Magnetic moments of spin $\frac{1}{2}^+$ and spin $\frac{3}{2}^+$ charmed baryons

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Abstract.
The magnetic moments of spin $\frac{1}{2}^+$ and spin $\frac{3}{2}^+$ charmed baryons have been calculated in chiral constituent quark model ($\chi$CQM). The effects of configuration mixing and quark masses have also been investigated. The results are not only in good agreement with existing experimental data but also show improvement over other phenomenological models.

Keywords: Magnetic moments, chiral constituent quark model, charmed baryons
PACS: 13.40.Em, 12.39.Fe, 14.20.Lq

INTRODUCTION

Heavy flavor baryons play an important role to understand the dynamics of light quarks in the bound state as well as to understand QCD at the hadronic scale [1]. The phenomenological implications of the heavy quark component in the nucleon have been investigated to estimate the possible size of intrinsic charm (IC) content of the nucleon [2] as well as to calculate the static properties like masses, magnetic moment etc. [3] which give valuable information regarding the internal structure of baryons.

The magnetic moments of spin $\frac{1}{2}^+$, spin $\frac{3}{2}^+$ charmed baryons have been considered in different approaches in literature. Calculations have been done in the non-relativistic quark model [4, 5], Skyrme model [6], bound state approach [7], relativistic three-quark model [8] etc. More recently, magnetic moments have been studied by considering the effective mass of the quark bound inside the baryon [9]. Calculations for the charmed baryon magnetic moments have also been done in QCD sum rule method (QCDSR) [10], QCD Spectral sum rule method (QSSR) [11] and light cone QCD sum rule method (LCQSR) [12, 13, 14]. However, there is little consensus among the different model predictions of the magnetic moments of charmed baryons.

The intrinsic heavy quarks are created from the quantum fluctuations associated with the bound state hadron dynamics and the process is completely determined by nonperturbative mechanisms [15]. It has been shown that one of the important model which finds application in the nonperturbative regime is the chiral constituent quark model ($\chi$CQM) [16, 17, 18]. The $\chi$CQM with spin-spin generated configuration mixing is able to give the satisfactory explanation for the spin and flavor distribution functions [19, 20], hyperon $\beta$ decay parameters [18], strangeness content of the nucleon [21], weak vector and axial-vector form factors [22], octet and decuplet baryon magnetic moments [23, 24, 25] etc. The successes of $\chi$CQM strongly suggest that constituent quarks and the weakly interacting Goldstone bosons (GBs) provide the appropriate degrees of freedom in the nonperturbative regime of QCD. Thus, the quantum fluctuations generated by broken chiral symmetry in $\chi$CQM should be able to provide a viable estimate of the heavier quark flavor, in particular the $c\bar{c}$ [15, 26].

The purpose of the present paper is to estimate the magnetic moments of spin $\frac{1}{2}^+$, spin $\frac{3}{2}^+$ charmed baryons in the SU(4) framework of $\chi$CQM. The generalized Cheng-Li mechanism [23] has been incorporated to calculate explicitly the contribution coming from the valence spin polarization, “quark sea” polarization and its orbital angular momentum. Further, it would also be interesting to examine the effects of the configuration mixing, symmetry breaking parameters, confinement effects, quark masses etc. on the magnetic moments.

SPIN STRUCTURE IN CHIRAL CONSTITUENT QUARK MODEL

In this section, we briefly review the essentials of the $\chi$CQM to calculate the spin structure of the baryons [23, 24, 25]. The basic process in the $\chi$CQM [16] is the internal emission of a Goldstone Boson by a constituent quark which further splits into a $q\bar{q}$ pair as $q\rightarrow GB^0 + q^0 \rightarrow (q\bar{q})^0 + q^0$; where $q\bar{q} + q^0$ constitutes the “quark sea” [18, 19, 20, 24]. The effective Lagrangian describing interaction between quarks and GBs is $\mathcal{L} = g_{15}q(\Phi)q$, where $g_{15}$ is the coupling

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The magnetic moment of a given baryon receives contributions from the valence quarks, "quark sea" and orbital fluctuations. The magnetic moment can be written as

\[
\mu(B) = \mu(B)_{\text{val}} + \mu(B)_{\text{sea}} + \mu(B)_{\text{orbit}}.
\]

where \(\mu(B)_{\text{val}}\) and \(\mu(B)_{\text{sea}}\) represent contributions from the valence quarks and the "quark sea" to the total magnetic moment due to spin polarizations. The term \(\mu(B)_{\text{orbit}}\) corresponds to the orbital angular momentum contribution of the "quark sea". In terms of quarks magnetic moments and spin polarizations, the valence, sea and orbital contributions can be written as

\[
\mu(B)_{\text{val}} = \sum_{q=u,d,s} \Delta q_{\text{val}} \mu_q; \quad \mu(B)_{\text{sea}} = \sum_{q=u,d,s} \Delta q_{\text{sea}} \mu_q; \quad \mu(B)_{\text{orbit}} = \sum_{q=u,d,s} \Delta q_{\text{val}} \mu(q^+!q^0).
\]

where \(\mu_q = \frac{e_q}{2M_q}\) (\(q = u,d,s,p,c\)) is the magnetic moment of quark, \(\mu(q^+!q^0)\) is the orbital moment for any chiral fluctuation. \(e_q\) and \(M_q\) are the electric charge and mass respectively for the quark \(q\).

The valence and quark sea spin polarizations \((\Delta q_{\text{val}})\) and \((\Delta q_{\text{sea}})\) can be calculated for the baryons using the spin structure discussed in the previous section. The orbital angular momentum contribution of each chiral fluctuation is given as [18, 25]

\[
\mu(q^+!q^0) = \frac{e_q}{2M_q} \mu(q^+!q^0) = \frac{e_q}{2M_q} \mu(q^+!q^0)
\]

where \(\mu(B) = \frac{M_B}{M_B + M_G}\) and \(\mu(B) = \frac{M_B}{M_B + M_G}\). The quantities \((l_q, l_{GB})\) and \((M_q, M_G)\) are the orbital angular momenta and masses of quark and GBs, respectively. The orbital moment of each process is then multiplied by the probability for such a process to take place to yield the magnetic moment due to all the transitions starting with a given valence quark

\[
[\mu(u^+!u)] = a \frac{1}{2} + \frac{\beta^2}{6} + \frac{\gamma^2}{8} + \frac{\gamma^2}{16} \mu(u^+!u) + \mu(u^+!d) + \alpha^2 \mu(u^+!s) + \gamma^2 \mu(u^+!c).
\]
and a small discrepancy in the case of and the orbital contributions separately and we find that our predictions are in agreement with their results. There is sum rule [10] and Light Cone QCD sum rule [14]. In this case also, we have presented the results for the valence, sea mass shell values of quarks and GBs [29, 30], we have calculated the magnetic moments of spin using the following set of magnetic moments. Our predicted value for baryons where the experimental data is available. In the case of charmed baryons, our results are consistent with the 

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RESULTS AND DISCUSSION

Using the following set of χCQM parameters (a, α, β, γ, ζ), quark masses (m_u, m_d, m_s, m_c) and GB masses (M_π, M_K, M_{K^*}, M_D, M_{D^*}, M_{K^*}),

\[
[\mu (d^+ u)] = \alpha \mu (d^+ u) + \frac{1}{2} + \frac{\beta^2}{6} + \frac{\zeta^2}{48} + \frac{\gamma^2}{16} \mu (d^+ u) + \alpha^2 \mu (d^+ u) + \alpha \gamma \mu (d^+ c) + \gamma^2 \mu (d^+ c); \tag{8}
\]

\[
[\mu (s^+ u)] = \alpha \mu (s^+ u) + \alpha^2 \mu (s^+ u) + \frac{1}{3} \beta^2 + \frac{\zeta^2}{48} + \frac{\gamma^2}{16} \mu (s^+ u) + \alpha \gamma \mu (s^+ c) + \gamma^2 \mu (s^+ c); \tag{9}
\]

and

\[
[\mu (c^+ c)] = \alpha \gamma \mu (c^+ c) + \gamma^2 \mu (c^+ c) + \frac{3}{16} \zeta^2 + \frac{9}{16} \gamma^2 \mu (c^+ c); \tag{10}
\]

The above equations can easily be generalized by including the coupling breaking and mass breaking terms and can be expressed in terms of the \( \chi \)CQM parameters \( a, \alpha', \beta, \gamma, \zeta \), quark masses \( (m_u, m_d, m_s, m_c) \) and GB masses \( (M_\pi, M_K, M_{K^*}, M_D, M_{D^*}, M_{K^*}) \).

In Table 2, we have compared our results for the spin \( \frac{1}{2}^+ \) baryons with other model calculations as well as with the available experimental data. Presently, only three experimental results are available for the decuplet baryons magnetic moments. Our predicted value for \( \mu_{\Lambda^+} \) is well within the experimental range [3]. Similarly, in the case of \( \mu_{\Sigma^+} \) and \( \Omega^+ \), our predicted values agree with the experimental values [31, 32]. In case of charmed baryons, there is no experimental information available, therefore, we have compared our results with the predictions of the QCD sum rule [10] and Light Cone QCD sum rule [14]. In this case also, we have presented the results for the valence, sea and the orbital contributions separately and we find that our predictions are in agreement with their results. There is a small discrepancy in the case of \( \Sigma^0 \) magnetic moment, which is due to the significant sea contribution. The “quark sea” and orbital contributions are quite large in magnitude for all the charmed baryons except in the case of \( \Omega^- \), \( \Omega^{*-} \), \( \Omega^{*+} \) and \( \Omega_{ccc}^{-} \). The measurements of the magnetic moments of charmed baryons represent an experimental challenge and several groups BTeV, SELEX Collaboration are contemplating the possibility of performing it in the near future which would test the success of present scheme.

SUMMARY AND CONCLUSION

We have calculated the magnetic moments of spin \( \frac{1}{2}^+ \) and \( \frac{3}{2}^+ \) baryons in the framework of SU(4) \( \chi \)CQM. Without taking any of the magnetic moment as input, a considerable good fit is achieved in the case of the octet and decuplet baryons where the experimental data is available. In the case of charmed baryons, our results are consistent with the
other approaches existing in the literature. The success of $\chi$CQM with the Cheng-Li mechanism and configuration mixing in achieving a fit to the magnetic moments suggest that constituent quarks and weakly interacting Goldstone Bosons provide the appropriate degree of freedom in the nonperturbative regime of QCD.

ACKNOWLEDGMENTS

H.D. would like to thank Department of Science and Technology, Government of India for financial support.

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### TABLE 1. Magnetic moment of spin \( \frac{1}{2} \) charmed baryons with configuration mixing (in units of \( \mu_N \)).

| Baryon | Data [3] | NRQM | QCDSR [10] | LCQSR [11] | Valence | Sea | Orbital | Total |
|--------|----------|------|------------|------------|---------|-----|---------|-------|
| \( p \) | 2.79 0.00 | 3    | 2.82 0.26 | 2.7 0.5    | 2.90    | 0.58| 0.47    | 2.80  |
| \( n \) | 1.91 0.00 | 2    | 1.97 0.15 | 1.8 0.35   | 1.85    | 0.18| 0.44    | 2.11  |
| \( \Sigma^+ \) | 2.458 0.010 | 2.88 | 2.31 0.25 | 2.2 0.4    | 2.51    | 0.51| 0.40    | 2.39  |
| \( \Sigma^0 \) | ... | 0.88 | 0.69 0.07 | 0.5 0.10   | 0.74    | 0.22| 0.02    | 0.54  |
| \( \Xi^- \) | 1.160 0.025 | 1.12 | 1.16 0.10 | 0.8 0.2    | 1.02    | 0.06| 0.36    | 1.32  |
| \( \Xi^0 \) | 1.250 0.014 | 1.53 | 1.15 0.05 | 1.3 0.3    | 1.29    | 0.14| 0.09    | 1.24  |
| \( \Omega^- \) | 0.6507 0.0025 | 0.53 | 0.64 0.06 | 0.7 0.2    | 0.59    | 0.03| 0.06    | 0.50  |
| CSGR  | 0.49 0.05 | 0.95 | 0.46      | 0.613 0.004 | 0.65 | 0.56| 0.15    | 0.7 0.2 | 0.59 | 0.06 | 0.01 | 0.66 |
| \( \Lambda^0 \) | ... | 2.54 | 2.1 0.3    | ...        | 2.32    | 0.52| 0.40    | 2.20  |
| \( \Sigma_c^+ \) | ... | 0.54 | 0.6 0.1    | ...        | 0.51    | 0.23| 0.02    | 0.30  |
| \( \Sigma_c^0 \) | ... | 1.46 | 1.6 0.2    | ...        | 1.30    | 0.06| 0.36    | 1.60  |
| \( \Xi_c^- \) | ... | 0.77 | ...        | 0.77       | 0.21    | 0.19| 0.76    | ...   |
| \( \Xi_c^0 \) | ... | 1.23 | ...        | ...        | 1.16    | 0.03| 0.19    | 1.32  |
| \( \Omega_c^- \) | ... | 0.99 | ...        | ...        | 0.93    | 0.04| 0.01    | 0.90  |
| \( \Lambda_c^0 \) | ... | 0.39 | 0.15 0.05  | 0.40 0.05  | 0.409   | 0.019| 0.002   | 0.392 |
| \( \Xi_{cc}^- \) | ... | 0.39 | ...        | 0.50 0.05  | 0.41    | 0.02| 0.01    | 0.40  |
| \( \Xi_{cc}^0 \) | ... | 0.39 | ...        | 0.35 0.05  | 0.29    | 0.0003| 0.01   | 0.28  |

### TABLE 2. Magnetic moments of the spin \( \frac{3}{2} \) charmed baryons (in units of \( \mu_N \)).

| Baryon | Data [3] | NRQM | QCDSR [10] | LCQSR [11] | Valence | Sea | Orbital | Total |
|--------|----------|------|------------|------------|---------|-----|---------|-------|
| \( \mu_{\Lambda^+} \) | 3.7 7.5 15 3 [31] | 6    | 4.13 1.30 | 4.4 0.8    | 4.53    | 0.97| 0.95    | 4.51  |
| \( \mu_{\Sigma^+} \) | 2.74 1.0 15 3 [31] | 3    | 2.07 0.65 | 2.2 0.4    | 2.27    | 0.61| 0.34    | 2.00  |
| \( \mu_{\Sigma^0} \) | ... | 0.0 0.0 | ...        | 0.0 0.0    | 0.0     | 0.25| 0.26    | 0.51  |
| \( \mu_{\Xi^-} \) | ... | 2.07 0.65 | 2.2 0.4    | 2.27    | 0.12| 0.87    | 3.02  |
| \( \mu_{\Xi^0} \) | ... | 3.35 | 2.13 0.82 | 2.7 0.6    | 2.74    | 0.67| 0.62    | 2.69  |
| \( \mu_{\Omega^-} \) | ... | 0.35 | 0.32 0.45 | 0.25 0.05  | 0.29    | 0.29| 0.02    | 0.02  |
| \( \mu_{\Xi_c^-} \) | ... | 2.65 | 1.66 0.73 | 2.28 0.5   | 2.16    | 0.11| 0.59    | 2.64  |
| \( \mu_{\Xi_c^0} \) | ... | 0.71 | 0.49 0.29 | 0.40 0.08  | 0.51    | 0.26| 0.29    | 0.54  |
| \( \mu_{\Omega_c^-} \) | ... | 2.29 | 1.51 0.52 | 2.0 0.4    | 1.64    | 0.08| 0.31    | 1.87  |
| \( \mu_{\Xi_{cc}^-} \) | ... | 2.02 | 0.06      | 1.94 1.49  | 1.65    | 0.35| 1.76    | 0.08  | 0.03 | 1.71  |
| \( \mu_{\Xi_{cc}^0} \) | ... | 1.94 | 1.49 0.45 | 1.65 0.35  | 1.76    | 0.08| 0.03    | 1.71  |

| Baryon | Data [3] | NRQM | QCDSR [10] | LCQSR [11] | Valence | Sea | Orbital | Total |
|--------|----------|------|------------|------------|---------|-----|---------|-------|
| \( \mu_{\Sigma_c^+} \) | ... | 4.39 | 4.81 1.22 | 4.09 0.80 | 0.63    | 3.92|
| \( \mu_{\Sigma_c^0} \) | ... | 1.39 | 2.00 0.46 | 1.30 0.36 | 0.03    | 0.97|
| \( \mu_{\Xi_c^-} \) | ... | 1.61 | 0.81 0.20 | 1.50 0.09 | 0.58    | 1.99|
| \( \mu_{\Xi_c^0} \) | ... | 1.74 | 1.68 0.42 | 1.67 0.39 | 0.31    | 1.59|
| \( \mu_{\Xi_{cc}^-} \) | ... | 1.26 | 0.68 0.18 | 1.21 0.08 | 0.30    | 1.43|
| \( \mu_{\Xi_{cc}^0} \) | ... | 0.91 | 0.62 0.89 | 0.85 0.05 | 0.02    | 0.86|
| \( \mu_{\Omega_c^-} \) | ... | 2.78 | ...       | ...       | 2.78    | 0.44| 0.32    | 2.66  |
| \( \mu_{\Omega_{cc}^-} \) | ... | 0.22 | ...       | ...       | 0.22    | 0.04| 0.29    | 0.47  |
| \( \mu_{\Omega_{cc}^0} \) | ... | 0.13 | ...       | ...       | 0.13    | 0.02| 0.01    | 0.14  |

| Baryon | Data [3] | NRQM | QCDSR [10] | LCQSR [11] | Valence | Sea | Orbital | Total |
|--------|----------|------|------------|------------|---------|-----|---------|-------|
| \( \mu_{\Xi_{cc}^+} \) | ... | 1.17 | ...       | ...       | 0.165   | 0.011| 0.002   | 0.155 |