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Rashba diamond in an Aharonov-Casher ring

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Spin interference due to Rashba spin-orbit interaction (SOI) in a ballistic two-dimensional electron gas ring conductor submitted to a bias voltage is investigated theoretically. We calculate the scattering matrices and differential conductance with lead-ring junction coupling as an adjustable parameter. Due to the interference of electronic waves traversing the ring, the differential conductance modulated by both bias voltage and SOI exhibits a diamond-shaped pattern, thus termed as Rashba diamond. This feature offers a supplementary degree of freedom to manipulate phase interference. © 2011 American Institute of Physics. [doi:10.1063/1.3647569]

Interference phenomena, such as the celebrated Aharonov-Casher (AC) effect, on a low-dimensional ring-shaped conductor patterned on a two-dimensional electron gas (2DEG) with a Rashba spin-orbit interaction (SOI) have attracted much attention. Rashba SOI, due to structure inversion symmetry breaking, is dominating in quantum wells made of narrow gap semiconductors and is among the most popular candidates to phase-coherent spintronic devices. Recently, a large (at the order of $10^{-11}$ eV m) while tunable Rashba parameter $\alpha$ (controlled by a gate voltage) has been achieved in InGaAs-based 2DEG systems.

We consider, in this letter, a one-dimensional (1D) ring conductor fabricated on a 2DEG with a Rashba SOI. In such a system, electrons experience an effective magnetic field $B_{eff} \propto \alpha p \times \hat{z}$ that is perpendicular to the momentum $p$ while in the 2DEG plane. Electron waves that traverse the ring along clockwise and counterclockwise directions accumulate different phases that depend on $x$ and the incident energy, which is reflected in the interference patterns of the conductance. Most studies were focusing on the conductance as a function of gate electric fields (therefore modulation of the energy of incident electrons), thus offering a supplementary degree of freedom to control. We also address the impact of the lead-ring junction transparency on the interference which is usually ignored in the AC ring literature.

In Fig. 1, the desired ring conductor is connected to two leads. At low temperature, when the conducting channel length is comparable to the mean free path of electrons, a phase-coherent transport is justified. Meanwhile, the Dyakonov-Perel spin relaxation mechanism is reduced in a two-dimensional strip. Since the width of the ring branches is much shorter than the dimension along transport direction, the energy level splitting due to transverse confinement is much larger than the energy spacing along transport direction, which supports a single-channel transport.

A static electric field is applied perpendicularly to the 2DEG plane and the magnetic field is absent. The 1D single-particle Hamiltonian for electrons in the ring is (in cylindrical coordinates with an in-plane angle $\phi$)

$$H_{1D} = -\frac{\hbar^2}{2m_e a^2} \frac{\partial^2}{\partial \phi^2} + i \alpha \left( \frac{1}{2} \hat{\sigma}_x + \hat{\sigma}_r \frac{\partial}{\partial \phi} \right),$$

where $m_e$ is the effective mass of the electrons and $a$ is the radius of the ring. In cylindrical coordinates, the Pauli matrices $\hat{\sigma}_x = \hat{\sigma}_x \cos \phi - \hat{\sigma}_y \sin \phi$ and $\hat{\sigma}_r = \hat{\sigma}_x \cos \phi + \hat{\sigma}_y \sin \phi$. The eigenvalues of $H_{1D}$ are given by

$$\epsilon_n = \frac{\hbar^2}{2m_e a^2} \left( n - \frac{\Phi_{AC}}{2\pi} \right)^2,$$

where the polarization index $s = \uparrow (+1)$, $\downarrow (-1)$ and the so-called AC phase $\Phi_{AC} = -\pi + \alpha \sqrt{\omega^2 + 1}$, given $\omega = 2m_e a \sqrt{E}/\hbar$. The corresponding normalized wave functions are $\Psi_{\uparrow(\downarrow)} = \chi_{\uparrow(\downarrow)} e^{i\phi}$, where the spinors are

$$\chi_{\uparrow} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos \theta \\ \sin \theta e^{i\phi} \end{pmatrix}, \quad \chi_{\downarrow} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sin \theta \\ -\cos \theta e^{i\phi} \end{pmatrix},$$

given $\sin \theta = \omega/\sqrt{\omega^2 + 1}$. When an electron of energy $E = \hbar^2 k^2/2m_e$ along the transport direction enters the ring (say, from the left lead), energy conservation requires

![FIG. 1. A one-dimensional ring connected to two leads. The amplitudes and the transport directions of electron waves are denoted using arrows. The ring and the leads are made of same 2DEG and the entire structure is under an electric field which tunes the Rashba SOI parameter $\alpha$.](image-url)
$\frac{\hbar^2 k^2}{2m_e} = \frac{\hbar^2}{2m_e} \left( n - \frac{\Phi_{AC}}{2\pi} \right)^2,$  

(4)

leading to $n_{x,1} = ak + \Phi_{AC}^x / (2\pi)$ and $n_{x,2} = -ak + \Phi_{AC}^x / (2\pi)$ for each polarization ($s$). The phase $\phi(n_{x,1,2})$ is the sum of the dynamical and the AC phases accumulated when travelling clockwise (counterclockwise).

Scattering matrix. The transport property of a coherent conductor is described by a scattering matrix. We derive a total scattering matrix $S$ that converts the incoming amplitudes ($\chi_L$ and $\chi_R$, see Fig. 1) to the outgoing ones ($\chi'_L$ and $\chi'_R$) for each eigenstate in Eq. (3). In the ballistic ring without a spin-flip scattering, we may treat $\Psi_0$ and $\Psi_\pm$ separately, and the total conductance is the sum of the contributions from these two states.

On each branch, a scattering matrix is assigned to each spin polarization, i.e., $S\delta$ on the upper branch and $S\delta'$ on the lower one (polarization index omitted for brevity). Since the spin-flip mechanism is suppressed, traversing a ring branch is equivalent to accumulating a phase shift. As an example, matching of the wave functions for spinor $\chi_L^0$ in the upper branch at two junctions gives the scattering matrix $S\delta_u$ satisfying

$$S\delta_u = S^T_{u*},$$

and $S\delta = S^T_{u*}$. Two components of the spinor $\chi_L^0$ differ by a phase factor that is unimportant, thus neglected. Same consideration applies to spinor $\chi_R^0$. At each junction (triangles as in Fig. 1) and for each polarization $s$, three incoming waves ($\chi_{LR}^0$, $\beta_L^0$, and $\gamma_{LR}^0$) are scattered to the outgoing ones ($\chi'_{LR}^0$, $\beta'_L^0$, and $\gamma'_{LR}^0$) by a symmetric scattering matrix

$$S_{LR}^c = \begin{pmatrix} -\cos \eta & 1 \sqrt{2} \sin \eta & 1 \sqrt{2} \sin \eta \\ 1 \sqrt{2} \sin \eta & -\sin^2(\eta/2) & \cos^2(\eta/2) \\ 1 \sqrt{2} \sin \eta & \cos^2(\eta/2) & -\sin^2(\eta/2) \end{pmatrix},$$

(6)

where the junction transparency $\eta \in [0, \pi/2]$ (Ref. 19). When $\eta = 0$, the ring is decoupled from the leads; when $\eta = \pi/2$, an incoming wave is fully transmitted to both branches with equal probability. To keep the essential physics, we consider two identical junctions.

Transmission probability. Time-reversal symmetry leads to a symmetric $S$ that satisfies $SS = 1$ as required by current conservation. Total conductance $G(E, \alpha) = (e^2/h)T(E, \alpha)$, proportional to the total transmission probability $T = T_\uparrow + T_\downarrow$, accounts contribution from two polarizations. When $\eta = \pi/2$, the total transmission probability is

$$T_{\pi/2} = \frac{16(1 - \cos \Delta) \sin^2(ak\pi)}{(1 + \cos \Delta)^2 + 8(1 - \cos \Delta) \sin^2(ak\pi)},$$

(7)

where $\Delta = \pi \sqrt{\omega^2 + 1}$ is half of the AC phase difference between two polarizations. When $\alpha = 0$ (SOI vanishes), $\Delta = \pi$ and $T_{\pi/2} = 2$ can be understood as $T_{\pi/2} = 1 + 1\phi_0$. In a fully transparent ring, the transmission probability of each polarization is unity. At a weak coupling $\eta = \pi/6$, the total transmission probability is

$$T_{\eta/6} = \frac{16\xi^2(1 - \cos \Delta) \sin^2(ak\pi)}{(2\xi^2 + 14\xi \cos(2ak\pi) - \cos \Delta)^2 + 4\xi^2 \sin^2(2ak\pi)},$$

(8)

where $\xi = 4\sqrt{3} - 7$. It is interesting to notice that the case in Ref. 17 corresponds to a transparency $\eta < \eta < \pi/2$.

$I$-$V$ curves. Applying a bias voltage ($V$) across the ring, the current (as a function of $V$, temperature ($T$), and $\alpha$) is given by an energy integration

$$I(V, \alpha, T) = \frac{e}{2\pi h} \int_0^{\infty} T(E, \alpha) \left[ f_L(E) - f_R(E) \right] dE,$$

where $f_L(E) = f(E - (E_F + eV))$ and $f_R(E) = f(E - E_F)$ are the Fermi-Dirac distribution functions of the leads and $E_F$ is the Fermi energy.

Fig. 2 show the step-like $I$-$V$ curves upon scanning the bias voltage, which is similar to the Coulomb blockade in quantum dot systems, except that the plateaus in this AC ring is determined by the destructive interference of phases rather than the Coulomb repulsion. Fixing $\alpha$, decreasing $\eta$ makes the steps more developed and the conductance peaks sharper. The width of the conductance peak, as broadened by the lead-ring coupling, reflects the lifetime of an electronic state inside the ring: The narrower is the peak the longer is the lifetime. When $\eta$ is small, the total transmission probability

$$T_c \approx 4\xi^2 \left( 1 - \cos \Delta \right) \sin^2(2ak\pi) \left( \sin \Delta + \cos(2ak\pi) \right)^2,$$

(9)

where $\epsilon \approx \eta/2$ as introduced in Ref. 10. The singularities of the transmission probability in Eq. (9) are determined by the solutions of $\cos \Delta + \cos(2ak\pi) = 0$ which is exactly the eigenvalue equation (Eq. (4)).

Rashba diamond. Fig. 3 shows the differential conductance ($dI/dV$) modulated by both $\alpha$ (horizontal axis) and $V$ (vertical axis). The presence of Rashba diamonds in panels (a) and (b), reminds us again the Coulomb diamonds in a system consisting of, e.g., quantum dots. For a given $\alpha$ and $\eta$, the bias fully controls the conductance, thus offering an alternative way to manipulate the interference-induced current

![FIG. 2. (Color online) Differential conductance (left $y$-axis, red solid lines) and $I$-$V$ curves (right $y$-axis, black dashed lines) at $\eta = \pi/2$ (strong coupling) and at $\eta = \pi/6$ (weak coupling). The conductance is in the unit of $e^2/h$. The radius of the ring $a = 1\mu m$. The Rashba parameter $\alpha = 1.8$ peV m, $E_F = 75$ meV, temperature $T = 15$ mK, and the effective mass $m_e \approx 0.05$ m.](https://example.com/figure2.png)
modulation. The panels (a) and (b) in Fig. 3 are for a quantum well based on InGaAs/InAlAs. In the weak coupling regime, the Rashba diamonds are more developed than in the strong coupling limit.

In a realistic 2DEG, the gate voltage modulates both \( \alpha \) and carrier density (therefore the Fermi level). We take the dependence of \( \alpha \) on the Fermi level is adjusted simultaneously when changing \( V \) and Fermi level is adjusted simultaneously when changing \( V \). The periodicity due topological phase interference (through \( \alpha \)) survives, which is particularly clear in the strong coupling limit.

To circumvent the experimental difficulty of applying a well-defined voltage bias, energy barriers formed using gate electrodes at the ring-lead junctions are suggested. The advantage of gate electrodes is the flexibility to tune the height of the barrier thus the junction transparencies. Non-magnetic tunnel barriers are good candidates as well since we do not expect significant spin-flip scattering at the junctions. Another experimentally relevant geometry consists of a ring conductor with two branches interrupted by tunnel barriers or gate electrodes, as in the electric Aharonov-Bohm experiment.

In conclusion, for an AC ring with a Rashba SOI, we have investigated the interference patterns due to topological phases. The scattering matrices, as parametrized by junction transparency, were obtained analytically. We have demonstrated the possibility to control the differential conductance by both a bias voltage and a gate electric field. A diamond-shaped pattern of the conductance is well developed in the weak coupling regime, which provides a supplementary degree of freedom to manipulate spins in the Rashba SOI based spintronic devices.