Decuplets, glueballs and nonet anomalies

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Abstract

A new approach to problem of glueball search is presented. It refers to early J. Rosner’s attempts to detect the tensor glueball. In the present description the glueball state is treated equally with singlet $q\bar{q}$ one. Mixing glueball with $q\bar{q}$ nonet creates decuplet. Glueball can be detected as its component. Our approach is based on hypothesis of vanishing exotic commutators described as VEC model. The model describes all multiplets of light mesons. This makes possible to compare the mass patterns of different multiplets. According to VEC description some abnormal nonets can be interpreted as incomplete decuplets. This makes possible to relate the anomaly of the nonet to a glueball component of decuplet. The model reflects rich diversity of strong interaction properties. The treatment presented is quite elementary: only masses of physical states are required to be known.
The pure glueball meson is not necessary

In current opinion the quark-gluon picture [1] well describes strong interactions. According to this picture the mesons are built out of quark (q) and antiquark (\(\bar{q}\)) which are coupled by gluon (g) exchange. It is supposed that this picture is valid for mesons having any signature \(J^{PC}\) and in any mass region. This feature is described as universality of the quark-gluon picture. The hypothetic g is quark-less, flavor-less electrically neutral particle. It has the property of self-interaction which implies the existence of bound states of two or more gluons. Such an object is called glueball (G) and is a singlet state of SU(3) symmetry. Hence, G can interfere with \(q\bar{q}\) singlet and isoscalar octet states.

The quark-gluon picture of strong interaction would be confirmed by the existence of G. This stimulates its experimental search. However, the hypothetic properties on G are too scanty for the needs of experimental investigation. The investigation turned out to be very difficult and during almost half a century did not provide satisfactory result. Desirable effect would be to detect a separate particle being pure G state. Such an object has not been found so far but during investigation considerable collection of particles which possibly are not \(q\bar{q}\) states was discovered. These particles are not pure G but probably include its considerable component (see [2] for the most recent reviews). The quest for G is still continued.

Although the pure G meson is not observed one cannot claim that G states do not exist. This may simply mean that the particles which are pure G states are unobservable. To be observable G must interact with other hadrons. For this to be the case it should have the ability to mix with larger unitary multiplet. Being mixed with nonet of \(q\bar{q}\) states it forms a decuplet. Then it is subjected to restrictions imposed on decuplet components by SU(3) symmetry G can be detected as one of the three interfering unphysical components creating the physical isoscalar states of decuplet. If it dominates one of the states then this state may be considered as the "G candidate". Hence, the existence of the G state can be established on the ground of unitary symmetry and observation of a pure G is not necessary.

The unitary symmetry is perceptible due to the property of mesons to form multiplets which are collections of particles having different but definite flavors and the same signature \(J^{PC}\). They form octets (O), nonets (N) and decuplets (D). Each multiplet is described by some representation of unitary symmetry SU(3). The SU(3) multiplets are analogical to the SU(2) ones describing electromagnetic interaction. However, the mass differentiation within SU(2) submultiplets of SU(3) multiplets is usually neglected, therefore, this symmetry is considered exact. In the strong interactions the
multiplets gather the particles having \emph{a priori different} masses. We call them “multiplets of broken $SU(3)$ symmetry”. It is believed that the symmetry breaking announces the interaction.

Several interactions can break $SU(3)$ symmetry. The most apparent effect of breaking is the difference between isotriplet and isodoublet masses of the particles belonging to the same multiplet (e.g. $\pi$ and $K$ mesons). This difference is attributed to nonperturbative $g$ interaction and cannot be calculated. However, the effect of this interaction can be described in the phenomenological approach by experimentally verifiable Gell-Mann - Okubo (GMO) formula for octet mesons. This formula fits the data and has the property of universality. We call this procedure the GMO-breaking $^1$.

Other breakings are much weaker and are masked by GMO one. They can be exhibited if the description of the multiplet complies with GMO requirement. Such a GMO-restricted multiplet is named the multiplet of \emph{flavor symmetry}. The effect of its breaking is described as \emph{anomaly}. The anomalies can be observed in the flavor multiplets larger than octet. Anomalies of flavor symmetry comprise information on the unknown interactions which we want to investigate. They can be observed through anomaly of multiplet mass pattern. However, they can be also seen due to pattern difference between two (or more) identical multiplets differing only by $J^{PC}$ signatures or belonging to various mass regions. Perhaps the investigation of anomalies is the most direct way from observation to understanding.

We begin with the question of how the anomalies can be recognized. In the next two sections we remind the VEC description of the light meson (LM) multiplets and define the benchmark multiplet which provides the pattern for anomalies search.

2 \hspace{0.5cm} \textbf{VEC description of light meson multiplets}

The approach refers to phenomenological investigations performed during the eighties of the twentieth century. They were devoted to the search of G as an object causing deformation of $2^{++}$ and $0^{-+}$ nonet structures $^3$. These attempts did not clarify much as they were premature. However, the very idea of such line of investigation cannot be questioned. The problem is in its actual realization. Now we have much larger sample of data and more tools for their analysis. Therefore, it is probably a right time to come back to these ideas.

$^1 K - a$ determines all differences between masses of the octet states as $3(x_8 - K) = K - a$
An opportunity for such an approach to be successful is provided by the model of VEC\(^2\) which describes all multiplets of LM using the system of master equations (ME) \([4, 5, 6]\)

\[
\sum_{i=1}^{n} l_i^2 x_i^r = \frac{1}{3} a^r + \frac{2}{3} b^r, \quad b \equiv 2K - a, \quad r = 0, 1, 2, \ldots
\]  

(1)

where \(r\) is power index. Particle symbols of \(x_i, a, K\) stand for the mass squared of physical mesons, \(l_i\) is the amplitude of octet content of isoscalar state \(x_i\):

\[
|x_8\rangle = \sum_{i=11}^{n} l_i |x_i\rangle.
\]  

(2)

Since isoscalar octet state \(x_8\) and physical \(x_i\) states describe uncharged states, the \(l_i\) are real numbers, hence

\[
l_i^2 > 0, \quad i = 1, 2, \ldots n.
\]  

(3)

The numbering of the isoscalar mesons is chosen so that \(i\) increases with growing mass:

\[
x_i < x_{i+1}.
\]  

(4)

ME are linear with respect to unknown \(l_i^2\)'s. The solutions of such systems of equations are well known. We are looking for solutions which satisfy the conditions of positivity (3). The \(l_i^2\)'s are functions of masses \(a, b, x_1, \ldots, x_n\). Conditions (3) which restrict these masses help to test the affiliation of the a given set of particles to the supposed multiplet. So they provide the criterion of relevant multiplet existence. Knowing the positive solution \(l_i^2\) of ME we can diagonalize the mass operator of the multiplet and determine its wave function. We thus have fully described the broken SU(3) multiplet in terms of physical masses \([6]\).

If we want to describe the flavor multiplet we should take into account the GMO-breaking. As argued above, this can be achieved at phenomenological level of description by accepting observed values of \(a\) and \(b\). This property distinguishes the \((a, b)\) from other masses and appoints them to the role of basic input. Below we use also property that for a given pair \((a, b)\) ME may predict the existence of multiplets including several number of states \(x_i\). Therefore, the masses \((a, b)\) can be viewed as "theory constants" while \(x_i\)'s as "parameters".

VEC predicts the existence of D \([6, 7]\). The mesons \(a, b, x_1, x_2, x_3\) belong to \(D\) if they fit criteria (3). Then the mass operator can be diagonalized

\[^2\text{Model of vanishing exotic commutators (formerly described as ECM model [4]).}\]
and the wave function of $D$ can be constructed. The wave function is expressed completely in terms of masses of $D$ particles. The $G$ state can be distinguished and its unphysical mass can be determined \[6\]

The determination of wave function requires very accurate data on masses. This is the merit of description, not its fault, as it transforms small mass differences into remarkable variation of the D shape. However, excessive sensitivity can weaken predictive power of the procedure. Therefore it is desirable to have also simpler criteria. They can be also formulated within the VEC model. We explain below how this can be done but begin from presenting some further features of ME which justify the procedure we propose.

3 Varieties of flavor multiplets

VEC predicts several multiplets which arise from solution of ME. The description of multiplets which include $n$ isoscalar mesons $x_i$ requires solving of the ME with respect to unknown quantities $l_i^2$. To calculate the $l_i^2$’s we need the set of equations ME for $r = 0, 1, \ldots, (n - 1)$. However, this is merely the minimal system of ME describing this multiplet.

The same multiplet can be described by larger ME system provided this system satisfies some solvability conditions. We use a particular form of such conditions which is suggested by open structure of the ME set. We take into account subsequent ME for $r = n, n + 1, \ldots$. The calculated $l_i^2$, expressed as the functions of the multiplet masses, should be inserted into these equations. It may happen that one of the equations (say, for $r = n$) is satisfied by these masses. Then this equation becomes the mass formula (MF) of the multiplet. Obviously, a multiplet may have more than one MF.

The MF arises due to restriction on the masses of $x_i$. Therefore, the number of MF cannot exceed $n$. The actual number $k$ of MF $(0 \leq k \leq n)$ should be determined from data fit for each multiplet separately. The number of ME to be considered for such a multiplet is $n + k - 1$.

The multiplets built on the same base $(a, b)$ and having the same $n$ but different $k$ are independent and have different patterns. They are considered as varieties of the same multiplet and marked by its currently used name indexed by $k$. The very existence of different varieties of the multiplet testifies the existence of various interactions influencing the structure of multiplet.

There may exist three $N$ multiplets $(N_0, N_1, N_2)$ and two $D$ multiplets $(D_0, D_1)$ which can be built out on the same $(a, b)$ basis.
4 Ideal nonet as a pattern

Nonet arises due to the mixing of octet isoscalar state with an SU(3) singlet and is described as ME multiplet for \( n = 2 \). Three old standing varieties of \( N \) are known:
- \( N_0 \) - known as Gell-Mann - Okubo (GMO-nonet) having no MF
- \( N_1 \) - described as Schwinger (S-nonet) having one MF
- \( N_2 \) - ideal (I-nonet) having two MF

The I nonet is described by system of ME:

\[
\begin{align*}
l_1^2 + l_2^2 &= 1, \quad (5a) \\
l_1^2 x_1 + l_2^2 x_2 &= \frac{1}{3}a + \frac{2}{3}b, \quad (5b) \\
l_1^2 x_1^2 + l_2^2 x_2^2 &= \frac{1}{3}a^2 + \frac{2}{3}b^2, \quad (5c) \\
l_1^2 x_1^3 + l_2^2 x_2^3 &= \frac{1}{3}a^3 + \frac{2}{3}b^3. \quad (5d)
\end{align*}
\]

Solving the first two equations we calculate \( l_1^2, l_2^2 \) as the functions of masses. Next, substituting \( l_i^2 \)'s into third and fourth equations we obtain two MF’s which determine the masses of mesons \( x_1, x_2 \). These MF’s can be transferred to more familiar form:

\[
x_1 = a, \quad x_2 = b, \quad l_1^2 = \frac{1}{3}, \quad l_2^2 = \frac{2}{3}; \quad |x_1 > = |a >, \quad |x_2 > = |b > . \quad (6)
\]

where \( |a >, |b > \) are the nonet basic states:

\[
a = \frac{1}{\sqrt{2}}(u \bar{u} + d \bar{d}), \quad b = s \bar{s}. \quad (7)
\]

This solution is determined by GMO breaking mechanism acting alone on the nonet states. It describes the shape of the nonet in a very simple way and is universal. Therefore, it can be used as a pattern for searching anomalies of flavor multiplets.

5 \( D_1 \) contents of the S-nonet components

- \( D \) includes three physical isoscalar states \( x_i \) which can be represented as a superpositions of the base states \( a, b, G \)
The multiplet \( D_1 \) is determined by the solution \( l_i^2 \) (\( i=1,2,3 \)) of the system of four ME

\[
l_1^2 + l_2^2 + l_3^2 = 1, \quad (8a)
\]
\[
l_1^2 x_1 + l_2^2 x_2 + l_3^2 x_3 = \frac{1}{3} a + \frac{2}{3} b, \quad (8b)
\]
\[
l_1^2 x_1^2 + l_2^2 x_2^2 + l_3^2 x_3^2 = \frac{1}{3} a^2 + \frac{2}{3} b^2, \quad (8c)
\]
\[
l_1^2 x_1^3 + l_2^2 x_2^3 + l_3^2 x_3^3 = \frac{1}{3} a^3 + \frac{2}{3} b^3. \quad (8d)
\]

One can show that \( D_1 \) can be represented as superposition of \( N_2 \) and some SU(3) singlet. The nature of the singlet is undetermined. The VEC requirements accept all "extending singlets" which are usually mentioned like G, hybrid or multiquark state. The singlet G is favored as only this state is supposed to have the property of universality. The MF is:

\[
(x_1 - a)(x_2 - a)(x_3 - a) + 2(x_1 - b)(x_2 - b)(x_3 - b) = 0. \quad (9)
\]

Combining (9) with the criterion (3) we find that the masses of \( D_1 \) are subjected to further restrictions: the masses of the \( D_1 \) have to satisfy the mass ordering rule (MOR) \[6\]

\[
x_1 < a < x_2 < b < x_3. \quad (MOR - D_1) \quad (10)
\]

MOR-\( D_1 \) divides accessible region of \( x_i \) mesons into three isolated subregions which are separated by \( a \) and \( b \). In each of the subregions the states \( x_i \) are uniformly dominated by \( a \), \( G \), \( b \).

\[
x_1 \sim a, \quad x_2 \sim G, \quad x_3 \sim b. \quad (11)
\]

Therefore, it is convenient to introduce another notation

\[
x_a \doteq x_1, \quad x_G \doteq x_2, \quad x_b \doteq x_3. \quad (12)
\]

which makes MOR-\( D_1 \) still more transparent:

\[
x_a < a < x_G < b < x_b. \quad (13)
\]

• S-nonet (\( N_1 \)) is considered to be a firmly established multiplet. It is announced for many \( J^{PC} \) mesons and dominates perception of LM spectroscopy. However, the constituents of its isoscalar components and diversity of S-nonet shapes remain vague. We argue that these problems arise from G mixing.
$N_1$ is described by the first three equations of the system (5). Its MF is

$$(a - x_1)(a - x_2) + 2(b - x_1)(b - x_2) = 0.$$  \hspace{1cm} (14)$$

As the S-nonet has one MF we need an extra information on the masses of $x_1$, $x_2$ mesons for evaluating $l_1^2$, $l_2^2$. One can use for that the known value of one of the masses and calculate the other one with the help of MF. We can see that the pair of masses determined this way is different from the values of masses of the I-nonet (6). The change of S-nonet masses $x_1$, $x_2$ relative to the I-nonet ones shows the anomaly. It is thus compatible with the existence of an extra state.

The components of $N_1$ have to comply with one of the two MOR conditions [8]

$$(a) \quad a < x_1 < b < x_2, \quad (MOR - N_{1(a)}),$$  \hspace{1cm} (15a)$$

$$(b) \quad x_1 < a < x_2 < b, \quad (MOR - N_{1(b)}).$$  \hspace{1cm} (15b)$$

These conditions determine two completely different nonets which we describe as $N_{1(a)}$ and $N_{1(b)}$ ones. Both of them are observed.

If $N_1$ is built out on the same $(a, b)$ base as $D_1$ then the MOR’s [13] may be considered as incomplete MOR-$D_1$ [13]. The comparison shows that

— if $x_1, x_2 \in N_{1(a)}$ then they are dominated by $(G, b)$ components of $D_1$,
— if $x_1, x_2 \in N_{1(b)}$ then they are dominated by $(a, G)$ components of $D_1$, respectively.

Both types of the $N_1$ include $G$ state. Therefore, the very existence of $N_1$ justifies the existence of $G$ which can be only seen as the state of $D_1$. This suggests that it plays an essential role in the structures of $D_1$. Perhaps within this multiplet the suitable $G$ is always “ready for use” since it is built of the gluons which mediate interactions between quarks which are present there. This is the way $G$ becomes a constituent of the isoscalar mesons of $D_1$.

The current description of $N$ does not explain the origin of the S-nonet anomalies. Moreover, $N$ themselves are distinguished by the results of biased experiments ignoring the possibility of $D$ appearance. Perhaps the results of these measurements should be reanalyzed. Also extending these experiments and increasing their accuracy is necessary. It is possible that the nature of these multiplets has been for a long time misunderstood. The explanation of this confusion may have far-reaching implications for meson spectroscopy. Some of the implications can be seen immediately.

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$^3$The current description of $N_1$ as S-nonet uses the mixing angle $\vartheta$ for determination the isoscalar states. The allowed regions of $\vartheta$ are: for $15a$ $\tan^2 \vartheta > \tan^2 \vartheta^{id}$ and for $15b$ $\tan^2 \vartheta < \tan^2 \vartheta^{id}$, where $\vartheta^{id} = 35.26^\circ$ is ideal mixing angle [9].
6 Unrecognised glueballs and missing mesons

• We have established that all S-nonets include $x_G$ dominated state. Reviewing the particle data \[10\] we find that the mesons $f_1(1285)$, $h_1(1380)$, $\eta(1405)$, $f_2(1430)$ should be $G$ dominated. The decay modes of $f_1(1285)$ and $h_1(1380)$ do not contradict these assignments; the $G$ dominated structure of $\eta(1405)$ meson established earlier \[11, 7\] is now confirmed; the $f_2(1430)$ should have $G$ dominated structure if it exists \[6\].

• The old standing puzzle of exceptional properties of $f_1(1230)$ $m = 1230 \pm 40 MeV$, $\Gamma = 250 \div 600 MeV$ \[17\] is solved by changing its affiliation from $N_1$ to $D_1$. If this signal belongs to $D_1$ then it should be attributed to two different particles: isosinglet meson $x_a$ and isotriplet meson $a_1$:

$$x_a, \quad a_1$$

which have similar modes of decay.

• The observed axial-vector mesons are collected into the $N_1$ multiplets having $1^{++}$ and $1^{-+}$ (described as $N_{1A}$ and $N_{1B}$), where instead of the physical $K_1(1270)$ and $K_1(1400)$ stand their C-even or C-odd combinations:

$$K_{1A} = K_1(1270)\cos\phi - K_1(1400)\sin\phi,$$

$$K_{1B} = K_1(1270)\sin\phi + K_1(1400)\cos\phi$$

Joint MF’s analysis of data on $N_{1A}$ and $N_{1B}$ gives the following values for these masses \[9\]

$$K_{1A} = (1340 \pm 8)^2 MeV^2,$$

$$K_{1B} = (1324 \pm 8)^2 MeV^2.$$
7 Call for new data

Anomalies of the S-nonets provide the evidence for the existence of further (beyond GMO) mechanisms of SU(3) breaking. The anomalies are caused by interactions which are unknown. It is just a purpose of the S-nonet investigation to recognize their nature. The anomalies provide much weaker breaking than the GMO one. This requires much more accurate data to make them observable. The present data (partly old and skimpy) enable us to select few nonets but are insufficient for completing decuplets. Therefore, for the sake of present and future development of meson spectroscopy it is necessary to increase accuracy of the data and extend the measurements to other $J^{PC}$ mesons.

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