Laboratory Constraints on a 33.9 MeV/c² Isosinglet Neutrino: Status and Perspectives

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Abstract

An anomaly in the time behaviour of the signals observed by the KARMEN Collaboration may be interpreted as the possible decay signature of a 33.9 MeV/c² mainly sterile neutrino. This note discusses the parameter space still open for the mixing of this hypothetical particle with the neutrinos of known leptonic flavour, considering the experimental results which became available recently, as well as those to be expected from forthcoming measurements. It is concluded that if no positive signature is observed, the envisaged laboratory experiments are not expected to close entirely the parameter space of mixing amplitudes. However, a proper reassessment of the ALEPH upper bound on the ντ neutrino mass including the possibility of τ flavour mixing, would certainly help in reducing the parameter space left open.

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1. Introduction. An excess of events observed in the KARMEN detector\cite{1} about 3.6 $\mu$s after the beam-on-target, was tentatively interpreted as the decay signature of a 33.9 MeV/c$^2$ neutral particle emitted in pion decay\cite{1}. The genuine physical reality of these events seems to be further confirmed by still more such events from KARMEN\cite{2}.

Ref.\cite{3} addressed specifically the tentative identification\cite{1} of the observed signal with a mainly isosinglet (sterile) neutrino emitted in pion decay through its admixture into the muonic flavour and decaying in flight principally into an electron-positron pair and an isodoublet neutrino. The conclusion reached in Ref.\cite{3} is that the observed decay rate implies the following constraint on the mixing amplitudes $U_{\alpha X} (\alpha = e, \mu, \tau)$ of the 33.9 MeV/c$^2$ mass eigenstate $X$ with the known leptonic flavours, provided the quantities $|U_{\alpha X}|^2 (\alpha = e, \mu, \tau)$ are small enough for this first order expanded approximation to a more complete analysis to be justified,

$$0.0285 K |U_{\mu X}|^2 \left[ 920 |U_{e X}|^2 + 210 |U_{\mu X}|^2 + 210 |U_{\tau X}|^2 \right] = 3 \cdot 10^{-11} \text{ s}^{-1} \ . \quad (1)$$

Here, the parameter $K$ takes the value $K = 1$ (resp. $K = 2$) for a Dirac (resp. Majorana) spinor describing the $X$ neutrino.

An isodoublet characterisation of the $X$ particle, and its identification with the dominant component of the $\nu_\tau$ neutrino, were rejected on the basis of cosmological and astrophysical arguments\cite{3} as well as the ALEPH bound of 24 MeV/c$^2$ (95 %C.L.) on the $\nu_\tau$ neutrino mass\cite{5}. It should be noted that the validity of that bound was questioned recently in the context of neutrino mixing scenarios\cite{3} in the $\tau$ flavour sector.

Independently of this controversy, the aim of this note is to investigate the impact of the negative outcome of recent searches for the $X$ particle\cite{7,8,9}, as well as that of a search soon to be pursued anew\cite{10}, and to identify possible laboratory experiments which could either confirm the existence of this particle, or else, exclude part or all of the parameter space implied by the KARMEN experiment on the basis of (1).

2. The Sterile Neutrino Scenario Revisited. Let us reconsider the sterile neutrino scenario as a possible explanation for the KARMEN anomaly\cite{1,3}, without any assumptions as to the smallness of mixing amplitudes. The neutrino flavour eigenstates $\nu_a$ are indexed by $(a = \alpha, 0)$ with $\alpha = e, \mu, \tau$. The $a = \alpha$ flavours are the known electroweak $SU(2)_L$ isodoublet neutrinos, while the $a = 0$ flavour is a $SU(2)_L \times U(1)_Y$ singlet. For simplicity, it is assumed that only one such isosinglet state is involved in the KARMEN anomaly, namely that the anomaly is dominated by only one such state.

Neutrino interaction eigenstates $\nu_a$ are mixtures of the neutrino mass eigenstates. These mass eigenstates are denoted $\nu_I$, the index $I = (i, X)$ taking the values $i = 1, 2, 3$ associated to the three known light neutrinos, and a fourth value $I = X$ corresponding to the $X$ particle of the KARMEN anomaly. All these states are related through a mixing matrix $U_{ai}$ such that,

$$\nu_a = U_{ai} \nu_I \ , \quad \nu_a^\dagger = \nu_I U^\dagger_{ai} \ . \quad (2)$$

Another interpretation within supersymmetric models has also been considered in Ref.\cite{4}. 

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If no other states are involved in such a mixing scenario, the matrix $U_{af}$ is unitary. However, if for example there were to exist additional singlet sterile neutrinos mixing with the usual neutrino flavours, but at a level such that their production in pion decay would be suppressed compared to that of the $X$ particle, the matrix $U_{af}$ would then not have to be unitary, but rather, be a sub-matrix of still a larger unitary mixing matrix. However, note that in the circumstance that the matrix $U_{af}$ is indeed unitary, one has the identity,

$$\sum_{i,j} \left| \sum_{a} U_{ia}^\dagger U_{af} \right|^2 = 3 \quad ,$$

in which the intermediate sum is over the isodoublet flavours $\alpha = e, \mu, \tau$ rather than over all four neutrino flavours $a$.

In the following, we shall make the approximation to ignore the masses of all ordinary neutrinos $\nu_i$ ($i = 1, 2, 3$) which are assumed to be sufficiently small in comparison to all other mass scales involved, including in particular the mass $m_X$ of the $X$ particle. It is then straightforward to compute some relevant decay rates.

First, the pion decay branching ratios are simply given by, in an obvious notation,

$$\frac{\Gamma(\pi^+ \rightarrow \ell^{+}_\alpha X)}{\Gamma(\pi^+ \rightarrow \ell^{+}_\alpha \nu_i)} = \frac{|U_{aX}|^2}{|U_{a\alpha}|^2} \frac{\lambda^{1/2}(m_{\pi}^2, m_{\ell}^2, m_X^2)}{m_\ell^2} \frac{m_{\ell}^2(m_{\ell}^2 + m_X^2) - (m_\pi^2 - m_X^2)^2}{(m_\pi^2 - m_\ell^2)^2} \quad ,$$

with the usual kinematical factor $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$.

Next, concerning $X$ decay modes, the electronic branch gives,

$$\Gamma(X \rightarrow \nu_i e^+ e^-) = \left\{ \frac{1}{8} \left| \sum_{\alpha} U_{i\alpha}^\dagger U_{aX} \right|^2 (1 + \rho^2) + \left| U_{i\nu}^\dagger U_{\nu X} \right|^2 - \frac{1}{2} \text{Re} \left[ \left( \sum_{\alpha} U_{i\alpha}^\dagger U_{aX} \right) \left( U_{i\nu}^\dagger U_{\nu X} \right)^* \right] (1 + \rho) \right\} K \Gamma_X \quad ,$$

where,

$$\Gamma_X = \frac{G_F^2 m_X^5}{192 \pi^3} \quad , \quad \rho = 1 - 4 \sin^2 \theta_W \quad ,$$

$G_F$ being Fermi’s coupling constant, and $\theta_W$ the electroweak gauge mixing angle such that $\sin^2 \theta_W = 0.2319$.

Similarly, for the neutrino branch, one has,

$$\Gamma(X \rightarrow \nu_i \nu_j \nu_k; i \neq j) = \frac{1}{4} \left| \sum_{\alpha} U_{i\alpha}^\dagger U_{aX} \right|^2 \left| \sum_{\beta} U_{j\beta}^\dagger U_{\beta k} \right|^2 K \Gamma_X \quad ,$$

$$\Gamma(X \rightarrow \nu_i \nu_j \nu_k; i = j) = \frac{1}{2} \left| \sum_{\alpha} U_{i\alpha}^\dagger U_{aX} \right|^2 \left| \sum_{\beta} U_{i\beta}^\dagger U_{\beta k} \right|^2 K \Gamma_X \quad .$$

Finally, the radiative branch is given by $\[13\]$

$$\Gamma(X \rightarrow \nu_i \gamma) \simeq \frac{27}{8} \frac{\alpha}{\pi} \left| \sum_{\alpha} U_{i\alpha}^\dagger U_{aX} \right|^2 K \Gamma_X \quad .$$
For later reference, the $Z_0$ neutrino decay rates are also useful,
\[
\Gamma(Z_0 \to \nu_I \bar{\nu}_J) = \frac{G_F M_Z^3}{12\sqrt{2}} \sum_\alpha U_{J\alpha}^* U_{I\alpha} \left[ \lambda^{1/2}(M_Z^2, m_I^2, m_J^2, \frac{m_I^2 + m_J^2}{2M_Z^2}, \frac{(m_I^2 - m_J^2)^2}{4M_Z^4}) \right],
\]
which, in the limit that the neutrino masses are neglected compared to the $Z_0$ mass $M_Z$, reduces to,
\[
\Gamma(Z_0 \to \nu_I \bar{\nu}_J) = \frac{G_F M_Z^3}{12\sqrt{2}} \left| \sum_\alpha U_{J\alpha}^* U_{I\alpha} \right|^2.
\]
Note that in this limit and when the matrix $U_{aI}$ is assumed unitary, the total invisible $Z_0$ decay width $\sum_{I,J} \Gamma(Z_0 \to \nu_I \bar{\nu}_J)$ reduces to the Standard Model result for three generations of electroweak neutrino isodoublets, as follows from (3).

The time characteristics of the KARMEN anomaly correspond[1] to a mass $M_X$ of
\[
M_X = 33.905 \text{ MeV}/c^2,
\]
which is indeed very close to the $\pi^+ - \mu^+$ mass difference of $33.91157 \text{ MeV}/c^2$. Correspondingly, the phase space factor appearing in the pion decay branching ratio (4) is then given by,
\[
Br = \frac{\Gamma(\pi^+ \to \mu^+ X)}{\sum_I \Gamma(\pi^+ \to \mu^+ \nu_I)} = \frac{0.0293 |U_{\mu X}|^2}{\left| U_{\mu 1} \right|^2 + \left| U_{\mu 2} \right|^2 + \left| U_{\mu 3} \right|^2 + 0.0293 |U_{\mu X}|^2},
\]
while the quantity $\Gamma_X$ takes the value,
\[
\Gamma_X = 1556 \text{ s}^{-1}.
\]

The quantity determined for the KARMEN anomaly is[1, 3],
\[
Br \Gamma_{\text{vis}} \simeq 3 \cdot 10^{-11} \text{ s}^{-1},
\]
where the visible decay width of the $X$ particle is defined by,
\[
\Gamma_{\text{vis}} = \sum_{i=1,2,3} \Gamma(X \to \nu_i e^- e^+) + \sum_{i=1,2,3} \Gamma(X \to \nu_i \gamma).
\]
Recent experimental results[7, 8, 9] imply upper bounds on the branching ratio $Br$, the most stringent of which is[7],
\[
Br < 2.6 \cdot 10^{-8} (95\% \text{ C.L.)}.
\]
A recent proposal[10] aims at improving this limit down to the level of $10^{-10}$. A further experimental constraint in the electronic sector is the upper bound on $|U_{eX}|^2$ from Ref.[13],
\[
|U_{eX}|^2 \lesssim 10^{-6}.
\]
Minimal Mixing. A minimal mixing hypothesis, involving a single sterile neutrino and compatible with the emission of the $X$ particle jointly with a muon, would imply a unitary mixing matrix of the form,

$$U_{at} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & e^{i\alpha} \cos\theta & 0 & e^{i\beta} \sin\theta \\
0 & 0 & 1 & 0 \\
0 & -e^{i\gamma} \sin\theta & 0 & e^{i(\gamma-\alpha+\beta)} \cos\theta
\end{pmatrix},$$

where $\theta$ is a mixing angle, and $\alpha$, $\beta$ and $\gamma$ arbitrary phase factors. In particular, note that this ansatz does not lead to mixing of the $X$ particle with the electron or tau flavours.

The KARMEN result \cite{15} then implies the values for the flavour mixing angle,

$$\text{Dirac case } K = 1 : \quad \sin^2 \theta = 2.2 \cdot 10^{-6}, \quad \sin 2\theta = 3.0 \cdot 10^{-3},$$

$$\text{Majorana case } K = 2 : \quad \sin^2 \theta = 1.6 \cdot 10^{-6}, \quad \sin 2\theta = 2.5 \cdot 10^{-3},$$

which in turn leads to the branching ratios,

$$\text{Dirac case } K = 1 : \quad Br = 6.5 \cdot 10^{-8},$$

$$\text{Majorana case } K = 2 : \quad Br = 4.6 \cdot 10^{-8}.$$

Hence, this result being larger than the recent upper bound \cite{17} by a factor 2.5 or 1.8 for $K = 1$ or $K = 2$, respectively, the minimal mixing scenario is already excluded on purely laboratory experimental grounds.

The Isodoublet Diagonal Mixing Approximation. The minimal mixing hypothesis shows that if the sterile neutrino interpretation of the KARMEN anomaly were to be viable, one would have to expect rather small mixing of the $X$ particle with the known leptonic flavours, but still large enough to account for the KARMEN constraint \cite{15}. This suggests the following approximation, namely that the admixture of the sterile neutrino flavour $a = 0$ into the known massive neutrino states is dominant compared to the mixing of the usual flavours $\alpha = e, \mu, \tau$ into the same states. Correspondingly, such an isodoublet diagonal mixing approximation amounts to the ansatz,

$$U_{at} = \begin{pmatrix}
\sqrt{1-x_e} & 0 & 0 & U_{eX} \\
0 & \sqrt{1-x_\mu} & 0 & U_{\mu X} \\
0 & 0 & \sqrt{1-x_\tau} & U_{\tau X} \\
-U_{eX}^* & -U_{\mu X}^* & -U_{\tau X}^* & \sqrt{1-x_e-x_\mu-x_\tau}
\end{pmatrix},$$

where

$$x_e = |U_{eX}|^2, \quad x_\mu = |U_{\mu X}|^2, \quad x_\tau = |U_{\tau X}|^2.$$

Even though such a mixing matrix is not unitary, leptonic flavour unitarity is violated only in the off-diagonal entries of the matrices $UU\dagger$ and $U\dagger U$, to order $U_{\alpha X}U_{\beta X}^*$ ($\alpha, \beta = e, \mu, \tau; \alpha \neq \beta$) for the $(3 \times 3)$ submatrices of the first three lines and columns of these
matrices, and to order $U_{\alpha X} U_{\beta X}^\dagger (\alpha, \beta = e, \mu, \tau; \alpha \neq \beta)$ for the last line and column of these matrices. The diagonal entries of $UU^\dagger$ and $U^\dagger U$ are all equal to unity. In particular, these properties of (22) imply that the identity (3) is violated by terms of order $x_{\alpha} x_{\beta} (\alpha, \beta = e, \mu, \tau; \alpha \neq \beta)$ only.

As a matter of fact, the choice in (22) is the closest one may come to a unitary mixing matrix given the isodoublet diagonal mixing approximation. In particular for $U_{eX} = 0$ and $U_{\mu X} = 0$, the ansatz (22) indeed defines a unitary matrix mixing the $a = \tau$ and $a = 0$ neutrino flavours only. Since the present experimental upper bounds on $|U_{eX}|^2$ and $Br$ imply values for $x_\tau$ and $x_\mu$ less than $10^{-6}$, the choice in (22) is thus in fact rather close to being unitary.

This is to be contrasted with the weak mixing approximation implicitly assumed in Ref.[3], corresponding in fact to the matrix (22) linearised to first order in $U_{\alpha X} (\alpha = e, \mu, \tau)$, namely,

$$U_{aI} \simeq \begin{pmatrix} 1 & 0 & 0 & U_{eX} \\ 0 & 1 & 0 & U_{\mu X} \\ 0 & 0 & 1 & U_{\tau X} \\ -U_{eX}^* & -U_{\mu X}^* & -U_{\tau X}^* & 1 \end{pmatrix} .$$

(24)

Clearly, since the sum of the modulus squared entries for each line and column is already larger than unity, such a choice is not unitary, nor can it be a submatrix of a still larger unitary matrix. Therefore, the weak mixing approximation (24) may be physically meaningful only when all three mixing amplitudes $U_{\alpha X} (\alpha = e, \mu, \tau)$ are sufficiently small\(^2\). In particular, it is only under this assumption that the constraint (11) does apply. In contradistinction to (22), the closer $x_\tau$ is to unity, the larger is the violation of leptonic unitarity in (24), even when $U_{eX} = 0$ and $U_{\mu X} = 0$. Compared to (22), (24) violates leptonic unitarity in the products $UU^\dagger$ and $U^\dagger U$ to order $U_{\alpha X} U_{\beta X}^* (\alpha, \beta = e, \mu, \tau)$, and the identity (3) as well to order $x_{\alpha} (\alpha = e, \mu, \tau)$.

Given the isodoublet diagonal mixing ansatz (22), the KARMEN result (15) reduces to the constraint,

$$K \frac{0.0293 x_\mu}{(1 - x_\mu)} + 0.0293 x_\mu \left[ 929 x_e (1 - x_e) + 208 x_\mu (1 - x_\mu) + 208 x_\tau (1 - x_\tau) \right] = 3 \cdot 10^{-11} .$$

(25)

Note that this relation does indeed reduce\(^3\) to (11) provided all three quantities $x_{\alpha} (\alpha = e, \mu, \tau)$ are sufficiently small.

On the other hand, the recent experimental upper bound (17) implies the limit,

$$x_\mu < 8.9 \cdot 10^{-7} \text{ (95\% C.L.)} .$$

(26)

\(^2\)As will appear later on, this is indeed the case for the parameters $x_\tau$ and $x_\mu$, but not necessarily for the parameter $x_\mu$, which on account of the KARMEN constraint (15), approaches the value unity the smaller $x_\mu$ is constrained to be.

\(^3\)The numerics in this expression are somewhat different from those in (11), presumably due to a value for $M_X$ slightly different from the unquoted one used in Ref.[3].
Fig. 1 displays the contraint (25) for a $X$ particle of Dirac or Majorana character. The curves represent the pairs of values for $(\log_{10} x_\mu, \log_{10} x_\tau)$ related through (25) for two extreme values of $x_e$, namely $x_e = 0$ for the dashed curve and $x_e = 10^{-6}$ for the dotted-dashed curve, corresponding to the range (18) presently allowed by laboratory experiments [13]. Note that for $x_\mu \leq 3 \cdot 10^{-8}$, these two curves become essentially undistinguishable.

The vertical lines in Fig. 1 also indicate the 95% C.L. upper bound (26) of $\log_{10} x_\mu = -6.05$ inferred from Ref. [7], as well as the limit of $x_\mu < 3.4 \cdot 10^{-9}$, corresponding to $\log_{10} x_\mu = -8.47$, to be reached by a new experiment [10] which aims at constraining the branching ratio $Br$ down to a level of $10^{-10}$ or better. The associated values for $x_\tau$ are also indicated as dashed or dotted-dashed horizontal lines, corresponding to $x_e = 0$ or $x_e = 10^{-6}$, respectively. Note that in the $K = 2$ Majorana case, the present upper bound of $x_\mu < 8.9 \cdot 10^{-7}$ implies that values for $x_e$ as large as $10^{-6}$ are already excluded on the basis of the KARMEN constraint (25). For the soon to be expected [10] upper bound of $x_\mu = 3.4 \cdot 10^{-9}$, the value for $x_\tau$ is independent of $x_e < 10^{-6}$. Note that these values for $x_\tau$ are lower bounds on that parameter, implied by the KARMEN constraint (25). The canonical upper bound on $x_\tau$ is of course the value unity, which in turn leads to the lower bound on $x_\mu$ of $2.0 \cdot 10^{-11}$ in the $K = 1$ Dirac case, or $9.8 \cdot 10^{-12}$ in the $K = 2$ Majorana case. Note that the existence of such non vanishing lower bounds on the mixing parameter $x_\tau$ also explains at post-eriori why the minimal mixing scenario of the previous section—in which $U_{\tau X} = 0$—is inconsistent with the presently most stringent upper bound (17) on $Br$.

From Fig. 1, one may thus conclude that, assuming the sterile neutrino interpretation [1, 3] of the KARMEN anomaly to be realised,

i) given the forthcoming measurements of Ref. [10], there is no interest, from the present point of view, in improving the upper limit on $x_e \lesssim 10^{-6}$ of Ref. [13];

ii) at present, the $x_\mu$ mixing parameter must be less than $8.9 \cdot 10^{-7}$ and larger than $2.0 \cdot 10^{-11}$ in the Dirac case or $9.8 \cdot 10^{-12}$ in the Majorana case;

ii) unless a positive signal is found, even the forthcoming measurements [10] will not be able to exclude the sterile neutrino scenario with the mixing amplitude squared $x_\mu$ in the range from $3.4 \cdot 10^{-9}$ to $2.0 \cdot 10^{-11}$ in the $K = 1$ Dirac case or $9.8 \cdot 10^{-12}$ in the $K = 2$ Majorana case, or correspondingly, values for $x_\tau$ between unity and $1.45 \cdot 10^{-3}$ in the Dirac case, or $7.24 \cdot 10^{-4}$ in the Majorana case. The latter conclusions are independent of the value for the mixing parameter $x_e < 10^{-6}$.

In this scenario, the $X$ particle of 33.9 MeV/c$^2$ would be emitted in pion decay due to its admixture into the muonic flavour, and would decay inside the detector essentially due to its admixture into the $\tau$ flavour via the neutral current coupling to an electron-positron pair. At present, it seems difficult to push further such limits on this interpretation of the KARMEN anomaly through laboratory experiments.
The $Z_0$ Invisible Decay Width and $\tau$ Decays. In order to reach more definite conclusions with regards to the sterile neutrino scenario of Ref.\[3\], it seems worthwhile to explore additional experimental laboratory constraints.

A first possibility which may come to mind is that of the invisible $Z_0$ decay width. In fact, since the isodoublet diagonal mixing ansatz (22) breaks leptonic unitarity, and consequently violates the identity (3) to order $x_\alpha x_\beta$ ($\alpha, \beta = e, \mu, \tau; \alpha \neq \beta$), a deviation from the Standard Model value for the $Z_0$ invisible width should involve a combination of these products of mixing parameters. Indeed, using the experimental values\[11\],

$$\Gamma_{\text{inv}}(Z_0) = 498.3 \pm 4.2 \text{ MeV}, \quad M_{Z} = 91.187 \pm 0.007 \text{ GeV}/c^2,$$

the result (10) implies the constraint,

$$x_e x_\mu + x_e x_\tau + x_\mu x_\tau = 0.0017 \pm 0.013.$$  \hspace{1cm} (28)

Quite clearly, given the present or even the forthcoming experimental upper bounds on the parameters $x_e$ and $x_\mu$ of $10^{-6}$ or less, even if the $\tau$ mixing parameter $x_\tau$ were to reach its maximal value of unity, this constraint is satisfied to many orders of magnitude.

In contradistinction, since the weak mixing approximation (24) violates the identity (3) to order $x_\alpha$ ($\alpha = e, \mu, \tau$), the $Z_0$ invisible decay width constraint reads in that case,

$$x_e + x_\mu + x_\tau = 0.0017 \pm 0.013.$$  \hspace{1cm} (29)

Taken at face value, this condition would imply the additional upper bound on $x_\tau$ of $x_\tau < 0.013$, or $\log_{10} x_\tau = -1.89$, thereby narrowing down further the window which is still open in the $(x_\mu, x_\tau)$ parameter space from the discussion of the previous section, even when the forthcoming limit of Ref.\[10\] is accounted for. However, for the reasons explained previously, the weak mixing ansatz (24) is not physically acceptable, and the constraint (29) can therefore not be included in any analysis.

Since the parameter $x_\tau$ is not constrained in the analysis of the previous section, another possibility which may be contemplated is that of $\tau$ decays. However, a simple consideration of the muonic and electronic branching ratios\[11\] within the isodoublet diagonal mixing approximation implies that the ratio of these two branching ratios, when properly accounting for leptonic mass contributions, should reduce to unity, whereas the experimental data\[11\] for the likewise mass corrected ratio lead to the result $1.009 \pm 0.017$. Hence, no additional constrained is to be derived from these leptonic branching ratios. Incidentally, within the weak mixing approximation, the same ratio provides for the constraint $(1 + x_\mu/1 + x_\tau) = 1.009 \pm 0.017$, which clearly, is easily accommodated given the present limits of $x_\mu < 8.9 \cdot 10^{-7}$ (95% C.L.) and $x_e \lesssim 10^{-6}$, independently of the fact that the weak mixing ansatz (24) is physically unsatisfactory.

It is conceivable nevertheless that $\tau$ decays may provide for a constraint on the mixing parameter $x_\tau$. Indeed, in Ref.\[3\], it is argued that when considering the possibility of mixing in the $\tau$ leptonic sector, the ALEPH upper bound\[4\] of 24 MeV/$c^2$ on the mass of the $\nu_\tau$ neutrino has to be reconsidered. However, within the isodoublet diagonal mixing ansatz (22) and when the parameter $x_\tau$ is very close to unity, a situation
still allowed on basis of the analysis of the previous section, it is mostly the \( X \) particle 
\[ 33.9 \text{ MeV/c}^2 \] which couples to the \( \tau \to 5\pi(\pi^0)\nu \) decay mode used by ALEPH, rather 
than the \( \nu_3 \) mass eigenstate. From the point of view of the kinematical analysis leading 
to the ALEPH mass limit, whether it is the \( X \) particle or the \( \nu_3 \) neutrino does not 
make a difference, so that the ALEPH result should in fact lead to an upper bound on 
\( x_{\tau} \) somewhat less that unity. However, the specific value for this upper bound requires 
a detailed analysis of the ALEPH hadronic spectra which would have to properly account 
for a mixing parameter \( x_{\tau} \) neither very close to unity, nor very small, along lines 
similar to those advocated in Ref.[6]. On the basis of the statistics accumulated[5], it 
seems reasonable to expect that such a reassessment—which is thus called for—of the 
ALEPH result, would be able to reach an upper bound on the order of \( 10^{-2} \) or better 
on \( x_{\tau} \), namely \( \log_{10} x_{\tau} = -2 \), thereby reducing further the window still open in the 
\((x_\mu, x_{\tau})\) parameter space.

As a matter of fact, if such a detailed reanalysis of the ALEPH data were to 
be completed, and if the sensitivity of the forthcoming experiment[10] aiming for an 
upper limit of \( 10^{-10} \) on \( Br \) were to be improved still to some extent, it may even 
become possible to confirm, or either refute definitely the interpretation of the KAR-
MEN anomaly as being due to a massive isosinglet neutrino mixing with the ordinary 
isodoublet neutrino flavours.

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Fig. 1: The parameter space for muon and tau leptonic flavour mixing with a sterile neutrino in the case of a Dirac (bottom) and of a Majorana (top) X particle. The dashed and dotted-dashed curves represent the KARMEN constraint (25) for $|U_{eX}|^2 = 0$ and $|U_{eX}|^2 = 10^{-6}$, respectively. The vertical and horizontal lines are discussed in the text; they represent different upper or lower bounds on the mixing parameters, with the shaded regions being the excluded ones.