Hadronic $W$ production and the Gottfried Sum Rule

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Abstract

The difference in production rate between $W^+$ and $W^-$ at hadron colliders is very sensitive to the the difference between up- and down-quark distributions in the proton. This sensitivity allows for a variety of useful measurements. We consider the difference $d_s(x, Q^2) - u_s(x, Q^2)$ in the sea distributions and the difference $\Delta u(x, Q^2) - \Delta d(x, Q^2)$ in the polarized parton distribution functions. In both cases we construct an asymmetry to reduce systematic uncertainties. Although we discuss measurements at the Tevatron and future hadron colliders, we find that the Brookhaven Relativistic Heavy Ion Collider (RHIC) is the most appropriate hadron collider for these measurements.
1 Introduction and Results

It has been appreciated for some time that useful information on the parton distribu-
tions of nucleons can be extracted from measurements of the charge asymmetry in
hadronic $W$ production\[1, 2, 3, 4\]. The asymmetry provides us with information on
the difference between the up- and down-quark structure of the proton, \textit{i.e.} on the
quantities $\delta q_v \equiv u_v - d_v$ and $\delta q_s \equiv d_s - u_s$. Here, as usual, the subscripts $v$ and $s$ refer
to the valence and sea components of the proton structure functions. The quantity $\delta q_v$
is studied in deep inelastic scattering experiments on proton and neutron targets and
is critical to the determination of the hadronic width of the $W$ from the ratio of the
$W$ and $Z$ cross sections in hadron collisions. While it is often assumed that $\delta q_s = 0$
on the basis of isospin symmetry, this does not have to be the case. As $u_v \neq d_v$,
evolution will inevitably result in $u_s \neq d_s$. Also, as there are more $u$- than $d$-quarks in
the proton, one might imagine that the further generation of $u\bar{u}$ pairs is suppressed by
Fermi statistics\[5\]. As we discuss further on, experimental evidence for the violation
of the Gottfried sum rule has been interpreted as being the result of the non-vanishing
of $\delta q_s$\[3, 6\].

Analogously for $W$ production with polarized beams, a double asymmetry in the
initial proton helicities and in the $W$ charge can provide us with useful information
on the up- and down-quark structure in the polarized proton. One of the theoretical
assumptions generally used in the interpretation on the EMC result on proton spin
is based on a combination of low energy experiment and isospin symmetry, \textit{i.e.} the
difference between the polarized $u$- and $d$ quark distributions. This quantity can be
directly measured at higher energy in $W$ production at a polarized proton-proton
collider.
The main point of this paper is that the $W$ charge asymmetry in hadron collisions measures both $\delta q_v$ and $\delta q_s$. We show, moreover, that the latter quantity can only be effectively probed in proton-proton colliders. $W$ production at RHIC thus has unique aspects. This includes the possibility of using polarized beams, which we discuss in some detail.

2 Gottfried sum rule in unpolarized production

The recent NMC\cite{7} result on the Gottfried\cite{8} sum rule

$$I_G(x) = \frac{1}{3} \int_x^1 dy [u_v(y) - d_v(y)] + \frac{2}{3} \int_x^1 dy [\bar{u}(y) - \bar{d}(y)]$$

(extrapolated to $x = 0$) of $I_G(0) = 0.240 \pm 0.016$ is significantly different from the result of $1/3$ expected if the usual assumptions are made, i.e. the proton consists of valence $uuu$ and $\bar{d}(y) = \bar{d}(y)$. This “violation” of the Gottfried sum rule is quite easy to understand — there are more $u$-quarks in the proton than $d$-quarks, so the Pauli exclusion principle suppresses the production of $u\bar{u}$ pairs in the sea relative to $d\bar{d}$ pairs\cite{5} — but an independent confirmation of this result is desirable.

The large number of $W$’s (we estimate 3000 for $\sqrt{s} = 200$ GeV and 300,000 for $\sqrt{s} = 500$ GeV) to be produced at the Brookhaven Relativistic Heavy Ion Collider (RHIC) will provide the necessary confirmation as has been suggested in the literature\cite{9}. Instead of considering the separate production of $W^+$ and $W^-$, consider the charge asymmetry $A_W$:\cite{1}:

$$N_W = \frac{d\sigma(W^+)}{dy_W} - \frac{d\sigma(W^-)}{dy_W}$$
$$D_W = \frac{d\sigma(W^+)}{dy_W} + \frac{d\sigma(W^-)}{dy_W}$$
$$A_W = \frac{N_W}{D_W}.$$  \hspace{1cm} (2)
It is straightforward to show that (to simplify the notation we suppress the dependence of the parton distribution functions on $Q^2$ in the following):

\[
N_W \propto \cos^2 \theta_c \{ u_s(x_2) [u_v(x_1) - d_v(x_1)] + u_v(x_2) [d_s(x_1) - u_s(x_1)] \} \\
+ \sin^2 \theta_c \{ u_v(x_1)s_s(x_2) - c_s(x_1)d_v(x_2) \} \\
+ (x_1 \leftrightarrow x_2)
\]  

(3)

and

\[
D_W \propto \cos^2 \theta_c \{ u_s(x_2) [u_v(x_1) + d_v(x_1)] + u_v(x_2) [d_s(x_1) + u_s(x_1)] + 2s_s(x_1)c_s(x_2) \} \\
+ \sin^2 \theta_c \{ u(x_1)s_s(x_2) + c_s(x_1)d(x_2) \} \\
+ (x_1 \leftrightarrow x_2),
\]  

(4)

where $\theta_c$ is the Cabibo angle. This gives the exact expression we use to calculate $A_W$.

In calculating these cross sections we include the usual $K$-factor for $W$ production, $K = 1 + 8\pi \alpha_s(M_W^2)/9$ (see, e.g., Ref. [1]).

To further illustrate our analysis, consider only the terms proportional to $\cos^2 \theta_c$. This is a relatively good approximation in any case as $\sin^2 \theta_c \sim 0.05$. In this limit, the numerator factor reduces to

\[
N_W \approx u_s(x_2)\delta q_v(x_1) + u_v(x_2)\delta q_s(x_1) + (x_1 \leftrightarrow x_2)
\]  

(5)

where \( \delta q_v(x) = u_v(x) - d_v(x) \) and \( \delta q_s(x) = d_s(x) - u_s(x) \). $W$ production is central and relatively flat in rapidity, so we examine $A_W$ at $y = 0$. Here, \( x_1 = x_2 = x_0 \equiv M_W \frac{M_W}{\sqrt{s}} \), and

\[
A_W \bigg|_{y=0} \approx \frac{u_s(x_0)\delta q_v(x_0) + u_v(x_0)\delta q_s(x_0)}{u_s(x_0) [u_v(x_0) + d_v(x_0)] + u_v(x_0) [d_s(x_0) + u_s(x_0)]}
\]  

(6)
A similar asymmetry can be formed at $p\bar{p}$ colliders. In this case, the numerator factor is

$$N_W \propto \cos^2 \theta_c \{u_s(x_2)\delta q_v(x_1) + \frac{1}{2} \delta q_v(x_1) (u_v(x_2) + d_v(x_2)) + u_v(x_1)\delta q_s(x_2)\}$$

$$+ \sin^2 \theta_c \{u_v(x_1)s_s(x_2) + c_s(x_1)d_v(x_2)\}$$

$$- (x_1 \leftrightarrow x_2).$$

(7)

The expression separates into terms proportional to $\delta q_v$ and $\delta q_s$ and another small term proportional to $\sin^2 \theta_c$, but it is not as simple as the expression (5) for the asymmetry at $pp$ colliders. More importantly, the asymmetry vanishes for $y = 0$ at $p\bar{p}$ colliders. In the central region of rapidity, where most of the $W$’s are produced, the asymmetry is small, severely limiting the usefulness of $p\bar{p}$ colliders in this measurement.

We have demonstrated the usefulness of $pp$ colliders in the measurement of the difference in the distributions $u_s$ and $d_s$. RHIC has been designed with a variable center-of-mass energy, ranging from 50-500 GeV. This gives an $x_0$ range $0.16 \leq x_0 \leq 1$ (though with low statistics at threshold). SSC and LHC are sensitive to $x_0 = 0.002$ and $0.005$ respectively (although moving away from $y = 0$ allows for a range of sensitivity about these values). At such small values of $x$, sea quarks dominate $W$ production. Referring to Eqn. (6), it is clear that the term $u_s(x_0)\delta q_v(x_0)$ will easily dominate the asymmetry, leading to very little sensitivity to $\delta q_s(x_0)$ at SSC and LHC.

This argument does not hold at RHIC since the larger $x$ region probed leads to a larger valence contribution, which in turn leads to a rather large contribution from the difference in the light sea quark distributions. Thus RHIC is the only $pp$ collider where this measurement can be performed.

Our conclusion thus far is that RHIC is uniquely suited for this measurement, because a) $A_W$ is large at small rapidity where the statistics are best, unlike all $p\bar{p}$
colliders, b) RHIC probes the correct $x$-range, unlike $pp$ supercolliders and c) RHIC has variable center-of-mass energy, which allows for measurement at different $x$ without measuring at different rapidity.

3 Results on the Gottfried Sum Rule

Consider the asymmetry $A_W$ at the Tevatron. In Fig. 1 we show the cross section for $W^+$ and $W^-$ production, for various parton distribution functions. Next, we show the numerator factor $N_W$ and the dominant contribution from $\delta q_s$ (Fig. 2). The contribution from a non-$SU(2)$ symmetric sea is very small compared to the contribution from the difference in valence distributions, $\delta q_v$. As we need to measure this asymmetry at large rapidity ($\sim 1.5$), we see that information on $d_s - u_s$ will be difficult to obtain from the Tevatron.

Some data exist\cite{10} on the leptonic asymmetry $A_\ell$ at the Tevatron. Here,

\begin{align*}
N_\ell &= \frac{d\sigma(\ell^-)}{dy_\ell} - \frac{d\sigma(\ell^+)}{dy_\ell} \\
D_\ell &= \frac{d\sigma(\ell^-)}{dy_\ell} + \frac{d\sigma(\ell^+)}{dy_\ell} \\
A_\ell &= \frac{N_\ell}{D_\ell} \quad (8)
\end{align*}

is the charge asymmetry in rapidity distribution for single lepton production. We show in Fig. 3 our result for $A_e$, the asymmetry for electrons and compare it to the data from CDF\cite{10}. In Fig. 4 we compare the contribution to $N_\ell$ from $\delta q_s$ with the total numerator.

Next, consider the asymmetry $A_W$ at hadron supercolliders. Our results for the SSC are also representative of the results that can be obtained at the LHC. In Fig. 5 we show the numerator factor $N_W$ and the dominant contribution from $\delta q_s$. It is clear
that the contribution from a non-$SU(2)$ symmetric sea is again very small compared to the contribution from the difference in valence distributions, $\delta q_v$. Although the statistics will be very good, the relative smallness of $\delta q_s$ will make this measurement difficult.

Finally, we consider the asymmetry $A_W$ at RHIC, a high luminosity ($\mathcal{L} = 2 \times 10^{32} \text{ cm}^{-2}\text{sec}^{-1} = 6000 \text{ pb}^{-1}/\text{yr}$) collider capable of producing proton-proton collisions for center-of-mass energies between 50 and 500 GeV. RHIC will also collide protons on heavy nuclei in order to connect to existing data and heavy ions on heavy ions. We assume a nominal running time of two months at full luminosity. In order to be somewhat conservative, we estimate event numbers based on 300 pb$^{-1}$ integrated luminosity. We assume a generic collider-type detector, and we require the photons and electrons observed to lie in the rapidity range $|y| \leq 2$. This simulates the acceptance of the proposed STAR detector at RHIC, level 2 for photons and electrons[12]. We do not consider the possibility of the detection of muons at RHIC. We show the numerator factor $N_W$, and the contribution to $N_W$ from $\delta q_s$ in Figs. 6a and 6b respectively for $\sqrt{s} = 200 \text{ GeV}$ and 500 GeV. At RHIC, the contributions from $\delta q_s$ are in no way suppressed; they can in fact dominate $N_W$. The parton distributions with a symmetric sea (e.g., HMRSB, EHLQ2 and MRSD0) generally give smaller asymmetries (especially at small $W$ rapidity, $y_W$) than the distributions with non-symmetric sea (e.g., MRSDM, CTEQ1L and CTEQ1M). Fig. 7 gives the asymmetry $A_W$ for $\sqrt{s} = 500 \text{ GeV}$ for various choices of the parton distribution functions. Here at last we can observe in the asymmetry the effects of the non-symmetric sea, although for this particular scenario, the $\delta q_s$ contribution using the CTEQ1L distributions accidentally small. The explanation for this phenomenon is given in Ref. [27] - the sign of $\bar{d} - \bar{u}$ changes with
$x$, and $W$ production at $\sqrt{s} = 500$ GeV is sensitive to values of $x$ near the crossover point for the CTEQ1L distributions.

Having determined that this measurement is in principle possible at RHIC, we study events with a single high-$p_T$ lepton and missing $p_T$. In Figs. 8a and 8b we show the rapidity distributions for production of $e^+$ and $e^-$ at center of mass energies of 200 and 500 GeV, respectively. Even in the worst case ($e^-$ production at 200 GeV), we expect about 15 events in the rapidity range $|y_e| < 0.5$, so the statistics should be sufficient. In Figs. 9 we compare the contributions to $N_\ell$. Again, the parton distributions with a symmetric sea give generally smaller asymmetries that the parton distributions with non-symmetric sea, as can be seen from the $\delta q_s$ contributions in Fig. 9, although the accidentally small $\delta q_s$ contribution from CTEQ1L distributions at $\sqrt{s} = 500$ GeV is apparent. Finally, in Fig. 10 we show the observable asymmetry $A_e$ for center-of-mass energies of 200 and 500 GeV. Here, especially at small electron rapidity $y_e$, the asymmetries from the symmetric sea parton distributions cluster together and are reasonably separated from the asymmetries from the non-symmetric sea parton distributions.

4 $\Delta u - \Delta d$ in polarized production

The interpretation of the EMC result on proton spin relies upon two low energy quantities, namely $(\Delta u - \Delta d) = g_A$ (from isospin invariance) and $(\Delta u + \Delta d - 2\Delta s) = 3 F - D$ (from $SU(3)$ symmetry) where $g_A$ is the axial-vector coupling in neutron $\beta$-decay and $F$ and $D$ are the invariant amplitudes for the axial-vector current in hyperon semileptonic decays. Here, in the usual notation

$$\Delta q(\mu^2) s_\alpha = \langle p, s \mid \bar{q} \gamma_\alpha \gamma_5 q \mid p, s \rangle |\mu^2, \quad (9)$$
where \( |p,s\rangle \) is a proton state with spin vector \( s_{\alpha} \). The quantity \( \Delta q \) is related to the polarized parton distribution function for quark \( q \)

\[
\Delta q(Q^2) = \int_0^1 \Delta q(x, Q^2) dx
\]

where \( \Delta q(x, Q^2) \) is the difference in the distributions of a quark \( q \) in a longitudinally polarized proton with the same and opposite helicity as the proton. As long as the assumptions of isospin invariance and \( SU(3) \) symmetry are valid, these particular (non-singlet) combinations of the \( \Delta q \)'s will not run with \( Q^2 \); they reduce to the difference and sum of the first moments of the polarized up- and down-valence distributions. Thus, the two quantities above can be used at higher energy even thought they are extracted from low energy data. It would, nonetheless, be comforting to extract one (or both) of these quantities at a higher energy as a consistency check.

Analogous to the earlier discussion of unpolarized \( W \) production we can derive the following for polarized \( W \) production:

\[
\frac{d\sigma^{++}(W^+)}{dy_W} - \frac{d\sigma^{+-}(W^+)}{dy_W} \propto \cos^2 \theta_c \Delta u(x_1)\Delta\bar{q}(x_2) + \sin^2 \theta_c \Delta u(x_1)\Delta s(x_2) + (x_1 \leftrightarrow x_2)
\]

\[
\frac{d\sigma^{++}(W^-)}{dy_W} - \frac{d\sigma^{+-}(W^-)}{dy_W} \propto \cos^2 \theta_c \Delta d(x_1)\Delta\bar{q}(x_2) + \sin^2 \theta_c \Delta \bar{q}(x_1)\Delta s(x_2) + (x_1 \leftrightarrow x_2).
\]

We assume that the charm content of the proton is zero, which is a very good approximation below Tevatron energies, and \( \Delta\bar{u} = \Delta\bar{d} = \Delta u_s = \Delta d_s \equiv \Delta \bar{q} \), and we again suppress the dependence of the parton distribution functions on \( Q^2 \). In the usual notation, \( \sigma^{++} (\sigma^{+-}) \) is the cross section for two protons with the same (opposite) helicities and the polarized parton distribution functions as described above. As before,
we include the standard $K$-factor in our calculation of the cross section. Just as the difference of cross sections for unpolarized $W$ production is the most interesting and useful quantity, we find that the difference

$$
\left\{ \left[ \frac{d\sigma^{++}(W^+)}{dy_W} - \frac{d\sigma^{-+}(W^+)}{dy_W} \right] - \left[ \frac{d\sigma^{++}(W^-)}{dy_W} - \frac{d\sigma^{-+}(W^-)}{dy_W} \right] \right\} \propto \left\{ \cos^2 \theta_c \left( \Delta u(x_1) - \Delta d(x_1) \right) \Delta \bar{q}(x_2) + \sin^2 \theta_c \Delta u_c(x_1) \Delta s(x_2) + (x_1 \leftrightarrow x_2) \right\}
$$

is particularly useful in isolating the differences we would like to measure.

Consider the quantity $A_{LL}$:

$$
A_{LL} = \left\{ \left[ \frac{d\sigma^{++}(W^+)}{dy_W} - \frac{d\sigma^{-+}(W^+)}{dy_W} \right] - \left[ \frac{d\sigma^{++}(W^-)}{dy_W} - \frac{d\sigma^{-+}(W^-)}{dy_W} \right] \right\}.
$$

(14)

This asymmetry is again dominated by the $\cos^2 \theta_c$ terms, not only because $\sin^2 \theta_c$ is small but also because $\Delta u(x)$ and $\Delta d(x)$ have opposite signs, so the interference is constructive. In the limit that we only consider the $\cos^2 \theta_c$ terms, this asymmetry reduces to

$$
A_{LL} \approx \left\{ \frac{[\Delta u(x_1) - \Delta d(x_1)]\Delta \bar{q}(x_2) + (x_1 \leftrightarrow x_2)}{[u(x_1) + d(x_1)]q(x_2) + (x_1 \leftrightarrow x_2)} \right\}.
$$

(15)

Finally, because the cross section for central $W$’s is large, we can examine this asymmetry at $y = 0$, which gives:

$$
A_{LL} \bigg|_{y=0} \approx \frac{[\Delta u(x_0) - \Delta d(x_0)]\Delta \bar{q}(x_0)}{[u(x_0) + d(x_0)]q(x_0)},
$$

(16)

where $x_0 = \frac{M_W}{\sqrt{s}}$ as before. Inclusion of the $\sin^2 \theta_c$ terms will slightly modify this result, but it should be possible to extract the combination $\Delta u(x) - \Delta d(x)$ from the data due to the large statistics. Furthermore, by running RHIC at several center-of-mass energies and moving away from $y = 0$, it will be possible to map the desired
polarized parton distributions for a range of $x$ values. We give the expressions above under the approximation $\sin^2 \theta_c = 0$ merely to simplify the discussion. In the figures we use the exact expressions.

5 Results on $\Delta u - \Delta d$

A program of polarized proton-proton collisions, at full energy and luminosity, is being discussed[11]. We show in Fig. 11, the $W$ rapidity distribution, $d\sigma/dy_W$, for $W^+$ and $W^-$ production at two representative RHIC energies, 500 GeV and 200 GeV. Given the large integrated luminosity from a two month run at RHIC ($\sim 300$ pb$^{-1}$), there will be a rather large number of $W$’s produced at RHIC. Next, we present our results for $A_{LL}$ as a function of the $W$ rapidity $y_W$ for RHIC at $\sqrt{s} = 200$ GeV (Fig. 12a) and at $\sqrt{s} = 500$ GeV (Fig. 12b). In Fig. 12, we use two extreme cases for the polarized parton distribution functions, one in which the polarized gluon is taken to be large (and consequently the polarized strange quark distribution is small - set $I_S=0$ of Bourrely, Guillet and Chiappetta (BGC)[13]) and one in which the polarized gluon is small (and the polarized strange quark distribution large - set $I_S=1$ of BGC). We note that the large difference in the asymmetries calculated using the different polarized parton distribution functions is due primarily to the large uncertainty in the polarized sea distribution. This uncertainty can be traced back to the different explanations of the EMC effect on proton spin. This uncertainty will be reduced using other measurements at RHIC, e.g., jet[14, 15, 17], direct photon[17, 18, 19], heavy quark[20] or charmonium production[21, 22, 23, 24], designed to measure the polarized gluon contribution to proton spin. Once this error is reduced, it will be possible to extract the desired quantity, $\Delta u - \Delta d$. Also, the polarized parton distribution set with the ‘smaller’
polarized sea quark gives a larger asymmetry at large rapidity for $\sqrt{s} = 200$ GeV. This is a consequence of the different large-$x$ behavior in the polarized parton distributions; the event rate at large rapidity is sufficiently small that for practical purposes, this region of large rapidity will contribute very little to the observed asymmetry.

Of course, the experiments at RHIC will not directly observe $W$’s, but rather will reconstruct them from the sample of single isolated lepton and missing $p_T$, so one should also study the possibilities including the decay. To simulate the acceptance of the STAR detector, we impose a cut of $|y| \leq 2$ on electrons; STAR will have no acceptance for muons\[12\]. In addition, we place a cut on the transverse mass of the reconstructed $W$ of 50 GeV, which will remove much of the background without significantly decreasing the signal. In Fig. 13 we show the rapidity distribution, $d\sigma/dy_e$, for single electrons and positrons for two representative center of mass energies, 500 and 200 GeV. Even including the $W$ decay there will still be a large number of events (about 130 for $\sqrt{s} = 200$ GeV and 13000 for $\sqrt{s} = 500$ GeV). We give, in Fig. 14 the asymmetry $A_{LL}^e$ including electronic decay of the $W$, with $\sqrt{s} = 200$ GeV (Fig. 14a) and with $\sqrt{s} = 500$ GeV (Fig. 14b). The curves follow the convention of Fig. 12. Here,

$$A_{LL}^e = \left\{ \frac{d\sigma^{++}(e^+)}{dy_e} - \frac{d\sigma^{+-}(e^+)}{dy_e} \right\} - \left\{ \frac{d\sigma^{++}(e^-)}{dy_e} - \frac{d\sigma^{+-}(e^-)}{dy_e} \right\} .$$

Again, the polarized parton distributions that give use a large polarized gluon to explain the EMC effect on the spin of the proton give give a small asymmetry due to the presence of a $\Delta \bar{q}$ factor in the asymmetry, and at $\sqrt{s} = 200$ GeV the different large-$x$ behavior of the polarized parton distributions is apparent in the crossings of the asymmetries. Once the uncertainties in the polarized gluon and sea distributions are reduced, it will be possible to extract $\Delta u - \Delta d$. 

\[12\]
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Figure Captions

Figure 1 Rapidity distribution for the production of $W^-$ and $W^+$ at the Tevatron. The different curves correspond to different parton distribution functions: dotted - HMRSB[25], double dotted - EHLQ set 2[26], solid - MRSD0[4], dashed - MRSDM[4], dot-dashed - CTEQ1L[27] and double dot-dashed - CTEQ1M[27].

Figure 2 Numerator factor $N_W$ (as defined in the text) versus $y_W$ at the Tevatron. The different curves follow the convention of Figure 1. Both the total $N_W$ and the contribution from $\delta q_s$ are shown.

Figure 3 Electron asymmetry $A_e$ at the Tevatron. The labeling of the different curves follows the convention of Figure 1. Data are from Ref. [10].

Figure 4 Numerator factor $N_e$ (as defined in the text) versus $y_e$ at the Tevatron. The different curves follow the convention of Figure 1. Both the total numerator factor and the contribution from $\delta q_s$ are shown.

Figure 5 As Fig. 2, at SSC.

Figure 6 As Figs. 2, 5, for a) RHIC at 200 GeV and b) RHIC at 500 GeV.

Figure 7 The charge asymmetry $A_W$ (as defined in the text) versus $y_W$ at RHIC for $\sqrt{s} = 500$ GeV. The different curves follow the convention of Figure 1.

Figure 8 The rapidity distribution for single $e^+$ and $e^-$ production at RHIC. The different curves follow the convention of Figure 1. Figure 8a shows the rapidity distribution for $\sqrt{s} = 200$ GeV and Figure 8b show the rapidity distribution for $\sqrt{s} = 500$ GeV.
Figure 9 As Fig. 4, for a) RHIC at 200 GeV and b) RHIC at 500 GeV.

Figure 10 The charge asymmetry $A_e$ in single lepton production (as defined in the text) versus $y_e$ at RHIC. The different curves follow the convention of Figure 1. Figure 10a shows $A_e$ at $\sqrt{s} = 200$ GeV and Figure 10b shows $A_e$ at $\sqrt{s} = 500$ GeV.

Figure 11 The rapidity distribution for production of $W^-$ and $W^+$ at RHIC, using EHLQ, set 226 parton distribution functions. The solid curve corresponds to $W^+$ production at $\sqrt{s} = 500$ GeV, the dashed curve corresponds to $W^-$ production at $\sqrt{s} = 500$ GeV, the dotted curve corresponds to $W^+$ production at $\sqrt{s} = 200$ GeV and the dot-dashed curve corresponds to $W^-$ production at $\sqrt{s} = 200$ GeV.

Figure 12 The asymmetry $A_{LL}$ (as defined in the text) versus $y_W$ at RHIC. Figure 12a shows $A_{LL}$ for $\sqrt{s} = 200$ GeV and Figure 12b shows $A_{LL}$ for $\sqrt{s} = 500$ GeV. We use EHLQ, set 226 for the unpolarized parton distributions, while in both cases, the solid curve uses the polarized parton distribution functions of BGC, set $I_S = 0$ and the dashed curve used the polarized parton distribution functions of BGC, set $I_S = 1$.

Figure 13 The rapidity distribution for production of $e^-$ and $e^+$ at RHIC, using EHLQ, set 226 parton distribution functions. The solid curve corresponds to $e^+$ production at $\sqrt{s} = 500$ GeV, the dashed curve corresponds to $e^-$ production at $\sqrt{s} = 500$ GeV, the dotted curve corresponds to $e^+$ production at $\sqrt{s} = 200$ GeV and the dot-dashed curve corresponds to $e^-$ production at $\sqrt{s} = 200$ GeV.

Figure 14 The asymmetry $A_{eLL}$ (as defined in the text) versus $y_e$ at RHIC. Figure 14a shows $A_{eLL}$ for $\sqrt{s} = 200$ GeV and Figure 14b shows $A_{eLL}$ for $\sqrt{s} = 500$ GeV. We use
EHLQ, set 2 for the unpolarized parton distributions, while in both cases, the solid curve uses the polarized parton distribution functions of BGC, set $I_S = 0$ and the dashed curve used the polarized parton distribution functions of BGC, set $I_S = 1$. 