Constraints on dark matter annihilation by Planck

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Abstract. We investigate the production of electrons and positrons in the Milky Way within the context of dark matter annihilation. Upper limits on the relevant cross-section are obtained by Planck data at different wavelengths with recent measurements of the positron spectra in the solar neighbourhood by AMS02. We consider synchrotron emission in the microwave bands. According to our results, the dark matter annihilation cross-section into electron-positron pairs should not be higher than the canonical value for a thermal relic if the mass of the dark matter candidate is smaller than a few GeV.

1. Introduction

Dark matter can be indirectly detected through the signatures of standard model particles produced by its annihilation or decay (see e.g. [1]). The recent results from indirect detection experiments in the solar neighbourhood have suggested the possibility that such a signature has been seen. In particular, the PAMELA experiment has pointed a significant excess of electrons and positrons above the expected smooth astrophysical background [2]. If these results are interpreted in terms of dark matter annihilation, then an abundant population of high-energy $e^\pm$ is being created everywhere in the Galactic dark matter halo, with the associated synchrotron emission in the Galactic magnetic field.

The present work focuses on the astrophysical signatures of dark matter annihilation into electron-positron pairs. We try to impose constraints on the relevant cross-section by comparing the predictions of an analytic model of particle propagation with a set of observational data obtained from Planck. In addition to the photon data, we also consider the recent measurements of the local positron spectra performed by AMS02 [3].

Rather than focusing on a particular dark matter candidate, we adopt a model-independent approach (see e.g. [4]), in which all the injected particles are created with the same initial energy $E_0$, of the order of the mass of the dark matter particle. Since this mass is usually much larger than the rest mass of the electron, electrons and positrons will be relativistic at the moment of their creation. However, they can efficiently lose their energy through different processes, such as inverse Compton scattering (ICS), synchrotron radiation, Coulomb collisions, bremsstrahlung, and ionization. We will often use the Lorentz factor $\gamma$ to express the energy $E = \gamma m_e c^2$ of the annihilation products, where $m_e$ denotes the rest mass of electron, and $c$ is the speed of light.
2. Model predictions

2.1. Electron-positron propagation

As in our previous work [4], the propagation of electrons and positrons through the interstellar medium (ISM) is determined by the diffusion-loss equation

\[
\frac{\partial}{\partial t} \frac{dn}{d\gamma}(x, \gamma) = \nabla \left[ K(x, \gamma) \nabla \frac{dn}{d\gamma}(x, \gamma) \right] + \frac{\partial}{\partial \gamma} \left[ b(x, \gamma) \frac{dn}{d\gamma}(x, \gamma) \right] + Q(x, \gamma). \tag{1}
\]

We assume a diffusion coefficient of the form

\[
K(\gamma) = K_0 \gamma^\delta. \tag{2}
\]

The energy loss rate

\[
b(x, \gamma) \equiv -\frac{d\gamma}{dt}(x, \gamma) = \sum_i b_i(x, \gamma) \tag{3}
\]

is a sum over the relevant physical processes, and the source term \(Q(x, \gamma)\) represents the instantaneous electron-positron injection rate.

Given enough time, the electron-positron population will approach a steady state distribution, \(\frac{\partial}{\partial t} \frac{dn}{d\gamma}(x, \gamma) = 0\). Assuming that \(b(x, \gamma)\) varies smoothly in space, the particle spectrum fulfills the relation

\[
\frac{\partial y(x, \gamma)}{\partial \gamma} + \frac{K(\gamma)}{b(\gamma)} \nabla^2 y(x, \gamma) = -Q(x, \gamma), \tag{4}
\]

where

\[
y(x, \gamma) \equiv b(\gamma) \frac{dn}{d\gamma}(x, \gamma). \tag{5}
\]

Imposing \(\frac{dn}{d\gamma}(x, \gamma) = 0\) at infinity, one obtains the Green’s function

\[
G(x, \gamma, x_s, \gamma_s) = \frac{\exp \left( -\frac{|x-x_s|^2}{2\Delta \lambda^2} \right)}{(2\pi \Delta \lambda^2)^{3/2}} \Theta(\gamma - \gamma_s) \tag{6}
\]

and either the image charges method or an expansion over the eigenfunctions of the linear differential operator may be used to derive the Green’s function for other boundary conditions (see e.g. [5]). The electron-positron spectrum is thus given by

\[
\frac{dn}{d\gamma}(x, \gamma) = \frac{1}{b(x, \gamma)} \int_0^\infty d\gamma_s \int d^3 x_s \frac{\exp \left( -\frac{|x-x_s|^2}{2\Delta \lambda^2} \right)}{(2\pi \Delta \lambda^2)^3} \frac{Q(x_s, \gamma_s)}{b(\gamma)}, \tag{7}
\]

where the quantity

\[
\Delta \lambda^2 = \lambda^2(\gamma) - \lambda^2(\gamma_s) \tag{8}
\]

is related to the characteristic diffusion length of the electrons and positrons, \(\gamma_s\) denotes their initial energy, and the variable \(\lambda\) is defined as

\[
\lambda^2(\gamma) = \int_\gamma^\infty \frac{2K(\gamma)}{b(\gamma)} d\gamma. \tag{9}
\]
Considering the dark matter halo as a spherically-symmetric source, the spatial integral can be reduced to one dimension, and the electron-positron spectrum is finally given by the expression

\[
\frac{dn}{d\gamma}(r, \gamma) = \frac{1}{b(\gamma)} \frac{\exp \left( -\frac{r^2}{2\Delta^2} \right)}{(2\pi r^2 \Delta^2)^{1/2}} \times \left\{ \int_\gamma^\infty d\gamma_s \int_0^\infty dr_s \, r_s \exp \left( -\frac{r_s^2}{2\Delta^2} \right) \times \left[ \exp \left( \frac{r r_s}{\Delta^2} \right) - \exp \left( -\frac{r r_s}{\Delta^2} \right) \right] Q(r_s, \gamma_s) \right\}.
\]

(10)

2.2. Loss rates

Electrons and positrons can lose their energy by several physical processes as they move through the ISM. We consider ICS of cosmic microwave background (CMB), starlight and infrared photons, synchrotron radiation, Coulomb collisions, bremsstrahlung, and ionization of neutral hydrogen atoms.

The energy loss rates depend on the energy of the particle. High-energy electrons and positrons mainly lose energy by ICS, e.g. [6]. We compute the total power radiated by a single electron using the formalism, based on the Klein-Nishina cross-section. In the non-relativistic regime, the loss function can be approximated as

\[
b_{\text{ICS}}(\gamma) = \frac{4}{3} \sigma_T \frac{m_e e^2}{c} \gamma^2 U_{\text{rad}},
\]

(11)

where \( \sigma_T \) is the Thomson cross-section. The combined radiation energy density of the CMB, starlight (SL), and infrared (IR) light from thermal dust emission (see e.g. [7], [8]) is represented by three grey bodies,

\[
U_{\text{rad}} = \frac{4\sigma_{\text{SB}}}{c} \left( T_{\text{CMB}}^4 + N_{\text{SL}} T_{\text{SL}}^4 + N_{\text{IR}} T_{\text{IR}}^4 \right)
\]

(12)

where \( T_i \) and \( N_i \) represent the effective temperature and the normalization of each component, respectively, and \( \sigma_{\text{SB}} \) is the Stefan-Boltzmann constant.

Synchrotron radiation is another important loss mechanism at high energies. The expression for the loss rate is similar to that of non-relativistic ICS, substituting the radiation energy density in equation (11) by the magnetic energy density,

\[
b_{\text{syn}}(\gamma) = \frac{4}{3} \sigma_T \frac{m_e e^2}{c} \gamma^2 U_B.
\]

(13)

For lower-energy electrons and positrons, Coulomb interactions with the thermal plasma must be taken into account. The loss rate is approximately [9]

\[
b_{\text{Coul}}(\gamma) \approx 1.2 \times 10^{-12} n_e \left[ 1 + \frac{\ln(\gamma/n_e)}{75} \right] \text{s}^{-1},
\]

(14)

where \( n_e \) is the number density of thermal electrons.

Collisions with thermal ions and electrons also produce radiation through bremsstrahlung. The loss rate due to bremsstrahlung can be approximated as [10]

\[
b_{\text{brem}}(\gamma) \approx 1.51 \times 10^{-16} n_e \gamma \left[ \ln(\gamma) + 0.36 \right] \text{s}^{-1}.
\]

(15)
Additional energy losses come from the ionization of hydrogen atoms. The loss rate is given in [11],

\[ b_{\text{ion}}(\gamma) = \frac{q_0^4 n_H}{8\pi \epsilon_0^2 m_e^2 c^3} \sqrt{1 - \frac{1}{\gamma^2}} \left( \ln \left( \frac{\gamma^2 - 1}{2} \right) - \left( \frac{2}{\gamma} - \frac{1}{\gamma^2} \right) \ln 2 + \frac{1}{\gamma^2} + \frac{1}{8} \left( 1 - \frac{1}{\gamma^2} \right)^2 \right), \tag{16} \]

where \( n_H \) is the number density of hydrogen atoms, \( q_0 \) is the electron charge, \( \epsilon_0 \) is the permittivity of free space, and \( I \) is the ionization energy of the hydrogen atom. The number density of thermal electrons and neutral atoms can be expressed in terms of the total ISM gas density \( \rho_g \) and the ionization fraction \( X_{\text{ion}} \) as

\[ n_e = \frac{\rho_g}{m_p} X_{\text{ion}} \tag{17} \]

and

\[ n_H = \frac{\rho_g}{m_p} (1 - X_{\text{ion}}), \tag{18} \]

respectively.

\section*{2.3. Source term}

Since the electrons and positrons in our model originate from the annihilation of dark matter particles, the instantaneous production rate at any given point can be expressed as

\[ Q(r, \gamma) = \eta n_{\text{dm}}(r) n_{\text{dm}}^*(r) \langle \sigma v \rangle_{e^\pm} \frac{dN_{e^\pm}}{d\gamma}(\gamma), \tag{19} \]

where \( n_{\text{dm}} \) and \( n_{\text{dm}}^* \) denote the number densities of dark matter particles and anti-particles, respectively, \( \langle \sigma v \rangle_{e^\pm} \) is the thermal average of the annihilation cross-section times the dark matter relative velocity, and \( \frac{dN_{e^\pm}}{d\gamma} \) is the injection spectrum of electrons and positrons in the final state. For self-conjugate dark matter particles, \( n_{\text{dm}} = n_{\text{dm}}^* = \frac{\rho_{\text{dm}}}{m_{\text{dm}}} \) and \( \eta = 1/2 \) in order to avoid double counting; else, \( n_{\text{dm}} = n_{\text{dm}}^* = \frac{\rho_{\text{dm}}}{2 m_{\text{dm}}} \) and \( \eta = 1 \).

We consider self-conjugate dark matter particles throughout this work and assume that each annihilation event injects one electron and one positron with roughly the same energy \( \gamma_0 \sim m_{\text{dm}}/m_e \),

\[ \frac{dN_{e^\pm}}{d\gamma}(\gamma) = 2 \delta(\gamma - \gamma_0), \tag{20} \]

where \( \delta(\gamma - \gamma_0) \) denotes a Dirac delta function. Although this is a rather coarse approximation, it has the advantage of being model-independent. For self-conjugate dark matter particles, we obtain

\[ Q(r, \gamma) = \left[ \frac{\rho_{\text{dm}}(r)}{m_{\text{dm}}} \right]^2 \langle \sigma v \rangle_{e^\pm} \delta(\gamma - \gamma_0). \tag{21} \]

We consider a spherically-symmetric halo, described by a density profile of the form

\[ \rho_{\text{dm}}(r) = \frac{\rho_s}{\left( \frac{r}{r_s} \right)^{3-\alpha}} \left( 1 + \frac{r}{r_s} \right)^{3-\alpha}, \tag{22} \]

where \( r_s \) and \( \rho_s \) denote a characteristic density and radius of the halo, respectively, and \( \alpha \) is the inner logarithmic slope of the density profile. Local inhomogeneities that would boost the expected signal, such as small-scale clumpiness or the presence of subhaloes, are not taken into account.
2.4. Surface brightness profile

Once the electron-positron spectrum is computed, the emission coefficient for photons of frequency $\nu$ is given by the integral

$$j_\nu(r, \nu) = \frac{1}{4\pi} \int_1^\infty \frac{dn}{d\gamma}(r, \gamma) l(\gamma, \nu) d\gamma$$

(23)

of the electron-positron spectrum $\frac{dn}{d\gamma}(r, \gamma)$ times the specific luminosity $l(\gamma, \nu)$ emitted at frequency $\nu$ by a single electron or positron with Lorentz factor $\gamma$. The intensity from any given direction in the sky is simply the integral along the line of sight of the emission coefficient. Since we assume a spherically-symmetric source and boundary conditions, it will only depend on the angular separation $\theta$ with respect to the Galactic centre,

$$I_\nu(\theta, \nu) = \int_0^\infty j_\nu(r, \nu) ds,$$

(24)

where $s$ represents the distance along the line of sight, and the radial distance $r$ to the centre of the Milky Way at any point along the ray is

$$r = \sqrt{x^2 + y^2},$$

(25)

with $x = s \sin \theta$, $y = s \cos \theta - R_\odot$, and $R_\odot = 8.5$ kpc (the distance of the Sun from the Galactic centre).

The contribution of synchrotron radiation, which dominates at low photon energies, can be estimated as [6]

$$l_{\text{syn}}(\gamma, \nu) = \frac{\sqrt{3} q_e^3 B}{m_e c^2} R[\chi(\gamma)],$$

(26)

where $m_e$ and $q_e$ denote the electron mass and charge, respectively, $B$ is the intensity of the magnetic field, and the function $R(\chi)$ is defined as [12]

$$R(\chi) \equiv 2\chi^2 \left[ K_{\frac{3}{4}}(\chi) K_{\frac{1}{4}}(\chi) - \frac{3}{5} \chi \left( K_{\frac{3}{4}}^2(\chi) - K_{\frac{1}{4}}^2(\chi) \right) \right].$$

(27)

In this expression, $K$ refers to the modified Bessel function, and the normalized frequency

$$\chi \equiv \frac{\nu}{3\gamma^2 \nu_c}$$

(28)

is expressed in terms of the cyclotron frequency

$$\nu_c = \frac{q_e B}{2\pi m_e c}.$$  

(29)

2.5. Astrophysical parameters

The photon intensity from the synchrotron depends on the astrophysical parameters that determine the propagation and energy losses of the relativistic particles. We calculate the electron-positron spectrum as described in expression (10), and then estimate the photon intensity according to expression (24).

Our canonical model assumes a dark matter density profile with $\alpha = 1$ [13], $r_s = 17$ kpc and $\rho_{\text{dm}} = 0.35$ GeV cm$^{-3}$, consistent with dynamical models of the Milky Way. The virial mass of the Galaxy is thus $10^{12}$ $M_\odot$, and the local dark matter density is $\rho_{\text{dm}}(r_\odot) c^2 = 0.3$ GeV cm$^{-3}$. The ISM is mainly composed of neutral hydrogen atoms ($X_{\text{ion}} = 0$) with number density
Table 1. The model of the diffusion coefficient, following the parameterization $K(\gamma) = K_0 \gamma^\delta$. The model MED has been proposed by [14].

| Model | $K_0$ [kpc$^2$ s$^{-1}$] | $\delta$ |
|-------|-----------------|--------|
| MED   | $1.76 \times 10^{-18}$ | 0.70   |

Table 2. Normalization of the grey-body models describing the interstellar radiation field, adopted from [15]. In our canonical model, we use the values appropriate for the Galactic centre in order to compute the ICS and synchrotron emission. For the electron-positron spectrum at the Solar neighbourhood, we use ISRF(I).

| Model   | $N_{\text{SL}}$ | $T_{\text{SL}} = 3481$ K | $N_{\text{IR}}$ | $T_{\text{IR}} = 40.6$ K |
|---------|-----------------|--------------------------|-----------------|--------------------------|
| ISRF (I)| $2.7 \times 10^{-12}$ | $7.0 \times 10^{-5}$     | canonical       | $1.7 \times 10^{-11}$ | $7.0 \times 10^{-5}$ |

\[ \rho_g/m_p \sim 1 \text{ cm}^{-3} \], and it is permeated by a tangled magnetic field whose intensity is $B \sim 6 \mu$G throughout the Galaxy.

For the diffusion coefficient (see equation 2), we consider the MED model discussed by [14], summarized in Table 1. We will also use a model of the ISRF (adopted from [15]) where the photon intensity is represented by grey-body components. The normalizations and effective temperatures of the light emitted by the Galactic stars and dust are quoted in Table 2.

3. Observational data

In order to constrain the production of relativistic electrons and positrons in the Milky Way, we consider observations of the whole sky at different wavelengths by using data from 9 channels of Planck observation at microwave wavelengths, which dominated by synchrotron emission. We take the full-resolution coadded temperature maps for each of the 9 frequency bands (30 GHz, 44 GHz, 70 GHz, 100 GHz, 143 GHz, 217 GHz, 353 GHz, 545 GHz and 857 GHz) [16].

Since we are interested in a spherically-symmetric component, we may follow a simple, conservative procedure in order to mask the emission from the Galactic disk and individual point sources without relying on any particular foreground model. For each frequency, we compute the average intensity $I(\theta)$ in 180 bins as a function of the angular separation $\theta$ from the Galactic centre. We also estimate the standard deviation $\sigma(\theta)$ within each bin, as well as the average standard deviation

\[ \sigma_{\text{ave}} = \frac{\sum_{i=1}^{n} \sigma(\theta_i)}{n}, \tag{30} \]

where $n = 180$ is the total number of the bins. We then start an iterative procedure, where all pixels more than $3\sigma_{\text{ave}}$ away from $I(\theta)$ are discarded until convergence is achieved.

The average intensity $I(\theta)$ for each wavelength are shown in Figures 1. Besides these observational data, we also consider the positrons spectrum in the solar neighbourhood by AMS02.

4. Constraints on the dark matter cross-section

Once the emission from the Galactic disc and the most prominent point sources is excluded, the remaining spherically-averaged component can be used to place upper limits on the cross-section for dark matter annihilation into electron-positron pairs.
We consider the injection energy (i.e. the mass of the dark matter particle) as a free parameter.

Recent measurements of the relativistic electrons and positrons in the solar neighbourhood. In particular, we consider the dependence of the observed spectrum, from the point of injection, and the surface brightness profile will become considerably shallower.

The maximum production rate allowed by the data can then be expressed as

\[ Q_0(r_\odot) < b(\gamma_0) \left[ \frac{dn}{dE} \right]_{\text{obs}}(\gamma_0), \] (31)

and one arrives to the condition

\[ (\sigma v)_{e^\pm}(\gamma_0) < \left( \frac{m_{\text{dm}}}{\rho_{\text{dm}}(r_\odot)} \right)^2 b(\gamma_0) \left[ \frac{dn}{dE} \right]_{\text{obs}}(\gamma_0) \] (32)

Figure 1. Planck spherically-averaged intensities. Solid red lines represent the original mean intensity \( I_0(\theta) \), while dotted blue lines correspond to the final intensity \( I(\theta) \) after discarding the outliers.
in order not to overproduce the observed signal.

From the point of view of particle physics, dark matter annihilation directly into electron-positron pairs is arguably not the most natural channel. In most models, dark matter annihilates into heavier products, and then these particles produce lower-energy electrons and positrons as secondaries. More precisely, we consider different source functions, replacing the Dirac delta in equation (20) by the appropriate injection spectrum. We used the electron-positron fluxes at production computed by [17], including electroweak corrections, for all the leptonic channels, as well as for annihilation into top and bottom quarks.

The upper limits on the annihilation cross-section into each channel are plotted in Figure 2. As can be readily seen in the figure, the tightest constraints are provided by the positron spectrum in the solar neighbourhood and synchrotron emission limit the production cross-section at lower energies. The constraints obtained for dark matter annihilation into muon-antimuon pairs are somewhat weaker than electron-positron pairs (by a factor of a few, especially at the interesting regime of low dark matter masses) due to the softer injection spectrum.

For annihilation into $\tau$ particles, the upper limits imposed synchrotron emission and the local positron spectrum are even less stringent. Similar conclusions may be reached for annihilation into top or bottom quarks.

5. Summary and conclusions
We have investigated the constraints on the dark matter annihilation cross-section into electron-positron pairs by comparing the predictions of an analytic model of particle propagation with a set of observational data obtained from Planck. We have compared the expected emission from synchrotron radiation within the Milky Way with 9 maps of the sky at different frequencies (30 GHz, 44 GHz, 70 GHz, 100 GHz, 143 GHz, 217 GHz, 353 GHz, 545 GHz and 857 GHz). A straightforward statistical criterion has been followed in order to mask the most obvious astrophysical signals (i.e. the emission from the Galactic disc and prominent point sources), and
observational upper limits are derived from the remaining spherically-symmetric component. In addition, we have also imposed that the predicted abundance of positrons in the solar neighbourhood does not exceed the measurements by AMS02.

According to our results, the dark matter annihilation cross-section into electron-positron pairs should not be higher than the canonical value for a thermal relic if the mass of the dark matter candidate is smaller than a few GeV. The upper limit on the cross-section is set by the local positron spectrum for low values of the injection energy.

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