EFFECT OF COLLECTIVE NEUTRINO OSCILLATIONS ON THE NEUTRINO MECHANISM OF CORE-COLLAPSE SUPERNOVAE

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ABSTRACT

In the seconds after collapse of a massive star, the newborn proto-neutron star (PNS) radiates neutrinos of all flavors. The absorption of electron-type neutrinos below the radius of the stalled shockwave may drive explosions (the “neutrino mechanism”). Because the heating rate is proportional to the neutrino energy, flavor conversion of higher-energy $\mu$ and $\tau$ neutrinos to electron-type neutrinos via collective neutrino oscillations (C$\nu$O) can increase the heating rate, and potentially drive explosions. We solve the steady-state boundary value problem of spherically-symmetric accretion between the PNS surface ($r_\text{PNS}$) and the shock ($r_\text{sh}$), for the first time including a scheme for flavor conversion via C$\nu$O. For a given $r_\text{PNS}$, PNS mass ($M$), and accretion rate ($\dot{M}$), we calculate the critical neutrino luminosity above which accretion is impossible and explosion results. We show that C$\nu$O decreases the critical luminosity by a factor of $\sim 1.5$, but only if the flavor conversion is fully completed inside $r_\text{sh}$. The effect is smaller for partial conversion. The shock radius and the physical scale for flavor conversion depend differently on the parameters of the problem. We quantify these dependencies and find that C$\nu$O lowers the critical luminosity substantially for small $M$ and $\dot{M}$, and large $r_\text{PNS}$. Thus, C$\nu$O can be important for shockwave revival if PNSs contract slowly, which may favor a stiff nuclear equation of state, and if progenitors reach low $M$ at early times after collapse, which favors the lowest-mass massive stars.

Subject headings: supernovae: general — neutrinos

1. INTRODUCTION

Core-collapse supernovae announce the deaths of massive stars. The explosion develops deep in the optically thick stellar core and the dynamics of the neutron star in formation reveals itself only through the emission of neutrinos, which are scarcely detected (e.g. Yüksel & Beacom 2007). Computer simulations are thus the primary means to probe the dynamics of the supernova explosions. Unfortunately, only the lowest-mass progenitors explode (Rampp & Janka 2000; Bruenn et al. 2001; Liebendorfer et al. 2001; Mezzacappa et al. 2001; Thompson et al. 2003; Kitaura et al. 2006; Janka et al. 2008). In particular, it proves difficult to revive the outward movement of the shockwave, which forms when the collapsing core reaches nuclear density, and which stops its progress due to neutrino emission losses and other effects (e.g. Burrows & Lattimer 1983; Bruenn 1989a,b). Accretion of matter through the stalled shock ensues and lasts for many dynamical times before explosion or eventual black hole formation.

The hot proto-neutron star (PNS) cools by emission of neutrinos of all flavors. Because $\nu_e$, $\bar{\nu}_e$, $\nu_\mu$, and $\bar{\nu}_\mu$ (hereafter collectively denoted as $\nu_\chi$) do not interact with the PNS matter through the charged-current interactions, they decouple from matter at smaller PNS radii and higher temperatures, and thus have higher average energy than $\nu_\chi$ and $\bar{\nu}_\chi$. A fraction of the $\nu_\chi$ and $\bar{\nu}_\chi$ are absorbed below the accretion shock, and the associated deposition of energy plays a significant role in the dynamics of the supernova and perhaps in the revival of the shockwave (e.g. Colgate & White 1966; Bethe & Wilson 1985). Specifically, the “neutrino mechanism”, as formulated by Burrows & Goshy (1993), states that the steady-state accretion through the shock turns into an explosion when the neutrino luminosity of the PNS ($L_{\nu_\chi,\text{PNS}}$) exceeds a critical value, $L_{\nu_\chi,\text{crit}}$. In Peicha & Thompson (2011) (hereafter Paper I) we showed that $L_{\nu_\chi,\text{crit}}$ is equivalent to reaching $\max(c_\text{S}^2/v_\text{esc}^2) \approx 0.19$ in the accretion flow, where $c_\text{S}$ is the sound speed and $v_\text{esc}$ is the local escape velocity. This “antasonic” condition is a manifestation of the inability of the flow to satisfy both the shock jump conditions and the Euler equations for the accretion flow simultaneously. We also determined the dependence of $L_{\nu_\chi,\text{crit}}$ on the key parameters of the problem, including the energies of the neutrinos over a wide range of parameter values. Specifically, and most importantly for this paper, we found that $L_{\nu_\chi,\text{crit}}$ is proportional to the inverse square of the $\nu_\chi$ and $\bar{\nu}_\chi$ energies.

Simulations of supernovae generally fail because the neutrino luminosities in the models never reach $L_{\nu_\chi,\text{crit}}$. For successful explosions, either (i) $L_{\nu_\chi,\text{crit}}$ needs to be decreased or (ii) $L_{\nu_\chi,\text{crit}}$ increased. As an example of the former, multi-dimensional effects like convection and SASI decrease $L_{\nu_\chi,\text{crit}}$ (Yamasaki & Yamada 2005, 2006; Murphy & Burrows 2008; Nordhaus et al. 2010) by making the heating more efficient (e.g. Herant et al. 1994; Burrows et al. 1995; Janka & Müller 1996; Fryer & Warren 2004; Buras et al. 2006) or cooling less efficient (Paper II). As an example of the latter, $L_{\nu_\chi,\text{crit}}$ can be enhanced by convection inside the PNS (e.g. Wilson & Mayle 1988; Bruenn & Dineva 1996; Keil et al. 1996).

Most of the heating below the shock occurs due to absorption of $\nu_\chi$ and $\bar{\nu}_\chi$ on neutrons and protons, while the more energetic $\nu_\chi$ escape without much interaction. Thus, because the luminosities in each flavor are similar (Janka 2001), $\sim 2/3$ of the total neutrino luminosity generated by the PNS is essen-
tially unused. However, due to the high density of neutrinos in this region, self-interaction between neutrinos becomes important and can lead to a range of phenomena called “collective neutrino oscillations” (e.g. Pantaleoni 1992; Duan et al. 2006, 2010). In particular, there is a possibility of an instability (Dasgupta et al. 2009) that exchanges part of the high-energy νe spectra with the νμ and ντ spectra, which may then produce significantly more heating than calculations neglecting neutrino oscillations. Chakraborty et al. (2011a,b) and Dasgupta et al. (2011) have investigated the relevance of C/O for the shock revival in the core-collapse simulations of several progenitor models.

Given that νν have significantly higher energy than νe and ντ, and that Lνν,core is proportional to inverse square of electron neutrino energy (Paper I), the flavor conversion of high energy νν to νe and ντ may have significant ramifications for supernova explosions. In this Letter, we quantify the changes to Lνν,core when the neutrinos are subject to flavor conversion due to collective neutrino oscillations (C/O). We model the neutrino conversion using the simplified analytical treatment summarized in Dasgupta et al. (2011) coupled to the code described in Paper I. In Section 2 we describe our calculations. In Section 3 we quantify the changes to Lνν,core and the shock radii, and compare the magnitude of the effect of C/O to other known pieces of physics. In light of our results, we assess on what timescale, and in what mass range of progenitors, C/O are likely to be important. In Section 4 we discuss and review our results.

2. METHOD

We extend the code developed in Paper I to include the effect of C/O. We calculate the structure of the steady-state accretion flow between the neutrinosphere at radius rν and the standoff accretion shock at rs assuming spherical symmetry. The key parameters of the problem are the mass accretion rate through the shock M, the PNS mass M, its radius rν, the νν luminosity of the PNS core Lνν,core, and the rms energies εν of the neutrinos. We assume that all six neutrino flavors have the same luminosity, Lνν,core. We solve the two-point boundary value problem composed of the Euler equations for the density ρ, velocity v, and temperature T along with the equation for the electron fraction Ye. The shock radius is determined self-consistently as an eigenvalue of the solution. We include heating and cooling due to the two most prominent charged-current interactions. Other interactions provide only a small change in Lνν,core (Paper I). Unlike Paper I, we assume that dLνν/dν = 0 for all neutrino flavors. The outer boundary conditions of the calculation are conservation of mass, momentum and energy through the shock. We assume pressureless free fall of iron with 1/4 free-fall acceleration upstream of the shock. We also assume that rν is the neutrinosphere of νν. We compare the effects of C/O to our fiducial calculation with electron neutrino rms energies εν,0 = 13 MeV and εν,0 = 15.5 MeV (Thompson et al. 2003).

According to Hannestad et al. (2006), the flavor conversion and effective increase of electron neutrino energies occur above a synchronization radius rν, which is defined as

\[ \mu = \frac{4 \sqrt{\nu_e}}{4 \sqrt{\nu_e}} \left( 1 + \frac{r^2}{4 \nu_e} \right) \]

(2)

The flavor conversion is more or less complete at radius rν defined as

\[ \mu = \frac{\mathcal{F}_e}{\mathcal{F}_e} \left( 1 + \frac{r^2}{4 \nu_e} \right) \]

(3)

Here, \( \mathcal{F}_e \) depends on the neutrino oscillation frequency and neutrino energy spectra and as in Dasgupta et al. (2011), we choose a typical value \( \mathcal{F}_e = 50 \text{ km}^{-1} \). The quantity \( \mathcal{F}_e / \mathcal{F}_e \) is the ratio of the net lepton asymmetry in the system to the neutrino flux available for oscillations, and is defined as

\[ \frac{\mathcal{F}_e}{\mathcal{F}_e} = \frac{\phi_{\nu_e} - \phi_{\bar{\nu}_e}}{\phi_{\nu_e} + \phi_{\bar{\nu}_e} - 2 \phi_{\nu_e} \phi_{\bar{\nu}_e}} \]

(4)

and similarly for \( \phi_{\bar{\nu}_e} \) and \( \phi_{\bar{\nu}_e} \). The collective potential \( \mu \) is defined as (Esteban-Pretel et al. 2007; Dasgupta et al. 2011)

\[ \mu(r) = \sqrt{2} \mathcal{F}_e \phi_{\nu_e} \left( \frac{r}{r_0} \right)^2 \left( \frac{r}{r_0} \right)^2 \]

(5)

where \( \mathcal{F}_e \) is the Fermi coupling constant and \( \phi_{\nu_e} = \phi_{\nu_e} + \phi_{\nu_e} + 4 \phi_{\nu_e} \). Equations (1) and (2) are solved for \( r_\nu \) and the neutrino number fluxes. We note here that the dependence of equation (4) on \( r_\nu \) introduces an absolute scale to equations (1) and (2), and therefore \( r_\nu \) and \( r_\nu \) do not scale linearly with \( r_\nu \). Instead, in the limit of \( r_\nu / r_\nu \gg 1 \), the scaling is

\[ \frac{r_\nu}{r_\nu} \propto L^{1/4}_{\nu_e, core} r_\nu^{-1/2} \]

(6)

for fixed neutrino energies. The same scaling holds for \( \nu_e \).

The collective neutrino flavor conversion effectively increases \( \nu_e \) and \( \nu_e \) between \( r_\nu \) and \( r_\nu \). Motivated by more complete studies of the physical extent of the flavor conversion, and as a numerical expedient, we adopt the following functional form for \( \nu_e \)

\[ \nu_e(r) = \nu_e(0) + \nu_e(r_\nu) \left\{ 1 + 1 \right\} \frac{r - (r_\nu + r_\nu)}{\sigma(r_\nu + r_\nu)} \]

(7)

where we use \( \nu_e = 20 \text{ MeV} \) (Thompson et al. 2003). We choose \( \sigma \approx 0.679 \) to have the term in curly brackets equal to 5% at \( r_\nu \) and 95% at \( r_\nu \). We adopt an analogous prescription for \( \nu_e \).

The neutrino energy self-consistently enters not only in the heating, but also in the reaction rates for the calculation of Ye and in the inner boundary condition on optical depth (see Paper I for details).

3. RESULTS

In the left panel of Figure 1 we present \( L^{\text{crit}}_{\nu_e, core} \) including C/O as a function of \( \dot{M} \) for \( \dot{M} = 1.2 \dot{M}_\odot \) and \( r_\nu = 60 \text{ km} \) (red dashed line) along with the fiducial calculation (black solid line). The critical curve in the fiducial calculation is approximately a power law (Paper I). We see that C/O lowers \( L^{\text{crit}}_{\nu_e, core} \) to \( 0.65 \) times the fiducial value for \( \dot{M} < 0.01 \dot{M}_\odot \text{ s}^{-1} \). For higher \( \dot{M} \), the critical curve turns upward and for \( \dot{M} \geq 0.02 \dot{M}_\odot \text{ s}^{-1} \), it essentially coincides with the fiducial calculation meaning that C/O have little effect.
The behavior seen in the left panel of Figure 1 is non-trivial even in our simple setup, because $r_{\text{sync}}$ and $r_{\text{end}}$ are a function of the boundary conditions. It can be understood by analyzing the position of $r_{\text{S}}$ relative to $r_{\text{sync}}$. We expect that $L_{\nu_{e,\text{core}}}$ will have an effect on $L_{\nu_{e,\text{core}}}^\text{crit}$ only if $r_{\text{sync}} < r_{\text{S}}$ and the full effect will be obtained for $r_{\text{end}} < r_{\text{S}}$. Because $r_{\text{S}}$ increases with $L_{\nu_{e,\text{core}}}$ (grey solid lines in Fig. 1 right panel) and reaches a maximum $r_{\text{S}}^\text{crit}$ at $L_{\nu_{e,\text{core}}}^\text{crit}$, the effect of $C_{\nu_{e}}$ is most prominent for $L_{\nu_{e,\text{core}}}$ close to $L_{\nu_{e,\text{core}}}^\text{crit}$. The right panel of Figure 1 shows the effect of $C_{\nu_{e}}$ on the shock radii. We see that $r_{\text{S}}^\text{crit}$ in the fiducial calculation (thick black solid line) closely follows the results from [Paper I] (dots) except that the calculations presented here have larger $r_{\text{S}}^\text{crit}$, because we set $dL_{\nu_{e}}/dr = 0$. The calculation with $C_{\nu_{O}}$ (red dashed line) closely follows the fiducial results for high $M$ and $L_{\nu_{e,\text{core}}}$, where $r_{\text{sync}} > r_{\text{S}}^\text{crit}$. Here, the $C_{\nu_{e}}$ effect is negligible and $L_{\nu_{e,\text{core}}}^\text{crit}$ is very close to the fiducial value. When $r_{\text{S}}^\text{crit} \approx r_{\text{sync}}$, which occurs for $L_{\nu_{e,\text{core}}}^\text{crit} \gtrsim 1.5 \times 10^{52}$ erg s$^{-1}$ ($M \approx 0.32$ $M_{\odot}$ s$^{-1}$), the $C_{\nu_{e}}$ effect starts to become important, and both $L_{\nu_{e,\text{core}}}$ and $r_{\text{S}}$ decrease relative to the fiducial calculation. For $L_{\nu_{e,\text{core}}}^\text{crit} \lesssim 2.5 \times 10^{54}$ ergs s$^{-1}$, which corresponds to $M \lesssim 0.035$ $M_{\odot}$ s$^{-1}$, $r_{\text{end}} < r_{\text{S}}^\text{crit}$ and $C_{\nu_{O}}$ affect the structure of the flow significantly and essentially saturate. $L_{\nu_{e,\text{core}}}^\text{crit}$ with $C_{\nu_{O}}$ is reduced to $\sim 0.65$ of the fiducial value for low $M$.

We have seen that the relative positions of $r_{\text{S}}$ and $r_{\text{sync}}$ determine the effect of $C_{\nu_{O}}$. It is known that multi-dimensional effects like convection and SASI consistently increase shock radii over the corresponding 1D value and decrease $L_{\nu_{e,\text{core}}}$ (e.g. Burrows et al. 1998; Ohnishi et al. 2006; Iwakami et al. 2008; Murphy & Burrows 2008; Marek & Janka 2009; Nordhaus et al. 2010). In Paper I, we parameterized these effects within our steady-state models and found that decreased cooling and increased heating have very similar consequences except that $r_{\text{S}}^\text{crit}$ increases only in the former case. For this reason, based on inspection of simulation results, we argued that the decrease of $L_{\nu_{e,\text{core}}}^\text{crit}$ seen in multi-dimensional simulations is the result of less efficient neutrino cooling. Thus, in order to evaluate the potentially stronger effect of $C_{\nu_{O}}$ in multi-dimensional simulations, we repeated our calculations, but with the normalization of the charged-current cooling rate decreased by a factor of 2. This decreases $L_{\nu_{e,\text{core}}}^\text{crit}$ by about 40% compared to the fiducial case, similar to the difference in $L_{\nu_{e,\text{core}}}^\text{crit}$ observed by Nordhaus et al. (2010) between 1D and 3D simulations. The blue dash-dotted line in the left panel of Figure 1 shows $L_{\nu_{e,\text{core}}}^\text{crit}$ with $C_{\nu_{O}}$ and reduced cooling. As expected, because of lower $L_{\nu_{e,\text{core}}}$ and higher $r_{\text{S}}$ at fixed $L_{\nu_{e,\text{core}}}$, the effect of $C_{\nu_{O}}$ starts to be apparent for $M \approx 0.5$ $M_{\odot}$ s$^{-1}$ and reaches full strength for $M \approx 0.04$ $M_{\odot}$ s$^{-1}$.

How does the strength of the $C_{\nu_{O}}$ effect scale with $M$ and $r_{\nu_{e}}$? In Paper I we showed that $r_{\text{S}}^\text{crit} / r_{\nu_{e}} \propto (L_{\nu_{e,\text{core}}}^\text{crit})^{-0.26}$ and that the dependencies on other parameters like $M$ and $r_{\nu_{e}}$ are much weaker. We plot values of $r_{\text{S}}^\text{crit}$ from Paper I as dots in Figure 1 right panel, for many different $M$ and $r_{\nu_{e}}$. We also showed in Paper I that the critical luminosity scales as $L_{\nu_{e,\text{core}}}^\text{crit} \propto M^{0.723} 1.84 r_{\nu_{e}}^{1.61}$. Thus, increasing $M$ or $M$ increases $L_{\nu_{e,\text{core}}}^\text{crit}$, which in turn decreases $r_{\text{S}}^\text{crit}$ and increases $r_{\text{sync}}$ and $r_{\text{end}}$ (eq. [6]) and the effect of $C_{\nu_{O}}$ will be weaker. Similarly, lower $r_{\nu_{e}}$ yields higher $L_{\nu_{e,\text{core}}}^\text{crit}$ and lower $r_{\text{S}}^\text{crit} / r_{\nu_{e}}$. At the same time, $r_{\text{sync}} / r_{\nu_{e}}$ and $r_{\text{end}} / r_{\nu_{e}}$ will increase (eq. [6]) and the $C_{\nu_{O}}$ effect will become important at smaller $M$ (smaller $L_{\nu_{e,\text{core}}}^\text{crit}$). Therefore, the collective oscillations will be most prominent for the sets of parameters that minimize $L_{\nu_{e,\text{core}}}^\text{crit}$: small $M$, $r_{\nu_{e}}$, and large $r_{\nu_{e}}$.

In Figure 1 the effect of $C_{\nu_{O}}$ becomes relevant at $M \sim 0.1$ $M_{\odot}$ s$^{-1}$. In fully consistent time-dependent calculations,
the values of $r_v = 60 \text{ km}$ and $M = 1.2M_\odot$ chosen for our calculation correspond to the very early stages after bounce, when $\dot{M}$ through the shock is still high. For lower $r_v$ and higher $M$, which correspond to later times after collapse, we plot in Figure 2 the ratio of $L_{\nu_e,\text{core}}$ including CvO to $L_{\nu_e,\text{core}}$ for a reference calculation ($\nu_{\text{red}}$) – the reduction factor of $L_{\nu_e,\text{core}}$ due to CvO (red lines). We note that in order to evaluate only the effect of CvO, we choose the reference calculation to have the same $M$, $r_v$, and microphysics. The blue dash-dotted line is obtained for a reference calculation with $M = 1.2 M_\odot$, $r_v = 60 \text{ km}$, and cooling reduced by a factor of 2. The two thick grey lines have reference calculations without radiation transport and with our fiducial neutrino cooling, respectively.

Figure 1 shows that to get a $\gtrsim 10\%$ reduction in $L_{\nu_e,\text{core}}$ for $r_v = 60 \text{ km}$ and $M = 1.2M_\odot$ due to CvO, we require $M \lesssim 0.1 M_\odot \text{ s}^{-1}$. To get the same reduction in $L_{\nu_e,\text{core}}$ for $M = 1.4M_\odot$ and $r_v = 40 \text{ km}$, Figure 2 shows that we require $M \lesssim 0.01 M_\odot \text{ s}^{-1}$. For $r_v = 20 \text{ km}$ and $M = 1.6M_\odot$, there is no noticeable reduction in $L_{\nu_e,\text{core}}$, because $r_{\nu_{\text{crit}}} \lesssim r_{\text{sync}}$ for the whole considered range of $M$. Looking at solar-metallicity supernova progenitor models of Woosley et al. (2002)\textsuperscript{4}, $M = 0.1 M_\odot \text{ s}^{-1}$ is reached $\sim 0.65 \text{ s}$ after the initiation of the collapse for an $11.2 M_\odot$ progenitor, but at $\sim 4 \text{ s}$ for a $15 M_\odot$ progenitor. At these times, the accreted masses are $M = 1.35$ and $2.2 M_\odot$ for the $11.2$ and $15.0 M_\odot$ progenitors, respectively. The lower limit of our calculations $M = 0.01 M_\odot \text{ s}^{-1}$ is reached only after $\sim 15 \text{ s}$ for the $11.2 M_\odot$ progenitor, when the PNS has almost fully cooled (Pons et al. 1999). From this investigation we conclude that the decrease of $L_{\nu_e,\text{core}}$ due to CvO is noticeable only for very low mass progenitors, which reach low $\dot{M}$ at early times, when $r_v$ is still potentially large. This would be possible for a stiff equation of state of dense nuclear matter, which would keep $r_v$ high\textsuperscript{5}. Multi-dimensional effects, which we model with decreased cooling efficiency in the flow, would increase $r_\nu$ and thus increase the $M$ where $\nu_{\text{red}}$ decreases as a result of CvO (blue dash-dotted line in Fig. 2).

Finally, we compare the CvO effect to other physical effects that have been shown to decrease $L_{\nu_e,\text{core}}$. In Figure 2 we plot with a thick grey line $\nu_{\text{red}}$ that was obtained by including the luminosity from the cooling of the accretion flow (“accretion luminosity”, $\Delta L_{\nu_e}/\Delta r \neq 0$) as was calculated in Paper I. We see that the decrease of $L_{\nu_e,\text{core}}$ by 10 to 20\% is somewhat smaller than the maximum effect from CvO, but is most prominent at high $\dot{M}$, and that it is of similar importance at small $\dot{M}$ and small $r_v$ (Paper I). The lower thick grey line shows $\nu_{\text{red}}$ for the

\textsuperscript{4}http://www.stellarevolution.org/data.shtml

\textsuperscript{5}Note however, that to get an explosion $L_{\nu_e,\text{core}}$ has to be reached by the actual core luminosity $L_{\nu_e,\text{core}}$, which depends on the equation of state in a more complicated way. Indeed, a softer equation of state generally leads to higher $L_{\nu_e,\text{core}}$ at early times after bounce and may thus be favorable for explosion via the neutrino mechanism (e.g. Marek & Janka 2009).
cooling rate reduced by a factor of 2, which we use as an approximation of multi-dimensional effects. The grey lines with dots illustrate the decrease in $L_{\nu_e,\text{core}}^{\text{crit}}$ due to multi-dimensional effects as calculated by Nordhaus et al. (2010). The relative drop when going from their 1D calculations to 2D is comparable to the effect of the accretion luminosity, and $\dot{f}_{\text{red}}$ in 3D is further decreased. At low $M$, the effect of Cr-O becomes comparable to that of increasing the dimension of the simulation from 1D to 3D, but only for fairly large $r_\nu$ and small $M$.

4. DISCUSSION & CONCLUSIONS

We investigate the effect of collective neutrino oscillations on the neutrino mechanism of core-collapse supernovae as parameterized by the critical neutrino luminosity $L_{\nu_e,\text{core}}^{\text{crit}}$. We assume that the $\nu_\alpha$ and $\bar{\nu}_\alpha$ energies increase to $\nu_\alpha$ between a synchronization radius $r_{\text{sync}}$ and end radius $r_{\text{end}}$, as summarized by Cascioli et al. (2011). We found that collective oscillations affect $r_{\nu_\alpha,\text{core}}$ if $r_{\text{sync}} < r_{\nu_\alpha}^{\text{crit}}$, where $r_{\nu_\alpha}^{\text{crit}}$ is the shock radius at $L_{\nu_\alpha,\text{core}}^{\text{crit}}$, and reduce $L_{\nu_\alpha,\text{core}}^{\text{crit}}$ by a factor of $\sim 1.5$ ($f_{\text{red}} = 0.65$, Fig. 2) if $r_{\text{end}} < r_{\nu_\alpha}^{\text{crit}}$. However, this occurs only for low $M$, small $M$, and large $r_\nu$ (see Fig. 1 left panel; Fig. 1 red lines). The reduction of $L_{\nu_\alpha,\text{core}}^{\text{crit}}$ depends on the assumed energy difference between $\nu_\alpha$ and $\bar{\nu}_\alpha$. If the energy $\varepsilon_{\nu_\alpha}$ was increased to 25 MeV or 35 MeV while keeping $\varepsilon_{\bar{\nu}_\alpha} = 13$ MeV and $\varepsilon_{\bar{\nu}_\alpha} = 15.5$ MeV, $L_{\nu_\alpha,\text{core}}^{\text{crit}}$ at low $M$ would be reduced by a factor of 2 and 3.5 ($f_{\text{red}} \approx 0.5$ and 0.3), respectively. The synchronization radius is rather insensitive to small changes in the neutrino energies, because $\mu(r) \approx r^{-4}$ (eq. 3) and thus any changes by a constant factor in equation (1) have a small effect on $r_{\text{sync}}$.

In order to get a significant decrease of $L_{\nu_\alpha,\text{core}}^{\text{crit}}$ due to Cr-O, the mass accretion rate needs to be sufficiently low while $r_\nu$ remains large. This is best achieved in the lowest mass progenitors, in which $M$ decreases very rapidly due to their steep density structure. However, for times $\lesssim 0.65$ s after the collapse is initiated, the mass accretion rate is likely still too high to cause a Cr-O-driven explosion in an 11.2 $M_\odot$ progenitor (see also Chakraborty et al. 2011; Dasgupta et al. 2011). At late times, $M$ decreases rapidly, but the effect of Cr-O on $L_{\nu_\alpha,\text{core}}^{\text{crit}}$ is likely offset by the simultaneous drop of $r_\nu$, as the PNS cools and the concomitant increase in $M$. If the decrease in $r_\nu$ is slow enough, perhaps due to a stiff equation of state, $M$ might drop enough so that the decrease in $L_{\nu_\alpha,\text{core}}^{\text{crit}}$ due to Cr-O becomes important before black hole formation or explosion via the ordinary neutrino mechanism.

Finally, we note that our implementation of Cr-O is quite favorable for having an effect on $L_{\nu_\alpha,\text{core}}^{\text{crit}}$ because we assume that the flavor conversion and the increase in the electron neutrino energy occurs for all neutrinos. The flavor conversion may not be complete for various reasons. In particular, refinements in the treatment of the neutrino radiation transport (multi-group and multi-angle) typically make the effect of Cr-O smaller, because physical scale of conversion is moved to larger radii (Esteban-Pretel et al. 2008; Chakraborty et al. 2011).

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