On Vertex Operators in Effective String Theory

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Abstract

In this note we construct vertex operators in effective string theory using the simplified covariant formalism, i.e. by embedding it in the Polyakov formalism supplemented by an anomaly term, and fixing to conformal gauge. These vertex operators represent off-shell background fields rather than dynamical string states. We construct vertex operators for nontrivial scalar, electromagnetic, and gravitational backgrounds. As an application, we compute a scalar form factor of a long string with length $R$, where the Fourier momentum $q$ of the external scalar field satisfies $q^2 \ll 1/\alpha'$, and we find the expected logarithmic dependence on the size of the string.

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1 Introduction

Relativistic string-like objects exist in many field theories such as QCD in which confining strings are stable in the large-$N$ limit. The dynamics of such relativistic strings at low energies is described by a two-dimensional effective field theory on the string worldsheet, which is called effective string theory. Effective string theory can be formulated in a manifestly Poincaré-invariant way [1], and can be further simplified by embedding it into the Polyakov formalism [2, 3], in which regularization and renormalization of ultraviolet divergences as well as the classification of gauge-invariant operators are much simplified. Various physical observables in effective string theory have been computed, including the spectrum of a static long string [1, 4, 5], the mass of a rotating string with large angular momentum [6], the worldsheet $S$-matrix [7, 8] and so on. Boundary operators in the covariant formalism of effective string theory have been classified in [9].

In this article we introduce off-shell vertex operators in effective string theory in the simplified covariant formalism. They are, as in fundamental string theory, crucial ingredients for the description of string interactions. In contrast to fundamental string theory, the vertex operators we construct correspond to external background fields rather than states of the string theory itself. Once we construct off-shell vertex operators, we have access to many interesting dynamical quantities such as form factors and structure functions, which are relevant for hadron dynamics in planar QCD.

Vertex operators in effective string theory are in general of the form

$$V[q] = \sum_i \mathcal{O}_i[q, X] e^{iqX},$$

(1.1)

where each $\mathcal{O}_i[q, X]$ has a definite $X$-scaling dimension\(^1\) under scaling of the target space coordinates $X^\mu$. For fixed Fourier momentum $q$, the spectrum of $X$-scaling dimensions of the set $\{\mathcal{O}_i[q, X]\}_i$ is always bounded above and continues downwards discretely towards $-\infty$. We would like to construct such vertex operators order by order in $X$-scaling dimension. The use of the Polyakov formalism with conformal gauge fixing is essential here; other gauges such as static gauge [10–14] are best adapted to study a static situation like a wound string or string stretched between quarks. Even in those situations, however, it is hard to construct vertex operators with the right covariance properties in a noncovariant gauge, particularly at the quantum level.

In constructing our vertices, regularization and renormalization are no problem – they can be done completely covariantly in the simplified formalism. In the simplified covariant formalism, the construction of vertex operators order by order is straightforward, particularly up to next-to-leading order in $\alpha'$. Up to and including NLO, we can safely ignore corrections coming from subleading terms in the effective string action and we are allowed to use the

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\(^1\) Here we are not discussing conformal dimension (or weight) under the Virasoro symmetry of the Polyakov formalism in conformal gauge. Unintegrated vertex operators in conformal gauge are always Virasoro primary operators of weight $(1, 1)$. 
free propagator and the free stress tensor, as was explained in [3] and which we shall review in the beginning of section 2. Using this formalism we shall construct various off-shell vertex operators for photon, scalar ("tachyon") and graviton perturbations up to and including NLO.

There are two sorts of quantum corrections. The first is to the properties of a single vertex operator in isolation, and all such corrections are suppressed by powers of \( \alpha'/|X|^2 \) and \( q\alpha'/|X| \). We will study these corrections in section 2. Such corrections to the form of a vertex operator can affect, for instance, form factors of a string with physical length \( R \), giving subleading terms in the large-\( R \) expansion. Here the physical size \( R \) can correspond to the length of a compact direction along which the string is wound; the distance between infinitely heavy static quarks between which the string is stretched; or to the physical length \( R \sim \sqrt{J\alpha'} \) of a rotating string with free endpoints.

The second sort of correction survives even in the leading term of the \( R \to \infty \) limit, and is proportional to \( q^2\alpha' \). Such corrections appear in correlation functions of vertex operators in an infinitely long string. We will see in section 3.1 that a controlled and calculable regime for off-shell-fields with nonzero momentum is given by \( |q| \lesssim \alpha'^{-1/2} \), a hierarchy of scales that allows us to apply the effective string framework. For photons or other external probes with energy lower than the QCD scale, the off-shell field is be unable to probe any of the short distance structure of QCD, such as quarks and gluons, and the effective string framework can be applied.

This article is organized as follows. In section 2 we construct vertex operators for various off-shell fields up to and including next-to-leading order. In section 3 we discuss the dependence of observables on the external momentum and compute a scalar form factor as a simple example. Our conclusions are presented in section 4.

## 2 Vertex operators for off-shell fields

### 2.1 The Polyakov formalism of effective string theory

We first briefly introduce the Polyakov formalism of effective string theory [3]. The notations described in [3] are used throughout the paper. The Polyakov formalism begins with the following partition function,

\[
Z = \int DX D\ge^{-S_{\text{Polyakov}} - S_{\text{PS}} - \cdots}.
\]  

(2.1)

Here, \( S_{\text{Polyakov}} \) is the Polyakov action [15],

\[
S_{\text{Polyakov}} := \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{|g|} g^{ab} \partial_a X^\mu \partial_b X_\mu,
\]  

(2.2)

and \( S_{\text{PS}} \) is the so-called Polchinski–Strominger (PS) term [1, 3],

\[
S_{\text{PS}} := \frac{26-D}{24\pi} \int d^2\sigma \sqrt{|g|} (g^{ab} \partial_a \varphi \partial_b \varphi - \varphi R^{(2)}), \quad \varphi := -\frac{1}{2} \log (g^{ab} \partial_a X^\mu \partial_b X_\mu).
\]  

(2.3)
The anomalous transformation of the path integral measure in (2.1) under a Weyl transformation is correctly cancelled by the classical transformation of $S_{PS}$, so that $\text{diff} \times \text{Weyl}$ invariance is maintained at the quantum level. The "○ ○ ○" in (2.1) stands for nonuniversal, $\text{diff} \times \text{Weyl}$-invariant operators with smaller $|X|$-scaling dimensions, which can be classified using the covariant calculus developed in [3] (see also [2]). The induced-curvature-squared term and the extrinsic-curvature-quartic term [16] are examples with the largest $|X|$-scaling, going as order $|X|^{-4}$ relative to the Polyakov / Nambu–Goto action.

We now review a point introduced in [3] on the operator product expansion of the stress tensor with $X$ and composites made from $X$, at next-to-leading order. Both the Lagrangian and the stress tensor receive corrections at NLO in the large-$|X|$ expansion, which is to say at relative order $\alpha'/|X|^2$, with coefficient $\beta$, which is proportional to the central charge deficit $D-26$. The two corrections conspire with each other in such a way that the OPE of the stress tensor with operators made from $X$ and its derivatives, is the same at NLO as it would be in the case $\beta = 0$. That is to say, the OPE of $T$ with $O[X]$, is just given by a sum of one and two free Wick contractions, modulo terms of relative order $|X|^{-4}$ and smaller. Therefore the conformal properties at NLO can be calculated in free field theory. This is a result of a Weyl-covariant path integral formulation of the theory in which the $X$-coordinates transform trivially\(^2\) under the Weyl symmetry.

This implies that when constructing off-shell vertex operators, we do not need to care about corrections associated with the PS term at the first subleading order in $|X|$. However, it is still a nontrivial problem: because of the double contractions involving the free stress tensor, naïve off-shell vertex operators one may write down are not primary even at relative $|X|^{-1}$ order, and therefore they must be modified by operators with lower $X$-scalings. Let us consider how the off-shell vertex operators are made primary at relative $|X|^{-1}$ order.

### 2.2 Off-shell Maxwell field

Suppose we have some worldsheet current $J^a$, which couples to a spacetime gauge field $A_\mu$. The simplest way to derive the coupling, including the normalization at zero momentum, is to work first in real space rather than Fourier space. The spacetime gauge field couples to the worldsheet through the pullback to an induced worldsheet gauge field

\[
A^\text{[ws]}_a = A_\mu \partial_a X^\mu, \tag{2.4}
\]

which has the induced gauge transformation

\[
\delta A^\text{[ws]}_a = (\partial_\mu \chi) \partial_a X^\mu = \partial_a \chi. \tag{2.5}
\]

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\(^2\) This is only in the simplified covariant formalism [3], but not the case in the old PS covariant formalism [1]. There is a nontrivial change of variables between the $X$ coordinates in the two papers, given in [3].

\(^3\) This may be extended to higher orders if a Weyl-invariant regularization and renormalization scheme is chosen with $X$ transforming trivially under the Weyl symmetry. We thank S. Dubovsky and I. Swanson for discussions on this point.
The low-energy coupling to the worldsheet is then
\[ S_{\text{photon}}^{\text{[ws]}} = \int d^2w A^a_{\text{[ws]}} J^a, \quad (2.6) \]
with unit coefficient. The unit coefficient of the low-energy coupling is forced by gauge-invariance under large gauge transformations. However this only controls the \( q \to 0 \) limit of the gauge vertex; away from zero momentum there is a \( q \)-dependence to the vertex that may differ from the naïve coupling.

If we decompose \( A_\mu \) in Fourier modes,
\[ A_\mu = \int d^Dq e_\mu(q) e^{iqX}, \quad (2.7) \]
then the naïve coupling to the worldsheet is of the form
\[ S_{\text{photon}}^{\text{[ws]}} = \int d^Dq e_\mu(q) \int d^2w e^{iqX} J^a \partial_a X, \quad (2.8) \]
which we can express as
\[ S_{\text{photon}}^{\text{[ws]}} = \int d^Dq \int d^2w V_{\text{classical photon}}^{\text{classical}}[e, q] J^a, \quad (2.9) \]
where
\[ V_{\text{classical photon}}^{\text{classical}}[e, q] \equiv e_\mu(q) \partial_a X^\mu e^{iqX}. \quad (2.10) \]
Working in unit gauge and using the notation \( V \equiv V_z \) and \( \tilde{V} \equiv V_{\bar{z}} \), it is simplest to work with one chirality at a time; these couple to the holomorphic current \( J^z = 2J_{\bar{z}} \equiv \tilde{J} \) and antiholomorphic current \( J^\bar{z} = 2J_z \equiv \bar{J} \), respectively.

For now we shall just work with the vertex operator \( V \) which couples to the antiholomorphic current \( \tilde{J} \):
\[ V_{\text{photon}}^{\text{classical}}[e, q] \equiv e_\mu e^{iqX} \partial X^\mu. \quad (2.11) \]
Since we will henceforth work only with one Fourier mode at a time, we can drop the argument \( q \) of the polarization vector.

For \( q^2 \) sufficiently small in units of \( \alpha'^{-1} \), worldsheet quantum effects can be treated as vanishingly small and this classical approximation is a good approximation to the quantum vertex operator. However, the expression (2.10) is not precisely correct for an off-shell photon even if \( q^2 \alpha' \) is small, so long as it is treated as nonzero and one wants to keep track of subleading effects in the \( q^2 \alpha' \) expansion. Quantum fluctuations alter the properties of the vertex operator, and the time-ordered exponential \( e^{iqX} \) is singular, and must be normal-ordered. So we are led to try the naïve normal-ordered vertex operator
\[ V_{\text{photon}}^{\text{photon}}[e, q] \equiv e_\mu :e^{iqX} \partial X^\mu:. \quad (2.12) \]
If we normal-order the operator, then it acquires an anomalous dimension and the integrated vertex is no longer invariant under the residual conformal symmetry.

The vertex operator can be made gauge-invariant in a simple and canonical way within the effective string framework, however. As explained in [3], the Weyl invariance of the Polyakov formalism is strongly spontaneously broken for large strings, by the expectation value of the operator \( I_{11} := \partial X^\mu \bar{\partial} X_\mu \). The logarithm of \( I_{11} \) functions as a Liouville field, whose gradient-squared is the PS anomaly-cancelling term [1].

The slightly less naïve photon vertex operator is therefore obtained by dressing the naïve vertex to conformality using an exponential of the effective Liouville field. We obtain a vertex operator of the form

\[
V_{\text{photon}}^\mu[e, q] \sim e^\mu e^{iqX} \partial X^\mu I_{11}^{-\frac{\alpha'}{4}},
\]

where the expression is understood to be normal-ordered. This has the right conformal weight but it is neither gauge-invariant nor primary. We may add a subleading term,

\[
V_{\text{photon}}^\mu[e, q] \sim V_{\text{photon}}^0[e, q] := e^\mu e^{iqX} \left( \partial X^\mu + i\frac{\alpha'}{4} q^\mu I_{11}^{-\frac{\alpha'}{4}} \right) I_{11}^{-\frac{\alpha'}{4}},
\]

where in general we define the invariants \( I_{pq} \equiv \partial^p X \cdot \bar{\partial}^q X \). For a longitudinal photon polarization \( e^\mu = q^\mu \) the vertex (2.14) becomes a total derivative,

\[
V_{\text{photon}}^0[q, q] = \partial \left( -ie^{iqX} I_{11}^{-\frac{\alpha'}{4}} \right).
\]

However, \( V_{\text{photon}}^0[e, q] \) still fails to be primary at the \( |X|^{-\frac{\alpha'}{2}} e^{iqX} \) order. The OPE of \( V_{\text{photon}}^0[e, q] \) with the free stress tensor \( T_{\text{free}} \) is given by

\[
T_{\text{free}}(w_1) \cdot V_{\text{photon}}^0[e, q](w_2) \sim \frac{1}{w_{12}^3} V_{\text{photon}}^0[e, q](w_2) + \frac{1}{w_{12}} \partial V_{\text{photon}}^0[e, q](w_2) + \frac{i\alpha'^2 q^2}{8w_{12}^3} (e \cdot \partial X)(q \cdot \bar{\partial} X) I_{11}^{-\frac{\alpha'}{4}} e^{iqX}(w_2) + O\left( |X|^{-\frac{\alpha'}{2}} e^{iqX} \right).
\]

The \( w_{12}^{-3} \) term can be canceled away by adding suitable terms to \( V_0[e, q] \). To obtain a vertex operator with the right conformal weight, we write

\[
V_{\text{photon}}^\mu[e, q] = V_{\text{photon}}^0[e, q] + \frac{\alpha'^2 q^2}{16} e^{iqX} I_{11}^{-\frac{\alpha'}{4}} \left( e \cdot \partial X \right) \left[ I_{21}(q \cdot \bar{\partial} X) + I_{12}(q \cdot \partial X) \right].
\]
For $e_\mu = q_\mu$ this is indeed a total derivative modulo terms of order $|X|^{-\frac{\alpha'}{4}q_\mu^2 - 1}e^{iqX}$,

$$V_{\text{photon}}^{[q,q]} = \partial \left[ -ie^{iqX} \mathcal{I}_{11}^{-\frac{\alpha'}{4}q_\mu^2} \mathcal{I}_{11}^{-\frac{\alpha'}{4}q_\mu^2 - 2} \mathcal{I}_{12} \partial e^{iqX} + \mathcal{I}_{12} \partial e^{iqX} \right] + O\left(|X|^{-\frac{\alpha'}{4}q_\mu^2 + 1}e^{iqX}\right).$$

(2.18)

Now it is clear that $V_{\text{photon}}^{[e,q]}$ satisfies the following OPE with the full stress tensor $T$,

$$T(w_1) \cdot V_{\text{photon}}^{[e,q]}(w_2) \sim \frac{1}{w_{12}^2} V_{\text{photon}}^{[e,q]}(w_2) + \frac{1}{w_{12}} \partial V_{\text{photon}}^{[e,q]}(w_2) + O\left(|X|^{-\frac{\alpha'}{4}q_\mu^2 + 1}e^{iqX}\right),$$

(2.19)

so that $\tilde{V} V_{\text{photon}}^{[e,q]}$ is indeed primary of correct weight $(1,1)$, modulo terms of order $|X|^{-\frac{\alpha'}{4}q_\mu^2 - 1}e^{iqX}$.

In the limit $R \to \infty$, note that all corrections to the form of the vertex operator vanish. This will not be the case for corrections to correlators of vertex operators. These correlators will have corrections proportional to $q_\mu^2 \alpha'$ even in the $R \to \infty$ limit, and we shall estimate those in later sections.

### 2.3 Off-shell tachyon field

We can also consider vertex operators whose undressed form is simply a normal-ordered exponential $e^{iqX}$. In the fundamental bosonic string this is called the "tachyon", since it corresponds via the state-operator correspondence to a state with negative mass squared. In the effective string, there is no state-operator correspondence, and the "tachyon" vertex operator need not correspond to a dynamical field at all. So, the "tachyon" vertex operator is a misnomer in this context, but we retain the term because the form of the undressed vertex operator is familiar to string theorists from the context of the fundamental bosonic string in the critical dimension.

One may write down naïvely an off-shell tachyon vertex operator as

$$V_{\text{tachyon}}^{[0,q]} = e^{iqX} \mathcal{I}_{11}^{-\frac{\alpha'}{4}q_\mu^2 + 1},$$

(2.20)

which has classically weight $(1,1)$ for any momentum $q_\mu$. However, when $q_\mu$ is off-shell, i.e. $q^2 \neq -4/\alpha'$, $V_{\text{tachyon}}^{[0,q]}$ is not primary at order $|X|^{-\frac{\alpha'}{4}q_\mu^2 + 1}e^{iqX}$. To see this explicitly, we compute the free OPE of $V_{\text{tachyon}}^{[0,q]}$ with the free stress tensor $T_{\text{free}}$,

$$T_{\text{free}}(w_1) \cdot V_{\text{tachyon}}^{[0,q]}(w_2) \sim \frac{1}{w_{12}^2} V_{\text{tachyon}}^{[0,q]}(w_2) + \frac{1}{w_{12}} \partial V_{\text{tachyon}}^{[0,q]}(w_2) + \frac{\alpha'}{2w_{12}^2} \left( \frac{\alpha'q_\mu^2}{4} - 1 \right) \mathcal{I}_{11}^{-\frac{\alpha'}{4}q_\mu^2} \partial e^{iqX} + O\left(|X|^{-\frac{\alpha'}{4}q_\mu^2}e^{iqX}\right).$$

(2.21)
We clearly see that $V^{\text{tachyon}}_0[q]$ fails to be primary because of the third term in the RHS of (2.21), which is of order $|X|^{-\frac{\alpha' q^2}{2}+\epsilon} e^{iqX}$. To cancel this term, one adds a subleading term to $V^{\text{tachyon}}_0[q]$,

$$V^{\text{tachyon}}[q] := V^{\text{tachyon}}_0[q] + V^{\text{tachyon}}_1[q],$$

where

$$V^{\text{tachyon}}_1[q] := -\frac{\alpha'}{4} \left( \frac{\alpha' q^2}{4} - 1 \right) T_{11}^{-\frac{\alpha' q^2}{4}-1} (T_{21} \partial e^{iqX} + T_{12} \partial e^{iqX}).$$

Then, $V^{\text{tachyon}}[q]$ is primary of weight $(1, 1)$ up to and including the $|X|^{-\frac{\alpha' q^2}{2}+\epsilon} e^{iqX}$ order. That is, the OPE of $V^{\text{tachyon}}[q]$ with the (full) stress tensor $T$ is that of a primary operator:

$$T(w_1) \cdot V^{\text{tachyon}}[q](w_2) \sim \frac{1}{w_{12}^2} V^{\text{tachyon}}[q] + \frac{1}{w_{12}} \partial V^{\text{tachyon}}[q](w_2) + O \left(|X|^{-\frac{\alpha' q^2}{2}} e^{iqX}\right).$$

In the effective string, the physical meaning of the tachyon vertex operator is simply any scalar background field that couples to the string worldsheet, in such a way that at vanishing momentum it is equivalent to adding $T_{11}$ to the worldsheet action, i.e., a change in the string tension. The simplest background field in planar Yang–Mills theory that would couple as the tachyon, then, would be a position-dependent gauge coupling defined at a fixed renormalization scale in the far ultraviolet, with wave-vector $q^\mu$. That is, if we take

$$g_{\text{YM}}^2(\mu) \equiv \left(1 + \epsilon e^{iqX}\right) (g_{\text{YM}}^2(\mu))^{\text{average}},$$

for sufficiently small $\epsilon$, then the dynamical scale $\Lambda_{\text{YM}}$ and the string tension $(2\pi \alpha')^{-1}$ will also be approximately constant, with small spatially varying perturbations proportional to $\epsilon e^{iqX}$, with coefficients of proportionality that depend on the details of the renormalization group flow from the asymptotically free regime to the confining regime.

### 2.4 Off-shell metric

Based on the discussion in section 2.2, let us begin with the following vertex operator for the off-shell graviton,

$$V^{\text{graviton}}_0[e, q] := e_{\mu\nu} \left( \partial X^\mu + i \frac{\alpha'}{4} q^\mu \frac{T_{21}}{T_{11}} \right) \left( \partial X^\nu + i \frac{\alpha'}{4} q^\nu \frac{T_{12}}{T_{11}} \right) e^{iqX} T_{11}^{-\frac{\alpha' q^2}{2}}.$$  

However, this fails to be transverse, which would be inconsistent with the principle of general relativity. Indeed, for $e_{\mu\nu} = q_\mu \xi_\nu + \xi_\mu q_\nu$, we get

$$V^{\text{graviton}}_0[q_\mu \xi_\nu + \xi_\mu q_\nu, q] = \partial \left[ \xi_\mu \left( \partial X^\mu + i \frac{\alpha'}{4} q^\mu \frac{T_{21}}{T_{11}} \right) e^{iqX} T_{11}^{-\frac{\alpha' q^2}{2}} \right] + \bar{\partial} \left[ \xi_\mu \left( \partial X^\mu + i \frac{\alpha'}{4} q^\mu \frac{T_{21}}{T_{11}} \right) e^{iqX} T_{11}^{-\frac{\alpha' q^2}{2}} \right]$$

$$- \frac{\alpha'}{2} (q \cdot \xi) \left( \frac{T_{22}}{T_{11}} - \frac{T_{21} T_{12}}{T_{11}^2} \right) e^{iqX} T_{11}^{-\frac{\alpha' q^2}{2}} + O \left(|X|^{-\frac{\alpha' q^2}{2}} e^{iqX}\right).$$


Here we have used the leading order equation of motion \( \partial \bar{\partial} X^\mu = O(|X|^{-1}) \). Equation (2.27) suggests to add the following term to \( V_0^{\text{graviton}}[e, q] \),

\[
V_1^{\text{graviton}}[e, q] := \frac{\alpha'}{4} e_{\mu \nu} \eta^{\mu \nu} \left( \frac{I_{22}}{I_{11}} - \frac{I_{21} I_{12}}{I_{11}^2} \right) e^{i q X} T_{11}^{-\frac{\alpha'}{2} q^2},
\]

(2.28)

so that for \( e_{\mu \nu} = q_{\mu} \xi_{\nu} + \xi_{\mu} q_{\nu} \), the sum

\[
V^{\text{graviton}}[e, q] := V_0^{\text{graviton}}[e, q] + V_1^{\text{graviton}}[e, q]
\]

(2.29)

becomes a total derivative modulo terms of order \( |X|^{-\frac{\alpha'}{2} - 1} e^{i q X} \). The discussion given in section 2.2 immediately tells us that \( V^{\text{graviton}}[e, q] \) is primary of weight \((1, 1)\) modulo terms of order \(|X|^{-\frac{\alpha'}{2} - 1} e^{i q X} \), so \( V^{\text{graviton}}[e, q] \) has all the desired properties.

### 2.5 Scaling of higher-order corrections

There are necessarily higher-order corrections to the form of the vertex operators themselves, due to interaction terms in the string worldsheet action. Due to the lemma of section 2.1, the conformal properties of a composite operator only change at NNLO in \( \alpha' \), and only due to two or more Wick contractions of fields in the composite operator.

For instance, consider a vertex operator of the form

\[
V = \exp\{i q X\}(\circ \circ \circ),
\]

(2.30)

where the \((\circ \circ \circ)\) represent the \( I_{11} \) dressing, and other internal parts of the vertex operator apart from the normal-ordered exponential carrying the momentum. Quantum corrections to the conformal properties of \( V \) can come, for instance, from a double Wick contraction between \( \exp\{i q X\} \) and the order \( \beta |X|^0 \) part of the stress tensor. The latter contains terms \([1, 3] \) such as \( \beta I_{31}/I_{11} \), and its double Wick contraction contains nonholomorphic, and also purely holomorphic terms. The nonholomorphic terms in the OPE between the stress tensor and anything else, must cancel order by order in \(|X|\) by holomorphy of the stress tensor. The holomorphic terms need not cancel, and the OPE contains uncanceled terms such as

\[
T^{[\beta, X^0]} \exp\{i q X\}(\circ \circ \circ) \ni \frac{\beta \alpha'^2 (q \cdot \bar{\partial} X)^2}{z^4 T_{11}^2} \exp\{i q X\}(\circ \circ \circ),
\]

(2.31)

where the stress tensor and vertex operator are understood to be inserted at \( z \) and the origin, respectively.

In order to maintain the primary conformal transformation of the vertex operator, we must supplement it with terms to cancel the order \( z^{-4} \) singularity. We can do this straightforwardly by adding to the vertex operator a piece

\[
V^{[\text{NNLO}]} \ni \frac{\beta \alpha'^2 I_{31} (q \cdot \bar{\partial} X)^2}{T_{11}^3} \exp\{i q X\}(\circ \circ \circ),
\]

(2.32)
which is of order $\beta q^2 \alpha'/|X|^2$ relative to the leading-order vertex operator.

Aside from the anomaly term at order $|X|^0$ whose coefficient is fixed by quantum Weyl invariance, the worldsheet Lagrangian also contains operators with adjustable coefficients such as the induced-curvature-squared term

$$
\Delta L = C_{ICS} \alpha' \frac{\hat{T}_{22}}{\hat{T}_{11}},
$$

where $C_{ICS}$ is a theory-dependent numerical coefficient, and $\hat{T}_{22}$ is the conformally covariantized $T_{22}$ defined in [3],

$$
\hat{T}_{22} \equiv T_{22} - \frac{\hat{T}_{12} T_{21}}{T_{11}}.
$$

The coefficient $C_{ICS}$ should be thought of as roughly $C_{ICS} \sim \frac{1}{M_{UV}^2}$, where $M_{UV}^2 \lesssim 1/\alpha'$ is the square of the strong coupling energy scale on the string worldsheet, corresponding to the masses of degrees of freedom which have been integrated out.

This curvature squared term adds terms to the stress-energy tensor at NNLO, such as

$$
T^{[ICS]} \ni C_{ICS} \alpha' \frac{T_{31} T_{22}}{T_{11}^3}.
$$

By the same logic as we encountered in the discussion of the effect of the anomaly term, the induced-curvature-squared term requires corrections $(\delta V)^{[ICS]}$ to the form of the vertex operator proportional to

$$
(\delta V)^{[ICS]} \ni C_{ICS} \alpha' \frac{T_{31} T_{22}}{T_{11}^5} (q \cdot \bar{\partial} X)^2 \exp\{iqX\} \{\circ \circ \circ\},
$$

whose one-contraction OPE with the free stress tensor, cancels the $q^2 z^{-4}$ singularity of the interacting stress tensor with the leading-order vertex operator.

## 2.6 Independent higher-derivative operator components

In addition to higher-derivative corrections forced by conformal invariance, there may also be independent operator components of vertex operators. Since both $\alpha'/|X|^2$ and $q^2 \alpha'$ contributions are suppressed, at leading order at large $|X|$ and low $q^2$, the conformal properties of composite operators are just the classical ones.

For scalar vertex operators, where transversality is not an issue, any conformal primary of weight $(1,1)$ and spacetime momentum $0$ in the $X$ sector, corresponds in a canonical way at leading-order to a conformal primary of weight $(1,1)$ and spacetime momentum $q$, obtained by multiplying by the momentum dependence and dressing factors, $e^{iqX} \frac{T_{11}}{T_{11}^{-\alpha' q^2/4}}$. 
Applying this to the operator $I_{11}$ itself, for instance, gives the leading order expression for the tachyon vertex operator,

$$I_{11} \rightarrow e^{iqX} I_{11}^{\alpha'q^2+1} \sim V_{\text{tachyon}}[q], \quad (2.37)$$

where the $\sim$ denotes both $q^2\alpha'$ corrections and $\alpha'/|X|^2$ corrections. But there are independent contributions such as

$$\frac{\hat{T}_{22}}{I_{11}} \rightarrow e^{iqX} \frac{\hat{T}_{22}}{I_{11}}^{\alpha'q^2-1}. \quad (2.38)$$

This is an allowed scalar vertex operator, up to contributions of relative order $q\alpha'/|X|$, which come from the condition that the operator be primary at the quantum level. The main conclusion here is that the scalar vertex operator is unique up to and including relative order $|X|^{-2}$. There are analogous independent components of the photon vertex operator as well.

### 3 Dependence on external momentum

For effective string theories in flat space with no electromagnetic or other background fields, the interesting observables studied in the literature so far are primarily properties of the spectrum, in particular the dependence of the spectrum on total angular momentum $J$ [6,17–19] or the length $R$ of a static string stretched between heavy quarks or wound around a compact direction [4, 5]. In the context of the spectrum, the case of strictly infinite strings is not so interesting, because it only captures the leading term in an asymptotic expansion, discarding finite-volume effects on the worldsheet. This is because there is only one dimensionless parameter on which the spectrum can depend, namely the ratio of the length of the string to the string scale $\sqrt{\alpha'}$.

In the presence of background fields, however, there is another dimensionless ratio, namely the ratio of the momentum-squared $q^2$ to the string tension. Dimensional analysis allows observables to have interesting dependence on this ratio even for an infinitely long string. As we have seen above, even the structure of the vertex operators themselves has a nontrivial dependence on $q^2\alpha'$.

We must be somewhat careful about how we define our observables because the background fields are nondynamical and as a result, some (sometimes but not always all) of the momentum-dependence of an individual vertex operator can be absorbed into a $q^2$-dependent normalization of the vertex operator itself. The result is that individual form factors, for a fixed string state and fixed momentum, may not be computable in the effective framework.

Processes involving multiple insertions of the background field, however, are uniquely determined by the form factors. Also, ratios of form factors for the same background field, do not suffer any ambiguity at all.
3.1 Corrections to correlation functions in the $R \to \infty$ limit

Scales and corrections in general

Higher-order corrections as functions of $q$, may vary from one microscopic theory to another. The effective theory breaks down in its own terms when $q^2 > M_{UV}^2$, where $M_{UV}^2$ is the mass of the lightest massive non-Goldstone excitations, such as the Liouville direction in [1], holographic direction [20] or other degree of freedom such as the worldsheet axion introduced in [21,22]. Typically such degrees of freedom would have masses order $M_{UV}^2 \sim \alpha'^{-1}$ but may be lighter in one ultraviolet completion or another. This sort of breakdown of the effective theory is due to the quantum fluctuations of the massive degrees of freedom. This mass scale sets the size of the nonuniversal higher-derivative corrections to the string worldsheet action such as the coefficient $C_{ICS}$ of the induced-curvature-squared term discussed in section 2.5,

$$C_{ICS} \sim \frac{1}{M_{UV}^2 \alpha'}.$$ (3.1)

One would only expect the effective string theory to make sense for distance scales parametrically longer than $\sqrt{\alpha'}$, or longer than the Compton wavelength $M_{UV}^{-1}$ of the lightest non-Goldstone excitation on the QCD string worldvolume if $M_{UV}^2 \lesssim 1/\alpha'$ as in [21,22]. Therefore we will choose a Wilsonian cutoff $\Lambda$ and always work in the limit where the momentum in the vertex operators satisfies

$$q^2 < \Lambda^2 \ll M_{UV}^2 \lesssim \frac{1}{\alpha'}.$$ (3.2)

We would like, however, to probe scales shorter than the physical size of the string itself, in order to learn anything interesting. So we should make our cutoff $\Lambda$ much larger than $R^{-1}$ so that we allow $q \gg R^{-1}$. Therefore we have

$$R^{-2} \lesssim q^2 < \Lambda^2 \ll M_{UV}^2 \lesssim \frac{1}{\alpha'}.$$ (3.3)

We must also cut off the range of vertex operator integrations. In order to treat the effective string theory in a perturbative Wilsonian framework, we should never let vertex operators approach each other more closely than the length cutoff $\Lambda^{-1}$, as measured by the induced metric $g_{ind}$, where we have chosen the cutoff to satisfy (3.3)

In unit gauge the coordinate size of the circle is fixed by a coordinate choice to be $O(1)$ (usually $2\pi$) and the the physical size is proportional to $R$, so the physical separation of vertex operators is of order $R|\Delta z|$ and so the cutoff on the coordinate separation $|\Delta z|$ is

$$|\Delta z|_{\text{min}} = \frac{1}{R\Lambda}.$$ (3.4)

So vertex operator integrations are cut off when the physical distance $R|\Delta z|$ between them satisfies

$$(R|\Delta z|)^{-2} < \Lambda^2 \ll M_{UV}^2 \lesssim \frac{1}{\alpha'}.$$ (3.5)
We would like to see concretely how quantum fluctuations are suppressed by these hierarchies between scales. In this section we shall look at some specific cases, and see that corrections are indeed suppressed by powers of $q^2 \alpha'$ and $\Lambda^2 \alpha'$.

We will now look at some examples. We will estimate the corrections to correlators of two integrated vertex operators. Particularly we focus on the largest possible corrections, namely those that come when two vertex operators approach one another as closely as our ultraviolet cutoff allows. While we will not compute higher-order corrections in detail, we would like to estimate how they scale at small $q\sqrt{\alpha'}$, with $R^2/\alpha'$ taken to infinity.

The absolute scaling of vertex operator correlation functions contains various complicated factors such as $R^{-\alpha' q^2/4}$ from the classical evaluation of the dressing and an $R^{-2}$ in each vertex operator expressing the Jacobian between coordinate measure $d^2 z$ and the integral over longitudinal positions in spacetime, $\sqrt{|g_{\text{induced}}|} d^2 z = d^2 x$. However we will only consider relative sizes of corrected and uncorrected contributions to correlation functions, and all these factors will cancel in the ratios.

**Connected tree correction to the two-tachyon correlator**

Let us consider the leading correction to the two-tachyon correlator, coming from the leading interaction term in the large-$R$ limit. The leading interaction is the anomaly term $\mathcal{L}_{\text{PS}} \propto I_{21} I_{12}/I_{11}^2$, which contains terms with zero, two, and higher $Y$-fluctuations, when we break up the operator $X$ as $X^\mu = E^\mu + Y^\mu$, where $E$ is any classical solution. For the particular case of a static string, the terms with zero, two, and three fluctuations, do not contribute to the $Y$ Lagrangian after integration by parts and a field redefinition (equivalently, using the leading-order equations of motion). The first contributing term is the one with four fluctuations, which is proportional to

$$
\mathcal{L}^{(4)}[Y] \propto (D - 26) \frac{(\partial Y \cdot \partial^2 Y)(\partial Y \cdot \bar{\partial}^2 Y)}{(\partial E \cdot \bar{\partial} E)^2}.
$$

(3.6)

Consider the effect of this term on the correlation function of two tachyon vertex operators $V_{\text{tachyon}} \propto e^{\pm i q X^I_1} I_{11}^{1-\frac{q^2}{4}}$. We can either contract terms in the dressing or in the exponentials. For the moment, let us consider contracting the terms in the exponentials; contracting terms in the dressing will also give suppressed contributions, as we shall see shortly.

A tree-level correction would therefore have to contain at least four powers of $q$. For small $q^2$, the order $q^4$ term comes from contraction of two pairs of $X$'s in each exponential. The disconnected, free-field contraction is proportional to $q^4 \alpha'^2$, with a logarithm-squared of $RA$. The connected piece goes as $\frac{\alpha'^4}{R} \int \frac{d^2 w}{(w-z_1)(w-z_2)} \frac{d^2 w}{(w-z_2)(w-z_2)}$. Rescaling the $z$'s and the $w$ coordinate to physical coordinates, $v \equiv R w, z_i \equiv R u_i$, the factors of $R$ cancel out, and we get an ultraviolet divergent integral

$$
\alpha'^4 q^4 \int \frac{d^2 v}{(v-u_1)(\bar{v}-\bar{u}_1)^2(v-u_2)^2(\bar{v}-\bar{u}_2)},
$$

(3.7)

with the integral cut off when $|v-u_i| < \Lambda^{-1}$. The result is of order $q^4 \alpha'^4 \Lambda^4$. 

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So the relative suppression of the connected tree diagram with a PS term vertex, compared to the disconnected two-contraction diagram, is of order $\alpha'^2\Lambda^4$. Despite the UV divergence, this term is small due to the hierarchy (3.5) between the cutoff $\Lambda$ and $\alpha'^{-\frac{3}{2}}$. Its renormalization can therefore be treated in perturbation theory.

**One-loop correction to the propagator**

The one-loop propagator correction works similarly. A single free propagator contracts two $X$'s, giving a contribution of $\alpha'q^2\log(R\Lambda)$ in the correlator of exponentials when expanded at low $q$. The leading correction to the propagator comes from the PS term, with the $\partial^2 Y$ contracted with the $\bar{\partial}^2 Y$ in the same vertex. This contraction goes as $\alpha'|z|^{-4} \propto \alpha'|R\Lambda|^{-4}$. The four powers of $R$ are then cancelled by the $I_{11}^2 \propto R^4$ in the denominator of the PS term. There is also a factor of $\alpha'^2$ from the two external propagators. The form of the effective vertex is $(4\pi)^{-1} \Delta(\alpha'^{-1}) \partial Y \cdot \bar{\partial} Y$, with $\Delta(\alpha'^{-1}) \propto \Lambda^4 \alpha'$. That is, the leading effect is a renormalization of the string tension. The resulting correlator is again $q^2 \alpha'^3 \Lambda^4 \log(R\Lambda)$, or $\alpha'^2 \Lambda^4$ relative to the free-field contribution.

**Contraction of fluctuations in the dressing factor**

We are taking $R$ much larger than every other scale in the problem, so it would seem naively that any contraction of fluctuations in the $I_{11}$ dressing, lowers the $R$-scaling and suppresses the term. This is not quite accurate, because these contractions also have derivatives in them, leading to more singular dependence on $|\Delta z_{\text{min}}|$, which when translated into physical distance can cancel the $R$-suppression.

For instance, consider the effects of contracting fluctuations in the dressing using the free propagator.

Expanding $X$ into background $E^\mu$ plus fluctuation $Y^\mu = X^\mu - E^\mu$, we have

$$
\mathcal{I}_{11}^{-\alpha'q^2/4} = (\partial E \cdot \bar{\partial} E)^{-\alpha'q^2/4} \left[ 1 - \frac{\alpha'q^2}{4} \frac{\partial E \cdot \bar{\partial} Y + \partial Y \cdot \bar{\partial} E}{\partial E \cdot \partial E} + O\left(\frac{\alpha'q^2|\partial Y|^2}{|\partial E|^2}\right) \right]. 
$$

Relative to the classical term in the two-point function, the term with one $Y$-fluctuation in each dressing, has an additional scaling of

$$
\frac{\langle VV \rangle_{\text{one-contraction}}}{\langle VV \rangle_{\text{no contractions}}} \propto \alpha'^2 q^4 |\partial E|^{-2} \langle \partial Y(z) \partial Y(z') \rangle \propto \alpha'^3 q^4 |\partial E|^{-2} |\Delta z|^{-2} \propto \frac{\alpha'^3 q^4}{R^2 |\Delta z|^2}. 
$$

Evaluating with the operators separated by the distance cutoff, we replace $|\Delta z|$ with $|\Delta z|_{\text{min}} = (R\Lambda)^{-1}$. Then we find that

$$
\frac{\langle VV \rangle_{\text{one-contraction}}}{\langle VV \rangle_{\text{no contractions}}} \bigg|_{\text{cutoff}} \propto \alpha'^3 \Lambda^2 q^4. 
$$
3.2 Momentum-dependent couplings of bulk fields

One can also consider explicitly momentum-dependent couplings of bulk fields to the worldsheet. These couplings can be important for some purposes, though much of the dependence can be absorbed into the definitions of the vertex operators. In the case of the tachyon vertex operator, one entire form factor’s worth of $q$-dependence can be absorbed into the vertex operator itself. Only ratios of form factors are physically meaningful.

Electromagnetic form factors have an intrinsic normalization that may be slightly more meaningful in certain respects. This is because the gauge field itself has a natural normalization at $q \to 0$. So unlike the case of the tachyon vertex, the physical normalization of the photon vertex at $q = 0$ is meaningful, and the physical meaning is transparent: $V^{\text{photon}}[e, q = 0]$ with $e^\mu = (1, 0, \cdots, 0)$ simply measures the total electric charge.

There are many allowed derivative couplings of the electromagnetic field to the worldsheet via the gauge-invariant field strength $F = dA$. Since $F$ is gauge-invariant, it can have arbitrary couplings that preserve all appropriate symmetries.

3.3 Form factors

Now then, we can use these off-shell vertex operators to compute structure functions and form factors of various kinds for long strings.

As mentioned earlier, all of the momentum dependence of the tachyon form factor can be absorbed into the tachyon vertex operator itself. Since there is no canonical normalization for the vertex operator, it can be rescaled arbitrarily as a function of $q$, so that an entire tachyon form factor’s worth of functional dependence can be absorbed. That is, if $\mathcal{F} \equiv \langle \Psi | V^{\text{tachyon}}[q] | \Psi' \rangle$, then under $V^{\text{tachyon}}[q] \to f(|q|) V^{\text{tachyon}}[q]$, we have $\mathcal{F} \to f(|q|) \mathcal{F}$, which we can use to set $\mathcal{F}$ to any function.

However we can only do this for one single state; we cannot independently rescale $V[q]$ for different states, for this would be inconsistent with the linearity of operators representing observables in quantum mechanics. So ratios of form factors are well-defined. That is, ratios such as

$$\mathcal{F}_2^1 \equiv \frac{\langle \Psi_1, +\frac{q}{2} V^{\text{tachyon}}[q] | \Psi_1, -\frac{q}{2} \rangle}{\langle \Psi_2, +\frac{q}{2} V^{\text{tachyon}}[q] | \Psi_2, -\frac{q}{2} \rangle} \tag{3.11}$$

are independent of the normalization of the vertex operator, including its $q$-dependence.

Similarly, if we take the form factor in a given state wound around a compact direction with radius $R$, then the ambiguous normalization factor $f(|q|)$ of the vertex operator must be $R$-independent, by virtue of spacetime locality.

Static string wound around a compact direction

To illustrate the idea, we would like to compute form factors, i.e., the amount by which the string state sources a given bulk background field as a function of momentum. Such
background fields can include the electromagnetic or gravitational field, or the tachyon which locally controls the string tension. The latter is technically the simplest, so we will pursue that example here. As noted above, the change in the form factor as a function of $R$, is observable and calculable within the effective theory, since the $R$-dependence cannot be absorbed into a rescaling of the vertex operator.

The in-state is the state with winding number $w = 1$ around direction $X_1 \sim X_1 + 2\pi R$, momentum $P^\mu_{\text{in}} \equiv P^\mu - q^\mu/2$, and no oscillators excited:

$$|\Psi, -\frac{q}{2}\rangle \equiv |0, w = 1; P^\mu - \frac{1}{2} q^\mu\rangle \quad (3.12)$$

where

$$P^\mu \equiv (\sqrt{M^2 + \frac{q^2}{4}, 0, \ldots, 0}), \quad M = \frac{R}{\alpha'} - \frac{D - 2}{12R} + O(R^{-3}), \quad q_1 = 0. \quad (3.13)$$

Similarly, the out-state is the same state, with $P^\mu_{\text{out}} = P^\mu + q^\mu/2$:

$$\langle \Psi, +\frac{q}{2} | \equiv \langle 0, w = 1; P^\mu + \frac{1}{2} q^\mu |. \quad (3.14)$$

The value of the mass $M$ is simply the string tension times its length, plus the (negative) Casimir contribution, up to corrections of order $R^{-3}$. This value has been derived directly within the old covariant formalism in [1]. Therefore the value of the energy $P^0$ is at first approximation $R\alpha'$. The corrections to this are of relative order $\alpha'/R$ from the Casimir term, and $q^2\alpha'^2/R^2$ from the small boost from the frame in which the string is static. Both are of subleading order in $R$ and we ignore them henceforth.

We need not have taken $q_1 = 0$, but then there would have had to be some momentum in the in-state along the winding direction $X_1$. We then would have had to choose $q_1$ to be quantized in units of $R^{-1}$, so that the $q$-expansion and $R^{-1}$-expansion would mix. Also, there would no longer be a canonical choice for the in-state; we would have had to excite $n_1 \equiv Rq_1$ units of oscillator energy, which in general can be done in many different ways. To avoid these complications we simply take the oscillator vacuum, and choose $q$ to be purely transverse, without loss of generality in the $X_2$ direction:

$$q^\mu = (0, 0, |q|, 0, \ldots, 0). \quad (3.15)$$

The form factor is

$$F^\Psi_{\text{tachyon}} \equiv \langle \psi, +\frac{q}{2} | \psi_{\text{tachyon}}[q] | \Psi, -\frac{q}{2} \rangle. \quad (3.16)$$

Let us compute up to order $q^2\alpha'$ and at leading order only in $\alpha'/|X|^2$. At this order we can compute in free field theory.

The vertex operator can be replaced with its zero mode part, up to a local normalization constant which only changes the overall normalization of the vertex operator in an
$R$-independent way. Suppressing the labels of the state and the background field (i.e. $\Psi$ and "tachyon"), we have

$$F = (\text{const.}) \, R^2 - \frac{\alpha'q^2}{2}. \quad (3.17)$$

The constant must contain dimensional factors of $\alpha'$ or $M$ to the power $\alpha'q^2/2$ to make up the units, but by locality the normalization constant cannot depend on $R$. The subleading terms in the tachyon vertex operator are of subleading order in $R$ and we drop them. The factor of $R^2$ is associated with the change of coordinates between $d^2 w$ and $dX^0dX^1$. For a three-point function the integration over $w$ can be eliminated, by gauge-fixing a residual conformal isometry, and the measure factor is independent of $R$.

The total form factor is divergent, due to the scalar field being sourced by every point on the string worldvolume, whose size goes to infinity in the $R \to \infty$ limit. So we calculate a "differential form factor"

$$\hat{F} \equiv \frac{d^2 F}{dX^0dX^1} = f(|q|) \, h(R) \left( \frac{R}{\sqrt{\alpha'}} \right)^{-\frac{\alpha'q^2}{2}}. \quad (3.18)$$

The $f(|q|)$ in front is an unknown (but $R$-independent) momentum dependence, which contains the arbitrariness in the dimensional coefficient normalizing $R$ inside the $-q^2\alpha'/2$ power. There is also a factor $h(R)$ multiplying the form factor density that can be absorbed into the state. This factor depends on the size of the $S^1$ and on the details of the state itself. It can only depend on the invariant mass $M = \frac{R}{\alpha} - \frac{D-2}{12R}$ and not on the momentum $q$ because our formalism is covariant, and therefore $h(R)$ can depend only on Lorentz-invariant properties of the state.

At low momenta the differential form factor scales as

$$\hat{F} \approx f(|q|) h(R) \left[ 1 - \frac{\alpha'q^2}{2} \log \left( \frac{R}{\sqrt{\alpha'}} \right) + O(|q|^4) \right]. \quad (3.19)$$

The factor $f(|q|)$ is nonuniversal and the factor $h(R)$ is physically meaningless as it can be absorbed into an overall normalization of the string state. The physically meaningful universal information is expressed as the combination

$$G(R, |q|) \equiv \frac{d^2 \log(\hat{F})}{dR \, d|q|} = \hat{F}^{-1} \, \frac{d^2 \hat{F}}{dR \, d|q|} - \hat{F}^{-2} \frac{d\hat{F}}{dR} \frac{d\hat{F}}{d|q|}, \quad (3.20)$$

in which the functions $f(|q|)$ and $h(R)$ drop out. In our example

$$G(R, |q|) = -\frac{\alpha'|q|}{R} + O(|q|^3). \quad (3.21)$$

The combination $G(R, |q|)$ is universal physically and meaningful. It could be calculated in principle in lattice Monte Carlo simulations. Similar quantities can be calculated for
stretched open strings and more experimentally relevant spinning string states with free endpoints. For these states, low energy electromagnetic and gravitational form factors would be of particular interest. The former would be an interesting model-independent arena for testing of the validity of the effective string theory of QCD in the confining phase; the latter would be of interest for deriving robust signatures of cosmic gravitational radiation produced by cosmic strings (See e.g. [23]).

4 Conclusions

We have constructed various off-shell vertex operators in the covariant formalism of effective string theory, up to and including next-to-leading order in large-|X| expansion. These operators provide a way to investigate interesting dynamical quantities relevant to hadron physics, such as form factors and structure functions. As required, they are all made primary of conformal weight (1,1), as well as gauge-invariant in the case of the photon and graviton vertex operators. We also discussed the regime of validity of our vertex operators as functions of momentum. As an example of how dynamical quantities can be obtained by using our vertex operators, we explicitly computed the scalar form factor’s dependence on the momentum q and radius R of the compact direction.

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References

[1] J. Polchinski and A. Strominger, "Effective string theory," Phys. Rev. Lett. 67 (1991) 1681–1684.

[2] N. D. Hari Dass and P. Matlock, "Covariant Calculus for Effective String Theories," Indian J. Phys. 88 (2014) 965–977, arXiv:0709.1765 [hep-th].

[3] S. Hellerman, S. Maeda, J. Maltz, and I. Swanson, "Effective String Theory Simplified," JHEP 09 (2014) 183, arXiv:1405.6197 [hep-th].
[4] J. M. Drummond, "Universal subleading spectrum of effective string theory," arXiv:hep-th/0411017 [hep-th].

[5] O. Aharony, M. Field, and N. Klinghoffer, "The effective string spectrum in the orthogonal gauge," JHEP 04 (2012) 048, arXiv:1111.5757 [hep-th].

[6] S. Hellerman and I. Swanson, "String Theory of the Regge Intercept," Phys. Rev. Lett. 114 no. 11, (2015) 111601, arXiv:1312.0999 [hep-th].

[7] S. Dubovsky, R. Flauger, and V. Gorbenko, "Effective String Theory Revisited," JHEP 09 (2012) 044, arXiv:1203.1054 [hep-th].

[8] S. Dubovsky and G. Hernandez-Chifflet, "Yang–Mills Glueballs as Closed Bosonic Strings," arXiv:1611.09796 [hep-th].

[9] S. Hellerman and I. Swanson, "Boundary Operators in Effective String Theory," arXiv:1609.01736 [hep-th].

[10] M. Luscher and P. Weisz, "String excitation energies in SU(N) gauge theories beyond the free-string approximation," JHEP 07 (2004) 014, arXiv:hep-th/0406205 [hep-th].

[11] O. Aharony and E. Karzbrun, "On the effective action of confining strings," JHEP 06 (2009) 012, arXiv:0903.1927 [hep-th].

[12] O. Aharony and N. Klinghoffer, "Corrections to Nambu-Goto energy levels from the effective string action," JHEP 12 (2010) 058, arXiv:1008.2648 [hep-th].

[13] O. Aharony and M. Dodelson, "Effective String Theory and Nonlinear Lorentz Invariance," JHEP 02 (2012) 008, arXiv:1111.5758 [hep-th].

[14] O. Aharony and Z. Komargodski, "The Effective Theory of Long Strings," JHEP 05 (2013) 118, arXiv:1302.6257 [hep-th].

[15] A. M. Polyakov, "Quantum Geometry of Bosonic Strings," Phys. Lett. B103 (1981) 207–210.

[16] S. Maeda, "Effective theory of stringy objects," Master’s thesis, The University of Tokyo, 2015.

[17] J. Sonnenschein and D. Weissman, "Rotating strings confronting PDG mesons," JHEP 08 (2014) 013, arXiv:1402.5603 [hep-ph].

[18] J. Sonnenschein and D. Weissman, "A rotating string model versus baryon spectra," JHEP 02 (2015) 147, arXiv:1408.0763 [hep-ph].
[19] J. Sonnenschein and D. Weissman, "Glueballs as rotating folded closed strings," *JHEP* **12** (2015) 011, arXiv:1507.01604 [hep-ph].

[20] M. Natsuume, "Nonlinear $\sigma$ model for string solitons," *Phys. Rev.* **D48** (1993) 835–838, arXiv:hep-th/9206062 [hep-th].

[21] S. Dubovsky, R. Flauger, and V. Gorbenko, "Evidence from Lattice Data for a New Particle on the Worldsheet of the QCD Flux Tube," *Phys. Rev. Lett.* **111** no. 6, (2013) 062006, arXiv:1301.2325 [hep-th].

[22] S. Dubovsky and V. Gorbenko, "Towards a Theory of the QCD String," *JHEP* **02** (2016) 022, arXiv:1511.01908 [hep-th].

[23] E. J. Copeland, R. C. Myers, and J. Polchinski, "Cosmic F and D strings," *JHEP* **06** (2004) 013, arXiv:hep-th/0312067 [hep-th].