The composite Fermion model of quantum Hall effect is internally inconsistent

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The composite fermion (CF) model of the quantum Hall effect which gives the correct series of charges is based on attachment of flux quanta to the electron. The construction of the series of charges leads to a field expression which requires that flux quanta are attached to the electron. The series based on the experimental data is correct but the field deduced from such a series is found to be incorrect. The size of the CF is compared with the electron radius and it is found that for the same density, the CF are internally inconsistent. The attachment of the flux quanta to the electron or detachment of flux quanta from CF is neither found experimentally nor is feasible theoretically.
1. Introduction

The fractional charges found in the experimental data of quantum Hall effect are described by $e_{\text{eff}} = \nu e$ where two expressions for $\nu$ appear to give the correct experimental values,

$$\nu_+ = \frac{\nu^*}{2p\nu^* + 1} \quad (1)$$
and

$$\nu_- = \frac{-\nu^*}{2p\nu^* - 1} \quad (2)$$

The above expressions are symmetric with respect to interchange of $\nu$ and $\nu^*$ so that,

$$\nu^* = \frac{\nu}{2p\nu + 1}, \quad \nu^* = \frac{-\nu}{2p\nu - 1}. \quad (3)$$

For $p=1$, the first of these series gives,

$$\nu^* = \frac{\nu}{2\nu + 1} \quad (4)$$

which is the same as found experimentally for $e_{\text{eff}} = \nu^* e$ and integer $\nu$. For $\nu = 1, 2, 3, 4, \text{etc.}$ correct values of the effective charge are found.

Jain(1989, 1998) introduced two types of quasiparticles, the composite fermions (CF) and the electrons. The magnetic field is quantized for both of these quasiparticles but they see different fields,

$$\nu = \frac{\rho \phi_o}{B} (\text{electrons}); \quad (5)$$
and

$$\nu^* = \frac{\rho \phi_o}{B^*} (\text{CFs}). \quad (6)$$

The two fields are related by the expression,

$$B^* = B - 2p\rho \phi_o. \quad (7)$$

If we put $B^*=0$, then B is quantized and when we put $B=0$, then $B^*$ is quantized. It is assumed that $\phi_o$ can be parallel as well as antiparallel to B. Here $p$ is an integer, $\nu$ and $\nu^*$ are also integers. By substituting integer $\nu$, fractional $\nu^*$ is produced. The number density of electrons per unit area is $\rho$ and $\phi_o = h\nu/e$. While $\nu^* = \nu/(2\nu + 1)$ gives a series
of experimentally measured values correctly, the field $B^*$ is not observed experimentally. Therefore, we wish to see if (7) is internally consistent. The factor $2p$ is an even number and it produces the correct series (4), i.e., the factor 2 in the denominator of (4) is correct. According to the expression (7), even number of flux quanta are attached to the electron, i.e.,

$$CF = e + 2p \text{ vortices.}$$

(8)

We can represent this quasiparticle by a picture showing an electron with two (even number) of flux quanta attached as shown in Fig.1.

2. Comments

(a). We find that the series (4) is experimentally observed but the field (7) is not observed experimentally. If there was a phase transition at which the even number of flux quanta were attached while cooling and detached while heating, then it was satisfactory to represent the field by (7) but no such phase transition is found experimentally.

The number density of electrons is given by $\rho = n_o/l^2$, where $l$ is the magnetic length, so that for $\nu=1$, 

$$l = (n_o \phi_o / B)^{1/2}.$$  \hspace{1cm} (9)

The size of the electron may be determined by the classical radius of the electron which is given by, $e^2/mc^2$, 

$$e^2/mc^2 = 2.8179 \times 10^{-13} \text{ cm}$$  \hspace{1cm} (10)

or by the Compton wave length which is the inverse mass,

$$\hbar/mc = 3.86159 \times 10^{-11} \text{ cm}$$  \hspace{1cm} (11)

For $h=6.6262 \times 10^{-27} \text{ erg s}$, $c=2.9979 \times 10^{10} \text{ cm/s}$ and $e = 4.80325 \times 10^{-10} \text{ esu}$, $\phi_o=\hbar c/e=4.13 \times 10^{-7} \text{ G cm}^2$ so that at $B=28.6 \text{ T}$, $n_o=1$,

$$l = 1.2 \times 10^{-4} \text{ cm.}$$  \hspace{1cm} (12)

Therefore, the magnetic length is seven orders of magnitude larger than the Compton wave length of the electron. Similarly, the magnetic length is nine orders of magnitude
larger than the classical radius of the electron. Therefore, if flux quanta are attached to the electron, the CF are going to be very large objects while the electrons are small. In the expressions (5) and (6) the density of CF is equal to the density of electrons. Why the very large CFs should have the same density as that of electrons? A small number of large objects can fill the same space as a large number of small objects but large objects can not fill the same space with the same number as the small objects unless the larger ones are squeezed, but there is no provision to squeeze in any of the formulas. Therefore, the expressions (5) and (6) are internally inconsistent with (7). There is not enough room to attach a lot of vortices to electrons. When a large number of vortices are attached to the electrons, we should require a lot more space than that occupied by electrons. Therefore CF model is internally inconsistent.

(b) When electrons get attached to the flux quanta, let us say 2, then the resulting CF is a large object so that they push some electrons out of the sample. Thus the charge leaks out of the sample resulting into flow of electrons or a current. No such current has been detected in the experiments. However, the leak current will disturb the resistivity measurement. If a constant voltage has to be maintained, the reduced current has to be balanced by an increase in resistivity in addition to that given by the classical Hall effect. Over and above the Hall resistivity, an additional increase in the resistivity should occur but no such increase in resistivity has been detected. Alternatively, the current drop should reflect in the voltage drop but no such voltage drop has been detected experimentally. Therefore, it can be said safely that composite fermions (CFs) have not been seen experimentally and the claims made are incorrect. According to the CF model, flux quanta are attached to the electron but it is found that no such flux quanta are attached to the electron. The attachment of flux quanta to electrons has not been found in the experimental data.

It has been pointed out by Kumada et al that there are two types of quasiparticles in the flanks of \( \nu=2/3 \). Hysteresis can arise because the energies of these quasiparticles having opposite spins depend on Zeeman energy. In fact "the opposite spin" aspect is
not a part of the CF model and this feature observed by Kumada et al is not contained in the CF model. In fact this experimental feature is obtained only in Shrivastava’s theory. Kumada et al make a case for CF on the basis of Fermi liquid, activation gap, polarization, cyclotron resonance, etc. but none of these experiments prove that flux quanta are attached to the electrons. The Fermi liquid does not require the flux attachment. The activation is an old barrier problem not connected to the flux attachment. The polarization where it has been measured correctly by NMR does not require flux attachment. Similarly, the cyclotron resonance can be done without attaching flux quanta to the electrons. The CF theory also violates the Biot-Savarts law of classical electrodynamics and hence CF is not consistent with Maxwell equations. The CF also do not obey the flux quantization correctly.

3. Proper theory

The correct theory of the quantum Hall effect is given by Shrivastava\(^4\). It has been pointed out that all of the experimental data on the quantum Hall effect agrees with the theory of Shrivastava.\(^4\)

We are not ignoring the award of Oliver Buckley Prize to Jain\(^1\) for the CF model but our paper is dated earlier. Some comments have appeared in the literature\(^6\). Willett’s\(^7\) data is also fully in agreement with the theory of Shrivastava\(^4\). Similarly, the ”opposite spin” feature pointed out by Kumada et al is clearly demonstrated in ref.4, but not in CF model.

4. Conclusions.

We find that the flux quanta are not attached to the electrons. Hence the CF model is incorrect. The flux attachment is neither found experimentally nor is feasible theoretically. Such experimentalists who are claiming to have observed the CF have observed only the series of charges but not the flux attachment. Usually it is a custom to agree with the decisions and proceed by assuming that the “awarded result” is correct but in the case of CF the award is clearly misplaced.
5. References

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Note: Ref.4 is available from:
Nova Science Publishers, Inc.,
400 Oser Avenue, Suite 1600,
Hauppauge, N. Y.. 11788-3619,
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Fig.1: Jain’s model of composite fermions is sketched showing flux quanta attached to one electron. It turns out that after attaching flux quanta, the resulting quasiparticle called CF will be too big to have the same density as that of electrons. This model is found to be internally inconsistent. Some experimentalists claim to have found the CF but such claims are “not true”.

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This figure "CF1FIG.jpg" is available in "jpg" format from:

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