An Experimental Study of the Wear at Hopper Walls*†

G.D. Corder
JK Tech, JKMRC Commercial Division*
R.B. Thorpe
University of Cambridge, Dept. of Chemical Engineering**

Abstract

In this paper we describe experiments in which the equivalent of 100 tonnes of abrasive sand have been passed through a pilot-scale wedge-shaped hopper of half angle 10° and outlet width 1 cm. The walls were then cut up and the change in wall thickness accurately measured. The rate of wear was found to be greatest at the outlet. In interpreting these results, we used the simple abrasive model for wear recommended by Johanson and Royal [1]. In this model, the rate of wear is simply proportional to both the force on a particle pressing on the wall and the speed with which it scrapes down that wall. In order to calculate these parameters, we used a measurement of the velocity in a granular material at the wall of the hopper and a prediction for the stress which is a modification of the well-known method of Janssen. The constant of proportionality (the wear coefficient) for the model was measured in a pin-on-disc experiment. A prediction for the wear profile was thus obtained which shows reasonable (within a factor of two) agreement with experiment. The apparent success of the pin-on-disc method of measuring the wear coefficient suggests that it can be used instead of the more expensive methods advocated by others.

1. Introduction

The flow of hard and abrasive materials from storage vessels such as hoppers and silos causes wear to the walls of the vessel. This wear is concentrated near the outlet. The rate of wear is greater in mass flow silos which are designed for the uniform flow desirable for other reasons. It is useful to be able to predict the wear profile and the rate of wear so that the silo can be reinforced with thicker walls where necessary. This problem has received little attention in the scientific and engineering literature. In a paper published in 1982, Johanson and Royal [1] give general guidelines on the matter which form the basis of advice they will give as part of a design package offered by the company of consultants for whom they work. The paper is short on scientific reasoning and gives no specific experimental results. The methodology used by Johanson and Royal is based on the simple abrasive model for wear. In this model, the rate of wear, \( dq/dt \), is simply proportional to both the force on a particle pressing on the wall and the speed, \( v_w \), with which it scrapes down that wall. Or in terms of an average contact stress between the particles and the wall:

\[
 dq/dt = C \sigma_w v_w
\]

Johanson and Royal use the well-known in-house methods of Jenike and Johanson Inc. to predict the stress and velocity at the wall. Despite the implication of flow, Johanson and Royal use the static stress profile without any argued justification. No measurements of wear from silos or hoppers are reported.

In order to complete their prediction of the wear profile down the walls of a silo, Johanson and Royal require an empirical constant of proportionality. This is obtained from their patented wear tester which consists of a full-pitch screw feeder that forces the granular material to be stored in the silo onto a rotating disc made of the material from which the silo walls are to be made. The speed of the disc and the normal force exerted on it by the granular material are recorded. The temperature of the material is monitored to ensure that it does not become too hot. After the tester has been run for an appropriate time, the disc of wall material is removed and the amount of wear is measured. This straightforwardly gives a value for the constant \( C \) in equation 1.

Another contribution to the study of the wear at hopper and silo walls originates from a group based
at the University of Newcastle in Australia. The simple wear model is again commended [2] and used to predict wear profiles, albeit with a warning that it may be only approximately correct. In a more recent publication [3], a linear wear test apparatus is proposed as an alternative to the Johanson and Royal tester. The linear wear tester consists of a conveyor belt onto which the appropriate bulk solid is fed from a hopper. A sample of wall material is then placed on top of the material on the belt and the sample is anchored to a load cell which is used to measure the shear force needed to hold the sample in position. The sample is then weighted to apply a normal force to the sample. The conveyor belt is then started and the belt speed measured. After a suitable length of time, the sample is removed and the amount of wear measured. A full set of tests was conducted in which the relation between the amount of wear, applied normal stress, belt velocity and time was obtained. In each case, a linear relation was measured which is in accordance with the simple model of abrasive wear (eqn. 1).

Results of tests where 2-mm beach sand was used to wear polyethylene (UHMW), mild steel and 304 stainless steel and bauxite was used to wear mild steel, bisalloy 360 and bisalloy 500 are reported. The results give the following values of C (as defined in eqn. 1) in the order listed in the previous sentence: 0.044, 0.047, 0.020, 0.53, 0.36, 0.35 TPa⁻¹. No measurements of in situ wear from hoppers or silos are reported.

We have also previously published some of the results of our research into the wear at hopper walls [4, 5, 6]. The work reported in the first two publications concerned experiments in which three kinds of sand were driven by air pressure through conical nozzles made from perspex and brass. The wear profiles were reasonably consistent with the simple model of abrasive wear (eqn. 1). Values for C for the wear of perspex were reported erroneously but later corrected [6] to: 3.5 TPa⁻¹ for an angular sand of 218 μm, 2.4 TPa⁻¹ for the sand used in the experiments reported in this paper, and 1.8 TPa⁻¹ for a sub-rounded sand of 165 μm particle size. The values for C from wear of brass were also incorrectly reported by the same factor, i.e. they should be 1.25, 0.5 and 0.35 TPa⁻¹, respectively, for the three sands. Further details of the work reported in the two papers [4, 5] and some of the work reported below is to be found in the PhD thesis [6].

2. Experimental apparatus

A relatively large wedge-shaped hopper (Figure 1) with perspex walls inclined at an angle (α) of 10° to the vertical was used in gravity flow experiments to measure the rate of wear of such walls by sand. The width of the slot outlet was 1 cm and its breadth was 20 cm. The sand used was Chelford 60 which is a common silica-based sand of medium angularity supplied by British Industrial Sand Ltd. The sand has a solid density of 2460 kg/m³ and a bulk density, ρ, of 1550 kg/m³. The volume-surface mean particle size is 211 μm. The sand has an angle of internal friction of 31° and an angle of wall friction against perspex of 20°.

![Fig. 1 Schematic diagram of the test hopper](image)

In the experiment for which results are reported below, the sand was allowed to flow through the apparatus under the influence of gravitational forces for the equivalent of 24 hours continuous running. This was achieved by recycling the sand periodically whilst ensuring that the level of the sand in the test hopper was always the same: further details are to be found in Corder [6]. The mass flow rate, W, varied slightly but was nearly constant at 1 ± 0.1 kg/s.
In a separate experiment, we asked a colleague in the Materials Science Department in Cambridge to use his wear apparatus to measure a value of $C$ for perspex as worn by sand. In Mercer's experiment [9], a pin of perspex was worn by a rotating piece of sandpaper at a stress of 60 kPa and a speed of 0.63 m/s. We understand that the pin-on-disc experiment is a common and cheap method of measuring wear rates when materials are being assessed for their resistance to abrasion. The value of $C$ was determined to be 93 TPa$^{-1}$, which for some time we thought to be unrealistically high—mainly because the value of $C$ from our nozzle experiments [5, 6] was much lower at 2.4 TPa$^{-1}$. However, the results outlined below lead us to believe that the air-driven nozzle experiments are not directly comparable to wear at the walls of a hopper in gravity flow. They do, however, remain relevant to wear during the discharge from pressurised systems.

3. Results

During operation, the speed of particle movement was measured by following the progress of dark grains down the walls. The resulting velocities are plotted against radial distance from the virtual apex of the hopper, $r$, in Figure 2. Also included in this figure are the velocities calculated by assuming that there is no profile of velocity across the hopper (i.e. the speed in the centre is the same as that at the wall), i.e. from the equation of mass continuity:

$$v = \frac{W}{2\rho arb} \quad (2)$$

The other line in Figure 2 is the more realistic best fit line which passes through the origin. This line is marked as semi-theoretical because the careful measurements of velocity profiles in hoppers of Cleaver and Nedderman [7] and their theoretical developments [8] more than justify such an epithet. It can be seen that the velocity at the walls is roughly a third that predicted from eqn. 2.

The perspex walls were removed from the apparatus and the thickness measured by means of a micrometre gauge. The results for the right-hand wall are shown in Figure 3: The thickness was measured in four places at each radial position at 2.5, 7.5, 12.5 and 17.5 cm from the front vertical wall of the hopper. It can be seen that the wear was uniform across the hopper with a roughly linear vertical variation: the greatest wear is at the outlet. The equivalent result for the left-hand wall is given in Figure 4. The differences between Figure 3 and Figure 4 are reassuringly minor given that the two walls are in symmetrical orientation to one another.
4. Theoretical prediction for conical hoppers

There is an extensive literature on the prediction of stress profiles in hoppers. These predictions can be divided into static and dynamic predictions. The static predictions assume that the material in the hopper is not moving and are generally speaking more sophisticated. The dynamic predictions include acceleration terms in a force-momentum balance which is used to predict both the mass flow rate of material and, as a consequence, the stress profile. It is usual to supply a lower boundary condition for the stress, and this has traditionally been that the stress falls to zero at the position of the outlet slot. A simple version of this approach is described by Thorpe [12] for a conical hopper.

The first theoretical prediction we present here is for the gravity flow of Chelford 60 sand through a 30° conical brass nozzle identical in size (outlet diameter of 2.24 mm) to that used in the experiments described by Corder and Thorpe [5]. When the predicted stress is multiplied by the velocity of the particles at the wall from a prediction based on a continuity equation analogous to equation 2, the result may be inserted in equation 1. By using the value of C reported for the nozzle experiments of Corder and Thorpe [5] of 0.5 TPa⁻¹, equation 1 was integrated with respect to time to give the shape of the wall of the hopper. The resulting positions for the right-hand wall after 10 and 20 years are given in Figure 5. It is helpful in understanding Figure 5 to note that the prediction after 0 years represents the position of the wall prior to any wear taking place. Corder [6] gives further details of how these predictions were obtained and how the stress and velocity profiles were adjusted as the position of the wall changed.

These results suggest that there should be little or no wear at the outlet. This is hardly surprising since the stress has been set to be zero at the outlet and as a result, the rate of wear given by eqn. 1 is zero. Such a shape is most unrealistic – as has been previously pointed out [4].

As was noted in the introduction, those who have published predictions for wear in hoppers have always used the value for stress obtained from a static analysis. This is fundamentally unsatisfactory because it does not make any allowance for the effect of acceleration – an effect which has been established in physics for three centuries. However, there has been speculation in the literature that the zero stress boundary condition at the outlet is in error [9, 10]. In order to avoid the highly complex approaches taken by some theoreticians on this matter, we have stuck to our simple one-dimensional model and set the stress at the outlet at such a value as makes the prediction of the flow rate from the hopper match the experimentally determined value. (The experimental value is usually 10-25% less than the value predicted from theories [12].) This approach is in part justified by the good agreement between predictions using the same approach and experiment reported by Corder [6] for the air-assisted wear of nozzles by sand. This stress at the outlet (i.e. that which correctly predicts the flow rate of the sand from the nozzle) lies between zero and the value which is obtained from a static analysis. Although our approach is somewhat speculative, we regard it as being better founded on fundamental understanding than any previous approach to the prediction of the wear at the walls of a hopper.

The stress profile generated in the manner indicated above was then substituted into equation 1 to obtain the rate of wear. This rate was integrated to obtain a prediction of the position of the wall after 10 and 20 years. These predictions are plotted in Figure 6, and are much more realistic than those generated by the fully dynamic method (i.e. those in Figure 5) with the wear being approximately uniform all the way up the hopper. However, this result of uniform wear is not in agreement with the predictions put forward in previous papers [1, 3, 4] which predict that the wear should be inversely proportional to the distance from the virtual apex (i.e., r). The reason for the difference lies in the details of the stress profile near the outlet, which we have already pointed out is neither fully dynamic nor static (as assumed in the previous papers). This is a significant result in that the rate of wear is greatest near the outlet where the difference between our prediction and that of others is greatest.

Fig. 5 Predicted positions of the wall after various wear times for a 2D hopper (zero stress at outlet)
5. Theoretical prediction for wedge-shaped Hoppers

The above result (of constant wear at all heights) for the wear of the walls of a conical hopper was predicted on very simple grounds by both ourselves [4] and others [1, 2, 3] for flow through the different geometry of a wedge-shaped hopper such as the one used in the experiments reported here. However, comparison with the experimental wear profiles (Figure 3) shows that although the prediction (of uniform wear) is better than that which would be obtained by using a stress profile with a zero value at the outlet (compare Figure 5), the wear obtained in the experiment is linear with distance from the outlet rather than uniform (i.e. the result of the previous simple prediction).

One of the main reasons why the wear profile is not constant is that the hopper is not infinite in breadth, b. This in turn affects the stress profile in that the shear stresses, $\tau_w$, generated on the front and back walls support the material and so reduce the vertical normal stress, $\sigma_{zz}$, assumed to be a constant over the top and bottom of the slice. This effect is variable because the fraction of the perimeter which is front and back wall as opposed to inclined perspex wall decreases from over a half towards the top of the experimental hopper to nearly zero at the outlet. The simplest way to account for this changing influence is to use the method of differential slices commonly attributed to Janssen. This approach involves a one-dimensional force balance in the vertical (z) direction over a differential slice with vertical sides: a diagram of the elemental slice used in this analysis is given in Figure 7. Because no momentum terms are involved this will lead to a static stress prediction. The resulting differential equation is:

$$\frac{d\sigma_{zz}}{dz} + \rho g = \frac{2\tau_w}{b} + \frac{\tau_{yz}}{ztana}$$

Taking the usual assumption of passive failure for the flow of a granular material in a converging duct, it is possible to relate the shear stresses to the normal stress and thus obtain an ordinary differential equation in $\sigma_{zz}$ and z only. This equation can be numerically integrated to give a stress profile, which when combined with the wall velocity and substituted into equation 1 using Mercer's coefficient, gives a new prediction of the wear rate, and this in turn on integration with respect to time gives the wear profile shown in Figure 8. Also shown in the graph are the experimental results from the left-hand wall — this time plotted without error bars. The prediction is clearly better than that of a constant wear rate, i.e. a constant value of $q$.

For a reason which will become obvious, the wear profile prediction is plotted in Figure 9 in the same way as above but with one difference. The difference
is that instead of passive failure of the granular material, active failure has been assumed throughout. The prediction is clearly further from the data and the shape of the curve is less like the shape of the data. In Figure 10, the experimental wear from both the left and right-hand walls are compared with the passive prediction of wear, but this time all three curves are normalised such that they agree on the amount of wear at the outlet: this is equivalent to using a value for $C$ in eqn. 1 that is about 50% higher than the value measured by Mercer. The agreement between prediction and experiment close to the outlet is encouragingly good. Reasons why agreement is not obtained further up the hopper are given in the next section.

The close agreement between the wear predicted by using Mercer’s pin-on-disc value for the wear coefficient $C$ in equation 1 suggests that this simple and cheap method may be an acceptable method for generating prediction of wear rates. This would be of benefit since it would obviate the need to use the more complex wear testers advocated by other authors [1, 3].

5. Discussion

It is possible that the velocity of the sand at the walls close to the outlet is not well predicted by any simple modification of eqn. 2. For example, it is plausible that the velocity of the particles at the wall near the outlet is greater than the value we have assumed in that the velocity distribution there is more uniform across the hopper. This would lead to an increase in velocity of up to threefold giving greater wear than predicted at the outlet. (It is worthy of note that the data plotted in Figure 2 are, for reasons of experimental difficulty, no closer to the outlet than 0.25 m.) This explanation, however, seems unlikely when it is noted from Figure 8 that the discrepancy between prediction and measurement seems to occur in a region 0.25 to 0.6 m around the outlet where measurements of velocity at the wall have been obtained.

The stress within a static granular material is understood to range between two limits. These limits are called active and passive. It is common to assume that the material towards the top surface in a silo is at or close to the active limit. This is because the material will have fallen into the silo from a filling device and such a movement is that typically associated with active failure. However, close to the outlet the material is assumed to be in passive failure. This is because it either is or has at some time been flowing through the hopper towards the outlet. This is a converging flow in which the walls have to push the material inwards; an action which is typically associated with passive failure. As a consequence, in some region within the hopper or silo there must be a transition from active to passive failure. If the transition is sudden it can give rise to large jumps in the horizontal normal stress as is illustrated in Figure 11. This phenomenon is known as the switch stress and is allowed for in the various national and international design codes for silos. However, the transition need not be sudden and, in the case of a hopper (as opposed to a conical-cylindrical silo)

![Figure 9: Comparison of experimental wear (○) with the active prediction of wear ( - )](image)

![Figure 10: Measured wear compared with the normalised prediction from the Janssen style analysis. Both having been normalised](image)

![Figure 11: Stress profiles in silos and hoppers](image)
such as the one used in the experiments reported here, the position of the switch is not fixed but can wander with time. This means that the time average normal stress on the inclined wall of the hopper will lie somewhere inbetween active and passive, being close to active towards the top of the material and almost certainly passive at the outlet. In Figure 12, we have plotted both the active and passive predictions for the wear profile for our hopper. Superimposed are three example sudden “switches” between active and passive. If there were to be a switch across a horizontal plane which did not move with time, it is clear that an unusual wear pattern would result. If on the other hand, the switch plane were to move around with time and/or change its orientation from the horizontal or become curved or, more likely, the switch from active to passive failure were to occur more gradually over a region representing as much as a half of the height of the hopper [13], the resulting wear profile would be for $q$ to be small for high values of $r$ (corresponding to the active limit) and then to rise up to join the passive curve as the outlet was approached. Reference to Figure 10 reveals that this is precisely the kind of wear profile we have measured in our experiments.

It is thus not possible to reject the hypothesis advocated previously by several authors [1, 3, 4, 5] that the wear of hopper and silo walls is consistent with the simple wear model (eqn. 1). However, a full understanding of the stress and velocity profiles is required for any prediction of wear to be accurate, some uncertainty remains over both the accuracy of any prediction of wear based on the simple model of wear and over that full validity of the model itself.

6. Conclusions

1) The wear caused by sand flowing under the influence of gravity to the walls of a hopper seems to match within reasonable limits the predictions of the simple model of abrasive wear advocated by several authors.

2) It is important (and not necessarily straightforward) to measure or be able to predict both the velocity of the particles at the wall and the normal stress on the wall.

3) The experimental wear profiles suggest that the stress at the wall is more accurately represented neither by the values usually associated with a static analysis nor by those associated with assuming a zero stress at the outlet. This is an interesting clue for those interested in modelling in detail the flow of material in the region of the outlet.

4) Wear in a wedge-shaped hopper is greatest at the outlet and this is not in agreement with the predictions of previous papers published on the wear at hopper walls.

5) It may be possible to measure the wear coefficient for gravity flow using a very simple and cheap pin-on-disc experiment. This would be a great saving in time and effort over the tests suggested by other authors.

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Nomenclature

$A$ Area of top of elemental slice ($m^2$)

$b$ Breadth of the wedge-shaped hopper in the $x$ direction (out of paper in all figures) ($m$)

$C$ Wear coefficient: constant of proportionality in equation 1 ($Pa^1$)

$g$ Acceleration due to gravity ($m/s^2$)

$q$ Reduction in the thickness of the wall ($m$)

$r$ Distance from the virtual apex of the hopper ("radial co-ordinate") ($m$)

$t$ Time (s)

$v_w$ Velocity of the particles at the wall ($m/s$)

$W$ Mass flow rate of sand (kg/s)

$z$ Vertical distance above a reference plane (i.e. the Cartesian co-ordinate) ($m$)

$\alpha$ Half angle of the hopper

$\rho$ Bulk density of sand ($kg/m^3$)
Stress on and at the wall (Pa)

\[ \sigma_{zz} \]

Average normal stress on a vertical plane (Pa)

\[ \tau \]

Shear stress on the front and back walls (Pa)

\[ \tau_{yz} \]

Shear stress on a vertical plane very close to the inclined walls of the hopper (Pa)

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Author’s short biography

**Dr. Glen David Corder**

In 1982, Glen Corder obtained first-class honours in chemical engineering from the University of Queensland in Australia, going on to obtain a masters degree in control engineering from the same university. He then went to the University of Cambridge in the United Kingdom and obtained his doctorate in chemical engineering in 1988. He then took up a job with KBC Process Automation in the UK before returning to Australia in 1992 to take up his present employment with JK Tech, and works on the analysis of particle grinding and classification plant.

**Dr. Rex Barry Thorpe**

In 1979, Rex Thorpe obtained first-class honours in chemical engineering from the University of Cambridge, UK. He went on to take the fourth year of the course in Cambridge to obtain a Master of Engineering degree. After undertaking a doctoral course and obtaining his PhD degree in 1984, he worked as a designer of oil and gas production platforms for Brown and Root (UK) Ltd. He then returned to Cambridge as a lecturer in the Department of Chemical Engineering. His research interests cover a number of topics in powder and multiphase flow.