Regge Trajectories of Exotic Hadrons in the Flux Tube Model

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We have investigated the Regge trajectories of exotic hadrons by considering different possible pentaquark configurations with finite quark mass in the flux tube model. Significant deviation is observed in the linear behavior of the Regge trajectories for pentaquark systems in view of the universal value of the Regge slope parameter for hadrons. The modified Regge trajectories are also compared with the available experimental and lattice data. It is observed that the non-linear Regge trajectories of such pentaquark systems can be well described by the relativistic corrections in view of the current quark masses and the high rotational speed of the quarks at the end of flux tube structure.

Keywords: Regge trajectory; meson; baryon; pentaquarks.

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1. Introduction

According to quark spectrum and color confinement different kind of color singlet quark configurations can exist in nature. Some of these configurations are exotic hadrons and the presence of such exotic hadrons in nature is believed to provide useful insights into the largely unknown dynamics of quark-gluon dynamics mainly with respect to the confinement and other closely related aspects. In general, the linear Regge trajectories of hadrons admit a linear confinement potential\(^1\) and therefore, it is also important to investigate the behavior of such trajectories for the case of exotic hadrons. Therefore, in order to further understand the quark-gluon interactions and properties of quark matter, it is useful to investigate the Regge trajectories of the exotic hadrons. It is worth noticing that many of the resonance states observed at Belle, Babar and CLEO experiments are quite difficult to interpret in terms of conventional mesonic and baryonic states\(^4\) and therefore, some other color singlet quark combinations are proposed viz. exotic mesons, tetraquarks, hybrid mesons etc. It is believed that these particles may actually be hybrids or
hadron molecules and some of the exotic hadrons also belong to pentaquark family which consists four quarks and one antiquark. The primary aim here is to further explore the properties of such pentaquark configurations. The theoretical prediction of such systems which are heavy and short lived states was first given by M. Jezabek and M. Praszalowicz in 1987. Initially, such states were observed as exotic baryons produced in different processes, but with the improved experimental facilities, the experimental observation of these particles was first reported by Diakonov, Petrov, and Polyakov in 1997. Since then many experiments have confirmed the existence of different pentaquark systems and their properties are discussed in detail based on lattice calculations. However, there are number of investigations to confirm the existence of such states including the recent one at CERN and DIANA collaborations. One may also notice that there are evidences for charmed pentaquarks in addition to the first observed pentaquark \( \Theta^+ \).

The properties of pentaquarks have been studied by using different models. The flux tube model has been very successful in predicting the classical mass as well as the angular momentum of hadrons and it has also been used to study the Regge trajectories of pentaquark configurations with diquark-antiquark-diquark clustering in rotating flux tube at different orbital excitations.

In the present work, we consider the flux tube model to extend our earlier work on Hadronic Regge trajectories to the case of pentaquarks system. A generalized formulation of the Regge trajectories of pentaquarks is developed by incorporating the mass of quarks. One-four and two-three quark/antiquark clustering is considered at the end of flux tube. It is found that the Regge trajectories of pentaquarks are highly non-linear and the results obtained are also compared with the experimental and lattice data.

2. The Classical Mass and Angular Momentum

In the flux tube model of hadrons, with the massless quarks lying at the end of the string, it is assumed that the endpoints of the string rotate with the speed of light. The mass of the hadron then emerges as the potential energy due to the string tension. Let length of the string is \( l \) and string tension is \( \sigma \). If the string is rotating about the mid point, the mass of hadron in the natural system of units (i.e., \( \hbar = 1 \) and \( c = 1 \)) is given as

\[
M = 2 \int_{0}^{\frac{\pi}{2}} \frac{\sigma \, dr}{\sqrt{1 - v^2}}. \tag{1}
\]

The angular momentum of the hadron is however, given as

\[
J = 2 \int_{0}^{\frac{\pi}{2}} \frac{\sigma v r \, dr}{\sqrt{1 - v^2}}. \tag{2}
\]
The inter-relation between $J$ and $M$ reads as

$$J = \alpha' M^2 + \alpha_0$$  \hspace{1cm} (3) $$

where $\alpha_0$ is a constant which is occurring due to the intrinsic spin of quarks and the Regge slope parameter $\alpha'$ is given by $\frac{1}{2\pi \sigma}$. The linear potential between quarks is given by $V(r) = \sigma r$ where $r$ is the separation between quarks.

Recently, we have studied the Regge trajectories of hadrons by using the massive quarks. Here, we consider different possible configurations of pentaquark systems as presented in Fig1 and Fig2 and if one extends the formulation in [26] for pentaquarks, the modified mass expression for the pentaquark for configuration (i) as in Fig1 reads as

$$M_{1i} = \frac{\sigma(M - m_1)l}{fM} \left( \int_0^f \frac{dv}{\sqrt{1 - v^2}} + \int_0^{m_1 f} \frac{dv}{\sqrt{1 - v^2}} \right)$$

$$+ \gamma_1 m_1 + \gamma_2 (M - m_1)$$  \hspace{1cm} (4) $$

where $M = m_1 + m_2 + m_3 + m_4 + m_5$, $\gamma_1 = \frac{1}{\sqrt{1 - f^2}}$ and $\gamma_2 = \frac{1}{\sqrt{1 - \frac{m_1 f^2}{(M - m_1)^2}}}$. Here $f$ is the fractional rotational speed (actual speed is $fc$) of the endpoint on string at position $r$ from the mid point of the string. The first two terms in the above expression show the flux tube contribution while the other two terms are relativistic masses of quarks. The Eq (4) after integration can be rewritten as follows,

$$M_{1i} = \frac{\sigma(M - m_1)l}{fM} \left( \sin^{-1} f + \sin^{-1} \frac{m_1 f}{(M - m_1)} \right)$$

$$+ \gamma_1 m_1 + \gamma_2 (M - m_1)$$  \hspace{1cm} (5) $$

The expression for the angular momentum of pentaquark ($J_{1i}$) is given by

$$J_{1i} = \frac{\sigma(M - m_1)^2l^2}{f^2M^2} \left( \int_0^f \frac{dv^2}{\sqrt{1 - v^2}} + \int_0^{m_1 f} \frac{dv}{\sqrt{1 - v^2}} \right)$$

$$+ \frac{m_1 l f}{M} \{ \gamma_1 (M - m_1) + \gamma_2 m_1 \}$$  \hspace{1cm} (6) $$

where the first two terms represent the angular momentum generated due to rotation of string and the other two terms represent the angular momentum generated due to motion of quarks. After, integration, the Eq (6) leads to the following form

$$J_{1i} = \frac{\sigma(M - m_1)^2l^2}{f^2M^2} \left( \frac{1}{2} \sin^{-1} f - \frac{f^2}{2} \sqrt{1 - f^2} + \frac{1}{2} \frac{m_1 f}{M - m_1} \right)$$

$$- \frac{m_1 f}{2(M - m_1)} \sqrt{1 - \frac{f^2 m_1^2}{(M - m_1)^2}}$$

$$+ \frac{m_1 l f}{M} \{ \gamma_1 (M - m_1) + \gamma_2 m_1 \}$$  \hspace{1cm} (7) $$
One can easily notice that the above expressions are directly dependent on \((M - m_1)\) and the configuration containing the heaviest quark at one end will have least contribution among all other configurations. Similarly for configuration (i) as presented in Fig2, the expression for mass reads as

\[
M_{2i} = \frac{\sigma(M - m_1 - m_2)}{fM} \left( \int_0^f \frac{dv}{\sqrt{1 - v^2}} + \int_0^{\frac{m_1 + m_2}{M - m_1 - m_2}} \frac{dv}{\sqrt{1 - v^2}} \right) \\
+ \gamma_3(m_1 + m_2) + \gamma_4(M - m_1 - m_2)
\]

where \(\gamma_3 = \frac{1}{\sqrt{1 - f^2}}\) and \(\gamma_4 = \frac{1}{\sqrt{1 - (\frac{m_1 + m_2}{M - m_1 - m_2})^2}}\). The string length is assumed same for all the configurations. The Eq (8), can be rewritten as follows after integration

\[
M_{2i} = \frac{\sigma(M - m_1 - m_2)}{fM} \left( \sin^{-1} f + \sin^{-1} \frac{(m_1 + m_2)f}{(M - m_1 - m_2)} \right) \\
+ \gamma_3(m_1 + m_2) + \gamma_4(M - m_1 - m_2)
\]

The modified angular momentum of pentaquark \((J_{2i})\) is given by

\[
J_{2i} = \frac{(m_1 + m_2)f}{M} \left( \gamma_3(M - m_1 - m_2) + \gamma_4(m_1 + m_2) \right)
\]

which can be simplified as follows after integration

\[
J_{2i} = \frac{\sigma(M - m_1 - m_2)^2 f^2}{fM^2} \left( \sin^{-1} f - \frac{f}{2} \sqrt{1 - f^2} + \frac{1}{2} \sin^{-1} \frac{(m_1 + m_2)f}{M - m_1 - m_2} \right) \\
- \frac{(m_1 + m_2)f}{2(M - m_1 - m_2)} \sqrt{1 - \frac{f^2(m_1 + m_2)^2}{(M - m_1 - m_2)^2}} \\
+ \frac{(m_1 + m_2)f}{M} \gamma_3(M - m_1 - m_2) + \gamma_4(m_1 + m_2)
\]

In Fig1 and Fig2 there are fifteen configurations in total which are equally probable. Therefore, the mass and angular momentum of the pentaquark should be averaged over all such configurations. The expressions of mass and angular momentum for configurations ‘1i’ and ‘2i’ are functions of \(\sin^{-1} \frac{m_1 f}{M - m_1}\) and \(\sin^{-1} \frac{(m_1 + m_2)f}{M - m_1 - m_2}\) respectively. Since \(\sin \theta \leq 1\) so \(f \leq \frac{M - m_1}{m_1}\) and \(f \leq \frac{M - m_1 - m_2}{m_1 + m_2}\). Further, according to the special theory of relativity \(f \leq 1\). Such conditions would also arise for all the configurations which shall satisfy simultaneously.

3. Summary and Conclusions

We have considered the mass of up, down, strange and charm quarks as \(m_u = 2.3\) MeV, \(m_d = 4.8\) MeV, \(m_s = 95\) MeV, and \(m_c = 1275\) MeV respectively, and the string tension \(\sigma = 0.2\) GeV\(^2\). The length of string should change for different
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quark masses in order to maintain the string tension such that the angular momentum remains constant. Therefore, the length of the string for different pentaquark configurations is taken different as given in table 1. In the last column of the table, the contribution of the pentaquark configuration when the heaviest quark is at one end of the string is shown. It is, therefore, clear that such configuration results are in close agreement with the actual ones.

The mass variation of pentaquarks with the rotational speed is presented in Fig3 which indicates almost the same pattern for all the pentaquark configurations. For lower speed, it is almost linear but for higher speed, it shows a highly non-linear behavior. The separation between each curve depends upon the current quark mass difference of pentaquarks. One may notice that there is a large gap between charm pentaquarks and non-charm pentaquarks as shown in Fig3. For the purpose of computation, the rotational speed of the low mass string end is taken as $f c < 1$ ($c = 1$) and mass as well as angular momenta are then calculated for different values of $f$. It is observed that for string length range 1.4 - 2.1 fm, our results as presented in table 1 are in good agreement with the experimental and lattice data. It is interesting to note that as the mass of pentaquarks increases the string length also increases. But from simple classical analysis, it can be proved that if the angular momentum and string tension are constant then the string length should decrease with mass. According to the special theory of relativity, at constant force a light mass system will be more relativistic than a heavier one which can easily be visualised in our data. It is found that for light pentaquarks nearly 90% mass is coming due to the relativistic correction and for heavier pentaquarks it is nearly 50%.

Fig4 shows the mass dependence of pentaquarks with string length. Here, we have considered the fractional rotational speed of lower mass end of pentaquark string as 0.5 which is an arbitrary number but seems to be reasonable for both the light and heavy pentaquarks. One can easily notice that the mass of the pentaquarks varies linearly with the string length. The intersection points on the mass axis denote the contribution of the current quark mass on the actual pentaquark mass. One can infer the string lengths for different pentaquarks from this graph by comparing it with the experimental data.

In order to perform the calculation of Regge trajectories, the averaged expressions of angular momentum ($J$) and mass ($M$) over all configurations are considered and the Fig5 represents a graph between averaged angular momentum and square of mass. In order to get the desired expressions, first the relation between $J$ and $M$ for any single quark configuration of pentaquark is calculated and then the average angular momentum over all the configurations in terms of $M$ is obtained. One can easily notice that it will not be proportional to the square of averaged mass of pentaquark (over all the quark configurations). On following an analysis for baryons by Nandan and Ranjan, it can easily be shown that the Regge trajectory for the pentaquark will not be linear. The non-linear Regge trajectories are found as depicted in Fig5 and it is interesting to note that for heavy pentaquarks the angular momen-
tum spectrum is richer than for light pentaquarks. Two Regge trajectories which intersect each other indicates that two different pentaquarks can have same angular momentum as well as mass. From the expressions of angular momenta and mass, it appears that the steepness of the Regge trajectories will depend upon the current quark mass and string length. The steepness of the trajectories increases with the increase in the string length, but for quarks’ current mass it is not so apparent. In the Fig 5, string length for pentaquarks $uudd\bar{c}$ and $uuds\bar{c}$ are same but still they intersect each other which means that the steepness of the Regge trajectory decreases with decrease in the current quark mass. From Fig 5, one may also conclude that the steepness of Regge trajectories decreases with mass of pentaquarks. This discussion becomes particularly important when one replaces the slope of the linear portion of the Regge trajectories by effective string tension.

Therefore, one may conclude that the Regge trajectories of pentaquarks is well described by the relativistic corrections with current quark masses. The mass of the pentaquark increases almost linearly with low rotational speed of the string but becomes highly non-linear at high rotational speed and also causes non-linearity in the Regge trajectories. If we wish to interpret the slope of the linear portion of the Regge trajectories as effective string tension then it will depend upon the string length and current quark mass. The interesting result we observed from our investigations is that two different pentaquarks can have equal mass as well as angular momentum. Still we need more experimental data to verify these results. In view of these results, one may expect the emergence of more/new pentaquarks from colliders in near future.

| S. No. | Pentaquark | Quark structure | Our results Mass(J) in MeV | Other’s results Mass(J) in MeV | String length in fm | Charm quark end contribution Mass(J) in MeV |
|--------|------------|-----------------|---------------------------|---------------------------|-------------------|---------------------------|
| 1.     | $\Theta^c_0$ | uudd\bar{c}     | 3020(1/2)                 | 3099(1/2)                 | 1.7               | 3024(1/2)                 |
| 2.     | $N^0_c$    | uuds\bar{c}     | 3110(1/2)                 | 3180(1/2)                 | 1.7               | 3115(0.511)               |
| 3.     | $\Xi^0_c$  | uuss\bar{c}     | 3604(1/2)                 | 3650(1/2)                 | 2.1               | 3606(0.508)               |
| 4.     | $\theta^+ c$ | uudds            | 1537(1/2)                 | 1540(1/2)                 | 1.4               | 1538(0.448)               |
| 5.     | $\Xi^- c$  | dds\bar{s}      | 1883(1/2)                 | 1860(1/2)                 | 1.6               | 1885.5(1.227)             |

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References

1. Mike Guidry, *Gauge Field Theories An Introduction with Applications*, (Wiley-VCH Verlag GmbH & Co. KGaA, 1991).
2. Ta-Pei Cheng, Ling-Fong Li, *Gauge theory of elementary particle physics*, (Claredon Press, Oxford 1982).
3. Wei Chen, T. G. Steele, Shi-Lin Zhu, *Universe* 2 13(2014), arXiv:1403.7457v1 [hep-ph].
4. X. L. Wang et al (Belle), Phys. Rev. Lett. 99, 142002(2007); M. Ablikim et al (BESIII Collaboration), Phys. Rev. Lett. 112 132001(2014), arXiv:1308.2760 [hep-ex]; W. Chen, T. Steele, M. L. Du, and S. L. Zhu, Eur. Phys. J C 74 2773(2014); C. Y. Cui, Y. L. Liu, and M. Q. Huang, Eur. Phys. J. C 73 2661(2013), arXiv:1308.3625 [hep-ph]; B. Aubert et al (BABAR), Phys. Rev. Lett. 95 142001(2005), arXiv:0506.081 [hep-ex]; Q. He et al (CLEO), Phys. Rev. D 74 091104(2006), arXiv:0611.021 [hep-ex]; C. Z. Yuan et al (Belle), Phys. Rev. Lett. 99 182004(2007).
5. M. Jezabek, M. Praszalowicz, *Proceedings of the Workshop on Skyrmions and Anomalies*, Krakow, Poland, 1987. World Scientific. p. 112.
6. T. Takano et al, Phys. Rev. Lett. 91 012002(2003), arXiv:hep-ex/0301020v2; V. V. Barmin et al, Phys. Atom Nucl. 66 1715(2003), arXiv:hep-ex/0304040v4; S. Stepanyan et al Phys. Rev. Lett. 91 252001(2003), arXiv:hep-ex/0307018v4; A. E. Asratyan, A. G. Dolgolenko, and M. A. Kubantsev, Phys. Atom. Nucl. 67 682(2004), arXiv:hep-ex/0300012v3; V. Kubarovsky et al, Phys. Rev. Lett. 92 032001(2004), Erratum-ibid.92:049902,2004, arXiv:hep-ex/031046v3; A. Airapetian et al, Phys. Lett. B 585 213(2004), arXiv:hep-ex/0312044v2; A. Aleev et al, Phys. Atom. Nucl. 68 974(2005), arXiv:hep-ex/0401024v5; Zeus Collaboration, Phys. Lett. B 591 7(2004), arXiv:hep-ex/0403051v2.
7. D. Diakonov, V. Petrov, and M. Polyakov, Z. Phys. A 359 305(1997), arXiv:hep-ph/9703373.
8. J. Barth et al, Phys. Lett. B 572 127(2003), arXiv:hep-ex/0307083v4; Daniel S. Carman, Eur. Phys. J. A 24 S1 15(2005), arXiv:hep-ph/0412074v1; Kenneth H. Hicks, Prog. Part. Nucl. Phys. 55 647(2005), arXiv:hep-ph/0504027v2; Michael Danilov and Roman Mizuk, Phys. Atom. Nucl. 71 vol 4 605(2008), arXiv:0704.3511v2 [hep-ex].
9. Sonia Kabana, J. Phys. G 31 S1155(2005), arXiv:hep-ex/0503019v1.
10. Tianbo Liu, Yajun Mao, and Bo-Qiang Ma, arXiv:1403.4455 v1 [hep-ex].
11. O. Jahn, J. W. Negele, and D. Sigaev, Proceedings of Science LAT2005 069(2006), arXiv:hep-lat/0509102v1.
12. A. Martinez Torres and E. Oset, *Proceeding for the CHIRAL 10 workshop*, Valencia (Spain), 2010, arXiv:1012.2967v1 [nucl-th].
13. Ken Hicks, arXiv:hep-ph/0703004v3.
14. S. M. Gerasyuta, V. I. Kochkir, and Xiang Liu, arXiv:1407.2702v1 [hep-ph].
15. http://home.web.cern.ch/about/updates/2015/07/discovery-new-class-particles-lhc; LHCb Collaboration, arXiv:hep-exp/1507.03414.
16. DIANA Collaboration, arXiv:hep-exp/1507.06001.
17. Fl. Stancu, Int. J. Mod. Phys. A 20 209(2005), arXiv:hep-ph/0408042v1.
18. Jiulun Ping et al, Phys. Rev. C 77 052501(2008), arXiv:0802.2891v1.
19. T. Regge, *Nuovo Cimento* 14 951(1959).
20. T. Regge, *Nuovo Cimento* 18 947(1960).
21. G. F. Chew and S. C. Frautschi, *Phys. Rev. Lett.* 8 41(1962).
22. Shuchi Bisht, Navjot Hothi, and Gaurav Bhakuni, *EJTP* 7 290(2010).
23. A. Inopin and G. S. Sharov, *Phys. Rev. D* 63 054023(2001), arXiv:hep-ph/9905499.
24. G. S. Sharov, *Phys. Rev. D* 62 094015(2000), arXiv:hep-ph/0004003.
25. Proceedings of the DAE Symp. on Nucl. Phys. 56 (2011) 706; ibid 56 (2011) 708.
26. Akhilesh Ranjan, Hemwati Nandan, Mod. Phys. Lett. A 27 vol. 8 1250047(2012).
27. Hemwati Nandan, T. Anna and H.C. Chandola, Euro Phys. Lett. 67(5) 746 (2004);
   Hemwati Nandan, H. C. Chandola and H. Dehnen, Int. J. Theor. Phys. 44(4) (2004)
   457 and references therein; Hemwati Nandan, AIP Conference Proceedings 93 17
   (2007).
28. B. V. Martemyanov et al, Phys. Rev. D 71 017502(2005), arXiv:hep-ph/0502021v1.
29. J. Beringer et al (Particle Data Group), Phys. Rev. D 86, 010001(2012).
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Fig. 1. Different configurations of pentaquarks with one quark at one end of the string.

Fig. 2. Different configurations of pentaquarks with two quarks at one end of the string.
Fig. 3. Mass (M) variation of different pentaquarks with variation in speed (f) of lighter end of pentaquark string.

Fig. 4. Mass (M) variation of different pentaquarks with variation in string length.
Fig. 5. Regge trajectories for different pentaquarks.