Tensor analysis of uncertainty in freight transport ULS-systems

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Abstract. The article is devoted to the application of the tensor basis for the analysis of uncertainties in decision-making in freight transport ULS-systems. The use of tensors for constructing a mathematical model of the coordinator in the decision-making system is shown. Tensor equations make calculation possible the system parameters for any change in the structure of connections in the basic mathematical model. This provides additional advantages when making decisions in an uncertain environment. The authors also demonstrate the use of a tensor apparatus for constructing a mathematical model of deep uncertainty, which is impossible with traditional methods of analysis. The article shows that tensor analysis can be used to model decision-making under conditions of deep uncertainty in complex conditions, such as a catastrophes. Representing a fuzzy variable as a tensor makes it possible to explore additional properties of this variable. Tensors describing additional properties of the fuzzy variable (deviator, spherical tensor, symmetric tensor, and antisymmetric tensor) can be obtained from the fuzzy variable tensor. Applying special calculations to a fuzzy tensor variable makes it possible to obtain the membership function with high accuracy and find the defuzzified output value of the membership function. Tensor analysis significantly expands the scope of ULS-systems research.

1. Introduction

The tasks that arise when making decisions differ significantly by subject area, by the type of information received and presented, by the number and types of criteria, as well as by the problem situations that the decisions should resolve. The features of such tasks include a high degree of vagueness and uncertainty of information. Traditional methods of analysis are not sufficiently suitable for decision-making tasks in the implementation of transport dispatching, since in real conditions neither the initial data nor the ideas about the project and problem situations and the goal that was set are completely accurate and correspond to reality, which is compounded by the need to predict the development of the subject area parameters. So decision making in uncertainty is the main task in decision-making in freight transport ULS-systems [1]. Such systems should ensure decision-making in difficult conditions such as catastrophe.

The theory of catastrophes deals with processes in which a smooth change in the system parameters is interrupted by an abrupt change (catastrophes). As a result, the mode of existence of the system changes or it is destroyed. Thus, catastrophes can be called abrupt changes that occur in the form of a sudden response of the system to a smooth change in external conditions (see figure 1).
Figure 1. Surface of the catastrophe.

Let us consider the solutions $\Phi_1(t, x; c_a), \Phi_2(t, x; c_a), \ldots$ of a system of $n$ equations defined in space $R^N$ with coordinates $x=(x_1, x_2, \ldots, x_N)$,

$$F_i\left(\Phi_i; c_a; \frac{d\Phi_i}{dx_1}, \frac{d^2\Phi_i}{dx_1dx_2}, \ldots, \frac{d^m\Phi_i}{dx_1^m}, \ldots, \right) = 0 \quad (1)$$

where $1 \leq i \leq n$, $1 \leq l$, $m \leq N$, $1 \leq a \leq k$.

The variables $x_i$ and $t$ can be considered spatial and temporal coordinates, respectively. Solutions of $F_i$ describe the state of a certain system, so they are called variable states. The equations $F_i = 0$ depend on $k$ parameters of $c_a$, i.e., they can qualitatively affect the properties of solutions of $F_i$, so the parameters of $c_a$ are control parameters. Consequently, the theory of catastrophes considers the equilibrium state $F_i(c_a)$ of the potential function $U(F_i; c_a)$, which changes when the control parameters of the CA change. The state variables on which the function $U(F_i; c_a)$ depends are essentially generalized coordinates of the system under consideration.

The main task of the dispatching service of a transport enterprise is to optimize traffic flows. The task of optimizing the decision-making process is a multi-criteria one. The automated cargo transport control system is proposed to be expressed as a two-level hierarchical system-coordinator. When the information support system is functioning, maximum coordination and coordination of its functioning processes should be achieved in order to achieve maximum efficiency of the system [2]. The coordination principle and the tasks of the control elements should be selected in such a way that the coordination principle is applicable and the transport information support system is coordinated based on the selected principle. The coordinator consists of the upper-level coordinating element $C_0$, which solves the problem of coordinating the entire system $D_0$, and the lower-level control elements $\{C^i, i \in M\}$, which solve the problems $\{D^i, i \in M^F\}$ of forming the optimal control signal for their functional subsystems (FS). The tasks $\{D^i, i \in M^F\}$ of the control elements of the lower level of the coordinator are to select such a control signal $\tilde{m}^i \in M^i$ that ensures the maximum efficiency of each subsystem.

There are two types of conflicts that can occur in the management process:

- inter-level conflict, when achieving the maximum efficiency of one of the subsystems prevents achieving the maximum efficiency of the system as a whole;

$$D: h_j(X) \geq 0, j=1..m, X - \text{desired solution},$$

$$q_i(X) (i=1..k) - \text{solution quality function } X,$$

$$h_j(X) - \text{restrictions that set the allowed range } D \text{ of possible changes to the solution } X.$$
– intra-level conflict, when an increase in the efficiency of one functional subsystem prevents the achievement of the maximum efficiency of another subsystem.

The coordinator consists of an top-level coordinating element $C_o$ that solves the problem of coordinating the entire $D_o$ system, and lower-level control elements $\{C^i, i \in M^{FS}\}$ that solve the tasks $\{D^i, i \in M^{FS}\}$ of generating the optimal control signal for their functional subsystems. The condition for the end of the iterative process in the coordinator is the choice of the optimal coordinating signal, i.e.:

$$A^i_k : \xi^i_{g+1} \rightarrow \xi^i_g, \ g = 1, 2, 3, ..., g_Z$$

The condition for the end of the iterative coordination process when optimizing parameters is the choice of the optimal control signal:

$$A^i_k : m^i_j \rightarrow m^i_{j+1}, \ l = 1, 2, 3, ..., l_Z$$

Tasks of the higher-level (coordinating) element and lower-level elements of the SIPD are formalized as tasks of predicting the state and maximizing the efficiency $E$ of functional subsystems and dispatching objects. The main idea of the proposed approach is to introduce two levels of description of dispatching control systems. The macro description level of the system corresponds to the description of the implemented distribution:

$$Q = \{g(x_P, y_E), x_P \in X_P, y_E \in Y_E\}, \ (5)$$

that is the probability $P$ with which the required transport flow $x_P$ will perform the $y_E$ task. Thus, many of the microstates leading to the same distribution, or macrostate. The most likely distribution $Q$ corresponds to a situation where there is the greatest uncertainty about the microstates of the system, since in this case there is the largest possible number of microstates leading to this distribution, assuming an equal probability of microstates.

2. Tensor basis for the system modeling of deep uncertainty

As noted above, the coordination process is complicated by the presence of various kinds of uncertainty. The computational complexity of the problem of processing fuzzy knowledge can be reduced through the use of tensor methodology [3]. The control process within the coordinator may be considered as a projection of the abstract control process in the space-structure of control flows. The dimension of space corresponds to the number of local control systems. The connection of local subsystems defines the coordinate system. The figure 2 shows a two-level management process, which is basic for building similar multi-level processes. The following designations are used in figure 2 below:

$Z$ – set of information signals from the managed process

$\Xi$ – set of signals from coordinator

$M$ – set of control actions on the process

$\vartheta$ – set of control flows,

$coal_{\alpha \beta}$ – coalition interaction between control flow $\alpha$ and control flow $\beta$,

$z_\alpha$ – signal of the controlled process to the control subsystem $\alpha$,

$\xi_\alpha$ – coordinating effect of the top-level coordinator on the control subsystem $\alpha$,

$m_\alpha$ – control influence of the subsystem $\alpha$ on the process.
The equations of the system can be written in tensor form:

\[ \varrho_\alpha = m_\alpha + \sum_\beta \text{coal}_{\alpha \beta}, \quad (6) \]

- control flow

\[ \text{coal}_{\alpha \beta} = \xi_{\alpha \beta} \cdot \varrho_\beta, \quad (7) \]

- the value of the coalition flow of the control subsystem \( \alpha \), required to form a unit \( \varrho_\beta \)

\[ \xi_{\alpha \beta} = \mu_{\alpha \beta} \cdot m_\alpha, \quad (8) \]

- coefficient that characterizes the value of the signal of the controlled process to the control subsystem \( \alpha \) necessary for forming the unit \( \varrho_\beta \)

\[ \xi_{\alpha \beta} = \eta_{\alpha \beta} \cdot \varrho_\beta, \quad (9) \]

- coefficient that characterizes the amount of coordinating influence on the subsystem \( \beta \), necessary for the formation of a unit \( \varrho_\beta \)

\[ m_\alpha = (d_{\alpha \beta} - a_{\alpha \beta}) \cdot \varrho_\beta \cdot (c_{\alpha \beta} - \mu_{\alpha \beta}) = (I - A) \cdot \Lambda - \text{matrix of } \xi_{\alpha \beta}, \quad (10) \]

Let's change the connection of local subsystems in any way (see figure 3). Each new connection is a new coordinate system. The invariant of the equation transformation is the value of the control action. Since there is a linear relationship between \( \text{coal}_{\alpha \beta} \), \( \xi_{\alpha \beta} \), and \( \varrho_\beta \), we can find the optimal value of the control action for various connections of control subsystems.
Figure 3. The examples of changing the link structure

So we got the equations for the new connection of control subsystems:

\[ \varphi_{p} = G_{\varphi}^{p} \varphi_{f} \] (12)

\[ G_{\varphi}^{p} = (\varphi_{mul} - \bar{\varphi}_{mul})^{-1} (\varphi_{conv} - \bar{\varphi}_{conv}) \] (13)

\[ \varphi_{df} = G_{\varphi}^{df} \varphi_{s} \] (14)

\[ \varphi_{df} = G_{\varphi}^{df} (\sum_{df mul} + \sum_{df conv}) \] (15)

These equations describe transformations for tensor equations (6), (7), (8), (9), (11). These equations can be used to calculate parameters for any option of connecting subsystems.

We use a tensor basis [4] to determine the fuzzy variables in the presented system. The set of values of a fuzzy variable \( m \) on a certain universe \( U \) can be represented as a matrix: \( m = \{ m_i \}, i \in N, m \in U, \mu_{F}(m) \) is a function of the membership of elements \( U \) to the fuzzy set \( F \).

\[ F = \{ < m, \mu_{F}(m) > \}, m \in U \] (16)

We define the matrix (17) and the tensor product \( T \) (18) (19) for the matrices \( m \) and \( \mu_{F}(m) \):

\[ \mu_{F}(m) = \{ \mu_{F}(m_i) \}, n \in N, i = l, n; \mu_{F}(m_i) \in F \] (17)

\[ T = m \otimes \mu_{F}(m) \] (18)

\[ T = \{ m_1, m_2 ... m_n \} \otimes \begin{bmatrix} \mu_{F}(m_1) \\ \mu_{F}(m_2) \\ \vdots \\ \mu_{F}(m_n) \end{bmatrix} = \begin{bmatrix} m_1 \ast \mu_{F}(m_1) & m_2 \ast \mu_{F}(m_1) & \cdots & m_n \ast \mu_{F}(m_1) \\ m_1 \ast \mu_{F}(m_2) & m_2 \ast \mu_{F}(m_2) & \cdots & m_n \ast \mu_{F}(m_2) \\ \vdots & \vdots & \ddots & \vdots \\ m_1 \ast \mu_{F}(m_n) & m_2 \ast \mu_{F}(m_n) & \cdots & m_n \ast \mu_{F}(m_n) \end{bmatrix} \] (19)
The dyad $T$ defines the fuzzy variable $m$. By converting from a tensor variable, we can get attached tensors, the analysis of which allows us to determine additional properties of the fuzzy variable.

### 3. Results of applying the tensor basis for the system modeling of deep uncertainty

Let's assume that for some system the following fuzzy values ($m_{\alpha}$, $\alpha=\{1,2,3,4\}$) of control actions of the subsystem $\alpha$ on the process are known: $m_1 = \text{fuzzy 15} = \{12, 15, 18\}$, $m_2 = \text{fuzzy 26} = \{23, 26, 29\}$, $m_3 = \text{fuzzy 34} = \{31, 34, 37\}$, $m_4 = \text{fuzzy 7} = \{4, 7, 10\}$. We heuristically define the membership functions $\mu_{m_{\alpha}}$ for $m_{\alpha}$ and get a graphical representation of fuzzy sets (see figure 4).

![Figure 4. The fuzzy variables $m_{\alpha}$](image)

Using equation (18) we obtain a tensor representation of fuzzy variables (see figure 5).

![Figure 5. The tensors of fuzzy variables $m_{\alpha}$](image)

Finally, for each tensor we obtain by transformations the set of ordered pairs $(SOP_{\alpha})$. We calculate the Frobenius norms for $SOP_{\alpha}$ and defuzzified values for $SOP_{\alpha}$. We perform a singular decomposition.
of the $T_{\alpha_c}$. Based on the results of the singular decomposition we define the $SOP_{\alpha_c}$. We calculate the Frobenius norms for $SOP_{\alpha_c}$ and defuzzified values for $SOP_{\alpha_c}$. The results of calculations are shown in table 1. Functions $trimf$, $kron$, $transpose$ in table 1 are MatLab’s functions.

### Table 1. Combined table of calculation results

| Fuzzy $15 = \{12, 15, 18\}$ | Fuzzy set $F_{15}$ | Tensor $T_{15}$ | Frobenius norm | Defuzzified value |
|-------------------------------|--------------------|----------------|----------------|-------------------|
| $m_1, \mu_{\alpha_c}$, $m_2 \in [12, 18]$, $\mu_{\alpha_c} = \text{trimf}([12:0.75:18],[12 15 18])$ | $F_{15} = \text{transpose}([12:0.75:18]), \transpose(\mu_{\alpha_c})$ | $T_{15} = m_1 \otimes \mu_{\alpha_c} = \text{kron}([12:0.75:18], \transpose(\mu_{\alpha_c}))$ | 45.4037 | 15 |

| Fuzzy $26 = \{23, 26, 29\}$ | Fuzzy set $F_{26}$ | Tensor $T_{26}$ | Frobenius norm | Defuzzified value |
|-------------------------------|--------------------|----------------|----------------|-------------------|
| $m_1, \mu_{\alpha_c}$, $m_2 \in [23, 29]$, $\mu_{\alpha_c} = \text{trimf}([23:0.75:29],[23 26 29])$ | $F_{26} = \text{transpose}([23:0.75:29]), \transpose(\mu_{\alpha_c})$ | $T_{26} = m_1 \otimes \mu_{\alpha_c} = \text{kron}([23:0.75:29], \transpose(\mu_{\alpha_c}))$ | 78.2336 | 26 |

| Fuzzy $34 = \{31, 34, 37\}$ | Fuzzy set $F_{34}$ | Tensor $T_{34}$ | Frobenius norm | Defuzzified value |
|-------------------------------|--------------------|----------------|----------------|-------------------|
| $m_1, \mu_{\alpha_c}$, $m_2 \in [31, 37]$, $\mu_{\alpha_c} = \text{trimf}([23:0.75:29],[31 34 37])$ | $F_{34} = \text{transpose}([31:0.75:37]), \transpose(\mu_{\alpha_c})$ | $T_{34} = m_1 \otimes \mu_{\alpha_c} = \text{kron}([31:0.75:37], \transpose(\mu_{\alpha_c}))$ | 102.1788 | 34 |

| Fuzzy $7 = \{4, 7, 10\}$ | Fuzzy set $F_{7}$ | Tensor $T_{7}$ | Frobenius norm | Defuzzified value |
|-------------------------------|--------------------|----------------|----------------|-------------------|
| $m_1, \mu_{\alpha_c}$, $m_2 \in [4, 10]$, $\mu_{\alpha_c} = \text{trimf}([4:0.75:10],[4 7 10])$ | $F_{7} = \text{transpose}([4:0.75:10]), \transpose(\mu_{\alpha_c})$ | $T_{7} = m_1 \otimes \mu_{\alpha_c} = \text{kron}([4:0.75:10], \transpose(\mu_{\alpha_c}))$ | 21.8518 | 7 |

From table 1, we see that the Frobenius norm of the $F_{\alpha_c}$ and Frobenius norm of the $T_{\alpha_c}$ completely coincide. Defuzzified value of the $F_{\alpha_c}$ and defuzzified value of the $T_{\alpha_c}$ also match. So the tensor fully reflects all the properties of the fuzzy variable. The representation of a fuzzy variable as a tensor allows us to investigate additional properties of uncertainty [5]. Since deviator, spherical tensor, symmetric tensor and antisymmetric tensor can be obtained from this tensor. Thus the representation of a fuzzy variable as a tensor is a representation of deep uncertainty [6] and opens up more possibilities for research.

### 4. Conclusion

The results obtained by the authors in the course of the study allow us to draw the following conclusions:

1. The mathematical model of the coordinator of the decision-making process under deep uncertainty can be implemented on the basis of a tensor basis. Tensor equations can be used to calculate system parameters for any change in the structure of connections in the basic mathematical model.

2. Representing a fuzzy variable as a tensor makes it possible to explore additional properties of this variable. Tensors describing additional properties of the fuzzy variable (deviator, spherical tensor, symmetric tensor, and antisymmetric tensor) can be obtained from the fuzzy variable tensor. Applying special calculations to a fuzzy tensor variable allows us to obtain the membership function with high accuracy and find the defuzzified output value for the membership function.
3. Tensor analysis can be used to model decision-making under conditions of deep uncertainty in complex conditions, such as catastrophes.

4. Tensor analysis significantly expands the field of research of ULS-systems.

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