Analysis to the Influence of Inertia on Parallel VSG's Stability

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Abstract. Virtual Synchronous Generator (VSG) is a flexible control approach for power electronic converters in the power system. In this paper, the nonlinear model for a pair of VSGs in parallel is built, and corresponding small-signal linear model considering each link is further established based on the second-order model of synchronous generator. Compared to the response curves of parallel model of inverter power supply with characteristics of synchronous generator in MATLAB / SIMULINK, the accuracy of the small signal model is verified. With the help of small signal model and its analysis method, the virtual moment of inertia, an important parameter of virtual synchronous machine, is studied in detail. For parallel VSGs, the universal selection principle of virtual moment of inertia $J$ is put forward considering different parameters of two machines, and the results of simulation show that the proposed principle of parameter selection can expand the range of parameter selection and the stability of the system can be improved.

1. Introduction

Facing the threat of depletion of energy and environmental deterioration, renewable energy generation has received unprecedented attention. Renewable energy is integrated into the grid by distributed generations. To improve the flexibility of system operation and control, distributed generations are usually connected to power grid by the inverter interface [1]. The VSG concept is put forward to make the inverters with the inertia and impedance of synchronous motors, and to improve the stability of system operation [2]. Differential compensation part is added to the forward channel of the conventional VSG control structure to ensure the steady-state precision of the output power, meanwhile, the dynamic response speed is accelerated and the system damping is increased [3]. Another new control strategy of variable virtual inertia is presented to fast suppress the frequency fluctuations by selecting different virtual inertia values in real time. However, theoretical principles for the selection of virtual inertia are not provided, and it is easy to cause system oscillation if the parameters selection is not appropriate [4]. VSG technology can improve the stability of micro source accessed to the power system. Its stability can be analyzed with small signal analysis methods [5], [6]. The small-signal model for VSG is necessary and conducive to further research the critical modes and the parameter design of VSG. The single VSG small-signal model was built and the influences of the parameters on the stability was analyzed in [7], [8]. But the analysis was too brief. A small-signal model for two parallel VSG was established and the influence of the sensitivity on each parameter was researched. However, the capacity and parameters of the studied object were identical [9].
In this paper, the parallel model for the inverter with characteristics of synchronous generators is built. The small-signal model considering each link is set up. The data of the nonlinear model is obtained to avoid solving equations inaccurately with improper selection of the initial value. Using the small signal model, the virtual moment of inertia is analyzed in detail, and the principle to select the virtual moment of inertia for the inverter with synchronous generator characteristics is given. The stability of the system is improved, and its correctness is verified by simulation.

2. Control strategy for parallel VSGs

Control structure for a pair of parallel VSGs is shown in Figure 1. Two inverters are respectively filtered by LC and then connected to the common bus via the transmission lines to supply the loads. The control for the interface adopts the VSG algorithm. It simulates the moment of inertia, synchronous reactance and stator resistance of synchronous generators. Combined with the droop control, the inverter switches are controlled by the modulation waves generated from voltage and current loops [10].

![Figure 1. Double-machine parallel integrated control structure](image)

3. Small-signal modelling for parallel VSGs

3.1. Coordinate Transformation

In this paper, the modeling is based upon the d-q rotation coordinate. The coordinate of a certain virtual synchronous generator is selected as the reference to unify the coordinate of each machine. The angle of the virtually internal potential of the nth VSG is ahead of $\delta_n$. The output formula of each VSG is transformed to the reference coordinate and is derived as follows:

$$
\delta_n = \int (\omega_n - \omega_d) dt
$$

$$
\begin{bmatrix}
g_D

g_Q
\end{bmatrix} =
\begin{bmatrix}
\cos(\delta_n) & -\sin(\delta_n)

\sin(\delta_n) & \cos(\delta_n)
\end{bmatrix}
\begin{bmatrix}
g_d

g_q
\end{bmatrix}
$$

(1)

Where $\omega_n$ is the rotational angular speed, $g_{d-q}$ is the d-q axis variable and $g_{D-Q}$ is the D-Q axis variable.

3.2. Power Control

The output voltage and current of the inverter are detected firstly and then the instantaneous active power and the reactive power are calculated [11]. After filtering, the average output power can be obtained as shown below:
\[
\begin{align*}
\frac{dP}{dt} &= \omega (p - P) \\
\frac{dQ}{dt} &= \omega (q - Q)
\end{align*}
\] (2)

Where \( p \) and \( q \) are the instantaneous active power and reactive power. \( H(s) \) is the transfer function for the low-pass filter. \( \omega_c \) is cut-off angle frequency. \( P \) and \( Q \) are average active and reactive power respectively.

By way of distributing the active power between inverters reasonably and improving the response speed to the fluctuation, the classical second-order equation control strategy for the synchronous generators is used. This strategy mainly deals with the active power and its equation is shown as follows:

\[
\frac{d\omega}{dt} = \frac{\omega_{n0} - \omega}{J \omega_{n0} m_p} + \frac{P_{\text{ref}} - P}{J \omega_{n0}}
\] (3)

Where \( \omega \) is the output angular frequency of the inverter. \( \omega_{n0} \) is the rated reference angular frequency. \( J \) is the virtual moment of inertia. \( m_p \) is the sag coefficient for active power and frequency. \( P_{\text{ref}} \) is the reference value of the active power.

The reactive power is controlled using the sag of reactive power and voltage [12]. The output reference voltage amplitude is set as the reference voltage of d axis and the reference voltage of q axis is set to 0.

\[
\begin{align*}
v_{\text{dpd}} &= v_{n0} - n_q Q \\
v_{\text{dpq}} &= 0
\end{align*}
\] (4)

Where \( v_{\text{dpd}} \) and \( v_{\text{dpq}} \) are the output d-q reference voltage of the sag controller respectively. \( v_{n0} \) is the reference voltage and \( n_q \) is the sag coefficient of reactive power.

3.3. Virtual impedance control

Inverters are commonly connected to the distribution power grid and the power control can cause the coupling of the active and reactive power. At the same time, in order to avoid the delay caused by the low pass filter, the virtual impedance control is used [13].

\[
\begin{bmatrix} v_{\text{od}}^* \\ v_{\text{oq}}^* \end{bmatrix} =
\begin{bmatrix} v_{\text{dpd}} \\ v_{\text{dpq}} \end{bmatrix} - \begin{bmatrix} r_v & -\omega L_v \\ \omega L_v & r_v \end{bmatrix} \begin{bmatrix} i_{\text{od}} \\ i_{\text{oq}} \end{bmatrix}
\] (5)

Where \( v_{\text{od}}^* \) and \( v_{\text{oq}}^* \) are the output voltages after the virtual impedance and also the given reference values of the voltage and current loops. \( r_v \) and \( L_v \) are the virtual resistance and the virtual inductance. \( i_{\text{od}} \) and \( i_{\text{oq}} \) are the output current of the inverter.

3.4. Control of double voltage and current loop

The double loop structure of voltage and current is adopted to make the output voltage of the inverter track the given voltage without difference and respond to the load quickly [14]. According to intima principles, The PI controller can achieve no static error in the d-q coordinate. The control block diagram is illustrated as follows.
3.5. The whole model

Combined with the parts of interface and load, then linearize the system at the working point, the state equations for the whole system can be obtained, as shown in (6).

\[
\begin{align*}
\Delta \dot{x}_{\text{state}} &= A \Delta x_{\text{state}} \\
\Delta x_{\text{state}} &= [\Delta \omega_1; \Delta P_1; \Delta Q_1; \Delta \phi_{i1}; \Delta \phi_q; \Delta Y_{q1}; \Delta Y_{q2}; \Delta Y_{dg}; \Delta I_{d1}; \Delta I_{d2}; \Delta I_{q1}; \Delta I_{q2}; \Delta I_{ld}; \Delta I_{ldq}]
\end{align*}
\]

(6)

Where \( A \) is the system state matrix. \( \Delta x_{\text{state}} \) contains 29 state variables. \( \Delta \) represents the small perturbations.

3.6. Solution for the steady-state working point

When the system parameters vary, the steady-state working point will change and the state matrix and the eigenvalue need to be updated [15]. Considering VSG1 is chosen as the reference coordinate, its internal voltage angle is the zero reference angle for the system, and its angular frequency becomes unknown. The system has six unknown variables: \( \Delta x_{\text{state}} = [U_{v1}, \alpha_1, U_{v2}, \delta_{v2}, U_3, \delta_3] \). Equations are listed in (7).

\[
\begin{align*}
\frac{f_{q_{in}}}{f_{q_{in}}} &= P_{in} - \frac{\omega}{m_i} + \frac{\omega_1}{m_{\omega}} - P_{i\omega} = 0 \\
\frac{f_{q_{in}}}{f_{q_{in}}} &= U_{in} - \frac{U_{in}}{n_{in}} - Q_{in} = 0 \\
\frac{f_{q_{in}}}{f_{q_{in}}} &= P_{im} + P_{a2} - P_{a3} = 0 \\
\frac{f_{q_{in}}}{f_{q_{in}}} &= Q_{im} + P_{a2} - Q_{a3} = 0
\end{align*}
\]

(7)

Where \( P_{in} \) and \( Q_{in} \) are the active and reactive power injected into the input bus from the \( i \)th inverter. \( P_{a3} \) and \( Q_{a3} \) are the active and reactive power provided by the bus to the load respectively.

It is easy to cause the system oscillation when the initial value is not properly selected. Here the initial value is derived from the model, which avoids the problems of manual parameter adjustment and solution error. In summary, the small signal model is obtained and the dynamic characteristics of the system are described. In order to verify the correctness of the deduced small-signal model, the same load disturbance is set. Taking the parallel connection of VSG with different capacities as an example and the curves of each variable of the two models are basically identical.

4. Influence of Virtual moment of inertia on the eigenvalue of State Matrix

The steady-state operating point is introduced into the system state matrix and all characteristic roots are obtained. Then the dynamic characteristics of the system under different working conditions and
different parameters can be analyzed. In this paper, the virtual moment of inertia introduced by the characteristics of synchronous motor is studied mainly, and the change of the characteristic root of the system is analyzed when $J$ is from 0.0003 kg•m² to 1.6 kg•m². Each characteristic root’s real part is negative, and the system small signal is stable.

![Figure 3. Control block diagram in the d-q coordinate system](image)

Figure 3. Control block diagram in the d-q coordinate system

When the moment of inertia $J$ changes from small to large value, the characteristic root locus is shown in Figure 3. The distance to imaginary axis of the characteristic root is much longer than other characteristic roots, and the oscillation frequency represented by the imaginary part is similar to that of the steady state of the system. In the process of parameter changes, the characteristic roots 3-14 and 22-29 are nearly unchanged, that is, the change of parameter $J$ has no effect on them, and the following analysis would not consider the above characteristic roots.

As shown in Figure 3(b), it is characteristic roots 15-29. The characteristic root 15 and 16 are two unequal negative solid roots at the beginning, showing over-damping characteristics. With the increase of the moment of inertia, the pair of characteristic roots moves to the right and changes from the solid axis to conjugate complex root, and the absolute value of the imaginary part has a process of increasing and then becoming smaller. The damping and oscillation frequencies change accordingly.

By controlling the physical structure of the inverter in d-q coordinates and the power are decoupled, but the intermediate variables in the equation of state are still coupled. Here, there are always two conjugate complex roots in the characteristic root 17-21, which are the closest to the virtual axis. The change trend is consistent, the real part moves to the right and the imaginary part decreases. The other characteristic roots move in the solid axis and never cross the imaginary axis into the right half-plane. The smaller the real part, the shorter the time for the system to be stable and the smaller the amplitude is. But if $J$ is too small, it will cause frequently fluctuation of system frequency. As an important parameter of virtual synchronous machine, $J$ should consider its function and select value according to the performance requirement of virtual machine, rather than finding a curve with the shortest response time. From the small signal analysis, it can be seen that if $J$ is bigger than a certain threshold, the system will be unstable, but there will be a trend of stability when it continues to increase. This puts forward the requirement for the parameter selection of the moment of inertia [16].

5. Inertia matching method of double-VSG parallel

5.1. Parallel of VSG with 1:1 capacity

5.1.1. The moment of inertia of two machines is same. In this case, the moment of inertia of the two machines is exactly the same, and there exists a stable threshold value ($J_t$) in the process of parameter changes. If $J$ exceeds the threshold value, the characteristic root will cross the virtual axis to the right half-plane, although there is a tendency of stabilization, there is no stability at last. The root locus should be below the threshold value with reference to Figure 4, otherwise the system will be unstable.

5.1.2. The moment of inertia of two machines is different. The moment of inertia of the same capacity VSG is not exactly the same. If $J_1<J_2$, $J_2$ increases gradually, in this case, all the characteristic roots’ real parts are on the left side of the imaginary axis, $J_t$=0.1 as shown in Figure 4(a), which can guarantee
the stable operation of the system and the oscillation frequency of conjugate complex roots decreases first and then increases slightly. If \( J_1 \geq J_s \), similar to the previous section, \( J_1 = 4 \) as shown in Figure 4(b), the damping ratio decreases with the increase of \( J_2 \), and the characteristic roots shifts to the right side of the imaginary axis. But when \( J_2 \) continues to increase, the real part of the characteristic root will return to a negative value, re-stabilizing the system and playing an indirect role in extending the range of virtual moment of inertia.

![Figure 4. Sample diagram of variable J2 characteristic root locus](image)

5.2. Parallel of VSG with 2:1 capacity

5.2.1. The moment of inertia of two machines is same. The proportional virtual moment of inertia will not increase the total inertia of two machines when the two capacities are different, so here still reference to the previous analysis method. When the moment of inertia of two machines is exactly the same and varies from small to large, the characteristic root locus is similar to Figure 3, which is the same as the equivalent capacity. The difference is that the stability threshold of virtual moment of inertia \( (J) \) is obviously reduced due to the difference of the two capacities. Of course, the moment of inertia is also different when VSG is in parallel with different capacities. So it is just to make use of the parameter consistency to determine \( J_s \).

5.2.2. The moment of inertia of two machines is different. The capacities of two machines are different, so is the moment of inertia. \( J_1 \) is fixed, \( J_2 \) varies from small to large, the characteristic root locus is similar to Figure 4, and the conclusion is also consistent: if \( J_1 < J_s \), all the characteristic roots are on the left side of the imaginary axis, and the system is stable all the time; If \( J_1 \geq J_s \), the characteristic root will gradually change to the right side of the virtual axis, the system will be unstable, with the \( J_2 \) increasing, the characteristic root will soon become negative, and then the system will be stable again. In fact, the significance of using the re-stable characteristic root to ensure the stability of the system is mainly for two machines with different capacities, because if the two machines with the same capacity are parallel, the \( J_2 \) will be too large after being stable again, and the stability time will be increased.

5.3. Simulation analysis

In parallel operation of VSGs, whether the capacity is the same or not, the stability threshold can be determined according to the consistency of \( J \), as long as the \( J \) of one machine is smaller than the threshold, the stability of the system can be guaranteed; And when the moment of inertia required in the system is large, increasing the \( J \) of a VSG can also ensure the stable operation of the system. This situation is more suitable for different capacity, because of its smaller re-stable value. The analysis of small signal stability guides the selection of the virtual rotation inertia \( J \) of parallel VSG, the principle of matching according to the uniform parameters of the double machine is given, and it is verified by simulation.
6. Conclusion

The small-signal model of parallel inverter power supply with the characteristics of synchronous generator is built in this article. Compared with the response curve of nonlinear parallel VSG model in MATLAB/SIMULINK, the accuracy of small signal model is verified. Under different capacity conditions, the important parameters of the virtual synchronizer, the virtual rotation inertia, are analyzed in detail, and the adjustment range of inertia is determined. The parameter selection principle is provided for the virtual rotation inertia under the adaptive parameters and so on, so as to realize more flexible adjustment and improve the stability of the system. For parallel VSG, whether or not the capacity of two machines is the same is the same, the stability threshold can be determined according to the consistency of $J$. The parameter $J$ is usually lower than the stability threshold, but the system can also be kept stable according to this principle of selection even if $J$ is greater than the stability threshold, which is divided into the following two cases. (1) As long as the $J$ of one machine is less than the threshold, the stability of the system can be guaranteed. (2) When the required rotational inertia of the system is greater than the stability threshold, the proper increase of the moment of inertia can also ensure the stable operation of the system, where the case is more suitable for different capacity, because the re-stability threshold is smaller. In this paper, the virtual rotation inertia is discussed with the help of small signal model, and the correctness of the proposed principle is verified by nonlinear simulation, and the model has important guiding significance for the selection of each parameter.
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