An exact solution for a partially clamped rectangle with a crack

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Abstract. The article deals with a boundary value problem for a rectangle whose horizontal sides are rigidly clamped, and the ends are free. In the centre of the rectangle, a vertical cut is made on which a discontinuity of the longitudinal displacements is given. An exact solution to the problem is constructed in the form of series in Papkovich–Fadle eigenfunctions. First, the corresponding boundary value problem for an infinite clamped strip is solved, then the solution for a rectangle is superimposed on this solution, with the help of which the boundary conditions at its ends are satisfied. Examples are given in which discontinuities of three types are considered which differ in the smoothness of the discontinuity contour near its ends.

1. Introduction

Many publications (for example, [1–9]) are devoted to problems for rectangular plates weakened by cracks with different boundary conditions both on the crack and the sides of the rectangle. Almost all the solutions were approximate. The exceptions are solutions to some problems in trigonometric Fourier series. In the general case, one has to deal with complex-valued Papkovich–Fadle eigenfunctions, the form of which is defined by homogeneous boundary conditions along two opposite sides of the rectangle (free sides, rigidly clamped, having stiffeners, etc.).

The solution constructed in this paper is based on the theory of expansions in Papkovich–Fadle eigenfunctions [10–13]. In this case, the boundary value problem is formulated as the problem of the junction of two half-strips located to the right and left of the line containing the cut. For this, two functions are introduced that are analytical to the right and to the left of the cut. The introduction of these functions is crucial for a successful solution to the problem for an infinite strip with a transverse crack [14]. Then, a rectangle is cut out of the strip by making two cross sections symmetrically located relative to the cut. The boundary conditions at its ends are satisfied by using the formulas given in the article [15].

2. Solution in a strip

Consider the infinite strip \( \{ \Pi : |x| < \infty, |y| \leq 1 \} \) with clamped sides,

\[
\begin{align*}
\varphi(x, \pm 1) &= \psi(x, \pm 1) = 0, \\
\end{align*}
\]

inside which there is the vertical cut \( \{ y : x = 0, |y| \leq \alpha, 0 < \alpha < 1 \} \) (even-symmetric deformation).
We will assume that a discontinuity of the longitudinal displacements is given on the cut $\gamma$:

$$U^+(0, y) - U^-(0, y) = 2u(y), \quad (2)$$

where $U^+(0, y)$ denote the displacements equal to $u(y)$ to the right and left of the cut, respectively.

The solution to the problem is sought in the form of the series

$$U(x, y) = \sum_{k=1}^{\infty} a_k \xi(\lambda_k, y)e^{\lambda_k x} + \bar{a}_k \tilde{\xi}(\bar{\lambda}_k, y)e^{\bar{\lambda}_k x},$$

$$V(x, y) = \sum_{k=1}^{\infty} a_k \chi(\lambda_k, y)e^{\lambda_k x} + \bar{a}_k \tilde{\chi}(\bar{\lambda}_k, y)e^{\bar{\lambda}_k x},$$

$$\sigma_z(x, y) = \sum_{k=1}^{\infty} a_k s_z(\lambda_k, y)e^{\lambda_k x} + \bar{a}_k \bar{s}_z(\bar{\lambda}_k, y)e^{\bar{\lambda}_k x},$$

$$\sigma_y(x, y) = \sum_{k=1}^{\infty} a_k s_y(\lambda_k, y)e^{\lambda_k x} + \bar{a}_k \bar{s}_y(\bar{\lambda}_k, y)e^{\bar{\lambda}_k x},$$

$$\tau_{xy}(x, y) = \sum_{k=1}^{\infty} a_k t_{xy}(\lambda_k, y)e^{\lambda_k x} + \bar{a}_k \bar{t}_{xy}(\bar{\lambda}_k, y)e^{\bar{\lambda}_k x},$$

in the Papkovich–Fadde eigenfunctions

$$\xi(\lambda_k, y) = \left(\frac{3 - \nu}{4\lambda_k} \sin \lambda_k - \frac{1 + \nu}{4} \cos \lambda_k\right) \cos \lambda_k y - \frac{1 + \nu}{4} y \sin \lambda_k \sin \lambda_k y,$$

$$\chi(\lambda_k, y) = \frac{1 + \nu}{4} (\cos \lambda_k \sin \lambda_k y - \sin \lambda_k \cos \lambda_k y),$$

$$s_z(\lambda_k, y) = \left(\frac{3 + \nu}{2} \sin \lambda_k - \frac{1 + \nu}{2} \lambda_k \cos \lambda_k\right) \cos \lambda_k y - \frac{1 + \nu}{2} \lambda_k \sin \lambda_k \sin \lambda_k y,$$

$$s_y(\lambda_k, y) = \left(-\frac{1}{2} \sin \lambda_k + \frac{1 + \nu}{2} \lambda_k \cos \lambda_k\right) \cos \lambda_k y + \frac{1 + \nu}{2} \lambda_k \sin \lambda_k \sin \lambda_k y,$$

$$t_{xy}(\lambda_k, y) = \left(-\sin \lambda_k + \frac{1 + \nu}{2} \lambda_k \cos \lambda_k\right) \sin \lambda_k y - \frac{1 + \nu}{2} \lambda_k \cos \lambda_k \cos \lambda_k y.$$  \quad \text{In formulas (2)--(4) the following notation is introduced: $U(x, y), V(x, y)$ are the longitudinal and transverse displacements, respectively, multiplied by the shear modulus; $\nu$ is Poisson’s ratio; $a_k, \bar{a}_k$ are the unknown expansion coefficients; the eigenvalues $\pm \lambda_k, \pm \bar{\lambda}_k$ are the set of roots of the characteristic equation:}

$$L(\lambda) = \frac{(3 - \nu) \sin 2\lambda}{8\lambda} - \frac{1 + \nu}{4} = 0,$$

and the first root $\lambda_1$ is real.

Following [14], we consider the problem of a strip with a transverse crack as the problem of the junction of the right (to the right of the cut) and left (to the left of the cut) half-strips. For this, we introduce two functions:

$$\Phi^*(x, y) = -\tau_{xy}(x, y) + i \left[ (1 + \nu) \frac{\partial V(x, y)}{\partial y} - \frac{1 - \nu}{2} \sigma_z(x, y) \right],$$

$$F^*(x, y) = -(1 + \nu) \frac{\partial U(x, y)}{\partial y} - \frac{1 - \nu}{2} \tau_{xy}(x, y) - i\sigma_y(x, y).$$

Substituting (3) into (6) gives (Re $\lambda_1 < 0$)

$$\Phi^*(x, y) = \sum_{k=1}^{\infty} 2 \text{Re} \left[ a_k \Phi(\lambda_k, y)e^{\lambda_k x} \right], \quad F^*(x, y) = \sum_{k=1}^{\infty} 2 \text{Re} \left[ a_k F(\lambda_k, y)e^{\lambda_k x} \right].$$
where
\[ \Phi(\lambda_k, y) = (1 + \nu)\lambda_k^2 (i \cos \lambda_k + y \sin \lambda_k) e^{i \lambda_k y}, \]
\[ F(\lambda_k, y) = (1 + \nu)\lambda_k [(\lambda_k \cos \lambda_k - \sin \lambda_k) + \lambda_k y \sin \lambda_k] e^{i \lambda_k y}. \]  
(8)

The expansion coefficients \( a_k \) are determined from the boundary conditions (2) on the cut as a solution to the problem of conjugation of the functions (7) analytic in the right and left half-strips. In this case, two complete minimal systems of basis functions will be involved to the right and left of the cut. Their union will not be minimal and, therefore, will not have a biorthogonal system. The purpose of introducing the functions (6) is to separate out from the non-minimal system of basis functions the minimal one to which it is possible to construct a biorthogonal system of functions and then to find, with its help, the coefficients \( a_k \).

Denote
\[ \omega_1 = \lambda_1, \omega_2 = -\lambda_1, \omega_3 = \lambda_2, \omega_4 = -\lambda_2, \ldots, a^+_k = A_k, a^-_k = -A_k. \]
(9)
Here \( a^+_k, a^-_k \) are the unknown expansion coefficients for the right and left half-strips, respectively.

Then, from the conjugation conditions at the junction of the half-strips, we obtain the two functional equations
\[ \sum_{k=1}^{\infty} 2 \Re[A_k \Phi(\omega_k, y)] = 0, \quad \sum_{k=1}^{\infty} 2 \Re[A_k F(\omega_k, y)] = 2 \frac{du(y)}{dy}. \]  
(10)

We construct the functions \( \Phi_k(y) \) and \( F_k(y) \) biorthogonal to the functions \( \Phi(\lambda_k, y) \) and \( F(\lambda_k, y) \), respectively, as solutions to the equations (see [12])
\[ \int_{\infty}^{y} \Phi(\lambda, y) \Phi_k(y) dy = \frac{\lambda L(\lambda)}{\lambda - \lambda_k}, \quad \int_{\infty}^{y} F(\lambda, y) F_k(y) dy = \frac{L(\lambda)}{\lambda - \lambda_k}. \]  
(11)

Based on the biorthogonality relations (which are a consequence of (11)), from (10) we obtain the system of algebraic equations
\[ \begin{cases} A_k \omega_k M_k + A_k \overline{\omega_k} M_k = 0; \\ A_k M_k + A_k \overline{M_k} = f_k + \overline{f_k}, \end{cases} \]  
(12)
where
\[ f_k = \frac{u_k}{\lambda_k}, \quad u_k = \int_{-\infty}^{\infty} u(y) u_k(y) dy, \quad u_k(y) = -\frac{\lambda_k \cos \lambda_k y}{2 \sin \lambda_k}, M_k = L(\lambda_k) / 2. \]  
(13)
The numbers \( u_k \) are the Lagrange coefficients (analogues of Fourier coefficients) of the expanded function \( u(y) \), the functions \( u_k(y) \) are the finite parts of the functions biorthogonal to the Papkovich–Fadle eigenfunctions \( \xi(\lambda_k, y) \).

Solving the system of equations (12) and taking into account the notation (9), we find
\[ a^+_k = -\frac{\overline{\lambda_k} (f_k + \overline{f_k})}{(\lambda_k - \overline{\lambda_k}) M_k}. \]  
(14)

Substituting (14) into (3) and separating the zero-series (as was done, for example, in [12, 13]), we obtain the solution to the problem for the clamped strip with the discontinuity (2). In particular, in the right half-strip (\( \Re \lambda_k < 0 \)) we have
\[ U^c(x, y) = -U^c(x, y) - \sum_{k=1}^{\infty} 2 \Re \left[ \frac{u_k \xi(\lambda_k, y)}{\lambda_k M_k} \frac{\Im(\overline{\lambda_k} e^{i \lambda_k y})}{\Im \lambda_k} \right], \]
\[ V^c(x, y) = V^c(x, y) + \sum_{k=1}^{\infty} 2 \Re \left[ \frac{u_k \chi(\lambda_k, y)}{\lambda_k M_k} \frac{\Im(\overline{\lambda_k} e^{i \lambda_k y})}{\Im \lambda_k} \right], \]  
(14)
\[ \sigma'_y(x, y) = -\sigma'_x(x, y) - \sum_{k=2}^{\infty} 2 \text{Re} \left\{ u_k \frac{s_y(\lambda_k, y)}{\lambda_k M_k} \frac{\lambda_k e^{i\phi_k}}{\text{Im} \lambda_k} \right\}, \]  
\[ \sigma'_x(x, y) = -\sigma'_y(x, y) - \sum_{k=2}^{\infty} 2 \text{Re} \left\{ u_k \frac{s_x(\lambda_k, y)}{\lambda_k^2 M_k} \frac{\lambda_k e^{i\phi_k}}{\text{Im} \lambda_k} \right\}, \]  
\[ \tau_{y_0}(x, y) = -\tau_{x_0}(x, y) - \sum_{k=2}^{\infty} 2 \text{Re} \left\{ u_k \frac{t_{y_0}(\lambda_k, y)}{\lambda_k M_k} \frac{\lambda_k e^{i\phi_k}}{\text{Im} \lambda_k} \right\}, \]

where

\[ U^i(x, y) = u_i \frac{\xi(\lambda_1, y)}{\lambda_1 M_1} (\lambda_1 x - 1)e^{i\phi_1}, \]
\[ V^i(x, y) = u_i \frac{\zeta(\lambda_1, y)}{M_1} (\lambda_1 x - 1)e^{i\phi_1}, \]
\[ \sigma^i_x(x, y) = u_i \frac{s_x(\lambda_1, y)}{M_1} (\lambda_1 x - 1)e^{i\phi_1}, \]
\[ \sigma^i_y(x, y) = u_i \frac{s_y(\lambda_1, y)}{\lambda_1^2 M_1} (\lambda_1 x + 1)e^{i\phi_1}, \]
\[ \tau_{y_0}^i(x, y) = u_i \frac{t_{y_0}(\lambda_1, y)}{\lambda_1 M_1} xe^{i\phi_1}. \]  

In formulas (15), the terms (16) corresponding to the real root \( \lambda_1 \) are taken outside the summation sign. The superscript “s” denotes the solution to the strip.

3. Solution in a rectangle

Consider the rectangle \( \{ P : |x| \leq d, |y| \leq 1 \} \) in which the horizontal sides are rigidly clamped, the ends are free, and the cut \( \gamma \) is made inside on which the discontinuity of longitudinal displacements (2) is given (figure 1).

![Figure 1. Geometry of the problem](image)

To obtain the solution for the rectangle \( P \) with the discontinuity (2), it is necessary to add to the solution (15) the corresponding solution for the rectangle with normal and tangential stresses given at the ends, equal in magnitude and opposite in sign to the stresses (15) in the cross sections \( x = \pm d \). In particular, for \( x = d \) we have
\[-\sigma_\nu^i(d, y) = \frac{u_k s_\nu(\lambda_k, y)}{M_k} \frac{1}{\lambda_k} \left. e^{\lambda_k d} \right|_0^\infty \sum_{k=2}^\infty 2 \text{Re} \left\{ \frac{u_k s_\nu(\lambda_k, y)}{M_k} \frac{\lambda_k}{\lambda_k} \right\} \lambda_k \Delta \]

\[-\tau_{\nu y}^i(d, y) = \frac{t_{\nu y}(\lambda_k, y)}{M_k} \frac{1}{\lambda_k} \left. de^{\lambda_k d} \right|_0^\infty \sum_{k=2}^\infty 2 \text{Re} \left\{ \frac{u_k t_{\nu y}(\lambda_k, y)}{M_k} \frac{\lambda_k}{\lambda_k} \right\} \lambda_k \Delta \]

This solution was obtained in [15]. Let us present it, taking the terms corresponding to the real root outside the summation sign:

(a) the solution relieving the normal stresses at the ends of the rectangle \( P \):

\[U(x, y) = U^i(x, y) + \sum_{k=2}^\infty 2 \text{Re} \left( \sigma_k \frac{\lambda_k}{\lambda_k} \right) \frac{\lambda_k}{\lambda_k} \Delta \]

\[V(x, y) = -V^i(x, y) - \sum_{k=2}^\infty 2 \text{Re} \left( \sigma_k \frac{\lambda_k}{\lambda_k} \right) \frac{\lambda_k}{\lambda_k} \Delta \]

\[\sigma_x(x, y) = \sigma_x^i(x, y) + \sum_{k=2}^\infty 2 \text{Re} \left( \sigma_k \frac{s_\nu(\lambda_k, y)}{M_k} \right) \frac{\lambda_k}{\lambda_k} \Delta \]

\[\sigma_y(x, y) = \sigma_y^i(x, y) + \sum_{k=2}^\infty 2 \text{Re} \left( \sigma_k \frac{s_\nu(\lambda_k, y)}{M_k} \right) \frac{\lambda_k}{\lambda_k} \Delta \]

where

\[U^i(x, y) = \left( \lambda_k \cosh \lambda_k d + \sinh \lambda_k d \right) \sinh \lambda_k x - \lambda_k x \cosh \lambda_k x \sinh \lambda_k d \sigma_i \frac{\lambda_k}{\lambda_k} \Delta \]

\[V^i(x, y) = \left( \lambda_k \cosh \lambda_k d + \sinh \lambda_k d \right) \cosh \lambda_k x - \lambda_k x \sinh \lambda_k x \sinh \lambda_k d \sigma_i \frac{\lambda_k}{\lambda_k} \Delta \]

\[\sigma_x^i(x, y) = \left( \lambda_k \cosh \lambda_k d + \sinh \lambda_k d \right) \cosh \lambda_k x - \lambda_k x \sinh \lambda_k x \sinh \lambda_k d \sigma_i \frac{s_\nu(\lambda_k, y)}{M_k} \Delta \]

\[\sigma_y^i(x, y) = \lambda_k^2 \left[ \left( \lambda_k \cosh \lambda_k d - \sinh \lambda_k d \right) \cosh \lambda_k x - \lambda_k x \sinh \lambda_k x \sinh \lambda_k d \sigma_i \frac{s_\nu(\lambda_k, y)}{M_k} \Delta \right] \]

\[\tau_{\nu y}^i(x, y) = \lambda_k^2 \left( \cosh \lambda_k d \sinh \lambda_k x - \cosh \lambda_k x \sinh \lambda_k d \right) \sigma_i \frac{t_{\nu y}(\lambda_k, y)}{\lambda_k} \Delta \]

(b) the solution relieving the tangential stresses at the ends of the rectangle \( P \):

\[U(x, y) = -U^i(x, y) - \sum_{k=2}^\infty 2 \text{Re} \left( \tau_k \frac{\lambda_k}{\lambda_k} \right) \frac{\lambda_k}{\lambda_k} \Delta \]

\[V(x, y) = V^i(x, y) + \sum_{k=2}^\infty 2 \text{Re} \left( \tau_k \frac{\lambda_k}{\lambda_k} \right) \frac{\lambda_k}{\lambda_k} \Delta \]

\[\sigma_x(x, y) = \sigma_x^i(x, y) + \sum_{k=2}^\infty 2 \text{Re} \left( \tau_k \frac{s_\nu(\lambda_k, y)}{M_k} \right) \frac{\lambda_k}{\lambda_k} \Delta \]

\[\sigma_y(x, y) = \sigma_y^i(x, y) + \sum_{k=2}^\infty 2 \text{Re} \left( \tau_k \frac{s_\nu(\lambda_k, y)}{M_k} \right) \frac{\lambda_k}{\lambda_k} \Delta \]
\[ \tau_{\psi}(x,y) = \tau_{\psi}^1(x,y) + \sum_{k=2}^{\infty} 2 \text{Re} \left( \frac{t_k(\lambda_k, y)}{\lambda_k M_1} \right) \text{Im}(\lambda_k \cosh \lambda_d \sinh \lambda_d x) \lambda_k \sinh \lambda_d x \cosh \lambda_d d), \]

where

\[ U^1(x,y) = \left[ (\cosh \lambda_d - \lambda_d d \sinh \lambda_d) \sinh \lambda_x + \lambda_x x \cosh \lambda_d \sinh \lambda_d x \right] \lambda_k \frac{s(\lambda_k, y)}{\lambda_k M_1 \Delta}, \]

\[ V^1(x,y) = \left[ (\cosh \lambda_d - \lambda_d d \sinh \lambda_d) \cosh \lambda_x + \lambda_x x \sinh \lambda_d \cosh \lambda_d x \right] \lambda_k \frac{s(\lambda_k, y)}{\lambda_k M_1 \Delta}, \]

\[ \sigma_x^1(x,y) = (-d \sinh \lambda_d \cosh \lambda_x + x \sinh \lambda_x \cosh \lambda_d d) \frac{s(\lambda_k, y)}{\lambda_k M_1 \Delta}, \]

\[ \sigma_y^1(x,y) = \lambda_k \left[ (2 \cosh \lambda_d - \lambda_d d \sinh \lambda_d) \cosh \lambda_x + \lambda_x x \sinh \lambda_d \lambda_x x \cosh \lambda_d d \right] \lambda_k \frac{s(\lambda_k, y)}{\lambda_k M_1 \Delta}, \]

\[ \tau_{\psi}^1(x,y) = \left[ (\cosh \lambda_d - \lambda_d d \sinh \lambda_d) \sinh \lambda_x + \lambda_x x \cosh \lambda_d \cosh \lambda_d x \right] \lambda_k \frac{t_k(\lambda_k, y)}{\lambda_k M_1 \Delta}. \]

The Lagrange coefficients \( \sigma_k \) and \( \tau_k \), in accordance with (17), will be as follows:

\[ \sigma_k = u_k \frac{\text{Im}(\lambda_k e^{j\omega})}{\lambda_k}, \quad \tau_k = u_k \frac{\text{Im}(e^{j\omega})}{\lambda_k} \ (k \geq 2), \]

\[ \sigma_i = u_i \frac{s(\lambda_i, y)}{\lambda_i M_1 (\lambda_i d - 1)e^{j\omega}} \tau_i = u_i \lambda_i \frac{t_k(\lambda_i, y)}{\lambda_i M_1} de^{j\omega}. \] (22)

Adding the solutions (15), (18) and (20), we obtain the complete solution to the problem under consideration.

Let us consider three types of discontinuities in the longitudinal displacements of different degrees of smoothness in the vicinity of the crack tip:

\[ u_1(y) = \begin{cases} \sqrt{\alpha^2 - y^2}, & |y| < \alpha; \\ 0, & \alpha < |y| \leq 1, \end{cases} \]

\[ u_2(y) = \begin{cases} 2 \left( \alpha^2 - y^2 \right), & |y| < \alpha; \\ 0, & \alpha < |y| \leq 1, \end{cases} \]

\[ u_3(y) = \begin{cases} 8 \left( \alpha^2 - y^2 \right)^2, & |y| < \alpha; \\ 0, & \alpha < |y| \leq 1. \end{cases} \] (23)

Accept \( \nu = 1/3, \alpha = 0.5, d = 2 \). We find by formulas (13) and (22) the Lagrange coefficients \( u_k \), \( \sigma_k \) and \( \tau_k \) for each of the expanded functions (23), and then substitute them into (15), (18) and (20). Figures 2 and 3 show the distribution of the normal stresses near the cut depending on the shape of the discontinuity. The solid curves correspond to \( u_1(y) \), dashed curves correspond to \( u_2(y) \), dotted curves correspond to \( u_3(y) \).

**Figure 2.** Distribution of stresses \( \sigma_x \) in the cross section \( x = 0.01 \) for each type of discontinuity

**Figure 3.** Distribution of stresses \( \sigma_y \) in the cross section \( x = 0.01 \) for each type of discontinuity
4. Conclusions

1. In the paper, we have obtained an exact solution to the boundary value problem of the theory of elasticity for a rectangle with a vertical cut on which a discontinuity of the longitudinal displacements is given. The horizontal sides of the rectangle are rigidly clamped, and the ends are free. Three types of discontinuities of different smoothness in the vicinity of the crack tip are considered, and a numerical comparison of the solutions is given. It can be shown that in the first case the stresses increase as they approach the crack tip like \( r^{-0.5} \), in the second case they increase like \( \ln r \), and in the third case they are finite. Here \( r \to 0 \) is the distance from the crack tip to any point inside the domain.

2. The solution to the problem is exact because the coefficients of the expansions into series in Papkovich–Fadle eigenfunctions have the form of simple Fourier integrals of the known shape of the discontinuity profile.

3. The scheme of solving the problem is preserved for other types of boundary conditions on the sides of the rectangle.

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