Spin-Gauge Theory of Gravity with Higgs-field Mechanism

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Abstract

We propose a Lorentz-covariant Yang-Mills spin-gauge theory, where the function valued Dirac matrices play the role of a non-scalar Higgs-field. As symmetry group we choose $SU(2) \times U(1)$. After symmetry breaking a non-scalar Lorentz-covariant Higgs-field gravity appears, which can be interpreted within a classical limit as Einstein’s metrical theory of gravity, where we restrict ourselves in a first step to its linearized version.
I. Introduction

Within the solar system and the binary pulsar PSR 1913 + 16 the classical gravitational interaction is described very well by Einstein’s general relativity. However, this theory - simultaneously the oldest non-abelian gauge-theory with the Poincargroup as gauge group - is not quantizable until now. On the other hand all the other fundamental interactions and their unifications are described successfully by quantizable Lorentz-covariant gauge theories with unitary gauge groups. Therefore the suspicion exists, that Einstein’s theory represents only a classical macroscopic description of gravity and that the fundamental microscopic gravitational interaction between elementary particles is also described by a unitary gauge group on the Minkowski space-time in such a way, that Einstein’s theory of macroscopic gravity is reached as an effective theory within a certain classical limit similarly as in the strong interaction the nuclear forces follow from the quantum chromodynamics.\(^1\) In this way the problem of quantization of gravity and its unification with the other interactions would be solvable.

In this connection the statement is of interest (Dehnen, Frommert and Ghaboussi, 1990), that the scalar Higgs-field of the elementary particle physics the basis of which are of course unitary transformation groups, mediates a Lorentz-invariant attractive gravitational interaction between those elementary particles which become massive by the spontaneous symmetry breaking, i.e. the Higgs-field has its source only in the mass and acts back only on the mass of the particles. The equivalence of inertial and gravitational mass is fulfilled automatically within this Higgs-field gravity. But if

\(^1\)For this general intention see also Stumpf, 1988.
the strength of this gravity shall be of the order of the Newtonian one, the mass of the gauge-bosons will be of the order of the Planck-mass.

For the last reason the standard Higgs-gravity, e.g. within the electroweak interaction (see Dehnen, Frommert, 1991), has presumably nothing to do with usual gravity. However, here the question arises, whether Einstein’s tensorial gravity may be a consequence of a more sophisticated Higgs-field, which is especially not a scalar one.

For this we extend back to a Yang-Mills $SU(2) \times U(1)$ spin-gauge theory of gravity on the Minkowski space-time of special relativity proposed by Dehnen et al. (Dehnen, Ghaboussi, 1985 and 1986; see also Chisholm, Farwell, 1989). In this theory, where a subgroup of the unitary transformations of Dirac’s $\gamma$-matrices between their different representations (internal spin group; see also Drechsler, 1988 and Bade, Jehle, 1953; cf. also Barut, 1984) is gauged, the $\gamma$-matrices became function valued but remained covariantly constant with respect to the internal spin group, whereas the gravitational interaction is mediated by the four gauge-bosons belonging to the group $SU(2) \times U(1)$ and the classical non-euclidian metric is constructed out of them as an effective field in a certain manner.

Here a modification in the sense of the Higgs-field gravity is indicated: Instead of considering the $\gamma$-matrices as covariantly constant it is possible to treat them as true field variables with a Higgs-Lagrange density, and this because also the $\gamma$-matrices possess a non-trivial ground-state, namely the usual constant standard representations. Because the $\gamma$-matrices can be understood as square root of the metric the gauge group is that of the square root of the metric; moreover, in consequence of this group the several spin
states (or particle-antiparticle states) are indistinguishable with respect to the interaction following from gauging the spin group (universality of the interaction). Both properties suggest that real gravity is involved.

In this way we get a quantizable unitary spin-gauge theory with Dirac’s $\gamma$-matrices as Higgs-fields; on this level an unification with all the other interactions may be possible. After spontaneous symmetry breaking a non-scalar Higgs-gravity appears, which can be identified in a classical limit with Einstein’s gravity, where we restrict ourselves in the first step for simplicity to the linear theory. The essential points are the following: the theory is from the beginning only Lorentz-covariant. After symmetry breaking and performing a unitary gauge the action of the excited $\gamma$-Higgs-field on the fermions in the Minkowski space-time is reinterpreted as if there would exist non-euclidean space-time connections and a non-euclidean metric (effective metric), in which the fermions move freely; then the deviation from the Minkowski space-time describes classical gravity. This happens, as usual, in the de Donder gauge and not in general coordinate covariance, which depends also on the fact that with the choice of the unitary gauge a gauge fixing is connected. In this way the gravitational constant is produced only by the symmetry breaking and the non-euclidian metric comes out to be an effective field, whereas the gauge-bosons get masses of the order of the Planck-mass and can be therefore neglected in the low energy limit; but in the high energy limit ($\simeq 10^{19}$ GeV) an additional ”strong” gravitational interaction exists. Simultaneously, our results give a new light on the role of the Higgs mechanism.

Finally we note, that as in the previous spin-gauge theory (c.f. Ghaboussi,
Dehnen and Israelit, 1987) a richer space-time geometrical structure results than only a Riemannian one. We find also an effective non-metricity, whereas an effective torsion does not appear. The question, whether it is possible to change the Lagrangian so that the non-metricity vanishes, will be clarified in a later paper.

II. The Model

In the beginning we repeat briefly the foundations of the previous work (Dehnen, Ghaboussi, 1986; see also Babu Joseph, Sabir, 1988) so far as necessary. Using 4-spinors it is appropriate to introduce the transformation matrices of the group $SU(2) \times U(1)$ in their $4 \times 4$-representation ($a = 0, 1, 2, 3$):

\begin{equation}
U = e^{i \lambda_a (x^\mu) \tau^a},
\end{equation}

where the $SU(2)$-generators are given by the Pauli matrices $\sigma^i$ as follows ($i = 1, 2, 3$):

\begin{equation}
\tau^i = \frac{1}{2} \begin{pmatrix}
\sigma^i & 0 \\
0 & \sigma^i
\end{pmatrix}.
\end{equation}

The $U(1)$-generator $\tau^0$ may be diagonal and commutes with (2.2); but its special form shall be determined only later. Thus the commutator relations for the generators $\tau^a$ are

\begin{equation}
[\tau^b, \tau^c] = i \epsilon^{bc}_a \tau^a,
\end{equation}

where $\epsilon^{bc}_a$ is the Levi-Civita symbol with the additional property to be zero, if $a$, $b$, or $c$ is zero.

\footnote{The explicite form of (2.2) is only used in (4.7).}
Then the 4-spinor $\psi$ and the Dirac matrices $\gamma^\mu$ transform as

$$\psi' = U\psi, \quad \gamma'^\mu = U\gamma^\mu U^{-1}$$

(2.4)

and the covariant spinor derivative reads

$$D_\mu \psi = (\partial_\mu + ig\omega_\mu)\psi$$

(2.5)

($g$ gauge coupling constant). The gauge potentials $\omega_\mu$ obey the transformation law

$$\omega'_\mu = U\omega_\mu U^{-1} + i g U|_\mu U^{-1}$$

(2.6)

and are connected with the real valued gauge fields $\omega_{\mu a}$ by

$$\omega_\mu = \omega_{\mu a} \tau^a.$$  

(2.7)

According to (2.4) Dirac’s $\gamma$-matrices become necessarily function valued, in consequence of which we need determination equations for them; as such ones we have chosen in our previous paper in analogy to general relativity:

$$D_\alpha \gamma^\mu = \partial_\alpha \gamma^\mu + ig [\omega_\alpha, \gamma^\mu] = 0, \quad \gamma^{(\mu\gamma^\nu)} = \eta^{\mu\nu} \cdot 1$$

(2.8)

($\eta^{\mu\nu} = \eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ Minkowski metric). Because the $\gamma$-matrices are the formal square root of the metric, the gauge transformations (2.4) are those, which are associated with the root of the metric. Therefore the concept described by the formulae (2.1) up to (2.7) may have to do something with gravity. And indeed, in our previous paper we could show, that

\[3\gamma^\mu \text{ are tensors with respect to the unitary transformations (2.1), but they are not elements of the adjoint representation.}\]

\[4|\mu \text{ denotes the partial derivative with respect to the coordinate } x^\mu.\]
a space-time geometrical interpretation of the theory results in an effective non-euclidian metric given by

\[ g_{\mu\nu} = \omega_{\mu a} \omega_{\nu b} \eta^{ab}. \]

However, the result (2.9) is connected with the condition that the gauge potentials \( \omega_{\mu a} \) do never vanish and possess a non-trivial ground-state representing according to (2.9) in the lowest order the Minkowski metric. This is an unusual feature; furthermore the conditions (2.8) are chosen for simplicity. Therefore it may be justified to give up the relations (2.8) and (2.9) and to consider Dirac’s \( \gamma \)-matrices as true field variables with a Higgs-Lagrange density, so that the non-trivial ground-state can be identified with the constant standard representations. It will come out, that after symmetry breaking the excited \( \gamma \)-Higgs-fields mediate a non-scalar Higgs-gravity, which results finally in Einstein’s metrical theory, where instead of (2.9) the connection between the effective non-euclidian metric and the \( \gamma \)-Higgs-field will be deduced from a space-time geometrical interpretation of the equation of motion for the 4-momentum of the fermions described by the spinor fields \( \psi \).

III. Lagrange Density and Field Equations

The translation of the model into a field-theoretical description results in a Lagrange density consisting of three minimally coupled Lorentz- and gauge-invariant real valued parts \((\hbar = 1, c = 1)\):

\[ \mathcal{L} = \mathcal{L}_M(\psi) + \mathcal{L}_F(\omega) + \mathcal{L}_H(\gamma). \]
Beginning with the last part, $\mathcal{L}_H(\gamma)$ belongs to the $\gamma$-Higgs-field and has the form:

$$
\mathcal{L}_H(\gamma) = \frac{1}{2} \text{tr} [(D_\alpha \tilde{\gamma}^\mu)(D^\alpha \tilde{\gamma}_\mu)] - V(\tilde{\gamma}) - \frac{i}{v} \tilde{\gamma}^\mu \tilde{\gamma}_\mu \psi,
$$

where

$$
V(\tilde{\gamma}) = \frac{\mu^2}{2} \text{tr}(\tilde{\gamma}^\mu \tilde{\gamma}_\mu) + \frac{\lambda}{4!} (\text{tr} \tilde{\gamma}^\mu \tilde{\gamma}_\mu)^2
$$

is the Higgs-potential. Herein $\tilde{\gamma}^\mu$ denotes from now the dynamic function valued $\gamma$-matrices, which obey the transformation law (2.4) and the ground-states of which are proportional to the constant standard representations $\gamma^\mu$ (bear this change of notation in mind). The last term on the right hand side of (3.2) represents the Yukawa-coupling term for generating the mass of the fermions by the $\gamma$-Higgs-field. In view of the electroweak interaction later on $\tilde{\gamma}^\mu$ must become isospin valued, which leads to the possibility of unification in a 8-dimensional spin-isospin space (c.f. chapt. 6).

The second term on the right hand side of (3.1) is that of the gauge-fields $\omega_\mu$:

$$
\mathcal{L}_F(\omega) = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} s^{ab},
$$

where $s^{ab}$ is the group-metric of $SU(2) \times U(1)$ and can be taken here as $\delta^{ab}$ (but compare the previous work). The gauge field strength are defined in the usual manner by

$$
F_{\mu\nu} = \frac{1}{ig} [D_\mu, D_\nu] = F_{\mu\nu a} \tau^a
$$

with

$$
F_{\mu\nu a} = \omega_{\nu a |\mu} - \omega_{\mu a |\nu} - g \epsilon_a^{jk} \omega_{\mu j} \omega_{\nu k}.
$$
The first Lagrangian in (3.1) concerns the fermionic matter fields and takes the form (\(\psi\) is only proportional to the Dirac spinor, see (4.4):

\[
L_M(\psi) = \frac{i}{2} \overline{\psi} \gamma^\mu D_\mu \psi - \frac{i}{2} \overline{(D_\mu \psi)} \gamma^\mu \psi.
\]

The adjoint spinor \(\overline{\psi}\) is given by

\[
\overline{\psi} = \psi^\dagger \zeta,
\]

wherein \(\zeta\) represents the \(SU(2) \times U(1)\)-covariant matrix with the property:

\[
(\zeta \gamma^\mu)^\dagger = \zeta \gamma^\mu.
\]

In view of the commutability of covariant derivative and multiplication with \(\zeta\) in (3.5) it is further necessary that

\[
D_\mu \zeta = 0.
\]

So long as (see chapt. 4)

\[
[\gamma^0, \tau^a] = 0,
\]

the equations (3.7) and (3.8) will be fulfilled only (up to a constant factor) by

\[
\zeta = \gamma^0 \quad (\zeta^\dagger = \zeta, \zeta^2 = 1),
\]

so that (3.6) yields as usual \(\overline{\psi} = \psi^\dagger \gamma^0\). Because of (3.9) the matrix \(\zeta\) is not only covariant but even invariant under gauge transformations. These results depend essentially on the relation (3.9), which may be not valid in a larger group (e.g. \(U(4)\)).

\[\text{A generalization of the theory to the full gauge group U(4) is in preparation (see also Drechsler, 1988).}\]
of (4.8), that all three expressions (3.2), (3.3) and (3.5) of the Lagrangian are real valued and contain no dimensional parameter with exception of $\mu^2$ in (3.2), which has the dimension of a mass square.

The field equations following from the action principle associated with (3.1) are given by the generalized Dirac-equation

\begin{equation}
 i \bar{\gamma}^\mu D_\mu \psi + \frac{i}{2} (D_\mu \bar{\gamma}^\mu) \psi - k \bar{\gamma}^\mu \bar{\gamma}_\mu \psi = 0
\end{equation}

as well as its adjoint equation, by the inhomogeneous Yang-Mills equation

\begin{equation}
 \partial_\nu F^{\nu \mu} + g \epsilon^{abc} F_{b}^{\nu \mu} \omega_{\nu c} = 4\pi j^\mu_a
\end{equation}

with the gauge currents

\begin{equation}
 j^\mu_a = j^\mu_a(\psi) + j^\mu_a(\gamma) = \frac{g}{2} \bar{\psi} \{ \bar{\gamma}^\mu, \tau_a \} \psi +
\end{equation}

\begin{equation}
 +ig \text{tr} ([\bar{\gamma}^\alpha, \tau_a] D^\mu \bar{\gamma}_\alpha)
\end{equation}

belonging to the matter and the Higgs-field respectively, and by the $\gamma$-Higgs-field equation: \(^6\)

\begin{equation}
 D_\alpha D^\alpha \bar{\gamma}_A^B + \left[ \mu^2 + \frac{\lambda}{6} \text{tr}(\bar{\gamma}^\alpha \bar{\gamma}_\alpha) \right] \bar{\gamma}_A^B =
\end{equation}

\begin{equation}
 = \frac{i}{2} \left[ \bar{\psi}^B \cdot (D^\mu \psi)_A - (D^\mu \bar{\psi})^B \cdot \psi_A \right] -
\end{equation}

\begin{equation}
 -k \left[ \bar{\psi}^B \cdot (\bar{\gamma}^\mu \psi)_A + (\bar{\psi} \bar{\gamma}^\mu)^B \cdot \psi_A \right].
\end{equation}

\(^6\)If $\bar{\gamma}^\mu$ is considered to be traceless, see (4.2) and (4.8), then also the traceless version of (3.13) is valid only.
Herein the lower capital latin index $A$ and the upper index $B$ denote the contragradientsly tranformed rows and columns of the spinorial matrices respectively. The homogeneous Yang-Mills equation following from the Jacobi-identity reads:

\[
\partial_{\mu} F_{\nu \lambda}^{\alpha} + g \omega_k \partial_{\mu} F_{\nu \lambda}^{\beta} \epsilon^{k\alpha} = 0. \tag{3.14}
\]

Finally we note the conservation laws valid modulo the field equations. First, from (3.12) the gauge current conservation follows immediately:

\[
\partial_{\mu}(j_{\mu}^{A} + g \frac{1}{4\pi} e_a^{bc} F^{\mu \nu}_{b} \omega_{\nu c}) = 0. \tag{3.15}
\]

Secondly, the energy-momentum law takes the form

\[
\partial_{\nu} T_{\mu \nu} = 0, \tag{3.16}
\]

where $T_{\mu \nu}$ is the gauge-invariant canonical energy-momentum tensor consisting of three parts corresponding to (3.1)

\[
T_{\mu \nu} = T_{\mu \nu}^{(\psi)} + T_{\mu \nu}^{(\omega)} + T_{\mu \nu}^{(\gamma)} \tag{3.17}
\]

with (modulo Dirac-equation):

\[
T_{\mu \nu}^{(\psi)} = \frac{i}{2} \left[ \psi \tilde{\gamma}^{\nu} D_{\mu} \psi - (D_{\mu} \psi) \tilde{\gamma}^{\nu} \psi \right], \tag{3.18a}
\]

\[
T_{\mu \nu}^{(\omega)} = -\frac{1}{4\pi} \left[ F_{\mu \alpha a} F^{\nu \alpha a} - \frac{1}{4} F_{\alpha \beta}^{a} F^{a \alpha \beta} \gamma_{\mu}^{\nu} \right], \tag{3.18b}
\]

\[
T_{\mu \nu}^{(\gamma)} = \text{tr} [(D_{\nu} \tilde{\gamma}_{\alpha})(D_{\mu} \tilde{\gamma}^{\alpha})] - \delta_{\mu}^{\nu} \left[ \frac{1}{2} \text{tr} [(D_{\alpha} \tilde{\gamma}_{\beta})(D^{\alpha} \tilde{\gamma}^{\beta})] - \frac{\mu}{2} \text{tr}(\tilde{\gamma}^{\alpha} \tilde{\gamma}_{\alpha}) \right].
\]
Because of the Yukawa-coupling term in (3.2) the trace of (3.18a) does not vanish. With the use of the Dirac-equation (3.11) and its adjoint equation one finds:

\begin{equation}
T_{\mu}^{\mu}(\psi) = k\overline{\psi} \gamma^\mu \tilde{\gamma}_\mu \psi.
\end{equation}

By insertion of (3.18) into (3.17) one obtains from (3.16) the equation of motion for the fermions. After substitution of the second covariant derivatives of the $\gamma$-Higgs-field using the field equation (3.13) one finds with the help of the Yang-Mills equations (3.12) and (3.14):

\begin{equation}
\partial_\nu T_{\mu\nu}(\psi) = -\frac{i}{2} \left[ \overline{\psi} (D^\mu \tilde{\gamma}^\alpha)(D_\alpha \psi) - (\overline{D_\alpha \psi}) (D^\mu \tilde{\gamma}^\alpha) \psi + + F_{\alpha\alpha} J^{\alpha\alpha}(\psi). \right]
\end{equation}

Integration over the space-like hypersurface $t = \text{const.}$ and neglection of surface integrals in the space-like infinity yield the momentum law for the 4-momentum $p^\mu = \int T^{\mu\nu}(\psi)d^3x$ of the fermions. On the right hand side of (3.20) one recognizes the Lorentz-forces of the gauge fields and the force of the $\gamma$-Higgs-field.

We finish with two remarks. First, the energy momentum tensor $T_{\mu}^{\nu}(\gamma)$, equ. (3.18c), does not vanish for the ground-state, see (4.2), but has the value:

\begin{equation}
T_{\mu}^{\nu}(\gamma) = \frac{3}{2} \frac{\mu^4}{\lambda} \delta_{\mu}^{\nu}.
\end{equation}

However this can be renormalized to zero by changing the Higgs-potential (3.2a) correspondingly; otherwise (3.21) will give rise within the complete
theory to a cosmological constant. Secondly, the $\gamma$-Higgs-field equation (3.13) contains as source for $\tilde{\gamma}^\mu$ the fermionic energy-momentum tensor $T^\mu_\nu(\psi)$ in its spinor valued form; and in this form it appears also in the $\gamma$-Higgs-field force of (3.20). This fact confirms the supposition, that the $\gamma$-Higgs-field equation results in Einstein’s field equation of gravitation (for the fermions) after a space-time geometrical interpretation of the $\gamma$-Higgs-field forces in (3.20) defining the effective space-time geometrical connection coefficients.

IV. Spontaneous Symmetry Breaking

Although one can recognize the gravitational structure already in equation (3.13) and (3.20) the space-time geometrical interpretation is only possible after symmetry breaking. The minimum of the energy-momentum tensor (3.18) in absence of matter and gauge fields is reached, when the Higgs-potential (3.2a) is in its minimum defined by

$$(4.1) \quad \text{tr} \left( \gamma^\mu \gamma^\mu \right) = -\frac{6\mu^2}{\lambda} = v^2 \quad (\mu^2 < 0).$$

Simultaneously, herewith all field equations (3.11) up to (3.14) are fulfilled. The ground-state $\gamma^\mu_0$ of the $\gamma$-Higgs-field must be proportional to the (constant) Dirac standard representation $\gamma^\mu$, i.e. $\gamma^\mu_0 = b\gamma^\mu$. Insertion into (4.1) results because of $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \cdot 1$ in $b = \frac{v}{\xi}$, so that we have for the ground-state:

$$(4.2) \quad \gamma^\mu_0 = \frac{v}{4\xi} \gamma^\mu.$$

$^7$Of course, global unitary transformations between the different standard representations and simultaneously of the generators are allowed.

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Herewith the Lagrange density (3.5) for the spinorial matter fields reads considering the $\gamma$-Higgs-field ground-state only:

\begin{equation}
\frac{i}{2} \overline{\psi} \frac{v}{4} \gamma^\mu \partial_\mu \psi + h.c.
\end{equation}

Comparison with the usual Dirac Lagrangian $\frac{i}{2} \overline{\psi} \gamma^\mu \partial_\mu \psi$ results in ($\psi_{DIR}$ Dirac spinor):

\begin{equation}
\psi = \frac{2}{\sqrt{v}} \overline{\psi}_{DIR}.
\end{equation}

Herewith the fermionic mass term in (3.2), identical with the trace (3.19) of the energy-momentum tensor $T_{\mu \nu}(\psi)$, takes the form for the groundstate $\gamma^0$:

\begin{equation}
T_{\mu}^{\mu}(\psi_{DIR}) = \overline{\psi}_{DIR} m \psi_{DIR}
\end{equation}

with the mass:

\begin{equation}
m = kv.
\end{equation}

On the other hand the Higgs-field gauge current $j_a^\mu(\gamma)$ gives rise after symmetry breaking to the mass of the gauge-bosons $\omega^\mu_a$. In the lowest order we find from (3.12a) with the use of (4.2):

\begin{equation}
-4\pi j_a^\mu(\gamma) = M^2_{ab} \omega^\mu_b, \quad M^2_{ab} = M^2_{a b \rho \nu} \eta^{\rho \nu},
\end{equation}

\begin{equation}
M^2_{a b \rho \nu} = -\frac{v^2}{\pi} g^2 tr ([\tau_a, \gamma^\rho] [\tau_b, \gamma^\nu]).
\end{equation}

Here it is convenient to choose the $U(1)$-generator explicitly. If we take the unit matrix, the gauge-boson $\omega^\mu_0$ remains massless of course (rest symmetry) and must be taken into account also in the low energy limit. In order
to avoid this, \(^8\) the first possibility consists in view of (3.9) in the choice \(\tau^0 = \frac{1}{2} \gamma^0\). Doing this we obtain from (4.6) with the use of (2.2) the diagonal mass matrix for the gauge-bosons:

\[
M_{00}^2 = 3\pi g^2 v^2, \\
M_{ij}^2 = 2\pi g^2 v^2 \delta_{ij}
\]

and zero otherwise. As we will see later the value of (4.7) is of the order of the square of the Planck-mass (\(\tilde{\Lambda} \approx 10^{19}\) GeV), so that all gauge-bosons can be neglected in the low energy limit. A second possibility for avoiding the \(\omega^\mu_0\)-boson exists in remaining the unit matrix for \(\tau^0\) but choosing the associated gauge-coupling constant \(g_1\) sufficiently small (\(g_1 << 1\)). This choice has the advantage, that the unitary \(SU(2) \times U(1)\) transformation (2.1) up to (2.4) is exactly identical with that of the 2-spinors (resulting from a decomposition of the chiral representation).

As one can prove easily, the general Higgs-field \(\tilde{\gamma}^\mu\) can be represented, if no spin orientation is present (classical limit), by

\[
\tilde{\gamma}^\mu(x^\alpha) = h^\mu_\lambda(x^\nu) U^{(0)} \tilde{\gamma}^\lambda U^{-1},
\]

so that it can be reduced within the unitary gauge as usual to the ground-state (4.2) in the following way

\[
\tilde{\gamma}^\mu(x^\nu) = h^\mu_\lambda(x^\nu) \tilde{\gamma}^{(0)}^\lambda,
\]

where \(^9\)

\[
h^\mu_\lambda(x^\nu) = \delta^\mu_\lambda + \epsilon^\mu_\lambda(x^\nu)
\]

\(^8\)It seems to us not suitable to identify this boson with the photon in view of the electroweak interaction.

\(^9\)Lifting and lowering of indices is performed always with \(\eta^{\mu\nu}\) and \(\eta_{\mu\lambda}\) respectively.
and \( \epsilon_{\mu\lambda}(x^\nu) \) describes the deviations from the ground-state, i.e. the excited Higgs-field. Herewith we are able to write down all field equations after symmetry breaking exactly in a non-matrix valued form. Of course, the \( h^\mu_{\lambda}(x^\nu) \) look like a tetrad-field, but their determination and connection with the effective non-euclidean metric follow only from the \( \gamma \)-Higgs-field equation after symmetry breaking.

V. Field Equations after Symmetry breaking and Gravitational Interaction

In this section we restrict ourselves in a first step for simplicity to the linearized theory, i.e. \(|\epsilon_{\mu\lambda}| \ll 1\) (weak field limit). We start in view of the gravitational aspect with the Higgs-field equation \(3.13\). Going over from a spinorial description to a Lorentz-tensorial equation we multiply \(3.13\) at first by \(\gamma_{\lambda BA}\). Then after insertion of \(4.2\), \(4.4\), \(4.5\), \(4.8a\) and \(4.8b\) we obtain linearized in \(\epsilon_{\mu\lambda}\) under neglection of the gauge-boson interaction because of \(4.7\) (low energy limit):

\[
\partial_\alpha \partial^\alpha \epsilon_{\mu\lambda} - \frac{\mu^2}{2} \eta_{\mu\lambda} = \frac{4}{v^2} \left[ T^\mu_{\lambda}(\psi_{\text{DIR}}) - \frac{1}{2} T(\psi_{\text{DIR}}) \eta_{\mu\lambda} \right],
\]

where

\[(5.1a)\quad T^\mu_{\lambda}(\psi_{\text{DIR}}) = \frac{i}{2} \left[ \overline{\psi}_{\text{DIR}} \gamma^\lambda D^\mu \psi_{\text{DIR}} - (D^\mu \overline{\psi}_{\text{DIR}}) \gamma^\lambda \psi_{\text{DIR}} \right]\]

is the usual (canonical) Dirac energy-momentum tensor. Obviously the antisymmetric and the traceless symmetric part of \(\epsilon_{\mu\lambda}\) remain massless, whereas the scalar trace \(\epsilon = \epsilon_{\mu\lambda} \eta_{\mu\lambda}\) possesses the Higgs-mass:

\[(5.1b)\quad M = \sqrt{-2\mu^2}.\]
It seems, that in the standard model of electroweak interaction only this scalar part of the total Higgs-field is taken into account. Furthermore, if (5.1) shall describe usual gravity, \( v^2 \sim G^{-1} \) (\( G \) Newtonian gravitational constant) must be valid, so that (4.7) is indeed of the order of the square of the Planck-mass \( M_{Pl} = \frac{1}{\sqrt{G}} \).

Before comparing (5.1) with Einstein’s field equations it is appropriate to interpret at first the Higgs-field forces in (3.20) geometrically, where in the low energy limit the Lorentz-forces of the gauge fields can be neglected. Insertion of (4.2), (4.4), (4.8a) and (4.8b) into (3.20) gives with respect to (3.18a) and (5.1a):

\[
\partial_{\nu} T^{\mu\nu}(\psi_{DIR}) = -\partial^{\nu} \epsilon_{\nu\rho} T^{\mu\rho}(\psi_{DIR}) - \partial^{\mu} \epsilon_{\rho\nu} T^{\rho\nu}(\psi_{DIR}) + \frac{1}{2} \partial^{\mu} T(\psi_{DIR})
\]

(5.2) linearized with regard to \( \epsilon^{\mu\lambda} \). The equations (5.1) and (5.2) describe the \( \gamma \)-Higgs-field interaction in its linearized version which is obviously very similar to that of general relativity.

Now, the comparison of (5.2) with the energy-momentum law of a classical affine geometrical theory with the affine connections \( \Gamma_{\mu \nu \rho} \)

\[
D_{\nu}^{(\Gamma)} T^{\mu\nu} = 0 \Rightarrow \partial_{\nu} T^{\mu\nu} = -\Gamma^{\nu}_{\mu \rho} T^{\mu\rho} - \Gamma^{\mu}_{\rho \nu} T^{\rho\nu}
\]

(5.3) is possible. Within this classical procedure we neglect all spin-influences, in consequence of which, c.f. (5.7), \( T^{\rho\nu}(\psi_{DIR}) \equiv T^{\rho\nu} = T^{\nu\rho} \) is a symmetric tensor; thus we find the unique identification:

\[
\Gamma^{\mu}_{\nu \rho} = \partial^{\mu}(\epsilon_{\rho\nu} - \frac{1}{2} \epsilon_{\rho\rho} + \frac{1}{14}(\partial_{\rho} \epsilon \delta_{\mu}^{\rho} + \partial_{\rho} \epsilon \delta^{\mu}_{\rho})
\]

(5.4)
if

\[(5.4a)\]

\[\partial^\nu \epsilon_{[\mu\nu]} = 0\]

is valid, see (5.6b). Herewith the forces of the excited \(\gamma\)-Higgs-field on the fermions in the Minkowski space-time are reinterpreted as the action of non-euclidean space-time geometrical connections.

Consequently, in the space-time geometrical limit the excited Higgs-field \(\epsilon^{\mu\lambda}\) or more precisely its derivatives play effectively the role of affine connections (effective connections). Their field equations are obtained in the following way: The equations (5.1) take the form assuming a negligible Higgs-mass (5.1b):

\[(5.5a)\]

\[\partial_\alpha \partial^\alpha \left[ \epsilon_{(\mu\nu)} - \frac{1}{2} \epsilon_{[\mu\nu]} \right] = \frac{4}{v^2} T_{(\mu\nu)}(\psi_{DIR})\]

and

\[(5.5b)\]

\[\partial_\alpha \partial^\alpha \epsilon_{[\mu\nu]} = \frac{4}{v^2} T_{[\mu\nu]}(\psi_{DIR}).\]

In the lowest order, which is considered here only, the right hand sides of (5.5) possess in view of (5.2) vanishing divergences. Therefore the following constraints hold in consequence of the field equations:

\[(5.6)\]

(a) \[\partial^\nu \left[ \epsilon_{(\mu\nu)} - \frac{1}{2} \epsilon_{[\mu\nu]} \right] = 0,\]

(b) \[\partial^\nu \epsilon_{[\mu\nu]} = 0,\]

the first of which has the structure of the de Donder condition and the second of which guarantees the fulfillment of the condition (5.4a).

Evidently the source of the antisymmetric part \(\epsilon_{[\mu\nu]}\) is the antisymmetric part of the fermionic energy-momentum tensor (5.1a), which can be written

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with the use of the Dirac-equation in the lowest order:

\[ T_{\mu\nu}(\psi_{\text{DIR}}) = \frac{1}{2} \left[ \overline{\psi}_{\text{DIR}} \gamma^{[\mu} \sigma_{\nu]} \lambda D_{\lambda} \psi_{\text{DIR}} + \right. \]

\[ + \left. (D_{\lambda} \psi_{\text{DIR}}) \sigma^{\lambda}{}_{[\mu} \gamma_{\nu]} \psi_{\text{DIR}} \right], \tag{5.7} \]

where \( \sigma^{\mu\nu} = i \gamma^{[\mu} \gamma^{\nu]} \) is the spin-operator. Therefore, if we neglect in the classical macroscopic limit all spin influences, the solution of (5.5b) is:

\[ \epsilon_{[\mu\nu]} \equiv 0, \tag{5.8} \]

whereby in the classical limit \( \epsilon_{\mu\nu} = \epsilon_{(\mu\nu)} \) is valid.

For discussion of the field equation (5.5a) for the symmetric part \( \epsilon_{(\mu\nu)} \) we compare directly with Einstein’s linearized field equations of gravity. Setting

\[ g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu} \tag{5.9} \]

and choosing the de Donder gauge, c.f. (5.6a)

\[ \partial^\nu (\gamma_{\mu\nu} - \frac{1}{2} \gamma \eta_{\mu\nu}) = 0 \tag{5.9a} \]

(\( \gamma = \gamma_{\mu\nu} \eta^{\mu\nu} \)) it is valid:

\[ \partial_\alpha \partial^\alpha \frac{1}{2} (\gamma_{\mu\nu} - \frac{1}{2} \gamma \eta_{\mu\nu}) = -8\pi G T_{(\mu\nu)} \tag{5.10} \]

The comparison with (5.5a) results immediately in:

\[ \epsilon_{(\mu\nu)} = \alpha \gamma_{\mu\nu} \tag{5.11} \]

and

\[ u^2 = (-4\pi G \alpha)^{-1} \tag{5.11a} \]
up to the proportional constant $\alpha$. Consequently the constraint (5.6a) is identical with the de Donder condition (5.9a) and Newton’s gravitational constant $G$ is correlated, as expected, with the Higgs-field ground-state value $v$. The constant $\alpha$ is adjusted in such a way, that the equation of motion (5.2) goes over in the lowest order into the Newtonian gravitational law; for this

\begin{equation}
\epsilon(\mu\nu) = -\frac{1}{4} \gamma(\mu\nu) \Rightarrow \alpha = -\frac{1}{4}, \quad v^2 = \frac{1}{\pi G}
\end{equation}

must be valid in view of (5.11) with $\gamma(\mu\nu) = 2\Phi \text{diag}(1,1,1,1)$ with respect to (5.10) ($\Phi$ Newtonian gravitational potential). Herewith the effective non-euclidian metric takes the form with respect to (5.9):

\begin{equation}
g_{\mu\nu} = \eta_{\mu\nu} - 4\epsilon(\mu\nu).
\end{equation}

Analyzing the affine geometric connections (5.4) we note that beside the Christoffel-symbols belonging to the metric (5.13)

\begin{equation}
\left\{ \frac{\alpha}{\mu\nu} \right\} = 2\eta^{\nu\lambda}(\partial_{\lambda}\epsilon(\mu\nu) - \partial_{\nu}\epsilon(\mu\lambda) - \partial_{\mu}\epsilon(\nu\lambda))
\end{equation}

there exists no effective torsion

\begin{equation}
\Gamma^{\nu}_{[\mu\rho]} \equiv 0
\end{equation}

but effective non-metricity:

\begin{equation}
Q_{\mu\nu\lambda} = -D_{\mu}^{(r)}g_{\nu\lambda} = 4\partial_{\mu}\epsilon(\nu\lambda) + \partial_{\nu}\epsilon(\mu\lambda) + \partial_{\lambda}\epsilon(\nu\mu) - \frac{3}{t}(\partial_{\nu}\epsilon_{\mu\lambda} + \partial_{\lambda}\epsilon_{\mu\nu} + \partial_{\mu}\epsilon_{\nu\lambda}) - \frac{1}{t}\partial_{\nu}\epsilon_{\mu\lambda}
\end{equation}

assuming $\epsilon_{[\mu\nu]} \equiv 0$ in all cases, c.f. (5.8).
The practical consequences of the appearance of non-metricity or even better its avoidance shall be investigated elsewhere.

Now we note the Dirac-equation for gravitational interaction according to the spin-gauge theory as well as the Yang-Mills equation for the very massive gauge fields. From (3.11) it follows immediately after insertion of (4.2), (4.4), (4.5a) and (4.8) under neglection of the gauge-boson interaction (low energy limit):

\[
(5.17) \quad i\gamma^\mu D_\mu \psi_{DIR} - m (1 + \frac{1}{2} \epsilon) \psi_{DIR} = 0
\]

with

\[
(5.17a) \quad D_\mu = \partial_\mu + \epsilon^\lambda_\mu \partial_\lambda + \frac{1}{2} (\partial_\lambda \epsilon^\lambda_\mu).
\]

In its non-relativistic limit this equation goes over into the Schrödinger-equation with usual Newtonian gravitational potential. Iteration of (5.17), elimination of all spin influences and linearization in \( \epsilon^\lambda_\mu \) give:

\[
(5.18) \quad D_\mu D^\mu \psi_{DIR} + \frac{m^2 c^2}{\hbar^2} (1 + \epsilon) \psi_{DIR} + \frac{i mc}{\hbar} \gamma^\mu (\partial_\mu \epsilon) \psi_{DIR} = 0.
\]

With the ansatz

\[
(5.19) \quad \psi_{DIR} = e^{-i \frac{mc^2}{\hbar} t} \varphi(x^\nu)
\]

we obtain from (5.18) under neglection of all terms up to the order of \( c^{-1} (\epsilon^\lambda_\mu \sim c^{-2}) \) the Schrödinger-equation

\[
(5.20) \quad \frac{\hbar^2}{2m} \Delta \varphi + \frac{mc^2}{2} (2\epsilon^{00} - \epsilon) \varphi = \frac{\hbar}{i} \partial_t \varphi
\]

\[10\] We use \( \hbar, c \) explicitely because of the ordering with respect to \( c^{-1} \).
Because of (5.12) $\epsilon^{00} = -\frac{1}{2} \Phi/c^2$, $\epsilon = \Phi/c^2$ is valid, so that (5.20) goes over into

$$\frac{\hbar^2}{2m} \Delta \varphi - m \Phi \varphi = \frac{\hbar}{i} \partial_t \varphi,$$

(5.21)

i.e. the usual Schrödinger-equation with classical gravitational potential $\Phi$.

We have shown this explicitly, because this quantum mechanical equation has been tested experimentally until now only for the gravitational interaction by the neutron-interference experiment of Collela, Overhauser and Werner (1975). It may be of interest however, that the Schrödinger-equation (5.21) does not guarantee, that atomic clocks and lengths measure the effective non-Euclidean metric; for this the influence of the gravitational field on the electric Coulomb potential between electron and nucleus of the atom is necessary (see e.g. Papapetrou, 1956), which is not yet included in our theory.

Finally, for the inhomogeneous Yang-Mills equation we obtain from (3.12) and (3.12a) with the use of (4.2), (4.4), (4.6) and (4.8):

$$\partial_\nu F^{\nu\mu} + g \epsilon_a^{bc} F^{\nu\mu} b c \omega_{\nu c} + M_{ab}^2 \omega^{\mu b} + 2 \epsilon_{(\rho\nu)} M_{ab}^2 \rho^\nu \omega^{\mu b} =$$

$$= 4\pi \left\{ \frac{g}{2} \psi^{\mu}_{\text{DIR}} \left\{ \gamma^\rho, \tau_a \right\} \psi_{\text{DIR}} (\delta^\mu_\lambda + \epsilon^\mu_\lambda) + \right\}$$

(5.22)

$$+ i g \frac{v^2}{16} (\partial^\mu \epsilon_{\rho\nu}) \text{tr} \left\{ [\gamma^\rho, \tau_a] \gamma^\nu \right\},$$

where we have restricted ourselves also to linearized interaction terms with respect to gravitation. On the left hand side we recognize the mass term and the interaction of the massive bosons with the gravitational potentials; on the right hand side we find as sources gravitationally influenced Dirac
gauge currents and a current associated with the gravitational field itself (remaining Higgs-field current). Because of this it may be justified to call the gauge-boson interaction as a "strong" but very massive gravitational interaction; its coupling constant $g$ remains however undetermined within our present theoretical approach.

VI. Final Remarks

In extension of a previous spin-gauge theory of gravity we have shown, that Dirac’s $\gamma$-matrices can be treated as a quantizable Higgs-field, in consequence of which Einstein’s metrical theory of gravitation follows as the classical macroscopic limit of the Higgs-field interaction after symmetry breaking.

In spite of this success there are several problems for the future. First, the effective space-time geometrical structure is not only a Riemannian one, but also non-metricity is present, which should be suppressed in the next step since no observational hint on it exists. This may be possible because the Lagrange density (3.2) for the Higgs-field is not yet unique but can be supplemented in its kinetic term, e.g. by $\text{tr}[(D^\alpha \tilde{\gamma}^\mu)(D_\mu \tilde{\gamma}^\alpha)]$. In connection with this it may also be attainable to avoid the constraint (5.6a), which corresponds to the de Donder condition, and perhaps in this way Einstein’s theory can be reached even exactly and not only in its linearized version as presented above.

Furthermore the theory, as it stands, contains only the gravitational interaction between the fermions. But the gravitational interaction with all bosons must be included within a complete and consistent theory of gravita-
tion; otherwise, as remarked above (c. f. Papapetrou, (1956)), atomic clocks and lengths do not measure the non-Euclidean effective metric. This may require however an unification with the other interactions on the microscopic level of unitary phase gauge transformations within a high dimensional (e.g. 8-dimensional) spin-isospin space describing gravitational and (electro-) weak interaction separately.

In this respect one could have a bold idea: Because in our theory the $\gamma$-matrices are treated as Higgs-field it could be possible to introduce the chiral asymmetry of the fermions with regard to the weak interaction, which is however present already in the $SU(5) - GUT$, by a special choice of the ground-state of the $\gamma$-Higgs-field in the course of the spontaneous symmetry breaking at approximately $10^{19}$ GeV connected with the separation of gravitational and electro-weak interaction.

References

Bade, W., Jehle, H., Rev. Mod. Phys. 25, 714 (1953); see also
Laporte, O., Uhlenbeck, B., Phys. Rev. 37, 1380 (1931)

Babu Joseph, K., Sabir, M., Mod. Phys. Lett. A 3, 497 (1988)

Barut, A.O., McEwan, J., Phys. Lett 135, 172 (1984); Lett. Math. Phys. 11, 67 (1986)

Chisholm, J., Farwell, R., J. Phys. A22, 1059 (1989), and the literature cited therein

Collela, R., Overhauser, A., Werner, S., Phys. Rev. Lett. 34, 1472 (1975)
Dehnen, H., Ghaboussi, F., Nucl. Phys. B262, 144 (1985)

Dehnen, H., Ghaboussi, F., Phys. Rev. D33, 2205 (1986)

Dehnen, H., Frommert, H., Ghaboussi, F., Int. J. theor. Phys. 29, 537 (1990)

Dehnen, H., Frommert, H., Int. J. theor. Phys. 30, 985 (1991)

Drechsler, W., Z. Phys. C41, 197 (1988)

Ghaboussi, F., Dehnen, H., Israelit, M., Phys. Rev. D35, 1189 (1987)

Ghaboussi, F., Il Nuov. Cim. 104A, 1475 (1991)

Papapetrou, A., Ann. der Phys. 17, 214, (1956)

Schouten, J., Ricci-Calculus, 2nd edition, Springer (Berlin, 1954)

Stumpf, H., Z. Naturforsch. 43a, 345 (1988)