Energy Resolution of Cryogenic Imaging Detectors

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Abstract. In this work, the theory of branching cascade processes is applied to the description of signal formation in cryogenic imaging detectors. The formula for the energy resolution of imaging detectors is derived. It is shown how the position information can improve the energy resolution of this type of detectors.

1. Introduction
Imaging detectors are under development for a variety of applications. As a rule, these detectors have an absorber with a large area that is read out by two or more sensors. In cryogenic imaging detectors sensors are usually superconducting tunnel junctions (STJs) placed at the perimeter. Quasiparticles generated by an incident particle in the absorber are registered by STJs. As signals of STJs depend on an incident particle absorption point of, so the imaging detector provides a position resolution of the events.

Although superconducting tunnel junctions (STJs) are small enough, position-sensitive STJ detectors [1] are still very attractive. A typical position-sensitive STJ detector consists of a superconducting absorber strip flanked by two STJs. Position-sensitive STJ detectors are being developed by the group of the Space Science Department of ESA (ESTEC) [2] and the group at Yale [3]. The ESTEC group calls their detector a Distributed Read-Out Imaging Device (DROID). They have achieved an intrinsic resolution of 2.1 eV at 500 eV on a DROID with Ta/Al based tunnel junctions and a 20x100x0.1 μm^3 Ta absorber. The Yale group has demonstrated a position-sensitive STJ detector in the optical, UV, and X-ray bands. They have achieved a resolution of 0.4 eV FWHM at 4.89 eV and 2.4 eV FWHM for UV devices, and 13.1 eV FWHM at 6 keV for 200x100 μm^2 absorber.

The group at Osaka Electro-Communication University have made a substrate-absorption-type imaging detector (SATID) that consist of the 5x5 mm^2 sapphire substrate and four series arrays each consisting of 96 superconducting tunnel junction (STJ) placed at the perimeter of the substrate [4, 5]. SATIDs combine advantages of STJs, which have an excellent energy resolution because of their small break-up energy for Cooper pairs, and the possibility of producing detectors with a large effective area and good effectiveness of registration by selecting proper substrate material.

In cryogenic imaging detector primary particles drop on an absorber with a large effective area, which is a superconducting strip in a DROID and a substrate in a SATID. In the processes of successive collisions, the energy of an incident particle transforms into the energy of excess quasiparticles. Quasiparticles propagate through the absorber and enter the STJs. For generality, we name quasi-electrons and phonons produced by a primary particle in an absorber quasiparticles. This means that the master quasiparticle that enters an STJ and determines the output signal is a quasi-electron in a DROID and a phonon in a SATID.
Quasiparticle entering an STJ produces current at its output. For distinguish the quasiparticles that enter an STJ and the quasiparticles that tunnel back and forth through the insulating barrier of an STJ we name quasiparticles in an STJ quasi-electrons. This means that quasi-electron, the master quasiparticle in a DROID, after entering an STJ has the probability to tunnel through the insulating barrier. In a SATID, a phonon that is the master quasiparticle after entering an STJ has the probability to break Cooper pair into quasi-electrons. Then, quasi-electron has the probability to tunnel through the insulating barrier and produces the current at the output of the STJ.

Integrating STJ’s currents, we obtain STJ’s charges. As STJ’s charges depend on the incident particle absorption point, so cryogenic imaging detector provides a spatial resolution of the events. By using the charges from STJs, we can obtain the position of the primary particle absorption point. The sum of the charges from STJs creates the energy spectrum of the incident particles.

2. Mathematical Model

This work is intended to describe statistical processes of particle registration in cryogenic imaging detectors. To answer the question of how the signal position dependence influences the energy resolution of an imaging detector, let us formulate the mathematical description of an X-ray quantum registration by a cryogenic imaging detector. As the process of signal formation at the output of the detector represents a random branching cascade process, the formalism of generating functions (GF) is the most adequate for its formulation [6]. The process of a primary particle energy transformation into the output signal of an imaging detector includes the following successive stages.

1. The stage of a primary particle interaction with an absorber. Let the energy of X-ray quanta to be so low that the dominant interaction with the matter is photoelectric absorption. As the size of an absorber is usually much greater than the down-conversion area, then we can consider that all the extra quasiparticles are produced at the same point \( r \). Let \( \rho(\bar{r}) \) is the distribution function of the primary particle interaction points with the absorber.

2. The stage of the photoelectron energy transformation into the energy of excess quasiparticles, i.e., into quasi-electrons in a DROID and into phonons in a SATID. Let \( f_v[s] \) be the GF of the number of quasiparticles produced by the primary particle, where \( s \) is the auxiliary variable of the generating function. The mean number of quasiparticles produced by primary particle is \( \bar{v} = E_{eff}/\varepsilon_{eff} \), where \( E \) is the energy of primary particle and \( \varepsilon_{eff} \) is the effective energy of quasiparticle creation, i.e., a quasi-electron in a DROID and a phonon with the energy higher than the energy bandgap of the STJ in a SATID. The variance of the number of quasiparticles obeys the low \( \sigma^2_v = F \cdot \bar{v} \), where \( F \) is the Fano factor. We suppose that the variance of the number of phonons with the energy higher than the energy bandgap obeys the same law as the variance of the number of quasi-electrons does.

3. The stage of quasiparticle’s propagation and transformation to the output signal of STJs. Let \( \tau_k(\bar{r}) \) is the probability of quasi-electron generated at the point \( \bar{r} \) to tunnel through the insulating barrier of the STJ number \( k \) in a DROID. In the case of a SATID, \( \tau_k(\bar{r}) \) is the probability of quasi-electron from Cooper pair beaked by phonon generated at the point \( \bar{r} \) to tunnel through the insulating barrier of the STJ number \( k \). The GF of this process has the form:

\[
f_{\tau_k(\bar{r})}[s] = 1 - \tau_k(\bar{r}) + \tau_k(\bar{r}) \cdot s.
\] (1)

These fluctuations have a quantum nature related to the wave-particle duality of a quasiparticle, as \( \tau_k(\bar{r}) \) has the sense of the modulus square of the transition amplitude of the quasiparticle from the point \( \bar{r} \) to the quasi-electron tunneled through the insulating barrier of the STJ number \( k \).

After the first tunneling, all quasi-electrons become identical and can tunnel back and forth. It can be shown that the GF of the numbers of quasi-electron’s multitunneling in the STJ number \( k \) has the form:

\[
f_{\eta_k}[s] = s \cdot (1 - p_{2k} + p_{2k} \cdot (1 - p_{1k}) \cdot s) / (1 - p_{1k} \cdot p_{2k} \cdot s^2).
\] (2)
From GF (2) the mean value and the variance of the numbers of quasi-electron’s multitunneling in the STJ number $k$ are:

$$m_k = (1 + p_{2k})/(1 - p_{1k} \cdot p_{2k})$$,

$$\sigma_{m_k}^2 = p_{2k} \cdot (1 - p_{2k} + 3p_{1k} + p_{1k} \cdot p_{2k})/(1 - p_{1k} \cdot p_{2k})^2$$,

where $p_{2k}$ and $p_{1k}$ are the probabilities of back and forth tunneling. In the absence of the backtunneling $p_{2k} = 0$, $m_k = 1$ and $\sigma_{m_k}^2 = 0$.

4. Let $f_{n_k}[s]$ be the GF of the STJ number $k$ electronic tract noise reduced to the amplifier input with the zero mean value and the variance $\sigma_{n_k}^2$.

For the model described above, the GF of the primary particle energy transformation into the signal at the imaging detector output takes the form:

$$f_Q[s] = \int_V dV \rho(\bar{r}) \cdot f_{n_k} \left( 1 - \sum_{k=1}^K \tau_k(\bar{r}) \right) + \sum_{k=1}^K \tau_k(\bar{r}) \cdot f_{m_k} \left[ \alpha_k(\bar{r}) \right] \prod_{k=1}^K f_{n_k} \left[ \alpha_k(\bar{r}) \right]$$,

where $K$ is the number of STJs.

For optimizing the energy resolution of the cryogenic imaging detector, we introduce in the GF (5) the correction factor $\alpha_k(\bar{r})$ for the STJ number $k$ signal when the incident particle is interacted at the point $\bar{r}$ of the absorber.

From the GF (5) the mean value $\overline{Q}$ of the output signal takes the form (e=1):

$$\overline{Q} = \int_V dV \rho(\bar{r}) \cdot \sum_{k=1}^K \overline{\alpha_k(\bar{r})} \cdot m_k \cdot \alpha_k(\bar{r}) = \int_V dV \rho(\bar{r}) \cdot \sum_{k=1}^K \overline{Q_k(\bar{r})} \cdot \alpha_k(\bar{r})$$

where $\overline{Q_k(\bar{r})}$ is the mean value of the STJ number k signal produced by the primary particle absorbed at the point $\bar{r}$.

The relative variance $\eta_Q^2$ related to the energy resolution of the detector by the expression $\Delta E/E = 2.355 \cdot \eta_Q$ is given by

$$\eta_Q^2 = \frac{\sigma_Q^2}{\overline{Q}^2} = \eta_n^2 + \left( \eta_n^2 - \frac{1}{\overline{V}} \right) \left( \eta_{sp}^2 + 1 \right) + \frac{1}{\overline{V} \cdot \left( \int_V dV \rho(\bar{r}) \sum_{k=1}^K \tau_k(\bar{r}) \cdot m_k \cdot \alpha_k(\bar{r}) \right)^2} \times \int_V dV \rho(\bar{r}) \sum_{k=1}^K \tau_k(\bar{r}) \left[ \sigma_{m_k}^2 + (m_k)^2 \right] \alpha_k^2(\bar{r}) + , (7)$$

$$\eta_{sp}^2 = \frac{1}{\overline{V}^2 \cdot \left( \int_V dV \rho(\bar{r}) \sum_{k=1}^K \tau_k(\bar{r}) \cdot m_k \cdot \alpha_k(\bar{r}) \right)^2} \times \int_V dV \rho(\bar{r}) \sum_{k=1}^K \sigma_{m_k}^2 \overline{m_k}^2 \alpha_k^2(\bar{r})$$

where $\eta_n^2$ is the relative variance of the number of quasiparticles produced by the primary particle, and

$$\eta_{sp}^2 = \frac{\int_V dV \rho(\bar{r}) \sum_{k=1}^K \tau_k^2(\bar{r}) \overline{m_k}^2 \alpha_k^2(\bar{r})}{\int_V dV \rho(\bar{r}) \sum_{k=1}^K \tau_k(\bar{r}) \overline{m_k} \alpha_k(\bar{r})}^2 - 1, (8)$$

represents the relative variance caused by spatial variations of the primary particle interaction point.
As \( \eta_v^2 = \frac{F}{\nu} \), then the second and the third terms in (7) are inversely proportional to the energy of the primary particle, and the fourth term is inversely proportional to the energy squared. As the first term in (7) is obviously independent on the energy of the primary particles, then the improvement in the energy resolution of the cryogenic imaging detectors can be achieved only by lowering the first term in (7).

The term \( \eta_{sp}^2 \) may be rewritten in the form:

\[
\eta_{sp}^2 = \left( \int dV \rho(\vec{r}) \sum_{k=1}^{K} (\overline{Q}_k(\vec{r}))^2 \cdot \alpha_k^2(\vec{r}) \right) - 1. \tag{9}
\]

Therefore, if we choose the correction factors inversely proportional to the mean values of the STJ output signals produced by the primary particles interacted at the point \( \vec{r} \) of the absorber:

\[
\alpha_k(\vec{r}) = \frac{1}{\overline{Q}_k(\vec{r})}, \tag{10}
\]

then the \( \eta_{sp}^2 \) is equal to zero. In this case, the energy resolution of the cryogenic imaging detector takes the form:

\[
\eta_Q^2 = \left( \eta_v^2 - \frac{1}{\nu} \right) + \frac{1}{\nu \cdot K^2} \int dV \rho(\vec{r}) \sum_{k=1}^{K} \left[ \eta_{m_k}^2 + 1 \right] / \tau_k(\vec{r}) + \frac{1}{\nu^2 \cdot K^2} \int dV \rho(\vec{r}) \sum_{k=1}^{K} \sigma_{m_k}^2 / \tau_k^2(\vec{r}), \tag{11}
\]

where \( \eta_{m_k}^2 = \sigma_{m_k}^2 / m_k^2 \).

Using this choice for the correction factors, the Osaka Electro-Communication University group improved the detector energy resolution for -particles from 11\% to 0.79\%.

3. Conclusion

This approach shows the significance of the fluctuations caused by spatial variations of the primary particle interaction point, and establishes the theoretical basis for the position correction method for improving the energy resolution of imaging detectors.

References

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