ABSTRACT Device-to-Device (D2D) communications underlaying cellular networks have emerged as a necessity for a substantial increase in the system throughput and the number of active devices for the future cellular networks. In underlay D2D networks, it is conventional to use different interference management (IM) techniques to allow D2D transmitters to reuse the cellular users’ subcarriers. Conventionally, those IM techniques pair a specific number (one or more) of D2D transmitters to each subcarrier and/or allow each D2D transmitter to transmit on a specific number of subcarriers simultaneously in order to achieve the target rates. Due to the mixed-integer nature of those IM techniques, convex optimization techniques cannot be used, and usually complex heuristic or game-theoretic approaches are exploited. In this paper, we introduce a reduced-constraints approach to seek sub-optimal joint power allocation and channel assignment solutions for two non-convex, mixed-integer, and non-linear programs (MINLP). Specifically, via the reduced-constraints approach and variable transformation techniques, we can exploit primal-dual algorithms to solve system power minimization and energy-efficiency maximization problems. Extensive numerical simulation results show that the proposed approach outperforms state-of-the-art techniques.

INDEX TERMS Device-to-Device, underlay cellular networks, multi-pair D2D, resources allocation, power minimization, energy efficiency, primal-dual algorithm, reduced-constraints.

I. INTRODUCTION

Recently, Device-to-Device (D2D) communication arose as a future technology to improve utilization and enhance spectral efficiency needed for a fully connected world. In D2D communication, devices in close proximity exchange information with each other directly rather than via the evolved Node Base (eNB), while the eNB takes control of resources allocation in order to efficiently manage interference. As a means for direct communication between devices in proximity underlaying cellular networks, D2D has emerged as a key technology with promises to improve utilization, energy efficiency (EE), and spectrum efficiency. Moreover, D2D communication could support some interesting features such as low power, short-range, long battery life, and massive connectivity for enabling Internet-of-Things (IoT) deployment underlaying cellular network [1], [2]. Additionally, D2D communication provides several benefits for the core network including load balancing, traffic offloading, better coverage for edge users, and satisfying the quality of service (QoS) requirements [3], [4]. The Long-Term Evolution (LTE) network standard has defined two D2D communication schemes. In the first one, D2D nodes reuse the available resources of the cellular users’ equipment (UE) in a non-orthogonal manner, which is known as the reuse or underlay mode, while in the second scheme, the available resources are divided between UEs and D2D nodes, which is known as dedicated or overlay mode [5]. Consequently, employing the non-orthogonal sharing concept in the underlay D2D scheme enhances the users’ connectivity and the network throughput by improving the spectrum...
efficiency of the network. The efficient utilization of the network resources can be achieved by careful resources allocation and interference management to guarantee successful decoding at the receivers [6]–[8]. In this paper, we focus on resources allocation for underlay D2D communication in the cellular network by concentrating on improving the EE of the network.

Although the D2D concept provides promising benefits to the core network, it exhibits some technical challenges such as D2D peer discovery, mode selection, and interference management between cellular users and D2D users. Peer discovery has been considered in several studies including [9] and the references therein, while the mode selection has been heavily investigated in the literature [5]. On the other hand, interference management and resource allocation have been considered from different perspectives. Recently, several investigations have focused on improving the performance of D2D networks by assigning different resources, such as the channel assignment and power allocation, to maximize the throughput or minimize the power consumption under different power and QoS constraints [6], [10]. To cope with the requirements of the green fifth-generation (5G) and beyond 5G (B5G) networks, EE is considered to be one of the most important metrics of the network. The maximization of EE has recently gained a lot of attention for different network architectures. In those schemes, the EE is targeted alone or as a joint objective with other performance metrics such as latency and sum-rate maximization. [11]–[15]. However, the EE maximization problem is a non-convex problem [11], in particular, fractional programming is used to solve this problem.

In this paper, we consider an underlay D2D communication model at which multi-pair of D2D coexists in an underlay scheme with a group of cellular users equipment (UEs) in the uplink scenario. Both D2D nodes and UEs are assumed to be uniformly deployed within the cell. Each cellular user uses only one resource block (RB), while each D2D transmitter is allowed to reuse the RB of any UE for the uplink purpose provided that all reuse partners can achieve their pre-defined QoS requirements. Generally, resource reuse can be done for the uplink as well as the downlink, which might be quite similar, with careful consideration of some basic differences. Those differences include (1) The interference from the eNB in the downlink is much higher than the interference from UE in the uplink, which limits the channel sharing availability. (2) The control is centralized at the eNB, which has higher processing capabilities than the UE, thus, allows better interference management. (3) The traffic load in the uplink is less than in the downlink, which gives higher spectrum efficiency when sharing the uplink resources rather than the downlink. Consequently, although utilizing the downlink is more challenging, it is a promising approach that needs further research. However, uplink sharing is still dominant in D2D communication and has been used in several recent research as in [16]–[18].

Usually, the formulations of interference management and resource allocation problems in D2D networks involve binary channel assignment parameters that lead to non-convex and mixed-integer, non-linear programs (MINLP). Additionally, several of those problems are non-deterministic polynomial-time hard (NP-hard) problems [1], [4], [19]. The objective of such a problem in the literature is to jointly assign subcarriers and powers to cellular and D2D users in order to optimize certain system metric subject to satisfying QoS Constraints. It is noteworthy that this type of problem is usually accompanied by some constraints on the maximum number of D2D pairs that can reuse the same subcarrier and the maximum number of subcarriers that can be reused by any D2D pair. Many research in the literature assumes that both constraints are set to unity to reduce the complexity of the assignment problem solution but on the expenses of losing possible improvement of the spectral efficiency and energy efficiency [1], [4], [6], [17], [20]–[22], while other techniques set certain values \( (w, v) \), which complicates the solution to get better performance by using heuristic [23], [24] and game-theoretic approaches [25]. One common approach to get a sub-optimal solution is to decompose the original problem into two sub-optimal problems for power allocation and channel assignment [6], [20], [21], [23].

Unlike the existing research [16]–[18], [24], [25], this paper investigates a generalized model that consider the performance of both UEs and D2D pairs as well. For the sake of making the best use of each RB and fully exploit the available degrees of freedom of D2D network, we introduce the idea of reduced-constraints optimization (RCO). In RCO, we reduce constraints that limit the number of D2D reuse partners on all RBs. In other words, we assume that all D2D transmitters are allowed to reuse all available RBs simultaneously, while all RBs can be reused by all D2D transmitters too under QoS constraints. Then, a variable transformation technique is applied to the reduced problem to get a convex form that can be solved for optimal power allocation using the Lagrangian function and the duality theory, which are typical in the field of optimization [22], [26], [27]. By solving the convex reduced problem, we get a sub-optimal subcarrier and power assignment for the original problem. The proposed technique leads to a simpler and more robust solution rather than the existing heuristic algorithms. The major contributions in this paper can be summarized as follows:

- Propose the reduced-constraints optimization technique.
- We applied RCO for solving two interesting optimization problems, \(^1\) namely, the power minimization and the EE maximization.
- A joint resource allocation scheme is proposed to achieve the minimum total power consumption. The proposed problem is characterized by a non-convex MINLP nature, which is solved by applying RCO and

\(^1\) For the sake of efficient resources utilization, we extend our work in [28], which investigated the throughput maximization and power minimization of the multi-pair D2D network, by concentrating on the EE perspective.
a variable transformation technique. The new problem is converted into a convex optimization problem that has a primal-dual optimal solution. Thus, we apply the duality theory and the Lagrangian function to propose a primal-dual algorithm to find the optimal power allocation that minimizes the total power consumption.

- A joint resources allocation problem for EE maximization, which again is a non-convex problem, is proposed. Then, we applied the RCO and Dinkelbach method to find the optimal EE. We propose a non-linear fractional programming algorithm based on primal-dual update equations for finding the optimal solution.

- The simulations results show that the proposed algorithms provide substantial power minimization and EE maximization compared to previous state-of-the-art algorithms in [6], [20], [21], [23], [24], [28].

The rest of the paper is organized as follows. In Section III, the overall system model is illustrated. Then, the power minimization and EE maximization problems are formulated and the proposed algorithms are introduced in Section IV and Section V, respectively. Moreover, simulation and discussion are presented in Section VI. Finally, the paper is concluded in Section VII.

Notations: Uppercase and lowercase boldface letters denote matrices and vectors, respectively. Moreover, \( A \succeq 0 \) stands for positive semi-definite matrix. The Gaussian distribution with mean \( \mu \) and variance \( \sigma^2 \) is denoted by \( \mathcal{N}(\mu, \sigma^2) \). The symbol \( [\cdot]^+ \) denotes max \((0, \cdot)\).

II. LITERATURE REVIEW
In the following, we shed the light on the research efforts in the literature that investigated resources sharing in D2D networks from different perspectives, various performance metrics, and different applications [1], [4], [6], [10]–[13], [19]–[21], [23], [29]–[32]. A single cell with one UE and one D2D pair is considered in [10], where the eNB performs optimal mode selection to maximize the sum-throughput under power and rate constraints. In particular, the eNB optimizes the transmission by selecting the proper mode, i.e. dedicated or underlay mode. In [6], a joint RB scheduling and power control problem is proposed to achieve high spectrum efficiency and network throughput. Then, the original problem is decomposed into two problems and then the problem is solved iteratively. Besides, a heuristic algorithm based on a column generation method is used to achieve near-optimal solutions efficiently. The sum-rate maximization problem is investigated in [29] for simultaneous spectrum access of machine-to-machine (M2M) and human-to-human (H2H) communications in uplink multi-pair underlaying cellular communication. In [20], a three-step scheme is proposed to solve resources allocation problem for maximizing the cell throughput while guaranteeing quality-of-service (QoS) requirements. Based on the minimum distance metric, D2D pairs are admitted. Then, powers are allocated for the admitted D2D pair and UE partners. Finally, a maximum weight bipartite matching is utilized to select a UE partner for each D2D pair to maximize the overall network throughput. The authors in [21] introduced a two-step low complexity matching algorithm for D2D pairing based on a “mutual selection” principle to maximize the cell sum throughput. A two-step resource sharing algorithm is proposed in [23] with adapted computational complexity according to the network conditions. Their proposed algorithm relies on Lagrangian dual decomposition, which is similarly adopted in this paper, to get the optimal power allocation that maximizes the network sum throughput.

A lot of research had been carried out in the domain of EE of D2D networks. The authors in [16] have adopted an EE maximization approach for a cooperative D2D network where each D2D pair can reuse only one RB and each RB can be reused only by one D2D pair. In [17], the authors investigated energy-efficient resource reuse strategies for downlink D2D communication underlaying cellular networks. They proposed an iterative algorithm, based on non-linear fractional programming and utilizing Karush-Kuhn-Tucker (KKT) conditions, to maximize the total EE of all the D2D links by joint power control and D2D-UE matching scheme. However, the allocated power to UEs is fixed and each RB can be reused only by one D2D pair, which reduces the overall cell energy efficiency. In [24], the authors have proposed a heuristic approach to iteratively optimize the uplink RBs assignment and power allocation in D2D underlaying cellular networks. Specifically, they divided the problem into two sub-problems. First, they allocate equal power for all RBs, and then they propose a many-to-many heuristic matching algorithm to assign RBs, which increases the complexity of the proposed algorithm. Then, they exploit the difference between concave functions to allocate power to all RBs.

The authors in [11] investigated different resources allocation schemes for EE maximization in various 5G wireless networks. In [12], the energy efficiency of a single cell D2D-cellular converged network is studied and the optimal D2D distance to maximize EE for video delivery is addressed. However, D2D overlay mode is assumed therein. Moreover, the effect of different network parameters, such as users’ density, eNBs’ density, and requested data rates on D2D communication in a multi-cell network is quantified in [13]. Then, an EE model is developed, where the EE is evaluated in both underlay and overlay modes with various offloading radius. In [33], the authors used a penalty function to eliminate part of constraints iteratively. Then, a two-layer approach is proposed, where optimal power and channel assignment are allocated iteratively. A generalized model is proposed in [34], where multi-cell and multiple bands are considered under a stochastic geometry framework. On the other hand, several game theory-based approaches have been proposed for EE resource allocation in D2D networks [18], [25]. In [18], a non-cooperative game theory-based resource allocation approach is used at which each D2D link selfishly adjusts its power and subcarrier allocation to maximize its own EE assuming a fixed resource allocation of other links. However, this approach certainly is not spectral-efficient.
Moreover, in [25] another coalition game is used to address the joint mode selection and resource allocation for both D2D and cellular links. Nevertheless, the authors assumed that D2D links only achieve their minimum required rates, which might not fully improve network performance.

III. SYSTEM MODEL

In this paper, the focus is on a single-cell uplink scenario, as shown in Fig. 1, where the eNB is surrounded by $M$ uniformly-distributed UEs within a cell of radius $r_{\text{max}}$ and $N$ uniformly-distributed D2D pairs assuming the maximum distance between each D2D pair is $r_{\text{max}}$. Each D2D pair is allowed to reuse any channel\(^2\) from those that are originally occupied by UEs forming a reuse partner for the uplink transmission, while each UE can use only one channel with interference among UEs provided that all reuse partners can achieve minimum pre-defined QoS requirements. Meanwhile, all nodes are equipped with a single antenna. Without loss of generality, the cell is assumed to be fully loaded, similar to [20], [23], which means that all UEs are active, and hence, all channels are used, i.e., the $m^{th}$ channel is assigned to the $m^{th}$ UE.

![Figure 1. System model: Multi-pair of D2D devices are reusing the uplink cellular resources of $M$ UEs.](image)

We consider a distance based path-loss model defined as $P_{\mu} = P_{\eta} g_{\eta,\mu} L d_{\eta,\mu}^{-\alpha}$, where $P_{\eta}$ and $P_{\mu}$ are the transmitted and the received powers, respectively, $g_{\eta,\mu}$ and $d_{\eta,\mu}$ are the Rayleigh fading channel power gain and the distance between node $\eta$ and node $\mu$, $\alpha$ is the path-loss exponent, and $L$ is the path-loss constant. Let $h_{\eta,\mu} = g_{\eta,\mu} L d_{\eta,\mu}^{-\alpha}$ be the channel power gain between node $\eta$ and node $\mu$. Further, denote by $h_{m}^{2b}$, $h_{m,n}^{2b}$, $h_{n,m}^{2b}$, and $h_{n,k,m}^{2d}$ the channel power gains between the $m^{th}$ UE and the eNB, the $n^{th}$ UE and the eNB, the $n^{th}$ D2D transmitter and the eNB, and the $n^{th}$ D2D transmitter and the $k^{th}$ D2D receiver in the $m^{th}$ channel, respectively.

Further, under the assumption that the channel state information (CSI) is available at the eNB, it is possible to assign different channels and allocate power to D2D pairs.

Let $x_{m}^{d}$ and $x_{n,m}^{d}$ be the transmitted signals of the $m^{th}$ UE and the $n^{th}$ D2D transmitter in the $m^{th}$ channel, assuming a normalized power such that $\mathbb{E}[|x_{m}^{d}|^2] = 1$ and $\mathbb{E}[|x_{n,m}^{d}|^2] = 1$.

The received signals at the eNB and the $k^{th}$ D2D receiver in the $m^{th}$ channel are given as follows:

$$
y_{m}^{d} = \sqrt{P_{m} h_{m}^{2b} x_{m}^{d}} + \sum_{n=1}^{N} a_{n,m} \sqrt{P_{n,m} h_{n,m}^{2b} x_{n,m}^{d}} + n_{m}^{b}, \quad (1)
$$

$$
y_{k,m}^{d} = \sqrt{P_{m} h_{m,k}^{2d} x_{m}^{d}} + \sum_{n=1}^{N} a_{n,m} \sqrt{P_{n,m} h_{n,k,m}^{2d} x_{n,m}^{d}} + n_{k,m}^{d}, \quad (2)
$$

where $P_{m}$ and $P_{n,m}$ are the transmitted power of the $m^{th}$ UE and the $n^{th}$ D2D transmitter in the $m^{th}$ channel, respectively, $n_{m}^{b}$ and $n_{k,m}^{d}$ are the additive white Gaussian noise (AWGNN) at the eNB and the $k^{th}$ D2D receiver in the $m^{th}$ channel, respectively, with a distribution $\mathcal{N}(0, N_{0})$, where $N_{0}$ is the noise power. The channel assignment coefficient is defined as follows:

$$
a_{n,m} \triangleq \begin{cases} 1, & \text{the } n^{th} \text{ D2D pair uses the } m^{th} \text{ channel}, \\ 0, & \text{otherwise}. \end{cases} \quad (3)
$$

Hence, the sum throughput is given by:

$$
T(A, P) = \sum_{m=1}^{M} \log_{2} \left(1 + \frac{P_{m} h_{m}^{2b}}{N_{0} + \sum_{n=1}^{N} a_{n,m} P_{n,m} h_{n,m}^{2b}} \right),
$$

where

$$
y_{m}^{a} = \frac{P_{m} h_{m}^{2b}}{N_{0} + \sum_{n=1}^{N} a_{n,m} P_{n,m} h_{n,m}^{2b}}, \quad (5)
$$

and

$$
y_{n,m}^{d} = \frac{P_{m} h_{n,k,m}^{2d}}{N_{0} + P_{n,m} h_{n,m}^{2d} + \sum_{k \neq n}^{N} a_{k,m} P_{k,m} h_{k,m}^{2d}}, \quad (6)
$$

are the signal-to-interference-and-noise-ratio (SINR) at the eNB and the $m^{th}$ D2D receiver in the $m^{th}$ channel, respectively.

The channel assignment matrix is $A \in \{0, 1\}^{N \times M}$, and $P = [p^{a}, p^{d}]$, $p^{a}$ and $p^{d}$ are the UE and the D2D transmit powers. It is worth noting that in [28], we proposed a primal-dual algorithm to maximize the sum-throughput.

IV. POWER MINIMIZATION PROBLEM

Resources allocation in such an underlay cellular network is a crucial process in optimizing the performance of the network. Therefore, suitable uplink resources allocation needs to be controlled by the eNB to manage the interference between

\(^2\)The assumption that each D2D pairs can share only one channel at maximum is to enforce fairness between D2D pairs or in some cases it is a hardware constraint.
UEs and D2D pairs and between D2D pairs themselves to achieve the pre-defined QoS constraints. In the following sub-sections, the total power optimization problem is formulated, then we apply the reduced-constraint transformation, followed by finding the optimal solution of the reduced problem using the Lagrange multipliers and duality theorems. In addition, a primal-dual algorithm is proposed to find the optimal primal-dual parameters.

The objective of the proposed joint channel and power allocation problem is to find the optimal transmit powers, \( P^* \), and the optimal assignment matrix, \( A^* \), that minimize the total cell transmit power and achieve the QoS constraints for all nodes. The constrained problem can be formulated as follows:

\[
\text{minimize}_{A, P \geq 0} \sum_{m=1}^{M} P_m + \sum_{m=1}^{M} \sum_{n=1}^{N} P_{n,m}
\]

subject to

\[
C_1 : \log_2 \left( 1 + \gamma^u_{m,n} \right) \geq R^u_m, \quad \forall m \in \mathcal{M},
\]

\[
C_2 : \sum_{m=1}^{M} a_{n,m} \log_2 \left( 1 + \gamma^d_{n,a,m} \right) \geq R^d_n, \quad \forall n \in \mathcal{N},
\]

\[
C_3 : P_m \leq P_{\text{max}}, \quad \forall m \in \mathcal{M},
\]

\[
C_4 : \sum_{n=1}^{N} P_{n,m} \leq P_{\text{max}}, \quad \forall n \in \mathcal{N},
\]

\[
C_5 : \sum_{n=1}^{N} a_{n,m} \leq w, \quad \forall m \in \mathcal{M},
\]

\[
C_6 : a_{n,m} \in \{0, 1\}, \quad \forall n \in \mathcal{N}, \quad \text{and} \quad \forall m \in \mathcal{M}, \quad (7)
\]

where \( R^u_m \) is the minimum required rate for the \( m^{th} \) UE and similarly, \( R^d_n \) is the minimum required aggregated rate for the \( n^{th} \) D2D pair. The minimum required rates are to guarantee fairness among UEs and D2D pairs, while \( P_{\text{max}} \) is the maximum transmit power for any UE or D2D transmitter. Constraints \( C_1 \) and \( C_2 \) ensure the minimum required rate of each UE and D2D pair. Meanwhile, constraints \( C_3 \) and \( C_4 \) limit the assigned powers for UEs and D2D pairs to the maximum allowable power level \( P_{\text{max}} \). Further, constraints \( C_5 \) and \( C_6 \) are usually used in the literature to choose the maximum number of D2D pairs that can reuse the same channel and the maximum number of channels that can be reused by any D2D pair to be \( w \) and \( v \), respectively. Although the objective function is linear, constraints make this problem a non-convex MINLP.

A. REDUCED-CONSTRAINTS OPTIMIZATION (RCO)

Instead of choosing the parameters \( w \) and \( v \) and using heuristic or game-theoretic approaches to solve (7), we propose another effective but simple approach for the total transmit power minimization. In this approach, we assume the same problem as (7), except that we reduce constraints \( C_5 \) and \( C_6 \) to all D2D pairs to reuse any available channel under the other QoS constraints. Exploiting this approach has manifold benefits such as 1) the problem is substantially simplified, 2) the problem can be easily transformed into a convex problem under a realistic assumption, 3) the duality theorem can be exploited to obtain a near-optimal solution, and 4) the cell capacity with D2D communication can be tightly reached. That is, the solution itself is systematic though it is not used before (because the original problem is a non-convex MINLP) to solve the power minimization problem or EE maximization problem in the next section for this model. In addition, the reduced problem leads to an analytic solution instead of the complex heuristic algorithms that exist in the literature. Additionally, in the following, we describe how to make the channel assignment based on the allocated power in each channel. Then, we relate the solution of the reduced problem to the solution of the original problem in (7).

B. CHANNEL ASSIGNMENT IN RCO

The channel assignment can be done by inspecting the optimal power allocation. The eNB assigns the \( m^{th} \) RB to the \( n^{th} \) D2D pair, i.e., \( a_{n,m} = 1 \), if any power level is allocated to the \( m^{th} \) RB of the \( n^{th} \) D2D pair, i.e., \( P_{n,m} > 0 \), otherwise, the \( m^{th} \) RB is not assigned to the \( n^{th} \) D2D pair, i.e., \( a_{n,m} = 0 \). This assignment is feasible since any D2D pair is allowed to reuse any number of RBs and any RB can be reused by any number of D2D pairs. Therefore, we can express our RCO-based transmit power minimization problem as follows:

\[
\text{minimize}_{P \geq 0} \quad P_T(P) = \sum_{m=1}^{M} P_m + \sum_{m=1}^{M} \sum_{n=1}^{N} P_{n,m}
\]

subject to

\[
C_1, C_3, C_4
\]

\[
\bar{C}_2 : \sum_{m=1}^{M} \log_2 \left( 1 + \gamma^d_{n,a,m} \right) \geq R^d_n, \quad \forall n \in \mathcal{N}, \quad (8)
\]

By comparing (7) and (8), the channel assignment parameters in \( \bar{C}_2 \) in the previous equation are removed in constraint \( C_2 \). It is noteworthy and we highlight that the reduced/reformulated problem in (8) is not equivalent to the original problem in (7) and the solution for both problems is not the same, neither the optimum power allocation nor the total power consumption (and also the channel assignment is different). In other words, we are proposing another approach for achieving a sub-optimal power minimization via using RCO. Thus, \( C_5 \) and \( C_6 \) in (7) are not necessarily satisfied due to constraints reduction.

C. ON THE RELATION TO THE ORIGINAL PROBLEM

The problem in (8) is a reduced version from the problem in (7). In the following, we want to give insights into the solution of the reduced problem and its relation to the original problem. First, the problem in (7) is a general problem that is considered in the literature, where \( 1 \leq w \leq N \) and \( 1 \leq v \leq M \). In [18], [23]–[25], each D2D pair can reuse
function in Appendix A, which is given as follows:

\[
\mathcal{L}(\mathbf{P}, \mathbf{\mu}, \lambda) = \sum_{m=1}^{M} (1 + \lambda_m^u) P_m + \sum_{m=1}^{M/N} (1 + \lambda_n^d) P_{n,m} - \sum_{m=1}^{M} \mu_m^u \log_2 (\gamma_m^u) + \sum_{m=1}^{M} \mu_m^d R_m^d - P_{\text{max}} \left( \sum_{m=1}^{M} \lambda_m^u + \sum_{n=1}^{N} \lambda_n^d \right) - \sum_{n=1}^{N} \sum_{m=1}^{M} \mu_n^d \log_2 (\gamma_{n,m}^d) + \sum_{n=1}^{N} \mu_n^d R_n^d. \tag{9}\]

Accordingly, the dual function is given by:

\[
\mathcal{D}(\mathbf{\mu}, \lambda) = \min_{P > 0} \mathcal{L}(\mathbf{P}, \mathbf{\mu}, \lambda). \tag{10}\]

The Lagrangian function \(\mathcal{L}(\mathbf{P}, \mathbf{\mu}, \lambda)\) is a convex function and we define the gradient of the Lagrangian function as:

\[
\nabla_{\mathbf{P}} \mathcal{L}(\mathbf{P}, \mathbf{\mu}, \lambda)|_{P=P^*} = 0. \tag{11}\]

Consequently, solving for \(\mathbf{P}^*\), the optimal power allocation for (8) is derived in Appendix B and is given by:

\[
P_m^* = \left[ \frac{\mu_m^u}{1 + \lambda_m^u + \sum_{n=1}^{N} \frac{\mu_n^d h_{n,k,m}^2}{N_0 + P_{n,m} h_{n,k,m}^2 + A_{n,m}}}, \right]^+, \tag{12}\]

\[
P_{n,m}^* = \left[ \frac{\mu_n^d}{1 + \lambda_n^d + \sum_{k=1}^{K} \frac{\mu_m^u h_{m,k,n}^2}{N_0 + P^{*}_{m,n} h_{m,k,n}^2 + B_{m,n}}}, \right]^+. \tag{13}\]

where

\[
A_{n,m} = \sum_{k=1}^{N} P_{k,m} h_{k,n,m}^2, \tag{14}\]

and

\[
B_{m,n} = \sum_{k=1}^{N} \frac{\mu_m^u h_{m,k,n}^2}{N_0 + P_{m,n} h_{m,k,n}^2 + \sum_{i=1}^{N} P_{i,m} h_{i,k,m}^2}. \tag{15}\]

Hence, the optimal dual solution is given by:

\[
\{\mathbf{\mu}^*, \lambda^*\} = \arg \max_{\mathbf{\mu}, \lambda \geq 0} \mathcal{D}(\mathbf{\mu}, \lambda) = \arg \max_{\mathbf{\mu}, \lambda \geq 0} \mathcal{L}(\mathbf{P}^*, \mathbf{\mu}, \lambda). \tag{16}\]

However, the closed-form solution of the set of Equations (12-16) is very difficult to attain. Therefore, a primal-dual algorithm is proposed, Algorithm 1, to find the optimal-dual pair solution. The update equations for the primal algorithm are given as follows:

\[
P_m[j+1] = \left[ \frac{\mu_m^u [j]}{1 + \lambda_m^u [j] + A_{m,n}[j]} \right]^+, \tag{17}\]

\[
P_{n,m}[j+1] = \left[ \frac{\mu_n^d [j]}{1 + \lambda_n^d [j] + B_{m,n}[j] + C_{m,n}[j]} \right]^+. \tag{18}\]
Algorithm 1

1: Initialize $P, \mu, \lambda, \epsilon, \Delta P$, and $j = 0$
2: while $\Delta P \geq \epsilon$
3: $j = j + 1$
4: Update the power allocation $P$ using (17) and (18)
5: Update the dual parameters $\lambda$ and $\mu$ using (22)-(25).
6: Update $\Delta P = P_T[j + 1] - P_T[j]$.
7: end
8: return $P^*_m = P_m[j + 1]$ and $P^*_{n,m} = P_{n,m}[j + 1]$
where

\[
P_{n,m}^* = \left[ \frac{1 + \mu_n^d}{\nu + \lambda_n^d + \frac{(1 + \mu_k^d)h_{n,k,m}^{2d}}{N_0 + \sum_{i=1}^{N} P_{n,m}^* h_{i,k,m}^{2d}} + A_{n,m}} \right]^+ ,
\]  
(32)

Hence, the optimal dual solution is given by:

\[
{\mu^*, \lambda^*} = \arg \min_{\mu, \lambda \geq 0} \mathcal{D}(\mu, \lambda) = \arg \min_{\mu, \lambda \geq 0} \mathcal{L}(P^*, \mu, \lambda).
\]  
(34)

A primal-dual algorithm is proposed to find the optimal dual pair solution. The update equations for the primal algorithm are given as follows:

\[
P_{m}[j + 1] = \left[ \frac{1 + \mu_m^u(j)}{\nu + \lambda_m^u(j) + A_{n,m}(j)} \right]^+ ,
\]

\[
P_{n,m}[j + 1] = \left[ \frac{1 + \mu_n^d(j)}{\nu + \lambda_n^d(j) + B_{n,m}(j) + C_{n,m}(j)} \right]^+ ,
\]  
(35)

where

\[
A_{n,m}(j) = \sum_{n=1}^{N} (1 + \mu_n^d(j))h_{n,m,n}^{2d} / N_0 + \sum_{k=1}^{N} P_{k,m}[j]h_{k,m,n}^{2d} + P_{m}[j]h_{m,n,m}^{2d} ,
\]

\[
B_{n,m}(j) = (1 + \mu_m^u(j))h_{m,n,m}^{2d} / N_0 + \sum_{n=1}^{N} P_{n,m}[j]h_{n,m,n}^{2d} ,
\]  
(36)

and

\[
C_{n,m}(j) = \sum_{k=1}^{N} (1 + \mu_k^d(j))h_{k,m,n}^{2d} / N_0 + P_{m}[j]h_{m,n,m}^{2d} + \sum_{n=1}^{N} P_{n,m}[j]h_{n,m,n}^{2d} ,
\]  
(38)

While the update equations for the dual algorithm are given as:

\[
\mu_n^u(j + 1) = \left[ \mu_n^u(j) + \alpha_n^u \left( R_n^u - \log_2 (1 + \gamma_n^u(j)) \right) \right]^+ ,
\]

\[
\mu_n^d(j + 1) = \left[ \mu_n^d(j) + \alpha_n^d \left( S_n^d - \log_2 (1 + \gamma_n^d(j)) \right) \right]^+ ,
\]

\[
\lambda_m^u(j + 1) = \left[ \lambda_m^u(j) + \beta_m^u \left( P_m[j] - P_{\text{max}} \right) \right]^+ ,
\]

\[
\lambda_n^u(j + 1) = \left[ \lambda_n^u(j) + \beta_n^u \left( \sum_{n=1}^{M} P_{n,m}[j] - P_{\text{max}} \right) \right]^+ ,
\]  
(39)

where \(v\) is given by Algorithm 2, similar to Algorithm 1 in [37], which iteratively solves for \(P^*\) using the primal-dual update equations and accordingly, updates \(v\) such that \(T(P^*) - v P_T(P^*) \leq \epsilon\) and \(\epsilon\) is the required accuracy, which also affects the convergence rate.

Remark 2: Similar to Remark 1, the minimum rate constraints should be in the rate achievable region to guarantee that the proposed primal-dual algorithm will converge. Moreover, when relaxing the minimum rate constrains, \(C_1\) and \(C_2\) in (26), the proposed EE maximization algorithm achieves the maximum sum-rate available under the EE maximization.

VI. PERFORMANCE ANALYSIS AND NUMERICAL RESULTS

The purpose of this section is to address that the proposed solutions achieve the required performance and give insights into different parameters that affect the performance. The simulation parameters are also adopted in [6], [20], [21], [23] for D2D models. The simulation parameters are listed in Table 1. Since we assumed that the cell is fully loaded, in Table 1, the number of UEs is \(M = 10\), which is the same number of channels. Note that the SINR for all UE and D2D pairs, \(S_n^u\) and \(S_n^d\), respectively, are chosen uniformly in the range \(0 : 10\) dB. Therefore, the minimum required rates are given by:

\[
R_n^u = \log_2 (1 + S_n^u) ,
\]

\[
R_n^d = \log_2 (1 + S_n^d) .
\]  
(40)

| Parameter      | Value        |
|----------------|--------------|
| \(M\)          | 10           |
| \(N\)          | 20           |
| UE SINR \(S_n^u\) | 0:10 dB      |
| D2D SINR \(S_n^d\) | 0:10 dB      |
| \(r_{\text{max}}\) | 500 m        |
| \(r_{\text{max}}^d\) | 20 m         |
| \(\eta\)       | 10^{-3}      |
| \(\epsilon\)   | 10^{-3}      |
| \(\alpha\)     | 4            |
| \(L\)          | 10^{-2}      |
| \(P_{\text{max}}\) | 24 dBm       |
| \(N_0\)        | -114, -174 dBm/Hz |
| RB             | 180 KHz      |

The performance of the proposed resources allocation algorithms is measured in terms of the total power consumption, energy efficiency, the total cell throughput gain, and the
access rate of D2D pairs. The cell throughput gain is the additional throughput that is achieved by D2D pairs when reusing the available RBs, while the access rate is the ratio between the number of the accessed D2D pairs to the number of UEs. Figs. 2-7 compare the performance of the proposed algorithms for different optimization objectives versus the energy-efficient D2D algorithm in [24]. The authors in [24] adopt the same model except that they do not assume a fully-loaded cell and they use the extra available RBs to be reused by D2D devices in addition to the used RBs. Further, we compare the performance of our proposed algorithms with the sum throughput maximization algorithms in [28] and in [23], which outperforms the algorithms in [6], [20], [21]. Interestingly, the proposed algorithms have a substantially enhanced performance when the noise power spectral density is decreased, which reflects the design criteria that is based on the high SINR assumption.
The proposed power minimization algorithm minimizes the total power consumption and outperforms all other algorithms at the low SNR, as shown in Fig. 2. Specifically, at an equal number of UEs and the accessed D2D pairs, the proposed power minimization algorithm achieves, approximately, 0.71 times the total power consumption of our proposed EE maximization algorithm and the throughput maximization algorithm in [28], and 0.4 times the total power consumption of the throughput maximization algorithm in [23]. However, its performance is limited by the noise, specifically, in the low SNR regime. Thus, minimizing the total power consumption is worthless in the low SNR regime when the number of the accessed D2D pairs is relatively larger than the number of UEs in the cell, approximately, the number of the accessed D2D pairs is 1.6 times the number of UEs. Besides, both proposed algorithms outperform the energy-efficient D2D algorithm in [24]. On the other hand, interestingly, the proposed EE maximization algorithm reduces the total power consumption at high SNR more than the proposed power minimization algorithm. Specifically, at an equal number of UEs and the accessed D2D pairs, the proposed EE maximization algorithm achieves, approximately, 0.66 times the total power consumption of our proposed power minimization algorithm, 0.42 times the total power consumption of the energy-efficient D2D algorithm in [24], 0.23 times the total power consumption of the throughput maximization algorithm in [28], and 0.18 times the total power consumption of the throughput maximization algorithm in [23].

In Fig. 3, the minimum rate constraints, $C_1$ and $C_2$ in (8) and (26), are removed to compare the total power consumption when the maximum sum-rate is achieved, see Remark 2. Without rate constraints, the proposed EE maximization algorithm outperforms other algorithms regardless of the SNR. Specifically, at low SNR and at an equal number of UEs and the accessed D2D pairs, the proposed EE maximization algorithm achieves, approximately, 0.48 times the total power consumption of the throughput maximization algorithms in [28] and in [23]. The power consumption curve (red dashed) of the proposed EE maximization algorithm goes down when increasing the number of D2D pairs. We suggest that this phenomenon results from the available diversity of D2D pairs and the channel conditions. Specifically, as the number of D2D pairs increases and the number of channels is fixed, the probability that there exists a D2D pair with better channel conditions increases. Thus, the total power consumption decreases. This phenomenon starts to appear when the number of D2D pairs is relatively larger than the number of RBs.

In Figs. 4 and 5, the proposed EE maximization algorithm outperforms other algorithms. Specifically, at high SNR and at an equal number of UEs and the accessed D2D pairs, the proposed EE maximization algorithm achieves, approximately, 2.75 times the EE of our proposed power minimization algorithm, 10 times the EE of the throughput maximization algorithm in [28], and 18.75 times the EE of the throughput maximization algorithm in [23]. It should be clear that the EE in Fig. 4 is normalized to 1 Watt and the typical allocated power is within mWatts and the maximum is 24 dBm, which corresponds to 250 mWatts. Besides, there are two different values for the noise power spectral density to differentiate between two regimes, the high and the low SNR. Thus, the high EE value is only an extreme case, and the noise power spectral density may differ in practice. It is worth noting that we did not plot the energy efficiency performance of the proposed EE maximization algorithm at high SNR in Fig. 5, i.e., in the case when there are no minimum rate constraints since it is approximately doubles and the plot of other algorithms will not be clear.

In Fig. 6, the throughput performance of the throughput maximization algorithms, and the proposed EE maximization algorithm outperform the proposed power minimization algorithm at low SNR since the power minimization algorithm is more sensitive to the interference at the low SNR regime. At an equal number of UEs and the accessed D2D pairs, the proposed power minimization algorithm achieves, approximately, 0.45 times the throughput gain of our proposed EE maximization algorithm, 0.42 times the throughput gain of the throughput maximization algorithm in [28], and 0.3 times the throughput gain of the throughput maximization algorithm in [28]. On the other hand, at high SNR, both the proposed power minimization algorithm and the throughput maximization algorithm in [28] outperform our proposed EE maximization algorithm and the throughput maximization algorithm in [23]. At an equal number of UEs and the accessed D2D pairs, the proposed power minimization algorithm and the throughput maximization algorithm in [28] achieve, approximately, 1.16 times the throughput gain of our proposed EE maximization algorithm, and 1.27 times the throughput gain of the throughput maximization algorithm in [23]. Without minimum rate constraints, the throughput maximization algorithms try to achieve the boundaries of the achievable sum-rate region and outperform the throughput maximization algorithm in [28].
performance of the proposed EE maximization algorithm regardless of the SNR.

Fig. 7 shows that minimizing the total power consumption reduces the number of accessed D2D pairs. The reason is that the power minimization tries to satisfy the minimum required rate and does not need to use more RBs, which reduces the number of accessed D2D pairs. Meanwhile, maximizing the throughput gives a priority to reusing more RBs, which gives a higher probability of increasing the accessed D2D pairs. This figure reveals that both the proposed EE maximization algorithm and the throughput maximization algorithm in [28] achieve approximately 100% access rate, while the proposed power minimization algorithm achieves approximately, 0.92 of the full access rate at an equal number of UEs and the accessed D2D pairs.

VII. CONCLUSION

In this paper, we investigated the problem of reducing power consumption and improving the energy efficiency of the multi-pair D2D communications underlaying cellular networks. The main focus is on exploiting the convex optimization techniques to derive the optimal power allocation and channel assignment coefficients to obtain the maximum achievable performance, in terms of the available energy efficiency and minimum transmit power, instead of the existing sub-optimal and heuristic algorithms. Specifically, we propose a primal-dual algorithm, where the objective is to minimize the total cell transmit power under the different quality of service constraints. Furthermore, a non-linear fractional programming approach is used to develop another primal-dual algorithm that maximizes energy efficiency. While these algorithms achieve the maximum available energy efficiency and minimum transmit power and are needed as a baseline for any comparison, they are limited by the complexity introduced as the number of UEs and D2D pairs increases. Simulations results show that efficient resource allocation can improve the spectrum utilization as well as the energy efficiency of the network. Thus, our proposed algorithms outperform state-of-the-art algorithms. At low SNR, the proposed power minimization algorithm achieves at most 0.71 times the total power consumption of other algorithms. On the other hand, the proposed EE maximization algorithm achieves at least 2.22 times the throughput gain of the proposed power minimization algorithm. At high SNR, the proposed power minimization algorithm achieves at least 1.16 times the throughput gain of other algorithms, while the proposed EE maximization algorithm achieves at most 0.66 times the total power consumption of other algorithms. Regardless of the SNR, the proposed EE maximization algorithm achieves at least 2.75 times the EE of other algorithms and the maximum access rate. As future research directions, the authors would like to investigate the problem of resources allocation for multi-cell D2D deployment as well as investigating further algorithms with reduced complexity for each objective.

APPENDIX A

THE LAGRANGIAN FUNCTION FOR THE POWER MINIMIZATION PROBLEM

We chose the Lagrange multipliers $\mu^u_m$ and $\mu^d_n$ for the rate constraints $C_1$ and $C_2$, respectively, and $\lambda^u_m$ and $\lambda^d_n$ for the power constraints $C_3$ and $C_4$, respectively. The Lagrangian function is represented in terms of the objective function and constraints multiplied by the associated Lagrange multiplier, i.e., a linear combination of the constraints. Since we assumed high SINR, the term $\log_2 (1 + SINR)$ can be approximated by $\log_2 (SINR)$. Thus, the Lagrangian function for the optimization problem in (8) is given as follows:

$$L(P, \mu, \lambda) = \sum_{m=1}^{M} P_m + \sum_{m=1}^{M} \sum_{n=1}^{N} P_{n,m}$$

$$+ \sum_{m=1}^{M} \mu^u_m (R^u_m - \log_2 (\gamma^u_{m}))$$

$$+ \sum_{n=1}^{N} \mu^d_n \left( R^d_n - \sum_{m=1}^{M} \log_2 (\gamma^d_{n,m}) \right)$$

$$+ \sum_{n=1}^{N} \lambda^u_m (P_m - P_{\text{max}})$$

$$+ \sum_{n=1}^{N} \lambda^d_n \left( \sum_{n=1}^{N} P_{n,m} - P_{\text{max}} \right)$$  (41)

Rearranging terms, the Lagrangian function for the optimization problem in (8) is given as follows:

$$L(P, \mu, \lambda) = \sum_{m=1}^{M} (1 + \mu^u_m)P_m + \sum_{m=1}^{M} \sum_{n=1}^{N} (1 + \lambda^d_n)P_{n,m}$$

$$- \sum_{m=1}^{M} \mu^u_m \log_2 (\gamma^u_{m}) + \sum_{m=1}^{M} \mu^u_m R^u_m$$

$$- P_{\text{max}} \left( \sum_{m=1}^{M} \lambda^u_m + \sum_{n=1}^{N} \lambda^d_n \right)$$

$$- \sum_{n=1}^{N} \sum_{m=1}^{M} \mu^d_n \log_2 (\gamma^d_{n,m}) + \sum_{n=1}^{N} \mu^d_n R^d_n.$$  (42)

APPENDIX B

OPTIMAL POWER ALLOCATION FOR THE POWER MINIMIZATION PROBLEM

To derive the optimal power allocation, $P_m$ and $P_{n,m}$, we calculate the derivative of the Lagrangian function with respect to $P_m$ and $P_{n,m}$. There are $(N+2)$ terms that are functions of $P_m$, which are $P_m, \gamma^u_m$, and $\gamma^d_{n,m}$ $\forall n$. First, we evaluate some terms as follows:

$$\frac{d}{dP_m} \log_2 (\gamma^u_m) = \frac{1}{P_m}.$$  (43)

and

$$\frac{d}{dP_m} \log_2 (\gamma^d_{n,m}) = \frac{-\mu^d_{n,m}}{N_0 + P_m h^d_{n,m} + \sum_{k=1}^{N} P_{k,m} h^d_{k,n,m}}.$$  (44)
Then, we calculate the derivative of the Lagrangian function with respect to $P_m$, as follows:

$$\frac{d \mathcal{L} (P, \mu, \lambda)}{dP_m} = 1 + \lambda_m^u - \frac{\mu_m^u}{P_m} + \sum_{n=1}^{N} \frac{\mu_n^d h_n^{2d} m,n}{N_0 + P_m h_n^{2d} m,n} + \sum_{k \neq n}^{N} \frac{\lambda_k^u h_k^{2d} m,n}{P_k m h_k^{2d} k,n,m}.$$  

Consequently, solving for $P_m^*$, the optimal power allocation for the $m^{th}$ UE is given by:

$$P_m^* = \left[ 1 + \lambda_m^u - \frac{\sum_{n=1}^{N} \mu_n^d h_n^{2d} m,n}{N_0 + P_m^* h_n^{2d} m,n} + A_{m,n} \right]^+, \quad (46)$$

where

$$A_{m,n} = \sum_{k=1}^{N} \frac{P_{n}^* h_{n,k}^{2d} m,n}{N_0 + P_m^* h_n^{2d} m,n}.$$  

Similarly, there are $(N + 2)$ terms that are functions of $P_{n,m}$, which are $P_{n,m}$, $\gamma_n^u$, and $\gamma_{n,m}^d \forall n$. First, we evaluate some terms as follows:

$$\frac{d}{dP_{n,m}} \log_2 \left( P_{n,m} \right) = -\frac{h_{n,m}^{2d}}{N_0 + \sum_{m=1}^{N} P_{n,m} h_{n,m}^{2d}}, \quad (48)$$

$$\frac{d}{dP_{n,m}} \log_2 \left( \gamma_{n,m}^d \right) = \frac{1}{P_{n,m}}, \quad (49)$$

and

$$\frac{d \log_2 \left( \gamma_{n,m}^d \right)}{dP_{n,m}} = -\frac{h_{n,k,m}^{2d}}{N_0 + P_m h_{n,k,m}^{2d} + \sum_{i=1}^{N} P_{i,m} h_{i,k,m}^{2d}}, \quad n \neq k. \quad (50)$$

Then, we calculate the derivative of the Lagrangian function with respect to $P_{n,m}$ as follows:

$$\frac{d \mathcal{L} (P, \mu, \lambda)}{dP_{n,m}} = 1 + \lambda_n^d + \frac{\mu_n^d h_n^{2d} m,n}{N_0 + \sum_{m=1}^{N} P_{n,m} h_{n,m}^{2d}} - \frac{\mu_n^d}{P_{n,m}} + \sum_{k=1}^{N} \frac{\mu_k^d h_k^{2d} m,n}{N_0 + P_m h_{n,k,m}^{2d} + \sum_{i=1}^{N} P_{i,m} h_{i,k,m}^{2d}}.$$  

Consequently, solving for $P_{n,m}^*$, the optimal power allocation for the $n^{th}$ D2D transmitter on the $m^{th}$ RB is given by:

$$P_{n,m}^* = \left[ 1 + \lambda_n^d + \frac{\mu_n^d h_n^{2d} m,n}{N_0 + \sum_{m=1}^{N} P_{n,m}^* h_{n,m}^{2d} m,n} + B_{m,n} \right]^+, \quad (52)$$

where

$$B_{m,n} = \sum_{k=1}^{N} \frac{\mu_k^d h_k^{2d} m,n}{N_0 + P_{n,m}^* h_{n,k,m}^{2d} m,n} + \sum_{i=1}^{N} \frac{P_{i,m}^* h_{i,k,m}^{2d}}{k \neq n}.$$  

**APPENDIX C**

**THE LAGRANGIAN FUNCTION FOR THE EE MAXIMIZATION PROBLEM**

Assuming high SINR, the Lagrangian function for the optimization problem in (27) is given as follows:

$$\mathcal{L} (P, \mu, \lambda) = \sum_{m=1}^{M} (1 + \mu_m^u) \log_2 \left( \gamma_{m}^u \right) - \sum_{m=1}^{M} (\lambda_m^u + \nu) P_m m,n + \sum_{n=1}^{N} \sum_{m=1}^{M} \log_2 \left( \gamma_{n,m}^d \right) - \nu \left( \sum_{m=1}^{M} P_m + \sum_{n=1}^{N} P_{n,m} \right) - \sum_{m=1}^{M} \mu_m^u (R_m^u - \log_2 \left( \gamma_{m}^u \right)) - \sum_{n=1}^{N} \mu_n^d \left( R_n^d - \sum_{m=1}^{M} \log_2 \left( \gamma_{n,m}^d \right) \right) - \sum_{m=1}^{M} \lambda_m^u (P_m - P_{\text{max}}) - \sum_{n=1}^{N} \lambda_n^d \left( \sum_{m=1}^{M} P_{n,m} - P_{\text{max}} \right). \quad (54)$$

Rearranging terms, the Lagrangian function is given as follows:

$$\mathcal{L} (P, \mu, \lambda) = \sum_{m=1}^{M} \left( 1 + \mu_m^u \right) \log_2 \left( \gamma_{m}^u \right) - \sum_{m=1}^{M} \left( \lambda_m^u + \nu \right) P_m m,n + \sum_{m=1}^{M} \left( \mu_m^u \right) R_m^u - \sum_{m=1}^{M} \mu_m^d \log_2 \left( \gamma_{n,m}^d \right) - \sum_{n=1}^{N} \mu_n^d \left( R_n^d - \sum_{m=1}^{M} \log_2 \left( \gamma_{n,m}^d \right) \right) - \sum_{m=1}^{M} \lambda_m^u \left( P_m - P_{\text{max}} \right) - \sum_{n=1}^{N} \lambda_n^d \left( \sum_{m=1}^{M} P_{n,m} - P_{\text{max}} \right). \quad (55)$$

**APPENDIX D**

**OPTIMAL POWER ALLOCATION FOR THE EE MAXIMIZATION PROBLEM**

Using (43)-(44), we calculate the derivative of the Lagrangian function with respect to $P_m$ as follows:

$$\frac{d \mathcal{L} (P, \mu, \lambda)}{dP_m} = \left( 1 + \mu_m^u \right) - (\lambda_m^u + \nu) - \frac{1}{P_m} + \frac{\mu_m^d h_m^{2d}}{N_0 + P_m h_m^{2d}} - \sum_{n=1}^{N} \frac{\mu_n^d h_n^{2d} m,n}{N_0 + P_m h_{n,m}^{2d} m,n}.$$  

$$- \sum_{n=1}^{N} \frac{P_{n,m}^* h_{n,m}^{2d} m,n}{N_0 + P_{n,m}^* h_{n,m}^{2d} m,n} + \sum_{k \neq n}^{N} \frac{1}{P_{n,m}^* h_{n,k,m}^{2d} m,n}.$$  

$$\frac{d \mathcal{L} (P, \mu, \lambda)}{dP_m} = \left( 1 + \mu_m^u \right) - (\lambda_m^u + \nu) - \sum_{n=1}^{N} \frac{\mu_n^d h_n^{2d} m,n}{N_0 + P_{n,m}^* h_{n,m}^{2d} m,n} + \sum_{k \neq n}^{N} \frac{P_{n,m}^* h_{n,k,m}^{2d} m,n}{N_0 + P_{n,m}^* h_{n,k,m}^{2d} m,n}. \quad (56)$$
Consequently, solving for $P_m^*$, the optimal power allocation for the $m^{th}$ UE is given by:

$$P_m^* = \left[ \frac{1 + \mu_m^d}{\sum_{n=1}^{N} N_0 + \sum_{k \neq n} \frac{P_{m,n,k,m}^{d2d}}{h_{n,k,m}} + \lambda_m^d + v} + \lambda_m^u + v \right]^{+}.$$  \hspace{1cm} (57)

Using (48)-(50), we calculate the derivative of the Lagrangian function with respect to $P_{n,m}$ as follows:

$$\frac{dL}{dP_{n,m}} = \left( 1 + \frac{\mu_n^d}{P_{n,m}} \right) - \left( \lambda_n^d + \frac{\mu_n^d}{P_{n,m}} \right) - \left( 1 + \frac{\mu_n^d}{P_{n,m}} \right) h_{n,k,m}^{d2d} =\hspace{1cm} (59)$$

Consequently, solving for $P_{n,m}^*$, the optimal power allocation for the $n^{th}$ D2T transmitter on the $m^{th}$ RB is given by:

$$P_{n,m}^* = \left[ \frac{1 + \frac{\mu_n^d}{P_{n,m}}}{\sum_{k \neq n} \frac{P_{n,m,k,m}^{d2d}}{h_{n,k,m}} + N_0 + \sum_{n=1}^{N} P_{n,m,n,k,m}^{d2d} + A_{n,m}} \right]^{+}, \hspace{1cm} (59)$$

where

$$A_{n,m} = \sum_{k \neq n}^{N} \frac{P_{n,m,k,m}^{d2d}}{h_{n,k,m}} + \sum_{i \neq k}^{N} P_{n,m,i,k,m}^{d2d}.$$  \hspace{1cm} (60)

**ACKNOWLEDGMENT**

This article was presented in part at the 2017 7th International Japan-Africa Conference on Electronics, Communications, and Computations (JAC-ECC 2019).

**REFERENCES**

[1] Y. Li, K. Chi, H. Chen, Z. Wang, and Y. Zhu, “Narrowband Internet of Things systems with opportunistic D2D communication,” IEEE Internet Things J., vol. 5, no. 3, pp. 1474–1484, Jun. 2018, doi: 10.1109/JIOT.2017.2783233.

[2] B. M. ElHalawany, O. Hashad, K. Wu, and A. S. Tag Eldien, “Uplink resource allocation for multi-cluster Internet-of-Things deployment underlying cellular networks,” Mobile Netw. Appl., vol. 25, no. 1, pp. 300–313, Jun. 2019, doi: 10.1007/s11036-019-01288-6.

[3] P. Mach, Z. Becvar, and T. Vanek, “In-band device-to-device communication in OFDMA cellular networks: A survey and challenges,” IEEE Commun. Surveys Tuts., vol. 17, no. 4, pp. 1885–1922, 4th Quart, 2015, doi: 10.1109/COMST.2015.2447036.

[4] M. M. Elmeslawy, “D2D communications for enabling Internet of Things underlying LTE cellular networks,” J. Wireless Networking Commun., vol. 6, no. 1, pp. 1–9, 2016, doi: 10.5923/j.jwnc.20160601.01.

[5] Z. Liu, T. Peng, S. Xiang, and W. Wang, “Mode selection for device-to-device (D2D) communication under LTE-advanced networks,” in Proc. IEEE Int. Conf. Commun. (ICC), Ottawa, ON, Canada, Jun. 2012, pp. 5563–5567, doi: 10.1109/ICC.2012.6364738.

[6] P. Phunchongharn, E. Hossain, and D. I. Kim, “Resource allocation for device-to-device communications underlying LTE-advanced networks,” IEEE Wireless Commun., vol. 20, no. 4, pp. 91–100, Aug. 2013, doi: 10.1109/MWC.2013.6590055.

[7] B. Wang, L. Chen, X. Chen, X. Zhang, and D. Yang, “Resource allocation optimization for device-to-device communication underlying cellular networks,” in Proc. IEEE 73rd Veh. Technol. Conf. (VTC Spring), Yokohama, Japan, May 2011, pp. 1–6, doi: 10.1109/VETECS.2011.5956157.

[8] B. M. ElHalawany, R. Ruby, and K. Wu, “D2D communication for enabling Internet-of-Things: Outage probability analysis,” IEEE Trans. Veh. Technol., vol. 68, no. 3, pp. 2322–2345, Mar. 2019, doi: 10.1109/TVT.2019.2891648.

[9] E. M. Mohamed, B. M. Elhalawany, H. S. Khalifa, M. Zareei, A. Zeb, and M. A. Abdelghany, “Relay probing for millimeter wave multi-hop D2D networks,” IEEE Access, vol. 8, pp. 30560–30574, Feb. 2020, doi: 10.1109/ACCESS.2020.297264.

[10] C.-H. Yu, K. Doppler, C. B. Ribeiro, and O. Tirkkonen, “Resource sharing optimization for device-to-device communication underlying cellular networks,” IEEE Trans. Wireless Commun., vol. 10, no. 8, pp. 2752–2763, Aug. 2011, doi: 10.1109/TWC.2011.060811.102120.

[11] L. D. Nguyen, “Resource allocation for energy efficiency in 5G wireless networks,” EAI Endorsed Trans. Ind. Netw. Intell., vol. 5, no. 14, Jun. 2018, Art. no. 154832, doi: 10.4108/eai.27-6-2018.154832.

[12] M. Jia, C. Zhang, and Y. Rui, “Energy efficiency optimization in D2D-cellular converged networks for video delivery,” Mobile Inf. Syst., vol. 2016, pp. 1–8, Jan. 2016.

[13] B. Shang, L. Zhao, K.-C. Chen, and G. Zhao, “Energy-efficient device-to-device communication in cellular networks,” in Proc. IEEE 83rd Veh. Technol. Conf. (VTC Spring), Nanjing, China, May 2016, pp. 1–5, doi: 10.1109/VTCspring.2016.7504229.

[14] L. D. Nguyen, H. D. Tuan, T. Q. Duong, O. A. Dobre, and H. V. Poor, “Downlink beamforming for energy-efficient homogeneous networks with massive MIMO and small cells,” IEEE Trans. Wireless Commun., vol. 17, no. 5, pp. 3386–3400, May 2018, doi: 10.1109/TWC.2018.2811472.

[15] Z. Sheng, H. D. Tuan, T. Q. Duong, and H. V. Poor, “Joint power allocation and beamforming for energy-efficient two-way multi-relay communications,” IEEE Trans. Wireless Commun., vol. 16, no. 10, pp. 6660–6671, Oct. 2017, doi: 10.1109/TWC.2017.2728059.

[16] X. Zhang, A. Khwaja, M. Naem, and A. Anpalagan, “Energy efficiency of cooperative D2D communications underlying LTE-A networks,” in Proc. 2nd Int. Conf. Mater. Adv. Manuf. Technol. (MEAMT), vol. 189, 2018, p. 03016.

[17] J. Hu, W. Heng, X. Li, and J. Wu, “Energy-efficient resource reuse scheme for D2D communications underlying cellular networks,” IEEE Commun. Lett., vol. 21, no. 9, pp. 2097–2100, Sep. 2017, doi: 10.1109/LCOMM.2017.2711490.

[18] Z. Zhou, M. Dong, K. Ota, J. Wu, and T. Sato, “Energy efficiency and spectral efficiency tradeoff in device-to-device (D2D) communications,” IEEE Wireless Commun. Lett., vol. 3, no. 5, pp. 485–488, Oct. 2014, doi: 10.1109/LWCL.2014.2351400.

[19] L. Liu and M. A. Abdelghany, “Relay probing for millimeter wave multi-hop D2D communications,” IEEE Commun. Lett., vol. 61, no. 8, pp. 5045–5058, Aug. 2018, doi: 10.1109/TWC.2018.2836937.

[20] D. Feng, L. Lu, Y. Yuan-Wu, G. Y. Li, G. Feng, and S. Li, “Device-to-device communications underlying cellular networks,” IEEE Trans. Commun., vol. 66, no. 5, pp. 2260–2275, May 2018, doi: 10.1109/TCOMM.2018.2821296.

[21] Y. Song, P.-Y. Kong, Y. Kim, S. Baek, and Y. Choi, “Cellular-assisted D2D communications for advanced metering infrastructure in smart grid,” IEEE Syst. J., vol. 13, no. 2, pp. 1347–1358, Jun. 2019, doi: 10.1109/JSYST.2019.2891719.

[22] H. H. Esmat, M. M. Elmeslawy, and I. I. Ibrahim, “Adaptive resource sharing algorithm for Device-to-Device communications underlying cellular networks,” IEEE Commun. Lett., vol. 20, no. 3, pp. 530–533, Mar. 2016, doi: 10.1109/LCOMM.2014.2351400.
OMNIA HASHAD (Student Member, IEEE) received the B.Sc. degree in electrical engineering (Electronics and Telecommunications) from the Faculty of Engineering at Shoubra, Benha University, Egypt, in 2013, where she is currently pursuing the master's degree in electrical engineering. She is also a Graduate Student with the Faculty of Engineering at Shoubra, Benha University. Her research interests include performance analysis, resource allocation, and optimization in wireless networks, device-to-device (D2D) communication, and the Internet-of-Things (IoT).

MOSTAFA M. FOUDA (Senior Member, IEEE) received the Ph.D. degree in information sciences from Tohoku University, Japan, in 2011. He has served as an Assistant Professor with Tohoku University, Japan. He was a Postdoctoral Research Associate with Tennessee Technological University, USA. He is currently an Assistant Professor with the Department of Electrical and Computer Engineering, Idaho State University, Pocatello, ID, USA. He is also an Associate Professor with Benha University, Egypt. He has published over 30 articles in IEEE conference proceedings and journals. His research interests include cyber security, machine learning, blockchain, the IoT, 5G networks, smart healthcare, and smart grid communications. He has served on the technical committees of several IEEE conferences. He is also a reviewer for several IEEE Transactions and Magazines and an Associate Editor of IEEE Access.

ADLY S. TAG ELDIEN (Member, IEEE) received the B.Sc., M.Sc., and Ph.D. degrees from Benha University, Egypt, in 1984, 1989, and 1993, respectively. He is the Ex-Head of the Network and Information Center, and the Head of the Electrical Engineering Department, Faculty of Engineering at Shoubra, Benha University. His research interests include robotics, networks, and mobile communication.

BASEM M. ELHALAWANY (Member, IEEE) received the master's degree from Benha University, Egypt, in 2011, and the Ph.D. degree from Egypt-Japan University of Science and Technology, Egypt, in 2014. He served as a Research Fellow with the Smart Sensing and Mobile Computing Laboratory, Shenzhen University, China, and with the EUJIST Center, Kyushu University, Japan. He is currently an Associate Professor with the Electrical Engineering Department, Faculty of Engineering at Shoubra, Benha University. His research interests include performance analysis, resource management, and optimization in wireless networks, non-orthogonal multiple access (NOMA), device-to-device (D2D) communication, physical-layer security (PLS), and machine-learning applications in communication.