Parameter identification of general damping model based on structural dynamic response

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Abstract. A general damping model suitable for linear systems is proposed, which is not limited by the traditional damping model, and the parameters of the damping model are identified by using the inversion algorithm based on the sensitivity of the structure dynamic response. Firstly, the expression of the general damping model is constructed, and the sensitivity formula of the dynamic response of the structure to the parameters of the damping model is deduced. Then, the identification equation of structural damping parameter inversion is constructed, and the regularization method is used to solve the equation in an iterative way. Finally, the applicability and identifiability of the general damping mathematical model are verified by taking a 12-story shearing frame as an example. The numerical simulation results show that the general damping mathematical model has higher accuracy than the classical damping model in damping parameter identification and dynamic calculation.

1. Introduction
Damping is an important parameter to characterize vibration attenuation and energy dissipation in vibration system, and its accurate description and characterization have a very important influence on the dynamic analysis of vibration system [1]. However, the energy losses of vibration system are affected by various factors. It is difficult to estimate the damping of the system according to the size and material of the system as well as the mass and stiffness. As a result, the construction of structural damping model and the identification of damping parameters for structural dynamics analysis in forward problem and inverse problem have always been research hotspots in field of vibration control, fault diagnosis and other engineering [2, 3]. Damping models can be divided into classical damping models and non-classical damping models. Existing studies have shown that the classical linear damping models can reasonably simulate the damping characteristics of the linear system, but the matrix of the linear damping system established according to different classical damping models is different [4, 5]. This shows that the classical damping systems still has their limitations. At present, the damping identification methods are mainly divided into three categories: the frequency domain identification methods represented by classical half-power bandwidth method, power spectrum method, transfer function method and energy ratio method; the time domain identification methods represented by free attenuation method, time series method, ITD and its improved method, dynamic response sensitivity method, and the time-frequency analysis methods represented by short-time Fourier transform, wavelet transform and Hilbert-yellow transform. Among them, the method based on dynamic response sensitivity has the characteristics of clear physical meaning and simple calculation, so it is a potential method for the damping identification of linear systems [6].

Based on the classical damping model, this paper firstly proposes a general damping model which is not limited by the traditional damping model and suitable for linear system. And then, the sensitivity
calculation formula of structural dynamic response to the parameters of the damping model is derived, the identification equation of structural damping parameter inversion is constructed, and the inversion algorithm based on structural dynamic response sensitivity is proposed. Finally, a numerical example is used to verify the identifiability and advantages of the general damping model.

2. Fundamental theory

2.1. Damping model of general linear system
At present, the damping models used in engineering have a certain approximation, such as the classical Rayleigh damping and Caughey damping, which abandon some damping factors. For example, the Rayleigh damping model only considers the first two modes of the Caughey damping model; the Caughey damping model considering the influence of all modal damping ratios only considers the coupling between the main diagonal elements (the interaction between the main modes), abandoning the coupling between the non main diagonal elements and the coupling between the main diagonal elements and the non main diagonal elements. These discards will inevitably affect the accuracy of structural damping. Based on the classical linear model, this paper proposes a general damping model of linear system as

$$\mathbf{C} = \begin{bmatrix}
    a_{11} & \cdots & a_{1n} \\
    \vdots & \ddots & \vdots \\
    a_{n1} & \cdots & a_{nn}
\end{bmatrix}$$  \hspace{1cm} (1)

where $a_{ij}$ is the damping parameter of the general damping model.

2.2. Sensitivity calculation of dynamic response to damping parameters
The finite element equation of motion of any damped multi-degree of freedom system can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F} \hspace{1cm} (2)$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the system; $\mathbf{x}$, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are the node displacement, velocity and acceleration vectors of the system respectively; $\mathbf{F}$ is the external incentive of the system.

The partial derivatives of the system motion equation (2) to the element $a_{ij}$ in the damping mapping matrix are obtained by

$$\mathbf{M}\left(\frac{\partial\ddot{\mathbf{x}}}{\partial a_{ij}}\right) + \mathbf{C}\left(\frac{\partial\dot{\mathbf{x}}}{\partial a_{ij}}\right) + \mathbf{K}\left(\frac{\partial\mathbf{x}}{\partial a_{ij}}\right) = -\left(\frac{\partial\mathbf{C}}{\partial a_{ij}}\right)\ddot{\mathbf{x}} \hspace{1cm} (3)$$

where $\frac{\partial\ddot{\mathbf{x}}}{\partial a_{ij}}$, $\frac{\partial\dot{\mathbf{x}}}{\partial a_{ij}}$ and $\frac{\partial\mathbf{x}}{\partial a_{ij}}$ are the sensitivity matrix of acceleration, velocity and displacement to unknown damping parameters $a_{ij}$; $\frac{\partial\mathbf{C}}{\partial a_{ij}}$ is the sensitivity matrix of general damping matrix to unknown damping parameters $a_{ij}$.

Because the velocity of the system $\dot{\mathbf{x}}$ can be obtained from equation (2), the right end of equation (3) can be regarded as equivalent force. Furthermore, the response sensitivity of the system can be obtained from equation (3) by using the Newmark algorithm.
2.3. Inverse identification algorithm for structural physical parameters

2.3.1 Definition and solution of parameter identification equation

It is an optimization process to identify structural physical parameters based on structural dynamic response sensitivity. In this process, the structural parameters to be identified are taken as variables, and the objective is to minimize the errors between the measured dynamic response and the calculated dynamic response. The identification equation of structural damping parameters can be defined as

\[ \delta \beta = S \cdot \delta \beta \]  

(4)

where \( \delta \beta \) is the difference between the measured acceleration response of the structure and the response of the measured degree of freedom calculated based on the finite element model; \( S \) is the sensitivity matrix of the acceleration response of the measuring point to the damping parameters of the structure; \( \delta \beta \) is the perturbation vectors of the parameters of the damping mapping matrix are unknown.

2.3.2 Iterative solution steps of inversion identification

Step 1: Select the initial value of damping parameter \( a_0 \) arbitrarily.

Step 2: Take the initial value \( k_0 \) as the damping parameter in equation (1), solve the dynamic response of the system including displacement, velocity and acceleration from equation (2), and then calculate the difference between the test acceleration and the calculated acceleration \( \delta \beta \).

Step 3: Using equation (3), the sensitivity matrix \( S \) of dynamic response to structural damping parameters is solved;

Step 4: Calculate the variation of damping parameters \( \delta \beta_k \) from equation (4) and calculate \( \beta_k = \beta_{k-1} + \delta \beta_k \);

Step 5: Adjust the identified damping parameter according to the constraint conditions in equation (4), assign the value not in the constraint interval back to the original value, and take the adjusted value as the initial value of the \( i + 1 \) iteration;

Step 6: The obtained parameters \( \beta_k \) are the identification values of damping parameters, until \( \left| \Delta \beta^i / \beta^i \right| < T_{id} \) is satisfied.

3. Numerical examples

3.1 Model description

A twelve story reinforced concrete frame structure is taken as an example here. The structure is simplified as an inter-laminar shear model. The mass of each layer of the structure is \( m_1 = 7097.4 \) kg, \( m_2 = 5723.7 \) kg, \( m_3 \sim m_{11} = 5387.0 \) kg and \( m_{12} = 4344.8 \) kg. The inter-layer stiffness of each layer is \( k_1 = 3.409 \times 10^5 \) kN/m, \( k_2 = 3.32 \times 10^5 \) kN/m, \( k_3 \sim k_5 = 3.989 \times 10^5 \) kN/m, \( k_6 \sim k_{11} = 3.662 \times 10^5 \) kN/m and \( k_{12} = 3.375 \times 10^5 \) kN/m. The random excitation with amplitude of \( 1.0 \times 10^4 \) kN is used to excite the top floor of the structure. The acceleration response of the structure is calculated by using the Newmark time history analysis method. The sampling frequency of the acceleration time history is 250Hz and the sampling time is 2 seconds. The calculated acceleration of each layer of the structure is used as the test response for inversion identification.

In order to verify the advantages of the general damping model proposed in this paper, the damping model of the structure is assumed to be the general damping model and the Caughey damping model respectively, and the identification acceleration response of the two models is compared. When taking the general damping model, the main diagonal elements of the damping matrix are assumed to be 0.022, the non main diagonal near the main diagonal is assumed to be a group of random numbers between 0.08 times of 0.022 and 0.12 times of 0.022, and the position far away from the main
diagonal is assumed to be 0. When the Caughey damping model is adopted, the modal damping ratio of each order is 0.022.

3.2 Algorithm validation

Figure 1 and Figure 2 compare the real value and identification value of general damping model and Caughey damping model respectively. It can be seen from Figure 1 that the lower order modal damping ratio in Caughey damping model can be identified relatively accurately, but the identification error of higher order modal damping parameters is large. The identification results in Figure 2 show that all damping parameters of general damping model can be identified accurately without considering noise, which shows that the identification accuracy of general damping model is much higher than that of Caughey damping model. However, because the identification parameters of the general damping model are more than those of the Caughey damping model, the computational efficiency of the general damping model is lower.

Figure 3 shows the comparison between the acceleration responses of the first layer of the structure calculated based on the identified damping parameters under the general damping model and Caughey model. It can be seen from Figure 3 that, without considering the noise, the identified acceleration response corresponding to the general damping model is completely coincident with the real response of the structure, while the identified acceleration response corresponding to the Caughey damping model can also reflect the main trend of the real dynamic response of the structure, but there is a certain difference with the real value, which further indicates that the identified acceleration response caused by the structural damping model is different. These results show that the dynamic response of the system is greatly affected by the system error, and this also has a potential negative impact on the accuracy of structural damage identification and vibration control based on the dynamic response.

Figure 1. Comparison between the identified and real values of Caughey damping parameters

Figure 2. Comparison of identified and real values of parameters of general damping model
Figure 3. Comparison between the accelerations of the first floor under the the identified and the real damping parameters with different damping models

4. Conclusion
This paper presents a general damping model which is not limited by the traditional damping model and suitable for linear system. Taking a twelve story shear structure as an example, the parameters of the damping model are identified by using the inversion algorithm based on the sensitivity of structural dynamic response. The results show that the general damping mathematical model is less efficient than the classical damping model, but it has higher accuracy in damping parameter identification and dynamic calculation.

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