Correlated coherent oscillations in coupled semiconductor charge qubits

Gou Shinkai, Toshiaki Hayashi, Takeshi Ota, and Toshimasa Fujisawa

1 NTT Basic Research Laboratories, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, 243-0198, Japan and
2 Department of Physics, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo, 152-8551, Japan

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We study coherent dynamics of two spatially separated electrons in a coupled semiconductor double quantum dot (DQD). Coherent oscillations in one DQD are strongly influenced by electronic states of the other DQD, or the two electrons simultaneously tunnel in a correlated manner. The observed coherent oscillations are interpreted as various two-qubit operations. The results encourage searching quantum entanglement in electronic devices.

Quantum coherence of a single particle and a few particles (qubits) has been investigated in various systems. Two-qubit unitary operations are key ingredients for performing quantum algorithms and correlating multiple qubits. Typical operations, such as controlled-rotation (CROT), which rotates the target qubit state conditionally on the control qubit state, and SWAP, which swaps quantum states of the two qubits, have been demonstrated. However usually one type of operations is realized depending on the type of coupling (Ising, Heisenberg, etc.) [2, 3, 4, 5]. Although other operations can in principle be designed in combination with some one-qubit gates, etc. [2, 3, 4, 5]. W e find that, even under the same coupling, two spatially separated electrons in the dots (DQDs), coherent oscillations of an electron in the first qubit can be controlled by the second qubit’s state and SWAP, and tunable charge qubits fabricated in a semiconductor nanostructure are suitable for the purpose, since various parameters can be controlled by external gate voltages. In the case of Ising (electrostatic dipole) coupling between charge qubits fabricated in two double quantum dots (DQDs), coherent oscillations of an electron in the first qubit can be controlled by the second qubit’s state (CROT operations) [9, 10]. We find that, even under the same coupling, two spatially separated electrons in the two DQDs change their locations coherently and collectively (the correlated coherent oscillations). These coherent dynamics can be used to design CROT, SWAP and other quantum operations, each in a single step.

We consider a system of two charge qubits, each of which possesses an excess electron in an orbital of the left or the right dot, $|L_i\rangle$ or $|R_i\rangle$, of $i$-th semiconductor DQD ($i = 1, 2$) [11, 12, 13]. The electrostatic coupling between the two qubits in the parallel geometry in Fig. 1(a) stabilizes anti-parallel configurations, $|LR\rangle$ ($\equiv |L_1\rangle|R_2\rangle$) and $|RL\rangle$, rather than parallel ones, $|LL\rangle$ and $|RR\rangle$. The coupling is expressed as an Ising-type Hamiltonian $J/4 \sigma_x^{(i)} \otimes \sigma_x^{(2)}$, where $\sigma_x^{(i)}$ are the Pauli matrices of the $i$-th qubit and $J > 0$ represents the strength of the coupling. The two-qubit system can be described by the Hamiltonian

$$H_{2q} = \frac{1}{2} \sum_i \left( \varepsilon_i \sigma_x^{(i)}(\downarrow) + \Delta_i \sigma_x^{(i)}(\uparrow) \right) + \frac{J}{4} \sigma_x^{(1)} \otimes \sigma_x^{(2)},$$

where the first term describes the energy offset $\varepsilon_i$ of the single-qubit states and the tunneling coupling energy $\Delta_i$ of the $i$-th qubit. A similar Hamiltonian can be found in various physical systems, such as superconducting qubits [6] and ultra-cold atoms in an optical lattice [14, 15]. Therefore, the following arguments can also be applied to those systems.

Figure 1(b) illustrates the charge configuration of the ground state in the $\varepsilon_1 - \varepsilon_2$ plane in the small but finite tunneling ($\Delta_1 = \Delta_2 \ll J$). The boundaries separating different charge states represent the resonant conditions, where some important quantum operations are expected. CROT operations are based on the coherent transitions of a target qubit controlled by a control qubit [4]. For example, a transition between $|LR\rangle$ and $|RR\rangle$ is understood as rotating the state of the first qubit when the second qubit state is $|R_2\rangle$. This transition is expected at $\varepsilon_1 = J/2$ [the right vertical line in Fig. 1(b)] and schematically shown in the energy diagram of the upper-left inset. Here, $|LL\rangle$ and $|RL\rangle$ should be out of resonance (separated by $J$) for the CROT operation. Another CROT operation for the transition between $|LL\rangle$ and $|RL\rangle$ appears at $\varepsilon_1 = -J/2$ [the left vertical line in Fig. 1(b) and corresponding energy diagram in the lower-left inset]. Similarly, CROT operations when the second qubit is the target are expected at $\varepsilon_2 = \pm J/2$ (the horizontal lines).

In contrast, the transition between $|LR\rangle$ and $|RL\rangle$ can not be expected as a first-order process, and requires simultaneous tunneling of two electrons via another state $|RR\rangle$ (or $|LL\rangle$) as illustrated by the solid (dashed) arrow in the upper-right inset of Fig. 1(b). This second-order process takes place when $\Delta_1$ and $\Delta_2$ are nonzero. When the first-order transitions are well suppressed ($|\varepsilon| \ll J/2 \gg \Delta_1, \Delta_2$), the second-order process dominates at $\varepsilon = \varepsilon_1 = \varepsilon_2$ [on the diagonal line in Fig. 1(b)]. In this condition, eigenstates of $H_{2q}$ are approximated to $\frac{1}{\sqrt{2}} (|LR\rangle \pm |RL\rangle)$, $|RR\rangle$ and $|LL\rangle$,
Ω = This determines the transition frequency between |T. The energy offsets, dots in DQD1 and DQD2. All measurements were performed and gold for metal gates). The circles represent quantum (blue and black respectively for unetched and etched surface, FIG. 1: (Color) (a) Colored SEM image of the control device |mate 1 |∆ 1 |mate 2, were independently controlled by changing some gate voltages simultaneously to compensate for electrostatic crosstalk. Tunneling rates are Γ_{L1} ∼ Γ_{R1} ∼ Γ_{L2} ∼ Γ_{R2} ∼ 1 GHz for the left (L) and right (R) barrier of the first (1) and second (2) DQD. (b) and (c) Charge diagram of the ground state at Δ_1 = Δ_2 = 0 in (b) and of the first excited state at Δ_1 = 13 μeV, Δ_2 = 25 μeV and J = 25 μeV in (c). Colors represent the charge state, LL (cyan), LR (magenta), RL (yellow) and RR (white). The resonant conditions are indicated by solid and dashed lines. Energy diagrams at some points (> < ⊕ ⊖) for CROTs, SWAP, and FLIP operations are shown in the insets. (d) Electrochemical potential of DQD1 and DQD2 in the steady state for initialization for large bias (Ini1 and Ini2), in the coherent evolution at zero bias (Evol1 and Evol2), and in the measurement at large bias (Meas1 and Meas2). A schematic of the voltage pulse is shown at the top. and the energy gap between \( \Omega = \frac{1}{\hbar} \Delta^2 J/|\varepsilon^2 - J^2/4| \) for equal coupling \( \Delta = \Delta_1 = \Delta_2 \). This determines the transition frequency between |LR⟩ and |RL⟩, and the approximate SWAP operation is expected in a single step of the half period. Approximate \( \sqrt{\text{SWAP}} \) operation of the quarter period is useful in preparation for correlated states \( \frac{1}{\sqrt{2}} (|LR⟩ ± i|RL⟩). \\

Similarly, another second-order transition between |LL⟩ and |RR⟩ [See energy diagram in the lower-right inset of Fig. 1(b)] is allowed at \( \varepsilon = \varepsilon_1 = -\varepsilon_2 \) [on the diagonal line ⊕ in Fig. 1(b)], which can be used to flip the total charge polarization of the two-qubit system in a single step (We call this process FLIP). The approximate \( \sqrt{\text{FLIP}} \) operation is useful in preparation of correlated states \( \frac{1}{\sqrt{2}} (|LL⟩ ± i|RR⟩). \) In this way, multiple two-qubit operations (CROTs, SWAP and FLIP) can be performed each in a single step.

We implemented a two-qubit system by integrating two sets of DQDs in a GaAs/AlGaAs heterostructure, as shown in the scanning electron micrograph (SEM) in Fig. 1(a) [16]. The two DQDs with individual source and drain electrodes are electrically isolated, and thus independent currents, \( I_1 \) and \( I_2 \), can be measured simultaneously. All qubit parameters can be controlled by 11 gate voltages. We have already confirmed the electrostatic coupling between the DQDs from the resonant tunneling characteristics, in which the resonant tunneling of the first DQD is switched by the charge state of the second DQD [16].

The pulse sequence consists of the following three steps, similar to our previous one-qubit experiment [12]. First, the system is initialized by setting DQD1 in the dissipative single-electron tunneling regime at \( V_{D1} = 700 \) μV, as illustrated in Ini1 of Fig. 1(d). An electron is prepared in the left dot with a high probability (long dwell time in the left dot) [17]. This initialization works at any energy offset \( \varepsilon'_1 \) (Hereafter, the prime is used for the biased situation). On the other hand, DQD2 is kept in the Coulomb blockade region with a small bias \( V_{D2} ∼ 10 \) μV (See Ini2), and thus the second qubit is relaxed in the steady state. Therefore, the system is initialized in |LL⟩ at \( \varepsilon'_2 ∼ -J/2 \) or |LR⟩ at \( \varepsilon'_2 > -J/2 \). Then, the system is suddenly brought into the Coulomb blockade condition by applying a square (negative) voltage pulse \( (V_{D1} = 0) \) with a rise time of about 0.1 ns for a period of \( t_p = 0.08 - 2 \) ns, where coherent time evolution is expected (See Evol1 and Evol2 illustrated for the SWAP action). The readout of the final state is performed by restoring the large bias \( (V_{D1} = 700 \) μV), where the electron in the right dot escapes to the drain and contributes to the current (Meas1). The above sequence is repeated at 100 MHz to obtain measurable current. We obtained the net electron numbers flowing per pulse, \( N_{P1} \) and \( N_{P2} \), for each DQD by using lock-in amplifiers [12]. Inelastic tunneling during the initialization period \( (10 \) ns \( - t_p) \) gives a background artifact proportional to \( t_p [-0.12 ∼ -0.16 \times t_p (\text{ns})] \), which was partially subtracted to highlight the coherent oscillations.

We first demonstrate the CROT operation of DQD1 using DQD2 as a control qubit. Figure 2(a) shows the coherent oscillations of DQD1 starting from the initial state \( |LR⟩ \) prepared at \( \varepsilon'_2 = +35 \) μeV. The overall behavior of the oscillations, including the dependence of the period, amplitude, and decoherence time on the detuning from the resonance \( (\varepsilon_1 = J/2) \), is consistent with our previous study on a single qubit [12] and with the two-qubit simulation, from which we obtained \( \Delta_1 = 13 \) μeV. When the initial state is prepared to be |LL⟩ at \( \varepsilon'_2 = -60 \) μeV, similar oscillations are observed as shown in Fig. 2(b) but with a resonance appearing at a significantly different value of \( \varepsilon_1 \). Crossover between the two
oscillations is depicted in Fig. 2(c), where \( N_{p1} \) measured at \( t_p = 0.25 \) ns (corresponding to \( \pi \)-pulse on resonance) is plotted in the \( \varepsilon_1 - \varepsilon_2 \) plane. The resonant conditions discussed in Fig. 1(b) are superimposed in Fig. 2(c) for clarity. The vertical patterns of the interference fringes are horizontally shifted discontinuously when the initial state is altered from \( |LR\rangle \) in the upper region to \( |LL\rangle \) in the lower region. The horizontal shift is a measure of the oscillations is depicted in Fig. 2(c), where \( N \) are horizontally shifted discontinuously when the initial state is superimposed in Fig. 2(c) for electrostatic coupling in the lower region. The horizontal shift is a measure of the electrostatic coupling \( J = 25 \) \( \mu \)eV. The large \( J \) as compared to \( \Delta_1 \) (13 \( \mu \)eV) ensures reasonable controllability of CROT.

The above CROT experiments were performed at small tunneling coupling of DQD2 (\( \Delta_2 \sim 3 \) \( \mu \)eV estimated from an independent measurement). As \( \Delta_2 \) is increased while keeping \( \Delta_1 \) constant, the second-order transitions appear as additional features (labeled by \( \square \)) elongated in the upper-right direction in Fig. 2(d) for \( \Delta_2 \sim 15 \) \( \mu \)eV and in Fig. 2(e) for \( \Delta_2 \sim 25 \) \( \mu \)eV (determined from the following analysis). The time evolution of \( N_{p1} \) is also investigated at various \( \varepsilon_1 \) and \( \varepsilon_2 \). Figure 2(f) shows the \( \varepsilon_1 \) dependence of \( N_{p1} \) (\( t_p \)) at \( \varepsilon_2 = 10 \) \( \mu \)eV [the dot-dashed line in Fig. 2(e)]. Two types of oscillations are resolved: one at \( \varepsilon_1 \sim 0 \) with the frequency comparable to that of the first-order process and the other at \( \varepsilon_1 \sim 25 \) \( \mu \)eV with a lower frequency of 1.3 GHz. Figure 2(h) shows typical \( N_{p1} \) plots of the two cases. We identify the slower oscillations to be correlated dynamics of the two qubits from the following analysis.

The resonant conditions (\( \varepsilon_1, \varepsilon_2 \)) were experimentally determined at the maximum \( N_{p1} \) for sufficiently long pulse (\( t_p \sim 1 \) ns) [for example, the dot-dashed line in Fig. 2(f)], and are plotted by crosses (\( \times \)) in Fig. 2(e). These conditions can be reproduced just by considering the eigenstates of \( H_{2q} \). The resonant conditions were numerically derived for the minimum energy gap between the eigenstates, and are shown by solid and dashed gray lines in Fig 2(e) with a fitted parameter \( \Delta_2 = 25 \) \( \mu \)eV. One of these line fits well with the experimental resonant conditions. The same resonance lines together with the charge state (shown by colors) of the first excited state are shown in Fig. 1(c) with the same parameters. As recognized by the neighboring charge states, the condition of interest (the thick line \( \alpha - \alpha' \)) represents the resonance of \( |LR\rangle \) and \( |RL\rangle \) (SWAP) for \( \varepsilon_2 \gg J/2 \), that of \( |LL\rangle \) and \( |RR\rangle \) (FLIP) for \( \varepsilon_2 \ll -J/2 \) and that of superpositions of four bases for \( \varepsilon_2 \sim 0 \). Namely, the straight resonant conditions in Fig. 1(b) for \( \Delta_1 = \Delta_2 \) are curved as in Fig. 1(c) for unequal coupling (\( \Delta_1 < \Delta_2 \)). The observed coherent dynamics at \( \varepsilon_2 \sim 0 \) involves complicated superposition of four two-qubit bases, and thus may be called correlated coherent oscillations rather than simpler SWAP or FLIP operations.

We also performed density-matrix simulations using the standard Lindblad master equation [17, 18]. In addition to the coherent processes described by \( H_{2q} \), incoherent tunneling transitions to/from the source and drain electrodes and spontaneous phonon emission in each DQD were included with realistic parameters. To reproduce the experimental pulse sequence, the time-dependent reduced density matrix \( \rho(t) \) in the Coulomb blockade region (at \( V_{D1} = 0 \)) was numerically calculated from the initial state \( \rho_0 \) prepared in the transport region (at finite \( V_{D2} \)). Expected tunneling electrons (measurement outcome) in the subsequent transport region after the pulse length \( t_p \) were evaluated. The simulated \( N_{p1} \) is plotted in Fig. 2(g) by using the same parameter in Fig. 2(f). Although the correlated oscillations are significantly degraded by other decoherence mechanisms (charge noise, finite rise-time of the pulse, etc.), the overall oscillation characteristics (fast and slow oscillations, \( \varepsilon_1 \)-dependent period around the resonance) are well reproduced in the simulations.

The correlated tunneling can be evidenced by measuring the signal in the DQD2. However, no readout signal in \( N_{p2} \) for DQD2 was obtained in the above measurements, since they were performed in the Coulomb...
blockade condition of DQD2 [See diagrams in Fig. 1(d)]. We confirmed a small but finite negative signal in \( N_{p2} \) (but positive in \( N_{p1} \)) when DQD2 was made closer to the conductive region by adjusting the electrochemical potential \( \delta_2 \) close to zero (data not shown). This indicates a fraction of SWAP action (transition from \(|LR\rangle\) to \(|RL\rangle\) produces negative current in DQD2). Since the second-order coupling decreases with increasing \(|\varepsilon_1|\), we could confirm coherent oscillations in the limited range of \(|\varepsilon_1| \lesssim 25 \mu eV\). Simpler SWAP and FLIP operations are expected at larger \(|\varepsilon_1|\).

We examined such conditions from dc measurement. Figure 3 shows (a) \( I_1 \) at \( V_{D1} = 700 \mu V \) and (b) \( I_2 \) at \( V_{D2} = 10 \mu V \). First-order tunneling current in DQD1 and DQD2 is seen as broad vertical lines [outside the plot range of (a)] and a horizontal line at around \( \varepsilon_2' = \pm J/2 \) in (b). The second-order cotunneling is simultaneously recorded in both currents and appears as very sharp current peaks \( \alpha, \beta, \) and \( \beta' \) running in the diagonal directions. Peak \( \alpha \) is associated with the two-qubit system of interest, and \( \beta - \beta' \) involves another orbital state (an excited state) neglected in the model. The negative current correlation (positive \( I_1 \) but negative \( I_2 \)) appearing in the upper region \( (\varepsilon_2' > 0) \) is understood as the cotunneling transition from the initial state \(|LR\rangle\) to the final state \(|RL\rangle\) at \( \varepsilon_1' \sim \varepsilon_2' > J/2 \) [See arrows in the upper-right inset of Fig. 1(b)]. On the other hand, the positive correlation (positive \( I_1 \) and \( I_2 \)) appearing in the lower region \( (\varepsilon_2' < 0) \) of Fig. 3 arises from the transition from \(|LL\rangle\) to \(|RR\rangle\) at \( \varepsilon_1' \sim -\varepsilon_2' > J/2 \) [See the lower-right inset of Fig. 1(b)]. The overall second-order tunneling peak constitutes the resonant condition \( \alpha - \alpha' \) in Fig. 1(c). Although coherent oscillations were confirmed only at \( \varepsilon_2 \sim 0 \), the appearance of cotunneling peak in the wide region in Fig. 3 suggests that coherent SWAP and FLIP operations are feasible.

Finally we’d like to make all data consistent. Data in Fig. 3 were taken at the same condition as in Fig. 2(b) but in the absence of a pulse \( (t_p = 0) \). Negative signals, where coherent dynamics vanishes, along the diagonal lines marked by \( \alpha, \beta, \) and \( \beta' \) in Fig. 2(e) coincide with the cotunneling currents in Fig. 3, and are attributed to the second-order cotunneling occurring in the initialization period. This spoils the initialization and measurement scheme and thus the coherent dynamics disappears. Such experimental details were all reproduced in the density-matrix simulation (not shown).

In summary, we have investigated first- and second-order tunneling processes by measuring pulse-induced current or current correlations. We expect that multiple two-qubit operations (CROT, SWAP and FLIP) can be induced just by changing parameters \( \varepsilon_1 \) and \( \varepsilon_2 \). Interestingly, the two electrons in our device have no spatial overlap of wavefunctions, and the expected quantum processes therefore allow us to correlate two electrons (a few hundred nanometers apart) without their touching each other. This will be an important test of quantum non-locality in mesoscopic electron devices and encourages the study of charge-based quantum information even under a constant coupling [18, 19, 20].

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\[\text{FIG. 3: Correlated cotunneling current in simultaneous measurement of } I_1 \text{ in (a) and } I_2 \text{ in (b) in the } \varepsilon_1' - \varepsilon_2' \text{ plane measured at } V_{D1} = 700 \mu V \text{ and } V_{D2} = 10 \mu V. \text{ Solid and dotted lines are current traces at } \varepsilon_2' = 50 \mu V \text{ and } -10 \mu V, \text{ respectively. Second-order tunneling peaks } \alpha, \beta, \text{ and } \beta' \text{ appear at the same marks in Fig. 2(e).}\]

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