Fourier analysis of real-time, high-statistics solar neutrino observations

G. L. Fogli, E. Lisi, and D. Montanino

Dipartimento di Fisica and Sezione INFN di Bari,
Via Amendola 173, I-70126 Bari, Italy

Abstract

Solar neutrino oscillations with wavelengths comparable to the Earth-Sun distance provide a viable explanations of the long-standing solar neutrino deficit. They imply a time-dependent modulation of the solar neutrino flux due to the eccentricity of the Earth orbit. Motivated by this testable prediction, we propose a Fourier analysis of the signal observable in real-time, solar neutrino experiments. We give the general expressions of the Fourier coefficients and of their correlated uncertainties in the presence of background. The expressions assume a particularly compact form in the case of two-flavor neutrino oscillations in vacuum. We discuss the sensitivity to the lowest harmonics of the new-generation, high-statistics experiments SuperKamiokande, Sudbury Neutrino Observatory, and Borexino.

PACS number(s): 26.65.+t, 13.15.+g, 14.60.Pq
The phenomenon of neutrino oscillations in vacuum [1] represents a possible solution [2–4] to the long-standing solar neutrino problem [5]. Considering for simplicity only the first two neutrino families $\nu_e$ and $\nu_\mu$, the $\nu_e$ survival probability $P$ at a distance $L$ from the Sun is given by

$$P(E) = 1 - \frac{\sin^2 2\theta}{2} \left( 1 - \cos \frac{\delta m^2 L}{2E} \right), \quad (1)$$

where $E$ is the neutrino energy, $\delta m^2$ is the neutrino squared mass difference, and $\theta$ is the vacuum mixing angle.

A tentative evidence for the oscillating term in Eq. (1) comes from the $E$-dependence of the solar neutrino deficit [6] as inferred from the four pioneering solar $\nu$ experiments [7]. The ellipticity of the Earth’s orbit implies a further, striking signature of the oscillation phenomenon, namely, the $L$-dependence of the observed flux (in addition to the trivial $1/L^2$ geometric factor) [8]. In particular, the survival probability $P$ in Eq. (1) is modulated in time by the periodic variation of $L$ which, at first order in the eccentricity ($\varepsilon = 0.0167$), is given by

$$L(t) = L_0 \left( 1 - \varepsilon \cos \frac{2\pi t}{T} \right) + O(\varepsilon^2) \quad (2)$$

with $L_0 = 1$ AU, $T = 1$ yr, and $t = 0$ at the perihelion. The first-generation experiments [9] have not collected enough statistics to test the $L$-dependence of the solar $\nu$ flux [9]. New-generation experiments should instead be able to probe the structure of the neutrino signal in the time domain [10].

In this work we propose to study the neutrino signal in the frequency domain through a Fourier analysis of the periodic variations associated to flavor oscillations in vacuum. This approach is particularly suited with real-time, high statistics experiments such as SuperKamiokande [11] (operating), the Sudbury Neutrino Observatory [12] (SNO, in construction), and Borexino [13] (in construction). It will be seen that the different sensitivity of each experiment to the lowest harmonics can help in discriminating the value of $\delta m^2$ (should the vacuum oscillation solution be confirmed). Compact expressions for the Fourier coefficients and for their uncertainties will be given.
In general, the neutrino event rate \( R(t) \) at the time \( t \) is the sum of a signal \( S(t) \) and of a (supposedly constant) background \( B \),

\[
R(t) = B + S(t) .
\]  

(3)

For symmetry reasons, the analysis can be restricted to the time interval \([0, T/2]\). It is understood that events collected in subsequent half-years must be symmetrically folded in this interval. The data sample consists then of \( N \) events collected at different times \( \{t_i\}_{1 \leq i \leq N} \), with \( t_i \in [0, T/2] \) and \( N \) equal to the total sum of background and signal events, \( N = N_S + N_B \). Notice that, in general, the background-signal separation can be performed on the average rates, but not on an event-by-event basis.

The expansion of the signal in terms of Fourier components \( f_n \) is defined as

\[
S(t) = S \left( 1 + 2 \sum_{n=1}^{\infty} f_n \cos \frac{2\pi nt}{T} \right) ,
\]

(4)

where \( S \) is the time-averaged signal

\[
S = \frac{2}{T} \int_0^{T/2} dt \ S(t) .
\]

(5)

The \( n \)-th harmonic corresponds to a period of \( 1/n \) yr. The explicit form of \( f_n \) reads

\[
f_n = \frac{2}{ST} \int_0^{T/2} dt \ R(t) \cos \frac{2\pi nt}{T},
\]

\[
= \frac{1}{N_S} \sum_{i=1}^{N_S+N_B} \cos \frac{2\pi nt_i}{T},
\]

(6)

(7)

where Eqs. (3) and (4) represent the theoretical definition and the experimental determination of the \( f_n \)’s, respectively. Although in Eq. (3) one can replace \( R(t) \) with \( S(t) \), the use of \( R(t) \) is more general, since background events do contribute to the statistical uncertainties. Notice that the sum in Eq. (7) does not require an event-by-event separation of signal and background, and does not involve any binning in time.

Each \( f_n \) is a linear combination of Poisson random variables \( \xi_t = R(t)dt \) [Eq. (1)], which represent the total number of events collected in the interval \([t, t + dt]\). If only statistical errors are considered, then \( \text{var}(\xi_t) = \xi_t \) and \( \text{cov}(\xi_t, \xi_{t'}) = 0 \), since fluctuations at different
times \( t \) and \( t' \) are uncorrelated \[14\]. The (co)variances of linear combinations of independent random variables are given by \[15\] \( \text{var}(\sum t \alpha_t \xi_t) = \sum t \alpha_t^2 \text{var}(\xi_t) \) and \( \text{cov}(\sum t \alpha_t \xi_t, \sum t \beta_t \xi_{t'}) = \sum t \alpha_t \beta_t \text{var}(\xi_t) \). It follows that

\[
\text{var}(f_n) = \int_0^{T/2} dt \, R(t) \left( \frac{2}{ST} \cos \frac{2\pi nt}{T} \right)^2 ,
\]

\[
\text{cov}(f_n, f_m) = \int_0^{T/2} dt \, R(t) \frac{4}{(ST)^2} \cos \frac{2\pi nt}{T} \cos \frac{2\pi mt}{T} .
\]

The final result for the one-sigma error \( \sigma_n \) affecting \( f_n \) and for the correlation \( \rho_{mn} \) \( (m \neq n) \) is:

\[
\sigma_n = \sqrt{1 + f_{2n} + B/S} ,
\]

\[
\rho_{mn} = \frac{f_{m+n} + f_{|m-n|}}{\sqrt{(1 + f_{2m} + B/S)(1 + f_{2n} + B/S)}} .
\]

The values of \( \sigma_n \) and \( \rho_{mn} \) can be expressed in terms of measured quantities with the substitutions \( ST/2 \rightarrow N_S \) and \( B/S \rightarrow N_B/N_S \).

In the standard (std) case, i.e. in the absence of oscillations, the Fourier transform of the signal

\[
S(t) \propto \left( \frac{L_0}{L(t)} \right)^2 = 1 + 2\varepsilon \cos \frac{2\pi t}{T}
\]

is trivial, only the first coefficient being nonzero and equal to the eccentricity \( \varepsilon \),

\[
f_n^{\text{std}} = \varepsilon \delta_{n1} .
\]

Moreover, in the standard case the statistical errors do not depend on \( n \),

\[
\sigma_n^{\text{std}} = \sqrt{\frac{N_B + N_S}{2N_S^2}} ,
\]

and thus form a “white noise” affecting all the harmonics. The error correlations, as derived from Eq. \[\text{11}\], are given by \( \rho_{mn}^{\text{std}} = (\varepsilon N_S/N)\delta_{m,n,\pm 1} \leq \varepsilon \) and thus are practically negligible.

In the case of two-flavor oscillations the signal is proportional to
\[ S(t) \propto \frac{L_0^2}{L(t)^2} \int dE \lambda(E) [\sigma_e(E)P + \sigma_\mu(E)(1 - P)], \quad (13) \]

where \( \lambda(E) \) is the neutrino energy spectrum, \( \sigma_e(E) \) and \( \sigma_\mu(E) \) are the \( \nu_e \) and \( \nu_\mu \) interaction cross sections, and the probability \( P \) is given by Eq. (11) with \( L \) as in Eq. (2). (It is understood that \( \sigma_e \) and \( \sigma_\mu \) are corrected for energy threshold and resolution effects in each detector.)

Given the signal \( S(t) \) as in Eq. (13), the time integration in the expression of the Fourier coefficients

\[ f_n = \frac{\int_0^{T/2} dt S(t) \cos \frac{2\pi nt}{T}}{\int_0^{T/2} dt S(t)} \]

can be performed analytically. The final result can be cast in the following, compact form:

\[ f_n = \frac{\varepsilon \delta_{n1} - \sin^2 2\theta D_n(\delta m^2)}{1 - \sin^2 2\theta D_0(\delta m^2)} \quad (n \geq 1), \quad (14) \]

where the detector-dependent functions \( D_n \) are given by

\[ D_n(\delta m^2) = \frac{\int dE \lambda(\sigma_e - \sigma_\mu) U_n}{2 \int dE \lambda \sigma_e} \quad (n \geq 0) \quad (15) \]

and the universal (i.e., detector-independent) functions \( U_n \) are given by

\[ U_n(z) = \delta_{n0} - u_n(z) - \varepsilon[u_{n+1}(z) + u_{n-1}(z) - \delta_{n1}], \]

\[ u_n(z) = \cos \left(z - \frac{n\pi}{2}\right) J_n(\varepsilon z), \]

where \( z = \delta m^2 L_0/2E \) and \( J_n \) is the Bessel function of order \( n \) [16]. Notice that, although our calculations are of \( O(\varepsilon) \), all orders in \( \varepsilon z \) are kept, since \( z \) may be large.

We apply now the Fourier analysis to the signal expected in the SuperKamiokande, SNO, and Borexino experiments. The SuperKamiokande and Borexino cross sections \( (\nu + e \rightarrow \nu' + e) \) are taken from [17], and the SNO cross section \( (\nu_e + d \rightarrow p + p + e) \) from [18]. These cross sections are corrected for energy threshold and resolution effects (see, e.g., [19]).

In particular, we consider a prospected threshold of 5 MeV for the recoil electron kinetic energy \( T_e \) in SuperKamiokande and SNO, and an analysis window \( T_e \in [0.25, 0.8] \) MeV for Borexino. The energy resolution is assumed to be 16% at 10 MeV for SuperKamiokande, 11% at 10 MeV for SNO, and 8% at 0.5 MeV for Borexino, scaling as \( \sqrt{T_e} \) for different recoil
energies. The Fourier components of the vacuum oscillation signal are calculated through Eq. (14) in the $\delta m^2$ range relevant for the solution of the solar neutrino problem and then plotted in Fig. 1.

Figure 1 shows the deviations of the Fourier components from their standard values, $f_n - \varepsilon \delta_{n1}$, as functions of $\delta m^2$ for maximal mixing ($\sin^2 2\theta = 1$). The deviations decrease with decreasing mixing (not shown), although not exactly in proportion to $\sin^2 2\theta$ [see Eq. (14)]. The three subfigures refer to SuperKamiokande, SNO, and Borexino (top to bottom). The gray, horizontal bands correspond to the $\pm 1\sigma$ statistical error relative to the standard (no oscillation) case for $N_S = 10^4$ events and $N_B = 0$ ($\sigma_n = 0.0071$). Error bands for different choices of $N_S$ or $N_B$ can be obtained through Eq. (12).

It can be seen from Fig. 1 that the SuperKamiokande and SNO experiments are sensitive only to the first harmonic, i.e., to variations of the signal with a period of 1 yr. The amplitude of the second harmonic is, in fact, much smaller than the $\pm 1\sigma$ error in both cases. This is in agreement with the results in [10], where the SNO and SuperKamiokande signals were shown to vary almost sinusoidally in time. A single parameter ($f_1$) is then sufficient to characterize the results of each of these two experiments. For $10^4$ events and no background, Fig. 1 shows that the genuine $n = 1$ oscillation component ($f_1 - \varepsilon$) may reach a significance of $\sim 3\sigma$ ($\sim 7\sigma$) in the most favorable case for SuperKamiokande (SNO). For a signal-to-background ratio equal to one ($N_B = N_S$), the statistical significance of the vacuum oscillation signal would be lower by a factor $\sqrt{2}$. This implies that the detection of an unmistakable signal for vacuum oscillation in the time or frequency domain will require very high statistics and good background rejection in both experiments. Even in the best conditions, there are some ranges of $\delta m^2$ where the expected time-modulation of the signal is small and undetectable (e.g., for $\delta m^2 \simeq 1.6 \times 10^{-10}$ eV$^2$ in Fig. 1) [10,20]. In such ranges, however, independent oscillation signals might show up in the energy domain as distortions of the recoil electron spectrum [10,19].

The situation is much more favorable for the Borexino experiment, since its signal is dominated by the monoenergetic $^7$Be solar neutrinos ($E = 0.862$ MeV), while the Su-
perKamiokande and SNO signals are smeared by the broad $^8$B neutrino spectrum ($E \lesssim 15$ MeV). The third panel of Fig. 1 shows, in fact, that at least one of the first three Fourier coefficients is larger than the (representative) statistical error in the $\delta m^2$ range of interest. Moreover, when one of the harmonics is small, another is large (and vice versa), so that no “holes” are left in the sensitivity to the $\delta m^2$ variable, provided that (at least) two of the first three Fourier components are measured. Notice also that the relative amplitude of the Fourier coefficients in Borexino is strongly dependent on $\delta m^2$, and thus the detection of two nonzero harmonics would be of great help in discriminating a preferred range of $\delta m^2$. The relative amplitude of the first Fourier coefficient in SuperKamiokande (or SNO) and Borexino is also dependent on $\delta m^2$ and, therefore, the combination of all the experiments will enhance the resolution in $\delta m^2$.

In conclusion, we have performed a Fourier analysis of the signal expected in the SuperKamiokande, SNO, and Borexino solar neutrino experiments. This method fully exploits the real-time features of the three detectors and requires no binning in time. Expressions for the correlated uncertainties of the Fourier components have been worked out in the general case and, in particular, for no oscillation. Compact expressions for the Fourier coefficients in the presence of $2\nu$ oscillations have been given. The method has been applied to the analysis of the signals expected in the new-generation experiments. SuperKamiokande and SNO are shown to be sensitive only to the first harmonic, while Borexino is also sensitive to the second and third harmonic. The combination of different Fourier component measurements is highly selective in $\delta m^2$.

We thank G. Bellini, M. G. Giammarchi, and A. Ianni for useful correspondence, and P. I. Krastev for helpful suggestions.
REFERENCES

[1] B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53, 1717 (1967) [Sov. Phys. JETP 26, 984 (1968)]; V. Gribov and B. Pontecorvo, Phys. Lett. 28B, 493 (1969).

[2] S. L. Glashow and L. M. Krauss, Phys. Lett. B 190, 199 (1987).

[3] P. I. Krastev and S. T. Petcov, Phys. Lett. B 285, 85 (1992); Phys. Rev. D 53, 1665 (1996).

[4] V. Barger, R. J. N. Phillips, and K. Whisnant, Phys. Rev. Lett. 69, 3135 (1992).

[5] J. N. Bahcall, Neutrino Astrophysics (Cambridge University Press, Cambridge, England, 1989).

[6] P. I. Krastev and S. T. Petcov, Phys. Lett. B 395, 69 (1997).

[7] The latest experimental results have been presented by K. Lande (Homestake Collaboration), Y. Suzuki (Kamiokande Collaboration), T. Kirsten (GALLEX Collaboration), and V. Gavrin (SAGE Collaboration), in Neutrino '96, 17th International Conference on Neutrino Physics and Astrophysics, Helsinki, Finland, 1996, to appear in the Proceedings.

[8] I. Pomeranchuk, as cited in Gribov and Pontecorvo [1].

[9] E. Calabresu, N. Ferrari, G. Fiorentini, and M. Lissia, Astropart. Phys. 4, 159 (1995).

[10] P. I. Krastev and S. T. Petcov, Nucl. Phys. B 449, 605 (1995).

[11] SuperKamiokande Collaboration, Y. Totsuka et al., in TAUP '95, Proceedings of the 4th International Workshop on Theoretical and Phenomenological Aspects of Underground Physics, Toledo, Spain, edited by A. Morales, J. Morales, and J. A. Villar [Nucl. Phys. B, Proc. Suppl. 48, 547 (1996)].

[12] SNO Collaboration, A. B. McDonald et al., in TAUP '95 [11], p. 357.
[13] Borexino Collaboration, G. Bellini et al., in TAUP ’95 [11], p. 363.

[14] We do not consider additional systematic effects that might be correlated in time, e.g., possible cyclic variations of the detector efficiency. The evaluation of these experimental uncertainties is premature and goes beyond the scope of this work.

[15] W. T. Eadie, D. Drijard, F. E. James, M. Roos, and B. Sadoulet, Statistical Methods in Experimental Physics (North Holland, Amsterdam and London, 1971).

[16] I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrals, Series, and Products (Academic Press, San Diego, CA, 1994).

[17] J. N. Bahcall, M. Kamionkowsky, and A. Sirlin, Phys. Rev. D 51, 6146 (1995).

[18] J. N. Bahcall and E. Lisi, Phys. Rev. D 54, 5417 (1996). See also S. D. Ellis and J. N. Bahcall, Nucl. Phys. A 114, 636 (1968).

[19] J. N. Bahcall, P. I. Krastev, and E. Lisi, Phys. Rev. C 55, 494 (1997).

[20] This situation is similar to the measurement of the near-far asymmetry $A_{NF}$ as defined in B. Faïd, G. L. Fogli, and E. Lisi, and D. Montanino, Phys. Rev. D 55, 1353 (1997). Indeed, it can be shown that $A_{NF} \simeq 4(f_1 - \epsilon)/\pi$ for SuperKamiokande and SNO. This approximate correspondence between $A_{NF}$ and $f_1$ does not hold for Borexino, due to nonnegligible higher harmonics.
FIGURES

FIG. 1. Deviations of the Fourier coefficients $f_n$ from their standard values $f_n^{\text{std}} = \varepsilon \delta_{n1}$, as functions of $\delta m^2$ for maximal $2\nu$ mixing ($\sin^2 2\theta = 1$). In each of the three panels (SuperKamiokande, SNO, and Borexino), the gray, horizontal band represents the $\pm 1\sigma$ statistical uncertainty associated to the standard predictions $f_n^{\text{std}}$ for $10^4$ events with no background. SuperKamiokande and SNO appear to be sensitive only to the first harmonic (solid curve), while Borexino is sensitive also to the second and third harmonic (dashed and dotted curve, respectively). Notice that the vertical scales are different, but the absolute width of the gray error band ($\sigma_n = \pm 0.0071$) is the same for the three experiments.
FIG. 1 Deviations of the Fourier coefficients $f_n$ from their standard values $f_n^{\text{std}} = \varepsilon_1 \delta_n$, as functions of $\delta m^2$ for maximal $2\nu$ mixing ($\sin^2 2\theta = 1$). In each of the three panels (SuperKamiokande, SNO, and Borexino), the gray, horizontal band represents the $\pm 1\sigma$ statistical uncertainty associated to the standard predictions $f_n^{\text{std}}$ for $10^4$ events with no background. SuperKamiokande and SNO appear to be sensitive only to the first harmonic (solid curve), while Borexino is sensitive also to the second and third harmonic (dashed and dotted curve, respectively). Notice that the vertical scales are different, but the absolute width of the gray error band ($\sigma_n = \pm 0.0071$) is the same for the three experiments.