Effect of Higher Transition Foundation on Dynamic Stability Boundaries of Simply Supported Beam

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Abstract: Accurate closed form solution to predict the dynamic stability behavior of simply supported (SS) beam on elastic foundation, subjected to an end axial periodic load, has been derived using simple single term trigonometric function which satisfies the geometric boundary conditions. The effect of higher transition foundation on dynamic stability regions is studied. The energy method is used in the present work to evaluate the dynamic stability boundaries. Euler - Bernoulli beam theory is used for the analysis. Numerical results are presented using proper non-dimensional parameters with varying foundation parameters below and above the second transition foundation value of elastic foundation, in both the digital and analogue forms for better comprehension of the dynamic stability behavior of the beams. The effect of the higher transition foundation value of the elastic foundation on the dynamic stability behavior is brought out in the present work.

Keywords: Dynamic stability, beams, periodic loads, elastic foundation, transition foundation.

1. INTRODUCTION

Prediction of dynamic stability boundaries of structural elements like beams subjected to periodic axial loads is an important input for the structural design engineers. Earlier studies on the dynamic stability of beams/columns are studied by Timoshenko and Gere [1]. The dynamic stability of structural elements is well discussed in the classic work of V.V. Bolotin [2]. In Ref. [3] the dynamic stability of slender bars, subjected to an axial periodic load at the free ends, is investigated using the FE method. It is shown that the dynamic stability curves obtained using the non-dimensional parameters are the same, for any boundary condition, as long as the vibration and stability mode shapes are similar. In Ref.[4], the dynamic stability of structural members subjected to a periodic loads is investigated by Rao et al. It is
shown that the boundaries/regions of the dynamic stability are obtained with suitable non-dimensional parameters are same for the structural members, where the mode shapes of the frequency and buckling are same or nearly the same, subjected to end periodic axial loads. In Ref. [5] one of author’s previous work, it is demonstrated that the deviation in assumed mode shapes with reference to exact mode shapes can be accurately assessed by means of $L_2$ norms of associated mode shapes. In Ref. [6] of author’s previous work, it is demonstrated that the deviation in assumed mode shapes with reference to exact mode shapes can be accurately assessed by means of $L_2$ norms of associated mode shapes.

Structural members like uniform beams and plates on elastic foundation are commonly encountered in many disciplines of engineering. The effect of elastic foundation on the regions of dynamic instability of beams is studied in Ref. [6]. An instability finding of Ref. [6] is that, as the foundation modulus (parameter) increases, the boundaries of dynamic instability are shifted away from the vertical axis and the instability region width decreased. It has been shown in Ref.[3] that the mode shapes of the buckling of columns resting on an elastic foundation depend on the value of the foundation stiffness parameter. A transition foundation value exists and beyond this transition foundation the mode shape of the buckling changes. For simply supported beam, the vibration modes shape and stability mode shape are the same (one-half sine wave), when the foundation parameter is below the first transition foundation parameter. The stability and vibration mode shapes are different (two-half sine waves for stability & one half-wave for free vibration problems respectively) when the foundation parameter is above the first transition value. For a simply supported column the value of this transition foundation parameters are found to be 4, 36, 144….. for (first, second, third……..) [1]. Furthermore, it does not exist for the vibration mode shapes of beams on an elastic foundation. Recently B. Subbaratnam et al investigated the dynamic stability of beams on elastic foundation [7]. In this study, it is shown that, as the elastic foundation increases, the width of the regions of dynamic stability decreases, thus the beam less sensitive to the dynamic stability phenomenon to periodic loads. Similar studies on SS rectangular plates subjected to biaxial periodic compressive load on elastic foundation have been reported in Ref. [8].

In Ref. [9] the dynamic instability of an axially loaded beam resting on an elastic foundation with damping is investigated. It is shown that, if the foundation stiffness or damping increases, the critical dynamic load also increases and the regions of instability shifts to a higher applied frequency. The dynamic stability of a non-uniform (tapered) cantilever beam resting on an elastic foundation was examined by Lee [10]. The effect of elastic foundation on natural frequencies, static buckling loads and regions of dynamic instability shapes of beams on an elastic foundation are studied by Yokoyama [11]. Subbaratnam et al [12], studied the two dimensional problems like rectangular plate on elastic foundation and the effect of elastic foundation on the regions of plates with uniform edge loads.

The main aim of the present proposed work is to investigate the effect of higher transition foundation on the regions of dynamic stability of SS beams resting on elastic foundation subjected to end axial periodic loads. The effect of elastic foundation, changing the mode shapes for stability and vibration on the dynamic stability boundaries, with increasing the values of foundation parameter, subjected to periodic forces is derived and discussed which highlights the simplicity and robustness of the proposed analytical work in obtaining the reasonably realistic solutions to dynamic instability problems.

2. FORMULATION

The mathematical formulation based on the variational energy principle and the evaluation of the dynamic stability boundaries is briefly discussed here for the simply supported beam resting on elastic foundation. A uniform SS beam, subjected to a concentrated axial compressive periodic load $P(t)$, resting on an elastic foundation, is shown in Fig.1. The total potential energy $\Pi$ of the beam is given by
where $U = $ Strain energy, $U_F = $ Energy stored in the elastic foundation, $T = $ Kinetic energy and $W = $ Work done by the periodic load. The expressions for $U$, $U_F$, $T$ and $W$ are given, by

$$
U = \frac{EI}{2} \int_0^L w'^2 \, dx 
$$

(2)

$$
U_F = \frac{k}{2} \int_0^L w^2 \, dx 
$$

(3)

$$
T = \frac{m\omega^2}{2} \int_0^L w^2 \, dx 
$$

(4)

and

$$
W = \frac{P(t) L}{2} \int_0^L w'^2 \, dx 
$$

(5)

where $'$ (prime) = first derivative, $''$ (double prime) = second derivative with respect to $x$, $E = $ Young’s modulus, $I = $ area moment of inertia, $m = $ mass per unit length, $w = $ lateral displacement , $P(t) = $ axial compressive periodic load and $\omega = $ natural frequency.

The load $P(t)$ is taken as

$$
P(t) = P_s + P_r \cos \theta t 
$$

(6)

where $P_s = $ constant part of $P(t)$, $P_r = $ periodic part of $P(t)$, $\theta = $ radian frequency of the compressive load $P_r$. The quantities $P_s$ and $P_r$ are expressed in terms of the buckling load parameter $P_{cr}$.

Substituting, Eqs. (2) to (5) in Eq. (1), the differential equation governing the dynamic stability problem of beams resting on elastic foundation is written in terms of the energies, as

$$
\Pi = \frac{EI}{2} \int_0^L w'^2 \, dx + \frac{k}{2} \int_0^L w^2 \, dx - \frac{P(t)}{2} \int_0^L w'^2 \, dx - \frac{m\omega^2 L}{2} \int_0^L w^2 \, dx 
$$

(7)

As $\omega = 0/2$, Eq. (7) becomes
Substituting the expression for $P(t)$ given in Eq. (6) in Eq. (8), we get

$$\Pi = \frac{EI}{2} \int_0^L w'^2 dx + \frac{k}{2} \int_0^L w'^2 dx - \frac{P(t)}{2} \int_0^L w'^2 dx - \frac{m}{2} \frac{\theta'^2}{4} \int_0^L w'^2 dx$$

(8)

As a first approximation the condition for the existence of these boundary solutions [2] is

$$\Pi = \frac{EI}{2} \int_0^L w'^2 dx + \frac{k}{2} \int_0^L w'^2 dx - \left( \frac{\alpha \pm \beta}{2} \right) \frac{P_c}{2} \int_0^L w'^2 dx - \frac{m}{2} \frac{\theta'^2}{4} \int_0^L w'^2 dx$$

(9)

(10)

where $\alpha$ and $\beta$ are the static load parameter ($= \frac{P_c}{P_c}$) and dynamic load parameter ($= \frac{P_d}{P_c}$) respectively.

Note that in Eq. (10), two conditions are combined under the plus or minus sign, thus yielding the two boundaries of the regions of instability. In this analysis static buckling load factor of the beam is considered as the reference.

2.1. Critical (Buckling) load and frequency parameters of SS beam resting on a foundation

For SS beam the distribution of lateral deflection, in the non-dimensional form is assumes as

$$W = a \sin m \pi X$$

(11)

where $a$ is the undetermined constant, $m$ is the mode number, $m_b$ for buckling and $m_f$ for free vibration problem, $W$ and $X$ are the non-dimensional lateral deflection and axial coordinate respectively.

Substituting Eq. (11), in the non-dimensional form of the Eq. (10) by the length of the beam and taking variation w. r. t $a$ and equating to zero. We get the buckling load parameter and the frequency parameter in terms of the mode number as

$$\lambda_b = \frac{P_c L^2}{EI} = \pi^2 \left( m^2 + \frac{\gamma}{m^2} \right)$$

(12)

and

$$\lambda_f = \frac{m \omega^2 L^4}{EI} = \pi^4 \left( m^2 + \frac{\gamma}{m^2} \right)$$

(13)

where $P_c$ is the critical load, $\omega$ is the natural frequency, $\gamma$ is the elastic foundation parameter ($= \frac{kL^4}{\pi^4 EI}$), $\lambda_b$ is the buckling/critical load parameter and $\lambda_f$ is the natural frequency parameter ($= \frac{m \omega^2 L^4}{EI}$).

2.2. Transition Values of the Foundation Parameter

The transition value $\gamma_{Ti}$ ($i = 1, 2, 3, \ldots$ and corresponding to the buckling mode number $m_i, m_f$) of the elastic foundation parameter $\lambda_i$ change of mode shape from $m_i$ to $(m_i + 1)$ half sine waves is evaluated from the equation

$$\lambda_{b(i+1)} = \lambda_b(m_i+1)$$

(14)

Then the value of $\gamma_{Ti}$ as
\[ \gamma_T = m_b^2 (m + 1) b^2, \quad (i, m_b = 1, 2, 3, 4 \ldots). \]  
(15)

The following two significant observations from the evaluation of the \( \gamma_T \) are:

a) \( \gamma_T \) for the buckling problem,

b) for the free vibration problems the phenomenon of transition does not exist.

The values of \( \gamma_T \) for the buckling problem are 4, 36, 144 \ldots for \( (i = 1, 2, 3 \ldots) \). In this present work the effect of the second transition foundation parameter \( \gamma_T = 36 \) and its effect on the dynamic stability regions of SS beam is studied in detail.

2.3. Variation of \( \lambda_b \) and \( \lambda_f \) for different values of \( m_b \) and \( m_f \)

Eq. (12) and Eq. (13) are the expressions for buckling load parameter (\( \lambda_b \)) and frequency parameter (\( \lambda_f \)) for the different values of \( m_b, m_f \) and \( \gamma \). Depending on these values of \( \gamma, m_b \) and \( m_f \), the expressions for \( \lambda_b \) and \( \lambda_f \) are evaluated concisely and presented here;

2.3.1 For \( \gamma < \gamma_T \) (\( m_b = m_f = 1 \))

The expressions for \( \lambda_b \) and \( \lambda_f \) are

\[ \lambda_b = \frac{P_c L^2}{EI} = \pi^2 \left[ 1 + \frac{\gamma}{4} \right] \]  
(16)

and

\[ \lambda_f = \frac{\bar{m} \omega^2 L^4}{EI} = \pi^4 (1 + \frac{\gamma}{4}) \]  
(17)

2.3.2 For \( \gamma > \gamma_T \) and \( \gamma < \gamma_{T2} \) (\( m_b = 2, m_f = 1 \))

These expressions are

\[ \lambda_b = \frac{P_c L^2}{EI} = \pi^2 \left[ 4 + \frac{\gamma}{2} \right] \]  
(18)

and

\[ \lambda_f = \frac{\bar{m} \omega^2 L^4}{EI} = \pi^4 (1 + \gamma) \]  
(19)

2.3.3 For \( \gamma > \gamma_T \) and \( \gamma < \gamma_{T3} \) (\( m_b = 2, m_f = 2 \))

The expressions for \( \lambda_b \) and \( \lambda_f \) are

\[ \lambda_b = \frac{P_c L^2}{EI} = \pi^2 \left[ 4 + \frac{\gamma}{2} \right] \]  
(20)

and

\[ \lambda_f = \frac{\bar{m} \omega^2 L^4}{EI} = \pi^4 (16 + \gamma) \]  
(21)

2.3.4 For \( \gamma > \gamma_{T2} \) and \( \gamma < \gamma_{T3} \) (\( m_b = 3, m_f = 1 \))

The expressions for \( \lambda_b \) and \( \lambda_f \) are

\[ \lambda_b = \frac{P_c L^2}{EI} = \pi^2 \left[ 9 + \frac{\gamma}{3} \right] \]  
(22)

and
\[ \lambda_f = \frac{\bar{m} \omega^2 L^4}{EI} = \pi^4 (1 + \gamma) \]  

(23)

2.3.5 For \( \gamma > \gamma_{T2} \) and \( \gamma < \gamma_{T3} \) (\( m_b = 3, m_f = 3 \))

The expressions for \( \lambda_b \) and \( \lambda_f \) are

\[ \lambda_b = \frac{P_{cr} L^2}{EI} = \pi^2 \left[ 9 + \frac{\gamma}{9} \right] \]  

(24)

and

\[ \lambda_f = \frac{\bar{m} \omega^2 L^4}{EI} = \pi^4 (81 + \gamma) \]  

(25)

The expressions for \( \lambda_b \) and \( \lambda_f \) for the above cases are given for the sake of completeness, for any combination of \( m_b, m_f \) and \( \gamma \) the buckling and frequency parameters can be easily obtained for a given value of these three parameters, taking care of the transition value of the foundation parameter for \( \gamma \).

3. DYNAMIC STABILITY FORMULAS

The dynamic stability formulas can be obtained, by applying the reference values of \( P_{cr} \) and \( \omega \) for different values of \( \gamma \) with the consideration of the proper values of \( m_b \) and \( m_f \). These formulas are to predict the dynamic instability boundaries of the SS beam resting on elastic foundation are:

a) Substituting the Eqs. (11), (16) and (17) in Eq. (10), half sine wave (first mode) for buckling and vibration \( (m_b = m_f = 1) \) as the reference values, where \( P_{cr} = \frac{\pi^2 EI}{L^2} \) and \( \bar{m} = \frac{\pi^4 EI}{\omega^2 L^4} \), without consideration of the effect of elastic foundation, then the dynamic stability formula is obtained, as

\[ 1 + \gamma - \left( \alpha + \frac{\beta}{2} \right) - \frac{\theta^2}{4 \omega^2} = 0 \]  

(26)

where \( \alpha = \frac{P_s}{P_{cr}} \) and \( \beta = \frac{P_f}{P_{cr}} \).

or

\[ \Omega = \frac{\theta}{\omega} = 2 \sqrt{(1 - \alpha)(1 \pm \mu) + \gamma} \]  

(27)

where \( \mu \) is the non-dimensional parameter, \( \mu = \frac{\beta}{2(1 - \alpha)} \).

b) Substituting Eqs. (11), (18) and (19) in Eq. (10) for \( m_b = 2 \) and \( m_f = 1 \) then the dynamic stability formula is written, as

\[ 1 - \left( \alpha + \frac{\beta}{2} \right) P_{cr} \frac{4 \pi^2}{L^2} = \frac{\bar{m} \theta^2}{4} \left[ \frac{\pi^4 EI}{L^4} \right] \left[ 16 + \frac{kl^4}{\pi^4 EI} \right] = 0 \]  

(28)

Substituting the values of \( P_{cr} = \frac{4 \pi^2 EI}{L^2} \) and \( \bar{m} = \frac{\pi^4 EI}{\omega^2 L^4} \) without consideration of foundation in Eq. (28) becomes
\[
\Omega = \frac{\theta}{\omega} = 2 \sqrt{\left(1 - \alpha \right)(1 \pm \mu) + \frac{\gamma}{16}\left(\frac{1 + \gamma}{1 + \gamma / 16}\right)}
\]  
(29)

c) In the similar way, substituting Eqs. (11), (22) and (23) in Eq. (10) for \( m_b = 3 \) and \( m_f = 1 \), the dynamic stability formula is obtained, as

\[
1 - \left(\frac{\alpha \pm \beta}{2}\right) P_{cr} \cdot \frac{9 \pi^2}{L^2} \frac{\pi^4 EI}{L^4} - \frac{\overline{m} \theta^2}{4} = 0
\]  
(30)

Substituting the reference values for \( P_{cr} = \frac{9 \pi^2 EI}{L^2} \) and \( \overline{m} = \frac{\pi^4 EI}{\omega^2 L^4} \) without consideration of foundation, Eq. (30) is written, as

\[
\Omega = \frac{\theta}{\omega} = 2 \sqrt{\left(1 - \alpha \right)(1 \pm \mu) + \frac{\gamma}{81}\left[\frac{1 + \gamma}{1 + \gamma / 81}\right]} \]  
(31)

The above three dynamic stability equations (formulas) are derived based on the mode numbers \( m_b \) and \( m_f \) for the values of the first and second transition foundation parameter \( \gamma \) lesser or greater than \( \gamma_{T1} \) and \( \gamma_{T2} \).

4. MASTER DYNAMIC STABILITY FORMULA

Substituting the Eq. (11), the values of reference buckling load \( P_{cr} \) and frequency \( \omega \) with consideration of foundation for half sine wave for stability and half sine wave vibration (\( m_b = 1, m_f = 1 \)), two half wave for stability and two- half wave for vibration (\( m_b = 2, m_f = 2 \)) and three half waves for both vibration and stability problems (\( m_b = 3, m_f = 3 \)), in Eq. (10), we get

\[
\Omega = \frac{\theta}{\omega} = 2 \sqrt{\left(1 - \alpha \right)(1 \pm \mu)}
\]  
(32)

It may be noted that the master dynamic stability formula shown in Eq. (32) does not contain the foundation parameter explicitly, unlike the earlier formulas where in the foundation parameters \( \gamma \) appear explicitly.

5. NUMERICAL RESULTS AND DISCUSSION

Figure 1 shows, SS beam resting on elastic foundation, subjected to an axial periodic load. The formula developed in the present study to investigate the dynamic stability boundaries of SS beam resting on an elastic foundation, subjected to axial periodic concentrated loads is very general. In the non-dimensional form the characteristic values of the beam such as the fundamental frequency and the static buckling load parameters do not appear explicitly. However these characteristic values appear implicitly in the definition of the non-dimensional parameters \( \mu \) and \( \Omega \).
### Table 1. Values of $\Omega_1$ and $\Omega_2$ for $\gamma = 5.0$ ($\gamma > \gamma_{T1}$)
(for stability two half sine waves and for vibration one half sine wave)

| $\mu$ | $\alpha = 0.6$ | $\alpha = 0.7$ | $\alpha = 0.8$ |
|-------|----------------|----------------|----------------|
|       | $\Omega_1$    | $\Omega_2$    | $\Omega_1$    | $\Omega_2$    | $\Omega_1$    | $\Omega_2$    |
| 0.0   | 3.6095         | 3.6095         | 3.3466         | 3.3466         | 3.0612         | 3.0612         |
| 0.1   | 3.5067         | 3.7094         | 3.2636         | 3.4276         | 3.0009         | 3.1204         |
| 0.2   | 3.4008         | 3.8067         | 3.1784         | 3.5067         | 2.9393         | 3.1784         |
| 0.3   | 3.2915         | 3.9016         | 3.0910         | 3.5840         | 2.8765         | 3.2355         |
| 0.4   | 3.1784         | 3.9942         | 3.0009         | 3.6598         | 2.8122         | 3.2915         |
| 0.5   | 3.0612         | 4.0848         | 2.9081         | 3.7340         | 2.7464         | 3.3466         |

### Table 2. Values of $\Omega_1$ and $\Omega_2$ for $\gamma = 15.0$ ($\gamma > \gamma_{T1}$)
(for stability two half sine waves and for vibration one half sine wave)

| $\mu$ | $\alpha = 0.6$ | $\alpha = 0.7$ | $\alpha = 0.8$ |
|-------|----------------|----------------|----------------|
|       | $\Omega_1$    | $\Omega_2$    | $\Omega_1$    | $\Omega_2$    | $\Omega_1$    | $\Omega_2$    |
| 0.0   | 6.6468         | 6.6468         | 6.3935         | 6.3935         | 6.1297         | 6.1297         |
| 0.1   | 6.5467         | 6.7455         | 6.3155         | 6.4705         | 6.0756         | 6.1834         |
| 0.2   | 6.4450         | 6.8427         | 6.2366         | 6.5467         | 6.0210         | 6.2366         |
| 0.3   | 6.3416         | 6.9386         | 6.1566         | 6.6219         | 5.9659         | 6.2893         |
| 0.4   | 6.2366         | 7.0331         | 6.0756         | 6.6963         | 5.9102         | 6.3416         |
| 0.5   | 6.1297         | 7.1265         | 5.9935         | 6.7699         | 5.8541         | 6.3935         |

### Table 3. Values of $\Omega_1$ and $\Omega_2$ for $\gamma = 25.0$ ($\gamma > \gamma_{T1}$)
(for stability two half sine waves and for vibration one half sine wave)

| $\mu$ | $\alpha = 0.6$ | $\alpha = 0.7$ | $\alpha = 0.8$ |
|-------|----------------|----------------|----------------|
|       | $\Omega_1$    | $\Omega_2$    | $\Omega_1$    | $\Omega_2$    | $\Omega_1$    | $\Omega_2$    |
| 0.0   | 8.9246         | 8.9246         | 8.6942         | 8.6942         | 8.4576         | 8.4576         |
| 0.1   | 8.8331         | 9.0151         | 8.6239         | 8.7640         | 8.4095         | 8.5054         |
| 0.2   | 8.7408         | 9.1047         | 8.5530         | 8.8331         | 8.3611         | 8.5530         |
| 0.3   | 8.6474         | 9.1934         | 8.4816         | 8.9018         | 8.3124         | 8.6003         |
| 0.4   | 8.5530         | 9.2812         | 8.4095         | 8.9699         | 8.2634         | 8.6474         |
| 0.5   | 8.4576         | 9.3683         | 8.3368         | 9.0375         | 8.2142         | 8.6942         |
Table 4. Values of $\Omega_1$ and $\Omega_2$ for $\gamma = 35.0$ ($\gamma > \gamma_{71}$)
(for stability two half sine waves and for vibration one half sine wave)

| $\mu$ | $\alpha = 0.6$ | $\alpha = 0.7$ | $\alpha = 0.8$ |
|-------|----------------|----------------|----------------|
|       | $\Omega_1$    | $\Omega_2$    | $\Omega_1$    | $\Omega_2$    | $\Omega_1$    | $\Omega_2$    |
| 0.0   | 10.8117        | 10.8117        | 10.6007        | 10.6007        | 10.3855        | 10.3855        |
| 0.1   | 10.7278        | 10.8950        | 10.5366        | 10.6645        | 10.3419        | 10.4289        |
| 0.2   | 10.6433        | 10.9776        | 10.4721        | 10.7278        | 10.2981        | 10.4721        |
| 0.3   | 10.5580        | 11.0596        | 10.4072        | 10.7908        | 10.2541        | 10.5151        |
| 0.4   | 10.4721        | 11.1410        | 10.3419        | 10.8534        | 10.2100        | 10.5580        |
| 0.5   | 10.3855        | 11.2218        | 10.2761        | 10.9157        | 10.1656        | 10.6007        |

Figure 2. Non-dimensional dynamic stability curves of SS beam resting on an foundation for $\gamma = 5.0$ ($\gamma > \gamma_{71}$)

Figure 3. Non-dimensional dynamic stability curves of SS beam resting on an foundation for $\gamma = 15.0$ ($\gamma > \gamma_{71}$)
Figure 4. Non-dimensional dynamic stability curves of SS beam resting on an foundation for $\gamma = 25.0$ ($\gamma > \gamma_{TI}$)

Figure 5. Non-dimensional dynamic stability curves of SS beam resting on an foundation for $\gamma = 35.0$ ($\gamma > \gamma_{TI}$)

Tables 1 to 4 gives the values of the stability boundaries $\Omega_2$ and $\Omega_3$ between which it is dynamically unstable with varying $\mu$ for $\alpha = 0.6$, 0.7 and 0.8 with the mode shape of two half sine wave for stability problem and one half sine wave for vibration problem for $\gamma > 4$ ($\gamma > \gamma_{TI}$). The instability boundaries $\Omega_2$ and $\Omega_3$ given in Tables 1 to 4 are for $\alpha = 0.6$, 0.7 and 0.8 with $\gamma = 5$ to $\gamma = 35$. Figures 2 to 5 shows the instability boundaries $\Omega_2$ and $\Omega_3$ given in Tables 1 to 4 for better visualization of the regions of dynamic stability. It can also be observed that with the increasing elastic foundation parameter, the dynamic instability regions are shifted away from the vertical axis. One significant observation in this
study is the regions of dynamic instability decreases for $\gamma = 5$ to $\gamma = 35.0$ for above the first transition foundation and below the second transition foundation parameter value.

### Table 5.
Values of $\Omega_1$ and $\Omega_2$ for $\gamma = 45.0$ ($\gamma > \gamma_{t2}$)
(for stability three half sine waves and for vibration one half sine wave)

| $\mu$ | $\alpha = 0.6$ | $\alpha = 0.7$ | $\alpha = 0.8$ |
|-------|----------------|----------------|----------------|
| 0.0   | 10.6314        | 10.6314        | 9.4536         |
| 0.1   | 10.4065        | 10.8517        | 9.3276         |
| 0.2   | 10.1767        | 11.0675        | 9.2000         |
| 0.3   | 9.9415         | 11.2793        | 9.0705         |
| 0.4   | 9.7006         | 11.4871        | 8.9391         |
| 0.5   | 9.4536         | 11.6912        | 8.8058         |

### Table 6.
Values of $\Omega_1$ and $\Omega_2$ for $\gamma = 55.0$ ($\gamma > \gamma_{t2}$)
(for three half sine waves for stability and one half sine wave for vibration)

| $\mu$ | $\alpha = 0.6$ | $\alpha = 0.7$ | $\alpha = 0.8$ |
|-------|----------------|----------------|----------------|
| 0.0   | 11.9980        | 11.9980        | 10.8291        |
| 0.1   | 11.7735        | 12.2184        | 10.7052        |
| 0.2   | 11.5446        | 12.4348        | 10.5798        |
| 0.3   | 11.3112        | 12.6476        | 10.4530        |
| 0.4   | 11.0728        | 12.8568        | 10.3246        |
| 0.5   | 10.8291        | 13.0627        | 10.1945        |

### Table 7.
Values of $\Omega_1$ and $\Omega_2$ for $\gamma = 60.0$ ($\gamma > \gamma_{t2}$)
(for stability three half sine waves and for vibration one half sine wave)

| $\mu$ | $\alpha = 0.6$ | $\alpha = 0.7$ | $\alpha = 0.8$ |
|-------|----------------|----------------|----------------|
| 0.0   | 12.6450        | 12.6450        | 11.4831        |
| 0.1   | 12.4213        | 12.8648        | 11.3604        |
| 0.2   | 12.1936        | 13.0809        | 11.2364        |
| 0.3   | 11.9614        | 13.2935        | 11.1109        |
| 0.4   | 11.7247        | 13.5027        | 10.9840        |
| 0.5   | 11.4831        | 13.7088        | 10.8557        |

Tables 5 to 7 gives the values of the stability boundaries $\Omega_1$ and $\Omega_2$, between which it is dynamically unstable for three-half sine waves for stability problem and one-half sine wave of vibration problem above the value of second transition foundation ($\gamma > 36$) for $\gamma = 45.0$ to $\gamma = 60.0$ respectively, obtained from the Eq. (31). It is observed that, the instability regions are initially wider above the second transition foundation value and the dynamic instability regions are shifted away from the vertical axis. The other significant observation is, the dynamic instability regions, decreasing with increasing foundation value for above the second transition foundation and below the third transition foundation

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value. Figures 6 to 8 shows the instability boundaries $\Omega_1$ and $\Omega_2$ given in Tables 5 and 7 for better visualization of the regions of dynamic stability.

**Figure 6.** Non-dimensional dynamic stability curves of SS beam resting on an foundation for $\gamma = 45.0$ ($\eta > \gamma_{T2}$)

**Figure 7.** Non-dimensional dynamic stability curves of SS beam resting on an foundation for $\gamma = 55.0$ ($\eta > \gamma_{T2}$)
Figure 8. Non-dimensional dynamic stability curves of SS beam resting on an foundation for $\gamma = 60.0$ ($\gamma > \gamma T_2$)

Figure 9 shows, the dynamic stability curves for two-half wave for both the stability and vibration problems, the reference critical load and reference frequency parameters are taken with consideration of elastic foundation, for the foundation parameter below the second transition value ($\gamma < 36$) and three half waves for both vibration and stability problems, both reference critical load and reference frequency parameters are with consideration of elastic foundation for the below the third transition value ($\gamma < 144$) are obtained from Eq. (32) are presented.

Figure 9. Master Dynamic stability curves
6. CONCLUSIONS

Analytical solutions are obtained to simplify the dynamic stability behavior of simply supported beams resting on an elastic foundation subjected to an end periodic concentrated axial load. Single term trigonometric admissible function, which satisfies the geometric boundary conditions, is used to obtain the feasible solution by means of classical Rayleigh – Ritz method. The effect higher transition foundation on the dynamic stability boundaries is established. The results, obtained with varying second transition foundation parameter, below and above this transition value, are presented in this work. As the elastic foundation increases, the width of the regions of dynamic instability decreases, thus making the beam less sensitive to the dynamic stability phenomenon to periodic loads. Finally, the use of the non-dimensional parameters derived in the present work is demonstrated, for the beams on elastic foundation, to obtain the master dynamic stability boundaries (curves) for the same mode for stability and vibration problems, the reference critical/buckling load and frequency parameters are obtained with consideration of foundation. No attempt is made here to validate the formulation as it has been established beyond doubt in the earlier studies of the author.

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