Radiating black holes in Einstein-Yang-Mills theory and cosmic censorship

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Exact nonstatic spherically symmetric black-hole solution of the higher dimensional Einstein-Yang-Mills equations for a null dust with Yang-Mills gauge charge are obtained by employing Wu-Yang ansatz, namely, HD-EYM Vaidya solution. It is interesting to note that gravitational contribution of YM gauge charge for this ansatz is indeed opposite (attractive rather than repulsive) that of Maxwell charge. It turns out that the gravitational collapse of null dust with YM gauge charge admit strong curvature shell focusing naked singularities violating cosmic censorship. However, there is significant shrinkage of the initial data space for a naked singularity of the HD-Vaidya collapse due to presence of YM gauge charge. The effect of YM gauge charge on structure and location of the apparent and event horizons is also discussed.

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I. INTRODUCTION

It is rather well established that higher dimensions provide a natural playground for the string theory and they are also required for its consistency. In fact, the first successful statistical counting of black-hole entropy in string theory was performed for a five dimensional black-hole [1]. This example provides the best laboratory for the microscopic string theory of black holes. The classical general relativity in more than four space-time dimensions has also been the subject of increasing attention [2]. Even from the classical standpoint, it is interesting to study the higher dimensional (HD) extension of Einstein’s theory, and in particular its black-hole solutions [3]. The physical properties of these solutions have been widely studied. Interest in HD black holes has been further intensified in recent years, thanks to the well known AdS/CFT correspondence which envisions a correspondence between string theory (or supergravity) on asymptotically locally anti-de Sitter backgrounds and the large N limit of certain conformal field theories defined on the boundary-at-infinity of these backgrounds [4-6]. There seems to be a general belief that endowing general relativity with a tunable parameter namely the space-time dimension should also lead to valuable insights into the nature of the theory, in particular into its most basic objects: black holes. For instance, four dimensional (4D) black holes are known to have a number of remarkable features, such as uniqueness, spherical topology, dynamical stability, and the laws of black-hole mechanics. One would like to know which of these are peculiar to four dimensions, and which are true more generally? At the very least, such probings into HD will lead to a deeper understanding of classical black holes and of what spacetime can do at its most extreme. There is a growing realization that the physics of HD black holes can be markedly different, and much richer than in four dimensions [7].

However, there are very few nonstatic black-hole solutions known; one of them is the Vaidya black-hole solution. The Vaidya solution [8] is a solution of Einstein’s equations with spherical symmetry for a null dust (radially propagating radiation) source (a Type II fluid) described by energy-momentum tensor, $T_{ab} = \psi n_a n_b$, $n_a$ being a null-vector field. Vaidya’s radiating star metric is today commonly used for two purposes: (i) as a testing ground for various formulations of the Cosmic Censorship Conjecture (CCC) and (ii) as an exterior solution for models of objects consisting of heat-conducting matter. Recently, it has also been employed with good effect in the study of Hawking radiation, the process of black-hole evaporation [9], and in the stochastic gravity program [10]. It has also advantage of allowing a study of the dynamical evolution of horizon associated with a radiating black-hole. Further, various situations involving spherically symmetric source as a mixture of a perfect fluid and null dust have been studied both in 4D [11, 12] as well as in HD [13, 14].

It is of interest to consider models based on different interacting fields including the Yang-Mills. In general, it is difficult to tackle Einstein-Yang-Mills (EYM) equations because of the nonlinearity both in the gauge fields as well as in the gravitational field. The solutions of the classical Yang-Mills fields depend upon the particular ansatz one chooses. Wu and Yang [20] found static spherically symmetric solutions of the Yang-Mills equations in flat space for the gauge group SO(3). A curved space-time generalization of these models has been investigated by several authors (see, e.g., [21]). Indeed Yasskin [21] has
presented an explicit procedure based on the Wu-Yang ansatz \cite{20} which gives the solution of EYM rather trivially.

Using this procedure, Mazharimousavi and Halilsoy \cite{22 24} have found a sequence of static spherically symmetric HD-EYM black-hole solutions. The remarkable feature of this ansatz is that the field has no contribution from gradient; instead, it has pure YM non-Abelian component. It, therefore, has only the magnetic part. It would be interesting to study the effect of pure YM field on gravitational collapse of null dust. That is the main motivation of this paper.

By employing the Wu-Yang ansatz, we shall present a class of HD nonstatic solutions describing the exterior of radiating black holes with null dust endowed with gauge charge. That is, we find analogue of the HD-Vaidya solution in EYM theory. We shall also consider the effect of YM gauge charge on the collapse of null dust described by the Vaidya solution onto a flat Minkowski cavity in HD. In the next section, we write the effective EYM equation in HD and find the generalized Vaidya solution namely HD-EYM Vaidya solution which would be followed in Sec. III by discussion of energy conditions and horizons. In Sec. IV, we shall study the gravitational collapse of null dust with YM gauge charge and then we conclude in Sec V.

II. HD-VAIDYA LIKE METRIC IN EINSTEIN-YANG-MILLS THEORY

We consider \((N - 1)(N - 2)/2\) parameter Lie group with structure constant \(C_{(β)(γ)}^{(α)}\). The gauge potentials \(A_a^{(α)}\) and the Yang-Mills fields \(F_a^{(α)}\) are related through the equation

\[
F_a^{(α)} = \partial_a A_b^{(α)} - \partial_b A_a^{(α)} + \frac{1}{2\sigma} C_{(β)(γ)}^{(α)} A_b^{(β)} A_a^{(γ)}. \tag{1}
\]

Then one can choose the gravity and gauge field action (Einstein-Yang-Mills), which in \(N\)-dimensions reads:

\[
I_G = \frac{1}{2} \int_M d^N x \sqrt{-g} \left[ R - \sum_{α=1}^{(N-1)(N-2)/2} F_a^{(α)} F_a^{(α)} + I_N \right]. \tag{2}
\]

Here, \(g = \det(g_{ab})\) is the determinant of the metric tensor, \(R\) is the Ricci Scalar \(\cite{22 23}\), and \(A_a^{(α)}\) are the gauge potentials. \(I_N\) is the action of null dust. We note that the internal indices \((α, β, γ, ...\) do not differ whether in covariant or contravariant form. We introduce the Wu-Yang ansatz in \(N\)-dimension \(\cite{22 24}\) as

\[
A^{(α)} = \frac{Q}{r^2} (x_i dx_j - x_j dx_i)
\]

where the super indices \(α\) is chosen according to the values of \(i\) and \(j\) in order and we choose \(σ = Q\) \(\cite{22 24}\). The Wu-Yang solution appears highly nonlinear because of mixing between space-time indices and gauge group indices. However, it is linear as expressed in the nonlinear gauge fields because purely magnetic gauge charge is chosen along with position dependent gauge field transformation \(\cite{21}\). The YM field 2 form is defined by the expression

\[
F^{(α)} = dA^{(α)} + \frac{1}{2Q} C_{(β)(γ)}^{(α)} A^{(β)} \wedge A^{(γ)}. \tag{4}
\]

The integrability conditions

\[
dF^{(α)} + \frac{1}{Q} C_{(β)(γ)}^{(α)} A^{(β)} \wedge F^{(γ)} = 0, \tag{5}
\]

as well as the YM equations

\[
dF^{(α)} + \frac{1}{Q} C_{(β)(γ)}^{(α)} A^{(β)} \wedge *F^{(γ)} = 0, \tag{6}
\]

are all satisfied. Here \(d\) is exterior derivative, \(\wedge\) stands for wedge product and \(*\) represents Hodge duality. All these are in the usual exterior differential forms notation.

Variation of the action with respect to the space-time metric \(g_{ab}\) yields the EYM equations

\[
G_{ab} = T_{ab}. \tag{7}
\]

The stress-energy tensor is written as

\[
T_{ab} = T_a^{G} + T_a^{N}, \tag{8}
\]

where the gauge stress-energy tensor \(T_a^{G}\) is

\[
T_a^{G} = \sum_{α=1}^{(N-1)(N-2)/2} \left[ 2F_{αa}^2 - \frac{1}{2} F_{βa}^2 \right], \tag{9}
\]

and the null dust energy-momentum tensor is

\[
T_a^{N} = ψ(v, r)n_an_b, \tag{10}
\]

with \(ψ(v, r)\), the nonzero energy-density and \(n_a\) is a null vector such that \(n_a = δ^0_a n_b δ^a_b = 0\).

Expressed in terms of Eddington coordinates, the metric of general spherically symmetric space-time in \(N\)-dimensional space-times \(\cite{13 14 23}\) is given by

\[
ds^2 = -A(v, r)^2 f(v, r) dv^2 + 2eA(v, r) dv dr + r^2 \left( dΩ_{N-2}^2 \right), \tag{11}
\]

where

\[
(dΩ_{N-2})^2 = dθ_1^2 + \sin^2(θ_1)dθ_2^2 + \sin^2(θ_2)sin(θ_3)dθ_3^2 + \ldots + \left( \prod_{j=1}^{N-2} \sin^2(θ_j) \right) dθ_{N-1}^2
\]

and \(\{x^a\} = \{v, r, θ_1, \ldots θ_{N-2}\}\). For null dust, \(T_v\) must be nonzero and \(T_v = T_r\) for null energy condition.
This would imply \( A(v, r) = g(v) \) which could be set to 1 without any loss of generality. It is useful to introduce a local mass function \( m(v, r) \) defined by \( f(v, r) = 1 - 2m(v, r)/(N-3)r^{(N-3)} \). For \( m(v, r) = m(v) \) and \( A = 1 \), the metric reduces to the \( N \)-dimensional Vaidya metric \([12]\). Therefore the entire family of solutions we are searching for is determined by a single function \( m(v, r) \) or \( f(v, r) \).

It may be noted that in view of Eq. (3), the gauge field has only the angular components, \( F^{\alpha \theta}_i \) with \( i \neq j \), nonzero and they go as \( r^{-2} \) which in turn makes \( T^\theta_\phi \) go as \( r^{-4} \) irrespective of \( N \geq 5 \). The null dust part will be given by \( T^\nu_\nu = \psi(r, v) \). Note that the second term in Eq. (9) is evaluated to read

\[
\sum_{\alpha = 1}^{(N-1)(N-2)/2} \left[ F^{(\alpha)}_{\lambda \sigma} F^{(\alpha)\lambda \sigma} \right] = \frac{(N-3)(N-2)Q^2(v)}{r^4},
\]

(12)

The last two equations are not independent and it suffices to integrate Eq. (13b) to give

\[
f(v, r) = \begin{cases} 
1 - \frac{M(v)}{r^{(N-3)}} - \frac{(N-3)Q^2(v)}{(N-5)r^{(N-5)}} & N > 5, \\
1 - \frac{M(v)}{r^2} - \frac{2Q^2(v)\ln r}{r^2} & N = 5.
\end{cases}
\]

(14)

where \( M(v) \) is an arbitrary function of \( v \). Since YM \( T^G_{ab} \) go as \( r^{-4} \) (the same as for Maxwell field in \( N = 4 \)), interestingly for all \( N \geq 5 \). That is why its contribution in \( f \) will be the same for all \( N \geq 5 \) as in 4-dimensional Reissner-Nordstrom (RN) static or Bonnor-Vaidya radiating black-hole \([26]\). The nonradiating limit of this would be HD-Yaskin black-hole and not HD analogue of Reissner-Nordstrom. In other way round, this is making the Yaskin YM-black-hole radiate.

There is however an important difference in the sign before \( Q^2 \) from the Maxwell case, which could be understood as follows \([27]\): gravitational potential \( \Phi \) at any \( r \) will go as

\[
\Phi = -(M - E(r))/r^{N-3},
\]

(15)

where \( E(r) \) is the YM energy lying between \( r \) and infinity \([37]\). It is easy to compute \( E(r) \)

\[
E(r) = \int_r^\infty (Q^2/r^4)r^{n-2}dr = -\frac{Q^2}{(N-5)r^{N-5}}
\]

(16)

Then the nonzero components would read as:

\[
T_\nu^\nu = \psi(r, v), \quad T_v^v = T_r^r = -(N-3)(N-2)Q^2(v)/2r^4 \quad \text{and} \quad T^\theta_\theta = T^\phi_\phi = \ldots = T^\theta_{\theta N-2} = -(N-3)(N-6)Q^2(v)/2r^4.
\]

It may be recalled that energy-momentum tensor (EMT) of a Type II fluid has a double null eigenvector, whereas an EMT of a Type I fluid has only one time-like eigenvector \([28]\).

For the energy-momentum tensor \([8]\) and with the metric \([11]\), the Einstein equations \([7]\) reduce to:

\[
\psi = -\frac{(N-2)}{2r} \frac{\partial f}{\partial v},
\]

(13a)

\[
r^2 \left[ \frac{\partial f}{\partial r} - (N-3)(1 - f) \right] + (N-3)Q^2(v) = 0,
\]

(13b)

\[
r^4 \left[ r^2 \frac{\partial^2 f}{\partial r^2} + (N-3) \left( 2r \frac{\partial f}{\partial r} - (N-4)(1 - f) \right) \right] - (N-3)(n-6)Q^2(v) = 0.
\]

(13c)

which would be negative for \( N > 5 \) and positive for \( N = 4 \). This is what is responsible for the opposite sign for \( Q^2 \) as that for the Maxwell case in \( N = 4 \) \([29]\). So we have for \( N > 5 \)

\[
\Phi = -\frac{M + (Q^2r^{N-5})}{r^{N-3}} = -\frac{M}{r^{N-3}} - \frac{Q^2}{r^2}.
\]

(17)

Unlike the Maxwell field, this is how YM field energy works in unison with mass and makes attractive contribution. For the Wu-Yang ansatz, the two fields are thus gravitationally distinguished.

From Eq. (13a), we obtain the energy density of the null dust with gauge charge as

\[
\psi(v, r) = \begin{cases} 
\frac{(N-2)}{2r^{N-2}} \frac{dM(v)}{dv} + \frac{(N-2)(N-3)}{(N-5)r^{(N-5)}} \frac{Q(v)\delta Q(v)}{d\psi}, & N > 5, \\
\frac{3}{2r^2} \frac{dM(v)}{dv} + \frac{3}{r} Q(v) \frac{dQ(v)}{d\psi} \ln(r), & N = 5.
\end{cases}
\]

(18)

and YM energy density and transverse stress are given by

\[
\zeta(v, r) = (N-3)(N-2) \frac{Q^2(v)}{2r^4},
\]

(19)

\[
P(v, r) = -(N-3)(N-6) \frac{Q^2(v)}{2r^4}.
\]

(20)
for $N \geq 5$. Note that $P(v, r) = 0$ for $N = 6$ where it changes sign from positive to negative. The solution obtained above represents a general class of non-static, $N$-dimensional spherically symmetric solution of EYM theory describing a radiating black-hole. It contains $N$-dimensional version of Vaidya [13, 19] solution.

In the static limit, $\dot{M} = 0$, it reduces to the black-hole solutions independently discovered in HD-EYM theory [21, 24].

Also with (13), the Kretschmann scalar ($\mathcal{K} = R_{abcd}R^{abcd}$) for the metric (11) reduces to

$$\mathcal{K} = \begin{cases} 
\frac{(N-3)(N-1)(N-2)^2}{r^4} \frac{M^2(v)}{2} + \frac{24(N-3)^2(N-2)}{(N-5)r^{N+3}} Q^2(v) \frac{M(v)}{2} + \frac{2(N-3)^2(N^2-N+16)}{(N-5)^2r^8} Q^4(v) & N > 5, \\
\frac{24}{5} M^2(v) + \frac{24(12\ln(r)-7)}{r^7} Q^2(v) \frac{M(v)}{2} + \frac{4(78\ln^2(r)-84\ln(r)+31)}{r^4} Q^4(v) & N = 5.
\end{cases}$$

(21)

where,

$$n_a = \delta_a^v, \quad l_a = \frac{1}{2} f(v, r) \delta_a^r + \delta_a^r, \quad (25a)$$

$$\gamma_{ab} = r^2 \delta_a^v \delta_b^v + r^2 \left[ \prod_{j=1}^{(-1)} \sin^2(\theta_j) \right] \delta_a^\theta \delta_b^\theta, \quad (25b)$$

$$l_a a^a = n_a n^a = 0 \quad l_a n^a = -1, \quad l^a \gamma_{ab} = 0; \gamma_{ab} n^b = 0, \quad (25c)$$

The optical behavior of null geodesics congruences is governed by the Raychaudhuri equation [22, 30]

$$\frac{d\Theta}{dv} = K \Theta - R_{ab} v^a v^b - \frac{1}{2} \Theta^2 - \sigma_{ab} \sigma^{ab} + \omega_{ab} \omega^{ab}, \quad (26)$$

with expansion $\Theta$, twist $\omega$, shear $\sigma$, and surface gravity $K$. Here $R_{ab}$ is the $N$-dimensional Ricci tensor, $\gamma^c_{ab}$ is the trace of the projection tensor for null geodesics. The expansion of the null rays parameterized by $v$ is given by

$$\Theta = \nabla_a v^a - K, \quad (27)$$

where the $\nabla$ is the covariant derivative and the surface gravity is

$$K = -n_a l^a \nabla_v l_a. \quad (28)$$

The apparent horizon (AH) is the outermost marginally trapped surface for the outgoing photons. The AH can be either null or spacelike, that is, it can "move" causally or acausally. As demonstrated by York [29], horizons can be obtained by noting that (i) apparent horizons are defined as surface such that $\Theta \simeq 0$ and (ii) event horizons are surfaces such that $d\Theta/dv \simeq 0$. It follows that apparent horizons are the zeros of

$$\begin{align*}
\frac{r(N-3) - M}{r(N-5)} Q^{N-3} r^{N-5} &= 0 \quad N > 5, \\
2 - M - 2Q^2 \ln(r) &= 0 \quad N = 5.
\end{align*}$$

(29)
For $Q^2 = 0$, we have Schwarzschild horizon $r_{AH} = (M)^{1/(N-3)}$. In general Eq. (29) admits a general solution

\[
\begin{align*}
\begin{cases}
\frac{r_{AH}}{2} + \frac{2Q^2}{\Delta} \\
\sqrt{Q^2 \pm \sqrt{Q^2 + M}} \\
\exp \left[ -\frac{1}{2} \frac{Q^2 \text{LambertW} \left( \frac{-e^{-M/Q^2}}{Q^2} \right) + M}{Q^2} \right]
\end{cases}
\end{align*}
\begin{align*}
N &= 6, \\
N &= 7, \\
N &= 5.
\end{align*}
\]

Here

\[
\Delta = \left( 4M \pm 4\sqrt{M^2 - 4Q^2} \right)^{1/2}.
\]

FIG. 1: Plot of $r_{AH}$ versus $M$ and $Q$

Thus for $N = 5$, the two roots of the Eq. (29) coincide, and there is only one horizon and for $N > 5$ there are two horizons, namely inner and outer horizons. Clearly, in the limit $Q = 0$, we have Schwarzschild horizon $r_{AH} = (M)^{1/(N-3)}$.

On the other hand, the future event horizon is a null surface which is the locus of outgoing future-directed null geodesic rays, that never manage to reach arbitrarily large distances from the black-hole, and is determined via the Raychaudhuri equation. It can be seen to be equivalent to the requirement that

\[
\left[ \frac{d^2r}{dv^2} \right]_{EH} \approx 0.
\]

The expression for the event horizon is the same as that for the apparent horizon with $M$ and $Q$ being respectively replaced by $M^*$ and $Q^*$ [18, 30], where $M^*$ and $Q^*$ are effective mass and charge defined as follows:

\[
M^*(v) = M(v) - \frac{L_M}{K},
\]

\[
Q^*(v) = Q(v) - \frac{L_Q}{K}.
\]

IV. GRAVITATIONAL COLLAPSE

In this section, we employ the above metric for investigation of YM gauge charge effect on formation of black-hole or naked singularity in collapse of null dust in the context of CCC. The physical situation is that of a radial influx of gauge charged null dust in an initially empty region of the HD-Minkowski space-time. The first shell arrives at $r = 0$ at time $v = 0$ and the final at $v = T$. A central singularity of growing mass is developed at $r = 0$. For $v < 0$ we have $M(v) = Q(v) = 0$, i.e., an empty HD-Minkowski metric, and for $v > T$, $M(v)$ and $Q^2(v)$ are positive definite constants. The metric for $v = 0$ to $v = T$ is the HD-EYM Vaidya (discussed above), and for $v > T$ we have the HD-EYM Schwarzschild [22, 22] solution.

In order to proceed further we would require the specific forms of the functions $M(v)$ and $Q^2(v)$, which we choose as follows [17, 32]:

\[
M(v) = \begin{cases}
0, & v < 0, \\
\lambda v (\lambda > 0), & 0 \leq v \leq T, \\
M_0(> 0), & v > T.
\end{cases}
\]

and

\[
Q^2(v) = \begin{cases}
0, & v < 0, \\
\mu^2 v^2 (\mu^2 > 0), & 0 \leq v \leq T, \\
Q_0^2(> 0), & v > T.
\end{cases}
\]
Then the space-time is self-similar, admitting a homothetic Killing vector

$$\xi^a = r \frac{\partial}{\partial r} + v \frac{\partial}{\partial v}, \quad (36)$$

which is given by the Lie derivative

$$L \xi g_{ab} = \xi_{;a;b} + \xi_{;b;a} = 2g_{ab}, \quad (37)$$

where $L$ denotes the Lie derivative. Let $K^a = dx^a/dk$ be the tangent vector to the null geodesics, where $k$ is an affine parameter. Then

$$g_{ab} K^a K^b = 0. \quad (38)$$

It follows that along null geodesics, we have

$$\xi^a K_a = r K_r + v K_v = C. \quad (39)$$

Following, we introduce

$$K^r = \frac{P}{r}, \quad (40)$$

and, from the null condition, we obtain

$$K^r = \left[ 1 - \frac{M(v)}{r(N-3)} \right] \frac{(N-3)Q^2(v)}{(N-5)r^2} \frac{P}{2r}. \quad (41)$$

To study the singularity we employ the method developed by Dwivedi and Joshi. Consider the equation for future-directed outgoing radial null geodesics

$$\frac{dr}{dv} = f = \frac{1}{2} \left[ 1 - \frac{M(v)}{r} - \frac{(N-3)Q^2(v)}{(N-5)r^2} \right]. \quad (42)$$

The region $f < 0$ is the trapped region and surface $f = 0$ represent trapping horizon. This is an ordinary differential equation with a singular point $v = 0$, $r = 0$. This singularity is (at least locally) naked if there are geodesics starting at it with a definite tangent. If no such geodesic exists, then singularity is not naked and strong CCC holds. To investigate the behavior near singular point, define

$$y = \frac{v}{r}. \quad (43)$$

Eq. (42), upon using Eqs. (34), (35) and (36), turns out to be

$$\frac{dr}{dv} = \frac{1}{2} \left[ 1 - \frac{\lambda y^{N-3}}{r} - \frac{(N-3)Q^2(v)}{(N-5)r^2} \right]. \quad (44)$$

If singularity is naked, there exists some value of $\lambda$ and $\mu$, such that at least one positive finite value $y_0$ exists which solves the algebraic equation

$$y_0 = \lim_{r \to 0} y = \lim_{r \to 0} \frac{v}{r}. \quad (45)$$

Table I: Variation of $\mu_c$ with $D$. For $\mu > \mu_c$, the end state of collapse is a black-hole for all $\lambda$ ($\lambda \geq 10^{-12}$).

| $D = N$ | Critical Value $\mu_c$ | Two equal Roots $\approx y_0$ |
|---------|------------------------|-----------------------------|
| 6       | 0.111111               | 3.0                         |
| 7       | 0.136081               | 3.0                         |
| 8       | 0.14907                | 3.0                         |
| 9       | 0.157133               | 3.0                         |
| 10      | 0.162648               | 3.0                         |

Using (44) and l’Hôpital’s rule we get

$$y_0 = \lim_{r \to 0} y = \lim_{r \to 0} \frac{v}{r} = \lim_{r \to 0} \frac{dv}{dr} \approx \frac{2}{1 - \lambda y_0^{(N-3)} - \frac{(N-3)Q^2}{(N-5)r^2} y_0^2} \quad (46)$$

which can be written in explicit form as,

$$\frac{(N-3)}{(N-5)} y_0^2 + \lambda y_0^{(N-2)} - y_0 + 2 = 0. \quad (47)$$

This algebraic equation is the key equation which governs the behavior of the tangent vector near the singular point. If the singularity is naked, Eq. (47) must have one or more positive roots $y_0$, i.e., at least one outgoing geodesic that will terminate in the past at the singularity. While the absence of positive roots indicates that the collapse will always lead to a black-hole. Any solution $y_0 > 0$ of the Eq. (47) would correspond to the naked singularity of the space-time, i.e., to a future-directed null geodesic emanating from the singularity ($v = 0$, $r = 0$).

The smallest such $y_0$ corresponds to the earliest ray emanating from the singularity and is called Cauchy horizon of the space-time. If $y_0$ is the smallest positive root of (47), then there are no naked singularity in the region $y < y_0$. It is easy to check that two roots of Eq. (47) are always positive if $\lambda \leq \lambda_C$ and the corresponding values of equal roots $y_0$ are shown in Table II. Thus, the occurrence of positive real roots implies that the strong CCC is violated, though not necessarily the weak CCC. The global nakedness of singularity can then be seen by making a junction onto HD-EYM Schwarschild space-time.

In the absence of positive real roots, the central singularity is not naked (censored) because in that case there are no outgoing future-directed null geodesics from the singularity. It can be seen that collapse always leads to black-hole if $\mu > \mu_c$ (Table I) for all $\lambda \leq 10^{-12}$. In the limit $\mu \to 0$ (i.e. when gauge charge is switched off) our results reduce to those previously obtained by us for HD-Vaidya collapse, and in that case, Eq. (47) admits positive roots for $\lambda \leq \lambda_C$. Hence singularities are naked for $\lambda \in (0, \lambda_C)$, and censored (black holes) otherwise. Thus $\lambda = \lambda_C$ is the critical value at which the
### TABLE II: Variation of critical parameter $\lambda_c$ and $y_0$ with $D$ in HD-Vaidya and HD-EYM Collapse

| $D = N$ | HD-Vaidya Collapse | HD-EM Collapse ($\mu = 0.1$) |
|---------|--------------------|-----------------------------|
|         | $\lambda_c^V = \frac{1}{N^2} \left( \frac{N-1}{N-2} \right)^{N-3}$ | $\lambda_c^\text{YM}$ |
| 6       | 27/2048 = 0.0132   | $\lambda_c^\text{EM}$ |
| 7       | 16/3125 = 0.00512  | $\lambda_c^\text{EM}$ |
| 8       | 3125/1492992 = 0.00206 | $\lambda_c^\text{EM}$ |
| 9       | 729/823543 = 0.00088 | $\lambda_c^\text{EM}$ |
| 10      | 823543/2147483648 = 0.000038 | $\lambda_c^\text{EM}$ |
|         | Double Roots($y_0 = \frac{2N-4}{N-3}$) | Double Roots($y_0$) |
| 6       | 8/3 = 2.6667       | 2.91                        |
| 7       | 5/2 = 2.5          | 2.69                        |
| 8       | 12/5 = 2.4         | 2.56                        |
| 9       | 7/3 = 2.34         | 2.48                        |
| 10      | 16/7 = 2.28        | 2.42                        |

All results concerning nakedness of singularity characterizing HD-Vaidya can be obtained from $\mu \to 0$ (see [16] for details).

### A. Strength of Naked Singularities:

From the physical point of view, one of the most important features of curvature singularity is its gravitational strength. A singularity is termed gravitationally strong or simply strong, if it destroys by crushing or stretching any object which falls into it. A sufficient condition [33] for a strong singularity as defined by Tipler [34] is that for at least one non spacelike geodesic with affine parameter $k$, in limiting approach to singularity, we must have

$$\lim_{k \to 0} k^2 \psi = \lim_{k \to 0} k^2 R_{ab} K^a K^b > 0 \quad (48)$$

where $R_{ab}$ is the Ricci tensor. Eq. (48), with the help of Eqs. (33), (40) and (41), can be expressed as

$$\lim_{k \to 0} k^2 \psi = \frac{(N - 2)(N - 3)}{2(N - 5)} \times \lim_{k \to 0} \left( (N - 5) \lambda y^{(N-4)} + 4 \mu^2 y \right) \left( \frac{k P}{r^2} \right)^2 \quad (49)$$

Our purpose here is to investigate the above condition along future-directed null geodesics coming out from the singularity. Eq. (39), because of Eqs. (33), (35), (40) and (41), yields

$$P = \frac{2C}{2 - y + \lambda y^{(N-2)} + \frac{(N-2)}{(N-5)} \mu^2 y^3} \quad (50)$$

and geodesics are completely determined. Further, we note that

$$\frac{dX}{dk} = \frac{1}{r} K^v - \frac{X}{r} K^r \quad (51)$$

which, on inserting the expressions for $K^r$ and $K^v$, become

$$\frac{dX}{dk} = \left( 2 - y + \lambda y^{(N-2)} + \frac{(N-2)}{(N-5)} \mu^2 y^3 \right) \frac{P}{2r^2} = \frac{C}{r^2}. \quad (52)$$

Using the fact that as singularity is approached, $k \to 0$, $r \to 0$ and $X \to a_+$ (a root of (47)) and using l'Hôpital's rule, we observe

$$\lim_{k \to 0} \frac{k P}{r^2} = \frac{y_0}{2} \quad (53)$$

when $\lim_{k \to 0} P = P_0 \neq \infty$ and hence Eq. (49) gives

$$\lim_{k \to 0} k^2 \psi = \frac{(N - 2)(N - 3)}{2(N - 5)} \times \left( (N - 5) \lambda y_0^{(N-4)} + 4 \mu^2 y_0 \right) \frac{y_0}{4} > 0. \quad (54)$$

Thus along radial null geodesics strong curvature condition is satisfied. Therefore, one may say that generically, the naked singularity is gravitationally strong [34]. Having seen that the naked singularity in our model is a strong curvature singularity, we also examine its scalar polynomial character. The singularity arising in the HD-Vaidya model was shown to be a strong curvature and also a scalar polynomial [12]. The presence YM gauge charge does not affect this feature which is evident from the divergence of Kretschmann scalar.

### V. CONCLUDING REMARKS

In conclusion, we have constructed nonstatic radiating black-hole solutions of the coupled EYM equations for a null dust with gauge charge in HD, namely HD-EYM Vaidya. The HD-EYM Vaidya solutions are obtained by employing HD curved-space generalization of Wu-Yang ansatz [21]. Thus we have an explicit nonstatic radiating black-hole solutions of Einstein equations for non-Abelian gauge theory. This yields in 4D, the same results as one would expect for charge null dust in the Abelian theory [20], i.e., in 4D the geometry is precisely of the Bonnor-Vaidya form and the charge that determines the geometry is YM gauge charge. However
in HD-EYM radiating black-hole solutions deviate from Bonnor-Vaidya solutions because here the term \(Q^2/r^2\) in the solution (14) is dimension independent while it would go as \(Q^2/r^{N-2}\) for the latter. This is also reflected in the divergence of the Kretschmann scalar in Eq. (21). Note that the last term diverges as \(r^{-8}\) while for HD solution with charge it would diverge as \(r^{-4(N-2)}\). This is the consequence of the Wu-Yang ansatz, which is also responsible for charge working in unison (attractive) with mass in its gravitational contribution.

The family of solutions discussed here belongs to Type II fluid. However, if \(M = Q = \text{constant}\) and the matter field degenerates to type I fluid, we can generate static black-hole solutions obtained in [22] by proper choice of these constants. In the static limit, this metric can be obtained from the metric in the usual spherically symmetric form

\[
ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2(d\Omega_{N-2})^2 \tag{55}
\]

by the coordinate transformation

\[
dv = A(r)^{-1}\left(dt + \epsilon\frac{dr}{f(r)}\right). \tag{56}
\]

In case of spherical symmetry, even when \(f(r)\) is replaced by \(f(t, r)\), one can cast the metric in the form (11) [35].

We have also used this solution to study the end state of collapsing star and showed that there exists a regular initial data which leads to naked singularity. The relevant question is what effect does the presence of the gauge charge have on formation or otherwise of a naked singularity. Our results imply that the presence of gauge charge leads to shrinking of the initial data space for naked singularity of the HD-Vaidya collapse. That is, it tends to favor black-hole. The gauge charge would contribute positively to gravity of the collapsing null dust. This should cover part of the parameter window in the initial data set for naked singularity. This is what has been demonstrated. That is, the parameter set which gave rise to naked singularity in HD-Vaidya collapse may now lead to black-hole in the presence of the gauge charge. There exists a threshold value for \(\mu\), as shown in Table. II, the parameter window gets fully covered ensuring formation of black-hole for all values of \(\lambda\). That is when \(\mu > \mu_C\) the CCC is always respected. The important point is that collapse of null dust with gauge charge would favor black-hole in comparison to naked singularity.

As final remarks it would be interesting to see how the results get modified in EYM theory with the Gauss-Bonnet combination of quadratic curvature terms and, in general, for the Lovelock polynomial [36].

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bridge, 1973)

[29] J. W. York Jr. Quantum Theory of Gravity: Essays in Honor of Sixtieth Birthday of Bryce S. DeWitt, edited by S. Christensen (Hilger, Bristol), (1984).

[30] R. L. Mallett Phys. Rev. D 33, 2201 (1986). B. D. Koberlein and R. L. Mallett, Phys. Rev. D 49, 5111 (1994).

[31] A spherically symmetric space-time is self similar if $g_{tt}(ct, cr) = g_{tt}(t, r)$ and $g_{rr}(ct, cr) = g_{rr}(t, r)$ for every $c > 0$. A self similar space-time is characterized by the existence of a homothetic Killing vector.

[32] P. S. Joshi Global Aspects in Gravitation and Cosmology (Clarendon Press, Oxford, 1993).

[33] C. J. S. Clarke and A. Krölak, J. Geom. Phys. 2 (1986) 127.

[34] F. J. Tipler, C. J. S. Clarke, and G. F. R. Ellis, in General Relativity and Gravitation, Plenum A. Held (Ed.), New York, 1980.

[35] A. B. Nielsen and M. Visser Class. Quantum Grav. 23, 4637 (2006).

[36] N. Dadhich and S. G. Ghosh (work in progress).

[37] In either case there is hidden electromagnetic contribution in mass $M$ that is not visible. What is visible is the $r$-dependent contribution from the electromagnetic field energy lying outside the black-hole. This has opposite contribution in the two cases because of the ansatz that makes the field go as $1/r^2$ always irrespective of the dimension of space-time.