Squeezing lepton pairs out of broken symmetries

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Abstract. We discuss two possible signatures of symmetry breaking that can appear in dilepton spectra, as measured in relativistic heavy ion collisions. The first involves scalar-vector meson mixing and is related to the breaking of Lorentz symmetry by a hot medium. The second is related to the breaking of Furry’s theorem by a charged quark-gluon plasma. Those signals will be accessible to upcoming measurements to be performed at the GSI, RHIC, and the LHC.

INTRODUCTION

Electromagnetic radiation, especially lepton pairs, represents probes of choice for the study of heavy ion reactions. Owing to the smallness of their final state interaction they travel essentially unscathed from creation to detection. Furthermore, via Vector Meson Dominance (VMD) [1], real and virtual photons are ideal as signatures of the in-medium behaviour of vector mesons [2]. This aspect has been highlighted in a theoretical interpretation [3] of the low dilepton invariant mass measurements performed by the CERES collaboration [4]. In short, it appears that the behaviour of hot and dense strongly interacting matter can indeed be revealed by electromagnetic observables [5].

In this work we shall outline how symmetry breaking effects can manifest themselves in dilepton spectra. The first of those involve the possibility of $\rho - a_0$ mixing in dense nuclear matter. The size of this contribution is density-dependent and goes to zero in vacuum, on account of Lorentz symmetry. We show that such a mixing opens up a new channel in dilepton production and will modify the spectrum in the $\phi$ invariant mass region. Secondly, we show that Furry’s theorem is no longer obeyed in a baryon rich quark gluon plasma. This has the consequence of permitting new processes like $gg \to l^+l^-$. We evaluate this contribution and compare it with $q\bar{q} \to l^+l^-$. We will consider only dielectrons here and the details of the calculations will only be sketched; the interested reader is invited to consult the references quoted in the text. The observables we discuss in this work are within reach of measurements soon to be performed by the HADES collaboration at the GSI, and by the PHENIX collaboration at RHIC.

DILEPTONS FROM MESON MIXING IN DENSE HADRONIC MATTER

There exists physical processes which are forbidden in free space but can take place in matter. Those are related to medium-induced symmetry breaking effects. The interaction respects the symmetry, but it is broken by the ground state. For instance, in matter, Lorentz symmetry is lost which leads to the mixing of different spin states even when the interaction Lagrangian respects all the required symmetry properties. A well-known example of this is $\sigma - \omega$ mixing in nuclear matter as discussed by Chin [6]. Another manifestation lies in the fact that different polarization states of vector fields can have different dispersion relations [2].

Our goal here is to identify the effects of scalar and vector meson mixing on dilepton production rates, which could be observed in high energy heavy ion experiments. The mixing of mesons considered here include both isoscalar and isovector channels. To be more specific, we estimate the rate of dilepton production from $\sigma - \omega$ and $\rho - a_0$ mixing. Some attention was paid to the former in the context of heavy-ion collisions [7, 8], and the importance of the later was shown recently [9].

In this first part we present thermal production rates of dileptons induced by scalar-vector mixing in the isoscalar and isovector channels. Those results are compared against $\pi\pi, K\bar{K}$ annihilation contributions, which are treated here as standard candles. Note that processes involving mixing of different G-parity states like $\sigma - \omega$ or $\rho - a_0$ are allowed only in matter which is not invariant under charge conjugation. Therefore, $\pi\pi \to e^+e^-$ through the coupling to nucleons via $\sigma$ and $\omega$, or $\pi\pi \to e^+e^-$ mediated by $\rho - a_0$ mixing, can take place in matter with a finite nucleon chemical potential.
Hence it is natural to look for such density-driven effects in experiments involving relatively low energy collisions where one expects to have a relatively large chemical potential.

The interaction Lagrangian used in the present model can be written as

$$L_{\text{int}} = g_{\rho} \bar{\psi}_s \phi_s \psi + g_{a_0} \bar{\psi}_a \phi_{a_0} \sigma \psi + g_{\omega NN} \bar{\psi}_\omega \psi \omega' + g_{\rho} \bar{\psi}_s \phi_s \omega' + \frac{\kappa_0}{2m_n} \bar{\psi} \sigma_{\mu\nu} \epsilon^{\mu\nu} \omega' |\rho_{a_0}',$$

where $\psi_s, \phi_{a_0}, \rho$ and $\omega$ correspond to nucleon, $\sigma, a_0$, $\rho$ and $\omega$ fields, and the $\tau's$ are Pauli matrices. From this point on we use $s$ and $v$ to denote scalar and vector mesons, i.e. $s = \sigma, a_0$ and $v = \omega, \rho$. It is understood that $\kappa_0 = 0.0$.

The polarization vector through which the scalar meson couples to the vector meson via the n-n loop is given by

$$\Pi_\mu(q_0, |q|) = 2i g_{s\rho} v \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[G(k)\Gamma_\mu G(k + q)].$$

where 2 is an isospin factor and the vertex for v-nn coupling is:

$$\Gamma_\mu = \gamma_\mu - \frac{\kappa_0}{2m_n} \sigma_{\mu\nu} q^\nu.$$ (4)

$G(k)$ is the in-medium nucleon propagator [10].

$\rho-a_0$ mixing opens up a new channel for dilepton production in dense and hot nuclear matter: $\pi + \eta \rightarrow e^+ + e^-$, accessible through n-n excitations. The Feynman diagram for this process is shown in Fig. 1. Similarly, the contribution for $\pi + \pi \rightarrow \sigma \rightarrow \omega \rightarrow e^+ + e^-$ can be evaluated from a diagram obtained from that in Fig. 1 by replacing one of the incoming $\pi's$ by $\eta$ and by performing the following substitutions: $a_0 \rightarrow \sigma$ and $\rho \rightarrow \omega$.

![FIGURE 1. The Feynman diagram for the process $\pi + \eta \rightarrow e^+ + e^-$.](image)

**Dilepton rates**

Once we obtain the relevant cross-section for the dilepton production channel, the thermal production rate of the lepton pairs (number of reactions per unit time, per unit volume $R_{12}^{ee} = dN_{e^+e^- \text{ pairs}}/d^4x$) can be estimated in the independent particle approximation of relativistic kinetic theory:

$$\frac{dR_{12}^{ee}}{dM^2} = \int \frac{d^3 k_1}{(2\pi)^3} f_1(k_1) \int \frac{d^3 k_2}{(2\pi)^3} f_2(k_2) \frac{d\sigma_{12}^{ee}}{dM^2}(s, M^2) v_{\text{rel}}$$

where $f_1, f_2$ are the thermal distributions of the 1,2 species and $v_{\text{rel}} = \lambda^{1/2}(s, m_1^2, m_2^2) \sqrt{\tau}$ with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the triangle function.

Fig. 2 shows the total dilepton yield with the combined effect of mixing in the isoscalar and isovector channels. We have chosen conditions that are representative of collisions to occur at the GSI, and we show the pion and kaon annihilation channels separately. It is important to see that rates induced by the mixing can be significant in certain windows of density and temperature. Particularly, mixing induces a higher yield of dileptons in the invariant mass region between the $\rho$ and $\phi$ peaks, and is strongly density dependent. The mixing in the isoscalar channel contribute essentially at the $\omega$ peak 1. On the other hand, the isovector channel seems to provide a better probe as it contributes

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1 Technical, the $\pi\pi$ annihilation channel should treat the $s-$ and the $p-$wave annihilation coherently. The interference effects have been found to be negligible.
in the tail region of the two pion annihilation contribution. It should be stressed here that broadening or dropping \( \rho \) meson mass would even favour our observation. In that case the \( \pi \pi \) background would be pushed more towards lower invariant mass region bringing the \( a_0 \) peak into a clearer relief. Importantly, the upcoming HADES experiment [12] has the necessary resolution, and will operate in the appropriate energy regime to adequately investigate the physics discussed here.

**DILEPTON EMISSION FROM A BARYON-RICH QUARK GLUON PLASMA**

At zero temperature, diagrams in QED that contain a fermion loop with an odd number of photon vertices are cancelled by an equal and opposite contribution coming from the same diagram with fermion lines running in the opposite direction (Furry’s theorem [13]). This statement can also be generalized to QCD for processes with two gluons and an odd number of photon vertices.

At finite temperature and density, the corresponding quantity to consider in connection with Furry’s theorem is

\[
\sum_n \langle n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | n \rangle e^{-\beta(E_n - \mu Q_n)}\]

where \( \beta = 1/T, \mu \) is a chemical potential, and \( A_{\mu} \) is an electromagnetic gauge field operator. Here, the effect of the charge conjugation operator \( C \) is such that

\[
C | n \rangle = e^{-i\phi} | -n \rangle,
\]

where \( | -n \rangle \) is a state in the ensemble with the same number of antiparticles as there are particles in \( | n \rangle \) and vice-versa. If \( \mu = 0 \), upon multiple insertions of the unit operator \( C^{-1} \), we get

\[
\sum_n \langle n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | n \rangle e^{-\beta E_n} = 0
\]  

(7)

and Furry’s theorem still holds.

However, if \( \mu \neq 0 \) (unequal number of particles and antiparticles), then

\[
\langle n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | n \rangle e^{-\beta(E_n - \mu Q_n)} = -\langle -n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | -n \rangle e^{-\beta E_n}.
\]  

(8)

The mirror term this time is \( -\langle n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | -n \rangle e^{-\beta(E_n + \mu Q_n)} \), with a different thermal weight. Thus

\[
\sum_n \langle n | A_{\mu_1} A_{\mu_2} \ldots A_{\mu_{2n+1}} | n \rangle e^{-\beta(E_n - \mu Q_n)} \neq 0.
\]  

(9)
One may say that the medium, being charged, manifestly breaks charge conjugation invariance and these Green’s functions are thus finite and will lead to the appearance of new processes in a perturbative expansion. Again, the appearance of processes that can be related to symmetry breaking in a medium has been noted before [6, 14].

\[
\begin{align*}
T^{pp\nu} &= \frac{1}{\beta} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} e^{2g} \text{tr}(\tau^a_{\alpha\beta}) \frac{d^3q}{(2\pi)^3} T_{\gamma\mu}(\gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\lambda) \left(\frac{q + p - k_a q_b + \rho_c}{(q + p - k)^2 q^2 (q + p)^2}\right), \\
T^{\nu\rho\mu} &= \frac{1}{\beta} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} e^{2g} \text{tr}(\tau^a_{\alpha\beta}) \frac{d^3q}{(2\pi)^3} T_{\gamma\mu}(\gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\lambda) \left(\frac{q + k - p_a q_b + \rho_c}{(q + k - p)^2 q^2 (q - p)^2}\right),
\end{align*}
\]

and they involve a trace over colour matrices, and a sum over Matsubara frequencies.

Let us now focus our attention on the diagrams of Fig. 3 for the case of two gluons and a photon attached to a quark loop. Such a process does not exist at zero temperature or even at finite temperature and zero density. At finite density this leads to a new source of dileptons: \(gg \to e^+e^-\). In order to obtain the full matrix element one must coherently sum contributions from both diagrams which have fermion number running in opposite directions. The amplitudes for \(T^{pp\nu}(= T^{\nu\rho\mu} + T^{\nu\rho\mu})\) are

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\end{align*}
\]

Dilepton rates

To calculate the contribution made by the diagram of Fig. 3 to the dilepton spectrum emanating from a quark gluon plasma we calculate the imaginary part of the photon self-energy containing the above diagram as an effective vertex (see [11] for details). We calculate in the limit of photon three momentum \(p = 0\). The imaginary part of the photon self-energy \(\Gamma^{\nu\rho\mu}\) contains the above diagram. For a finite chemical potential \(\mu\) and total momentum \(\beta = 0\) is given in terms of the discontinuity in the photon self-energy \(\phi\). We use the \(q\bar{q} \to e^+e^-\) Born process as a baseline.

The differential production rate for pairs of massless leptons with total energy \(E\) and total momentum \(\beta = 0\) is given in terms of the discontinuity in the photon self-energy \(\phi\). We use the \(q\bar{q} \to e^+e^-\) Born process as a baseline.

\[
E(x, \beta, \mu) = \frac{1}{4\sqrt{\beta}} \int_{-\infty}^{\infty} e^{2g} \text{tr}(\tau^a_{\alpha\beta}) \frac{d^3q}{(2\pi)^3} T_{\gamma\mu}(\gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\lambda) \left(\frac{q + p - k_a q_b + \rho_c}{(q + p - k)^2 q^2 (q + p)^2}\right),
\]


diff \int \frac{d^3q}{(2\pi)^3} T_{\gamma\mu}(\gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\lambda) \left(\frac{q + p - k_a q_b + \rho_c}{(q + p - k)^2 q^2 (q + p)^2}\right).
are therefore genuine in-medium manifestations, and are a testimony to the richness of the many-body problem. Theoretical refinements are needed before truly quantitative predictions can be made. In the lower energy sector, many-body theory has to be introduced in order to consistently extend the current vector meson spectral densities to include meson mixing effects [18]. At higher energies, we have begun a study of the effects of Hard Thermal Loop resummation [19, 20]. Finally, our rates will be inserted in appropriate dynamical models. As those works are in progress, it is stimulating to note that the signals discussed in this work are all accessible to the new generation of lepton pair measurements.

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