Decay of the vacuum energy into CMB photons

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ABSTRACT
We examine the possibility of the decay of the vacuum energy into a homogeneous distribution of a thermalized cosmic microwave background (CMB), which is characteristic of an adiabatic vacuum energy decay into photons. It is shown that observations of the primordial density fluctuation spectrum, obtained from CMB and galaxy distribution data, restrict the possible decay rate. When photon creation due to an adiabatic vacuum energy decay takes place, the standard linear temperature dependence \( T(z) = T_0 (1+z) \) is modified, where \( T_0 \) is the present CMB temperature, and can be parameterized by a modified CMB temperature dependence \( \mathcal{T}(z) = T_0 (1+z)^{1-\beta} \). From the observed CMB and galaxy distribution data, a strong limit on the maximum value of the decay rate is obtained by placing a maximum value \( \beta_{\text{max}} \approx 3.4 \times 10^{-3} \) on the \( \beta \) parameter.

Key words: galaxies: distances and redshifts – cosmic microwave background – cosmology: theory – cosmology: observations

1 INTRODUCTION
The present observed acceleration of the universe is due to a substance which we call dark energy, the nature of which is yet unknown. Bronstein (1933) was the first to introduce the idea of the possible decay of dark energy. A recent review of possible explanations for the nature of dark energy and its possible decay can be found, for example, in Peebles & Ratra (2003). As noted by Peebles & Ratra (2003), the evolution of the dark energy density and its related coupling to matter or radiation is, in general, assigned and not derived from an action principle. Discussions of this can be found in Pollock (1981); Kazanas (1984); Freese et al. (1987); Gasperini (1987); Sato, Terasawa & Yokoyama (1989); Bartlett & Silk (1990); Overduin et al. (1993); Matviash (1997); Birkel & Sarkar (1997).

In the present paper, we assume that the dark energy is the vacuum energy and investigate its possible decay. Some scalar field dark energy models, such as that of Peebles & Ratra (1988) and Ratra & Peebles (1988), were motivated, in part, by particle physics and used observational data to constrain the decay rate of dark energy. However, in general, almost all studies of the decay of the dark energy, assuming that it is the vacuum energy, as is done in this paper, are purely phenomenological and do not put strong limits on the decay from observational data (see, for example, Camuto et al. 1977; Endo & Fukui 1977; Bertolami 1986; Ozer & Tahla 1984; Freese et al. 1987; Gott & Rees 1987; Kolt 1989; Chen & We 1990; Pavon 1991; Carvalho, Lima & Waga 1992; Krauss & Schramm 1993; Overduin et al. 1993; Silveira & Waga 1994; Lima & Mai 1994; Maia & Silva 1994; Kalligas, Wesson & Everitt 1993; Lima & Trodden 1996; Garnavich et al. 1998; Shapir & Sol 2000; 2004; Shapir et al. 2003; Peebles & Ratra 2003; Overduin & Wesson 2004).

In a previous paper, we studied the limit put on the rate of a possible vacuum energy decay into cold dark matter (CDM) by the observed cosmic microwave background (CMB) and large galaxy survey data (Opher & Pelinson 2004). The observed temperature fluctuations of the CMB photons \( \delta T/T \) are approximately proportional to CDM density fluctuations \( \delta \rho/\rho \) (Padmanabhat 1993). CDM density fluctuations derived from the CMB data can be compared with those derived from the 2dF Galaxy Redshift Survey (2dFGRS) ( Lahav et al. 2004; Percival et al. 2004). It was found that the \( \delta \rho/\rho \) derived from the galaxy distribution data differs from the \( \delta \rho/\rho \) derived from the CMB data by no more than 10 per cent (see Cole et al. 2004 for the final data set of the 2dFGRS).

A vacuum energy decaying into CDM increases its total density, diluting \( \delta \rho/\rho \). In order to evaluate \( \delta \rho/\rho \) at the recombination era, when it created the \( \delta T/T \) of the CMB, its present measured value obtained from the galaxy distribution data must then be increased by a factor \( F \). Since the \( \delta \rho/\rho \) derived from the CMB and galaxy distribution data agree to 10 per cent, the maximum value for \( F \) is \( F_{\text{max}} = 1.1 \).
We found that the decay of the vacuum energy into CDM as a scale factor power law \( \rho_\Lambda \propto (1 + z)^\gamma \), gives a maximum value for the exponent \( \gamma_{\text{max}} \approx 0.06 \). For a parametrized vacuum decay into a CDM model with the form \( \rho_\Lambda(z, \nu) = \rho_\Lambda(z = 0) + \rho^0_\psi \left[ \nu / (1 - \nu) \right] \left[ (1 + z)^{\beta(1 - \nu) - 1} \right] \), where \( \rho^0_\psi \) is the present critical density, an upper limit on the \( \nu \) parameter was found to be \( \nu_{\text{max}} \approx 2.3 \times 10^{-3} \).

Here, we study the limit imposed on the rate of a possible decay of the vacuum energy into a homogeneous distribution of thermalized CMB photons. In this scenario, (\( \delta T/T \)) at the recombination epoch were diluted by photons created by a vacuum energy decay. Thus, (\( \delta T/T \)) at present are smaller than those existing at the recombination era. This implies larger \( \rho \) at the recombination era than those derived from the observed CMB data.

We generally assume that the CMB temperature \( T \) is proportional to \( (1 + z) \) in a Friedmann-Robertson Walker (FRW) universe. There is, however, little direct observational evidence for this relation despite considerable observational efforts to verify it. Lima et al. (2000) summarized some of the observational studies which have been made up to a redshift \( z \approx 4.5 \).

Although in the present paper, we investigate a possible decay of a homogeneous vacuum energy into a homogeneous distribution of photons, the vacuum energy may, in principle, not be homogeneous and its decay could then lead to an inhomogeneous distribution of photons. A vacuum energy depending on spatial position could be created, for example, by a Casimir effect, such as that described by Muller, Fagundes and Opher (2001, 2002). The inhomogeneous distribution of decay photons produced might be able to be detected in high precision CMB data. This possibility will be investigated in future studies.

In Section II, we put strong limits on the possible decay of the vacuum energy into a homogeneous distribution of thermalized CMB photons and its effect on (\( \delta \rho/\rho \)) derived from the observed CMB data. Our conclusions are presented in Section III.

### 2 VACUUM ENERGY DECAY INTO CMB PHOTONS

According to the standard model, (\( \delta T/T \)) were created at \( z_{\text{rec}} \sim 1100 \), the recombination epoch (Padmanabhan 1993). In the standard model, (\( \delta T/T \)) observed today are given by the expression

\[
\frac{\delta T}{T} \bigg|_{z_{\text{rec}}} = \mathcal{K} \frac{\delta \rho}{\rho} \bigg|_{z_{\text{rec}}},
\]

where \( \mathcal{K} \) is approximately constant and the temperature dependence of \( T(z) \) is

\[
T(z) = T_0 (1 + z),
\]

where \( T_0 \approx 2.75 \text{K} \) is the present CMB temperature. The present value of (\( \delta \rho/\rho \)) is given from the relation

\[
\frac{\delta \rho}{\rho} \bigg|_{z_{\text{rec}}} = D (z_{\text{rec}} \to z = 0) \frac{\delta \rho}{\rho} \bigg|_{z_{\text{rec}}},
\]

where \( D (z_{\text{rec}} \to z = 0) \) is the growth factor from the recombination era until the present time.

Let us examine a possible vacuum decay into photons. Assuming that the decay is adiabatic, the vacuum energy decays into a homogeneous distribution of thermalized black body CMB photons. This was shown by Lima et al. (2000), as follows. The conservation equation for the photon number density \( n \) is

\[
\dot{n} + 3nH = \psi,
\]

where \( H \) is the Hubble parameter and \( \psi \) is the photon source term. Taking Gibbs law into account, we have

\[
nT d\sigma = \frac{\rho + P}{n} d\nu,
\]

where \( \sigma \) is the specific entropy. Since \( d\sigma \) is an exact differential, we obtain the thermodynamic identity

\[
T \left( \frac{\partial P}{\partial T} \right)_n = \rho + P - n \left( \frac{\partial \rho}{\partial n} \right)_T.
\]

Using \( T \) and \( n \) as independent thermodynamic variables, we obtain

\[
\dot{T} = \left( \frac{\partial P}{\partial T} \right)_n \frac{\dot{n}}{n} - \frac{\psi}{nT} \frac{\partial \rho}{\partial T} \left[ \rho + P - n \frac{\dot{\rho}/\psi}{\psi} \right],
\]

\[
\dot{\sigma} = \frac{\psi}{nT} \left( \frac{\partial \rho}{\partial T} \right)_n \left[ \rho + P - n \frac{\dot{\rho}/\psi}{\psi} \right],
\]

where \( \dot{\rho}/\psi \) is the decay rate of the vacuum energy density (Lima & Trodden 1996). For photons, we have \( P = \rho/3 \). In order to have homogeneous equilibrium black body radiation, \( n \propto T^3 \), we must have

\[
\rho + P - n \frac{\dot{\rho}/\psi}{\psi} = 0
\]

or

\[
\dot{\rho}/\psi = \frac{4\rho}{3n}.
\]

We then have

\[
\dot{\sigma} = 0.
\]

Thus, when the decay of the vacuum energy is adiabatic and the specific entropy does not change, we obtain a homogeneous thermal distribution of black-body radiation. From the conservation equation for the photon density, \( \dot{n} + 3nH = \psi \), and the above relation between \( \dot{\rho}/\psi \) and \( \psi \), we have

\[
\frac{\dot{T}}{T} = -H + \frac{\psi}{3n}.
\]

For \( \psi = 0 \) (no photon creation), we obtain the standard FRW law relation, [Eq. (2)].

It is to be noted that previous studies of the decay of the vacuum energy assumed that the decay rate into photons is proportional to some power of \( H \) and/or the cosmic scale factor \( a \) (i.e., \( \psi \propto H^\alpha a^\gamma \), where \( \alpha \) and \( \gamma \) are constants). The combination of the values \( \alpha = 1 \) and \( \gamma = -3 \), is especially interesting as it indicates an adiabatic decay of the vacuum energy into a homogeneous distribution of thermalized CMB photons. Since \( n \) is inversely proportional to \( a^{-7} \), we have \( \psi \propto H^3 \). Defining \( \beta = \psi/3nH \), we obtain

\[
\frac{\dot{T}(z)}{T(z)} = T_0 (1 + z)^{1 - \beta}.
\]

According to Lima et al. (2000), the possible range of \( \beta \) is \( \beta \in [0, 1] \). The aim of the present article is to put a strong upper limit on the possible value of \( \beta \).
Two effects are produced by the decay of the vacuum energy into CMB photons:

1) There is a decrease in the observed \((\delta T/T)^2\) due to the increase of the homogeneous distribution of black body photons from the vacuum decay; and

2) When there is a vacuum energy decay into CMB photons, the value of the recombination redshift \(z_{\text{rec}}\) is higher than that of the standard model \(z_{\text{rec}}\) cooler at any given redshift. The recombination temperature thus occurs at a higher \(z\).

Due to the dilution of \((\delta T/T)^2\), instead of Eq. (10) of the standard model, we must use the relation

\[
F_1 \left( \frac{\delta T}{T} \right) \bigg|_{z_{\text{rec}}} = \mathcal{K} \frac{\delta \rho}{\rho} \bigg|_{z_{\text{rec}}}.
\]

We define

\[
F_1(z) \equiv \left[ \frac{T(z)}{T(z) - \Delta T(z)} \right] \bigg|_{z_{\text{rec}}}
\]

The difference between the recombination temperature \(T(z_{\text{rec}})\) predicted by the standard model and that of the model in which the vacuum energy decays into photons at temperature \(T(z_{\text{rec}})\) is

\[
\Delta T(z_{\text{rec}}) = T(z_{\text{rec}}) - T(z_{\text{rec}}).
\]

Using Eqs. (3), (11), and (12), we have

\[
F_1 = (1 + z_{\text{rec}})^{\beta}.
\]

From Eqs. (11) and (12), \(T(z)\) was lower than \(T(z)\) by \(\Delta T\) at \(z_{\text{rec}}\). Thus, the resultant recombination redshift \(z_{\text{rec}}\) was higher than that of the standard model \(z_{\text{rec}}\). Instead of Eq. (3), \((\delta \rho/\rho)\) at \(z \approx 0\) is now given by

\[
\left( \frac{\delta \rho}{\rho} \right) \bigg|_{z \approx 0} = D(z_{\text{rec}} \rightarrow z = 0) \frac{\delta \rho}{\rho} \bigg|_{z = z_{\text{rec}}},
\]

where \(D(z_{\text{rec}} \rightarrow z = 0)\) is the density fluctuation growth factor from the recombination era at \(z_{\text{rec}}\) until the present epoch, due to the decay of the vacuum energy into photons. Therefore, instead of Eq. (11), we have

\[
\left( \frac{\delta T}{T} \right) \bigg|_{z \approx 0} = \mathcal{K} \frac{\delta \rho}{\rho} \bigg|_{z = z_{\text{rec}}}.
\]

Using Eqs. (11) and (12), we have

\[
\left( \frac{\delta \rho}{\rho} \right) \bigg|_{z \approx 0} = \frac{F_1}{\mathcal{K}} D(z_{\text{rec}} \rightarrow z = 0) \left( \frac{\delta T}{T} \right) \bigg|_{z_{\text{rec}}}
\]

and from Eqs. (11) and (12),

\[
\left( \frac{\delta \rho}{\rho} \right) \bigg|_{z \approx 0} = \frac{F_1}{\mathcal{K}} D(z_{\text{rec}} \rightarrow z = 0) \left( \frac{\delta T}{T} \right) \bigg|_{z_{\text{rec}}}.
\]

Equations (11) and (12) give the correction factor \(F_2\) due to the change in the value of the recombination redshift,

\[
F_2 = \frac{D(z_{\text{rec}} \rightarrow z = 0)}{D(z_{\text{rec}} \rightarrow z = 0)}.
\]

We then find from Eq. (11)

\[
F_2 \simeq \frac{(1 + z_{\text{rec}})}{(1 + z_{\text{rec}})}.
\]

The temperature at \(z_{\text{rec}}\) in the standard model is

\[
T(z_{\text{rec}}) = T_0 (1 + z_{\text{rec}}).
\]

In order for the temperature at the recombination epoch \(z_{\text{rec}}\), when the vacuum energy is decaying into CMB photons, to be \(T(z_{\text{rec}})\), we must have, from Eq. (3),

\[
z_{\text{rec}} = (1 + z_{\text{rec}})^{1/(1-\beta)} - 1.
\]

From Eq. (13), we then have

\[
F_2 \simeq (1 + z_{\text{rec}})^{\beta/(1-\beta)}.
\]

The total factor \(F\) is composed of \(F_1\), due to the dilution of the CMB as a result of vacuum energy decay, and \(F_2\), due to the change in the redshift of the recombination epoch. Assuming that the effects described by \(F_1^2\) and \(F_2^2\) are independent and that the total factor \(F\) is the product of \(F_1^2\) and \(F_2^2\), we have

\[
F = F_1^2 F_2^2.
\]

Thus, from Eqs. (3), (11) and (12), the condition for the maximum value of \(\beta \in [0, 1]\) is

\[
\beta_{\text{max}} = \alpha \left[ 1 - \sqrt{1 - \frac{\ln(F_{\text{max}})}{2\alpha^2 \ln(1 + z_{\text{rec}})}} \right],
\]

where

\[
\alpha = 1 + \frac{\ln(F_{\text{max}})}{4 \ln(1 + z_{\text{rec}})}.
\]

As noted above, the maximum value of \(F\) from observations is \(F_{\text{max}} = 1.1\). A plot of \(\beta\) vs \(z_{\text{rec}}\) in the standard model is shown in Fig. 1 for \(F = F_{\text{max}} = 1.1\) from Eqs. (11) and (12). For \(z_{\text{rec}} \approx 1100\), we find a very small maximum value of the \(\beta\) parameter, \(\beta_{\text{max}} \approx 3.4 \times 10^{-3}\).

3 CONCLUSIONS

We show that the CMB data, together with the large galaxy survey data, put strong limits on the rate of a possible decay of the vacuum energy into a homogeneous distribution of thermalized CMB photons, between the recombination era and the present. Using the fact that the \((\delta \rho/\rho)^2\) derived from the CMB and galaxy distribution data do not differ by more than 10 per cent, we can place an upper limit on the \(\beta\) parameter for the decay of the vacuum energy into CMB photons, parametrized by a change in the CMB temperature at a given redshift \(z\) \(\Delta T(z) = T_0 (1 + z)^{1-\beta}\). We find that \(\beta_{\text{max}} \approx 3.4 \times 10^{-3}\).

In the above analysis, we assumed an Einstein-de Sitter CDM universe with a growth factor \(D(z) \approx 1 + z\) to obtain \(\beta_{\text{max}}\). This is true in a pressureless universe with an equation of state \(P/\rho \equiv w \approx 0\), where \(P\) is the pressure and \(\rho\) is the energy density. \(D(z)\) will change at small redshifts if \(w(z)\) becomes less than zero due to a vacuum energy contribution \((wV = -1)\), a quintessence energy contribution \((1 - w_Q < 0)\), or a phantom energy contribution \((w_P < -1)\). As a result, \(\beta_{\text{max}}\) will change somewhat. An investigation of these contributions is left for a future article.
Our results indicate that the rate of the decay of the vacuum energy into CMB photons is extremely small. They are consistent with a zero vacuum energy decay, $\beta = 0$.

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