HYBRID $SU(2) \times U(1)$ MODELS, ELECTRIC CHARGE NONCONSERVATION AND THE PHOTON MASS

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Abstract

Hybrid $SU(2) \times U(1)$ models are the models in which $SU(2) \times U(1)$ symmetry is broken down not only spontaneously (as in the Standard Model), but also explicitely by adding a hard mass term for the $U(1)$ field in the lagrangian. We study the issue of electric charge nonconservation and dequantization in these models. For this purpose we construct and analyze a series of hybrid models with different scalar contents. We show that some of these models posess an interesting property: the photon can remain massless (at least, at the tree level) even though the electric charge is not conserved.
I. INTRODUCTION

The electric charge conservation and the masslessness of photon are the two fundamental ingredients of the Standard Model. They have been tested experimentally many times with high precision and at present we have no evidence whatsoever that would question their validity.

Yet, over the past two decades there has been considerable interest in constructing and analysing models in which one or both of the above postulates do not hold true [1–9]. The purpose of these works was to understand better the reasons why the electric charge is conserved and the photon is massless. Also, without these works it would be very hard to think of new experimental ways of testing these laws.

One of the important discoveries made in those works was the closer relation between the two ideas: electric charge (non)conservation and electric charge (de)quantization [2,4,10].

All previous works on the subject have shared one common property: it was impossible to violate the electric charge conservation without giving the photon a (tiny) mass at the same time. This is not surprising at all if we consider Maxwell equations of classical electrodynamics. Since observational limit on the photon mass is very tight (it has to be less than $10^{-24}$ GeV or even $10^{-36}$ GeV [11]), model-building is severely restricted.

In this paper, our question is: can we make a model without that undesirable property? In other words, can one modify the Standard Model in such a way that the electric charge is not conserved but the photon is exactly massless? Based on all our previous experience with that kind of models, the answer would seem almost definitely, no.

However, in the present work we show how to construct a model in which the electric charge is not conserved but the photon \textit{is massless at the tree level}. \footnote{The important further questions if this property survives at higher orders of the perturbation theory and if the model is renormalizable, are left open.}

Briefly speaking, the main idea is as follows. We point out that the photon mass in the
most general case gets two contributions: the first from the mass term of the U(1) field (usually denoted by B) and the second from the spontaneous breaking of the electromagnetic symmetry. By the appropriate choice of parameters we can ensure that these two contributions cancel each other so that the photon remains massless (at least, at the tree level) despite the fact that the electric charge is not conserved.

The plan of the work is this: in Section 2 we reproduce some familiar formulas of the Standard Model in order to establish our notation and to make easier the comparison with the further material. Section 3 deals with the case when the Standard Model is extended by adding a hard mass term for the U(1) gauge field. In Section 4 we add to the model of Section 3 an electrically charged scalar singlet with non-zero vacuum expectation thus violating spontaneously the conservation of electric charge. In Section 5 we substitute the scalar singlet by the scalar doublet (with electric charge violating vacuum average) leaving the rest of the model the same as in Section 4. Our discussion and conclusions are contained in Section 6.

II. STANDARD MODEL

We shall restrict ourselves to only one generation of quarks and leptons (the addition of other generations will only clutter the notation without giving any new insights).

The part of the total Lagrangian which is relevant for our purposes is this

\[ \mathcal{L}_0 = \mathcal{L}_l + \mathcal{L}_q + \mathcal{L}_s \]

\[ \mathcal{L}_l = \bar{L}(i\partial + g\frac{\tau^a}{2} A^a - g'\frac{B}{2})L + \bar{e}_R(i\partial - g'B)e_R; \]

\[ \mathcal{L}_q = \bar{Q}(i\partial + g\frac{\tau^a}{2} A^a + g'\frac{3}{2}B)Q + \bar{u}_R(i\partial + \frac{2g'}{3}B)u_R + \bar{d}_R(i\partial - g'\frac{B}{3})d_R; \]

\[ \mathcal{L}_s = |(\partial_{\mu} - ig\frac{\tau^a}{2} A^a - i\frac{g'}{3} B_{\mu})\phi|^2 - h(\phi^0\phi - \frac{1}{2}v^2)^2. \]

where \( A^a = A^a_\mu \gamma^\mu \) and \( B^a = B_\mu \gamma^\mu \) are SU(2) and U(1) gauge fields,

\[ L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \quad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \]
After spontaneous symmetry breaking, 
\[
\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix},
\]
we are interested in the part of Lagrangian \( L_0 \) which is quadratic in the gauge fields \( A_i \) and \( B \):
\[
L_{0}^{\text{quad}} = \frac{1}{8} v^2 [g^2 (A^2_1 + A^2_2 + A^2_3) + g'^2 B^2_\mu - 2gg' A_3 B^\mu].
\]
(7)
This gives us the following mass matrix for the pair of fields \( A_3 \) and \( B \):
\[
M^0 = \begin{pmatrix} M^0_{33} & M^0_{34} \\ M^0_{43} & M^0_{44} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} g^2 v^2 & -\frac{1}{4} gg' v^2 \\ -\frac{1}{4} gg' v^2 & \frac{1}{4} g'^2 v^2 \end{pmatrix}.
\]
(8)
Diagonalizing this mass matrix we arrive at a pair of physical fields, \( A \) and \( Z \) which are identified with the photon and \( Z \)-boson:
\[
A_\mu^3 = Z_\mu \cos \theta + A_\mu \sin \theta
\]
(9)
\[
B_\mu = A_\mu \cos \theta - Z_\mu \sin \theta.
\]
(10)
Here, the Weinberg angle is given by the standard expression:
\[
\sin^2 \theta = \frac{g'^2}{g^2 + g'^2}.
\]
(11)
Now, changing the fields \( A_3, B \) into \( A, Z \) in our initial Lagrangian we finally obtain the electromagnetic interactions of quarks and leptons:
\[
L_{\text{em}} = L_{\text{em}}^l + L_{\text{em}}^q
\]
(12)
\[
L_{\text{em}}^l = A_\mu [\frac{1}{2} (g \sin \theta - g' \cos \theta) \bar{\nu}_L \gamma^\mu \nu_L - \frac{1}{2} (g \sin \theta + g' \cos \theta) \bar{e}_L \gamma^\mu e_L
\]
\[-g' \cos \theta \bar{e}_R \gamma^\mu e_R]
\]
(13)
\[
L_{\text{em}}^q = A_\mu [\frac{1}{2} g \sin \theta (\bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L) + g' \cos \theta (\frac{1}{6} \bar{u}_L \gamma^\mu u_L + \frac{1}{6} \bar{d}_L \gamma^\mu d_L + \frac{2}{3} \bar{u}_R \gamma^\mu u_R - \frac{1}{3} \bar{d}_R \gamma^\mu d_R)]
\]
(14)
From this formula we can read off the values of the electric charges of quarks and leptons:
\[ Q_\nu = \frac{1}{4} (g \sin \theta - g' \cos \theta) \] (15)
\[ Q_e = -\frac{1}{4} g \sin \theta - \frac{3}{4} g' \cos \theta \] (16)
\[ Q_e^5 = -\frac{1}{4} (g \sin \theta - g' \cos \theta) \] (17)
\[ Q_u = \frac{1}{4} g \sin \theta + \frac{5}{12} g' \cos \theta \] (18)
\[ Q_u^5 = \frac{1}{4} g \sin \theta - \frac{1}{4} g' \cos \theta \] (19)
\[ Q_d = -\frac{1}{4} g \sin \theta - \frac{1}{12} g' \cos \theta \] (20)
\[ Q_d^5 = -\frac{1}{4} g \sin \theta + \frac{1}{4} g' \cos \theta \] (21)

Consequently, the electric charges of neutron and proton are:
\[ Q_n = Q_u + 2Q_d = -\frac{1}{4} (g \sin \theta - g' \cos \theta) \] (22)
\[ Q_p = 2Q_u + Q_d = \frac{1}{4} g \sin \theta + \frac{3}{4} g' \cos \theta. \] (23)

At this point one may wonder why Eq. (13) to (23) do not look very familiar. The reason is this: in writing down Eq. (13) to (23) we have not taken into account the formula
\[ g \sin \theta = g' \cos \theta, \] (24)

which holds true in the Standard Model. We did not use this equation when deriving Eq. (13) to (23) because there exists a crucial distinction between them: Eq. (24) will not apply in the extended models to be considered below (Sections 3, 4, 5) whereas Eq. (13) to (23) will still be true in all those models (provided one puts in the modified value for \( \sin \theta \), see below.)

Of course, if one wants to stay within the Standard Model, than one has to put \( g \sin \theta = g' \cos \theta \) in Eq. (13) to (23) to recover the standard form for the electromagnetic Lagrangian:
\[ \mathcal{L}_{em} = A_\mu \sum_f Q_f \bar{f} \gamma^\mu f \] (25)

with the correct values of fermion charges \( Q_f \):
\[ Q_\nu = 0, \quad Q_e = -e, \quad Q_u = \frac{2}{3} e, \quad Q_d = -\frac{1}{3} e. \] (26)

Needless to say, all axial charges \( Q_5^i \) vanish identically in the Standard Model.
III. MINIMAL HYBRID $SU(2) \times U(1)$ MODEL

Let us consider a model which differs from the Standard Model only in one point: its lagrangian contains a mass term for the $U(1)$ gauge field $B$ (before spontaneous symmetry breaking):

\[ \mathcal{L}' = \mathcal{L}_0 + \frac{1}{2} m^2 B^2. \]  

(27)

After symmetry breaking, we obtain from this lagrangian the following mass matrix for the gauge fields:

\[ M' = M^0 + \Delta M' = \begin{pmatrix} M^0_{33} & M^0_{34} \\ M^0_{43} & M^0_{44} + m^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} g^2 v^2 & -\frac{1}{4} g g' v^2 \\ -\frac{1}{4} g g' v^2 & \frac{1}{4} g'^2 v^2 + m^2 \end{pmatrix}. \]  

(28)

Following the same path as in the standard case (Section 2), we diagonalize the mass matrix to obtain the physical fields. Although these fields are different from the standard fields \((9), (10)\), we keep the same notation for them: $A$ and $Z$ since we have to identify them with the observable particles: photon and Z-boson:

\[ A^3_\mu = Z_\mu \cos \theta' + A_\mu \sin \theta' \]  

(29)

\[ B_\mu = A_\mu \cos \theta' - Z_\mu \sin \theta'. \]  

(30)

The gauge boson masses acquire small corrections (assuming that $m$ is small); in particular, the photon gets non-zero mass:

\[ M_Z = \frac{1}{2} \sqrt{g^2 + g'^2 v} + O\left(\frac{m^2}{v}\right) \]  

(31)

\[ M_\gamma = g m + O\left(\frac{m^2}{v}\right). \]  

(32)

The Weinberg angle gets a small correction, too (to avoid misunderstanding, we note that we are working on the tree level throughout the paper so the word "correction" has nothing to do with perturbation theory):

\[ \sin^2 \theta' = \sin^2 \theta + \frac{m^2}{M_Z^2} \left(1 - \frac{e^2}{\sin^2 \theta} + e^2 - \sin^4 \theta\right) \approx 0.64 \frac{m^2}{M_Z^2}. \]  

(33)
This fact leads to a drastic consequence: the electric charge non-conservation. To show that, let us find the electromagnetic part of the lagrangian.

If we compare the ways of reasoning in Sections 2 and 3 we shall see that the same formulas, Eq. (13) and (14) applies also in the present case; the only change that should be made is to change \( \sin \theta \) to \( \sin \theta' \), the rest of the formulas being unchanged:

\[
\mathcal{L}^{'em} = \mathcal{L}^{'em}_l + \mathcal{L}^{'em}_q
\]

\[
\mathcal{L}^{'em}_l = A_\mu \left[ \frac{1}{2} (g \sin \theta' - g' \cos \theta') \bar{u}_L \gamma^\mu u_L - \frac{1}{2} (g \sin \theta' + g' \cos \theta') \bar{e}_L \gamma^\mu e_L \\
- g' \cos \theta' \bar{e}_R \gamma^\mu e_R \right]
\]

\[
\mathcal{L}^{'em}_q = A_\mu \left[ \frac{1}{2} g \sin \theta' (\bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L) + g' \cos \theta' \left( \frac{1}{6} \bar{u}_L \gamma^\mu u_L + \frac{1}{6} \bar{d}_L \gamma^\mu d_L + \frac{2}{3} \bar{u}_R \gamma^\mu u_R - \frac{1}{3} \bar{d}_R \gamma^\mu d_R \right) \right]
\]

Based on this formula, we can arrive at an important conclusion: as soon as the equality \( g \sin \theta = g' \cos \theta \) is broken, the electromagnetic current conservation is violated immediately. To avoid confusion, one essential point needs to be emphasized here. We have defined the electromagnetic current (and thereby the electric charge) as the current interacting with (i.e. standing in front of) the electromagnetic field \( A_\mu \). Naturally, one can ask about the standard fermion electromagnetic current of the form

\[
\mathcal{j}_\mu = e (\bar{\nu} \gamma_\mu \nu + \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d).
\]

Although this current is still conserved in the present model, it unfortunately becomes devoid of physical meaning, because all physical processes and experiments are based on the interaction between the charges and electromagnetic fields; therefore in the framework of the present model we have to attach physical meaning and reserve the name “electromagnetic current” for the current of Eq.(35) and (36), rather than that of Eq. (37).

To summarise, this theory features three fundamental deviations from the Standard Model: massiveness of photon, the electric charge dequantization, and the electric charge non-conservation.
Now, let us discuss the experimental limits on the parameter $m$ which result from the above three features.

In our case, the experimental upper bound on the photon mass gives, by far, the strongest constraint on the value of $m$. It has been established that the photon mass should be less than $10^{-24}$ GeV or even $10^{-36}$ \cite{11}. Therefore, from Eq. (32) we find that the parameter $m$ cannot exceed $2 \times 10^{-24}$ GeV or $2 \times 10^{-36}$ GeV. With such small values of the parameter $m$, the charge dequantization and charge non-conservation effects are expected to be too small to be observed. For instance, the best experimental limits on electric charge dequantization are given by the following figures: neutron charge: $Q_n < 10^{-21}$ \cite{12}; charge of an atom: $Q_a < 10^{-18}$ \cite{13} neutrino charge: $Q_\nu < 10^{-13}$ \cite{14} or $10^{-17}$ \cite{15} (for a detailed discussion of these and other constraints, see \cite{16}).

Thus we can conclude that the upper bound on the parameter $m$ imposed by the masslessness of photon makes all other predictions very hard to observe which limits our interest in this model.

Note that this model (with no fermions) was first suggested in Ref. \cite{17} under the name of "hybrid model". The authors of Ref. \cite{17} were motivated by the systematic search for renormalizable gauge models beyond the standard $SU(2) \times U(1)$ model. As concerns the renormalizability of the model which is certainly a very important issue, it has been proved in Ref. \cite{17} that the theory possesses the property called tree unitarity which is a weaker property than renormalizability. We are not aware of any work which would further address the problem of renormalizability of this type of models. Although it may appear to be of academical rather than phenomenological character, this work would certainly be very desirable because it would include or exclude a whole new class of gauge models from the set of renormalizable gauge theories. (Note that we do not share the belief that non-renormalizability of a theory automatically makes it physically uninteresting.)
IV. HYBRID MODEL WITH A SCALAR SINGLET

Let us now add to the Lagrangian of the previous section a piece containing the scalar singlet field $\phi_1$ with the electric charge $\epsilon$ (which coincides with the hypercharge in this case):

$$\mathcal{L}_1 = \mathcal{L}_0 + \frac{1}{2} m^2 B^2 + |(\partial_\mu - i \frac{g'}{2} B_\mu) \phi_1|^2 + P(\phi_1, \phi).$$

(38)

Now, assume that the field $\phi_1$ has non-zero vacuum expectation value $v_1$: $\langle \phi_1 \rangle = v_1$. Then, after spontaneous symmetry breaking the mass matrix of the system $A_3, B$ is:

$$M^1 = M^0 + \Delta M^1 = \begin{pmatrix} M^0_{33} & M^0_{34} \\ M^0_{43} & M^0_{44} + m^2 + \frac{1}{2} g^2 v_1^2 \epsilon_1^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} g^2 v^2 & -\frac{1}{4} gg' v^2 \\ -\frac{1}{4} gg' v^2 & \frac{1}{4} g^2 v^2 + m^2 + \frac{1}{2} g^2 v_1^2 \epsilon_1^2 \end{pmatrix}.$$ (39)

Performing the diagonalization as before, we obtain the mass of the physical photon to be:

$$M^2_\gamma = g^2 (m^2 + \frac{1}{2} g^2 v_1^2 \epsilon_1^2)$$ (40)

The formula for the photon mass (squared) consists of two contributions: the first is proportional to $m^2$ ("hard mass") and the second is proportional to $v_1^2$ ("soft mass"). Nothing seems to prevent us from considering negative values either for $m^2$ or for $v_1^2$. Thus we are led to a very interesting possibility: to choose these two parameters in such a way that they exactly cancel each other so that the photon remains massless\(^2\) (at least, at the tree level):

$$m^2 + \frac{1}{2} g^2 v_1^2 \epsilon_1^2 = 0.$$ (41)

Note that if this condition is satisfied, the Z-boson mass becomes exactly equal to that of the standard Z-boson:

\(^2\)Here, we disregard a possible appearance of a Nambu-Goldstone boson. One may expect that its manifestations would be sufficiently suppressed, but even if they were not, the model could be modified in analogy with Ref. \^8.
\[ M_Z = \frac{1}{2} \sqrt{g^2 + g'^2 v}. \]  \hspace{1cm} (42)

Now, do we obtain the electric charge non-conservation or dequantization in the fermion sector, in analogy with the result of Section 3? Unfortunately, the answer is: no. The reason is this: the calculation of the Weinberg angle in this model (denoted by $\theta_1$) shows that this angle is exactly equal to the Weinberg angle of the Standard Model:

\[ \sin^2 \theta_1 = \sin^2 \theta. \]  \hspace{1cm} (43)

Note that this exact equality has been obtained without assuming $m_2$ or $v_1^2$ to be small (but, of course, assuming that the condition of photon masslessness, Eq. \[ \Box \] holds.) From this equality it follows that the fermion electromagnetic current in this model remains exactly the same as in the Standard Model:

\[ j_\mu = e(-\bar{e}\gamma_\mu e + \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d). \]  \hspace{1cm} (44)

In other words any effects of the electric charge non-conservation or dequantization are absent in the fermion sector. Here we would like to stress an essential point: the absence of these effects in the fermion sector does not mean that they are absent altogether. One should not forget that giving the vacuum expectation to the charged scalar field $\phi_1$ leads to the electric charge non-conservation in the scalar sector. However, from the phenomenological point of view, these effects are much harder to observe. Such effects would be similar to those arising in a model with charged scalar field but without the $m^2$ term. Models of such type have been considered in the literature before and we do not intend to go into details here.

To conclude this Section, we see that in the context of the present model, vanishing of the photon mass leads to vanishing effects of charge non-conservation and charge dequantization in the fermion sector (but not in the scalar sector).
V. HYBRID MODEL WITH A SCALAR DOUBLET

In the previous section we have considered the model with a hard mass for the field $B$ and the scalar singlet violating the electromagnetic U(1) symmetry. In this Section, let us change the singlet into the scalar doublet, again violating U(1) symmetry; the rest of the model will be the same. Thus, the lagrangian of our new model reads:

$$L_2 = L_0 + \frac{1}{2} m^2 B^2 + \lvert (\partial_\mu - ig \frac{\tau^a}{2} A_\mu^a - ig' \frac{1}{2} (1 + \epsilon_2) B_\mu) \phi_2 \rvert^2 + P(\phi_2, \phi). \quad (45)$$

where the electric charges of the scalar doublet are:

$$Q(\phi_2) = \begin{pmatrix} 1 + \frac{\epsilon_2}{2} \\ \frac{\epsilon_2}{2} \end{pmatrix}. \quad (46)$$

We break the electromagnetic symmetry by assuming

$$\langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \quad (47)$$

After the spontaneous breakdown of symmetry the mass matrix of neutral gauge fields takes the form:

$$M^{(2)} = M^0 + \Delta M^{(2)} = \begin{pmatrix} M_{33}^0 + \frac{1}{4} g^2 v_2^2 & M_{34}^0 - \frac{1}{4} g g' (1 + \epsilon_2) v_2^2 \\ M_{43}^0 - \frac{1}{4} g g' (1 + \epsilon_2) v_2^2 & M_{44}^0 + m^2 + \frac{1}{4} g'^2 (1 + \epsilon_2)^2 v_2^2 \end{pmatrix}. \quad (48)$$

The condition for the photon to be massless is:

$$m^2 + \frac{1}{4} \epsilon^2 g^2 \frac{v^2 v_2^2}{v^2 + v_2^2} = 0 \quad (49)$$

From now on, we will assume that this condition is satisfied. Then, the mass of Z-boson is given by:

$$M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2 + \frac{1}{4} g^2 v_2^2 + \frac{1}{4} g'^2 (1 + \epsilon_2)^2 v_2^2 + m^2. \quad (50)$$

For the Weinberg angle we obtain, neglecting the terms of the order of $\epsilon_2^2$:

$$\sin^2 \theta_2 = \frac{g'^2 (v^2 + (1 + 2\epsilon_2) v_2^2)}{(g^2 + g'^2) v^2 + (g^2 + g'^2 (1 + 2\epsilon_2)) v_2^2} \quad (51)$$
Assuming that the vacuum expectation of the second doublet is much smaller than that of the Higgs doublet, we can write down a simpler expression:

\[ \sin^2 \theta_2 = \sin^2 \theta (1 + 2 \epsilon_2 \cos^2 \theta \frac{v^2_2}{v^2}) \tag{52} \]

where \( \theta \) is the Weinberg angle of the Standard Model. As before, the electromagnetic interaction is given by Eq. 12–14 in which \( \sin \theta \) has to be substituted by \( \sin \theta_2 \):

\[
L_{em}^2 = L_{em}^2 l + L_{em}^2 q \tag{53}
\]

\[
L_{em}^2 l = A_{\mu} \left[ \frac{1}{2} (g \sin \theta_2 - g' \cos \theta_2) \bar{\nu}_L \gamma^\mu \nu_L - \frac{1}{2} (g \sin \theta_2 + g' \cos \theta_2) \bar{e}_L \gamma^\mu e_L 
- g' \cos \theta_2 \bar{e}_R \gamma^\mu e_R \right] \tag{54}
\]

\[
L_{em}^2 q = A_{\mu} \left[ \frac{1}{2} g \sin \theta_2 (\bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L) + g' \cos \theta_2 (\frac{1}{6} \bar{u}_L \gamma^\mu u_L + \frac{1}{6} \bar{d}_L \gamma^\mu d_L + \frac{2}{3} \bar{u}_R \gamma^\mu u_R - \frac{1}{3} \bar{d}_R \gamma^\mu d_R) \right] \tag{55}
\]

We see that the charge dequantization and charge non-conservation effects are controlled by the parameter

\[ \delta = g \sin \theta_2 - g' \cos \theta_2. \tag{56} \]

This parameter measures the deviation of our theory from the Standard Model (in the latter \( g \sin \theta - g' \cos \theta = 0 \)). Up to the terms of the order of \( \frac{v^2_2}{v^2} \) we have:

\[ \delta = \epsilon \epsilon_2 \frac{v^2_2}{v^2}. \tag{57} \]

In terms of \( \delta \) we can conveniently express the dequantized lepton and quark charges.

The neutrino charge is:

\[ Q_{\nu} = \frac{1}{4} \delta. \tag{58} \]

The axial electron charge is equal to:

\[ Q^5_e = -\frac{1}{4} \delta. \tag{59} \]

Our normalization is such that the vector electron charge should coincide exactly with \( -e \), without any corrections:
\[ Q_e = -e. \]  

(60)

The vector \((Q_u)\) and the axial \((Q_u^5)\) charges of u-quark are given by:

\[ Q_u = \frac{2}{3} e + \frac{1}{12} \delta \]  

(61)

\[ Q_u^5 = \frac{1}{4} \delta. \]  

(62)

The charges of d-quark are equal to:

\[ Q_d = -\frac{1}{3} e - \frac{1}{6} \delta \]  

(63)

\[ Q_d^5 = -\frac{1}{4} \delta. \]  

(64)

Consequently, the vector charge of the neutron is:

\[ Q_n = Q_u + 2Q_d = -\frac{1}{4} \delta. \]  

(65)

The vector charge of the proton equals

\[ Q_p = 2Q_u + Q_d = e. \]  

(66)

Therefore, although the electric charge is dequantized in this model, nevertheless the following relations between the fermion charges hold true:

\[ Q_n + Q_\nu = 0; \quad Q_p + Q_e = 0. \]  

(67)

From various experiments testing the validity of electric charge quantization we can infer the following upper bounds on the parameter \(\delta\). From the upper bound (\[14,15\]) on the (electron) neutrino charge:

\[ \delta < 4 \times 10^{-13} or 4 \times 10^{-17}. \]  

(68)

From the constraint (\[12\]) on the neutron electric charge:

\[ \delta < 4 \times 10^{-21}. \]  

(69)

From the tests (\[13\]) of the neutrality of atoms:

\[ \delta < 4 \times 10^{-18}. \]  

(70)
VI. CONCLUSION AND OUTLOOK

To conclude, we have studied a series of hybrid $SU(2) \times U(1)$ models in which the symmetry is broken down not only spontaneously but also explicitly by adding a hard mass term for the $U(1)$ field in the lagrangian. We study the issue of electric charge nonconservation and dequantization in these models. For this purpose we construct and analyze a series of hybrid models with different scalar contents.

In the minimal hybrid model the electric charge is not conserved (even though there are no charge violating vacuum expectation values). The reason is that the mixing angle between the photon and the Z-boson gets changed as compared with the Standard Model, so that the electromagnetic current receives an additional non-conserved contribution. The same reason accounts for the fact that the electric charges become slightly different from their standard values (i.e., charge dequantization occurs). In this minimal model the photon acquires a non-zero mass which puts a tight limit on the allowed magnitude of the hard mass term for the $U(1)$ field.

Next, we added to the minimal hybrid model a scalar singlet with a non-zero electric charge. We then assumed that this singlet has a non-vanishing vacuum expectation thus violating spontaneously the electromagnetic symmetry. We showed that the photon mass (at the tree level) receives two contributions: the first from the hard mass term of the $U(1)$ field, and the second from the scalar vacuum expectation. The parameters can be chosen such that these two terms cancel against each other so that the photon remains massless (at the tree level). If this is done, the Weinberg angle is not changed, therefore the electric charges of the fermions remain the same and there is no electric charge non-conservation in the fermion sector (but in the scalar sector the electric charge is not conserved).

Finally, we presented a hybrid model with an extra scalar doublet (in addition to the Higgs doublet of the Standard Model). Again, we assumed that the doublet spontaneously violates the electromagnetic symmetry. In analogy with the previous model, the photon mass again consists of two terms: one due to the $U(1)$ hard mass term and the other due to
the new scalar doublet vacuum average. Also, we can arrange for these two terms to cancel.

However, here starts the difference with the scalar-singlet model and an interesting consequence arises. Although the photon mass is zero (at the tree level), the Weinberg angle does get modified, so that the electric charges of the fermions become \textit{dequantized} and, moreover, the electromagnetic current (defined as the current interacting with the electromagnetic field) is \textit{no longer conserved}.

From the results of the experiments measuring the electric charges of the neutron and atoms, and the astrophysical limits on the neutrino electric charge we have derived upper bounds on the parameter that governs the effects of charge dequantization and non-conservation in our model.

The next important step would be to consider the hybrid models beyond the tree level and address the problem of renormalizability of such models.

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