Secure Transmission for THz-Empowered RIS-Assisted Non-Terrestrial Networks

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Abstract—The non-terrestrial networks (NTNs) are recognized as a key component to provide cost-effective and high-capacity ubiquitous connectivity in the future wireless communications. In this paper, we investigate the secure transmission in a terahertz (THz)-empowered reconfigurable intelligent surface (RIS)-assisted NTN (T-RANTN), which is composed of a low-Earth orbit satellite transmitter, an RIS-installed high-altitude platform (HAP) and two unmanned aerial vehicle (UAV) receivers, only one of which is trustworthy. An approximate ergodic secrecy rate (ESR) expression is derived when the atmosphere turbulence and pointing error due to the characteristics of THz as well as the phase errors resulting from finite precision of RIS and imperfect channel estimation are taken into account simultaneously. Furthermore, according to the statistical and perfect channel state information of the untrustworthy receiver, we optimize the phase shifts of RIS to maximize the lower bound of secrecy rate (SR) and instantaneous SR, respectively, by using semidefinite relaxation method. Simulation results show that both the approximate expression for the ESR and the optimization algorithms are serviceable, and even when the jitter standard variance of the trustworthy receiver is greater than that of the untrustworthy one, a positive SR can still be guaranteed.

Index Terms—Reconfigurable intelligent surface (RIS), non-terrestrial networks (NTNs), ergodic secrecy rate (ESR), phase error, pointing error.

I. INTRODUCTION

IN RECENT years, with the global proposal and promotion of integrated space-Earth networks in the beyond fifth-generation (B5G) era, a number of advanced technologies involved, such as unmanned driving and transoceanic communication, have put forward higher requirements for future wireless communication networks, which include uninterrupted and ubiquitous connectivity, as well as ultra-high data rates and reliability [1]. Nevertheless, it is impractical to achieve these goals by intensively deploying a large number of terrestrial base stations due to their high installation and maintenance costs.

So far, the non-terrestrial networks (NTNs) may be one of the best solutions to this problem. Specifically, an NTN can consist of terrestrial and non-terrestrial nodes that are within the atmosphere or in the space. For example, a dense low-Earth orbit (LEO) satellite constellation was considered in [2], where an algorithm to dynamically establish the inter-plane inter-satellite links is proposed to maximize the sum rate of the constellation. The authors in [3] investigated the throughput of a LEO satellite-assisted Internet of vehicles composed of a LEO satellite constellation, multiple transmission control protocol sources and receivers, as well as an unmanned aerial vehicle (UAV) receiver. Therein, the high-data-rate free-space optics (FSO) and various optimization methods for optimal UV’s parameters were used to obtain a maximal system throughput. Furthermore, to serve users in remote areas, the idea of LEO satellites in combination with high-altitude platforms (HAPs) by utilizing millimeter wave (mmWave) was proposed in [4], where a two-tier matching algorithm is put forward to solve the problem of dynamic connection between HAPs and satellites resulting from the periodic motion of satellites to maximize the revenue in LEO satellites. In [5], [6], hybrid radio frequency (RF)/FSO systems, where the operating frequency could be switched according to the weather and pointing conditions, were studied. Instead of commonly used laser, mmWave and optical waves, the authors in [7] took advantage of the terahertz (THz) band for the wireless connectivity in airplanes, which may be far away from both the satellites and the ground base stations.

A detailed channel model for aerial THz communications was proposed and used to prove the feasibility of airborne THz link. It is worth noting that the THz wave supports higher throughput with less beam divergence than mmWave band, and offers a wider beam with an advantage of higher tolerance in pointing error caused by the sharp beam of high-frequency waves and the relative motion of two communicating equipments than the laser and optical waves counterparts [8], [9], [10], [11].

A line-of-sight (LoS) link is always required to ensure smooth connections. However, the high-frequency waves can be easily blocked by obstructions. The emerging technology, reconfigurable intelligent surface (RIS), is expected to be one of the most

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promising solutions to this issue [12]. From a macro perspective, RIS is a planar array consisting of a large number of elements, which are able to adaptively adjust their amplitudes and phase shifts to change the intensity and direction of the incident waves. It is more attractive that this operation is almost passive compared to traditional relays. Consequently, RIS-assisted NTNs (RANTNs) have been widely studied [13], [14], [15]. Specifically, in [14], an RANTN system was considered, where the source transmits the signal via RIS mounted on an UAV to the destination that cannot be directly reached, and the ergodic capacity of the system was derived. Moreover, the model of RIS assisting the communication of two LEO satellites, which may be on the same or different planes, was presented in [15]. Three types of RIS combination: an individual, multiple independent and multiple consecutive RIS, were considered, and their bit error rates were deduced respectively. In particular, the pointing error was taken into account in [14], [15].

Due to the broadcast nature of wireless communications, the confidential information is very likely to be wiretapped. The traditional solution is to encrypt the confidential message, but this is very inefficient [16]. In recent years, the physical layer security (PLS) appears to be an effective method to improve the secrecy of wireless networks. Furthermore, thanks to the occurrence of RIS, the system PLS has become more flexible. The authors in [17] minimized the transmit power under the constraint of the legitimate and wiretap users’ quality-of-service by jointly optimizing the precoding matrix and phase shifts of RIS. In [18], [19], [20], the artificial noise, which is in the null space of the legitimate channel, was made full use of to interrupt the wiretap channel to enhance the system security. However, despite the flexibility brought by RIS, the influence of phase error resulting from the finite precision of RIS or imperfect channel estimation on PLS cannot be neglected [21]. The secrecy outage probability and ergodic secrecy rate (ESR) of RIS-assisted end-to-end networks in the presence of phase error were analyzed in [22], [23], [24], where the phase error is modeled by uniform or Von Mises distribution. Especially, the authors in [24] considered two cases of colluding and non-colluding eavesdroppers.

As the deployment of air communication nodes is pushing forward worldwide, the RIS-assisted PLS can also play a significant role. A system model where the RIS covering the facade of a building bridges the communication between the UAV and the legitimate terrestrial user with an eavesdropper nearby was presented in [25]. The trajectory and power of UAV and the phase shifts of RIS were jointly optimized to maximize the SR. Following [25], the time division multiple access was applied for uplink and downlink communications to enhance the security of the system in [26]. Moreover, the method based on reinforcement learning can also be used to improve the system secrecy [27]. However, in the existing works, the influence of atmospheric environment on the links in the air was not tackled properly, which may cause wave attenuation and air vehicles’ instability. Besides, the phase error was not taken into account, which could be devastating to the system security once it occurs.

In this paper, we investigate the security issue of a THz-empowered RANTN (T-RANTN), which simultaneously suffers from atmosphere turbulence, pointing error and phase error. The main contributions of this paper are summarized as follows:

- It is the first work to investigate the security performance on T-RANTNs in the presence of atmosphere turbulence and pointing error due to the characteristics of THz as well as phase errors resulting from finite precision of RIS and imperfect channel estimation simultaneously.
- We derive an approximate expression for the ESR and optimize the phase shifts of RIS, given the statistical or perfect channel state information (CSI) of the untrustworthy receiver, to maximize the lower bound of SR and the instantaneous SR by using semidefinite relaxation method, respectively.
- Monte Carlo simulations validate the tightness of the approximate expression for ESR and the feasibility of the optimization algorithms. Besides, the relationships between ESR and variable parameters are also investigated, from which we arrive at the most important conclusion, namely all of the ESR, lower bound of SR and instantaneous SR can still be guaranteed to be positive even when the jitter standard variance of the trustworthy receiver is greater than that of the untrustworthy one.

The reminder of this paper is organized as follows. In Section II, we introduce the system model, in which the mathematical models of atmosphere turbulence, pointing error and phase error are described. The probability density functions (PDFs) of the received signal-to-noise ratios (SNRs) of two UAV receivers are given in Section III, and we derive an approximate expression for the ESR. In Section IV, the optimization algorithms for the phase shifts of RIS to maximize the lower bound of SR and instantaneous SR are proposed, under the assumption that the statistical or perfect CSI of the untrustworthy receiver is available, respectively. The simulation and theoretical results are shown in Section V, followed by the conclusion in Section VI.

**Notation:** The lowercase, lowercase bold, and uppercase bold characters $x$, $X$ represent scalars, vectors, and matrices, respectively. $f_x(x)$ and $f_{x,y}(x,y)$ represent the PDF of $x$ and the joint PDF of $x$ and $y$, respectively. $\Gamma(x)$ and $\Gamma(a,x)$ are...
the gamma function and the incomplete upper gamma function, respectively. $K_0(x)$, $I_0(x)$ and $\text{erf}(x)$ denote the modified Bessel function of the second kind with order $n$, the modified Bessel function of the first kind with order zero and the error function, respectively. $E[x]$ and $\text{arg}(x)$ are the expectation of real number $x$ and the phase of complex number $x$, respectively. $x \sim \mathcal{CN}(\mu, \sigma^2)$ means that the variable $x$ follows the complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$. $|x|$ denotes the modulus or absolute value of $x$, and $j$ is the imaginary unit. $F_p(a_0, \ldots, a_q; b_1, \ldots, b_q; x)$ is the generalized hypergeometric function. $[X]_{i,j}$ and $[x]_i$ are the element in the $i$th row and $j$th column of matrix $X$ and the $i$th element of vector $x$, respectively. $\mathbb{C}^{m \times n}$ denotes the space of $m \times n$ complex-valued matrices. $\text{tr}(X)$ represents the trace of matrix $X$.

II. SYSTEM MODEL

The system model of T-RANTN is shown in Fig. 1, where a LEO satellite (S) transmits the signal to a UAV (U) hovering at a low altitude; however, due to the complex surroundings or limited satellite coverage, S is not able to send information to U directly [28]. Therefore, an HAP (R), mounted with an $N$-element RIS, is utilized to connect S and U. At the same time, there is an untrustworthy UAV (E) located near U, who may want to obtain the confidential information sent from S. Besides, we assume that S, U and E are all equipped with a single antenna because of the hardware limitation [4].

According to the aviation regulations, the HAP is primarily located in the stratosphere, which enables itself to stay at a quasi-stationary position relative to the Earth. Therefore, the periodic movement of the LEO satellite around the Earth will definitely cause a connection from HAP within a certain period of time, and all channel links are assumed to be independent of each other and experience quasi-static fading during this period [4], [29]. Moreover, due to the severe path loss and high attenuation, the signals reflected by the RIS more than once have negligible power and hence can be ignored [12].

To achieve higher date rates, the THz band is made full use of in this system. Without loss of generality, we assume ideal LoS channels between S and R with merely path loss owing to the quasi-vacuum communication environment. Furthermore, since both R-U and R-E links are located high in the air with few obstructions, the free-space path loss model is applied. Referring to [30, Eq. (2.7)], the common path loss model for S-R, R-U and R-E links is generalized as

$$L_{\Pi} = G_{\Pi} \left( \frac{\lambda}{4\pi d_{\Pi}} \right)^2 , \quad \Pi \in \{s, u, e\} .$$

(1)

In contrast to the aforementioned atmosphere-free circumstance, the UAVs operate at low altitudes of a few hundred meters, where there is troposphere, so that the influence of atmosphere turbulence on THz cannot be ignored [10], [11]. In this case, instead of adopting the joint distribution of $\alpha$-$\mu$ fading and pointing error to illustrate the channel as [9], [31], we discuss the atmosphere turbulence and pointing error separately. To this end, the Gamma-Gamma distribution stands out as the model of general atmosphere turbulence fading as follows [14], [32]:

$$f_{\psi_{\varrho}} (\psi_{\varrho}) = \frac{2(\alpha_{\varrho}\beta_{\varrho})^{\alpha_{\varrho}+\beta_{\varrho}}}{\Gamma(\alpha_{\varrho}) \Gamma(\beta_{\varrho})} \frac{e^{-\psi_{\varrho}/\alpha_{\varrho}}}{\psi_{\varrho}^{\alpha_{\varrho}+\beta_{\varrho}}} K_{\alpha_{\varrho}-\beta_{\varrho}} \left( 2\sqrt{\alpha_{\varrho}\beta_{\varrho}/\psi_{\varrho}} \right) ,$$

$$\psi_{\varrho} > 0 , \quad \varrho \in \{u, e\} ,$$

(2)

where

$$\alpha_{\varrho} = \left[ \exp \left( \frac{0.49\varsigma_{\varrho}^2}{(1+1.11\varsigma_{\varrho}^{12/5})^{7/6}} \right) - 1 \right]^{-1} ,$$

$$\beta_{\varrho} = \left[ \exp \left( \frac{0.51\varsigma_{\varrho}^2}{(1+0.69\varsigma_{\varrho}^{12/5})^{5/6}} \right) - 1 \right]^{-1} ,$$

(3)

and $\varsigma_{\varrho}^2 = 1.23(2\pi/\lambda)^{7/6} d_{\varrho}^{11/6} C_{\varrho}^2$.

Due to air flow and hardware limitation of the device, there is relative motion between S and R, R and U as well as R and E definitely, whereas the sharp beam of THz waves requires a high precision to aim at the effective area of the receiver. Therefore, we must take full account of the pointing error for all links. However, the narrow beam divergence does not pose a threat to the S-R link, because the area of RIS is broad enough for covering the drifts caused by the vibration of R [14], [33]. Without loss of generality, we make a circular beam assumption for the receive antenna’s effective area and the transmit beam’s footprint. As shown in Fig. 2, $I$, $w$ and $g$ refer to the radius of the effective area of receive antenna, the beam waist at the receiver and the deviation between the centers of these two areas, respectively. Therefore, referring to [15], the PDF of the pointing error is expressed as

$$f_{\varrho_{\psi}} (\varrho_{\psi}) = \frac{2\exp\left( -\frac{1}{A_{\varrho_{\psi}}} \right) A_{\varrho_{\psi}}^{-1}}{A_{\varrho_{\psi}}^{\varrho_{\psi}}} , \quad 0 \leq \varrho_{\psi} \leq A_{\varrho_{\psi}} , \quad \varrho \in \{u, e\} ,$$

(4)

1 Since a large number of parameters are involved in this section, we summarize the upcoming symbols and definitions in Table I for readability. Note that the same subscripts are not repeated.

| Symbol | Definition |
|-------|------------|
| $\mathcal{L}_{\Pi}, \Pi \in \{s, u, e\}$ | Path loss on the S-R, R-U and R-E links |
| $\lambda$ | Wavelength |
| $G_{\Pi}$ | Gain of the antennas on S, U, and E |
| $d_{\Pi}$ | Distance between S and R, U, and E |
| $T_{\varrho, \varrho} \in \{u, e\}$ | Fading due to the atmosphere turbulence on the R-U and R-E links |
| $\sigma_\varrho$ | Large-scale scattering parameter of $T_{\varrho, \varrho}$ |
| $\sigma_{\psi}$ | Small-scale scattering parameter of $T_{\varrho, \varrho}$ |
| $C_{\varrho}^2$ | Index of refraction structure parameter |
| $p_{\varrho}$ | Fading brought by the pointing error on R-U and R-E links |
| $\rho_{\varrho}$ | Radius of the effective area of receive antennas on U and E |
| $\rho_{w}$ | Beam waist at U and E |
| $g_{\varrho}$ | Deviation between effective area and beam waist at U and E |
| $A_{\varrho}$ | Fraction of the collected power at U and E when there is no pointing error |
| $\sigma_{\psi, \varrho}$ | Jitter standard variance of $g_{\varrho}$ |
| $W_{\rho}^2$ | Equivalent beam width at U and E |

TABLE I

MAIN PARAMETERS IN THE FORMULAS
where

\[ A_\varphi = \left( \text{erf} \left( \frac{\sqrt{\pi} l_\varphi}{\sqrt{2} w_\varphi} \right) \right)^2, \quad \varphi_\varphi = W_\varphi/2\sigma_{g_\varphi}, \]

\[ W_\varphi^2 = w_\varphi^2 \frac{\varphi_\varphi}{w_\varphi} \exp \left( -\frac{\pi^2 l_\varphi^2}{w_\varphi^2} \right), \quad l_\varphi \lambda \sqrt{C_\varphi/(2\pi)}. \]

Moreover, it is intuitive to ignore the small-scale fading when U and E are remote area users as in [4]. Besides, even when they are located in complex surroundings, considering the features in THz communications that the power of scattering component is generally much lower (more than 20 dB) than that of LoS component, we can also ignore the small-scale fading and the channels between R and U as well as R and E should be dominated by the LoS component [34, 35, 36]. Let \( h_n^r = e^{j\theta_n^r} \), \( h_n^h = e^{j\theta_n^h} \), \( h_n^e = e^{j\theta_n^e} \), \( n \in \{1, 2, \ldots, N\} \) denote the normalized LoS channel coefficients of S-R-U and S-R-E links associated with the \( n \)th element, respectively. Then, with the parameters introduced above, the received signals at U and E can be expressed as follows:

\[ y_u = \sqrt{P} \left( \sum_{n=1}^{N} \sqrt{\xi_u} h_n^r g_n e^{j\nu_n} \sqrt{\xi_u} T_u P_u h_n^u \right) x + n_u, \quad (5) \]

\[ y_e = \sqrt{P} \left( \sum_{n=1}^{N} \sqrt{\xi_e} h_n^h g_n e^{j\nu_n} \sqrt{\xi_e} T_e P_e h_n^e \right) x + n_e, \quad (6) \]

where \( P \) and \( x \) denote the transmit power and the transmitted signal with \( \mathbb{E}[|x|^2] = 1 \), respectively, \( n_u \sim \mathcal{CN}(0, \delta_u^2) \) and \( n_e \sim \mathcal{CN}(0, \delta_e^2) \) are the additive white Gaussian noise (AWGN) with power \( \delta_u^2 \) at U and \( \delta_e^2 \) at E, respectively, and \( g_n \) and \( \nu_n \) are the amplitude and phase shift of the \( n \)th element, respectively. In particular, the amplitudes of all elements \( g_n, n \in \{1, 2, \ldots, N\} \) are defined as \( 1 \) to ensure the maximal reflect power [23]. Furthermore, we assume all elements can be adjusted independently.

Ideally, to maximize the received signal power at the target receiver U, the phase shifts of RIS ought to direct all signals toward U to the same direction \( \vartheta \). That is, the optimal phase shift of the \( n \)th element satisfies \( \nu_n = \vartheta - (\theta_n^r + \theta_n^h) \) without requisition of the CSI of S-R-E link. In the proposed system, U is a legitimate receiver known to S and R. Therefore, we can obtain perfect CSI of the S-R-U link as long as an effective channel estimation method is applied [37]. However, the finite adjustment precision of RIS makes only a limited number of phase shifts available [38]. As a result, the phase error, called quantization error, is inevitably introduced. We assume that only a discrete set of \( 2^b \) phases can be configured where \( b \in \{1, 2, \ldots, N\} \) stands for the quantization bit. Mathematically, the resulting phase shift of the \( n \)th reflecting element is most likely equal to

\[ \nu_n = \vartheta - (\theta_n^r + \theta_n^h) + \epsilon_n, \quad (7) \]

where \( \epsilon_n, n \in \{1, 2, \ldots, N\} \) is the phase error, which is assumed to be uniformly distributed with the PDF as

\[ f_{\epsilon_n} (\epsilon_n) = \begin{cases} \frac{1}{\pi b}, & \epsilon_n \in \left[ -\frac{\pi b}{2}, \frac{\pi b}{2} \right] \\ 0, & \text{otherwise} \end{cases} \quad (8) \]

Therefore, the SNRs at U and E are attained as

\[ \gamma_u = \frac{\sum_{n=1}^{N} T_u P_u \epsilon^{j\nu_n}}{\sqrt{\sum_{n=1}^{N} \xi_u^{\text{Th}}} + \sum_{n=1}^{N} \sigma^2 + \sum_{n=1}^{N} \text{Th}^2}, \quad (9) \]

\[ \gamma_e = \frac{\sum_{n=1}^{N} T_e P_e \epsilon^{j(\theta_n^r + \theta_n^h + \epsilon_n)}}{\sqrt{\sum_{n=1}^{N} \xi_e^{\text{Th}}} + \sum_{n=1}^{N} \sigma^2 + \sum_{n=1}^{N} \text{Th}^2}, \quad (10) \]

respectively, where \( \gamma_u = P L_s \xi_u^{\text{Th}}/\delta_u^2 \) and \( \gamma_e = P L_s \xi_e^{\text{Th}}/\delta_e^2 \).

### III. PERFORMANCE ANALYSIS

Both \( \gamma_u \) and \( \gamma_e \) in (9) and (10) can be disassembled into three parts: the constant terms \( \gamma_u^0 \xi_u^2 \) and \( \gamma_e^0 \xi_e^2 \), the combined effect of atmosphere turbulence and pointing error denoted by \( h_q = T_q P_q, q \in \{u, e\} \), and the equivalent channels of S-R-U and S-R-E links which are represented as \( r_u = \frac{1}{N} \sum_{n=1}^{N} e^{j\epsilon_n} \) and \( r_e = \frac{1}{N} \sum_{n=1}^{N} e^{j(\theta_n^r + \theta_n^h + \epsilon_n)} \), respectively. Therefore, as long as the distribution of \( h_q P_q, q \in \{u, e\} \) is derived, those of the received SNRs can be obtained. In the sequel, we will discuss them separately.

#### A. Distribution of \( h_q \)

The distribution of \( h_q, q \in \{u, e\} \) has been deduced in [39] by making use of the approximation of mixture Gamma turbulence model as

\[ h_q (h_q) = \frac{\varphi_\varphi^2}{A_\varphi^2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{q,i} \xi_q^{j-\alpha_q} \Gamma \left( \alpha_q - \xi_q - \frac{\xi_q - \theta_q^{\text{Th}}}{A_q} \right), \quad (11) \]

where \( \gamma_q > 0, q \in \{u, e\} \), and \( M \) is the order of the Gauss-Laguerre quadrature method, \( a_{q,i} = \frac{\eta_{q,i}}{\Gamma(\epsilon_q^{j-\alpha_q})} \), \( \xi_q^{j-\alpha_q} = \frac{\alpha_q \beta_q}{\varphi_\varphi^2}, \eta_{q,i} = \frac{(\alpha_q \beta_q)^{\epsilon_q^{j-\alpha_q}}}{\Gamma^{\alpha_q} \Gamma(\delta_q^{j-\alpha_q})} \), and \( \rho_{q,i} \) and \( a_{q,j} \) are the abscissas and weight factors for Laguerre integration [40, Table 25.9].

#### B. Distribution of \( r_u \)

Let us rewrite \( r_u \) as

\[ r_u = \frac{1}{N} \sum_{n=1}^{N} e^{j\epsilon_n} \left| C_u + jS_u \right|, \quad (12) \]
where $C_u = \frac{1}{N} \sum_{n=1}^{N} \cos(\varepsilon_n)$ and $S_u = \frac{1}{N} \sum_{n=1}^{N} \sin(\varepsilon_n)$. The expectations and variances of $C_u$ and $S_u$ can be derived as follows:

$$\mu_{u,c} = \mathbb{E} [C_u] = \mathbb{E} [\cos (\varepsilon_n)] = \varphi_1,$$
$$\mu_{u,s} = \mathbb{E} [S_u] = 0,$$

$$\chi^2_{u,c} = \frac{1}{N} \left( \mathbb{E} [\cos^2 (\varepsilon_n)] - \left( \mathbb{E} [\cos (\varepsilon_n)] \right)^2 \right) = \frac{1 + \varphi_2 - 2 \varphi_1^2}{2N},$$
$$\chi^2_{u,s} = \frac{1}{N} \left( \mathbb{E} [\sin^2 (\varepsilon_n)] - \left( \mathbb{E} [\sin (\varepsilon_n)] \right)^2 \right) = \frac{1 - \varphi_2}{2N},$$

where $\varphi_p, p = 1, 2$ is the characteristic function of $\varepsilon_n$,

$$\varphi_p = \mathbb{E} [e^{j p \varepsilon_n}] = \frac{2^b \sin \left( 2^{-b} p \pi \right)}{p \pi}.$$

With a view to the Gaussian distribution followed by $C_u$ and $S_u$ by means of the central limit theorem (CLT) when $N \to \infty$, the discussion of their independence is equivalent to that of their correlation with the Pearson’s correlation coefficient to be

$$\rho = \frac{\mathbb{E} [C_u S_u] - \mu_{u,c} \mu_{u,s}}{\chi_{u,c} \chi_{u,s}},$$

where

$$\mathbb{E} [C_u S_u] = \frac{1}{N^2} \left[ \sum_{i=1}^{N} \sum_{j=1}^{N} \cos (\varepsilon_i) \sin (\varepsilon_j) \right] = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \mathbb{E} [\cos (\varepsilon_i) \sin (\varepsilon_j)] + \frac{1}{N^2} \sum_{i=1}^{N} \mathbb{E} [\cos (\varepsilon_i) \sin (\varepsilon_i)].$$

After calculation we find that $\mathbb{E} [\cos (\varepsilon_i) \sin (\varepsilon_j)] = \mathbb{E} [\cos (\varepsilon_i) \sin (\varepsilon_i)] = 0$ always holds. As a result, $\rho = 0$, that is, $C_u$ and $S_u$ are independent. Based on the analysis above, referring to [21], $r_u$ has an approximate Nakagami distribution with PDF as

$$f_{r_u} (r_u) = \frac{2m^m}{\Gamma(m)} r_u^{2m-1} e^{-\frac{r_u^2}{\mu_{u,c}^2}} I_0 (2m \frac{r_u^2}{\mu_{u,c}^2}), r_u \geq 0,$$

for large $N$, where $m = \mu_{u,c}^2 / 4 \chi_{u,c}^2$ and $\Omega = \mu_{u,c}^2$.

### C. Distribution of $r_e$

Let $\theta_n^{\text{rec}} = \theta_n^e + \varepsilon_n$. Then, $r_e$ can be expressed as $r_e = \left| \sum_{n=1}^{N} e^{j \theta_n^{\text{rec}}} \right|$. To obtain the distribution of $r_e$, we need to derive the distribution of $\theta_n^{\text{rec}}$ first. As is generally assumed in the existing works, $\theta_e$ and $\theta_n$ are uniformly distributed in $[0, 2\pi]$; therefore, $\theta_n^e - \theta_n^m$ is also uniformly distributed in the same interval. Therefore, the PDF of $\theta_n^{\text{rec}}$ is equal to the convolution of those of $\theta_n^e - \theta_n^m$ and $\varepsilon_n$:

$$f_{\theta_n^{\text{rec}}} (\theta_n^{\text{rec}}) = \int_{-\infty}^{+\infty} f_{\theta_n^e - \theta_n^m} (\tau) f_{\varepsilon_n} (\theta_n^{\text{rec}} - \tau) d\tau.$$ (18)

Substituting $f_{\theta_n^e - \theta_n^m} (\tau)$ as

$$f_{\theta_n^e - \theta_n^m} (\tau) = \begin{cases} \frac{1}{2\pi}, & \tau \in [0, 2\pi), \\ 0, & \text{otherwise} \end{cases}$$

and (8) into (18) yields

$$f_{\theta_n^{\text{rec}}} (\theta_n^{\text{rec}}) = \begin{cases} \frac{2^{b-1} \theta_n^{\text{rec}} + \frac{1}{2b}}{\pi x}, & \theta_n^{\text{rec}} \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \\ \frac{2^{b-1} \theta_n^{\text{rec}} + 2^{b+1} + \frac{1}{4b}}{2\pi}, & \theta_n^{\text{rec}} \in \left[ 2\pi - \frac{\pi}{2}, 2\pi + \frac{\pi}{2} \right] \\ 0, & \text{otherwise} \end{cases}$$ (19)

Likewise, we re-define $r_e = \left| C_e + j S_e \right|$, where $C_e = \sum_{n=1}^{N} \cos (\theta_n^{\text{rec}})$ and $S_e = \sum_{n=1}^{N} \sin (\theta_n^{\text{rec}})$. The expectations and variances of $C_e$ and $S_e$ can be calculated as follows:

$$\mu_{e,c} = \mathbb{E} [C_e] = 0,$$
$$\mu_{e,s} = \mathbb{E} [S_e] = 0,$$

$$\chi^2_{e,c} = N \left( \mathbb{E} [\cos^2 (\theta_n^{\text{rec}})] - \left( \mathbb{E} [\cos (\theta_n^{\text{rec}})] \right)^2 \right) = \frac{N}{2},$$
$$\chi^2_{e,s} = N \left( \mathbb{E} [\sin^2 (\theta_n^{\text{rec}})] - \left( \mathbb{E} [\sin (\theta_n^{\text{rec}})] \right)^2 \right) = \frac{N}{2}.$$ (20)

Obviously, $C_e$ and $S_e$ follow the same zero-mean Gaussian distribution for a large value of $N$ in terms of CLT. Moreover, after a similar but more complex process to (15) and (16), we infer that $C_e$ and $S_e$ are also independent. Therefore, substituting the parameters in (20) into the PDF of the two-dimensional Gaussian distribution generates

$$f_{C_e, S_e} (x, y) = \frac{1}{2\pi \chi_{e,c} \chi_{e,s}} e^{-\frac{x^2}{\chi_{e,c}^2} - \frac{y^2}{\chi_{e,s}^2}}.$$ (21)

For the PDF of $r_e$, we can apply the transformation $f_{r_e, \phi} (r_e, \phi) = f_{r_e, S_e} (r_e, \phi)$ to (21), where $\phi$ is the phase of $\sum_{n=1}^{N} e^{j \theta_n^{\text{rec}}}$ satisfying $C_e = \left| r_e \cos (\phi) \right|$ and $S_e = \left| r_e \sin (\phi) \right|$. Therefore, (21) can be transformed into

$$f_{r_e, \phi} (r_e, \phi) = \frac{r_e}{2\pi \chi_{e,c} \chi_{e,s}} e^{-\frac{r_e^2 \cos^2 (\phi)}{2\chi_{e,c}^2} - \frac{r_e^2 \sin^2 (\phi)}{2\chi_{e,s}^2}}.$$ (22)

One of the marginal distributions is the PDF of $r_e$, which can be obtained as

$$f_{r_e} (r_e) = \frac{r_e}{\chi_{e,c}^2} e^{-\frac{r_e^2}{2\chi_{e,c}^2}}, r_e \geq 0.$$ (23)

### D. Ergodic Secrecy Rate

The last step before calculating the ESR is to derive the distributions of $H_u = h_u r_u$ and $H_e = h_e r_e$. The PDF of $H_u$ can be readily derived as

$$f_{H_u} (z) = \int_{0}^{\infty} f_{h_u} (x) f_{r_u} \left( \frac{z}{x} \right) dx = \sum_{i=1}^{M} A_{\alpha_i}^2 m^{m-1} I_{\alpha_i}^m \frac{2 m^m}{\Gamma(m)} \alpha_i \Omega \mu_{u,c}^2 \chi_{u,c}^2 I_1,$$ (24)
where \( I_1 \) is (25) shown at the bottom of this page. Similarly, the PDF of \( H_e \) can also be attained as

\[
    f_{H_e}(z) = \sum_{i=1}^{M} \frac{\omega^2}{A_e^2} \frac{\alpha_i}{\chi_{e,c}} I_2, \tag{26}
\]

where \( I_2 \) is (27) shown at the bottom of this page.

As stated before, the received SNRs of U and E in (9) and (10) can be re-expressed as

\[
    \gamma_u = \gamma^0_u N^2 (h_u r_u)^2 = \gamma^0_u N^2 H_u^2 \quad \text{and} \quad \gamma_e = \gamma^0_e (h_e r_e)^2 = \gamma^0_e H_e^2.
\]

Therefore, the ESR is generalized as

\[
    R_s = [E[R_u] - E[R_e]], \tag{28}
\]

denote the ergodic rates of U and E, which can be easily solved by mathematical softwares such as MATLAB through numerical integration.

\[
    I_1 = z^{2m-1} (I_1^2 + I_2^2 - I_3^3 + I_4^4),
\]

\[
    I_1^1 = \frac{1}{2} \left( \frac{\Omega}{m z^2} \right) \frac{\alpha}{\omega^2} \Gamma \left( \alpha - \omega^2 \right) \Gamma \left( m - \omega^2 \right),
\]

\[
    I_1^2 = \frac{1}{2} \left( \frac{\xi_{u,i}}{A_u} \right)^{2m-1} \frac{\alpha}{\omega^2} \Gamma \left( \alpha - \omega^2 \right) \Gamma \left( m - \omega^2 \right) F_3 \left( m - \omega^2 \frac{1}{2}, 1 - \alpha \frac{1}{2}, m \frac{1}{2}, 1 - \alpha \frac{1}{2}, m + 1 - \omega^2 \frac{1}{2}, 4A_u^2 \Omega \right),
\]

\[
    I_1^3 = \frac{\xi_{u,i}}{A_u} \frac{\alpha - \omega^2}{2} \frac{\alpha + m}{\omega^2} \Gamma \left( m - \omega^2 \right) F_3 \left( m - \omega^2 \frac{1}{2}, 1 - \alpha \frac{1}{2}, m \frac{1}{2}, 1 - \alpha \frac{1}{2}, m + 1 - \omega^2 \frac{1}{2}, 4A_u^2 \Omega \right),
\]

\[
    I_1^4 = \frac{\xi_{u,i}}{A_u} \frac{\alpha - \omega^2}{2} \frac{\alpha + m}{\omega^2} \Gamma \left( m - \omega^2 \right) F_3 \left( m - \omega^2 \frac{1}{2}, 1 - \alpha \frac{1}{2}, m \frac{1}{2}, 1 - \alpha \frac{1}{2}, m + 1 - \omega^2 \frac{1}{2}, 4A_u^2 \Omega \right). \tag{25}
\]

\[
    I_2 = z \left( I_2^1 + I_2^2 - I_3^3 + I_4^4 \right),
\]

\[
    I_2^1 = z^{2m-1} \frac{\alpha}{\omega^2} \frac{\alpha^2}{m z^2} \Gamma \left( \alpha - \omega^2 \right) \Gamma \left( m - \omega^2 \right),
\]

\[
    I_2^2 = \frac{1}{2} \frac{\xi_{e,c}}{A_e} \frac{\alpha - \omega^2}{\omega^2} \frac{\alpha}{m z^2} \Gamma \left( \alpha - \omega^2 \right) \Gamma \left( m - \omega^2 \right) F_3 \left( m - \omega^2 \frac{1}{2}, 1 - \alpha \frac{1}{2}, m \frac{1}{2}, 1 - \alpha \frac{1}{2}, m + 1 - \omega^2 \frac{1}{2}, 8A_e^2 \chi_{e,c}^2 \right),
\]

\[
    I_2^3 = \frac{1}{2} \frac{\xi_{e,c}}{A_e} \frac{\alpha - \omega^2}{\omega^2} \frac{\alpha}{m z^2} \Gamma \left( \alpha - \omega^2 \right) \Gamma \left( m - \omega^2 \right) F_3 \left( m - \omega^2 \frac{1}{2}, 1 - \alpha \frac{1}{2}, m \frac{1}{2}, 1 - \alpha \frac{1}{2}, m + 1 - \omega^2 \frac{1}{2}, 8A_e^2 \chi_{e,c}^2 \right),
\]

\[
    I_2^4 = \frac{1}{2} \frac{\xi_{e,c}}{A_e} \frac{\alpha - \omega^2}{\omega^2} \frac{\alpha}{m z^2} \Gamma \left( \alpha - \omega^2 \right) \Gamma \left( m - \omega^2 \right) F_3 \left( m - \omega^2 \frac{1}{2}, 1 - \alpha \frac{1}{2}, m \frac{1}{2}, 1 - \alpha \frac{1}{2}, m + 1 - \omega^2 \frac{1}{2}, 8A_e^2 \chi_{e,c}^2 \right). \tag{27}
\]

### E. Considering Both Quantization and Estimation Errors

According to [22], [23], [24], an imperfect channel estimation should also be considered in practical scenarios that arises from inaccurate channel estimation methods or synchronization errors caused by the mobility of communication nodes. The phase error it brings is called the estimation error. Different from the quantization error attached to \( \nu_n \), mentioned in (7), the estimation error comes with \( \mu^r_n \) and \( \mu^u_n \) during an imperfect channel estimation. In this subsection, we will analyze the effect of the superposition of these two kinds of phase errors on the ESR.

When the phase error, \( \nu_n \), is introduced in the imperfect channel estimation, it follows the Von Mises distribution with zero mean and concentration parameter \( \kappa \in \{1, 2, \ldots, N\} \), whose PDF is expressed as [36]

\[
    f_{\nu_n} (\nu_n) = \frac{e \cos(\nu_n)}{2\pi I_0(\kappa)}, \quad \nu_n \in [-\pi, \pi]. \tag{30}
\]

Simultaneously taking quantization and estimation errors into account, the total phase error \( \hat{\nu}_n \) is the sum of two independent variables subject to uniform and Von Mises distributions, respectively, namely \( \hat{\nu}_n = \nu_n + \nu_n \). Therefore, \( \hat{\nu}_n \) can be analogous to...
The analysis of the theoretical ESR is the same as that of (28), except for a few differences described below.

Firstly, the distribution of $\hat{\varepsilon}_n$ needs to be derived. Since the PDF of Von Mises distribution (30) is not integrable at the endpoints, we first need to make a transformation to it. Considering the case where $\kappa \gg 1$, we have $\log(\kappa) \approx \varepsilon^2/2\pi\kappa$. Meanwhile, when $\nu_n \to 0$, which is equivalent to $\kappa \gg 1$, $\cos(\nu_n) \approx 1 - \nu_n^2/2$ holds [41]. Substituting these two equations into (30), it is observed that when $\kappa \gg 1$ or $\nu_n \to 0$, the distribution of $\nu_n$ can be re-expressed as

$$f_{\nu_n}(\nu_n) \approx \sqrt{\frac{\kappa}{2\pi}} \exp \left(-\frac{\nu_n^2}{2}\right), \quad \nu_n \in [-\pi, \pi). \quad (31)$$

Convolving the PDFs of $\varepsilon_n$ and $\nu_n$ yields the PDF of $\hat{\varepsilon}_n$, which is (32) shown at the bottom of this page. Furthermore, since the distribution of $\hat{\varepsilon}_n$ is symmetric with respect to the vertical axis, its characteristic function must be in the real number field, that is,

$$\varphi_1 = E\left[e^{j\hat{\varepsilon}_n}\right] = E\left[\cos(\hat{\varepsilon}_n)\right]$$

$$\varphi_2 = E\left[e^{j2\hat{\varepsilon}_n}\right] = E\left[\cos(2\hat{\varepsilon}_n)\right]$$

which can be calculated by mathematical softwares and substituted into (13) to obtain $\hat{\mu}_{u,c}, \hat{\mu}_{u,s}, \hat{\lambda}_{u,c}^2$, and $\hat{\chi}_{u,s}^2$.

On the other hand, the phase related to the $r$th element of the S-R link in this case can be expressed as $\hat{\theta}_{avr} = \theta_{\nu} - \theta_{0} + \varepsilon_{n} + \nu_{n}$. It is observed that the PDF of $\hat{\theta}_{avr}$ can be derived by convolving those of $\theta_{0} - \theta_{\nu} + \varepsilon_{n}$ in (19) and $\nu_{n}$ in (30). Therefore, we can obtain the PDF of $\hat{\theta}_{avr}$ as

$$f_{\hat{\theta}_{avr}}(\hat{\theta}_{avr}) = \frac{e^{j\hat{\theta}_{avr}}}{4\pi} + \frac{\hat{\theta}_{avr}}{4\pi}, \quad \hat{\theta}_{avr} \in [-\pi, \pi)$$

$$f_{\hat{\theta}_{avr}}(\hat{\theta}_{avr}) = 0, \quad \hat{\theta}_{avr} \in [\pi, 3\pi). \quad (34)$$

Substitute $\hat{\theta}_{avr}$ for $\theta_{avr}$ in $r_e = \sum_{n=1}^{N} e^{j\theta_{avr}}$, and then the parameters $\hat{\mu}_{c,c}, \hat{\mu}_{c,s}, \hat{\chi}_{c,c}^2$, and $\hat{\chi}_{c,s}^2$ corresponding to those in (20) are deduced to be

$$\hat{\mu}_{c,c} = \mu_{c,c}, \hat{\mu}_{c,s} = 0,$$

$$\hat{\chi}_{c,c}^2 = \chi_{c,c}^2, \quad \hat{\chi}_{c,s}^2 = N \frac{2}{\sqrt{\kappa}} \exp \left(\sqrt{\frac{\kappa}{2\pi}}\right). \quad (35)$$

It should be noted that $C_u$ and $S_u$ as well as $C_c$ and $S_c$ are also independent in this case. To this end, the approximate ESR expression in the presence of both quantization and estimation errors can be also obtained as

$$\hat{R}_a = \left[E\left[\hat{R}_u\right] - E\left[\hat{R}_a\right]\right]^+, \quad (36)$$

with

$$E\left[\hat{R}_u\right] = \int_{0}^{\infty} \log_2 \left(1 + \gamma_u N^2 z^2\right) f_{H_u}(z) dz,$$

$$E\left[\hat{R}_a\right] = \int_{0}^{\infty} \log_2 \left(1 + \gamma_a z^2\right) f_{H_a}(z) dz.$$

Where $f_{H_u}(z)$ is attained by replacing $\mu_{u,c}$ and $\chi_{u,c}^2$ with $\hat{\mu}_{u,c}$ and $\hat{\chi}_{u,c}^2$ in (17) to get $f_{r_u}(r_u)$, respectively, and then use $f_{r_c}(r_c)$ in (24). Similarly, $f_{H_a}(z)$ is acquired by replacing $\chi_{c,c}^2$ with $\chi_{c,c}^2$ in (23) to get $f_{r_c}(r_c)$, and then use $f_{r_c}(r_c)$ in (26).

IV. PROBLEM FORMULATION AND OPTIMIZATION

In this section, we optimize the phase shifts of RIS to maximize the SR based on the statistical and perfect CSI of $E$, respectively.

A. Problem Formulation

Rewrite (5) and (6) as

$$y_u = \sqrt{PL_u C_u T_u P_u h_u^H \Theta_r + n_u}, \quad (38)$$

$$y_c = \sqrt{PL_c C_c T_c P_c h_c^H \Theta_r + n_c}, \quad (39)$$

where $h_u = [h_u^1, h_u^2, ..., h_u^N]^H \in \mathbb{C}^{N \times 1}$, $h_c^H = [h_c^1, h_c^2, ..., h_c^N]^H \in \mathbb{C}^{1 \times N}$, $\Theta = \text{diag}(t)$ and $t = [e^{j\gamma_1}, e^{j\gamma_2}, ..., e^{j\gamma_N}]^H \in \mathbb{C}^{N \times 1}$. Therefore, the SNR expressions (9) and (10) can also be re-expressed as

$$\gamma_u = \frac{\gamma_u^0}{\left|h_u^H \Theta_r \right|^2}, \quad (40)$$

$$\gamma_c = \frac{\gamma_c^0}{\left|h_c^H \Theta_r \right|^2}, \quad (41)$$

where $\gamma_u^0 = PL_u C_u T_u P_u / \delta_u^2$ and $\gamma_c^0 = PL_c C_c T_c P_c / \delta_c^2$, which can be regarded as constants for the variable $\Theta$. To this end, the problem is formulated as

$$\max_{\Theta} \left[\log_2 (1 + \gamma_u) - \log_2 (1 + \gamma_c)\right]^+, \quad (42)$$

s.t. $|\Theta|_{n,n} = 1, \ n = 1, 2, ..., N. \quad (42a)$

Next, we will discuss these two cases of $E$ separately.
B. Optimization With Statistical CSI of E

With the statistical CSI of E, the problem (42) can be transformed as follows:

\[
\max_{\Phi} \left\{ \frac{1 + \frac{\gamma_{\text{th}}}{\gamma_{\text{th}}^2} |h_u^H \Theta h_r|^2}{1 + \frac{\gamma_{\text{th}}}{\gamma_{\text{th}}^2} |h_c^H \Theta h_r|^2} \right\} \tag{43}
\]

\[
s.t. \quad \left| \Theta_{n,n} \right| = 1, \quad n = 1, 2, \ldots, N. \tag{43a}
\]

Apparently, the problem (43) no longer has any connection with the CSI of E; therefore, it is wise to compensate for the cascaded channel phases of S-R and R-U links, that is, let \( |\Theta^*|_{n,n} = \exp(-j \arg(h_r h_u^H + h_u h_c^H)) \), \( n \in \{1, 2, \ldots, N\} \), where \( \Theta^* \) is the optimal solution to problem (43). Then \( \Phi^* \) is obtained by \( \Phi^* = \Theta^* + \tilde{\Phi} \), \( \tilde{\Phi} \) is the quotient of two linear functions. Therefore, we continue to make \( \eta = 1/\text{tr}(\Lambda, \Psi) \). Meanwhile, to ensure that all constraints are convex, we temporarily relax (43a) first. As a result, the problem (43) can be re-expressed as

\[
\max_{\eta,\Phi} \text{tr} (\Lambda, \Phi) \tag{50}
\]

\[
s.t. \quad \eta > 0, \quad \Phi > 0, \tag{50a}
\]

\[
\text{tr} (\Lambda, \Phi) = 1, \tag{50b}
\]

\[
\left| \Phi_{n,n} \right| = \eta, \quad n = 1, 2, \ldots, N. \tag{50c}
\]

\[\Psi \geq 0, \tag{49b}\]

\[\text{rank} (\Psi) = 1. \tag{49c}\]

It is observed that the objective function in (49) is the quotient of two linear functions. Therefore, we continue to make \( \eta = 1/\text{tr}(\Lambda, \Psi) \). Meanwhile, to ensure that all constraints are convex, we temporarily relax (43a) first. As a result, the problem (43) can be re-expressed as

\[
\max_{\Lambda, \Psi} \text{tr} (\Lambda, \Psi) \tag{44}
\]

\[
s.t. \quad \left| \Theta_{n,n} \right| = 1, \quad n = 1, 2, \ldots, N. \tag{44a}
\]

where the objective function and constraint are all non-convex. Applying the identical equation \( \text{diag}(a)b = \text{diag}(b)a \) for two vectors \( a \) and \( b \), the above problem (44) can be re-expressed as

\[
\max_{\Phi} \frac{1 + \frac{\gamma_{\text{th}}}{\gamma_{\text{th}}^2} |h_u^H \Phi h_r|^2}{1 + \frac{\gamma_{\text{th}}}{\gamma_{\text{th}}^2} |h_c^H \Phi h_r|^2} \tag{45}
\]

\[
s.t. \quad \left| \Theta_{n,n} \right| = 1, \quad n = 1, 2, \ldots, N. \tag{45a}
\]

where \( \Phi \) is the maximal eigenvalue of \( \Psi^* \) and corresponding eigenvector to \( e_{\text{max}} \), respectively; otherwise, the eigenvalue \( e_{\text{uni}} \) of \( \Psi^* \) and corresponding eigenvector \( q_{\text{uni}} \) are unique. Therefore, \( t^* = \sqrt{e_{\text{uni}}q_{\text{uni}}} \) is the optimal solution. The discretization of \( t^* \) is the same as that in the previous subsection to obtain \( \hat{t}^* \).

Our proposed optimization algorithms are summarized in Algorithm 1. Since there is a specific optimal solution to problem (43) and the objective function of problem (50) is affine as well with a bounded variable, the convergence of Algorithm 1 is guaranteed. Additionally, regarding to the computational complexity, when only the statistical CSI of E is available, the phase shift of each element is derived in turn. Therefore, the problem (43) can be solved with a complexity of \( O(N^2) \); on the other hand, the problem (50) can be solved with a worst-case complexity of \( O(N^4) \) [42].

Remark: The optimization algorithms above are based on the assumption that there is only quantization error. Once the estimation error is also taken into account along with the quantization error, two independent phase errors subject to the Von Mises distribution should be imposed to the quantized optimal phase shifts before applied to S-R-U and S-R-E links, respectively. The related results will be shown in Section V.

V. SIMULATION RESULTS

To validate the theoretical analysis and evaluate the performance of the proposed algorithms, in this section, numerical results are presented under simulated channels. All simulations are run over 10000 times.

Unless otherwise specified, the parameters used in the simulations are set as follows: the operating wavelength \( \lambda = 500 \mu \text{m} \) (the corresponding frequency is 0.6 THz); the distance between two communicating nodes \( d_{sr} = 150 \text{ km}, d_{ru} = 19 \text{ km}, d_{ce} = 20 \text{ km} \); the beam waist at U and E \( w_u = w_c = w \); the
Algorithm 1: Phase Shifts Optimization Algorithm to Maximize the SR.

Input: Channels $h_r$, $h_a$, $h_c$; Constants $\gamma_0$ and $\gamma_1$.
Output: $t^*$

1: if the only the statistical CSI of $E$ is available then
2: \[ \Theta_i^{\star} = \exp(-j \arg([h_r]_n + [h_a]_n)), n = 1, 2, \ldots, N \]
3: \[ [t^*]_n = [\Theta_i^{\star}]_{n,n}, n = 1, 2, \ldots, N \]
4: else
5: solve problem (50) to find $\eta^{\star}$ and $\Phi^{\star}$
6: \[ \Psi^{\star} = \Phi^{\star}/\eta^{\star} \]
7: execute eigenvalue decomposition of $\Psi^{\star}$ to obtain its eigenvalues $\{\epsilon_i\}$, $1 \leq i \leq N$ and corresponding eigenvectors $\{q_i\}$, $1 \leq i \leq N$
8: if $1 < \text{rank}(\Psi^{\star}) \leq N$ then
9: \[ t^{\star} = \sqrt{\epsilon_{\max}} q_{\max} \]
10: else
11: \[ t^{\star} = \sqrt{\epsilon_{\text{uni}}} q_{\text{uni}} \]
12: end if
13: end if
14: discretize $t^{\star}$ based on the finite phase shift set $\Sigma = \left\{ 0, \frac{\pi}{2}, \ldots, \frac{(2^b-1)2\pi}{2^n} \right\}$ to yield $\tilde{t}^{\star}$
15: return $t^{\star}$

Fig. 3. Simulation and theoretical results for ESR versus the number of element $N$ with $b = 1$, $\kappa = 3$, $\sigma^u = 0.1$, $\sigma^e = 0.2$. $C^2_s = 10^{-13}$, $P/\delta^2 = 260$ dB.

antenna gains $G_s = 69$ dBi, $G_u = G_c = 55$ dBi (inferring the equal radius of the effective area of receive antennas on $U$ and $E$ $l_u = l_c = l$); the noise power at receivers $\delta^u = \delta^e = \delta^2$. Besides, for brevity, the single quantization error case and two-error case are represented by P1 and P2 in the legends of all figures, respectively.

Fig. 3 shows a comparison of Monte Carlo simulations and theoretical results for the ESR versus the number of reflective elements $N$. It can be seen that as $N$ increases, the gap between the theoretical ESR and the simulation results decreases, no matter there is P1 or P2. In addition, it is evident that the growth of $N$ brings an increase in ESR. For instance, under the case of P1 with $w/l = 6$, the ESR equals about 1.9 bps/Hz when $N = 10$, while it increases to around 6.6 bps/Hz when $N = 100$, indicating the diversity gain brought by RIS. In contrast, a larger ratio of $w$ to $l$ leads to a smaller ESR, which is because an excessively large $w/l$ reduces the energy per unit area of the wave reaching the receivers.

As depicted in Fig. 4, the index of refraction structure parameter $C^2_s$ going up means more intense atmosphere turbulence, and as a result the ESR eventually drops. Moreover, it is intuitive that the ESR can be generally guaranteed to be greater than 0 when the jitter standard variance of $U$ is smaller than that of $E$, as long as the other parameters are exactly the same. Whereas Fig. 4 shows that it is still practicable to maintain the ESR positive when $\sigma^u = \sigma^e$ and even when $\sigma^u > \sigma^e$ under the worst phase error case with $b = 1$ and $\kappa = 1$. Specifically, when $\sigma^e = 0.2$ and $\sigma^u = 0.1$, the ESR increases from near 0 bps/Hz to almost 1.4 bps/Hz as $N$ increases from 20 to 200. Therefore, by increasing the number of element, the system security can be ensured even in a hostile environment.

Additionally, we investigate the relationships between the quantization bit $b$ of uniform distribution, the concentration parameter $\kappa$ of Von Mises distribution, and the ESR in Fig. 5. Specifically, the ESR improves as $\kappa$ or $b$ increases, as a result of a decreasing phase error. However, it is worth noting that the larger the values of $b$ and $\kappa$, the weaker the effect of increasing them on system security. The main reason is that when there is a high-precision RIS or the knowledge of CSI is abundant ($b$ or $\kappa$ is big), it does not make much sense to continue to enhance them. Furthermore, Fig. 5(b) shows that the curves under P2 with $\kappa = 10$ and $b = 1$ is very close to that under P1 with $b = 1$, and it can be inferred from their relationship that these two curves will definitely coincide as long as $\kappa$ becomes large enough. Moreover, the coordinates in Fig. 5 imply a positive correlation between the ESR and transmit SNR.

Fig. 6 depicts the lower bounds of SR and instantaneous SR, obtained by the three different phase shift selection methods:
random phase shifts, phase shifts that align THz waves to U, and optimized phase shifts. We abbreviate these three methods as Rand, ToU, and Opt, respectively. As seen from this figure, the system is at serious secrecy risk when phase shifts of RIS are randomly selected. However, both the lower bound of SR and instantaneous SR are greatly improved with the help of our proposed optimization algorithms. Furthermore, when only the statistical CSI of E is available, as described in the previous section, the essence of Opt is ToU. Therefore, the lower bounds of SR obtained by both methods are exactly the same. Nevertheless, with a perfect CSI of E, as expected, Opt gives a larger gain to the instantaneous SR than ToU, except that this gain decreases as the phase error increases. To be more specific, the system achieves the best secrecy performance when the phase shifts of all elements are optimized without phase error. Moreover, when the number of quantization bits $b$ reaches 3, the optimized lower bound of SR obtained with a single quantization error is almost equal to the best result, and all of them tend to be stable as the transmit SNR increases. However, it needs a higher quantization precision to get closer to the best result for the instantaneous SR. Similarly, under P2 with $b = 1$, compared to the lower bound of SR, the instantaneous SR needs a larger $\kappa$ to reach the upper bound of it, which means more accurate CSI. Finally, Fig. 6 also reveals that the system can be maintained at a high level of security (both the lower bound of SR and instantaneous SR are far greater than 0) by taking advantage of the proposed optimization algorithms even when $\sigma_u^H > \sigma_e^H$.

VI. CONCLUSION

In this paper, we investigated a novel secure THz-empowered RANTN, where a LEO satellite indirectly send confidential information to a UAV in the troposphere, bridging by a RIS-mounted HAP situated in the stratosphere, while there is another untrustworthy UAV receiver near the trustworthy one. On account of the atmospheric environment, the sharp beam of THz wave, the relative motion of two communicating nodes, the finite adjustment precision of RIS and the imperfect channel estimation, the fading effects resulting from atmosphere turbulence, pointing error and phase errors were all taken into account simultaneously. Then the approximate ESR expression was derived. Besides, the phase shifts of RIS were optimized to enhance the security of the system with statistical and perfect CSI of the untrustworthy UAV, respectively. Finally, simulation results verified the effectiveness of our theoretical analysis and optimization algorithms. In the future, we will further optimize the trajectory of HAP to achieve greater performance gains.

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