OBTAINING COSMIC-RAY PROPAGATION PARAMETERS FROM DIFFUSE VERY HIGH ENERGY GAMMA-RAY EMISSION FROM THE GALACTIC CENTER RIDGE

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ABSTRACT

The recent discovery of diffuse, very high energy (VHE) γ-radiation from the Galactic center ridge by the HESS telescope allows for the first time the direct determination of the parameters of Galactic cosmic-ray-propagation models. Whereas this discovery showed that the diffuse γ-radiation can be explained by the interaction of VHE cosmic-ray (CR) protons with the interstellar gas located in several giant molecular clouds, we show in this paper that the associated diffusion coefficient for the protons depends on the epoch of activity of the central source of protons: Assuming that the supernova remnant (SNR) Sgr A East was responsible for the particle acceleration, we infer a diffusion coefficient for the Galactic center region of $\kappa = 1 - 5 \text{ kpc}^2 \text{ Myr}^{-1}$ for a mean proton energy of $\sim 3 \text{ TeV}$. More specifically, for impulsive injection in a $5 - 10 \text{ kyr SNR}$, we infer a value of $\kappa = 1 - 2 \text{ kpc}^2 \text{ Myr}^{-1}$, whereas for source activity timescales equal to the age of the SNR, the diffusion coefficient would increase to $\kappa \sim 5 \text{ kpc}^2 \text{ Myr}^{-1}$. These values are smaller than those inferred from local CR abundances. Finally, the above-mentioned values of $\kappa$ for impulsive injection are equally valid if the required transient source of protons was due to an earlier epoch of stellar infall into the black hole Sgr A*.

Subject headings: cosmic rays — diffusion — Galaxy: center — gamma rays: theory — ISM: clouds — supernova remnants

1. INTRODUCTION

The High Energy Stereoscopic System (HESS) of four telescopes currently offers the best angular resolution for the study of very high energy (VHE) γ-rays from cosmic sources (Aharonian et al. 2006). With an angular resolution of $\sim 0.08^\circ$, the HESS collaboration was able to resolve γ-rays associated with molecular clouds in the Galactic center region (Aharonian et al. 2006): Whereas a relatively good correlation was found between the γ-ray and CS (with the latter measured by Tsuboi et al. 1999) surface brightness distributions within 150 pc ($l \sim \pm 1^\circ$ along Galactic longitude) from the Galactic center, this correlation degraded at a distance of 200 pc from the Galactic center (i.e., at $l \sim 1.5^\circ$). A strong indicator that this diffuse component is indirectly associated with a source at the Galactic center is the similarity of the spectral indices of the point source HESS J1745–290 at the center, as well as this newly discovered diffuse extended emission. Furthermore, the relatively good correlation within $l \sim \pm 1^\circ$, but with degrading correlation beyond that, suggests that we are dealing with a source (e.g., the supernova remnant [SNR] Sgr A East or the central black hole Sgr A*) at the Galactic center (GC) that was active for some time in the past and that we are now seeing the high-energy particles diffusing from this central source (Aharonian et al. 2006). The most likely primary species responsible for the γ-ray emission then is protons, since electrons would have to compete against synchrotron losses, resulting in a spectral steepening toward large distances. Apart from slight effects of energy-dependent diffusion, protons, in a good approximation, do not lose energy within this environment, resulting in an approximate invariant spectral index with distance.

Whereas these results are important from an astrophysical viewpoint, they are also of importance from a cosmic-ray viewpoint, i.e., in view of the study of cosmic-ray propagation in our Galaxy: Aharonian et al. (2006) suggested that the diffusion coefficient for protons in the 4–40 TeV range should be less than $10^{30} \text{ cm}^2 \text{ s}^{-1}$ (or $3.5 \text{ kpc}^2 \text{ Myr}^{-1}$) as a result of enhanced turbulence and higher magnetic field strengths in the GC region. The HESS data therefore offer a unique possibility for measuring the diffusion coefficient in this part of the Galaxy and for comparing with other measurements of the Galactic diffusion coefficient. This is particularly important, as it has been shown (Büsching et al. 2005) that, given SNRs are the main sources of cosmic rays (CRs), the widely used method to obtain propagation parameters by fitting secondary to primary data is at least tainted, as the CR primary component then shows strong variations in space and time.

In this paper we model a transient source at the GC with an activity timescale in the past. By solving the transport equation for proton propagation along the Galactic plane, we obtain the range of diffusion coefficients that best fit the observed HESS profile. To do this we also model the gas distribution as traced by the CS emission.

Tsuboi et al. (1999) were the first to obtain full coverage of the Galactic center bow (GCB) and the molecular cloud structures in CS in the region of interest. Since the γ-ray surface brightness is reflected by the line-of-sight integral of the product of the cosmic-ray and gas densities, we introduce three-dimensional (3D) structures in the GC region (although not strictly modeling the GCB itself), such that the line-of-sight integrals through these model structures reproduce the true line-of-sight integrals through the gas density within $\sim 5\%$ accuracy, as described in more detail below. A detailed discussion about the systematic error due to this approach is given in § 2.2.

Using the diffusive model of CR propagation, together with observations made by HESS (Aharonian et al. 2006), it is possible to get an estimate of the diffusion coefficient controlling CR transport in the Galactic center region.

1.1. Very High Energy Gamma Radiation from the Galactic Center Region

In a recent paper, the HESS collaboration found extended, diffuse emission of VHE γ-rays in the GC region, closely matching...
the distribution of the interstellar gas in the region $|l| < 1^\circ$ (Aharonian et al. 2006). As they note, this VHE $\gamma$-ray emission is most probably caused by the interaction of hadronic CRs with the interstellar gas in these clouds. Given the rather hard spectrum of the observed diffuse $\gamma$-ray emission (spectral index of $-2.3$, compared to $-2.7$ for the local CR spectrum), and thus of the CR spectrum,\(^1\) they argue that these CRs must be near their most probably caused by the interaction of hadronic CRs with (Aharonian et al. 2006). As they note, this VHE is traced by the VHE $\gamma$-ray flux.\(^2\) Thus, assuming the local CR flux and spectrum are representative for the whole Galaxy, there must be an additional CR component in the GC region. Following Aharonian (2001) they estimate the energy losses. For the SMBH, Cheng et al. (2006) considered particle acceleration due to stellar infall on Sgr A* and found that the total energetics would scale with the mass of the infalling star. Whereas it is difficult to predict the mass of such an infalling star, the predicted CR energy output from a SNR is more firm (Aharonian et al. 1994), and the observed energetics is already compatible with this number of $E_{\text{CR}} \approx 0.1E_{\text{SNR}} \approx 10^{50}$ ergs; we thus adopt a SNR origin as proposed previously.

1.2. Gamma Rays from Pion Decay

The omnidirectional (i.e., integrated over solid angle) differential $\gamma$-ray source function $q_{\gamma^0}(E_\gamma, r)$ at the position $r = (l, b, r)$ for the decay $\pi^0 \rightarrow 2 \gamma$ is given by (Stecker et al. 1968)

$$q_{\gamma^0}(E_\gamma, r) = 2 \int_\pi^\infty \frac{Q_{\gamma^0}(\gamma_\pi, r)}{\sqrt{\gamma^2_\pi - 1}} \, d\gamma_{\pi},$$

where $\gamma_\pi$ is the pion Lorentz factor. The lower boundary of the integration is given by

$$\eta = \frac{E_\gamma}{m_\pi c^2} + \frac{m_\pi c^2}{4E_\gamma}.$$  

The pion source function then is

$$Q_{\gamma^0}(\gamma_\pi, r) = \rho_{\gamma\text{SNR}}(r)c \int_{\gamma_{\text{thr}}}^{\infty} \beta \sigma_{pp}^{\pi^0}(\gamma_\pi, \gamma_\pi) N_\pi(\gamma_\pi, r) \, d\gamma_{\pi},$$

where $\sigma_{pp}^{\pi^0}$ is the total cross section for pion production in $pp$ collisions and $N_\pi$ the CR proton spectrum. The differential photon flux from the decay of CR-induced $\pi^0$s from the direction $(l, b)$ is given by integrating equation (1) along the line of sight,

$$\frac{dN(E_\gamma, l, b)}{dE_\gamma \, d\Omega} = \frac{1}{4\pi} \int q_{\gamma^0}(E_\gamma, r) \, dr.$$  (4)

The above calculation is for pion production from $pp$ interactions only. The effect of the known chemical composition of the interstellar medium can be taken into account by increasing the total pion production cross section by a factor of 1.30 (Mannheim & Schlickeiser 1994).

2. REPRODUCING THE DIFFUSIVE GAMMA-RAY EMISSION FROM THE GALACTIC CENTER

As the observed photon index of the observed diffuse $\gamma$-radiation hardly varies over the whole emitting region (Aharonian et al. 2006), the spectral index of the CR in this region has to be almost constant. We can thus write for the CR proton spectrum

$$N_\pi(\gamma_p, r) \approx N_\pi(r)N_E(\gamma_p),$$

and thus we get from equations (1) and (3), for equation (4),

$$\frac{dN(E_\gamma, l, b)}{dE_\gamma \, d\Omega} = \frac{c}{2\pi} \int N_\pi(r)\rho_{\gamma\text{SNR}}(r) \, dr \cdot \int_{\pi}^{\infty} \frac{\beta \sigma_{pp}^{\pi^0}(\gamma_p, \gamma_p) N_\pi(\gamma_p, r)}{\sqrt{\gamma^2_\pi - 1}} \, d\gamma_{\pi}.$$  (5)

For our studies, we are primarily interested in the part of equation (6) that depends on the spatial coordinates, so with $r = r(l, b, r)$,

$$\frac{dN(E_\gamma, l, b)}{dE_\gamma \, d\Omega} \propto \epsilon(l, b) = \int N_\pi(r)\rho_{\gamma\text{SNR}}(r) \, dr,$$  (7)

where $\epsilon(l, b)$ is the relative emissivity from the direction $l, b$. The energy-dependent part of equation (6) is discussed in § 2.5.

In this paper, we reproduce the diffusive $\gamma$-ray emission calculating the line-of-sight integral equation (7) for various $(l, b)$ combinations to reproduce the distribution of diffuse $\gamma$-rays on the sky as seen by the HESS array. The derivation of the distribution of the interstellar gas $\rho_{\text{gas}}$ and the calculation of the spatial distribution of CR protons $N_\pi(r)$ dependent on the spatial diffusion coefficient is tackled in the sections below.

2.1. Gas Distribution near the Galactic Center

The inner $150 \text{ pc}$ region of the Galaxy contains interstellar H$_2$ gas of about $(2\pm5) \times 10^7 M_\odot$ (Tsutsui et al. 1999) in a rather complex setup of molecular clouds. For our analysis we assumed that the target material density function can be adequately described by the superposition of six spherical Gaussian functions on an asymmetric Gaussian base. These functions represent the molecular clouds associated with the radio arc of Sgr A and Sgr B, as well as the longitude-varying line-of-sight projection effect of the Galactic center bow as described by Tsutsui et al. (1999).

Even though we are able to reproduce the observed line-of-sight gas densities within about 5% from the observed values, as shown in Figure 1, uncertainties in the exact depth distribution along the $l$-coordinate in equation (7) are expected to result in a factor of 2 systematic uncertainty in the final estimate of the diffusion coefficient. We discuss this in more detail in the next section.
2.2. Cosmic-Ray Distribution near the Galactic Center

The CR distribution as a function of spatial coordinates is calculated using the diffusive model of CR propagation. We calculate the CR density assuming the CRs are coming from a single SNR event in the past, accelerating particles from \( t_{\text{on}} \) for a certain time \( \Delta t / C^2 \).

As was pointed out by Maeda et al. (2002) the SNR Sgr A East has an estimated age of 10 kyr, but younger ages have also been given for that SNR (Rockefeller et al. 2005). Thus, we also investigated the influence of the age of Sgr A East. For our study, we assume that Sgr A East has an age of \( 5 - 10 \) kyr and was accelerating CRs on various timescales less than its present age.

We are interested in the CR distribution near the source and thus neglect the effects of loss processes; calculations taking into account loss processes lead to similar values of the diffusion coefficient. We also neglect boundary effects by imposing boundary conditions for the CR propagation problem at infinity. The equation for the CR density thus becomes

\[
\frac{\partial N_r}{\partial r} - S = \kappa \nabla^2 N_r, \tag{8}
\]

where \( S = \Theta(t - t_{\text{on}}) - \Theta(t - t_{\text{on}} + \tau) \) describes the particle source, with \( \Theta \) denoting the Heaviside step function and \( \kappa \) the diffusion coefficient to be determined. Equation (8) is the ordinary heat equation, and one can find solutions for it in the literature, e.g., Morse & Feshbach (1953).

By using the integral in equation (7) the emission seen by HESS (Aharonian et al. 2006) can be recreated for different diffusion coefficients \( \kappa \) as shown in Figure 2 (top). As noted by Aharonian et al. (2006), the excess counts observed by HESS follow the known target material density fairly well, except for the region \( l \geq 1^\circ \). This provides a gauge for approximating the diffusion coefficient governing the CR propagation near the Galactic center.

To obtain better counting statistics, we compare our model and observed HESS excess counts, integrated over Galactic latitude. Integrating equation (7) over \( b \) thus shows that the excess counts observed by HESS then are proportional to the integral

\[
\epsilon(l) \propto \int_0^\infty \int_{-\pi/2}^{\pi/2} \rho(l, b, r) N_{\text{CR}}(l, b, r) r \, db \, dr, \tag{9}
\]

where the CR density \( N_{\text{CR}} \) has been calculated from equation (8) for different diffusion coefficients, as shown in Figure 2 (bottom).
The obtained fit is normalized to the total numbers of excess counts before calculating the corresponding $\chi^2$ value. The results of the analysis are shown in Figure 3, where we plotted the reduced $\chi^2$ for different source on times from 1 to 9 kyr, assuming an age of Sgr A East of 10 kyr. Our results for different source ages and on times are plotted in Figure 4. The resulting CR diffusion coefficients range from 1 to 4.8 kpc$^2$/Myr$^{-1}$. The diffusion coefficient estimated by Aharonian et al. (2006) fits quite well into this range.

We also note that the minimum reduced $\chi^2$ is 1.5 for ~25 degrees of freedom (doF), which indicates that the model agrees with the data within the ~5% confidence level. Even though this fit is still acceptable, one obvious source of systematic error is the above-mentioned assumptions that went into the 3D model for the gas clouds (see, e.g., Tsuboi et al. 1999): We assumed that peculiar motions near the Galactic center result in uncertainties on the precise line-of-sight distances to each cloud. We model this uncertainty as follows: By placing each cloud at the same distance from Earth (i.e., the distance to the GC, but maintaining the true locations on the plane of the sky), the average error ($\delta r$) on the radial distance ($r$) from the GC to each cloud can then be off by an rms amount of $\delta r/r \sim \sqrt{2} - 1 = 0.4$, which translates into a relative error on the diffusion coefficient of $\delta k/k = 2\delta r/r \sim 1$, with the extra factor of 2 on $\delta k$ arising from the square dependence of $k$ on the radial distance.

For almost all assumed source parameters we get a well-defined minimum reduced $\chi^2 \sim 1.5$ (see Fig. 3), leading to the above-mentioned measurement of $k$. This minimum value of $\chi^2$ increases slightly when the on time gets closer to the source age, which indicates that the HESS data are even consistent with source activity timescales equal to the assumed SNR age. A more sophisticated 3D model of the gas might result in improved $\chi^2$ values, but the improvement cannot be much more, since we have already reached the 5% confidence level with this simplified $\chi^2$ analysis. Furthermore, such a detailed 3D model for the gas is clearly beyond the scope of this paper, and even then the remaining systematic uncertainties should again be evaluated. We therefore estimate a final systematic uncertainty of a factor of 2 on the measured value of $k$.

2.3. The Influence of the Cosmic-Ray “Sea”

For the calculations presented above, we assumed that all the protons producing VHE $\gamma$-rays originate from the source in the direct vicinity of the GC, most probably Sgr A East. Nevertheless, as mentioned by (Aharonian 2001), one would expect that in the GC region there is also a background “sea” of CR originating from many sources. To investigate the effect of this CR background “sea,” we constructed a CR background by assuming that the $\gamma$-ray flux seen by HESS in the region $1 < l < 1.7$ is generated by this CR component. A result of this calculation is shown in Figure 5. From these calculations we find minimal values for the reduced $\chi^2$ that are similar to those presented above ($\sim 1.5$). The obtained values for the CR diffusion coefficient are about 40% smaller than those obtained from the calculations presented in the previous section. In this picture, the CR “sea” contributes $\approx 30\%$ to the observed $\gamma$-ray flux.

2.4. Total Energy in Cosmic Rays

By integrating the observed $\gamma$-ray spectrum from the region $|l| < 0.8^\circ, |b| < 0.3^\circ$ given by Aharonian et al. (2006) we find a

\[ \quad \]
\[ F(\gamma; > 400 \text{ GeV}) = 6.0278 \times 10^{-20} \frac{m_{\text{gas}}}{\text{cm}^{-2}} \frac{\text{s}^{-1}}{\text{G}^2}, \]  

(10)

where \( m_{\text{gas}} \) is the mass of the molecular gas in the region \( |l| < 0.8^\circ \) in units of solar masses. Assuming a mass of \((1.5 - 4.5) \times 10^7 \text{ M}_\odot\) in this region, we expect a \( \gamma \)-ray flux of \( 2.7 \times 10^{-12} \text{ cm}^{-2} \text{s}^{-1} \). If we take into account the fact that up to \( \approx 70\% \) of the \( \gamma \)-radiation from \( |l| < 0.8^\circ \) [i.e., \((6.3 - 11.4) \times 10^{-12} \text{ cm}^{-2} \text{s}^{-1}\)] originates from the CR component accelerated at the GC, then it is reasonable to assume that the latter CR component indeed originates from a single, powerful SN.

2.5. Mean Cosmic-Ray Energy

In the previous sections, we have shown that the diffuse \( \gamma \)-ray emission from the Galactic center ridge can be explained by CR hadrons whose propagation can be described by a diffusion coefficient, depending on source \( \text{on} \) time and age, in the range \( 1.0 - 4.8 \text{ kpc}^2 \text{ Myr}^{-1} \). This diffusion coefficient is valid for CRs generating the bulk of the \( \gamma \)-ray emission in the HESS energy range.

To compare this value with that obtained by fitting local CR data, as derived by, e.g., Moskalenko et al. (2002), Jones et al. (2001), or Maurin et al. (2002), we have to estimate the energy of the CRs probed here. For this investigation, we make use of the energy-dependent part of equation (6) and approximate the sensitivity of the HESS telescope array by a box function from 0.4 to 20 TeV. For the CR proton spectrum we assumed a power law with the observed photon index \( \Gamma = 2.29 \pm 0.27 \) (Aharonian et al. 2006). We used the Pythia (Sjöstrand et al. 2001a, 2001b) event generator package to calculate the number of \( \gamma \)-rays with \( E_\gamma > 0.4 \text{ TeV} \), for different CR proton energies. The result of this calculation is shown in Figure 6.

The maximum number of photons is produced by CR protons with energies of about 2.2 TeV, given a proton spectral index of 2.29. Note that the location of the maximum depends on the proton spectral index. Given the uncertainties stated by Aharonian et al. (2006) we find that CR protons with energies in the range 1.7 - 3 TeV contribute the most to the \( \gamma \)-rays seen by HESS.

Assuming a diffusion coefficient of the form

\[ \kappa = \kappa_0 \left( \frac{\xi}{\xi_0} \right)^{0.6}, \]

(11)

where \( \xi_0 = 1 \text{ GV/c} \) and \( \xi \) is the particle rigidity, we find \( \kappa_0 = 0.01 - 0.048 \text{ kpc}^2 \text{ Myr}^{-1} \), which is significantly smaller than the values found by fitting local CR data. However, \( \kappa \) for the latter range from 0.0535 to 0.201 \text{ kpc}^2 \text{ Myr}^{-1}, as compiled by Maurin et al. (2002 and references therein). This finding can be well explained by enhanced turbulence and a higher field strength of the interstellar magnetic field in the Galactic center region. We note, however, that the rigidity dependence of 0.6 can only apply to a limited energy (rigidity) range, since \( \kappa \) must always be larger than the Bohm limit.

3. SUMMARY AND DISCUSSION

We have shown that progress in the imaging Cerenkov technique now, for the first time, makes it possible to obtain the CR diffusion coefficient in other parts of the Galaxy. A diffusion coefficient in the range \( \kappa \sim 1.0 - 4.8 \text{ kpc}^2 \text{ Myr}^{-1} \) appears to be well determined from the data, although we have to add a factor of 2 systematic uncertainty arising from uncertainties in the actual 3D gas density distribution. We also demonstrated that the above values could be up to 40\% smaller due to the presence of a CR “sea.” We showed that because of uncertainties on the epoch when the central source activity started, which was assumed to be in the range 5 - 10 kyr, and because of the \( \text{on} \) time of the source, one is only able to give a range for the diffusion coefficient.

As we have shown above, based on the total energy of CRs in the GC region, the most convincing source for the CRs traced by the VHE \( \gamma \)-ray emission (apart from the CR background) is a powerful SN close to the GC, most probably Sgr A East. This is strengthened a posteriori by obtaining diffusion coefficients that have the expected order of magnitude (Aharonian et al. 2006). Nevertheless, we cannot rule out intermittent stellar capture by the central black hole Sgr A* (Cheng et al. 2006) as a source of variable particle acceleration at the GC, although we think that it is implausible that the SMBH accelerated CRs with the total energy expected from a SN, on a timescale not too different (unless there is a distinct difference in CR propagation in the GC region) from the age of the existing SNR at the GC, Sgr A East. Even in this case our calculations will still be applicable if we consider the \( \text{on} \) time to be much less than the time since the stellar infall (i.e., small \( \text{on} \) times in Fig. 4), which gives a narrower defined range for \( \kappa \) compared to the case in which the \( \text{on} \) time is comparable to the time since the event. A more detailed study of this effect will be treated elsewhere. However, if Sgr A East was the source of CRs, then we would have a relatively well defined range of \( \kappa \) as given in the text.

In general, by following the notion of Aharonian et al. (2006) of constraining the cosmic-ray diffusion coefficient \( \kappa \) from the spatial distribution of diffuse \( \gamma \)-rays resulting from impulsive injection of a CR source at some time in the past, we have shown that one can obtain unique values of the diffusion coefficient, provided that the gas density distribution and the epoch of onset and duration of the central source activity are known. We also showed that the CR diffusion coefficient in the Galactic center...
region is significantly smaller than that obtained by fitting local CR data. Our understanding of turbulence theory, however, is still too limited to understand how diffusion coefficients scale with the turbulence $\delta B$, the correlation length associated with this turbulence, the total magnetic field strength $B$, and rigidity dependence for perpendicular and parallel diffusion. We hope that our new obtained values of $\kappa$ at the GC will help to constrain results from turbulence theory.

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REFERENCES

Aharonian, F., et al. 2006, Nature, 439, 695
Aharonian, F. A. 2001, Space Sci. Rev., 99, 187
Aharonian, F. A., Drury, L. O'C., & Völk, H. J. 1994, A&A, 285, 645
Büssing, I., Kopp, A., Pohl, M., Schlickeiser, R., Perrot, C., & Grenier, I. 2005, ApJ, 619, 314
Cheng, K. S., Chernyshov, D. O., & Dogiel, V. A. 2006, ApJ, 645, 1138
Jones, F. C., Lukasiak, A., Ptuskin, V., & Webber, W. 2001, ApJ, 547, 264
Maeda, Y., et al. 2002, ApJ, 570, 671
Mannheim, K., & Schlickeiser, R. 1994, A&A, 286, 983
Maurin, D., Taillet, R., & Donato, F. 2002, A&A, 394, 1039
Morse, P. M., & Feshbach, H. 1953, Methods of Theoretical Physics (New York: McGraw-Hill)
Moskalenko, I. V., Strong, A. W., Ormes, J. F., & Potgieter, M. S. 2002, ApJ, 565, 280
Rockefeller, G., Fryer, C. L., Baganoff, F. K., & Melia, F. 2005, ApJ, 635, L141
Sjöstrand, T., Edén, P., Friberg, C., Lönndblad, L., Miu, G., Mrenna, S., & Norrbin, E. 2001a, Comput. Phys. Commun., 135, 238
Sjöstrand, T., Lönndblad, L., & Mrenna, S. 2001b, Pythia 6.2: Physics and Manual (Lund Univ. Rep. LU TP 01-21; Lund: Lund Univ.) (hep-ph/0108264)
Stecker, F. W., Tsuruta, S., & Fazio, G. G. 1968, ApJ, 151, 881
Tsukui, K., Toshihiro, H., & Ukita, N. 1999, ApJS, 120, 1
Wang, Q. D., Lu F. J., & Gotthelf, E. V. 2006, MNRAS, 367, 937