ABSTRACT

In this paper, I argue that the two versions of divided line (the first in Book VI and the recalled version in Book VII) create problems that cannot be solved — with or without the hypothesis that the objects belonging to the level of διάνοια on the divided line are intermediates. I also argue that the discussion of arithmetic and calculation does not fit Aristotle’s attribution of intermediates to Plato and provides no support for the claim that Plato had such intermediates in mind when he talked about διάνοια in the Republic. The upshot of my argument is negative: even if Aristotle’s report about Plato and intermediates is correct, there is no evidence for such objects provided in the passages I review from the Republic. If they are to be found in Plato, it will have to be elsewhere that they are found.

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I. THE PROBLEM

In Book VI of the Republic, Plato indicates that the proportions of the divided line are intended to indicate different degrees of clarity and truth:

There are four such conditions in the soul (παθήματα ἐν τῇ ψυχῇ), corresponding to the four subsections of our line: Understanding (νόησις) for the highest, thought for the second (διάνοια), belief (πίστις) for the third, and imaging (εἰκασία) for the last. Arrange them in a ratio, and consider that each shares in clarity (σαφήνεια) to the degree that the subsection it is set over shares in truth (ἀλήθεια). (Republic VI.511d6–e4)

In this paper I explore some of the problems that arise in Plato's discussions of clarity in the Republic, and whether these are solved by the hypothesis that Plato has the intermediates in mind when he talks about the subsection associated with διάνοια. As far as what Plato has in mind with respect to the role of clarity, it seems like this passage gives us as good a start as one could ever hope for: clarity applies to the παθήματα ἐν τῇ ψυχῇ—that is, to νόησις, διάνοια, πίστις, and εἰκασία, respectively; truth applies to whatever these παθήματα ἐν τῇ ψυχῇ are “set over” (ἐφ᾽ οἷς).

The παθήματα ἐν τῇ ψυχῇ, I take it, are cognitive conditions of some sort. And these are said to be “set over” whatever in the divided line passage is supposed to be evaluated in terms of degrees of ἀλήθεια. Now scholars have (correctly, I think) regarded ἀλήθεια as the measure that applies to the objects associated with each of the subsections of the line. But this was not obviously the initial way in which these two measures were associated with the proportions of the line’s subsections. Consider how Plato first divides the line (see appendix I for a representation):

It is like a line divided into two unequal sections. Then divide each section—namely, that of the visible and that of the intelligible—in the same ratio. In terms now of relative clarity and opacity (σαφήνεια καὶ ἀσαφεία), one subsection of the visible consists of images. And by images I mean, first, shadows, then reflections in water and in all close-packed, smooth, and shiny materials, and everything of that sort, if you understand. I do.

In the other subsection of the visible, put the originals of these images, namely, the animals around us, all the plants, and the whole class of manufactured things.

Consider them put.

Would you be willing to say that, as regards truth and untruth (ἀληθείᾳ τε καὶ μή), the division is in this proportion: As the opinable (τὸ δοξαστὸν) is to the knowable (τὸ γνωστὸν), so the likeness is to the thing that it is like?

Certainly. (Republic VI. 509d6–510b1)

I wish Plato had managed to make the appropriate connections between what is supposed to be measured by clarity and truth more consistent, but these two passages already reveal such a wish to be in vain. In the passage at 511d6–e4, it seemed obvious that clarity applied to cognitive conditions, but in this passage it seems that clarity applies to the objects with which the cognitive conditions (only named at the very end of the divided line passage, quoted above) are associated. Truth is brought in at 510a9, but applied to “the opinable” and “the knowable.” This distinction reminds us of
Plato’s discussion of knowledge, opinion, and ignorance at the end of Book V (to which I will turn in the next section), but what it seems to be referring to more immediately here is the original division of the line into two unequal parts, which Plato had initially associated with “the intelligible” (τὸ νοητὸν — 509d2) and “the visible” (ὁρατὸν). Whatever we are to say about these (intelligible and visible), it seems not to be an option to think of them as cognitive entities, but (qua intelligible and qua visible) as the kinds of entities to which cognitions might be applied. Thus, at the very beginning of the divided line passage, it appears that both clarity and truth are intended to apply to the kinds of entities to which cognitions might be applied, rather than to the cognitions themselves. This would seem to leave us with the uneasy interpretive option of either supposing that clarity and truth are just different names for the same measure, where the measure itself is a measure of some character of objects to which cognitions might be applied, or else that they are both measures of objects to which cognitions might be applied, but are nonetheless (somehow) different measures. But again, neither of these options strictly works for the explicitly different applications of clarity and truth that Plato gives at 511d6–e4.

So, my first problem in the association of clarity with truth has now been introduced: Plato seems to be somewhat less than clear in telling us precisely what truth and clarity are supposed to measure.

II. BACK TO BOOK V

Comparisons of the relative clarity of cognitions were first discussed in Book V, when Plato has Socrates and Glaucon compare the relative merits of knowledge (sometimes called ἐπιστήμη; sometimes called γνώσις), opinion (δόξα) and ignorance (ἀγνοια):
reference to these I call each the power it is: What is set over the same things and does the same I call the same power; what is set over something different and does something different I call a different one. Do you agree?
I do. (Republic V.477d1-7)

As everyone knows, he goes on to claim that knowledge is “set over” what is (τὸ ὄν; 477b11, 478a7, 478c3, 478d6), ignorance is “set over” what is not (478c3, 478d7), and so opinion, which has been shown to be intermediate between these others, is thus “set over” “what participates in both: what is and what is not” ἀμφοτέρων μετέχον, τοῦ εἶναι τε καί μὴ εἶναι — 478e1-2). Most of the remainder of Book V is thus spent on showing that “what is” consists in the forms, whereas what is and is not consists in such things as the “many beautiful things” (479a5-6), “just things” (479a6-7), “pious things” (479a7), and so on, all of which will be beautiful, just, or pious in some way, but also their opposites in some way, and will thus participate in both opposites (479b7).

Given this discussion, it seems just obvious to me that Plato is putting the cognitive powers into “set over” relationships with objects—and not at all with propositions or sentences that we think of as being the contents of cognitions. Rather, here in Book V, the “set over” relationship is between cognitions and the kinds of objects to which such cognitions are (naturally — see πέφυκε at 477b11) applied. Plato began this discussion by stating that philosophers are “those who love the sight of truth” (τοὺς τῆς ἀληθείας φιλοθεάμονας — 475e4). In the remainder of the passage, he does not mention truth; he only tells us that the object the cognitive powers are “set over” differ in terms of their degrees of being. But toward the end of his discussion, he also applies the language of clarity/brightness and unclarity/dimness (or darkness) to such objects:

Then do you know how to deal with them [sc. the things that both are and are not]? Or can you find a more appropriate place to put them than intermediate between being and not being? Surely that can’t be more than what is or not be more than what is not, for apparently nothing is darker than what is not or clearer than what is (οὔτε γάρ που σκοτωδέστερα μὴ ὄντος πρὸς τὸ μᾶλλον μὴ εἶναι φανήσεται, οὔτε φανότερα ὄντος πρὸς τὸ μᾶλλον εἶναι) (Republic V. 479c6-9)

So here again we find the same problem as the one with which we began: comparisons in terms of clarity (or brightness) are made between both cognitions and the sorts of objects to which cognitions are applied. I suggest, then, that we take the first sort of application of clarity/brightness comparisons to reflect the second — that is, the quality of cognitions is explicable in terms of the quality of the kinds of objects to which they are applied.

If my suggestion is correct, then if we use the distinction Plato provides in the passage with which we began (Republic VI.511d6-e4), it will mean that the clarity/brightness of cognitions will co-vary with the truth of the objects they are “set over.” At least Plato remains consistent (in the middle books of the Republic, at any rate) in applying measures of truth/untruth to the kinds of objects to which cognitions may be applied (as at 511d6-e4 and 510a9, both mentioned above, but see also 484c8, 508d10). Truth, in the simile of the sun, is the intelligible analog to light in the visible world, and different levels of each of these are said to co-vary with the clarity or obscurity of
the kinds of cognitions produced when applied to the objects “illuminated” by each.

But lest this summary of the relationship between clarity and truth seem like it will suffice to explain Plato’s proportions in the divided line, we should not conclude our discussion without paying attention to another very important passage that scholars have used to try to figure out how to understand this connection. Unfortunately, if we consult this other passage, we can actually manage to create an even greater problem.

III. THE NIGHTMARE AT 533D4-534A9

As he sums up his discussion of the role of dialectic in the higher education of the rulers of Kallipolis, Socrates has a few choice things to say about how dialectic compares with the practices of the mathematical studies, and then (incorrectly) recalls what he said about the divided line in Book VI (see appendix 2 for a representation):

From force of habit, we’ve often called these crafts sciences or kinds of knowledge (ἂς ἐπιστήμας), but they need another name, brighter than opinion but darker than knowledge (ἐναργεστέρου μὲν ἢ δόξης, ἀμυδροτέρου δὲ ἢ ἐπιστήμης). We called them thought (διάνοια) somewhere before. But I presume that we won’t dispute about a name when we have so many more important matters to investigate.

Of course not.

It will therefore be enough to call the first section knowledge (ἐπιστήμη), the second thought (διάνοια), the third belief (πίστις) and the fourth imaging (εἰκασία), just as we did before. The last two together we call opinion (δόξα), the other two, intellect (νόησις). Opinion (δόξα) is concerned with becoming (γένεσις), intellect (νόησις) with being (οὐσία). And as being (οὐσία) is to becoming (γένεσις), so intellect (νόησις) is to opinion (δόξα), and as intellect (νόησις) is to opinion (δόξα), so knowledge (ἐπιστήμη) is to belief (πίστις) and thought (διάνοια) to imaging (εἰκασία). But as to the ratios between the things these are set over and the division of the opinable (δοξαστόν) or the intelligible (νοητόν) sections into two, let’s pass them by, Glauc, lest they involve us in arguments many times longer than the ones we’ve already gone through.

To be frank, I don’t see how arguments of any length could pull Plato out of the hole into which he has dug himself here.

As I indicated above, Plato has emphatically not recalled things here “just as we did before.” Instead, he has not only changed several bits of terminology, but also done something to the proportion that calls for our attention. First, terminology: Notice that what had been the παθήμα provided for I₂ in the original version (νόησις) is now given as the name for I₁ and I₂ combined (which had originally been said to stand for τὸ νοητόν). Given the relation between these two terms (νόησις/νοητόν), it is perhaps understandable that Plato would have exchanged them here. But more troubling is that what used to be νόησις is now given as ἐπιστήμη — a term never used in the original divided line passage, but which obviously (again) recalls the cognitive power of the end of Book V. This is now contrasted to διάνοια, which is said to be darker than εἰκασία, but brighter than δόξα. But δόξα has also now been substituted for what had been τὸ ὁρατόν. Again, this may seem benign, especially given
the connections Plato makes between opinion and vision at the end of Book VI and also in the simile of the sun. Bringing in the two cognitive powers of Book V in order to place διάνοια between them in terms of brightness and darkness allows us to make sense of the relative cognitive merit of διάνοια, but if we try to put all of this together with what Plato actually did say about the divided line in Book VI, we run into problems.

For one thing, notice that Plato has also modified the proportion that he originally provided. (See the bottom left sections of the two appendices.) In both passages, Plato claims that the proportion expressed by the original division of the line (that is, between the combined top two subsections and the bottom two subsections) is the same as one that obtains between two of the subsections. But the two subsections he places into that proportion, here in Book VII, are different from the two he thus compared in Book VI. In Book VI, the proportion that was said to obtain between II + I2 and V1 + V2 (intelligible and visible, respectively) was the same as those to be found between I2 and I1 and also V2 and V1, that is:

\[ \frac{I1 + I2}{V1 + V2} = \frac{I2}{I1} = \frac{V2}{V1} \]

But here in the later passage (in Book VII), the proportion is different, viz.:

\[ \frac{I1 + I2}{V1 + V2} = \frac{I2}{V2} = \frac{I1}{V1} \]

In other words, Plato has interleaved the places of I1 and V2 in the proportions given. As a simple point of mathematics, this interchange would not yield the same proportion unless the length of I1 (representing διάνοια in both versions) is the same as that of V2 (which represents πίστις in both versions).

Scholars have divided over the question of whether or not Plato intended the proportions he gives to make the middle subsegments (I1 and V2) equal in length, though no one doubts that as a simple matter of mathematics, they must in fact be equal. On the one hand, Plato never actually explicitly says anything about this implication of his construction. But on the other, here in the recapitulation of the line, he alters the proportions in a way that would make no sense if he weren’t at least aware of this consequence.

One advantage that has been claimed for thinking that Plato really did intend to the middle segments to be equal is given by scholars who think the same objects are associated with each of them, though this has not been a view much shared by the many scholars who have written about this subject. Most scholars have argued that Plato surely would not have intended to make the two subsections equal in length, because this would imply that διάνοια would be equal in clarity to πίστις, which he surely does not accept, and which seems to be explicitly contradicted in the long quote just above, at 533d5–6. The problem is, again, that the modified proportions that Plato supplies here in Book VII — which, again, are supposed to recall the ones he provided in Book VI — require the very equality that most scholars have regarded as unintended. I do not see any persuasive solution to this problem, and in my earlier work, I confess to offering a rather strained speculation in response to it, which as far as I know, no one has ever actually accepted. I have no better explanation to offer even now.

In case this is not already enough of a problem, what Plato has to say about the relative merits of ἐπιστήμη, διάνοια, and δόξα actually makes things worse. Now it is strictly true that when he makes this comparison at
533d5-6, the relative merits are expressed in terms of brightness and darkness (ἐναργής and ἀμυδρός), but scholars have managed to agree on at least the understanding that this should be understood as a comparison of relative clarity (again, usually σαφήνεια). These, recall (together with relative degrees of ἀλήθεια) were said to be what the varying lengths of the line’s segments and subsegments were supposed to represent (again, see 509d9 and then 511e3). Most scholars have taken what Plato has to say here about ἐπιστήμη, διάνοια, and δόξα to show that he cannot regard διάνοια and πίστις to be equal in clarity, and so dismiss the equality of the middle segments as an aspect of the proportions of the line that Plato did not really intend or wish to call our attention to. But in order to represent διάνοια as clearer (or truer) than all of δόξα in the recapitulation of the line, it would have to be true that I₁ is longer than V₁ + V₂, which it plainly can’t be for the proportion to hold. For the proportion, again, it must be that I₁ = V₂ (in length). So it now looks like Plato is trying to tell us something about the epistemic merits of διάνοια that cannot be represented in the line as he has drawn it originally in Book VI, or as he recapitulates in here in Book VII. Now, it is presumably because of the superiority of διάνοια to πίστις and δόξα that scholars have proposed that Plato must have the intermediates in mind, since they would surely be truer than the visible originals that belong to the level of πίστις, or even the entire domain of visibles that belong collectively to δόξα. Just saying this seems to provide some advantage for the hypothesis. But many scholars have resisted the hypothesis, on two grounds: (1) Plato actually never manages to mention the intermediates in the divided line passage. Supporters of the intermediates here have claimed that they fulfill the requirements of the simile by being intelligible images of the forms. But the fact remains that while Plato does manage to associate the level belonging to διάνοια with images, the only images of forms he actually mentions in connection with this subsegment are the visible things drawn by the mathematicians — and these images are mentioned seven times in this very short passage (510b4-5, 510b7-9, 510b5-6, 510c1-1, 511a6-7, 511c1, 511c7-8). If he wanted us to have mathematical intermediates in mind when he identified the images belonging to this level, he could hardly have done a worse job of it. But in case this is not enough of a reason for doubting the hypothesis about the intermediates, there is another one, which I have already mentioned: (2) The way Plato makes his construction not only requires the middle two segments to be equal, his later “recollection” of what he had said absolutely requires that he be aware of that equality in reporting the proportions the way he does in Book VII. The alleged advantage to the hypothesis involving the intermediates makes sense only if we also reject the equality of the two middle subsegments by recognizing the ontological superiority of the mathematical intermediates — that is, their allegedly greater truth or reality (again, ἀλήθεια) which would be proportionate to the greater σαφήνεια of διάνοια, relative to the subsegments below it. The problem is that Plato’s construction does not and cannot represent these alleged ontological and epistemic superiorities. In fact, I think there is also a third problem with the hypothesis, which is that the mathematical intermediates are supposed to be perfect examples of their characteristics. But given the way Plato seems to measure ἀλήθεια, this would seem to make them no less true than the forms they supposedly image or participate in. Supporters could argue, I suppose, that their lack of uniqueness entails a lower degree of
ἀλήθεια, but again, I do not find this alleged measure of ἀλήθεια in our text. In any case, I will have more to say about this specific objection in the next section, where I talk about alleged intermediates in the higher education of the future rulers.

At any rate, the hypothesis about the intermediates does not help solve the problems I have noted with what Plato has to say about the divided line in terms of truth and clarity. Instead of clarifying or explaining what the text says, the hypothesis could perhaps help with one passage in Book VII, but then conflicts with what Plato actually says about his proportions and what they are supposed to represent — in both Books VI and VII.

As far as the problems with what Plato does say, these only get worse. In order for either version of the divided line to work, there must be a proportion between the two sections of the visible taken together, and the two sections of the intelligible taken together that can be applied to two of the subsections taken alone similarly compared. (Again, see the proportions given in the lower right of each appendix.) As far as I know, there has been no notice in the literature about the problem that this seems to create, namely, that V1 + V2 (that is, the entire lower section of the original division) must be clearer (and, as we soon learn, given the association of clarity and truth, also truer) than either V1 or V2 by themselves. But this seems to me to create nonsense: How can V1 + V2 be clearer or truer than either V1 or V2? Why would adding the relative lack of clarity (and truth) in V1 to whatever we find in V2 make V1 + V2 clearer (and truer) than V2 just by itself? Plato tells us that V1 consists in shadows and reflections in water and other reflective surfaces. Why would adding these to the visible originals give us a collection of things that is clearer or truer than the collection of visible originals without shadows and reflections added to that collection?

The same problem, obviously, clouds whatever we are supposed to make of the upper subsections of the line. If the lengths of the line segments are supposed to represent degrees of clarity and truth, it follows that I1 + I2 (the entire intelligible section of the original division) must be both clearer and truer than either I1 or I2 alone. But how can that be?

To go back to my troubles in the first section, the problem does not dissolve whether we take clarity or truth to measure objects to which cognitions apply or to measure some character of the cognitions themselves. The problem is that it seems absurd to think that visible originals taken together with their visible images (V1 + V2) will be clearer or truer than the visible originals alone (for example). Similarly, it seems absurd to think that taking the forms together with whatever objects we associate with διάνοια will yield greater clarity and truth than the clarity and truth of the forms alone. So, too, the epistemic deficiencies we are supposed to associate with the lower subsections of the line, relative to the subsections just above them in each of the original divisions, make it absurd to suppose that Plato intends whatever epistemic condition we should apply to the entire lower line (V1 + V2) — δόξα in the recapitulation — to be clearer and truer than either εἰκασία or πίστις by themselves. Why would adding the (less clear/true) εἰκασία to πίστις yield a clearer (or truer) cognitive condition (taken as a whole) than that enjoyed by πίστις alone? So, too, why would adding the (presumably inferior) clarity and truth associated with διάνοια to what νόησις provides yield a clearer (or truer) cognitive condition than νόησις by itself? The problem is that whatever we are to say about degrees of truth and clarity, it does not seem like these are
going to be *additive* in the way that a continuous line would suggest: plainly, the lengths of *either* of the main segments of the line will be longer than the lengths of the subsegments it contains. But if Plato’s proportions are supposed to work, the relative lengths of whatever parts of the line we are comparing are supposed to indicate the proportionate degree of clarity and truth of each part, whether in the original division or in the subsequent subdivisions.\(^{14}\) We are left with the unhappy result that Plato makes proportions of clarity and truth the focus of the comparisons he makes in the divided line passage, but in doing so, he creates an image that has both mathematical and also philosophical entailments that do not seem to represent views he would accept.

**IV. INTERMEDIATES IN THE MATHEMATICAL EDUCATION OF BOOK VII?**

At 524c13, Socrates reintroduces the distinction between the intelligible and the visible, which has run through all of the great “similes of light” in Books VI and VII. And then at 524d6, Socrates asks Glaucon to which of these do number and the one belong. When Glaucon first responds that he doesn’t know, Socrates reminds him of how summoners work — do things appear to be just one at any given time, or do they appear to be both one and also the opposite of one at the same time (524d8–a3). Glaucon is more nimble this time: he responds that “the sight of the one does possess this characteristic to a remarkable degree, for we see the same thing to be both one and an unlimited number at the same time” (525a4–6). Socrates then gets Glaucon to agree that the same is true for all numbers (525a7–9). Since “calculation and arithmetic are wholly concerned with numbers (525a10–11), “evidently they lead us towards truth” (525a13).

Here, too, in Plato’s discussion of numbers, scholars have tended to see Plato’s alleged commitment to such “mathematical intermediates.” Plato certainly encourages us to regard the numbers as belonging to the intelligible domain. But the question we should ask is whether invoking the “intermediates” makes better sense of the text than some other interpretation.

As a matter of fact, it seems to me that invoking the “intermediates” here actually adds an unnecessary complication to Plato’s discussion.\(^{15}\) Since Plato has Socrates insist that what he says about numbers is supposed to be understood in the very way he had characterized all of the summoners he mentioned immediately before, it would seem to follow that if the numbers to which he refers here are supposed to be intermediates between forms and sensibles, then the same would presumably apply to all of the other examples of summoners he has provided, including obviously bigness and smallness, thickness and thinness, hardness and softness, lightness and heaviness. The first and last of these pairs of contraries are included among the list of things that were also identified as part of “what is” in Book V. As I said earlier, one problem with intermediates is that they seem to qualify as wholly being what(ever) they are, but still managing to be intermediate between sensibles and forms. But the only characteristic Plato gives us of forms here in the *Republic* is that they are wholly what(ever) they are. So we are left with no reason to imagine that Plato has intermediates in mind when he discusses “what is” in Book V, and also no reason to suppose that he had them in mind at the end of Book VI, in the divided line. Here, too, I would say, that Plato is giving us the same characterization of “what is” as he has done.
consistently throughout his discussion of being in the *Republic*, and no reason is provided in the text for supposing that “what is” consists in anything other than the forms.

Plato has Socrates ask Glaucon whether “the one (τὸ ἕν) is adequately seen itself by itself (αὐτὸ καθ᾽ αὑτὸ) by the senses (at 524d8-­e1). A bit later, Socrates says that calculation “leads the soul forcibly upward and compels it to discuss the numbers themselves (αὐτῶν τῶν ἀριθμῶν, never permitting anyone to propose for discussion numbers attached to visible or tangible bodies” (525d5–8).

It seems the only good reason we might have to wonder if Plato has intermediates in mind here is what Aristotle reports about Plato’s view (without, however, saying that the view is reported in the *Republic*).

So let us see whether what Plato has to say here is a good fit with what Aristotle reports. Here is what Aristotle said:

He says that besides the sensible things and the forms, and between these, there exist the mathematical objects, differing from the sensible things in being eternal and immovable, and from the forms in that there are many alike whereas the form itself corresponding to these is only one. (Aristotle, *Metaphysics* 987b14-­18; trans. Apostle and Gerson [1991])

I argued earlier that the mathematical objects characterized here do not seem to be needed to explain what Plato has to say about any of the objects he mentioned in the divided line passage. I also noted that the introduction of these objects, as belonging to the subsection of the line associated with thought (*dianoia*), require interpreters to attribute to Plato a strange oversight, since such “intermediates” are never actually mentioned by Plato in that famous image. Neither are they mentioned here in Book VII, where Plato talks about numbers, for it is completely explicit that when Plato has Socrates tell Glauc0 about why numbers belong to the intelligible world, each one of the things he is talking about are “only one.” Each number belongs to the domain of intelligence because each one of them can be “adequately seen itself by itself,” but not by the senses. It may be that Plato thought there could be innumerably many perfect squares or triangles (or whatever) of different dimensions, whereas there is just one square itself and triangle itself (the forms of square and triangle, respectively, that is) and not many of each. So, the grounds that Aristotle gives for attributing the belief in “mathematical intermediates” to Plato plainly do not apply to what he has to say in this passage about the one or any of the other numbers. What he has to say about numbers in this passage, rather, seems to fit only with what Aristotle says applies to Plato’s forms. If Plato did intend to mention “mathematical intermediates” here in Book VII, he manages to do it in such a way that his own student, Aristotle, would have to count as a reference to forms, and not intermediates. This obviously does not count as an advantage for this proposed interpretation.

V. SUMMARY AND CONCLUSION

In this paper, I have discussed two of the passages in the *Republic* where scholars have been inclined to invoke the mathematical intermediates. I have found, however, no reason to support this hypothesis and several reasons to resist it. In brief, the intermediates do not solve the problems (which continue to me to seem unsolvable) in what Plato has to say about the divided line, and its proportions and what they signify. Moreover, where invoked to explain what Plato has to say about numbers in
Plato’s discussion of higher education in Book VII, the hypothesis fails even more obviously — since what Aristotle actually says about the intermediates does not seem to apply to the very things that supporters of the intermediates hypothesis want to explain in terms of them. Obviously, this does not prove that Aristotle was wrong or misreporting Plato’s thought. The most we can conclude from what I have said, at best, is just that the Republic gives no evidence for Aristotle’s claim.
Knowledge (ἐπιστήμη)

INTELLECT (νόησις)

Thought (διάνοια)

Belief (πίστις)

OPINION (δόξα)

Imaging (εἰκασία)

$\text{I}_1 + \frac{\text{I}_2}{\sqrt{\text{V}_1}} + \text{V}_2 = \frac{\text{I}_2}{\sqrt{\text{V}_2}} = \frac{\text{I}_1}{\sqrt{\text{V}_1}}$
have been a matter of great disagreement among scholars. See Smith 1996 for review and discussion, but I will have more to say about this issue below.

3 At least Plato has no problem being consistent in using this term: see 477b6, b9, c1-2, c6, d1, d3, d9, e1, e2, e3, 478a4, a14.

4 My complaint here is obviously with Fine 1990, who implausibly argues for a “veridical reading” of what Plato means by “what is,” “what is not,” and “what both is and is not.” See Smith 2000 and 2012 for my own criticisms of this view, and also the criticisms made by Gonzalez 1996 and Szaif 2007.

5 In Book II, Plato has Socrates talk about a false náthauroc év τῇ ὑπωγῇ (Republic II.382b8-c1), for which a “falsehood in words” is an image. I believe Plato would understand sentential falsehood (falsehood in words) in terms of objective falsehood (falsehood of objects), and not the other way around. The relevant passages here, I think, will be Plato’s discussion of truth early in Book V. See, for several examples, Republic V:485c3-4, c10, 485d2, 490a1, 490c2, 501d2, 508d4.

6 Translation modified here: Cooper 1997 translates the term as “clearer,” which may get the meaning right, but which ignores the problem I intend to discuss.

7 See, for examples, Bedu-Addo 1978: 116n15, 1979: 89-90, 105-108; Fogelin 1971: 381-382; Morrison 1977: 220-227; Ringbom 1968: 91-94.

8 Again, see Smith 1996 for a discussion and review of this vast literature.

9 I say “surely does not accept,” but perhaps any certainty on this entire subject is presumptuous. My reason for thinking that Plato “surely does not accept” this consequence is not just that it seems to be explicitly ruled out by what he says at 533d5-6, but also would seem to violate his insistence on the superiority of intellection and the intelligible realm to vision and the realm of the visible/opinable.

10 For the sake of full disclosure, here is what I said: “I am tempted to think that Plato might have woven this subtle flaw into the intricate fabric of his own image, because he wished to avoid the sin of perfection” (Smith 1996: 43).

11 I have discussed this problem in private correspondence with Damien Story, however, who had also noticed it prior to our discussion.

12 See note 2, above.

13 Again, not to mention the scholarly debates around the fact that Plato’s construction makes V2 equal in length to I1.

14 The problem becomes even more obvious when we compare the length of the entire line to any of its subsegments. Plato would surely not have wished us to understand that V1 + V2 + I1 + I2 taken as a whole would be clearer or truer than what we are supposed to find at I2 alone.

15 My argument here has a great deal in common with the one in Franklin 2012, who also thinks that intermediates do not appear in the Republic. I go further than Franklin, however, in denying that Plato does not
even recognize ideal mathematical entities as “theoretical fictions,” as Franklin describes them. Instead, I claim that only forms and sensible images of forms appear in Plato’s discussions of mathematical studies.

16 A good example of a distinguished scholar who sees “the one” and also “the numbers themselves” as examples of “intermediates” is to be found in James Adam’s justly famous edition of the Republic (Adam 1963 vol. 2, 114, note on 525D).

17 I eliminated the translators’ uses of upper case for “Forms” and “Mathematical Objects” because I have not used this convention in my own discussions. Otherwise the translation provided above as the one cited.