A Short-Term Optimal Dispatch Model Considering Uncertain Wind Power Output

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Abstract. Aiming at the security and economic problems caused by wind power connected to the grid, a short-term optimal dispatch model for power system considering the uncertain wind power output is proposed. Although wind power is an important clean energy source, wind power output is characterized by volatility and randomness, which damages the system power balance and brings difficulties to dispatching. In this paper, wind power output characteristics are described by uncertain sets of wind power output. Considering the positive and negative spinning reserve constraints, a bi-level optimization model is constructed. According to the dual principle, the bi-level optimization model can be transformed into a solvable robust optimization scheduling model. Finally, the numerical simulation shows that the proposed method can realize the utilization of wind power, rationally arrange the commitment operation and output of thermal power units, which has good robustness to wind power output fluctuations to ensure the safe and economic operation of the system.

1. Introduction

Environmental problems caused by the use of traditional fossil fuels have attracted much attention. In recent years, large-scale clean energy has been connected to the electric power system, which is more environmentally friendly than fossil fuels. Wind energy, characterized by green and high efficiency, is a representative clean energy and has become one of the large-scale new energy generation methods. Globally, the proportion of wind power connected to the system is increasing every year[1]. However, wind power is highly random and fluctuating and does not have the same dispatch capability as thermal power units. Different from load forecasting, the prediction accuracy of wind power is far lower than that of load forecasting, and the uncertainty of wind power brings challenges to the safe operation of power system, which is difficult to meet the requirements of dispatching operation[2]. While the uncertainty of wind power output will affect the safety of the system, it is very important to fully absorb wind power under wind access. Short-term optimal scheduling is to find the optimal unit commitment scheme and unit output within a short scheduling cycle, and ensure the safe operation of the system under the condition of power system balance. As a large number of random and volatile wind energy connected, the power system balance will be damaged without reasonably arranged scheduling plan. Considering the above background, it is crucial to make full use of wind power resources and make the economic operation of the system under the premise of system safety[3].

Stochastic programming[4] and robust optimization[5] are two common mathematical programming methods for optimal dispatch of wind power output uncertainty. Random programming uses random variables to describe uncertainty and usually requires a probability distribution function. In [6], a
stochastic unit combination model based on opportunistic constraint programming is established to describe the constraint conditions in the form of probability, which is converted into an internal and external two-layer optimization sub-problem solution. In [7], opportunity constraint planning is used to deal with the uncertainty of wind power output, and a multi-objective power grid optimization planning model including wind power plants is constructed by taking economic efficiency, load balance of power grid and other indicators into consideration. In [8], scenario analysis is used to deal with the uncertainty of wind turbine output, and a reactive power optimization model of distribution network is established. However, the scenario-based stochastic programming method will bring a huge amount of computation, and some scenarios with low probability of occurrence will be ignored after using the scenario reduction technique. Robust optimization represents the uncertainty of variables with an uncertain set and aims to develop the best scheme under the worst-case scenario. In [9], a general method of uncertainty set was proposed to solve the uncertainty problems. In [10], a theoretical framework for robust economic dispatching of power systems is proposed from the perspective of robust optimization.

2. Optimal dispatch model

Considering the uncertainty of wind power output, the short-term optimal dispatch model mainly achieves the following purposes: first, fully absorb wind power generation under wind power access, and ensure the safe operation of clean energy on grid; second, arrange the optimal output of thermal power units to ensure the economic operation of the system. Finally, short-term optimal dispatching is realized by considering the security and economy of the system.

(1) Objective function

Considering the short-term optimal scheduling model of thermal power and wind power, the objective function is to minimize the sum of the operating and start-stop costs of thermal power:

$$\min F = \sum_{i=1}^{N_T} \sum_{j=1}^{N_R} (a_i (P'_i)^2 + b_i P'_i + c_i d'_i + d_i (1 - d_i^{-1}) C_{i,j})$$

where $a_i$, $b_i$ and $c_i$ are the power generation coefficients of the thermal power unit; $P'_i$ is the power output of thermal power unit $i$ in time period $t$; $d'_i$ is the operating state of thermal power unit $i$ in time period $t$, where "0" indicates that the unit is shut down and "1" indicates that the unit is running; and $C_{i,j}$ is the starting cost of thermal power unit $i$ in time period $t$.

(2) Power balance constraint

$$\sum_{i=1}^{N_T} P'_i + \sum_{j=1}^{N_R} P'_j = P'_d$$

In the formula, $P'_i$, $P'_j$ and $P'_d$ are the thermal power output, wind power output and system load in the $t$ period respectively; $N_T$ and $N_R$ are the total number of thermal power generators and wind power generators respectively.

(3) Spinning reserve constraints

$$\min \left( \sum_{i=1}^{N_T} \overline{P}_{i} + \sum_{j=1}^{N_R} P_j \right) \geq P'_d (1 + L\%)$$

$$\max \left( \sum_{i=1}^{N_T} \underline{P}_{i} + \sum_{j=1}^{N_R} P_j \right) \leq P'_d (1 - L\%)$$

Equation (3) is the positive spinning reserve constraint and equation (4) is the negative spinning reserve constraint. In the formula, $\overline{P}_{i}$ and $\underline{P}_{i}$ are the maximum output and minimum output of the thermal power unit $i$ respectively; $L\%$ is the spinning reserve rate.

(4) Unit output constraint
\[ d_i^d P_i \leq P_i' \leq d_i^u \bar{P}_i \]  
(5) Unit ramp constraint

\[-P_{\text{down},i} \leq P_i' - P_i^{i-1} \leq P_{\text{up},i} \]  
(6) Minimum start/stop time constraints

In the formula, \( P_{\text{down},i} \) and \( P_{\text{up},i} \) are the climbing rate and landslide rate of unit \( i \) respectively.

\[ (d_i^{i-1} - d_i^u)(\bar{T}_{\text{on},i} - \bar{T}_{\text{off},i}) \geq 0 \]
\[ (d_i^u - d_i^{i-1})(\bar{T}_{\text{on},i} - \bar{T}_{\text{off},i}) \geq 0 \]  
(7) Continuous operation/down time constraints

\[ \bar{T}_{\text{on},i} = T_{\text{on},i}^{i-1}d_i^u + d_i^u \]
\[ \bar{T}_{\text{off},i} = T_{\text{off},i}^{i-1}(1 - d_i^u) + (1 - d_i^u) \]  
(8)

### 3. Dealing with uncertainty

With the development of wind power generation, wind power has been developing rapidly. However, wind power output is uncertain, it is very important to establish a reasonable wind power output model for dispatch optimization. Due to the randomness and volatility of wind power output, the accuracy of wind power output prediction is much lower than that of load forecasting. Therefore, the optimal dispatch based on the value of predicted wind power output can't guarantee the safe operation of the system. In this paper, the uncertainty set \( D \) is constructed based on the predicted value of wind power output to solve the uncertain problem considering the wind power output.

\[ D = \left\{ P_j' / \sum_{j=N_g} P_j' - \bar{P}_j' / \hat{P}_j' \leq \Gamma_i, P_j' \in \left[ \bar{P}_j' - \eta_j\hat{P}_j', \bar{P}_j' + \eta_j\hat{P}_j' \right], \forall j \in \{1,2, \ldots N_g\} \right\} \]  
(9)

where \( P_j' \) is the actual output of wind turbine \( j \) at \( t \) time, \( \bar{P}_j' \) is the forecast output of wind turbine \( j \) at \( t \) time, \( \hat{P}_j' \) is the deviation of actual and forecast output, \( \Gamma_i \) is the uncertainty budget at \( t \) time, \( \Gamma_i \in [0, N_g] \), and \( \eta_j \) is the coefficient, \( \eta_j \in [0,1] \).

Since Eq. (3)(4), the model is a two-layer optimization model, the problem cannot be solved directly. According to the dual principle, the dual transformation of Eq. (3)(4) yields to Eq. (10)(11).

\[ -\sum_{j=1}^{N_g} x_j' \bar{P}_j + \sum_{j=1}^{N_g} y_j' \bar{P}_j - \sum_{j=1}^{N_g} \alpha_j^\prime - \Gamma_i \beta_j^\prime \geq 0 \]
\[ P_d' (1 + L\%) - \sum_{i=1}^{N_g} \bar{P}_i \]  
(10)

\[ s.t \quad -x_j' + y_j' \leq 1 \]
\[ \hat{P}_j' (x_j' + y_j') - \alpha_j^\prime - \beta_j^\prime \leq 0 \]
\[ x_j', y_j', \alpha_j^\prime, \beta_j^\prime \geq 0 \]
According to the dual principle, the equation, \( x_j^*, y_j^*, \alpha_j^*, \beta_j^* \) are the dual variables corresponding to the constraint formed by the positive rotating backup constraint and the wind power output uncertainty set \( D \). \( \mu_j^*, \nu_j^*, \phi_j^*, \xi_j^* \) are the dual variables corresponding to the constraint formed by the negative rotating backup constraint and the wind power output uncertainty set \( D \).

### 4. Analysis of simulation results

To verify the feasibility of the model and method proposed in this paper, the simulation environment is established on a computer with an Intel Core i7-7700 CPU, 3.60 GHz, and 8 GB RAM, and the optimal scheduling model is solved by GAMS\([11]\) calling the Cplex\([12]\) solver.

#### 4.1. Testing Data

In this paper, the proposed method is validated by combining the standard 10-unit data and the data of three wind turbines. The traditional predictive value scheduling model and the robust optimization scheduling model are solved separately, and the scheduling scheme for 24 periods within one day is given.

The data for the Standard 10-unit example is shown in Table 1:

| Unit | \( P_{\text{max}} \) (MW) | \( P_{\text{min}} \) (MW) | \( \Delta P_{\text{on/off}} \) (MW) | \( T_{\text{on/off}}^{\text{min}} \) (h) | Cost ($) | \( a \) | \( b \) | \( c \) |
|------|-----------------|-----------------|-----------------|-----------------|--------|--------|--------|--------|
| 1    | 455             | 150             | 270             | 8               | 4500   | 0.00048| 16.19  | 1000   |
| 2    | 455             | 150             | 200             | 8               | 5000   | 0.00031| 17.26  | 970    |
| 3    | 130             | 20              | 50              | 5               | 550    | 0.00200| 16.60  | 700    |
| 4    | 130             | 20              | 50              | 5               | 560    | 0.00211| 16.50  | 680    |
| 5    | 162             | 25              | 75              | 6               | 900    | 0.00398| 19.70  | 450    |
| 6    | 80              | 20              | 40              | 3               | 170    | 0.00712| 22.26  | 370    |
| 7    | 85              | 25              | 45              | 3               | 260    | 0.00079| 27.74  | 480    |
| 8    | 55              | 10              | 20              | 1               | 30     | 0.00413| 25.92  | 660    |
| 9    | 55              | 10              | 20              | 1               | 30     | 0.00222| 27.27  | 665    |
| 10   | 55              | 10              | 20              | 1               | 30     | 0.00173| 27.29  | 670    |

The predicted output of the three wind turbines for the 24 hours prior to the day is shown in Fig. 1:
4.2. Calculation results

The robust optimal dispatch model and thermal power and wind power data constituted by equations (1)-(2), (5)-(8), (10)-(11) are used for the calculation, model using GAMS and solved by calling the solver. When the uncertainty budget in the proposed model is equal to zero, the operating costs of the traditional and robust models are solved separately for the conventional model using wind power forecast value calculations, and the operating costs of the two models are shown in Table 2:

| Operation cost ($) | Traditional model | Robust model |
|-------------------|------------------|--------------|
|                   | 477649.588       | 485996.687   |

The reason is that the robust optimization model takes into account the fluctuation of wind power output, and the larger the fluctuation, the more spinning spares there are, leading to an increase in cost. However, the robust optimization scheduling model proposed in this paper better deals with the impact of wind power output uncertainty on the system, thus, the model proposed in this paper has good robustness to power fluctuations and is more applicable to the power system scheduling problem of wind power access.

Figure 2 shows the 24 hours output and system load values of thermal power units obtained by robust optimal dispatching model in extreme scenarios. The results show the optimal scheduling scheme of the proposed model in this scenario.

![Figure 1. Forecast wind turbine output.](image1)

![Figure 2. Thermal power generating units 24 hours output and load values.](image2)
5. Conclusion
Aiming at the power system scheduling problem with uncertain wind power output, a robust optimal model is proposed. In this paper, the randomness and volatility of wind power generation are described by the uncertainty sets, and the bi-level optimal model of the original problem is transformed into a solvable optimal dispatch model through the dual principle. Finally, the simulation results prove the feasibility of the proposed robust optimal method, which can effectively deal with the uncertainty of wind power output and improve the security of system decision making in the case of wind power access.

Although the uncertainty of wind power output is modelled in this paper, the conservative problem of robust optimization results remains to be further studied, and the data-driven uncertainty modelling method is our future research work.

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