Optimization of STIRAP-based state transfer under dissipation

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Abstract

We quantify the influence of non-adiabatic leakage and system dissipation on the transfer fidelity with generic counter-intuitive pulses. By including the dissipation of all the parties, we find that the competition between non-adiabaticity and decoherence leads to an upper bound of the transfer fidelity. Using a systematic expansion valid in the desired high-fidelity limit, we derive the upper bound as a simple function of the system cooperativity. This approach also indicates how to reach the upper bound efficiently. Our results are widely applicable to quantum state engineering and are particularly significant for solid-state devices, which typically have non-negligible dissipation of the source and target systems.

1. Introduction

State transfer, where an arbitrary quantum state is transmitted from source to target, is a fundamental task in quantum state engineering. While sometimes the transfer can be achieved directly, in many circumstances it needs the mediation of a third party (a quantum bus), whose coherence property is critical for the transfer process. To mitigate the decoherence effect of the bus, a particularly interesting class of indirect transfer protocols based on the idea of stimulated Raman adiabatic passage (STIRAP) was developed, which involves negligible excitation of the quantum bus. If the source and target have negligible dissipation, the STIRAP transfer is performed in a decoherence-free subspace.

STIRAP was introduced more than two decades ago for population transfer in Λ-type atoms [1]. Due to the advantages of being robust, and efficient, this approach, as well as its theoretical extensions [2–9], has found applications in a variety of physical systems for different tasks (see e.g. [10–18] and for a most recent review, see [19]). Most recently, great improvements in the coherent control of solid-state devices has led to remarkable demonstrations of STIRAP in superconducting circuits [20] and NV centers [21–23]. STIRAP is also a promising choice for state transfer in optomechanics [24–26] and quantum dots [27–29]. However, unlike atomic systems, most solid-state quantum devices suffer significant dissipation and the desired decoherence-free subspace is absent. Under this scenario the prolonged operation time of STIRAP (required by adiabaticity) presents a severe drawback: even if the source and target are relatively weakly coupled to the environment, the accumulation of errors over a long transfer time could still significantly degrade the transfer fidelity. Thus, the trade-off between decoherence and non-adiabatic transitions is crucial for STIRAP under dissipation, but a quantitative study on the optimization of such a trade-off, as well as the resulting fundamental limitations, is still missing in the literature. This is the central issue of the present work.

Past work on the optimization of STIRAP for atomic population transfer was performed in the absence of decoherence, with the objective of reducing the non-adiabatic leakage and minimizing the pulse area or duration [30]. Other studies considered the effect of dissipation by including decoherence of the intermediate level [31–34], thus the crucial trade-off between decoherence and adiabaticity was not taken into consideration.

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Those treatments are often within a phenomenological approach [32, 35]. In studies of STIRAP in solid-state systems, the dissipation is numerically simulated (see e.g. [14, 20–22, 24–28, 36–38]). In order to provide physical insight and understand the power of STIRAP in a realistic scenario, we pursue here an analytical treatment with full consideration of the system dissipation (including that of the source and target). We solve the master equation with a perturbative approach which is accurate in the desired high-fidelity regime. The resulting upper bound of the fidelity is found as a simple function of the system cooperativities. We also provide a systematic optimization procedure of the time-dependent profiles, to reach this upper bound efficiently. Our results can be applied to a great variety of physical implementations including optomechanics, circuit QED, spin qubits, as well as hybrid systems combining different physical components [39].

This paper is organized as follows: the system and the basic physical process are described in section 2. In order to understand the relevant physics, we first provide in section 3 an estimation of the optimal fidelity by analyzing the unitary dynamics. Section 4 presents the perturbative treatment. Based on this method, the optimization analysis is elaborated in section 5, which contains the main results of our study. Finally, we give the summary and outlook in section 6.

2. System and STIRAP transfer

For quantum state engineering in solid-state devices, one of the most relevant scenarios of STIRAP-based state transfer is when a discrete mode (e.g., of a micromechanical resonator, a qubit, or an optical cavity) mediates the state transfer between two qubits (either of different type, such as the hybrid system of superconducting and spin qubits, or distant qubits of the same type). Furthermore, one or both of the qubits can be replaced by a cavity mode (see e.g. [40, 41]). We consider the setup schematically illustrated in figure 1(a), where two qubits interact resonantly with a common bosonic mode. Using the rotating wave approximation, the Hamiltonian in the interaction picture is

$$
H^I = \sum_{i=1,2} G_i(t) (a_i^\dagger \sigma_{x}^i + a_i \sigma_{x}^i),
$$

with $G_i(t)$ the tunable coupling strengths, $a$ the bosonic annihilation operator of the bus, and $\sigma_{x} = \sigma_x \pm i\sigma_y$. Such a Jaynes–Cummings Hamiltonian has been realized, for example, in cavity QED and various circuit QED architectures. Considering dissipation, the bus and qubits decay to their ground states with rates $\gamma_i$ and $\kappa_i$, respectively. The system dynamics is described by the master equation

![Figure 1.](image_url)
\[
\frac{\text{d}\rho}{\text{d}t} = -i[H^\dagger, \rho] + \mathcal{L}\rho,
\]
where \( \mathcal{L} = \mathcal{L}_m + \sum_i \mathcal{L}_i^{(i)} \) and the Lindblad dissipators are
\( \mathcal{L}_m = \gamma \mathcal{D}[a], \mathcal{L}_i^{(i)} = \kappa_i \mathcal{D}[\sigma_i^{(i)}] \), with \( \mathcal{D}[A] \rho = A \rho A^\dagger - \frac{1}{2} \{ A^\dagger A, \rho \} \). Pure dephasing terms \( \frac{1}{2} \gamma_i^{(i)} \mathcal{D}[\sigma_i^{(i)}] \) can also be included. This description of the dynamics is commonly adopted in cavity QED, superconducting qubits, optomechanical systems, and might capture the main features of other types of qubits as well, although their decoherence dynamics in some cases can be more complex (see e.g. [42]).

One fundamental task in quantum state engineering is to transfer an arbitrary state \( |\psi\rangle = c_0 |e\rangle + c_1 |g\rangle \) from qubit 1 to qubit 2. Considering a sufficiently low environmental temperature (to allow at most one excitation), the state transfer protocol is confined in a four-level subspace with basis: \( |1\rangle = |e^{(1)}, 0, g^{(2)}\rangle, |2\rangle = |g^{(1)}, 1, e^{(2)}\rangle, |3\rangle = |g^{(1)}, 0, e^{(2)}\rangle, |4\rangle = |e^{(1)}, 0, g^{(2)}\rangle \). It is clear that in this low-excitation limit, each party can be either a qubit or a bosonic mode, i.e., the discussion below is also applicable to the case of two qubits coupled through a third qubit [29, 36] or a solid-state qubit coupled to an optical cavity through a mechanical oscillator [43, 44] (as illustrated in figure 1(b)). An ideal transfer corresponds to \( c_0 |4\rangle + c_1 |1\rangle \rightarrow c_{\text{y}} |4\rangle + c_3 |3\rangle \)

At low temperature, state \( |4\rangle \) is stable and all the loss of fidelity comes from the transfer of the excited state \( |1\rangle \). Hence in the following we focus on \( c_{\text{y}} = 0 \), which is the most demanding case. Such a process is indicated by the green arrow in figure 1(c).

If the quantum bus has sufficiently good coherence properties, high-fidelity state transfer can be performed with Rabi oscillations via the bus. Instead, we will focus here on another common situation of a quantum bus suffering significant dissipation. In such case, a STIRAP protocol is usually a better choice (we will make a more detailed comparison later). In addition, STIRAP has the advantage of being robust to errors of the pulse timing.

The idea of STIRAP is to adiabatically tune \( G_1(t) \) from zero to a finite value and simultaneously tune \( G_2(t) \) from a finite value to zero, such that the system evolves from \( |1\rangle \) to \( |3\rangle \). In the whole process, the system adiabatically follows the instantaneous eigenstate (dark state):

\[
|\tilde{3}(t)\rangle = -\cos \theta(t)|1\rangle + \sin \theta(t)|3\rangle,
\]
with \( \theta(t) = G_1(t)/G_2(t) \) and

\[
\theta(0) = 0, \quad \theta(t) = \pi/2,
\]
where \( t_e \) is the final operation time and determines the overall speed of the transfer. The other instantaneous eigenstates \( \{|k(t)\rangle\} \) are related to the time-independent basis \( \{|k\rangle\} \) as follows:

\[
|\tilde{1}(t)\rangle = \frac{1}{\sqrt{2}} (\sin \theta(t)|1\rangle + |2\rangle + \cos \theta(t)|3\rangle),
\]

\[
|\tilde{3}(t)\rangle = \frac{1}{\sqrt{2}} (\sin \theta(t)|1\rangle + |2\rangle - \cos \theta(t)|3\rangle),
\]
while \( |\tilde{4}(t)\rangle = |4\rangle \). In this instantaneous basis, the density matrix \( \tilde{\rho} = U^\dagger(t)\rho U(t) \) (where \( U(t) = \sum_k |k(t)\rangle \langle k| \)) satisfies:

\[
\frac{\text{d}\tilde{\rho}(t)}{\text{d}t} = -i[H(t), \tilde{\rho}(t)] + \frac{\text{d}}{\text{d}t}\{[\mu, \tilde{\rho}(t)] + \bar{\mathcal{L}}\tilde{\rho}(t)\},
\]
where \( H(t) = G(t)|1\rangle \langle 1| - |3\rangle \langle 3| \), \( \mu = |2\rangle \langle 2| + |3\rangle \langle 2| \) - h.c., and \( G(t) = \sqrt{G_1^2(t) + G_2^2(t)} \). The transformed dissipator is defined by \( \bar{\mathcal{L}}\tilde{\rho} = U^\dagger \mathcal{L}(U\tilde{\rho}U^\dagger) U \) and its matrix form can be found in a straightforward way. The last two terms in equation (6) corrupt the desired transfer process: the first one represents the non-adiabatic leakage out of \( |\tilde{3}(t)\rangle \), which dominates for a fast-changing pulse; while the second term, i.e., the environmental dissipation, dominates for a slow-changing pulse. How to reach an optimal trade-off between the two effects will be the central issue in the following sections.

### 3. Optimized transfer fidelity: a first estimate

Before moving to a rigorous discussion, we analyze here a simple optimization of the fidelity based on an exact solution of the unitary dynamics, and compare the STIRAP scheme with other types of state-transfer protocols. First we notice that the first two terms on the right side of equation (6) yield an effective Hamiltonian

\[
\hat{H} + \frac{1}{\sqrt{2}} \frac{\text{d}}{\text{d}t} = G(t)I_z + \theta(t)J_z \equiv \tilde{M}(t) \cdot \vec{J},
\]
where we map \( \{|1\rangle, |2\rangle, |3\rangle\} \) to the spin-1 angular momentum states \( \{|+1\rangle, |0\rangle, |-1\rangle\} \), respectively. As seen, the time evolution is equivalent to the rotation of a spin in a time-dependent magnetic field (see the inset of figure 1(d)). For a coupling profile of the simple sin–cos form:

\[
\frac{\text{d}}{\text{d}t} = G(t)I_z + \theta(t)J_z = \tilde{M}(t) \cdot \vec{J},
\]
the magnetic field is constant and tilted from the $z$ direction by an angle $\phi = \arctan[\pi/(2G_0 t_f)]$. Since the initial state is an eigenstate of $I_z$ (i.e., is not aligned to $\vec{M}$), an oscillatory dynamics follows, with angular frequency given by $|\vec{M}| = \sqrt{G_0^2 + (\pi/2t_f)^2}$. The exact solution for the fidelity is:

$$F = \left(\frac{(G_0 t_f)^2 + (\pi/2)^2 \cos^2((G_0 t_f)^2 + (\pi/2)^2)}{(G_0 t_f)^2 + (\pi/2)^2}\right)^{\frac{1}{2}}.$$

Besides recovering $F \to 1$ in the adiabatic limit $t_f \gg 1/G_0$, we also notice that, even for a much shorter operation time $t_f \sim 1/G_0$, a perfect transfer is possible at the discrete values $G_0 t_f = 2\pi\sqrt{n^2 - (1/4)^2}$ ($n = 1, 2, \ldots$) as shown in figure 1(d). Such a condition can be matched by tuning the transfer time $t_f$ or the coupling strength $G_0$. Physically, these times correspond to the periodic return of the state to the $z$ direction.

It is certainly not surprising that an ideal state transfer can be realized in the absence of dissipation. In fact, more efficient and natural ways exist. Among these, we would like to mention the well-known transfer with sequential swaps (SSs), where a constant value of $G_1$ (while $G_2 = 0$) transfers $|1\rangle \to |2\rangle$ and then a second swap leads to $|2\rangle \to |3\rangle$. The total transfer time is $\pi/G_0$ (if both coupling strengths are equal: $G_1 = G_2 = G_0$), which is slightly shorter than the minimum transfer time $2\pi\sqrt{1 - (\pi/4)^2}/G_0 \approx 1.2\pi/G_0$ implied by equation (9).

Another way which is operationally simplest is to set both couplings to a constant $G_0$ and let the system evolve for a time $\pi/\sqrt{2}G_0$, which in the spin-1 language corresponds to a $\pi$-rotation around the $x$ axis. These strategies can be the optimal scheme in a specific parameter regime (see e.g. [25]).

Besides showing that a counter-intuitive coupling profile still allows for a perfect state transfer deep in the non-adiabatic regime (i.e., with $t_f \sim 1/G_0$), the exact solution of the unitary dynamics can serve as a useful reference for our discussion of the general case, including dissipation and more complicated choice for $\theta(t)$. In fact, we can use equation (8) to estimate the fidelity loss due to the decay of the intermediate system: Without dissipation, the population of the excited quantum bus due to the unitary time evolution is:

$$\langle 2|\rho^t(t)|2 \rangle = \left[\frac{\pi G_0 t_f}{2} - \frac{\cos(t\sqrt{G_0^2 + (\pi/2)^2})}{(G_0 t_f)^2 + (\pi/2)^2}\right]^2.$$

The population loss through the bus is approximately:

$$\gamma \int_0^{t_f} \langle 2|\rho^t(t)|2 \rangle dt \approx \frac{3\pi^2\gamma}{8G_0^2 t_f}.$$

As discussed later, this result is in agreement with the systematic perturbative treatment of the master equation. Combining this result with an estimate of the qubits damping $\kappa t_f$, one can optimize the transfer time and find the fidelity:

$$F_{\text{max}} \approx 1 - 1.22 \frac{2\pi}{\sqrt{C}},$$

where $C = 4G_0^2/\gamma\kappa$ is the system cooperativity. Thus, this simple example allows one to understand the relevant physics and typical time scales characterizing the state transfer process. Further optimization of $\theta(t)$ leads to a modest improvement of the fidelity as detailed in the following sections.

### 4. Method

A successful transfer based on STIRAP requires that the adiabatic dynamics plays a dominant role. Hence we can calculate the effect of the non-adiabatic leakage and dissipation in equation (6) with a perturbative method. To do this, the density matrix is expanded as $\hat{\rho}(t) = \hat{\rho}^{(0)}(t) + \hat{\rho}^{(1)}(t) + \hat{\rho}^{(2)}(t) + \cdots$, which yields a corresponding expansion for the transfer fidelity $F = \sum_{k=0}^{\infty} F^{(k)} = \sum_{k=0}^{\infty} \rho^{(k)}(t_f)$ (using the boundary condition equation (3)). It is straightforward to see that the lowest-order satisfies:

$$\frac{d}{dt}\hat{\rho}^{(0)}(t) = -i[\hat{H}(t), \hat{\rho}^{(0)}(t)].$$

The solution is simply $\hat{\rho}^{(0)}(t) = e^{-\hat{H}t}$, if qubit 1 is initially in the excited state. The zero order yields the ideal transfer fidelity $F^{(0)} = 1$. The higher-order contributions satisfy:

$^6$ After completing this work, we noticed that the same unitary dynamics is discussed in [36].
\[
\frac{d\hat{p}^{(k)}(t)}{dt} = -i [\hat{H}(t), \hat{p}^{(k)}(t)] + \frac{\hat{\theta}(t)}{\sqrt{2}} [\mu, \hat{p}^{(k-1)}(t)] + \hat{\mathcal{L}} \hat{p}^{(k-1)}(t),
\]
which can be solved iteratively. Integrating equation (14) gives:
\[
\hat{p}^{(k)}_{ab}(t) = \int_0^t d\tau e^{-i \int_0^\tau \Delta E_{ab}(\tau')d\tau'} \left( \frac{\hat{\theta}(t)}{\sqrt{2}} \hat{p}^{(k-1)}_{ab}(\tau) + L^{(k-1)}_{ab}(\tau) \right),
\]
where \(\varepsilon^{(k-1)}(\tau) \equiv [\mu, \hat{p}^{(k-1)}(\tau)], L^{(k-1)}_{ab}(\tau) \equiv (\hat{\mathcal{L}} \hat{p}^{(k-1)}(\tau))_{ab},\) and \(\Delta E_{ab}(t) = E_a(t) - E_b(t),\) with \(E_a(t) = G(t)(\delta_{a1} - \delta_{a3}).\)

Equation (15) yields the first order correction:
\[
F^{(1)} = -\int_0^{t_f} (\kappa_1 \cos^2 \theta(t') + \kappa_2 \sin^2 \theta(t') + \xi_\varphi \sin^2 2\theta(t')) dt',
\]
where \(\xi_\varphi = (\gamma_\varphi^{(1)} + \gamma_\varphi^{(2)})/2.\) The integrand has a rather transparent physical meaning: it simply reflects the decay and dephasing of the time-dependent dark state \([\hat{2}(t)].\) The second order correction can be also found as:
\[
F^{(2)} \simeq -2 \int_0^{t_f} dt' \theta(t') \int_0^{t'} dt'' \cos \left[ \int_{t''}^{t'} d\tau G(\tau) \right],
\]
which corresponds to the corruption of fidelity due to purely non-adiabatic leakage (i.e., the leakage out of the dark state \([\hat{2}(t)].\)). Here we have neglected the second-order contributions involving \(\kappa_1, \gamma_\varphi^{(1)},\) due to the fact that qubit dissipation already enters the first-order result and higher order terms involving \(\kappa_0, \gamma_\varphi^{(2)}\) should be much smaller. Finally, at third order we obtain:
\[
F^{(3)} \simeq -\gamma \int_0^{t_f} d\theta(t_1) \int_0^{t_1} dt_2 \sin \left( \int_{t_2}^{t_1} d\tau G(\tau) \right) \int_0^{t_2} dt_3 \theta(t_3) \sin \left( \int_{t_3}^{t_2} d\tau G(\tau) \right),
\]
which is the lowest-order correction from the damping of the bus \(\gamma.\) As shown by the comparison of analytical and numerical results in the next section, these first three corrections provide a sufficiently accurate description of the system dynamics in the high-fidelity regime.

The above formulas for \(F^{(2)}\) and \(F^{(3)}\) can be simplified in the relevant case of a sufficiently large \(G(\tau).\) In fact, they are given by integrals of the form \(\int_0^{t_f} dt' f(t') \exp \left[ \pm i \int_0^{t_f} d\tau G(\tau) \right],\) where \(f(t')\) is a relatively smooth function while \(\exp \left[ \pm i \int_0^{t_f} d\tau G(\tau) \right]\) is a fast oscillating factor. By performing multiple integrations by parts, a systematic expansion of such integrals in powers of \(G(\tau)^{-1}\) can be derived, yielding the non-adiabatic correction:
\[
F^{(2)} \simeq \frac{\theta(0)^2}{G(0)^2} - \frac{\theta(t_1)^2}{G(t_1)^2} + \frac{2\theta(0)\theta(t_1)}{G(0)G(t_1)} \cos \int_0^{t_f} d\tau G(\tau),
\]
which naturally vanishes when the system approaches the deep adiabatic limit \(\theta(t) \ll G(t).\) We also note that the last term of equation (19) has an oscillating dependence with respect to \(t_1,\) which is due to the spin-1 precession mentioned in section 3. Similar features appear in the simplified form of the third-order contribution:
\[
F^{(3)} \simeq -\gamma \int_0^{t_f} d\tau \frac{\theta(\tau)^2}{G(\tau)^2} - \gamma t_1 \frac{\theta(0)\theta(t_1)}{2G(0)G(t)} \cos \int_0^{t_f} d\tau G(\tau),
\]
since the quantum bus can only be populated through the second-order non-adiabatic leakage (see figure 1(c)). Therefore, this dissipation effect can also be suppressed by a long operation time and, not surprisingly, shows the same type of oscillating behavior of the non-adiabatic correction.

5. Optimizing the STIRAP operation

5.1. Parallel adiabatic passage (PAP)

Based on the perturbative treatment, we investigate the upper bound of the transfer fidelity and the optimization strategy. We start with coupling profiles of the form \(G_1(t) = G_0 \sin \theta(t)\) and \(G_2(t) = G_0 \cos \theta(t),\) i.e., assuming a constant energy splitting \(G(t) = G_0.\) This type of couplings is known as PAP [45] and is commonly adopted by STIRAP protocols, as it allows one to suppress leakage errors by avoiding anticrossing points (see, for example, [30, 46]). Due to the equal maximum couplings, PAP is a natural choice in the case of identical or similar qubits, which motivates us to take \(\kappa_1 = \kappa_2 = \kappa.\) The extension to the asymmetric case and general coupling profiles will be discussed later.

For PAP with negligible dephasing, \(F^{(1)} = -\kappa t_1\) so the dependence on \(\theta(t)\) vanishes and the fidelity is only determined by \(\dot{\theta}(t),\) which we decompose into a Fourier series:
$\theta(t) = \alpha_0 + \sum_{n>0} \alpha_n \cos(n\pi t/t_1)$. \hspace{1cm} (21)

Here $\alpha_0 = \pi/(2t_1)$ is fixed by the boundary condition equation (3) and $\alpha_n (n > 0)$ represent a set of optimization parameters. At a certain transfer time $t_1$, optimizing $\theta(t)$ requires $\partial F/\partial \alpha_n = 0$, which can be easily solved since the fidelity is a quadratic form of the $\alpha_n$. The solution is:

$$\alpha_n^{\text{opt}} = \begin{cases} \frac{-\pi/2t_1 - \gamma t_1}{4 + (\gamma t_1 - 4) \cos G_0 t_1} & \text{for } n \text{ even}, \\ 0 & \text{for } n \text{ odd}, \end{cases}$$

i.e. to obtain the optimal fidelity, the coefficients of the even terms are all equal; instead the odd-\textit{n} Fourier components vanish, which is a consequence of the symmetric setup ($\kappa_1 = \kappa_2$). In equation (22), $N_{\text{even}}$ is the total number of even-$n$ ($n > 0$) Fourier terms. Some examples of the resulting form of $\theta(t)$ are shown in figure 2(a).

The corresponding fidelity is obtained:

$$F \approx 1 - \kappa t_1 = \frac{\pi^2 \gamma}{8G_0^2 t_1 N_{\text{even}}} \left(2N_{\text{even}} + 1 - \frac{\gamma t_f}{4N_{\text{even}} + \gamma t_f + N_{\text{even}}(\gamma t_f - 4) \cos G_0 t_f} \right),$$

which is an increasing function of $N_{\text{even}}$, as including more optimization parameters improves the fidelity. In the limit $N_{\text{even}} \to \infty$, we get $\alpha_n^{\text{opt}} \approx -\pi/(2t_1 N_{\text{even}})$ and the optimized fidelity over coupling profiles is $F^{\text{opt}}(t_1) = 1 - \frac{\gamma^2 t_f}{4(2G_0^2 t_1)^2}$, which shows the competition between qubit decay and non-adiabatic transition with respect to the transfer time. If we further optimize over $t_1$, the maximum transfer fidelity is:

$$F_{\text{max}} = 1 - \frac{2\pi}{\sqrt{C}},$$

with the corresponding optimal transfer time $t_1^{\text{opt}} = \pi/(\kappa \sqrt{C})$. This expression provides the largest attainable fidelity when an arbitrary state is transferred with a STIRAP-like pulse in a symmetric configuration and for a given set of parameters $G_0, \kappa$ and $\gamma$ (which should be within the regime of validity of the perturbative approach). Defining the loss of fidelity $\Delta F = 1 - F$, we show in figure 2(b) that equation (24) is in excellent agreement with the numerically optimized fidelity, even with $\gamma \sim G_0$.

Equation (24) shows that the ultimate power of STIRAP is limited by the cooperativity. This however does not imply that high fidelity for the STIRAP transfer can be achieved even in the weak coupling regime, as the derivation of equation (24) assumes strong coupling ($\kappa \ll G$). On the other hand, the transfer fidelity of two sequential swaps, i.e., a first swap $|1\rangle \to |2\rangle$ followed by $|2\rangle \to |3\rangle$, is $F \sim 1 - (2\kappa + \gamma)/(2\pi G_0)$. This shows...
that a STIRAP transfer outperforms the SSs transfer when $\gamma > \kappa$, i.e., when the quantum bus suffers strong dissipation.

The maximum fidelity for a finite number of optimization parameters can also be found using equation (23). Assuming the condition $\gamma t_f < 4$, the highest fidelity is obtained at $\cos G_0 t_f = 1$ which reads:

$$F_{\text{max}}(N_{\text{even}}) = 1 - 2\pi \xi(N_{\text{even}})/C$$

(25)

with $\xi(N_{\text{even}}) \equiv (N_{\text{even}} + 3/2)/(N_{\text{even}} + 1)$. Thus, for a given setup, the goal of the fidelity (i.e., the acceptable deviation form the upper bound) determines $N_{\text{even}}$ which subsequently determines the coupling profiles and operation time. Systematic improvements of the pulse shape can be gained by progressively increasing the number $N_{\text{even}}$ of optimization parameters. However, one should also note that the gains in fidelity are progressively smaller when approaching the upper bound and eventually become insignificant, compared to the higher-order terms we have neglected in our perturbative treatment.

In the light of the last remark, it is worth discussing some subtle points which are reflected by the singular shape of the fully optimized pulse when $N_{\text{even}} \to \infty$. In this limit, equation (22) gives a $\hat{\theta}(t)$ which is zero at the boundaries, $\hat{\theta}(0) = \hat{\theta}(t_f) = 0$, but has the constant value $\pi/(2t_f)$ at intermediate times (see also figure 2(a)). This unphysical dependence is a consequence of neglecting higher-order terms which, as noted already, have a more important effect than extending the Fourier series to very high orders. Strictly speaking, we should introduce an effective cutoff $N_{\text{max}}$ (whose value depends on system parameters), such that we are only allowed to consider $N_{\text{even}} < N_{\text{max}}$. If one wants to further optimize the pulse shape and extend the Fourier series above $N_{\text{max}}$, it would be necessary to consider the high-order terms we have neglected so far. Note, however, that this more detailed treatment would only affect small time intervals around $t = 0$, $t_f$. Furthermore, the implicit cutoff is $N_{\text{max}} \gg 1$ in the considered parameter regime, justifying our derivation of equation (24): indeed, the accuracy of our upper bound is confirmed by the numerical calculations.

Finally, it is interesting to notice that even taking $N_{\text{even}} = 0$, i.e., adopting the simple dependence $\theta(t) = \pi t/(2t_f)$, the transfer fidelity is $F_{\text{max}}(N_{\text{even}} = 0) \approx 1 - 1.22(2\pi/C)$ at $t_f^\text{opt}(0) \approx 1.22\pi/(\kappa\sqrt{C})$, which is just slightly smaller than $F_{\text{max}}$. This observation suggests that, for a STIRAP-based state transfer under dissipation, the improvement of maximum fidelity by introducing a complicated pulse is not significant if the operation time can be precisely controlled. This is demonstrated in figure 2(c), where moderate values of $N_{\text{even}}$ are sufficient to achieve a small deviation from the upper bound. However, introducing more optimized harmonics leads to the beneficial effect of quickly smoothing out the strong oscillatory dependence obtained for $N_{\text{even}} = 0$. This is exhibited in figure 2(c) and can be seen from equation (23), where the first term in the large parenthesis dominates over the third (oscillating) term as $N_{\text{even}}$ increases. Due to the smoother dependence, the transfer operation should be robust to the error in pulse timing.

### 5.2. Generic coupling profiles beyond PAP

We now go beyond PAP and consider generic coupling profiles. Noticing that (see equations (19) and (20)) the key variable in $F^{(2)}$ and $F^{(3)}$ is $\hat{\theta}(t)/G(t)$, we can decompose it in Fourier series:

$$\hat{\theta}(t)/G(t) = \beta_0 + \sum_{n>0} \beta_n \cos(n\pi t/t_f).$$

(26)

Thus the optimization resembles the PAP case, as $F$ is a quadratic form of $\beta_n$. Following the same procedure as in the previous section, we find the stationary point with respect to the $\beta_{n>0}$ (i.e., for a given $\beta_0$):

$$\beta_{\text{even}}^\text{opt} = -\beta_0 \frac{4 + (\gamma t_f - 4)\cos \int_0^{t_f} G(\tau) d\tau}{4N_{\text{even}} + \gamma t_f + N_{\text{even}}(\gamma t_f - 4)\cos \int_0^{t_f} G(\tau) d\tau}. \quad (27)$$

And $\beta_{\text{odd}}^\text{opt} = 0$. The fidelity optimized over the coupling profiles reads:

$$F_{\text{opt}}(\beta_0, t_f) \leq 1 - \kappa t_f - \beta_0 \frac{\gamma t_f^2}{2N_{\text{even}}} \left(2N_{\text{even}} + 1 - \frac{\gamma t_f}{4N_{\text{even}} + \gamma t_f + N_{\text{even}}(\gamma t_f - 4)\cos \int_0^{t_f} G(\tau) d\tau}\right). \quad (28)$$

At this point, we notice two differences with respect to the PAP case. The first one is that not only $\hat{\theta}(t)/G(t)$, but also $\cos \int_0^{t_f} G(\tau) d\tau$ is affected by the coupling profiles (while in the PAP case this gives a simple constant, $\cos G_0 t_f$). In deriving equations (28) we have assumed that $\cos \int_0^{t_f} G(\tau) d\tau$ has a value independent of the $\beta_n$, which can be justified from the fact that $\hat{\theta}(t)$ and $G(t)$ are independent functions. However, it also turns out that the specific value of $\cos \int_0^{t_f} G(\tau) d\tau$ is not important for the upper bound. Since $F_{\text{opt}}(t_f)$ is (as expected) a monotonic function of $N_{\text{even}}$, the largest value is obtained by taking the limit $N_{\text{even}} \to \infty$, when the factor $\cos \int_0^{t_f} G(\tau) d\tau$ drops out of the final expressions of $\beta_\text{even}$ (since $\beta_{\text{even}}^\text{opt} \simeq -\beta_0/N_{\text{even}}$). The fidelity reads
\[ F_{\text{opt}}(\beta_0, t_f) \leq 1 - \kappa t_f - \beta_0^2 \gamma t_f. \]  

(29)

The second difference from PAP is that, while \( \alpha_0 = \pi/(2t_f) \) is fixed by the boundary condition, integration of equation (26) simply gives

\[ \beta_0 = \frac{1}{t_f} \int_0^{t_f} \sin(\beta_0) \frac{d\tau}{G(\tau)} \]  

(30)

and does not impose any obvious restriction on \( \beta_0 \), which remains as an additional optimization parameter. In light of equation (29), we would like to set \( \beta_0 \) as small as possible, thus we derive the lower bound imposed by external constraints on the magnitude of the couplings. In particular, we assume that \( G_i (i = 1, 2) \) cannot exceed a maximum value \( G_{i,\text{max}} \). Then, the maximum coupling at a given value of \( \theta = \arctan(G_1/G_2) \) is given by:

\[ G_{\text{max}}(\theta) = \begin{cases} G_{2,\text{max}} / \cos \theta & \text{if } 0 \leq \theta < \bar{\theta}, \\ G_{1,\text{max}} / \sin \theta & \text{if } \bar{\theta} < \theta \leq \pi/2, \end{cases} \]  

(31)

where \( \bar{\theta} \) is the angle with both couplings maximized (i.e., \( \bar{\theta} = \arctan(G_{1,\text{max}}/G_{2,\text{max}}) \)).

To proceed further, we notice that equation (27) implies \( \bar{\theta}(t) \geq 0 \) (this point is discussed in more detail in the appendix). Therefore, we can minimize \( \beta_0 \) by substituting \( G_{\text{max}}(\theta) \) in the denominator of equation (30). Furthermore, \( \bar{\theta}(t) \geq 0 \) implies there is a unique solution \( t = \bar{t} \) which satisfies \( \theta(t) = \bar{\theta} \). Hence equations (30) and (31) lead to

\[ \beta_0 \geq \frac{1}{t_f} \int_0^{\bar{t}} \cos(\beta_0) \frac{\sin(\beta_0)}{G_{2,\text{max}}} d\tau + \frac{1}{t_f} \int_{\bar{t}}^{t_f} \sin(\beta_0) \frac{\sin(\beta_0)}{G_{1,\text{max}}} d\tau. \]  

(32)

Using the boundary conditions \( \theta(0) = 0, \theta(t_f) = \pi/2 \), as well as elementary trigonometric relations to express \( \sin \bar{\theta}, \cos \bar{\theta} \) in terms of the \( G_{i,\text{max}} \), we get:

\[ \beta_0 \geq \frac{\sin \bar{\theta}}{t_f G_{2,\text{max}}} + \frac{\cos \bar{\theta}}{t_f G_{1,\text{max}}} = \frac{1}{t_f} \sqrt{\frac{G_2}{G_{1,\text{max}}} + \frac{G_1}{G_{2,\text{max}}}}, \]  

(33)

which, substituted in equation (29), yields the desired upper bound at fixed \( t_f \):

\[ F_{\text{opt}}(t_f) \leq 1 - \kappa t_f - \frac{2}{t_f} \left( \frac{1}{G_{1,\text{max}}^2} + \frac{1}{G_{2,\text{max}}^2} \right). \]  

(34)

An alternative derivation of equation (34) is given in the appendix. Finally, the maximum fidelity is obtained by further optimizing over \( t_f \):

\[ F_{\text{max}} = 1 - \frac{4}{\sqrt{\kappa_1} + \sqrt{\kappa_2}}, \]  

(35)

where we indicate with \( C_i = 4G_i^2 / \gamma \kappa_i \) the cooperativity of side \( i = 1, 2 \). In the limit \( C_1 = C_2 = C \), this result allows for a slightly larger fidelity than equation (24) \( (4\sqrt{2} / (2\pi) \approx 0.9) \). This can be attributed to the fact that here \( G(t) \) is an additional optimization parameter. Equation (35) also shows that in the limit when \( C_1 \) and \( C_2 \) are very different, the less coherent system (with lower \( C_i \)) dominates the fidelity loss. In figure 3(a) we compare our analytic expression to the numerical optimization, and show that equation (35) is indeed an accurate characterization of the maximum fidelity.

5.3. Optimization for unequal dissipation rate

Finally, we consider the case of a large difference in the qubit coherence (e.g., \( \kappa_1 \gg \kappa_2 \)). Since \( F^{(1)} \) depends on \( \theta(t) \) (see equation (16)), the previous analytic approach is not easily applicable when \( \kappa_1 \approx \kappa_2 \). Instead, a rigorous treatment is possible by applying the method of functional derivatives to our general formulas for the fidelity. This approach is currently under investigation, however it may not lead to a simple analytical form of the upper bound. Instead, to understand the general parametric dependence, we estimate the second term in equation (34) with \( F^{(1)} \sim - (\kappa_1 + \kappa_2) t_f / 2 \), which indicates that the relevant figure of merit is the smallest cooperativity

\[ C_{\text{min}} = \sqrt{\kappa_{\text{max}}} G_{2,\text{max}}^2, \]  

where \( \kappa_{\text{max}} = \max(\kappa_1, \kappa_2) \) and \( G_{\text{min}} = \min(G_{1,\text{max}}, G_{2,\text{max}}) \).

Note that large deviations from \( F^{(1)} \sim - (\kappa_1 + \kappa_2) t_f / 2 \) are possible if \( \theta(t) \) spends a significant fraction of the transfer time close to the initial or final angles \( \theta = 0, \pi/2 \). However, this situation is essentially equivalent to a shorter \( t_f \), thus we find that the final conclusion is generally in agreement with the numerics. For example, figure 3(b) shows that when \( G_{1,\text{max}} = G_{2,\text{max}} \) the maximum fidelity is well approximated by:
which is in agreement with equation (35) when $C_1 = C_2$. When $C_1$ is very different from $C_2$, equations (35) and (36) differ in the numerical prefactors but confirm the general argument that $\Delta F \sim 1/\sqrt{C_{\min}}$, for an optimized transfer time $t_{\text{opt}} \sim 1/(\kappa_{\max}\sqrt{C_{\min}})$.  

6. Conclusion and remarks

We have analyzed STIRAP-based state transfer under dissipation, which is a particularly relevant process in the quantum state engineering based on solid state systems. The trade-off between the non-adiabatic transitions and dissipation needs careful optimization of the time-dependent couplings and, instead of numerical optimization, we have developed an analytical treatment of the general parameter dependence. This approach has allowed us to reach a physical understanding of the optimal transfer time and derive the upper bound of the fidelity. We also show how to reach this bound efficiently. Our results provide a useful guideline for implementing such protocols in a variety of physical systems, as well as generalizations to alternative setups and more complicated scenarios, such as the composite adiabatic passage [6] and in the presence of detuning or non-Markovian noise.

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Appendix. Alternative derivation of the fidelity upper bound

We give here a slightly different and possibly more transparent derivation of equation (34). To this end, we consider in more detail the properties of the optimized $\dot{\theta}(t)/G(t)$ which, in the limit $N_{\text{even}} \to \infty$, assumes the following form:

$$\dot{\theta}(t)/G(t) \simeq \beta_0 \left( 1 - \frac{\cos(2\pi/t_f)}{N_{\text{even}}} \right) \equiv \beta_0 f(t).$$  

Note that $f(t)$ is positive, a property we used in equation (32). More interestingly, it is immediate to check that $f(0) = f(t_f) = 0$, showing that $F_{\text{opt}}^{(2)}$ vanishes for the optimal coupling profile (see equation (19)). Indeed, it is reasonable that the optimal time dependence satisfies this property, because $F_{\text{opt}}^{(2)}$ is the leading correction due to non-adiabaticity. This observation motivates us to optimize the fidelity under the assumption $\dot{\theta}(0) = \dot{\theta}(t_f) = 0$, when the general formula for $F$ simplifies to:
\[ F[\theta_0]=0 = 1 - \kappa t - \gamma \int_0^{t_f} dt \frac{\dot{\theta}(t)^2}{G(t)^2}. \]  \hfill (A.2)

We now directly minimize equation (A.2), without using the Fourier decomposition introduced in the main text. Clearly, we have:

\[ \int_0^{t_f} dt \frac{\dot{\theta}(t)^2}{G(t)^2} \geq \int_0^{t_f} dt \frac{\dot{\theta}(t)^2}{G_{\text{max}}(\dot{\theta}(t))^2}, \]  \hfill (A.3)

which, using equation (31) is written:

\[ \frac{1}{G_{\text{max}}^2} \int_0^{t_f} \left( \frac{d\sin \theta(t)}{dt} \right)^2 dt + \frac{1}{G_{\text{max}}^2} \int_0^{t_f} \left( \frac{d\cos \theta(t)}{dt} \right)^2 dt. \]  \hfill (A.4)

The two integrals are minimized when their integrands are constant, i.e., we can set \( d \sin \theta(t)/dt = \sin \dot{\theta}/\bar{t} \) and \( d \cos \theta(t)/dt = \cos \dot{\theta}/(\bar{t} - t) \). We conclude that equation (A.4) is larger or equal to:

\[ \frac{1}{\bar{t}} \left( \frac{\sin \dot{\theta}}{G_{\text{max}}} \right)^2 + \frac{1}{\bar{t} - \bar{t}} \left( \frac{\cos \dot{\theta}}{G_{\text{max}}} \right)^2 \geq \frac{1}{\bar{t}} \left( \frac{1}{G_{\text{max}}^2} \right) + \frac{1}{\bar{t} - \bar{t}} \left( \frac{1}{G_{\text{max}}^2} \right), \]  \hfill (A.5)

where in the last step we used the definition of \( \dot{\theta} \) and performed the minimization with respect to \( \bar{t} \), giving \( \bar{t} = t_f (\sin \dot{\theta})^2 \). Equation (A.5) is the desired result, in agreement with equation (34). This derivation also shows explicitly that it is possible to find a suitable time-dependence of \( \theta(t)/G(t) \) approaching the equality sign in equation (34).

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