Fermion Masses and Mixing in Four and More Dimensions

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Abstract

We give an overview of recent progress in the study of fermion mass and flavor mixing phenomena. Mass matrix ansatze are considered within the SM and SUSY GUTs where some predictive frameworks based on SU(5) and SO(10) are reviewed. We describe a variety of schemes to construct quark mass matrices in extra dimensions focusing on four major classes: models with the SM residing on 3-brane, models with universal extra dimensions, models with split fermions and models with warped extra dimensions. We outline how realistic patterns of quark mass matrices could be derived from orbifold models in heterotic superstring theory. Finally, we address the fermion mass problem in intersecting D-branes scenarios, and present models with D6-branes able to give a good quantitatively description of quark masses and mixing. The role of flavor/CP violation problem as a probe of new physics is emphasized.

1 Introduction

The origin of the quark and lepton masses, their mixing and three generation structure remains the major outstanding problem in particle physics. It has different aspects, questioning the origin of the family replication, the fermion mass spectrum, CP violation in weak interactions, suppression of the flavour changing neutral currents (FCNC), pattern of neutrino masses and oscillations, etc. The charged fermion masses and mixing angles derive from Yukawa couplings, which are arbitrary parameters within the Standard Model (SM), while the non-vanishing neutrino masses and mixings are direct evidence for physics beyond the SM. The experimental values of the fermion masses and mixings provide a best clue to this new physics. We require explanations for the large mass ratios between generations: $m_u \ll m_c \ll m_t$; $m_d \ll m_s \ll m_b$; $m_e \ll m_{\mu} \ll m_{\tau}$, and for the large mass splitting within the third (heaviest) generation: $m_{\tau} \sim m_b \ll m_t$. We need also an explanation for the smallness of the
off-diagonal elements of the quark weak coupling matrix $V_{CKM}$ and for the tiny neutrino masses and their large mixings as recent data suggest.

Grand Unified Theories (GUTs) have the power to fill the gap between theory and experiment. Indeed, within this framework the low energy group proceeds from the spontaneous breakdown of a single compact group. The simplest and most attractive grand unified theories are based on the unitary group $SU(5)$ or the orthogonal group $SO(10)$. Remarkably, all low energy fermion quantum numbers find a natural explanation within these theories. If the grand unified group breaks at very high energies to the standard model gauge group, an essential requirement is that the theory should be supersymmetric $^1$. In fact, supersymmetric theories have been the main extension of the SM for about 20 years. Not only supersymmetry (SUSY) provides a natural protection for the weak scale against any large scale as long as SUSY breaking is around $TeV$ (Hierarchy problem), but also, when incorporated in GUTs, it can predict the weak mixing angle in remarkably good agreement with the precise measurements at LEP $^2$ and leads to relations among quark and lepton masses. The most celebrated one is $m_t = m_\tau$ at the unification scale which is corroborated by experiment once the running is taken into account $^3$. In SUSY there are additional sources of CP violation to the SM source arising from the complex Yukawa couplings. This is due to the presence of new CP violating phases which arise from the complexity of the soft SUSY breaking terms and the SUSY preserving $\mu$-parameter. These new phases have significant implications and can modify the SM predictions, whence the experimental data on the CP asymmetries in K and B systems impose constraints on the SUSY GUT frameworks, and can help in picking up the right one (see $^4$ and references therein).

The other popular scenario for the solution of the hierarchy problem is large extra dimensions $^5$, where the observed weakness of gravity is due to the existence of new spatial dimensions large compared to the weak scale. The original scenario where the SM fields reside on a 3-brane, with a low fundamental cut-off and extra dimensions, allows having flavor physics close to the TeV scale $^6$. Small Yukawa couplings are generated by “shining” badly broken flavor symmetries from distant branes, and flavor and CP-violating processes are adequately suppressed by these symmetries. Other scenarios where extra dimensions are accessible to all the SM fields were proposed and referred to as universal extra dimensions (UED) $^9$. Here, the compactification scale can be lower than the previous scenarios because the Kaluza-Klein (KK) number in the equivalent four-dimensional theory is conserved, and thus the contributions to the electroweak observables arise only from loops. One can study the impact of UED on the values of the CKM parameters and whether there are interesting phenomenological implications on K and B decays $^{10}$. Also, we can envisage a scenario where the SM fields are confined to a thick wall in extra dimensions, with the fermions localized in specific points in the wall $^{55}$. In this so called ‘Split Fermions’ scenario, Yukawa couplings are suppressed due to the exponentially small overlaps of the fermions wave functions. This provides a framework for understanding both the fermion mass hierarchy and proton stability without imposing symmetries, but rather in terms of higher dimensional geometry.
Another mechanism to solve the hierarchy problem in extra dimensions is the R-S warped non-factorizable geometry \[12,13\]. The weak scale is generated from a large scale of order of the Planck scale through an exponential ‘warp’ factor which arises not from gauge interactions but from the background metric (which is a slice of AdS\(_5\) spacetime). One can then explore the phenomenology associated with this localized gravity model, and the KK tower of gravitons have strikingly different properties than in the factorizable case with large extra dimensions\[14\].

It is possible to embed the above scenarios within string theory, where fields confined on a brane are identified with open strings whose ends are attached to this brane. In fact, the true resolution to the flavor problem lies in the domain of the underlying fundamental theory of which the SM would be the low energy effective theory. Since at present Superstrings/“M”–theory is the only candidate for a truly fundamental quantum theory of all interactions, studies of the flavor structure of the Yukawa couplings within four-dimensional superstring models are well motivated. In particular, the couplings of the effective Lagrangian in superstring theory are in principle calculable and not input parameters, which allows to address the flavor problem quantitatively. Indeed, the structure of fermion masses has been studied in a number of semi-realistic heterotic string models such as orbifolds \[15,16,17\] which have a beautiful geometric mechanism to generate a mass hierarchy and the resulting renormalizable Yukawa couplings can be explicitly computed as functions of the geometrical moduli \[18,19,20\].

With the advent of Dirichlet D-branes, the phenomenological possibilities of string theory have widened in several respects, and the flavor problem within ‘intersecting D-branes’ models \[21,22\] seems promising. In these models, chiral fields to be identified with SM fermions live at different brane intersections and there is a natural origin for the replication of quark-lepton generations in that the branes would typically intersect a multiple number of times giving rise to the family structure. Moreover, the appearance of hierarchies in Yukawa couplings of different fermions comes naturally because these couplings are weighted exponentially with the area of the triangle shape in whose vertices lie the chiral fields, and thus different triangle areas corresponding to the various families could generate a hierarchical structure.

The structure of the review is as follows. We give in section 2 an overview of the quark-lepton spectrum and the generation of fermion masses and mixings in the SM. The hierarchy problem and its supersymmetric solution is outlined in section 3 where flavor issues are reviewed within the Minimal Supersymmetric Standard Model (MSSM). Fermion masses in SUSY GUTs are reviewed in section 4 where we examine models based on SU(5) and SO(10). Large extra dimensions as originally proposed or within the UED picture are reviewed in section 5 with regard to the flavor hierarchies. The implications of split fermions and warped geometry on the fermion masses and mixings are presented in sections 6 and 7 respectively. In section 8 we outline orbifold models studying the Yukawa structure. Section 9 is devoted to studies of Yukawa structure within intersecting D-branes models. Our conclusions are presented in section 10.
2 Fermion masses in the SM

The SM can be considered as a minimal theory of flavor. Being an internally consistent renormalizable gauge theory, it has been extremely successful in describing various experimental data. The physical masses of the charged leptons can be directly measured and correspond to the poles in their propagators:

\[ M_e = 0.511 \text{ MeV} \quad M_\mu = 106 \text{ MeV} \quad M_\tau = 1.78 \text{ GeV} \] (1)

However, due to confinement, the quark masses cannot be directly measured and have to be extracted from the properties of hadrons. Various techniques are used, such as chiral perturbation theory, QCD sum rules and lattice gauge theory. The light \( u \), \( d \) and \( s \) quark masses are usually normalised to the scale \( \mu = 1 \text{ GeV} \), corresponding to the non-perturbative scale of dynamical chiral symmetry breaking as follows:

\[
\begin{align*}
m_u(1 \text{ GeV}) &= 4.5 \pm 1 \text{ MeV} \\
m_d(1 \text{ GeV}) &= 8 \pm 2 \text{ MeV} \\
m_s(1 \text{ GeV}) &= 150 \pm 50 \text{ MeV} \end{align*}
\] (2)

However the renormalisation scale for the heavy quark masses is taken to be the quark mass itself:

\[
\begin{align*}
m_c(m_c) &= 1.25 \pm 0.15 \text{ GeV} \\
m_b(m_b) &= 4.25 \pm 0.15 \text{ GeV} \\
m_t(m_t) &= 166 \pm 5 \text{ GeV} \end{align*}
\] (3)

A remarkable feature of the SM is that the fermion and the gauge boson \( W^\pm, Z \) masses have a common origin, namely the Higgs mechanism. Fermions get masses through the Yukawa couplings to the Higgs doublet \( \phi \):

\[
\mathcal{L}_\text{Yuk} = \lambda^u_{ij} q_i C u^c_j \tilde{\phi} + \lambda^d_{ij} q_i C d^c_j \phi + \lambda^e_{ij} l_i C e^c_j \phi \quad (\tilde{\phi} = i \tau_2 \phi^*)
\] (4)

so, the fermion masses are related to the weak scale \( \langle \phi \rangle = v = 174 \text{ GeV} \). However, the Yukawa constants remain arbitrary: \( \lambda^{u,d,e} \) are general complex \( 3 \times 3 \) matrices. The fermion mass matrices \( \hat{m}_f \) can be brought to the diagonal form by the unitary transformations:

\[
V_f^L \hat{m}_f V_f^R = \hat{m}_f \text{diag}
\] (5)

Hence, quarks are mixed in the charged current interactions:

\[
\mathcal{L}_W = \frac{g}{\sqrt{2}} (u_1, u_2, u_3) L \gamma^\mu W^\mu_{\mu} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L = \sqrt{2} g (u, c, t) \gamma^\mu (1 + \gamma^5) W^\mu_{\mu} V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}
\] (6)
where \( V_{CKM} = V_u^L V_d^L \) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix which represents a measure of the difference between the unitary transformations \( V_u \) and \( V_d \) acting on the left-handed up-type and down-type quarks. It has been measured:

\[
|V_{CKM}| = \begin{pmatrix}
0.9734 \pm 0.0008 & 0.2196 \pm 0.0020 & 0.0036 \pm 0.0007 \\
0.224 \pm 0.016 & 0.96 \pm 0.013 & 0.0412 \pm 0.002 \\
0.0077 \pm 0.0014 & 0.0397 \pm 0.0033 & 0.9992 \pm 0.0002
\end{pmatrix}
\]  

(7)

Due to the arbitrariness in the phases of the quark fields, the mixing matrix \( V_{CKM} \) contains only one CP violating phase, which would be of order unity \( \sin^2 \delta_{CP} \sim 1 \) if the observed CP-violating phenomena is to be attributed largely to the CKM mechanism [23]. From solar and atmospheric neutrino oscillation data [24], we know the neutrino mass squared differences:

\[
\Delta m^2_{21} \sim 5 \times 10^{-5} \quad \Delta m^2_{32} \sim 3 \times 10^{-3}
\]  

(8)

However we do not know the absolute neutrino masses although there is an upper limit of \( m_{\nu_i} \leq 1 \text{ eV} \) from tritium beta decay and from cosmology. It is of key importance that the SM exhibits the natural suppression of the flavor changing neutral currents (FCNC), both in the gauge boson and Higgs exchanges[25]. However, the large CKM phase creates the strong CP problem since the overall phase of the complex Yukawa matrices would ultimately contribute to the \( \Theta \)-term in QCD and thus induce the CP violation in strong interactions, while data suggests the bound \( \Theta < 10^{-9} \).

Extensions of the SM are believed to be necessary in order to understand the origin of the fermion masses and CKM elements.

### 3 Fermion masses in MSSM

There is no symmetry within the SM model which protects the Higgs particle from acquiring a mass associated with physics beyond the SM. The Higgs boson mass-squared gets corrections depending quadratically on the SM ultra-violet cut-off \( \Lambda \) from the one-loop diagrams. Therefore the sum of the bare mass-squared term and the radiative corrections \( \sim \Lambda^2 \) of the one-loop diagrams must give a mass in the range \( 114 \text{ GeV} < M_h < 200 \text{ GeV} \), as indicated by precision data. This leads to a fine-tuning problem for \( \Lambda > 1 \text{ TeV} \) (the hierarchy problem).

The most popular approach to solving the hierarchy problem is based on SUSY. Essentially all the quadratically divergent loop diagrams have corresponding superpartner diagrams, involving stop, gaugino and higgsino loops. In the limit of exact SUSY, the diagrams cancel completely. However in the MSSM, it is supposed that SUSY is softly broken at a scale \( \mu = M_{SUSY} \sim 1 \text{ TeV} \). So, with typical superpartner masses of order \( M_{SUSY} \), the cancellation is incomplete and \( \Lambda \) is replaced by \( M_{SUSY} \).

The MSSM is consistent with the supersymmetric grand unification of the \( SU(3) \times SU(2) \times U(1) \) running gauge coupling constants at a scale \( \mu = \Lambda_{GUT} \sim \)
$3 \times 10^{16}$ GeV, and SUSY stabilizes the hierarchy between the weak scale $M_h$ and the grand unified one $\Lambda_{GUT}$, in the sense that once the ratio $\frac{M_h^2}{\Lambda_{GUT}^2}$ is set to be of order $10^{-28}$ at tree level, then it remains so to all orders of perturbation theory. However, SUSY does not explain why the gauge hierarchy exists in the first place.

In the MSSM the fermion masses emerge from the superpotential terms

$$W_{\text{Yuk}} = \lambda^u_{ij} q_i u^c_j \phi_2 + \lambda^d_{ij} q_i d^c_j \phi_1 + \lambda^e_{ij} l_i e^c_j \phi_1,$$

The Yukawa matrices $\lambda^{u,d,e,v}$ remain arbitrary in the MSSM, while the presence of two Higgses $\phi_1$ and $\phi_2$ with vacuum expectation values (VEVs) $v_1 = v \cos \beta$ and $v_2 = v \sin \beta$ ($v = 174$ GeV) involves also an additional parameter $\tan \beta = \frac{v_2}{v_1}$.

Due to the soft SUSY breaking terms, it is apparent that the unitary matrices which diagonalize the squark mass matrices are not, in general, the same as the CKM matrix which diagonalize the quark mass matrix, and unlike the SM, in the MSSM the rotations acting on the right-handed quarks are observables especially with non-universal soft breaking terms. These matrices work their way into the various squark-couplings and introduce flavor off-diagonal interactions and CP-violations beyond the CKM. These ‘new’ interactions are restricted by limits on rare decays, but they might be able to explain the possible discrepancy between the SM and experiment. In [26], supersymmetric contributions to the CP asymmetry of $B \to \phi K_S$ process were studied and limits for the mixing CP asymmetry $S_{\phi K_S}$ were obtained using the mass insertion approximation method. It was found that the LR or RL mass insertion, in the terminology of the conventional ‘super’-CKM basis [27], gives the largest contribution to $S_{\phi K_S}$, while the LL or RR contribution is small. Thus the large deviation between $S_{\phi K_S}$ and $S_{J/\psi K_S}$ observed by the B–factory experiments (Belle and BaBar) [28] can be attributed to SUSY models with large ($\sim 10^{-3}$) LR(RL) mass insertions. Similarly, the presence of non-universal A terms will be essential for embracing the gluonic contributions to the CP violation parameter $\epsilon'/\epsilon$ in Kaon physics, provided the L–R mass insertions are large [30], and the SUSY contribution to $\epsilon'/\epsilon$ can be of order $\sim 10^{-3}$ while respecting the severe bound on the electric dipole moment of the deuteron.

4 Fermion masses in SUSY GUTs

As we said above, in the SM & MSSM the Yukawa coupling matrices remain arbitrary, and there is no explanation to the origin of the strong hierarchy between their eigenvalues, nor to the allignment of the rotations acting on the up and down quarks such that the CKM mixing angles are small. In GUTs we have more symmetries and the question arises as to whether we gain predictivity for the fermion masses.

Softly broken SUSY at $m_S \sim 1$ TeV is the only plausible idea that can support the GUT against the gauge hierarchy problem [31]. On the other hand,
the MSSM without GUT is also not in best shape: unification at the string scale gives too small $\sin^2 \theta_W (M_Z)$. In SUSY GUTs we have the following paradigm: a basic fundamental theory below the Planck scale $M_P$ reduces to a SUSY GUT containing a compact subgroup (SU(5) or $SO(10)$), which then at $M_X \simeq 10^{16}$ GeV breaks down to the $SU(3) \times SU(2) \times U(1)$. That is, below the scale $M_X$ starts Great Desert with the MSSM spectrum (see, for example, [32] and references therein).

4.1 SUSY SU(5)

In minimal SU(5) model, the quarks and leptons of each family fit into the multiplets

$$\bar{5}_i = (d_i^c + l_i), \quad 10_i = (u_i^c + q_i + e_i^c); \quad i = 1, 2, 3$$

and the Higgs sector consists of chiral superfields in adjoint (24 dimensional) representation $\Sigma$ and fundamental ($5 + \bar{5}$) representations $H$ containing the MSSM Higgs doublets $\phi_{1,2}$ accompanied by Higgs colour triplets $\bar{T}, T$. The Higgs triplets would mediate unacceptably fast proton decay via $d = 5$ operators [33] unless their masses are $\sim M_X$, in contrast to the Higgs doublets weak scale mass (DT hierarchy problem). SUSY SU(5) offers a fine tuning solution to the DT problem which is stable against radiative corrections [31]. However, a natural solution can be provided by the “missing multiplet” mechanism [34].

As to the superpotential terms relevant for fermion masses, they are:

$$W_{\text{Yuk}} = \lambda^u_{ij} 10_i H 10_j + \lambda^d_{ij} 10_i \bar{H} \bar{5}_j$$

and we get automatically $m_b(M_X) = m_\tau(M_X)$ in agreement with experiment, while the other predictions $m_d/m_s(M_X) = m_\nu/m_\mu(M_X)$ fail phenomenologically by one order of magnitude. In addition, there is no explanation neither for the fermion mass hierarchy, nor for the CKM mixing pattern. So it is necessary to introduce a more complicated group theoretical structure. In fact one can consider higher order non-renormalizable operators cut-off by the Planck scale $M_P$ involving the 24-plet $\Sigma$, like:

$$\frac{1}{M_P} 10 \Sigma H 10 + \frac{1}{M_P} 10 \Sigma \bar{H} \bar{5}, \quad \frac{1}{M_P^2} 10 \Sigma^2 H 10 + \frac{1}{M_P^2} 10 \Sigma^2 \bar{H} \bar{5}, \quad \ldots$$

which contribute to the Yukawa couplings below the scale $M_X$ in powers of $\frac{M_X}{M_P}$. This suggests a way out, where the renormalizable Yukawa couplings fix only the third family masses, thus maintaining the $m_b = m_\tau$ unification, while masses of the lighter families emerge entirely from the higher dimensional operators, and one can avoid the wrong prediction $m_d, s = m_\nu, \mu$. This is analogous to a long ago speculated possibility that in the SM the neutrino mass is not zero, but of order $1/M$ where $M$ would correspond to new physics.
4.2 Mass matrix textures and their origin

The motivation for considering mass matrix ans"atze is to obtain testable relationships between fermion masses and mixing angles, which might reduce the number of parameters in the Yukawa sector and provide a hint to the physics beyond the SM. In particular, certain elements can be put to zero (so called “zero textures”), and the most famous one is the Fritzch ansatz [35]:

\[
\lambda_{u,d} = \begin{pmatrix}
0 & A & 0 \\
A^* & 0 & B \\
0 & B^* & C
\end{pmatrix}
\] (12)

with the assumed hierarchy of parameters

\[
|C| \gg |B| \gg |A| (13)
\]

Then, if we neglect the phase factors, the total number of parameters for each matrix \(\lambda_{u,d}\) is reduced to 3, i.e. just the number of quark species. This allows to express the quark mixing angles in terms of their mass ratios:

\[
s_{12} = \sqrt{\frac{\lambda_d}{\lambda_u}} - e^{i\delta}\sqrt{\frac{\lambda_u}{\lambda_c}}, \quad s_{23} = \sqrt{\frac{\lambda_b}{\lambda_c}} - e^{i\kappa}\sqrt{\frac{\lambda_c}{\lambda_t}}, \quad s_{13} = \frac{\lambda_u}{\lambda_c} s_{23} (14)
\]

where \(\delta\) is a CP-violating phase and \(\kappa\) is some unknown phase. In particular, when \(\delta \sim 1\), we obtain \(s_{12} \approx \sqrt{m_d/m_s}\) which fits the experimental value well. However, the \(s_{23}\) relation is excluded by the data for any value of the phase \(\kappa\). Consistency with experiment can, for example, be restored by introducing a non-zero \((\lambda_u)_{22}\) mass matrix element [36].

The origin of the textures can arise from a spontaneously broken horizontal symmetry between the fermion families. This flavor symmetry provides selection rules forbidding the transitions between the various ‘light’ left-handed and right-handed quark-lepton states. Then, these ‘light’ fermions can acquire mass through higher order operators induced by the horizontal Higgses and suitable intermediate heavy fermions mediating the forbidden transitions (Froggatt-Nielsen mechanism). In this way, effective SM Yukawa couplings are generated and can be small even though all fundamental Yukawa couplings of the underlying theory are of \(\mathcal{O}(1)\). Such a scheme could be implemented into SUSY GUTs (see [37] in the context of \(\text{SU}(5) \times \text{SU}(3)_H\), and zero textures would appear if appropriately superheavy states are absent (see [38] in the context of \(\text{SO}(10)\)).

4.3 SUSY SO(10)

SO(10) grand unified theory is an appealing candidate for the unification of quarks and leptons (family by family) and their interactions. This is because it is the smallest group in which each family of fermions, with a right handed neutrino, fits into a single irrep (the spinorial representation \(16\)). SO(10), thus, offers a natural explanation of the smallness of neutrino mass through the seesaw mechanism, and consequently it incorporates leptogenesis. Moreover, it
contains the Pati-Salam $SU(4) \times SU(2)_L \times SU(2)_R$ subgroup which is the prototype of quark-lepton unification. Higgs fields appearing in the $45_H$, $16_H$ and $\overline{16}_H$ are needed to break SO(10) to the SM. The two SM light Higgs doublets can be accommodated by a single $10_H$ of SO(10), which consists of a $5 + \overline{5}$ of SU(5) or a $(6, 1, 1) + (1, 2, 2)$ of Pati-Salam model. Doublet-triplet splitting of the Higgs fields can be achieved via the Dimopoulos-Wilczek mechanism \[39\] if the $\langle 45_H \rangle$ VEV points in the $B-L$ direction. One can introduce additional Higgs fields, for example, $126_H$, $\overline{126}_H$ pair which can help to stabilize the DT hierarchy. The $126$ Higgs representation was used in \[40\] to study a minimal SO(10) in the case of non–canonical see–saw, and it was shown that large neutrino mixing angles require $b-\tau$ unification. In fact, there are plenty of SO(10) models in the literature, and they differ by their choice of Higgs structure or the horizontal flavor symmetry (look at \[41\] for a recent review).

Albright and Barr \[38\] constructed an explicit SUSY SO(10) model based on a global $U(1) \times Z_2 \times Z_2$ flavor symmetry. The required Higgses are: one $45$, two $16 \oplus \overline{16}$ pairs, six $10$ and five $1$. The matter fields comprised superheavy ones in the representations: two $16 \oplus \overline{16}$ pairs, two $10$ and six $1$. Also there is a discrete $Z_2$ matter symmetry to avoid too rapid proton decay. The resulting mass matrices for the down quarks and leptons are

$$M_D = \begin{pmatrix}
\eta & \delta & \delta' e^{i\phi} \\
\delta & 0 & \sigma + \epsilon/3 \\
\delta' e^{i\phi} & -\epsilon/3 & 1
\end{pmatrix} m_b^0 \quad M_E = \begin{pmatrix}
\eta & \delta & \delta' e^{i\phi} \\
\delta & 0 & -\epsilon \\
\delta' e^{i\phi} & \sigma + \epsilon & 1
\end{pmatrix} m_b^0$$

and the up quark and Dirac neutrino mass matrices are:

$$M_U = \begin{pmatrix}
\eta & 0 & 0 \\
0 & 0 & \epsilon/3 \\
0 & -\epsilon/3 & 1
\end{pmatrix} m_t^0 \quad M_N = \begin{pmatrix}
\eta & 0 & 0 \\
0 & 0 & \epsilon \\
0 & -\epsilon & 1
\end{pmatrix} m_t^0$$

Several texture zeros appear in elements for which the flavor symmetry forces the absence of superheavy states needed to mediate, via Froggatt-Nielsen mechanism, the corresponding transitions. The antisymmetric $\epsilon$ terms arise from diagrams involving the adjoint $\langle 45_H \rangle$ Higgs VEV pointing in the $B-L$ direction. The lopsided nature of the large $\sigma$ terms in $D$ and $L$ arises from the appearances of diagrams involving the $\langle 5(16_H) \rangle$ Higgs VEV. The nine SM charged fermion masses, the three CKM mixing angles and CP violating phase are well-fitted with the 8 parameters in the above matrices, after renormalisation group evolution from the GUT scale.

5 Fermion masses in large extra dimensions

This represents a new framework for solving the hierarchy problem which does not rely on supersymmetry \[3\]. The gravitational and gauge interactions unite at the electroweak scale, and the observed weakness of gravity at long distances is due to the existence of large new spatial dimensions. This is particularly
plausible since it could be embedded in superstring/M theory which requires for its consistency 10/11 spacetime dimensions. However, the top-to-down approach from strings to SM (or MSSM) has not been particularly successful as far as phenomenology is concerned, and it may be useful to bridge between the low energy world of the SM and the string physics at the highest energy in a way where the presence of the extra dimensions is somehow clearly exploited. In this scenario, the fundamental (quantum gravity) scale $M_f$ in $4 + N$ dimensional setting can be brought far beneath the conventional Planck scale, perhaps as low as the electroweak scale $\sim 1$ TeV. The 'effective' 4-dimensional Planck scale becomes

$$M_{Pl}^2 \simeq M_{f}^{2+N} R^N$$

where $R^N = V_N$ denotes the characteristic volume of the N-dimensional compact space. With $M_f \sim 1$ TeV close to the electroweak scale, then $R$ chosen to reproduce the observed $M_{Pl}$ yields $R \sim 10^{-17}$. Since the gravitational force is currently being tested in the sub-millimeter region, this scenario is not rejected from $N = 2$ onwards.

5.1 SM confined to a brane

In the original scenarios, the SM fields were localized on a three-dimensional wall, or 3-brane, while the gravity alone propagates in the extra dimensions. Such scenarios can naturally be accommodated in string theory where the wall on which the SM fields live can be a D-brane [6]. Remarkably, no known lab, astrophysical or cosmological constraint excluded this scenario [7] (see also [12] for more stringent lower bounds on $M_f$ from supernova and neutron stars). A number of attempts was done in order to show how the existence of one or more extra dimensions can be exploited to yield possible resolutions of the fermion mass hierarchy [43], the apparent stability of the proton [5, 7, 44], and so on. As an example, not meant to be restrictive, a realistic theory of flavor in extra dimensions was constructed in [8] with $M_f$ in the region 5-10 TeV. It is based on $U(3)^5$ flavor symmetry in which the three Yukawa matrices are each promoted to a single ‘flavon’ field which propagates in the bulk and whose symmetry is strongly broken on its ‘source’ brane distant from us. Thus one can understand the small flavor parameters in our world from a hierarchy of distances in the extra dimensions. A simple brane configuration is suggested where our 3-brane and the three source 3-branes are located on a 4-brane, so that shining occurs in 1 dimension. This makes the calculation of the Yukawa matrix simple, and we get

$$\lambda_{u,d} \sim \begin{pmatrix} e' & e' & e' \\ e' & e & e \\ \ell' & \ell & A \end{pmatrix}_{u,d},$$

which features both a hierarchy of eigenvalues and small mixing angles. More generally, whenever we have source triplets with uncorrelated VEVs we are led to a CKM matrix with small mixing angles. This basic idea can be implemented
in a wide range of models taking into account the brane geometry and the directions in $U(3)^5$ space shone by the source triplets, and it was shown that in a simple grid model one could give the 9 quark masses and mixing angles quite successfully in terms of just 5 free parameters. Note that in this picture it is essential to have a flavor symmetry as well as a set of 'source' branes with a variety of sets of the VEVs, plus a sector of the bulk messenger fields charged under the flavor group. The SM are localized on the same brane and the hierarchy of the flavor symmetry breaking is caused by locating the 'source' branes at different distances. This is to be contrasted with the attempt suggested in [45] where the three SM families are identical but they happen to live in 3 different branes in the extra space. The Higgs gets a VEV in one separate 'source' brane, and it decays exponentially away from the source. In this way, the mass of the SM fermions will be determined by the overlap of their wave functions with the Higgs profile, and no need to postulate flavor symmetry, nor messenger fields are needed.

### 5.2 Universal extra dimensions: UED

One can study the effects of allowing some SM fields to propagate in the extra dimensions. This could allow gauge coupling unification [46], and provide new mechanisms for SUSY breaking [47] and the generation of fermion mass hierarchies [44], or even the existence of a Higgs doublet [48].

UED refer to a situation where the extra dimensions are accessible to all the SM fields. In this case, the upper bound from the electroweak data on the size of the extra dimensions is significantly larger than non-UED models. This is due to the conservation of momentum in the universal dimensions or, in the equivalent 4-dim theory, to the conservation of the KK number. In particular, there are no vertices involving only one non-zero KK mode, and consequently no tree-level contributions to the precision electroweak observables. The contributions arise first at one loop level. In the ACD model [9], we have one fifth universal extra dimension compactified over $S_1/Z_2$ orbifold. There are infinite KK modes of the SM particle with universal masses

$$m_{(n)}^2 = m_0^2 + \left(\frac{n}{R}\right)^2,$$

where $m_0$ is the mass of the zero mode which is the ordinary SM particle. It was shown that the bound on the compactification scale is rather large $\frac{1}{R} \geq 300$ GeV contrasted to the non-UED models bound ($\sim$ few TeV) [49].

Concerning the flavor/CP violation in this model, it is given, as in the SM, by the CKM matrix only. Therefore, one would not expect a significant deviation from the SM results in the CP asymmetries of the $B$ decays. In fact, within the UED scenario, the main effect of the KK modes on these processes is the modification of the Inami-Lim one loop functions, as was found for other processes [10]. In [54], it was shown that this modification is quite limited and cannot explain the $2.7\sigma$ deviation from $\sin 2\beta$ in the process $B_d \to \phi K_S$ announced by Belle and BaBar Collaborations [51, 52].
6 Split fermions scenario

Another class of extra dimensions which has drawn a lot of attention is the so-called 'split fermions' scenario proposed by Arkani-Hamed and Schalmzt (AS)\[53\]. According to this scenario, we live in a thick four dimensional subspace (thick brane) which is infinite in the usual four spacetime dimensions and possesses a finite volume in the extra dimensions. SM fields are constrained to live on this thick brane whilst gravity propagates in the whole extra dimensional spacetime; furthermore, the Higgs and the gauge fields are free to propagate in the entire thick brane, fermions, on the other hand, are ‘stuck’ at slightly different points in the extra dimensions. In this framework, the effective four dimensional Yukawa couplings are suppressed by exponential factors that depend on the distance among the different fermion fields localized in the brane. Direct coupling between fermions are exponentially suppressed by the small overlap of their wavefunctions which are given by narrow Gaussians, for e.g.

$$\psi_i \sim e^{-\frac{(y-y_i)^2}{\sigma^2}}$$

in the AS model of 5 dimensions, where $y_i$ is the position of the quark $i$ in the fifth dimension and $\sigma$ is the width of its wave function, with $\sigma \ll R$. Thus, the hierarchy of couplings does not come from symmetry but from geometry describing the localization of the fields in the brane with higher dimensional couplings assumed to be of order 1.

The quark mass matrices arise from the interaction of fermions and the VEV of the Higgs zero mode and are given by

$$\begin{align*}
(M_u)_{ij} &= \frac{v_0 (\lambda_u)_{ij}}{\sqrt{2}} e^{-\frac{(\Delta u_{ij})^2}{4\sigma^2}}, \\
(M_d)_{ij} &= \frac{v_0 (\lambda_d)_{ij}}{\sqrt{2}} e^{-\frac{(\Delta d_{ij})^2}{4\sigma^2}},
\end{align*}$$

(20)

where $\Delta_{ij} = |y_i - y_j|$ is the distance between flavor $i$ and $j$. The parameters $(\lambda_{u,d})_{ij}$ are the 5D Yukawa couplings, which are arbitrary matrices of order unity in general. The number of free parameters is larger than the number of the observed fermion masses and mixings, so it is easy to accommodate the various types of Yukawa textures with hierarchical or non-hierarchical features. Examples of hierarchical Yukawa couplings have been obtained in [54, 55, 56], which fit all the quark and lepton masses and mixing angles. For instance, for

$$\begin{align*}
y_{Q_L} \sim \sigma \begin{pmatrix} 0 \\ 14.2349 \\ 8.20333 \end{pmatrix}, \\
y_{d_R} \sim \sigma \begin{pmatrix} 19.4523 \\ 5.15818 \\ 10.1992 \end{pmatrix}, \\
y_{u_R} \sim \sigma \begin{pmatrix} 6.13244 \\ 20.092 \\ 9.64483 \end{pmatrix},
\end{align*}$$

(21)

and with $|v\lambda_{ij}^u| \sim 1.5$, $|v\lambda_{ij}^d| \sim 0.05$, one gets the correct fermion masses and mixings. However, the non-universality of the couplings with KK-gluon makes observable both left- and right-handed rotations $(V_L,V_R)$ that diagonalize the mass matrix. In general $V_R$ matrix has six phases, and these new phases might play an important role in CP violating asymmetries in the rare $B$ decays. Also, the non-universality in the fermion position leads to FCNC at tree level, which
makes it quite dangerous and strong bounds on the string scale were obtained: \( \frac{1}{R} > 10^4 \text{ GeV} \) \[57\]. In \[50\], the impact of the KK contributions to the \( B_d - \bar{B}_d \) mixing and CP asymmetry of \( B_d \to \phi K_S \) were analyzed, and split fermions models were shown to be able to accommodate the Belle results \[51\].

In \[58\], with approximately equal Yukawa strength for both the up and down quarks, it was shown that at least two extra dimensions were necessary in order to obtain sufficient CP violation while reproducing the correct mass spectrum and mixing angles.

7 Fermion masses in warped geometry

Randall and Sundrum \[12, 13\] have proposed a mechanism based on a non-factorizable geometry and which accounts for the ratio between the Planck scale and the weak scales without the need to introduce a large hierarchy between the ‘fundamental’ Planck scale and the volume of the compact space, as was the case in the original large extra dimension scenarios \[5, 6, 7\]. In \[12\], they assumed a 5-dimensional non-factorizable geometry based on a slice of AdS spacetime with two 3-branes residing at \( S_1/Z_2 \) orbifold fixed points. The bulk is only populated by gravity which is localized on one 3-brane while the SM lies on the other 3-brane. The 4-dimensional weak/Planck scale hierarchy is generated by an exponential function of the compactification radius, called a ‘warp’ factor:

\[ M_W / M_{Pl} \sim e^{-M_{Pl} R} \]

Later, this picture was extended to a situation where (some of) the SM particles reside in the five dimensional bulk \[59, 60, 61\]. It was realized that this situation may lead to a new flavor, and a possible geometrical interpretation for the hierarchy of quark and lepton masses might be generated from the warp factors. In \[62\], all the SM fields live in the 5D bulk with warped metric

\[ ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \]  \hspace{1cm} (22)

where \( \sigma(y) = \kappa |y| \) with \( \kappa \sim M_P \) is the curvature scale. However, the Higgs field must arise from a KK excitation that is localized by the AdS metric on the TeV-brane in order to obtain the observable masses of the \( W \) and \( Z \) gauge bosons. The fermion field can be decomposed into modes as:

\[ \Psi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \psi^{(n)}(x) e^{2\sigma(y)} f_n(y). \]  \hspace{1cm} (23)

where \( R \) is the radius of the compactified fifth dimension. Each fermion field has a bulk mass term parametrized by two free parameters \( c_{iL} \) and \( c_{jR} \), and the fermion zero modes develop an exponential profile:

\[ f_{0L(R)}(y) = \frac{e^{-c_{iL(R)} \sigma(y)}}{N_0}. \]  \hspace{1cm} (24)
where \( c = m_\psi / \kappa \) and \( m_\psi \) is the bulk mass term. This gives rise to four-dimensional Yukawa couplings

\[
Y_{ij}^{u,d} = \frac{\lambda_{ij}^{(5)} \sqrt{\kappa v_0}}{\pi \kappa R} f_{0L}^{u,d}(\pi R) f_{0R}^{u,d}(\pi R),
\]

(25)

where \( \lambda_{ij}^{(5)} \) are the 5-dimensional Yukawas. For suitable values of \( c_{iL} \) and \( c_{jR} \) one could accommodate the correct masses and mixings. Moreover, although having fermions localized at different points (by assuming they have different 5D masses \( c_i \)) leads normally to non-universal coupling with the KK gauge bosons, which gives rise to FCNC processes [63], that could be safely avoided in the model for the right choice of the \( c \)'s. However, no choice was possible to get masses and to suppress proton decay operators unless the Higgs is delocalized from the TeV-barne, and here one needs SUSY.

Ref. [50] also studied the flavor implications of the same scenario and the same conclusion is reached: Because there are plenty of free parameters (the \( \lambda_{ij}^{(5)} \) and the \( c \)'s) we can always in this class of models, as it was the case in Split fermions scenario, obtain any type of Yukawa texture. However, the non-universality of the gauge KK modes couplings to the fermions is less than the non-universality in Split fermions scenario, and it is not possible in warped geometry to deviate the value of \( S_{\phi K_s} \) from the value of \( S_{f/\psi K_s} \).

### 8 Fermion masses in heterotic string models

String theory is the prime candidate for the fundamental theory of particle physics from which the SM might be derived as a low-energy limit, and it is thought that it must be able, in principle, to tackle fundamental questions, such as the fermion masses and mixing, directly. In fact, in the context of superstrings one can calculate the Yukawa couplings in terms of scattering between the string states and certain VEVs that parameterize the string vacuum. Some realistic superstring derived standard-like models were constructed (e.g. [64, 65]) where a possible explanation of the top quark mass hierarchy and a successful prediction of the top quark mass were presented. However, the heterotic \( E_8 \times E_8 \) string theory [56] is still the most phenomenologically promising string scenario. It consists of a 10-dim right and left moving bosonic string, its right moving superpartner and left moving \( E_8 \times E_8 \) gauge strings whose momenta span the \( E_8 \times E_8 \) lattice. If we bosonize the right moving fermion string then its momenta span the \( SO(10) \) weight lattice. The theory has led to phenomenologically successful models since one \( E_8 \) factor can contain \( E_6 \) which in turn contains \( SO(10) \) with useful subgroups such as flipped \( SU(5) \times U(1) \) or \( SO(6) \times SO(4) \) while the other \( E_8 \) factor can be treated as a hidden sector gauge group. In fact the 248-dim adjoint gauge field of \( E_8 \) can be decomposed under \( E_6 \times SU(3) \) as \( 248 = (78, 1) + (1, 8) + (27, 3) + (27 + 3) \) which provides, in the first two components, gauge fields for the \( E_6 \times SU(3) \), and matter fields in \( (27, 3) \) with their antimatter particles. In turn the 27-dim representation
decomposes under $E_6 \supset SO(10)$ as $16 + 10 + 1$ providing the 16-dim spinor representation into which the SM falls. Different schemes for constructing classical string vacua have arisen, and it has been possible, by compactifying 6 dimensions out of the 10 dimensions of the theory on tori or Calaby-Yau manifolds, to build up four-dimensional strings that resemble the SM in many aspects, but perhaps, the most complete study of the Yukawa couplings has been carried out for orbifold compactifications \cite{13,15,19,20}. Abelian $Z_N$ orbifolds have been studied in depth, and we shall restrict our discussion to them noting, however, that other orbifolds like $Z_N \times Z_M$ have been studied \cite{67,68} with interesting phenomenology.

A $Z_N$ orbifold is constructed by dividing $R^6$ by a six-dimensional lattice $\Lambda$ modded by some $Z_N$ discrete symmetry, called the point group $P$. The space group $S$ is defined as $S = \Lambda \times P$. There are two types of closed strings on the orbifold. One of them is an untwisted string (U) which is closed on torus “unmodded” by the point group $P$ elements. The other is a twisted string (T) satisfying a boundary condition

$$x_\nu(\sigma = 2\pi) = gx_\nu(\sigma = 0),$$

where $\nu = 1, \ldots, 6$ and $g$ is an element of the space group whose point group component is non-trivial. The zero-mode, i.e. the centre of mass coordinate, of this twisted string satisfies the same boundary condition, and so is required to be at a fixed point of the corresponding space group element $g$. Physical twisted fields are associated with conjugation classes ($\{hgh^{-1}, h \in S\}$ is the conjugation class of $g$) of the space group rather than with particular elements, but for prime orbifolds ($Z_N$ where $N$ is prime) conjugation classes are in one-to-one correspondence with the fixed points of $P$. However, for non-prime orbifolds the situation is a bit more subtle since two different fixed points under one point group element may be connected by another element, then both of them correspond to the same conjugation class. We can write the space group element $g$ in the form $g = (\theta^k, e_\nu)$, where $\theta$ is the generator of $Z_N$ ($\theta^N = 1$), $e_\nu \in \Lambda$, and we have the corresponding six-dimensional ground state $|\theta^k, e_\nu\rangle$. We say here that the string belongs to the $\theta^k$ sector. The requirement of having $N = 1$ supersymmetry in four dimensions and the absence of tachyons restrict the number of possible point groups (look at \cite{20} for the complete list of the point groups corresponding to $Z_N$ orbifolds). The physical states must be invariant under a total $Z_N$ transformation which, besides the twist $\theta$ in the 6-dimensional space, includes a $Z_N$ gauge transformation, usually represented by shifts on $\Lambda_{E_8 \times E_8}$ and on $\Lambda_{SO(10)}$. Accordingly one has to construct for each $\theta^k$ sector linear combinations of states, associated with $\theta^k$ fixed points, that are eigenstates of $\theta$, such as $|\theta^k, e_\nu\rangle + \gamma^{-1}|\theta(\theta^k, e_\nu)\rangle + \cdots + \gamma^{-(m-1)}|\theta^{m-1}(\theta^k, e_\nu)\rangle$ where $\theta^m$ denotes the smallest twist fixing $(\theta^k, e_\nu)$ itself ($m < k$). These linear combinations have eigenvalues $\gamma = \exp[i2\pi p/m]$ under the $\theta$-twist with an integer $p = 0, \ldots, m - 1$. Couplings between physical states are calculated by using vertex operators which consist of several parts: the four-dimensional part, the six-dimensional $\theta$-eigenstate of the $\theta^k$ sector, oscillators on it, the bosonized
SO(10) part, the gauge part, and the ghost part. Nonvanishing couplings should be invariant under the symmetries of each part. Thus coupling terms are allowed if they are gauge invariant and space-group invariant. Furthermore, a product of eigenvalues $\gamma_a$ should satisfy $\prod_a \gamma_a = 1$, and the SO(10) momentum and the ghost number must be conserved. This leads to string election rules. In [19] three point vertices satisfying the invariance of point group and SO(10) gauge group were listed for the $Z_N$ orbifolds, and it showed that the only allowed couplings are UUU, UTT and TTT. For the orbifold $Z_3$, however, we have only pure untwisted (UUU) or twisted (TTT) couplings. The untwisted renormalizable coupling is proportional to $\epsilon^{ijk}$, then if it corresponds to the coupling of a Weinberg-Salam Higgs to quarks (or leptons) then one of the quarks (leptons) remains massless at this level and the other two acquire degenerate mass. Hence, there is no mass hierarchy for the two massive generations nor fermion mixing, even if non-canonical kinetic terms were present [16]. For other $Z_N$ orbifolds, the situation is very similar and one can exclude the possibility of getting mass hierarchy from untwisted matter. For the twisted coupling, and if we denote the space group element $g$ corresponding to the fixed point $f$ in the sector $\theta^k$ by the form $g = (\theta^k, (1-\theta^k)(f + v))$ where $v \in \Lambda$, then the space group invariance for a Yukawa coupling to be allowed implies that the product of the three relevant space group elements, say $g_1 g_2 g_3$, must contain the identity, whence two selection rules:

$$k_1 + k_2 + k_3 = 0 \mod N$$

$$(1-\theta^{k_1})(f_1 + v_1) + \theta^{k_1}(1-\theta^{k_2})(f_2 + v_2) - (1-\theta^{k_1+k_2})(f_3 + v_3) = 0, \; v_i \in \Lambda$$

The first one is called the point group selection rule, which implies that the coupling must be of the form $\theta^{k_1} \theta^{k_2} \theta^{-(k_1+k_2)}$. The second one is called the space group selection rule and was studied in [17, 19]. The lattice $\Lambda$ can get deformations compatible with the point group and these degrees of freedom correspond to the untwisted moduli surviving compactification. Twisted fields at different fixed points can communicate with each other only via world sheet instantons. The resulting renormalizable Yukawa couplings have been calculated in [15, 18] and they present a very rich range, which is extremely attractive as the geometrical origin of the observed variety and hierarchy of fermion masses. In fact, they contain suppression factors $e^{-ad}$ that depend on the relative positions $d$ of the fixed points to which the fields involved in the coupling are attached (i.e. $f_1, f_2, f_3$), and on the size and shape of the orbifold which are given by the VEVs of moduli fields. Thus the strength of the Yukawa is governed by the distance from the Higgs fixed point such that the light fermions are located further away from the Higgs. For prime orbifolds, the space–group selection rule is of the so–called diagonal type. This means that given two fields associated with two fixed points $f_1, f_2$, they can only couple to a unique third fixed point $f_3$. Let’s take $Z_3$ as an illustrative example. The action of the $Z_3$ generator $\theta$ on the $[SU(3)]^3$ root lattice basis is: $\theta e_i = e_{i+1}$, $\theta e_{i+1} = -(e_i + e_{i+1})$ with $i = 1, 3, 5$ denoting the three complex planes. In each 2–dim sublattice, the
twist $\theta$ acts as a rotation by $\frac{2\pi}{3}$ and we have three $\theta$-fixed points:

$$f^0_i = \frac{1}{3}(2e_i + e_{i+1}), \quad f^{+}_i = \frac{1}{3}(e_i + 2e_{i+1}), \quad f^{-}_i = 0.$$

Thus we have $27 \times 27 = 729$ allowed renormalizable Yukawa couplings because of the diagonal-type space group selection rule. In particular, the components of the three fixed points in each sublattice must be either equal or different. This shows that the Yukawa matrices derived from string theory have a very constrained flavor structure. In [20], it was shown that only 14 couplings out of the 729 allowed were different. Furthermore, the lattice deformation degrees of freedom are represented by three radii and six angles, and in the particular case of an orthogonal lattice there are only 8 distinct couplings which are reduced to only 4 when the radii are degenerate. In addition to the fact that the space and point groups selection rules are of diagonal type in prime orbifolds, the matter associated with a given fixed point in a $\theta^k$ sector is not degenerate, i.e. all fields have different gauge quantum numbers. Consequently, the mass matrices in prime orbifolds are diagonal at the renormalizable level. However, when the gauge group is spontaneously broken after compactification there appear new effective trilinear couplings coming from higher order non-renormalizable operators in which some of the fields get non-vanishing VEVs. These non-renormalizable couplings are no longer subjected to the trilinear selection rule. For even orbifolds, non-diagonal mass matrices at the renormalizable level could be obtained. The reason is twofold. First, Yukawa couplings are not necessarily of a unique $\theta^k_1 \theta^k_2 \theta^k_3$ type. Second, the space group selection rule for a given $\theta^k_1 \theta^k_2 \theta^k_3$ coupling is not, in general, of the diagonal type [69]. However, as shown in [17], the structure of the mass matrices is still strongly constrained by the selection rules, so that, as for prime orbifolds, no realistic prediction for the KM parameters can emerge at the renormalizable level and one has to call for non-renormalizable terms. Selection rules for non-renormalizable couplings in $\mathbb{Z}_N$ orbifolds were studied in [70], and were shown capable to lead to realistic quark mass matrices. Under the assumption of first generation mass coming from non-renormalizable couplings while renormalizable couplings being responsible for the second and third generations, we are led to the following natural ansatz for quark and lepton mass matrices [17]:

$$M = \begin{pmatrix} \epsilon & a & b \\ \hat{a} & A & c \\ \hat{b} & \hat{c} & B \end{pmatrix}$$

(28)

where $\epsilon, a, \hat{a}, b, \hat{b}, c, \hat{c}, A, B << B$ in magnitude. It was shown [10] [11] [20] that for a reasonable size and shape of the compactified space the $\mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}_6$-1, and possibly $\mathbb{Z}_7$ orbifolds can fit the physical quark and charged-lepton masses adequately. Under the same assumption, it was found in ref. [67] that the $\mathbb{Z}_2 \times \mathbb{Z}_6, \mathbb{Z}_2 \times \mathbb{Z}_6, \mathbb{Z}_3 \times \mathbb{Z}_6$ and $\mathbb{Z}_6 \times \mathbb{Z}_6$ orbifolds have the possibility to derive the Yukawa couplings for the second and third generations as well as the measured gauge coupling constants. References [71] [72] studied left-right symmetric quark
mass matrices whose up- and down-sectors have the same structure, and derived in a $Z_6$-II orbifold model with non-renormalizable couplings the texture:

\[
M_{u,d} = d_{u,d} \begin{pmatrix} 0 & \varepsilon^3_{u,d} & 0 \\ \varepsilon^3_{u,d} & \varepsilon^2_{u,d} & \varepsilon^2_{u,d} \\ 0 & \varepsilon^2_{u,d} & 1 \end{pmatrix} \]  

(10)

leading to a geometrical hierarchy $m_3 m_1 \approx m_2^2$ and which can give realistic mixing angles. In [74], two of the five Ramond-Roberts-Ross types for mass matrix, with five texture zeros in up and down quark sectors, have been derived from $Z_6$-II orbifold models and their phenomenological consequences on flavor mixings and CP violation were analyzed at the weak scale.

In [73], one could avoid having to use non-renormalizable terms by assuming two Wilson lines in $Z_3$ orbifold, which then automatically have three families of the SM particles, including Higgses. Having three light generations of supersymmetric Higgses introduces more Yukawa couplings, thus more flexibility in the computation of the mass matrices, and a completely geometrical explanation of masses and mixing is allowed. However, in order to get the correct masses and mixings entirely at the renormalizable level, one has to rely on the Fayet-Iliopoulos (FI) breaking. Despite the modifications due to three Higgs families and FI mixing, the model retains a large degree of predictivity.

9 Fermion masses in intersecting branes models

As we said in the introduction, the advent of D-branes, as allowed end points for open strings and which provide new ways of realizing non abelian gauge symmetry, has increased the phenomenological features of string theory in several respects. Type I and Type IIB orientifold models [74, 75], where the gauge groups of the effective low energy Lagrangian arises from sets of coincident D branes were proposed and investigations into their general phenomenological properties have been possible. Yukawa textures from D-branes at singularities were studied [76, 77]. Nevertheless, these studies proved to be unable to explain the experimental data, since they would generally lead to a variant of the “democratic” texture of Yukawa, and one has to break this democracy by higher order or non-renormalizable operators. However, recent studies of the flavor problem within ‘intersecting D-branes’ models [21, 22, 78] seemed more promising. In this scenario we have an interesting way to represent the SM massless chiral fermions: a fermion sitting at the intersection of a bunch of N Dp-branes and another set of M Dp-branes ($p > 3$), both containing the Minkowski space and intersecting at some angle in the ($p - 3$) extra dimensions, would transform as $(N, M)$ under the gauge group $U(N) \times U(M)$.

In [21, 22], D6-branes were considered in type IIA string theory compactified on a factorizable 6-torus $T^2 \times T^2 \times T^2$. If we denote the wrapping numbers of the D6a-brane on the i-th $T^2$ by $(n^i_a, m^i_a)$, then the number of times two branes $D6_a$ and $D6_b$ intersect in $T^6$ is given by the signed intersection number

\[
I_{ab} = (n^2_a m^1_b - n^1_a m^2_b) (n^3_a m^2_b - m^3_a n^2_b) (n^3_a m^3_b - m^3_a n^3_b) \]  

(29)
This gives a natural explanation of the family replication.

Two ways of embedding the SM gauge group were adopted, and both used four stacks $a, b, c, d$ of D6-branes (and their orientifold mirrors, where the mirror image of a cycle $(n,m)$ is obtained by reversing the second wrapping number $(n,-m)$). The branes are called respectively the Baryonic, Left, Right and Leptonic branes.

In model A \[21\], the initial gauge group is

\[
Model A : \quad U(3) \times U(2) \times U(1) \times U(1) \tag{30}
\]

with number of branes: $N_a = 3, N_b = 2, N_c = 1, N_d = 1$. In order to yield the desired SM spectrum, one could choose the following intersection numbers:

\[
\begin{align*}
I_{ab} &= 1 ; \quad I_{abs} = 2 \\
I_{ac} &= -3 ; \quad I_{acs} = -3 \\
I_{bd} &= 0 ; \quad I_{bds} = -3 \\
I_{cd} &= -3 ; \quad I_{cds} = 3
\end{align*}
\tag{31}
\]

all other intersections vanishing. The massless fermion spectrum is shown in Table 1 where the $N_R$ represents a right-handed neutrino. In this model one adopts the choice of splitting the left-handed quarks into one quark $(ab)$ and two quarks $(ab^*)$ in order to satisfy the gauge anomaly cancellation condition which requires the same number of doublets and antidoublets. One can consider

| Intersection | Matter fields | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ | $Y$ |
|--------------|---------------|------|------|------|------|-----|
| (ab)         | $Q_L$         | $(3,2)$ | 1    | -1   | 0    | 0   | 1/6 |
| (ab*)        | $q_L$         | $(3,2)$ | 1    | 1    | 0    | 0   | 1/6 |
| (ac)         | $U_R$         | $(3,1)$ | -1   | 0    | 0    | -2/3 |
| (ac*)        | $D_R$         | $(3,1)$ | -1   | 0    | -1   | 0   | 1/3 |
| (bd*)        | $L$           | $(1,2)$ | 0    | -1   | 0    | -1  | -1/2 |
| (cd)         | $E_R$         | $(1,1)$ | 0    | 0    | -1   | 1   | 1   |
| (cd*)        | $N_R$         | $(1,1)$ | 0    | 0    | 1    | 1   | 0   |

Table 1: Standard model spectrum and $U(1)$ charges in the first model (A)

the possibility of NS B-flux $b^i$ on a torus $T^i$. The existence of orientifolds on tori requires the quantization of the $B$ flux $b = 0, 1/2$. The introduction of non-zero NS B-flux background $b = 1/2$ on a torus modifies its complex structure and the effect is equivalent to changing a winding number from $(n,m)$ into $(n,m + n/2)$. The requirement to generate the SM matter field allows only the winding numbers denoted in Table 2 where $\beta^i = 1 - b^i$, $\epsilon = \pm 1$ and $\rho = 1, 1/3$ whereas $n_a^2$, $n_b^1$, $n_c^1$ and $n_d^2$ take integers. In order that the hypercharge $Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c + \frac{1}{3}Q_d$ remains massless the following condition should hold:

\[
n_c^1 = \frac{\beta^2}{2\beta^1}(n_a^2 + 3\rho n_d^2) \tag{32}
\]
Also, one can add some D-brane stacks which do not intersect with the first four stacks in order to satisfy the tadpole cancellation condition:

\[ \frac{3n^2_a}{\rho \beta^2} + \frac{2n^b_1}{\beta^2} + \frac{n^2_c}{\rho^2} = 16 \]  

(33)

For simplicity, we can take the specific D-brane configuration

\[ \beta^1 = \beta^2 = 1, \]

(34)

\[ \epsilon = -1, \quad \rho = 1, \]

(35)

\[ n^a_2 = 2, \quad n^b_1 = 0, \quad n^c_1 = 1, \quad n^2_d = 0. \]

(36)

and add 5 parallel D-branes with winding numbers \((1,0)\) \((1,0)\) \((1,0)\), not intersecting with any D-brane stack, in order to satisfy the tadpole condition so that we end up with the winding numbers in Table 3.

In model B [22], the initial gauge group is:

\[ Model B : \quad U(3) \times SU(2) \times U(1) \times U(1) \]  

(37)

with number of branes: \( N_a = 3, N_b = 1, N_c = 1, N_d = 1, \) but \( b = b^* \). The intersection numbers are:

\[ I_{ab} = 3, \]

\[ I_{ac} = -3, \quad I_{ac^*} = -3, \]

\[ I_{bd} = 3, \]

\[ I_{dc} = -3, \quad I_{dc^*} = -3, \]

(38)

and all others are vanishing. The massless fermion spectrum is shown in Table 4. The model is also anomaly free: there is no \( Q_b \) anomaly condition since...
doublets and antidoublets are the same in \( SU(2) \). For the wrapping numbers, one can take the values shown in Table 5 giving rise to the MSSM spectrum.

As to the Higgs sector, it is complicated in model A consisting of 4 doublets \((h_i,H_i)_{i=1,2}\) at the branes intersections of \((bc,bc^*)\) since we have two varieties of left quarks \((Q_L,q_L)\) and two varieties of right quarks \((U_R,D_R)\), while in model B we have two Higgs doublets \(H^a(bc),H^d(bc^*)\). The Yukawa coupling between fields \(i,j,k\) is given by

\[
Y_{ijk} \sim e^{-A_{ijk}} \sim \prod_{r=1}^{3} \theta \left[ \frac{\delta^{(r)}}{\phi^{(r)}} \right] (\kappa^{(r)}),
\]

with

\[
\delta^{(r)} = \frac{\lambda^{(r)}_a}{\lambda^{(r)}_b} + \frac{\lambda^{(r)}_c}{\lambda^{(r)}_d} + \frac{d^{(r)}_a \lambda^{(r)}_e + d^{(r)}_c \lambda^{(r)}_e + d^{(r)}_e \lambda^{(r)}_e}{d^{(r)}_a \lambda^{(r)}_e + d^{(r)}_c \lambda^{(r)}_e + d^{(r)}_e \lambda^{(r)}_e} + \frac{s^{(r)}_a \lambda^{(r)}_q + s^{(r)}_c \lambda^{(r)}_q + s^{(r)}_e \lambda^{(r)}_q}{s^{(r)}_a \lambda^{(r)}_q + s^{(r)}_c \lambda^{(r)}_q + s^{(r)}_e \lambda^{(r)}_q} = 0, \kappa^{(r)} = \frac{\lambda^{(r)}_a}{\lambda^{(r)}_b} \frac{\lambda^{(r)}_c}{\lambda^{(r)}_d} \frac{d^{(r)}_a \lambda^{(r)}_e + d^{(r)}_c \lambda^{(r)}_e + d^{(r)}_e \lambda^{(r)}_e}{d^{(r)}_a \lambda^{(r)}_e + d^{(r)}_c \lambda^{(r)}_e + d^{(r)}_e \lambda^{(r)}_e} \frac{s^{(r)}_a \lambda^{(r)}_q + s^{(r)}_c \lambda^{(r)}_q + s^{(r)}_e \lambda^{(r)}_q}{s^{(r)}_a \lambda^{(r)}_q + s^{(r)}_c \lambda^{(r)}_q + s^{(r)}_e \lambda^{(r)}_q} \in \mathbb{Z}
\]

where \(d^{(r)} = \text{g.c.d.} \left( \frac{\lambda^{(r)}_a}{\lambda^{(r)}_b}, \frac{\lambda^{(r)}_c}{\lambda^{(r)}_d}, \frac{\lambda^{(r)}_e}{\lambda^{(r)}_e} \right)\) and \(s^{(r)} = s(i^{(r)},j^{(r)},k^{(r)}) \in \mathbb{Z}^{3}\) is a linear function of the integers \(i^{(r)},j^{(r)},k^{(r)}\). \(\lambda^{(r)}\) represents the 'shifts' in the \(r^{th}\) torus. The exponent \(A_{ijk}\) in equation 39 represents the area of the
triangle at whose vertices lie the fermions $i, j,$ and $k$, and, thus, this gives a natural exponential hierarchy for the Yukawa couplings.

One can see that Yukawa couplings in model B are always in the form

$$Y_{ij} \sim \vartheta^{(1)} \begin{bmatrix} \delta(0) \\ 0 \end{bmatrix} \left( \kappa^{(1)} \right) \times \vartheta^{(2)} \begin{bmatrix} \delta(i) \\ 0 \end{bmatrix} \left( \kappa^{(2)} \right) \times \vartheta^{(3)} \begin{bmatrix} \delta(j) \\ 0 \end{bmatrix} \left( \kappa^{(3)} \right)$$

so it is of a ‘factorizable’ form $Y_{ij} \sim a_i b_j$ which always has two zero eigenvalues, and the phenomenology fails because of the way in which the family structure for the left-handed, right-handed quarks and the Higgses is ‘factorized’ among the different tori. One could check that putting the quarks in one torus with one Higgs doublet as in Table 6 is not enough since it lead to a Yukawa of the form

$$Y_{ij} \sim \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

with one degenerate mass eigenvalue.

Table 6: Alternative example of D6-brane wrapping numbers in the second model (B) leading to a chiral spectrum of the MSSM. The family structure of both the left-handed and right-handed quarks arises in the second torus.

In fact, most of intersecting brane models seem to have this acute problem, and it seems difficult to derive non-vanishing mixing angles as well as lighter fermion masses. However, there are several possibilities to overcome this difficulty.

First, one can try to construct models with some further matter content than SM fields by arranging the configuration of D6-branes. Second, we can modify the structure of the compactified space, either by leaving the factorizable form $T^6 = T^2 \times T^2 \times T^2$ and changing it into, say, $T^6 = T^4 \times T^2$, or by changing the metric on the compactified space by introducing a small warp factor on the tori. The third way is to change the origin of generation of Yukawa couplings so that it does not come from the multiple intersection of D6-branes as in conventional models.

As an example of the first way to remedy the problem, we take the model presented in [79] with three supersymmetric Higgs doublets. This model C is purely toroidal and has 4 stacks $N_a = 3$, $N_b = 2$, $N_c = 1$, $N_d = 1$ with intersection numbers

$$|I_{ab}| = 3 \quad \text{representing } Q_L \quad , \quad |I_{bc}| = 3 \quad \text{representing } H^u$$

$$|I_{ac}| = 3 \quad \text{representing } U_R \quad , \quad |I_{bd}| = 6 = 3 + 3 \quad \text{representing } H^d, L$$

$$|I_{ad}| = 3 \quad \text{representing } D_R \quad , \quad |I_{cd}| = 3 \quad \text{representing } E_R$$

(42)
and we can see from Table 7, where $\beta^2 = \gamma^2 = 1$ and $\alpha$ is arbitrary, that the model is anomaly free. The intersection numbers are obtained with the

| Intersection | Matter fields | $SU(3) \times SU(2)$ | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ | $Q_Y$ |
|-------------|---------------|----------------------|-------|-------|-------|-------|-------|
| (ab)        | $Q_L$         | 3(3,2)               | 1     | -1    | 0     | 0     | 1/6   |
| (ac)        | $U_R$         | 3(3,1)               | -1    | 0     | $\beta$ | 0     | -2/3  |
| (ad)        | $D_R$         | 3(3,1)               | -1    | 0     | 0     | $\gamma$ | 1/3 |
| (bd)        | $L$           | 3(1,2)               | 0     | +1    | 0     | $-\gamma$ | -1/2 |
| (bc)        | $H_u^a$       | 3(1,2)               | 0     | +1    | $-\beta$ | 0     | 1/2   |
| (cd)        | $E_R$         | 3(1,1)               | 0     | 0     | $-\beta$ | $\gamma$ | 1   |

Table 7: Standard model spectrum and $U(1)$ charges in the third model (C)

wrapping numbers in Table 8, and we see that the family structure of all the SM particles arise in the second torus. The quark Yukawa couplings for both

\[ Y_{ijk} \sim \begin{pmatrix} \delta(i, j, k) \\ 0 \end{pmatrix} \left( \kappa^{(2)} \right) \]

and we get the following textures:

\[ Y_{ij1} \sim \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & B \\ 0 & C & 0 \end{pmatrix}, \quad Y_{ij2} \sim \begin{pmatrix} 0 & 0 & C \\ 0 & A & 0 \\ B & 0 & 0 \end{pmatrix}, \quad Y_{ij3} \sim \begin{pmatrix} 0 & B & 0 \\ C & 0 & 0 \\ 0 & 0 & A \end{pmatrix}, \]

With $\epsilon = \epsilon'$ we get the quark mass matrices $M_{ij}^{u,d} = \begin{pmatrix} A v_1^{u,d} & B v_3^{u,d} & C v_2^{u,d} \\ C v_3^{u,d} & A v_2^{u,d} & B v_1^{u,d} \\ B v_2^{u,d} & C v_1^{u,d} & A v_3^{u,d} \end{pmatrix}^{(43)}$

with $\sum_{i=1}^{3} (v_i^u)^2 + (v_i^d)^2 = (174)^2 (GeV)^2$. With the following choice of parameters:

\[ v_1^u \simeq 63 \text{ MeV}, \quad v_2^u \simeq 0.95 \text{ GeV}, \quad v_3^u \simeq 174 \text{ GeV} \]
\[ v_1^d \simeq 8.5 \text{ MeV}, \quad v_2^d \simeq 136 \text{ MeV}, \quad v_3^d \simeq 4.2 \text{ GeV} \]
\[ \epsilon \simeq 0.002, \quad \text{area} \simeq 18.71. \]
In this expression, we put $\epsilon$ generated by the $\epsilon$-fact, the realistic hierarchical structure of fermion masses and a CKM mixing matrix. In input parameter then we reach the Yukawa matrices for the up-sector:

$$d_U = \{ m_t = 173.9 \text{ GeV}, \ m_e = 1.02 \text{ GeV}, \ m_u = 4.3 \text{ MeV} \}$$
$$d_D = \{ m_b = 4.19 \text{ GeV}, \ m_s = 136 \text{ MeV}, \ m_d = 8.2 \text{ MeV} \} \quad (45)$$

which are in the experimentally acceptable range, and a CKM matrix with diagonal elements near the unity, and $(V_{CKM})_{12} \simeq 0.216$. However, $(V_{CKM})_{13} \sim (V_{CKM})_{23} \sim 10^{-4} - 10^{-5}$. The choice $\alpha_{1,2} \sim v_1/v_3 \sim v_2/v_3 \sim 1$ leads to a nearly democratic Yukawa texture, while for complex vevs and a democratic texture, one gets the USY texture: $$Y_{u,d} = \lambda_{u,d} \begin{pmatrix} e^{i\epsilon_{13}} & 1 & e^{i\epsilon_{23}} \\ 1 & e^{i\epsilon_{13}} & e^{i\epsilon_{23}} \\ e^{i\epsilon_{23}} & e^{i\epsilon_{13}} & 1 \end{pmatrix}$$

which can generate the right spectrum when the phases are small.

Now we move to the second remedy by changing the complex structure of the internal space. In model $A$, the same orientifold model $A$ is taken but with a new feature in that a warp factor can play an important role in generating hierarchically suppressed fermion masses and CKM mixings. To illustrate this, let us consider the up-quark sector and define $A_{U,ij}^{(n)}$ as the area of a triangle formed by three intersection points $Q_{Li}$, $U_{Rj}$ and $X_{U,i}$ on the $n$-th torus, where $Q_{Li} = q_i$ ($i = 1, 2$), $Q_{L3} = Q_L$, $X_{U,i} = h_1$ ($i = 1, 2$) and $X_{U,3} = H_1$. Then the Yukawa matrix for the up sector is described as $Y_{U,ij} = (X_{U,i}) e^{-kA_{U,ij}}$, where $A_{U,ij} = A_{U,ij}^{(1)} + A_{U,ij}^{(3)}$. If there is no warping, as in model $A$, and owing to the degeneracy of the locations of quark doublets on the first torus, then $A_{U,ij}^{(1)}$ depends only on the index $j$ and can be called $A_{ij}^{U}$. On the other hand, on the third torus the right handed quarks stay at the same point, thus we define $A_{ij}^{Q} = A_{U,ij}^{(3)}$. Therefore, the Yukawa matrix for up-sector quarks is described in a factorizable form:

$$Y_{U,ij} \propto e^{-kA_{ij}^{Q}} e^{-kA_{ij}^{U}} \quad (46)$$

However, if we assume in this new model $D$ that the metric of the first torus depends on a coordinate on the third torus $x_8$, then the length of strings projected on the first torus becomes dependent on the coordinate $x_8$, and so does the area swept by the strings. In a rough approximation, we can take this effect of the $x_8$-dependence by modifying $A_{ij}^{U}$ into $A_{ij}^{U}(1 - \epsilon_i)$, where $\epsilon_i$ is a small quantity generated by the $x_8$-dependence. Surprisingly, this slight modification gives the realistic hierarchical structure of fermion masses and a CKM mixing matrix. In fact, the $\epsilon_i$ should be calculable in string theory, but if we treat it simply as an input parameter then we reach the Yukawa matrices for the up-sector:

$$Y_{ij}^{U} = e^{-kA_{ij}^{U}}, \quad (47)$$
$$A_{ij}^{U} = A_{ij}^{Q} + A_{ij}^{U}(1 - x \epsilon_i). \quad (48)$$

In this expression, we put $\epsilon_i$ in the form $\epsilon_i = xc_i$, where $x \ll 1$ and $c_i$ is of order 1. We define, besides $x$, two more small quantities $y$ and $z$:

$$y = \frac{e^{-kA_2}}{e^{-kA_3}}, \quad z = \frac{e^{-kA_1}}{e^{-kA_3}}, \quad (A_1 > A_2 > A_3) \quad (49)$$
and we solve the eigensystem of Yukawa matrices by series expansion with respect to $x$, $y$ and $z$ to finally obtain the leading order of the quark masses:

\[
m_{U/D,1}^2 = \frac{2}{9} \beta_U^4 \left\{ \left( c_3^{U/D} \right)^4 + \left( c_2^{U/D} \right)^4 y^2 \right\} x^4,
\]

\[
m_{U/D,2}^2 = \beta_0^{U/D} \alpha^{U/D} \left( c_2^{U/D} - c_3^{U/D} \right)^2 y^2 x^2,
\]

\[
m_{U/D,3}^2 = \beta_0^{U/D} (1 + y^2),
\]

and the CKM mixing matrix

\[
V_{CKM} \sim \begin{pmatrix} 1 - \epsilon^2 & \epsilon & \epsilon^3 \\ \epsilon & 1 - \epsilon^2 & \epsilon^3 \\ \epsilon^3 & \epsilon^3 & 1 \end{pmatrix}.
\]

where

\[
a^{U} = \beta_2^{U} - (\beta_1^{U})^2, \quad a^{D} = \beta_2^{D} - (\beta_1^{D})^2,
\]

\[
\beta_0^{U} = \sum_{j=1}^{3} (b_j^{U})^2, \quad \beta_0^{D} = \sum_{j=1}^{3} (b_j^{D})^2,
\]

\[
\beta_n^{U} = \sum_{i=1}^{3} (b_i^{U})^2 \left( \log b_i^{U} \right)^n / \beta_0^{U}, \quad \beta_n^{D} = \sum_{i=1}^{3} (b_i^{D})^2 \left( \log b_i^{D} \right)^n / \beta_0^{D}
\]

(for $n = 1, 2, \cdots$) (54)

and where we assume $x$, $y$ and $z$ of order $\epsilon$. In such a way, a hierarchical structure of a realistic CKM matrix is obtained. It is worth noting that both large mass hierarchy and small mixing angles can be originated from a warp factor on the internal manifold.

For the third way of treating the problem of factorization, we present the model E proposed in [82, 83]. It is based on supersymmetric composite fields constructed in type IIA $T^6/(Z_2 \times Z_2)$ orientifold with intersecting D6-branes. The D6-brane configuration of the model is given in Table 9. The D6$_2$-brane system consists of two parallel D6-branes with multiplicities six and two which are separated in the second torus in a consistent way with the orientifold projections. From the sector $aa$ of open strings ending on the same stack of branes we get gauge symmetries $U(2)_L \times U(1)_L$, $U(3)_c \times U(1)_c$ and $U(1)_1 \times U(1)_2$ corresponding to D6$_1$, D6$_2$ and D6$_3$ branes respectively. The hypercharge is defined as

\[
Y = \frac{1}{2} \left( \frac{Q_c}{3} - Q \right) + \frac{1}{2} (Q_1 - Q_2),
\]

while the additional non-anomalous $U(1)$ charge, $Q_R$, is defined as

\[
Q_R = Q_1 - Q_2.
\]

A schematic picture of the configuration of intersecting D6-branes of this model is given in figure 11.
| D6-brane | winding number | multiplicity |
|----------|---------------|--------------|
| D6₁     | (1, -1), (1, 1), (1, 0) | 4            |
| D6₂     | (1, 1), (1, 0), (1, -1) | 6 + 2        |
| D6₃     | (1, 0), (1, -1), (1, 1) | 2 + 2        |
| D6₄     | (1, 0), (0, 1), (0, -1) | 12           |
| D6₅     | (0, 1), (1, 0), (0, -1) | 8            |
| D6₆     | (0, 1), (0, -1), (1, 0) | 12           |

Table 9: Configuration of intersecting D6-branes in model (E). All three tori are considered to be rectangular (untitled). Three D6-branes, D6₄, D6₅ and D6₆, are on top of some O6-planes.

| sector | SU(3) × SU(2) × USp(8) × USp(12) | field |
|--------|----------------------------------|-------|
| D6₁ · D6₂ | (3^*, 2, 1, 1, 1)(-1/6, 0)(+1, −1, 0) x 2 | \( \hat{\tilde{q}}_i \) |
|         | (2, 1, 1, 1)(−1/2, 0)(+1, −1, 0) x 2 | \( \hat{\tilde{t}}_i \) |
| D6₁ · D6₄ | (1, 2, 1, 12, 1)(0, 0)(−1, 0, 0) | D     |
| D6₂ · D6₄ | (3, 1, 1, 12, 1)(+1/6, 0)(0, +1, 0) | C     |
|         | (1, 1, 1, 12, 1)(−1/2, 0)(0, +1, 0) | \( N \) |
| D6₁ · D6₃ | (1, 2, 1, 1, 1)(+1/2, +1)(−1, 0, +1) x 2 | \( H^{(1)}_i \) |
|         | (2, 1, 1, 1, 1)(−1/2, −1)(−1, 0, +1) x 2 | \( \tilde{H}^{(2)}_i \) |
| D6₁ · D6₅ | (1, 2, 8, 1, 1)(0, 0)(+1, 0, 0) | T     |
| D6₃ · D6₅ | (1, 1, 8, 1, 1)(+1/2, +1)(0, 0, −1) | \( T^{(+)} \) |
|         | (1, 1, 8, 1, 1)(−1/2, −1)(0, 0, −1) | \( T^{(-)} \) |
| D6₂ · D6₃ | (3, 1, 1, 1, 1)(−1/3, −1)(0, +1, −1) x 2 | \( d_i \) |
|         | (3, 1, 1, 1, 1)(+2/3, +1)(0, +1, −1) x 2 | \( \bar{d}_i \) |
|         | (1, 1, 1, 1, 1)(−1, 0, +1) x 2 | \( \bar{e}_i \) |
|         | (1, 1, 1, 1, 1)(+1, 0, +1) x 2 | \( \bar{\nu}_i \) |
| D6₂ · D6₆ | (3^*, 1, 1, 1, 12)(−1/6, 0)(0, −1, 0) | \( C \) |
|         | (1, 1, 1, 1, 12)(+1/2, 0)(0, −1, 0) | \( \bar{N} \) |
| D6₃ · D6₆ | (3^*, 1, 1, 1, 12)(−1/6, 0)(0, +1, 0) | \( D^{(+)} \) |
|         | (1, 1, 1, 1, 12)(+1/2, +1)(0, 0, +1) | \( \bar{D}^{(-)} \) |

Table 10: Low-energy particle contents before “hypercolor” confinement in model (E). The fields from aa sectors are neglected for simplicity.
Figure 1: Schematic picture of the configuration of intersecting D6-branes in model (E). This picture describes only the situation of the intersection of D6-branes, and the relative place of each D6-brane has no meaning. The number at the intersection point between D6_α and D6_β branes denotes intersection number $I_{\alpha\beta}$ with $a < b$.

There are no $ab' + b'a, aa' + a'a$ sectors of open string in this configuration. The massless particle contents are given in Table (10) where it is assumed that all twelve D6-branes of D6_4 are on top of one of eight O6-branes with the same winding numbers giving rise to USp(12)_{D6_4} gauge symmetry. The same is also assumed for eight and twelve D6-branes of D6_5 and D6_6. The USp's gauge symmetries are broken to the factors of USp(2) gauge symmetries whose interactions can be naturally stronger than any other unitary gauge interactions, and we call them “hypercolor” interactions.

In the left-handed sector (D6_1-D6_2-D6_4), the confinement of six USp(2)_{D6_4,\alpha} gauge interactions gives six generations of left-handed quark and lepton doublets:

$$C_\alpha D_\alpha \sim q_\alpha, \quad N_\alpha D_\alpha \sim l_\alpha,$$

where $\alpha = 1, 2, \cdots, 6$. Two of these six left-handed quark doublets and two of these six left-handed lepton doublets become massive through the string-level
Yukawa interactions of the form

\[ W_{\text{left}} = \sum_{i,\alpha} g_{ia}^{\text{left}-q} \bar{q}_i C_\alpha D_\alpha + \sum_{i,\alpha} g_{ia}^{\text{left}-1} \bar{N}_\alpha D_\alpha, \] (58)

where \( i = 1, 2 \). The values of masses are given as \( g_{ia}^{\text{left}-q} \Lambda_L \) and \( g_{ia}^{\text{left}-1} \Lambda_L \), where \( \Lambda_L \) denotes the dynamical scale of USp(2)_{D64,\alpha}.

The same kind of confinement happens in the right-handed sector (D62-D63-D64), and the Higgs sector (D61-D63-D65), where six USp(2)_{D64,\alpha} and two USp(2)_{D65,\alpha} gauge interactions are confined to give six generations of right-handed quarks and leptons and eight composite Higgs fields, respectively:

\[ C_\alpha \bar{D}^{(-)}_\alpha \sim u_\alpha, \quad C_\alpha \bar{D}^{(+)}_\alpha \sim d_\alpha, \quad \bar{N}_\alpha \bar{D}^{(-)}_\alpha \sim \nu_\alpha, \quad N_\alpha \bar{D}^{(+)}_\alpha \sim e_{\alpha} \] (59)

\[ T_\alpha T^{(+)} \sim H^{(1)}_a, \quad T_\alpha T^{(-)} \sim \bar{H}^{(1)}_a \] (60)

Four generations of right-handed quarks and leptons and four composite Higgs fields become massive through the string-level Yukawa interactions of the form:

\[ W_{\text{right}} + W_{\text{higgs}} = \sum_{i,\alpha} g_{ia}^{\text{right}-u} \bar{u}_i C \bar{D}^{(-)} + \sum_{i,\alpha} g_{ia}^{\text{right}-d} \bar{d}_i C \bar{D}^{(+)} + \sum_{i,\alpha} g_{ia}^{\text{right}-\nu} \bar{\nu}_i \bar{D}^{(-)} + \sum_{i,\alpha} g_{ia}^{\text{right}-e} \bar{e}_i \bar{D}^{(+)} \] (61)

The values of masses are given as \( g_{ia}^{\text{right}-u} \Lambda_R \), \( g_{ia}^{\text{right}-d} \Lambda_R \), \( g_{ia}^{(1)} \Lambda_H \), \( g_{ia}^{(2)} \Lambda_H \), where \( \Lambda_R \), \( \Lambda_H \) are the dynamical scales of USp(2)_{D66,\alpha} and USp(2)_{D65,\alpha}, respectively.

We stress here the most relevant fact about the model in that the origin of the generation is different from the conventional intersecting D-brane models. It is not the multiple intersection of D-branes, but the number of different D-branes with the same multiplicity and the same winding numbers.

The higher dimensional interactions in the superpotential which come from the recombination processes among open strings at six intersection points: (D62-D64) - (D64-D61) - (D61-D65) - (D63-D64) - (D63-D65) - (D65-D62), give rise to Yukawa interactions for the quark-lepton mass and mixing after the “hypercolor” confinement:

\[ \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g_{i\alpha}^{u}}{M_i} \bar{c}_i C_\alpha D_\alpha \bar{D}^{(-)}_\alpha \bar{T}^T_{\alpha} \rightarrow y_{\alpha}^{u} g_{i\alpha}^{u} \frac{\Lambda_{L} \Lambda_{R} \Lambda_{H}}{M_i} \sim g_{i\alpha}^{u} \] (62)

\[ \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g_{i\alpha}^{d}}{M_i} \bar{c}_i C_\alpha D_\alpha \bar{D}^{(+)}_\alpha \bar{T}^T_{\alpha} \rightarrow y_{\alpha}^{d} g_{i\alpha}^{d} \frac{\Lambda_{L} \Lambda_{R} \Lambda_{H}}{M_i} \sim g_{i\alpha}^{d} \] (63)

\[ \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g_{i\alpha}^{\nu}}{M_i} \bar{c}_i C_\alpha D_\alpha \bar{D}^{(-)}_\alpha \bar{T}^T_{\alpha} \rightarrow y_{\alpha}^{\nu} g_{i\alpha}^{\nu} \frac{\Lambda_{L} \Lambda_{R} \Lambda_{H}}{M_i} \sim g_{i\alpha}^{\nu} \] (64)

\[ \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g_{i\alpha}^{e}}{M_i} \bar{c}_i C_\alpha D_\alpha \bar{D}^{(+)}_\alpha \bar{T}^T_{\alpha} \rightarrow y_{\alpha}^{e} g_{i\alpha}^{e} \frac{\Lambda_{L} \Lambda_{R} \Lambda_{H}}{M_i} \sim g_{i\alpha}^{e} \] (65)
since all the scales of dynamics, $\Lambda_L$, $\Lambda_R$ and $\Lambda_H$ are of the order of the string scale $M_s$.

The Yukawa coupling matrices $g_{\alpha \beta a}^{\text{left-q}}$ in Eq. (68) and $g_{\alpha \beta a}^{\text{right-u}}$ and $g_{\alpha \beta a}^{\text{right-d}}$ in Eq. (69) can be evaluated using the area law (Eq. (39)) by:

\[
g^{\text{left-q}} = \begin{pmatrix}
\varepsilon_3 & \varepsilon_2 & 1 & \varepsilon_1^2 \varepsilon_2 & \varepsilon_2^2 \varepsilon_3 & \varepsilon_1^2 \varepsilon_3 & \varepsilon_2^2 \varepsilon_3 \\
\varepsilon_2^2 \varepsilon_3 & \varepsilon_1^2 \varepsilon_2 & \varepsilon_1^2 & \varepsilon_2 & 1 & \varepsilon_2^2 \varepsilon_3 & \varepsilon_1^2 \varepsilon_3 \\
\varepsilon_2 & \varepsilon_1 & \varepsilon_2^2 & \varepsilon_3 & \varepsilon_1^2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 \\
\varepsilon_1 & \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_2^2 & \varepsilon_3 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 \\
\varepsilon_1^2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 \\
\varepsilon_1 & \varepsilon_2 & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_1 & \varepsilon_2 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_1 & \varepsilon_2 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_1 & \varepsilon_2 & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_1 & \varepsilon_2 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_1 & \varepsilon_2 & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_1 & \varepsilon_2
\end{pmatrix}
\]

where $\varepsilon_1 = \exp(-A_i/2\pi a')$ and $A_i$ is the $1/8$ of the area of the $i$-th torus. As for $g_{\alpha \beta a}^u$ and $g_{\alpha \beta a}^d$ in Eqs. (67) and (68), it is not easy to estimate them from first principle, however one can also apply the area law to give approximately an order of estimation, and obtain:

\[
g^{u}_{\alpha \beta a=1} = g^{d}_{\alpha \beta a=1} = \begin{pmatrix}
\varepsilon_1^3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2
\end{pmatrix}_{\alpha \beta}
\]

This Yukawa matrix is nontrivial in the sense that it differs from the factorizable form, and one can check that with $\varepsilon_2 \ll \varepsilon_3(\sim 0.01) \ll \varepsilon_1(\sim 0.5)$ one is led to almost realistic structure of quark Yukawa coupling matrices.

10 Conclusions

The origin of the fermion masses and CKM elements is still one of the important open questions in particle physics. Within the SM and MSSM, we have moved a step forward in the way of understanding the origin of mass & relating it to the electroweak breaking mechanism. We have seen that the fermion mass structure in SUSY GUTs needs more analysis. Also, the other path of giving an entirely geometrical interpretation for the fermion masses in extra dimensions, superstrings and D-branes scenarios is quite an interesting possibility, but we are still far from a complete picture.

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