QCD Calculations of Radiative $B$ Decays

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The current status of $B \to X_s \gamma$ decay rate calculations is summarized. Missing ingredients at the NNLO level are listed. The global normalization factor and non-perturbative effects are discussed. Arguments are presented that results for the cutoff-enhanced perturbative corrections have been misused in Ref. [15] by applying them in the region $E_\gamma \in [1.0, 1.6]$ GeV, which means that the corresponding numerical effect on $\mathcal{B} (B \to X_s \gamma, E_\gamma > 1.6 \text{GeV})$ is unreliable.

1. Introduction

The motivation for precision studies of radiative $B$ decays is well known. First, they are sensitive to new physics loop effects that often arise at the same order in the electroweak couplings as the leading Standard Model (SM) contributions. Moreover, the inclusive $B \to X_s \gamma$ rate is well approximated by the perturbatively calculable radiative decay rate of the $b$-quark. The CLEO [1], BELLE [2] and BABAR [3] measurements have been combined by HFLAG [4] to get

$$\mathcal{B} (B \to X_s \gamma)_{\text{exp}} = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4}$$

for $E_\gamma > 1.6$ GeV. The corresponding SM prediction that was published two years ago\footnote{More recent contributions are discussed in Secs. 2 and 3.} reads [5]

$$\mathcal{B} (B \to X_s \gamma)_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}.$$ 

Its consistency with Eq. (1) provides strong constrains on many extensions of the SM (see, e.g., Ref. [6]).

Resummation of large logarithms ($\alpha_s \ln M_W^2/m_b^2)^n$ in the calculation of the decay rate is most conveniently performed after decoupling the electroweak bosons and the top quark. In the resulting effective theory, the relevant flavour-changing weak interactions are given by a linear combination of dimension-five and -six operators\footnote{The specific matrices $\Gamma_i$ and $\Gamma'_i$ can be found in Ref. [7].}

$$O_{1,2} = (s \Gamma_i)(c \Gamma'_i b), \quad \text{(current-current operators)}$$

$$O_{3,4,5,6} = (s \Gamma_i b) \sum_{q} (q \Gamma'_i q), \quad \text{(four-quark penguin operators)}$$

$$O_7 = \frac{e m_b}{16 \pi^2} s L a^{\mu \nu} b R F_{\mu \nu}, \quad \text{(photonic dipole operator)}$$

$$O_8 = \frac{g m_b}{16 \pi^2} s L a^{\mu \nu} T^{a} b R G_{\mu \nu}^{a}, \quad \text{(gluonic dipole operator)}$$

One begins with perturbatively calculating their Wilson coefficients $C_i$ at the renormalization scale $\mu_0 \sim (M_W, m_t)$. Next, the Renormalization Group Equations (RGE) are used for the evolution of $C_i$ down to the scale $\mu_b \sim m_b/2$. Finally, the operator on-shell matrix elements are calculated at $\mu_b$.

The Wilson coefficient RGE are governed by Anomalous Dimension Matrices (ADM’s) that are derived from ultraviolet divergences in the Feynman diagrams with operator insertions. Around 20000 four-loop diagrams with $O_{1,2}$ insertions have been calculated in Ref. [8] to make the large logarithm resummation complete up to $\mathcal{O} [\alpha_s^2 (\alpha_s \ln M_W^2/m_b^2)]$, i.e. at the Next-to-Next-to-Leading-Order (NNLO) in QCD. Including such corrections is necessary to suppress the theoretical uncertainty in Eq. (2) down to the level of the experimental error in Eq. (1). The numerical effect of the four-loop ADM’s on the branching ratio amounts to around $-4\%$ for $\mu_b = 2.5$ GeV. At present, all the relevant Wilson coefficients $C_i (\mu_b)$ are known at the NNLO [8,9]. However, evaluation of the matrix elements at this order is still in progress — see Sec. 4.

2. The global normalization factor

In order to reduce parametric uncertainties stemming from the CKM angles as well as from the $c$- and $b$-quark masses, one writes the branching ratio as follows [10]

$$\mathcal{B} (B \to X_s \gamma)_{E_\gamma > E_0} = \mathcal{B} (B \to X_c e^\nu)_{\text{exp}} \left| \frac{V_{cb}}{V_{ub}} \right|^2 \times$$

$$\times \frac{6\alpha_{\text{em}}}{\pi C} [P(E_0) + N(E_0)],$$

where $\alpha_{\text{em}} = \alpha_{\text{em}}^{\text{shell}}$, and $N(E_0)$ denotes the non-perturbative correction (see Sec. 3). The $m_c$-dependence of $B \to X_c e^\nu$ is accounted for by

$$C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma (B \to X_c e^\nu)}{\Gamma (B \to X_s \gamma)}.$$ 

while $P(E_0)$ is defined by the perturbative ratio

$$\frac{\Gamma (b \to X_s \gamma)_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2 \Gamma (b \to X_u e^\nu)} = \left| \frac{V_{ts}}{V_{tb}} \right|^2 \frac{6\alpha_{\text{em}}}{\pi} P(E_0).$$

The NNLO expression for the phase-space factor $C$ [5] is a known function of $m_c/m_b$ as well as non-perturbative parameters that affect $N(E_0)$, too. All
these quantities are determined in a single fit from the measured spectrum of the inclusive semileptonic decay $\bar{B} \to X_s e\bar{\nu}_e$. The fits are performed using either the $1S$ or the kinetic renormalization schemes. The corresponding results for $C$ and $m_c$ read

$$
\begin{align*}
C(m_c(m_c)) = & \begin{cases} 
0.582 \pm 0.016 & , \ 1S \ [11], \\
1.224 \pm 0.057 & , \\
0.546^{+0.023}_{-0.033} & , \ \text{kin.} \ [12, 13].
\end{cases}
\end{align*}
$$

(7)

The above $\overline{\text{MS}}$-scheme values of $m_c$ have been obtained from the $1S$- and kinetic-scheme ones using the three-loop and two-loop relations, respectively. The three-loop relation for the kinetic scheme is not yet known. The ratio $C$ is scheme-independent, but it is affected by the so-called weak annihilation contribution $B_{\text{WA}}$ that remains unknown. Since $B_{\text{WA}}$ cancels out in Eq. (4), fixing its value is a matter of convention in the present context. Here, we follow the convention of Ref. [12], namely $B_{\text{WA}}(\mu = m_b/2) = 0$.

The difference between the two determinations of $C$ amounts to 1.6σ when counted in terms of the upper error of the very recent kinetic-scheme result [12]. It is a consequence of using different experimental data sets, methodology and renormalization schemes. Fortunately, the effects of changing $C$ and $m_c$ partially compensate each other in Eq. (3) because $\partial/\partial m_c \ P(E_0) < 0$. For $E_0 = 1.6 \text{ GeV}$, I find

$$
B(\bar{B} \to X_s \gamma) = \begin{cases} 
(3.15 \pm 0.23) \times 10^{-4} & , \ 1S \ , \\
(3.25 \pm 0.24) \times 10^{-4} & , \ \text{kin.} \ ,
\end{cases}
$$

(8)

where the first result is just that of Ref. [3], while the second one has been obtained using the same code but with the input parameters from Refs. [12, 13].

The actual value of $B(\bar{B} \to X_s \gamma)$ in Ref. [12] is somewhat larger than $3.25 \times 10^{-4}$ because $P(E_0)$ was calculated there using the one-loop rather than two-loop determination of $m_c(m_c)$ from $m_c^{\text{kin}}$. In principle, using the one-loop relation is allowed at $O(\alpha_s^2)$ because $P(E_0)$ becomes $m_c$-dependent only at $O(\alpha_s)$.  

### 3. Cutoff-enhanced corrections

The perturbative ratio $P(E_0)$ in Eq. (9) depends on the cutoff energy $E_0$ via the dimensionless parameter

$$
\delta = 1 - \frac{2E_0}{m_b}.
$$

(9)

For very small $\delta$, i.e. close to the kinematical endpoint $E_0 = m_b/2$, the usual (“fixed-order”) perturbative expansion breaks down because the corrections behave like powers of $\ln \delta$. In that region, one needs to resum large logarithms of $\delta$. Such a resummation of the cutoff-enhanced corrections (i.e. corrections enhanced by powers of $\ln \delta$) has been performed up to the NNLO in Refs. [14, 15]. These results constitute a valuable contribution to our knowledge of the photon energy spectrum in the endpoint region. However, they need to be treated with extreme care further from the endpoint, where logarithms of $\delta$ no longer dominate. Naively, one might expect that resummation of small logarithms does not hurt, even if it is not an improvement. Unfortunately, this is not the case because the logarithmic and non-logarithmic terms undergo a strong cancellation away from the endpoint.

In order to illustrate this issue in a simple manner, let us consider only the dominant photonic dipole operator $O_7$ in Eq. (3). When all the other operators are neglected, the fixed-order expression for the cutoff-dependence of $P(E_0)$ is given by

$$
P(E_0) = P(0) \left[1 + \frac{\alpha_s}{\pi} \phi^{(1)}(\delta) + \frac{\alpha_s^2}{\pi^2} \phi^{(2)}(\delta) + \ldots \right]
$$

Each of the functions $\phi^{(k)}$ can be split into two parts: $\phi^{(k)}$ that is polynomial in $\ln \delta$, and $\phi^{(k)}$ that vanishes at the endpoint. The explicit expressions for $k = 1$ read [16]

$$
\begin{align*}
\phi^{(1)} &= \phi^{(1)}_L + \phi^{(1)}_N, \\
\phi^{(1)}_L(\delta) &= \frac{2}{3} \ln^2 \delta - \frac{7}{3} \ln \delta - \frac{31}{9}, \\
\phi^{(1)}_N(\delta) &= \frac{10}{3} \delta + \frac{1}{3} \delta^2 - \frac{2}{9} \delta^3 + \frac{1}{2} \delta^4 \ln \delta.
\end{align*}
$$

(10)

In Fig. 1, $\phi^{(1)}_L$, $\phi^{(1)}$ and their sum are plotted as functions of $E_0$. The endpoint is located at $m_b/2 \approx 2.34 \text{ GeV}$. One can see that $\phi^{(1)}_L$ begins to dominate around $E_0 = 2 \text{ GeV}$. On the other hand, already in the vicinity of $E_0 = 1.6 \text{ GeV}$, the cancellation of the two components of $\phi^{(1)}$ is very strong.
One may wonder whether similar cancellations occur at higher orders, too. A positive answer concerning $\phi^{(2)}$ is immediate because this function can easily be derived from the results of Ref. [17]. The corresponding plot is presented in Fig. 2 for the case when $\alpha_s = \alpha_s^{\text{eff}}(m_b)$ in Eq. (10).

No explicit results at order $\mathcal{O}(\alpha_s^2)$ are available. However, we know on general grounds that all the $\phi^{(k)}$ behave like $(2E_0/m_b)^4$ at small $E_0$. Two powers of $E_0$ originate from $F_{\nu\nu}$ in the vertex $O_7$ in Eq. (3), and additional powers come from the phase-space measure $dE/dE_\gamma$. Consequently, the Taylor expansions of $\phi^{(k)}_L$ and $\phi^{(k)}_N$ at small $E_0$ up to $\mathcal{O}(E_0^3)$ must exactly cancel each other. These Taylor expansions are shown by the dashed lines in Figs. 1 and 2. One can see that both $\phi^{(1)}_L$ and $\phi^{(2)}_L$ are well approximated by the dashed lines in the region below 1.6 GeV. It must also be the case at higher orders because $\phi^{(k)}_L$ are polynomial in $\ln \delta$ that is well approximated by the same expansion in the considered region (see Fig. 3). Thus, cancellations like those shown in Figs. 1 and 2 are expected to occur at any order in $\alpha_s$ (see “Note Added”).

In the approach of Refs. [14, 15], logarithms of $\delta$ have been resummed at the NNLO in $\phi^{(k)}_L$, while $\phi^{(k)}_N$ have been retained in the fixed order. More precisely, Eq. (10) has been effectively re-expressed as

$$P(E_0)/P(0) = X + \frac{\alpha_s(m_b)}{\pi} \phi^{(1)}_N + \frac{\alpha_s^2(m_b)}{\pi^2} \phi^{(2)}_N,$$

and $X$ has been calculated up to $\mathcal{O}(\alpha_s^2 \ln^m \delta)$, with $n = 0, 1, 2, \ldots$, and $m = n, n+1, \ldots, \text{max}(n)$. This is a reasonable approximation only in the region very close to the endpoint where no cancellation between the two components takes place. Elsewhere, it leads to overestimating the numerical effect of the $\mathcal{O}(\alpha_s^2)$ terms in Eq. (10) by a factor of order $|\phi^{(3)}_L/\phi^{(3)}_N| \sim (m_b/(2E_0))^4$ that amounts to around 4.6 for $E_0 = 1.6$ GeV, and around 30 for $E_0 = 1.0$ GeV.

Unfortunately, it was precisely the range $E_\gamma \in [1.0, 1.6]$ GeV where the authors of Ref. [17] applied their results to calculate the effect on $\mathcal{B}(B \to X_\gamma, E_\gamma > 1.6)$ GeV. They adopted the fixed-order result at $E_0 = 1.0$ GeV from Ref. [3]

$$\mathcal{B}(B \to X_\gamma, E_\gamma > 1.0) = 3.27 \times 10^{-4}, \quad (13)$$

and supplemented it with their own numerical value of $[P(1.0) - P(1.6)]/P(0)$ that follows from Eq. (12). That value is almost twice larger than in the fixed-order NNLO calculation. In the end, their result for the branching ratio with a cutoff at $E_0 = 1.6$ GeV was considerably lower than the one in Eq. (2). In view of the above remarks, their prediction should be considered unreliable.

4. Status of the NNLO QCD calculations of the matrix elements

On the l.h.s. of Eq. (12) that defines $P(E_0)$, the denominator is already known at the NNLO [18, 19]. In the expression for the numerator

$$\Gamma(b \to X_\gamma)_{E_\gamma > E_0} = \frac{G_F \alpha_{\text{em}} m_b^5}{32\pi^4} |V_{tb} V^*_{ts}|^2 \times \sum_{i,j=1}^8 C_i^{\text{eff}} C_j^{\text{eff}} G_{ij}(E_0), \quad (14)$$

the quantities $G_{ij}$ are determined by the matrix elements of $O_1, \ldots, O_8$. So far, only $G_{77}$ has been evaluated up to $\mathcal{O}(\alpha_s^2)$ in a complete manner [17, 20, 21]. The remaining $G_{ij}$ are fully known at the Next-to-Leading Order (NLO), i.e. up to $\mathcal{O}(\alpha_s)$ (see Ref. [22] for a complete list of references). At the NNLO, it is practically sufficient to restrict considerations to $G_{ij}$ with $i,j \in \{1, 2, 7, 8\}$ because the four-quark penguin operators have small Wilson coefficients. It is often convenient to apply the optical theorem, and calculate $G_{ij}$ by summing imaginary parts of the $b$-quark propagator Feynman...
diagrams. One can see in Fig. 4 that imaginary parts of three-, four- and five-loop diagrams occur at the NNLO in $G_{77}$, $G_{27}$ and $G_{22}$, respectively.

A relatively simple set of the NNLO contributions is given by diagrams with quark loops on the gluon lines. The quark in the loop is either massive (charm and bottom) or treated as massless (up, down and strange). Such contributions are already known for all the $G_{ij}$ with $i, j \in \{1, 2, 7, 8\}$, except for the massless case in $G_{18}$ and $G_{28}$. The BLM approximation for the complete NNLO correction is derived from the massless quark results.

In Ref. 27, the asymptotic behaviour for $m_c \gg m_b/2$ was calculated for all the non-BLM NNLO corrections to $G_{ij}$ with $i, j \in \{1, 2, 7, 8\}$, except for $G_{18}$ and $G_{28}$. Next, an interpolation in $m_c$ of these corrections was performed assuming that the interpolated quantities vanish at $m_c = 0$. The result of that procedure was an essential input for the NNLO estimate. The overall error of around 7% in the branching ratio was obtained by combining in quadrature four types of uncertainties: 5% non-perturbative, 3% parametric, 3% higher-order, and 3% due to the $m_c$-interpolation ambiguity.

The results in Eqs. (2) and (8) do not include several contributions to the branching ratio that are known at present. These additional effects are summarized in Tab. I. They sum up to around +1.6%, which is small when compared to the overall uncertainty of around 7%. Therefore, Eq. (8) can still be treated as an up-to-date SM prediction.

As the reader has already noticed, even in the $m_c$-interpolation approach of Ref. 27, there are still some missing NNLO ingredients, namely:

- the BLM contributions to $G_{18}$ and $G_{28}$.
- the large-$m_c$ results for $G_{78}$ and $G_{88}$.

Their numerical effect on the branching ratio is expected to remain within the estimated higher-order uncertainty of around 3%. The calculation of the most interesting $G_{78}$ is very advanced 30, and the results should become available soon (for any value of $m_c$). As far as $G_{88}$ is concerned, its calculation in the large-$m_c$ limit will automatically give the result for any value of $m_c$.

For the full NNLO calculation, the currently missing ingredients (apart from the ones listed above) are the non-BLM corrections to $G_{17}$ and $G_{27}$, $G_{11}$, $G_{12}$ and $G_{22}$, $G_{18}$ and $G_{28}$.

The calculation of (i) in the $m_c = 0$ limit is quite advanced, but also extremely difficult and time-consuming — see Ref. 31 for the status reports. Once it is finished, the main challenge will amount to finding (ii), even for $m_c = 0$. The corrections (iii) are expected to be numerically less important.

When the non-BLM corrections are known at $m_c = 0$ sometime in the future, the interpolation in $m_c$ is still going to be necessary. However, our error estimates should become more solid once the BLM approximation is no longer used at the boundary.

Finding all the non-BLM corrections for the actual value of $m_c \approx m_b/4$ is even more difficult, but it must be considered at some point, too. A calculation of the IR-divergent two-particle-cut contributions to (i) is being currently performed for arbitrary $m_c$ 31. The IR-convergent two-particle-cut contributions to (ii) for arbitrary $m_c$ are already known because they are given by products of the NLO corrections. The $(n \geq 3)$-particle-cut contributions to (ii) vanish at the endpoint, so their numerical relevance should be diminished by the high photon energy cutoff.

Apart from the NNLO corrections, there are other perturbative contributions to $\Gamma(b \to X_s \gamma)_{E_C > E_D}$ that have been neglected so far, namely tree-level diagrams with the $u$-quark analogues of $O_{3,4,5}$ and the four-quark penguin operators $O_{4,4,5,6}$. Such contributions are suppressed with respect to the leading term by $|C_{1,2}^u, C_{4,5,6,7}^{4}/C_7|^2 \leq 0.2$, where $C_{1,2}^u = (V_{us}^* V_{ub})/(V_{ts}^* V_{tb}) C_{1,2}$, as well as by the high photon energy cutoff. A quantitative verification of how small they really are should become available soon 32.
5. Non-perturbative effects

Let us begin with considering a simplified world where \(m_c = m_b\). There, in the decay of the \(\bar{B}\) meson, a high-energy photon (\(E \sim m_b/2\)) can be produced in four different ways:

1. Hard: The photon is emitted directly from the hard process of the \(b\)-quark decay.

2. Conversion: The \(b\)-quark decays in a hard way into quarks and gluons only. Next, one of the decay products scatters in a non-soft radiative manner with the remnants of the \(\bar{B}\) meson. This can be viewed as a parton-to-photon conversion in the QCD medium.

3. Collinear: In the process of hadronization, a collinear photon is emitted.

4. Annihilation: An energetic \(q\bar{q}\) state produced in the \(\bar{B}\) meson decay disintegrates radiatively.

The hard way would be the only one if no other operators but \(O_7\) were present in the effective theory. Non-perturbative effects in such a case were first analyzed in Ref. [28]. They arise as corrections of order \(\Lambda^2/m_b^2\) to \(\Gamma(b \to X_s\gamma)_{E>2E_0}\) when \((m_b - 2E_0) \sim m_h\). Moreover, these \(\mathcal{O}(\Lambda^2/m_b^2)\) terms cancel out in \(N(E_0)\) with the analogous non-perturbative corrections to the charmless semileptonic rate. Thus, we are left with the small \(\mathcal{O}(\Lambda^3/m_b^3)\) effects [33]. The \(\mathcal{O}(\Lambda^2/(m_b - 2E_0)^2)\) corrections are also small [32] for \(E_0 \leq 1.6\) GeV. All these terms are included in Eq. (5), and affect the branching ratio by around \(-0.7\%\).

The photon production via conversion is suppressed both by \(\alpha_s\) (due to the non-soft scattering) and by \(\Lambda/m_b\) (due to dilution of the target). The analysis in Ref. [28] shows that no other suppression factors occur. An effect on the branching ratio of roughly \(-1.5 \pm 1.5\%\) was found in that paper (see Tab. [1]).

In our simplified case \((m_c = m_b)\), the tree-level decay \(b \to (\text{quarks} \& \text{gluons})\) is possible only via the operators \(O_{3,4,5,6,8}\), and by the \(u\)-quark analogues of \(O_{1,2}\). Consequently, the collinear photon emission is suppressed either by \(\alpha_s|C_8/C_7|^2 \leq 0.08\), or by \(|(C_{1,2,4,5,6}^u)/(C_7)|^2 \leq 0.2\). Moreover, there is an additional suppression by products of the quark electric charges and, most importantly, by the high photon energy cutoff. The results of Ref. [29] lead to an estimate that the non-perturbative collinear effects due to \(O_8\) amount to around \(-0.2\%\) in the branching ratio for \(E_0 = 1.6\) GeV (see Tab. [1]).

As far as annihilation is concerned, photons originating from decays of \(\pi^0, \eta, \eta'\) and \(\omega\) are removed on the experimental side as (huge) background. Contributions from the other established \(q\bar{q}\) mesons are negligible. The corresponding perturbative diagrams are responsible for only around 0.1% of the total rate for \(E_0 = 1.6\) GeV.

Once the assumption \(m_c = m_b\) is relaxed, the \(\mathcal{O}(\Lambda^2/m_b^2)\) hard effects from \(O_{1,2}\) get replaced by a series of the form

\[
\frac{\Lambda^2}{m_b^2} \sum_{n=0}^{\infty} b_n \left(\frac{m_b}{m_c^2}\right)^n,
\]

with quickly decreasing coefficients \(b_n\). The calculable leading term has been included in Eq. (5). It affects the branching ratio by around +3.1%.

All the quantitatively estimated non-perturbative effects that have been mentioned so far sum up to

\[
(-0.7 - 0.2 - 1.5 + 3.1)\% = +0.7\%.
\]

However, since their evaluation is often very uncertain, and the knowledge of \(\mathcal{O}(\alpha_s\Lambda/m_b)\) contributions is by no means complete, a non-perturbative error of \(\pm 5\%\) has been assumed in Eq. (1). Probably the most interesting of all the unknown \(\mathcal{O}(\alpha_s\Lambda/m_b)\) effects originate from charm annihilation in the massive (\(\bar{c}s/\bar{q}c\)) intermediate states \((q = u\ or\ d)\), for the actual value of \(m_c \simeq m_b/4\).

One should remember that the error in Eq. (1) is affected by a non-perturbative theoretical uncertainty, too. It follows from the fact that the actual measurements are not performed with \(E_0 \simeq 1.6\) GeV. The most precise experimental results correspond to significantly higher photon energy cutoffs for which the \(\mathcal{O}(\Lambda^2/(m_b - 2E_0)^n)\) effects are no longer small. These effects are described by a non-perturbative shape function [37] that is constrained by the semileptonic data. The very recent analysis [38] of this function and its effects on the \(\bar{B} \to X_s\gamma\) photon spectrum can hopefully provide input for the future HFAG averages.

6. Conclusions

Thanks to the efforts of the past years, the uncertainties in \(\mathcal{B}(\bar{B} \to X_s\gamma)\) have reached the level of around \(\pm 7\%\) on both the experimental and theoretical sides. A significant progress in the perturbative calculations is expected in the near future. However, understanding the \(\mathcal{O}(\alpha_s\Lambda/m_b)\) non-perturbative effects remains the key issue.

7. Note Added

After the first version of this article was submitted to the arXiv, Einan Gardi pointed out to me that the approximate cancellation of logarithmic and non-logarithmic terms (Sec. [3] has already been discussed in Ref. [39].
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