Analysis on Water Quality Change Trend of the Municipal Sewer Network Based on the Mann-Kendall method

Z X Peng, H F Zhang1, Y Guo *, C Lang, C Y Wang, J Z Dong, K Gao, X M Song, YS Yang
Beijing Urban Drainage Monitoring Center Co., Ltd., Beijing 100031, China

Abstract. This paper used the Mann-Kendall method to analyse the water quality changes trend of the municipal sewer network in Beijing North Second Ring Road area from Oct. 2013 to Nov. 2017, which selected chemical oxygen demand (CODcr), ammonia nitrogen (NH3-N) and total phosphorus (TP) as three indicators, used the concentration value divided by the standard values as the pretreatment way, and then calculated the comprehensive index. The analysis result showed that the drainage water quality changes were divided into three phases since the “Decree on Urban Drainage and Sewage Treatment” was issued in this area. The first phase was from Oct. 2013 to Feb. 2014. It continued to decline. The second phase was from Mar. 2014 to May 2014. It rebounded and rose. The third phase was from Jun. 2014 to Nov. 2017, the shocks fell. In the end, the paper predicted that in Dec. 2017 the water quality of the sewer network in this area was still showed a downward trend but not significant. Through actual monitoring, it was found that the predicted results were in good agreement with the actual monitoring results.

1. Introduction
In order to strengthen the management of urban drainage and sewage treatment, ensure the safe operation of urban drainage and sewage treatment facilities, prevent and control urban water pollution and internal disasters, protect citizens’ lives, property and public safety, and protect the environment, the State Council of the People's Republic of China has issued the “Decree on Urban Drainage and Sewage Treatment” was issued on Oct. 2, 2013. After the Decree was issued, each local government has taken corresponding measures to strengthen drainage management. The Mann-Kendall method is a non-parametric statistical test method which is also called non-distribution test and its advantage is that the sample does not need to follow a certain distribution, but also does not interfere with a few abnormal values [1]. The Mann-Kendall method has been widely used as an effective means for trend testing and mutation point analysis [2], such as the Mann-Kendall method has been used to analyze the spatio-temporal changes of extreme continuous precipitation in Guangxi during 1951-2006 by Jianhong Li et al. [3].

Considering the complexity of the sewer system, this paper uses the Mann-Kendall method to analyze the change trend of the discharge water quality of the regional drainage households since the “Decree on Urban Drainage and Sewage Treatment” was issued in Beijing North Second Ring Road area.

2. Method introduction
2.1. Mann-Kendall mutation analysis method [4]
Set sequence as \( x_1, x_2, \cdots, x_n \). \( S_k \) represents the cumulative number of the \( i \)-th sample \( x_i > x_j \) (\( 1 \leq j \leq i \)). Define statistics:

\[
S_k = \sum_{i=1}^{k} \sum_{j=1}^{i} r_{ij},
\]

\( r_{ij} = \begin{cases} 1, & x_i > x_j \\ 0, & x_i \leq x_j \end{cases} \)

\( j = 1, 2, \cdots, i; k = 1, 2, \cdots, n \)

On the assumption that the time series is randomly independent, the mean and variance of \( S_k \) are:

\[
E[S_k] = k(k - 1)/4,
\]

\[
\text{var}[S_k] = k(k - 1)(2k + 5)/72 \quad (1 \leq k \leq n)
\]

Standardize \( S_k \):

\[
UF_k = \frac{S_k - E[S_k]}{\sqrt{\text{var}[S_k]}}, \text{ where } UF_1 = 0.
\]

Given \( \alpha \) as the significance level, if \( |UF_k| \geq U_\alpha \), it shows that the sequence has a clear trend. All \( UF_k \) can form a curve.

Apply this method to the inverse sequence \( x_n, x_{n-1}, \cdots, x_1 \), and repeat the same process on \( x_n, x_{n-1}, \cdots, x_1 \) and get \( UF_k' \), where \( UF_1' = 0 \). Define \( UB_k' \):

\[
UB_k = -UF_k' (k = n, n - 1, \cdots, 1)
\]

where \( UB_1 = 0 \)

### 2.2. Comprehensive index

Since the selected area is a residential area, there is basically no industrial drainage. Combined with the regulatory requirements for drainage permits, CODcr, NH3-N and TP are selected for analysis. In order to reflect the overall changes and trends in water quality in the area, comprehensive index of water quality pollution is calculated using three indicators.

#### 2.2.1. Pretreatment method

Due to the different magnitudes of CODcr, NH3-N and TP, pretreatment is needed. The pretreatment needs to reflect the relationship between the index data and the corresponding limits of the “Wastewater Quality Standards for Discharge to Municipal Sewers”, and is calculated as follows:

\[
P_{mi} = C_{mi}/C_i, \quad (m = 1, 2, \cdots, n, \ i \ \text{represents CODcr, NH}_3\text{-N, TP})
\]

\( P_{mi} \) indicates the \( i \)-th preprocessed pollutant value of the \( m \)-th time series; \( C_{mi} \) indicates the \( i \)-th pollutant monitor value of the \( m \)-th time series; \( C_i \) indicates the \( i \)-th pollutant standard limited value.

#### 2.2.2. Water pollution comprehensive index calculation

Water pollution comprehensive index is used to represent the pollution degree of the water quality; higher values indicate more serious pollution. The comprehensive index is calculated as follows:

\[
G_m = \sum w_i P_{mi},
\]

\( G_m \) is the \( m \)-th water pollution comprehensive index; \( w_i \) is the \( i \)-th pollutant weight; \( P_{mi} \) is the \( i \)-th preprocessed pollutant value of the \( m \)-th time series.

### 2.3. Mann-Kendall trend testing method

In the Mann-Kendall trend testing method, the original hypothesis \( H_0 \) is the time series data \( (x_1, x_2, \cdots, x_n) \), which are \( n \) independent, random samples of the same distribution; alternative hypothesis \( H_1 \) is a two-sided test. For all \( i, j \leq n \), and \( i \neq j \), the distribution of \( x_i \) and \( x_j \) is not the same. Define the test statistic \( S \):

\[
S = \sum_{i=2}^{n} \sum_{j=1}^{i-1} \text{sign}(x_i - x_j)
\]
where, \( \text{sign}() \) is a sign function as below:

\[
\text{sign}(x_i - x_j) = \begin{cases} 
-1, & x_i - x_j < 0 \\
0, & x_i - x_j = 0 \\
1, & x_i - x_j > 0 
\end{cases}
\]

\( S \) is a normal distribution with a mean of 0 and a variance of \( \text{Var}(S) = \frac{n(n-1)(2n+5)}{18} \).

When M-K Statistical Formula \( S \) is greater than, equal to, or less than 0, they are:

\[
\begin{align*}
Z &= \frac{S - 1}{\sqrt{\frac{n(n-1)(2n+5)}{18}}} & S > 0 \\
Z &= 0 & S = 0 \\
Z &= \frac{S + 1}{\sqrt{\frac{n(n-1)(2n+5)}{18}}} & S < 0
\end{align*}
\]

In the two-sided trend test, for a given confidence level \( \alpha \), if \( |Z| \geq Z_{1-\alpha/2} \), the original hypothesis \( H_0 \) is unacceptable, that is, at the confidence level \( \alpha \), there is a significant increase or decrease in time series data. A positive value of \( Z \) indicates an increasing trend, and a negative value indicates a decreasing trend. When the absolute value of \( Z \) is greater than or equal to 1.28, 1.64, and 2.32, it indicates that the reliability is 90\%, 95\%, and 99\% of significance, respectively.

3. Case studies

3.1. Research area overview

This study area is located in the northern part of Beijing North Second Ring Road, mainly including Bei Tai Ping Zhuang Street in Haidian District, Desheng Street in Xicheng District, Anzhen Street in Chaoyang District and Hepingli Street in Dongcheng District. The area is a residential area for residents. There are more edifices, restaurants, hotels, and comprehensive commercial buildings. The terminal of the sewer network monitoring point in the area is located in the red five-pointed star position in Figure 1.

![Fig. 1 the location of sewer network monitoring point](image)

3.2. Result analysis

In order to clarify the changes in the quality of sewage discharged from the area since the promulgation of the “Decree on Urban Drainage and Sewage Treatment”, the data from the Oct. 2013
to Nov. 2017 monitoring points at the Hepingli downstream site in the north of Beihuhe were selected for analysis.

3.2.1. Data pretreatment. Since the wastewater from the selected area is eventually recycled into the treatment plant as a reclaimed water plant, the pretreatment method for this paper uses A-level standard limits value of “Wastewater Quality Standards for Discharge to Municipal Sewers”. The pretreatment CODcr, NH3-N and TP sequence trends are shown in the figure 2.

![Fig. 2 the value of pre-treatment CODcr, NH3-N and TP](image)

3.2.2. Water pollution comprehensive index calculation. CODcr, NH3-N and TP have a greater impact on the environment. They are indicators of pollution reduction in the “Twelfth Five-Year Plan” period and are the basic indicators required for the detection of Beijing drainage permits. Thus, the importance of CODcr, NH3-N and TP are the same. The same weights are used to calculate the comprehensive index of water pollution which $w_i$ is equal to 1/3. The time series trend of water pollution comprehensive index after calculation is shown in Fig. 3.

![Fig. 3 the time series values of water pollution comprehensive index](image)
3.2.3. Mann-Kendall mutation analysis. The Mann-Kendall mutation analysis method only compares the size of the time series, preprocessing does not change the order of size within the time series. The Mann-Kendall mutation analysis was performed on the time series of the water pollution comprehensive index. The $U_F$ and $U_B$ curves are shown in Fig. 4.

The $U_F$ and $U_B$ curves showed the first mutation from Nov. to Dec. 2013, and $U_F$ continued to decrease. In Mar. 2014, there was a breakthrough threshold of $\alpha=0.05$, indicating that significant changes began to occur between Nov. and Dec. 2013. In Mar. 2014, there was a significant downward trend in pollution levels. The second mutation occurred from Mar. to Apr. 2014. $U_F$ turned positive from negative in May 2014. This indicates that the level of pollutants changed from Mar. to Apr. 2014 and caused an increase in pollutants pollution in May 2014. The third mutation occurred from Jun. to Jul. 2014, $U_F$ began to be negative; followed by Dec. 2014 to Jan. 2015, Feb. to Mar. 2015, Mar. to Apr. 2015, and Jun. to Jul. 2015, there was no change in $U_F$ was negative and breakthrough $\alpha=0.05$ threshold, indicating that the pollution began to decline from Jun. 2014, but the trend was not significant. There was the fourth mutation from Aug. to Sep. 2015. The $U_F$ continued to decline and broke through the threshold of $\alpha=0.05$ in Oct. 2016. This indicates that there has been a significant change from Aug. to Sep. 2015 and there was a significant decrease in pollution levels in Oct. 2016. After that, there were mutations from Feb. to Mar. 2017 and from Apr. to May 2017. However, there was no change in the negative $U_F$ value and breakthrough of the threshold value of $\alpha=0.05$. It showed that from Feb. 2017 the level of pollutant pollution began to continue downward trend but the trend is not significant.

In general, it can be divided into three phases. The first phase is from Oct. 2013 to Feb. 2014. It continues to decline. The second phase is from Mar. 2014 to May 2014. It rebounds and rises. The third phase From Jun. 2014 to Nov. 2017, the shocks fell.

3.2.4. Mann-Kendall trend test. Since the Mann-Kendall trend test also compares the size of the time series, preprocessing does not change the order of size within the time series. In order to predict more accurately, the most recent phase data which is the third phase, was selected to perform the forecast. The Mann-Kendall trend test was performed on the time series of the pretreated CODcr, NH3-N, TP, and water pollution comprehensive index of the third phase. The $Z$ value is shown in Table 1.
Table 1 the Z value of Mann-Kendall trend test

| class                        | Z value | threshold value ($\alpha=0.05$) | Trend                        |
|------------------------------|---------|---------------------------------|------------------------------|
| CODcr                        | -0.30   | -1.64                           | No significant decline      |
| NH$_3$-N                     | -0.55   | -1.64                           | No significant decline      |
| TP                           | -0.43   | -1.64                           | No significant decline      |
| water pollution comprehensive index | -0.28   | -1.64                           | No significant decline      |

From Table 1, it can be found that whether from the CODcr, NH$_3$-N, TP, or from water pollution comprehensive index, they all showed a downward trend, but below the significance level $\alpha=0.05$. The downward trend was not significant.

Through the comparison of monitoring data in Dec. 2017, the CODcr, NH$_3$-N, TP, and water pollution comprehensive index have all declined, shown in Fig. 5.

![Fig. 5 the value of pre-treatment CODcr, NH$_3$-N, TP and water pollution comprehensive index](image)

4. Concluding remarks
After the “Decree on Urban Drainage and Sewage Treatment” was issued, the degree of pollution of the drainage water quality in the northern part of Beijing North Second Ring Road was divided into three phases. The first phase is from Oct. 2013 to Feb. 2014. It continues to decline. The second phase is from Mar. 2014 to May 2014. It rebounds and rises. The third phase From Jun. 2014 to Nov. 2017, the shocks fell.

It was predicted that the drainage quality of the area showed a downward trend but not significant. Through actual monitoring, it was found that the predicted results were in good agreement with the actual monitoring results. It was showed that the Mann-Kendall method can be used to predict the change trend of the wastewater quality of the sewer network, on the other hand, it also provide technology support for the regulatory authorities to take adjusting measures.

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