The large ($10^2 - 10^5$) and strongly temperature dependent resistive anisotropy $\eta = (\sigma_{ab}/\sigma_c)^{1/2}$ of cuprates perhaps holds the key to understanding their normal state in-plane $\sigma_{ab}$ and out-of-plane $\sigma_c$ conductivities. It can be shown that $\eta$ is determined by the ratio of the phase coherence lengths $\ell_i$ in the respective directions: $\sigma_{ab}/\sigma_c = \ell_{ab}^2/\ell_c^2$. In layered crystals in which the out-of-plane transport is incoherent, $\ell_c$ is fixed, equal to the interlayer spacing. As a result, the $T$-dependence of $\eta$ is determined by that of $\ell_{ab}$, and vice versa, the in-plane phase coherence length can be obtained directly by measuring the resistive anisotropy. We present data for hole-doped $YBa_2Cu_3O_y$ ($6.3 < y < 6.9$) and $Y_{1-x}Pr_xBa_2Cu_3O_{y-\delta}$ ($0 < x \leq 0.55$) and show that $\sigma_{ab}$ of crystals with different doping levels can be well described by a two parameter universal function of the in-plane phase coherence length. In the electron-doped $Nd_{2-x}Ce_xCuO_{4-y}$, the dependence $\sigma_{ab}(\eta)$ indicates a crossover from incoherent to coherent transport in the $c$-direction.

We present data on the normal state in-plane $\sigma_{ab}$ and out-of-plane $\sigma_c$ conductivities of anisotropic layered crystals as diverse as hole-doped $YBa_2Cu_3O_y$ ($6.35 < y < 6.93$) and $Y_{1-x}Pr_xBa_2Cu_3O_{y-\delta}$ ($0 < x \leq 0.55$), and electron-doped $Nd_{2-x}Ce_xCuO_{4-y}$. The conductivities of oxygen deficient $YBa_2Cu_3O_y$ single crystals were measured using the four-point method as well as a multiterminal technique, in zero field and in a field of 14 $T$ in order to suppress superconductivity and reveal the normal state down to lower temperatures. The conductivities of $Y_{1-x}Pr_xBa_2Cu_3O_{y-\delta}$ and $Nd_{2-x}Ce_xCuO_{4-y}$ single crystals were measured using the multiterminal method.

Recently, it has been shown that the ratio of the conductivities is given by...
the ratio of the phase coherence lengths, hereafter called Thouless length (TL), in the respective directions:

\[ \frac{\sigma_{ab}}{\sigma_c} = \frac{\ell^2}{\ell_c^2} . \]  

(1)

Here \( \ell \) is the in-plane TL (subscript "ab" is omitted) and, for simplicity, we assume isotropic in-plane conduction, \( \sigma_a = \sigma_b \equiv \sigma_{ab} \). The incoherent out-of-plane transport means that the c-axis TL (\( \ell_c \)) is temperature independent, equal to the interlayer spacing \( \ell_0 \). Consequently, the in-plane TL is given directly by the measured anisotropy; i.e., \( \ell = (\rho_c/\rho_{ab})^{1/2} \ell_0 \). Therefore, \( \sigma(\ell) \) can be obtained by plotting \( \sigma_{ab} \) vs \( (\rho_c/\rho_{ab})^{1/2} \ell_0 \).

Figures 1(a) and 1(b) are plots of \( \sigma_{ab} \) vs \( (\rho_c/\rho_{ab})^{1/2} \ell_0 \) (\( \ell_0 = 11.7 \text{Å} \)) of \( YBa_2Cu_3O_y \) and \( Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta} \), respectively. The open symbols represent raw data corresponding to different doping. The dashed curves are “isotherms” joining points corresponding to the same temperature. All data were taken with in the T-range \( T_c < T \leq 300 \text{ K} \). Notice that, at a given temperature, the anisotropy changes nonmonotonically as a function of doping, reaching a maximum around \( y \approx 6.42 \) in \( YBa_2Cu_3O_y \) and \( x \approx 0.42 \) in \( Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta} \). According to Eq. (1), the anisotropy is determined by the in-plane TL which, in turn, correlates with the localization length on the insulating side and the correlation length on the metallic side of the metal-insulator transition (MIT). Both of these length scales increase on approaching the MIT and so does the anisotropy, indicating that the MIT takes place close to \( y = 6.42 \) in \( YBa_2Cu_3O_y \) and \( x = 0.42 \) in \( Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta} \).

An idea outlined in Ref. 2 is that variation with doping of the number of carriers and of the amount of disorder do not alter fundamentally the \( \sigma(\ell) \) dependence. The effect of these changes can be incorporated into two constants, one of which, \( \bar{\sigma} \), normalizes the magnitude of the in-plane conductivity, and the other, \( \bar{\ell} \), normalizes the in-plane Thouless length:

\[ \frac{\sigma_{ab}}{\bar{\sigma}} = f \left( \frac{\ell}{\bar{\ell}} \right) ; \quad f(1) = 1. \]  

(2)

Here \( f(y) \) is the same function for a given class of single crystals, such as \( YBa_2Cu_3O_y \). Varying dopant concentration only changes the values of the normalization constants \( \bar{\sigma} \) and \( \bar{\ell} \), shifting the respective segments of \( \sigma_{ab}(\ell) \) along a common trajectory.

The continuous curves indicated in Figs. 1(a) and 1(b) by the filled symbols were generated by shifting the segments corresponding to different oxygen and Pr concentrations, respectively, parallel to themselves (as indicated by the arrows), matching the values and the slopes of the overlapping segments. The data for the \( y = 6.93 \) and \( x = 0.53 \) samples were used as references in Figs. 1(a) and 1(b), respectively. On a log-log plot, such shifts are equivalent to a change of the normalization constants \( \bar{\sigma} \) and \( \bar{\ell} \). Through this procedure, we obtained the trajectory \( \sigma_{ab}(\ell) \) given by Eq. (2) with the values of \( \bar{\sigma} \) and \( \bar{\ell} \) corresponding to the reference samples. The branching point in Fig. 1(a) reflects the MIT. The branching point is absent in Fig. 1(b) due to the lack of data for samples close to the MIT, on its “metallic side”.

Fig. 1. In-plane conductivity $\sigma_{ab}$ of (a) $YBa_2Cu_3O_y$ and (b) $Y_{1-x}Pr_xBa_2Cu_3O_{2-\delta}$ single crystals plotted vs phase coherence length defined as $\ell = (\rho_c/\rho_{ab})^{1/2}l_0$, with $l_0 = 11.7\text{Å}$. The open symbols correspond to raw data. The filled symbols which form a trajectory [Eq. (2)] are obtained by shifting the open symbol segments parallel to themselves as indicated by arrows. The dashed lines are “isotherms”, joining points corresponding to the same temperature. For all samples, anisotropy monotonically increases with decreasing temperature.

The fact that the hole-doped cuprates follow so well the scaling relationship (2) indicates that, indeed, the out-of-plane TL is T-independent in these crystals. However, there must also be systems for which a crossover from incoherent to coherent out-of-plane transport takes place. In such an intermediate regime, the TL in the c-direction becomes T-dependent, but is much smaller and increases at a smaller rate with decreasing temperature than the in-plane TL. As a result, the anisotropy $\sigma_{ab}/\sigma_c = \ell_{ab}^2/\ell_c^2$ would still increase with decreasing $T$ over an extended T-range. We looked for an example of such a regime in the data of electron-doped $Nd_{2-x}Ce_xCuO_{4-y}$. These crystals have very large anisotropy ($\sigma_{ab}/\sigma_c \sim 10^4 - 10^5$), similar to the most anisotropic hole-doped cuprates, however, both $\rho_{ab}(T)$ and $\rho_c(T)$ are metallic at all temperatures $T > T_c$. The T-dependence of $\rho_{ab}$ is quadratic, similar to conventional 3D metals described by Boltzmann-Landau transport theory. We studied several single crystals with two nominal Ce concentrations of $x = 0.08$ and 0.29, which correspond to underdoped and overdoped regime, respectively. Within each group, the resistivities varied due to uncontrolled variations in the oxygen content.

When the out-of-plane TL becomes temperature dependent, $\eta \equiv (\rho_c/\rho_{ab})^{1/2}$ no longer reflects the $T$-dependence of in-plane TL, as in the case of hole-doped systems. Let us assume that both TLs are determined by diffusion, so that

$$\ell_{ab}^2 = D_\parallel \tau_\varphi + \xi^2; \quad \ell_c^2 = D_\perp \tau_\varphi + \ell_0^2.$$  

(3)
Here $D_{\parallel}$ and $D_{\perp}$ are the components of the diffusion tensor, $\tau_{\varphi}(T)$ is the decoherence time, and $\xi$ and $\ell_0$ are cutoffs. These empirical cutoffs ensure that, in the high temperature limit [the limit of small $\tau_{\varphi}(T)$], $\ell_{ab}$ does not scale to zero with decreasing $\tau_{\varphi}$ and $\ell_c$ saturates at the value of the interlayer distance $\ell_0$ (Refs. 2, 3). The anisotropy is then given by

$$\frac{\sigma_{ab}}{\sigma_c} = \frac{D_{\parallel} \tau_{\varphi} + \xi^2}{D_{\perp} \tau_{\varphi} + \ell_0^2}.$$  \hspace{1cm} (4)$$

The fact that the anisotropy in $Nd_{2-x}Ce_xCuO_4-y$ increases with decreasing temperature [Fig. 2(a)] indicates that $D_{\parallel} \gg D_{\perp}$ and $D_{\perp} \tau_{\varphi} \sim \ell_0^2$. In the limit $T \to 0$, when $D_{\perp} \tau_{\varphi} \gg \ell_0^2$, the anisotropy should saturate at the level $D_{\parallel}/D_{\perp}$.

The Drude-type conductivity is given by $\sigma_{ab} = k\tau$ ($k = \text{const}$), where $\tau$ is the relaxation time of the distribution function. Assuming that $\tau$ and $\tau_{\varphi}$ are proportional to each other [$\tau_{\varphi} = A\tau$ ($A = \text{const}$)], one gets from Eq. (4) the following expression for $\sigma_{ab}(\eta)$:

$$\sigma_{ab} = \sigma_0 \frac{\eta^2 - (\xi^2/\ell_0^2)}{1 - \epsilon \eta^2},$$ \hspace{1cm} (5)$$

where $\epsilon = D_{\perp}/D_{\parallel} \ll 1$, and $\sigma_0 = k\ell_0^2/D_{\parallel}A$. Equation (5) shows that the signature of crossover to coherent out-of-plane transport is a more rapid than quadratic increase of $\sigma_{ab}$ with $\eta$. Note that the incoherent regime ($\ell_c = \ell_0$) would lead to $\sigma_{ab} \propto \eta^2$ for diffusive motion and to linear $\sigma_{ab}(\eta) \propto \eta - (\xi/\ell_0)$ for ballistic motion of the electrons (which is the case in optimally doped $YBa_2Cu_3O_7$) (see Ref. 3). The
divergence of $\sigma_{ab}(\eta)$ when $\eta^2 \to 1/\epsilon$ corresponds to the saturation of the anisotropy at low temperatures at the level $D_\parallel/D_\perp$ and, correspondingly, to the fully developed coherent transport in all directions. Our analysis, therefore, is especially useful to identify the intermediate regime of out-of-plane coherence when the anisotropy is still strongly T-dependent as is the case of $Nd_{2-x}Ce_xCuO_{4-y}$.

All curves (except perhaps sample #1) in Fig. 2(b) display stronger than quadratic $\sigma_{ab}(\eta)$ dependence indicating that, indeed, in these samples there is a crossover to coherent out-of-plane transport. Note that the overdoped crystals (#5 and 6) have appreciably lower anisotropy and slower rate of its increase with decreasing temperature than the underdoped crystals (#1 – 4) [Fig. 2(a)]. This reflects that the ratio $\epsilon = D_\perp/D_\parallel$ increases with overdoping. Correspondingly, the rate of increase of $\sigma_{ab}(\eta)$ is much higher in overdoped than in underdoped crystals [Fig. 2(b) and Eq. (5)]. Therefore, even though the underdoped $Nd_{2-x}Ce_xCuO_{4-y}$ crystals are already in the intermediate regime of coherence (unlike the hole doped crystals), the development of interlayer coherence accelerates visibly with overdoping. However, by fitting the data to Eq. (5) we estimate that even in the overdoped samples, the coherence barely extends from a given $CuO_2$ layer to its next-nearest neighbor even at temperature as low as 50 K.

In summary, we have shown that both $YBa_2Cu_3O_y$ and $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ systems are characterized by a universal functional dependence of the in-plane conductivity on the in-plane TL, with doping dependent normalization parameters. This dependence reflects the relationship between the relaxation time, which determines the in-plane conductivity, and the phase coherence length. The data on the electron-doped $Nd_{2-x}Ce_xCuO_{4-y}$ indicate that these crystals are in an intermediate regime in which the c-axis TL is no longer T-independent but is still much smaller and changes with temperature at a smaller rate than the in-plane TL.

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