Hidden and self-excited attractors in Chua circuit: SPICE simulation and synchronization

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Abstract

Nowadays various chaotic secure communication systems based on synchronization of chaotic circuits are widely studied. To achieve synchronization, the control signal proportional to the difference between the circuits signals, adjust the state of one circuit. In this paper the synchronization of two Chua circuits is simulated in SPICE. It is shown that the choice of control signal is be not straightforward, especially in the case of multistability and hidden attractors.

1 Introduction: hidden and self-excited attractors in Chua circuit

Since the first chaotic behavior in dynamical systems was revealed by numerical integration [1], the researchers started to be interested in circuit implementation of chaos (see, e.g. [2, 3] and others), which allows one to ensure that the pseudo-orbits can be traceable by actual orbits (see, e.g. the corresponding discussion on shadowing in [4, 5]). At the same time various engineering perspectives of chaotic circuits application have been found [6].

The Chua circuit, invented in 1983 by Leon Chua [7, 8], is the simplest electronic circuit exhibiting chaos. Consider one of classical Chua circuits shown in Fig. 1.

![Chua circuit](image)

The circuit consists of passive resistors ($R$ and $R_0$), capacitors ($C_1$ and $C_2$), conductor $L$, and one nonlinear element with characteristics $f(\cdot)$, called Chua diode. It is described by the following equations

\[
\begin{align*}
\frac{dv_1}{dt} &= \frac{1}{C_1} \left( \frac{1}{R} (v_2 - v_1) - f(v_1) \right), \\
\frac{dv_2}{dt} &= \frac{1}{C_2} \left( \frac{1}{R} (v_1 - v_2) + i_L \right), \\
\frac{di_L}{dt} &= -\frac{1}{L} (v_2 + R_0 i_L), \\
f(v_1) &= \begin{cases} 
G_b v_1 + (G_b - G_a)E_1, & \text{if } v_1 \leq -E_1, \\
G_a v_1, & \text{if } |v_1| < E_1, \\
G_b v_1 + (G_a - G_b)E_1, & \text{if } v_1 \geq E_1.
\end{cases}
\end{align*}
\]

Here $v_1$ and $v_2$ are voltages across capacitors $C_1$ and $C_2$, respectively, $i_L$ is a current through conductor $L$, function $f(v_1)$ is volt-ampere characteristics of Chua’s diode. In the following discussion we choose $C_2$ such that $RC_2 = 1$.

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and put $E_1 = 1$. By the introduction of new variables

$$
\begin{align*}
x &= v_1, \quad y = v_2, \quad z = Ri_L, \\
\alpha &= \frac{1}{RC_1}, \quad \beta = \frac{R}{L}, \quad \gamma = \frac{R_0}{L} \\
\psi(x) &= Rf(x), \quad RG_a = m_0, \quad RG_b = m_1,
\end{align*}
$$

system (1) is transformed to the following form

$$
\begin{align*}
\frac{dx}{dt} &= \alpha(y - x - \psi(x)), \\
\frac{dy}{dt} &= x - y + z, \\
\frac{dz}{dt} &= - (\beta y + \gamma z), \\
\psi(x) &= m_1 x + (m_0 - m_1) \text{sat}(x),
\end{align*}
$$

(3)

Until recently there had been found Chua attractors, which are excited from unstable equilibria only and, thus, can be easily computed (see, e.g. a gallery of Chua attractors in [9]). Note that L. Chua [8], analyzing various cases of attractors in Chua’s circuit, did not admit the existence of attractors of another type — so called hidden attractors, being discovered later in his circuits. An attractor is called a self-excited attractor if its basin of attraction intersects an arbitrarily small open neighborhood of equilibrium, otherwise it is called a hidden attractor [10, 11, 12, 13]. Hidden attractor has basin of attraction which does not overlap with an arbitrarily small vicinity of equilibria.

For example, hidden attractors are attractors in systems without equilibria or with only one stable equilibrium (a special case of multistability and coexistence of attractors). The hidden vs self-excited classification of attractors was introduced in connection with the discovery of the first hidden Chua attractor [14, 15, 10, 16, 17]. The Leonov-Kuznetsov’s classification of attractors as hidden or self-excited is captured much attention of scientists from around the world and hidden Chua attractors have become intensively studied (see, e.g. [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. In Fig. 2 is shown an example of self-excited and hidden Chua attractors visualized by numerical integration of system (3) in MATLAB.

## Visualization of hidden Chua attractor in SPICE

Nowadays various Simulation Programs with Integrated Circuit Emphasis (SPICE) are widely used to analyze and design analog circuits [33]. Consider simulation of hidden Chua attractor in SIMetrix SPICE\(^1\) for the following parameters: $C_1 = 118.2u$, $C_2 = 1m$, $L = 82.8281$, $R_0 = 430.7m$, $R = 1k$, $G_b = -0.0001768$, $G_a = -0.0011468$ ($\alpha = 8.4562$, $\beta = 12.0732$, $\gamma = 0.0052$, $m_0 = -0.1768$, $m_1 = -1.1468$).

```
1+SIMETRIX
2R 1 L, P 0 430m
3X$psi psi.OUT psi.OUT $$arbsourcepsi pinnames: N1 OUT
4 .subckt $$arbsourcepsi N1 OUT
5BI OUT 0 1=-0.0011468*V(N1) + (-0.00017680+0.0011468)*LIMIT(V(N1),-1,1)
6.ends
7C1 psi.OUT 0 118.2u 1C=2 BRANCH={IF (ANALYSIS=2,1,0)}
8C2 C2.P 0 1m IC=1 BRANCH={IF (ANALYSIS=2,1,0)}
9L L,P C2.P 82.8281 1C=-1m BRANCH={IF (ANALYSIS=2,0,1)}
10R psi.OUT C2.P 1k
11.graph "XY(C2.P, psi.OUT)" initxlims=false
12.TRAN 0 50 0 10n UIC
```

In Fig. 3 is shown corresponding SPICE realization of Chua circuit (see Fig. 1). Here $L$, $R_0$, $R$, $C_1$, $C_2$ correspond to the elements of Chua circuit, and element $XYProbe$ is used to measure the voltage on capacitors (and plot projection of trajectories on $(v_1, v_2)$-plane). The Chua diode with characteristic $f(v_1)$ is realized as “Arbitrary Source” (voltage-controlled current source) $\psi$.

\(^1\)https://www.simetrix.co.uk/
Figure 2: Left subfigure: two symmetric self-excited attractors (blue) excited from zero unstable equilibrium (parameters $\alpha = 15$, $\beta = 28$, $\gamma = 0$, $m_0 = -5/7$, $m_1 = -8/7$). Right subfigure: two symmetric hidden chaotic attractors (blue), trajectories (red) from unstable manifolds $M_{\text{unst}}^\pm$ of two saddle points $S^\pm$ are either attracted to locally stable zero equilibrium $F_0$, or tend to infinity; trajectories (black) from stable manifolds $M_{\text{st}}^0, \pm$ tend to $F_0$ or $S^\pm$ (parameters $\alpha = 8.4562$, $\beta = 12.0732$, $\gamma = 0.0052$, $m_0 = -0.1768$, $m_1 = -1.1468$).

For the considered values of parameters there are three equilibria in the system: the zero equilibrium $F_0 = (0, 0, 0)$ is a stable focus-node and two symmetric saddle equilibria $S^\pm$. To check that an attractor is hidden, we have to demonstrate that the trajectories from certain small vicinities of equilibria are not attracted by the attractor. Figs. 4 and 5 show SPICE simulation\(^2\) of trajectory in a vicinity of zero equilibrium and a trajectory with initial condition corresponding to the unstable manifold of the saddle (in both cases the considered trajectories are attracted to the zero equilibrium). Projection of twin symmetric chaotic attractors on the plane $(v_1, v_2)$ and attraction to the zero equilibrium are shown in Fig. 6.

In Fig. 7 is shown simulation self-excited Chua attractor in SPICE for the parameters $C_1 = 66.667u$, $C_2 = 1m$, $L = 35.7143$, $R_0 = 0$, $R = 1k$, $G_b = -0.0071429$, $G_a = -0.0011468$. In this case the trajectory with initial data in a vicinity of zero equilibrium is attracted by the self-excited attractor.

3 Synchronization of Chua circuits

Nowadays various chaotic secure communication systems based on Chua circuits and other electronic generators of chaotic oscillations are of interest [34, 35, 36, 37, 38, 39]. The operation of such systems is based on the synchronization chaotic signals of two chaotic identical generators (transmitter and receiver) for different initial data. The control signal proportional to the difference between the circuits signals, adjust the state of one receiver. The multistability and existence of hidden attractors may lead to improper workreceiver of such systems.

\(^2\)In our experiment “Max time step” is set to 10ns.
Consider now two \( x \)-coupled Chua systems

\[
\begin{align*}
\frac{dx}{dt} &= \alpha(y - x - \psi(x) + \delta(\bar{x} - x)), \\
\frac{dy}{dt} &= x - y + z, \\
\frac{dz}{dt} &= -(\beta y + \gamma z), \\
\frac{d\bar{x}}{dt} &= \alpha(\bar{y} - \bar{x} - \psi(\bar{x}) + \delta(x - \bar{x})), \\
\frac{d\bar{y}}{dt} &= \bar{x} - \bar{y} + \bar{z}, \\
\frac{d\bar{z}}{dt} &= -(\beta \bar{y} + \gamma \bar{z}),
\end{align*}
\] (4)

where \( \delta \) is a coupling factor (\( \delta = \frac{R}{\mathcal{R}_{\text{coupling}}} \)), and the corresponding circuit (see Fig. 8).
Figure 5: Vicinity of saddle point: \( v_2(0) = -2.80804m, \ v_1(0) = -6.5224, \ i_L(0) = 6.5196m \)

Figure 6: Voltage \( v_1 \) vs voltage \( v_2 \). Initial condition for blue trajectory (attracted to the stable zero equilibrium): \( v_2(0) = -2.80804m, \ v_1(0) = -6.5224, \ i_L = 6.5196m \). Initial condition for red trajectory (visualizing a hidden attractor – projection on the plane): \( v_2(0) = 1, \ v_1(0) = 2, \ i_L = -4m \). Initial condition for green trajectory (attracted to the stable zero equilibrium): \( v_2(0) = 2.80804m, \ v_1(0) = 6.5224, \ i_L = -6.5196m \).

Figure 8: Two resistor-coupled Chua circuits
Figure 7: Voltage $v_1$ vs voltage $v_2$. Initial condition for red trajectory (visualizing a self-excited attractor projection on the plane): $v_2(0) = 10m$, $v_1(0) = 0$, $i_L = 0$.

SIMetrix SPICE realization of the coupled Chua circuits is shown in Fig. 9. Here the passive elements of the first circuit: $L, R_0, R, C1, C2$, are identical to the corresponding elements of the second circuit: $L1, R1, R2, C3, C4$. Elements $XY$ Probe are used to measure voltages on capacitors and plot on $(v_1, v_2)$-plane. To simulate nonlinear elements with characteristic $f(v_1)$, we use SPICE elements “Arbitrary Source” (voltage-controlled current source) psi and psi1. Resistor $R_3$ connects two circuits and characterizes the distance between transmitter and receiver. Below it is shown that critical coupling value (i.e. maximum value for which synchronization takes place) is different for different choice of initial data because of the multistability and hidden attractors. In our experiments the simulation time is 100 seconds and minimal value of $R3$ is 1000.

**Example 1.** Consider initial data of the circuits on one of the symmetrical hidden attractors: Chua circuit 1 — $v_1(0) = -3.7727, v_2(0) = -1.3511, i_L(0) = 0.0047$; Chua circuit 2 — $v_1(0) = -3.6, v_2(0) = -1.3511, i_L(0) = 0.0047$.

**Example 2.** Consider initial data of the circuits on two symmetrical hidden attractors: Chua circuit 1: $v_1(0) = -3.7727, v_2(0) = -1.3511, i_L(0) = 0.0047$; Chua circuit 2: $v_1(0) = 3.7727, v_2(0) = 1.3511, i_L(0) = -0.0047$.

**Example 3** Consider initial data of the circuits on one of the symmetrical hidden attractors and stable zero equilibrium: Chua circuit 1 — $v_1(0) = -3.7727, v_2(0) = -1.3511, i_L(0) = 0.0047$; Chua circuit 2 — $v_1(0) = 0, v_2(0) = 0, i_L(0) = 0$.

**Example 4** Consider the case of symmetrical self-excited attractors for the parameters $\alpha = 15, \beta = 28, \gamma = 0, m_0 = -5/7, m_1 = -8/7$ (corresponding SPICE parameters: $C_1 = 6.667 \cdot 10^{-5}, C_2 = 0.001, G_a = -7.1429 \cdot 10^{-4}, G_b = -1.1429, L = 35.7134, R_0 = 0$). Take the following initial data of the circuits on one of the symmetrical self-excited attractors and unstable zero equilibrium: Chua circuit 1 — $v_1(0) = -0.780, v_2(0) = -0.525, i_L(0) = 3.5m$; Chua circuit 2 — $v_1(0) = 0, v_2(0) = 0, i_L(0) = 0$.

Figure 10: Initial data of the circuits on the same hidden attractor. Critical coupling value $R3 = 2.348 \cdot 10^3$. 

**Example 2.** Consider initial data of the circuits on two symmetrical hidden attractors: Chua circuit 1: $v_1(0) = -3.7727, v_2(0) = -1.3511, i_L(0) = 0.0047$; Chua circuit 2: $v_1(0) = 3.7727, v_2(0) = 1.3511, i_L(0) = -0.0047$.

**Example 3** Consider initial data of the circuits on one of the symmetrical hidden attractors and stable zero equilibrium: Chua circuit 1 — $v_1(0) = -3.7727, v_2(0) = -1.3511, i_L(0) = 0.0047$; Chua circuit 2 — $v_1(0) = 0, v_2(0) = 0, i_L(0) = 0$.

**Example 4** Consider the case of symmetrical self-excited attractors for the parameters $\alpha = 15, \beta = 28, \gamma = 0, m_0 = -5/7, m_1 = -8/7$ (corresponding SPICE parameters: $C_1 = 6.667 \cdot 10^{-5}, C_2 = 0.001, G_a = -7.1429 \cdot 10^{-4}, G_b = -1.1429, L = 35.7134, R_0 = 0$). Take the following initial data of the circuits on one of the symmetrical self-excited attractors and unstable zero equilibrium: Chua circuit 1 — $v_1(0) = -0.780, v_2(0) = -0.525, i_L(0) = 3.5m$; Chua circuit 2 — $v_1(0) = 0, v_2(0) = 0, i_L(0) = 0$. 

6
Coupling resistor
Chua circuit 1
Chua circuit 2

-0.0011468*V(N1) + (-0.00017680 + 0.0011468)*LIMIT(V(N1),-1,1)

-0.0011468*V(N1) + (-0.00017680 + 0.0011468)*LIMIT(V(N1),-1,1)

Figure 9: SIMetrix SPICE model of two coupled Chua’s circuits
Figure 11: Initial data of the circuits on two symmetrical hidden attractors. Critical coupling resistor value is $R_3 = 5.110 \cdot 10^3$.

Figure 12: Initial data of the circuits on a hidden attractor and stable zero equilibrium. Critical coupling resistor value is $R_3 = 2.300 \cdot 10^3$.

Figure 13: Initial data of the circuits on a self-excited attractor and unstable zero equilibrium. Critical coupling resistor value is $R_3 = 2.870 \cdot 10^3$. 
Conclusion

In conclusion we note that the simulation of circuit behavior by software, as well as the numerical integration of its dynamical model, is subject to numerical errors due to time discretization step (see, e.g. the corresponding examples with phase-locked-loop based circuits in [40, 41, 42, 43]). Observation of hidden Chua attractors in physical experiments is discussed, e.g., in [23, 44, 45, 47, 46].

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References

[1] E.N. Lorenz, Deterministic nonperiodic flow, J. Atmos. Sci. 20 (1963), pp. 130–141.

[2] R. Tokunaga, M. Komuro, T. Matsumoto, and L.O. Chua, lorenz attractor from an electrical circuit with uncoupled continuous piecewise-linear resistor, International Journal of Circuit Theory and Applications 17 (1989), pp. 71–85.

[3] F. Robinson, Analogue electronic model of the Lorenz equations, International Journal of Electronics 68 (1990), pp. 803–819.

[4] S. Pilyugin, Theory of pseudo-orbit shadowing in dynamical systems, Differential Equations 47 (2011), pp. 1929–1938.

[5] R. Lozi, chap. Can we trust in numerical computations of chaotic solutions of dynamical systems?, Topology and Dynamics of Chaos (Ed.: Ch. Letellier), World Scientific Series in Nonlinear Science Series A, 2013, pp. 63–98.

[6] G. Chen and T. Ueta, Chaos in Circuits and Systems, World Scientific, 2002.

[7] T. Matsumoto, A chaotic attractor from Chua’s circuit, IEEE Transaction on Circuits and Systems 31 (1990), pp. 1055–1058.

[8] L. Chua, A zoo of strange attractors from the canonical Chua’s circuits, Proceedings of the IEEE 35th Midwest Symposium on Circuits and Systems (Cat. No.92CH3099-9) 2 (1992), pp. 916–926.

[9] E. Bilotta and P. Pantano, A gallery of Chua attractors, Vol. Series A. 61, World Scientific, 2008.

[10] G. Leonov, N. Kuznetsov, and V. Vagaitsev, Localization of hidden Chua’s attractors, Physics Letters A 375 (2011), pp. 2230–2233.

[11] G. Leonov and N. Kuznetsov, Hidden attractors in dynamical systems. From hidden oscillations in Hilbert-Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractors in Chua circuits, International Journal of Bifurcation and Chaos 23 (2013), art. no. 1330002.

[12] G. Leonov, N. Kuznetsov, and T. Mokaev, Homoclinic orbits, and self-excited and hidden attractors in a Lorenz-like system describing convective fluid motion, Eur. Phys. J. Special Topics 224 (2015), pp. 1421–1458.

[13] N. Kuznetsov, Hidden attractors in fundamental problems and engineering models. A short survey, Lecture Notes in Electrical Engineering 371 (2016), pp. 13–25, (Plenary lecture at International Conference on Advanced Engineering Theory and Applications 2015).

[14] G. Leonov and N. Kuznetsov, Localization of hidden oscillations in dynamical systems (plenary lecture), in 4th International Scientific Conference on Physics and Control, URL http://www.math.spbu.ru/user/leonov/publications/2009-PhysCon-Leonov-plenary-hidden-oscillations.pdf#page=21, 2009.

[15] N. Kuznetsov, G. Leonov, and V. Vagaitsev, Analytical-numerical method for attractor localization of generalized Chua’s system, IFAC Proceedings Volumes 43 (2010), pp. 29–33.

[16] G. Leonov, N. Kuznetsov, and V. Vagaitsev, Hidden attractor in smooth Chua systems, Physica D: Nonlinear Phenomena 241 (2012), pp. 1482–1486.
[17] N. Kuznetsov, O. Kuznetsova, G. Leonov, and V. Vagaitsev, *Analytical-numerical localization of hidden attractor in electrical Chua's circuit*, Informatics in Control, Automation and Robotics, Lecture Notes in Electrical Engineering, Volume 174, Part 4 174 (2013), pp. 149–158.

[18] Q. Li, H. Zeng, and X.S. Yang, *On hidden twin attractors and bifurcation in the Chua’s circuit*, Nonlinear Dynamics 77 (2014), pp. 255–266.

[19] I. Burkin and N. Khien, *Analytical-numerical methods of finding hidden oscillations in multidimensional dynamical systems*, Differential Equations 50 (2014), pp. 1695–1717.

[20] C. Li and J.C. Sprott, *Coexisting hidden attractors in a 4-D simplified Lorenz system*, International Journal of Bifurcation and Chaos 24 (2014), art. num. 1450034.

[21] Z. Zhusubaliyev, E. Mosekilde, A. Churilov, and A. Medvedev, *Multistability and hidden attractors in an impulsive Goodwin oscillator with time delay*, European Physical Journal: Special Topics 224 (2015), pp. 1519–1539.

[22] A. Kuznetsov, S. Kuznetsov, E. Mosekilde, and N. Stankevich, *Co-existing hidden attractors in a radio-physical oscillator system*, Journal of Physics A: Mathematical and Theoretical 48 (2015), p. 125101.

[23] M. Chen, M. Li, Q. Yu, B. Bao, Q. Xu, and J. Wang, *Dynamics of self-excited attractors and hidden attractors in generalized memristor-based Chua’s circuit*, Nonlinear Dynamics 81 (2015), pp. 215–226.

[24] V. Semenov, I. Korneev, P. Arinushkin, G. Strelkova, T. Vadivasova, and V. Anishchenko, *Numerical and experimental studies of attractors in memristor-based Chua’s oscillator with a line of equilibria. Noise-induced effects*, European Physical Journal: Special Topics 224 (2015), pp. 1553–1561.

[25] T. Menacer, R. Lozi, and L. Chua, *Hidden bifurcations in the multislipral Chua attractor*, International Journal of Bifurcation and Chaos 26 (2016), art. num. 1630039.

[26] I. Zelinka, *Evolutionary identification of hidden chaotic attractors*, Engineering Applications of Artificial Intelligence 50 (2016), pp. 159–167.

[27] M.F. Danca, N. Kuznetsov, and G. Chen, *Unusual dynamics and hidden attractors of the Rabinovich–Fabrikant system*, Nonlinear Dynamics 88 (2017), pp. 791–805.

[28] M.F. Danca, *Hidden transient chaotic attractors of Rabinovich–Fabrikant system*, Nonlinear Dynamics 86 (2016), pp. 1263–1270.

[29] Z. Wei, V.T. Pham, T. Kapitaniak, and Z. Wang, *Bifurcation analysis and circuit realization for multiple-delayed Wang–Chen system with hidden chaotic attractors*, Nonlinear Dynamics 85 (2016), pp. 1635–1650.

[30] V.T. Pham, C. Volos, S. Jafari, S. Vaidyanathan, T. Kapitaniak, and X. Wang, *A chaotic system with different families of hidden attractors*, International Journal of Bifurcation and Chaos 26 (2016), p. 1650139.

[31] S. Jafari, V.T. Pham, S. Golpayegani, M. Moghtadaei, and S. Kingni, *The relationship between chaotic maps and some chaotic systems with hidden attractors*, Int. J. Bifurcat. Chaos 26 (2016), art. num. 1650211.

[32] D. Dudkowski, S. Jafari, T. Kapitaniak, N. Kuznetsov, G. Leonov, and A. Prasad, *Hidden attractors in dynamical systems*, Physics Reports 637 (2016), pp. 1–50.

[33] J. Williams, *Analog Circuit Design: Art, Science, and Personalities*, Elsevier Science, 2016.

[34] T. Kapitaniak, *Chaotic Oscillators: Theory and Applications*, World Scientific, 1992.

[35] M. Ogorzalek, *Chaos and Complexity in Nonlinear Electronic Circuits*, World Scientific, 1997.

[36] T. Yang, *A survey of chaotic secure communication systems*, International Journal of Computational Cognition 2 (2004), pp. 81–130.

[37] M. Eisencraft, R. Attux, and R. Suyama, *Chaotic Signals in Digital Communications*, CRC Press, 2013.

[38] N. Kuznetsov and G. Leonov, *Hidden attractors in dynamical systems: systems with no equilibria, multistability and coexisting attractors*, IFAC Proceedings Volumes 47 (2014), pp. 5445–5454.

[39] N. Kuznetsov, G. Leonov, T. Mokaev, and S. Seledzhi, *Hidden attractor in the Rabinovich system, Chua circuits and PLL*, AIP Conference Proceedings 1738 (2016), art. num. 210008.
[40] G. Leonov, N. Kuznetsov, M. Yuldashev, and R. Yuldashev, *Hold-in, pull-in, and lock-in ranges of PLL circuits: rigorous mathematical definitions and limitations of classical theory*, IEEE Transactions on Circuits and Systems–I: Regular Papers 62 (2015), pp. 2454–2464.

[41] G. Bianchi, N. Kuznetsov, G. Leonov, S. Seledzhi, M. Yuldashev, and R. Yuldashev, *Hidden oscillations in SPICE simulation of two-phase Costas loop with non-linear VCO*, IFAC-PapersOnLine 49 (2016), pp. 45–50.

[42] R. Best, N. Kuznetsov, G. Leonov, M. Yuldashev, and R. Yuldashev, *Tutorial on dynamic analysis of the Costas loop*, Annual Reviews in Control 42 (2016), pp. 27–49.

[43] N. Kuznetsov, G. Leonov, M. Yuldashev, and R. Yuldashev, *Hidden attractors in dynamical models of phase-locked loop circuits: limitations of simulation in MATLAB and SPICE*, Commun Nonlinear Sci Numer Simulat (2017), doi: 10.1016/j.cnsns.2017.03.010.

[44] B. Bao, F. Hu, M. Chen, Q. Xu, and Y. Yu, *Self-excited and hidden attractors found simultaneously in a modified Chua’s circuit*, International Journal of Bifurcation and Chaos 25 (2015), art. num. 1550075.

[45] M. Chen, J. Yu, and B.C. Bao, *Hidden dynamics and multi-stability in an improved third-order Chua’s circuit*, The Journal of Engineering (2015), doi: 10.1049/joe.2015.0149.

[46] B.C. Bao, Q.D. Li, N. Wang, and Q. Xu, *Multistability in Chua’s circuit with two stable node-foci*, Chaos: An Interdisciplinary Journal of Nonlinear Science 26 (2016), art. num. 043111.

[47] M. Chen, J. Yu, and B.C. Bao, *Finding hidden attractors in improved memristor-based Chua’s circuit*, Electronics Letters 51 (2015), pp. 462–464.