SHOT NOISE IN MESOSCOPIC DIFFUSIVE ANDREEV WIRES

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Abstract We study shot noise in mesoscopic diffusive wires between a normal and a superconducting terminal. We particularly focus on the regime, in which the proximity-induced reentrance effect is important. We will examine the difference between a simple Boltzmann-Langevin description, which neglects induced correlations beyond the simple conductivity correction, and a full quantum calculation. In the latter approach, it turns out that two Andreev pairs propagating coherently into the normal metal are anti-correlated for \( E \lesssim E_c \), where \( E_c = \hbar D/L^2 \) is the Thouless energy. In a fork geometry the flux-sensitive suppression of the effective charge was confirmed experimentally.

Keywords: Shot noise, counting statistics, Andreev reflection, proximity effect

1. Introduction

Fluctuations of the current in mesoscopic conductors originate from the quantum scattering of the charge carriers, and are sensitive to their interference, statistics and interaction. This makes the theoretical and experimental study of noise in small electronic circuits interesting and challenging (for recent reviews, see Refs. [1, 2]).

Statistical correlations in the transport of fermions have led to a number of interesting predictions. For example, the noise of a single-channel quantum contact of transparency \( T \) at zero temperature has the form \( S_I = 2|eI|(1 - T) \) [3–6]. The noise is thus suppressed below the Schottky value \( 2|eI| \) of uncorrelated charge transfer. The suppression is a direct consequence of the Pauli principle, which prevents two electrons from tunneling together. It is convenient to measure the deviation from the Schottky result by the so-called Fano factor \( F = S_I/2|eI| \). For a num-
ber of generic conductors, it turns out that the suppression of the Fano factor is universal, i.e., it does not depend on details of the conductor like geometry or impurity concentration. In particular, a diffusive metal with elastic scattering leads to \( F_{\text{diff}}^N = 1/3 \) \([7, 8]\), which is independent on the concrete shape of the conductor \([9]\) and has been confirmed experimentally \([10, 11]\).

If superconductivity comes into play the fundamental charge transport mechanism at energies below the superconducting gap is Andreev reflection. Two electrons enter the superconductor simultaneously and form a Cooper pair, which can propagate in the superconductor. Thus, in this process a charge transfer of \( 2e \) occurs, but with a reduced probability, since two particles have to tunnel. The shot noise is proportional to the charge of the elementary processes, and one thus naively expects a doubling of the shot noise, which was indeed found theoretically \([12, 13]\) and experimentally \([14, 15]\) for diffusive conductors. It is remarkable that this doubling occurs for diffusive conductors, whereas it is not found for other conductors like, e.g., single-channel contacts \([3, 12, 16-18]\), double tunnel junctions \([19-22]\), or diffusive junctions with a tunneling barrier \([23, 24]\). A doubling of the full Schottky noise was recently observed experimentally \([25]\).

In this article we address the counting statistics and the noise in diffusive structures with normal and/or superconducting terminals. In particular we concentrate on the energy- and phase-dependent shot noise in an Andreev interferometer. First, we briefly review the theory of full counting statistics using the Keldysh Green’s function approach. We derive the counting statistics of diffusive conductors for various limits. In Sec. 3 we obtain generic results for the shot noise in diffusive conductors in some limiting cases. Finally, in Secs. 4 and 5 we discuss the information contained in the energy- and phase-dependence of the shot noise in diffusive wires and Andreev interferometers. A good qualitative agreement of experimental results and our full quantum calculation is demonstrated.

2. Current Statistics

Consider the following Gedanken experiment. A constant voltage bias is applied to a mesoscopic conductor for a certain time interval \( t_0 \). During this time interval we count the number of charges \( N \) passing the conductor. Due to quantum-mechanical uncertainty the outcome of the experiment is described by a probability \( P_{t_0}(N) \) that \( N \) charges have passed the conductor. This is the so-called full counting statistics (FCS), which is the most natural description of quantum transport. Alterna-
tively we may study the characteristic function \( \Phi(\chi) = \sum_N P_t N e^{i N \chi} \). The \( k \)th coefficient in the expansion of the characteristic function in powers of \( \chi \) yields the moments of the counted charge \( \langle N^k \rangle \). An equivalent description is to obtain the cumulant generating function (CGF) \( S(\chi) = \ln \Phi(\chi) \), which gives the cumulants directly. This will be the quantity which we obtain below. The cumulants are directly connected to the (measurable) zero-frequency current correlations functions.

One route to counting statistics is the Keldysh-Green’s function approach in combination with the circuit theory of mesoscopic transport developed by Nazarov [26–28]. For details, we refer to several articles in [2]. In this approach, terminals are described by 4\( \times \)4 Green’s function matrices. For a normal terminal (N) with occupation \( f \) we introduce the 2\( \times \)2 matrix \( \hat{f} = \text{diag}(f(E), f(-E)) \) and have

\[
\hat{G}_N(\chi) = \begin{pmatrix}
\hat{\tau}_3 (1 - 2\hat{f}) & -2\hat{\tau}_3 e^{i\chi\hat{\tau}_3}\hat{f} \\
-2\hat{\tau}_3 e^{-i\chi\hat{\tau}_3}\hat{f} & \hat{\tau}_3 (2\hat{f} - 1)
\end{pmatrix}.
\] (1)

For a superconducting terminal (S) at equilibrium and for \( E \ll \Delta \) we have

\[
\hat{G}_S = \begin{pmatrix}
\hat{\tau}_1 & 0 \\
0 & \hat{\tau}_1
\end{pmatrix}.
\] (2)

For a general contact described by a scattering matrix with corresponding transmission eigenvalues \( \{T_n\} \) the counting statistics is obtained as [29]

\[
S(\chi) = \frac{t}{\hbar} \int dE \sum_n \text{Tr} \ln \left[ 1 + \frac{T_n}{4} \{\hat{G}_L, \hat{G}_R\} - 2 \right] .
\] (3)

To give a simple example, we evaluate the counting statistics of a quantum contact between two normal terminals and obtain in agreement with the scattering matrix approach [30, 31] (see Appendix for a short summary)

\[
S(\chi) = \frac{t}{\hbar} \int dE \sum_n \ln \left[ 1 + T_n f_L (1 - f_R)(e^{i\chi} - 1) + T_n f_R (1 - f_L)(e^{-i\chi} - 1) \right].
\] (4)

Thus, the statistics is a simple multinomial form of the two possible events of left and right transfer of charges. The formula (3) includes in addition the statistics of SN- and SS-contacts [16, 29].

In diffusive conductors we have to find the general counting statistics by a different method. The quantum kinetic equation for the diffusive wire is the so-called Usadel equation [32]

\[
\hbar \frac{\partial}{\partial x} D(x) \hat{G}(x) \frac{\partial}{\partial x} \hat{G}(x) = -i \left[ E\hat{\tau}_3, \hat{G}(x) \right],
\] (5)
with continuous boundary conditions at both ends of the conductor. The right-hand side of Eq. (5) accounts for the decoherence of electrons and holes during the propagation in the normal metal at finite energies $E$. A full solution has so far been only obtained numerically, and we will discuss the implications of the decoherence on the shot noise later.

For diffusive wires between normal terminals or with one superconducting terminal (see Fig. 1a and Fig. 1b for small energies $eV, k_B T \ll E_c$) the right hand side of Eq. (5) can be neglected. We obtain the general solution \cite{27, 33, 34}

$$S(\chi) = \frac{t_0 G_N}{8h} \int dE \text{Tr } \cosh^2 \left( \frac{1}{2} \{ \tilde{G}_L, \tilde{G}_R \} \right).$$

The same result can be obtained by averaging Eq. (3) over the transmission eigenvalue distribution of a diffusive scatterer, i.e. the bimodal distribution \cite{35}

$$\rho(T) = \frac{G_N}{2G_Q} \frac{1}{T \sqrt{T^2 - 1}}.$$  

Another drastic simplification can be made if the proximity effect is negligible, i.e. the right hand side of Eq. (5) is dominant. In Ref. \cite{36} it was shown, that the diffusive wire can be mapped onto a series of two diffusive wires contacted by normal terminals, which constitute the electron and hole propagation (see Fig. 1c). For the special case of diffusive connectors the counting statistics is independent of the geometry. As a consequence, the counting statistics is exactly given by that of a normal conductor, with halved conductance and a negative counting field for the hole terminal.
The energy dependence resulting from the right hand side of Eq. (5) leads to interesting effects related to the quantum propagation of electron-hole pairs [37]. Below, we will address in detail the dependence of the shot noise on voltage and temperature, and compare our theoretical predictions, based on (5) with available experimental results.

3. Shot Noise in Diffusive Conductors

To obtain the shot noise from the counting statistics we calculate

$$S_I = -\frac{2e^2}{t_0} \frac{\partial^2}{\partial \chi^2} S(\chi) \bigg|_{\chi=0}.$$  \hspace{1cm} (8)

From Eq. (3) we obtain for a general scatterer between normal terminals

$$S_{iNN} = \frac{2e^2}{\hbar} \sum_n \int dE \left[ T_n (1 - T_n) (f_L(E) - f_R(E))^2 \right.$$  
$$+ T_n \left[ f_L(E) (1 - f_L(E)) + f_R(E) (1 - f_R(E)) \right].$$  \hspace{1cm} (9)

Averaging (9) over the transmission eigenvalue distribution (7) we find

$$S_{iNN} = 2G_N \int dE \left[ f_L(E) (1 - f_L(E)) + f_R(E) (1 - f_R(E)) \right.$$  
$$+ \frac{1}{3} (f_L(E) - f_R(E))^2 \bigg].$$  \hspace{1cm} (10)

The energy integration can be done using the formulas in the appendix. As result we obtain

$$S_{iNN} (T, V) = \frac{4}{3} G_N k_B T + \frac{2}{3} eG_N V \coth \left( \frac{eV}{2k_B T} \right),$$  \hspace{1cm} (11)

where we introduced the conductance $G_N = \frac{e^2}{h} \sum_n T_n$.

Using the result (6) we can obtain at $E = 0$ the general formula for the noise

$$S_I = -\frac{G}{4\hbar} \int dE \text{Tr} \left[ \frac{\partial^2}{\partial \chi^2} \{ \hat{G}_L, \hat{G}_R \} - \frac{2}{3} \left( \frac{\partial}{\partial \chi} \{ \hat{G}_L, \hat{G}_R \} \right)^2 \right]_{\chi \to 0}.$$  \hspace{1cm} (12)

For a diffusive wire between a normal and a superconducting terminal at $eV, k_B T \ll E_c$ we obtain

$$S_{iNS} (V) = \frac{2G_N}{3} \int dE [f(E) + f(-E)] [2 - f(E) - f(-E)].$$  \hspace{1cm} (13)
Evaluating the energy integration we find
\[
S_{I}^{NS}(V) = \frac{8}{3} G_N k_B T + \frac{4}{3} e G_N \coth \left( \frac{e V}{k_B T} \right).
\] (14)

This result can, indeed, be inferred from the normal-state result Eq. (12) by the replacement \( G_N \to 2G_N \) and \( e \to 2e \).

While the derivations presented previously are valid in the limit \( eV, k_B T \ll E_c \), we can also obtain the noise in the limit \( eV \gg E_c \) or \( k_B T \gg E_c \). Here we employ the incoherent Andreev circuit theory approach [36]. According to the mapping rules we obtain the noise from the normal result by the replacement \( G_N \to G_N/2, V \to 2V \) and \( S_I \to 4S_I \). By applying these substitutions to Eq. (11) we again obtain the result (14).

We see that the shot noise (as well as the full counting statistics) in the discussed regimes is universal. First, at low energies \( eV, k_B T \ll E_c \) universality means, that the noise depends only on the normal-state conductance \( G_N \) and is independent of the detailed geometry. Furthermore, it turns out that in the incoherent regime the full counting statistics and, therefore the current noise is also the same. The universality of the noise is quite surprising, since the transport mechanisms differ quite drastically in both limits. This remarkable coincidence holds, however, only for diffusive conductors. For double tunnel junctions, chaotic cavities, or other combinations of scatterers the transport properties differ (see Ref. [34] and references therein).

4. Energy-dependent current noise

The full quantum-mechanical description requires the solution of the Usadel equation (5) to first order in the counting field. We will discuss these results later. Let us first note, that one can obtain an approximate expression for the energy dependence of the shot noise by a generalized Boltzmann-Langevin approach. We recall, that the kinetic equation for the average distribution function has the form
\[
\frac{\partial}{\partial x} \sigma(E, x) \frac{\partial}{\partial x} (1 - f(E, x) - f(-E, x)) = 0.
\] (15)

The local energy-dependent conductivity incorporates the effect of the proximity induced coherence and is given by \( \sigma(E, x) = \sigma_N \cosh(\text{Re} \theta(E, x)) \), where the spectral angle \( \theta \) obeys \( hD (\partial^2/\partial x^2) \theta(E, x) = -iE \sin(\theta(E, x)) \) with appropriate boundary conditions (see Ref. [38] for details). Solving Eq. (15) the current (per unit area of the contact) is
\[
I(V, T) = \frac{1}{e} \int \text{d}E G(E) \left[ 1 - f_N(E) - f_N(-E) \right].
\] (16)
Shot noise of a diffusive proximity wire. Both the differential conductance and the noise show a reentrant behaviour. The effective charge reveals that the correlated Andreev pair transport suppresses the noise below the uncorrelated Boltzmann-Langevin result.

Here we have defined the spectral conductance

$$\frac{1}{G(E)} = \int_0^L dx \frac{1}{L \sigma(E, x)},$$

which have written here for a one-dimensional wire of length $L$ with a uniform cross section. For an arbitrary geometry, the spectral conductance has to be found from the solution of a diffusion equation.

Guided by the kinetic equation for the average transport, we may try to find the current noise by generalizing the Boltzmann-Langevin approach [13] to include the energy- and space-dependent conductivity. In fact, this task has already been performed. In Ref. [27] it was shown, that the counting statistics of a diffusive normal wire only depends on the conductance, defined in exactly the same way as (17). In this proof an energy-independent conductivity $\sigma(x)$, but an arbitrary form of the diffusive metal was taken into account. However, it is obvious, that the same result is obtained if a spectral conductivity $\sigma(E, x)$ is assumed. As a result the current would be given by Eq. (16).

Next, we combine this observation with the results for incoherent Andreev transport obtained in Ref. [36]. In this work it was shown that the counting statistics of a diffusive wire between a normal and a superconducting terminal is mapped onto a series of two diffusive wires. Again, this holds equally well if we assume ad hoc an energy-dependent conductance of both diffusive wires. Due to electron-hole symmetry the
spectral conductance of both wires and therefore of the total wire is the same.

Collecting these observations, we obtain the current noise from Eq. (10) by inserting the spectral conductance (17) inside the energy integral and multiplying by a factor of 2. In this way we obtain the result of the modified Boltzmann-Langevin approach

\[
S_{BL}^I = 4 \int dE G(E) \left[ f(E)(1 - f(E)) + f(-E)(1 - f(-E)) \right. \\
+ \frac{1}{3}(f(E) - f(-E))^2 \left. \right].
\] (18)

Evaluating the energy integral with the help of the formulas in the appendix we find

\[
S_{BL}^I = \frac{8}{3} G(V,T)k_B T + \frac{4}{3} eI(V,T) \coth \left( \frac{eV}{k_B T} \right).
\] (19)

Here we introduced the temperature-dependent differential conductance \( G(V,T) = \frac{dI(V,T)}{dV} \). The same result was obtained by a different method recently in Ref. [39]. At zero temperature, we obtain the result [40]

\[
S_{BL}^I(V) = \frac{4}{3} eI(V).
\] (20)

This result represents the starting point for our further considerations. It was derived neglecting correlations of scattering events between electrons and holes in the normal metal. Thus, in the following we will be specifically interested in the deviations of the noise from the simple Boltzmann-Langevin result (20) and introduce the effective charge \( q_{eff}(V) = (3/2)\partial S_I^I/\partial I \) [41].

The full quantum-mechanical calculation of the energy-dependent shot noise was performed in Ref. [37] and the results are shown in Fig. 2. A direct comparison of the differential shot noise and the differential conductance (for zero temperature) shows the difference in the energy dependence. The effective charge defined above displays the clear deviation of the quantum noise from the Boltzmann-Langevin result of \( 2e \). At energies below the Thouless energy \( E_c = \hbar D/L^2 \) the effective charge is suppressed below \( 2e \). This shows that the correlated Andreev pair transport suppresses the noise below the uncorrelated Boltzmann-Langevin result.

5. Phase-dependent shot noise

To experimentally probe the pair correlations in diffusive superconductor-normal metal-heterostructures it is most convenient to use an
Andreev interferometer. An example is shown in the left part of Fig. 3. A diffusive wire connected to a normal terminal is split into two parts, which are connected to two different point of a superconducting terminal. By passing a magnetic flux through the loop one can effectively vary the phase difference between the two connections to the superconductor. Such a structure has been experimentally realized by the Yale group [41]. In Fig. 3 we present a direct comparison between our theoretical predictions and the experimentally obtained effective charge. Note that we have included the experimental temperature in the theoretical modelling. The finite temperature explains the strong decrease of the effective charge in the regime $|eV| \leq k_BT$, where the noise is fixed by the fluctuation-dissipation theorem. The disagreement between theory and experiment in this regime stems solely from differences in the measured temperature-dependent conductance from the theoretical prediction. We attribute this to heating effects. The qualitative agreement in the shot-noise regime $|eV| \geq k_BT$ is satisfactory, if one takes into account, that we have no free parameters for the theoretical calculation. Both, experiment and theory show a suppression of the effective charge for some finite energy, which is of the order of the Thouless energy and depends on flux in a qualitative similar manner.
integer flux the effective charge is completely flat, in contrast to what
one would expect from circuit arguments based on the conductance dis-
tribution in the fork geometry. Currently we have no explanation for
this behaviour, and therefore more work is needed in this direction.

6. Conclusions

Shot noise in diffusive heterostructures between normal and super-
conducting terminal provides valuable information on the correlated An-
dreev pair transport. We have examined the difference between a simple
Boltzmann-Langevin description, which neglects these correlations, and
a full quantum calculation. Examining the effective charge we have
shown that two Andreev pairs propagating coherently into the normal
metal are anti-correlated. The phase sensitivity of the suppression of
the effective charge was confirmed experimentally.

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Appendix: Scattering Approach

One of the most general approaches to quantum transport is the scattering ma-
trix approach. Following Levitov and Lesovik [30, 31] the counting statistics can
be found from a slight modification of the standard scattering matrix approach.
For a two terminal structure we label the scattering states in the leads by
ψ = (c_L^1, ..., c_L^n, c_R^1, ..., c_R^n). The modified scattering matrix and the occupation matrix
read

\[ S_\chi = \begin{pmatrix} r & t e^{i\chi/2} \\ t' e^{-i\chi/2} & r' \end{pmatrix} \quad f = \begin{pmatrix} f_L & 0 \\ 0 & f_R \end{pmatrix} \].

The counting statistics is then given by

\[ \Phi(E) = \det \left( 1 - f + f S_\chi S_\chi^\dagger \right) \quad S_\chi(\chi) \equiv \ln \Phi(E) = \frac{t_0}{\hbar} \int dE \ln \Phi(E(\chi)) \].

Using the standard polar decomposition for the scattering matrix we obtain the count-
ing statistics (12).

Appendix: Integrals

In the derivations in this article we encounter integral expressions of the forms

\[ I_1(U) = \int dE (E) \left[ f_1(E)(1 - f_1(E)) + f_2(E)(1 - f_2(E)) \right] \],

\[ I_2(U) = \int dE (E) \left[ f_1(E)(1 - f_2(E)) + f_2(E)(1 - f_1(E)) \right] \].

Here we introduced \( f_1(E) = f_D(E + U) \) and \( f_2 = f_D(E - U) \), where \( f_D(E) = (\exp(E/k_B T) + 1)^{-1} \) is the Fermi-Dirac distribution. The goal is to reduce the integrals
to expressions related to the current \( I(U) = \int dE (E)(f_1(E) - f_2(E)) \). The first
integral is solved by noting that $-k_B T \left( \partial / \partial U \right) f_{i(2)}(E) = \pm f_{i(2)}(E) (1 - f_{i(2)}(E))$ and we find for the first integral

$$I_1(U) = k_B T \frac{\partial I(U)}{\partial U}. \quad (B.3)$$

With the help of the identity

$$f_1(E)(1 - f_2(E)) + f_2(E)(1 - f_1(E)) = (f_1(E) - f_2(E)) \coth \left( \frac{U}{k_B T} \right), \quad (B.4)$$

we find for the second integral

$$I_2(U) = I(U) \coth \left( \frac{U}{k_B T} \right). \quad (B.5)$$

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