Performance Analysis for Physical Layer Security in Multi-Antenna Downlink Networks with Limited CSI Feedback

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Abstract—Channel state information (CSI) at the transmitter is of importance to the performance of physical layer security based on multi-antenna networks. Specifically, CSI is not only beneficial to improve the capacity of the legitimate channel, but also can be used to degrade the performance of the eavesdropper channel. Thus, the secrecy rate increases accordingly. This letter focuses on the quantitative analysis of the ergodic secrecy sum-rate in terms of feedback amount of the CSI from the legitimate users in multiuser multi-antenna downlink networks. Furthermore, the asymptotic characteristics of the ergodic secrecy sum-rate in two extreme cases is investigated in some detail. Finally, our theoretical claims are confirmed by the numerical results.

Index Terms—Physical layer security, CSI feedback, ergodic secrecy sum-rate, asymptotic characteristics.

I. INTRODUCTION

Due to the open nature of wireless channel, information transmission security is always a critical issue in wireless communications. Traditionally, secure communication is realized by using cryptography technology. With the development of interception technology, cryptography technology becomes more and more complex, resulting in high computation In fact, it has been proven by information theory that wireless security can be guaranteed by physical layer technology, namely physical layer security [1] [2]. The advantage of physical layer security lies in that it is independent of the interception ability of the eavesdropper, so it is appealing in secure communications with low-complexity nodes.

The performance of physical layer security depends on the difference between the legitimate channel capacity and the eavesdropper channel capacity, namely secrecy rate [3]. Intuitively, multi-antenna techniques can improve the legitimate channel capacity and degrade the performance of the eavesdropper channel simultaneously by exploiting the spatial degree of freedom. Thus, physical layer security based on multi-antenna system draws considerably attentions [4] [5]. In [6], the problem regarding the maximum secrecy rate in multiple-antenna system was addressed by designing an optimal transmit beam, assuming that full CSI related to the legitimate and eavesdropper channels is available at the transmitter. However, in practical systems, the eavesdropper is usually passive and hidden, so the transmitter is difficult to obtain the eavesdropper CSI. Under such a condition, it seems impossible to provide a steady secrecy rate, and thus the ergodic secrecy rate is adopted as a useful and intuitive metric to evaluate security. In [7], the ergodic secrecy rate for Gaussian MISO wiretap channels was analyzed and the corresponding optimal transmit beam was presented. The work shown in [8] considered the scenario where all nodes have multiple antennas, and the ergodic rate for such an MIMO secrecy scenario has been developed. Another advantage of the multi-antenna secrecy system lies in that it supports concurrent transmission of multiple legitimate users. On one hand, multiuser transmission can improve the ergodic secrecy sum-rate. On the other hand, the inter-user interference can weaken the interception effect. In [9], the ergodic secrecy sum-rate for orthogonal random beamforming with opportunistic scheduling was presented. Yet, in multiuser downlink networks, orthogonal random beamforming may suffer performance loss if the number of users is not so large. In [10], the authors proposed to convey the quantized CSI from the legitimate user to the transmitter for transmit beam design in a single receiver secure scenario.

Combining the advantages of multiuser transmission and CSI conveyance, this letter proposes to perform limited feedback zero-forcing (ZFBF) at the transmitter in multiuser multi-antenna downlink networks [11]. We focus on the quantitative analysis of the ergodic secrecy sum-rate, and derive its closed-form expression in terms of the feedback amount. Furthermore, the asymptotic characteristics of the ergodic secrecy sum-rate is analyzed, so as to provide an explicit insight on system parameter optimization for physical layer security in multiuser multi-antenna downlink networks.

The rest of this letter is organized as follows. We first provide an overview of the multiuser secure system model in Section II, and then the ergodic secrecy sum-rate is derived in Section III. We investigate the asymptotic characteristics of the ergodic secrecy sum-rate in Section IV. Some numerical results are given to show the accuracy of the theoretical analysis in Section V. Finally, the whole letter is concluded in Section VI.

II. SYSTEM MODEL

We consider a homogeneous multiuser multi-antenna downlink network employing physical layer security for secure communication, where a base station (BS) equipped with $N_t > 1$
antennas communicates with $K$ single antenna legitimate users (LU), while a passive single antenna eavesdropper attempts to intercept the transmission information, as shown in Fig.1. In this letter, we only consider the case of $K = N_t$, since the multi-antenna system can admit $N_t$ users at most for each time slot. Note that if $K > N_t$, user scheduling can be adopted to select $N_t$ users. We use $h_k$ to denote the $N_t$ dimensional legitimate channel vector from the BS to the $k$th LU with independent and identically distributed (i.i.d.) zero mean and unit variance complex Gaussian entries. In addition, we use $\alpha g$ to denote the $N_t$ dimensional eavesdropper channel vector from the BS to the eavesdropper, where $\alpha$ is the relative path loss defined as the ratio of path loss of the eavesdropper channel and that of the legitimate channel, and $g$ is the channel fast fading distributed as $CN(0, I_{N_t})$.

The whole network is operated in slotted time. At the beginning of each time slot, the LU first estimates the CSI related to its legitimate channel, and then chooses an optimal codeword to quantize the CSI from a predetermined quantization codebook $H_k = \{h_{k,1}, \cdots, h_{k,2^B}\}$ of size $2^B$ according to the following selection criterion by assuming perfect CSI at the LU:

$$h_{k,\text{opt}} = \arg \max_{h_{k,i} \in H_k} \left| h_{k,i}^H h_{k,\text{opt}} \right|^2$$

(1)

where $h_{k,\text{opt}} = \frac{h_{k,\text{opt}}}{\|h_{k,\text{opt}}\|}$ is the channel direction vector. Next, the index of the optimal codeword is conveyed by the $k$th LU and $h_{k,\text{opt}}$ is recovered at the BS from the same codebook as the instantaneous CSI of the $k$th legitimate channel. Based on the $N_t$ LUs’ feedback information, the BS determines the optimal transmit beam $w_k$ for the $k$th LU by making use of zero-forcing beamforming (ZFBF) design method. Specifically, given $h_{k,\text{opt}}$, we first construct its complementary matrix

$$\hat{H}_k = [h_{k,\text{opt}}, \cdots, h_{k-1,\text{opt}}, h_{k+1,\text{opt}}, \cdots, h_{N_t,\text{opt}}]$$

Taking singular value decomposition (SVD) to $\hat{H}_k$, if $V_k^\perp$ is the matrix composed of the right singular vectors with zero singular values, then we randomly choose a unit norm vector from the space spanned by $V_k^\perp$ as the transmit beam $w_k$. Since $V_k^\perp$ is the null space of $\hat{H}_k$, we have

$$h_{k,\text{opt}}^H w_k = 0, i \neq k$$

(2)

Thus, the receive signals at the $k$th LU and the eavesdropper can be expressed as

$$y_k = P h_k^H w_k x_k + \sqrt{P} h_k^H \sum_{i=1,i\neq k}^{N_t} w_i x_i + n_k$$

(3)

and

$$y_0 = \sqrt{P} \alpha g \sum_{i=1}^{N_t} w_i x_i + n_0$$

(4)

respectively, where $x_k$ is the normalized Gaussian distributed transmit signal for the $k$th LU, $P$ is the transmit power, $n_k$ is the additive Gaussian white noise with variance $\sigma^2$. In this context, the ergodic secrecy sum-rate can be expressed as

$$R = \sum_{k=1}^{N_t} E[\log_2(1 + \gamma_k)]$$

$$= \sum_{k=1}^{N_t} E[\log_2(1 + \gamma_k)] - \sum_{k=1}^{N_t} E[\log_2(1 + \zeta_k)]$$

(5)

where

$$\gamma_k = \frac{|w_k^H h_k|^2}{\sum_{i=1,i\neq k}^{N_t} |w_i^H h_k|^2 + \sigma^2/P}$$

and

$$\zeta_k = \frac{|w_k^H h_k|^2}{\sum_{i=1,i\neq k}^{N_t} |w_i^H h_k|^2 + \sigma^2/\alpha P}$$

are the SINR related to the $k$th LU’s signal at the $i$th LU and the eavesdropper. (5) follows the fact that the legitimate and eavesdropper channels are independent of each other.

The focus of this letter is on the quantitative analysis of the ergodic secrecy sum-rate in terms of feedback amount $B$ and BS antenna number (LU number) $N_t$.

III. PERFORMANCE ANALYSIS OF PHYSICAL LAYER SECURITY

In this section, we intend to derive the ergodic secrecy sum-rate $R$ in a multiuser multi-antenna downlink network in presence of a passive eavesdropper. As seen in (5), the key of computing the ergodic secrecy sum-rate is to obtain the distributions of the SINRs $\gamma_k$ and $\zeta_k$. In what follows, we first give an investigation of their distributions employing limited feedback ZFBF. According to the theory of random vector quantization (RVQ) [12], the relationship between the original and the quantized channel direction vectors is given by

$$h_k = \sqrt{1-a} h_{k,\text{opt}} + \sqrt{a} s_k$$

(6)

where $a = \sin^2\left(\frac{\theta}{2} (h_k, h_{k,\text{opt}})\right)$ is the magnitude of the quantization error, and $s_k$ is an unit norm vector isotropically distributed in the nullspace of $h_{k,\text{opt}}$, and is independent of $a$.

Substituting (6) into the definition of the SINR $\gamma_k$, we have

$$\gamma_k = \frac{|w_k^H h_k|^2}{\|h_k\| \sum_{i=1,i\neq k}^{N_t} |s_i^H w_k|^2 + \sigma^2/P}$$

(7)

$$d = \frac{\chi^2}{\Gamma(N_t-1, \delta) \sum_{i=1,i\neq k}^{N_t} \beta(1, N_t-2) + \sigma^2/P}$$

(8)
where \( d \) denotes the equality in distribution, and \( \delta = 2 - \frac{2a}{\alpha - 1} \).
Eq. (7) holds true by applying the property of limited feedback ZFBF in [3]. In (9), \( \mathbf{w}_k \) is designed regardless of \( \mathbf{h}_k \), so \( \|\mathbf{h}_k^H \mathbf{w}_k\|^2 \) is \( \chi^2_2 \) distributed with the probability density function (pdf) of \( \exp(-x) \). According to the theory of quantization cell approximation [13], the product of channel gain \( \|\mathbf{h}_k\|^2 \) and the magnitude of quantization error \( a \) is \( \Gamma(N_t - 1, \delta) \) distributed with the pdf of \( \frac{\delta x^{N_t-1} \exp(-\delta x)}{\Gamma(N_t-1)} \). Additionally, since \( \mathbf{s}_k \) and \( \mathbf{w}_k \) are i.i.d. isotropic vectors in the \( N_t - 1 \) dimensional null space of \( \mathbf{h}_k, \), \( \|\mathbf{h}_k^H \mathbf{w}_k\|^2 \) is \( \beta(1, N_t - 2) \) distributed with pdf of \( (N_t - 2)(1-x)^{N_t-3} \) [12]. In (9), the product of a \( \Gamma(N_t - 1, \delta) \) distributed random variable and a \( \beta(1, N_t - 2) \) distributed random variable is distributed \( \delta \chi^2_2 \) distributed random variables is \( \frac{\delta x^{N_t-1} \exp(-\delta x)}{\Gamma(N_t-1)} \). Let \( y \sim \chi^2_2 \) and \( z \sim \chi^2_2 \), the cumulative distribution function (cdf) of \( \gamma_k \) is derived as

\[
F(x) = P_t \left( \frac{\gamma}{\gamma + \sigma^2 / P} \leq x \right)
\]

\[
= \int_0^\infty F_{Z|Y}(x(\gamma y + \sigma^2 / P)) f_Y(y) dy
\]

\[
= \int_0^\infty \left( 1 - \exp\left( -x(\gamma y + \sigma^2 / P) \right) \right) \frac{y^{N_t-1} \exp(-y)}{\Gamma(N_t-1)} dy
\]

\[
= 1 - \frac{\exp(-x \sigma^2 / P)}{(1 + \delta x)^{N_t-1}} \int_0^\infty \frac{\exp(-y)}{\Gamma(N_t-1)} \frac{y^{N_t-1} \exp(-y)}{\Gamma(N_t-1)} dy
\]

\[
= 1 - \frac{\exp(-x \sigma^2 / P)}{(1 + \delta x)^{N_t-1}}
\]

(10)

where \( F_{Z|Y}(\cdot) \) is the conditional cdf of \( z \) for a given \( y \), \( f_Y(\cdot) \) is the probability density function (pdf) of \( y \), and \( \Gamma(\cdot) \) is the Gamma function. (10) holds true since \( \exp(-x(\gamma y + \sigma^2 / P)) \) is the pdf of \( (1 + \delta x) y \).

Examining the definition of \( \zeta_k \), it can be considered as a special \( \gamma_k \) without CSI feedback, namely \( B = 0 \) or \( \delta = 1 \), so the cdf of \( \zeta_k \) can be derived based on (10) as

\[
G(x) = 1 - \frac{\exp(-x \sigma^2 / \alpha^2 P)}{(1 + x)^{N_t-1}}
\]

(11)

Substituting (10) and (11) into (5), we have

\[
R = N_t \int_0^\infty \log_2(1 + x) \left( \frac{F(x)}{G(x)} - G'(x) \right) dx
\]

\[
= N_t \log_2(e) \int_0^\infty \left( \ln(1 + x) - \frac{G(x)}{1 + x} \right) dx
\]

\[
= N_t \log_2(e) \int_0^\infty \left( \frac{1 - F(x)}{1 + x} - \frac{G(x)}{1 + x} \right) dx
\]

\[
= N_t \log_2(e) \left[ -\frac{\exp(-x \sigma^2 / P)}{(x + 1) (x + \delta - 1)^{N_t-1}} \right]_0^\infty dx
\]

\[
= N_t \log_2(e) \left[ \frac{\exp(-x \sigma^2 / \alpha^2 P)}{(x + 1)^{N_t-1}} \right]_0^\infty dx
\]

\[
= N_t \log_2(e) \left( \frac{\exp(-x \sigma^2 / \alpha^2 P)}{(x + 1)^{N_t-1}} \right)_0^\infty dx
\]

\[
= \frac{N_t \log_2(e)}{\delta N_t-1} I_1 \left( \sigma^2 / P, \delta^{-1}, N_t - 1 \right)
\]

IV. ASYMPTOTIC PERFORMANCE ANALYSIS

In this section, for the sake of evaluating the performance easily, we analyze the asymptotic characteristics of ergodic secrecy sum-rate in some extreme cases.

A. Interference-Limited Case

If transmit power \( P \) is quite high or LU number is so large, the receive noise is negligible with respect to the inter-user interference. Thus, the SINRs are reduced as \( \gamma_k = \frac{\|\mathbf{h}_k^H \mathbf{w}_k\|^2}{\sum_{i=1,i\neq k}^N \|\mathbf{h}_i^H \mathbf{w}_i\|^2} \) and \( \zeta_k = \frac{\|\mathbf{h}_k^H \mathbf{w}_k\|^2}{\sum_{i=1}^N \|\mathbf{w}_i\|^2} \), respectively. In this case, the cdfs can be expressed as

\[
F(x) = 1 - \frac{1}{(1 + \delta x)^{N_t-1}}
\]

(13)

and

\[
G(x) = 1 - \frac{1}{(1 + x)^{N_t-1}}
\]

(14)

Similar to (12), the ergodic secrecy sum-rate can be computed as

\[
R = \frac{N_t \log_2(e)}{\delta N_t-1} \int_0^\infty \frac{1}{(x + 1)(x + \delta - 1)^{N_t-1}} dx
\]

\[
= \frac{N_t \log_2(e)}{\delta N_t-1} \int_0^\infty \frac{1}{(x + 1)^{N_t-1}} dx
\]

\[
= N_t \log_2(e) B(1, N_t - 1) 2 F_1 \left( N_t - 1, 1; N_t; 1 - \delta \right)
\]

\[
= N_t \log_2(e) / (N_t - 1)
\]

(15)
where $B(x, y)$ is the Beta function and $2F_1(\alpha, \beta; \gamma; z)$ is the Gauss hypergeometric function. Eq. (15) is derived according to [eq. 3.1971, 15]. It is found that in interference-limited case, the ergodic secrecy sum-rate is independent of transmit power $P$ and channel condition $\alpha$. Given $N_t$ and $B$, the ergodic secrecy sum-rate is a constant.

**B. Noise-Limited Case**

If the inter-user interference is negligible due to low transmit power or strong noise, the SINRs can be approximated as $\gamma_k = \frac{P}{\sigma^2} |h_k^H w_k|^2$ and $\zeta_k = \frac{P_{o2}}{\sigma^2} |g_k^H w_k|^2$, respectively. As analyzed earlier, both $|h_k^H w_k|^2$ and $|g_k^H w_k|^2$ are $\chi_2^2$ distributed, so the cdfs of the SINRs are given by

$$F(x) = 1 - \exp(-\sigma^2/Px)$$

and

$$G(x) = 1 - \exp(-\sigma^2/(P\alpha^2)x)$$

In this case, the ergodic secrecy sum-rate can be computed as

$$R = N_t \log_2(e) \left( \exp(\sigma^2/P) \text{Ei}(\sigma^2/P) - \exp(\sigma^2/(P\alpha^2)) \text{Ei}(\sigma^2/(P\alpha^2)) \right)$$

(18)

It is found that the ergodic secrecy sum-rate in noise-limited case is independent of feedback amount. In other words, the ergodic secrecy sum-rates with different $B$s asymptotically approach the same value as $P$ decreases.

**V. NUMERICAL RESULTS**

To examine the accuracy of the derived ergodic secrecy sum-rate in multiuser multi-antenna downlink networks, we present several numerical results in the following scenarios: we set $N_t = 5$ and $B = 4$. In addition, we define $\text{SNR} = 10 \log_{10} \frac{P}{\sigma^2}$ as the transmit SNR.

**VI. CONCLUSION**

This letter focused on the performance analysis for physical layer security in multiuser multi-antenna downlink networks with quantized CSI feedback, and derived the closed-form expression of ergodic secrecy sum-rate in terms of feedback amount and channel condition. Through asymptotic analysis, we obtained the relatively simple expression of ergodic secrecy sum-rate in two extreme scenarios. Numerical simulation reconfirmed the high accuracy of the derived theoretical results.

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