Interference of Degenerate Polariton Condensates in Microcavities

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We demonstrate theoretically that the interaction of two degenerate condensates of exciton-polaritons in microcavities leads to polarization dependent parametric oscillations. The resonant polariton-polariton scattering is governed by the polarization of the condensates and can be fully suppressed for certain polarizations. A polarization controlled solid state optical gate is proposed.

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Introduction. With the recent experimental confirmation of Bose-Einstein condensation (BEC) in semiconductors microcavities [1], the use of quantum interference to construct new devices becomes a real possibility. The potentiality of microcavities for the realization of micron-sized optical parametric oscillators (OPOs) has been recently revealed [2]. In these systems, photons in the cavity are strongly coupled to excitons in the interior quantum wells, forming new normal modes known as (exciton-)polaritons [3]. Optical parametric oscillations take place due to resonant polariton-polariton scattering processes, which maintain coherence in the system. Polaritons are optically injected in the initial (pump) state and are then scattered into states with lower and higher frequencies, referred to as signal and idler respectively. Since coherence is preserved, the reverse scattering rebuilds the population of the pump state allowing oscillations to develop in which polaritons fluctuate between pump, signal and idler states.

Very recently, a degenerate OPO based on a microcavity has been studied experimentally [4]. The cavity was pumped by two continuous wave lasers having the same energy but opposite in-plane wavevectors. In this configuration, the signal and idler states are symmetric with respect to the pump states and have the same energy (Fig. 1a), which is expected to allow for much longer coherence times than in the conventional microcavity OPO [5, 6]. In this Letter we present a quantum theory of the degenerate OPO, which accounts for the polarization of light. We show that the distribution of polaritons in reciprocal space and their scattering are governed by the polarization of the two pumps, so that a polarization controlled optical gate of a micron size can be realized.

Polaritons are composite bosons and have two allowed spin projections on the axis of the cavity (±1). Their polarization (linear, circular or elliptical) can be fully described by a 3D vector called pseudospin [7]. Theoretically, the effect of polariton-polariton interactions is accounted for using the zero-range interaction and mean-field approximations that lead to the Gross-Pitaevskii equations. In previous work these equations were used to study the scattering of polaritons by a single impurity [8], the spatial structure formed by microcavity parametric oscillator polaritons [9] and the dispersion of polariton superfluids [10]. In this work we consider the interference of polarized polariton liquids in the degenerate OPO. From classical arguments one would expect polaritons to scatter equally around an elastic circle in reciprocal space (Fig. 1b). Indeed, all the scattering trajectories shown in Fig. 1b are equally allowed by energy-momentum conservation laws and the scattering matrix element can be assumed wavevector independent with a good accuracy [7, 8, 9, 10]. However, experiments [4] and the theory presented here reveal that a non-uniform intensity can appear. This symmetry breaking of scattered polaritons takes place due to quantum interference. Specifically, if the pumps are co-linearly polarized then polaritons preferentially scatter to the wavevectors perpendicular to the pumps' [Fig. 1b]. Contrary, if the pumps are cross-polarized then the scattering to perpendicular directions is greatly suppressed.

Theoretical Model. A coherent ensemble of polaritons is described by two coupled wavefunctions, χ(⃗r) and...
\( \vec{\sigma}(\vec{x}) \), which represent excitons and photons in a microcavity respectively. Each wavefunction is a vector with two components representing two orthogonal linear polarizations (say \( x \) and \( y \)). Following a standard mean field treatment the evolution of the wavefunctions is described by the Gross-Pitaevskii equations \[10\]:

\[
\begin{align*}
\hbar \frac{\partial \vec{\chi}(\vec{x}, t)}{\partial t} &= - \frac{\hbar^2 \nabla^2}{2m_x} \vec{\chi}(\vec{x}, t) + \Omega \vec{\sigma}(\vec{x}) + V_0 \vec{\chi}'(\vec{x}) \cdot \vec{\chi}(\vec{x}) \\
&\quad - V_1 \vec{\chi}'(\vec{x}) \cdot \vec{\chi}(\vec{x}) - \frac{\hbar}{2\tau_x} \vec{\chi}(\vec{x}) \\
\theta \frac{\partial \vec{\phi}(\vec{x}, t)}{\partial t} &= - \frac{\hbar^2 \nabla^2}{2m_\phi} \vec{\phi}(\vec{x}, t) + \Omega \vec{\chi}(\vec{x}) + \vec{f}(\vec{x}, t) \\
&\quad + \frac{\hbar}{2\tau_\phi} \vec{\phi}(\vec{x}, t)
\end{align*}
\]  

(1)

\( m_x \) and \( m_\phi \) are effective masses assigned to the parabolic dispersion of excitons and cavity photons with respect to the in-plane wavevector. \( \Omega \) is the exciton-photon coupling energy, which is related to the quality factor and exciton decay rate \[11\] \[12\]. \( V_0 \) and \( V_1 \) are constants determining the strength of the non-linear interactions \[10\]. \( \tau_x \) and \( \tau_\phi \) are the lifetimes of excitons and photons, which account for the inelastic scattering and radiative decay of polaritons. \( \vec{f}(\vec{x}, t) \) represents an optical pumping, which for a single, continuous wave, Gaussian pump is given by the Fourier integral:

\[
\vec{f}(\vec{x}, t) = \left( \frac{A_x}{A_y} \right) \int \frac{d\vec{k}}{(2\pi)^3/2} \left( \frac{i\Gamma}{E_{LP}(\vec{k}) - E_p - i\Gamma} \right) e^{-iE_p/t} e^{-L^2(k_p)^2/4}
\]

(3)

\( A_x \) and \( A_y \) define the amplitudes of the two linearly polarized components of the pump. \( E_p \) is the pump energy, \( k_p \) is the pump in-plane wavevector and \( L \) is the width of the laser spot in real space. The pump energy should be chosen higher (0.35meV in our calculation) than the energy of the bare lower polariton branch to compensate for the energy renormalization (blue shift) caused by forward scattering processes \[13\]. The fraction \( \Omega / \left( E_{LP}(\vec{k}) - E_p - i\Gamma \right) \) is introduced to account for non-uniform absorption of the Gaussian optical pump. The fraction is derived from a Lorentz oscillator model where \( \Gamma \) is the homogeneous oscillator (HWHM) linewidth \[13\], \( E_{LP}(\vec{k}) \) is the bare dispersion of the lower polariton branch. If there is more than one pump then \( \vec{f} \) can be constructed by a superposition of individual pumps. \( \vec{f}_b(\vec{x}, t) \) is a Langevin noise term much smaller in magnitude (\( \times 10^{-4} \)) than the pump. We take this term as a randomly changing white noise with no correlation between each point in space and time. The noise term is necessary to seed the polariton scattering in the case of exactly co-polarized pumps.

Eqs. \[11\] \[12\] completely determine the dynamics of interacting polaritons once initial wavefunctions and parameters are defined. Note that in our model we have ignored the longitudinal-transverse splitting of polaritons as it is not essential in the experiment \[4\]. We also neglected the disorder induced elastic scattering of polaritons, which is a linear effect. It is dominated by the polariton-polariton scattering at high pumping intensities.

**Microcavity Interference Device.** We consider the generation of polaritons by two spatially overlapping linearly polarized pumps with opposite in-plane wavevectors. We assume both pumps have the same amplitude and phase. We solved Eqs. \[11\] \[12\] numerically using a (fifth-order) Adams-Bashforth-Moulton predictor-corrector method \[15\]. The wavefunctions were initially set to zero at all points in space and the evolution calculated for 2000ps. Figure 2 shows the time integrated Fourier transform of \( \vec{\phi}(\vec{x}) \), that is, the distribution of the photonic component in reciprocal space. The photonic component of the polariton wavefunction can be directly measured in optical experiments. When the two pumps are co-linearly polarized there is strong scattering to the wavevectors perpendicular to the pumps’. Note that this result is in sharp contrast to what we expected classically. When the two pumps are cross-linearly polarized the scattering to the signal states (with wavevector perpendicular to the pumps’) is suppressed. The device thus functions as an optical gate where the signal depends on the relative polarization of the input light. Figure 4 illustrates the dependence of the pseudospin vector around a circle in reciprocal space for different mutual orientations of the two pump polarizations. The pseudospin vector is the quantum analogue of the Stokes vector; its components are given by:

\[
\begin{align*}
S_x &= \frac{|\phi_x|^2 - |\phi_y|^2}{S_0} \quad S_y = \frac{\phi_x^* \phi_y + \phi_y^* \phi_x}{S_0} \\
S_z &= \frac{i \phi_x^* \phi_y - \phi_y^* \phi_x}{S_0} \quad S_0 = |\phi_x|^2 + |\phi_y|^2
\end{align*}
\]

From the symmetry of the pump polariton pseudospins in the case of cross-polarized pumps (Fig. 3i) one sees that the pseudospins of the signal states should not have any preferential direction. From this we should expect the suppression of scattering to the signal states. In the case of co-polarized pumps, we observe stronger scattering to the cross-polarized states. This is a consequence of the difference in sign between polariton-polariton interaction constants in singlet and triplet configurations as discussed in Ref. \[7\] \[16\].

The signal state amplitudes also exhibit a non-trivial time dependence (Fig. 4). In the case of co-polarized pumps we can see an oscillatory behaviour with period \( T \approx 135ps \). The oscillations become irregular and eventually vanish if we increase the difference between the
and the pump. The Gross-Pitaevskii equation describing
glect here the polarization degree of freedom, the lifetime
c space,
these oscillations we use a 1D analytic model in which
\(V\). (a-d) show the
FIG. 2: Time-averaged photon intensity in reciprocal space.
(a, e) show the \(x\) linearly polarized component; (e-h) show the \(y\) polarized component. Different rows correspond to differ-
ent pump polarizations; in (a, e) the pumps are co-linearly
polarized component. Different rows correspond to differ-
y polarized component; (e-h) show the
FIG. 3: (color) The variation of the (time-averaged) pseudo-
scattering dynamics is:
\[
\frac{i \hbar}{\partial t} \frac{\partial \rho(\theta)}{\partial t} = \hbar \omega \chi(\theta) + V' \chi^*(\theta - \pi) \int_{-\pi}^{\pi} \chi(\theta') \chi(\theta') d\theta'
\]
where \(V' = (V_0 - V_1) |\vec{k}_p|^2 / (16\pi^4)\) and \(\hbar \omega\) is the (renor-
malized) exciton energy. Eq. (4) accounts for the elastic
scattering processes where two excitons with wavevectors
given by \(\theta'\) and \(\theta' - \pi\) scatter to states with wavevectors
given by \(\theta\) and \(\theta - \pi\). Let us now introduce the variables:
\[
\rho(\theta) = \rho(\theta - \pi) = \chi(\theta - \pi) \chi(\theta)
\]
\[
n(\theta) = \chi^*(\theta) \chi(\theta)
\]
The evolution of these variables can be derived from Eq. (4). In the symmetric case, \(n(\theta) = n(\theta - \pi)\):
\[
\frac{i \hbar}{\partial t} \frac{\partial \rho(\theta)}{\partial t} = 2\hbar \omega \rho(\theta) + 2V' n(\theta) \int_{-\pi}^{\pi} \rho(\theta') d\theta'
\]
\[
\frac{i \hbar}{\partial t} \frac{\partial n(\theta)}{\partial t} = V' \left[ \rho^*(\theta) \int_{-\pi}^{\pi} \rho(\theta') d\theta' - \rho(\theta) \int_{-\pi}^{\pi} \rho^*(\theta') d\theta' \right]
\]
Eq. (8) shows that the total number of excitons is con-
served, that is, \(\frac{\partial n(\theta)}{\partial t} \int_{-\pi}^{\pi} n(\theta) d\theta = 0\). To proceed we inte-
grate Eq. (7) with respect to \(\theta\) and \(t\), which gives:
\[
\int_{-\pi}^{\pi} \rho(\theta) d\theta = A_0 e^{-2i(\omega + V') t / \hbar}
\]
where \(A_0 = \int_{-\pi}^{\pi} \rho(\theta) d\theta\) and \(N = \int_{-\pi}^{\pi} n(\theta) d\theta\). In-
trouding the variables \(\tilde{\rho}(\theta) = \rho(\theta) e^{2i(\omega + V') t / \hbar}\) and
where the scale on the $S_0$ axis is different in the lower and upper regions. Slight asymmetries appear between the two pump states and two signal states due to asymmetry in the Langevin noise. (a-c) use the same pump polarizations as in Fig.2 (a-c). Panels (d-f) show the corresponding evolution of the signal state horizontal polarization degree, $S_x$.

To compare this period to the numerical results we calculate the constants $A_0$ and $N$ from the distribution of total intensity, $S_0$, around the elastic circle. Eq. 13 then estimates the period as $T = 160\text{ps}$, which is close to the value obtained numerically.

**Conclusion.** We have demonstrated theoretically that the quantum interference between polaritons created by two spatially overlapping, degenerate, co-polarized laser beams, with opposite in-plane wavevectors, results in enhanced scattering to perpendicular directions. This symmetry breaking effect is in contrast to what one expects from classical considerations. The scattering is suppressed in the case of excitation with cross-polarized lasers, giving the basis for a polarization controlled solid state optical gate. Furthermore, the characteristic parametric oscillations in intensity and polarization between pump and signal states, that appear in the co-polarized case, can be fully suppressed by increasing the difference between pump polarizations.

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