Influence of the Cut-off Elevation Angle and Elevation-Dependent Weighting on Parameter Estimates: A Case of CONT05

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Abstract

In this paper, results are presented on studies which have been performed to investigate the impact of the cut-off elevation angle (CEA) and elevation-dependent weighting (EDW) on the EOP estimates and baseline length repeatability. For this test, CONT05 observations were processed with different CEA and EDW, keeping all other options the same as used during the routine processing. Uncertainties and biases, as well as correlations between estimated parameters have been investigated. It has been shown that small CEA, up to about 8–10 degrees does not have large impact on the results, and applying EDW allows us to get better result (smaller errors). However, this result has been proven with standard geodetic VLBI observations, where rather few observations were made at low elevations. Perhaps, special R&D sessions with more uniform distribution of observations over elevation may be useful for more detailed study on the subject.
1 Introduction

It is well known that precision and accuracy of astronomical observations, both optical and radio, made through the Earth’s atmosphere depend on the elevation at which the object is observed. These errors grow with decreasing of the elevation due to larger air mass and difficulties in modelling of refraction effects at low elevation. From this point of view observations should be made in the near-zenith zone when possible.

On the other hand, inclusion in processing of observations made at low elevations is important when definite groups of highly correlated parameters, for instance station coordinates and zenith troposphere delays, are estimated simultaneously. In such a case using observations made at in a widest range of elevation allows us to mitigate the correlations between unknowns and improve the solution.

To meet these mutually exclusive requirements, proper elevation-dependent weighting (EDW) of observations is used. In a special case of step-like weighting function, i.e. rejection of the observations made at the elevation less than the given limit, such a limit usually is called cut-off elevation angle (CEA).

It was shown in many studies that elevation-dependent weighting may have a significant impact on the results of processing of the space geodesy observations. In particular, several studies of this effect was made by the Goddard and Vienna VLBI analysis groups in the framework of the IVS VLBI2010 Committee activity\footnote{href="http://ivscc.gsfc.nasa.gov/pipermail/ivs-v2c/"}. They investigated an influence of CEA and EDW on geodetic results such as Earth orientation parameters (EOP), baseline length repeatability, troposphere parameters, and station heights. Those results were based on simulation. Gipson in [1] used another approach to EDW. He applied elevation dependent additive noise to the measurement error instead of using a weighting factor as it is usually being made. He tested his method with the actual CONT05 observations. Results of both mentioned and other results are sometimes contradictory. This gave an impulse to the present work where some results are presented of investigation of the impact of the CEA and EDW on the baseline length repeatability and EOP estimates.

2 Test description

For this test, CONT05A observations were processed making use of OCCAM software with different EDW functions including CEA, keeping all other options as follows:

- Kalman filter mode (KF),
- random walk model for clocks, PSD=1.5 ps$^2$/s,
- random walk model for ZTD, PSD=0.25 ps$^2$/s,
- one NS and EW troposphere gradient estimate for the session.

For the continuous EDW mode (continuous weighting function), the measurement error is multiplied by a factor

$$W_e = (\sin e_0 / \sin e)^p,$$

(1)
Figure 1: Actual values of the hydrostatic (HMF), wet (WMF) mapping functions for the C0509 (05SEP20XA) session (one point correspond to one observation). Solid line corresponds to function $1/\sin e$. Data for the full elevation range (left) and zoomed data for low elevations (right) are shown.

where $e_0$ and $p$ are EDW parameters, $e$ is the source elevation. Such a weighting function provides a smooth stepless change in weight for any $e_0$. One can see that a case of $e_0 = 90^\circ$, $p = 1$ gives merely $W_e = 1/\sin e$, which is close to actual wet mapping function used in the last works by MacMillan and Gipson (private communications). Figure 1 shows actual hydrostatic and wet mapping functions along with approximation $1/\sin e$ for a typical CONT05 session. One can see that these functions are close enough, and either of them can be used for the EDW without significant impact on the result.

EDW mode with $e_0 = 10$, $p = 2$ was implemented in the OCCAM/GROSS software [2] for routine data processing. It will be referred hereafter as ”normal mode”.

In a case of CEA we have

$$W_e = \begin{cases} 
1, & \text{if } e \geq e_0 \\
10^8, & \text{otherwise}
\end{cases}$$

The latter line corresponds to the KF realization used in OCCAM.

For VLBI delay, measurement error coming from correlator is multiplied by two $W_e$ values computed for both the stations. In our test, $e_0 = 3(2)25^\circ$ were used for CEA test, and $e_0 = 10, 25, 45, 90^\circ$, $p = 1, 2$ were used for continuous EDW mode.

### 3 Test Results

#### 3.1 Baseline length repeatability

Test results obtained with different CEA are shown in Figure 2. The case of $e_0 = 3^\circ$ includes all the observations without weighting, since no CONT05 observations were made at the elevation less than $4^\circ$.

Table I shows EDW test results. Different EDW modes are denoted as w$_e$p, where $e$ and $p$ are $e_0$ and $p$ in Eq. (1). Test results are given for quadratic approximation in percent.
Figure 2: Dependence of the baseline length repeatability on the cut-off elevation angle (quadratic regression): all tested modes (left) and data for low elevations (right).
Table 1: Comparison of the baseline repeatability obtained with different EDW models. See explanation in text.

| EDW mode | Baseline length, 10^3 km |
|----------|--------------------------|
|          | 3 | 6 | 9 | 12 |
| w_10.1   | 98.0 | 97.2 | 97.3 | 97.6 |
| w_10.2   | 95.3 | 95.0 | 95.5 | 96.1 |
| w_25.1   | 102.6 | 92.8 | 87.2 | 83.8 |
| w_25.2   | 95.7 | 91.8 | 91.4 | 91.9 |
| w_45.1   | 101.7 | 90.0 | 84.6 | 81.9 |
| w_45.2   | 104.0 | 97.2 | 101.8 | 107.6 |
| w_90.1   | 101.3 | 90.5 | 85.4 | 82.8 |
| w_90.2   | 109.6 | 127.3 | 161.1 | 191.1 |

Table 2: EOP statistics for different EDW models. See explanation in text.

| Statistics         | All | 10.1 | 10.2 | 25.1 | 25.2 | 45.1 | 45.2 | 90.1 | 90.2 |
|--------------------|-----|------|------|------|------|------|------|------|------|
| Xp uncertainty     | 26  | 26   | 25   | 26   | 28   | 27   | 37   | 30   | 51   |
| Yp uncertainty     | 25  | 24   | 24   | 24   | 26   | 25   | 32   | 26   | 43   |
| UT1 uncertainty    | 1.1 | 1.1  | 1.1  | 1.1  | 1.0  | 1.3  | 1.1  | 1.6  |
| Xc uncertainty     | 21  | 21   | 20   | 20   | 19   | 20   | 22   | 22   | 27   |
| Yc uncertainty     | 19  | 18   | 18   | 18   | 18   | 18   | 21   | 20   | 27   |
| Xp bias w.r.t. IGS | -79 | -75  | -79  | -77  | -73  | -80  | -86  | -81  | -123 |
| Yp bias w.r.t. IGS | +135 | +135 | +128 | +132 | +129 | +134 | +127 | +129 | +92  |
| Xp wrms w.r.t. IGS | 77  | 72   | 76   | 69   | 65   | 68   | 69   | 64   | 86   |
| Yp wrms w.r.t. IGS | 73  | 74   | 72   | 69   | 61   | 68   | 63   | 71   | 83   |
| mean              | 75  | 73   | 74   | 70   | 63   | 68   | 66   | 68   | 84   |

with respect to the case of CEA with $\epsilon_0 = 3^\circ$. One can see that several EDW modes show about the same improvement in the baseline length repeatability.

### 3.2 EOP

Figure 3 shows EOP statistics for different CEA. Notation used is the following: $X_p$, $Y_p$ - terrestrial pole coordinates, $X_c$, $Y_c$ - celestial pole coordinates. Weighted Allan deviation is computed as described in [3]. All the results related to $X_p$, $Y_p$, $X_c$ and $Y_c$ are given in $\mu$as, the results related to UT1 are given in $\mu$s. Comparison with IGS time series is shown in Figure 4. $X_p$ and $Y_p$ wrms with respect to IGS EOP series are computed after removing the bias.

Table 2 shows the main EDW test results. Notation of EDW modes is the same as in Table 1 with the 'w-' prefix omitted. 'All' column corresponds to inclusion of all observations without weighting. One can see again that several EDW modes show about the same EOP precision and accuracy.
Figure 3: Statistics of EOP obtained from CONT05.
Figure 4: Comparison of EOP obtained from CONT05 with the IGS EOP series.
4 Conclusions

The preliminary conclusions from this test are the following.

- The baseline length repeatability steadily grows with the CEA increasing, remaining practically the same in the cut-off angle range from $3^\circ$ (i.e. no cut-off for the CONT05) to $9^\circ$.
- The best result is obtained when the EDW elevation-depending weighting is applied to the low-elevation observations. However, the test results are not always unambiguous. Further adjustment of the weighting method may be fruitful.
- The Xp, Yp and UT1 uncertainties grow with the increasing cut-off angle after about $10^\circ$. Most probably, this reflects the fact that only about 6% of the total number of CONT05 observations were made at the elevations below $10^\circ$. The Xc and Yc uncertainties and scatter depend on the CEA much less.
- Xp bias w.r.t. IGS slightly depends on the CEA, except the maximum tested CEA values, evidently unrealistic. In contrast, Yp bias substantially changes with increasing CEA. Most probably, this can be explained by the CONT05 network orientation, for which the longitude of the central meridian $\lambda_0 = 265^\circ$ just corresponds to the Y direction of the terrestrial coordinate system.
- Some statistics such as the uncertainty and the scatter of the Xc and Yc, as well as the WRMS of Xp and Yp w.r.t. IGS have the minimum at the CEA around $15^\circ$, which is interesting and deserves a supplement investigation.
- As one can expect, the correlations between EOP comprising Xp and Yp grow with increasing CEA, but remain small due to good CONT05 network geometry. The same can be expected for the IVS2010 network. The correlation between Xc and Yc remain practically the same for all tested CEA, except the maximum tested CEA value, evidently unrealistic.

Finally, we can conclude that inclusion of the low-elevation observations, properly weighted, improves the baseline length repeatability and EOP results. On the contrary, filtering the observations using the cut-off elevation method may lead to degradation of geodetic results. However, this should be mentioned that the conclusions drawn from the result obtained in this paper has been proven with standard geodetic VLBI observations, where rather few observations were made at low elevations, as mentioned above. Perhaps, special R&D sessions with more uniform distribution of observations over the sky, including observations at very low elevations, may be useful for more detailed study on the impact and optimal processing of the low-elevation observations on geodetic parameters obtained from VLBI observations.

It ought be mentioned that all the EDW modes considered in this paper in fact modify only diagonal elements of the corresponding covariance matrix. According to Gipson’s work [1] best result can be achieved in a case of account also for correlations between observations. It seems to be interesting to investigate how this approach will work in KF estimator.
References

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