Period Color and Amplitude Color relations for MACHO project LMC RR Lyraes

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Received XX XXX 2004 / Accepted XX XXX 2004

ABSTRACT
In this paper, we analyze period color and amplitude color relations at minimum, mean and maximum V band light for 6391 RRab stars in the Large Magellanic Cloud obtained by the MACHO project. Specifically, we find that color and amplitude are nearly independent of period at minimum light but that there exists a definite relation between period and color and amplitude and color at maximum light. These two properties are easily explained by the application of the Stefan Boltzmann law and the interaction of the photosphere and hydrogen ionization front at minimum light. When we examine the slope of the period color relation as a function of phase, we find that the slope varies significantly with phase and is small for a wide range of phases around minimum light. This suggests that another factor that needs to be considered when trying to understand RR Lyrae observed properties is their behavior at different phases during a pulsation cycle.

Key words: RR Lyraes – Stars: fundamental parameters

1 INTRODUCTION
Kanbur and Ngeow (2004, hereafter KN) and Kanbur, Ngeow and Buchler (2004, hereafter KNB) and Ngeow and Kanbur (2004b) analyzed Cepheid light curves in the Galaxy, LMC and SMC to derive period color (PC) and amplitude color relations (AC) as a function of phase. They analyzed their results in the context of the work of Simon Kanbur and Mihalas (1993, hereafter SKM). SKM presented the following equation, derived from an application of the Stefan Boltzmann law at maximum and minimum light as-
\[ \log L_{\text{max}} - \log L_{\text{min}} = 4(\log T_{\text{max}} - \log T_{\text{min}}) + \text{const.}, \]
(1)
where \( L_{\text{max}} \), \( L_{\text{min}} \), \( T_{\text{max}} \), \( T_{\text{min}} \) are photospheric luminosities and temperatures at these phases. If we consider wavelength bands which are good indicators of temperature, say \( X \), \( Y \), so that \( \log T = a + b(X - Y) \), then equation (1) becomes
\[ X_{\text{min}} - X_{\text{max}} = -10.b((X - Y)_{\text{max}} - (X - Y)_{\text{min}}) + \text{const.}, \]
(2)
where \( (X - Y)_{z} = X_{z} - Y_{z} \), and \( Y_{z} \) denotes the value of \( Y \) at \( X_{z} \). If, for some physical reason, there is a flat relation between period and either \( T_{\text{max}} \) or \( T_{\text{min}} \), then equations (1) and (2) predict that there will be a relation between amplitude and the \( T_{\text{min}} \) or \( T_{\text{max}} \) respectively. Assuming our observations are in bands which are good indicators of temperature, then a flat relation between period and color at max/min is related to a relation between amplitude and color at min/max light.

Whilst Cepheids display a flat relation between period and color at maximum light (KN, KNB), Sturch (1967) and Clementini et al (1995) published evidence that fundamental mode RR Lyrae stars (RRab) are such that they follow a flat relation between period and effective temperature at minimum light. Kanbur (1995) and Kanbur and Phillips (1996) again used radiative hydrodynamical models of RRab stars to provide credible evidence that the physical reason for this was again the interaction of the HIF and photosphere. For RRab stars, this interaction occurs at minimum light. This difference, between Cepheids and RR Lyraes, in the phase at which an interaction between the photosphere and HIF occurs is caused by the hotter temperatures and lower \( L/M \) ratios relevant for RRab stars (Kanbur 1995). However, these previous studies did not look for evidence of amplitude-color (AC) relations. Extensive data in two wavelength bands for 6391 RR Lyrae stars in the LMC discovered by the MACHO project (Alcock et al, 2003) provides an ideal opportunity both to verify earlier observational results and look for AC relations in RRab stars. This is the main thrust of this paper.
2 THE DATA

The data considered were obtained by the MACHO project and initially analyzed in Alcock et al (2003). These data consist of, on average, some 400-800 points per star in both MACHO V and \( R \) bands and were kindly supplied by Kovacs (2004). There are 6391 stars considered in our sample. We used the periods published in Alcock et al (2003). In a future paper, we will compare these periods with those calculated by the Lomb-Scargle method (Lomb 1976, Scargle 1982). We then used this published period to fit a sixth order Fourier series to both V and \( R \) band data. Figure 1 shows a representative star from our sample. The solid line displays the Fourier fit and the open squares the original V band data. We estimated quantities at maximum, mean, minimum V band light and colors using the Fourier fits to both V and \( R \) band data. We removed stars whose periods did not lie between 0.3 and 0.9 days and whose \( V - R \) color did not lie between 0 and 0.6. We also removed stars not classified as fundamental mode RRab stars by Alcock et al (2003). This left 4830 stars which we studied further. We leave the detailed treatment of the remaining stars for another paper. We emphasize that we did not correct our \( V - R \) colors for extinction.

3 RESULTS

Our period color (PC) and amplitude-color relations at maximum, mean and minimum V band light are given in figures 2-4 and figures 5-6 respectively. The two rows of table 1 provide the slopes, together with their standard errors, of straight line fits to these data for PC and AC relations. We see that the PC relation at minimum light has a slope close to zero (0.041 \( \pm \) 0.012) whilst the PC relation at maximum light has a significantly different slope (0.423 \( \pm \) 0.015). The AC relation also has a slope close to zero at minimum light (\( -0.0005 \pm 0.003 \)). This slope becomes significantly different at maximum light (\( -0.180 \pm 0.002 \)). In both PC and AC cases, the behaviour at mean light is intermediate between the behaviour at maximum/minimum light. The large number of stars means that the slopes are quite accurately determined. These results are robust to changes in the color cuts adopted in the previous section. For example, the difference in the slopes of the PC and AC relations at maximum and minimum light is still present even when we exclude stars whose \( V - R \) colors do not lie between 0.2 and 0.4. Our results are similarly robust to changes in the adopted period cuts. Figures 2-7 and table 1 are completely consistent with the theory developed in SKM, Kanbur (1995), Kanbur and Phillips (1996) and Kanbur, Ngeow and Buchler (2004).

Figure 8 provides a plot of the slope in a PC relation against the phase at which the PC relation is evaluated, where we specify that phase 0 is minimum V band light. We see clearly that there is a large range in slope and that this slope decreases to its lowest value at minimum light and is in fact smaller than 0.1 for a range of phases (\( \approx 0.4 \)) around minimum light. Further, the slope at mean light given in table 1 is very close to the figure obtained when taking the straight average of the slope at each phase point in figure 8. Hence PC relations at mean light will be affected by the behaviour of PC relations at other phases and in particular by the physics of RRab stars at minimum light. Figure 8 not only provides further evidence for our results but also implies that in order to fully understand observed properties of RRab stars, it is sometimes necessary to study those properties as a function of phase.

Could not correcting for extinction be affecting our results? Were this to be true, figures 2-7 imply that this omission would have a rather drastic and systematic nature. For example in order to use this to "make" the PC or AC relation at maximum light flat, would mean that extinction of the order of 0.1 - 0.2 mags affects about half of these stars. A proper accounting for extinction would probably make our relations tighter.

Could an amplitude bias due to crowded field photometry affect our results? Alcock et al (2004) have discussed how to estimate such a bias. They used these estimates to correct estimates of the mean magnitude of first overtone RRC stars in the MACHO LMC sample. Their estimate of amplitude bias ranged from -0.1 to -0.2 mags. Even if a large fraction of the stars in figure 5 had amplitudes underestimated by 0.2 mags, this would not be enough to remove the non-zero slope in this figure.

We have produced similar PC/AC plots at these three phases for all the other types of RR Lyrae stars identified by Alcock et al (2003). BL1 and BL2 type Blazhko variables display similar behaviour. Results for the 36 RRc stars in this sample are inconclusive. The stars classified as binary stars do not exhibit the behaviour displayed in figures 3-7.

4 CONCLUSIONS AND DISCUSSION

By studying the large sample of RRab stars in the LMC provided by the MACHO project, and using the MACHO \( V - R \) colors, we have found compelling evidence that RRab stars in the LMC display a PC and AC relation that is flat at minimum light but that has a significant slope at maximum light. By considering the PC and AC relations at minimum and maximum light in the context of SKM, KN and KNB, we also found strong evidence that these observational features of RRab stars are caused by the interaction of the photosphere and hydrogen ionization front though we caution that final confirmation must await the construction of new hydrodynamic models of RRab stars in the LMC. In particular we show convincing evidence of a relation between V band amplitude and \( V - R \) color at maximum light such that higher amplitude stars are driven to hotter temperatures at V band maximum light. Galactic Cepheids display the "opposite" property: a flat PC relation at maximum light and an AC relation such that higher amplitude stars are driven to cooler temperatures at minimum light (SKM, KN, KNB). Note that this is more than just saying brightness fluctuations are caused predominantly by temperature variations. This is indeed true for classical Cepheid variables, but in addition, given a way to get a flat PC relation at max/min light, then we will have an AC relation at the opposite max/min phase. Further, since the PC relation for RRab stars is flat for a large range of phases around minimum light, this work suggests that the study of the way the photosphere interacts with the HIF as a function of phase and metallicity will provide deeper insights into the observed
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Figure 1. Fourier decomposition curve together with original V band data for a representative star in the sample.

Figure 2. Period against V-R color at maximum V band light.

Figure 3. Period against V-R color at mean V band light.

Figure 4. Period against V-R color at minimum V band light.

properties of both RR Lyraes and Cepheids. These projects are currently under way.

ACKNOWLEDGMENTS
This paper utilizes public domain data obtained by the MACHO project, jointly funded by the US Department of Energy through the University of California, Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48, by the National Science Foundation through the Center for Particle Astrophysics of the University of California under cooperative agreement AST-8809616, and by the Mount

Table 1. Period color and Amplitude color slope with error at maximum, mean and minimum V band light

|       | Minimum          | Mean            | Maximum         |
|-------|------------------|-----------------|-----------------|
| PC    | $0.0411 \pm 0.012$ | $0.118 \pm 0.011$ | $0.423 \pm 0.015$ |
| AC    | $-0.0005 \pm 0.003$ | $-0.030 \pm 0.003$ | $-0.180 \pm 0.002$ |
Figure 5. $V$ band amplitude against $V-R$ color at maximum $V$ band light.

Figure 6. $V$ band amplitude against $V-R$ color at mean $V$ band light.

Figure 7. $V$ band amplitude against $V-R$ color at minimum $V$ band light.

Figure 8. Plot of fitted phase against PC slope.

Stromlo and Siding Spring Observatory, part of the Australian National University.

SMK thanks Geza Kovacs for providing the raw data and to D. Welch, K. Cook and D. Alves for stimulating discussions. IF thanks the Massachusetts Space Grant Consortium for funding a FCRAO summer internship in 2004 when this work was completed.

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