Precise calibration of a rotation photogrammetric system

Yongjun ZHANG*, Kun HU, Zuxun ZHANG, Tao KE and Shan HUANG
School of Remote Sensing and Information Engineering, Wuhan University, 129 Luoyu Road, Wuhan, 430079, China
(Received 7 January 2012; final version received 17 November 2012)

Rotation photogrammetric systems are widely used for 3D information acquisition, where high-precision calibration is one of the critical steps. This study first shows how to derive the rotation model and deviation model in the object space coordinate system according to the basic structure of the system and the geometric relationship of the related coordinate systems. Then, overall adjustment of multi-images from a surveying station is employed to calibrate the rotation matrix and the deviation matrix of the system. The exterior orientation parameters of images captured by other surveying stations can be automatically calculated for 3D reconstruction. Finally, real measured data from Wumen wall of the Forbidden City is employed to verify the performance of the proposed calibration method. Experimental results show that this method is accurate and reliable and that a millimetre level precision can be obtained in practice.

Keywords: rotation photogrammetric system; calibration; deviation matrix; overall adjustment

1. Introduction

The calibration of camera system parameters is a necessary step for 3D information extraction from 2D images (1). As one of the commonly used 3D vision systems in the fields of close range photogrammetry and computer vision, a rotation photogrammetric system is composed of a digital camera fixed on a horizontal and vertical rotation platform. The vertical and horizontal rotation angles of the camera are automatically recorded by the system. By high-precision calibration, the relationship between the angles and the exterior orientation parameters of images can be established. Therefore, the calibration method of the camera system needs to be investigated.

Camera calibration in the context of 3D machine vision is the process of determining the internal camera geometric and optical characteristics (intrinsic parameters) and/or the 3D position and orientation of the camera frame relative to a certain world coordinate system (extrinsic parameters) (2). The commonly used calibration methods are divided into traditional calibration and self-calibration methods. The traditional approaches take advantage of a calibration pattern with precisely known structure. The parameters of the camera model are determined by conjugate points in the image space and the object space (3–5). The self-calibration method utilizes correspondences between small numbers of points in two or more views of a moving camera (6). Research has been done, based on the assumption that rotational motions of the camera is only considered and the rotation centre is its optical centre (7, 8). Since the location of the optical centre is difficult to be exactly known, the translational offset was considered in self-calibration (9). However, this method requires strict constraint of the camera movements.

The main task of the calibration method proposed in this study is to determine the fixed rotation matrix and deviation matrix in the object space coordinates system. It is similar to hand-eye calibration of a robotic camera (10–12), the orientation and position of the object in relation to the camera needs to be translated to those in relation to the platform, and homogeneous matrix equation of the form \( AX_1 = XB \) needs to be solved.

The basic structure of the system is illustrated and the rotation and deviation models of the system are presented in Section 1. Section 2 derives the methods to solve the rotation matrix and deviation matrix by overall adjustment, and Section 3 calculates the exterior orientation parameters of images for 3D reconstruction. Experiments are carried out in Section 4 to measure the performance of the proposed method. Finally, conclusions and future work are discussed.

2. Basic structure and mathematical models

2.1. Basic structure of the system

The rotation photogrammetric system is mainly composed of a digital camera and a horizontal and vertical rotation platform. As shown in Figure 1, the camera is fixed on the upper part of the platform, with the axis of lens perpendicular to the cylinder \( A_U \). In the rotation coordinate system \( PC = XYZ \), \( PC \) is the central point of intersection of the horizontal rotation cylinder \( A_H \) and...
the vertically rotation cylinder $A_V$, the $X$-axis and $Y$-axis lie on the central lines of $A_H$ and $A_U$, respectively, and the $Z$-axis is parallel to the axis of lens. $s$, $d$ and $h$ are the deviations of the principal point of image $O$ with respect to the $X$-axis, the $Y$-axis and the $Z$-axis, respectively. $f$ is the focal length. 

When the platform is rotating around the rotation cylinders $A_H$ and $A_V$, the exterior orientation parameters of the image captured by the camera are changing. The geometric relationship of the related coordinate systems is shown in Figure 2. $P - X_0Y_0Z_0$ is the standard position coordinate system where the camera is directly facing the target, with the $Y_0$-axis pointing to the zenith, the $Z_0$-axis perpendicular to the target and the $X_0$-axis perpendicular to the former two. If the camera is being rotated around $A_H$ and $A_V$ with the angles $\theta_V$ then, $\theta_H$, $P_C - X_0Y_0Z_0$ will be coincident with $P_C - XYZ$. $S - X_GY_GZ_G$ is the object space coordinate system.

### 2.2. Rotation model of the system

According to the basic structure of the rotation photogrammetric system and the geometric relationship of the related coordinate systems, the rotation model of the system in relation to the object space coordinate system can be written as:

$$R = R_{SC} R_{HV} R_I$$

(1)

Where $R$ is the rotation matrix of the exterior orientation parameters in the object space coordinate system. The standard approach of constructing $R$ is to use three angular elements $\phi$, $\omega$ and $\kappa$ (13), then $R$ can be expressed as follows:

$$R = R_\phi R_\omega R_\kappa = \begin{bmatrix} \cos \phi & 0 & - \sin \phi \\ 0 & 1 & 0 \\ \sin \phi & 0 & \cos \phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \omega & - \sin \omega \\ 0 & \sin \omega & \cos \omega \end{bmatrix} \begin{bmatrix} \cos \kappa & - \sin \kappa & 0 \\ \sin \kappa & \cos \kappa & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2)

Where $R_{SC}$ is the rotation matrix between the object space coordinate system and the standard position coordinate system. With regard to the images captured by a fixed survey station, the value of $R_{SC}$ is invariant. The approach of constructing $R_{SC}$ is similar to that of $R$, which can be expressed by three sequential rotations $\phi_{SC}$, $\omega_{SC}$ and $\kappa_{SC}$ as follows:

$$R_{SC} = R_{\phi_{SC}} R_{\omega_{SC}} R_{\kappa_{SC}} = \begin{bmatrix} \cos \phi_{SC} & 0 & - \sin \phi_{SC} \\ 0 & 1 & 0 \\ \sin \phi_{SC} & 0 & \cos \phi_{SC} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \omega_{SC} & - \sin \omega_{SC} \\ 0 & \sin \omega_{SC} & \cos \omega_{SC} \end{bmatrix} \begin{bmatrix} \cos \kappa_{SC} & - \sin \kappa_{SC} & 0 \\ \sin \kappa_{SC} & \cos \kappa_{SC} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3)

Where $R_{HV}$ is the rotation matrix between the standard position coordinate system and the rotation coordinate system. $R_{HV}$ is constructed by $\theta_H$ and $\theta_V$, and its value varies according to the imaging attitude among different images. The basic form of $R_{HV}$ is:
Where $R_I$ is the rotation matrix between the rotation coordinate system and the image space coordinate system. With regard to all the images captured by different survey stations, the value of $R_I$ is invariant, so $R_I$ is the intrinsic rotation parameter of the system which needs to be calibrated. The approach of constructing $R_I$ is also similar to that of $R$, which can be expressed by three sequential rotations $\phi_I$, $\omega_I$ and $\kappa_I$ as follows:

$$R_I = R_{\phi_I}R_{\omega_I}R_{\kappa_I} = \begin{bmatrix}
\cos \phi_I & -\sin \phi_I & 0 \\
\sin \phi_I & \cos \phi_I & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \omega_I & -\sin \omega_I \\
0 & \sin \omega_I & \cos \omega_I
\end{bmatrix} \begin{bmatrix}
\cos \kappa_I & -\sin \kappa_I & 0 \\
\sin \kappa_I & \cos \kappa_I & 0 \\
0 & 0 & 1
\end{bmatrix} \tag{5}
$$

### 2.3. Deviation model of the system

The deviation model of the rotation photogrammetric system in relation to the object space coordinate system is:

$$P = R_{SC}R_{HI}T_I + P_C \tag{6}$$

Where $P = (X_I, Y_I, Z_I)^T$ is the coordinate matrix of linear elements $X_I$, $Y_I$ and $Z_I$ of the exterior orientation parameters, $T_I = (s, h, d)^T$ is the coordinate matrix of the original point $O$ of the image space coordinate system in the rotation coordinate system. With regard to all the images captured by different survey stations, the value of $T_I$ is invariant, so $T_I$ is the intrinsic deviation parameter of the system which needs to be calibrated. $P_C = (X_C, Y_C, Z_C)^T$ is the coordinate matrix of $P_C$ in the object space coordinate system. With regard to images captured by a fixed survey station, the value of $P_C$ is invariant. Hence, $R_I$ and $T_I$ are the parameters of the rotation photogrammetric system that need to be calibrated.

### 3. Calibration method

#### 3.1. Calibration of the rotation matrix

If $N(N \geq 3)$ images are captured by the first survey station, the exterior orientation parameters of the images are recorded as $(\phi_{Ij}, \omega_{Ij}, \kappa_{Ij}, X_{S_{Ij}}, Y_{S_{Ij}}, Z_{S_{Ij}})$, the vertical rotation angle and horizontal rotation angle are recorded as $\theta_{HIj}$ and $\theta_{1j}$, respectively; where, the subscript 1 and $j$ ($j = 1, 2, \ldots, N$) identifies the station number and the image number.

According to Equation (2), the rotation matrix of the exterior orientation parameters $R_{Ij}$ are constructed by the angular elements $\phi_{Ij}, \omega_{Ij}, \kappa_{Ij}$ ($j = 1, 2, \ldots, N$). The coordinate matrix of the exterior orientation parameters $P_{Ij}$ are constructed by the linear elements $X_{S_{Ij}}, Y_{S_{Ij}}, Z_{S_{Ij}}$ ($j = 1, 2, \ldots, N$).

The initial value of $T_I$ and $R_{SC1}$ (i.e. the rotation matrix between the object space coordinate system and the standard position coordinate system of the first survey station) can be calculated by the rotation models (refer to Equation (1)) of the first three images $R_{Ij} = R_{SC1}R_{HI1}R_{I1}$ ($j = 1, 2, 3$). If $R_I$ is eliminated from the three equations, for example, these equations can be written as:

$$
\begin{align*}
(R_{I1}^{-1}R_{SC1})R_{I1} &= R_{SC1}(R_{HI1}R_{I1}^{-1}) \\
(R_{I2}^{-1}R_{SC1})R_{I1} &= R_{SC1}(R_{HI2}R_{I1}^{-1}) \\
(R_{I3}^{-1}R_{SC1})R_{I1} &= R_{SC1}(R_{HI3}R_{I1}^{-1})
\end{align*} \tag{7}
$$

Setting the parameter at the lower right corner of the matrix $R_{SC1}$ to be 1, the other eights to be unknowns, Equation (7) is expanded into eighteen equations about the eight unknowns. Since the initial value of $R_{SC1}$ need not be calculated precisely, the correlations of the nine parameters of $R_{SC1}$ are not considered in this step. The normal equations are constructed and calculated on the basis of least squares adjustment (14).

$$X = (A^TA)^{-1}A^TL \tag{8}$$

Where $X$ is the unknown matrix, $A$ is the coefficient matrix and $L$ is the constant matrix.

The initial value of $R_{SC1}$ is constructed and normalized by the unknowns and the initial value of $R_I$ can be calculated based on the rotation model (refer to Equation (1)). $R_{SC1}$ and $R_I$ can be decomposed to be the rotation angles of $\phi_{SC1}$, $\omega_{SC1}$, $\kappa_{SC1}$ and $\phi_I$, $\omega_I$, $\kappa_I$, respectively (13).

Then, the rotation models (refer to Equation (1)) of all the images captured by the first survey station are converted into error equations and linearized using the Taylor series expansion as follows:

$$V = \frac{\partial F}{\partial \phi_{SC1}}d\phi_{SC1} + \frac{\partial F}{\partial \omega_{SC1}}d\omega_{SC1} + \frac{\partial F}{\partial \kappa_{SC1}}d\kappa_{SC1}
+ \frac{\partial F}{\partial \phi_{Ij}}d\phi_{Ij} + \frac{\partial F}{\partial \omega_{Ij}}d\omega_{Ij} + \frac{\partial F}{\partial \kappa_{Ij}}d\kappa_{Ij} + V_0 \tag{9}$$

Where $V$ is the correction terms of the observations, $d\phi_{SC1}$, $d\omega_{SC1}$, $d\kappa_{SC1}$, $d\phi_{Ij}$, $d\omega_{Ij}$ and $d\kappa_{Ij}$ are the correction terms of $\phi_{SC1}$, $\omega_{SC1}$, $\kappa_{SC1}$, $\phi_{Ij}$, $\omega_{Ij}$ and $\kappa_{Ij}$, and $V_0$ are the first-order partial derivatives of $F = R_I - R_{SC1}R_{HI1}R_I$ ($j = 1, 2, \ldots, N$).
in accordance with the unknowns, \( V_0 \) is the constant matrix calculated by the approximate values of the unknowns.

The matrices in Equation (9) are in three columns and rows. Therefore, the corresponding items of all the matrices are extracted to compose nine error equations for convenience of calculation. Taking the initial values of \( \phi_{SC1}, \omega_{SC1}, \kappa_{SC1}, \phi_1, \omega_1 \) and \( \kappa_1 \) as iterated initial values of unknowns, the normal equations are constructed and calculated by overall adjustment as Equation (8). The precise value of \( R_{SC1} \) and \( T_I \) are constructed by the final iterated results.

### 3.2. Calibration of the deviation matrix

To calculate the deviation matrix \( P_{C1} \), the deviation models (refer to Equation (6)) of images captured by the first survey station can be rewritten as follows:

\[
P_{ij} = \left( R_{SC1} R_{HV1} P_{ij} \right) E \left( T_I P_{C1} \right)_{j=1,2,...,N} \quad (10)
\]

Where \( E \) is the identity matrix, \( P_{C1} \) is the coordinate matrix of \( P_C \) in the object space coordinate system of the first survey station.

Taking \( X = (T_I, P_{C1})^T \) as unknowns, the normal equations are constructed and calculated by overall adjustment as Equation (8). Then, the precise value of \( T_I \) is acquired. The flow chart of calibrating the rotation matrix and deviation matrix is shown in Figure 3.

### 4. Calculation of exterior orientation parameters

Once the rotation matrix and deviation matrix are calibrated by the data of images of the first survey station, the exterior orientation parameters of the images of other survey stations can be automatically calculated taking advantages of the vertical and horizontal rotation angles. The angles are recorded as \( \theta_{Hij} \) and \( \theta_{Vij} \), respectively; where, the subscript \( i \) and \( j (i,j=1,2,...,N) \) identifies the station number and the image number. The exterior orientation parameters of the first image of each other station is recorded as \( (\phi_{11}, \omega_{11}, \kappa_{11}, X_{S11}, Y_{S11}, Z_{S11}) \) \((i=1,2,...,N)\). The rotation matrix of the exterior orientation parameters \( R_{ij} \) and the coordinate matrix of the exterior orientation parameters \( P_{ij} \) are constructed according to Equation (2). The rotation matrix between the standard position coordinate system and the rotation coordinate system \( R_{HVij} \) is constructed according to Equation (4).

The rotation matrix between the object space coordinate system and the standard position coordinate system of the \( i \) survey station \( R_{SC1} (i=1,2,...,N) \) is calculated as follows:

\[
R_{SC1} = R_{ij} R_{ij}^{-1} R_{HV1}^{-1}
\]  

(11)

The coordinate matrix of \( P_C \) in the object space coordinate system of the \( i \) survey station \( P_{C1} (i=1,2,...,N) \) is calculated as follows:

\[
P_{C1} = P_{ij} - R_{SC1} R_{HV1} T_I \quad (12)
\]

The rotation matrix \( R_{ij} (i=1,2,...,N) \) and the coordinate matrix \( P_{ij} (i=1,2,...,N) \) of the exterior orientation parameters of other images of the \( i \) survey station are calculated according to Equations (1) and (6). They are decomposed to be the six exterior orientation parameters, finally. The flow chart of calculating the exterior orientation parameters is shown in Figure 4.

### 5. Experiments and results

To verify the performance of the calibration method proposed in this study, survey data from Wumen wall of the Forbidden City were used for experiments. A Canon digital camera with a focal length of 50 mm is mounted on the platform of the rotation photogrammetric system. Eight survey stations were set up along the wall at a distance of about 25 m. Four to eight images were captured by the system at each survey station from different directions. The horizontal and vertical rotation angles of each image were automatically recorded. The

---

Figure 3. Flow chart of calibrating the rotation matrix and deviation matrix.
physical pixel size of the camera is 0.0064 mm; thus, the
ground sample distance is about 3.2 mm. The exterior
orientation parameters of the images are precisely calcu-
lated by block adjustment. The unit weight root mean
square (RMS) error of bundle adjustment is 0.0018 mm
(equal to 0.28 pixels).

Four images of the first survey station were used for
calibration. The horizontal and vertical rotation angles
and the exterior orientation parameters of the first station
are taken as known during calibration. The solutions to
\( \phi, \omega, \kappa \) were 0.056692, -0.027153 and
0.000622 rad, respectively. The RMS error was
0.000123 rad and their theoretical accuracy was about
0.000349 rad. The solutions to \( s, h \) and \( d \) were
0.002695, 0.070068, and -0.050236 m, respectively. The
RMS error is 0.002848 m and the theoretical accuracy
was about 0.003696 m.

After precise calibration, the exterior orientation
parameters of the first images of a certain survey stations
are taken as known to predict the exterior orientation
parameters of the rest images. Based on the calibration
method proposed in this study, the actual accuracies of
the exterior orientation parameters of other images of
each station were calculated by the calibrated parameters
and are shown in Table 1.

The actual accuracy of the angular elements \( \phi, \omega \)
and \( \kappa \) reaches to a few ten thousandth of a radian, with
the maximum and minimum errors to be 0.0011 and
0.000054 rad, 0.000936 and 0.000072 rad, 0.000258 and
0.000035 rad, respectively. The actual accuracy of the
linear elements \( X_s, Y_s \) and \( Z_s \) are at a millimetre level,
with the maximum and minimum errors to be 0.008292
and 0.000075 m, 0.008982 and 0.000082 m, 0.005932 and
0.000203 m, respectively. The exterior orientation
parameters of other images are calculated precisely based
on the calibration method proposed in this study. Hence,
this method is meaningful for high-precision 3D
reconstruction in practice.

6. Conclusions and future work

According to the detailed analysis of the structure of the
rotation photogrammetric system, this study promotes a
novel calibration method utilizing the exterior orientation
parameters, horizontal and vertical rotation angles of
images captured by one survey station. High precision
orientation and positioning of the system are realized by
overall adjustment with multi-images. Supposing that the
exterior orientation parameters of the first image of a cer-
tain survey station are known, the exterior orientation
parameters of the other images can be automatically cal-
culated making use of the calibration result. The exterior orientation parameters of the other images can be automatically calculated making use of the calibration result.

The real data experiments of the Wumen wall of the
Forbidden City verify the excellent performance of the
proposed method. The study is meaningful for high-precision calibration and 3D reconstruction by the rotation pho-
togrammetric system. Furthermore, the relative orientation
and position between the survey stations can be measured
during the layout of the survey network. The exterior or-
ientation parameters of the first image of other survey sta-
tions are no longer needed, which makes the 3D
reconstruction work more practical and convenient. Thus,
the modified calibration method and error propagation regulation need to be further investigated in the future.

Acknowledgements

This manuscript was supported by the National Basic Research Programme of China with project number 2012CB719904, the National Natural Science Foundation of China with Project No. 4111292 and 41071233 and the National Key Technology Research and Development Programme with Project No. 2011BAH12B05. We are very grateful also, for the comments and contributions of anonymous reviewers and members of the editorial team.

Notes on contributors

Yongjun Zhang is a professor and major in digital photogrammetry and remote sensing.

Kun Hu is a post-graduate student and major in photogrammetry and remote sensing.

Zuxun Zhang is a professor and major in digital photogrammetry and remote sensing.

Tao Ke is an associate professor and major in digital photogrammetry and remote sensing.

Shan Huang is a post-graduate student and major in digital photogrammetry.

References

(1) Zhang, Z.Y. A Flexible New Technique for Camera Calibration. *IEEE Trans. Pattern Anal. Mach. Intell.* **2000**, *22* (11), 1330–1334.

(2) Tsai, R.Y. A Versatile Camera Calibration Technique for High-Accuracy 3D Machine Vision Metrology Using off-the-Shelf TV Cameras and Lenses. *IEEE Trans. Robot. Autom.* **1987**, *RA-3* (4), 323–344.

(3) Niem, W. Automatic Reconstruction of 3D Objects Using a Mobile Camera. *Image Vis. Comput.* **1999**, *17* (2), 125–134.

(4) Zhang, Y.J.; Zhang, Z.X.; Zhang, J.Q. Camera Calibration Using 2D-DLT and Bundle Adjustment with Planar Scenes. *Geomatics Inform. Sci. Wuhan Univ.* **2002**, *2* (6), 566–571.

(5) Zhang, Y.J.; Zhang, Z.X.; Zhang, J.Q. Camera calibration technique with planar scenes. Presented at proceedings of the SPIE, Machine Vision Applications in Industrial Inspection XI, SPIE 5011: 291–296, Santa Clara, CA, January 20, 2003.

(6) Maybank, S.J. A theory of self-calibration of a moving camera. *Int. J. Comput. Vision* **1992**, *8* (2), 123–151.

(7) de Agapito, L.; Hartley, R.I.; Hayman, E. Linear calibration of a rotating and zooming camera. Presented at IEEE Computer Society Conference on Computer Vision and Pattern Recognition, Fort Collins, CO, June 23, 1999.

(8) Hartley, R.I. Self-Calibration from Multiple Views with a Rotating Camera. *Int. J. Comput. Vision* **2001**, *45* (2), 107–127.

(9) Ji, Q.; Dai, S.T. Self-Calibration of a Rotation Camera with a Translational Offset. *IEEE Trans. Robot. Autom.* **2004**, *20* (1), 1–14.

(10) Horaud, R.; Dornaika, F. Hand Eye Calibration. *Int. J. Robot. Res.* **1995**, *14* (3), 195–210.

(11) Allen, P.K.; Timcenko, A.; Yoshimi, B.; Michelman, P. Automated Tracking and Grasping of a Moving Object with a Robotic Hand-Eye System. *IEEE Trans. Robot. Autom.* **1993**, *9* (2), 152–165.

(12) Andreff, N.; Horaud, R.; Espiau, B. Robot Hand-Eye Calibration Using Structure-From-Motion. *Int. J. Robot. Res.* **2001**, *2* , 228–248.

(13) Mikhail, E.M.; Bethel, J.S.; McGlone, J.C. *Introduction to Modern Photogrammetry*; John Wiley & Sons: New York, NY, 2001.

(14) Fan, H. *Theory of Errors and Least Squares Adjustment*; Royal Institute of Technology, Division of Geodesy: Stockholm, 2005.