The total cross section for double ionization of lithiumlike ions by a high-energy photon is calculated in leading order of the nonrelativistic perturbation theory. The partial contributions due to simultaneous and sequential emissions of two electrons are taken into account. The cross section under consideration is shown to be related to those for double photoeffect on the ground and excited $2^1S$ states of heliumlike ions. The double-to-single ionization ratio is equal to $R = 0.288/Z^2$ for lithiumlike ions with moderate nuclear charge numbers $Z$. However, even for the lightest three-electron targets such as Li and Be$^+$, analytical predictions are found to be in good agreement with the numerical calculations performed within the framework of different rather involved approaches.

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Recent progress in developments of intense sources of the synchrotron radiation has raised considerable interest in the theoretical and experimental investigations of the double ionization of atoms and ions following by the absorption of a single photon. The double photoionization is a process of fundamental importance, because it is entirely caused by the electron correlations. As a target, it became usual to choose neutral helium or heliumlike ions, which represent the simplest multi-electron systems. The extensive studies of the double photoeffect on helium atom in the ground state allowed one to deduce detailed information about the inner-shell electron correlations \[1, 2, 3\].

The growing number of theoretical calculations has been recently devoted to the double photoeffect on excited metastable \(2^1S\) states of the helium isoelectronic sequence \[4, 5, 6, 7, 8, 9, 10, 11, 12, 13\]. The problem is also of particular interest, because it is concerned with the study of inter-shell electron correlations. However, due to lack of intense sources of the metastable helium, there has been no experimental work on the topic. In addition, the direct measurements on heliumlike ions with moderate values of nuclear charge number \(Z \gg 1\) are hampered in this case due to absence of long-living excited states.

Another direction of present experimental and theoretical investigations is focused on the double and even triple photoionization of three-electron targets in the ground state \[14, 15, 16, 17, 18, 19, 20, 21, 22\]. At present, the lightest targets, namely, the lithium atom or the \(Be^+\) ion, are used only. For lithium, the double-to-single ionization ratio \(R = \sigma^{++}/\sigma^+\) has been measured for photon energies \(\omega < 1\) keV, which is still essentially below the high-energy nonrelativistic asymptotic limit. After reaching the maximum value of about 4.6% around \(\omega \approx 240\) eV, the ratio \(R\) declines slowly up to 3.9% at \(\omega \approx 910\) eV \[17\].

In the high-energy nonrelativistic limit, the theoretical calculations of double and triple photoionizations of Li have been first performed with the use of B-spline basis sets \[19\]. The asymptotic double-to-single ionization ratio is predicted to be 3.37(3)%, which is consistent with the available experimental data. The high-energy limits for the ratio \(R\) were also calculated within the framework of the multiconfigurational Hartree-Fock method \[20\] and with the use of fully correlated variational wave functions \[21\]. The value of 3.36% reported by Yan \[21\] is in perfect agreement with the calculation performed by van der Hart and Greene \[19\], while the ratio of 1.81% predicted by Cooper \[20\] is significantly lower. In the work \[21\], the calculation is also extended to the ground state of \(Be^+\) ion, which yields \(\sigma^{++}/\sigma^+ = 1.97\)%. Recently, the double and triple photoionization of Li have been calculated for the near-threshold energy range, employing the time-dependent close-coupling approach \[22\].

2
In contrast to the sophisticated numerical methods mentioned above, we shall present an analytical evaluation of the total cross section for double photoionization of lithiumlike ions within the high-energy nonrelativistic limit. The cross section under investigation is also shown to be related to those for double photoeffect on the ground and excited $2^1S, 3^1S$ states of heliumlike ions with the same nuclear charge number $Z$. Accordingly, experiments with stable three-electron targets can allow one to test the theoretical predictions for low-lying excited states of heliumlike ions.

The present study is performed for asymptotic photon energies $\omega$ characterized by $I \ll \omega \ll m$, where $I = m(\alpha Z)^2/2$ is the binding energy of the K-shell electron, $m$ is the electron mass, and $\alpha$ is the fine-structure constant ($\hbar = 1$, $c = 1$). The ejected electrons are considered as being nonrelativistic. Accordingly, the Coulomb parameter is supposed to be sufficiently small, that is, $\alpha Z \ll 1$. Since the photon interacts with a single electron, the emission of two electrons is mediated only via the electron-electron interaction. The latter is taken into account within the framework of perturbation theory. As a zeroth approximation, Coulomb wave functions are employed. Although the formal parameter of the perturbation theory is $1/Z$, the actual expansion turns out to converge extremely fast for any value of the nuclear charge number $Z$ [23]. Accordingly, the present consideration applies to the lithium isoelectronic sequence within the range $3 \leq Z \leq 30$.

In contrast to the heliumlike targets, the double photoionization of lithiumlike ions can proceed via three channels, namely, double ionization of both K-shell electrons, double ionization of K- and L-shell electrons, and single ionization of one K-shell electron with excitation of the other K-shell electron. In the latter case, the remaining ion results to be in the doubly excited state, which can decay afterwards either due to the Auger effect or due to radiative emission. For multicharged ions with not too large values of $Z$, the Auger decay accompanied by electron ejection is the dominant process. The first two channels represent the simultaneous (direct) ionization, while the third channel can be referred to as the sequential (indirect) ionization. In a real experiment, one usually measures the number of ions, the charge of which has been increased by two units. Therefore, all three channels contribute to the total cross section of double ionization of lithiumlike ions

$$
\sigma^{++}(\text{Li}) = \sigma^{++}_{\text{sim}}(\text{Li}) + \sigma^{++}_{\text{seq}}(\text{Li}).
$$

(1)

Let us first consider the direct photoionization. Our calculations are performed in leading order of the nonrelativistic perturbation theory with respect to the electron-electron interaction. In this approximation, one has to consider only the Feynman diagrams with one-photon exchange. Accordingly, it is sufficient to take into account the interaction only between two active electrons, which are involved in real transitions. The interaction with the third electron (spectator) is ne-
glected, since it can first contribute only in the next-to-leading order of the perturbation theory. Therefore, the cross section $\sigma_{KK}^{++}(\text{Li})$ for double ionization of the K-shell electrons in lithiumlike ion is equal to the cross section $\sigma^{++}(\text{He})$ for double photoeffect on heliumlike ion in the ground state. The latter can be cast into the following analytical form \[24, 25\]

$$\sigma^{++}(\text{He}) = \sigma^{+}(\text{He})Z^{-2}B_1. \quad (2)$$

Here $B_1 = 0.090$ and $\sigma^{+}(\text{He})$ is the total cross section for single photoeffect on heliumlike ion in the high-energy limit. Within the approximations employed here, one can write

$$\sigma^{+}(\text{He}) = 2\sigma_{K}^{+}, \quad (3)$$

where

$$\sigma_{K}^{+} = \frac{2^7\pi\alpha}{3m\omega} \left(\frac{I}{\omega}\right)^{5/2} \quad (4)$$

is the total cross section for single photoionization of the K-shell electron in the Born high-energy limit \[26\].

Now we shall consider in more details the simultaneous ejection of two electrons from different shells (K and L) following by the absorption of a single photon. In the high-energy nonrelativistic limit, the double photoionization is known to proceed mainly due to the electron-electron interaction in the initial state, providing the Coulomb gauge is employed \[24\]. Another contribution to the amplitude of the process, which is due to the electron-electron interaction in the final state, turns out to be of about the factor $I/\omega$ smaller and, therefore, can be neglected. In addition, if $\omega \gg I$, the photon energy is distributed among the ejected electrons extremely nonuniformly \[24\]. The main contribution to the cross section arises from the edge domains of the electron energy spectrum, where the energy of one electron is much larger than that of the second electron, that is, either $E_{p1} \sim \omega$ and $E_{p2} \sim I$ or $E_{p1} \sim I$ and $E_{p2} \sim \omega$. In the following, we shall label the fast and slow electrons by the indices 1 and 2, respectively. Then there is just one edge domain characterized by the restriction $p_1 \gg p_2$ on the momenta of escaping electrons at infinity. Accordingly, one needs to take into account only the Feynman diagrams depicted in Fig. 11. The total amplitude of the process is given by

$$M = M_a - M_b, \quad (5)$$
\[ M_a = w^*_{\lambda_1} w^*_{\lambda_2} A_a w_{\mu_1} w_{\mu_2}, \quad (6) \]
\[ M_b = w^*_{\lambda_1} w^*_{\lambda_2} A_b w_{\mu_2} w_{\mu_1}, \quad (7) \]
\[ A_a = \langle \psi_{p_1} \psi_{p_2} | \hat{V}_\gamma G_C(E) V_{12} | \psi_{1s} \psi_{2s} \rangle, \quad (8) \]
\[ A_b = \langle \psi_{p_1} \psi_{p_2} | \hat{V}_\gamma G_C(E) V_{12} | \psi_{2s} \psi_{1s} \rangle. \quad (9) \]

Here \( w_\mu \) denotes the Pauli spinor characterized by the spin projection \( \mu \) relative to a quantization axis, \( G_C(E) \) is the nonrelativistic Coulomb Green’s function with the energy \( E = E_{1s} + E_{2s} - E_{p_2} \), \( E_{p_2} \) is the energy of slow electron in the continuous spectrum, and \( E_{1s} \) and \( E_{2s} \) are the single-electron energies of the orbitals \( 1s \) and \( 2s \), respectively. In the coordinate representation, the operators \( \hat{V}_\gamma \) and \( V_{12} \), which describe the electron-photon and electron-electron interactions, read
\[ \hat{V}_\gamma = -\frac{i}{m} \frac{\sqrt{4\pi \alpha}}{\sqrt{2\omega}} e^{i(k \cdot r)} (e \cdot \nabla), \quad (10) \]
\[ V_{12} = \frac{\alpha}{|r_1 - r_2|}, \quad (11) \]

where \( r_1 \) and \( r_2 \) are the electron coordinates, \( k \) is the photon momentum, and \( \omega = |k| = k \) and \( e \) are the energy and the polarization vector of a photon, respectively. The latter obeys the conditions \( (e \cdot k) = 0 \) and \( (e^* \cdot e) = 1 \). Since the operators \( \hat{V}_\gamma \) and \( V_{12} \), and the nonrelativistic Green’s function do not contain the spin matrices, the amplitudes \( (6) \) and \( (7) \) take the forms
\[ M_a = A_a \delta_{\lambda_1 \mu_1} \delta_{\lambda_2 \mu_2}, \quad (12) \]
\[ M_b = A_b \delta_{\lambda_1 \mu_2} \delta_{\lambda_2 \mu_1}. \quad (13) \]

Analytical expressions for the amplitudes \( (8) \) and \( (9) \) have been obtained in Ref. \[7\]. In the high-energy limit, both amplitudes are real.

The fivefold differential cross section for double photoionization of the KL electron pair of lithiumlike ion is given by
\[ d^5\sigma_{KL}^{++}(Li) = \frac{2}{4} \sum_{\lambda_1, \lambda_2} \sum_{\mu_1, \mu_2} |M|^2 d^5\Gamma, \quad (14) \]
\[ d^5\Gamma = mp_1 \frac{d\Omega_1}{(2\pi)^2} \frac{d^3p_2}{(2\pi)^3}, \quad (15) \]

where \( d\Omega_1 \) is the solid angle of fast electrons and the amplitude \( M \) is defined by Eqs. \( (6) \), \( (12) \), and \( (13) \). The expression \( (14) \) involves summation over the electron polarizations in the final state and averaging over those of the initial state. The factor 2 accounts for the existence of two K-shell
electrons; each of them can be coupled with the L-shell electron. Inserting the amplitude $M$ in Eq. (14), one obtains

$$d^5 \sigma_{KL}^{++}(Li) = 2(\mathcal{A}_a^2 + \mathcal{A}_b^2 - \mathcal{A}_a \mathcal{A}_b) \, d^5 \Gamma. \quad (16)$$

According to the results obtained in the work [7], the cross sections for double photoionization of heliumlike ions in the $2^1S$ and $2^3S$ states can be also expressed through the same amplitudes $\mathcal{A}_a$ and $\mathcal{A}_b$ as follows

$$d^5 \sigma_{s}^{++}(He^*) = (\mathcal{A}_a^2 + \mathcal{A}_b^2 + 2 \mathcal{A}_a \mathcal{A}_b) \, d^5 \Gamma, \quad (17)$$

$$d^5 \sigma_{t}^{++}(He^*) = (\mathcal{A}_a^2 + \mathcal{A}_b^2 - 2 \mathcal{A}_a \mathcal{A}_b) \, d^5 \Gamma. \quad (18)$$

The subscripts "s" and "t" refer to the singlet and triplet states, respectively, while the notation $He^*$ implies the heliumlike ion in the excited $1s2s$ state. Taking into account the statistical weights for the singlet and triplet states, one obtains the averaged cross section for double photoeffect on the metastable heliumlike ion

$$d^5 \sigma^{++}(He) = \frac{1}{4} d^5 \sigma_{s}^{++}(He^*) + \frac{3}{4} d^5 \sigma_{t}^{++}(He^*) = (\mathcal{A}_a^2 + \mathcal{A}_b^2 - \mathcal{A}_a \mathcal{A}_b) \, d^5 \Gamma. \quad (19)$$

Comparing Eqs. (16) and (19), one finds a simple relation between the cross sections of KL-ionization of Li- and He*-like ions

$$d^5 \sigma_{KL}^{++}(Li) = 2 d^5 \sigma^{++}(He*). \quad (20)$$

Adding the expression (20) to the partial contribution for the direct KK-ionization, one obtains the differential cross section for the simultaneous double photoionization of lithiumlike ion

$$d^5 \sigma_{sim}^{++}(Li) = d^5 \sigma_{KK}^{++}(Li) + d^5 \sigma_{KL}^{++}(Li) = d^5 \sigma^{++}(He) + 2 d^5 \sigma^{++}(He*). \quad (21)$$

Integrating Eq. (21) over the solid angles of electron ejections and averaging it over the photon polarizations, one arrives at the energy distribution of slow electrons

$$d\sigma_{sim}^{++}(Li) = d\sigma^{++}(He) + 2 d\sigma^{++}(He*) = \sigma^{++}(Li) Z^{-2} \beta(\varepsilon_2) d\varepsilon_2,$$

$$\beta(\varepsilon_2) = \frac{16}{17} \left\{ Q_1(\varepsilon_2) + \frac{1}{4} K_s(\varepsilon_2) + \frac{3}{4} K_t(\varepsilon_2) \right\}, \quad (23)$$

where $\varepsilon_2 = E_{p2}/I$ denotes the dimensionless energy of the low-energy electron. The explicit expression for the function $Q_1(\varepsilon_2)$ reads

$$Q_1(\varepsilon_2) = \frac{8 J^2(\varepsilon_2)}{1 - \exp(-2\pi/\sqrt{\varepsilon_2})}. \quad (24)$$
together with
\[ J(\varepsilon_2) = \frac{8\zeta^2}{(1+\zeta)^2} \left\{ \frac{J_1}{\varepsilon_2 + 1} - \frac{J_2}{\varepsilon_2 + 2} \right\}, \quad \zeta = (\varepsilon_2 + 2)^{-1/2}, \] (25)
\[ J_1 = \exp\left(-\frac{2}{\sqrt{\varepsilon_2}} \arctan\sqrt{\varepsilon_2}\right) \int_0^1 \frac{t^{-\zeta}(1-t)}{(1+qt)^2} dt, \] (26)
\[ J_2 = \int_0^1 \frac{t^{-\zeta}(1-t)^3}{(1+qt)^3} \Phi_1(t)\Phi_2(t) dt, \quad q = \frac{1-\zeta}{1+\zeta}, \] (27)

where
\[ \Phi_1(t) = \exp\left(-\frac{2}{\sqrt{\varepsilon_2}} \arctan\frac{\sqrt{\varepsilon_2}(1-t)}{a+bt}\right), \quad a = \sqrt{\varepsilon_2 + 2} + 2, \] (28)
\[ \Phi_2(t) = \frac{(3\zeta^2 + 1)(1-t)^2 + 6\zeta(1-t^2) + 2(1+t)^2}{[(2\zeta^2 + 1)(1-t^2) + 4\zeta(1-t^2) + (1+t)^2]^2}, \quad b = \sqrt{\varepsilon_2 + 2} - 2. \] (29)

In Eq. (23), the functions \( K_{s,t}(\varepsilon_2) = (9/8)R_{s,t}(\varepsilon_2) \) are also expressed via the definite integrals over the elementary functions [7]
\[ R_{s,t}(\varepsilon_2) = \frac{27}{9} \left\{ \lambda \int_1^\infty \left( \frac{y+1}{y-1} \right)^{1/\lambda} \left[ \chi_1(y) \pm \chi_2(y) \right] dy \right\}^2, \quad \lambda = \sqrt{\varepsilon_2 + 5/4}, \] (30)
\[ \chi_1(y) = (x-1/2)^{-3} \left\{ \varphi(1/2,1) + \varphi(1/2,2)/2 - (x^2 - 2x + 5/4)\varphi(x,2) + + 2(x-1/2)^2(x-1)\varphi(x,3) - \varphi(x,1) \right\}, \] (31)
\[ \chi_2(y) = (x-1)^{-4}(x-5/2) \left\{ \varphi(1,1) - \varphi(x,x) - (x-1)^2\varphi(x,2) \right\} + + (2x-1)(x-1)^{-1}\varphi(x,3), \] (32)
\[ \varphi(\nu,k) = (\nu^2 + \varepsilon_2)^{-k} \exp\left(-\frac{2}{\sqrt{\varepsilon_2}} \arctan\sqrt{\varepsilon_2} \right), \quad x = \lambda y + 3/2. \] (33)

In Eq. (30), the signs ”+“ and ”−“ correspond to the indices ”s“ and ”t“, respectively.

Within our approximations, the cross section \( \sigma^+(\text{Li}) \) for single photoeffect on lithium-like ions in the ground state is related to those on hydrogen-like and helium-like ions as follows
\[ \sigma^+(\text{Li}) = 2\sigma^+_K + \sigma^+_L = 17/8 \sigma^+_K = 17/16 \sigma^+(\text{He}). \] (34)

In addition, the cross section \( \sigma^+(\text{He}^*) \) for single ionization of helium-like ions in the excited 1s2s state is given by
\[ \sigma^+(\text{He}^*) = \sigma^+_K + \sigma^+_L = 9/8 \sigma^+_K. \] (35)

The resulting energy function \( \beta(\varepsilon_2) \) defined by Eq. (23) is depicted in Fig. 2. Integrating Eq. (22) over the energy \( \varepsilon_2 \) of the slow electron yields the total cross section for the direct double photoionization
\[ \sigma_{\text{sin}}^{++}(\text{Li}) = \sigma^{++}(\text{He}) + 2\sigma^{++}(\text{He}^*) = \sigma^{+}(\text{Li})Z^{-2}B, \] (36)
\[ B = \frac{16}{17} \left\{ B_1 + \frac{9}{8} \left[ \frac{1}{4} B_s + \frac{3}{4} B_t \right] \right\}, \] (37)

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where
\[ B_1 = \int_0^\infty Q_1(\varepsilon_2) \, d\varepsilon_2, \quad B_{s,t} = \int_0^\infty R_{s,t}(\varepsilon_2) \, d\varepsilon_2. \] (38)

In general, the upper limit in the integrals (38) is the finite number \( \varepsilon_\gamma/2 \), where \( \varepsilon_\gamma = \omega/I \) is the dimensionless energy of the incoming photon. It should be chosen in such a way to fulfill the condition \( \varepsilon_\gamma \gg \varepsilon_2 \). However, since the integrands decrease very rapidly for increasing \( \varepsilon_2 \), the upper limit can be trend to infinity. Then the numerical values for the constants (38) are equal to \( B_1 = 0.090 \) [24, 25], \( B_s = 0.318 \) [7], and \( B_t = 0.0575 \) [7]. In fact, the energy integrals are already saturated at \( \varepsilon_2 \approx 1 \). In view of the relation (37), one finally obtains \( B = 0.215 \). This is our first result.

Now we shall consider the indirect double photoionization of lithiumlike ions in the ground state. For light multicharged ions, the corresponding cross section \( \sigma_{\text{seq}}^{++}(\text{Li}) \) coincides with that \( \sigma_{\text{KK}}^{++}(\text{Li}) \) for single ionization with excitation. In this case, one K-shell electron leaves the ion, while the another one undergoes a transition into a higher-lying state forming together with the 2s-electron the doubly excited states, which afterwards decay via emission of the Auger electron. To leading order of the nonrelativistic perturbation theory, electron excitations can occur only into the \( ns \) states with the principal quantum numbers \( n \geq 2 \) [27]. Since the 2s state is occupied just by half in Li-like ions and it is vacant in He-like ions, the cross section \( \sigma_{\text{KK}}^{++}(\text{Li, 2s}) \) for the single ionization of lithiumlike ions with excitation into the 2s state is half as large as that \( \sigma^{++}(\text{He, 2s}) \) for heliumlike ions. In the case of transitions into other excited states, the cross sections for three-electron systems are equal to those for two-electron ions. Accordingly, one can write down the following chain of relations

\[ \sigma_{\text{seq}}^{++}(\text{Li}) = \sigma_{\text{KK}}^{++}(\text{Li}) = \sum_{n \geq 2} \sigma_{\text{KK}}^{++}(\text{Li, } ns) = \frac{1}{2} \sigma^{++}(\text{He, 2s}) + \sum_{n \geq 3} \sigma^{++}(\text{He, } ns). \] (39)

To leading order of the perturbation theory with respect to the electron-electron interaction, the cross sections \( \sigma^{++}(\text{He, } ns) \) for the single ionization of heliumlike ions with excitation into the \( ns \) states read [27]

\[ \sigma^{++}(\text{He, } ns) = \sigma^+(\text{He})Z^{-2}Q(n), \] (40)
\[ Q(n) = \frac{48}{n^3} \left\{ \kappa \int_1^\infty \left( \frac{y+1}{y-1} \right)^{1/\kappa} \chi(y)(1+\kappa y)^{-3} dy \right\}^2, \] (41)
\[ \chi(y) = \phi(1, 1) - \phi(v, 1) - (v-1)^2 \phi(v, 2), \quad \kappa = \sqrt{2-n^{-2}}, \] (42)
\[ \phi(\nu, k) = \frac{(\nu-n^{-1})^{n-k}}{(\nu+n^{-1})^{n+k}}, \quad v = 2 + \kappa y. \] (43)
For the lowest principal quantum numbers $n$, the numerical values of the function are equal to $Q(2) = 9.11 \cdot 10^{-2}$, $Q(3) = 1.71 \cdot 10^{-2}$, $Q(4) = 0.62 \cdot 10^{-2}$, $Q(5) = 0.30 \cdot 10^{-2}$, $Q(6) = 0.17 \cdot 10^{-2}$, $Q(7) = 0.10 \cdot 10^{-2}$, $Q(8) = 0.7 \cdot 10^{-3}$, $Q(9) = 0.5 \cdot 10^{-3}$. In the asymptotic limit $n \gg 1$, the following relation

$$n^3 Q(n) |_{n \gg 1} \simeq 0.334$$

holds.

Inserting Eq. (40) into Eq. (39) and taking into account the relation (34) yields as our second result

$$\sigma^{++}_{\text{seq}}(\text{Li}) = \sigma^+(\text{Li}) Z^{-2} C,$$

$$C = \frac{16}{17} \left\{ \frac{1}{2} Q(2) + \sum_{n \geq 3} Q(n) \right\}.$$ 

Employing the numerical data for the function $Q(n)$, we obtain $C = 0.073$.

The double-to-single photoionization ratio $R$ for lithiumlike ions, which is usually measured experimentally, is given by

$$R = \frac{\sigma^{++}(\text{Li})}{\sigma^+(\text{Li})} = \frac{B + C}{Z^2} = \frac{0.288}{Z^2},$$

where Eqs. (1), (36), and (45) have been employed. As follows from the ratio $C/B \simeq 1/3$, about one third of the total cross section for double photoeffect is gained on account of the channel of sequential ionization. The universal scaling law (47) is our third result. The highest absolute accuracy for the universal curve can be expected for lithiumlike multicharged ions with moderate nuclear charge numbers $10 \lesssim Z \lesssim 25$, since the higher-order corrections omitted in the present calculations are of minor importance for the total cross sections. Nevertheless, for the particular cases of the neutral lithium and the Be$^+$ ion, our predictions for the ratios $R$ are equal to 3.2% and 1.8%, respectively, which are already in good agreement with the numerical results obtained within the framework of rather complicated methods in works [19, 21].

Concluding, we have established the universal high-energy behavior of the double-to-single photoionization ratio for lithiumlike ions in the ground state. The partial contributions due to the direct and indirect ionization channels are taken into account. We have also found relations between the cross sections for double photoeffect on Li- and He-like ions with the same nuclear charge number $Z$. These relations can be employed for experimental tests of theoretical predictions concerning the double photoionization of low-lying excited states of He-like ions.
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FIG. 1: Feynman diagrams for ionization of the K- and L-shell electrons following by the absorption of a single photon. Solid lines denote electrons in the Coulomb field of the nucleus, dashed line denotes the electron-electron Coulomb interaction, and the wavy line denotes an incident photon. The line with a heavy dot corresponds to the Coulomb Green’s function.

FIG. 2: The function $\beta(\varepsilon_2)$ is calculated with respect to the dimensionless energy $\varepsilon_2$ of the slow electron according to Eq. (23).
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