Networked Control Systems Secured by Quantum Key Distribution

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Abstract—Cyber-security has become vital for modern networked control systems (NCS). In this paper, we propose that the emerging technology of quantum key distribution (QKD) can be applied to enhance the privacy and security of NCS up to an unbreakable level. QKD can continuously distribute random secret keys with much higher privacy between communication parties, and thus enable the one-time pad encryption that cannot be truly applied in classical networks. We show that the resulting overall security of NCS can be essentially improved, and present a composable definition of security based on the analysis of the key generation and management processes. Moreover, because the security is mainly determined by quantum keys rather than the complexity of encryption algorithms, the control performance can be improved as well by reducing the time delay using simpler algorithms. These advantages are demonstrated by the example of a remotely controlled servo system, showing that the introduction of QKD to NCS can simultaneously improve the security and performance by using the simplest encryption algorithm XOR. Furthermore, we propose a novel Kalman-filter embedded communication protocol that can more efficiently use the raw keys generated by QKD.

Index Terms—cyber-security; networked control system; QKD; Kalman Filter; one-time pad

I. INTRODUCTION

C YBER PHYSICAL SYSTEMS (CPS) are ubiquitous in modern industries [1]. As a typical application, many control actions are performed over CPS where the controller, the sensor and the plant are connected through high speed Ethernet or field bus. The resulting networked control systems can be flexibly designed based on time-driven or event-driven feedback, and the communication data, if necessary, can be stored and processed in the cloud [2], [3]. However, accompanied with the benefit is the high risk of cyber-security, because data protection is fragile or even absent in many traditional industrial control networks [4]. Data transmission in such networks can be easily wiretapped or adversely changed by cyber attacks whose threat can be much severer than physical attacks [5].

The cyber-security of NCS has drawn intense attention in the literature [6]. According to the manner of attacks, cyber-attacks can be classified into denial-of-service (DoS) attack, replay attack, and deception attack. Various attack detection methods have been proposed [4], such as Bayesian detection with binary hypothesis by testing with prior probabilities [7] and weighted least square approaches by comparing the data of observer with a predetermined threshold [8]. From a control theoretical point of view, the detection of cyber-attacks can be modeled and analyzed in terms of stability or detectability problems [9] and filtering methods can be applied to improve the performance of NCS [10].

Comparing with control theoretic cyber-protection methods, strong data encryption is more straightforward and effective. Although data encryption is often restricted by limited (or sometimes unavailable) computation and communication resources, it is inevitable in critical infrastructures such as nuclear power plants where security is of the highest priority. Generally speaking, any encryption method involves an encryption algorithm for processing data and keys that are held by the users. The strength of data encryption, symmetric or asymmetric, is determined by the security of keys. However, because the keys cannot be frequently updated under most circumstances, almost all encryption methods used so far have to be strengthened by high algorithmic complexity (e.g., AES, DES, IDEA, RC2, RSA) [11]. Such algorithms either use long keys to form large-size cipher blocks, which increases the burden of communication, or use short keys to form small-size cipher blocks, which releases the communication burden but increases the complexity of computation [12], [13].

One can certainly develop new encryption algorithms with higher complexity, but they are still under risk with more powerful computing technologies, not to mention the increased communication and computation burden. Especially, many currently used encryption algorithms (e.g., RSA) can be easily cracked by a quantum computer that is expected to be realizable in foreseeable future [14]. Under this circumstance, one may have to resort to one-time pad (OTP, i.e., each key is used only once) because of its provable absolute security that does not rely on the mathematical complexity of decryption algorithms [15], [16]. However, OTP is usually impossible over classical networks because the distribution of the unlimited amount of random keys between communication parties poses an equally difficult problem as the secure data transmission.

Nevertheless, the obstacle that hinders the application of OTP can be overcome by the emerging technology of quantum key distribution (QKD) [17]. In the most well-known BB84 protocol, random keys can be continuously generated and shared between communication parties using single photons (i.e., a so-called quantum channel). In terms of Shannon’s [15] definition on perfect secrecy, it can be proved that BB84 is absolutely secure [18]–[21]. Moreover, a remarkable advantage

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of QKD is that malicious attacks on the quantum channel can be detected because of quantum no-cloning property [22].

After two decades’ intense research and development, QKD system has become commercially available. Demonstrated applications have appeared in financial and government fields [23] in which security is highly demanded. To the authors’ knowledge, no applications have been reported in networked control of cyber-physical systems where security is also critical. Despite of the potential high cost that can be gradually reduced in the future, QKD can bring at least three benefits when being applied to CPS:

1) The privacy security of data transmission can be essentially enhanced to an unbreakable level that is impossible with any classical communication system.
2) Cyber attacks can be more easily detected based on OTP that is enabled by QKD, because any adverse operation on the cipher texts will cause uncontrollable changes that can be discovered when being decrypted with synchronized encryption keys.
3) Guaranteed by strongly secure keys, it is in principle unnecessary to use complex encryption algorithms to secure the data transmission. Without sacrificing the security, simpler algorithms can be used to reduce the burden of computation and communication, and hence improve the control performance.

This paper will demonstrate the above benefits based on our proposed integrated system structure for the QKD-based NCS. To evaluate the influence of QKD to security, network throughput and NCS control performance, we present measures for quantifying the improved trade-off, then the control performance can be greatly improved without sacrificing security. In addition to the system design and performance analysis, we also develop a novel encryption algorithm that pertains to the data transmission in NCS, which saves the amount of secret keys and further improves the security level.

The remainder of this paper is as follows. In Section II we design a practical NCS based on network communication and QKD. In Section III we introduce the basic confidential security issues, study the factors that influence security and give a definition of security for networked control systems. In Section IV we analyze the induced time delay caused by network, which will further influence the performance of NCS. In Section V we study the performance of NCS using different encryption algorithms and the tradeoff between security and performance. In Section VI we propose a new filtering based security strategy directly encrypting with the raw quantum keys that contain error bits, then the quantum keys can be used more efficiently. Finally, in Section VII conclusions are made.

II. QKD SECURED NETWORKED CONTROL SYSTEMS

In this section, we will introduce the control and communication structure of a networked control system equipped with a QKD device. The key generation methods and relevant encryption algorithms will be introduced as well.

A. Structure of Networked Control Systems

Consider a point-to-point NCS in which the plant is remotely controlled by a controller. For illustration, we use a mechanical servo as the example of plant. Any other dynamical control plants can be applied to the system introduced in the following. As illustrated in Fig.1 the mechanical servo, Bob, is remotely controlled by Alice, a proportional-integral (PI) controller associated with another IP. Alice receives and decrypts the position data of the servo that is encrypted by Bob and sends back encrypted real-time control commands to Bob. The data transmission between Alice and Bob is based on communication protocol such as TCP/IP, UDP, etc.

![Fig. 1. Schematic diagram of a QKD based networked control system.](image-url)

To secure the data transmission, a QKD system is integrated into the network. It operates independently with the NCS, and supplies secret keys for the above data encryption and decryption of position data and control commands. The terminologies involved in this process are summarized in Table I.

| TABLE I | TABLE OF TERMINOLOGIES |
|---------|------------------------|
| $K_i$   | the i-th key used for encryption and decryption |
| $T_{K_i}$ | the encryption matrix of the key $K_i$ |
| $T_{K_i}^{-1}$ | the decryption matrix of the key $K_i$ |
| $u(t)$ | the plain text of the control command sent from Alice |
| $u'(t)$ | the cipher text of the control command received by Bob |
| $y(t)$ | the plain text of the position sent from Bob |
| $y'(t)$ | the cipher text of the position received by Alice |
| $\tau_1$ | the one-way delay from Bob to Alice |
| $\tau_2$ | the one-way delay from Alice to Bob |
| $r(t)$ | the reference signal |
| $e(t)$ | the error signal |

In detail, Alice generates the control signal according to the error signal $e(t)$ as

$$e(t) = r(t) - y(t - \tau_1),$$

(1)
where \( r(t) \) is the reference signal and \( y(t - \tau) \) is the position signal of the servo sent from Bob after a time delay \( \tau \). Assume that Alice adopts a PI controller, the generated control command is

\[
u(t) = K_P e(t) + K_I \int_0^t e(t)dt,
\]

where \( K_P \) and \( K_I \) are the parameters of the controller.

Then, Alice encrypts the control command \( u(t) \) with the key \( K_i \), which produces the cipher text

\[
u'(t) = T_{K_i} u(t),
\]

where \( T_{K_i} \) represents the transformation posed by the encryption algorithm using the key \( K_i \). The cipher text \( u'(t) \) is then sent to Bob, which, after another time delay \( \tau_2 \), arrives and is decrypted using the key \( K_i' \) as follows

\[
u(t + \tau_2) = T_{K_i'}^{-1} u'(t).
\]

Note that in symmetric encryption schemes, the keys \( K_i \) and \( K_i' \) are supposed to be identical. However, errors can occur during the distribution of keys between Alice and Bob (see Sec. II-B), i.e.,

\[
K_i' = K_i + \Delta_i,
\]

where \( \Delta_i \) represents the difference between keys.

In practical networked control systems, the real-time performance is largely determined by the communication latencies of closed control loop. As is shown in Fig. 2, the time delay includes software, hardware and network delays. So that the servo can only receive a command made according to its previous status which will deteriorate the performance.

![Fig. 2. The time delays in a typical sampling control period of NCS.](image)

The closed-loop control period of the NCS is defined as the round-trip time delay, which is usually random. Consider the control process in the \( i \)th control period, we summarize the following generalized state-space model for the encrypted dynamics of the networked control system:

\[
\begin{align*}
x_i &= Ax_{i-1} + Bu_{i-1}, \\
y_i &= Cx_i, \\
u_i &= T_{k_i}^{-1} + T_{k_i} \bar{u}_i, \\
y_i &= T_{k_i'}^{-1} + T_{k_i'} \bar{y}_i, \\
\bar{u}_i &= F \cdot (r_i - \bar{y}_i),
\end{align*}
\]

where \( x_i \) is the state vector, \( \bar{u}_i \) and \( \bar{y}_i \) are the original control and output of the plant, \( u_i \) and \( y_i \) are transmitted control and output after encryption and decryption, and \( r_i \) is the reference signal. The first three equations represent the standard linear dynamics and measurement of the plant, in which the control command \( u_{i-1} \) is received by the plant after encryption with key \( k_i \) and decryption with key \( k_i + \Delta_i \), where \( \Delta_i \) represents potential errors in the key. The fourth equation describes the process that the measurement data \( y_i \) is encrypted (using key \( k_i' \)) by Bob and decrypted by Alice (using key \( k_i' + \Delta_i' \) with possible error \( \Delta_i' \)). The received data is then processed by the feedback matrix \( F \) to produce the next control command.

### B. Generation of symmetric secret keys with QKD

The function of QKD is to safely and continuously generate symmetric keys between Alice and Bob. In this paper, we will use the BB84 protocol, which is the earliest and also the most broadly used protocol in practical systems.

The major difference of QKD from its classical counterpart is that the secrecy of generated keys is physically guaranteed by quantum mechanics. In detail, the protocol can be divided into the following parts.

1) **Key Sifting:** Alice prepares and sends a bit encoded by single photons that are randomly polarized along two sets of bases, each of which having two orthogonal directions. Bob receives a bit after measuring the received photon along one of the two bases he randomly chooses. According to quantum mechanics, the measurement outcome is deterministic if and only if it is performed along the right basis, otherwise the outcome is random. Therefore, Alice and Bob can tell each other through classical communication which basis they choose instead of the bits themselves, and keep those bits that are sent and measured under the same basis. These bits are thus symmetrically distributed between Alice and Bob, forming the set of raw keys for encryption and decryption.

2) **Error correction:** Because the raw keys may contain errors due to the imperfection of single photon detector or losses during the transmission, a cascade protocol is used to locate the wrong keys by comparing the check bit and halving the key strings. This process is continued until the exact positions of wrong bits are found, which will be corrected or discarded.

3) **Privacy amplification:** To avoid the information leakage in the classical communication for basis exchange and error correction, a privacy amplification protocol is applied using a Hash function, which guarantees the absolute security of the final key in an asymptotic manner. As a price, the capacity of keys will be further reduced during this process.

### C. Encryption Algorithms

The security of a cryptographic system is essentially determined by that of secret keys. Ideally, the data transmission is absolutely secure if every key is used only once (i.e., one-time pad). However, because it is usually impossible to distribute unlimited number of keys over classical communication networks, one has to enhance the security by strong encryption algorithms.
We will briefly introduce some widely used symmetric encryption algorithms. These algorithms all belong to the category of block cipher, which divides the plain text into identical-size blocks [24]. These blocks are processed to generate the cipher text via a series of basic operations (e.g., ByteSub, ShiftRow, MixColumn and RoundKeyAddition, see details in [25]) using a secret key.

1) **XOR:** the simplest method for encryption, which is performed by bit-by-bit XOR operation on the plain text and keys, namely the RoundKeyAddition.

2) **DES (Data Encryption Standard):** the algorithm based on rounds of Feistel operations. The standard DES consists of 16 Feistel units, each of which halves the i-th round text into $L_i$ and $R_i$, and updates them as:

$$L_{i+1} = R_i, \quad R_{i+1} = L_i \oplus F(R_i, K_{i+1}),$$

(7)

where $K_i$ is the sub-key used in this round and $F(R_i, K_i)$ is the result of last Feistel round. The sub-keys used in each round is generated by the ByteSub operation from the original key, and a simple MixColumn is used during the exchange of left and right halves according to (7). DES is actually an extension of XOR, but the security is much higher owing to the complicated bit position operation.

3) **AES (Advanced Encryption Standard):** the algorithm based on rounds of operations that uses the Substitution-Permutation Network (SPN) to generate round keys. The text is processed by cycling bit operations in each round. AES is much more complex than DES because of the increased key length as well as more complex round function (See Tab. [IV]). Presently, AES(128) with 128-bit key and 10 SPN rounds, AES(192) with 192-bit key and 12 SPN rounds, and AES(256) with 256-bit key and 14 SPN rounds, are most popular.

### D. Advantages of QKD-Based NCS

Before expanding our studies, we make some remarks on how QKD may improve the security against cyber attacks by enabling the OTP.

1) **DoS attack:** in such attack, Eve blocks the transmission of cipher texts by maliciously overloading the communication network. Although none of plain texts are cracked, the communication can be interrupted between the actuator and controller and thus deteriorate the performance. Note that the DoS or the dropout can be detected with TCP/IP protocol by handshaking, but they may not be efficiently detectable with other protocols such as UDP.

When one-time pad and QKD are applied, it is easy to detect the occurrence of DoS no matter which network communication protocol is used. This is because every data block Bob receives must be decrypted with the same key for encryption. Let $E$ be the set of admissible plain texts (e.g., the range of control commands or sampled measurement data). If some blocks are missing during the transmission, and Bob uses the key that is supposed to be used on the missing block to decrypt some other blocks he receives later, he cannot get a meaningful message because the key does not match, namely $T^{-1}_{K_i}(T_{K_{i+j}}u_{i+j}) \notin E$, where $K_i$ and $K_{i+j}$ are the desynchronized keys used for decryption and encryption due to DoS attack. In this way, DoS can be detected.

2) **Replay attack:** in such attack, Eve replaces the transmitted messages from Alice to Bob with some previously recorded ones, in order to deceive Bob by the old messages that are still meaningful to him. When the messages are encrypted, the deception can be successful if the key is not changed, which is often the case in classical networks. However, it will not work when every key is used only once (i.e., one-time pad) because the replayed cipher text will not be decrypted by the key that is only shared between Alice and Bob. Thus, when Eve performs the replay attack, Bob will not obtain a meaningful plain text as $T^{-1}_{K_i}(T_{K_i}u_i) \notin E$, where $K_i$ and $K_{i+j}$ are the desynchronized keys used for encryption and decryption due to the replay attack, and this can be used to detect the occurrence of the attack.

3) **Deception attack:** in such attack, Eve injects malicious data in the communication to deteriorate the performance of NCS, e.g., by changing several bits of the cipher text. Similar to the case of replay attack, the injected data will not be able to convey a meaningful and spurious message if the cipher text is changed before decrypted by Bob as $T^{-1}_{K_i}(T_{K_i}u_i + \Delta) \notin E$, where $\Delta$ represents Eve's modification on the cipher text. Therefore, the deception attack can also be easily detected from the resulting messy code when OTP is enabled by QKD. One can also determine whether the attack is a replay attack or a deception attack by checking the repetition of the cipher texts.

### III. Security Measure of QKD-based Networked Control Systems

For the purpose of control systems analysis and design, a quantitative evaluation of the overall cyber-security is necessary. Generally speaking, the security of networked control systems is determined by the security of keys (i.e., how the keys are generated and distributed), the security of key management (i.e., how frequently every key is reused) and the security of the encryption algorithm (i.e., how hard the algorithm can be cracked).

Regarding these factors, we firstly define the security $S_A$ of an encryption algorithm. Suppose that the length of the key is $N$ and $R$ is number of rounds performed in the algorithm. We define the security strength of the encryption algorithm as follows (see the derivation in Appendix A):

$$S_A = \eta R \log_2 N,$$

(8)

where $\eta$ represents the complexity of each round operation (e.g., the Feistel of DES or the SPN of AES). In Table [I] the security strength of several encryption algorithms are calculated according to the measure (8). Because AES(256) is recognized as the currently strongest symmetric encryption algorithm, we normalize the measure by that of AES(256).

Based on $S_A$, the overall security of the QKD-based NCS is quantified as follows:

$$S = 1 - \varepsilon r (1 - S_A),$$

(9)

where $r$ is the average times ($r = 1$ for OTP and otherwise $r > 1$) for the keys to be reused before being discarded, and $\varepsilon$ represents the insecurity of the keys generated by QKD.
after privacy amplification. The insecurity parameter \( \varepsilon \) can be arbitrarily small under properly designed QKD protocol.

Equation (9) shows that the security can be arbitrarily strong when the keys are sufficiently safe, under which circumstance the strength of encryption algorithms is not as important as before.

### IV. Round-trip Delay of QKD-Based Networked Control

The real-time performance of practical networked control systems can be seriously deteriorated or even destabilized by the time delay of the closed loop control over the communication network. In this section, we analyze and quantify this effect, especially how it is affected by data encryption and decryption processes.

The end-to-end (one-way) delay can be decomposed into four types of delays: software delay for the generation of the control command, encryption-decryption delay, network delay for data transmission and hardware delay caused by the plant and other devices in the system. Among these elements, the encryption-decryption delay is dependent with the complexity of the adopted algorithm, the transmission delay varies according to the communication load as well as available bandwidth, and the other delays are relatively short and fixed during the control process.

We are mainly concerned with the time delay caused by data encryption and decryption, which is controllable by the choice of algorithms. Highly complicated encryption algorithms may increase the time for computation, but such difference can be minor under powerful computational hardware. The more important factor is the size of the cipher text blocks that directly influences the communication load of network. Therefore, the time delay caused by encryption-decryption is essentially determined by the length of the key that is proportional to the size of the cipher text blocks. For example, AES(256), one of the most secure algorithms, uses 256-bit keys.

In the QKD-based NCS, keys are important resources. To guarantee that there are always sufficient secure keys for one-time pad encryption, the consumption rate of keys by NCS must be lower than the generation rate of keys by QKD. Complex encryption algorithms usually generate long cipher texts using long keys. Consequently, more communication is needed for generating such keys in BB84, which induces larger time delays. This factor is important in the tradeoff between security and performance.

Let \( N \) be the length of cipher text and \( l_{EC} \) be the number of bits that must be exchanged for error correction. In the.Cascade protocol of BB84, \( l_{EC} \) varies with \( N \) and quantum bit error rate (QBER) \( e \). Practically, QBER is evaluated by checking some bits of raw keys at the beginning, and it asks the QBER must be less than 11% \([19]\). Then the keys are rearranged and divided into identical-size blocks so that there is only one error bit in each block. Let \( l_0 \) be the length of block, which is required to be no less than \([2]\). Alice and Bob then independently calculate the check bit of keys and compare the results with each other. During the halving process of error correction in Cascade, a total number of \( \log_2 l_0 \) bits are transferred for each single block \([16]\). While the process of privacy amplification will decrease the capacity of keys from \( n \) to \( m \), so the total number bits transferred for error correction is:

\[
l_{EC} = \left\lfloor \frac{nN}{ml_0} \right\rfloor \log_2 l_0. \tag{10}\]

So the total number of bits exchanged between Alice and Bob is:

\[
l = N + l_{EC} = N + \left\lfloor \frac{nN}{ml_0} \right\rfloor \log_2 l_0. \tag{11}\]

Take QBER \( e = 10\% \) as an example, \( l_0 \) is \( \lfloor \frac{11}{10} \rfloor = 10 \). Assume \( \frac{n}{m} = 0.2 \), it can be calculated from (11) that \( l = 21.29 \) for XOR, \( l = 170.30 \) for DES, and \( l = 681.21 \) for AES(256). It can be seen that \( l \) increases when using more complex encryption algorithms.

#### TABLE II

**Security Measure of Various Encryption Algorithms**

| Algorithm   | \( \eta \) | \( R \) | \( \log_2 N \) | \( S_A \) |
|-------------|------------|--------|---------------|---------|
| XOR         | 0.0625     | 1      | 3             | 0.0017  |
| n-Feistel   | 0.2083     | \( n \) | 6             | 0.0112n |
| DES         | 0.2083     | 16     | 6             | 0.1785  |
| AES(128)    | 1          | 10     | 7             | 0.6250  |
| AES(192)    | 1          | 12     | 7.58          | 0.8121  |
| AES(256)    | 1          | 14     | 8             | 1       |

![Fig. 3. The round-trip delay of NCS encrypted by XOR and AES.](image-url)

To demonstrate how the NCS time delays depend on the complexity of encryption algorithms, we build an experimental networked control servo system and test the XOR and AES algorithms. The round-trip delay is measured by checking the arrival time of two adjacent measurement signals \( y(t) \) at Alice’s part. As is shown shown in Fig[3] the experimental round-trip delay increase when the communication load of network starts to be heavier form the 35th control period and the time delay with more complex encryption algorithms (i.e., AES) tend to be longer. The difference is not visible when the communication is not busy at the beginning, but as the experiment goes, complex algorithms collect more data.
(i.e., cipher text) and consumed more keys. As a result the performance of NCS is degraded.

The round-trip delay of NCS during each control period is highly influenced by the complexity of encryption algorithms, which, according to [26], [27], can be calculated as follows:

\[ \tau = T_0 + \frac{2l}{C} + \Delta \tau \]

\[ = T_0 + 2C^{-1} \left( N + \left\lfloor \frac{nN}{m_l} \right\rfloor \log_2 l_0 \right) + \Delta \tau, \tag{12} \]

where \( T_0 \) is the total time delay without data encryption-decryption and \( l/C \) describes the queueing delay. In our networked control system, both the command and realtime position are transmitted through ethernet, then the queueing delay should be doubled. \( \Delta \tau \) is the small variable delay caused by hardware, which is much smaller than the delay caused by communication.

The parameters \( T_0 \) and \( C \) in (12) can be identified from the results of the above encrypted NCS experiments using XOR, DES and AES algorithms, which turns out to be: \( T_0 = 0.055s, C = 18000s^{-1}. \)

The following theoretical analysis on the tradeoff between the security and the performance will be based on this identified model.

V. TRADEOFF BETWEEN SECURITY AND PERFORMANCE

The security of QKD-based NCS is determined by the QKD, the algorithm complexity and the one-time pad. According to (9), the compound security can be high even with simple encryption algorithms, so that the length of cipher text can be reduced, which will greatly release the communication burden. As a consequence, the induced time delay caused by classical communication will be reduced as well. In this section, we show how the performance is preserved by simple encryption algorithms without sacrificing too much security.

A. Performance of NCS Encrypted with Different Algorithms

We evaluate the performance by following the mean square error as:

\[ P = \int_0^\infty e^2(t)dt. \tag{13} \]

Secured by QKD, we test the performance under simple and complex encryption algorithms, including XOR, DES and AES. Due to the time delay (12) induced by the data transmission, the performance with these algorithms are more or less deteriorated, as shown in Fig4. Both the overshoots and the resetting time increase, in which XOR has the best performance and AES(256) has the worst performance.

As illustrated above, the communication required for QKD increases the time delay and thereby influences the performance. While the time delay caused by encryption and decryption is usually fixed and negligible because the hardware computing power is usually sufficiently high, so the performance is mainly influenced by the induced time delay caused by communication.

B. The Tradeoff between Security and Performance

When QKD is used in the NCS, the overall security can be arbitrarily high according to (9) when \( \varepsilon \) is sufficiently small. The security parameter is almost independent with the algorithm complexity \( S_A \). Since the network time delay is shorter when using simple encryption algorithms, there is no need to use complex encryption algorithms that tend to worsen the control performance.

Take \( \varepsilon = 0.1 \) as an example, the security of different encryption algorithms can be enhanced and listed in Table III

| Algorithm | \( S_A \) | \( S \) | average delay(s) | \( P \) |
|-----------|--------|------|----------------|------|
| XOR       | 0.0017 | 0.9002 | 0.0645 | 0.4044 |
| 1-Feistel | 0.0112 | 0.9011 | 0.0731 | 0.4254 |
| 8-Feistel | 0.0896 | 0.9089 | 0.0731 | 0.4254 |
| DES       | 0.1792 | 0.9179 | 0.0731 | 0.4254 |
| AES(128)  | 0.6250 | 0.9625 | 0.0978 | 0.4969 |
| AES(192)  | 0.8121 | 0.9812 | 0.1118 | 0.5340 |
| AES(256)  | 1      | 1     | 0.1307 | 0.6300 |

It can be seen that when encrypted with quantum keys, the security is enhanced no matter which algorithm is used, and the improvement is greater especially for simple algorithms such as XOR and 1-Feistel. The resulting security of different algorithms is much closer to each other. For example, the security of XOR is only about 10% less than that of AES(256), implying that the algorithm complexity is less important. So simple algorithms such as XOR can be used in QKD based network to improve performance without sacrificing the privacy security.

VI. ENHANCED SECURITY OF NCS WITH RAW QUANTUM KEY DISTRIBUTION

As illustrated above, the control performance can be effectively improved by using simple encryption algorithms, while the NCS security level is maintained at a high level. In
practice, this requires sufficient supply of secret keys by the QKD system, which may not be realistic because most sifted raw keys are discarded during the post processing (including the error correction and privacy amplification) and the amount of final keys is very limited. The post processing can also decrease the security of the keys if not being well designed.

In this section, we will show that these discarded raw keys can be more efficiently used. This is because the errors of the transmitted data (usually numerical values) caused by the key errors can be taken as an equivalent noise. Since most closed-loop control processes can tolerate noises at a certain level, the raw keys may not have to be corrected if the equivalent noise is tolerable. In this way, the QKD protocol can be greatly simplified because the succeeding privacy amplification will be also unnecessary, as its function is to remedy the information leakage during error correction. This will bring at least three advantages: (1) the sifted raw keys, which can be fast generated, may be more efficiently used instead of being wasted; (2) the security of keys is the highest without information leakage in error correction; (3) more communication bandwidth can be saved without having to preform the post processing.

To implement the above idea, we need to make sure that the key error induced noise in the transmitted data is suppressible. From the point of view of cryptography, the avalanche effect of the encryption algorithm, namely the sensitivity of the decrypted plain text with the change of the cipher text, should be as weak as possible, otherwise a single one-bit error may cause uncontrollable large deviation. In this regard, XOR is the best choice, under which every one-bit error in the key only affects one bit in the message transmitted from Alice to Bob.

The avalanche effect can be also understood from the control model [6]. When the key error \( \Delta_i \) occurs, the avalanche effect is evaluated by the difference

\[
\nu_i = \tilde{y}_i - \bar{y}_i,
\]

between the original plain text \( \bar{y}_i \) and decrypted command \( \tilde{u}_i \) after encryption-decryption. It is desired that the encryption matrix \( T_{k_i} \) is chosen such that \( d_i \) is as insensitive as possible to the key error \( \Delta_i \), and here we choose the bit-by-bit XOR encryption. If the random key errors all occur to the lower digits of the plain text, it is reasonable to take the resulting deviation of \( \bar{u}_i \) as a white noise.

Similarly, the error in the key \( k_i' \) used for the encryption of feedback position of Bob will also lead to an equivalent noise as:

\[
\nu_i = y_i - \bar{y}_i.
\]

This leads to the following equivalent stochastic control model:

\[
\begin{align*}
x_i &= Ax_{i-1} + B(u_{i-1} + \omega_{i-1}) \\
y_i &= Cx_i + \nu_i,
\end{align*}
\]

where \( y_i \) is the measurement result that Alice receives through ethernet.

If the error occurs in the higher digits, the resulting deviation can be filtered out by additional detection and smoothing operations. Concretely, we can check the value of \( d_i = \|y_i - y_{i-1}\| \) between adjacent received signals \( y_i \) and \( y_{i-1} \). Once \( d_i \) exceeds some prescribed threshold value, say \( \delta \), a key error is deemed to occur to the high digits. We can then simply smooth it by making all high digits of \( y_i \) identical with those of \( y_{i-1} \), which is shown in Fig. 5.

Note that this strategy is very effective when the signal to be transmitted is slowly varying. For violently changing signals, one should still use the corrected final keys to encrypt the high digits.

![Fig. 5. Equivalent noise in the position data of the servo systems that is induced by the errors (with error rate \( \approx 10\% \)) in the raw keys. (a) the original noise; (b) the noise after removing errors in high digits.](image)

Now we only need to consider the error in the lower digits that is not detectable with the above method. Take them as white Gaussian noises, whose variance is proportional to the threshold value \( \delta \). In the beginning, we use the corrected final keys for encryption-decryption to guarantee that no errors occur to the high digits of the data. Afterwards, we encrypt and decrypt the data all with raw quantum keys, and the noises induced by key errors can be attenuated by the Kalman Filter [28]. Let \( \hat{\xi}_{i-1} \) be the a priori estimation of state vector at the end of the \( i \)-th control period. It can be calculated by

\[
\hat{\xi}_{i-1} = A\hat{\xi}_{i-1} + Bu_{i-1},
\]

based on the posteriori state estimation \( \hat{\xi}_{i-1} \) and its resulting control command \( u_{i-1} = F \cdot (r_{i-1} - C\hat{\xi}_{i-1}) \) at the previous period, following which the posteriori state estimation in the \( i \)-th control period is made as follows:

\[
\hat{\xi}_i = \hat{\xi}_{i-1} + K_i(y_i - C\hat{\xi}_{i-1}),
\]

where matrices \( P_{i-1}, P_i \) and \( K_i \) are calculated as follows:

\[
P_{i-1} = AP_{i-1}A^T + Q,
\]

\[
P_i = (I - K_iC)P_{i-1},
\]

\[
K_i = P_{i-1}C^T(\text{det}(CP_{i-1}C^T + R))^{-1},
\]

with \( Q \) being the covariance matrices of the process noise \( \omega_i \) and \( R \) being the covariance matrices of measurement noise \( \nu_i \).

In Fig. 6, we simulate the dynamical response of the NCS with and without the Kalman Filter. Compared with the
previous simulation using corrected keys, it can be seen that
the use of raw keys introduces additional noises to the output
signal, but the transient response is much faster because the
induced time delay gets shorter without post processing. The
introduction of the Kalman Filter can not only smooth out the
fluctuation but also suppress the overshoot, showing that both
good control performance and high security can be achieved
with raw keys.

VII. CONCLUSION

To conclude, we proposed a novel scheme for improving
the cyber-security of networked control systems by using the
one-time pad encryption that is enabled by the emerging
technology of quantum key distribution. We analyzed the
factors that affect the overall security of the NCS, and derived
a security measure for the evaluation and design of secure
networked control systems. Based on this measure, we studied
the tradeoff between the security and the control performance
using various encryption algorithms from the simplest XOR
and the most complicated AES, showing that QKD can greatly
improve the performance while maintaining the security at
a high level. We further propose a scheme for more efficiently
exploiting the raw keys that contain errors, and introduce the
Kalman Filter to attenuate the error-induced noises.

To our knowledge, this is the first study that integrated QKD
with NCS for security enhancement. The provably secure keys
generated by QKD enables the use of one-time pad without
having to use complex encryption algorithms. The control
framework can be applied to many systems, especially when
security is highly demanded. Although the current cost of
QKD is high, it can be decreased to be affordable under many
critical circumstances.

This work lays a fundamental framework for QKD-based
NCS. The extension from point-to-point control to multi-
agent control over a complex communication network is more
challenging. There are many interesting topics, such as the
modeling of the QKD network and evaluation of the overall
network security, as well as the following control problems
such as consensus. These topics are to be studies in future.

APPENDIX A

SECURITY MEASURE OF QKD BASED NCS

To evaluate the security strength of an encrypted NCS, we
firstly introduce the following cryptographic terminologies:

1) Plain text space $\mathcal{M}$, the set of all plain texts that
may be decrypted to be. In our system, it is a combination
of practical secure communication data and another subset of
possible decrypted data that only occurs under cyber attacks.

2) Cipher text space $\mathcal{C}$, the set of cipher texts after en-
cryption, which will be transmitted through the communication
network.

3) Key space $\mathcal{K}$, the set of secret keys, which are usually
binary strings.

A well defined plain text space will add the difficulty for
eavesdropper to crack the message. In our example system,
it contains the digitized values of PI command, the real-time
position of servo and other symbols, which will enlarge the
confusion effect of encryption.

The overall security is mainly determined by the security
of keys, the security of key management and the security of
encryption algorithms.

A. Security of Keys

The security of an encryption system can be defined by the
mutual information $I(\mathcal{M}, \mathcal{C})$ between the plain text space $\mathcal{M}$
and the cipher text space $\mathcal{C}$. Ideally, it should be zero, meaning
that the plain text space $\mathcal{M}$ is absolutely independent from the
cipher text space $\mathcal{C}$. So Eve is unable to get any meaningful
message from the cipher text. In practical system, the mutual
information between plain text and cipher text can not be
zero but the value can approach to zero after post processing
methods.

The security of QKD is due to the fact that the information
about keys that Eve can get is exponentially small. In [29],
Renner proposed the composable security of QKD determined
by the process of error correction and privacy amplification. So
the security discussion on QKD can be divided into following
two parts.

1) Information leakage of Cascade: Once the raw keys are
distributed by measuring the phase of photons and exchanging
the information about bases, there are still some error bits,
namely some bits of raw keys are different between Alice and
Bob because of the error of measurement. So it is necessary
to distill the raw keys and delete the error bits. When Cascade
is used in error correction, there is still some information
leakage.

2) Security of privacy amplification: Because the fact that
Eve can get some formation about sifted keys by tapping
the communication of error correction, the process of privacy
amplification will repair the leakage by decreasing the key
capacity.

We assume that Alice and Bob have shared a key string $W$
and the length of $W$ is $n$. Eve has some information about

Fig. 6. Performance of Raw QKD encrypted NCS with or without Kalman
Filter.
W which can be defined as V and the length of V is t. It is reasonable to assume that:

\[ 0 < I(K, V) < I(K, W), \]  

(22)

where the information that Eve can get about keys can be calculated by mutual information as \( I(K, V) \). Alice and Bob can randomly share a hash function, namely \( g : (0,1)^n \rightarrow (0,1)^m \). As proved in [18], the mutual information that Eve can get about keys after privacy amplification \( I(K, GV) \) is limited as follows:

\[ I(K, GV) \leq \frac{2^{-n+t+m}}{\ln 2}. \]  

(23)

Define \( s = n - t - m \), then

\[ I(K, GV) \leq \frac{2^{-s}}{\ln 2}, \]  

(24)

where \( s \) represents the reduction ratio of the capacity of keys during privacy amplification. As a result, the information that Eve can get from keys after privacy amplification is exponentially small.

### B. Security of Key Management

One-time pad is the only provably absolutely secure encryption method even under unlimited computation power. The security of one-time pad has two aspects. On the one hand, give a copy of cipher text, the eavesdropper gains no more information about the plain text even after trying every possible key, namely,

\[ P[M = m_0 | C = c] = P[M = m_0], \]  

(25)

which can be called information-theoretic security [16].

On the other hand, when one-time pad cannot be ensured due to the limited supply of keys, some keys may be reused, this circumstance can be described as:

\[ m_i \oplus k_i = c_i, \]  

(26)

where \( r \) is the times that a single key \( k_i \) is reused. The information that Eve can get from the cipher text on the plain text can then be valued by the conditional probability. Namely on knowing the cipher text is \( C = c_i \), the probability that plain text is \( M = m_{i+1} \) can be calculated according to the Bayes Rule as:

\[ P[M = m_{i+1} | C = c_i] = \frac{P[M = m_{i+1} \cap C = c_i]}{P[C = c_i]}. \]  

(27)

Additionally assume that the times a single key is reused is limited as \( r < 2^t \), other than the key can be regarded as unchangeable.

The derivation shows that when the keys are reused, the information that Eve can get from cipher text is increased, which degrades the security of practical systems.

The security of one-time pad is also influenced by the updated keys. While in our system, the randomness is well ensured by the physically randomly generated single photons.

### C. Security of Encryption Algorithms

To define the security of encryption algorithms, the parameters should be reasonable rather than measured from the cipher text, and they should be connected with the complexity of algorithms and the difficulty for brute force. Avalanche effect and linear cryptanalysis, which are measured according to the results of cipher text, can be easily affected by the choosing of plain text and keys. So we use the following parameters to define the security of encryption algorithms:

1) **Length of keys \( N \)**: The length of keys will influence the number of keys that should be used for brute force. For example, if the length of keys is \( N \), then there will be \( 2^N \) keys need to be tried. While this is the maximum time of testing for brute force attack. The security of encryption should be defined according to the most efficient method of cracking and considering usually concurrent computation is used in hardware, we use \( \log_2 N \) to define the security contributed by the length of keys and this definition is introduced more in detail in [30].

2) **Number of rounds \( R \)**: Most encryption algorithms have its own round function, for example DES is 16 rounds of Feistel, and the adding of rounds will increase the complexity of encryption.

3) **Complexity of round operation**: Round operation is the encrypting unit of a complex encryption algorithm, and it will encrypt the plain text by XOR with the keys, substituting bytes, shifting rows, mixing columns and so on. All the operations are reversible and will change the states of plain text.

The security of round operations is a combination of the security of subkeys and the security of calculation operations. According to key expanding algorithm, a subkey is decided by that of upper round and updating method, and the latter is public. So the security of subkey is equivalent to the security of original keys, namely the security of quantum key distribution, which has been defined before. Based on this, the security...
of round function is equivalent to the security of calculation operations.

According to Table IV, we can define the complexity of round function \( \eta \) by adding the security of ByteSub \( S_B \), the security of ShiftRow \( S_S \), the security of MixColumn \( S_M \) and the security of RoundKeyAddition \( S_R \) as follows:

\[
\eta = S_B + S_S + S_M + S_R.
\] (29)

Now we can come up with the following conclusions:
(a) ByteSub can enhance the security by substituting the plain text according to S-box, a designed method mapping the bytes of plain text to the elements of S-box. The security of ByteSub is influenced by the size of S-box.

(b) ShiftRow, MixColumn and RoundKeyAddition are three methods of changing the value of bits and they are equivalent when functioning on the plain text. While the complexity can be different because of the designed rule and that will influence the security.

To quantitatively compare the difference of varied encryption algorithms, we assume the complexity of SPN is 1 composed by four parts of the same weight and define the complexity of other round functions by their relative complexity of operations, namely the time of operations compared with SPN according to Table IV, namely,

\[
\eta(*) = \sum \frac{C^*}{C_{SPN}} S_{SPN},
\] (30)

where \( C_{SPN} \) means the complexity of AES (SPN), \( C^* \) means the complexity of other algorithms.

| Algorithms           | XOR | Feistel (DES) | SPN (AES) |
|----------------------|-----|---------------|-----------|
| ByteSub              | 0   | 4 × 16        | 16 × 16   |
| ShiftRow             | 0   | 1             | 3         |
| MixColumn            | 0   | 0             | 1         |
| RoundKeyAddition     | 32  | 32            | 128       |
| \( S_B \)            | 0   | 0.0625        | 0.25      |
| \( S_S \)            | 0   | 0.0833        | 0.25      |
| \( S_M \)            | 0   | 0             | 0.25      |
| \( S_R \)            | 0.0625 | 0.0625 | 0.25      |
| \( \eta \)           | 0.0625 | 0.2083 | 1         |

So we can define the security of encryption algorithms as:

\[
S_A = \eta R \log_2 N.
\] (31)

Based on the analysis above, we give the security definition of QKD based NCS as (5)(4).

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