Robust Resource-Aware Self-triggered Model Predictive Control

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Abstract—The wide adoption of wireless devices in the Internet of Things requires controllers that are able to operate with limited resources, such as battery life. Operating these devices robustly in an uncertain environment, while managing available resources, increases the difficulty of controller design. This paper proposes a robust self-triggered model predictive control approach to optimize a control objective while managing resource consumption. In particular, a novel zero-order-hold aperiodic discrete-time feedback control law is developed to ensure robust constraint satisfaction for continuous-time linear systems.

I. INTRODUCTION

The operation of devices in the Internet of Things (IoT) networks and wireless sensing systems are strongly impacted by resource factors, including battery life and hardware longevity. In order to avoid unnecessary resource consumption caused by extra device triggers/updates, such as the cold-boot power of a sensor, the controllers with aperiodic triggers can be deployed under self-triggered and event-triggered control schemes [1]–[4]. In particular, the control action under an event-triggered scheme is updated reactively by monitoring a trigger condition, whereas a self-triggered scheme updates proactively by planning the next trigger instant in advance, leaving sensors and controllers in idle mode. Due to the limitation of the resource factors, especially battery life, a self-triggered scheme can be preferable and is, therefore, the research object of this work. More applications refer to [5] [2, Section 4].

On top of the decision of triggering time sequences, the operation of a self-triggered device in an uncertain environment requires extra consideration of uncertainty propagation due to the lack of state measurement between two consecutive triggers. Most works decouple the triggering time decision from uncertainty propagation. For example, a tube-based method [6], [7] and a Lipschitz constant based method [8] have been applied to quantify the uncertainty evolution. In discrete-time systems, the triggering time sequence has been chosen to maximize the duration of open-loop operation [7] or by monitoring the discrepancy between nominal performance and actual performance [9]. Similar strategies have been applied to continuous time systems, where either the system is discretized [10] or the actual state is compared to a nominal state with continuous state measurements [8].

In this work, we consider a resource-aware self-triggered control problem for an uncertain continuous-time linear system, where, to the best of our knowledge, no existing results can be directly applied in a numerical reliable way. Notice that because the system is confined by limited resource factors such as battery life, continuous state measurement is impractical. Instead, we unify the triggering time sequence decision, feedback gain and the control input selection within one optimization problem. The main contributions of this work are summarized as follows:

1. a novel decomposition of the dynamics into those linked to process noise and those to feedback dynamics is proposed. Accordingly, we present the continuous-time ellipsoidal set propagation dynamics driven by a discretetime affine feedback control;
2. a robust resource-aware self-triggered MPC scheme is proposed that enables a unified decision of triggering time sequence and robust control input sequence;
3. numerically stable implementation details are provided.

Outline Even though this paper study uncertain linear dynamics, to better introduce the concept, Section II starts with reviewing deterministic resource-aware model predictive control in a generic form, alongside differential inequalities and some necessary results from ellipsoidal calculus. The main results are elaborated in Section III where dynamics driven by the proposed control law are summarized in Lemma II. A numerical validation of the proposed controller is given in IV followed by a conclusion in Section V.

Notation The Minkowski sum of two sets $X, Y \subset \mathbb{R}^n$ is denoted by $X \oplus Y = \{x + y \mid x \in X, y \in Y\}$. The set of symmetric positive (semi-)definite matrices in $\mathbb{R}^{n \times n}$ is denoted by $(\mathbb{S}_+^n)_{sym}$. An ellipsoid in $\mathbb{R}^n$ centered at $q \in \mathbb{R}^n$ is defined as $\mathcal{E}(q, Q) := \{q + Q^{1/2}v \mid v^{	op}v \leq 1\}$ with $Q \in \mathbb{S}_+^n$. The support function of a convex set $X \subset \mathbb{R}^n$ is defined by $V[X](c) := \max_{x \in X} \{c^{	op}x\}$ for all $c \in \mathbb{R}^n$. The notation $\mathbb{Z}^{a,b} = \{z \in \mathbb{Z} \mid a \leq z \leq b\}$ is used to denote integer ranges and we use the notation $\mathbb{0}$ to denote the zero matrix. The set of compact subsets of $\mathbb{R}^n$ is denoted by $\mathbb{K}^n$, and the subset of compact convex subsets $\mathbb{K}_C^n$.

II. PRELIMINARY

A. Deterministic Resource-aware Model Predictive Control

The dynamic of a linear time invariant (LTI) system in continuous time is given by

$$\forall t \in [0, \infty), \quad \frac{dx(t)}{dt} = Ax(t) + Bu(t)$$

(1)
with coefficient matrices $A \in \mathbb{R}^{n_x \times n_x}$ and $B \in \mathbb{R}^{n_x \times n_u}$, state $x(\cdot) : [0, \infty) \to \mathbb{R}^{n_x}$, control input $u(\cdot) : [0, \infty) \to \mathbb{R}^{n_u}$. Here, the state and control inputs are subject to constraints

$$\forall t \in [0, \infty), \ x(t) \in X, \ u(t) \in U$$

with constraint sets $X \subseteq \mathbb{R}^{n_x}$ and $U \subseteq \mathbb{R}^{n_u}$.

In the context of self-triggered control scheme, the control inputs are changed at triggering time instances $\{t_k\}_{k=0}^{N-1}$. Therefore, one can represent the zero-hold control inputs by using the direct optimal control approach [11] over the time horizon $[0, t_N]$, i.e.,

$$u(t) = \sum_{k=0}^{N-1} v_k \cdot \zeta_k(t, t_k, t_k+1),$$

with $v_k \in \mathbb{R}^{n_u}$ the coefficients, and $\zeta_k \in L^2[t_0, t_N], \ k \in \mathbb{Z}_0^{N-1}$ model the triggering property with a piece-wise constant function

$$\zeta_k(t, t_k, t_k+1) = \begin{cases} 1 & t \in (t_k, t_{k+1}] \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The update of the control input at each triggering time is confined by a resource factor [12], in practice, which can model the battery and bandwidth of the network. This resource defined by $r \in \mathbb{R}^{n_r}$ is recharged at a constant rate $\rho$ until saturation, i.e.,

$$\forall t \in [t_k, t_{k+1}), \ \dot{r}(t) = h(\tau - r(t))\rho,$$

where $\tau$ is a saturation value and $h(\cdot)$ is the heaviside function with $h(s) = 1$ if $s > 0$ and 0 elsewhere. When the agent is triggered to update the control input, the resource is discharged by an amount $\eta(\Delta_k)$ to pay the update cost. Thus, the resource at triggering time instants $\{t_k\}_{k=0}^{N-1}$ is

$$r(t) = \begin{cases} r_0 & t = t_0 \\ \lim_{t \to t_k^-} r(t) - \eta(\Delta_k) & t \in \{t_k\}_{k=0}^{N-1} \tag{4} \end{cases}$$

with an initially available resource $r_0$ at $t_0$. Here, $t \to t_k^-$ represents the left limits, i.e., $t \to t_k$ and $t < t_k$. Moreover, the resource $r$ is further lower bounded by $r \in [\underline{r}, \overline{r}]$. For the sake of compactness, we use the notation $v \in \mathbb{R}^{n_x n_u} := [v_0^T, v_1^T, \ldots, v_{N-1}^T]^T$ to stack the control coefficients, and define the triggering time interval $\Delta_k := [t_k, t_{k+1})$. We and denote $g(r(t_k), \Delta_k) := \min \{r(t_k) + \rho \Delta_k - \eta(\Delta_k), \tau\}$.

Accordingly, the resource-aware model predictive control (MPC) problem \cite{2} can be summarized as

$$\min_{x(\cdot), v(\cdot), \Delta} M(x(t_N)) + \sum_{k=0}^{N-1} \int_{t_k}^{t_{k+1}} l(x(\tau), v(\tau)) d\tau \tag{5a}$$

s.t.

$$x(t_0) = x_0, \ r(t_0) = r_0, \tag{5b}$$

$$\forall t \in [t_0, t_N], \ \frac{dx(t)}{dt} = Ax(t) + Bu(t), \tag{5c}$$

$$\forall t \in [t_0, t_N], \ x(t) \in X, \ u(t) \in U, \tag{5d}$$

$$\forall k \in \{0, 1, \ldots, N-1\}$$

$$r(t_{k+1}) = g(r(t_k), \Delta_k), \tag{5e}$$

$$r(t_{k+1}) \in [\underline{r}, \overline{r}], \tag{5f}$$

$$\Delta_k \in [\underline{\Delta}, \overline{\Delta}], \tag{5g}$$

where $l(\cdot, \cdot)$ and $M(\cdot)$ in \eqref{5a} are the stage and terminal costs respectively, the saturated resource dynamics \eqref{5a} is a simplified yet equivalent formulation of the resource dynamics \cite{2}. The constraints of the triggering time interval in \eqref{5g} protect the system from becoming Zeno/frozen, meaning that the triggering time $\Delta$ is zero/infinite. The initial state and resource are given by \eqref{5a}.

In the receding horizon scheme, a resource-aware self-triggered agent can update its control input when its resource is sufficiently high to stay above the lower bound $\underline{r}$. Otherwise, it must wait until enough resource is available. Once the controller is triggered at the current time instance, the resource-aware self-triggered controller solves \eqref{5} to plan the next triggering time and the associated control input.

\section{B. Differential Inequality}

Let us consider the uncertain continuous time autonomous dynamics $\dot{x}(t) = f(x(t), w(t))$ perturbed by $w \in W$ for the compact set $W \subseteq \mathbb{R}^{n_w}$. For a given set of initial states $X_0$ at $t_1$, we denote the reachable set at time $t_2 > t_1$ as

$$X(t_2) := \left\{ \xi \in \mathbb{R}^{n_x} | \exists \ w(t) \in W, \ \forall t \in [t_1, t_2], \ \dot{x}(t) = f(x(t), w(t)), \ x(t_1) \in X_0, \ x(t_2) = \xi \right\}. \tag{6}$$

Moreover, we define the set-valued mapping

$$\Gamma_f(c, X) := \{ f(x, w) | c^T \xi = V[X](c), \ x \in X, w \in W \}. \tag{7}$$

The convex enclosure of the reachable set can be characterized by the following theorem.

\begin{theorem} \text{[13, Theorem 3]} \label{thm1} \end{theorem}

Let $Y : [t_1, t_2] \to \mathbb{R}^{n_x}$ be a set-valued function such that

1) the function $V[Y(\cdot)](c)$ is Lipschitz continuous on $[t_1, t_2]$ for all $c \in \mathbb{R}^{n_x}$ and

2) the set-valued function $Y$ satisfies the differential inequality

\begin{align*}
\text{a.e.} \quad & t \in [t_1, t_2], \quad \dot{V}[Y(t)](c) \geq V[\Gamma_f(c, X)](c) \\
& \text{with } V[Y(t_1)](c) \geq V[X_1](c) \text{ for all } c \in \mathbb{R}^{n_x}. \tag{8}
\end{align*}

Then, $Y$ is an enclosure of the reachable tube of $X(t)$ for all $t \in [t_1, t_2]$, i.e., $X(t) \subseteq Y(t), \ \forall t \in [t_1, t_2]$.

\section{C. Ellipsoidal Calculus}

This section recap some useful results from ellipsoidal calculus \cite{14}. The support function of an ellipsoid $E(q, Q)$ is given by

$$V[E(q, Q)](c) = c^T q + \sqrt{c^T Q c}. \tag{9}$$

This value is obtained at the boundary of the ellipsoid as

$$Z[E(q, Q)](c) := \arg \max_z \{ c^T x | x \in E(q, Q) \} = \frac{Qc}{\sqrt{c^T Q c}}. \tag{10}$$
The Minkowski sum of two ellipsoids is not necessarily an ellipsoid, and it can be outer approximated by \( \forall \lambda \in (0, 1), \)
\[
\mathcal{E}(q_1, Q_2) \oplus \mathcal{E}(q_2, Q_2) \subseteq \mathcal{E}(q_1 + q_2, \frac{Q_1}{\lambda} + \frac{Q_2}{1 - \lambda}).
\] (6)

III. MAIN RESULTS

In this paper, we consider the following uncertain linear time-invariant dynamics
\[
\frac{dx(t)}{dt} = Ax(t) + B_u w(t) + B_w w(t),
\] (7)
with matrices \( B_u \in \mathbb{R}^{n_x \times n_u}, B_w \in \mathbb{R}^{n_x \times n_w}, \) and uncertainty \( w(t) \in \mathcal{E}(0, Q_w(t)) \) for \( Q_w(\cdot) \in \mathbb{S}_{++}^{n_w} \). In the following, we will derive the continuous-time dynamics of the ellipsoidal outer approximation of the reachable set \( X(\cdot) \) driven by a discrete-time feedback control law. Notice that while the ellipsoidal outer approximation of a reachable set under continuous-time feedback control has been widely explored [15]–[18]. However, because the triggering time is a decision variable in a self-triggered scheme, we observe that a direct application of most previous work is numerically unstable when used in an optimization algorithm. This motivates us to adopt differential inequality in this work.

In self-triggered schemes, the control input is only allowed to change when the system is triggered. In particular, if the system is triggered at \( t_k \) with its state contained in the reachable set \( X(t_k) \), which in turn is bounded within \( \mathcal{E}(q_k, Q_k) \), we propose to update its input via a nominal term \( v_k \) and a feedback term \( K \) as \( \forall t \in (t_k, t_{k+1}] \),
\[
Q_k(t, x) = v_k + K x, \quad x \in \mathcal{E}(q_k, Q_k).
\] (8)

This control input must then remain constant until its next trigger at \( t_{k+1} \). Before delving into the details of the proposed controller, we summarize the mechanism to first give an intuitive general viewpoint. Given any sequence of \( \{t_1 \}_{i=0}^{N-1}, \{t_i \}_{i=0}^{N} \) and a feedback control law \( K \), we can define the chain of reachable sets depicted in Figure 1.

**The reachable set outer-approximation at triggering time instances:** Consider the trigger at \( t_1 \), the reachable set developed over the interval \( [t_0, t_1) \) is outer approximated by an ellipsoid \( \mathcal{E}(q_1, Q_1) \) (Big green ellipsoid \( Q_1 \) in Figure 1).

Based on the discussion above, characterization of the reachable set propagation between two consecutive triggers is vital to the MPC design. The propagation of the reachable set outer approximation between two consecutive triggers \( (t_k, t_{k+1}] \) is stated in the following.

**Lemma 1** Let \( X(t_k) \subseteq \mathcal{E}(q_k, Q_k) \) and dynamics (7) be driven by control law (8). The reachable set \( X(t) \) for all \( t \in [t_k, t_{k+1}] \) is outer bounded by
\[
X(t) \subseteq \mathcal{E}(q_k(t), Q_{fb,k}(t)) \oplus \mathcal{E}(0, Q_{op,k}(t, \lambda_k(t))),
\] (9)
where \( \lambda_k(\cdot) : \mathbb{R} \to \mathbb{R}^+ \) is any positive real-valued function on \( [t_k, t_{k+1}] \) and the shape of the outer approximation is characterized by
\[
\begin{align}
\frac{dQ_{fb,k}(t)}{dt} &= A Q_{fb,k}(t) + B V_k \\
\frac{dQ_{op,k}(t, \lambda_k(t))}{dt} &= A Q_{op,k}(t, \lambda_k(t)) + B K Q_k \\
\frac{dQ_{cr,k}(t)}{dt} &= A Q_{cr,k}(t) + B K Q_k
\end{align}
\] (10)
with \( Q_{fb,k}(t_k) = Q_{cr,k}(t_k) = Q_k, \) \( Q_{op,k}(t_k, \lambda_k(t_k)) = 0. \)

**Proof.** In our proof, we first derive the decomposition in (10) and then, work out the dynamics (10) for \( k \)-th interval \( [t_k, t_{k+1}] \).

**Reachable set decomposition:** Note that the reachable set driven by control law (8) is
\[
X(t) := \left\{ \begin{array}{ll}
\exists \tau \in [t_k, t_{k+1}] \in \mathbb{R}^{n_x}, & x(t) = A x(t) + B_u (v_k + K \hat{x}) + B_w \tau, \\
\hat{x}(t) = \hat{x}(q_k(t), Q_k), & x(t) = \xi
\end{array} \right\}
= \left\{ \begin{array}{ll}
\exists \tau \in [t_k, t_{k+1}] \in \mathbb{R}^{n_x}, & x(t) = A x(t) + B_u (v_k + K \hat{x}) + B_w \tau, \\
\hat{x}(t) = \hat{x}(q_k(t), Q_k), & x(t) = \xi
\end{array} \right\}
\] (11)
where Equality (11) follows the linearity of the dynamics (7).
Dynamics of feedback component: The set \( X_{fb,k}(t) \) can be rewritten as

\[
X_{fb,k}(t) := \left\{ \xi \in \mathbb{R}^{n_x}, \exists \bar{z} \in \mathbb{R}^{n_z}, z^T\bar{z} \leq 1, \forall \tau \in [t_k, t] \middle| x(\tau) = Ax(\tau) + B_k(v_k + K\bar{z}) \right\}
\]

\[
= \left\{ e^{At}\xi_k + \int_0^t e^{A(t-\tau)}B_k v_k d\tau \middle| z^Tz \leq 1 \right\}, \tag{12}
\]

where (12) utilizes the explicit solution of \( \dot{x}(\tau) = Ax(\tau) + B_k(v_k + K\bar{z}) \). Notice that, by definition of an ellipsoid, \( X_{fb,k}(t) \) is also an ellipsoid, whose center is given by the term (a) and the shape is defined by the term (b). For simplification, we denote it by \( E(q_k(t), Q_{fb,k}(t)) \). In particular,

\[
q_k(t) = e^{At}\xi_k + \int_0^t e^{A(t-\tau)}B_k v_k d\tau
\]

which is the dynamics (10a). Moreover, we denote \( \bar{B}(t) := \int_0^t e^{A(t-\tau)}d\tau B_k \) such that

\[
Q_{fb,k}(t) = \bar{B}(t)Q_k \bar{B}(t)^T
\]

\[
\Rightarrow \frac{dQ_{fb,k}(t)}{dt} = AQ_{fb,k}(t) + Q_{fb,k}(t)A^T + B_kQ_k \bar{B}(t)K \bar{B}(t)^T + B_kQ_k \bar{B}(t)K Q_k \bar{B}(t)^T \tag{10b}
\]

which recovers (10b) and (10c).

Dynamics of open-loop component: The remaining proof will construct an outer approximation of \( X_{op,k}(t) \) with ellipsoid \( E(0, Q_{op,k}(t, \lambda_k(t))) \). Notice that the autonomous dynamics considered in the reachable set \( X_{op,k}(t) \) are

\[
\dot{x}(t) = f_w(x, w) := Ax(t) + B_w w(t).
\]

In order to apply Theorem 1 we introduce support function

\[
V[\tilde{f}_w(c, E(0, Q_{op,k}(t, \lambda_k(t))))(c)]
= \max_{w \in E(0, Q_{op,k}(t))} c^T \begin{pmatrix} A\tilde{E}(0, Q_{op,k}(t, \lambda_k(t)))c \\ B_w w \end{pmatrix}
= \max_{w \in E(0, Q_{op,k}(t))} c^T \begin{pmatrix} A \frac{Q_{op,k}(t, \lambda_k(t))c}{\sqrt{c^T Q_{op,k}(t, \lambda_k(t))c}} + B_w w \end{pmatrix}
= c^T A \frac{Q_{op,k}(t, \lambda_k(t))c}{\sqrt{c^T Q_{op,k}(t, \lambda_k(t))c}} + c^T B_w Q_{op,k}(t, \lambda_k(t))c.
\]

We apply Theorem 1 to outer approximate the shape of the \( E(0, Q_{op,k}(t, \lambda_k(t))) \), which yields

\[
\dot{V}[E(0, Q_{op,k}(t, \lambda_k(t))))](c) \geq \max_{w \in E(0, Q_{op,k}(t, \lambda_k(t)))} c^T \begin{pmatrix} A \frac{Q_{op,k}(t, \lambda_k(t))c}{\sqrt{c^T Q_{op,k}(t, \lambda_k(t))c}} + B_w w \end{pmatrix}
\]

\[
= \frac{1}{2} c^T Q_{op,k}(t, \lambda_k(t))c \geq c^T \frac{1}{2} A Q_k c + \sqrt{c^T B_w Q_{op,k}(t, \lambda_k(t))c} \sqrt{c^T Q_{op,k}(t, \lambda_k(t))c}.
\]

By applying the tight arithmetic-geometric mean inequality [19], we reformulate the second inequality above as

\[
\frac{1}{2} c^T Q_{op,k}(t, \lambda_k(t))c 
\geq c^T A Q_k c + \inf_{\lambda > 0} \frac{1}{2\lambda} c^T B_w Q_{op,k}(t, \lambda_k(t))c + \frac{\lambda}{2} c^T Q_{op,k}(t, \lambda_k(t))c.
\]

According to Theorem 1 we can construct an ellipsoidal outer approximation of \( X_{op,k}(t) \) by enforcing the following inequality

\[
\frac{1}{2} \left( c^T Q_{op,k}(t, \lambda_k(t))c + \sqrt{c^T Q_{op,k}(t, \lambda_k(t))c} \right)^T \geq \inf_{\lambda > 0} \frac{1}{2\lambda} c^T B_w Q_{op,k}(t, \lambda_k(t))c + \frac{\lambda}{2} c^T Q_{op,k}(t, \lambda_k(t))c.
\]

where the decomposition in (d) is used to build a symmetric form of \( Q_{op,k}(t) \) from the asymmetric form that appeared in (13). The final step is to get rid of the inequality and the infimum operator. Note that if there exists \( \lambda_k(t) : \mathbb{R} \rightarrow \mathbb{R}^+ \) such that the following dynamics is satisfied, then the inequality (13) is satisfied.

\[
\frac{dQ_{op,k}(t, \lambda_k(t))}{dt} = AQ_{op,k}(t, \lambda_k(t)) + Q_{op,k}(t, \lambda_k(t))A^T + B_w Q_{op,k}(t, \lambda_k(t))A^T + \frac{B_w Q_{op,k}(t, \lambda_k(t))B_w^T}{\lambda_k(t)}
\]

This recovers the dynamics (10c), which results in the inclusion \( X_{op,k}(t) \subset E(0, Q_{op,k}(t)) \) by construction, we thus have

\[
\mathcal{X}(t) \subseteq E(q_{fb,k}(t), Q_{fb,k}(t)) \cap X_{op,k}(t) \subset E(q_{fb,k}(t), Q_{fb,k}(t)) \cap E(0, Q_{op,k}(t, \lambda_k(t)))
\]

for any \( t \in [t_k, t_{k+1}] \), which concludes the decomposition (9).

\[\square\]

Remark 1: Lemma 1 indicates that the evolution of the uncertainty between two consecutive triggers \([t_k, t_{k+1}]\) can be decomposed into two dynamic parts. Both are independently driven by the zero-order-hold feedback generated at \( t_k \) and by the open-loop accumulation of the disturbance \( w(t) \), respectively. The former corresponds to \( E(q_k(t), Q_{fb,k}(t)) \) and the latter is an outer approximation given by \( E(0, Q_{op,k}(t, \lambda_k(t))) \). Moreover, as the trigger occurs at \( t_k \), \( Q_{cr,k}(t) \) can be perceived as the correlation between the uncertainty at \( t \) and \( t_k \), see (10c). This, in terms, reflects the self-triggered property.

A. Robust Resource-Aware MPC

This section summarizes a robust MPC controller that incorporates the dynamics derived in the last section into the self-triggered MPC scheme. In particular, the controller optimizes the nominal performance while ensuring a robust input/state constraint satisfaction. In general, the nominal inputs \( \{v_k\}_{k=0}^{N-1} \), the feedback control \( K \), and the triggering time instances \( \{t_k\}_{k=1}^N \) are determined by solving the following problem:

\[
\begin{aligned}
\min_{K, v, \Delta q_{fb,k}(\cdot), Q_{fb,k}(\cdot), Q_{op,k}(\cdot)} & M(q(t_N)) + \sum_{k=0}^{N-1} \int_{t_k}^{t_{k+1}} l(q_{fb,k}(\tau), v_k) d\tau \\
\text{subject to} & 
\end{aligned}
\]
subject to

$$X(t_0) = \mathcal{E}(q_0, Q_0), \quad r(t_0) = r_{00},$$

(14a)

$$\forall t \in (t_0, t_{\text{fin}}), \forall k \in \mathbb{Z}_0^{N-1},$$

$$X(t) \leq X_k, Q_k X(t_k) \leq U, \quad r(t_{k+1}) \in [\Delta, \bar{\Delta}],$$

(14b)

$$\forall t \in [t_k, t_{k+1}], \forall k \in \mathbb{Z}_0^{N-1},$$

$$X(t) \subseteq \mathcal{E}(q_k(t), Q_{f,b,k}(t)) + \mathcal{E}(0, Q_{op,k}(t, \lambda_k(t))),$$

(14c)

$$\frac{dQ_{f,b,k}(t)}{dt} = Aq_{f,b,k}(t) + B_{\text{op},k} q_{f,b,k}(t) = q_k,$$

$$\frac{dQ_{op,k}(t, \lambda_k(t))}{dt} = AQ_{op,k}(t, \lambda_k(t)),$$

$$+ Q_{op,k}(t, \lambda_k(t)) A^T + \lambda_k(t) Q_{op,k}(t, \lambda_k(t)),$$

$$+ \frac{B_w Q_{op,k}(t, \lambda_k(t))}{\lambda_k(t)}, \quad Q_{op,k}(t_k, \lambda_k(t_k)) = 0,$$

$$Q_{f,b,k}(t_k) = Q_{cr,k}(t_k) = Q_k,$$

$$Q_k = \frac{Q_{f,b,k-1}(t_k)}{\kappa_k} + \frac{Q_{op,k-1}}{1 - \kappa_k}, \quad \kappa_k \in (0, 1).$$

(14d)

The initial conditions are enforced by (14a) with potentially uncertain measurements. The dynamics summarized in Lemma 1 is enforced in constraints (14c). (14d) models how the reachable sets from adjacent intervals connect to each other (see the over bound arrow in the middle of Figure 1). In particular, the reachable set in time interval $[t_k, t_{k+1}]$ links to the reachable set at $t_k$, where the ellipsoid $Q_k$ is used to generate the outer approximation of

$$\mathcal{E}(q_k(t), Q_{f,b,k}(t)) + \mathcal{E}(0, Q_{op,k}(t, \lambda_k(t))).$$

Finally, we summarize a few important notes to enable an efficient implementation of the proposed MPC controller (14).

- To solve the problem within a direct optimal control scheme, the integration of the ordinary differential equations can be achieved by numerical integration methods such as the Runge-Kutta method or the collocation method [20]. In this case, the collocation method is preferable because the integration is linear with respect to the triggering time difference $\{\Delta_k\}_{k=0}^{N-1}$, while other numerical integration methods depend on high order terms of $\{\Delta_k\}_{k=0}^{N-1}$, which results in low numerical stability.

- When the feasible sets $X, U$ are ellipsoidal, equation (6) can be used to determine the satisfaction of both constraints. If these sets are polytopic, then calculus of support functions can be applied. Each linear constraint can be rewritten as

$$\forall x \in \mathcal{E}(q_k(t), Q_{f,b,k}(t)) + \mathcal{E}(0, Q_{op,k}(t, \lambda_k(t))), \quad c^T x \leq C,$$

$$\implies c^T q_k + V[\mathcal{E}(0, Q_{f,b,k}(t))](c)$$

+ $$V[\mathcal{E}(0, Q_{op,k}(t, \lambda_k(t)))](c) \leq C.$$
This paper proposes a novel resource-aware robust self-triggered MPC, which generalizes resource-aware self-triggered MPC to an uncertain environment. The dynamics ellipsoidal outer approximation of the reachable sets that are governed by a discrete-time feedback control law, is derived to accommodate a continuous-time uncertain disturbance. This feedback law is intentionally designed to be compatible with a self-triggered control scheme. Finally, the proposed scheme is validated through a numerical example.

V. Conclusion

To show the advantage of the proposed scheme over a controller with a fixed sample period \( \Delta \), a second experiment has been conducted with a disturbance 10 times stronger, \( w(t) \in [-0.4, 0.4] \), whose output and resource responses are shown in Figure 4 and 5, respectively. In both cases, the proposed controller shows comparable performance against the open-loop controller. However, these problems can be fixed by forcing triggering the agent when a reference change is detected.

![Fig. 3: Comparison: open-loop vs closed-loop robust self-triggered MPC](image)

![Fig. 4: Output of the proposed controller with larger disturbance](image)

![Fig. 5: Resource response of the proposed controller with larger disturbance](image)

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