Generalized Structural Equations Approach in the of Elderly Self-rated Health

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Abstract. To model classified and ordinal data, the Generalized Structural Equation Model (GSEM), which is based on the integration of two generalized linear model (GLM) and Structural Equation Modeling (SEM) algorithms, is applied. Unlike the SEM, this model does not require the normality assumption. The main purpose of this paper is to introduce and compare weighted (WLSMV) and unweighted (ULSMV) least squares mean and variance adjusted methods, two of the most applicable estimators of GSEM, for studying factors affecting the elderly self-rated health in 2015 in Tehran, Iran. 600 elderly people aged 60 years and above from 22 regions of Tehran were selected using multi-stage sampling. Self-rated health of the elderly variable (a 5-point Likert scale) was analyzed as an ordinal variable and was modeled considering the variables of social support, financial and environment security, spirituality, mental and physical health, functional health and health-related behaviors by Mplus software. The results showed that WLSMV outperformed ULSMV according to the smaller values of RMSEA and larger values for CFI and TLI indexes (RMSEA WLSMV = 0.04, CFI WLSMV = 0.965, and TLI WLSMV = 0.936). To prevent concluding invalid results in studying ordinal data due to considering them as a continues variable, it is important selecting correct statistical method according to the type of variables.

1. Introduction
Likert-type scale items for operationalizing unobserved constructs by using more manageable observed variables in the social and behavioural sciences are employed. Structural equation modelling (SEM) is a group of statistical techniques that investigate the relationships among observed and latent variables. Researchers start with a hypothesized model representing the inter-relationships among variables, and then evaluate if the hypothesized model fits the data well. A model is said to fit the data if the relationships specified in the model adequately reproduce the relationships existing in the data. In SEMs and under the normal theory, continuity and multivariate normality distributed scores of the observed variables are assumed [1]. However, survey data are not frequently collected with these preferred properties and are usually gathered on a Likert scale. So, these kinds of categorical data naturally are non-continuous and often show some kinds of non-normality [2]. The relationship between observed and factor scores for ordered categorical data is non-linear and should be studied by generalized structural equation model (GSEM). If the ordered categorical data are analysed as continuous, the distributions of observed scores can not reveal the true underlying distributions [1], mainly in small number of category conditions like as two-categories. The violation of normality assumption for
observed variables happens when a few response categories exist for the analysed data [3]. Under these circumstances, maximum likelihood (ML) estimation method as normal-theory-based SEM methods is not appropriate. In this case, not only ML estimates suffer from the precision and accuracy lacks, but also misleading conclusions may be drawn from empirical data. If severe non-normality due to categorization exists, in previous simulation studies have been showed that ML has resulted in inflating chi-square statistics, downward-biasing of factor loadings, and biasing standard errors to some degree [4-5]. Moreover, underestimation of the true relationship among variables might happen by computing Pearson correlations for the ordered categorical data [6]. To estimate the true relationship among variables, polychoric correlations for more than two categories and tetrochoric correlations for two categories can be computed.

For ordered categorical data, different SEM methods have been suggested [6-9]. When ordinal data are analysed, Robust maximum likelihood (MLR) [10], diagonally weighted least squares (DWLS) or weighted least squares mean-and-variance-adjusted (WLSMV) [11], and unweighted least squares mean-and-variance-adjusted (ULSMV) [12] estimators with statistical corrections to standard errors and chi-square statistics have been proposed to act better than ML. When continuous observed variable distributions differ from normality, MLR has been extensively applied. Though for categorical observed data (binary or ordinal variables), when either the normality assumption or the continuity property are not full fill, WLSMV and ULSMV are specifically designed.

DWLS does not consider any distributional assumptions for observed variables, though for each observed categorical variable a normal latent distribution is assumed. This estimator uses a diagonal weight matrix in parameter estimation which is much less computationally demanding than the weighted least squares (WLS) estimator which uses the full weight matrix [13]. The full weight matrix in WLS contains the asymptotic variances and covariances of the thresholds and polychoric correlations. The diagonal weight matrix in DWLS utilizes only the asymptotic variances of the thresholds and polychoric correlations. Comparing to ML and WLS, simulation studies showed that the DWLS estimator is superior when data are ordered and categorical, regarding parameter and standard error estimation accuracy [6,12-14]. Since this estimator employs a diagonal weight matrix, its chi-square statistic deviates from the central target central chi-square distribution under correctly specified models. Adjusting the chi-square statistic to be more consistent with the target chi-square distribution, one of the robust methods that is called the mean- and variance-adjusted weighted least squares have been suggested [7, 12]. When the model is properly identified, WLSMV modifies the uncorrected chi-square statistic in the way that the revised chi-square has its mean and variance equal to those of the target central chi-square distribution asymptotically. Savalei & Rhemtulla [15] indicated that WLSMV is the most common method in the SEM literature to analyze ordered categorical variables.

As an alternative to the WLSMV estimator for analysing ordered categorical data in SEM, the unweighted least squares (ULS) estimator has also been recommended [16]. Similar to WLSMV, ULS provides model parameter estimations based on the polychoric correlation matrix and thresholds. However, the weight matrix in the ULS fit function is further simplified to be an identity matrix. The mean and variance-adjusted ULS (ULSMV) is the robust ULS estimator where the chi-square statistic from ULS is corrected so that its mean and variance equal to those of the target central chi-square distribution when the hypothesized model is correctly specified. Although ULSMV has been less applied in substantive research, previous studies found that it provides parameter and standard error estimations as accurate as, or slightly better than, those from WLSMV under many conditions [14, 16-17]. Savalei and Rhemtulla [15] found that the robust chi-square statistic from ULSMV outperformed that from WLSMV regarding Type I error rates and power. The major disadvantage is that ULSMV encounters a higher model non-convergence rate than WLSMV [16].

To study people assessment of their health which is an individual experience of physical, mental and social events that affect a person's sense of well-being at a given time, self-rated health is an appropriate indicator [18]. In this study, the self-rated health of the Tehran elderlies was considered by the question of "How do you assess your overall physical health?". This question had Likert scale with 5 possible
response category which is considered as ordinal data which analysing it WLSMV and ULSMV methods is the main purpose of this study.

2. Research Methods
To inspect how well latent factors are measured by observed items, and how much the latent factors correlate with each other, the confirmatory factor analysis (CFA) model in the framework of SEM is applied. If \( m \) represents the number of observed variables and \( q \) is the number of latent factors, a typical CFA model is expressed as [19]:

\[
\Sigma = \Lambda \Phi \Lambda' + \Psi
\]  

(1)

where the \( m \times m \) population variance-covariance matrix for the observed variables is \( \Sigma \), the \( m \times q \) loading matrix is \( \Lambda \), the \( q \times q \) variance-covariance matrix among the factors is \( \Phi \), and the \( m \times m \) residual variance-covariance matrix is \( \Psi \). \( \theta \) also could be considered as the vector containing model parameters. The loading parameters in \( \Lambda \), variances and covariances among the latent factors in \( \Phi \), and residual variances and covariances in \( \Psi \) are estimated by a CFA model. Some parameters are typically fixed. To examine whether the hypothesized model fits the sample data, SEM analysis could be applied. To quantify the difference between the observed variance-covariance matrix and the model-implied variance-covariance matrix, a fit function is also used. If \( S \) represents the sample covariance matrix, \( S (\theta) \) indicates the model-implied covariance matrix governed by \( \theta \), \( s \) and \( \sigma (\theta) \) shows the vectorizations of the non-duplicated elements in \( S \) and \( \Sigma (\theta) \), respectively, the fit function, \( F \) is defined as:

\[
F = (S - \sigma (\theta)) W^{-1} (S - \sigma (\theta))'
\]

(2)

where \( W \) is a positive definite weight matrix. By minimizing the fit function from a sample of \( n \) observations as \( \hat{F} \), the model parameters in \( \theta \) are estimated. Assuming that data follow a multivariate normal distribution and the model is appropriately identified, the \( T \) statistic computed as \( T = (n - 1) \hat{F} \) follows a central chi-square distribution with the expectation being the model degrees of freedom [1]. The chi-square statistic is thus compared against the target chi-square distribution to test the null hypothesis that the unstructured covariance matrix equals to the model-implied covariance matrix. One of the most frequently applied estimators to estimate the parameters in \( \theta \) is ML [1], where \( W \) is a function of the model-implied covariance matrix \( \Sigma (\theta) \).

When data are ordered and categorical, the unobserved continuous variable is categorized into the observed ordinal variable by applying a set of thresholds. To estimate the polychoric correlation coefficient for ordered data, Olsson (1979) proposed the two-step ML method which is implemented in WLSMV or ULSMV in Mplus [20]. If \( x \) and \( y \) are two observed ordered categorical variables, and \( \xi \) and \( \eta \) be their corresponding underlying continuous variables, respectively. The joint distribution of \( \xi \) and \( \eta \) is bivariate normal with \( \rho^* \) being the polychoric correlation. \( x \) and \( y \) have \( s \) and \( r \) categories, respectively. \( x \) and \( y \) are then obtained by categorizing \( \xi \) and \( \eta \) using a set of thresholds, such that:

\[
x = i, \text{ if } a_{i-1} \leq \xi \leq a_{i} \\
y = j, \text{ if } b_{j-1} \leq \eta \leq b_{j}
\]

(3)

where \( i \) can be 1, 2..., \( s \), \( j \) can be 1, 2..., \( r \), and \( a_{i} \) and \( b_{j} \) are the thresholds for categorization. For the two-step ML estimation method [20], the first step is to estimate the thresholds from the cumulative marginal proportions. If \( \Phi_{i} \) be the univariate standard normal cumulative distribution function, the thresholds are estimated as:

\[
a_{i} = \Phi_{i}^{-1}(P_{i}) \\
b_{j} = \Phi_{j}^{-1}(P_{j})
\]

(4)

where \( P_{i} \) and \( P_{j} \) are the observed cumulative marginal proportions. The second step is to estimate the polychoric correlation coefficient. If \( \pi_{ij} \) be the probability corresponding to the observed scores of \( x = i \) and \( y = j \), the sample likelihood can then be expressed as:

\[
L = C \prod_{i} \prod_{j} n_{ij} \pi_{ij}^{n_{ij}}
\]

(5)

where \( C \) is a constant and \( n_{ij} \) is the number of observations that have \( x = i \) and \( y = j \). Considering the bivariate standard normal cumulative distribution function with correlation \( \rho^* \) for \( a_{i} \) and \( b_{j} \) in the
definition of \( \pi_{ij} \) and finding the derivation of \( \ln \), log-likelihood by \( \rho^* \) results in approximation \( \rho^* \) applying an iterative method such as the Newton-Raphson algorithm [20].

2.1. Estimation methods for CFA with ordered categorical data

The weighted least squares (WLS) estimator [21] from Equation (2) could be obtained by considering \( \sigma(\theta) \) as a vector containing the thresholds and the polychoric correlations, \( s \) as the corresponding sample estimates, the weight matrix \( W \) as the asymptotic variance-covariance matrix of the thresholds and polychoric correlations [20].

If \( \Gamma \) be the asymptotic covariance matrix of the elements in \( s \), and \( \hat{\Gamma} \) be its sample estimate. For the WLS estimator, \( W = \hat{\Gamma} \) which can result in statistical and computational problems due to very large values of \( \hat{\Gamma} \) when the model is complex and the number of ordered categorical variables is large [22]. \( \hat{\Gamma} \) based on small to moderate sample sizes can be very inaccurate, its inversion can increase the computational burden substantially with the dimension incensement, and it is very likely to be singular for small sample sizes with some score patterns having very low or high probabilities [12].

Based on a Taylor expansion, the asymptotic covariance matrix for the sample estimate of \( \theta \) can be shown as [22]:

\[
aV(\theta) = n^{-1}(\Delta \hat{W}^{-1})^{-1} \Delta \hat{W} \Gamma \hat{W}^{-1} \Delta (\Delta \hat{W}^{-1})^{-1}
\]

(6) where \( \Delta = \hat{\sigma}(\theta) / \partial \theta \). In equation (6), \( W \) doesn’t have to be \( \Gamma \), Muthén (1993) suggested using a simpler weight matrix for \( W \) so that the inversion of \( \Gamma \) in equation (2) for WLS estimator can be avoided [22]. He suggested \( W = I \) which results in the unweighted least squares (ULS) estimator or \( W = \text{diag}(\Gamma) \) leads to the DWLS estimator [23].

2.2. Robust Corrections for the DWLS and ULS Chi-square Statistics

The chi-square statistic for the WLS estimator [21], is calculated as:

\[
T_{wls} = (n - 1)F_{wls}
\]

(7) It asymptotically follows the central chi-square distribution with its expectation equal to the model degrees of freedom under correctly specified models. For DWLS, because \( W \) only utilizes the diagonal elements of \( \Gamma \), the uncorrected chi-square statistic as:

\[
T_{DWLS} = (n - 1)F_{DWLS}
\]

(8) no longer follows the target chi-square distribution asymptotically, where \( F_{DWLS} \) is the fit function with \( W = \text{diag}(\Gamma) \). Similar to [11] robust correction for the chi-square statistic, \( T_{DWLS} \) can be corrected so that for WLSMV, equation (8) could be written as:

\[
T_{WLSMV} = a_{DWLS}(n - 1)F_{DWLS} + b_{DWLS}
\]

(9) where \( a_{DWLS} \) and \( b_{DWLS} \) are the scale and shift factors respectively such that \( a_{DWLS} = \left( \frac{d}{\text{tr}(\hat{U}_{DWLS})^2} \right)^{1/2} \), \( b_{DWLS} = d - (d. \text{tr}(\hat{U}_{DWLS})^2) / \text{tr}(\hat{U}_{DWLS}^2) \) with \( \hat{U}_{DWLS} = (W^{-1} - \hat{\Gamma} \Delta_{DWLS} \Delta_{DWLS}^{-1} \hat{\Gamma})^{-1} \hat{\Gamma} \Delta_{DWLS} W^{-1} \) where \( d \) is the model degrees of freedom. For ULSMV, \( T_{ULS} \) can be similarly corrected so that the robust chi-square statistic can be calculated from Equation (9) by substituting \( T_{ULSMV}, a_{ULS} \) and \( b_{ULS} \) instead of \( T_{WLSMV}, a_{DWLS} \) and \( b_{DWLS} \), respectively. \( T_{WLSMV} \) and \( T_{ULSMV} \) both have their mean and variance equal to the mean \((d)\) and variance \((2d)\), respectively, of the target chi-square distribution asymptotically [7]. Previous studies focused on parameter and standard error estimations showed that WLSMV in general outperformed ML and ULS, and the superiority of WLSMV occurs especially under conditions with small sizes and less than four ordered categories [6, 12-14].

2.3. Robust model-fit indexes for WLSMV and ULSMV

In almost every SEM application, the chi-square statistic is reported which tests the null hypothesis that the model explains the data in the population. However, the chi-square statistic only follows the central chi-square distribution under correctly specified models. For mis-specified models, the chi-square statistic performance can be problematic because increasing the sample size causes to inflate, the chi-
square statistic even when the model is only slightly mis-specified. Since hypothesized models are always mis-specified in reality, the application of the chi-square statistic is not as meaningful as it intends to be [19].

Along with the chi-square statistic, RMSEA, CFI, and TLI are three widely applied model-fit indexes in the SEM literature. These indexes all defined as functions of the fit functions in the population, and thus also functions of the model chi-square statistic. RMSEA is specified as an absolute measure of the model-data fit which quantifies how great the hypothesized model is from the perfect fit to the data [24]. If $F$ represent a general fit function, which can be either $F_{DWLS}$ or $F_{ULS}$. $H$ be the hypothesized model and $F_H$ and $d_H$ represent the fit function and degrees of freedom from the hypothesized model, respectively, $B$ denotes the baseline model, The population RMSEA ($RMSEA_{pop}$), is defined as:

$$RMSEA_{pop} = (F_H - F_B) \frac{d_B}{d_H}$$  \hspace{1cm} (10)

RMSEA measures conceptually the degree of misfit per model degree of freedom. The sample estimate of $RMSEA_n$ given a sample size of $n$ is calculated as:

$$RMSEA_n = (\max \left(1 - O^2(n-1)\frac{F_H - F_B}{(n-1)d_H}\right)) \frac{d_H}{d_B}$$  \hspace{1cm} (11)

CFI and TLI compare the fit of the hypothesized model with the baseline model and referred to as incremental fit indexes [24]. The population CFI and TLI are defined as:

$$CFI_{pop} = 1 - \frac{F_H}{d_H}$$  \hspace{1cm} (12)
$$TLI_{pop} = 1 - \frac{d_H}{d_B}$$  \hspace{1cm} (13)

CFI and TLI represent how much the fit and the fit per degree of freedom, respectively, improve when comparing the hypothesized model with the baseline model which yields the worst model-data fit.

The sample estimates of $CFI_n$ and $TLI_n$ are calculated as:

$$CFI_n = 1 - \frac{(n-1)F_H - d_H}{(n-1)d_B}$$  \hspace{1cm} (14)
$$TLI_n = 1 - \frac{(n-1)F_H - d_B}{(n-1)d_B}$$  \hspace{1cm} (15)

All the mentioned indexes are functions of the chi-square statistic given finite sample sizes, when WLSMV or ULSMV are applied, it is conceptually necessary to replace the uncorrected chi-square statistic with the robust chi-square statistic. Population-corrected (PR) model-fit indexes are the model-fit indexes that are calculated in this way [25]. The PR model-fit indexes are calculated simply by replacing $T_H = (n-1)F_H$ by $T_H = a_H(n-1)F_H + b_H$ for the hypothesized model, and $T_B = (n-1)F_B$ by $T_B = a_B(n-1)F_B + b_B$ for the baseline model. The PR model-fit indexes are calculated by current software programs like as, Mplus and are reported in almost every study in substantive areas which applies WLSMV or ULSMV as the estimation method.

3. Results and Discussion

The data of this study are taken from a mixed quantitative and qualitative research in 2015 in Tehran, which was conducted with the aim of investigating the dimensions of successful aging, as one of the most important dimensions of aging health [26-28]. In a quantitative stage, using multi-stage stratified sampling, 600 elderly aged 60-year-olds and more from 22 regions of Tehran in 1394 were sampled. The results of exploratory factor analysis showed that the indicators determining the health of self-rated of the elderly are six factors of social support, financial and environment security, spirituality, functional health, mental and physical health, and health-related behaviors (Cronbach’s alpha = 0.93).

The outcome of the study, health-rated was asked by the question of “How do you assess your overall physical health?” with 5 categories Likert scale (1=very good, 2=good, 3=middle, 4=bad, 5=very bad). According to the purpose of this study factors of social support (1-Receiving emotional supports from family, 2-Family taking care in sickness. 3- Providing family support for well and enjoyable aging.
4-Children satisfaction. 5- Feeling peaceful (calm) and secure in home and family life. 6- Feeling loved and respected by surrounding people. 7- Having good relationship with grandchildren, son, or daughter in law. 8- The amount of family’s and friends’ acceptance of opinion (suggestion). 9- Marriage and family life satisfaction. 10- Feeling loneliness). financial and environment security (1-assessing financial and environment security. 2- Covering all expenses by family income. 3- Having enough financial savings for old age. 4- Concerning about the medical expenses of yourself and your family at sickness time. 5- After aging/retirement, decreasing amount of ability in purchasing and managing living costs. 6- Availability of hospital and clinics in neighborhood. 7- Appropriating of parks and recreational facilities in neighborhood for the elders. 8- Appropriating of home for old ages including the number of stairs, and slippery floors. 9- Liking neighborhood), spirituality (1-Helping thanksgiving to feel calm in old age. 2- Bearing difficulties and problems due to aging by trusting in God. 3- Helping spirituality to feel calm in this age. 4- Helping religious rituals or spiritual practices like as praying, visiting (a place of worship) church or mosque or participating in religious events to feel relaxed in old age), functional health (1-Performing independently outside home activities/chores like as shopping or visiting physician’s office. 2- Performing independently personal activities like as grooming and taking a shower/bath. 3- Having hearing problems. 4- Having vision problems. 5- Concerning about getting old in terms of not being able to manage personal activities), mental and physical health (1-Feeling stressed and anxious during the past 7 days. 2-Feeling usually sad. 3- Feeling hopeless and disappointed during the past 7 days. 4- Feeling fatigued during the past 7 days 5- Sleeping easily at night. 6- Having physical pain like as back pain or leg pain during the past 7 days. 7- Suffering from chronic physical conditions), and health-related behaviors (1-Considering important regular medical examination and check-ups. 2- Caring about maintaining your health. 3- Considering important to have fruit and vegetables in daily meal. 4- Choosing a healthy diet for example with low in fat, sugar, and salt) as influential factors on health rated of elderlies are analyzed using GSEM in Mplus software.

51.3 percent and 48.7 percent of the sample were female and male, respectively and the most of them were in 60-65 year-old (38.1 percent). To study the influential factors of elderlies self-rated health, two of the most known methods of WLSMV and ULSMV for ordinal data were applied. Table (1) indicated goodness of fit indexes of fitted models for self-rated health of elderly influential factors. According to the results, WLSMV method outperforms ULSMV method according to the less values for RMSEA and larger values for CFI and TLI indexes. Thus, the results of this method will be reported in Table (2). Figure (1) indicates the diagram of estimation results of fitting WLSMV on self-rated health of elderlies in Tehran.

| Table 1. Goodness of fit indexes for self-rated health of elderly influential factors |
|---------------------------------|-----------------|-----------------|
|                                 | WLSMV           | ULSMV           |
| RMSEA                           | 0.040           | 0.043           |
| CFI                             | 0.965           | 0.943           |
| TLI                             | 0.936           | 0.910           |

| Table 2. WLSMV estimate for self-rated health of elderly influential factors |
|---------------------------------|-----------------|-----------------|-----------------|
|                                 | Standardized estimate | Standard error | P-value         |
| Social support                  | -0.053          | 0.062           | 0.395           |
| Financial and environment security | 0.232           | 0.117           | 0.047*a         |
| Spirituality                    | 0.084           | 0.038           | 0.026*b         |
| Functional health               | 0.168           | 0.061           | 0.006*b         |
| Mental and physical health      | 0.382           | 0.078           | 0.000*b         |
| Health-related behaviors        | 0.064           | 0.077           | 0.405           |

*a*significant at 0.05, *b*significant at 0.01

The results of Table (2) shows that factors of financial and environment security and spirituality at 0.05 confidence level and factors of functional health and mental and physical health at 0.01 confidence level had significant influence on self-rated health of elderlies in Tehran.
4. Conclusion
The maximum likelihood (ML) method, which is used to calculate estimates in the Structural Equation Model (SEM), is not a convenient method to study non-continuous observed variables, such as ordered variables that are often used in educational, social, and behavioral sciences to construct latent variables. Instead, two of the most frequently used estimators of GSEM, diagonally weighted least squares (DWLS or WLSMV), and unweighted least squares mean-and variance-adjusted (ULSMV) estimators, have been suggested for estimating ordinal data. The main aim of this article was to compare WLSMV and ULSMV estimators in analysing self-rated health variable in Tehran. The results of this revealed the goodness of WLSMV comparing to ULSMV and insisted in selecting the suitable method for investigation the ordered variables in the study in order to preventing misleading conclusions.

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