Coherent analysis of quantum optical sideband modes

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We demonstrate a device that allows for the coherent analysis of a pair of optical frequency sidebands in an arbitrary basis. We show that our device is quantum noise limited and hence applications for this scheme may be found in discrete and continuous variable optical quantum information experiments.

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Quantum information can be encoded on light using two orthogonal optical modes \( \hat{a} \). Polarization is a good example and can be used to encode both discrete quantum bits (qubits) \(^2\) and continuous variable quantum information \(^3\). A key feature of polarization is the ability to analyse in any basis using a series of half and quarter-wave plates (or a rotatable Babinet compensator), a polarizing beamsplitter and intensity measurements. By analysing in a complete set of bases, say linear, diagonal and circular, the amplitudes and relative phase of the polarization modes can be determined. Another example encoding scheme is a pair of optical sideband modes separated from an average or carrier frequency by a radio or microwave frequency. Much like polarisation analysis, full characterisation of this encoding scheme would reveal the amplitude and relative phase of the sidebands. However unless here is equal power in the two frequency modes, homodyne detection of the in-phase and out-of-phase quadratures is insufficient to fully characterize the system.

In this work we propose and demonstrate a device designed to achieve full characterization of optical sideband modes. This "rf-analyser" may be used to analyse the average optical power in each optical sideband mode and the phase relationship between them. The rf-analyser is directly analogous to a polarisation analysis system comprising a rotatable Babinet compensator followed by a polarising beamsplitter \(^3\). We show that our device is quantum noise limited and thus could be used as a new analysis tool for non-classical states of light both in the quantum noise limited and thus could be used as a new analysis tool for non-classical states of light both in the continuous variable and the discrete variable \(^2\) domains.

**Theory:** Fig. 1 schematically illustrates the system. It is essentially an unequal arm-length Mach-Zehnder interferometer (UMZI) followed by an acousto-optic modulator (AOM) with the two UMZI outputs interferometrically combined at the AOM. In the Heisenberg picture this system has two inputs: \( A_{\text{in}} \) and \( v_{\text{in}} \); and two outputs: \( A_{\text{out1}} \) and \( A_{\text{out2}} \). Let us define the average or carrier frequency to be \( \omega_0 \) and the radio or microwave sideband frequencies of interest to be \( \pm \Omega \). The annihilation operators for the outputs of the rf-analyser at the frequencies of interest are

\[
\begin{align*}
A_{\text{out1}}(\Omega) &= \cos \theta A_{\text{in}}(\Omega) - e^{i\phi} \sin \theta A_{\text{in}}(-\Omega) \\
A_{\text{out1}}(-\Omega) &= i \cos \theta v_{\text{in}}(-\Omega) - i e^{i\phi} \sin \theta v_{\text{in}}(3\Omega) \\
A_{\text{out2}}(\Omega) &= -e^{i\phi} \cos \theta A_{\text{in}}(\Omega) - \sin \theta v_{\text{in}}(3\Omega) \\
A_{\text{out2}}(-\Omega) &= i e^{i\phi} \cos \theta A_{\text{in}}(-\Omega) + i \sin \theta A_{\text{in}}(\Omega)
\end{align*}
\]

where the absence of hats indicate operators in the Fourier domain and the modulation frequency applied to the AOM is \( 2\Omega \). The diffraction efficiency of the AOM is \( \sin^2 \theta \) and, assuming 100% maximum diffraction efficiency, \( \theta \) is proportional to the amplitude of the radio-frequency signal driving the AOM \(^7\). The UMZI is designed to have a differential time delay such that \( \omega_0 \tau = \pi/2 \) and \( \pm \Omega \tau = \pm \pi/2 \) where \( \tau = \Delta l/c \) for \( \Delta l \) the differential path length and \( c \) the speed of light in the UMZI. The differential time delay between the two paths from the outputs of the UMZI to the AOM is \( \tau_2 \), such that \( \omega_0 \tau_2 = \phi \) and \( \Omega \tau_2 \approx 0 \). All beamsplitters have power transmission of 50% and the symmetric phase convention has been used \(^8\).

Eqs. 1 are the key theoretical results of this paper. To illustrate with an explicit example, a single photon state \( |\psi\rangle = (\mu |1\rangle - \Omega + \nu \mid 1\rangle + \Omega) \otimes |0\rangle \) would be transformed to

\[ |\psi\rangle' = \left( (i \mu e^{i\phi} \cos \theta + i \nu \sin \theta) |1\rangle - \Omega, A_{\text{out2}} \right) \]
+ (ν cos θ − μe^{iφ} sin θ | 1 \rangle_{+Ω,A_{out1}} \otimes | 0 \rangle). If one or both output ports of the device illustrated in Fig. 1 are monitored, the input state may be analysed as a function of both the diffraction efficiency and the optical phase at the AOM. The two degrees of freedom, θ and φ, allow complete analysis of both the amplitude and relative phase of the projections of the input state onto the two bases.

**Experiment:** The rf-analysers have been constructed and tested experimentally using a commercial Nd:YAG laser operating at 1064 nm. Phase modulation (PM) sidebands were imposed on the laser beam at 90.5 MHz with a commercial electro-optic modulator (EOM). In the absence of modulation, the output of the laser was quantum noise limited in both amplitude and phase quadratures at this frequency. The differential path length in the UMZI was limited in both amplitude and phase quadratures at this frequency. The fringe visibility was 98%. All the results based on homodyne detection are shown. Both experimental results show the desired sinusoidal behaviour with φ. The visibility of the fringe based on spectral measurements is 97% and the visibility of the homodyne fringe is 96%. All the results based on homodyne detection are slightly lower than predicted by theory. This is due to the finite demodulating efficiencies in the experiment to which the homodyne detection system was particularly sensitive. The insets to Fig. 2 show examples of how these measurements were made. In the spectral measurements, the horizontal axis of the inset represents the optical power of the Ω modulation sidebands respectively. Similar notational definitions apply for the ensemble average photon numbers. All input variances except \( V_{\text{in}} \) are set to the QNL.

![FIG. 2: Measurements of \( P_{+Ω,\text{out1}}/P_{Ω,\text{in}} \) and \( n_{+Ω,\text{out1}}/n_{Ω,\text{in}} \) as a function of the optical phase φ. Also shown are theoretical predictions for these measurements. The error bars for the spectral measurements are approximately 0.005 and 12% of the nominal value for the homodyne measurements.](image-url)
amplitude of the rf drive signal to calibrate the horizontal axis. That is, $V_{\text{in}}^- = 4V_{\text{in},\text{det}}^- - 3$ where $V_{\text{in},\text{det}}^-$ is the measured phase quadrature variance.

A true rf-analyser should be able to distinguish between different input states. Two additional modulation states were used as inputs to test this - lower (upper) sideband modulation, LSB (USB) comprising, nearly, a single modulation sideband at $-\Omega$ (+$\Omega$). Both the LSB and USB inputs were derived experimentally by transmitting a PM field through an additional “state preparation” UMZI prior to the rf-analyser. Our state preparation UMZI had $\Omega \tau \approx 1.33$ radians and was locked to $\omega_0 \tau = \pm \pi/2$. An UMZI with this value of $\Omega \tau$ would generate $P_{-\Omega,\text{in}} = 0.985P_{\Omega,\text{in}}$ and $P_{+\Omega,\text{in}} = 0.015P_{\Omega,\text{in}}$ for the LSB input and vice versa for the USB input [8]. The two sidebands would have a relative phase of approximately 1.33 radians.

Fig. 3 shows measurements of $P_{+\Omega,\text{out1}}/P_{\Omega,\text{in}}$ and $n_{+\Omega,\text{out1}}/n_{\Omega,\text{in}}$ for the PM, LSB and USB input states as a function of the drive power to the AOM for a fixed $\phi = 0$. This measurement is analogous to polarisation analysis in the linear basis. The theoretical predictions for these measurements are shown as solid lines. Fig. 3 shows that all three input states may be clearly distinguished on the basis of these measurements. The results in Fig. 3 show good agreement between theory and experiment up to an AOM drive power of approximately 0.25W (i.e. $(0.5\sqrt{W})^2$). Beyond this, the experimental results and theoretical predictions diverge from one another. This is primarily because the AOM had a measured maximum diffraction efficiency of 85% whereas Eq. 4 assumes 100% maximum diffraction efficiency.

**Conclusion:** In summary, we have proposed and demonstrated a device which may be used to perform arbitrary rotations of states in optical sideband modes and to then coherently separate photons from different sideband modes into separate spatial modes. This device is the “frequency basis” analogue of a Babinet compensator followed by a polarising beamsplitter. Such a device may be put to all of the same uses as it’s polarisation analogue in both discrete and continuous variable quantum optical experiments.

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[1] E. Knill and L. Laflamme and G. J. Milburn, Nature 409, 46 (2001).
[2] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White Phys. Rev. A 64, 052312 (2001).
[3] N. Korolkova, G. Leuchs, R. Loudon, T. C. Ralph, and C. Silberhorn Phys. Rev. A 65, 052306 (2002).
[4] W. P. Bowen, N. Trepas, R. Schnabel, and P. K. Lam Phys. Rev. Lett. 89, 253601 (2002).
[5] E. Hecht, *Optics*, 4th Ed, Addison Wesley Publishing, USA (2003).
[6] E. H. Huntington and T. C. Ralph, Phys. Rev. A, 69, 042318 (2004).
[7] E. H. Young, S.-K. Yao, Proc. IEEE, 69, 54 (1981).
[8] D. F. Walls, and G. J. Milburn, *Quantum Optics*, Springer, Berlin (1995).
[9] K.J. Resch, S. H. Myrskog, J. S. Lundeen, and A. M. Steinberg, Phys. Rev. A, 64, 056101 (2001).
[10] H. Kogelnik and T. Li, App. Opt., 5, 1550 (1966).
[11] H. A. Bachor and T. C. Ralph, *A Guide to Experiments in Quantum Optics 2nd Ed.*, Wiley-VCH, Weinheim (2003).
[12] R. J. Glauber, Phys. Rev., 130, 2529 (1963).
[13] T. C. Ralph, W. J. Munro, and R. E. S. Polkinghorne, Phys. Rev. Lett., 85, 2035 (2000).