Analytical Model of Spin-Polarized Semiconductor Lasers

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We formulate an analytical model for vertical-cavity surface-emitting lasers (VCSELs) with injection (pump) of spin-polarized electrons. Our results for two different modes of carrier recombination allow for a systematic analysis of the operational regimes of the spin-VCSELs. We demonstrate that threshold reduction by electrically-pumped spin-polarized carriers can be larger than previously assumed possible. Near the threshold, such VCSELs can act as effective non-linear filters of circularly-polarized light, owing to their spin-dependent gain.

Spin-dependent properties have been successfully used in metallic magnetic multilayers for a variety of device applications exploiting magnetoresistive effects. Unfortunately, much less is known about practical paths to implement spin-controlled devices that would go beyond magnetoresistance. One such encouraging development is demonstrated with spin-polarized vertical cavity surface-emitting lasers (spin-VCSELs). Spin-polarized carriers created by circularly-polarized photo-excitation or electrical injection can enhance the performance of spin-VCSELs as compared to their conventional (spin-unpolarized) counterparts. This work has demonstrated threshold current reduction and independent modulation of optical polarization and intensity. While numerical results using the rate equation (RE) description of spin-VCSELs were already presented, a large number of materials parameters makes it difficult to systematically elucidate how the spin-dependent effects modify the device operation. Consequently, even the widely accepted theoretical limit of maximum 50% threshold current reduction needs to be re-examined.

For clarity of our approach we formulate a simple RE model of spin-VCSELs which, while similar to prior numerical work, can be solved analytically. We consider a quantum well (QW) as the active region in the VCSEL. Spin-resolved electron and hole densities are \( n_\pm \), \( p_\pm \), where \(+(-)\) denotes the spin up (down) component; the total carrier densities are \( n = n_+ + n_- \), \( p = p_+ + p_- \). Analogously, for photon density we write \( S = S^+ + S^- \), where \(+(-)\) is the right (left) circularly-polarized component. Electrically or optically injected/pumped spin-polarized electrons into the QW can be represented by current density \( P \). Electromagnetically, we consider a continuous wave injection \( \propto \tau \). The spin relaxation time of holes is much shorter than for electrons, \( \tau^h \ll \tau^e \), implying that the holes can be considered unpolarized with \( p_\pm = p/2 \).

In conventional VCSELs, the optical gain term, describing stimulated emission, can be simply modeled as \( g(n, S) = \frac{g_0(n - n_{\text{tr}})}{1 + \epsilon S} \), where \( g_0 \) is the density-independent coefficient, \( n_{\text{tr}} \) is the transparency density, and \( \epsilon \) is the gain compression factor. We generalize this relation in the spin-polarized case as \( g(n, S) \rightarrow g_\pm(n_\pm, p_\pm, S^\pm) = \frac{g_\pm(n_\pm + p_\pm - n_{\text{tr}})}{(1 + \epsilon S)} \). This form differs from the previously employed gain expressions as it explicitly contains the hole density, but it coincides with the rigorously derived gain expression from semiconductor Bloch equations.

The charge neutrality \( p_\pm = p/2 = n/2 \), allows us to recover the spin-unpolarized limit for the gain expression, as well as to decouple the REs for electrons from those for holes. The spin-polarized REs for electrons thus become

\[
dn_\pm/\text{dt} = J_\pm - g_\pm(n_\pm, S^\mp) S^{\mp} - (n_\pm - n_{\text{tr}})/\tau^e - R^\pm_\p, \tag{1}
\]

\[
dS^{\mp}/\text{dt} = \Gamma g_\mp(n_\mp, S^{\mp}) S^{\pm} - S^{\pm}/\tau_{\text{ph}} + \beta TR^3_\p, \quad \tag{2}
\]

where \( \tau_{\text{ph}} \) is the photon lifetime, \( \Gamma \) is the optical confinement coefficient, \( \beta \) is the spontaneous-emission coupling coefficient, and \( R^\pm_\p \) is the radiative spontaneous recombination rate. For the spin-unpolarized case, \( R^\pm_\p \) is typically assumed to be quadratic in carrier densities, \( R^\pm_\p = Bn_{\pm}^2/2 \), where \( B \) is a temperature-dependent constant. Another simple form, \( R^\p_\p = n/\tau_r \), \( \tau_r \) is the recombination time, is a good approximation at high \( n_\mp \) but was not considered in the prior work on spin-VCSELs. For spin-polarized electrons these forms are generalized as \( R^\p_\p = Bn_{\pm}^2 \). Electrically injection in QWs, using Fe or FeCo, allows for \( |P_n| \sim 0.3 - 0.7 \) with similar values for \( |P| \), while \( |P_n| \rightarrow 1 \) is attainable optically at room temperature. In the unpolarized limit \( J_r = J_\mp, n_\mp = n_{\text{tr}}, S^+ = S^- \) or \( P_r = P_n = P_{\text{ph}} = 0 \). As in the recent experiments, we consider a continuous wave operation of VCSEL and look for steady-state solutions of Eqs. (1,2). Guided by the experimental range \( 0 < \beta < 10^{-5} \), we mostly focus on the limit \( \beta = 0 \), for which all the operating regimes of spin-VCSELs can be simply described, and, additionally, consider \( \epsilon = 0 \), relevant for moderate pumping intensities.

We first show analytical results for unpolarized \( (P_r = 0) \) VCSEL and LR in the inset of Fig. (1) Injection current density is normalized to the unpolarized threshold value, \( J_r = N_{\text{tr}}/\tau_r \), with \( N_{\text{tr}} \) denoting the total electron density at (and above) the threshold, \( N_{\text{tr}} = n(J > J_r) = \)
regimes of a spin-VCSEL. (i) For $J < J_1$, still in a spin-LED regime, the photon densities ($S =$ $S^+ + S^-$) is shown for spin-unpolarized laser ($p_J = 0$) and two spontaneous-emission coupling coefficients ($\beta = 0, 0.002$), as well as a spin laser with $p_J = 0.5$, $\beta = 0$.

The unpolarized case has two regimes: for $J < J_T$, the device behaves as a light-emitting diode (LED), with negligible stimulated emission; for $J > J_T$, it is a fully lasing VCSEL.

With finite $p_J$ and $\tau_n^u \to \infty$, we reveal a more complicated behavior further explored in Fig. 1 main panel. The two threshold currents, $J_{T1}$ and $J_{T2}$, the equalities hold only when $p_J = 0$, delimit three regimes of a spin-VCSEL. (i) For $J < J_{T1}$, it operates as a spin-LED (ii) For $J_{T1} \leq J \leq J_{T2}$, there is mixed operation: lasing only with left-circularly polarized light $S^-$ (we assume $J_+ > J_-$), which can be deduced from the spin-dependent gain term in Eqs. (1,2), while $S^+$ is still in a spin-LED regime ($S^+ \to 0$ for $\beta \to 0$). (iii) For $J \geq J_{T2}$, it is fully lasing with both $S^+ > 0$.

RE description of spin-unpolarized lasers reveals that the carrier densities are clamped above $J_T$. We have found a related effect in the spin-polarized case. For $J > J_{T1}$, Eq. (2) for $S^-$ can be divided by $S^-$, showing that the quantity $n_+ + p_+ = n_+ + n/2$ (we recall the charge neutrality condition and $\tau_n^u \to 0$) will be clamped at $N_T$. If $J_{T1} < J < J_{T2}$ then neither $n_+$ nor $n_-$ are separately clamped. If $J > J_{T2}$, then $n_+ + n/2 = N_T$ must hold in addition the previous condition $n_+ + n/2 = N_T$.

These two conditions yield $n_+ = N_T/2$, independent of $p_J$. Thus, above $J_{T2}$, the sum of Eqs. (3) for $S^-$ and $S^+$ reduces to the usual unpolarized equation, and $S$ is independent of $p_J$ (inset of Fig. 1). However, if $p_J \neq 0$ then we still find $S^- \neq S^+$ (Fig. 1). J > $J_T$.

Most of the results discussed above do not change qualitatively for a finite $\tau_n^u$ or QR. The general features, such as the existence of three regimes of operation, remain the same. However, the photon and carrier densities (for a given $J$) as well as the threshold $J_{T1}$ depend quantitatively on the spin-flip rate and the recombination form. We investigate this dependence in Fig. 2 which shows the evolution of photon and carrier polarizations with the injection current. We consider both LR and QR forms and express our results using the ratio of the radiative recombination and spin relaxation times: $t = \tau_s/\tau_n^u$. For QR the unpolarized threshold is $J_T = BN_T^2/2$, and we define $\tau_s = N_T/J_T$ by analogy with the LR case.

When $0 < J < J_{T1}$, then $P_n(J)$ depends on the recombination form; $P_n$ is constant for LR, while for QR it grows monotonically. Only for a very long spin relaxation time, $t \to 0$, $P_n(J < J_{T1}) \to P_J$. At $\beta = 0$ there is no stimulated emission, so that $P_S \equiv 0$. When $J_{T1} < J < J_{T2}$, $P_S = -1$ for any $t$, recall Fig. 1. With only a partially polarized electron current, the spin-VCSEL emits fully circularly-polarized light, analogous to the spin-filtering effect in magnetic materials. In the same regime, $P_n$ decreases with $J$, because, while $n_-$ grows, $n_+$ must drop in order to maintain $n_+ + n/2 = 3/2n_+ + 1/2n_- = N_T$.

For $p_J \neq 0$, the $J_{T2} - J_{T1}$ interval is widest for $\tau_n^u \to \infty$. When $\tau_n^u$ decreases then the lower threshold $J_{T1}$ grows towards its unpolarized counterpart $J_T$, and $J_{T2} - J_{T1}$ contracts, Fig. 2. We find $J_{T2} = J_T/(1 - p_J)$.
n_/ N_T, so that } ~ 10^{-4} can effectively drive the device to the fully lasing regime. For typical values of } \sim 5, setting } = 0 is an accurate assumption. At }p_2, we recall } = N_T/2 so that the spin-flip term in Eq. 1 for } is zero. The equation reduces to } \equiv 1/2(1 - }_J)J = R^\text{sp}_p, explaining why }p_2 depends only on }_J, but not on }. For the same reason, }_S and }_n are independent of } when } \geq }p_2. The quantity }_S increases (|}_S| decreases) with } as }_S = -}_p J/(J - }_J) to the asymptotic value }_S = -}_p. Thus }_p can be inferred from }_S only at sufficiently high injection. In all three regimes, defined by Fig. 1, we find that 0 \leq }_n < }_p, (only for }_n \to \infty and } < }_p). For } > }_p we find }_n = 0. Therefore }_n is not enhanced in the spin-VCSEL’s active region.

Experiments have demonstrated that injecting spin-polarized carriers reduces the threshold current in a VCSEL, i.e., }_p( }_J \neq 0) < }_p. We quantify this threshold-current reduction with } = (} - }_p)/}_, obtained analytically from REs

\begin{align*}
\text{LR: } & \quad } = |} |/(2 + |} | + 4), \\
\text{QR: } & \quad } = 1 - 2/ (2 + |} |)^2 \times \left[1 + 2|} | + 4t^2 + (1 - 2t) \sqrt{(1 + 2t)^2 + 4|} | t}\right],
\end{align*}

and we find } > 0, for any } \neq 0, }_n > 0 and both LR and QR. As expected, } \to 0 for }_n \to 0 (t \to \infty), independent of the recombination form, since we have always assumed }_n = 0. We illustrate further } for both LR and QR in Fig. 3. We choose } = 1.5, close to the typical values }_s \sim 1 \text{ ns and }_n \sim 100 \text{ ps for QWs at room temperature.} From Fig. 3 or Eqs. 3,4 one can see that } = |} | for LR is always smaller than for QR. For } = 1 and }_n \to \infty (but still }_n = 0) we obtain the maximum current reductions

\begin{align}
\text{LR: } & \quad } = 1/3, \quad \text{QR: } = 5/9.
\end{align}

Remarkably, for QR the maximum } is larger than previously assumed possible } = 1/2, even though holes are completely unpolarized (} = 0). This can be explained as follows. First we calculate } from Eq. 2 for }^- . The threshold density is reached when }_n + n/2 = N_T, which gives } = (2/3)N_T for }_J = 1, i.e., for }_n = n. Thus the threshold electron density is reduced by only }/3. Assuming } = 0 at the threshold, the threshold current from Eq. 1 is given by }_s = Bn^2/2. If }_J = 1 then } = }_s = Bn^2/2 = (2/3)^2}_. Thus } \to 0 as } \to \infty, a direct consequence of the quadratic dependence of recombination on } (note that for LR both }_T and }_J are reduced only by }/3).

For comparison, we calculate } for QR in a priori “ideal” case, where } = } for any injection } and both }_n, }_n \to \infty. Therefore the gain and QR terms in Eq. 1 for }_n (} ) and in Eq. 2 for }^- (}^+) must be modified by replacing }/2 with }_n (} ). Surprisingly, we obtain } = 1/2, a smaller reduction than in the case of unpolarized holes. A simple calculation yields } = N_T/2, i.e., the reduction of the threshold electron density is larger than for }_n = 0. However, the interband recombination }^+ is more efficient, because none of the holes undergo spin-flip. Thus we obtain }_s = }_T = Bn^2 = BN^2/4 = }_J/2. For }_n = 0 and }_J = 1, half of the holes are inaccessible for recombination with the fully spin-polarized electrons. This lowers the injection necessary to overcome the recombination losses and the interplay of stimulated and spontaneous recombination leads to a smaller } in the “ideal” case.

In this work we show that even a simple rate equation model can reveal surprising trends for operation of spin-VCSELs. The maximum threshold current reduction is not achieved for infinite electron and hole spin relaxation times, but rather when the hole spin lifetime vanishes. The corresponding threshold reduction exceeds the previously considered theoretical limit. We expect that the transparency of our analytical approach will help to elucidate the operation of spin-controlled lasers, which continue to be actively studied.15

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Threshold current reduction } as a function of injection polarization }_J. Shown are both linear and quadratic recombination (LR and QR) for various }_n, the ratio of recombination time and electron spin relaxation time. The hole spin relaxation time }_s = 0, except for the dashed line and what was previously assumed “ideal” case, in which }_n \to \infty.
\end{figure}

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