Trimaximal lepton mixing with a trivial Dirac phase

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Abstract

We present a model which employs the seesaw mechanism with five right-handed neutrinos, leading to trimaximal and $CP$-conserving lepton mixing. Tri-bimaximal mixing is a natural limiting case of our model which occurs when one particular vacuum expectation value is real and preserves the $\mu-\tau$ interchange symmetry of the Lagrangian. Our model allows for leptogenesis even in the case of exact tri-bimaximal mixing.
1 Introduction

It is well known that the Standard Model (SM) of the electroweak interactions is incomplete, because (among other reasons) it offers neither an explanation for the exceptionally small neutrino masses nor for the generation of a baryon asymmetry of the Universe (BAU) of the observed size. One possible way out of these problems consists in the introduction of gauge-invariant right-handed neutrinos. Their Majorana mass terms are not proportional to the vacuum expectation values (VEVs) which break the gauge symmetry and may, therefore, be of a very high mass scale. This leads to a seesaw mechanism \([1]\) generating masses for the standard left-handed neutrinos—masses which are, as a consequence of the high mass scale, strongly suppressed. The extra-heavy right-handed neutrinos also allow for the generation of the BAU through the mechanism of leptogenesis \([2,3]\), which is based on \(CP\)-violating asymmetries in their Yukawa-interaction-mediated decays.

On the experimental side \([4]\), it is by now established that there is neutrino mixing in the weak charged current. This should, according to theory, be parameterized by a \(3 \times 3\) unitary mixing matrix \(U_{\text{PMNS}}\). Although the data are not sufficiently precise yet, it appears that this matrix is close to the tri-bimaximal mixing (TBM) form \([5]\):

\[
U_{\text{PMNS}} \approx U_{\text{TBM}} \equiv \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix},
\]

where we have omitted the possible presence of non-zero Majorana phases multiplying \(U_{\text{TBM}}\) on the right. A milder assumption is that \(U_{\text{PMNS}}\) only displays trimaximal mixing (TM), which is defined as the second column of \(U_{\text{PMNS}}\) being \((1,1,1)^T/\sqrt{3}\).

There exist in the literature models based on the seesaw mechanism which predict TBM \([6,7,8]\) or TM \([9,10]\). Unfortunately, models predicting TBM \([6]\) usually do not allow for leptogenesis, as was recently pointed out by Jenkins and Manohar \([11]\). For recent studies of the relationship between TBM (or rather deviations from TBM) and leptogenesis see \([12]\).

In this paper we propose a generalization of a previous model of two of us \([8]\) which predicted TBM. The generalization leads to TM with the additional prediction of a real (except for Majorana phases) \(U_{\text{PMNS}}\), thus allowing for greater predictivity than just TM. We moreover demonstrate that our model, both in its TM and in its TBM versions, allows for leptogenesis.

2 The model

The model discussed here was introduced in \([8]\) and is an extension of the SM based on the gauge group \(SU(2) \times U(1)\). Using the index \(\alpha = e, \mu, \tau\), the fermion sector consists of \(SU(2)\) doublets \(D_{\alpha L}\), \(SU(2)\) singlets \(\alpha_R\) with electric charge \(-1\) and \(SU(2)\) singlets \(\nu_{\alpha R}\) with electric charge \(0\). There are two additional right-handed neutrino singlets \(\nu_{jR}\) \((j = 1, 2)\), so that the total number of right-handed neutrinos is five. The scalar sector consists of four Higgs doublets \(\phi_0\) and \(\phi_\alpha\), all of them with weak hypercharge \(1/2\), and a complex gauge singlet \(\chi\).
The family symmetries of our model are the following:

- Three $U(1)$ symmetries associated with the family lepton numbers $L_\alpha$. All scalar fields have $L_\alpha = 0$ for all $\alpha = e, \mu, \tau$. The fermion multiplets $D_\beta L$, $\beta_R$ and $\nu_{\beta R}$ have lepton number $L_\alpha = 1$ if $\beta = \alpha$ and $L_\alpha = 0$ otherwise.

- Three $\mathbb{Z}_2$ symmetries $[7, 13]$
  \[ \mathbb{Z}_2^{(\alpha)} : \quad \alpha_R \rightarrow -\alpha_R, \quad \phi_\alpha \rightarrow -\phi_\alpha, \]  
  for $\alpha = e, \mu, \tau$.

- The permutation symmetry $S_3$ of the indices $e, \mu, \tau$. With respect to this $S_3$ the gauge multiplets are arranged in triplets, doublets and one singlet as
  \[
  \begin{pmatrix}
  D_{e L} \\
  D_{\mu L} \\
  D_{\tau L}
  \end{pmatrix},
  \begin{pmatrix}
  e_R \\
  \mu_R \\
  \tau_R
  \end{pmatrix},
  \begin{pmatrix}
  \nu_{e R} \\
  \nu_{\mu R} \\
  \nu_{\tau R}
  \end{pmatrix},
  \begin{pmatrix}
  \phi_e \\
  \phi_\mu \\
  \phi_\tau
  \end{pmatrix},
  \begin{pmatrix}
  \nu_{1 R} \\
  \nu_{2 R}
  \end{pmatrix},
  \begin{pmatrix}
  \chi \\
  \chi^*
  \end{pmatrix},
  \phi_0,
  \]
  respectively. We view $S_3$ as being generated by the $\mu-\tau$ interchange $I_{\mu \tau}$ and the cyclic permutation $C_{e \mu \tau}$, which are represented by
  \[
  I_{\mu \tau} \rightarrow \begin{pmatrix}
  1 & 0 & 0 \\
  0 & 0 & 1 \\
  0 & 1 & 0
  \end{pmatrix},
  C_{e \mu \tau} \rightarrow \begin{pmatrix}
  0 & 1 & 0 \\
  0 & 0 & 1 \\
  1 & 0 & 0
  \end{pmatrix}
  \]  
  (3)
  in the case of triplets and by
  \[
  I_{\mu \tau} \rightarrow \begin{pmatrix}
  0 & 1 & 0 \\
  1 & 0 & 0
  \end{pmatrix},
  C_{e \mu \tau} \rightarrow \begin{pmatrix}
  \omega & 0 & 0 \\
  0 & \omega^2
  \end{pmatrix}
  \]  
  (5)
  for the doublets, where $\omega = \exp(2i\pi/3)$.

The symmetries defined here generate a group which has the structure of a semidirect product, with $S_3$ acting upon the three $U(1)$ and the three $\mathbb{Z}_2$ symmetries—for details see [8]. Under this group the multiplets of (3) are irreducible.

In the model there is both soft and spontaneous symmetry breaking. The soft breaking proceeds stepwise as specified in table 1. The symmetry $I_{\mu \tau}$ is not broken softly. The lepton numbers are softly broken at high energy, i.e. at the seesaw scale [14] where the right-handed neutrino singlets acquire Majorana mass terms. The symmetries $\mathbb{Z}_2^{(\alpha)}$ and $C_{e \mu \tau}$ are softly broken at low energy, i.e. at the electroweak scale. All the symmetries except the family lepton numbers are spontaneously broken.

The multiplets and symmetries uniquely determine the Yukawa Lagrangian

\[
L_{\text{Yukawa}} = -y_1 \sum_{\alpha=e,\mu,\tau} \bar{D}_\alpha L_\alpha \nu_{\beta R} \phi_\beta
\]  
(6a)
| symmetry | dimension | Lagrangian |
|----------|-----------|------------|
| $U(1)_{L_{\alpha}}$ | 3 | $L_{\text{Majorana}}$ |
| $\mathbb{Z}_2^{(\alpha)}$ | 2 | $V$ |
| $C_{e\mu\tau}$ | 2, 1 | $V$ |

Table 1: Soft breaking of the family symmetries of our model. In the first column the symmetry is indicated, the second column gives the dimension of the terms responsible for the soft breaking and the third column specifies the part of the Lagrangian where the soft breaking occurs. $L_{\text{Majorana}}$ refers to the mass terms of the right-handed neutrino singlets; $V$ refers to the scalar potential. $I_{\mu\tau}$ is not softly broken, only spontaneously.

\[-y_2 \sum_{\alpha=e,\mu,\tau} D_{\alpha L} \nu_{\alpha R} (i\tau_2 \phi_0^*) \]  
\[+ \frac{y_3}{2} (\chi \nu^T R C^{-1} \nu_{1R} + \chi^* \nu^T _{2R} C^{-1} \nu_{2R}) + \text{H.c.} \quad \text{(6b)} \]

Note that this Yukawa Lagrangian has a minimal number of couplings. The charged-lepton masses $m_\alpha = |y_1 v_\alpha|$ ($\alpha = e, \mu, \tau$) are different because of the different VEVs $v_\alpha \equiv \langle \phi_\alpha^0 \rangle_0$; these different VEVs of course break spontaneously both $I_{\mu\tau}$ and $C_{e\mu\tau}$.

Taking into account the pattern of soft symmetry breaking outlined in table 1, the Majorana mass terms of the right-handed neutrino singlets are given by

\[L_{\text{Majorana}} = \frac{M_0^*}{2} \sum_{\alpha=e,\mu,\tau} \nu^T_{\alpha R} C^{-1} \nu_{\alpha R} \]  
\[+ M_1^* \left( \nu^T _{eR} C^{-1} \nu_{\mu R} + \nu^T _{\mu R} C^{-1} \nu_{\tau R} + \nu^T _{\tau R} C^{-1} \nu_{e R} \right) \]  
\[+ M_2^* \left[ \nu^T _{1R} C^{-1} \left( \nu_{e R} + \omega \nu_{\mu R} + \omega^2 \nu_{\tau R} \right) + \nu^T _{2R} C^{-1} \left( \nu_{e R} + \omega^2 \nu_{\mu R} + \omega \nu_{\tau R} \right) \right] \]  
\[+ M_4^* \nu^T _{1R} C^{-1} \nu_{2R} + \text{H.c.} \quad \text{(7d)} \]

3 Neutrino masses and lepton mixing

It is easy to derive the $5 \times 3$ neutrino Dirac mass matrix $M_D$ from (6b) and to derive the $5 \times 5$ right-handed-neutrino Majorana mass matrix $M_R$ from (6c) and (7). One obtains

\[M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M_R = \begin{pmatrix} M_0 & M_1 & M_1 & M_2 & M_2 \\ M_1 & M_0 & M_1 & \omega^2 M_2 & \omega M_2 \\ M_1 & M_1 & M_0 & \omega M_2 & \omega^2 M_2 \\ M_2 & \omega^2 M_2 & \omega M_2 & M_N & M_4 \\ M_2 & \omega M_2 & \omega^2 M_2 & M_4 & M_N^* \end{pmatrix}, \]

where $a \equiv y_2^* v_0$ ($v_0 \equiv \langle \phi_0^0 \rangle_0$) and

\[M_N \equiv y_3^* \nu^* _{\chi}, \quad M_N' \equiv y_3^* v_{\chi} \quad \text{with} \quad v_{\chi} \equiv \langle \chi \rangle _0. \]
We assume that \( v_\chi \), and hence \( M_N \) and \( M'_N \), are of the same (very high) scale as the bare Majorana masses \( M_{0,1,2,4} \). We apply the seesaw formula [1] to obtain a light-neutrino mass matrix

\[
M_\nu = -M_D^T M_R^{-1} M_D \begin{pmatrix} x + y + t & z + \omega^2 y + \omega t & z + \omega y + \omega^2 t \\ z + \omega^2 y + \omega t & x + \omega y + \omega^2 t & z + y + t \\ z + \omega y + \omega^2 t & z + y + t & x + \omega^2 y + \omega t \end{pmatrix} .
\] (10)

The precise formulas for \( x \) and \( z \) are found in [8]; here it suffices to know that \( x \) and \( z \) are independent of \( y \) and \( t \). On the other hand [8],

\[
y = -a^2 \frac{(M_0 + 2M_1)M_2^2}{\det M_R} M'_N, \tag{11a}
\]
\[
t = -a^2 \frac{(M_0 + 2M_1)M_2^2}{\det M_R} M_N. \tag{11b}
\]

From (9) and (11) one deduces that \( y/t = e^{2i\xi} \), where \( \xi \equiv \arg v_\chi \). We therefore use the parameterization

\[
y = e^{i\xi} u, \quad t = e^{-i\xi} u, \tag{12}
\]

with a complex \( u \).

Since the phase of \( x \) may be removed from the \( M_\nu \) of equation (10) without destroying its form, that mass matrix has six real degrees of freedom: three moduli \( |x|, |z| \) and \( |u| \), and three phases \( \arg z, \arg u \) and \( \xi \).

If the VEV \( v_\chi \) of \( \chi \) is real, then \( I_\mu \tau \) is not spontaneously broken at the seesaw scale and TBM ensues [8]. We generalize that situation in this paper to allow for a general \( \xi \). As we shall next see, this leads to TM with the added bonus of a real lepton mixing matrix (but for possible Majorana phases).

It is easy to check that the \( M_\nu \) of (10) is diagonalized by

\[
U^T M_\nu U = \text{diag}(\mu_1, \mu_2, \mu_3) , \tag{13}
\]

with

\[
U = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2s/\sqrt{6} \\ -c/\sqrt{6} + s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6} - c/\sqrt{2} \\ -c/\sqrt{6} - s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6} + c/\sqrt{2} \end{pmatrix} , \tag{14}
\]

where \( c \equiv \cos(\xi/2) \) and \( s \equiv \sin(\xi/2) \). It is remarkable that, despite the occurrence of three phases in \( M_\nu \), the diagonalization matrix \( U \) is real, i.e. there is no Dirac phase. The eigenvalues of \( M_\nu \), though, are complex, i.e. there are Majorana phases:

\[
\mu_1 = x - z + 3u, \tag{15a}
\]
\[
\mu_2 = x + 2z, \tag{15b}
\]
\[
\mu_3 = x - z - 3u. \tag{15c}
\]

They have the same dependence on the three complex parameters as in the TBM case. Since in our model the charged-lepton mass matrix is automatically diagonal, \( U = U_{\text{PMNS}} \) but for the Majorana phases which result from rendering \( \mu_{1,2,3} \) real and positive.
Table 2: The solar and atmospheric mixing angles as functions of $|U_{e3}|^2$ in our model.

| $|U_{e3}|^2$ | $\tan^2 \theta_\odot$ | $\cos^2 2\theta_{\text{atm}}$ |
|-------------|-----------------|-----------------|
| 0           | 0.5             | 0               |
| 0.01        | 0.5076          | 0.0201          |
| 0.02        | 0.5155          | 0.0404          |
| 0.03        | 0.5236          | 0.0609          |
| 0.04        | 0.5319          | 0.0816          |
| 0.05        | 0.5405          | 0.1025          |

If we define

$$\tan^2 \theta_\odot \equiv \left| \frac{U_{e2}}{U_{e1}} \right|^2,$$

(16a)

and

$$\cos 2\theta_{\text{atm}} \equiv \frac{|U_{\tau 3}|^2 - |U_{\mu 3}|^2}{|U_{\tau 3}|^2 + |U_{\mu 3}|^2},$$

(16b)

then we see that in our model these quantities are functions of $|U_{e3}|^2$:

$$\tan^2 \theta_\odot = \frac{1}{2 - 3 |U_{e3}|^2},$$

(17a)

$$\cos^2 2\theta_{\text{atm}} = \frac{|U_{e3}|^2 - 2 - 3 |U_{e3}|^2}{(1 - |U_{e3}|^2)^2}. $$

(17b)

As always when mixing is trimaximal, $\tan^2 \theta_\odot$ cannot be smaller than $1/2$ [9, 15]; this is slightly disfavoured by experiment [16, 17]. Equations (17) are translated into numeric form in table 2 which we should compare to the experimental values [17]

$$|U_{e3}|^2 \leq 0.056 \ (3\sigma),$$

(18a)

$$\tan^2 \theta_\odot = 0.437^{+0.047}_{-0.033},$$

(18b)

$$\cos^2 2\theta_{\text{atm}} \leq 0.02 \ (1\sigma).$$

(18c)

We see that, in the context of our model, the experimental bound on $\theta_{\text{atm}}$ places significant constraints on both $|U_{e3}|$ and $\theta_\odot$.

As argued before, the mass matrix (10) has six parameters; these correspond to the three neutrino masses, which are free in our model, the modulus $|U_{e3}|$ and the two Majorana phases. If one wishes, the Majorana phases can be enforced to be trivial because
our model is compatible with the $CP$ transformation

\[
CP : \begin{cases}
D_{\alpha L} & \rightarrow i S_{\alpha \beta} \gamma^0 \overline{C}_{D_{\beta L}}^T, \\
\alpha_R & \rightarrow i S_{\alpha \beta} \gamma^0 \overline{C}_{\alpha_R}^T, \\
\nu_{\alpha R} & \rightarrow i S_{\nu \beta} \gamma^0 \overline{C}_{\nu_{\beta R}}^T, \\
\nu_{j R} & \rightarrow i \gamma^0 \overline{C}_{\nu_{j R}}^T, \\
\phi_\alpha & \rightarrow S_{\alpha \beta} \phi_\beta^* \rightarrow \phi_0^* \\
\chi & \rightarrow \chi^*
\end{cases}
\]  

with $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.  \hfill (19)

The effect of this $CP$ symmetry is to render $y_{1,2,3}$ in (5) and $M_0,1,2,4$ in (7) real. However, in that case we would have $M_N^* = M_N$ which would make leptogenesis impossible, as one can deduce from the computations in the next section. Therefore, we will not impose this $CP$ symmetry in the following.

### 4 Leptogenesis

In order to study leptogenesis we need the masses $\hat{M}_k \ (k = 1, \ldots, 5)$ of the heavy neutrinos and the diagonalization matrix $V$ of $M_R$: 

\[
V^T M_R V = \text{diag} \left( \hat{M}_1, \hat{M}_2, \hat{M}_3, \hat{M}_4, \hat{M}_5 \right), \hfill (20)
\]

where $V$ is $5 \times 5$ unitary and the $\hat{M}_k$ are real and positive. The relevant matrix for leptogenesis is then $R \equiv V^T M_D M_D^T V^*$.

Given the form of $M_D$ in (8), we find

\[
R = |a|^2 \overline{V}^T \overline{V}, \hfill (21)
\]

where $\overline{V}$ is the upper $3 \times 5$ submatrix of $V$:

\[
V = \begin{pmatrix} \overline{V} \\ \hat{V} \end{pmatrix}, \quad \overline{V} : 3 \times 5 \text{ matrix}, \quad \hat{V} : 2 \times 5 \text{ matrix}. \hfill (22)
\]

We make the Ansatz

\[
V = V_u V_b, \quad V_u = \begin{pmatrix} U_{\text{TBM}} & 0 \\ 0 & U_{\text{BM}} \end{pmatrix}, \quad \text{where } U_{\text{BM}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}. \hfill (23)
\]

It is clear that

\[
R = |a|^2 \overline{V}_b^T \overline{V}_b^*, \hfill (24)
\]

where

\[
V_b = \begin{pmatrix} \overline{V}_b \\ \hat{V}_b \end{pmatrix}, \quad \overline{V}_b : 3 \times 5 \text{ matrix}, \quad \hat{V}_b : 2 \times 5 \text{ matrix}. \hfill (25)
\]
$V_b$ is the unitary matrix that diagonalizes

$$V_u^T M_R V_u = \begin{pmatrix} M_0 - M_1 & 0 & 0 & \tilde{M}_2 & 0 \\ 0 & M_0 + 2M_1 & 0 & 0 & 0 \\ 0 & 0 & M_0 - M_1 & 0 & -i\tilde{M}_2 \\ \tilde{M}_2 & 0 & 0 & \tilde{M}_N + M_4 & \tilde{M}_N \\ 0 & 0 & -i\tilde{M}_2 & \tilde{M}_N & \tilde{M}_N - M_4 \end{pmatrix}, \quad (26)$$

where $\tilde{M}_2 \equiv \sqrt{3}M_2$, $\tilde{M}_N \equiv (M'_N + M_N)/2$ and $\hat{M}_N \equiv (M'_N - M_N)/2$. One sees that $\hat{M}_1 = |M_0 + 2M_1|$. The matrix (26) neatly separates into two $2 \times 2$ submatrices in the special case $M'_N = M_N$ which corresponds to TBM; we shall from now on restrict ourselves to that special case. From (26) one deduces that $\bar{V}_b$ has the structure

$$\bar{V}_b = \begin{pmatrix} e^{i\varphi_1} \cos \psi_1 & 0 & 0 & e^{i\varphi_2} \sin \psi_1 & 0 \\ 0 & e^{i\theta} & 0 & 0 & 0 \\ 0 & 0 & e^{i\varphi_3} \cos \psi_2 & 0 & e^{i\varphi_4} \sin \psi_2 \end{pmatrix}, \quad (27)$$

hence

$$R = |a|^2 \begin{pmatrix} \cos^2 \psi_1 & 0 & 0 & n_1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \cos^2 \psi_2 & 0 & n_2 \\ n_1^* & 0 & 0 & \sin^2 \psi_1 & 0 \\ 0 & 0 & n_2^* & 0 & \sin^2 \psi_2 \end{pmatrix}, \quad (28)$$

where $n_1 \equiv e^{i(\varphi_1 - \varphi_2)} \sin \psi_1 \cos \psi_1$ and $n_2 \equiv e^{i(\varphi_3 - \varphi_4)} \sin \psi_2 \cos \psi_2$.

Let us now suppose that $\hat{M}_1 \ll \hat{M}_{2,3,4,5}$. Then, the $CP$-violating asymmetry relevant for leptogenesis is

$$\epsilon_1 = \frac{1}{8\pi |v_0|^2} R_{11} \sum_{k=1}^{5} \text{Im} [(R_{1k})^2]. \quad (29)$$

It is clear that the only non-zero contribution to the sum occurs for $k = 4$, which means that in this case leptogenesis involves only two heavy neutrinos, and

$$\epsilon_1 = \frac{|a|^2}{8\pi |v_0|^2} \sin \psi_1 \sin (2\varphi_1 - 2\varphi_2) = \frac{|y_2|^2}{8\pi} \sin \psi_1 \sin (2\varphi_1 - 2\varphi_2). \quad (30)$$

Notice that this $CP$-violating asymmetry crucially depends on the difference $2\varphi_1 - 2\varphi_2$ between the Majorana phases of the two heavy neutrinos with masses $\hat{M}_1$ and $\hat{M}_4$.

In our model we are able to obtain a non-zero leptogenesis even in the case, treated above, of TBM, i.e. even when $M'_N = M_N$. This does not conform with the observation in [11] that the models in the literature that generate exact TBM using a flavour symmetry do not have leptogenesis. The crucial point is that our model has more than
three (specifically, five) right-handed neutrinos and this leads to a matrix $\tilde{V}$ in (21) which is not a unitary $3 \times 3$ matrix—in which case $R \propto 1$ would be trivial and leptogenesis would not be possible—but rather a $3 \times 5$ submatrix of a unitary $5 \times 5$ matrix. Thus, our model serves as an example of how it is possible to evade the limitation pointed out in [11]: having more than three right-handed neutrinos in the seesaw mechanism allows one to reconcile TBM with leptogenesis.

It is well known that in the most general case the phases responsible for $CP$ violation at low energies and those responsible for leptogenesis are not related [18]. This is the case in the present model: the Majorana phases in the diagonalization matrix of $M_\nu$, which contribute to neutrinoless double-$\beta$ decay, and those in the diagonalization matrix $V$ of $M_R$ are not directly related. This is in contrast to the $\mu-\tau$-symmetric model of [19] with a $3 \times 3$ matrix $M_R$ where those phases are identical. Nevertheless, if we apply the $CP$ transformation (19) to the present model, one can read off from the matrix (26) that both sets of phases, for neutrinoless double-$\beta$ decay and for leptogenesis, become trivial.

5 Conclusions

In this paper we have generalized the model introduced in [8] by allowing for a complex VEV $v_\chi$ of the scalar gauge singlet $\chi$. This generalization leads to trimaximal lepton mixing, with tri-bimaximal mixing as the limiting case when $v_\chi$ is real. Although the phase $\xi$ of $v_\chi$ shows up in the light-neutrino mass matrix $M_\nu$, it does not induce a non-trivial Dirac phase in the PMNS matrix but rather a non-zero element $U_{e3}$. This is a special feature of the model due to its symmetries and to the five right-handed neutrino singlets in the seesaw mechanism. Another property of the model, resulting from five instead of three right-handed neutrinos, is the possibility of leptogenesis even in the limit of exact TBM. It is worth noting that, like its precursor in [8], the model has a minimal number of Yukawa couplings; the price to pay is that the different charged-lepton masses have to be generated by different VEVs, which in turn needs a sufficiently rich scalar potential. As for the renormalization-group evolution of $M_\nu$, it is possible to achieve stability under the one-loop evolution through a slightly different choice of symmetries, in the same way as discussed in [8, 10].

Since the model predicts trimaximal mixing and thus $\sin^2 \theta_\odot \geq 1/3$, where $\theta_\odot$ is the solar mixing angle, there is a slight tension with the present fits to the neutrino-oscillation data and the model can probably be tested in the near future, also by taking into account the correlations, given in (17), between the three mixing angles.

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