Fast Simulation Method for Ocean Wave Base on Ocean Wave Spectrum and Improved Gerstner Model with GPU

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Abstract. For the randomness and complexity of ocean wave, and the simulation of large-scale ocean requires a great amount of computation, but the computational efficiency is low, the real-time ability is poor, a fast method of wave simulation is proposed based on the observation and research results of oceanography, it takes advantage of the grid which combined with the technique of LOD and projection, and use the height map of ocean which is formed by retrieval of ocean wave spectrum and directional spectrum to compute with FFT, and it uses the height map to cyclic mapping for the grid on GPU which combined with the technique of LOD and projection to get the dynamic height data and simulation of ocean. The experimental results show that the method is vivid and it conforms with randomness and complexity of ocean wave, it effectively improves the simulation speed of the wave and satisfied with the real-time ability and fidelity in simulation system of ocean.

1. Introduction
In recent years, with the maturity simulation of ocean wave, it is applied more and more. Such as reconstruction of Ocean Battlefield Environment, visualization of ocean prediction, training of navigation simulation, special effect production of film, 3D game and so on. Ocean wave is a kind of complicated stochastic process, at present, many scholars have made a lot of achievements for simulation of ocean wave in three areas[1]: The first is based on the physical characteristics of the wave of the sea to simulate, not directly simulates the movement of ocean wave but gets the dynamic state of each inside particle in wave through by calculation of Navier-Stokes equation, such as the method was proposed by Kass is similar to the reality but with a bad real-time effect, there are less application use the method. The second is based on the geometry of shape of ocean wave which constructed by mathematical formula of sines or cosines, it artificially sets wave parameters to simulate a approximately real surface of the sea[2], such as the method was proposed by Laibing Yan is easy to simulate and the real-time ability is better but with a poor authenticity and obviously artificial mechanisms, its suitable range is narrow and often used in the 3D game or animation. The third is based on the observational results of sea which use the appropriate ocean wave spectrum to simulate the surface of sea[3], such as the method was proposed by Baolong Wang with better ability of real-time and fidelity, the effect of simulation is well but the peaks of wave is smooth which is relative in realistic ocean in our life.

On the other hand, if each grid node requires calculation in each frame in the simulation, the height of wave in the complexity of ocean wave needs large computational amount, the CPU load is high and
it is the most important influencing factors to the real-time ability for simulation of ocean, moreover, large scale simulation of sea needs the technique of LOD[4], but it requires frequent grid computing when the viewpoint in the roaming status, it will affect the rendering efficiency.

This paper presents a method to simulate ocean which aimed at the shortage of simulation in ocean wave spectrum which with peaks of wave is not realistic and the poor ability of real-time in large scale simulation of ocean, it combines with improved Gerstner model and algorithm of FFT to simulate the sea, and it uses the grid which combined with the technique of LOD and projection to cyclic texture mapping to get the dynamic height data, so as to realize the simulation of ocean wave. The experimental results show that the method has the characters of randomness and complexity, it effectively reduces the occupancy rate of CPU and satisfied with the ability of real-time and fidelity for simulation of sea in the real-time simulation system.

2. Gerstner model

Gestner-wave[5] is the first method for Simulation of sea of Computer Graphics(1986 Fourier and Reeves), using the geometric model which is constructed by sines or cosines to simulate ocean is not satisfied with the authenticity of the simulation system, but the Gerstner model can better simulate the growing wave and micro motion in Ocean scene, compared to the sine wave, the peaks of Gerstner model is more sharp when the vertex of wave front move to the peaks, it well makes up for the fault of simulation which is used sine wave. In addition, the model is also proposed the approximate approach to solve the equations of fluid mechanics, it has two obvious advantages:

The normal vector of Gerstner model is easier to solve and it is conducive to rendering and operating to luminous efficiency in simulation of sea.

According to the habit of establishment of three-dimensional coordinates, assume that the static sea level is $X_0Z$ plane, $Y$-axis is vertically upwards, at time $t$, with the amplitude of $A$, the equations (2-1),(2-2) of motion of water particle which in the progress of wave propagation with the amplitude of $A$ as follow[6]:

\[
\begin{align*}
\vec{x}(x_0, t) &= \vec{x}_0 - (\vec{k}/k)A \sin(\vec{k} \cdot \vec{x}_0 - \omega t) \\
y(x_0, t) &= A \cos(\vec{k} \cdot \vec{x}_0 - \omega t)
\end{align*}
\] (2-1)

\[
\vec{x}_0 \text{ is the horizontal maker on the surface of sea when there is have no external forces, } \vec{x}_0 = (x_0, z_0),
\]

$y(x_0, t)$ is the height of sea surface, $\vec{k}$ is the vector which has same direction with the spread wave, $k$ is the number of waves which wavelength is $\lambda$, $k = 2\pi/\lambda$.

The image (Figure.1) below shows the Gerstner-wave with different parameters:

![Gerstner-wave with different parameters](image)

**Figure 1.** Gerstner-wave with different parameters

Moreover, on the basis of the above equations, the wave is regarded as the superposition of $N$ waves of Gerstner, and the equations of motion of the point on the sea level are as follows(2-3),(2-4)[7]:
\[ x(x_0, t) = x_0 + \sum_{i=1}^{N} \frac{\vec{k}_i}{k_i} A_i \sin(\vec{k}_i \cdot x_0 - \omega t + \phi_i) \]  
\[ y(x_0, t) = \sum_{i=1}^{N} A_i \cos(\vec{k}_i \cdot x_0 - \omega t + \phi_i) \]  

(2-3) (2-4)

3. Ocean wave spectrum
In the early 1940s, Neumann and Pierson are proposed to study ocean wave by using stochastic process, as the so-called method of spectral analysis, this is the beginning of spectral analysis to analyse the waves [8]. Ocean wave spectrum is also called energy spectrum which describes the relationship between the energy of the waves and the various components of the waves. And the ocean wave spectrum can be divided into two types: the first is the one-dimensional spectrum which shows the relationship between the wave energy and the frequency of the wave, it also called spectrum of ocean waves. The second is the two-dimensional spectrum which expresses the relationship between the wave energy and the frequency and direction of the waves, it also called directional spectrum of ocean waves.

Our commonly used now one-dimensional spectrum are as: Neumann spectrum, P-M spectrum, Phillips spectrum, Bretschneider spectrum [9], and so on. And the two-dimensional spectrum is generally used the commonly extension function to obtain, such as the function of ITTC(International Towing Tank Conference), ISSC( International Ship Structure Conference), SWOP(Stereo Wave Observation Project), and so forth. Ocean wave spectrum is also called energy spectrum which describes the relationship between the wave energy and the various components of the waves.

In general, if only consider about the factor of wind, the several ocean wave spectra have been proposed have the following standard form (3-1) when describing the waves:

\[ S(\omega) = \frac{A}{\omega_p} \exp\left[-\frac{\omega^2}{\omega_p^2}\right] \]  

(3-1)

A and B are the environmental parameters, in different observation area, wind speed, wind direction, the parameter A, B are rely on these measurements to determine their value, and the value of \( q, p \) is mainly considered the weight of high frequency and low frequency.

In addition, the magnitude and direction of the wind are random to led the waves travel in different directions, and even if the wind direction is fixed, there are some deviation about the direction of wave, In other words, the energy of wave is always distributed in different directions. In order to more accurately describe the random wave, introducing the dissipative information of wave in each direction, this is the direction spectrum. Assume \( \theta \) is the angle which deflected the prevailing wind direction. \( D(\theta, \omega) \) is the function of distributed direction, \( S(\theta, \omega) \) is the directional spectrum which generally defined as follow (3-2):

\[ S(\omega, \theta) = S(\omega) \cdot D(f, \theta) \]  

(3-2)

And the function of distributed direction should meet the following conditions (3-3):

\[ \int_{-\theta_0}^{\theta_0} D(\omega, \theta) \theta \, d\theta = 1 \]  

(3-3)

\(-\theta_0 \sim \theta_0\) is the range of direction which in Energy distribution. \( \omega \) is the Angular frequency.

4. Simulation method of ocean wave
The speed of simulation of ocean based on ocean wave spectrum is fast and the visual effect of large area of the sea is also better, but the shape of peaks of wave generated by algorithm of FFT is quite smooth, in fact, the shape of peaks is more sharp and the shape of troughs in practical situations even if the weather is calm and flat, and the phenomenon is more obvious in the abominable weather. On the other hand, the Gerstner model has bad performance for simulation of ocean and if the parameters of equations of motion are satisfied with some certain conditions, it could creates the phenomenon of
wave interaction as follows (Figure 2)[10], and this situation is not going to happen in the ocean scene. But the shape of peak is actual in the simulation of ocean which based on the method of Gerstner model.

![Figure 2. phenomenon of wave interaction](image)

To solve these two problems, this paper combines with advantages of high authenticity of ocean wave spectrum and fidelity of peaks of Gerstner model, considering that the real-time ability of large scale of ocean modeling, it proposes a improved method to simulate the ocean wave.

4.1. Improved gerstner model with controlled factor

In order to prevent the phenomenon of wave interaction which based on simulation of Gerstner model and control the steepness of the peak, this paper add a control factor $\mu$, the improved formula as follow (4-1,4-2):

$$
\begin{align}
    x(x_0, t) &= x_0 - \mu(k/k)A\sin(\frac{k \cdot x_0}{k} - \omega t) \\
    y(x_0, t) &= A\cos(\frac{k \cdot x_0}{k} - \omega t)
\end{align}
$$

(4-1)

(4-2)

The simulation of waveforms which uses the above model are shown below (Figure 3)

![Figure 3. simulation of Gerstner model with Increased](image)

Through repeated experiments, it is found regularity between $k \cdot A$, $\mu$: The condition of simulation without wave interaction is $k \cdot A > 1$, thus, the critical value $\mu$ shall conform to the relationship: $\mu \leq 1/(k \cdot A)$. And in order to generate the complex waveforms in two-dimensional plane, it makes the superposition of N Gerstner models, the equations of motion of the point on the sea level are as follows (4-3,4-4):
In this way, the profile of waveform generated by the superposition of N waves is shown in the figure (Figure 4) below, the abscissa represents the point on the sea level, the ordinate represents the amplitude about the point, therefore, using the improved Gerstner with controlled factor can generate the more sharp waveform, this is more in line with the random pattern of ocean waves in the scene.

\[
\begin{align*}
\vec{x}(x_0, t) &= x_0 - \mu \sum_{i=1}^{N} (\vec{k}_i / k_i) A_i \sin(\vec{k}_i \cdot \vec{x}_0 - \omega t + \phi_i) \\
y(x_0, t) &= \sum_{i=1}^{N} A_i \cos(\vec{k}_i \cdot \vec{x}_0 - \omega t + \phi_i)
\end{align*}
\]

(4-3)

(4-4)

4.2. FFT operation of gerstner model and ocean wave spectrum

It has been set up the improved Gerstner with controlled factor in 3.1, next, it should consider how to get the characteristic value of each wave from the sea wave spectrum, first to establish the grid of sea surface, using the coordinate values of each grid point to calculate the amplitude of wave to realize the simulation of the waves. This paper use the P-M spectrum as the frequency spectrum and use the function of SWOP as the extension function to construct the directional spectrum without loss of generality.

P-M spectrum is based on the Neumann spectrum, its observed data of the North Atlantic were analyzed by the method of spectral analysis, the spectrum shows the full development of the ocean wave, the formula is shown as the following (4-5) [11]:

\[
S_{\text{p-m}}(\omega) = S_p(\omega) \exp\left(-\frac{S}{4} \left(\frac{\omega}{\omega_p}\right)^4\right)
\]

(4-5)

In the formula, \(S_p(\omega) = \omega g / \omega_p^3\), \(g\) is the gravity acceleration, \(g=9.8\text{m/s}^2\), \(\omega_p\) is the Peak frequency of frequency spectrum, \(\omega_p=0.855\text{g/U5}\), U5 is the wind speed at 5 m from the sea level, using the international unit m/s, \(\alpha\) is the constant of Phillips, \(\alpha=0.0081\).

Due to the P-M spectrum is one-dimensional, and the sea surface is three-dimensional, the above formula can only be described as the distribution of energy with frequency, it needs the function of distributed direction to reflect the distribution of wave energy with frequency in different directions. The method applies function of SWOP as follows (4-6, 4-7, 4-8) [12]:

\[
D_{\text{SWOP}}(\omega, \theta) = \frac{1}{\pi} \left[1 + p \cos(2\theta) + q \cos(4\theta)\right] 0 \leq \theta \leq \pi / 2
\]

(4-6)

\[
p = 0.5 + 0.82 \exp\left[-\frac{1}{2} \left(\frac{\omega}{\omega_p}\right)^4\right]
\]

(4-7)

\[
q = 0.32 \exp\left[-\frac{1}{2} \left(\frac{\omega}{\omega_p}\right)^4\right]
\]

(4-8)

According to the formula combined with P-M spectrum to get the spectrum of wave number as follows:
Then, the $S_{PM-SWOP}$ needs to discretize which means the frequency range and direction range of wave motion are divided, obtaining the interval of angular frequency $\Delta \omega$ and direction $\Delta \theta$, determining the angular frequency $\omega_i$ and direction angle, the relationship between wave spectrum and amplitude is as follows [13]:

$$A_j = \sqrt{2S(\omega_i, \theta_i)\Delta \omega \Delta \theta}$$

(4-10)

By using this formula, it can get the wave amplitude of different frequency and direction angle based on $S_{PM-SWOP}$, at the time, bringing the acquired amplitude $A_{ij}$ of discrete points on the grid of sea level to the improved Gerstner with controlled factor to FFT calculation, and taking the value of calculation to the grid of sea to obtain the ocean wave with high authenticity.

4.3. Texture mapping of grid based on gpu

In the large simulation system of ocean, the CPU resources of computer are not exclusive to the ocean simulation when the system is in a state of multiple tasks, the speed is bound to be affected if the algorithm of ocean simulation is complex. This paper presents a fast method to accelerate the simulation of ocean, firstly, according to the reference, setting up the projection grid in the three-dimensional space, combining with the view rule of human eyes and technique of LOD, replacing the grid in reference to the grid whose center and surrounding is dense and sparse, as figure (Figure. 5) below shows:

![Figure 5. grid combined with the technique of LOD and projection](image)

In the three dimensional world, the grid of concentric circle which uses S as the center, it makes N concentric circles with the radius of increased the distance and M partitions for i-th (1 ≤ i ≤ N) concentric circle, each isotonic point is the vertex of the concentric circle for the grid model. The formula of each radius of the concentric circle as follows (4-11, 4-12):

$$\begin{cases} R_i = 2R_{i-1}, & i = 1, 2, \cdots, N-1 \\ R_i = R_{\text{max}}, & i = N \end{cases}$$

(4-11)

(4-12)

In the formula, Ri is the radius of i-th concentric circle, Rmax is the max distance for the sea level. And it sets the j-th vertex coordinates (x, y, z) for the i-th concentric circle is (4-13, 4-14, 4-15):

$$\begin{cases} x = R_i \cos(2\pi/M) \\ y = l_{i,j} \sin \theta + H, \text{if } y < (y_{\text{max}} + y_{\text{min}})/2 \\ z = R_i \sin(2\pi/M) \end{cases}$$

(4-13)

(4-14)

(4-15)

$\theta$ is the angle between the sea level and the projection grid, H is the minimum height from sea level to projection grid, $l_{i,j}$ is the distance between concentric circle of N and i.
The grid combined with the technique of LOD and projection could reduce substantially the load on the CPU, but if the movement of each viewpoint needs to be recalculated the grid of sea, the rendering speed is affected with the frequent calculation of grid. On the other hand, with the emergence and continuous development of mode of GPU programmable, it is necessary to move the rendering and calculating operation of vertex to GPU, it is an effective method to reduce the computational load of CPU. Based on the above two points, a method of texture mapping for vertex on GPU is proposed.

After the grid is established, making normalization for the data of height field which is from the method from 3.2 and mapping them to the number between $[0,1]$, it saves the form of grayscale in the bitmap file to build the texture map of height, in order to ensure the mapping result is not distorted and do not take up extra storage space, the texture size is calculated by using the following (4-16, 4-17):

$$\begin{align*}
\text{Amount}_{\text{tex}} &= \max(a, b) \\
\text{Size} &= \text{Amount}_{\text{tex}}^2
\end{align*}$$

In the formula, $a$ is the number of rings in the radial grid and $b$ is Number of segmentation of rings, Size represents the texture size, a single texture can only show a moment of the sea, in order to realize the dynamic change of wave height, it needs to generate the height maps in a orderly sequence of time, then using the height map to cyclic mapping for the grid on the ocean to realize the dynamic of the ocean wave. The specific approach is: Setting $n$ sampling point of time $f_1, f_2, ..., f_n$ in time $t$ and using the improved method of this paper to obtain the wave shape. To reduce the number of time sampling and keeping smooth transition for sampling points which is adjacent, using method of interpolation for the the texture map with adjacent time to generate the new texture map. Assuming it needs to interpolate $k$ times between the adjacent point, the calculation formula of the elements of texture is as follows (4-18):

$$h = h_i \left(1 - i / k\right) + h_{i+1} \times i / k$$

$h$ represents the value of the texture element after interpolation, $h$ and $h_{t+1}$ represent two elements on a height image of texture in adjacent time, respectively, $i$ is the number sequence number of interpolation, $1 \leq i \leq k$, eventually it will generate a sequence of $k \times n$ images of a high degree of texture. The flow chart of this method is as follows (Figure.6):
5. Experiment and result analysis
For testing the validity of the proposed method, the paper does the simulation experiments and gives the result of data and graphs. Based on the improved model, using the matlab and osg as development platform to simulate the ocean wave on ordinary PC (I3 3220, 3.3GHz, 4G of memory, gtx650 graphics).

According to the method described in the last section, this paper using the simulation software of matlab to simulate the ocean wave which used the method, the simulation results are shown below (Figure.7):

(a) wind speed 10, direction angle 0
(b) wind speed 20, direction angle 0
(c) wind speed 10, direction angle 90
(d) wind speed 20, direction angle 90

Figure 7. simulation of different wind speed and direction angle

The following figure (Figure.8) is the simulation with wind speed 5 and direction 0 on the OSG platform which using the sequence of rendered image of high texture map based on the technique of texture mapping for GPU vertex.

(a) wind speed 5, direction angle 0
(b) wind speed 15, direction angle 0

Figure 8. simulation of the improved method on

To verify the rendering speed of the simulation, this paper designs 8 simulation experiments with different grid resolutions, the grid resolutions in each set of experiments are as follows:
(1) $64 \times 48$  (2) $128 \times 96$  (3) $192 \times 144$  (4) $256 \times 192$
(5) $320 \times 240$  (6) $384 \times 288$  (7) $448 \times 336$  (8) $512 \times 384$

Each experiment adopts the method of circulation of texture and dynamic rendering of LOD to simulate the ocean with different grid resolutions, and it compares the average rendering speed (Figure 9) and average CPU utilization (Figure 10) at once for the experiments.

From the above diagram, it is easy to see the result which uses resolution of $512 \times 384$ based on the method of circulation still adapt to the real-time ability with FPS 30, but the simulation which uses the dynamic rendering of LOD could not maintain the real-time ability with FPS 30. In the respect of CPU utilization, in the test time of five minutes, the CPU utilization is almost below 10% which uses the method of circulation of texture; conversely, the CPU utilization stays in a high position based on the method of dynamic rendering of LOD. From here, it could use the method of circulation of texture to simulate the ocean which with the high efficiency and real-time ability.

6. Conclusions
This paper goes deep into analyzing the traditional Gerstner model and ocean wave spectrum and on this basis proposed an fast method to simulate the ocean wave on the simulation platform of matlab and OSG, using the height map of discrete point of grid which based on the calculation of FFT of interpolation of SPM-SWOP spectrum and improved Gerstner with controlled factor on the grid combined with the technique of LOD and projection, the obtained simulation of ocean waves with high authenticity. Moreover, it uses the height map to cyclic mapping for the grid on GPU to get the dynamic height data from the grid. The most significant advantage of this method is that it retains the real sense of modeling method of the ocean wave, freeing the CPU from the heavy burden of calculation and it greatly enhance the speed and efficiency of the simulation. The experimental results
show this method not only can keep the real ocean wave effect, but also could achieve a higher rendering speed, the method is suitable for complex simulation systems of ocean wave modeling.

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