Stability and Evolution of Galactic Discs

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Abstract. In this review, I discuss just three aspects of the stability
and evolution of galactic discs. (1) I first review our understanding of the
bar instability and how it can be controlled. Disc galaxies in which the
orbital speed does not decrease much towards the centre have no difficulty
avoiding bars, even when dark matter makes an insignificant contribution
to the inner part of the rotation curve. (2) I then briefly discuss inter-
actions between disturbances in the discs of galaxies and the spherical
components, which generally exert a damping effect through dynamical
friction. The fact that bars in real galaxies appear to rotate quite rapidly,
seems to require dark matter halos to have large, low-density cores. (3) In
the remainder of the article, I consider the theory of spiral structure. The
new development here is that the distribution function for stars in the
Solar neighbourhood, as measured by HIPPARCOS, is far less smooth
than most theoretical work had previously supposed. The strong vari-
ations in the values of the DF over small ranges in angular momentum
have the appearance of having been caused by scattering at Lindblad res-
onances with spiral patterns. This result, if confirmed when the radial
velocity data become available, supports the picture of spiral patterns as
dynamical instabilities driven by substructure in the DF. The details of
how decaying patterns might seed conditions for a new instability remain
unclear, and deserve fresh attention.

1. Bar instability

1.1. Mechanism

The mechanism for the bar instability in galaxy discs was clearly described by
Toomre (1981) and is also reviewed by Binney & Tremaine (1987, chapter 6).
The key idea is that waves can reflect from both the centre of a galaxy and
the corotation circle allowing a standing wave to be set up. As for all resonant
cavities (organ pipes, guitar strings, etc.) the phase change around a complete
loop is a multiple of $2\pi$ only for certain values of the frequency, or pattern speed
in the case of a galaxy; the spectrum of modes is therefore discrete.

The standing-wave pattern is, as usual, the super-position two travelling
waves. The direction of propagation of small-amplitude wave packets depends
on a number of factors; for the relevant cases (short-wavelength branch of the
dispersion relation inside corotation), leading spiral waves propagate outwards while trailing waves travel inwards. As waves bounce off the centre, they reflect from trailing to leading, and at corotation they switch back to trailing.

The important difference between bar-forming modes and other more familiar standing wave patterns is that as incident leading waves reflect off the corotation circle they are swing-amplified into stronger trailing waves. The amplification process was first described in the early papers by Goldreich & Lynden-Bell (1965) for gaseous discs and by Julian & Toomre (1966) for stellar discs and was reviewed by Toomre (1981 and this conference). Since the reflected wave has larger amplitude than the incident wave, conservation of wave action requires that there also be a transmitted wave; waves inside corotation are negative-energy, negative-angular momentum disturbances (Lynden-Bell & Kalnajs 1972) whereas these quantities are both positive for the transmitted wave outside corotation.

1.2. Strategies for stabilising discs

Toomre’s mechanism for the instability suggests three distinct methods by which it can be prevented, as summarised by Binney & Tremaine (1987, §6.3).

The simplest to understand, is to make the disc dynamically hot; the radial velocity dispersion of the stars, \( \sigma_u \), is measured by Toomre’s parameter

\[
Q \equiv \frac{\sigma_u}{\sigma_{u,\text{crit}}} = \frac{\sigma_u \kappa}{3.36 G \Sigma},
\]

where \( \Sigma \) is the disc surface density and \( \kappa \) is the epicyclic frequency. If \( Q \gtrsim 2 \), collective density waves become very weak and growth rates of all instabilities are reduced to the point that the disc is effectively stable (Sellwood & Athanassoula 1986). This is unlikely to be how real spiral galaxies are stabilised, however, since a high \( Q \) would both require the disc to be unrealistically thick (Sellwood & Merritt 1994 and references therein) and would also inhibit spiral patterns.

A second strategy, proposed by Ostriker & Peebles (1973), is to immerse the disc in a dynamically hot bulge/halo. In swing-amplification parlance, this strategy works by increasing the parameter

\[
X \equiv \frac{\lambda_y}{\lambda_{\text{crit}}} = \frac{2\pi R}{m} \frac{\kappa^2}{4\pi^2 G \Sigma},
\]

where the spiral arm multiplicity \( m = 2 \) for a bar. The effect of adding halo can be thought of either as increasing \( \kappa \) while holding \( \Sigma \) fixed or reducing \( \Sigma \) while holding \( \kappa \) fixed. Either way, if \( X \) is increased to the point where it exceeds 3 (for a flat rotation curve) the swing-amplifier is tamed and the global bar instability is suppressed. The disadvantage of this strategy is that the swing-amplifier simply prefers higher values of \( m \) instead; galaxies should then exhibit mostly multi-arm spiral patterns (Sellwood & Carlberg 1984). While it is hard to quantify the number of spiral arms in a galaxy, the overall impression from the majority of spiral galaxies is of an underlying bi-symmetry, which is inconsistent with the Ostriker-Peebles strategy for global stability. Furthermore, there is no evidence for a difference in halo fraction between barred and unbarred galaxies (Sellwood 1999).
A third, and probably the most promising, strategy was advocated by Toomre (1981). A key aspect of the instability mechanism is that amplified, ingoing, trailing waves are able to reach the centre where they can reflect into outgoing, leading waves. Toomre therefore proposed that if the centre of the galaxy should be made inhospitable for density waves, the feed-back loop would be cut and the disc would avoid this particularly virulent instability.

### 1.3. Hard or soft centres

The Lin-Shu-Kalnajs (Lin & Shu 1966; Kalnajs 1965) dispersion relation for collisionless particle discs indicates that small-amplitude density waves are able to propagate only between corotation and the Lindblad resonances on either side. The system is unable to sustain waves beyond the Lindblad resonances because particles cannot oscillate at frequencies higher than $\kappa$.

In fact, the last few frames of Toomre’s (1981) Figure 8, aptly dubbed “dust-to-ashes,” provide a graphic illustration of the ultimate fate of an amplified wave packet that encounters a Lindblad resonance – it is damped “as a wave on a beach” in the manner predicted by Mark’s (1974) second order treatment.

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1Lovelace, Jore & Haynes (1997) point out that there are higher frequency solutions to the dispersion relation, near higher order resonances, which are analogous to Bernstein waves in plasmas. These solutions have a severely limited frequency ranges, lie much further from corotation and are inaccessible to waves propagating on the fundamental branches except through non-linear effects; they therefore seem unlikely to be of importance for our purposes.
Figure 2. Left: A snapshot of the stable disc constructed by Sellwood & Moore (1999). The model supports strong, 2-arm spiral patterns but it has no tendency to form a bar. Right: The rotation curve at this time, showing the total circular speed (solid curve) and the separate contributions of the unresponsive central mass (dotted) and halo (dashed).

The stability of the disc is therefore profoundly influenced by whether the centre is hard or soft, as illustrated in Figure 1. A galaxy with a hard centre, such as the Mestel (V = const.) disc shown in the top panels, has a high central density which keeps the rotation speed high to close to the centre, so that an inner Lindblad resonance must be present for every reasonable disturbance frequency. For the soft-centred isochrone disc, on the other hand (lower panels), disturbances with angular frequencies in the range $0.06 < \Omega_p < 0.5$ (in units of $GM/a^3$, with $M$ the disc mass and $a$ the length scale) do not have inner Lindblad resonances, and are therefore able to reflect off the centre. The dramatically different stability properties of these two models (Kalnajs 1978; Zang 1976; Evans & Read 1998) can therefore be understood.

These last-cited global stability analyses are far from straightforward, but they have been confirmed, at least for unstable models, by quiet start $N$-body simulations. Earn & Sellwood (1995) were able to construct the mode shapes and determine the eigenfrequencies from the time evolution of their models, obtaining essentially perfect agreement with linear theory. Confirmation that the Mestel disc is linearly stable is proving more difficult, but it seems likely that particle noise, which can never be completely eliminated in a system having a finite number of particles, is once again responsible for the discrepancy (Sellwood & Evans, in preparation).

Sellwood & Moore (1999) have, at last, provided a robust example of an $N$-body disc that is stabilised by a hard centre. In their model (shown in Figure 2), a small dense bulge-like mass in the centre is able to prevent an almost fully self-gravitating disc from developing a bar whilst the disc is able to support a sequence of large-amplitude 2- and 3-arm spiral disturbances. (I discuss the possible origin of these spiral patterns in §3.) The rotation curve of their model is
not unlike those of many of the Sc galaxies observed by Rubin, Ford & Thomnard (1980) and the absence of bars in real galaxies having steeply rising inner rotation curves can therefore be understood without invoking either a high $Q$ or a massive halo.

2. Dynamical friction on bars

It was customary, in early work, to assume that dynamically hot spheroidal components, especially dark matter halos, could be modelled as rigid, unresponsive mass distributions. In a test of this assumption (Sellwood 1980), I found that it was inadequate once a bar had formed, but I also suggested that it may not be too bad an approximation for spiral waves. Recent work (Dubinski & Kuijken 1995; Nelson & Tremaine 1995; Binney, Jiang & Dutta 1998) has shown that the dynamics of warps is also profoundly influenced by a responsive halo. These experiences caution that treating a halo as a rigid mass distribution may be inadequate in other contexts also.

Dynamical friction between a bar and a live halo was studied by Tremaine & Weinberg (1984). In a follow-up paper, Weinberg (1985) estimated the frictional force for reasonable parameters and concluded that it could be strong enough to stop a bar from rotating altogether on a time scale of a few initial bar periods!

This rather surprising prediction has only recently been confirmed in fully self-consistent disc-bulge N-body simulations with adequate spatial resolution (Debattista & Sellwood 1996; Athanassoula 1996). They found that bars which formed in discs embedded in a dense halo (but not so dense as to suppress the bar instability entirely, §2.2), were slowed dramatically by the strong frictional forces predicted by Weinberg.

A convenient dimensionless, and therefore distance independent, estimate of the angular speed of a bar is the ratio $D_L/a_B$, where $a_B$ is the semi-major axis of the bar and $D_L$ is the distance from the centre to the major-axis Lagrange point (corotation). Direct estimates of this ratio are $D_L/a_B \simeq 1.4 \pm 0.3$ for NGC 936 (Merrifield & Kuijken 1995 and this volume) and a similar value for NGC 4596 (Gerssen 1998). By modelling the gas flow pattern in a 2-D rotating potential derived from near IR surface photometry, Lindblad et al. (1996) for NGC 1365 and Weiner (1998) for NGC 4123 concluded that this ratio should be about 1.3 in both galaxies. Athanassoula (1992) argued that the morphology of dust lanes in barred galaxies requires $D_L/a_B \simeq 1.2$. Other, still more model dependent, estimates of bar pattern speeds can be made from the locations of rings (e.g. Buta & Combes 1996 for a review). While the data are meagre, there are no credible estimates which suggest $D_L/a_B \gtrsim 1.5$ for any galaxy.

These values differ from those found in simulations having moderately dense halos. Debattista & Sellwood (1998), report that $D_L/a_B$ rose from just greater than unity at about the time the bar formed, to significantly more than two by the time dynamical friction against the halo effectively ceased, which occurred in about 20 rotation periods in the inner galaxy. While their result confirms the prediction from perturbation theory, it appears to be quite inconsistent with real galaxies. However, they also found that in models in which the central halo density was much lower, such that the disc contribution to the circular speed at two disc scale lengths was $\gtrsim 85\%$ of the total, friction was reduced to the
level at which the bar could continue to rotate with the Lagrange point at a distance \(< 1.5a_B\). (The conclusion is not dramatically different for anisotropic and rotating halos – Debattista & Sellwood, in preparation.) Debattista & Sellwood (1998) used this result to argue that real dark matter halos must have large, low-density cores – in apparent contradiction with the predictions from cosmological simulations (e.g. Navarro 1998).

3. Spiral structure

There have been no major developments in the theory of spiral structure in recent years, yet there is still no consensus that we have reached a basic understanding of the phenomenon. Since passing companions (Toomre 1981 and this conference) do excite a swing-amplified response, and some spiral patterns also appear to be driven by bars, the most insistent problem remains for spiral patterns in unbarred and isolated galaxies.

3.1. Long-lived or transient spiral waves?

There is still considerable disagreement over the lifetime of spiral waves; C. C. Lin and his co-workers favour quasi-stationary patterns while Toomre, myself and others prefer to think of spirals as short-lived. Unfortunately, direct observational evidence to determine the lifetimes of spiral patterns is unobtainable.

Bertin et al. (1989) imagine that the equilibrium model (their “basic state”) is a cool disc \((Q \gtrsim 1)\) with a smooth DF. They seek global instabilities having a low growth rate, and suggest that a “quasi-steady” wave can be maintained when various non-linear effects, such as shock damping, are taken into account. The pattern must evolve slowly due to secular changes. A key ingredient of the instabilities they favour is a “\(Q\) barrier” in the inner galaxy that shields the waves from the inner Lindblad resonance.

In the alternative picture of recurrent, short-lived spiral patterns, the random motions of the stars rise steadily over time as a direct result of the non-adiabatic potential fluctuations from the spiral patterns themselves (Barbanis & Woltjer 1967; Carlberg & Sellwood 1986; Jenkins & Binney 1990). Some cooling is therefore required to keep the disc responsive \((Q \lesssim 2)\) and the spiral patterns active (Sellwood & Carlberg 1984; Toomre 1990). All possible cooling mechanisms involve dissipation in the gas component, which therefore accounts immediately for the absence of spiral arms in S0 galaxies which lack gas. The most efficient cooling mechanism is through infall of fresh gas to the disc, but dissipation in the existing gas, mass loss from old stars, etc. can also be important.

The origin of the fluctuating spiral patterns is less well understood, however. Toomre (1990) and Toomre & Kalnajs (1991) argue that chaotic spiral patterns in galaxies result from the vigorous response of the disc to co-orbiting mass clumps within the disc, such as giant molecular clouds. The spiral patterns therefore change shape and amplitude continuously on a time-scale of less than an orbital period. These authors do not expect strong Lindblad resonances to be present, essentially because the large-amplitude waves do not have well-defined pattern speeds.
Short-lived patterns, with fresh spirals appearing in rapid succession, have been observed in \(N\)-body simulations for several decades (e.g. Lindblad 1960; Hohl 1970; James & Sellwood 1978). Sellwood & Carlberg (1984) showed that such patterns appear to be swing-amplified but from a level that seemed too high to be consistent with the above shot noise interpretation for their finite number of particles – the amplitude seemed independent of \(N\). Further analysis by Sellwood (1989) showed that the transient spirals resulted from the superposition of a small number of somewhat longer lived waves, which had density maxima near corotation and for which the Lindblad resonances were not shielded. Sellwood & Lin (1989) also showed that the DF did not remain smooth and that resonant scattering by one wave seeded a new instability, at least in their low-mass disc. Some echoes of this idea have been found in higher mass discs with more realistic rotation curves (Sellwood 1991) but the details of exactly how instabilities recur remain obscure.

3.2. Could resonant scattering be observed?

Before investing more effort to try to unravel the behaviour of the \(N\)-body simulations, it seemed appropriate to ask whether some observational consequence could be found to indicate whether or not these ideas were on the right track. With the HIPPARCOS mission already underway, I proposed (Sellwood 1994) that the data on the full space motions of Solar neighbourhood stars be examined for evidence of resonant scattering peaks in the local DF.

Stars interacting with a steady non-axisymmetric potential disturbance rotating at angular rate \(\Omega_p\) conserve neither their energy nor their angular momentum, but the combination

\[
I_3 \equiv E - \Omega_p L,
\]

known as the Jacobi invariant, is conserved. Here, \(E\) and \(L\) are the instantaneous energy and angular momentum per unit mass. Thus the changes in these quantities are related as

\[
\Delta E = \Omega_p \Delta L.
\]

For a steady wave, scattering occurs only at the principal resonances for a pattern (Lynden-Bell & Kalnajs 1972); the resonances are somewhat broadened when the pattern has a finite lifetime. We will be most interested in the change in random energy of a star – the excess energy the star has over one on a circular orbit with the same angular momentum. Since \(dE/dL = \Omega\) for circular orbits, we expect no change in random motion at corotation, where \(\Omega = \Omega_p\). Stars losing (gaining) angular momentum at inner (outer) Lindblad resonances, gain random energy and move onto more eccentric orbits, as shown in the Lindblad diagram Figure 3(a).

We now put ourselves in the position of an observer who is able to measure both \(E_{\text{tan}}\) and \(L\) for many stars in a small region of the galaxy. The distribution of local stars in the space of these two variables might reveal the presence of a scattering peak. In Figure 3(b), \(L_0\) is the angular momentum of a circular orbit at the position of the observer and stars in the shaded areas would never visit the neighbourhood of the observer. The density of stars in this plot will decrease for higher \(E_{\text{tan}}\) (since the DF is likely to be a decreasing function of energy) and the asymmetric drift implies there will be an excess of stars with \(L < L_0\).
Figure 3. Left: The Lindblad diagram for an idealised galaxy model having a flat rotation curve of unit velocity with a small core. No stars can be found in the shaded area which is bounded by the locus of circular orbits. The arrows show the effect of angular momentum exchanges for an imagined 3-arm pattern; the plus symbols mark the positions for circular orbits of corotation and the inner and outer Lindblad resonances. The assumed pattern speed determines the slopes of the arrows which show that the exchange of energy and angular momentum moves stars at both resonances onto eccentric orbits. Right: The same changes viewed by a local observer positioned in the disc at $R = 1$, just inside the ILR of the pattern. If the observer measures $E_{\text{ran}}$ and $L$ for many stars, those which were scattered from circular orbits at the ILR would be spread along the dashed curve depending on $\Delta L$. 
The ILR of a spiral wave will scatter stars upwards and to the left in this plot. Since the density of stars is higher for small $E_{\text{ran}}$ (nearly circular orbits), we might hope to be able to observe an excess of stars along some trajectory, such as the dashed curve shown; the precise location of this trajectory will depend upon the value of $\Omega_p$. Such scattering peaks are observed in $N$-body simulations (Sellwood 1994).

3.3. HIPPARCOS stars

Local kinematics of stars in the HIPPARCOS sample have been studied in some detail by Binney & Dehnen (1998). The satellite determined the position on the sky, a parallactic distance and the two components of proper motion transverse to the line of sight. The only one of the six phase space coordinates lacking, therefore, is the radial component of velocity, which is also needed for the above analysis. Dehnen (1998) deduced the missing component in a statistical sense, reasoning that the full, intrinsic distribution of velocities of local stars should be identical over the whole sky. With this assumption, differing viewing directions give us different projections of the same intrinsic velocity distribution, which can be combined to yield the missing information.

With this technique, Dehnen (private communication) has kindly computed $E_{\text{ran}}$ and $L$, for his samples B4 and GI of the HIPPARCOS catalogue, which includes $\sim 14000$ mostly main sequence stars of a broad range of ages, and prepared the plot shown in Figure 4. The asymmetric drift is clearly visible.

The local DF manifestly is not smooth; the density contours in this plane show significant and coherent distortions from that expected if the velocity distributions were closely Gaussian. Dehnen (1998) interprets the substructure at small $E_{\text{ran}}$ as confirmation of the star streams and moving groups, but the structure at high $E_{\text{ran}}$ has not been seen before.

There is a clear hint of at least one scattering line, and maybe a second, with the morphology of that expected from a strong ILR. These features, if confirmed when the radial velocities become available, support the picture of recurrent transient spiral patterns and are quite inconsistent with the idea that ILRs are shielded by a $Q$-barrier.

3.4. Effect of small scale features in DF

Resonant scattering depopulates the DF at small $E_{\text{ran}}$ over a narrow range of $L$ and moves these stars to higher $E_{\text{ran}}$ and smaller $L$. Strong density variations over narrow ranges of angular momentum are likely to be destabilising (Lovelace & Hohlfeld 1978). As these modes are excited by phase-space density gradients at corotation, the mechanism could be described as Landau excitation.

The simplest such instability to understand is the “groove mode,” which was described by Sellwood & Kahn (1991) using both $N$-body simulations and local theory. They showed that a half-mass Mestel disc with $Q = 1.5$, which was globally stable when the DF was smooth, became strongly unstable to global 2- and 3-arm spiral modes when they removed particles over a narrow range in $L$. They referred to the narrow feature as a “groove,” but owing to random motion in the disc the surface density is imperceptibly reduced over a broad radial range – the feature is narrow only in integral space.
Figure 4. The density of Solar neighbourhood stars in \((E_{\text{ran}}, L)\)-space. The figure was constructed by Dehnen using \(\sim 14\,000\) stars selected from the HIPPARCOS sample. Apart from the skew to lower \(L\), which is caused by the asymmetric drift, a smooth DF would have produced a featureless plot, whereas the contours show one or more distinct ridges.
Figure 5. The mechanism for a groove mode in the shearing sheet viewed from a frame at rest in the groove centre. Top: The unperturbed edges of the groove in surface density are marked by dotted lines (its width is greatly exaggerated) and possible perturbing wave-like disturbances are marked by the solid curves. The heavy arrows in the groove centre indicate the gravitational stresses acting on the shaded over-densities. Bottom: The over-densities within the groove (shaded) and the swing-amplified supporting response from the surrounding disc (contours). The supporting response was calculated using the methods described by Julian & Toomre (1966) for $Q = 1.8$, $\Gamma = 1$ and $X = 2$; the Lindblad resonances for this pattern are marked by the dashed lines.
The mechanism for the instability is illustrated in Figure 5. The top panel shows a local patch of the disc, a shearing sheet, with a groove in which the surface density is lower between the dotted lines – the groove width is greatly exaggerated and, for simplicity, the blurring effects of random motions have been ignored. The diagram is drawn in a frame which co-rotates with the centre of the groove. (For definiteness, we will assume the galactic centre is far down the page and that the mean angular momentum of material in the sheet therefore increases up the page.) If wave-like disturbances are present on the edges of the groove as shown, the shaded areas mark regions where a larger density excess is created by the wave. If the two waves on opposite sides of the groove have a phase difference, the density excesses created by each attract the other; the azimuthal components of these force vectors are marked by the heavy arrows. Angular momentum is therefore exchanged between the density excesses, which cause the density maxima to grow if the phase difference has the sign in the illustration. To understand why the density excesses grow, focus first on the lower edge; material in the shaded density excess is urged forward by the forces from the density excess on the other edge and therefore gains angular momentum. Increased angular momentum causes the home radius of this material to increase causing the bulge to grow. Similarly, material in the bulge on the upper edge loses angular momentum causing it to sink further into the groove. In the absence of the other wave, each edge wave would be neutrally stable, but they aggravate each other through their mutual interaction to make the combined disturbance unstable.

If this were all that occurs, the instability would be mild and inconsequential, but the instability develops in a background disc that responds enthusiastically to orbiting density inhomogeneities. A possible example of the supporting response is contoured in the lower panel. In our case, the density disturbance is periodic along the groove and the supporting response is therefore also periodic with the same wavelength and extends as far as the Lindblad resonances on either side (shown by the dashed lines). Once again, the response is due to swing-amplification and, as shown by Julian & Toomre (1966), the disturbance in the supporting response is considerably more massive that the co-orbiting mass clump, unless $Q \gg 2$. The supporting response therefore converts a mild local disturbance into a large-scale spiral instability. In principle, the groove supports instabilities of many possible wavelengths, but the strongest spirals will be for those at which the swing-amplifier is most responsive.

The distribution of stars in the Galaxy is clearly more complicated than a smooth distribution with a single “groove,” but almost any narrow feature in the density of stars as a function of $L$ is destabilising (Sellwood & Kahn 1991). These modes therefore seem promising candidates for the generation of spiral patterns in the Milky Way.

4. Conclusions

The mechanism for the bar mode, which is the dominant global mode of a smooth disc, and ways in which it can be suppressed are well understood. We now believe that real galaxy discs, which possess most of the mass in the inner parts of spiral galaxies, avoid the bar-forming instability by having a dense centre.
Recent work has indicated that the usual rigid halo approximation is often inadequate and that a responsive halo strongly influences the mechanics of barred and/or warped galaxies. A moderately dense live halo slows a galactic bar through dynamical friction on a very short time-scale and the apparent generally high pattern speeds of real bars therefore require that the halo has a large core and that the halo central density can be little more than the minimum required to prevent it from being hollow.

The theory of spiral structure seems set for a major step forward now that the HIPPARCOS data indicate that it is probably wrong to assume a smooth DF. Spiral structure is likely to result from local instabilities caused by small-scale variations in the DF which give rise to large-scale spiral patterns with the assistance of the swing-amplifier. While the mechanism for linear instabilities of this form is already reasonably clear, exactly how the structure in the DF arose is not. The existence of resonant scattering peaks suggests that these are at least one of the processes which sculptrues the DF, but it is unclear whether it is the only, or even the dominant, source of local inhomogeneities in the DF. The HIPPARCOS data have provided a much needed pointer to the way forward in this erstwhile stalled area for research and suddenly there is plenty to do!

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