Nonlinear transport through a dynamic impurity in a strongly interacting one-dimensional electron gas

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We analyze the transport properties of a Luttinger liquid with an imbedded impurity of explicitly time-dependent strength. We employ a radiative boundary condition formalism to describe the coupling to the voltage sources. Assuming the impurity time dependence to be oscillatory we present a full analytic perturbative result in impurity strength for arbitrary interaction parameter calculated with help of Coulomb gas expansion (CGE). Moreover, a full analytic solution beyond the above restriction is possible for a special non-trivial interaction strength which has been achieved independently by full resummation of CGE series as well as via refermionization technique. The resulting nonlinear current-voltage characteristic turns out to be very rich due to the presence of the additional energy scale associated with the impurity oscillation frequency. In accordance with the previous studies we also find an enhancement of the linear conductance of the wire to values above the unitary limit \( G_0 = 2e^2/h \).

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I. INTRODUCTION

Unlike their higher-dimensional counterparts the one-dimensional metals are known to constitute a distinguished universality class of so-called Luttinger liquids (LL).\(^2\) Probably the most prominent of their properties is the zero-bias anomaly (ZBA) which reveals itself as a vanishing electronic density of states at the Fermi edge. Among other things it leads to zero conductance of an LL quantum wire in the low energy sector as long as at least one static impurity is present.\(^2\)\(^3\) This has been convincingly demonstrated in a number of recent experiments on single-walled carbon nanotubes (SWNTs)\(^4\)\(^5\) whose electronic degrees of freedom had been shown to be adequately described by the LL theory\(^6\)\(^7\).

The way towards these spectacular findings was not smooth and has been marked by a number of controversies, almost all of which seem by now to be resolved. Probably the most prominent of them is the linear conductance of the clean LL. According to early theoretical studies it should be equal to \( G = gG_0 \), where \( G_0 = 2e^2/h \) is the universal conductance quantum while

\[
g = 1/\sqrt{1 + U_0/(\pi v_F)}
\]

is the LL interaction parameter produced by the bare interaction amplitude \( U_0 \). However, the experimental results favored the picture where \( G = G_0 \).\(^2\) The subsequent theoretical efforts could reveal the physics behind this phenomenon. The clue to the problem’s solution is the proper inclusion of the voltage sources into the theory, which can either be done by imposing boundary conditions on the particle densities, or using the so-called \( g(x) \)-model where the interaction parameter is supposed to extrapolate outside of the sample to the non-interacting value \( g = 1 \) in the particle reservoirs modeling the voltage sources.\(^10\)\(^11\)\(^12\)\(^13\)\(^14\) Both approaches yield matching results also for the impurity scattering thereby surpassing the previously widely used ‘voltage drop’ approach, where the voltage is supposed to change abruptly at the impurity site.

In view of future technological applications in, e.g., the emerging field of nanoelectronics, the obvious way forward in understanding the properties of such one-dimensional systems appears to be the study of their transport properties in presence of time-dependent perturbations. In general, these can be realized in form of AC voltages or oscillating impurities. Such a situation is quite interesting from the physical point of view as the additional energy scale associated with the time evolution may change the phase diagram of the system. Probably the simplest question one can answer is whether the zero-bias anomaly survives under such conditions. Thus far a number of different approaches have been used and quite interesting predictions have been made.\(^15\)\(^16\)\(^17\)\(^18\)\(^19\)\(^20\)\(^21\)\(^22\)\(^23\)\(^24\)\(^25\)\(^26\) One spectacular observation is the dynamic conductance enhancement (DCE) reported by Feldman and Gefen.\(^20\) Most of the actual studies including the just mentioned one are using the voltage drop approach, which requires much care as soon as it is being used in connection with the perturbation expansion in impurity strength. One attempt to improve and enhance the study has been undertaken,\(^22\) where the authors have used the \( g(x) \)-model. One is, however, still quite far from complete understanding of the problem. For the major part because up to now only perturbative calculations have been undertaken and no analytic solutions for arbitrary impurity strength, voltages and temperature exist. We would like to close this gap. For one thing we employ the more justified approach (from our point of view) to take care of the applied voltage by means of the radiative boundary condition formalism.\(^13\) This method in connection with the Coulomb gas expansion enables one to construct an alternative full perturbative series in impurity strength. One particular advantage of this series is that for a special interaction parameter \( g = 1/2 \) it can easily be summed up,
thereby yielding a first exact analytic solution in the full parameter range. In order to cross check these results we also performed the same calculation in the refermionization representation.

The structure of the paper is as follows: in Section II we are going to present the necessary ingredients of the LL picture and show how to incorporate voltage sources in the presence of a dynamic impurity within the boundary condition framework. The ensuing investigation in the path integral formalism (Section III) will then allow us to derive a perturbative result valid for small impurity strength and for all interaction parameters \( g < 1/2 \) (Section IV). Furthermore, in Section V we shall derive an exact result in the special case \( g = 1/2 \). This result will be confirmed by an alternative calculation based on refermionization. A short summary of results and discussion of predictions for experiments concludes the paper.

II. LUTTINGER LIQUID

The universality class of strongly interacting, one-dimensional electron systems in the low energy sector is that of the so-called Luttinger liquid. The corresponding generic electron-electron interaction potential is extremely short ranged and is basically given by \( U(x-y) = U_0 \delta(x-y) \). This particular feature facilitates a diagonalization of the model Hamiltonian for a clean spinless system (realistic electrons with spin and other systems are discussed in Conclusions) of infinite length in terms of canonically conjugate bosonic fields \( \theta(x) \) and \( \partial_x \phi(x) \), which describe the plasmon like excitations of the system. Then (from now on we set \( \hbar = e = k_B = 1 \))

\[
H_{LL} = \frac{v}{2} \int dx \left[ g \left( \partial_x \phi \right)^2 + g^{-1} \left( \partial_x \theta \right)^2 \right],
\]

where \( g \) is given in Eq. 11. Values \( g < 1 \) correspond to a repulsive interaction, whereas \( g > 1 \) describes an attractive potential and \( g = 1 \) reproduces the non-interacting Fermi gas case. The sound velocity \( v = v_F/g \) is generated by renormalization of the Fermi velocity \( v_F \) of the bare non-interacting system. In this bosonization representation the ordinary fermion field operators are given by simple exponentials of linear combinations of the fields \( \theta(x) \) and \( \phi(x) \).

The relation between the electron density and the phase fields is even simpler. The electron density in the wire can be written as a sum of contributions for left- and right-moving electrons, \( \rho(x) = \rho_L(x) + \rho_R(x) \) where \( \rho_{L,R}(x) = \psi_{L,R}^\dagger(x) \psi_{L,R}(x) \), \( \psi_{L,R}(x) \) being the field operators of left/right moving electrons. Then, the phase field \( \theta(x) \) has a simple physical interpretation as it is related to the electron density operator and the current operator \( I(x, t) \) via

\[
\rho(x, t) = \rho_L + \rho_R = \frac{1}{\sqrt{\pi}} \partial_x \theta(x, t),
\]

\[
I(x, t) = v_F [\rho_L - \rho_R] = -\frac{1}{\sqrt{\pi}} \partial_t \theta(x, t).
\]

An insertion of a local pointlike scatterer at \( x = 0 \) results in two different contributions to the Hamiltonian. The first one is the forward scattering, proportional to \( \psi_L^\dagger(0) \psi_L(0) + \psi_R^\dagger(0) \psi_R(0) \). In the case of a static impurity it can be gauged away and does not affect the conductance in any way. In the dynamic case its influence can be investigated using the conventional equation of motion method. From now on we concentrate on the local backward scattering

\[
H_{sc} = \lambda(t) \left[ \psi_L^\dagger(0) \psi_R(0) + \text{h.c.} \right],
\]

which is known to change the static transmission profoundly. Using the bosonization identity one finds it to translate into

\[
H_{sc} = \lambda(t) \cos \left[ 2 \sqrt{\pi} \theta(x = 0) \right].
\]

Here, \( \lambda(t) \) is the impurity strength which we assume to depend explicitly on time. Such a coupling can be realized in experiments in many different ways. The best controlled way is probably an STM tip placed in immediate vicinity of the LL. A time-dependent voltage difference between the tip and the wire then acts as an impurity of time-dependent strength. Another possibility could be a setup where the wire is locally subjected to a laser radiation of changing intensity. The third option is a locally enhanced coupling to the internal phonon mode of the wire itself, as long as the back action of the impurity onto the phonon degree of freedom is negligible.

As outlined in the Introduction, the question of how to incorporate an applied voltage \( U \) has been subject to controversy. Early works assumed a local drop in chemical potential at the impurity site but the result was a wrong prediction for quantities as basic as the linear conductance. It turned out that the reason for this failure was that only a part \( V \) of the applied voltage \( U \) drops at the impurity. In this picture, the voltage drop assumption, which has been in use for quite a while, corresponds to \( V = U \). It was shown though, that this assumption is true only if the impurity energy scale is sufficiently large, which renders perturbative approaches in \( \lambda \) based on the voltage drop assumption questionable. There are different remedies to this shortcoming of which we favor the radiative boundary conditions (RBC) approach.

For a static impurity this method leads to boundary conditions for the phase field \( \theta \) at the ends of the wire. It turns out that this technique can easily be generalized to the case of a time-dependent impurity. If we restrict our analysis to periodic impurity strength variations and calculate time-averaged quantities, then all RBC equations are very conveniently replaced by the ones averaged
over a period $T = 2\pi/\Omega$ of the impurity oscillation. For the boundary condition itself one then obtains

$$\frac{1}{T} \int_0^T dt \left( \frac{1}{g^2} \partial_x^2 + \frac{1}{v_F} \partial_t \right) \langle \theta(x = \mp L/2) \rangle = \pm \frac{U}{2\sqrt{\pi v_F}}$$

(7)

which is, in fact, a weaker requirement than the original one. Nevertheless, its implementation in order to compute the non-linear current-voltage characteristic using turns out to be more cumbersome than the static calculation.

### III. PATH INTEGRAL DESCRIPTION

In order to solve the impurity scattering problem in an LL using the RBC approach, it turns out to be beneficial to describe the system in terms of a real-time bosonic path integral. Translating the Hamiltonian $H_{LL} + H_x$ we find that the action functional of the system consisting of the LL and the time-dependent impurity reads

$$S[\theta] = \frac{1}{2g} \int_C \int dx \left[ \frac{1}{v} (\partial_x \theta)^2 - v (\partial_x \theta)^2 \right]$$

$$- \int_C dt \lambda(t) \cos \left[ 2\sqrt{\pi} \theta(x = 0, t) \right] \quad (8)$$

The time integration has to be performed on the Keldysh contour $C$ as we are investigating nonequilibrium properties of the system. The partition function is then given by $Z = \int D\theta e^{iS[\theta]}$ where the path integration runs over all phase fields $\theta(x, t)$ compatible with the boundary conditions \(\theta(x, t)\). This can be achieved by decomposing $\theta(x, t)$ into a homogeneous solution $\theta_h$ satisfying equilibrium boundary conditions and a particular solution $\theta_p$ satisfying \(\theta_p\). The latter reads

$$\theta_p(x, t) = -\frac{g^2 V}{2\sqrt{\pi v_F}} |x| - \frac{U - V}{2\sqrt{\pi v_F}} t$$

(9)

where $V$ is an arbitrary parameter. In the case of a static impurity, it was shown that $V$ can be interpreted as the four-terminal voltage, i.e. the voltage drop at the impurity site. Generally, $V < U$, but for large impurity strength $V \to U$. In the case of a dynamic impurity this interpretation must be slightly altered. In analogy to the static impurity case, we fix the parameter $V$ by the requirement

$$\frac{1}{T} \int_0^T dt \langle \partial_t \theta_h(x, t) \rangle = 0.$$ 

(10)

The average current is then calculated by means of \(\langle \theta \rangle\). Our choice of $V$ ensures that the only contribution comes from $\theta_h$ and one finds the time-averaged current

$$\bar{I} = G_0(U - V). \quad (11)$$

This result suggests the interpretation of $V$ as the average four-terminal voltage and the current $I_{BS} = G_0 V$ as a backscattering current.\cite{1}

Inserting the minimizing action $\theta = \theta_p + \theta_h$ into the action functional \(S[\theta]\), using \(\theta_p\), one obtains an action functional only depending on $\theta_h$,

$$S[\theta_h] = \frac{1}{2g} \int_C dt \int dx \left[ \frac{1}{v} (\partial_t \theta_h)^2 - v (\partial_x \theta_h)^2 \right]$$

$$- \frac{eV}{\sqrt{\pi}} \int_C dt \theta_h(x = 0, t)$$

$$- \int_C dt \lambda(t) \cos \left[ 2\sqrt{\pi} \theta_h(x = 0, t) - (U - V)t \right].$$

(12)

Having derived this effective action, we define the generating functional

$$Z[\eta] = \int D\theta_h \exp \left\{ iS[\theta_h] + i\sqrt{\pi} \int_C dt \eta(t) (\partial_t \theta_h) \right\}$$

(13)

which enables us to rewrite \(\langle \theta \rangle\) in terms of a functional derivative as

$$\frac{1}{T} \int_0^T dt \frac{\delta Z[\eta]}{\delta \eta(t)} |_{\eta = 0} = 0.$$ 

(14)

We assume for simplicity that the auxiliary field $\eta(t)$ is the same on both branches of the Keldysh contour, $\eta(t-) = \eta(t+)$. The task of calculating the current has thus been reduced to the calculation of the partition function \(Z[\eta]\).

With the exception of the additional time-dependent factor $\lambda(t)$, the ensuing calculation can be performed in a similar fashion as in the static case. The final result reads

$$Z[\eta] = \exp \left\{ - \int dt dt' \eta(t) \tilde{C}(t - t') \eta(t') - igV \int dt \eta(t) \right\} \left( 1 + \sum_{m=1}^{\infty} Z_m[\eta] \right)$$

(15)
where

$$Z_m[\eta] = (i\lambda)^{2m} \int D_{2m} t \sum \exp \left\{ \sum_{j<k} u_j u_k C(t_j - t_k) + \sum_j u_j \left[ \int dt \eta(t) \dot{C}(t - t_j) - i(U - V + gV) t_j \right] \right\}$$

$$\times \mu(t_{2m}) \sin[\pi \eta(t_{2m})] \prod_{j=1}^{2m-1} \mu(t_j) \sin \left\{ \pi g \left[ \eta(t_j) + \sum_{k=j+1}^{2m} u_k \right] \right\}.$$  \quad (16)

Here, for convenience in the following perturbative expansion, we assumed the time dependence to be of the form $\lambda(t) = \lambda \mu(t)$ with $\max |\mu(t)| = 1$. The system can be regarded as a one-dimensional array of discrete charges $u_i = \pm 1$ located at positions $t_j$ ($0 < i < 2m$) on the real time axis which interact via the potential $\mu(t_j)$.

We would like to stress that thus far no approximations have been made, the results $\langle i \rangle$ and $\langle 1 \rangle$ are exact for arbitrary $g$ and $\lambda(t)$.

\section{IV. Perturbation Theory in $\lambda$}

Equation $\langle 1 \rangle$ is very well suited for a perturbative expansion in $\lambda$. We limit ourselves to the leading order which corresponds to $m = 1$ and which is proportional to $\lambda^2$. Using an oscillating impurity $\mu(t) = \cos(\Omega t)$, one can use $\langle 1 \rangle$ and $\langle 2 \rangle$ and the self-consistency condition $\langle 3 \rangle$ reads

$$V = \pi \lambda^2 \left( \frac{\pi}{\beta \Delta} \right)^2 \sin(\pi g)$$

$$\times \int_0^\infty dr \frac{\cos(\Omega r)}{\sinh^{2\eta}(\pi r / \beta)} \sin[(U - V + gV) t].$$  \quad (19)

For future convenience we introduce dimensionless quantities by measuring all energies in units of $\beta = 1/T$. Hence, defining $u = \beta U$, $v = \beta V$, $\alpha' = \beta \lambda$, $\Delta' = \beta \Delta$ and $\Omega' = \beta \Omega$, one obtains

$$v = \alpha \sin(\pi g) \int_0^\infty dx \frac{\cos(\Omega' x)}{\sinh^{2\eta}(\pi x)} \sin[ux - (1 - g)vx].$$  \quad (20)

where the prefactor was absorbed in the renormalized impurity strength $\alpha = \pi \lambda^2 (\pi / \Delta')^2 g$. In the following, we shall use this self-consistency equation to determine the time-averaged differential conductance $G_d = d\bar{I}(U)/dU$ at $U = 0$ as well as the time-averaged conductance $\bar{G} = \bar{I}/\bar{U}$ at finite $U$.

By virtue of $\langle 1 \rangle$, the differential conductance is given by

$$G_d = G_0[1 - V'(U)] = G_0[1 - v'(u)]$$  \quad (21)

as a function of the derivative $v'(u)$. At $u = 0$, this function can easily be calculated from $\langle 2 \rangle$ by deriving both sides with respect to $u$ and using $v(0) = 0$. One obtains

$$v'(0) = \frac{1}{\alpha} + 1 - g$$  \quad (22)

where

$$\kappa(\Omega') = \alpha \sin(\pi g) \int_0^\infty dx \frac{x \cos(\Omega' x)}{\sinh^{2\eta}(\pi x)}.$$  \quad (23)

This integral converges for strong interaction $0 < g < 1/2$ and can be calculated analytically. It turns out that the zero bias anomaly, which is always present in static systems for $T = 0$, vanishes as soon as the impurity acquires its own dynamics for $\Omega \neq 0$. It appears that the finite oscillation frequency plays the role of the effective temperature. In order to investigate the interplay between the effects of finite $\Omega$ and $T$, the normalization to $1/\beta$ is very convenient.

Using the last two equations one can analyze the differential conductance. The resulting graph is shown in Fig. 1. As expected, in the limit $\Omega \to 0$ the remnants of the zero bias anomaly are more pronounced for smaller $g$. The qualitative behavior of $G_d$ changes dramatically as soon as $g$ becomes smaller than $1/2$. Then at some threshold value $\Omega_0$ the differential conductance $G_d$ exceeds the unitarity limit $G_0$. This dynamical conductance enhancement (DCE) is in accordance with previous findings. The actual dependence of $\Omega_0$ on the interaction strength is quite spectacular. While it is nearly linear for small $g$ (s. the inset of Fig. 1), it diverges logarithmically towards $g = 1/2$. We find qualitatively the same behavior for the maximal $G_d$ as a function of the interaction strength. By an explicit calculation of the next order $\lambda$ contributions we made sure that DCE is not a perturbation theory artefact.
As a next step, we calculate the conductance for finite bias voltage $U$. The integral in (20) can be solved analytically for $g < 1/2$ and the resulting self-consistency equation can easily be solved numerically, leading to a unique solution $v = v(u)$. By virtue of (11) this leads to the current-voltage characteristic shown in Fig. 2. The generic feature of all curves is that for voltages higher than the oscillation frequency the voltage dependence of the conductance approximately follows that of the static system. This is not surprising as in this case the excitations with energies of the order $\Omega$ dominate the system’s transport properties rendering all other energy scales irrelevant. Similar behavior has been found in the context of resonant tunneling in LLs.

Again, for low voltages the conductance exceeds the conductance quantum $G_0$ above $\Omega_0$. This behavior was predicted in the voltage drop regime. The authors also gave an explanation of the effect: in the weak scattering regime $\lambda \ll 1$, the system can be regarded as consisting of two chiral LLs coupled by weak tunneling. Due to the $g$-dependent tunneling density of states, scattering between left- and right-moving particles may be strongly enhanced for $\Omega \approx U$. For $g < 1/2$ this can lead to a negative backscattering current. While in the voltage drop regime this effect shows up whenever $\Omega > U$, our results show that the occurrence is restricted to a smaller parameter range. From Fig. 1 and Fig. 2 we see that $G$ exceeds $G_0$ whenever $\Omega > \max(\Omega_0, U)$.

V. EXACT SOLUTION AT $g = 1/2$

From the above results it became apparent that the interaction strength $g = 1/2$ is very special and in need of more thorough investigation. Luckily, in this situation the summation of (15) can be performed analytically and an exact result can be derived. On the other hand, the corresponding Hamiltonian can be rewritten in terms of some new fermionic fields (the so-called refermionization procedure) thereby acquiring a quadratic shape. This offers an alternative way to calculate the transport properties of the system.

A. Coulomb gas expansion resummation

The ‘collapsed-dipole-approximation’ which after slight modifications continues to work in the time-dependent case, becomes exact and using (14), one obtains the self-consistency equation

$$V = \frac{2\pi \lambda_B}{\beta T} \int_0^T dt \mu(t) \int_0^\infty ds \frac{\sin((U-V/2)s)}{\sin[\pi s/\beta]}$$

where $\lambda_B = \pi \lambda^2 / \Delta$. In this equation, $G(s)$ is defined by

$$G(s) = \mu^2(t-s) \exp(-\lambda_B [G(s) - G(0)])$$

and $V = \frac{\pi u}{\lambda_B^2}$. In this case, $G(s)$ is determined by

$$G(s) = \frac{\pi u}{\lambda_B^2}$$

It was also found that for the static case, all energies occurring in (24) can be measured in units of the impurity energy scale $\lambda_B$. It turns out that this property is maintained also the case of an oscillating impurity. We therefore introduce the dimensionless quantities $v = V/\lambda_B, u = U/\lambda_B, \vartheta = 1/(\beta \lambda_B)$ and $\epsilon = \Omega/\lambda_B$. Now, the $t$-integration in (24) can be performed and leads to

$$v = \pi \theta \int_0^\infty dx \frac{\sin((u-v/2)x)}{\sinh(\pi \theta x)} e^{-x/2}$$

$$\times \left\{ \cos(\epsilon x) I_0 \left[ -\frac{\sin(\epsilon x)}{2\epsilon} \right] + I_1 \left[ -\frac{\sin(\epsilon x)}{2\epsilon} \right] \right\}$$

where $I_n(x)$ denotes the modified Bessel function. The remaining convergent integral can easily be evaluated numerically and the resulting self-consistency equation can be solved and yields a unique voltage $v = v_C(\epsilon, u, \vartheta)$. By virtue of (11), one obtains the current-voltage characteristic shown in Fig. 3. As the first prominent feature of
of the Heisenberg equations of motion. In the clean parts of the system Hamiltonian $H = H_{LL} + H_{ac}$ can be rewritten in the following way

$$H = -iv \int dx \Psi^\dagger(x) \partial_x \Psi(x) + \lambda(t) a \left[ \Psi(0) - \Psi^\dagger(0) \right]$$

(26)

where $\Psi^\dagger(x)$ and $\Psi(x)$ are new chiral fermionic fields which depend on $\theta(x)$ and $\phi(x)$. The operator $a$ is an auxiliary Majorana fermion satisfying $\{a, a^\dagger\} = 2$ which does not affect physical results. Insertion of such an object facilitates the derivation of equations of motion for this Hamiltonian. Moreover, one can define a formal particle density

$$\tilde{\rho}(x) := \langle \Psi^\dagger(x) \Psi(x) \rangle$$

(27)

where $\langle \cdots \rangle$ denotes normal ordering with respect to the equilibrium $U = 0$ state. This makes it possible to express the boundary conditions (7) in terms of this new density operator. As it is time-dependent, we take again the time average over the oscillation period $\mathcal{T}$. Defining $\bar{\rho}(x) = (1/\mathcal{T}) \int_0^\mathcal{T} \tilde{\rho}(x,t) \, dt$, the boundary condition (7) becomes

$$5\bar{\rho}(-L/2) - 3\bar{\rho}(L/2) = \frac{U}{\pi v_F}.$$  

(28)

The Hamiltonian (26) can still be diagonalized by means of the Heisenberg equations of motion. In the clean parts this result one recognizes the gradual disappearance of the zero bias anomaly. As in the perturbative result, as soon as the applied voltage exceeds $\Omega$, the current-voltage characteristic nearly reproduces the static behavior.\(^{13}\)

### B. Refermionization

The above results can be recovered by an alternative technique. The starting point is the observation that the system Hamiltonian $H = H_{LL} + H_{ac}$ can be rewritten in the following way

$$H = -iv \int dx \Psi^\dagger(x) \partial_x \Psi(x) + \lambda(t) a \left[ \Psi(0) - \Psi^\dagger(0) \right]$$

(26)

where $\Psi^\dagger(x)$ and $\Psi(x)$ are new chiral fermionic fields which depend on $\theta(x)$ and $\phi(x)$. The operator $a$ is an auxiliary Majorana fermion satisfying $\{a, a^\dagger\} = 2$ which does not affect physical results. Insertion of such an object facilitates the derivation of equations of motion for this Hamiltonian. Moreover, one can define a formal particle density

$$\tilde{\rho}(x) := \langle \Psi^\dagger(x) \Psi(x) \rangle$$

(27)

where $\langle \cdots \rangle$ denotes normal ordering with respect to the equilibrium $U = 0$ state. This makes it possible to express the boundary conditions (7) in terms of this new density operator. As it is time-dependent, we take again the time average over the oscillation period $\mathcal{T}$. Defining $\bar{\rho}(x) = (1/\mathcal{T}) \int_0^\mathcal{T} \tilde{\rho}(x,t) \, dt$, the boundary condition (7) becomes

$$5\bar{\rho}(-L/2) - 3\bar{\rho}(L/2) = \frac{U}{\pi v_F}.$$  

(28)

The Hamiltonian (26) can still be diagonalized by means of the Heisenberg equations of motion. In the clean parts of the wire ($x \neq 0$) the solutions are plane waves with dispersion relation $\omega = kv$. Due to the impurity potential, the function acquires a discontinuity at $x = 0$. Therefore, one makes the ansatz

$$\Psi(x,t) = \frac{1}{\sqrt{T}} \sum_k \exp \{ik(vt - x)\} \begin{cases} a_k, & \text{for } x < 0; \\ b_k, & \text{for } x > 0. \end{cases}$$

(29)

The anti-commutation relation then requires $\Psi(0,t) = (1/2\sqrt{\mathcal{T}}) \sum_k e^{ikvt}(a_k + b_k)$ at $x = 0$. Inserting (29) into the Heisenberg equations of motions and using the oscillating impurity strength $\lambda(t) = \lambda \cos(\Omega t) = \lambda \cos(vKt)$, one can derive a scattering matrix equation for the fermions $a_k$ and $b_k$. Due to the inelastic scattering, an operator $b_k$ will couple to operators $a_{k'}$ and $a^\dagger_{k'}$ where $k' = k + 2nK$ for all integers $n \in \mathbb{Z}$. Therefore, it is convenient to introduce a vector notation and to define

$$a(k) := \begin{pmatrix} a_k \\ a_{k-2K} \\ \vdots \\ a_k \\ a_{k+2K} \\ \vdots \end{pmatrix}$$

(30)

and analogously for $b(k)$. Moreover, we define the vector $a^S(k)$ which is $a(k)$ for $K \to -K$. The scattering of $a_k$ fermions to $b_k$ fermions is then described by the equation

$$b^S(-k) = -N(k) [2N(k) - iI]^{-1} \times \left[ iN(k)^{-1} a^S(-k) - 2a^\dagger(k) \right]$$

(31)

where $I_{mn} = \delta_{mn}$ is the unity matrix and $N$ is defined by

$$N(k)_{mn} = \frac{\lambda_B}{8v} \left\{ \sum_{q=\pm} \frac{\delta_{m,n+q}}{k + (2n + q)K} + \frac{\delta_{mn}}{k + (2n + 1)K} + \frac{\delta_{mn}}{k + (2n - 1)K} \right\}$$

(32)

where we introduced the impurity energy scale $\lambda_B = 2\lambda^2/v$. This matrix is tridiagonal and can be inverted for any finite dimension. Using a finite-dimensional matrix of size $(n+1) \times (n+1)$ amounts to using $2n\Omega$ as a cut-off frequency. Therefore, although (31) is exact, numerical results which base on a finite-dimensional matrix $N(k)$ will be more accurate for large oscillation frequencies $\Omega$.

The $a_k$ operators describe free fermions which constitute a Fermi sea filled up to the chemical potential $\mu$. For the time-averaged current, we obtain formally the same result as for the static case,\(^{29}\)

$$\bar{I} = \frac{v_F}{4} \sum_k \left\langle (a_k^\dagger + b_k^\dagger)(a_k + b_k) \right\rangle.$$  

(33)

Going to a system of infinite length, the sum over $k$ can be replaced by an integral and the self-consistency
VI. CONCLUSION

We have investigated the transport characteristics of a voltage biased Luttinger liquid with an imbedded time-dependent impurity. Using the radiative boundary condition formalism we derived an exact analytic expansion for the nonlinear time-averaged current-voltage characteristics. The transport properties of the system were calculated exactly in all regimes. This effect of the dynamic conductance enhancement (DCE) relies heavily on the special mathematical property of the backscattering operator, namely on its scaling dimension $\alpha$ being equal to $g$ in the spinless case. Moreover, it is then actually the local operator with the smallest scaling dimension the theory can support and the only one which can have $\alpha$ smaller than 1/2. This is the requirement to for observation of DCE. The situation changes slightly when one is dealing with a spinful system. In that case the corresponding Hamiltonian is constructed out of two sets of conjugated fields describing the charge and spin density fields $\theta_{c,s}$ and $\phi_{c,s}$. The electron field operator is given by

$$\psi_{\sigma=\uparrow,\downarrow} \sim e^{i\sqrt{\pi/2}[\theta_{c,s}+\phi_{c,s}]-[\theta_{c,s}+\phi_{c,s}]}.$$

The corresponding Hamilton operators for the participating fields are given by two copies of (2) but with $g = 1$ in the spin channel. Hence the ‘physical’ backscattering operator as defined in (6) would only have the scaling dimension $\alpha = (1 + g)/2$ and therefore unable to generate DCE. Nevertheless, there is an operator which has exactly the same shape as (3) but contains the $\psi_{c,s}$ field only and which has a scaling dimension $2g$. It corresponds to the more complicated scattering of the kind $\psi_{L,\sigma}^\dagger(0)\psi_{L,\sigma}(0)\psi_{R,\sigma}^\dagger(0)\psi_{R,\sigma}(0)$. Not only would such a kind of scattering be present in any spinful system with local scattering, it will also be the most dominant one in the low energy regime because of its small scaling dimension. Thus, for a spinful system with $g < 1/4$ we expect our predictions to hold even quantitatively as long as $\Omega$ is not too high.

Probably the most spectacular realizations of LL physics are found in the SWNTs. Contrary to the ordinary spinful LL, the nanotube’s electronic degrees of freedom can be adequately described by a four-channel LL. Three of them are free and only one of them is genuinely interacting. Following the line of reasoning from the last paragraph we shall find that the operator with the smallest scaling dimension $\alpha = 4g$ can be constructed out of 8 electron field operators and is given in Eq. (6.10) of [31]. The critical value for the interaction constant is then $g = 1/8$. As a rule, the correlation strengths measured in the experiments vary between 0.1 and 0.25 (see e.g. [4]). Therefore, we expect the DCE to be visible in the SWNT-based setups, where the oscillating impurity is either realized by an STM tip brought into the vicinity of the tube or by a local coupling of the SWNT to a strong laser radiation.

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