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Title: Hacking detection based on the elastic properties of liquid crystals in different phases

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Supplement 2

From the experimental data in Fig. 6(b), the anchoring coefficient on each substrate was estimated. In the experiment, the substrates of PI and P3 are rubbed uniformly at first. After cell fabrication in parallel orientation, photo-alignment rotates the director from rubbing direction to perpendicular to the rubbing. So the rubbing and photo-alignment processes of P3 should be considered.

So, firstly we considered the anchoring coefficient of rubbed and photo-aligned P3 substrate. We checked with the twist cell with perpendicularly rubbed PI and P3. The cell showed deviation angle from the rubbing or photo-aligned direction. $W_{PI\_rub}$ and $W_{P3\_rub}$ are the anchoring coefficients of the lower (PI) and upper (P3) substrates due to the rubbing and $W_{P3\_pho}$ is the anchoring coefficient of P3 due to the photo-alignment. With consideration of $W_{PI\_rub} >> W_{P3\_rub}$ and $W_{P3\_pho}$, we could estimate the anchoring coefficient of rubbed P3 substrate ($W_{P3\_rub}$ or $W_{P3\_pho}$) with the $W_{P3\_rub} = K_2 (\Delta \theta_{easy}/d)$. The equation is obtained with the minimization of free energy consisted of twist deformation and anchoring at both substrates. $\theta_{easy}$ is the rubbing or photo-aligned direction on the P3 and it is $\pi/2$ here. $\Delta \theta_{easy}$ is the deviation angle of director from the easy axis on the P3 substrate. $K_2 (3 \times 10^{-12}$ N) is the elastic constant of the twist mode and $d$ (2.8 $\mu$m) is the cell gap [1]. We obtained $W_{P3\_rub} \approx 1.8 \times 10^{-5}$ J/m$^2$ and $W_{P3\_pho} = 5.2 \times 10^{-6}$ J/m$^2$. The $W_{P3\_rub}$ and $W_{P3\_pho}$ are considered as the $W_{P3\_rub, o}$ and $W_{P3\_pho, o}$ respectively in the below calculation.

Now consider the LC cell irradiated with light. First, consider the deformation in NP. Since there is only twist deformation in the NP, the free energy $F$ per unit area can be expressed as follows [2].

$$F = \frac{1}{2} K_2 (\theta_{2} - \theta_{1})^2 + \frac{1}{2} W_{PI\_rub} (\theta_{e,1} - \theta_{1})^2 + \frac{1}{2} W_{P3\_rub} (\theta_{e,1} - \theta_{2})^2 + \frac{1}{2} W_{P3\_pho} (\theta_{e,2} - \theta_{2})^2 \ldots (1)$$

In the above equation, $\theta_{e,1}$ and $\theta_{e,2}$ are easy axis direction of bottom and upper substrates respectively. In this experiment, $\theta_{e,1} = 0$ and $\theta_{e,2} = \pi/2$. $\theta_1$ and $\theta_2$ are director directions in NP on the bottom and upper substrates respectively. The angles deviated from the easy axis on the surface are satisfied the relation $|0 - \theta_1| \ll |\pi/2 - \theta_2|$ due to the difference in anchoring coefficient in the experiment. And it can be set to $\theta_1 \sim 0$. And as the rubbed P3 surface is
modified by the another photo-alignment, we consider that the anchoring coefficients of rubbed
and photo-aligned are changed exponentially as function of irradiated energy [3]. So we put as
\[ W_{P3,\text{rub}} = W_{P3,\text{rub},o} \exp \left( -E_n/E_\tau \right) \quad \text{and} \quad W_{P3,\text{pho}} = W_{P3,\text{pho},o} \left( 1 - \exp \left( -E_n/E_\tau \right) \right). \]
Here \( W_{P3,\text{rub},o} \) and \( W_{P3,\text{pho},o} \) are the maximum anchoring coefficients on P3 by rubbing
and photo-alignment. And \( E_n \) is the energy of irradiated light and \( E_\tau \) is the characteristic
energy of anchoring coefficient response. Then, we obtain the
\[ \theta_2 = \frac{\theta_1 K_2/d + \theta_{e,1} W_{P3,\text{rub}} + \theta_{e,2} W_{P3,\text{pho}}}{(K_2/d + W_{P3,\text{rub}} + W_{P3,\text{pho}})} \]
as function of irradiation energy. We fitted the angle adjusting \( E_\tau \) and we obtained \( E_\tau = 56 \text{ J/cm}^2 \). In Fig.
6(b), we plotted the fitted \( \theta_2 \) as function of irradiated energy. There are limitations in the above
calculation: We assumed the anchoring coefficient changes exponentially with irradiated energy
independent of the processes. We considered that coefficients of decay of rubbing and growth
of photo-alignment are the same even the different background of the alignment.

In the SP, twist of the director is not allowed in the cell, and the directors in each
substrate are lined up in an average direction that minimizes the free energy. Actually there is
striped pattern and the average direction seems to also reflect the defects. If the direction is \( \theta_s \),
from Eq. (1),
\[ W_{PI,\text{rub}} (\theta_{e,1} - \theta_s) + W_{P3,\text{rub}} (\theta_{e,1} - \theta_s) + W_{P3,\text{pho}} (\theta_{e,2} - \theta_s) = 0 \]
is satisfied. Substituting the saturated experimental value \( \theta_s \) when light is sufficiently irradiated,
\( W_{PI,\text{rub}} \) is approximately 8 times of \( W_{P3,\text{pho},o} \) and is about \( 4.2 \times 10^{-5} \text{ J/m}^2 \).

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