COSMOLOGICAL
PHASE TRANSITIONS

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**Preface**

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This Thesis is dedicated to all thinking creatures in our Universe.

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*Janne Ignatius*
Abstract

It is generally believed that several phase transitions have taken place in the early Universe. The effects of cosmological phase transitions may well have been crucial for the evolution of the Universe, and thus for the existence of life as we know it.

The cosmological phase transitions investigated here are related to strong and electroweak interactions. When the Universe was about $10^{-11}$ seconds old and the horizon radius equaled one centimeter, symmetry between weak and electromagnetic interactions was broken. It is quite possible that the baryon asymmetry, one of the most important properties of our Universe, was generated in this transition. Later happened the phase transition from the quark–gluon plasma to the hadron matter. At the quark–hadron transition the size of a causally connected region of space, the horizon radius, was ten kilometers and the age of the Universe $10^{-5}$ seconds.

The new $Z(3)$ phase transition suggested in this work is situated to a temperature or energy two orders of magnitude above the electroweak scale. At that time the Universe was roughly $10^{-15}$ seconds old and the horizon radius was of the order of one micrometer. This hypothetical phase transition is caused purely by the high-temperature properties of strong interactions.

The discussion of the different phase transitions is based on the assumption that they are first order. It is pointed out that especially the onset of a cosmological phase transition shows a universal behavior. General methods are presented, applicable to an analysis of the sequence of events taking place in cosmological first-order phase transitions.

An equation of state is derived for the electroweak matter near the phase transition point. The thermodynamically allowed region for the velocities of the phase transition front is determined.

The nucleation rate of bubbles of the broken-symmetry phase is computed in generic first-order cosmological phase transitions. The radial dependence of the Helmholtz free energy of the bubbles is also discussed.
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Original Papers
List of Papers

This Thesis consists of an introductory review part, followed by three research publications in chronological order:

I: K. Enqvist, J. Ignatius, K. Kajantie and K. Rummukainen,

*Nucleation and Bubble Growth in a First-Order Cosmological Electroweak Phase Transition*,

Physical Review D 45 (1992) 3415–3428.

II: J. Ignatius, K. Kajantie and K. Rummukainen,

*Cosmological QCD Z(3) Phase Transition in the 10 TeV Temperature Range?*,

Physical Review Letters 68 (1992) 737–740.

III: J. Ignatius,

*Bubble Free Energy in Cosmological Phase Transitions*,

Physics Letters B 309 (1993) 252–257.
1 Introduction

Cosmological phase transitions offer a rich variety of physical phenomena for investigation, and some of their effects may be observable in the present Universe. In this Thesis the mechanisms of cosmological phase transitions are studied, concentrating on transitions which are related to strong and electroweak interactions.

All the phase transitions to be discussed are—or are assumed to be—first order. In the first-order transitions a metastable phase may exist alongside a stable one for some temperature range. In the thermodynamical limit, \( i.e., \) in an infinitely large system in which the temperature is changed with an infinitely slow rate, the phase transition takes place at a certain critical temperature \( T_c \).

If the rate of the temperature decrease is finite, as is the case in the expanding Universe, the phase transition temperature differs from the equilibrium value \( T_c \). At the critical temperature of a phase transition nothing happens, the high-temperature phase just moves into a supercooled state. At a somewhat lower temperature bubbles of the new phase begin to nucleate. The bubbles grow and convert the space to the new phase. The new phase has a lower energy density than the old phase. This means that in the phase transition the Universe is heated up to a certain temperature not higher than \( T_c \). After the transition is completed, the Universe starts to cool again in the usual way.

The supercooling is crucial for the scenarios in which the baryon asymmetry of the Universe is generated at the electroweak scale. There is more matter than antimatter in the Universe, and more than a billion photons for every baryon. This fundamental cosmological fact, crucial for us human beings, should be explained in a satisfactory way.

A few years ago it was realized that at temperatures above the critical temperature of the electroweak theory, certain electroweak processes mediated by the
so-called sphalerons destroy any pre-existing baryon plus lepton number asymmetry [Kuzmin, Rubakov and Shaposhnikov 1985]. It became thus necessary to understand how the baryon asymmetry of the Universe could have been created at the electroweak phase transition. In principle, this is possible since all the three necessary conditions for generation of baryon asymmetry [Sakharov 1967] could have been satisfied during the transition. Firstly, due to the anomaly in the electroweak theory ['t Hooft 1976a, 1976b], baryon-number violating reactions were taking place. Secondly, CP–symmetry was violated because of fundamental gauge and Higgs interactions of quarks. The third condition, a departure from thermal equilibrium, was well satisfied because of the supercooling, provided the phase transition is first order.

In order to obtain any quantitative estimates for the amount of baryon asymmetry created, one must have a detailed understanding of how the electroweak phase transition proceeded. Motivated by this, we have investigated in this Thesis mechanisms of the electroweak phase transition.

The other phase transitions considered in this work are related to quantum chromodynamics. The possible observable consequences of the cosmological quark–hadron phase transition are due to the density inhomogeneities produced during the transition. If the length scale of these inhomogeneities had been large enough, they could have later affected the nucleosynthesis. This effect could be observed in the abundance of light elements in the present-day Universe. However, it seems probable that the length scale was too small for that, as will be discussed later on. In the case of the Z(3) phase transition suggested in this Thesis, it might in principle be that the density inhomogeneities generated could have affected later processes like the electroweak phase transition.

In addition to those questions considered in this work there are several other interesting topics related to cosmological phase transitions. For example, we have not studied the possible phase transition of a grand unified theory, which may have been cosmologically important as a driving source of inflation. Likewise, the topological defects, like monopoles or cosmic strings, which could have been created in cosmological first-order phase transitions are not discussed.
This Thesis is organized as follows. We first give a brief summary of the contents of the original research papers, which are appended. In Section 2 the main events in the evolution of the Universe are described. In Section 3 cosmological first-order phase transitions are discussed on a general level, without specifying the physical model. In Section 4 the general methods presented in the previous Section are applied to two physical cases, to the electroweak and the quark–hadron phase transition. Finally, in Section 5 we present conclusions and point out directions for the future work.

Summary of the Original Papers

Paper I: Nucleation and Bubble Growth in a First-Order Cosmological Electroweak Phase Transition. In this paper the thermodynamical properties of electroweak matter near the critical temperature are systematically investigated for the first time. Assuming a quartic form for the Higgs potential (to be discussed in Subsection 4.1 of this introductory review part), we derive an equation of state that describes the electroweak phase transition, and compare the electroweak transition with the quark–hadron transition. The nucleation rate of bubbles of the broken-symmetry phase is computed by solving numerically the field equation. We present a useful expression for the volume fraction not touched by the bubbles, slightly different from those given previously by other authors. We perform numerical simulations of bubble nucleation and growth which confirm our analytical calculations. Finally, we also study what velocities of the phase front are allowed assuming only that the general conditions of energy-momentum conservation and entropy increase are valid. Based on these considerations, we claim that in the cosmological electroweak phase transition the bubbles most likely grew as weak deflagrations.

Paper II: Cosmological QCD $Z(3)$ Phase Transition in the 10 TeV Temperature Range? Using as a starting point the earlier observation that in the QCD there are metastable vacua at high temperatures, we develop a cosmological scenario
which leads to a phase transition, not known before, at a temperature two orders of magnitude above the electroweak scale. Qualitatively, this phase transition differs from the usual ones in that the pressure difference between the stable and metastable vacua is huge, and in that there were only relatively few bubbles nucleated inside the horizon. Our scenario is based on the hypothesis that at very early times domains of metastable vacua were created and underwent an inflationary expansion due to some processes which could be related for instance to the breaking of the grand unified symmetry. This hypothesis is the main uncertainty in our scenario. Later on it has been also claimed that the metastable vacua should only be interpreted as field configurations that contribute to the Euclidean path integral, not as physically accessible states [Belyaev et al. 1992; Chen, Dobroliubov and Semenoff 1992]. In a more recent investigation it has been, however, argued that the metastable vacua do represent physically realizable systems [Gocksch and Pisarski 1993].

**Paper III: Bubble Free Energy in Cosmological Phase Transitions.** In this paper the free energy of spherical bubbles is studied in order parameter or Higgs field models having the same quartic potential as used in Paper I. A numerical function with a good accuracy is given for the nucleation action. Using this nucleation free energy of critical bubbles as an input, the general free energy is solved as a function of the bubble radius and the temperature. The calculation is based on the approximation that all the temperature dependence in the free energy comes from the volume term. This approximation should be valid if one is not too far from the limit of small relative supercooling. The bubble radius and curvature-dependent interface tension are discussed in detail. The results of this study are applicable for the case of the electroweak phase transition, and probably for the quark–hadron transition as well.
2 Short History of the Universe

The big bang model provides a general framework for describing the evolution of the Universe. The success of the model is based on only a few—but fundamental—astronomical observations: the redshift in the spectra of distant galaxies, the existence of the 2.7 Kelvin cosmic microwave background radiation, and the abundance of light elements in the Universe. The redshift is believed to be caused by the expansion of the Universe. Additional evidence for the big bang model was presented in April 1992, when it was announced that the observations from the COBE satellite show an anisotropy of $\Delta T / T \approx 6 \times 10^{-6}$ in the background radiation [Smoot et al. 1992].

In principle, we are able to obtain (semi)direct information of the early Universe by observing electromagnetic or gravitational radiation, or exotic relics like very massive particles or small black holes. So far, the only cosmological messengers we have been able to detect in our instruments are the photons. This means that in direct observations we have to limit ourselves to a Universe older than a few hundred thousand years. At that time the temperature was somewhat less than the binding energies of electrons in light atoms. The electrons and light nuclei were able to form stable atoms, and the Universe became transparent for photons. This is the epoch we are looking at when observing the cosmic microwave background radiation.

The abundance of the light elements, as observed in the present-day Universe, gives us indirect but firm evidence of the time of the primordial nucleosynthesis. The major part of the nucleosynthesis took place when the Universe was a few minutes old [see e.g. Applegate, Hogan and Scherrer 1987]. This is the earliest epoch of which we have more or less certain information.

The study of the very young Universe requires an extrapolation of the cosmological model to even earlier times. In addition to Einstein’s theory of gravitation, what is needed for that is a knowledge of interactions between elementary particles at high energies. This knowledge, the standard model of particle physics, is based on laboratory
experiments done at colossal particle accelerators.

It is believed that when the Universe was approximately $10^{-5}$ seconds old and temperature was of the order of 100 MeV, a phase transition from the quark–gluon plasma to the hadron matter took place. In the quark–gluon plasma phase the quarks and gluons were free, whereas in the hadron phase they became confined and formed mesons like the pions, and baryons like the proton and the neutron. The existence of quark–gluon plasma has not yet been confirmed experimentally, but several groups are trying to detect it by using heavy-ion collisions. As mentioned in the Introduction, probably the scale of the density inhomogeneities produced in the cosmological quark–hadron phase transition was too short to have observable consequences (see Subsection 4.2).

Further back in time, the electroweak phase transition took place at $t \approx 10^{-11}$ s. The temperature was then about 100 GeV which roughly corresponds the highest energies available in particle accelerators. Hence this transition presents the earliest time in the history of the Universe of which we have a moderately detailed knowledge. At the classical level of the electroweak theory, the mass terms of the quarks and gauge bosons vanish in the high-temperature symmetric phase. In the phase transition their mass terms, which are proportional to the vacuum expectation value of the Higgs field, become non-vanishing. It is believed that the electroweak phase transition had a significant effect on the baryon asymmetry observed in the present Universe.
Of still earlier times we presumably know at least how temperature evolved during most of the time. Extrapolating the theories of strong and electroweak interactions up over ten decades in energy, one is tempted to believe in grand unification at the scale of $10^{14}$ GeV. However, the huge energy gap between the electroweak and the grand unified scales may very well hide new phenomena. One of the most speculated issues is supersymmetry; if it exists, the symmetry between bosons and fermions was restored above certain temperature. The hypothetical transition suggested in Paper II is also situated to the gap between the electroweak and grand unified scales; its transition temperature is two orders of magnitude above that of the electroweak phase transition.

The most important idea, connected to the (hot) big bang model and related to these very early times, is the hypothesis of inflation. The inflation, or an exponential expansion of the scale factor at very early times, would resolve the smoothness problem—why the temperature of the microwave background is almost uniform over scales bigger than the horizon scale when the photons last scattered. Furthermore, the inflation predicts that the Universe expands eternally but with an always slowing rate, the alternative we are astonishingly close to according to observations of the average mass density of the present Universe.

The extreme limit for the validity of the standard cosmology is the Planck scale. Quantum mechanics tells that if we inspect any system on a scale small enough, the physical quantities fluctuate strongly. For gravity this fluctuation length scale is given by $1/M_{\text{Pl}}$, where the value of the Planck mass is $M_{\text{Pl}} \approx 1.2 \times 10^{19}$ GeV. The horizon radius, the size of a causally connected region of space, was equal to this fluctuation scale at the Planck time when the age of the Universe was $10^{-44}$ s. At the Planck scale general relativity is not valid any more, one would need a quantized theory of gravity.

Let us finally present the fundamental equations governing the evolution of the Universe in the big bang model. A more detailed treatment can be found for instance in the textbook by Kolb and Turner [1990]. A beautiful way to formulate the Einstein

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1In this Thesis the system of so-called natural units is used: $\hbar = c = k_B = 1$. However, numerical values for time and length are often given in SI-units.
equations is to use the action
\[ S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{A,\phi,\psi} - \frac{1}{16\pi G} R_{\text{sc}} \right\}, \] (1)
where \( g \) is the determinant of the metric tensor, \( \mathcal{L}_{A,\phi,\psi} \) is the Lagrangian density for all
the gauge and matter fields in the standard model, \( G \) is the gravitational constant and \( R_{\text{sc}} \) the curvature scalar. The Einstein equations now follow by demanding that the
variation of the action with respect to the metric vanishes. The metric that produces
homogeneous and isotropic three-spaces is the Robertson–Walker metric,
\[ ds^2 = dt^2 - R^2(t) \left[ \frac{d\tilde{r}^2}{1 - k\tilde{r}^2} + \tilde{r}^2 d\theta^2 + \tilde{r}^2 \sin^2 \theta d\phi^2 \right], \] (2)
where \( R(t) \) is the cosmic scale factor, \( \tilde{r} \) is a dimensionless scaled coordinate and the
constant \( k \) takes values of +1, 0 or −1 for three-spaces of positive, zero or negative
curvature, respectively.

The full stress-energy tensor, the variation of the action of \( \mathcal{L}_{A,\phi,\psi} \) with respect
to the metric, is a complicated object. However, to obtain the general evolution of
the Universe it is enough to assume that it is given by the simple perfect fluid form
\[ T^\mu_\nu = \text{diag}(\varepsilon, -p, -p, -p), \] where \( \varepsilon \) is the energy density and \( p \) the pressure. The
independent Einstein equations can in this case be chosen as
\[ \frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3} \varepsilon - \frac{k}{R^2}, \] (3)
\[ d(\varepsilon R^3) = -p d(R^3). \] (4)
The upper one, or Friedmann equation, gives the time dependence of the scale fac-
tor. The lower equation can also be derived from the local conservation of ener-
momentum of the perfect fluid.

In the case of the very early Universe, when matter behaves like radiation, pressure
and energy density are given by
\[ p = \frac{1}{3} \varepsilon = \frac{\pi^2}{90} g_* T^4, \] (5)
where \( g_* \) is the effective number of relativistic degrees of freedom. Every relativistic
bosonic and fermionic degree of freedom increases the value of \( g_* \) with unity and 7/8,
respectively. The relation (5) is an idealization, valid for free particles. Inclusion of the interactions between particles modifies it slightly even at temperatures far away from any phase transition [see e.g. Enqvist and Sirkka 1993].

When the early Universe was at local thermal equilibrium, entropy was conserved. This can be seen from eq. (4) using standard thermodynamics combined with the extreme smallness of all chemical potentials. The conservation of entropy implies that $RT = \text{constant}$, as long as $g_*$ does not change. The curvature term (the one with $k$) is negligible in the Friedmann equation (3) and we can neglect it. Now it follows from substituting the expression for the energy density in eq. (5) to the Friedmann equation and expressing the gravitational constant as $G = 1/M_{Pl}^2$ that

$$T^2 t = \sqrt{\frac{45}{16\pi^3}} \frac{M_{Pl}}{\sqrt{g_*}}. \quad (6)$$

This important equation gives the relation between temperature and time in a radiation-dominated Universe.
3 First-Order Phase Transitions

3.1 Nucleation Rate in Field Theory

Quantum field theory forms the theoretical framework for describing microscopic relativistic processes at zero temperature. The theory is elegantly formulated in terms of the Feynman path integrals. This formulation can be generalized to non-zero temperatures [Bernard 1974; Gross, Pisarski and Yaffe 1981].

The thermodynamical properties of a physical system at nonzero temperature can be calculated from the partition function

$$Z = \text{tr} e^{-\beta H},$$

where $\beta$ is the inverse temperature and $H$ the Hamilton operator.

The operator $\exp(-\beta H)$ in eq. (7) is analogous to the quantum-mechanical time evolution operator $\exp(-iHt)$, if one makes the identification $it = 1/T$. Indeed, in Euclidean space-time the path-integral representation for the partition function of a scalar field is

$$Z = \int_{\beta-\text{periodic}} \left[ d\phi(\tau, x) \right] \exp \left\{ -\int_0^\beta d\tau \int d^3x \, L_E(\phi, \partial \phi) \right\},$$

where the imaginary time is denoted by $\tau$ and $L_E$ is the Euclidean Lagrangian density.

We can see that along the imaginary-time direction the field $\phi$ propagates over a distance $\beta$. Because of the trace in the definition of the partition function in eq. (7), the physical states have to be identical at “times” 0 and $\beta$. Thus the scalar field must obey

$$\phi(\tau + \beta, x) = \phi(\tau, x) \quad \forall \tau, x.$$

The subscript ‘$\beta$—periodic’ in eq. (8) refers to this periodic boundary condition.

For fermion fields the boundary conditions are not periodic but antiperiodic. This is due to the anticommuting nature of fermions, which is reflected in the definition of
the time-ordering operator. For gauge fields Gauss’s law (known in electrodynamics as Maxwell’s first law) must be imposed on physical states. Furthermore, periodicity of the temporal gauge field $A_0$ is just the most natural choice, not an automatic consequence.

At this point, two comments related to the partition function are in order. Firstly, by formulating the theory in Euclidean space-time we loose information on the real-time evolution of the system. The imaginary-time formalism is applicable only for describing processes which occur at or near thermal equilibrium. Most of the time the evolution of the Universe probably happened very near thermal equilibrium, but not during the first-order phase transitions. However, the moment of the onset of a first-order phase transition can still be calculated using these methods. Secondly, we have assumed that the chemical potentials for different particle species vanish. This is usually justified when one calculates quantities like pressure or energy density because of the very small baryon and (presumably) lepton asymmetry [see e.g. Kajantie and Kurki-Suonio 1986].

Next, we will discuss the nucleation rate in phase transitions. We consider a single scalar field, for which the Euclidean effective action is given by

$$ S = \int_0^{\beta} \! d\tau \int d^3x \left[ \frac{1}{2} \left( \frac{\partial \phi}{\partial \tau} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi, T) \right]. \quad (10) $$

The potential $V(\phi, T)$ should be understood as an effective potential. Besides the classical part, it includes both zero-temperature quantum corrections and finite-temperature thermal corrections. We assume that the corrections do not change the derivative part of the action. For a discussion on this point see for instance [Brahm 1992].

In fig. 2, the qualitative behavior of the potential is shown. At high temperatures, the only vacuum of the scalar field model is at $\phi = 0$. At some lower temperature, another local minimum starts to develop. With decreasing temperature, the local minimum soon becomes the true vacuum, and the vacuum at origin becomes metastable. At still lower temperatures the metastable vacuum vanishes, and the system has again only one vacuum. It is the wall between the old metastable vacuum and the new true vacuum that causes the phase transition to be first order. The thermodynamical transition temperature $T_c$ is the temperature at which the two vacua are degenerate. In cosmology the system supercools because of the expansion of the Universe and because the transition rate is only finite, and therefore the transition takes place at a
Figure 2: Qualitative behavior of the effective potential for the scalar field, $V(\phi, T)$. The potential is plotted at four different temperatures which obey $T_4 < T_3 < T_2 < T_1$ and is normalized to vanish at $\phi=0$ for all values of $T$.

temperature somewhat lower than $T_c$.

We assume that the mechanism for the phase transition is homogeneous nucleation. However, one should note that the presence of relic fluctuations or exotic objects like magnetic monopoles, cosmic strings, black holes or very massive particles might modify the mechanism. In the phase transition, bubbles of the stable phase are created. Nucleated bubbles which are bigger than so-called critical bubbles begin to grow. When the bubbles grow, the Universe is gradually converted to the new phase.

The decay rate of a metastable vacuum has in the context of statistical field theory been calculated by Langer [1969]. For relativistic quantum field theory at zero temperature the decay rate has been evaluated by Coleman [1977] and Callan and Coleman [1977], and the same semiclassical methods can be applied also at finite temperatures [Affleck 1981; Linde 1977, 1981, 1983; Arnold and McLerran 1987]. At high temperatures, the mechanism for bubble creation is, instead of quantum tunnelling, thermal over-barrier nucleation. The four-dimensional action can be approximated with the three-dimensional one:

$$S \approx \frac{1}{T} \int d^3x \left[ \frac{1}{2}(\nabla \phi)^2 + V(\phi, T) \right] \equiv \frac{S_3}{T}. \quad (11)$$

In the case of weakly first-order phase transitions the three-dimensional action is at least a very good approximation, if not even exact.
The so-called critical bubble is a non-vanishing solution of the field equation:

\[
\frac{\delta S_3}{\delta \phi} \bigg|_{\tilde{\phi}} = -\nabla^2 \tilde{\phi} + V'(\tilde{\phi}, T) = 0,
\]

where the prime means derivative with respect to \( \phi \). The boundary conditions are that the derivative of the solution must vanish at the center of the bubble, chosen to be at the origin, and that at infinity the solution must be in the metastable vacuum. It has been shown that the critical bubble is spherically symmetric [Coleman, Glaser and Martin 1978]. An example of the critical bubble solution \( \tilde{\phi}(r) \) is given later on in fig. 6 in Subsection 4.1.

The decay rate of the metastable vacuum can be calculated from the imaginary part of the partition function. The extremum action corresponding to the critical bubble will be denoted by \( \tilde{S}_3(T) \), and the potential is normalized as in fig. 2 so that the vacuum action of the high-temperature phase vanishes. The probability of nucleation per unit time per unit volume is

\[
p(T) = \frac{\omega_-}{2\pi} \left( \frac{\tilde{S}_3(T)}{2\pi T} \right)^{3/2} \left| \frac{\text{det}'[-\partial^2 + V''(\tilde{\phi}(r), T)]}{\text{det}[-\partial^2 + V''(0, T)]} \right|^{-1/2} \exp \left[-\tilde{S}_3(T)/T\right],
\]

where \( \text{det}' \) means that the zero eigenvalues resulting from the three translations of the bubble center should be omitted when calculating the functional determinant. The real quantity \( \omega_- \) is the (angular) frequency of the unstable mode of small fluctuations around the bubble solution.

The dominant factors in the nucleation rate (13) are the exponential part and the dimensional part of the prefactor. By prefactor we mean the product of all the factors that multiply the exponential part. If one transforms the functional determinant to dimensionless units, the dimensional quantity that factorizes out is \( M^3(T) \), where \( M(T) \) denotes the mass of the quadratic term of the potential in the symmetric phase. It has been shown [Brihaye and Kunz 1993] that the frequency of the unstable mode can be approximated well—at least for some set of parameters of the quartic potential—with the thin-wall formula \( \omega_- = \sqrt{2}/R_{Tcr} \), where \( R_{Tcr} \) is the radius of maximal tension of the critical bubble (see Paper III). The remaining dimensionless determinant is estimated to differ from unity at most by a few orders of magnitude. This expectation gains some confidence from a somewhat similar case, namely from investigations of the sphaleron.
transition rate where the corresponding determinant has been calculated both numerically [Carson and McLerran 1990] and analytically [Carson et al. 1990]. Very recently, the fluctuation determinant has been evaluated in the case of a critical bubble as well [Baacke and Kiselev 1993]. The result, though renormalization scheme dependent, is not very far from unity for realistic values of $T/M$.

Changing the value of the prefactor with even several orders of magnitude would not significantly affect the cosmological nucleation calculations, as we shall see later on. We may rewrite the nucleation rate in the form

$$p(T) = b T^4_c e^{-S_3(T)/T},$$

(14)

where the value of the slowly changing function $b(T)$ is not essential compared with the exponential part. Besides the simple estimates $T^4$ and $T^4_c$, prefactors like $M^4(T)$ [McLerran et al. 1991] and the Laplace radius of the critical bubble $R_{Lcr}$ multiplied by some other dimensional factors [Csernai and Kapusta 1992a] have been proposed. ($R_{Lcr}$ will be defined in eq. (30) in Subsection 3.3.)

### 3.2 Bubble Nucleation and Growth

A measure telling how the phase transition proceeds is the fraction of space converted to the new phase. In cosmology, the formula for the volume fraction remaining in the old phase given by Guth and Tye [1980] is usually employed [see also Guth and Weinberg 1981]. For time scales much shorter than the Hubble time the expansion of the Universe can be neglected. This approximation should be valid for the whole electroweak phase transition and for the initial stages of the quark–hadron phase transition. The fraction of space still in the metastable phase at time $t$ is given by

$$f_{ms}(t) = e^{-I(t)}; \quad I(t) = \int_{t_c}^t dt' p(T(t')) V(t',t),$$

(15)

where $t_c$ is the time when temperature is equal to $T_c$ and $V(t',t)$ is the volume that a bubble nucleated at time $t'$ occupies at time $t$. 

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A different expression for $f_{\text{ms}}$ has been proposed by Csernai and Kapusta [1992b]. In their approach, the metastable fraction is given by the integral equation

$$f_{\text{ms}}(t) = 1 - \int_{t_c}^{t} dt' p(T(t')) f_{\text{ms}}(t') V(t', t) .$$  

(16)

The two expressions (15) and (16) coincide in the very beginning of the phase transition when $1 - f_{\text{ms}} \ll 1$, and always if the bubbles do not grow. However, they give different results when growing bubbles begin to overlap. The underlying assumption of the Guth–Tye formula (15) is that when two bubbles come into contact they simply grow inside each other without any effect on those parts of the bubbles which do not touch yet. As a consequence the phase transition is never fully completed in an infinite volume; there always remains a non-zero fraction of space in the old phase. The exponentiation of the naive fraction $I(t)$ takes care of bubble overlap and of the fact that the volume fraction where new bubbles can nucleate decreases with time. The latter feature is explicitly taken into account in the Csernai–Kapusta formula (16), whereas the volume occupied by two bubbles which have come into contact with each other is still given as a sum of their individual volumes.

Before going further, the concept of bubble in this context should be clarified. Nucleated bubbles grew probably as deflagrations both in the electroweak and in the quark–hadron phase transition. The effect a deflagration bubble has on the cosmic fluid is illustrated in fig. 3. The coordinate system is chosen to be initially at rest with respect to the cosmic fluid. First, the fluid is hit by a supersonic shock front which heats it and accelerates it to the velocity $v_{\text{fluid}}$ in the same direction as the shock. Later on, a subsonic deflagration front, which is the actual phase boundary, propagates into the moving fluid transforming it to the new phase. The fluid is cooled down and stays again at rest. Depending on the situation, the bubble surface should be identified in the calculation as either the phase boundary or as the shock front. In the latter case the expressions (15) and (16) give, instead of the fraction of space in the old phase, the fraction not touched by the shocks.

When bubbles begin to touch several things can happen. If the deflagration fronts expand very slowly, as is the case in the quark–hadron phase transition after the system has reheated back to $T_c$ (Subsection 4.2), they rearrange the surface quickly
Figure 3: World lines of cosmic fluid near a deflagration preceded by a shock front. In this idealized figure space-time is 1+1 dimensional, and the initial bubble, located at the origin, is infinitesimally small. Dashed lines show how two fluid points move. In the old phase, temperature is $T_f$ outside the shock, and $T_q$ in the region between the shock and the interface. In the new phase temperature is equal to $T_h$. Slopes of the lines are the inverse velocities of the shock front, the phase boundary (“wall”), and the fluid in the intermediate region.

to a spherical shape so that the total volume stays approximately constant. However, when the radii of the bubbles exceed a characteristic length $R_{\text{fus}}(t)$, the rearrangement process is too slow compared with the cosmic expansion to take place [Witten 1984]. Finally, when the stable phase fills most of the space, the interface has arranged itself in such a way that the high-temperature phase is located in isolated droplets. For slow deflagration fronts in the electroweak phase transition the process should be similar, except that the characteristic length $R_{\text{fus}}(t)$ is smaller than the one which follows directly from Witten’s argument since the time scale of the whole phase transition is much shorter than the Hubble time (Subsection 4.1). The case of shock collisions which is relevant for studying the reheating is qualitatively different, because shock fronts do not fuse together.

When comparing the Guth–Tye and Csernai–Kapusta expressions for the volume fraction $f_{\text{ms}}(t)$, one may come to the conclusion that neither of them is perfect. For
phase boundaries, the Guth–Tye formula overestimates the time needed to complete the phase transition because it assumes that the bubbles grow inside each other without any interference; the Csernai–Kapusta formula underestimates the completion time since after two bubbles have fused together the single volume grows in reality more slowly than the total volume of two bubbles had they been separated. For shock fronts, the Guth–Tye formula should be rather accurate unless reflection plays a significant role in the front collisions. In actual calculations the Guth–Tye formula is easier to handle. We will adopt the Guth–Tye formula (15) for further use.

Let us now inspect more closely that stage of the phase transition during which most of the bubbles are nucleated. We will assume that the nucleation is not active any more inside the bubbles. This assumption is too strong only if we consider slow deflagrations preceded by shocks, and if the shocks are, on one hand, too weak to reheat the plasma enough to stop the nucleation and, on the other hand, not so weak that their effect on the nucleation rate could be neglected. In the last case, if one identified the bubble boundary as the deflagration front and not as the shock, there would be no complications.

The actual period of nucleation is short. During it the prefactor in the expression (14) for the nucleation rate changes only slowly compared with the exponentiated nucleation action, as does the bubble wall velocity \( v \). We may hence approximate them as being constants. By writing the nucleation rate in the form

\[
p(t) = p_0 e^{-S(t)},
\]

we obtain the following expression for the metastable volume fraction:

\[
f_{ms}(t) = \exp\left[-\frac{4\pi}{3} v^3 p_0 \int_{t_c}^{t} dt' e^{-S(t')} (t - t')^3\right].
\]

The size of the bubble when it is first nucleated is very small, as will be shown later on, and therefore it is left out of this expression. We define \( t_f \) as the time when the bubbles occupy a significant fraction of the space. For simplicity \( t_f \) is always called the phase transition time, even if the bubbles are defined in terms of the shock spheres. The above expression for \( f_{ms} \) can be evaluated by noticing that all the essential change in it takes place when \( (t - t_c)/(t_f - t_c) \) is close to unity [Fuller, Mathews and Alcock 1988]. After
expanding the nucleation action as a Taylor series, the naive fraction \( I(t) \) which after exponentiation gives the fraction of space still in the old vacuum, \( \exp[-I(t)] = f_{ms}(t) \), can be expressed as

\[
I(t) = I(t_f) \, e^{-\beta(t_f-t)} ,
\]

where

\[
\beta = -\frac{dS}{dt} \mid_{t_f} .
\]

The positive factor \( \beta \) turns out to be an important scale-setting parameter for the phase transition.

The cosmological phase transition discussed in Paper II differs qualitatively from the ‘normal’ cases. It is exceptional in the sense that in thermal equilibrium the same phase is the true vacuum at all temperatures. The approximation presented in last paragraph is valid both for this ‘exceptional’ phase transition and for the electroweak and the quark–hadron transition when

\[
\beta (t - t_c) \gg 1 .
\]

In normal transitions, the nucleation action is proportional to the square of \( 1/(t-t_c) \), and decreases very rapidly after the critical temperature \( T_c \) has been reached. As a consequence, the validity condition is very clearly fulfilled in both electroweak and quark–hadron transition: the left-hand side of eq. (21) is during actual nucleation bigger than unity by two orders of magnitude. In the transition considered in Paper II, the nucleation action behaves as \( \log^{3/2}(1/t) \). (In this case \( t_c \) in eq. (21) means the time when the metastable vacua were created.) Due to the slowness of the decrease of the action, the left-hand side of eq. (21) is not more than about 6. However, this value can still be considered to lie within the allowed range.

As in Paper I, we will define \( t_f \) to be the time when the volume fraction occupied by the bubbles equals \( 1/e \), in other words, we put \( I(t_f) = 1 \) in eq. (19). The phase transition time \( t_f \) can then be solved from the equation

\[
1 = I(t_f) = \frac{8 \pi \nu^3}{\beta^4} P(t_f) .
\]

In principle, the Guth–Tye formula may not be any more fully valid at \( t_f \) due to bubble
Figure 4: A two-dimensional simulation of bubble growth, taken from Paper I. The three frames of size $(40v/\beta)^2$ show the bubble configuration at times $t_f - 4.5/\beta$, $t_f - 2.5/\beta$ and $t_f - 0.5/\beta$. Bubble collisions were neglected in this simulation, but new bubbles were not allowed to nucleate inside existing ones.

collisions. However, the inaccuracy that this causes is negligible when determining for example how much the system supercools.

As an interlude, let us approximate eq. (22) dimensionally as $p(t_f)t_c^4 \approx 1$. The purpose of this zeroth-order approximation is to give an estimate of the scales involved. Utilizing the relation between time and temperature in radiation-dominated Universe, eq. (3), we obtain

$$S(t_f) = \frac{\bar{S}_3(T_f)}{T_f} \approx 4 \log \frac{M_{Pl}}{T_c},$$

(23)

where $T_f$ stands for the temperature in those parts of space which have not been affected by the bubbles yet (see fig. 3). Now we can see why uncertainties of several orders of magnitude, e.g. in the prefactor of the nucleation rate, are not important in cosmology: the value of the critical nucleation action is very large—as a result of the very slow expansion rate of the Universe or the weakness of the gravitational interaction, which is seen as the large value of the Planck mass. For the transition temperatures of the electroweak and the quark–hadron phase transition, the right-hand side of eq. (23) is equal to 150–160 or 180–185, respectively. This also means that the validity of the semiclassical or WKB approximation, employed in the calculation of the nucleation rate, should in this regard be on firm footing.
Let us return to the general, more accurate analysis. As in Paper I, we define an effective nucleation rate $\psi(t)$ as follows:

$$\psi(t) = p(t)f_{\text{ms}}(t) = -\frac{p(t_f)}{\beta} \frac{df_{\text{ms}}(t)}{dt}.$$ (24)

The decay rate per unit volume per unit time is corrected with the metastable fraction, since as the phase transition proceeds there is less space available for the bubbles of the new phase to nucleate. Integrating the effective nucleation rate we obtain the number of bubbles in unit volume as a function of time:

$$n(t) = \int_{t_c}^t dt' \psi(t') = \frac{p(t_f)}{\beta} [1 - f_{\text{ms}}(t)].$$ (25)

The number density increases linearly with the fraction of space in the low-temperature vacuum.

A good estimate for the final number density of bubbles $n_{\text{final}}$ is obtained by setting $f_{\text{ms}} = 0$ in eq. (25) and using eq. (24). If several collisions of the shocks are needed to produce notable reheating (Subsection 4.2), the true final number density is somewhat larger than the one obtained from eq. (25). The average distance of nucleation centers $R_{\text{nucl}}$, defined as $n_{\text{final}} = 1/R_{\text{nucl}}^3$, is given by

$$R_{\text{nucl}} = 2\pi^{1/3} \frac{\beta}{\beta},$$ (26)

where $\beta$ is defined in eq. (20). Increase in the rate of change of the action means a decrease in the nucleation distance because then more bubbles nucleate during a given time interval after the nucleation has effectively been turned on.

In both the electroweak and the quark–hadron phase transition there were a vast number of bubbles nucleated inside the horizon. This was partially due to the rapid change in the nucleation action, and partially due to the smallness of the dimensionless ratio $\sigma^{3/2}/L\sqrt{T_c}$, where $\sigma$ is the interface tension and $L$ the latent heat of the transition (see eq. (31) in Subsection 3.3). On the other hand, in the case of the phase transition suggested in Paper II the nucleation action decreases so slowly that there were only a few hundred nucleated bubbles inside one horizon.

Let us now consider the distribution of bubble sizes. This has been recently discussed by Turner, Weinberg and Widrow [1992]. The bubble size distribution is easily
Figure 5: Bubble size distribution. The area to the right of the right vertical line is the bubble size distribution at time \( t = t_f - 3/\beta \), and the area to the right of the left vertical line is the bubble size distribution at time \( t_f \). In other words, the vertical lines correspond bubbles of zero-radius at those times.

obtained after realizing that a bubble which has a radius \( \rho \) at time \( t \), was nucleated at time \( t - \rho/v \). Here we made again the approximation that at nucleation the bubble radius is so small that it can be neglected (see Subsection 3.3). The distribution of bubble sizes is hence given by

\[
g_t(\rho) = \frac{1}{v} \psi(t - \rho/v) . \tag{27}
\]

The number of bubbles in unit volume at time \( t \) with radius between \( \rho \) and \( \rho + d\rho \) is equal to \( g_t(\rho)d\rho \). More explicitly, the bubble size distribution is given by

\[
g_t(\rho) = \frac{p(t_f)}{v} e^{-u} \exp \left[-e^{-u}\right] ; \quad u = \beta(t_f - t + \rho/v) . \tag{28}
\]

When inspecting the spatial scale of density inhomogeneities, the distribution of distances between nucleation sites of bubbles near each other could likewise be used. Meyer et al. [1991] have studied it extending the naive fraction approximation for the whole nucleation period.

The distribution of bubble sizes given in eq. (28) is presented in fig. 5. When time \( t \) is in the vicinity of the phase transition time \( t_f \), our approach loses its reliability. Therefore the distribution at time \( t_f \), the area to the right of the left vertical line in the figure, may well be somewhat distorted near the origin, whereas the other distribution
corresponding to \( t = t_f - (3/\beta) \) can be expected to be quite accurate. The bubble size distribution is universal in the sense that if the validity of the assumptions is equally good in two different phase transitions, the appropriately scaled distributions should look the same.

### 3.3 Thin-Wall Limit

In the so-called thin-wall limit, or small relative supercooling limit, the equations for the amount of supercooling and the distance between nucleation sites can be expressed in a simple form and solved with a good accuracy. Let us consider a thin-walled bubble of radius \( R \). Let the whole interior of the bubble be in the vacuum of the new phase and the whole exterior in the old vacuum, and let the interface between be infinitely thin. The free energy density difference between the two phases is, in the absence of any relevant conserved charge, equal to minus the pressure difference \( \Delta p \). By difference we mean the value of the quantity in the low-temperature phase minus that in the high-temperature phase. Hence the free energy of the bubble is

\[
F(R) = -\frac{4\pi}{3} \Delta p R^3 + 4\pi \sigma R^2,
\]

(29)

where \( \sigma \) is the interface tension. A generalization of this expansion is discussed in detail in Paper III. The radius of the critical bubble and the corresponding free energy density are found by maximizing \( F(R) \) with respect to \( R \), with the result

\[
R_{Lcr} = \frac{2\sigma}{\Delta p}, \quad F_{cr} = \frac{16\pi}{3} \frac{\sigma^3}{(\Delta p)^2}.
\]

(30)

Above \( R_{Lcr} \) is Laplace’s definition for the radius of the critical bubble. The probability of fluctuation in which a critical bubble is formed is proportional to the Boltzmann factor \( \exp(-F_{cr}/T) \).

For the normal phase transitions the pressure difference can be written as \( \Delta p(T) \approx L(1-T/T_c) \), as given in Section 4 in eq. (37). Using eq. (30), the nucleation action appearing in eq. (17) can be expressed as follows:

\[
S(T) = \frac{C^2}{(1-T/T_c)^2} \quad C = 4\sqrt{\frac{\pi}{3}} \frac{\sigma^{3/2}}{L\sqrt{T_c}}.
\]

(31)
where $L$ is the latent heat of the transition. In the case of the exceptional phase transition discussed in Paper II, one had to use the exact expression for the pressure difference. Thanks to the simple form of the expression this would not give rise to any difficulties in the thin-wall calculation. In the rest of this Subsection, we will analyze only the normal phase transitions.

Utilizing eq. (6) and using the fact that $T$ is very close to $T_c$, we can write the derivative $\beta$ of the action in eq. (20) as

$$t_c \beta = S(T_f) / (1 - T_f/T_c).$$

Substituting the expression (14) for the nucleation rate, the rightmost part of eq. (22) then becomes

$$8\pi v^3 (t_c T_c)^4 b e^{-C^2/(1-T_f/T_c)^2} = \exp \left[ A - \frac{C^2}{(1-T_f/T_c)^2} - 8 \log \frac{C}{(1-T_f/T_c)^{3/2}} \right],$$

where

$$A = 4 \log \left( \frac{M_{Pl}}{T_c} \right) + 2 \log \left( \frac{45}{16\pi^3 g_s |t_c|} \right) + \log(8\pi v^3 b),$$

and $b$ is the dimensionless factor introduced in eq. (14). The phase transition temperature is obtained by demanding that the argument of the exponential function on the right-hand side of eq. (32) vanishes. A good accuracy is achieved by iterating the resulting equation once. This procedure gives the following expression for the amount of supercooling [Kajantie 1992]:

$$1 - \frac{T_f}{T_c} = \frac{C}{\sqrt{\tilde{A}}}; \quad \tilde{A} = A - 4 \log(A^{3/2}/C).$$

The phase transition temperature $T_f$ given by this equation should be close to the equilibrium transition temperature $T_c$, otherwise the approximation used is invalid.

In the expression for $A$ in eq. (33) the first term is by far the dominant one, while for $\tilde{A}$ the second term does have some significance; for example in realistic estimates for the quark–hadron transition its value is 30% of that of the first term in the expression (34). The physical reason for such a large value of this correction term is that the growth time of bubbles, although tremendously longer than the time scale of microscopic interactions, is still very far from the Hubble time.

Now we can determine the average distance between nucleation centers. We compare it with the horizon radius $R_{\text{hor}} = 2t_c$:

$$\frac{R_{\text{nucl}}}{R_{\text{hor}}} = \frac{\pi^{1/3} v C}{A^{3/2}} = 6.00 \frac{v}{A^{3/2}} \frac{\sigma^{3/2}}{L \sqrt{T_c}}.$$
The later a phase transition takes place, the more bubbles there tends to be inside the horizon. For example, in a phase transition at the grand unified scale there should be less bubbles created than in a phase transition at the QCD scale. Of course, in actual cases the dependence of $R_{\text{nucl}}/R_{\text{hor}}$ on the microscopic ratio $\sigma^{3/2}/L$ may well cancel the dependence on the factor $\log(M_{\text{Pl}}/T)$ related to the cosmic background.

It is also interesting to calculate the radius of the critical bubble $R_{\text{cr}}$, \textit{i.e.}, the bubble size immediately after the nucleation. It can be determined from Laplace’s relation (30) and is given by

\begin{equation}
R_{\text{cr}}(T_f) T_c = \sqrt{\frac{3 \tilde{A}}{4\pi \sigma / T_c^3}}.
\end{equation}

Here $R_{\text{cr}}$ is compared with $1/T_c$, which gives the length scale for microscopic processes.

From eqs. (35) and (36) we can observe that a natural length scale for $R_{\text{nucl}}$ is roughly set by the horizon radius, and for $R_{\text{cr}}$ by the microscopic length $1/T_c$. We can thus conclude that the bubbles are indeed microscopic when first nucleated, but grow to a macroscopic size before colliding with other bubbles (unless the interface tension is extremely small).
4 Phenomenology of Cosmological Phase Transitions

In this Section the general methods described above are applied to the case of the electroweak and the quark–hadron phase transition. We present a partially quantitative description of the different events that took place during these cosmological transitions.

We consider in this introductory review part only the first-order electroweak and quark–hadron phase transitions and their cosmological consequences, but whether these transitions in Nature really are first order is by no means known with certainty. A second-order phase transition would have had less cosmological significance.

A fundamental ingredient for the study of a first-order phase transition is the thermodynamical equation of state. The absence of chemical potential simplifies the thermodynamics, since the physical quantities become functions of only one variable, the temperature. The pressure $p(T)$, now equal to minus the free energy density, is taken as the basic quantity.

When the temperature is in the vicinity of the thermodynamical transition temperature $T_c$, the pressure difference between the stable and the metastable phases can be expanded as

$$\Delta p(T) = L \left(1 - \frac{T}{T_c}\right) + \cdots. \quad (37)$$

The quantity $\Delta p(T)$ is defined as the pressure in the low-temperature phase minus the pressure in the high-temperature phase. However, this expansion is not valid for the exceptional phase transition suggested in Paper II because no critical temperature $T_c$ exists there.

Given the pressure $p(T)$, the entropy and the energy density can be determined from

$$s(T) = \frac{dp}{dT}, \quad \varepsilon(T) = Ts - p. \quad (38)$$
These relations hold in both phases.

4.1 The Electroweak Phase Transition

Although the concept of symmetry restoration at high temperatures has been known for long [Kirzhnits 1972; Kirzhnits and Linde 1972; Dolan and Jackiw 1974; Weinberg 1974], the electroweak phase transition has remained poorly understood until recent times. The interest in it was renewed some time after the observation that nonperturbative processes, mediated by the sphalerons, have a significant effect on the baryon asymmetry of the Universe [Kuzmin, Rubakov and Shaposhnikov 1985]. Since then, several authors have studied the dynamics of the first-order electroweak phase transition [McLerran et al. 1991; Turok 1992; Paper I; Anderson and Hall 1992; Dine et al. 1992; Liu, McLerran and Turok 1992; Carrington and Kapusta 1993].

The minimal standard model of electroweak interactions contains one scalar doublet. The order parameter of the electroweak phase transition is the field corresponding to the low-temperature physical Higgs particle, usually taken as the real part of that component of the doublet which acquires a vacuum expectation value.

The Higgs boson has never been experimentally detected, and guesses for its mass cover a wide range. Even its existence may be questioned; for instance, a condensate of heavy quarks could effectively act as a Higgs particle [Lindner 1992]. Furthermore, it might be that the real electroweak theory contains more scalar fields than just one doublet. In the case of two doublets a finite temperature potential of the same form as in the case of one doublet can be used as a reasonable approximation for that combination of scalar fields which drives the transition [McLerran et al. 1991].

The first-order electroweak phase transition is commonly described by the following effective quartic potential [Linde 1983]:

$$V(\phi, T) = \frac{1}{2} \gamma (T^2 - T_0^2) \phi^2 - \frac{1}{3} \alpha T \phi^3 + \frac{1}{4} \lambda \phi^4,$$

where the order parameter field $\phi(x)$ is a real scalar function. The qualitative (but
The thermodynamical transition temperature is

\[ T_c = \frac{T_0}{\sqrt{1 - \frac{2 \alpha^2}{3 \lambda \gamma}}}, \]  

and the lowest temperature where the symmetric vacuum can exist is \( T_0 \).

The potential \( V(\phi, T) \) could in principle be derived from the full microscopic theory. The question how to best determine the effective potential beyond the naive one-loop level is currently under an active study. One method is to employ effective three-dimensional theories, with [Bunk et al. 1993; Kajantie, Rummukainen and Shaposhnikov 1993; Farakos et al. 1993] or without [Shaposhnikov 1993] lattice Monte Carlo simulations. These studies seem to indicate that the electroweak phase transition would not be as weakly first order as expected on the basis of perturbative analysis. (There are arguments telling that the existence of supersymmetry could also make the transition more strongly first order [Espinosa, Quirós and Zwirner 1993].)

The point of view taken here is that the potential in eq. (39) should be regarded as a phenomenological one, valid in the vicinity of \( T_c \). The parameters \( T_0, \gamma, \alpha \) and \( \lambda \) are to be chosen so that the potential quantitatively correctly describes the phase transition. In principle, the values of these parameters could be determined by experiment or observation. In practice, first-principles calculations, even if not fully satisfactory, are employed to obtain an estimate of the relevant scale of the parameters.

Next, we will discuss the equation of state near the electroweak phase transition. Let us inspect two configurations where the field \( \phi \) is spatially uniform with a value corresponding either to the minimum at the origin or the other minimum of \( V(\phi, T) \). Since the potential vanishes at origin, the difference in the free energy densities between these two configurations is equal to the value of the effective potential at the other minimum. We denote it as \( V(v(T), T) \equiv -\tilde{B}(T) \), where \( v(T) \) is the value of the field at the other minimum. This leads to the following expressions for the pressure (Paper I):

\[ p_q(T) = aT^4, \quad p_h(T) = aT^4 + \tilde{B}(T). \]  

The same factor \( a \) is used in both radiative terms, because at \( T_c \) the pressures of both

\footnote{For simplicity the subscripts ‘q’ and ‘li’, adopted from the quark–hadron transition, are used also here to denote the high- and low-temperature phase, respectively.}
phases must be equal and $\tilde{B}(T)$ vanishes. The term $\tilde{B}(T)$ was attached to the low-temperature phase since both $\tilde{B}(T)$ and the low-temperature phase exist only up to a certain temperature somewhat above $T_c$. Moreover, one might add a constant $-\tilde{B}(0)$ to each of the phases in order to make $p_h(T)$ vanish at zero temperature. However, this is unnecessary since the above equation of state is invalid at low temperatures anyway (Paper I).

From the relation (3) one observes that the scale for the constant $a$ in eq. (41) is apart from the factor $\pi^2/90$ set by $g_*$, the effective number of relativistic degrees of freedom. At very high temperatures all the particles of the minimal standard model contribute to $g_*$ giving

$$g_* = \frac{2}{\gamma} \left( e^\mu, \nu \right) + \frac{3 \times 2}{g} + \frac{4}{\Phi} + \frac{7}{8} \left[ \frac{3 \times 4}{e, \mu, \tau} + \frac{3 \times 2}{v} + \frac{6 \times 3 \times 4}{q} \right] = 106.75 . \tag{42}$$

At the phase transition all three weak gauge bosons increase their degrees of freedom with unity by eating the altogether three would-be Goldstone bosons from the scalar doublet. This does not change the value of $g_*$ significantly, because slightly below $T_c$ the masses of the gauge bosons and the physical Higgs particle are presumably small compared with the temperature. However, the top–quark, the other yet undetected particle predicted by the minimal standard model, might decrease the value of $g_*$ significantly—the contribution from a single quark to the value of $g_*$ is about 10%.

At the phase transition, the jump in $g_*$, folded together with the change in the interactions causing deviation from the radiative free-gas behavior, produces latent heat of the transition. For the equation of state (41) the latent heat is given by $L = -T_c \tilde{B}'(T_c)$, and its value is hence reflected in the parameters of the potential (19). One should note that latent heat $L$ includes not only the effect of the Higgs particle but also of all the other particles, since in our order parameter model all the other fields have been integrated out.

As we have now specified the potential $V(\phi, T)$ we will shortly return to the nucleation process. It is convenient to express the temperature dependence of various quantities by using the function $\bar{\lambda}(T)$,

$$\bar{\lambda}(T) = \frac{9 \lambda \gamma}{2 \alpha^2} \left( 1 - \frac{T_c^2}{T^2} \right) , \tag{43}$$
which satisfies \( \bar{\lambda}(T_0) = 0, \bar{\lambda}(T_c) = 1 \). The nucleation action is written in terms of the function \( \bar{\lambda}(T) \) as follows:

\[
S(\bar{\lambda}(T)) = \frac{2^{9/2} \pi}{3^5} \frac{\alpha}{\lambda^{3/2}} \frac{f(\bar{\lambda})}{(1 - \lambda)^2}.
\]  

(44)

The function \( f(\bar{\lambda}) \) has been determined in Papers I and III by solving numerically the field equation (12) for the potential (39). It is a smoothly behaving function with the special value \( f(1) = 1 \). In fig. 6, the bubble solution of the field equation is shown for two illustrative values of \( \bar{\lambda} \). For \( \bar{\lambda} = 0.9 \) the solution somewhat resembles a thin-walled bubble, but for \( \bar{\lambda} = 0.6 \) the bubble core is far away from the true vacuum.

From eq. (22) one can solve for \( t_f \), the age of the Universe when the bubbles had filled the space. Explicitly, the equation determining the amount of supercooling is, in analogy with demanding that the right side of eq. (32) equals unity, as follows:

\[
A - S(\bar{\lambda}_f) + 4 \log \left[ \frac{1}{(T_c/T_0)^2 - 1} \frac{dS}{d\bar{\lambda}} \bigg|_{\bar{\lambda}_f} \right] = 0, \]

(45)

where \( A \) is given by eq. (33) and \( \bar{\lambda}_f \equiv \bar{\lambda}(T_f) \). The thin-wall limit, or small relative supercooling scenario, considered in Subsection 3.3 follows as a special case if the solution \( \bar{\lambda}_f \) of the above equation is close to unity. For example, the expression (31) for the nucleation action follows from eq. (44) in the limit \( \bar{\lambda}_f \to 1 \).
Let us now inspect in more detail how the cosmological electroweak phase transition is assumed to have proceeded. The sequence of events is illustrated in fig. 7. The numerical values of various quantities presented in the figure correspond to the following values of the parameters:

\[
T_c = 100 \text{ GeV}, \quad \gamma = 0.1309, \\
\alpha = 0.0162, \quad \lambda = 0.0131.
\]  

(46)

The values of \(\gamma, \alpha\) and \(\lambda\) are those obtained by Huet et al. [1993] by substituting the zero-temperature masses \(M_t = M_W, M_h = 40 \text{ GeV} \simeq M_W/2\) into the perturbatively evaluated effective potential. Here the unrealistically low mass for the Higgs–particle is more crucial than that for the top-quark, as will be discussed shortly. The parameter values (46), used here just for illustration, imply that the phase transition is very weakly first order. This is indicated by the small latent heat (= \(0.086 T_c^4\)) and the long correlation length (= \(15.0/T_c\) in both phases at \(T_c\)).

Trust in the perturbative calculations leads to complications here. The Higgs mass used among others by Huet et al. [1993] is close to the upper limit of the allowed region if one requires that the baryon asymmetry of the Universe was created in the electroweak transition. A necessary condition for the electroweak baryon asymmetry generation is that the sphaleron transitions should have been frozen out in the low-temperature phase already at \(T_c\). For that to have happened the Higgs scalar must not be heavier than about 40 GeV according to perturbative analysis [e.g., Huet et al. 1993]. On the other hand, the Higgs mass of such a low value seems to be ruled out by LEP experiments [Davier 1992]. As a solution to this dilemma some authors have assumed the existence of additional scalar fields. In multi-Higgs cases the parameter \(M_h\) is not the true zero temperature mass of a physical Higgs particle and is hence not constrained by the mass measurements [see Liu, McLerran and Turok 1992, and references therein].

In our phenomenological approach the parameter values quoted in eq. (46) do not pose any problems. Even within the minimal standard model these values do not imply a Higgs mass which were ruled out experimentally, as long as one does not try to extend the validity of the potential down to zero temperatures.
Figure 7: Schematic 1+1 dimensional figure of the electroweak phase transition. Grey area presents the high-temperature and white the low-temperature phase, and the separate pictures at right give two-dimensional snapshots of the transition. The numbers are for the parameter values in eq. (46). The idea for this figure is borrowed from Rummukainen [1990].

In fig. 7, three essentially different time scales can be seen. One is the Hubble time at the transition. Another is the duration of time the Universe stayed in the supercooled metastable state, $\Delta t_{sc} = t_f - t_c$. The phase transition time $t_f$ can be determined from eq. (45) by utilizing for the nucleation action the numerical function given in Paper III. At the nucleation, the value of the action turns out to be only 106 instead of the naive value which according to eq. (45) equals $A (= 145)$, i.e., the last term in that equation does have some significance. The shortest time scale appearing in fig. 7 is the growth time of the shocks, defined as $\Delta t_{ah} = R_{nucl}/v$. The average distance of nucleation centers $R_{nucl}$ is defined in eq. (26). In numerical calculations the dimensionless prefactor $b$ of the nucleation rate in eq. (14) was taken to be unity and the free-gas value $v = 1/\sqrt{3}$ was used for the shock velocity.

The phase transition was completed at time $t_c$ when the actual bubbles of new
phase, expanding as deflagrations behind the shocks, had met and coalesced fully. (Detonations, the other type of explosive processes, would require much more supercooling than eq. (15) indicates for the present parameters.) The growth time of the bubbles of the new phase cannot be solved, because the velocity of the deflagration front is not known. However, it seems probable that the difference of the velocities of the deflagration and shock fronts was clearly less than one order of magnitude [Ignatius, Kajantie, Kurki-Suonio and Laine 1993], which indicates that the growth time of deflagrations did not differ drastically from $\Delta t_{sh}$. This also means that the inaccuracies in the estimates of the time scales are not significant (cf. Subsection 4.2).

Mutual collisions of the shocks reheat the system to a certain temperature $T_{rh}$. As will be demonstrated below, the reheating temperature $T_{rh}$ was in the electroweak phase transition less than the critical temperature $T_c$. Because of this and the fastness of the phase transition, $T_{rh}$ can be estimated by going to the extreme case where the whole space were converted instantaneously from the old to the new phase at time $t_f$. DeGrand and Kajantie [1984] called this scenario "abrupt transition". The reheating temperature can be obtained from the equation

$$\varepsilon_h(T_{rh}) = \varepsilon_q(T_f).$$

(47)

For the parameter values given in eq. (16) the reheating turns out to be only 13% on the scale where 100% would mean reheating back to the critical temperature. The fact that the critical temperature $T_c$ is not reached during the reheating is crucial for the scenario of baryon asymmetry generation discussed by Liu, McLerran and Turok [1992]. If the reheating temperature $T_{rh}$ had been close to $T_c$, the velocity of deflagration fronts would have decreased substantially (see Subsection 4.2). In that case the baryon number produced in the front would have had time to diffuse to the old phase, where it would have been washed out by the sphalerons.

At the final stage of the phase transition the shrinking droplets of the old phase produced rarefaction waves; however, their effect was shadowed by the presence of the remnants of shocks. Recently Huet et al. [1993] have demonstrated that the deflagra-

\[\text{3In condensed matter physics a first-order phase transition with less than 100% reheating is called "hypercooled" instead of supercooled [see for example Leggett and Yip 1990].}\]
Figure 8: Relation between temperature and time in the electroweak (EW) and in the quark–hadron (QH) phase transition. Dotted curve denotes the reheating period during which temperature was far from being spatially uniform.

Inhomogeneities did not develop any instabilities while expanding. Thus the length scale of inhomogeneities is given by the usual expression \( R_{\text{nucl}} \) for the average distance of nucleation centers. However, it is hardly probable that the density inhomogeneities produced in the electroweak phase transition could have been of any importance. For instance, for the parameters \( Q^2 \) one can estimate the maximal relative pressure differences to be only \( \Delta p/p \approx 4 \times 10^{-5} \). In comparison, for the storms in the atmosphere of Earth this quantity can be three orders of magnitude larger.

In fig. 8 the behavior of temperature versus time both in the electroweak and in the quark–hadron transition is shown schematically in a log–log plot. From the figure we can see that the electroweak phase transition did not last long. Afterwards, the usual relation between temperature and time, given in eq. (1), became soon valid again.
4.2 The Quark–Hadron Phase Transition

Investigations of the cosmological quark–hadron phase transition started over a decade ago [Olive 1981; Suhonen 1982]. After these early studies it was soon realized that the onset of the supposedly first-order transition required supercooling [Hogan 1983]. Although much progress was made during the subsequent years in understanding various features of the transition [Witten 1984; DeGrand and Kajantie 1984; Applegate and Hogan 1985; Kajantie and Kurki-Suonio 1986; Fuller, Mathews and Alcock 1988], several questions are still unanswered, partially due to insufficient knowledge of the properties of thermal quantum chromodynamics. (For a more complete reference list see Bonometto and Pantano [1993].)

Lattice Monte Carlo simulations provide the best tool currently available for the study of the equation of state in QCD near the transition point. But even with this method one has not been able to solve the order of the transition for the cosmologically relevant case, i.e., when the chemical potential vanishes and there are two light (u,d) and one intermediate-mass (s) quark species. What is known is that in the case of pure glue, corresponding to infinitely heavy quarks, the transition is first order, and so it is with four light quarks. (For a review on lattice results see [Petersson 1992].) Also is known that there is a substantial jump in the energy density within a temperature interval of less than 10 MeV around the critical temperature. However, for working out the consequences of the phase transition in cosmology this information is not sufficient as long as one is not able to distinguish between a first-order, and a second-order or non-existing transition. If the transition is not first order, no supercooling can occur, even if the equation of state gave rise to a very rapid change in the energy density. This is due to the extremely slow expansion of the Universe.

Guided by the recent lattice calculations, we use in numerical estimates the following values for the physical quantities of the quark–hadron phase transition:

\[
T_c = 150 \text{ MeV} , \quad L = 2 T_c^4 , \quad \sigma = 0.02 T_c^3 .
\] (48)

Here \(T_c\) is the critical temperature, \(L\) the latent heat and \(\sigma\) the interface tension. The true value of the critical temperature lies very probably somewhere between 100 and
250 MeV, and 150 MeV may be a good guess for its value [Petersson 1992]. For the other two quantities there is currently no lower limit, since they vanish if the phase transition is not first order. The values of $L$ and $\sigma$ given in eq. (48) are based on pure glue lattice simulations: the value of the latent heat is taken from Iwasaki et al. [1992], and the interface tension has been determined in computer studies by Grossmann and Laursen [1993], where the length of the lattice in the imaginary time direction was 2 lattice points, and by Iwasaki et al. [1993], where it was 4 and 6 lattice points.

The simplest analytical model for the QCD equation of state is that of the MIT bag model, which is often employed in the cosmological context. In this model the pressures of quark and hadron phases are given by

$$p_q(T) = a_q T^4 - B, \quad p_h(T) = a_h T^4.$$  
(49)

The value of the bag constant $B$ is determined from the condition of equal pressure at $T_c$: $B = (a_q - a_h)T_c^4$. The bag equations (49) also follow as a special case from the more general equation of state presented in Subsection 4.1. Approximating $\tilde{B}(T) = (L/4)(1 - T^4/T_c^4)$, which in the limit of small supercooling is equivalent to eq. (57), and substituting this to the equation of state (41) with the constant $-\tilde{B}(0)$ added in both sides, one recovers the bag equations:

$$p_q(T) = a T^4 - \frac{L}{4}, \quad p_h(T) = (a - \frac{L}{4T_c^4}) T^4.$$  
(50)

The equation of state of the naive bag model, which follows from counting the particle species and utilizing eq. (33), gives an approximate upper limit for the latent heat. Somewhat above the critical temperature $T_c$ the strongly interacting relativistic particles are the gluons, and u– and d–quark. (Depending on its mass, also the s–quark could be counted.) For temperatures somewhat below $T_c$ the only strongly interacting particles that we include within this naive approach are the pions, the lightest hadrons. The other low-massed and massless particles—the photon, electron, muon and neutrinos—are present in both phases. The effective number of relativistic degrees of freedom in the quark–gluon and in the hadron phase is thus given by $g_{sq} = 51.25$ and $g_{sh} = 17.25$, respectively.

In fig. 9, the energy density of the cosmic fluid is schematically plotted both for the naive bag model, and for a more realistic equation of state consistent with the lattice
Figure 9: Behavior of energy density in QCD. Thin curve is for the naive bag equation of state, and thick curve for a weaker first-order transition. Dotted lines denote the metastable branches.

simulations. We clearly see how the naive bag model exaggerates the value of latent heat, $L_{\text{bag}} = 14.9 T_c^4 \gg L$. The parameter values of the bag model can be corrected to reproduce a desired latent heat. However, the corrected bag model does not mimic well the realistic equation of state over the whole range. But in the vicinity of $T_c$ it reproduces the true equation of state with a first-order accuracy in the pressure and zeroth-order accuracy in the energy density, which is sufficient for determining the onset of nucleation. In this case the use of the bag model with improved parameter values is in practice equivalent to employing directly the thin-wall approximation discussed in Subsection 3.3, except that changing the equation of state also affects the relation between time and temperature of the Universe.

Validity of the thin-wall approximation is violated if the correlation length associated with the transition is not clearly smaller than the radius of the critical bubble. It is not easy to tell what the relevant correlation length is in QCD near the transition temperature. However, since the transition is just weakly first order, if first order at all, it is quite possible that the correlation length is quite large.

In a case where the thin-wall approximation is inapplicable, one could employ in nucleation calculations the order parameter model presented in Subsection 4.1. There is a one-to-one correspondence between the four parameters of the potential (39) of

\[ \text{(39)} \]
that model and the four physical parameters of the transition [Paper I; Kajantie 1992]:

\[ [T_0, \gamma, \alpha, \lambda] \leftrightarrow [T_c, \sigma, L, l_c], \tag{51} \]

where \( l_c \) is correlation length. (In the order parameter model with a quartic potential the correlation lengths in both phases are equal to \( l_c \) at the critical temperature.) Once the values of the physical parameters are known, also the parameters of the potential are completely fixed.

It is not clear how one should interpret the order parameter field in QCD because the theory does not have any classical potential driving the transition. If the latent heat is not small, the order parameter represents several degrees of freedom. Then it would seem more natural to identify the \( \phi \)-field with a thermodynamical quantity like the energy density, in the same manner as was done by Csernai and Kapusta [1992a].

From now on we will assume that the nucleation in the cosmological quark–hadron phase transition took place under conditions which were close to the thin-wall limit. For the values of the physical parameters presented in eq. (48), eq. (36) gives for the critical bubble a very large value of the radius, \( R_{ct}(T_f) = 38/T_c = 51 \) fm. This shows that the thin-wall approximation would remain valid even if the correlation length were large.

The main events of the quark–hadron phase transition are shown in fig. 10 in the same way as was done in fig. 7 for the electroweak case. Inspecting first the early stages of the phase transition, we note that in the quark–hadron phase transition the supercooling was smaller than in the electroweak transition. Secondly, we may compare the growth time of shocks with the duration of supercooling utilizing for example eqs. (34) and (35):

\[ \frac{\Delta t_{sh}}{\Delta t_{sc}} = \frac{\pi^{1/3}}{A}. \tag{52} \]

It is interesting to note that this ratio is completely determined by the nucleation action, or approximately by the age of the Universe. In other words, at any temperature there is the definite relation (52) between the degree of supercooling and the nucleation distance (assuming that the value of the shock velocity is a constant). This result is in principle valid only in the thin-wall limit. However, from fig. 7 one can infer that the
prediction holds rather well also for the electroweak transition in the example case we considered.

The values derived for the two time scales $\Delta t_{sc}$ and $\Delta t_{sh}$ may be somewhat erroneous, since in three dimensions the shocks are weak, especially if the deflagration front is very slow [Kurki-Suonio 1985]. It seems that the deflagration front velocity was indeed quite small, probably at least one order of magnitude smaller than the velocity of the shock front [Kajantie 1992; Ignatius, Kajantie, Kurki-Suonio and Laine 1993]. The weakness of the shocks could make the estimates of the nucleation process inaccurate, because new bubbles could possibly nucleate to a region already touched by a shock. This inaccuracy can be at least partially cured by using in the calculations an effective velocity $v$, which is smaller than the true velocity of the deflagration front.

The main difference between the electroweak and the quark–hadron transition is that only in the latter transition the Universe was reheated back to the critical temperature (see fig. 8). This is due to the much larger value of the latent heat in the quark–hadron transition. After the reheating, the phase transition proceeded very slowly, and almost in thermal equilibrium. The expansion of the Universe did not cause any cooling; instead, the denser quark–gluon matter was transformed to the more dilute hadron matter. As is discussed in Paper I, the duration of this period can be approximately determined from the relation

$$
\frac{t_e}{t_c} - 1 \approx \frac{L}{2 \varepsilon_q(T_c)}.
$$

This approximation holds if the resulting value is clearly smaller than unity. In the case of this period of slow burning the expansion rate of the Universe determines the typical velocities of deflagration fronts, too. The velocities are roughly given by $R_{\text{nucl}}/R_{\text{hor}}$ divided by $L/4\varepsilon_q$. The numerical value that this gives for the velocity is of the order of $10^{-4}$.

At the final stages of the phase transition the decaying quark droplets produced rarefaction waves [Kajantie and Kurki-Suonio 1986]. This led to the creation of density inhomogeneities, which in principle could have significantly affected the nucleosynthesis and could be observed in the present-day Universe. However, if the parameter values in eq. (48) are roughly correct, the distance scale of density inhomogeneities was too short.
Figure 10: Schematic 1+1 dimensional figure of the phase transition from the quark–gluon (grey) to the hadron phase (white). This figure was presented originally by Rummukainen [1990]; the current version is a modified one. For clarity only the effect of the dying quark droplets is shown in the world lines of the cosmic fluid (thin dotted lines), the effect of an expanding hadron bubble has been illustrated earlier in fig. 3. The numbers correspond to the values of physical quantities in eq. (48).
for this to happen. Only a distance scale $R_{\text{nucl}}$ of at least one meter at the quark–hadron transition temperature could have had later an effect on the abundance of light elements [Applegate, Hogan and Scherrer 1987; Kurki-Suonio et al. 1990; Mathews et al. 1990]. Redshifted to the present Universe, the length $R_{\text{nucl}} \approx 4 \text{ cm}$ is less than the distance from Earth to Sun. In the cosmic scale this is a very short distance.

If the interface tension were the same as in eq. (48) but the latent heat much smaller, the distance scale $R_{\text{nucl}}$, given in eq. (35) and proportional to $\sigma^{3/2}/L$, would correspondingly be much larger. This possibility cannot presently be ruled out, because the values of the physical quantities $\sigma$ and $L$ are not known. However, it is tempting to think that the values of these quantities would not be arbitrary if the phase transition were very weakly first order; that instead in this limit they would show some sort of universal behavior compared with other physical transitions. By weakly first order transition we mean a transition in which the values of the thermodynamical quantities $L$, $\sigma$ and $1/l_c$ are in dimensionless units small when compared with unity.

A phase transition which can be treated analytically is the one between ordered and disordered phases in the two-dimensional $q$–state Potts model [see e.g. Wu 1982]. This model is a generalization of the Ising model to $q$ spins, and the transition is for $q > 4$ first order, the stronger the larger $q$ is. In the case of the transition of the two-dimensional Potts model there are no additional parameters besides $q$. The values of the latent heat [e.g., Wu 1982] and more recently, the interface tension [Borgs and Janke 1992], have been calculated analytically. In the limit where the phase transition becomes weaker and weaker, that is, when $q$ approaches 4 from above, the interface tension vanishes more rapidly than the latent heat. (A similar behavior can be seen in the quartic order parameter model presented in Subsection 4.1, if one believes in the naive way of weakening the phase transition: by decreasing the value of $\alpha$ and keeping the other parameters appearing in the potential (39) constant one observes that $\sigma^{3/2}/L \propto \alpha^{5/2}$.) The scaling argument coming from the two-dimensional Potts model seems to hint that if a cosmological first-order phase transition is made weaker, the distance scale between nucleation sites gets smaller—a quite natural behavior.

It seems that in the cosmological quark–hadron phase transition the expanding deflagration bubbles were on the borderline between stability and instability.
[Huet et al. 1993], if one assumes that latent heat was carried away from the front by hydrodynamic flow and not by neutrinos. The opposite assumption has also been made [Freese and Adams 1990; Adams, Freese and Langer 1993], and it easily leads to instabilities. However, according to Applegate and Hogan [1985] the relative importance of neutrinos as heat carriers vanishes in the limit of small supercooling.

Finally, let us mention two exotic topics related to the cosmological quark–hadron phase transition. Firstly, Mardor and Svetitsky [1991] made the observation that in the MIT bag model small hadron bubbles exist already above $T_c$. But this does not imply that the same would be true for the real QCD. Indeed, employing the quartic order parameter model the normal behavior is recovered (Paper III). Secondly, in some circumstances it might be possible for the quark droplets to survive over the transition [Witten 1984]. If stable, these lumps of strange quark matter could then in principle be observed in the present-day Universe.
5 Conclusions and Future Prospects

The goal of this Thesis has been to achieve an insight to the physical processes that occurred in the Universe during different first-order phase transitions.

We have pointed out that especially the onset of a cosmological phase transition shows a universal behavior allowing for a general approach. We have presented in this work some general methods which can be applied to an analysis of the sequence of events taking place in cosmological first-order phase transitions.

In the case of the cosmological electroweak or quark–hadron phase transition, the quantitative description has not yet reached a fully satisfactory level. The reason for this is the lacking knowledge of the correct input values of the physical parameters. The case with the phase transition suggested in Paper II is different in the sense that there the main uncertainty comes from the basic assumptions of the scenario itself.

In the future one will hopefully learn the correct values of the physical quantities related to the electroweak and quark–hadron phase transitions. With their improving accuracy lattice simulations should provide this information for QCD with physical quark masses, as well as for the electroweak theory.

Combining lattice simulations with analytical calculations in effective three-dimensional theories, it seems to be possible to derive for the electroweak theory a potential incorporating even non-perturbative effects [Shaposhnikov 1993; Farakos et al. 1993]. Such a potential could then be employed for studying the electroweak phase transition.

In order to understand in detail the dynamics of cosmological phase transitions one must have a good knowledge of the bubble growth at microscopic level. At present we are investigating the growth of bubbles using a model in which there is a friction-like coupling between the order parameter field and a cosmic fluid field. By employing this model one is able to numerically simulate for instance the collisions between bubbles.
Moreover, the velocity of a deflagration wall can be determined exactly as a function of the friction coefficient. So far, we have applied the model in 1+1 dimensions [Ignatius, Kajantie, Kurki-Suonio and Laine 1993]. In the future, the computations should be extended to include spherically symmetric three-dimensional bubbles. Furthermore, one could try to obtain a good estimate for the value of the friction coupling that determines the velocity of the bubble wall.

By combining all these developments, one is led to the conclusion that it is possible to achieve an accurate quantitative description of different cosmological phase transitions in the not too distant future. It would then also be possible to calculate the value of the baryon asymmetry of the Universe from the first principles.
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