Study of the Decay $D_s^+ \rightarrow \pi^+ \pi^+ \pi^- \eta$ and Observation of the W-annihilation Decay $D_s^+ \rightarrow a_0(980) + p^0$
T. J. Zhu, W. J. Zhu, Y. C. Zhu, Z. A. Zhu, B. S. Zou, J. H. Zou (BESIII Collaboration)

Institute of High Energy Physics, Beijing 100049, People’s Republic of China
Beihang University, Beijing 100191, People’s Republic of China
Beijing Institute of Petrochemical Technology, Beijing 102617, People’s Republic of China
Bochum Ruhr-University, D-44780 Bochum, Germany
Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
Central China Normal University, Wuhan 430079, People’s Republic of China
China Center of Advanced Science and Technology, Beijing 100190, People’s Republic of China
COMSATS University Islamabad, Lahore Campus, Defence Road, Off Raiwind Road, 54000 Lahore, Pakistan
Fudan University, Shanghai 200433, People’s Republic of China
G.I. Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia
GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany
Guangxi Normal University, Guilin 541004, People’s Republic of China
Guangxi University, Nanning 530004, People’s Republic of China
Hangzhou Normal University, Hangzhou 310036, People’s Republic of China
Henan Institute of Science and Technology, Luoyang 471003, People’s Republic of China
Hunan Normal University, Xinyang 453007, People’s Republic of China
Hunan University, Changsha 410081, People’s Republic of China
Hunan University, Changsha 410082, People’s Republic of China
Indian Institute of Technology Madras, Chennai 600036, India
Indiana University, Bloomington, Indiana 47405, USA
INFN Laboratori Nazionali di Frascati, (A)INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy; (B)INFN Sezione di Perugia, I-06100, Perugia, Italy; (C)University of Perugia, I-06100, Perugia, Italy
INFN Sezione di Ferrara, (A)INFN Sezione di Ferrara, I-44122, Ferrara, Italy; (B)University of Ferrara, I-44122, Ferrara, Italy
Institute of Modern Physics, Lanzhou 730000, People’s Republic of China
Institute of Physics and Technology, Peace Ave. 54B, Ulaanbaatar 13330, Mongolia
Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
Justus-Liebig-Universitaet Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany
Lanzhou University, Lanzhou 730000, People’s Republic of China
Liaoning Normal University, Dalian 116029, People’s Republic of China
Liaoning University, Shenyang 110036, People’s Republic of China
Nanjing Normal University, Nanjing 210023, People’s Republic of China
Nanjing University, Nanjing 210093, People’s Republic of China
Nankai University, Tianjin 300071, People’s Republic of China
North China Electric Power University, Beijing 102206, People’s Republic of China
Peking University, Beijing 100871, People’s Republic of China
Qufu Normal University, Qufu 273165, People’s Republic of China
Shandong Normal University, Jinan 250014, People’s Republic of China
Shandong University, Jinan 250100, People’s Republic of China
Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China
Shanxi Normal University, Linfen 041004, People’s Republic of China
Shanxi University, Taiyuan 030006, People’s Republic of China
Sichuan University, Chengdu 610064, People’s Republic of China
Southeast University, Suzhou 215006, People’s Republic of China
South China Normal University, Guangzhou 510006, People’s Republic of China
State Key Laboratory of Particle Detection and Electronics, Beijing 100049, People’s Republic of China
Sun Yat-Sen University, Guangzhou 510275, People’s Republic of China
Suranaree University of Technology, University Avenue 111, Nakhon Ratchasima 30000, Thailand
Tsinghua University, Beijing 100084, People’s Republic of China
Turkish Accelerator Center Particle Factory Group, (A)Istanbul Bilgi University, 34060 Eqip, Istanbul, Turkey; (B)Near East University, Nicosia, North Cyprus, Mersin 10, Turkey
University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
University of Groningen, NL-9747 AA Groningen, The Netherlands
University of Hawaii, Honolulu, Hawaii 96822, USA
University of Jinan, Jinan 250022, People’s Republic of China
The decay $D_s^+ \to \pi^+\pi^+\pi^-\eta$ is observed for the first time, using $e^+e^-$ collision data corresponding to an integrated luminosity of 6.32 fb$^{-1}$, collected by the BESIII detector at center-of-mass energies between 4.178 and 4.226 GeV. The absolute branching fraction for this decay is measured to be $B(D_s^+ \to \pi^+\pi^+\pi^-\eta) = (3.12 \pm 0.13_{\text{stat}} \pm 0.09_{\text{syst}})\%$. The first amplitude analysis of this decay reveals the sub-structures in $D_s^+ \to \pi^+\pi^+\pi^-\eta$ and determines the relative fractions and the phases among these sub-structures. The dominant intermediate process is $D_s^+ \to a_1(1260)^+\gamma$, $a_1(1260)^+ \to \rho(770)^0\pi^+$ with a branching fraction of $(1.73 \pm 0.14_{\text{stat}} \pm 0.08_{\text{syst}})\%$. We also observe the W-annihilation process $D_s^+ \to a_0(980)^+\rho(770)^0\eta$ with a branching fraction of $(0.21 \pm 0.08_{\text{stat}} \pm 0.05_{\text{syst}})\%$, which is larger than the branching fractions of other measured pure W-annihilation decays by one order of magnitude.

PACS numbers:
state, $D_s^+ \to \pi^0 \pi^+$, is less than 0.037% [5], and that of the $\bar{V}P$ final state, $D_s^+ \to \rho(770)^0 \pi^+$, is 0.019% [2], while the BP of the WA process with an $SP$ final state, $D_s^+ \to a_0(980)^0 \pi^0(\eta)$, $a_0(980)^0(\eta) \to \pi^0(\eta)$, is about 1.46% [7]. Here, $V$, $P$, and $S$ denote vector, pseudoscalar, and scalar mesons, respectively. Since the direct production of the $a_0(980)^0(\eta)$ system via the $c\bar{s} \to W^+ \to u\bar{d} \to a_0(980)^0(\eta)$ transition violates G-parity conservation [8], the WA process $D_s^+ \to a_0(980)^0(\eta)$ is suppressed. The origin of the abnormally large BF for the $SP$ decay mode is still controversial. Reference [9] argues that the $SP$ transition of the abnormally large BF for the $\tau\nu$ decay originates from the tension between theoretical calculations and experimental measurements at LHCb have a large systematic uncertainty from the limited knowledge of the inclusive $D_s^+ \to \pi^+\pi^+\pi^-X$ decay [12]. A precise measurement of the decay $D_s^+ \to \pi^+\pi^+\pi^-\eta$ may provide useful input to this problem.

This Letter reports the first observation of the $D_s^+ \to \pi^+\pi^+\pi^-\eta$ decay, along with an amplitude analysis and a BF measurement of this decay. This analysis uses $e^+e^-$ collision data samples corresponding to an integrated luminosity of $6.32 \text{ fb}^{-1}$ collected at the center-of-mass energies $(E_{cm})$ between 4.178 and 4.226 GeV. Throughout this Letter, charged-conjugate modes and exchange symmetry of two identical $\pi^+$ are always implied.

The BESIII detector and the upgraded multi-gap resistive plate chambers used in the time-of-flight systems are described in Refs. [13, 14], respectively. Simulated data samples produced with a GEANT4-based [15] Monte Carlo (MC) program, which includes the geometric description of the BESIII detector and the detector response, are used to determine detection efficiencies and to estimate backgrounds. The simulation models the beam energy spread and initial state radiation (ISR) in the $e^+e^-$ annihilations with the generator KKMC [16]. The inclusive MC sample includes the production of open charm processes modeled with CONEX [17], the ISR production of vector charmonium(-like) states, and the continuum processes incorporated in KKMC [16]. The known decay modes are modeled with EVTGEN [18] using BFs taken from the PDG [2], and the remaining unknown charmonium decays are modeled with LUNDCHARM [19]. Final state radiation (FSR) from charged final-state particles is incorporated using PHOTOS [20].

We employ the double-tag (DT) technique [21] to select $D_s^+ \to \pi^+\pi^+\pi^-\eta$ decays in $e^+e^-$ samples in $D_s^+ \to \gamma D_s^+$ events. The single-tag (ST) $D_s^-$ candidates are reconstructed from eight hadronic decay modes (tag side): $D_s^- \to K_{S}^{0}K_{S}^{0}$, $K_{S}^{0}K_{S}^{0}$, $K_{S}^{0}K_{S}^{0}$, $K_{S}^{0}K_{S}^{0}$, $K_{S}^{0}K_{S}^{0}$, $K_{S}^{0}K_{S}^{0}$, $K_{S}^{0}K_{S}^{0}$, $K_{S}^{0}K_{S}^{0}$, and $K_{S}^{0}K_{S}^{0}$. These tag modes are combined to perform an amplitude analysis. A DT candidate is selected by reconstructing the $D_s^+ \to \pi^+\pi^+\pi^-\eta$ decay (signal side) from the remaining particles that are not used in the ST reconstruction. Here, the $K_{S}^{0}$, $\pi^0$, $\eta$, and $\eta'$ mesons are reconstructed from $K_{S}^{0}$, $\pi^0$, $\eta$, and $\eta'$ decays, respectively. The selection criteria for charged and neutral particle candidates are identical to those used in Ref. [22]. For the decay mode $D_s^- \to K^-\pi^+\pi^-$, we exclude the di-pion mass range [0.487, 0.511] GeV/$c^2$ to avoid overlap with the $D_s^- \to K_{S}^{0}K_{S}^{0}$ mode.

The invariant mass of the tag (signal) $D_s^+(\pm)$ candidate $M_{tag}$ ($M_{sig}$) is required to be within the range [1.87, 2.06] GeV/$c^2$. We calculate the recoiling mass $M_{rec} = \{|E_{cm} - (\vec{p}_{D_s}^2 + m_{D_s}^2)^{1/2} - |\vec{p}_{D_s}^2|^{1/2}\}^{1/2}$ in the $e^+e^-$ center-of-mass system, where $\vec{p}_{D_s}$ is the momentum of the reconstructed $D_s$ and $m_{D_s}$ is the known mass of the $D_s$ meson [2]. The value of $M_{rec}$ is required to be in the range [2.05, 2.18] GeV/$c^2$ for the data sample collected at 4.178 GeV to suppress the non-$D_s^\pm D_s^\mp$ events. The $M_{rec}$ ranges for the other data samples are the same as those in Ref. [23].

A seven-constraint kinematic fit is applied to the $e^+e^- \to D_s^+D_s^\mp \gamma$ candidates, where $D_s$ re-
cays to one of the tag modes and $D_\ell^+$ decays to the signal mode. In addition to the constraints of four-momentum conservation in the $e^+e^-$ center-of-mass system, the invariant masses of $\eta$, tag $D_s^+$, and $D_\ell^+$ candidates are constrained to their individual PDG values [2]. If there are multiple candidates (in $\approx 15\%$ of the selected events) in an event, the one with the minimum $\chi^2$ of the seven-constraint kinematic fit is accepted.

To suppress the background from $D_s^+ \to K_S^0\pi^+\eta$ decays, a secondary vertex fit [24] is performed on the $\pi^+\pi^-\pi^-$ pair. If its invariant mass is in the range [0.487, 0.511] GeV/$c^2$ and the flight distance between the interaction point (IP) [24] and the decay point is two times greater than its uncertainty, we reject these candidates. To suppress the background from the decay $D_\ell^+ \to \pi^+\eta$ with $\eta \to \pi^+\pi^-\pi^-$, we reject candidates with $M_{\pi^+\pi^-\pi^-} < 1$ GeV/$c^2$. To suppress the background where individual photons from random $\pi^0$s feed into the $\eta \to \gamma\gamma$ reconstruction, we define two invariant masses $M(\gamma\gamma\pi^0)$ and $M(\gamma\gamma\pi^0_{\text{other}})$, where $\gamma\gamma$, $\pi^0$, and $\pi^0_{\text{other}}$ denote the photon of $\eta$ from the signal side, the photon of $\pi^0$ from the tag side, and the other photons including the transition photon from $D_s^+\to\pi^+\eta$ respectively. We reject events with $M(\gamma\gamma\pi^0)$ or $M(\gamma\gamma\pi^0_{\text{other}})$ in the range [0.115, 0.150] GeV/$c^2$.

We further reduce the background by using a gradient-boosted decision tree (BDT) implemented in the TMVA software package [25]. The BDT takes four discriminating variables as inputs: the recoiling mass of $D_s^+$, the momentum of the lower-energy photon from $\eta$, the invariant mass of the two photons used to reconstruct $\eta$, and the energy of the transition photon from $D_s^+\to\pi^+\eta$. We place a requirement on the output of the BDT to ensure the samples have a purity greater than 85%: (87.0 $\pm$ 3.8)$\%$, (85.6 $\pm$ 4.9)$\%$, and (89.2 $\pm$ 9.2)$\%$ at $E_{\text{cm}} = 4.178, 4.189-4.219$, and 4.226 GeV, respectively.

An unbinned maximum likelihood method is adopted in the amplitude analysis of the $D_s^+ \to \pi^+\pi^-\pi^-$ decay. The likelihood function is constructed with a probability density function (PDF) in which the momenta of the four final-state particles are used as inputs. The total likelihood is the product of the likelihoods for all data samples. The total amplitude model is expressed as a coherent sum over all intermediate processes $M(p_j) = \sum \rho_n e^{i\phi_n} A_n(p_j)$, where $\rho_n e^{i\phi_n}$ is the coefficient of the $n^{\text{th}}$ amplitude with magnitude $\rho_n$ and phase $\phi_n$. The $n^{\text{th}}$ amplitude $A_n(p_j)$ is given by $A_n = P_1^n P_2^n S_n F_1^n F_2^n F_3^n$, where the indices 1, 2 and 3 correspond to the two subsequent intermediate resonances and the $D_\ell^+$ meson, $F_3^n$ is the Blatt-Weisskopf barrier factor [26, 27] and $P_j^n$ is the propagator of the intermediate resonance. The function $S_n$ describes angular momentum conservation in the decay and is constructed using the covariant tensor formalism [27]. The relativistic Breit-Wigner (RBW) [28] function is used to describe the propagator for the resonances $\eta(1405)$, $f_1(1420)$ and $a_1(1260)$. The resonance $\rho(770)^+$ is parameterized by the Gounaris-Sakurai [29] lineshape, and the resonances $a_0(980)$ and $f_0(980)$ are parameterized by a coupled Flatté formula, and the parameters are fixed to the values given in Ref. [30] and Ref. [31], respectively. We use the same parameterization to describe $f_0(500)$ as Ref. [32]. The masses and widths of the intermediate resonances, except for $a_0(980)$, $f_0(980)$ and $f_0(500)$, are taken from Ref. [2].

The background PDF $B(p_j)$ is constructed from inclusive MC samples by using a multi-dimensional kernel density estimator [33] with five independent Lorentz invariant variables ($M_{\pi^+\pi^-\pi}^*$, $M_{\pi^+\pi^-\pi^-}$, $M_{\pi^+\pi^-\pi^0}$, and $M_{\pi^+\pi^-\pi^0}$). As a consequence, the combined PDF can be written as

$$
\epsilon R_4 \left[ \frac{|M(p_j)|^2}{\int |M(p_j)|^2 R_4 dp_j} + (1 - \epsilon) \frac{B_4(p_j)}{\epsilon B_4(p_j) R_4 dp_j} \right],
$$

where $\epsilon$ is the acceptance function determined with phase-space (PHSP) MC samples generated with a uniform distribution of the $D_s^+ \to \pi^+\pi^-\pi^-$ decay over PHSP, $B_4$ is $B/\epsilon$, and $R_4 dp_j$ is the element of four-body PHSP. The normalization integral in the denominator is determined by a MC technique as described in Ref. [34].

In the initial amplitude fit, we include a few obvious components. Then, further amplitudes are added one at a time to the fit, and the statistical significance of the new amplitude is calculated with the change of the log-likelihood, after taking the change of the degrees of freedom into account. Only amplitudes with significance larger than $5 \sigma$ are chosen for the optimal set. The dominant CF amplitude to this final state is $D_s^+ \to a_1(1260)^+\eta$, $a_1(1260) \to [\rho(770)^0 \pi^+\eta]_S$, where the subscript $S$ means that the angular momentum of $\rho^0\pi^+$ combination is zero (S-wave). Thus, we choose this amplitude as the reference and its phase is fixed to 0. The $D_\ell^+ \to a_0(980)^+\rho(770)^0$, $a_0(980)^+ \to \pi^+\eta$ decay is observed with a significance larger than $7 \sigma$. We also consider some possible amplitudes involving the resonances $f_0(500)$, $f_0(980)$, $f_1(1285)$, $\eta(1295)$, $\eta(1405)$, $\eta(1475)$, $f_1(1420)$, $f_1(1510)$ and $\pi(1300)$, as well as non-resonant components. Moreover, charge conjugation for $D_s^+ \to \eta(1405)(f_1(1420))^{\pi^+}$ with $\eta(1405)(f_1(1420)) \to a_0(980)^+\pi^-$ and $\eta(1405)(f_1(1420)) \to a_0(980)^-\pi^+$ requires their magnitudes and phases to be the same. Finally, eleven amplitudes are retained in the nominal fit, as listed in Table I. The mass projections of the fit are shown in Fig. 2. For the $n^{\text{th}}$ component, its contribution relative to the total BF is quantified by the fit fraction (FF) defined by $\text{FF}_n = \int |\rho_n A_n(p_j)|^2 R_4 dp_j / \int |M|^2 R_4 dp_j$. The final amplitudes, their phases and FFs are listed in Table I. The sum of the FFs of all the components is $(95.0 \pm 4.9)%$. A $\chi^2$ value is calculated to quantify the quality of the fit, as defined in Ref. [34]. The goodness of fit is $\chi^2/\text{NDOF} = 153.2/133 = 1.15$ and the p-value is 11.1%.

In addition, 300 sets of signal MC samples with the same size as the data samples are generated to validate the fit performance, as described in Ref. [22]. No significant bias is found in our fit.
We determine the systematic uncertainties by taking the differences between the values of $\phi_i$ and $F_{\text{FF}}$ found by the nominal fit and those found from fit variations. The masses and widths of the intermediate states are varied by $\pm 1 \sigma$ [2]. The masses and coupling constants of $a_0(980)$ and $f_0(500)$ are varied within the uncertainties reported in Ref. [30] and Ref. [31], respectively. The barrier radii for $D_s^+$ and the other intermediate states are varied by $\pm 1$ GeV$^{-1}$. The uncertainties related to line-shape are estimated by using an alternative RBW function for $f_0(500)$ with mass and width fixed to 526 MeV/$c^2$ and 535 MeV [35]. The uncertainties from detector effects are investigated with the same method as described in Ref. [22]. The uncertainty related to the background is estimated by varying the background yield within statistical uncertainty, and constructing the background PDF with the other five independent variables. The total uncertainties are obtained by adding all the contributions in quadrature, and are listed in Table I.

Further, we measure the BF of $D_s^+(\rho(770)^0\pi^+)\eta$ with the DT technique. To improve the statistical precision, the DT candidates are selected without reconstructing the transition photon from $D_s^+$ and without the BDT requirement. We use the same eight tag modes as used in the amplitude analysis. For each tag mode, if there are multiple tag $D_s^+$ candidates, the one with $M_{\text{cc}}$ closest to the known mass of $D_s^+$ [2] is retained. For each tag mode, a DT candidate with the average mass $\left(M_{\text{sig}} + M_{\text{tag}}\right)/2$ closest to the known mass of $D_s^+$ [2] is retained. The ST yield ($Y_{\text{tag}}$) and DT yield ($Y_{\text{sig}}$) in data are determined from fits to the $M_{\text{tag}}$ and $M_{\text{sig}}$ distributions, respectively, as shown in Fig. 3. The signal shape is modeled with the MC-simulated shape convolved with a Gaussian function, and the background is parameterized as a second-order Chebyshev function.

These fits result in a total ST yield of $Y_{\text{tag}} = 479.093 \pm 1952$ and a signal yield of $Y_{\text{sig}} = 2139 \pm 78$ events. An updated inclusive MC sample based on our amplitude analysis results is used to determine the ST efficiencies ($\epsilon_{\text{ST}}^i$) and DT efficiencies ($\epsilon_{\text{DT}}^i$). Inserting these numbers in $B(D_s^+ \rightarrow \pi^+\pi^+\pi^-\eta) = Y_{\text{tag}}(B(\eta \rightarrow \gamma\gamma) \times \sum_i \frac{\alpha_i}{i} Y_{\text{tag}} \epsilon_{\text{DT}}^i / \epsilon_{\text{ST}}^i)$, where $i$ denotes the $i^{th}$ tag mode and $\alpha$ denotes the $\alpha^{th}$ center-of-mass energy point, we obtain $B(D_s^+ \rightarrow \pi^+\pi^+\pi^-\eta) = (3.12 \pm 0.13)\%$, where the uncertainty is statistical only.

The systematic uncertainties for the BF measurement are described next. The uncertainty of the signal yield and total ST yield is assigned to be 1.4% by examining the changes of the fit yields when varying the signal and background shapes. The $\pi^\pm$ tracking (PID) efficiencies are studied using samples of $e^+e^- \rightarrow K^+K^-\pi_\pm\pi^-\pi^\mp$ ($e^+e^- \rightarrow K^+K^-\pi_\pm\pi^-\pi^\mp$ and $\pi^+\pi^-\pi^+\pi^-\pi^\pm$) events. The corresponding systematic uncertainties are estimated as 0.3% (0.4%). The uncertainty due to the $\eta$ reconstruction efficiency is 2.0% [7]. The uncertainty from the amplitude model is estimated to be 0.4%, which is the change of signal efficiency when the parameters are varied according to the covariance matrix in the nominal amplitude fit. The uncertainty due to MC simulation sample size is 0.3%, and that from the BF of $B(\eta \rightarrow \gamma\gamma)$ is 0.5% [2]. Adding these uncertainties in quadrature gives a total systematic uncertainty of 2.9%.

In summary, using $e^+e^-$ annihilation data equivalent to an integrated luminosity of 6.32 fb$^{-1}$ recorded with the BESIII detector at $E_{\text{cm}} = 4.178-4.226$ GeV, we observe the $D_s^+ \rightarrow \pi^+\pi^+\pi^-\eta$ decay for the first time. The absolute BF of this decay is measured to be $B(D_s^+ \rightarrow \pi^+\pi^+\pi^-\eta) = (3.12 \pm 0.13_{\text{stat}} \pm 0.09_{\text{syst}})\%$. The first amplitude analysis of this decay is also performed. The obtained intermediate processes, phases and FFs are summarized in Table I. The BF for the intermediate processes are calculated with $B_{\text{BF}} = F_{\text{BF}} \times B(D_s^+ \rightarrow \pi^+\pi^+\pi^-\eta)$. The $D_s^+ \rightarrow a_1(1260)^+\eta$, $a_1(1260)^+ \rightarrow \rho(770)^0\pi^+$ decay is dominant with a BF of $(1.73 \pm 0.14_{\text{stat}} \pm 0.08_{\text{syst}})\%$.

Our results offer critical input for estimating the $D_s^+ \rightarrow \pi^+\pi^+\pi^-X$ background contribution in tests of the lepton
flavor universality with semileptonic $B$ decays. Assuming that $B(a_1(1260)^{++} \rightarrow \rho(770)^{0}\pi^0) = B(a_1(1260)^{++} \rightarrow \rho(770)^{0}\pi^+\pi^-)$, the BF of $D_s^{+} \rightarrow \pi^{+}\pi^{0}\pi^+\eta$ decay is expected to be comparable to the one of $D_s^{+} \rightarrow \pi^{+}\pi^{+}\pi^-\eta$. In this case, the missing inclusive hadronic $\eta$ decay fraction of $D_s^{+}$ is reduced to $(2.3 \pm 3.2)\%$, thereby indicating that there is no large room for unobserved exclusive $D_s^{+} \rightarrow \eta X$ decays. Furthermore, we observe the WA decay $D_s^{+} \rightarrow a_0(980)^{++}\rho(770)^0$, $a_0(980)^{++} \rightarrow \pi^+\eta$ with BF of $(0.21 \pm 0.08_{\text{stat.}} \pm 0.05_{\text{syst.}})\%$. This BF and the one of the decay $D_s^{+} \rightarrow a_0(980)^{++}\pi^0\pi^0$ obtained in Ref. [7] are both larger than those of the pure WA processes $D_s^{+} \rightarrow \rho(770)^0\pi^+$ and $D_s^{+} \rightarrow \rho(770)^0\pi^+$ by one order of magnitude. These measurements indicate that long-distance weak annihilation may play an essential role, and provide a good opportunity to study the final-state rescattering in the WA process [1, 9, 10].

ACKNOWLEDGMENTS

The BESIII collaboration thanks the staff of BEPCII and the IHEP computing center for their strong support. This work is supported in part by National Key Research and Development Program of China under Contracts Nos. 2020YFA0406400, 2020YFA0406300; National Natural Science Foundation of China (NSFC) under Contracts No. 11625523, 11635010, 11735014, 11822506, 11835012, 11935015, 11935016, 11935018, 11961141012; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; Joint Large-Scale Scientific Facility Funds of the NSFC and CAS under Contracts Nos. U1732263, U1832107, U1832207; CAS Key Research Program of Frontier Sciences under Contract No. QYZDJ-SSW-LH040; 100 Talents Program of CAS; INPAC and Shanghai Key Laboratory for Particle Physics and Cosmology; ERC under Contract No. 758462; European Union Horizon 2020 research and innovation programme under Contract No. Marie Sklodowska-Curie grant agreement No 894790; German Research Foundation DFG under Contracts Nos. 443159800, Collaborative Research Center CRC 1044, FOR 2359, GRK 214; Istituto Nazionale di Fisica Nucleare, Italy; Ministry of Development of Turkey under Contract No. DPT2006K-120470; National Science and Technology fund; Olle Engkvist Foundation under Contract No. 200-0605; STFC (United Kingdom); The Knut and Alice Wallenberg Foundation (Sweden) under Contract No. 2016.0157; The Royal Society, UK under Contracts Nos. DH140054, DH160214; The Swedish Research Council; U. S. Department of Energy under Contracts Nos. DE-FG02-05ER41374, DE-SC-0012069.

[1] H. Y. Cheng and C. W. Chiang, Phys. Rev. D 81, 074021 (2010).
[2] P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).
[3] L. L. Chau and H. Y. Cheng, Phys. Rev. D 36, 137 (1987).
[4] J. L. Rosner, Phys. Rev. D 60, 114026 (1999).
[5] H. Mendez et al. (CLEO Collaboration), Phys. Rev. D 81, 052013 (2010).
[6] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 79, 032003 (2009).
[7] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 123, 112001 (2019).
[8] N. N. Achasov and G. N. Shestakov, Phys. Rev. D 96, 036013 (2017).
[9] R. Molina et al., Phys. Lett. B 803, 135279 (2020).
[10] Y. K. Hsiao et al., Eur. Phys. J. C 80, 895 (2020).
[11] Y. Amhis et al. (Heavy Flavor Averaging Group), Eur. Phys. J. C 77, 895 (2017); updated results available at https://hflav-eos.web.cern.ch/hflav-eos/semi/spring19/html/RDsDsstar/RDRDs.html.
[12] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 120, 171802 (2018).
[13] M. Ablikim et al. (BESIII Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 614, 345 (2010).
[14] X. Wang et al., J. Instrum. 11, C08009 (2016).
[15] S. Agostinelli et al. (GEANT4 Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 506, 250 (2003).
[16] S. Jadach, B. F. L. Ward and Z. Was, Phys. Rev. D 63, 113009 (2001).
[17] R. G. Ping, Chin. Phys. C 38, 083001 (2014).
[18] D. J. Lange, Nucl. Instrum. Methods Phys. Res., Sect.
A 462, 152 (2001); R. G. Ping, Chin. Phys. C 32, 243 (2008).
[19] J. C. Chen, G. S. Huang, X. R. Qi, D. H. Zhang, and Y. S. Zhu, Phys. Rev. D 62, 034003 (2000); R. L. Yang, R. G. Ping, and H. Chen, Chin. Phys. Lett. 31, 061301 (2014).
[20] P. Golonka and Z. Was, Eur. Phys. J. C 45, 97 (2006).
[21] R. M. Baltrusaitis et al. (Mark III Collaboration), Phys. Rev. Lett. 56, 2140 (1986).
[22] M. Ablikim et al. (BESIII Collaboration), arXiv:2011.08041.
[23] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 103, 092006 (2021).
[24] M. Xu et al., Chin. Phys. C 33, 428 (2009).
[25] A. Hocker et al., Proc. Sci. ACAT2007 040 (2007).
[26] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley & Sons, New York, 1973).
[27] B. S. Zou and D. V. Bugg, Eur. Phys. J. A 16, 537 (2003).
[28] J. D. Jackson, Nuovo Cimento 34, 1644 (1964).
[29] G. J. Gounaris and J. J. Sakurai, Phys. Rev. Lett. 21, 244 (1968).
[30] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 95, 032002 (2017).
[31] M. Ablikim et al. (BESIII Collaboration), Phys. Lett. B 607, 243 (2005).
[32] D. V. Bugg, A. V. Sarantsev and B. S. Zou, Nucl. Phys. B 471, 59 (1996).
[33] K. S. Cranmer, Comput. Phys. Commun. 136, 198 (2001).
[34] M. Artuso et al. (CLEO Collaboration), Phys. Rev. D 85, 122002 (2012).
[35] M. Ablikim et al. (BESII Collaboration), Phys. Lett. B 598, 149 (2004).