Influence of boundary conditions on the aero-thermo-elastic stability of a closed cylindrical shell

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Abstract. In this paper, in a linear formulation, the stability problem of a closed cylindrical shell under the influence of an inhomogeneous temperature field and a supersonic gas stream flowing around the shell is considered. The stability conditions for the unperturbed state of the aero-thermo-elastic system under consideration are obtained. It was shown for different boundary conditions that by the combined action of the temperature field and the flowing stream, the stability process can be controlled and the critical flutter velocity can be substantially changed using the temperature field. The following most significant results were obtained: 1) in the case of a homogeneous temperature field, if the edges of the shell freely move in the longitudinal direction: a) a constant temperature field practically does not affect the value of the critical velocity $\nu_{cr}$; b) the critical velocity function $\nu_{cr}$, depending on the number of circumferential waves $n$, has a minimum point; 2) in the case of a homogeneous temperature field, if the edges of the shell are fixed: a) for negative values of $T_0$, the lower the temperature, the wider the stability region; b) for positive values of $T_0$ up to a certain temperature value the stability region narrows, after which, with increasing temperature it expands; c) starting from a certain temperature value $T_0^*$ for all $T_0 > T_0^*$ the system is unstable for any $0 < \nu < \nu_{cr}$, and with increasing speed ($\nu > \nu_{cr}$) the system becomes stable, and the larger the radius of the shell, the smaller this value $T_0^*$; 3) in the case of a temperature field inhomogeneous over the thickness of the shell, if the edges of the shell move freely in the longitudinal direction: a) when $\Theta > 0$ the critical velocity increases significantly and the minimum point of the function $\nu_{cr}(n)$ moves towards the lower values of $n$; b) when $\Theta < 0$ the opposite is observed; 4) in the case of a temperature field inhomogeneous over thickness of the shell, if the edges of the shell are fixed: a) the stability region expands with increasing $|\Theta|$; b) for a fixed value of the gradient $\Theta$, an increase in the radius of the shell $R$ leads to an expansion of the stability region; 5) fixing the edges of the shell leads to a significant increase in the value of the critical velocity of the flowing stream.

1. The problem statement

Let us consider a thin isotropic closed circular cylindrical shell of constant thickness $h$ and radius $R$, in an inhomogeneous temperature field $T$. The shell is assigned to cylindrical coordinates $x, \varphi, r$, the coordinate lines $x$ and $\varphi$ coincide with the curvature lines of the middle surface of the shell ($x$ — along the generatrix, $\varphi$ — along the arc of the cross section). There is a supersonic
gas stream on the outside of the shell with an unperturbed velocity $U$ directed along the axis $0x$. The stability issues of this aero-thermo-elastic system are investigated. The investigation is based on the following well-known assumptions:

a) the Kirchhoff–Love hypothesis of non-deformable normal [1];

b) the “law of plane sections” for determining the aerodynamic pressure [2]:

$$ p = p_\infty \left(1 + \frac{\alpha - 1}{2} \frac{v_3}{a_\infty}\right)^{2\alpha/(\alpha-1)}, $$  

(1)

where $p$ is the gas pressure on the surface of the shell, $a_\infty$ is the speed of sound in the unperturbed gas ($a_\infty^2 = \kappa p_\infty \rho_\infty^{-1}$), $p_\infty$ and $\rho_\infty$ are the gas pressure and density in the unperturbed state, $\alpha$ is the isentropic gas coefficient, $v_3$ is the normal component of the speed of the shell’s surface points;

c) the linear law of temperature change over the thickness of the shell [3]: $T = T_0(x, \varphi) + (R - r)\Theta(x, \varphi)$;

d) the Neumann hypothesis on the absence of displacements from the temperature changes [4].

For simplicity and clarity, it is assumed that from the front surfaces of the shell ($r = R \pm h/2$) there occurs a heat exchange with the medium according to the Newton-Richmann law (constant temperatures $T_+$ and $T_-$ are maintained on the surfaces), and the side surfaces ($x = 0$ and $x = a$) are heat-insulated.

Under the action of the temperature field inhomogeneous over the thickness, the shell buckles (with an axisymmetric deflection $w_T(x)$) and, as a result, aeroelastic pressure appears. The specified state is accepted as unperturbed [5], and its stability under the influence of the temperature field and the pressure of the flowing gas stream is studied.

1.1. Determination of aero-thermo-elastic stresses of the unperturbed state

Taking into account the accepted assumptions, from the basic equations, relations and boundary conditions of the theory of thermoelasticity of thin shells, in a similar way as in [5], the following expressions are obtained for the internal forces of the unperturbed state:

$$ T_{11}^0 = \delta \left[ \frac{E\mu}{1 - \mu^2} \frac{h}{a} \int_0^a \frac{w_T(x)}{R} \, dx - \frac{Eh\alpha}{1 - \mu} T_0 \right], $$

$$ T_{22}^0 = Eh \left[ \frac{w_T}{R} - (1 - \delta)\alpha T_0 + \delta \left( \frac{\mu^2}{1 - \mu^2} \frac{1}{Ra} \int_0^a w_T(x) \, dx - \frac{\alpha T_0}{1 - \mu} \right) \right], $$

$$ T_0 = T_+ + T_- \frac{2}{k h - 2\lambda}. $$  

(2)

Here $w_T$ is the solution of the following equation:

$$ D \frac{d^4 w_T}{dx^4} + \frac{12}{Rh^2} \left[ \frac{1 - \mu^2}{R} w_T + \frac{\delta \mu^2}{Ra} \int_0^a w_T(x) \, dx + (1 - \delta)\alpha \mu (1 + \mu) T_0 \right] + \kappa p_\infty M \frac{dw_T}{dx} = 0, $$

(3)

which satisfies the following boundary conditions:

$$ w_T = 0, \quad \frac{d^2 w_T}{dx^2} + \alpha (1 + \mu) \Theta = 0 \quad \text{for} \ x = 0, \ x = a. $$

(4)

For obtaining (4) it is assumed that the edges of the shell $x = 0, \ x = a$ are either hinge supported and are free to move along the axis $0x$, or hinge supported and fixed.
In (2)–(4) $M = U a_{\infty}^{-1}$ is the Mach number for the unperturbed flow, $D = Eh^3/12(1 - \mu^2)$, $E$ is the elastic modulus, $\mu$ is the Poisson’s ratio, $\alpha$ is the coefficient of linear thermal expansion, $\lambda$ is the thermal conductivity of the shell material, $k$ is the heat transfer coefficient,

$$\delta = \begin{cases} 
0, & \text{when the edges of the shell move freely,} \\
1, & \text{when the edges of the shell are fixed.} 
\end{cases}$$

1.2. Basic equations and boundary conditions of the perturbed state

In this paper, we will consider: a) the shell stability problem with hinge supported and freely moving edges, and b) the axisymmetric shell stability problem, when the edges are hinge supported and fixed.

On the basis of the theory of thermoelasticity of isotropic bodies, the Kirchhoff–Love hypothesis, and accepted assumptions, similarly to [6], the following system of linear differential stability equations of the aero-thermo-elastic system is obtained:

$$\frac{\partial^2 u}{\partial x^2} + \frac{1 - \mu \partial^2 u}{2 \partial \varphi^2} + \frac{1 + \mu}{2} \frac{\partial^2 v}{\partial x \partial \varphi} + \frac{\mu}{R \partial x} \frac{1}{E R h} \frac{(1 - \mu^2)T^0_{22}}{R^2 \partial x} \frac{\partial w}{\partial x} = 0,$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{1 - \mu \partial^2 v}{2 \partial \varphi^2} + \frac{1 + \mu}{2} \frac{\partial^2 u}{\partial x \partial \varphi} + \frac{1}{R \partial x} \frac{\partial w}{\partial \varphi} - \frac{h^2}{12R \partial x} \frac{\partial}{\partial x} \left( \frac{\Delta w + w}{R^2} \right) = 0,$$

$$D \left[ \Delta w + \frac{\mu}{R^2 \partial x^2} \frac{\partial^2 w}{\partial x^2} + \frac{1}{R^2 \partial \varphi^2} \frac{\partial^2 w}{\partial \varphi^2} + \frac{12}{R h^2} \frac{(\partial v}{\partial \varphi} + \mu \frac{\partial u}{\partial x} + \frac{w}{R} \right] + \rho_0 \frac{\partial^2 w}{\partial t^2} - T^0_{11} \frac{\partial^2 w}{\partial x^2}$$

$$- T^0_{22} \left( \frac{\partial^2 w}{\partial x^2} + \frac{w}{R^2} \right) + \left( \rho_0 h \varepsilon + \frac{\varepsilon p_\infty}{a_\infty} \right) \frac{\partial w}{\partial t} + \varepsilon p_\infty M \frac{\partial w}{\partial x} + \frac{\varepsilon (x + 1)}{2} p_\infty M^2 \frac{\partial w}{\partial x} \frac{\partial w}{\partial x} = 0,$$

where $u(x, \varphi, t)$, $v(x, \varphi, t)$, and $w(x, \varphi, t)$ are the perturbations of displacements of the middle surface points of the shell, $\rho_0$ is the shell material density, $\varepsilon$ is the linear attenuation coefficient.

When solving specific problems of stability, boundary conditions with respect to perturbations, following from the conditions for the edges fixing type, are added to system (5). If the edges of the shell are hinge supported, freely move in the longitudinal direction and motionless in the arc direction, the boundary conditions are presented in the form:

$$v = 0, \quad \frac{\partial u}{\partial x} = 0,$$

$$w = 0, \quad \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{for } x = 0 \text{ and } x = a. \quad (6)$$

In the case of the axisymmetric problem, the stability of the aero-thermo-elastic system with fixed edges reduces to solving the equation

$$D \left[ \frac{\partial^4 w}{\partial x^4} + \frac{12}{R h^2} \left( \frac{1 - \mu^2}{R} w - \mu \frac{\partial w}{\partial x} \frac{\partial w}{\partial x} + \frac{\mu^2}{R a} \int_0^a w \, dx + \mu \frac{\partial w}{\partial x} \frac{\partial \int_0^a w \, dx}{\partial x} \frac{\partial w}{\partial x} \right) \right] - \frac{E h}{1 - \mu} \alpha T^0 \frac{\partial^2 w}{\partial x^2}$$

$$+ \rho_0 h \varepsilon p_\infty \frac{\partial w}{\partial t} + \varepsilon p_\infty M \frac{\partial w}{\partial x} + \frac{\varepsilon (x + 1)}{2} p_\infty M^2 \frac{\partial w}{\partial x} \frac{\partial w}{\partial x} = 0 \quad (7)$$

with boundary conditions:

$$w = 0, \quad \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{for } x = 0 \text{ and } x = a. \quad (8)$$

Let us turn to the study of stability issues based on the boundary value problems formulated in section 1. The cases of the shell’s edges fixing type in the longitudinal directions are considered separately.
Table 1. The values of the critical velocity $\nu_{cr}$ at different values of temperature $T_0$ and number of circumferential waves $n$.

| $n$ | $T_0 = -500$ | $T_0 = 0$ | $T_0 = 400$ |
|-----|--------------|------------|-------------|
| 0   | 0.1160       | 0.1160     | 0.1160      |
| 5   | 1.1287       | 1.1671     | 1.1233      |
| 10  | 0.9076       | 0.9084     | 0.9091      |
| 15  | 0.6363       | 0.6392     | 0.6415      |
| 17  | 0.6161       | 0.6192     | 0.6217      |
| 20  | 0.6565       | 0.6596     | 0.6620      |
| 25  | 0.8514       | 0.8540     | 0.8562      |
| 30  | 1.1507       | 1.1529     | 1.1546      |

2. The case of free edges
Here we will solve the problem (5), (6). The solution of system (5) satisfying conditions (6) can be represented as

$$u(x, \varphi, t) = \sum_{i=1}^{\infty} u_i(t) \cos(\lambda_i x) \cos(n \theta),$$

$$v(x, \varphi, t) = \sum_{i=1}^{\infty} v_i(t) \sin(\lambda_i x) \sin(n \theta),$$

$$w(x, \varphi, t) = \sum_{i=1}^{\infty} w_i(t) \sin(\lambda_i x) \cos(n \theta) \left( \lambda_i = \frac{i\pi}{a} \right),$$

(9)
where $u_i(t), v_i(t), w_i(t)$ are the unknown functions of time $t$, $n$ is the number of waves in the circumferential direction.

Substituting (9) into system (5) and using the orthogonalization process, we obtain a linear system of ordinary differential equations for the dimensionless functions $x_m = w_m/h$. Representing the solution of this system in the form $x_m = y_m e^{\lambda \tau}$, we obtain a homogeneous system of linear algebraic equations with respect to $y_m$. From the condition for the existence of a nontrivial solution, we obtain a characteristic equation with respect to $\lambda$.

The undisturbed shape of the shell is stable if the real parts of the roots of the characteristic equation are negative.

Using the Hurwitz theorem, we determine the stability regions of the aero-thermo-elastic system in the parameter space $\nu$, $T_0$, $\Theta$, where $\nu = Mh/a$. On this basis, the critical flow velocity at which the shell loses stability is determined. Special cases of the temperature field are considered.

2.1. The case of a constant temperature field ($T_0 \neq 0$, $\Theta = 0$, $w_T = 0$)
In this case, if we neglect the influence of attenuation, the characteristic equation is biquadratic and according to the Hurwitz theorem, the stability conditions are:

$$T_0(\varphi_{11}^{(1)} + \varphi_{22}^{(1)}) > 0,$$

$$-\frac{16}{9}K^2\nu^2 + [\varphi_{11} + \varphi_{22} - T_0(\varphi_{11}^{(1)} + \varphi_{22}^{(1)})]^2 > 0,$$

$$\frac{4}{9}K^2\nu^2 + (\varphi_{11} + T_0\varphi_{11}^{(1)})(\varphi_{22} + T_0\varphi_{22}^{(1)}) > 0,$$

(10)
where

\[ \varphi_{11} = \frac{1}{12\pi^4 \xi_0 (1 - \mu^2) + (n^2 + \pi^2 \xi^2)^4 \psi^2} \left\{ \psi^2 \left[ n^4 (n^2 - 1)^2 + \pi^2 \xi^2 (4n^6 + 6n^4 \pi^2 \xi^2 + \pi^6 \xi^6) \right] \\
+ \pi^2 \xi^2 (4n^2 - \mu) \right\} + 12 \xi^2 \left[ \pi^2 \xi^2 (n^2 \mu - \pi^2 \xi^2) + (n^2 + \pi^2 \xi^2) \mu^2 \right] \\
- n^2 \pi^2 \xi^2 \psi^2 \left[ 2 + \pi^2 \xi^2 (2 \mu^2 - \mu - 3) + n^2 (\mu^2 + 2 \mu - 5) \right] \right\}, \]

\[ \varphi_{22} = \frac{1}{12\pi^4 \xi_0 (1 - \mu^2) + \psi^2 (n^2 + \pi^2 \xi^2)^4} \left\{ \psi^2 \left[ n^4 (n^2 - 1)^2 \right] \\
+ 16 \pi^2 \xi^2 (n^6 + 6n^4 \pi^2 \xi^2 + 16 \pi^6 \xi^6) + 64 \pi^6 \xi^6 (4n^2 - \mu) \right\} - 12 \xi^2 \left[ 4 \pi^2 \xi^2 (n^2 \mu - 4 \pi^2 \xi^2) \right] \\
+ (n^2 + 4 \pi^2 \xi^2) \mu^2 \right\} - 4n^2 \pi^2 \xi^2 \psi^2 \left[ 2 + 4 \pi^2 \xi^2 (2 \mu^2 - \mu - 3) + n^2 (\mu^2 + 2 \mu - 5) \right] \right\}, \]

\[ \varphi_{12} = -\frac{12 \alpha (1 - \mu^2) \xi^2 [(n^2 - 1)(n^4 + \pi^4 \xi^4) + n^2 \pi^2 \xi^2 (2n^2 - 3)]}{12\pi^4 \xi_0 (1 - \mu^2) + \psi^2 (n^2 + \pi^2 \xi^2)^4}, \]

\[ \varphi_{21} = -\frac{12 \alpha (1 - \mu^2) \xi^2 [(n^2 - 1)(n^4 + 16 \pi^4 \xi^4) + 4n^2 \pi^2 \xi^2 (2n^2 - 3)] (n^2 + \pi^2 \xi^2)^2}{12\pi^4 \xi_0 (1 - \mu^2) + \psi^2 (n^2 + \pi^2 \xi^2)^4}, \]

\[ \xi = \frac{h}{a}, \quad \psi = \frac{R}{a}, \quad \lambda = 210 \text{ W/(m}^2\text{C)}, \quad \mu = 0.34; \quad a = 1 \text{ m}, \quad h/a = 1/100, \quad R/a = 2. \]

For fixed values \( T_0 \), by numerical solution of inequality (10) the critical velocity values at which the aero-thermo-elastic System under consideration loses stability are determined. For calculation here and further: \( \alpha = 23.8 \times 10^{-6} \text{ C}^{-1}, \quad k = 1200 \text{ W/(m}^2\text{C}) \), \( \lambda = 210 \text{ W/(m}^2\text{C}) \), \( \mu = 0.34; \quad a = 1 \text{ m}, \quad h/a = 1/100, \quad R/a = 2. \) The results of numerical calculations are shown in figure 1 and table 1.

The above figure and table show that: a) if the edges of the shell freely move in the longitudinal direction, then a constant temperature field practically does not affect the value \( \nu_{cr} \); b) depending on the number of waves \( n \), the critical velocity \( \nu_{cr} \), as expected [7, 8], has a minimum point.
2.2. The case of a temperature field inhomogeneous over the thickness of the shell \((T_0 = 0, \Theta \neq 0, w_T \neq 0)\)

In this case, similarly to the previous one, the stability conditions are:

\[
\Theta \varphi_{11}^{(2)} + \varphi_{22}^{(2)} > 0,
\]

\[
- \frac{16}{9} K^2 \nu^2 + 4\Theta \left[ -\frac{2}{3} K \nu \varphi_{21}^{(2)} + \varphi_{12}^{(2)} \left( \frac{2}{3} K \nu + \varphi_{21}^{(2)} \right) \right] + \left[ \varphi_{11} - \varphi_{22} + \Theta (\varphi_{11}^{(2)} - \varphi_{22}^{(2)}) \right] > 0,
\]

\[
\frac{4}{9} K^2 \nu^2 - \Theta \left[ \frac{2}{3} K \nu (\varphi_{11}^{(2)} - \varphi_{22}^{(2)}) + (\varphi_{12}^{(2)} \varphi_{21}^{(2)} + \Theta \varphi_{11}^{(2)} ) (\varphi_{22} + \Theta \varphi_{22}^{(2)}) > 0,
\]

where

\[
\varphi_{11}^{(2)} = -a\alpha (1 + \mu) \left[ 9\pi^4 \xi^2 \psi^2 (1 - \mu^2) \left( 4\pi^6 \xi^2 \psi^2 \left( 3 + \pi^2 \right) \left[ (n^4 + \pi^4 \xi^4) (n^2 - 1) + 2n^4 \pi^2 \xi^2 \right] ight.ight. \\
- 3n^2 \pi^2 (1 + \pi^2) \xi^2 \{ 3 \left[ 2048 + 17 \pi^2 (3 + \pi^2)] \left[ 2n^4 \pi^2 \xi^2 + (n^2 - 1) (n^4 + \pi^4 \xi^4) \right] \\ \\
- \left. n^2 \pi^2 (5120 + 1 \pi^2 + 5 \pi^4) \xi^2 \right) \} \{ \pi^4 \left( (n^2 + \pi^2 \xi^4) \psi^2 + 12 \pi^4 \xi^6 (1 - \mu^2) \right) \}^{-1},
\]

\[
\varphi_{22}^{(2)} = -a\alpha (1 + \mu) \left[ \left( \frac{3\pi^4 \xi^2 \psi^2 (1 - \mu^2) \left( 4\pi^6 \xi^2 \psi^2 \left( 3 + \pi^2 \right) \left( n^4 + \pi^4 \xi^4) (n^2 - 1) + 2n^4 \pi^2 \xi^2 \right) ight)}{\pi^4 \{ (n^2 + \pi^2 \xi^4) \psi^2 + 12 \pi^4 \xi^6 (1 - \mu^2) \}} \right]^{-1}
\]

In this case, the critical velocity values are calculated from the stability conditions for different cases of temperature field gradient. The results are shown in figure 2.

Figure 2 and table 2 show that the inhomogeneity of the temperature field, in contrast to the previous case, has a significant effect both on the value of the critical velocity and on the nature of the dependence \(\nu_{cr}(n)\). Namely: a) when \(\Theta \geq 0\) the critical velocity significantly increases and the minimum point of the function \(\nu_{cr}(n)\) moves towards the lower values of \(n\); b) when \(\Theta < 0\) the opposite is observed.
**Figure 2.** Dependence of the critical velocity $\nu_{cr}$ on the number of circumferential waves $n$ in the case of a temperature field inhomogeneous over the thickness of the shell.

**Table 2.** The values of the critical velocity $\nu_{cr}$ at $T_0 = 0$, different values of $\Theta$ and number of circumferential waves $n$.

| $n$ | $-1000$ | $-500$ | $0$, $w_T = 0$ | $500$ | $1000$ |
|-----|---------|--------|----------------|------|-------|
| 0   | 0.1150  | 0.1155 | 0.1160         | 0.1165 | 0.1170 |
| 5   | 1.0155  | 1.0644 | 1.1257         | 1.2075 | 1.3308 |
| 10  | 0.8302  | 0.8658 | 0.6815         | 0.7102 | 0.7444 |
| 15  | 0.5861  | 0.6110 | 0.6392         | 0.6716 | 0.7099 |
| 20  | 0.5900  | 0.6220 | 0.6596         | 0.7025 | 0.7544 |
| 25  | 0.7435  | 0.7941 | 0.8540         | 0.9284 | 1.0282 |
| 30  | 0.9765  | 1.0542 | 1.1529         | 1.2899 | 1.5251 |

2.3. The general case ($T_0 \neq 0$, $\Theta \neq 0$, $w_T \neq 0$)

Using the Hurwitz theorem, the following stability conditions for the aero-thermo-elastic system are obtained

$$2\chi^2 + \varphi_{11} + \varphi_{22} + T_0(\varphi_{11}^{(1)} + \varphi_{22}^{(1)}) + \Theta(\varphi_{11}^{(2)} + \varphi_{22}^{(2)}) > 0,$$

$$- \frac{16}{9} K^2 \nu^2 - \frac{8}{3} K \nu \varphi_{21}^{(2)} + \Theta \varphi_{12}^{(2)} \left( \frac{8K\nu}{3} + 4\Theta \varphi_{21}^{(2)} \right) + (\varphi_{11} + T_0 \varphi_{11}^{(1)} + \Theta \varphi_{11}^{(2)})^2$$

$$+ 2(\varphi_{11} + T_0 \varphi_{11}^{(1)} + \Theta \varphi_{11}^{(2)})(\chi^2 - \varphi_{22} - T_0 \varphi_{22}^{(1)} - \Theta \varphi_{22}^{(2)})$$

$$+ 2\chi^2(\varphi_{22} + T_0 \varphi_{22}^{(1)} + \Theta \varphi_{22}^{(2)}) + (\varphi_{22} + T_0 \varphi_{22}^{(1)} + \Theta \varphi_{22}^{(2)})^2 > 0,$$

$$\frac{4K^2 \nu^2}{9} - \Theta \left[ \frac{2}{3} K \nu(\varphi_{12}^{(2)} - \varphi_{21}^{(2)} - \varphi_{12}^{(2)} \varphi_{21}^{(2)}) + (\varphi_{11} + T_0 \varphi_{11}^{(1)} + \Theta \varphi_{11}^{(2)})(\varphi_{22} + T_0 \varphi_{22}^{(1)} + \Theta \varphi_{22}^{(2)}) \right] > 0.$$

Table 3 shows that: a) inhomogeneity of the temperature field has a significant effect on the value of the critical velocity (if $\Theta < 0$, then $\nu_{cr}$ decreases, if $\Theta > 0$, then $\nu_{cr}$ increases); b) the function $\nu_{cr}(n)$ has extremum points depending on the value of the temperature field gradient $\Theta$. Here, in contrast to the previous case and the cases of absence of a temperature field, the function $\nu_{cr}$ has a maximum point.
Table 3. The values of the critical velocity $\nu_{cr}$ at $T_0 = -100$, different values of $\Theta$ and number of circumferential waves $n$.

| $n$ | $\Theta$ | $-500$ | $0, w_T = 0$ | $100$ | $1000$ |
|-----|----------|--------|--------------|-------|--------|
| 0   |          | 0.1154 | 0.1160       | 0.1161| 0.1170 |
| 5   |          | 1.0649 | 1.1263       | 1.1407| 1.3317 |
| 10  |          | 0.8657 | 0.9073       | 0.9179| 1.0313 |
| 15  |          | 0.6105 | 0.6389       | 0.6447| 0.7092 |
| 20  |          | 0.622  | 0.6589       | 0.6670| 0.7536 |
| 25  |          | 0.7936 | 0.8535       | 0.8670| 1.0275 |
| 30  |          | 1.0539 | 1.1525       | 1.1759| 1.5243 |

Figure 3. The stability regions in the case of a homogeneous temperature field ($\Theta = 0$)

3. The case of fixed edges

Here, for simplicity and clarity, the axisymmetric problem is considered, i.e. we solve the problem (7), (8). Then the solution of equation (7) satisfying conditions (8) can be represented as

$$w(x, t) = \sum_{i=1}^{\infty} f_i(t) \sin(\lambda_i x) \quad \left(\lambda_i = \frac{i\pi}{a}\right)$$

where $f_i(t)$ are the unknown function of time $t$.

In this paper we consider the case of the binomial approximation:

$$w = f_1(t) \sin(\lambda_1 x) + f_2(t) \sin(\lambda_2 x),$$

Substituting (14) into equation (7) and using the orthogonalization process, we obtain a homogeneous system of ordinary differential equations with respect to $f_i(t)$ ($i = 1, 2$) with constant coefficients. Representing the solution of that system in the form $f_i(t) = c_i e^{pt}$ ($c_i$ are constants), we obtain a homogeneous system of linear algebraic equations with respect to $c_i$. From the condition for the existence of a nontrivial solution, we arrive at a characteristic equation with respect to $p$. On this basis, the stability regions on planes $(U, T_0)$ (the case of a homogeneous temperature field, $\Theta = 0$) and $(U, \Theta)$ (the case of an inhomogeneous temperature field, $T_0 = 0$) are constructed. Figures 3 and 4 show the indicated regions constructed when $\chi = 0$ and for the initial data used above.

3.1. The case of a constant temperature field ($T_0 \neq 0$, $\Theta = 0$, $w_T = 0$)

Figure 3 shows that:
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3.2. The case of a temperature field inhomogeneous over the thickness of the shell \((T_0 = 0, \Theta \neq 0, w_T \neq 0)\)

Figure 4 shows that:

a) with \(|\Theta|\) increasing the stability region expands;

b) for a fixed value of the gradient \(\Theta\), an increase in the radius of the shell \(R\) leads to an expansion of the stability region;

c) in the absence of flow, if \(T_0 = 0\), then under the influence of the temperature field of the type \(T = \Theta(r - R)\) there is no loss of stability.

3.3. The general case

The general case \((T_0 \neq 0, \Theta \neq 0)\) is also considered. Table 3 shows the numerical results only for a fixed \(T_0\). Calculations were also made to study the effect of geometric parameters on the critical velocity value. Table 4 is devoted to that study and shows that: a) at a fixed temperature, the thinner the shell, the lower the critical velocity of the flow, b) at a fixed shell thickness, an increase in the temperature field leads to a decrease in the critical velocity of the flow, c) an increase in the parameter \(R/a\) leads to an increase in the critical velocity.

Table 5 shows, in addition to the above conclusions, that the combined effect of the parameters of the inhomogeneous temperature field on the value of the critical velocity is more noticeable.
Table 5. The value of the critical velocity $\nu_{cr}$ at $T_0 = -100$ and different values of gradient $\Theta$.

| $h/a$  | $\Theta$ | $R/a = 2$ | $\Theta$ | $R/a = 2$ | $\Theta$ | $R/a = 2$ | $\Theta$ | $R/a = 2$ |
|-------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1/70  | 0.1712   | 0.1704   | 0.1696   | 0.1688   | 0.1680   |
| 1/100 | 0.0697   | 0.0694   | 0.0691   | 0.0688   | 0.0685   |
| 1/200 | 0.0150   | 0.0149   | 0.0149   | 0.0148   | 0.0147   |
| $R/a$ | $h/a = 1/100$ | $h/a = 1/100$ | $h/a = 1/100$ | $h/a = 1/100$ | $h/a = 1/100$ |
| 3     | 0.0839   | 0.0838   | 0.0837   | 0.0836   | 0.0834   |
| 5     | 0.1022   | 0.10222  | 0.1023   | 0.10232  | 0.1024   |
| 10    | 0.1611   | 0.1615   | 0.1620   | 0.1624   | 0.1629   |

A comparison of tables 1 and 3 with 2 and 4, respectively, leads to the conclusion that fixing the edges of the shell leads to a significant increase in the critical velocity of the flow.

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