Radiative Heat Transfer in Buildings

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Abstract. In the field of building physics, heat and moisture transports are often studied. These transports represent core problem in building performances. The heat transport is often investigated as heat conduction through building envelopes but the heat transfer inside buildings usually remains overlooked. The heat transfer in closed spaces may consist of conduction, convection and radiation. In thermal equilibrium, these transports compensate not only heat losses going through building envelopes but they may influence the temperatures of interior surfaces that occasionally suffer from the condensation of water vapors. So far, the thermal building technology has investigated heat losses prevalently as simple heat conduction through building envelopes along with ventilation (infiltration or exfiltration). Such an approximation avoids considering an alternative procedure taking into account direct radiative and convective heat flows from interior heating sources towards exterior spaces. For this purpose, it is necessary to have a computational formalism capable of determining the radiative heat flows established in interiors of buildings. The recent monographs provide such formalism only for gray or black bodies but not for their combinations. In this contribution, the general formalism for the computation of radiative heat flows between black and grey bodies is derived. The formalism consists of the system of equations specifying radiosities, heat fluxes and heat flows related to each surface of the interior. It is shown that this general system of equations may be reduced to two particular systems holding separately for black and grey bodies. In this way, the universality of the developed computational formalism is documented.

1. Introduction
So far, the thermal building technology has investigated heat loss prevalently as simple heat conduction through building envelopes along with ventilation [1-3]. Such an approximation does not take into account direct radiative heat flows inside interiors and exteriors. In this conference contribution, we derive general equations for radiative heat transfers in interior spaces of buildings. The equations are based on the radiosity concept [4 – 6]. Recent monographs present this concept in the form of equations that are convenient for diffusive gray surfaces but when they are applied to black surfaces, the heat fluxes show uncertain expressions 0/0. For this reason, it is desirable to derive more general equations that are valid not only for gray surfaces but also for black surfaces as well as for their combinations. The generalized formalism consists of the system of equations specifying radiosities, heat fluxes and heat flows related to each surface of the interior. In addition, it is shown that this general system of equations may be reduced to two particular systems holding separately for black and grey bodies.
2. General equations for radiative heat transfer

The radiative heat flux $q_i$ of the $i$-th surface is defined as a difference between the radiosity $W_i$ and the radiative energy flux $H_i$ coming from all surfaces to the $i$-th surface (see Figure 1):

$$q_i = W_i - H_i \quad \text{(Watt/m}^2\text{)}$$

(1)

The radiosity $W_i$ is introduced as follows

$$W_i = \varepsilon_i E_{bi} + \rho_i H_i$$

(2)

where $\varepsilon_i$ is surface emissivity, $\rho_i$ surface reflectance ($\varepsilon_i + \rho_i = 1$) and $E_{bi}$ is the Stefan-Boltzmann term $E_{bi} = 5.67 \times 10^{-8} T^4$. The total radiative energy flow $S_i H_i$ coming to the $i$-th surface may be expressed as the sum of radiative heat flows of all surfaces:

$$S_i H_i = \sum_{j=1}^{N} S_j F_{ji} W_j$$

(3)

where $F_{ji}$ is the view factor \([4-6]\) related to the $i$-th and $j$-th surfaces and $S_i$, $S_j$ are their areas. By using the rule of symmetry \([4-6]\)

$$S_j \cdot F_{ji} = S_i \cdot F_{ij}$$

(4)

Eq. (3) may be modified as follows:

$$S_i H_i = \sum_{j=1}^{N} S_j F_{ij} W_j = S_j \sum_{j=1}^{N} F_{ij} W_j$$

(5)

$$H_i = \sum_{j=1}^{N} F_{ij} W_j$$

(6)
By inserting relation (6) into Equations (1) and (2), general equations for radiative heat transfers between arbitrary types of surfaces may be specified:

\[ W_i = \varepsilon_i E_{bi} + \rho_i \sum_{j=1}^{N} F_{ij} W_j \]  \hspace{1cm} \text{(Watt/m}^2) \tag{7} \\
\[ q_i = W_i - \sum_{j=1}^{N} F_{ij} W_j \]  \hspace{1cm} \text{(Watt/m}^2) \tag{8} \\
\[ \Phi_i = S_i q_i \]  \hspace{1cm} \text{(Watt)} \tag{9} \\

From these general equations, all other particular equations for radiative heat transfers among grey or black bodies can be derived as is shown in the following sections 3 and 4.

3. Particular system of equations for black bodies

The surfaces of absolute black bodies have zero reflectance \( \rho = 0 \) and their emissivity equals one \( \varepsilon = 1 \). By implementing these constants into the system of Equations (7) – (9), the following relations emerge:

\[ W_i = E_{bi} \]  \hspace{1cm} \text{(Watt/m}^2) \tag{10} \\
\[ q_i = E_{bi} - \sum_{j=1}^{N} F_{ij} E_{bi} \]  \hspace{1cm} \text{(Watt/m}^2) \tag{11} \\
\[ \Phi_i = S_i q_i \]  \hspace{1cm} \text{(Watt)} \tag{12} \\

4. Particular system of equations for gray bodies

By inserting Relation (2) into (1), we obtain the heat flux \( q \) for grey bodies \( (\varepsilon_i + \rho_i = 1) \) in a usual form (see Equation (16)):

\[ q_i = \varepsilon_i E_{bi} + \rho_i H_i - H_i = \varepsilon_i E_{bi} + (\rho_i - 1)H_i = \varepsilon_i E_{bi} - \varepsilon_i H_i = \]
\[ = \varepsilon_i (E_{bi} - H_i) = \varepsilon_i (E_{bi} - H_i) = \varepsilon_i \left( E_{bi} - \frac{W_i - \varepsilon_i E_{bi}}{\rho_i} \right) = \]
\[ = \frac{E_i}{\rho_i} \left( \rho_i E_{bi} - W_i + \varepsilon_i E_{bi} \right) = \frac{E_i}{\rho_i} \left[ \left( \rho_i + \varepsilon_i \right) E_{bi} - W_i \right] = \frac{E_i}{\rho_i} \left( E_{bi} - W_i \right) \tag{15} \]
5. Results and discussion

In Section 2 a generalized system of equations for radiative heat transfer has been derived. This system is valid for grey surfaces, for black surfaces as well as for their arbitrary combinations. In Sections 3 the general system of equations has been reduced to those holding only for radiative heat transfers between black bodies. In Section 4 the general system of equations has been reduced to those holding only for grey bodies. The reason why the generalized system of equations has been derived consists in the fact that the common system for grey bodies, especially Equation (17), cannot be applied to black bodies since it yields an uncertain expression 0/0 as is shown in the following lines:

Equation (17) applied to black body ( \( \rho = 0 \), \( \varepsilon = 1 \), \( W_i = E_{bi} \) ) gives

\[
q_i = \frac{\varepsilon_i}{\rho_i} (E_{bi} - W_i) = \frac{1}{0} (E_{bi} - E_{hi}) = \frac{0}{0}
\]  

The generalized system of Equations (7) – (9) does not suffer from such a drawback and can be used for any types of surfaces and their combinations.

6. Conclusions

In this conference contribution, a generalized system of equations for radiative heat transfer has been derived. It can be used for arbitrary combinations of gray and black surfaces without the risk that an uncertain expression will appear and this is a welcome property. This positive property has been obtained at the expense of a larger numerical laboriousness. For example, when Equations (17) and (8) are compared, it can be seen that Equation (17) may be calculated very quickly whereas the solution of Equation (8) requires computations of a large sum \( \sum_{j=1}^{N} F_{ij} W_j \). However, this does not depreciate the value of the generalized system of Equations (7) - (9). The system is quite universal and this is very valuable and useful property.

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