Study of thermal monopoles in lattice QCD

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Motivation:
To study in SU(3) gluodynamics and QCD
– thermal monopole properties and their role in the quark-gluon plasma
– magnetic currents properties near the confinement-deconfinement transition
Dual superconductor - one of the most popular ideas about nature of confinement
t’ Hooft ’75, Mandelstam ’76
Confinement in QCD is due to condensation of color-magnetic monopoles
Respective effective theory - dual Abelian Higgs model (dual superconductor)

Problem: how to determine monopoles in QCD
 t’ Hooft ’81:
Partial gauge fixing
\( SU(N) \to U(1)^{N-1} \)
Very successful application of the MA gauge to define monopoles on a lattice

\[
\sum_{c \neq 3, 8} \left( \partial_\mu \delta_{ac} + \sum_{b=3, 8} f_{abc} A_\mu^b(x) \right) A_\mu^c(x) = 0, \quad a \neq 3, 8
\]

extremums (over \( g \)) of the functional \( F_{\text{MAG}}[A^g] \)

\[
F_{\text{MAG}}[A] = \frac{1}{V} \int d^4x \sum_{a \neq 3, 8} [A_\mu^a(x)]^2
\]

Abelian projection:
\( A_\mu^a(x) T^a \rightarrow A_\mu^3(x) T^3 + A_\mu^8(x) T^8 \)

on lattice
\[
F(U) = \frac{1}{V} \sum_{x, \mu} \left( |U_\mu(x)^{11}| + |U_\mu(x)^{22}| + |U_\mu(x)^{33}| \right),
\]
\( U_\mu(x) \rightarrow u_\mu(x) \in U(1)^2 \)
Magnetic currents definition:

\[ j_{\mu}^{(a)} \equiv \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} \partial_{\nu} \Theta_{\rho\sigma}^{(a)} = -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial_{\nu} m_{\rho\sigma}^{(a)}, \ a = 1, 2, 3 \]

satisfy the constraint

\[ \sum_a j_{\mu}^{(a)}(x) = 0, \]

on any link \( \{x, \mu\} \) of the dual lattice

Magnetic currents form closed loops
Simulations details:
SU(3) gluodynamics, standard Wilson action, mostly $32^3 \times 6$ lattices

$N_f = 2$ lattice QCD at $T > 0$, configurations produced by DIK collaboration, 2009
- Wilson action for the gauge field
- the non-perturbatively $O(a)$ improved Wilson fermionic action $S_F$:

$$S_F = S_F^{(0)} - \frac{i}{2} \kappa g c_{sw} a^5 \sum_s \bar{\psi}(s) \sigma_{\mu\nu} F_{\mu\nu}(s) \psi(s)$$

- Lattice size $12 \times (32)^3$
- Crossover at $T_c = 200$ MeV, $m_\pi = 400$ MeV
- Variation of temperature by variation of $L_t$
Nonpercolating monopole cluster average size - ’susceptibility’ $\chi_{cl}$.
Percolation of magnetic currents and phase transition

![Graph showing percolation of magnetic currents for SU(3), 32$^3$ x 6.](image)

Same for SU(3)
There are proposals suggesting that the color-magnetic monopoles contribution can explain strong coupling property of QGP near transition

Chernodub and Zakharov 2006, Liao and Shuryak 2006,

Chernodub and Zakharov:
Thermal monopoles are related to clusters of magnetic currents wrapped in $T$ dimension

Figure from D’Alessandro, D’Elia and Shuryak, 2010
Wrapping number for given cluster:

\[
N_{wr}^a = \frac{1}{3L_t} \sum_{j_4(x) \in \text{cluster}} j_4^a(x)
\]

\[
\rho = \frac{\langle \sum_{\text{clusters}, a} \left| N_{wr}^a \right| \rangle}{3L_s^3 a^3}
\]
First lattice study in SU(2) by VB, Mitrjushkin, Muller-Preussker, 1992

Comprehensive lattice study in SU(2) by D’Alessandro and D’Elia 2007

Subsequent work, also in SU(2): VB, Braguta, 2011; VB, Kononenko, 2012

First results for SU(3) and QCD: VB, Kononenko, Mitryushkin, presented at Confinement X, 2012
Liao and Shuryak

**Magnetic scenario:**
- magnetic monopoles are weakly interacting $(\alpha_M \sim 1/\alpha_E)$ near $T_c$ and are dominating fluctuations
- strongly influence QGP property, in particular reduce its viscosity

**Alternative approach to study of monopoles:** Classical molecular dynamics simulations for system with mixture of magnetic and electric charges
- Remarkably, good qualitative agreement with lattice results for density-density correlation functions
- Magnetic coupling $\alpha_M$ was computed from (lattice) correlation functions
- $\alpha_M$ increases with temperature
\( \alpha_M \) (blue symbols) extracted by Shuryak and Liao from lattice data obtained by D’Alessandro and D’Elia
Coulomb plasma parameter

\[ \Gamma = \alpha_m \left( \frac{4\pi \rho}{3T^3} \right)^{1/3} \]

- $\Gamma > 1$, i.e. strongly coupled plasma
- $\Gamma$ increases up to about 5 with increasing temperature
Total thermal monopoles density in $SU2$
Total thermal monopoles density $\rho$ vs $T/T_0$
Bose-Einstein condensation of the thermal monopoles

First study in SU(2) theory by D’Alessandro, D’Elia and Shuryak, 2010

a trajectory wrapping $k$ times in a time direction represents a set of $k$ monopoles permuted cyclically for non-relativistic noninteracting bosons

$$\rho_k = \frac{e^{-\hat{\mu}k}}{\lambda^3 k^{5/2}}$$ (1)

$\hat{\mu} \equiv -\mu/T$ is a chemical potential

$\lambda$ is the De Broglie thermal wavelength

the condensation temperature $T_{BEC}$ is determined by the vanishing of the chemical potential

$T_{BEC} \approx T_c$ D’Alessandro, D’Elia and Shuryak, 2010 confirmed in VB, Kononeko, 2012
Thermal monopoles density $\rho_k$ vs $T/T_c$ for $SU(2)$
Bose-Einstein condensation

Thermal monopoles density $\rho_k$ vs $T/T_c$ for $SU(3)$
Thermal monopole interactions

\[
g_{\text{MM}}(r) = \frac{\langle \rho^a_M(0)\rho^a_M(r) \rangle}{2\rho^b_M\rho^b_M} + \frac{\langle \rho^a_A(0)\rho^a_A(r) \rangle}{2\rho^b_A\rho^b_A}
\]

\[
g_{\text{AM}}(r) = \frac{\langle \rho^a_A(0)\rho^a_M(r) \rangle}{2\rho^b_A\rho^b_M} + \frac{\langle \rho^a_M(0)\rho^a_A(r) \rangle}{2\rho^b_A\rho^b_M}
\]

\[
g_{\text{MM,AM}}(r) = e^{-U(r)/T}
\]

\[
U(r) = \frac{\alpha_m}{r} e^{-m_D r}
\]
Thermal monopole interactions

Thermal monopoles correlation functions for $T/T_c = 2$
Table of results for $T/T_c = 2$

|       | $\alpha_M$ | $m_D/T$ | $\Gamma$  |
|-------|------------|---------|-----------|
| $SU(2)$ | 2.61(15)  | 1.75(12)| 1.95(16) |
| $SU(3)$ | 2.8(6)    | 1.8(2)  | 2.2(4)   |
| QCD    | 1.4(2)    | 1.8(1)  | 1.4(2)   |
Conclusions

We present new evidence on the percolation transition at $T_c$ in $SU(3)$ gluodynamics.

Our numerical results indicate

- Density of thermal monopoles in $SU(3)$ gluodynamics is similar to that in $SU(2)$ gluodynamics.

- In QCD it is substantially higher.

- Qualitative confirmation of the Bose-Einstein condensation in $SU(3)$ gluodynamics.

- Magnetic coupling $\alpha_m$ and screening mass $m_D/T$ in $SU(3)$ are close to those in $SU(2)$.

- $\alpha_m$ in QCD is lower by factor 2, $m_D/T$ is somewhat lower.