Gigantic Enhancement of Magneto-Chiral Effect in Photonic Crystals

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We theoretically propose a method to enhance dramatically a magneto-chiral (MC) effect by using the photonic crystals composed of a multiferroic material. The MC effect, the directional birefringence even for unpolarized light, is so small that it has been difficult to observe experimentally. Two kinds of periodic structures are investigated; (a) a multilayer and (b) a stripe composed of a magneto-chiral material and air. In both cases, the difference in reflectivity between different magnetization directions is enhanced by a factor of hundreds compared with a bulk material.

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Optical effects in media with broken symmetry have been of great interest from both scientific and application viewpoints. Broken inversion symmetry $\mathcal{I}$ and time-reversal symmetry $\mathcal{T}$ give rise to polarization-dependent properties: natural optical activity and magneto-optical effects, respectively. Polarization-independent properties require breaking of both $\mathcal{I}$ and $\mathcal{T}$. When both $\mathcal{I}$ and $\mathcal{T}$ are simultaneously broken, we have directional birefringence even for unpolarized light, which is non-reciprocal. Such directional birefringence is conventionally called a magneto-chiral (MC) effect or an optical magnetoelectric (OME) effect, if the microscopic structure of the medium is helical or polar, respectively. Nevertheless, as the polarization-independent non-reciprocal effects, the MC effect and OME effect are quite similar, and we call both the effects “MC effects” for simplicity. The MC effect has been mostly studied in chiral media in external fields. On the other hand, only recently has it been investigated in condensed materials without external fields. Such materials should be multiferroic, i.e., without $\mathcal{I}$ and $\mathcal{T}$. Multiferroic materials such as GaFeO$_3$ exhibit the spontaneous MC effect even in the absence of the external fields. Optical properties in multiferroics can be described by a toroidal moment defined by $\vec{T} \equiv \sum_i \vec{r}_i \times \vec{S}_i \simeq \vec{P} \times \vec{M}$, where $\vec{r}_i$ is the displacement from the center position of atoms, $\vec{S}_i$ is a magnetic moment at $i$-th site, $\vec{P}$ is the spontaneous electric polarization, and $\vec{M}$ is the magnetization, respectively. The dielectric function depends on whether light propagation is parallel or antiparallel to the toroidal moment.

Unfortunately the MC effect is usually too small to be observed experimentally. In this letter we theoretically propose a method to enhance the MC effect by using a photonic crystal. A photonic crystal is a periodic array of dielectric materials. Light propagation in a photonic crystal described by a wave equation is analogous to electron motion in a solid. In this analogy the wave equation with a spatially modulated dielectric constant corresponds to the Schrödinger equation with a potential. However, most studies on photonic crystals do not utilize this analogy to electronic systems; instead they aim only to realize the localization of light or high $Q$-values. In condensed materials, magnetism has been one of the central issues, while much less attention has been paid to magnetic photonic crystals. In light of rich physics and applications arising from magnetism in condensed materials, magnetic photonic crystals should have potential importance both for fundamental physics and for application. Thus it is highly desirable to construct a theory describing light propagation in materials without time-reversal symmetry.

We theoretically found that the difference in reflectivity for opposite directions of the toroidal moment is magnified by hundreds of times in photonic crystals than that in a bulk. The enhancement is due to the Bragg reflection and a photonic band gap, which are absent in a bulk or a Fabry-Perot cavity. In this respect photonic crystals are of great promise for better interferometers in place of Fabry-Perot cavities.

We solve the following wave equation derived from Maxwell equations, $\frac{1}{\varepsilon(\vec{r})} \nabla \times \left[ \nabla \times \vec{E}(\vec{r}) \right] = (\hat{\sigma})^2 \vec{E}(\vec{r})$, where $\varepsilon(\vec{r})$ is a dielectric function and $\vec{E}$ is an electric field vector. The direction of the toroidal moment is postulated to be parallel to the $x$ axis, and the MC effect can be introduced in the dielectric function of the medium 1 as $\varepsilon_1 = \varepsilon_1^0 + \alpha k_x$, where $\varepsilon_1^0$ is the static dielectric constant, $\alpha$ is a strength of the MC effect and $k_x$ is an operator representing the $x$ component of the unit wavevector. We set the dielectric constants of the two media as $\varepsilon_0^0 = 15, \alpha = 10^{-6}$ and $\varepsilon_2 = 1$, and consider the normal incidence throughout this letter. In order to solve the equation above, we use a transfer matrix method for multilayer structures shown in Fig. 1(a), and a plane-wave expansion method for...
stripe structures shown in Fig. (b). The polarization change, i.e., magneto-optical effects and natural optical activity, are neglected throughout this letter, and we only focus on the MC effect, which is polarization-independent. We calculate frequency dependence of the difference in reflectivity for opposite directions of the toroidal moments. It should be noted that the difference is zero in a conventional material, and non-zero only in a magneto-chiral medium. First of all, we calculate reflectivity without the periodic modulation of $\varepsilon(f)$. Let us consider light reflection of a magneto-chiral material with the normal incidence: 
$$R_{\text{bulk}}^\uparrow - R_{\text{bulk}}^\downarrow = \left(\frac{\sqrt{15+10-\pi}}{\sqrt{15+10+\pi}}\right)^2 - \left(\frac{\sqrt{15-10-\pi}}{\sqrt{15-10+\pi}}\right)^2 \sim 10^{-8},$$
where the indices $\uparrow$ and $\downarrow$ denote whether $\hat{T}$ is parallel or antiparallel to the $x$-axis. This value should be compared with that in a periodic structure. A MC effect in a material is originated from the spin-orbit interaction that is nothing but a relativistic effect, and is difficult to control microscopically. In contrast, we can enhance the effect by the artificial periodic modulation with the size of the order of light wavelength.

(a) Magneto-chiral multilayers

The multilayered structure (16 layers) composed of the MC medium and air is shown in Fig. 1(a). The ratio of the thickness between the MC medium and air is put to be 0.2 : 0.8. We solve the wave equation by means of the transfer matrix method. The boundary conditions and the phase change through the propagation can be described as matrices, and we can solve the equation simply by multiplying the matrices.

We calculate reflectivity $R_{\uparrow,\downarrow}$ where the arrows represent the directions of $\hat{T}$ whether it is parallel or antiparallel to the $x$ axis. Figure 2 shows the frequency dependence of the difference in reflectivities for opposite directions of $\hat{T}$. Here we introduced small absorption. There are two distinct behaviors in the figure. One is the region characterized by "gap" in the figure corresponds to the photonic band gap. The other is the region where $R_{\uparrow} - R_{\downarrow}$ oscillates, corresponding to a Fabry-Perot cavity that has nothing to do with the periodic structure. At the edge of the gap, we have a remarkable enhancement. This enhancement is interpreted as follows. The different toroidal moment direction has the different refractive index, leading to different reflectivity. Hence the band gaps for the opposite toroidal moment directions are at different frequency regions. At the gap edge, it can happen that the frequency is in the band gap for $\hat{T} \parallel \hat{e}_x$, and in the conduction band for $\hat{T} \parallel -\hat{e}_x$, because we have the nonreciprocal dispersion $\omega(\vec{k}) \neq \omega(-\vec{k})$. Therefore the difference in reflectivities between the two toroidal moment directions is significantly magnified. In other words, this difference in reflectivities behaves qualitatively similar to $\frac{d^2 \omega}{d \varepsilon^2}$, and the differential is singular at the edge of the gap, resulting in the enhancement. The magnification at the gap edge is robust under small changes of incident angles and toroidal moment direction, whereas the values in the Fabry-Perot moment region are fragile against these changes. We note that if there is no absorption in a MC medium, the value of $R_{\uparrow} - R_{\downarrow}$ at the gap edge is the order of $10^{-5}$, an order of magnitude larger.

(b) Magneto-chiral stripes

In the stripe structures shown in Fig. 1(b), reflectivity changes as the direction of the toroidal moment is reversed. We calculate $R_{\uparrow}^1 - R_{\downarrow}^1$, which is the difference between reflectivities of the first-order Bragg reflected waves, denoted by the subscript 1. We solve the equation by Fourier transformation in the region $0 < z < L$,

$$E = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} A_{nm} e^{ik_{a} x} e^{i\frac{2\pi z}{L}}, \quad (1)$$

where $k_{a} = \frac{2\pi n}{L}$, $L$ is the thickness of the stripe, and we have a sinusoidal modulation $\varepsilon = \frac{1}{2}(\varepsilon_1 - \varepsilon_2) + \frac{1}{2}(\varepsilon_1 - \varepsilon_2) \cos \frac{2\pi x}{a}$, where $a$ a periodicity along the $x$ direction. It should be noted that the structure shown in Fig. 1(b) is only a schematic one. In the calculation, we use the sinusoidal dielectric modulation instead of the rectangular one, because the higher order Fourier components give no essential contribution to the results. We choose the dielectric constant of the substrate as $\varepsilon_3 = 15$. We take 606 Fourier components, $n = -1, 0, 1$, $m = 0-200$, which are enough for convergence. Figure 3 represents the frequency dependence of the difference in intensities of Bragg reflected waves for $L = 0.5a$, $a$, and $2a$. In the region $\frac{2\pi n}{L} \leq 1.0$, the wavenumber is imaginary, implying that the reflected light is an evanescent wave. Therefore we should focus on the region $\frac{2\pi n}{L} \geq 1.0$. In the region just above $\frac{2\pi n}{L} = 1.0$, the reflection angle is close to $90^\circ$, and it is experimentally difficult to detect the beam. Let us consider the case $L = a$ at first. We find an enhancement at $\frac{2\pi n}{L} \approx 1.3$ corresponding to the Bragg reflection angle $\theta = 63^\circ$, which can be observed experimentally. Moreover, even away from $\frac{2\pi n}{L} \approx 1.3$, $R_{\uparrow}^1 - R_{\downarrow}^1$ is of the order of $10^{-6}$, which is two orders of magnitude larger than that in a bulk. Although there is no band gap in this system, the optical effect is remarkably enhanced when the wavelength of light is comparable to the periodicity of the photonic crystals. When the thickness of the stripe changes, $L = 0.5a$ and $2a$, the order of $R_{\uparrow}^1 - R_{\downarrow}^1$ remains the same. It is because this enhancement is due to the periodicity along the $x$ direction and has nothing to do with the value of the thickness.

Let us discuss the relation and the difference between (a) and (b) in Fig. 1. In the MC multilayers, a periodic structure and the non-reciprocal dispersion $\omega(\vec{k}) \neq \omega(-\vec{k})$ play essential roles. On the other hand, in the MC
FIG. 1: Schematic illustration of (a) a magneto-chiral multilayer and (b) a stripe structure whose constituent materials are multiferroics and air. The vector $\vec{G}$ in (b) denotes a reciprocal lattice vector.

stripes without any band gap, a periodic structure brings about multiple scatterings, and only the waves that satisfy the Bragg conditions can be diffracted. In both cases, $\tilde{R}^\dagger - \tilde{R}^\uparrow$ is roughly a differential of $\tilde{R}$ with respect to $\omega$, where $\tilde{R}^\dagger$ is $R^\dagger$ or $R^\uparrow$. The multiple scatterings induce the oscillation of $\tilde{R}$ and the large $\frac{\partial R}{\partial \omega}$, resulting in a remarkable enhancement of $\tilde{R}^\dagger - \tilde{R}^\uparrow$. In order to magnify this effect, it is necessary to array the materials parallel to the toroidal moment direction. It should be noted that the enhancement factor of hundreds is not specific to these cases ($\alpha = 10^{-6}$) but valid for another value of $\alpha$.

In conclusion, we have investigated an enhancement of the magneto-chiral effect in photonic crystals. The difference in intensities of two waves for different toroidal moment directions is hundreds of times enhanced from that in a bulk. The direction of the toroidal moment can be reversed by changing the directions of the polarization or the magnetization by external fields. Our theory is in principle applicable for a wide range of materials, by changing dielectric constants and thickness of the layers at one’s disposal.

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