The masses of $D^{*}_{sj}(2317)$ and $D^{*}_{sj}(2463)$ in the MIT bag model

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The masses of the $D^{*}_{sj}(2317)$ and $D^{*}_{sj}(2463)$ were found in the MIT bag model in a reasonable agreement with the experimental values. The spectrum of $0^-, 1^-$ for $D$ and $B$ mesons is presented. The equalities between the mass splitting $(1^+ - 1^-)(c\bar{q}) \approx (1^+ - 1^-)(\bar{c}s)$ and $(0^+ - 0^-)(c\bar{q}) \approx (0^+ - 0^-)(\bar{c}s)$, where $q = u, d$ are confirmed despite of the fact that $SU(3)_L \times SU(3)_R$ chiral symmetry is broken by the non-zero strange quark mass.

I. INTRODUCTION

A new scalar meson with the $c\bar{s}$ flavor content and a mass around 2.32 GeV has been recently discovered by the BaBar collaboration [1] as a narrow resonance in the $D_s \pi^0$ decay channel and confirmed by the CLEO collaboration [2]. This state aroused a particular interest because of a plain conflict with the theoretical calculations which predict the mass about 100 – 300 MeV above its experimental value [3]. This discrepancy opened a possibility for more refine explanations of the new state [4]. However, it is not clear, wea ther the difference between the predictions and experiments rises a real issue or it is rather a problem of the models. Besides all that, there are successful explanations of the new state [4].

II. MASS CALCULATION

The mass of the bag can be written in the form [13]:

$$M = E_Q + E_q + E_{bag} + E_0 + E_M + E_E$$

(1)

where $E_Q(q)$ is the energy of the heavy(light) quark, $E_{bag} = \frac{4}{3}\pi BR^3$ is the volume energy of the spherical bag of the radius $R$, $E_0 = Z/R$ is the Casimir energy and the last two terms describe the energies of the color-magnetic and color-electric interactions between quarks. Let us first notice that for the heavy-light systems the center of mass of the bag is well defined. It coincides with the heavy quark position and the corrections due to the c.m. motion can be neglected. The energy of the heavy quark is approximated by the formula $E_Q = m_Q + P_{light}^2/2m_Q$, where $m_Q$ is the mass of the heavy quark and the second term is the non-relativistic kinetic energy. The total momentum of the bag in the c.m. frame should be zero thus the values of the heavy and light quark momenta are equal. Let us notice that the non-relativistic kinetic energy term in $E_Q$ was neglected in the previous calculations [12, 13]. This is justified if one wishes to study the lowest energy states. However for exited states, the contribution of the kinetic energy of the heavy quark is more important and should be included. The energy of the color-magnetic interactions between the heavy and light quarks is equal to [12, 13]

$$E_M = \frac{\lambda A}{m_Q R^2} \langle \vec{s}Q \vec{s}_q \rangle$$

(2)

where $\lambda = \{1, 2\}$ for barions and mesons respectively, $A$ is calculated from the light quark wave-function and depends on its mass. The constant

$$\alpha_s = \frac{0.47}{\log(1 + 1/\Lambda R)}, \quad \Lambda = 0.42 \text{ GeV}$$

(3)

is inspired by the QCD running coupling formula [13, 14]. The spin-spin interaction term $\langle \vec{s}Q \vec{s}_q \rangle = \{-3, 1\}$ for scalar and vector mesons respectively. There are two sources which contribute to the color-electric energy: the interactions between quarks and their self-interactions. For mesons with one heavy quark it can be written as [12]:

$$E_E = -\frac{4\alpha_s}{3R} \left( C + \frac{1}{2} \right)$$

(4)
TABLE I: Masses of pseudoscalar and vector mesons.

| flavors | $J^P$ | Exp. mass MeV | bag mass GeV | flavors | $J^P$ | Exp. mass MeV | bag mass GeV |
|---------|-------|---------------|--------------|---------|-------|---------------|--------------|
| $D$     | $c\bar{q}$ | 0$^-$         | 1.868        | $B$     | $b\bar{q}$ | 0$^-$         | 5.278        |
| $D^*$   | $c\bar{q}$ | 1$^-$         | 2.008        | $B^*$   | $b\bar{q}$ | 1$^-$         | 5.327        |
| $D_0$   | $c\bar{s}$ | 0$^-$         | 1.969        | $B_0$   | $b\bar{s}$ | 0$^-$         | 5.363        |
| $D_0^*$ | $c\bar{s}$ | 1$^-$         | 2.099        | $B_0^*$ | $b\bar{s}$ | 1$^-$         | -            |

TABLE II: Masses of scalar and pseudovector mesons.

| flavors | $J^P$ | Exp. mass MeV | bag mass GeV | flavors | $J^P$ | Exp. mass MeV | bag mass GeV |
|---------|-------|---------------|--------------|---------|-------|---------------|--------------|
| $D_1$   | $c\bar{q}$ | 0$^+$         | 2.200        | $B$     | $b\bar{q}$ | 0$^+$         | 5.576        |
| $D_1^*$ | $c\bar{q}$ | 1$^+$         | 2.383        | $B_1^*$ | $b\bar{q}$ | 1$^+$         | -            |
| $D_{1s}$| $c\bar{s}$ | 0$^+$         | 2.288        | $B_s$   | $b\bar{s}$ | 0$^+$         | -            |
| $D_{1s}^*$ | $c\bar{s}$ | 1$^+$         | 2.465        | $B_{1s}^*$ | $b\bar{s}$ | 1$^+$         | -            |

where $C$ can be calculated and depends on the mass of the light quark. The second term in the parenthesis comes from the self-interaction of the heavy quark. The heavy quark self-interaction is divergent and the infinite term is omitted in $^{11}$ $^{12}$. The radius of the bag is determined by minimizing the formula $E' = E_{bag} + E_0 + E_q$ with respect to $R$.

The value of the bag constant $B^{1/4} = 0.1383$ and the Casimir parameter $Z = -0.73$ are taken from the paper $^{13}$. The masses of the quarks $m_u = m_d = 0$, $m_s = 0.198$ GeV, $m_c = 1.824$ GeV, $m_b = 5.21$ GeV were determined to reproduce the masses of the mesons $D, D_s, B$. Using model parameters in the $0^+, 1^-$ sector one finds the light quark energy $E_q = 2.04/R$ for the $u, d$ quarks and $E_q = \sqrt{m_q^2 + (3.15)^2}/R$ for the strange quark. The radius of the non-strange meson is $R = 4.1$ GeV$^{-1}$ and for the strange meson $R_s = 4.2$ GeV$^{-1}$. The color-magnetic interaction parameter is $A(m_q = 0) = 0.52$ and $A(m_s) = 0.5$ whereas for the color-electric part one finds $C(m_q = 0) = 0.52$ and $C(m_s) = 0.62$. These numbers together with the equations $^{11}$ $^{12}$ give the spectrum written in the Table 1.

For the scalar and pseudovector mesons, where the light quark is in the $1P_{1/2}$ state, the light quark energy is $E_q = 3.81/R$ for the $u, d$ quarks and $E_q = \sqrt{m_q^2 + (3.91)^2}/R$ for the strange quark. The radii are $R = 5.09$ GeV$^{-1}$ for the non-strange and $R_s = 5.08$ GeV$^{-1}$ for the strange mesons. The color-magnetic interaction parameter is $A(m_q = 0) = 0.87$ and $A(m_s) = 0.84$ whereas the color-electric parameter takes values $C(m_q = 0) = 0.88$ and $C(m_s) = 0.79$. The masses of the $0^+, 1^+$ states are given in the Table 2.

The mass splitting relations $\Delta M_{1+} = \Delta M_{0+}$ $^{2}$ $^{3}$ (where $\Delta M_{1+} = M_{1+} - M_{1-}$ and so on) and $\Delta M_{1+0+} = \Delta M_{1-0}$ $^{2}$ $^{3}$ should hold in the chiral limit. Indeed the experiment finds $^2$ $\Delta M_{1+} - \Delta M_{0+} = 1.2 \pm 2.1$ MeV and $\Delta M_{1+} - \Delta M_{1-} = 1.6 \pm 2.1$ MeV for the $c\bar{s}$ mesons. The MIT bag model fails to reproduce the mass splitting equalities at this level of accuracy. This can be expected because the MIT bag model does not handle the chiral symmetry in the appropriate way. In spite of that the model predicts that the following relations hold (at the level of 10 – 20 MeV) independently of the flavor of the light quark:

$$\Delta M_{1+} - (c\bar{q}) \approx \Delta M_{1+} - (c\bar{s})$$  \hspace{1cm} (5)
$$\Delta M_{1+} + (c\bar{q}) \approx \Delta M_{1+} + (c\bar{s})$$

where $q = u, d$. These relations follow from the $SU(3)_L \times SU(3)_R$. However this symmetry is broken by the non-zero, strange quark mass and it is not clear why it should work so well. Let us stress that from the relations $^3$ only the last one is confirmed by the experiments at the accuracy of 1 – 2 MeV. Other relations are the assumptions based on the $SU(3)_L \times SU(3)_R$ chiral symmetry or follow from the model calculations.

Finally, to be clear, let us notice that the considered MIT bag model joins several features of works $^{12}$ $^{13}$. The parameters $B, Z$ as well as the description of the coupling $\alpha_s$ comes from the paper $^{13}$. The treatment of the infinity in the electric heavy quark interaction follows paper $^{12}$. The heavy quark kinetic term was also added. The quark masses are adjusted anew to reflect the changes in the model.
III. CONCLUSIONS

The result of the previous paragraph, at the face value, can be regarded as a success of the MIT bag model. The description of the meson spectrum (similar for baryons [11, 12, 13]) at the level of 4-5 per cent is already at the edge of the reliability of the model.

However the more careful scrutiny leads to additional questions. Let us first notice that the quark masses are not treated on an equal footing. The \( m_u, m_d, m_s \) are current masses whereas \( m_c, m_b \) are constituent masses. This follows from the treatment of the heavy quark self-interaction in the color-electric energy. The infinite term is absorbed to the mass of the heavy quark. Secondly, the coupling constant at the energy scale \( 1/R = 0.25 - 0.1 \) GeV takes values \( \alpha_s = 1.0 - 1.2 \). This is certainly not a small parameter. Indeed one can try to fit a new set of the bag model parameters with current quark masses and a small coupling constant. First, the infinite term could be cut-off by the strange quark mass and it is not obvious why the relations (5) should hold.

In the further work one can try to include the light quark in the \( 1 \pm \) state. However the calculation of the \( 1P_{3/2} \) state is more involved and require reconsideration of the model assumptions (e.g. sphericity of the bag) thus demanding much more extensive elaboration then this short note assumed.

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\[ \text{[19]} \text{ One can include the kinetic part of the heavy quark energy in } E'. \text{ The change in the radius can be absorbed by the changes of the parameters of the model with no important impact on the spectrum. The discussion of the interaction treatment for } R \text{ determination can be found in 12.} \]
\[ \text{[20]} \text{ For example } B^{1/2} = 1.383 \text{ GeV, } Z = -1.85, m_u = m_d = 0, m_s = 0.143 \text{ GeV, } m_c = 1.482 \text{ GeV, } m_b = 4.6 \text{ GeV gives a relatively good fit to the lowest energy hadron spectrum. The energy scale for the heavy-light quarks interactions is set to } \sqrt{m_Q E_q}. \text{ The infinite term is regularized by the Compton wavelength of the heavy quark. The author tried different}} \]
sets of parameters with reasonable assumptions about the energy scale of the interactions and regularization methods with similar success.