Light fermions in quantum gravity

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Abstract. We study the impact of quantum gravity, formulated as a quantum field theory of the metric, on chiral symmetry in a fermionic matter sector. Specifically we address the question of whether metric fluctuations can induce chiral symmetry breaking and bound state formation. Our results based on the functional renormalization group indicate that chiral symmetry is left intact even at strong gravitational coupling. In particular, we found that asymptotically safe quantum gravity where the gravitational couplings approach a non-Gaußian fixed point generically admits universes with light fermions. Our results thus further support quantum gravity theories built on fluctuations of the metric field such as the asymptotic-safety scenario. A study of chiral symmetry breaking through gravitational quantum effects may also serve as a significant benchmark test for other quantum gravity scenarios, since a completely broken chiral symmetry at the Planck scale would not be in accordance with the observation of light fermions in our universe. We demonstrate that this elementary observation already imposes constraints on a generic UV completion of gravity.
1. Introduction

Any phenomenologically relevant theory of quantum gravity has to satisfy a number of physical requirements. In addition to internal or mathematical consistency, observations demand that such a theory has to provide for the existence of a semi-classical limit. Another phenomenological requirement is the existence of light matter (compared with the Planck scale). Upon coupling matter degrees of freedom to dynamical quantum gravity, it is not \textit{a priori} clear whether all the observed low-energy properties of matter remain unaffected. This provides for an observational window into the quantum gravity regime, since a quantum gravity theory proposed to describe our universe must not change any of the observed properties of matter. Here, a specific requirement is that dynamical quantum gravity must allow for the existence of light fermionic matter. In many studies of quantum gravity, matter is either ignored, or treated rather as a kinematic degree of freedom. For instance, in string compactifications, information about the existence of light matter is drawn from anomaly-cancelation arguments.

As soon as matter and interactions are taken into account dynamically, the existence of light matter is far from being self-evident: from a bottom-up viewpoint, the regime of quantum gravity is naturally defined as the domain where gravity fluctuations become as relevant as matter fluctuations. If gravity becomes even strongly interacting, its dynamical influence on the matter sector may even be similar to strong matter correlations as induced by the other forces of particle physics.

This question becomes particularly paradigmatic in the case of fermions. In standard particle physics scenarios, fermions are light because their mass is protected by chiral symmetry. So far, the mass of all fermionic matter content in the universe is associated with the phenomenon of chiral symmetry breaking. This mass generation is a consequence of strong correlations among fermions. In the electroweak sector, these strong correlations are provided by the Higgs sector, whereas gluon-induced interactions are responsible for mass generation in the strong interactions.

Gravity at first sight seems to have some similarity to these particle physics examples, as it is similar to the Yang–Mills theories in many respects. Therefore, this work is devoted to a first investigation of whether gravitational fluctuations can induce chiral symmetry breaking in
a chiral fermion sector. If such a mechanism exists, any theory of quantum gravity would have to control it or evade it in a natural way. Since such a mechanism would be active at or above the Planck scale, it would naturally force fermions to have masses of the order of the Planck scale in contrast to observation.

In this work, gravitational fluctuations are parameterized as fluctuations of the metric field. This is at least an effective description below or even near the Planck scale. In the context of Weinberg’s asymptotic safety scenario for quantum gravity [1], this can even be a consistent description up to arbitrarily short distances. As asymptotic safety is based on the existence of a non-Gaussian fixed point (NGFP) in the gravitational couplings, i.e. an interacting ultraviolet (UV) limit, the interplay between the gravitational sector and the chiral fermion sector is of particular interest.

Evidence for the existence of such a fixed point has been provided in different approaches [2–6]. Also causal dynamical triangulations (see e.g. [7] for a review) may be interpreted as providing evidence for this scenario. In particular, the functional renormalization group (RG), following the pioneering work of Reuter [8], has facilitated numerous studies supporting asymptotic safety in gravity [9–34]; for reviews see [35–38]. First steps in the investigation of the compatibility of asymptotically safe quantum gravity with quantized matter have been reported in [39, 40]. Here, the backreaction of fermionic and bosonic matter onto the gravitational fixed-point properties was investigated. The requirement for the existence of a physically admissible fixed point with a positive value for the Newton coupling then imposes constraints on the matter content of the universe. Most importantly, the matter content of the standard model of particle physics is compatible with asymptotically safe quantum gravity within the investigated truncation [39]. (For studies on the effect of the gravitational fixed point on gauge theories, see [41–44]. Studies dealing with a possible solution of the triviality problem in the Higgs sector through the coupling to gravity have been described in [45–47].)

In this work, we explore the gravity-induced interactions and potentially strong correlations in a chiral fermion sector for the first time using the functional RG. As gravity shares some features with the Yang–Mills theories, we are motivated by recent advances in QCD, where chiral symmetry breaking can be understood as the consequence of a critical dynamics among the chiral fermions which is triggered by gluon-induced strong correlations [48, 49]; for the successful determination of the critical temperature for chiral symmetry breaking in QCD using the functional RG, see [50–53]. It is tempting to speculate that gravity might facilitate a similar mechanism in a strong-coupling regime. If so, such a mechanism might exhibit a dependence on control parameters of the theory such as the fermion flavor number $N_f$. In fact, chiral quantum phase transitions as a function of $N_f$ have been observed in many systems such as many-flavor QCD, QED$_3$ or the three-dimensional (3D) Thirring model [49, 54–61].

The present study of the gravitationally stimulated chiral dynamics is based on a truncation of the full quantum effective action that concentrates on a Fierz-complete basis of chiral fermionic four-point functions in the point-like limit. Again, this is motivated by analogous studies on other theories, where such an ansatz provides for both an intuitive and quantitatively meaningful approach to chiral symmetry breaking.

As our main result, we do not find any indications of gravitationally stimulated chiral symmetry breaking within this ansatz. Whereas the Gaussian fermion matter fixed point turns into an interacting non-Gaussian one, the universality properties of this fixed point receive rather small modifications in the asymptotic-safety scenario if fermionic degrees of freedom dominate the matter sector. As a general pattern, gravitational binding which would favor chiral symmetry
breaking is compensated for by gravitational contributions to anomalous scaling of the fermion interactions. Within this minimal truncation, we can therefore conclude that asymptotic safety is well compatible with the existence (and observation) of light fermions despite an interacting UV sector that stimulates fermion self-interactions.

This paper is structured as follows: we will introduce the functional RG (FRG) as a tool for our investigation in section 2 and introduce the system under study in section 3. Results concerning the asymptotic-safety scenario as well as for general classes of effective theories of quantum gravity are presented in section 4. Finally, we conclude in section 5. Technical details can be found in appendix.

2. Functional renormalization group

The functional RG facilitates the non-perturbative evaluation of full correlation functions. Within the formulation used here, we focus on the scale-dependent effective average action, which is the generating functional of 1PI correlators that include all fluctuations from the UV down to the infrared (IR) scale $k$. At $k = 0$, $\Gamma_k$ coincides with the standard effective action $\Gamma = \Gamma_{k=0}$. The scale dependence of the effective average action is governed by the Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr}\{[\Gamma_k^{(2)} + R_k]^{-1}(\partial_t R_k)\}. \quad (1)$$

Here, $\partial_t = k \partial_k$, $\Gamma_k^{(2)}$ is the second functional derivative of $\Gamma_k$ with respect to the fields and $R_k$ is an IR regulator function. Accordingly, the right-hand side of the Wetterich equation depends on the full (field-dependent) regularized propagator $(\Gamma_k^{(2)} + R_k)^{-1}$, which is matrix-valued in field space. The supertrace STr contains a trace over the spectrum of the full propagator in all appropriate indices (i.e. on a flat background in the absence of classical background fields it translates into a momentum integral and a trace over Lorentz and internal indices). For Grassmann-valued fields the supertrace involves an additional negative sign. For reviews on the functional RG and the Wetterich equation, see [63–68].

Since quantum fluctuations generate all possible operators compatible with the symmetries and the field content of the microscopic action, the effective (average) action lives in theory space that is spanned by all these operators. Expanding $\Gamma_k = \sum_n g_n(k)\mathcal{O}_n$ into the infinite sum of all operators $\mathcal{O}$ with running couplings $g_n(k)$ allows us to rewrite the Wetterich equation as an infinite tower of coupled differential equations. In practice, a truncation of theory space to a smaller subspace is necessary. In the case of asymptotically safe quantum gravity, numerous studies have shown a high degree of stability of truncations of the Einstein–Hilbert type and beyond. The results appear to converge under generalizations of the truncation in various ways, as well as under a change of the regularization scheme, thus providing strong support for the existence of the NGFP in full theory space [8–19, 21–31, 34].

The perturbative non-unitarity of theories involving higher derivative operators does not directly apply here: indeed unitarity has to be reinvestigated within the non-perturbative setting and can remain intact within the asymptotic-safety scenario (see e.g. the corresponding discussions in [15, 27, 36, 37]).

In the following, we investigate the compatibility of the asymptotic-safety scenario with the existence of light fermionic matter. More generally, our results can be applied to generic effective theories of quantum gravity formulated in terms of fluctuations of the metric field and their interplay with chiral fermions.
3. Chiral fermions in quantum Einstein gravity

In the case of QCD-like theories, many studies based on functional methods suggest that chiral symmetry is broken for gauge couplings larger than a critical value \([48, 49, 69–71]\). In direct analogy it is tempting to expect that there exists a critical value for the Newton coupling at which metric fluctuations break chiral symmetry. This would agree with the picture that gravity is always attractive and thus should support fermionic binding phenomena.

We investigate this scenario in a specific fermionic system with a chiral SU\((N_f)_L \times SU(N_f)_R\) symmetry. We parameterize this system by an action of the form

\[
\Gamma_{\text{F}} = \int \! d^4x \sqrt{g} \bar{\psi}^i \gamma^\mu \nabla_\mu \psi^i + \frac{1}{2} \int \! d^4x \sqrt{g} \left[ \bar{\lambda}_-(k)(V - A) + \bar{\lambda}_+(k)(V + A) \right],
\]

where

\[
V = (\bar{\psi}^i \gamma_\mu \psi^j)(\bar{\psi}^j \gamma^\mu \psi^i),
\]

\[
A = -(\bar{\psi}^i \gamma_\mu \gamma^5 \psi^j)(\bar{\psi}^j \gamma^\mu \gamma^5 \psi^i).
\]

These fermionic self-interactions form a complete basis of four-fermion operators in the point-like limit which are invariant under the chiral symmetry. The parentheses indicate expressions with fully contracted Dirac indices. The \(\gamma\) matrices are understood to live in curved spacetime, being related to their flat-space cousins \(\gamma^a\) by the vielbein: \(\gamma^\mu = e_\mu^a \gamma^a\). Flavor indices are denoted by Latin letters \(i, j, \ldots\) and run from 1 to \(N_f\). The covariant derivative \(\nabla_\mu\) is given by \(\nabla_\mu = \partial_\mu + \frac{1}{16}[\gamma^a, \gamma^\mu] \omega_{\mu ab} \bar{\psi} \psi\), where \(\omega_{\mu ab}\) denotes the spin connection. The latter can be determined in terms of the Christoffel connection by demanding that \(\nabla_\mu e_\mu^a = 0\).

All other non-derivative SU\((N_f)_L \times SU(N_f)_R\) symmetric four-fermion operators, e.g. a flavor non-singlet scalar-pseudo-scalar interaction, can be transformed into some combination of the above ones by a Fierz transformation. Including all of the basis operators implies that we cover all possible channels for chiral symmetry breaking in the point-like, i.e. momentum-independent, limit. This is important, as gravity might pick one specific channel to induce the breaking of chiral symmetry. This ansatz of operators in the chiral sector is strongly motivated by similar lines of reasoning in QCD-like theories or other strongly correlated fermionic systems. There, the ansatz is capable of describing the approach to chiral symmetry breaking qualitatively as well as quantitatively. Of course, gravity may choose to break chiral symmetry in a fashion differing from Yang–Mills theory; potential further mechanisms will be briefly outlined below.

In \(d > 2\)D spacetime, four-fermion interactions are perturbatively non-renormalizable. In RG language, this translates into the fact that these couplings are irrelevant at the Gaußian fixed point. Accordingly, they have to be set to zero at the UV scale in a fundamental theory in a perturbative setting. Even though zero initially, such couplings are generated by interactions in the flow towards the IR, for instance, in the context of QCD or also when coupled to gravity. The flow of such fermionic self-interactions can then provide indications of chiral symmetry breaking. Beyond perturbation theory, non-Gaußian fixed points may exist that could allow us to construct a non-perturbatively renormalizable (asymptotically safe) theory with non-vanishing four-fermion interactions. In \(2 < d < 4\) dimensions, for instance, the Gross–Neveu model provides for a simple and well-understood example of asymptotic safety \([72]\).
At this point, we can already discuss the relation between chiral symmetry breaking and the fixed-point structure of the four-fermion couplings $\lambda_{\pm}$ [49]. For this, we introduce the dimensionless renormalized couplings $\lambda_{\pm}$ and the fermionic anomalous dimension $\eta_\psi$:

$$
\lambda_{\pm} = \frac{k^2 \lambda_{\pm}}{Z_\psi}, \quad \eta_\psi = -\partial_t \ln Z_\psi.
$$

(5)

Due to the one-loop form of the Wetterich equation, the $\beta$ functions for $\lambda_{\pm}$ have the generic form

$$
\beta_{\lambda_{\pm}} = (2 + \eta_\psi) \lambda_{\pm} + a \lambda_{\pm}^2 + b \lambda_{\pm} \lambda_{\mp} + c \lambda_{\mp}^2 + d \lambda_{\pm} + e.
$$

(6)

Herein the first term arises from dimensional (and anomalous) scaling. The quadratic contributions follow from a purely fermionic two-vertex diagram (cf diagram (2c) in figure 2). A tadpole contribution $\sim d \lambda_{\pm}$ may also exist, as well as a $\lambda_{\pm}$-independent part $\sim e$ which results from the coupling to other fields, for instance, arising from the covariant derivative in the kinetic term in equation (2). The numerical values for $a$, $b$ and $c$ depend on the regulator; the contributions $d$ and $e$ will also depend on further couplings. Specific representations will be given below.

Fixing all other couplings, the $\beta$ function of a given fermionic coupling $\beta_{\lambda_{\pm}} = \partial_t \lambda_{\pm}$ as a function of $\lambda_{\pm}$ is a parabola with two fixed points $\lambda_{\pm}^\pm$ where $\beta_{\lambda_{\pm}}(\lambda_{\pm}^\pm) = 0$. The coupled system of two fermionic $\beta$ functions then admits $2^2$ fixed points which need not necessarily all be real.

In order to illustrate the relevance of this fermionic fixed-point structure, let us concentrate on the $\lambda_+$ channel and perform a Fierz transformation to the standard scalar-pseudo-scalar channel,

$$
\lambda_+[\left(\bar{\psi}^i \gamma_\mu \gamma_5 \psi^i\right)^2 - \left(\bar{\psi}^i \gamma_\mu \gamma_5 \psi^i\right)^2] = \lambda_+ \left[\left(\bar{\psi}^i \gamma^\mu \psi^i\right)^2 - \left(\bar{\psi}^i \gamma^\mu \psi^i\right)^2\right],
$$

(7)

where $\lambda_+ \equiv \bar{\psi}^i \psi^i$ and similarly for the pseudo-scalar channel. Equation (7) is an exact Fierz identity if the couplings satisfy

$$
\lambda_+ = -\frac{1}{2} \lambda_+.
$$

(8)

The fixed-point structure in the $(V + A)$ channel hence implies a corresponding fixed-point structure in the standard scalar-pseudo-scalar channel, where chiral symmetry breaking is expected to be visible. In figure 1, the $\beta$ function $\beta_{\lambda_+} = \partial_t \lambda_+$ for this chiral channel is sketched for vanishing gravitational coupling. The two crossings of the parabola with the $\lambda_+$ axis indicate the two fixed points. The Gaußian fixed point at $\lambda_+^0 = 0$ is IR attractive, whereas the non-Gaußian fixed point $\lambda_+^c = \lambda_{c,cr} > 0$ is UV attractive (arrows indicate the flow towards the IR). If the system is in the attractive domain of the Gaußian fixed point, fermionic correlations die out quickly during the RG flow and the system remains in the chirally symmetric phase.

If the system starts to the right of the non-Gaußian fixed point $\lambda_+ > \lambda_{c,cr}$, the coupling runs towards very large values and actually diverges at a finite RG scale. Whereas the point-like fermionic truncation breaks down here, a transition to a bosonized description with a boson being related to a fermion bilinear identifies the chiral coupling with the mass term of a chiral bosonic field $\lambda_+ \sim 1/m_\psi^2$. The divergence of the fermionic coupling therefore indicates that the bosonic mass term drops below zero, which is a signature of symmetry breaking. It characterizes the onset of the effective chiral potential to acquire a Mexican-hat form. In NJL-type models, the non-Gaußian fixed point is identical to the critical value of the coupling $\lambda_{c,cr}$ required to put the system into the chiral symmetry broken phase.
Figure 1. Sketch of the $\beta$ function for the chiral channel $\lambda_\sigma$. In the absence of further interactions, the parabola-type $\beta$ function exhibits two fixed points: the Gaußian fixed point at $\lambda_\sigma^* = 0$ and a non-Gaußian fixed point at $\lambda_\sigma^* = \lambda_{\sigma, cr} > 0$. Arrows indicate the RG flow toward the IR. Further interactions can shift the parabola (dashed line) and lead to an annihilation of the two fixed points (dot-dashed line). This annihilation is an indication of an approach to chiral criticality. Such a scenario typically occurs in QCD-like theories [49, 73–75].

A new feature comes about by coupling the fermionic system to another interaction, e.g. a gauge theory. Then, an originally weakly coupled fermion sector near the Gaußian fixed point can dynamically be driven to chiral criticality. For this, the Gaußian fixed point needs to annihilate with the non-Gaußian fixed point, such that the corresponding $\beta$ function for $\lambda_\sigma$ becomes completely negative, as indicated by the dashed and dot-dashed lines in figure 1. Fixed-point annihilation is thus an indication of an approach to chiral criticality. This scenario is indeed realized, e.g., in QCD-like and other theories [49, 73–75]. With regard to the structure of equation (6), the last two terms together with the gravitational contribution to $\eta_\psi$, in principle, have the potential to destabilize the fermionic fixed points and drive the fermionic system to criticality. Whether or not this happens is a quantitative question that has to be addressed by integrating out gravitational fluctuations in a given quantum gravity theory.

For the gravitational part we work in the background field formalism [76], where the full metric is split according to

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu},$$

(9)

where this split does not imply that we consider only small fluctuations around, e.g., a flat background. Within the FRG approach we have access to physics also in the fully non-perturbative regime. This formalism, being highly useful in non-Abelian gauge theories (see, e.g., [65, 66]), is mandatory in gravity, since the background metric allows for a meaningful notion of ‘high-momentum’ and ‘low-momentum’ modes as implied by the spectrum of the background covariant Laplacian.

The desired feature of background independence naively seems spoiled in this way; however, the background formalism turns out to be merely a technical tool, leaving physical results unaffected and thus independent of the background [35].

Following standard approximations we work within a single-metric truncation, i.e. we set $g_{\mu\nu} = \bar{g}_{\mu\nu}$ after evaluating $\Gamma_k^{(2)}$. As suggested by recent studies [31, 77], qualitative results in the Einstein–Hilbert sector are not affected by this approximation.
On a general (curved) spacetime our truncation then reads

$$\Gamma_k = \Gamma_{k\text{-EH}} + \Gamma_{k\text{-gf}} + \Gamma_{k\text{-F}},$$

where the Einstein–Hilbert term and the gauge-fixing term are given by

$$\Gamma_{k\text{-EH}} = 2\tilde{\kappa}^2 Z_N(k) \int d^4x \sqrt{\bar{g}} (- R + 2\tilde{\lambda}(k)).$$

$$\Gamma_{k\text{-gf}} = \frac{Z_N(k)}{2\alpha} \int d^4x \sqrt{\bar{g}} \bar{g}^{\mu\nu} F_\mu[\bar{g}, h] F_\nu[\bar{g}, h].$$

with

$$F_\mu[\bar{g}, h] = \sqrt{2\tilde{\kappa}} \left( \bar{D}^\nu h_{\mu\nu} - \frac{1+\rho}{4} \bar{D}_\mu h^\nu \right).$$

Herein, $\tilde{\kappa} = (32\pi G_N)^{-1/2}$ is related to the bare Newton constant $G_N$. We denote the cosmological constant by $\tilde{\lambda}$ without any subscript. It should not be confused with the four-fermion couplings $\tilde{\lambda}_\pm$.

A ghost sector corresponding to Faddeev–Popov gauge fixing is implicitly understood here. Minimally coupled matter is at first sight not coupled to Faddeev–Popov ghosts; however, such non-standard couplings will be generated during the RG flow even within a minimally coupled truncation. In the present study, we neglect such matter–ghost interactions.

For the vielbein, we work in the symmetric vielbein gauge [78, 79] such that $O(4)$ ghosts do not occur. This gauge also allows us to reexpress vielbein fluctuations purely in terms of metric fluctuations.

Details of the second functional derivative of the effective action can be found in appendix.

The fermionic self-interactions $\sim \tilde{\lambda}_\pm$ do not directly contribute to the pure gravity flow. Technically this is true, because no one-loop diagram containing a fermionic four-point vertex can be formed that has only gravitons on external legs. Hence, the Einstein–Hilbert sector receives contributions only from the minimally coupled kinetic fermion term as determined in [39], where the approximation $Z_\psi = 1$ has been used.

Our new task here is to compute the gravitationally stimulated flow of the fermion interactions $\tilde{\lambda}_\pm$. For this, a flat-background calculation, setting $\bar{g}_{\mu\nu} = \delta_{\mu\nu}$, is fully sufficient and technically favorable.

In the following, we ignore a nontrivial running of the fermion kinetic term by setting $Z_\psi = 1$. In the Yang–Mills theory, this is justified in the Landau gauge $\alpha \to 0$, where this flow of $Z_\psi$ vanishes in a similar truncation [74]. In gravity, however, the flow of $Z_\psi$ does receive nontrivial contributions even in the Landau–deWitt gauge $\rho \to \alpha \to 0$. This marks a first difference between gravity and the Yang–Mills theory in this context. We keep track of this difference by maintaining the dependence of the flow on the fermion anomalous dimension $\eta_\psi$. Whereas $\eta_\psi = 0$ in the present approximation, we will later treat $\eta_\psi$ as a free parameter to explore possible consequences of this difference between the Yang–Mills theory and gravity.

We decompose the metric fluctuations $h_{\mu\nu}$ into a transverse traceless tensor, a transverse vector, a scalar and the trace part. We then specialize to the Landau–deWitt gauge, $\rho \to \alpha \to 0$, which implies that only the transverse traceless tensor $h_{\mu\nu}^{TT}$ and the trace mode $h = \bar{g}^{\mu\nu} h_{\mu\nu}$ can contribute to the flow of the fermionic couplings (see also [29]).

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Figure 2. Contributions to the running of the four-fermion couplings, sorted according to the number of vertices they contain. The diagrams containing curly lines receive contributions only from the trace mode; the diagrams with spiraling lines exist only for the TT mode. The functional RG equation receives contributions from these diagram types with all internal lines and all vertices denoting full regularized propagators and vertices, respectively. The right-hand side of the flow is given by the \( \tilde{\partial}_t \) derivative of these diagrams, yielding corresponding regulator insertions in the internal propagators; cf equation (14).

\[
\partial_t \Gamma_k = \frac{1}{2} \text{Str} \{ [\Gamma_k^{(2)} + R_k]^{-1} (\partial_t R_k) \} = \frac{1}{2} \text{Str} \tilde{\partial}_t \ln P_k + \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \text{Str} \tilde{\partial}_t (P_k^{-1} \mathcal{F}_k)^n,
\]

where the derivative \( \tilde{\partial}_t \) in the second line by definition acts only on the \( k \) dependence of the regulator, \( \tilde{\partial}_t = \int \partial_t R_k \frac{\delta}{\delta R_k} \). Since each factor of \( \mathcal{F}_k \) contains a coupling to external fields, this expansion simply corresponds to an expansion in the number of vertices. As we are interested in the flow of the four-fermion coupling, we can neglect terms with more than four vertices. The contributing terms are then given by the diagrams in figure 2.

Diagrams (2c), (3a), (4a) and (4b) occur when the Yang–Mills theory is coupled to fermions minimally. The additional diagrams can be traced back to the fact that the volume element containing \( \sqrt{g} \) generates additional graviton–fermion couplings. Also the covariant derivative in
Table 1. Fixed-point values for the gravitational couplings in the Einstein–Hilbert sector as a function of the number of chiral Dirac fermions, as computed in [39].

| $N_f$ | 1   | 2   | 3   | 4   | 5   | 10  | 25  |
|-------|-----|-----|-----|-----|-----|-----|-----|
| $g_*$ | 0.52| 2.30| 3.67| 3.84| 3.79| 3.11| 1.84|
| $\lambda_*$ | 0.21| -1.38| -4.34| -6.18| -7.64| -12.33| -17.90|

the kinetic term generates not only one- but also two-graviton fermion couplings. An additional triangular diagram, built from a two-fermion two-graviton vertex and two vertices coupling the graviton to one external and one internal (anti)fermion, vanishes in the Landau–deWitt gauge. The former vertex exists only for the transverse traceless, whereas the latter couples only to the trace mode. Since the metric propagator is diagonal in these modes for all choices of gauge parameters $\alpha$ and $\rho$, a non-vanishing diagram of this type cannot be constructed.

For the following discussion, we introduce the dimensionless renormalized gravitational couplings

$$
\frac{g}{Z_N} = \frac{G_N k^2}{Z_N} = \frac{k^2}{32\pi \kappa^2 Z_N}, \quad \frac{\lambda}{Z_N} = \frac{\bar{\lambda}}{k^2},
$$

and the corresponding anomalous dimension for the metric

$$
\eta_N = -\partial_t \ln Z_N.
$$

Considering metric fluctuations within a general effective theory of quantum gravity, these coupling constants can acquire a wide range of values near the Planck scale. By contrast, these couplings are tightly constrained in the asymptotic-safety scenario in the deep UV by the properties of a UV fixed point. In fact, these fixed-point values are a result of the formalism and not a parameter of the theory. In this aspect, asymptotic safety is distinct from many other approaches to quantum gravity, since here the microscopic action is a prediction and not an assumption of the theory. The anomalous dimension of the background metric yields $\eta_N = -2$ as a necessary requirement for the existence of the non-Gaussian fixed point [8]. The fixed point values $g_*, \lambda_*$ of the gravitational couplings depend on the regularization scheme and also on the matter content. Within the regularization scheme and gauge choice used in [39], the fixed-point values given in table 1 were obtained that will be used in our analysis for illustration.

In the following, we investigate the influence of metric fluctuations on the chiral fermion sector taking into account both types of scenarios for quantum gravity.

4. Flow of the fermion sector

Let us now discuss the flow in the fermion sector as induced by the various diagrams sketched in figure 2.

First we observe a cancelation between the two box diagrams (4a) and (4b). This is not specific to gravity, but occurs whenever a Yukawa-type fermion–antifermion–$\phi$ vertex with a scalar field $\phi$ without flavor (or internal-symmetry) and/or Dirac indices exists in the theory. As these box diagrams involve only the trace mode, this cancelation mechanism between ladder
and crossed-ladder topologies is at work here. A similar mechanism is active in a non-chiral Yukawa coupling of the type $\phi \psi^i \psi^i$.

This marks an important difference to Yang–Mills interactions, as corresponding box diagrams are the only contribution that generate the four-fermion interaction even if they are set to zero initially. Since gravity gives rise to a larger number of vertices from a minimally coupled kinetic fermion term, the diagram (2a) in figure 2, being absent in the Yang–Mills theory, will create this interaction here. Therefore the $\beta$ functions, as given by equation (6), contain similar types of terms as in the Yang–Mills theory.

Schematically, the $\beta$ functions for the dimensionful couplings $\tilde{\lambda}_{\pm}$ for a general regulator $R_k(p^2)$ are given by

$$\beta_{\tilde{\lambda}_{\pm}} = \partial_{\tilde{\lambda}_{\pm}} = 2 \left[ \tilde{\lambda}_{\pm} \frac{1}{2} f[0, 0, 1] - \tilde{\lambda}_{\pm} \frac{3}{8} f[0, 1, 0] \mp \frac{15}{312} f[0, 2, 0] - \tilde{\lambda}_{\pm} \frac{3}{16} f[1, 0, 1] 
+ \tilde{\lambda}_{\pm} \frac{3}{4} \left( \frac{p}{k} \right)^{2} f[2, 0, 1] + \text{fermion loops} \right].$$

The fermionic contribution indicated by the fermion-loop term corresponds to the diagram class (2c) in figure 2 and was first calculated in [74]. The regulator-dependent dimensionful threshold functions are defined by

$$I[n_t, n_{TT}, n_{h}] = \tilde{\theta} \int \frac{d^4 p}{(2\pi)^4} \left( p^2 \right)^n \frac{1}{(Z_p p^2(1 + r_k(t^2)))^{n_t}} \times \frac{1}{(\Gamma_k^{(2)}(1 + r_k(t^2)))^{n_{TT}}},$$

where $n = n_{TT} + n_t + n_{h} - 1$. In the above notation, we have already used regulators of the type $R_k = \Gamma_k^{(2)}(Y_k)$, with a dimensionless regulator shape function $r_k(y)$ and $y = p^2/k^2$. Specializing to linear regulators of the type

$$r_k^{\text{grav}}(p^2) = \left( \frac{\Gamma_k^{(2)}(k^2), \Gamma_k}{\Gamma_k^{(2)}(p^2)} - 1 \right) \theta(k^2 - p^2), \quad r_k^{\text{term}}(p^2) = \left( \sqrt{\frac{k^2}{p^2}} - 1 \right) \theta(k^2 - p^2),$$

the fermionic flow equations for the dimensionless couplings $\lambda_{\pm}$ can be determined explicitly:

$$\partial_t \lambda_- = 2(1 + \eta_\phi) \lambda_- + 2 \left[ -\frac{5g(\eta_N - 6)}{24\pi(1 - 2\lambda)^2} \lambda_- - \frac{g(-6 + \eta_N)}{4\pi(3 - 4\lambda)^2} \lambda_- - \frac{5g^2(\eta_N - 8)}{128(-1 + 2\lambda)^3} \right] = \frac{\lambda_-}{\lambda_-} \frac{g(36\eta_N - 7(34 - 24\lambda + \eta_N(-3 + 4\lambda)))}{35\pi(3 - 4\lambda)^2} \lambda_- - \frac{9g(21\eta_N + 24(-14 + \eta_N) - 32(-7 + \eta_N)\lambda)}{448\pi(3 - 4\lambda)^2} \lambda_- \right]$$

$$+ (-5 + \eta_\phi) \frac{\lambda_-^2 - N_\lambda^2 - N_\lambda^2}{40\pi^2},$$

(19)

$$\partial_t \lambda_+ = 2(1 + \eta_\phi) \lambda_+ + 2 \left[ -\frac{5g(\eta_N - 6)}{24\pi(1 - 2\lambda)^2} \lambda_+ - \frac{g(-6 + \eta_N)}{4\pi(3 - 4\lambda)^2} \lambda_+ + \frac{5g^2(\eta_N - 8)}{128(-1 + 2\lambda)^3} \right] = \frac{\lambda_+}{\lambda_+} \frac{g(36\eta_N - 7(34 - 24\lambda + \eta_N(-3 + 4\lambda)))}{35\pi(3 - 4\lambda)^2} \lambda_+ - \frac{9g(21\eta_N + 24(-14 + \eta_N) - 32(-7 + \eta_N)\lambda)}{448\pi(3 - 4\lambda)^2} \lambda_+ \right]$$

$$+ (-5 + \eta_\phi) \frac{-2\lambda_- \lambda_+ - 2N_\lambda \lambda_- + 3\lambda_+^2}{40\pi^2}.$$  

(20)
Herein, the single terms correspond to the diagrams in figure 2 in the following sequence: the first terms are the dimensional scaling terms of $\lambda_\pm$. The first term in square brackets corresponds to the transverse traceless tadpole (1a) and the second to the conformal tadpole (1b). The third term in square brackets that enters the two $\beta$ functions with a different sign is represented by the two-vertex diagram (2a) with internal metric propagators only. The mixed two-vertex diagram (2b) results in the fourth term in square brackets. Finally, the three-vertex diagram (3a) corresponds to the last term in square brackets. The fermion-loop contributions (2c) are represented in the two different last terms; they agree with [74].

We find four pairs of (real) non-Gaussian fixed points for $\lambda_\pm$ as a function of $(g, \lambda, N_f, \eta_N, \eta_\psi)$. For gravity approaching a non-Gaussian fixed point, $g_* \neq 0$, the fermionic Gaussian fixed point is shifted and also becomes non-Gaussian. If all four fixed points persist beyond this truncation, each one defines a UV universality class of the fermionic matter sector. The fixed points can quantitatively be classified by their number of relevant directions and the corresponding critical exponents.

The universal critical exponents can be read off from the linearized form of the $\beta$ functions for general couplings $g$, in the vicinity of the fixed point $g_*$,

$$\partial_t g_i = \sum_j B_{ij} (g_j - g_j^*) + \cdots,$$  \hspace{1cm} (21)

where the stability matrix $B_{ij}$ is defined by

$$B_{ij} = \frac{\partial \beta_{g_i}}{\partial g_j} \bigg|_{g=g_*}. \hspace{1cm} (22)$$

Equation (21) is solved by

$$g_i(k) = g_i^* + \sum_n C_n V^n_i \left( \frac{k}{k_0} \right)^{-\theta_n}. \hspace{1cm} (23)$$

Herein the critical exponents $\{\theta\} = -\text{spect}(B_{ij})$ are minus the eigenvalues of the stability matrix and $V^n$ are the (right) eigenvectors of $B_{ij}$. The scale $k_0$ is a reference scale and $C_n$ are constants of integration.

In order for the flow to hit a fixed point in the UV, all $C_n$ pertaining to irrelevant directions with $\theta_n < 0$ have to be set to zero. By contrast, the $C_n$ for relevant directions with $\theta_n > 0$ are free physical parameters that determine the long-range physics. As a consequence, a non-Gaussian fixed point can be used to construct a predictive fundamental theory if it has a finite number of relevant directions.

In the absence of gravity, the Gaussian fixed point has two irrelevant directions, both with critical exponent $\theta_{\text{Gau}} = -2$, corresponding to the standard power-counting canonical dimension of the fermion interactions in $d = 4$ dimensions. The other three non-Gaussian fixed points all have at least one relevant direction with critical exponent $\theta = 2$, as can be proven on general grounds [74]. Two of these fixed points have an additional irrelevant direction with negative critical exponent. The last fixed point has another relevant direction with positive critical exponent. Let us now discuss the effect exerted by metric fluctuations on the chiral fixed-point structure.
4.1. Asymptotically safe quantum gravity

Let us analyze the fermionic flow in the asymptotic-safety scenario near the gravitational UV fixed point where $\eta_N = -2$ and $g$ and $\lambda$ approach their fixed point values $g_\ast, \lambda_\ast$ as a function of the number of fermions $N_f$ as determined in [39]; cf table 1. We also set $\eta_\psi = 0$ here, as is consistent with our truncation.

As one of our main results, the fermionic fixed-point structure persists under the inclusion of metric fluctuations. Since the four-fermion couplings do not couple back into the flow of the Einstein–Hilbert sector, the stability matrix has a $2 \times 2$ block of zeros off the diagonal. Therefore the gravitational and fermionic critical exponents are determined by the eigenvalues in the Einstein–Hilbert sector and the fermionic subsector separately. Accordingly, the gravitational critical exponents are given by the well-known exponents with positive real parts in the Einstein–Hilbert sector (see [38] for an overview of typical values without the effect of the minimally coupled fermions) and the two real critical exponents from the fermionic subsector.

Again we have four different fermionic fixed points at our disposal, each defining its own matter universality class, which have two, one or no relevant directions. The dependence of the critical exponents on $N_f$ at each of the four fixed points is shown in figure 3. These critical exponents are determined by inserting the fixed point values of the gravitational couplings taken from [39] into the fermionic part of the stability matrix. As these fixed point values are determined within a slightly different regularization scheme, this scheme-dependent error adds to the systematic error of our truncation. Nevertheless, the general chiral fixed point structure is rather insensitive to variations of the non-universal input.

The critical exponents approach the limiting values of the purely fermionic system for large $N_f$. This is due to the following mechanism at work here: as shown in [39], the backreaction of a minimally coupled fermion sector onto the Einstein–Hilbert sector shifts $\lambda_\ast$ to increasingly negative values as a function of $N_f$. In the propagators, a negative value for $\lambda$ acts similarly to a mass term for the metric. This suppresses the contribution from metric fluctuations to $\beta_{\lambda,\pm}$ for large $N_f$. This decoupling mechanism induced by an increasingly negative cosmological constant ensures that the properties of the matter sector will not be strongly altered by metric fluctuations.

This chiral fixed-point structure is illustrated in figure 4, where the fixed point positions and the RG flows towards the IR are depicted in the $(\lambda_+, \lambda_-)$ plane for $N_f = 2$. The pure fermionic flow in the upper panel differs very slightly from the corresponding flow including the metric fluctuations in the gravitational fixed point regime (lower panel). Apart from minor shifts of the fixed point positions, the flow diagrams are very similar.

It should be emphasized that the decoupling mechanism due to a negative cosmological constant is only active in theories with a dominant number of fermionic degrees of freedom, as is the case for the standard model. Since minimally coupled scalars shift the fixed-point value for the cosmological constant towards $\lambda_\ast \to -\frac{1}{2}$ [39], a larger number of scalars even results in an enhancement of metric fluctuations. As a consequence, even at the shifted Gaussian fixed point the fermionic system can develop strong correlations, since the fixed point values for $\lambda_{\pm}$ can then become quite large (cf figure 7). Accordingly, in theories with a supersymmetric matter content and low-scale supersymmetry breaking, such a decoupling mechanism might not occur or occur only in a much weaker fashion. Supersymmetric theories with a quantum gravity embedding may thus have to satisfy stronger constraints as far as the initial conditions of their RG flow are concerned.
Figure 3. Critical exponents in the fermionic subsector as a function of $N_f$, where we always plot one critical exponent with red dots and the second one with blue squares. The upper plot corresponds to the shifted GFP and therefore has irrelevant directions only.
Figure 4. Flow lines towards the IR in the chiral \((\lambda_+, \lambda_-)\) plane for \(N_f = 2\). The upper panel shows the flow with \(g = 0 = \lambda\) and the lower one with \(g \approx 2.3\) and \(\lambda \approx -1.38\). Both panels differ only slightly, since the decoupling mechanism due to the negative cosmological constant induces a strong suppression of the metric fluctuations. Dots represent the shifted Gaussian fixed point with two irrelevant directions and diamonds/squares depict non-Gaussian fixed points with one/two relevant directions.

4.2. General effective quantum gravity theories

Let us broaden our viewpoint slightly by considering a wider range of effective theories of quantum gravity. Consider an unknown UV completion of quantum gravity, which may differ strongly from a description in a QFT framework, e.g. by introducing a physical discreteness scale for spacetime, possibly including the matter sector in a unified manner. Still, effective metric degrees of freedom may be expected to become relevant at a scale \(k_0\) close to or below the Planck scale (see also [80]). The underlying microscopic theory may effectively reduce to a quantum field theory for matter and effective metric degrees of freedom with gravity rapidly
Figure 5. Flow toward the IR in the $\lambda_+, \lambda_-$-plane for $\eta_N = 0, \eta_\psi = 0, g = 0.1, \lambda = 0.1$ and $N_f = 6$. For initial values to the right of the red lines, the chiral system is in the universality class of the (shifted) Gaußian fixed point. Any microscopic theory that would put the effective quantum field theory to the left of the red lines would generically not support light fermions.

becoming semi-classical toward the IR. Without any knowledge of the underlying theory, the effective quantum field theory may be at any point in theory space.

For scales $k < k_0$, accordingly our present framework becomes applicable even for these different approaches to quantum gravity. Within this framework, we thus allow for any value of the chiral couplings and admissible values of the gravitational couplings (i.e. a positive Newton constant, and $\lambda < 1/2$ in order to avoid the potentially artificial propagator poles). The anomalous dimensions $\eta_N$ and $\eta_\psi$, representing inessential couplings, will be determined by the effective field theory. Nevertheless, in our truncated framework, we allow them to acquire a priori unknown values of $O(1)$.

Following the RG flow further towards the IR, the fermionic sector should approach the Gaußian fixed point in standard scenarios in order to be weakly correlated on scales where metric fluctuations become unimportant, and gauge boson and matter fluctuations start to dominate the picture. This requires that the initial values of the four-fermion couplings, which can (in principle) be determined from the underlying microscopic theory, have to lie within the basin of attraction of the Gaußian fixed point. This presents a nontrivial requirement that has to be fulfilled by any theory of quantum gravity.

As a specific example, we depict the basin of attraction for $\eta_N = 0, \eta_\psi = 0, N_f = 6, g = 0.1$ and $\lambda = 0.1$ in figure 5.

For generic values of $g$ and $\lambda$, the system can be altered considerably. As a first observation, the parabolas characterizing the fermionic $\beta$ functions broaden as a function of $g$ for a fixed value of $\lambda$, as shown in figure 6. In this example, we plot the $\beta$ function for $\lambda_+$ for fixed $N_f = 2$, $\eta_N = -2, \eta_\psi = 0$ and $\lambda = 0$. We set $\lambda_-$ on the shifted Gaußian fixed point value.

A particularly strong effect can be observed for positive values of $\lambda$. Here, the contribution from the metric sector is further enhanced for $\lambda > 0$. Indeed the $\beta$ functions show the well-known divergence for $\lambda = \frac{1}{2}$. Whereas the divergence is likely to be an artifact of the simple

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Figure 6. $\beta_\lambda$ as a function of $\lambda_+$ for different values of $g$. The various curves correspond to $g = 0$ (full blue), $g = 0.2$ (red dashed), $g = 0.5$ (purple dotted) and $g = 2$ (dotdashed). The inlay shows the region around the Gaußian fixed point, illustrating a significant shift from the non-interacting limit.

Figure 7. Fixed-point value of $\lambda_+$ at the shifted Gaußian fixed point for $N_f = 2$, $\eta_\psi = 0$ and $\eta_N = 0$ as a function of $\lambda$ for $g = 1$ (full blue line), $g = 0.5$ (red dotted line) and $g = 0.2$ (magenta dashed line).

Einstein–Hilbert truncation in the IR, the response of the gravitational propagators naturally suggests an enhancement of metric fluctuations for positive cosmological constant $\lambda$.

As an example, we plot the value of $\lambda_+$ at the shifted Gaußian fixed point in figure 7.

We observe that the Gaußian fixed point can be shifted to considerably larger values of $(\lambda_-, \lambda_+)$. This implies that the system will be strongly interacting in this sector even at the fixed point that corresponds to the Gaußian fixed point in the absence of gravity. This may exert a strong influence on the physical properties of this shifted Gaußian universality class. As an example of such a significant deformation, we find that a negative anomalous dimension can in principle induce a crucial change in the fixed point structure. In particular, the shifted Gaußian fixed point ‘collides’ with a fixed point with one relevant direction at a negative value for $\eta_\psi_{\text{crit}}$ (see figure 8). At this point, one critical exponent of each fixed-point approaches zero.
Figure 8. At $\eta_{\psi} \approx -1.0592$ (for the specific parameter values $N_f = 6$, $g = 0.1$, $\lambda = 0.1$, $\eta_N = -2$), the Gaußian fixed point and a fixed point with one relevant direction fall on top of each other (in the lower right quadrant).

For values $\eta_{\psi} < \eta_{\psi}^{\text{crit}}$ the two fixed points move apart again. Assuming that in the regime of strong gravity fluctuations, these induce a large negative value for $\eta_{\psi}$, the flow towards the semiclassical IR would then cross $\eta_{\psi}^{\text{crit}}$. Accordingly, points that lie in what at a first glance seems to be the basin of attraction for the shifted Gaußian fixed point can leave this region during the flow. Precise constraints for such quantum gravity theories and the associated chiral sector then depend on the dynamical details of the full flow.

4.3. Metric versus gauge boson fluctuations

At first sight, the absence of gravity-stimulated chiral symmetry breaking seems surprising. Since gravity leads to attractive forces between matter, it is plausible to expect that binding phenomena are enhanced upon the inclusion of gravity. It is therefore instructive to confront our results with the chiral symmetry breaking mechanism in QCD-like theories.

Several technical differences have already become obvious: (i) the fermion anomalous dimension $\eta_{\psi}$ does not vanish in gravity in the Landau–deWitt gauge (even though we have set it to zero in the present approximation). (ii) Ladder and crossed-ladder box diagrams ((4a) and (4b) in figure 2) cancel in gravity, but play an important role in the approach to chiral symmetry breaking in QCD.

Further differences can be read off from the above analysis. For the fixed-point annihilation to occur (cf figure 1), the terms $\sim g^2$ in the fermionic flows ((2a) in figure 2) have to dominate. In the case of gravity, they are however outweighed by the tadpole terms $\sim g\lambda_{\pm}$.

In more physical terms, the $\sim g^2$ terms describe the attractive nature of gravity, whereas the tadpole terms $\sim g\lambda_{\pm}$ play the role of a gravity contribution to the (anomalous) scaling of the fermion couplings, $\partial_t \lambda_{\pm} = +2(1 + \eta_{\psi} + \cdots g)\lambda_{\pm} + \cdots$. Quite generally, the fixed point structure in the fermionic flows for $d > 2$ arises from a balancing between dimensional scaling $\partial_t \lambda_{\pm} \sim \lambda_{\pm}$ and fermion fluctuations $\sim \lambda_{\pm}^2$. Whereas gauge-field fluctuations support...
the fermionic fluctuation channels, metric fluctuations also have a strong influence on the anomalous dimensional scaling which counteracts the general attractive effect of gravity.

This viewpoint is further supported by other technical observations: whereas gravity is channel blind with respect to the scaling terms, i.e. \( \partial_t \lambda_i \sim g \lambda_i \), gauge boson fluctuations with coupling \( g_{\text{gb}} \) also give rise to terms \( \partial_t \lambda_i \sim g_{\text{gb}}^2 \lambda_j \) with \( i \neq j \) that rather behave like the above-mentioned fluctuation terms. Finally, we should mention that there are further examples that fluctuations of attractive forces do not necessarily support binding phenomena: for example an effective flavored chiral Yukawa interaction in QCD-like theories contributes via box diagrams with a sign opposite to that of gauge bosons to the fermionic flow.

Still, it should be kept in mind that the present analysis is carried out in a restricted truncation of the effective action that—although meaningful for QCD-like theories—might not be sufficient for gravity. Further operators that could potentially be relevant for the interplay between gravity and a chiral fermion sector are discussed below.

5. Conclusions

We have investigated the quantum interplay between chiral fermions and metric fluctuations in quantum gravity. In contrast to QCD-like systems, where gluon fluctuations can induce strong fermionic correlations leading to chiral symmetry breaking, metric fluctuations do not support this mechanism in an analogous framework. Our result thus indicates that the existence and observations of light fermions are well compatible with a regime, e.g. near or above the Planck scale, where quantum gravity effects in the form of sizeable metric fluctuations set in.

More specifically, light fermions are compatible with the asymptotic-safety scenario of quantum gravity which provides a UV completion of quantum gravity within quantum field theory. In particular, we observe a decoupling mechanism for metric fluctuations: as is known from [39], the dominant effect of fermionic fluctuations is a shift of the gravitational non-Gaußian fixed point towards increasingly negative values of the dimensionless renormalized cosmological constant for larger numbers of flavors \( N_f \). As a consequence, metric fluctuations decouple from the matter sector as such a cosmological constant acts like a mass term in the propagators of the metric modes and thus suppresses metric fluctuations. Apart from a slight shift of the Gaußian matter fixed point to small but non-vanishing values of the fermion interactions, the universality class of the Gaußian fixed point which is supposed to describe the fermionic matter content of the universe is left rather unaffected. In particular, the critical exponents remain very close to the Gaußian values.

Such a compatibility scenario between matter and asymptotically safe gravity holds at least as long as fermions remain the dominant degrees of freedom. For larger numbers of bosonic modes, in particular for a supersymmetric matter content, the whole system may behave differently.

Our results are also applicable to scenarios where quantized metric fluctuations are considered as part of an effective quantum field theory at an intermediate scale below or near the Planck scale. Here the underlying UV completion of gravity, which may use different degrees of freedom for gravity than the metric or even leave the local QFT framework completely, will determine the initial conditions for the RG flow at this intermediate scale. Also, from this more general viewpoint, we do not find any indications of gravity-stimulated chiral symmetry breaking. Still, our results can be used to decompose the accessible theory space into those branches where the chiral sector remains symmetric and those branches where the
chiral sector becomes critical and typically generates heavy fermion masses. The distribution of these branches in theory space depends on the gravitational couplings. In particular, the universality properties of the shifted Gaussian matter fixed point can substantially vary. This analysis provides general constraints on the Planck scale behavior of any microscopic theory of quantum gravity: the existence and observation of light fermions potentially excludes those branches of theory space where the chiral sector is critical at the Planck scale.

Of course, a variety of further aspects could modify our results quantitatively and qualitatively, as our analysis is performed in a limited and comparatively small hypersurface in theory space. Still, the interplay between sizeable metric fluctuations and chiral symmetry has the potential to provide relevant phenomenological constraints for any theory of quantum gravity also in a more complete and quantitatively reliable investigation. Let us try to list some issues that need to be addressed in further studies.

Whereas we have used flat-space calculations mainly as a technical tool, the full theory of quantum gravity will predict an (effective) manifold as its solution to the equations of motion. This resulting background may have an influence on the chiral status of the matter sector itself, as it may screen or enhance fermionic long-range fluctuations that lead to chiral criticality. Screening mechanisms for chiral symmetry breaking of this type have already been studied in various chiral models, such as two- and three-dimensional (gauged) Thirring models [81–83] or the 4D gauged NJL model [84]. Depending on its sign, curvature can act like an IR cutoff that screens the critical IR fluctuations.

Also, if the question of chiral symmetry breaking and restoration is considered in a cosmological context, the thermal evolution of the universe may play an essential role. Broken chiral symmetry could be restored during reheating if the corresponding temperature is sufficiently high compared with the scale of critical fermion dynamics—independently of whether it is stimulated by metric fluctuations or by another mechanism.

Let us also discuss the possibility of gravity-stimulated chiral symmetry breaking which would strongly differ from the scenario arising from QCD-like theories. As gravity supports a richer structure of operators, chiral criticality could be triggered by operators that are typical of gravity, but do not occur for other theories.

Up to canonical dimensions four and five, only explicitly symmetry breaking terms (such as e.g. \( \int d^{4}x \sqrt{g} R \bar{\psi} \gamma^i \psi^i \)) exist.

At dimensions six (for two-fermion terms) and eight (for four-fermion terms), we encounter a variety of new terms that are not forbidden by explicit chiral symmetry breaking, for instance

\[
\text{dim}6 : \int d^{4}x \sqrt{g} R \bar{\psi} \vec{\nabla} \psi, \quad \int d^{4}x \sqrt{g} R_{\mu\nu} \bar{\psi} \gamma^\mu \nabla^\nu \psi, \quad (24)
\]

\[
\text{dim}8 : \int d^{4}x \sqrt{g} R(V^2 \pm A^2),
\int d^{4}x \sqrt{g} R_{\mu\nu} \left( (\bar{\psi}^i \gamma^\mu \psi^i)(\bar{\psi}^j \gamma^\nu \psi^j) - (\bar{\psi}^i \gamma^\mu \gamma^5 \psi^i)(\bar{\psi}^j \gamma^\nu \gamma^5 \psi^j) \right). \quad (25)
\]

At higher dimensionalities the number of terms increases considerably, as then e.g. contractions involving the Riemann tensor will also be possible. Furthermore, couplings involving \( \vec{\nabla} \) or higher powers of the curvature are possible. Distinguishing between the background and the fluctuation metric leads to an even larger ‘zoo’ of possible operators.
Figure 9. Two and four-fermion couplings are generated from metric fluctuations. Here we have not drawn external graviton lines/couplings to a nontrivial background curvature; these will be generated by taking derivatives of the above diagrams with respect to the desired metric structure. The results then correspond to dimensions six and eight operators of the type listed in equations (24) and (25). The upper self-energy diagram on flat space also contributes to the fermion anomalous dimension $\eta_\psi$. Similar diagrams occur at higher order in the fermion field.

Several comments are in order here: the effect of metric fluctuations implies that none of these couplings will have a Gaussian fixed point, as the antifermion–fermion two-graviton vertex generically generates these couplings even if they are set to zero. The corresponding diagrams are indicated in figure 9.

These couplings raise several issues: in order for the asymptotic-safety scenario to work, the complete system of gravitational couplings, four fermion-couplings and mixed couplings such as the above ones has to admit a non-Gaussian fixed point (even though this does not necessarily require $g_{\ast\ast} \neq 0$ for all possible couplings). As the above couplings couple nontrivially into the flow of the Einstein–Hilbert action and the four-fermion couplings, they may change our findings in this sector.

Since, in the truncation that we have studied here, chiral symmetry breaking is avoided as the gravitational contribution to the anomalous scaling of the fermionic couplings outweigths the contribution that triggers bound-state formation, we expect that, in particular, the dimension six non-minimal kinetic terms in equation (24) represent an interesting extension of our truncation specific to gravity. This is because these generate further contributions $\sim g^2$ and do not contribute to the anomalous scaling. As we expect that these couplings are nonzero at any UV fixed point, they constitute a non-vanishing contribution to the $\beta$ functions of the four-fermion couplings. In other words, these dimension six terms genuine to gravity have the potential to act in a structurally identical manner in the fermionic flows as the fermionic self-interaction terms considered so far.

Another question that has remained unaddressed so far is the question of gravity-induced symmetry breaking patterns. In this work, we have imposed a rather standard chiral $\text{SU}(N_f)_L \times \text{SU}(N_f)_R$ (with additional $\text{U}(1)$ factors of particle number and axial symmetry) and implicitly assumed its breaking in a QCD-like fashion, i.e. to a remaining mesonic $\text{SU}(N_f)$ symmetry. Other breaking patterns are conceivable, including an originally larger symmetry that may break into the standard chiral symmetry upon large metric fluctuations. This work represents only a first step in this direction. In principle, it seems worthwhile to think not only about
gravitationally stimulated symmetry breaking and corresponding condensates, but also about corresponding bound states or excitations on top of condensates. If a gravitationally stimulated symmetry breaking transition with a remnant standard chiral symmetry occurred near the Planck scale, stable bound states (analogous to hadrons in QCD) may have remained and (if equipped with the right quantum numbers) could contribute to the dark matter in the universe.

Furthermore, in analogy to recent ideas in QCD, where a quarkyonic phase with confinement but intact chiral symmetry supports a spectrum of bound states, bound states may form that correspond to bosonized operators, e.g. of the form equation (25). These might form at a scale where quantum gravity is strongly interacting and may then become massive at the much lower scale of chiral symmetry breaking. Supporting a stable bound state over such a large range of scales requires, of course, a highly nontrivial interplay between gravity and matter.

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**Appendix. Variation of the effective action**

We expand the vierbein around a (flat) background:

\[ e_{\mu a} = \tilde{e}_{\mu a} + \delta e_{\mu a}, \]  

(A.1)

where higher orders are not needed in our calculation. In the following, we choose the Lorentz symmetric gauge with gauge-fixing functional \[78, 79\], as then all vierbein fluctuations can be rewritten in terms of metric fluctuations without ghosts due to the \( O(4) \) gauge fixing:

\[ F_{ab} = e_{\mu a} \tilde{g}^{\mu \nu} \tilde{e}_{\nu b} - e_{\mu b} \tilde{g}^{\mu \nu} \tilde{e}_{\nu a}. \]  

(A.2)

This allows us to write

\[ \delta e_{\mu a} = \frac{1}{2} h_\mu^a \tilde{e}_\kappa a, \]  

(A.3)

\[ \delta e_{\kappa b} = -\frac{1}{2} h_\mu^b \tilde{e}^{\mu \kappa}. \]  

(A.4)

We also have that

\[ [\gamma^a, \gamma^b] \delta \omega_{\mu ab} = [\gamma^\lambda, \gamma^\nu] D_\nu h_\lambda. \]  

(A.5)

From (A.5) we can deduce for constant external fermions, where total derivatives can be discarded, that

\[ [\gamma^a, \gamma^b] \delta^2 \omega_{\mu ab} = [\gamma^\lambda, \gamma^\nu] \left( -h_\lambda^\gamma D_\nu h_\mu - h_\mu^\gamma D_\nu h_\lambda + \frac{1}{2} h_\kappa \gamma D_\mu h_\kappa \right) \]  

(A.6)

where we have set \( g_{\mu \nu} = \tilde{g}_{\mu \nu} \) and \( e_{\mu a} = \tilde{e}_{\mu a} \) and then dropped the bar on the covariant derivative.
We then go over to Fourier space

\[ \psi(x) = \int \frac{d^4p}{(2\pi)^4} \psi(p)e^{-ipx}, \quad h_{\mu\nu}(x) = \int \frac{d^4p}{(2\pi)^4} h_{\mu\nu}(p)e^{-ipx}, \quad \bar{\psi}(x) = \int \frac{d^4p}{(2\pi)^4} \bar{\psi}(p)e^{ipx}, \]

(A.7)

where \( \psi(x) \) and \( \psi(p) \) denote Fourier transforms of each other.

Now we may evaluate the mixed fermion–graviton vertices, where our conventions are

\[ \Gamma = \frac{\delta}{\delta \Phi^T(-p)} \frac{\delta}{\delta \Phi(q)} \]

(A.8)

where the collective fields

\[ \Phi^T(-q) = \left( h^{TT}_{\kappa\lambda}(-q), h(-q), \psi^T_i(-q), \bar{\psi}_i(q) \right), \]

\[ \Phi(q) = \left( h^{TT}_{\mu\nu}(q), h(q), \psi_j(q), \bar{\psi}^T_j(-q) \right). \]

(A.9)

Here the second line should be read as a column vector. The symbol T refers to transposition in Dirac space and in field space. As we work in the Landau–deWitt gauge, only the transverse traceless and the trace mode can contribute.

The first variation of the kinetic fermion term with respect to the metric is given by

\[ \delta \Gamma_{\text{kin}} = iZ\psi \int d^4x \bar{\psi}^j \left( \delta_d(\sqrt{g}) \gamma^\mu \nabla_\mu + \sqrt{g} \delta_d \gamma^\mu \nabla_\mu + \sqrt{g} \gamma^\mu \delta \nabla_\mu \right) \psi^i. \]

(A.11)

To read off the trace-mode-fermion vertices we Fourier-transform the first variation of the kinetic term with respect to the metric to get (in agreement with [45])

\[ \delta \Gamma_{\text{kin}} = Z\psi \int \frac{d^4p}{(2\pi)^4} \left( \left( \frac{3}{16} h(p) \bar{\psi}^i(p) \bar{\psi}^i - \frac{3}{16} \bar{\psi}^i \bar{\psi}^i(-p) h(p) \right) \right). \]

(A.12)

In this notation, \( \bar{\psi} \) and \( \psi \) are the constant background fields, whereas the momentum-dependent fluctuation fields are distinguished by carrying an appropriate argument. This allows us to evaluate the following vertices:

\[ V_{h\bar{\psi}^iT} = \delta \frac{\delta}{\delta h(-p)} \Gamma_{\text{kin}} \bar{\psi}^iT = \frac{3}{16} Z\psi \bar{\psi}^iT \bar{\psi}^i, \]

(A.13)

\[ V_{h\psi^i} = \frac{3}{16} Z\psi \bar{\psi}^i \bar{\psi}^i, \]

(A.14)

\[ V_{h\bar{\psi}^iT} = \frac{3}{16} Z\psi \bar{\psi}^iT \bar{\psi}^i, \]

(A.15)

\[ V_{h\bar{\psi}^i} = \frac{3}{16} Z\psi \bar{\psi}^i \]

(A.16)

where the momentum is always the momentum of the incoming graviton.

The corresponding vertices with the TT mode vanish, as the first term in (A.11) contains only the trace mode, the second term vanishes by transversality for constant external fermion fields and the last term vanishes as the contraction \( \gamma^\mu [\gamma^\nu, \gamma^\kappa] D_\kappa h^{TT}_{\mu\nu} = 0 \).
The second variation of the kinetic fermion term with respect to the metric contains only a TT contribution, as the trace contribution is always of the form $\bar{\psi} i h \slashed{\partial} h \psi i$, which can be rewritten as a total derivative for constant external fermions.

From the fact that we have constant external fermions, at least one of the variations has to hit the covariant derivative $\nabla_\mu$ and hence produce a $[\gamma^\mu, \gamma^b] \delta \omega_{\mu \alpha \beta}$. Accordingly, the second variation will necessarily contain $\gamma^\mu [\gamma^\kappa, \gamma^\lambda]$. As there is one derivative in the kinetic term, the vertex has to be proportional to the momentum of one of the gravitons. The only possible structure that cannot be rewritten into a total derivative is then $\gamma^\mu [\gamma^\kappa, \gamma^\lambda] h_{\kappa \lambda} D_\mu h^\lambda_\kappa$. Our explicit calculation now only has to fix the sign and the numerical factor of the vertex. From (A.6) and (A.5) we deduce that

$$\delta^2 \Gamma_{\text{kin}} = i Z_\psi \int d^4x \sqrt{g} \bar{\psi} i \left( -\frac{1}{16} \right) (h^\mu_\lambda \gamma^\nu [\gamma^\kappa, \gamma^\lambda] D_\mu h_{\kappa \lambda}) \psi i. \quad (A.17)$$

The vertex that results from this expression is given by

$$V_{h_{\kappa \lambda} T^T h} = \frac{-1}{128} p_\psi \tilde{V} \{ \gamma^\rho, \gamma^a \} T^T \psi (\delta_{\mu \nu} \delta_{\kappa \alpha} \delta_{\lambda \beta} + \delta_{\mu \nu} \delta_{\kappa \alpha} \delta_{\lambda \beta} + \delta_{\mu \nu} \delta_{\kappa \alpha} \delta_{\lambda \beta} + \delta_{\mu \nu} \delta_{\kappa \alpha} \delta_{\lambda \beta} - \delta_{\mu \nu} \delta_{\kappa \alpha} \delta_{\lambda \beta})$$

$$\tilde{V}_h = \frac{-1}{2} \tilde{\lambda}_- + \frac{1}{2} \tilde{\lambda}_+ (\bar{\psi} i \gamma^\mu \psi i) \psi i T^T \gamma^\mu T^T, \quad \tilde{V}_h = \frac{-1}{2} \tilde{\lambda}_- + \frac{1}{2} \tilde{\lambda}_+ (\bar{\psi} i \gamma^\mu \psi i) \psi i T^T \gamma^\mu T^T, \quad \tilde{V}_h = \frac{-1}{2} \tilde{\lambda}_- + \frac{1}{2} \tilde{\lambda}_+ (\bar{\psi} i \gamma^\mu \psi i) \psi i T^T \gamma^\mu T^T, \quad \tilde{V}_h = \frac{-1}{2} \tilde{\lambda}_- + \frac{1}{2} \tilde{\lambda}_+ (\bar{\psi} i \gamma^\mu \psi i) \psi i T^T \gamma^\mu T^T.$$

The tadpole receives contributions from both the TT and the trace mode. The corresponding vertices are given by

$$V_{4T} = \frac{1}{16} (\tilde{\lambda}_- (V - A) + \tilde{\lambda}_+ (V + A)), \quad V_{4T h} = -\frac{1}{8} (\delta_{\mu \kappa} \delta_{\nu \lambda} + \delta_{\mu \lambda} \delta_{\nu \kappa}) (\tilde{\lambda}_- (V - A) + \tilde{\lambda}_+ (V + A)). \quad (A.21)$$
The variations of the four-fermion terms with respect to the fermions read as follows:

\[
V_{\psi f}^{\psi_i T \bar{\psi}_f} = \frac{-\bar{\lambda}_- + \bar{\lambda}_+}{2} \left[ 2 \left( \gamma^\mu T \bar{\psi}_f \gamma^\mu (\psi_i T \gamma^\mu T) - 2 \gamma^\mu T \delta^i_j \left( \bar{\psi}_f \gamma^\mu \psi_f \right) \right) + \frac{-\bar{\lambda}_- - \bar{\lambda}_+}{2} \left[ 2 \left( \gamma^5 \gamma^\mu T \bar{\psi}_f \gamma^5 \gamma^\mu (\psi_i T \gamma^5 \gamma^\mu T) - 2 \gamma^5 \gamma^\mu T \delta^i_j \left( \bar{\psi}_f \gamma^5 \gamma^\mu \psi_f \right) \right) \right],
\]

\[
V_{\bar{\psi} f}^{\psi_i T \bar{\psi}_f} = -\frac{\bar{\lambda}_- + \bar{\lambda}_+}{2} \left( \gamma^\mu T \bar{\psi}_f \gamma^\mu (\psi_i T \gamma^\mu T) - \frac{-\bar{\lambda}_- - \bar{\lambda}_+}{2} \left( \gamma^5 \gamma^\mu T \bar{\psi}_f \gamma^5 \gamma^\mu (\psi_i T \gamma^5 \gamma^\mu T) \right) \right),
\]

\[
V_{\bar{\psi} f}^{\bar{\psi} j T \psi_f} = -\frac{\bar{\lambda}_- + \bar{\lambda}_+}{2} \left( \gamma^\mu T \bar{\psi}_f \gamma^\mu (\psi_j T \gamma^\mu T) - \frac{-\bar{\lambda}_- - \bar{\lambda}_+}{2} \left( \gamma^5 \gamma^\mu T \bar{\psi}_f \gamma^5 \gamma^\mu (\psi_j T \gamma^5 \gamma^\mu T) \right) \right),
\]

\[
V_{\bar{\psi} f}^{\bar{\psi} j T \psi_f} = \frac{-\bar{\lambda}_- + \bar{\lambda}_+}{2} \left[ 2 \delta^i_j \gamma^\mu \left( \bar{\psi}_f \gamma^\mu \psi_f \right) + 2 \left( \gamma^\mu \psi_f \right) \left( \bar{\psi}_f \gamma^\mu \psi_f \right) \right] + \frac{-\bar{\lambda}_- - \bar{\lambda}_+}{2} \left[ 2 \delta^i_j \gamma^5 \left( \bar{\psi}_f \gamma^5 \psi_f \right) + 2 \left( \gamma^5 \psi_f \right) \left( \bar{\psi}_f \gamma^5 \psi_f \right) \right].
\]

(A.22)

Here we suppress the Dirac index structure; by round brackets we indicate the way in which the Dirac indices of the terms have to be contracted.

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