Embeddings for Non-Critical Superstrings

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Abstract

It was previously shown that at critical central charge, \(N\)-extended superstrings can be embedded in \((N + 1)\)-extended superstrings. In other words, \((N = 0, c = 26) \rightarrow (N = 1, c = 15) \rightarrow (N = 2, c = 6) \rightarrow (N = 3, c = 0) \rightarrow (N = 4, c = 0)\). In this paper, we show that similar embeddings are also possible for \(N\)-extended superstrings at non-critical central charge. For any \(x\), the embedding is \((N = 0, c = 26 + x) \rightarrow (N = 1, c = 15 + x) \rightarrow (N = 2, c = 6 + x) \rightarrow (N = 3, c = x) \rightarrow (N = 4, c = x)\). As was conjectured by Vafa, the \((N = 2, c = 9) \rightarrow (N = 3, c = 3)\) embedding can be used to prove that \(N = 0\) topological strings are special vacua of \(N=1\) topological strings.
It has recently been noticed \[1, 2, 3, 4\] that starting with a critical \(N\)-extended stress tensor and with a twisted set of \(N\)-extended “ghosts”, it is possible to construct a critical \((N + 1)\)-extended stress tensor (this procedure differs from that of refs. \[3, 4\] in not requiring the existence of a \(U(1)\) current). The BRST charge constructed out of this \((N + 1)\)-extended stress tensor has the same cohomology as the BRST charge constructed out of the original \(N\)-extended stress tensor \[7, 4\]. Furthermore, it was shown for the \((N = 0, c = 26) \to (N = 1, c = 15)\) and \((N = 1, c = 15) \to (N = 2, c = 6)\) embeddings that if the \((N + 1)\)-extended stress tensor corresponds to the matter sector of a critical \((N + 1)\)-extended string, the \((N + 1)\)-extended prescription for calculating scattering amplitudes gives the same result as the original \(N\)-extended prescription \[3\].

In this paper, it will be shown that for \(N\)-extended stress tensors with non-critical central charge, it is also possible to use a twisted set of \(N\)-extended ghosts to construct an \((N + 1)\)-extended stress tensor. Although the \((N + 1)\)-extended stress tensor has non-critical central charge, the difference between the central charges of the \(N\)-extended and \((N + 1)\)-extended stress tensors is always equal to the difference of the critical central charges. In other words, we will describe the following embeddings:

\[
(N = 0, c = 26 + x) \to (N = 1, c = 15 + x) \to (N = 2, c = 6 + x) \to (N = 3, c = x) \to (N = 4, c = x),
\]

where \(x\) is arbitrary. Note that as in the critical case, these embeddings do not require the existence of a \(U(1)\) current.

There are various interesting features of these non-critical embeddings that will be described. For \(N = 0 \to N = 1\) and \(N = 1 \to N = 2\), it is possible to generalize the critical embeddings of ref. \[3\] by simply changing some of the coefficients in the stress tensor. However, for \(N = 2 \to N = 3\) and \(N = 3 \to N = 4\), the non-critical \((N + 1)\)-extended stress tensor requires an infinite number of terms, unlike the critical case described in ref. \[3\]. Surprisingly, it is not difficult to determine the explicit structure of these infinite terms.

Another interesting point is that there are two very different ways to construct the \(N = 1 \to N = 2\) non-critical embedding. The first construction has only a finite number of terms but requires bosonization of the twisted ghosts. The second construction shares the structure of the the other \(N \to N + 1\) embeddings but requires an infinite number of terms for the critical, as well as the non-critical case.

As an important application of our non-critical embeddings, it was conjectured by Vafa that our result on the \((N = 2, c = 9) \to (N = 3, c = 3)\) embedding can be used to prove the equivalence of the scattering amplitudes of \(N = 0\) and \(N = 1\) topological strings. This will be discussed briefly after presenting our non-critical embeddings, and will be described in more detail in ref. \[8\].

We begin this paper by giving a general procedure for turning a critical embedding into a non-critical embedding, and will then give explicit expressions for the non-critical embeddings. Except for the \(N = 0 \to N = 1\) embedding (which was also found independently by Amit Giveon), these embeddings
were constructed with the aid of the computer program of [9]. The $N = 1 \to N = 2$ non-critical embedding will be described after the others because it has some fundamentally different features. We will conclude with some comments on the significance of our results.

The $N \to (N + 1)$ critical embedding for $N \geq 2$ is [4, 10]

$$T_{\text{crit}} = T_{m}^{\text{crit}} + T_{g}, \quad G_{\text{crit}} = B + j,$$  \hspace{1cm} (2)

where all objects are $N$-extended superfields which depend on $z$ and on $N$ grassmann parameters, $T_{\text{crit}}$ and $G_{\text{crit}}$ are the components of the critical $(N + 1)$-extended stress tensor, $T_{m}^{\text{crit}}$ is a critical $N$-extended stress-tensor, $B$ and $C$ are the twisted $N$-extended ghosts, $T_{g}$ is the $N$-extended stress tensor for the twisted ghosts, and $j$ is the integrand of the $N$-extended BRST charge. For the $N = 0 \to N = 1$ critical embedding, there exist total derivative correction terms to $T_{\text{crit}}$ and $G_{\text{crit}}$, however these correction terms do not affect the construction described below. The $N = 1 \to N = 2$ critical embedding described in refs. [2, 3] is fundamentally different from the other cases and will be discussed separately.

To construct a non-critical $N \to (N + 1)$ embedding from the critical embedding of eq. (2), one first introduces a new $(N + 1)$-extended stress tensor, $\hat{T}$ and $\hat{G}$, where $\hat{T}$ and $\hat{G}$ describe a $c = x$ representation which is unrelated to $T_{m}^{\text{crit}}$ and $T_{g}$. It is clear that if $T_{\text{crit}}$ and $G_{\text{crit}}$ describe an $(N + 1)$-extended stress tensor with $c = c_{\text{crit}}$, then $T' = T_{\text{crit}} + \hat{T}$ and $G' = G_{\text{crit}} + \hat{G}$ describe an $(N + 1)$-extended stress tensor with $c = c_{\text{crit}} + x$.

The next step is to find a similarity transformation that removes all explicit dependence of $\hat{G}$ from the non-critical stress tensor described by $T'$ and $G'$. This can be done inductively by first defining

$$T'' = e^{-\int C\hat{G}} T' e^{\int C\hat{G}}, \quad G'' = e^{-\int C\hat{G}} G' e^{\int C\hat{G}}$$  \hspace{1cm} (3)

where $\int$ signifies a super-integration over $z$ and over the $N$ grassmann parameters.

It is easy to use eq. (2) to check that $T'' = T'$ and $G'' = G' - \hat{G} + CT + ...$, where $\ldots$ signifies terms with at least ghost-number 2 (the ghost number operator is $\int CB$). Therefore, after the similarity transformation,

$$T'' = (T_{m}^{\text{crit}} + \hat{T}) + T_{g}, \quad G'' = B + (j + CT) + ...$$  \hspace{1cm} (4)

Let $Y$ be all terms in $\ldots$ with ghost-number 2 containing either $\hat{T}$ or $\hat{G}$. Since the OPE of $G''$ with $G''$ has only ghost-number zero terms ($T''$ has ghost-number zero), the symmetrized OPE of $B$ with $Y$ must be non-singular. Therefore,

$$[\int CY , B] = - [B , \int C] Y - [\int CB , Y] = -3Y.$$  \hspace{1cm} (5)

So after performing the similarity transformation,

$$T''' = e^{\frac{1}{3} \int CY} T'' e^{-\frac{1}{3} \int CY} = T'', \quad G''' = e^{\frac{1}{3} \int CY} G'' e^{\frac{1}{3} \int CY} = G'' - Y + ...,$$  \hspace{1cm} (6)
all terms containing \( \hat{G} \) have at least ghost-number 3. By repeating this inductive procedure, one can construct the \((N+1)\)-extended stress tensor with central charge \( c = c_{\text{crit}} + x \):

\[
T = (T_{\text{crit}}^m + \hat{T}) + T_g, \quad G = B + (j + C\hat{T}) + ..., \tag{7}
\]

where the ... depends only on the \(N\)-extended ghosts. Since \( T_{\text{crit}}^m \) and \( \hat{T} \) only appear in the combination \( T_{\text{crit}}^m + \hat{T} \), this combination can be replaced by an arbitrary non-critical \(N\)-extended stress tensor \( T_m \) which is then embedded into the non-critical \((N+1)\)-extended stress tensor \( T \).

The explicit non-critical \( N \rightarrow (N+1) \) embeddings constructed using this inductive procedure are as follows:

\[
N = 0 \rightarrow N = 1: \quad T = T_m - \frac{3}{2} B\partial C - \frac{1}{2} \partial B C + \frac{1}{2} \partial^2 (C\partial C), \quad G = B + C T_m + B C \partial C - \frac{x}{24} C \partial C \partial^2 C + \frac{15 + x}{6} \partial^2 C, \tag{8}
\]

which satisfy the \( N = 1 \) OPE:

\[
T(z)T(w) \sim \frac{1}{4}(15 + x) \frac{1}{(z - w)^4} + \frac{2T}{(z - w)^2} + \frac{\partial T}{z - w},
\]

\[
T(z)G(w) \sim \frac{4G}{(z - w)^2} + \frac{\partial G}{z - w},
\]

\[
G(z)G(w) \sim \frac{4}{3}(15 + x) \frac{1}{(z - w)^8} + \frac{2T}{z - w}. \tag{9}
\]

\[
N = 2 \rightarrow N = 3: \quad T = T_m - \frac{1}{2} \partial (B C) + (D C)(D B) + (\bar{D}C)(DB),
\]

\[
G = B + C T_m + C (D C)(D B) + (\bar{D}C)(DB) - B(D\bar{C})(DC) - \left( \frac{x}{3} \right) [D, \bar{D}]C
\]

\[
+ \sum_{n=1}^{\infty} \frac{x}{n} \left[ -C(D\bar{C})(D C) + C(D\bar{C})(D \partial C) - 2(D\bar{C})(DC) [D, \bar{D}]C
\]

\[
- (2n - 1)C(D\bar{D}C)(\bar{D}DC) \right] [(DC)(DC)]^{n-1}, \tag{10}
\]

where \( D = \partial_0 - \frac{1}{2} \partial \bar{\psi}_2 \) and \( \bar{D} = \partial_{\bar{0}} - \frac{1}{2} \bar{\partial} \psi_2 \) are the usual \( N = 2 \) fermionic derivatives, all fields are \( N = 2 \) superfields, \( C(Z_1)B(Z_2) \sim \theta_{12}\bar{\theta}_{12}/z_{12}, \theta_{12} \equiv \theta_1 - \theta_2, z_{12} \equiv z_1 - z_2 + \frac{1}{2}(\theta_1 \bar{\theta}_2 + \bar{\theta}_1 \theta_2) \), and the \( N = 3 \) OPE is

\[
T(Z_1)T(Z_2) \sim \frac{4}{z_{12}^2} \theta_{12}\bar{\theta}_{12}T + \frac{\theta_{12}DT + \bar{\theta}_{12}\bar{D}T + \theta_{12}\bar{\theta}_{12}\partial T}{z_{12}},
\]

\[
T(Z_1)G(Z_2) \sim \frac{1}{2} \theta_{12}\bar{\theta}_{12}G + \frac{\theta_{12}DG + \bar{\theta}_{12}\bar{D}G + \theta_{12}\bar{\theta}_{12}\partial G}{z_{12}},
\]

\[
G(Z_1)G(Z_2) \sim \frac{4}{z_{12}^2} + \frac{\theta_{12}\bar{\theta}_{12}}{2}, \tag{11}
\]

\[
N = 3 \rightarrow N = 4:
\]

\[
T = T_m - \frac{1}{2} C\partial B + \frac{1}{2} (D_1C)(D_1B),
\]

3
\[ G = B + CT_m - \frac{1}{4} (D(C))^2 B + \frac{1}{2} C(D(C))(D(B)) + \frac{1}{2} C\partial CB - \frac{x}{36} \int \epsilon^{ijk} D_i D_j D_k C \]

\[ - \sum_{n=1}^{\infty} \frac{x \epsilon^{ijk}}{36(2n+1)^{4n}} \left( \int \partial D_i D_j D_k \left[ C((D(C))^2)^n \right] - 3C(D_i D_j C)(D_k C)((D(C))^2)^{(n-1)} \right), \]  

(12)

where the sum over \( i, j, k, l \) is understood, \( \int z \) stands for an ordinary integration up to the point \( z \), \( D_i = \partial_\theta + \theta \partial_\zeta \) are the usual N=3 fermionic derivatives, \( C(Z_1)B(Z_2) \sim \theta_1^{j_1} \theta_1^{j_2} \theta_1^{j_3} / z_1 \), and the \( N=4 \) OPE is

\[ T(Z_1)T(Z_2) \sim \frac{-1/4 x + 1/4 \epsilon_{ijk} \theta_{12} \theta_{12} D_k T + \theta_{12} \theta_{12} \theta_{12} \theta_{12} \partial T}{z_1} + \frac{1}{2} \frac{\theta_{12} \theta_{12} \theta_{12} T}{z_1^2}, \]

\[ T(Z_1)G(Z_2) \sim \frac{1}{2} \epsilon_{ijk} \theta_{12} \theta_{12} D_k G + \theta_{12} \theta_{12} \theta_{12} \partial G}{z_1}, \]

\[ G(Z_1)G(Z_2) \sim \frac{x}{3} \ln z_1 + \frac{\theta_{1} \theta_{1} \theta_{1} \theta_{1}}{z_1^2} 2 T. \]  

(13)

Note that the \( \int z \) in \( G \) causes the lowest component of the non-critical \( N=4 \) stress-tensor to be non-local, however this is expected since the OPE of the lowest component with itself goes like \( \log(z_1 - z_2) \).

The critical \( N=1 \rightarrow N=2 \) embedding described in ref. [3] has the form

\[ T_{\text{crit}} = T_{\text{crit}}^m + T_g, \quad G_{\text{crit}} = b, \quad G_{\text{crit}}^+ = j, \quad J_{\text{crit}} = cb - \xi \eta, \]  

(14)

where \( T_{\text{crit}}^m \) is a \( c = 15 \) stress-tensor, \( b \) and \( c \) are the fermionic Virasoro ghosts, \( \beta = \partial \xi e^{-\phi} \) and \( \gamma = \eta e^\phi \) are the bosonic super-Virasoro ghosts, \( T_g \) is the \( c = -9 \) stress-tensor for the twisted ghosts, and \( j \) is the \( N=1 \) BRST current (it includes total derivative correction terms). Note that this embedding requires manifest \( N=1 \) supersymmetry to be broken and also requires bosonization of the super-Virasoro ghosts.

A non-critical \( N=1 \rightarrow N=2 \) embedding can be obtained using a similar procedure as before. One first introduces a new \( N=2 \) stress tensor, \( (\hat{T}, \hat{G}^-, \hat{G}^+, \hat{J}) \), which has central charge \( c = x \) and which is unrelated to \( T_{\text{crit}}^m \) and \( T_g \). Adding this \( N=2 \) stress-tensor to the critical stress tensor of eq. (12) produces an \( N=2 \) stress-tensor, \( (T = T_{\text{crit}} + \hat{T}, G^- = G_{\text{crit}} + \hat{G}^-, G^+ = G_{\text{crit}}^+ + \hat{G}^+, J = J_{\text{crit}} + \hat{J}) \), with \( c = 6 + x \). One then has to find a similarity transformation that removes all explicit dependence on \( \hat{G}^+ - \hat{G}^- \) and \( \hat{J} \).

The resulting non-critical \( N=1 \rightarrow N=2 \) embedding is:

\[ T = T_m - \frac{3}{2} b \partial c - \frac{1}{2} \partial cb - \frac{3}{2} \eta \partial \xi - \frac{1}{2} \partial \eta \xi - \frac{1}{2} (\partial \phi)^2 - \partial^2 \phi, \]

\[ G^+ = \eta e^\phi G_m + cT_m - \frac{1}{2} c(\partial \phi)^2 - \frac{12 + x}{12} c \partial^2 \phi - b \eta \partial \eta e^{2\phi} + \frac{6 + x}{6} \partial c \xi \eta \]

\[ + \frac{24 + x}{12} c \partial \xi \eta + \frac{12 + x}{12} c \partial \eta \xi + c \partial cb + \frac{6 + x}{6} \partial^2 c - \frac{x}{6} \partial c \partial \phi, \]

\[ G^- = b, \]

\[ J = cb - \frac{6 + x}{6} \xi \eta + \frac{x}{6} \partial \phi, \]  

(15)
where the $N = 2$ OPE is

$$
T(z)T(w) \sim \frac{\frac{1}{2}(6 + x)}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}, \\
T(z)G^\pm(w) \sim \frac{\frac{3}{2}G^\pm(w)}{(z-w)^2} + \frac{\partial G^\pm(w)}{z-w}, \\
T(z)J(w) \sim \frac{J(w)}{(z-w)^2} + \frac{\partial J(w)}{z-w}, \\
G^+(z)G^-(w) \sim \frac{\frac{1}{3}(6 + x)}{(z-w)^3} + \frac{J(w)}{(z-w)^2} + \frac{T(w) + \frac{1}{2}\partial J(w)}{z-w}, \\
J(z)G^\pm(w) \sim \pm G^\pm(w) + \frac{1}{2}(6 + x), \\
J(z)J(w) \sim \frac{\frac{1}{3}(6 + x)}{(z-w)^2}. 
$$

(16)

It is natural to ask if there also exists an $N = 1 \rightarrow N = 2$ embedding which preserves manifest $N = 1$ supersymmetry and does not require bosonization. In fact, we have found such an embedding using the computer and inspired guesswork, and have explicitly checked that the $N = 2$ OPE’s are satisfied up to terms with ghost-number greater than 11. This second type of $N = 1 \rightarrow N = 2$ embedding can be written in $N = 1$ superfield notation as:

$$
T = T_m + \frac{1}{2}(DB)(DC) - B\partial C - \frac{1}{2}\partial BC + \sum_{n=1}^{\infty} \frac{1}{4^n} \partial \left[ C(D\partial C)(DC)^{2n-2} \right], \\
G = B + CT_m + \frac{1}{2}B\partial CC - \frac{1}{4}(DC)^2B + \frac{1}{2}DCCDB - \frac{6 + x}{6}D\partial C \\
+ \sum_{n=1}^{\infty} \frac{1}{4^n} \left[ (1 + (n-1)\frac{6 + x}{3})C\partial C(D\partial C)(DC)^{2n-2} \\
+ \frac{6 + x}{6}C\partial^2 C(DC)^{2n-1} - \frac{6 + x}{6}D\partial C(DC)^{2n} \right], 
$$

(17)

where $D = \partial_0 + \theta \partial_z$ is the $N = 1$ fermionic derivative and the $N = 2$ OPE is

$$
T(Z_1)T(Z_2) \sim \frac{\frac{1}{2}(6 + x)}{z_{12}^2} + \frac{3\theta_{12} T}{z_{12}^2} + \frac{\frac{1}{2}DT + \theta_{12}\partial T}{z_{12}}, \\
T(Z_1)G(Z_2) \sim \frac{\theta_{12} G}{z_{12}^2} + \frac{\frac{1}{2}DG + \theta_{12}\partial G}{z_{12}}, \\
G(Z_1)G(Z_2) \sim \frac{\frac{1}{2}(6 + x)}{z_{12}^2} + \frac{\theta_{12}2T}{z_{12}}. 
$$

(18)

Note that the embedding requires an infinite number of terms even when $c = c_{crit} = 6$ (i.e. $x = 0$).

Although the expressions for $G$ in eqs. (10), (12), and (17), contain terms of arbitrarily high ghost number, the structure of these terms is fixed by the requirement that they have the correct OPE with $T$. It is easy to show that at each ghost-number, this completely determines the terms up to an overall coefficient. The overall coefficient at each ghost number can be determined by analyzing the OPE of $G$ with $G$. 

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Because these embeddings are for non-critical string theories, it is not straightforward to compare scattering amplitudes using the $N$-extended and $(N+1)$-extended prescriptions. However there is one special case where the non-critical embedding does allow a comparison of scattering amplitudes. If one starts with a $(N=2, c=9)$ matter sector and twists the $N=2$ stress tensor in the usual way, $N=0$ topological string amplitudes can be calculated by pretending it is a bosonic string theory with the $b$ ghosts replaced by the spin 2 fermionic generator. Similarly, if one starts with a $(N=3, c=3)$ matter sector and twists with respect to one of the $SO(3)$ generators, $N=1$ topological amplitudes can be calculated by pretending it is an $N=1$ superstring with the $b$ and $\beta$ ghosts replaced by spin 2 and spin 3/2 generators [6]. So given a $(N=2, c=9)$ matter sector, one can either calculate $N=0$ topological amplitudes, or use the non-critical $N=2 \rightarrow N=3$ embedding to construct a $(N=3, c=3)$ matter sector and calculate $N=1$ topological amplitudes.

It was conjectured by Cumrun Vafa that these two scattering amplitudes coincide, and it is straightforward to use the properties of the non-critical embedding to prove his conjecture correct. The proof of equivalence for $N=0$ and $N=1$ topological amplitudes is very similar to the proof of ref. [3] for $N=0$ and $N=1$ ordinary amplitudes. As will be discussed in more detail in ref. [8], one needs to insert picture-changing operators in the $N=1$ prescription (which for topological strings are the products of spin 3/2 fermionic generators and the delta-functions of spin 3/2 bosonic generators), as well as beltrami differentials sewn with the spin 2 fermionic generators. If the $(N=3, c=3)$ matter sector comes from the non-critical $N=2 \rightarrow N=3$ embedding described in eq. (10), these spin 3/2 generators depend linearly on the twisted ghosts which were added to the original $(N=2, c=9)$ matter sector. It is easy to check that these picture-changing operator insertions absorb the zero modes of the twisted ghosts and that the non-zero modes of the twisted ghosts cancel each other out. So after integrating over the twisted ghosts, only the spin 2 fermionic generators remain in the functional integral, and the $N=1$ topological prescription reduces to the original $N=0$ topological prescription.

In this paper, we have shown that the embeddings of critical superstrings found in earlier works can be generalized to embeddings of non-critical superstrings. Because our constructions of these new embeddings are rather complicated, it is natural to ask if there is an underlying principle that guarantees their existence. Certainly at the classical level, it is always possible to embed a system with less symmetry into a system with more symmetry by simply adding “artificial” gauge degrees of freedom. However at the quantum level, things are not so simple.

For example, it appears that without the presence of a $U(1)$ current, it is only possible to embed the $N$-extended string into an $(N+1)$-extended string if the difference of the central charges is equal to the difference of the critical central charges. Although this allows the $N=0$ topological string to be embedded into the $N=1$ topological string, a similar embedding is not possible for the $N=1$
topological string into the $N = 2$ topological string. The reason is that an $N = 2$ topological string comes from twisting an $N = 4$ superstring with vanishing central charge (the central charge is three times the anomaly of the ghost-number current, which is zero for the $N = 2$ string). However the $N = 3 \rightarrow N = 4$ embedding of eq. (12) maps the $(N = 3, c = 3)$ superstring into the $(N = 4, c = 3)$ superstring, rather than the desired $(N = 4, c = 0)$ superstring. It would be interesting to learn if there is another $N = 3 \rightarrow N = 4$ non-critical embedding which embeds the $N = 1$ topological string into the $N = 2$ topological string, or if there is a fundamental obstruction to such an embedding.

Another interesting question is why the $N = 1 \rightarrow N = 2$ embedding looks so different from all other $N \rightarrow N + 1$ embeddings. As was shown in eqs. (13) and (17), it is only possible to embed $N = 1$ into $N = 2$ using a finite number of terms if one bosonizes the twisted $(\beta, \gamma)$ ghosts. Without such a bosonization, even the critical embedding requires an infinite number of terms. The $N = 1 \rightarrow N = 2$ embedding is of special significance because it allows the Ramond-Neveu-Schwarz description of the ten-dimensional superstring to be related to the manifestly spacetime-supersymmetric Green-Schwarz description [2].

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