A simple model illustrating the impossibility of measuring off-shell effects

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We consider a simple model and make use of field transformations in Lagrangian field theories to illustrate the impossibility of measuring off-shell effects in nucleon-nucleon bremsstrahlung and related processes.

1. INTRODUCTION

The issue of how to describe the interaction of a particle which is not on its mass shell has a long history. For example, in their derivation of the low-energy theorem of Compton scattering in 1954, Gell-Mann and Goldberger \cite{1} took into account that the electromagnetic vertex of an off-shell nucleon is more complicated than that of a free nucleon. In case of the electromagnetic interaction, some restrictions on the general form of the off-shell vertex result from the Ward-Takahashi identity \cite{2}. It is a natural and legitimate question to ask whether it is possible to extract the off-shell behavior of particular interaction vertices from empirical information similarly as one, say, extracts the electromagnetic form factors of the nucleon from elastic electron scattering. In this context one might think of the electromagnetic interaction of a bound nucleon \cite{3}. Another example would be an investigation of the off-shell nucleon-nucleon amplitude entering the nucleon-nucleon-bremsstrahlung process \cite{4}.

It has only been recently that the relevance of field transformations in the framework of Lagrangian field theories \cite{5} has been emphasized in addressing this question. For the case of pions, Compton scattering \cite{6} and pion-pion bremsstrahlung \cite{7} have been considered using chiral perturbation theory. It was shown that off-shell effects both depend on the model used and on the choice of representation for the fields. From that we concluded that off-shell effects are not only model dependent but also representation dependent, making a unique extraction of off-shell effects impossible. In practice, the freedom of choosing appropriate field variables has been used in Refs. \cite{8} to obtain the most general effective Lagrangian describing low-energy (virtual) Compton scattering.

Here, we will extend our previous discussion \cite{6,7} to the investigation of a spin one-half system \cite{9} which should remove any uncertainty that the results of the previous works somehow depended on the simplicity of a spin-zero process.
2. THE MODEL

As our toy model we take
\[ L_0 = \overline{\Psi}(i\partial - m)\Psi - \frac{e\kappa}{4m} \overline{\Psi}\sigma_{\mu\nu}F^{\mu\nu}\Psi + \overline{\Phi}(i\partial - m)\Phi + g\overline{\Psi}\Psi\overline{\Phi}, \] (1)

where \( D_{\mu}\Psi = (\partial_{\mu} + ieA_{\mu})\Psi \) is the covariant derivative of the proton field, \( e \) and \( \kappa \) are the proton charge and anomalous magnetic moment respectively, \( A_{\mu} \) is the photon field, and \( F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \) is the electromagnetic field strength tensor. The fields \( \Psi \) and \( \Phi \) refer to protons and neutrons, respectively. For calculational simplicity, we neglect the electromagnetic coupling to the neutron magnetic moment. In the framework of Eq. (1) it is straightforward to calculate the Born diagrams of proton-neutron bremsstrahlung:

\[ M_{NN\gamma}^0 = ieg\overline{\pi_n}(p_4)u_n(p_2)\overline{\pi_p}(p_3) \left[ \left( \frac{ie\kappa}{2m}\sigma_{\mu\nu}e^{\mu\nu}\right) \frac{\slashed{p}_3 + \slashed{k} + m}{(p_3 + k)^2 - m^2} \right. \]
\[ + \left. \frac{\slashed{p}_1 - \slashed{k} + m}{(p_1 - k)^2 - m^2} \left( \frac{ie\kappa}{2m}\sigma_{\mu\nu}e^{\mu\nu}\right) \right] u_p(p_1), \] (2)

which corresponds to the usual choice for nucleon-nucleon bremsstrahlung for the electromagnetic parts. Obviously, for pedagogical reasons, Eq. (2) has a much simplified interaction for the strong part.

3. FIELD TRANSFORMATIONS

Next, we perform a change of variables leading to different off-shell behavior in the nucleon-nucleon amplitude as well as the photon-nucleon vertex, and study the effect on the total \( pn \) bremsstrahlung amplitude. For that purpose we consider the transformation

\[ \Psi \rightarrow \Psi + \tilde{a}g\overline{\Phi}\Phi\Psi + \tilde{b}e\sigma_{\mu\nu}F^{\mu\nu}\Psi, \] (3)

where \( \tilde{a} \) and \( \tilde{b} \) are real constants, determining the overall strength of the transformation. Equation (3) results in a class of equivalent Lagrangians

\[ L(\tilde{a}, \tilde{b}) = L_0 + \Delta L(\tilde{a}, \tilde{b}) = L_0 + \Delta L_1(\tilde{a}, \tilde{b}) + \Delta L_2(\tilde{a}, \tilde{b}), \] (4)

where the explicit expressions of \( \Delta L_1 \) and \( \Delta L_2 \) can be found in Ref. [9]. Consider, for example, \( \tilde{a} \neq 0 \) and \( \tilde{b} = 0 \), in which case the \( \tilde{a} \) term will generate an off-shell dependence in the proton legs of the \( pn \) amplitude

\[ i\tilde{a}g[\slashed{p}_3 - m] + (\slashed{p}_1 - m)\overline{\pi_n}(p_4)u_n(p_2), \] (5)

in conjunction with a \( pn\gamma \) contact term

\[ -2i\tilde{a}eg\overline{\pi_p}(p_3) \left( \frac{ie\kappa}{2m}\sigma_{\mu\nu}e^{\mu\nu}\right) u_p(p_1)\overline{\pi_n}(p_4)u_n(p_2). \] (6)

One can show that in the amplitude the off-shell dependence from Eq. (3) precisely cancels the contact term of Eq. (3). In the present context, such a cancellation for the charge, but not the magnetic, parts is simultaneously enforced by gauge invariance. A similar observation was also made in Ref. [10] for real Compton scattering off a charged pion.
However, the implication of field transformations is even more general, because it also has consequences for terms which are not fixed by gauge invariance, such as the magnetic terms above. The general case $\tilde{a} \neq 0 \neq \tilde{b}$ is discussed in Ref. [9].

The above result is a simple illustration of the equivalence theorem of Lagrangian field theory [3], according to which all Lagrangians of Eq. (4) result in identical $S$-matrix elements. One can also make use of this observation in order to show that what appears as an off-shell effect in an $S$-matrix element for one Lagrangian may originate in a contact term from an equivalent Lagrangian [1].

4. CONCLUSION

We conclude that off-shell effects cannot in any unambiguous way be extracted from an $S$-matrix element.

REFERENCES

1. M. Gell-Mann and M.L. Goldberger, Phys. Rev. 96 (1954) 1433.
2. J.C. Ward, Phys. Rev. 78 (1950) 182; Y. Takahashi, Nuovo Cim. 6 (1957) 371.
3. H.W.L. Naus and J.H. Koch, Phys. Rev. C 36 (1987) 2459; H.W.L. Naus, S.J. Pollock, J.H. Koch, and U. Oelfke, Nucl. Phys. A509 (1990) 717; P.C. Tiemeijer and J.A. Tjon, Phys. Rev. C 42 (1990) 599; X. Song, J.P. Chen, and J.S. McCarthy, Z. Phys. A 341 (1992) 275.
4. See e.g. V.R. Brown, P.L. Anthony, and J. Franklin, Phys. Rev. C 44 (1991) 1296; V. Herrmann and K. Nakayama, Phys. Rev. C 46 (1992) 2199; A. Katsogiannis and K. Amos, Phys. Rev. C 47 (1993) 1376; M. Jetter and H.V. von Geramb, Phys. Rev. C 49 (1994) 1832; G.H. Martinus, O. Scholten, and J.A. Tjon, Phys. Lett. B 402 (1997) 7; R.L. Workman and H.W. Fearing, Phys. Rev. C 34 (1986) 780.
5. J.S.R. Chisholm, Nucl. Phys. 26 (1961) 469; S. Kamefuchi, L. O’Raifeartaigh, and A. Salam, Nucl. Phys. 28 (1961) 529; S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177 (1969) 2239; S. Scherer and H.W. Fearing, Phys. Rev. D 52 (1995) 6445.
6. S. Scherer and H.W. Fearing, Phys. Rev. C 51 (1995) 359; S. Scherer, in Proceedings of the Conference on Perspectives in Nuclear Physics at Intermediate Energies, Trieste, 1995 [nucl-th/9506028].
7. H.W. Fearing, Phys. Rev. Lett. 81 (1998) 758; Few-Body Syst. Suppl., in press, nucl-th/0006040.
8. A.I. L’vov, Int. J. Mod. Phys. A 8 (1993) 5267; S. Scherer, A.Yu. Korchin, and J.H. Koch, Phys. Rev. C 54 (1996) 904; H.W. Fearing and S. Scherer, Few-Body Syst. 23 (1998) 111.
9. H.W. Fearing and S. Scherer, accepted for publication in Phys. Rev. C (2000) [nucl-th/9909076].
10. A.E. Kaloshin, Phys. Atom. Nucl. 62 (1999) 1899 [Yad. Fiz. 62 (1999) 2049].