Magnetic-Flux-Flow Diagrams for Design and Analysis of Josephson Junction Circuits

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Abstract—Josephson junction circuits, such as superconducting quantum interference devices (SQUIDs) and single-flux-quantum circuits, have been successfully applied in both the analog and digital domains. Due to phase-based working principles, their analyses are not well supported by charge-based analysis methods and conventional circuit diagrams. This article introduces a type of magnetic-flux-flow (MFF) diagrams to simplify the design and analysis of Josephson junction circuits. MFF diagrams are the graphical expression of circuit equations derived by loop analysis methods. They vividly depict the functions of Josephson junctions and superconducting loops and visualize how Josephson junctions transfer flux quanta among superconducting loops. The applications of MFF diagrams in the design and analysis of typical Josephson junction circuits are demonstrated case by case; it is shown that MFF diagrams are complementary to conventional circuit diagrams for both Josephson junction circuits and normal electric circuits.

Index Terms—Josephson junction circuit, magnetic-flux-flow diagram, single flux quantum (SFQ), superconducting quantum interference device (SQUID).

I. INTRODUCTION

Josephson junctions have been used to develop various ultralow-noise, power-efficient, and high-speed superconducting circuits [1] that are successfully applied in both the analog and digital domains. Superconducting quantum interference devices (SQUIDs) are the first type of practical Josephson junction circuits [2]; they are ultrasensitive flux-to-voltage converters widely used in magnetic field measurements [3], [4]. Single-flux-quantum (SFQ) circuits are ultrahigh-speed and low-power Josephson junction digital circuits [5], [6], promising for the next-generation signal processing and computing systems [7]. Moreover, Josephson junctions have also enabled quantum electromagnetic circuits for quantum computing [8], [9].

Typical Josephson junction circuits, such as the dc SQUID and the basic SFQ logic circuits [10], are illustrated in Fig. 1. They are Josephson junction networks connected through superconducting wires. Due to the Josephson effects [11] and fluxoid quantization phenomena [12], they are distinct from normal resistor–inductor–capacitor (RLC) circuits and are usually analyzed by using macroscopic phases as variables. A nodal voltage \( v_n \) \((n = 1, 2, \ldots)\) is expressed with a nodal macroscopic phase \( \varphi_n \), namely \( d\varphi_n/dt = 2\pi v_n/\Phi_0 \). Accordingly, the voltage-based modified nodal analysis (MNA) method [13] is transformed into the phase-based MNA method [14] by using nodal phases or the integrals of nodal voltages as variables. With this phase-based MNA method, the simulation program with integrated circuit emphasis (SPICE) tools [15], [16], [17], [18] special for Josephson junction circuits are developed on the basis of the conventional SPICE. These tools can set up circuit equations with the netlist captured from circuit diagrams automatically and calculate the real-time voltages and currents of all the Josephson junctions with numerical solutions.

In the results calculated by the MNA method, nodal phases in SFQ circuits keep increasing with the voltage pulses of Josephson junctions, whereas the voltage pulses are transient signals that remain zero in steady states. A single nodal phase cannot directly indicate the number of flux quanta trapped in loops, and a single voltage pulse cannot indicate the “0” and “1” states of SFQ logics according to the voltage levels. SFQ logics have to be identified from voltage pulses with the synchronization of clock pulses [19], which are unfamiliar to electronic engineers.
who get used to the logic defined with voltage levels in the complementary metal–oxide–semiconductor (CMOS) circuits.

The above-mentioned problems are caused by the mismatch between Josephson junction circuits and conventional circuit diagrams. According to the charge–flux duality [20], one electric circuit has two pictures [21], as illustrated in Fig. 2; it is viewed as an electric charge distribution system where electric charges are flowing from node to node and is also viewed as a magnetic-flux distribution system where magnetic fluxes are transferred from loop to loop. Josephson junction circuits achieve unique flux-effect functions because of the fluxoid quantization effect in loops and are working more like magnetic-flux distribution systems. However, conventional circuit diagrams emphasize the connections of circuit elements at nodes; to cooperate with the MNA method, they deconstruct circuit loops into branches and internalize the flux inputs of loops, such as the trapped fluxes, externally applied fluxes, and mutually coupled fluxes, using virtual sources, inductors, and transformers in branches. For Josephson junction circuits, conventional circuit diagrams and the MNA method focus on depicting the electric charge exchanges between superconducting loops less obvious.

In this article, we introduce a type of magnetic-flux-flow (MFF) diagrams to depict the dynamics of Josephson junction circuits working as the magnetic-flux distribution system. The MFF diagram of a Josephson junction circuit is the graphical expression of the circuit equations derived by loop analysis methods; it depicts the routes of MFFs inside the Josephson junction circuit and visualizes the behaviors of Josephson junctions transferring the flux quanta among loops. MFF diagrams of typical Josephson junction circuits are illustrated in Section III; they vividly interpret how the external flux adjusts the flux-quantum flow in dc SQUID and how the Boolean logics are implemented by the flux-quantum flows inside SFQ circuits.

II. THEORY

A. Circuit Equations of Josephson Junction Circuits

According to the loop analysis methods [20], [22], all the Josephson junction circuits shown in Fig. 1 can be decomposed into a group of independent superconducting loops inserted with Josephson junctions. Assuming that a given Josephson junction circuit consists of $P$ loops and $Q$ Josephson junctions ($P$ and $Q$ are integers), as shown, Fig. 3(a) and (b), we define each loop, namely Loop-$i$ ($i = 1, 2, \ldots, P$), with a loop-current $i_{m_i}$ and a total flux coupled in the loop $\Phi_{m_i}$, corresponding to the external flux input $\Phi_{e_i}$.

The total magnetic fluxes coupled in those superconducting loops are written in matrix as follows:

$$\Phi_m = L_m \cdot i_m - \Phi_e$$  \hspace{1cm} (1)$$

where the vectors $\Phi_m = [\Phi_{m_1}, \ldots, \Phi_{m_P}]^T$, $\Phi_e = [\Phi_{e1}, \ldots, \Phi_{eP}]^T$, $i_m = [i_{m_1}, \ldots, i_{m_P}]^T$, and $L_m$ is the loop-to-loop inductance matrix that describes the self-inductance and mutual inductance of those mutually coupled loops, as illustrated in Fig. 3(c).

Each Josephson junction, namely AJ-$j$ ($j = 1, 2, \ldots, Q$), is an active junction (AJ), which is already biased with a current $I_{bj}$, as illustrated in Fig. 4(a). It generates a phase difference $\theta_j$ at two terminals, corresponding to the branch current $i_{bj}$. Here, we use a nominal flux [21] $\Phi_{bj}$ to redefine the phase difference $\theta_j$ and the voltage $v_{bj}$ as follows:

$$\Phi_{bj} = \frac{\Phi_0 \theta_j}{2\pi}; v_{bj} = \frac{d\Phi_{bj}}{dt}$$  \hspace{1cm} (2)$$

where $\Phi_0$ is the flux quantum, $\Phi_0 = 2.07 \times 10^{-15}$ Wb.

The equivalent circuit of AJ-$j$ is accordingly drawn as shown in Fig. 4(b), referring to the resistively and capacitively shunted junction model [23]. With $\Phi_{bj}$ as the nominal flux output of AJ-$j$, the current-to-flux function of AJs in the matrix is written...
as follows:

\[ F_j(\Phi_j, i_{bj}, I_{b_j}) = 0 \iff \\
\text{C} \cdot \frac{\partial \Phi_j}{\partial t} + \text{G} \cdot \Phi_j + I_0 \cdot \sin \frac{2\pi \Phi_j}{\Phi_0} + I_n = i_{bj} + I_{b_j} \tag{3} \]

where \( \Phi_j = [\Phi_{b1}, \ldots, \Phi_{bQ}]^T, \ i_0 = [i_{b1}, \ldots, i_{bQ}]^T, \ I_n = [I_{n1}, \ldots, I_{nQ}]^T, \ C = \text{diag}\{C_{11}, \ldots, C_{QQ}\}, \)

\( \ G = \text{diag}\{1/R_{11}, \ldots, 1/R_{QQ}\}, \) and \( I_0 = \text{diag}\{I_{b1}, \ldots, I_{bQ}\} \) are the diagonal matrices defined with circuit parameters of AJs. The inputs and outputs of loops and AJs are complied with two circuit laws.

First, the \( \Phi_0 \) of AJs and the \( \Phi_m \) coupled in loops satisfy the flux quantization law \cite{12}, namely

\[ \Phi_m + \sigma \cdot \Phi_0 = n\Phi_0 \tag{4} \]

where \( n = [n_1, \ldots, n_P]^T, \) and \( n_i \) is the number of flux quanta trapped in Loop-\( i \); \( n \) remains constant after loops are cooled in the superconducting state; \( \sigma \) is the loop-to-junction incidence matrix, as defined in Fig. 3(d), which describes how AJs are connected in loops.

Second, the relation between \( i_m \) and \( i_0 \) is defined by Kirchhoff’s current law as follows:

\[ i_0 = \sigma^T \cdot i_m \tag{5} \]

where \( \sigma^T \) is the transpose of \( \sigma \).

### B. Transfer Functions of Loops and AJs

By synthesizing the circuit equations from (1)–(5), we can draw a general system model of Josephson junction circuits, as shown in Fig. 5. In this system diagram, with its \( i_{th} \) supplied by loops, AJ-\( j \) works as a flux pump and generates \( \Phi_{th} \) for the loops it is inserted; the \( v_{th} \) records exactly the flow rate; meanwhile, Loop-\( i \) works as a flux container; it preserves \( \Phi_{mi} \) contributed by the outputs of AJs and generates loop-current \( i_{mi} \) to drive the AJs in the loop.

The transfer function diagram of the current-to-flux function of AJ-\( j (j = 1, 2, \ldots, Q) \) defined in (3) is shown in Fig. 6(a). This transfer function is analogous to the one of a classical particle rolling in a washboard potential \cite{13} \( U_j \), as illustrated in Fig. 7; the potential \( U_j \) is derived as follows:

\[ U_j = \int_0^{\Phi_{th}} (I_{0j} \sin \frac{2\pi \Phi_j}{\Phi_0} - I_{bj}) \cdot d\Phi_{th} \]

\[ = \frac{\Phi_0 I_{0j}}{2\pi} (1 - \cos \frac{2\pi \Phi_j}{\Phi_0}) - I_{bj} \Phi_{th} \tag{6} \]

where the linear part of \( U_j \) is set by the bias current \( I_{bj} \), whereas the cosine part is decided by the critical current \( I_{th} \).

In Fig. 7, the particle in Case-I with \( I_{bj} = 0.3I_{th} \) is stepping forward among the periodical potential valleys, which depicts the behavior of AJs in SFQ circuits. The particle in Case-II with \( I_{bj} = 1.1I_{th} \) is rolling downhill in a sharp decline potential, which simulates the AJs in SQUID circuits.

In Case-I, AJ-\( j \) stays still on the floor of potential valleys if \( i_{th} < I_{TH,j} \), and it will step forward as long as \( i_{th} > I_{TH,j} \). Thus, AJ-\( j \) is turned ON and OFF like a diode, and \( I_{TH,j} \) is its threshold current that is determined by the maximum \( dU_j/d\Phi_{th} \) in (6) as follows:

\[ I_{TH,j} = I_{0j} - I_{bj} \tag{7} \]

This threshold current \( I_{TH,j} \) is adjusted by \( I_{bj} \).

Referring to (1) and (4), the transfer function diagram of Loop-\( i \) is shown in Fig. 6(b), where the flux \( \Phi_{mi} \) is composed of the trapped flux \( n_i \Phi_0 \) and the algebraic sum of the output of AJs
inserted in Loop-i; $\Phi_{mi}$ is normalized with $\Phi_0$ as $\chi_i$, and

$$\chi_i \equiv \frac{\Phi_{mi}}{\Phi_0} = n_i - \sum_{j=1}^{Q} \sigma_{ij} \frac{\Phi_{0j}}{\Phi_0}. \quad (8)$$

To maintain the $\chi_i$, Loop-i generates loop current $i_{mi}$ to cancel the external flux input $\Phi_{ei}$ and the coupled fluxes from other loops. Thus, Loop-i achieves a flux-to-current function as follows:

$$i_{mi} = \frac{1}{L_i} \left( \Phi_{ei} + \chi_i \Phi_0 + \sum_{j=1}^{P} M_{ij} i_{mj} \right); M_{ij} = 0 \quad (9)$$

where $M_{ii} = 0$ is defined to ignore the input $M_{ii} i_{mi}$ in the function diagram shown in Fig. 6(b).

In SFQ circuits, $\chi_i$ is close to an integer since $\Phi_{0j}$ is periodically located in the potential valleys, as illustrated in Fig. 7. Therefore, the “0” and “1” states of SFQ logics, similar to how the “low” and “high” voltage levels indicate the states of CMOS logics.

C. MFF Diagrams

We can see that the diagrams in Fig. 6 are interconnected for the nonzero elements of $\sigma$ and $L_m$. With $\sigma_{ij} \neq 0$, $i_{mi}$ of Loop-i is sent to the input of AJ-j, and $\Phi_{0j}$ of AJ-j is connected to the input of Loop-i; similarly, with $M_{ik} \neq 0$ as the weight, $M_{ik} i_{mi}$ is the input of Loop-k, and $M_{ik} i_{mi}$ is the input of Loop-i.

We introduce a kind of MFF diagrams to graphically depict the interactions between AJs and loops inside Josephson junction circuits. The symbols and connections for MFF diagrams are illustrated in Fig. 8.

First, the function enclosed in the dashed box in Fig. 6(a) is symbolized with a brick-shaped block, as shown in Fig. 8(a); the one shown in Fig. 6(b) is represented with a circle, as shown in Fig. 8(b). The outer loop [20], with loop current fixed as zero, is represented with a bar, as shown in Fig. 8(c), which is similar to the “ground” in conventional circuit diagrams.

Second, the nonzero elements of $\sigma$ and $L_m$ that connect AJs and loops are symbolized with two types of directed lines, as illustrated in Fig. 8(d). The input with an arrow entering in AJs or loops is configured with a plus sign (+) and otherwise is assigned with a minus sign (−). For instance, for $\sigma_{ij} = 1$, the directed line that connects Loop-i and AJ-j has an arrow leaving Loop-i and entering AJ-j; for $\sigma_{kj} = -1$, the directed line has an arrow leaving AJ-j and entering Loop-k. Meanwhile, for the nonzero $M_{ik}$, Loop-i and Loop-k are connected by a double-arrow line with two arrows entering loops. Moreover, the external flux $\Phi_{ei}$ and the trapped flux $n_i \Phi_0$ are two independent inputs entering Loop-i.

By marking the value of $\chi_i$ in the circle to visualize the state of Loop-i, we can vividly depict the flux-quantum flows inside Josephson junction circuits with MFF diagrams. For example, in the MFF diagram shown in Fig. 8(d), AJ-j generates $-\Phi_{0j}$ in Loop-i and $+\Phi_{0j}$ in Loop-k, and it is meanwhile pushed by Loop-i and blocked by Loop-k for their loop currents; once $i_{0j}$ is larger than $I_{TH,j}$, AJ-j will be triggered to increase the $\Phi_{0j}$; one flux-quantum increase in $\Phi_{0j}$ will change $\chi_i$ to $\chi_i - 1$ and the $\chi_k$ to $\chi_k + 1$, which looks like AJ-j transfers a flux quantum from Loop-i to Loop-k along the directed lines.

Typical MFF diagrams and their working principles are demonstrated in the Appendix. We can see that MFF diagrams depict the topology of Josephson junction circuits with the routes of flux-quantum flows and visualize the dynamics of the flux-quantum flows altered by AJs inside the circuits. MFF diagrams picture the dynamics of electric circuits working as the magnetic-flux distribution system.

D. Applications of MFF Diagrams

By using MFF diagrams, the SFQ logic bits are visualized with loops and their values of $\chi_i$; they are as understandable as the CMOS logic bits depicted with the nodes and voltages in conventional circuit diagrams. For the analysis of Josephson junction circuits, MFF diagrams are more easily understood than conventional circuit diagrams.

Therefore, we can use MFF diagrams to enable conceptual design before the conventional procedure for Josephson junction circuits, as illustrated in Fig. 9. MFF diagrams make it easy to describe the design specifications with MFFs and verify the design through simulations based on the general system model.
The verified MFF diagrams can be directly transformed into conventional schematics for physical implementations.

### III. APPLICATION EXAMPLE

#### A. DC Superconducting Quantum Interference Device

With the concept of AJs, the equivalent circuit of the dc SQUID circuit shown in Fig. 1(a) is redrawn and shown in Fig. 10(a). We extract two AJs and one loop and draw the MFF diagram, as shown in Fig. 10(b). In the MFF diagram, the dc SQUID consists of a flux-quantum generator and a flux-quantum absorber; being activated by \( i_{m1} \), AJ-1 keeps pumping flux quanta into Loop-1, and AJ-2 keeps drawing the flux quanta out of Loop-1. The average voltage measured at two terminals of dc SQUIDs is exactly the average flow-rate of flux quanta passing through AJ-1 or AJ-2; it is altered by the external flux \( \Phi_e \), which modulates the \( i_{m1} \); this is why the current–voltage characteristics of dc SQUIDs are modulated by the external flux.

#### B. TTL-to-SFQ

The transistor-to-transistor logic (TTL)-to-SFQ converter is used to transform a TTL clock into a flux quantum for SFQ logics, as shown in Fig. 11(a), which is actually a dc SQUID with \( I_{TH1} > 0 \) and \( I_{TH2} > 0 \). Its working principle is clearly exhibited in the MFF diagram shown in Fig. 11(b). First, a high-level TTL signal generates a negative \( \Phi_e \) to Loop-1 and induces a negative \( i_{m1} \); this \( i_{m1} \) will turn on the AJ-1 to pump one flux quantum into Loop-1; after the TTL signal returns to the low level, the flux quantum in Loop-1 will induce a positive \( i_{m1} \) to turn on the AJ-2 and is finally ejected to the subsequent loops by AJ-2.

#### C. Josephson Transmission Line

The Josephson transmission line (JTL) circuit shown in Fig. 1(b) is redrawn with AJs and shown in Fig. 11(c); its MFF diagram is shown in Fig. 11(d), which intuitively exhibits that the JTL transfers flux quanta from loop to loop, like a flux-quantum pipeline.

#### D. D-Type Flip-Flop Circuit

The D-type flip-flop (DFF) circuit shown in Fig. 1(c) is redrawn by defining AJs and loops and shown in Fig. 12(a). Its MFF diagram is shown in Fig. 12(b), where AJ-2, AJ-3, and Loop-2 constitute an unbuffered flux-quantum pipeline; Loop-1, Loop-3, and AJ-4 form a flux-quantum merger. Therefore, any flux quantum in Loop-2 will be transferred to empty Loop-3 by the unbuffered pipeline and stored in Loop-3, until it is merged by AJ-4 with the flux quantum pumped in by the clock signal in Loop-1. The DFF logic is used to synchronize the input signal with the clock signal.

#### E. Synchronous and Gate Circuit

For the AND gate circuit shown in Fig. 1(d), its MFF diagram is drawn and shown in Fig. 13, where, the subdiagram between In1 and Loop-7 and the one between In2 and Loop-8 are two DFFs; they synchronize the two flux-quantum inputs with the clock signals pumped into Loop-1 and Loop-3; two flux quanta are pumped out by AJ-7 and AJ-8 and are finally merged by the AND-type merger consisting of Loop-7, Loop-8, and AJ-11. The working principle of an AND-type merger is demonstrated in Fig. 19.
IV. CONCLUSION

We introduced a kind of MFF diagrams for the design and analysis of Josephson junction circuits. MFF diagrams are the graphical expressions of the circuit equations derived by the loop analysis method; they are mathematically equivalent to the circuit equations derived by the MNA method and can be equivalently converted into conventional circuit diagrams for physical implementations. Compared with the conventional circuit diagrams, the advantages of MFF diagrams are summarized as follows.

1) MFF diagrams intuitively depict the dynamics of Josephson junction circuits working as the magnetic-flux distribution system. Pictured in MFF diagrams, Josephson junctions are the diode-like magnetic flux pumps that are turned ON and OFF according to the threshold current; loops are the flux-quantum containers that preserve flux quanta with loop currents; the dc SQUID is a flux-quantum transfer system with two AJs continuously pumping flux quanta in and out of the loop; the SFQ circuits are the networks that use loops to preserve the logic bits and AJs to alter the logic states.

2) All the circuit variables in MFF diagrams achieve physical meanings and are easily understood by electronic engineers. For instance, the nominal flux $\Phi_{\theta_j}$ records the total flux flowing through AJ-$j$, and the voltage $v_{\theta_j}$ is the flow rate; $\chi_i$ is the number of flux quanta in Loop-$i$; its value defines the state of SFQ logic bits. Furthermore, the bias current $I_{bj}$ is used to set the threshold current of AJ-$j$, whereas the external flux $\Phi_{ei}$ is used to alter the offset of $i_{m_i}$ inside Loop-$i$.

3) MFF diagrams are suitable for both Josephson junction circuits and normal RLC circuits, as well as their hybrids, because AJ-$j$ with $I_{bj} = 0$ becomes a normal component of resistor and capacitor in parallel, and Loop-$i$ also represents a non-superconducting loop by setting $n_i$ with any value. MFF diagrams bridge the gap between superconducting and normal electric circuits.

MFF diagrams are usually used to depict the flux-quantum interactions inside Josephson junction circuits, whereas conventional circuit diagrams are well supported by electronic design automation tools for physical implementations. Two kinds of diagrams are complementary in the design and analysis of Josephson junction circuits.

APPENDIX

A. Flux-Quantum Generator and Absorber

**Flux-Quantum generator:** It is implemented by connecting AJ-1 to Loop-1 with $\sigma_{11} = -1$, as illustrated in Fig. 16(a). If Loop-1 is empty, AJ-1 will be turned ON by $-i_{m_1}$ to fill one flux quantum to Loop-1.

**Flux-quantum absorber:** It is implemented by connecting AJ-1 to Loop-1 with $\sigma_{11} = 1$, as illustrated in Fig. 16(b). If Loop-1 contains one flux-quantum, AJ-1 will be turned ON by $i_{m_1}$ to dump the flux quantum in Loop-1.
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Fig. 16. (a) Flux-quantum generator, where AJ-1 will generate one flux quantum for Loop-1 when \(-i_{m1} > I_{TH1}\). (b) Flux-quantum absorber, where AJ-1 will absorb one flux quantum from Loop-1 when \(i_{m1} > I_{TH1}\). The value of \(\chi\) is marked in loops and highlighted in black to visualize the variations of logic states, in the analysis of SFQ circuits.

Fig. 17. (a) Flux-quantum pipeline, where AJ-1 is connected between Loop-1 and Loop-2 and transfers one flux-quantum from Loop-1 to Loop-2 when \(i_{\theta1} > I_{TH1}\). (b) Flux-quantum splitter, where AJ-1 eliminates one flux quantum from Loop-1 and adds one flux quantum to both Loop-2 and Loop-3 when \(i_{\theta1} > I_{TH1}\). (c) Flux-quantum merger, where AJ-1 eliminates one flux quantum from both Loop-1 and Loop-2 and adds one flux quantum to Loop-3 when \(i_{\theta1} > I_{TH1}\).

B. Flux-Quantum Transmitters

There are four basic flux-quantum transmitters in Josephson junction circuits.

**Flux-quantum pipeline:** It transfers flux quanta from one loop to another, through the AJ connected between two loops, as illustrated in Fig. 17(a).

**Flux-quantum splitter:** It duplicates and transfers one flux quantum in the input loop to two output loops through one AJ, as depicted in Fig. 17(b).

**Flux-quantum merger:** It merges two flux quanta in two input loops and pumps one flux quantum to the third loop through one AJ, as illustrated in Fig. 17(c).

**Unbuffered flux-quantum pipeline:** It is a flux-quantum pipeline with a flux-quantum absorber connected at the input loop, as shown in Fig. 18. If Loop-2 is empty, AJ-1 will pump the flux quantum from Loop-1 to Loop-2, as illustrated in Fig. 18(a); otherwise, AJ-2 will dump the flux quantum in Loop-1, as illustrated in Fig. 18(b).

C. Basic Logic Circuits

There are three basic logic components for the design of SFQ circuits.

**AND-type merger:** It is a two-input merger with two flux-quantum absorbers configured at input loops; its flux-quantum flow implements exactly the AND-logic, as shown in Fig. 19. The \(\chi_3\) will be changed from “0” to “1,” only when two input loops receive a flux quantum at the same time, as shown in Fig. 19(b); otherwise, \(\chi_3\) remains unchanged, and the input flux quantum will be dumped by the flux-quantum absorbers, as shown in Fig. 19(a) and (c).

**OR-type merger:** It is a two-input merger implementing the OR-logic, in which two input loops are coupled with a negative \(M_{12}\), and each input loop is configured with a flux-quantum generator and a flux-quantum absorber, as shown in Fig. 20. With the aid of \(M_{12}\), the flux quantum in one input loop will decrease the loop current of another input loop and thereby trigger the flux-quantum generator to fill the empty input loop; if Loop-3 is empty, two flux quanta in the input loops will be merged to change the \(\chi_3\) from “0” to “1,” as illustrated in Fig. 20(a); otherwise, they will be both dumped by AJ-1, as shown in Fig. 20(b).

**Flux-quantum multiplexer:** It is used to switch the flux-quantum flow according to the control signal, as illustrated in Fig. 21. The flux quantum in Loop-2 is the control signal; if Loop-2 is filled, the flux quantum pumped into Loop-1 will flow through Loop-3 to the outer loop, as illustrated in Fig. 21(a); it will flow through Loop-4 to the outer loop if Loop-2 is empty, as demonstrated in Fig. 21(b).
Fig. 20. OR-type merger, where Loop-1 and Loop-2 are coupled with a negative $M_{23}$. (a) Flux quantum in Loop-1 will decrease $i_{m3}$ and then turn on AJ-4 to induce one flux quantum to Loop-2; two flux quanta in Loop-1 and Loop-2 will be merged and transferred to Loop-3 when Loop-3 is empty. (b) Two flux quanta are absorbed by AJ-1, when Loop-3 is already occupied.

Fig. 21. Flux-quantum multiplexer, where Loop-2 and Loop-3 are coupled through a negative $M_{23}$. (a) Flux quantum in Loop-2 will decrease $i_{m3}$ through $M_{23}$ to make $i_{m3}$ less than $i_{m4}$; if $\chi_2 = 1$, one flux quantum received by Loop-1 is set to be first transferred by AJ-1 to Loop-3 and finally absorbed by AJ-3. (b) If Loop-2 is empty before Loop-1 receives a flux quantum, $i_{m3}$ will be larger than $i_{m4}$, and therefore, AJ-3 is set to be first triggered to transfer the flux quantum in Loop-1 to Loop-4; the flux quantum in Loop-4 will be finally absorbed by AJ-5 in Loop-2.

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