Novel Bose-Einstein Interference in the Passage of a Fast Particle in a Dense Medium

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Abstract.
When an energetic particle collides coherently with many medium particles at high energies, the Bose-Einstein symmetry with respect to the interchange of the exchanged virtual bosons leads to a destructive interference of the Feynman amplitudes in most regions of the phase space but a constructive interference in some other regions of the phase space. As a consequence, the recoiling medium particles have a tendency to come out collectively along the direction of the incident fast particle, each carrying a substantial fraction of the incident longitudinal momentum. Such an interference appearing as collective recoils of scatterers along the incident particle direction may have been observed in angular correlations of hadrons associated with a high-\(p_T\) trigger in high-energy AuAu collisions at RHIC.

1. Introduction
In the collision of an energetic particle \(p=(p_0, \mathbf{p})\) with \(n\) medium particles, how dense must the medium be for the multiple collisions to become a single coherent \((1+n)\)-body collision, instead of a sequence of \(n\) incoherent 2-body collisions? It is instructive to find out the conditions on the medium density and the energy of the incident particle that determine whether the set of multiple collisions are coherent or incoherent [1]. For such a purpose, we consider a binary collision between the incident fast particle \(p\) and a medium scatterer \(a_i\) with the exchange of a boson, in the medium center-of-momentum frame. The longitudinal momentum transfer \(q_z\) for the binary collision can be obtained from the transverse momentum transfer \(q_T\) by \(q_z \sim q_T^2/2p_0\). The longitudinal momentum transfer is associated with a longitudinal coherence length \(\Delta z_{\text{coh}} \sim \hbar/q_z \sim 2\hbar p_0/q_T^2\) that specifies the uncertainties in the longitudinal locations at which the virtual boson is exchanged between the fast particle and the scatterer.

The nature of the multiple scattering process can be inferred by comparing the longitudinal coherence length \(\Delta z_{\text{coh}}\) with the mean free path \(\lambda\) of the jet in the dense medium that depends not only on the density of the medium but also on the binary collision cross section. If \(\Delta z_{\text{coh}} \ll \lambda\), then a single binary collision is well completed before another binary collision begins, and the multiple collision process consists of a sequence of \(n\) incoherent two-body collisions.

If \(\Delta z_{\text{coh}} \gg \lambda\), then a single binary collision is not completed before another one begins, and the multiple collision process consists of a set of coherent collisions as a single \((1+n)\)-body collision. For a set of initial and final states in such a coherent \((1+n)\)-body collision, there are

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n! different trajectories in the passage of collisions along $\Delta z_{\text{coh}}$ at which various virtual bosons are exchanged. By Bose-Einstein symmetry, the total Feynman amplitude is then the sum of the amplitudes for all possible interchanges of the exchanged virtual bosons. The summation of these Feynman amplitudes and the accompanying interference constitute the Bose-Einstein interference in the passage of the fast particle in the dense medium.

There is another important effect that accompanies the coherent $(1+n)$-body collision and changes the nature of the collision process. A sequence of $n$ incoherent two-body collisions contains only $2n$ degrees of freedom, which can be chosen to be the transverse momentum transfers $\{q_{1T}, q_{2T}, q_{3T}, \ldots, q_{nT}\}$. The longitudinal momentum transfer $q_z$ after each individual jet-parton collision is then a dependent variable, depending on the corresponding transverse momentum transfer as $q_z \sim |q_{1T}|^2/2p_0$. In contrast, the coherent $(1+n)$-body collision links the incident particle with $n$ scatterers as a single collisional unit. There are $3n-1$ degrees of freedom in this coherent $(1+n)$-body collision, after the $3(n+1)$ degrees of freedom are reduced by the constraints of the conservation of energy and momentum. The set of longitudinal momentum transfers, $\{q_{iz}, i = 1, 2, \ldots, n\}$, can also be independent variables with their own probability distribution functions. The longitudinal momentum of the incident fast particle can flow to the $n$ scatterers, which can then carry a substantial fraction of the longitudinal momentum of the incident fast particle. Thus, the degrees of freedom increases from $2n$ for incoherent collisions to $3n-1$ for a coherent collision, and this increase changes the character of the collision process.

It is of interest to inquire whether there are fast particles energetic enough, scatterer media dense enough, and binary cross section large enough, for coherent collisions to occur. In high energy central collisions between heavy nuclei such as those at RHIC and LHC, both jets (mini-jets) and a dense medium are produced after each collision. The jets will collide with partons in the dense medium, and these collisions may satisfy the condition for coherent collisions. The longitudinal coherent length $\Delta z_{\text{coh}}$ is of order 25 fm, for a typical transverse momentum transfer $q_T \sim 0.4$ GeV/c from a jet of momentum $p_0 \sim 10$ GeV/c to a medium parton in a binary collision at RHIC [2]. The longitudinal coherent length $\Delta z_{\text{coh}}$ is much greater than the radius $R$ of a large nucleus. On the other hand, the away side jet is quenched by the dense medium in the most central AuAu collisions at RHIC and LHC [3, 4, 5], and the near-side jet collides with about 4-6 medium partons [6]-[13]. Therefore, one can infer that the mean-free path $\lambda$ for the collision of the jet with medium partons is much smaller than the nuclear radius $R$. Hence, $\Delta z_{\text{coh}} \gg R \gg \lambda$ in high-energy central nuclear collisions at RHIC and LHC, and the multiple collision process of a jet with medium partons along its path constitutes a set of coherent collision.

2. Bose-Einstein Interference of Feynman Amplitudes

As an example of the interference of Feynman amplitudes in a coherent collision, we consider

$$p + a_1 + a_2 \rightarrow p' + a'_1 + a'_2,$$  \hspace{1cm} (1)

the collisions of an energetic fermion $p$ with two fermion scatterers $a_1$ and $a_2$ of rest mass $m$ in the Abelian gauge theory. The Feynman amplitude for diagram 1(a) is given by [14]

$$M_a = \frac{1}{\not p - \not q_1 - m + i\epsilon} \tilde{u}(p')\gamma_\nu \frac{1}{\not q_2} \tilde{u}(a'_2)\gamma_\nu u(a_2) \frac{1}{\not q_1} \tilde{u}(a'_1)\gamma_\nu u(a_1).$$  \hspace{1cm} (2)

If the spatial separation between the scatterers is so large that $\lambda \gg \Delta z_{\text{coh}}$, then diagram 1(a) can be cut into two disjoint pieces and the collision process consists effectively of a set of two incoherent two-body collisions.

On the other hand, if the collision process is characterized by $\lambda \ll \Delta z_{\text{coh}}$, it comprises a set of coherent $(1+2)$-body collision. There is an additional Feynman amplitude $M_b$ for diagram
with the emission and absorption of virtual bosons of momenta \( q_1 \) and \( q_2 \). By Bose-Einstein symmetry, the total amplitude \( M_b \) obtained by making a symmetrized permutation of the bosons in diagram 1(a),

\[
M_b = -g^4 \bar{u}(p') \gamma_\mu \frac{1}{\not{p} - \not{q}_2 - m + i\epsilon} \gamma_\nu u(p) \frac{1}{q_2^2} \bar{u}(a_2') \gamma_\nu u(a_2) \frac{1}{q_1^2} \bar{u}(a_1') \gamma_\mu u(a_1).
\]

(3)

The trajectories for diagram 1(a) and 1(b) are both possible paths in a coherent collision, leading from the a set of initial states to a set of final states. By Bose-Einstein symmetry, the total amplitude \( M \) for coherent collisions is the symmetrized sum of \( M_a \) and \( M_b \).

We consider the high-energy limit and assume the conservation of helicity with

\[ \{p_0, |p|, p'_0, |p'|\} \gg \{|a_i|, q_0, |q_i|\} \gg m, \text{ for } i = 1, 2. \]

(4)

In this limit, we have approximately [14]

\[
\bar{u}(a') \gamma_\mu u(a) \sim \sqrt{\frac{a_0 + m}{a_0 + m + 2m}} \frac{a_\mu'}{a_0 + m + 2m} \frac{a_\mu}{a_0 + m + 2m} = \tilde{a}_\mu,
\]

(5)

\[
\frac{1}{\not{p} - \not{q}_2 - m + i\epsilon} \gamma_\nu u(p) \sim \frac{\not{p} - \not{q}_1 + m}{2p \cdot q_1 - i\epsilon} \gamma_\nu u(p),
\]

(6)

\[
\bar{u}(p) \gamma_\nu (\not{p} - \not{q}_1 + m) \gamma_\mu u(p) \sim \frac{2p_0 p_\mu}{m},
\]

(7)

where \( \epsilon \) is a small positive quantity. We shall be interested in the case in which the fermion \( p' \) after the collision remains on the mass shell. The mass shell condition can be expressed as

\[
(p - q_1 - q_2)^2 - m^2 \sim -2p \cdot q_1 - 2p \cdot q_2 \sim 0.
\]

(8)

The symmetrized sum of the Feynman amplitudes \( M_a \) and \( M_b \) in the high-energy limit is

\[
M \sim \frac{g^4}{2m} \frac{2p \cdot \tilde{a}_1}{m} \frac{2p \cdot \tilde{a}_2}{m} \frac{1}{q_2^2 q_1^2} \left( \frac{1}{2p \cdot q_1 - i\epsilon} + \frac{1}{2p \cdot q_2 - i\epsilon} \right).
\]

(9)

Note that the amplitudes \( M_a \) and \( M_b \) correlate with each other because of the mass-shell condition (8). The real parts of the amplitudes destructively cancel, and the imaginary parts interfere and add constructively, to result in sharp distributions at \( p \cdot q_1 \sim 0 \) and \( p \cdot q_2 \sim 0 \),

\[
M \sim \frac{g^4}{2m} \frac{2p \cdot \tilde{a}_1}{m} \frac{2p \cdot \tilde{a}_2}{m} \frac{1}{q_2^2 q_1^2} \left\{ i\pi \Delta(2p \cdot q_1) + i\pi \Delta(2p \cdot q_2) \right\}, \quad \Delta(2p \cdot q_1) = \frac{1}{\pi} \frac{\epsilon}{(2p \cdot q_1)^2 + \epsilon^2},
\]

(10)
where the function $\Delta(2p \cdot q_i)$ approaches the Dirac delta function $\delta(2p \cdot q_i)$ in the limit $\epsilon \to 0$.

Generalizing to the case of the coherent collision of a fast fermion with $n$ fermion scatterers, the total Feynman amplitude is

$$M = \frac{g^{2n}}{2m} \left( \prod_{i=1}^{n} \frac{2p \cdot \hat{a}_i}{mq_i^+} \right) \mathcal{M}(q_1, q_2, \ldots, q_n),$$

where $\mathcal{M}(q_1, q_2, \ldots, q_n)$ is the sum of $n!$ amplitudes involving symmetric permutations of the exchanged bosons given by

$$\mathcal{M}(q_1, q_2, \ldots, q_n) = \prod_{j=1}^{n-1} \frac{1}{\sum_{i=1}^{j} 2p \cdot q_i - i\epsilon} + \text{symmetric permutations}. \quad (12)$$

The above sum involves extensive cancellations. Remarkably, it can be shown that this sum of $n!$ permutations turns out to be a product of sharp distributions centered at $2p \cdot q_i$.

$$\mathcal{M}(q_1, q_2, \ldots, q_n) \Delta(\sum_{i=1}^{n} 2p \cdot q_i) = (2\pi i)^{n-1} \prod_{i=1}^{n} \Delta(2p \cdot q_i), \quad (13)$$

which indicates that in coherent collisions, there is a destructive interference of the Feynman amplitudes in most regions of the phase space but a constructive interference in some other regions of the phase space, leading to sharp distributions at $2p \cdot q_i \sim 0$.

Equations (11)-(13) for the total Feynman amplitude as a product of delta functions of $2p \cdot q_i$ for a coherent collision are similar to previous results obtained for the emission of many real photons or gluons in bremsstrahlung, and for the sum of ladder and cross-ladder amplitudes in the collision of two fermions [14]-[21].

3. Consequences of the BE Interference on the Recoils of Fermion Scatterers

The differential cross section for the coherent collision $p + a_1 + \ldots + a_n \to p' + a'_1 + \ldots + a'_n$ is

$$d^n \sigma = \sum_{f_{j_1}f_{j_2}\ldots f_{j_n}T_1T_2\ldots T_n} |M|^2 (2\pi)^4 \delta^4(p' + \sum_{i=1}^{n} q_i - p) \frac{d^4p'}{(2\pi)^4 2p'_0} \prod_{i=1}^{n} \frac{d^4a'_i}{(2\pi)^4 2a'_0} D_i(a'_i), \quad (14)$$

where $f_{j_i}$ is the dimensionless flux factor between the fast particle $p$ and scatterer $a_i$.

$$f_{j_i} = \frac{\sqrt{p \cdot a_i - m \cdot m_T}}{m_i m}, \quad (15)$$

and $\{T_i, i = 1, \ldots (n-1)\}$ are the collision times for the intermediate state of the fast particle $p$ to collide with the scatterers after the $i$th collision, and $m_T = \sqrt{m_i^2 + a_i^2}$. The function $D_i$ describes the mass-shell conditions of medium scatterers after collision,

$$D_i(a'_i) = \frac{\Gamma_i/2}{\pi \{[a'_i - \sqrt{(a'_i - A)^2 + (m_i + S)^2 + A_0^2]}^2 + \Gamma_i^2/4\}}, \quad (16)$$

where $A = \{A, A_0\}$ and $S$ are the vector and scalar mean fields experienced by the medium scatterer $a'_i$ after collision, respectively, and $\Gamma_i$ is the width of $a'_i$. As the mean fields and scatterer widths increase with density and are presumably quite large and dominant for a dense
The denominators of the boson propagator in Eq. (17) becomes
\[ \frac{g^{4n}(2m)^{n-1}}{4\pi^{n+1}} \left( \prod_{i=1}^{n} \frac{D_{ij}}{p_{ij}} \right) \left( \prod_{i=1}^{n} \Delta(2p \cdot q_i) \right) \left( \prod_{i=1}^{n} \Delta(2p \cdot q_i) dq_i \right) \left( \frac{n}{m^2 q_i^2} \right) \frac{dq_i dq_{iz}}{2a_{i0}^2}. \] (17)

The distribution \( \Delta(p \cdot q_i) \) can be written as
\[ \Delta(2p \cdot q_i) = \frac{1}{p_0 + p_z} \Delta(q_i - q_{iz} - (p_0 - p_z)(q_{i0} + q_{iz}) + 2p_T \cdot q_T), \] (18)

which provides the constraint \( q_{i0} - q_{iz} \sim 0 \). As a consequence, the integral of \( |\Delta(p \cdot q_i)|^2 dq_{i0} \) can be carried out, with the collision time \( T_c \), canceling one of the factors in \( \Delta(2p \cdot q_i) \) for \( p_{i0} \to 0 \).

The denominators of the boson propagator in Eq. (17) becomes
\[ q_{i0}^2 = (q_{i0} + q_{iz})(q_{i0} - q_{iz}) - |q_{iT}|^2 \approx -|q_{iT}|^2. \] (19)

We obtain
\[ d^n \sigma = \frac{\alpha^2}{p_z m} \left( \frac{\alpha^2}{p_z^2} \right)^{n-1} \left( \prod_{i=1}^{n} \left[ 2mD_{ij} \right] p_{ij}^{-1} \right) \left( \prod_{i=1}^{n} \frac{(2p \cdot \tilde{a}_i)^2}{m^2 2a_{i0}^2 |q_T|^2} dq_{iz} dq_{iT} \right), \] (20)

where \( \alpha = g^2/4\pi \), the last two factors are dimensionless, and \( d^n \sigma \) has the dimension of \( (\alpha^2/p_z^2)^n \).

To find the probability distribution for the longitudinal momentum transfer \( q_{iz} \), we introduce the fractional longitudinal momentum kick
\[ x_i = \frac{q_{iz}}{p_z}, \quad dq_{iz} = p_z dx_i. \] (21)

To investigate the \( x_i \) dependence of the factor \( (2p \cdot \tilde{a}_i)/2a_{i0}^2 \) in Eq. (20), we note from Eq. (6) that \( \tilde{a}_i \) can be written as a function of \( q_i \) and \( a_i \),
\[ \tilde{a}_i \sim \frac{a_{i0} + m q_i}{a_{i0}^2 + m^2} + \frac{a_{i0} + m + a_{i0} + m}{\sqrt{(a_{i0}^2 + m)(a_{i0}^2 + m^2)}}. \] (22)

Because of the \( \Delta(2p \cdot q_i) \) constraint, the factor \( (2p \cdot \tilde{a}_i)^2/2a_{i0}^2 \) in Eq. (20) becomes
\[ \frac{(2p \cdot \tilde{a}_i)^2}{2a_{i0}^2} \sim \frac{(a_{i0} + a_{i0})^2 (p \cdot a_i)^2}{2(a_{i0}^2)^2} \equiv \frac{\kappa_i (p \cdot a_i)^2}{a_{i0}}. \] (23)

We obtain from Eq. (25)
\[ d^n \sigma = \left\{ \frac{\alpha^2}{p_z m} \left( \frac{\alpha^2}{p_z^2} \right)^{n-1} \frac{p_z}{m^2} \left( \prod_{i=1}^{n} \left[ 2mD_{ij} \right] p_{ij}^{-1} \right) \frac{\kappa_i (p \cdot a_i)^2}{a_{i0}} \right\} \frac{dx_1 dx_2 \ldots dx_n dq_{1T} dq_{2T} \ldots dq_{nT}}{|q_{1T}|^4 |q_{2T}|^4 \ldots |q_{nT}|^4}. \] (24)

The fermion scatterers can possess different initial energies \( a_{i0} \) at the moment of their collisions with the energetic jet. In the case when \( a_{i0} < q_{i0} \), the factor \( \kappa_i \) approaches \( 1/2 + O(a_{i0}/q_{iz}) \) with \( (a_{i0}/q_{iz}) \ll 1 \). In the other extreme when \( a_{i0} > q_{i0} \), the factor \( \kappa \) approaches \( 2 + O(q_{iz}/a_{i0}) \) with \( (q_{iz}/a_{i0}) \ll 1 \). The dependence of \( \kappa_i \) on \( x_i \) is weak in either limits and can be neglected in our approximate estimate. We obtain then approximately
\[ d^n \sigma \sim \left\{ \text{constant factor} \right\} \frac{dx_1 dx_2 \ldots dx_n dq_{1T} dq_{2T} \ldots dq_{nT}}{|q_{1T}|^4 |q_{2T}|^4 \ldots |q_{nT}|^4}. \] (25)
where the constant factor is approximately independent of \( x_i \) and \( q_{iT} \). By symmetry, the fraction of momentum transfers \( x_i \) for different scatterers should be approximately the same on the average, and \( x_i^{\text{max}} \sim 1/n \). Then as far as \( x_i \) is concerned, the average distribution is

\[
\frac{dP}{dx_i} \bigg|_{x_i=x_j=1, \ldots, n} \sim n \Theta \left( \frac{1}{n} - x_i \right),
\]

and the average longitudinal momentum fraction is

\[
\langle x_i \rangle \sim \frac{1}{2n} \quad \text{or} \quad \langle q_{iz} \rangle \sim \frac{p_z}{2n}.
\]

The above results in Eqs. (25)-(27) indicate that in the passage of an energetic fermion making coherent collisions with medium partons, the reaction has a high probability for the occurrence of small values of \( |q_{iT}| \). The singularities at \( |q_{iT}| \sim 0 \) in Eq. (25) correspond to the case of infrared instabilities that may be renormalized, and a momentum cut-off \( \Lambda \) may be introduced. The scatterers acquire an average longitudinal momentum \( \langle q_{iz} \rangle \sim p_z/2n \) that is expected to be much greater than \( |q_{iT}| \). Thus, there is a collective quantum many-body effect arising from Bose-Einstein interference such that the fermion scatterers emerge in the direction of the incident particle, each carrying a fraction of the forward longitudinal momentum of the incident particle that is inversely proportional to twice the number of scatterers, \( \langle q_{iz} \rangle \sim p_z/2n \).

### 4. Bose-Einstein Interference for Coherent Collisions in Non-Abelian Theory

The above considerations for the Abelian theory can be extended to the non-Abelian theory. As an example, we consider a quark jet \( p \) making coherent collisions with quarks \( a_1 \) and \( a_2 \) in the reaction \( p + a_1 + a_2 \rightarrow p' + a'_1 + a'_2 \), in the non-Abelian theory. We shall neglect four-particle vertices and loops, which are of higher-orders. The Feynman diagrams are then the same as those in Fig. 1. One associates each quark vertex with a color matrix \( T^{(p,1,2)}_{\alpha,\beta} \) where the superscript \( p, 1, \) or \( 2 \) identifies the quark \( p, a_1, \) or \( a_2, \) and the subscripts \( \alpha \) or \( \beta \) give the \( SU(3) \) color matrix index. The Feynman amplitude \( M_a \) for diagram 1(a) is

\[
M_a = -g^4 \bar{u}(p')T^{(p)}_{\beta} \gamma_{\mu}u(p) \frac{1}{\not{p} - \not{q}_1 - m + i\epsilon} T^{(p)}_{\alpha} \gamma_{\mu}u(p) \frac{1}{q^2_2} \bar{u}(a'_2)T^{(2)}_{\beta} \gamma_{\mu}u(a_2) \frac{1}{q^2_1} \bar{u}(a'_1)T^{(1)}_{\alpha} \gamma_{\mu}u(a_1).
\]

The Feynman amplitude \( M_b \) for diagram 1(b) is

\[
M_b = -g^4 \bar{u}(p')T^{(p)}_{\alpha} \gamma_{\mu}u(p) \frac{1}{\not{p} - \not{q}_2 - m + i\epsilon} T^{(p)}_{\beta} \gamma_{\mu}u(p) \frac{1}{q^2_2} \bar{u}(a'_2)T^{(2)}_{\beta} \gamma_{\mu}u(a_2) \frac{1}{q^2_1} \bar{u}(a'_1)T^{(1)}_{\alpha} \gamma_{\mu}u(a_1).
\]

In the high-energy limit for coherent collisions, the sum of the Feynman amplitudes is

\[
M \sim \frac{g^4}{2m} \left( \frac{2p \cdot \tilde{a}_1}{m} \frac{2p \cdot \tilde{a}_2}{m} \frac{1}{q^2_2 q^2_1} \left( \frac{T^{(p)}_{\beta}T^{(p)}_{\alpha}}{2p \cdot q_1 - i\epsilon} + \frac{T^{(p)}_{\alpha}T^{(p)}_{\beta}}{2p \cdot q_2 - i\epsilon} \right) \right).
\]

We can rewrite the product of the color matrices for the quark jet \( p \) as

\[
T^{(p)}_{\beta}T^{(p)}_{\alpha} = \frac{1}{2} \left( [T^{(p)}_{\beta}T^{(p)}_{\alpha}]_+ + [T^{(p)}_{\beta}T^{(p)}_{\alpha}]_+ \right),
\]

\[
T^{(p)}_{\alpha}T^{(p)}_{\beta} = \frac{1}{2} \left( [T^{(p)}_{\beta}T^{(p)}_{\alpha}]_+ - [T^{(p)}_{\beta}T^{(p)}_{\alpha}]_+ \right).
\]

where the integral is

\[
\int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4} \left( \frac{1}{q^2_1} + \frac{1}{q^2_2} \right) \left( \frac{T^{(p)}_{\beta}T^{(p)}_{\alpha}}{2p \cdot q_1 - i\epsilon} + \frac{T^{(p)}_{\alpha}T^{(p)}_{\beta}}{2p \cdot q_2 - i\epsilon} \right).
\]
For coherent collisions, the Feynman amplitude is then

\[ M \sim \frac{q^4}{2m} \frac{4p\cdot q_1}{m} \frac{2p\cdot q_2}{q_3^2 q_1^2} \left\{ (\mathcal{M}_1 + \mathcal{M}_2) \left[ \frac{T^{(p)} T^{(p)} T^{(1)}}{2} \right] + (\mathcal{M}_1 - \mathcal{M}_2) \left[ \frac{T^{(p)} T^{(p)} - T^{(2)} T^{(1)}}{2} \right] \right\}, \tag{31} \]

\[ \mathcal{M}_1 + \mathcal{M}_2 = i \Delta(2p \cdot q_1) + i \Delta(2p \cdot q_2), \quad \text{and} \quad \mathcal{M}_1 - \mathcal{M}_2 = \frac{4p\cdot q_1}{(2p\cdot q_1)^2} q_2. \tag{32} \]

From the above analysis, we find that the color degrees of freedom in QCD bring in additional properties to the Feynman amplitudes. Bose-Einstein symmetry with respect to the interchange of gluons in QCD involves not only the space-time exchange symmetry but also color index exchange symmetry. The total exchange symmetry can be attained with symmetric space-time of gluons in QCD involves not only the space-time exchange symmetry but also color index exchange symmetry. The total exchange symmetry can be attained with symmetric space-time antisymmetric and color index antisymmetric, as in the \((\mathcal{M}_1 - \mathcal{M}_2)\) term in Eq. (31).

For the space-time symmetric and color index exchange symmetric component, the Feynman amplitude is equal to the Abelian Feynman amplitude multiplied by a color factor. It will exhibit the same degree of Bose-Einstein interference as in the Abelian theory. Previous analysis on the longitudinal momentum transfer of recoiling fermions in the Abelian theory can be applied to the non-Abelian theory for this space-time symmetric and color index exchange symmetric component. There is thus a finite probability for the presence of constraints to lead to recoiling quarks receiving significant momentum kicks along the direction of the incident quark jet.

The above considerations for jets and scatterers can be extended to cases involving gluon jets and gluon scatterers. The results are similar but with a small modification on the distribution for gluon scatterers [1].

5. Signatures for Bose-Einstein Interference in the Passage of a Jet

From the results in the above sections, the occurrence of the Bose-Einstein interference in the coherent collisions of a jet with medium partons possesses the following characteristics:

1. The Bose-Einstein interference is a quantum many-body effect. It occurs only in the coherent multiple collisions of the fast jet with two or more scatterers, \(n \geq 2\).
2. Each scatterer has a transverse momentum distribution of the type \(1/|q_T|^4\), which peaks at small values of \(|q_T|\).
3. Each scatterer acquires a longitudinal momentum kick \(q_z\) along the incident jet direction that is approximately inversely proportional to twice the number of scatterers, with \(\langle q_z \rangle \sim p_z/2n\).
4. As a consequence, the final effect is the occurrence of collective recoils of the scatterers along the jet direction.

In a high energy central collision of heavy nuclei, both jets and a dense medium are produced soon after the collision and jets propagate in the dense medium. The mean free path for the jet in the medium may be small compared with the longitudinal coherent length to provide the appropriate environment for coherent collisions. It is of interest to search for evidence for Bose-Einstein interference in the passages of the jet in the dense medium. In such a search, we need to separate out jet fragments from recoiling scatterers among the detected hadrons. Such a separation is indeed kinematically possible in the \(\Delta \phi-\Delta \eta\) correlation measurements where \(\Delta \phi = \phi(\text{associated}) - \phi(\text{trigger})\) is the azimuthal angle difference between the produced hadron pair, and \(\Delta \eta = \eta(\text{associated}) - \eta(\text{trigger})\) is the pseudorapidity difference. We show in Fig. 2 the \(\Delta \phi-\Delta \eta\) correlation data from the STAR Collaboration [23] for the most central AuAu collision at RHIC with a high \(p_T\) trigger [23]. Jet fragments are distributed in a small cone of \((\Delta \phi, \Delta \eta)\sim 0\) [22]-[25]. Medium particles are distributed in the “ridge” part at \(\Delta \phi \sim 0\) (Fig. 2) along the
\( \Delta \eta \) axis. In particular, ridge particle with \(|\Delta \eta| > 0.6 \) and \( \Delta \phi \sim 0 \) along the ridge (Fig. 2) can be identified as medium scatterer partons for the following reasons:

(i) The yield of the ridge particles increases approximately linearly with the number of participants [23].

(ii) The yield of the ridge particles is nearly independent of (i) the flavor content, (ii) the meson/hyperon character, and (iii) the transverse momentum \( p_T \) (above 4 GeV) of the jet trigger [23, 24].

(iii) The ridge particles have a temperature (inverse slope) that is similar (but slightly higher) than that of the inclusive bulk particles, but lower than the temperature of the near-side jet fragments [23]

(iv) The baryon/meson ratio of the ridge particles is similar to those of the bulk hadrons and is quite different from those in the jet fragments [25].

(v) The Wigner function analysis indicates that the momentum distribution of medium partons has a rapidity plateau structure even at the early stage of flux tube fragmentation [11].

With the medium scatterers (ridge particles) separated from the incident high-\( p_T \) jet fragments, the occurrence of the Bose-Einstein interference can be characterized by the collective recoils of the scatterers (the ridge particles) along the jet direction. The collective recoils will lead to the \( \Delta \phi \sim 0 \) correlation of the ridge particles with the high-\( p_T \) trigger. As shown in Fig. 2, such collective recoils of the medium particles have indeed been observed in \( \Delta \phi, \Delta \eta \) correlations of produced hadrons in AuAu collisions at RHIC by the STAR Collaboration [22]-[25]. Similar \( \Delta \phi, \Delta \eta \) correlations have also been observed by the PHENIX Collaboration [2, 30, 31], and the PHOBOS Collaboration [32]. Quantitatively, the collective recoils of the kicked medium partons have been encoded into the longitudinal momentum kick \( \langle q_z \rangle \) of the momentum kick model analysis that yields the observed \( \Delta \phi, \Delta \eta, \) and \( p_T \) dependencies of the angular correlations [6]-[13].

It is of interest to examine the relationship \( \langle q_z \rangle \sim p_z/2n \) between \( n \) and \( \langle q_z \rangle \), when such a collective momentum kick occurs. For the most central AuAu collisions at \( \sqrt{s_{NN}} = 200 \) GeV at RHIC, we previously found from the momentum kick model analysis that \( \langle n \rangle \sim 6 \) and \( q_z \sim 1 \) GeV [8]. In another momentum kick model analysis for the highest multiplicity \( pp \) collisions at \( \sqrt{s_{NN}} = 7 \) TeV at the LHC, we previously found that \( \langle n \rangle \sim 2.4 \) and \( q_z \sim 2 \) GeV. The experimental data give a longitudinal momentum transfer \( \langle q_z \rangle \) that is approximately inverse proportional to the number of scatterers \( \langle n \rangle \), in rough agreement with the signatures discussed above.

With regard to the threshold \( n \geq 2 \) for the occurrence of the Bose-Einstein interference, the presence of a threshold implies a sudden increase of the ridge yield as a function of centrality,
as represented by the number of participants. Although the experimental data with a high $p_T$ trigger appear to be consistent with the presence of thresholds, the large error bars and the scarcity of the number of data points in the threshold regions preclude a definitive conclusion.

However, threshold effects for the ridge yield (2D Gaussian yield) as a function of the number of participants, $N_{\text{part}}$, have been observed in another angular correlation measurements with a low-$p_T$ trigger from the STAR Collaboration [26, 27, 28, 29]. We note previously that a fast jet parton possesses low-$p_T$ jet fragments and a minimum-$p_T$-biased low-$p_T$ trigger can also indicate the passage of a fast parent jet [13]. As a consequence, ridge particles will also be associated with a low-$p_T$ trigger. The sudden increase of the amplitude and the peak $\eta$ width of the ridge yield as a function of $N_{\text{part}}$ may indicate the presence of a threshold for the ridge yield as a function of centrality [1].

6. Conclusions and Discussions

In the collision of a fast particle with many medium particles, the set of multiple collisions can occur incoherently as a sequence of $n$ two-body collisions or coherently as a single $(1+n)$-body collision. From the binary collision data, one obtains the longitudinal correlation length $\Delta z_{\text{coh}} \sim 2h p_0/\sqrt{s}$. If the medium is so dilute, the binary cross section so small, and the incident particle so slow that $\lambda \gg \Delta z_{\text{coh}}$, then the set of multiple collisions consists of a sequence of incoherent collisions. On the other hand, if the medium is so dense, the binary cross section so large, and the incident particle is so energetic such that $\Delta z_{\text{coh}} \gg \lambda$, then the set of multiple collisions consists of a single coherent $(1+n)$-body collision.

In central collisions of heavy nuclei at RHIC energies, the environment of the produced medium favors the occurrence of coherent collisions between the jet and medium particles. The dynamics is governed by Feynman diagrams linking the incident jet with the scatterers as a connected unit. For a set of initial states and a set of final states in such a coherent collision, there are many different trajectories in the sequences of collisions along $\Delta z_{\text{coh}}$ at which various virtual bosons are exchanged. By Bose-Einstein symmetry, the total Feynman amplitude is the sum of all Feynman amplitudes with all interchanges of the virtual boson vertices.

Remarkably, in high-energy collisions, the summation of these Feynman amplitudes from different ways of ordering the virtual boson vertices interfere with each other, resulting in the cancellation of some parts of the Feynman amplitudes and leaving only the other parts of sharp distributions. The longitudinal momentum transfer to each scatterer is then constrained to be the same as the energy transfer. There is a substantial flow of the longitudinal momentum from the jet to the scatterers. Furthermore, the transverse momentum distribution favors small values of $q_T$. As a consequence, the scatterers recoil collectively along the jet direction, leading to a correlation of the scatterers and the jet around $\Delta \phi \equiv \phi(\text{scatterer}) - \phi(\text{trigger jet}) \sim 0$.

For the coherent collision of an energetic parton with parton scatterers in non-Abelian cases, we find that the complete Bose-Einstein symmetry in the exchange of virtual gluons consists not only of space-time exchange symmetry but also color index exchange symmetry. There is always a space-time symmetric and color-index symmetric component of the Feynman amplitude that behaves in the same way as the Feynman amplitude in the Abelian case, and the corresponding recoiling partons behave in the same way as in the Abelian case. There is thus a finite probability for the parton scatterers to emerge collectively along the incident trigger jet direction, each carrying a significant fraction of the longitudinal momentum of the incident jet.

The collective recoils will lead to the $\Delta \phi \sim 0$ correlation of the ridge particles with the high-$p_T$ trigger. Such a signature of the Bose-Einstein interference may have been observed in the $\Delta \phi \sim 0$ correlation in the angular correlation measurements of produced hadron pairs in central AuAu collisions at RHIC [22]-[31]. The centrality dependence of the ridge yield in AuAu collisions at $\sqrt{s_{NN}} = 200$ GeV with a low-$p_T$ trigger [26, 27, 28, 29] may also be consistent with the presence of a ridge threshold at $n = 2$, as expected in the quantum many-body effect of
Bose-Einstein interference.

The collective recoils of the scatterers from the Bose-Einstein interference may be the origin of the longitudinal momentum kick along the jet direction postulated in the momentum kick model that has been quite successful in the analysis of the angular correlations of hadrons produced in high-energy heavy-ion collisions [6]-[13]. It is interesting to note that the present Bose-Einstein interference in coherent collisions is consistent with previous results on the Bose-Einstein interference in the emission of real photons and gluons in high-energy interactions and in the sum of the ladder and cross-ladder loop diagrams in the collision of two particles [15]-[21].

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