Adaptive neural network control for nonlinear non-strict feedback time-delay systems

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ABSTRACT
This paper focuses on adaptive neural control for a class of non-strict feedback nonlinear systems with state delays and input delay. By combining integral transformation with adaptive neural control approach, a backstepping-based adaptive neural control scheme is proposed. The suggested control schemes guarantees that the tracking error converges to a small neighbourhood of the origin, meanwhile, all the closed-loop signals remain bounded. Simulation examples are used to verify the effectiveness of the proposed method.

ARTICLE HISTORY
Received 10 July 2020
Accepted 5 October 2020

KEYWORDS
Adaptive neural control; non-strict feedback systems; time delay; backstepping

1. Introduction
In the past three decades, backstepping has been developed an useful method for nonlinear system controller design (see Kanellakopoulos et al., 1991; Schwartz et al., 1999; Yao & Tomizuka, 1997). Particularly, adaptive neural or fuzzy control approach is combined with backstepping to cope with control design of nonlinear systems. In Tong et al. (2020) and Tong and Li (2020), the problem of adaptive fuzzy output feedback control is discussed for a class of strict-feedback nonlinear systems. The designed adaptive fuzzy controllers guarantee achievement of output tracking issue and boundedness of all the closed-loop signals. For more results on approximation-based adaptive control of nonlinear systems, please see Rubio and Yu (2007), M. Wang et al. (2010), Ge et al. (2004) and Ho et al. (2005).

Note that delays are usually the source of system instability or performance degradation. Recently, the control systems with input delay have also received more and more attention. In Liu et al. (2018), the stabilization of linear interconnected systems with input delays is studied. Decentralized controllers are designed based on the solvability of linear matrix inequalities. In Lin et al. (2020), the problem of finite-time stabilization is addressed for linear switched systems with input delays. By converting the time continuous system into a discrete system, a digital state feedback and a digital output feedback controllers are proposed respectively. In Zhu et al. (2012), the authors introduce the integral transformation in the control design to deal with the input delay. Furthermore, an adaptive neural controller is constructed for strict-feedback nonlinear systems with input delay. This way is applied to multi-agent systems with input delays in Y. M. Li et al. (2020).

However, the existing results on adaptive control for nonlinear input delay systems are mainly presented for strict-feedback systems. Theoretically, these control strategies cannot be directly applied to the systems with non-strict feedback form, which include the strict-feedback form as a special case. Even though there are some backstepping-based adaptive neural or fuzzy control schemes of non-strict feedback systems to be reported, in these existing control designs the proposed virtual control signals involve the information on the state variables of the subsequent subsystems. That generates the algebraic loop phenomenon and does not meet the backstepping design rule.

In the current paper, we will consider the adaptive neural control for non-strict feedback systems with state and input delays. In controller design, the integral transformation is utilized to cope with the input delay in order to ensure the feasibility of controller design. The structural feature of the basis vector functions of RBF NNs is used to design the virtual control signals involve the information on the state variables of the subsequent subsystems. That generates the algebraic loop phenomenon and does not meet the backstepping design rule.

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2. Preliminaries and problem formulation

Consider a nonlinear non-strict feedback system, which is represented by the following differential equations:

\[
\begin{align*}
\dot{x}_i &= f_i(x) + g_i(x)x_{i+1} + q_i(x(t - \tau_i)), \\
\dot{x}_n &= f_n(x) + g_n(x) D(u(t - \tau)) + q_n(x(t - \tau_n)),
\end{align*}
\]

where \( x = [x_1, x_2, \ldots, x_n]^T \), \( u \in R \) and \( y \in R \) are system's state, input and output variables, respectively, \( f_i(\cdot), g_i(\cdot) \) and \( q_i(\cdot) \) are unknown smooth nonlinear functions, with \( f_i(0) = 0 \) and \( q_i(0) = 0 \), and input delay \( \tau \) is known positive constant, state delay \( \tau_i \) is unknown. In addition, the system function \( f_i(x) \) is required not to contain independent factors \(-g_i(x)x_{i+1} \). \( D(u(t - \tau)) \in R \) denotes the control input subject to saturation nonlinearity described by

\[
D(u) = \begin{cases} 
|u|, & \text{if } |u| \leq u_{\text{max}} \\
\text{sign}(u)u_{\text{max}}, & \text{if } |u| > u_{\text{max}} 
\end{cases}
\]

where \( u_{\text{max}} \) is known upper bound of \( u(t) \). According to H. Wang et al. (2013), introduce the function \( g(u) \) as follows

\[
g(u) = u_{\text{max}} \times \tanh\left(\frac{u}{u_{\text{max}}}\right).
\]

Then \( D(u) \) can be expressed in the following form:

\[
D(u) = g(u) + d(u),
\]

where \( d(u) = D(u) - g(u) \) is a bounded function, and its bound can be obtained as

\[
|d(u)| = |D(u) - g(u)| \leq u_{\text{max}}(1 - \tanh(1)) = D_m.
\]

In addition, by the mean value theorem, there exists \( \mu(t) \) such that

\[
g(u) = g(u_0) + g_{u\mu}(u - u_0),
\]

where \( g_{u\mu} = \frac{\partial g(u)}{\partial u}|_{u = u_0} \). By choosing \( u_0 = 0 \), (5) can be written as

\[
g(u) = g_{u\mu}u.
\]

In order to facilitate the control design, the following assumptions and lemmas are given.

**Assumptions 2.1:** The control gain function \( g_i \) (1 ≤ i ≤ n) is bounded and meets the inequality:

\[
0 < g_s \leq g_i \leq g_t,
\]

where \( g_s \) and \( g_t \) are unknown constant.

**Assumptions 2.2:** For the function \( g_{u\mu} \) in (6), there is unknown constant \( g_j \) such that

\[
0 < g_j \leq g_{u\mu} \leq 1.
\]

**Assumptions 2.3:** The reference signal \( y_d \) and its nth order time derivatives are continuous and bounded. Therefore, we assume that there exist positive constant \( d \) such that \( |y_d(t)| \leq d \) and \( |y_d^{(k)}(t)| \leq d \) for \( k = 1, 2, \ldots, n \), where \( y_d^{(k)}(t) \) denotes the kth order derivatives of \( y_d(t) \).

**Assumptions 2.4:** There exist strictly increasing smooth functions \( \varphi_i(\cdot) : R^+ \to R^+ \), with \( \varphi_i(0) = 0 \), such that for \( i = 1, 2, \ldots, n \)

\[
|\varphi_i(x)| \leq \varphi_i(\|x\|).
\]

**Remark 2.1:** From the above assumption, we know that there are smooth functions \( h_i(s) \) make \( \varphi_i(s) = sh_i(s) \), and the following inequality holds

\[
\varphi_i \left( \sum_{i=1}^{n} a_i \right) \leq \sum_{i=1}^{n} na_i h_i(na_i).
\]

**Definition 2.1:** To construct the controller by using backstepping method, the following transformation is defined

\[
z_i = x_i - \alpha_{i-1}, \quad (1 \leq i \leq n - 1)
\]

\[
z_n = x_n - \alpha_{n-1} - g_n(x) \int_{t-\tau}^{t} D(u(\theta)) d\theta,
\]

where \( \alpha_i \) is the virtual control signal of the ith subsystem and has the form

\[
\alpha_i = -a_i^2 z_i - \bar{\theta}_i \varphi_i(\tilde{x}_i) \tanh(\tilde{U}_i)
\]

where \( \alpha_0 = y_d \), \( U_i = \frac{\varphi_i(\tilde{x}_i)}{a_i} \) and \( \bar{\theta}_i \) are common positive constants, \( \tilde{U}_i \) is the estimation of \( \theta_i \), an unknown constant that will be shown below. The estimation error is defined as \( \tilde{z}_i = z_i - \tilde{\theta}_i, \varphi_i(x) = (1 + \|S_i(x_i)\|), S_i(x) \) is the basic function vector of neural network \( i = 1, 2, \ldots, n \), \( \tilde{x}_i = [x_1, \ldots, x_i]^T \).

**Lemma 2.1 (Su et al., 2016):** Let \( \tilde{x}_q = [x_1, \ldots, x_q]^T \) and \( S(\tilde{x}_q) = [S_1(\tilde{x}_q), \ldots, S_q(\tilde{x}_q)]^T \) be the input variables and basic function vector of a RBFN NN respectively. Then, for any positive integer \( k \leq q \), the following inequality holds:

\[
\|S(\tilde{x}_k)\| \leq \|S(\tilde{x}_k)\|.
\]

**Lemma 2.2 (Chen et al., 2014):** For \( z_i = x_i - \alpha_{i-1}, (1 \leq i \leq n) \), the following holds:

\[
\|x\| \leq \sum_{i=1}^{n} \tilde{\alpha}_i |z_i| + d,
\]

with \( \tilde{\alpha}_i = 1.5 + a_i^2 + \frac{2}{\bar{\theta}_i} \) and \( d \) is the upper bound of \( y_d \).
Lemma 2.3 (Tong et al., 2012): For any $w \in \mathbb{R}$ and $\delta > 0$, the following holds:

$$0 \leq |w| - w \tanh \left( \frac{w}{\delta} \right) \leq k\delta$$

with $k = 0.2785$.

3. Controller design

This section will propose an adaptive neural network control scheme based on backstepping. The design process is divided into $n$ steps. For convenience, we substitute $x(t)$, $z_j(t)$ and $q_i(x(t))$ for $x(t - \tau_j)$, $z_j(t - \tau_j)$ and $q_i(x(t - \tau_j))$.

Step 1. For the first subsystem, define the Lyapunov function as

$$V_1 = \frac{1}{2} z_1^2 + \frac{g_s}{2\gamma_1^2} \hat{\theta}_1^2,$$  

(13)

by (10), we have

$$\dot{z}_1 = g_1(x_1)(z_2 + \alpha_1) + f_1(x) + q_1(x_1) - \dot{y}_d.$$  

(14)

The derivative of $V_1$ is given by,

$$\dot{V}_1 = z_1 z_2 g_1 + z_1 g_1 \alpha_1 + z_1 f_1(x) + z_1 q_1(x_1) - z_1 \dot{y}_d$$

$$- \frac{g_s}{\gamma_1^2} \hat{\theta}_1 \dot{\hat{\theta}}_1.$$  

(15)

By Lemma 2.2 and Assumption 2.4, for $z_1 q_1(x_1)$ of (15), one has:

$$z_1 q_1(x_1) \leq |z_1| \phi_1 \left( \|x_1\| \right)$$

$$\leq \sum_{j=1}^n \left\{ \frac{1}{2} (n + 1)^2 \alpha_j^2 z_1^2 + \frac{1}{2} z_1^2 (\tau_1) h_j^2 \right\}$$

$$+ \frac{1}{2\beta_1} z_1^2 (n + 1)^2 \alpha_j^2 h_j^2 + \frac{1}{2\beta_1} d^2$$

$$= \frac{1}{2} C_1 z_1^2 + \sum_{j=1}^n \frac{1}{2} z_1^2 (\tau_1) h_j^2 + \frac{1}{2\beta_1} d^2,$$  

(16)

where $C_1 = \sum_{j=1}^n (n + 1)^2 \alpha_j^2 + \frac{1}{\beta_1} (n + 1)^2 \alpha_j^2 h_j^2$. $\beta_1$ are given parameters. To simplify control design, introduce functions $h_{t_1}$ and $H_{t_1}$. $h_{t_1}$ will be defined below, $H_{t_1}$ is defined as:

$$H_{t_1} = \sum_{j=1}^n \frac{1}{2} z_1^2 (\tau_1) h_j^2.$$  

(17)

Substituting (16) and (17) into (15) yields

$$\dot{V}_1 \leq z_1 z_2 g_1 + z_1 g_1 \alpha_1 + z_1 f_1(x) - z_1 h_{t_1} + H_{t_1}$$

$$+ \frac{1}{2} \beta_1 d^2 - \frac{g_s}{\gamma_1^2} \hat{\theta}_1 \dot{\hat{\theta}}_1,$$  

(18)

where $\tilde{f}_1$ is:

$$\tilde{f}_1 = f_1(x) + \frac{1}{2} C_1 z_1 - \dot{y}_d + h_{t_1}.$$  

(19)

The function $\tilde{f}_1$ can not be used to design virtual control signal because it contains unknown functions. By the approximation ability of RBF NNs, the following formula holds

$$\tilde{f}_1 = W_1^T S_1(z_1) + \delta(z_1), \quad |\delta(z_1)| \leq \varepsilon_1$$  

$$Z_1 = [x, y_d, \dot{y}_d]^T, \quad x = [x_1, x_2, \ldots, x_n]^T$$  

(20)

where $\delta(z_1)$ is estimation error, $\varepsilon_1$ is known accuracy of estimation. Because the virtual control $\alpha_1$ can not contain the state after the first subsystem, the state vector in the neural network is changed to the current state vector in the first system by Lemma 2.1. By the above discussion and Lemma 2.3, we can get:

$$z_1 f_1 = z_1 g_s \left\{ \frac{\|W_1\|^2}{g_s} + \frac{\varepsilon_1}{g_s} \right\} + k \delta_1 \Phi_1 g_s,$$  

(21)

where

$$\Phi_1(x_1) = (1 + \|S_1(x_1)\|).$$

The parameter $\delta_1$ is positive constant.

Substituting (19) and (21) into (18) produces:

$$\dot{V}_1 \leq z_1 \left( \alpha_1 g_1 \Phi_1(x_1) \tanh \left( \frac{z_1 \Phi_1(x_1)}{\delta_1} \right) + g_1 \alpha_1 \right)$$

$$+ k \delta_1 \Phi_1 g_s + z_1 z_2 g_1$$

$$- z_1 h_{t_1} + H_{t_1} + \frac{1}{2} \beta_1 d^2 - \frac{g_s}{\gamma_1^2} \hat{\theta}_1 \dot{\hat{\theta}}_1.$$  

(22)

We can get virtual control:

$$\alpha_1 = -a_1^2 z_1 - \hat{\alpha}_1 \Phi_1(x_1) \tanh(U_1).$$  

(23)

where $U_1 = \frac{z_1 \Phi_1(x_1)}{\delta_1}$, then, the virtual control is substituted into the inequality (22) and expressed as:

$$\dot{V}_1 \leq -a_1^2 z_1^2 g_s + \hat{\alpha}_1 \frac{g_s}{\gamma_1^2} \left( z_1 \Phi_1(x_1) \tanh(U_1) \gamma_1^2 - \dot{\hat{\alpha}}_1 \right)$$

$$+ k \delta_1 \Phi_1 g_s + z_1 z_2 g_1 - z_1 h_{t_1} + H_{t_1} + \frac{1}{2} \beta_1 d^2.$$  

(24)

And it turns out that the adaptive law $\dot{\hat{\alpha}}_1$ is

$$\dot{\hat{\alpha}}_1 = -a_1^2 \hat{\alpha}_1 + \gamma_1^2 z_1 \Phi_1(x_1) \tanh(U_1)$$  

(25)

then substituting adaptive law into (24), we have:

$$\dot{V}_1 \leq -a_1^2 z_1^2 g_s - \frac{g_s}{\gamma_1^2} a_1^2 \hat{\alpha}_1 \dot{\hat{\alpha}}_1 + z_1 z_2 g_1 + k \delta_1 \Phi_1 g_s$$

(26)
Combine (28) and (29), take the derivative of the
\[ \partial \alpha \]
\[ z_i \]
For \( z_i \) by transformation we can have:
\[ \dot{z}_i = f_i(x) + g_i(x)(z_{i+1} + \alpha_i) + q_i(x_i) - \dot{\alpha}_{i-1} \] (28)
where
\[ \dot{\alpha}_{i-1} = \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} (f_k + g_k x_{k+1} + q_k(x_{k})) + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \theta_k} \hat{\theta}_k + \sum_{k=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^k} y_{d}^{k+1} \] (29)
Combine (28) and (29), take the derivative of the \( V_i \)
\[ \dot{V}_i = z_i \left[ f_i(x) - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} f_k \right. \\
- \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} g_k x_{k+1} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \theta_k} \dot{\theta}_k \\
- \sum_{k=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^k} y_{d}^{k+1} + z_{i+1} g_i + g_i \alpha_i + z_{i-1} g_{i-1} \\
\left. + z_i \left[ q_i(x_i) - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} q_k(x_{k}) \right] \right] \] (30)
For the delay term \( z_i[q_i(x_i) - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} q_k(x_{k})] \), for notational simplicity, make \( \frac{\partial \alpha_{i-1}}{\partial x_k} = -1 \), by Assumption 2.4 and Lemma 2.2, one has:
\[ z_i \left[ q_i(x_i) - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} q_k(x_{k}) \right] \leq \frac{1}{2} \text{C}_i z_i^2 \] (31)
where \( \text{C}_i = \sum_{j=1}^{n} (n + 1)^2 a_j^2 + \frac{1}{\beta_k} (n + 1)^2 a_j^2 h_j^2 \), \( \beta_k \) are the given parameters. For convenience, substitute \( h_j^2, h_j \) for \( h_j^2 ((n + 1) a_j |z_j(\tau_k)|), h_j^2 [(n + 1) a_j |z_j d] \) respectively. Similar to the first step, introduce functions \( h_i \) and \( H_i \), \( H_i \) will be defined below, \( H_i \) is defined as:
\[ H_i = \sum_{k=1}^{n} \sum_{j=1}^{i} \frac{1}{2} a_j^2 (\tau_k) h_j^2. \] (32)
Substituting (31) and (32) into (30) yields
\[ \dot{V}_i = z_i \dot{h}_i + z_i g_i \alpha_i + z_i z_{i+1} g_i + H_i + \frac{1}{2} \sum_{k=1}^{i} \beta_k d^2 \\
- z_i \dot{h}_i - z_{i-1} z_i g_{i-1} - \frac{g_i}{\gamma_i^2} \hat{\theta}_i \hat{\theta}_i \] (33)
where
\[ \tilde{h}_i = f_i(x) - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} f_k - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} g_k x_{k+1} \\
- \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \theta_k} \dot{\theta}_k - \sum_{k=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^k} y_{d}^{k+1} \\
+ \frac{1}{2} \text{C}_i z_i \sum_{k=1}^{i} \left( \frac{\partial \alpha_{i-1}}{\partial x_k} \right)^2 + z_{i-1} g_{i-1} + h_i \] (34)
By neural networks approach \( \tilde{h}_i \),
\[ \tilde{h}_i = W_i^T S_i (z_i) + \delta (z_i), |\delta (z_i)| \leq \epsilon_i, \quad z_i = [x_i, \hat{\theta}_{i-1}, \hat{\gamma}_d]^T \]
\[ x = [x_1, x_2, \ldots, x_n]^T, \quad \hat{\theta}_{i-1} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{i-1}]^T, \]
\[ \hat{\gamma}_d = [\gamma_{d1}, \gamma_{d2}, \ldots, \gamma_{d_l}]^T \] (35)
By Lemma 2.3 we get:
\[ z_i \tilde{h}_i \leq z_i \theta_i g_i \phi_i (\hat{x}_i) \tanh \left( \frac{z_i \phi_i (\hat{x}_i)}{\delta_i} \right) + k \delta_i \theta_i g_i \] (36)
where
\[ \theta_i = \max \left\{ \frac{\epsilon_i}{g_s}, \frac{1}{g_s} \right\}, \quad \phi_i (\hat{x}_i) = (1 + \|S_i (\hat{x}_i)\|), \]
\[ \hat{x}_i = [x_1, x_2, \ldots, x_n]^T. \]
Substituting (34) into (33) and then using (36) result in
\[ \dot{V}_i \leq z_i \left[ \theta_i g_i \phi_i (\hat{x}_i) \tanh \left( \frac{z_i \phi_i (\hat{x}_i)}{\delta_i} \right) + g_i \alpha_i \right] \\
- z_i \dot{h}_i + H_i + k \delta_i \theta_i g_i + \frac{1}{2} \sum_{k=1}^{i} \beta_k d^2 \\
+ z_i z_{i+1} g_i - z_{i-1} z_i g_{i-1} - \frac{g_i}{\gamma_i^2} \hat{\theta}_i \hat{\theta}_i \] (37)
we can get virtual control \( \alpha_i \) as:
\[ \alpha_i = -a_i^2 \dot{z}_i - \hat{\theta}_i \phi_i (\hat{x}_i) \tanh (U_i), \] (38)
where \( U_i = \frac{z_i \phi_i (\hat{x}_i)}{\delta_i}, \quad a_i > 0 \) and \( \delta_i > 0 \) are design parameters, \( \hat{\theta}_i \) is the estimation of \( \theta_i \).
Thus, replacing $\alpha_i$ into (37) gives

$$
\dot{V}_i \leq -\sigma_i^2 \dot{z}_i^2 g_s - z_i \dot{h}_i + H_i \dot{z}_i + k_\delta \dot{z}_i g_s \\
+ \dot{\theta}_i \frac{g_s}{\gamma_i} (z_i \phi_i(\bar{x}_i) \tanh(U_i) \gamma_i^{-1} - \dot{\theta}_i) \\
+ \frac{1}{2} \sum_{k=1}^i \beta_k d^2 + z_i \dot{z}_{i+1} g_i - z_{i-1} \dot{z}_{i-1} g_{i-1}
$$

(39)

We get

$$
\dot{\theta}_i = -\sigma_i^2 \dot{\theta}_i + \gamma_i^2 z_i \phi_i(\bar{x}_i) \tanh(U_i).
$$

(40)

Then, substituting (40) into (39) produces:

$$
\dot{V}_i \leq -\sigma_i^2 \dot{z}_i^2 g_s - \frac{g_s}{2 \gamma_i} \sigma_i^2 \dot{\theta}_i^2 + k_\delta \dot{z}_i g_s - z_i \dot{h}_i \\
+ H_i \dot{z}_i + z_i \dot{z}_{i+1} g_i - z_{i-1} \dot{z}_{i-1} g_{i-1} \\
+ \frac{g_s}{2 \gamma_i} \sigma_i^2 \dot{\theta}_i^2 + \frac{1}{2} \sum_{k=1}^i \beta_k d^2.
$$

(41)

**Step n−1.** For the $(n−1)$th subsystem, consider a Lyapunov function $V_{n−1}$ as

$$
V_{n−1} = \frac{1}{2} \dot{z}_{n−1}^2 + \frac{g_s}{2 \gamma_{n−1}} \dot{\theta}_{n−1}^2.
$$

(42)

For $z_{n−1}$, by transformation, it can be obtained that

$$
\dot{z}_{n−1} = g_{n−1}(x) \left( z_{n−1} + \alpha_{n−1} - g_n \int_{t−\tau}^t D(u(\theta)) d\theta \right) \\
+ f_{n−1} + q_{n−1}(x_{n−1}) - \dot{\alpha}_{n−2},
$$

(43)

where

$$
\dot{\alpha}_{n−2} = \sum_{k=1}^{n−2} \frac{\partial \alpha_{n−2}}{\partial x_k} (f_k + g_k x_{k+1} + q_k(x_{n−2})) \\
+ \sum_{k=1}^{n−2} \frac{\partial \alpha_{n−2}}{\partial \theta_k} \dot{\theta}_k + \sum_{k=0}^{n−2} \frac{\partial \alpha_{n−2}}{\partial y_d} \dot{y}_{d,k+1}.
$$

(44)

The time derivative of $V_{n−1}$ is given by

$$
\dot{V}_{n−1} = z_{n−1} \left[ g_{n−1} \left( z_{n−1} + \alpha_{n−1} - g_n \int_{t−\tau}^t D(u(\theta)) d\theta \right) \\
+ f_{n−1} + q_{n−1}(x_{n−1}) - \sum_{k=1}^{n−2} \frac{\partial \alpha_{n−2}}{\partial x_k} q_k(x_{n−1}) \\
- \sum_{k=1}^{n−2} \frac{\partial \alpha_{n−2}}{\partial x_k} (f_k + g_k x_{k+1}) - \sum_{k=1}^{n−2} \frac{\partial \alpha_{n−2}}{\partial \theta_k} \dot{\theta}_k \\
- \sum_{k=0}^{n−2} \frac{\partial \alpha_{n−2}}{\partial y_d} \dot{y}_{d,k+1} + z_{n−2} g_{n−2} \right] \right. \\
- z_{n−1} \dot{z}_{n−2} g_{n−2}
$$

(45)

The integral term and time delay terms are dealt with as following:

For the integral term $-z_{n−1} g_{n−1} g_n \int_{t−\tau}^t D(u(\theta)) d\theta$, by mean value theorem

$$
\int_{t−\tau}^t D(u(\theta)) d\theta = \tau u(\xi), \quad \xi \in [t−\tau, t],
$$

(46)

since the control signal has a saturation-limited nature, there is a positive constant $D_n$ that makes $\tau |u(\xi)| \leq \tau D_n$, we get:

$$
- z_{n−1} g_{n−1} g_n \int_{t−\tau}^t D(u(\theta)) d\theta \\
\leq z_{n−1} g_{n−1} g_n \tau D_n \tanh \left( \frac{z_{n−1} g_{n−1} g_n \tau D_n}{\delta_{n−1,1}} \right) + k_\delta \dot{\alpha}_{n−1,1}.
$$

(47)

For the time delay term, one has

$$
\dot{z}_{n−1} \left[ g_{n−1}(x_{n−1}) - \sum_{k=1}^{n−2} \frac{\partial \alpha_{n−2}}{\partial x_k} q_k(x_{n−1}) \right] \\
\leq \frac{1}{2} C_{n−1} z_{n−1}^2 \sum_{k=1}^{n−1} \left( \frac{\partial \alpha_{n−2}}{\partial x_k} \right)^2 + \sum_{k=1}^{n−1} \sum_{j=1}^{n−1} \frac{1}{2} Z_j^2 (\tau_k) h_j^2 \\
+ \frac{1}{2} \sum_{k=1}^{n−1} \beta_k d^2
$$

(48)

where $C_{n−1} = \sum_{j=1}^n (n+1)^2 \tilde{a}_j^2 + \frac{1}{\gamma_n} (n+1)^2 \tilde{a}_n^2 h_{n−1}$, $\beta_k$ are given parameters.

Let $h_j^2$, $h_{n−1}^2$ denote $h_j^2((n+1) \bar{a}_j | z_j(x_{k,j}))$, $h_{n−1}^2((n+1) \bar{a}_n | d_{n−1})$ respectively. Similarly, introduce functions $h_{n−1,1}, h_{n−1,1}, h_{n−1,1}$, will be defined below and $h_{n−1,1}$ is

$$
H_{n−1} = \sum_{k=1}^{n−1} \sum_{j=1}^{n−1} \frac{1}{2} Z_j^2 (\tau_k) h_j^2.
$$

(49)

By using (47)–(49), (45) can be rewritten as:

$$
\dot{V}_{n−1} \leq z_{n−1} z_{n−2} g_{n−1} g_n - z_{n−1} g_{n−1} \alpha_{n−1} + k_\delta \dot{\alpha}_{n−1,1} \\
+ \frac{1}{2} \sum_{k=1}^{n−1} \beta_k d^2 - z_{n−1} z_{n−2} g_{n−2} \\
+ \int_{t−\tau}^t D(u(\theta)) d\theta \\
+ z_{n−1} \dot{z}_{n−1} g_{n−1} g_n \tau D_n \tanh \left( \frac{z_{n−1} g_{n−1} g_n \tau D_n}{\delta_{n−1,1}} \right) + k_\delta \dot{\alpha}_{n−1,1} + h_{n−1,1}
$$

(50)

where

$$
\dot{f}_{n−1} = f_{n−1} - \sum_{k=1}^{n−2} \frac{\partial \alpha_{n−2}}{\partial \theta_k} (f_k + g_k x_{k+1})
$$
Replacing (51)–(53) into (50) gives:

\[
\dot{\hat{\theta}}_{n-1} = \gamma_{n-1}^2 \phi_n(\hat{x}_{n-1}) \tanh(U_{n-1}) - \sigma_{n-1}^2 \dot{\hat{\theta}}_{n-1}.
\] 

(56)

So, we select adaptive law as:

\[
\dot{\hat{\theta}}_{n-1} = \gamma_{n-1}^2 \phi_n(\hat{x}_{n-1}) \tanh(U_{n-1}) - \sigma_{n-1}^2 \dot{\hat{\theta}}_{n-1}.
\] 

(57)

Substituting (56) into (55)

\[
V_{n-1} \leq -\sigma_{n-1}^2 \zeta_{n-1}^2 g_s - \frac{g_s}{2\gamma_{n-1}^2} \sigma_{n-1}^2 \dot{\hat{\theta}}_{n-1}^2 + k\delta_{n-1,1}
\] 

\[
- \zeta_{n-1}^2 g_s \zeta_{n-1} g_{n-1} + z_n \zeta_{n-1} g_{n-1} + H_{\delta_{n-1,1}}
\] 

\[
+ \frac{1}{2} \sum_{k=1}^{n-1} \beta_k d^2 - z_{n-1} h_{n-1} + H_{\delta_{n-1,1}}
\] 

\[
= \frac{g_s}{2\gamma_{n-1}^2} \sigma_{n-1}^2 \dot{\hat{\theta}}_{n-1}^2 + k\delta_{n-1,2} \dot{\hat{\theta}}_{n-1} g_s
\] 

(58)

Step n. For the nth subsystem, consider \( V_n \) as

\[
V_n = \frac{1}{2} \zeta_{n-1}^2 + \frac{g_s}{2\gamma_{n}^2} \dot{\hat{\theta}}_{n}^2,
\] 

(59)

For \( z_n \), it can be obtained that

\[
\dot{z}_n = f_n(x) + q_n(x_{n-1}) - \dot{\alpha}_{n-1} + \sum_{j=1}^{n-1} \frac{\partial g_j}{\partial x_j} \tau u(\xi)
\] 

\[+ g_n D(u(t)),
\] 

(60)

where

\[
\dot{\alpha}_{n-1} = \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} (f_k + g_k x_{k+1}) + q_k(x_{k-1})
\] 

\[+ \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_{d_k}} + \sum_{k=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_{d_k}} k_{n-1}.
\] 

(61)

According to (60) and (61), the time derivative of \( V_n \) is given by

\[
\dot{V}_n \leq z_n \left[ f_n(x) + q_n(x_{n-1}) - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} q_k(x_{k-1})
\right]
\] 

\[+ \sum_{j=1}^{n-1} \frac{\partial g_j}{\partial x_j} \tau u(\xi) + g_n D(u(t))
\] 

\[+ z_{n-1} g_{n-1} - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} f_k - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} (g_k x_{k+1})
\] 

we take \( \alpha_{n-1} \) as

\[
\alpha_{n-1} = -\sigma_{n-1}^2 \zeta_{n-1} - \dot{\alpha}_{n-1} \phi_n(\hat{x}_{n-1}) \tanh(U_{n-1})
\] 

(55)

where \( U_{n-1} = \frac{\zeta_{n-1} \phi_n(\hat{x}_{n-1})}{\delta_{n-1}} \). Then, the following inequality can be obtained:

\[
\dot{V}_{n-1} \leq -\sigma_{n-1}^2 \zeta_{n-1}^2 g_s + k\delta_{n-1,1} - \zeta_{n-1}^2 \zeta_{n-1} g_{n-2}
\] 

\[+ z_n \zeta_{n-1} g_{n-1} + k\delta_{n-1,2} \dot{\hat{\theta}}_{n-1} g_s
\] 

\[+ \frac{1}{2} \sum_{k=1}^{n-1} \beta_k d^2 - z_{n-1} h_{n-1}
\] 

\[+ H_{\delta_{n-1,1}} + \dot{\theta}_{n-1} \frac{g_s}{\gamma_{n-1}^2}
\] 

\[\times \left( \zeta_{n-1} \phi_n(\hat{x}_{n-1}) \tanh(U_{n-1}) \zeta_{n-1} - \dot{\hat{\theta}}_{n-1} \right)
\] 

(54)

By the approximation ability of RBF NNs,

\[
\tilde{f}_{n-1} = W_{n-1}^T S_{n-1}(Z_{n-1}) + \delta(Z_{n-1}),
\]

\[|\delta(Z_{n-1})| \leq \varepsilon_{n-1}, \quad Z_{n-1} = [\hat{x}_{n-1}, \tilde{y}_{d_{n-1}}] \]

\[x = [x_1, x_2, \ldots, x_n]^T, \quad \tilde{\theta}_{n-2} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{n-2}]^T,
\]

\[\tilde{y}_{d_{n-1}} = [y_d y_1 \ldots, y_d^{(n-1)}]^T.
\]

where \( \delta(Z_{n-1}) \) is estimation error, \( \varepsilon_{n-1} \) is a given estimated accuracy. Use Lemmas 2.2 and 2.3 again to get

\[
z_{n-1} \tilde{f}_{n-1} \leq z_{n-1} \zeta_{n-1} g_s \phi_n(\tilde{x}_{n-1})\]

\[\tanh \left( \frac{z_{n-1} \zeta_{n-1} \phi_n(\tilde{x}_{n-1})}{\zeta_{n-1,2}} \right) + k\delta_{n-1,2} \zeta_{n-1} g_s
\]

(53)

where

\[\tilde{\theta}_{n-1} = \max \left\{ \varepsilon_{n-1} \left\| W_{n-1} \right\| \right\},
\]

\[\phi_{n-1} \left( \tilde{x}_{n-1} \right) = (1 + \left\| S_{n-1}(\tilde{x}_{n-1}) \right\|),
\]

\[\tilde{x}_{n-1} = [x_1, x_2, x_3, \ldots, x_n]^T.
\]

Replacing (51)–(53) into (50) gives:

\[
\dot{V}_{n-1} \leq z_n \left[ \phi_n(\tilde{x}_{n-1}) \tanh \left( \frac{z_{n-1} \zeta_{n-1} \phi_n(\tilde{x}_{n-1})}{\zeta_{n-1,2}} \right) \right.
\]

\[+ g_n \zeta_{n-1} \zeta_{n-1} g_s + k\delta_{n-1,1} - z_{n-1} \zeta_{n-1} g_{n-1}
\] 

\[+ k\delta_{n-1,2} \zeta_{n-1} g_s + \frac{1}{2} \sum_{k=1}^{n-1} \beta_k d^2
\] 

\[+ \frac{g_s}{\gamma_{n-1}^2} \zeta_{n-1} \zeta_{n-1} g_{n-2} + z_n \zeta_{n-1} g_{n-1}
\] 

\[+ \sum_{k=1}^{n-1} \frac{\partial g_{n-1}}{\partial x_k} \tau u(\xi)
\] 

\[+ g_n D(u(t)),
\]

(60)
Substituting (63)–(65) into (62) results in:
\[
- z_n z_{n-1} g_{n-1} + \frac{g_n}{\gamma_n^2} \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \theta_k} \hat{\theta}_k
= - z_n z_{n-1} g_{n-1} + \frac{g_n}{\gamma_n^2} \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \theta_k} \hat{\theta}_k
\]
(62)

where
\[
\begin{align*}
& z_n \left[ q_n(x_{tn}) - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} q_k(x_{tn}) \right] \\
& \leq \frac{1}{2} C_n z_n^2 \sum_{k=1}^{n} \left( \frac{\partial \alpha_{n-1}}{\partial x_k} \right)^2 + \frac{1}{2} n \sum_{k=1}^{n} \sum_{j=1}^{n} \tau_k h_n^2
+ \frac{1}{2} n \beta_k d^2,
\end{align*}
\]
(63)

where \( C_n = \sum_{j=1}^{n} (n+1)^2 a_j^2 + \frac{1}{\rho_n} (n+1)^2 \beta_j^2 \), substitute \( h_n^2 \) and \( h_n^2 \) for \( h_n^2 = h_n^2 + (n+1)\bar{a}_j z_j(t_k) \) and \( h_n^2 = (n+1)\bar{a}_d d \) respectively. Introduce functions \( h_{tn}, H_{tn}, \) and \( h_{tn} \) will be defined below, is, \( H_{tn} \)
\[
H_{tn} = \sum_{k=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \tau_j h^2_j,
\]
(64)

besides, one has
\[
z_n \sum_{j=1}^{n} \frac{\partial g_n}{\partial x_j} \tau (\xi) \leq z_n \sum_{j=1}^{n} \frac{\partial g_n}{\partial x_j} \tau D_n
\times \tanh \left( \frac{\sum_{j=1}^{n-1} \frac{\partial g_n}{\partial x_j} \tau D_n}{\delta_{n,1}} \right) + k \delta_{n,1}.
\]
(65)

Substituting (63)–(65) into (62) results in:
\[
\dot{V}_n \leq z_n \tilde{f}_n + z_n \left( g_n D(u(t)) - z_n h_{tn} + H_{tn} \right)
+ \frac{1}{2} \sum_{k=1}^{n} \beta_k d^2 + k \delta_{n,1}
- z_n z_{n-1} g_{n-1} - \frac{g_n}{\gamma_n^2} \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \theta_k} \hat{\theta}_k,
\]
(66)

define assistant function \( \tilde{f}_n \):
\[
\tilde{f}_n = f_n(x) + \frac{1}{2} C_n z_n^2 \left( \frac{\partial \alpha_{n-1}}{\partial x_k} \right)^2
+ \sum_{j=1}^{n} \frac{\partial g_n}{\partial x_j} \tau D_n \tanh \left( \frac{\sum_{j=1}^{n-1} \frac{\partial g_n}{\partial x_j} \tau D_n}{\delta_{n,1}} \right)
- \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} (g_k x_{k+1}) - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} f_k
\]
(72)

By the approximation ability of RBF NNs to model the unknown \( \tilde{f}_n \) and get
\[
\tilde{f}_n = W_n^T S_n (Z_n) + \delta (Z_n), \quad |\delta (Z_n)| \leq \epsilon_n,
\]
\[
Z_n = [x, \tilde{\theta}_{n-1}, \tilde{y}_{dn}]^T, \quad x = [x, x_2, \ldots, x_n]^T,
\]
\[
\tilde{\theta}_{n-1} = [\tilde{\theta}_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_{n-1}]^T, \quad \tilde{y}_{dn} = [y_d, \tilde{y}_d, \ldots, \tilde{y}_{dn}]^T
\]
(68)

By Lemma 2.3, we have the following inequalities:
\[
z_n \tilde{f}_n \leq z_n \theta_n g_s g_n (\tilde{x}_n) \tanh \left( \frac{z_n \phi_n (\tilde{x}_n)}{\delta_n} \right) + k \delta_n g_s g_s,
\]
(69)

where
\[
\theta_n = \max \left\{ \epsilon_n, \frac{\| W_n \|^2}{g_s} \right\}, \quad \phi_n (\tilde{x}_n) = (1 + \| S_n (x_n) \|),
\]
\[
\tilde{x}_n = [x, x_2, x_3, \ldots, x_n]^T.
\]

Thus, (66) can be expressed as
\[
\dot{V}_n \leq z_n \left[ g_n D(u(t)) + \theta_n g_s g_n (\tilde{x}_n) \tanh \left( \frac{z_n \phi_n (\tilde{x}_n)}{\delta_n} \right) \right]
+ k \delta_n g_s g_s - z_n h_{tn} + H_{tn} + \frac{1}{2} \sum_{k=1}^{n} \beta_k d^2 + k \delta_{n,1}
- z_n z_{n-1} g_{n-1} - \frac{g_n}{\gamma_n^2} \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \theta_k} \hat{\theta}_k.
\]
(70)

Regarding saturation input, it can be obtained as follows
\[
\begin{align*}
\dot{z}_n g_n D(u(t)) &= z_n g_n (g(u) + d(u)) = z_n g_n (g_n u + d(u)) \\
\dot{z}_n g_n d(u) &\leq \frac{1}{2} g_n z_n^2 + \frac{1}{2} g_n D_n^2.
\end{align*}
\]
(71)

It follows from (70) that
\[
\dot{V}_n \leq z_n \left[ \theta_n g_s g_n (\tilde{x}_n) \tanh \left( \frac{z_n \phi_n (\tilde{x}_n)}{\delta_n} \right) \right]
+ g_n g_{nu} + \frac{1}{2} g_n z_n
+ \frac{1}{2} g_n D_n^2 + k \delta_n g_s g_s - z_n h_{tn} + H_{tn}
+ \frac{1}{2} \sum_{k=1}^{n} \beta_k d^2 + k \delta_{n,1} - z_n z_{n-1} g_{n-1} - \frac{g_n}{\gamma_n^2} \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \theta_k} \hat{\theta}_k.
\]
(72)
So, we can get real control as

$$u = - \left( a_n^2 + \frac{1}{2} \right) z_n - \tilde{\theta}_n \phi_n (\tilde{x}_n) \tanh(U_n),$$  \hfill (73)

where $U_n = \frac{z_n \phi_n(\tilde{x}_n)}{\delta}$. By substituting (73) into (72), one has

$$\dot{V}_n \leq \tilde{\theta}_n \frac{g_s}{\gamma_n} (z_n \phi_n (\tilde{x}_n) \tanh(U_n) g_n)^2 - \hat{\theta}_n$$

$$- a_n^2 z_n g_n g_l + \frac{1}{2} g_n g_n D_n^2 + k\delta_n \theta_n$$

$$- z_n h_t + H_t + \frac{1}{2} \sum_{k=1}^{n} \beta_k d^2 - z_n z_{n-1} g_{n-1}$$

$$+ k\delta_n \theta_n g_l g_s$$  \hfill (74)

The adaptive law is defined as the solution to the following differential equation:

$$\dot{\theta}_n = c_n z_n \phi_n (\tilde{x}_n) \tanh(U_n) - \sigma_n^2 \dot{\theta}_n$$  \hfill (75)

where $c_n = g_i^{1/2}$. Then, by taking (75) into account, (74) can be rewritten as

$$\dot{V}_n \leq - a_n^2 z_n^2 g_n g_l - \frac{g_s}{2\gamma_n} \sigma_n^2 \dot{\theta}_n + \frac{1}{2} g_n g_n D_n^2 + k\delta_n \theta_n g_l$$

$$- z_n h_t + H_t + \frac{1}{2} \sum_{k=1}^{n} \beta_k d^2 + k\delta_n \theta_n$$

$$- z_n z_{n-1} g_{n-1} + \frac{g_s}{2\gamma_n} \sigma_n^2 \theta_n.$$  \hfill (76)

In order to eliminate the influence of the time delay term, for all of these systems, define Lyapunov function

$$V = \sum_{i=1}^{n} V_i + \sum_{i=1}^{n} W_i,$$  \hfill (77)

where the function $W_i$ is defined as:

$$W_i = \sum_{k=1}^{n} e^{-(t-s_k)} \int_{s_k}^{s} c(n,k) e^{s^2} (s) h_i^2$$

$$\times ((n + 1) \tilde{\alpha} | z_i(s) |) ds.$$  \hfill (78)

The derivative of the $V$ shows

$$\dot{V} \leq - \left( \sum_{i=1}^{n-1} a_i^2 z_i^2 g_i + \sum_{i=1}^{n-1} \frac{g_i}{2\gamma_i} \sigma_i^2 \dot{\theta}_i^2 \right)$$

$$- \left( a_n^2 z_n^2 g_n g_l + \frac{g_s}{2\gamma_n} \sigma_n^2 \dot{\theta}_n^2 \right) + \sum_{i=1}^{n} k\delta_i \theta_i$$

$$+ k\delta_{n-1,2} \theta_{n-1,2} g_s + k\delta_n \theta_n g_s + \sum_{i=1}^{n} \frac{g_i}{2\gamma_i} \sigma_i^2 \dot{\theta}_i^2$$

$$+ k\delta_n \theta_n g_l g_s + \frac{1}{2} g_n g_n D_n^2 + k\delta_n \theta_n g_s$$

$$+ \frac{1}{2} \sum_{k=1}^{n-1} \beta_k d^2 - \sum_{i=1}^{n} W_i \leq -a_0 V + b_0.$$  \hfill (82)

where

$$a_0 = \min \left\{ 2a_i^2 g_i, \sigma_i^2 : 1 \leq i \leq n - 1, 2a_n^2 g_n g_l, \sigma_n^2, 1 \right\},$$

$$b_0 = \sum_{i=1}^{n-1} k\delta_i \theta_{i-1} g_s + k\delta_{n-1,2} + k\delta_n \theta_n g_s$$

$$+ \sum_{i=1}^{n} \frac{g_i}{2\gamma_i} \sigma_i^2 \dot{\theta}_i^2 + k\delta_n \theta_n.$$
Define \( V = \max \{ V_0, \frac{b_0}{a_0} \} \). Therefore, we have \( \| z \| = \sqrt{2V} \), that means that all the signals are bounded in the closed-loop system. In addition

\[
|y - y_r| \leq \sqrt{2V},
\]

\[
\lim_{t \to \infty} |y - y_r| \leq \sqrt{2\frac{b_0}{a_0} = k^* \delta^*},
\]

where \( k^* = \sqrt{2\frac{b_0}{a_0}} \). It is shown that the tracking error is bounded and the limit tends to a small enough set.

### 4. Simulation example

To illustrate the validity of the control method proposed in this paper and further compare with other control methods, consider the following two simulation examples:

**Example 4.1:** Consider the following second-order non-strict feedback nonlinear system:

\[
\begin{align*}
\dot{x}_1 &= x_1x_2^2 + (1 + e^{x_1^2}) x_2 + x_2^2 (t - \tau_1) \sin (x_2 (t - \tau_1)), \\
\dot{x}_2 &= x_1^2 x_2 + (1 + 0.5 \sin (x_1 x_2)) u (t - \tau) + x_1 (t - \tau_2) \\
&\quad \times \sin (x_1 (t - \tau_2) x_2 (t - \tau_2)), \\
y &= x_1,
\end{align*}
\]

where \( x_1, x_2 \) denote the state variables, \( y \) is the system output, and the input saturation limits are chosen as \( u_{max} = 5 \) and \( u_{min} = -5 \). The system satisfies Assumptions 2.1–2.4, by selecting parameters \( a_1 = 2, a_2 = 5, \delta_1 = \delta_2 = 2, \sigma_1 = \sigma_2 = 1, \gamma_1 = \gamma_2 = 1, g_1 = 5 \). The simulation is carried out with the initial conditions \( x_1(0) = 0.5, x_2(0) = 0.5, \dot{\theta}_1 = \dot{\theta}_2 = 0 \), the reference signal is assumed to be \( y_d = \sin(x) \), state delay \( \tau_1 = \tau_2 = 2 \), input delay \( \tau = 0.1 \). \( \psi(t) = 0, (i = 1, 2), \phi(t) = 0 \) when \( t < 0 \). Each control signal is as follows

\[
\begin{align*}
\alpha_1 &= -a_1 z_1 - \dot{\theta}_1 \phi_1 (\tilde{x}_1) \tanh \left( \frac{z_1 \phi_1 (\tilde{x}_1)}{\delta_1} \right), \\
u &= \left( a_2 + \frac{1}{2} \right) z_2 - \dot{\theta}_2 \phi_2 (\tilde{x}_2) \tanh \left( \frac{z_2 \phi_2 (\tilde{x}_2)}{\delta_2} \right) \\
\dot{\theta}_1 &= \gamma_1 z_1 \phi_1 (\tilde{x}_1) \tanh \left( \frac{z_1 \phi_1 (\tilde{x}_1)}{\delta_1} \right) - \sigma_1 \dot{\theta}_1,
\end{align*}
\]

Figures 1–4 illustrate the simulation results. Figure 1 shows the input signal of the system can well track the given reference signal. Figures 2 and 3 show that all the closed-loop signals are bounded. Figure 4 displays the saturation control input signal \( u \). Figures 1–4 show that the proposed control method can be applied to a class of non-strict feedback systems.

**Example 4.2:** Consider the following nonlinear system, which is taken from H. Li et al. (2017),

\[
\begin{align*}
\dot{x}_1 &= x_2 + \frac{-x_1}{1 + x_1^2} + 0.1 \sin(x_1) \\
\dot{x}_2 &= -x_2 e^{-x_1^2} x_1^2 + u(t - \tau) + 0.1 \cos(x_2) \\
y &= x_1
\end{align*}
\]
The work of the original literature is to use the Pade approximation method to deal with the input delay. The virtual control, real control law and adaptive law are designed as

\[\alpha_1 = -c_1 z_1 - \frac{b_1 z_1}{2a_1^2 (k_{d1}^2 - z_1^2)} \theta_1 \phi_1^T (Z_1) \phi_1 (Z_1)\]
\[\alpha_2 = -c_2 z_2 - \frac{b_2 z_2}{2a_2^2 (k_{d2}^2 - z_2^2)} \theta_2 \phi_2^T (Z_2) \phi_2 (Z_2)\]
\[u = -c_2 z_2 - \frac{b_2 z_2}{2a_2^2} \phi_2^T (Z_2) \phi_2 (Z_2) - \frac{b_2 z_2}{2r_2}\]
\[\dot{\theta}_1 = -\frac{\eta_1 b_1 z_1^2}{2a_1^2 (k_{d1}^2 - z_1^2)} \phi_1^T (Z_1) \phi_1 (Z_1) - \sigma_1 \theta_1\]
\[\dot{\theta}_2 = \frac{\eta_2 b_2 z_2^2}{2a_2^2} \phi_2^T (Z_2) \phi_2 (Z_2) - \sigma_2 \theta_2\]

According to the original article, the given tracking signal is \(y_d = \sin(t)\), the initial conditions of the system are \([x_1(0), x_2(0)]^T = [0.07, 0.2]^T\), \([\theta_1(0), \theta_2(0)]^T = [0, 0]^T\), select control parameters as \(a_1 = 2, a_2 = 3, c_1 = 50, c_2 = 54, b_1 = 0.08, b_2 = 0.09, \sigma_1 = 1, \sigma_2 = 2, r_1 = 1, r_2 = 2, k_{d1} = 0.12, \eta_1 = 0.4, \eta_2 = 0.2\). When the input delay is \(\tau = 0.0043\), system output can track the given reference signal well. The tracking effect is shown in Figure 5. When the input delay is selected as 0.1, the system output can no longer track the given reference signal, as shown in Figure 6.

Under the same reference signal and initial conditions, the method proposed in this paper is used in the control system (85). The control parameters are \(a_1 = a_2 = 12, \delta_1 = \delta_2 = 0.1, \gamma_1 = \gamma_2 = 5, \sigma_1 = \sigma_2 = 0.03, g_t = 0.3\). When the input delay is \(\tau = 0.0043\), the simulation is...
shown in Figure 7, when $\tau = 0.1$, the simulation effect is shown in Figure 8. The simulation results show that the method proposed in this paper is more effective for the control of the system (85).

5. Conclusion

This paper addresses the problem of adaptive tracking control for a class of SISO nonlinear systems with state and input delays. The controller is constructed by backstepping adaptive neural control approach, which ensures that all signals in the closed-loop system are bounded and the tracking error converges to a small neighbourhood around the origin. In the future research, we hope further develop some new approaches to nonlinear systems with input delay. In addition, finite-time control issue is also considered for nonlinear systems with state and input delays.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by the National Natural Science Foundation of China [grant numbers 61873137, 61673227].

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