Nonuniversal behavior of scattering between fractional quantum Hall edges

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Among the predicted properties of fractional quantum Hall states are fractionally charged quasi-particles and conducting edge-states described as chiral Luttinger liquids. In a system with a narrow constriction, tunneling of quasi-particles between states at different edges can lead to resistance and to shot noise. The ratio of the shot noise to the backscattered current, in the weak scattering regime, measures the fractional charge of the quasi-particle, which has been confirmed in several experiments. However, the non-linearity of the resistance predicted by the chiral Luttinger liquid theory was apparently not observed in some of these cases. As a possible explanation for these discrepancies, we consider a model where a smooth edge profile leads to formation of additional edge states. Coupling between the current carrying edge mode and the additional phonon like mode can lead to nonuniversal exponents in the current-voltage characteristic, while preserving the ratio between shot noise and the back-scattered current, for weak backscattering. For special values of the coupling, one may obtain a linear I-V behavior.

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The physics of interacting charge carriers confined to two spatial dimensions and subject to a magnetic field has turned out to be surprisingly rich and continues to pose ever new questions and to unveil new phenomena. In the regime of the fractional quantum Hall effect (FQHE), the effect of interactions is so strong that the bulk of the system turns into an incompressible quantum liquid and the edges carry strongly interacting excitations described as a chiral Luttinger liquid (LL) [1]. In addition, the elementary excitations themselves are very different from the original charge carriers in that they have a fractional charge [2].

The fractional charge of FQHE quasi-particles can be detected in shot noise measurements [3]. While the resistance of a single chiral edge state is not renormalized by scattering centers, as the excitations in a chiral LL move in one direction only, scattering from one edge of a sample to the other edge induces both resistance and shot noise. In the weak scattering limit, scattering events are independent of each other and are described by a Poisson process. Then, the variance of the number of scattering events is equal to their average, and the strength of current fluctuations $\langle (\Delta I)^2 \rangle$ is proportional to the product of backscattered current and the quasi-particle charge.

Shot noise experiments carried out by several groups have confirmed the predicted fractional charge of FQHE particles at filling fractions $\nu = 1/3$ and $\nu = 2/5$ [3]. In at least some of these cases, however [3][4], it appears that the samples did not exhibit the strongly nonlinear resistance expected for scattering between chiral LLs [5]. In this letter we suggest that the scattering of fractionally charged quasi-particles may indeed give rise to a non-universal current-voltage characteristic over an interesting range, while preserving the ratio of shot noise to backscattered current, if the chiral edge states couple to additional degrees of freedom. In the presence of a smooth confining potential, edge reconstruction may occur [6][7] and give rise to additional phonon like modes coupling to the original chiral LL. In our model we assume that there is coulomb coupling between the additional modes and the chiral edge states, but there is no tunneling of charge between the modes, or between the additional modes and the contacts. The additional modes therefore carry no net electric current, but can change the dynamics of backscattering between the two edges.

Specifically, we discuss the following setup. From two reservoirs at voltage $\pm V/2$ current is injected into the chiral edges at the boundary of an incompressible quantum Hall state with filling factor $\nu$. We specialize on a simple situation with only one edge state but expect that our general conclusions hold true in more complex situations as well. In the center of the sample, a constriction in the Hall bar allows for scattering of charge $\nu$ quasi-particles from one edge to the other.
We assume that in a region of length $L$ about the constriction, there exists at each edge a pair of extra branches of excitations, moving both to the left and to the right, which can happen if the confining potential is relatively soft. For example, if there were a one-dimensional (fluctuating) Wigner crystal of electrons in the low-density region between the edge of the bulk quantized Hall state and the fully-depleted edge of the sample [10], then the extra modes would arise as acoustic phonon modes of the electron crystal. Equivalently, one could envisage at each edge a very narrow strip of electrons in a fractional Hall state with lower filling fraction than the bulk [10]. The extra modes in this case would correspond to the left and right-moving edges of the low-density strip. In any case, we shall refer to the extra modes as "phonon modes" throughout this paper. The model geometry is indicated schematically in Figure 1.

We assume that disorder can be ignored, as a first approximation, and we ignore all scattering of electrons between the various modes, other than backscattering at the constriction. The phonon mode at each edge is coupled via a short-range density-density interaction to the charge on the adjacent chiral edge state of the bulk system. (We assume that the long-range part of the Coulomb interaction is screened by an adjacent metal gate.) The phonon modes themselves may be characterized by the sound velocity they would have in the absence of coupling to the chiral edge state and by an effective Luttinger liquid interaction parameter determined by the mass density and one-dimensional compressibility; alternatively, the Luttinger parameter can be characterized by an effective filling factor $\tilde{\nu}$ for the low-density strip, which we assume to be smaller than the bulk $\nu$.

Our analysis shows that for frequencies larger than $\omega_L = v_F/L$ ($v_F$ is the Fermi velocity in the chiral states), this coupling renormalizes the LL parameter $\nu$ of the chiral edge to $\nu F > \nu$. The strength of this renormalization depends on the coupling and on the ratio of the velocities of chiral and phonon modes. As a result, the scattering dynamics for voltages and temperatures larger than $\omega_L$ is described by a nonuniversal exponent depending on the interaction strength.

For frequencies smaller than $\omega_L$, this coupling is not effective and the dynamics stays unchanged. In the experiments, shot noise was measured at frequencies in the kilohertz or low megahertz range; we assume that this is small compared to $\omega_L$. As a result, the quasi-particle charge is not renormalized and can be extracted from a noise measurement. In the limit $V \gg T$, it is given by the ratio of shot noise and backscattered current.

We describe the chiral edges and scattering between them by the finite temperature action

$$S = \frac{\hbar}{2\nu} \int dx dx' dt d\tau \Theta(x, \tau) G^{-1}(x, x'; \tau - \tau') \Theta(x', \tau') + \lambda \int d\tau \cos \left[2\sqrt{\pi} \Theta(0, \tau) \right].$$

Here, $\nu$ is the filling factor of the bulk with $1/\nu$ an odd integer, and the inverse Green function $G^{-1}$ describes the dynamics of the chiral edge states in the presence of the phonon modes. As the coupled system is quadratic in all densities, $G$ can be calculated exactly. The left and right moving charge densities are given by $\rho_{\pm}(x, \tau) = 1/\sqrt{\pi}(\partial_x \pm (i/v_F) \partial_x) \Theta(x, \tau)$, and the coupling to the reservoirs is described by radiative boundary conditions [14]

$$\rho_{\pm}(\pm \infty, \tau) = \pm \nu e V/2 \pi \hbar v_F.$$

As we are interested in calculating quasi-particle scattering, we integrate over all fields away from the impurity site and obtain an effective action for $\Theta(x = 0, \tau)$. Its dynamics is governed by $G(0, 0; \omega_n)$, where $G$ is the Green function of the $\Theta$-field coupled to a phonon system of length $L$. We have calculated $G$ in real space as a solution of the saddle point equation with a delta-function inhomogeneity. We find that it shows a crossover between two continuum theories: for low-frequencies the coupling to the phonons is not effective, whereas in the high-frequency regime one finds the same dynamics as for an infinite length phonon system,

$$G(0, 0; \omega_n) \sim \frac{\nu}{2|\omega_n|}, \quad |\omega_n| \ll v_F/L \quad (3)$$

$$G(0, 0; \omega_n) \sim \frac{\nu F(u, \gamma)}{2|\omega_n|}, \quad |\omega_n| \gg v_F/L \quad (4).$$

![FIG. 2. Dynamical renormalization factor $F(u, \gamma)$ plotted against the coupling strength $\gamma$ for the special value of the phonon velocity $u = 0.61$.](image)

The renormalization factor $F(u, \gamma)$ is most easily calculated by integrating out the phonon systems in a continuum theory,
\[ F(u, \gamma) = \int_{-\infty}^{\infty} \frac{dx}{\pi} \frac{1 - \gamma^2 \frac{2u^2}{1+x^2}}{1 + x^2 \left( 1 - \gamma^2 \frac{2u^2}{1+x^2} \right)^2} , \]  

which describes the backscattering probability in the linear transport regime. As the microscopic parameters \( \lambda \) and \( \tau_c \) are not readily available for experimental systems, it is useful to treat \( R \) as an effective parameter when comparing to experiments. We define the noise as 

\[ \langle \Delta I(\omega) \Delta I(\omega') \rangle = 2\pi \delta(\omega + \omega') S(\omega) . \]

In the limit of \( \omega \) going to zero we find

\[ S = RT \nu \frac{e^2}{\pi \hbar} \frac{\Gamma(1-\nu F)}{\Gamma(\nu F)} \frac{\sqrt{\nu F - i\nu eV/2\pi T}}{\Gamma(1-\nu F - i\nu eV/2\pi T)} \]

\[ -2T \left( \frac{\partial I_B}{\partial V} - \nu \frac{e^2}{2\pi \hbar} \right) . \]

In Figure 3, the total noise is plotted against the transmitted current for a quasi-particle charge \( e/3 \) and different values of the renormalization factor \( F \). For \( F = 1 \), we find LL behavior with suppressed shot noise for large currents, as the impurity becomes more and more transparent. For a Fermi liquid (FL), which would correspond to \( F = 3 \), the shot noise grows linearly with the current. As an example for an intermediate situation with a nonuniversal noise–current characteristic we show the result for \( F = 2 \).

To test the accuracy of our perturbative calculation, we have compared it for the special value \( F = 1 \) (no renormalization of scattering dynamics) to the exact solution in \([12,13]\). We find good agreement for small reflection probabilities, e.g. for a reflection coefficient \( R = 0.1 \) the perturbative result for the shot noise deviates less than 10% from the exact solution.
We note that the coupling to the phonon mode changes the exponent characterizing the scattering dynamics from $\nu$ to $\nu F$ in Eqs. (8,10), while the fractional charge $\nu e$ is not multiplied by $F$ and not renormalized. As a consequence, in the limit of dc voltage much larger than the temperature, we find for the ratio of the shot noise to backscattered current
\[ S - 2 T \nu \frac{e^2}{2 \pi} = \nu e + O(T/V) \]  \[ (11) \]

Thus the effective charge defined by this ratio coincides with the fractional charge $\nu e$.

As stated above, we have ignored any effects of disorder or impurities at the edge of the sample. If there is disorder with Fourier components commensurate with the reciprocal of the spacing between electrons in the low-density edge strip, this can lead to pinning of the Wigner crystal and an effective gap $\omega_d$ in the phonon dispersion relation. (Equivalently, we may say that impurity-induced backscattering between the opposite moving edge modes of the low-density strip is relevant at low energies, and can lead to localization. Scattering of electrons between the chiral edge state and the low-density strip should be irrelevant at low energies, but it might also need to be taken into account if the disorder is not sufficiently weak.) If there is an effective energy gap for the phonons, our analysis should still be valid, provided that $eV \gg \hbar \omega_d$, while the shot noise is measured at frequencies low compared to $\omega_d$ and $\omega_L$.

How do the results for our model compare with experiments? In Ref. [6], at $\nu = 1/3$, shot noise was found to correspond to quasi-particles of charge $e/3$, while a linear I-V dependence was observed over the voltage-range where the data was taken. This result could be understood directly in terms of our model if we assume an appropriate value for the coupling between a chiral edge-state and additional phonon-like modes. In Ref. [6], shot noise corresponding to charge $e/5$ was observed (as expected) for the case of $\nu = 2/5$, while the I-V curves appear again to be more linear than one would expect from a chiral LL model. Since the case $\nu = 2/5$ requires at least two chiral edge-modes, our model cannot be applied directly; nevertheless, we expect that similar considerations apply.

The $\nu = 1/3$ measurements of Ref. [6] by contrast, do show a markedly nonlinear I-V characteristic, in the range where the shot noise was measured. Similarly, there appears to be a nonlinear I-V characteristic in the data exhibited in Ref. [6] at $\nu = 2/5$. In Ref. [6] the nonlinearity was compared explicitly with the LL predictions and found to deviate in the regime of small temperatures and low voltages where backscattering is strong, but there is no quantitative comparison in the regime where the backscattering is weak. In all these cases the data were taken over an extended range of the reflectivity $R$, not just in the limit of weak back scattering, and the shot noise was analyzed using a formula with an additional factor $(1-R)$, typical of non-interacting fermions. As our calculation is only valid to lowest-order in $R$, we cannot make a statement about the appearance of such a factor.

In summary, we have studied the dynamics of scattering between fractional quantum Hall edges in the presence of additional phonon like edge modes. We find that the coupling between these additional modes and the current carrying edge mode leads to a nonuniversal current-voltage characteristic which may explain the deviations from Luttinger liquid behavior observed in experiments. The fractional charge of FQHE quasi-particles, however, can still be measured as the quotient of shot noise and backscattered current, in the limit of weak backscattering.

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