Slotted rectangular waveguide with dielectric sandwich structure inside

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Abstract. This paper continues the series of works devoted to the investigation of leaky-wave antenna based on layered rectangular waveguide with periodic transverse slots in broad face. Previously developed wavenumber calculation technique has been adapted for analysis of slotted sandwich waveguide with three layers at least. The paper provides the numerical results of velocity factor dependencies for partially filled slotted rectangular waveguide containing a dielectric slab in the middle position inside or an air gap between two dielectric slabs. Additionally, dispersion properties are also considered for multilayer waveguide with linear laws combinations of thickness and permittivity. This allows recognizing the trends to develop new prospective antennas with complex patterns of tilt angle change. All numerical results obtained are confirmed with the in-situ measurements of transmission coefficient phase.

1. Introduction
Recently the new model of leaky-wave antenna based on rectangular waveguide with transverse slots in broad face and sandwich structure inside has been suggested in [1]. The technique of composing the dispersion equation has been developed. The numerical results of its solution have been obtained in [2]: frequency dependencies of velocity factor, radiation pattern tilt angle, and beamwidth. The experiments described in [3] have confirmed the validity of the calculation and mathematical model.

However, these studies were limited only by consideration of two-layered structure and analysis of waveguide filling factor when dielectric was immediately at the face parallel to the slot plane. This fact does not show a complete picture of the process inside the partially filled slotted waveguide.

The use of three layers in the model can be helpful for analysis of the dielectric slab position impact on the antenna performance. Using the bigger number of layers allows establishing required patterns of change in permittivity, permeability, and thickness between adjacent slabs to solve the various tasks. For instance, it can provide the linear law of angle-frequency performance variation, which is important for possibility of these antennas application in airborne radars.

2. Investigated Antenna Model
The model of investigated antenna is presented in figure 1. This is the endless slotted rectangular waveguide with cross section dimensions $a \times b$. The slotted face is combined with an infinite top ground plane. Transverse slots are located at the distance of $X_0$ from narrow face and $p$ from one another. Length and width of slots are $L$ and $W$, respectively. The waveguide contains the sandwich structure inside, whose layers are parallel to the ground plane. Every slab has its own thickness $d_p$. 


permittivity $\varepsilon_p$, and permeability $\mu_p$. For the convenience wall thickness is taken as infinite small and wall material is a perfect electric conductor. Coordinate axes are chosen as follows: $y$ is along the waveguide axis, $z$ is normal to the slot plane, and $x$ is oriented to complete the right-hand system.

This system is supposed to support the propagation with wavenumber $k_z = \beta - ja$, where $\beta$ is a phase constant and $a$ is an attenuation constant. The complex propagation constant $k_z$ is defined from a transcendental equation by matching the tangential components of magnetic field from different sides of the slot plane according to the technique offered in [1] and [4].

The investigated antenna is divided into two subspaces (external and internal), either of which is described by equivalent lines model. This model uses the Green’s function method and allows taking into account the structure inhomogeneity. Generally, dispersion equation for leaky-wave antenna based on multilayer waveguide looks like equation for antenna on the basis of two-layered waveguide:

$$
\sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{1}{k_z^2 + k_{ym}^2} \left[ \frac{\cos\left(k_z L/2\right)}{\left(\pi/L\right)^2 - k_z^2} \right]^2 dk_x = \frac{2\pi}{a} \sum_{n=-\infty}^{\infty} \frac{1}{\left(\pi n/a\right)^2 + k_m^2} \left[ \frac{\sin\left(\pi nx/a\right) \cos\left(\pi mL/2a\right)}{\left(\pi m/a\right)^2 - \pi/L} \right]^2,
$$

where $k_z$ is wavenumber along the axis $x$ in space region; $k_{ym} = k_y + 2\pi n/p$, where $n$ is an order of space Floquet harmonic; $\varepsilon_m$ is Kronecker delta taking a value $\varepsilon_m = 1$, if $m = 0$, and $\varepsilon_m = 2$, if $m \neq 0$; $Y_{\text{ext,int}}^H(0), \ Y_{\text{ext,int}}^E(0)$ are external and internal input modal admittances from the slot plane in $H$-line and $E$-line, respectively. However, the one substantial difference takes a place: the bigger the number of layers, the bigger iterations quantity is required for calculation of input admittances (figure 2) [5]:

$$
\tilde{Y}_{\text{E,H}}^E, H\left(z_p\right) = Y_{\text{p+1}}^E, H \frac{\tilde{Y}_{\text{p+1}}^E, H\left(z_p\right) \cot\gamma_{p+1} d_{p+1} + jY_{\text{p+1}}^E, H}{Y_{\text{p+1}}^E \cot\gamma_{p+1} d_{p+1} + jY_{\text{p+1}}^E, H\left(z_{p+1}\right)},
$$

where $Y_p^E = \gamma_p/\omega \mu_p$ and $Y_p^H = \omega \varepsilon_p/\gamma_p$ are equivalent wave admittances of internal layers; $\gamma_p = \sqrt{k_p^2 - \left(\pi n/a\right)^2}$ is a longitudinal propagation constant of every single dielectric slab in chosen coordinate system, $m$ is a number of half waves along the broad side of waveguide.

**Figure 1.** The leaky-wave antenna based on rectangular waveguide with sandwich structure

**Figure 2.** For the admittances recalculation.

3. Numerical Results

The equation (1) is transcendental since it contains nonalgebraic functions, particularly trigonometric. That is why its solution is only possible by means of numerical methods. The standard parameters we used are following: $a = 23$ mm, $b = 10$ mm, $L = 8$ mm, $W = 0.5$ mm, $p = 2$ mm, $x_0 = a/2 = 11.5$ mm.
3.1. Partially Filled Rectangular Waveguide

In previous works [2] and [3] it has been shown the structure has bigger value of velocity factor if the dielectric slab is located near the slotted wall in comparison with position near the opposite one. However, phase velocity dependencies for waveguide with dielectric in the middle position were unknown. Figure 3 shows the plot of velocity factor for leaky-wave antenna based on rectangular waveguide partially filled with Teflon ($\varepsilon = 2$) at frequency of 8 GHz. Figure 3a illustrates the dependence for dielectric slab shifted from the face opposite to slots, whereas figure 3b demonstrates the results for inverted structure, i.e. for an air gap between two dielectric slabs. Really, the closer dielectric to slots, the smaller value of phase velocity. In both cases curves are seen to have nonlinear patterns of change with dielectric (or gap) shift directional extremums. And the more the dielectric layer thickness, the more dependencies distortion.

![Figure 3](image)

**Figure 3.** Two-dimensional velocity factor dependence of slotted waveguide on thickness and position of (a) dielectric slab; (b) gap between two dielectric slabs.

3.2. Multilayer Slotted Rectangular Waveguide

The following part of paper is devoted to the investigation and discussion of different laws of change in thickness and permittivity between adjacent layers inside the waveguide, and definition of their influence on the antenna performance. At the first stage of research only simple linear laws have been considered. Figure 4 shows dispersion dependencies for leaky-wave antenna based on rectangular waveguide filled with five dielectrics whose parameters follow in different orders. Permittivity changes from 1 to 5, more dark shading corresponds to bigger values of permittivity. Figure 5 shows analogous situation with the change in dielectric layer thickness.

![Figure 4](image)

**Figure 4.** Dispersion curves for multilayer waveguide with periodic transverse slots at the linear increase (a) and decrease (b) of dielectric layer thickness.

![Figure 5](image)

**Figure 5.** Dispersion curves for multilayer waveguide with periodic transverse slots at the linear increase (a) and decrease (b) of dielectric layer permittivity.

The results obtained for multilayer waveguide confirm and complete the trends obtained for partially filled waveguides. The curves from figures 4 and 5 look quite similar and make it possible to conclude
following. An arithmetic mean value of permittivity inside the waveguide lies behind the system cut-off frequency. At the same time the bigger the permittivity of layer located near the slots, the higher the slope of dispersion curve at a point of cut-off frequency.

4. Experimental Results

In order to prove the correctness of developed calculation technique and obtained numerical results we have conducted experimental research, which implies some in-situ measurements of transmission coefficient $S_{21}$. We used the same workpiece of slotted waveguide and the same dielectric stripes from Teflon and PCT-5 as it had been in [3]. We placed it in different order inside the waveguide and built the velocity factor dependencies recalculated from a phase of transmission coefficient $S_{21}$. Figure 6 shows the studied cases of cross section and corresponding experimental dispersion curves in comparison with the theoretical data. It is seen they have a good agreement.

![Figure 6. Comparison of experimental and theoretical data. Solid lines – measured values; dashed lines – calculated dependencies. More dark shading in sketches corresponds to PCT-5 ($\varepsilon = 5$), light shading is to Teflon ($\varepsilon = 2$). Some inaccuracy takes a place in a low part of studied frequency band because of very high slope of dispersion curve such the phase changes so quickly, whereas a frequency step is too low. The other possible reason is a feature of calibration with coaxial-to-waveguide transition.](image)

5. Conclusion

The velocity factor calculation technique has been adapted for analysis of leaky-wave antenna based on rectangular waveguide with periodic transverse slots and internal sandwich structure. The dielectric layer position influence on the leaky-wave antenna propagation constant is considered in case of partially filled waveguide. The dielectric slab thickness significantly impacts on the dispersion curves distortion. Also, antenna based on five-layered structure has been investigated. All theoretical results are confirmed with experimental research. The combination of dielectric layers with various parameters allows obtaining arbitrary performance of periodic leaky-wave antenna.

Acknowledgements

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References

[1] Abdullin R R, Knyazev S T, Lesnaya L L and Shabunin S N 2013 Analysis of partially dielectric-filled rectangular waveguide with transverse slots using Green’s function method Proc 7th European Conference on Antennas and Propagation (Gothenburg) pp 3453–57.

[2] Abdullin R R, Mitelman Yu E and Shabunin S N 2014 Radiation pattern of leaky-wave antenna based on partially-filled rectangular waveguide. Proc Loughborough Antenna & Propagation Conference (Loughborough) pp 516–518.

[3] Abdullin R R 2016 Experimental research of leaky-wave antenna based on partially-filled rectangular waveguide with transverse slots Antennas no 3 (Moscow: Radiotechnics) pp 46–54.

[4] Liu J, Jackson D R and Long Y 2011 Modal analysis of dielectric-filled rectangular waveguide with transverse slots IEEE Trans. Antennas and Propag., vol. 59, pp 3194-3203.

[5] Panchenko B A, Nefedov E I 1986 Microstrip antennas. (Moscow: Radio & Connection).