Principle of Mach, the Equivalence Principle and concepts of inertial mass

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Abstract

A study of kinematics of a 2-body system is used to show that the Mach principle, previously rejected by general relativity, can still serve as an alternative to the concept of absolute space, if one takes into account that the background of distant stars (galaxies) determines both the inertial and the gravitational masses of a body.

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It is well known that in classical mechanics there were two distinct concepts of inertial mass and absolute space: that of Newton (in which inertial mass is a property of a body with respect to absolute space), and that of Mach (in which inertial mass is the property determined by the masses of distant stars) [1]. It is also known that when constructing general relativity, Einstein started with the Mach principle, but had to reject it thereafter (e.g., see [2]) because of its disagreement with the Equivalence principle. However, there still is no general agreement among scientists about the necessity of total rejection of the Mach principle (e.g., see [3-7]).

Let there be a two-body system with the inertial and gravitational masses \( m_i, M_i \) and \( m_g, M_g \), accordingly. Let it be surrounded by some collection of distant stars, at rest with respect to the center of inertia (CI) of the system. Let these two bodies rotate around their common CI with some angular velocity \( \omega \). To be more specific, let us assume that the distance between the CI of the system and the nearest star is much greater than the distance \( R \) between the bodies, which, in its turn, is much greater than the size of the bodies; this will allow us to consider them as material points. Let us also assume that their velocities are much smaller than the speed of light, which will allow us to use the equations of classical mechanics. We shall further assume that the angular velocity of rotation \( \omega \) is such that the bodies can get closer to each other only because of the loss of energy due to gravitational radiation, i.e., in the absence of such radiation, the distance between the two bodies would remain constant.

Let us analyze the situation arising in the hypothetical case in which the distant stars disappear, assuming that at that instant the bodies rotated. In such setup, there are only two logically possible situations:

a) either the kinematics of the relative motion of the system changes, i.e., an observer located on one of the bodies perceives a picture different from the one he would see in the presence of fixed distant stars;

b) or the observer does not perceive any difference in the kinematic pattern of the relative motion of the bodies.

The situation (a) can happen only if the Mach principle holds in the form in which it
has been known so far [1], since in Newton description by definition there is no change in the state of the system under a removal of sufficiently distant stars (we set up that the gravitational influence of this stars is infinitesimal). A change in the state of the system would be possible only due to a change in the relationship between the inertial and gravitational masses (by virtue of the Mach principle, the absence of distant stars must lead to a strong decrease in the values of inertial masses, and, therefore, to sharp increase in the relative acceleration of their mutual approach). Such a situation would be contradict the principle of equality of inertial \( m_i \) and gravitational \( m_g \) masses but, generally speaking, we cannot consider the principle of equivalence like one of underlying axioms of general relativity\(^1\). The case (a) was studied already rather well (profound analysis of this case is given by P. Graneau [9] and A. Assis [10,11]). Following these works we have to infer that if the case (a) happens we can either say that the gravitational constant \( \gamma \) or the inertial mass \( m_i \) of the test body will be a function of the amount and distribution of distant bodies (stars and galaxies).

However, we cannot test neither the case (a) nor the case (b) (although a direct check-up of the Mach principle can be realized in the laboratory, the effect will be too small to be detected). Therefore, we still must consider the case (b) and case (a) like enjoying equal rights. In this paper we shall prove that if would be realized the case (b) the Mach principle can still remain an alternative to the Newton concept of absolute space, on the one hand, and allows for the equality of the inertial and gravitational masses, on the other hand (at least, in classical mechanics).

For detailed analysis of the case (b) we are going to consider two hypothetical situations: distant bodies (the rest of stars of the Universe) do not exist and the Universe only consists of two bodies \( m \) and \( M \); distant stars (galaxies) exist.

In the case (b) both concepts of inertial mass could be valid - the Newton and modified\(^2\) Mach ones (see below). Notice that in this reasoning we have to assume that aside from

\(^1\)See, e.g., brilliant work of M. Sachs “On the Logical Status of Equivalence Principles in General Relativity Theory” [8]. That is, there is no \( \alpha \pi \) reason why it should necessarily follow that \( m_i = m_g \), even though experimental observations confirm this equality to high accuracy.

\(^2\)Generally accepted Mach principle can be realized in the case (a) only (see above and [9-11])
“local action” (in the Faraday terminology) and independently of it there is “action-at-a-distance” (instantaneous action) in nature.

So, in order to modify the Mach principle and still keep the kinematic equivalence of the two concepts of space for a circular motion, one has to assume that distant stars determine both inertial y gravitational properties of a body. We shall call such a concept “quasi-Mach”. Notice that in quasi-Mach case of situation (b), the masses cannot be equal to zero, because each body serves as a background for the second one.

Notice that these bodies may turn around their common CI, following either elliptic or circular orbits. In our Gedanken experiment we chose the circular orbits. Such a selection of circular orbits may seem unfounded at the first sight, but in fact it can be easily explained: in order to obtain the relationships in which we are interested (see below), we have to choose such kind of motion whose relative kinematics does not depend on whether it is Newton or the quasi-Mach concept is true. In the case of elliptic motion, if the stars disappear, the observer will see either (i) “oscillations” of bodies with respect to each other (aphelion-perihelion), which would automatically signify the validity of Newton’s concept, or (ii) these “oscillations” will cease, which would mean that there is an influence of stars on the masses of the bodies. Thus in both cases (i) and (ii) we would obtain a unique answer in favor of one of the two concepts. In reality such an experiment is naturally impossible. However, in the case of circular orbits there is no unique choice of a valid concept for the observer, which will allow us to retain the assumption that the two concept are equally justified.

If we choose as the “true” concept the quasi-Mach one, then by equating kinematic properties of the same type in the presence and in the absence of distant stars, we can obtain a relationship between the “old” (in the presence of the rest of the matter) and the “new” (in its absence, accordingly) masses.

We shall use the equations of classical mechanics for the “new” masses, while for the “old” masses we shall use the equation for gravitational radiation of a system of two bodies rotating around their common CI.

\[\text{3}^{\text{Below we shall prove that this assumption has got rather strict arguments, at least, in classical mechanics (see Theorem)}}\]
In the case of “new” masses, we consider two bodies in an absolutely empty space, which come closer to each other under the influence of the gravitational force. Remember that speaking about a rotation of such a system has no meaning any more since both the stars and the notion of an absolute space are absent in this concept, so that the only “real” coordinate here is the distance between the two bodies. The second law of Newton and the inverse-square law lead to a relative acceleration in the two-body system:

$$\ddot{x} = \gamma \frac{M_g m_g}{M_1 m_1} \left( \frac{M_1 + m_1}{x^2} \right),$$  

where $x$ is the distance between the bodies; $M_1, m_1$ and $M_g, m_g$ are the inertial and gravitational masses of the bodies $M$ and $m$, accordingly.

In the case of “old” masses, the two bodies rotate around their common CI in the presence of stars (or in the absolute space, which in this case is the same). The potential energy of the system has the form

$$\varepsilon_{pot} = \gamma \frac{M_g m_g}{r},$$

where $r$ is the distance between bodies. From the condition of equality forces and the rotational frequencies, we can obtain the expressions for the linear velocities of the bodies:

$$V_M^2 = \gamma \frac{m_g M_g m_1}{M_1 (M_1 + m_1) r}; \quad V_m^2 = \gamma \frac{m_g M_g M_1}{m_1 (M_1 + m_1) r}.$$

Substituting them into the equations for the kinetic energy of the bodies, we find

$$\varepsilon_{k}^{tot} = \gamma \frac{M_g m_g}{2r},$$

where $\varepsilon_{k}^{tot}$ is the total kinetic energy of the system (remember that we consider these bodies as material points). Notice that for a circular motion, the total kinetic energy of bodies depends only on their gravitational masses, rather than inertial ones. This fact allows us to prove following theorem: In the framework of classical mechanics the gravitational mass determines the “inertia” of material body.

**Proof:** So, let some body $m$ (with inertial mass $m_i$) to move along a straight line with constant velocity $V$. Its kinetic energy is:

$$K = \frac{m_i V^2}{2} \quad (2)$$

From kinematic point of view the movement with constant velocity along a straight line and (with constant linear velocity) along a circumference of infinity radius are equivalents.
We can consider also the movement of the similar body (with same inertial mass $m_i$) as circumference movement around the other body $M$. Now we can require that kinetic energy and linear velocity of body $m$ to be equal to that of the case (2). Nothing can forbid us to do it. Now from the equivalence force conditions we have:

$$\frac{m_i V^2}{R} = \gamma \frac{m_g M_g}{R^2}$$

(3)

where $m_g, M_g$ are gravitational masses of bodies $m$ and $M$. $R$ is distance between $m$ and $M$. Expressing "$V^2$" from (3) and substituting it into the formula of the kinetic energy of the body $m$ obtain:

$$K = \gamma \frac{m_g M_g}{2R}$$

(4)

Let us now to increase $M_g$ and $R$ conserving the same time value of $K$. In this case $M_g(R)$ and $R(M_g)$ are one-to-one functions. It is obvious that if $K, m_g$ and $\gamma$ are constants, $M_g(R)$ and $R$ will be linear dependent functions, i.e.

$$M_g(R) = C \cdot R,$$

where $C$ is some suitable dimensional constant. After tending $R$ to infinity (conserving the same time values of $K, m_g$ and $\gamma$) we obtain from (4)

$$K = m_g \frac{\gamma C}{2}$$

(5)

where "$\gamma C$" has dimension of the "$V^2$". It means that (5) can be rewritten as

$$K = m_g \beta \frac{V^2}{2}$$

(6)

here $\beta$ is non-defined constant.

Recalling above-mentioned notice (equivalence between straight line movement and the same one along the circumference of the infinite radius), we conclude that kinetic energy of the body moving along the straight line with the constant velocity is proportional to the gravitational mass. Comparing (6) and (2) we obtain the equivalence

$$m_i = m_g \beta,$$

(7)

where $\beta$ is a constant non-defined in frames of the above-mentioned considerations.

Now we can assumed that $\beta = 1$ in (6) and as result, the total mechanical energy of the system (see above) is

$$\epsilon = -\gamma \frac{M m}{2r}.$$  

(8)

Here and below we shall skip the indices "$i$" and "$g$" according to the meaning of the problem.
The radiation rate of the gravitational energy during a circular motion has the form [12]:

\[- \frac{d\varepsilon}{dt} = 32\gamma \left( \frac{mM}{m + M} \right) \frac{r^4\omega^6}{5c^2}. \quad (9)\]

From (8) we have

\[\frac{d\varepsilon}{dt} = \gamma \frac{Mm}{2r^2} \frac{dr}{dt}. \quad (10)\]

Then we obtain

\[\omega^6 = \gamma^3 \left( \frac{m + M}{r^9} \right)^3. \quad (11)\]

Substituting (10) and (11) into (9) and differentiating the resulting expression with respect to time, we can find the relative acceleration of the mutual approach of the bodies:

\[\frac{d^2r}{dt^2} = -3\gamma^6 \left[ \frac{64}{5c^5} mM(m + M) \right] r^{-7}. \quad (12)\]

Now, denoting in (1) both masses by the index “n” and in (12), by “o” (“new” and “old” masses), and comparing these two expressions, we obtain the desired relationship

\[M_n + m_n = \alpha \frac{m_o^2 M_o^2 (m_o + M_o)^2}{R^5}. \quad (13)\]

Here we denoted all constants coefficients by \(\alpha\), while \(R\) is the distance between the bodies.

Now let us consider another situation: distant stars (galaxies) exist “again” (remember that according to Mach, it is irrelevant whether it is the background or the bodies that rotate). Then \(m_n\) becomes \(m_o\), and \(M_n\) becomes \(M_o\). That is, \(m_n\) and \(M_n\) get multiplied by some factors \(A\) and \(B\) (\(Am_n = m_o; BM_n = M_o\)) which are functions of the masses: \(A = A(M_o, \Phi), B = B(m_o, \Phi)\), where \(\Phi\) are masses of the rest of stars. Then (13) can be written in the form

\[M_n + m_n = \alpha \frac{m_o^2 M_o^2 A^2 B^2 (Am_n + BM_n)^2}{R^5}. \quad (14)\]

Using the fact that \(\Phi \gg m_o, M_o\), we can expand \(A(M_o, \Phi), B(m_o, \Phi)\) in a power series in small parameters. In the zeroth approximation,

\[A \cong A(0, \Phi); \quad B \cong B(0, \Phi).\]
Since it is clear that the functional form of \( A \) and \( B \) must be same, we can write

\[
A(0, \Phi) \equiv B(0, \Phi) = A(\Phi). \tag{15}
\]

We do not possess more information about the form of the function \( A(\Phi) \), but we shall assume that when \( \Phi \to \infty, A \to \text{const} \), i.e., we shall assume that \( A \) is a constant specific for our Universe. Substituting (15) into (14), we obtain

\[
\alpha A^6 M_n^2 (m_n + M_n) = R^5. \tag{16}
\]

The constant \( A \) cannot be determined within the framework of this problem. We have only shown the necessity of its existence \textit{if the quasi-Mach concept is true}. Now let us note that there exists such \( p \) that \( M_n = p m_n \). The Eq.(16) implies that

\[
[p^2(1 + p)]^{1/5} m_n = (\alpha A^6)^{-1/5} R. \tag{17}
\]

Thus we have shown that Newton solution of the problem of absolute space and inertial mass, taking into account the requirements of general relativity, is \textit{non-unique} if the Mach principle is modified in a suitable way. We have also shown that if we choose the quasi-Mach concept as \textit{true}, then in our approximation the masses of two bodies in an empty Universe are proportional to the distance between them (17). When the bodies approach each other, their masses tend to zero (we mast not forget that our computation are based on the bodies being pointlike, though this restriction is not a matter of principle). Notice that if the masses of the bodies vanish when they come close to each other, this would not signify that matter disappear, and therefore such vanishing of the masses would mean that there is some conserved property of matter in nature which, perhaps, is unrelated to the space-time structure of the Universe. In the framework of the problem under consideration we are unable to say anything more specific about this property.

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