Gravitational waves and fundamental physics

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I give an overview of the motivations for gravitational-wave research, concentrating on the aspects related to “fundamental” physics.

1 Introduction

I am particularly glad to contribute to this Volume in honour of the 70th birthday of Adriano Di Giacomo. Adriano has been my teacher, and I learned much from him. Exactly twenty years ago, I was working for my Diploma thesis under his supervision, and then I did my PhD with him. The atmosphere of enthusiasm and energy that we had in his group will certainly stay among my fond memories.

With Adriano we worked on problems that belong to “fundamental” physics. In particular, at the center of our interests was the problem of quark confinement, and more generally the non-perturbative aspects of QCD. Presently, I am mostly working on gravitational-wave (GW) physics. Then, I decided to use the opportunity of my contribution to this Volume to collect some ideas about what can we hope to learn about fundamental physics from the observation and the study of GWs, with the forthcoming generation of GW experiments.

Of course, astrophysics can certainly be considered fundamental in its own right, and the direct detection of GWs emitted by fascinating objects such as neutron stars or black holes would be a fundamental discovery in itself. Here however I will discuss “fundamental” physics, in the sense that is usually given to this word in high-energy physics, i.e. something which has to do with the basic laws that govern the interaction of matter.

Rather than performing a systematic discussion of all the situations where GWs can carry informations on fundamental physics, I will concentrate on three examples that I consider especially interesting.

2 Gravitational waves and quantum chromodynamics

What could possibly GWs have to do with QCD? The connection is provided by neutron stars. Neutron stars are remarkable objects from many points of view. They are the final state of stars which, after the exhaustion of their nuclear fuel and the subsequent explosion and ejection of the external layers, remain with a core more massive than the Chandrasekhar mass $M_{\text{Ch}} \approx 1.4M_\odot$ (but still the core must be lighter than a critical value $M_{\text{bh}} < O(2 - 3)M_\odot$, beyond which a black hole instead forms). When $M_{\text{Ch}} < M_{\text{core}} < M_{\text{bh}}$, the core of the star collapses under its own weight, until it reaches a radius $R \approx 10$ km, where the self-gravity of the star is now balanced by the neutron degeneracy pressure. As a consequence, the nuclear matter inside the star is compressed to extreme densities. The internal structure of neutron stars depends strongly on the equation of state; however in the inner core, say $R < O(1)$ km, the density reaches values of order $1\text{ GeV/fm}^3$. We are therefore in a regime governed by QCD at high density. This is a non-perturbative regime, which can be used to ask important questions about QCD.
In particular, what is the true ground state of QCD at low temperatures? In general, at low temperatures we expect that quarks and gluons are confined into hadrons, mainly neutrons and protons, which in turn are bound together in nuclei. Following this line of reasonings, one concludes that the true ground state is given by the nucleus with the largest binding energy per nucleon, which is $^{56}$Fe. However, the abundance of $^{56}$Fe in the Universe is very low. This is explained by the fact that, when the temperature of the Universe dropped below the deconfinement temperature, quarks and gluons first were bound together into neutrons and protons. To synthesize heavier nuclei, it is necessary that protons overcome their repulsive Coulomb interaction. This energy barrier is larger for heavier nuclei, so in this primordial nucleosynthesis only the lightest elements could be synthesized. Elements heavier than $^7$Li can only be created in stellar nucleosynthesis, where the huge pressures in the stellar cores forces the light nuclei to get sufficiently close, overcoming their Coulomb barrier. Even so, only the heaviest stars can burn their nuclear fuel up to $^{56}$Fe. This shows that the true ground state of QCD does not necessarily correspond to the state of matter that we see around us. The true ground state is the state of minimum energy, but this can be separated from the initial state of the Universe by an energy barrier that can be overcome only under exceptional conditions.

The conditions inside the core of a neutron star (NS) are however so extreme, with a density of order $1 \text{ GeV/fm}^3$, that an energy barrier of order of the QCD scale can be overcome, and the true ground state can then be revealed. A particularly interesting possibility is the strange-quark matter hypothesis, which states that the true ground state of strong interactions is a deconfined mixture of $u, d, s$ quarks, approximately in equal proportions. We typically expect that, energetically, deconfined two-flavor quark matter lies about O(100) MeV per nucleon above the energy per nucleon in nuclei, since this is a typical QCD scale; on the other hand, in strange-quark matter this could be over-compensated by the fact that now we have three different Fermi seas among which a given baryon number can be shared (this is completely analogous to the fact that in nuclei it is energetically favorable to have approximately an equal number of neutrons and protons, despite the fact that the neutrons is heavier than the proton). Back-of-the-envelope estimates suggest that what we gain opening up a third Fermi sea is again of the order of 100 MeV per nucleons. So, a quantitative assessment of the strange matter hypothesis is difficult, since it comes from a delicate balance between the precise numerical values of non-perturbative quantities. If this hypothesis is correct, however, in the core of NS weak interactions would convert about one third of the $u, d$ quarks into $s$ quarks. A star with a quark core is called a hybrid star. Actually, once initiated, it is quite possible that this process would extend to the whole star, transforming it into a quark star.

So, the interior of neutron stars is a good place to look for fundamental issues of QCD, and the question is how can we access it. GWs are a unique probe of this interior structure, for the following reasons.

A neutron star is a very rigid object, whose vibrations are characterized in terms of its normal modes. These normal modes can be excited, for instance, after a supernova explosion, when the newborn NS settles down. Furthermore, NSs occasionally undergo catastrophic rearrangements of their internal structure (crustquake and possibly corequakes). A very interesting example of this phenomenon is provided by magnetars, which are neutron stars with huge magnetic fields, of order $10^{14} - 10^{15}$ G, i.e. 100 to 1000 times stronger than in ordinary pulsars. It is believed that magnetars provide an explanation for the phenomenon of soft gamma repeaters (SGR), x-ray sources that occasionally emit
Figure 1: The frequency of GWs from the $f$-mode for different equations of state. The curve labeled SS2 corresponds to a quark star, and the (yellow) curve BBS2 to a star whose matter composition includes the strange baryons $\Sigma^-$ and $\Lambda^0$. (From Benhar, Ferrari and Gualtieri, Ref. [7]).

huge bursts of soft $\gamma$-rays. The mechanism invoked to explain the burst activity is that the magnetic field lines in magnetars drift through the liquid interior of the NS, stressing the crust from below and generating strong shear strains. For magnetic fields stronger than about $10^{14}$ G, these stresses are so large that they cause the breaking of the 1 km thick NS crust, whose elastic energy is suddenly released in a large starquake, which generates a burst of soft gamma rays. In has been estimated that these starquakes can radiate in GWs an energy $\Delta E_{\text{rad}} \sim 10^{-10} - 10^{-9} M_\odot c^2$. For NS at typical galactic distances, bursts of this type could then be detectable already with the next generation of GW detectors. These GWs provide a remarkable probe of the NS interior, for two basic reasons.

- When an excited normal mode relaxes by GW emission, GWs are emitted (by current quadrupole radiation) at a frequency equal to the frequency of the normal mode. In turn, this normal mode frequency depends, in a calculable way, on the equation of state in the NS interior. Therefore the value of the GW frequency carries important informations on the internal NS structure.

- Because of the smallness of gravitational cross-sections, for GWs even an object such as the core of a NS is basically transparent. Therefore, GWs generated inside the core, for instance as a consequence of a corequake, travel unaffected outside the NS. This is of course very different from electromagnetic waves, for which the NS interior is totally opaque, and is an excellent example of the fact that GW astronomy can potentially open up a completely new window on the Universe, unaccessible to electromagnetic observations.

In Fig. 1 we show the GW frequency emitted by the fundamental mode (or f-mode) of a compact star, for different equations of state, as a function of the star mass. It is
particularly interesting to see how GWs emitted by a star made entirely of strange quark matter differs from the result for NS with more conventional equations of state. The observation of a burst of GWs from a NS at a frequency \( f \) in the range \( 2.6 - 3 \) kHz would be a very strong indication in favor of the existence of strange quark matter. Estimates of the strength of the GW emission from \( f \)-modes suggest that present detectors do not have the sensitivity required for detecting these waves, unless the source is in our galactic neighborhood: for a source at a typical galactic distance \( r = 10 \) kpc, these GWs could however be accessible at advanced detectors.\(^7\)

3 Compact binaries and the expansion history of the Universe

A binary system made of two compact stars such as neutron stars and/or black holes is a particularly clean system for gravitational-wave physics. The stars emit GWs because of their orbital acceleration. The emission of GWs costs energy, which is drawn from the orbital energy; then the system slowly spirals inward. As a consequence, the orbital rotational frequency increases, and therefore also the GW frequency increases with time. This in turns raises the power emitted in GWs so, on a timescale of millions of years, this is a runaway process that ends up in the coalescence of the system.

3.1 The Hulse-Taylor binary pulsar

Experimentally, the decrease of the orbital period of the binary because of GW emission has been beautifully tested in the famous Hulse-Taylor binary pulsar (PSR 1913+16). This is a system made of two neutron stars, one of which is observed as a pulsar. Pulsars are extraordinarily stable clocks, which rival in stability with the best atomic clocks. The arrival time of the pulses on Earth is however modulated by a number of effects due to special and general relativity, due to the orbital motion of the pulsar around its companion (as well as to the motion of the detector, on Earth, around the Sun or, more precisely, around the Solar System Barycenter), and to the propagation of the waves in the gravitational fields of the two NS, and of the Solar System. These effects, such as Römer, Einstein and Shapiro time delays, can be accurately computed as a function of the parameters of the binary system, obtaining the so-called timing formula.\(^8\) Fitting the timing residuals, measured by now over a span of 27 years, to this timing formula, the parameters of the orbit of this binary system have been determined with great accuracy. To have an idea of the remarkable experimental precision, let us mention that the five so-called Keplerian parameter of the orbit are determined as follows.\(^9\)

\[
\begin{align*}
a_p \sin \iota &= 2.3417725(8) \text{ s}, \\
T_0 &= 52144.90097844(5) \text{ (MJD),} \\
\phi_0 &= 292.54487(8) \text{ deg}, \\
P_b &= 0.322997448930(4) \text{ days}.
\end{align*}
\]

Here \( a_p \) is the semimajor axis of the pulsar (in seconds), \( \iota \) the inclination angle of the orbit with respect to the line of sight, \( e \) the eccentricity of the orbit, \( T_0 \) a time of passage at periastron (Mean Julian Day), \( \phi_0 \) is the periastron angle measured with respect to the line of nodes, and \( P_b \) the orbital period. Furthermore, three so-called post-Keplerian parameters have been obtained from the timing residuals: the rate of advance of the position of the periastron \( \dot{\phi}_0 \) and the Einstein parameter \( \gamma \) (which enters in the expression
for the difference between coordinate time and the pulsar proper time) are given by

\[ \dot{\phi}_0 = 4.226595(5) \text{ deg/yr} , \quad \gamma = 0.0042919(8) , \]

and the rate of change of the orbital period is

\[ \dot{P}_b = -2.4184(9) \times 10^{-12} . \]

Assuming the validity of General Relativity, from the values of \( \dot{\phi}_0 \) and \( \gamma \) one can obtain the values of the pulsar mass, \( m_p \), and of the companion, \( m_c \). The result is

\[ m_p = 1.4414(2) M_\odot , \quad m_c = 1.3867(2) M_\odot . \]

Having the masses, and with such a remarkable precision, General Relativity now predicts the value of \( \dot{P}_b \) due to the emission of gravitational radiation. The ratio between the experimental value \( \dot{P}_b^{\exp} \) and the value \( \dot{P}_b^{GR} \) predicted by General Relativity turns out to be

\[ \frac{\dot{P}_b^{\exp}}{\dot{P}_b^{GR}} = 1.0013(21) . \]

This provides a wonderful confirmation of General Relativity, as well as of the existence of GWs. A confirmation at the level of 30% comes from another binary pulsar, PSR 1534+12. Recently, it has been discovered the first double NS binary in which both neutron stars are observed as pulsars, PSR J0737-3039. This system is the most relativistic binary NS system known, with an orbital period of only 2.4 hr. After just two years of observation, this system provides confirmation of GW emission at a level comparable to the Hulse-Taylor binary pulsar. Furthermore, more post-Keplerian parameters are measurable, providing a confirmation of General Relativity in strong fields at the 0.1% level.

### 3.2 Coalescing binaries as standard candles

Beside providing the first evidence for GWs, binary neutron stars are of the greatest importance for GW research because the last stage of the coalescence is among the most interesting sources for GW detectors. In particular, the frequency of the GW emitted enters into the bandwidth of ground-based interferometers (\( f > O(10) \text{ Hz} \)) about 17 min before coalescence, and sweeps up in frequency, until the kHZ, where the NS-NS binary coalesce. A NS-NS merging is a very rare event, on a galactic scale. The expected rate (determined from the observed population of NS-NS binaries, and dominated by the recently discovered double pulsar, since this is the system with the shortest time to merging, 85 Myr), is found to be \( 80_{-70}^{+210} \text{ Myr}^{-1} \) per galaxy. From this one finds that the expected rate for the initial LIGO and VIRGO is \( 35_{-30}^{+90} \times 10^{-3} \text{ yr}^{-1} \), while for advanced interferometers one gets an extremely interesting rate, \( 190_{-150}^{+470} \text{ yr}^{-1} \), that is, between two events per day and one event per week!

A remarkable fact about binary coalescence is that it can provide an absolute measurement of the distance to the source, something which is extremely rare, and important, in astronomy. This point can be understood looking at the waveform of an inspiraling binary; as long as the system is not at cosmological distances (so that we can neglect the
expansion of the Universe during the propagation of the wave from the source to the observer) the waveform of the GW, to lowest order in \( v/c \), is

\[
h_+(t) = \frac{4}{r} \left( \frac{G M_c}{c^2} \right)^{5/3} \left( \frac{\pi f(t_{\text{ret}})}{c} \right)^{2/3} \left( \frac{1 + \cos^2 \iota}{2} \right) \cos \Phi(t_{\text{ret}}),
\]

\[
h_\times(t) = \frac{4}{r} \left( \frac{G M_c}{c^2} \right)^{5/3} \left( \frac{\pi f(t_{\text{ret}})}{c} \right)^{2/3} \cos \iota \sin \Phi(t_{\text{ret}}),
\]

(6)

where \( h_+ \) and \( h_\times \) are the amplitudes for the two polarizations of the GW, \( \iota \) is the inclination of the orbit with respect to the line of sight,

\[
M_c = \left( \frac{m_1 m_2}{m_1 + m_2} \right)^{3/5}
\]

is a combination of the masses of the two stars known as the chirp mass, and \( r \) is the distance to the source; \( f \) is the frequency of the GW, which evolves in time according to

\[
\dot{f} = \frac{96}{5} \pi^{8/3} \left( \frac{G M_c}{c^3} \right)^{5/3} f^{11/3},
\]

(8)

\( t_{\text{ret}} \) is retarded time, and the phase \( \Phi \) is given by

\[
\Phi(t) = 2\pi \int_{t_0}^{t} dt' f(t').
\]

(9)

For a binary at a cosmological distance, i.e. at redshift \( z \), taking into account the propagation in a Friedmann-Robertson-Walker Universe, these equations are modified in a very simple way: (1) The frequency that appears in the above formulae is the frequency measured by the observer, \( f_{\text{obs}} \), which is red-shifted with respect to the source frequency \( f_s \), i.e. \( f_{\text{obs}} = f_s/(1+z) \), and similarly \( t \) and \( t_{\text{ret}} \) are measured with the observer’s clock. (2) The chirp mass \( M_c \) must be replaced by \( M_c = (1+z)M_c \). (3) The distance \( r \) to the source must be replaced by the luminosity distance \( d_L(z) \).

Then, the signal received by the observed from a binary inspiral at redshift \( z \), when expressed in terms of the observer time \( t \), is given by

\[
h_+(t) = h_c(t_{\text{ret}}) \frac{1 + \cos^2 \iota}{2} \cos \Phi(t_{\text{ret}}),
\]

(10)

\[
h_\times(t) = h_c(t_{\text{ret}}) \cos \iota \sin \Phi(t_{\text{ret}}),
\]

(11)

where

\[
h_c(t) = \frac{4}{d_L(z)} \left( \frac{G M_c(z)}{c^2} \right)^{5/3} \left( \frac{\pi f_{\text{obs}}(t)}{c} \right)^{2/3}.
\]

(12)

Let us recall that the luminosity distance \( d_L \) of a source is defined by

\[
F = \frac{\mathcal{L}}{4\pi d_L^2}.
\]

(13)
where \( F \) is the flux (energy per unit time per unit area) measured by the observer, and \( L \) is the absolute luminosity of the source, i.e. the power that it radiates in its rest frame. For small redshifts, \( d_L \) is related to the present value of the Hubble parameter \( H_0 \) and to the deceleration parameter \( q_0 \) by

\[
\frac{H_0 d_L}{c} = z + \frac{1}{2} (1 - q_0) z^2 + \ldots
\]  

(14)

The first term of this expansion gives just the Hubble law \( z \simeq (H_0/c) d_L \), which states that redshifts are proportional to distances. The term \( O(z^2) \) is the correction to the linear law for moderate redshifts. For large redshifts, the Taylor series is no longer appropriate, and the whole expansion history of the Universe is encoded in a function \( d_L(z) \). As an example, for a spatially flat Universe, one finds

\[
d_L(z) = c (1 + z) \int_0^z \frac{dz'}{H(z')},
\]

(15)

where \( H(z) \) is the value of the Hubble parameter at redshift \( z \). Knowing \( d_L(z) \) we can therefore obtain \( H(z) \). This shows that the luminosity distance function \( d_L(z) \) is an extremely important quantity, which encodes the whole expansion history of the Universe.

Now we can understand why coalescing binaries are standard candles. Suppose that we can measure the amplitudes of both polarizations \( h_+, h_\times \), as well as \( \dot{f}_{\text{obs}} \) (for ground-based interferometers, this actually requires correlations between different detectors). The amplitude of \( h_+ \) is \( h_c (1 + \cos^2 \iota) / 2 \), while the amplitude of \( h_\times \) is \( h_c \cos \iota \). From their ratio we can therefore obtain the value of \( \cos \iota \), that is, the inclination of the orbit with respect to the line of sight. On the other hand, eq. (8) (with the replacement \( M \rightarrow M_c \) mentioned above) shows that if we measure the value of \( \dot{f}_{\text{obs}} \) corresponding to a given value of \( f_{\text{obs}} \), we get \( M_c \). Now in the expression for \( h_+ \) and \( h_\times \) all parameters have been fixed, except \( d_L(z) \). This means that, from the measured value of \( h_+ \) (or of \( h_\times \)), we can now read \( d_L \). If, at the same time, we can measure the redshift \( z \) of the source, we have found a gravitational standard candle, and we can use it to measure the Hubble constant and, more generally, the evolution of the Universe.\(^\[14\]

In a sense, this gravitational standard candle is complementary to the standard candles that one has in conventional astrophysics (i.e. from sources detected electromagnetically). In conventional astrophysics, the determination of the redshift \( z \) of a source is straightforward (one simple measures the redshift of some spectral line) while the determination of the absolute distance to the source is “the” great problem. Here, on the contrary, there is no theoretical uncertainty on the distance \( d_L \), so for the distance the only error comes from the experimental accuracy in the measure of the GW. However, the determination of the redshift of the source can be more difficult. Various possibilities have been proposed. The simplest, of course, is to see an optical counterpart. In particular a NS-NS coalescence is also expected to emit a \( \gamma \)-ray burst. In this case, gravitational observations give the luminosity distance \( d_L \) of the source while electromagnetic observations could provide its redshift \( z \). Alternatively, for NS-NS coalescence, we can use the observational fact that the NS mass spectrum is strongly peaked around \( 1.4 M_\odot \). Then,\(^\[*\]

\(*\)It is important that the ellipticity of the orbit does not enter; it can in fact be shown that, by the time that the stars approach the coalescence stage, angular momentum losses have circularized the orbit to great accuracy.
from the measured value of $M_c = (1 + z)M_*$ and assuming $m_1 = m_2 = 1.4M_\odot$, which gives $M_c \simeq 1.2M_\odot$, we get an estimate of $z$.

Besides, statistical methods combining observations of different coalescences have been proposed.

What sort of “fundamental” informations can we get from these measurements? At the advanced level, Virgo and LIGO are expected to detect at least tens of NS-NS coalescences per year, up to distances of order 2 Gpc, measuring the chirp mass with a precision that can be better than 0.1%. The masses of NSs are typically of order $1.4M_\odot$. Stellar-mass black holes, as observed in x-ray binaries, are in general more massive, typically with masses of order $10M_\odot$, and therefore emit an even more powerful GW signal during their inspiral and coalescence. The coalescence of two black holes, each one with $10M_\odot$, could be seen by advanced Virgo and advanced LIGO up to redshifts $z \sim 2 - 3$. Furthermore, the space interferometer LISA, which is expected to fly in about 10 years, is sensitive to GWs in the mHz region, which corresponds to the wave emitted by supermassive BH with masses up to $10^6M_\odot$. Nowadays, supermassive BH with masses between $10^6$ and $10^9M_\odot$ are known to exist in the center of most (and probably all) galaxies, including ours. The coalescence of two supermassive black holes, which could take place for instance during the collision and merging of two galaxies or of pre-galactic structure at high redshifts, would be among the most luminous events in the Universe. Even if the merger rate is poorly understood, observations from the Hubble Space Telescope and from x-ray satellites such as Chandra have revealed that these merging are not at all uncommon, out to cosmological distances. LISA could detect them out to $z \sim 10$, and is expected to measure at least several events over its mission.

The most important issue that can be addressed with a measure of $d_L(z)$ is to understand “dark energy”, the quite mysterious component of the energy budget of the Universe that manifests itself through an acceleration of the expansion of the Universe at high redshift. This has been observed, at $z < 1.7$, using Type Ia supernovae as standard candles. A possible concern in these determinations is the absence of a solid theoretical understanding of the source. After all, supernovae are complicated phenomena. In particular, one can be concerned about the possibility of an evolution of the supernovae brightness with redshift, and of interstellar extinction in the host galaxy and in our Galaxy, leading to unknown systematics. Gravitational-wave standard candles could lead to completely independent determinations, and complement and increase the confidence in other standard candles, as well as extending the result to higher redshifts. In particular, it is of great importance to measure the equation of state of this dark energy component. This can be parametrized in terms of the ratio of pressure to density $w = p/\rho$, which can in general be a function of redshift, $w(z)$. The value $w(z) = -1$ corresponds to a cosmological constant while different values can arise, for instance, from evolving fields. At present, from Type Ia supernovae, we have a bound on the average value of $w(z)$ over values of $z$ up to $z \simeq 1.7$, given by $\langle w(z) \rangle < -0.55$. In any case, the answer will have profound implications both in cosmology and in particle physics.

4 Stochastic backgrounds and the Big-Bang

Another possible target of gravitational-wave experiments is given by stochastic backgrounds of cosmological origin. These are the gravitational analog of the 2.7 K microwave photon background and they can carry unique informations on the state of the
very early Universe and on physics at correspondingly high energies. To understand this point, it is important to realize that a background of relic particles gives a snapshot of the state of the Universe at a very precise moment, that is, at the time when these particles decoupled from the primordial plasma.

In particular, if these particles never reached thermal equilibrium with the primordial plasma, they still carry the imprint of the mechanism that created them. In fact, suppose for instance that a stochastic background is generated at some cosmological epoch, e.g. by a phase transition in the early Universe. If the particle species in question thermalizes and comes to equilibrium with the rest of the primordial plasma, any peculiar feature, for instance of its energy spectrum, which could have revealed informations on the mechanism that generated it, will be erased, so for instance the energy spectrum will become a “dull” black-body spectrum. If instead these particles immediately decouple from the primordial plasma, they will arrive to us still carrying all their information, just undergoing a redshift because of the expansion of the Universe.

The smaller is the cross section of a particle, the earlier it decouples. Therefore particles with only gravitational interactions, like gravitons and possibly other fields predicted by string theory, decouple much earlier than particles which have also electroweak or strong interactions. The condition for decoupling is that the interaction rate of the process that maintains equilibrium, \( \Gamma \), becomes smaller than the characteristic time scale, which is given by the Hubble parameter \( H \),

\[
\Gamma \ll H \Rightarrow \text{decoupled} \quad (16)
\]

(we set \( \hbar = c = 1 \)). A simple back-of-the-envelope computation shows that, for gravitons,

\[
\frac{\Gamma}{H} \sim \left( \frac{T}{M_{\text{Pl}}} \right)^3, \quad (17)
\]

so that gravitons are decoupled below the Planck scale \( M_{\text{Pl}} \sim 10^{19} \text{ GeV} \), i.e., already \( 10^{-44} \) sec after the big-bang. This means that a background of GWs produced in the very early Universe encodes still today all the informations about the conditions in which it was created. In principle, if a sufficiently strong stochastic background has been created during the Big-Bang, its detection could literally provide us with a snapshot of the Big-Bang itself!

For comparison, the photons that we observe in the CMB decoupled when the temperature was of order \( T \simeq 0.2 \text{ eV} \), or \( 3 \times 10^5 \) yr after the Big Bang. Therefore, the photons of the CMB give us a snapshot of the state of the Universe at \( t \sim 3 \times 10^5 \) yr. This difference in scales simply reflects the difference in the strength of the gravitational and electromagnetic interactions.

### 4.1 Existing bounds

The intensity of a stochastic background of GWs can be characterized by the dimensionless quantity

\[
\Omega_{gw}(f) = \frac{1}{\rho_c} \frac{d\rho_{gw}}{d \log f}, \quad (18)
\]

where \( \rho_{gw} \) is the energy density of the stochastic background of gravitational waves, \( f \) is the frequency, \( \rho_c = 3H_0^2/(8\pi G_N) \) is the present value of the critical energy density for closing the Universe, and the present value of the Hubble parameter \( H_0 \) is usually
written as \( H_0 = h_0 \times 100 \text{ km/(sec–Mpc)} \), where \( h_0 = 0.71^{+0.04}_{-0.03} \) parametrizes the existing experimental uncertainty\(^6\).

At present, we have three major bounds on \( h_0^2 \Omega_{gw} \), illustrated in Fig. 2. On the horizontal axis we plot the GW frequency, covering a huge range of frequencies. The lowest value, \( f = 10^{-18} \) Hz, corresponds to a wavelength as large as the present Hubble radius of the Universe; the highest value shown, \( f = 10^{10} \) Hz, has instead the following meaning: if we take a graviton produced during the Planck era, with a typical energy of the order of the Planck or string energy scale, and we redshift it to the present time using the standard cosmological model, we find that today it has a frequency of order \( 10^{10} - 10^{12} \) Hz. These values therefore are of order of the maximum possible cutoff of spectra of GWs produced in the very early Universe. The maximum cutoff for astrophysical processes is of course much lower, of order 10 kHz. So this huge frequency range encompasses all the GWs that can be considered. The three bounds in the figure come out as follows.

**Nucleosynthesis bound.** The outcome of big-bang nucleosynthesis (BBN) depends on a balance between the particle production rates and the expansion rate of the Universe, measured by the Hubble parameter \( H \). Einstein equation gives \( H^2 \sim G \rho \), where \( \rho \) is the total energy density, including of course \( \rho_{gw} \). Nucleosynthesis successfully predicts the primordial abundances of deuterium, \(^3\)He, \(^4\)He and \(^7\)Li assuming that the only contributions to \( \rho \) come from the particles of the Standard Model, and no GW contribution. Therefore, in order not to spoil the agreement, any further contribution to \( \rho \) at time of nucleosynthesis, including the contribution of GWs, cannot exceed a maximum value. The bound is usually written in terms of an effective number of neutrino species \( N_\nu \) (i.e. any extra contribution to energy density is normalized to the energy density that would be carried by one neutrino species in thermal equilibrium). Thus, in the Standard Model, \( N_\nu = 3 \), and any extra form of energy density gives further contributions to \( N_\nu \) (of course, not in general integer, since the energy of a thermal neutrino is just an arbitrary normalization scale). In terms of \( N_\nu \), the bound reads\(^{23}\)

\[
\int_{f=0}^{f=\infty} d(\log f) \ h_0^2 \Omega_{gw}(f) \leq 5.6 \times 10^{-6} (N_\nu - 3).
\]  

(19)

The upper limits on \( N_\nu \) depends on the assumptions made about nucleosynthesis, and can be as stringent as \( N_\nu < 3.2 \). However, if one invokes a chemical potential for the electron neutrino, this can be relaxed up to \( N_\nu < 7.1 \).\(^{26}\)

**Bounds from millisecond pulsar.** Millisecond pulsars are an extremely impressive source of high precision measurements\(^{13}\). For instance, the observations of the first msec pulsar discovered, B1937+21, give a period of 1.557 806 468 819 794 5(4) ms. As a consequence, pulsars are also a natural detector of GWs, since a GW passing between us and the pulsar causes a fluctuation in the time of arrival of the pulse, proportional to the GW amplitude. The highest sensitivities can then be reached for a continuous source, such as a stochastic background, after one or more years of integration, and therefore for

\(^6\)Clearly, it is not very convenient to normalize \( \rho_{gw} \) to a quantity, \( \rho_c \), which has an experimental uncertainty: this uncertainty would appear in all the subsequent formulas, although it has nothing to do with the uncertainties on the GW background itself. Therefore, it is customary to rather characterize the stochastic GW background by the quantity \( h_0^2 \Omega_{gw}(f) \), which is independent of \( h_0 \).
Figure 2: Bounds on $h_0^2\Omega_{gw}$ from nucleosynthesis with $N_\nu = 7.1$ and $N_\nu = 3.2$ (dashed lines, black), from pulsar timing (wedge-shaped, red) and from CMB (green), and the sensitivity to stochastic GWs of LISA, and of the correlation between two ground-based interferometers such as VIRGO and LIGO: (I) two interferometers of first generation. (II) two advanced interferometers. (III) two $3^{rd}$ generation interferometers.
$f \sim 1/T$ with $T$ equal to a few years, i.e. $f \sim 10^{-9} - 10^{-8}$ Hz. This gives the bound labeled “pulsar timing” in Fig. 2.

**Bounds from CMB.** Another important constraint comes from the measurement of the fluctuation of the temperature of the cosmic microwave background radiation (CMB). The idea is that a strong background of GWs at very long wavelengths produces a stochastic redshift on the frequencies of the photons of the 2.7K radiation, and therefore a fluctuation in their temperature. Indeed, it is quite possible that the observed anisotropies are partly due to GWs. The condition that a stochastic background of GWs does not produce anisotropies in excess of those observed gives the bound labeled CMB in Fig. 2.

### 4.2 Sensitivity of GW detectors to $h_0^2\Omega_{gw}$, and theoretical expectations.

The optimal strategy to detect a stochastic background of GWs is to correlate the output of two detectors for a time as long as possible. With the present sensitivities, correlating for four months, and at 90% confidence level, the two LIGO I detectors would reach $h_0^2\Omega_{gw} \simeq 3.5 \times 10^{-6}$ (comparable sensitivities would be reached correlating Virgo with a second nearby interferometer, such as GEO). At the advanced LIGO level one obtains $h_0^2\Omega_{gw} \simeq 5.1 \times 10^{-9}$ while for so-called third generation detectors, which are currently under investigation, one could get $h_0^2\Omega_{gw} \simeq 3.7 \times 10^{-11}$ (of course this last figure is more hypothetical). LISA, on the other hand, thanks to the fact that it works at lower frequencies, can reach excellent sensitivities even as a single detector, with $h_0^2\Omega_{gw} \simeq 10^{-11}$. These sensitivities are shown, together with the existing bounds, in Fig. 2. From this figure we see that, while first generation ground-based interferometers have marginal chances of detection, at the level of advanced interferometer and for the space interferometer LISA, we are penetrating quite deeply into an unexplored region.

The next question, of course, is whether there are theoretical predictions of stochastic backgrounds of GWs accessible with these sensitivities. Actually, most predictions of stochastic backgrounds of cosmological origin unavoidably face uncertainties due to our ignorance of early Universe cosmology and/or physics beyond the Standard Model. However, many examples shows that there are plausible mechanisms for generating detectable background,[23] for instance in string cosmology,[27,28] in a certain range of parameters of the model, is produced a GW background that can be observable both with ground-based interferometers and with LISA, while at the electroweak phase transition, in extensions of the Standard Model, one finds backgrounds that can be detectable at LISA.[29]

The exploration of this new territory might provide us with great rewards.

### References

1. A. R. Bodmer, *Phys. Rev.* **D4** (1971) 1601.
2. E. Witten, *Phys. Rev.* **D30** (1984) 272.
3. N.K. Glendenning, *Compact Stars*, Springer-Verlag 2000.
4. R. C. Duncan and C. Thompson, *Ap. J.* **392** (1992) L9.
5. J. A. de Freitas Pacheco, *Astron. Astrophys.* **336** (1998) 397.

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*To discriminate the GW contributions from the effect of scalar perturbations it would be necessary to detect, beside the already observed E-polarization, also the $B$-polarization of the CMB. This is a target of forthcoming CMB experiments.*
6. E. Coccia, F. Dubath and M. Maggiore, *Phys. Rev.* **D70** (2004) 085418.
7. O. Benhar, V. Ferrari and L. Gualtieri *Phys. Rev.* **D70** 124015 (2004).
8. T. Damour and N. Deruelle, *Ann. Inst. H. Poincaré* **44** 263 (1986).
9. J. M. Weisberg and J. H. Taylor, astro-ph/0407149.
10. I. H. Stairs, S. E. Thorsett, J. H. Taylor and A. Wolszczan, *Ap. J.* **581** (2002) 501.
11. M. Burgay *et al.*, *Nature* **426**, 531 (2003).
12. M. Kramer *et al.*, astro-ph/0503386.
13. D. R. Lorimer, *Living Rev. Rel.* **8** (2005) 7 [arXiv:astro-ph/0511258].
14. B. F. Schutz, *Nature*, **323**, 310 (1986).
15. D. Markovic, *Phys. Rev.* **D48** 4738 (1993).
16. Cutler, C. and Flanagan, E. E. (1994). *Phys. Rev. D* **49**, 2658.
17. S. A. Hughes, *MNRAS* **331** 805 (2002).
18. A. Vecchio *Phys. Rev. D* **70**, 042001 (2004).
19. D. Riess *et al.*, *Astronom. J.* **116**, 1009 (1998).
20. S. Perlmutter *et al.*, *Ap. J.* **517**, 565 (1999).
21. D. E. Holz and S. A. Hughes, astro-ph/0504616.
22. P. M. Garnavich *et al.* [Supernova Search Team Collaboration], *Astrophys. J.* **509** (1998) 74 [arXiv:astro-ph/9806396].
23. M. Maggiore, *Phys. Rept.* **331**, 283 (2000).
24. A. Buonanno, “TASI Lectures on Gravitational Waves from the Early Universe,” arXiv:gr-qc/0303085.
25. M. Maggiore, “Stochastic backgrounds of gravitational waves,” in ICTP Lectures Notes, V. Ferrari, J. C. Miller and L. Rezzolla eds., arXiv:gr-qc/0008027.
26. See the chapter on Big-Bang Nucleosynthesis of the Particle Data Group, S. Eidelman *et al.* *Phys. Lett.* **B592**, 1 (2004).
27. M. Gasperini and G. Veneziano, *Phys. Rept.* **373** (2003) 1.
28. R. Brustein, M. Gasperini, M. Giovannini and G. Veneziano, *Phys. Lett.* **B361** (1995) 45.
29. R. Apreda, M. Maggiore, A. Nicolis and A. Riotto, *Nucl. Phys. B* **631** (2002) 342.