Holographic Responses of Fermion Matter

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Abstract

We consider the $D4-D8-\overline{D8}$ brane system which serves as ultraviolet completion of the Nambu-Jona-Lasinio model, where the only degrees of freedom carrying baryon charge are fermions. By turning on chemical potential for this charge one may expect the formation of the Fermi liquid ground state. At strong coupling we use the dual holographic description to investigate the responses of the system to small perturbations. In the chirally symmetric phase we find that the density dependent part of the heat capacity vanishes linearly with temperature. We also observe a zero sound excitation in the collisionless regime, whose speed is equal to that of normal sound in the hydrodynamic regime. Both the linear dependence of the heat capacity and the existence of zero sound are properties of the Fermi liquid ground state. We also compute the two-point function of the currents at vanishing frequency but do not find any singularities at finite values of the momentum.

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1. Introduction

Understanding QCD at finite baryon density or finite chemical potential is an important and difficult problem. Lattice computations are complicated by the sign problem, and one largely has to rely on phenomenological models. At large values of the chemical potential asymptotic freedom ensures that the physics is determined by the dynamics of the quarks near the Fermi surface. The physics simplifies in the planar limit, since some of the perturbative instabilities are suppressed (see [1] and references therein for the recent work in this direction).

It is interesting to compare the predictions of perturbative QCD with those coming from string theoretic models. Consider the holographic model of QCD [2,3] at finite temperature $T$ and baryon chemical potential $\mu$. An incomplete list of references where this setup was analyzed includes [4-15]. The holographic model is by no means equivalent to QCD, and in fact lacks asymptotic freedom. In the holographic model it is possible to adjust the glueball mass scale to be lower than the meson mass scale. This leads to the existence of the deconfined but chirally broken phase. The phase diagram then contains a line of first order phase transitions between a chirally symmetric phase at larger values of $\mu$ and $T$ and a broken phase at smaller values of these parameters [5,6]. In addition, at large values of $\mu$ there is a phase with condensed baryons [10,12], which is not what we naively expect from perturbative QCD. One may ask whether the physics of the system resembles that of perturbative QCD, and, in particular whether one can see signatures of Fermi liquid formation.

The question is actually more general than this\textsuperscript{1}. Generic attractive interactions are believed to destabilize the Fermi surface, leading to the formation of a gap and a superconducting ground state. (For a holographic description of superconductivity see [22-31] and references therein.) In the ’t Hooft limit the perturbative quark-quark interactions are suppressed, and there is an opportunity for the Fermi liquid ground state to survive, even though the coupling is strong. In this paper we attempt to analyze the fate of the fermion matter from the $D4-D8$ strings by studying its responses to small external perturbations. Since our main interest is not QCD dynamics and the Yang-Mills degrees of freedom simply provide the strong interaction between quarks, we consider the (decompactification) limit which corresponds, in the field theoretic regime, to a certain UV completion of the Nambu-Jona-Lasinio model [32]. (See [33] for more details.) We turn on chemical potential

\textsuperscript{1} Recent work on holographic Fermi systems includes [16-20], see also [21].
which is expected to lead to the formation of charged matter, whose ingredients, at least in the field theoretic regime are fundamental fermions. We consider the phase where chiral symmetry is restored, which corresponds to the $D8 - \overline{D8}$ branes falling into the horizon of the black hole. Such a phase is perturbatively stable for a wide range of $\mu$ and $T$.

We show that the density dependent part of the heat capacity at low temperature is linear in $T$. This is the behavior expected for systems with a Fermi surface, where only a fraction of quasiparticles is excited at small temperatures. We also observe that at arbitrarily low temperatures there exists a massless excitation (zero sound) whose speed is equal to the speed of normal sound in the hydrodynamic regime. The existence of zero sound is also a feature of the Fermi liquid and corresponds to the deformation of the Fermi surface away from the spherical shape. We then compute the current-current two-point function at vanishing frequency $\omega = 0$ and finite spacial momentum $q$ using the holographic techniques. Such a two-point function should be sensitive to the finite gap in the distribution function at the Fermi momentum $q = q_F$. We do not find any singular features however. As we discuss below, this might be related to a non-generic dispersion relation of the quasiparticles near the Fermi surface.

The rest of the paper is organized as follows. In the next section we review the $D4 - D8 - \overline{D8}$ system at finite chemical potential, including phase structure and thermodynamics. We compute the speed of normal sound in the hydrodynamic regime and show that the density dependent part of the heat capacity vanishes linearly with $T$ at small temperatures. We then study small fluctuations at small temperature (collisionless regime) and find the massless excitation (zero sound) in section 3. The speed of zero sound is equal to the speed of normal sound in the hydrodynamic regime. In section 4 we compute the current-current two-point function and show that no visible singularities are present. We discuss our results in section 5. Appendix A contains the derivation of the speed of normal sound in the $D3 - D7$ system. Appendix B is devoted to the derivation of the boundary conditions in the limit of vanishing $\omega$.

2. Thermodynamics

2.1. Review

In this section we will review the system studied in [33] at zero temperature. The configuration we are interested in consists of three kinds of branes: $D4$, $D8$ and $\overline{D8}$ branes. The $N_c$ $D4$ branes are extended along the $x^0, \cdots, x^4$ directions whereas the $N_f$
$D8$ and $\overline{D8}$ branes are parallel and located a distance $L$ apart from each other along the $x^4$ direction. The only massless excitations of the fundamental strings stretching between the $D8$ and $D4$ branes are spacetime Weyl fermions. In particular, $4 - 8$ strings contribute left-handed $q_L$ fermions localized at the $x_4 = -\frac{L}{2}$ intersection while $4 - 8$ strings provide right-handed $q_R$ fermions localized at $x_4 = \frac{L}{2}$. Studying this system at strong coupling, i.e., $\lambda \gg L \gg l_s$ where $\lambda$ is the five-dimensional 't Hooft coupling constant, requires taking the near horizon limit of the $D4$-branes. Assuming $N_f \ll N_c$, the back-reaction of the $D8$ branes on the geometry can be neglected. In this case, it suffices to explore the physics of the fundamental matter through the DBI (Dirac-Born-Infeld) action governing their dynamics as they propagate in the near horizon geometry of the $N_c D4$ branes.

In the following, we summarize some of the results of [6] concerning the $D4 - D8 - \overline{D8}$ system at zero temperature but non-zero chemical potential. This requires turning on an electric field on the worldvolume of the $D8$ branes. The value of the chemical potential is then read off from the asymptotic value of the gauge field on the brane.

Let us start by considering the near horizon region of $D4$-branes

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{4}} (-dt^2 + dx_i^2 + dX_4^2) + \left(\frac{U}{R}\right)^{-\frac{3}{4}} (dU^2 + U^2 d\Omega_4^2)$$

$$e^\Phi = g_s \left(\frac{U}{R}\right)^{\frac{3}{4}} \quad F_4 = dC_3 = \frac{2\pi N_c}{\Omega_4} \omega_4$$

(2.1)

Here $t$ is time, $i = 1, 2, 3$ are the three spatial dimensions of the worldvolume of the $D4$-brane, $U$ is the radial direction and $d\Omega_4^2$ the metric of the unit four-sphere. $\Omega_4$ and $\omega_4$ denote the volume and volume form of the unit four-sphere respectively. The parameter $R$ in (2.1) is defined as

$$R^3 = \pi g_s N_c l_s^3 = \pi \lambda$$

(2.2)

where $\lambda$ is the 't Hooft coupling constant. Note that here and in the rest of the paper we set $\alpha' = 1$.

$D8$ branes propagating in the geometry (2.1) wrap $R^{3,1} \times S^4$ and their embedding profile is specified by a single function $X_4(U)$. Assuming chiral symmetry is restored $\partial_U X_4 = 0$ and the induced metric takes the following form

$$ds^2_{D8} = \left(\frac{U}{R}\right)^{\frac{3}{4}} (-dt^2 + dx_i^2) + \left(\frac{U}{R}\right)^{-\frac{3}{4}} (dU^2 + U^2 d\Omega_4^2)$$

(2.3)
Studying the system at finite chemical potential, requires introducing non-trivial flux $F_0U \neq 0$ along the brane worldvolume. The action density (DBI) reads

$$S_{D8} = -\mathcal{N} \int dUU\sqrt{\frac{2}{3} (1 - (\partial_U A_0)^2)}$$

where $\mathcal{N} \equiv \frac{\mu g_s}{\sqrt{\frac{3}{2} (2\pi)^{11/2}}} \frac{N c N_f}{\sqrt{\lambda}}$. Note that we made the gauge choice $A_U = 0$ and rescaled $A_0$ according to

$$A_0 \rightarrow 2\pi A_0$$

The conserved charge associated with $A_0$ is given by

$$U^{\frac{2}{3}} \frac{\partial_U A_0}{\sqrt{1 - (\partial_U A_0)^2}} = d$$

It follows that the electric field on the brane worldvolume satisfies

$$\partial_U A_0 = \frac{d}{\sqrt{d^2 + U^5}}$$

We then determine the chemical potential $\mu$ from the asymptotic value of $A_0$ as

$$\mu = \frac{1}{2\pi} \int_0^\infty \partial_U A_0 = \frac{1}{2\pi} \gamma d^{\frac{2}{3}}$$

where $\gamma$ is defined by

$$\gamma \equiv \frac{\Gamma\left[\frac{3}{10}\right] \Gamma\left[\frac{1}{5}\right]}{5\sqrt{\pi}}$$

2.2. The speed of sound

Holography relates the brane action to the grand canonical potential $\Xi = F - \mu \rho$, where $F$ the free energy density, $\mu$ the chemical potential and $\rho$ the charge density. The precise identification at zero temperature is $S_{D8} = -\Xi$. Evaluating the DBI action on the solution (2.7) we deduce that the grand canonical potential for this phase is given by

$$\Xi = -S_{D8} = -\frac{2}{7} \mathcal{N} \gamma d^{\frac{2}{3}}$$

where $d$ can be expressed in terms of $\mu$ through (2.8). Note that to arrive at (2.10) we renormalized the action by subtracting the contribution from the configuration with straight branes and no flux.
Knowledge of the grand canonical potential allows us to determine several thermodynamic quantities of the system. The charge density for instance can be shown to be

$$\rho = -\frac{\delta \Xi}{\delta \mu} = 2\pi dN$$  \hspace{1cm} (2.11)

Another interesting property is the speed of sound. This is given by

$$u^2 = \left( \frac{\partial P}{\partial \epsilon} \right)_\rho$$  \hspace{1cm} (2.12)

where $P$ and $\epsilon$ denote the pressure and energy density respectively while the derivative is taken at constant volume and particle number. At zero temperature, the pressure is equal and opposite to the grand canonical potential $P = -\Xi$ while the energy density $\epsilon$ is equal to the free energy and is given by

$$\epsilon = \Xi + \mu \rho = \frac{5}{4}N\gamma d^2 = \frac{5}{2}P$$  \hspace{1cm} (2.13)

Using (2.12) we find that

$$u^2 = \frac{2}{5}$$  \hspace{1cm} (2.14)

2.3. Specific Heat

The specific heat capacity $C_v$ is an important physical property of matter which often reveals the nature of quasiparticle excitations. The specific heat of a fermionic liquid for instance, exhibits linear behavior at low temperatures. On the other hand, one finds for a bosonic gas in $3 + 1$ dimensions that $C_v \sim T^3$. In the following we will see that the heat capacity of the $D4 - D8 - \overline{D8}$ system varies linearly with the temperature in accord with the predictions of the theory of Fermi Liquids [34].

The $D4 - D8 - \overline{D8}$ at finite temperature and chemical potential has been studied in [6]. Here, we will briefly review the results of [6] necessary for the computation of the specific heat. The relevant background geometry is given by

$$ds^2 = \left( \frac{U}{R} \right)^{\frac{1}{2}} \left( -f(U)dt^2 + dx_i^2 + dX_4^2 \right) + \left( \frac{U}{R} \right)^{-\frac{1}{4}} \left( \frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right)$$  \hspace{1cm} (2.15)

$$e^\Phi = g_s \left( \frac{U}{R} \right)^{\frac{1}{2}} \hspace{1cm} F_4 = dC_3 = \frac{2\pi N_c}{\Omega_4} \omega_4$$
with \( f(U) = 1 - \frac{U^3}{U^2} \). The temperature \( T \) is related to the minimum value \( U_T \) of \( U \) as

\[
T = \frac{3U_T^2}{4\pi R^2} \quad \Rightarrow \quad U_T = \left( \frac{4\pi}{3} \right)^2 R^3 T^2 \tag{2.16}
\]

The induced metric on the straight D8–branes is then

\[
ds_{D8}^2 = \left( \frac{U}{R} \right)^{\frac{3}{2}} (-f(U) dt^2 + dx_i^2) + \left( \frac{U}{R} \right)^{-\frac{3}{2}} \left( \frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right) \tag{2.17}
\]

while the DBI action for this configuration at finite chemical potential reads

\[
S_{D8} = -\frac{N V_3}{T} \int dU U^5 \sqrt{1 - (\partial U A_0)^2} \tag{2.18}
\]

where \( V_3 \) denotes the volume of \( R^3 \). The equation of motion following from (2.18) has the exact same form as the one at zero temperature. Eq. (2.7) is therefore still valid, however the chemical potential \( \mu \) is now given by

\[
\mu = \frac{1}{2\pi} \int_{U_T}^\infty \partial U A_0 = \frac{d}{3\pi U_T^2} F \left[ \frac{1}{2}, \frac{3}{10}, \frac{13}{10}, -\frac{d^2}{U_T^5} \right] \tag{2.19}
\]

with \( F \left[ \frac{1}{2}, \frac{3}{10}, \frac{13}{10}, -\frac{d^2}{U_T^5} \right] \) the standard hypergeometric function.

Evaluating the specific heat, as well as the charge density, requires knowledge of the grand canonical potential \( \Xi \) of the system. At finite temperature, the latter is related to the DBI action through \( \Xi = -TS_{D8} \). As usual however, the action evaluated on the solution (2.7) is infinite. Holographic renormalization [35], [36] is then required for a consistent removal of the divergences. Nonetheless, for the purposes of calculating the charge density dependent terms of the grand canonical potential any renormalization scheme will suffice. This is because turning on a non-trivial chemical potential does not introduce additional divergences into the action. The appropriate counterterms are thus fixed and independent of the charge density or chemical potential of the system\(^2\).

We therefore proceed to renormalize the action by subtracting the contribution of straight branes with no flux. The result is

\[
\Xi = NV_3 \int_{U_T}^\infty dU \left( \frac{U^5}{\sqrt{U^5 + d^5}} - U^\frac{7}{2} \right) = NV_3 \frac{2}{7} U_T^\frac{7}{2} \left[ 1 - F \left[ \frac{1}{2}, -\frac{7}{10}, \frac{3}{10}, -\frac{d^2}{U_T^5} \right] \right] \tag{2.20}
\]

\(^2\) This can be mainly attributed to the flat brane embedding considered in this paper. We thank M.Taylor for explaining this to us.
We are interested in the density dependent term of eq. (2.20) which we subsequently define as
\[
\Delta \Xi = -N V^3 \frac{2}{7} U^7 T^\frac{3}{7} \left[ \frac{1}{2}, -\frac{7}{10}, \frac{3}{10}, -\frac{d^2}{U^5} \right]
\] (2.21)

It is now straightforward to compute the charge density \( \rho \) from
\[
\rho \equiv -\frac{\partial \Delta \Xi}{\partial \mu} = 2\pi N d
\] (2.22)

The specific heat on the other hand is defined through
\[
C_v \equiv T \left( \frac{\partial S}{\partial T} \right)_{\rho, V}
\] (2.23)
where the entropy \( S \) is given by
\[
S \equiv -\left( \frac{\partial \Delta \Xi}{\partial T} \right)_{\mu} = -\left[ \left( \frac{\partial \Delta \Xi}{\partial T} \right)_d + \left( \frac{\partial \Delta \Xi}{\partial d} \right)_T \left( \frac{\partial d}{\partial T} \right)_{\mu} \right]
\] (2.24)

Using (2.19) along with the identity
\[
\left( \frac{\partial d}{\partial T} \right)_{\mu} \left( \frac{\partial \mu}{\partial d} \right)_T \left( \frac{\partial T}{\partial \mu} \right)_d = -1
\] (2.25)
we find that the density-dependent part of the specific heat\(^3\) behaves at low temperatures like
\[
C_v \simeq \alpha T + \mathcal{O}(T^{11}) \quad \alpha \equiv \frac{16\pi^2}{9} \lambda \rho V_3
\] (2.26)

3. Small Fluctuations and Zero Sound

In this section we will compute the massless excitation coupled to the density operator in the \( D4 - D8 - \overline{D8} \) system at strong coupling. This requires analyzing the linearized equations of motions which follow from the action describing the dynamics of \( D8 \) and \( \overline{D8} \) branes. The full action consists of a Dirac-Born-Infeld (DBI) and a Chern-Simons (CS) term.

The DBI part of the action can be expressed as
\[
S_{DBI,D8} \sim \int d\Omega_4 \int d^4 x \int dU e^{-\Phi} \sqrt{-\det[G + \Phi]}
\] (3.1)

\(^3\) The density independent part behaves like \( T^6 \).
where $\Phi$ is the dilaton, $G$ the induced metric and $F$ the gauge field strength. We will consider fluctuations which are independent of the coordinates of $S^4$. Given that metric and gauge field perturbations decouple, it suffices to set $G = G^{(0)}$ and expand the gauge field $A_0 = A_0^{(0)} + A_0, A_i = A_i$ and the gauge field strength $F_{0U} = F_{0U}^{(0)} + F_{0U}, F_{ij} = F_{ij}$. Here $i, j = 0, 1, 2, 3$ and the superscript $(0)$ denotes the background values of the corresponding fields. Moreover, we set $A_\theta = 0, \forall \theta \in S^4$ and choose the gauge $A_U = 0$.

The DBI part of the action for the fluctuating fields is then given by

$$S_{\text{DBI,fl}} = -\frac{1}{2} N \int d^4 x \int dU g(U) \left[ \sum_i F_{iU}^2 - f_1(U) F_{0U}^2 - f_2(U) R^3 \sum_i F_{0i}^2 + f_3(U) R^3 \sum_{i<j} F_{ij}^2 \right]$$

(3.2)

where the functions $g(U), f_i(U)$ are defined as

$$g(U) = \sqrt{U^5 + d^2} \quad f_1(U) = \frac{U^5 + d^2}{U^5}$$

$$f_2(U) = \frac{1}{U^3} \quad f_3(U) = \frac{1}{U^3} \frac{U^5}{U^5 + d^2}$$

(3.3)

Not all of these functions are independent from each other. In particular, $f_2 = f_1 f_3$.

We now turn to the contribution of the CS term to the action

$$S_{\text{CS}} = i \frac{\mu_8}{3!} \int_{D_8} F_4 \wedge A \wedge F \wedge F = i \frac{\mu_8 (2\pi)^3}{3!} N_c \int A \wedge F \wedge F$$

(3.4)

where in the last equality we integrated the RR four form $F_4$ over the four sphere. Recall that the gauge field components $A_I$ with $I = 0, 1, 2, 3, U$ are rescaled according to $A_I \rightarrow 2\pi A_I$. Expanding (3.4) to quadratic order in the fields we arrive at

$$S_{\text{CS,fl}} = i 4dR^4 \frac{1}{2} N \int d^4 x \int dU g(U) \frac{f_3(U)}{U^2} [A_1 F_{23} + A_2 F_{31} + A_3 F_{12}]$$

(3.5)

With the help of (3.5) and (3.2) we can finally write the full action for the D8 branes as

$$S_{D_8,fl} = -\frac{1}{2} N \int d^4 x \int dU g(U) \left[ \sum_i F_{iU}^2 - f_1(U) F_{0U}^2 - f_2(U) R^3 \sum_i F_{0i}^2 + f_3(U) R^3 \sum_{i<j} F_{ij}^2 - i 4dR^4 \frac{f_3(U)}{U^2} (A_1 F_{23} + A_2 F_{31} + A_3 F_{12}) \right]$$

(3.6)

It is convenient to express the equations of motion in the momentum space representation where

$$A_M(x^\mu, r) = \int \frac{d^4 k}{(2\pi)^4} e^{ik_\mu x^\mu} \tilde{A}_M(k^\mu, r)$$

(3.7)
Choosing $k_\mu = (-\omega, 0, 0, q)$ we arrive at the following set of equations

$$
\begin{align*}
\partial_U [g(U)f_1(U)(\partial_U \tilde{A}_0)] - q^2 R^3 g(U)f_2(U)\tilde{A}_0 - q\omega R^3 g(U)f_2(U)\tilde{A}_0 &= 0 \\
\partial_U [g(U)(\partial_U \tilde{A}_0)] + \omega^2 R^3 g(U)f_2(U)\tilde{A}_0 + q\omega R^3 g(U)f_2(U)\tilde{A}_0 &= 0 \\
\partial_U [g(U)(\partial_U \tilde{A}_{\perp,1})] + g(U)R^3 [\omega^2 f_2(U) - q^2 f_3(U)]\tilde{A}_{\perp,1} - 4dR^2 qg(U)\frac{f_3(U)}{U^2} - \tilde{A}_{\perp,2} &= 0 \\
\partial_U [g(U)(\partial_U \tilde{A}_{\perp,2})] + g(U)R^3 [\omega^2 f_2(U) - q^2 f_3(U)]\tilde{A}_{\perp,2} + 4dR^2 qg(U)\frac{f_3(U)}{U^2} - \tilde{A}_{\perp,1} &= 0
\end{align*}
$$

(3.8)

To fix the residual gauge invariance we additionally impose Gauss law:

$$
\tilde{q}\tilde{A}_0' + \tilde{\omega} f_1 \tilde{A}_0' = 0
$$

(3.9)

Henceforth we will focus on the longitudinal modes of the gauge field. Note that the CS part of the action affected the field equations for the transverse components only.

Computing the quasinormal spectrum requires working with the gauge invariant combination

$$
E = q\tilde{A}_0 + \omega\tilde{A}_0
$$

(3.10)

We therefore use Gauss Law to express the first derivative of $\tilde{A}_0$ in terms of $E$

$$
\tilde{A}_0' = \frac{q}{q^2 - \omega^2 f_1} E'
$$

(3.11)

and combine the equations for the longitudinal modes in one

$$
E'' + \left(\frac{g'}{g} + \frac{f_1}{f_1} \frac{q^2}{q^2 - \omega^2 f_1} \right) E' + R^3 f_3 (\omega^2 f_1 - q^2) E = 0
$$

(3.12)

Primes indicate differentiation with respect to the variable $U$. Note that to arrive at (3.12) we also used the identity $f_2 = f_1 f_3$.

It is convenient to make a change of variables

$$
y = 2\sqrt{\frac{R^3}{U}} \quad U = \frac{4R^3}{y^2}
$$

(3.13)

to express eq. (3.12) as

$$
\ddot{E} + \left(\frac{3}{y} + \frac{\dot{g}}{g} + \frac{\dot{f}_1}{f_1} \frac{q^2}{q^2 - \omega^2 f_1} \right) \dot{E} + \frac{1}{f_1} (\omega^2 f_1 - q^2) E = 0
$$

(3.14)
Dots denote differentiation with respect to \( y \) while
\[
\frac{\dot{g}}{g} = -\frac{5}{y} \left( \frac{1}{1 + \tilde{\mu}^{10} y^{10}} \right) \quad f_1 = 1 + \tilde{\mu}^{10} y^{10} \quad \frac{\dot{f}_1}{f_1} = \frac{10}{y} \left( \frac{\tilde{\mu}^{10} y^{10}}{1 + \tilde{\mu}^{10} y^{10}} \right)
\] (3.15)
Here \( \tilde{\mu} \) is defined as
\[
\tilde{\mu} \equiv \frac{dR}{2R^{\gamma_2}} = \sqrt{\frac{\mu}{2\gamma \lambda}}
\] (3.16)
In the vicinity of the horizon (3.14) reduces to a Bessel-type differential equation
\[
\ddot{E} + \frac{3}{y} \dot{E} + \omega^2 E = 0
\] (3.17)
with general solution in terms of Hankel functions
\[
E(y) = A \frac{H_1^{(1)}(y)}{y} + B \frac{H_1^{(2)}(y)}{y}
\] (3.18)
Imposing the incoming wave boundary condition at the horizon [37-40] singles out one of the solutions
\[
E(y) = A \frac{H_1^{(1)}(y)}{y}
\] (3.19)
where \( H^{(1)} \) denotes the Hankel function of first kind. In the limit of small frequencies, or to be precise for \( \omega y \ll 1 \), (3.19) further reduces to
\[
E(y) \simeq \frac{A}{y^2} + A \omega^2 \left[ \frac{1}{4} \left( 1 - 2\tilde{\gamma} + i\pi + 2 \ln 2 \right) - \frac{1}{2} \ln \omega y \right]
\] (3.20)
with \( \tilde{\gamma} \) the Euler number \( \tilde{\gamma} \approx 0.5772 \).

On the other hand, for sufficiently small \( \omega \) and \( q \) the last term in eq. (3.14) can be neglected. This yields
\[
\ddot{E} + \left( \frac{3}{y} \frac{\dot{g}}{g} + \frac{\dot{f}_1}{f_1} \frac{q^2}{q^2 - \omega^2 f_1} \right) \dot{E} = 0
\] (3.21)
Eq. (3.21) is analytically tractable. Its general solution can be expressed in terms of hypergeometric functions as follows
\[
E(y) = C_0 + C_1 y^3 \left( \frac{1}{\sqrt{1 + \tilde{\mu}^{10} y^{10}}} - \frac{5}{3} \left( \frac{\omega^2}{q^2} - \frac{2}{5} \right) F \left[ \frac{3}{10}, \frac{1}{2}, \frac{13}{10}, -\tilde{\mu}^{10} y^{10} \right] \right)
\] (3.22)
Near the boundary in particular (3.22) reduces to
\[
E(y) \approx C_0 + \frac{5}{3} C_1 \left( 1 - \frac{\omega^2}{q^2} \right) y^3
\] (3.23)
Imposing normalizability translates to \( C_0 = 0 \). Hence, the spectrum of quasinormal modes will be obtained as a solution to this equation.

In the vicinity of the horizon the behavior of (3.22) is

\[
E(y) \simeq (C_0 + bC_1) + \frac{aC_1}{y^2} \tag{3.24}
\]

with \( a \) and \( b \) defined as follows

\[
a = \frac{5}{2\tilde{\mu}^5} \frac{\omega^2}{q^2} \quad b = \frac{5}{2\tilde{\mu}^3} \gamma \left( \frac{2}{5} - \frac{\omega^2}{q^2} \right) \quad \frac{b}{a} = \frac{\tilde{\mu}^2 q^2}{\omega^2} \left( \frac{2}{5} - \frac{\omega^2}{q^2} \right) \tag{3.25}
\]

Our next step would be to match the near horizon solution (3.24) to (3.20). Observe however that a logarithmic term present in (3.20) is absent from (3.24). The apparent inconsistency is resolved by computing the first order correction to the solution given by (3.24). In the vicinity of the horizon this is a pretty simple task. It essentially requires solving the following inhomogeneous differential equation

\[
\ddot{E} + \frac{3}{y} \dot{E} = -\omega^2 \left( C_0 + bC_1 + \frac{aC_1}{y^2} \right) \tag{3.26}
\]

The solution for large \( y \) is now given by

\[
E(y) = (C_0 + bC_1) + \frac{aC_1}{y^2} - \frac{1}{2} aC_1 \omega^2 \ln y + \mathcal{O}(y^2) \tag{3.27}
\]

which we can readily compare with (3.20) to arrive at

\[
C_0 = A \left[ \frac{1}{4} \left( 1 - 2\gamma + i\pi - 2 \ln \frac{\omega}{2} \right) \omega^2 - \frac{b}{a} \right] \quad C_1 = \frac{A}{a} \tag{3.28}
\]

We are interested in the quasinormal mode with linear dispersion relation in the regime of small \( \omega \) and \( q \). Imposing \( C_0 = 0 \) implies

\[
\frac{1}{4} \left( 1 - 2\gamma + i\pi - 2 \ln \frac{\omega}{2} \right) \omega^4 - \frac{2}{5} \tilde{\mu}^2 q^2 + \tilde{\mu}^2 \omega^2 = 0 \tag{3.29}
\]

Neglecting higher order terms in \( \omega \) and \( q \) finally leads to

\[
\omega^2 = u_0^2 q^2 \quad u_0^2 = \frac{2}{5} \tag{3.30}
\]

This is exactly equal to the (usual) sound mode computed in section 2.

As a final note, let us consider corrections to the dispersion relation (3.30). Making the substitution \( \omega \rightarrow \omega = \sqrt{\frac{2}{5}} q + \delta \omega \) and expanding eq. (3.29) to first order in \( \delta \omega \) yields the following solution

\[
\delta \omega = -\frac{1 + 2\gamma + \ln \frac{q^2}{16}}{10 \tilde{\mu}^2} q^3 - i \frac{\pi}{10 \tilde{\mu}^2} q^3 + \mathcal{O}(q^5) \tag{3.31}
\]

Notice that the imaginary part of the dispersion relation is of the type \(-iq^3\) in contrast to both the typical Fermi liquid behavior [41] and what was observed in [42], [43].
4. Is there a sharp Fermi surface?

In the previous section we observed a sound-like excitation in the regime of vanishing temperature. It is known that Fermi liquids possess such zero sound mode in the collisionless regime. It is associated with the deformation of the Fermi surface away from the spherical shape. The natural question is whether we can observe the existence of the Fermi surface directly. From the theory of normal Fermi liquids we learn that the jump in the distribution function can indeed be observed as a singularity in the retarded current-current Green’s function. More precisely, the two point function at $\omega = 0$ behaves like

$$G(\omega = 0, q) \sim \left( \frac{q}{2q_F} - 1 \right) \log \left( \frac{q}{2q_F} - 1 \right)$$ (4.1)

as $q$ approaches the value of twice the Fermi momentum $q_F$. In the previous section, we focused on the massless excitation, but the equations can be easily adapted to the case of $\omega = 0$, $q$ finite.

In this section we analyze equation (3.12) for $\omega = 0$. It will be convenient to use the variable $x = \tilde{\mu} y$, where $\tilde{\mu}$ is defined in (3.16) and introduce the rescaled momentum $\tilde{q}$ via $\tilde{q} = q/\tilde{\mu}$. Then eq. (3.12) takes the form

$$\partial_x^2 E + \frac{1}{x} \frac{13x^{10} - 2}{1 + x^{10}} \partial_x E - \frac{\tilde{q}^2}{1 + x^{10}} E = 0$$ (4.2)

In the near-horizon region, $x \gg 1$ the solution of eq. (4.2) is a linear combination of the two solutions,

$$E^{(1)} \sim 1, \quad E^{(2)} \sim \frac{1}{x^{12}}$$ (4.3)

We would like to argue that it is the second solution which is physical. Indeed, as the value of $q$ is taken to zero, fluctuation of the electric field on the D8 brane corresponds to the infinitesimal change of the value of $\mu$. Yet, in this static situation the boundary condition at the horizon imposes $E(x \to \infty) = 0$. In fact, in Appendix B we show that $E(x \to \infty) = 0$ condition smoothly connects to the incoming wave boundary conditions at the horizon, as $\omega$ is taken to be nonzero.

We can now use initial conditions at $x_{max} \to \infty$ to integrate eq. (4.2) numerically all the way to the boundary. Near the boundary $x = 0$ the solution of (4.2) is given by

$$E = AF_I(x) + BF_{II}(x)$$ (4.4)
Fig 1. Ratio $A/B$ [see eq. (4.4)] as a function of $\tilde{q}$.

where

$$F_I(x) = \left(1 - \frac{\tilde{q}^2 x^2}{2} - \frac{\tilde{q}^4 x^4}{8}\right) + \ldots, \quad F_{II}(x) = x^3 + \ldots \quad (4.5)$$

The two-point function is proportional to the ratio $A/B$, which can be extracted from the numerical solution with the help of (4.5). The result is shown in Fig.1. As evident from this figure, we do not observe any characteristic structure in the wide region of $\tilde{q}$. Note that this is the phase which exhibited such characteristic features of the Fermi liquid as linear heat capacity and zero sound. We discuss the significance of these observations in the next section.

5. Discussion

In this paper we consider the $D4 - D8 - \overline{D8}$ system in the limit $R_4 \to \infty$ which corresponds to a certain UV completion of the NJL model at weak coupling. Chemical potential for the fermions is turned on, and from the field theoretic point of view formation of a Fermi liquid is a realistic possibility. The holographic description is applicable at large coupling, and a phase transition from the Fermi liquid to some other ground state may in principle happen as coupling is varied. However we observed a couple of features that suggest that such a phase transition does not happen and we are dealing with a strongly coupled Fermi liquid-type system.

The most significant indication that this is the case is the linear behavior of the density-dependent part of the heat capacity at low temperatures (2.26). This is a very generic
feature of the systems which have a step-like distribution function at zero temperature. At small temperature the number of excited states is proportional to $T$, as well as the average energy of the excitation, leading to $E \sim T^2$ and $C_v \sim T$. Note that this behavior is in sharp contrast with that of $D3-D7$ system at small temperature, where $C_v \sim T^6$ [42]. This difference may be related to the existence of charged bosons in the case of the $D3-D7$ system. Their condensation is presumably responsible for the different low temperature behavior of $C_v$. It is impressive that the physics seems to vary smoothly between the field theoretic and holographic regimes.

In section 3 we found a sound mode at vanishing temperature, the zero sound. Again, such a feature is characteristic of Fermi liquids, where zero sound is associated with the deformation of the Fermi surface away from the spherical shape. It is interesting that the speed of zero sound, given by (3.30) is equal to the speed of normal sound propagating due to the fermion matter in the hydrodynamic regime [see eq. (2.14).] Note that the speed of this “brane sound” differs from that of the sound propagating on the stack of $D4$ branes by a factor of $\sqrt{2}$. (The general formula for the speed of sound on the $Dp$ branes is $v_s^2 = (5 - p)/(9 - p)$, see e.g. [44-46].) This is not a surprise since the fermionic degrees of freedom are localized on a 3+1 dimensional defect.

In fact, as we show in appendix A, the value of the speed of zero sound coincides with the one of first sound for the $D3-D7$ system as well. There, the two expressions match as functions of the ratio between the hypermultiplet mass and the chemical potential. The fact that the speed of sound in these two regimes is the same is in principle consistent with the Fermi liquid theory. As reviewed in [43], when only the first two Landau parameters $F_0$ and $F_1$ are turned on, the ratio of zero to first sound velocities goes to one as $F_0$ and $F_1$ are taken to infinity. For a generic large interaction $F(\vartheta)$, the ratio still goes to a constant. We leave the study of interpolation of the sound mode between the collisionless and hydrodynamic regimes to future work.

It is interesting that the damping term in the dispersion relation for the zero sound behaves like $-iq^3$. This should be contrasted with the $-iq^2$ damping observed in [42],[43]. Possible microscopic explanation of this behavior might be related to the non-linear dispersion relation near the Fermi surface. In fact\(^4\) this may also explain the absence of the apparent singularity in the current-current two-point function at $\omega = 0$, computed in section 4. Indeed, the absence of the singularity, which one would generically expect at

\(^4\) We thank Edward Shuryak for pointing this out.
\( q = 2 q_F \) is one of the main puzzles that we encounter. It would be nice to have a phenomenological theory that would explain the observations of the present paper. It is clear that such a description must account for strong coupling, since the relevant dimensionless parameter \( \lambda \mu \) is necessarily large for the DBI analysis to be applicable. One obvious difficulty in applying Landau’s theory of Fermi liquids is the assumption that the number of particles is conserved as the strength of the interaction is varied. It is not obvious that this should be the case here.

Finally, we would like to make a comment about the dimensions of the D-branes. In this paper we considered the \( D4 - D8 - \overline{D8} \) system, which has problems with the dilaton tadpole beyond the probe brane approximation for the \( D8 \) branes. However we expect the physics to be similar for other D-brane systems, for example obtained from the \( D4 - D8 - \overline{D8} \) by the T-duality. It would be interesting to investigate these cases in more detail.

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**Appendix A. \( D3-D7 \) system and speed of sound.**

In this appendix we compute\(^5\) the speed of sound for the system studied in [43]. As already mentioned in section 2, the speed of (normal) sound in liquids is defined as

\[
\nu^2 = \left( \frac{\partial P}{\partial \varepsilon} \right)_\rho
\]

with \( P \) the pressure and \( \varepsilon \) the energy density. Evaluating the speed of sound is then a trivial exercise once the grand canonical potential of the system is known. Within the framework of gauge/gravity duality, the grand canonical potential is identified with the DBI action evaluated on the specific configuration. Here we are interested in the black hole embedding

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\(^5\) After this Appendix had been written we received [47] which contains similar result.
of the D3-D7 system at zero temperature. Analytic results in this case, including evaluation of the action, are given in [48]. According to [48], but in the conventions of [43]

\[ S_{\text{ren.}} = -N \int_0^\infty \left( \frac{r^6}{\sqrt{r^6 + \mu^2 - c^2}} - r^3 \right) = \frac{1}{4} N \gamma (\mu^2 - m^2)^2 \] (A.2)

where \( N, \gamma, \mu \) and \( m \) are defined in section 3 of [43].

At zero temperature the pressure is equal and opposite to the grand canonical potential \( P = -\Xi \) while the energy density is equal to the free energy and given by

\[ \epsilon = \Xi + \mu \rho = -S_{D7} + \mu \frac{\partial S_{D7}}{\partial \mu} = \frac{1}{4} N \gamma (\mu^2 - m^2)(3\mu^2 + m^2) \] (A.3)

Given that pressure and energy density are functions of the chemical potential only\(^6\) we can rewrite (A.1) as

\[ u^2 = \left( \frac{\partial P}{\partial \epsilon} \right) = \left( \frac{\partial P}{\partial \mu} \right) \left( \frac{\partial \mu}{\partial \epsilon} \right) \] (A.4)

and using (A.2) as well as (A.3) we conclude that

\[ u^2 = \frac{\mu^2 - m^2}{3\mu^2 - m^2} \] (A.5)

Observe that the speed of zero sound computed in [43] is exactly equal to (A.5).

**Appendix B. The boundary condition at the horizon in the limit \( \omega \rightarrow 0 \).**

In section 4, we numerically computed the retarded Green’s function for the density operator and investigated its behavior for different values of \( q \). Given that the singularity which signals the presence of a Fermi surface sits at \( \omega = 0 \), the calculation was performed by setting \( \omega = 0 \) in (3.14). However, the appropriate boundary conditions at the horizon when \( \omega = 0 \) are not known. Here, we will show that in the limit \( \omega \rightarrow 0 \) the incoming wave boundary condition reduces to

\[ \lim_{y \to \infty} E(y) = 0 \] (B.1)

Let us investigate the region close to the horizon henceforth defined as the regime where \( \tilde{\mu} y \gg 1 \). Given that we want to study the behavior of the solution at very small frequencies, we will not require \( \omega^2 \tilde{\mu}^{10} y^{10} \) to be large. Eq. (3.14) in this case reduces to

\[ \tilde{E} + \left( \frac{3}{y} + \frac{10}{y} \frac{1}{1 - \omega^2 \tilde{\mu}^{10} y^{10}} \right) \tilde{E} + \omega^2 \left( 1 - \frac{1}{\omega^2 \tilde{\mu}^{10} y^{10}} \right) E = 0 \] (B.2)

\(^6\) Note that we take the hypermultiplet mass \( m \) as a fixed parameter of our ensemble.
Observe that when \( \frac{\omega^2}{q^2} \mu^{10} y^{10} \gg 1 \), (B.2) further reduces to (3.17). In this case, the incoming wave boundary condition singles out one of the solutions, given by (3.19). If in addition \( \omega y \ll 1 \), (3.19) behaves like

\[
E(y) \simeq \frac{A}{y^2} + A\omega^2 \left[ \frac{1}{4} (1 - 2\gamma + i\pi + 2 \ln 2) - \frac{1}{2} \ln \omega y \right]
\]  
(B.3)

On the other hand, when \( \frac{\omega^2}{q^2} \mu^{10} y^{10} \ll 1 \), eq. (B.2) becomes

\[
\ddot{E} + \frac{13}{y} \dot{E} = 0
\]  
(B.4)

This is precisely (3.14) in the vicinity of the horizon for \( \omega = 0 \). Its general solution is expressed as

\[
E(y) = -\frac{D}{y^{12}} + F
\]  
(B.5)

Clearly, one of the solutions vanishes for large \( y \) while the other behaves as a constant. Which of the two corresponds to (B.3)?

It is not difficult to see, that there exists an additional, intermediate region where (B.2) is analytically tractable. Consider for instance the behavior of the last term for both large and small \( y \).

\[
\omega^2 \left( 1 - \frac{1}{\frac{\omega^2}{q^2} \mu^{10} y^{10}} \right) \simeq \begin{cases} 
\omega^2 & \frac{\omega^2}{q^2} \mu^{10} y^{10} \gg 1 \\
- \frac{q^2}{\mu^{10} y^{10}} & \frac{\omega^2}{q^2} \mu^{10} y^{10} \ll 1
\end{cases}
\]  
(B.6)

It is obvious that this term is negligible so long as

\[
\left( \frac{q}{\mu} \right)^4 \frac{1}{\mu} \ll y \ll \frac{1}{\omega}
\]  
(B.7)

Consequently, eq. (B.2) reduces to

\[
\ddot{E} + \left( 3 \frac{10}{y} + \frac{1}{y} \frac{1 - \frac{q^2}{\omega^2} \mu^{10} y^{10}}{1 - \frac{\omega^2}{q^2} \mu^{10} y^{10}} \right) \dot{E} = 0
\]  
(B.8)

\[ ^7 \text{Here we tacitly assumed that } \left( \frac{q}{\mu} \right) > 1. \text{ If this is not the case, eq. (A.8) extends its regime of validity in the region } \frac{\omega^2}{q^2} \mu^{10} y^{10} \ll 1 \text{ as well. Repeating the same analysis we can easily see that the final result remains unaltered.} \]
with general solution

\[ E(y) = \frac{B}{y^2} \left( 1 - \frac{1}{6} q^2 \frac{1}{\omega^2 \tilde{\mu}^{10} y^{10}} \right) + C \]  

(B.9)

For large \( y \) we can neglect the term which behaves like \( y^{-12} \) to get

\[ E(y) \simeq \frac{B}{y^2} + C \]  

(B.10)

Similarly, in the region of small \( y \) (B.9) becomes

\[ E(y) \simeq -B \frac{1}{6} q^2 \frac{1}{\omega^2 \tilde{\mu}^{10} y^{12}} + C \]  

(B.11)

In principle, we should proceed to match eq. (B.10) to (B.3) and eq. (B.11) to (B.5). Observe however, that (B.10) does not contain the logarithmic term present in (B.3). To recover the logarithmic behavior we need to solve (B.8) in the near horizon region to next order in \( \omega \). We should therefore consider the following equation

\[ \ddot{E} + \frac{3}{y} \dot{E} = \omega^2 \left( \frac{B}{y^2} + C \right) \]  

(B.12)

Solving (B.12) yields

\[ E(y) = \frac{B}{y^2} + C - \frac{1}{2} B \omega^2 \ln y + \mathcal{O}(y^2) \]  

(B.13)

and combining all the above we deduce that

\[ \frac{F}{D} = 6 \omega^4 \tilde{\mu}^{10} \frac{1}{q^2} \frac{1}{4} \left( 1 - 2 \tilde{\gamma} + i \pi - 2 \ln \frac{\omega}{2} \right) \]  

(B.14)

It is clear, that (B.14) vanishes when \( \omega = 0 \) and the other parameters are kept fixed. Note that taking the limit \( \omega \to 0 \) is completely justified given that the matching technique is valid in the regime

\[ \omega \ll \tilde{\mu} \left( \frac{\tilde{\mu}}{q} \right)^{\frac{1}{4}} \]  

(B.15)
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