MEASURING DARK MATTER ELLIPTICITY OF A901/902 USING PARTICLE-BASED LENSING

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ABSTRACT

We present a non-parametric measure of the ellipticity and the alignment of the dark matter halos in the A901/902 supercluster. This supercluster is a system of four separate peaks in a $0.5 \times 0.5$ field of view. We map the mass distribution of each individual peak using an improved version of Particle-Based Lensing (PBL) and measure the ellipticity of the dark matter halos associated with two of the peaks directly from the mass map and by fitting them to a singular isothermal ellipse. The parametric and non-parametric measurements are consistent for A901b while the position angle for the Southwest Group is different for the two techniques. We account for this discrepancy to substructure present in the Southwest Peak. We estimate an axis ratio of $0.37 \pm 0.1$ for A901b and $0.54^{+0.08}_{-0.09}$ for the Southwest Group.

Key words: galaxies: clusters: general – gravitational lensing: weak – methods: analytical – methods: statistical

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1. INTRODUCTION

Clusters of galaxies are the largest virialized structures in the universe. Their mass distribution reveals properties of the primordial density field (Sheth & Tormen 1999; Frenk et al. 1990; Press & Schechter 1974), and their mass function is sensitive to predictions of $\Lambda$CDM cosmology (Francis et al. 2009; Bartelmann et al. 2006; Eke et al. 1996). As suggested by their low gas fraction, the baryons in clusters have little influence on the mass distribution beyond the core. Thus, clusters can be well approximated as dark matter halos. These halos produce spectacular arcs and multiple images (Zitrin et al. 2009; Zitrin & Broadhurst 2009; Limousin et al. 2008; Goldberg et al. 2009) and cause weak distortions of background galaxies.

Weak lensing inversion techniques study this distortion pattern to infer properties of the mass distribution of the cluster independent of its physical/dynamical state (Clowe et al. 2006; Okabe & Umetsu 2008). Weak lensing has been used to measure dark matter density profiles in clusters (Broadhurst et al. 2008, 2005; Umetsu & Broadhurst 2008; Mandelbaum et al. 2008) to test predictions of $N$-body simulations of the standard $\Lambda$CDM model (Spergel et al. 2007; Komatsu et al. 2009). However, weak lensing has systematic and statistical errors. We have to understand and reduce these errors in order to constrain the mass, size, and shape of galaxy clusters.

Gravitational lensing mass reconstruction of clusters have been studied for two decades (Okabe et al. 2010; King & Corless 2007; Pedersen & Dahle 2007; Cypriano et al. 2001; Williams et al. 1999; Allen 1998; Wu & Hammer 1995; Tyson et al. 1990; Oguri et al. 2009; Bardeau et al. 2007; Bradač et al. 2006; Broadhurst et al. 2005; Clowe & Schneider 2001; Bacon et al. 2006) and many different methods have been developed in the process (Kaiser 1995; Seitz & Schneider 1995; Bartelmann 1995; Schneider & Bartelmann 1997; Seitz & Schneider 2001). Almost all of them perform reasonably well in detecting massive peaks (primarily associated with the brightest cluster galaxy (BCG)) in the field of view. This is because massive peaks produce significant lensing signals which can be detected by the simplest weak lensing technique. The rapid improvement in the quality of observations has lead to extensive research on lens mass reconstruction techniques (Seitz et al. 1998; Geiger & Schneider 1999; Marshall et al. 2002; Lombardi & Bertin 1999; Diego et al. 2007; Bradač et al. 2005; Merten et al. 2009; Bradač et al. 2009; Mandelbaum et al. 2009).

In this paper, we have improved Particle-Based Lensing (PBL; Deb et al. 2008) by smoothing the ellipticities prior to mass reconstruction and evaluated the covariance of the resulting mass distribution. PBL is a mass reconstruction technique where the mass (or potential) is calculated at the location of each image. There is a weight function associated with each image with a kernel having a width that varies according to the local number density.

We aim to measure the ellipticity of the dark matter halos of the supercluster A901/902 using weak gravitational lensing. A901/902 is a complicated system with several sub-clusters spread around a field of view of $0.5 \times 0.5$ ($\sim 5 \ Mpc^2$) at a redshift of $z = 0.165$. This field was surveyed to study how galaxy evolution gets affected by the density of the environment (Gray et al. 2009). The weak lensing analysis (Heymans et al. 2008, hereafter H08) of this field reconstructs the large-scale dark matter distribution for this supercluster. The mass estimates of each peak were made using Navarro–Frenk–White (NFW) profiles and non-parametric maximum likelihood methods. Comparison with the light profile has shown that the substructure in the dark matter peaks is closely followed by the substructure in galaxy groups.

The mass and morphology of the dark matter halos of these galaxies play an important role in their response to surrounding environment. Since this field of view has three clusters and a group of galaxies, it is a very good laboratory to study large-scale structure and its influence on galaxy transformation.

The paper is organized in the following way. In Sections 2–4, we discuss the technical aspects of mass reconstruction and noise covariance analysis using PBL. In Section 5, we describe measuring ellipticity of the dark matter halo using parametric and non-parametric techniques. In Section 6, we give a brief description of the data and in Section 7 we report the estimated ellipticity for the individual peaks of the supercluster A901/902. In the last section, we discuss our results and possible directions of future work.
2. WEAK LENSING RECONSTRUCTIONS

Weak lensing is a statistical measure of small distortions in background galaxies caused by a cluster. The distribution of intrinsic ellipticities in source galaxies follows a Gaussian distribution with zero mean and a width of 0.2–0.3 (Mellier 1999). In order to extract a lensing signal from the weak distortion of image galaxies, several of them have to be averaged to smooth out the intrinsic noise. The ellipticities of the background image galaxies are the weak lensing observables. In the linear regime ($\kappa \ll 1$), the ellipticity can be approximated by the shear of the lensing field. The shear has two components given by

$$\gamma \equiv \gamma^1 + i\gamma^2. \quad (1)$$

The shear $\gamma^i$ and the dimensionless surface mass density $\kappa$ are linear combinations of the second derivatives of the potential $\psi$ in angular coordinates,

$$\gamma^1 \equiv \frac{\psi_{,xx} - \psi_{,xy}}{2}, \quad (2)$$

$$\gamma^2 \equiv \psi_{,xy}, \quad (3)$$

$$\kappa \equiv \frac{\psi_{,xx} + \psi_{,yy}}{2}. \quad (4)$$

where $\psi_{,xx}, \psi_{,xy}, ...$ refers to $\frac{\partial^2 \psi}{\partial x^2}, \frac{\partial^2 \psi}{\partial x \partial y}, ...$

As we approach a semi-strong regime, the ellipticity can no longer be represented by the shear. In regions where $(1 - \kappa)^2 - |\gamma|^2 > 0$, the ellipticity induced by the lens is given by

$$\varepsilon^i = g^i \equiv \frac{\gamma^i}{(1 - \kappa)}. \quad (5)$$

and in regions where $(1 - \kappa)^2 - |\gamma|^2 < 0$ (the strong lensing regime) the induced ellipticity is given by the inverse conjugate of the reduced shear. Here, $i = 1, 2$ represents the two components of ellipticity or shear. In this paper, we will be addressing the weak lensing regime only. The above expressions are written for a source plane at infinity. The redshift dependence for convergence and shear is included by

$$\kappa(\theta, z) = Z(z)\kappa(\theta) \quad \gamma^i(\theta, z) = Z(z)\gamma^i(\theta), \quad (6)$$

where $Z(z)$ is the redshift weight function for background images given by

$$Z(z) = \frac{(D_{ls}/D_s)(z)}{(D_{ls}/D_s)(z \rightarrow \infty)}. \quad (7)$$

2.1. Non-parametric Mass Reconstruction Techniques

Non-parametric weak lensing mass reconstruction techniques are broadly divided into two categories: convergence techniques (Kaiser & Squires 1993, KS93) and potential techniques (Bartelmann et al. 1996; Deb et al. 2008). Traditionally, convergence techniques measure $\kappa$ directly by applying a convolution on the measured ellipticity (with the exception of Diego et al. 2007), whereas potential techniques do a $\chi^2$ minimization. The advantage of convergence methods is that they are very fast and useful for testing the data. The disadvantage is that these techniques reconstruct the semi-strong regimes less accurately. Hence the potential techniques gain importance as we approach the strong lensing regime. The supercluster A901/902 is sub-critical with $\kappa \ll 1$. In this case, the advantage of using PBL, a potential technique, is having an estimate of the covariance matrix. In potential techniques, a likelihood function is written between the observed ellipticities and the reduced shear and maximized iteratively to obtain the best solution

$$\mathcal{L} = \exp \left[ - \frac{\sum_{mn} (\varepsilon_{mn} - \psi_{mn}^{(v)}(\theta_m)) C_{mn}^{-1} (\varepsilon_{mn} - \psi_{mn}^{(v)}(\theta_m))}{2} \right], \quad (8)$$

where $\varepsilon_{mn}$ are the observables. Using finite differencing or PBL, the derivatives of the potential can be written as

$$\psi_{mn}^{(v)} = D_{mn}^{(v)} \psi_m, \quad (9)$$

where $\psi_{mn}^{(v)}$ represents derivatives of the potential, $v$ is the order of the derivative. For first derivatives $\nu = (x, y)$, for second derivatives $\nu = (xx, xy$ and $yy)$ and so on. The likelihood function given by Equation (8) is linearized and written as a function of $D_{mn}^{(v)}$, the data correlation, the potential, and the constraints at each step of the minimization and the potential is solved iteratively at the maximum of the likelihood.

3. METHOD OPTIMIZATION

Weak lensing is a statistical measure of small distortions in background galaxies caused by a cluster. In order to extract a lensing signal from the weak distortion of image galaxies, several of them have to be averaged to smooth out the intrinsic noise. Smoothing is an integral part of any problem with noisy data and low signal-to-noise ratio. The error budget of weak lensing is controlled by the scale at which the data are smoothed. Optimizing this scale is necessary independent of the technique that is used. In this section, we will introduce smoothing, describe the inversion technique used to create mass maps from measured ellipticities, and lay down a foundation for the choice of the optimal smoothing scale that will produce minimum reconstruction errors in the recovered mass.

In order to give equal weight to all ellipticity measurements around a single image, we use an azimuthally symmetric smoothing function. We choose a normalized Gaussian for this purpose.

$$\hat{\varepsilon}_m^i = Q_{mn} \varepsilon_n^i, \quad (10)$$

where $\hat{\varepsilon}_m^i$ represents smoothed ellipticity field and $Q$ is the smoothing function. Here, $m, n$ represent the background image index. In this paper, we have smoothed the data with a normalized Gaussian given by

$$Q_{mn} = \frac{\exp(-r_{mn}^2/2\zeta^2)}{\sum_{mn} \exp(-r_{mn}^2/2\zeta^2)}. \quad (11)$$

Smoothing of data prior to mass reconstruction has been done by several groups (Bartelmann 1995; Seitz & Schneider 1996, 1998, 2001), the covariance due to this smoothing is given by

$$C_{mn} = Q_{nm} Q_{mn} \sigma_n^2 \quad (12)$$

Here, $\sigma_n$ is the noise due to intrinsic ellipticities. This covariance between the constraints is used in the likelihood analysis in Equation (8).
3.1. Fitting the Error

We have defined a smoothing function and described the inversion procedure for creating mass maps. The input parameter to this method is the smoothing scale. In this section, we fit the weak lensing reconstruction error as a function of the smoothing scale, measurement error, number density, and the length scale at which structure can be resolved.

3.1.1. Toy Problem: A Sine Wave

In order to understand how errors propagate into the reconstructed mass, we will study a simple problem. The convergence \( \kappa \) for this field is defined by

\[
\kappa = A \sin \left( \frac{2\pi x}{\lambda} \right). \tag{13}
\]

where \( \lambda \) is the wavelength; it represents the scale at which structure can be resolved. We do a weak lensing reconstruction of this wave and determine the scale at which the reconstruction error is minimal. We generate the mock data by adding noise to the shear (given by Equation (13)) drawn from a Gaussian of width \( \sigma_\kappa \). This noise is varied over a wide range to determine a fitting formula for the error in the reconstructed \( \kappa \). We define the error that we are trying to measure as the difference between the true and reconstructed mass. We take into account these two effects and fit them to the error in the reconstructed convergence from the mock data.

\[
\sigma_\kappa^2 = A_0 \left( \frac{\xi}{\lambda} \right)^4 + B_0 \frac{\sigma_\kappa^2}{(\kappa^2 n) (\xi/\lambda)^2} + C_0. \tag{15}
\]

Here, \( n \) is the number density per unit wavelength of background galaxy images. The first term represents the second-order bias due to smoothing of small-scale structure. The error introduced by this term is proportional to the square of the smoothing length. The second term is the contribution from external error; in the case of weak lensing this is the error due to the intrinsic ellipticities of the background galaxies. \( \sigma_\kappa^2/(\kappa^2) \) represents the noise-to-signal ratio. The contribution from this term is inversely proportional to \( (\xi/\lambda)^2 n \), implying that the external errors get washed out as the number density of images increases or as the area of smoothing increases.

Figure 1 shows a fit for Equation (15) to the observed error for increasing amount \( \sigma_\kappa^2 \). We have plotted two cases, the solid (dashed) line represents the fitted value and the triangles (squares) represent error for \( \sigma_\kappa^2 = 1(5) \) measured from the reconstruction. Each data point (triangles and squares) on the graph represents a different mass reconstruction corresponding to a different smoothing scale and noise. As it is evident from the plot, Equation (15) is valid around the minima of the curves which defines the optimal smoothing scale for mass reconstruction. This happens because Equation (15) was constructed by considering the first term in the Taylor expansion with non-zero contribution to errors. As we move away from the minima, higher order terms start dominating. Since we are interested in computing the smoothing scale at which the error is minimum, we do not need the higher order terms. This is a demonstration with a toy model, we generalize this form and use Equation (15) to determine the scale at which we smooth ellipticities prior to the \( \chi^2 \) minimization.

4. COVARIANCE

In this section, we will compute the covariance due to smoothing in the reconstructed mass, which will be used to compute the ellipticity of the mass distribution. The covariance in the likelihood function has been studied extensively in the context of cosmic shear, Eifler et al. (2008b) have studied the covariance between cosmological parameters to determine its effect on the likelihood analysis for cosmological parameter estimation. Eifler et al. (2008a) use the covariance among the data when comparing the information content in aperture mass and two-point correlation function. However, not much importance has been given to the role of the covariance in cluster mass reconstructions with some exceptions, such as the work of Bridle et al. (1998) who derive an analytic expression for the covariance and compare it with the covariances from Monte Carlo simulations. Also, van Waerbeke (2000) has shown that a maximum likelihood lensing mass reconstruction has a noise that follows a Gaussian random field. Merten et al. (2009) have also used covariance between the mass bins for computing mass maps.
We have defined the covariance due to smoothing of error in Section 3 in the ellipticities. The covariance of any linear function $f(\theta)$ sampled at positions $\theta_n$ between any two positions $\theta_n$ and $\theta_m$ is given by

$$\text{Cov}(f; \theta_n, \theta_m) = \langle (f_n - f)(f_m - f) \rangle. \quad (16)$$

For the sake of simplicity of notation in the rest of the paper, we will denote $\text{Cov}(f; \theta_n, \theta_m) \equiv \text{Cov}_{\theta_n\theta_m}$, where $n, m$ represent either the total number of image galaxies in the case of PBL or the total number of grid cells in the case of a grid based method. We will derive the expression for the covariance matrix in the linear regime. This calculation is applicable to any technique where the derivatives of the potential can be expressed as a matrix times a potential. Hence, it is applicable to PBL and finite-differencing techniques. In the very weak lensing regime where $\kappa \ll 1$, the ellipticity can be approximated by the shear, $\gamma$,

$$\epsilon_i = \gamma_i, \quad (17)$$

where $i = 1, 2$ represents the two components of ellipticity and shear. In this case, we can write a likelihood between the observed ellipticity and the measured shear. Using Equation (8), we can write the likelihood as

$$\mathcal{L} = \exp \left[ -\frac{1}{2} \sum_{mn} \left( \hat{\epsilon}_m - G_{mp} \psi_p \right) C_{mn}^{-1} \left( \hat{\epsilon}_n - G_{nq} \psi_q \right) \right], \quad (18)$$

where $G^i$ is matrix that relates the potential to $\gamma^i$, $C$ is the covariance due to smoothing, and $\hat{\epsilon}$ is the smoothed ellipticity field. Using Equation (9)

$$K = \frac{D^{xx} + D^{yy}}{2} \quad (19)$$

$$G^1 = \frac{D^{xx} - D^{yy}}{2} \quad (20)$$

$$G^2 = D^{yy}. \quad (21)$$

Setting the first derivative of the log of the likelihood to zero we get,

$$\sum_i (G^i)^T C^{-1} G^i \psi = \sum_i (G^i)^T C^{-1} \hat{\epsilon}^i. \quad (22)$$

From this equation, we can solve for the potential. Once we know the potential there is a linear relationship between the measured ellipticities and $\kappa$. In this case, the estimated $\hat{\kappa}$ is given by

$$\hat{\kappa} = KM^{-1} \sum_i (G^i)^T C^{-1} \hat{\epsilon}^i \approx \sum_i V^i \hat{\epsilon}^i, \quad (23)$$

where $K$ is the matrix that relates the potential to $\kappa$, $M = \sum_i (G^i)^T C^{-1} G^i$ and

$$V^i = KM^{-1} (G^i)^T C^{-1}. \quad (24)$$

The covariance in $\hat{\kappa}$ follows Equation (16). This can be rewritten as

$$\text{Cov}_{\theta_n\theta_m} = V \text{Cov}_{\hat{\epsilon}} V^T, \quad (25)$$

where $\text{Cov}_{\hat{\epsilon}}$ is the covariance in the ellipticity. There are several effects that contribute to this covariance. As discussed in

| M       | X | Y = MX | Cov^T |
|---------|---|--------|--------|
| $G^1$   | $\psi$ | $\gamma_1$ | $G^{1\psi}Cov_{\theta}^\psi G^{1\psi}^T$ |
| $G^2$   | $\psi$ | $\gamma_2$ | $G^{2\psi}Cov_{\theta}^\psi G^{2\psi}^T$ |
| $\kappa$ | $\psi$ | $\kappa$ | $Cov_{\theta}^\psi K^T$ |
| $\kappa$ | $\kappa$ | $\kappa$ | $K Cov_{\theta}^\psi K^T$ |

Notes. The last column gives the covariance among the observable in the third column. The last row is the final expression for the covariance in the reconstructed $\kappa$.

Section 3.1, one term is due to smoothing of small-scale structure and the other is due to averaging the error. We have derived the effect of smoothing on errors due to intrinsic alignment of galaxies and used it for the maximum likelihood analysis. Here, we are going to derive a more general expression for the covariance in the final reconstructed mass. The covariance in measured ellipticity is given by

$$\text{Cov}_{\theta_n\theta_m} = \langle (\hat{\epsilon}_n - \bar{\gamma}_n)(\hat{\epsilon}_m - \bar{\gamma}_m) \rangle = \delta_{mn} \sigma_n^2. \quad (26)$$

where $\bar{\gamma}_p$ is the true shear for the lens. After smoothing, this becomes

$$\text{Cov}_{\theta_n\theta_m} = \langle (\hat{\epsilon}_n - \bar{\gamma}_n)(\hat{\epsilon}_m - \bar{\gamma}_m) \rangle \quad (27)$$

Let us define

$$\bar{\gamma}_m = Q_{mp} \sigma_p^2 + \bar{\gamma}_m \gamma_m + \bar{\gamma}_n \gamma_m - \bar{\gamma}_n Q_{mp} \gamma_p - \bar{\gamma}_n Q_{mp} \gamma_p. \quad (28)$$

Using this notation, we can write the above equations as

$$\text{Cov}_{\theta_n\theta_m} = Q_{mp} \sigma_p^2 + \langle (\bar{\gamma}_n - \gamma_n)(\bar{\gamma}_m - \gamma_m) \rangle. \quad (29)$$

The first term is the same as Equation (12). Since we do not know the true shear, we replace $\bar{\gamma}$ with the reconstructed shear. Replacing this in Equation (25), we have an expression for the covariance in the linear regime. The generalization of the covariance calculation in the non-linear regime is given in Appendix B. These relations are summarized in Table 1.

5. MEASURING THE ELLIPTICITY OF GALAXY CLUSTERS

It has long been established from simulations (Jing & Suto 2002; Rahman et al. 2006) that galaxy clusters are triaxial. Oguri et al. (2005) have used triaxial dark matter halos to study the steep mass profile of A1689 and arc statistics using semi-analytic models (Oguri et al. 2003) for triaxial dark matter halos and have explained the abundance of gravitationally lensed arcs for a sample of clusters. Cypriano et al. (2004) have measured the ellipticity and the position angle of a sample of X-ray selected clusters parametrically using gravitational lensing and found that the dark matter halos were aligned with the BCGs. In this paper, we will compute the ellipticities of dark matter halos non-parametrically. This model-free estimation is done by calculating the quadrupole moments of the mass map using the noise matrix derived in the previous section.

There is a strong dependence of ellipticity on amplitude of mass fluctuations $\sigma_8$ (Ho et al. 2006). A higher value of $\sigma_8$
indicates that cluster formation has started earlier and hence the measured ellipticity of clusters in the local universe would be lower. Clusters are formed through hierarchical merging of smaller dark matter halos. Thus at their infancy, they have more infalling matter and are more elliptical. As they virialize, they become more and more spherical. Thus, we expect clusters at higher redshift to be more elliptical than low redshift clusters. This has been confirmed by measuring higher order moments of the X-ray gas distribution (Jeltema et al. 2005; Buote & Tsai 1995). Following the procedure that we lay down in this paper, we can measure the ellipticity of cluster halos from lensing for a large sample of clusters distributed in redshift. This will make the contribution of the errors due to projection smaller. Since gravitational lensing probes the projected mass of the lens, it is very difficult to constrain whether the halo is oblate or prolate.

We measure the shape of the lens by calculating the second-order moments of the mass distribution, this will give us the eccentricity and the position angle for an elliptical mass distribution. We will also fit the ellipticity of the lens parametrically, for a truly elliptical lens this will give a very good description of the shape of the underlying dark matter halo.

5.1. Measuring Cluster Shapes Non-parametrically

In the previous sections, we have derived the covariance of a mass map in the linear regime. The supercluster A901/902 is a subcritical cluster, hence for the error analysis we assume linearity. It is important to note that linearity is not assumed for doing the mass reconstruction. Since we have a correlated mass map with a covariance that is singular, we will perform a singular value decomposition as explained in Appendix A and consider a few eigenmodes that are significant. We then transform to a basis where the eigenmodes are independent. This is done by the transformation

$$\kappa' = (\kappa) + U^T(\kappa - (\kappa)),$$

where $U$ is the orthogonal matrix from singular value decomposition defined in Equation (A1). The $\kappa'$s are independent. We will express all quantities in terms of $\kappa'$. Here, $(\kappa)$ is the mean density of the field, it is calculated as follows:

$$<(\kappa)> = \frac{\sum_{mn} C_{mn}^{-1} \kappa_m}{\sum_{mn} C_{mn}^{-1}}.$$

The ellipticity of the dark matter is defined in terms of the quadrupole moments. The simple definition for quadrupole moments for the $\kappa$ field is given by

$$I_{ij} = \frac{\sum_m \kappa_m w_m x_i^{(i)} x_j^{(j)}}{\sum_m \kappa_m}.$$

We know that $\kappa$ is correlated and hence we write the above expression in terms of $\kappa'$ and inverse weight it by $s$, where $s$ is defined in Equation (A1). The errors in the quadrupole moments are given by the octopole moments and are calculated in a similar fashion. Here, $w$ is the weight function, we vary the width of the weight function to evaluate the moments at different radius.

The ellipticities of the lens are defined by

$$e_1 = \frac{(I_{xx} - I_{yy})}{I_{xx} + I_{yy} + 2(I_{xx}I_{yy} - I_{xy}^2)^{1/2}},$$

$$e_2 = \frac{2I_{xy}}{I_{xx} + I_{yy} + 2(I_{xx}I_{yy} - I_{xy}^2)^{1/2}}.$$  

5.2. Measuring Shapes Parametrically

One of the first measurements of dark matter ellipticity from parametric techniques was done in Hoekstra et al. (2004) for the cluster CL1358+62 using strong lensing. In our analysis, we will fit singular isothermal ellipse to the dark matter halos to constrain the shape of the dark matter halos. The convergence and the shear for this profile (Kormann et al. 1994) are given by

$$\kappa = \frac{1}{2} \theta_E f \left[ \cos (\phi - \alpha) + f^2 \sin (\phi - \alpha) \right]^{-1},$$

where $\theta_E$ is the Einstein radius given by

$$\theta_E = 4\pi \frac{\sigma_v^2}{c^2} \frac{D_l}{D_d},$$

where $\sigma_v$ is the velocity dispersion of the cluster. Here, $f$ is the axis ratio and $\alpha$ is the position angle. This relationship is degenerate with $\alpha = \alpha + \pi/2$ and $f = 1/f$. The complex ellipticity is related to the axis ratio via

$$e = \left( \frac{1 - f}{1 + f} \right) e^{2i\alpha}.$$
This model is fitted by minimizing a $\chi^2$ of the form

$$\chi^2 = \sum_{m,i} \frac{(\varepsilon_m^i - g_m^i)^2}{\sigma_e^2},$$  \hspace{1cm} (38)

where $i = 1, 2$ is the two components of ellipticity, $\varepsilon_m^i$ is the ellipticity of the $m$th galaxy and $g_m^i$ is the reduced shear and $\sigma_e$ is the error in the tangential ellipticity. This form of the mass model has been investigated by King & Schneider (2001) and applied to a sample of X-ray luminous clusters (Cypriano et al. 2004).

6. DATA

The shear catalog for this cluster is generated following the algorithms described in Kaiser et al. (1995) and Heymans et al. (2006). The modeling of the temporal variation of the point-spread function (PSF) is outlined in Heymans et al. (2005). The A901/902 field of view has several tens of good stars for stellar modeling of the PSF, the stars were chosen to maximize the signal to noise of the stellar ellipticity function and minimize the temporal variation. The charge transfer efficiency of the Advanced Camera for Surveys has degraded over the years, this is corrected using the methods proposed by Rhodes et al.
of view have been detected by H08. We zoom into each of the peaks and reconstruct a mass map and an error map for the peaks. It is evident from the mass maps that these galaxy clusters are sub-critical, typically around the peaks the average $\kappa = 0.1$ and the noise from the intrinsic ellipticity is ~0.3, hence the signal to noise around the mass peaks is ~0.3. Using this value in Equation (15), we get an approximate smoothing scale of 0.5, below which we will not be able to measure any significant structure. Because the number density of source varies across the field, the ellipticities are smoothed with a Gaussian with a width of $0.5/\sqrt{n/\sigma^3}$, where $n$ is the number density of the images. These mass maps are used to calculate the ellipticity of the individual peaks. The BCG of each cluster is considered to be the center for these calculations.

**A901b.** First, we discuss A901b, the most compact peak with a single halo. The reconstruction parameters are reported in Table 2. The ellipticity of the peak is $0.45^{+0.11}_{-0.10}$ and the position angle is $90^\circ$ suggesting that it is elongated in the north–south direction. The top left panel of Figure 2 is a map of A901b and the right panel shows the error map for the same field of view. We have plotted the ellipticity of the dark matter halo versus radial distance in the upper panel of Figure 3. The squares connected by dot-dashed line represent the ellipticity of the dark matter halo. The ellipticity does not change significantly with radius.

**Southwest Group.** The bottom left panel of Figure 2 is the dark matter reconstruction of the Southwest Group. This peak clearly shows that it is not spherically symmetric, it is elongated in the northwest direction with significant substructure suggesting that the group is not completely virialized. We measure an ellipticity of $0.3 \pm 0.07$ and a position angle of $120^\circ \pm 4^\circ$. Since the dark matter distribution has multiple maxima, an elliptical dark matter halo may not be the best description for the Southwest Group. We have plotted the ellipticity versus radial distance for the Southwest Group in the lower panel of Figure 3. The ellipticity decreases as we move away from the center.

**A901a and A902.** A901a has two distinct peaks. This is clear from the dark matter map of the top left panel of Figure 4. It has non-zero quadrupole moments, however, it is not possible to describe it using an ellipse.

If we compare Figures 2 and 4 with H08, we see that the dark matter distribution is very similar. In this analysis, we have smoothed the data prior to reconstruction and included the covariance due to smoothing in the $\chi^2$ minimization and in calculation of covariance of the final mass reconstruction. This makes the errors of the reconstruction well understood. The error maps in the right hand panels of Figures 2 and 4 are computed by taking the square root of the diagonal of the covariance matrix of $\kappa$. The central peak of A901b is detected at 7$\sigma$ significance, the two sub-peaks of the Southwest Group is detected at 4$\sigma$. The central peak of A901a is a 5$\sigma$ detection the secondary peak is a 2$\sigma$ detection. The peak of A902 is detected at 4$\sigma$. The mass measured within 1̲ $^\circ$ of the center of each peak is listed in Table 2. We measure the ellipticity of A901b and the Southwest Peak since these two peaks are relatively less disturbed. A901a has two distinct peaks and A902 is very disturbed even in the outer regions. Hence, it is difficult to represent these peaks with ellipses.

In order to understand the errors of our reconstruction, we used a Monte Carlo simulation with no intrinsic signal for a hundred realizations for the same field of view as one of the sub-peaks and did a noise reconstruction using the same smoothing scale. Only 9.2% of these reconstructions had peaks detected at 2$\sigma$ and 24.7% of them had 1.5$\sigma$ peaks. We did not detect

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**Figure 3.** Ellipticity vs. radial distance. The dot-dashed line represents the ellipticity of the dark matter halos and the dotted line represents the ellipticity of the light distribution. The upper panel is a plot for A901b and the lower panel is a plot for the Southwest Group. For both sub-clusters, the ellipticity of the light distribution decreases with radial distance. For A901b, the ellipticity of the dark matter does not vary much with radial distance. The ellipticity of the dark matter also decreases with radial distance for the Southwest Group.

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(2007). Furthermore, magnitude cuts are applied to the galaxy catalog to eliminate cluster member galaxies and foreground galaxies. The magnitudes are chosen such that $23 < m_{HST} \leq 27.5$ and the signal to noise for each galaxy are chosen to be $> 5$ with a size greater than 3 pixels. The total sample size is ~60,000 galaxies with 65 galaxies per square arcminute. For this sample, the contamination due to cluster member galaxies is low since the redshifts of the background galaxies are chosen such that the median redshift $z_{m} > 0.6$. This redshift is calculated using a magnitude-dependent redshift relation given by Equation (9) in H08. As shown by Oguri et al. (2010), the weak lensing dilution should have negligible effect on cluster ellipticity. The ellipticity of the dark matter halo depends on the ellipticity of the shear pattern and not on its amplitude. For A901/902, the redshift for 90% of the source galaxies was not known. However, the redshift weight function does not have a very strong dependence on the redshift for $z_{g} > 1$ and $z_{l} = 0.165$. Hence, we assume the sources to be at a single redshift of 1.4 which is the estimated median redshift of the background sources. A more detailed description of the data and tests for systematics can be found in H08.

**7. RESULTS**

We have outlined a recipe for choosing the smoothing scale and using PBL for mass reconstruction. We apply this technique to the A901/902 field of view. The dark matter peaks in this field
any peaks with higher significance. In the reconstructed mass maps of A901/902, we have detected the most significant peak at $7\sigma$ which is a robust detection. The least significant peak is detected at $2\sigma$ which has a 90% probability of being real.

We have four distinct sub-clusters in this field-of-view separated by less than $3\, h^{-1}\, \text{Mpc}$. Hence, we also investigate whether the clusters to be aligned to each other (Hopkins et al. 2005). We test the alignment by looking at the cosine of the angle between the major axis of the dark matter halos. We plot the value of this angle as a function of distance in Figure 5. A901a, A901b, and A902 are very closely aligned. They are plotted in the upper panel of the figure. The cosine of the angle between the major axis of these three peaks is very close to unity. The lower panel represents the alignment between the Southwest Group and the other peaks. The angle made by the major axis of the Southwest Group deviates most from A901b. This is seen in the dotted line. The misalignment is less obvious for A901a and A902, especially when we go out radially. Even though the Southwest
Figure 5. Cosine of the angle between the major axis of the four sub-clusters with radial distance. A901a, A901b, and A902 have major axis pointing in almost the same direction, hence the cosine of the angle between their major axis is very close unity. This is plotted in the upper panel. The major axis of the Southwest Group is misaligned with the other clusters, this effect being most pronounced for A901b, the cosine of angle between A901b and the Southwest Group deviates most from unity. This is seen in the lower panel of the plot.

Peak shows some misalignment, it is well within expectations from simulations (Hopkins et al. 2005).

7.1. Parametric Fitting

From the non-parametric reconstruction, it is clear that A901a and A902 are double peaked systems, while A901b and the Southwest Peak are closer to a single halo with substructure. We have fitted this data to a singular isothermal ellipse described in Section 5.2. We fit a single halo centered on the BCG in each cluster for consistency with the non-parametric case. Since the aim of this study is to measure the ellipticity of the lens, we do this fit for the Southwest Peak and A901b only because A901a and A902 have irregular structure and hence cannot be modeled as an ellipse. The field of view contains four distinct peaks, so we consider a patch of 178 arcmin² around each peak. This ensures that the shear signal is not contaminated by the other dark matter halos. The constraints on the velocity dispersion, ellipticity, and position angle are listed in Table 2. The constraints on the velocity dispersion are consistent with Gray et al. (2002). We estimate the ellipticity of A901b to be $0.39 \pm 0.09$ and a position angle to be 90° implying that the dark matter halo is elliptical in the vertical direction. The dark matter map of the Southwest Peak is clearly not spherical. This peak is well fitted by the singular isothermal ellipse and the ellipticity of this peak is $0.4^{+0.13}_{-0.16}$. In Figure 6, we have plotted the joint 1(2)$\sigma$ error probability distribution between the axis ratio and the position angle. It is not possible to constrain the ellipticity of A901a and A902 with an elliptical mass model. In fact a parametric fit is consistent with the spherical model with high error bars. It is clear from the mass reconstruction that the peaks are disturbed. As a matter of fact A901a has two distinct peaks, the second peak coincides with an infalling X-ray group. A902 also has another galaxy group in the background at a redshift $z = 0.46$.

Figure 6. The two panels represent the joint 1(2)$\sigma$ error probability distribution for A901b and the Southwest Peak derived from the parametric modeling, described in Section 5.2. The y-axis $f$ is the axis ratio and the x-axis $\alpha$ is the position angle defined in section Section 5.2 in radians. For both plots, we have plotted the 1$\sigma$ and 2$\sigma$ contours. Both peaks have non-zero ellipticity at 2$\sigma$ level.

(A color version of this figure is available in the online journal.)
7.2. Comparison between Dark Matter and Light Distribution

In this section, we compare the ellipticities of the light distribution and the dark matter distribution. We map the light distribution by using the stellar masses of the cluster member galaxies. The field of view for A901/902 is very large with a total of 60,000 galaxies. This data set is also too large for spectroscopy. Most galaxies in this field of view were too faint for calculating photometric redshifts, hence a magnitude-dependent redshift relation is used to ascertain the median redshift (Schrabback et al. 2007).

$$z_m = 0.29(m_{F606} - 22) + 0.31.$$  \hfill (39)

The cluster galaxies were selected assuming that the photometric redshift of clusters follows a Gaussian (see Figure 13 in Gray et al. 2009) and the galaxy number density follows the average number density $n(z, R)$ ($R$-band magnitude) and varies smoothly with magnitude and redshift. The mean redshift of the cluster galaxies is $z_{\text{phot}} = 0.17$. The detailed data analysis for this
We have measured the ellipticity of the light distribution by measuring quadrupole moments of the cluster member galaxies weighted by their stellar masses. The results are listed in Table 2. The light distribution and the dark matter distribution are coincident for A901b. In the Southwest Peak, the light distribution is coincident with one of the sub-peaks. The light distribution is less elliptical compared to the mass distribution.

The results from the non-parametric reconstruction of A901b indicate that the major axis of the light distribution and the dark matter distribution are not coincident. Figure 7 shows a comparison of the light and dark matter distribution. The background map represents the stellar mass of the cluster member in units of \( M_\odot \). The contours represent the dark matter distribution.

We also compare the ellipticity of the dark matter and light distributions in Figure 8. The light distribution becomes more circular with increasing radius and the dark matter ellipticity does not change significantly with radial distance. For the Southwest Group, the ellipticity of both the dark matter and the light distribution decreases with radial distance. In Figure 8, we plot the alignment between the light and the dark matter ellipticity as a function of radius. For A901a, A902, and the Southwest Group the light and the dark matter are well aligned.

However, A901b the orientation of the light distribution is misaligned with the dark matter distribution. There are a number of possible explanations for this misalignment. There can be contamination from field galaxies and the cluster galaxy sample may not be completed. The cluster galaxies also sample the dark matter potential at finite positions making the error in the light distribution measurements higher. The presence of line-of-sight structure can also bias the orientations of the light distribution and the dark matter distribution differently. There could also be further complexity in galaxy formation and evolution in the cluster environment causing the light distribution to be significantly different from the dark matter distribution. Misalignments among the cluster member galaxy distribution and dark matter distribution is also seen in the cluster sample studied by Oguri et al. (2010).

7.3. Comparison between Parametric and Non-parametric Results

We have computed the ellipticity and the position angle for A901b and the Southwest Peak parametrically and non-parametrically. The results for A901b using both methods are consistent. The position angle for the Southwest Peak is inconsistent between the measurements. This peak has an irregular mass distribution, hence an elliptical mass model does not provide a full description of its shape. The values of the measured ellipticity are very close to that expected from simulations (Ho et al. 2006; Bailin & Steinmetz 2005) and previous observations of dark matter galaxy halo ellipticities from the Red Sequence Cluster Survey (Hoekstra et al. 2004). The measured velocity dispersion for the peaks is consistent with the results from ground-based investigations (Gray et al. 2002). The error in the minor to major axis ratio is quite high since weak shear is a noisy estimator of the dark matter distribution, however, it is the only way to uniquely measure the dark matter halo shape.

8. Discussion and Future Work

In this paper, we have measured the projected ellipticity of dark matter halos non-parametrically. This is done using an improved PBL by including smoothing of the ellipticity field. We have also calculated the covariance of the resulting mass distribution, making the errors of the reconstruction well understood. We have applied this technique to the supercluster A901/902 and reconstructed each of the peaks individually and measured the ellipticity of A901b and Southwest Peak from the PBL reconstruction and using parametric models. The other two peaks A901a and A902 have a lot of substructure and cannot be modeled as ellipses. We have not considered the line-of-sight ellipticity of the dark matter halos of A901/902. This is because lensing probes the projected mass distribution. Corless et al. (2009) have fitted triaxial NFW to galaxy clusters with high errors on the concentration and mass suggesting that more information form strong lensing, X-rays, and the Sunyaev–Zel’dovich effect is required to constrain parameters pertaining to the line of sight. A joint analysis using X-rays and lensing has been done by Morandi et al. (2010) to constrain the ellipticity along the line of sight.

In the future, we will measure the ellipticity of the projected dark matter halos for a large sample of clusters and compare them to simulations (Hopkins et al. 2005). The error introduced due to the line-of-sight ellipticity for a large sample will be insignificant. A similar study has been done in X-rays by Jeltema et al. (2005) by studying the ratio of the higher order moments to the zeroth-order moments.

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Appendix A

Inverting the Covariance Matrix for \( \chi^2 \) Minimization

In order to do a \( \chi^2 \) minimization, the inverse of the covariance matrix is important. Before doing the inverse, it is important to understand the properties of the covariance matrix. It is essential to remember that every image location is not independent. This is because we are smoothing the data, this makes the number of image positions larger than the number of “resolution units” for a given smoothing scale. This situation has been encountered before (Pan & Szapudi 2005; Gaztañaga & Scoccimarro 2005) when the data vector was larger than the number of independent information content. This makes input data degenerate, i.e., for a smoothing scale considerably higher than the interparticle separation two positions spatially close to each other essentially represent the same information. This makes the covariance matrix singular. The number of independent components of the covariance matrix is inversely proportional to the area under the smoothing kernel.

We invert this singular matrix by using the Singular Value Decomposition (SVD) implementation of Tikhonov regularization. The traditional SVD matrix inversion is given by

\[
M^{-1} = VSU^T.
\] (A1)
Here, $U$ and $V$ are orthogonal matrices, since $C$ is symmetric $U = V$. $S$ is a diagonal matrix, the diagonal elements are given by $[1/s]$. Here, $s$ represents the eigenvalues of $C$. In the case of a singular matrix, some of the eigenvalues are zero. Usually, $s$ is written in descending order and the first $l$ non-zero eigenvalues are used for the matrix inversion. For the $n-l$, eigenvalues $1/s$ are replaced by zero. In real cases, it is commonly seen that the eigenvalues are not zero, rather they are numerically very small, and hence dominated by round-off error. These problems are termed as ill-conditioned problems. In order to deal with this situation, a truncated SVD is used to do the matrix inversion, i.e., eigenvalues below a certain threshold are considered to be zero. Choice of this threshold is depend on the particular problem. If the singular values of a matrix can be distinguished from the non-singular ones in a well-defined fashion then the choice of the threshold becomes simple. This is the case when there is a substantial gap between the largest singular eigenvalue and the smallest non-singular eigenvalue. However, in this particular problem where the covariance is due to Gaussian smoothing of the data there is no such distinction. As a matter of fact, the eigenvalues smoothly asymptote toward zero. Hence, the problem does not direct us toward any obvious choice of the threshold value. Since the problem is severely ill-posed, we use Tikhonov regularizarion to invert the covariance matrix. This is equivalent to

$$M^{-1} = Vf_{m}U^{T},$$

(A2)

where $f_{m} = s_{m}/s_{\alpha}$ are the filter factors. For $s_{m} \gg \alpha$, $f_{m} = 1/s_{m}$ and for $s_{m} \ll \alpha$, $f_{m} = 0$. The presence of the regularization ensures that there is a smooth transition between $f_{m} = 1/s_{m}$ and $f_{m} = 0$ instead of an abrupt cutoff threshold. The regularization parameter $\alpha$ is given by

$$\alpha = C\chi^{2}.$$

(A3)

As $\alpha$ decreases more and more eigenvalues are included in matrix inversion. As the smoothing scale is increased the area becomes higher, $\alpha$ becomes higher and the number of modes used in the matrix inversion becomes lower.

APPENDIX B

GENERALIZATION TO THE NON-LINEAR REGIME

In Section 4, we have evaluated the covariance for the reconstructed $\kappa$ in the linear regime. Inverse techniques like PBL are often able to reconstruct the semi-strong regime of weak lensing clusters within a couple of iterations. Hence, we write down a general formalism in the semi-strong region.

In the second iteration, the potential is given by

$$\psi^{(1)} = (G)^{T}C^{-1}\hat{\xi} = L(\epsilon^{(0)} - \hat{\xi}),$$

(B1)

where $\epsilon^{(0)}$ is the modeled ellipticity from the first step of the minimization. In order to get the correct solution for the potential, both components of the ellipticity will be summed; for the sake of simplicity of notation we have not included it in Equation (B1). We assume that we are in the semi-strong region and $\kappa < 1$, hence the modeled ellipticity is given by

$$\langle \epsilon_{m}^{(0)} \rangle = \frac{\langle \epsilon_{m}^{(0)} \rangle}{(1 - \kappa_{m}^{(0)})} = \frac{\epsilon_{m}^{(0)}}{1 + \kappa_{m}^{(0)}}.$$  (B2)

The reconstructed $\kappa$ in the second iteration is given by

$$\kappa^{(1)} = K\psi^{(0)} + K\psi^{(1)}.$$  (B3)

The expectation value for the ellipticity is given by

$$\langle \epsilon_{m} \rangle = \gamma_{m}^{(1)}(1 + \kappa_{m}^{(1)}).$$  (B4)

The covariance in $\kappa$ after the first iteration is given by

$$\text{Cov}_{\kappa}^{(1)} = \langle K \psi^{(0)} \psi^{(0)T} K^{T} + 2K \psi^{(1)} \psi^{(1)T} K^{T} + K \psi^{(1)} \psi^{(1)T} K^{T} \rangle.$$

(B5)

We have already calculated the covariance due to the first term in the previous section. We have to evaluate the second term and the third term. The second term is given by

$$\text{term}2 = 2KL(\langle \hat{\xi}\epsilon^{(0)} \rangle - \langle \hat{\xi}\hat{\epsilon} \rangle)L^{T}K^{T}$$

(B6)

and the third term is given by

$$\text{term}3 = KL(\langle \epsilon^{(0)}\hat{\epsilon} \rangle - 2\langle \hat{\xi}\hat{\epsilon} \rangle)L^{T}K^{T}. (B7)$$

Now, we will evaluate the covariance the terms $\langle \epsilon^{(0)}\hat{\epsilon} \rangle$, $\langle \hat{\xi}\hat{\epsilon} \rangle$.

$$\langle \hat{\epsilon}_{m}\epsilon_{n}^{(0)} \rangle = \gamma_{m}^{(0)}(1 + \kappa_{m}^{(0)})\gamma_{n}^{(0)}(1 + \kappa_{n}^{(0)}).$$

(B8)

Using these expressions for term2 and term3, we can evaluate the covariance in the reconstructed mass after two iterations. We can do a similar calculation for the next steps iteratively and keep higher order terms in the expansion of Equation (B2) and obtain an expression for the covariance of the reconstructed mass.

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