Abstract

We investigate non-commutative gauge theories in homogeneous spaces $G/H$. We construct such theories by adding cubic terms to IIB matrix model which contain the structure constants of $G$. The isometry of a homogeneous space, $G$ must be a subgroup of $SO(10)$ in our construction. We investigate $CP^2 = SU(3)/U(2)$ case in detail which gives rise to 4 dimensional non-commutative gauge theory. We show that non-commutative gauge theory on $R^4$ can be realized in the large $N$ limit by letting the action approach IIB matrix model in a definite way. We discuss possible relevances of these theories to the large $N$ limit of IIB matrix model.
1 Introduction

Matrix models which are dimensional reductions of 10 dimensional super Yang-Mills theory first appeared as low energy effective theories of $N$ coincident D-branes. They may also be regarded as matrix regularization of light-cone membrane action [1] or Green-Schwarz superstring action [2]. Our hope is that they may provide us second quantized string theory in the large $N$ limit. It is because they may contain arbitrary numbers of strings represented as block-diagonal matrices.

Non-commutative D-branes such as the following are formal classical solutions of matrix models in the large $N$ limit

$$A_{1}^{cl} = \hat{p}, \quad A_{2}^{cl} = \hat{q},$$

$$[\hat{p}, \hat{q}] = -i, \quad (1.1)$$

since they solve the equation of motion

$$[A_{\mu}, [A_{\mu}, A_{\nu}]] = 0. \quad (1.2)$$

Non-commutative (NC) gauge theory is obtained from matrix models around NC space-time [3] [4] [5]. The advantage of matrix model construction of NC gauge theory is that it maintains the manifest gauge invariance under $U(N)$ transformations

$$A_{\mu} \rightarrow UA_{\mu}U^{\dagger}. \quad (1.3)$$

The gauge invariant observables of NC gauge theory, the Wilson lines were constructed through matrix models [6] [7]. NC gauge theory exhibits UV-IR mixing which is a characteristic feature of string theory [8].

In this paper we investigate NC gauge theory on homogeneous spaces through matrix models. It is an interesting problem on its own to study NC gauge theories on curved manifolds. A homogeneous space is realized as $G/H$ where $G$ is a Lie group and $H$ is a closed subgroup of $G$. We further assume that they are symmetric spaces which are invariant under space reflection (parity). At Lie algebra level in symmetric spaces, the following commutation relations hold

$$g = h + m,$$

$$[h, h] \subset h, \quad [h, m] \subset m, \quad [m, m] \subset h, \quad (1.4)$$
where $g$ and $h$ are the generators of $G$ and $H$ respectively.

We construct NC gauge theories on homogeneous spaces by adding cubic couplings to IIB matrix model with a large but finite $N$. The isometry of a homogeneous space $G/H$ is the group $G$. The group $G$ has to be a subgroup of $SO(10)$ which is the symmetry of IIB matrix model. We investigate $CP^2 = SU(3)/U(2)$ case in detail which gives rise to 4 dimensional NC gauge theory. We show that NC gauge theory on $R^4$ can be realized in the large $N$ limit by letting the action approach IIB matrix model in a definite way. Although SUSY is broken in general with finite $N$, we argue that SUSY is locally resurrected in such a limit. We hope that our construction is useful to investigate 4 dimensional NC super Yang-Mills theory nonperturbatively. Since the strength of the cubic couplings formally vanish in the large $N$ limit, our results may be relevant to investigate the large $N$ limit of IIB matrix model.

In section 2, we construct fuzzy homogeneous spaces $G/H$ as the orbit of a state in a definite representation of dimension $N$ which is invariant under $H$ modulo the overall $U(1)$ phase. We then construct NC gauge fields as bi-local states. Such a construction facilitates matrix model realizations since gauge fields are naturally represented by $N \times N$ Hermitian matrices. In sections 3, we construct NC gauge theories on homogeneous spaces $G/H$ through matrix models. For this purpose we deform IIB matrix model by adding cubic terms in $A_\mu$ which contain structure constants of $G$. We investigate $CP^2 = SU(3)/U(2)$ case in detail which gives rise to 4 dimensional NC gauge theory. We show that NC gauge theories on $R^4$ can be realized in the large $N$ limit by letting the action approach IIB matrix model in a definite way. We investigate quantum effects of 4 dimensional NC gauge theory and its formal infrared limit. We conclude in section 4 with discussions. We discuss possible relevances of these theories to the large $N$ limit of IIB matrix model.

2 Non-commutative Spacetime

In this section, we formulate a generic procedure to construct fuzzy homogeneous spaces $G/H$ and gauge fields on them. We pick a state $|0>\>$ in a definite representation of $G$ which is invariant under $H$. In this construction, we identify the states which only differ by their $U(1)$ phases since they are equivalent as quantum states. The set of all states which can be reached by multiplying elements of $G$ to $|0>\>$ is called the orbit of $|0>\>$. Fuzzy homogeneous spaces $G/H$ are constructed as the orbit of $|0>\>$. It is represented by the
irreducible representation which is descended from $|0>$. The dimension of the representation $N$ can be interpreted as the volume of fuzzy $G/H$ in the unit of non-commutativity scale. We recall that the basic degrees of freedom in NC gauge theory are bi-local fields which can be interpreted as zeromodes of open strings\[9\]. We then construct NC gauge fields by forming the tensor product of the relevant irreducible representation and its complex conjugate. Such a construction facilitates matrix model realizations since gauge fields are naturally represented by $N \times N$ Hermitian matrices.

We first recall a fuzzy flat manifold $R^2$. We have non-commutative coordinates $\hat{x}, \hat{y}$ which satisfy the canonical commutation relation

$$[\hat{x}, \hat{y}] = i. \quad (2.1)$$

Obviously we cannot realize such a commutation relation with finite size matrices. We can construct the creation and annihilation operators $\hat{a}, \hat{a}^\dagger$ out of them:

$$\hat{a} = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{y}), \quad \hat{a}^\dagger = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{y}),$$

$$[\hat{a}, \hat{a}^\dagger] = 1. \quad (2.2)$$

A localized state $|0>$ at the origin can be defined as

$$\hat{a}|0> = 0. \quad (2.3)$$

We can construct surrounding states by applying the creation operators to $|0>$ as

$$\hat{a}^\dagger|0>, (\hat{a}^\dagger)^2|0>, \cdots. \quad (2.4)$$

The states which can be constructed from $|0>$ by applying $\hat{a}^\dagger$ finite times are localized around the origin. They are uniformly distributed with the density $1/2\pi$ in accordance with the semiclassical quantization condition.

The conjugate momentum operators are identified with

$$\hat{p}_x = \hat{y}, \quad \hat{p}_y = -\hat{x}. \quad (2.5)$$

since they satisfy the canonical commutation relationship with $\hat{x}, \hat{y}$. They generate the translations along fuzzy plane. We subsequently introduce the adjoint operators $P_\mu = [\hat{p}_\mu, ]$. |\[10\]|2This commutation relation is realized by the guiding center coordinates of electrons in magnetic field. In fact NC gauge theory may be realized in quantum Hall system\[10\].
Since $P_x$ and $P_y$ commute, we can simultaneously diagonalize them. The eigen states can be constructed as

$$P_\mu \exp(ik_x \hat{x} + i k_y \hat{y}) = [\hat{p}_\mu, \exp(ik_x \hat{x} + i k_y \hat{y})] = k_\mu \exp(ik_x \hat{x} + i k_y \hat{y}).$$

(2.6)

They are the analogous of the plane waves in non-commutative space. They can also be interpreted as the bi-local fields or dipoles with the length $k$.

We next review $S^2 = SU(2)/U(1)$ case as the first example of a curved manifold. In $SU(2)$, we have the Hermitian operators $\hat{j}_x, \hat{j}_y, \hat{j}_z$ which satisfy the commutation relations of the angular momentum.

$$[\hat{j}_x, \hat{j}_y] = i \hat{j}_z.$$  

(2.7)

Contrary to $R^2$ case, such a commutation relation can be realized with finite size matrices since $S^2$ is compact. The raising and lowering operators $\hat{j}^+, \hat{j}^-$ can be formed from $\hat{j}_x, \hat{j}_y$ which satisfy

$$\hat{j}^\pm = \frac{1}{\sqrt{2}} (\hat{j}_x \pm i \hat{j}_y),$$

$$[\hat{j}^+, \hat{j}^-] = \hat{j}_z,$$

$$[\hat{j}_z, \hat{j}^\pm] = \pm j^\pm.$$  

(2.8)

We consider the $N = 2l + 1$ dimensional representation of spin $l$. Since we can diagonalize $\hat{j}_z$, we can label the states with the eigenvalue of $\hat{j}_z$ as $|m>$ where

$$\hat{j}_z |m> = m|m>.$$  

(2.9)

We consider the semiclassical limit where $l$ is assumed to be large. In such a situation we expect to recover a large smooth sphere. We can further expect that a flat fuzzy plane is realized in the neighborhood of a particular point such as the north-pole. In fact the notion of locality can be introduced in the large $l$ limit as follows.

As a localized states at the north-pole, we can choose $|l>$. We can subsequently construct other states by using the lowering operators

$$\hat{j}^- |l>, (\hat{j}^-)^2 |l>, \cdots.$$  

(2.10)

Since we have assumed that $l$ is large, these states which are constructed from $|l>$ with finite operations of $\hat{j}^-$ are all localized around the north-pole.
For these states, we can approximate \( \hat{j}_z \sim l \). By rescaling the operators, we can obtain the following commutation relations from (2.8),

\[
\hat{a} = \frac{1}{\sqrt{l}} \hat{j}^+, \quad \hat{a}^\dagger = \frac{1}{\sqrt{l}} \hat{j}^-, \quad \hat{l} = \frac{1}{l} \hat{j}_z
\]

\[
[\hat{a}, \hat{a}^\dagger] = \hat{l}, \quad [\hat{l}, \hat{a}] = \frac{1}{l} \hat{a}, \quad [\hat{l}, \hat{a}^\dagger] = -\frac{1}{l} \hat{a}^\dagger.
\] (2.11)

Since we obtain the identical algebra with (2.2), we conclude that flat fuzzy plane is realized around the north-pole of \( S^2 \) in the large \( l \) limit. The density of the states is \( 1/(2\pi l) \) in the original coordinate and the area of \( S^2 \) is \( 4\pi l^2 \). Thus the total number of the states is \( 2l \) semiclassically which is consistent with the spin \( l \) representation of \( SU(2) \).

The local coordinates which satisfy the canonical commutation relationship can be identified as

\[
\tilde{x} = \frac{1}{\sqrt{l}} \hat{j}_x, \quad \tilde{y} = \frac{1}{\sqrt{l}} \hat{j}_y.
\] (2.12)

The conjugate momentum operators are

\[
\tilde{p}_x = \frac{1}{\sqrt{l}} \hat{j}_y, \quad \tilde{p}_y = -\frac{1}{\sqrt{l}} \hat{j}_x.
\] (2.13)

In the local patch, the eigen functions of the adjoint \( \tilde{P}_\alpha = [\tilde{p}_\alpha, \cdot] \) can be constructed just like the flat space case as \( \exp(ik_x \tilde{x} + ik_y \tilde{y}) \). We point out here that adjoint operators of \( J_{\mu} = [\hat{j}_{\mu}, \cdot] \) also satisfy the commutation relation of \( SU(2) \). Using the commutation relations in (2.11) which are realized around the north-pole, we find that \( \tilde{P}^2 \) can be related to the Casimir operator of \( SU(2) \) in a local patch as follows

\[
\frac{J^2}{l} \exp(ik_x \tilde{x} + ik_y \tilde{y}) = \tilde{P}^2 \exp(ik_x \tilde{x} + ik_y \tilde{y}) + O(\frac{1}{l}).
\] (2.14)

The homogeneous space \( S^2 \) is generated by \( \hat{j}_x \) and \( \hat{j}_y \) starting from the state \( |l> \). The state \( |l> \) only changes its \( U(1) \) phase under the rotation around the \( z \) axis. As quantum states, we may identify the states which differ only by their \( U(1) \) phases. Therefore fuzzy \( S^2 \) can also be realized as \( G/H \) where \( G = SU(2) \) and \( H = U(1) \).

We can parameterize \( CP^1 \) or \( S^2 \) by two complex coordinates \( u_\alpha \) such that

\[
u_\alpha^* u_\alpha = 1.
\] (2.15)

Fuzzy \( S^2 \) can be represented by the following states with spin \( l \).

\[
u_{\alpha_1} \cdots u_{\alpha_2l}.
\] (2.16)
We construct gauge fields on fuzzy $S^2$ as bi-local fields. Hence the bi-local fields are represented as

$$u_{\alpha_1} \cdots u_{\alpha_l} u_{\beta_1}^* \cdots u_{\beta_l}^*.$$  

(2.17)

They can be decomposed into the irreducible representations with spin $n$

$$\sum_{n=0}^{2l} (u_{\alpha_1} \cdots u_{\alpha_n} u_{\beta_1}^* \cdots u_{\beta_n}^*)$$

(2.18)

which are traceless under the contractions of $\alpha$ and $\beta$ indices.

The adjoint generators of $SU(2)$ transformations are represented as

$$J^i = -\frac{1}{2} \left[ u_{\beta} \sigma^{i}\alpha u_{\alpha} \frac{\partial}{\partial u_{\alpha}} - u_{\alpha}^* \sigma^{i}\beta u_{\beta}^* \frac{\partial}{\partial u_{\beta}^*} \right]$$

(2.19)

which satisfy the $SU(2)$ algebra

$$[J_i, J_j] = i\epsilon_{ijk} J_k.$$  

(2.20)

The Casimir operator acts on the bi-local fields as

$$J^2 = \frac{1}{2} \left( u_{\alpha} \frac{\partial}{\partial u_{\alpha}} \right)^2 + \frac{1}{2} u_{\alpha} \frac{\partial}{\partial u_{\alpha}} + c.c. = n^2 + n.$$  

(2.21)

Therefore the eigenvalues of the Laplacian on $S^2$, $J^2/l$ are $n(n+1)/l$ with $n$ integers. Since $\sum_{n=0}^{2l}(2n+1) = (2l+1)^2$, a group of representations with spins up to $2l$ form the complete basis of $N \times N$ Hermitian matrices as it is evident from our construction.

After reviewing the well-known example of fuzzy $S^2$ case, we can formulate a general procedure to construct fuzzy homogeneous space $G/H$ as follows. We first consider a representation of $G$ which contains a (highest weight) state which is invariant under $H$ modulo the over all $U(1)$ phase. We also require that fuzzy $R^n$ where $n$ is the dimension of $G/H$ is realized in the local patch around such a state. We are thus restricted to symplectic manifolds. It is because the * product on such a manifold can be reduced to that of a flat manifold (Moyal product) locally by choosing the Darboux coordinates. Since the Kähler form serves as the symplectic form, Kähler manifolds such as $CP^n$ satisfy this requirement [11].

We embed the Lie generators of $G$ into $N$ dimensional Hermitian matrices where $N$ is the dimension of the representation. The gauge fields on fuzzy $G/H$ are constructed as bi-local fields. The bi-local fields are the tensor products of the relevant representation and the complex conjugate of it. Since they are reducible, we can decompose them into the irreducible representations. They are guaranteed to form the complete basis of $N \times N$
Hermitian matrices by construction. In what follows we discuss several concrete examples in higher dimensions following this general procedure.

Our main interest in this paper is \( \mathbb{CP}^2 = SU(3)/U(2) \). \( SU(3) \) is generated by 8 Hermitian operators \( t^a \) which satisfy

\[
[t^a, t^b] = i f^{abc} t^c. \tag{2.22}
\]

The structure constants of \( SU(3) \) are

\[
\begin{align*}
  f_{123} &= 1, \\
  f_{147} &= f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2}, \\
  f_{458} &= f_{678} = \frac{\sqrt{3}}{2},
\end{align*}
\]  \( \tag{2.23} \)

The irreducible representations of \( SU(3) \) can be classified by the Young Tableaux which is specified with a pair of integers \((p, q)\). \(^3\)

We consider the representation \((p, 0)\) which can be realized by totally symmetrizing the following states

\[
u_{\alpha_1} u_{\alpha_2} \cdots u_{\alpha_p}, \tag{2.24}\]

where \(\sum_{\alpha=1}^{3} u_{\alpha}^* u_{\alpha} = 1\). Let us consider the state \( u_{\alpha_i} = \delta_{\alpha_i,3} \) for all \( i \). Such a state \(|p\rangle\) is a singlet of \( SU(2) \) subgroup of \( SU(3) \) which are generated by \( t^1, t^2, t^3 \) and the eigen state of \( t^8 \) with the eigenvalue \( p/\sqrt{3} \).

The commutation relations among \( t^4 \cdots t^7 \) are

\[
\begin{align*}
  [t^4, t^5] &= \frac{i}{2} \sqrt{3} t^8 + i \frac{1}{2} t^3, \\
  [t^6, t^7] &= \frac{i}{2} \sqrt{3} t^8 - i \frac{1}{2} t^3. \tag{2.25}
\end{align*}
\]

We may introduce the raising and lowering operators:

\[
\begin{align*}
  u^\pm &= \frac{1}{\sqrt{2}} (t^6 \pm it^7), \\
  v^\pm &= \frac{1}{\sqrt{2}} (t^4 \pm it^5). \tag{2.26}
\end{align*}
\]

As \( S^2 \) case, we can construct descendent states by applying lowering operators \( u^-, v^- \) to \(|p\rangle\).

\[
|p\rangle, u^-|p\rangle, v^-|p\rangle, (u^-)^2|p\rangle, (v^-)^2|p\rangle, u^- v^-|p\rangle, \cdots. \tag{2.27}
\]

\(^3\)There are \( p + q \) boxes in the first low and \( q \) boxes in the second low.
In the large $p$ limit, the states which are obtained from $|p\rangle$ with finite actions of the lowering operators form a localized patch.

After rescaling $u, v \rightarrow \sqrt{p/2} \tilde{u}, \sqrt{p/2} \tilde{v}$ we obtain the following commutation relation which are realized in such a local patch,

$$\begin{align*}
[\tilde{u}^+, \tilde{u}^-] &= 1 + O(1/\sqrt{p}) \\
[\tilde{v}^+, \tilde{v}^-] &= 1 + O(1/\sqrt{p}).
\end{align*}$$

(2.28)

We thus conclude that flat fuzzy $R^4$ is realized in a local patch of $CP^2$ in the large $p$ limit. The density of the states is $1/\pi^2 p^2$ in the original coordinate and the volume of $CP^2$ is $\pi^2 p^4/2$. Thus the total number of the states is $p^2/2$ semiclassically which is consistent with the $(p,0)$ representation of $SU(3)$.

The local coordinates which satisfy the canonical commutation relationship can be identified as

$$\begin{align*}
\tilde{x} &= \frac{\sqrt{2}}{\sqrt{p}} t^4, \\
\tilde{y} &= \frac{\sqrt{2}}{\sqrt{p}} t^5, \\
[\tilde{x}, \tilde{y}] &= i, \\
\tilde{w} &= \frac{\sqrt{2}}{\sqrt{p}} t^6, \\
\tilde{z} &= \frac{\sqrt{2}}{\sqrt{p}} t^7, \\
[\tilde{w}, \tilde{z}] &= i.
\end{align*}$$

(2.29)

We also find

$$\begin{align*}
[t^1, \tilde{x}] &= \frac{i}{2} \tilde{z}, \\
[t^1, \tilde{y}] &= -\frac{i}{2} \tilde{w}, \\
[t^1, \tilde{w}] &= \frac{i}{2} \tilde{y}, \\
[t^1, \tilde{z}] &= -\frac{i}{2} \tilde{x}, \\
[t^2, \tilde{x}] &= \frac{i}{2} \tilde{w}, \\
[t^2, \tilde{y}] &= \frac{i}{2} \tilde{z}, \\
[t^2, \tilde{w}] &= -\frac{i}{2} \tilde{x}, \\
[t^2, \tilde{z}] &= -\frac{i}{2} \tilde{y}, \\
[t^3, \tilde{x}] &= \frac{i}{2} \tilde{y}, \\
[t^3, \tilde{y}] &= -\frac{i}{2} \tilde{x}, \\
[t^3, \tilde{w}] &= -\frac{i}{2} \tilde{z}, \\
[t^3, \tilde{z}] &= \frac{i}{2} \tilde{w}.
\end{align*}$$

(2.30)

We may thus identify the $SU(2)$ subgroup formed by $t^1, t^2, t^3$ as a subgroup of $SO(4)$ as

$$t^1 = \frac{-i}{2} (L^{14} - L^{23}), \\
t^2 = \frac{-i}{2} (L^{13} + L^{24}), \\
t^3 = \frac{-i}{2} (L^{12} - L^{34}).$$

(2.31)

The conjugate momentum operators are

$$\tilde{p}_x = \tilde{y}, \\
\tilde{p}_y = -\tilde{x}, \\
\tilde{p}_w = \tilde{z}, \\
\tilde{p}_z = -\tilde{w}. $$

(2.32)

In the flat space limit, the eigen functions of the adjoint operators $\tilde{P}_\alpha = [\tilde{p}_\alpha, ]$ can be constructed as $exp(ik \cdot x) \equiv exp(ik_x \tilde{x} + ik_y \tilde{y} + ik_w \tilde{w} + ik_z \tilde{z})$. We introduce here the adjoint
operators $T^a = [t^a, ]$ which also satisfy the commutation relation of $SU(3)$. Using the commutation relations in (2.29) which are realized around the north-pole, we find that $\tilde{P}_\alpha^2$ can be related to the Casimir operator of $SU(3)$ in a local patch

$$\frac{2}{p}(T^a)^2 \exp(ik \cdot x) = \tilde{P}_\alpha^2 \exp(ik \cdot x) + O(\frac{1}{p}).$$

(2.33)

Thus the Laplacian on $CP^2$ which reduces to that of flat $R^4$ in a local patch in the large $N$ limit is $2(T^a)^2/p$. The eigenvalues $2(T^a)^2/p$ are $2n(n+2)/p$ for the representation $(n,n)$.

It is because the bi-local states are represented as

$$u_{\alpha_1} \ldots u_{\alpha_p} u^*_{\beta_1} \ldots u^*_{\beta_p}.$$ (2.34)

They can be decomposed into the irreducible representations of $(n,n)$ type

$$\sum_{n=0}^{p} (u_{\alpha_1} \ldots u_{\alpha_n} u^*_{\beta_1} \ldots u^*_{\beta_n})$$ (2.35)

which are traceless under the contractions of any pairs of $\alpha$ and $\beta$ indices. The adjoint generators of $SU(3)$ transformations are represented as

$$T^a = - \left[ u^*_{\alpha \beta} \frac{\partial}{\partial u_{\alpha}} - u_{\alpha \beta} \frac{\partial}{\partial u^*_{\beta}} \right]$$ (2.36)

which satisfy the $SU(3)$ algebra. The Casimir operator acts on the bi-local fields as

$$T^2 = \frac{1}{2}(u_{\alpha} \frac{\partial}{\partial u_{\alpha}})^2 + u_{\alpha} \frac{\partial}{\partial u_{\alpha}} + c.c. = n^2 + 2n.$$ (2.37)

The dimension of the representation $(n,n)$ is $(n+1)^3$. Since $\sum_{n=0}^{p}(n+1)^3 = (p+1)^2(p+2)^2/4$, a group of the irreducible representations $(n,n)$ with $n$ up to $p$ form a complete basis of $N \times N$ Hermitian matrices where $N = (p+1)(p+2)/2$ is the dimension of the representation $(p,0)$.

Our final example in this section is $CP^3 = SO(5)/U(2) \cong [13] \sim [18]$. $SO(5)$ are generated by 10 anti-symmetric matrices $t_{\mu \nu}$ which satisfy:

$$[t_{\mu \nu}, t_{\rho \sigma}] = i\delta_{\mu \rho} t_{\nu \sigma} - i\delta_{\nu \rho} t_{\mu \sigma} - i\delta_{\mu \sigma} t_{\nu \rho} + i\delta_{\nu \sigma} t_{\mu \rho}.$$ (2.38)

Our investigation is constrained to the homogeneous spaces which can be realized through IIB matrix model. $G$ is maximal in this case since the number of its generators cannot exceed 10 which is the number of bosonic matrices $A_\mu$ in IIB matrix model. $SO(5)$ contains a subgroup $SO(4) = SU(2) \times SU(2)$ which is generated by

$$j_1 = \frac{1}{2}(t_{33} + t_{14}), \quad j_2 = \frac{1}{2}(t_{31} + t_{24}), \quad j_3 = \frac{1}{2}(t_{12} + t_{34}),$$

$$k_1 = \frac{1}{2}(t_{33} - t_{14}), \quad k_2 = \frac{1}{2}(t_{31} - t_{24}), \quad k_3 = \frac{1}{2}(t_{12} - t_{34}).$$ (2.39)
We can simultaneously diagonalize \( t^2, j^2, k^2, j_z, k_z \).

The irreducible representations of \( SO(5) \) are represented by the Young Tableaux with a pair of integers \((m, n)\) with \( m \geq n \). We consider the representation \((p/2, p/2)\) which decomposes into the representations of \( SO(4) = SU(2) \times SU(2) \) as follows

\[
\left( \frac{p}{2}, \frac{p}{2} \right)_5 = (0, \frac{p}{2})_4 + \left( \frac{1}{2} p - 2 \right)_2 \frac{p}{2} + \cdots + \left( \frac{p}{2}, 0 \right)_4.
\] (2.40)

As a localized state \(|p>\), we consider such a state in \((p/2, 0)\) which is invariant under \( U(2) \) modulo \( U(1) \) phases:

\[
\begin{align*}
 j^2|p> &= (\frac{p}{2}) (\frac{p}{2} + 2) |p>, \\
 j_3|p> &= (\frac{p}{2}) |p>, \\
 k^2|p> &= 0, \\
 t^2|p> &= (\frac{p}{2} + 4p) |p>.
\end{align*}
\] (2.41)

In a local patch around \(|p>\), the following commutation relations are realized

\[
\begin{align*}
 [j_1, j_2] &= ij_3 \sim \frac{i p}{2}, \\
 [t_{15}, t_{25}] &= it_{12} \sim \frac{i p}{2}, \\
 [t_{35}, t_{45}] &= it_{34} \sim \frac{i p}{2}.
\end{align*}
\] (2.42)

We thus find that fuzzy \( R^6 \) is realized in the large \( p \) limit in the local patch just like the previous examples.

The bi-local fields are obtained by considering the direct product of

\[
(p/2, p/2) \otimes (p/2, p/2)
\] (2.43)

which can be decomposed into the irreducible representations of \( SO(5) \) as

\[
\sum_{m=0}^{p} \sum_{n=0}^{m} (m, n).
\] (2.44)

Since the dimension of the \((m, n)\) representation is

\[
D(m, n) = (1 + m - n)(1 + 2n)(2 + m + n)(3 + 2m)/6,
\] (2.45)

we can check that the dimension of the bi-local fields agrees with those of \( N \times N \) Hermitian matrices

\[
\sum_{m=0}^{p} \sum_{n=0}^{m} D(m, n) = D(p/2, p/2)^2.
\] (2.46)
\section{Matrix Model Realization}

In this section, we construct NC gauge theories on homogeneous spaces $G/H$ through matrix models. We propose to deform IIB matrix model action as follows generalizing the $S^2 = SU(2)/U(1)$ case\textsuperscript{[19]}:\footnote{See also\textsuperscript{[21]}.}

\begin{equation}
S_{IIB} \rightarrow S_{IIB} + \frac{i}{3} \alpha f_{\mu\nu\rho} Tr[A_{\mu}, A_{\nu}] A_{\rho},
\end{equation}

where $f_{\mu\nu\rho}$ is the structure constant of $G$. Since there are 10 Hermitian matrices $A_\mu$ in IIB matrix model, the number of the Lie generators of $G$ cannot exceed 10 in this construction. This action does not preserve SUSY unless $G = SU(2)$. However we show that NC gauge theory on $R^4$ can be realized in the large $N$ limit by letting the action approach IIB matrix model in a definite way. Although SUSY is broken in general with finite $N$, we argue that SUSY is locally resurrected in such a limit. Since this model possesses the translation invariance

\begin{equation}
A_\mu \rightarrow A_\mu + c_\mu
\end{equation}

and also

\begin{equation}
\psi \rightarrow \psi + \epsilon,
\end{equation}

we remove these zero-modes by restricting $A_\mu$ and $\psi$ to be traceless.

The equation of motion is

\begin{equation}
[A_\mu, [A_\mu, A_\nu]] + i \alpha f_{\mu\nu\rho} [A_\mu, A_\rho] = 0.
\end{equation}

The nontrivial classical solution is

\begin{equation}
A^c_\mu = \alpha t^a, \text{ other } A^c_\mu = 0,
\end{equation}

where $t^a$'s satisfy the Lie algebra of $G$. Although diagonal matrices also solve the equation of motion, the nontrivial solution (3.3) minimizes the classical action. In $SU(2)$ case, it is evaluated for the irreducible representation of spin $l$

\begin{equation}
-\frac{\alpha^4}{6} l(l+1)(2l+1).
\end{equation}

For a large but fixed $N$, the irreducible representation of spin $l$ where $N = 2l+1$ is selected by minimizing the classical action\textsuperscript{[22]}. There is no quantum corrections to worry about thanks to SUSY.
Let us investigate the analogous problem in $CP^2 = SU(3)/U(2)$ case. The classical action for the irreducible representation $(p, q)$ of $SU(3)$ is evaluated as

$$-\frac{\alpha^4}{4} C_2(p, q) \text{dim}(p, q),$$

(3.7)

where $C_2(p, q)$ is the Casimir and $\text{dim}(p, q)$ is the dimension of the representation

$$C_2(p, q) = \frac{1}{3} [p(p + 3) + q(q + 3) + pq],$$

$$\text{dim}(p, q) = \frac{(p + 1)(q + 1)(p + q + 2)}{2}. \quad (3.8)$$

We can see that the classical action is minimized for the $(p, 0)$ type representation with the fixed $\text{dim}(p, q) = N$. We also note that reducible representations do not minimize the classical action. We conclude that the desired representation of $(p, 0)$ type which is relevant to fuzzy $CP^2$ is selected by minimizing the classical action. Therefore this model realizes $U(1)$ NC gauge theory on $CP^2$ with finite $N$.

In the large $N$ limit, the fuzzy $R^4$ is realized locally as it has been shown in the previous section. We expand the action around the classical solution as $A_\mu = \alpha \sqrt{p/2} (\hat{p}_\mu + \hat{a}_\mu)$. In this parameterization, the non-commutativity scale is fixed to be 1. After using the Moyal-Weyl correspondence,

$$\hat{a} \rightarrow a(x),$$

$$\hat{a} \hat{b} \rightarrow a(x) \star b(x),$$

$$\text{Tr} \rightarrow \left(\frac{1}{2\pi}\right)^2 \int d^4 x,$$

(3.9)

we obtain the following NC gauge theory from (3.1)

$$-\alpha^4 \left(\frac{p}{2}\right)^2 \left(\frac{1}{2\pi}\right)^2 \int d^4 x \left( \frac{1}{4} [D_\alpha, D_\beta]^2 + \frac{1}{2} [D_\alpha, \phi_i]^2 + \frac{1}{4} [\phi_i, \phi_j]^2 ight. + \frac{1}{2} \bar{\psi} \Gamma_\alpha [D_\alpha, \psi] + \frac{1}{2} \bar{\psi} \Gamma_i [\phi_i, \psi] \bigg), \quad (3.10)$$

The cubic terms are suppressed by $\sqrt{2/p}$. In this way, we identify the coupling constant of NC gauge theory as

$$g^2_{NC} = \left(\frac{4\pi}{p\alpha^2}\right)^2.$$  \quad (3.11)

The analogous relations hold for 2 and 6 dimensional cases respectively as

$$g^2_{NC_2} = 2\pi \left(\frac{1}{l\alpha^2}\right)^2, \quad g^2_{NC_6} = (2\pi)^3 \left(\frac{2}{p\alpha^2}\right)^2.$$  \quad (3.12)
The classical action assumes the following value in the large $N$ limit

$$-rac{2\pi^2 p^2}{3g_{NC}^2} \sim -\frac{4\pi^2 N}{3g_{NC}^2}.$$  \hspace{1cm} (3.13)

We need to choose $g_{NC} \sim 1$ to obtain interacting NC gauge theory. For this purpose, we may choose $\alpha$ such that $g_{NC} \sim 1$ for a fixed $N$. We therefore generally need to choose $\alpha^2 \sim 1/p$. In 4 dimensions, we find $N \sim p^2/2$ for the $(p,0)$ representation which minimizes the classical action. If we let $N$ large in this way, we find that $\alpha$ vanishes in the large $N$ limit as $\alpha^2 \sim O(\sqrt{1/N})$. From (3.10), we can see that SUSY is locally recovered in this limit. We have argued that NC gauge theory on $R^4$ can be obtained by expanding IIB matrix model around fuzzy $R^4$ \cite{4}. Our matrix model construction makes such a statement more precise. Although we can formulate NC gauge theories nonperturbatively through unitary matrices \cite{23}, our construction may be useful for supersymmetric gauge theories.

We move on to investigate quantum theory. After the gauge fixing, the quadratic action for $\hat{a}_\mu$ is simply given by

$$\frac{1}{2} \text{Tr} \log \left( \hat{a}_\mu P_\mu \hat{a}_\nu \right).$$  \hspace{1cm} (3.14)

As we have described in the preceding section, we may identify the eigenstates of $P_\mu^2 = 2T^2/p$ with $(n,n)$ representations of $SU(3)$ with the eigenvalues

$$2n(n+2)/p.$$  \hspace{1cm} (3.15)

In a local patch, we can adopt the four dimensional approximation for $\hat{p}_\mu$ as in (2.32). In such an approximation, the eigenvalues $\tilde{P}_\alpha^2$ are $2n^2/p$. They are uniformly distributed over the momentum space with the density $p^2/8\pi^2$.

The one loop correction to the classical action is

$$\frac{1}{2} \text{Tr} \log (P^2\delta_{\mu\nu}) - \text{Tr} \log (P^2) - \frac{1}{4} \text{Tr} \log \left( (P^2 + iF_{\mu\nu}\Gamma_{\mu\nu})(\frac{1+\Gamma_{11}}{2}) \right),$$  \hspace{1cm} (3.16)

where $F_{\mu\nu} = -\sqrt{2/p}f_{\mu\nu\rho}P^\rho$. It differs from that of IIB matrix model by

$$\frac{1}{2} \text{Tr} \log (P^2\delta_{\mu\nu}) - \frac{1}{2} \text{Tr} \log (P^2\delta_{\mu\nu} - 2iF_{\mu\nu}).$$  \hspace{1cm} (3.17)

In dimensions higher than 2, we can expand this expression into the power series of $F_{\mu\nu}$. The leading correction is found to be

$$3Tr \frac{1}{T^2} = 3\sum_{n=1}^{p} (n+1)^3 \frac{1}{n(n+2)} \sim 3N.$$  \hspace{1cm} (3.18)
We find that the one loop correction is of the opposite (positive) sign with the classical action. It is unlike the $CP^1 = SU(2)/U(1)$ case where the quantum corrections vanish due to SUSY. Nevertheless the quantum correction in $CP^2$ case is proportional to $N$ and hence the volume of the manifold. As a catch phrase, one might say that the quantum correction to the cosmological constant is finite in this model. The contribution to (3.18) is dominated by the states with large Casimir eigenvalues. Such states represent nonlocal states connecting the opposite sides of spacetime. In this sense we believe that this quantum correction is the signature of cosmic scale SUSY breaking and there is no contradiction to the notion of local recovery of SUSY.

We investigate formal semiclassical (or infrared) limit of NC gauge theory on $CP^2$. In this case, we fix the size of spacetime to be 1. In a semiclassical approximation, we can identify

\[
Tr \to \frac{1}{(2\pi)^2} \left(\frac{p}{2}\right)^2 \int d^4x \sqrt{g},
\]

\[
P_\mu P_\mu \to -\frac{1}{\sqrt{g}} \partial_a \sqrt{g} g^{ab} \partial_b.
\]  

(3.19)

We may further identify

\[
P_\mu \to -iK_\mu^a(x) \partial_a,
\]

(3.20)

where the Killing vectors in homogeneous spaces are related to the inverse metric as

\[
\sum_\mu K_\mu^a K_\mu^b = g^{ab}.
\]  

(3.21)

Since $P_\mu$ satisfy the Lie algebra of $SU(3)$, we find

\[
K_\mu^a \partial_a K_\nu^b - K_\nu^a \partial_a K_\mu^b = -f_{\mu\nu\rho} K_\rho^b.
\]  

(3.22)

We can also parameterize

\[
\hat{a}_\mu \to K_\mu^a(x) b_a(x) + N_\mu^i(x) \phi_i(x),
\]

(3.23)

where $N_\mu^i$ are orthogonal to the Killing vectors

\[
\sum_\mu K_\mu^a N_\mu^i = 0.
\]  

(3.24)

In order that (3.24) is consistent with (3.22), $N_\mu^i$ must satisfy

\[
K_\mu^a \partial_a N_\nu^i - K_\nu^a \partial_a N_\mu^i = -f_{\mu\nu\rho} N_\rho^i.
\]  

(3.25)
We may define the inverse metric in the ‘transverse’ space from $N^i_\mu$.

$$\sum_\mu N^i_\mu N^j_\mu = g^{ij}. \quad (3.26)$$

From (3.22) and (3.25), we can show that $g^{ij}$ is invariant under the isometry

$$K^a_\mu \partial_a g^{ij} = 0. \quad (3.27)$$

We can subsequently conclude that the transverse space is flat, namely $g^{ij} = \delta^{ij}$.

Since

$$P_\mu a_\nu - P_\nu a_\mu = -iK^a_\mu K^b_\nu (\partial_a b_b - \partial_b b_a) + i f^{ab}_\mu K^c_\rho b_c$$

$$- i(K^a_\mu N^i_\nu - K^a_\nu N^i_\mu) \partial_a \phi_i + i f^{ab}_\mu N^i_\rho \phi_i, \quad (3.28)$$

we find

$$F_{\mu \nu} = -iK^a_\mu K^b_\nu (\partial_a b_b - \partial_b b_a) - i(K^a_\mu N^i_\nu - K^a_\nu N^i_\mu) \partial_a \phi_i. \quad (3.29)$$

The bosonic part of the action (3.1) gives the following result in the semiclassical limit.

$$\int d^4 x \sqrt{g} \left( \frac{1}{4} g^{ac} g^{bd} (\partial_a b_b - \partial_b b_a) (\partial_c b_d - \partial_d b_c) + \frac{1}{2} g^{ab} \partial_a \phi_i \partial_b \phi_i \right. \right.$$  

$$- \left. \frac{1}{2} f^{a}_\mu K^b_\mu K^i_\rho (\partial_a b_b - \partial_b b_a) \phi_i \right). \quad (3.30)$$

We thus obtain gauge theory on the four dimensional curved manifold $CP^2$.

The fermionic part is

$$\frac{1}{2} Tr \bar{\psi} \Gamma_\mu [A_\mu, \psi]. \quad (3.31)$$

In the semiclassical limit, we obtain

$$\frac{1}{2} \int d^4 x \sqrt{g} \left( -i \bar{\psi} \gamma^a \partial_a \psi \right), \quad (3.32)$$

where $\gamma^a = \Gamma_\mu K^a_\mu$ and $\gamma^i = \Gamma_\mu N^a_\mu$. They satisfy the following commutation relations

$$\{ \gamma^a, \gamma^b \} = g^{ab}, \quad \{ \gamma^i, \gamma^j \} = \delta^{ij}, \quad \{ \gamma^a, \gamma^i \} = 0. \quad (3.33)$$

We note that the fermionic kinetic term is invariant under $SU(3)$ transformations

$$Tr \bar{\psi} \Gamma_\mu [\hat{a}_\mu, \psi] \rightarrow \int d^4 x \sqrt{g} ( -i \bar{\psi} \Gamma_\mu K^a_\mu \partial_a \psi ). \quad (3.34)$$
It is because an $SU(3)$ rotation
\[ \delta \hat{p}_\mu = i \alpha f_{\mu \nu \rho} \hat{p}_\nu = [\hat{p}_\rho, \hat{p}_\mu] \] (3.35)
can be undone by the $SO(10)$ transformation of $\psi$ as
\[ \delta \psi = -\frac{i}{4} \alpha f_{\mu \nu \rho} \Gamma^{\mu \nu} \psi. \] (3.36)

In symmetric homogeneous spaces, we can locally choose a flat metric for $g^{ab}$ with the vanishing spin connection. Since our fermionic action is valid locally, it must be valid globally due to $SU(3)$ invariance.

Before concluding this section, we make a brief comment on NC gauge theory on $CP^3$ case. The classical action is evaluated for the irreducible representation $(m, n)$ of $SO(5)$ as
\[ -\frac{\alpha^4}{2} C_2(m, n) \text{dim}(m, n), \] (3.37)
where
\[ C_2(m, n) = m^2 + n^2 + 3m + n, \]
\[ \text{dim}(m, n) = \frac{(m + n + 2)(m - n + 1)(3 + 2m)(1 + 2n)}{6}. \] (3.38)

It is not minimized by an $(m, m)$ type representation but rather minimized by an $(m, 0)$ type representation for the fixed $N = \text{dim}(m, n)$. Unfortunately the $(p/2, p/2)$ representation which is relevant to $CP^3$ as it is explained in section 2 is metastable in our construction. The quantum correction around the $(p/2, p/2)$ state is found to be much larger than the classical action since
\[ 6 Tr \frac{1}{T^2} = 6 \sum_{m=0}^{p} \sum_{n=0}^{m} \text{dim}(m, n) \frac{1}{C_2(m, n)} \sim \frac{3}{4} (\log(4) - 1) p^4, \] (3.39)
while $N \sim p^3/6$. So the one loop correction to the cosmological constant is infinite in this model.

## 4 Conclusions and Discussions

We have investigated NC gauge theories on fuzzy homogeneous spaces $G/H$ through matrix models. We have considered deformed IIB matrix models with finite $N$. In our construction, the isometry of a homogeneous space, $G$ must be a subgroup of $SO(10)$ which is the symmetry
of IIB matrix model. A local patch and the coordinates can be introduced semiclassically when \( N \) is large. We require that a fuzzy flat hyper plane is realized in the local patch. Although the Hermitian matrices \( A_\mu \) have been interpreted as coordinates in matrix models, we have interpreted them as Killing vectors of spacetime.

We have investigated 4 dimensional NC gauge theory on fuzzy \( CP^2 \) in detail. We have shown that NC gauge theory on \( R^4 \) is realized by letting the cubic coupling vanish in the large \( N \) limit. We have therefore made the relation between matrix models and NC gauge theory more precise. Our construction may be useful for nonperturbative investigations of supersymmetric NC gauge theories. In string theory, NC gauge theory is realized by introducing constant \( B_{\mu\nu} \) field\(^{24}\)\(^{25}\). Let us consider a localized state \(|p\rangle \) on \( CP^2 \) as it is discussed in section 2. In the local patch around it, \( \{t^4, t^5, t^6, t^7\} \) can be interpreted as the local coordinates. \( f_{458} \) and \( f_{678} \) can be interpreted as constant \( B_{12} \) and \( B_{34} \) fields since the deformation term behaves locally like

\[
if_{458} \frac{(2\pi)^2}{g_{NC}} \text{Tr} \left( \frac{2}{p} \hat{t}_8 \left( [\hat{a}_4, \hat{a}_5] + [\hat{a}_6, \hat{a}_7] \right) \right)
\]

\[
\rightarrow \frac{i}{g_{NC}^2} \int d^4x (a_4 \ast a_5 - a_5 \ast a_4 + a_6 \ast a_7 - a_7 \ast a_6).
\]  

(4.1)

This is consistent with the coupling of \( B_{\mu\nu} \) field in IIB matrix model\(^{26}\).

We hope to draw possible implications for the large \( N \) limit of IIB matrix model based on our results. We recall that the cubic terms we have added to IIB matrix model formally vanish in the large \( N \) limit. Nevertheless they affect the theory since different NC gauge theories are realized based on different homogeneous spaces \( G/H \). The situation is analogous to the magnetic systems where different polarization directions are realized depending on the directions of a tiny external magnetic field. We should hence clarify under what conditions unique physics is realized in the large \( N \) limit of IIB matrix model. Another related issue is the possibility that the cubic terms may be dynamically generated in the large \( N \) limit of IIB matrix model. We recall here that four dimensional distributions for \( A_\mu \) are found to be favored in the mean field approximation\(^{27}\)\(^{28}\). It is interesting to study which homogeneous space minimizes the free energy of IIB matrix model under the mean field approximation.

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