Validated Objects: Specification, Implementation, and Applications

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Abstract

Guaranteeing the validity of concurrent operations on distributed objects is a key property for ensuring reliability and consistency in distributed systems. Usually, the methods for validating these operations, if present, are wired in the object implementation. In this work, we formalize the notion of a validated object, decoupling the object operations and properties from the validation procedure. We consider two types of objects, satisfying different levels of consistency: the validated totally-ordered object, offering a total ordering of its operations, and its weaker variant, the validated regular object. We provide conditions under which it is possible to implement these objects. In particular, we show that crash-tolerant implementations of validated regular objects are always possible in an asynchronous system with a majority of correct processes. However, for validated totally-ordered objects, consensus is always required if a property of the object we introduce in this work, persistent validity, does not hold. Persistent validity combined with another new property, persistent execution, allows consensus-free crash-tolerant implementations of validated totally-ordered objects. We demonstrate the utility of validated objects by considering several applications conforming to our formalism.

2012 ACM Subject Classification Computing methodologies → Distributed algorithms

Keywords and phrases Validation, Concurrent objects, Fault-tolerance, Distributed computing.

Funding Antonio Fernández Anta: Partially supported by Spanish State Research Agency (AEI) ECID project PID2019-109805RB-I00, cofunded by FEDER.

1 Introduction

Motivation. In distributed computing research, there is an important line of work on the formalization and implementation of distributed concurrent objects. A fundamental challenge of these implementations is making sure the operations that modify the state of an object never drive it into an incorrect or inconsistent state. In most proposals, the operations (and their arguments) invoked on the object have been assumed to be always valid, or ensuring this validity has been delegated to the application layer. With the popularization of public data structures (due to the wide usage and vast application scope of distributed ledger technologies), there is a growing interest on algorithms and objects capable of tolerating non-compliant user behavior. In this context, the implementation of an object cannot assume anymore that the operations invoked in the object will be well formed and respect any specification rule. Hence, the implementation of the object must be cautious, and validate operations before applying them. The direct way to do this is to introduce validation tests into the code that implements the object, so that an invalid operation execution is interrupted before it damages the object’s state.
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In this paper we explore the possibility of separating an object’s implementation from the validation of the operations invoked in the object, and the implications of this separation. This approach is inspired by aspect-oriented programming [19], which aims in modular programming by separating cross-cutting concerns, i.e., cohesive areas of functionality. The idea is to add specific checks (advices as called) without changing the code of a program (object in our case). Our work is meant to be a first step on understanding how the application requirements and properties impact the algorithms that implement a distributed object through the introduction of a validation predicate valid() that wraps the application logic of the object.

Our approach and contributions. We employ a modular approach in which the characteristics and methods to validate the operations of an object are not “wired” in the object implementation. In particular, given a concurrent object \( O \) and its supported set of operations \( OP \), we recast this object as a validated object via an apply() function. This function includes a validation filter, so that a specific operation \( op \in OP \) is validated before it is executed, in accordance to an associated validation predicate valid(). Different validation predicates can be enforced via the apply() function without affecting the core code of the object.

Consider the following example. Let \( O \) be a simple R/W register supported by two operations, read(), which returns the value of the register, and write\((v)\), which changes the value of the register into \( v \). Say that we would like to impose that only positive numbers are written on the register. One approach would be to include a test directly in the code of the write function (see Code 1). However, should a different or an additional rule be needed, the code would have to be changed again, possibly jeopardizing the implementation correctness (especially in the case of complex objects).

With our approach, we separate the test from the code implementing the object. Processes invoke the desired operation via an apply() function. The process passes to apply() the same parameters as it would do in the “normal” case, and the apply function invokes a valid() predicate that has incorporated the desired validation test (i.e., in the case of a write operation, that \( v \) is positive, see Code 2). In case it is true, it then invokes execute(), which applies the operation on the object (i.e., it sets \( v \) as the value of the register). In case the validation fails (e.g., a negative value was intended to be written), apply() will return a NACK, signaling the violation of the imposed restriction (see Code 3). Should we require a different validation (e.g., we want a Boolean register), we would only replace the test in the valid() predicate (e.g., \( v > 0 \) in Line 5 becomes \( v \in \{\text{True, False}\} \)), without making any change on the object’s implementation (i.e., in function execute()).

A particular challenge of our approach is to implement the validated version of a given object on a decentralized setting while guaranteeing certain level of consistency. In this work, we consider two types of validated objects, each providing a different level of consistency, the validated regular object and the totally-ordered one. Intuitively, a regular object provides consistency guarantees similar to a regular register [20], while the totally-ordered property is similar to linearizability [17]. We are now ready to summarize the contributions of this work.

We introduce the formalization for a generic validated object \( O \), along with the two men-
tioned consistency types, on which the application-specific operations are called (Section 2).

- We provide an algorithm to implement validated regular objects in crash-prone asynchronous distributed systems (Section 3).

- We provide an algorithm to implement validated totally-ordered objects under crash or Byzantine failures using the corresponding version of consensus [21, 3] (Section 4.1).

- Then in Section 4.2 we define a property of a validity predicate, which we call persistent validity, and in Section 4.3 we show that validated totally-ordered objects without persistent validity can be used to solve consensus.

- In Section 4.4, we introduce an additional property, that we call persistent execution, which allows a validated totally-ordered object to be implemented without consensus.

- Finally, in Section 5 we present some applications (such as a punching system and a cryptocurrency) that conform to the formalism we provided, demonstrating its usability.

**Related work.** The impact of a validation function has been already treated by previous work, according, however, to specific use cases.

In [12], a validity property called forward acceptability is defined, which enables the operations of a generic application to be commutative. This work only considers eventually consistent objects with this property. It provides an algorithm for a specific case, a PC-Ledger, that is implemented in a consensus-free system. On our side, we have a wider focus, including any object, characterizing its validity function and going in detail with the different consistency properties we are able to guarantee.

In [4], the authors introduce and solve the notion of Validated Byzantine Agreement to ensure that the decided value is one proposed by a non-faulty process. To do so, they enhance the system with an external validity condition, which requires that the agreement value is valid according to a global, polynomial-time computable predicate, known to all processes and it is application-determined. Hence, each process proposes a value that should satisfy this predicate. Such an external validity condition could be implemented using our approach via an appropriate apply() function and valid() predicate (which would implement the required predicate).

In blockchain systems, records are usually validated after the total order is globally agreed, validating and executing transaction according to the agreed sequence. As an example, Ethereum [26] first constructs a block, and then network nodes sequentially run the Ethereum Virtual Machine on each transaction to validate it, and update the global state if valid. This brings to the acceptance and inclusion in the block of invalid transaction inside the global order, in order to gain time in the consensus challenge of the system. A mitigation to this problem is brought by [5], that is build on top of [7], where validation is run by a subset of nodes before the proposal is broadcast to the whole network, in order to not overload nodes. In our work we abstract and generalize these behaviors, mapping them to validated objects with different consistency criteria.

In [1], the authors define a Validated Distributed Ledger Object. The validation is only taken into account in respect to Ledger Objects limiting the scope to that particular kind of data structure. Furthermore, the authors do not investigate or characterize the properties required by validation; they only assume the existence of a validation predicate.
2 Validated Objects

2.1 Concurrent Objects and Histories

We recall the general definition of object formalized in [1] where an object type T specifies 
(i) the set of values (or states) that any object O of type T can take, and (ii) the set of 
operations that a process can use to modify or access the value of O. An object O of type T 
is a concurrent object if it is a shared object accessed by multiple processes [24] [3]. Each 
operation on an object O consists of an invocation event and a response event, that must 
occur in this order. A history of operations on O, denoted by \( H_O \), is a sequence of invocation 
and response events, starting with an invocation event. (The sequence order of a history 
reflects the real time ordering of the events.) An operation \( \pi \) is complete in a history \( H_O \), if 
\( H_O \) contains both the invocation and the matching response of \( \pi \), in this order. A history 
\( H_O \) is complete if it contains only complete operations; otherwise it is partial [24]. As 
in [24], we convert a partial history to a complete one by, for each incomplete operation \( \pi \), 
either removing the invocation of \( \pi \) or completing \( \pi \) with a response event. From this point 
onward, we consider only complete histories. An operation \( \pi_1 \) precedes an operation \( \pi_2 \) (or 
\( \pi_2 \) succeeds \( \pi_1 \)), denoted by \( \pi_1 \rightarrow \pi_2 \), in \( H_O \), if the response event of \( \pi_1 \) appears before the 
invocation event of \( \pi_2 \) in \( H_O \). Two operations are concurrent if none precedes the other. A 
run R of a distributed system that implements object \( O \) generates a (potentially infinite) 
history \( H_O \).

2.2 Validated Object Types

In this work we consider validated objects. These are 
concurrent objects in which the operations executed 
and how they are interleaved are filtered with a predicate \( \text{valid}() \). This predicate has as argument the 
state of the object \( S \) and a new operation \( op \) issued by process \( i \), and it determines whether \( op \) is valid in 
the light of \( S \). The state \( S \) is given by an ordered set of operations that have been executed in the object 
(and are valid). (The operations in \( S \) could be concurrent with \( op \) but have been “applied” in the object 
before it.) A second function \( \text{execute}() \) has the same arguments as \( \text{valid}() \) and, if the operation \( op \) is valid, 
is used to obtain the value that \( op \) returns. We will use the term \( \text{operation} \) and the symbol \( op \) for custom 
object logic unknown to our formalism, and we refer 
to \( \text{functions} \) when referring to the primitives of the objects we define, e.g., \( \text{valid} \) or \( \text{execute} \).

In order to use a validated object, a client \( i \) invokes a function \( \text{apply}(op, i) \), which checks whether the operation \( op \) invoked by \( i \) is valid, and if so it applies it in the object by executing 
\( op \). If the operation is not valid, the call \( \text{apply}(op, i) \) returns \( \text{NACK}, \bot \). If the operation is 
valid, \( \text{apply}(op, i) \) returns \( \text{ACK}, r \), where \( r \) is the value that \( op \) returns. Code 3 shows a 
centralized implementation of the function \( \text{apply}() \), executed at a single central server. This 
code is provided for illustration purposes. The operator \( |\_| \) that defines how the operations 
are combined into the state \( S \) is not detailed on purpose.

\[
\text{Code 3} \text{ Centralized implementation of the apply function for a validated object } O. \text{ Code executed by the central server. Function execute(S, op, i) provides the result of operation op by process i in state S. The operator } |\_| \text{ combines the new operation with the previous valid executed operations.}
\]

1: \( S \leftarrow \varnothing \) is the state of the object 
2: function \( \text{apply}(op, i) \) 
3: if \( \text{valid}(S, op, i) \) then \( r \leftarrow \text{execute}(S, op, i) \) 
4: \( S \leftarrow S \upharpoonright op \) 
5: return \( (ACK, r) \) 
6: else return \( (\text{NACK}, \bot) \)}
Observe that the history \( H_O \) of a run \( R \) of a validated object \( O \) contains only the operations \( op \) that are found valid and are in fact executed. These are the operations for which \( \text{apply}(op, i) \) returns \((\text{ACK}, r)\). We denote the set of complete operations in history \( H_O \) generated in run \( R \) by \( C(R) \).

In the following we assume that \( \text{valid()} \) and \( \text{execute()} \) have the following arguments:

1. A strict partially ordered set of operations, given as a pair \((P, \prec)\). \( P \) is a set of operations and \( \prec \) is a strict partial order defined in \( P \). In the special case in which \( \prec \) is a total order, denoted as \( \preceq \), the first argument can be provided as a sequence of operations.
2. The operation \( op \) to be considered.
3. The process \( i \) that issued \( op \).

In this work we consider two types of validated objects.

\textbf{Definition 1.} A validated object specified with functions \( \text{valid()} \) and \( \text{execute()} \) is a validated regular object if in every run \( R \) a partial order \( \prec \) among the set \( C(R) \) of complete operations can be defined, such that,

1. \( \forall op, op' \in C(R), op \rightarrow op' \implies op \prec op' \);
2. \( \forall op \in C(R), \text{let } P(op) = \{op' : op' \in C(R) \land \text{op' \prec op}\} \text{ and client } i \text{ the issuer of } op. \text{ Then, } \text{validate}(\langle P(op), \prec \rangle, op, i) = \text{true} \text{ and } op \text{ returns in its response event the value } \text{execute}(\langle P(op), \prec \rangle, op, i). \)

The following is a stronger version in which operations are totally ordered.

\textbf{Definition 2.} A validated object specified with functions \( \text{valid()} \) and \( \text{execute()} \) is a validated totally-ordered object\footnote{Note that if \( \text{valid()} \) is considered as the sequential specification of the object, then the totally-ordered property is a form of linearizability defined over the executed operations. However, to avoid confusion, we prefer to use a different name since we do not include in the object histories the operations that were rejected with \((\text{NACK}, -)\) by \( \text{apply()} \).} if in every run \( R \) a total order \( \preceq \) among the set \( C(R) \) of complete operations can be defined, such that,

1. \( \forall op, op' \in C(R), op \rightarrow op' \implies op \preceq op' \);
2. \( \forall op \in C(R), \text{let } P(op) = \{op' : op' \in C(R) \land \text{op' \preceq op}\} \text{ and client } i \text{ the issuer of } op. \text{ Then, } \text{validate}(\langle P(op), \preceq \rangle, op, i) = \text{true} \text{ and } op \text{ returns in its response event the value } \text{execute}(\langle P(op), \preceq \rangle, op, i). \)

In a run \( R \) of a validated totally-ordered object, the set \( C(R) \) is totally ordered by \( \preceq \). We will denote the resulting sequence of all the operation of \( R \) by \( S(R) \).

\section{Algorithms Implementing Validated Regular Objects}

We present algorithms implementing validated regular objects in an asynchronous system.

\subsection{Model}

We assume a distributed system composed of \( n \) processes with unique identities from the set \( I = \{1, \ldots, n\} \). Processes are asynchronous and crash prone, i.e., they advance their execution at arbitrary speed and can stop permanently (i.e., crash) at any point during their execution. Each process \( i \) has write access to a linearizable SWMR Distributed Ledger Object (DLO)\footnote{\cite{SWMR}} \( L_i \) denoted \( L_i \). All processes can read all DLOs. A DLO \( L_i \) has a state, which is a totally ordered sequence \( S \) of records, initially empty, and has two operations:

\begin{itemize}
  \item \( L_i.\text{get}() \), which returns the current state (sequence of records) \( S \) of the DLO,
  \item \( L_i.\text{append}(r) \), which adds record \( r \) to the end of the sequence \( S \).
\end{itemize}
These DLOs are reliable in the sense that any invocation to these operations by a correct process eventually completes \[\text{1}\]. Reliable linearizable SWMR DLOs can be implemented in an unreliable asynchronous system. This follows from the work of Imbs et al. \[18\], in which they implement SWMR atomic h-registers, which are registers that, when read, return the whole history of written values. Each of these h-registers trivially implements a SWMR DLO. Moreover, their implementation is on a distributed system with \(n\) servers, out of which up to \(f < n/3\) can be Byzantine. Hence, the implementation is Byzantine-tolerant with optimal resilience. In this section we also observe that, with minor changes, the algorithm proposed by Imbs et al. \[18\] implements SWMR atomic h-registers for \(n\) crash-prone processes, out of which \(f < n/2\) can fail.

As usual, we assume the following well-formedness property: A process \(i\) does not invoke a call to the function \(\text{apply}(op, i)\) of the object being implemented before the previous one has finished.

### 3.2 Crash-tolerant Algorithm for Validated Regular Objects

Let us consider a validated regular object \(O\) specified by the functions \(\text{valid}(\cdot)\) and \(\text{execute}(\cdot)\). Code 4 presents an implementation of the \(\text{apply}(\cdot)\) function to be run by each of the processes of the distributed system in order to implement an instance of the object \(O\). For technical reasons, we assume that the invocation action of a valid operation \(op\) issued by \(i\) occurs when it enters the loop in Line 2 and invokes \(L_i.\text{get}(\cdot)\), and that it completes when the \(L_i.\text{append}(\langle ts, op \rangle)\) operation in Line 9 completes. What happens before and after these two actions respectively in the execution of \(\text{apply}(op, i)\) is local to process \(i\), and not visible outside the client. This assumption removes the uncertainty of whether an operation has been completed if the process crashes after Line 9 but never executes Line 10.

We proceed by demonstrating that Code 4 implements a validated regular object as defined in Definition 3. Observe from Code 4 that every operation \(op\) issued by process \(i\) is assigned a timestamp \(ts(op) = (i, T_i, 1, \ldots, T_i, n)\), which is appended as part of the record of \(op\) in ledger \(L_i\) if it is valid and completes. The value \(T_j\) in the timestamp is the number of records found in ledger \(L_j\) in the loop of Line 2. These timestamps are used to define the partial order \(<\) among completed operations.

▶ **Definition 3.** Given any two completed operations \(op, op' \in C(R)\), with respective timestamps \(ts(op) = (i, T_i, 1, \ldots, T_i, n)\) and \(ts(op') = (k, T'_k, 1, \ldots, T'_k, n)\), \(i, k \in [1, n]\), then \(op < op'\) if and only if \(T_i < T'_k\).

We show now that \(<\) is a strict partial order as required.

▶ **Lemma 4.** If \(op < op'\) it cannot happen that \(op' < op\). Hence, \(<\) is a strict order.

**Proof.** Assume for contradiction that \(op < op'\) and \(op' < op\). Let \(ts(op) = (i, T_1, \ldots, T_n)\) and \(ts(op') = (k, T'_1, \ldots, T'_n)\). Then \(\text{apply}(op, i)\) finds \(T_i\) records in ledger \(L_i\) and \(T_k\) records in ledger \(L_k\), while \(\text{apply}(op', k)\) finds \(T'_k\) records in \(L_i\) and \(T'_k\) records in ledger \(L_k\). By assumption, we have that \(T_i < T'_k\) and \(T_k > T'_k\). From \(T_i < T'_k\) and the linearizability of \(L_i\),
Implementation of the `apply` function for a validated totally-ordered object \( O \) that uses an Atomic Broadcast service. The code is for process \( i \in [1, n] \).

```
1: \( S \leftarrow \emptyset \) > \( S \) is a sequence of operations
2: function `apply`(\( op, i \))
3: \( ret \leftarrow \perp \)
4: `AB.broadcast`([\( op, i \)])
5: wait until \( ret \neq \perp \)
6: `return` \( (ret) \)
7: upon \( (AB.deliver([op, j])) \) do
8: if `valid`(\( S, op, j \)) then
9: \( r \leftarrow \text{execute}(S, op, i) \)
10: `S` |\( \| \) `op`
11: if \( j = i \) then \( \text{ret} \leftarrow (ACK, r) \)
12: else
13: if \( j = i \) then \( \text{ret} \leftarrow (NACK, -) \)
```

The append operation in `apply`(\( op, i \)) precedes or is concurrent with the \( L_i.get() \) operation in `apply`(\( op', k \)). Hence, \( i \) executed \( L_k.get() \) before \( k \) invoked \( L_k.append([ts(op'), op']) \). By the linearizability of \( L_k \), it is not possible that \( T_k > T' \), and we have a contradiction.

**Lemma 5.** \( op \rightarrow op' \implies op \prec op' \).

**Proof.** Let us assume \( op \) was issued by process \( i \) and \( op' \) was issued by process \( k \). Let \( ts(op) = (i, T_1, \ldots, T_i, \ldots, T_n) \). From \( op \rightarrow op' \), the response action of \( op \) happened before the invocation action of \( op' \). So, the execution of the append operation \( L_i.append([ts(op), op']) \) in the call `apply`(\( op, i \)) was completed before the \( L_i.get() \) call in `apply`(\( op', k \)). Then, because of the linearizability of the ledgers, the length of ledger \( L_i \) found in `apply`(\( op', k \)) is \( T' \geq T_i + 1 \) (since the append operation increased its length). Hence, \( op \prec op' \) from Definition 3.

The proof of the next lemma is given in Appendix A.

**Lemma 6.** For each complete operation \( op \) (issued by \( i \)), \( \text{valid}((P(op), \prec), op, i) = True \). Moreover, \( op \) returns in its response event the value `execute`((\( P(op), \prec \), \( op, i \)).

**Theorem 7.** Code 4 implements a validated regular object as defined in Definition 2 in a crash-prone asynchronous system with linearizable SWMR DLOs.

From the fact that reliable linearizable SWMR DLOs can be implemented in a crash-prone asynchronous system [18], we have the following corollary.

**Corollary 8.** It is possible to implement validated regular objects in an asynchronous system with \( n \) crash-prone processes from which up to \( f < n/2 \) can fail.

## 4 Validated Totally-ordered Objects

### 4.1 Implementing Validated Totally-ordered Objects with Consensus

We consider now the set of validated totally-ordered objects as a whole. The first observation is that an object without validation can be seen as a validated object in which the `valid()` predicate always holds. Hence, an object with consensus number \( k \) [16] will also have at least consensus number \( k \) in its validated version.

Code 5 shows an algorithm that can be used to implement a validated totally-ordered object using an Atomic Broadcast service, which is known to be equivalent to Consensus [25] (and to MWMR Distributed Ledger Objects [1]). An Atomic Broadcast (AB) service [10, 6, 9, 22], ensures reliable and total ordering of the messages exchanged. Such a communication abstraction is based on appropriate crash-tolerant or Byzantine-tolerant consensus algorithms [10, 25].
The service has two operations, \( AB\text{.}broadcast(m) \) used by a process to broadcast a message \( m \) to all other processes, and \( AB\text{.}deliver(m) \) used by the service to deliver a message \( m \) to a process. From a user point of view, the AB service is defined by the following properties:

- **Validity**: if a correct process \( AB\text{.}broadcast \)s a message, it eventually \( AB\text{.}deliver \)s it.
- **Agreement**: if a correct process \( AB\text{.}deliver \)s a message, all correct processes will eventually \( AB\text{.}deliver \) that message.
- **Integrity**: a message is \( AB\text{.}deliver \)ed by a correct process at most once, and only if it was previously \( AB\text{.}broadcast \).
- **Total Order**: the messages \( AB\text{.}deliver \)ed by the correct processes are totally ordered (i.e., if a correct process \( AB\text{.}deliver \)s message \( m \) before message \( m' \), every correct process \( AB\text{.}deliver \)es these message in the same order).

Note that if the AB service used is a crash-tolerant one, then Code 5 provides crash-tolerant implementation of the \( apply \) function, whereas if a Byzantine-tolerant AB service is used, then we have a Byzantine-tolerant implementation of \( apply \). It follows that Code 5 implements a validated totally-ordered object defined by the \( valid() \) and \( execute() \) functions.

▶ **Theorem 9.** Code 5 implements a validated totally-ordered object as defined in Definition 2 in a fault-prone asynchronous system with an Atomic Broadcast service.

**Proof.** The claim holds from the following observations. Firstly, from the Agreement and Total Order properties of the AB service, all correct processes \( AB\text{.}deliver \) the same tuples \( \langle \text{op}, j \rangle \) in the same order. This guarantees (by induction) that the sequence \( S \) maintained in all correct processes is the same. Moreover, for every \( op \in S \) it holds that \( valid(S, op, j) = True \), where \( S(op) \) is the subsequence preceding \( op \) in \( S \). Finally, for every invocation \( apply(op, i) \) by a correct process \( i \), the Validity of the AB service guarantees that the tuple \( \langle op, i \rangle \) will be \( AB\text{.}deliver \)ed to \( i \). Let \( S_i(op) \) be the local value of the sequence \( S \) when \( op \) is \( AB\text{.}deliver \)ed to \( i \). Then, the call \( apply(op, i) \) returns \((NACK, -)\) if \( valid(S_i(op), op, i) = False \), and it returns \((ACK, execute(S_i(op), op, i)) \) if \( valid(S_i(op), op, i) = True \).

▷

### 4.2 Persistent Validity

With Code 5 we have shown that all validated totally-ordered objects can be implemented with a Consensus / Atomic Broadcast service. In this section we explore conditions in the \( valid() \) and \( execute() \) functions that may allow a validated totally-ordered object to be implemented without consensus. We first define a property of some objects that we call **persistent validity**.

▶ **Definition 10.** Given a validated totally-ordered object together with its \( valid() \) predicate, we say that the object satisfies persistent validity iff for every run \( R \), with order \( \prec \), every prefix \( S \) of \( S(R) \) and every operation \( op \notin S \), if \( valid(S, op, i) = True \) then \( \forall op_j \notin S, j \neq i : valid(S, op_j, j) = True \land valid(S|op_j, op_i, i) = False \).

Persistent validity informally says that once an operation is valid, then it cannot be made invalid by operations issued by the other processes. In Section 4.3 we show how persistent validity can help in the implementation of different objects according to different consistency criteria. But first, we investigate the implications of a validation predicate \( valid() \) for which the persistent validity does not hold.

---

2 Recall that \( S(R) \) is the sequence of operations in \( C(R) \) totally ordered by \( \prec \).
Then, there is a run

Let prefix

We demonstrate that a validated totally-ordered object whose


Lemma 12. Let $O$ be a validated totally-ordered object without persistent validity, and let prefix $S$, processes $i$ and $j$, and operations $op_i, op_j \notin S$ as in Observation 11. Moreover,
assume that valid(S|op_i, op_j, j) = False. Then Code 6 allows processes i and j to reach consensus.

Proof. Process i first writes its proposed value v_i in its own register \texttt{cons_register}_i and then calls \texttt{O.apply(op_i, i)}. Process j does the same with register \texttt{cons_register}_j and call \texttt{O.apply(op_j, j)}. By assumption, only one of the operations \texttt{op}_j and \texttt{op}_j is found valid. Then, if process i receives an ACK from \texttt{apply(op_i, i)}, it can safely decide v_i, knowing that process j will receive NACK and decide v_j as well. On the other hand, if process i receives NACK and process j receives ACK, value v_j is decided by both processes.

Let us now assume that one process crashes; wlog, process j. If process j never issued the call \texttt{O.apply(op_j, j)} or the call was issued but \texttt{op}_j was found invalid, then \texttt{O.apply(op_j, i)} will return ACK and process i will decide v_i. If, on the other hand, j issued the call \texttt{O.apply(op_j, j)} and \texttt{op}_j was found valid, then process i receives a NACK, and reads \texttt{cons_register}_j. Since the value v_j was written in \texttt{cons_register}_j by j before calling \texttt{O.apply(op_j, j)}, the read operation completes and returns v_j, which is the value decided by process i. Process j cannot decide a different value, since \texttt{O.apply(op_j, j)} returns ACK.

Operation \texttt{op}_j does not invalidate operation \texttt{op}_j. We now deal with the case in which \texttt{op}_j makes \texttt{op}_i invalid, but \texttt{op}_j does not invalidate \texttt{op}_j. Observe that Code 6 does not solve this case because, since \texttt{op}_j is always valid, the value returned by call \texttt{O.apply(op_j, j)} does not allow process j to know whether \texttt{op}_j was found valid. Notice that we use the validated totally-ordered object as a black box. Therefore, process j does not have direct access to the totally-ordered sequence of operations in the object. Thus, for process j to know whether \texttt{op}_j is found valid some extra work needs to be done. The key of the solution is the use of the shared memory available in the system to log the calls \texttt{O.apply()} and the values they return. To do so, a generic process k has a SWMR vector \texttt{oplist}_k through with \texttt{apply()} call are issued. The result of the call is written by object O in a SWMR vector \texttt{reslist}_k from where k can read it. This process is encapsulated in the side of the generic caller process k in the function \texttt{LoggedApply()} presented in Code 7.

On its side, object O is waiting for \texttt{apply()} calls being issued via the vector \texttt{oplist}_k, and when one appears it applies it and writes in \texttt{reslist}_k the corresponding result. This can be implemented with one concurrent task for each process k as presented in Code 8. Note that, since the object O and the shared memory are both reliable, if an \texttt{apply()} call is written by process k in \texttt{oplist}_k, eventually the corresponding response will be written in \texttt{reslist}_k, even if k has crashed in the mid time.

With this logged method of using the object, the algorithm that processes i and j can use to solve consensus is presented in Code 9. Observe that the code for process i is similar to the one in Code 6, replacing the call \texttt{O.apply(op_i, i)} with call \texttt{LoggedApply(op_i, i)}. However,
the code for process $j$ is different, since it has to access $\text{oplist}_i$ and $\text{reslist}_i$ to determine whether $\text{op}_i$ was found valid.

Lemma 13. Let $O$ be a validated totally-ordered object without persistent validity, and let prefix $S$, processes $i$ and $j$, and operations $\text{op}_i, \text{op}_j \notin S$ as in Observation 11. Moreover, assume that valid$(S||\text{op}_i, \text{op}_j) = \text{True}$. Then Codes 8, 7, and 9 allow processes $i$ and $j$ to reach consensus.

Proof. Without crashes, both processes $i$ and $j$ start by writing their proposed values $v_i$ and $v_j$ in their respective $\text{cons\_register}_i$ and $\text{cons\_register}_j$. Then, they call $\text{LoggedApply}(\cdot)$ with their operations $\text{op}_i$ and $\text{op}_j$. As in Code 7, process $i$ waits for response and decides $v_i$ or $v_j$ depending on whether $\text{op}_i$ was found valid or not. This is determined from the value returned by the $\text{LoggedApply}(\text{op}_i, i)$ call.

On its hand, process $j$ always receives $\text{ACK}$ from the $\text{LoggedApply}(\text{op}_j, j)$ call, since operation $\text{op}_j$ is found valid by hypothesis. So, it can not use this to know whether $\text{op}_i$ precedes $\text{op}_j$ and was hence found valid. Instead, it first checks if process $i$ submitted $\text{op}_i$ via a $\text{LoggedApply}(\text{op}_i, i)$ call by searching in the $\text{oplist}_i$ vector. If $\text{op}_i$ was not submitted, then process $j$ can safely decide $v_2$ (line 24), because if it is submitted now it will be found invalid. Note that process $i$ will decide $v_2$ as well.

If $\text{op}_i$ is found in $\text{oplist}_i$ (line 16), then process $j$ needs to wait for the result of $\text{LoggedApply}(\text{op}_i, i)$ by reading from register $\text{reslist}_i$. As mentioned, because of the reliability of the object and the shared memory, the result will eventually be written there. At this point, if the result of $\text{LoggedApply}(\text{op}_i, i)$ is $\text{ACK}$ then it means that $\text{op}_i$ was ordered before $\text{op}_j$, and the value to be decided is $v_i$. If it is $\text{NACK}$ then $\text{op}_j$ has been ordered before $\text{op}_i$, $\text{op}_i$ was invalid, and the value to be decided is $v_j$. In either case, the decided value is consistent with the one decided by process $i$, solving consensus between the two processes.

The correctness for the case when process $j$ crashes is as in the proof of Lemma 12. If process $i$ crashes before writing $\text{op}_i$ in $\text{oplist}_i$, then $j$ decide $v_j$ as described above. Otherwise, $\text{op}_i$ will be processed by $O$ and found valid (and $v_1$ will be the decided value in both processes) or invalid (and $v_2$ will be the decided value).

Definition 14. Given a validated totally-ordered object together with its associated $\text{valid()}$ predicate and $\text{execute()}$ function, we say that the object satisfies persistent execution iff
1. it satisfies persistent validity and
2. for every run $R$, with order $\preceq$, every prefix $S$ of $S(R)$, and every pair of operations $op_i, op_j \notin S$ from processes $i \neq j$, if $\text{valid}(S, op_i, i) = \text{True} \land \text{valid}(S, op_j, j) = \text{True}$ then $\text{execute}(S, op_i, i) = \text{execute}(S || op_j, op_i)$.

\textbf{Theorem 15.} Let $O$ be a validated totally-ordered object without persistent validity, then $O$ can be used to solve consensus in a crash-prone asynchronous system with $n \geq 3$ processes in which at most one process can crash.

\textbf{Proof.} From Lemmas 12 and 13 we have that two processes $i$ and $j$ can solve consensus between them. To make the solution applicable to the $n$ processes, and allow any of the $n$ values proposed to be decided, we have each process writing its proposed value in a SWMR register $\text{prop}_k$ in the shared memory. Processes $i$ and $j$ wait until $n-1$ such registers are filled, and choose one value from these values. Then, they run the consensus algorithm between them, proposing the chosen values. As soon as one of the two processes decides, it writes the decision in the shared memory. They use SWMR registers $\text{decision}_i$ and $\text{decision}_j$. Since at least one process $i$ or $j$ is correct, the value decided is eventually written in at least one of these registers. Then, the other processes can read it from there and also decide. ▶

Observe that in a crash-prone asynchronous system in which one process can crash consensus cannot be solved [11]. Thus, Theorem 15 implies that any validated totally-ordered object without persistent validity is as strong as a consensus object [24] in such a system.

### 4.4 Consensus-free Total Order with Persistent Execution

The previous result shows that persistent validity is required in order to be able to implement a validated totally-ordered object without consensus. Unfortunately this is not enough, as can be trivially observed from the fact that a valid() predicate that is always True satisfies persistent validity. To be able to implement the object without consensus, some additional condition must be imposed. The following is an instance of such a condition.

An object with persistent execution has significant flexibility for reordering concurrent operations to obtain different total orders $\preceq$ that satisfy the conditions of Definition 2. Consider a validated totally-ordered object $O$ and a finite run $R$. Let $K$ be a set of concurrent operations issued by different processes $k$, such that $\forall op_k \in K$, it holds that $op_k \notin S(R)$, $\exists op \in S(R) : op_k \rightarrow op$, and $\text{valid}(S(R), op_k, k) = \text{True}$.

\textbf{Lemma 16.} If the validated totally-ordered object $O$ satisfies persistent execution, then $R$ can be extended with all the operations in $K$, in any order, satisfying Definition 2. Moreover, $\forall op_k \in K$, the value returned by $O.\text{apply}(op_k, k)$ is $(\text{ACK}, \text{execute}(S(R), op_k, k))$.

\textbf{Proof.} Any extension $R'$ as described will respect property (1) of Definition 2 because the operations in $K$ do not precede in real time order those in $S(R)$ and they are concurrent among themselves. Regarding property (2), from the assumption that $\forall op_k \in K, \text{valid}(S(R), op_k, k) = \text{True}$, that the operations in $K$ are issued by different processes, and persistent validity, we have that all operations in $K$ will be valid in the extension of $R$. Finally, the value returned for $op_k$ will be $(\text{ACK}, \text{execute}(S(R), op_k, k))$ from property (2) of Definition [14].

From this lemma, we can derive that Code [4] implements a validated totally-ordered object $O$ when persistent execution is satisfied. The total order $\preceq$ of a run of $O$ has to be an extension of the order from Def. 3, imposing an order among those operations that are not ordered by $\preceq$. One possibility is to order complete operations in a run by the real time...
order of their response events in the history of the run. This total order is consistent with
≺ because Def. 3 and Code 4 guarantee that (1) if \( op \prec op' \) then \( op \) completes before \( op' \) and that (2) if \( op \) completes before \( op' \) and \( op \neq op' \) then \( op \) and \( op' \) are concurrent. Hence the following result, which implies that consensus is not required to implement validated totally-ordered objects with persistent execution.

\[ \textbf{Theorem 17.} \text{ It is possible to implement a validated totally-ordered object } O \text{ that satisfies persistent validity and persistent execution in an asynchronous system with } n \text{ crash-prone processes from which up to } f < n/2 \text{ can fail.} \]

5 Applications of Validated Objects

To demonstrate the usefulness of validated objects, in this section we present a number of possible applications providing the exact properties that each application satisfies. For each application, we present both a relaxed version, i.e., one that uses regular validated objects, and a strict version, i.e., one that uses totally-ordered validated objects, and we analyze what validity properties are required for the applications being realized. (One more application is given in Appendix B.)

5.1 Punching System

A punching system is an object that can be used by a process to log its activity. It essentially allows a process to signal the start of an activity and then signal that activity’s end. One practical such system is used for tracking employee arrival and departure in various organisations. Such object may have the following two operations:

- \textbf{punch-in}(t, i), that can only be invoked by process \( i \), to mark his arrival at time \( t \),
- \textbf{punch-out}(i), that can only be invoked by process \( i \), to mark his departure and return the hours worked since he last punched-in

Notice that the \textbf{punch-in}(t, i) operation for \( i \) is only valid if the last operation from \( i \) was a \textbf{punch-out}(i) operation and vice-versa.

This object has both the persistent validity and persistent execution properties, as whenever \( i \) recorded a \textbf{punch-in} operation the \textbf{punch-out} operation remains valid no matter of the operations executed by any other process \( j \). Persistent execution also holds since the value of the object at \( i \) remains the same until \( i \) performs its \textbf{punch-in} or \textbf{punch-out} operations.

Notice that since the process \( i \) is restricted to obtain its own working hours (i.e., invoke only \textbf{punch-out}(i)) then by well-formedness the relaxed version of the punching system is equivalent with the strict version. Thus, the system may be implemented without consensus utilizing the functions defined in Code 10. Recall in this code that \( P \) is the set of complete operations that precede \( op \) using the order \( \prec \). Note also that \( \prec \) orders all the operations from the same process (from well-formedness and property (1) of Definitions 1 and 2), so \( lop_i \) is well defined.

\begin{verbatim}
1: function valid((P, \prec), op, i)
2:     if (i is not the issuer of op) then
3:           return(False)
4:     lop_i \leftarrow \{op' : op' the last operation of i in P\}
5:     if (op = punch-out(k)) then
6:           return \((i = k \land lop_i = \{\text{punch-in}(t, i)\})\)
7:     else \(\triangleright op = \text{punch-in}(t, k)\)
8:           return \((i = k \land op = \text{punch-in}(t, k) \land \{lop_i = \emptyset \lor op \prec op'\})\)
9: function execute((P, \prec), op, i)
10:    if (op = punch-out(i)) then
11:        lt_i \leftarrow \{t : op' = \text{punch-in}(t, i) \land \\n12:                  \exists \text{op''} \in P \text{ s.t. } op' \prec op''\}
13:        return \((\text{hours(now())} - \text{lt}_i)\)
14: else return (⊥)
\end{verbatim}

5.2 Cryptocurrency

In this section we implement a cryptocurrency (asset transfer) \[15\]. For that, a validated object is created, which holds an account for each process in \([1, n]\). For simplicity we assume that each process has initially a balance of ibalance tokens. The object has only two operations as described in \[15\]:

- \(\text{transfer}(i, k, x)\), that can only be invoked by process \(i\), transfers \(x > 0\) tokens from the account of the issuing process \(i\) to the account of process \(k\), and
- \(\text{read}(k)\), which returns an estimate of the current balance of the account of process \(k\).

We assume that the operations are cryptographically signed by the issuer. As usual, it is not allowed that a process ever has negative balance. Hence, an operation \(\text{transfer}(i, k, x)\) is valid and can be executed only when the balance of \(i\) is higher than the amount \(x\) to be transferred. In \[15\] this validation is embedded of the operation execution, while here validation and execution are separated in different functions \(\text{valid}()\) and \(\text{execute}()\) (see Code \[11\]).

We can also get this object in two flavors. In the relaxed version of the object the value returned by the \(\text{read}(k)\) operation must include all operations that precede the \(\text{read}(k)\) in real time ordering, but may not include some of the \(\text{transfer}()\) operations that are concurrent with the call. Thus, the operation may return a lower bound of the actual balance (including the concurrent operations). On the other hand, in the strict version (i.e., if we use a validated totally-ordered object) then the balance operations will return the exact amount of the balance. The same applies to \(\text{transfer}()\) operations. In the relaxed version some of them may be found invalid because incoming funds in concurrent transfers are not accounted for.

In order to implement the relaxed version of this object, it is enough to use the functions \(\text{valid}()\) and \(\text{execute}()\) as defined in Code \[11\] and use them in Code \[4\]. Observe that the cryptocurrency object satisfies the property of persistent validity but it does not satisfy the property of a persistent execution. Therefore, in order to implement the strict version of this object, one may combine the functions of Code \[11\] with Code \[5\] which uses the Atomic Broadcast service.

 verso.

\[\text{Theorem 19.} \] Code \[11\] combined with Code \[4\] or with Code \[5\] implement the relaxed and strict cryptocurrency (asset transfer), respectively.
5.3 Do-All: Task Execution

Do-All is an object in which a set of processes execute tasks taken from a set of jobs. Any process can take any task from the job set. In respect to the number of jobs that processes should execute, the specification of the Do-All can be traced to (1) a strict validated totally-ordered object, Definition 2, if a specific number $T$ of job’s executions must be respected, or (2) to a relaxed validated regular object, Definition 1, if executions of jobs beyond the threshold $T$ can be tolerated when some conditions are met, e.g., if they were initiated in a batch of concurrent operations.

Specifically, the Do-All object supports the following operations:

- $\text{Do}(t, i)$: process $i$ claims and performs task $t$.
- $\text{Completed}(t, i)$: process $i$ reports the completion of task $t$.

$\text{Do}()$ is not valid if a certain number of processes performed the task (say 3 for redundancy). Notice that, as mentioned, we can have the strict version (i.e., totally-order version) of the object, in which exactly 3 processes can do a task, and the relaxed version (regular version) in which at least 3 do it. Observe that this object does not satisfy the persistent validity property, nor the persistent execution one. Code 12 shows an implementation for the valid and execute predicates to realize the Do-All object in both cases. The following result holds.

$\triangleright$ Theorem 20. Code 12 combined with Code 4 or with Code 5 implement the relaxed and strict Do-All object, respectively.

6 Conclusions

In this paper we have formalized the notion of a validated object, decoupling the object operations and properties from the validation procedure. We have focused on two type of objects, satisfying different levels of consistency: the validated totally-ordered object, offering a total ordering of its operations, and its weaker variant, the validated regular object. For both types, we have provided crash-tolerant implementations. Note that these implementations only attempt to prove that it is possible to implement different types of validated objects with and without consensus. Our objective was not to make them as efficient as possible; this is the subject of future work.

For validated totally-ordered objects, we further considered the persistent validity and persistent execution properties and their impact on the object’s implementation. Our investigation has shown that (i) in the absence of persistent validity, the object is as strong as consensus, and (ii) persistent validity is not enough to implement a validated totally-ordered object without consensus; persistent execution was needed. An interesting future direction
is to investigate whether there exists a weaker property than persistent execution, that together with persistent validity would yield consensus-free implementations of validated totally-ordered objects.

Furthermore, this investigation could be extended for Byzantine failures. We believe that with certain adjustments, a Byzantine-tolerant implementation of validated regular objects can be obtained from the one presented in Section 3. Observe that the consensus-based implementation of validated totally-ordered objects presented in Section 4.1 can tolerate Byzantine failures, when a Byzantine-tolerant Atomic Broadcast service is used. Also, the negative result of Section 4.3 trivially applies to Byzantine failures. What remains to be investigated are the conditions under which it is possible to obtain Byzantine-tolerant consensus-free implementations of validated totally-ordered objects. Finally, other consistency levels for validated objects can be defined, beyond regular and totally-ordered, and their implementability in different distributed system models be explored.

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A Proof of Lemma 6

Statement. For each complete operation \( op \) (issued by \( i \)), valid(\( \{P(\langle op \rangle, \prec \rangle, op, i) = \text{True} \). Moreover, \( op \) returns in its response event the value execute(\( \{P(\langle op \rangle, \prec \rangle, op, i) \).

Proof. The claim follows if we show that the set \( P \) created in Line 6 of Code 4 is the same as \( P(\langle op \rangle) \). Recall that \( ts(\langle op \rangle) = (i, T_1, \ldots, T_k, \ldots, T_n) \). Observe that in a ledger \( L_k \) the timestamp \( ts = (k, T'_{1}, \ldots, T'_{k}, \ldots, T'_{n}) \) in the \( j \)th record in the ledger has \( T'_k = j - 1 \). Then, it holds that \( P \subseteq P(\langle op \rangle) \), because for each \( k \), for each record \( \langle ts', op' \rangle \in G_k \), the timestamp \( ts' = (k, T'_{1}, \ldots, T'_{k}, \ldots, T'_{n}) \) satisfies that \( T'_k < T_k = \lvert G_k \rvert \).

Let us assume there is an operation \( op' \in P(\langle op \rangle) \) (hence, \( op' \prec op \)) and \( op' \notin P(\langle op \rangle) \). Assume \( op' \) was issued by process \( k \), and \( ts(op') = (k, T'_1, \ldots, T'_k, \ldots, T'_n) \). Then, by linearizability of the ledgers and \( op' \notin P(\langle op \rangle) \) was appended in ledger \( L_k \) after the \( L_k.g\text{et}(\langle \rangle) \) of apply(\( \langle op, i \rangle \)) found \( T_k \) records in the ledger. Hence, \( op' \) is the \( j \)th record in ledger \( L_k \), where \( j > T_k \). Note from Code 4 that \( T'_k = j - 1 \), since by well-formedness the \( j \)th operation of process \( k \) finds \( j - 1 \) records in ledger \( L_k \). Then, \( T'_k \geq T_k \), and hence it cannot happen that \( op' \prec op \). ◀

B Versioned Read/Write Objects

A versioned object is a read/write object with the difference that each value written is associated with a version from a totally-ordered set of versions. A write operation succeeds only if it attempts to write a value with a version higher than any of the versions used by previous write operations; otherwise the write operation fails. In particular the object was introduced in [23], and supports two operations:

- \( \text{write}(\langle \langle \text{ver}, v \rangle, x \rangle) \): process \( i \) attempts to write value \( v \) with version \( \text{ver} \) on object \( x \).
- \( \text{read}(x) \): process \( i \) attempts to read the latest value and version of the object \( x \).

In the strict case only the writes that satisfy the total ordering may be executed and thus this will ensure a strict order on the version of the writes. Therefore, we will obtain a single consistent sequence of versions. On the other hand on the relaxed case, multiple writes promoting the same version may conflict allowing multiple writes to be executed. In such a case only some of those will succeed by the operation definition, thus ensuring the properties of Coverability as presented in [23].

A versioned object does not satisfy persistent validity, neither persistent execution. Thus, in order to implement the strict version of the object we use consensus. The following result holds.

Theorem 21. Code 13 combined with Code 4 or with Code 5 implement the relax and strict versioned R/W object, respectively.
Algorithm 13 Functions valid() and execute() to implement a R/W versioned object.

1: function valid((\(P, \prec\), \(op, i\))
2: if (\(i\) is not the issuer of \(op\)) then
3: \(\text{return}(\text{False})\)
4: if (\(op = \text{read}(x)\)) then
5: \(\text{return}(\text{True})\)
6: \(\text{else}\)
7: if \(op = \text{write}(\langle \text{ver}, v \rangle, x)\) then
8: \(\text{ver}_{\text{max}} \leftarrow \max \{\text{ver} : \text{write}(\langle \text{ver}, * \rangle, x) \in \mathcal{P} \}\)
9: \(\text{return}(\text{ver} > \text{ver}_{\text{max}})\)
10: \(\text{else}\)
11: \(\text{return}(\text{False})\)

12: function execute((\(P, \prec\), \(op, i\))
13: if \(op = \text{read}(x)\) then
14: \(\text{ver}_{\text{max}} \leftarrow \max \{\text{ver} : \text{write}(\langle \text{ver}, * \rangle, x, j) \in \mathcal{P} \}\)
15: \(v_{\text{max}} \leftarrow \{v : \text{write}(\langle \text{ver}_{\text{max}}, v \rangle, x, j) \in \mathcal{P} \}\)
16: \(\text{return}(\text{ver}_{\text{max}}, v_{\text{max}})\)
17: \(\text{else}\)
18: \(\text{return}(\perp)\)