Solution of the Problem of the Associated Petroleum Gas Injection into a Porous Medium Saturated with Methane and Ice

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Abstract. A system of nonlinear differential equations is proposed for describing the process of the gas hydrate formation during the injection of methane into a porous reservoir of finite length. This reservoir initially saturated with methane and ice. The case of phase transformations “ice + gas → gas hydrate” occurring in an extended region is considered. It has been established that over time the extended region of hydrate formation degenerates into a frontal surface. It is shown that at low pressures of the injected gas it is possible to stop the phase transition boundary.

1. Introduction
One of the probable causes of global warming is an increase in the concentration of greenhouse gases in the atmosphere. This growth is due to different circumstances, among which the combustion of associated petroleum gas can be distinguished. This is primarily due to the fact of absence necessary infrastructure for the gas collection and transportation. Therefore, a sufficiently large amount of this natural resource (mainly methane) is lost, as well as environmental pollution occurs.

One way to collect associated petroleum gas is its burial in porous reservoirs in gas hydrate form [1,2]. Since, under the same conditions, a unit volume of a gas hydrate contains significantly more gas than in free state, the gas hydrate state allows storing large amounts of methane at relatively low pressures [3-6]. This reduces the risk of methane release (which is a greenhouse gas) to the surface and actually increases the capacity of underground storage tanks for associated petroleum gas. In this case, the most effective is the creation of underground gas storage facilities in the conditions of permafrost [7]. Assesments of the possibility of creating underground gas storage facilities in the gas hydrate state in the permafrost zones were carried out in [8]. The intensity of the gas hydrate formation can be increased in various ways, the description of which is presented in a number of works, for example, [9-13].

For realization the technology of storing associated petroleum gas in porous reservoirs in gas hydrate form, preliminary studies are needed. At the same time, it is worth noting the importance of mathematical modeling, since the results obtained during theoretical studies significantly reduce the amount of necessary experimental and field data [14].

Mathematical modeling of the gas hydrate formation during gas injection into a porous medium, initially saturated with gas and ice, was carried out in [15]. In this work, the process of the hydrate formation was considered in the self-similiar approximation for a semi-infinite reservoir. However, the real reservoirs are of sufficient but limited length. Therefore, self-similar solutions are valid only for
the initial times of the process of the hydrate formation, when the right boundary influence is not significant.

In this paper, system of nonlinear differential equations is proposed for describing the gas hydrate formation during the injection of methane into a porous reservoir of finite length, saturated in the initial state with methane and ice. A numerical study of the gas hydrate formation in a porous medium has been carried out. The cases of hydrate formation both on the frontal surface and in the extended region are considered.

2. Mathematical model

Consider a porous reservoir of length $L$ saturated at the initial time with gas (methane) and ice, the pressure $p_0$ and temperature $T_0$ of which corresponds to the conditions for their existence in free (no gas hydrate) state. Let us suppose that the associated petroleum gas (methane) begins to be pumped through the left boundary of the reservoir with temperature $T_e$ and pressure $p_e$. Moreover, these parameters correspond to the conditions for the stable existence of methane hydrate. We assume that the reservoir is homogeneous and isotropic; also, we neglect the influence of the upper and lower boundaries. In this regard, we can assume that the task is one-dimensional and the process parameters depend only on the spatial coordinate and time [16].

Considering the instantaneous decrease in temperature at this boundary to the value of $T_e$, we will assume that three characteristic regions are formed in the reservoir. In the near region adjacent to the left reservoir boundary, gas (methane) and its hydrate are in the pores, and gas and ice are in the far region, which is bordering to the right reservoir boundary. The intermediate region separating two above-mentioned regions contains in its pores gas, hydrate and ice, which are in state of thermodynamic equilibrium. Accordingly, there are two moving boundaries: $x = x_{(b)}$, separating the near and intermediate regions, and $x = x_{(d)}$, separating the intermediate and far regions.

When describing the process of heat and mass transfer in a porous media, we assume that this process is one-temperature. We will assume that the porosity is constant; the skeleton of the porous medium, hydrate and ice are incompressible and immobile, and the gas is calorically perfect:

$$m, \rho_{sk}, \rho_h, \rho_i = const; \quad \nu_{sk} = \nu_h = \nu_i = 0; \quad p = \rho_g R_g T.$$ 

Here $m$ is the porosity; $\rho_j$ and $\nu_j$ ($j = sk, h, i$) are the true density and velocity of the $j$th phase, respectively; $p$ is the pressure; $T$ is the temperature; $R_g$ is the gas constant. The subscripts $sk, h, i, g$ correspond to the parameters of the skeleton, gas hydrate, ice and gas. Methane hydrate is a two-component system with a mass concentration of gas $G$.

The system of nonlinear differential equations for describing the process of the gas hydrate formation includes the equations of mass conservation (gas and ice), the equation of heat influx and the Darcy equation. This system in the plane-parallel case taking into account the assumptions has the form [15, 17-19]:

$$\frac{\partial}{\partial t}(mS_g \rho_g) + \frac{\partial}{\partial x}(mS_g \nu_g \rho_g) = -mG \rho_h \frac{\partial S_h}{\partial t},$$  

$$\frac{\partial}{\partial t}(mS_i \rho_i) = -m(1 - G) \rho_h \frac{\partial S_h}{\partial t},$$  

$$\rho_c \frac{\partial T}{\partial t} + \rho_g c_g mS_g \nu_g \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + m \rho_h L_h \frac{\partial S_h}{\partial t},$$  

$$mS_g \nu_g = -\frac{k_g}{\mu_g} \frac{\partial p}{\partial x}.$$  

Here $x$ is the coordinate; $t$ is the time; $S_j$ ($j = g, i, h$) is the pore saturation with the $j$th phase; $\mu_g$ is the gas phase dynamic viscosity; $L_h$ is the heat of formation of methane hydrate; $\rho_c$ and $\lambda$ are, respectively, the specific volumetric heat capacity and the thermal conductivity of the “porous reservoir – saturating substance” system.

When writing the heat inflow equation (3), the barothermal effect is neglected. The specific
volumetric heat capacity $\rho c$ and the thermal conductivity coefficient $\lambda$ will be considered constant values, since they are determined mainly by the parameters of the skeleton.

The dependence of the phase permeability coefficient for gas on the initial gas saturation is set as follows:

$$k_g = k_0 S_g^3,$$

where $k_0$ is the absolute reservoir permeability.

In the region of coexistence of gas, hydrate and ice, temperature and pressure are related by the condition of phase equilibrium [14]:

$$T = T_0 + k \ln \left\{ \frac{p}{p_{s0}} \right\},$$

(5)

where $T_*$ and $p_{s0}$ are the empirical parameters depending on the gas hydrate type.

Since there are regions in the reservoir in which gas, hydrate and ice are in different states, it is necessary to introduce relationships that follow from the laws of conservation of mass and heat at the moving boundaries of the phase transition:

$$mS_g \rho g (u_g - \dot{x}(j)) - mG \rho g S_h \dot{x}(j) = 0,$$

$$m(S_g \rho_g G \rho g S_h \dot{x}(j)) = 0,$$

$$\left[ \lambda \frac{\partial T}{\partial x} \right] = \left[ m \rho g L_h S_h \dot{x}(j) \right].$$

Here $[\psi]$ is the parameter $\psi$ jump at the phase transition boundary $x = x(j)$ ($j = n, d$); $\dot{x}(j)$ is the movement speed of this boundary.

In the initial state, the reservoir is saturated with gas and ice, so the initial conditions will take the form:

$$t = 0: \quad p = p_0, \quad T = T_0 \quad (0 \leq x \leq L).$$

(7)

The conditions on the left and right reservoir boundaries can be presented in the form:

$$t > 0: \quad p = p_x, \quad T = T_e \quad (x = 0),$$

$$t > 0: \quad p = p_0, \quad \frac{\partial T}{\partial x} = 0 \quad (x = L).$$

(8)

The equations of pressure and heat conduction for the near and far regions, which follow from the system (1) and (3), are constructed taking into account the Darcy law (4) and the gas state equation. We assume that the term, taking into account the variability of temperature, is small if the characteristic temperature drops in the filtration area are small:

$$\frac{\partial p}{\partial t} = \frac{k_g}{m \mu_g S_g} \frac{\partial}{\partial x} \left( p \frac{\partial p}{\partial x} \right),$$

(9)

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \kappa^{(T)} \frac{\partial T}{\partial x} \right) + c_g k_g \frac{p}{\mu_g \rho c R_g T} \frac{\partial p}{\partial x} \frac{\partial T}{\partial x},$$

(10)

where $\kappa^{(T)} = \lambda / \rho c$ is the thermal reservoir diffusivity.

The equations for finding the distributions of pressure, temperature, and hydrate saturation in the intermediate region are:

$$\frac{\partial p}{\partial t} = \frac{k_g}{m \mu_g S_g} \frac{\partial}{\partial x} \left( p \frac{\partial p}{\partial x} \right) - \frac{\mathcal{R} T}{\Delta T} \frac{\partial T}{\partial t} + \kappa^{(T)} \frac{\mathcal{R} T}{\Delta T} \frac{\partial^2 T}{\partial x^2} + c_g k_g \frac{p}{\mu_g \rho c \Delta T} \frac{\partial p}{\partial x} \frac{\partial T}{\partial x},$$

(11)

$$\frac{\partial S_h}{\partial t} = - \frac{c_g k_g}{\mu_g \rho c R_g T} \frac{p}{\Delta T} \frac{\partial p}{\partial x} \frac{\partial T}{\partial x},$$

(12)
where \( \Re = \frac{G \rho_f R_g}{1 - S_{l0}} \), \( \Delta T = \frac{m \rho_h L_h}{\rho c} \).

We will assume that at the gas injection into the reservoir on the surface \( x = x_{(n)} \) separating the near and intermediate regions, there is a jump of hydrate saturation from \( S_{h(n)}^- = S_{h(l)}^- > 0 \) to \( S_{h(n)}^+ > 0 \).

Then conditions (6) can be represented as:

\[
\begin{align*}
\frac{k_g^{+}(n)}{\mu_g} \left( \frac{\partial p}{\partial x} \right)_{(n)}^+ - \frac{k_g^{-}(n)}{\mu_g} \left( \frac{\partial p}{\partial x} \right)_{(n)}^- &= m x_{(n)} \left[ S_{g(n)}^- - S_{g(n)}^+ + \frac{\rho_h G(S_{h(n)}^- - S_{h(n)}^+)}{\rho_g(n)} \right], \\
\lambda \left( \frac{\partial T}{\partial x} \right)_{(n)}^- - \lambda \left( \frac{\partial T}{\partial x} \right)_{(n)}^+ &= m \rho_h L_h \left( S_{h(n)}^- - S_{h(n)}^+ \right) x_{(n)},
\end{align*}
\]

(13)

(14)

Where \( S_{h(n)}^- \) is the value of hydrate saturation in the near region. At the border \( x = x_{(dp)} \), which separates the far and intermediate regions and on which the process of hydrate formation ends, conditions (6) can be represented as:

\[
\left( \frac{\partial p}{\partial x} \right)_{(d)}^- = \left( \frac{\partial p}{\partial x} \right)_{(d)}^+,
\]

\[
\left( \frac{\partial T}{\partial x} \right)_{(d)}^- = \left( \frac{\partial T}{\partial x} \right)_{(d)}^+.
\]

(15)

(16)

In this paper, numerical implementation of the mathematical model was carried out using the finite-difference approximation of constructed system of differential equations. The solution of problem (9) - (16) with initial-boundary conditions (7) and (8) was carried out on the basis of the method of catching the front in the grid node with a step along the coordinate \( h = L/n \), where \( n \) is the number of splitting points.

3. Calculations results

Fig. 1 shows the temperature and hydrate saturation distributions for the case of an open right boundary with \( p_e = 2.5 \) MPa. For other parameters characterizing the system, the following values are assumed: \( T_e = 265 \) K, \( L = 50 \) m, \( k_0 = 10^{-14} \) m², \( m = 0.2 \), \( G = 0.12 \), \( \nu = 0.35 \), \( \mu_g = 10^{-5} \) Pa·s, \( \lambda = 2 \) W/(m K), \( L_h = 1.66 \times 10^6 \) J/kg, \( \rho_h = 900 \) kg/m³, \( \rho_i = 900 \) kg/m³, \( T_0 = 271 \) K, \( T^* = 30 \) K, \( p_0 = 2 \) MPa, \( p_{ao} = 2.38 \) MPa [2,6].

Fig. 1 shows that over time the far boundary \( x = x_{(dp)} \) moves backward, towards the near boundary \( x = x_{(n)} \). In this case, a decrease in the hydrate saturation value in the extended region and a slight decrease in temperature in the extended region are observed. Thus, in the zone of three-phase equilibrium, partial decomposition of the previously formed gas hydrate occurs. This is due to convective drift of the heated gas due to the heat released during the gas hydrate formation at the boundary \( x = x_{(n)} \) during filtration in the extended region. The merging of two boundaries means that the process of hydrate formation occurs already on the frontal surface \( x = x_{(n)} \). In fig. 1 such mode is shown for the time \( t = 1500 \) days.
Figure 1. The distribution of the temperature and hydrate saturation on the $x$-coordinate for the different time points: 1 – $t = 30$ days; 2 – $t = 280$ days; 3 – $t = 1500$ days.

Fig. 2a shows the dependence of the phase transition boundary coordinate on time when gas is injected under pressure $p_c = 2.3$ MPa. It follows from the figure that the phase transition surface moves only up to a certain position $x_{(n)}^*$, and then its movement stops. At the same time in the reservoir area $x > x_{(n)}^*$ contains only gas and ice, and the area $x < x_{(n)}^*$ contains only gas and hydrate. This is explained by the fact that from the moment of stopping the phase transition boundary, the reservoir temperature becomes equal to the equilibrium temperature of hydrate formation.
Figure 2. a – the dependence of the phase transition boundary coordinate on time; b – temperature distribution (solid line) and equilibrium temperature distribution (dotted line) at the time of stopping the hydrate formation front.

In this case, the equilibrium temperature, as follows from fig. 2b, has the form of a straight line, decreasing deeper into the reservoir. Therefore, there is a point \( x = x^*_e \) at which the equilibrium temperature of hydrate formation coincides with the reservoir temperature. In this case, in the region \( x > x^*_e \) the parameters of the system will correspond to the condition of stable existence of gas and ice, and in the region \( x < x^*_e \) – of gas and hydrate. Thus, to hydrate a porous medium without stopping the phase transition boundary, it is necessary that the injected gas temperature \( T_e \) satisfies the condition:

\[
T_e < T_0 + T_s \ln \left( \frac{P_0}{P_{s0}} \right).
\]

**Conclusion**
Mathematical model is proposed for the problem of the methane hydrate formation in a porous reservoir of finite length initially saturated with gas and ice. It is shown that hydrate formation can occur both in the extended zone and at the frontal boundary. It has been established that a part of the gas hydrate formed in the extended region can be further decomposed to gas and ice. In this case, the extended region of hydrate formation will degenerate into the frontal surface. If the injected gas temperature is higher than the equilibrium hydrate formation temperature corresponding to the initial pressure, then the regime with the stop of the phase transition boundary is realized.

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