Pion-nucleon elastic scattering amplitude within covariant baryon chiral perturbation theory up to $\mathcal{O}(p^4)$ level

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Abstract

The $\mathcal{O}(p^4)$ calculation on pion-nucleon elastic scattering amplitude in EOMS scheme within covariant baryon chiral perturbation theory is reviewed. Numerical fits to partial wave amplitudes up to $\sqrt{s} = 1.13$GeV and 1.20GeV are performed and the results are compared with previous studies.

Keywords: $\pi-N$ scattering, chiral perturbation theory, partial wave analysis

1. Introduction

Many efforts have been made in studying $\pi-N$ scatterings at low energies. However, unlike the successfullness of chiral perturbation theory in pure mesonic sector, a chiral expansion in $\pi-N$ scattering amplitude suffers from the power counting breaking (PCB) problem in the traditional subtraction $\overline{\text{MS}} - 1$ scheme. Many proposals have been made to treat this problem, e.g., heavy baryon chiral perturbation theory $[3]$, infrared regularization scheme $[5]$, extended on mass shell (EOMS) scheme $[4]$, etc.. The EOMS scheme provides a good solution to the PCB problem, e.g., see $[5]$, in the sense that it faithfully respects the analytic structure of the original amplitudes and being scale independent.

In this talk we will present our work on the $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ calculation on $\pi-N$ scattering amplitude in EOMS scheme and will compare it with previous results in the literature.

2. NNLO and NNNLO calculations

We start from the following effective lagrangian at $\mathcal{O}(p^3)$ level (extendable to $\mathcal{O}(p^4)$ $[4]$):

$$\mathcal{L}_{\text{eff}} = \overline{N} \left[ i\not{\partial} - m + \frac{g_A}{2} \not{\gamma}_5 + c_i O_i^{(2)} + d_i O_i^{(3)} \right] N + \frac{f^2}{4} (\not{d}' u_\alpha + X_\alpha) + \frac{f_4}{8} (\not{d}' u_\alpha)(\not{X}_4) + \frac{\ell_s}{16} (\not{X}_s),$$

where $O_i^{(2)}$ and $O_i^{(3)}$ are relevant operators of $\mathcal{O}(p^2)$ and $\mathcal{O}(p^3)$ respectively, $i \in (1, 2, 3, 4)$ and $j \in (0, 1, 2, 3, 5, 14, 15, 16, 18)$ $[6]$.

Decomposition of $\pi-N$ amplitude is standard,

$$T^{\mu N}_{SS} = \delta_{\mu N} T^+ + \frac{1}{2} [\tau_{\mu N}, T^-].$$

$$T^+ = \overline{u}(p, \bar{s}) \left[ D^+ + i \frac{\epsilon}{2m_N} \sigma^\mu q_{\bar{s}} B^+ \right] u(p, s).$$

To carry out the calculation in EOMS scheme one firstly perform $\overline{\text{MS}} - 1$ subtraction to remove ultraviolet divergencies, then additional subtraction (A.S.) to absorb PCB terms. Taking the nucleon mass renormalization for example, one has,

$$m_N = m - 4c_1 M^2 - \frac{3m^2}{2f^2} \left[ \Delta_N - M^2 I(m^2) \right]$$

$$= \tilde{m} - 4c_1 M^2 + \frac{3M^2 g^2}{2f^2} (I(m^2)) \quad (\overline{\text{MS}} - 1)$$

$$= \tilde{m} - 4c_1 M^2 + \frac{3M^2 g^2}{2f^2} (m^2) - \frac{3mM^2 g^2}{2f^2} (A.S.)$$

where $\tilde{m}$ is the nucleon mass in chiral limit. The last term on the r.h.s. of the third equality is opposite to the PCB term which is absorbed by redefining $c_i$ as: $c_i = c_i - \frac{3M^2 g^2}{2f^2}$.

Definitions of all functions appeared here follow from $\text{Appendix A}$.

Another example is the calculation of the axial-vector coupling $g_A$:

$$g_A = g + 4d_i M^2 - \frac{g^3 m^2}{32f^2} \Delta_N + \frac{g(4 - g^2)}{2f^2} \Delta_A - \frac{g^2 + g^2}{2f^2} \Delta_A + \frac{g^3}{4f^2} J_0(0) - \frac{g(8 - g^2)M^2}{4f^2} I(m^2) - \frac{g^2 M^4}{4f^2} I_N(0)$$

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where \( g_A \) is the axial charge in the chiral limit. Ultraviolet divergencies are treated by \( MS - 1 \) subtraction. If we start with \( g_A \), there are no PCB terms to be extracted. The PCB effects are included in \( g_A \). If we start with a bare \( g \), we need to redefine it as, 
\[
\bar{g} = g' = \bar{g}^e = g + \frac{g^e}{\pi^2} R. 
\]
We prefer the latter hereafter, i.e. starting with bare parameters.

Similar to \( m_N \) and \( g_A \) renormalization, the calculation of scattering amplitude up to \( O(p^4) \) in EOMS scheme is straightforward, if the PCB terms in functions \( D \) and \( B \) for loop amplitudes are known,

\[
D_{PCB}^s = \frac{1}{64f^2m^2\sigma^2} \left\{ 6g^2m^2\sigma^2 + 2\sigma^4 \right\} + \frac{g^4}{f^2} \left\{ 10m^4 + 7M^2 + t + \bar{t} \right\} + 3m^2 \left( 3t - 7M^2 \right) \sigma^2 + \sigma^4 \right\}
\]

\[
D_{PCB}^c = \frac{g^4m}{64f^2\pi\sigma^2} \left\{ \sigma^2 \left( t - 2M^2 + 2\sigma \right) - 2m^2 \left( 2M^2 - t \right) \left( 2M^2 - t + 2\sigma \right) \right\}
\]

\[
B_{PCB} = \frac{g^4m^4}{8f^2\pi^2\sigma^2} (2M^2 - t + 2\sigma) \right\}
\]

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D_{PCB}^c = \frac{g^4m}{64f^2\pi\sigma^2} \left\{ \sigma^2 \left( t - 2M^2 + 2\sigma \right) - 2m^2 \left( 2M^2 - t \right) \left( 2M^2 - t + 2\sigma \right) \right\}
\]

\[
B_{PCB} = \frac{g^4m^4}{8f^2\pi^2\sigma^2} (2M^2 - t + 2\sigma) \right\}
\]

where \( \sigma = s - m^2 \). After mass and \( g_A \) renormalization, the PCB terms above can be absorbed by redefining \( c_i's \):

\[
c'_i = c_i = c_i - \frac{3g^4m}{128f^2\pi^2} \right\}
\]

\[
c'_i = c_i = c_i + \frac{g^4m}{32f^2\pi^2} \right\}
\]

\[
c'_i = c_i = c_i - \frac{9g^4m}{4f^2\pi^2} \right\}
\]

\[
c'_i = c_i = c_i + \frac{g^4}{64f^2\pi^2} (5 + g^2) m \right\}
\]

and the \( \bar{c}_i's \) are determined by fitting data. Theoretically, the NNLO amplitudes keep good analytic, correct power counting and scale-independent properties.

In the following we further extend the above calculation to \( O(p^4) \) level:

\[
m_N = m + \cdots - 2(8e_3m + e_115 + e_116)M^4 + \frac{3M^2\Delta_A}{f^2} \left( 2c_1 - c_4 \right) - \frac{c_6}{d},
\]

\[
g_A = g + \cdots - \frac{2g}{m^2} \left( c_3 \left( 4M^2 + m^2\Delta_A - M^2f^2\right)(m^2) \right)
\]
we plot the fit up to are fixed at their our fits. To let the fitted LECs same as [13], also listed our calculations give a reasonable description to data and the numerical studies given in Ref. [7–9]. Data being fitted are up to \( O(p^3) \) results, replacement of \( m \) in nucleon propagator with \( m_2 = m - 4c_1M^2 \), namely making Dyson resummation to renormalize \( m \) to \( m_2 \) first, will simplify calculations greatly [3]. The \( O(p^3) \) part in Eq. (6) doesn’t contribute PCB terms, while the one in Eq. (7) does and \( g' \) is now redefined as \( g' = g^{\prime \prime} - \frac{g^{\prime \prime \prime}m^2}{16\pi^2} \). The \( O(p^3) \) term in Eq. (7) doesn’t contribute PCB terms as well as the full amplitude are also shown explicitly on the \( O(p^3) \) level we have performed two fits, the first one denoted by, e.g., \( \Delta x = I_{10} \) and \( \Delta y = I_{10} \), characterized by the \( N\Delta \) axial coupling \( h_A \), Fit results are summarized in Table 1 where we have also listed the results from Refs. [7] and [8] for comparison. We see that, in general, our fit results at \( O(p^3) \) level are in good agreement with that of Refs. [7, 9], except the \( d_5 \) parameter. We also listed our \( O(p^3) \) results from the best solutions in our fits. To let the fitted LECs same as [13], \( d_1 \) and \( h_b \) are fixed at their \( O(p^3) \) fitting results. In Figures 1 and 2 we plot the fit up to \( \sqrt{s} = 1.13 \text{GeV} \) and \( 1.20 \text{GeV} \), respectively. We find that, both \( O(p^4) \) and \( O(p^5) \) calculations give a reasonable description to data and the \( O(p^5) \) calculation improves the fit quality.

\[
-4m^2 \left[(c_3 + c_4) t^2(m^2) + h_4 \left(M^2 - F^2(m^2)\right)\right].
\]

\[
(7)
\]

Only \( O(p^4) \) parts are shown explicitly on the r.h.s. of Eqs. (6), (7), and ellipses represent lower order contributions given by Eqs. (3), (4). It is worth noticing that when obtaining the \( O(p^5) \) results, replacement of \( m \) in nucleon propagator with \( m_2 = m - 4c_1M^2 \), namely making Dyson resummation to renormalize \( m \) to \( m_2 \) first, will simplify calculations greatly [3]. The \( O(p^5) \) part in Eq. (6) doesn’t contribute PCB terms, while the one in Eq. (7) does and \( g' \) is now redefined as \( g' = g^{\prime \prime} - \frac{g^{\prime \prime \prime}m^2}{16\pi^2} \). The \( O(p^3) \) term in Eq. (7) doesn’t contribute PCB terms as well as the full amplitude are also shown explicitly on the \( O(p^3) \) level we have performed two fits, the first one denoted by, e.g., \( \Delta x = I_{10} \) and \( \Delta y = I_{10} \), characterized by the \( N\Delta \) axial coupling \( h_A \), Fit results are summarized in Table 1 where we have also listed the results from Refs. [7] and [8] for comparison. We see that, in general, our fit results at \( O(p^3) \) level are in good agreement with that of Refs. [7, 9], except the \( d_5 \) parameter. We also listed our \( O(p^3) \) results from the best solutions in our fits. To let the fitted LECs same as [13], \( d_1 \) and \( h_b \) are fixed at their \( O(p^3) \) fitting results. In Figures 1 and 2 we plot the fit up to \( \sqrt{s} = 1.13 \text{GeV} \) and \( 1.20 \text{GeV} \), respectively. We find that, both \( O(p^4) \) and \( O(p^5) \) calculations give a reasonable description to data and the \( O(p^5) \) calculation improves the fit quality.

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Appendix A. Definition of loop integrals

- 1 meson: \( \Delta x = I_{10} \)
  \[
  \Delta x = \frac{1}{\pi} \int \frac{d^4k}{(2\pi)^4} \frac{1}{M^2 - k^2} \frac{1}{k^4 - k^0 k^0}.
  \]
  \[
  \gamma^4 \left[ 1, k', k'' k'' \right] = \frac{1}{\pi^2} \frac{1}{M^2 - k^2} \frac{1}{M^2 - (\Sigma - k)^2}.
  \]
- 1 nucleon: \( \Delta y = I_{10} \)
  \[
  \Delta y = \frac{1}{\pi} \int \frac{d^4k}{(2\pi)^4} \frac{1}{m^2 - (k - P)^2} \frac{1}{m^2 - (k - P')^2}.
  \]
- 2 nucleons: \( J_N = I_{10} \)
  \[
  J_N = \frac{1}{\pi} \int \frac{d^4k}{(2\pi)^4} \frac{1}{m^2 - (k - P)^2} \frac{1}{m^2 - (k - P')^2}.
  \]
- 1 mesons, 2 nucleons: \( I_A = I_{10} \)
  \[
  I_A = \frac{1}{\pi} \int \frac{d^4k}{(2\pi)^4} \frac{1}{m^2 - (k - P)^2} \frac{1}{m^2 - (P - P')^2}.
  \]

After removing part proportional to \( R = \frac{1}{\pi} + \gamma_E = 1 - \ln 4\pi \), the remaining scalar integrals are finite and denoted by, e.g., \( \bar{I}(s), J_N(t), I_A(t) \), etc..

References

[1] J. Gasser, M. E. Sainio and A. Svarc, Nucl. Phys. B 307, 779 (1988).
[2] E. E. Jenkins and A. V. Manohar, Phys. Lett. B 255, 558 (1991).
[3] T. Becher and H. Leutwyler, Eur. Phys. J. C 9, 643 (1999).
[4] J. P. Fuchs, J. Gegea, G. Japaridze and S. Scherer, Phys. Rev. D 68, 056005 (2003).
[5] L. S. Geng, J. Martin Camalich, L. Alvarez-Ruso and M. J. Vicente Vacas, Phys. Rev. Lett. 101, 220202 (2008).
[6] N. Fettes, U.-G. Meissner, M. Mojzis and S. Steininger, Annals Phys. 283, 273 (2000).
[7] J. M. Alcaron, J. Martin Camalich and J. A. Oller, Prog. Part. Nucl. Phys. 67, 375 (2012).
[8] J. M. Alcaron, J. Martin Camalich, J. A. Oller and L. Alvarez-Ruso, Phys. Rev. C 83, 055205 (2011).
[9] J. Martin Camalich, J. M. Alcaron and J. A. Oller, Prog. Part. Nucl. Phys. 67, 327 (2012).
[10] Y. Ch. Chen, D. L. Yao, H. Q. Zheng, in preparation.
[11] R. A. Arndt, W. J. Briscoe, I. I. Strakovsky, R. L. Workman, Phys. Rev. C 74, 045205 (2006); R. A. Arndt, et al. The SAD program, http://gwdac.phys.gwu.edu.
[12] V. Pascalutsa and D. R. Phillips, Phys. Rev. C 67, 055202 (2003).
[13] N. Fettes and U.-G. Meissner, Nucl. Phys. A 676, 311 (2000).
Figure 1: (Color online) Fit up to 1.13 GeV. The fourth- and third-order fits are presented by the solid(blue) and dash(red) lines respectively.

Figure 2: (Color online) Fit up to 1.20 GeV. The fourth- and third-order fits are presented by the solid(blue) and dash(red) lines respectively.