A POSSIBLE APPROACH TO THREE-DIMENSIONAL COSMIC-RAY PROPAGATION IN THE GALAXY. IV. ELECTRONS AND ELECTRON-INDUCED $\gamma$-RAYS

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ABSTRACT

Based on the diffusion-halo model for cosmic-ray (CR) propagation, including stochastic reacceleration due to collisions with hydromagnetic turbulence, we study the behavior of the electron component and the diffuse $\gamma$-rays (D$\gamma$’s) induced by them. The galactic parameters appearing in these studies are essentially the same as those appearing in the hadronic CR components, while we additionally need information on the interstellar radiation field, taking into account dependences on both the photon energy, $E_{\gamma}$, and the position, $r$. We compare our numerical results with the data on hadrons, electrons, and D$\gamma$’s, including the most recent results from Fermi, which gives two remarkable results: (1) the electron spectrum falls with energy as $E_{\gamma}^{-3}$ up to 1 TeV, and does not exhibit prominent spectral features around 500 GeV, in contrast to the dramatic excess appearing in both ATIC and PPB-BETS spectra and (2) the EGRET GeV-excess in the D$\gamma$ spectrum is neither due to an astronomical origin (much harder CR spectrum in the galactic center) nor a cosmological one (dark matter annihilation or decay), but due to an instrumental problem. In the present paper, however, we focus our interest rather conservatively upon the internal relation between these three components, using common galactic parameters. We find that they are in reasonable harmony with each other within both the theoretical and experimental uncertainties, apart from the electron-anomaly problem, while some enhancement of D$\gamma$’s appears in the high galactic latitude with $|b| > 60^\circ$ in the GeV region.

Key words: cosmic rays – Galaxy: structure – gamma rays: diffuse background

Online-only material: color figures

1. INTRODUCTION

Although the electron component is only a small fraction of all cosmic-ray (CR) components, around 1% of the proton intensity around 10 GeV, it plays a key role in understanding the structure of our Galaxy and the galactic phenomena occurring within it. This is because electrons have electromagnetic interactions with the interstellar radiation field (ISRF), such as photons and magnetic fields, resulting in drastic energy loss during propagation through the Galaxy, in contrast to the hadronic component.

This peculiar nature yields valuable information for the study of CR astrophysics, which cannot be obtained by the hadronic components alone. Namely, due to the rapid energy-loss rate, proportional to $E_{\gamma}^2$ in the high energy (HE) region, from the inverse Compton (IC) scattering off photons and synchrotron radiation in magnetic fields, the lifetime of TeV electrons is at most $10^5$ yr, indicating that detected electrons have originated in nearby sources, less than 1 kpc from the solar system (SS). Therefore, accurate observations of TeV electrons will provide a direct signature of nearby CR sources as well as the mechanism of the CR acceleration, while depending on the environment of the SS. Therefore, accurate observations of TeV electrons will provide a direct signature of nearby CR sources as well as the mechanism of the CR acceleration, while depending on the interstellar medium. The most recent data for the age and distance of each supernova remnant near the SS, although the statistics of HE electron data are currently too poor to identify sources definitely.

Particle identification and energy determination of HE electrons are, however, quite difficult, while direct observation of low energy (LE) electrons is relatively easy using, for instance, magnetic spectrometers, and has been performed by several groups (Golden et al. 1994; Boezio et al. 2000; DuVernois et al. 2001; Aguilar et al. 2002).

Although the statistics are not sufficient, the only group that succeeded in observing directly TeV electrons is Nishimura et al. (1980; see also Kobayashi et al. 1999) with the use of the balloon-borne emulsion chamber. It should be noted that they actually observe event by event the vertex point of the electron with subsequent $e^+ e^-$-pair due to bremsstrahlung, with no uncertainty from proton contamination. The precision in the energy determination is approximately 10% for electrons in the energy region larger than 50 GeV, based on both the three-dimensional cascade theory (Nishimura 1964) and the simulations (Kasahara 1985; Okamoto & Shibata 1987), which have been well established by the use of accelerator beams (Hotta et al. 1980; Sato & Sugimoto 1979).

Recent development in HE electron observations is indeed remarkable, particularly those of ATIC (Chang et al. 2008) and PPB-BETS (Torii et al. 2006), which showed an anomaly in the electron spectrum with a significant bump around 500 GeV. Both groups point out that the excess indicates either a nearby source of energetic electrons or those coming from the annihilation of dark matter (DM) particles.

On the other hand, the most recent results obtained by the Fermi Large Area Telescope (Fermi-LAT; Abdo et al. 2009) present no prominent excess, with the electron spectrum falling with energy as $E_{\gamma}^{-3.04}$ up to 1 TeV, which is not inconsistent with the emulsion chamber data (Kobayashi et al. 1999) within the statistical errors. The H.E.S.S. ground-based telescope (Aharonyan et al. 2008, 2009) also shows no indication of structure in the electron spectrum, but rather a power-law.
spectrum with $E^{-3.0±0.1±0.3}$ (0.1: stat. error, 0.3: syst. error), albeit this being an indirect observation.

Nevertheless, looking carefully at Fermi data around the anomaly energy, they still show systematically an enhancement as large as 30% compared to the numerical results (Abdo et al. 2009; Strong et al. 2004; see also Figure 14 in this paper), so that we cannot exclude the possibility of an additional component such as local sources and/or the DM scenario, while strength of the anomaly compared to the background diffuse electrons is not as dramatic as presented by ATIC and PPB-BETS.

In any case, both observational and theoretical studies for HE electrons are becoming increasingly important not only for astrophysics, but also for particle physics and cosmology. It is, therefore, desirable to find a reasonable model for electron propagation in the Galaxy, which must explain consistently and simultaneously all CR observables and not just electrons, using common galactic parameters with the smallest number of variables possible. In the sense, the recent review article by Strong et al. (2007) is a useful survey of both the theory and relevant experimental data for the propagation of CRs, comprehensively summarizing the current landscape and open questions, although it was published just before the anomaly problem mentioned above.

Under these situations, we have studied the three-dimensional CR propagation model analytically, and found excellent agreement with the experimental data for various hadronic components, stable primaries, secondaries such as boron and sub-iron elements ($Z = 21–23$), isotopes such as $^{10}$Be, and antiprotons as well, in four papers (Shibata et al. 2004, 2006, 2008; Shibata & Ito 2007), hereafter referred to as Papers I, II, IV, and III, respectively.

We have applied our model further to the studies of diffuse $γ$-rays (DY’s; Shibata et al. 2007; hereafter Paper V), and found that all these components are generally in agreement with each other using the same galactic parameters, within the uncertainties in the experimental data and various kinds of cross sections used for the numerical calculations. However, in Paper V, we use the simulation results for electron-induced $γ$-rays provided by Hunter et al. (Bertsch et al. 1993; Hunter et al. 1997), where the modeling of CR propagation and the galactic parameters assumed are somewhat different from ours. So we have yet to see complete internal consistency among all CR components—hadrons, electrons, and DY’s—using the same galactic parameters in our propagation model.

In the present paper, we extend it to the electron component, based on the diffusion-halo model proposed by Ginzburg et al. (1980), taking the reacceleration process into account. However, we focus in the present work on diffuse electrons in the steady state without discriminating those produced by nearby sources from those of distant ones, and present the intensity of the DY’s produced by them in the energy range, $E_γ = 30$ MeV–100 GeV, covered by EGRET and Fermi. Comparison with radio and TeV-$γ$ data will be reported separately in the near future.

In order to apply our model to the electron component and electron-induced DY’s, we need information on the ISRF in addition to the interstellar matter (ISM), particularly their spatial gradients for the study of the $(l, b)$-distribution of DY’s ($l$: galactic longitude; $b$: galactic latitude). Nowadays the most advanced and standard code for the ISM and ISRF models is GALPROP, extensively developed by Strong & Moskalenko (1998), incorporating the latest survey data in the very wide wavelength range from ultraviolet to radio. In the present work, we assume empirical density distributions for the ISM and ISRF, smoothing the numerical data given by GALPROP available most recently (Porter et al. 2008), in order to combine with our analytical solution for electron-induced $Dγ$’s.

In Section 2, we discuss the interstellar environment provided by GALPROP, focusing on the spatial distribution of both matter (atomic, molecular, and ionized hydrogen) and photons (ultraviolet, visible, infrared, mid- and far-infrared, and cosmic microwave background (CMB) radiations), and in Section 3 we present the relevant elementary processes for electrons, focusing on the energy losses due to ionization, bremsstrahlung, synchrotron, IC, and on the energy gain due to the reacceleration.

In Section 4, we present the diffusion equation, and give its solution explicitly in the steady state, $N_e(r; E_γ)$, where the Klein–Nishina effect is quite important in the electron energy spectrum in the HE region, $>10$ GeV. In Section 5, we present the emissivity of electron-induced $γ$’s, $q_γ(r; E_γ)$, with use of realistic spatial distributions of ISM and ISRF as discussed in Section 2, and show the numerical results at several observational points, where those of the hadron-induced $γ$’s are presented as well. In Section 6, we first summarize the galactic parameters and their explicit values expected from the CR data, and then compare our numerical results of electron flux $Dγ$’s with recent observational data, including those most recently obtained by Fermi and H.E.S.S. Finally in Section 7, we summarize the results and discuss several remaining open questions, while we do not touch upon the so-called electron anomaly.

2. INTERSTELLAR ENVIRONMENT OF OUR GALAXY

2.1. Interstellar Matter

First, we consider the ISM for two processes, ionization and bremsstrahlung. In Figure 1, we plot histograms of column density for H1 and H2 in the galactic plane (GP) given by GALPROP, where we also plot the empirical curves used in the present work,

$$\ln \rho_{H_1}(r) = \rho_{H_1}^{(0)} + \rho_{H_1}^{(1)} r + \rho_{H_1}^{(2)} r^2 \ln r + \rho_{H_1}^{(3)} r^3, \quad (1a)$$

$$\ln \rho_{H_2}(r) = \rho_{H_2}^{(0)} + \rho_{H_2}^{(1)} r + \rho_{H_2}^{(2)} r^2 \ln r + \rho_{H_2}^{(3)} r^3, \quad (1b)$$

with $r$ in kpc and $\rho_h$ ("h" $\equiv$ H1, H2) in $10^{20}$ H atoms cm$^{-2}$. The numerical values of the coefficients are summarized in Table 1.
However, the choice of the above empirical form is not critical, and other choices may be possible.

The H$_2$ gas is strongly confined to the GP and its vertical structure is modeled by a Gaussian distribution with a width of approximately 70 pc, while the H1 gas lies in a flat layer with an FWHM of 230 pc in 3.5 kpc < r < $r_0$ (= 8.5 kpc), and is approximated by the sum of two Gaussians and an exponential tail (Ferriere 2001; Moskalenko et al. 2002). Taking these situations into account, we assume the following spatial distribution for the ISM gas density, corresponding to Equations (1a) and (1b),

$$\frac{n_H(r) - n_H^0}{n_H^0} = \frac{\rho_H (r)}{\rho_H (0)} \exp \left[ -\frac{(z - z_0)^2}{2z_0^2} \right] + \frac{\rho_H (r)}{\rho_H (0)} \exp \left[ -\frac{(z - z_0)^2}{2z_0^2} \right] + \frac{\rho_H (r)}{\rho_H (0)} \exp \left[ -\frac{(z - z_0)^2}{2z_0^2} \right],$$

(2)

where $n_H^0$ is the gas density of H1 (H2) at the SS with typically $n_H^0 \approx n_H^0 \approx 0.5$ H atoms cm$^{-3}$. See Table 2 for the explicit forms of $\Sigma_H$ and $\Sigma_H$.

For the ionized hydrogen gas, H I, we use the two-component model of Cordes et al. (1991), $n_{H_1}(r) = n_{H_1}^{(1)}(r) + n_{H_1}^{(2)}(r)$, and both components are modeled by a Gaussian-type distribution for the radial structure, and by a simple exponential one for the vertical structure. The explicit forms of the two components at the SS, [$n_{H_1}^{(1)}(r_0), n_{H_1}^{(2)}(r_0)$], are [0.025, 0.013] cm$^{-3}$ respectively (Cordes et al. 1991; Strong & Moskalenko 1998). So the contribution of H1 is much smaller than those of H2 and H2 is not important in the present work.

2.2. Interstellar Radiation Field

First, we consider the medium—virtual photons induced by the static magnetic field—for the synchrotron radiation. It is approximately given by an exponential-type gradient, while the scale height is not yet clear. Practically, for the study of synchrotron radiation, we need the energy density of virtual photons at $r$, $\epsilon_B (r)$, and assume in the present work

$$\epsilon_B (r) = \epsilon_{B,0} \exp \left[ -(r/r_B + |z|/z_B) \right],$$

(3)

with

$$\epsilon_{B,0} = B_0^2/(8\pi),$$

where $B_0$ is the magnetic field at the galactic center (GC), and $\epsilon_{B,0}$ is its energy density, for instance $\epsilon_{B,0} \approx 1$ eV cm$^{-3}$ for

![Figure 2. Interstellar radiation field at two galactocentric distances obtained by GALPROP, $r = 0$ (GC; square symbols) and 8 kpc (near SS; circle symbols). Open marks correspond to maximum metallicity gradient, and filled ones to the minimum metallicity gradient. Dotted curves are given by Equation (6) with parameters summarized in Table 3 for each population $i$, while the solid ones are those superposing them. CMB radiation (solid curve) is also shown for reference.](image)

$B_0 = 6$ $\mu$G, and typically $[2r_B, 2z_B] \approx [10, 2]$ kpc (Strong et al. 2000).

On the other hand, the photon gas for the IC process is somewhat different from those discussed above. Namely, we need the number density of the photon gas in the ISRF, $n_{ph}(r; E_{ph})$, as a function of the target photon energy $E_{ph}$ at $r$. Separating it into two parts, an $r$-dependent energy-density term, $\epsilon_{ph}(r)$, and an $r$-independent term, $W_{ph}(k)$ with $k = E_{ph}/(k_B T_{ph})$, we rewrite $n_{ph}(r; E_{ph})$ as

$$E_{ph} n_{ph}(r; E_{ph}) dE_{ph} = \epsilon_{ph}(r) W_{ph}(k) dk \ln k,$$

(4)

where $k_B$ is the Boltzmann constant and $T_{ph}$ is the characteristic temperature of the ISRF.

There are three main radiation sources in the photon gas, (1) the 2.7 K CMB radiation, (2) stellar radiation with wavelengths of 0.1–10 $\mu$m (ultraviolet–visible–near-infrared), and (3) re-emitted radiation from dust grains at 10–1000 $\mu$m (mid-to-far-infrared). We classify them further into six wavelength bands, each labeled with $i = 0$ for (1), $i = 1, 2, 3$ for stellar-1, -2,
Equation (7).

\[ z = (1983; \text{filled gray circles}). \text{Solid curves are the empirical ones obtained by gradient (open circles), where also shown are those obtained by Mathis et al.} \]

\[ \text{also presented are those of } r^{-3} \text{ in (2), and } r^2 \text{ in (3) (see Figure 2). Needless to say, there is no spatial gradient in the CMB (i.e., 0), which is distributed uniformly in space, } \epsilon_{\text{ph}}(r) \equiv \epsilon_{\text{ph}}^0 = 0.261 \text{ eV cm}^{-3}, \text{ and the normalized spectrum, } W_{\text{ph}}^{(0)}(k), \text{ is given by the familiar Planck formula with } T_{\text{ph}} = 2.73 \text{ K.} \]

On the other hand, for (2) and (3) in the wavelength range \( \lambda = 0.1-1000 \mu \text{m}, \) the energy density, \( \epsilon_{\text{ph}}^{(i)}(r) \) (\( i = 1-5 \)), must depend on \( r \), and \( W_{\text{ph}}^{(i)}(k) \) is unlike the simple CMB spectrum, and is very complicated. In the following discussions, we often omit the suffix \( i \) for simplicity unless otherwise specified.

In the present work, we assume a Gaussian-type distribution in \( \ln k \) for \( W_{\text{ph}}(k) \),

\[ W_{\text{ph}}(k) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(\ln k - \ln \lambda_0)^2}{2\sigma^2}}, \quad k = \lambda_0 / \lambda, \]

so that the mean radiation intensity, \( I_{\text{ph}} \), is given by

\[ \frac{4\pi \lambda I_{\text{ph}}(r; \lambda)}{c \epsilon_{\text{ph}}(r)} = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(\ln(\lambda_0 / \lambda))^2}{2\sigma^2}\right], \]

where \( \lambda_0 \) is the peak wavelength for each radiation with \( k_B T_{\text{ph}} = 2\pi c \lambda_0 \).

In Figure 2, we present examples of the mean radiation intensity (multiplied by \( 4\pi \lambda \)) for the maximal metallicity gradient (filled symbols) and no metallicity gradient (open symbols) at two radial distances, \( r = 0 \) (squares) and 8 kpc (circles) in the GP given by GALPROP, where also drawn are curves expected from the right-hand side of Equation (6) for \( W_{\text{ph}}(k) \), assuming

\[ \epsilon_{\text{ph}}(r) = \epsilon_{\text{ph,0}} e^{-|r/r_{\text{ph}} + |z|/z_{\text{ph}}|}, \]

for \( r \geq 3 \text{ kpc} \), and see the caption of Table 3 otherwise. In Table 3, we summarize numerical values of \( [\lambda_0^{(i)}, T_i, \sigma_i; \epsilon_{\text{ph,0}}^{(i)}] \) (\( i = 0-5 \)) with \( r_{\text{ph}} = 3.2 \text{ kpc} \) irrespective of the population \( i \), and also presented are those of \( \epsilon_{\text{ph}}^{(i)}(r) \) for \( r \lesssim 3 \text{ kpc} \), while they are independent of \( r \) except \( \epsilon_{\text{ph}}^{(2)}(r) \).

\[ \text{Figure 3. ISRF energy density as a function of galactocentric distance } r \text{ at } \zeta = 0 \text{ for the stellar and the re-emission from dust grains, each with the maximum metallicity gradient (filled black circles) and minimum metallicity gradient (open circles), where also shown are those obtained by Mathis et al. (1983; filled gray circles). Solid curves are the empirical ones obtained by Equation (7).} \]

-3 in (2), and \( i = 4, 5 \) for dust-1, -2 in (3) (see Figure 2). Needless to say, there is no spatial gradient in the CMB (i.e., 0), which is distributed uniformly in space, \( \epsilon_{\text{ph}}(r) \equiv \epsilon_{\text{ph}}^0 = 0.261 \text{ eV cm}^{-3} \), and the normalized spectrum, \( W_{\text{ph}}^{(0)}(k) \), is given by the familiar Planck formula with \( T_{\text{ph}} = 2.73 \text{ K.} \)

The Astrophysical Journal in the HE region; see Appendix A for the explicit forms of the IC-all (heavy solid curve), synchrotron + IC's (dotted curve), and by CMB (broken dotted curve), together with the sums, IC-stellar + IC-dust + IC-CMB (thin solid curve), and synchrotron + IC-all (heavy solid curve).

Let us demonstrate the energy density separately for the stellar and the dust radiation, \( \sum_{i=3}^5 \epsilon_{\text{ph}}^{(i)} \) and \( \sum_{i=4}^5 \epsilon_{\text{ph}}^{(i)} \) respectively, against the galactocentric distance \( r \) in Figure 3, where we also plot numerical data given by Mathis et al. (1983; filled gray symbols). Two curves for the stellar emission and the dust re-emission are drawn by the use of the parameterization summarized in Table 3, where we do not take the difference in the choice of metallicity gradient into account, as it is effective only near the GC for the dust re-emission and is approximately one order of magnitude smaller than the stellar radiation.

For the latitudinal scale height, \( z_{\text{ph}} \), in Equation (7), we assume \( z_{\text{ph}} \approx r_{\text{ph}}/8 = 0.4 \text{ kpc} \), referring to the speculation by Freudenreich (1998) based on the DIRBE (Diffused Infrared Background Experiment) survey, while the surveys of the diffuse FIR/sub-millimeter emission for the latitudinal direction at various radial distances \( r \) are not sufficient to construct a reliable model.

\[ \Delta E_r \]

\[ \frac{\Delta E_r}{\Delta r} \]

\[ \text{Figure 4. Energy losses per unit time of CR electrons in ISRF at SS (r = 8.5 kpc), shown separately for four components, synchrotron (heavy dotted line), IC's by stellar radiation (broken curve), by re-emission from dust grains (dotted curve), and by CMB (broken dotted curve), together with the sums, IC-all = IC-stellar + IC-dust + IC-CMB (thin solid curve), and synchrotron + IC-all (heavy solid curve).} \]

3. ENERGY LOSS AND GAIN

3.1. Energy Loss in ISM and ISRF

The energy loss processes for the electron component are dramatically different from those for the hadronic components, with four main processes: bremsstrahlung (≡ “rad”), ionization (≡ “ion”), synchrotron, and IC (together ≡ “sic”). For the bremsstrahlung (Koch & Motz 1959; Gould 1969; Ginzburg 1979),

\[ \frac{1}{E_r} \left( \frac{\Delta E_r}{\Delta r} \right)_{\text{rad}} \simeq n(r) w_{\text{rad}}(E_r) \left[ 1 + O \left( \frac{H_{\text{He}}}{n} \right) \right], \]

with

\[ n(r) = n_{\text{H}}(r) + n_{\text{H\text{I}}} + n_{\text{H\text{II}}}(r), \]

where \( w_{\text{rad}}(E_r) \gg m_e c^2 \equiv w_{\text{rad}}^{(\infty)} \approx 7.30 \times 10^{-16} \text{ cm}^3 \text{ s}^{-1} \), independent of \( E_r \) with the complete screening cross section in the HE region; see Appendix A for the explicit forms of \( w_{\text{rad}}(E_r) \), and Section 2.1 for \( n_{\text{H\text{I}}} \) (≡ H\text{I}, H\text{II}, H\text{III}).

Similarly for the ionization, we use the Bethe–Bloch formula (Ginzburg 1979)

\[ \left( \frac{\Delta E_{e^-}}{\Delta r} \right)_{\text{ion}} \simeq n(r) w_{\text{ion}}(E_r) \left[ 1 + O \left( \frac{H_{\text{He}}}{n} \right) \right], \]
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with

\[ w_{\text{ion}}(E_e) = w_{\text{ion}}^{(0)} \left[ \ln \frac{E_e}{\text{GeV}} + 13.8 \right] , \]

and \( w_{\text{ion}}^{(0)} = 0.229 \times 10^{-16} \text{ cm}^3 \text{ s}^{-1} \).

On the other hand, the energy losses due to the synchrotron (abbreviated as “SY”) for subscripts appearing in the following equations) and IC are rather complicated, in addition to the energy-dependent cross section of the Klein–Nishina formula,

\[ \frac{\Delta E_e}{\Delta t}_{\text{SY}} = \frac{\Delta E_e}{\Delta t}_{\text{IC}}, \]

where

\[ -\frac{1}{w_T E_e} \frac{\Delta E_e}{\Delta t}_{\text{SY}} = \epsilon_B(r), \] (12a)

\[ -\frac{1}{w_T E_e} \frac{\Delta E_e}{\Delta t}_{\text{IC}} = \sum_{i=0}^{5} \epsilon_{\text{ph}}^{(i)}(r) \Lambda(E_e, T_i), \] (12b)

with \( w_T = 1.018 \times 10^{-16} \text{ cm}^3 \text{ s}^{-1} \). See Section 2.2 and Table 3 for \( [T_i; \epsilon_{\text{ph}}^{(i)}(r)] \) (i = 0–5), and \( \Lambda(E_e, T_i) \) is given by Equation (A6), which comes from the Klein–Nishina cross section. In Figure 4, we present the energy loss divided by \( E_e^2 \) at the SS against \( E_e \) separately for individual (virtual) photon fields as well as for superposed ones, \( -\frac{\Delta E_e}{\Delta t}_{\text{SY}} \) and \( \frac{\Delta E_e}{\Delta t}_{\text{IC}} \), where we assume \( B_1 = 5 \mu \text{G} \), corresponding to \( \epsilon_B = 0.93 \text{ eV cm}^3 \), for the magnetic field, and use \( \epsilon_{\text{ph}}^{(i)}(r) \) presented in the second line from the bottom of Table 3 with \( r = r_0 \) for the photon gas field.

For \( E_e \lesssim 1 \text{ GeV} \), \( \Lambda(E_e, T_i) \approx 1 \), i.e., the Thomson cross section is valid, so that Equation (11) is separable in \( r \) and \( E_e \), leading to a simple expression, \( \epsilon(r) w_T E_e^2 \), with \( \epsilon(r) = \epsilon_B(r) + \sum_{i=0}^{5} \epsilon_{\text{ph}}^{(i)}(r) \). In practice, we find that it is reproduced well by the following form over a wide energy range:

\[ -\frac{\Delta E_e}{\Delta t}_{\text{SY}} \approx \epsilon(r) w_T E_e^{2-\delta}, \quad \delta = 0.075, \] (13)

while \( \epsilon(r) \) depends on \( E_e \), very weakly.

In Figure 5, we demonstrate the energy loss of individual processes separately, those due to “rad,” “ion” and “sic” at the SS against the kinetic energy of the electron \( E_e \) with \( \epsilon_{\text{rad}} = 2 \text{ eV cm}^3 \) and \( \epsilon_{\text{ion}} = 1 \text{ H atoms cm}^3 \), where we plot the above empirical relationship (13) (dotted curve) and the energy gain due to the reacceleration (≡ “rea”; see the next subsection) together. One finds that it reproduces satisfactorily the exact one (Equation (11)) with Equations (12a) and (12b).

### 3.2. Energy Gain due to the Reacceleration

In Paper II, we present the energy gain per unit time due to the reacceleration

\[ \frac{1}{E_e} \left[ \frac{\Delta E_e}{\Delta t} \right]_{\text{rea}} = n(r) w_{\text{rea}}[E_e/\text{GeV}]^{-\alpha}, \] (14)

with

\[ w_{\text{rea}} = c \zeta_0; \quad \zeta_0 \approx \frac{4}{9} \frac{v_M^2}{n_0^* D_0^*}, \] (15)

where \( w_{\text{rea}} = 15.0 \times 10^{-16} \text{ cm}^3 \text{ s}^{-1} \) in the case of, for instance, \( \zeta_0 = 50 \text{ millibarn (mbarn)} \), corresponding to the choice of a parameter set with \( v_M = 20–30 \text{ km s}^{-1} \) (AlfVén velocity), \( n_0^* = 0.06–0.14 \text{ H atoms cm}^{-3} \), and \( D_0^* = 2 \times 10^{28} \text{ cm}^2 \text{ s}^{-1} \). The smallness of the gas density with \( n_0^* \ll 1 \text{ H atoms cm}^{-3} \) indicates that the reacceleration process occurs even at some distance from the GP.

The fluctuation in the energy gain due to the reacceleration is given (Gaisser 1990; Paper II) by

\[ \frac{1}{E_e} \left[ \frac{\Delta E_e^2}{\Delta t} \right]_{\text{rea}} = \frac{1}{2} n(r) w_{\text{rea}}[E_e/\text{GeV}]^{-\alpha}. \] (16)
level of at most $10^{-3}$ at energies of 1–100 TeV (Sakakibara 1965; Nagashima et al. 1989; Cutler & Groom 1991). This is the reason why even the simplest leaky-box model and/or the simplified diffusion model such as, for instance, constant gas density and constant diffusion coefficient without spatial gradient, reproduces the CR hadronic components so well (Berezinskii et al. 1990).

Now, corresponding to the simplification (Equation (19)) for $n(r)$, we assume the following simple exponential type form for $\epsilon(r)$ as well:

$$\bar{\epsilon}(r) = \bar{\epsilon}_0 \exp[-(r/r_\epsilon + |z|/z_\epsilon)],$$

(20)

where $\bar{\epsilon}_0$ is the (interpolated) average energy density of the ISRF at the GC and two parameters, $r_\epsilon$ and $z_\epsilon$, correspond to the scale heights for the spatial gradients, almost independent of the energy. Typically $[\bar{\epsilon}_0; r_\epsilon, z_\epsilon] \approx [16 \text{ eV cm}^{-3}; 4 \text{ kpc}, 0.75 \text{ kpc}]$ (Ishikawa 2010).

However, while the simplifications given by Equations (19) and (20) are applied for electrons (and hadrons), we stress here that those presented in Section 2 are actually used for $D_\gamma$’s as discussed in Section 6, namely $n_\gamma(r)$ with “$h$” ≡ H1, HII, H2 for $n(r)$, and $\epsilon_\gamma^{(i)}(r)$ ($i = 0–5$) for $\epsilon(r)$ with weak energy dependences in $\epsilon_\gamma^{(i)}$ as presented in Table 3. This is because $D_\gamma$’s produced by CR hadrons and electrons are directly affected by the environment of ISM and ISRF around the birth site of the produced $\gamma$’s.

4. DIFFUSION EQUATION FOR ELECTRON COMPONENT

4.1. Basic Equation

The transport equation for the electron density, $N_e(r; E_e, t)$, is given by (Berezinskii et al. 1990),

$$\left[ \frac{\partial}{\partial t} - \nabla \cdot D(r; E_e) \nabla + \Delta E \right] N_e(r; E_e, t) = Q(r; E_e, t),$$

(21)

with

$$\Delta E = -\frac{1}{2} \frac{\partial^2}{\partial E_e^2} \left( \int \Delta E_{\|}^2 \frac{\Delta E_{\|}^2}{\Delta t} \right),$$

(22)

see Equations (19) and (20) for the average energy loss (gain) in all the processes, with the replacement of $[n(r), \epsilon(r)]$ in Equation (17) by $[\bar{n}(r), \bar{\epsilon}(r)]$, and Equation (16) for the fluctuation of the energy gain in the reacceleration process, respectively. For the diffusion coefficient and the source spectrum, we assume (note $\nu \approx c$ and $R_e \approx E_e$)

$$D(r; E_e) = E_e^\nu D(r), \quad Q(r; E_e, t) = E_e^{-\gamma} Q(r; t),$$

(23)

with

$$D(r) = D_0 \exp(r/r_D + |z|/z_D),$$

(24a)

$$Q(r; t) = Q_0(t) \exp[-(r/r_Q + |z|/z_Q)].$$

(24b)

In Table 4, we summarize parameters related to the scale heights, $r_D$, $z_D$, ..., which often appear in the present paper.

Now, remembering $\mathcal{W}_i(E_e) \gg \mathcal{W}_i(E_e)$ in the HE region, say, $E_e \gtrsim E^+_* (\approx 7 \text{ GeV})$, and vice versa in the LE region, $E_e \lesssim E^-_*$. 

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**Figure 6.** Numerical values of $\mathcal{W}_i(E_e) \equiv w_{\text{rad}}(E_e)E_e + w_{\text{ion}}(E_e) - w_{\text{rea}}E_e^{1-\alpha}$ against $E_e$ for $\zeta_0 = 40$–55 mbar, where the screening effect for $w_{\text{rad}}$ is taken into account.

**3.3. Total Energy Loss and Gain**

As discussed in the last two subsections, we have the total average energy loss and energy gain per unit time

$$\left\langle \frac{\Delta E_e}{\Delta t} \right\rangle_{\text{all}} = n(r)\mathcal{W}_n(E_e) + \epsilon(r)\mathcal{W}_\epsilon(E_e),$$

(17)

where

$$\mathcal{W}_n(E_e) = w_{\text{rad}}(E_e)E_e + w_{\text{ion}}(E_e) - w_{\text{rea}}E_e^{1-\alpha},$$

(18a)

and

$$\mathcal{W}_\epsilon(E_e) \equiv w_{\text{rad}}^2E_e^{-\delta}, \quad \delta = 0.075.$$  

(18b)

One might note from Equation (18a) that there exist two energies, $E_e^-$ and $E_e^+$, at which the first term proportional to $n(r)$ on the right-hand side of Equation (17) becomes null. In Figure 6, we demonstrate $\mathcal{W}_n(E_e)$ against several choices of $\zeta_0$, and find $[E_e^-, E_e^+] \approx [0.1, 7] \text{ GeV}$ in the case of $\zeta_0 = 50 \text{ mbar}$. Namely, the synchrotron-IC is dominant for $E_e < E_e^+$, while the reacceleration is effective for $E_e^+ \lesssim E_e \lesssim E_e^\gamma$ and the ionization for $E_e \lesssim E_e^-$. As discussed in Section 2, the total number density of the ISM gas, $n(r)$, and the total energy density of the ISRF, $\epsilon(r)$, have complicated spatial distributions coming from local irregularities, which are not yet well established. On the other hand, in our previous papers, we have assumed a simple exponential-type form for $n(r)$, smearing out the local irregularities,

$$\bar{n}(r) = \bar{n}_0 \exp[-(r/r_\alpha + |z|/z_\alpha)],$$

(19)

greatly simplifying the complicated distributions given by Equations (1) and (2) with Tables 1 and 2, where $\bar{n}_0$ is the (interpolated) average gas density with approximately 1.5 H atoms cm$^{-3}$ at the GC, and $[r_\alpha, z_\alpha] \approx [20, 0.2] \text{ kpc}$.

In spite of such a simplification, we have found that our model reproduces remarkably well the experimental data on hadronic components. This tells us that charged CR components are well mixed during their propagation in the Galaxy over a residence time of approximately $10^7 \text{ yr}$, effectively smearing the local inhomogeneous structure of the ISM. In fact, it is well established that the anisotropy amplitude of CRs is of the order of $10^{-3}$ at energies of 1–100 TeV (Sakakibara 1965; Nagashima et al. 1989; Cutler & Groom 1991). This is the reason why even the simplest leaky-box model and/or the simplified diffusion model such as, for instance, constant gas density and constant diffusion coefficient without spatial gradient, reproduces the CR hadronic components so well (Berezinskii et al. 1990).
the energy loss given by Equation (17) is written as

$$\left( \frac{\Delta E_e}{\Delta t} \right)_{\text{all}} \approx \left\{ \begin{array}{l} \tilde{\epsilon}(r) \mathcal{W}_e(E_e) + O(\tilde{n}(r) \mathcal{W}_n(E_n)), E_e \gtrsim E_e^*, \\
\tilde{n}(r) \mathcal{W}_n(E_n) + O(\tilde{\epsilon}(r) \mathcal{W}_e(E_e)), E_e \lesssim E_e^*, \end{array} \right. \tag{25a}$$

so that in the following discussion, we first solve the diffusion Equation (21) in the HE region, regarding \( \tilde{n}(r) \mathcal{W}_n \) as a perturbative term, where we can neglect the fluctuation term due to the reacceleration. Next we give the solution in the LE region, regarding \( \tilde{\epsilon} \mathcal{W}_e \) as a perturbative term by contrast, which is completely the same as the former one after replacing \( [\tilde{\epsilon}, \mathcal{W}_e] \) with \([\tilde{n}, \mathcal{W}_n]\) (and vice versa), while we have to take the fluctuation term, \( \left( \Delta E_e^2 \right)_{\text{reac}} \), into account in this case.

Thus for the steady state \((\partial/\partial t) = 0\), the solution of Equation (21) in the HE region is divided into three

$$N_{e,\epsilon} \simeq N_{e,\epsilon}^{(0)} + \tilde{N}_{e,\epsilon}^{(0)} + N_{e,\epsilon}^{(1)}, \tag{26a}$$

where the first term is a principal one coming from \( \tilde{\epsilon} \mathcal{W}_e \), the second term corresponds to the perturbative term from \( \tilde{n} \mathcal{W}_n \), and the third term to the fluctuation term due to the reacceleration given by the second term of the right-hand side in Equation (22), while it is negligible in practice, \( N_{e,\epsilon}^{(0)} \approx 0 \).

The solution in the LE region is similarly given by replacing the suffix \( \epsilon \) with \( n \) (and vice versa), but we cannot neglect the fluctuation term \( N_{e,n}^{(1)} \) in contrast,

$$N_{e,n} \simeq N_{e,n}^{(0)} + \tilde{N}_{e,n}^{(0)} + N_{e,n}^{(1)}, \tag{26b}$$

The first term in Equation (26a) is written immediately as

$$N_{e,\epsilon}^{(0)}(r; E_e) = \int_0^\infty \Pi_{e,\epsilon}^{(0)}(r; y) f_{e,\epsilon}^{(0)}(0; y; E_e) dy, \tag{27}$$

where \( \Pi_{e,\epsilon}^{(0)} \) and \( f_{e,\epsilon}^{(0)} \) satisfy

$$\left[ \tilde{\epsilon}(r) \frac{\partial}{\partial y} - \nabla \cdot (D(r) \nabla) \right] \cdot \Pi_{e,\epsilon}^{(0)}(r; y) = Q(r) \delta(y), \tag{28a}$$

$$\left[ c E_e \frac{\partial}{\partial y} - \frac{\partial}{\partial E_e} \mathcal{W}_e(E_e) \right] \cdot f_{e,\epsilon}^{(0)}(y; E_e) = 0, \tag{28b}$$

with \( f_{e,\epsilon}^{(0)}(0; E_e) = E_e^{-\gamma - \alpha} \).

4.2. Solution in the Steady State

It is possible to solve exactly Equation (28a) with use of the procedure presented in Paper I, after replacing \( \tilde{n}(r) \) by \( \tilde{\epsilon}(r) \), and we present here only the critical term related to \((r, z, \gamma)\), omitting constant terms such as \( Q_0 \) and \( \epsilon_0 \) (see Appendix B for the full form),

$$\Pi_{e,\epsilon}^{(0)}(r; y) \propto \exp[-\tilde{\epsilon}_r(y - |z|/z_D)], \tag{29}$$

$$\tilde{\epsilon}_r \simeq \frac{D_e}{\tilde{\epsilon}_r c z_D} \left( 1 + \frac{1}{\kappa} \right), \quad \kappa = 1 + \frac{z_D/\tilde{\epsilon}_D}{z_d}, \tag{30}$$

with \( D_e \equiv D(r, 0), \tilde{\epsilon}_r \equiv \tilde{\epsilon}(r, 0) \). As \( \Pi_{e,\epsilon}^{(0)} \) is of the form of \( e^{-\tilde{\epsilon}_r \gamma} \), the Laplace transform of \( f_{e,\epsilon}^{(0)} \) with respect to \( \gamma \), \( F_{e,\epsilon}^{(0)}(E_e) \), is sufficient for our purpose to obtain the electron density,

$$F_{e,\epsilon}^{(0)}(E_e) = \int_0^\infty e^{-\tilde{\epsilon}_r \gamma} f_{e,\epsilon}^{(0)}(y; E_e) dy,$$

thus we have immediately from Equation (28b)

$$F_{e,\epsilon}^{(0)}(E_e) = \frac{c}{\mathcal{W}_e(E_e)} \int_{E_0}^\infty dE_0 E_0^{-\gamma} e^{-Y_{r,e}(E_e, E_0)}, \tag{31}$$

with

$$Y_{r,e}(E_e, E_0) = c \tilde{\epsilon}_r \int_{E_0}^{E_e} \frac{E^{\alpha}}{\mathcal{W}_e(E)} dE.
$$

In the HE limit, \( E_e \gg 1 \) GeV, using Equation (18b), we find

$$F_{e,\epsilon}^{(0)}(E_e) \simeq \frac{c E_e^{-(\gamma + 1 - \delta)}}{(1 - \alpha - \delta) w_T} \left[ 1 - \frac{\gamma + \alpha - \delta - 2}{c \tilde{\epsilon}_r E_e^2 w_T} + \cdots \right].$$

giving a spectral index with \( \gamma + 1 - \delta \), where \( \delta(= 0.075) \) comes from the effect of the Klein–Nishina cross section. Practically, however, it must be softer than the above index because of the exponential cutoff with \( e^{-E_e/E_{\text{cut}}} \) in the electron injection spectrum somewhere around 20 TeV (Reynolds & Keohane 1999; Hendrick & Reynolds 2001; Yamazaki et al. 2006).

Now the principal term, \( N_{e,\epsilon}^{(0)} \), in Equation (26a) for the electron density in the HE region, \( E_e \gtrsim E_e^* \), is given by

$$N_{e,\epsilon}^{(0)}(r; E_e) \propto F_{e,\epsilon}^{(0)}(E_e) e^{-|z|/z_D}, \tag{32}$$

while the perturbative term, \( \tilde{N}_{e,\epsilon}^{(0)} \), is obtained by the use of the iteration method as presented in Appendix B.1, giving.
$N_{e,n}^{(1)}/N_{e,n}^{(0)} \sim 10\%$ with the first iteration for $E_e \gtrsim 1$ GeV at SS as shown in Figure 24(a). In practice, we perform only the first iteration, neglecting the second and higher iterations. Full form of $N_{e,n}(r; E_e)$ is given by Equation (B3).

The numerical procedure in the LE region is similar to that in the HE region mentioned above by replacing the suffix “$e$” with “$n$” (and vice versa), while we have to take into account the third term in Equation (26b), $N_{e,n}^{(1)}$, corresponding to the fluctuation. We find again that the perturbative term, $\tilde{N}_{e,n}^{(0)}$, is obtained by the use of the iteration method as presented in Appendix B.2, giving $\tilde{N}_{e,n}^{(0)}/N_{e,n}^{(0)} \sim 10\%$ with the first iteration for $E_e \lesssim 10$ GeV, as shown in Figure 24(b).

On the other hand, the numerical procedure in the fluctuation effect due to the reacceleration is a little bit cumbersome, which is presented in Appendix B.3. We give an example of the ratio, $N_{e,n}^{(1)}/[N_{e,n}^{(0)} + N_{e,n}^{(1)}]$, at SS for the first iteration in Figure 7, where $\tilde{\eta}$ is the effective ratio of the energy density to the gas density defined by Equation (B6), approximately with 2 eV. One finds that it is significant around 0.3–1.5 GeV in the case of $\tilde{\eta} = 50$ mbarn, boosting the solution without the fluctuation, $N_{e,n}^{(0)} + N_{e,n}^{(1)}$, by approximately 25%. So we perform only the first iteration also for $N_{e,n}^{(1)}$ in the LE region, as the contribution coming from the second and higher iterations is at most of the magnitude of a few percent or less (Ishikawa 2010). Full form of $N_{e,n}(r; E_e)$ is given by Equation (B15).

Finally, we give the electron density covering all energies so that it continues smoothly at the energy $E_e$ between the HE and LE regions at the SS ($r = r_0$), with $E_e \approx E_e^c$ in practice, but not always $E_e = E_e^c$.

$$\frac{N_{e}(r; E_e)}{N_{e,0}} = \begin{cases} \frac{F_{\odot}(E_e)}{F_{\odot}(E_e^c)} e^{-|z|/z_D} : & \text{for } E_e \geq E_e^c, \\ \frac{F_{\odot,n}(E_e)}{F_{\odot,n}(E_e^c)} e^{-|z|/z_D} : & \text{for } E_e \leq E_e^c, \end{cases}$$

(33a)

$$\left. \frac{\partial}{\partial E_e} F_{\odot,n}(E_e) \right|_{E_e = E_e^c} = \left. \frac{\partial}{\partial E_e} F_{\odot,n}(E_e) \right|_{E_e = E_e^c},$$

(33b)

see Equations (B4) and (B13) for $F_{\odot}(E_e)$ and $F_{\odot,n}(E_e)$, respectively, and $N_{e,0}$ is determined by the normalization with the experimental data as discussed in Section 5.

In Figure 8, we show the numerical results of $N_{e}(r_0; E_e)/N_{e,0}$ in two cases, (1) $[\tilde{\eta}_0, \tilde{\eta}_0, \tilde{\eta}_0] = [50, 180, 30 \text{ eV}^{-1}]$ mbarn with $\alpha = \frac{1}{4}$ (reacceleration with Kolmogorov-type spectrum in hydromagnetic turbulence) and (2) $[0, 90, 15 \text{ eV}^{-1}]$ mbarn with $\alpha = \frac{1}{4}$ (no reacceleration with Kraichnan-type spectrum) for $\tilde{\epsilon}_0/\tilde{n}_0 = 2 \text{ eV}$ with $\beta (\equiv \gamma + \alpha) = 2.6, 2.7,$ and 2.8, see Equation (30) with $r = r_0$ for $\tilde{\eta}_0$, where we assume $E_{\text{cut}} = 20 \text{ TeV}$ (Reynolds & Keohane 1999; Hendrick & Reynolds 2001; Yamazaki et al. 2006) in the electron injection spectrum with $E_{\gamma}^{-\gamma} e^{-E_{\gamma}/E_{\gamma,r}}$, and the results show the use of both Klein–Nishina (solid curves) and Thomson (dotted curves) cross sections.

We find two critical points in Figure 8. First, those by the former cross section give approximately 40–50% (20%–30%) larger than those by the latter at 1 TeV (100 GeV), where the density is normalized at $E_e = 10$ GeV, leading to significantly harder spectra than those with the Thomson cross section, as expected. Similar results are also recently reported by Delahaye et al. (2010), while their main purpose is to study the nearby sources of electron and the positron excess problem as well, which are outside the scope of the present paper.

Second, the reacceleration effect is significant in the energy region less than 10 GeV as compared to the curves without the reacceleration process. Unfortunately, however, it is difficult to observe such a signal in the direct experimental data on the electron component because of the modulation effect in the LE region $\lesssim 5$ GeV, which masks the electron flux boosted by the reacceleration, see Figure 13, even if it actually occurs.
radial distance \( r \) for the (1) electron and (2) proton components, both normalized at the SS for four energies, 0.1, 1, 10, and 100 GeV, with \( \beta = 2.7 \), where the scale heights are set as \( [\tilde{r}_n, \tilde{r}_e] = [30, 8] \) kpc and \( [z_D, z_e] = [3, 0.2, 0.75] \) kpc (see Table 4 for \( \tilde{r}_n \) and \( \tilde{r}_e \)). We plot the results of Hunter et al. (1997; square symbols) and Strong et al. (1988; thin filled histogram) together, where the former are based on the assumption that the CR density is coupled to the gas density at each galactocentric quadrant, and plotted separately for four galactocentric quadrants, I, II, III, and IV. We find that the radial dependence of the electron density, \( N_e(r; E_e) \), is much stronger than that of the proton density, \( N_p(r; E_p) \), in the energy region of 1–100 GeV as expected, while the other two authors assume no spatial dependence in the energy spectrum, namely the shape of the energy spectrum at the SS is the same everywhere in the Galaxy.

5. ELECTRON-INDUCED \( \gamma \)-RAY SPECTRUM

For convenience in the following discussion, we summarize two cross sections in Table 5, \( \sigma_{EB}(E_e, E_\gamma) \) and \( \sigma_{IC}(E_e, E_\gamma; E_{ph}) \), each for the bremsstrahlung (abbreviated as “EB” for subscript attached here and in the following) and the IC processes respectively, where \( E_{ph} \) is the energy of target photon before scattering. For the bremsstrahlung process, we present the cross section in the case of only one-electron atoms (\( Z = 1 \)); see Gould (1969) for two-electron atoms (\( Z = 2 \)).

### Table 5

| Bremsstrahlung (EB) | Inverse Compton (IC) |
|---------------------|-----------------------|
| \( \sigma_{EB}(E_e, E_\gamma; E_{ph}) = \sigma_{EB}^{(0)} \phi_{EB}(x, E_\gamma) \frac{dx}{x} \) | \( \sigma_{IC}(E_e, E_\gamma; E_{ph}; dE_\gamma) = \phi_{IC}(x, q) \frac{dx}{X} \) |
| \( \sigma_{EB}^{(0)} = 4\alpha_j Z(Z + 1) \left( \frac{e^2}{m_e c^2} \right)^2 \left( \alpha_j = \frac{1}{137} \right) \) | \( \phi_{IC}(x, q) = 8\pi \phi_{ph}(x, q) \) |
| \( \phi_{EB}(x, E_\gamma) = (1 + (1 - x^2) \phi_2(x) - \frac{2}{3} (1 - x) \phi_2(x) \) | \( q \equiv q(x, X) = x \frac{1}{1 - x} \) |
| \( \phi_1(x) = 1 + \int_x^1 \phi_0(y) \left( 1 - \frac{y}{x} \right)^2 \frac{dy}{y} \) | \( X \equiv X(E_e, E_{ph}) = \frac{k}{\sigma_{IC}^2} = \frac{E_{ph} E_e}{m_e c^2/2} \) |
| \( \phi_2(x) = \frac{5}{6} + \int_x^1 \phi_0(y) \left( 1 + \ln \frac{y^2}{y^2 - 1 - \frac{2}{1 + x}} \right) \frac{dy}{y} \) | \( k \equiv k(E_{ph}, T_{ph}) = \frac{E_{ph} k_{ph}}{E_{ph}} \) |
| \( \phi_0(y) = 1 - \left( \frac{1 + y^2/(2a_j Z)^2}{1 + y^2/(2a_j Z)^2} \right)^2 \) | \( \phi_4(x) \equiv \phi_4(E_e, T_{ph}) = \frac{m_e c^2/2}{\sqrt{k_{ph} E_{ph}}} \) |

### Notes

1. With \( x = E_\gamma/E_e \), where \( E_e \) is the incident energy of electrons, \( E_\gamma \) is the energy of the produced \( \gamma \)-ray’s, and \( E_{ph} \) is the energy of the target photon before electron scattering.

2. To do so, we use the observational data on the electron intensity at the SS, \( dI_e^{*}/dE_e \), which is...
related to the electron density by

$$\frac{dI_e^\circ}{dE_e}(E_e) = \frac{c}{4\pi} N_e(r_\odot; E_e).$$

In practice, we normalize the electron density at $E_e = 10$ GeV with use of the most recent data (see Figure 14), where the solar modulation effect is negligible,

$$cN_e^\circ \equiv cN_e(r_\odot; E_e) = 2.26 \text{ m}^2 \text{s}^{-1} \text{ GeV}^{-1},$$

corresponding to $dI_e^\circ/dE_e = 0.180 \text{ m}^{-2} \text{s}^{-1} \text{ GeV}^{-1}$ at $E_e = 10$ GeV in Figure 14, while $E_e = 100$ GeV (per nucleon) for the hadron-induced $\gamma$'s ($\pi^0 \rightarrow 2\gamma$) with $cN_p(r_\odot; E_e) = 6.16 \text{ m}^{-2} \text{s}^{-1} \text{ GeV}^{-1}$ (Paper V). One should keep in mind that the uncertainty in the normalization is of magnitude as large as 10%.

Thus taking care of the terms related to $r$, we have

$$q_{\text{EB}}(r; E_\gamma) = \frac{\epsilon_{\text{EB}}(r, E_\gamma) I_{\text{EB}}(r)}{n(r) w_{\text{EB}}^\circ N_e^\circ} = e^{-|\ell|/z_0} \int_0^1 \Phi_{\text{EB}}(x, E_\gamma; E_e) F_i(E_e) \frac{dx}{F_\odot(E_e) x^2},$$

for $E_\gamma \geq E_e$ with $E_e = E_\gamma/x$, and $w_{\text{EB}}^\circ = c\epsilon_{\text{EB}}^\circ = 1.39 \times 10^{-16} \text{ cm}^3 \text{s}^{-1}$ for the hydrogen gas ($z = 1$), where one should take care of the energy range $E_e \leq E_\gamma$.

Next we consider the emissivity of $\gamma$'s coming from the IC process, which is somewhat complicated, as there are several kinds of target photons with different energy density as well as with different scale heights in the spatial gradient. Here we present a result taking into account the six wavelength bands in $\epsilon_{\text{ph}}^{(i)}(r)$ ($i = 0-5$) (see Equation (7) and Table 3),

$$q_{IC}^{(i)}(r; E_\gamma) = \frac{\epsilon_{IC}^{(i)}(r, E_\gamma) I_{IC}(r)}{\epsilon_{\text{ph}}(r) w_{\text{IC}}^\circ N_e^\circ} = e^{-|\ell|/z_0} \int_0^1 \Phi_{\text{IC}}^{(i)}(x, E_\gamma; E_e) F_i(E_e) \frac{dx}{F_\odot(E_e) x^2},$$

with $\epsilon_{\text{ph}}^{(i)}(r)$ in eV cm$^{-3}$, where $\Phi_{\text{IC}}^{(i)}(x, E_\gamma; E_e)$ is given by Equation (C2), see Appendix C for the details.

Let us show the numerical results for two cases of emissivity in Figure 10, (1) $r/r_\odot = 0.5, 1.0, 1.5$ with $z = 0$ in the GP, and (2) $z = 0.2, 0.4, 0.6$ kpc with $r = r_\odot$ normal to the GP at SS, assuming $\beta = \gamma + \alpha = 2.7$, where we present separately those coming from $\pi^0$ (solid curves), EB (broken curves), and IC (dotted curves). One finds that EB-$\gamma$'s and $\pi^0$-$\gamma$'s are comparable around 50 MeV, and IC-$\gamma$'s and $\pi^0$-$\gamma$'s around two energies, $\sim 20$ MeV and $\sim 1$ TeV.

SEE Paper V for the emissivity originating in $\pi^0$, $q_{\pi^0}(r; E_\gamma)$, while we use more realistic gas density, $n(r)$, in the present paper. Note also in $q_{\gamma}$ that the semi-empirical production cross section of $\gamma$'s, $\sigma_{pp-\gamma}(E_p, E_\gamma)$, in proton–proton collision we use is valid over very wide energy ranges, 1 GeV–1 PeV, nicely reproducing various kinds of physical quantities such as pseudo-rapidity, energy spectrum, multiplicity, etc., obtained by both the accelerator and CR experiments with local target layer (Suzuki et al. 2005).

Once we have the emissivity of $\gamma$'s induced by the interaction between the electrons and the media of ISM and ISRF, we can obtain immediately the intensity of $\gamma$’s observed at the SS ($r = r_\odot$), coming from the direction $\theta(l, b)$,

$$\frac{d^3I_\odot^\circ}{dE_\gamma dl ds(\sin b)} = \frac{1}{4\pi} \int_0^\infty q_\gamma(r; E_\gamma) ds,$$

with

$$q_\gamma(r; E_\gamma) = q_{\text{EB}}(r; E_\gamma) + \sum_{i=0}^5 q_{IC}^{(i)}(r; E_\gamma),$$

where the integration with respect to $s$ is performed along the arrival direction of $\gamma$’s, $\theta(l, b)$, at the SS, and $r(r, z)$ is bound to $(s; l, b)$ as follows:

$$r(s; l, b) = \sqrt{r_\odot^2 + s^2 \cos^2 b - 2r_\odot s \cos b \cos l},$$

$$z(s; b) = s \sin b.$$

6. COMPARISON WITH THE OBSERVATIONAL DATA

6.1. Critical Parameters

We assume that the source distribution of electron component, $Q(r; E_e)$, is the same as that of the hadronic component except for the cutoff electron energy, for instance $E_{\text{cut}} \approx 20$ TeV, with the supernova remnants as the main energy supply, while the pulsars and pulsar wind nebulae might contribute to them as well, particularly to positrons and electrons (for instance, Delahaye et al. 2010). So the galactic parameters used in the
present work are essentially the same as those appearing in Papers I–V, and we summarize them briefly in the following.

The recent observational data on the energy spectra of CR hadronic components give indices with $2.74 \pm 0.08$ for proton (Derbina et al. 2005), and with a common value of $\sim 2.7$ for nuclei between the oxygen and iron (Müller 2009), whereas there still remains uncertainty for helium, for instance with $2.68 \pm 0.05$ by JACEE (Asakimori et al. 1998) in contrast to $2.78 \pm 0.20$ by RUNJOB (Derbina et al. 2005). Note that PAMELA (Picozza et al. 2007) reports recently a common index of 2.73 in both the proton and helium spectra, albeit the energy region is limited below 500 GeV. Any way, the spectrum index $\beta$ of proton must lie well within 2.7–2.8 in the HE region at $\beta = 2.73$ in both the proton and helium spectra, albeit the energy to 2.

Critical parameter only three critical parameters, $\{\beta, \alpha, \bar{\sigma}_0\}$ needed to compare with the observational data, aside from two critical indices, $\{\alpha, \beta\}$, that of $\bar{\sigma}_0$: normalized average path length, $\bar{\sigma}_0$: normalized lifetime of an isolate with $10^6$ yr.

For electron components, the additional parameter newly appears, $\bar{s}$, given by Equation (30), physical meaning of which is essentially the same as $\bar{\sigma}$; i.e., while the inverse of $\bar{\sigma}$ gives the average path length, $\bar{s}$, in units of $\text{cm}^2$ in ISM as discussed in Paper I, that of $\bar{s}$ corresponds to the average path length, $\bar{s}$, in units of eV $\text{cm}^{-2}$ in ISRF, namely the total amount of photon-gas energy that CR has passed through the ISRF.

Now from Equation (33), one should remark that there appear only three critical parameters, $\{\bar{s}, \bar{\sigma}_0, \bar{s}_0\}$, in $F_{r,s}(E_s)$ and $F_{r,\alpha}(E_\alpha)$ needed to compare with the observational data, aside from two critical indices, $\{\alpha, \beta\}$, note that various galactic parameters such as the diffusion constant, gas density, energy density, their scale heights, etc. are all involved implicitly in these three ones.

6.2. Charged Components

6.2.1. Hadron Components

As we have presented the experimental results on CR hadron components in the past papers (Papers I–IV), we give here only three kinds of secondary-to-primary ratio with new data, $B/C$, sub-Fe/Fe, and $\bar{\rho}/\rho$, that have since become available. See Paper III for the secondary unstable nuclei, while new data are still not available.

In Figure 11, we present $B/C$ and sub-Fe/Fe, plotted together with new ones from CREAM (Ahn et al. 2008) and TRACER (Müller 2009), where we plot also RUNJOB (Derbina et al. 2005) data for reference, while the data quality is rather poor with large atmospheric correction. We compare our numerical results with the data for two models, (1) reacceleration with the set of $\{\bar{s}, \bar{\sigma}_0\} = [50, 150–300] \text{mbarn}$, for $\alpha = \gamma$, and (2) no reacceleration with $[0, 75–150] \text{mbarn}$, for $\alpha = \frac{1}{10}$. It is still not clear which model reproduces the experimental data more satisfactorily. As is well known, the advantage of the former explains naturally the drop of the ratio in the lower energy region around ACE/CRIS (Davis et al. 2000) without assuming an ad hoc drop in the path length distribution.

Next we present $\bar{\rho}/\rho$ in Figure 12, plotted together with new data from PAMELA (Adriani et al. 2010), where we present numerical curves with several sets of modulation parameters, 0.2–1.5 GV, for the reacceleration model shown in Figure 11(a). One finds that our result is in good agreement with the PAMELA in the HE region around 100 GeV, where the modulation effect is absolutely negligible.

6.2.2. Electron Component

Let us present the electron data separately before and after Fermi, where “electron” denotes both electron and positron. First in Figure 13 we present the electron energy spectrum before Fermi, while the experimental data are presented for those reported in the period from 1994 to 2008 alone (Golden
et al. 1994; Kobayashi et al. 1999; DuVernois et al. 2001; Torii et al. 2001, 2006; Aguilar et al. 2002; Chang et al. 2008), and also plotted are the data (filled purple squares) for reference after applying a demodulated correction to HEAT data, HEAT-LIS, (DuVernois et al. 2001) using the force-field approximation with the modulation parameter of 755 MV (670 MV) for the 1994 (1995) data.

The numerical curves are normalized at 10 GeV with two indices, $\beta = 2.7, 2.8$, assuming two models—(1) reacceleration and (2) no reacceleration each with the same parameter sets as those used in Figure 11, while we assume additionally two cases of $\zeta_0$, [20, 30] eV$^{-1}$ m barn for the reacceleration (1), and [10, 15] eV$^{-1}$ m barn for no reacceleration (2). Aside from the prominent spectral features around 500 GeV appearing in ATIC (Chang et al. 2008) and PPB-BETS (Torii et al. 2006) data, our model with the reacceleration reproduces the data well in the higher energy region, $\geq 10$ GeV, in Figure 13(a), where the solar modulation effect is small. On the other hand, the model without reacceleration in Figure 13(b) is somewhat difficult to fit to the demodulated HEAT-LIS data.

Now, in Figure 14 we present the most recent data obtained by Fermi (Abdo et al. 2009; Ackermann et al. 2010) and H.E.S.S. (Aharonian et al. 2009) together with those presented in Figure 13, where numerical curves are the same as shown in Figure 13(a). We find that both Fermi and H.E.S.S. data do not exhibit the prominent bump around 500 GeV reported by ATIC and PPB-BETS, with both giving a spectrum falling with energy as $E^{-3}$ up to 1 TeV, which is not inconsistent with emulsion chamber data (Kobayashi et al. 1999) within the statistical errors. Looking Figure 14, however, we find that Fermi and H.E.S.S. data seem to deviate systematically from numerical curves with an enhancement by 20%–30% around 500 GeV, indicating still some additional local sources of HE CR electrons, which will be discussed again in Section 7.

6.3. Diffuse $\gamma$-ray Component

6.3.1. Isotropic Background $\gamma$-rays

$\gamma$'s near the GP are mainly hadron-induced ($\pi^0 \rightarrow 2\gamma$) and electron-induced (EB + IC). In addition to these two components, we have isotropic background $\gamma$'s (BGs) with various origins such as extragalactic sources (EGs), unidentified sources, instrumental sources, DM, etc., so that the BGs depend on individual detectors with different sensitivity in energy and the angular resolution, while depending on the propagation model as well. Therefore it is not an easy task to estimate the extragalactic $\gamma$'s, while its origin is one of the fundamental problems in astrophysics, studied in so many papers with various candidates—unresolved blazars (e.g., Stecker & Salamon 1996; Chiang & Mukherjee 1998; Mücke & Pohl 2000), intergalactic shocks produced by the assembly of large-scale structures (e.g., Loeb & Waxman 2000; Totani & Kitayama 2000; Miniati et al. 2000; Gabici & Blasi 2003), DM annihilation (e.g., Bergström 2000; Ullio et al. 2002; Ahn et al. 2007), etc. In the present paper, however, we use the acronym “BGs” all together for $\gamma$'s other than those induced by $\pi^0$, EB (bremsstrahlung) and IC, while acknowledging EGRET and Fermi teams have estimated very carefully the EG-$\gamma$ intensity.

In Figure 15, we present an example of EGRET data (histogram, source-subtracted; Hunter et al. 1997) together...
with numerical curves on the latitudinal distribution averaged over full longitude ranges, $0^\circ$–$360^\circ$ with the energy interval 300–500 MeV, where we give the contributions of $\pi^0$ (solid red), EB (dotted red), IC (broken red), BG (solid black), and total flux, $\pi^0$+EB+IC+BG (heavy solid red), assuming $[\bar{\sigma}_0, \bar{\sigma}_0, \bar{\sigma}_0] = [50, 180, 30$ eV$^{-1}]$ mbarn with $[\alpha, \beta] = [4/3, 2.7]$. Here we draw a horizontal line for BG by the use of the least square method so that the histogram is well reproduced, where the fitting is applied for $|b| \leq 60^\circ$ as there remain considerable uncertainties in the latitudinal distribution for both the ISM and ISRF far distant from the GP, see $\Sigma_h(r, z)$ (“$h^\prime = H_1, H_2$” in Equation (2)) and the scale height $z_{ph}$ in Equation (7). It is remarkable that the numerical curve is in good agreement with the data not only in shape, but also in absolute value, except the high latitude around the galactic pole.

In Figure 16, we summarize the intensity of BGs obtained by past works, Kappadath et al. (1996) for COMPTEL, Sreekumar et al. (1998) for EGRET, Strong et al. (2004) for EGRET (revised), and Abdo et al. (2010) for Fermi, where also plotted are those estimated by the present work (see Figures 15 and 17). Dotted line is given by Abdo et al., and the solid curve is used in the present work (see the text), modifying it slightly in the LE region.

(A color version of this figure is available in the online journal.)

Figure 16. BG spectrum obtained by COMPTEL (Kappadath et al. 1996), EGRET (Sreekumar et al. 1998), EGRET (revised; Strong et al. 2004), and Fermi (Abdo et al. 2010), where also plotted are those estimated by the present work using EGRET and Fermi data (see Figures15 and 17). Dotted line is given by Abdo et al., and the solid curve is used in the present work (see the text), modifying it slightly in the LE region.

(A color version of this figure is available in the online journal.)

6.3.2. Spatial Distribution

We present two examples of the latitudinal distributions for EGRET and Fermi (Porter 2009) with the energy interval around [300–500] MeV in Figures 17(a) and (b) respectively, together with our numerical results taking the BG contribution (broken dotted lines) into account mentioned above, $dI_{BG}/dE_{\gamma}$, where
plotted are three curves for each figure with $\beta = 2.6$ (green), 2.7 (red), and 2.8 (blue). One finds that the agreement between the data and the curves is excellent except for the high latitude $|b| \gtrsim 60^\circ$. In these calculations, we take the point-spread function (PSF) with the energy dependence into account, for instance, with $7^\circ$ (HWHM) at 30-50 MeV (Hunter et al. 1997).

Corresponding to the latitudinal distributions as shown in Figures 17(a) and (b), we demonstrate the longitudinal distributions near the GP in Figures 18(a) and (b), where numerical curves are shifted by $\Delta l = +10^\circ$ in both EGRET and Fermi so that experimental data are reproduced more satisfactorily. Again we find that the numerical results are in nice agreement with the data in both shape and absolute value, and consistent with $\beta \sim 2.7$.

### 6.3.3. Energy Spectrum

First, in Figure 19(a) we present the energy spectrum of $\gamma$’s averaged over the field of view with $-60^\circ < l < 60^\circ$ and $|b| < 10^\circ$, where numerical curves with the reacceleration are also presented separately for those coming from $\pi^0$, EB, and IC (all with colored thin solid curves), BG (heavy black solid curve), and total (colored heavy solid curves). Here and in the following, we omit EGRET data in GeV region because of the instrumental problem in detection of $\gamma$’s (Stecker et al. 2008). One finds that the curve with $\beta = 2.7$ (red) is in good agreement with the data in the energy region below 1 GeV, while they deviate slightly from the curve above 1 GeV, with approximately 20% enhancement.

Figure 19(b) reproduces Figure 19(a), but for curves without reacceleration, corresponding to Figures 11(b) and 13(b) (see also Figure 8(b)). The fit is not as good as for the reacceleration model, particularly in the LE region, $E_\gamma \lesssim 200$ MeV, with ~40% enhancement, while with ~20% in the HE region, $E_\gamma \gtrsim 1$ GeV, giving nearly the same enhancement as in the case of (a) with the reacceleration.

Second, in Figures 20 and 21, we present the energy spectra of $\gamma$’s for different sky views (Abdo et al. 2010; see also supplementary material at http://link.aps.org/supplemental/10.1103/PhysRevLett.104.101101). Figure 20 shows those averaged over independent galactic latitude ranges covering low, mid, and high galactic latitudes, (1) $10^\circ < |b| < 20^\circ$, (2) $20^\circ < |b| < 60^\circ$, and (3) $60^\circ < |b| < 90^\circ$ respectively. Figure 21 shows those averaged over different hemispheres, which are (1) centered at the north ($b \geq 0^\circ$; open squares) and south ($b < 0^\circ$; filled squares) galactic poles, (2) the GC ($270^\circ \leq l \leq 90^\circ$), and (3) anticenter ($90^\circ \leq l \leq 270^\circ$), all with the galactic latitudes excluding $|b| < 10^\circ$. In these figures, we subtract $\gamma$’s coming from point sources based on the Fermi catalog.

It is remarkable in Figure 20 that EGRET and Fermi data agree pretty well with each other, overlapping nicely around 0.2–1 GeV, in all latitude ranges. One finds that the numerical curves with $\beta = 2.7$ reproduce generally well both the EGRET and Fermi data in Figures 20 and 21 but Figure 20(c), taking account of the uncertainties in various galactic parameters, particularly in those related to the ISM and ISRF.

On the other hand, in Figure 20(c) for the high latitude, $|b| > 60^\circ$, we have a noticeable enhancement in Fermi with approximately 70% as compared to the numerical curves. To see the deviation more clearly, we present them all together in Figure 22, where we show additionally numerical curves (dotted colors) using EGRET-BG obtained by Sreekumar et al. (1998) for reference (see Figure 16). Figure 23 reproduces Figure 21 with numerical curves using Fermi-BG (dotted colors) in addition to those using Fermi-BG (solid colors).
One finds that the spectrum shapes with EGRET-BG are quite different from the data in the HE region, although the enhancement is rather improved in the energy region \( \lesssim 1\text{ GeV} \), which is discussed again in the next section.

7. DISCUSSION AND SUMMARY

We have studied the diffusion-halo model with stochastic reacceleration, comparing it with the most recent data on hadronic, electronic, and D\(\gamma\) components. We have two particular interests: to find a unified model for CR acceleration and propagation from the viewpoint of astrophysics, and to search for a signal of novel sources such as the primordial black hole (PBH) and/or DM from the viewpoints of particle physics and cosmology. Both are of course closely connected with each other in the sense that the knowledge of the former is decisive in confirming the latter. While several groups (Torii et al. 2006; Chang et al. 2008) have reported the possibility of annihilation and/or decay of DM particles, giving a significant bump in electron flux around 500 GeV, Fermi (Abdo et al. 2009) and H.E.S.S. (Aharonian et al. 2009) give a rather flat spectrum up to 1 TeV without the prominent excess. In the present paper,
however, we have focused our interest rather conservatively on the internal consistency among various CR components from the view point of astrophysics, leaving the puzzle of the possible electron/positron excess to further observations and mutual cross-checks in data analysis among individual groups.

In our past works on the hadronic component, we concluded that the diffusion-halo model with the reacceleration with the parameter set, \([\zeta_0, \sigma_0] = [50, 180] \text{ mbarn} \) with \([\alpha, \beta] = [11, 2.7–2.8]\), is in harmony with the CR hadron data presently available. The most recent data on the B/C ratio by CREAM and TRACER (Figure 11) as well as on the \(\bar{p}/p\) ratio by PAMELA (Figure 12) also support the present model. However, it is worth mentioning here that our interpretation for the energy dependence of the B/C ratio is somewhat different from that by CREAM (Ahn et al. 2008) and TRACER (Müller 2009).

They claim that the index \(\alpha\) favors 0.5–0.6 instead of \(\frac{1}{3}\), resulting in a rapid decrease with energy for the interstellar propagation path length. In contrast to their interpretation, we would like to point out that the value of 0.5–0.6 is not fundamental, but is rather accidental due to the reacceleration effect, namely it is boosted upward around the GeV region by the energy gain, resulting coincidentally in the soft slope with 0.5–0.6 in the energy region 1–100 GeV. The intrinsic one must be \(\frac{1}{3}\) (Kolmogorov type for wave number spectrum in hydromagnetic turbulence), leading to (1) a natural drop in path length distribution in the LE region \(\lesssim 1\) GeV without introducing an artificial break there, as originally proposed by Simon et al. (1986) and (2) a reasonable amplitude in the anisotropy of CR’s with the level of 10–3 in TeV region nowadays established experimentally.

We apply the diffusion-halo model with and without the stochastic reacceleration for the electron and \(\gamma\)-ray components. Apart from the electron anomaly around 500 GeV, we find that the parameter set with the reacceleration, \([\alpha, \beta; \zeta_0, \sigma_0]\), expected from hadron component reproduces rather well both the spectrum shape and the absolute value in both the electron (Figure 13(a)) and \(\gamma\)-ray (Figures 21–24) components, assuming the additional parameter \(\bar{\zeta}_0\) with 20–30 eV\(^{-1}\) mbarn. Physical meanings of the numerical set with \([\zeta_0, \sigma_0] = [50, 180] \text{ mbarn}\) are discussed in Papers I–III in connection with the diffusion constant \(D_\odot\), gas density \(n_\odot\), their scale heights, \(z_D, z_n\), etc., giving reasonable values matched with the observational data.

Let us consider the physical meaning of 20–30 eV\(^{-1}\) mbarn in \(\bar{\zeta}_0\). The relation between \(\bar{\zeta}_0\) and \(\tilde{\zeta}_0\) is given by

\[
\bar{\zeta}_0 = \frac{\bar{n}_\odot + z_D/z_n}{\bar{\epsilon}_\odot} = \frac{\bar{n}_\odot}{\bar{\epsilon}_\odot} + \frac{z_D}{z_n},
\]

see Section 4.2 for \(\tilde{\zeta}_0\) and \(\bar{\zeta}_0\) with \(r = r_\odot\), and Table 4 for \(\bar{\epsilon}_\odot\) and \(\kappa\). Namely, it is closely related to the ratio of the energy density \(\bar{\epsilon}_\odot\) to the gas density \(\bar{n}_\odot\) at the SS, for the smeared energy density, smeared gas density, respectively, and three latitudinal scale heights, \(z_n, z_\odot,\) and \(z_D\). As discussed in Sections 2 and 3.1, we have \(n_\odot = n_{\odot H}, n_{\odot H}, n_{\odot He} = 1.14\) H atoms cm\(^{-3}\), and \(\epsilon_\odot = \epsilon_{\odot H} + \epsilon_{\odot He} \approx 2.3\) eV cm\(^{-3}\), leading to \(\epsilon_\odot/n_\odot \approx 2\) eV. Rememering that the scale heights used in the present paper are \([\zeta_n, \zeta_\odot, z_D] = [0.2, 0.75, 3.0]\) kpc, we find \(\bar{\zeta}_0 = 32\) eV\(^{-1}\) mbarn for \(\bar{\zeta}_0 = 180\) mbarn, giving a consistent result, while the latter with 180 mbarn is expected from the relation, \(\bar{\zeta}_0 \approx D_\odot ([\bar{n}_\odot z_D z_n])\) with a reasonable set \([D_\odot, \bar{n}_\odot] = [3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}, 1 \text{ cm}^{-3}]\) and \([z_D, z_n] = [3, 0.2]\) kpc as discussed in Papers I and II.

As mentioned above, the electron spectra currently available are generally in agreement with those expected from the hadron spectra, considering the uncertainties inherent in both the experimental data and the numerical parameters, but not quite satisfactory, with the Fermi data giving the excess by 20%–30% around several hundred GeV compared to the numerical results as seen in Figure 14. It might be related to the positron excess around 10–100 GeV observed by PAMELA (Adriani et al. 2009), indicating some nearby sources and/or exotic ones from DM annihilation or decay, while beyond the subject of the present paper. In fact most recently Delahaye et al. (2010) show that the electron spectra with Fermi, H.E.S.S., and PAMELA are reproduced rather well by the standard astrophysical processes, assuming two sources separately, the distant and local nearby ones, whereas they stress that there remain too large theoretical uncertainties to build a standard model for CR electrons. So it is critical to study the \(\gamma\)-ray’s and diffuse radio emissions simultaneously in order to reduce the uncertainties inherent in the galactic parameters assumed for the numerical calculations.
We compared our numerical results on the energy spectrum of Dy’s with EGRET and Fermi data for several sets of the field of view (Figures 17–21), and found that overall, the CR data, hadron (Figure 6 in Paper V) and electron (Figure 14) components, reproduce rather satisfactorily Dy’s for both EGRET and Fermi, considering the fact that we have uncertainties with at least 10%–20% in the galactic parameters assumed here as well as in the flux normalization of the hadron and electron components. Small enhancements of Dy’s in GeV region (Figures 19, 20), albeit they are still within the uncertainties, may indicate those from nearby sources such as the supernova remnants, pulsars, and pulsar wind nebulae.

We found, however, that Fermi data give the significant excess with approximately 70% or more in the high latitude (Figure 22), well beyond the uncertainties, against the numerical results in GeV region. This result may indicate a signature of very large electron–halo far distant from the GP, with, for instance, as large as 25 kpc (Keshet et al. 2004), and/or something else coming from the cosmological origin. We are also concerned if the excess here discussed relates to those appearing in the electron spectrum between 100 and 1000 GeV observed by Fermi and H.E.S.S. (Figure 14) and in the positron spectrum around several tens GeV by PAMELA. To make clear the correlation between these excesses, Dy’s in the high latitude and the electrons/positrons around several tens to hundred GeV, crucially important is the anisotropy study for the HE electron, which will be discussed elsewhere in the near future.

Finally, we briefly argue the electron spectrum obtained by Fermi from the observational point of view, aside from the prominent bumps indicated by ATIC and PPB-BETS. Fermi is indeed excellent in the observation of γ-rays, although we have some concerns about the separation of electrons from hadrons as well as their energy determination in the HE region, while acknowledging the team have studied very carefully the reliability from various kinds of checks, with both beam tests and simulational analyses.

Nevertheless, one should keep in mind that Fermi does not provide purely direct observations for electrons, but quasi-direct ones in the sense that electron events are selected by statistical analysis based on simulations for the spread of electron showers, where a small number of electrons are statistically selected from a large proton background. In contrast to these quasi-direct experiments, the PAMELA apparatus consists of a permanent magnetic spectrometer with a silicon tracking system, providing good identification between electrons and positrons, though limited to a maximum detectable rigidity of 100 GV.

Anyway, we await further studies and mutual cross-checks among the groups from various points of view to get a firm conclusion for the electron excess around 500 GeV, while acknowledging the team have studied very carefully the separation of electrons from hadrons as well as their energy determination in the HE region, while acknowledging the team have studied very carefully the reliability from various kinds of checks, with both beam tests and simulational analyses.

1 After submitting the present paper, we find that they decided to use a permanent magnet in place of the super-conducting magnet, which will be carried into space by the last flight of the space shuttle scheduled for the end of 2011 February (Kounine 2010). & Martinez 2010), the threshold energies of which are now overlapping with the Fermi satellite data.

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APPENDIX A

ENERGY LOSS OF ELECTRONS IN ISM and ISRF

The energy-loss rate due to the bremsstrahlung in the gas density n(r) is given by

\[ \frac{\Delta E_e}{\Delta t} = n(r)w_{\text{rad}}(E_e)E_e, \]

where

\[ w_{\text{rad}}(E_e) = \int_0^1 \phi_{\text{rad}}(x, E_e)dx, \]

with

\[ w_{\text{rad}}^{(0)} = 4c\alpha_e Z(Z + 1) \left( \frac{e^2}{m_e c^2} \right)^2 = 1.39 \times 10^{-16} \text{ cm}^3 \text{s}^{-1}, \]

for hydrogen atoms, and see the left-hand side of Table 5 for \( \phi_{\text{rad}}(x, E_e) \).

For \( E_e \gg m_e c^2 \), we can use the complete screening cross section, leading to the well-known result

\[ \frac{\Delta E_e}{\Delta t} = n(r)w_{\text{rad}}^{(\infty)} E_e, \quad w_{\text{rad}}^{(\infty)} = 7.30 \times 10^{-16} \text{ cm}^3 \text{s}^{-1}. \]

On the other hand, the energy-loss rate due to the IC is given, taking into account the energy spectrum of the target photon at \( r, n_{\text{ph}}(r, E_{\text{ph}}) \), by

\[ \frac{\Delta E_e}{\Delta t} = 2 \int_0^\infty dE_{\text{ph}} \int_{E_{\text{ph}}}^{E_{\text{ph}}(r, E_{\text{ph}})} \phi_{\text{ph}}(E_{\text{ph}}) \sigma_{\text{IC}}(E_e, E_{\gamma}) \frac{dE_{\gamma}}{dE_{\gamma}}, \]

with

\[ E_M := E_M(E_e, E_{\text{ph}}) := E_e \frac{X}{1 + X}; \]

\[ X := (X_0, E_{\text{ph}}) := \frac{4E_{\text{ph}}E_e}{(m_e c^2)^2}; \]

see the right-hand side of Table 5 for \( \sigma_{\text{IC}}(E_e, E_{\gamma}; E_{\text{ph}}) \). Here we omit the suffix i introduced in Section 2.2 for simplicity. For \( E_e \gg m_e c^2 \), equivalently \( X \gg 1 \), one finds a reasonable result, \( E_M \approx E_e \), leading to \( E_{\text{ph}} \ll E_{\gamma} \ll E_e \).

From Equation (4) in the text

\[ n_{\text{ph}}(r, E_{\text{ph}})dE_{\text{ph}} = \left[ \frac{\phi_{\text{ph}}(r)}{k_B T_{\text{ph}}} \right] W_{\text{ph}}(k) \frac{dk}{k^2}, \quad k = \frac{E_{\text{ph}}}{k_B T_{\text{ph}}}, \]

where \( W_{\text{ph}}(k) \) is the Planck function for the 2.7 K CMB, and the Gaussian function given by Equation (5) for the stellar radiation and the re-emission from the dust grains.
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The integration with respect to $E_y$ is given (Jones 1965, 1968) by, (see Equation (A3) for $X$)  

$$ \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} E_y \sigma_{\text{IC}}(E_x, E_y; \phi_{\text{ph}}) dE_y \approx \sigma_{\text{IC}}^{(0)}(E_x) S_{\text{IC}}(X)/X^2, $$

with  

$$ \sigma_{\text{IC}}^{(0)} = 3 \sigma_T = 2.00 \times 10^{-24} \text{cm}^2 \quad (\sigma_T: \text{Thomson cross section}), $$

and  

$$ S_{\text{IC}}(X) = \left( \frac{X}{2} + 6 + \frac{6}{X} \right) \ln(1 + X) - \frac{11 X^3/12 + 6X^2 + 9X + 4}{(1 + X)^2} - 2 - 2 \int_0^X \frac{\ln(1 + t)}{t} dt. $$

One should note that the approximation used above is only $E_x \gg m_e c^2$, readily satisfying the condition in the energy region of interest, $E_x \gtrsim 10$ MeV.

Now, we have the energy-loss rate due to the IC scattering in a compact form after integrating over the energy $\phi_{\text{ph}}$ of the target photon in Equation (A2),

$$ - \left( \frac{\Delta E_x}{\Delta t} \right)_{\text{IC}} = \epsilon_{\phi_{\text{ph}}} (r) \omega_{\text{ph}} \Lambda(E_x, \phi_{\text{ph}}) E_x^2, $$

where $E_x$ is in units of GeV and $\epsilon_{\phi_{\text{ph}}}$ in eV cm$^{-1}$, and

$$ \omega_{\text{ph}} = \frac{4}{3} \frac{c \sigma_T \times 10^{-9}}{m_e c^2/\text{GeV}} = 1.018 \times 10^{-16} \text{ cm}^3 \text{s}^{-1}, $$

$$ \Lambda(E_x, \phi_{\text{ph}}) \equiv \Lambda(\theta_x) = 9 \int_0^\infty S_{\text{IC}}(X) W_{\phi_{\text{ph}}} (\theta_x^2 X) dX/X^4, $$

with

$$ \theta_x = \theta_x(E_x, \phi_{\text{ph}}) = \frac{m_e c^2 / \sqrt{4(k_B \phi_{\text{ph}}) E_x}}. $$

The above discussions are applicable also for the synchrotron radiation, since it is caused by the collision between an electron and the virtual photon induced by the magnetic field. Practically, however, we have the condition $h\omega_k \Gamma_e \ll m_e c^2$ ($\Gamma_e$: Lorentz factor of electron) with $\omega_k = e H_\perp/m_e c^2$, and we can use the Thomson scattering cross section, namely $\Lambda(E_x, \phi_{\text{ph}}) \rightarrow 1$. Hence we obtain Equation (12a).

**APPENDIX B**

**CONTRIBUTION OF PERTURBATIVE TERMS IN THE TRANSPORT EQUATION**

**B.1. High Energy Region $E_x \gtrsim E_x^*$**

Since we can neglect the fluctuation due to the reacceleration in the HE region, we take here the second term in Equation (25a) alone, omitting the second term in Equation (22). The transport equation for the electron density in the HE region, $N_{e,0}^{(0)}(r; E_x, t)$, without the perturbative term, is given by

$$ \left[ \frac{\partial}{\partial t} - \nabla \cdot D(r; E_x) \nabla - \tilde{e}(r) \frac{\partial}{\partial E_x} W_{\text{IC}}(E_x) \right] \cdot N_{e,0}^{(0)}(r; E_x, t) = Q(r; E_x, t). $$

As discussed in Section 4.1, we regard $N_{e,0}^{(0)}(r; E_x, t)$ as the solution of the first-order approximation for Equation (21), so that we have the following equation with the perturbative term, $\tilde{n}(r)[W_{\text{ph}}(E_x) N_{e,e}^{(0)}(r; E_x, t)]$, moving it to the right-hand side,

$$ \left[ \frac{\partial}{\partial t} - \nabla \cdot D(r; E_x) \nabla - \tilde{e}(r) \frac{\partial}{\partial E_x} W_{\text{IC}}(E_x) \right] \cdot N_{e,e}(r; E_x, t) = Q(r; E_x, t) + \tilde{n}(r) \frac{\partial}{\partial E_x} W_{\text{IC}}(E_x) N_{e,e}(r; E_x, t). $$

Now, we rewrite the solution

$$ N_{e,e}(r; E_x, t) = N_{e,e}^{(0)}(r; E_x, t) + \tilde{N}_{e,e}^{(0)}(r; E_x, t), $$

leading to

$$ \left[ \frac{\partial}{\partial t} - \nabla \cdot D(r; E_x) \nabla - \tilde{e}(r) \frac{\partial}{\partial E_x} W_{\text{IC}}(E_x) \right] \cdot \tilde{N}_{e,e}^{(0)}(r; E_x, t) = \tilde{n}(r) \frac{\partial}{\partial E_x} W_{\text{IC}}(E_x) N_{e,e}^{(0)}(r; E_x, t). $$

Thus for the steady state ($\partial/\partial t = 0$), we have the solution of the second-order approximation

$$ \tilde{N}_{e,e}^{(0)}(r; E_x) = \int_0^\infty \tilde{N}_{e,e}^{(0)}(r; y) f_{n}^{(0)}(y; E_x) dy, $$

with

$$ \left[ \tilde{e}(r) \frac{\partial}{\partial y} - \nabla \cdot D(r; E_x) \nabla \right] \cdot \tilde{N}_{e,e}^{(0)}(r; y) = \tilde{Q}^{(0)}(r) \delta(y), $$

where $\tilde{Q}^{(0)}(r)$ and $f_{n}^{(0)}(0; E_x)$ are given by replacing $Q^{(0)}(r)$ [$\equiv Q(r)$] and $f_{n}^{(0)}(0; E_x)$ (see Equations (24b) and (28b)) with $Q^{(0)}(r) = Q_{\text{ph}} e^{-r/\zeta_Q} \Rightarrow \tilde{Q}^{(0)}(r)$, the scale heights in the source, $r_Q$ and $\zeta_Q$, must be replaced as

$$ \frac{1}{r_Q} \Rightarrow \frac{1}{r_n}, \quad \frac{1}{\zeta_Q} \Rightarrow \frac{1}{\zeta_n} + \frac{1}{\zeta_D} = \frac{2}{\zeta_n}, $$

leading to the following replacements,

$$ \omega_k = \left( \frac{1}{\zeta_Q} - \frac{1}{2\zeta_n} \right) \Rightarrow \left( \frac{2}{\zeta_n} - \frac{1}{2\zeta_n} \right) \equiv \tilde{\omega}_k, $$

and

$$ \frac{1}{\zeta_n} \Rightarrow \frac{1}{\zeta_Q}. $$
while the radial scale height in the source, \( r_0 \), does not appear explicitly in this procedure.

Now the Laplace transform of \( \tilde{\epsilon}_n^0(y; E_e) \) is immediately given (see Equation (31)) by

\[
\tilde{F}_{\epsilon,n}^0(E_e) = \int_0^\infty dE_0 \left[ \mathcal{W}_n(E_0) \tilde{F}_{r,n}^0(E_e) \right]' e^{-\bar{\epsilon}_r(E_e, E_0)}, \tag{B1}
\]

where \([\cdots]'\) denotes the differential with respect to \( E_0 \), and we obtain

\[
\frac{\tilde{N}_{e,n}^0(r; E_e)}{N_{e,n}^0(r; E_e)} = \frac{2}{\bar{\eta}_0} \frac{\tilde{F}_{\epsilon,n}^0(E_e)}{F_{r,n}^0(E_e)}, \tag{B2}
\]

with

\[
\bar{\eta}_r = \bar{\epsilon}_r / \bar{\eta}_r = \bar{\eta}_0 e^{-2r(1/\bar{r}_r-1/r_0)}; \quad \bar{\eta}_0 = \bar{\epsilon}_0 / \bar{\eta}_0.
\]

In Figure 24(a), we show \( \tilde{N}_{e,n}^0(r; E_e) / N_{e,n}^0(r; E_e) \) against \( E_e \) at the SS with \([\bar{\epsilon}_0, \bar{r}_0] = [2 \text{ eV}, 8 \text{ kpc}] \), corresponding to \([\bar{\epsilon}_0, \bar{r}_0] = [2 \text{ eV cm}^{-3}, 1 \text{ cm}^{-3}] \). Then one finds that the perturbative contribution due to the energy change in proportion to the gas density, \( \tilde{\eta}_n(r) \), is less than 10% in the energy region \( E_e \gtrsim E_e^* \).

We finally obtain

\[
N_{e,n}(r; E_e) = \frac{2Q_0}{\bar{\epsilon}_0} \frac{J_{\epsilon}(\bar{\epsilon}_e, U_e)}{J_e(U_e)} \tilde{F}_{r,n}^0(E_e) e^{-|z|/z_D}, \tag{B3}
\]

with

\[
F_{r,\epsilon}(E_e) = F_{r,\epsilon}^0(E_e) + \frac{2}{\bar{\eta}_0} \frac{J_{\epsilon}(\bar{\epsilon}_e, U_e)}{J_e(U_e)} \tilde{F}_{r,\epsilon}^0(E_e). \tag{B4}
\]

**B.2. Low Energy Region** \( E_e \lesssim E_e^* \)

In the LE region, the fluctuation term due to the reacceleration, \( (\Delta E_e/\Delta t)_{\text{reac}} \), becomes now effective as compared to the average energy-loss term due to the synchrotron-IC effect, \( (\Delta E_e/\Delta t)_{\text{IC}} \), in proportion to the energy density, \( \bar{\epsilon}(r) \). So Equation (25b) is approximately written as

\[
\bar{\epsilon}(r) \mathcal{W}_n(E_e) + O[\bar{\epsilon}(r) \mathcal{W}_n(E_e)] \simeq \bar{\epsilon}(r) [\mathcal{W}_n(E_e) + \bar{\eta}_n \mathcal{W}_n(E_e)], \tag{B5}
\]

with

\[
\bar{\epsilon}(r)/\bar{\eta}_n(r) \approx (\bar{\epsilon}(r)/\bar{\eta}_n(r))_{\text{eff}} \equiv \bar{\eta}_* , \tag{B6}
\]

where the effective value of \( \bar{\eta}_* \) is of the magnitude of \([1-5] \text{ eV} \), and for instance \( \bar{\eta}_* = \bar{\eta}_n \approx 2 \text{ eV} \) at the SS.

Neglecting the second term in Equation (B5), we have the solution for the principal term, corresponding to Equation (32) (see Section 3.3 and Table 4 for \( v, U_{\epsilon} \))

\[
N_{e,n}^{0}(r; E_e) = \frac{2Q_0}{\bar{\epsilon}_0 c} \frac{J_{\epsilon}(\bar{\epsilon}_e, U_e)}{J_e(U_e)} F_{r,\epsilon}^{(0)}(E_e) e^{-|z|/z_D}, \tag{B7}
\]

with

\[
F_{r,\epsilon}^{(0)}(E_e) = \frac{c}{|\bar{\mathcal{W}}_n(E_e)|} \int_{E_{\text{min}}}^{E_{\text{max}}} dE_0 E_0^{-\gamma} e^{-Y_{\epsilon,n}(E_e, E_0)}, \tag{B8}
\]

and

\[
Y_{\epsilon,n}(E_e, E_0) = c\bar{\sigma}_r \int_{E_e}^{E_0} \frac{E^u}{\bar{\mathcal{W}}_n(E)} dE. \tag{B9}
\]

Now putting

\[
\mathcal{V}_n^\alpha(E_e) = \mathcal{W}_n(E_e) + \bar{\eta}_n \mathcal{W}_n(E_e),
\]

the electron density with the synchrotron-IC effect (perturbative term here) is immediately

\[
N_{e,n}^{0}(r; E_e) = \frac{2Q_0}{\bar{\epsilon}_0 c} \frac{J_{\epsilon}(\bar{\epsilon}_e, U_e)}{J_e(U_e)} F_{r,\epsilon}^{(0)}(E_e) e^{-|z|/z_D}, \tag{B10}
\]

where \( F_{r,\epsilon}^{(0)}(E_e) \) is given by replacing \( \mathcal{W}_n \) with \( \mathcal{V}_n^\alpha \) in Equations (B8) and (B9), and it is related to the perturbative term, \( \tilde{N}_{e,n}^0 \), discussed in Section 4.1 as

\[
N_{e,n}^{*}(r; E_e) = N_{e,n}^{0}(r; E_e) + \tilde{N}_{e,n}^{0}(r; E_e). \tag{B11}
\]

Here one should be careful of the integral range of \( E_0 \) in Equation (B8), \([E_{\min}, E_{\max}] \), since we have two zero points in \( \mathcal{V}_n^\alpha(E_e) \) at two energies, \( E_{-}^* \) and \( E_{+}^* \), for instance, \([E_{-}^*, E_{+}^*] \approx [0.1, 1] \text{ GeV} \) for \( \zeta_0 = 50 \text{ mbarn} \) and \( \bar{\eta}_* = 2 \text{ eV} \), and

\[
N_{e,n}^{*}(r; E_e) = \begin{cases} \left[ E_e, E_e^- \right] & \text{for} \; E_e < E_e^- , \\ \left[ E_e, E_e^+ \right] & \text{for} \; E_e^- < E_e < E_e^+ , \\ \left[ E_e, +\infty \right] & \text{for} \; E_e > E_e^+ . \end{cases}
\]

We present \( \tilde{N}_{e,n}^{0}/N_{e,n}^{0} \) against \( E_e \) for several sets of \([\bar{\eta}_n, z_n, z_D] \) at the SS in Figure 24(b), and one finds that it is much less than 10% in the LE region \( E_e \lesssim E_e^* \).
B.3. Contribution from the Fluctuation in the Reacceleration

Once we confirm that the contribution of \( O[\bar{\varepsilon}(r)\mathcal{V}_n(E_e)] \) is approximately given by Equation (B5) with the energy loss proportional to \( \bar{n}(r) \), it is possible to use the path length distribution, \( I_n(r; x) \), as presented in Paper I,

\[
I_n(r; x) \simeq \frac{2Q_0}{\bar{n}_0c} \frac{J_n(u_0)}{J_n(u_n)} e^{-\bar{\sigma}_n|z|/z_0}, \tag{B12}
\]

but the slab equation is now slightly cumbersome,

\[
\left[ c\bar{\sigma}_r E^{\alpha} - \frac{\partial}{\partial E_e} W_n(E_e) - c\xi_0 \frac{1}{4} \frac{\partial^2}{\partial E^2} E_e^{2-\alpha} \right] F_{r,n}(E_e) = c E_e^{-\gamma}.
\]

Remembering that \( F_{r,n}^{(0)}(E_e) \) is a solution of the equation

\[
\left[ c\bar{\sigma}_r E^{\alpha} - \frac{\partial}{\partial E_e} W_n(E_e) \right] F_{r,n}^{(0)}(E_e) = c E_e^{-\gamma},
\]

we can write \( F_{r,n}(E_e) \) as

\[
F_{r,n}(E_e) = F_{r,n}^{(0)}(E_e) + F_{r,n}^{(1)}(E_e), \tag{B13}
\]

with

\[
\left[ c\bar{\sigma}_r E^{\alpha} - \frac{\partial}{\partial E_e} W_n(E_e) \right] F_{r,n}^{(1)}(E_e) = c\xi_0 \frac{1}{4} \frac{\partial^2}{\partial E^2} E_e^{2-\alpha} F_{r,n}^{(0)}(E_e).
\]

The solution for Equation (B14) is immediately obtained after the following replacement in Equation (28b)

\[
c E_e^{-\gamma} \Rightarrow c\xi_0 \frac{1}{4} \frac{\partial^2}{\partial E^2} E_e^{2-\alpha} F_{r,n}^{(0)}(E_e),
\]

and the explicit form is given by

\[
F_{r,n}^{(1)}(E_e) = c\xi_0 \frac{1}{4} \int_0^\infty dE_0 \frac{E_0^{2-\alpha} F_{r,n}^{(0)}(E_0)}{|W_n^2(E)|} e^{-\bar{\sigma}_e(E,e)},
\]

where \( \bar{\sigma}_e(E,e) \) is given by replacing \( W((E,E)) \) in Equation (B9) with \( |W_n^2(E)| \).

Now, from Equations (B12) and (B13), the electron density in the LE region is given by

\[
N_{e,n}(r; E_e) = \frac{2Q_0}{\bar{n}_0} \frac{J_n(u_0)}{J_n(u_n)} e^{-\bar{\sigma}_n|z|/z_0} F_{r,n}(E_e), \tag{B15}
\]

where \( F_{r,n}(E_e) \) is given by Equation (B13).

Corresponding to Equation (26b), we rewrite Equation (B15), dividing into three terms as (see also Equation (B11))

\[
N_{e,n}(r; E_e) = \tilde{N}_{e,n}^{(0)}(r; E_e) + \tilde{N}_{e,n}^{(1)}(r; E_e) + \tilde{N}_{e,n}^{(2)}(r; E_e),
\]

and in Figure 7 in the text, we present \( N_{e,n}^{(1)}/[\tilde{N}_{e,n}^{(0)} + \tilde{N}_{e,n}^{(1)}] \) against \( E_e \). One finds that the contribution of the fluctuation is effective around GeV region, boosting the electron density without the fluctuation by approximately 25%.

APPENDIX C
EMISSIVITY OF \( \gamma \)'S COMING FROM THE INVERSE COMPTON PROCESS

In this appendix, we omit the suffix \( i \) for simplicity. The production rate of \( \gamma \)'s per unit time due to the bremsstrahlung,
