Quantitative analysis of sustained oscillation associated with saturation non-linearity in a grid-connected voltage source converter

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Funding information
National Natural Science Foundation of China, Grant/Award Numbers: U1866601, 51737007, 51925701; Science and Technology Project of State Grid Corporation of China, Grant/Award Number: SGZJ0000KXJS1900418

Abstract
The interaction between the voltage source converter (VSC) and weak grid causes a negatively damped or diverging oscillation (DVO). The diverging oscillation generally evolves into a sustained oscillation (STO) when governed by the nonlinearities in the VSC's control system, such as the saturation of pulse width modulation (PWM). The conventional small-signal impedance/admittance modelling (SSIM/SSAM) and stability analysis methods are not suitable for the characteristic analysis of STO. This paper presents a large-signal impedance/admittance model (LSIM/LSAM)-based quantitative analysis method to better understand the mechanism, characteristics, and occurrence conditions of STO. The LSAM extends the SSAM by incorporating the nonlinearity of PWM saturation using the describing function method. First, a unique time-varying characteristic of the oscillation associated with nonlinearity is investigated. Then, two LSIM/LSAM-based occurrence criteria for STO are established based on an equivalent RLC circuit model of the system. Three critical pieces of information, including the occurrence condition, frequency, and magnitude, associated with STO are revealed by the criteria. Finally, the LSAM-based analysis method is applied to investigate the time-varying admittance of the VSC and to assess the magnitude and frequency of STO. Both the criteria and LSAM-based analysis results have been verified through time-domain simulations on a typical grid-connected VSC.

1 | INTRODUCTION

With the rapid development of power-electronic based renewable generation, emerging electromagnetic oscillations caused by the interaction between the voltage source converter (VSC) and the weak grid have raised great concerns [1–3]. Such oscillations endanger the system's stability, reliability and trigger other power quality problems [4–6].

A negatively damped oscillation continuously grows until the protection relays are triggered and the VSCs or wind farms participating in the interaction are disconnected. The frequency and damping of the oscillation are determined by linear components in the converter control system [7, 8]. However, in many practical cases, the diverging oscillation (DVO) usually evolves into a sustained oscillation (STO) with a constant magnitude governed by the non-linearities in the VSC’s control system, such as pulse width modulation (PWM) saturation [4]. The features, e.g. magnitude, associated with such STO cannot be determined using the linearized modelling and analysis techniques. The information about the magnitude and frequency characteristics of the STO is quite meaningful for the system operators to take appropriate measures for the safe operation of the system.

Impedance/admittance model (IM/AM)-based analysis has been widely used to evaluate the oscillatory stability due to its advantages in modelling “black-box” devices and manipulating complicated networks [7, 9–19]. By dividing the target system into source and load subsystems and representing them as

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independent IMs or AMs, the oscillatory stability can be evaluated using the IM-based Nyquist criterion [8, 20] or Bode diagram [21]. For example, with Bode plots of the source and load IMs, the oscillation frequency is obtained by finding the intersection point of the magnitudes of impedances, and the stability is inferred by the phase difference at that frequency. However, most of the existing IM/AMs are developed in the small-signal sense or representations of linearized systems. Also, the small-signal IM/AM-based Nyquist analysis can be used to evaluate the stability of a linearizable system, but it is not suitable for the quantitative analysis of the STO governed by non-linearities [22].

Previously, modelling and stability analysis methods based on describing function and phase plane have been proposed to quantify the STO's parameters [23]. In [24], the STO in a simple DC-DC converter was analyzed by modeling the dead-band non-linearity using describing function. In [25], a simple feedback control system was divided into a linear transfer function and a non-linear describing function, and the STO's magnitude and frequency were determined by the intersections of the transfer function and describing function in the complex plane. The work in [26] extended the analysis method and results of [25] by considering the multi-deadband non-linearity. However, the conventional describing function-based or phase plane-based methods can provide analytical solutions only for systems with fairly simple control structures and low-order characteristics. For those complex power electronic converter based systems, such as the three-phase AC-DC converter [9] or VSC-HVDC [27], they usually have multiple controllers and complex networks. Such complex systems are higher-order and can hardly be divided into independent linear and non-linear parts for the conventional describing function based modelling and analysis. Owing to these challenges, earlier investigations on the STO mainly relied on the electromagnetic transient (EMT) models and time-domain simulations [4, 6, 28] or experimental measurements [5]. For instance, the work in [4] established an EMT model of a grid-connected VSC considering the PWM saturation non-linearity and used fast Fourier transform (FFT) to compute the magnitudes and frequencies of STO under different system conditions. However, EMT models are not easily accessible due to the manufacturer's commercial concerns. The experimental analysis requires special hardware, which is costly and impractical at large-scale.

Recently, some efforts have been made to investigate the STO caused by non-linearity in complex control systems using analytical analysis methods [29–32]. In [29], the STO caused by imperfect compensation of the valve control non-linearity in steam-turbine generators was investigated. The non-linearity was approximated using a quintic polynomial when developing the state-space model for the system. However, the resulting state-space model becomes fairly complex and cannot be expressed in an explicit way. As a result, the analytical model is suitable for qualitative analysis only. The mechanism, occurrence condition, and parameters of the STO still have to be analyzed through numerical simulations. In [30], the detailed state-space model of a three-phase VSC was established considering the current saturation non-linearity caused by hard limiters in the dq-frame current control loop. The reference currents in the d- and q-axis were expressed using piecewise functions when plotting the eigenvalue trajectory. Then, the work in [31] extended the analysis in [30] using the Lyapunov function and the mixed potential theory. Both [30] and [31] provided a theoretical stability boundary for the system, where the STO would not occur. However, little attention was paid to the dynamic characteristics of the STO, as well as the computation of its magnitude and frequency. In [32], it was found that the current saturation caused by hard limiters could lead to STO and even transient angle instability. Nevertheless, the stability analysis method proposed in [32] was based on a quasi-static model, where the small-signal dynamics of the outer and inner loop controls were completely ignored. Therefore, the proposed model and analysis method can hardly be used for quantifying the parameters of the STO.

The frequency-domain methods, such as IM, are generally more intuitive and convenient than the above time-domain methods or the energy-function-based methods for modelling and analysis of high-order power electronic based systems [33]. Shah et al. proposed an analytic modelling approach, named as the large-signal impedance/admittance modelling (LSIM/LSAM), to account for the non-linearities in frequency-domain impedance/admittance modelling [34]. The authors mainly focused on designing mitigation control strategies, while paid little attention to the detailed mathematical derivations of the LSIM/LSAM. The utilization of LSIM/LSAM for the characteristic analysis of STO caused by the non-linearities requires more rigorous investigations.

This paper presents a detailed quantitative analysis of STO in a grid-connected VSC, including the mechanism, characteristics, occurrence criteria, and assessment of the magnitude and frequency of the STO. The contributions are as follows.

1. A unique time-varying characteristic of the oscillation associated with non-linearity is revealed. The source of non-linearity and root-cause of the STO are identified.
2. Two LSIM/LSAM-based occurrence criteria are established based on the equivalent RLC circuit model of the grid-tied VSC. The criteria contain three vital information, including the occurrence condition, frequency, and magnitude of the STO.
3. A step-by-step procedure to establish the LSAM of VSC is presented with a detailed mathematical derivation for modelling the non-linearity of PWM saturation. The proposed criteria and LSAM-based analysis method are applied to assess the magnitude and frequency of the STO.

The rest of this paper is organized as follows. In Section 2, the characteristics of the STO in a typical grid-tied VSC are investigated using time-domain simulations. In Section 3, the concept of LSIM/LSAM is introduced to model the non-linearity from PWM saturation. The LSIM/LSAM-based criteria for the quantitative analysis of STO are established. The STO's magnitude and frequency are assessed using the criteria and the measurement-based LSAM. Section 4 elaborates the step-by-step procedure to construct the LSAM in an analytical way.
The VSC operates in grid-following mode. The synchronization between the VSC and grid is achieved through a synchronous reference frame (SRF)-PLL, as shown in Figure 3. The outputs of the PWM will get limited if the modulation index $m_{abc}$ becomes higher than unity. This effect is called the over-modulation or PWM saturation [4], which can be modelled by a saturation function:

$$f(x) = \begin{cases} Kx & (|x| \leq a) \\ Ka & (x > a) \\ -Ka & (x < -a) \end{cases}$$

where $K = 1$ and $a = 1$ for the following analysis.

### 2.2 Time-varying characteristic of oscillation associated with non-linearity

Previous researches have demonstrated that the variations in the system’s operating conditions or converter control parameters introduce time-varying features in the triggered oscillations [7, 9, 16]. This subsection aims to investigate a new type of time-varying characteristic that is associated with the non-linearity in the VSC’s control system. This type of time-varying characteristic is different from the previous ones. To demonstrate this, an oscillation in a typical grid-connected VSC is triggered by a sudden change in system strength by varying grid inductance from 0.5 to 0.55 mH at $t = 10$ s. These certain grid inductance values are chosen intentionally, so that the magnitude of the triggered oscillation diverges until a signal somewhere in the converter control system gets saturated. The phase-A voltage measured at the PCC bus is shown in Figure 4(a). To get a better view, the oscillatory component is extracted and shown in Figure 4(b). A gradual increase in the oscillatory component can be observed after the change of system strength. Such growing oscillation is hereinafter termed as a diverging oscillation (DVO). About 23 s later, the magnitude of the DVO stopped growing and maintained at a constant magnitude, named as sustained oscillation (STO).

The position of control saturation is located by monitoring the output signals of PWM and current proportional-integral (PI) controllers during the STO event. As shown in Figure 5(a), after the DVO turned into STO, the modulation index was saturated, i.e. the upper and lower peaks of $m_{abc}$ got clipped. On the other hand, the PI outputs remained below their hard limits (see Figure 5(b)). This implies that the non-linearity of PWM saturation is the root-cause of the STO, whereas, the PI controller saturation has no role in that STO. It is because the outputs of PI controls are relatively smaller and rarely saturate under the above control scheme (see Figure 2). This observation is consistent with the analysis results presented in [4].

The magnitude and frequency of the oscillatory component of the PCC voltage are obtained using short-time Fourier transform (STFT). The magnitude and frequency variations with respect to time are shown in Figure 6. Evidently, the oscillation magnitude continuously grew until it reached a saturation...
value of about 0.065 kV (0.17 p.u.) at \( t = 33 \) s. The oscillation frequency remained fixed at about 432.4 Hz before the PWM saturated (i.e. remained fixed within the DVO region in Figure 6). However, as the DVO turned into STO, the frequency went through a transient region and quickly decreased to 430.6 Hz. This clearly suggests that the frequencies of the DVO and STO should be viewed differently. The mechanism of this new type of oscillation characteristic associated with the PWM saturation has not been reported by previous works.

After applying the STFT on the oscillatory components of the PCC voltage and current, the impedance of the VSC is obtained by dividing the voltage with the current at the oscillation frequency. The time-domain resistance and reactance plots of the resulting impedance are presented in Figure 7. It can be seen that the VSC exhibited a negative resistance and capacitive reactance both in the DVO and in the STO regions. Moreover, in the transient region, the resistance changed from more negative to a less negative value, indicating a slight improvement in the oscillation damping. A similar step change is also observed in VSC’s reactance. These step changes in the VSC’s impedance again indicate the unique time-varying characteristic of the oscillation that is associated with the PWM saturation.
2.3 Characteristics of the sustained oscillation under different system strengths

The magnitude and frequency of the STO are greatly affected by the grid inductance. This is because the level of PWM saturation is different at different system strengths. For instance, Figure 8 compares the modulation indices before and after PWM saturation (i.e. $m_a$ and $m_a'$ shown in Figure 2) for two grid inductance values, that is 0.55 and 0.65 mH. A higher $m_a$ is observed for larger inductance (weaker system connection), and hence the clipped region of $m_a'$ is wider, indicating a severer saturation. Governed by the converter closed-loop control, the magnitude of the STO varies in response to different levels of PWM saturation. Besides, the phase difference in modulation indices for different grid inductances indicates that the frequency of STO is also different. To further explore the effect of grid inductance on the magnitude and frequency, the grid inductance is varied from 0.5 to 0.7 mH with a step increment of 0.01 mH. The STO’s magnitude and frequency for different grid inductance values are computed using FFT, and plotted against grid inductance, as shown in Figure 9. The results show that the increase in the grid inductance leads to relatively higher magnitudes and lower frequencies of the STO. Thus, the system strength has an evident impact on the characteristics of the STO that occurs in the grid-connected VSC.

To sum up, the saturated oscillatory voltage and unique time-varying characteristic of the oscillation are associated with the PWM saturation. The magnitude and frequency of the STO are also affected by the PWM saturation. The conventional small-signal impedance/admittance model (SSIM/SSAM) can hardly incorporate the saturation function due to the existence of non-differentiable points (e.g. $x = \pm a$ in Equation (1)). Also, the conventional SSIM/SSAM-based analysis methods are not suitable for the characteristic analysis of the STO, including its occurrence condition, frequency, and magnitude. This paper attempts to address these challenges by utilizing a LSIM/LSAM and presenting different criteria for the characteristic analysis of STO.

3 MEASUREMENT-BASED LSAM AND QUANTITATIVE ANALYSIS OF STO

In this section, the large-signal impedance/admittance model (LSIM/LSAM) is introduced to investigate the time-varying IM/AM responses caused by the PWM saturation. This section first establishes two LSIM/LSAM-based occurrence criteria for the STO. Then, the LSAMs are obtained using the measurement-based method. Finally, the proposed criteria and the obtained LSAMs are used to assess the magnitude and frequency of the STO.
3.1 LSIM/LSAM-based occurrence criteria for STO

The step change of the VSC impedance caused by PWM saturation can be viewed as a magnitude sensitive characteristic. The small-signal impedance model (SSIM) cannot capture such characteristic as it is defined as the function of the oscillation frequency without any relation to the oscillation magnitude. Therefore, the core problem for modelling the PWM saturation lies in how to incorporate the oscillation magnitude into the VSC’s IM. This can be addressed by introducing the following large-signal impedance model (LSIM) [34], defined as:

$$Z_{LSIM}(s, \Delta V_s) = \frac{\Delta V_s(f_s)}{\Delta I_s(f_s)}, s = j 2\pi f_s$$ (2)

where $\Delta V_s$, $\Delta I_s$ are the oscillatory voltage and the resultant current at the PCC bus, both in phasor form; the scalar quantity $\Delta V_s$ denotes the magnitude of the oscillatory voltage; $f_s$ is the oscillation frequency.

The reciprocal of LSIM is defined as the large-signal admittance model (LSAM), that is,

$$Y_{LSAM}(s, \Delta V_s) = \frac{\Delta I_s(f_s)}{\Delta V_s(f_s)}, s = j 2\pi f_s$$ (3)

When the oscillation is in the DVO region, the PWM has not saturated because the magnitude of the oscillatory voltage $\Delta V_s$ is of relatively low value. Thus, in this region, the LSIM and SSIM of VSC can be viewed as the same. However, the PWM saturates in the transient and the STO region, the LSIM starts changing with the growing magnitude of the oscillation due to the magnitude sensitive characteristic. As for the impedance at the STO region (shown in Figure 7), it is essentially an LSIM saturated because the magnitude of the oscillatory voltage $\Delta V_s$ diverges, perturbation voltage with different magnitudes is injected at the PCC bus to represent the magnitude variation of the oscillation. Therefore, the scalar quantity $\Delta V_s$ in Equations (2) and (3) can also be viewed as the magnitude of the external perturbation. Its value ranges from zero to the magnitude of the STO. It is pertinent to mention that the LSIM of the grid has no such magnitude sensitive characteristic because it is modelled by an RL branch without any non-linearity. That is to say, the LSIM of the grid presents no difference with its SSIM.

For a VSC-grid system, the LSIM of the whole system is the sum of LSIMs of the VSC and the grid, that is,

$$Z_{sys}(s, \Delta V_s) = \frac{\Delta V_s}{\Delta I_s} = \frac{\Delta V_s(f_s)}{\Delta I_s(f_s)}, s = j 2\pi f_s$$ (4)

where $Z_{LSIM}$ and $Z_g$ are the LSIMs of the VSC and the grid, respectively; $R_g$ and $X_g$ are the lumped resistance and reactance, respectively.

Based on Equation (4), the overall LSIM of the system can be represented by an equivalent second-order RLC series circuit, as shown in Figure 10.

![Figure 10](image.png)

To investigate how the parameters of the above RLC circuit vary as the oscillation evolves, the real part $R_g$ and imaginary part $X_g$ of the system’s LSIM are computed at the oscillation frequency, as displayed in Figure 11. The resistance $R_g$ remains negative in the DVO region. However, as the magnitude of oscillation grows to a threshold value due to PWM saturation, the resistance $R_g$ experiences a rapid rise and settles at zero in the STO region. On the other hand, the reactance $X_g$ remains at zero for both DVO and STO regions, except for a short-lived fluctuation in the transient region. The above observations indicate that: the DVO is characterized by a negative resistance and a zero reactance, while the STO is characterized by zero resistance as well as zero reactance. The occurrence of DVO can be explained from the equivalent series RLC circuit, that is, the equation $X_g = X_{LSIM} + Z_g = 0$ holds true at the oscillation frequency. The divergence of oscillation indicates a negative damping or negative resistance, namely $R_g = R_{LSIM}$ and $R_g < 0$. However, as the effect of PWM saturation appears, the resistance of VSC experiences a rapid rise and becomes less negative. Thus, the STO occurs when the total resistance of the system becomes zero, indicating a null-damping condition. Therefore, a zero value of the system’s LSIM, or $Z_{sys}(s, \Delta V_s) = 0$, can be used to establish the STO’s occurrence criterion.

Based on the above interpretation, two LSIM/LSAM-based occurrence criteria are proposed as follows:
Criterion I: A DVO enters the STO region when the total LSAM of the system satisfies:

\[ Z_{\text{sys}}(s, \Delta V_c) = Z_{\text{LSAM}}(s, \Delta V_c) + Z_g(s) = 0 \]

\[ \Rightarrow \begin{cases} R_G = R_{\text{LSAM}} + R_g = 0 \\ X_G = X_{\text{LSAM}} + X_g = 0 \end{cases} \]  \hspace{1cm} (5)

Criterion I can be applied when conducting the IM-based reactance-frequency crossover analysis [35], where the STO occurs if the zero-crossing point of the lumped reactance \( X_G \) coincides with that of the lumped resistance \( R_G \).

Criterion II: Equation (5) can be rearranged to form another criterion, which is based on the magnitude and phase information of the LSAM, as shown below.

\[ Z_{\text{LSAM}}(s, \Delta V_c) + Z_g(s) = 0 \]

\[ \Rightarrow 1 + \frac{Z_g(s)}{Z_{\text{LSAM}}(s, \Delta V_c)} = 0 \Rightarrow \frac{Z_g(s)}{Z_{\text{LSAM}}(s, \Delta V_c)} = -1 \Rightarrow |Y_{\text{LSAM}}(s, \Delta V_c)| \frac{\Delta V_c}{Y_g(s)} \]

\[ = -1 \]  \hspace{1cm} (6)

The STO occurs if the following equation holds:

\[ \frac{Y_{\text{LSAM}}(s, \Delta V_c)}{Y_g(s)} = -1 \iff \left\{ \begin{array}{l} |Y_{\text{LSAM}}(\omega, \Delta V_c)| = |Y_g(\omega)| \\ \angle Y_{\text{LSAM}}(\omega, \Delta V_c) - \angle Y_g(\omega) = 180^\circ \end{array} \right. \]  \hspace{1cm} (7)

According to Criterion II, the STO happens if the phase difference between the LSAMs of the VSC and grid equals \( 180^\circ \) at the magnitude intersection. This criterion provides three crucial information for the STO: (1) the occurrence condition; (2) the frequency; and (3) the magnitude.

3.2 | Quantitative analysis of STO using LSAM and criteria

The LSAM of VSC can be obtained by either perturbation-based frequency scanning method [7] or analytical derivation. The former method relies on measuring the driving-point admittance by injecting perturbation voltages of different frequencies and magnitudes. The latter method directly computes the LSAM based on the steady-state operating points and control parameters. Either of the methods produces the same LSAM result. However, the perturbation-based method is preferred when the analytical derivations are complex and VSC’s control structure and parameters are unknown. Here, we present a detailed procedure for obtaining the measurement-based LSAM and determining the STO’s magnitude and frequency using the established criterion.

Figure 12 displays the admittance-frequency curves (magnitudes and phases in the Bode plot) in the concerned frequency range of (420 Hz, 445 Hz). In the figure, the LSAM of the grid is represented by a solid black line, which is not affected by the magnitude of the perturbation. The LSAMs of VSC under different perturbation magnitudes 0–0.14, 0.15, 0.16, and 0.17 p.u. are also plotted in Figure 12. From Figure 12, the following observations can be made:

1. The LSAMs of VSC coincide with one another when the perturbation magnitude varies from 0 to 0.14 p.u. This is because the PWM has not saturated within this relatively low magnitude range. However, as the PWM becomes increasingly saturated due to larger perturbation magnitudes (from 0.15 to 0.17 p.u.), the LSAM changes with slightly increasing magnitudes (within \( f_s < 439 \text{ Hz} \)) and slightly decreasing phases. This suggests a less negative resistance and reactance of the VSC. The above observations are consistent with the results presented in Figure 7.

2. As the perturbation magnitude increases, the magnitude intersection point of the VSC’s and grid’s LSAM moves leftward, thus the oscillation frequency decreases. Besides, the phase difference decreases from above \( 180^\circ \) to \( 180^\circ \) in this process. This indicates the system changes from highly unstable to marginally stable as the oscillation evolves from DVO to STO. Specifically, the rightmost intersection with a phase difference of \( 191^\circ (>180^\circ) \) implies instability or DVO. As magnitudes of the oscillation (or perturbation) increase to 0.15, 0.16, and 0.17 p.u., the frequencies decrease to 434, 432, and 431 Hz, respectively. The phase differences associated with the above three sets of frequencies are \( 187^\circ, 183^\circ \), and \( 180^\circ \), respectively. The trend agrees with the results displayed in Figure 6.

3. The LSAM of VSC with the magnitude of 0.17 p.u. intersects the LSAM of grid at a critical point (431 Hz), where the phase difference is \( 180^\circ \). According to Criterion II, it is determined that for such a grid-tied VSC affected by PWM saturation, the DVO has become a STO with a magnitude of 0.17 p.u. and a frequency of 431 Hz. These results are consistent with those obtained from EMT simulations, as shown in Figure 6.
Figure 12 has illustrated a straightforward procedure to quantitatively assess the frequency and magnitude of the STO. First, the LSAMs of the VSC, which are generally non-linear functions of the frequency and perturbation magnitude, is obtained or established. Second, the LSAM or SSAM of the grid is computed by aggregating all network components. Finally, the frequency and magnitude of STO is identified by finding the critical point that satisfies the Criterion II. The above analysis can also be performed using Criterion I with the IM-based reactance-frequency crossover analysis since Criteria I and II are equivalent [35].

In the above example, the LSAMs of VSC were given by the perturbation-based frequency scanning method. This method works effectively when the rough values of STO’s magnitude and frequency are known beforehand. In such case, the LSAMs obtained within a relatively narrow range of magnitude and frequency are sufficient for assessing the STO’s parameters. Also, the workload of measurement is moderate. However, for a general system without any prior knowledge of the STO, such perturbation-based identification and the following trial-and-error searching for the critical points could bring much heavier workload. Therefore, an analytically derived LSAM is required to improve the accuracy and computational efficiency of the above LSAM-based analysis method for the assessment of STO’s parameters.

4 DEVELOPMENT OF THE ANALYTICAL LSAM

4.1 Frequency-domain representation of PWM saturation

The time-domain relationship between the modulation indices before and after saturation ($m_{abc}$ and $m_{abc}'$) can be established using a non-linear saturation function, $m_{abc}' = f(m_{abc})$, where $f$ is given by Equation (1). Take phase-A of the modulation index as an example, it is expressed in the time-domain as:

$$m_a(t) = M_1 \cos(\omega_1 t + \phi_{m1}) + \Delta M_s \cos(\omega_s t + \phi_{ms})$$  (8)

where $M_1$ and $\phi_{m1}$ are the magnitude and phase of the fundamental component; $\Delta M_s$ and $\phi_{ms}$ are the magnitude and phase of the oscillatory component with frequency $\omega_s$. When establishing the following LSAM, $\Delta M_s$ can be viewed as a perturbation component.

To compute the saturated modulation index $m_{abc}'$, the multi-input describing function method can be used [23], according to which, the output signal of the saturation function can be approximated by its first harmonic component(s). In this way, the modulation index after saturation $m_{abc}'$ is given as:

$$m_a'(t) = M_1' \cos(\omega_1 t + \phi_{m1}') + \Delta M_s' \cos(\omega_s t + \phi_{ms}')$$  (9)

According to the multi-input describing function method, the saturation function affects the magnitude of the input signal but keeps its phase unchanged [23]. Therefore, by comparing Equations (8) and (9), it can be obtained that $\phi_{m1}' = \phi_{m1}, \phi_{ms}' = \phi_{ms}$. Further, to quantitatively investigate the magnitude variation of each frequency component, two saturation gains are defined as follows:

$$N_{m1} = M_1'/M_1, \quad N_{ms} = \Delta M_s'/\Delta M_s$$  (10)

where $N_{m1}, N_{ms}$ are the saturation gains for the fundamental component $f_1$ and the associated perturbation component $f_s$, respectively; and $N_{m1}, N_{ms} \leq 1$.

Since $M_1$ and $M_1', \Delta M_s$ and $\Delta M_s'$ are in phase, $N_{m1}$ and $N_{ms}$ are real variables without phase shifts. That is to say, the saturations (8) and (9), it can be obtained that

$$m_a(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j \Omega) \cdot e^{j\Omega t} \cdot m_a'(t) d\Omega$$  (11)

where $F(j\Omega)$ is the frequency-domain Fourier transform of the saturation function, given by [36]:

$$F(j\Omega) = -2jK \cdot \sin (\omega \Omega)/\Omega^2$$  (12)

The exponential components in Equation (11) can be expanded to series form, which gives:

$$m_a(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j \Omega) \cdot E(M_1, \Delta M_s) d\Omega$$  (13)

with

$$E(M_1, \Delta M_s) = \sum_{m=0}^{\infty} \varepsilon_m f_{2m}(M_1 \Omega) \cos(2m \omega_1 t + \frac{\pi}{2}) + 2j \sum_{m=0}^{\infty} \varepsilon_m f_{2m+1}(M_1 \Omega) \sin(2m \omega_1 t + \frac{\pi}{2})$$  (14)

where $f_n$ is the $n$-th order first kind Bessel function [36]; $\varepsilon_i$ satisfies $\varepsilon_1 = 1$ and $\varepsilon_i = 2$ ($i = 2, 3, \ldots, n$).

By keeping only the first harmonic components $\omega_1$ and $\omega_s$ in Equation (14), Equation (13) can be rewritten as:

$$m_a'(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j \Omega) \left[ 2j \cdot f_1(M_1 \Omega) \cdot f_s(\Delta M_s \Omega) \cos(\omega_1 t) \right] d\Omega$$  (15)

$$+ \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j \Omega) \left[ 2j \cdot f_0(M_1 \Omega) \cdot f_s(\Delta M_s \Omega) \cos(\omega_1 t) \right] d\Omega$$
By combining Equations (8), (10), and (15), the saturation gains defined in Equation (10) are explicitly obtained as:

\[
\begin{align*}
N_{m1}(M_1, \Delta M_s) &= \frac{j}{\pi M_1} \int_{-\infty}^{+\infty} F(\Omega) J_0(\Omega M_1) \Delta M_s j \Omega d\Omega \\
N_{ms}(M_1, \Delta M_s) &= \frac{j}{\pi \Delta M_s} \int_{-\infty}^{+\infty} F(\Omega) J_0(\Omega M_1) \Delta M_s j \Omega d\Omega
\end{align*}
\]

(16)

If the magnitudes \( M_1, \Delta M_s \) are known, \( F \) and \( j \) in Equation (16) become the function of \( \Omega \). Then, \( N_{m1} \) and \( N_{ms} \) can be easily computed using the built-in tools available in commercial software packages, for example, the “integral” function in MATLAB.

4.2 Iterative computation of LSAM

From Equations (2) or (3), the perturbed voltage and current at the PCC bus can be written in the form of:

\[
\begin{align*}
e_s(t) &= V_1 \cos(\omega_1 t + \phi_1) + \Delta V_s \cos(\omega_s t + \phi_s) \\
i_s(t) &= I_1 \cos(\omega_1 t + \phi_1) + \Delta I_s \cos(\omega_s t + \phi_s)
\end{align*}
\]

(17)

With Equations (16) and (17) the circuit equations associated with the saturated modulation index, voltage, and current are obtained by applying the Kirchhoff’s voltage law:

\[
\begin{align*}
(R + sL)I_1 &= \frac{V_{dc}}{K_m} N_{m1} M_1 - V_1 \quad s_1 = j2\pi f_1 \\
(R + sL)\Delta I_s &= \frac{V_{dc}}{K_m} N_{ms} \Delta M_s - \Delta V_s \quad s = j2\pi f_s
\end{align*}
\]

(18)

where \( V_1, I_1, \) and \( M_1 \) are fundamental signals; \( \Delta V_s, \Delta I_s, \) and \( \Delta M_s \) are perturbation signals, all in phasor forms.

From the second equation in Equation (18), the LSAM of VSC can be derived as:

\[
Y_{LSAM}(s \Delta V_s) = \frac{\Delta I_s(f_s)}{\Delta V_s(f_s)} = \frac{G_{s11}(s \Delta V_s)}{G_{ii1}(s \Delta V_s)}, t = j2\pi f_s
\]

(19)

with

\[
\begin{align*}
G_{s11} &= R + sL - 0.5K_m V_{dc} K_0i N_{m1}[-G_{d} (\Delta \omega_s) - G_e (\Delta \delta_s) + j2\omega_1 I_1] \\
G_{ii1} &= -1 + K_m V_{dc} K_0i N_{m1} [1 + G_0(\Delta \delta_s) + G_e (\Delta \omega_s) I_1] \\
&+ K_m V_{dc} K_0i \phi_1 G_0(\Delta \delta_s) j I_1 \cos \phi_1 G_1 (\Delta \delta_s) \\
&+ j I_1 \sin \phi_1 G_1(\Delta \omega_s)
\end{align*}
\]

(20)

where \( G_0(\theta) = 0.5H_{PLL}(\theta)/(1 + \frac{1}{V_1 H_{PLL}(\theta)}) \) is the closed-loop gain of PLL, \( \Delta \delta_s = \frac{j2\pi f_0 I_1 - f_0}{2}; \) other notations and their meanings are given in Table 1.

From Equations (19) and (20), the saturation gain \( N_{ms} \) exists in both the numerator and denominator of the LSAM expression, which significantly affects the LSAM’s magnitude and phase. Note that if the saturation gain in Equation (20) is not considered, the LSAM becomes the SSAM, of which the derivation process can be found in [7]. Therefore, the computation of the saturation gain is critical to get the LSAM. Equation (16) shows that the saturation gain \( N_{ms} \) is the function of \( M_1 \) and \( \Delta M_s \). For a preset value of the perturbation magnitude \( \Delta V_s \), both \( M_1 \) and \( \Delta M_s \) can be obtained by substituting Equation (19) into (18), which leads to the following expression:

\[
\begin{align*}
M_1 &= \left( \frac{(R + sL)I_1 + V_1}{K_m V_{dc} N_{m1}} \right) \Delta M_s
\end{align*}
\]

(21)

According to Equations (16) and (21), to obtain the saturation gains \( N_{m1} \) and \( N_{ms} \), we need to first compute \( M_1 \) and \( \Delta M_s \), which, in turn requires the values of \( N_{m1} \) and \( N_{ms} \). Therefore, Equations (16) and (21) are coupled equations, which can be numerically computed in a few iterations [34]. Initially, set \( N_{m1}^{(0)} = N_{ms}^{(0)} = 1 \) and then Equation (21) gives \( M_1^{(0)}, \Delta M_s^{(0)} \), which are next substituted into Equation (16) to get \( N_{m1}^{(1)}, N_{ms}^{(1)} \). Repeat the iteration process until the \( k \)-step gains \( M_1^{(k)}, \Delta M_s^{(k)} \) satisfy:

\[
\max \left\{ \left| M_1^{(k)} - M_1^{(k-1)} \right|, \left| \Delta M_s^{(k)} - \Delta M_s^{(k-1)} \right| \right\} \leq \delta
\]

(22)

where \( \delta \) is the preset error tolerance.

As the iteration comes to an end, the obtained \( M_1^{(k)}, \Delta M_s^{(k)} \) are substituted into Equation (16), and then Equation (20) to get \( Y_{LSAM} \).

5 CASE STUDIES

In this section, the magnitude and frequency of STO are assessed using the derived analytical LSAM. The characteristics of STO under different system strengths and VSC’s loading levels are investigated. The analysis results obtained from the measurement-based LSAM and analytical LSAM are compared.

5.1 Quantitative analysis of STO using the derived analytical LSAM

Contrary to the measured LSAM in Sections 3.2, here the LSAMs corresponding to perturbation magnitudes of 0–0.14, 0.15, 0.16, and 0.17 p.u. are directly obtained using the analytical model given by Equations (19) and (20). The LSAMs corresponding to the above perturbation magnitudes are plotted in Figure 13. As expected, consistent with the results presented in Sections 3.2, the LSAM did not change for the magnitude ranging from 0 to 0.14 p.u. as the PWM was not saturated yet. The rightmost intersection at 433.8 Hz with a phase difference of 190° is an indication of a DVO at 433.8 Hz. For the larger perturbation magnitudes, the intersection point moves leftward and settles at 430.4 Hz with critical phase difference of 180°. According to Criterion II, the DVO turns into STO when the frequency and magnitude of the oscillation do not move...
FIGURE 13 Analytical LSAM-based analysis for the assessment of STO’s magnitude and frequency

Further and reach constant values of 430.4 Hz and 0.17 p.u., respectively. These analytical LSAM-based results are consistent with those obtained from measurement-based LSAMs (see Figure 12).

Compared to the measurement-based LSAM, assessing the magnitude and frequency of STO using the analytical LSAM has the following advantages: (1) The LSAM is directly obtained from the control parameters and operating conditions; (2) The rough values of the STO’s magnitude and frequency are not necessary to be known beforehand; (3) The trial-and-error searching for critically stable points is carried out with greater efficiency and accuracy.

5.2 Analytical LSAM-based quantitative analysis under different system strengths

It has already been illustrated that the system strength, defined by the short-circuit ratio (SCR), has a great impact on the magnitude and frequency of the STO (see Figure 9). By varying the grid inductance \(L_g\), the STO’s magnitudes and frequencies are assessed using analytical LSAM and EMT simulation under different SCRs, as listed in Table 2. For quantitative comparison, the relative errors (REs) in the STO’s magnitudes for the LSAM-based analysis with respect to EMT simulation are also given in Table 2. The comparative results show that the magnitudes and frequencies obtained using LSAM coincide with those from EMT simulation. The accuracy is fairly good (RE < 5%) when the system strength is strong or moderate (SCR > 3). Compared with LSAM-based analysis, the SSAM-based analysis is unable to assess the STO’s magnitude. If approximating the STO’s frequency with the interaction frequency of the VSC’s and grid’s SSAM, the accuracy would reduce greatly. As the system strength becomes much weaker, for instance when SCR = 2.33, RE increases to 8%. This is probably because the impact from other factors, such as the coupling effects [10] or the non-linearity introduced by PLL become more significant as the system strengths are lowered.

The LSAM results for a particular scenario when \(L_g = 0.7\) mH or SCR = 3 are displayed in Figure 14. For the sake of better visualization, only the LSAM curve with the perturbation magnitude of 0.205 p.u. is plotted. Then, the EMT simulation is carried out to validate the above theoretical results. At \(t = 3\) s, \(L_g\) is increased from 0.5 to 0.7 mH to trigger the oscillation. Figure 15 shows the three-phase voltage at the PCC bus and its FFT result. Obviously, the STO occurs almost without experiencing the DVO region because of a relatively large negative damping. The FFT result of the PCC voltage indicates that the magnitude

| \(L_g\) (mH) | SCR | \(f_{int}\) (Hz) | \(f_s\) (Hz) | \(V_{LSAM}\) (p.u.) | \(V_{sim}\) (p.u.) | RE (%) |
|---|---|---|---|---|---|---|
| 0.50 | 4.19 | S | S | 0 | S | 0 | 0 |
| 0.55 | 3.81 | 435 | 430 | 0.170 | 431 | 0.163 | 4.30 |
| 0.60 | 3.49 | 432 | 424 | 0.175 | 422 | 0.184 | 4.89 |
| 0.70 | 3.00 | 426 | 417 | 0.205 | 413 | 0.216 | 5.09 |
| 0.80 | 2.62 | 417 | 405 | 0.240 | 399 | 0.235 | 7.33 |
| 0.90 | 2.33 | 405 | 395 | 0.300 | 388 | 0.324 | 7.40 |

\(f_{int}\): Interaction frequency of the VSC’s and grid’s SSAM; \(f_s\): Frequency of STO in Hz; \(V_{LSAM}\) and \(V_{sim}\): Magnitude of STO in p.u.; RE = \(\left|\frac{V_{sim} - V_{LSAM}}{V_{sim}}\right| \times 100\%\): Relative error in the STO’s magnitude for the LSAM-based analysis with respect to EMT simulation, in percent; S: Stable.

FIGURE 14 Analytical LSAM-based analysis with SCR = 3

FIGURE 15 The PCC voltage and its FFT result during the STO
5.3 Analytical LSAM-based quantitative analysis under different loading levels

This scenario tests the methodology at different loading levels. The loading level of VSC is increased from the previous 0.35 to 0.6 MW (about 71%) by changing the $d$-axis reference current. The grid inductance is set to a fixed value 0.55 mH. The corresponding LSAM results are displayed in Figure 16, followed by the EMT simulation results shown in Figure 17. The RE in the STO’s magnitudes obtained from LSAM-based analysis (0.155 p.u.) and EMT simulation (0.148 p.u.) is under 5%, which is tolerable in engineering practice.

6 CONCLUSION

This paper presented a detailed quantitative analysis of the STO in a grid-connected VSC. The impact of PWM saturation on the STO’s characteristics was fully considered using the large-signal impedance/admittance model (LSIM/LSAM). Extensive theoretical and EMT simulation analyses were conducted for quantitative investigations on the STO’s characteristics, including the occurrence condition, frequency, and magnitude. The following conclusions have been drawn:

1. The unique time-varying characteristic of the oscillation, the magnitude sensitive characteristic of the impedance and the occurrence of STO were found to be caused by the non-linearity of PWM saturation.
2. By representing the grid-connected VSC system with an equivalent second-order RLC circuit model, two LSIM/LSAM-based occurrence criteria for STO were established. Further, the magnitude and frequency of the STO were assessed using the LSIM/LSAM-based criteria and quantitative analysis.
3. The effects of various influencing parameters on the characteristics of the STO were investigated. The results showed that both the magnitude and frequency of the STO were greatly affected by the system strength and the VSC’s loading level. Further, the theoretical results were verified through EMT-based modelling and simulation analysis.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China (U1866601, 51737007, 51925701), and the Science and Technology Project of State Grid Corporation of China (SGZJ0000KXJS1900418).

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