Numerical simulation of Quantum Teleportation in a chain of three nuclear spins system taking into account second neighbor iteration

G.V. López and L. Lara
Departamento de Física, Universidad de Guadalajara
Apartado Postal 4-137, 44410 Guadalajara, Jalisco, México

PACS: 03.67.-a, 03.67.Lx, 03.67.Dd, 03.67.Hk

ABSTRACT

For a one-dimensional chain of three nuclear spins (one half), we make the numerical simulation of quantum teleportation of a given state from one end of the chain to the other end, taking into account first and second neighbor interactions among the spins. It is shown that a well defined teleportation protocol is achieved for a ratio of the first to second neighbor interaction coupling constant of $J'/J \geq 0.04$. We also show that the optimum Rabi’s frequency to control the non-resonant effects is dominated by the second neighbor interaction coupling parameter $(J')$. 
1. Introduction

Quantum teleportation is a technique which allows us to move a quantum state of our quantum system from one location to another location using quantum computation or quantum information [1]. Quantum computation or quantum information uses quantum bits (called qubits) to handle the information on the quantum system. A qubit is a superposition of any two quantum basic states of the system (called \(|0\rangle\) and \(|1\rangle\)), \(\Psi = C_0|0\rangle + C_1|1\rangle\), such that \(|C_0|^2 + |C_1|^2 = 1\). One may call \(|0\rangle\) and \(|1\rangle\) the basic qubits of the system. The \(L\)-tensorial product of \(L\)-basic qubits form an \(L\)-register of \(L\)-qubits, \(|x\rangle = |i_{L-1}, \ldots, i_0\rangle\) with \(i_j = 0, 1\) for \(j = 0, \ldots, L - 1\) ("0" for ground state and "1" for exited state). The set of these \(L\)-registers makes up the \(2^L\)-dimensional Hilbert space where the quantum computer and quantum information work. A typical element of this space is \(\Psi = \sum C_x|x\rangle\), where \(\sum |C_x|^2 = 1\), and \(|C_x|^2\) gives us the probability of having the state \(|x\rangle\) after measurement.

We are interested in studying the phenomena of quantum teleportation in a solid state quantum computer formed by a one-dimensional chain of nuclear spins [2], where a preliminary study of this technique was done using two-qubit registers and considering first neighbor interaction between spins [3]. In this paper, we will consider also the second neighbor interaction among the spins in a chain of three nuclear spins system. Thus, the phenomena of quantum teleportation from one end of the chain of spins to the other end is studied numerically. Adding a quantum state (qubit) to our chain of three nuclear spins means to add another nuclear spin at one end, having, then, a quantum computer working with four-qubit registers. We study the well performance of the quantum teleportation algorithm through the fidelity parameter and determine the minimum value of the second neighbor interaction coupling constant to do this. Finally, we also see the modification that the so called \(2\pi k\)-method could have with the consideration of second neighbor iteration and its implication in our simulation.

2. Equation of Motion

Consider a one-dimensional chain of four equally spaced nuclear-spins system (spin one half) making an angle \(\cos \theta = 1/\sqrt{3}\) with respect the z-component of the magnetic field (chosen in this way to kill the dipole-dipole interaction between spins) and having an rf-magnetic field in the transversal plane. The magnetic field is given by

\[
\mathbf{B} = (b \cos(\omega t + \varphi), -b \sin(\omega t + \varphi), B(z)) ,
\]

where \(b\), \(\omega\) and \(\varphi\) are the amplitude, the angular frequency and the phase of the rf-field, which could be different for different pulses. \(B(z)\) is the amplitude of the z-component of the magnetic field. Thus, the Hamiltonian of the system up to second neighbor interaction is given by

\[
H = -\sum_{k=0}^{3} \mu_k \cdot \mathbf{B}_k - 2J\hbar \sum_{k=0}^{2} I_k^x I_{k+1}^x - 2J'\hbar \sum_{k=0}^{1} I_k^x I_{k+2}^x ,
\]

where \(\mu_k\) represents the magnetic moment of the kth-nucleus which is given in terms of the nuclear spin as \(\mu_k = \hbar \gamma (I_k^x, I_k^y, I_k^z)\), being \(\gamma\) the proton gyromagnetic ratio. \(\mathbf{B}_k\) represents the magnetic field at the location of the kth-spin. The second term
at the right side of (2) represents the first neighbor spin interaction, and the third
term represents the second neighbor spin interaction. \( J \) and \( J' \) are the coupling
constants for these interactions. This Hamiltonian can be written in the following
way
\[
H = H_0 + W ,
\]
where \( H_0 \) and \( W \) are given by
\[
H_0 = -\hbar \left\{ \sum_{k=0}^{3} \omega_k I_k^z + 2J(I_{0}^{z}I_{1}^{z} + I_{1}^{z}I_{2}^{z} + I_{2}^{z}I_{3}^{z}) + 2J'(I_{0}^{z}I_{2}^{z} + I_{1}^{z}I_{3}^{z}) \right\} =
\]
and
\[
W = -\frac{\hbar \Omega}{2} \sum_{k=0}^{3} \left[ e^{i\omega t I_k^+} + e^{-i\omega t I_k^-} \right],
\]
where \( \omega_k = \gamma B(z_k) \) is the Larmore frequency of the kth-spin, \( \Omega = \gamma b \) is the Rabi’s
frequency, and \( I_k^\pm = I_k^x \pm iI_k^y \) represents the ascend operator (+) or the descend
operator (-). The Hamiltonian \( H_0 \) is diagonal on the basis \( \{ |i_3i_2i_1i_0 \rangle \} \), where \( i_j = 0,1 \) (zero for the ground state and one for the exited state),
\[
|H_0|i_3i_2i_1i_0 \rangle = E_{i_3i_2i_1i_0} |i_3i_2i_1i_0 \rangle .
\]
The eigenvalues \( E_{i_3i_2i_1i_0} \) are given by
\[
E_{i_3i_2i_1i_0} = -\frac{\hbar \omega_k}{2} \sum_{k=0}^{3} \left[ e^{i\omega t I_k^+} + e^{-i\omega t I_k^-} \right] .
\]
The term (3c) of the Hamiltonian allows to have a single spin transitions on the
above eigenstates by choosing the proper resonant frequency.

To solve the Schrödinger equation
\[
i\hbar \frac{\partial \Psi}{\partial t} = H \Psi ,
\]
let us propose a solution of the form
\[
\Psi(t) = \sum_{k=0}^{15} C_k(t)|k \rangle ,
\]
where we have used decimal notation for the eigenstates in (4a), \( H_0|k \rangle = E_k|k \rangle \).
Substituting (6) in (5), multiplying for the bra \( \langle m | \), and using the orthogonality
relation \( \langle m | k \rangle = \delta_{mk} \), we get the following equation for the coefficients
\[
i\hbar \dot{C}_m = E_m C_m + \sum_{k=0}^{15} C_k \langle m | W | k \rangle \quad m = 0, \ldots, 15. \tag{7}
\]
Now, using the following transformation
\[
C_m = D_m e^{-iE_m t/\hbar} ,
\]
the fast oscillation term $E_mC_m$ of Eq. (7) is removed (this is equivalent to going to the interaction representation), and the following equation is gotten for the coefficients $D_m$

$$i\dot{D}_m = \frac{1}{\hbar} \sum_{k=0}^{15} W_{mk} D_k e^{i\omega_{mk} t}, \quad (9a)$$

where $W_{mk}$ denotes the matrix elements $\langle m | W | k \rangle$, and $\omega_{mk}$ are defined as

$$\omega_{mk} = \frac{E_m - E_k}{\hbar}. \quad (9b)$$

Eq. (9a) represents a set of sixteen real coupling ordinary differential equations which can be solved numerically, and where $W'_{mk}$s are given by

$$(W) = -\frac{\hbar \Omega}{2} \times \{0, z, "or" z^*\}, \quad (9c)$$

where $z$ is defined as $z = e^{i(\omega t + \varphi)}$, and $z^*$ is its complex conjugated.

A pulse of time length $\tau$, phase $\varphi$, and resonant frequency $\omega = \omega_{i,j}$ will be denoted by $R_{i,j}(\Omega \tau, \varphi)$, and it is understood that such a pulse implies the solution of Eq. (9a) with given initial conditions.

3. Quantum teleportation and numerical simulation

The basic idea of quantum teleportation [1] is that Alice (left end qubit in our chain of three qubits) and Bob (the other end qubit) are related through and entangled state,

$$\Psi_e = \frac{1}{\sqrt{2}} \left( |0A00B\rangle + |1A01B\rangle \right). \quad (10)$$

So, neglecting decoherence effects which could destroy this entangled state, we adjoin to Alice an arbitrary state $\Psi_x$,

$$\Psi_x = C^x_0 |0\rangle + C^x_1 |1\rangle, \quad (11)$$

resulting the quantum state ($\Psi_1 = \Psi_x \otimes \Psi_e$)

$$\Psi_1 = \frac{1}{\sqrt{2}} \left[ C^x_0 |0000\rangle + C^x_0 |0101\rangle + C^x_1 |1000\rangle + C^x_1 |1101\rangle \right]. \quad (12)$$

Then, one applies a Controlled-Not (CN) operation in this added state and Alice,

$$CN_{23}[i_3, i_2, i_1, i_0] = |i_3, i_2 \oplus i_3, i_2, i_0\rangle, \quad (13)$$

where $i_2 \oplus i_3 = i_2 + i_3 (mod \ 2)$, resulting the state ($\Psi_2 = CN_{23}\Psi_1$)

$$\Psi_2 = \frac{1}{\sqrt{2}} \left[ C^x_0 |0000\rangle + C^x_0 |0101\rangle + C^x_1 |1100\rangle + C^x_1 |1001\rangle \right]. \quad (14)$$

Finally, one applies a superposition (Hadamard) operation in the added state location,

$$A_3 \left\{ \begin{array}{c} |0000\rangle = \frac{1}{\sqrt{2}} \left( |0000\rangle + |1000\rangle \right), \\ |1000\rangle = \frac{1}{\sqrt{2}} \left( |0000\rangle - |1000\rangle \right). \end{array} \right.$$
resulting, after some rearrangements, the state \((\Psi_3 = A_3 \Psi_2)\)
\[
\Psi_3 = \frac{1}{2} \left\{ |00\rangle \otimes |0\rangle \left( C_0^x |0\rangle + C_1^x |1\rangle \right) + |01\rangle \otimes |0\rangle \otimes \left( C_0^x |1\rangle + C_1^x |0\rangle \right) + |10\rangle \otimes |0\rangle \otimes \left( C_0^x |0\rangle - C_1^x |1\rangle \right) + |11\rangle \otimes |0\rangle \otimes \left( C_0^x |1\rangle - C_1^x |0\rangle \right) \right\}.
\]
(16)

When Alice measures both of her qubits, there are four possible cases \((|00\rangle, |01\rangle, |10\rangle, \text{ and } |11\rangle)\), and for each case Bob will get the original \(\Psi_x\) state by applying a proper operation to his state: identity, Not \((\hat{N})\), \(\sigma_z\), or \(\sigma_z \hat{N}\), where the action of these operators are as follows: \(\hat{N} |0\rangle = |1\rangle\), \(\hat{N} |1\rangle = |0\rangle\), \(\sigma_z |0\rangle = |0\rangle\), and \(\sigma_z |1\rangle = -|1\rangle\). The most important features in this algorithm are that Alice and Bob do not need to know the state \(\Psi_x\), and there are always four possibilities for Alice measurement.

Now, the algorithm we need to implement this quantum teleportation technique in our one-dimensional three nuclear spins system varies a little from the scheme presented above since our three-qubits quantum computer will start from the ground state
\[
\Psi_{00} = |000\rangle.
\]
(17a)

Therefore, one needs first to attach the unknown state (11) to (17a) to get the initial four-spins wave function \((\Psi_0 = \Psi_x \otimes \Psi_{00})\),
\[
\Psi_0 = C_0^x |0000\rangle + C_1^x |1000\rangle,
\]
(17b)
where \(|C_0^x|^2 + |C_1^x|^2 = 1\). The entangled state (12) is obtained with the following three pulses
\[
\Psi_1 = R_{4,5}(\pi, -3\pi/2) R_{8,12}(\pi/2, -\pi/2) R_{0,4}(\pi/2, -\pi/2) \Psi_0.
\]
(16c)

The Controlled-Not operation \(\hat{C}\hat{N}_{23}\Psi_1\) is gotten through the following two pulses
\[
\Psi_2 = R_{9,13}(\pi, 3\pi/2) R_{8,12}(\pi, -3\pi/2) \Psi_1.
\]
(17d)

Finally, the wave function (16) is gotten after the application of the following two pulses to the above wave function
\[
\Psi_3 = R_{4,12}(\pi/2, -3\pi/2) R_{1,9}(\pi/2, -3\pi/2) \Psi_2.
\]
(17e)

To make the numerical simulation of this algorithm, we have chosen the following parameters in units of \(2\pi \times \text{Mhz}\),
\[
\omega_0 = 100, \ \omega_1 = 200, \ \omega_2 = 400, \ \omega_3 = 800, \ J = 10, \ J' = 0.4, \ \Omega = 0.1.
\]
(18a)

These parameters were chosen in this way to have a clear definition on our Zeeman spectrum and transitions among them. Of course, our main results are applied to the actual current design parameters of reference [4]. On the other hand, we selected the state (11) with the following coefficients
\[
C_0^x = \frac{1}{3} \quad \text{and} \quad C_1^x = \frac{\sqrt{2}}{3}.
\]
(18b)
Fig. 1 shows the Zeeman spectrum of the four nuclear spins system with the transitions used during our teleportation simulation. Fig. 2a shows the probabilities $|C_0|^2$, $|C_5|^2$, $|C_8|^2$ and $|C_{13}|^2$ during the first three pulses, where the formation of the entangled wave function (12) is shown, from the initial state (17b). Fig. 2b shows the probabilities $|C_0|^2$, $|C_5|^2$, $|C_9|^2$ and $|C_{12}|^2$ during the following two pulses to get at the end the wave function (14). Fig. 2c shows the probabilities $|C_0|^2$, $|C_8|^2$, $|C_{13}|^2$, $|C_{12}|^2$, $|C_4|^2$, $|C_9|^2$ and $|C_1|^2$ during the last two pulses to get at the end the desired function (16). Note that at the end of our algorithm one expects the following values for these probabilities

$$|C_0|^2 = |C_8|^2 = |C_5|^2 = |C_{13}|^2 = \frac{|C_0|^2}{4},$$  \hspace{1cm} (19a)

and

$$|C_{12}|^2 = |C_4|^2 = |C_9|^2 = |C_1|^2 = \frac{|C_1|^2}{4}.$$  \hspace{1cm} (19b)

To get a better feeling what is going on on each process, we calculate the z-component of the expected values of the spin for each qubit. These expected values are given by

$$\langle I_5^z \rangle = \frac{1}{2} \sum_{k=0}^{15} (-1)^k |C_k(t)|^2,$$  \hspace{1cm} (20a)

$$\langle I_1^z \rangle = \frac{1}{2} \left\{ |C_0|^2 + |C_1|^2 - |C_2|^2 - |C_3|^2 + |C_4|^2 + |C_5|^2 - |C_6|^2 - |C_7|^2 + |C_8|^2 + |C_9|^2 - |C_{10}|^2 - |C_{11}|^2 + |C_{12}|^2 + |C_{13}|^2 - |C_{14}|^2 - |C_{15}|^2 \right\},$$  \hspace{1cm} (20b)

$$\langle I_2^z \rangle = \frac{1}{2} \left\{ |C_0|^2 + |C_1|^2 + |C_2|^2 + |C_3|^2 - |C_4|^2 - |C_5|^2 - |C_6|^2 - |C_7|^2 + |C_8|^2 + |C_9|^2 + |C_{10}|^2 + |C_{11}|^2 - |C_{12}|^2 - |C_{13}|^2 - |C_{14}|^2 - |C_{15}|^2 \right\},$$  \hspace{1cm} (20c)

and

$$\langle I_5^x \rangle = \frac{1}{2} \sum_{k=0}^{7} |C_k|^2 - \frac{1}{2} \sum_{k=8}^{15} |C_k|^2.$$  \hspace{1cm} (20d)

Figs. 3a, 3b, and 3c show these expected values during entangled formation, controlled-not operation, and final teleportation. As one can see, these behavior is what one could expected for each process. Fig. 4a shows the probabilities (19a) and (19b) at the end of the teleportation (wave function (16)), and Fig. 4b shows the probabilities of the non-resonant states involved in the dynamics.

To see the values of the second neighbor interaction coupling parameter from which one could have a well defined teleportation algorithm, we study the fidelity parameter [5],

$$F = \langle \Psi_{expected} | \Psi \rangle,$$  \hspace{1cm} (21)
where $\Psi_{\text{expected}}$ is the ideal wave function (16). Fig. 5 shows this fidelity parameter as a function of the ratio of second to first neighbor coupling parameters ($J'/J$). As one can see, a well defined teleportation algorithm is gotten for $J'/J \geq 0.04$.

4. Quantum teleportation and the $2\pi k$-method

One of the important results from the consideration of first neighbor interaction and the selection of the parameter as $J/\Delta \omega \ll 1$ and $\Omega/\Delta \omega \ll 1$ is the possibility to choose the Rabi’s frequency $\Omega$, in such a way that the non-resonant effects are eliminated. This procedure is called the $2\pi k$-method [4], and this Rabi’s frequency is chosen as $\Omega = |\Delta|/\sqrt{4k^2 - 1}$ for a $\pi$-pulse, where $k$ is an integer number, $\Delta$ is the detuning parameter ($\Delta = (E_p - E_m)/\hbar - \omega$) between the states $|p\rangle$ and $|m\rangle$ when the resonant frequency is $\omega$, and this detuning parameter is proportional to the first neighbor coupling constant $J$. Let us see how this detuning parameter could be modified due to second neighbor interaction. Assuming that the states $|p\rangle$ and $|m\rangle$ are the only ones involved in the dynamics, from Eq. (9a), one has

$$i\dot{D}_m = \frac{W_{mp}}{\hbar} D_p e^{i\omega_m t}, \quad \mathrm{and} \quad i\dot{D}_p = \frac{W_{pm}}{\hbar} D_m e^{i\omega_m t}. \quad (22)$$

Thus, given the initial conditions $D_p(0) = C_p(0)$ and $D_m(0) = C_m(0)$, the solution is readily given by

$$D_p(t) = \left\{ C_p(0) \left( \cos \frac{\Omega_e t}{2} - i \frac{\Delta}{\Omega_e} \sin \frac{\Omega_e t}{2} \right) + i \frac{\Omega}{\Omega_e} C_m(0) \sin \frac{\Omega_e t}{2} \right\} e^{it\Delta t}, \quad (23a)$$

and

$$D_m(t) = \left\{ C_m(0) \left( \cos \frac{\Omega_e t}{2} - i \frac{\Delta}{\Omega_e} \sin \frac{\Omega_e t}{2} \right) + i \frac{\Omega}{\Omega_e} C_p(0) \sin \frac{\Omega_e t}{2} \right\} e^{-it\Delta t}, \quad (23b)$$

where $\Omega_e$ is defined as $\Omega_e = \sqrt{\Omega^2 + \Delta^2}$. For a $\pi$-pulse ($t = \tau = \pi/\Omega$), one can select the term $\Omega_e \pi/2\Omega$ to be equal to any multiple of $\pi$, $\Omega_e \pi/2\Omega = k\pi$, to get the condition $\Omega = |\Delta|/\sqrt{4k^2 - 1}$. This condition gets rid of the non-resonant terms since from Eqs. (20a) and (20b) one gets

$$D_p(\tau) = (-1)^k C_p(0) e^{i\Delta\pi/2\Omega}, \quad \mathrm{and} \quad D_m(\tau) = (-1)^k C_m(0) e^{-i\Delta\pi/2\Omega}.$$

For a $\pi/2$-pulse ($t = \tau = \pi/2\Omega$), one can select the term $\Omega_e \pi/4\Omega$ to be equal to any multiple of $\pi$, $\Omega_e \pi/4\Omega = k\pi$, to get the condition $\Omega = |\Delta|/\sqrt{16k^2 - 1}$. This condition gets rid also of the non-resonant terms.

Now, if for example one selects a resonant transition which contains the Larmor frequency $\omega_0$, these frequencies could be $\omega_0 + J + J'$, $\omega_0 - J + J'$, $\omega_0 - J - J'$ or $\omega_0 + J - J'$ which correspond to the transitions (decimal notation) $|0\rangle \leftrightarrow |1\rangle$ ($|10\rangle \leftrightarrow |11\rangle$), $|2\rangle \leftrightarrow |3\rangle$, $|6\rangle \leftrightarrow |7\rangle$, and $|2\rangle \leftrightarrow |3\rangle$. So, all of these states are pertubated, and the frequency difference $\Delta$ may have the values $2J, 2J', 2J + 2J'$, or $2J - 2J'$. For other Larmor frequencies the additional values of the detuning parameter are $4J$, and $4J + 2J'$. Thus, let us denote by $\Omega^{(k)}_\Delta$ the Rabi’s frequency selected by this method,

$$\Omega^{(k)}_\Delta = \frac{|\Delta|}{\sqrt{4k^2 - 1}}, \quad \pi\text{-pulse} \quad (24a)$$

7
and

\[ \tilde{\Omega}_{\Delta}^{(k)} = \frac{|\Delta|}{\sqrt{16k^2 - 1}} , \quad \pi/2\text{-pulse} \]  

(24b)

where \( \Delta \) can have the values \( 4J + 2J' \), \( 4J \), \( 2J + 2J' \), \( 2J \) or \( 2J' \). To see the dependence of our teleportation algorithm with respect to the Rabi’s frequency, we use again the fidelity parameter (21). With the same values for our parameters as (18a) but \( \Omega \), Fig. 6 shows the fidelity parameter as a function of the Rabi’s frequency. Dashed vertical lines mark the omega values where the peaks occur. These peaks correspond to some specific omega defined through the \( 2\pi k \)-method. For example, the line (1), (2) and (3) correspond to the following Rabi’s frequency values

\[ \Omega_{4J+2J'}^{(202)} \approx \Omega_{4J}^{(199)} \approx \Omega_{2J+2J'}^{(103)} \approx \Omega_{2J}^{(100)} \approx \Omega_{2J}^{(4)} = 0.10079 , \]

\[ \Omega_{4J+2J'}^{(305)} \approx \Omega_{4J}^{(304)} \approx \Omega_{2J+2J'}^{(156)} \approx \Omega_{2J}^{(150)} \approx \Omega_{2J_1'}^{(6)} = 0.066889 , \]

and

\[ \Omega_{4J+2J'}^{(407)} \approx \Omega_{4J}^{(400)} \approx \Omega_{2J+2J'}^{(208)} \approx \Omega_{2J}^{(200)} \approx \Omega_{2J_1'}^{(8)} = 0.050098 . \]

As one can see in Fig. 7a, where we have plotted \( \Omega_{\Delta}^{(k)} \) (for the detuning values mentioned above) and where the corresponding dashed vertical lines of Fig. 6 have been drawn, around these lines there are several other values of \( \Omega_{4J+2J'}^{(k)} \), \( \Omega_{4J}^{(k)} \), \( \Omega_{2J+2J'}^{(k)} \), and \( \Omega_{2J'}^{(k)} \) which, in principle, should cause a peak in the fidelity parameter (because they belong to the \( 2\pi k \)-method). However, they do not appear at all on Fig. 6. This means that the peaks values on the fidelity parameter are fully dominated by the second neighbor coupling interaction parameter (\( J' \)). On the other hand, the reason why only even numbers of \( k \) appears for these peaks for \( \Omega_{2J'}^{(k)} \) can be seen in Fig. 7b where \( \Omega_{2J'}^{(k)} \) and \( \tilde{\Omega}_{2J'}^{(k)} \) have been plotted as a function of \( k \). On this plot one sees that the jth-dashed lines correspond to \( \tilde{\Omega}_{2J'}^{(j)} = \Omega_{2J'}^{2j} \). Therefore, the peaks on fidelity correspond to \( 2\pi k \)-method dominated by \( J' \) and by \( \pi/2 \)-pulses. This seems reasonable since our teleportation algorithm starts with \( \pi/2 \)-pulses and finishes with \( \pi/2 \)-pulses.
5. Conclusions

We have made a numerical simulation of teleportation in a solid state quantum computer modeled by one-dimensional chain of three nuclear spins (one half), and considering first and second neighbor interactions. We have shown that a good teleportation algorithm can be gotten if the ratio of second to first neighbor interaction constants is chosen such that $J'/J \geq 0.04$. We also studied the effect of the second neighbor interaction on the detuning parameter which is used in the so called $2\pi k$-method to eliminate non-resonant transitions, and we have shown that the application of this method in our teleportation algorithm is not so simple since the detuning parameter varies with both parameters $J$ and $J'$ (first and second neighbor coupling interactions). However, the peaks on the fidelity parameter are dominated by the second neighbor coupling parameter and the $\pi/2$-pulses.

Acknowledgements

This work was supported by SEP under the contract PROMEP/103.5/04/1911 and the University of Guadalajara.
Figure Captions

Fig. 1 Energy levels and resonant frequencies used within the algorithm.

Fig. 2 Probabilities $|C_k|^2 : (k)$. (a) Formation of the entangled state, wave function (12). (b) Formation of the wave function (14). (c) Formation the wave function (16).

Fig. 3 Expected values $\langle J^z_k \rangle$: (k=0,1,2,3). (a) During formation of wave function (12). (b) During formation of the wave function (14). (c) During formation of the wave function (16).

Fig. 4 Probabilities $|C_k|^2$. (a) For the expected registers $k = 0, 8, 5, 13, 12, 4, 9, 1$. (b) For the non-resonant states $k = 2, 3, 6, 7, 10, 11, 14, 15$.

Fig. 5 Fidelity parameter as a function of $J'/J$.

Fig. 6 Fidelity parameter as a function of $\Omega$.

Fig. 7 (a): Rabi frequency $\Omega^{(k)}_{\Delta}$ as a function of $k$ for $\Delta = 4J + 2J'$ [1], $\Delta = 4J$ [2], $\Delta = 2J + 2J'$ [3], $\Delta = 2J$ [4], $\Delta = 2J'$ [5]. Dashed lines $(j)$ for $j = 1, \ldots, 8$ correspond to Fig. 6. (b): Rabi frequency $\overline{\Omega}^{(k)}_{\Delta}$ [1] and $\overline{\Omega}^{(k)}_{\Delta}$ [2] as a function of $k$. Dashed lines $(j)$ for $j = 1, \ldots, 8$ correspond to Fig. 6.
References

1. C.H. Benneth, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. Wootters
   Phys. Rev. Lett., 70 (1993) 1895.
2. S. Lloyd, Science, 261 (1995) 1569.
   S. Lloyd, Sci. Amer., 273 (1995) 140.
   G.P. Berman, G.D. Doolen, D.J. Kamenev, G.V. López, and V.I. Tsifrinovich
   Phys. Rev. A, 6106 (2000) 2305.
3. G.P. Berman, G.D. Doolen, G.V. López, and V.I. Tsifrinovich,
   quant-ph/9802015 (1998).
4. G.P. Berman, G.D. Doolen, D.J. Kamenev, G.V. López, and V.I. Tsifrinovich
   Contemporary Mathematics, 305 (2002) 13.
5. A. Peres, Phys. Rev. A 30 (1984) 1610.
   B. Schumacher, Phys. Rev. A, 51 (1995) 2738.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 7