Thermal convection in fluidized granular systems

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Thermal convection is observed in molecular dynamic simulation of a fluidized granular system of nearly elastic hard disks moving under gravity, inside a rectangular box. Boundaries introduce no shearing or time dependence, but the energy injection comes from a slip (shear-free) thermalizing base. The top wall is perfectly elastic and lateral boundaries are either elastic or periodic. The observed convection comes from the effect of gravity and the spontaneous granular temperature gradient that the system dynamically develops.

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In the study of granular systems, convection has attracted particular attention. Many of the experimental and simulation studies of granular convection focus their attention on the effects of a vibrating base. Particularly care has been taken to determine the different patterns that appear depending on the amplitude and frequency of the vibrating base, the form of the interaction force between the grains, the effects caused by the roughness and inclination of the walls and the role of voids in this vibrating-base-plus-wall convection. Some theories based on hydrodynamic continuum equations for this type of convection have been developed. Also the effect of the internal shear bands in the system has been pointed out as a source of convection.

The purpose of this letter is to point out the existence of a convective regime in granular systems when there is no vibrating base, or any effect caused by the roughness or the angle of the walls, or any shearing effect caused by the walls themselves. The source of this convection stems from gravity and the dissipative nature of the granular collisions, and some hint about its existence was already mentioned in [1]. In the present model system, each wall is represented by a shear-free and time-independent boundary condition.

More precisely the bottom wall gives a stochastic normal component to the velocity of each particle that collides with it. The tangential velocity is unchanged, thus a shear-free thermal boundary condition is imposed. The top wall is perfectly elastic while the lateral walls are either perfectly elastic or they correspond to a periodic boundary. Particles are subjected to the acceleration of gravity $g$.

When energy is pumped into a granular system through a boundary—the bottom wall in the present case—a temperature gradient develops spontaneously. Since the particle-particle collisions are dissipative the system is hotter close to the injecting boundary and colder away from it. There is then an energy flux from the base upwards which is dissipated in the bulk through collisions. If the system is almost perfectly elastic it remains macroscopically static.

Performing molecular dynamic simulations of inelastic hard disk systems in the presence of gravity, a transition from a hydrostatic (purely conductive regime) to a convective regime is observed by increasing the dissipative parameter present in the particle-particle collision rule.

In standard fluids the onset of thermal convection is roughly determined by the ratio between the characteristic times of the processes against convection (viscous and thermal diffusion) and favorable to convection (buoyancy), through which the externally imposed temperature gradient plays a role. The idea that underlies this letter is that in a granular system there are similar mechanisms which lead to a transition from a conductive to a convective regime.

There is an important difference, though. The temperature gradient is not externally imposed but rather it is dynamically created by the system itself. This means that whenever energy is injected into the system a temperature gradient develops (due to the dissipative nature of the particle-particle collisions) and not three but four ingredients compete inhibiting or fostering the appearance of convection: viscous and thermal diffusion, buoyancy and dissipation.

This letter reports the results about convection appearing in a 2D system of $N$ hard disks with mass $m = 1$ and diameter $\sigma = 1$, which collide inelastically with the rule

$$v_{12} \cdot \hat{n} = -(1 - 2q) \left( v_{12} \cdot \hat{n} \right), \quad v_{12}' \cdot \hat{t} = v_{12}' \cdot \hat{t}, \quad (1)$$

where $v_{12} = \bar{c}_1 - \bar{c}_2$ is the relative velocity between the colliding particles, the primed and unprimed variables re-
fer to the post and precollisional velocities, \( \hat{n} \) and \( \hat{t} \) are
the unit vectors normal and tangential to the contact
plane, and \( q \) is the dissipative coefficient \( q = (1 - r)/2 \),
\( r \) being the normal restitution coefficient. Only trans-
lational degrees of freedom are present. The simulation
results reported below come from our event-driven molec-
ular dynamic simulations [15], and the careful measuring
routines developed in [16].

As already mentioned, the system is maintained in a
fluidized state by the injection of energy from a thermal
(stochastic) base \((y = 0)\) at temperature \( T(0) = T_{\text{base}} = 1 \)
in energy units, while the top boundary is a perfectly
elastic wall at \( y = L \). Gravity enters the problem through
the Froude number,

\[
Fr = \frac{mgL}{T_{\text{base}}}.
\]

Although there is a wide range of parameters for which
convection occurs, all the simulations being reported cor-
respond to \( N = 2300 \) particles, fraction of occupied area
\( \rho_A = 0.18 \), and relative gravity strength \( Fr = 0.1 \). The
only parameter that varies from one simulation to an-
other is \( q \), even though for convenience the product \( qN \)
is used in what follows.

a. Convection inside a box. Consider first the granu-
lar system inside a squared box with a thermal base, and
perfectly elastic upper and lateral walls. As the elastic
walls do not produce any direct shear on this system and
the bottom base is stochastic but preserving the hori-
zontal component of the velocity, the boundaries intro-
duce neither spatial nor temporal macroscopic correla-
tions. It can be said that none of the usual conditions
under which convection has been studied in granular sys-
tems are present, nevertheless convection does appear, as
observed in Fig. 1.

The hydrodynamic stationary solution for the associ-
ated conservative system \((q = 0)\) is simply a constant
temperature system with no heat flux and density de-
creasing with height. Including a small amount of energy
dissipation in each collision, one and even multiple rolls
may develop into the system. Hence the convection we
observe is due solely to gravity and the dissipative nature
of the collisions between particles.

To understand the origin of this convection, we have
plotted (see Fig. 2) the dependence on the dissipative pa-
rameter \( qN \) of the observed granular temperature dif-
ference between the bottom \((y = 0)\) and the top of the
system \((y = L)\), \( \Delta = T(0) - T(L) \). For small values
of \( qN \) this difference increases with increasing \( qN \). This
situation corresponds to the conductive regime, in which
\( \Delta \) can be written as an expansion on the small param-
eter \( qN\rho_A \) [17]. This difference \( \Delta \) reaches a maximum,
at about \( qN \approx 4 \), and from then on it decreases, prov-
ing that a mechanism favoring energy transport from the
base upwards appears at this value. This fact is also cor-
roborated by the amount of energy per unit time (call it
heat flux) \( Q(0) \), entering the system through the base,
plotted in the same figure in arbitrary units. It is seen
that the heat flux is steeper about the same \( qN \) for which
\( \Delta \) reaches its maximum: more energy per unit time is re-
quired to keep stable the temperature at the base.

From these observations we can conclude that there
exists a threshold \( qN \) value from which the convection is
triggered. Furthermore, this also supports the idea that
convection starts when a critical value of the temperature
difference between the bottom and the top of the system
is reached.

![Fig. 1. Averaged velocity field for different values of \( qN \). At the left \( qN = 6 \) and at the right \( qN = 34 \).](image)

![Fig. 2. Differences between the bottom and top temperatures (solid circles) and rescaled heat flux (open circles) versus \( qN \).](image)
are plotted in Fig. 3 clearly showing a supercritical transition at $qN \approx 4$ from the conductive to the convective regime with one convection roll. Due to the symmetry of the problem, rolls with both signatures are equally probable and they appear in our simulations, depending only on the initial condition.

It can also be seen that from $qN \approx 34$ up there is a coexistence of regimes with zero and non zero circulation which corresponds to the competition of one and two convective rolls, this is, a subcritical transition from the one-roll to the two-rolls regime. As the dissipation coefficient $q$ continues increasing, other transitions to multi-rolls patterns can be observed with difficulty but since the system starts getting denser all convective movement eventually disappears.

Although no theory has been presented yet, it seems that the appearance of the two and multi-rolls regimes could be due to a change in the effective aspect ratio of the system: as dissipation increases, there are regions where density rises considerably, lowering the average height occupied by the system.

It is worth mentioning that when two rolls were observed they always appeared as shown in Fig. 1, namely, the fluid goes up in the middle of the box and comes down along the walls. This privileged signature seems to have its origin in the local increase of density that walls cause. Higher density implies more collisions and therefore more dissipation, hence lower granular temperature: the system is heavier near the walls.

**b. Convection with horizontal periodic boundary conditions.** Any effect that the elastic lateral walls could have on the onset of convection in the previous case is discarded when periodic lateral boundary conditions are imposed on the system.

The container is a periodic channel, and in this case a transition to a convective regime is found again, although, due to the absence of lateral boundaries, the convective rolls appearing in the system travel now along the channel. This was observed even though the simulations were carefully initialized with zero total horizontal momentum $P_x$, namely with zero horizontal mass flux. Since the boundary conditions do not change the horizontal component of the velocities, $P_x$ remains zero during the evolution, as was confirmed in the simulation.

In order to detect this pattern, we performed time averages of the mass flux field. Because of the roll movement, this averaging time must be larger than the microscopic time and smaller than the time needed for the roll to travel a significant distance. We chose this time to be much smaller than the ideal gas thermal diffusion time which, in our units, is of order $N \sqrt{\pi / T_{\text{base}}}$, but large enough to contain multiple particle-particle collisions. The observed rolls persisted for times longer than the macroscopic time, resulting in an hydrodynamic pattern.

![Mass flux field averaged in circles of 250 collisions per particle. The figures a), b), c) and d) correspond to cycles 100,112,124 and 136 respectively. A big roll can be observed moving to the left side of the system while a small roll appears varying its size.](image)
observed to have one large roughly circular roll accompanied by a smaller one.

The reason for this movement seems to be found in vortex dynamics. It is well known that a vortex near a fixed isolating wall behaves as if it were in front of a twin vortex which ensures the condition of null hydrodynamic velocity \( v_y = 0 \) at the top elastic wall [18]. This twin vortex would induce a movement parallel to the wall and a sense determined by the sign of the circulation \( \Phi \) as when a vortex near a fixed isolating wall behaves as if it were in front of a twin vortex.

In the present case, to detect the onset of convection, it was not practical to measure the circulation \( \Phi \) as when a vortex near a fixed isolating wall behaves as if it were in front of a twin vortex. Instead the system was between hard lateral walls. Instead the transition was detected measuring a space velocity correlation near the transition due to the finite size of the system. The dashed line corresponds to the curve \( C_{tot} = 0.0013 \sqrt{qN} \).

\[ C_{tot} = \frac{1}{8} \sum_{i,j} \vec{v}(i,j) \cdot \vec{v}(i',j'), \]

where the cells \((i', j')\) refer to the eight first neighbors of the \((i, j)\) cell. This observable has the advantage of being insensitive to the displacements of the convective pattern. The total correlation is defined as \( C_{tot} = \frac{1}{N_{cells}} \sum_{i,j} C(i,j) \). It measures how similar, on the average, are the velocities in neighboring cells. When there is no convective pattern in the system, then the time averaged value of \( C_{tot} \) is nearly zero, while as soon as a convective current develops, \( C_{tot} \) takes distinctly positive value.

Figure 5 shows the evolution of \( C_{tot} \) with \( qN \). The transition from conductive to convective regime is clearly seen. It takes place roughly at \( qN = 18 \).

In conclusion it can be stated that bidimensional granular systems present convective regimes when there is gravity and a thermal base even though no shearing is introduced through the boundary conditions. It has been argued that such convection owes its existence only to gravity and the dissipative nature of the particle-particle collisions. If the system approaches the elastic limit such convection disappears as there is no spontaneous creation of a vertical temperature gradient. This gradient plays the key role in this kind of convection.

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[1] H. J. Herrmann. Physica A, 191, 263 (1992).
[2] K.M. Aoki, T. Akiyama, Y. Maki and T. Watanabe. Phys. Rev. E, 54, 874 (1996).
[3] E.E. Ehrichs, H.M. Jaeger, G.S. Karczmar, J.B. Knight, V.Yu. Kuperman and S.R. Nagel. Science, 267, 1632 (1995).
[4] C. Bizon, M.D. Shattuck, J.R. de Bruyn, J.B. Swift, W.D. Mc Cormick, and H.L. Swinney. J. Stat. Phys., 93, 7210 (1998).
[5] J.B. Knight, E.E. Ehrichs, V.Yu. Kuperman, J.K. Flint, H.M. Jaeger and S.R. Nagel. Phys. Rev. E., 54, 5726 (1996).
[6] H.K. Pak, E. van Doorn and R.P. Behringer. Phys. Rev. Lett., 74, 4643 (1995).
[7] C. Saluena and T. Poeschel. cond-mat/9807071.
[8] S.G.K. Tennakoon, L. Kondic and R.P. Behringer. Europhys. Lett. 99, 1 (1998).
[9] E. L. Grossman. Phys. Rev. E, 56, 3290 (1997).
[10] M. Bourzutschky and J. Miller. Phys. Rev. Lett., 74, 2216 (1995).
[11] H. Hayakawa, S. Yue and D.C. Hong. Phys. Rev. Lett., 75, 2328 (1995).
[12] H. Hayakawa and D.C. Hong. Powders & Grains 97 (Balkema, Rotterdam, 1997).
[13] J.B. Knight. Phys. Rev. E., 55, 6016 (1997).
[14] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, ( Dover, 1981).
[15] M. Marín, D. Risso, and P. Cordero J. Comp. Phys. 109, 306 (1993)
[16] D. Risso, PhD. Thesis, Universidad de Chile (1994).
[17] R. Ramírez, PhD. Thesis, Universidad de Chile (1999).
[18] P.G. Saffman. Vortex Dynamics in Cambridge Monographs on Mechanics and Applied Mathematics (Cambridge University Press, 1992).