Improved genetic algorithm with two-level multipoint approximation for complex frame structural optimization

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Abstract. In this paper, an improved structural topology and sizing optimization method is developed for the fast and efficient engineering design of complex frame structures where beam elements are mainly used in the structures. Discrete and continuous variables are included that the elimination or existence of beam elements are treated as discrete variables (0,1), and the continuous sizing variables of beam cross sections are considered to be continuous variables. To solve the mixed variable problem, the paper introduces a two-level multipoint approximation strategy (TMA). The first-level approximate problem is established by using the branched multipoint approximate function, which includes both two types of variables. Genetic algorithm (GA) is used to determine the absence or presence of beam members. The second-level approximate problem that only involving retained continuous size variables is made on this basis, which uses Taylor expansion and dual methods to solve the inner layer continuous optimization problem. Meanwhile, a strategy of adding a new complementary design point is adopted to expend the search scopes and improve the precision. Temporal deletion techniques are used to temporarily remove redundant constraints and local vibration modes processing techniques are used for continuum topology optimization under frequency constraints. Several representative examples are investigated to validate the effectiveness of the improved method.

1. Introduction

Frame structures are a commonly-used typical form in aerospace and civil construction. Different from the traditional truss structures, frame structures constructed from beam elements need to consider both axial and flexural loads. In the early, the frame structure models are more likely to be considered as a continuum problem. When design variables are evaluated at small values or zero with mathematical programming, the corresponding bars are removed from the structure and obtained the optimal topology finally. However, not until Sved and Ginos [1] found the singular solution phenomenon in the process of topology optimization was it discovered that some solutions seemed to be not valid and the method is no longer effective in some cases, since this problem is not entirely continuous. A further research is given by Cheng et. al [2][3][4] who pointed out the root cause of the singular solution, the stress constraints are not continuous when the cross sectional areas of the bars reached zero or extremely small. Further studies conclusively showed that the nodal displacement and local buckling constraints have the same problem. Chen and Guo proposed a relaxation method solved this problem well. However, it still remains a poorly problem that it is difficult for this method to deal with the frequency constraints. And besides the static constraints, dynamic response in structural design such as structural modal frequency are always need to be considered, which would also along with the singular solution. Take three-bar frame structure for example [5], the feasible region of minimum
structural quality with frequency constraint consists of disconnected subdomains. Due to the existence of singular solution, it is difficult to solve with traditional optimization method. Ni et al [5] introduces the relaxation method to construct the PLMP algorithm. And Du et al [6] proposes ICM algorithm. Both of them use intermediate variables or intermediate function to solve specific constraint problems. Unfortunately, this type of approach would induces low interoperability among these algorithm.

As the most common form of discrete structure, the truss optimization problem is well solved by two-level multipoint approximation combined with genetic algorithm (TMA-GA) [7]. The initial structure consists of as many bars as possible, and then the elimination or existence of truss elements are treated as discrete variables which is determined by discrete optimization technique. The sizing variables of truss are considered to be continuous variables. As an extension of the truss structures, frame structures are also one of the discrete structures. Previous studies of An [8] developed TMA-GA for laminate stacking sequence optimization, which cooperate GA with dual methods to solved the discrete/continuous topology optimization problem and obtained good effect. However, due to the complexity of the frame structures, the algorithm might be trapped around the local optimal or even could be difficult to converge. Based on studies showing that lacking the screening mechanism of initial points and shrinking of search range along with the increasing of iterations may lead to the problem.

In view of this thought, an improved genetic algorithm with two-level multipoint approximation for complex frame structural optimization is proposed in this paper. For the randomness of GA which appears in calculation process, the validity check for frame structure is applied to avoid ineffective individuals in GA. To guide the right optimization process, the temporal deletion techniques is conducted to remove constraints when the corresponding nodes or beams are deleted from the initial model [9]. Moreover, this paper puts forward a strategy of adding new complementary design points for the first time to enhance the ability of searching for globally optimal solutions.

Several representative examples are investigated to validate the effectiveness of the improved algorithm. The following of the paper is organized as follows: in Section 2, the problem formulation for frame structural optimization is presented. The construction of two-level multipoint approximate function and GA are elaborated in section 3. And the details of the particular technology for frame structural optimization are given in section 4. Some verifiable example are followed in section 5 and the conclusion is made in the section 6.

2. Problem formulation and solution process

The topology optimization of frame structures starts from an initial structure. Then, the optimal configuration and the corresponding cross-sectional dimensions of the bar is determined by GA and dual method. For the deleted members, this paper gives small value to avoid numerical singularity. Based on the above assumptions, the complex frame structural topology / size optimization is formulated in (1).

\[
\begin{align*}
\text{find} & \quad X = \{x_{11}, x_{12}, ..., x_{1l_1}, ..., x_{1l_1'}, ..., x_{i1}, x_{i2}, ..., x_{i l_i'}, ..., x_{n1}, x_{n2}, ..., x_{n l_n'}, ..., x_{1l_n}\} \\
& = \{x_1, x_2, ..., x_m\}\\
\text{min} & \quad f(x, \alpha)\quad \text{ subject to } \quad g_j(X, \alpha) \leq 0 \quad \text{ for } j = 1, ..., J_0 \\
& \quad \alpha_i = 0 \text{ or } \alpha_i = 1 \\
& \quad x_{il_i}^L \leq x_{il_i} \leq x_{il_i}^U \quad \text{ if } \alpha_i = 1 \\
& \quad x_{il_i} = x_{il_i}^b \quad \text{ if } \alpha_i = 0 \\
& \quad l_i = 1, ..., L_i \quad i = 1, ..., n
\end{align*}
\]

Where \(X = \{x_1, x_2, ..., x_m\}\) is the continuous size variables, which consists of the section size of bar members; \(x_{il_i}^L\) and \(x_{il_i}^U\) are respectively the upper and lower limits of continuous size variables; \(x_{il_i}^b\) represents the cross-sectional dimensions corresponding to the removable bars (in this paper,
\( x_{i|l}^{b} = 0.01x_{i|l0}, \) \( x_{i|l0} \) is the initial value of the size variables; \( \alpha_i \) is the discrete topological variables, \( \alpha_i = 1 \) or 0 represents the existence (1) or elimination (0) of related bar members; \( n \) is the number of discrete variables, which is also the number of structural components; \( m \) is the number of continuous variables, \( m = \sum_{i=1}^{n} L_i \), \( L_i \) is the number of cross-sectional dimensions for \( i \)th bar member (for example, if the section shape of \( i \)th bar member is a rectangle, both of the length and width are continuous variables for this bar, so the \( L_i = 2; \) \( f(x, \alpha) \) is the objective function and \( g_j(x, \alpha) \) is the constraint function, \( f_0 \) is the number of the constraint functions.

Above, there are two number fields \( X_k^1 \) and \( X_k^2 \) for design variables according to the values of continuous variables.

\[
X_k^1 = \{ x_{i|l_1}^L | x_{i|l_1}^L \leq x_{i|l_1} \leq x_{i|l_1}^U \}, \quad k = 1, ..., m \tag{2}
\]

\[
X_k^2 = \{ x_{i|l_1}^L | x_{i|l_1} = x_{i|l_1}^b \}, \quad k = 1, ..., m \tag{3}
\]

The \( X_k^1 \) represents that the value of the corresponding continuous variables can change in this interval, and the other domain \( X_k^2 \) indicates that the related continuous variables are set to a fixed small value. When the value of \( \alpha_i \) is 1, the related bar member need to be retained, and the cross-sectional dimensions for relevant \( i \)th bar member can change from the given range. The value of \( \alpha_i \) is 0 means the \( i \)th bar needs to be deleted. To avoid strangeness, the related variables are given a small value.

By defining the \( X_k^1 \) and \( X_k^2 \), the complex frame structural topology/size optimization built in Eq.(1) is formulated in a new form:

\[
\begin{align*}
\text{find} & \quad X = \{x_1, x_2, ..., x_m\}^T \\
\alpha & = \{\alpha_1, \alpha_2, ..., \alpha_n\}^T \\
\text{min} & \quad f(x, \alpha) \\
\text{S.t.} & \quad g_j(X, \alpha) \leq 0 \quad j = 1, ..., f_0 \\
& \quad \alpha_i = 0 \text{ or } \alpha_i = 1 \quad i = 1, ..., n \\
& \quad x_k^L \leq x_k \leq x_k^U \quad \text{if } x_k \subseteq X_k^1 \\
& \quad x_k = x_k^L \quad \text{if } x_k \subseteq X_k^2 \\
& \quad k = 1, ..., m
\end{align*} \tag{4}
\]

In Eq.(4), \( X_k^1 \) and \( X_k^2 \) are defined in Eq.(2) and (3), which are the value domains; \( x_k^U \) and \( x_k^L \) are the upper and lower limits for continuous size variables; \( x_k^b \) is the small value when the discrete variable \( \alpha_i = 0 \).

3. Optimization scheme

As the first part shows, the problem contains both discrete and continuous variables is hard to be solved with routine method. Investigating the previous work [10], the two-level multipoint approximation method cooperates with GA have been proved to be a very effective approximation method to solve the mixed variable problem. The optimization problem is approximated by using the branched multipoint approximate functions and the initial information of the structural responses is obtained with the use of the finite element software MSC.Patran/Nastran. In this case, it converts to an explicit problem. Then GA is adopted to optimize the approximate problem, where the discrete 0/1 variables is optimized to determine the existence (1) or elimination (0) of the related bar members. At last, the second-level approximate with dual method are constructed to optimize the cross-sectional dimensions for retained bar members after assessing the individual fitness in GA.

3.1. The first-level approximate problem

In Eq. (4), the objective function \( f^{(p)}(x, \alpha) \) and the constraint function \( g_j^{(p)}(x, \alpha) \) are approximated with the branched multipoint approximate technique, which are substituted with the function \( w^{(p)}(x, \alpha) \) as follow:

\[
w^{(p)}(x, \alpha) = \sum_{t=1}^{H} w(X_t, \alpha_t) + \sum_{k=1}^{m} w_{k|\ell}(X, \alpha) \tag{3.1}
\]
Where
\[ \overline{w}_{k,t}(X, \alpha) = \begin{cases} \frac{1}{r_{o,t}} \frac{\partial w(X_k, \alpha_t)}{\partial x_k} x_{k,t} \left( 1 - r_{o,t} \right) \left( x_{k,t} - x_{k,t} \right) & \text{if } x_k \not\in X_k^1 \\ \frac{1}{r_{m,t}} \frac{\partial w(X_k, \alpha_t)}{\partial x_k} \left( 1 - e^{-r_{m,t}(x_k - x_{k,t})} \right) & \text{if } x_k \not\in X_k^2 \\ \right. \]
\[ h_t(X, \alpha) = \frac{\tilde{h_t}(X, \alpha)}{\sum_{l=1}^{L} h_l(X, \alpha)} \quad t = 1, \ldots, H \]
\[ \tilde{h_t}(X, \alpha) = \prod_{k=1}^{n} (X - X_k)^T (X - X_t) \quad (3.4) \]

In Eq. (3.1)-(3.4), \( X_t, \alpha_t \) are the t-th known point; \( w^{(p)}(x, \alpha) \) and \( \partial w^{(p)}(X_t, \alpha_t)/\partial x_k \) are the function values and the partial derivatives of p-th stage; \( H \) is the number of the known points with the upper limit \( H_{\text{max}} \). This paper only counts the last \( H_{\text{max}} \) known points. When \( H > H_{\text{max}}, r_{o,t} \) and \( r_{m,t} \) are the adaptive parameters which are used to decide the nonlinearity of the approximation. Eq. (3.5) is the governing equation:
\[ \begin{cases} \text{Min} \sqrt{\sum_{k=1}^{H} \left[ w(X_k) - w(X_t) - \sum_{l=1}^{n} \bar{w}_{k,t}(X_t) \right]^2} \\ \text{s.t. } r_{0,t}^L \leq r_{o,t} \leq r_{0,t}^U, r_{m,t}^L \leq r_{m,t} \leq r_{m,t}^U, t = 1, \ldots, H \end{cases} \]

Where \( r_{0,t}^L, r_{0,t}^U, r_{m,t}^L, r_{m,t}^U \) are the lower and upper bounds on \( r_{o,t} \) and \( r_{m,t} \), respectively. If there is only one known point, \( r_{o,t} \) and \( r_{m,t} \) are assigned the initial values \( r_{o,t,0} \) and \( r_{m,t,0} \). In this paper, \( r_{o,t,0} = -1, r_{m,t,0} = 3.5, r_{0,t}^L = -5, r_{0,t}^U = 5, r_{m,t}^L = -5, r_{m,t}^U = 5 \) \{11\}[12].

It has to be stressed that the bar would not be deleted when the related \( \alpha_t = 0 \), but the cross-sectional dimensions are set to a minimum value to keep the continuity, so the objective function and the constraint function are still differentiable.

### 3.2 Optimize the discrete variables by GA

This paper uses GA to optimize the discrete variables to determine the elimination or existence of beam elements on the infrastructure. The GA code, initial population, fitness calculation and reproduction operator proceed as follows:

1. The coding scheme of GA

   According to the layered optimization strategy, the external layer optimization only includes the presence or absence of a component. So, encoded string consists of 0-1 discrete variables binary coding is suitable. With 0 means the related bars are removed and 1 means the bars are kept. The code can be written as: \( \tilde{\alpha} = \alpha_1, \alpha_2, \ldots, \alpha_n \), \( \alpha_n \) represents the discrete variable \((0, 1)\).

2. Initial population

   The initial population generates randomly because of lack of experience. However, once get the optimal individuals of the first generation, every generation after that would generate in terms of the elite of former generations. So, there are three parts in the generation:

   (a) The optimal individuals of former generations;

   (b) Members generated from those in (1) by making \( \alpha \) approach 0 with a greater probability if the corresponding optimal continuous variables are small enough \{11\};

   (c) Members generated randomly;

3. The calculation of fitness

   The value of the individual fitness determines the chance of inheritance. Since GA is used for solving the unconstrained problem, the suitable fitness function is established with exterior penalty technology to remove all constraints.

If \((X^*, \alpha^*)\) is feasible:
\[ F_1 = f^{(p)}(X^*, \alpha^*) - f^{(p)}(X, \alpha) \epsilon \max \left[ g_1^{(p)}(X^*, \alpha^*) \ldots g_{l_1}^{(p)}(X^*, \alpha^*) \right] \] (3.6)

Otherwise:
\[ F_1 = \tilde{f}^{(p)}(X, \alpha) + f^{(p)}(X, \alpha) \left[ 1 + \sum_{j=1}^{l_1} \frac{g_j^{(p)}}{\sum_{l=1}^{l_1} g_l^{(p)}} (X^*, \alpha^*)^2 - 1 \right] \] (3.7)
\[ f^{(p)}(X, \alpha) = \begin{cases} f^{(p)}(X^*, \alpha^*) & \text{if } f^{(p)}(X^*, \alpha^*) > f^{(p)}(X, \alpha) \\ f^{(p)}(X, \alpha) & \text{if } f^{(p)}(X^*, \alpha^*) < f^{(p)}(X, \alpha) \end{cases} \] \quad (3.8)

\[ g_j^{(p)} \] is the tasks violating the limitation of the related constraint, \( f^{(p)}(X, \alpha) \) is the average of the \( p \)-th generation target function; \( f^{(p)}(X^*, \alpha^*) \) and \( g_j^{(p)}(X^*, \alpha^*) \) are the objective values and constraint values, which could be obtained by solving the inner continuum optimization problem in Eq. (3.13). Unitary method have been introduced to improve on suitable fitness function as (3.9), \( F_2 \) is the new expressions:

\[ F_2 = \left[ 1 - \frac{F_1 - \text{min}(F_2)}{\text{max}(F_2) - \text{min}(F_2)} \right]^s \quad (3.9) \]

Where \( s \) is the normalized exponents (in this paper, \( s = 2 \)); \( \text{max}(F_2) \) and \( \text{min}(F_1) \) are the maximum and minimum of all individuals in the population. Besides, the scaling operation was performed for the value of suitable fitness to avoid the premature phenomena existing in usual genetic algorithm. This paper defines that each individual could produce at most two offspring, and the final fitness function is expressed as follows:

\[
\text{Fitness} = a F_2 + b
\]

\[ a = \begin{cases} \frac{\text{avg}(F_2)}{\text{[avg}(F_2)-\text{min}(F_2])} & \text{min}(F_2) \leq 2\text{avg}(F_2) - \text{max}(F_2) \\ \frac{\text{avg}(F_2)}{\text{[max}(F_2)-\text{avg}(F_2])} & \text{Other} \end{cases} \quad (3.10) \]

\[ b = \begin{cases} (\text{a} \times \text{min}(F_2)) & \text{min}(F_2) \leq 2\text{avg}(F_2) - \text{max}(F_2) \\ (\text{a} \times [\text{max}(F_2) - 2\text{avg}(F_2)]) & \text{Other} \end{cases} \quad (3.12) \]

It can be seen that each type of structure need to be performed further size optimization before calculating the fitness to determine the optimal structure information.

After that, this paper uses the simulated roulette wheel selection method to determine the reproduction of individual, which followed by cross-mutation and recombination operations of genes. The iteration does not stop until evolutionary algebra reaches the upper bounds MaxG.

### 3.3 The second-level approximate problem

The discrete variables in the first-level approximate problem have been determined as \( \alpha_{i,p}^* \) by GA. The cross-sectional dimensions of members whose related \( \alpha^*_p \) are 0 would be fixed to a small value and removed from the second-level approximate problem. So, the corresponding constrain \( g_j(X, \alpha) \) do not exist.

However, the first-level approximate problem is explicit and nonlinear with a large number of continuous variables, which lead the problem difficult to solve with dual method. Thus, the first-level approximate problem (3.1) are converted to linearization expressions by making use of taylor series expression in the variable space \( X \), which is called the second-level approximate problem.

In the \( q \)-th, the approximate problem is formed as follows:

\[
\begin{align*}
\text{find} & \quad \hat{X} = \{\hat{x}_1, \hat{x}_2, ..., \hat{x}_l\} \\
\min & \quad \hat{f}^q(\hat{X}) = f(\hat{X}_q) + \sum_{k=1}^{l} \frac{\partial f(\hat{x}_k)}{\partial \hat{x}_k} (\hat{x}_k - \hat{x}_{k(q)}) \\
s. t. & \quad \tilde{g}^q_j(\hat{X}) = \tilde{g}_j(\hat{X}_q) - \sum_{k=1}^{l} \tilde{x}_{k(q)} \frac{\partial \tilde{g}_j(\hat{x}_k)}{\partial \hat{x}_k} \left( \frac{1}{\hat{x}_k} - \frac{1}{\tilde{x}_{k(q)}} \right) \leq 0 \\
& \quad j = 1, ..., j_f \quad k = 1, ..., l \\
& \quad \tilde{x}_{k(q)}^L \leq \hat{x}_k \leq \tilde{x}_{k(q)}^U
\end{align*}
\]

(3.13)

Where

\[
\begin{align*}
\tilde{x}_{k(q)}^U &= \min\{x_{k(p)}^U, \tilde{x}_{k(q)}^U\} \\
\tilde{x}_{k(q)}^L &= \min\{x_{k(p)}^L, \tilde{x}_{k(q)}^L\}
\end{align*}
\]

(3.14)

(3.15)

\[ \hat{f}^q(\hat{X}_q) \] and \[ \tilde{g}^q_j(\hat{X}_q) \] respectively are the objective function and constraint function in the \( q \)-th step. \( \tilde{x}_{k(q)}^U \) and \( \tilde{x}_{k(q)}^L \) are the upper and lower bounds of move limit; \( \hat{x}_{k(q)}^U \) and \( \hat{x}_{k(q)}^L \) are the upper and lower bounds of \( \hat{x}_k \).
The dual method is stated as follow:

\[
\begin{align*}
\text{find} & \quad \lambda = (\lambda_1, \lambda_2, ..., \lambda_{l_2})^T \\
\text{max} & \quad l(\lambda) = \min_{\bar{x} \in R_k(x)} \left\{ \hat{f}^q(\bar{x}_q) + \sum_{j=1}^{l_2} \lambda_j \hat{g}_j^q(\bar{x}_q) \right\} \\
\text{s.t.} & \quad \lambda_j \geq 0 \quad j = 1, ..., l_2
\end{align*}
\]  

(3.16)

Where, \( R(\bar{x}) = \{ \bar{x}_k | \bar{x}_{k(q)}^L \leq \bar{x}_k \leq \bar{x}_{k(q)}^U, k = 1, ..., I \} \). The relationship between design variables and dual variables are stated as Eq. (3.17):

\[
\bar{x}_k = \left\{ \begin{array}{ll}
\bar{x}_{k(q)}^L & \bar{x}_k \leq \bar{x}_{k(q)}^L \\
\bar{x}_k & \bar{x}_{k(q)}^L < \bar{x}_k < \bar{x}_{k(q)}^U \\
\bar{x}_{k(q)}^U & \bar{x}_k \geq \bar{x}_{k(q)}^U
\end{array} \right.
\]  

(3.17)

\[
x_k' = \left\{ \begin{array}{ll}
\sqrt{x_k} & x_k' > 0 \\
0 & x_k' < 0
\end{array} \right.
\]  

(3.18)

\[
x_k' = \frac{\sum_{j=1}^{l_2} \lambda_j \bar{x}_{k(q)}^j}{\frac{\partial f(\bar{x}_q)}{\partial x_k}}
\]  

(3.19)

The dual expression has been transformed into an unconstrained optimization problem, which could be solved efficiently by the BFGS method [13]. The optimal solution of the Eq. (3.13) in the q-th step could be obtained when the dual problem is solved. And the iteration stop would not stop until the second-level approximate problem converges. At this point, the continuum optimization of inner layer has been finished.

3.4 Flow block diagram for the two-level multipoint approximation method with GA

The flow block diagram for complex frame structural optimization is presented in detail in figure 1. MAXG is the maximum generation numbers, and structure sensitive analysis is solved by software MSC. Patran/Nastran. \( x_k^L \) is the lower limits of continuous size variables; \( x_k^b \) is the small value when the related bar members are removed; The strategy of adding a new complementary design point is executed after the first-level approximate problem converged.

4. The particular technology for frame structural optimization

4.1. The strategy of adding a new complementary design point

The approximation function is constructed from a small number of initial points, which may lead to the relatively low accuracy of the approximation function at the non-initial points. This paper puts forward a strategy of adding a new complementary design point after the termination of the optimization process at the first time. The variables corresponds to the removed members are changed from zero or extremely small value to the lower bounds of continuous variable, on the contrast, the remained variables performs the opposite action. By using this strategy, the new approximation function is constructed with more uncorrelated design points, which may improve the precision as well as reduce the drawback of falling into a local optimal solution. The corresponding formulation is shown as follow:

\[
x_k = \begin{cases} 
  x_k^L & \text{if } x_k \in X_k^1 \rightarrow \alpha_i = 1 \\
  x_k^b & \text{if } x_k \in X_k^2 \rightarrow \alpha_i = 0
\end{cases}
\]

(4.1)

Where \( x_k^L \) is the lower bound of continuous size variables, \( x_k^b \) is the extremely small value when the related bar members are delated; the cross-sectional dimensions of the remained bar members are set to \( x_k^L \) and the value of the deleted bar member are set to \( x_k^L \) to expand the search scopes. It should be noted that the operation is contrary to normal settings at this step.
4.2. Temporary deletion techniques

In order to maintain the structural continuity, the cross-sectional dimensions of the deleted bars are set to a small value rather than zero. The stress of the deleted bars would increase a lot and let the related cross-sectional dimensions increase. It may mislead the optimization process. To avoid such problems, the displacement and stress constraints on the removed members should be deleted to guide the right optimization process. The specific operation is shown as follows:

(a) The displacement and stress constraints related to the removed components as well as node are deleted temporarily from the second-level approximation function Eq. (3.13). So, the number of the constraints narrows to \( J_2 \) in Eq. (4) from \( J_0 \) in Eq. (3.13) in this way.
The displacement and stress constraints related to the removed components as well as node are deleted temporarily when come to the convergence judgment module of the optimization problems. And the formulation of appropriate criteria is stated as Eq. (4.2).

\[
\left\{ \begin{array}{l}
\frac{|f^{(p)}(x,\alpha)-f^{(p-1)}(x,\alpha)|}{f^{(p)}(x,\alpha)} \leq \xi_1 \\
\max \{g_1^{(p)}(x,\alpha), g_2^{(p)}(x,\alpha), \ldots, g_{J_0}^{(p)}(x,\alpha)\} \leq \xi_2
\end{array} \right.
\] (4.2)

Where \(\xi_1\) and \(\xi_2\) are the control precision of convergence judgment; The number of the constraints reduces from \(J_0\) to \(J_0'\).

### 4.3. Local vibration modes processing techniques for frequency constraints

The assumption of small cross-sectional of the deleted members is accepted widely in topology optimization for frame structures. Whereas the final optimization results consist of necessary components and redundancy slender rod may generate the local vibration modes. It is noted that this phenomenon always appears in small cross-sectional area especially related to the deleted elements for frequency-constrained topology optimization. Through analysis, the bending rigidity of the frame structures is generally a quadratic function to the cross-sectional area while the mass matrix are linear to the cross-sectional area. The different decay speed result in the local vibration modes.

This paper puts forward an effective strategy that the material density is forced to set to an extremely small value for the removed beams while the density of the remained beams keep the initial value. The conversion would let the local vibration mode only appear in the higher order mode frequency and produce no effect on normal results. The mathematical expression is established as follow:

\[
\rho_{x_i} = \begin{cases} 
\rho_{x_i}^b & \text{if } x_k \subseteq X_k^1 \rightarrow \alpha_i = 1 \\
\rho_{x_i}^0 & \text{if } x_k \subseteq X_k^2 \rightarrow \alpha_i = 0 
\end{cases}
\] (4.3)

\(\rho_{x_i}\) is the material density of the \(i\)-th beam; \(\rho_{x_i}^b\) is the extremely small value to represent the \(i\)-th deleted components (\(\rho_{x_i}^b = 10^{-5}\rho_{x_i}^0\) in this paper); \(\rho_{x_i}^0\) is the initial value of the material density.

### 5. Numerical examples

Several representative examples are investigated to validate the effectiveness of the improved genetic algorithm with two-level multipoint approximation method for complex frame structural optimization.

#### 5.1. Example 1: 12-beam planar frame structure

For this example, the optimal design of the 12-beam frame structure (Fig.2) is conducted with the optimization target of reducing structural weight for fundamental frequency constraints. The structure parameters are shown as follow: The lumped mass load on the free nodes is 454kg. The density of material is 2770kg/m\(^3\) and the elastic modulus of the beams are all 68.9Gpa with a poission’s ratio of 0.29. The cross sections of all the beams are round with initial radius \(R_i = 0.5m\). The final topology result is shown in Figure 3.

![Figure 2. The model of twelve-beam frame structure.](image-url)
As shown in [14], the fundamental frequency is set to more than 14Hz in the optimization problem. Using this optimal result, the lower bound of the fundamental frequency from the analysis of Nastran is 14.13Hz. So, the fundamental frequency constraint is set to 14.13Hz in this paper. The upper and lower limits respectively are 2.5m and 0.2m, the comparison results of two algorithms are given in Table 1.

| Variables | Reference[14] | Traditional TMA-GA | Improved TMA-GA |
|-----------|---------------|--------------------|-----------------|
| $R_1$ (m) | 0.4745        | 0.5391             | 0.4995          |
| $R_2$ (m) | 0.2345        | 0.2000             | 0.2001          |
| $R_3$ (m) | 0.4745        | 0.3337             | 0.3344          |
| $R_4$ (m) | 0.4745        | 0.4262             | 0.5028          |
| $R_5$ (m) | 0.3145        | 0.2188             | 0.2782          |
| $R_6$ (m) | 0.2345        | 0.2000             | 0.2146          |
| $R_7$ (m) | 0.4745        | 0.5105             | 0.5076          |
| $R_9$ (m) | 0             | 0                  | 0               |
| $R_{10}$ (m) | 0.3645     | 0.4133             | 0.3334          |
| $R_{11}$ (m) | 0              | 0                  | 0               |
| $R_{12}$ (m) | 0              | 0                  | 0               |

| Weight/kg | 110602* | 105160* | 103413* |
|-----------|---------|---------|---------|
| Fundamental frequency/Hz | 14.00* | 14.08   | 14.08   |
| No. of S.A | 2000*  | 52      | 75      |

*Analysis results from [14]

It can be seen from the optimization results that the topological configurations obtained by the two algorithms are consistent, whereas, the values of optimal radii are different. And the optimal structural mass obtained by the improved method is lighter than that in [14]. In addition, the number of the structural analysis have dropped dramatically from 2000 to 75 in this work, which means the algorithm proposed in this paper is far more efficient than pure genetic algorithm. Compared with traditional TMA-GA, the adding design point strategy decreases the weight of the structure without costing too much computing time.

5.2. Example 2: 72-beam spatial frame structure

The model of 72-beam spatial frame structure is shown in Figure 3, which is the typical example in topology optimization. In order to simplify calculating, the overall structure takes the form of four standard layers with identical configuration as shown in Figure 4. The sequences of the node and beam number are arranged from the top layer to the ground. The material properties are set the same as the last example.
This 72-beam spatial frame structure is optimized under two different constraint conditions. The first one is the fundamental frequency constraint problem, in which two optimal results are compared between the traditional TMA-GA and the improved TMA-GA. The other is the nodal displacement constraint problem with an extra contrast of pure GA.

Figure 4. (a) 72-beam structure model        (b) One of the standard layers

(a) The constraint of fundamental frequency

The minimum fundamental frequency of the structure has been stipulated to be more than 10Hz. The initial radius $R_i = 0.02m$, of which the upper and lower bounds are respectively 0.05m and 0.02m. The relationship between topology variables and beam members are shown in Table 2. We can see from the table that there are 16 topology variables who corresponding to the different continuous variables of 72 beams. For example, the first topology variable is set to link up with the cross-sectional areas of beams for 1-4.

Table 2. Design variable linking for 72-beam frame structure.

| Variable number | Beam number | Variable number | Beam number |
|-----------------|-------------|-----------------|-------------|
| 1               | 1-4         | 9               | 37-40       |
| 2               | 5-12        | 10              | 41-48       |
| 3               | 13-16       | 11              | 49-52       |
| 4               | 17-18       | 12              | 53-54       |
| 5               | 19-22       | 13              | 55-58       |
| 6               | 23-30       | 14              | 59-66       |
| 7               | 31-34       | 15              | 67-70       |
| 8               | 35-36       | 16              | 71-72       |

In this example, whether or not performs the strategy of adding a new complementary design point is the key. This paper compares the running efficiency and optimization results in these two situations. And the operation results are shown in Table 3.
Table 3. Optimization results for constraint of fundamental frequency.

| Variable number | Traditional TMA-GA | Improved TMA-GA |
|-----------------|--------------------|-----------------|
| 1               | 0.022              | 0.030301        |
| 2               | 0.02               | 0.02            |
| 3               | 0.020306           | 0.02            |
| 4               | 0.020439           | 0.02            |
| 5               | 0                  | 0               |
| 6               | 0.022              | 0               |
| 7               | 0                  | 0               |
| 8               | 0.030267           | 0               |
| 9               | 0.02               | 0.02            |
| 10              | 0.021415           | 0.02            |
| 11              | 0.02979            | 0.029412        |
| 12              | 0.02               | 0.02814         |
| 13              | 0                  | 0               |
| 14              | 0.03025            | 0               |
| 15              | 0                  | 0               |
| 16              | 0                  | 0               |

| Weight/kg       | 1725.3000*         | 1603.1999       |
| Fundamental freq/Hz | 10.023*       | 10.048          |
| No. of S.A      | 30*               | 45              |

*Analysis results of traditional TMA-GA

When using the improved strategy of adding a complementary design point, the structural weight is lighter than that in traditional TMA-GA method without increasing too much computation time. It is perfectly acceptable that the number of the structural analyses only increases from 30 to 45. The final topology configuration between the two algorithms are different as shown in figure 5 and figure 6. We can see that the final topology configuration of improved TMA-GA is more reasonable and lighter than the other one.

![Figure 5. The final topology configuration of traditional TMA-GA](image_url)
Figure 6. The final topology configuration of improved TMA-GA.

(b) The constraint of nodal displacement
The maximum nodal displacement shall not exceed 0.00635m for node 1-4 in all directions as shown in Figure 4 (b), the allowable stresses for all the beams are not exceeding 175Mpa. The initial parameters are the same as example (a). The main forces are loaded in z direction for nodes 1-4 as show in Table 4. The final optimization results are put in Table 5. The first column of the result corresponds to the pure genetic algorithm, the second column is the result of the traditional TMA-GA method and the rest part is the result of the improved TMA-GA method. All of the three algorithms use the same model with unified parameters.

Table 4. Loading condition for displacement constraint problem

| Node | $P_x(N)$ | $P_y(N)$ | $P_z(N)$ | $M_x(N \cdot m)$ | $M_y(N \cdot m)$ | $M_z(N \cdot m)$ |
|------|----------|----------|----------|------------------|------------------|------------------|
| 1    | 0        | 0        | -22241   | 0                | 0                | 0                |
| 2    | 0        | 0        | -22241   | 0                | 0                | 0                |
| 3    | 0        | 0        | -22241   | 0                | 0                | 0                |
| 4    | 0        | 0        | -22241   | 0                | 0                | 0                |
| 5    | 2224.1   | 2224.1   | -22241   | 0                | 0                | 0                |

Table 5. Optimization results for constraint of nodal displacement.

| Variable number | Pure GA[15] | Traditional TMA-GA | Improved TMA-GA |
|-----------------|-------------|--------------------|-----------------|
| 1               | 0.01949     | 0.01877            | 0.0185          |
| 2               | 0.01665     | 0.016045           | 0.015398        |
| 3               | 0.00982     | 0.009241           | 0.009441        |
| 4               | 0.00891     | 0.008662           | 0.007718        |
| 5               | 0           | 0                  | 0               |
| 6               | 0           | 0                  | 0               |
| 7               | 0           | 0                  | 0               |
| 8               | 0.01040     | 0.006765           | 0.004621        |
| 9               | 0.01101     | 0.009122           | 0.009649        |
| 10              | 0.01070     | 0.00917            | 0.009582        |
| 11              | 0.00958     | 0.009528           | 0.009992        |
### Table 5: Analysis results of traditional TMA-GA and improved TMA-GA0.12013

| Weight/kg | Maximum displacement/m | No. of S.A |
|-----------|-------------------------|------------|
|           |                         |            |
| 165.062   | 0.00605                 | 14000      |
| 145.420   | 0.00634                 | 23         |
| 137.590   | 0.00634                 | 25         |

*a* Analysis results of traditional TMA-GA  
*b* Analysis results of improved TMA-GA0.12013

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#### Figure 7. The final topology configuration

As this case very clearly demonstrates, all of three methods got the same topology configuration. However, the TMA-GA method could obtain the better results than pure GA with far less computation and evolving time as shown in Table 5. Compared with traditional TMA-GA, the adding design point strategy improves the accuracy and increases the probability of obtaining the global optimal solution without reducing efficiency too much. The whole mass of the structure has reduced to about 137 kg with only three more structural analyses. It is also worth noting that maximum displacement is significantly different between TMA-GA and pure GA. The constraint values of the TMA-GA has reached 0.00634m which could be count as the critical constraints. It means the results obtained by TMA-GA is closer to the optimal solution.

According to the obtained results, the precision as well as validity of the algorithm are improved by using the proposed method and the final topology configuration is shown in Figure 7.

#### 6. Conclusion

This paper extends the traditional TMA-GA method to solve the complex frame structural optimization. Based on the initial ground structure method, the integrated topology/size optimization model is established, and then simplified by the two-level multipoint approximation strategy. The hierarchic optimization strategy is adopted where discrete variables are optimized with GA and the inner layer continuous optimization problem are solved by dual methods. Some unique strategies are developed to solve the problems emerged in frame structural optimization. The strategy of adding a new complementary design point is proposed to improve the precision as well as reduce the drawback of falling into a local optimal solution. By introducing the temporary deletion techniques, the
phenomenon of incorrect development of components would be contained. With the use of local vibration modes processing techniques, the frequency-constrained problems could be well resolved. As pointed out in the introduction to this paper, the first example shows that the improved algorithm is far more efficient than the pure GA while the second example indicates the precision and better global property of the adding point strategy compared with the traditional TMA-GA. In general, the results of practical cases studies indicate that this improved algorithm can decrease iteration number significantly, and obtain a solution much closer to global optimal one.

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