Chargino contribution to the rare decay $b \rightarrow ssd\bar{d}$

Xiao-Hong Wu* and Da-Xin Zhang†

Institute of Theoretical Physics,
School of Physics, Peking University, Beijing 100871, China

February 6, 2004

Abstract

The rare decay $b \rightarrow ssd\bar{d}$ is studied in the supersymmetric standard model by considering the contribution from the chargino box diagrams. We find that this contribution amounts to $10^{-9}$ in the branching ratio.

PACS: 12.15.Mm, 13.90.+i, 14.40.Lb, 14.40.Nd
A very important aspect in the study of B physics is to expose possible virtual effects due to physics beyond the Standard Model (SM). These virtual effects are likely to be hidden in the processes induced by flavor changing neutral current (FCNC) interactions, since the SM contributions come from loop diagrams so that they do not always dominate over the new physics contributions. Furthermore, since the SM contributions serve as the background, the rare decays with unobservable SM contributions are more suitable to probe the new physics signals. As was pointed out in [1] and [2], the rare decay $b \rightarrow ss\bar{d}$ is a clean channel with less SM background. The quark-level transition materializes as decays of B mesons into $S = 2$ final states, of which a fraction $1/4$ are states with two charged kaons which can be identified at future experiments. Several exclusive channels of $b \rightarrow ss\bar{d}$ transition have been searched by different groups [3]. More data will improve the bounds on these transitions and on the possible new physics.

The rare decay $b \rightarrow ss\bar{d}$ in the SM proceeds with a box diagram which is strongly suppresses by the second order weak interactions and by the Glashow-Maiani-Illiopoulous (GIM) cancellations. In the SM this branching ratio is too small to be observable. This may help to expose the possible signal of new physics clearly from the background of the SM. Motivated by this observation, several new physics models has been studied for this process[1, 2, 4].

In this Letter we will consider the chargino contribution to $b \rightarrow ss\bar{d}$ within the Minimal Supersymmetric Standard Model (MSSM). In the MSSM, the charged Higgs contribution is negligible[2]. The gluino-squark contribution has been studied in [1] which can be as large as $10^{-8}$ in the branching ratio, depending on the parameter space. The chargino contribution, which is usually another important source of the FCNC interactions, is to be studied below.

In the MSSM, the chargino contribution to $b \rightarrow ss\bar{d}$ is from the chargino and the up-type squark diagrams. These up-type squarks are not degenerate and do not align with the up-type quarks in the super-multiplets. We use the mass insertion approach to estimate the chargino contribution. Following the notations of [5], the couplings of the left-handed down-type quarks, the left-handed up-type squarks and
the charginos are flavor diagonal. In this basis, the mass-squared matrix for the up-type squarks is not diagonal. The off-diagonal elements, which will be denoted as $\Delta_{ij}^{LL}$ ($i, j = 1, 2, 3$), are taken as the two-point vertices. Consequently, the insertions of these two-point vertices on the up-type squarks propagators inside the loops induce the transitions between different external down-type quarks. Similarly, since the right-handed stop couples with the higgsino components of the charginos with a strength which is proportional to large Yukawa coupling of the top quark, there are also the diagrams with the insertions of the left-handed up-type squarks and the right-handed stop $\Delta_{3i}^{LR}$ ($i = 1, 2, 3$) inside the loops. Note that the possible large splitting in the masses of the left-handed and the right-handed stops induces another source of FCNC.

The relevant effects are parameterized as

$$
R_{sb}^{LL} = \frac{K_{is}^* K_{jb} \Delta_{ij}^{LL}}{K_{ts}^* K_{tb} \tilde{m}^2}, \quad R_{sd}^{LL} = \frac{K_{is}^* K_{jd} \Delta_{ij}^{LL}}{K_{ts}^* K_{td} \tilde{m}^2},
$$

$R_{tb}^{RL} = \frac{K_{ib} \Delta_{3i}^{RL}}{K_{tb} \tilde{m}^2}, \quad R_{st}^{LR} = \frac{K_{is}^* \Delta_{3i}^{LR}}{K_{ts}^* \tilde{m}^2},$

$R_{td}^{RL} = \frac{K_{id} \Delta_{3i}^{RL}}{K_{td} \tilde{m}^2},$

(1)

where $K$ is the CKM matrix, and $\tilde{m}$ is the averaged up-type squark mass. The repeating indices $i, j$ in the same equation indicate sum over from 1 to 3. The quantities $R$’s are constrained directly by several experimental observables, e.g. $R_{sb}^{LL}, R_{st}^{LR}$ and $R_{tb}^{RL}$ are constrained by the mass difference $\Delta M_{Bs}$ and by the branching ratio of $b \to s\gamma$, and $R_{sd}^{LL}, R_{st}^{LR}, R_{td}^{RL}$ are constrained by the mass difference $\Delta M_K$.

The Feynman diagrams relevant to the process $b \to ss\bar{d}$ are shown in Fig. 1. We calculate the decay width from the chargino contribution to be

$$
\Gamma = \frac{m_b^5}{48(2\pi)^3} \sum_{i, j} \frac{|g^4|}{64\pi^2} K_{ib} K_{is}^* K_{ts} K_{td} \frac{1}{m_{\tilde{g}_i}^2} [V_{i1} V_{j1} V_{j1} x_j^2 f(x_j, x_j, x_j, x_j, x_{ij}) R_{sb}^{LL} R_{sd}^{LL}]
$$

$$- h_1 V_{i1} V_{j1} V_{j2} x_j f(x_j, x_j, x_j, x_{Rj}, x_{ij}) (R_{tb}^{RL} R_{sd}^{LL} + R_{st}^{LR} R_{sd}^{LL} + R_{sb}^{LL} R_{st}^{LR} + R_{sb}^{LL} R_{td}^{RL})
$$

$$+ h_2^2 V_{i1} V_{i2} V_{j1} V_{j2} x_j f(x_j, x_j, x_{Rj}, x_{Rj}, x_{ij}) (R_{sb}^{LL} + R_{sd}^{LL})
$$

$$+ x_j^2 f(x_j, x_j, x_{Rj}, x_{Rj}, x_{ij}) (V_{i1} V_{i2} V_{j1} V_{j2} (R_{st}^{LR} R_{st}^{LR} + R_{tb}^{RL} R_{td}^{RL}))
$$

3
\[ + V_{1i} V_{2j} V_{2j} R_{ib} (R_{tb} R_{st} + R_{st} R_{td}) \]
\[- h^2 V_{1i} V_{2j} V_{2j} x_j f(x, x, R_j, x, i, x) (R_{tb} R_{st} + 2 R_{st} + R_{td}) \]
\[ + h^4 V_{1i} V_{2j} V_{2j} f(x, R_j, x, i, x)^2 \]

with \( x_j = \frac{\tilde{m}^2}{m_{\chi_j^\pm}}, x_{Rj} = \frac{m_{\tilde{i}R}}{m_{\chi_j^\pm}}, x_{ij} = \frac{m_{ij}}{m_{\chi_j^\pm}}, V \) is the matrix to diagonalize the chargino mass matrix \( X_{\text{chargino}} \) with \( U X_{\text{chargino}} V^{-1} = \text{diag}(m_{\chi_1^\pm}, m_{\chi_2^\pm}) \), and the Yukawa couplings are defined as \( h_t = m_t/(\sqrt{2}m_W \sin \beta), h_b = m_b/(\sqrt{2}m_W \cos \beta) \). The loop functions are [5]

\[ f(x, y, z) = x^2 \ln x/((x - y)(x - z)(x - 1)) + y^2 \ln y/((y - x)(y - z)(y - 1)) \]
\[ + z^2 \ln z/((z - x)(z - y)(z - 1)), \]
\[ f(x, y, p, q) = [f(x, p, q) - f(y, p, q)]/(x - y), \]
\[ f(x, y, p, q, m) = [f(x, p, q, m) - f(y, p, q, m)]/(x - y). \]

We would like to mention that the chargino contribution of the box diagrams is proportional to \( 1/m_{\chi_j^\pm}^2 \). If both charginos are heavy, their contribution to \( \text{Br}(b \rightarrow ss \bar{d}) \) is small, so the main contribution comes from the lighter chargino \( \tilde{\chi}_1^\pm \) with a small mass (say < 200GeV).

Numerically we take into account the constraints of \( \Delta M_{BS} \geq 14.4\, \text{ps}^{-1} \) [6], \( 2 \times 10^{-4} \leq \text{Br}(B \rightarrow X_s \gamma) \leq 4.5 \times 10^{-4} \), and the lower bounds on the superparticles [7]. As for the constraint of \( \Delta M_K \), we demand the chargino contribution to \( \Delta M_K \) does not exceed the experimental value of \( \Delta M_K^{\exp} = (3.489 \pm 0.008) \times 10^{-15}\text{GeV} \).

The calculation of this process contains 14 parameters, 6 of them are \( \Delta_{ij}^{LL} \) with \( ij = 11, 12, 13, 22, 23, 33 \) which contribute to \( R_{sb,sd}^{LL} \), 3 of them are \( \Delta_{ij}^{LR} \) with \( ij = 13, 23, 33 \) which contribute to \( R_{tb,td}^{RL} \) and \( R_{st}^{LR} \). The other parameters are \( \tan \beta, M_2, \mu \) in chargino mass matrix, a common up-type squark mass \( \tilde{m} \) and the mass of the (light) right-handed stop \( m_{\tilde{i}R} \). We scan the parameter space in the region \( 4 \leq \tan \beta \leq 50, 250\text{GeV} \leq \tilde{m} \leq 500\text{GeV}, 90\text{GeV} \leq m_{\tilde{i}R} \leq 200\text{GeV}, 100\text{GeV} \leq M_2 \leq 500\text{GeV}, -500\text{GeV} \leq \mu \leq 500\text{GeV} \). All the 9 \( \Delta \)'s are normalized by \( \tilde{m}^2 \) varying in the range between -1 and 1. We find in the calculation that a common feature is that \( \text{Br}(b \rightarrow \)
$ssd$) does not depend sensitively on $\tan \beta$, but rather strongly on the mass parameters $\tilde{m}$, $m_{\tilde{t}_R}$ and $m_{\tilde{\chi}^\pm_1}$, the lighter chargino mass.

We find that the contribution to $\text{Br}(b \rightarrow ssd)$ from $\Delta^{LL}_{ij}$’s (with $ij = 11, 12, 13, 22, 23, 33$) and thus from $R_{sb,sd}^{LL}$ is always below $10^{-11}$ if we use the experimental constraints mentioned above. The dominant contribution to $\text{Br}(b \rightarrow ssd)$ comes from $\Delta^{LR}_{13,23}$, while $\Delta^{LR}_{33}$ is less important. We give in Tab. 1 two representative points with non-vanished $\Delta^{LR,LL}_{23}$ to show the main contribution of $\Delta^{LR}_{ij}$. We show the $\Delta^{LR}_{13,23,33}$ contribution in Fig. 2. From Fig. 2, $\text{Br}(b \rightarrow ssd)$ can be as large as $10^{-9}$. In some region, with the increasing of $\text{Br}(b \rightarrow ssd)$, $\Delta M_{B_s}$ can be as large as $60 \text{ps}^{-1}$.

Table 1: Two points of the scan with $\tan \beta = 10$. The other $\Delta$’s which are not presented in the table are taken as zero.

| $\Delta (\tilde{m}^2)$ | $\tilde{m}$ (GeV) | $m_{\tilde{t}_R}$ (GeV) | $m_{\tilde{\chi}^\pm_1,2}$ (GeV) | $\text{Br}(b \rightarrow ssd)$ | $\Delta M_{B_s}$ (ps$^{-1}$) |
|------------------------|------------------|------------------------|------------------------|-------------------------|------------------------|
| $\Delta^{LR}_{23} = -0.79$ | 493,124 | 149,273 | $2.4 \times 10^{-10}$ | 27 |
| $\Delta^{LL}_{23} = -0.46$ | 322,103 | 176,500 | $1.7 \times 10^{-12}$ | 19 |

In conclusion, we have studied the chargino contribution to the rare decay $b \rightarrow ssd$. We find that the dominant contribution is from $\Delta^{LR}_{13,23}$ and the branching ratio can amount to $10^{-9}$. This may be an important source of $b \rightarrow ssd$ in the MSSM.

We thank P. Singer for pointing out several mistakes. The work of DXZ is supported in part by the National Natural Science Foundation of China (NSFC) under the grant No. 90103014 and No. 10205001, and by the Ministry of Education of China. And XHW’s work is supported in part by the China Postdoctoral Science Foundation.
References

[1] K. Huitu, C. D. Lu, P. Singer and D. X. Zhang, Phys. Rev. Lett. 81, 4313 (1998).

[2] K. Huitu, C. D. Lu, P. Singer and D. X. Zhang, Phys. Lett. B 445, 394 (1999) [arXiv:hep-ph/9812253].

[3] G. Abbiendi et al. [OPAL Collaboration], Phys. Lett. B 476, 233 (2000) [arXiv:hep-ex/0002008]; J. Damet, P. Eerola, A. Manara and S. E. Nooij, Eur. Phys. J. directC 3, 7 (2001) [arXiv:hep-ex/0012057]; A. Garmash et al. [Belle Collaboration], Phys. Rev. D 65, 092005 (2002) [arXiv:hep-ex/0201007]; A. Garmash et al. [Belle Collaboration], arXiv:hep-ex/0307082.

[4] S. Fajfer and P. Singer, Phys. Rev. D 65, 017301 (2002) [arXiv:hep-ph/0110233]; S. Fajfer and P. Singer, Phys. Rev. D 62, 117702 (2000) [arXiv:hep-ph/0007132]; Z. j. Xiao, W. j. Li, L. b. Guo and G. g. Lu, Eur. Phys. J. C 18, 681 (2001) [arXiv:hep-ph/0011175]; B. Dutta, C. S. Kim and S. Oh, Phys. Lett. B 535, 249 (2002) [arXiv:hep-ph/0202019]; E. J. Chun and J. S. Lee, arXiv:hep-ph/0307108; J. P. Saha and A. Kundu, arXiv:hep-ph/0307259.

[5] A. J. Buras, A. Romanino, L. Silvestrini, Nucl. Phys. B520 (1998) 3.

[6] A. Stocchi, Nucl. Phys. Proc. Suppl. 117 (2003) 145.

[7] K. Hagiwara et al., Particle Data Group, Phys. Rev. D66 (2002) 010001.
Figure 1: The Feynman diagrams for process $b \rightarrow s\bar{s}d$. Crossing diagrams are not shown.
Figure 2: $\text{Br}(b \to ss\bar{d})$ vs $\Delta M_{B_s}$ with the contribution from $\Delta^{LR}_{13,23,33}$. 