Modeling of High-Speed Motor Drive Systems Considering Two Kinds of Vibrations

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Abstract. In order to realize energy-saving motor drive systems, the energy consumed by the motor drive must be reduced as much as possible. To this end, we propose a new and integrated model for motor drives including both the bending vibration and the torsional vibration as well as the gyroscopic effect in this paper. Special attention is paid to the coupling of these two kinds of vibrations. The approach is based on Lagrangian mechanics and the precise computation of the energies in the bending vibration and the torsion vibration from the viewpoint of flexible material mechanics.

1. Introduction
In modern industries, rotary machines play a very important role in manufacturing, which are driven by AC or DC motors. In many applications a high-speed operation is desired in order to enhance the operation efficiency. Such demand is particularly high in middle-to-high power machines such as compressors, air blowers in mines and fly-wheels in energy storages. The major challenges in high-power, high-speed operation of rotary machines are: safety, energy efficiency.

In this project, the magnetic bearing (MB) is adopted for safety and energy-saving. In addition, to lower the energy consumed by the motor drive itself, it is necessary to reduce the inertia of the shaft. That is, either a short or a slender shaft should be used. In many applications it may not be able to use a short shaft in the motor drive. Meanwhile, a slender shaft may bend easily at high-speed rotations because the critical speed gets lower. This bending vibration poses a great danger to the MB and must be avoided by suitable control. To this end, a model that captures the complicated behavior of the flexible shaft is a must.

Traditionally, the bending vibration and the torsional vibration are modeled and controlled separately [3, 4]. However, in high-speed operation of a flexible shaft the bending vibration is coupled with the torsional vibration. So, a model capturing this dynamics coupling is indispensable for realizing an energy-saving control.

This paper aims at building an integrated dynamic model for high-speed flexible shafts accounting for these two kinds of vibrations and their coupling, thus paving the road for the design of an integrated vibration control system. The approach is to apply the Lagrangian
dynamics and compute all energies in the bending and torsional vibrations from the viewpoint of flexible material mechanics.

2. Motor drive system
The motor drive system is a shaft supported by a 5-degree-of-freedom magnetic bearing as shown in Figure 1. The origin of the frame is put at the midpoint between the two radial MBs and along the rotation center line of the shaft. The $z$-axis conforms to the shaft axis, the $x$-axis and $y$-axis conform to the directions of the attractive forces of MB. $l_i$ denotes the distance from the coordinate origin to the radial bearings and assumed a constant. This is ensured by a suitable control of the axial MBs.

Further, due to manufacturing and assembling errors, the shaft has a decentering $\varepsilon$ as well as an angle $\tau$ between the $z$-axis and the principal axis of shaft inertia.

3. Modeling of coupled bending and torsional vibrations under shaft rotation
The motion equations of the bending-torsional vibration at high-speed motor drive with a decentering are derived by using Euler-Lagrange equation.

The shaft is divided into $n$ finite elements. Figure 2 illustrate the $i$th element with a decentering. Note that $O$ is the center of the static frame, $S$ is the center of the shaft, and $G$ is the center of gravity. Further, $\rho(= OS)$ is the displacement along the radial direction and caused by the bending vibration, and $\varepsilon(= SG)$ is the distance between the decentering and the shaft center.

The kinetic energy $V_i$ of the $i$th finite element is the sum of shaft kinetic energy in each axis direction:

$$V_i = \frac{1}{2} \{m(x_{Gi}^2 + y_{Gi}^2) + J_r(\dot{\theta}_{zGi}^2 + \dot{\theta}_{yGi}^2) + J_{ai}(\dot{\phi}^2 + \phi_i^2)\}. \quad (1)$$

Moreover, the potential energy $T_i$ of the $i$th finite element is given as follows by using the stiffness parameters

$$T_i = \frac{1}{2} \{k_{11}(x_i^2 + y_i^2) + k_{22}(\theta_{yi}^2 + \theta_{xi}^2) + k_\phi \phi_i^2\} \quad (2)$$

where $k_{11} = 12EI_{ri}/l_i^3$, $k_{22} = 4EI_{ri}/l_i$, $k_\phi = GK/l_i$. 

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**Figure 1.** 5 DOF AMB

**Figure 2.** Eccentric position in coupled tortional and bending vibrations
Substitution of these energy functions into Euler-Lagrange equation yields

\[
\begin{align*}
    & m\ddot{x}_i + k_{11}x_i = m\varepsilon\{(\omega_z + \dot{\phi})^2 \cos(\omega_z t + \phi) + \ddot{\phi}\sin(\omega_z t + \phi)\} \quad (3) \\
    & J_i\ddot{\theta}_yi + k_{22}\theta_yi = J_i\tau\{(\omega_z + \dot{\phi})^2 \sin(\omega_z t + \phi) - \ddot{\phi}\cos(\omega_z t + \phi)\} \quad (4) \\
    & m\ddot{y}_i + k_{11}y_i = m\varepsilon\{(\omega_z + \dot{\phi})^2 \sin(\omega_z t + \phi) - \ddot{\phi}\cos(\omega_z t + \phi)\} \quad (5) \\
    & J_i\ddot{\theta}_{xi} + k_{22}\theta_{xi} = J_i\tau\{(\omega_z + \dot{\phi})^2 \cos(\omega_z t + \phi) + \ddot{\phi}\sin(\omega_z t + \phi)\} \quad (6) \\
    & J_{ai}\ddot{\phi}_i + k_{phi}\phi_i \approx m\varepsilon\{\ddot{x}_i \sin(\omega_z t + \phi) - \ddot{y}_i \cos(\omega_z t + \phi)\} + J_i\tau\{\ddot{\theta}_{xi} \sin(\omega_z t + \phi) - \ddot{\theta}_{yi} \cos(\omega_z t + \phi)\}. \quad (7)
\end{align*}
\]

The right sides are the internal forces and torques stemming from the decentering.

For the MB, the magnetic force and stiffness of the input work come in. When the gyro moments of the shaft are considered, the motion equations of vibration become

\[
\begin{align*}
    \begin{cases}
    M_i\ddot{x} + C\ddot{x} + (K_i + K_a)x = u_i + w_i, \quad (8a) \\
    M_i\ddot{\phi} + K_i\phi = u_i + w_i. \quad (8b)
    \end{cases}
\end{align*}
\]

Here \(x\) is the vector of displacements and bending angles, \(\phi\) is the vector of torsional angles, \(w_i, w_i\) are the vectors of internal forces and torques, \(u_i\) is the MB input vector and \(u_i\) is the motor torque input. Note that \(j\)th and \(k\)th elements are controlled by the forces of the MB.

4. Numerical example

The motor drive system used in this example is similar to [3], except that the elasticity is considered. The material of the shaft is iron. The number of finite elements is \(n = 6\). A decentering \(\varepsilon = 0.01\) mm and a difference angle \(\tau = 0.01\) rad from the principal axis of shaft inertia are assumed at the 3rd central position. The finite elements on which the electromagnets act are the 1st and 5th.

As the boundary condition, the translational displacements of the 0th and 6th finite elements are assumed as 0 for the bending vibration. For the torsional vibration, the angle of the 0th finite element is taken as the base, and those of all other elements are illustrated by their relative displacement from the 0th torsional angle.

The subsequent figures indicate the behavior of each element. The displacement response of a finite element is illustrated by a specific color and Table 1 shows the correspondence.

| Table 1. Correspondence between element number and color |
|--------------------------------------------------------|
| Color        | • | • | • | • | • | • |
| Element number | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

By an eigenvalue analysis on the equations (8a) and (8b), the primary bending vibration velocity is obtained as \(\omega_z = 3861\) rad/s. The vibration at primary bending vibration velocity is shown from Figure 3 to Figure 6. Figure 3 shows the \(x\) displacement of each finite element in the coupled bending and torsional vibration, in which the 3rd element has the largest amplitude because the decentering is placed at it. The 1st and 5th elements have the second largest amplitudes that are almost the same. Figure 5 is the torsional angle between adjacent elements in the same coupled motion. The amplitudes increase when the element gets far away from the motor side (0th element). Meanwhile, Figures 4 and 6 illustrate the difference between the displacements in the coupled bending and torsional vibrations and the pure bending or pure
torsional vibrations. As can be conceived from the (8a) and (8b), since the internal forces $w_t$ and $w_b$ depend on the displacement of the bending deformation, a difference in vibration occurs both in bending and torsion at the bending critical speed. These two figures demonstrate the coupling effect. When a slender shaft is used, this coupling will get more severe.

![Figure 3. $x$ displacement (bending-torsion)](image1)

![Figure 4. $x$ difference](image2)

![Figure 5. $\phi$ displacement (bending-torsion)](image3)

![Figure 6. $\phi$ difference](image4)

On the other hand, at any higher but non-critical speed the displacements are much smaller and ignorable.

5. Conclusion
In this paper, we established a new and integrated dynamic model for high-speed motor drives consisting of magnetic bearing and low-inertia shaft. This integrated model includes the bending vibration, torsional vibration and gyroscopic motion of the shaft as well as their dynamic couplings. Numerical simulations validated that when the motor speed got closer to the critical speed, the coupling between the bending vibration and the torsional vibration was excited. This model forms the first step toward the construction of a safe, energy-saving and high performance motor drive control system for high-speed and high-power motors.

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