Dark Matter from Late Invisible Decays to/of Gravitinos

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In this work, we sift a simple supersymmetric framework of late invisible decays to/of the gravitino. We investigate two cases where the gravitino is the lightest supersymmetric particle or the next-to-lightest supersymmetric particle. The next-to-lightest supersymmetric particle decays into two dark matter candidates and has a long lifetime due to gravitationally suppressed interactions. However, because of the absence of any hadronic or electromagnetic products, it satisfies the tight bounds set by big bang nucleosynthesis and cosmic microwave background. One or both of the dark matter candidates produced in invisible decays can contribute to the amount of dark radiation and suppress perturbations at scales that are being probed by the galaxy power spectrum and the Lyman-alpha forest data. We show that these constraints are satisfied in large regions of the parameter space and, as a result, the late invisible decays to/of the gravitino can be responsible for the entire dark matter relic abundance.

I. INTRODUCTION

There are various lines of evidence for the existence of dark matter (DM) in the universe \cite{1}, but its identity is still unknown and remains one of the most important problems at the interface of cosmology and particle physics. Given the many ongoing direct and indirect DM detection experiments, along with collider searches trying to pin down the nature of DM, this puzzle is expected to be solved in the foreseeable future.

Supersymmetric (SUSY) extensions of the standard model (SM) with R-parity conservation provide a natural candidate for DM. In these models, the lightest supersymmetric particle (LSP) is stable and hence can account for the DM particle. In the minimal supersymmetric standard model (MSSM), the LSP is either the lightest neutralino $\tilde{\chi}_1^0$ or the gravitino $\tilde{G}$.

The presence of the gravitino results in important cosmological constraints on SUSY models. If gravitino is not the LSP, then it will decay to the LSP and its SUSY partner. If gravitino is the LSP, then the next-to-lightest supersymmetric particle (NLSP) will decay to the gravitino. Due to Planck suppressed interactions of $\tilde{G}$ with other particles \cite{2}, these decays have long lifetimes. In particular, if $m_{\tilde{G}} < 40$ TeV, the decay to/of the gravitino will occur after the onset of the big bang nucleosynthesis (BBN). Such decays are tightly constrained by cosmological considerations from BBN and cosmic microwave background (CMB). Late decays of neutral or charged particles through electromagnetic and hadronic channels are severely restricted by BBN constraints \cite{3,4}. Moreover, late decays that release energy in the electromagnetic mode can give rise to a chemical potential for the cosmic microwave background (CMB) photons \cite{5,6}, which is constrained by observations \cite{7}. Late injection of energetic neutrinos, which can produce secondary particles, is constrained by BBN bounds as well as CMB limits on the amount of extra radiation and structure formation \cite{8}. These bounds severely constrain the abundance of the NLSP and, through that, put tight limits on the reheating of the universe. These studies in the context of axion/axino production from invisible decays have been discussed in Ref.\cite{9}. For other recent dark radiation setups see Ref.\cite{10}.

In this paper, we investigate a SUSY scenario that can accommodate invisible decay to/of the gravitino. The model is a minimal extension of the MSSM and has two DM candidates. One of the DM candidates is the LSP and the other one is an R-parity even fermion $N$ with $O(\text{GeV})$ mass that is a singlet under the SM gauge group. The LSP in this model can be either the gravitino or the SUSY partner of $N$ (denoted by $\tilde{N}$). The model is well motivated due to its ability to generate the baryon abundance of the universe at temperatures well below the electroweak scale, and to explain the apparent coincidence between the the observed DM and baryon energy densities \cite{11,12}. The invisible decays involving the gravitino are are $N \rightarrow \tilde{G} + N$ and $\tilde{G} \rightarrow \tilde{N} + N$ that respectively take place for a gravitino NLSP and $\tilde{N}$ NLSP and vice versa. Although both decays involve gravitationally suppressed interactions, and hence have a long lifetime, they circumvent the severe BBN and CMB constraints since they do not include electromagnetic or hadronic products.

However, depending on the mass ratio between $\tilde{N}$ and $\tilde{G}$, it is possible that one or both of the decay products are relativistic during the epoch of matter-radiation equality. They may then contribute to the amount of dark radiation and suppress DM perturbations scales that are probed by the galaxy clustering and the Lyman-alpha forest data. We will show that for the current data these constraints are satisfied in large regions of the parameter space. As a result, the late decays can be responsible for the entire DM relic abundance in this model. Moreover, in a broader context, late invisible decays to/of the
gravitino considerably relax the constraints on reheating of the 
universe in SUSY models [13].

The structure of this paper is as follows. In Section II we 
discuss the model. In Section III we discuss the late invisible 
decays that involve the gravitino. We discuss production of 
dark radiation in Section IV, and constraints from structure 
formation in Section V. We present our results in Section VI. 
Finally, we close this paper by concluding it in Section VII.

II. THE MODEL

The model is an extension of the MSSM that contains iso-
singlet color-triplet superfields $X$ and $\bar{X}$ with respective hy-
percharges $+4/3$ and $-4/3$, and a singlet superfield $N$. The 
superpotential of this model is given by [14]

$$W = W_{\text{MSSM}} + W_{\text{new}},$$

$$W_{\text{new}} = \lambda_i N u_i^c + \lambda_{ij} \bar{X} d_i^c d_j^c + M_X X \bar{X} + \frac{M_N}{2} N N.$$  \hspace{1cm} (1)

Here $i, j$ denote flavor indices (color indices are omitted for 
simplicity), with $\lambda_{ij}$ being antisymmetric under $i \leftrightarrow j$. 
We assign quantum number $+1$ under $R$-parity to the scalar com-
ponents of $X$, $\bar{X}$ and the fermionic components of $N$. As 
shown in [11] [12] [14], with two (or more) copies of $X$, $\bar{X}$ one 
can generate the baryon asymmetry of the universe from 
the interference of tree-level and one-loop diagrams in decay 
processes governed by the $X$, $\bar{X}$ interactions.

The exchange of $X$, $\bar{X}$ particles in combination with the 
Majorana mass of $N$ lead to double proton decay $pp \rightarrow 
K^+K^+$. Current limits on this process from the Super-
Kamiokande experiment [15] require that $|\lambda_1\lambda_{12}|^2 \leq 10^{-10}$ 
for $M_N \sim 100$ GeV. This is also enough to satisfy constraints 
from $K^0-L$ and $B^0_s-B_s^0$ mixing and neutron-antineutron 
oscillations [11].

Assuming that $M_N \ll M_X$, one finds an effective four-
fermion interaction $N u_i^c d_j^c d_k^c$ after integrating out scalars 
$\tilde{X}, \tilde{\bar{X}}$. This results in decay modes $N \rightarrow p + e^- + \nu_e, N \rightarrow 
\bar{p} + e^+ + \nu_e$, which are kinematically open as long as $M_N > 
m_p + m_e$ (with $m_p$ and $m_e$ being the proton mass and the 
electron mass respectively). It is seen that $N$ becomes absol-
utely stable if $M_N \lesssim m_p + m_e$. However, in this case, 
we will have catastrophic proton decay via $p \rightarrow N + e^+ + \nu_e$ if 
m_p > M_N + m_e. Therefore a viable scenario with stable $N$ 
arises provided that

$$m_p - m_e \leq M_N \leq m_p + m_e.$$  \hspace{1cm} (2)

The important point to emphasize is that stability of $N$ is 
not related to any new symmetry. It is the stability of the pro-
on, combined with the kinematic condition in Eq. (2), that 
ensures $N$ is a stable particle in the above mass window. This 
leads to a natural realization of GeV DM with or without 
SUSY [16], which provides a suitable framework to address 
the DM-baryon coincidence puzzle. Possible signatures of 
such an $\mathcal{O}(\text{GeV})$ GeV DM particle in collider and indirect 
searches have been studied in [17] [18].

When $R$-parity is conserved, which we assume to be the 
case here, the LSP is also a DM candidate. In consequence, if $M_N \approx \mathcal{O}(\text{GeV})$, a multi-component DM scenario can be 
realized in this model as both the LSP and $N$ are stable in this 
case.

After SUSY breaking, the real and imaginary parts of $\bar{N}$ 
acquire different masses:

$$m_{N_{1,R}}^2 = M_N^2 + \bar{m}^2 \pm B_N M_N,$$  \hspace{1cm} (3)

where $\bar{m}$ is the soft SUSY breaking mass of $\bar{N}$ and $B_N M_N$ 
is $B$-term associated with the $M_N N^2/2$ superpotential term. 
Depending on the sign of $B_N$, $\bar{N}_R$ or $\bar{N}_I$ will be the lighter 
of the two mass eigenstates. In the special case that $|B_N M_N| \ll 
\bar{m}^2$, $\bar{N}_R$ and $\bar{N}_I$ are approximately degenerate. It is clear that 
there are regions in the parameter space where $\bar{N}$ (or one of its 
components) is either the LSP or the NLSP. As we will show 
below, particularly interesting scenarios can arise with $\bar{N}$ LSP 
and gravitino NLSP and vice versa.

Before closing this section, we briefly comment on the 
prospects for the detection of $N$ and $\bar{N}$ as DM candidates in 
this model. $N$ interacts with nucleons via its coupling to the 
up quark that is mediated by the $X$ scalar, see Eq. (1). The 
resulting spin-independent scattering cross section is extremely 
small as pointed in [16]. The spin-dependent scattering cross 
section is $\sigma_{SD}^{(N-p)} \sim |\lambda|^2 m_p^2/64\pi m_X^4$ [16], where $m_X$ denotes 
the mass of $X$ scalar. For $M_X \sim \mathcal{O}(\text{TeV})$ and $|\lambda| \sim 1$, 
we have $\sigma_{SD}^{(N-p)} \sim 10^{-42}$ cm$^2$. This is much below the current 
bounds from direct detection experiments [19], but within 
the LHC future reach [20]. The situation is more promis-
ing for $\bar{N}$. It interacts with nucleons via exchange of the $X$ 
fermion with up quarks, which results in an spin-independent 
scattering cross section $\sigma_{SD}^{(\bar{N}-p)} \sim |\lambda|^2 m_p^2/16\pi M_X^4$ [12]. For 
$M_X \sim \mathcal{O}(\text{TeV})$ and $|\lambda|^{-1} \leq 10^{-1}$, we have $\sigma_{SD}^{(\bar{N}-p)} < 10^{-45}$ 
cm$^2$, which is well within the range currently being probed by 
direct detection searches [21].

III. LATE INVISIBLE DECAYS

As mentioned earlier, late decays that involve the gravitino 
are subject to very tight cosmological constraints from BBN and 
CMB [3][3]. Interestingly, however, the model given in 
Eq. (1) can result in invisible decays to/of the gravitino 
thereby circumventing these tight cosmological constraints.

The two interesting scenarios, pointed out before, are: (1) $\bar{N}$ 
NLSP and gravitino LSP, and, (2) $\bar{N}$ LSP and gravitino NLSP. 
The decays $\bar{N} \rightarrow G + N$ (in the former case) and $G \rightarrow \bar{N} + N$ 
(in the latter case) do not produce charged particles, hadrons, 
or neutrinos. They are therefore totally invisible and, as a re-

1 Invisible decays of/to gravitinos involving axino and axion have been discussed in [22].
2 Secondary production of these particles from the interaction of $N$ and $\bar{N}$ 
with nucleons is totally negligible since the corresponding cross section 
is several orders of magnitude smaller than that of the weak interactions for 
typical values of the model parameters.
The late decay is evaded the above BBN and CMB bounds. We note, however, that these decays can produce relativistic DM quanta. Cosmological constraints on the model then come from the effective number of neutrinos $N_{\text{eff}}$ and from structure formation, which we will discuss in detail later on.

Here we describe different possibilities for a late invisible decay that can arise in our model in more detail:

1. $\tilde{N}$ NLSP and gravitino LSP, $m_\tilde{N} > M_N \gg m_\tilde{G}$. The late decay is $\tilde{N} \to \tilde{G} + N$ with the corresponding width

$$\Gamma_{\tilde{N} \to N + \tilde{G}} = \frac{1}{4\pi} \frac{m_\tilde{N}^5}{M_P^2 m_\tilde{G}^2} \left( 1 - \frac{m_\tilde{G}^2}{m_\tilde{N}^2} \right)^4. \quad (4)$$

For $M_N \approx 1$ GeV, which we consider here, both of the decay products are stable and contribute to the DM relic abundance. However, since $m_{3/2} \ll \mathcal{O}(\text{GeV})$, the contribution of the gravitino is subdominant. In this case, the dominant component of DM is $N$, while relativistically produced gravitinos from $\tilde{N}$ decay may contribute to the dark radiation. Gravitinos much lighter than GeV can be realized in models of gauge-mediated SUSY breaking (GMSB). We note that such light gravitinos do not affect stability of N as the only $R$-parity conserving decay mode $N \to \tilde{G} \tilde{G}$ is forbidden by Lorentz invariance.

2. $\tilde{N}$ NLSP and gravitino LSP, $m_\tilde{N} > m_\tilde{G} \gg M_N$. The late decay is $\tilde{N} \to \tilde{G} + N$ and the corresponding decay width is

$$\Gamma_{\tilde{N} \to N + \tilde{G}} = \frac{1}{4\pi} \frac{m_\tilde{N}^5}{M_P^2 m_\tilde{G}^2} \left( 1 - \frac{m_\tilde{G}^2}{m_\tilde{N}^2} \right)^4. \quad (5)$$

Both of $\tilde{N}$ and $N$ contribute to the DM relic density. However, since $m_\tilde{G} \gg \mathcal{O}(\text{GeV})$, gravitinos constitute the dominant component of DM, and relativistically produced $N$ quanta from the decay may contribute to the dark radiation.

3. $\tilde{G}$ NLSP and $\tilde{N}$ LSP, $m_\tilde{G} > m_\tilde{N} \gg M_N$. The late decay is $\tilde{G} \to \tilde{N} + N$ and has the following decay width

$$\Gamma_{\tilde{G} \to \tilde{N} + N} = \frac{1}{192\pi} \frac{m_\tilde{G}^3}{M_P^2} \left( 1 - \frac{m_\tilde{N}^2}{m_\tilde{G}^2} \right)^4. \quad (6)$$

Both of $\tilde{N}$ and $N$ to the DM relic abundance. However, since $m_\tilde{N} \gg \mathcal{O}(\text{GeV})$, the dominant component of DM is $\tilde{N}$, while relativistically produced $N$ quanta from $\tilde{G}$ decay may contribute to the dark radiation.

Since $M_N \approx 1$ GeV is set by the stability condition of $N$, the parameter space relevant for late decay in all the three cases is two dimensional, namely the $m_\tilde{N} - m_\tilde{G}$ plane. We will discuss in detail the allowed regions of the parameter space for each of these cases later on.

Some comments are in order before closing this section. Even though the late decays mentioned above are invisible, they are inevitably accompanied by higher-order processes that produce hadrons and charged particles. One notable channel, shown in Fig. 1, is NLSP decay to three quark final states mediated by an off-shell $N$ and $X, \tilde{X}$ scalars. The branching ratio for this mode is given by

$$\text{Br}_{\tilde{N} \to quarks} \approx \frac{1}{(10\pi^2)^2} \cdot 3 \cdot |\lambda\lambda'|^2 \left( \frac{B_X M_X m_\tilde{N}^2}{m_X^4} \right)^2. \quad (7)$$

Here $B_X M_X$ is the $B$-term associated with $X X \bar{X}$ term in Eq. (1), and we have assumed $m_{\text{NLSP}} \gg m_{\text{LSP}}$. The first factor on the right-hand (RH) side of Eq. (7) is the ratio of the phase space factors for four-body and two-body decays respectively, and 3 denotes the color multiplicity factor. For $M_X, m_X \lesssim \mathcal{O}(\text{TeV})$ (to be compatible with the LHC bounds on the colored particles) and $|B_X| \sim \mathcal{O}(\text{TeV})$, moderately small values of $|\lambda\lambda'|$ and $m_{\text{NLSP}} \lesssim 100$ GeV will be enough to push down $\text{Br}_{\tilde{N}}$ below $10^{-10}$. Such a small value of $\text{Br}_{\tilde{N}}$ easily satisfies the tight constraints from BBN and CMB bounds mentioned above [5–6, 8].

Finally, there are other potentially dangerous decay modes that need to be considered. The lightest SUSY particle in the MSSM sector can decay to $\tilde{N}$ and to the gravitino. These decays can be dangerous if the corresponding lifetime exceeds 1 second. To avoid this, it suffices if the more efficient decay takes place before the onset of BBN. The lightest SUSY particle in the MSSM sector can be the lightest neutralino $\tilde{\chi}_1^0$, the sneutrino $\tilde{\nu}$, or the slepton $\tilde{l}$. The neutralino $\tilde{\chi}_1^0$ undergoes two-body decay to $\tilde{N} N$ and four-body decay to $\tilde{u} \tilde{u} N \tilde{N}$. The two-body decay occurs via a loop diagram and is dominant. Fig. 2 shows typical diagrams for the decay of a Bino-type $\tilde{\chi}_1^0$. The decay width receives contributions from SUSY preserving and SUSY breaking interactions. The latter dominates by a factor of $(A_X/M_N)^2$, where $A_X$ is the A-term associated with the $X Nu^c$ superpotential term, see Eq. (1). The resulting decay width is:

$$\Gamma_{\tilde{\chi}_1^0} \sim \frac{4}{3} \frac{1}{(4\pi)^4} \frac{1}{8\pi} |\lambda|^4 \left( \frac{m_\tilde{N}^3 A_X^2}{M_X^2} \right). \quad (8)$$

Here $\alpha$ is the electroweak fine structure constant, factors of $4/3$ and $1/(4\pi)^4$ on the RH side take the the color multiplicity and hypercharge of the up-type quarks and the loop factor into account respectively, and we have assumed $m_{\tilde{\chi}_1^0} \gg m_N$. As mentioned before, only the combination $|\lambda\lambda'|$ is subject to phenomenological constraints in our model, and hence $|\lambda|$ does not need to be very small. For $A_X, M_X \sim 1$ TeV, $m_{\tilde{\chi}_1^0} \approx 100$ GeV, and $|\lambda| \sim 10^{-2}$, we find $\tau_{\tilde{\chi}_1^0} \ll 10^{-6}$ sec. Combination of SUSY breaking and electroweak breaking interactions lead to similar decay widths for Wino-type and Higgsino-type $\tilde{\chi}_1^0$ via one-loop diagrams. The sneutrino $\tilde{\nu}$ and slepton $\tilde{l}$ decay to $\nu \tilde{N} \tilde{N}$ and $L \tilde{N}$ final states via an off-shell Bino, and the corresponding decay widths are still $\ll 1$ sec. We therefore see that for reasonable choice of parameters the lightest SUSY particle in the MSSM sector decays early enough to easily avoid any potential danger.
FIG. 1. The hadronic decay mode of $\tilde{N}$ in cases 1 and 2. The branching ratio for this channel, given in Eq. (7), is typically very small and easily satisfies the tightest BBN bounds. A similar diagram exists for hadronic decay of $\tilde{G}$ in case 3 with the role of $\tilde{N}$ and $\tilde{G}$ being reversed.

FIG. 2. Typical diagrams for the decay of a Bino-type $\chi_1^0$ into $\tilde{N}N$. Additional diagrams that are obtained by switching $u \leftrightarrow X$ and $\bar{u} \leftrightarrow \bar{X}$ in the loop. The SUSY breaking contributions from the diagram on the right dominate giving rise to the decay width in Eq. (8).

IV. DARK RADIATION CONSTRAINTS ON LATE INVISIBLE DECAYS

The amount of dark radiation in the early universe is parametrized by the effective number of neutrinos $N_{\text{eff}}$. The present observational bound on $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - N_{\text{eff,SM}}$ (where $N_{\text{eff}} = 3.04$) from Planck+WMAP9+ACT+SPT+BAO+HST at 2$\sigma$ is $\Delta N_{\text{eff}} = 0.48^{+0.48}_{-0.45}$ [23], which implies that $\Delta N_{\text{eff}} = 0.96$ at 2$\sigma$. The value precise of $N_{\text{eff}}$ depends on Hubble constant where the Planck data and HST measurements differ [24]. The reconciliation can occur using a non-zero $\Delta N_{\text{eff}}$ [25]. Setting aside the dust contamination in the BICEP2 results, the tension in the CMB tensor polarization measurement between the recent BICEP2 [26] and Planck data can also be reconciled with $\Delta N_{\text{eff}}=0.81\pm0.25$ at more than $3\sigma$ confidence level in a joint analysis [27], disfavoring $\Delta N_{\text{eff}} = 0$. Since the presence of dark radiation is debatable, here we take a more conservative approach. We use the data the derive bounds on frameworks that naturally may induce a non-negligible dark radiation component through non-thermal DM production.

In order to relate the energy density associated with non-thermally, relativistically produced DM with the effective number of neutrinos, we start by calculating the ratio between the respective energy densities. Since the cold dark matter (CDM) and neutrino energy densities are redshifted like $\rho_{\text{DM}} \propto \Omega_{\text{DM}} a^{-3}$ and $\rho_\nu \propto \Omega_\nu a^{-4} N_\nu/3$, the ratio between the neutrino and DM energy densities at the matter-radiation equality is

$$\frac{\rho_\nu}{\rho_{\text{DM}}} = \frac{\Omega_\nu}{\Omega_{\text{DM}}} \frac{N_\nu}{3} a_{\text{eq}} \frac{1}{\Omega_{\text{DM}}} \frac{1}{3} a_{\text{eq}},$$

where $\gamma_\nu(0) = 4.84 \times 10^{-5}$, $\Omega_{\text{DM}} \sim 0.227$, $N_\nu$ is the number of neutrinos. For $N_\nu = 1$, we thus find that the energy density of one neutrino is $\sim 16\%$ of the DM density. As a result, if DM particles had a kinetic energy equivalent to $\gamma_\nu \sim 1.16$ at $a_{\text{eq}}$, this fraction would produce the same effect as an extra neutrino species in the expansion of the universe at that time [28]. However, as we will discuss below, constraints stemming from structure formation require the fraction of DM particles with appreciable kinetic energy to be $\ll 1$. Therefore, in order to still mimic one neutrino species a small fraction of DM particles have to be relativistically produced.

In the general decay setup where a heavy particle with mass $M$ decays at rest to two particles with masses $m_1$ and $m_2$ at time $t_{\text{dec}}$, the boost factors of the daughter particles follows

$$\gamma_1(t)^2 = 1 + \frac{a_{\text{dec}}^2}{a(t)} \frac{p^2}{m_1^2},$$

$$\gamma_2(t)^2 = 1 + \frac{a_{\text{dec}}^2}{a(t)} \frac{p^2}{m_2^2}.$$  

where

$$p = \left[ \frac{(M^2 - (m_1 + m_2)^2)}{2M} \right]^{1/2} \left( M^2 - (m_1 - m_2)^2 \right)^{1/2},$$

(11)
FIG. 3. The free-streaming scale $k_{fs}$ for N & $\tilde{G}$ for the decay process $\bar{N} \rightarrow \tilde{G} + N$ (case 1) as a function of scale factor $a$. Each panel is for different value of $m_\tilde{G}$ and $m_\tilde{N}$. For the left panel the mass is set to $m_\tilde{G} = 5$ GeV and $m_\tilde{N} = 0.02$ GeV; in the right panel masses are set to $m_\tilde{G} = 20$ GeV and $m_\tilde{N} = 0.2$ GeV. The mass of $m_N$ is 1 GeV. The black solid line is the size of the horizon at a given scale factor. The solid color lines are for N, and dash-dotted lines are for $\tilde{G}$. The density perturbation is suppressed for $k > k_{fs}$. Between $k_{fs}$ and the horizon the density perturbation will grow.

is the momentum of the daughter particles at the time of production.

If both of the daughter particles are stable, they both contribute to the DM relic density. If $f$ is the fraction of the energy density in DM that is produced from the late decay, the amount of dark radiation that is mimicked by the kinetic energy of the daughter particles is found to be

$$\Delta N_{\text{eff}} = \left[ \frac{(\gamma_{1,eq} - 1)m_1 + (\gamma_{2,eq} - 1)m_2}{0.16(m_1 + m_2)} \right] f.$$  \hspace{1cm} (12)

We note that the normalization factor 0.16 above appears due to the neutrino-DM energy density fraction at the matter radiation equality according to Eq. (9).

If $m_1 \ll m_2$, then the likely scenario is that species 1 is the dominant DM component, hence $m_{DM} \simeq m_1$, while species 2 makes the major contribution to the dark radiation. In this case, assuming that $\gamma_{2,eq} \gg 1$, we have

$$\Delta N_{\text{eff}} \simeq 4.87 \times 10^{-3} \left( \frac{t_{\text{dec}}}{10^6 \text{s}} \right)^{1/2} \left( \frac{p}{m_{DM}} \right) f,$$  \hspace{1cm} (13)

where $p$ is given in Eq. (11).

V. STRUCTURE FORMATION CONSTRAINTS ON LATE INVISIBLE DECAYS

In this section, we discuss large scale structure constraints and present our results for dark radiation in the model described in section II. The median speed of the decay products at a given time $t$ for $M \rightarrow m_1 + m_2$ is described by

$$v_{1,2,\text{med}}(t) \sim \frac{a_{\text{dec}}/a(t)p}{\sqrt{(a_{\text{dec}}/a(t)p)^2 + m_{1,2}^2}}.$$  \hspace{1cm} (14)

The scaling of the free-streaming distance can be understood in terms of the Jeans wavenumber:

$$k_{fs}(a) = \frac{\sqrt{\rho}a^2/2M_P^2}{v_{\text{med}}(a)} = \frac{3}{2} \frac{aH(a)}{v_{\text{med}}(a)},$$  \hspace{1cm} (15)

where for $k > k_{fs}$, the density perturbation is suppressed.

Correlation of the galaxy distribution probes the matter power spectrum on scales of $0.02 \text{ h Mpc}^{-1} \lesssim k \lesssim 0.2 \text{ h Mpc}^{-1}$ at $z \sim 0$ [34]. Indeed one of the best cosmological probes of constraining massive standard model neutrinos, as a class of “hot dark matter” (HDM), is galaxy power spectrum. The current neutrino mass limits from SDSS galaxy clustering is about $\Sigma m_\nu < 0.3-0.62$ eV [34]. The abundance of HDM that is allowed by current galaxy power spectrum is given by

$$\Omega_\nu h^2 = \frac{\Sigma m_\nu}{94.1 \text{eV}}.$$  \hspace{1cm} (16)

This predicts $\Omega_\nu \lesssim 0.007-0.01$, which gives the ratio of “DM” and “HDM” $\Omega_\nu/\Omega_{DM} \lesssim 0.03-0.06$. Therefore the amount

3 The case when the same species is responsible for DM and dark radiation has been studied in detail in [35][36].
of “HDM” that suppresses structure growth at scale \( k \sim 0.02 \, \text{h Mpc}^{-1} \), either from the subdominant part or the dominant part of the dark matter, is limited to be less than 3-6% of the total dark matter.

Lyman-alpha forest data probes the matter power spectrum on smaller scales, \( 0.1 \, \text{h Mpc}^{-1} \lesssim k \lesssim 2 \, \text{h Mpc}^{-1} \) at \( z \sim 2-4 \) \cite{32, 33}. For current Lyman-alpha forest data, the error of measurement is roughly in the range of 5-10% \cite{32, 33}. In \cite{33}, these authors utilize numerical simulations to study Lyman-alpha forest limits for the warm+cold dark matter models. They found that, with fraction of sterile neutrino \( \Theta \), at \( k \sim 0.1 \) \hMpc\(^{-1} \), the contours for the decay lifetime \( m \) in the range they studied is allowed by the data. So the amount of “WDM” which suppresses structure growth at scale \( k \sim 0.1 \) \hMpc\(^{-1} \) cannot be more than 10-35% of the total dark matter.

In Fig. (3), we plot the free-streaming scale \( k_{fs} \) for \( N \) and \( \tilde{G} \) for the decay process \( \bar{N} \to \tilde{G} + N \) (case 1) as a function of scale factor \( a \). Each panel is for different value of \( m_{\tilde{G}} \) and \( m_{\tilde{N}} \). We find that \( k_{fs} \) for the decay products, depending on the masses, can suppress scales that can be probed by the large scale structure data.

We note that constraints on our late invisible decay models from estimates of the free-streaming length are meant to provide rough estimates. To understand the power spectrum suppression scale accurately, one will need to solve the Boltzmann equations and derive the perturbation evolution \cite{56, 57}. However, this is beyond the scope of this study, and we will leave this for future work.

VI. RESULTS

In this section, we present our results. In figures 4-6, we include the constraints from structure formation and plot \( \Delta N_{\text{eff}} \) in the \( m_{NLSP} - m_{LSP} \) parameter space. The figures depict contours for the decay lifetime \( t_{\text{dec}} \) of the NLSP and bands representing the value of \( \Delta N_{\text{eff}} \). We have shown LSP masses up to 1 TeV ; for larger masses the SUSY particles will be too heavy to have a realistic prospect for their detection at the LHC. We also note that the region \( m_{NLSP} \lesssim m_{LSP} + M_N \), where \( M_N \approx 1 \) GeV, is kinematically forbidden.

Fig. (4) shows the results for case 1, \( \bar{N} \to N + \tilde{G} \) decay with \( m_{\tilde{N}} > m_N \gg m_{\tilde{G}} \). In this case \( N \) is the dominant component of DM, and gravitino quanta from \( N \) decay make the main contribution to dark radiation. The corresponding decay width is given by Eq. (4) where \( M_N \approx 1 \) GeV. Since the decay creates the same number of \( N \) and \( \tilde{G} \) quanta, the contribution of gravitinos to the total DM density is \( f m_{\tilde{G}}/1 \) GeV, where \( f \) is the ratio of \( N \) number density from \( N \) decay to its total value. We take \( f = 1 \) henceforth, which results in the tightest bounds from structure formation and \( \Delta N_{\text{eff}} \). If we use smaller values of \( f \), the constraints will become weaker, but we also need to have additional source of DM.

For \( m_{\tilde{N}} \gg 1 \) GeV, Eqs. (4)-(13) result in \( \Delta N_{\text{eff}} \propto m_{\tilde{G}}^{1/2} \), implying that \( \Delta N_{\text{eff}} = \text{const} \) bands lie along the curves \( m_{\tilde{N}}^2 \propto \Delta N_{\text{eff}}^2 m_{\tilde{G}}^2 \). We note, however, that \( m_{\tilde{G}} \) eventually catches up with \( m_{\tilde{N}} \), at which point the decay becomes kinematically impossible. Therefore, the \( \Delta N_{\text{eff}} = \text{const} \) bands have a turning point where they bend to the left as seen in the figure.

We find that the Lyman-alpha forest data is effective in constraining the parameter space for \( m_{\tilde{G}} > 100 \) GeV, while the galaxy power spectrum sets a stronger constraint for \( m_{\tilde{N}} < 10 \) GeV. In the latter case, the gravitino mass needs to be smaller than 0.01 GeV. We see that it is still possible to get \( \Delta N_{\text{eff}} \sim 0.5 \) in the allowed region of the parameter space for \( m_{\tilde{N}} < 2 \) GeV.

Fig. (5) shows the results for case 2, \( \bar{N} \to N + \tilde{G} \) decay with \( m_{\tilde{G}} > m_N > M_N \). In this case gravitino is the dominant component of DM, and \( N \) quanta from \( N \) decay make the main contribution to dark radiation. The corresponding decay width is given by Eq. (5) where \( M_N \approx 1 \) GeV.

Eqs. (5)-(13) result in \( \Delta N_{\text{eff}} \propto m_{\tilde{G}}^{1/2}(m_{\tilde{N}}^2 - m_{\tilde{G}}^2) \). This implies that \( \Delta N_{\text{eff}} = \text{const} \) bands are concentrated around the line \( m_{\tilde{N}} = m_{\tilde{G}} \) for large values of \( m_{\tilde{N}} \) as seen in the figure.

We find that the Lyman-alpha forest data is effective in constraining the parameter space for \( m_{\tilde{G}} > 100 \) GeV, while the galaxy power spectrum sets a stronger constraint for \( m_{\tilde{N}} < 100 \) GeV. Combining both constraints, we find that \( \Delta N_{\text{eff}} \lesssim 0.1 \) in the allowed region of the parameter space.

Fig. (6) shows the results for case 3, \( \bar{N} \to \tilde{N} + N \) decay with \( m_{\tilde{N}} > m_N > M_N \). In this case \( \tilde{N} \) is the dominant component of DM, and \( N \) quanta from gravitino decay make the main contribution to dark radiation. The corresponding decay width is given by Eq. (6) where \( M_N \approx 1 \) GeV. For \( m_{\tilde{G}} \gg m_N \) Eqs. (6)-(13) result in \( \Delta N_{\text{eff}} \propto m_{\tilde{G}}^{1/2} m_{\tilde{N}}^{1/2} \). Therefore \( \Delta N_{\text{eff}} = \text{const} \) bands lie along the curves \( m_{\tilde{N}}^2 \propto \Delta N_{\text{eff}}^2 / m_{\tilde{G}} \). Along this curve \( m_{\tilde{N}}/m_{\tilde{G}} \) decreases as \( m_{\tilde{G}} \) becomes smaller. Hence, in order for the decay to be kinematically allowed, the \( \Delta N_{\text{eff}} = \text{const} \) bands should eventually bend to the right. On the other hand, when \( m_N \sim m_{\tilde{G}} \), we have \( \Delta N_{\text{eff}} \propto 1/m_{\tilde{G}}^{1/2}(m_{\tilde{G}} - m_{\tilde{N}}) \). This implies that the \( \Delta N_{\text{eff}} = \text{const} \) bands join together around the line \( m_{\tilde{N}} = m_{\tilde{G}} \) line after turning to the right as seen in the figure.

We find that the Lyman-alpha forest data is effective in constraining the parameter space for \( m_{\tilde{G}} < 150 \) GeV, while the galaxy power spectrum sets a stronger constraint for \( m_{\tilde{N}} < 100 \) GeV. Combining both constraints, we find that \( \Delta N_{\text{eff}} \lesssim 0.5 \) in the allowed region of the parameter space.

VII. CONCLUSION

In this work, we investigated a simple extension of the MSSM that accommodates late invisible decays to/of the gravitino. The model includes new iso-singlet color-triplet superfields \( X \) and \( \tilde{X} \) and a singlet superfield \( N \). Such extension allows us to explain the baryon asymmetry of the universe, and can also address the DM-baryon coincidence puzzle. In addition to the LSP, this model has an \( R \)-parity even DM candidate when the singlet fermion \( N \) has \( O(\text{GeV}) \) mass.

Interesting cases arise when the singlet scalar \( \bar{N} \) and the
gravitino $\tilde{G}$ are the NLSP and LSP, respectively, and vice versa. The resulting decays $\tilde{N} \rightarrow \tilde{G} + N$ and $\tilde{N} \rightarrow \tilde{G} + N$ have long lifetimes as they involve gravitationally suppressed interactions. However, since both of the outgoing particles are invisible, these late decay are not subject to the tight BBN and CMB constraints on the hadronic and electromagnetic channels. On the other hand, depending on the mass ratios of the daughter and parent particles, it is possible that one or both of the DM candidates contribute to the amount of dark radiation or suppress perturbations at scales that are being probed by the galaxy cower spectrum and the Lyman-alpha forest data. We performed a detailed study of the $m_{\tilde{N}} - m_{\tilde{G}}$ parameter space in light of these constraints and showed that the entire DM content of the universe can be produced from the late invisible decays to/of the gravitino. Such decays have very important consequences in a broader context as they considerably relax the constraints on reheating of the universe in SUSY models.

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FIG. 6. The same as Fig. 4 but for case 3 ($\tilde{G} \rightarrow \tilde{N} + N, m_{\tilde{N}} \gg M_N \approx 1$ GeV).

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