SM Higgs mass bounds from theory

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Abstract

The two-loop Higgs mass upper bounds are reanalyzed. Previous results for a cutoff scale $\Lambda \approx$ few TeV are found to be too stringent. For $\Lambda = 10^{19}$ GeV we find $M_H < 180 \pm 4 \pm 5$ GeV, the first error indicating the theoretical uncertainty, the second error reflecting the experimental uncertainty due to $m_t = 175 \pm 6$ GeV. We also summarize the lower bounds on $M_H$. We find that a SM Higgs mass in the range of 160 to 170 GeV will certainly allow for a perturbative and well-behaved SM up to the Planck-mass scale $\Lambda_{Pl} \simeq 10^{19}$ GeV, with no need for new physics to set in below this scale.

It is well known that the high energy behavior of the Standard Model (SM) Higgs quartic coupling is unsatisfactory at high energies. For a heavy Higgs boson it manifests itself in the (one-loop) Landau pole \cite{1} when using a perturbative approach, or in large cutoff effects when performing lattice calculations \cite{2,3}. If the Higgs boson is light the Higgs running coupling may become negative at high energy, giving rise to the problem of vacuum stability \cite{4}. Defining a cutoff scale $\Lambda$ both problems are avoided if the Higgs boson mass $M_H$ is constrained from below and above, with no need for introduction of physics beyond the SM.

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Previous work \cite{5, 6, 7, 8} extensively investigated the dependence of the $M_H$ bounds on the top quark mass $m_t$. By the time the next generation of colliders are operating, the good experimental knowledge of $m_t$ will make the theoretical bounds on the Higgs mass a single function of the cutoff scale $\Lambda$. However, there are theoretical uncertainties which remain in the calculation of the $M_H$ bounds. In the case of the $M_H$ lower bound these uncertainties have been recently addressed in Ref. \cite{9, 10, 11}. Here we review the sensitivity of the upper bound on $M_H$ with regard to various cutoff criteria, the inclusion of matching corrections, and the choice of the matching scale $\mu_0$ using a two-loop perturbative approach \cite{12}. We also summarize the recent results for the lower bounds \cite{9, 10, 11}. If a SM Higgs boson is found, the future measurement of its mass can immediately be used to determine up to which maximal energy scale the Standard Model could be valid.

The high-energy evolution of the SM running couplings is determined by the beta functions of the theory. The value of the Higgs and top quark $\overline{\text{MS}}$ running couplings are fixed at low energies through the matching conditions

\begin{align}
\bar{\lambda}(\mu_0) &= \frac{M_H^2}{2v^2} [1 + \delta_H(\mu_0)], \\
\bar{g}_t(\mu_0) &= \frac{\sqrt{2}m_t}{v} [1 + \delta_t(\mu_0)],
\end{align}

where $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV, and $\mu_0$ is the matching scale. The definitions of the tree level couplings are obtained by setting the matching corrections $\delta$ equal to zero, thus fixing our notation.

In the present analysis, we restrict ourselves to two-loop beta functions \cite{6, 13} and the corresponding one-loop matching conditions (see Ref. \cite{14} for $\delta_H$ and Ref. \cite{15} for $\delta_t$) in the $\overline{\text{MS}}$ scheme. For the $\overline{\text{MS}}$ electroweak and strong couplings we take the input values $\bar{g}(M_Z) = 0.651$, $\bar{g}'(M_Z) = 0.357$, and $\alpha_s(M_Z) = 0.118$.

For sufficiently large initial coupling $\lambda(\mu_0)$ the two-loop running coupling $\lambda(\mu)$ approaches the (meta-stable) fixed-point value $\lambda_{FP} = 12.1\ldots$ at some high energy scale $\mu_\ast$. Perturbation theory, however, ceases to be meaningful before reaching the fixed point. In addition, lattice calculations indicate that such large values for the coupling are inconsistent with the requirement of small cutoff effects \cite{3}. To

\footnote{This is in contrast to the one-loop running coupling $\lambda(\mu)$ which approaches a Landau singularity at high energies.}
accommodate these results we define two different cutoff conditions for the running coupling at the cutoff scale $\Lambda$:

$$\lambda_c(\Lambda) = \lambda_{FP}/4 \quad \text{and} \quad \lambda_c(\Lambda) = \lambda_{FP}/2. \quad (3)$$

The first choice corresponds to a running coupling which is definitely perturbative at scale $\Lambda$ \cite{16}, giving rise to a modest 25% two-loop correction to the one-loop beta function $\beta_\lambda$ of the Higgs quartic coupling. This condition gives $M_H$ upper bounds below which the SM is certainly well-defined and perturbative. The second choice, $\lambda_c(\Lambda) = \lambda_{FP}/2$, causes a 50% two-loop correction to $\beta_\lambda$, and its value is comparable with upper bounds on $\lambda(\Lambda)$ which can be obtained from lattice calculations \cite{3}. It is also close to the upper bound of the perturbative regime \cite{16}. Hence these two choices for $\lambda_c(\Lambda)$ summarize the uncertainty connected to the precise formulation of the cutoff condition. Carrying out a numerical analysis \cite{12} we find the uncertainty of the $M_H$ upper bound related to the cutoff condition to be about $\pm 50$ GeV for TeV cutoff scales, and less than $\pm 3$ GeV for cutoff scales approaching $\Lambda_{PL} \simeq 10^{19}$ GeV.

A far more important point is the choice of the matching scale $\mu_0$ used in the definition of the couplings, Eqs. (1) and (2). On first sight, it seems to be convenient to take $\mu_0 = M_Z$ since the gauge couplings have also been defined at this scale. This choice, however, introduces large corrections in the matching correction $\delta_H$ if the Higgs mass is larger than $\approx 400$ GeV, and it results in the strange effect that the Higgs coupling is restricted to a maximal value of 1.2 which is obtained for $M_H = 495$ GeV \cite{12}. This point can be identified as low-scale end-point of the short-dashed curve in Fig. 1. Clearly the choice $\mu_0 = M_Z$ is inappropriate for perturbative calculations involving large values of $M_H$. Neglecting the matching conditions (setting $\delta_i = 0, i = H, t$), the choice $\mu_0 = M_Z$ also leads to unreliable results, now resulting in a too stringent bound on $M_H$ for low cutoff scales (dotted curve in Fig. 1). This confirms that the appropriate choices for the Higgs and top quark matching scales are $\mu_{0,H} \simeq \max\{m_t, M_H\}$ and $\mu_{0,t} \simeq m_t$, respectively, as described in \cite{12}.

Using the choice $\mu_{0,H} = M_H$, we can also estimate the uncertainty in the Higgs mass upper bound due to higher order corrections to the matching corrections $\delta_H$ and $\delta_t$. This is done by comparing the solid line (which includes matching corrections) and the long-dashed line (without matching corrections) in Fig. 1. We find that
Figure 1: Choosing two-loop RG evolution and cutoff condition $\lambda_c(\Lambda) = \lambda_{FP}/2$, the upper bound on $M_H$ is calculated. The running Higgs and Yukawa couplings, $\lambda(\mu)$ and $g_t(\mu)$, are fixed by the physical masses $M_H$ and $m_t$ using matching conditions with or without one-loop matching corrections. In addition, the Higgs matching scale is varied to be $\mu_{0,H} = M_H$ or $M_Z$. The top-quark mass is fixed at $m_t = 175$ GeV, and $\mu_{0,t} = m_t$.

the difference of the two results exceeds 100 GeV at small embedding scale $\Lambda$, but reduces to less than about 6 GeV at large scale.

The sum of all theoretical uncertainties in the Higgs upper bound is summarized in Fig. 2 taking $m_t = 175$ GeV. In that figure we also show the results for the lower bounds using the results from [9, 10, 11], taking $m_t = 175$ GeV and $\alpha_s(M_Z) = 0.118$. We use two-loop beta functions and appropriately choose the matching scale to be $\mu_{0,H} = M_H$. We vary the cutoff condition between $\lambda_c(\Lambda) = \lambda_{FP}/4$ and $\lambda_{FP}/2$ as discussed above, and we carry out numerical calculations both with and without inclusion of one-loop matching corrections $\delta_H$ and $\delta_t$. For low cutoff scales, the $M_H$ lower bound and its uncertainties are taken from [11], and for larger cutoff scales we use the condition $\lambda_c(\Lambda) = 0$ [9]. Hence we are able to identify an “allowed” area for which we find a well-behaved, perturbative and stable Higgs sector of the SM, and we can identify “disallowed” areas for which the SM Higgs sector is an inconsistent or unstable theory. The black area identifies the transition zone between these regions, indicating the uncertainties related to the choice of the various cutoff criteria and their perturbative implementation.
Figure 2: Summary of the uncertainties connected to the bounds on $M_H$. The upper solid area indicates the sum of theoretical uncertainties in the $M_H$ upper bound for $m_t = 175$ GeV [12]. The upper edge corresponds to Higgs masses for which the SM Higgs sector ceases to be meaningful at scale $\Lambda$ (see text), and the lower edge indicates a value of $M_H$ for which perturbation theory is certainly expected to be reliable at scale $\Lambda$. The lower solid area represents the theoretical uncertainties in the $M_H$ lower bounds derived from stability requirements [9, 10, 11] using $m_t = 175$ GeV and $\alpha_s = 0.118$.

Looking at Fig. 2 we conclude that a SM Higgs mass in the range of 160 to 170 GeV results in a SM renormalisation-group behavior which is perturbative and well-behaved up to the Planck scale $\Lambda_{Pl} \simeq 10^{19}$ GeV.

The remaining experimental uncertainty due to the top quark mass is not represented here and can be found in [9, 10, 11] for lower and upper bound, respectively. In particular, the result $m_t = 175 \pm 6$ GeV leads to an upper bound

$$M_H < 180 \pm 4 \pm 5 \text{ GeV} \quad \text{if} \quad \Lambda = 10^{19} \text{ GeV}, \quad (4)$$

the first error indicating the theoretical uncertainty, the second error reflecting the residual $m_t$ dependence [12].
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