Full counting statistics of generic spin entangler with quantum dot-ferromagnet detectors

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Abstract – Entanglement between spatially separated electrons in nanoscale transport is a fundamental property, yet to be demonstrated experimentally. Here we propose and analyse theoretically the transport statistics of a generic spin entangler coupled to a hybrid quantum dot-ferromagnet detector system. We show that the full distribution of charges arriving at the ferromagnetic terminals provides complete information on the spin state of the particles emitted by the entangler. This provides means for spin entanglement detection via electrical current correlations, with optimal measurement strategies depending on the a priori knowledge of ferromagnet polarization and spin-flip rates in the detector dots. The scheme is exemplified by applying it to Andreev and triple dot entanglers.

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Introduction. – Entanglement between spatially separated quantum systems constitutes an indispensable resource for quantum information processing [1]. In nanoscale electronic systems, a promising arena for quantum information and computation, the ultimate carriers of quantum information are individual electrons. Controlled creation, spatial separation and detection of entangled electrons thus constitute key elements in nanoscale quantum information processing. During the last one and a half decade, a large number of schemes for transport generation and detection of electronic entanglement have been proposed [2,3]. However, a clear-cut experimental demonstration is still lacking. The main reason is the paramount difficulty to, in a single nanosystem, generate, coherently control and unambiguously detect the entanglement.

An early key proposal for spin entanglement generation is the quantum-dot–based Andreev entangler [4]; a superconductor coupled to two quantum dots, further coupled to normal leads. In the transport state, Cooper pairs, electron spin singlets, tunnel out from the superconductor, via the dots, into the leads. At ideal operation, each Cooper pair is coherently split without altering the spin properties. This gives a source of pairs of spatially separated, maximally spin entangled electrons in the leads. Subsequent theoretical works extended on or further analysed different properties of the entangler [4–8].

Recently, important steps were taken towards an experimental demonstration of an Andreev entangler. In a series of experiments [9–14], splitting of Cooper pairs into two quantum dots, formed in semiconductor nanowires or carbon nanotubes, was reported. Efficient splitting of the pairs was clearly demonstrated by current [13] and cross-correlation measurements [12]. The experiments
spurred further theoretical investigations on various aspects of Cooper pair splitters [15–21].

Importantly, in none of the experiments reported [9–14] were the spin properties of the emitted pairs directly investigated. To verify that the Cooper pair splitters also work as Andreev entanglers, emitting spin singlets, non-local spin sensitive detection is necessary. To this aim, albeit challenging, a natural extension of the experiments would be to couple the dots to ferromagnetic (FM) leads, see fig. 1. By performing a set of current cross-correlation measurements [22–26] with non-collinear FM-polarization [22,23,27] the entanglement can be tested by a Bell inequality or even quantified by spin state tomography [28,29]. To facilitate such an experiment under realistic conditions, in the presence of spurious tunneling processes, spin-flip scattering in the dots and limited magnitude of the polarization, several questions need to be carefully addressed. Most importantly: i) How are the spin properties of the pair emitted by the entangler manifested in the cross-correlators of the currents at the FM-leads? And, if possible, ii) how can system parameters and detector settings be optimized to allow for an unambiguous detection of the entanglement of the emitted state?

In this work we provide answers to these questions by considering the full statistics of charge transfer between the entangler and the FM-leads. To make the scheme applicable beyond Cooper pair splitters we consider a generic entangler-detector setup, shown in fig. 1. The entangler emits arbitrary single- and two-particle spin states into the dots. The two dots together with the FM-leads constitute the detector. To avoid cross-talk between the two detector dots as well as back-action of the detector on the entangler, we consider a weak entangler-detector coupling. Importantly, working within a spin-dependent quantum master equation formalism [30–35] we can treat both charging effects and spin-flip scattering in the dots, extending on earlier works [15,27,36] on statistics of entanglers coupled, via a single non-interacting dot, to FM-electrodes.

The charge transfer statistics allows us to identify the individual particle tunneling events [37] as well as their spin properties. Based on this statistics we show how the current cross-correlations provide direct information on the spin properties of the emitted, entangled two-particle state. In line with earlier work [23–26] we find that spurious single-particle tunneling does not affect the correlations. Moreover, depending on how well the FM-polarizations and the spin-flip rate are characterized, we propose measurement strategies to optimize the entanglement detection. To demonstrate the versatility of our scheme we apply it both to the Andreev entangler [4] and a triple-quantum-dot entangler [38].

Entangler-detector system. — The combined entangler-detector system is shown in fig. 1. The detector subsystem consists of two quantum dots, $A$ and $B$, with each dot $\alpha = A,B$ coupled to two FM-leads $\alpha^+, \alpha^-$ via tunnel barriers with rates $\Gamma_{\alpha^+} = \Gamma_{\alpha^-} = \Gamma_{\alpha}/2$. Each dot has a single spin degenerate level, at energy $\varepsilon_\alpha$ and $\varv_\alpha$, respectively. Double occupancy of dot $A$ or $B$ is prevented by strong on-site Coulomb interaction. Coulomb interaction between the dots is neglected. The FM-leads have polarizations $\vec{p}_{\alpha} = -\vec{p}_{\alpha} = \mp p \hat{n}_\alpha$ with identical magnitude $p$ and unit vectors $\hat{n}_A$ and $\hat{n}_B$ non-collinear.

The generic entangler is acting as a source of both single electrons and split pairs of spin-correlated electrons, see fig. 1. The pair emission process is characterized by a $4 \times 4$ rate matrix $\gamma_{AB}$, with elements $\gamma_{\sigma^0 \sigma^0,\tau^0 \tau^0}$ where $\sigma, \sigma', \tau, \tau' = \uparrow, \downarrow$. This describes emission of pairs with a spin density matrix $\hat{\gamma}_{AB}/\text{Tr}[\hat{\gamma}_{AB}]$ at a rate $\text{Tr}[\hat{\gamma}_{AB}]$. As a key example, emission of spin singlets $|\Psi_S\rangle = (|\uparrow\downarrow\rangle \mp |\downarrow\uparrow\rangle)/\sqrt{2}$ with a rate $\gamma$ gives $\tilde{\gamma}_{AB} = \gamma |\Psi_S\rangle \langle \Psi_S|$. The emission of single particles is correspondingly described by $2 \times 2$ rate matrices $\gamma_{\alpha}$ with matrix elements $\gamma_{\sigma^0 \sigma'}$. Throughout the paper we consider $\gamma_{AB} = \gamma_{\sigma^0 \sigma^0} = \gamma_{\sigma^\sigma' \sigma^\sigma'} = \gamma_{\sigma^\sigma' \sigma^\sigma'} = \gamma_{\alpha \alpha}$, achievable for relevant entanglers [4,38]. In addition, we account for spin-flip scattering in the dots with a rate $\eta_{\alpha}$ taken to be the same for $A$ and $B$.

The FM-leads are all kept at the same potential. Moreover, a large bias is applied between the entangler and the FM-leads, in order to have all detector-entangler energy levels well inside the bias window. The temperature of the leads is much smaller than the bias as well as the distance from the entangler-detector energy levels to the edges of the bias window. This allows us to neglect back-tunneling from the FM-leads into the dots, known to complicate the entanglement detection [39,40].

Full transport statistics. — As we describe in detail below, in this high-bias regime the transport properties can be described exactly within a quantum master equation approach to the reduced spin density matrix of the dots. The full distribution of charge transferred to the FM-leads (during a long measurement time) is conveniently characterised [37] by a cumulant generating function $F_\chi$, where $\chi = \{\chi_A, \chi_B\}$ denotes the set of lead counting fields. To leading order in the rate matrices we find

$$F_\chi = \sum_{\alpha,m} \text{Tr} \left[ \hat{Q}_\alpha^{\chi}\hat{n}_\alpha \right] (e^{\chi_{\alpha m}} - 1) + \sum_{\alpha,m} \text{Tr} \left[ \hat{Q}_\alpha^{\chi_m} \otimes \hat{Q}_{\beta m} \hat{n}_{\alpha} \hat{n}_{\beta} \right] (e^{\chi_{\alpha m} + \chi_{\beta m}} - 1),$$

(1)

where $\otimes$ denotes the direct product, $\hat{Q}_\alpha = (1/2)[1 + \zeta_\alpha \vec{n}_{\alpha m} \cdot \vec{S}]$ is a $2 \times 2$ detector matrix with $\vec{S} = \langle \vec{\sigma}_x, \vec{\sigma}_y, \vec{\sigma}_z \rangle$ a vector of Pauli matrices and $0 \leq \zeta_\alpha \leq 1$ the detection efficiency. The efficiency $\zeta_\alpha = p_{\alpha}(1 - \eta_\alpha)$ is a product of the FM-lead polarization $p_{\alpha}$ and $1 - \eta_\alpha$, where $\eta_\alpha = \eta/(\Gamma_{\alpha} + \eta)$ is the dimensionless spin-flip rate in dot $\alpha$, ranging from 0 for negligible spin-flip scattering to 1 for complete spin randomization.
Equation (1) is the key technical result of our paper. It allows for a compelling and physically clear picture of the transport statistics through the entangler-detector system and provides means to identify the spin properties of the emitted pairs. The generating function $F_\chi$ in eq. (1) describes a set of independent Poisson transfer processes of single and pairs of particles:

- Each term $1 - e^{iQ_{Am} + Q_{Bn}}$ describes a pair of particles arriving, one particle to lead $A_n$ and one to $B_n$, with a transfer rate $\text{Tr}[(\hat{Q}_{Am}^\dagger \otimes \hat{Q}_{Bn}^\dagger)\hat{\gamma}_{AB}]$. The rate depends on the spin properties of the emitted pair, via the rate matrix $\hat{\gamma}_{AB}$. In particular, for an emitted singlet $\hat{\gamma}_{AB} = \gamma[\Psi_S]\langle\Psi_S|$ we have the two-particle rate $(\gamma/4)[1 - \zeta A\zeta B\hat{n}_{Am} \cdot \hat{n}_{Bn}]$. This rate depends on the relative orientation of the polarizations via $\hat{n}_{Am} \cdot \hat{n}_{Bn}$, demonstrating the non-local character of the spin-correlations.

- Each term $1 - e^{iQ_{Am}}$ describes a single particle arriving at terminal $Am$, with a transfer rate $\text{Tr}[\hat{Q}_{Am}^\dagger \hat{\gamma}_A]$. Similar to the two-particle term, the transfer rate depends on the spin properties of the emitted particle via $\hat{\gamma}_A$.

**Detector efficiency vs. entanglement suppression.** As is clear from eq. (1), both finite spin-flip scattering $n_\alpha > 0$ and non-unity polarization $p_\alpha < 1$ lead to a reduced efficiency $\zeta < 1$. Importantly, the transfer rates in $F_\chi$ can be rewritten as follows, providing a different picture: Making use of the formal quantum operation approach [1] we can write the detector matrix as $Q_{Am}^\dagger = E_A(Q_{am})$, where $E_A(\hat{q}) = \gamma_A \hat{q} + (1 - \zeta_A)\text{Tr}[\hat{q}]/2$ is the depolarization operation for a $2 \times 2$ matrix $\hat{q}$ and $Q_{am} = (1/2)[1 + \hat{n}_{am} \cdot \vec{\sigma}]$ the ideal detector matrix, for efficiency $\zeta_A = 1$. Noting $E_A(\hat{q}) = [1 + 3\zeta_A]/4(1 - \zeta_A)\hat{q} + [1/4](1 - \zeta_A)(\sigma_x \hat{q} \sigma_x + \sigma_y \hat{q} \sigma_y + \sigma_z \hat{q} \sigma_z)$, we can write the single-particle transfer rate as $\text{Tr}[E_A(Q_{am})\hat{\gamma}_A] = \text{Tr}[Q_{am}^\dagger E_A(\hat{\gamma}_A)],$ describing perfect detection of a depolarized rate matrix $\hat{\gamma}_A$. This is readily extended to the two-particle, which can be written as

$$\text{Tr}[E(\hat{Q}_{Am}^\dagger \otimes \hat{Q}_{Bn}^\dagger)\hat{\gamma}_{AB}] = \text{Tr}[(\hat{Q}_{Am}^\dagger \otimes \hat{Q}_{Bn}^\dagger)E(\hat{\gamma}_{AB})],$$

where $E = E_A \otimes E_B$ describes two independent, local depolarization operations. For clarity, the depolarized two-particle rate matrix can be written explicitly:

$$E(\hat{\gamma}_{AB}) = \zeta_A \zeta_B \hat{\gamma}_{AB} + \frac{\zeta_A (1 - \zeta_A)}{2} \text{Tr}_{B}[\hat{\gamma}_{AB}] \otimes \hat{1} + \frac{\zeta_B (1 - \zeta_B)}{2} \text{Tr}_{A}[\hat{\gamma}_{AB}] \otimes \hat{1} + \frac{(1 - \zeta_A)(1 - \zeta_B)}{4} \hat{1} \otimes \hat{1},$$

where Tr$_{[\ldots]}$ denotes a partial trace over the spin degrees of freedom in dot $\alpha$. Taking again the example of (maximally entangled) spin singlets emitted with a rate $\gamma$, i.e. $\hat{\gamma}_{AB} = \gamma[\Psi_S]\langle\Psi_S|$, the depolarized rate $E(\hat{\gamma}_{AB}) = \gamma[\zeta_A \zeta_B[\Psi_S]\langle\Psi_S| + [1/4](1 - \zeta_A \zeta_B)\hat{1} \otimes \hat{1}]$ describes emission of Werner states [41], entangled only for $\zeta_A \zeta_B > 2/3$. This clearly illustrates the following: the two-particle transfer rate is the same for maximally entangled states detected with reduced efficiency as for partially entangled states detected with unit efficiency. As we now discuss, this insight greatly helps to develop measurement strategies for an unambiguous entanglement detection.

**Cross-correlations and entanglement detection.** From $F_\chi$ the different low-frequency cumulants are obtained by successive derivatives with respect to the counting fields. For the average electrical current at terminal $A_m$ (and similarly for $B_n$) we have $I_{Am} = -ie\partial_{X_{Am}} F_\chi|_{X = 0}$ giving

$$I_{Am} = e\text{Tr} \left[ \hat{Q}_{Am}^\dagger (\hat{\gamma}_A + \text{Tr}_B[\hat{\gamma}_{AB}]) \right].$$

The average current provides information about the single-particle processes through $A$ via $\hat{\gamma}_A$ as well as the local, reduced single-particle properties of the emitted pairs, via $\text{Tr}_B[\hat{\gamma}_{AB}]$. Consequently, $I_{Am}$ and $I_{Bn}$ are local quantities and cannot provide full information on the emitted two-particle state, in particular not on the entanglement.

Turning instead to the non-local cross-correlations between currents at reservoirs $am$ and $bn$, obtained as $S_{Am,Bn} = -e^2\partial_{X_{Am}} \partial_{X_{Bn}} F_\chi|_{X = 0}$, we have

$$S_{Am,Bn} = e^2\text{Tr} \left[ (\hat{Q}_{Am}^\dagger \otimes \hat{Q}_{Bn}^\dagger)\hat{\gamma}_{AB} \right].$$

From eqs. (5) and (1) it is clear that $S_{Am,Bn}$ is directly proportional to the corresponding two-particle emission rate. In particular, $S_{Am,Bn}$ provides direct information about the spin properties of the individual pairs, via $\hat{\gamma}_{AB}$. Moreover, $S_{Am,Bn}$ does not contain any information about the single-particle emission or correlation between emitted pairs (contributes only to next order in $\zeta_0/T_3; \zeta_{AB}/T_3$). This illustrates in a compelling way that a long-time measurement, with a large number of emitted pairs collected in the leads, effectively [23–26] constitutes an average over a large number of identically prepared pair spin states.

Importantly, the form of the cross-correlator in eq. (5) allows in principle for entanglement detection via, e.g. [28] a complete tomographic reconstruction of $\hat{\gamma}_{AB}$ or a test of a Bell inequality [23–26]. In both cases, one needs to perform a set of measurements with different polarization settings $\hat{n}_A$ and $\hat{n}_B$. However, the interpretation of the measurement result, in particular the answer to the question “is the emitted state entangled?”, depends both on the method of detection as well as an accurate knowledge of the detector efficiencies. This is clearly illustrated by considering separately two cases:

- When the detector efficiencies $\zeta_A$ are accurately known, i.e. both the FM-polarizations $p_\alpha$ and the spin-flip rates $n_\alpha$ can be faithfully determined, a quantum spin tomography is in principle viable [29] for arbitrary $\zeta_A$. In contrast, a Bell inequality test can only be performed for a limited range of efficiencies. Interestingly, as was discussed by Eberhardt already two decades ago [42], an a priori knowledge
about $\zeta_\alpha$ allows one to optimize polarization settings $\bar{n}_\alpha$, increasing the efficiency range for which a Bell inequality violation is possible.

When the efficiencies $\zeta_\alpha$ are not known, a quantum spin tomography can give an incorrect two-particle state. In particular, the reconstructed state can have an entanglement larger than the emitted state, opening up for the incorrect conclusion that entanglement has been detected. This “false detection” scenario can be illustrated by considering emission of Werner states $\kappa |\Psi_S\rangle\langle\Psi_S| + [1/4] (1 - \kappa) \otimes I$. An overestimation of the detector joint efficiency $\zeta_A\zeta_B$ by a factor $1/\kappa$ will then lead to tomographically reconstructed singlet state $|\Psi_S\rangle\langle\Psi_S|$, maximally entangled. In contrast, a Bell test with unknown detector efficiencies cannot lead to “false detection” of entanglement\(^1\). However, for unknown or ill-characterized efficiencies it is difficult to identify detector settings for an optimal violation, making a Bell test experimentally more demanding.

**Quantum master equation.** – We now turn to the derivation of the transport statistics, in terms of the reduced density operator of the state in the dots, $\rho = \rho(t)$. In the high-bias limit considered, the dynamics of $\rho$ can be described exactly by a Liouville equation on Lindblad form

$$\frac{d\rho}{dt} = \mathcal{L}_H(\rho) + \mathcal{L}_1(\rho) + \mathcal{L}_2(\rho) + \mathcal{L}_\eta(\rho) + \mathcal{L}_{TFM}(\rho).$$

Here the term $\mathcal{L}_H(\rho) = -i[H, \rho]$ describes the free evolution of the dot state, with $H = \sum_{\alpha\sigma} \varepsilon_\alpha d^\dagger_\alpha d_\alpha + d^\dagger_\alpha (d_\alpha)$ creating (annihilating) electrons in dot $\alpha$, with spin $\sigma$. The terms $\mathcal{L}_1(\rho)$ and $\mathcal{L}_2(\rho)$ describe the injection, from the entangler to the dots, of single- and two-particle states, respectively, and are given by

$$\mathcal{L}_1(\rho) = \sum_{\alpha\sigma'} \gamma_{\alpha\sigma'}^{\sigma\sigma'} \left[ d^\dagger_\alpha \rho d_\alpha d^\dagger_{\alpha\sigma'} - \frac{1}{2} \{ d^\dagger_{\alpha\sigma'}, d_\alpha \} \right]$$

and

$$\mathcal{L}_2(\rho) = \sum_{\tau\tau', \sigma} \gamma_{\tau\tau'}^{\sigma\sigma} \left[ d^\dagger_{\sigma\tau'} \rho d_{\sigma\tau'} - \frac{1}{2} \{ d_{\sigma\tau'}, d^\dagger_{\sigma\tau} \} \right].$$

To preserve the trace and ensure positivity of the density matrix the emission rate matrices must be Hermitian $\gamma_\alpha = \gamma_\alpha^\dagger$, $\gamma_{\alpha\beta} = \gamma_{\beta\alpha}^\dagger$. Entangler examples with detailed derivations of the one- and two-particle rate matrices are given below. Spin-flip scattering in the dots, with a rate $\eta$, is accounted for by the term

$$\mathcal{L}_\eta(\rho) = \eta \sum_{\alpha \sigma} \left[ d^\dagger_\alpha \rho d_\alpha d^\dagger_{\alpha\sigma} - \frac{1}{2} \{ d^\dagger_{\alpha\sigma}, d_\alpha \} \right].$$

The last term $\mathcal{L}_{TFM}(\rho)$ accounts for the coupling to the FM-reservoirs. In order to describe the full charge transfer statistics we have included counting fields $\chi_{\alpha m}$, with $m = \pm$, in the terms describing tunnelling out to the FM-reservoirs. This gives

$$\mathcal{L}_{TFM}(\rho) = \sum_{\alpha m \sigma \sigma'} \Gamma_\alpha \left[ d^\dagger_\alpha \rho d_\alpha \sigma d^\dagger_{\alpha m \sigma'} \chi_{\alpha m} - \frac{1}{2} \{ d^\dagger_{\alpha m \sigma}, d_\alpha \sigma \} \right],$$

where $Q_{\alpha m}^{\sigma\sigma'} = (Q_{\alpha m})_{\sigma\sigma'}$.

Working in the local spin-Fock basis $\{|0\rangle, |\sigma_A\rangle, |\sigma_B\rangle, |\sigma_{A\beta}\rangle\}$, with $\sigma, \tau = \uparrow, \downarrow$, eq. (6) can be written as a linear matrix equation $d\tilde{\rho}_\chi/dt = \mathcal{M}_\chi \tilde{\rho}_\chi$. Here $\mathcal{M}_\chi$ is a $\chi$-dependent transition rate matrix and the ($\chi$-dependent) vector $\tilde{\rho}_\chi = [\rho_0, \tilde{\rho}_A, \tilde{\rho}_B, \tilde{\rho}_{AB}]$, where $\rho_0$ is the matrix element for both dots empty and $\tilde{\rho}_A(\tilde{\rho}_B)$ a vector with the elements for only dot $\alpha$ (both dot $A$ and $B$) occupied, including one (two-)particle spin coherences.

Following ref. [30] the generating function $F_\chi$ can then be obtained from the eigenvalue problem

$$\mathcal{M}_\chi \tilde{\rho}_\chi = F_\chi \tilde{\rho}_\chi.$$  

To leading order in $\gamma_{\alpha\sigma'}/\Gamma_\alpha, \gamma_{\alpha\beta}/\Gamma_{\alpha\beta}$, it is possible to solve eq. (11) analytically, giving eq. (1) above.

**Transfer rate matrices.** – We now turn to a discussion of the single- and two-particle transfer rate matrices $\tilde{\gamma}_\alpha$ and $\tilde{\gamma}_{AB}$. As pointed out above, we consider an entangler subsystem which is weakly coupled to the dots $A$ and $B$. Together with the high-bias limit, this implies that single and pairs of particles which have tunneled out of the entangler will only tunnel out to the FM-leads, and never back to the entangler. In addition, we make the assumption that the many-body state of the entangler has a well-defined energy $E_c$. We can then evaluate the rate matrices within a $T$-matrix formulation of time-dependent many-body perturbation theory [43]. Discussing explicitly the key quantity, the two-particle rate matrix $\tilde{\gamma}_{AB}$, we have the spin-dependent golden rule result,

$$\tilde{\gamma}_{AB} = \frac{\Gamma_A + \Gamma_B}{(E_A + E_B - E_c)^2 + (\Gamma_A + \Gamma_B)^2/4}. $$

The spin dependence is contained in $\tilde{T}$, which has elements

$$(\tilde{T})_{\sigma\sigma', \tau\tau'} = Tr \left\{ \xi_{\alpha} H^{(2)}_{TF} |\sigma_A \tau\rangle \langle \sigma'_{B}\tau' | H^{(2)}_{TF} \right\},$$

where $\rho_\alpha$ is the density matrix of the isolated entangler, $H^{(2)}_{TF}$ the effective two-particle entangler-dot tunneling Hamiltonian and the trace is running over the degrees of freedom both of the entangler and the dots. The second factor in eq. (12) is $\int dE_A dE_B \nu_A(E_A) \nu_B(E_B) \delta(E_A + E_B - E_c)$, where $\nu_\alpha(E_\alpha) = (\Gamma_\alpha/e_\alpha^2 + 1/4)^{-1}$ is the density of states of dot $\alpha = A, B$, broadened by the coupling to the FM-leads. The single-particle rate matrices $\tilde{\gamma}_\alpha$ can be calculated in a similar manner. Note that in evaluating eq. (12), the polarization and tunnel rate

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\(^1\)Note that in electronic entanglers there is no “detection loop-hole”, all particles contribute to the cross-correlators.
Full counting statistics of generic spin entangler with quantum dot-ferromagnet detectors

Andreev entangler. – We first consider an Andreev entangler, of large interest due to the recent Cooper pair splitter experiments [9–14], discussed above. For completeness we show a schematic of the Andreev entangler-dot detector system in fig. 2, including relevant energies and tunneling rates. In line with our earlier assumptions we here consider the case where the dominating two-particle process is emission of split Cooper pairs into the normal reservoir. For the same reason, spin-flip scattering in the dots has no effect on the result in eq. (12).

The expression (12) opens up for a treatment of a wide range of two-particle spin entanglers coupled to quantum dot detectors, including entanglers with a possibly mixed spin state or spin-dependent entangler-dot tunneling. The only information required to evaluate the transfer rate matrices is the effective one- and two-particle tunneling rates and the energy and spin properties of the isolated entangler state. To demonstrate the viability of our approach we analyse two quantum-dot-based spin entanglers.

The rate matrix in eq. (12) as

$$\tilde{\gamma}_{\alpha\sigma} = \frac{2J^2(2\Gamma_A + 2\Gamma_B)}{(\varepsilon_A + \varepsilon_B)^2 + (2\Gamma_A + 2\Gamma_B)^2/4} |\Psi_S\rangle \langle \Psi_S|.$$  (14)

along the same lines one can obtain $\tilde{\gamma}_\alpha$. With $\tilde{\gamma}_{AB}$ and $\tilde{\gamma}_\alpha$ we can then evaluate via eq. (1) the full transport statistics for the Andreev entangler. We stress that from the expressions for the current, eq. (4), and cross-correlations, eq. (5), we reproduce known results [4,6] in the parameter limits corresponding to our assumptions.

Triple-dot entangler. – As a second example we consider the triple-dot entangler proposed in ref. [38]. Here the entangler consists of a quantum dot with a single, spin degenerate level at energy $\epsilon_d$ and an on-site interaction strength $U$, coupled to a normal lead. The entangler dot is further coupled to the detector dots via tunnel barriers with rates $t_A = t_B = t$. A schematic of the entangler-detector system is shown in fig. 3. As stated above, double occupancy of the detector dots $A$ and $B$ is prohibited by strong on-site interactions. To have a two-particle rate larger than or of the order of the single-particle rates, the level energies $\epsilon_d, \epsilon_A$ and $\epsilon_B$ are tuned to meet the two-particle resonance condition $2\epsilon_d + U \approx \epsilon_A + \epsilon_B$. Moreover, single-particle resonances, $\epsilon_d \approx \epsilon_A, \epsilon_d \approx \epsilon_B, U$, are avoided.

Under these assumptions, together with the high-bias condition, the state of the isolated entangler is the superconducting ground state, with an energy $E_0$ here taken to be zero. Moreover, the effective two-particle tunneling Hamiltonian can be conveniently written [44] as $H^{(2)}_T = J b_0 (d_{A0}^\dagger d_{B0}^\dagger - d_{A0}^\dagger d_{B0}^\dagger + \text{h.c.})$. Here $J$ is the tunneling element, depending on the properties of the superconductor and the coupling to the dots, and $b_0$ the destruction operator of a Cooper pair in the superconductor with the properties $\langle b_0 \rangle = \langle b_0^\dagger \rangle = \langle b_0^\dagger b_0 \rangle = 1$, where the average is taken with respect to the superconducting ground state. We then directly obtain $\hat{T} = J^2 |\Psi_S\rangle \langle \Psi_S|$ with $|\Psi_S\rangle$ the spin singlet state and, writing out explicitly, the two-particle rate matrix in eq. (12) as

$$\tilde{\gamma}_{AB} = \frac{2J^2(2\Gamma_A + 2\Gamma_B)}{(\varepsilon_A + \varepsilon_B)^2 + (2\Gamma_A + 2\Gamma_B)^2/4} |\Psi_S\rangle \langle \Psi_S|.$$  (15)
where $\delta \epsilon = 2\epsilon d + U - (\epsilon_A + \epsilon_B)$, the energy away from two-particle resonance. Along the same lines one can obtain $\gamma_n$. From the expressions for the current, eq. (4), we reproduce known results [38] in the parameter limits corresponding to our assumptions.

Efficiency bounds. – For the two examples discussed, it is instructive to analyse bounds on the detector efficiency for a test of a given Bell inequality. Here we consider the commonly used CHSH inequality [45], taking the emitted state to be a singlet, in line with eqs. (14) and (15). This gives the inequality $\zeta_{AB} \geq 1/\sqrt{2}$, i.e. for equal efficiencies $\zeta_A = \zeta_B \geq 0.84$ [22], which can be violated only for large FM-polarizations $p$ and small spin-flip scattering rates $\eta_A, \eta_B$. We also note that for an emitted Werner state, with singlet probability $p_S \leq 1$, the inequality becomes even stronger, $p_S \zeta_{AB} \geq 1/\sqrt{2}$.

Conclusions. – We have investigated the full counting statistics of spin-dependent single- and two-particle transfer in a generic entangler coupled to quantum dot-ferromagnet detectors. From the full statistics we have identified individual charge transfer events. Moreover, we have demonstrated that the current cross-correlators can be used to determine the two-particle spin state even in the presence of spin-flip scattering and limited ferromagnet polarization. For the future, it would be interesting to investigate in more detail how the obtained results are modified when relaxing assumptions such as equal dot-ferromagnet couplings and ferromagnet polarizations as well as single occupancy of detector dots, in order to strengthen the connection to experimentally realizable systems.

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Additional remark: During the preparation of the present manuscript we became aware of the recent work [46] where related aspects of entanglement detection in Cooper pair splitters were discussed.

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