Efficiencies of Magnetic Field Amplification and Electron Acceleration in Young Supernova Remnants: Global Averages and Kepler’s Supernova Remnant

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Abstract
Particle acceleration to suprathermal energies in strong astrophysical shock waves is a widespread phenomenon, generally explained by diffusive shock acceleration. Such shocks can also amplify the upstream magnetic field considerably beyond simple compression. The complex plasma physics processes involved are often parameterized by assuming that shocks put some fraction \( \epsilon_e \) of their energy into fast particles and another fraction \( \epsilon_B \) into the magnetic field. Modelers of shocks in supernovae, supernova remnants (SNRs), and gamma-ray bursters, among other locations, often assume typical values for these fractions, presumed to remain constant in time. However, it is rare that enough properties of a source are independently constrained that values of the epsilons can be inferred directly. SNRs can provide such circumstances. Here we summarize results from global fits to spatially integrated emission in six young SNRs, finding \( 10^{-4} \lesssim \epsilon_e \lesssim 0.05 \) and \( 0.001 \lesssim \epsilon_B \lesssim 0.1 \). These large variations might be put down to the differing ages and environments of these SNRs, so we conduct a detailed analysis of a single remnant, that of Kepler’s supernova. Both epsilons can be determined at seven different locations around the shock, and we find even larger ranges for both epsilons, as well as for their ratio (thus independent of the shock energy itself). We conclude that unknown factors have a large influence on the efficiency of both processes. Shock obliquity, upstream neutral fraction, or other possibilities need to be explored, while calculations assuming fixed values of the epsilons should be regarded as provisional.

Unified Astronomy Thesaurus concepts: Shocks (2086); Supernova remnants (1667); Magnetic fields (994); Cosmic rays (329)

1. Introduction

Strong shocks in supernovae, supernova remnants (SNRs), and gamma-ray bursters (GRB sources) are widely observed to produce nonthermal particle distributions and to amplify ambient magnetic fields (e.g., van Paradijs et al. 2000; Weiler et al. 2002; Reynolds et al. 2008). The total energy in these nonthermal components is generally small compared to bulk thermal and kinetic energies; to produce the Galactic cosmic rays, roughly 10% of supernova energy is adequate. The processes responsible for particle acceleration and magnetic field amplification are fairly well understood in broad terms (see any of various reviews, such as Blandford & Eichler 1987; Malkov & Drury 2001; Ressler et al. 2014), but details remain frustratingly elusive. Even as basic an issue as the fraction of shock energy eventually winding up in particles and the magnetic field is difficult to predict from first principles and may evolve with changing conditions.

Observational constraints on these quantities are also surprisingly hard to come by. An early attempt at using the spatial structure of radio images of young SNRs to discriminate among models for magnetic field evolution (Reynolds & Chevalier 1981) compared theoretical profiles of SNR radio emission assuming uniform postshock magnetic field strength (shock expanding into constant magnetic field, with only compression increasing the postshock value), with a model with magnetic energy density \( u_B \) amplified to a value proportional to postshock pressure \( P_2 \). (The postshock pressure in a strong shock from the Rankine–Hugoniot jump conditions is \( P_2 = \left(2/(\gamma + 1)\right)P_0v^2_s \) or \( 3P_0v^2_s/4 \) for adiabatic index \( \gamma = 5/3 \), where \( P_0 \) is the upstream gas density and \( v_s \) the shock speed. \( P_2 \) is also proportional to the total thermal energy density \( \epsilon_2 \): \( \epsilon_2 = P_2/(\gamma - 1) \), so that \( \rho_0v^2_s = [(\gamma + 1)/(\gamma - 1)/2]P_2 = (8/9)\epsilon_2 \) for \( \gamma = 5/3 \). The convention has been to absorb the factor 8/9 into the definitions of \( \epsilon \) efficiency factors, and we shall follow that convention here: \( \epsilon_e \equiv u_e/\rho_0v^2_s \) and \( \epsilon_B \equiv (B^2/8\pi)/\rho_0v^2_s \).

Reynolds & Chevalier (1981) found substantial differences between predictions of their two models for the magnetic field: amplified-field models predicted that the radio profile should rise from the shock inward all the way to the contact discontinuity between shocked ejecta and shocked ambient material. However, various confounding effects made this prediction difficult to test. A similar test (Chomiuk & Wilcots 2009) used collective luminosity functions of extragalactic SNRs to try to discriminate between these models, finding that (assuming a constant cosmic-ray energy density) the swept-up model was less favored by the data. However, the assumption of a non-evolving relativistic-electron density is not supported by more recent studies of diffusive shock acceleration. See, for instance, Riquelme & Spitkovsky (2011).

The dependence of the synchrotron luminosity on both the energy density of relativistic electrons and that of the magnetic field plagues attempts to use synchrotron emission as a diagnostic in contexts as disparate as active galactic nuclei, GRB sources, the interstellar medium of normal galaxies, supernovae, nova and SNRs, pulsar wind nebulae, and elsewhere. By now the assumption of equipartition of energy between particles and field has been largely abandoned. Originally introduced to obtain a lower limit to the energy required to explain extended emission in radio galaxies (in particular M87; Burbidge 1956), the assumption took on a life of its own, supported by no more than a vague sense that nature ought to put equal amounts of energy in each available
form. The difficulty of deciding how much energy might reside in relativistic protons has meant that apart from providing absolute lower limits for energy arguments, in which case one ignores energy in protons, the assumption has substantial model dependence, and its utility and predictive power are limited.

The assumption of constant fractions of shock energy going into relativistic electrons ($\epsilon_e$) and magnetic field ($\epsilon_B$) is implicit in Reynolds & Chevalier (1981) and made explicitly in Chevalier (1984). It appears to have been the standard assumption in early GRB modeling, which seems to be where the $\epsilon$ notation originated (e.g., Sari et al. 1998). The notation and the assumption that these fractions remain constant as the shock wave evolves have become widespread in modeling radio emission from supernovae and SNRs, as well as GRBs (e.g., Lundqvist et al. 2020). Typical assumed values for these fractions are in the range 0.01–0.1 for GRBs (e.g., Nava et al. 2014). However, Panaitescu & Kumar (2001) fit a multiparameter model for GRB afterglows to eight events, finding values ranging over an order of magnitude for $\epsilon_e$ ($10^{-2}$–$10^{-1}$) and three orders of magnitude for $\epsilon_B$ ($10^{-4}$–$10^{-1}$). In general, both the values and the assumption of constancy lack the observational support one would like in such fundamental quantities. Additionally, the relativistic shocks inferred in GRB afterglows may have qualitatively different effects in both magnetic field amplification and electron acceleration than would be found in nonrelativistic shocks in SNRs. Numerical simulations (e.g., Crumley et al. 2019) show substantial shock velocity dependence in $\epsilon_e$, as well as dependence on other shock parameters. In SNR studies, more recent work has attempted to simulate the acceleration of electrons and amplification of magnetic field based on analytic prescriptions, so that the values of $\epsilon_e$ and $\epsilon_B$ can evolve (e.g., Sarbadhycary et al. 2017; Pavlović et al. 2018). However, these studies also assume parameters such as a fixed ratio of cosmic-ray electron to ion energy, or a fixed injection efficiency into the acceleration process, that affect the behavior of the $\epsilon$ factors.

It is the goal of this paper to determine as well as possible the values of $\epsilon_e$ and $\epsilon_B$ in young SNRs, using the best observational data available, and making minimal theoretical assumptions. We discuss methods for determining magnetic field strengths in Section 2 and for extracting relativistic-electron energy densities in Section 3. We then apply these to estimate average $\epsilon$ factors for several young SNRs in Section 4. The core of the paper is in Sections 5 and 6, where we obtain spatially resolved values of the $\epsilon$ factors at seven locations around the rim of Kepler’s SNR. The results are discussed in Section 7 and summarized in Section 8.

2. Magnetic Field Determinations

There are various methods for obtaining independent measures of magnetic field in compact synchrotron sources. One interesting method, applicable to radio supernovae, produces some valuable information on the values and evolution of $u_B$ and $\epsilon_B$ and so shall be described here. While this method cannot be applied to SNRs, it does allow inferences of those quantities over time as a supernova evolves. The unsettling results (at least for SN 1993J) set the stage for our considerations of SNRs.

2.1. Synchrotron Self-absorption in Supernovae

This method, applicable whenever the process operates, relies on synchrotron self-absorption (SSA): the observation of the frequency at which a source becomes optically thin to synchrotron radiation, as well as the flux at that frequency, allows the determination of two out of the three quantities: source size, magnetic field, and electron energy density. However, the operation of SSA at observable radio frequencies requires conditions in a diffuse source that are fairly restrictive. Using the notation of Pacholczyk (1970), the absorption coefficient for a homogeneous synchrotron source with magnetic field $B$ and electron energy spectrum $N(E) = KE^{-\alpha}$ electrons cm$^{-3}$ erg$^{-1}$ can be written as

$$\kappa_\nu = c_6(s)(1.25 \times 10^{10})^{(s+4)/2}c_9(s + 1) \times KB^{(s+2)/2} \nu^{-(s+4)/2} \text{ cm}^{-1},$$

(1)

where the numerical constants are given in Pacholczyk (1970). For a typical electron energy index $s = 2.5$ (synchrotron spectral index $\alpha = (s-1)/2 = 0.75$, with $S_\nu \propto \nu^{-\alpha}$), this is

$$\kappa_\nu = 6.13 \times 10^2 KB^{9/4} \nu^{-13/4} \text{ cm}^{-1}.$$  

(2)

If $s > 2$, the energy density in electrons $u_e$ depends only on the lower energy limit to the spectrum $E_i$. Synchrotron emission basically requires $E_i \gtrsim 10m_e c^2$. An estimate of the required conditions for observable SSA can be made by characterizing both $K$ and $B$ in terms of energy densities. If $u_e$ and $u_B$ are both equal to some nonthermal energy density $u_{\text{month}}$, then a source of line-of-sight extent $L$ will become opaque to SSA below a frequency of about

$$\nu_1 \sim 2 \times 10^6 L^{4/13} u_{\text{month}}^{-17/26} \text{ Hz}$$

(3)

(still for $s = 2.5$), and for that frequency to exceed 100 MHz, the source extent must satisfy

$$L \gtrsim 3 \times 10^5 u_{\text{month}}^{-17/8} \text{ cm}.$$  

(4)

For a source extent less than 1 pc, the nonthermal energy density must exceed about $10^{-6}$ erg cm$^{-3}$, or about 1 MeV cm$^{-3}$. Thus, SSA is an important effect only for very high energy density circumstances, such as supernovae—but not SNRs.

Most radio supernovae show evolving spectra with a peak at some frequency that moves lower with time, attributed to synchrotron emission with some opacity setting in at lower frequencies. Some combination of free–free absorption, either coincident or foreground, and SSA is likely responsible (e.g., Chevalier 1984; Weiler et al. 2002). An extensive study by Chevalier (1998) attributes low-frequency absorption in 8 of 13 radio supernovae to SSA. He reduces the three required source parameters to two by assuming a fixed ratio (not necessarily one) of $u_e$ to $u_B$ and derives source sizes and magnetic fields on the assumption that $u_B \propto \nu_\nu^{2}$, the postshock pressure. Magnetic field strengths depend fairly weakly on all parameters and are in the range 0.1–0.6 G at the time of the emission peak.

In one case, SN 1993J, the radio observations are sufficiently frequent and the frequency coverage so extensive as to allow a detailed determination of $u_e$ and $u_B$ independently and as a function of time (Fransson & Björnsson 1998), since the very long baseline interferometry observations of a more or less constant expansion velocity (for the first ~100 days) of $2 \times 10^4$ km s$^{-1}$ (Bartel et al. 1994) allow the radius to be
inferred. They find that \( u_e \propto \rho_{\rm v}^2 \) describes the data well, and much better than \( u_e \propto \rho \) alone (here \( \rho \) is the upstream, i.e., circumstellar medium (CSM), density). They determine \( \epsilon_e \sim 5 \times 10^{-4} \). For magnetic field, the apparent deceleration beginning around day 100, with \( R \propto t^m \) and \( m = 0.74 \), allows a discrimination between a model with \( u_B \propto \rho_{\rm v}^2 \) and one with \( B \propto 1/R \), with the latter description providing a better fit to data. Before day 100, they find \( u_B/\rho_{\rm v}^2 \sim 0.14 \), independent of time. The suggestions of nonconstant \( \epsilon_B \) and, worse, of a change in the very dependence of \( \epsilon_B \) on supernova parameters are worrisome hints that the simple picture of constant epsilons is a poor description of the processes of shock acceleration and magnetic field amplification. While we can only observe SNRs evolve through a small fraction of their lifetimes, the results of SN 1993J should put us on notice that results for SNRs may fail to tell a complete picture.

The detailed supernova inferences rely on a simple one-zone emission model (though with spectral sophistication; Fransson & Björnsson 1998 evolve the electron distribution under both Coulomb and synchrotron losses) and on the operation of SSA as an absorption mechanism. They also apply to very high shock velocities and dense CSM. The importance of the processes of magnetic field amplification and particle acceleration is sufficiently great that testing assumptions such as the scaling of \( u_e \) and \( u_B \) with density and shock velocity, and possible evolution of efficiencies with time, in different regions of parameter space is a high priority. It is fortunate that methods exist to allow this in SNRs, especially to replace SSA for magnetic field determinations.

2.2. Magnetic Field Determinations in Supernova Remnants

SNRs are far too diffuse for SSA to be an important mechanism, so inferences of magnetic field strengths rely on different techniques. Reynolds et al. (2012a) review these. Of particular interest is the “thin rims” argument, based on observations of X-ray synchrotron emission from young SNRs (Bamba et al. 2003; Vink & Laming 2003; Parizot et al. 2006), in particular, on the commonly observed morphology of thin tangential rims at the shock front (located, for instance, by H\( \alpha \) observations). This method relies on the assumption that the disappearance of emission a short distance downstream results from synchrotron losses on the emitting electrons as they are advected (or diffuse) downstream. A thorough treatment is given by Ressler et al. (2014), who also include a discussion of an alternative explanation for thin rims, decay of magnetic turbulence downstream (Pohl et al. 2005). Under the simplest assumptions (electron transport by pure advection, ignoring magnetic field damping, the delta-function approximation for the single-electron emitted spectrum), a straightforward relation can be derived between rim thickness and magnetic field strength (Parizot et al. 2006):

\[
B \approx 210 \left( \frac{v_s}{1000 \, \text{km} \, \text{s}^{-1}} \right)^{2/3} \left( \frac{w}{0.01 \, \text{pc}} \right)^{-2/3} \mu \text{G}, \tag{5}
\]

where \( w \) is the filament width in the radial direction. (We have assumed a small geometric correction factor \( (4\bar{P}/\bar{P}_{\text{comp}}) \) to be unity, where \( \bar{P}_{\text{comp}} \) is the compression ratio and \( \bar{P} = 1 \) for a perfect sphere. Since we will always be employing this relation in small regions at the very edge of the remnant, the locally spherical approximation should be quite good.) However, more elaborate treatments produce somewhat different values; Ressler et al. (2014) collect published values for the remnant SN 1006 ranging from 65 to 130 \( \mu \)G, with one report of 14 \( \mu \)G. All agree, however, in requiring amplification of magnetic field beyond simple shock compression. The situation is made more complicated by the presence in a few cases of thin radio rims, produced by electrons with energies far too low to be affected by synchrotron losses, and thus requiring magnetic damping, a process with few theoretical or observational constraints.

Given \( B \), we then have the magnetic field energy density \( u_B \equiv B^2/8\pi \). We note that if \( B \) is determined in this way, a prediction results for \( \epsilon_B \propto B^2/\rho_{\rm v}^2 \propto v_{\rm c}^{-2/3} \), other things being equal. We shall apply Equation (5) in the analysis below.

3. Relativistic-electron Energy Densities

We shall use observations of radio synchrotron intensity to obtain relativistic-electron energy densities, assuming simple power-law electron energy distributions. We take \( N(E) = K E^{-\alpha} \) electrons cm\(^{-3} \) erg\(^{-1} \). The electron energy density is then

\[
\frac{u_e}{s-2} = \int_{E_i}^{E_h} K E^\alpha dE = \frac{K}{s-2} \frac{E_h^{2-s} - E_i^{2-s}}{2-s} \tag{6}
\]

The energy ranges are somewhat arbitrary; for true synchrotron emission, \( E_i \sim 10^{-5} E_h \) is a reasonable estimate, while since we shall have \( s > 2 \) and \( E_h \gg E_i \), \( u_e \) is essentially independent of the value of \( E_h \). So we approximate

\[
\frac{u_e}{s-2} \approx \frac{K}{s-2} E_i^{2-s} \tag{7}
\]

Our strategy for finding \( K \) and hence \( u_e \) will be to observe synchrotron intensities, which in the optically thin limit are just proportional to the synchrotron emissivity \( j_e \). The synchrotron emissivity can be written as (again in the notation of Pacholczyk 1970)

\[
\frac{j_e}{\nu} = c_2 s(2c_1)^\nu KB^{(\alpha+1)/2} \nu^{-\alpha} \text{ erg cm}^{-3} \text{ s}^{-1} \text{ Hz}^{-1} \text{ sr}^{-1} \tag{8}
\]

Here \( c_2 \) is a function of the electron index \( s \): \( c_2 \approx 10^{-24} \) cgs for \( 2 < s < 3 \), and \( c_1 \equiv 6.27 \times 10^{18} \) cgs. The constants can be conveniently lumped together as

\[
\frac{c_j}{c_2(2c_1)^\nu} \tag{9}
\]

For \( s = 2.42 \), its value for Kepler, \( c_j(2.42) \equiv c_2(2.42)(2c_1)^{0.71} = 3.69 \times 10^{-10} \) cgs. Then,

\[
K = c_j^{-1} B^{-(\alpha+1)/2} \nu^{\alpha} \frac{j_e}{\nu} \tag{10}
\]

We neglect SSA for SNRs, so the intensity \( I_\nu \) on any line of sight is just given by \( \frac{j_e}{\nu} d\ell \).

We can infer global average values of the emissivity by measuring the total remnant flux at frequency \( \nu \), \( S_\nu \). For spatially resolved measurements, we shall measure local values of \( I_\nu \) and estimate (short) line-of-sight depths \( L \) so \( j_e = \langle \nu I_\nu \rangle L \). For a homogeneous source of volume \( V_{\text{em}} \) at distance \( d \), we can write (assuming isotropic synchrotron emission, that is, disordered magnetic field)

\[
S_\nu = \langle 4\pi j_\nu \rangle \frac{V_{\text{em}}}{4\pi d^2} \tag{11}
\]

We will first apply this to obtain a mean \( u_e \) for a spherical remnant of radius \( R \) where we take the emitting volume to be \( V_{\text{em}} = (1/\bar{P}) 4\pi R^3/3 \), where \( \bar{P} \) is the shock compression.
Finally, we obtain the assumed value of $\rho_0$. All quantities on the right are observable with the exception of $\rho_0$, which we infer by dividing those values by an assumed compression ratio $r_{\text{comp}}$ of 4 and assume 5 kpc here. However, the main import of our results will be in comparing inferences at different points, hence independent of distance.

5. Spatially Resolved Efficiencies in Kepler’s Supernova Remnant

The surprisingly large range of values of the various efficiency parameters listed in Table 2 applies across rather different SNRs, of different ages and properties. The environments of these SNRs are likely very different as well; one might hope to attribute the results of Table 2 to this heterogeneity. However, it is possible to use similar methods to analyze the conditions at different locations around the rim of a single remnant. This technique has obvious benefits in removing dependence on many quantities that vary between remnants, chiefly but not exclusively distance. Here we describe a spatially resolved analysis of the efficiencies of electron acceleration and magnetic field amplification at different locations around the periphery of Kepler’s SNR (“Kepler” hereafter). The distance to Kepler is uncertain, with values ranging from 4 kpc (Sankrit et al. 2005) to 8 kpc (e.g., Millard et al. 2020). Recent optical determinations suggest 5 kpc (Sankrit et al. 2016), and we shall assume 5 kpc here. However, the main import of our results will be in comparing inferences at different points, hence independent of distance.

5.1. Observational Strategy

To fix the efficiencies of shock energy deposition into magnetic field and relativistic electrons, one requires independent local measurements of density, shock velocity, magnetic field strength, and electron energy density. Such measurements have substantial uncertainties for an SNR. X-ray diagnostics of plasma density are fraught with uncertainties, requiring spectral modeling and assumptions about filling factors. Shock velocity determinations from X-ray spectra suffer from uncertainty about prompt electron heating in the shock. These difficulties require alternative methods for determining each quantity.

An excellent diagnostic for the density of a hot plasma is the temperature of grains embedded in it (e.g., Dwek & Arendt 1992), which are collisionally heated by the plasma. We have previously employed this method to obtain postshock densities of SNRs in the Large Magellanic Cloud (Borkowski et al. 2006; Williams et al. 2006) and of Kepler (Williams et al. 2012). The latter study involved analysis of a complete spectral mapping of Kepler with the Spitzer IRS and extraction of spectra between 7.5 and 38 $\mu$m at several locations. These spectra were fit with a dust-heating shock model in which postshock grains were heated as they were advected downstream in a constant-temperature, constant-density plasma (Borkowski et al. 2006). This model provided good descriptions of observed IR spectra, allowing extraction of postshock densities at various points in Kepler. These densities are listed in Table 3. We require $\rho_0$, which we infer by dividing those values by an assumed compression ratio $r_{\text{comp}}$ of 4 and assuming cosmic abundances (mean mass per hydrogen atom $\mu = 1.4$).

For shock velocities, we use proper motions of expansion, as measured from Chandra X-ray data (Katsuda et al. 2008), in 14 regions around the periphery. We then deduce magnetic field strengths using the simple thin-rim analysis of Equation (5), measured from the 741 ks Chandra image (Reynolds et al. 2007), the same image used for the second epoch of proper-motion measurements (Katsuda et al. 2008). Then, the electron

| Object     | $P=06$ | VBK05 | RP12 Loss | RP12 Damp | $T=15$ |
|------------|--------|-------|-----------|-----------|--------|
| Cas A      | 210–230| 500   | 520       | 115–260   |        |
| Kepler     | 170–180| 200   | 250       | 80–135    |        |
| Tycho      | 200–230| 300   | 310       | 85–150    | 50–400 |
| SN 1006    | 57–90  | 140   | 130       | 64–65     | 40–200 |
| RCW 86     | 10     |       |           |           |        |

*Table 1: Magnetic Field Strengths in Young Remnants*

Note. Magnetic field strengths are in $\mu$G. References: $P=06$—Parizot et al. (2006); VBK05—Völk et al. (2005); RP12—Rettig & Pohl (2012); $T=15$—Tran et al. (2015).
Table 2
Derived Properties of Remnants

| Remnant     | B (μG) | S_v (Jy) | d (kpc) | α^a | R (arcsec) | u_e | u_B | u_e/u_B | n_0 (cm^{-3}) | v_sh (km s^{-1}) | p^c | c_e (10^{-3}) | c_B (10^{-3}) | References |
|-------------|--------|----------|---------|-----|------------|-----|-----|---------|---------------|-----------------|-----|-------------|-------------|------------|
| G1.9+0.3    | 320^a  | 0.6      | 8.5     | 0.6 | 50         | 0.22| 41  | 0.0055  | 0.02          | 14,000          | 0.91| 0.25        | 45           | 1          |
| Cas A       | 220    | 2300     | 3.3     | 0.77| 150        | 0.29| 19  | 0.15    | 1             | 5800            | 3.3 | 86          | 5            | 2          |
| Kepler      | 175    | 18       | 5       | 0.71| 117.7      | 3.6 | 12  | 0.30    | 3             | 4000            | 11  | 0.58        | 1.1          | 3          |
| Tycho       | 215    | 50       | 2.4     | 0.6 | 240        | 1.1 | 18  | 0.060   | 0.2           | 2000            | 0.19| 5.9         | 100          | 4          |
| SN 1006     | 74     | 19       | 2.2     | 0.6 | 900        | 0.050| 2.1 | 0.023   | 0.05          | 5000            | 0.29| 0.17        | 7.4          | 5          |
| RCW 86      | 100^b  | 49       | 2.5     | 0.6 | 1260       | 0.025| 4.0 | 0.0063  | 0.5           | 600             | 0.042| 0.60        | 96           | 6          |

Notes. Magnetic field values from Parizot et al. (2006) except as noted.

References: (1) Carlton et al. 2011, (2) Chevalier & Oishi 2003, (3) Williams et al. 2012, (4) Williams et al. 2013, (5) Winkler et al. 2014, (6) Williams et al. 2011a.

^a From Green (2019).
^b In units of 10^{-10} erg cm^{-3}.
^c From Katsuda et al. 2008, scaled to a 5 kpc distance.
^d Obtained from a new measurement. See text.
^e Obtained from Völk et al. (2005).

Table 3
Observed Properties of Rim Regions

| Region^a | PA (deg) | Density n_B | v_sh | Width | Length | ⟨I_B⟩ |
|----------|---------|-------------|------|-------|--------|------|
| 1        | 16      | 26 (22, 31)| 2580 ± 490 | 3.3  | 10 | 0.54 ± 0.06 |
| 2        | 32      | 32 (29, 39)| 2470 ± 520 | 2.3  | 33 | 0.13 ± 0.04 |
| 3        | 75      | 11 (10, 13)| 3530 ± 550 | 2.4  | 33 | 0.17 ± 0.05 |
| 4        | 96      | < 9 (8, 10)| 4530 ± 520 | 4.0  | 17 | 0.35 ± 0.04 |
| 5        | 102     | < 9 (8, 10)| 4880 ± 540 | 3.2  | 28 | 0.13 ± 0.01 |
| 6        | 135     | 5.3 (3.6, 7.0)| 7160 ± 1200 | 5.4  | 15 | 0.53 ± 0.04 |
| 7        | 155     | 3.0 (2.6, 3.6)| 4220 ± 810 | 3.9  | 15 | 1.1 ± 0.03 |
| 7b^c     | 164     | 2.1 (1.1, 3.1)| 4980 ± 810 | 2300 | 4900 ± 700 | 5.4  | 15 | 0.53 ± 0.04 |
| 8        | 172     | < 1.0 (0.6, 1.4)| 5740 ± 700 | 2.5  | 24 | 0.04 ± 0.02 |
| 9        | 230     | 3.0 (2.5, 3.5)| 4880 ± 630 | 2.4  | 53 | 0.13 ± 0.01 |
| 10       | 242     | 3.0 (2.6, 3.6)| 3840 ± 650 | 3.9  | 15 | 0.11 ± 0.03 |
| 12       | 258     | < 2.0 (1.5, 2.5)| 4270 ± 740 | 2.5  | 24 | 0.04 ± 0.02 |
| 13       | 319     | 48 (41, 62)| 1800 ± 850 | 2000 | 2700 ± 500 | 5.4  | 15 | 0.53 ± 0.04 |
| 14       | 345     | 22 (20, 26)| 2700 ± 500 | 2.4  | 53 | 0.13 ± 0.01 |

Notes. Values for Regions 4, 5, 8, and 12 are determinations, but upper limits for densities at the extreme edge.

^a Numbering in Katsuda et al. (2008).

^b Postshock density.

^c From Katsuda et al. (2008), scaled to a 5 kpc distance.

^d Assumed the same as Region 4.

^e Between K08 regions 7 and 8 (see Figure 2).

energy density can be extracted from intensities measured from a radio synchrotron image (5 GHz VLA; DeLauney et al. 2002) using Equation (13).

Kepler’s SNR is an excellent target with which to attempt this determination. The very strong N–S brightness gradient at all wavelengths indicates that the shock wave is expanding into highly asymmetric material; Blair et al. (2007) find that the density to the north is 4–9 times that to the south (see also Williams et al. 2012), and the shock velocities show up to a factor 3 of variation (Katsuda et al. 2008). These wide variations allow us to examine the dependence of efficiencies on several factors. Thin X-ray films with nonthermal spectra can be seen in several locations around the periphery. Below we outline the quantitative results of our investigation.

5.2. Density Measurements

The determinations of postshock density were based on observations of Kepler with the Spitzer IRS instruments: both orders of the low-resolution (LL) module (14–38 μm), and order 1 (7.5–14 μm) of the short-wavelength low-resolution (SL) module. The entire remnant was mapped with the LL instrument, and selected regions with the SL module. The observations and analysis are described in Williams et al. (2012). Figure 3 illustrates the sensitivity of model spectra to gas density. Models depend weakly on both ion temperature kT_e and electron temperature T_e (see Blair et al. 2007; Williams et al. 2012, for details). Observed electron temperatures in young SNRs are typically a few keV (Reynolds et al. 2007); we assume kT_e = 1.5 keV but set uncertainties by allowing a range between 1 and 2 keV.

Broadly, variations of a factor of 30 were found in density between the faint southern rim and bright north, but values were obtained at many regions around the periphery. (Averaging over larger regions reduces the contrast to the range of 4–9 quoted in Blair et al. 2007). Figure 1 shows regions from which spectra were extracted, models fit, and densities obtained. Those densities are tabulated in Table 3. Statistical
The uncertainties of the fits are much smaller than the spread of values possible by changing the assumed electron temperature between 1 and 2 keV; that spread is shown in Figure 4.

While we believe that the emission from each region is well characterized by the densities listed in Table 3, in a few regions the IR emission is so faint that it is not clear whether the densities listed there describe the immediate postshock density. Figure 1 shows that for Regions 4, 5, 8, and 12, detectable IR is only at the innermost edge. Both figures 1 and 2 show the location of the blast wave as indicated by radio and nonthermal X-rays; the absence of immediate IR emission there suggests density variations along the line of sight, with IR appearing only once the density is somewhat larger (and shocks somewhat slower). For those regions, we have chosen to regard the densities of Figure 3 as upper limits. Higher spatial resolution IR observations, possible with JWST, will be required to improve our knowledge of the immediate postshock density in all locations.

5.3. Shock Velocities

Proper motions of expansion were measured by Katsuda et al. (2008) comparing Chandra images obtained in 2000 and 2006, uncertainties of the fits are much smaller than the spread of values possible by changing the assumed electron temperature between 1 and 2 keV; that spread is shown in Figure 4.

While we believe that the emission from each region is well characterized by the densities listed in Table 3, in a few regions the IR emission is so faint that it is not clear whether the densities listed there describe the immediate postshock density. Figure 1 shows that for Regions 4, 5, 8, and 12, detectable IR is only at the innermost edge. Both figures 1 and 2 show the location of the blast wave as indicated by radio and nonthermal X-rays; the absence of immediate IR emission there suggests density variations along the line of sight, with IR appearing only once the density is somewhat larger (and shocks somewhat slower). For those regions, we have chosen to regard the densities of Figure 3 as upper limits. Higher spatial resolution IR observations, possible with JWST, will be required to improve our knowledge of the immediate postshock density in all locations.

5.3. Shock Velocities

Proper motions of expansion were measured by Katsuda et al. (2008) comparing Chandra images obtained in 2000 and 2006,
using images between 1.0 and 8.0 keV. We have converted them, and their uncertainties, into velocities for our nominal distance of 5 kpc. (Regions shown in Figure 1 correspond to whole Spitzer IRS pixels. They overlap, but are not identical with, the corresponding regions used by Katsuda et al. 2008.) Figure 4 shows both densities and shock velocities of various regions. As expected, these quantities anticorrelate. Densities vary fairly smoothly around the periphery of Kepler, with lowest values in the south and highest in the north, well correlated with the brightness in radio or X-rays, while the shock velocity (with larger errors) also varies fairly smoothly in the opposite sense. Postshock pressures \( P_2 = \rho v^2 \) (again, eliding the factor \( 2/(\gamma - 1) \)) are shown in Figure 4, where we have assumed \( \rho_0 = 1.4 n_H m_H/\epsilon_{\text{comp}} \) appropriate for neutral gas upstream. We take \( \epsilon_{\text{comp}} = 4 \).

The pressure varies by about a factor of a few around the periphery of Kepler, not unexpected given the very strong gradient in external density, with highest pressure to the north, where the external density is much larger. This magnitude of pressure variation is comparable to the radial pressure gradient in the interior of a Sedov blast wave, where the dynamical timescale is comparable to the age.

For Regions 4, 5, 8, and 12, we regard our density determinations as upper limits, hence upper limits on the pressure. The variations in pressure we find, however, are relatively small (Figure 4), suggesting that the true densities in those regions are unlikely to be far below those in closely neighboring regions.

### 5.4. Magnetic Field Measurements

Regions shown in Figure 2 were selected for the presence of a “thin rim” of nonthermal X-rays, from which a magnetic field strength could be extracted, making the simplest assumptions about thin-rim physics as embodied in Equation (5). Radial profiles (averaged azimuthally over the region’s azimuthal dimensions) were extracted; two examples are shown in Figure 5. Widths were measured at the intensity level halfway between the rim peak and the interior minimum. Uncertainties in this process were estimated to be \( 0^\prime 2 \), although, as we shall see below, the exact value turns out not to be critical. At 5 kpc, \( 10^{-2} \text{ pc} = 0^\prime 41 \), so for a radial width \( w \) of a rim in arcsec,

\[
B = 116 (v_{\text{sh}}/10^8 \text{ cm s}^{-1})^2/3 (w/\text{arcsec})^{-2/3} \mu G.
\]

Note that since our shock velocities are obtained from angular proper motions, the ratio \( v_{\text{sh}}/w \) is independent of distance; we have also assumed 5 kpc to obtain the values for \( v_{\text{sh}} \) in Table 3. Values of magnetic field inferred from Equation (14) are listed in Table 4. Not all the rim locations of Table 3 showed distinct rims, so we restrict our analysis to the seven listed in Table 4.

### 5.5. Radio Intensities

We determine mean intensities over each small region assuming that it is homogeneous and that the line-of-sight depth \( L \) is equal to the transverse extent of the filament being sampled (so generally larger than the region’s radial width). Flux densities in each region are measured from the 5 GHz radio image (DeLaney et al. 2002), then corrected for background taken from comparable or larger regions outside the remnant, divided by the solid angle \( \Delta \Omega \), and extrapolated to 1 GHz assuming \( j_\nu \propto \nu^{-0.33} \) everywhere. The results are listed as \( \langle I_\nu \rangle \) in Table 3, and the derived electron energy densities are given in Table 4. Quoted uncertainties result from the off-source rms fluctuation level of 0.14 mJy (DeLaney et al. 2002), scaled by the square root of the extraction area.

### 6. Results

The globally averaged values for efficiencies for our six remnants are given in Table 2. The range is extraordinary: a factor of 500 in \( \epsilon_e \) and 90 for \( \epsilon_B \). Their ratio (also equal to...
Figure 6. Left: efficiencies of relativistic-electron acceleration $\epsilon_e$ and magnetic field amplification $\epsilon_B$ for seven locations around the perimeter of Kepler. The values and their ratio scatter over orders of magnitude. Right: efficiencies vs. shock velocity. In both cases, error bars are not formal uncertainties but simply illustrate a range of a factor of 2. See text.

### Table 4

| Region | $B$ (\(\mu G\)) | $u_e$ (\(10^{-10}\) erg cm\(^{-3}\)) | $u_B$ (\(10^{-10}\) erg cm\(^{-3}\)) | $u_e/u_B$ | $P$ (10\(^{-\nu}\) dyn cm\(^{-2}\)) | $\epsilon_e$ (10\(^{-\nu}\)) | $\epsilon_B$ (10\(^{-\nu}\)) | $u_e/u_{Bav}$ |
|--------|----------------|---------------------------------|---------------------------------|-------------|---------------------------------|----------------|----------------|----------------|
| 3      | 105            | 53                             | 4.36                            | 12.2        | 7.9                             | 68             | 56             | 7.1            |
| 4      | 157            | 0.74                           | 9.84                            | 0.076       | < 11                           | > 0.69         | > 9.1          | 0.20           |
| 5      | 161            | 2.44                           | 10.3                            | 0.24        | < 12                           | > 2.0          | > 8.3          | 0.68           |
| 7      | 121            | 15.9                           | 5.8                             | 2.75        | 3.1                             | 52             | 19             | 2.7            |
| 8      | 148            | 2.54                           | 8.7                             | 0.29        | < 2.1                          | > 12           | > 42           | 0.61           |
| 10     | 80             | 55.6                           | 2.5                             | 2.6         | 0.69                            | > 20           | 9.9            | 4.7            |
| 13     | 60             | 189                            | 1.4                             | 133         | 9.0                             | 210            | 1.6            | 9.7            |

Note.

\(^a\) Assuming the median value of $B_{nv} = 121 \mu G$ and $u_{Bav} = 5.83 \times 10^{-10}$ erg cm\(^{-3}\) for all regions.

The interior of Kepler is in rough pressure equilbrium, that is, the postshock pressure $P_2$ is constant within a factor of a few. Certainly $L_i$ is constant. Then, we have no way to explain the factor of 20 variation in mean intensity recorded in Table 3 other than strong variations of the epsilons. Certainly the line-of-sight depth $L$ required to obtain $j_\nu$ from $(I_0)$ is unlikely to vary by this much.

The quantitative results of Table 4, plotted in Figure 6, make this point clearly. Error bars shown there are not formal uncertainties but simply illustrate an assumed factor of 2 (100\%) range. Section 7.1 discusses uncertainties in detail. But Figure 6 shows that the efficiencies of magnetic field amplification and relativistic-electron acceleration vary by a far greater amount, orders of magnitude, around the periphery of Kepler. Their ratio, completely independent of the pressure determination (and therefore of the density upper limits in some regions), varies by over three orders of magnitude, both larger and smaller than 1. Even removing the two extreme regions 4 and 13, the range is a factor of 90. In particular, adjoining regions show no particular correlation in efficiencies; the smooth trends of density or pressure with azimuth shown in Figure 4 are not evident. Figure 6 also plots the efficiencies versus shock velocity; while a weak trend appears to be present, it is almost entirely due to one region, 13, with a much lower shock velocity.

We also plot the dimensionless ratio $u_e/u_B$ in Figure 7. These values also scatter widely, ranging over three orders of magnitude around Kepler’s periphery, with values both larger and smaller than 1. Now the inference of $u_e$ from observations depends strongly on the inferred magnetic field strength (Equation (13)), inducing a strong indirect dependence on shock speed through Equation (5): $u_e \propto B^{-1.71}$. Any intrinsic dependence of the observed synchrotron intensity $I_\nu$ on shock speed can be isolated by plotting the dimensionless ratio $u_e/u_{Bav}$ that is, using the median value of $B$ to calculate both $u_e$ and $u_B$. This quantity, Column (9) of Table 4, is plotted versus shock speed in Figure 7. As for $\epsilon_B$, there is the suggestion of a trend with velocity, here to lower $\epsilon_B$ with higher shock velocity; however, the scatter is large. Figure 7 also illustrates ranges of 50\% for $u_e/u_B$ (left panel) and 30\% for $u_e/u_{Bav}$ (right panel). Since the normalization by postshock pressure is absent, several of the uncertainties documented in Section 7.1 are not relevant for either ratio, while all variation of magnetic field is removed in the right panel. Future...
observational and theoretical studies should address the possibility of trends with shock velocity. Other young SNRs such as Tycho are susceptible to a spatially resolved analysis such as this.

7. Discussion

Our startling results require closer examination. First, the values of $\epsilon_B$, including the value for average remnant properties, spread over two orders of magnitude for remnant averages, and for Kepler in particular are far lower than the 0.1–0.01 often assumed for nonrelativistic shocks (e.g., Lundqvist et al. 2020). A more sophisticated model for $\epsilon_B$ (Sarbadhicary et al. 2017) predicts values that, while not constant, range only over a factor of 3. Most significant for constraining these efficiencies is the range of values around the rim of Kepler. Table 4 and Figure 6 show a particularly large range (factor of up to 300) in $\epsilon_e$, while $\epsilon_B$ varies by a smaller but still large factor of 35.

7.1. Possible Sources of Uncertainty

The magnitude of the variations among regions shown in Table 4 means that uncertainties of even factors of several cannot change the qualitative result of strong variations of efficiency. However, it is worth considering various possible sources of uncertainty.

7.1.1. Distance

If shock velocities are obtained from angular proper motions, Equation (5) shows that magnetic fields inferred from the angular widths of azimuthal thin rims are independent of distance. Electron energy densities inferred from Equations (12) and (13) depend only on the distance-independent intensity $I_e$ and the line-of-sight depth $L$; if the latter is estimated, as we do here, by the angular dimensions of the emitting region in the plane of the sky, we have $u_e \propto d^{-1}$. Alternatively, since $u_e \propto j_e$, if an observed radio flux and angular size are used in Equation (11), we have $v_{em} \propto d^3$, so again $u_e \propto d^{-1}$. Thus, $u_e/u_B \propto d^{-1}$. Our method of obtaining the density at different locations around the rim of Kepler is independent of distance, so the postshock pressure $P_2 \propto d^2$, giving $\epsilon_e \propto d^{-3}$ and $\epsilon_B \propto d^{-2}$.

The range of values of $u_e$ in Table 2 is about $10^4$, although if Cas A is removed, the range drops to 180. These values are so large compared to reasonable distance uncertainties of at most a factor of 2, that it is hard to imagine that such uncertainties could mask or remove the dramatic trends of Table 2. The spread in values of $\epsilon_e$ and $\epsilon_B$ is considerably smaller, but $\epsilon_e$ also varies by a factor of 500 (or 51 without Cas A).

7.1.2. Density Determinations

The quoted uncertainties in densities determined from fitted dust temperatures reflect primarily the range in assumed proton temperatures; the statistical errors in the fits are much smaller, for a particular model of grain composition, structure, and size distribution. The models (described in detail in Williams et al. 2011b) do make assumptions about these quantities, but they are constrained to some extent by the spectra. The most generous allowance for uncertainties in these quantities is unlikely to exceed a factor of 2 in any case.

For the regions for which we feel that the true immediate postshock density may be less than that for the emission we detect and fit in those regions (regions 4, 5, 8, and 12), there is a possibility that $\epsilon_e$ is considerably larger than the lower limits shown in Figure 6. Since these are the lowest values, it is conceivable that the true densities could be low enough to reduce the scatter considerably. This would require, however, more than an order of magnitude difference, reducing the inferred pressure in those regions to values considerably below those in neighboring regions, well outside the scatter in pressure found elsewhere, unless blast wave velocities are substantially lower as well. Such density variations could reflect the propagation of the blast wave into an inhomogeneous upstream medium, seen in projection. We believe it unlikely that large variations are present, but without more sensitive infrared observations at higher spatial resolution, we cannot rule out the possibility. In any case, the large scatter in $\epsilon_e/\epsilon_B$ is, of course, unaffected.

7.1.3. Shock Compression Ratio

The compression ratio enters into the determination of the magnetic field (Equation (5)) and into the upstream density as determined from collisionally heated dust downstream: $B \propto r_{\text{comp}}^{-2/3}$ and $\rho_0 \propto r_{\text{comp}}^{-1}$. In numerical results above, we have assumed a compression ratio of 4. We know from the presence of synchrotron rims that the blast wave in Kepler is accelerating electrons to TeV energies, and it is conceivable that the effective compression ratio is larger than 4, due either to particle escape or to an energetically significant population of relativistic ions (Jones & Ellison 1991), but it is very unlikely to be larger than the relativistic limit of 7, since ample
postshock thermal emission indicates that the shock is not dominated by highly relativistic particles.

So \( u_B \propto \frac{r_{\text{comp}}}{E_{\mu}^{3/5}} \) and \( \epsilon_B \propto \frac{r_{\text{comp}}}{E_{\mu}^{3/5}} \). Then, \( u_e \propto B^{1+\alpha} \propto r_{\text{comp}}^{-2(1+\alpha)/3} \) and \( \epsilon_e \propto r_{\text{comp}}^{-0.14} \). For our assumed radio spectral index \( \alpha = 0.71 \), we have \( u_e \propto r_{\text{comp}}^{-1} \) and \( \epsilon_e \propto r_{\text{comp}}^{-3} \). Then, we would reduce \( u_B \) by a factor of 0.47 and \( \epsilon_B \) by 0.83, while \( u_e \) would drop by 0.53 and \( \epsilon_e \) by 0.92. So at most a factor of 2 uncertainty results from this range of compression ratios.

### 7.1.4. Magnetic Fields

As pointed out above, large uncertainties accompany the estimates of magnetic field from filament widths. However, all estimates agree on requiring substantial magnetic field enhancement over a factor of at most 4 (or 7) increase due to shock compression alone. To estimate the effects of this uncertainty, we have assigned all regions a single value \( B_{\Sigma} = 122 \mu G \), the median of the values shown in Table 4, which gives \( u_B = 5.92 \times 10^{10} \) erg cm\(^{-3} \). The last columns of Table 4 give \( u_e/u_B \) and \( \epsilon_e/\epsilon_B \) for this value of \( B \). The spread in \( u_e/u_B \) is reduced from about 1800 to 50, which is the spread in \( u_e \) alone.

### 7.1.5. Radio Properties

The radio spectral index of Kepler between 1.4 and 5 GHz is observed to vary with location between about 0.65 and 0.8 over most of the remnant, including all of the regions we have measured (DeLaney et al. 2002). This corresponds to a range in electron energy index \( s \) of 2.3–2.6. In terms of \( \alpha = (s-1)/2 \), the spectral index dependencies of the electron energy density \( u_e \) are

\[
u_e \propto E_j^{1-2\alpha}(2\alpha-1)^{-1}c_j(\alpha)B^{-1(1+\alpha)}\nu^\alpha.\tag{16}\]

Evaluating this expression for the median magnetic field of 122 \( \mu G \), \( E_j = 8.2 \times 10^{-7} \) erg, and \( \nu = 1 \) GHz, for \( \alpha = 0.65 \) and 0.8, gives

\[
u_e(\alpha = 0.8) \over \nu_e(\alpha = 0.65) = 2.4.\tag{17}\]

Very little of the scatter in the values of \( u_e \) at different locations in Kepler can be due to variations of \( \alpha \).

It is also unlikely that huge variations in the minimum energy of the electron distribution occur at different locations. Our fiducial value of 100\( m_e c^2 \) simply reflects the energy at which electrons are relativistic enough for the synchrotron formulae on which our analysis rests to apply.

The radio flux measurements for the different regions carry uncertainties, as listed in Table 3. Those were obtained from the off-source rms values of 4.8 GHz radio flux in the image of DeLaney et al. (2002), scaled by the square root of the extraction area. The signal-to-noise ratio for the regions ranges from 3 for region 3 to 31 for region 13. These cannot explain the orders-of-magnitude spread evident in Table 4 and exhibited in Figure 6.

Finally, we obtain emissivities by assuming a line-of-sight depth of our regions, which we take to be the longer dimension of our extraction region. Again, this can easily be in error by a factor of a few, but not by orders of magnitude.

Unfortunately, we cannot calculate formal uncertainties in \( u_e \) or \( u_B \), or their ratio. While we report uncertainties in density and shock velocity, these cannot be propagated to functions of those quantities without knowledge of their distributions, quite unlikely to be Gaussian. We know even less about the uncertainties in magnetic field. The variations among authors illustrated in Table 1 are not statistical or systematic errors but results of applications of slightly different versions of the basic argument embodied in Equation (5). A better estimate is represented by the spread shown in Table 1 for the reference whose values we have used, Parizot et al. (2006)—of order 10%. But again, we do not know enough about the distribution of the uncertainties to be able to apply a simple error propagation formalism. Figures 6 and 7 illustrate what different levels of uncertainty would look like in the distributions; the values we have chosen represent our estimates of reasonable ranges based on the discussion above.

### 7.2. Consequences

We are forced to conclude that the various sources of uncertainty are dwarfed by the enormous spread of values of \( u_e \), \( \epsilon_B \), \( \epsilon_e \), and \( \epsilon_B \) at different locations on Kepler’s periphery. The primary source of the spread in \( u_e/u_B \) results from the radio brightness variations (Equation (13)). We conjecture that the missing physics required to explain the spread has to do with acceleration of particles, electrons in particular, which evidently has a more complex dependence on parameters than we have included. No monotonic relation between \( u_e \) and shock velocity is apparent in the values in Tables 3 and 4; another likely possibility, the shock obliquity angle \( \theta_{\text{SNR}} \) between the shock velocity and upstream magnetic field, may have a highly nonlinear effect on the ultimate population of \( \sim \) GeV electrons producing the radio emission.

What has become of equipartition, the time-honored principle still often used to infer properties of synchrotron sources? First, in the SNR context, SNRs are very inefficient at producing either magnetic field energy or relativistic-particle energy. Compared, say, to extragalactic radio sources, relatively little of the total supernova energy ever winds up in nonthermal forms (that is, both \( \epsilon_B \) and \( \epsilon_e \) are always small). It is easier to imagine wide variations in \( u_e/u_B \) when a much larger pool of thermal energy is available for any of various processes to produce electrons or magnetic field. In addition, absence of direct information on relativistic protons means that we simply have no idea what the total nonthermal energy density is (though very broad inferences from observed Galactic cosmic rays seem to require about 10% of total supernova energy winding up in relativistic baryons). So electron energy densities are a small fraction of a fairly small fraction, and we should not be astonished if there is no clear relation between the energy in relativistic electrons and other pools of energy. As for magnetic energy, when SNR magnetic fields are inferred using equipartition arguments, they essentially serve as proxies for the SNR mean surface brightness \( \Sigma \), as can be seen from, for instance, Pacholczyk (1970), Equation (7.14), where \( B_{\text{equip}} \propto (\Sigma/D)^{2/7} \), \( D \) being the source diameter. There is no obvious mechanism that could operate to transfer energy to or from the magnetic field based on the energy in relativistic electrons. Figure 7 shows that the density-independent ratio \( u_e/u_B \) varies by orders of magnitude in regions of comparable shock velocity, emphasizing this point.

A related consequence of our results for Kepler is the problematic nature of the traditional \( \Sigma-D \) relation as a diagnostic tool. The large azimuthal variations we find suggest that global averages can be very misleading. While the average values of \( \epsilon_e \) and \( \epsilon_B \) we obtain from the global properties for Kepler (Table 2) do fall within the large range of values at particular locations, it is not clear that those global values
represent any kind of mean of the physical properties. Great care should be taken in drawing conclusions based on such global values.

One might hope to extract from this exercise some clue as to what additional parameters might be required to explain the range of efficiencies. The absence of smooth trends of efficiencies with position angle around the remnant disfavors explanations such as a smoothly varying shock obliquity angle $\theta_{\text{sh}}$, as one might expect for a blast wave encountering a uniform upstream magnetic field. The large scatter for adjoining regions shown in Figure 6 requires relatively small-scale variations in properties important for particle acceleration and/or magnetic field amplification. Our attempt to extract information relatively independent of magnetic field inferences is shown in the right panel of Figure 7, where a noisy trend is visible as lower electron energy density with higher shock velocity. We do not claim the unambiguous existence of such a relation since the apparent trend is strongly dependent on one or two points, but the data might offer a clue toward identifying the additional physics evidently necessary to fully understand electron acceleration in strong shock waves.

8. Conclusions

We have used a combination of radio, infrared, and X-ray data to estimate nonthermal energy densities in relativistic electrons and magnetic field, first, in global averages over six young SNRs and, second, for seven regions around the periphery of Kepler’s SNR. For the various remnants, we find enormous ranges of these quantities, with $\eta_{\text{e}/\text{B}}$ varying by a factor of over 3000 (or among the Type Ia remnants, i.e., excluding Cas A, by a factor of 70). The efficiencies range from $10^{-4}$ to 0.05 for $\epsilon_{\text{e}}$ and from 0.002 to 0.1 for $\eta_{\text{B}}$. There are no clear trends with age or shock speed evident in Table 2.

Of course, the collection of remnants is quite inhomogeneous, and the objects themselves are inhomogeneous, so describing them by global average values may conceal systematic trends in these quantities. We have therefore extracted densities, shock speeds, magnetic field strengths, and radio fluxes from seven regions around Kepler and find similarly large spreads in the energy densities, their ratio, and the efficiencies. Figures 6 and 7 summarize the results.

The variations in our inferred values are so large that even conservative assessments of sources of error or uncertainty are quite unable to account for them. There seems no alternative but to conclude that additional parameters must influence the amplification of magnetic field and acceleration of electrons. Various possibilities come to mind: the neutral fraction of upstream gas, the obliquity angle between the local shock normal and the upstream magnetic field direction, or downstream variations in, for instance, magnetic turbulence. Evidence for local variations of shock acceleration physics at different points around SNR peripheries (through the dimensionless diffusion coefficient $\eta \equiv \lambda_{\text{diff}}/L_{\text{sh}}$, the ratio of mean free path to Larmor radius) has recently been presented by Tsuji et al. (2021); the origins of the variations of $\eta$ they find presumably rely on some of these additional parameters.

Given our evident lack of understanding of all the factors that contribute to shock acceleration of particles and turbulent amplification of magnetic field, it would seem that assumptions of, for instance, constant values of the efficiency as a function of time or local conditions in synchrotron sources should be made with extreme caution and borne in mind as possible sources of systematic error or completely incorrect inference when analyzing spectra of such objects as gamma-ray bursters or radio supernovae.

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