Abstract

We perform a systematic analysis of globally consistent D-brane quivers which realize the MSSM and analyze them with respect to their Yukawa couplings. Often, desired couplings are perturbatively forbidden due to the presence of global $U(1)$ symmetries. We investigate the conditions under which D-brane instantons will induce these missing couplings without generating other phenomenological drawbacks, such as R-parity violating couplings or a $\mu$-term which is too large. Furthermore, we systematically analyze which quivers allow for a mechanism that can account for the small neutrino masses and other experimentally observed hierarchies. We show that only a small fraction of the globally consistent D-brane quivers exhibits phenomenology compatible with experimental observations.
1 Introduction

Intersecting D-brane models have been proven to be a fruitful playground for realistic model building (for reviews on this subject see [1–3]). In these string compactifications, the gauge groups arise from stacks of D6-branes that fill out four-dimensional spacetime and wrap three-cycles in the internal Calabi-Yau threefold. Chiral matter appears at the intersection in the internal space of different cycles wrapped by the D6-brane stacks. The multiplicity of chiral matter between two stacks of D6-branes is given by the topological intersection number of the respective three-cycles.

Over the last decade, many semi-realistic MSSM-like and supersymmetric GUT-like realizations have been constructed based on intersecting branes, mostly using toroidal orbifold compactifications. Once the spectrum of a particular string compactification is determined, finer details can be investigated, such as the Yukawa couplings for the chiral matter fields. In intersecting brane compactifications such Yukawa couplings can be extracted from string amplitudes [15–18]. These amplitudes are suppressed by open string world-sheet instantons connecting the three intersecting branes [6, 19] and thus could potentially give rise to hierarchies observed in nature. However, often times these desired couplings are forbidden due to the violation of global U(1) symmetries. The latter are remnants of the Green-Schwarz mechanism, which is crucial for the cancellation of abelian, mixed and gravitational anomalies.

Recently, it has been realized that D-brane instantons can break these global symmetries and generate otherwise forbidden couplings [25–28]. For type IIA compactifications the relevant objects are so called E2-instantons, which wrap a three-cycle in the internal manifold and are point-like in space-time. They are charged under the global \( U(1) \)’s, where the charge is given by

\[
Q_a(E2) = -N_a \pi_{E2} \circ (\pi_a + \pi'_a) .
\]

(1)

Here the subscript \( a \) refers to the global \( U(1)_a \) arising from the stack of \( N_a \) D6 branes which wrap the cycle \( \pi_a \). The instanton itself wraps the cycle \( \pi_{E2} \) and \( \pi'_a \) denotes the orientifold image cycle of \( \pi_a \).

A superpotential term which violates global \( U(1) \) symmetries is perturbatively

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1 For original work on non-supersymmetric intersecting D-branes, see [4–7], and for chiral supersymmetric ones, see [8, 9]. For supersymmetric MSSM realizations, see [10–12] and for supersymmetric MSSM-like constructions within type II RCFT’s, see [13, 14].

2 As demonstrated in [20] (see also [21–24]) allowing for additional Higgs pairs might give rise to all Yukawa couplings, and, for appropriate values of the open and closed string moduli, give rise to desired Yukawa hierarchies.

3 For a recent review on the D-instanton effects see [29] and also [30,31].

4 Note that, in contrast to [25], there is an additional minus sign in (1), which is due to the fact that a positive intersection number \( I_{E2a} \) corresponds to the transformation behavior \( (E2, \pi) \) of the charged fermionic zero modes, rather than \( (E2, \pi) \).
forbidden, but can be generated non-perturbatively
\[ W^{np} = e^{-S_{E2}} \prod \Phi_{a_i, b_i}, \tag{2} \]
if the E2 instanton compensates for the global U(1) charge carried by the product \( \prod \Phi_{a_i, b_i} \). The exponential suppression factor due to the classical instanton action depends on the volume of the three-cycle wrapped by the instanton and is given by
\[ e^{-S_{cl}^{E2}} = e^{-\frac{2\pi}{\ell_3} Vol_{E2}}. \tag{3} \]
Note that the instanton is independent of the 4D gauge couplings and therefore has no interpretation as a gauge instanton. Rather, it is purely stringy. Due to this property, D-instantons not only induce perturbatively forbidden couplings but also may give a natural explanation for hierarchies, which are poorly understood from a field theoretical point of view.

Apart from the charged instanton modes, which arise from strings attached to the instanton and a D6-brane, a generic D-instanton also exhibits the so called uncharged instanton modes. The latter consist of the four bosonic modes \( x^\mu \), which are associated with breakdown of four-dimensional Poincaré invariance, and the fermionic modes \( \theta^\alpha \) and \( \bar{\tau}^{\dot{\alpha}} \), which indicate the breakdown of the \( \mathcal{N} = 2 \) supersymmetry preserved by the Calabi-Yau manifold down to \( \mathcal{N} = 1 \) supersymmetry. Additional zero modes appear if the instanton wraps a non-rigid three-cycle in the internal manifold. In the presence of multiple instantons, there may arise zero modes at intersections of two instantons.

The non-perturbative contribution to the superpotential is given by the path integral over all instanton zero modes. Thus, in order to give rise to superpotential terms, one has to ensure that all uncharged zero modes, apart from \( x^\mu \) and \( \theta^\alpha \), are projected out or lifted. There are various mechanisms to ensure the saturation of these additional undesired zero modes, such as lifting via fluxes [32–35], saturation via multi-instanton configurations [32,36–38], or via additional interaction terms arising when the instanton wraps a cycle which coincides with one of the spacetime filling D6-branes [39–42].

If the E2-instanton wraps a rigid orientifold invariant cycle, the zero modes are subject to the orientifold projection and the \( \tau^{\dot{\alpha}} \) modes are projected out [43–46]. This is referred to as a rigid \( O(1) \) instanton and can contribute to the superpotential, due to the absence of the \( \tau^{\dot{\alpha}} \) modes. For simplicity and clarity in this analysis, we will assume that rigid \( O(1) \) instantons generate the missing couplings. For an \( O(1) \) instanton the \( E2a \) and \( E2a' \) sector are identified, which modifies the charge of the instanton under \( U(1)_a \) to
\[ Q_a(E2) = -N_a \pi_{E2} \circ \pi_a. \tag{4} \]
We emphasize, though, that any other instanton configuration with the same
charged zero mode structure is equally good and the analysis performed applies analogously.

It has been explicitly shown that D-instantons carrying the correct zero mode structure can generate Majorana masses for right-handed neutrinos \([25,27,46–50]\), induce a \(\mu\) term for the Higgs pair \([25, 27, 46]\), and generate the \(10\ 10\ 5\) Yukawa coupling in \(SU(5)\)-GUT-like models \([51, 52]\). Moreover, these nonperturbative effects are relevant for supersymmetry breaking \([53–59]\) and moduli stabilization \([60–68]\).

In \([69]\) the authors analyzed specific semi-realistic four-stack D-brane quivers in which all of the matter content transforms as bifundamentals of the respective gauge groups, and thus these setups do not exhibit any matter transforming as symmetric or antisymmetric. For these quivers, they analyzed under what circumstances perturbatively missing couplings can be generated via D-instantons and investigated their phenomenological implications. A potential problem for this class of D-brane quivers is that the instantons which induce the Yukawa couplings lead to phenomenologically undesirable effects, such as a \(\mu\)-term which is too large or too much family mixing. As also shown in \([69]\), these phenomenological drawbacks can be circumvented if one allows for a second Higgs pair.

In this work we follow a slightly different path. Here, instead of looking at concrete semi-realistic realizations of the standard model, we focus on the whole class of all globally consistent D-brane quivers which exhibit the standard model gauge symmetry and the exact MSSM matter spectrum plus three right-handed neutrinos \([14]\). We investigate these quivers with respect to their Yukawa couplings and analyze in each case whether instantons can generate perturbatively forbidden, but desired, couplings without inducing tadpoles or R-parity violating couplings. In addition, we require that these quivers exhibit a mechanism which accounts for the smallness of the neutrino masses, give rise to a small \(\mu\)-term, and give Yukawa textures compatible with experimental observations without too much fine-tuning. Let us emphasize that this analysis is completely independent of any concrete global realization and is thus a bottom-up analysis \([70–72]\).

We show that only a small subset of the globally consistent D-brane quivers exhibits such phenomenology. The whole analysis is independent of any concrete realization and thus the results serve as a good starting point for future model building. Furthermore, even though this analysis is performed in the type IIA corner of the string theory landscape, the results also apply to the T-dual type I framework as well as to type IIB with D-branes on singularities.

This paper is organized as follows. In chapter 2 we describe the criteria and conditions for our MSSM realizations. We start by investigating the implications of the top-down constraints, which include tadpole cancellation and presence of a massless \(U(1)\gamma\), for D-brane quivers and then proceed to discuss a number of

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5This holds not true for instanton configurations where zero modes are lifted via fluxes. The latter contribute to the tadpole equations and thus modify the analysis.
bottom-up constraints, motivated by experimental observations. In section 3.1 we present all solutions to our constraints arising from three-stack models where the D-branes wrap generic cycles. We discuss phenomenological properties of each solution in detail, including the role played by instantons. We see that some of these setups exhibit phenomenological drawbacks, such as too much family mixing or the presence of R-parity violating couplings, once non-perturbative effects are taken into account. In section 3.2 we analyze four stack models, which give rise to a much richer structure. We embed further bottom-up constraints motivated by the lessons learned in section 3.1 and in [69]. We present the subset of D-brane quivers compatible with these phenomenological considerations. This subset represents a good starting point for concrete MSSM realizations with realistic Yukawa structure. In section 3.3 we consider D-brane quivers where the $SU(2)_L$ of the MSSM arises from a stack of D-branes wrapping an orientifold invariant cycle and perform an analysis similar to those in sections 3.1 and 3.2. We conclude in section 4 with a summary of our results and a brief outlook.

2 Constraints

In this chapter we discuss criteria and conditions that we require of the MSSM realizations. Some conditions, such as tadpole cancellation and the presence of a massless hypercharge, are top-down constraints. Others, such as conditions on the spectrum and on the superpotential, are bottom-up constraints.

Before discussing these criteria and conditions in detail let us describe the generic setup of MSSM quivers based on three and four stacks of D-branes. The standard model matter content arises at intersections of three or four stacks of D-branes which give rise to the gauge symmetry

$$ U(3)_a \times U(2)_b \times U(1)_c \times U(1)_Y. $$

The tadpole conditions imply the vanishing of non-abelian anomalies, while abelian and mixed anomalies are cancelled via the Green-Schwarz mechanism. Generically, the anomalous $U(1)$'s acquire a mass and survive only as global symmetries, which forbid various couplings on the perturbative level. Since the standard model gauge symmetries contain the abelian symmetry $U(1)_Y$, we require that a linear combination

$$ U(1)_Y = \sum_x q_x U(1)_x, $$

remains massless. Thus, the resulting gauge group in four-dimensional spacetime is

$$ SU(3)_a \times SU(2)_b \times U(1)_Y. $$

\footnote{The $SU(2)$ can be also realized as $Sp(2)$. This case will be discussed in chapter 3.3}
Let us briefly comment on the origin of the MSSM spectrum. The left-handed quarks \( q_L \) are localized at intersections of brane \( a \) and \( b \) or its orientifold image \( b' \), while the right-handed quarks, \( u_R \) and \( d_R \), arise at intersections of brane \( a \) with one of the \( U(1) \) branes or its orientifold image. Depending on the hypercharge, the right-handed quarks can also transform as antisymmetric of \( SU(3) \). The left-handed leptons and the Higgs fields, \( H_u \) and \( H_d \), are charged under the \( SU(2) \) and neutral under \( SU(3) \), and thus appear at intersections between brane \( b \) and one of the \( U(1) \) branes. Finally, the right-handed electrons \( E_R \) and neutrinos \( N_R \), which are singlets under \( SU(3) \) and \( SU(2) \), can arise at intersections of two \( U(1) \) branes or at the intersection \( bb' \), in which case they would transform as antisymmetric of \( SU(2) \). We emphasize that the actual origin of the matter fields crucially depends on the choice of hypercharge \( U(1)_Y \).

We impose constraints on the Yukawa couplings and neutrino masses according to experimental observations. Specifically, we require that the Yukawa couplings \( q_L H_u u_R, q_L H_d d_R, \) and \( L H_d E_R \) give rise to three massive families of up-quarks, down-quarks, and electrons. Furthermore, we forbid the presence of R-parity violating couplings and we require the presence of a mechanism which accounts for the observed smallness of the neutrino masses.

We start by analyzing the top-down constraints, tadpole cancellation and the presence of a massless \( U(1) \). Both constraints, which are conditions on the cycles the D-branes wrap, imply restrictions on the transformation properties of the chiral spectrum. This will be discussed in the sections 2.1 and 2.2. The bottom-up constraints, which contain conditions on the spectrum and superpotential, will be subject in sections 2.3, 2.4, and 2.5. Specifically, we require that the chiral spectrum arising from the three or four stacks reproduces the MSSM spectrum plus three right-handed neutrinos and does not allow for any additional chiral exotics. In 2.3 we define precisely what we mean by the latter. In section 2.4 we discuss the constraints arising from the MSSM superpotential and in section 2.5 we present two mechanisms which can account for small neutrino masses.

2.1 Tadpole Condition

The tadpole condition is a constraint on the cycles the D-branes wrap and reads in type IIA

\[
\sum_x N_x (\pi_x + \pi'_x) - 4\pi_{O6} = 0 .
\]

Multiplying this equation with the homology class of the cycle that is wrapped by a stack \( a \) gives, after a few manipulations

\[
\sum_{x \neq a} N_x (\pi_x \circ \pi'_a - \pi_x \circ \pi_a) + \frac{N_a - 4}{2} (\pi_a \circ \pi'_a + \pi_a \circ \pi_{O6}) + \frac{N_a + 4}{2} (\pi_a \circ \pi'_a - \pi_a \circ \pi_{O6}) = 0 ,
\]
which constrains the transformation behavior of the chiral matter fields. Given
the relations displayed in Table 1, one gets
\[ \#(a) - \#(\overline{a}) + (N_a - 4)\#(\begin{array}{c}\square \end{array}) + (N_a + 4)\#(\begin{array}{c}\square \end{array}) = 0, \quad (6) \]
where \#(a) gives the total number of fields transforming as fundamental under
\( SU(N_a) \) and analogously for \#(\overline{a}), \#(\begin{array}{c}\square \end{array}) \) and \#(\begin{array}{c}\square \end{array})). Equation (6) is nothing
else than the anomaly cancellation for non-abelian gauge theories for \( SU(N_a) \)
gauge groups with rank \( N_a > 2 \). It holds still true for \( N_a = 2 \), i.e \( SU(2) \), where
\( 2 = \mathfrak{Z} \), but note that string theory distinguishes between 2 and \( \mathfrak{Z} \).

Let us turn to the case \( N_a = 1 \). The \( U(1) \) does not give rise to any antisym-
metrics, thus for \( N_a = 1 \) the constraint (6) reduces to
\[ \#(a) - \#(\overline{a}) + 5\#(\begin{array}{c}\square \end{array}) = 0 \quad \text{mod} \ 3. \quad (7) \]

Summarizing, tadpole cancellation implies constraints on the transformation
properties of the chiral matter fields. For \( N_a > 2 \) it is the usual non-abelian
anomaly cancellation. Since the chiral spectrum is the exact MSSM plus three
right-handed neutrinos the constraint for \( U(3) \) is trivially satisfied. On the other
hand for \( U(2) \) and the \( U(1) \) tadpole cancellation puts non-trivial constraints,
given by (6) and (7), on the transformation behavior of the matter fields.

2.2 Massless U(1)’s

Generically, the \( U(1) \) gauge bosons acquire a mass term via the Green-Schwarz
mechanism, which ensures the cancellation of pure abelian, mixed and gravita-
tional anomalies. The massive \( U(1) \)'s are not part of the low energy effective
gauge symmetry, but remain as unbroken global symmetries at perturbative level
and thus may forbid various desired couplings.

A linear combination
\[ U(1) = \sum_x q_x U(1)_x \quad (8) \]
remains massless if the condition [5]
\[ \sum_x q_x N_x (\pi_x - \pi'_x) = 0 \quad (9) \]
is satisfied. Here $x$ denotes the different D-brane stacks present in the model. Since the standard model contains the $U(1)_Y$ hypercharge as a gauge symmetry, we require the presence of massless $U(1)$ which can be identified with $U(1)_Y$. As we will see in chapter\textsuperscript{3}, only for a few combinations (8) do all the matter particles have the proper hypercharge [14].

Analogously to the analysis performed for the tadpole constraints we multiply equation (9) with the homology class $\pi_a$ of the cycle wrapped by the D-brane stack $a$. After a few manipulations and applying the relation given in Table 1, we obtain

$$-q_a N_a \left( \#(\square_a) + \#(\square_b) \right) + \sum_{x \neq a} q_x N_x \#(a, x) - \sum_{x \neq a} q_x N_x \#(a, x) = 0 . \quad (10)$$

Note that (10) gives a constraint for every D-brane stack present in the model. Thus for three and four stack models we expect three and four additional constraints respectively, due to the presence of a massless hypercharge.

Due to the absence of antisymmetric matter for $U(1)$’s the condition (10) applies only to nonabelian gauge symmetry. For abelian gauge groups, the condition reads

$$-q_a \frac{\#(a) - \#(\pi) + 8\#(\square_b)}{3} + \sum_{x \neq a} q_x N_x \#(a, x) - \sum_{x \neq a} q_x N_x \#(a, x) = 0 . \quad (11)$$

### 2.3 Chiral and Non-chiral Spectrum

Here we discuss the origin of the MSSM spectrum and give a precise definition of chiral exotics. In order to do so, let us split the whole class of D-brane stacks into two disjoint classes $O$ and $H$. Here $O$, the observable class, corresponds to the three or four D-brane stacks and their orientifold images from which the MSSM matter arises. Generically, additional hidden D-brane stacks are present to ensure the cancellation of the RR-charges. The latter are elements of the subclass $H$.

In this work we require that all MSSM fields plus the three right-handed neutrino $N_R$ are only charged under the subclass $O$\textsuperscript{7}. Thus we do not allow for any MSSM matter field to appear at intersections between a brane of class $O$ and a hidden brane of class $H$. Moreover, we forbid any MSSM matter fields arising from two D-branes of type $H$, and we require the absence of any chiral fields charged under any D-brane stacks in the observable sector in addition to the MSSM spectrum. Thus all matter fields charged under $O$ and $H$ appear as vector-like state and potentially receive a large mass once the open string moduli are fixed. We emphasize again that the $OO$ sector contains only MSSM matter fields, so that chiral exotics can only appear within the hidden sector.

\textsuperscript{7}Note that this is in contrast to the analysis of [14], where the authors allowed for right-handed neutrinos from the hidden sector.
These assumptions require, as an immediate consequence, that the constraints on the transformation behavior arising from the tadpole cancellation as well as from the appearance of a massless hypercharge $U(1)_Y$ have to be satisfied within the observable class $O$. This is due to the fact that non-chiral matter does not enter the constraints derived in sections 2.1 and 2.2. Thus $x$ in the constraints (6), (7), (10) and (11) is an element of $O$ and runs from 1 to the number of stacks.

2.4 Yukawa Couplings

The superpotential of the MSSM contains the terms

$$q_L H_u u_R \quad q_L H_d d_R \quad L H_d E_R \quad L H_u N_R \quad H_u H_d$$

(12)

Any realistic string vacua has to exhibit such terms in its superpotential. If the smallness of the neutrino masses is due to the type I seesaw mechanism, then the presence of a mass term

$$N_R N_R$$

(13)

for the right-handed neutrinos is required. All these couplings have to be realized either perturbatively or via D-instanton effects, as discussed in the introduction. The MSSM allows for additional couplings

$$u_R d_R d_R \quad q_L L d_R \quad L L E_R \quad L H_u$$

(14)

which are invariant under the standard model gauge symmetries. These are referred to as R-parity violating couplings and could lead to a rapid proton decay, which is in contradiction with experiments. In this work we require the absence of any of these couplings, both perturbatively and non-perturbatively. Let us mention that the experimental constraints on just $L$- or $B$-number violating couplings are not as stringent, as long as it is ensured that the setup preserves one of the two symmetries. Nevertheless, such couplings have to be suppressed compared to the MSSM Yukawa couplings (12). This implies that R-parity violating couplings should be perturbatively absent. Moreover, if some of the couplings in (12) are perturbatively forbidden, and thus have to be induced via instantons, we require the very same instanton which induces the desired Yukawa coupling does not generate any R-parity violating couplings (14). Otherwise the latter are expected to be of the same order as the instanton induced Yukawa coupling, which is not compatible with experimental observations. Also, in the absence of R-parity violating couplings, the lightest supersymmetric particle cannot decay and could therefore be a dark matter candidate.

Let us briefly comment on R-parity violating couplings involving the right-handed neutrinos. The Majorana mass term (13) violates R-parity, but its presence does not lead to any contradictions with experiments. The other gauge
invariant couplings,
\[ N_R N_R N_R \quad H_u H_d N_R \] (15)
which are R-parity violating, may affect the Higgs potential and thus the Higgs VEV. We do not forbid the presence of such terms.

Finally, we require that instantons required to generate Yukawa couplings do not also induce terms of the form
\[ N_R \] (16)
in the superpotential. The latter is a tadpole and would indicate an instability of the string vacuum.

2.5 Neutrino Masses

Finally, let us discuss the neutrino masses. Experimental observations indicate that the neutrino mass is very small compared to the masses of any other chiral matter. Here we allow for two different mechanisms which explain this hierarchy. The first one is the well known Type I seesaw mechanism. A necessary ingredient for this mechanism is the presence of a Majorana mass term for the right-handed neutrinos, in addition to the usual Dirac mass term. The second scenario is string inspired [73] and assumes that the Dirac mass term is perturbatively forbidden. An instanton is used to induce this term with high suppression, and thus explains the hierarchy between neutrino mass and all other standard model particle masses. For various quivers we will also encounter hybrids of these two mechanisms.

Let us briefly comment on the Weinberg operator, which provides another potential explanation for the smallness of the neutrino masses. Such an operator is generically perturbatively forbidden, but could in principle be generated via D-instantons, giving rise to a term
\[ e^{-S_{E2}} \frac{L H_u L H_u}{M_s} \] (17)
Assuming that \( M_s \) is of the order \( 10^{18} \) GeV and the Higgs VEV is around 100 GeV leads to neutrino masses which are too small, even with a negligible suppression factor\(^8\) [46].

3 MSSM Quivers and Their Phenomenology

In this chapter, we will study the three and four stack models which give rise to the exact MSSM plus three right-handed neutrinos. As we discussed in section

\(^8\)In principle the string mass can be lower if we allow for large extra dimensions.
this puts severe constraints on the transformation behavior of the matter fields. Matching the MSSM also requires a massless hypercharge, which can only occur if equations (10) and (11) are satisfied.

We study setups which satisfy these constraints with respect to their Yukawa couplings. To be more precise, we investigate which Yukawa couplings are perturbatively realized and under what circumstances non-perturbative effects can induce perturbatively forbidden couplings. Furthermore, we analyze the setups with respect to R-parity violating couplings and investigate if the setups allow for a $\mu$-term and neutrino masses of the observed order.

We start by investigating quivers based on three stacks of D-branes. For these we will discuss in detail all setups which match the MSSM spectrum, satisfy the constraints (6), (7), (10) and (11), and do not give rise to R-parity violating couplings on the perturbative level. We will investigate under what circumstances D-brane instantons can generate perturbatively forbidden, but desired, couplings and analyze further phenomenological implications of the presence of such instantons. As we will see, these additional effects cause some of the quivers to be unrealistic.

In section 3.2 we allow for an additional $U(1)$ D-brane stack. This enlarges the number globally consistent D-brane quivers. We perform a systematic analysis for all these quiver with respect to the constraints studied in chapter 2 and embed further bottom-up conditions motivated by the lessons learned in section 3.1. All constraints are summarized in detail at the beginning of section 3.2. It turns out that only a small subset of the globally consistent D-brane quivers exhibit a desirable phenomenology. For each choice of hypercharge we list these quivers, which serve as starting point for future model building explorations.

Later in section 3.3 we study quivers where the $SU(2)_L$ is realized as an $Sp(2)$ gauge group. Sticking to the exact MSSM and 3 right-handed neutrinos, it is easy to prove that there exists only one quiver which does not give rise to R-parity violating couplings. This particular quiver was locally realized and analyzed in [19, 74]. All the MSSM Yukawa couplings are perturbatively present and we show that a D-instanton which induces a large Majorana mass term can account for small neutrino masses.

### 3.1 Three-stack Models

The most economical way to realize the MSSM spectrum is to use three stacks of D-branes wrapping generic cycles, which give rise to the gauge symmetry $U(3)_a \times U(2)_b \times U(1)_c$ in four-dimensional space-time. Generically, the abelian parts of $U(3)_a$ and $U(2)_b$, as well as $U(1)_c$ itself, are anomalous. The Green-Schwarz mechanism, which ensures the cancellation of these anomalies, promotes the abelian gauge symmetries to global $U(1)$ symmetries which are respected by all perturbative couplings. In order to match the MSSM gauge symmetry, we
require that a linear combination

\[ U(1)_Y = q_a U(1)_a + q_b U(1)_b + q_c U(1)_c, \]  

(18)

identified as the hypercharge \( U(1)_Y \), does not acquire a St"uckelberg mass term, and thus remains massless. One finds two different linear combinations\(^9\) for the hypercharge which are consistent with the MSSM hypercharge assignments, namely

\[ U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c \quad U(1)_Y = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b . \]  

(19)

In the following we analyze the two different cases. We present all possible realizations of the MSSM for each choice of hypercharge and investigate each quiver with respect to its Yukawa couplings. If they are perturbatively forbidden, we examine under what conditions they can be generated by D-instantons.

3.1.1 \( U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c \)

We start by examining the first case. For this choice of hypercharge, the right-handed electron arises at intersections between the brane \( c \) and its orientifold image \( c' \). The right-handed neutrino is located at intersections between \( b \) and \( b' \) and transforms as antisymmetric under \( SU(2) \). The right-handed d-quarks \( d_R \) have two potential origins, since they can arise from the sector \( aa' \) or sector \( ac \). Similarly, the left-handed quarks \( q_L \) have two potential origins, coming either from the sector \( ab \) or the sector \( ab' \). The right-handed u-quarks \( u_R \) are localized at intersections between stack \( a \) and \( c' \).

Below we summarize the potential origins of all the matter fields. Here the \( a \) and \( \overline{a} \) correspond to fundamental and antifundamental representations of the gauge group \( U(3)_a \), and similarly for the other stacks. The Young diagrams \( \Box \) and \( \overline{\Box} \) denote fields transforming as symmetric and antisymmetric representations of the respective gauge symmetry.

\[
\begin{align*}
q_L & : \quad (a, \overline{b}), \quad (a, b) \\
u_R & : \quad (\overline{a}, \overline{c}) \\
d_R & : \quad (\overline{a}, c), \quad \overline{\Box}_b \\
L & : \quad (b, \overline{c}), \quad (\overline{b}, \overline{c}) \\
E_R & : \quad \Box_c \\
N_R & : \quad \Box_b, \quad \overline{\Box}_b \\
H_u & : \quad (\overline{b}, c), \quad (b, c) \\
H_d & : \quad (b, \overline{c}), \quad (\overline{b}, \overline{c})
\end{align*}
\]

\(^9\)Up to minus sign in front of \( U(2)_b \) and \( U(1)_c \).
Out of all possible MSSM setups based on the above transformation behavior, there are 16 which are tadpole free and give rise to the massless hypercharge. Only two of these do not give rise to any R-parity violating couplings on the perturbative level. Tables 2 and 3 display for these two setups the origin and the transformation behavior of the MSSM matter content. We analyze each case individually with respect to their Yukawa couplings.

| Sector | Matter fields | Transformation | Multiplicity | Hypercharge |
|--------|---------------|----------------|--------------|-------------|
| $ab$   | $q_L$         | $(a, b)$       | 3            | $\frac{1}{6}$ |
| $ac'$  | $u_R$         | $(\bar{a}, \bar{c})$ | 3            | $-\frac{2}{3}$ |
| $aa'$  | $d_R$         | $\mathbb{I}$  | 3            | $\frac{1}{3}$ |
| $bc$   | $L$           | $(b, c)$       | 3            | $-\frac{2}{3}$ |
| $bc'$  | $H_u + H_d$   | $(b, c) + (\bar{b}, \bar{c})$ | 1            | $\frac{1}{3} - \frac{1}{2}$ |
| $bb'$  | $N_R$         | $\mathbb{I}$  | 3            | 0           |
| $cc'$  | $E_R$         | $\mathbb{I}$  | 3            | 1           |

Table 2: Spectrum for setup 1 with $U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c$

Table 2 depicts the origin, transformation behavior, and multiplicity of the respective matter fields for the first setup. One can easily check that the constraints arising from tadpole cancellation and from the presence of the massless hypercharge are satisfied. Moreover, there are no R-parity violating couplings on the perturbative level.

The perturbatively allowed Yukawa couplings are

$$< q_{L(1,-1,0)}^I H_{u(0,1,1)}^J u_R^{R(1,0,-1,0)} > < L_{(0,1,1)}^I H_{d(0,1,-1,0)}^J E_R^{R(0,0,0)} >$$

$$< L_{(0,1,-1)}^I H_{u(0,1,1)}^J N_R^{R(0,-2,0)} > < H_{u(0,1,1)}^I H_{d(0,-1,1)} > .$$

Here the capital letters $I$ and $J$ denote the family index and the subscript indicates the charge under the global $U(1)_a, U(1)_b$ and $U(1)_c$. On the other hand the phenomenologically desired Yukawa coupling

$$< q_{L(1,-1,0)}^I H_{d(0,1,-1,0)}^J d_R^{R(2,0,0)} >$$

is perturbatively forbidden. An instanton carrying the charges

$$Q_a(E_2) = -3 \quad Q_b(E_2) = 2 \quad Q_c(E_2) = 1$$

under the global $U(1)$’s can compensate for the overshooting in the coupling $q_L H_d d_R$. Thus, in accord with [14], a rigid $O(1)$ instanton exhibiting the intersection pattern

$$I_{E2a} = 1 \quad I_{E2b} = -1 \quad I_{E2c} = -1$$

Note that setups with this hypercharge have an additional symmetry under $b \rightarrow b'$.

In [75] the author discusses a three-stack quiver similar to the ones we analyze here.
induces the coupling $q_L H_d d_R$. Let us analyze the path integral in more detail. Apart from the generic uncharged zero modes $x^\mu$ and $\theta^\alpha$, there are also three $\bar{\lambda}_a$, two $\lambda_b$ and one $\lambda_c$ zero modes. The path integral takes the form

$$\int d^4 x d^2 \theta d^3 \bar{\lambda}_a d^2 \lambda_b d \lambda_c \ e^{-S_{E2}^{gl}} < \bar{\lambda}_a q^I_L \lambda_b > < \lambda_b H_d \lambda_c > < \bar{\lambda}_a d^J_R \lambda_a > e^{Z'} , \quad (23)$$

where the three point amplitudes depicted in Figure 1 can be calculated applying CFT techniques [47]. Performing the integral over the charged zero modes gives the superpotential contribution

$$\int d^4 x d^2 \theta Y^{IJ}_{q_L H_d d_R} q^I_L H_d d^J_R . \quad (24)$$

Here $Y^{IJ}_{q_L H_d d_R}$ contains the suppression factor $e^{-S_{E2}^{gl}}$ of the instanton, the regularized one loop amplitude $e^{Z'}$, as well as the world-sheet instanton contributions arising from the three disc amplitudes. Note, though, that the induced $3 \times 3$ Yukawa matrix factorizes

$$Y^{IJ}_{q_L H_d d_R} = Y^I Y^J . \quad (25)$$

This is due to the fact that the disk amplitudes do not contain both matter fields $q^I_L$ and $d^J_R$ simultaneously. Thus, in order to generate non-vanishing masses for all three $d$-quarks, one needs three different instantons with the intersection pattern (22). Note that this not only explains the hierarchy between the $u$-quark Yukawa couplings, but also can account for the hierarchy between the $d$-quark families. To match experimental observations the suppression factors should lie in the range of $10^{-5} - 10^{-2}$ \footnote{Note that the disk diagrams in (23) are suppressed by worldsheet instantons which may also contribute to the hierarchies.}
The Dirac neutrino masses are perturbatively realized and therefore are expected to be of the same order as the masses for the other leptons. The presence of a large Majorana mass for the right-handed neutrinos would give a natural explanation for the smallness of the neutrino masses via the seesaw mechanism. On the perturbative level such a term is forbidden

\[ M_{N_R} N_R^{(0,-2,0)} N_R^{(0,-2,0)} . \]  

(26)

A rigid \( O(1) \)-instanton with the intersection pattern

\[ I_{E2a} = 0 \quad I_{E2b} = -2 \quad I_{E2c} = 0. \]  

(27)

and a suppression factor of the instanton of order \( 10^{-5} \) to \( 10^{-2} \) induces such a Majorana mass term in the range \( 10^{13} \text{ GeV} < M_{N_R} < 10^{16} \text{ GeV} \). Assuming that the Higgs VEV is around \( 100 \text{ GeV} \), one gets seesaw masses in the observed range \( (10^{-3} - 1) \text{ eV} \).

Finally, let us mention that for this quiver another linear combination

\[ U(1)^{B-L} = -\frac{1}{6} U(1)_a - \frac{1}{2} U(1)_b + \frac{1}{2} U(1)_c \]  

(28)

satisfies the constraints (10) and (11) and might survive as local symmetry in the low energy effective action. This linear combination can be interpreted as \( U(1)^{B-L} \) and if indeed present would forbid the generation of the Majorana mass term. In that case the quiver is unrealistic, since their is no other mechanism to explain the smallness of the neutrino masses. Note though that the conditions (10) and (11) are just necessary constraints for the presence of a massless \( U(1) \). Whether such a linear combination is indeed massless depends on the concrete realization.

We now turn to setup 2. Table 3 displays the origin and transformation behavior of the MSSM matter content. Here the perturbatively allowed couplings

| Sector | Matter fields | Transformation | Multiplicity | Hypercharge |
|--------|---------------|----------------|--------------|-------------|
| \( ab \) | \( q_L \) | \((a, b)\) | 1 | \( \frac{1}{6} \) |
| \( ab' \) | \( Q_L \) | \((a, b)\) | 2 | \( \frac{1}{6} \) |
| \( a \) | \( u_R \) | \((\bar{a}, \bar{c})\) | 3 | \( -\frac{1}{3} \) |
| \( a' \) | \( d_R \) | \((\bar{a}, \bar{c})\) | 3 | \( \frac{1}{3} \) |
| \( b \) | \( L \) | \((b, \bar{c})\) | 3 | \( -\frac{1}{2} \) |
| \( b' \) | \( H_u + H_d \) | \((b, c) + (b, \bar{c})\) | 1 | \( \frac{1}{2} - \frac{1}{2} \) |
| \( b \) | \( N_R \) | \((\bar{b}, \bar{c})\) | 3 | 0 |
| \( c \) | \( E_R \) | \((\bar{b}, \bar{c})\) | 3 | 1 |

Table 3: Spectrum for setup 2 with \( U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c \)
are
\[ < q_{L(1,1,0)} H_{u(0,1,1)} u_{R(-1,0,-1)}^I > < L_{(0,1,-1)}^I H_{d(0,-1,-1)} E_{R(0,0,2)}^J > \]
\[ < H_{u(0,1,1)} H_{d(0,-1,-1)} > . \]
The perturbatively forbidden, but phenomenologically desired couplings are
\[ < Q_{L(1,1,0)} H_{u(0,1,1)} u_{R(-1,0,-1)}^J > < Q_{L(1,1,0)} H_{d(0,-1,-1)} d_{R(0,0,0)}^J > \]
\[ < q_{L(-1,0,1)} H_{d(0,-1,0)} d_{R(2,0,0)}^I > < L_{(0,1,-1)}^I H_{u(0,1,1)} N_{R(0,2,0)}^J > . \]
Let us first discuss the u-quark Yukawa couplings. Before including non-perturbative effects, the u-quark Yukawa coupling matrix takes the form
\[
Y_{uR} = \begin{pmatrix}
A_{11} & A_{12} & A_{13} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\] (29)
and thus only one family acquires a mass. We identify this family with the heaviest generation. In order to generate masses for the other two families we have to fill the zero entries in the Yukawa coupling matrix \([29]\). There are potentially two different instantons which could generate the missing coupling \(Q_{L} H_{uR}\). Their intersection pattern is given by
\[
I_{E_{2}a}^1 = 0 \quad I_{E_{2}b}^1 = 1 \quad I_{E_{2}c}^1 = 0 \quad I_{E_{2}'a}^2 = 0 \quad I_{E_{2}'b}^2 = 1 \quad I_{E_{2}'c}^2 = 0 \quad I_{E_{2}c}^{N=2} = 1 .
\]
While the first instanton \(E_{2}^1\) exhibits only two charged zero modes, namely two \(\lambda_b\) modes, the other one \(E_{2}'^1\) has two additional charged zero modes \(\lambda_c\) and \(\bar{\lambda}_c\). Figure 2 displays how these charged zero modes are saturated via one or two disc diagrams, respectively. For both types of instantons the induced Yukawa matrix does not factorize, so one instanton can give masses to both families. If both types of instantons are present, the one which wraps the smaller three-cycle in the internal manifold and thus exhibits the smaller suppression factor gives the dominant contribution.

We now turn to the d-quark Yukawa coupling. Similarly to the previous setup, the coupling \(q_{L} H_{d} d_{R}^J\) gets generated by an instanton with the intersection pattern
\[
I_{E_{2}a}^2 = 1 \quad I_{E_{2}b}^2 = -1 \quad I_{E_{2}c}^2 = -1 .
\]
The coupling \(Q_{L} H_{d} d_{R}^J\) is induced by an instanton with the intersection pattern
\[
I_{E_{2}a} = 1 \quad I_{E_{2}b} = 0 \quad I_{E_{2}c} = -1 .
\] (30)
Figure 2: Instanton induced Yukawa coupling $Q^I_L H_u u^I_R$ for setup 2.

The disk diagrams necessary to saturate the charged zero modes are depicted in the Figures 1 and 3. Analogously to the previous setup, the fact that the Yukawa matrix is factorizable requires at least two different instantons with the intersection pattern (30).

Figure 3: Instanton induced Yukawa coupling $Q^I_L H_d d^I_R$ for setup 2.

Looking at the quark Yukawa matrices after taking into account perturbative and non-perturbative contributions, we see that all of quark masses except the top mass are generated by instantons and are therefore suppressed according to the volume of the instanton wrapped cycles. Thus this quiver gives a natural explanation for the observed mass hierarchy of the top quark relative to all the other quarks.

Finally, let us discuss the Dirac mass term $L^I H_u N^I_R$, which can be generated via two types of instantons with intersection patterns

$$I^a_{E2} = 0 \quad I^b_{E2} = 2 \quad I^c_{E2} = 0$$
$$I^a_{E2} = 0 \quad I^b_{E2} = 2 \quad I^c_{E2} = 0 \quad I^{N=2}_{E2} = 1.$$ (31) (32)

As discussed in [73], the Dirac neutrino mass term is often perturbatively forbidden, but can be generated by non-perturbative effects. Such a term would be
suppressed and would therefore give an intriguing explanation for the smallness of the neutrino mass. For this quiver, however, the nonperturbative generation of Dirac mass term via the instanton $E2_4$ cannot account for smallness of the neutrino mass since $E2_4$ also generates a Majorana mass term. Taking both terms into account, the mass matrix takes the form (here simplified for only one family)

$$m_{\nu} = \begin{pmatrix} 0 & e^{-S_{E24} \langle H_u \rangle} \\ e^{-S_{E24} \langle H_u \rangle} & e^{-S_{E24} M_s} \end{pmatrix},$$

where $\langle H_u \rangle$ denotes the VEV of the Higgs field and $M_s$ is the string mass. The mass eigenvalues of $m_{\nu}$ are of the order

$$m_{\nu}^1 = e^{-S_{E24} \langle H_u \rangle^2 M_s} \quad \text{and} \quad m_{\nu}^2 = e^{-S_{E24} M_s}$$

Taking $M_s \simeq 10^{18}$ GeV, $\langle H_u \rangle \simeq 100$ GeV, and a negligible instanton suppression factor, we get neutrino masses of order $10^{-5}$ eV, which is lower than the experimental value. Thus we need a suppression factor which is bigger than 1, which is of course impossible. This is very similar to non-perturbative generation of the Weinberg operator (17), where the generic values for $\langle H_u \rangle$ and $M_s$ give a contribution to the neutrino masses which is significantly smaller than the observed one.

On the other hand the instanton $E2_4'$, with the intersection pattern (32), does not generate a Majorana mass term for the right-handed neutrinos. Thus if only $E2_4'$ is present and it has a suppression factor of $10^{-14}$ to $10^{-11}$, then the nonperturbative generation of the Dirac neutrino mass term indeed gives a natural explanation for the smallness of the neutrino masses.

Note, though, that the instantons $E2_1$, $E2_1'$ and $E2_3$ generically induce the R-parity violating couplings $L L E_R$, $L H_u$, $u_R d_R d_R$ and $q_L L d_R$ of the same order as the induced Yukawa couplings $Q_L H_u u_R$ and $Q_L H_d d_R$. Thus this setup is ruled out as unrealistic.

### 3.1.2 $U(1)_Y = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b$

Now we turn to the other potential hypercharge choice for three-stack models. For this hypercharge, the right-handed electron arises at intersections between the brane $b$ and its image $b'$ and transforms as antisymmetric of $SU(2)$. The right-handed neutrino is located at intersections between $c$ and $c'$ and transforms as symmetric under $U(1)_c$. The right-handed $u$-quarks transform as antisymmetric of $SU(3)$, and thus appear at intersection of stack $a$ with its orientifold image $a'$. The right-handed $d$-quarks have potentially two origins, since they can arise from the sector $ac$ or the sector $ac'$. In contrast to the previous case, the left-handed

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13As for the Weinberg operator in principle one can lower the string scale if one allows for large extra dimensions.
quarks $q_L$ have only one possible origin, as they must arise from the sector $ab$. Below we summarized the possible transformation behaviors of the respective matter fields.

$q_L : (a, b)$

$u_R : (a, c)$

$d_R : (a, c), (\bar{a}, \bar{c})$

$L : (b, \bar{c}), (b, c)$

$E_R : (b, \bar{c})$

$N_R : (b, c)$

$H_u : (\bar{b}, c), (\bar{b}, \bar{c})$

$H_d : (b, \bar{c}), (b, c)$

If we require tadpole cancellation, masslessness of the hypercharge, and the absence of any R-parity violating couplings, then we get again two different setups. We discuss each individually.

For the first setup, the origin and transformation behavior of the matter fields is given in Table 4.

| Sector | Matter fields | Transformation | Multiplicity | Hypercharge |
|--------|---------------|----------------|--------------|-------------|
| $ab$   | $q_L$         | $(a, b)$       | 3            | $\frac{1}{6}$ |
| $ac'$  | $d_R$         | $(\bar{a}, \bar{c})$ | 3            | $\frac{1}{3}$ |
| $aa'$  | $u_R$         | $(b, \bar{c})$ | 3            | $-\frac{2}{3}$ |
| $bc'$  | $L$           | $(b, \bar{c})$ | 3            | $-\frac{1}{2}$ |
| $bc'$  | $H_u + H_d$   | $(b, \bar{c}) + (b, c)$ | 1            | $\frac{1}{2} - \frac{1}{2}$ |
| $bb'$  | $E_R$         | (b)           | 3            | 0           |
| $cc'$  | $N_R$         | (b)           | 3            | 1           |

Table 4: Spectrum for setup 1 with $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$

Here, the perturbatively allowed couplings are

\[
< q^I_{L(1,-1,0)} H^I_{d(0,1,1)} d^J_{R(-1,0,-1)} > \quad < L^I_{(0,1,-1)} H^I_{d(0,1,1)} E^J_{R(0,-2,0)} >
\]

\[
< L^I_{(0,1,-1)} H^I_{u(0,-1,-1)} N^J_{R(0,0,2)} > \quad < H^I_{d(0,1,1)} H^I_{u(0,-1,-1)} >
\]

and the u-quark Yukawa coupling

\[
< q^I_{L(1,-1,0)} H^I_{u(0,-1,-1)} u^J_{R(2,0,0)} > \quad (35)
\]
is violating the global $U(1)$ symmetries and is therefore perturbatively forbidden. It can be induced by an instanton with the intersection pattern

$$I_{E2_1a} = 1 \quad I_{E2_1b} = -1 \quad I_{E2_1c} = -1.$$  
(36)

Due to the factorization of the instanton induced Yukawa matrix, one again needs three different instantons to generate mass terms for all three generations.

The Dirac neutrino mass is perturbatively realized and expected to be of the same order as the lepton masses. A large Majorana mass

$$M_{N_R} N_{R(0,0,2)} N_{R(0,0,2)}$$

could account for the smallness of the neutrino masses via the seesaw mechanism. Such a mass term can be generated by an instanton with the intersection pattern

$$I_{E2_2a} = 0 \quad I_{E2_2b} = 0 \quad I_{E2_2c} = 4.$$ 

In order to get neutrino masses compatible with experiments, the suppression factor should be in the range $10^{-5}$ to $10^{-2}$.

Both instantons, $E2_1$ and $E2_2$, do not generate any of the dangerous $R$-parity violating couplings $d_R d_R u_R$, $L L E_R$, $q_L L d_R$ or $L H_u$. Note, though, that the $u$-quark coupling is realized non-perturbatively while the $d$-quark couplings is perturbatively allowed. This suggests that the $u$-quark masses are significantly smaller than the $d$-quark masses, which contradicts experimental observations.

This quiver may possess another massless $U(1)$ which is given by

$$U(1)^{B-L} = -\frac{1}{6} U(1)_a - \frac{1}{2} U(1)_b + \frac{1}{2} U(1)_c$$  
(37)

and can be interpreted as $U(1)^{B-L}$. If this symmetry is present then the Majorana mass term cannot be generated and the quiver does not exhibit a mechanism which could account for the small neutrino masses. Analogously to setup 1 in 3.1.1, the presence of such a massless linear combination depends crucially on the concrete realization.

The second setup with this hypercharge which does not give rise to any $R$-parity violating couplings differs from setup 1 only in the transformation behavior of the right-handed neutrino $N_R$. Table 5 summarizes the origin as well as the transformation behavior of the MSSM matter content. The perturbatively allowed couplings are

$$<q^I_{L(1,-1,0)} H_{d(0,1,1)} d^I_{R(-1,0,-1)}> \quad <L^I_{(0,1,-1)} H_{d(0,1,1)} E^I_{R(0,-2,0)}> \quad <H_{u(0,-1,-1)} H_{d(0,1,1)}> ,$$

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Table 5: Spectrum for setup 2 with $U(1)_Y = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b$

while here the Dirac neutrino mass term is perturbatively forbidden, due to the different transformation behavior of the right-handed neutrino. As in the first setup, the $u$-quark Yukawa coupling is perturbatively absent. The perturbatively forbidden, but desired couplings are

$$< q_L^{(1,-1,0)} H_u^{(0,-1,-1)} u_R^{(2,0,0)} > < L_I^{(0,1,-1)} H_u^{(0,-1,-1)} N_R^{(0,0,-2)} > .$$ \hspace{1cm} (38)

The discussion of the non-perturbative generation of the $u$-quark Yukawa coupling is analogous to setup 1. The coupling $q_L H_u u_R$ can be induced by an instanton with the intersection pattern

$$I_{E2a} = 1 \quad I_{E2b} = -1 \quad I_{E2c} = -1 ,$$ \hspace{1cm} (39)

and one needs three different instantons with such an intersection pattern in order to generate masses for all three families. Again, note that the non-perturbative generation of the top Yukawa coupling is not favorable and a huge fine-tuning is required to match experimental observations.

Let us turn to the Dirac neutrino mass term $L H_u N_R$. It can be induced by an instanton with the intersection pattern

$$I_{E2a} = 0 \quad I_{E2b} = 0 \quad I_{E2c} = -4 .$$ \hspace{1cm} (40)

Note, though, that the same instanton also induces a Majorana mass term

$$M_{N_R} N_R^{(0,0,-2)} N_R^{(0,0,-2)}$$ \hspace{1cm} (41)

for the right handed neutrino, an effect similar to one seen in the second setup in section [3.1.1]. In contrast to the quiver displayed in Table 3, this setup does not exhibit an instanton which only generates the Dirac mass term. Thus in this quiver we cannot account for the smallness of the neutrino masses.

Let us summarize the phenomenological drawbacks of this setup once more. Though the instantons required to generate the desired, but perturbatively forbidden, couplings do not induce any R-parity violating couplings, this setup still has
two major flaws. First, the u-quark couplings are generated non-perturbatively and are thus suppressed relative to the d-quark couplings, which are perturbatively allowed. Second, the instanton inducing the Dirac mass term for the neutrinos also generates a Majorana mass term for the right-handed neutrinos. Thus, the seesaw mechanism gives neutrino masses which are much smaller than the observed values.

3.2 Four-stack Models

Another very natural way of realizing the MSSM is to embed the matter content at intersections of four stacks of D-branes, which wrap generic cycles and give rise to the gauge symmetry

\[ U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d \]

The left-handed quarks \( q_L \) are localized at the intersection of brane \( a \) with brane \( b \) or its orientifold image \( b' \). The right-handed quarks, \( u_R \) and \( d_R \), arise at intersections of brane \( a \) with one of the \( U(1) \) branes or its orientifold image, or at the intersection of stack \( a \) with its orientifold image, in which case the right-handed quarks would transform as antisymmetric of \( SU(3) \). The left-handed leptons are charged under the \( U(2) \) and are neutral under \( U(3) \), and thus appear at intersections of brane \( b \) with one of the \( U(1) \) branes or its orientifold image. Finally, the right-handed electron \( E_R \) and the right-handed neutrino \( N_R \), both singlets under \( U(3) \) and \( U(2) \), arise at the intersection of two \( U(1) \) branes or at the intersection of \( b \) with \( b' \), in which case they would transform as antisymmetric of \( SU(2) \).

For the four-stack models, the hypercharges compatible with the MSSM hypercharge assignment are [14]

- \( U(1)_Y = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b \)
- \( U(1)_Y = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b - \frac{1}{2} U(1)_d \)
- \( U(1)_Y = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b + U(1)_d \)
- \( U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c \)
- \( U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c - \frac{1}{2} U(1)_d \)
- \( U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c - \frac{3}{2} U(1)_d \)

Before discussing the different hypercharge setups in detail, let us summarize what we call a phenomenologically favorable setup:

- All the MSSM matter content and the right-handed neutrinos, apart from the Higgs fields, appear as chiral fields at an intersection between two D-branes. We emphasize again that all the MSSM matter and the right-handed neutrinos have to appear at intersections between the four D-branes.
and their orientifold images. Moreover, there are no further chiral fields charged under the gauge groups of the four D-branes.

- As discussed in chapter 2, the tadpole condition puts restrictions on the transformation behavior of the matter fields. We require these constraints to be satisfied.

- In chapter 2 we derived constraints on the transformation properties of the matter fields arising from the presence of a massless $U(1)$ gauge symmetry. We require that these constraints, given by equations (10) and (11), are satisfied by the hypercharge $U(1)_Y$.

- We forbid any R-parity violating couplings on perturbative level. These include the couplings $L L E_R$, $u_R d_R d_R$, $q_L L d_R$ and $L H_u$.

- All the Yukawa couplings which are missing, due to being perturbatively forbidden, are generated by instantons. The desired couplings are:

$$q_L H_u u_R \quad q_L H_d d_R \quad L H_d E_R \quad L H_u N_R \quad H_u H_d .$$

We require that all three families of the u-quark, d-quark, and electron acquire a mass. This translates into the condition that the associated Yukawa matrices, $Y^{u}_{qL \rightarrow u_R}$, $Y^{d}_{qL \rightarrow d_R}$ or $Y^{d}_{L H_d \rightarrow E_R}$ have non-zero eigenvalues, where the entries of the respective Yukawa matrix can be perturbatively or non-perturbatively generated. For the neutrinos we allow one generation to be light, or even massless. Thus the Yukawa matrix $Y^{L}_{L H_u \rightarrow N_R}$ may exhibit one zero eigenvalue. The other two eigenvalues must be non-vanishing.

- Often times an instanton which is required to generate the Yukawa couplings also induces a tadpole $N_R$ and thus an instability for the setup. We rule out any setup which requires the presence of such an instanton.

- As seen in 3.1.1 an instanton which is required to generate the Yukawa couplings might also generate an R-parity violating coupling of the same order. Quivers which require such instantons are ruled out as unrealistic.

- We rule out setups which lead to a large family mixing in the quark Yukawa couplings. For example, this might happen if the left-handed quark has two possible origins, namely the $ab$ sector and the $ab'$ sector. As encountered in [69,75] the quark Yukawa matrices, taking into account only perturbative contributions, take the form

$$Y^{P}_{u_L \rightarrow H_u u_R} = \begin{pmatrix}
A_{u_{11}}^{u} & A_{u_{12}}^{u} & A_{u_{13}}^{u} \\
A_{u_{21}}^{u} & A_{u_{22}}^{u} & A_{u_{23}}^{u} \\
0 & 0 & 0
\end{pmatrix} \quad Y^{P}_{d_L \rightarrow H_d d_R} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
A_{d_{31}}^{d} & A_{d_{32}}^{d} & A_{d_{33}}^{d}
\end{pmatrix} .$$

(43)
Such a Yukawa texture suggests that the u-quark mass for the two heaviest families is much larger than the one for the lightest family after also taking into account instanton effects which will fill the zero entries in the matrix $Y^P_{uLH_u uR}$. On the other hand it also suggests that the family with the lightest u-quark also contains the heaviest d-quark, in contradiction with experimental observation. Setups with such Yukawa matrix texture lead to a large mixing between different families which is not observed in nature. Therefore we rule out such setups, since they are phenomenologically unrealistic.

- If the $\mu$-term is perturbatively forbidden, we require it to be generated non-perturbatively. If an instanton simultaneously generates a perturbatively forbidden, but desired, Yukawa coupling and the $\mu$ term, the latter is generically too large. Such a situation was encountered in [69], where it was shown that this problem can be overcome if one allows for a second Higgs pair. In this work we do not allow for a second Higgs pair, since we restrain ourselves to the exact MSSM matter content plus three right-handed neutrinos. We leave it for future work to make a systematic analysis where one allows for a second Higgs pair.

- We require that a phenomenologically realistic setup exhibits a natural explanation for the smallness of the neutrino mass. As discussed in section 2.5 and in [46], the non-perturbative realization of the Weinberg operator does not give neutrino masses in the desired range unless the string mass is significantly lowered.

In [73], the authors present an intriguing mechanism which is based on the non-perturbative generation of the Dirac mass term for the neutrinos. We saw a realization of this mechanism in the second setup of section 3.1.1. For this mechanism to work, instantons inducing a Majorana mass term for the right-handed neutrinos must be absent. In particular, the same instanton which generates the Dirac mass term must not also generate the Majorana mass term. Otherwise the neutrino masses are not in the observed range, as we saw in section 3.1.2.

Another possibility to obtain small neutrino masses is via the seesaw mechanism. A necessary ingredient for this mechanism is a large Majorana

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14 Analogously we rule out other setups which exhibit Yukawa textures that also lead to similar problems, for instance the following Yukawa textures:

$$
Y^P_{uLH_u uR} = \begin{pmatrix}
A^u_{11} & 0 & 0 \\
0 & A^u_{22} & 0 \\
0 & 0 & 0
\end{pmatrix},
$$

$$
Y^P_{dLH_d dR} = \begin{pmatrix}
0 & 0 & 0 \\
A^d_{21} & 0 & 0 \\
A^d_{31} & 0 & 0
\end{pmatrix}.
$$

lead to a large, undesired family mixing.
mass term for the right-handed neutrinos, which can be generated non-perturbatively [25, 27, 46–49]. We impose the same constraints on the Majorana mass generating instantons that we impose on instantons which generate Yukawa couplings. Specifically, we require these instantons do not generate R-parity violating couplings, $N_R$ tadpoles, or give rise to a $\mu$-term that is too large.

- Due to the fact that the top quark mass is so much larger than all other masses of the elementary particles, we require that its Yukawa coupling is realized perturbatively. The non-perturbative suppression of other couplings relative to the top Yukawa coupling then gives a natural explanation for the observed hierarchies, without too much fine-tuning. We also allow for quivers where none of the Yukawa couplings $q_L H_u u_R$, $q_L H_d d_R$ and $L H_d E_R$ is perturbatively realized. For such quivers one can still obtain a natural hierarchy between the top quark mass and all other matter fields if the instanton inducing the top Yukawa coupling is least suppressed.

For every choice of hypercharge, we present the potential transformation behavior under the gauge groups and give the number of setups which satisfy the various phenomenological conditions listed above. The small subset of quivers which are compatible with all these constraints are listed in tables. The latter represent a good starting point for concrete string realizations of the MSSM with realistic Yukawa textures.

3.2.1 $U(Y) = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b - \frac{1}{2} U(1)_d$

$q_L : (a, \bar{b})$

$u_R : \square_b$

$d_R : (\bar{a}, c), (\bar{a}, \bar{c})$

$L : (b, \bar{c}), (b, c)$

$E_R : \square_b, \square_u$

$N_R : \square_c, \square_c$

$H_u : (\bar{b}, c), (\bar{b}, \bar{c})$

$H_d : (b, \bar{c}), (b, c)$

There exist a few solutions that satisfy the tadpole and masslessness conditions. For all these solutions none of the matter fields is charged under $U(1)_d$. Thus they correspond to the three-stack quivers analyzed in section 3.1.2. For this and all following choices of hypercharge we require that all four D-branes are populated. Therefore for this choice of hypercharge we do not find any solutions which satisfy the tadpole and masslessness constraints.
3.2.2 $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c$

\[
\begin{aligned}
q_L & : & (a, \bar{b}), & (a, b) \\
u_R & : & (\bar{a}, \bar{c}) \\
d_R & : & (\bar{a}, c), & \emptyset \\
L & : & (b, \bar{c}), & (\bar{b}, \bar{c}) \\
E_R & : & \emptyset \\
N_R & : & \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \\
H_u & : & (\bar{b}, c), & (b, c) \\
H_d & : & (b, \bar{c}), & (\bar{b}, \bar{c})
\end{aligned}
\]

This hypercharge exhibits 8 tadpole free setups with the exact MSSM matter content and a massless $U(1)_Y$. Out of these, only one does not give rise to R-parity violating couplings on the perturbative level. If one includes the instanton required to induce the desired Yukawa couplings, the same instanton will also generate R-parity violating couplings, and thus this setup is unrealistic.

3.2.3 $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{3}{2}U(1)_d$

\[
\begin{aligned}
q_L & : & (a, \bar{b}), & (a, b) \\
u_R & : & (\bar{a}, \bar{c}) \\
d_R & : & (\bar{a}, c), & \emptyset \\
L & : & (b, \bar{c}), & (\bar{b}, \bar{c}) \\
E_R & : & \emptyset, & (\bar{c}, \bar{d}) \\
N_R & : & \emptyset, \emptyset, \emptyset, \emptyset \\
H_u & : & (\bar{b}, c), & (b, c) \\
H_d & : & (b, \bar{c}), & (\bar{b}, \bar{c})
\end{aligned}
\]

For this hypercharge, one obtains 16 models with the exact MSSM matter content which also satisfy the constraints arising from tadpole cancellation and the masslessness of the hypercharge. Only 2 of them do not give rise to any R-parity violating couplings on the perturbative level. Moreover, one of these is unrealistic since the instanton generating the desired, but perturbatively forbidden, Yukawa couplings also induces R-parity violating couplings. The only surviving setup is displayed in Table 6.\(^{15}\)

\(^{15}\)Let us briefly explain how to read the table. Any given row specifies one solution, and the
This quiver allows for an additional massless $U(1)$, satisfying (10) and (11), which is given by

$$U(1)^{add} = \frac{2}{3} U(1)_a + U(1)_b.$$  \hspace{1cm} (44)

Note though that the constraints (10) and (11) are just necessary conditions, and whether $U(1)^{add}$ is indeed massless needs to be checked for a concrete realization. Assuming $U(1)^{add}$ is massless the gauge symmetries in four dimensional spacetime is

$$SU(3) \times SU(2) \times U(1)_Y \times U(1)^{add},$$  \hspace{1cm} (45)

which would forbid the generation of a Majorana mass term for the right-handed neutrinos. Since the Dirac neutrino mass is perturbatively realized, $U(1)^{add}$ being massless implies that the seesaw does not work, and there is no mechanism to account for the smallness of the neutrino mass.

Subsequently, for the other choices of hypercharge, we will encounter for a few quivers a similar scenario, where the presence of an additional massless $U(1)$ might forbid desired couplings. We will denote such quivers with a †. Let us emphasize again that even though the linear combination (44) does satisfy the constraints (10) and (11), this is not sufficient to conclude that $U(1)^{add}$ is indeed massless for a concrete realization.

| Solution # | $q_L$ | $d_R$ | $u_R$ | $L$ | $E_R$ | $N_R$ | $H_u$ | $H_d$ |
|------------|-------|-------|-------|-----|-------|-------|-------|-------|
| 1†         | $0$   | $3$   | $0$   | $3$ | $3$   | $0$   | $3$   | $0$   |

Table 6: Spectrum for the solution with $U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c - \frac{3}{2} U(1)_d$.

Columns display the potential transformation behaviors of the MSSM matter content giving rise to right charge under the Standard model gauge groups. For every solution the table indicates how many matter fields have the respective transformation behavior. Note that the choice of hypercharge may lead to some symmetries within the MSSM spectrum. For instance any solution for $U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c - \frac{3}{2} U(1)_d$ is also a solution under the exchange of $b \rightarrow b'$. We take into account these types of symmetries and only present nonequivalent solutions.
3.2.4 \( U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b \)

\[
q_L : \quad (a, \bar{b})
\]

\[
u_R : \quad \begin{array}{c}
\begin{array}{c}
\text{SU}(3)
\end{array}
\end{array}
\]

\[
d_R : \quad (\bar{a}, c), \quad (\bar{a}, \bar{c}), \quad (\bar{a}, d), \quad (\bar{a}, \bar{d})
\]

\[
L : \quad (b, \tau), \quad (b, c), \quad (b, d), \quad (b, d)
\]

\[
E_R : \quad \begin{array}{c}
\begin{array}{c}
\text{SU}(3)
\end{array}
\end{array}
\]

\[
N_R : \quad (c, \bar{d}), \quad (\bar{c}, d), \quad (c, d), \quad (\bar{c}, d), \quad \begin{array}{c}
\begin{array}{c}
\text{SU}(3)
\end{array}
\end{array}
\]

\[
H_u : \quad (\bar{b}, c), \quad (\bar{b}, \bar{c}), \quad (\bar{b}, d), \quad (\bar{b}, \bar{d})
\]

\[
H_d : \quad (b, \tau), \quad (b, c), \quad (b, d), \quad (b, d)
\]

Allowing for an additional D-brane compared to the three-stack setups with the same hypercharge, discussed in section 3.1.2, gives rise to 782 models which satisfy the tadpole cancellation and hypercharge masslessness constraints. 135 of these setups do not exhibit any R-parity violating couplings on the perturbative level. Only 44 models turn out to be realistic once we take into account non-perturbative constraints on the instantons which generate the desired Yukawa couplings.

Note that the u-quarks for this hypercharge appear as antisymmetric of \( SU(3) \), and therefore the top Yukawa coupling is never realized perturbatively. However, there are 12 models in which all the Yukawa couplings \( q_L H_u \nu_R, q_L H_d d_R \) and \( L H_d E_r \) are perturbatively forbidden. For these setups it is still possible to generate a hierarchy between the top Yukawa coupling and all other couplings without too much fine-tuning.

It turns out that 3 of these 12 models require a large fine-tuning to generate neutrino masses in the desired range. This is due to the fact that the instantons which induce the Dirac mass terms also generate Majorana mass terms. This was already encountered in section 3.1.2 where we have shown that in such a case the seesaw mechanism generates neutrino masses which are too small. All twelve solutions are displayed in Table 7, where we indicate with a \( \bullet \) the 3 solutions which need a large fine-tuning to obtain neutrino masses in the desired range\(^{16}\).

Four quivers, denoted by \( \dagger \), may exhibit additional massless \( U(1) \)'s which forbid the non-perturbative generation of perturbatively forbidden, but desired couplings. In this case, such setups have to be ruled out as unrealistic.

\(^{16}\)In Table 7 we omit columns which have zero entries for all solutions.
3.2.5 \( U(1)_Y = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b + U(1)_d \)

With this choice of hypercharge there are 144 tadpole free setups with exactly the MSSM matter content and a massless \( U(1)_Y \). Out of these, only 30 do not exhibit any R-parity violating couplings on the perturbative level. Moreover, if we also take into account non-perturbative effects, an additional 8 setups are ruled out. Thus we obtain 22 solutions which not only are tadpole free and do not generate any R-parity violating couplings, but also can generate a \( \mu \) term of the desired order. In order to explain the hierarchy between the top-quark mass and all other matter fields we further require the absence of the Yukawa couplings \( q_L H_a d_R \) and \( L H_d E_R \) on the perturbative level. We obtain 14 models that are displayed in Table 8. The last two quivers in Table 8 may give rise to an additional massless \( U(1)_Y \) which if indeed present forbids some of the desired Yukawa couplings.
Table 8: Spectrum for the solutions with $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d$.

### 3.2.6 $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d$

- $q_L : (a, b), (a, b)$
- $u_R : (\bar{a}, \bar{c}), (\bar{a}, d)$
- $d_R : (a, c), (\bar{a}, \bar{d})$
- $L : (b, \bar{c}), (\bar{b}, \bar{c}), (\bar{b}, d), (b, d)$
- $E_R : (c, \bar{d}), (\bar{c}, \bar{d})$
- $N_R : (\bar{c}, \bar{b}), (c, d), (\bar{c}, \bar{d})$
- $H_u : (\bar{b}, c), (b, c), (b, \bar{d}), (\bar{b}, \bar{d})$
- $H_d : (b, \bar{c}), (\bar{b}, \bar{c}), (\bar{b}, d), (b, d)$

For this choice of massless hypercharge we obtain 3974 tadpole free models, only 480 do not give rise to any R-parity violating couplings at the perturbative level. Including non-perturbative effects the number of realistic models is 51. Moreover, if we require that the top Yukawa coupling is perturbatively present or, in case it is absent, that the d-quark- and the electron Yukawa couplings are also perturbatively forbidden, we get 45 realistic models. For 11 of these 45 models, a large fine-tuning is required to avoid large family mixing and to obtain neutrino masses in the desired range. Note that all setups for which the left-handed quarks $q_L$ arise from two sectors, namely $ab$ and $ab'$, suffer under a too large mixing between different families. To overcome such a mixing a large amount of fine-tuning is required. Nevertheless we display all 45 solutions in Table 9. We indicate the 11 solutions which generically give rise to a unrealistic
CKM matrix with a ♣. Of these 45 solutions, there are 18 models in which the μ-term is perturbatively forbidden and gets generated by an instanton with appropriate charge. In order to get a μ-term of the desired order \(10^2 - 10^3\) GeV the suppression factor is expected to be in the range \(10^{-16} - 10^{-15}\). In 2 of these quivers, the instanton that is needed to generate the μ-term would also induce the R-parity violating coupling \(q_L L d_R\). Since the suppression factor of the μ term generating instanton is highly suppressed we do not exclude these setups. They are marked with a ♣.

Furthermore, 14 quivers may give rise to additional massless \(U(1)'s\) which forbid some of the desired Yukawa couplings. All these are indicated with a †. Again, if the potential massless \(U(1)'s\) become massive via the Green-Schwarz mechanism, then these quivers exhibit realistic Yukawa textures. If massless, though, the additional \(U(1)'s\) might prevent the generation of some desired Yukawa couplings.

Finally, 3 of these quivers run into the same issue discussed in section [3.1.1] where the instanton which generates the Dirac neutrino mass term \(LH_u N_R\) also generates the Majorana mass term \(N_R N_R\). In such a case, the seesaw masses are far below the experimentally observed order. Again we mark these quivers with a •.

### 3.3 \(SU(2)\) Realized as \(Sp(2)\)

As discussed previously, a stack of \(N\) coincident D-branes in general gives rise to a \(U(N)\) gauge group. If the stack wraps an orientifold invariant cycle, however, then the gauge group is \(Sp(2N)\). Since \(Sp(2)\) is isomorphic to \(SU(2)\), we can realize the \(SU(2)_L\) of the MSSM as a \(U(2)\) from a D-brane stack on a generic cycle or as an \(Sp(2)\) arising from a D-brane stack wrapping an orientifold invariant cycle. The former was the subject in sections [3.1] and [3.2] and here we focus on the latter. In this case, all representations are real and it is easy to see that the tadpole equations do not impose any condition on the transformation behavior under the \(Sp(2)\). This suggests that there might be more solutions than for the \(U(2)\) realization of \(SU(2)_L\) considered in the previous chapter.

This conclusion turns out to be too naive for two reasons. First, the \(Sp(2)\) does not exhibit a \(U(1)_b\) which could contribute to the hypercharge. This restricts the possible number of hypercharge choices to the subset of the previously analyzed ones which do not have any contribution from \(U(1)_b\), namely

- \(U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c\)
- \(U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c - \frac{3}{2} U(1)_d\)
- \(U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c - \frac{1}{2} U(1)_d\).

Here the first choice can be realized as three or four stack model while the latter two are quivers based on four stacks of D-branes.
The second reason why there are only a few MSSM realizations is due to the fact that the stack $b$ is identified with its image stack $b'$. This limits the potential origins of fields charged under the $SU(2)_L$. For instance, for the first two hypercharges the leptons and the Higgs pair arise from the same sector $bc$. Thus for these setups the $\mu$-term is perturbatively realized, but also the R-parity violating coupling $LH_u$ is present, which make these configurations unrealistic.

This leaves us with the hypercharge $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d$, which we now discuss.

### Table 9: Spectrum for the solutions with $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d$.
3.3.1 \( U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c - \frac{1}{2} U(1)_d \)

\[
\begin{align*}
q_L &: (a, b) \\
u_R &: (\bar{a}, \bar{c}), (\bar{a}, \bar{d}) \\
d_R &: (\bar{a}, c), (\bar{a}, d), \quad \Box_a \\
L &: (b, \bar{c}), (b, d) \\
E_R &: (c, \bar{d}), \quad \Box_{c,d}, \quad \Box_{c,d} \\
N_R &: (c, d), (\bar{c}, \bar{d}) \\
H_u &: (b, c), (b, \bar{d}) \\
H_d &: (b, \bar{c}), (b, d)
\end{align*}
\]

For this choice of massless hypercharge, we obtain 100 tadpole free models, 8 of which don’t give rise to R-parity violating couplings. Taking into account non-perturbative effects, we are left with one setup, which also has a perturbatively present top Yukawa coupling. This setup is presented in Table 10.

| Solution # | \( Q_L \) | \( d_R \) \((\bar{a}, c)\) | \( u_R \) \((\bar{a}, \bar{c})\) | \( L \) \((c, d)\) | \( E_R \) \((\bar{c}, \bar{d})\) | \( N_R \) | \( H_u \) | \( H_d \) |
|-----------|-----------|-----------------|-----------------|-----------------|-----------------|--------|--------|--------|
| 1 \(\dagger\) | 3         | 3               | 3               | 3               | 3               | 3      | 1      | 1      |

Table 10: Spectrum for the solution with \( U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c - \frac{1}{2} U(1)_d \).

This quiver, whose local realization has been studied in [19, 74], exhibits all desired Yukawa couplings on the perturbative level

\[
\begin{align*}
< q^I_{L(1,0,0)} H_{u(0,1,0)} &> & < q^I_{L(1,0,0)} u^I_{R(-1,-1,0)} > & < q^I_{L(1,0,0)} d^I_{R(-1,1,0)} > \\
< L^I_{(0,0,1)} H_{d(0,-1,0)} &> & < L^I_{(0,0,1)} E^I_{R(0,1,-1)} > & < L^I_{(0,0,1)} N^I_{R(0,-1,-1)} > \\
& & < H_{u(0,1,0)} H_{d(0,-1,0)} >
\end{align*}
\]

(46)

Therefore, the mass hierarchy between the quarks and leptons, as well as the hierarchies within the families, are due to the worldsheat instantons rather than spacetime instantons.

Since the Dirac neutrino masses are perturbatively realized, they are expected to be of the same order as the masses for the other leptons. As in 3.1.1 a Majorana mass term of the right order can account for the measured smallness of neutrino masses via the type I seesaw mechanism. Such a term can be induced by an instanton with the intersection pattern

\[
I^a_{E2a} = 0 \quad I^b_{E2b} = 0 \quad I^c_{E2c} = -2 \quad I^d_{E2d} = -2,
\]

(47)

where the suppression factor should be in the range \(10^{-5}\) to \(10^{-2}\) to account for the observed neutrino masses. Note that this instanton does not induce any
undesired R-parity violating couplings. This setup may give rise to an additional massless $U(1)$

\[ U(1)^{\text{add}} = U(1)_c , \]

which if indeed present would forbid the Majorana mass term for the right-handed neutrino.

4 Conclusion

In this work we systematically investigate MSSM D-brane quivers, arising from three and four stacks of D-branes, with respect to their Yukawa structure. For almost all quivers, various desired Yukawa couplings are perturbatively forbidden due to global $U(1)$ selection rules. D-brane instanton effects can generate these missing couplings and may also account for various hierarchies. Here we analyzed the implications of such non-perturbative effects for the phenomenology of the respective quivers. We find that often times the desired Yukawa coupling inducing instanton also leads to phenomenologically undesired effects. The latter include the generation of R-parity violating couplings, of $N_R$ tadpoles, of a $\mu$-term which is too large, or too large family mixing. Subsequently, such quivers are ruled out as unrealistic.

Furthermore, we require that a viable quiver exhibits a mechanism which can account for the smallness of the neutrino masses. In this work we considered two different scenarios. The first scenario is the well known type I seesaw mechanism, which requires the presence of a large Majorana mass term for the right-handed neutrinos. Such a mass term is perturbatively forbidden, but can be induced by D-instantons with the right zero mode structure. The second scenario assumes that the Dirac mass term violates global $U(1)$ selection rules and thus is perturbatively forbidden. A D-instanton with high suppression factor which compensates for the global $U(1)$ charge carried by the Dirac mass term can account for the small neutrino masses.

In section 3.1 we analyze in detail D-brane quivers with respect to these constraints. We find only one realistic quiver, displayed in Table 2. All others suffer from some phenomenological drawbacks. Either the Yukawa coupling inducing instanton generates R-parity violating couplings, or the top Yukawa coupling is perturbatively forbidden, while some d-quark couplings are perturbatively realized. The latter is in contradiction with observations, which suggest the opposite hierarchy. We also encounter one quiver that does not allow for neutrino masses in the observed range. This is due to the fact that the Dirac mass generating instanton also induces the Majorana mass term for the right-handed neutrinos. Thus the seesaw mass is effectively a non-perturbatively generated Weinberg operator which generically gives too small neutrino masses [46].
Equipped with what we learned from the three stack quivers, we perform a systematic search for phenomenologically viable D-brane quivers based on four D-brane stacks. We show that only a small subset of the D-brane quivers that satisfy the two top-down constraints, tadpole cancellation and presence of a massless hypercharge, give rise to phenomenology compatible with experimental observations. We display these quivers in the tables 6, 7, 8, 9 and 10. These quivers serve as a starting point for future quests for concrete MSSM realizations with realistic Yukawa texture.

Some of these quivers potentially exhibit additional massless $U(1)$'s. These additional symmetries have to be preserved by the superpotential and thus often times various desired Yukawa couplings are forbidden, even at the non-perturbative level. However, the conditions derived in section 2.2 which are constraints on the transformation behavior of the matter fields arising from the masslessness condition, are only necessary conditions. The actual masslessness condition is a constraint on the cycles. Thus, it needs to be checked for a concrete realization if such additional $U(1)$'s are indeed present.

Some quivers discussed in sections 3.1 and 3.2 may exhibit dimension five operators which could lead to rapid proton decay, unless they are sufficiently suppressed. We leave it for future work [76] to analyze the constraints arising from the considerations of these operators (for a similar analysis for $SU(5)$ orientifolds, see [77]).

This bottom-up analysis shows that family splitting, namely that different families of the same matter field arise from different sectors is phenomenologically disfavored. This splitting has been used in [24] as a mechanism to explain the different mass hierarchies in the MSSM. We expect that increasing the number of D-brane stacks allows for MSSM quivers which exhibit such family splitting while not containing phenomenological drawbacks, thus giving a mechanism to explain observed mass hierarchies. It would be interesting to extend the current analysis by performing a detailed analysis of Yukawa textures for higher-stack quivers [76].

We would also like to point out that the approach presented here has broader applications to other corners of the string landscape. While the concrete analysis has been carried out explicitly in the Type IIA context, it has a straightforward map to Type I constructions with magnetized D9-branes. Analogous studies can be carried out along the same lines in the Type IIB context with D-branes at singularities. In such a case, however, the analysis of the corresponding quivers is carried out in a geometric regime of closed sector moduli which are T-dual to a non-geometric regime of Type IIA analysis (and vice versa).

In this work we restrict the spectrum to be the exact MSSM spectrum plus three right-handed neutrinos. As shown in [69], allowing for an additional Higgs pair can overcome the problem of a too large $\mu$-term. We leave it for future work to systematically analyze such D-brane quivers. Furthermore, one might entertain the idea of allowing additional singlets under the standard model gauge
groups, which could acquire a VEV and then induce some of the desired Yukawa couplings \cite{24} via higher order couplings. It would be interesting to see if the splitting of standard model families is still phenomenologically disfavored in an analysis which allows for additional chiral singlets.

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