Lateral-Torsional Buckling Resistance of Crane Runway Girders

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Abstract. The paper presents the method of design crane girders of a mono- and bisymmetrical cross-section. In the paper authors investigate unrestrained crane beams. Approach presented in Seeßelberg [5] was complemented with possibility of easy hand calculation according to alternative method [2]. Results coming from different sources were compared. The focus was on monosymmetric sections of I-rolled profiles reinforced by welded angles, which are commonly used in crane beams designs. Their biggest advantage is high stiffness with respect to the weaker axis. As a result, the horizontal deflection of the beam is reduced, but verification of load capacity of such an element is quite complicated. The paper presents the relationships that allow design of these elements using simple formulas, which may be implemented in a spreadsheet. Simultaneously, a simplified method has been proposed, in which, with a reduced workload, results with an error at an acceptable level are obtained. Additionally, structural verification of the load capacity of the crane girder can then be performed only using the free LTBeam program, used to determine the critical moment of lateral torsional buckling.

1. Introduction

Crane beams of overhead travelling cranes are subject to biaxial bending and torsion due to the crane operation with the wheel loads are the eccentric horizontal loads. The crane runway girders are prone to lateral-torsional buckling due to this loading condition. In the case, where hot rolled sections are used, the required lateral stiffness of the top flange is often not sufficient, even for light crane operations and medium span lengths. In order to increase the lateral stiffness, on the top flanges of the beams, there are welded angle profiles on both sides of the top flange. This type of construction is a widespread application for crane runway beams, especially for medium and heavy crane operations. However, the assessment method for lateral-torsional buckling is for this type of crane runway beam more difficult because of the monosymmetric cross-section together with the loading condition (biaxial bending and torsion). Within the scope of this article, the different possibilities for the assessment methods of lateral-torsional buckling are investigated for crane girders built from hot rolled profiles combined with angle sections as mentioned.

Crane runway beams are subject to biaxial bending and torsion due to the Vlasov’s theory. Method of assessment the lateral-torsional buckling resistance of crane girders with mono- and bisymmetrical cross-sections is included in EN 1993-6: 2007 [2]. In the method presented there, we neglect the impact of displacements when determining the cross-sectional forces. In contrast, guidelines for the design of
crane beams, included in the study of the British Institute of Steel Constructions [1], require both determin- 
ing the bending moment with respect to the axis z and the horizontal force in deformed configuration (after twisting the section by angle $\phi$). The result of such approach is an increase of both cross-sectional 
forces, but it complicates the calculations [3].

Formulas for design of crane runway beams having the combined cross-section are complicated 
due to the terms coming from the Vlasov’s theory. The purpose of the work was, first, to simplify cal-
culation of torsional characteristics of cross-section. Thanks to this, one can implement them in any 
spreadsheet. Secondly, the goal was to propose a way to check the bearing capacity of crane girders, 
neglecting the terms coming from Vlasov’s theory. Fig. 1a presents an example of a monosymmetric 
cross-section and an additional bending moment with respect to a weaker axis (z) in a deformed config-
uration.

\begin{equation}
M_{z,Ed} = M_{z,Ed} + \phi_d M_{y,Ed}
\end{equation}

\begin{equation}
H_k = H_k + \phi_k Q_k
\end{equation}

where: $M_{z,Ed}, M_{y,Ed}$ – the design values of the maximum moments about the $z$- and $y$- 

axes respectively; $\phi_d, \phi_k$ – twisting angles; $Q_k, H_k$ – vertical and horizontal forces respectively (lower index $d$ denotes design and $k$ characteristic values).

The horizontal deflection results from both the horizontal force $H_k$ and the twisting of the cross-

section determined on basis of Vlasov’s theory:

\begin{equation}
f_y = \max_{0 \leq x \leq L} \left| \sum_i \left[ \frac{\eta_i(x) + e_z \phi_k(x)}{600} \right] \right| \leq \frac{L}{600}
\end{equation}

where: $\eta_i(x)$ – displacement due to horizontal load $H_k$, $e_z$ – ordinate of the load application point relative to the shear center $S$ (Fig. 1a).
2. Lateral-torsional buckling resistance of crane mono- and bisymmetric girders

2.1 Alternative method according to EN 1993-6:2007

The calculation model described below will be denoted as EN2 method. The crane girder’s verification condition should satisfy the following interactive formula [2]:

\[
\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk} / \gamma_{MI}} + \frac{C_{mc} M_{z,Ed}}{M_{z,Rk} / \gamma_{MI}} + k_{zw} k_w k_{\alpha} \frac{B_{w,Ed}}{M_{w,Rk} / \gamma_{MI}} \leq 1 \tag{4}
\]

where:

\[
k_{zw} = 1 - \frac{M_{z,Ed}}{M_{z,Rk} / \gamma_{MI}}, \quad k_w = 0.7 - 0.2 \frac{B_{w,Ed}}{M_{w,Rk} / \gamma_{MI}}, \quad k_{\alpha} = \frac{1}{1 - M_{y,Ed} / M_{cr}},
\tag{5abc}
\]

and \( M_{y,Ed}, M_{z,Ed}, B_{w,Ed} \) – the design values of the maximum moments about the \( y \)-\( y \) and \( z \)-\( z \) axis and warping torsional moment respectively, \( M_{y,Rk}, M_{z,Rk}, B_{w,Rk} \) – characteristic values of the resistance moments of the cross-section about its \( y \)-\( y \) and \( z \)-\( z \) axis and warping torsional resistance moment respectively, \( M_{cr} \) – elastic critical lateral-torsional buckling (LTB) moment, \( \chi_{LT} \) – is reduction factor for lateral-torsional buckling, \( C_{mc} \) – equivalent uniform moment factor for bending about the \( z \)-\( z \) axis, \( \gamma_{MI} \) – partial safety factor. This method is valid under conditions: \( I_{z,t} / I_{z,c} \geq 0.2 \) (\( I_{z,c} \) and \( I_{z,t} \) are the second moments of area about the \( z \)-\( z \) axis for the compression and tension flanges respectively) and \( B_{w,Ed} (B_{w,Rk} / \gamma_{MI}) \leq 0.3 \).

Another calculation model, denoted further as SCI takes into account the dependencies (1÷3). This comes from British guidance [3] for beams under torsion. As this approach leads to increased values of internal forces, automatically the verification formulas are slightly more conservative.

2.2 Assessment of twist angle and warping torsional moment for common cases of crane girders

In case shown in Fig. 2a (simple supported beam, single load) Table 1 presents the functions of twisting angle \( \varphi(x) \) and warping torsional moment \( B_w(x) \) for the thin-wall cross-section beam.

| Table 1. Formulas for twisting angle \( \varphi(x) \) and warping torsional moment \( B_w(x) \) (Fig. 2a) |
|---------------------------------------------------------------|
| Left side \((0 \leq x \leq a)\)                                    | Right side \((a < x \leq L)\)                                           | Value for \( x = a \)                                       |
| \( \varphi(x) = \frac{T}{kG_lT} \left( \frac{b}{L} \cdot \frac{shkL}{shkL} \right) \) | \( \varphi(x) = \frac{T}{kG_lT} \left( \frac{a}{L} \cdot \frac{shkL}{shkL} \right) \) | \( \varphi(x = a) = \frac{T}{kG_lT} \left( \frac{ab}{L} \right) \) |
| \( B_w(x) = \frac{T}{k} \cdot \frac{shkL}{shkL} \)               | \( B_w(x) = \frac{T}{k} \cdot \frac{shkL}{shkL} \)                      | \( B_w(x = a) = \frac{T}{k} \cdot \mu \)                  |

where: \( x = L - x \), \( k = \sqrt{G_lT / EI_w} \), \( I_T \) – St. Venant torsion constant, \( I_w \) – warping moment of inertia, \( E \) – Young’s modulus, \( G \) – shear modulus, \( \mu = shkL \cdot shkL / shkL \).
In case shown in Fig. 2b (simple supported beam, double load) from Table 1 results:

\[ \varphi(x = a) = \frac{T}{kG}\left(\frac{k}{L}\mu + \mu_1\right), \quad B_w(x = a) = \frac{T}{k}\left(\mu - \mu_1\right), \]  

(7ab)

where: \( \mu_1 = \frac{shk}{shk(b-c)/shkL} \).

Formulas for verification of load capacity of double span crane runway beams (Fig. 2c) are becoming little more complicated. In this case, you can take functions from table 1 as for a single-span beam.

For investigated simplified method, marked as SP, verification formula is as follows:

\[ \frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk} / \gamma_{M1}} + \frac{2C_{me} M_{z,Ed}}{M_{z,Rk} / \gamma_{M1}} \leq 1 \]  

(6)

2.3 Simplified method according to EN 1993-6:2007

The calculation model described below will be denoted as EN1 method and it refers to simplified assessment of lateral torsional buckling resistance of a simply supported beam. In this alternative standard method, section of girder is reduced to compression flange plus one fifth of the web and is verified against flexural buckling as a compression member. Verification should be performed with respect to axial compressive force equal to the bending moment divided by the distance between the centroids of the flanges.

Fig. 3. Assumptions for simplified method

\[ \frac{N_{OG,Ed}}{\chi_z \cdot A_{OG} \cdot f_y / \gamma_{M1}} + k_{zz} \frac{M_{z,Ed}}{W_{OG,z} \cdot f_y / \gamma_{M1}} \leq 1 \]  

(12)

\[ N_{OG,Ed} = M_{y,Ed} \frac{S_{OG,y}}{I_y} \]  

(13)

where \( h_w \) - height of the web, \( A_{OG} \), \( W_{OG,z} \), \( S_{OG,y} \) - section area, elastic modulus with respect to z axis, and first moment of area with respect to y for OG section, \( L_{ex,z} = L \) for single span beam and \( L_{ex,z} = 0.85L \) for double span beam. Other parameters are determined from formulas:

\[ n_z = \frac{N_{OG,Ed}}{\chi_z \cdot A_{OG} \cdot f_y / \gamma_{M1}}, \quad k_{zz} = \begin{cases} C_{me} \left[1 + (2\bar{x} - 0.6)n_z\right] \leq C_{me} (1+1.4n_z) & \text{Class 1,2} \\ C_{me} (1+0.6\bar{x}n_z) \leq C_{me} (1+0.6n_z) & \text{Class 3} \end{cases} \]  

(14ab)

where: \( C_{me} = 0.9 \). For I-section reinforced with angle sections, class 3 of the section is assumed.

2.4 Parameters of asymmetry of a monosymmetric section

When dimensioning the gantry beam with a cross-section made of I-section reinforced with angles, problems start already while trying to determine its torsional parameters. The algorithm for estimation the asymmetry parameters of the monosymmetric cross section (notation according to Fig. 2) is shown below.
First, we calculate the coordinate \( z_M = (z_A - (1/I_z) \int \omega_A y dA) \) of the shear center \( M \):

\[
z_M = h_2 - L_{z1} h_0 - L_{z3} \cdot \frac{c}{2} \quad \frac{I_z}{I_z}
\]

where: \( L_{z1} = t_f b_1^{3/2} /12, L_{z3} = c b_2^{3/2} /2 \) (\( t_f, t \) Fig.1, \( b_1, b_2, h_0, c \) Fig.4a).

The warping moment of inertia \( I_w = ( \int \omega_A z dA ) \) equals to [9]:

\[
I_w = I_{wA} - (h_2 - z_M)^2 \cdot I_z, \quad I_{wA} = I_{z1} \cdot h_0^2 + I_{z3} \cdot c^2 / 3
\]

Sectorial coordinates related to shear center \( M \) (Fig. 4b) in points 1 and 2 are:

\[
\omega_1 = \frac{d_1 b_1}{2}, \quad \omega_2 = -\frac{d_2 b_2}{2} - \frac{c b_2}{2} \quad \frac{1}{17a-b}
\]

The parameter of asymmetry \( \beta_z = (0.5 \int (z^2 + y^2) dA_0) / I_y z_M \) is determined from the formula:

\[
\beta_z = \frac{p_0 + p_1 + p_2 + 2p_3}{2I_y} - z_M \quad \frac{18}{19a-d}
\]

where: \( t_w \) web thickness, \( h_1-h_2-t_f/2, h_2-h_2-t_f/2, A_1 = t_f b_1, A_2 = t_b y_2 + (t_f - t) b_1, I_{z2} = t b_2^3 / 12 + (t_f - t) b_1^3 / 12. \)

For monosymmetric, welded L-section we have:

\[
\begin{align*}
z_M &= h_2 - L_{z1} \cdot h_0 \quad \frac{I_z}{I_z} \quad \frac{20a-c}{L_{z2}} \\
I_w &= I_{wA} = \frac{I_{z1} \cdot I_{z2}}{I_z} \quad \frac{h_0^2}{h_0^2} \quad \frac{p_0 + p_1 + p_2}{2I_y} - z_M \quad \frac{19a-d}{19a-d}
\end{align*}
\]

where: \( A_1 = t_f b_1 I_{z1} = t_f b_1^3 / 12, A_2 = t_f b_2, I_{z2} = t_f b_2^3 / 12 \) (\( t_f, t_f \) - thickness of the bottom and top flange, respectively)

Let’s consider a crane girder with a section built from HEA450 I-beam reinforced with two angles L100x16 [2]. Table 2 compares torsional cross-section properties, determined using the PropSection program [10] with those calculated on basis of formulas above. In the FEM program [10], we determine the torsional characteristics by solving the St. Venant pure torsion problem (Neumann problem) using triangular finite elements.
Table 2. Comparison of torsional properties determined using the PropSectionFEM programme [10] and analytical rules (15÷18).

| Property   | FEM [10] | Eq. (15÷19) | Relative error |
|------------|----------|-------------|----------------|
| $I_w \cdot 10^3$ | 8297.6   | 8417        | 1.4%           |
| $I_T$      | 309.6    | 294         | -5%            |
| $z_M$      | 12.99    | 13.2        | 1.7%           |
| $\beta_z$  | -15.34   | -15.48      | 0.9%           |

3. Example

For simplicity, it was neglected dead weight of crane girder and the rail. It was assumed the crane girder beam’s section is composite profile built from HEA450, reinforced with two angles L100x16. Span of the beam is equal to $L=7.5$ m, distance between points, where load is applied is assumed to be $c=3$m. Cross-section properties are given in Table 2 – analytical formulas. The following data has been adopted for the calculations (Fig.4a): $h_0=419$ mm, $h_2=163$ mm, $A_1=63$ cm$^2$, $I_{z1}=4725$ cm$^4$, $A_2=92.4$ cm$^2$, $I_{z2}=16240$ cm$^4$, $A_3=14.7$ cm$^2$, $I_{z3}=17240$ cm$^4$, $I_z=38490$ cm$^4$, $I_y=79760$ cm$^4$, $e_z=127$ mm). In order to determine the lateral buckling moment they were used LTBeam [11] programme.

Table 3. Cross-section resistance cross-sectional forces

| $G I_T$ [kNm$^2$] | $E I_w$ [kNm$^4$] | $k$ [1/m] | $M_{y,Rk}$ [kNm] | $M_{z,Rk}$ [kNm] | $B_{w,Rk}$ [kNm$^2$] | $c\over L$ |
|-------------------|-------------------|----------|------------------|------------------|----------------------|----------|
| 238.1             | 17678             | 0.367    | 703.1            | 361.8            | 34.0                 | 0.4      |

The simple supported beam was considered. Geometry and loads are shown in Fig 5. Three load cases (load groups) were verified, as they seem the most onerous for crane girders of such type.

Fig. 5 The basic load cases for crane girders a) load group 1, b, c) load group 5 (characteristic value: $H_T=17.5$ kN, $T_T=2.22$ kNm, $H_5=20$ kN, $H_5=2.54$ kNm)

Estimation of load capacity for given load groups is shown in tables 4-6 below.

Table 4. Load group 1 (Fig.5a)

| $M_{y,Ed}$ [kNm] | $M_{z,Ed}$ [kNm] | $M_{z,Ed}$ [kNm] | $B_{w,Ed}$ [kNm] | $M_{y}$ [kNm] | $a$ [m] | SCI | EN2 | SP |
|------------------|------------------|------------------|------------------|---------------|--------|-----|-----|----|
| 324              | 28.35            | 29.6             | 2.75             | 3610          | 3.00   | 0.76| 0.76| 0.78|
Table 5. Load group 5 (Fig.5b)

| $M_{y,Ed}$ | $M_{z,Ed}$ | $M_{z,Ed}$ | $B_{w,Ed}$ | $M_{cr}$ | $a$ | SCI | EN2 | SP |
|------------|------------|------------|------------|----------|----|-----|-----|----|
| [kNm]      | [kNm]      | [kNm]      | [kNm]      | [kNm]    | [m] |     |     |    |
| 275.4      | 48.6       | 51.2       | 4.1        | 3610     | 3.00| 0.75| 0.75| 0.80|

Table 6. Load group 5 (Fig.5c)

| $M_{y,Ed}$ | $M_{z,Ed}$ | $M_{z,Ed}$ | $B_{w,Ed}$ | $M_{cr}$ | $a$ | SCI | EN2 | SP |
|------------|------------|------------|------------|----------|----|-----|-----|----|
| [kNm]      | [kNm]      | [kNm]      | [kNm]      | [kNm]    | [m] |     |     |    |
| 258.2      | 50.63      | 53.2       | 4.2        | 3880     | 3.75| 0.72| 0.72| 0.78|

4. Conclusions

Both methods (EN2, SP) proposed to check the bearing capacity of crane girders of a monosymmetric cross section are quite easy to implement in any spreadsheet. However, neglecting terms coming from twisting according to Vlasov’s theory makes this task extremely easy. Verification of the load capacity of the crane girder can be done with the help of any FEM program for beams. It does not cause additional complications, for instance in the case of a double-span beams. As it can be easily noted the most conservative method is simplified one (SP), but the difference of load capacity estimated according to SP is quite close to results obtained employing methods SCI or EN2. Thus at cost of little bigger margin of safety, we are gaining great simplification of calculation. However, it should be noted, that difference is not fundamental for typical designs, what will be illustrated with examples.

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