Supersymmetry and DLCQ

The SDLCQ Collaboration

Abstract

In this talk we describe the application of discrete light cone quantization (DLCQ) to supersymmetric field theories. We find that it is possible to formulate DLCQ so that supersymmetry is exactly preserved in the discrete approximation and call this formulation of DLCQ, SDLCQ. It combines the power of DLCQ with all of the beauty of supersymmetry. We have applied SDLCQ to several interesting supersymmetric theories and discussed zero modes, vacuum degeneracy, massless states, mass gaps, and theories in higher dimensions. Most recently we have used it to discuss the Maldacena conjecture.
1 Introduction

In the last decade there have been significant improvements in our understanding of gauge theories and important breakthroughs in the nonperturbative description of supersymmetric gauge theories [24, 25]. In the last few years various relations between string theory, brane theory and gauge fields [12, 1] have also emerged. While these developments give us some insight into strongly coupled gauge theories [25], they do not offer a direct method for non-perturbative calculations. In this talk we discuss some recent developments in the light cone quantization approach to non-perturbative problems. We have found that these methods have the potential to expand our understanding of strongly coupled gauge theories in directions not previously available.

The original idea of light cone quantization was formulated half of a century ago [11], but apart from several technical clarifications [19] it remained mostly undeveloped. The first change came in the mid 80’s when the Discrete Light Cone Quantization (DLCQ) was suggested as a practical way for calculating the masses and wavefunctions of hadrons [21]. Although the direct application of the method to realistic problems meets some difficulties (for review see [9]), DLCQ has been successful in studying various two dimensional models. Given the importance of supersymmetric theories, it is not surprising that light cone quantization was ultimately applied to such models [16, 6, 7]. In these early works the mass spectrum was shown to be supersymmetric in the continuum and a great deal of information about the properties of bound states in supersymmetric theories was extracted. However the straightforward application of DLCQ to the supersymmetric systems had one disadvantage: the supersymmetry was lost in the discrete formulation. The way to solve this problem was suggested in [20], where the alternative formulation of DLCQ was introduced. Namely it was noted that since the supercharge is the ”square root” of the Hamiltonian one can define a new DLCQ procedure based on the supercharge. We will discuss this formulation (called SDLCQ) in this talk.

2 Supersymmetric Yang–Mills Theory in the Light–Cone Gauge.

We will consider here the bound state problem for various supersymmetric matrix models in two dimensions. These models may be constructed by dimensional reduction of supersymmetric Yang–Mills theory in higher dimensions. Before we begin the a discussion
of bound state problem for a specific systems it is worthwhile to summarize some basic ideas of Discrete Light Cone Quantization, for a complete review see [9].

Let us consider general relativistic systems in two dimensions. In the usual canonical quantization of such systems one imposes certain commutation relations between coordinates and momenta at equal time. However, as was pointed out by Dirac long ago [11], this is not the only possibility. Another scheme of quantization treats the light like coordinate \( x^+ = \frac{1}{\sqrt{2}} (x^0 + x^1) \) as a new time and then the system is quantized canonically. This scheme (called light cone quantization) has both positive and negative aspects. The main disadvantage of light cone quantization is the presence of constraints. Even systems as simple as free bosonic field has one: from the action

\[
S = \int d^2 x \partial_+ \phi \partial_- \phi
\]

one can derive the constraint relating coordinate and momentum:

\[
\pi = \partial_- \phi.
\]

For more complicated systems the constraints are also present and in general they are hard to resolve.

The main advantage of the light cone is the decoupling of positive and negative momentum modes. This property is crucial for DLCQ. In the Discrete Light Cone Quantization one considers the theory on the finite circle along the \( x^- \) axis: \(-L < x^- < L\). Then all the momenta become quantized and the integer number measuring the total momentum in terms of "elementary momentum" is called the harmonic resolution \( K \). Due to the decoupling property one may work only in the sector with positive momenta and as a result there is a finite number of states for any finite value of resolution. Of course the full quantum field theory in the continuum corresponds to the limit \( L \to \infty \), and in this limit the elementary bit of momentum goes to zero, as the harmonic resolution goes to infinity and the infinite number of degrees of freedom are restored. It is believed that the "quantum mechanical" approximation is suitable for describing the lowest states in the spectrum. Note that the problem of constraints in DLCQ is a quantum mechanical one and thus it is easier to solve. Usually this problem can be reformulated in terms of zero modes and the solution can be found for any value of the resolution.

DLCQ is mainly used to solve the bound state problem and we will formulate this problem for a general two dimensional theory. The theory in the continuum has full Poincare symmetry, thus the states are naturally labeled by the eigenvalues of Casimir
operators of the Poincare algebra. One such Casimir is the mass operator: $M^2 = P^\mu P_\mu$. Another Casimir is related to the spin of the particle and is generally not used. After compactifying the $x^-$ direction one looses Lorentz symmetry, but not the translational invariance in $x^+$ and $x^-$ directions. Thus $P^+$ and $P^-$ are still conserved charges, and the states are characterized by both $P^+$ and $P^-$. If we consider DLCQ as an approximation to the continuum theory, we anticipate that in the limit of infinite harmonic resolution (or $L \to \infty$) the full Poincare symmetry is restored. Thus the aim would be to study the value of $M^2$ as function of $K$ and to extrapolate the results to the $K = \infty$.

The usual way to define $M^2$ in DLCQ is based on separate calculation of $P^+$ and $P^-$ in matrix form and then bringing them together:

$$M^2 = 2P^+P^-.$$  \hfill (3)

Usually one works in the sector with fixed $P^+$, but the calculation of light cone Hamiltonian $P^-$ is a nontrivial problem. Important simplifications occur for supersymmetric theories [20].

Supersymmetry is the only nontrivial extension of Poincare algebra compatible with the existence of an S matrix [26]. Namely in addition to usual bosonic generators of symmetries, fermionic ones are allowed and the full (super)algebra in two dimensions reads:

$$\{Q^I_I, Q^J_J\} = 2\delta^{IJ}\gamma^{\mu}_{\alpha\beta}P_\mu + \varepsilon_{\alpha\beta}Z^{IJ},$$  \hfill (4)

$$[P_\mu, P_\nu] = 0, \quad [P_\mu, Q^I_I] = 0.$$  \hfill (5)

In this expression $\varepsilon$ is the antisymmetric $2 \times 2$ matrix, $\varepsilon_{12} = 1$ and $Z^{IJ}$ is the set of c-numbers called central charges. In this talk we will put them equal to zero. It is convenient to choose two dimensional gamma matrices in the form: $\gamma^0 = \sigma^2; \gamma^1 = i\sigma^1$, then one can rewrite (4) in terms of light cone components:

$$\{Q^+_I, Q^+_J\} = 2\sqrt{2}\delta^{IJ}P^+, \quad \{Q^-_I, Q^-_J\} = 2\sqrt{2}\delta^{IJ}P^-,$$

$$\{Q^+_I, Q^-_J\} = 2Z_{IJ}.$$  \hfill (6) \hfill (7) \hfill (8)

As we mentioned before, in DLCQ diagonalization of $P^+$ is trivial and the construction of Hamiltonian is the main problem. The last set of equations suggests an alternative way of dealing with this problem: one can first construct the matrix representation for
the supercharge $Q^-$ and then just square it. This version of DLCQ first suggested in [20] appeared to be very fruitful. First of all it preserves supersymmetry at finite resolution, while the conventional DLCQ applied to supersymmetric theories doesn’t. The supersymmetric version of DLCQ (SDLCQ) also provides the better numerical convergence.

To summarize, we have two procedures for studying the bound state spectrum: DLCQ and SDLCQ. To implement the first one we should construct the light cone Hamiltonian and diagonalize it, while the second approach requires the construction of the supercharge. Of course the SDLCQ method is appropriate only for the theories with supersymmetries, although it can be modified to study models with soft supersymmetry breaking.

2.1 Reduction from Three Dimensions.

Let us start by defining a simple supersymmetric system in two dimensions. It can be constructed by dimensional reduction of SYM from three to two dimensions. Our starting point is the action for SYM in 2 + 1 dimensions:

$$S = \int d^3x \text{tr} \left( -\frac{1}{4} F_{AB} F^{AB} + i \bar{\Psi} \gamma^A D_A \Psi \right).$$  (9)

The system consists of gauge field $A_A$ and two–component Majorana fermion $\Psi$, both transforming according to adjoint representation of gauge group. We assume that this group is either $U(N)$ or $SU(N)$ and thus matrices $A_A^i$ and $\Psi_{ij}$ are hermitian. Studying dimensional reduction of $SYM_D$ we introduce the following conventions for the indices: the capital latin letters correspond to $D$ dimensional spacetime, greek indices label two dimensional coordinates and the lower case letters are used as matrix indices. According to this conventions the indexes in (9) go from zero to two, the field strength $F_{AB}$ and covariant derivative $D_A$ are defined in the usual way:

$$F_{AB} = \partial_A A_B - \partial_B A_A + ig[A_A, A_B],$$
$$D_A \Psi = \partial_A \Psi + ig[A_A, \Psi].$$  (10)

Dimensional reduction to 1 + 1 means that we require all fields to be independent on coordinate $x^2$, in other words we place the system on the cylinder with radius $L_\perp$ along the $x^2$ axis and consider only zero modes of the fields. We consider this reduction as a formal way of getting a two dimensional matrix model. In the reduced theory it is convenient to introduce two dimensional indices and treat the $A^2$ component of gauge
field as a two dimensional scalar $\phi$. The action for the reduced theory has the form:

$$S = \int d^2 x \, \text{tr} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_{\mu} \phi D^{\mu} \phi + i \bar{\Psi} \gamma^\mu D_\mu \Psi - 2ig\phi \bar{\Psi} \gamma_5 \Psi \right), \quad (11)$$

We choose the special representation of three dimensional gamma matrices:

$$\gamma^0 = \sigma^2, \quad \gamma^1 = i\sigma^1, \quad \gamma^2 = i\sigma^3, \quad (12)$$

then it is natural to write the spinor $\Psi$ in terms of its components:

$$\Psi = (\psi, \chi)^T. \quad (13)$$

Taking all these definitions into account one can rewrite the dimensional reduction of (9) as:

$$S = L_\perp \int d^2 x \left( \frac{1}{2} D_{\mu} \phi D^{\mu} \phi + i\sqrt{2} \bar{\psi} D_+ \psi + i\sqrt{2} \bar{\chi} D_- \chi + 2g\psi \{\psi, \chi\} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right). \quad (14)$$

The covariant derivatives here are taken with respect to the light cone coordinates:

$$x^\pm = \frac{x^0 \pm x^1}{\sqrt{2}}. \quad (15)$$

Note that by rescaling the fields and coupling constant $g$ we can take the constant $L_\perp$ to be equal to one.

The bound state problem for the system (14) was first studied in [20]. The supersymmetric version of the discrete light cone quantization was used in order to find the mass spectrum, and the zero modes were neglected [20]. We have found that while zero modes are not very important for calculations of massive spectrum, they play crucial role in the description of the vacuum of the theory.

Let us consider (14) as the theory in the continuum. In this case one can choose the light cone gauge:

$$A^+ = 0, \quad (16)$$

then equations of motion for $A^-$ and $\chi$ give constraints:

$$-\partial_-^2 A^- = gJ^+, \quad (17)$$

$$\sqrt{2}i\partial_- \chi = g[\phi, \psi], \quad (18)$$

$$J^+(x) = \frac{1}{i} [\phi(x), \partial_- \phi(x)] - \frac{1}{\sqrt{2}} \{\psi(x), \psi(x)\}. \quad (19)$$
Solving this constraints and substituting the result back into the action one determines the Lagrangian as function of physical fields $\phi$ and $\psi$ only. Then using the usual Noether technique, we can construct the conserved charges corresponding to the translational invariance:

$$P^+ = \int d^3x^+ \text{tr} \left( (\partial_- \phi)^2 + i \sqrt{2} \psi \partial_- \psi \right),$$  

(20)$$

$$P^- = \int d^3x^- \text{tr} \left( -\frac{g^2}{2} J^+ \frac{1}{\partial_-^2} J^+ + \frac{ig^2}{2\sqrt{2}} [\phi, \psi] \frac{1}{\partial_-} [\phi, \psi] \right).$$  

(21)

We can also construct the Noether charges corresponding to the supersymmetry transformation. However the naive SUSY transformations break the gauge fixing condition $A^+ = 0$, so they should be accompanied by compensating a gauge transformation:

$$\delta A_\mu = \frac{i}{2} \bar{\varepsilon} \gamma_\mu \Psi - D_\mu \frac{i}{2} \bar{\varepsilon} \gamma_- \frac{1}{\partial_-} \Psi,$$  

(22)

$$\delta \Psi = \frac{1}{4} F_{\mu\nu} \gamma^{\mu\nu} \varepsilon - \frac{g}{2} \{\bar{\varepsilon} \gamma_- \frac{1}{\partial_-} \Psi, \Psi\}.$$

The resulting supercharges are:

$$Q^+ = 2 \int d^3x^+ \text{tr} (\psi \partial_- \phi),$$  

(23)

$$Q^- = -2g \int d^3x^- \text{tr} \left( J^+ \frac{1}{\partial_-} \psi \right).$$  

(24)

Consider now the general reduction of SYM$_D$ to two dimensions. By counting the fermionic and bosonic degrees of freedom one can see that the SYM can be defined only in a limited number of spacetime dimensions, namely $D$ can be equal to 2, 3, 4, 6 or 10. The last case is the most general one: all other system can be obtained by dimensional reduction and appropriate truncation of degrees of freedom. So we will concentrate on the reduction $10 \rightarrow 2$, and the comments on four and six dimensional cases will be made in the end.

As in the last subsection we start from ten dimensional action:

$$S = \int d^3x^\mu \text{tr} \left( -\frac{1}{4} F_{AB} F^{AB} + i \bar{\Psi} \gamma^A D_A \Psi \right).$$  

(25)

According to our general conventions the indexes in (25) go from zero to nine, $\Psi$ is the ten dimensional Majorana–Weyl spinor. A general spinor in ten dimensions has $2^{10/2} = 32$ complex components, if the appropriate basis of gamma matrices is chosen.
then the Majorana condition makes all the components real. Since all the matrices in such representation are real, the Weyl condition
\[ \Gamma_{11} \Psi = \Psi \] (26)
is compatible with the reality of \( \Psi \) and thus it eliminates half of its components. In the special representation of Dirac matrices:
\[ \Gamma^0 = \sigma_2 \otimes 1_{16}, \] (27)
\[ \Gamma^I = i\sigma_1 \otimes \gamma^I, \quad I = 1, \ldots, 8; \] (28)
\[ \Gamma^9 = i\sigma_1 \otimes \gamma^9, \] (29)
the \( \Gamma_{11} = \Gamma^0 \cdots \Gamma^9 \) has very simple form: \( \Gamma_{11} = \sigma_3 \otimes 1_{16} \). Then the Majorana spinor of positive chirality can be written in terms of 16–component real object \( \psi \):
\[ \Psi = 2^{1/2} \begin{pmatrix} \psi^0 \\ \psi \end{pmatrix}. \] (30)

Let us return to the expressions for \( \Gamma \) matrices. The ten dimensional Dirac algebra
\[ \{ \Gamma_\mu, \Gamma_\nu \} = 2g_{\mu\nu} \]
is equivalent to the spin(8) algebra for \( \gamma \) matrices: \( \{ \gamma_I, \gamma_J \} = 2\delta_{IJ} \) and the ninth matrix can be chosen to be \( \gamma^9 = \gamma^1 \ldots \gamma^8 \). Note that the 16 dimensional representation of spin(8) is the reducible one: it can be decomposed as \( 8_s + 8_c \)
\[ \gamma^I = \begin{pmatrix} 0 & \beta_I \\ \beta^T_I & 0 \end{pmatrix}, \quad I = 1, \ldots, 8. \] (31)
The explicit expressions for the \( \beta_I \) satisfying \( \{ \beta_I, \beta_J \} = 2\delta_{IJ} \) can be found in [?]. Such choice leads to the convenient form of \( \gamma^9 \):
\[ \gamma^9 = \begin{pmatrix} 1_8 & 0 \\ 0 & -1_8 \end{pmatrix}. \] (32)

So far we have found nonzero components of the spinor given by (30). However as we saw before not all such components are physical in the light cone gauge, so it is useful to perform the analog of decomposition (13). In ten dimension it is related with breaking the sixteen component spinor \( \psi \) on the left and right–moving components using the projection operators
\[ P_L = \frac{1}{2}(1 - \gamma^9), \quad P_R = \frac{1}{2}(1 + \gamma^9). \] (33)
After introducing the light–cone coordinates \( x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^9) \) the action (25) can be rewritten as
\[
S_{9+1}^{LC} = \int dx^+ dx^- d\mathbf{x} \text{tr} \left( \frac{1}{2} F_+^2 + F_+ F_- - \frac{1}{4} F_{IJ}^2 \\
+ i\sqrt{2} \psi_R^T D_+ \psi_R + i\sqrt{2} \psi_L^T D_- \psi_L + 2i\psi_L^T \gamma^I D_I \psi_R \right),
\]
where the repeated indices \( I, J \) are summed over \( 1, \ldots, 8 \). After applying the light–cone gauge \( A^+ = 0 \) one can eliminate nonphysical degrees of freedom using the Euler–Lagrange equations for \( \psi_L \) and \( A^- \):
\[
\partial_- \psi_L = -\frac{1}{\sqrt{2}} \gamma^I D_I \psi_R,
\]
\[
\partial_2 A_+ = \partial_- \partial_I A_I + gJ^+ \tag{36}
\]
\[
J^+ = i[A_I, \partial_- A_I] + 2\sqrt{2} \psi_R^T \psi_R. \tag{37}
\]

Performing the reduction to two dimensions means that all fields are assumed to be independent on the transverse coordinates: \( \partial_\perp \Phi = 0 \). Then as before one can construct the conserved momenta \( P^\pm \) in terms of physical degrees of freedom:
\[
P^+ = \int dx^- \text{tr} \left( (\partial_- A_I)^2 + i\sqrt{2} \psi_R \partial_- \psi_R \right), \tag{38}
\]
\[
P^- = \int dx^- \text{tr} \left( -\frac{g^2}{2} J^+ \frac{1}{\partial^2} J^+ + \frac{ig^2}{2\sqrt{2}} [A_I, \psi_R^T] \beta_I \frac{1}{\partial_-} \beta_J [A_J, \psi_R] \right) \\
-\frac{1}{4} \int dx^- \text{tr} \left( [A_I A_J]^2 \right). \tag{39}
\]

We can also construct the Noether charges corresponding to the supersymmetry transformation (22). As in the three dimensional case it is convenient to decompose the supercharge in two components:
\[
Q^+ = P_L Q, \quad Q^- = P_R Q.
\]
The resulting eight component supercharges are given by
\[
Q^+ = 2 \int dx^- \text{tr} \left( \beta_I^T \psi_R \partial_- A_I \right), \tag{40}
\]
\[
Q^- = -2g \int dx^- \text{tr} \left( J^+ \frac{1}{\partial_-} \psi_R + \frac{i}{4} [A_I A_J] (\beta_I \beta_J - \beta_J \beta_I) \psi_R \right). \tag{41}
\]

Finally we make a short comment on dimensional reduction of \( SYM_{3+1} \) and \( SYM_{5+1} \). These systems can be constructed repeating the procedure just described. However there
is an easier way to construct the Hamiltonian and supercharges for the dimensionally reduced theories, namely one has to truncate the unwanted degrees of freedom in the ten dimensional expressions. This is especially easy for the bosonic coordinates: one simply considers indices $I$ and $J$ running from one to two (for $D = 4$) or to four (for $D = 6$). The fermionic truncation can also be performed by requiring the spinor $\psi_R$ to be 2– or 4–component. Then the only problem is the choice of $2 \times 2$ or $4 \times 4$ beta matrices satisfying

$$\{\beta_I, \beta_J\} = 2\delta_{IJ}, \quad (42)$$

which is an easy task.

**Conclusion.**

While we are still far from completely solving the bound state problem in three and four dimensional theories, we can already make some statements about these theories:

We have studied the structure of bound states for two dimensional supersymmetric models obtained by dimensional reduction from SYM$_{2+1}$. For this theory we have proven that any normalizable bound state in the continuum must include a contribution with arbitrarily large number of partons and we have shown that this is the general property of supersymmetric matrix models [30]. This scenario is to be contrasted with the simple bound states discovered in a number of, 1+1 dimensional theories with complex fermions, such as the Schwinger model, the t’Hooft model, and a dimensionally reduced theory with complex adjoint fermions [4, 22]. We also study the massless states of SYM$_{2+1}$ in DLCQ. Some of them are constructed explicitly and the general formula for the number of massless states as function of harmonic resolution is derived for the large $N$ case [31].

Some of our results on zero modes in 2 dimensions are discussed in [3]. They can be used to describe the vacuum structure of SYM$_{2+1}$ on a cylinder because only zero modes contributes to such structures. Thus studying the dimensionally reduced theory in 1 + 1 provides all the necessary information. We found that two dimensional models also determine the behavior of bound states at weak coupling in three dimensions and determine the exact number of massless states. We performed such counting only for $(1,1)$ theory. The theory with $(2,2)$ supersymmetry theory [4] and the even more interesting case of the $(8,8)$ theory [2], which is known to have a mass gap have not yet been addressed.

The bound state problem we have studied so far is the traditional one for DLCQ.
However this is not the only calculation that can be done us this method. The problem of computing of correlation functions, more traditional for conventional quantum field theory, can also be addressed in the light cone quantization. Unlike the usual methods of QFT, DLCQ calculations are valid beyond perturbation theory and thus can be used for testing the duality between gauge theory and supergravity.

There has been a great deal of excitement during this past year following the realization that certain field theories admit concrete realizations as a string theory on a particular background \[17\]. By now many examples of this type of correspondence for field theories in various dimensions with various field contents have been reported in the literature (for a comprehensive review and list of references, see \[1\]). However, attempts to apply these correspondences to study the details of these theories have only met with limited success so far. The problem stems from the fact that our understanding of both sides of the correspondence is limited. On the field theory side, most of what we know comes from perturbation theory where we assume that the coupling is weak. On the string theory side, most of what we know comes from the supergravity approximation where the curvature is small. There are no known situations where both approximations are simultaneously valid. At the present time, comparisons between the dual gauge/string theories have been restricted to either qualitative issues or quantities constrained by symmetry. Any improvement in our understanding of field theories beyond perturbation theory or string theories beyond the supergravity approximation is therefore a welcome development.

We have studied the field theory/string theory correspondence \[27\] motivated by considering the near-horizon decoupling limit of a D1-brane in type IIB string theory \[15\]. The gauge theory corresponding to this theory is the Yang-Mills theory in two dimensions with 16 supercharges. Its SDLCQ formulation was recently reported in \[2\]. This is probably the simplest known example of a field theory/string theory correspondence involving a field theory in two dimensions with a concrete Lagrangian formulation.

A convenient quantity that can be computed on both sides of the correspondence is the correlation function of gauge invariant operators \[13, 29\]. We focused on two point functions of the stress-energy tensor. This turns out to be a very convenient quantity to compute for many reasons. Some aspects of this as it pertains to a consideration of black hole entropy was recently discussed in \[14\]. There are other physical quantities often reported in the literature. In the DLCQ literature, the spectrum of hadrons is often reported. This would be fine for theories in a confining phase. However, we expect
the SYM in two dimension to flow to a non-trivial conformal fixed point in the infra-red \[13, 10\]. The spectrum of states will therefore form a continuum and will be cumbersome to handle. On the string theory side, entropy density \[8\] and the quark anti-quark potential \[8, 23, 18\] are frequently reported. The definition of entropy density requires that we place the field theory in a space-like box which seems incommensurate with the discretized light cone. Similarly, a static quark anti-quark configuration does not fit very well inside a discretized light-cone geometry. The correlation function of point-like operators do not suffer from these problems.

Let us now mention the immediate challenges to DLCQ following from our consideration. First of all it is straightforward to extend the numerical results for the correlator to higher resolution and thus to test the Maldacena conjecture. The only problem here is the limits in one’s computing resources. The better computer power may also help to extend our analysis of three dimensional system to larger values of transverse truncation and it might be possible to extrapolate the results to continuum. The simple transverse truncation we have used so far do not provide much information about behavior of the spectrum as function of transverse resolution. Another serious disadvantage of our approach is that we study the theory on the cylinder and thus the structure of the topological excitations of our system differs from the one of SYM$_{2+1}$ in infinite spacetime. Such excitations can become important in the strong coupling limit. However, even the spectrum for the theory on the cylinder is of a great interest and we leave the detailed numerical study of this system for future work. Finally solving for bound states of four dimensional theories is still the greatest challenge for DLCQ.

In this talk we have reviewed some of the progress in the application of discrete light cone quantization to the supersymmetric systems. Studying such systems is especially interesting because the cancellation between bosonic and fermionic loops make these theories much easier to renormalize than models without supersymmetry. Although we didn’t need this advantage when considering two dimensional systems, it becomes crucial in higher dimensions. From this point of view it is desirable to have exact SUSY in discretized theories to simplify the renormalization in DLCQ.

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References

[1] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, “Large N field theories, string theory and gravity,” [arXiv:hep-th/9905111].

[2] F. Antonuccio, O. Lunin, S. Pinsky, H. C. Pauli, and S. Tsujimaru, “The DLCQ spectrum of N=(8,8) superYang-Mills,” Phys. Rev. D58 (1998) 105024, [arXiv:hep-th/9806133].

[3] F. Antonuccio, O. Lunin, S. Pinsky, and S. Tsujimaru, “The Light cone vacuum in (1+1)-dimensional superYang-Mills theory,” [arXiv:hep-th/9811254].

[4] F. Antonuccio, H. C. Pauli, S. Pinsky, and S. Tsujimaru, “DLCQ bound states of N=(2,2) super Yang-Mills at finite and large N,” Phys. Rev. D58 (1998) 125006, [arXiv:hep-th/9808120].

[5] F. Antonuccio, S.S. Pinsky, ”Matrix theories from reduced SU(N) Yang–Mills with adjoint fermions,” Phys.Lett B397 (1997) 42, [arXiv:hep-th/9612021].

[6] G. Bhanot, K. Demeterfi, I.R. Klebanov, ”(1+1)–Dimensional Large N QCD Coupled to Adjoint fermions,” Phys. Rev. D48 (1993) 4980, [arXiv:hep-th/9307111].

[7] J. Boorstein, D. Kutasov, ”Symmetries and mass splittings in QCD$_2$ coupled to adjoint fermions,” Nucl.Phys. B421 (1994) 263, [arXiv:hep-th/9401044].

[8] A. Brandhuber, N. Itzhaki, J. Sonnenschein, and S. Yankielowicz, “Wilson loops, confinement, and phase transitions in large N gauge theories from supergravity,” JHEP 06 (1998) 001, [arXiv:hep-th/9803263].

[9] S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, “Quantum chromodynamics and other field theories on the light cone,” Phys. Rept. 301 (1998) 299, [arXiv:hep-ph/9705477].

[10] R. Dijkgraaf, E. Verlinde, and H. Verlinde, “Matrix string theory,” Nucl. Phys. B500 (1997) 43, [arXiv:hep-th/9703030].

[11] P. A. M. Dirac, “Forms of relativistic dynamics,” Rev. Mod. Phys. 21 (1949) 392.

[12] A. Giveon, D. Kutasov, ”Brane dynamics and gauge theory,” [arXiv:hep-th/9802067].

[13] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, “Gauge theory correlators from noncritical string theory,” Phys. Lett. B428 (1998) 105, [arXiv:hep-th/9802109].
[14] A. Hashimoto and N. Itzhaki, “A Comment on the Zamolodchikov c function and the black string entropy,” [hep-th/9903067].
[15] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, “Supergravity and the large N limit of theories with sixteen supercharges,” Phys. Rev. D58 (1998) 046004, [hep-th/9802042].
[16] D. Kutasov, ”Two Dimensional QCD Coupled to Adjoint Matter and String Theory,” Phys. Rev. D48 (1993) 4980, [hep-th/9306013].
[17] J. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231, [hep-th/9711200].
[18] J. Maldacena, “Wilson loops in large N field theories,” Phys. Rev. Lett. 80 (1998) 4859, [hep-th/9803002].
[19] T. Maskawa and K. Yamawaki, “The problem of $p^+=0$ mode in the null plane field theory and Dirac’s method of quantization,” Prog. Theor. Phys. 56 (1976) 270.
[20] Y. Matsumura, N. Sakai, and T. Sakai, “Mass spectra of supersymmetric Yang-Mills theories in (1+1)-dimensions,” Phys. Rev. D52 (1995) 2446–2461, [hep-th/9504150].
[21] H. C. Pauli and S. J. Brodsky, “Discretized light cone quantization: solution to a field theory in one space one time dimensions,” Phys. Rev. D32 (1985) 1993, 2001.
[22] S. Pinsky, ”The Analog of the t’Hooft Pion with Adjoint Fermions,” Invited talk at New Nonperturbative Methods and Quantization of the Light Cone, Les Houches, France, 24 Feb - 7 Mar 1997. [hep-th/9705242].
[23] S.-J. Rey and J. Yee, “Macroscopic strings as heavy quarks in large N gauge theory and anti-de Sitter supergravity,” [hep-th/9803001].
[24] N. Seiberg, ”Electric-magnetic duality in supersymmetric non-Abelian gauge theo- ries,” Nucl.Phys. B435 (1995) 129
[25] N. Seiberg, E. Witten, ”Monopoles, duality and chiral symmetry breaking in N=2 supersymmetric QCD,” Nucl.Phys. B431 (1994) 484.
[26] J. Wess, J. Bagger, Supersymmetry and supergravity, Princeton University Press(1992).
[27] F. Antonuccio, A. Hashimoto, O. Lunin, and S. Pinsky, *JHEP* **9907:029**, 1999, 
hep-th/9906087.

[28] E. Witten, ”Bound states of strings and p–branes,” *Nucl.Phys.* **B460**, (1996) 335,
hep-th/9510135.

[29] E. Witten, “Anti-de Sitter space and holography,” *Adv. Theor. Math. Phys.* **2** (1998) 253, 
hep-th/9802150.

[30] F. Antonuccio, O. Lunin, and S. Pinsky, “Bound States of Dimensionally Reduced SYM(2+1) at Finite N” Phys. Lett. **B429**: 327-335, 1998, 
hep-th/9803027.

[31] F. Antonuccio, O. Lunin, and S. Pinsky, “Nonperturbative Spectrum of Two-Dimensional (1,1) Superyand-Mills at Finite and Large N” Phys. Rev. **D58**: 085009, 1998. 
hep-th/9803170.