Non-perturbative effects in the transverse momentum distribution of electroweak bosons at the LHC∗

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The transverse momentum of electroweak bosons in a Drell-Yan process is an important quantity for the experimental program at the LHC. The new model of non-perturbative gluon emission in an initial state parton shower presented in this note gives a good description of this quantity for the data taken in previous experiments over a wide range of CM energy. The model’s prediction for the transverse momentum distribution of $Z$ bosons for the LHC is presented and used for a comparison with other approaches.

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1. Introduction

The Drell-Yan process has been widely studied in many past [1] and present [2, 3] hadron collider experiments and played a significant role in

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the development of our understanding of QCD and electroweak (EW) interactions, both from the experimental and theoretical point of view. Certainly this will also be the case for the LHC experiments, especially because it will soon become the unique $W$ and $Z$-boson production factory which is expected to collect 300 million $W$ and 20 million $Z$ events per year of its operation at energies $\sqrt{s} = 14$ TeV and the luminosity of $10^{33} \, cm^{-2} s^{-1}$.

Among various distributions of $W$ and $Z$ observables, the transverse momentum spectrum of vector bosons in a Drell-Yan process is a very useful and important quantity for the experimental program at the LHC. In the case of $W$ production, the uncertainty in the shape of the spectrum directly affects the measurement of the $W$ mass \cite{4} and its mass charge asymmetry $M_{W^+} - M_{W^-}$ \cite{5}. It also helps to understand the signature for Higgs boson production at either Tevatron or LHC \cite{6}. Although the experiments measure the $Z$ transverse momentum distribution and use this to infer that of the $W$ boson, the extent to which the effects are non-universal limits the ultimate accuracy of the measurement, unless elaborate tricks, as proposed in Ref. \cite{7} are used. For these reasons, it is of utmost importance to predict the $W$ and $Z$ observables with as high as possible theoretical precision. The sources of uncertainty in the theoretical predictions of observables, such as the transverse momentum of electroweak bosons discussed here, are of perturbative and non-perturbative origin. In this short note we will concentrate on the modelling of the latter in the framework of a backward evolution parton shower approach \cite{8} which is widely used in general purpose Monte Carlo Generators such as Herwig \cite{9}, Pythia \cite{10} or Sherpa \cite{11}. During the parton shower evolution, which terminates at some scale of typical hadron mass, the recoil from the emitted gluons builds up a transverse momentum for the $W/Z$. In order to fit existing data, the conventional backward evolution parton shower approach needs to be supplemented by the so-called intrinsic (or ‘primordial’) transverse momentum $k_T$ distribution of partons initiating the shower. The physical motivation behind this additional non-perturbative ingredient is the Fermi motion of partons within a hadron. Therefore, its average value per parton can be estimated based solely on the proton size and uncertainty principle to be of the order of $0.3 - 0.5$ GeV. But the values extracted from data first of all are too large and secondly grow with collision energy which cannot be explained by Fermi motion. For example, in Herwig++ its value grows from $k_T = 0.9$ GeV, which is needed to describe the data taken at the energy $\sqrt{s} = 62$ GeV (experiment R209), to 2.1 GeV which, is needed at the Tevatron energies ($\sqrt{s} = 1800$ GeV). This motivated us to propose a model for backward evolution in which an additional non-perturbative component at low transverse momentum pro-

\footnote{Together with other backward-evolution steps, such as an incoming sea-quark being evolved back to an incoming gluon by emitting a corresponding antiquark.}
vides additional smearing at each step of the evolution. By construction we expect more non-perturbative smearing for longer parton shower evolution ladder which might cure the problem of dependence on centre of mass energy as well as on the size of needed intrinsic smearing, which in our studies is kept, according to the Fermi motion argumentation fixed at 0.4 GeV.

In the following sections we will first briefly describe the model, then present how it fits the existing data sets, and at the end of this note we will demonstrate the model’s predictions for the LHC energies which we use for a comparison with other approaches.

2. Model

The implementation of transverse momentum production in which non-perturbative smearing takes place throughout the perturbative evolution, was achieved by a simple modification to an initial-state parton shower algorithm. The model was implemented in the framework of Herwig++ [12] in which the Sudakov form factor for backward evolution from some scale \( \tilde{q}_{\text{max}} \) down to \( \tilde{q} \) takes the form

\[
\Delta(\tilde{q}; p_{\perp\text{max}}, p_{\perp0}) = \exp\left\{ -\int_{\tilde{q}^2}^{\tilde{q}_{\text{max}}^2} \frac{d\tilde{q}^2}{\tilde{q}^2} \int_{z_0}^{z_1} dz \frac{\alpha_s(p_{\perp}) x' f_b(x', \tilde{q}^2)}{2\pi} x f_a(x, \tilde{q}^2) P_{ba}(z, \tilde{q}^2) \right\},
\]

with \( x' = x/z \), for further details cf. Ref. [13].

The argument of the strong coupling \( \alpha_s \) in Eq. (1) is the transverse momentum \( p_{\perp} \) of an emission\(^2\). The cut-off scale represented by \( p_{\perp0} \) is needed to avoid divergence of the strong coupling. Below the cut-off scale \( \alpha_s \) is equal to zero and consequently the derivative of the Sudakov form factor is equal to zero which translates to zero probability of the gluon emission below \( p_{\perp0} \). Therefore, the two arguments of the Sudakov form factor, \( p_{\perp\text{max}} \) and \( p_{\perp0} \), are not the evolution variables but only explicitly specify the available phase-space of an emission.

In order to populate the phase-space below \( p_{\perp0} \) by additional non-perturbative emissions we introduce the additional Sudakov form factor \( \Delta_{NP} \) such that

\[
\Delta(\tilde{q}; p_{\perp\text{max}}, 0) = \Delta_{\text{pert}}(\tilde{q}; p_{\perp\text{max}}, p_{\perp0}) \Delta_{NP}(\tilde{q}; p_{\perp0}, 0)
\]

\(^2\)In Herwig++, the argument of \( \alpha_s \) is a slightly simplified expression, equal to the transverse momentum to the required accuracy, but not exactly. We have tested the implementation of our model with this simplified expression and the exact expression for transverse momentum, and find very similar results. We therefore use the default expression.
We achieve this by extending $\alpha_s(p_{\perp})$ into the non-perturbative region using the following model

$$\alpha_s(p_{\perp}) = \begin{cases} \varphi(p_{\perp}), & p_{\perp} < p_{\perp 0}, \\ \alpha_s^{(\text{pert})}(p_{\perp}), & p_{\perp} \geq p_{\perp 0}. \end{cases} \quad (3)$$

In order to explore the possibility of a reasonable description of experimental data, we have studied in a greater detail two simple choices of the non–perturbative function $\varphi(p_{\perp})$: flat continuation of $\alpha_s(p_{\perp} < p_{\perp 0})$, with a constant value $\varphi_0$, $\alpha_s(p_{\perp} < p_{\perp 0}) = \varphi_0$ and a quadratic interpolation between the two values $\alpha_s(p_{\perp 0})$ and $\varphi_0 = \varphi(0)$:

$$\alpha_s(p_{\perp} < p_{\perp 0}) = \varphi_0 + (\alpha_s(p_{\perp 0}) - \varphi_0)p_{\perp}^2/p_{\perp 0}^2.$$ 

In both cases our model is determined by two free parameters $p_{\perp 0}$ and $\varphi_0$.

3. Data sets and fitting results

In this section we present some new results of the model which were obtained after important improvements of Herwig++’s parton shower, released with version 2.3.1 of the program. The main change in the program was a fix for a wrongly applied PDF veto in the parton shower $\bar{q} \to \bar{q} g$ splittings which, by construction of our model could have influence on previously presented results [14]. Therefore, we have repeated the procedure described in detail in [14] and have fitted the two parameters of our model to the Drell-Yan data from three experiments: the fixed target $p$–Cu Fermilab E605 [15] $\sqrt{s} = 38.8$ GeV, CERN ISR $p$–$p$ collisions experiment R209 [16] at $\sqrt{s} = 62$ GeV and CDF Tevatron Run I experiment with energies at $\sqrt{s} = 1800$ GeV [2]. These experiments cover the whole spectrum of centre mass energy for the Drell-Yan process data sets which are interesting for our studies.

3.1. Parton-level study

In the case of purely parton-level shower, with all the light-quark and gluon effective masses and cutoffs set to zero\(^4\), with our model for the low-scale $\alpha_s$ as the only non-perturbative input the fitting procedure gave the optimal value for the quadratic extrapolation with $\alpha_s(0) = 0.0$ and $p_{\perp 0} = 3$.

\(^3\) There are more data available but all at even lower CM energies.

\(^4\) For technical reasons, it is not possible to set them exactly to zero. However, we have confirmed that if they are small enough their precise values become irrelevant and have very little effect on the results.
Fig. 1. The comparisons of the parton-level results from the non–perturbative model with the data from E605 with $\chi^2/\text{bin} = 0.88$ (left), R209 $\chi^2/\text{bin} = 0.76$ (middle) and CDF $\chi^2/\text{bin} = 1.0$ (right). The Monte Carlo results come from our parameter set with $\varphi_0 = 0.0$, $p_{\perp 0} = 0.70 \text{GeV}$.

0.7. The resulting low-$p_T$ distributions for the new values are presented on top of the data sets in Fig. 1. The $\chi^2/\text{bin}$ values are a little higher than before the parton shower improvements, nevertheless the agreement with data remains at a high-level; $\chi^2$ for all the experimental data sets are below or equal one. If we are only interested in the $W/Z$ transverse momentum distribution, it is enough to use a parton-level study, however, if one needs to simulate fully exclusive events then a hadronization model has to be used.

### 3.2. Hadron-level results

The hadronization model used in Herwig++ requires termination of the shower using non-perturbative effective parton masses tuned to $e^+e^-$ data. Therefore, we performed the same analysis as above but this time with restored tuned effective parton masses. In this case the best and most stable situation was found for $\alpha_s(0) = 4$ and $p_{\perp 0} = 2.5 \text{GeV}$, giving the $\chi^2$ per degree of freedom of 0.80 for CDF, 0.66 for R209 and the worst 8.6 for E605. We should stress that the used parameter set may not be the optimal choice for each experiment or CM energy but rather the best compromise between the three experiments. As the fixed target data in our analysis do not even include the systematic errors quoted to be around 5–10%, we have deliberately put a bit more emphasis on the Tevatron results.
Fig. 2. Comparison of the hadron level results from the non–perturbative model with data from E605 with $\chi^2/bin = 8.6$ (left), R209 $\chi^2/bin = 0.66$ (middle) and CDF $\chi^2/bin = 0.80$ (right). The Monte Carlo results are from our parameter set with $\varphi_0 = 4.0, p_{\perp 0} = 2.5$ GeV.

3.3. Remarks

The first remark is that the new parameter choices for both the parton and hadron-level models, are not very different from the ones obtained using the old version of Herwig++. Before parton shower improvements our best choices were, for the parton-level mode: $\alpha_s(0) = 0.0, p_{\perp 0} = 0.75$ GeV, and for the hadron-level case: $\alpha_s(0) = 3$ and $p_{\perp 0} = 3.0$ GeV [14].

We have also checked how the results depend on the intrinsic momentum $k_{\perp}$ by varying its value with $\delta k_{\perp} = \pm 0.1$ GeV around our fixed value $k_{\perp} = 0.4$, which is in the range permitted by the Fermi motion. We have repeated the fitting procedure and observed that for both intrinsic momenta, $k_{\perp \pm} = k_{\perp} + \delta k_{\perp}$, we are able to find a pair of parameters for which our model gives equally good description of data sets as for the central value of $k_{\perp} = 0.4$. Moreover, we have observed that the value of $\alpha_s(0)$ parameter for all studied intrinsic momenta remains the same but the $p_{\perp 0}$ value is shifted for a bigger intrinsic momentum to a higher scale and for a smaller one to a lower scale. Therefore, by changing the intrinsic momentum from 0.4 to 0.5 GeV we can obtain exactly the same best model’s parameters set as in [14] and the same shape of $\alpha_s$ as presented in Figure 4 from [14]. In that case the comparison of the shape of $\alpha_s$ in the non–perturbative region of the parton-level study are in good agreement with other approaches to modelling non–perturbative corrections to inclusive observables with a modified coupling in the soft region [17] [18].

The last remark is that using our model as the only non-perturbative
ingredient in the simulation, i.e. removing the non-perturbative constituent parton masses that usually cut off the parton shower in Herwig++, gives a somewhat better description of the data. This lays open the speculation that perhaps, in some way, the two approaches could be combined. One could for example use our model for the initial-state radiation, and the usual model, tuned to describe the final states of $e^+e^-$ annihilation, for final-state radiation.

4. Predictions for LHC and comparison with other approaches

At the end of this note we would like to compare the results for a transverse momentum distribution of the $Z$ boson at the LHC energies using the nonperturbative gluon emission model and two other approaches: ResBos [19] and the Gaussian intrinsic $k_\perp$ extrapolation. But first let us compare our prediction of the parton level, marked as the filled histogram in Fig. 3, and of the hadron level, dot–dashed blue line. Both histograms, as expected, give a consistent extrapolation.

The result from ResBos in Fig. 3 (solid, black) shows a slightly different behaviour from our predictions. We predict a slightly more prominent peak...
and a stronger suppression towards larger transverse momenta. Both computations match the data well at large transverse momenta as they rely on the same hard matrix element contribution for a single hard gluon emission. Let us stress the remarkable feature that we predict the same peak position with these models which is very important from the experimental point of view. This feature is quite understandable as both models are built on the same footing: extra emissions of soft gluons. A comparison of ResBos to data from experiments at various energies including the experiments E605 and R209 was done in [20].

Furthermore, we see the Herwig++ result from only using intrinsic $\langle k_\perp \rangle = 5.7 \text{ GeV}$ (dashed, red) as recommended in [12]. This large value stems from an extrapolation from lower energy data with the assumption that the average $k_\perp$ will depend linearly on $\ln(M/\sqrt{s})$. The peak is seen to lie at a considerably higher value of the transverse momentum. It would clearly be of interest to have experimental data to distinguish these two models of non-perturbative transverse momentum.

### 5. Conclusion

We consider the model based on soft-gluon radiation, much like the resummation program ResBos, to have a more meaningful physics input than simply extrapolating the Gaussian smearing of a primordial transverse momentum. The model implemented in the improved parton shower of Herwig++ (release 2.3.1) gives a good (and very similar to the older version of Herwig++) description of data. On the other hand, the fitting procedure shows that the best values for the model’s parameters are slightly different than the previous ones.

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5 This recommendation has been changed and in the latest version of Herwig++ its new value $\langle k_\perp \rangle = 2.2 \text{ GeV}$ is adjusted to be in agreement with the prediction of the non-perturbative model presented in Fig. 3.
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