Thermodynamics of viscous dark energy in RSII braneworld

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We show that for a RSII braneworld filled with interacting viscous dark energy and dark matter, one can always rewrite the Friedmann equation in the form of the first law of thermodynamics, $dE = T_h dS_h + W dV$, at apparent horizon. In addition, the generalized second law of thermodynamics can fulfilled in a region enclosed by the apparent horizon on the brane for both constant and time variable 5-dynamical Newtons constant $G_5$. These results hold regardless of the specific form of the dark energy. Our study further support that in an accelerating universe with spatial curvature, the apparent horizon is a physical boundary from the thermodynamical point of view.

I. INTRODUCTION

Observational data indicates that our universe is currently under accelerating expansion $^{1,2}$. It seems that some unknown energy components (dark energy) with negative pressure are responsible for this late-time acceleration $^3$. However, understanding the nature of dark energy is one of the fundamental problems of modern theoretical cosmology $^4$. An alternative approach to accommodate dark energy is modifying the general theory of relativity on large scales. Among these theories, scalar-tensor theories $^5$, f(R) gravity $^6$, DGP braneworld gravity $^7$ and string-inspired theories $^8$ are studied extensively.

The cosmological models with non-viscous cosmic fluid has been studied widely in the literature. Early treatises on viscous cosmology are given in $^9$. The viscous entropy production in the early universe and viscous fluids on the Randall-Sundrum branes have been studied respectively in $^{10}$. A special branch of viscous cosmology is to investigate how the bulk viscosity can influence the future singularity, commonly called the Big Rip, when the fluid is in the phantom state corresponding to $w_D < -1$. A lot of works have been done in this direction $^{11,12}$. In particular, it was first pointed out in $^{11}$ that the presence of a bulk viscosity proportional to the Hubble expansion $H$ can cause the fluid to pass from the quintessence region into the phantom region and thereby

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inevitably lead to a future singularity.

In the present work we are interested to investigate the interacting viscous dark energy and dark matter in RSII braneworld, from the thermodynamic point of view. In particular, we desire to examine under what conditions the underlying system obeys the generalized second law of thermodynamics, namely the sum of entropies of the individual components, including that of the background, to be positive. Then we extend our analysis with considering the time variable 5D Newton’s constant $G_5$. Until now, in most the investigated dark energy models a constant Newton’s “constant” $G$ has been considered. However, there are significant indications that $G$ can by varying, being a function of time or equivalently of the scale factor $\frac{\dot{G}}{G}$ [13]. In particular, observations of Hulse-Taylor binary pulsar [14, 15], helio-seismological data [16], Type Ia supernova observations [1] and astroseismological data from the pulsating white dwarf star G117-B15A [18] lead to $|\dot{G}/G| \lesssim 4.10 \times 10^{-11}\,yr^{-1}$, for $z \lesssim 3.5$ [19]. Additionally, a varying $G$ has some theoretical advantages too, alleviating the dark matter problem [20], the cosmic coincidence problem [21] and the discrepancies in Hubble parameter value [22].

There have been many proposals in the literature attempting to theoretically justified a varying gravitational constant, despite the lack of a full, underlying quantum gravity theory. Starting with the simple but pioneering work of Dirac [23], the varying behavior in Kaluza-Klein theory was associated with a scalar field appearing in the metric component corresponding to the 5-th dimension [24] and its size variation [25]. An alternative approach arises from Brans-Dicke framework [26], where the gravitational constant is replaced by a scalar field coupling to gravity through a new parameter, and it has been generalized to various forms of scalar-tensor theories [27], leading to a considerably broader range of variable-$G$ theories. In addition, justification of a varying Newton’s constant has been established with the use of conformal invariance and its induced local transformations [28]. Finally, a varying $G$ can arise perturbatively through a semiclassical treatment of Hilbert-Einstein action [29], non-perturbatively through quantum-gravitational approaches within the “Hilbert-Einstein truncation” [30], or through gravitational holography [31, 32].

II. BASIC EQUATIONS

Our starting point is the four-dimensional homogenous and isotropic FRW universe on the brane with the metric

$$ds^2 = h_{\mu\nu}dx^\mu dx^\nu + \tilde{r}^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

(1)
where \( \tilde{r} = a(t)r \), \( x^0 = t, x^1 = r \), the two dimensional metric \( h_{\mu\nu} = \text{diag} \left( -1, a^2/(1 - kr^2) \right) \). Here \( k \) denotes the curvature of space with \( k = 0, 1, -1 \) corresponding to open, flat, and closed universes, respectively. A closed universe with a small positive curvature (\( \Omega_k \simeq 0.01 \)) is compatible with observations \([33]\). The dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is determined by the relation \( h^{\mu\nu} \partial_\mu \tilde{r} \partial_\nu \tilde{r} = 0 \), which implies that the vector \( \nabla \tilde{r} \) is null on the apparent horizon surface. The apparent horizon was argued as a causal horizon for a dynamical spacetime and is associated with gravitational entropy and surface gravity \([34, 35]\). For the FRW universe the apparent horizon radius reads

\[
\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}.
\]  

(2)

The associated surface gravity on the apparent horizon can be defined as

\[
\kappa = \frac{1}{\sqrt{-h}} \partial_a \left( \sqrt{-h} h^{ab} \partial_b \tilde{r} \right),
\]

(3)

thus one can easily express the surface gravity on the apparent horizon

\[
\kappa = \left( 1 - \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A} \right). \]

(4)

The associated temperature on the apparent horizon can be expressed in the form

\[
T_h = \frac{|\kappa|}{2\pi} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A} \right).
\]

(5)

where \( \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A} < 1 \) ensures that the temperature is positive. Recently the Hawking radiation on the apparent horizon has been observed in \([36]\) which gives more solid physical implication of the temperature associated with the apparent horizon.

The Friedmann equation for 3-dimensional Randall-Sundrum (RS) II brane embedded in an 5-dimensional AdS bulk can be written \([37]\)

\[
H^2 + \frac{k}{a^2} - \frac{\kappa_5^2 \Lambda_5}{6} - \frac{C}{a^4} = \frac{\kappa_5^4}{36} \rho^2.
\]

(6)

where

\[
\kappa_5^2 = 8\pi G_5, \quad \Lambda_5 = -\frac{6}{\kappa_5^2 \ell^2},
\]

(7)

\( \Lambda_5 \) is the 5-dimensional bulk cosmological constant, and \( \ell \) is the AdS radius of the bulk spacetime. Here \( \rho = \rho_m + \rho_D \) where \( \rho_m \) and \( \rho_D \) are, respectively, the energy density of dark matter and dark energy confined to the brane and \( H = \dot{a}/a \) is the Hubble parameter on the brane. The constant \( C \) comes from the 5-dimensional bulk Weyl tensor. In this paper we are interested in AdS bulk
spacetimes, so the bulk Weyl tensor vanishes and thus we set $\mathcal{C} = 0$ in the following discussions. The energy-momentum tensor of the matter and energy content on the brane is as,

$$ T_{\mu\nu} = \rho u_\mu u_\nu + \tilde{p}_D(g_{\mu\nu} + u_\mu u_\nu), \quad (8) $$

where $u_\mu$ is the four-velocity vector, and

$$ \tilde{p}_D = p_D - 3H\xi, \quad (9) $$

is the effective pressure of dark energy and $\xi$ is the viscosity coefficient. The condition $\xi > 0$ guaranties a positive entropy production and, in consequence, no violation of the second law of the thermodynamics [38]. The total energy density on the brane satisfies a conservation law

$$ \dot{\rho} + 3H(\rho + \tilde{p}_D) = 0. \quad (10) $$

However, since we consider the interaction between dark matter and dark energy, $\rho_m$ and $\rho_D$ do not conserve separately, they must rather enter the energy balances

$$ \dot{\rho}_m + 3H\rho_m = Q, \quad (11) $$

$$ \dot{\rho}_D + 3H\rho_D(1 + w_D) = 9H^2\xi - Q. \quad (12) $$

where $w_D = p_D/\rho_D$ is the equation of state parameter of viscous dark energy and $Q = \Gamma \rho_D$ denotes the interaction between the dark components. We also assume the interaction term is positive, $Q > 0$, which means that there is an energy transfer from the dark energy to dark matter. Hereafter we assume that the brane cosmological constant is zero (if it does not vanish, one can absorb it in the stress-energy tensor of fluid on the brane).

III. FIRST LAW OF THERMODYNAMICS IN VICIOUS BRANEWORLD

In this section we are going to examine the first law of thermodynamics on the brane. In particular, we show that for a closed universe filled with viscous dark energy and dark matter the Friedmann equation can be written directly in the form of the first law of thermodynamics at apparent horizon on the brane. Using Eq. (7) the Friedmann equation (6) can be written as

$$ \sqrt{H^2 + \frac{k}{a^2}} + \frac{1}{\ell^2} = \frac{4\pi G_5}{3} (\rho_m + \rho_D). \quad (13) $$

In terms of the apparent horizon radius we have

$$ \rho_m + \rho_D = \frac{3}{4\pi G_5} \sqrt{\frac{1}{r_A^2} + \frac{1}{\ell^2}}. \quad (14) $$
Taking differential form of equation (13) and using Eqs. (11) and (12), we can get the differential form of the Friedmann equation

\[ H \left[ \rho_D (1 + u + w_D) - 3H\xi \right] dt = \frac{\ell}{4\pi G_5 \bar{r}_A^2 \sqrt{\bar{r}_A^2 + \ell^2}} \frac{d\bar{r}_A}{\bar{r}_A^2 + \ell^2}. \]  

(15)

where \( u = \rho_m/\rho_D \) is the ratio of energy densities. Multiplying both sides of the equation (22) by a factor \( 4\pi \bar{r}_A^3 \left( 1 - \frac{\dot{\bar{r}}_A}{2H\bar{r}_A} \right) \), and using the expression (1) for the surface gravity, after some simplification one can rewrite this equation in the form

\[- \frac{\kappa}{2\pi} \frac{2\pi \ell}{G_5} \frac{\bar{r}_A^2 d\bar{r}_A}{\sqrt{\bar{r}_A^2 + \ell^2}} = 4\pi \bar{r}_A^3 H \left[ \rho_D (1 + u + w_D) - 3H\xi \right] dt \]

\[-2\pi \bar{r}_A^2 \left[ \rho_D (1 + u + w_D) - 3H\xi \right] d\bar{r}_A. \]  

(16)

\( E = (\rho_m + \rho_D)V \) is the total energy content of the universe inside a 3-sphere of radius \( \bar{r}_A \) on the brane, where \( V = \frac{4\pi}{3} \bar{r}_A^3 \) is the volume enveloped by 3-dimensional sphere with the area of apparent horizon \( A = 4\pi \bar{r}_A^2 \). Taking differential form of the relation \( E = (\rho_m + \rho_D) \frac{4\pi}{3} \bar{r}_A^3 \) for the total matter and energy inside the apparent horizon, we get

\[ dE = 4\pi \bar{r}_A^2 (\rho_m + \rho_D) d\bar{r}_A + \frac{4\pi}{3} \bar{r}_A^3 (\dot{\rho}_m + \dot{\rho}_D) dt. \]  

(17)

Using Eqs. (11) and (12), we obtain

\[ dE = 4\pi \bar{r}_A^2 \rho_D (1 + u) d\bar{r}_A - 4\pi \bar{r}_A^3 H \left[ \rho_D (1 + u + w_D) - 3H\xi \right] dt. \]  

(18)

Substituting this relation into (16), after some simplifications one can rewrite this equation in the form

\[ dE - W dV = \frac{\kappa}{2\pi} \frac{2\pi \ell}{G_5} \frac{\bar{r}_A^2 d\bar{r}_A}{\sqrt{\bar{r}_A^2 + \ell^2}}. \]  

(19)

where

\[ W = \frac{1}{2} [\rho_m + \rho_D - \bar{p}_D] = \frac{1}{2} \rho_D \left[ 1 + u - w_D + 3H\xi \right], \]

is the matter work density [34]. The work density term is regarded as the work done by the change of the apparent horizon, which is used to replace the negative pressure if compared with the standard first law of thermodynamics, \( dE = T dS - pdV \). For a pure de Sitter space, \( \rho_m + \rho_D = -\bar{p}_D \), then our work term reduces to the standard \(-\bar{p}_D dV \). Expression (19) is nothing, but the first law of
thermodynamics at the apparent horizon on the brane, namely \(dE = T_h dS_h + W dV\). We can define the entropy associated with the apparent horizon on the brane as

\[
S_h = \frac{2\pi \ell}{G_5} \int_0^{\tilde{r}_A} \frac{\tilde{r}_A^2}{\sqrt{\tilde{r}_A^2 + \ell^2}} d\tilde{r}_A.
\]

(20)

After the integration we have

\[
S_h = \frac{2\pi \tilde{r}_A^3}{3G_5} \times 2F_1 \left( \frac{3}{2}, \frac{1}{2}; \frac{5}{2}; -\frac{\tilde{r}_A^2}{\ell^2} \right),
\]

(21)

where \(2F_1(a, b, c, z)\) is the hypergeometric function. It is worth noticing when \(\tilde{r}_A \ll \ell\), which physically means that the size of the extra dimension is very large if compared with the apparent horizon radius, one recovers the 5-dimensional area formula for the entropy on the brane \([39–42]\). This is due to the fact that because of the absence of the negative cosmological constant in the bulk, no localization of gravity happens on the brane. As a result, the gravity on the brane is still 5-dimensional. In this way we show that for a non-flat universe filled with viscous dark energy and dark matter the Friedmann equation can be written in the form of the first law of thermodynamics at apparent horizon in RSII braneworld.

IV. GSL AND INTERACTING VISCOS DEATH ENERGY

Our aim here is to investigate the validity of the generalized second law of thermodynamics in a region enclosed by the apparent horizon on the brane. Taking the derivative of Eq. (14) with respect to the cosmic time and using Eqs. (11) and (12), one gets

\[
\dot{\tilde{r}}_A = \frac{4\pi}{\ell} G_5 H \tilde{r}_A^2 \left[ \rho_D (1 + u + w_D) - 3H \xi \right] \sqrt{\tilde{r}_A^2 + \ell^2}.
\]

(22)

Next we turn to calculate \(T_h \dot{S}_h\):

\[
T_h \dot{S}_h = \frac{1}{2\pi \tilde{r}_A} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A} \right) \frac{d}{dt} \left[ \frac{2\pi \tilde{r}_A^3}{3G_5} \times 2F_1 \left( \frac{3}{2}, \frac{1}{2}; \frac{5}{2}; -\frac{\tilde{r}_A^2}{\ell^2} \right) \right]
\]

\[
= \frac{1}{2\pi \tilde{r}_A} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A} \right) \frac{2\pi \ell}{G_5} \frac{\tilde{r}_A^2 \dot{\tilde{r}}_A}{\sqrt{\tilde{r}_A^2 + \ell^2}}.
\]

(23)

Using Eq. (22), after some simplification we obtain

\[
T_h \dot{S}_h = 4\pi H \left[ \rho_D (1 + u + w_D) - 3H \xi \right] \tilde{r}_A^3 \left( 1 - \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A} \right).
\]

(24)

As we argued above the term \(1 - \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A}\) is positive to ensure \(T_h > 0\), however, in an accelerating universe the equation of state parameter of dark energy may satisfy the condition \(w_D < -1 - u + \).
This implies that the second law of thermodynamics, $\dot{S}_h \geq 0$, does not hold. However, as we will see below the generalized second law of thermodynamics, $\dot{S}_h + \dot{S}_m + \dot{S}_D \geq 0$, is still fulfilled throughout the history of the universe. The entropy of the viscous dark energy plus dark matter inside the apparent horizon, $S = S_m + S_D$, can be related to the total energy $E = (\rho_m + \rho_D)V$ and pressure $\tilde{p}_D$ in the horizon by the Gibbs equation:

$$TdS = d[(\rho_m + \rho_D)V] + \tilde{p}_DdV = V(d\rho_m + d\rho_D) + [\rho_D(1 + u + w_D) - 3H\xi]dV,$$

where $T = T_m = T_D$ and $S = S_m + S_D$ are, respectively, the temperature and the total entropy of the energy and matter content inside the horizon, and $V = \frac{4\pi}{3} \tilde{r}_A^3$ is the volume enveloped by the apparent horizon. Here we assumed that the temperature of both dark components are equal, due to their mutual interaction. We also limit ourselves to the assumption that the thermal system bounded by the apparent horizon remains in equilibrium so that the temperature of the system must be uniform and the same as the temperature of its boundary. This requires that the temperature $T$ of the viscous dark energy inside the apparent horizon should be in equilibrium with the temperature $T_h$ associated with the apparent horizon, so we have $T = T_h$. This expression holds in the local equilibrium hypothesis. If the temperature of the fluid differs much from that of the horizon, there will be spontaneous heat flow between the horizon and the fluid and the local equilibrium hypothesis will no longer hold. This is also at variance with the FRW geometry. In general, when we consider the thermal equilibrium state of the universe, the temperature of the universe is associated with the apparent horizon. Therefore from the Gibbs equation (25) we obtain

$$T_h(\dot{S}_m + \dot{S}_D) = 4\pi \tilde{r}_A^2 [\rho_D(1 + u + w_D) - 3H\xi] \dot{\tilde{r}}_A - 4\pi H \tilde{r}_A^3 [\rho_D(1 + u + w_D) - 3H\xi].$$

To check the generalized second law of thermodynamics, we have to examine the evolution of the total entropy $S_h + S_m + S_D$. Adding equations (24) and (26), we get

$$T_h(\dot{S}_h + \dot{S}_m + \dot{S}_D) = 2\pi \tilde{r}_A^2 [\rho_D(1 + u + w_D) - 3H\xi] \dot{\tilde{r}}_A = \frac{A}{2} [\rho_D(1 + u + w_D) - 3H\xi] \dot{\tilde{r}}_A.$$

where $A = 4\pi \tilde{r}_A^2$ is the area of the apparent horizon on the brane. Substituting $\dot{\tilde{r}}_A$ from Eq. (22) into (27) we reach

$$T_h(\dot{S}_h + \dot{S}_m + \dot{S}_D) = \frac{2\pi}{\ell} G_5 A \tilde{r}_A^2 \sqrt{\tilde{r}_A^2 + \ell^2} H [\rho_D(1 + u + w_D) - 3H\xi]^2.$$

The right hand side of the above equation cannot be negative throughout the history of the universe, which means that $\dot{S}_h + \dot{S}_m + \dot{S}_D \geq 0$ always holds. This indicates that the generalized second law of thermodynamics is fulfilled in the RS II braneworld embedded in the AdS bulk.
V. GSL AND WITH VARIABLE 5D NEWTON’S CONSTANT

In this section we would like to perform the above analysis with considering the time variable 5D Newton’s constant $G_5$. There is some evidence of a variable $G_5$ through numerous astrophysical observations \[43\]. Models with variable Newton’s constant can fix some of the hardest problems in cosmology like the age problem, cosmic coincidence problem and finding the value of the Hubble parameter \[44\].

Taking the derivative of Eq. (14) with respect to the cosmic time and using Eqs. (11) and (12), one gets

$$\dot{\tilde{r}}_A = \left( 4\pi G_5 H \rho_D (1 + u + w_D) - 3H \xi \right) - PG_5 \frac{\tilde{r}^2_A}{\ell} \sqrt{\tilde{r}^2_A + \ell^2}. \quad (29)$$

where we have defined

$$P = \frac{\sqrt{\tilde{r}^2_A + \ell^2}}{\tilde{r}_A G_5 \ell}. \quad (30)$$

Next we calculate $T_h \dot{S}_h$:

$$T_h \dot{S}_h = \frac{1}{2\pi \tilde{r}_A} \left(1 - \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A}\right) \left[2\pi \ell \frac{\tilde{r}^2_A \tilde{r}_A}{G_5} \sqrt{\tilde{r}^2_A + \ell^2} - \frac{\dot{G}_5}{G_5} S_h \right]. \quad (31)$$

Next we examine the evolution of the total entropy $S_h + S_m + S_D$. Adding equations (31) and (26), and using Eq. (29) we reach

$$T_h (\dot{S}_h + \dot{S}_m + \dot{S}_D) = 2\pi \tilde{r}^2_A \rho_D (1 + u + w_D) - 3H \xi \tilde{r}_A
- \frac{\dot{G}_5}{G_5} P \tilde{r}^3_A + \frac{S_h}{2\pi \tilde{r}_A} \left(1 - \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A}\right) \right). \quad (32)$$

Substituting $\dot{\tilde{r}}_A$ from Eq. (29) into (32) we reach

$$T_h (\dot{S}_h + \dot{S}_m + \dot{S}_D) = \frac{2\pi}{\ell} G_5 A \tilde{r}^2_A \sqrt{\tilde{r}^2_A + \ell^2} H \rho_D (1 + u + w_D) - 3H \xi \tilde{r}_A
- \frac{A}{2} PG_5 \rho_D (1 + u + w_D) - 3H \xi \frac{\tilde{r}^2_A}{\ell} \sqrt{\tilde{r}^2_A + \ell^2}
- \frac{\dot{G}_5}{G_5} \left[ P \tilde{r}^3_A + \frac{S_h}{2\pi \tilde{r}_A} \right] \left(1 - \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A}\right). \quad (33)$$

For $\dot{G}_5 = 0$, we obtain the result of the previous section. In this case the validity of GSL depend to the sign of $\dot{G}_5$, if $\dot{G}_5 < 0$, and $\rho_D (1 + u + w_D) > 3H \xi$, then $T_h (\dot{S}_h + \dot{S}_m + \dot{S}_D) > 0$. 
VI. SUMMARY AND DISCUSSIONS

In the present paper we have showed that the Friedmann equations on a RSII braneworld filled with interacting viscous dark energy and dark matter can be written directly in the form of the first law of thermodynamics at apparent horizon. Then we examined the validity of the generalized second law of thermodynamics, we studied the time evolution of the total entropy, including the entropy associated with the apparent horizon and the entropy of the viscous dark energy inside the apparent horizon. Our study have shown that the generalized second law of thermodynamics is always protected in a RSII braneworld filled with interacting viscous dark energy and dark matter in a region enclosed by the apparent horizon. Then we extended our study to the case time variable 5-dynamical Newtons constant $G_5$. According to the our calculations the generalized second law of thermodynamics is valid if $\dot{G}_5 < 0$, and $\rho_D (1 + u + w_D) > 3 H \xi$. These results hold regardless of the specific form of the dark energy.

Acknowledgments

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