Understanding Deep MIMO Detection

Qiang Hu, Feifei Gao, Fellow, IEEE, Hao Zhang, Geoffrey Ye Li, Fellow, IEEE, and Zongben Xu

Abstract—Incorporating deep learning (DL) into multiple-input multiple-output (MIMO) detection has been deemed as a promising technique for future wireless communications. However, most of the DL-based MIMO detection algorithms are lack of interpretation on internal mechanisms. In this paper, we analyze the performance of the DL-based MIMO detection to better understand its strengths and weaknesses. We investigate and compare two different models: data-driven DL detector with neural networks activated by rectifier linear unit (ReLU) function and model-driven DL detector based on traditional detection algorithms. We show that the data-driven DL detector asymptotically approaches to the maximum a posterior (MAP) detector in various scenarios but requires a large amount of training samples to converge in time-varying channels. On the other hand, the model-driven DL detector utilizes the expert knowledge to alleviate the impact of channels and achieves relatively high detection accuracy with a small set of training data. Simulation results confirm our analytical results and demonstrate the effectiveness of the DL-based MIMO detection for both linear and nonlinear signal systems.

Index Terms—Explainable deep learning, MIMO detection, deep neural network, model-driven.

I. INTRODUCTION

MULITIPLE-INPUT multiple-output (MIMO) technology is vital for future wireless communication systems to support explosively growing throughput requirement [1], [2], [3]. The maximum a posterior (MAP) detector delivers the optimal detection performance but is computationally intractable [4]. Alternatively, suboptimal detection algorithms are implemented to achieve a better tradeoff between accuracy and complexity. The linear detectors, such as zero-forcing (ZF) and linear minimum mean-squared error (LMMSE), are with low complexity but exhibit poor detection performance.

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Qiang Hu and Hao Zhang are with the Department of Electrical Engineering, Tsinghua University, Beijing 100084, China (e-mail: hq760001570@163.com; haozhang@mail.tsinghua.edu.cn). Feifei Gao is with the Institute for Artificial Intelligence, the State Key Laboratory of Intelligent Technologies and Systems, the Beijing National Research Center for Information Science and Technology (BNRist), and the Department of Automation, Tsinghua University, Beijing 100084, China (e-mail: feifeigao@ieee.org).

Geoffrey Ye Li is with the Department of Electrical and Computer Engineering, Imperial College London, South Kensington Campus, SW7 2AZ London, U.K. (e-mail: Geoffrey.Li@imperial.ac.uk).

Zongben Xu is with the School of Mathematics and Statistics, Xi’an Jiaotong University, Xi’an 710049, China (e-mail: zxu@mail.xjtu.edu.cn).

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The iterative detection algorithms, e.g., approximate message passing (AMP) [5], [6], sphere decoding (SD) [7], soft interference cancellation (SIC) [8], [9], can achieve relatively good performance with moderate complexity. However, many iterative detectors are model-specific and require complete knowledge of channel state information (CSI).

Over the last decade, deep learning (DL) has made profound technical revolution to many areas, such as computer vision [10], speech recognition [11], and auto driving [12]. Recently, DL has been applied to the design of communication systems, including physical layer processing [13], [14], [15] and resource management [16], [17], etc. Among all these applications, MIMO detection is one of the most crucial and fundamental issues. The DL-based detectors could achieve better performance than the traditional detection algorithms in some complex environments [18].

Generally, the existing DL-based detectors can be divided into two categories: data-driven DL detectors based on deep neural networks (DNNs) and model-driven DL detectors from unfolding iterative detection algorithms. The data-driven DL detectors use DNNs for detection and can recover transmitted symbols at high precision without model knowledge [19], [20]. The model-driven DL detectors are designed from the traditional iterative detection algorithms, each layer of which represents a single iteration with added trainable parameters [21], [22], [23]. The resulting detectors tend to have better performance and faster convergence compared to the original iterative detection algorithms [23].

Despite their great success, data-driven DL detectors are considered as black boxes for signal reception and only experimental evaluation is available to demonstrate their performance. In fact, there exists a lot of literature on analyzing the internal mechanisms of DNNs. The pioneering works in [24] and [25] have proved that any continuous function defined on a compact set can approximated with any precision by a DNN with sigmoid activation function. Recently, it has been proved in [26] and [27] that DNNs with rectified linear units (ReLU DNNs) can also approximate to a large family of functions. Furthermore, the DL-based channel estimation has been proved to converge to the minimum mean-squared error (MMSE) estimator at the size of training set increases [28]. However, the analysis above can not be directly generalized to the MIMO detection and little literature has been dedicated to the interpretation of the DL-based MIMO detection, not say its advantages and disadvantages.

In this paper, we analyze the performance of the DL-based MIMO detection from the theoretical perspective. Our contributions are listed as follows:
We prove that the data-driven DL detector with ReLU DNNs can well approximate the MAP detector under sufficiently large training set in MIMO systems. The rate of convergence of the data-driven DL detector to the MAP detector scales at least polynomially fast with the size of training samples.

We show that the data-driven DL detector requires no CSI to achieve the MAP comparable performance in time-invariant channels and are robust to CSI uncertainty. For time-varying channels, the data-driven DL detector requires perfect CSI for convergence to the MAP detector and are sensitive to CSI uncertainty.

We propose a model-driven DL detector, referred to as DisNet, that combines a SIC detector and a discriminative net. The resulting DL detector improves error rate compared to the original SIC detector and approaches the MAP detector at affordable complexity.

The rest of this paper is organized as follows. The system model and the traditional MIMO detection algorithms are introduced in Section II. The performance analysis of the data-driven and the model-driven DL detectors is presented in Sections III and IV, respectively. Simulation results are provided in Section V followed by the conclusions in Section VI.

Notations: We use lowercase letters and capital letters in boldface to denote vectors and matrices, respectively. The positive integer set, natural number set, real number set, and complex number set are denoted by \( \mathbb{N} \), \( \mathbb{Z} \), \( \mathbb{R} \), and \( \mathbb{C} \), respectively. The real and imaginary parts of a complex matrix or vector are denoted by \( \Re \{ \cdot \} \) and \( \Im \{ \cdot \} \), respectively. Notation \( \{ \cdot \} \) denotes the transpose and Hermitian of a matrix or vector, respectively. \( \mathbb{E} \{ \cdot \} \) denotes the expectation, \( \| \cdot \|_2 \) and \( \| \cdot \|_\infty \) represent the 2-norm and supremum-norm of a vector or a matrix, respectively. Notation \( \lceil \cdot \rceil \) represents the ceiling of a real number. Notation \( 1_A(x) \) is an indicator function of set \( A \), where \( 1_A(x) = 1 \) if \( x \in A \) and \( 1_A(x) = 0 \) if \( x \notin A \).

II. PRELIMINARIES ON MIMO DETECTION

Let us consider a standard linear MIMO system with \( N_t \) transmit and \( N_r \) receive antennas. The \( N_r \times 1 \) received signal vector at the BS is

\[
\bar{x} = \bar{H}s + \bar{n},
\]

where \( \bar{H} \in \mathbb{C}^{N_r \times N_t} \) is the channel matrix, \( \bar{s} \in \bar{\mathbb{S}}^{N_t} \) is the transmitted symbol vector with mutually independent elements drawn from a discrete constellation \( \bar{\mathbb{S}} \), and \( \bar{n} \in \mathbb{C}^{N_r} \) is an independent zero-mean Gaussian noise vector with element-wise variance \( \sigma_n^2 \).

To facilitate DL processing, we re-organize (1) into a real-valued signal model

\[
x = \mathcal{R}(\bar{H})s + \mathcal{R}(\bar{n}),
\]

where

\[
x = \begin{bmatrix} \mathcal{R}(\bar{x}) \\ \Re \{ \mathcal{R}(\bar{s}) \} \\ \Im \{ \mathcal{R}(\bar{n}) \} \end{bmatrix}, \quad s = \begin{bmatrix} \mathcal{R}(\bar{s}) \\ \Im \{ \mathcal{R}(\bar{s}) \} \end{bmatrix}, \quad n = \begin{bmatrix} \mathcal{R}(\bar{n}) \\ \Im \{ \mathcal{R}(\bar{n}) \} \end{bmatrix},
\]

and

\[
H = \begin{bmatrix} \Re \{ \mathcal{R}(\bar{H}) \} \\ \Im \{ \mathcal{R}(\bar{H}) \} \end{bmatrix}.
\]

Assume that the real part and the imaginary part of \( \bar{S} \) are the same, i.e. \( \Re \{ \bar{S} \} = \Im \{ \bar{S} \} = 3 \{ \bar{S} \} \). Then, we have \( s \in \mathbb{S}^{2N_t} \), \( x \in \mathbb{R}^{2N_r} \), and \( n \in \mathbb{R}^{2N_r} \) in (2).

Let \( p_o(s|x) \) be the conditional probability of \( s \) given \( x \). The MAP detector is optimal in terms of minimizing the error probability of symbol detection [29], i.e.,

\[
s_{\text{MAP}} = \arg \max_{s \in \mathbb{S}^{2N_t}} p_o(s|x).
\]

However, there are two reasons that prevent the MAP detector from practical applications: 1) an exhaustive search of \( \mathbb{S}^{2N_t} \) input combinations is required to solve the MAP optimization in (4) and is computationally infeasible when \( N_t \) is large; 2) the accurate knowledge of \( p_o(s|x) \) is difficult to obtain especially when unknown distortions are presented, e.g., quantization error of analog-to-digital converter (ADC) [30].

III. DATA-DRIVEN DL DETECTOR

In this section, we first introduce the basic setting of data-driven DL detector in Section III-A. Then, the performance of the data-driven DL detector is analyzed via statistical learning theory in Section III-B. Finally, the limits of the data-driven DL detector is discussed in Section III-C.

A. Basic Setting of Data-Driven DL Detector

Let us consider a data-driven DL detector, \( D \), with a fully-connected ReLU DNN, where only the received signal \( x \) is available. A major concern for \( D \) is that the constellations in communication systems are generally not taken as the targets of DNNs. To comply with standard processing in DL methods, we need to re-parameterize \( s \) using one-hot mapping. Let \( s_i \in \mathbb{S}^2 \) be the 2-dimensional vector of real-valued symbols transmitted at the \( i \)-th antenna. Stacking all the symbols at transmitted antenna, we can express \( s \) as

\[
s = (s_1, s_2, \ldots, s_{j_{N_t}}),
\]

where \( \{ j_1, \ldots, j_{2N_t} \} \in \mathbb{N}^{2N_t} : 0 \leq j_i, j_{N_t+i} \leq (|\bar{S}| - 1) \forall i = 1, \ldots, N_t \) and \( s_i = (s_{j_i}, s_{j_{N_t+i}}) \) for \( s_{j_i} \in \bar{\mathbb{S}} \) and \( s_{j_{N_t+i}} \in \bar{\mathbb{S}} \).

For notation convenience, we associate a unit vector \( u \in \mathbb{R}^{2N_t} \) with each \( s \in \mathbb{S}^{2N_t} \), where the index of nonzero element of \( u \) can be derived from \( \sum_{i=1}^{N_t-1} (j_{N_t+i}+j_i)|S| = \begin{bmatrix} j_{N_t+i} \end{bmatrix} \).

Here, \( u \) is a bijective transformation of \( s \) with \( p_o(u|x) = p_o(u|x) \) and can be set as the target for MIMO detection. The input-output sample set of \( D \) is then defined by

\[
Z = \{ (x_m, \mathbf{u}_m) | x \in \mathbb{R}^{2N_r}, \mathbf{u} \in \mathbb{R}^{2N_t}, m = 1, \ldots, |Z| \}
\]

and the samples in \( Z \) are independent and identically distributed (i.i.d.).

Let \( l \in \mathbb{N} \) be the number of hidden layers and the neuron assignment of \( D \) is given by \( d = (d_0, d_1, \ldots, d_l, d_{l+1}) \in \mathbb{N}^{l+2} \).

1 Zero is the first index.
with $d_0 = 2N$, and $d_{l+1} = |\Sigma|^{2N}$. The depth, width, and size of $D$ are defined by $l$, $\max\{d_1, \ldots, d_l\}$, and $d_u = \sum_{i}^l d_i$, respectively.

Let

$$\Theta = \{ \theta = (\text{vec}(W_0), b_0, \ldots, \text{vec}(W_{l}), b_{l}) \in \mathbb{R}^{d_l} \}$$

be the set of all parameters of $D$, where $d_s = \sum_{i=0}^l d_{i+1} \times (d_i + 1)$, $W_i \in \mathbb{R}^{d_{i+1} \times d_i}$ is the weight matrix connecting the $i$-th layer to the $(i+1)$-th layer, and $b_i \in \mathbb{R}^{d_{i+1}}$ is the bias vector of the $(i+1)$-th layer for $i \in \{0, \ldots, l\}$.

For a fixed $\theta$, $p_\theta(x) = \psi_{d_{l+1}} \circ A_l \circ \varphi_{d_l} \circ A_{l-1} \circ \varphi_{d_{l-1}} \circ \ldots \circ \varphi_{d_1} \circ A_0(x)$

is the underlying function of $D$, where $\circ$ denotes the function composition, i.e., $\varphi_{d_i}(A_i(\cdot))$ represented by $\varphi_{d_i} \circ A_{i-1}$, $\psi_{d_{i+1}} : \mathbb{R}^{d_{i+1}} \rightarrow \mathbb{R}^{d_i}$ is the entry-wise softmax function, $A_i : \mathbb{R}^{d_i} \rightarrow \mathbb{R}^{d_{i+1}}$ is the affine transformation with weight $W_i$ and bias $b_i$, and $\varphi_{d_i} : \mathbb{R}^{d_i} \rightarrow \mathbb{R}^{d_i}$ is the entry-wise ReLU activation function for $i \in \{0, \ldots, l\}$. In fact, ReLU DNNs have been widely used by the most of the DL embedded communication systems [31], [32]. Some state-of-the-art networks, e.g., ResNet and GoogleNet, involve the use of ReLU activation functions and the convolutional layers are variant forms of dense layer with weights regularized. Therefore, we choose ReLU DNNs as an example to illustrate the DL based MIMO detection.

Denote $\tilde{x}_i = [x_{i1}, \ldots, x_{i_{d_i}}]^T$ as the output of the $i$-th layer. Let $A_i(\tilde{x}_i) = [x_{i1}, \ldots, x_{i_{d_{i+1}}}]^T \in \mathbb{R}^{d_{i+1}}$ be the preactivation of the $(i+1)$-th layer. The ReLU activation function and softmax function can be expressed as

$$\varphi_{d_{i+1}}(A_i(\tilde{x}_i)) = (\max\{0, x_{iA_1}\}, \ldots, \max\{0, x_{iA_{d_{i+1}}}\}),$$  

and

$$\psi_{d_{i+1}}(A_i(\tilde{x}_i)) = \left( \frac{\exp(x_{iA_1})}{\sum_{j=1}^{d_{i+1}} \exp(x_{iA_j})}, \ldots, \frac{\exp(x_{iA_{d_{i+1}}})}{\sum_{j=1}^{d_{i+1}} \exp(x_{iA_j})} \right),$$

respectively.

From (9), the neurons in $D$ have only two states: zero output or replicating input. All possible states of neurons in $D$ can be represented by a set $\mathcal{K} \subseteq \{0, 1\}^{d_k}$. Each element in $\mathcal{K}$ is a $d_k$-dimensional vector with its entries being either 0 or 1. Similar to [26], the input space of $D$ is partitioned into linear regions according to different states. Denote $\mathcal{X}$ as the input space of $D$ and we have

$$\mathcal{X}_k \subseteq \mathcal{X}, \quad k = 1, \ldots, |\mathcal{K}|,$$

$$\mathcal{X} = \bigcup_{k = 1}^{|\mathcal{K}|} \mathcal{X}_k,$$

where $\mathcal{X}_k$ is the input region corresponding to the $k$-th state. For $x \in \mathcal{X}_k$, $A_i(\tilde{x}_i)$ in (8) satisfies

$$A_i(\tilde{x}_i) = \begin{cases} W_0 x + b_0, & i = 0, \\ W_i A_{i-1}(\tilde{x}_{i-1}) + b_i, & i \geq 1, \end{cases}$$

where $W_i = W_i A_i$ and $A_i$ is an $\mathbb{R}^{d_i \times d_i}$ diagonal matrix whose diagonal element is either 0 or 1. Moreover, $A_0 = I_{d_0}$ and $W_0 = W_0 A_0$. By expanding $A_i(\tilde{x}_i)$ recursively, we further obtain

$$A_i(\tilde{x}_i) = \prod_{j=0}^{i} \tilde{W}_j x + \sum_{j=0}^{i-1} \left( \prod_{p=0}^{j} \tilde{W}_{i-p} \right) b_{i-1-j} + b_i,$$

$$= \tilde{W}_i x + \tilde{b}_i,$$

where $\tilde{W}_i = \prod_{j=0}^{i} W_j$ and $\tilde{b}_i = \sum_{j=0}^{i-1} \left( \prod_{p=0}^{j} \tilde{W}_{i-p} \right) b_{i-1-j} + b_i$. Let $f_\theta(x) = A_i(\tilde{x}_i)$ be the input at the last layer and

$$f_\theta(x) = \tilde{W}_x x + \tilde{b}_x,$$

turns into a piecewise linear function for any $x \in \mathcal{X}$ with $\tilde{W}_x, \tilde{b}_x$ are i.i.d. multinomial random variables.

Denote

$$p_\theta(u|x) = \prod_{i=1}^{d_{l+1}} p_{\theta,i}(x|u)^u_i(u)$$

as the estimated conditional probabilities of $u$ given $x$, where $p_{\theta,i}(x|u)$ is the $i$-th entry of $p_\theta(x)$.

To find $\theta$ that leads to the optimal data-driven DL detector, we need a loss function to measure the distance between $p_\theta(u|x)$ and $p_\theta(u|x)$. Typically, the KL divergence of $p_\theta(u|x)$ and $p_\theta(u|x)$ is adopted, which is defined as

$$D_{KL}(p_\theta, p_\theta) = \mathbb{E}\{\ln p_\theta(u|x) / p_\theta(u|x)\},$$

Note that $D_{KL}(p_\theta, p_\theta)$ is non-negative and is equal to zero if and only if $p_\theta(u|x) = p_\theta(u|x)$ [33]. Though the KL information is not a distance function, its convergence often implies the same trend in other metrics [34].

Let $\Theta_R = \{ \theta \mid |\theta|_\infty \leq R, R \geq 1 \}$ be the bounded subset of $\Theta$ and the performance of the data-driven DL detector will be evaluated within $\Theta_R$. Let

$$J(\rho_o) = \mathbb{E}\{\ln p_\theta(u|x)\}$$

be the expectation of $\ln p_\theta(u|x)$. The optimal data-driven DL detector that minimizes $D_{KL}(p_\theta, p_\theta)$ within $\Theta_R$ can be expressed as

$$\theta_o = \arg \min_{\theta \in \Theta_R} D_{KL}(p_\theta, p_\theta) = \arg \max_{\theta \in \Theta_R} J(\rho_o),$$
where $J(p_\theta) = \mathbb{E}\{\ln p_\theta(u|x)\}$. However, the optimization over $J(p_\theta)$ in (19) is difficult to implement in practice. Generally,

$$J_Z(p_\theta) = \frac{1}{|Z|} \sum_{(u_m,x_m) \in Z} \ln p_\theta(u_m|x_m)$$  \hfill (20)

is applied to optimize $\theta$ with respect to (w.r.t.) $Z$ and

$$\theta_Z = \arg \max_{\theta \in \Theta_R} J_Z(p_\theta)$$  \hfill (21)

is the parameter vector of the corresponding maximum log-likelihood detector.

Then, the detected symbol of the data-driven DL detector can be expressed as

$$s_{DL} = \arg \max_{s \in \mathbb{R}^{7 \times 1}} p_{\theta_Z}(f_{\text{map}}(s)|x),$$  \hfill (22)

where $f_{\text{map}}(s)$ is one-to-one mapping from $s$ to $u$.

According to (17), $D_{KL}(p_o, p_{\theta_Z})$ can be decomposed into

$$D_{KL}(p_o, p_{\theta_Z}) = \left| J(p_o) - J(p_{\theta_o}) \right| + \left| J(p_{\theta_o}) - J(p_{\theta_Z}) \right|.$$  \hfill (23)

The first term $\left| J(p_o) - J(p_{\theta_o}) \right|$ in (23) is referred to as the approximation error, which is non-negative and is independent of $Z$. The second term $\left| J(p_{\theta_o}) - J(p_{\theta_Z}) \right|$ in (23) is called as the generalization error and is also non-negative.

Denote $f(x)$ as an $\mathbb{R}^d \rightarrow \mathbb{R}$ function and let $\ell_2$ be the finite 2-norm space of $f(x)$ with

$$\|f(x)\|_2 = \left[ \mathbb{E}\{f^2(x)\} \right]^{1/2} < +\infty.$$  \hfill (24)

The following theorem, proved in Appendix A, demonstrates that the approximation error in (23) can be narrowed down with any precision by ReLU DNNs.

**Theorem 1:** If $\ln p_o(u|x) \in \ell_2$, then there exists an optimized DL estimator built on a ReLU DNN of $\theta \in \Theta_R$ with sufficiently large $R$ and at most $\lceil \log_2(d_o + 1) \rceil$ hidden layers such that

$$J(p_o) - J(p_{\theta_o}) \leq \varepsilon$$  \hfill (25)

for any $\varepsilon > 0$.

**Remark 1:** Theorem 1 indicates that the data-driven DL detector is model independent and can approximate most of target distributions, which is in accordance with the results in [35]. Therefore, the data-driven DL detector is more flexible and capable compared to other model-based MIMO detection algorithms [36].

Next, we will discuss the convergence of the generalization error in (23). The following two auxiliary lemmas, proved in Appendixes B and C, respectively, are presented first.

**Lemma 1:** Let $\alpha = R\|d\|_{\infty}$, $\beta = \alpha/(\alpha - 1)$, and

$$\nu = \mathbb{E}\left\{\left[\ln d_{i+1} + 2(\alpha^{i+1}(\|x\|_2 + \beta) - \beta)\right]^2\right\}.$$  \hfill (26)

Assume that $\mathbb{E}\{\|x\|_2^2\}$ and $\nu$ are finite. For any $\varepsilon > 0$ and $|Z| \geq 4\nu/\varepsilon^2$, we have

$$\mathbb{P}\left(\sup_{\theta \in \Theta_R} |J_Z(p_\theta) - J(p_\theta)| > \varepsilon\right) \leq 4\mathbb{P}\left(\sup_{\theta \in \Theta_R} |J_Z^2(p_\theta)| > \frac{\varepsilon}{4}\right),$$  \hfill (27)

where $P$ is the distribution of training samples in $Z$ and

$$J^*_Z(p_\theta) = \frac{1}{|Z|} \sum_{m=1}^{|Z|} \omega_m \ln p_\theta(u_m|x_m)$$  \hfill (28)

with $\{\omega_1, \ldots, \omega_{|Z|}\}$ a Rademacher sequence.

**Lemma 2:** Assume $\frac{1}{|Z|} \sum_{m=1}^{|Z|} \|x_m\|^2_2 \leq \delta^2$. For any $\theta, \lambda \in \Theta_R$, we have

$$|J_Z(p_\theta) - J_Z(p_\lambda)| \leq 4\delta^2 + \alpha^2(\delta - \beta)^2.$$  \hfill (29)

If there exits a collection of functions $p_1(x), \ldots, p_C(x) \in D$ with their parameters belonging to $\Theta_R$ for $C \in \mathbb{N}$, the functions in this collection satisfy

$$|J_Z(p_\theta) - J_Z(p_j)| \leq \varepsilon, \forall j \in \{1, \ldots, C\},$$  \hfill (30)

for any $p_\theta(x) \in D$ with $\theta \in \Theta_R$ and $\varepsilon > 0$, where

$$p_j(u|x) = \prod_{i=1}^{d_{i+1}} p_{j,i}(x)^{1_{a_i(u)}}$$  \hfill (31)

and $p_{j,i}(x)$ is the $i$-th entry of $p_j(x)$. Define the covering number $C(\varepsilon, \Theta_R)$ as the smallest value of $C \in \mathbb{N}$ that satisfies (30). The following lemma, proved in Appendix D, indicates the upper bound of $\ln C(\varepsilon, \Theta_R)$.

**Lemma 3:** With the settings of Lemma 2, for any $\varepsilon > 0$, it holds that

$$\ln C(\varepsilon, \Theta_R) \leq d_s \ln \left[ \left( \frac{4\delta^4 + \alpha^2(\delta + \beta)^2}{\varepsilon} \right) \right].$$  \hfill (32)

Following Lemma 3, the next theorem, proved in Appendix E, demonstrates the rate of convergence of the generalization error in (23).

**Theorem 2:** Let $\alpha = R\|d\|_{\infty}$, $\beta = \alpha/(\alpha - 1)$, $\mu = \mathbb{E}\{\|x\|_2^2\}$, and

$$\nu = \mathbb{E}\left\{\left[\ln d_{i+1} + 1\right](\alpha^{i+1}(\|x\|_2 + \beta) - \beta)\right]^2\right\}.$$  \hfill (33)

Denote $\sigma$ as the variance of $\|x\|_2^2$. Let $d_1 = [\ln d_{i+1} + 1](\alpha^{i+1}\delta + \beta) - \beta)^2$ and $d_2 = 3\alpha^2(\delta + \beta)$ for any $\delta^2 \geq \mu$. For any $\varepsilon > 0$, we have

$$\mathbb{P}(\left|J(p_{\theta_o}) - J(p_{\theta_Z})\right| > \varepsilon) \leq 8\exp\left(-\frac{|Z|^2 \varepsilon^2}{1024d_1}\right) + \frac{4\sigma^2}{|Z|(d^2 - \mu)^2}$$  \hfill (34)

if $|Z| \geq 16\mu/\varepsilon^2$ and $|Z| \geq (1024d_1d_s \ln \frac{\delta^2}{\varepsilon})/\varepsilon^2$.

**Remark 2:** Theorem 2 demonstrates that the convergent rate of the generalization error will decrease with the training sample size while increase with the network size. Therefore, the training size and the network size are the key factors for detection performance. A tradeoff between the generalization error and the approximation error should be carefully balanced when implementing the data-driven DL detector.

The following corollary, proved in Appendix F, presents our main conclusion on the performance of the data-driven DL detector.

**Corollary 1:** For any $\varepsilon > 0$ and sufficiently large $R$, there exists a DL detector built on a ReLU DNN with at most $\lceil \log_2(d_o + 1) \rceil$ hidden layers and $\theta \in \Theta_R$ such that

$$\lim_{|Z| \to +\infty} \mathbb{P}(D_{KL}(p_o, p_{\theta_Z}) > \varepsilon) = 0.$$  \hfill (35)
For different transmitted symbols, identical in both training data and deployed environment. With the time-varying channels, significantly in the varying channels [22].

C. Limits of Data-Driven DL Detector

The data-driven DL detector derived in (22) is model independent and requires no CSI to learn a detection mapping that can approach MAP performance. However, it is only applicable when the channels are static and degrades significantly in the varying channels [22].

According to Corollary 1, \( p_o(u|x) = p_o(Hs + n)s)p(s) \) is the target distribution for the data-driven DL detector. With the time-invariant channels, \( H \) remains deterministic and is identical in both training data and deployed environment. For different transmitted symbols, \( p_o(Hs + n)s \) are different and we can directly recover \( s \) using the maximum likelihood (ML) rule. With the time-varying channels, \( H \) is a zero-mean random matrix and \( p_o(Hs + n)s \) are the same for different transmitted symbols. As a consequence, \( s \) is indistinguishable using the ML rule.

To alleviate the impact of the time-varying channels, let us consider the DL detector where both \( H \) and \( x \) are taken as its input. The target distribution is converted into

\[
p_o(u|x, H) = p_o(Hs + n)s)p(s). \tag{36}
\]

According to Corollary 1, we can also prove that the data-driven DL detector with CSI can well approximate the MAP detector in (36). Therefore, CSI is essential to the data-driven DL detector to detect \( s \) over all possible realizations of \( H \). However, the data-driven DL detector tends to have a large network structure by taking \( H \) as the input and thus requires enormous train samples to converge, as shown in Theorem 2. On the other side, the data-driven DL detector with CSI yields the optimal accuracy but its complexity is prohibitive for large-scale MIMO systems.

IV. MODEL-DRIVEN DL DETECTOR

In this section, we propose a model-driven DL detector that combines the SIC algorithm with an alternative detector through a discriminative net, also referred as to the DisNet. We first introduce the structure of the DisNet in Section IV-A and then discuss its pros and cons in Section IV-B.

A. DisNet Receiver Architecture

To reduce the training overhead for the DL-based MIMO detection, an effective way is to use the model-driven DL detector that integrates the model knowledge into the network structure. Traditional model-driven DL detectors, e.g., DetNet [22] and OAMP-Net [23], are based on the iterative detection algorithms and can achieve improved error rate by optimizing the enlarged parameter space using the data knowledge. However, these model-driven DL detectors are determined by the underlying iterative algorithms, which are generally far from the MAP performance. Consequently, their performance is suboptimal and some other traditional detectors may yield better detection accuracy.

Algorithm 1 DisNet Detector

1: Input: Received signal \( x \) and channel \( H \).
2: SIC detector: Set the iteration number as 2 and generate an initial guess of the detected symbol \( s_{SIC} \).
3: Discriminate net: Given the 2-norm loss \( \| x - Hs_{SIC} \|_2 \), determine whether \( s_{SIC} \) is correct. If so, go to Step 4. Otherwise, go to Step 5.
4: Output \( s_{SIC} \).
5: Use the alternative detector and output another guess of the detected symbol \( s_{alt} \).

Suppose that the MIMO channel model is linear as shown in (2) and \( H \) is known at the receiver. Here, we present a DisNet architecture that consists of a SIC detector, an alternative detector and a connecting discriminative net, as depicted in Fig. 1. The SIC detector is capable of achieving the MAP comparable performance with controllable complexity and thus is selected as the basic infrastructure for designing the DisNet. This allows the DisNet to improve the performance over previously proposed model-driven DL detectors [20]. Generally, the SIC detector requires 4 or 5 iterations to guarantee its convergence. But in most of cases, the SIC detector only needs 2 or 3 iterations to converge and then output the corrected symbols. Hence, most of iterations of the SIC detector are unnecessary. To reduce the number of iterations, the DisNet combines the SIC detector with an alternative detector through a discriminative net as shown in Fig. 1. The discriminative net and the alternative detector are used to detect and correct the error symbols generated from the SIC detector. If the output symbol of the SIC detector is classified to be correct by the discriminative net, the DisNet directly outputs the symbol. Otherwise, the DisNet replaces the SIC detector with the alternate detector to detect the symbol. In this way, the SIC detector of the DisNet does not require too many iterations. The discriminative net and the alternative detector can correct the erroneous symbol from the SIC detector. Specifically, the discriminative net of the DisNet is established by a fully-connected classification DNN, and the alternative detector can be an arbitrary detection algorithm other than the SIC detector.

Let \( s_{SIC} \) and \( s_{alt} \) denote the detected symbols of the SIC and the alternative detectors, respectively. In the DisNet, the alternate detector and the discriminative net are used to reduce...
the number of iterations and correct the erroneous symbols of the SIC detector. The initial iteration number of the SIC detector is set to be 2 in the DisNet. The input to the discriminative net is the 2-norm loss \( ||x - Hs_{SIC}||_2 \). The discriminative net serves as a classifier to determine whether \( s_{SIC} \) is correct based on \( ||x - Hs_{SIC}||_2 \). If so, the DisNet directly outputs \( s_{SIC} \) as the final detected symbol. Otherwise, the alternative detector is applied and outputs \( s_{alt} \) to replace \( s_{SIC} \). Since the error rate of the SIC detector is generally at a low level, the frequency of using the alternative detector is essentially not high. The MAP or SD detector can be used as the alternative detector, even though their computational complexity is relatively high. Furthermore, the error rate of the SIC detector decreases slightly with the number of iterations. Empirically, 2 iterations is enough for the SIC detector of the DisNet. The overall joint detection scheme is summarized in Algorithm 1. 

B. Discussion

The DisNet introduces a fully connected discriminative net to improve the performance of detection. The interpretation on the DisNet follows the same way with that of the data-driven DL detector. According to Corollary 1, the discriminative net is capable of accurately computing conditional probability that \( s_{SIC} \) is correct given \( ||x - Hs_{SIC}||_2 \). Once trained properly, the discriminative net can detect error symbols with high precision under relatively high signal-to-noise ratio (SNR). Furthermore, the DisNet preserves the structure of the SIC detector and only uses a small-sized discriminative net to detect error symbols. In contrast to the data-driven DL detector, the DisNet has fewer parameters to learn and can be trained by a small amount of data according to Theorem 2, allowing it to be quickly deployed in large-scale MIMO systems.

The DisNet is also capable of mitigating the error propagation of the SIC detector. If error symbols of the SIC detector are detected, then it is very possible for the alternative detector to correct these symbols due to their different mechanisms. In this way, the DisNet avoids the error propagation and provides better detection performance with less computational costs than the SIC detector. Furthermore, the DisNet can adjust the decision threshold of the discriminator according to different accuracy requirements. Hence, the DisNet is a better and more flexible classifier compared to the SIC detector.

Similar to other model-driven DL detectors, the DisNet is customized for linear MIMO systems and thus does not capture the model independent property of the DL methods. Nevertheless, the DisNet is not established on a specific DNN-embedded detection algorithm and can be regarded as a modular system with the underlying SIC or the alternative detectors changed seamlessly. If the SIC detector is unsuitable for some scenarios, then the DisNet can replace it by another specifically designed detector. In this way, the DisNet provides a more flexible networking configuration than other model-driven DL detectors.

V. SIMULATION RESULTS

In Section V-B and Section V-C, computer simulation is provided to verify that the data-driven DL detector asymptotically approaches to the MAP detector under linear and nonlinear MIMO systems. We also evaluate the performance of the DisNet in Section V-D. Simulation results demonstrate that the DisNet outperforms some model-driven DL detectors and can achieve the MAP comparable performance with affordable complexity.

A. Simulation Settings

The SNR is defined as

\[
SNR = \frac{E[|Hs|^2]}{E[|n|^2]}. \tag{37}
\]

Theorem 1 has not indicated the relation between the BER performance and the network size. The underlying network of the data-driven DL detector is empirically set, which has 4 layers and each hidden layer is equipped with the same number of neurons. Denote \( d = 100 \) as the width of the data-driven DL detector. The QPSK constellation is adopted at the transmitter and all the transmitted symbols are generated with equal probability.

The simulation results of the data-driven DL detector are evaluated under 2 \( \times \) 2 MIMO systems with time-invariant and time-varying channels, respectively. For the time-invariant channel, 40,000 independently samples are generated to train the data-driven DL detector. For the time-varying channels, the data-driven detector is trained for 100,000 iterations with a batch size of 2,000 training samples. In both cases, the data-driven DL detector is trained on different SNRs and tested over 20,000 samples. The network of the DL detector is trained using the Adam optimizer with the learning rate set to be 0.001. The MAP detector in (4) serves as the benchmark of the performance evaluation. We also consider the case of imperfect CSI generated from the channel estimation error. The imperfect CSI satisfies the additive model and the entries of \( H \) are corrupted by additive Gaussian noise whose variance is 10% the magnitude of its corresponding entry [20].

The performance of the DisNet is evaluated under a 8 \( \times \) 8 MIMO system with time-varying channel. We train DisNet with 50,000 samples and test over 20,000 samples. The correct symbol of the SIC detector is set to be the positive instance. The MAP detector in (4) is also used as the benchmark.

B. DL-Driven DL Detector in Linear Systems

In this subsection, we investigate the bit error rate (BER) performance and convergence of the data-driven DL detector under a linear MIMO model in (2).

1) Time-Invariant Channel: Fig. 2 compares the BER performance of the data-driven DL detector with the model-based ZF, AMP, SD, MAP detectors versus SNR over a 2 \( \times \) 2 time-invariant Gaussian channel. We assume that perfect CSI is available for the ZF, AMP, and SD detectors, and the MAP detector is evaluated under both perfect and imperfect CSI, respectively, while the data-driven DL detector has no CSI. As shown in Fig. 2, the data-driven DL detector can well approximate the MAP detector under perfect CSI and significantly outperforms other traditional model-based detectors,
which confirms that $p_{\Theta}(u|x) \approx p_{\Theta}(u|x)$ in Corollary 1. Specifically, the data-driven DL detector is immune to CSI uncertainty under the time-invariant channel and outperforms the MAP detector under imperfect CSI.

Fig. 3(a) shows the BER performance of the data-driven DL detector versus the network width and $d$ under fixed SNRs over a $2 \times 2$ time-invariant Gaussian channel. The BERs of the MAP detectors derived at the same SNRs are used as the benchmark. The approximation error determines the BER performance of the DL estimator under a sufficiently large training set. When $d$ is small, the dimension of the parameter space, $\Theta_R$, is not big enough to fit with $p_{\Theta}(u|x)$ and the BER curves of the data-driven DL detector are significantly higher than those of the MAP detector. As $d$ increases, the dimension of the parameter space of $\Theta_R$ is enlarged and the approximation error decreases until both BERs converge, which verifies the main conclusion in Theorem 1.

Fig. 3(b) shows the BER performance of the data-driven DL estimator versus the size of training samples, $|Z|$, over a $2 \times 2$ time-invariant Gaussian channel. The SNRs are fixed and $d = 100$. In Fig. 3(b), the BER curves of the MAP detector are used as the benchmark. Similarly, the generalization error is the main factor that affects the BER performance under large $d$. When $|Z|$ is small, the BER curves of the data-driven DL detector do not converge and are significantly higher than those of the MAP estimator. As $|Z|$ increases, the BERs of the data-driven DL detector gradually approach to those of the MAP detector, which verifies the main conclusion in Theorem 2.

2) Time-Varying Channel: Fig. 4 compares the BER performance of the data-driven DL detector with the model-based ZF, AMP, SD, MAP detectors versus SNR over a $2 \times 2$ time-varying Gaussian channel. We assume that the ZF, AMP, and SD detectors have perfect CSI while both the DL and MAP detectors are evaluated under perfect and imperfect CSI, respectively. Perfect CSI represents the case when the estimation error can be ignored at high SNRs, while imperfect CSI corresponds to the case of the channel estimation at low SNRs. As illustrated in Fig. 5(a), the data-driven DL detector manages to achieve the MAP comparable BER performance by incorporating $H$ and substantially outperforms the other model-based detectors. However, the BER performance of the data-driven DL detector is severely deteriorated by imperfect CSI in the time-varying channel and can only approach to the MAP detector with imperfect CSI.

C. Data-Driven DL Detector in Nonlinear Systems

In this subsection, we evaluate the BER performance of the data-driven DL detector under a nonlinear MIMO system. We will demonstrate that the data-driven DL detector is applicable to a broader range of scenarios than the traditional MIMO detection algorithms. Furthermore, we only compare the data-driven DL detector to the ZF and MAP detectors.

Consider a MIMO system corrupted by the quantization error of ADC. We assume that each element of the channel output undergoes an entry-wise $B$ bit uniform quantizer $Q_c$. The channel model in (2) can be rewritten as

$$x = Q_c(Hs + n). \quad (38)$$

Each real-valued input of $Q_c$ is mapped to one of $2^B$ bins, which are defined by the set of $2^B - 1$ thresholds $\{r_1, r_2, \ldots, r_{2^B - 1}\}$ such that $-\infty < r_1 < r_2 < \ldots < r_{2^B - 1} < \infty$. Specifically, we define $r_0 = -\infty$ and $r_{2^B} = \infty$. The threshold $r_b$ is given by

$$r_b = \sqrt{B}(-2^{B-1} + b)2^{-B}, \quad \text{for } b = 1, \ldots, 2^B - 1, \quad (39)$$

where the quantization output of $Q_c$ is $r_b - \Delta/2$ when the input falls in the interval $(r_{b-1}, r_b)$.²

In Fig. 5, we compare the BER performance of the data-driven DL detector with the ZF and MAP detectors versus SNR over a quantized $2 \times 2$ time-invariant Gaussian channel. The quantization bits are set to be 4 and 8, respectively. Perfect CSI is assumed to be available at the receiver. In Fig. 5, the BER performance of the data-driven DL detector is close to that of the MAP detector under both 4-bit and 8-bit quantizers, while the BER performance of the ZF detector degrades significantly. Hence, the data-driven DL detector is also able to provide the MAP comparable BER performance in the quantized Gaussian channels and verifies the model independence of the data-driven DL detector.

D. Model-Driven DL Detector

In Fig. 6, we compare the BER performance of the DisNet with the DetNet, OAMP-Net, SIC, and MAP detectors versus SNR over a $8 \times 8$ time-varying Gaussian channel. We assume that perfect CSI is available at the receiver. In Fig. 6, the alternate detector of the DisNet is set as the MAP detector and the SIC detector is evaluated under 2 and 5 iterations, respectively. The BER curve of the SIC detector derived from 2 iterations serves as the baseline for measuring the detection performance of the DisNet and the BER curve under 5 iterations is used to illustrate the detection performance of the SIC detector after convergence. In Fig. 6, the BER performance of the DisNet is significantly better than those of the DetNet, OAMP-Net, and SIC detectors and approaches to $2^B$.

²If $b = 2^B$, the output of $Q_c$ is $\sqrt{B}(-2^{B-1} - 1)2^{-B}$.
that of the MAP detector. Specifically, both the BERs of the DisNet and MAP detectors are equal to zero under 17.5 dB and 20 dB. Fig. 6 also demonstrates that the SIC detector is better than the DetNet and OAMP-Net in terms of the BER performance. Therefore, the selection of detection algorithms is more important than trainable variables in improving the BER performance of the model-driven DL detectors.

Table I presents the classification results of the discriminative net of the DisNet. In Table I, threshold is the minimum probability of the discriminative net to determine whether its input symbol is correct and $\gamma$ denotes the proportion of the number of detected error symbols over the size of text data. At high SNRs, the discriminative net can detect the error symbols from the SIC detector perfectly and $\gamma$ is extremely low. In this case, the DisNet avoids the frequent use of the MAP detector and significantly improves the performance of the SIC detector by correcting few detected error symbols. However, the number of the detected error symbols increases sharply at low SNRs and it is difficult for the DisNet to determine whether the input symbol is correct. In particular, the DisNet directly classifies all the inputs as the error symbols at 0 dB. Consequently, the MAP detector becomes a significant overhead when being used too frequently and the DisNet is
under the SD detector but is lower than that of the SIC detector. This shows that the BER curve of the DisNet is higher than that of the SIC detector. Fig. 7 evaluates the BER performance of the DisNet under different alternate detection algorithms and the semidefinite relaxation decoder [38]. Since the DFE and SDR represent the decision feedback equalizer and the semidefinite relaxation decoder [38], they are no longer an effective method to enhance the performance of the SIC detector.

To avoid high complexity of the MAP detector, we compare the BER performance of the DisNet under different alternate detectors versus SNR over a $8 \times 8$ time-varying Gaussian channel in Fig. 7. The alternate detectors of the DisNet in Fig. 7 are the SD, SDR, and DFE detectors, respectively, where the DFE and SDR represent the decision feedback equalizer algorithm and the semidefinite relaxation decoder [38]. Since Fig. 7 evaluates the BER performance of the DisNet under different alternative detector, the DetNet and OAMP-Net have no alternate detectors and are not presented in Fig. 7. The computational complexity for these detection algorithms is presented in Table II, where $l_{det}$ and $l_{sic}$ are the numbers of the required iterations of the SD and SIC detectors for convergence and $N_{alt}$ denotes the complexity of the alternative detector. In Fig. 7, the BER curve of the DisNet under the SD detector is overlapped with that under the MAP detector. Meanwhile, the SD detector has much lower complexity than the MAP detector as illustrated in Table II and thus is a better choice for the alternative detector of the DisNet. Fig. 7 also shows that the BER curve of the DisNet is higher than that under the SD detector but is lower than that of the SIC detector under 5 iterations at high SNRs. Since the SDR detector has linear complexity as shown in Table II, its complexity is generally not larger than that of the SIC detector, which is an effective replacement for the SD detector at high SNRs. In Table II, the DFE detector has the lowest complexity but its BER performance degrades significantly as shown in Fig. 7. Nevertheless, the DFE detector achieves greater accuracy than the SIC detector under 2 iterations and is a suitable choice if low complexity is required.

VI. Conclusion

In this paper, we have made the first attempt on interpreting the DL-based MIMO detection with two different deep architectures: the DNN embedded data-driven DL detector and the model-driven DisNet. We have showed that the data-driven DL detector can converge to the MAP detector in various scenarios under suitably configured structure and sufficiently large training set. Moreover, the data-driven DL detector is robust to CSI uncertainty in time-invariant channels and suffers from imperfect CSI in time-varying channels. The data-driven DL detector is, however, ineffective in large-scale time-varying MIMO systems due to its requirement on a large number of training samples. On the other hand, the proposed model-driven DisNet successfully addresses this problem by combining the SIC algorithm with an alternative detector through a discriminative net and achieves the MAP comparable performance with only a small training set.
Since $\ln p_0(u|x)$ has finite 2-norm for every possible $u$, $\ln p_0(\hat{u}|x)$ can be approximated by a ReLU DNN with at most $\lceil \log_2(d_0 + 1) \rceil$ hidden layers [27, 28]. We simply put these ReLU DNNs in parallel and combine their outputs together to compose a single ReLU DNN. As a result, there exits a DL detector with $\theta_\varepsilon \in \Theta_R$ and at most $\lceil \log_2(d_0 + 1) \rceil$ layers such that

$$\left| f_{\theta_\varepsilon}(x) - \ln p_0(\hat{u}|x) \right| \leq \varepsilon$$

(43)

for any $\varepsilon > 0$ and $i \in \{1, \ldots, d_{i+1}\}$. From (43) and (42), $|g_{\theta_\varepsilon}(x)|$ satisfies

$$|g_{\theta_\varepsilon}(x)| \leq \left| \ln p_0(\hat{u}|x) - f_{\theta_\varepsilon}(x) \right| + \ln \left[ \sum_{j=1}^{d_{i+1}} \exp(f_{\theta_{l,j}}(x)) \right] \leq \varepsilon$$

(44)

Specifically, $\sum_{j=1}^{d_{i+1}} \exp(f_{\theta_{l,j}}(x))$ in (44) is upper bounded by

$$\sum_{j=1}^{d_{i+1}} \exp(f_{\theta_{l,j}}(x)) \leq e^\varepsilon \sum_{j=1}^{d_{i+1}} p_0(\hat{u}|x) = e^\varepsilon$$

(45)

and is lower bounded by $\sum_{j=1}^{d_{i+1}} \exp(f_{\theta_{l,j}}(x)) \geq e^{-\varepsilon}$. Then, $\left| \ln \left[ \sum_{j=1}^{d_{i+1}} \exp(f_{\theta_{l,j}}(x)) \right] \right| \leq \varepsilon$ and $|g_{\theta_\varepsilon}(x)| \leq 2\varepsilon$ hold. As a result,

$$J(p_\theta) - J(p_\vartheta) \leq 2\varepsilon \sum_{i=1}^{d_{i+1}} \int f_{de}(\hat{u}_i, x) dx = 2\varepsilon.$$  

(46)

From (19), $p_{\theta}(u|x)$ has the lowest KL information for all $\theta \in \Theta_R$ and we have

$$J(p_\theta) - J(p_\vartheta) \leq J(p_\vartheta) = J(p_\theta).$$  

(47)

It is then easy to derive (25) from (47) since $\varepsilon$ is an arbitrary positive value.

APPENDIX B

PROOF FOR LEMMA 1

According to Symmetrization Lemma in [39], the inequality in (27) holds if $P(\{|JZ(p_\theta) - J(p_\vartheta)| > \frac{\varepsilon}{2}\} \leq \frac{1}{\varepsilon}$ for all $\theta \in \Theta_R$. Let $\sigma^2(p_\vartheta)$ be the variance of $\ln p_0(u|x)$ and

$$P(\{|JZ(p_\theta) - J(p_\vartheta)| \geq \frac{\varepsilon}{2}\} \leq \frac{4\sigma^2(p_\vartheta)}{|Z|^2}$$

(48)

for all $\theta \in \Theta_R$. Specifically, $\sigma^2(p_\vartheta)$ satisfies

$$\sigma^2(p_\vartheta) \leq E \{ \ln p_0(u|x)^2 \}$$

$$\leq \sum_{i=1}^{d_{i+1}} \int \left[ f_{\theta_{l,i}}(x) - \ln \left( \sum_{j=1}^{d_{i+1}} \exp(f_{\theta_{l,j}}(x)) \right) \right]^2 f_{de}(\hat{u}_i, x) dx$$

$$\leq \sum_{i=1}^{d_{i+1}} \int \left[ f_{\theta_{l,i}}(x) + \ln \left( \sum_{j=1}^{d_{i+1}} \exp(f_{\theta_{l,j}}(x)) \right) \right]^2 f_{de}(\hat{u}_i, x) dx.$$  

(49)

Assume that $\mathcal{X}$ follows the partition in (11). The triangle inequality assures that

$$|f_{\theta_{l,i}}(x)| \leq \|w_{\mathcal{X}_{k,i}}x + b_{\mathcal{X}_{k,i}}\|_2 \leq \|w_{\mathcal{X}_{k,i}}\|_2 + \|b_{\mathcal{X}_{k,i}}\|_2$$

(50)

for $x \in \mathcal{X}_k$, where $w_{\mathcal{X}_{k,i}}$ and $b_{\mathcal{X}_{k,i}}$ are the $i$-th row and the $i$-th entry of $W_{\mathcal{X}_k}$ and $b_{\mathcal{X}_k}$, respectively.

From (13) and (14), $\|w_{\mathcal{X}_{k,i}}\|_2$ and $\|b_{\mathcal{X}_{k,i}}\|_2$ in (50) are upper bounded by

$$\|w_{\mathcal{X}_{k,i}}\|_2 = \|\tilde{w}_{l,i} \prod_{j=0}^{l-1} W_j\|_2 = \|\tilde{w}_{l,i} A_l \prod_{j=0}^{l-1} W_j A_j\|_2$$

$$\leq \|\tilde{w}_{l,i} A_l\|_2 \prod_{j=0}^{l-1} \|W_j A_j\|_2 \leq \|\tilde{w}_{l,i}\|_2 \prod_{j=0}^{l-1} \|W_j\|_2$$

(51)

and

$$\|b_{\mathcal{X}_{k,i}}\| = \left| \sum_{j=0}^{l-1} \tilde{w}_{l,i} \left( \prod_{q=0}^{j-1} W_{l-1-q} \right) b_{l-1-j} + b_{l,i} \right|$$

$$\leq \left| \sum_{j=0}^{l-1} \tilde{w}_{l,i} \left( \prod_{q=0}^{j-1} W_{l-1-q} \right) b_{l-1-j} \right| + \|b_{l,i}\|$$

$$\leq \left| \sum_{j=0}^{l-1} \tilde{w}_{l,i} \left( \prod_{q=0}^{j-1} W_{l-1-q} \right) \right| \|b_{l-1-j}\| + \|b_{l,i}\|$$

$$\leq \left| \sum_{j=0}^{l-1} \tilde{w}_{l,i} \left( \prod_{q=0}^{j-1} W_{l-1-q} \right) \right| \|b_{l-1-j}\| + \|b_{l,i}\|,$$  

(52)

respectively, where $\tilde{w}_{l,i}$ and $w_{l,i}$ are the $i$-th rows of $W_l$ and $W_i$ and $b_{l,i}$ is the $i$-th entry of $b_i$. Since $\|W_j\|_2 \leq R \|d\|_\infty = \alpha$, $\|b_j\|_2 \leq \sqrt{R} \|d\|_\infty \leq \alpha$, $\|w_{l,i}\|_2 \leq \sqrt{R} \|d\|_\infty \leq \alpha$, and $\|b_{l,i}\| \leq \|d\|_\infty \leq \alpha$ for $j \in \{0, 1, \ldots, l-1\}$, (51) and (52) can be further bounded by $\|w_{\mathcal{X}_{k,i}}\|_2 \leq \alpha^{l+1}$ and

$$\|b_{\mathcal{X}_{k,i}}\| \leq \sum_{i=0}^{l-1} \alpha^{|i+1|} \alpha + \frac{\alpha^{l+2} - \alpha}{\alpha - 1} \leq \beta(\alpha^{l+1} - 1),$$  

(53)

respectively. Then, we have

$$|f_{\theta_{l,i}}(x)| \leq \alpha^{l+1}(\|x\|_2 + \beta) - \beta.$$  

(54)

Using (54), $\ln \left[ \sum_{i=1}^{d_{i+1}} \exp(f_{\theta_{l,i}}(x)) \right]$ in (49) is upper bounded by

$$\ln \left[ \sum_{i=1}^{d_{i+1}} \exp(f_{\theta_{l,i}}(x)) \right] \leq \ln d_{l+1} + \|\alpha^{l+1}(\|x\|_2 + \beta) - \beta\|.$$  

(55)

Combining (54) and (55), we have

$$\sigma^2(p_\vartheta) \leq \sum_{i=1}^{d_{i+1}} \int \left[ \ln d_{l+1} + 2(\alpha^{l+1}(\|x\|_2 + \beta) - \beta) \right]^2 f_{de}(\hat{u}_i, x) dx$$

$$= E \{ \ln d_{l+1} + 2(\alpha^{l+1}(\|x\|_2 + \beta) - \beta) \}^2 \} = \nu.$$  

(56)

Replacing $\sigma^2(p_\vartheta)$ in (48) by its bound in (56) and letting $|Z| \geq 4\nu/\varepsilon^2$, we obtain the inequality in (27), which completes the proof.
APPENDIX C
PROOF FOR LEMMA 2

Let \( Z_i \) denote the index set of training samples in \( Z \) where
\[
u_j = \bar{u}_i, \quad \forall j \in Z_i
\]
for \( i \in \{1, \ldots, d_{i+1}\} \). Then, \( |J_Z(p_\theta) - J_Z(p_\lambda)| \) is upper bounded by
\[
|J_Z(p_\theta) - J_Z(p_\lambda)| \leq \frac{1}{|Z|} \sum_{i=1}^{d_{i+1}} \sum_{j \in Z_i} |f_{\theta,i}(x_j) - f_{\lambda,i}(x_j)|
\]
for \( j \in \{0, \ldots, l\} \) and prove it by induction. First, the base case \( j = 1 \) holds as
\[
e_1 = \| (W_0 x + b_0) - (V_0 x + p_0) \|_2 \leq r \| d \|_\infty (\| x \|_2 + 1).
\]
At the induction step, we assume that (62) is valid for \( j \in \{1, \ldots, l-1\} \). It implies by (61) that
\[
e_{j+1} = 3 \alpha e_\beta + r \| d \|_\infty (y_j + 1) = r \| d \|_\infty (\| x \|_2 + 1)
\]
\[
+ \sum_{q=0}^{j-2} (3\alpha)^{q+1} (y_{j-q} + 1) + (y_{j+1})
\]
\[
= r \| d \|_\infty (3\alpha)^{j}(\| x \|_2 + 1) + \sum_{q=0}^{j-1} (3\alpha)^{q}(y_{j-q} + 1).
\]
(64)

Therefore, our claim (62) holds for \( j \in \{1, \ldots, l\} \).

From (61), \( |f_\theta(i, x) - f_{\lambda,i}(x)| \) is upper bounded by
\[
|f_\theta(i, x) - f_{\lambda,i}(x)| \leq r \| d \|_\infty \left( \sum_{q=0}^{l-1} 3^q \alpha^q \| x \|_2 + (3\alpha)^l \right.
\]
\[
+ \sum_{q=0}^{l-1} (3\alpha)^q \beta (\beta - 1) \bigg) \bigg) \leq \frac{3r}{2} \| d \|_\infty (3\alpha)^l (\| x \|_2 + \beta) = \xi r,
\]
(65)

where \( \xi = \frac{3}{2} \| d \|_\infty (3\alpha)^l (\| x \|_2 + \beta) \). From (65), the upper bound on \( \ln(\sum_{i=1}^{d_{i+1}} \exp(f_{\theta,i}(x))) \) can be expressed as
\[
\ln \left( \sum_{i=1}^{d_{i+1}} \exp(f_{\theta,i}(x)) \right) \leq \ln \left( \sum_{i=1}^{d_{i+1}} \exp(f_{\lambda,i}(x) + \xi r) \right)
\]
\[
= \xi r + \ln \left( \sum_{i=1}^{d_{i+1}} \exp(f_{\lambda,i}(x)) \right),
\]
(66)

Similarly, \( \ln(\sum_{i=1}^{d_{i+1}} \exp(f_{\theta,i}(x))) \) is lower bounded by
\[
\ln \left( \sum_{i=1}^{d_{i+1}} \exp(f_{\theta,i}(x)) \right) \geq \ln \left( \sum_{i=1}^{d_{i+1}} \exp(f_{\lambda,i}(x)) \right) - \xi r.
\]
(67)

Therefore, we have
\[
\ln \left( \sum_{i=1}^{d_{i+1}} \exp(f_{\theta,i}(x)) \right) - \ln \left( \sum_{i=1}^{d_{i+1}} \exp(f_{\lambda,i}(x)) \right) \leq \xi r.
\]
(68)

Using (65) and (68), we derive the upper bound on \( |J_Z(p_\theta) - J_Z(p_\lambda)| \) as
\[
|J_Z(p_\theta) - J_Z(p_\lambda)| \leq \frac{1}{|Z|} \sum_{m=1}^{|Z|} 3r \| d \|_\infty (3\alpha)^l (\| x_m \|_2 + \beta)
\]
\[
\leq 3^{l+1} \| d \|_\infty \alpha^l (\delta + \beta) \| \theta - \lambda \|_\infty.
\]
(69)
APPENDIX D  
PROOF FOR LEMMA 3

Choose a collection of parameters \( \theta_1, \ldots, \theta_C \in \Theta_R \) such that balls centered at \( \theta_j \) with radius

\[
r_b = \epsilon \frac{3^{l+1}\|d\|_\infty \alpha_j'(\delta + \beta)}{3^{l+1} \|d\|_\infty \alpha_j'(\delta + \beta)}
\]

(70)

cover \( \Theta_R \) for \( j \in \{1, \ldots, C\} \). For \( j \in \{1, \ldots, C\} \), there exists \( \|\theta - \theta_j\|_\infty \leq r_b \) for any \( \theta \in \Theta_R \). Let \( C_b = \ln C \) and \( C_b \leq d_n \ln (4R/r_b) \) satisfies [40]. It implies by Lemma 2 and (70) that

\[
|J_Z(p_\theta) - J_Z(p_{\theta_j})| \leq 3^{l+1}\|\theta - \theta_j\|_\infty \|d\|_\infty \alpha_j'(\delta + \beta) \leq \epsilon.
\]

(71)

The upper bound of \( \ln C(\epsilon, \Theta_R) \) is given by

\[
\ln C(\epsilon, \Theta_R) \leq C_b \leq d_n \ln (4R/r_b) = d_n \ln \left[ \frac{3^{l+1}4\|d\|_\infty \alpha_j'(\delta + \beta)}{\epsilon} \right].
\]

(72)

APPENDIX E  
PROOF FOR THEOREM 2

From (19) and (21), we can bound \( J(p_{\theta_{\alpha}}) - J(p_{\theta_{\beta}}) \) by

\[
0 \leq J(p_{\theta_{\alpha}}) - J(p_{\theta_{\beta}}) = |J(p_{\theta_{\alpha}}) - J(p_{\theta_{\beta}})| - |J_Z(p_{\theta_{\alpha}}) - J_Z(p_{\theta_{\beta}})| \leq \frac{1}{\|Z\|_\infty} \sum_{m=1}^{\|Z\|_\infty} |m_{\alpha} - m_{\beta}|.
\]

(73)

According to (73) and Lemma 1, we know the following inequalities

\[
P\left(|J(p_{\theta_{\alpha}}) - J(p_{\theta_{\beta}}})| > \epsilon \right) \leq P\left(\sup_{\theta_{\alpha}, \theta_{\beta}}|J_Z(p_{\theta_{\alpha}}) - J_Z(p_{\theta_{\beta}})| > \frac{\epsilon}{8} \right) \leq 4P\left(\sup_{\theta_{\alpha}, \theta_{\beta}}|J_Z(p_{\theta_{\alpha}})| > \frac{\epsilon}{8} \right).
\]

(74)

holds if \( |Z| \geq 16\epsilon^2 \).

Assume that \( Z \) is fixed with \( \frac{1}{\|Z\|_\infty} \sum_{m=1}^{\|Z\|_\infty} |m_{\alpha} - m_{\beta}| \leq \delta^2 \). Let \( C = C(\epsilon/16, \Theta_R) \) and choose a collection of functions \( p_1(x), \ldots, p_C(x) \in \mathcal{D} \) such that

\[
|J_Z(p_{\theta_{\alpha}}) - J_Z(p_{\theta_{\beta}})| \leq \frac{\epsilon}{16}, \quad \forall j \in \{1, \ldots, C\},
\]

(75)

for any \( p_{\theta}(x) \in \mathcal{D} \) and \( \theta \in \Theta_R \). Let \( p^* \) represent \( p_j \) at which the minimum value in (75) is achieved. Since \( |J_Z(p_{\theta_{\alpha}})| = \frac{1}{\|Z\|_\infty} \sum_{m=1}^{\|Z\|_\infty} \omega_p J_Z(p_{\theta_{\alpha}}) \leq |J_Z(p_{\theta_{\alpha}})| \), we have

\[
P\left(\sup_{\theta_{\alpha}, \theta_{\beta}}|J_Z(p_{\theta_{\alpha}})| > \frac{\epsilon}{8} \right) \\
\leq P\left(\sup_{\theta_{\alpha}, \theta_{\beta}}|J_Z(p^*) + |J_Z(p_{\theta_{\alpha}}) - J_Z(p^*))| > \frac{\epsilon}{8} \right) \\
\leq P\left(\max_{j \in \{1, \ldots, C\}}|J_Z(p_j)| > \frac{\epsilon}{16} \right) \\
\leq \sum_{j=1}^{C} P\left(|J_Z(p_j)| > \frac{\epsilon}{16} \right).
\]

(76)

For each \( f_j(x) \), Hoeffding’s Inequality [41] gives the following bound

\[
P\left(|J^2_Z(p_j)| > \frac{\epsilon}{16} \right) = P\left(\left|\sum_{m=1}^{\|Z\|_\infty} \omega_m p_j u_m x_m \right| > \frac{\epsilon}{16} \right) \\
\leq 2\exp\left[-2\left(\frac{\epsilon}{16}\right)^2 / \left(\sum_{m=1}^{\|Z\|_\infty} (\ln p_j (x_m))^2 \right) \right].
\]

(77)

From (49), we know \( \sum_{m=1}^{\|Z\|_\infty} (\ln p_j (x_m))^2 \) is upper bounded by

\[
\sum_{m=1}^{\|Z\|_\infty} (\ln p_j (x_m))^2 \\
\leq \sum_{m=1}^{\|Z\|_\infty} |\ln d_t + 1| (\|x_m\|_2 + \beta)^2 \\
\leq |Z| |\ln d_t + 1| (\|x_m\|_2 + \beta)^2 = |Z| \delta_1.
\]

(78)

Replacing \( \sum_{m=1}^{\|Z\|_\infty} (\ln p_j (x_m))^2 \) in (77) by the upper bound in (78) and substituting (77) into (76) yields

\[
P\left(\sup_{\theta_{\alpha}, \theta_{\beta}}|J_Z(p_{\theta_{\alpha}})| > \frac{\epsilon}{8} \right) \leq 2\exp\left(-\frac{|Z| \epsilon^2}{512\delta_1^2} \right).
\]

(79)

According to Lemma 3, \( \ln C \) is upper bounded by

\[
\ln C \leq d_n \ln \left[ \frac{3^{l+1}4\|d\|_\infty \alpha_j'(\delta + \beta)}{\epsilon} \right] \leq d_n \ln \frac{\delta_2}{\epsilon}.
\]

(80)

If \( |Z| \geq (1024\delta_1^2 d_n \ln \frac{\delta_2}{\epsilon})/\epsilon^2 \), then we have \( \ln C \leq |Z| \epsilon^2/(1024\delta_1^2) \) and

\[
P\left(\sup_{\theta_{\alpha}, \theta_{\beta}}|J_Z(p_{\theta_{\alpha}})| > \frac{\epsilon}{8} \right) \leq 2\exp\left(-\frac{|Z| \epsilon^2}{1024\delta_1^2} \right).
\]

(81)

Integrating out \( P(\sup_{\theta_{\alpha}, \theta_{\beta}}|J_Z(p_{\theta_{\alpha}})| > \frac{\epsilon}{8} \right) \) over \( Z \) in (81) produces

\[
P_{\mathcal{Z}}\left(\sup_{\theta_{\alpha}, \theta_{\beta}}|J_Z(p_{\theta_{\alpha}})| > \frac{\epsilon}{8} \right) \leq 2\exp\left(-\frac{|Z| \epsilon^2}{1024\delta_1^2} \right) + P_{\mathcal{Z}},
\]

(82)

where \( P_{\mathcal{Z}} = P\left(\frac{1}{|Z|} \sum_{m=1}^{\|Z\|_\infty} \omega_m \|x_m\|_2^2 - \mu \geq (\delta^2 - \mu) \right) \leq \frac{\sigma^2}{|Z|(\delta^2 - \mu)^2}.
\]

(83)

Combining (74) and (82), we obtain

\[
P\left(|J(p_{\theta_{\alpha}}) - J(p_{\theta_{\beta}})| > \epsilon \right) \\
\leq 8\exp\left(-\frac{|Z| \epsilon^2}{1024\delta_1^2} \right) + \frac{4 \sigma^2}{|Z|(\delta^2 - \mu)^2}.
\]

(84)
APPENDIX F

PROOF FOR COROLLARY 1

According to (23), $D_{KL}(p_o, p_{θz})$ is decomposed into the approximation and generalization errors. Theorem 1 demonstrates that there exists an optimized data-driven DL estimator $p_{θz}(\mathbf{x})$ with at most $\lceil \log_2(d_o + 1) \rceil$ hidden layers and sufficiently large $R$ such that the approximation error $J(p_o) - J(p_{θz}) \leq ε$ for any $ε > 0$.

Moreover, Theorem 2 implies that the generalization error $J(p_o) - J(p_{θz})$ satisfies

$$\lim_{|Z| \to +∞} \mathbf{P}(|J(p_o) - J(p_{θz})| > ε) = 0 \quad (85)$$

for any $ε > 0$. Combining Theorems 1 and Theorem 2, we have

$$\lim_{|Z| \to +∞} \mathbf{P}(D_{KL}(p_o, p_{θz}) > ε) = 0 \quad (86)$$

for any $ε > 0$, which completes the proof.

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Qiang Hu received the B.S. degree in applied physics and the M.S. degree in telecommunication engineering from the Beijing University of Posts and Communications (BUPT), China, in 2013 and 2016, respectively, and the Ph.D. degree in telecommunication from Tsinghua University in 2022. His research interests include machine learning theory, statistical signal processing, and convex optimization.

Feifei Gao (Fellow, IEEE) received the B.Eng. degree from Xi’an Jiaotong University, Xi’an, China, in 2002, the M.Sc. degree from McMaster University, Hamilton, ON, Canada, in 2004, and the Ph.D. degree from the National University of Singapore, Singapore, in 2007. He was a Research Fellow with the Institute for Infocomm Research (I2R), A*STAR, Singapore, in 2008; and an Assistant Professor with the School of Engineering and Science, Jacobs University, Bremen, Germany, from 2009 to 2010. In 2011, he joined the Department of Automation, Tsinghua University, Beijing, China, where he is currently an Associate Professor. He has authored/coauthored more than 150 refereed IEEE journal articles and more than 150 IEEE conference proceeding papers. His research interests include communication theory, signal processing for communications, array signal processing, and convex optimizations, with particular interests in MIMO techniques, multi-carrier communications, cooperative communication, and cognitive radio networks. He has also served as the Symposium Co-Chair for 2019 IEEE Conference on Communications (ICC), 2018 IEEE Vehicular Technology Conference Spring (VTC), 2015 IEEE Conference on Communications (ICC), 2014 IEEE Global Communications Conference (GLOBECOM), and 2014 IEEE Vehicular Technology Conference Fall (VTC); and a technical committee member for many other IEEE conferences. He has served as an Editor for IEEE TRANSACTIONS ON COMMUNICATIONS, IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, IEEE SIGNAL PROCESSING LETTERS, IEEE COMMUNICATIONS LETTERS, IEEE WIRELESS COMMUNICATIONS LETTERS, and China Communications; and a Senior Editor for IEEE SIGNAL PROCESSING LETTERS and IEEE COMMUNICATIONS LETTERS.

Hao Zhang received the B.S. and M.S. degrees in applied mathematics and the Ph.D. degree in electronic engineering from Tsinghua University, Beijing, China, in 1995, 1997, and 2001, respectively. Since August 2001, he has been with the Department of Electronic Engineering, Tsinghua University, where he is currently an Associate Professor. His research interests include high-resolution spectral analysis, array processing, and advanced statistical and intelligent techniques applied to signal processing.

Geoffrey Ye Li (Fellow, IEEE) is currently the Chair Professor with Imperial College London, U.K. Before joining Imperial College London, in 2020, he was a Professor with the Georgia Institute of Technology, USA, for 20 years; and the Principal Technical Staff Member with AT&T Laboratories–Research, NJ, USA, for five years. His general research interests include statistical signal processing and machine learning for wireless communications. In these areas, he has published over 600 journals and conference papers, in addition to over 40 granted patents and several books. His publications have been cited over 60,000 times with an H-index is 110. He has been recognized as a Highly Cited Researcher by Thomson Reuters, almost every year. He was an IET fellow for his contributions to signal processing for wireless communications. He has won several prestigious awards from IEEE Signal Processing, Vehicular Technology, and Communications Societies, including the IEEE ComSoc Edwin Howard Armstrong Achievement Award in 2019. He has organized and chaired many international conferences, including the Technical Program Vice-Chair of the IEEE ICC’03; and the General Co-Chair of the IEEE GlobalSIP’14, the IEEE VTC’19 Fall, the IEEE SPAWC’20, and IEEE VTC’22 Fall. He has been involved in editorial activities for over 20 technical journals, including the Founding Editor-in-Chief of IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS Special Issue on Machine Learning in Communications and Networking.

Zongben Xu received the Ph.D. degree in mathematics from Xi’an Jiaotong University, China, in 1987. He is currently the Chief Scientist of the National Basic Research Program of China (973 Project) and the Director of the Institute for Information and System Sciences of the university. He was the Vice President of Xi’an Jiaotong University, from 2003 to 2014. His current research interests include intelligent information processing and applied mathematics. He was elected as a member of the Chinese Academy of Science in 2011. He received the Tan Kan Kee Science Award in Science Technology in 2018, the National Natural Science Award of China in 2007, the National Award on Scientific and Technological Advances of China in 2011, the CSIAM Su Buchin Applied Mathematics Prize in 2008, and the ITIQAM Richard Price Award. He delivered a 45 minute talk on the International Congress of Mathematicians in 2010.