One-loop renormalisation of $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory in the adjoint representation

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We construct a superpotential for the general $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory coupled to chiral matter in the adjoint representation, and investigate the one-loop renormalisability of the theory.
1. Introduction

$\mathcal{N} = \frac{1}{2}$ supersymmetric theories (i.e. theories defined on non-anticommutative super-space) have recently attracted much attention\[1\]–\[4\]. Such theories are non-hermitian and only have half the supersymmetry of the corresponding $\mathcal{N} = 1$ theory. These theories are not power-counting renormalisable\[2\] but it has been argued\[7\]–\[10\] that they are in fact nevertheless renormalisable, in other words only a finite number of additional terms need to be added to the lagrangian to absorb divergences to all orders. In previous work we have confirmed this renormalisability at the one-loop level. In particular we have shown that although divergent gauge non-invariant terms are generated at the one-loop level, they can be removed by divergent field redefinitions leading to a renormalisable theory in which $\mathcal{N} = \frac{1}{2}$ supersymmetry is preserved at the one-loop level in both the pure gauge case\[11\] and in the case of chiral matter in the fundamental representation\[12\]. On the other hand, the authors of Ref.\[13\] obtained the one loop effective action for pure $\mathcal{N} = \frac{1}{2}$ supersymmetry using a superfield formalism. Although they found divergent contributions which broke supergauge invariance, their final result was gauge-invariant without the need for any redefinition. In subsequent work\[14\] it was shown that the $\mathcal{N} = \frac{1}{2}$ superfield action requires modification to ensure renormalisability, which is consistent with our findings in the component formulation\[12\].

It was pointed out in Ref.\[4\] that an $\mathcal{N} = \frac{1}{2}$ supersymmetric theory can also be constructed with matter in the adjoint representation. Our purpose here is to repeat the analysis of Ref.\[12\] for the adjoint case, then proceed to consider the addition of superpotential terms, which will turn out to be a non-trivial task. The adjoint action of Ref.\[4\] was written for the gauge group $U(N)$. As we noted in Refs.\[11\], \[12\], at the quantum level the $U(N)$ gauge invariance cannot be retained. In the case of chiral matter in the fundamental representation we were obliged to consider a modified theory with the gauge group $SU(N) \otimes U(1)$. In the adjoint case with a trilinear superpotential, it will turn out that the matter fields must also be in a representation of $SU(N) \otimes U(1)$. However, for simplicity of exposition we shall start by considering the adjoint case without a superpotential, in other words adapting the calculations of Ref.\[12\] to the adjoint case.

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1 See Refs.\[3\]\[4\] for other discussions of the ultra-violet properties of these theories.
The classical action without a superpotential may be written

\[
S_0 = \int d^4x \left[ -\frac{1}{4} F^{\mu\nu A} F^{A}_{\mu\nu} - i \bar{\lambda}^A \sigma^\mu (D_\mu \lambda)^A + \frac{1}{2} D^A D^A \
- \frac{1}{2} i C^{\mu
u} \epsilon^{ABC} F^{A}_{\mu\nu} \bar{\lambda}^B \lambda^C \
+ \frac{1}{5} g^2 |C|^2 d^{abde} (\bar{\lambda}^a \bar{\lambda}^b) (\bar{\lambda}^c \bar{\lambda}^d) + \frac{1}{4N} g^2 |C|^2 (\bar{\lambda}^a \bar{\lambda}^a) (\bar{\lambda}^b \bar{\lambda}^b)
\right.
\]

\[
+ \bar{F} F - i \bar{\psi} \sigma^\mu D_\mu \psi - D^\mu \bar{\phi} D_\mu \phi
\]
\[
+ g \bar{\phi} D F \phi + ig \sqrt{2} (\bar{\phi} \lambda^F \psi - \bar{\psi} \lambda^F \phi)
\]
\[
+ d^{abc} g C^{\mu\nu} \left( \sqrt{2} D_\mu \bar{\phi}^a \lambda^b \sigma^\nu \psi^c + i \bar{\phi}^a F^{b}_{\mu\nu} F^c \right)
\]
\[
+ d^{ab0} g_0 C^{\mu\nu} \left( \sqrt{2} D_\mu \bar{\phi}^a \lambda^b \sigma^\nu \psi^0 + \bar{\phi}^a F^{b}_{\mu\nu} F^0 \right)
\]
\[
+ d^{000} g_0 C^{\mu\nu} \left( \sqrt{2} D_\mu \bar{\phi}^0 \lambda^a \sigma^\nu \psi^0 + \bar{\phi}^0 F^{a}_{\mu\nu} F^0 \right)
\]
\[
+ d^{ab0} g_0 C^{\mu\nu} \left( \sqrt{2} D_\mu \bar{\phi}^a \lambda^b \sigma^\nu \psi^0 + i \bar{\phi}^a F^{b}_{\mu\nu} F^0 \right)
\]
\[
- \frac{1}{4} g^2 |C|^2 \bar{\phi} \lambda^F \lambda^F \bar{F} F
\].

Here

\( \lambda^F = \lambda^a \bar{F}^a \), \((\bar{F}^A)^{BC} = i f^{BAC}\),

and we have

\[
D_\mu \phi = \partial_\mu \phi + i g A^F_{\mu} \phi,
\]

\[
F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g f^{ABC} A^B_\mu A^C_\nu,
\]

with similar definitions for \( D_\mu \psi, D_\mu \lambda \). If one decomposes \( U(N) \) as \( SU(N) \odot U(1) \) then our convention is that \( \phi^a \) (for example) are the \( SU(N) \) components and \( \phi^0 \) the \( U(1) \) component. (For later convenience we also define \( g_A \) similarly to encompass both \( g_a = g \) and \( g_0 \).) Of course then \( f^{ABC} = 0 \) unless all indices are \( SU(N) \). We note that \( d^{ab0} = \sqrt{\frac{2}{N}} \delta^{ab} \), \( d^{000} = \sqrt{\frac{2}{N}} \). (Useful identities for \( U(N) \) are listed in Appendix B.) We also have

\[
e^{abc} = g, \quad e^{a0b} = e^{ab0} = e^{000} = g_0, \quad e^{0ab} = \frac{g^2}{g_0}.
\]

We have written the \( \bar{\phi} \lambda \bar{\lambda} F \) term as it is given starting from the superspace formalism. We note that it has the opposite sign from that given in Ref. [4]. This term is \( \mathcal{N} = \frac{1}{2} \) supersymmetric on its own and so the exact form chosen should not affect the renormalisability.
of the theory. It is easy to show that Eq. (1.1) is invariant under

\[
\begin{align*}
\delta A^\mu & = - i \bar{\lambda}^\alpha \sigma_\mu \epsilon \\
\delta \lambda^\alpha_\alpha & = i \epsilon_\alpha D^A + (\sigma^{\mu\nu} \epsilon)_\alpha [F^A_{\mu
u} + \frac{1}{2} i C_{\mu
u} \epsilon^{ABC} d^{ABC} \lambda^B \bar{\lambda}^C], \quad \delta \bar{\lambda}^\alpha_\alpha = 0, \\
\delta D^A & = - \epsilon \sigma^\mu D_\mu \bar{\lambda}^A, \\
\delta \phi & = \sqrt{2} \epsilon \psi, \quad \delta \bar{\phi} = 0, \\
\delta \psi^\alpha & = \sqrt{2} \epsilon^\alpha F, \quad \delta \bar{\psi}_\dot{\alpha} = - i \sqrt{2} (D_\mu \bar{\psi}) (\epsilon \sigma^\mu)_{\dot{\alpha}}, \\
\delta F^a & = \delta F^0 = 0, \\
\delta \bar{F}^a & = - i \sqrt{2} D_\mu \bar{\psi}^a \sigma^\mu \epsilon - 2 g (\bar{\psi} \epsilon \lambda F)^a \\
& + 2 g C^{\mu\nu} D_\mu (\bar{\psi}^b \epsilon \sigma_\nu \bar{\lambda}^c d^{bca} + \bar{\psi}^b \epsilon \sigma_\nu \bar{\lambda}^0 d^{b0a}) + 2 g C^1_{\mu\nu} D_\mu (\bar{\psi}^0 \epsilon \sigma_\nu \bar{\lambda}^b d^{b0a}), \\
\delta \bar{F}^0 & = - i \sqrt{2} D_\mu \bar{\psi}^0 \sigma^\mu \epsilon \\
& + 2 g C^2_{\mu\nu} D_\mu (\bar{\psi}^a \epsilon \sigma_\nu \bar{\lambda}^b d^{aba}) + 2 g_0 C^{\mu\nu} D_\mu (\bar{\psi}^0 \epsilon \sigma_\nu \bar{\lambda}^0 d^{000}).
\end{align*}
\]

In Eq. (1.1), \( C^{\mu\nu} \) is related to the non-anti-commutativity parameter \( C^{\alpha\beta} \) by

\[
C^{\mu\nu} = C^{\alpha\beta} \epsilon_\alpha \sigma^{\mu\nu} \gamma, \quad (1.6)
\]

where

\[
\begin{align*}
\sigma^{\mu\nu} & = \frac{1}{4} (\sigma^\mu \sigma^\nu - \sigma^\nu \sigma^\mu), \\
\bar{\sigma}^{\mu\nu} & = \frac{1}{4} (\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu),
\end{align*}
\]

and

\[
|C|^2 = C^{\mu\nu} C_{\mu\nu}. \quad (1.8)
\]

Our conventions are in accord with [3]; in particular,

\[
\sigma^\mu \bar{\sigma}^\nu = - \epsilon^{\mu\nu} + 2 \sigma^{\mu\nu}. \quad (1.9)
\]

Properties of \( C \) which follow from Eq. (1.6) are

\[
\begin{align*}
C^{\alpha\beta} & = \frac{1}{2} \epsilon^{\alpha\gamma} (\sigma^{\mu\nu})_\gamma^{\beta} C^{\mu\nu}, \quad (1.10a) \\
C^{\mu\nu} \sigma^{\alpha\beta} & = C^{\alpha\gamma} \sigma^{\mu\nu} \gamma^{\beta}, \quad (1.10b) \\
C^{\mu\nu} \bar{\sigma}^{\alpha\beta} & = - C^{\beta\gamma} \sigma^{\mu\nu} \gamma^{\alpha}. \quad (1.10c)
\end{align*}
\]

In Eqs. (1.1), \( C^{\mu\nu}_{1,2} \) will be identical to \( C^{\mu\nu} \) at the classical level; but we have distinguished them to allow for the possibility of different renormalisations (in practice an
overall numerical factor) at the quantum level; so that $C_{1,2}^{\mu\nu}$ will obey properties analogous to Eqs. (1.6), (1.8) and (1.10). It is important to note that this is only compatible with $\mathcal{N} = \frac{1}{2}$ supersymmetry due to the fact that the $\partial_\mu \bar{\phi}^0 \bar{\lambda}^a \bar{\sigma}_\nu \psi^b$ term in Eq. (1.11) contains no gauge field; and the variation of the gauge field in $D_\mu \bar{\phi}^a \bar{\lambda}^b \bar{\sigma}_\nu \psi^c$ gives zero. This implies that the variations of the terms containing either $C_{1}^{\mu\nu}$ or $C_{2}^{\mu\nu}$ respectively are self-contained. (By contrast, the variation of the gauge field in the $D_\mu \bar{\phi}^b \bar{\lambda}^a \bar{\sigma}_\nu \psi^b$ term is cancelled by the $C^{\mu\nu}$ term in the variation of the $\lambda$ in the $\bar{\phi} \lambda \psi$ term, which forces the $C^{\mu\nu}$ in the 6th line of Eq. (1.1) to be equal to that in the pure gauge terms, and similarly for that in the 7th line; the terms in the 8th line do not get renormalised at all.)

We use the standard gauge-fixing term

$$S_{gf} = \frac{1}{2\alpha} \int d^4x (\partial.A)^2$$

with its associated ghost terms. The gauge propagators for $SU(N)$ and $U(1)$ are both given by

$$\Delta_{\mu\nu} = -\frac{1}{p^2} \left( \eta_{\mu\nu} + (\alpha - 1) \frac{p_\mu p_\nu}{p^2} \right)$$

(omitting group factors) and the gaugino propagator is

$$\Delta_{\alpha\dot{\alpha}} = \frac{p_\mu \sigma^\mu_{\alpha\dot{\alpha}}}{p^2},$$

where the momentum enters at the end of the propagator with the undotted index. The one-loop graphs contributing to the “standard” terms in the lagrangian (those without a $C^{\mu\nu}$) are the same as in the ordinary $\mathcal{N} = 1$ case, so anomalous dimensions and gauge $\beta$-functions are as for $\mathcal{N} = 1$. Since our gauge-fixing term in Eq. (1.11) does not preserve supersymmetry, the anomalous dimensions for $A_\mu$ and $\lambda$ are different (and moreover gauge-parameter dependent), as are those for $\phi$ and $\psi$. However, the gauge $\beta$-functions are of course gauge-independent. The one-loop one-particle-irreducible (1PI) graphs contributing to the new terms (those containing $C$) are depicted in Figs. 1–6. With the exception of Fig. 6 (which gives zero contributions in the case of chiral fields in the fundamental representation) these diagrams are the same as those considered in Ref. [12]. The divergent contributions from these and other diagrams considered later are listed in Appendix A.
2. Renormalisation of the adjoint $SU(N)$ action

The renormalisation of $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory presents certain subtleties. The bare action is given by

$$
S_B = S_{0B} + \frac{1}{N} \gamma_1 g_0^2 |C|^2 (\overline{\lambda}^a \lambda^a) (\overline{\lambda}^0 \lambda^0)
$$

where $S_{0B}$ is obtained by replacing all fields and couplings in $S_0$ (in Eq. (1.1)) by their bare versions, given below. The terms involving $\gamma_1 - 3$ are separately invariant under $\mathcal{N} = \frac{1}{2}$ supersymmetry. Those with $\gamma_1, \gamma_2$ must be included at this stage to obtain a renormalisable lagrangian; those with $\gamma_3$ will be required when we introduce a superpotential but could be omitted at present.

We found in Refs. [11], [12] that non-linear renormalisations of $\lambda$ and $\bar{F}$ were required; and in a subsequent paper [13] we pointed out that non-linear renormalisations of $F$, $\bar{F}$ are required even in ordinary $\mathcal{N} = 1$ supersymmetric gauge theory when working in the uneliminated formalism. Note that in the $\mathcal{N} = \frac{1}{2}$ supersymmetric case, fields and their conjugates may renormalise differently. The renormalisations of the remaining fields and couplings are linear as usual and given by

$$
\bar{\lambda}^a_B = Z_1^{\frac{1}{2}} \lambda^a, \quad A^a_{\mu B} = Z_2 A^a_{\mu}, \quad D^a_B = Z_3^2 D^a, \quad \phi_B^a = Z_5^2 \phi^a,
$$

$$
\psi_B^a = Z_4^2 \psi^a, \quad \bar{\phi}_B^a = Z_6^2 \bar{\phi}^a, \quad \bar{\psi}_B^a = Z_7^2 \bar{\psi}^a, \quad F_B = Z_{1F} F, \quad g_B = Z_{1g} g,
$$

$$
C_{\mu \nu}^B = Z_{C_{\mu \nu}} C_{\mu \nu}, \quad |C|^2 = Z_{|C|^2} |C|^2, \quad C_{1,2B}^{\mu \nu} = Z_{C_{1,2}} C_{1,2}^{\mu \nu}, \quad \gamma_{1-3B} = Z_{1-3}.
$$

The corresponding $U(1)$ gauge multiplet fields $\bar{\lambda}^0$ etc are unrenormalised (as are the $U(1)$ chiral fields $\phi^0$ etc in the case with no superpotential); so is $g_0$. The auxiliary field $F$ is also unrenormalised, i.e. $Z_F = 1$ (though again this will no longer be the case when we later introduce a superpotential). In Eq. (2.2), $Z_{1-3}$ are divergent contributions, in other words we have set the renormalised couplings $\gamma_{1-3}$ to zero for simplicity. The other renormalisation constants start with tree-level values of 1. As we mentioned before, the renormalisation constants for the fields and for the gauge coupling $g$ are the same as in the
ordinary $\mathcal{N} = 1$ supersymmetric theory (for a gauge theory coupled to an adjoint chiral field) and are therefore given up to one loop by

\begin{align}
Z_\lambda &= 1 - g^2 NL(2\alpha + 2), \\
Z_A &= 1 + g^2 NL(1 - \alpha) \\
Z_D &= 1 - 2NLg^2, \\
Z_g &= 1 - 2g^2 NL, \\
Z_\phi &= 1 + 2g^2 (1 - \alpha) LN, \\
Z_\psi &= 1 - 2g^2 (1 + \alpha) LN,
\end{align}

where (using dimensional regularisation with $d = 4 - \epsilon$) $L = \frac{1}{16\pi^2 \epsilon}$. The renormalisation of $\lambda^A$ is given by

\begin{align}
\lambda_B^0 &= Z_\lambda^\frac{1}{2} \lambda^a - \frac{1}{2} NLg^2 C^{\mu\nu} d^{abc} c_\mu \bar{\lambda}^c A^b_{\nu} - NLg^2 g_0 C^{\mu\nu} d^{ab0} c_\mu \bar{\lambda}^0 A^b_{\nu} \\
&\quad + i\sqrt{2}\rho_4 NLg^3 d^{abc} (C_\psi)^b \bar{\phi}^c + i\sqrt{2}\rho_5 NLg^3 d^{ab00} (C_\psi)^b \bar{\phi}^0, \\
\lambda_B^0 &= \lambda_0 i\sqrt{2}\rho_6 NLg^2 g_0 d^{ab0} (C_\psi)^a \bar{\phi}^b,
\end{align}

where $(C_\psi)^a = C^{a\beta} \psi^\beta$. The replacement of $\lambda$ by $\lambda_B$ produces a change in the action given (to first order) by

\begin{align}
S_0(\lambda_B) - S_0(\lambda) = NLg^2 \int d^4 x \Big\{ &\rho_4 g [ig d^{abc} f^{cde} \bar{\phi}^a \bar{\phi}^b \psi^c (C_\psi]^d) \\
&+ \sqrt{2} C^{\mu\nu} d^{abc} \phi^a \bar{\lambda}^b \gamma^\mu \psi^c + \sqrt{2} C^{\mu\nu} d^{abc} D_\mu \bar{\phi}^a \bar{\lambda}^b \gamma^\nu \psi^c] \\
&+ \rho_5 \sqrt{2} g C^{\mu\nu} d^{ab0} (\bar{\phi}^a \bar{\lambda}^b \gamma^\mu \bar{\sigma}_\nu \psi^0 + D_\mu \bar{\phi}^a \bar{\lambda}^b \gamma^\nu \sigma^0 \psi^0) \\
&+ \rho_6 \sqrt{2} g_0 C^{\mu\nu} d^{ab0} (\bar{\phi}^a \bar{\lambda}^0 \gamma^\mu \bar{\sigma}_\nu \psi^b + D_\mu \bar{\phi}^a \bar{\lambda}^0 \gamma^\nu \sigma^b \psi^b) + \ldots \Big\},
\end{align}

where the ellipsis indicates the terms not involving $\rho_{4-6}$ (which were given previously in Ref. [12]). The value of $\rho_4$ will be chosen so as to cancel the divergent contributions from Fig. 6; $\rho_{5,6}$ will be specified later when we renormalise the theory with a superpotential.
We now find that to render finite the contributions linear in $F$ we require

$$\tilde{F}_B^a = Z_F \tilde{F}^a + i C^{\mu\nu} \Lambda g^2 \left\{ g N \left[ (5 + 2\alpha) \partial_\mu A^b_\nu - \frac{1}{4}(11 + 4\alpha) g f^{bde} A^{d}_{\mu} A^{e}_{\nu} \right] \phi^c d^{abc} \\
+ \sqrt{2N} g \left[ 2 ((4 + \alpha) - z_{C_1}) \partial_\mu A^a_\nu - \left( \frac{1}{2}(9 + 2\alpha) - z_{C_1} \right) g f^{abc} A^b_\mu A^c_\nu \phi^0 \right] \\
+ 2\sqrt{2N} g_0 \left( (1 - \alpha) + z_3 \right) \partial_\mu A^0_\nu \phi^0 \right\} \\
+ \frac{1}{8} L g^4 |C|^2 \left[ (2(1 - \alpha) N f^{aee} f^{bde} - 11 N d^{abc} d^{cde} + 4(\delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd})) \phi^b \bar{\chi}^c \bar{\chi}^d \right] \tag{2.6} \\
- L g^3 |C|^2 \left\{ d^{abc} \sqrt{2N} \left[ g \phi^c \bar{\chi}^c \bar{\chi}^0 + 3 g_0 \phi^c \bar{\chi}^c \bar{\chi}^0 \right] + 4 g_0 \phi^0 \bar{\chi}^0 \bar{\chi}^0 \right\},$$

$$\tilde{F}_B^0 = Z_F \tilde{F}^0 + i \sqrt{2N} L (2 + z_2 - z_{C_2}) C^{\mu\nu} F^{a}_{\mu\nu} \phi^0 \\
- 2 d^{abc} g^4 L |C|^2 \sqrt{2N} \phi^0 \bar{\chi}^c \bar{\chi}^0 - 8 g^3 g_0 L |C|^2 \phi^0 \bar{\chi}^0 \bar{\chi}^0.$$}

Writing $Z^{(n)}_C$ for the $n$-loop contribution to $Z_C$ we set

$$Z^{(1)}_C = z_C N L g^2 \tag{2.7}$$

with similar definitions for $Z_{|C|^2}$, $Z_{C1}$, $Z_{C2}$, $Z_{C3}$. We now find that with

$$z_C = z_{|C|^2} = 0, \quad z_{C1} = -z_{C2} = 2, \quad z_1 = -3, \quad \rho_4 = 1, \quad \rho_5 = z_2 - z_{C2}, \quad \rho_6 = z_3, \tag{2.8}$$

the one-loop effective action is finite, for arbitrary $z_2$, $z_3$.

3. The superpotential

We now consider the problem of adding superpotential terms to the lagrangian Eq. (1.1). The following potential terms are $\mathcal{N} = \frac{1}{2}$ invariant at the classical level:

$$S_{\text{int}} = \int d^4x \tr \left\{ y \left[ \phi^2 \tilde{F} - \psi^2 \phi \right] + \bar{\phi}^2 \tilde{F} - \bar{\psi}^2 \phi + \frac{4}{3} i g C^{\mu\nu} \bar{\phi}^3 \tilde{F}_{\mu\nu} + \frac{2}{3} C^{\mu\nu} D_{\mu} \tilde{\phi} D_{\nu} \tilde{\phi} \right\} \tag{3.1} \\
+ m \left[ \phi \tilde{F} - \frac{1}{2} \psi \psi + \bar{\phi} \tilde{F} - \frac{1}{2} \bar{\psi} \bar{\psi} + i C^{\mu\nu} \bar{\phi} \tilde{F}_{\mu\nu} \tilde{\phi} - \frac{4}{3} g^3 |C|^2 \phi \bar{\phi} \bar{\chi}^0 \tilde{\chi}^0 \right].$$

Here in the interests of conciseness we have written the superpotential in index-free form, so that

$$\phi = \phi^A R_A, \quad \psi = \psi^A R_A, \quad \hat{A}_\mu = g A^a_{\mu} R^a + g_0 A^0_{\mu} R^0; \tag{3.2}$$

it then follows that $\tilde{F}_{\mu\nu} = g A^A_{\mu} R^A$, with $F_{\mu\nu}$ defined as in Eq. (1.3). The group matrices are normalised so that $\text{Tr}[R^A R^B] = \frac{1}{2} \delta^{AB}$; in particular, $R^0 = \sqrt{\frac{1}{2N}} 1$. It is easy to
check that $S_{\text{int}}$ is $\mathcal{N} = \frac{1}{2}$ invariant. Except for the last mass term, this superpotential is most readily derived directly from the superspace formalism. Denoting an adjoint chiral superfield as $\Phi_A$, we have that under a gauge transformation

$$\Phi_A \rightarrow \Omega \ast \Phi_A \ast \Omega^{-1}, \quad \overline{\Phi}_A \rightarrow \overline{\Omega} \ast \overline{\Phi}_A \ast \overline{\Omega}^{-1},$$

so that the gauge interactions are written in superfield form as

$$\int d^4 \theta \text{tr} \left[ \overline{\Phi}_A \ast e^V \ast \Phi_A \ast e^{-V} \right].$$

The following superpotential terms are manifestly also invariant:

$$\int d^2 \theta \text{tr} \left[ \frac{1}{2} m \Phi_A \ast \Phi_A + \frac{1}{3} y \Phi_A \ast \Phi_A \ast \Phi_A \right]$$

$$+ \int d^2 \overline{\theta} \text{tr} \left[ \frac{1}{2} m \overline{\Phi}_A \ast \Phi_A + \frac{1}{3} y \overline{\Phi}_A \ast \Phi_A \ast \Phi_A \right]. \tag{3.3}$$

Expanded in component fields we have

$$\Phi_A(y, \theta) = \phi(y) + \sqrt{2} \theta \psi(y) + \theta \theta F(y) \tag{3.4a}$$

$$\overline{\Phi}_A(\overline{y}, \overline{\theta}) = \overline{\phi}(\overline{y}) + \sqrt{2} \overline{\theta} \overline{\psi}(\overline{y})$$

$$+ \overline{\theta} \left( F(\overline{y}) + ig C^{\mu \nu} \partial_\mu \{ \phi, A_\nu \}(\overline{y}) - \frac{g^2}{2} C^{\mu \nu} [A_\mu, \{ A_\nu, \phi \}](\overline{y}) \right), \tag{3.4b}$$

where $\overline{y}^\mu = y^\mu - 2i \theta \sigma^\mu \overline{\theta}$. Note the modification of the $\overline{\theta} \theta$-term.[4]

If we substitute Eq. (3.4) in Eq. (3.3) we obtain Eq. (3.1) except for the last term. (This can also be expressed in superfields but in a more unwieldy form). The coefficient of this final term is arbitrary since it is separately $\mathcal{N} = \frac{1}{2}$ invariant; the reason for our particular choice will be explained later (after Eq. (A.18) in Appendix A). A similar set of mass terms is admissible in the case of the fundamental representation, with mass terms coupling the fundamental and anti-fundamental representation fields[17]. However, no trilinear term is possible in the $\mathcal{N} = \frac{1}{2}$ case for the fundamental representation. If we have both adjoint and fundamental (antifundamental) representations $\Phi(\overline{\Phi})$ we can construct $\mathcal{N} = 2$-type invariants, of the form

$$y \left[ \int d^2 \theta \overline{\Phi} \ast \Phi_A \ast \Phi + \int d^2 \overline{\theta} \Phi_A \ast \overline{\Phi}_A \ast \overline{\Phi} \right]. \tag{3.5}$$

At the classical level $\phi$ may be considered as forming a representation of $U(N)$. However, just as we saw in Ref. [12] for the gauge group, the $U(N)$ structure is not preserved.
at the quantum level. The $\phi^a$ renormalise differently from the $\phi^0$ and this means that, for instance, there must be a different mass parameter ($m$, say) for the $\phi^a F^a$, $\psi^a \psi^a$ terms than for the $\phi^0 F^0$, $\psi^0 \psi^0$ terms ($m_0$, say). In the case of the mass terms this does not present serious difficulty since we can separate the mass terms in Eq. (3.1) into separately $\mathcal{N} = \frac{1}{2}$ invariant sets of terms involving either $m$ or $m_0$. However, in the case of the trilinear superpotential terms, we need to invoke three separate couplings, one ($y$, say) for $\phi^a \phi^b F^c$ terms, one ($y_1$, say) for $\phi^a \phi^b F^0$, $\phi^a \phi^0 F^b$ etc and one ($y_2$, say) for $\phi^0 \phi^0 F^0$. In the $\mathcal{N} = 1$ case the theory would, of course, be renormalisable, with each of $y$, $y_{1,2}$ renormalising differently. By contrast, in the $\mathcal{N} = \frac{1}{2}$ case many of the $\bar{\phi}^3 A_\mu$ terms are linked by $\mathcal{N} = \frac{1}{2}$ transformations to more than one of these groups of terms and so cannot be assigned a unique coupling out of $y$, $y_{1,2}$. So in the presence of trilinear superpotential terms, the $\mathcal{N} = \frac{1}{2}$ invariance cannot be maintained at the quantum level. It is this linking of different groups of terms, specifically those corresponding purely to $SU(N)$ with those containing $U(1)$ fields, which implies that we cannot have an $\mathcal{N} = \frac{1}{2}$ theory with a superpotential if the chiral fields belong to $SU(N)$ alone.
4. The renormalised action with superpotential

As we explained in the previous section, many of the individual terms with couplings \( m \) or \( y \) in Eq. \((3.1)\) will renormalise differently and hence need to be assigned their own separate couplings. For renormalisability, Eq. \((3.1)\) needs to be replaced by

\[
S_{\text{int}} = \int d^4 x \left\{ \frac{1}{4} y d^{abc}(\phi^a \phi^b F^c - \psi^a \psi^b \phi^c) + \frac{1}{4} y_1 d^{abc} \phi^{00}(\phi^a \phi^b F^{0}) - \psi^a \psi^b \phi^c - 2 \psi^a \psi^0 \phi^0 \right\}
\]

\[
+ \frac{1}{4} y_2 d^{000}(\phi^0 \phi^0 F^{0}) - \psi^0 \psi^0 \phi^0 \right\} + \frac{1}{4} y_4 F_{\mu \nu}^d \left( \frac{1}{4} y d^{abc} d^{d e f} \phi^a \phi^b \phi^c + \frac{1}{3} y_3 \frac{1}{N} \delta^{a b c} \phi^a \phi^b \phi^c
\]

\[
+ \frac{1}{2} y_4 \sqrt{\frac{2}{N}} d^{abc} \bar{\phi}^a \phi^b \phi^c + y_5 \frac{1}{N} \phi^0 \bar{\phi}^0 \phi^0 \right\} \right)
\]

\[
+ i g C^{\mu \nu} F_{\mu \nu}^d \left( \frac{1}{6} y d^{abc} d^{d e f} \phi^a \phi^b \phi^c + y_1 \frac{1}{N} \phi^0 \bar{\phi}^0 \phi^0 + \frac{1}{3} y_2 \frac{1}{N} \phi^0 \bar{\phi}^0 \phi^0 \right)
\]

\[
+ \frac{1}{6} i g C^{\mu \nu} f^{a b c}(D_{\mu} \bar{\phi}^a)(D_{\nu} \phi^b) \phi^c
\]

\[
+ \frac{1}{N} \left[ \phi^a F^a - \frac{1}{2} \psi^0 \phi^a + \bar{\phi}^a \bar{\phi}^a - \frac{1}{2} \bar{\psi}^0 \phi^a
\]

\[
+ \frac{1}{2} i g C^{\mu \nu} d^{abc} F_{\mu \nu}^c \phi^a \phi^b + \frac{1}{2} i g C_{1} d^{abc} F_{\mu \nu}^a \phi^b \phi^c + \frac{1}{2} i g C_{1} d^{abc} F_{\mu \nu} \phi^a \phi^b
\]

\[
+ m_0 \left[ \phi^0 \bar{F}^0 - \frac{1}{2} \psi^0 \phi^0 + \bar{\phi}^0 \bar{F}^0 - \frac{1}{2} \bar{\psi}^0 \psi^0
\]

\[
+ \frac{1}{2} i g C_{2} d^{abc} F_{\mu \nu} \phi^0 \phi^b + \frac{1}{2} i g C_{2} d^{abc} F_{\mu \nu} \phi^0 \phi^c
\]

\[
+ \frac{1}{2} i g C_{2} d^{abc} F_{\mu \nu} \phi^0 \phi^c
\]

\[
+ |C|^2 \left[ - \frac{1}{6} \phi^a \bar{\phi}^a \lambda^0 + \frac{1}{2} \phi^a \phi^a \lambda^c \lambda^d
\]

\[
+ \frac{1}{2} \phi^a \phi^a \lambda^0 \right] \right)
\]

Each of the coefficients \( m, y, \) etc above will renormalise separately. However, for simplicity when we quote the results for Feynman diagrams, we will use the values of the coefficients as implied by Eq. \((3.1)\), i.e. \( y_{1-5} = y, m_0 = \mu_1 = m, \mu_{2-5} = 0, \) so that these are effectively the renormalised values of these couplings. Note that the \( g_0 C^{\mu \nu} F_{\mu \nu}^0 \sqrt{2} d^{abc} \bar{\phi}^a \phi^b \phi^c \) and \( \frac{1}{N} g_0 C^{\mu \nu} F_{\mu \nu}^0 \phi^a \phi^a \phi^0 \) terms only mix with the \( d^{abc} \bar{\phi}^a \phi^b \phi^c \) or \( d^{abc} \phi^a \phi^b \phi^c \) fields respectively and hence can be assigned the coupling \( y \) or \( y_1 \) respectively.
The renormalisation constants $Z_{\phi, \psi}$, $Z_F$ now acquire $y$-dependent contributions, so we have

\begin{align}
Z_{\phi} &= 1 + \left[-\frac{1}{4}y^2 + 2g^2(1 - \alpha)\right]LN, \\
Z_{\psi} &= 1 + \left[-\frac{1}{4}y^2 - 2g^2(1 + \alpha)\right]LN, \\
Z_{\phi^0} &= Z_{\psi^0} = 1 - \frac{1}{4}y^2LN, \\
Z_F &= 1 - \frac{1}{4}y^2LN.
\end{align}

(4.2)

Here we write $\phi_B^0 = Z_{\phi^0} \frac{1}{2} \phi^0$, etc, since the $U(1)$ chiral fields are now renormalised. Now for the bare action we also need to replace $m_B = Z_m m_B$, $y_B = Z_y y_B$ etc in addition to the replacements given earlier. These renormalisation constants are given according to the non-renormalisation theorem by

\begin{align}
Z_m &= Z_{\phi}^{-1}, \\
Z_{m_0} &= Z_{\phi^0}^{-1}, \\
Z_y &= Z_{\phi}^{-\frac{3}{2}}, \\
Z_{y_1} &= Z_{\phi}^{-1} Z_{\phi^0}^{-\frac{1}{2}}, \\
Z_{y_2} &= Z_{\phi^0}^{-\frac{3}{2}},
\end{align}

(4.3)

where $Z_{\phi}$, $Z_{\phi^0}$ are the renormalisation constants for the chiral superfield $\Phi$ given by

\begin{align}
Z_{\phi} &= 1 + \left[-\frac{1}{4}y^2 + 4g^2\right]LN, \\
Z_{\phi^0} &= 1 - \frac{1}{4}y^2LN.
\end{align}

(4.4)

The redefinitions of $F$ and $\bar{F}$ found in Ref. [11] need to be modified in the presence of mass terms and the $U(1)$ gauge group. This is easily done following the arguments of Ref. [15]: there are no one-loop diagrams giving divergent contributions to $m\phi F$ or $m\bar{\phi}\bar{F}$ although there are counterterm contributions from $m_B \phi_B F$, $m_B \bar{\phi}_B \bar{F}$. At one loop we have

\begin{align}
\bar{F}_B' &= \bar{F}_B + (\alpha + 3)g^2NL (m\bar{\phi}^a + \frac{1}{4}yd^{abc}\bar{\phi}^b \bar{\phi}^c) + \frac{1}{2}(\alpha + 3)yg^2N\bar{d}^{ab0} \bar{\phi}^b \bar{\phi}^0, \\
\bar{F}_B^0 &= \bar{F}_B^0, \\
F_B' &= Z_F F_B + (\alpha + 3)g^2NL (m\phi^a + \frac{1}{4}yd^{abc}\phi^b \phi^c) + \frac{1}{2}(\alpha + 3)yg^2N\phi^{ab0} \phi^b \phi^0, \\
F_B^0 &= Z_F F_B^0.
\end{align}

(4.5)
Here $\tilde{F}_B^a$ etc are as given in Eq. (2.4), though of course using the non-zero $Z_F$ given in Eq. (1.2). The new $C$-dependent diagrams in the presence of a superpotential are depicted in Figs. 7–11, and their divergent contributions in the corresponding Tables. We omit diagrams giving contributions of the form $A_\mu A_\nu \bar{\phi}^3$ which complete the $F_{\mu\nu}$ in $F_{\mu\nu} \bar{\phi}^3$ contributions; we already have ample evidence that gauge invariance, even when apparently violated, can be restored by making divergent field redefinitions. We also omit diagrams of the form $\bar{\phi}^3 \bar{\lambda}^2$; these are separately $N = \frac{1}{2}$ invariant and are not going to give any more information about the preservation of $N = \frac{1}{2}$ supersymmetry.

We now choose the renormalisation constants at our disposal to ensure finiteness. In order to ensure renormalisability of the action in Eq. (4.1), we find we now need to impose specific values for the hitherto arbitrary coefficients $z_2, z_3$, namely

$$z_2 = -4, \quad z_3 = 4. \quad (4.6)$$

We find moreover

$$Z_{y_3} = 1 - 6LN g^2,$$
$$Z_{y_4} = 1 - 4LN g^2,$$
$$Z_{y_5} = 1 - 2LN g^2,$$
$$Z_{\mu_1} = 1 + \frac{32}{3N} L g^2 \left( 1 - \frac{g^2}{g_0^2} \right),$$
$$Z_{\mu_2} = - \frac{4}{g_0^2} L,$$
$$Z_{\mu_3} = 0,$$
$$Z_{\mu_4} = 2LN g^2,$$
$$Z_{\mu_5} = 4LN g^2. \quad (4.7)$$
5. The eliminated formalism

It is instructive and also provides a useful check to perform the calculation in the eliminated formalism. In the eliminated case Eq. (4.1) is replaced by

\[
\tilde{S}_{\text{mass}} = \int \! d^4 x \left\{ -\frac{1}{4} y d^{abc} \psi^a \psi^b \phi^c - \frac{1}{4} y_1 d^{ab0} (\psi^a \psi^b \phi^0) + 2 \psi^a \psi^0 \phi^b + \frac{1}{2} y_2 d^{000} \psi^0 \psi^0 \phi^0 - \frac{1}{2} y_2 d^{000} \psi^0 \psi^0 \phi^0 \\
- \frac{1}{4} y d^{abc} \bar{\phi}^a \psi^b \phi^c - \frac{1}{4} y_1 d^{ab0} (\bar{\phi}^a \psi^b \phi^0) - \frac{1}{4} y y_1 d^{ab0} (\bar{\phi}^a \psi^b \phi^0) - \frac{1}{4} y_2 d^{000} \psi^0 \phi^0 \phi^0 \\
+ m^2 \bar{\phi}^a \phi^a - \frac{1}{2} m \psi^a \psi^a - \frac{1}{2} m \bar{\phi}^a \phi^0 - \frac{1}{2} m_0 \bar{\phi}^0 \phi^0 - \frac{1}{2} m_0 \bar{\phi}^0 \phi^0 \\
- (y d^{ab} \phi^a \phi^b + 2 y_1 d^{ab0} \phi^a \phi^0) \left( y d^{ab} \bar{\phi}^a \phi^b + 2 y_1 d^{ab0} \bar{\phi}^a \phi^0 \right) \\
- (y d^{ab} \phi^a \phi^b + y_2 d^{00} \phi^0 \phi^0) \left( y_1 d^{ab} \bar{\phi}^a \phi^b + y_2 d^{000} \bar{\phi}^0 \phi^0 \right) \\
+ \frac{1}{6} i y C^{\mu\nu} F^{ab}_{\mu\nu} (D_\mu \bar{\phi})^a (D_\nu \phi)^b \phi^c \\
+ \frac{1}{6} i g F^{d}_{\mu\nu} \left( -\frac{1}{2} y C^{\mu\nu} F^{d}_{\mu\nu} d^{abc} \bar{\phi}^a \phi^b \phi^c + 2 y_3 C^{\mu\nu} - 3 y_1 (1 - Z_2) C^{\mu\nu} \right) \frac{1}{N} \delta^{a\bar{b}} \delta^{c\bar{d}} \bar{\phi}^a \phi^b \phi^c \\
+ 3 [(y_4 - y_1) C^{\mu\nu} - \frac{1}{2} y C^1_{\mu\nu}] \sqrt{\frac{2}{N}} d^{abcd} \bar{\phi}^a \phi^b \phi^c \\
+ 6 [y_5 C^{\mu\nu} - y_1 C^1_{\mu\nu}] \frac{1}{2} y_2 (1 - Z_2) C^{\mu\nu} \frac{1}{N} \bar{\phi}^a \phi^0 \phi^0 \\
- \frac{1}{12} y_0 C^{\mu\nu} F^{\mu\nu} \left( y (1 - 3 Z_3) \sqrt{\frac{2}{N}} d^{abc} \bar{\phi}^a \phi^b \phi^c + 6 y_1 (1 - 2 Z_3) \frac{1}{N} \bar{\phi}^a \phi^0 \phi^0 \\
+ 2 y_2 \frac{1}{N} \bar{\phi}^a \phi^0 \phi^0 \right) \\
- \frac{1}{2} \left\{ g m d^{abc} C^{\mu\nu} F^{c}_{\mu\nu} \bar{\phi}^a \phi^b + g m_0 \left( 1 - 2 Z_3 \right) C^{\mu\nu} d^{ab0} F^{0}_{\mu\nu} \bar{\phi}^a \phi^b \\
+ g \left[ m C^1_{\mu\nu} + m_0 (1 - 2 Z_2) C^{\mu\nu} \right] d^{ab0} F^{b}_{\mu\nu} \bar{\phi}^a \phi^0 \right\} \\
+ g^2 C^2 \left[ -\frac{1}{8} (\mu_1 - 2 m) f^{ace} f^{bde} + \mu_2 \frac{2}{N} \delta^{ab} \delta^{cd} \right] \bar{\phi}^a \phi^b \bar{\lambda}^c \lambda^d \\
+ g d^{abc} \sqrt{2N} C^2 \bar{\phi}^a \lambda^b \left( \mu_3 g_0 \bar{\phi}^a \lambda^0 + g \mu_4 \bar{\phi}^0 \lambda^c \right) + \frac{1}{N} \mu_5 g_0 |C|^2 \bar{\phi}^a \phi^0 \lambda^a \lambda^0 \right\}
\right\}
\end{array}
\right.
\right\} (5.1)

while we simply strike out the terms involving \( F, \bar{F} \) in Eq. (4.1). Once again note that in quoting diagrammatic results we set \( y_{1-5} = y, m_0 = \mu_1 = m, \mu_2 = 0 \), so that these are effectively the renormalised values of these couplings. In Table 7, the contributions from Figs. 7(f-k) are now absent while those from Figs. 7(l-r) change sign. Similarly, in Table 8, the contributions from Figs. 8(e-p) are now absent while those from Figs. 8(q-dd) change sign. In Table 9, the contributions from Figs. 9(f-n) are now absent while those from Figs. 9(o-z) change sign. In Table 10, the contribution from Fig. 10(d) is now absent. In Table 11, the contributions from Figs. 11(j-o) are now absent while those from Figs. 11(p-v) which contain two factors of \( d^{abc} \) acquire an additional factor of \( (-\frac{1}{2}) \). The
loop level, and also that certain groups of trilinear terms for which
N sup2
fields and so at the quantum level we are obliged to consider
quantum corrections, the
N sup2
invariant set of mass terms and an
invariance of the trilinear terms requires the chiral matter be in the adjoint
representation of
N sup2
in transformation mix superpotential terms with
SU
sup2
invariant set of trilinear terms for this case.

\[
\Gamma^{(1)\text{pole}}_{71\text{Pelim}} = i g^2 C^{\mu \nu} \left[ -\frac{1}{2} (7 + 5\alpha) N g^{ab} \partial_\mu \phi^a \bar{\phi}^b \bar{\phi}^c \\
+ 3(1 - \alpha) g \sqrt{2N} \partial_\mu \phi^a \bar{\phi}^a \bar{\phi}^0 - 2(5 + \alpha) g_0 \sqrt{2N} \partial_\mu \phi^0 \bar{\phi}^a \bar{\phi}^0 \right],
\]

\[
\Gamma^{(1)\text{pole}}_{81\text{Pelim}} = i g^4 C^{\mu \nu} f^{ab} A^b_\mu A^b_\nu \left[ \frac{1}{2} (5 + 3\alpha) N d^{cde} \bar{\phi}^c \bar{\phi}^d + 2\alpha \sqrt{2N} \phi^e \bar{\phi}^0 \right],
\]

\[
\Gamma^{(1)\text{pole}}_{91\text{Pelim}} = |C|^2 m_L \left\{ \frac{2 g^2}{\sqrt{2N}} \delta^{ab} \delta^{cd} + \left\{ \frac{1}{2} N (3 + \alpha) + \frac{4}{N} \left( 1 - \frac{2}{a} \right) \right\} f^{ace} f^{bde} \right\} g^{4 \bar{\phi}^a \bar{\phi}^b \bar{\phi}^c \bar{\phi}^c} \\
- 2g^4 \bar{g}^{abc} \sqrt{2N} \phi^a \bar{\phi}^b \bar{\phi}^c \bar{\phi}^d - 8g^3 \bar{g}^a \bar{\phi}^0 \bar{\phi}^a \bar{\phi}^0 \bar{\phi}^0 \right\},
\]

\[
\Gamma^{(1)\text{pole}}_{101\text{Pelim}} = \Gamma^{(1)\text{pole}}_{101\text{Pelim}},
\]

\[
\Gamma^{(1)\text{pole}}_{111\text{Pelim}} = i C^{\mu \nu} \lambda g^2 \left\{ -\frac{1}{2} g \left( 3 + \frac{3}{2} \alpha \right) N f^{ab} f^{cde} \partial_\mu \bar{\phi}^a \bar{\phi}^b \bar{\phi}^c A^d_{\nu} \\
+ \left\{ - \left( \frac{3}{4} + \frac{7}{12} \alpha \right) d^{ab} d^{cd} + ( \frac{5}{2} - \frac{7}{6} \alpha ) \delta^{ab} \delta^{cd} \right\} g \bar{\phi}^a \bar{\phi}^b \bar{\phi}^c \partial_\mu A^d_{\nu} \\
- \frac{1}{4} (7 + 5\alpha) g \sqrt{2N} a^{abc} \bar{\phi}^0 \bar{\phi}^a \bar{\phi}^b \partial_\mu A^c_{\nu} + \frac{3}{2} (1 - \alpha) g \bar{\phi}^0 \bar{\phi}^0 \bar{\phi}^a \partial_\mu A^a_{\nu} \\
- \frac{1}{2} (5 + \alpha) g_0 \sqrt{2N} a^{abc} \bar{\phi}^0 \bar{\phi}^a \bar{\phi}^c \partial_\mu A^0_{\nu} - 2(5 + \alpha) g_0 \bar{\phi}^0 \bar{\phi}^a \bar{\phi}^0 \partial_\mu A^0_{\nu} \right\},
\]

(5.2)

respectively. The results in Eq. (1.7) are unchanged, which is a very good check on the calculation.

6. Conclusions

We have repeated our earlier one-loop analysis of \( \mathcal{N} = \frac{1}{2} \) supersymmetry for the case of chiral matter in the adjoint representation. We have constructed an \( \mathcal{N} = \frac{1}{2} \) invariant set of mass terms and an \( \mathcal{N} = \frac{1}{2} \) invariant set of trilinear terms for this case. The \( \mathcal{N} = \frac{1}{2} \) invariance of the trilinear terms requires the chiral matter be in the adjoint representation of \( U(\mathcal{N}) \) rather than \( SU(\mathcal{N}) \) at the classical level. However, once we consider quantum corrections, the \( U(1) \) chiral fields will renormalise differently from the \( SU(\mathcal{N}) \) fields and so at the quantum level we are obliged to consider \( SU(\mathcal{N}) \otimes U(1) \) rather than \( U(\mathcal{N}) \). On the other hand, the \( \mathcal{N} = \frac{1}{2} \) transformations mix superpotential terms with different kinds of field (\( SU(\mathcal{N}) \) or \( U(1) \)) and so it is clear that the \( \mathcal{N} = \frac{1}{2} \) invariance of the trilinear terms cannot be preserved at the quantum level. We have shown that the \( \mathcal{N} = \frac{1}{2} \) supersymmetry of the mass terms is preserved under renormalisation at the one-loop level, and also that certain groups of trilinear terms for which \( \mathcal{N} = \frac{1}{2} \) supersymmetry does not mix the different gauge groups remain \( \mathcal{N} = \frac{1}{2} \) supersymmetric at one loop. However the renormalisability is assured by making a particular choice of the parameters
\( \gamma_2, \gamma_3 \) (in Eq. 2.1), as determined by Eq. (4.6). This also implies (through Eq. (2.8)) a particular choice of renormalisation for the gaugino \( \lambda \), parametrised by \( \rho_5 \) (in Eq. 2.4). The necessity for these choices seems somewhat counterintuitive as these renormalisations are all present in the theory without superpotential and yet there appeared to be nothing in the theory without superpotential to enforce these choices. It would be reassuring if some independent confirmation could be found for these particular values. Presumably the necessity for the non-linear renormalisations we are compelled to make lies in our use of a non-supersymmetric gauge (the obvious choice when working in components, of course). So the answer to this puzzle might lie in a close scrutiny of the gauge-invariance Ward identities. Of course a calculation in superspace would also be illuminating. It is always tempting to investigate whether the behaviour at one loop persists to higher orders but the proliferation of diagrams in this case would almost certainly be prohibitive.

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Appendix A. Results for one-loop diagrams

In this Appendix we list the divergent contributions from the various one-loop diagrams.

The contributions from the graphs shown in Fig. 1 are of the form

\[
\sqrt{2} N g^2 g_B L C^{\mu \nu} d^{ABC} \left( \partial_\mu \bar{\phi}^A X_1^{ABC} \bar{\lambda}^B \bar{\sigma}_\nu \psi^C + \bar{\phi}^A Y_1^{ABC} \bar{\lambda}^B \bar{\sigma}_\nu \partial_\mu \psi^C \right)
\]

(A.1)

where \( X_1^{ABC} \) and \( Y_1^{ABC} \) consist of a number \( X_1, Y_1 \) multiplying a tensor structure formed of a product of terms like \( c^A \) or \( d^A \), where \( c^A = 1 - \delta^{A0} \), \( d^A = 1 + \delta^{A0} \). The \( X_1, Y_1 \) and the tensor structures are given separately in Table 1. (The contributions from Figs. 2–4, 7, 8 also involve tensors \( X_i^{ABC} \), \( Y_i^{ABC} \) etc (for Fig. \( i \)) which can be decomposed similarly and will be similarly presented.)
Table 1: Contributions from Fig. 1

The sum of the contributions from Table 1 can be written in the form

$$\Gamma^{(1)\text{pole}}_{11\Pi} = Ng^2 \sqrt{2} LC^{\mu\nu} \left[ (2 + 3\alpha) gd^{abc} \partial_{\mu} \bar{\phi}^a \bar{\lambda}^b \sigma_{\nu} \psi^c - gd^{abc} \bar{\phi}^a \bar{\lambda}^b \sigma_{\nu} \partial_{\mu} \psi^c ight. \\
\left. + 2(1 + \alpha) gd^{ab0} \partial_{\mu} \bar{\phi}^a \bar{\lambda}^b \sigma_{\nu} \psi^0 - 2gd^{ab0} \bar{\phi}^a \bar{\lambda}^b \sigma_{\nu} \partial_{\mu} \psi^0 \\
+ 2\alpha g_0 d^{ab0} \partial_{\mu} \bar{\phi}^a \bar{\lambda}^b \sigma_{\nu} \psi^b \\
+ 2(1 + \alpha) gd^{a0b} \partial_{\mu} \bar{\phi}^a \bar{\lambda}^0 \sigma_{\nu} \psi^b \right] \quad (A.2)$$

The contributions from the graphs shown in Fig. 2 are of the form

$$\sqrt{2} g^3 gC NLC^{\mu\nu} A^A_{\mu} \bar{\phi}^B \bar{\lambda}^C \sigma_{\nu} \psi^D \left( X_2^{ABCD} f^{BAE} d^{CDE} + Y_2^{ABCD} f^{DAE} d^{CBE} + Z_2^{ABCD} f^{BDE} d^{CAE} \right) \quad (A.3)$$

where $g_e \equiv g$. The $X_2$, $Y_2$, $Z_2$ and tensor products in the decomposition of $X_2^{ABCD}$, $Y_2^{ABCD}$ and $Z_2^{ABCD}$ (as described earlier) are shown in Table 2:
| Fig. | $X_2$       | $Y_2$       | $Z_2$       | Tensor                                      |
|------|-------------|-------------|-------------|---------------------------------------------|
| 2a   | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $c^A_c^B_c^C_d^D$                          |
| 2b   | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$  | $c^A_c^B_c^C_d^D$                          |
| 2c   | 1           | -1          | 1           | $c^A_c^B_c^C_d^D$                          |
| 2d   | -1          | -1          | -1          | $c^A_c^B_c^C_d^D$                          |
| 2e   | 1           | 0           | 0           | $c^A_c^B_c^C_c^D$                          |
| 2f   | $-\frac{1}{4}(1 - \alpha)$ | $\frac{1}{4}(1 - \alpha)$ | $-\frac{1}{4}(1 - \alpha)$ | $c^A_c^B_d^C_c^D$                          |
| 2g   | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$  | $c^A_c^B_c^C_c^D$                          |
| 2h   | $\frac{1}{2}\alpha$ | 0         | 0           | $c^A_c^B$                                  |
| 2i   | $\frac{3}{4}\alpha$ | 0         | 0           | $c^A_c^B$                                  |
| 2j   | $-\frac{3}{4}(3 + \alpha)$ | 0         | 0           | $c^A_c^B$                                  |
| 2k   | $\frac{1}{8}\alpha$ | $-\frac{1}{8}\alpha$ | $\frac{1}{8}\alpha$ | $c^A_c^B_d^C_c^D$                           |
| 2l   | $-\frac{3}{8}(1 - \alpha)$ | $-\frac{1}{8}(1 - \alpha)$ | $\frac{1}{8}(1 - \alpha)$ | $c^A_c^B_d^C_c^D$                           |
| 2m   | $\frac{1}{2}\alpha$ | $\frac{1}{2}\alpha$ | $-\frac{1}{2}\alpha$ | $c^A_c^B_d^C_c^D$                           |
| 2n   | $\frac{1}{7}\alpha$ | $-\frac{1}{7}\alpha$ | $\frac{1}{7}\alpha$ | $c^A_c^B_c^C_d^D$                           |
| 2o   | $\frac{1}{4}\alpha$ | $\frac{1}{4}\alpha$ | $\frac{1}{4}\alpha$ | $c^A_c^B_d^C_c^D$                           |
| 2p   | $\frac{3}{8}(3 + \alpha)$ | $-\frac{1}{8}(3 + \alpha)$ | $\frac{1}{8}(3 + \alpha)$ | $c^A_c^B_c^C_c^D$                           |
| 2q   | $\alpha$   | 0           | 0           | $c^A_c^B_c^C_c^D$                          |
| 2r   | $-\frac{1}{4}\alpha$ | $\frac{1}{4}\alpha$ | $-\frac{1}{4}\alpha$ | $c^A_c^B_c^C_d^D$                           |
| 2s   | $\frac{3}{8}(1 + \alpha)$ | $\frac{3}{8}(1 + \alpha)$ | $-\frac{3}{8}(1 + \alpha)$ | $c^A_c^B_c^C_c^D$                           |
| 2t   | $-\frac{1}{2}\alpha$ | $-\frac{1}{2}\alpha$ | $-\frac{1}{2}\alpha$ | $c^A_c^B_c^C_c^D$                           |
| 2u   | $\frac{1}{2}\alpha$ | $\frac{1}{2}\alpha$ | $\frac{1}{2}\alpha$ | $c^A_c^B_c^C_d^D$                           |
| 2v   | $-\frac{3}{8}\alpha$ | $-\frac{3}{8}\alpha$ | $\frac{3}{8}\alpha$ | $c^A_c^B_c^C_d^D$                           |
| 2w   | $-\frac{1}{4}(3 + \alpha)$ | $-\frac{1}{4}(3 + \alpha)$ | $-\frac{1}{4}(3 + \alpha)$ | $c^A_c^B_d^C_c^D$                           |
| 2x   | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$  | $c^A_c^B_d^C_c^D$                          |
| 2y   | 1           | -1          | 1           | $c^A_c^B_d^C_c^D$                          |

Table 2: Contributions from Fig. 2
The sum of the contributions from Table 2 can be written in the form

\[
\Gamma_{21\text{pole}}^{(1)} = \sqrt{2} g^3 L C^{\mu\nu} A^a_{\mu} \left[ \left( \frac{7}{2} (1 + \alpha) f_{bae} d_{cde} - f_{dace} d_{cbe} + \frac{1}{2} f_{bde} d_{cae} \right) N g^b_{\bar{\phi}} \bar{\chi}^{\nu} \bar{\sigma}_{\nu} \psi^d - \frac{3}{2} (1 + 5 \alpha) \sqrt{2} Ng_{\bar{\phi}} f_{abc} \bar{\chi}^{0} \bar{\sigma}_{0} \psi^c \right] \quad (A.4)
\]

The contributions from Fig. 3 are of the form

\[
i g^3 N L C^{\mu\nu} (\partial_{\mu} A^A_{\nu} \bar{\phi} X_3^{ABC} F^C + A^{A}_{\nu} \partial_{\mu} \bar{\phi} Y_3^{ABC} F^C) d^{ABC} \quad (A.5)
\]

where the \( X_3 \), \( Y_3 \) and tensor products in the decomposition of \( X_3^{ABC} \) and \( Y_3^{ABC} \) are given in Table 3:

\[
\begin{array}{cccc}
\text{Fig.} & X_3 & Y_3 & \text{Tensor} \\
3a & 0 & 3 & c^A c^B d^C \\
3b & 0 & -2 & c^A c^B d^C \\
3c & 1 & 1 & c^A c^B d^C \\
3d & -(5 + \alpha) & 0 & c^A \\
3e & 2\alpha & -2 & c^A c^B d^C
\end{array}
\]

Table 3: Contributions from Fig. 3

The contributions from Table 3 add to

\[
\Gamma_{31\text{pole}}^{(1)} = i N g^3 L C^{\mu\nu} \left[ -(4 - \alpha) d^{abc} \bar{\phi} \partial_{\mu} A^a_{\nu} F^c - 3 (1 - \alpha) d^{ab0} \bar{\phi} \partial_{\mu} A^b_{\nu} F^0 - (5 + \alpha) d^{abc} \bar{\phi} \partial_{\mu} A^a_{\nu} F^c \right]. \quad (A.6)
\]

The contributions from Fig. 4 are of the form

\[
i g^4 N L C^{\mu\nu} A^A_{\mu} A^B_{\nu} (X_4^{ABCD} f^{ABE} d^{CDE} + Y_4^{ABCD} f^{ACE} d^{BDE}) \bar{\phi}^C F^D \quad (A.7)
\]
where the $X_4$ and $Y_4$ and tensor products in the usual decomposition are given in Table 4:

| Fig. | $X_4$ | $Y_4$ | Tensor |
|------|-------|-------|--------|
| 4a   | $-\frac{3}{4}\alpha$ | 0     | $e^A e^B e^C d^D$ |
| 4b   | $\frac{1}{2}\alpha$  | $\alpha$ | $e^A e^B e^C d^D$ |
| 4c   | $-\frac{1}{2}\alpha$  | $-\alpha$ | $e^A e^B e^C d^D$ |
| 4d   | 0     | 0     | $e^A e^B e^C d^D$ |
| 4e   | $\frac{1}{4}(2 + \alpha)$ | $2 + \alpha$ | $e^A e^B e^C d^D$ |
| 4f   | $-\frac{1}{2}$      | 1     | $e^A e^B e^C d^D$ |
| 4g   | $-\frac{3}{2}\alpha$ | 0     | $e^A e^B$ |
| 4h   | $\frac{3}{2}(1 + \alpha)$ | 0     | $e^A e^B$ |
| 4i   | $-\frac{1}{4}(3 + \alpha)$ | $-(3 + \alpha)$ | $e^A e^B e^C d^D$ |
| 4j   | $\frac{1}{2}\alpha$  | 0     | $e^A e^B e^C d^D$ |
| 4k   | $-\frac{3}{4}\alpha$ | 0     | $e^A e^B e^C d^D$ |
| 4l   | 0     | 0     | |

Table 4: Contributions from Fig. 4

The contributions from Table 4 add to

$$
\Gamma_{41\Pi}^{(1)\text{pole}} = ig^4 L C^{\mu\nu} A_\mu^a A_\nu^b \left( \frac{1}{4}(3 - 4\alpha)N f^{abef} d^{cde} \bar{\phi}^c F^d \right) \\
- 2\alpha \sqrt{2N} f^{abef} \bar{\phi}^c F^0 + \frac{3}{2} \sqrt{2N} f^{abef} \bar{\phi}^0 F^c \right). \tag{A.8}
$$

The contributions from Fig. 5 are of the form

$$
X_5^{ABCD} |C|^2 g^2 g_{CD} L \bar{\phi}^A \bar{\lambda}^C \bar{\lambda}^D F^B \tag{A.9}
$$

where $X_5^{ABCD}$ is given in Table 5. In Table 5 we have introduced the notation $(\tilde{D}^A)^{BC} = d^{ABC}$. Using results from the Appendix, the contributions from Table 5 add to

$$
\Gamma_{51\Pi}^{(1)\text{pole}} = g^4 L |C|^2 \left[ -\frac{1}{2}(3 + \alpha)N f^{acef} f^{bde} + \frac{11}{8} N d^{abef} d^{cde} \\
- \frac{1}{2}\delta^{ab}\delta^{cd} - \frac{1}{2}\delta^{ac}\delta^{bd} \right] \bar{\phi}^a \bar{\lambda}^c \bar{\lambda}^d F^b \\
+ d^{abef} g^3 L |C|^2 \sqrt{2N} \left[ g_0 \bar{\phi}^0 \bar{\lambda}^b \bar{\lambda}^c F^e + 3g_0 \bar{\phi}^a \bar{\lambda}^b \bar{\lambda}^0 F^c + 2g_0 \bar{\phi}^a \bar{\lambda}^b \bar{\lambda}^c F^0 \right] \\
+ 4g^3 g_0 L |C|^2 \left( \bar{\phi}^0 \bar{\lambda}^a \bar{\lambda}^0 F^a + 2 \bar{\phi}^a \bar{\lambda}^0 \bar{\lambda}^0 F^a \right). \tag{A.10}
$$
The divergent contributions to the effective action from the graphs in Fig. 6 are of the form

$$iLN g^3 X_6 C^\alpha \beta d^{ab} f^{cde} \bar{\phi}^a \bar{\phi}^b \psi^c \psi^d$$

(A.11)

where the contributions from the individual graphs to $X_6$ and the associated tensors in the usual decomposition are given in Table 6:

| Fig. | $X_5^{ABCD}$ |
|------|--------------|
| 5a   | 0            |
| 5b   | $4 \text{tr}[\tilde{F}^A \tilde{F}^C \tilde{D}^B \tilde{D}^D]$ |
| 5c   | $-2 \text{tr}[\tilde{F}^A \tilde{D}^C \tilde{F}^D \tilde{D}^B]$ |
| 5d   | $-\alpha N d^C c^D c^X d^{ABX} d^{CDX}$ |
| 5e   | $(1 + \alpha) N c^X d^{ABX} d^{CDX}$ |
| 5f   | $-\frac{1}{2} N \alpha c^A d^B c^X d^{ABX} d^{CDX}$ |
| 5g   | 0            |
| 5h   | $2 \alpha \text{tr}[\tilde{F}^C \tilde{F}^A \tilde{D}^B \tilde{D}^D]$ |
| 5i   | $-\frac{1}{2} (3 + \alpha) \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^D \tilde{F}^C]$ |
| 5j   | $\frac{1}{2} \alpha (\text{tr}[\tilde{F}^C \tilde{F}^A \tilde{F}^D \tilde{F}^B] - \frac{1}{2} N f^{XAC} f^{XBD})$ |
| 5k   | $(\text{tr}[\tilde{F}^A \tilde{F}^C \tilde{D}^B \tilde{F}^D] + \frac{1}{2} N f^{XAC} f^{XBD})$ |

Table 5: Contributions from Fig. 5

Table 6: Contributions from Fig. 6

The contributions from Table 6 add to

$$\Gamma_{61P_1}^{(1)\text{pole}} = -iLN g^3 C^\alpha \beta d^{ab} f^{cde} \bar{\phi}^a \bar{\phi}^b \psi^c \psi^d$$

(A.12)
The divergent contributions to the effective action from the graphs in Fig. 7 are of the form

\[ \text{Im} L N g^2 g_A X_7^{ABC} C^{\mu \nu} d^{ABC} \partial_\mu A_\nu \bar{\phi}^B \bar{\phi}^C \]  

(A.13)

where the contributions from the individual graphs to \( X_7 \) and the associated tensors in the usual decomposition are given in Table 7:

| Fig. | \( X_7 \) | Tensor |
|------|------------|--------|
| 7a   | 2          | \( c^A c^B d^C \) |
| 7b   | -1         | \( c^A c^B d^C \) |
| 7c   | -1         | \( c^A c^B d^C \) |
| 7d   | 0          |        |
| 7e   | -4         | \( d^A c^B c^C \) |
| 7f   | -2\alpha   | \( d^A c^B c^C \) |
| 7g   | -2         | \( d^A c^B c^C \) |
| 7h   | -\( \frac{1}{2} \) | \( c^A c^B d^C \) |
| 7i   | -1         | \( c^A c^B d^C \) |
| 7j   | -(1 + 2\alpha) | \( c^A c^B d^C \) |
| 7k   | \( \frac{3}{2} \) | \( c^A c^B d^C \) |
| 7l   | -\( \frac{1}{2} (5 + \alpha) \) | \( c^A \) |
| 7m   | \( \alpha \) | \( d^A c^B c^C \) |
| 7n   | 1          | \( d^A c^B c^C \) |
| 7o   | \( \frac{1}{2} \) | \( c^A c^B d^C \) |
| 7p   | 1          | \( c^A c^B d^C \) |
| 7q   | 1 + 2\alpha | \( c^A c^B d^C \) |
| 7r   | -\( \frac{3}{2} \) | \( c^A c^B d^C \) |

Table 7: Contributions from Fig. 7

These results add to

\[
\Gamma_{11\text{pole}}^{(1)} = -\frac{1}{2} (5 + \alpha) i L g^2 C^{\mu \nu} m \left[ 3 N g d^{abc} \partial_\mu A_\nu \bar{\phi}^a \bar{\phi}^b \bar{\phi}^c + 2 g \sqrt{2} N \partial_\mu A_\nu \bar{\phi}^a \bar{\phi} \right] + 4 g_0 \sqrt{2} N \partial_\mu A_\nu \bar{\phi}^a \bar{\phi}^a \bar{\phi}^a \right].
\]  

(A.14)
(Note that the contributions from Figs. 7(h-k) cancel those from Figs. 7(o-r) respectively.)

The divergent contributions to the effective action from the graphs in Fig. 8 are of the form

\[ im\Lambda g^A X_8^{ABCD} C^{\mu\nu} f^{ABE} d^{CDE} A_\mu A_\nu \phi^C \phi^D \]

(A.15)

where the contributions from the individual graphs to \(X_8\) and the associated tensors in the usual decomposition are given in Table 8:

| Fig. | \(X_8\) | Tensor |
|------|---------|--------|
| 8a   | -2      | \(c^A c^B c^C c^D\) |
| 8b   | 1       | \(c^A c^B c^C c^D\) |
| 8c   | 1       | \(c^A c^B c^C c^D\) |
| 8d   | 2       | \(c^A c^B c^C c^D\) |
| 8e   | \(\alpha\) | \(c^A c^B c^C c^D\) |
| 8f   | 1       | \(c^A c^B c^C c^D\) |
| 8g   | \(-\frac{1}{4}(3 + \alpha)\) | \(c^A c^B c^C d^D\) |
| 8h   | 0       |        |
| 8i   | 0       |        |
| 8j   | 1       | \(c^A c^B c^C d^D\) |
| 8k   | \(\frac{3}{4}\alpha\) | \(c^A c^B c^C d^D\) |
| 8l   | \(-\frac{1}{2}\alpha\) | \(c^A c^B c^C d^D\) |
| 8m   | \(\frac{3}{4}\alpha\) | \(c^A c^B c^C d^D\) |
| 8n   | \(\frac{1}{4}(2 + \alpha)\) | \(c^A c^B c^C d^D\) |
| 8o   | 0       |        |
| 8p   | 0       |        |
| 8q   | \(-\frac{3}{4}\alpha\) | \(c^A c^B\) |
| 8r   | \(\frac{3}{4}(1 + \alpha)\) | \(c^A c^B\) |
| 8s   | \(-\frac{1}{2}\alpha\) | \(c^A c^B c^C c^D\) |
| 8t   | \(-\frac{1}{2}\) | \(c^A c^B c^C c^D\) |

Table 8: Contributions from Fig. 8
These results add to

$$\Gamma_{81\text{PI}}^{(1)\text{pole}} = i g^4 C^{\mu\nu\rho\sigma} m f^{ab} A^{\alpha}_{\mu} A^{\beta}_{\nu} \left[ \frac{1}{4} (13 + 2\alpha) N d^{cde} \bar{\phi}^{e} \bar{\phi}^{d} + \frac{3}{2} \sqrt{2N} \bar{\phi}^{e} \bar{\phi}^{0} \right]$$  \hspace{1cm} (A.16)

(Note that the contributions from Figs. 8(g-p) cancel those from Figs. 8(u-dd) respectively.)

The contributions from the individual graphs to $X_{9}^{ABCD}$ are given in Table 9. The results in Table 9 add to

$$X_{9}^{ABCD} g^2 g_c g_D m L |C|^2 \bar{\phi}^A \bar{\phi}^B \bar{\lambda}^C \bar{\lambda}^D.$$  \hspace{1cm} (A.17)

The contributions from the individual graphs to $X_{9}^{ABCD}$ are given in Table 9. The results in Table 9 add to

$$\Gamma_{91\text{PI}}^{(1)\text{pole}} = |C|^2 m L \left\{ \frac{11}{8} N d^{a b c d e} - \frac{1}{2} \left( 1 - 4 \frac{g^2}{g_0^2} \right) \delta^{a b} \delta^{c d} - \frac{1}{2} \delta^{a d} \delta^{b c} + \frac{4}{N} \left( 1 - \frac{g^2}{g_0^2} \right) f^{a c e} f^{b d e} \right\} g^4 \bar{\phi}^a \bar{\phi}^b \bar{\lambda}^c \bar{\lambda}^d \hspace{1cm} (A.18)

+ g^3 d^{a b c} \sqrt{2N} \bar{\phi}^a \bar{\lambda}^b \left( 3g_0 \bar{\phi}^c \bar{\lambda}^0 + g\bar{\phi}^0 \bar{\lambda}^c \right) + 4g^3 g_0 \bar{\phi}^a \bar{\phi}^0 \bar{\lambda}^a \bar{\lambda}^0 \right\}.

(Note that the contributions from Figs. 9(h–m) cancel those from Figs. 9(u–z); this is analogous to the situation with Figs. 7 and 8, and is a consequence of our choice of coefficient for the last term in Eq. (3.1).)
| Fig. | $X_9^{ABCD}$ |
|------|-------------|
| 9a   | $\frac{1}{2} \alpha \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D]$ |
| 9b   | $\frac{1}{2} \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D]$ |
| 9c   | $\frac{1}{2} (3 + \alpha) \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D]$ |
| 9d   | $-\alpha \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D]$ |
| 9e   | $N d^{ABE} d^{CDE} c^{A} c^{B} c^{C} c^{D} c^{E} - 2c^{C} c^{D} \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D] + \frac{4}{N} f^{ACE} f^{BDE} + \frac{2a^2}{g^2} (c^{A} c^{B} c^{C} c^{D} \delta^{AB} \delta^{CD} - \frac{2}{N} f^{ACE} f^{BDE})$ |
| 9f   | $\frac{1}{2} \alpha \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D]$ |
| 9g   | $\frac{1}{2} \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D]$ |
| 9h   | $-\text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D] - \frac{1}{2} N f^{ACE} f^{BDE}$ |
| 9i   | $-\frac{1}{2} \alpha \left( \text{tr}[\tilde{F}^A \tilde{F}^C \tilde{F}^B \tilde{F}^D] - \frac{1}{2} N f^{ACE} f^{BDE} \right)$ |
| 9j   | $-2\alpha \text{tr}[\tilde{F}^C \tilde{F}^A \tilde{D}^B \tilde{D}^D]$ |
| 9k   | $-4\text{tr}[\tilde{F}^C \tilde{F}^A \tilde{D}^B \tilde{D}^D]$ |
| 9l   | $\frac{1}{2} \alpha N c^{A} d^{B} c^{E} d^{ABE} d^{CDE}$ |
| 9m   | $2\text{tr}[\tilde{F}^A \tilde{D}^C \tilde{F}^D \tilde{D}^B]$ |
| 9n   | 0 |
| 9o   | 0 |
| 9p   | $-\frac{1}{2} \alpha N d^{ABE} d^{CDE} c^{D} c^{E}$ |
| 9q   | $\frac{1}{2} (1 + \alpha) N d^{ABE} d^{CDE} c^{E}$ |
| 9r   | $-\frac{1}{2} (3 + \alpha) \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D]$ |
| 9s   | $-\frac{1}{2} \alpha \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D]$ |
| 9t   | $-\frac{1}{4} \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D]$ |
| 9u   | $\text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D] + \frac{1}{2} N f^{ACE} f^{BDE}$ |
| 9v   | $\frac{1}{2} \alpha \left( \text{tr}[\tilde{F}^A \tilde{F}^C \tilde{F}^B \tilde{F}^D] - \frac{1}{2} N f^{ACE} f^{BDE} \right)$ |
| 9w   | $2\alpha \text{tr}[\tilde{F}^C \tilde{F}^A \tilde{D}^B \tilde{D}^D]$ |
| 9x   | $4\text{tr}[\tilde{F}^C \tilde{F}^A \tilde{D}^D \tilde{D}^B]$ |

**Table 9:** Contributions from Fig. 9
The results from Fig. 10 are of the form

\[ N_g g^2 L X_{10} C^\mu_\nu f^{abc} \partial_\mu \bar{\phi}^a \partial_\nu \bar{\phi}^b \bar{\phi}^c \]  

and the contributions from the individual graphs to \( X \) are given in Table 10.

\[ \Gamma^{(1)\text{pole}}_{10} = \frac{1}{2} N g g^2 L C^\mu_\nu (1 + \alpha) f^{abc} \partial_\mu \bar{\phi}^a \partial_\nu \bar{\phi}^b \bar{\phi}^c \]  

We have not explicitly drawn most of the diagrams (labelled Fig. 11(a,b...)) giving contributions of the form

\[ i C^\mu_\nu g g^2 g_D L (X^{ABC}_D \partial_\mu \bar{\phi}^A \bar{\phi}^B \bar{\phi}^C A^D_\nu + Y^{ABC}_D \bar{\phi}^A \bar{\phi}^B \bar{\phi}^C \partial_\mu A^D_\nu) , \]

since they can be obtained by adding external scalar lines to the diagrams of Fig. 7. Thus Figs. 11(e-o) are obtained from Figs. 7(a-k) by adding an external scalar (\( \bar{\phi} \)) line at the position of the cross. Figs. 11(p-v) are obtained from Figs. 7(l-r) by adding an external
scalar ($\tilde{\phi}$) line at the position of the dot. The remaining Figs. 11(a-d) are depicted in Fig. 11. The individual contributions to $X_{11}^{ABCD}$ and $Y_{11}^{ABCD}$ in Eq. (A.21) are given in Table 11.

| Fig. | $X_{11}^{ABCD}$ | $Y_{11}^{ABCD}$ |
|------|-----------------|-----------------|
| 11a  | $-\alpha tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D]$ | 0 |
| 11b  | $-tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D]$ | 0 |
| 11c  | $\frac{1}{2}\alpha (tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D] - \frac{1}{2}Nf^{ABE}f^{CDE})$ | 0 |
| 11d  | 0 | 0 |
| 11e  | $-4tr[\tilde{D}B\tilde{D}^A\tilde{F}^C\tilde{F}^D]$ | 0 |
| 11f  | $2tr[\tilde{D}^A\tilde{D}B\tilde{F}^C\tilde{F}^D]$ | 0 |
| 11g  | $2tr[\tilde{D}^A\tilde{F}^B\tilde{D}^C\tilde{F}^D]$ | 0 |
| 11h  | $-2Nf^{ABE}f^{CDE}$ | 0 |
| 11i  | 0 | $-4tr[\tilde{D}^B\tilde{F}^C\tilde{D}^D\tilde{F}^A]$ |
| 11j  | 0 | $-2\alpha tr[\tilde{F}^B\tilde{F}^C\tilde{D}^D\tilde{D}^A]$ |
| 11k  | 0 | $-2tr[\tilde{F}^B\tilde{F}^C\tilde{D}^D\tilde{D}^A]$ |
| 11l  | $tr[\tilde{D}^A\tilde{D}B\tilde{F}^C\tilde{F}^D]$ | 0 |
| 11m  | $2tr[\tilde{F}^D\tilde{F}^A\tilde{D}^B\tilde{D}^C]$ | 0 |
| 11n  | $2tr[\tilde{D}^B\tilde{D}^C\tilde{F}^D\tilde{F}^A]$ | $-2\alpha tr[\tilde{D}^B\tilde{D}^C\tilde{F}^D\tilde{F}^A]$ |
| 11o  | $-3tr[\tilde{D}^B\tilde{D}^C\tilde{F}^D\tilde{F}^A]$ | 0 |
| 11p  | 0 | $\frac{1}{6}(5 + \alpha) \left( tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D] - Nc^Dd^{ABE}d^{CDE} \right)$ |
| 11q  | $\frac{3}{2}\alpha \left( tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D] - \frac{1}{2}Nf^{ABE}f^{CDE} \right)$ | $\frac{1}{3}\alpha \left( 4tr[\tilde{F}^A\tilde{F}^B\tilde{D}^C\tilde{D}^D] + Nc^Dc^Ed^{ADE}d^{BCE} \right)$ |
| 11r  | $tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D]$ | $\frac{1}{3} \left( tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D] \right)$ |
| 11s  | $\frac{1}{3} \left( (3 + \alpha)Nf^{ABE}f^{CDE} - tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D] \right)$ | $\frac{1}{6}(1 + \alpha)tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D]$ |
| 11t  | $\frac{1}{6} \left( 4tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D] - \alpha Nf^{ABE}f^{CDE} \right)$ | $-\frac{1}{3}\alpha tr[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D]$ |

Table 11: Contributions from Fig. 11
Table 11: Contributions from Fig. 11 (continued)

The results sum to

$$\Gamma_{111\Pi}^{(1)} = i C^{\mu \nu} y g^2 L \left( -\frac{1}{2} g \left( 3 + \frac{7}{3} \alpha \right) N f^{abc} f^{cde} \partial_{\mu} \bar{\phi}^a \phi^b \phi^c A^d_{\nu} \right. $$

$$+ \left[ - \left( \frac{5}{4} - \frac{1}{3} \alpha \right) N d^{abc} d^{cde} + \left( 3 + \frac{7}{3} \alpha \right) \delta^{ab} \delta^{cd} \right] g \bar{\phi}^a \phi^b \phi^c \partial_{\mu} A^d_{\nu} $$

$$- \frac{1}{3} (9 + \alpha) g \sqrt{2} N \bar{d}^{abc} \phi^a \phi^b \phi^c \partial_{\mu} A^d_{\nu} - (5 + \alpha) g \bar{\phi}^a \phi^b \phi^c \partial_{\mu} A^d_{\nu} $$

$$- 2 g_0 \sqrt{2} N \bar{d}^{abc} \phi^a \phi^b \phi^c \partial_{\mu} A^0_{\nu} - 8 g_0 \bar{\phi}^a \phi^b \phi^c \partial_{\mu} A^0_{\nu} \right). \quad (A.22)$$

Appendix B. Group identities for $U(N)$

The basic commutation relations for $U(N)$ are (for the fundamental representation):

$$[R^a, R^b] = i f^{abc} R^c, \quad \{R^A, R^B\} = d^{ABC} R^C, \quad (B.1)$$

where $d^{ABC}$ is totally symmetric. Defining matrices $\tilde{F}^A$, $\tilde{D}^A$ by $$(\tilde{F}^A)^{BC} = i f^{BAC}, \quad (\tilde{D}^A)^{BC} = d^{ABC},$$ useful identities for $U(N)$ are

$$\text{Tr}[\tilde{F}^A \tilde{F}^B] = N \delta^{AB}, \quad \text{Tr}[\tilde{D}^A \tilde{D}^B] = N \delta^{AB},$$

$$\text{Tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C] = N d^{ABC} c^A c^B d^C, \quad \text{Tr}[\tilde{F}^A \tilde{D}^B \tilde{D}^C] = i N f^{ABC},$$

$$f^{ABE} d^{CDE} + f^{ACE} d^{DBE} + f^{ADE} d^{BCE} = 0, \quad (B.2)$$

$$f^{ABE} f^{CDE} = d^{ACE} d^{BDE} - d^{ADE} d^{BCE},$$

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and also

\[
\text{Tr}[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{F}^D] = c^A c^B c^C c^D \left[ \frac{1}{2} \delta(AB \delta CD) \right. \\
\left. + \frac{N}{4} \left( d^{ABE} d^{CDE} + d^{ADE} d^{BCE} - d^{ACE} d^{BDE} \right) \right],
\]

\[
\text{Tr}[\tilde{F}^A\tilde{F}^B\tilde{F}^C\tilde{D}^D] = - \frac{N}{4} i \left( d^{ABE} f^{CDE} + f^{ABE} d^{CDE} \right) c^A c^B c^C d^D,
\]

\[
\text{Tr}[\tilde{F}^A\tilde{F}^B\tilde{D}^C\tilde{D}^D] = \left[ \frac{1}{2} c^A c^B c^C c^D \left( \delta^{AB} \delta^{CD} - \delta^{AC} \delta^{BD} - \delta^{AD} \delta^{BC} \right) \right. \\
\left. + \frac{N}{4} c^A c^B c^D \left( d^{ABE} d^{CDE} + d^{ADE} d^{BCE} - d^{ACE} d^{BDE} \right) \right],
\]

\[
\text{Tr}[\tilde{F}^A\tilde{D}^B\tilde{F}^C\tilde{D}^D] = c^A c^B c^C c^D \left[ \frac{1}{2} \left( \delta^{AC} \delta^{BD} - \delta^{AB} \delta^{CD} - \delta^{AD} \delta^{BC} \right) \right. \\
\left. + \frac{N}{4} c^A c^B c^D \left( d^{ABE} d^{CDE} + d^{ADE} d^{BCE} - d^{ACE} d^{BDE} \right) \right].
\]

(B.3)
Fig. 1: Diagrams with one gaugino, one scalar and one chiral fermion line; the dot represents the position of a $C$. 
Fig. 2: Diagrams with one gaugino, one scalar, one chiral fermion and one gauge line; the dot represents the position of a $C$. 
Fig. 2 (continued).
Fig. 3: Diagrams with one gauge, one scalar and one auxiliary line; the dot represents the position of a $C$. 
Fig. 4: Diagrams with two gauge, one scalar and one auxiliary line; the dot represents the position of a $C$. 
Fig. 5: Diagrams with two gaugino, one scalar and one auxiliary line; the dot represents the position of a $C$ or a $|C|^2$. 
Fig. 6: Diagrams with two scalar and two chiral fermion lines; the dot represents the position of a $C$. 
Fig. 7: Diagrams with two scalar, one gauge line; a dot denotes a $C$, a cross a mass and a crossed circle a vertex with both a mass and a $C$. 
Fig. 7 (continued)
Fig. 8: Diagrams with two scalar, two gauge lines; a dot denotes a C, a cross a mass and a crossed circle a vertex with both a mass and a C.
Fig. 8 (continued)
Fig. 9: Diagrams with two scalar, two gaugino lines; a dot denotes a $C$, a cross a mass and a crossed circle a vertex with both a mass and a $C$. 
Fig. 9 (continued)
Fig. 10: Diagrams with three scalar lines; a dot represents the position of a $C$, a cross a superpotential vertex without a $C$ and a crossed circle a superpotential vertex with a $C$.

Fig. 11: Diagrams with three scalar lines and one gauge line; a crossed circle represents a superpotential vertex with a $C$. 
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