Determination of the weak phase $\gamma = \text{arg}(V_{ub}^*)$

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Abstract: Various strategies for extracting or constraining the weak phase $\gamma$ with controlled theoretical uncertainties are reviewed. Measurements of the rates for the hadronic decays $B^{\pm} \to \pi K$ provide largely model-independent information on $\gamma$. Hadronic uncertainties enter only at the level of nonfactorizable contributions to the decay amplitudes that are power-suppressed in $\Lambda/m_b$ and, simultaneously, violate $\text{SU}(3)$ flavor symmetry or are doubly Cabibbo suppressed. In addition, these decays have a rich potential for probing physics beyond the Standard Model. Alternative proposals for learning $\gamma$ are also discussed briefly.

The main objectives of the $B$ factories are to explore the physics of CP violation, to determine the flavor parameters of the electroweak theory, and to probe for signs of physics beyond the Standard Model. This will test the Cabibbo–Kobayashi–Maskawa (CKM) mechanism, which predicts that all CP violation results from a single complex phase in the quark mixing matrix. Facing the announcement of evidence for a CP asymmetry in the decays $B \to J/\psi K_S$ by the CDF Collaboration [1], the confirmation of direct CP violation in $K \to \pi\pi$ decays by the KTeV and NA48 groups [2, 3], and the successful start of the charged $B$ factories at SLAC and KEK, the year 1999 has been an important step in achieving this goal.

The determination of the sides and angles of the “unitarity triangle” $V_{ub}^*V_{ud}+V_{cb}^*V_{cd}+V_{tb}^*V_{td} = 0$ plays a central role in the $B$-factory program. Adopting the standard phase conventions for the CKM matrix, only the two smallest elements in this relation, $V_{ub}$ and $V_{td}$, have nonvanishing imaginary parts (to an excellent approximation). In the Standard Model the angle $\beta = -\text{arg}(V_{td})$ can be determined in a theoretically clean way by measuring the mixing-induced CP asymmetry in the decays $B \to J/\psi K_S$. The preliminary CDF result implies $\sin 2\beta = 0.79^{+0.41}_{-0.44}$ [1]. Without any dynamical assumption, the hadronic uncertainties in the description of the interference terms relevant to the determination of $\gamma$ are of relative magnitude $O(\lambda^2)$ or $O(\epsilon_{\text{SU}(3)}/N_c)$, where $\lambda = \sin \theta_c \approx 0.22$ is a measure of Cabibbo suppression, $\epsilon_{\text{SU}(3)} \approx 20\%$ is the typical size of $\text{SU}(3)$...
breaking, and the factor \(1/N_c\) indicates that the corresponding terms vanish in the factorization approximation. Factorizable SU(3) breaking can be accounted for in a straightforward way.

Recently, the accuracy of this description has been further improved when it was shown that nonleptonic \(B\) decays into two light mesons, such as \(B \to \pi K\) and \(B \to \pi \pi\), admit a heavy-quark expansion [3]. To leading order in \(\Lambda/m_b\), but to all orders in perturbation theory, the decay amplitudes for these processes can be calculated from first principles without recourse to phenomenological models. The QCD factorization theorem proved in [3], which emerges as the leading term in the heavy-quark limit. With the help of this theorem the irreducible theoretical uncertainties in the description of the \(B^\pm \to \pi K\) decay amplitudes can be reduced by an extra factor of \(O(\Lambda/m_b)\), rendering their analysis essentially model independent. As a consequence of this fact, and because they are dominated by FCNC transitions, the decays \(B^\pm \to \pi K\) offer a sensitive probe to physics beyond the Standard Model [3,10,11,12,14,18], much in the same way as the “classical” FCNC processes \(B \to X_s \gamma\) or \(B \to X_s l^+l^-\).

In the following three sections we discuss how, in the context of the Standard Model, \(B^\pm \to \pi K\) rate measurements can be used to derive bounds on \(\gamma\) and to extract its value with controlled theoretical uncertainties. We then explore how these analyses could be affected by New Physics. Finally, we briefly summarize alternative methods to determine \(\gamma\) using other decay modes.

1. Theory of \(B^\pm \to \pi K\) decays

The hadronic decays \(B \to \pi K\) are mediated by a low-energy effective weak Hamiltonian, whose operators allow for three distinct classes of flavor topologies: QCD penguins, trees, and electroweak penguins. In the Standard Model the weak couplings associated with these topologies are known. From the measured branching ratios one can deduce that QCD penguins dominate the \(B \to \pi K\) decay amplitudes [14], whereas trees and electroweak penguins are subleading and of a similar strength [13]. The theoretical description of the two charged modes \(B^\pm \to \pi^\pm K^0\) and \(B^\pm \to \pi^0 K^\pm\) exploits the fact that the amplitudes for these processes differ in a pure isospin amplitude \(A_{3/2}\), defined as the matrix element of the isovector part of the effective Hamiltonian between a \(B\) meson and the \(\pi^\pm K^0\) eigenstate with \(I = \frac{3}{2}\). In the Standard Model the parameters of this amplitude are determined, up to an overall strong phase \(\phi\), in the limit of SU(3) flavor symmetry [3]. Using the QCD factorization theorem proved in [3], the SU(3)-breaking corrections can be calculated in a model-independent way up to nonfactorizable terms that are power-suppressed in \(\Lambda/m_b\) and vanish in the heavy-quark limit.

A convenient parameterization of the decay amplitudes \(A_{+0} \equiv A(B^+ \to \pi^+ K^0)\) and \(A_{0+} \equiv -\sqrt{2}A(B^+ \to \pi^0 K^+)\) is

\[
A_{+0} = P \left(1 - \varepsilon_a e^{i\gamma} e^{i\eta}\right),
A_{0+} = P \left[1 - \varepsilon_a e^{i\gamma} e^{i\eta} - \varepsilon_{3/2} e^{i\phi} (e^{i\gamma} - \delta_{\text{EW}})\right],
\]

where \(P\) is the dominant penguin amplitude defined as the sum of all terms in the \(B^+ \to \pi^+ K^0\) amplitude not proportional to \(e^{i\gamma}\), \(\eta\) and \(\phi\) are strong phases, and \(\varepsilon_a\), \(\varepsilon_{3/2}\) and \(\delta_{\text{EW}}\) are real hadronic parameters. The weak phase \(\gamma\) changes sign under a CP transformation, whereas all other parameters stay invariant.

Based on a naive quark-diagram analysis one would not expect the \(B^+ \to \pi^+ K^0\) amplitude to receive a contribution from \(\bar{b} \to \bar{u}u\bar{s}\) tree topologies; however, such a contribution can be induced through final-state rescattering or annihilation contributions [10,11,18,13,20,21]. They are parameterized by \(\varepsilon_a = O(\Lambda^2)\). In the heavy-quark limit this parameter can be calculated and is found to be very small, \(\varepsilon_a \approx -2\%\) [22]. In the future, it will be possible to put upper and lower bounds on \(\varepsilon_a\) by comparing the CP-averaged branching ratios for the decays \(B^\pm \to \pi^\pm K^0\) and \(B^\pm \to K^\pm K^0\) [20]. Below we assume \(|\varepsilon_a| \leq 0.1\); however, our results will be almost insensitive to this assumption.

The terms proportional to \(\varepsilon_{3/2}\) in \([1,3]\) parameterize the isospin amplitude \(A_{3/2}\). The weak phase \(e^{i\gamma}\) enters through the tree process \(\bar{b} \to \bar{u}u\bar{s}\), whereas the quantity \(\delta_{\text{EW}}\) describes the effects of electroweak penguins. The parameter
\( \varepsilon_{3/2} \) measures the relative strength of tree and QCD penguin contributions. Information about it can be derived by using SU(3) flavor symmetry to relate the tree contribution to the isospin amplitude \( A_{3/2} \) to the corresponding contribution in the decay \( B^+ \to \pi^+\pi^0 \). Since the final state \( \pi^+\pi^0 \) has isospin \( I = 2 \), the amplitude for this process does not receive any correction from QCD penguins. Moreover, electroweak penguins in \( b \to d\bar{q}q \) transitions are negligibly small. We define a related parameter \( \bar{\varepsilon}_{3/2} \) by writing

\[
\bar{\varepsilon}_{3/2} = R_1 \left| \frac{V_{us}}{V_{td}} \right| \left[ \frac{2B(B^+ \to \pi^+\pi^0)}{B(B^+ \to \pi^+K^0)} \right]^{1/2}.
\]

SU(3)-breaking corrections are described by the factor \( R_1 = 1.22 \pm 0.05 \), which can be calculated in a model-independent way using the QCD factorization theorem for nonleptonic decays [22]. Using preliminary data reported by the CLEO Collaboration [23] to evaluate the ratio of the CP-averaged branching ratios in (1.2) we obtain

\[
\bar{\varepsilon}_{3/2} = 0.21 \pm 0.06_{\text{exp}} \pm 0.01_{\text{th}}.
\]

With a better measurement of the branching ratios the uncertainty in \( \bar{\varepsilon}_{3/2} \) will be reduced significantly.

Finally, the parameter

\[
\delta_{\text{EW}} = R_2 \left| \frac{V_{ub}^* V_{cs}}{V_{ub} V_{us}} \right| \frac{\alpha}{8\pi} \frac{x_t}{\sin^2 \theta_W} \left( 1 + 3 \ln \frac{x_t}{x_t - 1} \right)
\]

\[
= (0.64 \pm 0.09) \times 0.085 \left| \frac{V_{ub}^*}{V_{ub}} \right|,
\]

with \( x_t = (m_c/m_W)^2 \), describes the ratio of electroweak penguin and tree contributions to the isospin amplitude \( A_{3/2} \). In the SU(3) limit it is calculable in terms of Standard Model parameters [3, 24]. SU(3)-breaking as well as small electromagnetic corrections are accounted for by the quantity \( R_2 = 0.92 \pm 0.09 [3, 22] \). The error quoted in (1.4) includes the uncertainty in the top-quark mass.

Important observables in the study of the weak phase \( \gamma \) are the ratio of the CP-averaged branching ratios in the two \( B^\pm \to \pi K \) decay modes,

\[
R_* = \frac{B(B^\pm \to \pi^\pm K^0)}{2B(B^+ \to \pi^0 K^\pm)} = 0.75 \pm 0.28,
\]

and a particular combination of the direct CP asymmetries,

\[
\bar{A} = \frac{A_{\text{CP}}(B^+ \to \pi^0 K^\pm)}{R_*} - A_{\text{CP}}(B^\pm \to \pi^\pm K^0)
\]

\[
= -0.52 \pm 0.42.
\]

The experimental values of these quantities are derived using preliminary CLEO data reported in [23]. The theoretical expressions for \( R_* \) and \( \bar{A} \) obtained using the parameterization in (1.1) are

\[
R_*^{-1} = 1 + 2 \bar{\varepsilon}_{3/2} \cos \phi (\delta_{\text{EW}} - \cos \gamma)
\]

\[
+ \bar{\varepsilon}_{3/2}^2(1 - 2 \delta_{\text{EW}} \cos \gamma + \delta_{\text{EW}}^2) + O(\varepsilon_{3/2}^2\varepsilon_a),
\]

\[
\bar{A} = 2 \bar{\varepsilon}_{3/2} \sin \gamma \sin \phi + O(\varepsilon_{3/2}^2\varepsilon_a).
\]

Note that the rescattering effects described by \( \varepsilon_a \) are suppressed by a factor of \( \bar{\varepsilon}_{3/2} \) and thus reduced to the percent level. Explicit expressions for these contributions can be found in [3].

2. Lower bound on \( \gamma \) and constraint in the \( (\bar{\rho}, \bar{\eta}) \) plane

There are several strategies for exploiting the above relations. From a measurement of the ratio \( R_* \) alone a bound on \( \cos \gamma \) can be derived, implying a nontrivial constraint on the Wolfenstein parameters \( \bar{\rho} \) and \( \bar{\eta} \) defining the apex of the unitarity triangle [3]. Only CP-averaged branching ratios are needed for this purpose. Varying the strong phases \( \phi \) and \( \eta \) independently we first obtain an upper bound on the inverse of \( R_* \). Keeping terms of linear order in \( \varepsilon_a \) yields

\[
R_*^{-1} \leq (1 + \bar{\varepsilon}_{3/2} |\delta_{\text{EW}} - \cos \gamma|)^2 + \bar{\varepsilon}_{3/2}^2 \sin^2 \gamma
\]

\[
+ 2\bar{\varepsilon}_{3/2} \varepsilon_a \sin^2 \gamma.
\]

Provided \( R_* \) is significantly smaller than 1, this bound implies an exclusion region for \( \cos \gamma \) which becomes larger the smaller the values of \( R_* \) and \( \bar{\varepsilon}_{3/2} \) are. It is convenient to consider instead of \( R_* \) the related quantity [3]

\[
X_R = \frac{\sqrt{R_*^{-1} - 1}}{\bar{\varepsilon}_{3/2}} = 0.72 \pm 0.98_{\text{exp}} \pm 0.03_{\text{th}}.
\]
Because of the theoretical factor $R_1$ entering the definition of $\bar{\varepsilon}_{3/2}$ in (1.2) this is, strictly speaking, not an observable. However, the theoretical uncertainty in $X_R$ is so much smaller than the present experimental error that it can be ignored. The advantage of presenting our results in terms of $X_R$ rather than $R_\ast$ is that the leading dependence on $\bar{\varepsilon}_{3/2}$ cancels out, leading to the simple bound $|X_R| \leq |\delta_{\text{EW}} - \cos \gamma| + O(\bar{\varepsilon}_{3/2}, \varepsilon_o)$.

Figure 1: Theoretical upper bound on the ratio $X_R$ versus $|\gamma|$ for $\varepsilon_o = 0.1$ (solid line) and $\varepsilon_o = 0$ (dashed line). The horizontal line and band show the current experimental value with its 1σ variation.

In Figure 1 we show the upper bound on $X_R$ as a function of $|\gamma|$, obtained by varying the input parameters in the intervals $0.15 \leq \bar{\varepsilon}_{3/2} \leq 0.27$ and $0.49 \leq \delta_{\text{EW}} \leq 0.79$ (corresponding to using $|V_{ub}/V_{cb}| = 0.085 \pm 0.015$ in (1.3)). Note that the effect of the rescattering contribution parameterized by $\varepsilon_o$ is very small. The gray band shows the current value of $X_R$, which clearly has too large an error to provide any useful information on $\gamma$. The situation may change, however, once a more precise measurement of $X_R$ will become available. For instance, if the current central value $X_R = 0.72$ were confirmed, it would imply the bound $|\gamma| > 75^\circ$, marking a significant improvement over the indirect limit $|\gamma| > 37^\circ$ inferred from the global analysis of the unitarity triangle including information from $K$–$\bar{K}$ mixing (3).

So far, as in previous work, we have used the inequality (2.1) to derive a lower bound on $|\gamma|$. However, a large part of the uncertainty in the value of $\delta_{\text{EW}}$, and thus in the resulting bound on $|\gamma|$, comes from the present large error on $|V_{ub}|$. Since this is not a hadronic uncertainty, it is appropriate to separate it and turn (2.1) into a constraint on the Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$. To this end, we use that $\cos \gamma = \bar{\rho}/\sqrt{|\bar{\rho}^2 + \bar{\eta}^2|}$ by definition, and $\delta_{\text{EW}} = (0.24 \pm 0.03)/\sqrt{|\bar{\rho}^2 + \bar{\eta}^2|}$ from (1.3). The solid lines in Figure 2 show the resulting constraint in the $(\bar{\rho}, \bar{\eta})$ plane obtained for the representative values $X_R = 0.5, 0.75, 1.0, 1.25$ (from right to left), which for $\bar{\varepsilon}_{3/2} = 0.21$ would correspond to $R_\ast = 0.82, 0.75, 0.68, 0.63$, respectively. Values to the right of these lines are excluded. For comparison, the dashed circles show the constraint arising from the measurement of the ratio $|V_{ub}/V_{cb}| = 0.085 \pm 0.015$ in semileptonic $B$ decays, and the dashed-dotted line shows the bound implied by the present experimental limit on the mass difference $\Delta m_s$ in the $B_s$ system (4). Values to the left of this line are excluded. It is evident from the figure that the bound resulting from a measurement of the ratio $X_R$ in $B^{\pm} \to \pi K$ decays may be very nontrivial and, in particular, may eliminate the possibility that $\gamma = 0$. The combination of this bound with information from semileptonic decays and $B$–$\bar{B}$ mixing alone would then determine the Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$ within narrow ranges, and in the context of the CKM model would prove the existence of direct CP violation in $B$ decays.

\footnote{An observation of CP violation, such as the measurement of $\varepsilon_K$ in $K$–$\bar{K}$ mixing or $\sin 2\beta$ in $B \to J/\psi K_S$ decays, is however needed to fix the sign of $\bar{\eta}$.}
3. Extraction of $\gamma$

Ultimately, the goal is of course not only to derive a bound on $\gamma$ but to determine this parameter directly from the data. This requires to fix the strong phase $\phi$ in (1.7), which can be achieved either through the measurement of a CP asymmetry or with the help of theory. A strategy for an experimental determination of $\gamma$ from $B^\pm \to \pi K$ decays has been suggested in \cite{22}. It generalizes a method proposed by Gronau, Rosner and London \cite{25} to include the effects of electroweak penguins. The approach has later been refined to account for rescattering contributions to the $B^\pm \to \pi^\pm K^0$ decay amplitudes \cite{7}. Before discussing this method, we will first illustrate an easier strategy for a theory-guided determination of $\gamma$ based on the QCD factorization theorem for nonleptonic decays \cite{8}. This method does not require any measurement of a CP asymmetry.

3.1 Theory-guided determination

In the previous section the theoretical predictions for the nonleptonic $B \to \pi K$ decay amplitudes obtained using the QCD factorization theorem were used in a minimal way, i.e., only to calculate the size of the SU(3)-breaking effects parameterized by $R_1$ and $R_2$. The resulting bound on $\gamma$ and the corresponding constraint in the $(\rho, \bar{\eta})$ plane are therefore theoretically very clean. However, they are only useful if the value of $X_R$ is found to be larger than about 0.5 (see Figure 1), in which case values of $|\gamma|$ below $65^\circ$ are excluded. If it would turn out that $X_R < 0.5$, then it is possible to satisfy the inequality (2.2) also for small values of $\gamma$, however, at the price of having a very large strong phase, $\phi \approx 180^\circ$. But this possibility can be discarded based on the model-independent prediction that \cite{8}

$$\phi = O[\alpha_s(m_b), \Lambda/m_b].$$

(3.1)

A direct calculation of this phase to leading power in $\Lambda/m_b$ yields $\phi \approx -11^\circ$ \cite{22}. Using the fact that $\phi$ is parametrically small, we can exploit a measurement of the ratio $X_R$ to obtain a determination of $|\gamma|$ — corresponding to an allowed region in the $(\rho, \bar{\eta})$ plane — rather than just a bound. This determination is unique up to a sign. Note that for small values of $\phi$ the impact of the strong phase in the expression for $R_\ast$ in (1.7) is a second-order effect. As long as $|\phi| \ll \sqrt{2|\Delta \varepsilon_{3/2}/\varepsilon_{3/2}|}$, the uncertainty in $\cos \phi$ has a much smaller effect than the uncertainty in $\varepsilon_{3/2}$. With the present value of $\varepsilon_{3/2}$ this is the case as long as $|\phi| \ll 43^\circ$. We believe it is a safe assumption to take $|\phi| < 25^\circ$ (i.e., more than twice the value obtained to leading order in $\Lambda/m_b$), so that $\cos \phi > 0.9$.

![Figure 3: Allowed regions in the $(\rho, \bar{\eta})$ plane for fixed values of $X_R$, obtained by varying all theoretical parameters inside their respective ranges of uncertainty, as specified in the text. The sign of $\bar{\eta}$ is not determined.](image)

Solving the equation for $R_\ast$ in (1.7) for $\cos \gamma$, and including the corrections of $O(\varepsilon_a)$, we find

$$\cos \gamma = \delta_{EW} - \frac{X_R + \frac{1}{2}\varepsilon_{3/2}(X_R^2 - 1 + \delta_{EW}^2)}{\cos \phi + \varepsilon_{3/2}\delta_{EW}}$$

$$+ \frac{\varepsilon_a \cos \eta \sin^2 \gamma}{\cos \phi + \varepsilon_{3/2}\delta_{EW}},$$

(3.2)

where we have set $\cos \phi = 1$ in the numerator of the $O(\varepsilon_a)$ term. Using the QCD factorization theorem one finds that $\varepsilon_a \cos \eta \approx -0.02$ in the heavy-quark limit \cite{22}, and we assign a 100% uncertainty to this estimate. In evaluating the result (3.2) we scan the parameters in the ranges $0.15 \leq \varepsilon_{3/2} \leq 0.27$, $0.55 \leq \delta_{EW} \leq 0.73$, $-25^\circ \leq \phi \leq 25^\circ$, and $-0.04 \leq \varepsilon_a \cos \eta \sin^2 \gamma \leq 0$. Figure 3 shows the allowed regions in the $(\rho, \bar{\eta})$ plane for the representative values $X_R = 0.25$, 0.75, and 1.25 (from right to left). We stress that with this method a useful constraint on the Wolfenstein parameters is obtained for any value of $X_R$.

3.2 Model-independent determination

It is important that, once more precise data on $B^\pm \to \pi K$ decays will become available, it will
be possible to test the prediction of a small strong phase $\phi$ experimentally. To this end, one must determine the CP asymmetry $\tilde{A}$ defined in (1.6) in addition to the ratio $R_\ast$. From (1.7) it follows that for fixed values of $\xi_{3/2}$ and $\delta_{EW}$ the quantities $R_\ast$ and $\tilde{A}$ define contours in the $(\gamma, \phi)$ plane, whose intersections determine the two phases up to possible discrete ambiguities [6,7]. Figure 4 shows these contours for some representative values, assuming $\xi_{3/2} = 0.21$, $\delta_{EW} = 0.64$, and $\varepsilon_a = 0$. In practice, including the uncertainties in the values of these parameters changes the contour lines into contour bands. Typically, the spread of the bands induces an error in the determination of $\gamma$ of about $10^\circ$ [6]. In the most general case there are up to eight discrete solutions for the two phases, four of which are related to the other four by a sign change $(\gamma, \phi) \rightarrow (-\gamma, -\phi)$. However, for typical values of $R_\ast$ it turns out that often only four solutions exist, two of which are related to the other two by a sign change. The theoretical prediction that $\phi$ is small implies that solutions should exist where the contours intersect close to the lower portion in the plot. Other solutions with large $\phi$ are strongly disfavored. Note that according to (1.7) the sign of the CP asymmetry $\tilde{A}$ fixes the relative sign between the two phases $\gamma$ and $\phi$. If we trust the theoretical prediction that $\phi$ is negative [22], it follows that in most cases there remains only a unique solution for $\gamma$, i.e., the CP-violating phase $\gamma$ can be determined without any discrete ambiguity.

Consider, as an example, the hypothetical case where $R_\ast = 0.8$ and $\tilde{A} = -15\%$. Figure 4 then allows the four solutions where $(\gamma, \phi) \approx (\pm 82^\circ, \mp 21^\circ)$ or $(\pm 158^\circ, \mp 78^\circ)$. The second pair of solutions is strongly disfavored because of the large values of the strong phase $\phi$. From the first pair of solutions, the one with $\phi \approx -21^\circ$ is closest to our theoretical expectation that $\phi \approx -11^\circ$, hence leaving $\gamma \approx 82^\circ$ as the unique solution.

4. Sensitivity to New Physics

In the presence of New Physics the theoretical description of $B^\pm \rightarrow \pi K$ decays becomes more complicated. In particular, new CP-violating contributions to the decay amplitudes may be induced. A detailed analysis of such effects has been presented in [13]. A convenient and completely general parameterization of the two amplitudes in (1.1) is obtained by replacing

$$P \rightarrow P', \quad \varepsilon_a e^{i\gamma} e^{i\eta} \rightarrow i\rho e^{i\phi},$$

$$\delta_{EW} \rightarrow a e^{i\phi_a} + ib e^{i\phi_b}, \quad (4.1)$$

where $\rho, a, b$ are real hadronic parameters, and $\phi, \phi_a, \phi_b$ are strong phases. The terms $i\rho$ and $ib$ change sign under a CP transformation. New Physics effects parameterized by $P'$ and $\rho$ are isospin conserving, while those described by $a$ and $b$ violate isospin symmetry. Note that the parameter $P'$ cancels in all ratios of branching ratios and thus does not affect the quantities $R_\ast$ and $X_R$ as well as any CP asymmetry. Because the ratio $R_\ast$ in (1.5) would be 1 in the limit of isospin symmetry, it is particularly sensitive to isospin-violating New Physics contributions.

New Physics can affect the bound on $\gamma$ derived from (2.1) as well as the extraction of $\gamma$ using the strategies discussed above. We will discuss these two possibilities in turn.

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**Figure 4:** Contours of constant $R_\ast$ (“hyperbolas”) and constant $|\tilde{A}|$ (“circles”) in the $(|\gamma|, |\phi|)$ plane. The sign of the asymmetry $\tilde{A}$ determines the sign of the product $\sin \gamma \sin \phi$. The contours for $R_\ast$ refer to values from 0.6 to 1.0 in steps of 0.1, those for the asymmetry correspond to 5%, 15%, and 25%, as indicated.
4.1 Effects on the bound on $\gamma$

The upper bound on $R^\tau_\pi$ in (2.1) and the corresponding bound on $X_R$ shown in Figure 1 are model-independent results valid in the Standard Model. Note that the extremal value of $R^\tau_\pi$ is such that $|X_R| \leq (1 + \delta_{\text{EW}})$ irrespective of $\gamma$. A value of $|X_R|$ exceeding this bound would be a clear signal for New Physics [6, 10,13].

Consider first the case where New Physics may induce arbitrary CP-violating contributions to the $B \to \pi K$ decay amplitudes, while preserving isospin symmetry. Then the only change with respect to the Standard Model is that the parameter $\rho$ may no longer be as small as $O(\varepsilon_a)$. Varying the strong phases $\phi$ and $\phi_\rho$ independently, and allowing for an arbitrarily large New Physics contribution to $\rho$, one can derive the bound [13]

$$|X_R| \leq \sqrt{1 - 2 \delta_{\text{EW}} \cos \gamma + \delta_{\text{EW}}^2} \leq 1 + \delta_{\text{EW}}.$$  \hspace{1cm} (4.2)

Note that the extremal value is the same as in the Standard Model, i.e., isospin-conserving New Physics effects cannot lead to a value of $|X_R|$ exceeding $(1 + \delta_{\text{EW}})$. For intermediate values of $\gamma$ the Standard Model bound on $X_R$ is weakened; but even for large values $\rho = O(1)$, corresponding to a significant New Physics contribution to the decay amplitudes, the effect is small.

If both isospin-violating and isospin-conserving New Physics contributions are present and involve new CP-violating phases, the analysis becomes more complicated. Still, it is possible to derive model-independent bounds on $X_R$. Allowing for arbitrary values of $\rho$ and all strong phases, one obtains [14]

$$|X_R| \leq \sqrt{(|a| + |\cos \gamma|)^2 + (|b| + |\sin \gamma|)^2} \leq 1 + \sqrt{a^2 + b^2} \leq \frac{2}{\varepsilon_{3/2}} + X_R,$$  \hspace{1cm} (4.3)

where the last inequality is relevant only in cases where $\sqrt{a^2 + b^2} \gg 1$. The important point to note is that with isospin-violating New Physics contributions the value of $|X_R|$ can exceed the upper bound in the Standard Model by a potentially large amount. For instance, if $\sqrt{a^2 + b^2}$ is twice as large as in the Standard Model, corresponding to a New Physics contribution to the decay amplitudes of only 10–15%, then $|X_R|$ could be as large as 2.6 as compared with the maximal value 1.8 allowed in the Standard Model. Also, in the most general case where $b$ and $\rho$ are nonzero, the maximal value $|X_R|$ can take is no longer restricted to occur at the endpoints $\gamma = 0^\circ$ or $180^\circ$, which are disfavored by the global analysis of the unitarity triangle [4]. Rather, $|X_R|$ would take its maximal value if $|\tan \gamma| = |\rho| = |b/a|$. The present experimental value of $X_R$ in (2.2) has too large an error to determine whether there is any deviation from the Standard Model. If $X_R$ turns out to be larger than 1 (i.e., at least one third of a standard deviation above its current central value), then an interpretation of this result in the Standard Model would require a large value $|\gamma| > 91^\circ$ (see Figure 1), which would be difficult to accommodate in view of the upper bound implied by the experimental constraint on $B_s - \bar{B_s}$ mixing, thus providing evidence for New Physics. If $X_R > 1.3$, one could go a step further and conclude that the New Physics must necessarily violate isospin [13].

4.2 Effects on the determination of $\gamma$

A value of the observable $R_\pi$, violating the bound [2] would be an exciting hint for New Physics. However, even if a future precise measurement will give a value that is consistent with the Standard Model bound, $B^\pm \to \pi K$ decays provide an excellent testing ground for physics beyond the Standard Model. This is so because New Physics may cause a significant shift in the value of $\gamma$ extracted using the strategies discussed in Section 4, leading to inconsistencies when this value is compared with other determinations of $\gamma$.

A global fit of the unitarity triangle combining information from semileptonic $B$ decays, $B$–$\bar{B}$ mixing, CP violation in the kaon system, and mixing-induced CP violation in $B \to J/\psi K_S$ decays provides information on $\gamma$ which in a few years will determine its value within a rather narrow range [6]. Such an indirect determination could be complemented by direct measurements of $\gamma$ using, e.g., $B \to DK^{(*)}$ decays (see below), or using the triangle relation $\gamma = 180^\circ - \alpha - \beta$ combined with a measurement of $\alpha$. We will assume that a discrepancy of more than 25$^\circ$ between the “true” $\gamma = \arg(V_{ub}^*)$ and the value $\gamma_{\pi K}$ extracted in $B^\pm \to \pi K$ decays will be observable.
after a few years of operation at the $B$ factories. This sets the benchmark for sensitivity to New Physics effects.

![Figure 5: Contours of constant $X_R$ versus $\gamma$ and the parameter $a$, assuming $\gamma > 0$. The horizontal band shows the value of $a$ in the Standard Model.](image)

In order to illustrate how big an effect New Physics could have on the extracted value of $\gamma$ we consider the simplest case where there are no new CP-violating couplings. Then all New Physics contributions in (4.1) are parameterized by the single parameter $a_{NP} = a - \delta_{EW}$. A more general discussion can be found in [13]. We also assume for simplicity that the strong phase $\phi$ is small, as suggested by (3.1). In this case the difference between the value $\gamma_{\pi K}$ extracted from $B^\pm \rightarrow \pi K$ decays and the “true” value of $\gamma$ is to a good approximation given by
\[
\cos \gamma_{\pi K} \simeq \cos \gamma - a_{NP} \, . \tag{4.4}
\]

In Figure 5 we show contours of constant $X_R$ versus $\gamma$ and $a$, assuming without loss of generality that $\gamma > 0$. Obviously, even a moderate New Physics contribution to the parameter $a$ can induce a large shift in $\gamma$. Note that the present central value of $X_R \approx 0.7$ is such that values of $a$ less than the Standard Model result $a \approx 0.64$ are disfavored, since they would require values of $\gamma$ exceeding $10^6$, in conflict with the global analysis of the unitarity triangle [3].

### 4.3 Survey of New Physics models

In [13], we have explored how physics beyond the Standard Model could affect purely hadronic FCNC transitions of the type $\bar{b} \rightarrow \bar{c}q\bar{q}$ focusing, in particular, on isospin violation. Unlike in the Standard Model, where isospin-violating effects in these processes are suppressed by electroweak gauge couplings or small CKM matrix elements, in many New Physics scenarios these effects are not parametrically suppressed relative to isospin-conserving FCNC processes. In the language of effective weak Hamiltonians this implies that the Wilson coefficients of QCD and electroweak penguin operators are of a similar magnitude. For a large class of New Physics models we found that the coefficients of the electroweak penguin operators are, in fact, due to “trojan” penguins, which are neither related to penguin diagrams nor of electroweak origin.

Specifically, we have considered: (a) models with tree-level FCNC couplings of the $Z$ boson, extended gauge models with an extra $Z'$ boson, supersymmetric models with broken $R$-parity; (b) supersymmetric models with $R$-parity conservation; (c) two-Higgs–doublet models, and models with anomalous gauge-boson couplings. Some of these models have also been investigated in [13, 14]. In case (a), the resulting electroweak penguin coefficients can be much larger than in the Standard Model because they are due to tree-level processes. In case (b), these coefficients can compete with the ones of the Standard Model because they arise from strong-interaction box diagrams, which scale relative to the Standard Model like $(\alpha_s/\alpha)(m_W^2/m_{\text{SUSY}}^2)$. In models (c), on the other hand, isospin-violating New Physics effects are not parametrically enhanced and are generally smaller than in the Standard Model.

For each New Physics model we have explored which region of parameter space can be probed by the $B^\pm \rightarrow \pi K$ observables, and how big a departure from the Standard Model predictions one can expect under realistic circumstances, taking into account all constraints on the model parameters implied by other processes. Table 1 summarizes our estimates of the maximal isospin-violating contributions to the decay amplitudes, as parameterized by $|a_{NP}|$. They are the potentially most important source of New Physics effects in $B^\pm \rightarrow \pi K$ decays. For comparison, we recall that in the Standard Model $a \approx 0.64$. Also shown are the corresponding maximal values of the difference $|\gamma_{\pi K} - \gamma|$. As noted above, in models with tree-level FCNC couplings New Physics effects can be dramatic, whereas in
2.0 180°
extra Z' boson
14° 180°
SUSY without R-parity
14° 180°
SUSY with R-parity
0.4 25°
1.3 180°
two-Higgs–doublet mod.
0.15 10°
anom. gauge-boson coupl.
0.3 20°

Table 1: Maximal contributions to $\alpha_{NP}$ and shifts in $\gamma$ in extensions of the Standard Model. Entries marked with “*” are upper bounds derived using (4.3). For the case of supersymmetric models with R-parity the first (second) row corresponds to maximal right-handed (left-handed) strange–bottom squark mixing. For the two-Higgs–doublet models we take $m_{H^\pm} > 100$ GeV and $\tan \beta > 1$.

supersymmetric models with R-parity conservation isospin-violating loop effects can be competitive with the Standard Model. In the case of supersymmetric models with R-parity violation the bound (4.3) implies interesting limits on certain combinations of the trilinear couplings $\lambda'_{ijk}$ and $\lambda''_{ijk}$, as discussed in [13].

5. Alternative approaches and recent developments

We will now review recent developments regarding other approaches to determining $\gamma$, focusing mainly on proposals that can be pursued at the first-generation $B$ factories.

5.1 Variants of the $B^{\pm} \to \pi K$ strategy

The first proposal to constraining $\gamma$ using CP-averaged $B \to \pi K$ branching ratios was made by Fleischer and Mannel [27], who suggested to consider the ratio

$$R = \frac{\tau(B^+)}{\tau(B^0)} \frac{B(B^0 \to \pi^+ K^0)}{B(B^{\pm} \to \pi^{\pm} K^0)} = 1.11 \pm 0.33.$$  

(5.1)

Neglecting the small rescattering contribution to the $B^{\pm} \to \pi^{\pm} K^0$ decay amplitudes as well as electroweak penguin contributions yields

$$R \simeq 1 - 2 \varepsilon_T \cos \phi_T \cos \gamma + \varepsilon_T^2$$

$$= \sin^2 \gamma + (\varepsilon_T - \cos \phi_T \cos \gamma)^2 + \sin^2 \phi_T \cos^2 \gamma$$

$$\geq \sin^2 \gamma,$$  

(5.2)

where $\varepsilon_T$ is a real parameter of similar magnitude as $\varepsilon_{3/2}$, and $\phi_T$ is a strong phase. If the ratio $R$ was found significantly less than 1, the above inequality would imply an exclusion region around $\gamma = 90^\circ$.

Unlike the parameter $\varepsilon_{3/2}$ in $B^{\pm} \to \pi K$ decays, the quantity $\varepsilon_T$ is not constrained by SU(3) symmetry and cannot be determined experimentally. The strategy proposed in [27] is to eliminate this quantity in deriving a bound on $\gamma$. This weakens the handle on the weak phase except for the particular case where $\varepsilon_T \approx \cos \phi_T \cos \gamma$. The neglect of electroweak penguin and rescattering corrections is questionable and has given rise to some criticism [18, 19, 20, 21]. Yet, although the bound (5.2) is theoretically not as clean as the corresponding bound (2.1) on the ratio $R$, a precise measurement of the ratio $R$ can provide for an interesting consistency check. Various refinements and extensions of the original Fleischer–Mannel strategy are discussed in [28].

Some authors have suggested to eliminate the small rescattering contribution to the $B^{\pm} \to \pi^{\pm} K^0$ decay amplitudes, parameterized by $\varepsilon_\alpha$ in (1.1), by assuming SU(3) symmetry and exploiting amplitude relations connecting $B \to \pi K$ and $B \to \pi \pi$ decays with other decay modes, such as $B^+ \to K^+ \bar{K}^0$, $B^+ \to \bar{\pi}^{+} \eta^{(0)}$, $B_s \to K \pi$ [29], or $B^+ \to K^+ \bar{\eta}^{(0)}$, $B_s \to \pi^{0} \eta^{(0)}$ [30]. Note that approaches based on $B_s$ decays have to await second-generation $B$ factories at hadron colliders. Besides relying on the assumption of SU(3) flavor symmetry the above proposals suffer from theoretical uncertainties related to $\eta^{'}-\eta$ mixing. Given that the rescattering contribution in question is expected to be very small [22], and that this expectation can be tested experimentally using $B^{\pm} \to K^{\pm} \bar{K}^0$ decays [20], neglecting $\varepsilon_\alpha$ or putting an upper bound on it will most likely be a better approximation than neglecting the potentially much larger SU(3)-breaking corrections in the above strategies.

5.2 SU(3) relations between $B_d$ and $B_s$ decay amplitudes

Fleischer has recently suggested to use the U-spin $(d \leftrightarrow s)$ subgroup of flavor SU(3) to derive relations between various decays into CP eigenstates.
such as \[31\]

\[
\begin{align*}
B_d \to J/\psi K_S & \leftrightarrow B_s \to J/\psi K_S, \\
B_d \to D^+_d D^-_d & \leftrightarrow B_s \to D^+_s D^-_s, \\
B_d \to \pi^+\pi^- & \leftrightarrow B_s \to K^+K^-.
\end{align*}
\] (5.3)

The first two relations provide access to the weak phase \(\gamma\), while the third one is sensitive to both \(\beta\) and \(\gamma\). Although this strategy involves \(B_s\) decays and thus cannot be pursued at the first-generation \(B\) factories, we discuss it because of its general nature.

Consider the example of \(B_d, B_s \to J/\psi K_S\) decays, which are governed by an interference of tree and penguin topologies. The sensitivity to \(\gamma\) arises from the presence of top- and up-quark penguins. A general parameterization of the decay amplitudes \(A_{d,s} \equiv A(B_{d,s} \to J/\psi K_S)\) is

\[
\begin{align*}
A_d &= A' \left(1 + \tan^2 \theta_C \epsilon e^{i\phi} e^{i\gamma}\right), \\
A_s &= A \left(1 + \epsilon e^{i\phi} e^{i\gamma}\right),
\end{align*}
\] (5.4)

where \(\theta_C\) is the Cabibbo angle, \(\epsilon^{(i)}\) are strong phases, and the penguin contributions are proportional to parameters \(\epsilon^{(i)} \sim 0.2\). Exact U-spin symmetry would imply \(A' = A, \epsilon' = \epsilon,\) and \(\phi' = \phi\). In that limit \(\gamma\) could be determined (with discrete ambiguities) by measuring the direct and mixing-induced CP asymmetries \(A^\text{dir}_{CP}(B_s \to J/\psi K_S)\) and \(A^\text{mix}_{CP}(B_s \to J/\psi K_S)\) as well as the ratio of the CP-averaged \(B_d, B_s \to J/\psi K_S\) decay rates. These observables would suffice to fix \(\epsilon, \phi, \) and \(\gamma\). Assuming \(A' = A, 3\) this parameter cancels out.

This approach is interesting in that it is general and can be applied to several different decay modes \[41\]. Assuming the theoretical uncertainties related to U-spin breaking can be controlled, it will provide several independent determinations of \(\beta\) and \(\gamma\). However, a question that needs to be addressed in future work is how important the SU(3)-breaking corrections leading to \(A' \neq A\) are, given that \(\epsilon \sim 0.2\) is expected to be a small parameter.

5.3 Dalitz-plot analysis in \(B^\pm \to \pi^\pm\pi^+\pi^-\) decays

There have been several proposals for obtaining information on the weak phases \(\alpha\) and \(\gamma\) from an analysis of the Dalitz plot in \(B \to 3\pi\) decays. In fact, the approach of Quinn and Snyder \[32\] based on the decays \(B \to \rho\pi \to 3\pi\) is considered to offer one of the most promising ways to measure \(\alpha\) \[43\]. The strategy of Bediaga et al. \[34\] (see also \[35\]) for learning \(\gamma\) is to fit the measured Dalitz distribution \(d^2\Gamma/dm_1^2 dm_2^2\) in the decays \(B^+ \to \pi^+\pi^+\pi^-\), where \(m_1^2 = (p_{\pi^+} + p_{\pi^-})^2\), to an ansatz of the form

\[
\left| \sum_i a_i e^{i\theta_i} F_i(m_1^2, m_2^2) \right|^2.
\] (5.5)

\(F_i(m_1^2, m_2^2)\) are appropriate kinematic functions for resonance or continuum contributions. From the fit one extracts a set \(\{a_i, \theta_i\}\) of real amplitudes and complex phases. Performing a similar fit to the CP-conjugated decays \(B^- \to \pi^-\pi^-\pi^+\) gives parameters \(\{a_i, \theta_i\}\). The complex phases are sums of strong and weak phases, and the latter ones are determined from the differences \(\phi_i = \frac{1}{2}(\theta_i - \bar{\theta}_i)\). The weak phase \(\gamma\) enters through the interference of the \(\chi_{c}^0\pi^\pm\) resonance state with other resonance channels (e.g., \(\rho^0\pi^\pm, f_0^0\pi^\pm\), etc.) and nonresonant contributions. The associated CKM factors are \(V_{cb}^*V_{cd}\) and \(V_{ub}^*V_{ud}\), respectively.

A theoretical problem inherent in this approach is the “penguin pollution”, i.e., the fact that \(\bar{b} \to d\bar{q}q\) penguin transitions contaminate the analysis. If the penguin/tree ratio is assumed to be at most 20\%, then the resulting error in the extraction of \(\gamma\) is bound to be less than 11\% \[34\]. Unfortunately, the recent data on \(B \to \pi K\) and \(B \to \pi\pi\) decays reported by the CLEO Collaboration \[28\] suggest that the penguin/tree ratio may be significantly larger than 20\%.

The feasibility of this method profits from the fact that no tagging is required (only charged \(B\)-meson decays are considered), the final state consists of three charged pions (no \(\pi^0\) reconstruction is needed), and a Dalitz plot analysis typically does not require very large data samples. The authors of \[34\] estimate that with only 1000 events one could obtain a resolution of \(\Delta \gamma \approx 20^\circ\). Potential problems of the approach are that the size of the interference term depends on the yet unknown \(B^\pm \to \chi_{c}\pi^\pm \to \pi^\pm\pi^+\pi^-\) branching ratio, and that contamination from nonresonant channels may, in the end, require larger data samples.
5.4 Extracting $\gamma$ in $B \to DK^{(*)}$ decays

$B \to DK^{(*)}$ decays were originally considered to be the “classical” way for determining $\gamma$. Later, it was realized that this is a very challenging route, which poses high demands to experiment and theory. We discuss this strategy because it provides an extraction of $\gamma$ from tree processes alone, which is unlikely to be affected much by New Physics.

The original idea of Gronau and Wyler (see also [31, 32]) was to use the interference of the amplitudes for the decays $B^+ \to D^0 K^+$ and $B^+ \to D^0 K^+$ occurring if the charm meson is detected as a CP eigenstate $D_1^0 = \frac{1}{\sqrt{2}}(D^0 + D^0)$. The first decay is due to the quark transition $\bar{b} \to \bar{c}u s$ proportional to $V_{cs}^\ast V_{us}$, whereas the second one is due to the decay $\bar{b} \to \bar{u}c s$ proportional to $V_{us}^\ast V_{cs}$. The relative phase of these two combinations of CKM elements is $\gamma$. Ideally, one would measure the six rates for the decays $B^+ \to \bar{D}^0 K^+$, $B^+ \to D^0 K^+$, $B^+ \to \bar{D}^0 K^+$, and their CP conjugates. Then, using isospin triangles, $\gamma$ could be determined in a theoretically clean way.

This strategy is hindered by the fact that it is not possible experimentally to determine the rate of the doubly Cabibbo-suppressed decay $B^+ \to D^0 K^+$ followed by $D^0 \to K^- \pi^+$, because its combined branching ratio is similar to that of the transition $B^+ \to \bar{D}^0 K^+$ followed by the doubly Cabibbo-suppressed decay $\bar{D}^0 \to K^- \pi^+$ [33]. Several approaches have been suggested to circumvent this problem [33, 40]; however, they are challenging from an experimental point of view because precise measurements of very small decay rates are required.

Recently, some authors have suggested to use isospin relations combined with certain dynamical assumptions (the neglect of annihilation contributions relative to color-suppressed tree amplitudes) to eliminate the “difficult” $B^+ \to D^0 K^+$ and $B^- \to \bar{D}^0 K^-$ rates in favor of six other $B, \bar{B} \to DK$ rates [11, 12]. Unfortunately, it is difficult to gauge the accuracy of the assumptions made in these proposals.

Considering the various options that have been discussed it appears that measuring $\gamma$ at the first-generation $B$ factories using $B \to DK^{(*)}$ decays is very challenging [43], and more demanding than a determination based on $B^\pm \to \pi K$ decays. On the other hand, to have two independent determinations of $\gamma$ from these two classes of decays would be extremely important. Whereas $\gamma$ measured in $B \to DK^{(*)}$ decays is likely to be the “true” phase of the CKM matrix, the angle $\gamma_{\pi K}$ determined in $B^\pm \to \pi K$ decays probes loop processes and may easily be affected by New Physics. As discussed in Section 4 and summarized in Table 1, comparing the two determinations would provide a very sensitive probe for physics beyond the Standard Model.

6. Conclusions

Among the strategies for determining the weak phase $\gamma = \arg(V_{ub}^\ast)$ of the quark mixing matrix, approaches based on rate measurements in $B \to \pi K$ decays play an important role and have received a lot of attention recently. The corresponding branching ratios are “large” and the final states are “easy” to detect experimentally. Using isospin, Fierz and flavor symmetries together with the fact that nonleptonic $B$ decays into two light mesons admit a heavy-quark expansion, a largely model-independent description of certain observables concerning the charged modes $B^\pm \to \pi K$ can be obtained. Various proposals exist for extracting information on $\gamma$ and on the Wolfenstein parameters $\rho$ and $\eta$ using these decays. In the future, this will allow us to determine $\gamma$ with a theoretical uncertainty of about 10°. When combined with an alternative measurement of $\gamma$ using other decays such as $B \to DK^{(*)}$, this will provide for a sensitive probe of physics beyond the Standard Model.

Acknowledgments

It is a pleasure to thank the SLAC Theory Group for the hospitality extended to me during my visit earlier this year. I am grateful to M. Beneke, G. Buchalla, Y. Grossman, A. Kagan, J. Rosner and C. Sachrajda for collaboration on parts of the work reported here. This research was supported by the Department of Energy under contract DE–AC03–76SF00515.
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