On the effect of the central body small deformations on its satellite trajectory in the problem of the two-body gravitational interaction

Dmitry G. Kiryan | George V. Kiryan

Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences, Saint-Petersburg, Russia

Correspondence
Dmitry G. Kiryan, Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences V.O., Bolshoj pr., 61 Saint-Petersburg, 199178, Russia.
Email: diki.ipme@gmail.com

The problem of the two-body gravitational interaction has been solved numerically based on the classical mechanics principles. One of the bodies is a deformable three-axis ellipsoid (central body) and the other is a material point (satellite). The relationship of the angular discrepancy between the calculated and actual positions of the satellite pericenter with central body’s gravity anomaly has been established.

1 PROBLEM DEFINITION AND JUSTIFICATION

Analysis of the influence of nonsphericity of a massive central body upon its satellite trajectory is a well-known problem of satellite-based gravity prospecting. However, it has not been yet set out with respect to the Sun and Mercury, and, thus, is of a specific interest. The point of interest is to try, in terms of the classical mechanics, to discover the nature of the so-called “anomalous” shift of the Mercury’s perihelion1 during its synodicyear2 by angle $\Delta \psi$.

The Mercury’s perihelion “anomalous” shift was found out as a result of many-year diligent work of French astronomer Urbain Jean Joseph Le Verrier who observed the Mercury at the Paris Observatory during almost half a century and elaborated the theory of its motion taking into account all the gravitational perturbations he knew at that time [1]. Le Verrier revealed that the calculated Mercury’s trajectory was somewhat different from the optical observations, namely, the perihelion was being displaced faster than predicted by his theory. The obtained discrepancy appeared to be $\Delta \psi \approx 0.1''$ for the Mercury synodic period. To facilitate comparison of this small quantity in analyzing different hypotheses explaining the nature of discrepancy $\Delta \psi$, it was decided to express it in terms of time periods equal to 100 Earth years. Different authors report the discrepancy values ranging from $38''$ to $43''$.

Unfortunately, the classical mechanics has “almost officially” refused further investigation of the $\Delta \psi$ nature by accepting, without proper criticism, in the early 20th century, the Einstein’s formula [2–4] for the “anomalous” Mercury’s perihelion shift looking like

$$\Delta \psi = \frac{6\pi G M_{\odot}}{c^2 a (1 - e^2)} \quad \text{or} \quad \Delta \psi = \frac{24\pi^3 a^3}{c^2 T^2 (1 - e^2)}, \quad (1)$$

where $G$ is the gravitational constant, $M_{\odot}$ is the mass of the Sun as a material point, $c$ is the light velocity in vacuum, $T$ is the Mercury’s synodic period, $a$, $e$ are the semi-major axis and eccentricity of the unperturbed Mercury’s orbit, respectively.

Relationship (1) suggested by Einstein gave the required value of the Mercury’s perihelion shift $\Delta \psi \approx 0.1''$, however, at the cost of refusing the classical Newton’s law on gravitational interaction. At that, any search for gravitation anomalies

---

1 The minimal distance between the satellite and center of mass of the central body.
2 The object’s period of revolution about the Sun.
FIGURE 1 “Anomalous” shift of the Mercury’s perihelion $\Delta \psi$ during its synodic period. Digits 1, 2, 3 mark the sequence of the Mercury’s perihelia

or other physical processes able to explain the $\Delta \psi$ nature was finished. Formula (1) was accepted by the scientific community as the only possible explanation of the Le Verrier’s discrepancy between the calculated and actual locations of the Mercury’s perihelion equal to $\Delta \psi \approx 0.1''$ per one revolution about the Sun.

Analysis of the postulates and hypotheses formula (1) for constant $\Delta \psi$ is based on is not the objective of this study. All the history of search for sources of the Mercury’s perihelion “anomalous” shift is given in detail in the book by N. T. Roseveare “Mercury’s perihelion: from Le Verrier to Einstein” [5].

To our mind, the classical mechanics made haste to give up its position. The Le Verrier’s theory and Einstein’s formula (1) consider the Sun as a material point. But are there sufficient grounds to assume the Sun to be a material point in calculating the Mercury’s orbit? The minimal distance between the Mercury and Sun is about 66 Sun radii or 153 light seconds, which is very close to the Sun on the cosmic scale. Thus it is possible to regard the Sun not as a material point but as a deformable body whose shape may differ from a sphere.

Thus, we have a closed system of two gravitating bodies: the Sun and Mercury. Estimates obtained based on observations show that the Sun’s self-rotation\(^3\) causes its polar compression, which means that the Sun’s polar radius is shorter than the equatorial one by $\approx 6.3$ km. In the case of an ellipsoid of revolution, including the case of the Sun, the compression estimate will be introduced via the oblateness\(^4\). The existence of the compression/oblateness means that the assumption of the Sun deformability is well-grounded. Now, the Sun is a deformable body while the Mercury is a material point (proofmass).

As a component of the Solar System, the Sun participates in two motions, namely, its self-rotation and revolution about the barycenter\(^5\). The effect of the Sun bulges caused by its self-rotation upon the Mercury’s orbit is similar to that of the Earth’s bulges upon the trajectories of its artificial satellites [6, 7]. However, since the Sun’s oblateness ($1/9000000$) is significantly (approximately 30, 200 times) lower than that of the Earth ($1/298$), the gravitational effect of the Sun’s bulges on the Mercury’s orbit may be ignored. A similar assumption was made in Bowler [[8], Chapter 7].

Thus, the only remaining type of rotation is revolution of the Sun as a deformable body about the Solar System barycenter whose location continuously changes with respect to the Sun’s center of mass (Figure 2).

Like all the Solar System material objects, the Sun is affected by centrifugal forces in rotating about the Solar System barycenter. Therefore, it stands to reason that the Sun as a deformable body obtains under the action of centrifugal forces a shape different from the sphere. In the first approximation, this shape may be assumed to be an ellipsoid whose semi-major axis lies in the ecliptic plane and is directed to the Solar System barycenter.

Let us estimate the effect of each planet on the center of mass $O$ position relative to the Sun (Figure 3). The $r_O$ calculations for each planet\(^6\) are given in Table 1.

---

\(^3\) The self-rotation period of the Sun or, more exactly, of its surface layers, is 25–34 days.

\(^4\) This a quantity characterizing the extent of deformation of a circle or sphere diameter with formation of an ellipse or ellipsoid. The oblateness is typically designed as $f$. It is defined as $f = \frac{a-b}{a}$, where $a$ and $b$ are the ellipse semi-major and semi-minor axes. Deformation (oblateness) of planets depends on their structures and self-rotation velocities.

\(^5\) Instantaneous Solar System center of mass.

\(^6\) The planet characteristics have been taken from http://nssdc.gsfc.nasa.gov
FIGURE 2  The picture has been taken from book [9]. “Figure 22. The Sun’s center motion relative to the Solar System center of mass from 1940 to 2000”

FIGURE 3  Rotation of the Sun-Planet System about center of mass O

TABLE 1  The ratio between \( r_\odot \) and average Sun radius \( R_\odot \) for each Solar System planet. 
\( L \) is the distance between the Sun and planet in light minutes, \( e \) is the orbit eccentricity, \( T \) is the planet synodic period.

|       | \( L \) [min] | \( e \)   | \( T \) [day] | \( r_\odot / R_\odot \) |
|-------|--------------|---------|--------------|------------------------|
| Mercury | 3.219        | 0.20566 | 87.969       | 0.00001                |
| Venus  | 6.016        | 0.00675 | 224.701      | 0.00038                |
| Earth  | 8.317        | 0.01675 | 365.256      | 0.00065                |
| Mars   | 12.671       | 0.09347 | 686.980      | 0.00011                |
| Jupiter| 43.284       | 0.04887 | **4332.589** | **1.06733**            |
| Saturn | 79.695       | 0.05649 | 10759.220    | 0.58876                |
| Uranus | 159.692      | 0.04566 | 30685.400    | 0.18025                |
| Neptune| 249.898      | 0.01126 | 60189.000    | 0.33275                |
| Pluto  | 328.359      | 0.24881 | 90560.000    | 0.00006                |
Table 1 demonstrates that the main factor determining the radial displacement of the Solar System barycenter relative to the Sun’s center of mass are the giant planets, namely, Jupiter, Saturn, Uranus, and Neptune. Certainly, allowing for the current giant planet locations in determining the barycenter coordinates relative to the Sun will introduce some corrections. However, the very fact of the existence of the variable field of centrifugal forces acting upon the Sun in the planes parallel to the ecliptic plane cannot be ignored. Since among all the planets only Jupiter produces the maximal displacement of center of mass \( \mathbf{O} \) with respect to the Sun, let us consider Jupiter as the main source of the Sun’s centrifugal deformation that, as we believe, will finally allow revealing the nature of the Mercury’s perihelion “anomalous” shift \( \Delta \psi \approx 0.1'' \).

The period of the Sun–Jupiter pair revolution about \( \mathbf{O} \) is determined by Jupiter’s synodic period. Hence, in rotating about the center of mass of the Sun–Jupiter system, the Sun as a deformable body will be affected by relevant centrifugal forces that form, in the first approximation, a three-axis ellipsoid \(^7\) with semi-major axis \( a \) lying in the ecliptic plane and oriented on Jupiter.

Actually, we consider a three-body system: Sun, Mercury, and Jupiter, but in two stages. At the first stage, we justify the Sun’s nonsphericity by the existence of centrifugal forces when the Sun–Jupiter pair revolves about the center of mass with the period equal to the Jupiter’s synodic period. At the second stage, we analyze the Mercury’s motion about the center of mass of the ellipsoid of Sun. The self-rotation periods of the Sun–Jupiter and Sun–Mercury pairs are quite different. During one Mercury’s revolution about the Sun, the Sun itself moves with respect to center of mass \( \mathbf{O} \) of the Sun–Jupiter pair by the angle equal to

\[
\frac{2\pi}{T_\phi} \cdot \left( \frac{180}{\pi} \right) \approx 7.3^\circ ,
\]

where \( T_\phi \) and \( T_\psi \) are the synodic periods of Jupiter and Mercury. The angle is quite small. In the first approximation, we can assume that the Sun is a motionless ellipsoid from the Mercury’s point of view.

Thus, the subject of research is a closed system consisting of the Sun and Mercury. Mercury as a material point revolves about the motionless ellipsoid of the Sun. Our goal is to reveal the possible relationship between the Sun’s ellipsoid parameters and “anomalous” shift of Mercury’s perihelion \( \Delta \psi \) provided such relationship indeed exists.

2 | A SYSTEM OF TWO GRAVITATING BODIES

Consider a classical problem of motion of two gravitating bodies obeying the Newton’s mechanics laws. Let us inject into consideration a motionless orthogonal coordinate system \( \mathbf{O}xyz \) (Figure 4). Radius-vectors \( \vec{r}_A \) and \( \vec{r}_B \) define the current positions of bodies \( A \) and \( B \) with masses \( m_A \) and \( m_B \), respectively. Let bodies \( A \) and \( B \) be centrally symmetric.

Behavior of gravitationally interacting bodies \( A \) and \( B \) in the Figure 4 left panel may be described by a set of second-order differential equations

\[
\begin{align*}
\begin{cases}
m_A \ddot{r}_A = \vec{f}_B \\
m_B \ddot{r}_B = \vec{f}_A
\end{cases}
\end{align*}
\text{or}
\begin{align*}
\begin{cases}
m_A \ddot{r}_A = \vec{E}_B m_A \\
m_B \ddot{r}_B = \vec{E}_A m_B
\end{cases}
\end{align*}
\]

\(^7\) In reality, the “liquid” Sun shape is much more complicated \([10, 11]\) since there is also the self-rotation as well as continuously varying position of the Solar System barycenter. However, this is beyond the scope of our study.
Here $\vec{f}_b$ is the gravitational force acting on body $A$ from body $B$, which is a product of the gravity field intensity $\vec{E}_b$ and mass $m_A$. Force $\vec{f}_A$ is defined in the same way.

Let us simplify task (3) by subtracting the upper equation from the lower one. Taking into account that

$$\vec{r} = \vec{r}_B - \vec{r}_A$$  \hspace{1cm} (4)

obtain the equation of the body $B$ motion with respect to the motionless centrally symmetric body $A$

$$\vec{\ddot{r}} = \vec{E}_A - \vec{E}_B .$$  \hspace{1cm} (5)

Gravity field intensities $\vec{E}_A$ and $\vec{E}_B$ will be expressed through corresponding potential gradients $U_A$ and $U_B$

$$U_A = G \frac{m_A}{r}, \quad \vec{E}_A = (\nabla U_A \cdot \vec{e}_r) \vec{e}_r \Rightarrow \vec{E}_A = - \frac{G m_A}{r^2} \vec{e}_r .$$  \hspace{1cm} (6)

$$U_B = G \frac{m_B}{r}, \quad \vec{E}_B = (\nabla U_B \cdot \vec{e}_r)(-\vec{e}_r) \Rightarrow \vec{E}_B = - \frac{G m_B}{r^2} (-\vec{e}_r) .$$  \hspace{1cm} (7)

Here $G$ is the gravitational constant, $r = |\vec{r}|$ is the distance between point masses $m_A$ and $m_B$, $\nabla$ is the nabla operator that is a vector differential operator in the spherical coordinate system

$$\nabla = \frac{\partial}{\partial r} \vec{e}_r + \frac{1}{r} \frac{\partial}{\partial \varphi} \vec{e}_\varphi + \frac{1}{r \cos \varphi} \frac{\partial}{\partial \lambda} \vec{e}_\lambda ,$$  \hspace{1cm} (8)

where $\vec{e}_r, \vec{e}_\varphi, \vec{e}_\lambda$ are unit vectors of the spherical coordinate system (Figure 4).

Let us express relations (6) and (7) obtained for the bodies $A$ and $B$ gravity field in terms of potentials and substitute them into Equation (5):

$$\vec{\ddot{r}} = (\nabla(U_A + U_B) \cdot \vec{e}_r) \vec{e}_r .$$  \hspace{1cm} (9)

Substituting Equations (6) and (7) into Equation (5) or (9) and taking into account the fact that $m_A \gg m_B$ under the task condition, obtain

$$\vec{\ddot{r}} = - G \frac{m_A + m_B}{r^2} \vec{e}_r \quad \Rightarrow \quad \vec{\ddot{r}} \approx - G \frac{m_A}{r^2} \vec{e}_r .$$  \hspace{1cm} (10)

Thus, excluding from Equation (10) mass $m_B$, we exclude from consideration the body $B$ gravity field itself.

However, obtained Equation (10) is valid only for point masses or centrally symmetric bodies. To factor in the influence of the central body $A$ (mass $m_A$) geometry on the trajectory of point mass $m_B$, let us use in the first approximation the expression for the three-axis ellipsoid potential. In general case, the three-axis ellipsoid potential is a convergent series expressed in terms of Legendre polynomials [12, 13]. For convenience and without compromising the essence, let us orient the ellipsoid so that the axes of the Cartesian coordinate system Ox,yz coincide with the ellipsoid’s principal axes, namely, the ellipsoid semi-axis $a$ coincides with axis Ox, semi-axis $b$ coincides with axis Oy, and semi-axis $c$ lies on axis Oz. The coordinate system origin (point O) coincides with the center of mass of the homogeneous ellipsoid (Figure 5).

Let us restrict ourselves to three expansion terms of the three-axis ellipsoid potential. This is sufficient to allow for the shape of central body $A$ in calculating the satellite $B$ trajectory, that is,

$$U = \frac{G m_A}{r} + \frac{G}{r^3} \left( \left( J_c - \frac{J_a + J_b}{2} \right) \frac{1 - \sin^2 \varphi}{2} + \frac{(J_b - J_a) 3 \cos^2 \varphi \cos 2\lambda}{4} \right) ,$$  \hspace{1cm} (11)

where

$$J_a = \frac{m_A}{5} (b^2 + c^2) , \quad J_b = \frac{m_A}{5} (a^2 + c^2) , \quad J_c = \frac{m_A}{5} (a^2 + b^2) .$$  \hspace{1cm} (12)

Here $J_a, J_b, J_c$ are the ellipsoid’s principal moments of inertia, $G$ is the scale-dimension factor (gravitational constant), $m_A$ is the ellipsoid’s gravitating mass, $r, \varphi, \lambda$ are the spherical coordinates of material point $B$, that is, distance, latitude, and longitude, respectively.
Let us make some assumptions that will help simplify expression (11) for the potential of central body \( A \), which is a three-axis ellipsoid

1. \( \varphi(t) \equiv 0 \) (trajectory \( B \) lies in plane \( Oxy \))
2. \( a > b = c \) (ellipsoid of revolution)

As a result, expression (11) takes a simpler form:

\[
U(r,\lambda) = G \frac{m_A}{r} \left( \frac{1}{r^2} + \frac{1 + 3 \cos 2\lambda}{r^2} \left( \frac{a^2 - b^2}{20} \right) \right).
\]

(13)

Thus, in our task, the Equation (9) potentials will be as follows:

\[
\frac{\delta U}{\delta r} \gg \frac{1}{r} \frac{\delta U}{\delta \lambda} \implies U_A = U(r), \quad U_B = 0.
\]

(14)

Substituting Equation (14) into (9), obtain the motion equation for material point \( B \) having radius-vector \( \vec{r} \) with respect to motionless central body \( A \) shaped as an ellipsoid with semi-axes \( a > b = c \) :

\[
\vec{\ddot{r}} = -G \frac{m_A}{r^2} \left( 1 + \frac{3}{r^2} \left( \frac{a^2 - b^2}{20} \right) \right) \frac{1 + 3 \cos 2\lambda}{r^2} \vec{e}_r.
\]

(15)

The ellipsoid deformation is defined by changes in its semi-major axis \( a \). Semi-axes \( b \) and \( c \) will be recalculated under the condition of invariance of the ellipsoid volume \( V_A \) in the process of its deformation. Hence, when semi-major axis \( a \) is varied, semi-axes \( b \) and \( c \) are to vary correspondingly:

\[
V_A(k) = \frac{4}{3} \pi abc = Const \quad \Rightarrow \quad a = kR_A, \quad b = \frac{R_A}{\sqrt{k}}, \quad c = b.
\]

(16)

Here, \( R_A \) is the radius of the undeformed spherical body \( A \), \( k > 1 \) is the real factor defining the extent of the ellipsoid transformation via variation in the length of its semi-axis \( a \).

3 | RESEARCH RESULTS

In this study, we have solved numerically Equation (15) where central body \( A \) is assumed to be the Sun and material point \( B \) represents Mercury. The initial conditions for Mercury are as follows:

\[
\lambda(t) \bigg|_{t=0} = \lambda_0, \quad r(t) \bigg|_{t=0} = r_{\text{perihelion}}, \quad \dot{r}(t) \bigg|_{t=0} = v_{\text{perihelion}}.
\]

(17)
The effect of the angle $\lambda_0$ value on the Mercury’s perihelion shift $\Delta \psi$. $\lambda_*$ is the critical angle at which the perihelion shift direction changes.

The procedure for solving Equation (15) with initial conditions (17) is a sequence of calculating the Mercury’s trajectory at different values of semi-axis $a$ of the ellipsoid, that is, the Sun:

$$a(k) = k R_\odot, \quad \text{where} \quad k \in \mathbb{R}, \; k > 1.$$  \hspace{1cm} (18)

Among a great variety of solutions at $\lambda_0 = 0$, the following value of the deformation coefficient $k$ was chosen:

$$k \approx 1.00044$$  \hspace{1cm} (19)

at which the so-called “anomalous” shift of the Mercury perihelion $\Delta \psi = 0.1''$ takes place. This means that the Sun’s oblateness in the ecliptic plane is about $1/1516$, namely, semi-major axis $a$ exceeds semi-axis $b$ by $\approx 459 \text{ km} \approx 0.00066R_\odot$. As compared with the Sun size, this is a quite small value, $\approx 0.066\%$ of the Sun’s radius. However, just this difference in the Sun’s ellipsoid semi-axes gives rise to the missing Mercury’s anomaly.

In addition, in solving the problem, we have revealed a dependence of the Mercury’s perihelion direction on the initial angle $\lambda_0$ in Equation (17). This result is shown graphically in Figure 6.

The plot shows that when $\lambda_0$ belongs to ranges

$$\lambda_* < \lambda_0 < \pi - \lambda_* \quad \text{and} \quad \pi + \lambda_* < \lambda_0 < 2\pi - \lambda_*,$$  \hspace{1cm} (20)

the reverse (clockwise) motion of the Mercury’s perihelion is observed only conceptually.

### 4 CONCLUSIONS

The performed study of the effect of the Sun’s nonsphericity on the Mercury’s trajectory has shown that the so-called “anomalous” shift of the Mercury’s perihelion, that is, the angular discrepancy, is a result of the fact that Le Verrier assumed in his calculations that the Sun is a material point.

The Mercury’s perihelion shift $\Delta \psi \approx 0.1''$ is determined by the Sun’s nonsphericity characterized by the ratio between the ellipsoid semi-axes $a$, $b$, and $c$. The source of the Sun’s nonsphericity (when this is not about the polar compression) is the Sun deformation in its rotation about the Solar System barycenter.

The practice of satellite-based gravity prospecting of the Earth, Moon, and Mars shows that the artificial satellite trajectory is quite sensitive to gravity anomalies of the central body. In our case, the role of the satellite indicating the Sun’s gravity anomalies is played by Mercury. The real indicator of the Sun’s gravity anomaly is the Mercury’s angular discrepancy $\Delta \psi \approx 0.1''$.

The fact that Mercury in its motion is sensitive to the Sun’s gravity anomalies makes it clear to us that, if peculiar features of the Sun deformation are taken into account in the theoretical model of Mercury’s motion, then the “anomalous” shift detected by Urbain Le Verrier merely disappears. Note that this was revealed without involving metaphysics or modifying the Newton’s law on the gravitational interaction of material point.
Actually, the problem of interpretation of “anomalous” shift $\Delta \psi$ reduces to the inverse satellite-based gravity prospecting problem [14] as applied to the Sun and its satellite Mercury.

AFTERWORD

In solving this problem with a “great” history, as well as in solving other problems, we followed the fact that in a case of an unsolvable, at first glance, problem, one should not hurry to involve metaphysics and construct something abstract and immaterial. A Scientist should in any situation remain on the materialistic position and base himself on facts confirmed by independent experiments. May be it looks routinely, but it is indeed necessary to be critical on the existing metaphysical interpretations of observed phenomena, which further become a basis for one or another metaphysical concept, for example, “dark matter.”

If the problem remains unsolved for decades or even centuries, then one should return to the initial point, to the beginning of the way, and try to find new or previously missed facts, conduct test experiments and observations, and check the correctness of the problem definition.

If you desire to return to the truth you do not need to search for the road.
You know it. You came down that way. Retrace your footsteps.

BERNARD OF CLAIRVAUX

Diligently following this principle, we succeeded, as we hope, in solving several problems with history, namely, in revealing the sources of Chandler’s wobble of the Earth’s rotation pole [15], in answering the question of why nobody succeeded in refining the gravitation constant [16], in proving the real existence of the gravitational dipole and formulating the definition of the gravitating mass [17], in explaining physical nature of the “plateau” in the galaxy rotation curve without using such a metaphysical concept as “dark matter” [18].

CONFlict of interest

The authors declare no potential conflict of interests.

ORCId

Dmitry G. Kiryan https://orcid.org/0000-0001-6518-0178

REFERENCES

[1] Le Verrier, U.J.: Théorie du mouvement de Mercure. Annales de l’Observatoire de Paris 5, 1 (1859)
[2] Einstein, A.B.: Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie. In: Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin, Sitzung der physikalisch-mathematischen Classe, pp. 831–839. (1915)
[3] Einstein, A., Infeld, L.: The Evolution of Physics: The Growth of Ideas From the Early Concepts to Relativity and Quanta. Cambridge University Press, Cambridge (1938)
[4] Pauli, W.: Relativitätstheorie. Teubner, Lpz. (1921)
[5] Roseveare, N.T.: Mercury’s Perihelion, from Le Verrier to Einstein. Oxford University Press, USA (1982)
[6] Levantovsky, V.I.: Elementary Space Flight Mechanics. NAUKA, Moscow (1980)
[7] Aksenov, E.P.: Theory of Motion of Artificial Earth Satellites. NAUKA, Moscow (1977)
[8] Bowler, M.G.: Gravitation and Relativity. Pergamon Press, Oxford, New York (1976)
[9] Kulikovsky, P.G.: Handbook for Amateur Astronomer. Gos. Izdat. Tekh.-Teor. Liter, Moscow (1953)
[10] Poincaré, H., Dreyfus, L.: Figures D’équilibre D’une Masse Fluide: Leçons Professées à la Sorbonne en 1900. Cours de la Faculté des sciences de Paris. Cours de physique mathématique. In: Naud, C. (ed.). Gauthier-Villars, Paris (1902)
[11] Krat, V.: Equilibrium Figures of Celestial Bodies. Gos. Izdat. Tekh.-Teor. Liter, Moscow (1950)
[12] Grushinsky, N.P.: Basic Principles of Gravimetry. NAUKA, Moscow (1983)
[13] Torge, W.: Gravimetry. Walter de Gruyter GmbH, Berlin, New York (1989)
[14] Mironov, V.S.: Course of Gravity Prospecting. NEDRA, Leningrad (1980)
[15] Kiryan, D., Kiryan, G.: Moon’s periogee mass as a missing component of the Earth’s precession-nutation theory. PAMM 14, 49–60 (2014)
[16] Kiryan, D.G., Kiryan, G.V.: Decimal-by-decimal analysis of the gravitational constant value as exemplified by torsion balance. E-prints https://vixra.org/pdf/1708.0245v3.pdf 12, (2017)
[17] Kiryan, D.G., Kiryan, G.V.: Gravitational interaction in the medium of non-zero density. E-prints https://vixra.org/pdf/1406.0128v2.pdf (2014)

[18] Kiryan, D.G., Kiryan, G.V.: Modeling the Evolution of a gravitating bodies cluster based on absolutely inelastic collisions. E-prints https://arxiv.org/abs/1811.06372v5 (2018)

How to cite this article: Kiryan, D.G., Kiryan, G.V.: On the effect of the central body small deformations on its satellite trajectory in the problem of the two-body gravitational interaction. Z. Angew. Math. Mech. 102, e202000326 (2022). https://doi.org/10.1002/zamm.202000326