Vacuum Instability in Low-Energy Supersymmetry Breaking Models

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Abstract

We show that for the simplest models of gauge mediated supersymmetry breaking, including all existing models, the true vacuum will not preserve QCD if it communicates supersymmetry breaking to the visible sector. We show that the desired supersymmetry breaking color preserving vacuum can nonetheless be stable cosmologically, but only if certain couplings are sufficiently small. We also present simple modifications to the sector which communicates supersymmetry breaking in which the true vacuum is acceptable, suggesting desirable properties to be sought in dynamical models of supersymmetry breaking.

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1 Introduction

Dynamical supersymmetry breaking (DSB) offers a natural explanation for the hierarchy between the electroweak scale and the Planck scale. In this context, the “DSB sector” contains the gauge interactions responsible for supersymmetry breaking and the chiral superfields charged under this group, while the visible sector includes the supersymmetric standard model. There are several known models which break supersymmetry dynamically [1, 2, 3, 4], and can be used as the DSB sector. But a model of supersymmetry breaking is not sufficient; the supersymmetry breaking must be communicated to the visible sector. At present, there are two scenarios. Supersymmetry breaking can be transmitted from the DSB sector to the visible sector either by supergravity or by gauge interactions. The latter scenario has the virtue of ensuring a natural suppression of flavor-changing neutral currents [5]. Also, if supersymmetry breaking is gauge mediated, the soft breaking parameters are potentially determined in terms of a few coupling constants.

However, it is difficult to make a phenomenologically successful model of this type. It is essential to give the gauginos a mass. Current models [2, 4] do so by first breaking supersymmetry in one sector, then generating a VEV for the $F$ component of a singlet field, coupling this singlet field to messenger quarks, leading to a one loop gluino mass of order of the electroweak scale. This is not very economical, and ideally, one would like to construct a model in which the communication of supersymmetry breaking is more direct. However, constructing more directly connected models can prove to be difficult.

Were one to try to introduce the messenger quarks into the DSB sector, asymptotic freedom would typically be lost [1]. One could try to eliminate the scalar in favor of a fermion bilinear but that seems not to fit well into models of DSB so far constructed [7]. One might introduce the singlet directly into the DSB sector [4, 8]. But it is not clear that even a locally stable minimum can be constructed which breaks supersymmetry and preserves color.

The phenomenology of gauge mediated supersymmetry breaking is the subject of several recent studies [9]. In this paper, we explore another question, namely the stability of phenomenologically successful minima. Our results regarding the properties of the global minima in the simplest known models of this type are presented in the next section. In section 3 we consider the possibility that we live in the false vacuum. More complicated models are discussed in section 4. A summary can be found in section 5.
2 The Vacua of the Messenger Sector

In the existing models of gauge mediated supersymmetry breaking there is a sector which breaks supersymmetry dynamically at a scale $\Lambda_s \sim 10^2 - 10^4$ TeV. This DSB sector has a global symmetry $G_m$ which is weakly gauged so that it may be considered a small perturbation to the DSB mechanism. Supersymmetry breaking is transmitted from the DSB sector to the visible sector through a “messenger sector”, which consists of a gauge singlet, $S$, a pair of messenger quarks, $q$ and $\bar{q}$, in the 3 and $\bar{3}$ of the color $SU(3)_C$ group, and a pair of chiral superfields, $N$ and $P$, in vector-like representations of $G_m$. If there is a $(G_m)^3$ anomaly in the DSB sector, the messenger sector should also include other superfields, $E_i$, to cancel the anomaly. The superpotential of the messenger sector includes the most general dimension-3 operators:

$$W_{\text{mes}} = \kappa S\bar{q}q + \frac{\lambda}{3} S^3 + \lambda_1 PNS + W_1(P, N, E_i).$$  \hspace{1cm} (2.1)

The $PNS$ term communicates supersymmetry breaking from the fields charged under the messenger group to the gauge singlet, and the $S\bar{q}q$ term links the gauge singlet with the messenger quarks. Couplings of $P$ and $N$ to the other charged chiral superfields are contained in $W_1$. A discrete or $R$ symmetry is required to forbid linear or quadratic terms in $S$ and a $\bar{q}q$ term, which otherwise would have coefficients of order the Planck scale. We choose a phase definition of the fields such that the coupling constants, $\kappa, \lambda, \lambda_1$, are positive.

We focus on the case $G_m \equiv U(1)$. Let $g$ be the $U(1)$ gauge coupling in the normalization where $P, N$ and $E_i$ have charges $+1, -1$ and $y_i$, respectively. Upon integrating out the DSB sector, the only presence of supersymmetry breaking in the messenger sector is through an effective potential for the charged scalars [3]:

$$V_{\text{SB}} = M_1^2 \left( |P|^2 - |N|^2 + y_i|E_i|^2 \right) + M_2^2 \left( |P|^2 + |N|^2 + y_i^2|E_i|^2 \right) + \ldots$$  \hspace{1cm} (2.2)

where the mass parameters $M_k, k = 1, 2$ are related to the supersymmetry breaking scale by

$$M_k^2 = c_k \Lambda_s^2 \left( \frac{g^2}{(4\pi)^2} \right)^k.$$  \hspace{1cm} (2.3)

Here $c_k (j = 1, 2)$ are coefficients of order unity (in explicit calculations they can be larger, but perturbation breaks down if they are too large) with a sign dependent on the content of the DSB sector. The two terms shown in (2.2) give masses to the charged scalars and
are due to an one-loop Fayet-Iliopoulos term and a two-loop contribution, respectively. The ellipsis in (2.2) stand for higher dimensional terms and contributions from more loops.

The scalar potential of the messenger sector includes also a contribution from the U(1) $D$-term,

$$ V_D = \frac{g^2}{2} \left( |P|^2 - |N|^2 + y_t |E_i|^2 \right)^2 , $$

and $F$ terms:

$$ V_F = |F_S|^2 + \kappa^2 |S|^2 \left( |q|^2 + \bar{q}^2 \right) + \left| \lambda_1 NS + \frac{\partial W_1}{\partial P} \right|^2 + \left| \lambda_1 PS + \frac{\partial W_1}{\partial N} \right|^2 + \sum_i \left| \frac{\partial W_1}{\partial E_i} \right|^2 , $$

where

$$ F_S = \kappa \bar{q} q + \lambda S^2 + \lambda_1 PN . $$

The SU(3)$_C D$ term contribution to the classical potential vanishes if $q = \bar{q}$.

The desired minimum of the scalar potential of the messenger sector,

$$ V_{\text{mes}} = V_D \left( |P|^2, |N|^2, |E_i|^2 \right) + V_{\text{SB}} \left( |P|^2, |N|^2, |E_i|^2 \right) + V_F \left( P, N, S, q, \bar{q}, E_i \right) , $$

is at $S = \langle S \rangle, F_S = \langle F_S \rangle$ and $q = \bar{q} = 0$, where the fermion components of $q$ and $\bar{q}$ have a Dirac mass $\kappa |\langle S \rangle|$, and the eigenvalues of the mass squared matrix of the scalar components is $\kappa^2 |\langle S \rangle|^2 \pm \kappa |\langle F_S \rangle|$. As a result, if $\langle F_S \rangle$ is non-zero, a one loop gluino mass and two-loop squark masses are produced, leading to highly predictive models.

However, the $S \bar{q} q$ term in the superpotential is dangerous because it tends to induce VEVs for $q$ and $\bar{q}$ in order to minimize the $|F_S|^2$ term in the scalar potential. It is important to determine whether $q$ and $\bar{q}$ actually vanish at the global minimum of $V_{\text{mes}}$. We first analyze this problem in the simplest known models of this type [2].

### 2.1 Minimal Models

We start with the class of minimal models, defined by

$$ \frac{\partial W_1}{\partial P} = \frac{\partial W_1}{\partial N} = 0 . $$

This includes models without the “extra” fields $E$ (i.e. the fields in the messenger sector which cancel the potential anomaly of the messenger group from the DSB sector), models with $W_1 = 0$ [2], and models with $E$ fields which do not couple to $P$ or $N$. An examination of eqs. (2.4)-(2.7) shows that the global minimum of $V_{\text{mes}}$ is at $S = 0$ and $\bar{q} q = -\frac{\lambda_1}{\kappa} PN$, so that $F_S = 0$. Therefore, the condition $q = \bar{q} = 0, F_S \neq 0$ could be realized at most at
a local minimum, i.e. any vacuum that preserves color and breaks supersymmetry in the visible sector is unstable. Note that this proof does not refer to the actual expression of $V_{SB}$, such that it remains true even if the messenger group, $G_m$, is non-Abelian.

To decide whether there are viable local minima, one has to consider explicit models. The model presented in [2] is the only example of this type constructed so far. The messenger $U(1)$ has no anomaly in the DSB sector, and so there is no $E$ field. This model, based on a $SU(6) \times U(1)$ supercolor gauge group, is constructed such that the Fayet-Iliopoulous term vanishes ($M_1 = 0$). As a result, there is only one scale in the messenger sector, $M_2^2 < 0$, produced at two loops, and one expects that the gauge singlet VEV can be produced without fine-tuning. The scalar potential of the messenger sector is given by

$$V_{mes} = \frac{g^2}{2} \left( |P|^2 - |N|^2 \right)^2 + \left( M_2^2 + \lambda_1 |S|^2 \right) \left( |P|^2 + |N|^2 \right) + \kappa^2 |S|^2 \left( |q|^2 + |\bar{q}|^2 \right) + \kappa \bar{q}q + \lambda S^2 + \lambda_1 PN \right|^2. \quad (2.9)$$

This potential is unbounded from below, but higher order terms in $V_{SB}$ (see eq. (2.2)) will rectify this, resulting in a deep global minimum far away in field space. As shown below eq. (2.8), in this true vacuum the visible sector is supersymmetric.

To search for viable local minima, we set $q = \bar{q} = 0$ in (2.9). The remaining potential has only quadratic and quartic terms in $P, N$ and $S$, so that the vacuum manifold can be found immediately. At the minimum $S \neq 0$ only if

$$\lambda > \lambda_1 . \quad (2.10)$$

These minima are placed at

$$q = \bar{q} = 0 , \ |P|^2 = |N|^2 = -M_2^2 \frac{\lambda}{\lambda_1^2(2 - \lambda_1/\lambda)} ,$$

$$|S|^2 = -M_2^2 \frac{1 - \lambda_1/\lambda}{\lambda_1^2(2 - \lambda_1/\lambda)} , \ \text{Arg}(PNS^*2) = -1 . \quad (2.11)$$

and the condition of stability is (see the Appendix)

$$\lambda_1^3 \leq 2\lambda g^2 . \quad (2.12)$$

These are also local minima of $V_{mes}$ only if the following conditions are satisfied at the points (2.11):

$$\frac{\partial^2 V_{mes}}{\partial |q|^2} + \frac{\partial^2 V_{mes}}{\partial |\bar{q}|^2} \geq 0 \quad (2.13)$$
\[
\frac{\partial^2 V_{\text{mes}}}{\partial |q|^2} \frac{\partial^2 V_{\text{mes}}}{\partial |\bar{q}|^2} \geq \left( \frac{\partial^2 V_{\text{mes}}}{\partial |\bar{q}| \partial |q|} \right)^2 .
\]  

(2.14)

Condition (2.13) is automatically satisfied, while condition (2.14) requires

\[
\lambda_1 \leq \frac{\kappa \lambda}{\kappa + \lambda} .
\]  

(2.15)

In conclusion, the messenger sector of the model \cite{2} has a viable false vacuum (see eq. (2.11)) if and only if (2.10), (2.12) and (2.15) are satisfied. In section 3 we study the stability of this false vacuum.

Note that the inclusion of a pair of vector-like leptons which couple to \( S \), for giving mass to the usual sleptons and gauginos \cite{3}, does not change the position of the false vacuum.

## 2.2 Models with One \( E \) Field

We have shown so far that in the models without extra fields, \( E \), coupled to \( P \) or \( N \), the deepest minimum is not viable. Although models with extra fields might seem less attractive, their existence will change the vacuum structure, and can conceivably stabilize the preferred minimum. In the remainder of this section we study whether it is possible to have a viable true vacuum in models where there is only one charged superfield, \( E \), in addition to \( P \) and \( N \) \cite{3}.

There are two ways of including \( E \) in the renormalizable superpotential (\( \lambda_2 > 0 \)):

\[
W_1 = \frac{\lambda_2}{2} E N^2
\]  

(2.16)

if \( E \) has charge \( y = 2 \), or

\[
W_1 = \frac{\lambda_2}{2} E^2 N
\]  

(2.17)

if \( y = 1/2 \). The cases \( y = -2 \) and \( y = -1/2 \) can be derived from these ones by putting \( P \leftrightarrow N, M_1^2 \leftrightarrow -M_1^2 \).

For generic parameters, \( V_{\text{mes}} \) cannot be minimized analytically. However, the global minimum can be identified for a large range of parameters. To this end, we split \( V_{\text{mes}} \) into two pieces:

\[
V_{\text{mes}} = V_a + V_b
\]  

(2.18)

where

\[
V_a \equiv V_D + V_{SB} + \left| \frac{\partial W_1}{\partial E} \right|^2
\]  

(2.19)
and

\[ V_b \geq 0. \quad (2.20) \]

When \( W_1 \) is given by (2.16), \( V_b = 0 \) if and only if \( S = 0, NE = 0 \) and \( \bar{q}q = -\frac{\alpha_1}{\kappa}PN \). Hence, if the global minimum of \( V_a \) is at \( NE = 0 \), then the global minimum of \( V_{\text{mes}} \) is at \( S = 0, F_S = 0 \). A straightforward minimization of \( V_a \) shows that \( NE \neq 0 \) only if

\[ M_2^2 < 0 \quad (2.21) \]

and

\[ \frac{M_1^2}{|M_2^2|} < 2 + \frac{6g^2}{\lambda_2^2}. \quad (2.22) \]

This range of parameters, which might allow supersymmetry breaking in the visible sector, can be further reduced. It would be useful to see for what range of parameters the minima of \( V_{\text{mes}}(q = \bar{q} = 0) \) are lower than the global minimum of \( V_{\text{mes}}(S = 0) \), because this is a necessary condition for the existence of a viable true vacuum. However, \( V_{\text{mes}}(q = \bar{q} = 0) \) contains quartic, quadratic and linear terms in \( S, P, N \) and \( E \), and cannot be minimized analytically. Our strategy is to find a function that bounds \( V_{\text{mes}}(q = \bar{q} = 0) \) from below and can be minimized analytically, and to compare its minimum with the global minimum of \( V_{\text{mes}}(S = 0) \). Consider

\[ V_1 \equiv V_{\text{mes}}(q = \bar{q} = 0) - |F_S|^2. \quad (2.23) \]

It is easy to find the minimum of \( V_1 \) with respect to \( S \):

\[ V_1(P, N, E)_{S_{\text{min}}} = V_D + V_{SB} + \frac{\lambda_2^2}{4}|N|^4 + \lambda_2^2|NE|^2 - \frac{\lambda_2^2|NP E|^2}{|N|^2 + |P|^2}. \quad (2.24) \]

Again, this potential cannot be minimized analytically, so we need to bound it from below. For any \( N, P \) and \( E \),

\[ V_1(P, N, E)_{S_{\text{min}}} \geq V_2 \equiv V_a + \frac{\lambda_2^2}{4}|E|^2 \left(3|N|^2 - |P|^2 \right), \quad (2.25) \]

the equality being satisfied if \( |N| = |P| \) or \( E = 0 \).

Now we have a simple potential, \( V_2 \), which is smaller than \( V_{\text{mes}}(q = \bar{q} = 0) \) at any point in field space, and we can compare it with \( V_{\text{mes}}(S = 0) \) at \( \bar{q}q = -\frac{\alpha_1}{\kappa}PN \):

\[ V_{\text{mes}}(S = 0)_{\bar{q}q_{\text{min}}} = V_a + \lambda_2^2|NE|^2. \quad (2.26) \]

They are equal at their global minima provided

\[ |E|^2 \left(|N|^2 + |P|^2 \right) = 0 \quad (2.27) \]
at the global minimum of $V_2$. If this condition is satisfied, then

$$V_{\text{mes}}(S = 0)_{\text{min}} < V_{\text{mes}}(q = \bar{q} = 0)_{\text{min}}$$

whenever $F_S \neq 0$, so that the global minimum of $V_{\text{mes}}$ breaks color or preserves supersymmetry.

For $\lambda_2^2 < 16g^2$, $V_2$ is bounded from below and it can be minimized analytically. When $M_2^2 < 0$, the result is that eq. (2.27) is not satisfied only if

$$\frac{M_1^2}{|M_2^2|} \leq \begin{cases} 
1 & \text{, for } \frac{g^2}{\lambda_2^2} \geq \frac{1}{8} \\
2 - \frac{8g^2}{\lambda_2^2} & \text{, for } \frac{1}{8} > \frac{g^2}{\lambda_2^2} > \frac{1}{16}
\end{cases}$$

(2.29)

For $\lambda_2^2 > 16g^2$, eq. (2.22) requires

$$\frac{M_1^2}{|M_2^2|} < \frac{19}{8}.$$ 

(2.30)

These are severe constraints on the DSB sector. For instance, the model of this type constructed in [6] gives

$$\frac{M_1^2}{|M_2^2|} = \frac{\pi^2}{2g^2}.$$ 

(2.31)

Thus, a necessary (but not sufficient) condition for a stable viable vacuum in this model is $g > 1.4$ and $\lambda_2 > 5.7$, or $g > 1.8$. However, for such large values of $g$, the DSB and messenger sectors cannot be treated separately. One has to minimize the complete scalar potential and decide whether the global minimum is well behaved.

Note that whenever the messenger sector has a $U(1)_3$ anomaly, as in this class of models, cancelled by the DSB sector, no symmetry can forbid a Fayet-Iliopoulos term. We expect $|M_2^2| \gtrsim \pi^2|M_1^2|$, so that other models in this class might be viable only if they yield a negative $M_1^2$, which is not the case of the DSB sectors whose spectra have been investigated [6].

Finally, consider the case where $W_1$ is given by (2.17). It is easy to see that $V_b = 0$ if and only if $S = 0$, $\bar{q}q = -\frac{\Lambda}{\kappa}PN$ and $E = 0$. For $M_2^2 \geq 0$, $V_a$ is minimum at $E = 0$. For $M_2^2 < 0$, ignoring the higher order terms in $V_{\text{SB}}$ (the ellipsis in eq.(2.2)) we find: $V_a \to -\infty$ for $|N| \to \infty$ and $|E|^2 < -2M_2^2/\lambda_2^2$, but $V_a(E = 0) < V_a(E \neq 0)$. Thus, if the perturbative expansion (2.2) is valid, then $V_a$ and $V_b$ are minimum at $E = S = F_S = 0$, so that supersymmetry is unbroken in the low energy sector.
3 Vacuum Tunnelling

So far we have shown that for a large class of models, including all the models existing in
the literature, there are vacua with viable properties for some range of parameters, but
they are unstable. Thus, these models can describe nature only if:
i) the early universe was placed in the false vacuum by a phase transition;
ii) thermal fluctuations did not cause a transition at a later time from the false vacuum
to the true one;
iii) the lifetime of the false vacuum is at least of the order of the present age of the
universe.
Here we study only the restrictions imposed by condition iii).

A false vacuum will eventually decay to the true vacuum by tunnelling, but its lifetime
could be arbitrarily long if some coupling constants are fine-tuned. However, if condition
iii) restricts the parameter space too much, then these models become unsatisfactory
because the goal of DSB is to avoid fine-tuning.

The zero temperature vacuum tunnelling rate may be computed semi-classically \[10\]. The transition probability per unit volume per unit time is given by:
\[
\frac{\Gamma}{V} = A e^{-S_E[\phi]},
\]
where \(S_E[\phi]\) is the Euclidean action for the “bounce” configuration \(\phi\). \(A\) has the dimen-
sions of \((\text{mass})^4\) and is of the order of \((10\text{ TeV})^4\) in the present case. The space-time
four-volume available for the transition is about \(t^4\) where \(t \approx 10^{10}\) years is the age of the
universe. For the stability of the false vacuum it is reasonable to demand that \(t^4 \Gamma/V \leq 1\),
which implies
\[
S_E[\phi] \geq 400.
\]

A reliable estimate of the bounce action can be obtained only numerically in most
cases (see \[10\] for an analytical approximation in the “thin wall” case). This computation
is simple if there is only one scalar field. The bounce solution is an \(O(4)\) invariant non-
trivial field configuration which is an extremum of the Euclidian action, and obeys the
boundary conditions: \((d\phi(r)/dr)_{r=\infty} = 0\), \(\phi(r \rightarrow \infty) = \phi_f\), where \(\phi\) is the scalar field and
\(\phi_f\) is its value at the false vacuum. In this case one can solve the Euclidian equations of
motion from some initial point \(\phi(0)\) and look at the limiting value \(\phi(\infty)\). For arbitrary
values of \(\phi(0)\) the value of \(\phi(\infty)\) is either an “overshoot” \((\phi(\infty) > \phi_f)\) or an “undershoot”
\((\phi(\infty) < \phi_f)\). Since the correct value of \(\phi(0)\) (called the “escape point”) must lie between
two trial values which end in an overshoot and an undershoot, the search converges rapidly by bisections.

When two or more scalar fields are involved, the bracketing property of the overshoot and the undershoot is lost making the search for the bounce a much more difficult task. It is however possible to find an upper bound on the bounce action by replacing the multi-field potential by a suitably chosen single field potential.

We apply this procedure to the minimal model \[2\], whose viable false vacuum has the coordinates given by eq. (2.11), by choosing a convenient direction in the field space and replacing the multidimensional space by a one dimensional space extending in that direction and passing through the false vacuum. Then we find the escape point in this one dimensional case by bisections of undershoots and overshoots. The solution obtained by this method is not a bounce solution of the multidimensional case, but it gives an upper bound on the least bounce action. We have further refined our bound on the least bounce action by repeating the above procedure for different directions in field space to find the direction that gives the least bounce action. Finally we search for the bounce in the full multidimensional space in a neighborhood of the straight line trajectory in the “best” direction. The last step improves the upper bound set on \(\lambda\) by a factor of 1/2.

When searching for the bounce in the multidimensional space, we use methods similar to ref. \[11\].

The one dimensional cross section in the field space that we finally chose as the initial guess for the trajectory for the multidimensional search of the bounce connects the false vacuum (2.11) to the point with \(|P| = |N|\) unchanged and

\[
S = 0, \quad |q|^{2} = |\bar{q}|^{2} = -M_{2}^{2} \frac{\lambda}{\kappa \lambda_{1}^{2}(2 - \lambda_{1}/\lambda)}.
\]  

(3.3)

It can be verified that the potential at the latter point is less than the potential at the false vacuum, hence the one dimensional cross section has a bounce solution.

Our numerical results are shown in Figure 1. We have searched for the bounce for values of \(\lambda, \lambda_{1}\) and \(\kappa\) ranging from 0.1 to 1. The graphs are plotted for \(\kappa = 0.4\) and \(\kappa = 1.0\). For a given value of \(\kappa\), the region above the dashed curve in Figure 1 is ruled out analytically because there is no local minimum (see eq. (2.15)), while the region above the solid line is ruled out by our numerical results (the bounce action is less than 400). From the graphs one can see that in this model the false vacuum can not be long lived when \(\lambda_{1} > 0.1\). Note that these bounds are independent of \(g\).
Fig. 1. Vacuum Stability. The region above the solid (dashed) lines is ruled out numerically (analytically).

In conclusion, the numerical analysis of the vacuum tunnelling indicates that in the interesting range of parameters where the theory can be treated pertubatively, at least one coupling is less than 0.1. This may not be regarded as a small enough number to qualify the theory as unnatural. Thus, viable and long lived false vacua in this theory are not ruled out at this stage. However, a more robust numerical analysis might constrain the parameter space even further.

4 Enlarging the Messenger Sector

Although it is conceivable that the universe is at present in a false vacuum, it would be preferable if the messenger sector had viable stable vacua.

If new fields are added to the messenger sector, then there will be more $F$-type terms contributing to the scalar potential, so that the position of the true vacuum is likely to change. One may hope that in some cases the true vacuum is viable. Broadly speaking, the minimal messenger sector can be modified in three ways:

(i) modifying/enlarging the matter content transforming under SU(3)$_C$

(ii) enlarging the sector transforming non-trivially under the messenger group $G_m$ ($P, N, E$ etc.),

(iii) enlarging the singlet sector ($S$).

In the following we consider modifications of only one of the above kinds at a time.
Some of our results can be extended in a straightforward manner to the cases when several parts of the messenger sector are changed at once.

(i) SU(3)$_C$ colored sector.

The most severe constraint on this sector is the preservation of asymptotic freedom for QCD. In supersymmetric QCD, apart from the chiral multiplets corresponding to the usual quarks, there is still room for either up to three $(3 + \overline{3})$s or one 8. With this chiral content the results of section 2 will not change.

(ii) The $G_m$ colored sector.

We have not explored the many possibilities when $G_m$ is non-Abelian. Note, however, that our result regarding the true vacuum in minimal models (see section 2.1) is true for any $G_m$ if there are no new terms in the superpotential.

In the simplest case, when $G_m$ is a U(1), one can enlarge the minimal messenger sector by introducing additional charged fields. Although the potential is hard to minimize in the general case, considerable simplification is achieved by taking some of the coupling constants to be small. For instance, consider enlarging this sector by introducing two fields $E_1$ and $E_2$ with charges 2 and $-\frac{1}{2}$ respectively. The superpotential is given by (2.1) with,

$$W_1(P, N, E_i) = \frac{\lambda_2}{2} N^2 E_1 + \frac{\lambda_3}{2} P E_2^2.$$  

When $\lambda_1 = 0$ it is possible to have a global minimum of the potential at $S = q = \overline{q} = E_1 = 0$ and $P, N, E_2 \neq 0$. In particular, this happens when

$$1 + \frac{4g^2}{\lambda_2^2} - \frac{3g^2}{4\lambda_3^2} < -\frac{M_2^2}{M_1^2} < 1 + \frac{4g^2}{\lambda_2^2},$$

with $M_2^2 < 0$ and $M_1^2 > 0$. When $\lambda_1$ is given a small positive value, this minimum is shifted. One can parametrize the small changes in the coordinates of the minimum by $\delta \phi = l_\phi x$, where $\phi$ stands for $P, N, E_1, E_2, S$ and $q$, $l_\phi$ are complex numbers, and $x > 0$.

The change in the potential is a quartic polynomial in $x$ and one can optimize the choice $l_\phi$ in the limit of $\lambda_1 \to 0$. We have found that the optimal choice for $l_q$ is 0, i.e the minimum can be perturbed to move to $q = \overline{q} = 0$ and $P, N, E_1, E_2, S, F_S \neq 0$, provided $\lambda_3$ and $\lambda_1/\lambda_3^2$ are small and $\kappa > \lambda$.

One can also search for parameters with a good vacuum in which $\lambda_1$ is not small. For example, we find that $\lambda_1 = 0.3, \lambda_2 = 1.0, \lambda_3 = 0.8, \lambda = 1.0, \kappa = 2.0, g = 1.0$ and $|M_1/M_2| = 10$ yields a viable vacuum at $q = \overline{q} = 0, |S| = 2.89, |N| = 7.45, |P| = 0.221, |E_1| = 0.014, |E_2| = 6.21$, in units of $|M_2|$, and $|F_S| = 6.19|M_2|^2$. 

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This example demonstrates that color symmetric vacua may be obtained in the messenger sector if there are at least two $E$ fields. It remains a challenge to construct a DSB model in which anomalies cancel and the ratio $|M_1/M_2|$ is suitable.

(iii) The singlet sector.

Consider now a messenger sector with two gauge singlets, $S$ and $S'$. The most general superpotential, up to transformations which leave the minimal Kahler potential invariant, includes the coupling of only one singlet to $PN$, and the couplings of the both singlets to $\bar{q}q$:

$$W = (\kappa S + \kappa' S')\bar{q}q + \frac{\lambda}{3} S^3 + \frac{\lambda'}{3} S'^3 + \lambda_0 S^2 S' + \lambda_0' S S'^2 + \lambda_1 S P N + W_1(P, N, E_i) .$$

We will consider the case where there is one $E$ field in order to use the simplest DSB sector, i.e. the $SU(3) \times SU(2)$ model, as described in [6].

The case $W_1 = 0$ (which corresponds to a messenger charge of $E, y$, different than $\pm 2, \pm 1/2$) can be analyzed as in the one singlet models (see section 2.1). The global minimum is at $S = S' = 0$, $F_S = F_{S'} = 0$, so that supersymmetry is unbroken in the visible sector.

The case $W_1 = \frac{\lambda_2}{2} E N^2$ (with a single $E$ field) is already complicated. There are 8 coupling constants in the superpotential, and only 5 phases can be absorbed by field transformations. Thus, together with $g$ there are 12 parameters that can be varied when one searches for a minimum of the 11 dimensional field space. Note that the DSB sector remains the same as in ref. [3], so that the one- and two-loop masses of the charged scalars satisfy eq. (2.31).

Analytically, using the method described in the case of the model with two $E$ fields, we have found that close to the point $\lambda_1 = \lambda_2 = 0$ in the parameter space it is possible to obtain a global minimum of the potential at $q, \bar{q} = 0$ and $S, S', P, N, F_S \neq 0$.

We have numerically minimized the potential and have found that, for a range of parameters, the deepest minimum is viable. For example, when

$$g = 0.8, \ \lambda_1 = 0.3, \ \lambda_2 = 0.2, \ \lambda = 1.0, \ \lambda' = 0.5$$

$$\lambda_0 = 0.25, \ \lambda_0' = 0.5, \ k = 1.0, \ k' = 2.0,$$

the vacuum is at

$$q = \bar{q} = 0, \ S \approx 1.906, \ S' \approx -3.982$$

The two singlet model with no $E$ field is discussed in ref. [2].
\[ P \approx 3.688, \quad N \approx -7.792, \quad E \approx 4.295, \quad (4.5) \]

in units of \(|M_2|\). In fact this is a set of degenerate vacua connected by phase transformations which preserve the following relations:

\[ \text{Arg}(SS') = \text{Arg}(PNS'^2) = \text{Arg}(SPN^*E^*) = \pi. \quad (4.6) \]

At these minima, \(|F_S| \approx 0.856|M_2^2|\), \(|F_{S'}| \approx 1.249|M_2^2|\), and the vacuum energy density of the messenger sector is

\[ V_{\text{min}} \approx -113.5 M_2^4. \quad (4.7) \]

In the absence of analytical constraints it is not possible to be absolutely sure that a minimum found numerically is the global one. However, for this model it is possible to find analytically the minimum when \(S = S' = 0\). When the parameters are given by eq. (4.4), the minimum value of the potential is

\[ V(S = S' = 0)_{\text{min}} = -\frac{3M_2^4}{6g^2 - \lambda_2^2}\left(\frac{M_1^4}{M_2^4} - 2\frac{M_1^2}{|M_2|^2} + 6\frac{g^2}{\lambda_2^2}\right) \]

\[ = -110.6 M_2^4. \quad (4.8) \]

Thus, the viable minimum (4.5), found numerically, is deeper than the minimum at \(S = S' = 0\). The values (4.7) and (4.9) are close to each other because the bulk of the two potentials at the minimum comes from the same \(M_1^2|N|^2\) term. Although we have not ruled out analytically a deeper minimum at \(S, S', q \neq 0\), this is unlikely to exist due to the \(F_q\) and \(F_{\bar{q}}\) terms which can be large.

Note that a similar two singlet model, with an additional \(S'H_uH_d\) coupling of the Higgs superfields, was discussed in ref. [6] as a source for a \(\mu\) term. However, in that case the \(\lambda_0\) and \(\lambda'_0\) coupling constants have to be small, to allow the hierarchy between the electroweak scale and the messenger quark masses. It is easy to see that under these conditions both \(V_{\text{mes}}(q = \bar{q} = 0)\) and \(V_{\text{mes}}(S = 0)\) have the global minimum at \(S' \approx 0\) and \(F_{S'} \approx 0\). Therefore, these two potentials can be compared exactly as in section 2.2. The result of this analysis is that the true vacuum of the model used to produce a \(\mu\) term is not viable (at least in the case of a weakly gauged messenger group).

## 5 Summary and Conclusions

Many of the naturalness problems of the standard model and its supersymmetric versions can be solved within models with dynamical supersymmetry breaking at low en-
ergy. However, the predictive power of this type of models is accompanied by important phenomenological restrictions. We have analyzed the properties of the vacua of all the potentially realistic models with DSB existing in the literature and their simple extensions and generalizations. The main issue is that the messenger quarks should not develop VEVs while learning of supersymmetry breaking from the gauge singlet which couples to the $P$ and $N$ fields, which are charged under the messenger group. Here we summarize our main conclusions.

1. In the case of the minimal models, where $P$ and $N$ couple only to the gauge singlet (such as the one in [2]), the deepest minimum does not break supersymmetry in the visible sector. For a range of parameters, the model of ref. [2] (in which there is no $E$ field and no Fayet-Iliopoulos term) has a false vacuum with viable properties. The lifetime of this vacuum is longer than the present age of the universe provided some coupling constants are sufficiently small. The condition $\lambda_1 < 0.1$ (see Fig. 1), found numerically, is necessary but probably not sufficient; the actual bound might be much stronger.

2. In the models where there is only one charged field, $E$, in addition to $P$ and $N$, the true vacuum typically breaks color or preserves supersymmetry. A necessary (but not necessarily sufficient) condition to evade this conclusion is given by the inequality (2.30) involving the one- and two-loop masses of the charged fields. The models of ref. [6] do not satisfy this condition unless the messenger gauge coupling is large, which is in contradiction with the requirement of a weakly gauged symmetry of the DSB sector. It would be useful to build models which have a negative Fayet-Iliopoulos term such that the necessary condition (2.30) is satisfied while keeping weak the messenger gauge interaction.

3. Simple extensions of the models with one $E$ field, which include additional charged or gauge singlet chiral superfields, have viable vacua for a range of parameters. This is the case of a model with two charged fields in addition to $P$ and $N$. However, it will be a non-trivial task to find a DSB sector which cancels the U(1)$^3$ and the mixed U(1) - gravitational anomalies of the messenger sector for this model. A model with two gauge singlets and a charged field, $E$, has a viable true vacuum in the limit where some coupling constants are small. A numerical search has yielded a viable vacuum for a natural range of parameters. This result is encouraging since the DSB sector is simple, as in ref. [6]. However, the second singlet cannot be used to produce a $\mu$ term at the global minimum.

We conclude by emphasizing the need for extensive studies of the properties of diverse DSB sectors, and for further searches for messenger sectors with viable vacua. It is worth noting that if one accepts the possibility of living in a false vacuum, it suggests new model
building alternatives, since the DSB sector itself might be such that the supersymmetry breaking vacuum is only a local minimum. Nevertheless, it would be desirable to find viable models in which the universe is in the true vacuum.

Note

After the completion of this work we were informed about ref. \[12\] where the minima of the scalar potential of the messenger sector given in \[2\] are briefly discussed.

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Appendix

In this Appendix we derive conditions of stability for the color preserving vacua of the minimal model discussed in section 2.1. We seek the minima of $V_{\text{mes}}$ (see eq. (2.9)) at $q = \bar{q} = 0$ and $S \neq 0$. These satisfy $\text{Arg}(PNS^{*2}) = -1$, so that we need to minimize a potential which does not depend on complex phases:

$$V_0(|S|, |P|, |N|) \equiv V_{\text{mes}}(q = \bar{q} = 0, F_S = \lambda|S|^2 - \lambda_1|PN|) .$$

Its minima are amongst the solutions of

$$\frac{\partial V_0}{\partial |S|} = 2|S| \left\{ 2\lambda^2|S|^2 - \lambda_1 \left[ 2\lambda|PN| - \lambda_1 \left( |P|^2 + |N|^2 \right) \right] \right\} = 0$$

(A.2)
\[
\frac{\partial V_0}{\partial |P|} = 2|P| \left[ g^2 \left( |P|^2 - |N|^2 \right) + M_2^2 + \lambda_1 \left( |S|^2 + |N|^2 \right) \right] - 2\lambda\lambda_1 |S|^2 |N| = 0 \tag{A.3}
\]
\[
\frac{\partial V_0}{\partial |N|} = 2|N| \left[ g^2 \left( |N|^2 - |P|^2 \right) + M_2^2 + \lambda_1 \left( |S|^2 + |P|^2 \right) \right] - 2\lambda\lambda_1 |S|^2 |P| = 0 \tag{A.4}
\]

From (A.2) we see that \( S \neq 0 \) only if \( \lambda > \lambda_1 \).

If \( |P| \neq |N| \), then it can be shown that the set of equations (A.2)-(A.4) has solutions with \( |S|, |P| \) and \( |N| \) positive (for \( M_2^2 < 0 \)) only if
\[
\frac{\lambda_3}{2\lambda} \leq g^2 < \frac{\lambda_2}{2}.
\tag{A.5}
\]

Computing the second derivatives one can see that these solutions are not minima but saddle points: they are local maxima either in the \( |S| - |P| \) or in the \( |S| - |N| \) subspaces.

Therefore, we have to solve (A.2)-(A.4) with \( |N| = |P| \), and the result is given by eq. (2.11). These extrema of \( V_0 \) are minima provided the matrix \( \partial^2 V_0 / (\partial \phi_i \partial \phi_j) \), with \( \phi_i = |S|, |P|, |N| \), has only positive eigenvalues. These eigenvalues are given by
\[
\Delta_{1,2} = \frac{1}{2} \left[ A + B + C \pm \sqrt{(A-B-C)^2 + 8D^2} \right]
\]
\[
\Delta_3 = C - B
\tag{A.6}
\]
where
\[
A \equiv \left. \frac{\partial^2 V_0}{\partial |S|^2} \right|_{|N| = |P|}, \quad B \equiv \left. \frac{\partial^2 V_0}{\partial |P| \partial |N|} \right|_{|N| = |P|}\]
\[
C \equiv \left. \frac{\partial^2 V_0}{\partial |P|^2} \right|_{|N| = |P|}, \quad D \equiv \left. \frac{\partial^2 V_0}{\partial |S| \partial |P|} \right|_{|N| = |P|}
\tag{A.7}
\]

At the points (2.11) \( \Delta_{1,2} \) are always positive, while \( \Delta_3 \geq 0 \) if and only if \( \lambda_3 \leq 2\lambda g^2 \). This is the sufficient and necessary condition for the extremum of \( V_0 \) at \( |N| = |P| \) to be a local minimum.

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