V773 Cas, QS Aql, AND BR Ind: ECLIPSING BINARIES AS PARTS OF MULTIPLE SYSTEMS

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ABSTRACT

Eclipsing binaries remain crucial objects for our understanding of the universe. In particular, those that are components of multiple systems can help us solve the problem of the formation of these systems. Analysis of the radial velocities together with the light curve produced for the first time precise physical parameters of the components of the multiple systems V773 Cas, QS Aql, and BR Ind. Their visual orbits were also analyzed, which resulted in slightly improved orbital elements. What is typical for all these systems is that their most dominant source is the third distant component. The system V773 Cas consists of two similar G1-2V stars revolving in a circular orbit and a more distant component of the A3V type. Additionally, the improved value of parallax was calculated to be 17.6 mas. Analysis of QS Aql resulted in the following: the inner eclipsing pair is composed of B6V and F1V stars, and the third component is of about the B6 spectral type. The outer orbit has high eccentricity of about 0.95, and observations near its upcoming periastron passage between the years 2038 and 2040 are of high importance. Also, the parallax of the system was derived to be about 2.89 mas, moving the star much closer to the Sun than originally assumed. The system BR Ind was found to be a quadruple star consisting of two eclipsing K dwarfs orbiting each other with a period of 1.786 days; the distant component is a single-lined spectroscopic binary with an orbital period of about 6 days. Both pairs are moving around each other on their 148 year orbit.

Key words: binaries: eclipsing – binaries: spectroscopic – stars: fundamental parameters – stars: individual (V773 Cas, QS Aql, BR Ind)

Supporting material: machine-readable tables

1. INTRODUCTION

Eclipsing binaries and multiple systems play a crucial role in our understanding of the universe. Eclipsing binaries are being used for the precise derivation of stellar parameters, such as the stellar radius, mass, or luminosity. On the other hand, multiple systems play an important role in our calibrations of models of star formation and evolution, because the presence of triple, quadruple, or even higher-order systems can serve as a very sensitive indicator in these models and simulations. Finally, this distant component can play a crucial role during the evolution of the system: it offers the possibility to study the role of the so-called Kozai cycles in tidal friction (see, e.g., Eggleton & Kiseleva-Eggleton 2001) or to detect a slow precession of both inner and outer orbits.

With modern techniques, large telescopes, automatic surveys, and satellite observatories, the borders of astrophysical front-line research are continually moving toward more distant and fainter targets. This is quite a logical process, but we have to be very careful when saying anything about the completeness of our knowledge of bright and close systems. As already stated in several recently published papers, relatively bright targets among eclipsing binaries located within one hundred parsecs of the Sun can also bring us new, surprising results (see, e.g., Mérand et al. 2011 or Nemčak et al. 2016).

Therefore, we focused our efforts on three rather seldom-investigated systems (namely, V773 Cas, QS Aql, and BR Ind), which have besides an inner eclipsing pair a more distant, third component, detected via interferometry, and have orbital periods of several decades to hundreds of years (hence, the ratio of their periods is very high). Moreover, all of these stars show an Algol-like light curve, and no spectroscopic or photometric study of them has been done yet. As already published earlier, e.g., by Zasche et al. (2009), a list of such systems with eclipsing components among visual doubles, where both the inner and outer orbits are known, is still limited to only several dozens across the entire sky.

Statistics of triple and multiple systems are still rather limited, but what can surely be said is that there is a lack of systems with a higher-mass tertiary among triple stars. This was shown, e.g., by Tokovinin (2008) on spectroscopic triple stars or by Borkovits et al. (2016) on Kepler eclipsing binaries. Only a small fraction of systems have a tertiary more massive than the eclipsing pair itself. This forms the basis for our contribution to the topic.

2. THE DATA

Spectroscopy was obtained in two observatories. Most of the data points for these systems came from the Ondřejov observatory and its 2 m telescope (resolution $R \sim 12500$).
Additionally, data on BR Ind and some data on QS Aql were obtained with the FEROS instrument mounted on the 2.2 m MPG telescope located in La Silla Observatory in Chile (\(R \sim 48000\)). The individual exposure times were chosen according to the quality of the particular night and the specifications of the instrument to achieve a S/N between several dozens and a few hundreds.

The original FEROS data were reduced using the standard routines. The final radial velocities (hereafter, RV) used for the analysis were derived via a technique comparing both the direct and flipped profiles of the spectral lines manually on the computer screen to achieve the best match using the program SPEF0 (Horn et al. 1996, Škoda 1996) on several absorption lines in the measured spectral region (usually Fe, Ca, or Si lines). The derived radial velocities are given in tables in the appendix (see Table 9).

Photometry for these three systems was collected over the time span of 2008 to 2016. However, some older data used only for the minimum times were already published earlier, but the complete light curves (hereafter, LC) are published here for the first time. All of the data were obtained in the Johnson–Cousins photometric system Bessell (1990); in particular, data on the system V773 Cas were obtained in \(BVRI\), while those on the systems QS Aql and BR Ind were obtained in \(BVRI\) filters.

Owing to the relatively high brightness of the targets, only rather small telescopes were used for these photometric observations. The system V773 Cas was observed (by one of the authors, PS) with a 34 mm refractor at a private observatory in Brno, Czech Republic, using an SBIG ST-7XME CCD camera. The star QS Aql was monitored (by one of the authors, RU) with a similar instrument at a private observatory in Jilové u Prahy, Czech Republic, using a G2-0402 CCD camera. The only southern star, BR Ind, was observed with the FRAM telescope (Prouza et al. 2010), installed and operated at the Pierre Auger Observatory at Malargüe, Argentina. For the observations, only a small Nikkor lens with a 107 mm diameter and a CCD camera of the G4-16000 type (which was mounted on the 30 cm FRAM telescope itself) were used. All the measurements were processed by the software C-MUNIPACK,\(^7\) which is based on aperture photometry and uses the standard DAOPHOT routines (Tody 1993).

3. THE ANALYSIS

This entire work is based on classical techniques of photometry and spectroscopy together with the analysis of positional measurements of the visual double in the sky obtained over a much longer time span (more than a century). Combining these methods together, one can obtain not only reliable orbital and stellar parameters but also the structure of the system and its long-term evolution. The advantage is also the fact that having complete information about the mass, inclinations, periods, etc., we can fill in still rather incomplete statistics of triple and quadruple systems, compared to models of formation of binaries and multiple systems (Tokovinin 2008).

The visual orbit based on already published interferometric data was first analyzed. However, the orbits of systems analyzed within this study were published quite recently. Hence, our new recalculations led to only slight improvements of the fits. Data were downloaded from the already published papers and the Washington Double Star Catalog (hereafter, the WDS\(^8\); Mason et al. 2001). The orbits were calculated in accordance with Zasche and Wolf (2007), but the coverage of the orbits was usually not perfect and only parts of long orbits are covered by data nowadays.

Both the photometry and spectroscopy were studied in the standard manner. The obtained photometric data and the radial velocities were analyzed with the program PHOEBE (Prša & Zwitter 2005), which uses the classical Wilson–Devinney algorithm (Wilson & Devinney 1971) and its later modifications and allows us to fit the relevant parameters of the eclipsing components and their relative orbit. For the modeling, we used several assumptions. First, the primary temperature was set to the value corresponding to the particular spectral type (see, e.g., calibrations by Pecaut & Mamajek 2013 and the updated web site\(^9\)). The limb-darkening coefficients were obtained through interpolation in tables by van Hamme (1993). The albedo coefficients \(A_i\) and the gravity darkening coefficients \(g_i\) were fixed at their suggested values according to the temperatures of the components. As all studied eclipsing binaries are members of multiple systems, third light from the remaining components was taken into account.

Finally, if we have the LC+RV solution and the mass of both eclipsing binaries is known, we can proceed to the combined analysis of the visual orbit and period changes of the eclipsing pair. The method itself was introduced in Zasche and Wolf (2007), and its usage was presented, e.g., in Zasche et al. (2012) and Zasche et al. (2014). The most crucial for the whole analysis seem to be the quality of the input observations in both methods and the data coverage of the long orbit. These are usually problematic in cases, such as ours, where the third-body orbit is too long and only a small fraction of the orbit is covered. On the other hand, if we have good data coverage in both methods, we can even calculate independently the distance to the system.

4. V773 CAS

The first system in our sample of stars is the northern-hemisphere V773 Cas (=HIP 8115, HD 10543, \(V_{\text{max}} = 6.18\) mag), an eclipsing binary discovered on the basis of Hipparcos data (Perryman & ESA 1997, Kazarovets et al. 1999). However, many years before the discovery of its photometric variability, the star was recognized as a visual binary. Its most recent orbital solution is the one published by Hartkopf and Mason (2009), who derived a period of about 193 years, an orbital eccentricity of about 0.77, and a semimajor axis of about 0.9. The spectral type was usually stated as A3V (Jaschek 1978) or A2V (Palmer et al. 1968). However, as noted by Cvetković and Ninković (2010), there arises a large discrepancy between the astrophysical and dynamical total mass of the system. From its spectral types the total mass should be about 3 \(M_\odot\), while from the orbital solution arises a \(M_{\text{dyn}} = 11.9\ M_\odot\). This strange discrepancy led to our initial interest in the star.

First of all, we found out that for V773 Cas, three new interferometric measurements had been obtained since the last

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\(^{7}\) See [http://c-munipack.sourceforge.net/](http://c-munipack.sourceforge.net/).

\(^{8}\) [http://ad.usno.navy.mil/wds/](http://ad.usno.navy.mil/wds/)

\(^{9}\) [http://www.pas.rochester.edu/~emamajek/EEM_dwarf_UBVIJK_colors_Teff.txt](http://www.pas.rochester.edu/~emamajek/EEM_dwarf_UBVIJK_colors_Teff.txt)
Figure 1. Light and radial velocity curve fits of V773 Cas based on the PHOEBE fitting.

visual orbit calculation was published by Hartkopf and Mason (2009). We added these three data points and re-ran the fitting procedure, which resulted in an only slightly different orbital solution. However, our solution would not be complete enough if we did not try to also incorporate the photometric monitoring and our results from the LC+RV analysis.

The primary temperature was fixed at a value of 5900 K, which resulted from our spectral estimations and also from the primary mass. The results of our LC+RV solution are shown in Figure 1, where one can see the light curve in BVR filters together with the RV curve based on the Ondřejov data. In total, there were 20 obtained spectrograms and 15 nights of photometry. The radial velocities were mostly derived from the Ca I and Fe I lines. The parameters of the least-squares fitting procedure, which resulted in an only slightly different solution, the third body is the brightest member of the system, whose movement is negligible over the time span of the observed spectra. However, as one can see from the relatively high value of the third light resulting from the LC solution, the third body is the brightest member of the system, and it is probably responsible for the spectral classification of V773 Cas as A2-3 in the past.

The above-mentioned solution was derived using the PHOEBE code and the LC+RV fitting. However, we also tried a different approach to the problem. Using the available 20 spectrograms, we applied the code PYTERPOL.10 (Nemravová et al. 2016). This program determines kinematic and radiative properties of binary components through comparison of observed spectra to synthetic ones obtained through interpolation in pre-calculated grids of synthetic spectra (Palacios et al. 2010 and de Laverny et al. 2012). Using this approach, we obtained the solution presented in Table 2. This result is in very good agreement with the PHOEBE solution presented above as well as with the observed magnitude difference between the two visual components (of about 1–2 mag from the WDS catalog).

The linear ephemerides written in Table 1 are the best suitable elements for prospective observations of V773 Cas in the upcoming years. However, these elements will change significantly due to the orbital motion of the eclipsing pair around a common barycenter with the third component. The most significant change of the orbital elements of the inner pair will take place near the periastron passage, which will occur in 2021. We plotted the predicted period variation of the V773 Cas eclipsing pair in the O–C diagram in Figure 2. This diagram was constructed in accordance with the visual orbit of the double derived from our combined analysis. The orbit of the third component is given in Figure 2, and the parameters of such a fit are given in Table 3. A list of the minimum times used for the analysis is given in the appendix (see Table 8).

The problem is that to achieve such a self-consistent solution, we cannot use the Hipparcos parallax as an input parameter. The spectral classification of about A3V for the third component comes not only from the already published papers but also from our findings about the spectra (the lines indicate a spectral type of about A3) as well as from photometric indices of the third

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**Table 1**

| Parameter | Primary | Secondary | Tertiary |
|-----------|---------|-----------|----------|
| HJD$_0$  | 2448500.9209 ± 0.0003 | ... | ... |
| $P$ [day] | 2.587332 ± 0.000002 | ... | ... |
| $a$ [R$_\odot$] | 9.96 ± 0.06 | ... | ... |
| $v_c$ [km s$^{-1}$] | 7.11 ± 0.30 | ... | ... |
| $q$ = $M_2/M_1$ | 1.00 ± 0.05 | ... | ... |
| $i$ [deg] | 84.7 ± 2.2 | ... | ... |
| $K$ [km s$^{-1}$] | 97.1 ± 0.9 | 97.0 ± 1.6 | ... |
| $T$ [K] | 5900 (fixed) | 5842 ± 50 | ... |
| $M$ [M$_\odot$] | 0.99 ± 0.03 | 0.99 ± 0.04 | ... |
| $R$ [R$_\odot$] | 1.05 ± 0.05 | 1.05 ± 0.05 | ... |
| $M_{tot}$ [mag] | 4.55 ± 0.10 | 4.58 ± 0.10 | ... |
| $L_B$ [%] | 10.0 ± 0.9 | 9.5 ± 0.9 | 80.5 ± 0.9 |
| $L_V$ [%] | 12.5 ± 0.7 | 12.1 ± 0.7 | 75.4 ± 0.6 |
| $L_R$ [%] | 15.0 ± 0.6 | 14.6 ± 0.6 | 70.4 ± 0.5 |

**Table 2**

| Parameter | Primary | Secondary | Tertiary |
|-----------|---------|-----------|----------|
| $T$ [K] | 5933 ± 131 | 5693 ± 161 | 8522 ± 38 |
| $v$ sin $i$ [km s$^{-1}$] | 32.17 ± 2.32 | 49.10 ± 7.46 | 84.55 ± 1.42 |
| $L$ [%] | 13.1 ± 0.8 | 11.1 ± 0.5 | 75.8 ± 0.6 |

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10 https://github.com/chrysante87/pyterpol/wiki
body derived from the LC+RV solution. Hence, the total mass of the three components should be about $4 \, M_\odot$, which is in contradiction to the mass computed using the *Hipparcos* parallax, $\pi_{\text{HIP}} = 11.77 \pm 0.67$ mas. Hence, the parallax needed for our combined solution to be self-consistent is about $\pi_{\text{new}} = 17.6 \pm 1.5$ mas, but its uncertainty is still rather high because it is based only on a mass estimation. Such a situation is nothing novel, because *Hipparcos* data sometimes produce spurious results for close double stars (see, e.g., Docobo et al. 2008).

5. QS AQL

The second eclipsing system analyzed in the present study is QS Aql (=HIP 96840, HD 185936, $V_{\text{max}} = 6.01$ mag), which is the brightest one and also the most massive one among the studied systems. Jaschek (1978) determined its spectral type to be B5V, while others, like Cannon & Pickering (1923) and Lu (1991), published its type as B3. Moreover, its variability was first detected by Millman (1928), but its eclipsing nature was confirmed by Guthnick (1931), who also gave its correct orbital period of about 2.5 days. Some 40 years later, Knipe (1971) discovered a rapid period change, which occurred at about 1964 (his suggestion) and was caused by the periastron passage in the wide orbit around the barycenter. The period change was so rapid that the eccentricity of the wide orbit must have been very high. On the other hand, the first astrometric observations are more than 80 years old, but their accuracy is questionable due to the small angular separation of the components. Many reliable speckle interferometric observations have been obtained since 1976. The most recent orbital solution was computed by Docobo and Ling (2007), who derived an orbital period of 61.72 years and a surprisingly high eccentricity of about 0.966. The total mass of the system was estimated to be about $20 \, M_\odot$, with rather high uncertainty. Mayer (2004) noted that the combined analysis of period changes and the visual orbit is still problematic due to the poor coverage of data by both methods.

We started the photometric monitoring of this interesting system in 2007 and the collection of new spectroscopy in 2012. Since the last calculation of its visual orbit by Docobo and Ling (2007), one new observation of the visual double has been...
published. The system is known to be a single-lined spectroscopic binary; secondary and tertiary component lines were not detected in the spectra. For a discussion of the individual RV solutions, see below.

Light curve analysis was carried out using the data obtained in BV/R’ filters in the Czech Republic in 2009 and 2010. The results are shown in Figure 3, while the parameters corresponding to the best-fitting synthetic light curve are given in Table 4. In Figure 3 you can also see some small variability in the residuals after subtraction of the light curve. However, these deviations are only caused by poor photometric conditions in some of the nights and our decision not to remove outlying points on the light curves. The results of the RV fitting are shown in Figure 3, where one can see that the secondary velocities have also been derived but are affected by much larger errors than those of the primary ones.

As one can see from the parameters given in Table 4, the eclipsing components are rather different, but the most luminous one seems to be the third distant member. What is rather surprising is the fact that the amplitude of the RV variations for the primary component resulted in about 74 km s\(^{-1}\), while previous studies have given a \(K_1\) value between 40.8 km s\(^{-1}\) (Abt et al. 1990) and 58.4 km s\(^{-1}\) (Holmgren 1987). Other solutions have also given rather low values of \(K_1\), about 47.3 km s\(^{-1}\)—see Hill (1931) and Lucy and Sweeney (1971). This discrepancy probably comes from the fact that the lines are very broad and are blended together with the third (dominant) component, which remains on almost the same position over the whole time interval. Because of this, the lines are rather asymmetric—previous authors probably measured the wide wings of the lines instead of the cores. If we measure the wings, the amplitude will really be lower. However, because of the high-dispersion FEROS spectra, we were able to both confidently identify the eclipsing components for the first time (hence, SB1 becomes SB2) and derive the amplitude \(K_1\) with greater conclusiveness.

If we had tried to obtain a combined solution of the visual orbit and the period variation of the eclipsing pair, the analysis would still have been rather problematic. For a reliable fit of the visual orbit, only data obtained since 1976 were taken into account. On the other hand, older times of the minima were also used because these define the rapid period change near the

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### Table 4

| Parameter | Primary | Secondary | Tertiary |
|-----------|---------|-----------|----------|
| HJD0     | 244044.5442 ± 0.0015 | ... | ... |
| \(P\) [d] | 2.5132987 ± 0.0000075 | ... | ... |
| \(\omega\) [deg] | 13.78 ± 0.11 | ... | ... |
| \(v\) [km s\(^{-1}\)] | −16.13 ± 0.62 | ... | ... |
| \(q = M_2/M_1\) | 0.37 ± 0.02 | ... | ... |
| \(i\) [deg] | 83.6 ± 1.3 | ... | ... |
| \(K\) [km s\(^{-1}\)] | 73.98 ± 0.33 | 201.76 ± 2.09 | ... |
| \(T\) [K] | 14500 (fixed) | 7910 ± 78 | ... |
| \(M\) [M\(_\odot\)] | 4.07 ± 0.09 | 1.49 ± 0.05 | ... |
| \(R\) [R\(_\odot\)] | 4.08 ± 0.15 | 1.65 ± 0.20 | ... |
| \(M_{bol}\) [mag] | −2.31 ± 0.18 | 2.29 ± 0.14 | ... |
| \(L_\odot\) [\%] | 47.6 ± 2.9 | 1.4 ± 0.4 | 51.0 ± 3.1 |
| \(L_V\) [\%] | 47.4 ± 1.2 | 2.0 ± 0.3 | 50.6 ± 1.4 |
| \(L_R\) [\%] | 49.2 ± 3.4 | 2.3 ± 0.2 | 48.5 ± 3.5 |
| \(L_I\) [\%] | 48.7 ± 2.3 | 2.7 ± 0.2 | 48.6 ± 2.5 |

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### Figure 4

Orbit of QS Aql in the sky and the \(O–C\) diagram. See Figure 2 for a description.

### Table 5

| Parameter | Value |
|-----------|-------|
| \(p_1\) [year] | 77.0 ± 4.3 |
| \(T_0\) [year] | 1962.3 ± 2.3 |
| \(e\) | 0.947 ± 0.038 |
| \(R\) [arcsec] | 0.111 ± 0.045 |
| \(i\) [deg] | 61.2 ± 3.6 |
| \(\Omega\) [deg] | 144.5 ± 5.1 |
| \(\omega\) [deg] | 336.8 ± 4.7 |
| \(M_1\) [M\(_\odot\)] | 4.04 ± 0.86 |
| \(\pi\) [mas] | 2.89 ± 0.55 |

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### Figure 5

Gamma velocities of QS Aql based on individual published RV solutions.

Year 1960 quite well. For the results of our fitting, see Figure 4. The parameters of our solution are presented in Table 5. As a by-product of the fitting, the gamma velocity changes are also plotted in accordance with the third-body orbit. Also
plotted are the individual gamma velocities from individual studies published over the last century. Our result is shown in Figure 5. Unfortunately, some individual systemic velocities are affected by large uncertainties, and the predicted RV variation is rather flat during most of the p3 period.

However, a discussion of this combined solution is necessary. The presented final fits are still rather preliminary, and the astrometry especially suffers from many deviating points. This is probably caused by a rather large eccentricity and an inclined orbit. Several much more reliable observations would be very useful in the upcoming years. Heintze et al. (1989) discussed the spectroscopic orbit by Holmgren (1987) and concluded that the third light should be about 1.2 times as strong as the combined light of the eclipsing pair. Their conclusion implies a tertiary mass of 4.3 $M_\odot$ and a spectral type of B5-6 V. However, this assumption is contradicted by the last interferometric observations, which indicate that both visual components are of similar brightness (i.e., the combined light from the eclipsing pair is roughly the same as the light from the third star)—this agrees with our new LC+RV solution as well. From this information, we can derive that the third component is probably of about the same spectral type as the primary, i.e., B6V, with a mass of about 4 $M_\odot$. Exactly the same result was obtained from our combined analysis of period changes and the visual orbit (see Table 5).

Given the relatively well-derived amplitudes of both phenomena (the semimajor axis for the visual orbit as well as the semimagnitude of the period variations in the $O$–$C$ diagram), we also tried to independently determine distance to the system. Quite interesting is how the parallax of QS Aql changed from the original Hipparcos value of 1.98 ± 0.82 mas (Perryman & ESA 1997), while the new one was recalculated to 0.49 ± 0.62 mas (van Leeuwen 2007). On the other hand, Docobo & Andrade (2006) presented two different possible values of parallax based on two different methods and the Hipparcos parallax—namely, 1.8 and 3.1 mas. No other parallax estimation has been found in the literature. However, with our solution, the parallax value has to be larger than the Hipparcos ones due to the amplitudes in both methods, of about 2.89 mas. Future space missions, like Gaia (Perryman et al. 2001), would solve this problem; however, the star may be too bright and close to the bright limit of the satellite.

6. BR IND

The last system in our analysis, BR Ind (=HIP 104604, HD 201427, $V_{\text{max}}$ = 7.07 mag), is rather neglected and only seldom investigated. Its photometric variability was discovered on the basis of Hipparcos data (Perryman & ESA 1997), giving an orbital period of 0.89277 days (a short note about its possible double value was also added therein). Its spectral type, F8V, was published by Houk (1978); however, it is not clear to which component this classification belongs. The star is also known to be a visual double, having a time span of positional observations of more than 100 years. The most recent orbital solution was published by Seymour et al. (2002), who derived a period of 167 years, a semimajor axis of 0′′894, and an eccentricity of 0.521. However, since then five new observations have been obtained, and the orbit should be recalculated.

We collected available photometry of BR Ind to find out which of the orbital periods is the correct one (0.89 or 1.78 days). However, the photometry from surveys like ASAS (Pojmanski 1997) and Pi of the sky (Burd et al. 2005) was not able to distinguish between these two periods. Therefore, photometry for BR Ind in $BVRI$ filters was obtained in 2014 and 2015 using the FRAM telescope. With these data we finally confirmed that the correct orbital period of the inner pair is really double, i.e., of about 1.786 days.

On the other hand, we also obtained spectroscopy of BR Ind with the FEROS spectrograph in La Silla. However, after four nights of observations (and after obtaining 27 échelle spectra), we were not able to detect the 1.8 day period on the most prominent lines. Instead, the lines followed a longer periodicity of several days. Hence, we applied for more observing time using the Tycho Brahe proposals for the 2.2 m MPG telescope and the FEROS instrument again. For four consecutive seasons, we obtained 14 more spectra of BR Ind, which finally led to the solution.

The most prominent lines, which were also used to derive the radial velocities, were the Fe and Ca lines. These were analyzed, leading to the detection of a 6 day variation. Only much weaker lines were detected as the lines came from the primary and secondary components of the eclipsing pair and followed a 1.786 day variability. Hence, for the subsequent analysis, we denoted the 6 day orbit as the “B” component and consistently designated the eclipsing pair as “A.” The results of our RV fitting are plotted in Figures 6 and 7. The resulting parameters of pair B are given in Table 6. As one can see, the orbit is only slightly eccentric. The structure of BR Ind is plotted in Figure 8.

Light curve fitting was carried out together with radial velocity analysis in PHOEBE. The results are plotted in Figure 7, while the parameters are given in Table 6. We can see that both components are very similar to each other (therefore, the problems with the 0.89 versus 1.78 day period). The gamma velocities of both the eclipsing pair and the B pair are similar to each other and close to zero. This indicates that the orbit is very close to a face-on orientation, which was confirmed via the fitting of the astrometry (see below). We also collected available photometry to derive the times of the eclipses and constructed the $O$–$C$ diagram plotted in Figure 9. No visible variation can be seen there during these approximately 25 years of observations. This also indicates that the period of the visual pair is rather long or the orbit is nearly face-on.

The available positional measurements were analyzed, leading to slight improvement of the fit published by Seymour et al. (2002). The parameters are given in Table 7 and the plot of the orbit in Figure 9. As one can see, the period is a bit shorter and the eccentricity higher. The orientation of the orbit
is really close to face-on \((i = 155^\circ)\), which is in agreement with the discussion in the previous paragraph.

From the fit of the visual orbit we can also derive the total mass of the whole system. This resulted in about 3.34 \(M_\odot\). Such mass can be used to derive the individual masses of pair B in the system. If we accept the orbital solution derived from our LC+RV analysis, the mass of pair B has to be about 1.65 \(M_\odot\).

Due to the fact that the B subsystem is only an SB1-type binary, we can only estimate its individual masses. For the SB1-type binary we can calculate a so-called mass function (Kallrath & Milone 2009),

\[
f(M)_B = \frac{1}{2\pi G} K^3 P_B (1 - e_B^2)^{3/2} = \frac{(M_{bb} \sin i_B)^3}{(M_{bb} + M_{bb})^2},
\]

**Figure 7.** Light and radial velocity curve fits of the inner pair of BR Ind based on the PHOEBE fitting.  

**Figure 8.** Structure of BR Ind derived from our analysis.  

**Figure 9.** Orbit of BR Ind in the sky and the \(O-C\) diagram. See Figure 2 for a description.

**Table 6**  
The Parameters from The LC+RV Fitting of BR Ind

| Parameter | Pair A | Pair B |
|-----------|--------|--------|
| Primary   |        |        |
| Secondary |        |        |
| HJD0      | 2448500.4755 ± 0.0002 | 2456563.186 ± 0.052 |
| \(P\) [day] | 1.7855618 ± 0.0000015 | 6.0009949 ± 0.0000020 |
| \(a\) \([\text{R}_\odot]\) | 7.37 ± 0.04 | 9.551 ± 0.026 |
| \(v_c\) \([\text{km s}^{-1}]\) | -2.66 ± 0.08 | -3.131 ± 0.089 |
| \(e\) | ... | 0.190 ± 0.003 |
| \(\omega\) \([\text{deg}]\) | ... | 161.83 ± 0.94 |
| \(q = M_2/M_1\) | 0.96 ± 0.02 | ... |
| \(i\) \([\text{deg}]\) | 85.17 ± 1.8 | ... |
| \(K\) \([\text{km s}^{-1}]\) | 101.9 ± 0.4 | 106.4 ± 0.4 |
| \(T\) \([\text{K}]\) | 5170 (fixed) | 5203 ± 75 |
| \(M\) \([\text{M}_\odot}\) | 0.86 ± 0.02 | 0.83 ± 0.02 |
| \(R\) \([\text{R}_\odot}\) | 1.23 ± 0.03 | 0.95 ± 0.02 |
| \(M_{bol}\) \([\text{mag}]\) | 4.78 ± 0.05 | 5.32 ± 0.06 |
| \(L_B\) \([\%]\) | 16.5 ± 2.0 | 10.2 ± 2.0 | 73.3 ± 3.5 |
| \(L_V\) \([\%]\) | 24.3 ± 2.3 | 14.9 ± 1.7 | 60.8 ± 4.0 |
| \(L_R\) \([\%]\) | 26.3 ± 2.4 | 16.0 ± 1.6 | 57.7 ± 4.2 |

**Table 7**  
The Parameters of the Visual Orbit of BR Ind

| Parameter | Our Solution | Seymour et al. (2002) |
|-----------|--------------|-----------------------|
| \(p_3\) \([\text{year}]\) | 147.9 ± 2.5 | 167.0 |
| \(T_0\) \([\text{year}]\) | 2050.3 ± 1.9 | 2056.0 |
| \(e\) | 0.711 ± 0.021 | 0.521 |
| \(a\) \([\text{arcsec}]\) | 0.864 ± 0.045 | 0.894 |
| \(i\) \([\text{deg}]\) | 154.6 ± 8.2 | 141.9 |
| \(\Omega\) \([\text{deg}]\) | 220.8 ± 11.8 | 142.8 |
| \(\omega\) \([\text{deg}]\) | 80.1 ± 9.9 | 178.4 |

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and from knowledge of the total mass of pair B we can derive the product \((M_{Bb} \cdot \sin i_B) = 0.47 \: M_\odot\). Obviously, we do not know the inclination of the pair, but at least some estimation can also be done with these values. Based on the LC solution, the third light of pair B is higher than the combined light coming from the two eclipsing components of pair A. Hence, this finding is in excellent agreement with the mass derived from the SB1 binary of pair B and the total mass—but only with the assumption that the inclination is close to 90°. Hence, the two components of the B pair should be about F8V+M2V. With such a configuration the individual luminosity levels and their ratios as well as the non-detection of the Bb component in the spectra will be explained. Also, the spectral classification is now clarified: the Ba component instead of the eclipsing pair. Finally, this solution indicates that the two visual components have their individual magnitudes shifted to about 1.1 mag, which is in agreement with the magnitude differences published in the WDS catalog (Mason et al. 2001). However, the whole discussion is solely based on the assumption that the \(Hipparcos\) parallax (van Leeuwen 2007) of 20.65 mas is correct.

7. DISCUSSION AND CONCLUSIONS

Although a lot of work has been done on theoretical modeling as well as on observations over the last decades, the formation of systems of higher multiplicity remains an open question. Multiplicity itself is the most promising mechanism for producing close binaries with short orbital periods below 1 day. A third component may cause Kozai cycles, and then the tidal friction between the binary components will cause orbital shrinkage and circularization—see, e.g., Eggleton & Kiseleva-Eggleton (2001) and Borkovits et al. (2016). Nevertheless, this is not the only possible scenario of the origin of such systems; several other competing theories are still being discussed. Truly existing systems were probably produced by a combination of several different mechanisms—see, e.g., Tokovinin (2008). Numerical simulations that include only particular mechanisms are able to explain only some statistical properties of multiple systems and fail to explain others. That is a matter of intensive investigation in recent years, and each newly discovered multiple system with its known orbital and physical parameters should help us to improve the statistical properties of the sample and provide us with new observational constraints.

The study of the three selected eclipsing multiple systems provides us with only a piece of information needed for the construction of the theory of stellar formation and evolution. Despite this fact, it is still a valuable contribution to the topic in several aspects. First, the three systems represent the most typical multiple systems today (containing a close inner pair with a period of a few days and a distant component with a much longer period). Second, their physical and orbital properties are now well determined and can be placed into the broader context of theoretical modeling (e.g., the ratio of the periods of the inner and outer orbits, the mass ratios, and the eccentricity values)—see, e.g., Halbwachs et al. (2003) or Tokovinin (2008).

Third, each of the studied systems is interesting and deserving of attention. \(V773\) Cas was found to be much closer to the Sun than originally assumed on the basis of \(Hipparcos\) data. \(QS\) \(Aql\) is rather massive, but moves around a common barycenter with the third component in an orbit with a very high eccentricity of about 0.95. Also, \(BR\) \(Ind\) was found to be a rare quadruple system consisting of eclipsing and non-eclipsing pairs. This detection of higher-order multiplicity among stars of such a late type is also rather remarkable, because their multiplicity fraction is generally very low—see, e.g., Duchêne & Kraus (2013). A common characteristic of these systems is that the most massive and most luminous star is the distant third component (except in \(BR\) \(Ind\), where it is a binary). And as already shown (e.g., by Tokovinin 2008 and Borkovits et al. 2016), such systems are still rather rare.

As a by-product, possible mutual inclinations for these three systems and their orbits have also been derived. Due to the fact that the longitude of the ascending node of the inner eclipsing pair is not known, we can only estimate the ranges within which the mutual inclination should lie. This resulted in 48°–142° for \(V773\) Cas, 22°–145° for \(QS\) \(Aql\), and 69°–120° for \(BR\) \(Ind\). As one can see, the ranges are still rather large, and the uncertainty should be lower when knowing the longitude of the ascending node. However, this can only be achieved by resolving the inner eclipsing pair via interferometry, which is very problematic (due to a luminous third star and the small angular separation of the eclipsing components). The most promising in this sense seems to be \(V773\) Cas, where the predicted angular distance was about 0.8 mas.

Finally, such a study is also viable from an observational point of view. We can see that there still exist many systems that have never been analyzed and whose parameters are not known, despite the fact that observations exist and are easily obtainable. At this point it would be suitable to mention that the photometric data on \(V773\) Cas and \(QS\) \(Aql\) were obtained using only 34 mm sized telescopes, while those on \(BR\) \(Ind\) were obtained with a 107 mm one.

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APPENDIX A

TABLE OF MINIMA

This section contains the list of minimum timings for the systems used for the analysis.
This section contains the list of radial velocities for the systems used for the analysis.

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