SYSTEMATIC INVESTIGATION OF SOLAR MODULATION OF GALACTIC PROTONS FOR SOLAR CYCLE 23 USING A MONTE CARLO APPROACH WITH PARTICLE DRIFT EFFECTS AND LATITUDE DEPENDENCE

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ABSTRACT

A propagation model of galactic cosmic protons through the heliosphere was implemented using a two-dimensional Monte Carlo approach to determine the differential intensities of protons during solar cycle 23. The model includes the effects due to the variation of solar activity during the propagation of cosmic rays from the boundary of the heliopause down to Earth’s position. Drift effects are also accounted for. The simulated spectra were found to be in agreement with those obtained from experimental observations carried out by the BESS, AMS, and PAMELA collaborations. In addition, the modulated spectrum determined with the present code for the year 1995 exhibits the latitudinal gradient and equatorial southward offset minimum found by the Ulysses fast scan in 1995.

Key words: cosmic rays – solar–terrestrial relations – solar wind – Sun: heliosphere

1. INTRODUCTION

During the last two decades—using balloon flights and spaceborne missions—the fluxes of galactic cosmic rays (GCRs) and their energy distributions were observed in different phases of solar activity. These data allow one to attempt a better understanding of processes related to the transport of GCRs through the heliosphere. Furthermore, the study of propagation properties—the effect of solar modulation on the fluxes—of GCRs may, in turn, provide a tool to determine demodulated local interstellar spectra (LISs) of GCR components, for instance, protons, light nuclei, electrons, positrons, antiprotons, etc., and thus, a further understanding of processes of generation, acceleration, and diffusion within the Milky Way (e.g., see Boella et al. 1998; Strong et al. 2007; Evoli et al. 2008; Putze et al. 2009).

In addition, an accurate determination of demodulated spectra may allow one to untangle features due to new physics—dark matter (e.g., see Bottino et al. 1998; Cirelli & Cline 2010; Ibarra et al. 2010; Salati 2011; Weniger 2011, and references therein)—or astrophysical sources (e.g., see Chang et al. 2008; Abdou et al. 2009; Adriani et al. 2009a; Cernuda 2011; Mertsch & Sarkar 2011, and references therein).

Recently, spectra of GCRs were obtained using dedicated spectrometers on space-borne missions (e.g., see Alcaraz et al. 2000a, 2000b, 2000c; 2000d; Aguilar et al. 2002, 2007; Adriani et al. 2009a, 2009b, 2010) and balloon flights (e.g., see Boezio et al. 1999; Menon et al. 2000; Haino et al. 2004; Shikaze et al. 2007; Abe et al. 2008; Mitchell et al. 2008). These spectra were measured (1) with an accuracy down to about or less than 30% and (2) covering a time duration longer than a solar cycle, i.e., these spectra were measured under largely different solar conditions. These data can be hopefully exploited to determine a general treatment of solar modulation in the inner heliosphere to be used for different phases of solar activity and a better understanding of the space radiation environment close to Earth (e.g., see Leroy & Rancoita 2007, and references therein). In the near future even more accurate and systematic data will be available from AMS-02. This spectrometer is operational on board the International Space Station from 2011 May and is expected to collect data for more than a solar cycle (Battiston 2010; Bobik et al. 2010a). These observations will allow one to obtain accurate spectra with different solar activity conditions from some hundreds of MeV up to very high energy (a few TeV); in addition, using the same experimental apparatus, systematic errors on measured fluxes are expected to be minimized. Furthermore, observations made by the Ulysses spacecraft (Simpson et al. 1992) in the inner heliosphere could determine a latitudinal dependence of GCR (mostly protons) intensity with an equatorial southward offset minimum and a north polar excess (e.g., see Simpson et al. 1996). Finally, it has be remarked that modulation phenomena were observed at low energies (i.e., lower than 500 MeV/nucleon) in the outer heliosphere (e.g., see Webber et al. 2008) and are currently being investigated, for instance, by Langner et al. (2003), Langner & Potgieter (2004), Bobik et al. (2008b), and Potgieter (2008) (see also references cited by these authors).

In the present model, a two-dimensional (2D)—depending on the helio-colatitude and radial distance from the Sun (Bobik et al. 2003, 2008a, 2010a)—Monte Carlo approach is adopted to solve the transport equation of propagation of GCRs down to the inner heliosphere, without addressing CR modulation observed in the outer heliosphere. The model exhibits a slow time dependence because of the (almost) monthly averages of solar activity parameters adopted for (1) the solar wind speed \( V_{sw} \), (2) the tilt angle \( \alpha_0 \) of the neutral sheet, and (3) the diffusion parameter \( K_0 \) (discussed in Section 2.1). Furthermore, one has to remark that the solar wind usually takes of the order of or more than one year to reach the border of the heliosphere. As a consequence, the above parameters are locally evaluated within the heliosphere, allowing the modulation treatment to better (or dynamically) account for the effects of solar activity as a function of the distance from the Sun. In addition, the current treatment accounts for effects due to the charge sign of particles (i.e., the so-called particle drift effect), e.g., those related, for instance, to (1) the curvature and gradient of the interplanetary magnetic field (IMF) and
the extension of the neutral current sheet inside the heliosphere. Thus the model introduces a dependence on the sign of the solar-field polarity \( A \) (e.g., see Clem et al. 1996; Boella et al. 2001). The present code allows the fluxes of protons (as well as antiprotons) and helium nuclei to be modulated from the border of the heliosphere down to Earth—but outside Earth’s magnetosphere (Bobik et al. 2006)—in order to compare them with the available experimental observations. Furthermore, modulated electron and positron spectra can be derived accounting for the additional collision, radiative and inverse Compton energy losses (see Bobik et al. 2011c).

In the next sections, the heliosphere, drifts, diffusion tensor, determination of the diffusion parameter, dependence of both the solar wind and IMF on the radial distance and helio-colatitude, and neutral current sheet are discussed (Sections 2–4). Then, the implementation of the mathematical model and the parameterization with the dynamical treatment of the heliosphere are treated (Sections 5 and 6). Finally, comparisons between the modulated spectra of differential intensities obtained with those observed experimentally are performed and discussed (Section 7).

2. HELIOSPHERE AND DRIFT MECHANISMS

The transport of galactic protons (GPs) inside the heliosphere was initially treated by Parker (1965), who demonstrated that—in the framework of statistical physics—the random walk of the cosmic-ray particles is a Markov process, describable by a Fokker–Planck equation (hereafter FPE; e.g., see also Axford 1965; Fisk 1976; Potgieter et al. 1993, and also Sections 4.1.2.4 of Leroy & Rancoita 2011, and references therein). Thus (at the time \( t \)), the number density\(^6\) \( N \) of GPs per unit interval of particle energy \( T \) (the so-called differential density) can be obtained from the solution of the FPE

\[
\frac{\partial U}{\partial t} = \frac{\partial}{\partial x_i} \left( K_{ij} \frac{\partial U}{\partial x_j} \right) + \frac{1}{3} \frac{\partial V_{sw,i}}{\partial x_i} \frac{\partial}{\partial T} (\alpha_{rel} T U) \nonumber \\
- \frac{\partial}{\partial x_i} \left( V_{sw,i} U \right) - \frac{\partial}{\partial x_i} (v_{d,i} U) \quad (1)
\]

(e.g., see Jokipii et al. 1977, Equation (4.75) in Section 4.1.2.6 of Leroy & Rancoita 2011 and references therein); here \( V_{sw} \) is the solar wind velocity along the axis \( x \),

\[
v_{d,i} = \frac{\partial K_{ij}^A}{\partial x_j} \quad (2)
\]

is the drift velocity (e.g., see Jokipii et al. 1977; Jokipii & Levy 1977 and also Bobik et al. 2010b, and references therein). \( K_{ij}^A \) and \( K_{ij}^S \) are the antisymmetric and symmetric part of the diffusion tensor, respectively,

\[
\alpha_{rel} = \frac{T + 2m_e c^2}{T + m_e c^2}, \quad (3)
\]

and \( m_e \) is the rest mass of the proton. The number density \( U \) is related to the differential intensity \( J \) as

\[
J = \frac{v U}{4\pi}, \quad (3)
\]

where \( v \) is the speed of the GCR particle. Equation (1)—as is well known—describes (1) the diffusion of GCRs by magnetic irregularities, (2) the so-called adiabatic energy changes associated with expansions and compressions of cosmic radiation, (3) the convection effect resulting from the solar wind with velocity \( \vec{V}_{sw} \), and (4) the drift effects related to the drift velocity \( \vec{v}_d \).

In turn, the drift velocity is determined by the antisymmetric part of the diffusion tensor (see Equation (2) and Section 4) which accounts for gradient, curvature, and current sheet drifts of particles in the IMF; i.e., it depends on the charge sign of the particles.

Furthermore—as discussed by Jokipii & Levy (1977)—one can rewrite Equation (1) as

\[
\frac{\partial U}{\partial t} = \frac{\partial}{\partial x_i} \left( K_{ij}^S \frac{\partial U}{\partial x_j} \right) + \frac{1}{3} \frac{\partial V_{sw,i}}{\partial x_i} \frac{\partial}{\partial T} (\alpha_{rel} T U) \nonumber \\
- \frac{\partial}{\partial x_i} \left( V_{sw,i} + v_{d,i} \right) U. \quad (4)
\]

Thus, one obtains that drift effects are accounted for by a convection velocity in which the drift velocity is added to the solar wind velocity. In this way, the resulting effective convection velocity may non-negligibly differ from that due to the solar wind; but—as remarked by Jokipii & Levy (1977)—noting that \( \vec{V} \cdot \vec{v}_d = 0 \), one finds that drift effects do not contribute to the adiabatic energy changes (second right-hand term of Equations (1) and (4)). Even if drift effects are included\(^7\) in Equations (1) and (4), some modulation models\(^8\) neglected it (e.g., see Jokipii et al. 1977; Usoskin et al. 2005, and references therein). Gradients of particle density can also result from the convection effect. Drift mechanisms can modify both the radial and (solar) latitude dependence of the gradient magnitude. For instance, drift motions can affect modulated GCR spectra by redirecting particles within the heliosphere (Jokipii et al. 1977). When the particle Larmor radius is much shorter than the magnetic field scale length, drift effects can be taken into account by evaluating the average distance in which a relevant field variation occurs. Drift effects affect particle motions over large distances due to the large-scale variation of the IMF strength. Different intensities of GCR modulation were observed in time periods with opposite field polarity, for instance, by Emerson & Meyer (1984), Garcia-Munoz et al. (1986), Clem et al. (2000), and Boella et al. (2001). Thus, it is necessary to explicitly consider particle drifts inside the equation of propagation of GCR.

As is well known for a reference system with the third coordinate along the average magnetic field, the symmetric part of the diffusion tensor (or coefficient)—for isotropic perpendicular diffusion—includes both the transverse \( K_{\perp} \) and parallel \( K_{\parallel} \) components (e.g., see Jokipii 1971; Potgieter & Moraal 1985; Potgieter & Le Roux 1994). In turn, for a standard Parker field (Equation (15)) these two components are related to the radial component in heliocentric spherical coordinates as

\[
K_{rr} = K_{\parallel} c^2 \cos^2 \psi + K_{\perp} \sin^2 \psi, \quad (5)
\]

with \( \psi \) the angle between the radial and magnetic field directions—the so-called spiral angle (Equation (16))—(e.g., see Fisk 1976; Potgieter & Le Roux 1994) and \( K_{\theta \theta} = K_{\perp} \).
where $\theta$ is the polar angle (Potgieter et al. 1993). The general transformations of the symmetric and antisymmetric parts of the diffusion tensor from field-aligned to heliospheric (spherical) coordinates can be found in Burger et al. (2008). Furthermore, a general discussion about the role of parallel and perpendicular diffusion is available in Giacalone & Jokipii (1999).

Potgieter & Le Roux (1994) (see also Potgieter et al. 1993) suggested that the parallel diffusion coefficient is given by

$$K_\parallel \approx \beta k_1(r, t) K_P(P, t) \left[ \frac{B_\|}{3B} \right]$$  \hspace{1cm} (6)

with $\beta = v/c$, $v$ the particle velocity and $c$ the speed of light; the diffusion parameter $k_1$ accounts for the dependence on the solar activity and is treated in Section 2.1; $B_\parallel$ (typically $\approx 5$ nT) is the value of the IMF at Earth’s orbit but it varies as a function of time; $B$ is the magnitude of the large-scale IMF (discussed in Section 3) and thus depends on the heliospheric region (Section 5) through which GCRs are transported; finally, the term $K_P$ takes into account the dependence on the rigidity $P$ of the GCR particle and is usually expressed in GV. To a first approximation, one can assume that

$$K_P \approx P$$  \hspace{1cm} (7)

for particle rigidities above a threshold value $P_\text{th}$ within the rigidity range (0.4–1.015) GV, as commonly supposed by many authors (e.g., see Gloeckler & Jokipii 1966; Gleeson & Axford 1968; Perko 1987; Potgieter & Le Roux 1994; Strauss et al. 2011). In the present model, $K_P$ is assumed to be equal to the value of the rigidity ($P$) above the upper limit of the $P_\text{th}$ range, i.e., for proton kinetic energies $\geq 0.44$ GeV (see Sections 7.2 and 7.2.1). Below $P_\text{th}$, it can usually be approximated to a constant (e.g., see Perko 1987; Potgieter & Le Roux 1994; Wibberenz et al. 2001; Strauss et al. 2011). Nowadays treatments resulting in a more complex dependence of the diffusion tensor on rigidity are proposed by several authors (e.g., see Ferreira et al. 2001; Pei et al. 2010a, and references therein). Some of these studies are motivated from dealing with magnetohydrodynamic turbulence in the expanding solar wind and/or accounting for observations carried out on data of low-energy electrons collected using spacecraft (for instance, (3–10) MeV from Ulysses in Ferreira et al. 2001 and 16 MeV from Pioneer 10 in Potgieter & Ferreira 2002).

In heliocentric spherical coordinates, the perpendicular diffusion coefficient has two components, one along the radial direction, $K_{\perp r}$, and the other for the polar direction $K_{\perp \theta}$. $\rho_0$ is the ratio between the perpendicular (in the radial direction) and parallel diffusion coefficients, i.e., $K_{\perp r} = \rho_0 K_{\parallel}$. In the present model, we use $\rho_0 = 0.05$: this value is in the middle of the range suggested by Palmer (1982; see also Giacalone 1998 and Section 6.3 of Burger et al. 2000). The value of the perpendicular diffusion coefficient in the polar direction ($K_{\perp \theta}$) can be assumed to be almost equal to that in the radial direction ($K_{\perp r}$); e.g., see Potgieter 2000, and references therein). However, Potgieter (2000) suggested the usage of an enhanced $K_{\perp \theta}$ in the polar regions in order to reproduce the amplitude and rigidity dependence of the latitudinal gradients of GCR differential intensities for protons and electrons (e.g., see Potgieter 1997; Heber et al. 1998). He introduced a sharp transition (via a transition function, e.g., Figure 7 in that article) in the colatitude regions $120^\circ \leq \theta \leq 130^\circ$ and $60^\circ \geq \theta \geq 50^\circ$. He also derived that $K_{\perp \theta}$ has to be increased by a factor of about (or larger than) 10; Ferreira & Potgieter (2004) used a factor of 8. In the current code, $K_{\perp \theta}$ is given by

$$K_{\perp \theta} = \begin{cases} 10 K_{\perp r}, & \text{in the polar regions,} \\ K_{\perp r}, & \text{in the equatorial region,} \end{cases}$$  \hspace{1cm} (8)

where the polar regions correspond to colatitudes with $\theta \lesssim 30^\circ$ or $\theta \gtrsim 150^\circ$, while the equatorial region corresponds to colatitudes with $30^\circ \lesssim \theta \lesssim 150^\circ$. The solar colatitudes of $30^\circ$ and $150^\circ$ correspond to those at which the solar wind (SW) speed becomes constant in periods not dominated by high solar activity (Equation (17)). The usage of the transition function can be fully implemented in the current treatment, but is not required with the present overall code accuracy. In fact, the results obtained from the so-called “L” model (i.e., the one with a better agreement with data, see Sections 7.2 and 7.2.1) indicate that only in periods not dominated by high solar activity does the enhancement of $K_{\perp \theta}$ (Equation (8)) slightly improve the overall agreement with data by a few percent. Finally, in the Appendix the diffusion coefficients in heliocentric polar coordinates are expressed in terms of those parallel and perpendicular to the IMF.

It should be noted that the diffusion tensor (1) is not well determined during solar maxima and (2) can be adapted to better account for the complex structure of the IMF—which depends on the solar activity—found with the Ulysses spacecraft (e.g., see Burger et al. 2008, and references therein). For instance, Potgieter et al. (2001; see also references therein) discussed the so-called propagating diffusion barriers and suggested a time-dependent model for the diffusion coefficients. The latter are supposed to be $\propto [B_\parallel/B(t)]^n$, where $B(t)$ is the IMF magnitude at the time $t$ and $B_\parallel = 5$ nT is the average IMF magnitude during minimum modulation conditions at Earth (Potgieter & Ferreira 2001; Potgieter et al. 2003); $n$ is the ratio between the actual tilt-angle value (Section 3) and that close to solar minimum ($7^\circ–15^\circ$; e.g., see Potgieter & Ferreira 2001; Potgieter et al. 2001). However, in the current model the time dependence of the diffusion coefficients is taken into account using a diffusion parameter, which is treated in Section 2.1. The agreement with data obtained during high solar activity is discussed in Section 7.2.

### 2.1. Diffusion Parameter in the Framework of the Force Field Model

In the FFM (e.g., see Gleeson & Axford 1968; Gleeson & Urch 1971 and also Section 4.1.2.4 of Leroy & Rancoita 2011), Gleeson & Axford (1968) assumed that, at the time $t$, (1) modulation effects can be expressed with a spherically symmetric modulated differential number density $U$ of GCRs, (2) the diffusion coefficient reduces to a scalar given by a separable function of $r$ (the radial distance from the Sun) and $P$ (the particle rigidity in GV):

$$K(r, t) = \beta k_1(r, t)K_P(P, t)$$  \hspace{1cm} (9)

with $K_P$ from Equation (7) for particle rigidities above $\approx 1$ GV, and (3) the modulation occurs in a steady-state condition, i.e., the relaxation time of the distribution is short with respect to the solar cycle duration so that one can assume that the partial derivative of $U$ with respect to time is zero. They derived that

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9 While in Equation (1), it is expressed by a tensor with a symmetric and an antisymmetric part (see discussion in Section 2).
the differential intensity (Equation (3)) at a radial distance \( r \) is given by the expression

\[
J(r, E_t, t) = J(r_{\text{in}}, E_t + \Phi_p) \left[ \frac{E_t^2 - m_e^2 c^4}{(E_t + \Phi_p)^2 - m_e^2 c^4} \right],
\]

(10)

where \( J(r_{\text{in}}, E_t + \Phi_p) \) is the undisturbed intensity beyond the solar wind termination located at a radial distance \( r_{\text{in}} \) from the Sun, \( E_t \) is the total energy of the particle with rest mass \( m_t \), and, finally, \( \Phi_p \) is the so-called force-field energy loss (Gleeson & Axford 1968; Gleeson & Urch 1971). When modulation is small (\( \Phi_p \ll m_e c^2, T \); Gleeson & Axford 1968; Gleeson & Urch 1971, 1973), they determined that

\[
\Phi_p = -\frac{Ze P}{K_p(P, t)} \phi_p(r, t) \approx Ze \phi_p(r, t),
\]

where \( Ze \) is the particle charge and \( \phi_p(r, t) \) is the so-called modulation strength (or modulation parameter) usually expressed in units of GV (or MV). Assuming that \( V_{sw} \) (the solar wind speed) and \( k_1 \) are almost constant, \( \phi_p(r, t) \) is linearly dependent on \((r_{\text{in}} - r)\) (e.g., see Equation (4.64) of Leroy & Rancoita 2011), from which one gets that the diffusion parameter is given by

\[
k_1(t) \approx \frac{V_{sw}(t)(r_{\text{in}} - r)}{3\phi_p(r, t)},
\]

(11)

i.e., \( k_1 \) (similarly to \( \phi_p(r, t) \)) is linearly dependent on \((r_{\text{in}} - r)\). As already mentioned, in the FFM the diffusion coefficient \( K(r, t) \) is a scalar quantity and does not account for effects related to the charge sign of the transported particles. \( \phi_p(r, t) \) is independent of the species of GCR particles (e.g., see discussion on page 1014 of Gleeson & Axford 1968 or Equation (1) of Usoskin et al. 2005, and also Bobik et al. 2011a, 2011b). Usoskin et al. 2005 determined the values of the modulation strengths \( \phi_p(r_{\text{Earth}}) \) monthly for the time period from 1951 up to 2004 using measurements of neutron monitors (i.e., located at \( r_{\text{Earth}} = 1 \) AU); the values of solar wind speeds are available from NASA/GSFC’s OMNI data set through OMNIWeb.

To determine \( \phi_p(r_{\text{Earth}}) \), Usoskin et al. (2005) used an approximated expression of the LIS for protons from Burger et al. (2000). In practice, their spectrum differs from that due to Burger et al. (2000) by about or more than 5% at kinetic energies lower than about 117 MeV. Furthermore, in the present work we found that the error-weighted average of the differential spectral index, \( \gamma_{sw} \) (Equation (31) and discussion in Section 7.1), of the proton LIS is only compatible, within one standard deviation, with the differential spectral index \( (\gamma = 2.78) \) of the spectrum from Burger & Potgieter (1989) or Burger et al. (2000). It has to be remarked that the latter spectral index is the one used by Usoskin et al. (2005). Usoskin et al. (2005, see Appendix A in that article) also found that using other commonly adopted LISs their corresponding values of the modulation strengths follow a linear relation with respect to \( \phi_p(r_{\text{Earth}}) \). However, the differential spectral indexes of these spectra are not compatible within three or more standard deviations with that found in Section 7.1. Moreover, it has to be noted that the response of neutron monitors has to be evaluated by combining (1) the effects of both the geomagnetic cutoff rigidity (Usoskin et al. 2005) which results in a reduced sensitivity of detection apparatus and (2) the so-called atmospheric yield function (Clem & Dorman 2000). Thus, one finds that (1) the contribution of the GCRs with rigidities below 2 GV amounts to about or less than 1.1% of the total neutron monitor counts due to particles with energies up to about 50 GV and (2) the maximum neutron monitor sensitivity—the maximum of the response function (see Figure 7 of Clem & Dorman 2000)—occurs in the rigidity interval (3–15) GV. In addition, Boella et al. (2001) determined—using IMP8 satellite data during the period 1973–1995—that charge effects (discussed in Sections 1 and 2) result in a variation of proton or helium fluxes during solar minima with opposite magnetic field polarities of 14% ± 6% at ≈ 300 MeV/nucleon. This variation steadily decreases with increasing energy (e.g., see Figure 4.13 on page 378 of Leroy & Rancoita 2011). As a consequence, \( \phi_p(r_{\text{Earth}}) \) is expected to be marginally affected by drift effects.

\( k_1 \) (Equation (11)) depends on the value of the solar wind termination located at a radial distance \( r_{\text{in}} \) related, in turn, to the solar wind speed (e.g., see Chapter 7 of Meyer-Vernet 2007, and Sections 4.1.2.3, 4.1.2.4 of Leroy & Rancoita 2011). In the present simulation code, the effective heliosphere assumes that the solar wind termination is located at 100 AU (see a further discussion in Sections 5 and 7.3). Therefore, from the diffusion parameter \( k_1 \) one has to derive that \( (K_0) \) for an effective heliosphere with a radial extension of 100 AU. In practice, for a radial extension of 100 AU the diffusion parameter \( K_0 \) (Equation (12)) replaces \( k_1 \) in Equation (6) (for instance, see the Appendix) and allows one to obtain similar modulation effects on the differential intensities of GCRs with respect to those obtained using \( k_1 \) when the heliosphere has a variable radial extension \( r_{\text{in}} \). Using Equation (11) one obtains

\[
k_0 \approx k_1 \frac{99 \text{AU}}{(r_{\text{in}} - r_{\text{Earth}})} = 99 \text{AU} \frac{V_{sw}}{3\phi_p(r_{\text{Earth}})},
\]

(12)

where 99 AU (as already mentioned) is the distance of the Earth from the border of the effective heliosphere used in the current simulation code. In Figure 1, the diffusion parameter \( K_0 \)—obtained from Equation (12)—is shown as a function of the corresponding value of the smoothed sunspot number, SSN (SSN 2010). The \( K_0 \) data had to be subdivided into four sets, i.e., ascending and descending phases for both negative and positive solar magnetic-field polarities. For each set, the data could be fitted with a practical relationship (see Figure 1) between \( K_0 \) and SSN values for \( 10 \lesssim \text{SSN} \lesssim 165 \), i.e., finding

\[
K_F = c_1 + c_2 \times \text{SSN}^{-1} + c_3 \times \text{SSN} + c_4 \times \text{SSN}^2
\]

(13)

with the parameters \( c_i \) shown in Table 1. In addition, the data were found to exhibit a Gaussian distribution of percentage differences \( R_{\text{perc}} \) of \( K_0 \) values from the corresponding fitted values \( K_F \), with

\[
R_{\text{perc}} = \frac{K_F - K_0}{K_F}.
\]

(14)

The rms values of the Gaussian distributions were found to be \( \approx 0.1339, 0.1254, 0.1040 \), and 0.1213 for the phases ascending with \( A < 0 \), descending with \( A < 0 \), ascending with \( A > 0 \), and descending with \( A > 0 \), respectively. From the practical relationship found (Equation (13)), we can use the estimated SSN values to obtain the diffusion parameter \( K_0 \) at times beyond 2004. This procedure allows one to extend the \( \approx 40 \) year period by exploiting the practical relationship between the fitted \( K_0 \) values and the SSN values (one of the main parameters related to the solar activity). In addition, we introduced in our code a Gaussian random variation of \( K_0 \) with rms corresponding to those found for each subset of data. Results of the simulation with and without the Gaussian variation are consistent within the uncertainties of the code. Furthermore, it can be noted that
Figure 1. Diffusion parameter $K_0$ (left side) and percentage differences $R_{\text{perc}}$ (Equation (14); right side) as a function of the SSN value; the central continuous lines are obtained from a fit of $K_0$ with respect to SSN values in the range $10 \lesssim \text{SSN} \lesssim 165$; the dashed and dotted lines are obtained adding (top) or subtracting (bottom) one standard deviation from the fitted values.

Table 1

| Parameters and Phase | $A < 0$ Ascending | $A < 0$ Descending | $A > 0$ Ascending | $A > 0$ Descending |
|---------------------|-------------------|-------------------|-------------------|-------------------|
| $c_1$               | +0.0001686        | +8.872 x $10^{-5}$| +2.39708 x $10^{-4}$| +2.28037 x $10^{-4}$|
| $c_2$               | +0.001488         | +0.001874         |                   |                   |
| $c_3$               |                   |                   | -8.28987 x $10^{-7}$| -1.00984 x $10^{-6}$|
| $c_4$               | -3.164 x $10^{-9}$|                   |                   |                   |

$K_0$ provides an overall increase (for $r_{\text{tm}}$ lower than 100 AU) or decrease (for $r_{\text{tm}}$ larger than 100 AU) in the modulation effects. A tuning of the effective extension of the heliosphere and its dependence on the solar activity is likely to be obtained using the experimental data from long-duration accurate observations, like those from the AMS-02 spectrometer.

3. IMF DEPENDENCE ON SOLAR WIND AND LATITUDE

Parker (1958) suggested that the solar corona is stationary and expanding due to an outflow of the coronal plasma—generating the so-called solar wind—with a spherically symmetric velocity. In his model, the solar wind speed ($V_{\text{sw}}$) becomes almost constant beyond a radial distance from the Sun $r_{\phi} \approx (0.3–0.4) \text{ AU}$ (e.g., see Figure 1 of Parker 1958). Furthermore, the magnetic-field lines are frozen in the streaming particles that comprise the solar wind. Thus, beyond $r_{\phi}$, in a spherical reference frame rotating with the Sun the components of the outward velocity of a plasma element carrying the magnetic field are: $V_r = V_{\text{sw}}$, $V_\theta = 0$, and $V_\phi = \omega (r - r_{\phi}) \sin \theta$ with $\omega$ the angular velocity of the Sun. The streamline has the shape of an Archimedean spiral (termed Parker spiral).
In heliocentric spherical coordinates, the standard Parker spiral field can be expressed as (e.g., see Equation (2) of Hattingh & Burger 1995):

\[
\vec{B}_p = \frac{A}{r^2} (\vec{e}_r - \Gamma \vec{e}_\phi) [1 - 2H(\theta - \theta')], \tag{15}
\]

where \(A\) is a coefficient that determines the field polarity and allows \(|\vec{B}_p|\) to be equal to \(B_p\) (Section 2), i.e., the value of the IMF at Earth’s orbit as extracted from NASA/GSFC's OMNI data set through OMNIWeb (King & Papatashili 2005); \(\vec{e}_r\) and \(\vec{e}_\phi\) are unit vector components in the radial and azimuthal directions, respectively; \(\theta\) is the colatitude (polar angle); \(\theta'\) is the polar angle determining the position of the HCS (Jokipii & Thomas 1981); \(H\) is the Heaviside function: thus, \([1 - 2H(\theta - \theta')]\) allows \(\vec{B}_p\) to interchange the sign in the two regions—above and below the heliospheric current sheet (HCS)—of the heliosphere; finally,

\[
\Gamma = \tan \psi = \frac{\omega (r - r_\odot) \sin \theta}{V_{sw}} \tag{16}
\]

with \(\psi\) the spiral angle. In the present model, \(\omega\) is assumed to be independent of the heliographic latitude and equal to the sidereal rotation at the Sun’s equator. However, the simple representation of the Parker spiral (Equations (15) and (16)) based on a constant solar wind speed needs to be complemented with the present knowledge of the dependence of the speed \(V_{sw}\) on solar latitude. Large variations of the solar wind structure were observed for solar latitudes up to \(80^\circ\) by the Ulysses spacecraft (Wenzel et al. 1992). For instance, during a period of low solar activity the solar wind speed increases by almost a factor two from the ecliptic plane to the poles, thus subdividing the heliosphere into two regions with slow and fast solar wind (Mc Comas et al. 2000). To represent the observed speeds, Fichtner et al. (1996) suggested that the solar wind speed may be proportional to \((1 + \cos^2 \theta)\). In the present model we use

\[
V_{sw}(\theta) = \begin{cases}
V_{sw_{\text{max}}}, & \text{for } \theta \lesssim 30^\circ \text{ and } \theta \geq 150^\circ, \\
V_{sw_{\text{max}}} \times (1 + \cos \theta), & \text{for } 30^\circ < \theta < 150^\circ \tag{17}
\end{cases}
\]

with \(V_{sw_{\text{max}}} \approx 760 \text{ km s}^{-1}\) (e.g., see Mc Comas et al. 2000) and \(V_{sw_{\text{max}}}\) is the corresponding value extracted from NASA/GSFC’s OMNI data set through OMNIWeb (King & Papatashili 2005). Equation (17) exhibits a slightly better agreement with observed data than that proposed by Fichtner et al. (1996). Jokipii & Kota (1995) and Pommois et al. (2001) proposed other functions for such periods. However, these functions depend on an additional parameter related to the latitudinal extension of the region with a slow solar wind. This parameter can be determined only using measurements to be performed largely outside the ecliptic plane, like those made by the Ulysses spacecraft. Thus, Equation (17) has the advantage of allowing one to more generally treat periods of low solar activity. Furthermore, Mc Comas et al. (2000) observed that during the Sun’s approach to solar maximum (1) the coronal structure becomes increasingly complex and (2) the magnetic field becomes less dipolar. In the present model, for the solar wind we assume a speed independent of the colatitude in periods characterized by high solar activity. As previously, the value of the speed is extracted from NASA/GSFC’s OMNI data set through OMNIWeb (King & Papatashili 2005).

Potgieter et al. (1989) pointed out how classical drift modulation models—based on the Parker magnetic field up to the polar region—encounter difficulties (see also Section 7.4) in accounting for the significantly lower latitudinal dependence of the CR intensity. Simpson (1996) subsequently observed this phenomenon using Ulysses data collected in the inner heliosphere. Heber et al. (1998) remarked that (1) one needs to assume an anisotropy of the perpendicular diffusion coefficient and enhancement in the latitude direction (as already treated in Section 2) and (2) Parker’s IMF has to be modified as proposed by Jokipii and Kota (1989). In the present model, the magnitude of the magnetic field (Equation (15)) is enhanced by introducing a small latitudinal component (e.g., see Langner 2004; Langner & Potgieter 2004)

\[
B_\theta = \frac{A}{r r_\odot} \delta(\theta), \tag{18}
\]

with \(r_\odot\) the solar radius, and

\[
\delta(\theta) = \frac{8.7 \times 10^{-5}}{\sin \theta}; \tag{19}
\]

for \(\theta \gtrsim 17^\circ\) and \(\theta \gtrsim 178.3^\circ\), \(\delta\) is \(\approx 3 \times 10^{-3}\) (Fichtner et al. 1996). It has to be noted that Equations (18) and (19) allow one to obtain \(V \cdot \vec{B} = 0\). The magnitude of the magnetic field used in the current model is given by (Jokipii & Kota 1989)

\[
B = \frac{A}{r^3} \sqrt{1 + \Gamma^2 + \left(\frac{r}{r_\odot}\right)^2 \delta^2(\theta)}. \tag{20}
\]

In Figure 2, the magnitude \(|B|\) of the IMF from Jokipii & Kota (1989)—computed using Equations (18)–(20)—compared with that from Parker (solid line; Equation (15)) at 1, 5, and 10 AU as a function of the colatitude. For the purpose of this calculation, at 1 AU and \(90^\circ\) \(|B| = 5 \text{ nT}|\).
structure of the solar magnetic field is more or less axially symmetric, dominated by the dipole component. These periods are characterized by corresponding low values of the tilt angle ($\alpha_i < 30^\circ$, Potgieter et al. 2001). As the solar activity increases, the dipolar structure inclines more and more with respect to the rotation axis and the effect of higher multipoles becomes more relevant (Sanderson et al. 2003). During the years of high activity, the structure of the solar magnetosphere is very complex and the dipole component is very tilted (e.g., see Sanderson et al. 2003; Wang & Sheeley 2002). These periods are characterized by corresponding large values of the tilt angle ($\alpha_i > 75^\circ$, Potgieter et al. 2001). Finally, one can remark that, dealing with neutron-monitor measurements, Cliver & Ling (2001) remarked that the obser-
vation of cosmic-ray particles due to drift mechanisms. In a coordinate system with the third coordinate along the average IMF, one finds (e.g., see Potgieter & Moraal 1985; Burger & Hattingh 1995)

$$K_A = \frac{p v}{3 Z e |B|},$$

where $p$, $v$, and $Z e$ are the momentum, velocity and charge of the cosmic-ray particle, respectively. Thus, the antisymmetric elements of the diffusion-tensor matrix (Section 2) are

$$K_{ij}^A = K_A \epsilon_{i,j,k} \frac{B_k}{|B|}$$

with $\epsilon_{i,j,k}$ the Levi–Civita symbol (e.g., see Equation (10) of Parker 1965).

As already mentioned in Section 2, the drift velocity $\vec{v}_d$ (Equation (2)) accounts (1) for effects due to gradient and curvature drifts experienced by cosmic-ray particles transported through the IMF, (2) net drift effects occurring close to the HCS, where the IMF changes polarity (e.g., see Parker 1957; Burger et al. 1985; Potgieter & Moraal 1985), and (3) can be calculated using the antisymmetric part of the diffusion tensor (e.g., see Parker 1965; Jokipii et al. 1977; Potgieter & Moraal 1985; Burger & Hattingh 1995, and references therein).

Burger et al. (2000; see also references therein) remarked that the obser-
vational results (carried out below 5 GV) are consistent with a small (or very small) ratio of the perpendicular to parallel diffusion coefficients. As discussed by Parker (1965), a small value of that ratio indicates that cosmic-ray particles are practically moving through several gyro-orbits between each scattering event, i.e., drift motion is weakly affected by scattering. In addition, for cosmic-ray particles with rigidities $\gtrsim (10–15)$ GV and an IMF expressed by Equations (15) and (20), the particle gyro-radius is smaller (or much smaller) than any local (i.e., inside the heliosphere) scale variation of magnetic field $L \equiv |(1/B)(\partial B_i/\partial x_i)|^{-1}$. In this way, for regions outside that of the HCS, Isenberg & Jokipii (1979) remarked that $\vec{v}_d$ is determined by the terms due to the gradient and curvature drifts (e.g., see also Parker 1957; Armstrong et al. 1985).

Potgieter & Moraal (1985) treated the modulation of GCRs for steady-state conditions with relevant drift effects including that due to a wavy HCS (WHCS). They succeeded in formulating a 2D description (of the WHCS), which—as discussed by Burger & Hattingh (1995)—is equivalent to the treatment of transport in a three-dimensional heliosphere with the assumption of an axis-symmetric particle distribution. Thus, they allowed one to neglect the azimuthal dependence. The effect of a WHCS was included via an appropriate modification of the antisymmetric part of the diffusion tensor. In this 2D modeling, the WHCS is described as a wide region whose width depends on the rigidity of cosmic-ray particles and actual value of the tilt angle $\alpha_i$. The resulting drift velocity in heliocentric polar coordinates—as used in the current model—is given by (e.g., see Equation (6) of Burger & Hattingh 1995):

$$\vec{v}_d = f(\theta) \nabla \left( K_A \frac{\vec{B}}{|B|} \right) + \left( \frac{\partial f(\theta)}{\partial \theta} \right) \frac{K_A}{r} \vec{e}_\theta \times \left( \frac{\vec{B}}{|B|} \right)$$

(22)

where $K_A$ is from Equation (21), $\theta$ is the colatitude, $f(\theta)$ is a transition function that accounts for the effects of a wavy neutral sheet (Potgieter & Moraal 1985), and $\vec{e}_\theta$ is the unit vector along the latitudinal direction. $f(\theta)$ is expressed as (e.g., see Equation (14) of Potgieter & Moraal 1985):

$$f(\theta) = \begin{cases} (1/a_h) \arctan \left( 1 - \frac{2 \pi}{\pi} \tan(a_h) \right), & \text{if } a_h < \frac{\pi}{2}, \\ \left( 1 - 2 H (\theta - (\pi/2)) \right), & \text{if } a_h = \frac{\pi}{2} \end{cases}$$

with $H$ the Heaviside function,

$$a_h = \arccos \left( \frac{\pi}{2 c_h} - 1 \right)$$

(e.g., see Equation (15) of Potgieter & Moraal 1985),

$$c_h = \frac{\pi}{2} - \frac{1}{2} \sin(\alpha_i + \Delta \theta_{\text{HCS}})$$

(e.g., see Equation (23) of Burger & Potgieter 1989),

$$\Delta \theta_{\text{HCS}} = \frac{2 r_p}{r}$$

($r_p$ is the particle gyro-radius, e.g., see Hattingh & Burger 1995 and also Section 4.2 of Burger & Hattingh 1995), and finally $f(c_h) = 0.5$ and $f(\pi/2) = 0$. $\Delta \theta_{\text{HCS}}$ is determined from the maximum distance that a particle drifting along the neutral sheet can be away from this sheet (Burger & Potgieter 1989). The first term ($\vec{v}_d$) of Equation (22) accounts for the gradient and curvature drifts, the second ($\vec{v}_{\text{HCS}}$) for drift in the region affected by a WHCS. The transition function sets the rate at which the first term of Equation (22) goes to 0 on the ecliptic plane ($\theta = \pi/2$; Potgieter & Moraal 1985).
5. PARAMETERS OF THE EFFECTIVE HELIOSPHERE USED IN THE CURRENT MODEL

As discussed by Potgieter (2008, see also references therein), until recently the heliosphere was assumed to be spherical in most modulation models with an outer boundary at radial distances beyond ≈100 AU. Presently, the heliospheric structure is considered latitudinally asymmetric (particularly) during solar minimum conditions mostly because the SW depends on the latitude and solar activity (Section 3). As a consequence, the position of the termination shock (TS; where the SW ram pressure is balanced by interstellar pressure) can exhibit a latitudinal asymmetry.

Using solar wind speeds observed from *Ulysses*, Whang and collaborators (e.g., see Whang & Burlaga 2000; Whang et al. 2003, 2004) could estimate the radial position of the TS on and outside the ecliptic plane. They found that (1) on the ecliptic the radial distance of the TS is about 80 AU on average (without large variation between low and high solar activities), (2) near the ecliptic the radial distance varies by less than 20 AU, and (3) outside the ecliptic plane (e.g., at a latitude of 35°) the location of the TS increases by more than or about 50 AU (Whang et al. 2003). In addition, Whang and collaborators estimated that the averaged value over a 26-year period of the radial distance of the TS increases with latitude (see Table 2 of Whang et al. 2003). It is worth noting that ≈100 AU is the averaged value over the corresponding solid angle of the TS location, which can be obtained from Table 2 of Whang et al. (2003). Furthermore (e.g., see Stone et al. 2005, 2008), *Voyager 1* and 2 reached the TS in 2004 and 2007 located at about 94.0 AU and 83.7 AU, respectively, in agreement with the predictions from Whang and collaborators. Langner & Potgieter (2005) treated symmetric and asymmetric TS models and concluded that for the A > 0 cycle for solar minimum no significant difference occurs; for the A < 0 cycle differences remain insignificant in the nose direction while, approaching the tail direction, some differences can be appreciated at proton energies below (1–1.5) GeV. However, Langner & Potgieter (2005) and Potgieter (2008) suggested that, in general, a symmetric TS with a radial distance of ≈100 AU is still a reasonable assumption.

In the present model (as already discussed in Section 2.1), the effect of the modulation is obtained for GCR propagation through a symmetric effective heliosphere with a radius of 100 AU. The diffusion parameter K0 is determined (following the procedure described in Section 2.1) using the values of the modulation strength, SSN values (SSN 2010), and radius of the effective heliosphere. Furthermore, (see discussion in Section 2.1) the atmospheric yield function results in a diffusion parameter related to modulated intensities of GCRs (mostly protons) with rigidities above 2 GV.

Other parameters (which depend on the solar activity) are the tilt angle α, the HCS, the magnetic field polarity (related to the sign of the coefficient A in Equation (15)), the magnetic field magnitude (B0), and the solar wind velocity (Vsw). The latter two parameters are measured at Earth’s orbit. The polarity of the magnetic field and B0 determine the IMF described by means of Equations (15), (16), and (18)–(20). α, and the field polarity are used to deal with the drift velocity (as discussed in Section 4), which modifies the overall convection velocity (Equation (4)). The drift contribution is relevant during low solar activity—e.g., for α < 30° (Section 3)—and decreases with increasing solar activity. α values are obtained from Wilcox Solar Observatory (Hoeksema 1995; WSO 2010) and are calculated using two different models called “R” and “L.” Ferreira & Potgieter (2003, 2004) suggested that the “R” model accounts for GCR observations during periods of increasing solar activity (for instance, 1987.4–1990.0 and 1995.5–2000.0), while the “L” model accounts for periods of decreasing solar activity (for instance, 1990.0–1995.5 and 2000.0–2010.0). The implementation of the “R” and “L” models in the current code is further treated in Sections 7.2 and 7.2.1. Finally, the latitudinal dependence (e.g., see Section 3) of the solar wind (Equation (17)) depends (at low solar activity) on the values (averaged over 27 days) of the SW speed and on the ecliptic at Earth’s orbit.

The time spent by the SW to cover the distance from the outer corona up to the boundary of the effective heliosphere can be expressed in units of the time needed for a sidereal rotation on the equator of the Sun (about 25 days; e.g., see page 77 of Aschwanden 2006 and also Brajša et al. 2001; for a survey see Ruždjak et al. 2005). For instance, depending on the wind speed, on the ecliptic the SW spends the corresponding amount of time needed to complete from 12 up to 20 sidereal solar rotations to reach the outer boundary. In the present code, the effective heliosphere (with a radius of 100 AU) was subdivided into 15 spherical regions. In each region, the parameters (e.g., SW speed, K0, B0, α, etc.) are determined at the time of the solar wind ejection.

6. THE MONTE CARLO CODE HelMod

It is worthwhile to note that Equations (1) and (4) can be solved analytically only by treating a simplified transport of GCRs through the heliosphere (e.g., see Section 2.1 and also Gleeson & Axford 1968; Caballero-Lopez & Moraal 2004). Complex configurations regarding the transport inside the heliosphere were proposed using numerical methods such as finite-difference integration (e.g., Burger & Potgieter 1989).

As implemented in the HelMod code version 1.5, the current approach (1) follows that of Yamada et al. (1998), Gervasi et al. (1999), Zhang (1999), Alanko et al. (2003), Pei et al. (2010b), and Strauss et al. (2011) and (2) exploits a Monte Carlo technique to determine the number density U (Section 2) using the set of approximated stochastic differential equations (SDEs)

| Observations | “L” Model | “R” Model | No Drift | Diagonal Approx. | Scalar Approx. |
|--------------|-----------|-----------|----------|------------------|----------------|
| BESS–1999    | 8.7       | 8.0       | 14.6     | 32.0             | 29.7           |
| BESS–2000    | 16.2      | 15.8      | 13.0     | 23.6             | 26.7           |
| BESS–2002    | 12.7      | 15.0      | 12.2     | 34.8             | 33.2           |
treated in the Appendix for a 2D approximation (radial distance and colatitude). For (1) an IMF described by the standard Parker field (Equation (15)) and (2) both solar wind and drift velocity in the region of the WHCS directed radially (e.g., \(V_{sw,r} = V_{sw}\) and \(v_{HCS,r} = v_{HCS}\), the SDEs approximated in terms of the increments \(\Delta r\), \(\Delta \mu(\theta)\), \(\Delta T\), and \(\Delta t\) (with \(\mu(\theta) \equiv \cos(\theta)\)) are (see the Appendix):

\[
\Delta r = \frac{1}{r^2} \left[ \frac{\partial}{\partial r} \left( r^2 K_{r,r}^S \right) \right] \Delta t + \left( V_{sw} + v_{dr,r} + v_{HCS} \right) \Delta t + \omega_r \sqrt{2 K_{r,r}^S} \Delta t,
\]

\[
\Delta \mu(\theta) = \frac{1}{r^2} \left\{ \frac{\partial}{\partial \mu(\theta)} \left[ (1 - \mu^2(\theta)) K_{\mu,\mu}^S \right] \right\} \Delta t - \frac{v_{dr,\theta}}{r} \sqrt{1 - \mu^2(\theta)} \Delta t + \omega_\mu(\theta) \sqrt{2 K_{\mu,\mu}^S (1 - \mu^2(\theta))} \Delta t,
\]

\[
\Delta T = -\frac{2}{3} \frac{\alpha_{rel} V_{sw} T}{r} \Delta t.
\]

The procedure to integrate the SDEs is as follows: (1) events are isotropically generated on the outer border of the effective heliosphere; (2) each event is integrated over the time evolution of a pseudoparticle and is processed forward in time until it reaches either the outer (inner) border of the effective heliosphere located at 100 AU (r0) or the pseudoparticle energy becomes lower than a minimum threshold (which depends on the set of experimental data taken into consideration), then a new particle is generated; (3) when a pseudoparticle reaches a particular region (for instance that corresponding to Earth position) its injection energy, statistical weight, etc., are recorded; (4) finally, the number density \(U\) results from the normalized distribution function obtained using a procedure from Pei et al. (2010b; see Section 4.3 in this article). The forward-in-time approach allows one to reproduce rigorously processes occurring inside the heliosphere.

In the present code, \(\Delta t\) varies as \(r^2/K_{r,r}\), thus allowing an increase of the accuracy in the inner heliosphere, but keeping the appropriate precision up to regions close to the outer border of the effective heliosphere. Furthermore, this condition ensures that the diffusion process is dominant (see Section 4.1 of Kruells & Achterberg 1994).

7. RESULTS

The current modulation code (Section 6) provides a modulated differential intensity for protons using an LIS of protons. In the following, we will discuss (1) the LIS used (Section 7.1), (2) the comparison of simulated (modulated) differential intensities with those obtained from the measurements of BESS, AMS, and PAMELA spectrometers during solar cycle 23 (Sections 7.2 and 7.2.1), and (3) the dependence of the present results on the treatment of the heliosphere extension (Section 7.3). Furthermore (Section 7.4), the simulated fluxes obtained with the HelMod code are compared with (and found to reproduce the features of) the experimental data from the Ulysses fast scan in 1995 (Simpson et al. 1996).

7.1. Local Interstellar Spectrum

Recently, Herbst et al. (2010) reviewed different proton LISs published in the literature and determined that—as can be seen in Figure 2(b) in that article—these spectra agree well with each other for proton energies above 10 GeV. For this comparison, they used, among others, the LIS from Burger et al. (2000; BPH-LIS) in the form of the approximated analytical expression from Usoskin et al. (2005). Over the past years, Moskalenko, Strong, and collaborators, using GALPROP, provided an LIS for protons (e.g., see Moskalenko et al. 2002; Strong & Moskalenko 2004; Trovati et al. 2011; see also Langner 2004; Langner et al. 2003); the latest calculation agrees with the BPH-LIS above 1 GV (e.g., see Trovati et al. 2011); it should be noted that the GALPROP spectrum is constrained by a few measured quantities (for instance, the B/C and other isotopes and/or nuclei ratios), some of which will be (accurately) re-determined in the coming years using data from the PAMELA and AMS-02 missions.

In units of \((\text{sr}^{-1} \text{m}^2 \text{s GeV}^{-1})\)—(Burger et al. 2000; see also Usoskin et al. 2002)—the BPH-LIS is expressed as

\[
J_{\text{BPH}}(T) = \begin{cases} 
0 & \text{for } r \geq 7, \\
\exp\left[9.472 - 1.999 \ln R - 0.6938 (\ln R)^2 \right] + 0.2988 (\ln R)^3 - 0.04714 (\ln R)^3, & \text{for } R < 7,
\end{cases}
\]
Figure 3. Left: spectral index ($\gamma$) obtained (see the text) for AMS–1998, BESS–1998, BESS–2002, and PAMELA–2006/08. Right: normalization constant ($J_0$) for BESS–1997, AMS–1998, BESS–1998, BESS–1999, BESS–2002, and PAMELA–2006/08. The dotted lines represent the values of the spectral index ($\gamma_{wa}$) and normalization constant ($J_{0,wa}$), respectively; the continuous lines represent the error-weighted averages of spectral index ($\gamma_{wa}$) and normalization constant ($J_{0,wa}$).

Table 3

For BESS–1999, BESS–2000, and BESS–2002, $\eta_{wa}$ (%) Obtained from Equation (34) without Any Enhancement of the Diffusion Tensor along the Polar Direction Using “L” and “R” Models for the Tilt Angle and for No Drift Approximation, Diagonal Approximation and, finally, Scalar Approximation (see the text)

| Observations | “L” Model | “R” Model | No Drift | Diagonal Approx. | Scalar Approx. |
|--------------|------------|------------|----------|------------------|----------------|
| BESS–1999    | 6.8        | 8.1        | 24.3     | 30.5             | 30.2           |
| BESS–2000    | 11.3       | 10.2       | 10.8     | 26.2             | 26.2           |
| BESS–2002    | 13.0       | 15.7       | 12.7     | 33.9             | 33.2           |

with

$$R \equiv R(T) = \frac{P(T)}{P_0},$$

where

$$P(T) = \sqrt{\frac{T + 2E_{\text{ext}}}{e}}$$

(e.g., see Equation (4.94) in Leroy & Rancoita 2011) is the proton rigidity in GV with $E_{\text{ext}} = m_p c^2$, $m_p$ is the rest mass of the proton in GeV/c$^2$, $T$ is the proton kinetic energy in GeV, $e$ is the electron charge, $c$ is the speed of light, $\gamma_{\text{BPH}} = 2.78$ is the spectral index, $J_{0,\text{BPH}} = 1.9 \times 10^4$ (sr m$^2$ s GeV)$^{-1}$ is a normalization constant, and, finally, $P_0 = 1$ GV.

Above (10–20) GeV the differential proton intensities are slightly or marginally affected by modulation. The BPH-LIS (first line of Equation (30)) was compared to experimental spectra available in the literature and collected during solar cycle 23. These observations also account for data in the energy range where modulation is relevant, e.g., AMS–1998 (Aguilar et al. 2002), BESS–1998 (with data only in the range (20–117) GeV) (Sanuki et al. 2000), BESS–2002 (Haino et al. 2004), and PAMELA–2006/08 (Adriani et al. 2011). In Figure 3, the spectral indexes ($\gamma$) of AMS–1998 and PAMELA–2006/08 are those from Aguilar et al. (2002) and Adriani et al. (2011), respectively, while for BESS–1998 and BESS–2002 the spectral indexes were obtained from a fit to the published data of the differential proton intensities. It has to be noted that the rigidity-independent part of the spectral index found by PAMELA–2006/08 is $\gamma_{\text{PAMELA}} = 2.790 \pm 0.008$ (stat) $\pm 0.001$ (syst). Adriani and collaborators (2011) found that the spectral index depends on rigidity as expressed in Equation (19) with a maximum variation of the order of the previously quoted uncertainties in the rigidity range (30–200) GV. Furthermore, the spectral index (2.79 ± 0.08) found by Caprice–1994 (Boezio et al. 1999) is in agreement with those found by the experiments discussed in this section, but the quoted errors are larger.

The normalization constants $J_0$ (Figure 3) (1) depend on the set of experimental observations, e.g., BESS–1997 (Shikaze et al. 2007), BESS–1998 (Shikaze et al. 2007; Sanuki et al. 2000), AMS–1998 (Aguilar et al. 2002), BESS–1999 (Shikaze et al. 2007), BESS–2002 (Haino et al. 2004), and PAMELA–2006/08 (Adriani et al. 2011) and (2) were obtained from a fit using $\gamma_{\text{BPH}}$ as spectral index to the experimental data. For BESS–2000 (Shikaze et al. 2007), the experimental observations did not exceed 21.5 GeV, i.e., an energy region of proton differential intensity which might (marginally) still be affected by modulation in a period of high solar activity; thus, the normalization constant used for these data was the one obtained from BESS–2002 (Haino et al. 2004) data.

The weighted averages of both the spectral index ($\gamma$) and normalization constant ($J_0$) and their errors were determined following the procedure indicated on pages 14–15 of PDB et al. (2010). The error-weighted averages found are

$$\gamma_{wa} = 2.783 \pm 0.009 \quad (31)$$

and

$$J_{0,wa} = (1.76 \pm 0.01) \times 10^4 \text{ (sr m}^2 \text{ s GeV)}^{-1}. \quad (32)$$

$\gamma_{wa}$ is in good agreement with that ($\gamma_{\text{BPH}}$) suggested by Burger et al. (2000; Equation (30)). $J_{0,wa}$ and $\gamma_{wa}$ are represented with the continuous lines in Figure 3; in the same figure the dotted lines refer to the values of the BPH-LIS (Equation (30)). We note that the value of $J_0$ found from a fit to Caprice–1994 data above 20 GeV (Boezio et al. 1999) is $1.44 \pm 0.02$: this value differs by more than 5 standard deviations from $J_{0,wa}$ (Equation (32)).
In Sections 7.2 and 7.2.1, using the current modulation code the observed proton spectra are compared with the modulated differential intensities obtained from an interstellar differential (per unit of kinetic energy) proton intensity \( J_{\text{HelMod}}(T) \) given by

\[
J_{\text{HelMod}}(T) = J_{\text{BHP}}(T) \left( \frac{J_0}{J_{0,\text{HHP}}} \right) \text{(sr m}^2 \text{ s GeV)}^{-1}.
\]

\( J_{\text{HelMod}}(T) \) keeps the same spectral index for \( P(T) \gtrsim 7 \text{ GV} \) as in Equation (30) and depends linearly on \( J_0 \), which accounts for the slight variations in absolute fluxes between the observations.

7.2. Comparison with Observations Obtained During Solar Cycle 23

We used the present code for quantitative comparisons (using Equations (34) and (35)) with experimental data (discussed later in this section) collected during solar cycle 23, in periods with high solar activity, i.e., when the solar magnetic field becomes increasingly complex and less dipolar (Sections 2 and 3). This code allowed us to investigate how the modulated (simulated) differential intensities are affected by the (1) particle drift effect (Sections 2 and 4), (2) polar enhancement of the diffusion tensor along the polar direction \( K_\perp \theta \) (Equation (8)), and, finally, (3) values of the tilt angle \( \alpha_t \) calculated following the approach of the "R" and "L" models (Section 5 and Hoeksema 1995; WSO 2010). The magnetic field is modified with respect to Parker’s magnetic field in the polar region as proposed by Jokipii & Kota (1989; Section 3).

The effects related to particle drift were investigated (a) via the suppression of the drift velocity, i.e., under the assumption that \( K_A = 0 \) (Section 4), thus no drift convection was accounted for, (b) in a pure diffusion approximation with a diagonal diffusion tensor (termed diagonal approximation), where \( K_{rr} = K_\parallel \) and \( K_{\theta\theta} = p_\perp K_\parallel \) (Section 2), and, finally, (c) in a pure diffusion approximation with components both equal to \( K \) (called scalar approximation; as in Equation (9)). The case (a) accounts for the hypothesis that magnetic drift convection is almost completely suppressed during solar maxima. In addition, cases (b) and (c) allow one to assume that the diffusion propagation is independent of magnetic structure.

Each modulated (simulated) differential intensity was obtained using a diffusion tensor (Sections 2, 2.1, 4 and Appendix) whose elements depend on the actual value of the diffusion parameter \( K_\parallel \). Furthermore, the modulated spectra were derived from a LIS (Equations (30) and (33)) whose normalization constant \( (J_0) \) depends on the experimental set of data (see discussion in Section 7.1). In addition, these
differential intensities were calculated (1) for a polar-increased value of $K_{\perp \varnothing}$ (Equation (8)) and also with $K_{\perp \varnothing} = K_{\perp \alpha}$, and (2) accounting for particles inside two heliospheric regions where solar latitudes are lower than $[5^\circ \text{ to } 7^\circ]$ and $[30^\circ]$, respectively. As discussed in Section 2.1, in the present model $K_P$ is assumed to be equal to the value of the rigidity $P$ (Equation (7)) above proton kinetic energies of $\approx 444$ MeV (e.g., Gloeckler & Jokipii 1966; Gleeson & Axford 1968; Perko 1987; Potgieter & Le Roux 1994). However, it should be noted that a systematic investigation of its dependence below that value and the shape of the low-energy part of the LIS (Equations (30) and (33)) was not attempted using the modulated intensities obtained from the HelMod code. In fact, this investigation is likely to be carried out using the experimental data from accurate observations over a long duration, like those from the AMS-02 spectrometer which will allow one to reconstruct the particle trajectory. The reconstructed particle trajectory will also permit an untangling of GCRs coming from outside the magnetosphere at large geomagnetic latitudes ($\Theta_M$) where less energetic particles can enter the magnetosphere. For instance, inside the highest geomagnetic region with $1 < \Theta_M < 1.1$ rad (e.g., see Figure 2(c) in Alcaraz et al. 2000a and Figure 8 in Bobik et al. 2006) the AMS-1998 data indicate that (1) the effective geomagnetic cutoff prevents primary protons (i.e., CR protons) from being fully observed with energies below $\approx (0.5$–$0.6)$ GeV and (2) secondary particles largely contribute to the overall differential intensity. In addition, it should be noted that the BESS observations were usually performed at large geomagnetic latitudes with $\Theta_M$ close to $1.13$ rad.

The last period of high solar activity was during solar cycle 23; the BESS collaboration took data in the years 1999, 2000, and 2002 (see sets of data in Shikaze et al. 2007). These data were compared with those obtained by means of the HelMod code using the error-weighted root mean square ($\eta_{\text{rms}}$) of the relative difference ($\eta$) between experimental data ($f_{\text{exp}}$) and those resulting from simulated differential intensities ($f_{\text{sim}}$). For each set of experimental data and with the approximations and/or models described above, we determined the quantity

$$\eta_{\text{rms}} = \sqrt{\frac{\sum_i (\eta_i/\sigma_{\eta,i})^2}{\sum_i 1/\sigma_{\eta,i}^2}}$$

where $T_i$ is the average energy of the $i$th energy bin of the differential intensity distribution and $\sigma_{\eta,i}$ are the errors including the experimental and Monte Carlo uncertainties; the latter account for the Poisson error of each energy bin. The simulated differential intensities are interpolated with a cubic spline function.

In Tables 2 and 3, the values of the parameter $\eta_{\text{rms}}$ (%) are shown; they were obtained in the energy range $^{12}$ from 444 MeV up to 30 GeV using the “L” and “R” models for the tilt angle $\alpha_t$ (Section 5 and Hoeksema 1995; WSO 2010), for the no drift, diagonal and scalar approximations (discussed previously in this section), with (Table 2) and without (Table 3) the enhancement of the diffusion tensor along the polar direction ($K_{\perp \varnothing}$; Equation (8)). The simulated differential intensities were obtained for a heliospheric region where solar latitudes are lower than $[30^\circ]$. From inspection of Tables 2 and 3, one can note that (1) the no drift approximation is more appropriate than the diagonal and scalar approximations, (2) the “L” model for calculating the value of the tilt angle $\alpha_t$ is slightly to be preferred to the “R” model (although the overall differences between these two models are marginal), (3) the results obtained accounting for drift effects using tilt angles from the “L” model are in better agreement with experimental data with respect to the no drift approximation, and, finally, (4) the minimum difference with the experimental data occurs when $K_{\perp \varnothing} = K_{\perp \alpha}$ is assumed independently of the latitude (Table 3).

$^{12}$ Above 30 GeV the differential intensity is marginally (if at all) affected by modulation.
In addition, the results obtained for a heliospheric region where solar latitudes are lower than $|5.7^\circ|$, exhibit a behavior similar to those lower than $|30^\circ|$, but with values of $\eta_{\text{rms}}$ (%) larger by about several percent. In Figures 4–6 the differential intensities determined with the HelMod code are shown and compared with the experimental data of BESS–1999, BESS–2000, and BESS–2002, respectively; in the same figures, the dashed line is the LIS (Equations (30) and (33)) with normalization constants $J_0$ treated in Section 7.1. These modulated intensities are the ones calculated for a heliospheric region where solar latitudes are lower than $|30^\circ|$, using $K_{\perp} = K_{\perp\parallel}$ independently of the latitude and including particle drift effects with the values of the tilt angle from the “L” model.

Finally, it is concluded that the present code combining diffusion and drift mechanisms is suited to describe the modulation effect in periods with high solar activity (e.g., see Ferreira & Potgieter 2004; Ndiitwani et al. 2005).

### 7.2.1. Periods Not Dominated by High Solar Activity

In periods where the solar activity is no longer at maximum, the solar magnetic field becomes increasingly dipolar (Sections 2 and 3). We used the present code to compare the simulated differential intensities with experimental data obtained during periods not dominated by high solar activity in solar cycle 23, i.e., BESS–1997 (Shikaze et al. 2007), AMS–1998 (Aguilar et al. 2002), BESS–1998 (Shikaze et al. 2007; Sanuki et al. 2000), and PAMELA–2006/08 (Adriani et al. 2011). As discussed in Section 7.2, the simulated spectra were calculated including the effects due to particle drift—expected to be relevant (Sections 2 and 4)—with values of the tilt angle ($\alpha_t$) calculated following the approach of the “R” and “L” models (Section 5 and Hoeksema 1995; WSO 2010), with and without the polar enhancement of the diffusion tensor along the polar direction ($K_{\perp\parallel}$; Equation (8)). Similarly to the treatment for periods with high solar activity (Section 7.2), the effects related to particle drift were also investigated (1) via the suppression of the drift velocity (no drift), (2) with the diagonal approximation, and, finally, (3) with the scalar approximation.

In Tables 4 and 5, the values of the parameter $\eta_{\text{rms}}$ (%) are shown. They were obtained in the energy range from 444 MeV up to 30 GeV using the “L” and “R” models for the tilt angle $\alpha_t$ (Section 5 and Hoeksema 1995; WSO 2010), for the no drift, diagonal, and scalar approximations (discussed in this Section 7.2), with (Table 4) and without (Table 5) the enhancement of the diffusion tensor along the polar direction ($K_{\perp\parallel}$; Equation (8)). The simulated differential intensities were obtained for a heliospheric region where solar latitudes are lower than $|5.7^\circ|$. From inspection of Tables 4 and 5, one can note that (1) the diagonal approximation is more appropriate than the no drift and scalar approximations, (2) the “L” model for tilt angles ($\alpha_t$) is slightly to be preferred to “R” model, and, finally, (3) the minimum difference with the experimental data occurs when the enhancement of the diffusion tensor along the polar direction ($K_{\perp\parallel}$; Equation (8)) is taken into account (Table 4).

In addition, the results obtained for a heliospheric region where solar latitudes are lower than $|30^\circ|$, exhibit a behavior similar to those lower than $|5.7^\circ|$, but with values of $\eta_{\text{rms}}$ (%) larger by about several percent. In Figures 7–10, the differential intensities determined with the HelMod code are shown and compared to the experimental data of BESS–1997, AMS–1998, BESS–1998, and PAMELA–2006/08, respectively; in the same figures, the dashed line is the LIS (Equations (30) and (33)) with normalization constants $J_0$ treated in Section 7.1. These modulated intensities are the ones calculated for a heliospheric region where solar latitudes are lower than $|5.7^\circ|$, using the enhancement of the diffusion tensor along the polar direction ($K_{\perp\parallel}$; Equation (8)) and including particle drift effects with values of the tilt angle from the “L” model.

Finally, it is concluded that the present code combining diffusion and drift mechanisms is also suited to describe the modulation effect in periods when the solar activity is no longer at the maximum.
the tilt angle simulated intensities were obtained (1) using the “L” model for a heliospheric region where solar latitudes are lower than \( f_{\text{th}} \) energy bin (above 30 GeV), \( \sigma_{\hat{\eta},i,h} \) is the error due to Monte Carlo uncertainties for the \( \hat{\eta}_{\text{rms}} \) and 120 AU. The sensitivity of this approach was estimated from the differences of the simulated intensities with radii of 80, 90, 110, and 120 AU with that with a radius of 100 AU for protons with energies above 30 GeV, i.e., for an energy region in which the spectrum is unaffected by modulation and, thus, no difference is expected. For this purpose, we defined the quantity (see also Equations (34) and (35))

\[
\hat{\eta}_{\text{rms},h} = \sqrt{\frac{\sum_i \left( \hat{\eta}_{i,h}/\sigma_{\hat{\eta},i,h} \right)^2}{\sum_i 1/\sigma_{\hat{\eta},i,h}^2}}
\]  

with

\[
\hat{\eta}_{i,h} = \frac{f_h(T_i) - f_{100\text{AU}}(T_i)}{f_{100\text{AU}}(T_i)}
\]

where \( f_h(T_i) \) is the differential intensity of the \( i \)th energy bin (above 30 GeV), \( \sigma_{\hat{\eta},i,h} \) is the error due to Monte Carlo uncertainties for the \( i \)th energy bin, \( f_{100\text{AU}}(T_i) \) is the differential intensity computed with a radius of 100 AU, and, finally “h” indicates 80, 90, 110, and 120 AU. \( \hat{\eta}_{\text{rms}},h \) is equal to about 2.3\% for heliospheres with radii of 80, 90, 110, and 120 AU. Thus, the modulated intensities for heliospheres with radii of 80, 90, 110, and 120 AU are compatible with that with a radius of 100 AU; slightly larger values of \( \hat{\eta}_{\text{rms}} \) were obtained for a spherical heliosphere with a radius of 120 AU.

The radial distance of the heliosphere was varied from 80 up to 120 AU. The corresponding simulated differential intensities were compared to the experimental data from BESS–2002 (data collected during high solar activity (Section 7.2) and PAMELA–2006/08 (data collected when the solar activity was no longer large (Section 7.2.1)), i.e., when the heliosphere is expected to be smaller or larger (and possibly no longer spherical) than 100 AU, respectively.

The values of \( \hat{\eta}_{\text{rms}} \) (Equation (34)) in percent calculated for spherical heliospheres with radii of 80, 90, 110, and 120 AU are shown in Table 6 and compared with those calculated with a radius of 100 AU (see Tables 3 and 4). For BESS–2002, the simulated intensities were obtained (1) using the “L” model for the tilt angle \( \alpha_t \) (Section 5 and Hoeksema 1995; WSO 2010), (2) with \( K_{L,0} = K_{L,0} \) independent of the latitude, and (3) inside a heliospheric region where solar latitudes are lower than \( 30^\circ \).

### Table 4
For BESS–1997, AMS–1998, BESS–1998, and PAMELA–2006/08, \( \hat{\eta}_{\text{rms}} \) (%) obtained from Equation (34) with enhancement of the diffusion tensor along the polar direction using “L” and “R” models for the tilt angle and for no drift approximation, diagonal approximation and, finally, scalar approximation (see Section 7.2).

| Observations | “L” Model | “R” Model | No Drift | Diagonal Approx. | Scalar Approx. |
|--------------|------------|------------|----------|-----------------|----------------|
| BESS–1997    | 9.2        | 17.7       | 10.4     | 9.5             | 17.6           |
| AMS–1998     | 4.6        | 7.9        | 12.9     | 5.4             | 17.3           |
| BESS–1998    | 9.1        | 14.1       | 9.3      | 4.7             | 13.6           |
| PAMELA–2006/08 | 7.1        | 13.4       | 5.9      | 17.5            | 52.5           |

### Table 5
For BESS–1997, AMS–1998, BESS–1998, and PAMELA–2006/08, \( \hat{\eta}_{\text{rms}} \) (%) obtained from Equation (34) without any enhancement of the diffusion tensor along the polar direction using “L” and “R” models for the tilt angle and for no drift approximation, diagonal approximation and, finally, scalar approximation (see Section 7.2).

| Observations | “L” Model | “R” Model | No Drift | Diagonal Approx. | Scalar Approx. |
|--------------|------------|------------|----------|-----------------|----------------|
| BESS–1997    | 13.4       | 20.6       | 14.2     | 11.3            | 12.0           |
| AMS–1998     | 6.1        | 11.3       | 11.4     | 6.0             | 3.7            |
| BESS–1998    | 11.1       | 17.7       | 7.3      | 4.1             | 7.1            |
| PAMELA–2006/08 | 11.0       | 24.7       | 5.4      | 12.3            | 30.6           |

### Table 6
\( \hat{\eta}_{\text{rms}} \) (%) calculated for a spherical heliosphere with radius of 80, 90, 110, and 120 AU and compared with those obtained with a radius of 100 AU for BESS–2002 (see Table 3) and for PAMELA–2006/08 (see Table 4).

| Observations | 80 AU | 90 AU | 100 AU | 110 AU | 120 AU |
|--------------|-------|-------|--------|--------|--------|
| BESS–2002    | 14.5  | 12.0  | 12.2   | 13.0   | 14.5   |
| PAMELA–2006/08 | 5.7   | 6.1   | 7.1    | 7.7    | 10.8   |

7.3. Dependence on the Extension of the Heliosphere

In Sections 7.2 and 7.2.1, the simulated differential intensities were obtained from an LIS (described by Equations (30) and (33)) propagating through a spherical heliosphere with a radius of 100 AU down to Earth. However, the physical dimensions of the heliosphere also depend on the speed of the solar wind. In the HelMod code, the simulated modulated intensities are determined by the properties of the diffusion tensor (Sections 2, 2.1, 4 and Appendix), whose elements are related to the actual value of the diffusion parameter. \( K_0 \) acts as a scaling factor for the overall modulation effect. It was indirectly determined from neutron monitor measurements; thus, it is expected to be sensitive to the overall modulation effect (from the heliosphere boundary down to Earth), but almost independent of the variation in heliosphere dimensions.

The radial distance of the heliosphere was varied from 80 up to 120 AU. The corresponding simulated differential intensities were compared to the experimental data from BESS–2002 (data collected during high solar activity (Section 7.2) and PAMELA–2006/08 (data collected when the solar activity was no longer large (Section 7.2.1)), i.e., when the heliosphere is expected to be smaller or larger (and possibly no longer spherical) than 100 AU, respectively.

The values of \( \hat{\eta}_{\text{rms}} \) (Equation (34)) in percent calculated for spherical heliospheres with radii of 80, 90, 110, and 120 AU are shown in Table 6 and compared with those calculated with a radius of 100 AU (see Tables 3 and 4). For BESS–2002, the simulated intensities were obtained (1) using the “L” model for the tilt angle \( \alpha_t \) (Section 5 and Hoeksema 1995; WSO 2010), (2) with \( K_{L,0} = K_{L,0} \) independent of the latitude, and (3) inside a heliospheric region where solar latitudes are lower than \( 30^\circ \).
related to the variation of the physical dimensions of the heliosphere within the present approximations.

7.4. Dependence on Heliospheric Latitude

Observations made by the Ulysses spacecraft (Simpson et al. 1992) in the inner heliosphere could determine a latitudinal dependence of GCR (mostly protons) intensity with an equatorial southward offset minimum and a north polar excess. This dependence was also discussed in terms of modulation models which included particle drift effects (e.g., see Simpson 1996; Heber et al. 1998). For protons with energies larger than 100 MeV, Simpson et al. (1996) expressed their results in terms of the solar latitude and found that (1) the latitudinal gradient is \( \approx (0.33 \pm 0.02)\% \text{ deg}^{-1} \), (2) the counting rate minimum is nearly constant in a latitudinal region of \( \approx (15^\circ - 5^\circ) \) at \( \approx 1.35 \text{ AU} \) (e.g., see Simpson 1996; Heber et al. 1998), and (3) the rate at the minimum is about \( \approx 80\% \) with respect to that at \( \approx 80^\circ \). Ferreira et al. (2003) have shown that the latitudinal dip—found with the Ulysses fast scan—can be reproduced in a model using the Parker standard field and a polar enhancement of the diffusion tensor.

Using the HelMod code, we could investigate the latitudinal dependence of the differential intensity at 1 AU, above 444 MeV (as so far treated) up to 200 GeV. The heliosphere was subdivided into 20 regions equally spaced with respect to the colatitude parameter \( \mu(\theta) \) (Section 6). The total fluxes obtained in each region were divided by the maximum flux occurring at the north pole; thus, \( R_{\text{lat}} \) represents the normalized flux (to that at the north pole) as a function of the colatitude. In addition, the values of \( R_{\text{lat}} \) were calculated for periods of opposite magnetic polarities and compatible with Ulysses pole-to-pole fast scans, i.e., for the years 1995 with \( A > 0 \) and 2007 with \( A < 0 \). \( R_{\text{lat}} \) can be equivalently expressed as a function of the solar latitude for a comparison with the results obtained by Simpson et al. (1996). \( R_{\text{lat}} \) as a function of the solar latitude is shown in Figure 11 for the years 1995 (left-hand side) and 2007 (right-hand side). \( R_{\text{lat}} \) is also shown as a function of the minimum kinetic energy accounted for protons \( (T_e) \), i.e., (1) \( T_e > 444 \text{ MeV} \), (2) \( T_e > 600 \text{ MeV} \), (3) \( T_e > 1200 \text{ MeV} \), and (4) \( T_e > 2100 \text{ MeV} \).

By inspection of Figure 11, for the year 1995 one can see that \( R_{\text{lat}} \) has (1) a latitudinal gradient of \( (0.23 \pm 0.01)\% \text{ deg}^{-1} \), (2) an equatorial southward offset minimum in the latitudinal region of \( \approx (15^\circ - 5^\circ) \) with \( T_e > 444 \text{ MeV} \), (3) at this minimum, the flux is about \( \approx 80\% \) of that at the north pole. Thus, the simulated fluxes reproduce the features of the experimental data (above 100 MeV) exhibited in Figures 2 and 3 of Simpson et al. (1996; see also Heber et al. 2008) regarding the period of the 1995 Ulysses fast scan. However, with increasing \( T_e \), the latitudinal gradient decreases (Figure 11, left-hand side). In the year 2007, a similar minimum is exhibited for \( T_e > 444 \text{ MeV} \) with a \( \approx 10\% \) north–south pole asymmetry; with increasing \( T_e \), this asymmetry gradually disappears and the flux reduction on the equatorial region is less pronounced down to \( \approx 88\% \) with \( T_e > 2100 \text{ MeV} \) (Figure 11, right-hand side).

It is worthwhile to note that in the HelMod code the magnetic field structure is treated similarly in the north and south hemispheres approximating Parker’s magnetic field with that suggested by Jokipii & Kota (1989; Section 3). However, the current 2D model uses the complete \( 2 \times 2 \) diffusion tensor (see Sections 3, 6 and Appendix) which contains both symmetric and antisymmetric components in the off-diagonal...
terms. The symmetric component of the off-diagonal terms (Equation (A13)) is determined by the divergence-free IMF used, which exhibits a latitudinal component arising from the modification by Jokipii & Kota (1989; Equations (15), (18), and (19)). In the framework of the present 2D model, the north polar excess and equatorial southward offset minimum shown in Figure 11 originate from the non-zero symmetric component of the off-diagonal terms. The actual extension of the dip is related to both the enhancement of the diffusion tensor in the polar regions (Equation (8)) and drift effects.

8. CONCLUSIONS

A systematic investigation of the solar modulation effect on the propagation of cosmic protons through the heliosphere down to the Earth was carried out by comparing experimental observations performed during solar cycle 23 and simulated differential intensities obtained using the HelMod code. The simulated spectra were derived from a LIS (Equations (30) and (33)) whose normalization constant ($J_0$) depends on the experimental set of data (see discussion in Section 7.1). The stochastic 2D Monte Carlo (HelMod) code includes (1) a fully treated diffusion tensor with symmetric and antisymmetric off-diagonal elements, (2) a diffusion parameter which is a function of the intensity of solar activity and varies with solar polarity and phase (Sections 2.1 and 5), and (3) a magnetic field which is modified with respect to Parker’s magnetic field in the polar region as proposed by Jokipii & Kota (1989; Section 3).

For observations performed during high solar activity, the simulated intensities (which have better agreement with the experimental data) were obtained (1) using the “L” model for the tilt angle $\alpha_t$ (Section 5 and Hoeksema 1995; WSO 2010), (2) with $K_{L0} = K_{LR}$ independent of the latitude, and (3) inside a heliospheric region where solar latitudes are lower than $5^\circ$. For observations performed when solar activity is no longer at the maximum, the simulated intensities (which have better agreement with the experimental data) were obtained (1) using the “L” model for the tilt angle $\alpha_t$ (Section 5 and Hoeksema 1995; WSO 2010), (2) with an enhancement of the diffusion tensor along the polar direction ($K_{L0}$; Equation (8)), and (3) inside a heliospheric region where solar latitudes are lower than $5^\circ$.7.

In addition (within 2.3%), the simulated differential intensities for spherical heliospheres with radii of 80, 90 and 110 AU (and also 120 AU for BESS-2002) are compatible with that with a radius of 100 AU; a slightly lower agreement was obtained for a spherical heliosphere with a radius of 120 AU for PAMELA–2006/2008. These results indicate that, within the present approximations, the diffusion parameter almost accounts for effects related to the variation of the physical dimensions of the heliosphere.

The simulated modulated spectrum determined for the year 1995 exhibits a latitudinal gradient of $(0.23 \pm 0.01)\% \text{deg}^{-1}$, an equatorial southward offset minimum in the latitudinal region $\approx - (18^\circ - 5^\circ)$ with $T_e > 444$ MeV, and at this minimum the flux is about $\approx 80\%$ of that at north pole. Thus, the simulated fluxes reproduce the features of the experimental data from the Ulysses fast scan in 1995 (Simpson et al. 1996).

Although the treatment is highly simplified with respect to the complexity of the physical mechanisms responsible for modulation effects, the overall satisfactory agreement found allows us to conclude that the choice of parameters regarding the structure of the IMF, diffusion tensor, diffusion parameter, and tilt angle is almost appropriate to describe the experimental data. Finally, the experimental data from accurate observations over a long duration (like those from the AMS-02 spectrometer) will allow us to undertake a deeper systematic investigation of solar modulation effects over a period longer than a solar cycle. Thus, possibly, further advancements can be put forward in the present approximations on the transport of GCRs through the heliosphere, for instance those at low rigidities, the spatial and rigidity properties of diffusion tensor.

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APPENDIX

DIFFUSION TENSOR AND STOCHASTIC DIFFERENTIAL EQUATIONS

In a reference frame with the third coordinates along the average magnetic field, the matrix of the diffusion tensor used in Equations (1) and (4) is given by (e.g., see Jokipii 1971):

$$K_{ik} = \begin{bmatrix} K_{lr} & -K_A & 0 \\ K_A & K_{L0} & 0 \\ 0 & 0 & K_{||} \end{bmatrix}.$$ 

In heliocentric spherical coordinates ($r, \theta, \phi$), for instance those used in Equations (A25)–(A27), the matrix elements of the $3 \times 3$ tensor are found, for instance, in Equation (17) of Burger et al. (2008). In a 2D approximation, the matrix elements of the resulting tensor are

$$K_{rr} = K_{L0} \sin^2 \xi + \cos^2 \xi (K_{||} \cos^2 \psi + K_{lr} \sin^2 \psi),$$  \hspace{1cm} (A1)

$$K_{\theta \theta} = K_{L0} \cos^2 \xi + \sin^2 \xi (K_{||} \cos^2 \psi + K_{lr} \sin^2 \psi),$$  \hspace{1cm} (A2)

$$K_{r \theta} = -K_A \sin \psi + \sin \xi \cos \xi (K_{||} \cos^2 \psi + K_{lr} \sin^2 \psi - K_{L0}),$$  \hspace{1cm} (A3)
\[ K_{\theta r} = K_A \sin \psi + \sin \xi \cos \xi (K_{||} \cos^2 \psi + K_{\perp r} \sin^2 \psi - K_{\perp \theta}) \]  

(A4)

with \( \tan \psi = -B_\phi/(\sqrt{B_r^2 + B_\phi^2}) \) and \( \tan \xi = B_\theta/B_r \) (see Figure 6 of Burger et al. 2008), where \( \psi \) is the spiral angle (for a standard Parker IMF \( \tan \psi \) reduces to Equation (16) and \( \tan \xi = 0 \)), and with

\[ K_{||} = \beta k_i(r, t) K_P(P, t) \left[ \frac{B_\theta}{3B} \right], \]

(A5)

\[ K_{\perp r} = \rho_k K_{||}, \]

(A6)

\[ K_{\perp \theta} = \iota(\theta) \rho_k K_{||}, \]

(A7)

where \( \rho_k = 0.05 \) and \( \iota(\theta) \) is a step function that is 1 in the equatorial region and 10 in the polar region (Section 2). The diffusion parameter \( k_i(r, t) \) is replaced by \( K_0 \) for an effective heliosphere of 100 AU (Section 2.1). Furthermore, the matrix elements of the later tensor consist of a symmetric \((K_{ik}^S)\) and antisymmetric \((K_{ik}^A)\) part:

\[ K_{ik} = K_{ik}^A + K_{ik}^S \]

(A8)

with the antisymmetric part related to drift velocity (Equation (22)), \( \vec{v}_d \), and treated in Section 4. Finally, using Equations (A5)–(A8), Equations (A1)–(A4) can be rewritten as

\[ K_{rr} = K_{r r}^S = \beta K_0 K_P(P, t) \left[ \frac{B_\theta}{3B} \right] \left[ (\iota(\theta) \rho_k \sin^2 \xi + \cos^2 \xi (\cos^2 \psi + \rho_k \sin^2 \psi)) \right], \]

(A9)

\[ K_{\theta \theta} = K_{\theta \theta}^S = \beta K_0 K_P(P, t) \left[ \frac{B_\theta}{3B} \right] \left[ (\iota(\theta) \rho_k \cos^2 \xi + \sin^2 \xi (\cos^2 \psi + \rho_k \sin^2 \psi)) \right], \]

(A10)

\[ K_{\theta r} = K_{\theta r}^A + K_{\theta r}^S, \]

(A11)

\[ K_{\theta \theta} = K_{\theta \theta}^A \]

(A12)

with

\[ K_{\theta r}^S = \beta K_0 K_P(P, t) \left[ \frac{B_\theta}{3B} \right] \left[ \sin \xi \cos \xi [\cos^2 \psi + \rho_k \sin^2 \psi - (\iota(\theta) \rho_k)]] \right]. \]

(A13)

Equations (1, 4) can be re-expressed in heliocentric spherical coordinates as

\[ \frac{\partial U}{\partial \tau} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 K_{rr}^S \frac{\partial U}{\partial r} + r K_{r \theta}^S \frac{\partial U}{\partial \theta} + \frac{r}{\sin \theta} K_{\theta \theta}^S \frac{\partial U}{\partial \phi} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta K_{\theta r}^S \frac{\partial U}{\partial r} + \sin \theta \frac{\partial U}{r} + K_{\theta \theta}^S \frac{\partial U}{\partial \theta} + \frac{1}{r^2} K_{\theta \phi}^S \frac{\partial U}{\partial \phi} \right) \]

(A15)

where \( U \) is the number density of GCRs (Section 2) and \( T \) is the kinetic energy. In turn, in a 2D (radial distance and colatitude) approximation, Equation (A15) can be re-expressed as

\[ \frac{\partial U}{\partial \tau} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 K_{rr}^S \frac{\partial U}{\partial r} + r K_{r \theta}^S \frac{\partial U}{\partial \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta K_{\theta r}^S \frac{\partial U}{\partial r} + \sin \theta \frac{\partial U}{r} + K_{\theta \theta}^S \frac{\partial U}{\partial \theta} \right) - \frac{1}{r^2} \frac{\partial^2 V_r U}{\partial r^2} - \frac{1}{r \sin \theta} \frac{\partial \sin \theta V_r U}{\partial \theta} - \frac{1}{r \sin \theta} \frac{\partial V_r U}{\partial \phi} \]

(A16)

\[ - \frac{1}{r^2} \frac{\partial^2 v_{d,r} U}{\partial r^2} - \frac{1}{r \sin \theta} \frac{\partial \sin \theta v_{d,r} U}{\partial \theta} - \frac{1}{r \sin \theta} \frac{\partial v_{d,r} U}{\partial \phi} + \frac{1}{3} \left( \frac{1}{r^2} \frac{\partial^2 V_r U}{\partial r^2} + \frac{1}{r \sin \theta} \frac{\partial \sin \theta V_r U}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial V_r U}{\partial \phi} \right) \frac{\partial}{\partial T}(\alpha_{\text{eq}} T U), \]
Let us define the variable \( \mu = \cos(\theta) \); then we obtain
\[
\partial \theta = -(1 - \mu^2)^{-0.5} \partial \mu.
\]  
(17)

In addition, one can introduce the function
\[
F = U r^2.
\]  
(18)

Using Equations (A16)–(A18), for an SW radially propagating (i.e. \( V_{sw,r} = V_{sw} \)) we find
\[
\frac{\partial F}{\partial t} = - \frac{\partial}{\partial r} \left[ \frac{F}{r^2} \frac{\partial}{\partial r} \left( r^2 K_{rr}^S r \right) \right] - \frac{\partial}{\partial \mu} \left[ F \frac{\partial}{\partial \mu} \left( \frac{K_{\theta\theta}^S \sqrt{1 - \mu^2}}{r} \right) \right] - \frac{\partial}{\partial r} \left[ F (V_{sw} + v_{d,r}) \right]
\]
\[
- \frac{\partial}{\partial \mu} \left[ - F \left( \frac{\partial}{\partial \mu} \left( \frac{K_{\theta\theta}^S (1 - \mu^2)}{r^2} \right) \right) \right] - \frac{\partial}{\partial r} \left[ F v_{d,\theta} \frac{\sqrt{1 - \mu^2}}{r} \right] - \frac{\partial}{\partial T} \left[ - F \frac{v_{d,\theta} T \partial V_{sw} r^2}{3r^2} \right]
\]
\[
+ \frac{1}{2} \frac{\partial}{\partial r} \frac{\partial}{\partial \mu} \left[ 2K_{rr}^S F \right] + \frac{1}{2} \frac{\partial}{\partial \mu} \frac{\partial}{\partial r} \left[ - \frac{2K_{rr}^S \sqrt{1 - \mu^2}}{r} \right] + \frac{1}{2} \frac{\partial}{\partial \mu} \frac{\partial}{\partial r} \left[ - \frac{2K_{rr}^S (1 - \mu^2)}{r^2} \right] + \frac{1}{2} \frac{\partial}{\partial \mu} \frac{\partial}{\partial \mu} \left[ \frac{2K_{\theta\theta}^S (1 - \mu^2)}{r^2} F \right] .
\]  
(19)

Furthermore, following the treatment discussed in Sections 4.3–4.3.5 of Gardiner (1985), one can (1) express the Fokker–Planck equation involving \( F \)—which, in turn, is a function of \( q = (r, \mu, T) \)—as
\[
\frac{\partial}{\partial t} F = - \sum_i \frac{\partial}{\partial q_i} [A_i(q, t) F] + \frac{1}{2} \sum_{i,j} \frac{\partial}{\partial q_i} \frac{\partial}{\partial q_j} \left[ (D(q, t))_{ij} F \right]
\]  
(20)

with \( \dot{D} = \dot{L} L^T \) and (2) obtain the equivalent set of differential equations
\[
dq = A(q, t) dt + \dot{L}(q, t) dW(t),
\]  
(21)

where \( A(q, t) dt \) accounts for the so-called advective processes (e.g., Kruehl & Achterberg 1994), \( L(q, t) dW(t) \) is the stochastic term containing \( dW(t) \) which is the increment of the so-called Wiener process (e.g., Section 4.3 of Gardiner 1985). Equations (21) are termed stochastic differential equations (SDEs).

Furthermore, one can note that (1) the first right-hand term of Equation (20) is equal to those included in the first three lines of Equation (19) and (2) the second right-hand term of Equation (20) is equal to those included in the fourth and fifth lines of Equation (19). Thus, using Equations (19) and (20) one derives:
\[
A = \left[ \begin{array}{c} \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 K_{rr}^S \right) - \frac{\partial}{\partial \mu} \left( \frac{K_{\theta\theta}^S \sqrt{1 - \mu^2}}{r} \right) + V_{sw} + v_{d,r} \\
- \frac{1}{r^2} \frac{\partial}{\partial r} \left( r K_{\theta\theta}^S \sqrt{1 - \mu^2} \right) + \frac{2}{r^2} \frac{\partial}{\partial \mu} \left( \frac{K_{\theta\theta}^S (1 - \mu^2)}{r} \right) - \frac{v_{d,\theta} \sqrt{1 - \mu^2}}{r} \end{array} \right].
\]  
(22)

and
\[
\dot{D} = \left[ \begin{array}{cc} 2K_{rr}^S & -\frac{2K_{rr}^S \sqrt{1 - \mu^2}}{r} \\
-\frac{2K_{rr}^S}{r} & \frac{2K_{rr}^S (1 - \mu^2)}{r^2} \end{array} \right].
\]  
(23)

As discussed by Pei et al. (2010b, see Section 2 therein), the matrix \( \dot{D} \) can be downgraded to a \( 2 \times 2 \) matrix, because second-order acceleration mechanisms are not considered in Equations (1) and (4).

As already shown by Pei et al. (2010b, see Appendix B therein), the matrix \( \dot{L} \) is not unique. However, there is only one positive definite, i.e.,
\[
\dot{L} = \left[ \begin{array}{cc} \left( K_{rr}^S - K_{\theta\theta}^S \right) \left( 0.5 K_{\theta\theta}^S \right)^{1/2} & -K_{rr}^S \left( \frac{2}{0.5 K_{\theta\theta}^S} \right)^{1/2} \\
0 & \frac{2K_{rr}^S (1 - \mu^2)}{r^2} \left( \frac{2}{0.5 K_{\theta\theta}^S} \right)^{1/2} \end{array} \right].
\]  
(24)

Finally, for a 2D model (like that treated in Sections 3 and 4), from Equations (21), (22), and (24) one finds the following SDEs:
\[
dr = \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 K_{rr}^S \right) - \frac{\partial}{\partial \mu} \left( \frac{K_{\theta\theta}^S \sqrt{1 - \mu^2}}{r} \right) + V_{sw} + v_{d,r} \right] dt + \sqrt{K_{rr}^S \left( \frac{K_{rr}^S K_{\theta\theta}^S - (K_{\theta\theta}^S)^2}{0.5 K_{\theta\theta}^S} \right)^{1/2}} dW_r - K_{rr}^S \sqrt{2} dW_\mu.
\]  
(25)
\[
d\mu = \left\{ \begin{align*}
&-\frac{1}{r^2} \frac{\partial}{\partial r} \left( r K_r^S \sqrt{1 - \mu^2} \right) + \frac{\partial}{\partial \mu} \left[ K_{r0}^S \left( 1 - \mu^2 \right) \right] - \frac{v_{d, r}}{r} (1 - \mu^2) \right\} dt + \sqrt{\frac{2 K_{r0}^S (1 - \mu^2)}{r^2}} dW_{\mu}, \\
&\Delta T = -\frac{2}{3} \alpha_{rel} V_{sw} T dt
\end{align*} \]

with \( dW_i (i = r, \mu(\theta)) \) the increment of the Wiener process. It should be noted that the above usage of Equation (A24) ensures imaginary terms do not appear in Equations (A25)–(A30). For an IMF described by a standard Parker field (Equation (15)) requiring \( K_{r0}^S = 0 \), the above SDEs reduce to:

\[
dr = \frac{1}{r^2} \left[ \frac{\partial}{\partial r} \left( r^2 K_r^S \right) \right] dt + (V_{sw} + v_{d, r}) dt + \sqrt{2 K_r^S} dW_r,
\]

\[
d\mu = \frac{1}{r^2} \left[ \frac{\partial}{\partial \mu} \left( (1 - \mu^2) K_{r0}^S \right) \right] dt - \frac{v_{d, \theta}}{r} (1 - \mu^2) dt + \sqrt{2 K_{r0}^S (1 - \mu^2)} dW_{\mu},
\]

\[
\Delta \mu = \left\{ \begin{align*}
&\frac{1}{r^2} \frac{\partial}{\partial r} \left( r K_{r0}^S \sqrt{1 - \mu^2} \right) + \frac{\partial}{\partial \mu} \left[ K_{r0}^S (1 - \mu^2) \right] - \frac{v_{d, \theta}}{r} (1 - \mu^2) \right\} \Delta t + \omega_{r} \sqrt{\frac{K_r^S K_{r0}^S - (K_{r0}^S)^2}{0.5 K_{r0}^2} \Delta t} - \omega_{\mu} K_{r0}^S \sqrt{\frac{2 K_{r0}^2}{K_{r0}^S} \Delta t}, \\
\end{align*} \]

\[
\Delta T = -\frac{2}{3} \alpha_{rel} V_{sw} T \Delta t.
\]

Equations (A25)–(A27) can be approximated by

\[
\Delta r = \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 K_r^S \right) \right] - \frac{\partial}{\partial \mu} \left[ K_{r0}^S (1 - \mu^2) \right] + V_{sw} + v_{d, r} + v_{HCS} \right\} \Delta t + \omega_{r} \sqrt{\frac{K_r^S K_{r0}^S - (K_{r0}^S)^2}{0.5 K_{r0}^2} \Delta t} - \omega_{\mu} K_{r0}^S \sqrt{\frac{2 K_{r0}^2}{K_{r0}^S} \Delta t},
\]

\[
\Delta \mu = \left\{ \begin{align*}
&\frac{1}{r^2} \frac{\partial}{\partial r} \left( r K_{r0}^S \sqrt{1 - \mu^2} \right) + \frac{\partial}{\partial \mu} \left[ K_{r0}^S (1 - \mu^2) \right] - \frac{v_{d, \theta}}{r} (1 - \mu^2) \right\} \Delta t + \omega_{r} \sqrt{\frac{2 K_{r0}^S (1 - \mu^2)}{r^2} \Delta t} - \omega_{\mu} \Delta t,
\end{align*} \]

\[
\Delta T = -\frac{2}{3} \alpha_{rel} V_{sw} T \Delta t.
\]

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