Dependence of QCD hadron masses on the number of dynamical quarks

Dong Chen* and Robert D. Mawhinney**

*Department of Physics, Columbia University, New York, NY 10027, USA

We have studied the hadron spectrum while varying the number of light dynamical quarks when the physical lattice spacing and volume are held fixed relative to the rho mass. For two and zero flavors of staggered fermions, we find the nucleon to rho mass ratios (extrapolated to zero valence quark mass) are very similar. However, for four flavors the ratio is 7% (2 σ) above the two flavor result.

1. INTRODUCTION

The effects of dynamical quarks in zero temperature simulations of QCD have generally been found to be quite small for ratios of physical quantities. In particular, quenched and two flavor dynamical simulations, for similar lattice spacings and volumes, have yielded similar results for the nucleon to rho mass ratio. Currently we can not probe the differences between zero and two flavor calculations as the continuum limit is approached, due to limitations in computer power. Instead, we have undertaken a comparison of quenched, two flavor and four flavor QCD at zero temperature, while holding the lattice spacing and volume fixed in physical units, to look for dynamical quark effects.

At Lattice ‘95, the Columbia group reported a spectrum study with two flavors of staggered quarks on a $16^3 \times 40$ lattice [1]. Since then, simulations for four and zero flavors on $16^3 \times 32$ volumes at Columbia and zero flavors at Ohio State have been done. We have chosen our simulation parameters so that the valence rho mass, extrapolated to zero valence quark mass ($m_{\text{val}} = 0$), is independent of the number of dynamical quarks. (We have achieved this to the 3% level.) All the simulations consist of large data samples (by current standards), affording a precise comparison.

2. SIMULATION DETAILS

Table [1] lists the parameters for our simulations. The Columbia simulations were all done on the 256-node, 16 Gigaflop computer at Columbia, now completing its seventh year of full-time calculations. The two flavor simulation was done with the inexact R hybrid molecular dynamics algorithm of [2]; the four and 0 flavor simulations used an exact hybrid Monte Carlo algorithm, employing the Φ hybrid molecular dynamics algorithm of [2] and a Monte Carlo accept/reject step. The quenched simulations of OSU were done on the Ohio Supercomputer Center T3D using a mixture of over-relaxed and Metropolis steps.

From Table [1] it is apparent that the two and four flavor simulations had very similar parameters for the evolution. For the four flavor calculation, we used a tighter conjugate gradient stopping condition in the evolution, so that the accept/reject step of the exact algorithm would be based upon as accurate a value for the 5-dimensional Hamiltonian as possible. The stopping condition we used, and a test condition a factor of 3 smaller, both gave the same sequence of lattices for about 10 time units when evolved from the same starting lattice.

The four flavor run had an acceptance rate of 95%. This high acceptance rate is reassuring for the choice of the running conditions for our inexact, two flavor run. In particular this is evidence that the errors in our two flavor run due to using an inexact algorithm are small. This is important, since we want to compare physics from the two different algorithms.
Table 1
Simulation parameters for the four runs presented. The run length, thermalization, hadron measurement frequency and jackknife block size are in time units for the CU runs and sweeps for the OSU runs.

|                      | $N_f = 4$ (CU) | $N_f = 2$ (CU) | $N_f = 0$ (CU) | $N_f = 0$ (OSU) |
|----------------------|---------------|---------------|---------------|---------------|
| volume               | $16^3 \times 32$ | $16^4 \times 40$ | $16^4 \times 32$ | $16^4 \times 32$ |
| $\beta$              | 5.4           | 5.7           | 6.05          | 6.05          |
| $m_{\text{dynamical}}$ | 0.01         | 0.01          | 0.01          | 0.01          |
| evolution            | HMC           | HMD           | HMC           | OR + Metropolis |
| run length           | 4450          | 4870          | 187,125       | 787,500       |
| thermalization       | 250           | 250           | 375           | 25,000        |
| acceptance rate      | 0.95          | 0.91          | 0.91          | 0.91          |
| trajectory length    | 0.5           | 0.5           | 0.75          | 0.75          |
| step size            | 0.0078125     | 0.0078125     | 0.025         | 0.025         |
| CG stopping condition| $1.13 \times 10^{-6}$ | $1.01 \times 10^{-5}$ | $1.01 \times 10^{-5}$ | $1.01 \times 10^{-5}$ |
| total run time       | 5 months      | 7.5 months    | 1.7 months    | 25k node-hours |
| hadron source        | $16^4$ wall, all coordinates $(x, y, z)$ even |
| valence quark masses | 0.01, 0.015, 0.02, 0.025 |
| hadron measurement frequency | 5 | 6 | 187.5 | 2500 |
| number of lattices   | 840           | 770           | 996           | 306           |
| measurements per lattice | 4 | 5 | 1 | 4 |
| jackknife block size | 50            | 60            | 2250          | 7500          |
| number of blocks     | 84            | 77            | 83            | 102           |

3. RESULTS

Figure 1 shows $\pi$, $\rho$ and nucleon effective masses for the four simulations, where the effective masses are found from fits to time slices $t$ to $t + 3$. The wall source is fixed to Coulomb gauge and the simple local staggered sinks are used. While the rho masses for all cases are very similar, the four flavor nucleon mass is significantly larger than the others. Notice there is little difference between zero and two flavors.

Figure 2 shows the dependence of the nucleon to rho mass ratio on $m_{\text{val}}$. The lines are extrapolations of the ratios, while the data points for $m_{\text{val}} = 0$ are the ratio of the extrapolated masses. The ratio of the extrapolated masses gives $m_N/m_\rho$ equal to: 1.591(41) for $N_f = 4$, 1.489(31) for $N_f = 2$, 1.470(29) for CU $N_f = 0$ and 1.443(28) for OSU $N_f = 0$. Note the 1 $\sigma$ agreement between the two quenched calculations and the closeness of the two flavor result. The four flavor result is 2 $\sigma$ above the two flavor result.

The error on the four flavor ratio of $m_N/m_\rho$ is larger than for two flavors, while the error on the individual masses is quite similar. We have studied this in some detail and observe that for
Figure 1. The effective masses for the $\pi$, $\rho$ and $N$ for $m_{\text{val}} = 0.01$.

Four flavors, for the block size we are using, there is less correlation between fluctuations in the rho and nucleon propagators than for the other simulations. When the ratio is calculated, there is less cancellation between these uncorrelated fluctuations, leading to the larger error quoted.

In conclusion, we have seen a 7% difference (2 $\sigma$) in $m_N/m_\rho$ for $m_{\text{val}} = 0$ depending on whether two flavors or four flavors of dynamical fermions were used in the evolution of the configurations. Notice that the increased number of flavors is making the unphysically large value for the ratio increase. We are anxious to understand if this effect persists for larger volumes and when the four flavor dynamical fermion mass is varied.

We are currently checking for the effects of four dynamical fermions in other observables.

REFERENCES
1. D. Chen, Nucl. Phys. B (Proc. Suppl.) 42 (1995), 312; Nucl. Phys. B (Proc. Suppl.) 47 (1996), 382.
2. S. Gottlieb, et. al., Phys. Rev. D 35 (1987), 2531.

Figure 2. $m_\rho$ and $m_N$ extrapolated to $m_{\text{val}} = 0$.

Figure 3. $m_N/m_\rho$ versus $m_{\text{val}}$. The lines are extrapolations in the ratios and the data points at $m_{\text{val}} = 0$ are ratios of extrapolated masses.