMs can be expressed as [19]

\[
F_r = \sum_{x} F_{\text{Re}x} \sin \left[ v_r \left( p \theta - \omega t \right) + \varphi^{x} \right]
\]

where \( p \) is the number of pole pairs, \( \theta \) is the angular position, \( t \) is the time, \( \omega \) is the angular frequency, \( \varphi^{x} \) and \( F_{\text{Re}x} \) are the initial phase angle and amplitude of the \( v_r \)-th harmonic of MMF, respectively. When \( t=0 \), the instantaneous value of each harmonic is 0. Therefore, considering the periodicity of trigonometric functions, \( \varphi^{x} = 0/\pi \). It should be emphasized that the spatial order and temporal order of MMF are the same, and \( v_r \) is defined as the temporal order in this article, the value of which can be given as

\[
v_r = 2k_r - 1 \quad (k_r = 1, 2, 3, \ldots)
\]

With the consideration of the slot effect, the equivalent air gap permeance can be written as [20]

\[
\lambda_{x} (\theta, t)=\lambda_{0} + \sum_{k} \lambda_{x k} \cos (k_z Z \theta) \quad (k_z = 1, 2, 3, \ldots)
\]

where \( \lambda_{0} \) average air gap permeance, \( \lambda_{x k} \) is the magnitudes of the \( k_z \)-th harmonic permeance caused by slot effect, and \( Z \) is the slot number.

Multiplying (1) and (3), the radial air-gap flux density generated by PMs can be expressed as [21]

\[
B_{x z} = F_{r x} \lambda_{x z} (\theta, t)
\]

\[
= \sum_{x} \lambda_{0 x} F_{\text{Re}x} \sin \left[ v_r \left( p \theta - \omega t \right) + \varphi^{x} \right] +
\]

\[
\frac{1}{2} \sum_{x} \sum_{x} F_{\text{Re}x} \lambda_{x z} \sin (v_r p \pm k_z Z \theta - v_r \omega t + \varphi^{x})
\]  

It can be found from (4) that the permeance can affect the spatial distribution of the air-gap magnetic field, but the frequency is constant. Considering only the difference in the temporal order, the air-gap flux density close to the teeth at the center line of the teeth can be rewritten as [22]

\[
B_{x z} = \sum_{x} F_{\text{Re}x} \sin (v_r \omega t + \varphi^{x})
\]

B. THE FORCE-HARMONIC COUPLING MODEL CONSIDERING THE FORCE PHASE ANGLE CHARACTERISTIC

The electromagnetic force acting on the ferromagnetic substance can be calculated by Maxwell stress tensor method. Ignoring the tangential component of magnetic flux density, the radial force density on the stator is approximated by [23]-[24]

\[
f_r = \frac{1}{2 \mu_0} B_{\text{Re}z}^2
\]

where \( \mu_0 \) is the permeability of air, the value of which is \( 4\pi \times 10^{-7} \) H/m.

Substituting (5) into (6), the equation for \( f_r \) considering the force phase angle can be derived as

\[
f_r = \frac{1}{2 \mu_0} \sum_{x} \sum_{\nu} B_{\text{Re}x} B_{\text{Re}z} \cos \left[ (v_{r1} \pm v_{r2}) \omega t + (\varphi^{x} \pm \varphi^{z}) \right]
\]

\[
= \frac{1}{2 \mu_0} \sum_{x} \sum_{\nu} B_{\text{Re}x} B_{\text{Re}z} \cos \left[ (v_{r1} \pm v_{r2}) \omega t \right] \varphi^{x} \pm \varphi^{z} = 0, 2\pi
\]

\[
\pm \frac{1}{2 \mu_0} \sum_{x} \sum_{\nu} B_{\text{Re}x} B_{\text{Re}z} \cos \left[ (v_{r1} \pm v_{r2}) \omega t \right] \varphi^{x} \pm \varphi^{z} = -\pi, \pi
\]

According to (7), it can be found that the phase angle has a significant impact on the coupling relationship among the force components. \( r \) is defined as the temporal order of force wave in this paper. Under the condition of \( \varphi^{x} \pm \varphi^{z} = 0/2\pi \), when \( r = v_{r1} + v_{r2} \), the contribution of corresponding force components is negative, and when \( r = v_{r1} - v_{r2} \), the contribution of corresponding force components is positive. Under the condition of \( \varphi^{x} \pm \varphi^{z} = -\pi/\pi \), when \( r = v_{r1} + v_{r2} \), the contribution of corresponding force components is positive, and when \( r = v_{r1} - v_{r2} \), the contribution of corresponding force components is negative. In other words, the phase angle has an important impact for the contribution effect of the force component, and \( f_r \) can be rewritten as a series of the combinations of the positive and negative force components

\[
f_r = f_{\text{Total}} + f_{\text{Total}}
\]

where, \( f_{\text{Total}} \) and \( f_{\text{Total}} \) are the combinations of positive and negative force components, respectively.

Therefore, considering the phase angle characteristic, there is not only superposition effect among force components, but also compensation effect among force components, which is different from the traditional analytical model of radial force wave.

IV THE ANALYSIS AND REDUCTION OF THE SLOT FREQUENCY FORCE HARMONIC BASED ON THE PROPOSED FORCE-HARMONIC COUPLING MODEL

A. THE ANALYTICAL MODEL OF THE SLOT FREQUENCY FORCE HARMONIC

When \( v_{r1} \pm v_{r2} = k_z Z / p \), the equation of the slot frequency force harmonic can be deduced as
where,  

\[
\begin{align*}
    f_+^{\text{Total}} \left( k_Z \frac{Z}{p} \right) &= \pm \frac{1}{2\mu_0} \sum_{y_1 \times y_2} \sum_{k} B_{R1}^{y_1} B_{R2}^{y_2} \cos \left[ k_Z \frac{Z}{p} \omega t \right] \left( y_1 \times y_2 \times k \frac{Z}{p} \right) \\
    f_-^{\text{Total}} \left( k_Z \frac{Z}{p} \right) &= \pm \frac{1}{2\mu_0} \sum_{y_1 \times y_2} \sum_{k} B_{R1}^{y_1} B_{R2}^{y_2} \cos \left[ k_Z \frac{Z}{p} \omega t \right] \left( y_1 \times y_2 \times k \frac{Z}{p} \right)
\end{align*}
\]

\[
\begin{align*}
    \phi^{x_1} + \phi^{x_2} &= 0, \quad 2\pi \\
    \phi^{x_1} + \phi^{x_2} &= -\pi, \quad \pi
\end{align*}
\]

(9)

According to the 3D structure in Fig. 1, the 2D finite element model of the prototype is established, as shown in Fig. 3. Combined with the finite element model and applied the finite element software of Maxwell, the air-gap flux density and its FFT result are calculated, as shown in Fig. 4. It can be seen that temporal orders of the air-gap flux density are \(2kR-1\) (1, 3, 5…), which are coincide with the (2). In the Fig. 4b, the phase angle of each harmonic is distinguished by the positive and negative amplitudes. When the amplitude is positive, the initial phase angle of the corresponding harmonic is 0, and when the amplitude is negative, the initial phase angle of the corresponding harmonic is \(\pi\).

**FIGURE 4.** The air-gap flux density and its FFT result. (a) Curve, (b) FFT result.

According to (7), it can be found that the F(1,11), F(5,7) and F(3,9) are positive components, and the F(1,13) and F(3,15) are negative components. In other words, the \(12f\) component of the radial force wave can be approximately...
equal to
\[
f_{12} = f'_{12} = F(1,11) - F(1,13) + F(3,9) - F(3,15) + F(5,7)
\]
\[
= -\frac{1}{2\mu_0} \left( B^1_{Rm} B^1_{Rn} - B^1_{Rm} B^3_{Rn} + B^3_{Rm} B^9_{Rn} - B^3_{Rm} B^{15}_{Rn} + B^7_{Rn} B^7_{Rn} \right) \cos(12\omega t) = f_{rTotal}^+(12) + f_{rTotal}^-(12)
\]  

where \( f'_{12} \) is the 12f component of the radial force harmonic because of the interaction among \( F(1,11), F(1,13), F(3,9), F(3,15), \) and \( F(5,7) \). \( f_{rTotal}^+(12) \) and \( f_{rTotal}^-(12) \) are the combinations of positive and negative force components in (11), respectively.

**C. THE CALCULATION OF THE SLOT FREQUENCY RADIAL FORCE HARMONIC**

Fig. 5 shows the spatial-temporal distribution of radial force density and its 2-D FFT result. The slot frequency force harmonics are 12 and 24 times the fundamental frequency, and the 12 component is more obvious, which corresponds exactly to this situation of \( k_g = 1 \) in equation (9). It can be found that the slot frequency component of the slot number order force wave is the most significant, and the slot number order force wave is the main source of the slot frequency force harmonic, which is the root of the root of the zeroth mode vibration [10]. Fig. 6 shows the radial force density curve and its corresponding temporal orders.
f(12) and f(12) are compared in Fig. 8, and the maximum values of both curves are shown in Table 2. f(12) is calculated by FEM, and the point close to the teeth surface and on the center is taken as the sampling point. The error of and is caused by ignoring the high harmonics. It can be found that the relative error is below 2%, which can be deemed acceptable. Therefore, f(12) can be replaced by f′(12), and the coupling relationship of (11) is verified, which demonstrates the importance of considering the phase angle of the force wave. Based on the coupling effect of force component, the slot frequency force harmonic can be reduced effectively by adjusting the negative components of force component, the slot frequency force harmonic can be reduced effectively by adjusting the negative components to offset the positive components, which is not available in traditional analysis.

When the number of magnetic pole segment is i in the axial direction and each rotor segment is shifted sequential at the same angle α in the circumferential direction, the air-gap flux density generated by the PMs in the kth segment expressed as [11]

\[
B_{Rn} = \sum_{v_k} B_{Rn}^{v_k} \sin v_k \left[ p\alpha (k-1) - \omega t \right] \tag{12}
\]

where \(B_{Rn}^{v_k}\) is the amplitude of the \(v_k\)-th harmonic of the air-gap flux density.

Due to the phase displacements caused by the rotor-step skewing, the air-gap flux density is distributed unevenly along the axial direction. The synthetic air-gap flux density across all segments can be expressed as

\[
B'_{RTotal} = \frac{1}{i} \sum_{i=1}^{i} B_{Rn}^{v_k} \sin v_k \left[ p\theta - p\alpha (k-1) - \omega t \right] \tag{13}
\]

According to (13), it can be found that the amplitudes of harmonics of the air-gap flux density change with the change of the segment number and skewed angle. Therefore, choosing the appropriate the segment number and skewed angle can theoretically achieve the mutual compensation of the positive components and the negative components in (11). In other words, the principle of the rotor-step skewing is further explained as a compensation strategy.

When the number of magnetic pole segment is 3, based on FEM and equation (11), the amplitudes of the \(f(12)\) at different skewed angle are shown in Fig. 10. It can be found that \(f(12)\) is the smallest when the skewed angle is 3.33°, and this is the same as the traditional analysis, which further verifies the correctness of the coupling model in this paper.

The radial force density with rotor-step skewing and its FFT result calculated by FEM are shown in Fig. 11. Compared with the original motor, the 12\(f\) component of the optimized prototype with rotor-step skewing is reduced effectively, which confirms the effectiveness of the optimization method.

### Table 2 The amplitudes of \(T(12)\) and \(T(12)\)

| Specifications           | \(f_{12}^c (\text{N/m}^2)\) | \(f_{12} (\text{N/m}^2)\) | Relative error |
|-------------------------|----------------------------|--------------------------|----------------|
| Maximum values and error| 9510.93                   | 9387.32                  | 1.38%          |

**D. REDUCTION OF SLOT FREQUENCY RADIAL FORCE HARMONIC WITH ROTOR-STEP SKewing**

In this section, the rotor-step skewing is adopted to reduce the cogging torque, due to the advantages of simple structure and low cost [25]. Fig. 9 shows the structure of the rotor-step skewing.
$f'(12)$ at different skewed angle.

Fig. 10. $f'(12)$ at different skewed angle.

The radial force density with rotor-step skewing and its FFT result.

(a) Curves, (b) FFT result.

Fig. 11. The radial force density with rotor-step skewing and its FFT result.

E ANALYSIS OF SLOT FREQUENCY VIBRATION

Fig. 12 shows the vibration calculation procedure of the prototype. Based on FEM, the radial force waves on the stator teeth are calculated. Combined with the modal analysis, the vibration acceleration of the prototype can be obtained through the modal superposition method [25]. The modal and modal superposition method are analyzed by the software of Ansys.

Fig. 13 shows the comparison of the simulated vibration acceleration of the motors at the rated speed. The dominating frequency components are $12f$, $24f$, and $36f$, which called first-, second-, third-order slot frequency, respectively. Compared with the original motor, the slot frequency vibration is effectively reduced, which confirms the correctness of the force-harmonic coupling model and the effectiveness of the optimization method.

V EXPERIMENTAL VERIFICATION OF THE PROTOTYPE

In order to verify the accuracy of the simulation, the experiment platform of the prototype with the segment number of 3 and skewed angle of $3.33^\circ$ is set up, as shown in Fig. 14. The experiment is carried out at rated speed on no-load, and the frequency of controller is set to 75 Hz. The vibration tensor is placed in the middle of the prototype in the axial direction, and the vibration acceleration is collected through data acquisition system produced by DEWEsoft.

Fig. 15 shows the no-load line back EMF of the prototype and its FFT results, and it can be found that the curve and harmonics amplitude of the simulation and measurement are in good agreement. The amplitude of fundamental wave is the biggest, and the amplitude of other harmonics is much smaller than that of the fundamental wave, which can be ignored. The error between the
simulation results and the experimental results is caused by the test equipment and test condition, which can be deemed acceptable.

The experimental results of the vibration at rated speed are compared with simulation results as shown in Fig. 16. It can be found that the measured results are similar as that of the simulation, and the main characteristics points at 900, 1800 and 2700 Hz are consistent. These frequencies are related to the stator slot number, and they are called first-, second-, and third-order slot frequency, respectively. The measured acceleration at 900 Hz is 64.3 mm/s², which is considerably close to the simulated result 68.4 mm/s². There are some differences between the two results; however, with the error in noise results being deemed acceptable.

Since the project only evaluated the electromagnetic noise, the measured noise may include other vibration sources such as rotor eccentricity, bearing noise and friction noise, which inevitably bring about the difference between calculated values and measured values. However, the simulation results are basically consistent with the experimental results at main slot frequencies, and generally meet the required accuracy. Based on the comparison between the simulated and the measured results, the precision of the simulation models can be verified.

**VI CONCLUSION**

In this paper, the force-harmonic coupling model considering the force phase angle characteristic has been established, and the slot frequency force harmonic of a 6-poles 36-slots PMSM has been analyzed and reduced based on the proposed model. The conclusions are listed as follows

1) The phase angle has a significant impact on the coupling relationship among the force components. Under the condition of \( \phi_{x1} \pm \phi_{x2} = 0/2\pi \), when \( r = v_{R1} + v_{R2} \), the contribution of corresponding force components is negative, and when \( r = v_{R1} - v_{R2} \), the contribution of corresponding force components is positive. Under the condition of \( \phi_{x1} \pm \phi_{x2} = -\pi/\pi \), when \( r = v_{R1} + v_{R2} \), the contribution of corresponding force components is positive, and when \( r = v_{R1} - v_{R2} \), the contribution of corresponding force components is negative. Considering the phase angle, there is not only superposition effect among force components, but also compensation effect among these force components, which is different from the traditional analytical model of the radial force wave.

2) Based on the proposed force-harmonic coupling model, the principle of the rotor-step skewing is further
explained as the mutual compensation effect of the positive and negative force components. The slot frequency force harmonic and vibration can be effectively offset through choosing the appropriate segment number and skewed angle.

REFERENCES

[1] G. Liu, G. Qiu, J. Shi, and F. Zhang, “Study on Counter-Rotating Dual-Rotor Permanent Magnet Motor for Underwater Vehicle Propulsion,” IEEE Trans. Appl. Supercond., vol. 28, no. 3, pp. 1-5, April 2018.

[2] I. D. Chasiotis, and Y. L. Karnavas, “A Generic Multi-Criteria Design Approach Toward High Power Density and Fault-Tolerant Low-Speed PMSM for Pod Applications,” IEEE Trans. Transport. Electrific., vol. 5, no. 2, pp. 356-370, June 2019.

[3] F. Chai, Y. Li, Y. Pei, and Y. Yu, “Analysis of Radial Vibration Caused by Magnetic Force and Torque Pulse in Interior Permanent Magnet Synchronous Motors Considering Air-Gap Deformations,” IEEE Trans. Ind. Electron., vol. 66, no. 9, pp. 6703-6714, Sept. 2019.

[4] J. Xu, and H. Zhang, “Random Asymmetric Carrier PWM Method for PMSM Vibration Reduction,” IEEE Access, vol. 8, pp. 109411-109420, 2020.

[5] C. Lee, H. Seol, J. Lee, S. Lee, and D. Kang, “Optimization of Vibration and Noise Characteristics of Skewed Permanent Brushless Direct Current Motor,” IEEE Trans. Magn., vol. 53, no. 11, pp. 1-5, Nov. 2017, Art no. 8210605.

[6] A. Cassat, et al., “A Practical Solution to Mitigate Vibrations in Industrial PM Motors Having Concentric Windings,” IEEE Trans. Ind. Appl., vol. 48, no. 5, pp. 1526-1538, Sept.-Oct. 2012.

[7] J. Hong, S. Wang, Y. Sun, and H. Cao, “A PM Pole With Axial Varied Width for Vibration Mitigation in PM Brush DC Motors,” IEEE Trans. Ind. Electron., vol. 66, no. 5, pp. 3595-3604, May 2019.

[8] S. Wang, J. Hong, Y. Sun, J. Shen, H. Cao, and Z. Yang, “Analysis and Experimental Verification of Electromagnetic Vibration Mode of PM Brush DC Motors,” IEEE Trans. Ind. Electron., vol. 33, no. 3, pp. 1411-1421, Sept. 2018.

[9] S. Wang, J. Hong, Y. Sun, and H. Cao, “Analysis and Reduction of Electromagnetic Vibration of PM Brush DC Motors,” IEEE Trans. Ind. Appl., vol. 55, no. 5, pp. 4605-4612, Sept.-Oct. 2019.

[10] S. Wang, J. Hong, Y. Sun, and H. Cao, “Analysis of Zeroth-Mode Slot Frequency Vibration of Integer Slot Permanent-Magnet Synchronous Motors,” IEEE Trans. Ind. Electron., vol. 67, no. 4, pp. 2954-2964, April 2020.

[11] X. Wang, X. Sun, P. Gao, “Study on the effects of rotor step skewing on the vibration and noise of a PMSM for electric vehicles,” IET Electr. Power Appl., vol. 14, no. 1, pp. 131-138, Jan. 2019.

[12] S. Wang, J. Hong, Y. Sun, and H. Cao, “Effect Comparison of Zigzag Skew PM Pole and Straight Skew Slot for Vibration Mitigation of PM Brush DC Motors,” IEEE Trans. Ind. Electron., vol. 67, no. 6, pp. 4752-4761, June 2020.

[13] J. Hong, S. Wang, Y. Sun, X. Cao, and H. Cao, “Piecewise Staggered Poles With Continuous Skew Edge for Vibration Reduction in Surface-Mounted PM Synchronous Machines,” IEEE Trans. Ind. Electron., vol. 68, no. 9, pp. 8498-8506, Sept. 2021.

[14] J. Hong, S. Wang, Y. Sun, and H. Cao, “An effective method with copper ring for vibration reduction in permanent magnet brush DC motors,” IEEE Trans. Magn., vol. 54, no. 11, pp. 1-5, Nov. 2018.

[15] J. Jung, D. Kim, J. Hong, G. Lee, and S. Jeon, “Experimental Verification and Effects of Step Skewed Rotor Type IPMSM on Vibration and Noise,” IEEE Trans. Magn., vol. 47, no. 10, pp. 3661-3664, Oct. 2011.

[16] S. Wang, J. Hong, Y. Sun, Z. Zheng, and H. Cao, “Filling force valley with interpoles for pole-frequency vibration reduction in Surface-mounted PM synchronous machines,” IEEE Trans. Ind. Electron., vol. 67, no. 6, pp. 6709-6720, Aug. 2020.

[17] S. G. Min, and B. Sarielloglu, “Modeling and Investigation on Electromagnetic Noise in PM Motors With Single- and Double-Layer Concentrated Winding for EV and HEV Application,” IEEE Trans. Transport. Electrific., vol. 4, no. 1, pp. 292-302, March 2018.

[18] J. K. Jia, L. Kang, Y. S. Zhan, Y. B. Sun, and M. Y. Guo, “The Model of Pole Slot Radial Force Wave Compensation for Surface-Mounted Three-Phase Permanent Magnet Synchronous Motor and Parameter Identification,” Trans. China Electrotech. Soc., vol. 36, no. 8, pp. 1596-1606, Apr. 2021.

[19] M. Cheng, P. Han and W. Hua, “General Airgap Field Modulation Theory for Electrical Machines,” IEEE Trans. Ind. Electron., vol. 64, no. 8, pp. 6063-6074, Aug. 2017.

[20] Z. Xing, W. Zhao, X. Wang, and Y. Sun, “Reduction of radial electromagnetic force waves based on PM segmentation in SPMSMs,” IEEE Trans. Magn., vol. 56, no. 2, pp. 1-7, Feb. 2020.

[21] M. Cheng, X. Zhu, Y. Wang, R. Wang, and W. Wang, “Effect and Inhibition Method of Armature-Reaction Field on Superconducting Coil in Field-Modulation Superconducting Electrical Machine,” IEEE Trans. Energy Convers., vol. 35, no. 1, pp. 279-291, March 2020.

[22] F. Lin, S. Zuo, W. Deng and S. Wu, “Modeling and analysis of electromagnetic force, vibration, and noise in permanent-magnet synchronous motor considering current harmonics,” IEEE Trans. Ind. Electron., vol. 67, no. 5, pp. 7455-7466, Dec. 2016.

[23] H. Yang, and Y. Chen, “Influence of Radial Force Harmonics with Low Mode Number on Electromagnetic Vibration of PMSM,” IEEE Trans. Energy Convers., vol. 29, no. 1, pp. 38-45, March 2014.

[24] J. Zou, H. Lan, Y. Xu and B. Zhao, “Analysis of global and local force harmonics and their effects on vibration in permanent magnet synchronous machines,” IEEE Trans. Energy Convers., vol. 32, no. 4, pp. 1523-1532, Dec. 2017.

[25] W. Fei and Z. Q. Zhu, “Comparison of cogging torque reduction in permanent magnet brushless machines by conventional and herringbone skewing techniques,” IEEE Trans. Energy Convers., vol. 28, no. 3, pp. 664-674, Sep. 2013.

[26] Hu S, Zuo S, and Liu M, et al. “Modeling and analysis of radial electromagnetic force and vibroacoustic behavior in switched reluctance motors.” Mech. Syst. Signal Proc., vol. 142, Aug. 2020. DOI: 10.1016/j.ymssp.2020.106778.

ZHIYAN GUO received a B.S. degree in electrical automation from the school of Nanchang University, Nanchang, China, in 2007 and an M.S. degree in electrical engineering from Northeast University, Shenyang, China, in 2014. He is currently pursuing a Ph.D. degree with the School of Electrical Engineering, Shenyang University of Technology, China. His research interests include high-precision servo systems in machine tools, vibration abatement and motion control systems.

JIANKUAN XIA received B.S., M.S., and Ph.D. degrees in electrical engineering from Shenyang University of Technology, China, in 1986, 1997, and 2006, respectively. He is currently a professor at the School of Electrical Engineering, Shenyang University of Technology, China. His research interests include artificial intelligence and modern control theory, power electronics and power transmission, large synchronous motor excitation system design and research, and special motor design and control.

ZEXING LI received a B.S. degree in electrical automation from the school of Shenyang University of Technology, Shenyang, China, in 2017. He is currently pursuing M.S. and Ph.D. degrees with school of electrical engineering at
Shenyang University of Technology, China. His research interests are motor design and control.

**Chen Zhao** received a B.S. degree in Mechanism design, manufacturing, and automatization from the school of Shenyang Jianzhu University, China, in 2011 and an M.S. degree in mechanical engineering from Shenyang Jianzhu University, Shenyang, China, in 2018. He is currently a researcher and asset manager at the School of Shenyang Institute of Engineering, China. His research interests include mechanical-electrical integration system design and ultra-precision manufacturing technology.