Oscillation Mode Variability in Evolved Compact Pulsators from Kepler Photometry. II. Comparison of Modulation Patterns between Raw and Corrected Flux

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Abstract

We present the second results of an ensemble and systematic survey of oscillation mode variability in compact pulsators observed with the original Kepler mission. Two types of flux calibrations, raw and corrected, collected on two hot B subdwarf stars, KIC 2438324 and KIC 11179657, are thoroughly examined with the goal of evaluating the difference in patterns when oscillation modes modulate in amplitude (AM) and frequency (FM). We concentrate on AMs and FMs occurring in seven multiplet components in each star as representative frequencies. The analysis shows that FM measurements are independent of the flux calibration we choose. However, if flux contamination by nearby stars is large, AMs may be significantly different between raw and corrected flux. In addition, AMs suffer, to some extent, from a systematic modulation pattern, which is most likely induced by instrumental effects and differs from one star to another. Our results indicate that stars with no contamination are better candidates to quantitatively compare modulation patterns with theory and should be given a higher priority for such studies, since light contamination will destroy real AM patterns.

Unified Astronomy Thesaurus concepts: Photometry (1234); Stellar oscillations (1617); B subdwarf stars (129); Pulsation modes (1309); Pulsation frequency method (1308)

1. Introduction

This series of papers is devoted to an ensemble and systematic survey of oscillation mode variability in pulsating hot B subdwarf (sdB) and white dwarf stars using Kepler’s photometry, particularly for stars with over 2 yr of monitoring. We recall, in the first paper of this series (Zong et al. 2018, hereafter Paper I), that a particular sdB star, KIC 3527751, was thoroughly analyzed with a total of 204 frequencies resolved from a nearby contiguous 38 month long light curve. We investigated mode variability and stability for 143 frequencies with relatively large amplitudes (i.e., with acceptable significance level in just a portion of the entire light curve) and found that all of those frequencies show evident variations, with regular or irregular modulation patterns. We showed that these variations are likely reminiscent of nonlinear weak mode interactions predicted by the resonant mode coupling formalism (e.g., Buchler et al. 1997), where the oscillation mode under certain resonance conditions may exhibit temporal amplitude modulations (AMs) and frequency modulations (FMs) of various patterns, or remain stable over time. Such observed modulation patterns will eventually be compared to theoretical ones, once calculations of nonlinear coupling coefficients involving many modes (Buchler et al. 1995) become available. The direct comparison of AMs and FMs has only been achieved for a few pulsating stars (see, e.g., Kovacs & Buchler 1989), thus far. Therefore, the current and most urgent step is to provide intrinsic AM and FM measurements, as this opportunity is now offered by Kepler. Recent results show that pulsating sdB and white dwarf stars may be among the best candidates on this front (see, e.g., Zong et al. 2016a).

In the original Kepler field, the satellite collected exquisite high-quality photometry for 18 pulsating sdB stars and six white dwarf stars (see Paper I, and references therein). Most of these objects have been intensively observed in the competitive short-cadence (58.85 s) mode over a duration of 2 yr, due to their rapid oscillation timescales (from a few minutes to a few hours) and scientific significance. Kepler pipelines provide available light curves in two forms; the raw flux, also referred to as the Simple Aperture Photometry (SAP), and the corrected flux, also called Pre-search Data Conditioning SAP (PDC-SAP; see, e.g., Jenkins et al. 2010). The latter data impose cotrending basis vectors to correct the discontinuities and contamination of raw flux over different quarters. Although the corrected data contain cleaner light curves, this process may bring extra or modify intrinsic astrophysical signatures of some particular targets (see Murphy 2012). In the context of precisely characterizing modulation patterns in oscillation modes, the impact of using two different types of flux has never been examined. Concretely evaluating these differences for some particular stars might be necessary before attempting quantitative comparisons with theoretical calculations.

With this purpose in mind, we chose two long-period g-mode pulsating sdB stars, KIC 2438324 and KIC 11179657, as representative objects to check if using two different kinds of fluxes leads to significant differences in the modulation patterns. KIC 2438324, or B4 in NGC 6791, has a mean brightness $K_P = 18.267$, effective temperature $T_{\text{eff}} = 24,786 \pm 655$ K, and surface gravity $\log g = 5.30 \pm 0.09$ dex (Reed et al. 2012). The detection of pulsations in this very faint star required the acquisition of over 6 months of Kepler photometry (Pablo et al. 2011), which established its variability. KIC 11179657, or USNO-A2.0 1350-10140904, has a mean brightness $K_P = 17.065$, $T_{\text{eff}} = 26,000 \pm 800$ K, and $\log g = 5.14 \pm 0.13$ dex (Østensen et al. 2010). For that star, the presence of pulsations was revealed during the first year survey phase, on the basis of an ~30 day light curve (Østensen et al. 2010). Both stars are found to be members of a binary system with low-mass main-sequence companions, and both are not synchronized, as suggested by the seismic rotation periods, which are longer than the orbital periods (Pablo et al. 2011, 2012).
In this paper, we first assess the robustness of our error estimates for the amplitude, frequency, and phase from quantitative tests in Section 2. Section 3 is dedicated to the thorough analysis of the Kepler photometry collected for these two sdB stars. The extracted modulation patterns for representative mode frequencies and comparisons with the orbital signals are presented in Sections 4 and 5, followed by a discussion in Section 6 and a conclusion.

2. Testing the Robustness of Error Estimates

Considering the importance of error estimates for our scientific goals, we made a series of simulations to assess their robustness for frequencies extracted from the Kepler photometry. These tests constitute an extension of those described in (Zong et al. 2016b). A variant of such simulations was also carried out recently by Silvotti et al. (2018), in order to check the reliability of errors for measured frequencies that suggest the presence of a giant planet around V391 Peg.

We briefly describe how this series of simulations was done: (1) 50 artificial light curves containing Gaussian white noise were generated using a time sampling of 58.85 s and a duration of 200 days without interruption; (2) in each light curve, 1000 frequencies of constant amplitude (details are provided in Zong et al. 2016b) were injected, with amplitude values varying from light curve to light curve; (3) the code FELIX3 was used to detect and extract automatically the injected frequencies in each simulated light curve; and (4) two estimates for the normalized errors were calculated, using different methods. One method was to derive analytically the errors following (Montgomery & Odonoghue 1999, hereafter MO99), with the relations for amplitude, frequency, and phase uncertainties:

\[
\sigma_A = \sqrt{2/N} \sigma_m, \tag{1a}
\]

\[
\sigma_f = \frac{\sqrt{3}}{\pi T A} \sigma_A, \tag{1b}
\]

\[
\sigma_\phi = \frac{\sigma_A}{A}, \tag{1c}
\]

where \(N\) and \(T\) are the number of data points and the total duration of the time series, respectively. \(A\) and \(\sigma_m\) are the measurable amplitude of a frequency peak and the rms deviation of the magnitude in the light curve, respectively.

The other method that has been used thus far in the code FELIX directly measures \(\sigma_A\) as the median value of the noise around a detected peak in the Fourier transform of the light curve. \(\sigma_f\) is then calculated with Equation (1b) and \(\sigma_\phi\) comes from the covariance matrix of the nonlinear least-squares fit of the light curve. The normalized errors for the amplitude, frequency, and phase are defined as

\[
\Delta_A = (A_{\text{pre}} - A_{\text{inj}})/\sigma_A, \tag{2a}
\]

\[
\Delta_f = (f_{\text{pre}} - f_{\text{inj}})/\sigma_f, \tag{2b}
\]

\[
\Delta_\phi = (\phi_{\text{pre}} - \phi_{\text{inj}})/\sigma_\phi, \tag{2c}
\]

respectively. Subscripts “pre” and “inj” refer to prewhitened (measured) and injected values.

Figure 1 shows the results of our simulations from a total of 50,000 injected sinusoidal waves. By using normalized errors, one expects to obtain normal distributions of zero mean and standard deviation one, \(N \sim (0, 1)\), if the values of \(\sigma_A, \sigma_f,\) and \(\sigma_\phi\) are correctly estimated. A narrower distribution than \(N \sim (0, 1)\) gives an overestimation of error. We clearly see that all prewhitened values are very similar to the injected ones, as revealed by the vertical dashed lines near the center. This indicates that there is no bias in the frequency extraction method. Amplitude and frequency uncertainties evaluated from MO99 equations and from our implemented method are both found to be in agreement with the normal distribution in general. However, ours slightly overestimates \(\sigma_A\) and \(\sigma_f\) by about 10% and 5%, while MO99 slightly underestimate these errors by about 2% and 5%, respectively. We note that there is possibly a slight dependence on the signal-to-noise ratio (S/N) of the normalized frequency deviations (i.e., the dashed lines at the low and high S/N ends stand at two sides of the solid vertical lines). The phase deviations, however, show two different behaviors. The one derived from the covariance matrix of the nonlinear fit presents a clear dependence on the S/N as it becomes gradually overestimated with increasing S/N (or amplitude). For this part, MO99 gives a constant overestimate by a factor of about 3 for all S/Ns. We note that Equation (1c) is obtained by averaging the observed time. A change in the zero-point of time will introduce a constant factor to that equation that could correct for the observed difference. While investigating further the reasons behind these differences could be of interest, we stay with our main goal here, which is to calibrate our method to estimate errors accurately. Based on the tests presented above, we thus validated our method to compute uncertainties for the amplitude and frequency, and we implemented a normalized method (MO99 formula divided by a constant value 3) to estimate phase uncertainties.

3. Kepler Photometry

In this work, all data were obtained from the Kepler space telescope during its initial survey phase. We only analyzed short-cadence (58.85 s) light curves distributed through the MAST4 website. This exposure allows us to detect rapid oscillations in compact pulsating stars (see, e.g., Charpinet et al. 2011). KIC 2438324 was continuously observed from Q6.1–Q17.2, for a total duration of 35 months. KIC 11179657 was observed in several segments, Q2.3, Q5.1–Q7.3, Q9.1–Q11.3, Q13.1–Q15.3, and Q17.1–Q17.2, because this star fell on the nonfunctional CCD Module 3 every four quarters, due to the onboard photometer being rolled by 90° every ~93 days. This rotation of the instrument may induce fluctuations of the contamination factor of some targets, as images of these stars relocate onto different CCD modules. The initial 1 month run Q2.3 was not considered further, due to its long disconnection with the main part of the Kepler observations. Table 1 lists the contamination values reported at different quarters for 3.2 Kepler 438324 and KIC 11179657. The KIC 2438324 light curve was contaminated by nearby stars with a factor varying from 0.007–0.418. In contrast, the KIC 11179657 light curve has almost never suffered pollution from the light of nearby stars, with a maximum factor of 0.0002.

In the following, both raw and corrected light curves are analyzed. These were produced through the standard Kepler Science Processing Pipeline provided by Jenkins et al. (2010). As in Paper I, we performed additional data detrending and 3σ clipping to remove residual drifts and data point outliers. After

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3 Frequency Extraction for Lightcurve eXploitation (see the details in Charpinet et al. 2010, 2011 and Zong et al. 2016b).

4 The Mikulski Archive for Space Telescopes: http://archive.stsci.edu/kepler/.
these operations, the raw and corrected light curves for KIC 11179657 consist of 1,413,308 and 1,399,858 points, with duty cycles of 91.6% and 90.7%, respectively, over a period of $\sim$2.88 yr. For KIC 2438324, the light curves contain 1,227,891 and 1,235,149 measurements for the corrected and raw flux, corresponding to duty cycles of 72.9% and 73.3%, respectively, over $\sim$3.14 yr. The two light curves are shown in their entirety in the top panels of Figure 2. A clear feature related to KIC 2438324 is that light variations in the corrected flux are significantly higher than that of the raw flux when the contamination factor is large (see Table 1). Parts of the light curves are expanded in the bottom panels where the brightness variations from the binary reflection effect dominate over the pulsations. The latter are however clearly seen in the Lomb–Scargle Periodogram (LSP; Lomb 1976; Scargle 1982) represented in Figure 3. The two stars show very similar LSPs with oscillations mostly in the low-frequency g-mode domain between 100 and 400 $\mu$Hz, and few low-amplitude modes in the 400–600 $\mu$Hz range. However, the noise level for KIC 11179657 is much lower than that of KIC 2438324, although their main frequencies have very similar amplitudes, $\sim$2 ppt. We note that the amplitudes of oscillations in KIC 2438324 differ in the two types of fluxes, mainly due to contaminating light from the nearby star.

Frequency extraction follows the same procedure as in Paper 1, which is based on a standard prewhitening and nonlinear least-squares fitting approach (Deeming 1975). We provide in Tables 2 and 3 the lists of detected peaks for KIC 2438324 and KIC 11179657, respectively, along with their fitted attributes: frequency (in nanohertz and period in seconds), amplitude (in ppt, i.e., parts per thousand) relative to the mean brightness of the star and the S/N of the detection. Each attribute has an associated error: $\sigma_f$, $\sigma_p$, and $\sigma_A$. The “ID” column uniquely identifies a detected frequency with a sequence number that indicates its rank by order of decreasing amplitude. The “Modulation” column indicates if amplitude only (AM; for characterization of systematic modulations) or amplitude/frequency modulations (AFMs; in representative rotational multiplets) are presented here for a given frequency. These modes are further discussed in the following sections.

Figure 1. Distribution of normalized amplitude, frequency, and phase deviations between the extracted and injected signals (from left to right panels). Top panels correspond to the default method to compute errors used with the code FELIX (see the text) and the bottom panels correspond to errors determined analytically following Montgomery & Odonoghue (1999). The (central) dashed lines indicates the mean deviations calculated from the distribution, which are all very close to zero. The dotted curves indicate $\pm 3\times$ the standard deviations as a function of S/N. In all panels, except the top-right one, S/N-averaged values for $3\sigma$ are represented by vertical solid lines. In the top-right panel, the solid curve is fitted using a reciprocal model. Color coding refers to the occurrence (using a logarithmic scale) of measurements at specific deviation and S/N values.

| Quarter | KIC 2438324 | KIC 11179657 |
|---------|-------------|--------------|
| 5       | ...         | 0.0001       |
| 6       | 0.046       | 0            |
| 7       | 0.414       | 0            |
| 8       | 0.007       | ...          |
| 9       | 0.031       | 0.0002       |
| 10      | 0.047       | 0            |
| 11      | 0.416       | 0            |
| 12      | 0.007       | ...          |
| 13      | 0.032       | 0.0001       |
| 14      | 0.046       | 0            |
| 15      | 0.418       | 0            |
| 16      | 0.007       | ...          |
| 17      | 0.032       | 0            |

Table 1
Contamination Values per Quarter for KIC 2438324 and KIC 11179657 during the Main Campaign
Table 2 lists 22 detected frequencies in KIC 2438324 associated with stellar oscillations and two additional low-frequency peaks of orbital nature. All of them are above an adopted threshold of $5.6 \times \sigma$ (as tested in Zong et al. 2016a). There are 16 frequencies attributed to components of two triplets near 217 and 317 $\mu$Hz, and five doublets near 229, 291, 342, 372, and 406 $\mu$Hz. One component, $f_{16}$, of the doublet near 290 $\mu$Hz falls close to 10 times the orbit frequency, $f_{\text{orb}} \sim 29 \mu$Hz. We note that all frequencies are found in the 100–500 $\mu$Hz range. For KIC 11179657 (Table 3), we detected 33 independent frequencies above 5.6$\sigma$, all attributed to oscillations. We identify three triplets near 195, 284, and 307 $\mu$Hz, and four doublets near 185, 206, 337, and 368 $\mu$Hz, and a possible quintuplet near 260 $\mu$Hz with two missing components.  

Table 3 also contains a frequency, $f_{32}$, that is close to the difference of $f_{05}$ and $f_{01}$. We further note that two weak peaks (of S/N $\sim$ 5; i.e., not provided in Table 3) are seen at 336.37 and 262.88 $\mu$Hz. These could be additional components belonging to the doublet near 337 $\mu$Hz and the 260 $\mu$Hz quintuplet, respectively. In addition to the oscillations, we also detected a binary signal, $f_{\text{orb}}$, at a frequency of 29.341850 $\mu$Hz and its first harmonic 2$f_{\text{orb}}$. Like for KIC 2438324, most of the frequencies of KIC 11179657 are found in the 100–400 $\mu$Hz frequency range, except the frequency $f_{27}$ at 558.2 $\mu$Hz.
4. Modulation Patterns

Following the same strategy as in Paper I to analyze amplitude and AMs in these two stars, we focus on frequencies of highest amplitudes, in particular those being components of rotational multiplets. We note that, for both stars, there are relatively few frequencies compared to KIC 3527751 as rotational multiplets. We note that, for both stars, there are amplitudes and AMs in these two stars, we focus on frequencies sliding LSPs due to two specific limitations: (1) In general, we find that most frequencies are close to stability and do not show obvious variations in these diagrams, as they are not sensitive enough. (2) They are of limited use to precisely measure modulation patterns for comparison between raw and corrected fluxes (the main objective of the present analysis). In the following, we therefore only present the modulations obtained by prewhitening the frequencies in various parts of the light curve. The 35 month light curve of KIC 2438324 was divided into a series of adjacent pieces using a time step of about 10 days and a filtering window of width 180 days. Each part was analyzed using the same prewhitening technique applied to the whole light curve. This provided 95 measurements for all frequencies with amplitudes above ∼0.7 ppt. More details on how to obtain measurements from each light-curve segment can be found in Section 4.2 of Paper I. For KIC 11179657, the light curve was divided into pieces of a time step of ∼15 days and window width of 120 days. The difference in time step (10 days versus 15 days) does not change the general patterns of the AMs and FMs, but using the longer time step was necessary to reduce computation time. The time window size of 120 days was chosen to ensure sufficient resolution in frequency and to minimize contamination from side-band frequencies associated with the orbital signal with very large amplitude that suffers instrumental AMs. Since this star experienced two long interruptions of its monitoring, a few measurements were discarded when they were very close to large interval gaps. We finally obtained 45 measurements for the frequencies of amplitude above ∼0.3 ppt.

4.1. Representative Frequencies in KIC 2438324

In the following, we do not provide AMs and FMs for frequencies with an S/N lower than ∼12, due to their small number of exploitable measurements. We end up with 16 frequencies that contain more than 50 measurements each. In this section, we focus on AMs and FMs occurring in one triplet and two doublets as representative modes to illustrate the differences encountered with raw and corrected photometric data. For the other modes, the uncovered patterns will be discussed in Section 5.

Figure 4 shows the AMs and FMs detected in the triplet components near 216.9 μHz. The LSP of the full light curve shows rather simple peak structures suggesting that the three components experience only weak modulations. This is confirmed by the precise measurements of the modulation patterns presented in the middle and right panels. The retrograde ($m = -1$) component of the triplet displays frequency variations within ∼±2 nHz, i.e., comparable to the uncertainties. The other two components show larger FMs of ∼±10 nHz. Besides, during most of the observing run, the FMs appear to be somewhat antiphased between the $m = 0$ and $m = -1$ components. Comparing now the two data reduction levels (raw versus corrected flux), we find that the FMs of the three components are very similar. However, the two different types of AM patterns show some differences. For instance, the retrograde mode evolves in an apparent antiphase over most of
Concerning measured frequencies, the observations. Despite these differences, we find that these AMs show very similar modulation patterns in each type of flux, overall, which is further illustrated in Figure 8.

Figure 5 illustrates the AMs and FMs disclosed in the components forming two doubllets near 343 and 373 μHz, respectively. The structures in the LSP near these two doublets have, again, relatively simple forms, similar to that of the observing run, with almost the same values extracted from both types of the observing run, with a magnitude of ±10 nHz. Concerning measured AMs, they appear clearly different in the raw and corrected fluxes, respectively. However, despite these differences, all of them seem to follow similar patterns. We note that the amplitude uncertainty of the 344μHz component is relative large compared to the other three modes.

4.2. Representative Frequencies in KIC 11179657

Similar to KIC 2438324, AMs and FMs are provided for one triplet and two doubllets as representative frequencies observed in KIC 11179657. In this case, we measured AMs and FMs for modes with S/N values down to about 20, a bit higher than for KIC 2438324. This establishes 11 modes with measured modulations out of 34 detected frequencies. This higher S/N limit is chosen because the light curves run across two large interval gaps. Details on these 11 frequencies with precise AMs and FMs are presented in Section 5.

Figure 6 shows the AMs and FMs disclosed in the components belonging to the triplet near 284.6 μHz. The nearly equidistant triplet reveals broadened structures in the LSP for each component, due to aliasing introduced by the large gaps in the light curves. A noticeable feature is that both AMs and FMs show almost the same patterns whatever the flux calibration used. The $m = \pm 1$ components show regular frequency variations evolving in antiphase for some events. The quasiperiodic FMs can be roughly estimated to be on a timescale of 1 yr, with a magnitude of ±10 nHz. In contrast, the
The central ($m = 0$) component exhibits a less obvious FM, with a magnitude of ±4 nHz. The AMs happening in this triplet reveal relatively simple patterns, e.g., the central component following a roughly linear increase in amplitude during the whole observation run. We also note that the $m = -1$ and $m = 0$ components have very similar AMs during the last

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Figure 4. AMs and FMs of the triplet components near 217 μHz in KIC 2438324. Left panel: the LSP shows nearly equidistant structures whose frequency spacing values are given in the text. The dashed line is our typical detection limit of 5.6σ above local median noise. The corresponding FMs and AMs of each component, measured from two different correction levels of the photometric data, are presented in the middle and right panels, respectively. Note that the FMs are shifted to their averaged values.

Figure 5. Same as Figure 4, but for an example of two doublets near 343 and 373 μHz in KIC 2438324 (from the top to bottom panels), respectively.
observation segment. However, in the first segment, the similar AMs are found between the $m=+1$ and $m=0$ components.

Figure 7 shows the AMs and FMs obtained for the components forming two doublets near 196 and 308 $\mu$Hz, respectively. These doublets, similar to the triplet previously described, show no difference in the measurements of amplitude and AMs from the raw and corrected fluxes. The 196 $\mu$Hz doublet has regular frequency variations, which first evolve in phase (the first segment),...
then gradually switch to antiphase (the last segment) between the two components. The FMs measured in the 308 \(\mu\)Hz doublet show the opposite, by first evolving somewhat in antiphase, then gradually becoming in phase, if we roughly estimate the modulation patterns. All these four components are measured with relatively small FMs, typically within the range of \(\pm 10\) nHz. A closer look at the AMs of these components suggests possibly regular modulation patterns. The 196 \(\mu\)Hz doublet possibly exhibits anticorrelation between its two components. However, the 308 \(\mu\)Hz doublet does not show clear correlations in the variations occurring between the two components.

### 5. Comparison with an Orbital Signal

As described in the above section, the representative frequencies may have very similar modulation patterns in their amplitudes, which intuitively suggests instrumental effects from the satellite itself. To carefully check for such effects, reliable references should be used as calibrators for the amplitude. Binary signals, such as observed in KIC 2438324, can be good candidates for such references, due to their usually high amplitudes (enough precision) and several harmonics (systematic variation) showing up in the LSP. The orbital period is normally stable in a sdB+dM binary system over a time baseline of \(\sim 3\) yr (see, e.g., Lee et al. 2009). It is therefore natural to use this binary signal to gauge the variations observed in modes with AMs and FMs.

Figure 8 shows the comparison of AMs measured in the orbital frequency \(f_{\text{orb}}\) and its first harmonic \(2f_{\text{orb}}\) with AMs obtained for 13 independent pulsation frequencies of KIC 2438324. This clearly establishes that \(f_{\text{orb}}\) (and its harmonics) are subject to almost exactly the same modulation.
patterns, both in the raw and corrected flux data sets. Moreover, in general, the AMs of the independent pulsation frequencies are very similar to that of \( f_{\text{orb}} \), since all measurements are consistent with the AM of \( f_{\text{orb}} \) within the uncertainties. The AMs of \( f_{\text{orb}} \) are measured with peak-to-peak variations of \( \sim 10\% \) and \( \sim 5\% \) using the raw and corrected fluxes, respectively. These values also correspond to the magnitude of the variations observed in the 13 independent modes. We also note that the modulation patterns are different, when comparing raw and corrected fluxes, both in terms of amplitudes and phases of the effect. Another observation worth noting is that the corrected AMs of the independent frequencies follow less closely to that of \( f_{\text{orb}} \) than the raw AMs (as revealed by a careful comparison of the two panels of Figure 8).

Figure 9 compares AMs of \( f_{\text{orb}} \) and its first harmonic with the other three frequencies, \( f_02 \), \( f_{12} \), and \( f_{15} \), with available measurements in KIC 2438324. We find that the modulation patterns are significantly different in these cases, in particular for \( f_{12} \), whose AM exhibits a large \( \sim 50\% \) peak-to-peak variation. This frequency seems to experience three local maxima and minima (with two missing measurements due to the amplitude becoming too low). Moreover, we note that the two other frequencies, \( f_02 \) and \( f_{15} \), have AMs that are not following the general trend of \( f_{\text{orb}} \) in general, although there is some overlap between the modulations.

Figure 10 shows the comparison of AMs between the orbital frequency, \( f_{\text{orb}} \), and 11 independent frequencies of KIC 11179657. We only provide the raw AM patterns for these frequencies considering that this star is only slightly contaminated (also see Figures 6 and 7). In contrast with what we observe for KIC 2438324, we find that \( f_{03} \) presents a very similar modulation pattern compared to \( f_{\text{orb}} \) over the entire observation run. However, if we compare the modulation patterns in each observational segment, then almost all frequencies show similar AM patterns in at least one segment, such as \( f_{02} \) (in the first part) and \( f_{05} \) (in the middle and last parts), considering the uncertainties of the measurements. In addition, we find that the two independent frequencies may have similar AMs at some observational segment. For instance, AM patterns of \( f_09 \) and \( f_{12} \) are found to be somewhat similar to the AM of \( f_{02} \) in the middle observation.

Figure 9. Same as Figure 8, but for another three independent frequencies in KIC 2438324. Note that these AMs are significantly different from those in Figure 8.

Figure 10. AM comparison of 11 independent frequencies with the orbital frequency in the sdB star KIC 11179657. Each of these frequencies are normalized by their averaged amplitudes and shifted to the values where the legends indicate. Note that the errors for the orbital frequency \( f_{\text{orb}} \) are smaller than the symbol itself.
segment. Whether this is a coincidence or due to calibration issues needs further investigation.

6. Discussion

The results presented in this paper are the continuation of our project to analyze AMs and FMs in compact pulsators by exploiting the full data sets available from Kepler. A natural explanation for such phenomena is the weak nonlinear interaction between resonant modes, which can produce diverse modulation patterns (Buchler et al. 1995, 1997). However, our detailed studies of KIC 2438324 and KIC 11179657 led to the discovery of systematic AMs for several of the independent pulsation modes identified in these two stars. This behavior is different from previous modulations characterized in several compact pulsating stars (Zong et al. 2016a, 2016b, Paper I). It is plausible that this kind of AMs may be induced by instrumental effects onboard or by the data reduction pipeline. This finding adds further complexity to the goal of precisely measuring the intrinsic amplitudes of oscillations or other signals (e.g., transits of planets).

6.1. Comparison of Frequency Content with Previous Results

KIC 2438324 and KIC 11179657 show a much lower number of frequencies than that detected in KIC 3527751 (Paper I), i.e., a few tens compared to more than 200. In both stars, the frequency contents are similar with modes detected in the [100, 600] μHz range. In KIC 2438324, most of the frequencies already detected in Pablo et al. (2011) are recovered, except one low-amplitude (suspected) frequency at 405.45 μHz although we have analyzed a light curve more than six times longer than theirs. From our analysis, we only have four additional low-amplitude frequencies not seen in Pablo et al. (2011), which complete the triplet near 319 μHz and the independent frequency around 290 μHz that is now a doublet. Compared to Pablo et al. (2012), we have recovered all their frequencies from our 5.6σ level, except one at 262.8 μHz which is nonetheless obtained if we allow S/N ~ 5.1 to be a trustworthy detection. As the noise level decreases, we find eight additional weak frequencies, which complete the former doublet at 195 μHz to make it a triplet, and two former independent frequencies near 206 and 369 μHz, which are now doublets.

Both stars show a rich set of multiplets interpreted as rotational splittings and relatively few independent frequencies. These may prove to be good candidates for further seismic modeling (see, e.g., Charpinet et al. 2019). At present, there is no detailed seismic result obtained for sdB pulsators with the full Kepler photometry yet. Considering their similar frequency contents and orbital periods, it will be interesting to compare their internal structural and dynamical properties through the technique of asteroseismology (see, e.g., Charpinet et al. 2008).

6.2. Modulation Patterns

All representative pulsation modes illustrated in this paper are discovered with variations in amplitude and frequency. Considering that our evaluations of the uncertainties are proven to be robust (Section 2), we believe that the values determined for the amplitudes and frequencies are of high confidence. Besides, AMs and FMs from the two different flux calibrations (raw and corrected) can serve as a way to double check the determined values. Our results suggest that the FMs show consistent patterns, when measured alternatively from the raw and corrected fluxes, for both stars (see Figures 4–7). However, the correction of the flux introduces a significant difference in AM patterns when the fraction of contaminating light coming from nearby background or foreground objects is large. For instance, in KIC 2438324 we not only observe that the values of peak-to-peak variation are different, but also that the phase of the AMs shifts between raw and corrected data for the same frequency. The corrected AMs also follow that of f_{orb} less closely than the raw AMs. All these observations suggest that the correction of raw flux brings an extra signal to AMs, as the contamination factor increases. FMs remain unaffected by this effect because we extract the highest peaks even if the AMs induce symmetric (weak) sidebands around those peaks in the LSP.

We recall that nonlinear resonant interactions between three modes forming a triplet can produce various AMs and FMs (Buchler et al. 1995, 1997; Goupil et al. 1998) featuring a stable, periodic, or irregular modulation pattern. All the representative frequencies discussed previously satisfy this triplet resonance condition (doublets are incomplete triplets with an undetected component). Considering the uncertainties, the detected AMs and FMs in the representative frequencies might be associated with the intermediate regimes of the triplet resonance (see such examples in Zong et al. 2016a, 2016b). We clearly see quasiperiodic FMs occurring on timescales of several months to well over a year, as shown in Figure 11, which is comparable to the timescale that corresponds to the frequency mismatch of ~0.01 and ~0.02 μHz in the triplets 217 μHz (KIC 2438324) and 284 μHz (KIC 11179657), respectively. From this figure, we can clearly see various configurations, which indicates that their modulation patterns are indeed different (as roughly displayed in Figures 4–7). Nevertheless, as nonlinear resonant coupling theory predicts, similar timescales of FMs within the same multiplet are observed. Several resonant components, whose frequencies evolve in phase or antiphase during the observations, show similar modulation patterns to those found between the components of the quintuplet Q1 in KIC 3527751 (Paper I). We note that most of the FM timescales are not close to that of Keplor’s orbit and its resonances, which was recently reported on phase modulations in non-Blazhko RR Lyrae stars (Benkő et al. 2019). However, the measured AMs, particularly in KIC 2438324, are contaminated by a systematic modulation pattern (discussed later) and by the correction of the light fraction associated with the target. This pollution can seriously impair the recovery of intrinsic AM patterns of oscillation modes, which is the most straightforward way to characterize nonlinear effects on stellar pulsations. These systematic patterns will need to be removed prior to any detailed quantitative comparison with theoretical predictions. Our suggestion, in the meantime, is that stars without contaminating light should be given a higher priority for such studies. Finally, we point out that the modulations may become more complicated as the number of the detected frequency increases, comparing AMs and FMs detected in KIC 2438324, KIC 11179657, KIC 10139564, and KIC 3527751.

6.3. Instrumental Effects

At odds with what was found for the star KIC 3527751, we discovered systematic AM patterns in several independent modes and in the orbital signals of the two stars considered in
The orbital signal, expected to be constant in amplitude, was used as a reference to calibrate intrinsic modulation patterns. This is the first time that this systematic is characterized to better measure AMs and FMs of oscillation modes in pulsating compact stars. In previous studies, we had not encountered this kind of modulation pattern although we had processed more than 300 frequencies (see Zong et al. 2016a, 2016b, and Paper I). Systematic AMs are likely different from star to star, but they can be distinguished from the various AMs induced by nonlinear mode interactions. A key feature on these systematic modulations is that they have a timescale identical to Kepler’s orbital period, \( P \sim 372.5 \) days.

Figure 12 shows a phase diagram constructed on the 372.5 days period for the AM of \( f_{orb} \) in KIC 2438324. The folded curves clearly show that the right period was used for the raw and corrected AMs (with different zero-points in phase). To confirm the period, we used the phase dispersion minimization to evaluate the period, which gives a value of 372.7 and 371.5 days for the raw and the corrected AM, respectively. Similarly, a value of 373.1 days was obtained from an LSP of the raw AM curve. This modulation pattern of timescale identical to the orbital period of Kepler cannot be coincidental and is most likely induced by instrumental effects onboard.

This finding shows that one should be cautious when exploiting Kepler’s light curves to measure long-term AMs, particularly with periods near to the spacecraft orbit. For instance, a similar small-amplitude fluctuation was uncovered in M-giant stars based on Kepler’s photometry (Bányai et al. 2013), with several stars showing a systematic brightness fluctuation.

**Figure 11.** LSPs of frequency modulations concentrating on the representative frequencies in KIC 2438324 (a) and KIC 11179657 (b). The vertical line indicates the Kepler orbital frequency, \( f_K = 0.0311 \) \( \mu \)Hz, and its harmonics. The dashed horizontal line represents the typical 4\( \sigma \) level of local noise. The detected frequencies above that level are found with the values where the vertical segments stand. Note that the LSPs in (b) are resolved with sidelobes due to observation gaps.

**Figure 12.** Phase diagram of the amplitude variation of the orbital frequency in KIC 2438324: (Top panel) raw flux and (bottom panel) corrected flux.
variation on a Kepler-year timescale of unclear origin. Such variations can impair many scientific objectives, such as determining the transit depth for wide-orbit exoplanets, which can be used to measure oblateness of transiting planets (see, e.g., Biersteker & Schlichting 2017). As the spacecraft rolls by 90° every season, the fraction of contaminating light changes every quarter, which could possibly be related to the Kepler-year periodic AMs seen in KIC 2438324. However, the AMs are still showing a Kepler-year period (Figure 12) in the corrected fluxes, besides the fact that these have been rectified by the contamination factor. This indicates that the contamination factor may not be estimated precisely enough within and across quarters. In addition, quantum efficiency could also differ when stellar light is acquired from different pixels. Moreover, the point-spread function of the stars will also change as the telescope roll angle changes every quarter. Further investigation will need to be carried out with target pixel photometry, which may help us remove these systematic amplitude variations. A plausible reason might be that the neighboring pixels around that target still contain a very small fraction of the flux (see, e.g., Pápics et al. 2017), meaning that we do not count the right amount of flux for the target when using the standard Kepler reduction pipeline. We finally recall that this kind of systematic in AMs also needs to be given careful inspection for the ongoing mission the Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2014), especially for targets observed over several sectors.

6.4. General Prospects

Space missions, such as Kepler (Borucki et al. 2010) and TESS (Ricker et al. 2014), bring opportunities to measure frequencies in pulsating stars with accuracy of tens of microhertz down to a few nanohertz. We are presently capable of measuring frequency variations on very tiny scales, allowing us to study new physics, such as how to precisely calculate FM patterns induced by nonlinear effects on pulsation modes. In particular, progress in this direction would allow securely determining secular rates of frequency variations due to stellar evolution, or detecting with a higher confidence the presence of small orbiting objects, with residual modulations after the nonlinear FM is correctly removed. This has received particular interest in compact pulsators these past decades, with the possibility of constraining the rate of pulsation period change (Kepler et al. 2005; Vauclair et al. 2011; Hermes et al. 2013) and uncover periodic phase variations caused by the presence of planetary companions (Silvotti et al. 2007, 2018). In the context of the two close binary systems, KIC 2438324 and KIC 11179657, discussed in this paper, after correcting for the complex FMs due to nonlinear interactions, the residual modulation could also be used to measure the rate of spin-up induced by tides from the companion, if this acceleration is rapid enough, or set upper limits otherwise.

Even though flux can be measured with high precision from space instruments, consecutive long-term observations are not free of systematic uncertainties that can affect such measurements. We have shown that thorough inspections of the obtained amplitudes must be carried out if the goal is to accurately characterize amplitude variations over long timescales (to study, e.g., the physics of nonlinear mode interactions inside the stars in Goupil & Buchler 1994; Buchler et al. 1995). In the context of nonlinear physics, any instrumental bias to the intrinsic values of AMs lead to results far away from the real modulation patterns, which will likely affect the critical physical quantities from the calculations of amplitude equations.

7. Conclusion

In this paper, the second of a series devoted to a systematic survey of pulsations in evolved compact stars (hot B subdwarf and white dwarf stars) as observed from the Kepler spacecraft, we focus on the comparison of AMs and FMs of oscillation modes as measured from the two types of flux calibrations, raw and corrected, delivered by the standard data reduction pipeline. This is done for two pulsating sdB stars, KIC 2438324 and KIC 11179657, which are both primary components of a binary system. The first goal was to precisely measure the intrinsic AMs and FMs of oscillations in those compact stars, further paving the way to theoretical and quantitative calculations within the framework of nonlinear stellar oscillation theory, which could rely on such observations.

We first extended our estimation of uncertainties in measuring amplitudes and frequencies by testing 50,000 artificial signals, a large improvement over the more limited set of 1000 signals originally used by Zong et al. (2016b). These new simulations agree well with the previous results of Zong et al. (2016b), and demonstrate that amplitude and frequency errors are all measured accurately, independent of the S/N of the signal. However, uncertainties in the phase determination may be overestimated depending on the S/N (Figure 1). The latter, however, are not used for our objectives, yet. With these quantitative tests, we could precisely and confidently extract frequencies from the light curves of the two sdB stars using the two different flux calibrations, raw (SAP) and corrected (PDC-SAP). Since the photometry is longer than that in previously published literature about these objects, we were able to resolve 22 and 34 frequencies in KIC 2438324 (Table 1) and KIC 11179657 (Table 2), respectively. These bring an additional four and eight low-amplitude modes to the formerly available lists of detected pulsations in these stars. We mention that both stars show very similar and relative simple frequency contents, and could be of high interest for future comparative seismic analyses.

We then precisely measured AMs/FMs for frequencies with amplitudes down to ~0.7 and ~0.3 ppt in KIC 2438324 and KIC 11179657, respectively. By comparing modulation patterns measured from two different kinds of flux, we find that AM patterns change significantly for the modes observed in KIC 2438324, which suffer from significant light contamination by nearby stars (Figure 8), but not for the modes in KIC 11179657 whose contamination factor is almost zero (Figure 10). Differing from our previous studies, we identify clear systematic AMs of frequencies in KIC 2438324 through inspection of many independent modes and binary signals. Several methods have been used to determine the timescale of this regular AM, leading to a periodicity of about 372.5 days, which is identical to Kepler’s orbital period around the Sun (Figure 12). We argue that this systematic AM presents an additional difficulty, which to some extent can impair an accurate determination of the intrinsic AMs needed to constrain theoretical calculations of nonlinear couplings of pulsation modes, but also, more generally, for the study of long-period variables or transiting exoplanets. However, we stress that this effect can be corrected provided that some reference of normally constant amplitude exist, which is the case for
pulsating stars in binary systems. In summary, these findings suggest that stars without contamination could much more easily rectify the systematic AMs because the correction of light pollution could destroy the real AM patterns.

Although the AMs evolve the following different patterns in KIC 2438324, the FM patterns are found to be unaffected from these two independent measurements with different types of flux. All the representative frequencies exhibit frequency variations on a relatively narrow scale (Figures 4–7), typically of 10–20 nHz, which is of the same order as the frequency mismatch found in the triplets at 217 μHz in KIC 2438324 (Figure 4) and 284 μHz in KIC 11179657 (Figure 6). These FMs show somewhat (anti-)correlations between different components within the same multiplet, with peak-to-peak periodicities ranging from months to years. This suggests that nonlinear weak mode interaction happens between the involved components, as expected from the nonlinear mode coupling theory (e.g., Buchler et al. 1995). We recall that similar results have been obtained in our previous studies, such as in KIC 3527751 (Paper I). Observation of these FMs are of particular interest for future comparisons with nonlinear calculations using, e.g., the amplitude equation formalism (Goupil & Buchler 1994).

In forthcoming work, we will concentrate on the other pulsating sdB stars with appropriate data available for such kind of study in order to provide a more general statistical view on these modulations. We stress that the comparison described in this paper also applies to the photometry gathered from the TESS mission, since these data are processed through a very similar pipeline to the one used for Kepler (Jenkins et al. 2016). We expect that AMs/FMs of oscillation modes in compact pulsators from Kepler observations, as well as from the ongoing TESS monitoring (Charpinet et al. 2005), will ultimately provide a solid base for future theoretical investigations of nonlinear effects in stellar pulsations, which may lead to new insight into the physics of stars and their oscillations, through, e.g., the determination of linear growth rates of modes.

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Software: FELIX (Charpinet et al. 2010).

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References

Bányai, E., Kiss, L. L., Bedding, T. R., et al. 2013, MNRAS, 436, 1576
Benkő, J. M., Jurcsik, J., & Derekas, A. 2019, MNRAS, 485, 5897
Biersteker, J., & Schlichting, H. 2017, AJ, 154, 164
Borucki, W. J., Koch, D., Basri, G., et al. 2010, Sci, 327, 977
Buchler, J. R., Goupil, M.-J., & Hansen, C. J. 1997, A&A, 321, 159
Buchler, J. R., Goupil, M. J., & Serre, T. 1995, A&A, 296, 407
Charpinet, S., Grassard, P., Fontaine, G., et al. 2019, A&A, 632, 90
Charpinet, S., Fontaine, G., Grassard, P., Green, E. M., & Chayer, P. 2005a, A&A, 437, 575
Charpinet, S., Green, E. M., Baglin, A., et al. 2010, A&A, 516, L6
Charpinet, S., Van Grootel, V., Fontaine, G., et al. 2011, A&A, 530, A3
Charpinet, S., Van Grootel, V., Reese, D., et al. 2008, A&A, 489, 377
Deeming, T. J. 1975, Ap&SS, 36, 137
Goupil, M.-J., & Buchler, J. R. 1994, A&A, 291, 481
Goupil, M. J., Dziembowski, W. A., & Fontaine, G. 1998, BaltA, 7, 21
Hermes, J. J., Montgomery, M. H., Mullally, F., Winget, D. E., & Bischoff-Kim, A. 2013, ApJ, 766, 42
Jenkins, J. M., Caldwell, D. A., Chandrasekaran, H., et al. 2010, ApJL, 713, L87
Jenkins, J. M., Twicken, J. D., McCauliff, S., et al. 2016, Proc. SPIE, 9913, 99133E
Kepler, S. O., Costa, J. E. S., Castanheira, B. G., et al. 2005, ApJ, 634, 1311
Kovács, G., & Buchler, J. R. 1989, ApJ, 346, 898
Lee, J. W., Kim, S.-I., Kim, C.-H., et al. 2009, AJ, 137, 3181
Lomb, N. R. 1976, ApSS, 39, 447
Montgomery, M. H., & Odonoghue, D. 1999, DSSN, 13, 28
Murphy, S. J. 2012, MNRAS, 422, 665
Östensen, R. H., Silvotti, R., Charpinet, S., et al. 2010, MNRAS, 409, 1470
Pablo, H., Kawaler, S. D., & Green, E. M. 2011, ApJL, 740, L47
Pablo, H., Kawaler, S. D., Reed, M. D., et al. 2012, MNRAS, 422, 1343
Pápics, P. I., Tkachenko, A., Van Reeth, T., et al. 2017, A&A, 598, A74
Reed, M. D., Baran, A., Östensen, R. H., Telting, J., & O’Toole, S. J. 2012, MNRAS, 427, 1245
Ricker, G. R., Winn, J. N., Vanderspek, R., et al. 2014, Proc. SPIE, 9143, 914320
Scargle, J. D. 1982, ApJ, 263, 835
Silvotti, R., Schuh, S., Janulis, R., et al. 2007, Natur, 449, 189
Silvotti, R., Schuh, S., Kim, S.-I., et al. 2018, A&A, 611, A85
Vauclair, G., Fu, J.-N., Solheim, J.-E., et al. 2011, A&A, 528, A5
Zong, W., Charpinet, S., Fu, J.-N., et al. 2015, ApJ, 853, 98
Zong, W., Charpinet, S., & Vauclair, G. 2016b, A&A, 594, A46
Zong, W., Charpinet, S., Vauclair, G., Giammichele, N., & Van Grootel, V. 2016a, A&A, 585, A22