Abstract

We review recent lattice computations relevant for $D$ and $B$ decays. Decay constants $f_{D,D_s}$, $f_{B,B_s}$, $D \rightarrow K(K^*)$, $B \rightarrow \pi, \rho$ semi-leptonic form factors together with the slope of the Isgur-Wise function calculations are presented. Some recent results of $B \rightarrow K^*\gamma$ form factors will be given. $1/M$ corrections to the asymptotic scaling laws are discussed.

1 Introduction

$B$ meson physics is now an active field (where several thousand of physicists are engaged) in which lattice QCD takes advantage from recent improvements increasing the statistics and improving the control of the systematics through theoretical progress (improved actions · · ·). In addition to phenomenological predictions, lattice QCD can test the scaling laws predicted by HQET (Heavy Quark Effective Theory). Up to now, precise predictions can be made only in the quenched approximation for which the systematic errors cannot be evaluated; however, results obtained in the unquenched theory[1] suggest that the quenching effect is small when one deals with heavy quarks. In general, comparison with experiment shows an agreement with lattice data inside error bars which are still sizable although decreasing. This is a review of recent calculations done in the quenched approximation using the Wilson and clover (continuum limit improved) actions on heavy-light meson decays.

2 Strategy to study $B$ meson on the lattice

Since the inverse lattice spacing $a^{-1}$ ranges from 2 to 4 GeV, one cannot study $B$ meson directly on the lattice. Indirect informations can be obtained through the following:

- One uses a set of relatively heavy mesons with masses up to $0.7a^{-1}$, i.e. heavier than the $D$ but lighter than the $B$ meson ("fictitious $D$ mesons"). We call this mass region the "moving quark" region.
- On the other hand, a method proposed by Eichten[3] allows one to put a quark with infinite mass on the lattice and the latter is considered in this approach as a static source of color. We call this mass region the "static quark" one (where heavy flavours are studied at lowest order in $1/M$ expansion, $M$ being the heavy meson mass).
A physical quantity computed in these two mass regions is interpolated to the $B$ meson with the help of the scaling laws of the HQET. The value in the static limit reduces the uncertainties due to the extrapolation. This method has shown to be very effective (Fig.1) in the estimation of the decay constants. In semi-leptonic decays, the calculation in the static limit is not yet available, but one can study the scaling behaviour and try an extrapolation to the $B$ meson. The predictions concerning the $B$ meson semi-leptonic decays remain at a semi-quantitative level but it shows that the extrapolation may be done and improvements in the near future are expected. The next section is an illustration of the method explained above.

3 Leptonic decays $D(B) \rightarrow \ell\nu$

A hadron mass can be obtained from the study of an appropriate Euclidean correlation function as the coefficient of its exponential time dependence:

$$G(t) = \int d^3x \langle \bar{u}(x,t)\gamma_0\gamma_5c(x,t)\bar{c}(0,0)\gamma_0\gamma_5u(0,0) \rangle \approx t \rightarrow \infty \frac{f_2m_D}{2} e^{-m_D t}$$

The determination of the expectation value in eq.1 is a non-perturbative problem which can be solved numerically. The second approach (static) is based on the expansion of the heavy quark ($H$) propagator in inverse powers of the quark mass as proposed by Eichten\cite{3}; the $H$ is static and does not live effectively on the lattice but the quantity $f_H \sqrt{M_H}$ can be measured and is predicted to be independent of the heavy mass. The confrontation between the two methods is presented in Fig.1. The HQET tells us that when $M_H \rightarrow \infty$, the vector ($V$) and pseudoscalar ($P$) decay constants scale with the mass of the heavy quark, $M_H$, \cite{3-5} ($M = M_P = M_V = M_H, \beta_0 = 11 - \frac{2}{3}N_f$) as:

$$\frac{M}{f_V} = \frac{f_P}{\sqrt{M}} = \frac{C}{\alpha_s(M)^{2/\beta_0}}.$$
Table 1: \( W \) (C) refers to Wilson (clover) action.

| Ref.       | \( \beta \) | \( f_D \) (MeV) | \( f_{D_s} \) (MeV) |
|------------|-------------|-----------------|---------------------|
| ELC(W) \[4\] | 6.4         | 210 ± 15        | 227 ± 15            |
| APE (C) \[6\] | 6.0         | 218 ± 9         | 240 ± 9             |
| BLS(W) \[7\] | 6.3         | 208(9) ± 35 ± 12| 230(7) ± 30 ± 18    |
| UKQCD (C) \[8\] | 6.2         | 185\( ^+_{14} \)\( ^+_{12} \) | 212\( ^+_{14} \)\( ^+_{12} \) |
| WA75 \[9\] | -           | 232 ± 45 ± 39   |                     |
| CLEO2 \[10\] | -           | 344 ± 37 ± 52   |                     |
| ARGUS \[11\] | -           | 267 ± 28        |                     |

Figure 1. Linear and quadratic fit in \( 1/M \) are shown, the vertical line shows the physical \( B \) meson.

In Figure 1, we notice the consistency between the moving quark results and the static ones. It appears that there are large corrections to the asymptotic scaling behaviour (2). The lattice results compared to the experimental ones are reported in Table 1(2) concerning the \( D(B) \) meson.

From Table 1, one can see that the different lattices agree more or less. Up to 5\% - 10\% we find: \( f_D \sim 210\text{MeV} \) and \( f_{D_s} \sim 230\text{MeV} \). \( f_{D_s} \) which has been predicted by lattice since several years, will provide an important check since the large experimental errors may be substantially reduced in the future.

In Table 2, the general tendency is \( f_B \sim 200\text{MeV} \) (up to 20\%), in agreement with QCD Sum Rules calculations.

The \( B \)--parameter: the predictions for the \( B - \bar{B} \) mixing depend on the \( B \)--parameter of the heavy light \( \Delta B = 2 \) four quark operator. ELC gives \[8\]: \( B_{\ell \ell} = 1.05 \pm 0.08 \), \( B_{\ell\ell} = 1.16 \pm 0.07 \) and \( \frac{B_{\ell\ell}}{B_{D_s}} \approx \frac{B_{\ell\ell}}{B_{D_s}} = 1.02 \pm 0.02 \); the prediction for the physically relevant combination in \( B - \bar{B} \)
mixing and CP violation is: $f_{B_d}\sqrt{B_{B_d}} = 220 \pm 40 \text{MeV}$.

## 4 Semi-leptonic decays $D(B) \to K, K^*(\pi, \rho)\ell\nu$

In this study, we first calculate $D \to K, K^*$ then extrapolate to $B \to \pi, \rho$. The amplitudes are expressed in terms of four form factors $f^+, V$ and $A_{1,2}$ and we need to know them at momentum transfer $q^2 = 0$ (where we have the maximum of phase space). The present lattices run in the close vicinity of $q^2 = 0$ for the $D$ meson, far from so for the $B$ one. In the latter case we proceed in two steps:

- The first is to extrapolate to the $B$ meson mass at fixed momentum near the no recoil point ($q_{\text{max}}^2$) and this is doable with the help of HQET: when $M \to \infty$ at fixed $\vec{q}$ and $||\vec{q}|| \ll M$, the form factors scale as [16]: $f^+, V, A_2 \sim \frac{1}{M^{1/2}}$, and $A_1 \sim \frac{1}{M^{1/2}}$.

- The 2\textsuperscript{nd} and difficult step is to extrapolate to $q^2 = 0$; people often use the Nearest Pole Dominance approximation VMD ($F(q^2) = F(0)\frac{1}{1 - q^2/M_t^2}$, $M_t$ is the exchanged meson mass in the t-channel) which has no firm theoretical grounding.

### D meson study: lattice results compared to model predictions and experimental data

The Isgur-Wise function $\xi(x)$: when the final meson is heavy, semi-leptonic form factors are expressed in terms of one universal function, $f_{B_d}\sqrt{B_{B_d}} = 220 \pm 40 \text{MeV}$. 

$$
\begin{array}{|c|c|c|c|}
\hline
\text{Ref.} & \beta & f_B(\text{MeV}) & \frac{f_{B_d}}{f_{B_d}} \\
\hline
\text{ELC(W)}[4] & 6.4 & 205 \pm 40 & 1.08 \pm 0.06 \\
\text{APE (S-C)}[12] & 6.2 & 290 \pm 15 \pm 45 & 1.11(3) \\
\text{APE (S)}[14] & 6.0 & 350 \pm 40 \pm 30 & 1.14(4) \\
\text{APE (S-C)}[14] & 6.0 & 328 \pm 36 & 1.19(5) \\
\text{UKQCD (C)}[8] & 6.2 & 160^{+6}_{-6}^{+15}_{-19} & 1.22 \pm 0.04 \\
\text{UKQCD (S-C)}[8] & 6.2 & 253^{+16}_{-15}^{+105}_{-14} & 1.14^{+4}_{-3} \\
\text{BLS (S)}[7] & 6.3 & 235(20) \pm 21 & 1.11 \pm 0.05 \\
\text{Allton (S-W)}[13] & 6.0 & 310 \pm 25 \pm 50 & 1.09 \pm 0.04 \\
\text{HEMCGC}[1] & 5.6 & 200 \pm 48 & - \\
\hline
\end{array}
$$

Table 2: $S$ refers to Static limit. The HEMCGC unquenched result agree with those obtained in the quenched approximation.
Table 3: $D \to K, K^*$ semi-leptonic form factors. EXP, LAT, QM and SR refer to experimental average, lattice, quark model and QCD sum rules calculations respectively.

| Ref. | $f^*(0)$ | $V(0)$ | $A_1(0)$ |
|------|----------|--------|----------|
| EXP 24 | 0.77(4)  | 1.16 ± 0.16 | 0.61(5) |
| APE 18 | 0.72(9)  | 1.00 ± 0.20 | 0.64 ± 0.11 |
| UKQCD 14 | 0.67 ± 17 | 0.98 ± 19 | 0.70 ± 17 |
| ELC 17 | 0.60 ± 15 ± 0.07 | 0.86 ± 0.24 | 0.64 ± 0.16 |
| APE 15 | 0.63 ± 0.08 | 0.86 ± 0.10 | 0.53 ± 0.03 |
| BLS 20 | 0.90 ± 0.08 ± 0.21 | 1.43 ± 0.45 ± 0.49 | 0.83 ± 0.14 ± 0.28 |
| SR 23 | 0.60 ± 0.10 | 1.10 ± 0.25 | 0.50 ± 0.15 |
| QM.1 21 | 0.76 | 1.23 | 0.88 |
| QM.2 22 | 0.8 | 1.1 | 0.8 |

| Ref. | $A_2(0)$ | $V(0)/A_1(0)$ | $A_2(0)/A_1(0)$ |
|------|----------|----------------|------------------|
| EXP 24 | 0.45(9) | 1.90 ± 0.25 | 0.74 ± 0.15 |
| APE 18 | 0.46 ± 0.34 | 1.59 ± 0.29 | 0.73 ± 0.45 |
| UKQCD 14 | 0.68 ± 11 | 1.3 ± 0.2 | 0.6 ± 0.3 |
| ELC 17 | 0.40 ± 0.28 ± 0.04 | 1.6 ± 0.2 | 0.4 ± 0.4 |
| APE 15 | 0.19 ± 0.21 | 1.99 ± 0.22 ± 0.33 | 0.7 ± 0.16 ± 0.17 |
| BLS 20 | 0.60 ± 0.15 | 2.2 ± 0.2 | 1.2 ± 0.2 |
| SR 23 | 0.76 ± 10 | 1.4 | 1.3 |
| QM.1 21 | 0.8 | 1.4 | 1.0 |
| QM.2 22 | 0.8 | 1.4 | 1.0 |

Unknown apart from the no recoil point $\xi(1) = 1$. In practice we try to measure its slope around this point $\rho^2 = -\xi'(1)$. In the $D$ mass region, UKQCD found $\rho^2 = 1.2^{+0.3}_{-0.3}$ while CLEO2 gives $\rho^2 = 1.01 ± 0.15 ± 0.09$ [20]. A preliminary analysis (UKQCD-Martinelli [27]) gives $\rho^2 = 1.7 ± 0.2$ for masses $m \geq m_D$.

5 Radiative Decay $B \to K^*\gamma$

The decay rate is expressed in terms of $T_{1,2}(q^2)$ where for a real photon ($q^2 = 0$), we have the exact condition $T_1(0) = T_2(0)$; the method used to extract $T_{1,2}$ is the same as the semi-leptonic one:

- Use the HQET scaling rules near $q^2_{\text{max}}$ (at leading order, $T_1/\sqrt{M}$, $T_2/\sqrt{M}$) to extrapolate to $M_B$. With the clover action, APE($\beta = 6.0$) [28] and UKQCD($\beta = 6.2$) [29] find for $T_2(q^2_{\text{max}})$: 0.21(2) and 0.269±17 respectively.

- The problem is: how to extrapolate to $q^2 = 0$? If we apply VMD on the two factors simultaneously, we find that they scale differently at $q^2 = 0$ ($T_1(0) \sim M^{-1/2}$ and $T_2(0) \sim M^{-3/2}$) in contradiction with the fact that they must be equal at $q^2 = 0$. So for which one is VMD better?

* Applying VMD on $T_2$, APE [28], UKQCD [29] and Bernard et al. [30] find for $T_2(0)$: 0.084(7), 0.112±7±7, and 0.10 ± 0.01 ± 0.03 respectively.

* Applying VMD on $T_1$, APE [28] finds a larger value $T_1(0) = 0.20(7)$.

There is thus a contradiction between the two approaches; it seems to us that the higher value is favoured because indications from lattices (APE and UKQCD) show that the $q^2$ dependence of $T_2$ is much weaker
than would be predicted by VMD. Using the 1st approach, UKQCD finds
BR(\(B \rightarrow K^*\gamma\)) = (1.7±0.6(stat)\(\pm^{11}_{9}\)(sys)\(\times10^{-5}\) while APE, when applying VDM on \(T_1\) finds a value closer (preliminary) to CLEO\[31\] result: \(BR = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}\).

6 Conclusion

There have been good quantitative studies for \(f_{D_s} (\sim 230\text{MeV})\), \(f_D (\sim 210\text{MeV})\), \(f_B (\sim 200\text{MeV} (20\%))\) and for \(D \rightarrow K^{(*)}\ell\nu\) (agreement up to 10\%). The study of \(\frac{1}{M}\) corrections (which are found to be large) to HQET are now under control. Concerning the \(B \rightarrow \pi(\rho)\ell\nu\) and \(B \rightarrow K^*\gamma\), since the \(q^2\) behaviour is still largely unknown, the predictions at \(q^2 = 0\) are only qualitative.

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