Remark on the variational principle in the AdS/CFT correspondence for the scalar field

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Abstract

We discuss how the variational principle can be used as a criterion for choosing, among scalar field actions implying the same equation of motion, the appropriate one for the AdS/CFT correspondence.

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1 Introduction

The important physical consequences of Maldacena conjecture [1] on the equivalence (or duality) of the large $N$ limit of $SU(N)$ superconformal field theories in $n$ dimensions and supergravity on anti de Sitter space-time in $n + 1$ dimensions, $AdS_{n+1}$, attracted a lot of interest in the recent literature. By supergravity here, one means the tree level approximation of string or M-theory defined on $AdS_{n+1} \times M_d$, where $M_d$ is some $d$-dimensional compactification space. This conjecture was further elaborated in the works of Gubser, Klebanov and Polyakov [2] and Witten [3]. In this, so called, AdS/CFT correspondence, field correlators of a conformal field theory on the $n$ dimensional boundary of an anti de Sitter space of dimension $n + 1$ are defined by the dynamics of fields living inside this space. The boundary values of these fields, whose dynamics is defined in the bulk, have the role of sources for the field correlators of the conformal theory on the boundary.

The presence of a boundary with non vanishing fields together with the fact that the metric is singular there may lead to some non standard results, comparing with usual quantum field theory, defined in spaces where the fields and field derivatives either vanish or have finite limiting values on the boundary. Classical actions that differ by total derivative terms (surface terms), having thus the same equations of motion, may play different roles in the AdS/CFT correspondence. In the case of fermionic fields, if one starts simply with the Dirac action as governing the dynamics inside $AdS_{n+1}$

$$\int d^{n+1}x \sqrt{g} \bar{\psi} (\not{D} - m) \psi,$$

and imposes the equation of motion, $(\not{D} - m) \psi = 0$, one obtains a vanishing on shell action. This would imply a trivial mapping between the bulk theory and the boundary correlators. A solution to this problem was found by Henningson and Sfetsos [4], who showed that a non trivial mapping is possible if one introduces a surface term in the classical action. Soon after that, an interesting interpretation for the role played by this surface term was found by Henneaux [5], showing that the action is minimized by solutions of the equation of motion only if the surface term is added. Thus, the appropriate use of the variational principle furnishes, in the fermionic case a criterion for choosing the

\footnote{For recent reviews with a wide list of references see [4] and [5].}
appropriate boundary term to be included in the action in the AdS/CFT correspondence.

One important point to be remarked is the relation between the form of the fermionic action and the need for the surface term. As the fermionic Lagrangian is linear in the field derivatives, the fields $\psi$ and $\bar{\psi}$ will be canonically conjugate variables. So, as we are interpreting this fermionic fields as quantum operators one can not arbitrarily fix both of them on the boundary. That is why the action is minimized by the equation of motion only if one includes an additional surface term. Things are different for the scalar field whose Lagrangian is quadratic in the field derivatives. In this case there is no problem in requiring that a solution of the equation of motion (just the field itself not the field derivatives) has some arbitrary limiting behavior in the boundary. However, as we will see, the fact that one can not fix the field and field time derivative simultaneously will make it possible again to use the variational principle as a guide for choosing the appropriate, among different actions that imply the same equation of motion in the bulk, for the AdS/CFT correspondence. For massless scalar fields, if one chooses the standard form of the Klein Gordon action in AdS (curved) spacetime

$$\frac{1}{2} \int d^{n+1}x \sqrt{g} \partial_\mu \phi \partial^\mu \phi ,$$

one does not need to introduce surface terms in the classical action in order to obtain the AdS/CFT correspondence, as we are going to review on section 2. If instead of this action, one chooses

$$-\frac{1}{2} \int d^{n+1}x \sqrt{g} \phi \nabla_\mu \nabla^\mu \phi ,$$

(where $\nabla_\mu$ is the covariant derivative in AdS space) one would get a vanishing generating functional for the correlators on the boundary. Both actions have the same Euler Lagrange equation of motion $\nabla_\mu \nabla^\mu \phi = 0$, and would thus describe the same dynamics in spaces with fields and field derivatives vanishing on the boundary. However only the first one gives the appropriate mapping in the AdS/CFT correspondence. The aim of this letter is to show how the variational principle can be used also in the scalar field case so as to select the appropriate action in the AdS/CFT context. In section 2 we review the AdS/CFT correspondence in the scalar field case and also make some remarks about the regularizations that must be introduced in order to have a well defined meaning to some singular quantities that show up, in particular in the discussion of the variational
principle. In section 3 we discuss the case of the (would be) partially integrated action, show why this action is not minimized by the equations of motion and extend these results for the case of massive fields. Then, in section 4 we present some concluding remarks. We also included an appendix where we describe the behavior of the field time derivative which is useful in our discussion of the variational principle.

2 Massless Scalar Fields on AdS/CFT and the Variational Principle

Let us first consider a \((n+2)\)-dimensional pseudo-Euclidean space-time with coordinates \((y^a) = (y^0, y^1, \ldots, y^n, y^{n+1})\) and metric \(\eta_{ab} = \text{diag}(+, -, -, \ldots, - , +)\), so that the measure

\[
y^2 = (y^0)^2 + (y^{n+1})^2 - \sum_{i=1}^{n} (y^i)^2,
\]

is preserved under the transformations of the Lorentz group \(\text{SO}(2,n)\). The \(\text{AdS}_{n+1}\) can then be defined as a submanifold of this \((n+2)\)-dimensional pseudo-Euclidean space-time such that \(y^2 = b^2 = \text{constant}\). A pair of “light cone” coordinates can be defined as

\[
u = y^0 + i y^{n+1}, \quad v = y^0 - i y^{n+1}
\]

so that \(y^2 = \nu \bar{\nu} - \bar{y}^2 = b^2\). Then following Witten \cite{3}, we set \(b = 0\) and define \((x^0, \vec{x}) \equiv (u^{-1}, \vec{x})\), so that Anti de Sitter space-time can be characterized by the measure

\[
ds^2 = \frac{(dx^0)^2}{(x^0)^2} + \frac{(d\vec{x})^2}{(x^0)^2} \equiv \frac{1}{(x^0)^2} \sum_{\mu=0}^{n} (dx^\mu)^2
\]

in the upper half space \(x^0 \geq 0\) and \(x^i \equiv x_i\), \((i = 1, \ldots, n)\) are the coordinates in \(n\)-dimensional Euclidean space, \(E^n\), in the boundary of AdS space-time defined by \(x^0 = 0\) plus a point at the infinity \(x^0 = \infty\). The metric in the \(\text{AdS}_{n+1}\) space is taken as

\[
g_{\mu\nu} = \frac{1}{(x^0)^2} \delta_{\mu\nu}, \quad \sqrt{g} = \frac{1}{(x^0)^{n+1}}, \quad g^{\mu\nu} = (x^0)^2 \delta^{\mu\nu}.
\]

For massless scalar fields in the AdS/CFT correspondence one assumes that the field \(\phi(x^0, x^i)\), where \(i = 1, \ldots, n\) on \(\text{AdS}_{n+1}\) has some boundary value \(\phi_0(x^i)\) as \(x_0 \to 0\).

\[\frac{\text{We follow closely the notation and definitions of Petersen} \cite{4}.}\]

\[\frac{\text{Alternatively, the coordinates on the boundary could be defined to be an} \ n\text{-dimensional Minkowski space.}}\]
This object will be coupled to a field $\mathcal{O}(x^i)$, representing the CFT on the boundary. The correlation functions for the $\mathcal{O}(x^i)$ field will then be calculated from the generator

$$Z[\phi] = \langle \exp \int d^nx\, \phi_0(x^i)\, \mathcal{O}(x^i) \rangle .$$

(5)

where the integral is defined on the boundary $E^n$.

The mapping between the spaces is realized then by associating this generator of CFT correlation functions on the boundary with the on shell value of the classical action of $AdS_{n+1}$. One introduces the action that governs the dynamics inside the $AdS_{n+1}$ space as the standard Klein Gordon action in curved space-time:

$$I_1[\phi] = \frac{1}{2} \int d^{n+1}x\sqrt{g}\, \partial_\mu \phi \partial^\mu \phi .$$

(6)

The variation of the action $I_1$ when we change the field $\phi$ by a small amount $\delta \phi$ is formally

$$\delta I_1[\phi] = -\int d^{n+1}x\, \partial_\mu (\sqrt{g}\, \partial^\mu \phi) \, \delta \phi + \int d^{n+1}x\, \partial_\mu [\sqrt{g} (\partial^\mu \phi) \, \delta \phi]$$

$$= -\int d^{n+1}x\, \sqrt{g} (\nabla_\mu \partial^\mu \phi) \, \delta \phi + \int d^n x\, \sqrt{g} (\partial^0 \phi) \, \delta \phi .$$

(7)

However, the metric and also the field derivative, as we will see in the next section, are singular at $x^0 = 0$. Thus we need some prescription in order to have a well defined meaning to these expressions. We consider the last integral in (7) to be calculated near the boundary, at $x^0 = \epsilon$. Moreover, the variations $\delta \phi$ of the fields in the bulk are subject to the condition

$$\delta \phi \big|_{x^0=\epsilon} = 0 .$$

(8)

This condition does not mean that $\phi$ reaches the limiting value $\phi_0$ at $x^0 = \epsilon$ but rather that we are not varying the field configuration for $x^0 < \epsilon$ with respect to the classical solution. Assuming these conditions to hold, the last integral in (7) vanishes and thus the action is stationary with respect to variations of the field if the Euler Lagrange equation of motion is satisfied:

$$\nabla_\mu \nabla^\mu \phi = \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} \partial^\mu \phi) = 0 .$$

(9)

The field $\phi(x^0, \vec{x})$ inside the $AdS_{n+1}$ space is related to the field on the boundary by calculating the Green’s function that represents the evolution of the field from it’s
"initial" value at $x^0 = 0$ to the values inside $AdS_{n+1}$. Following [3] one finds
\[ \phi(x^0, \vec{x}) = c \int d^n x' \frac{(x^0)^n}{((x^0)^2 + (\vec{x} - \vec{x}')^2)^n} \phi_0(\vec{x}'). \] (10)

The action $I_1[\phi]$, Eq. (3), with the metric [3] can be rewritten as
\[ I[\phi] = \frac{1}{2} \int d^{n+1} x \partial_\mu ( (x^0)^{-n+1} \phi \partial_\mu \phi) - \frac{1}{2} \int d^{n+1} x \phi \partial_\mu ( (x^0)^{-n+1} \partial_\mu \phi). \] (11)

So, considering the on shell value of the action, the last term vanishes by the equation of motion whereas the first one leads to a surface term. Using the Green function solution (10) we can write the action as
\[ I[\phi] = -\frac{cn^2}{2} \int d^n x d^n x' \frac{\phi_0(\vec{x}') \phi_0(\vec{x})}{(\vec{x} - \vec{x}')^{2n}} \] (12)

that generates the appropriate two point functions for the operators $O(\vec{x})$ on the boundary:
\[ \langle O(\vec{x})O(\vec{x}') \rangle \sim \frac{1}{(\vec{x} - \vec{x}')^{2n}}. \] (13)

3 Partially Integrated Scalar Field Action

Now let us see how the previous discussion on AdS/CFT is modified if instead of action $I_1[\phi]$, Eq. (3), we decide to consider the action
\[ I_2[\phi] = -\frac{1}{2} \int d^{n+1} x \sqrt{g} \phi \nabla_\mu \nabla^\mu \phi . \] (14)

The Euler Lagrange equation of motion associated with this action is also given by $\nabla_\mu \nabla^\mu \phi = 0$, Eq. (3), i.e., it is the same as that corresponding to the action (3). In the standard field theory case, where one usually assumes that the space does not have a boundary, both actions would describe the same dynamics.

However, let us see what happens if we repeat the procedure described in the previous section in order to find the generator of correlation functions from action $I_2[\phi]$. Considering that the solution for the fields on $AdS_{n+1}$ in terms of the boundary values would be the same Green function as given by eq. (10), since the equation of motion for the fields described by $I_2[\phi]$ is the same as that for the previous action $I_1[\phi]$ action we can rewrite
\[ I_2[\phi] = -\frac{1}{2} \int d^{n+1} x \phi \partial_\mu ( (x^0)^{-n+1} \partial_\mu \phi), \] (15)
and this vanishes on shell. So we would find a vanishing generator of correlation functions if we use this action, analogously to the fermionic case discussed in the introduction.

Let us now interpret this result in the light of the ideas of reference \[7\] of appropriately using the variational principle to take into account the effects of the presence of the boundary. In the fermionic case the complete solutions for the fields $\psi$ and $\overline{\psi}$ can not have well defined values on the boundary of AdS. The same thing does not happen in the scalar case, where we can assume the field to have a well defined value $\phi_0$ on the boundary, at least for the massless case. There is however a difference in the action $I_2[\phi]$, Eq. (13), with respect to the previously considered action $I_1[\phi]$, Eq. (7), concerning the variational principle. The point is that the action $I_2$ involves second order field derivatives. For field theories whose dynamics is governed by a classical action involving just first order field derivatives, the variational principle states that considering all field configurations with fixed values (of just the field itself) on a space time border, the action is minimized by the one that satisfies the Euler Lagrange equation. However, when one considers field theories involving higher order field derivatives one needs in general to fix also some of the derivatives of the fields on the border \[3, 7\]. Let us see what is the variation of action $I_2$ when we make a small variation $\delta \phi$ in the field $\phi$:

$$
\delta I_2[\phi] = -\int d^{n+1}x \sqrt{g} (\nabla_\mu \partial^\mu \phi \delta \phi) + \frac{1}{2} \int d^n x [(x^0)^{-n+1} (\partial_0 \phi \delta \phi]
- \int d^n x [(x^0)^{-n+1} \phi \delta (\partial_0 \phi)] .
$$

So, that action would be stationary if, besides satisfying the equation of motion, we could make $\delta \phi$ and also $\delta (\partial_0 \phi)$ vanish on the boundary. This would correspond to look at solutions with well defined values of the field and also of its "time" derivative on the boundary. Assuming that the field has a well defined non vanishing boundary value, let us look at the derivative of $\phi$. Considering the field in the AdS$_{n+1}$ space as given by the Green function, Eq. (10), we calculate $\partial_0 \phi$:

$$
\partial_0 \phi(x^0, \vec{x}) = \frac{nc}{x^0} \int d^n x' \frac{(x^0)^n}{(x^0)^2 + (\vec{x} - \vec{x}')^2} \phi_0(\vec{x}')
- 2nc \int d^n x' \frac{(x^0)^{n+1}}{(x^0)^2 + (\vec{x} - \vec{x}')^2} \phi_0(\vec{x}') .
$$
Choosing the normalization constant to be $c = (\sqrt{n/\pi})^n$, we find (see the Appendix):

$$
\partial_0 \phi(x^0, \vec{x}) \bigg|_{x^0 \to 0} = \frac{n}{x^0} \phi_0(\vec{x}) - 2n \left( \frac{n}{n+1} \right)^{\frac{n}{2}} \phi_0(\vec{x}) ,
$$

so that the derivative of the field $\phi$ behaves as $(x^0)^{-1}\phi_0$ near the boundary. So, as we require the field $\phi$ to have a well defined finite value $\phi_0$ on the boundary, the derivative $\partial_0 \phi$ is not defined there essentially because of the singular nature of the metric at $x^0 \to 0$.

A regularized meaning to $\delta I_2[\phi]$, Eq. (14), is possible if consider again the surface integrals to be defined at $x^0 = \epsilon$ and assume the condition $\delta \phi|_{x^0=\epsilon} = 0$, Eq. (8), to hold. If we were dealing with just classical objects there would be no objection to adding to (8) the extra condition

$$
\delta(\partial_0 \phi)|_{x^0=\epsilon} = 0 .
$$

In this case the usual prescription of the variational principle including derivatives of the fields up to second order [8] would be satisfied and thus the action $I_2$ would be minimized by the solutions of the Euler Lagrange equations. However, inserting the equation of motion in the action $I_2[\phi]$ we would find a vanishing boundary term contribution. So that this action would lead to vanishing correlators in the AdS/CFT correspondence.

The point is that we are considering the classical action just as an approximation for the quantum action. Thus the field $\phi$ and its ”time” derivative $\partial_0 \phi$ are to be taken as quantum operators corresponding to a canonical pair of non commuting variables. So we can not impose that $\delta \phi$ and $\delta \partial_0 \phi$ vanish at the same time, which would violate the uncertainty principle. As we are fixing the limiting value of the field, we can not assume the condition $\delta(\partial_0 \phi) = 0$ to hold on the boundary and the last term of equation (16) is non vanishing. Therefore the action $I_2[\phi]$, Eq. (14), is not minimized by the configurations that satisfy the equation of motion $\nabla_\mu \nabla^\mu \phi = 0$, Eq. (8), as long as we consider the field and its derivative as quantum operators. If one decides, as in the fermionic case [7], to add to the action $I_2$ a surface term that compensates for the variation of the action one would find that the appropriate one is just

$$
M[\phi] = \frac{1}{2} \int d^{n+1}x \sqrt{g} \nabla_\mu \left( \phi \nabla^\mu \phi \right) .
$$

That gives trivially $I_2 + M = I_1$ showing then that the analysis of the variational principle indicates that indeed the action $I_1$ is the appropriate one for calculating the field correlators on the boundary in the AdS/CFT correspondence.
It is interesting to mention that alternatively, one could have imposed that the derivative $\partial_0 \phi$ to be well defined on the boundary with the price that the field $\phi_0$ itself would not be fixed, so one would still find non-vanishing boundary terms. This alternative case should correspond to Neumann boundary conditions. It is even possible to discuss a generalized boundary condition where the condition is imposed on a linear combination of $\phi_0$ and $\partial_0 \phi$. The calculations of the AdS Green’s functions for these two cases have been discussed recently by Minces and Rivelles [10].

Now, let us see if the presence of mass would change our conclusions. If we introduce a mass term

$$I_{M}[\phi] = \frac{1}{2} \int d^{n+1}x \ m^2 \phi^2,$$

into actions $I_{1}[\phi]$, Eq. (6), and $I_{2}[\phi]$, Eq. (14), the equation of motion is now the massive Klein Gordon and one finds the solution [3, 4]

$$\phi(x^0, \vec{x}) = c \int d^n x' \frac{(x^0)^{n+\lambda_+}}{(x^0)^2 + (\vec{x} - \vec{x}')^2)^{n+\lambda_+}} \phi_0(\vec{x}')$$

$$= (x^0)^{-\lambda_+} c \int d^n x' \frac{(x^0)^{n+2\lambda_+}}{(x^0)^2 + (\vec{x} - \vec{x}')^2)^{n+\lambda_+}} \phi_0(\vec{x}') ,$$

where $\lambda_+$ is the highest root of $\lambda(\lambda + n) = m^2$.

Note that the factor of $\phi_0(\vec{x}')$ in the integrand of the above equation in the limit $x^0 \to 0$ corresponds to an $n$-dimensional delta function, since

$$\frac{\epsilon^\beta}{(\epsilon^2 + \vec{x}^2)^\alpha}$$

corresponds to $\delta^n(\vec{x})$ up to a normalization, if one guarantees that $2\alpha - n = \beta > 0$. So we see that on the boundary, $x^0 \to 0$, the field $\phi(x^0, \vec{x})$ has the asymptotic behavior

$$\phi(x^0, \vec{x}) \to (x^0)^{-\lambda_+} \phi_0(\vec{x}).$$

(23)

Now, the two-point correlation functions for the operators $O_{\Delta}(\vec{x})$ with conformal dimension $\Delta = n + \lambda_+$ on the boundary are given by:

$$\langle O_{\Delta}(\vec{x})O_{\Delta}(\vec{x}') \rangle \sim \frac{1}{(\vec{x} - \vec{x}')^{2n+2\lambda_+}}.$$

(24)

As the mass term does not involve field derivatives, there will be no extra surface term in $\delta I_2$. Then all the previous discussion about the variational principle remains the same and action $I_2$ is ruled out by the impossibility of fixing at the same time $\partial_0 \phi$ and $\phi$ (up to a scale factor) on the boundary.
4 Conclusions

We have seen here that the idea of reference [7] of using the minimum action principle as a criterion for selecting a classical action that will describe the mapping between the AdS space and the conformal theory on the boundary also works properly in the scalar field case. In contrast to the fermionic case, the field can be completely fixed on the boundary. However, partially integrated actions involving the second order field derivative would be minimized only if one requires the additional condition of fixing the field derivative on the boundary. This condition is not possible for canonically conjugate quantum fields. So the variational principle, together with the quantum ingredient of the uncertainty principle rules out all the Lagrangians that would lead to vanishing correlation functions.

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Appendix

The integrand of the first term in (17) behaves as an $n$-dimensional “delta function” times $\phi_0(\vec{x}')$ which implies a behavior like $(1/x^0)\phi(x^0, \vec{x})$. The second term has the same behavior in the limit $x^0 \to 0$, although this may be not transparent in the above expression. To have a clue of this, let us recall a representation for the one-dimensional “delta function”

$$\delta(x) = \frac{1}{\pi} \lim_{y \to 0} \frac{y}{x^2 + y^2}$$

and consider (up to a constant) the product of rational functions in the $x^i$ variables

$$\frac{x^0}{(x^0)^2 + (x^1)^2} \frac{x^0}{(x^0)^2 + (x^2)^2} \cdots \frac{x^0}{(x^0)^2 + (x^n)^2}$$

$$\frac{(x^0)^n}{(x^0)^{2n} + (x^0)^{2(n-1)}(x^1)^2 + (x^2)^2 + \ldots + (x^n)^2 + \ldots}.$$  

If we compare it with the term that shows up in (17)

$$\frac{(x^0)^n}{(x^0)^2 + (x^1)^2 + (x^2)^2 + \ldots + (x^n)^2}.$$
\[
\frac{(x^0)^n}{(x^0)^{2n} + n(x^0)^{2(n-1)}(x^1)^2 + \ldots + (x^n)^2 + \ldots}, \tag{27}
\]

where we are considering \(x^i\), \((i = 1, 2, \ldots, n)\) to be small compared to \(x^0\), since the “delta functions” are non vanishing only for \(x^1 = x^2 = \ldots = x^n = 0\), we find

\[
\lim_{x^0 \to 0} \frac{(x^0)^n}{((x^0)^2 + (x^1)^2 + (x^2)^2 + \ldots + (x^n)^2)^{n+1}} = (\pi)^n \delta(\sqrt{n} x^1) + \delta(\sqrt{n} x^2) \ldots \delta(\sqrt{n} x^n), \tag{28}
\]

While, for the second term in eq. \((17)\) we have

\[
\frac{(x^0)^{n+1}}{(x^0)^{2n} + (n + 1)(x^0)^{2n-1}(x^1)^2 + \ldots + (x^n)^2 + \ldots} = \frac{(x^0)^{n+1}}{(x^0)^{2n} + (n + 1)(x^0)^{2n-1}(x^1)^2 + \ldots + (x^n)^2 + \ldots} = \frac{1}{x^0 (x^0)^{2n} + (n + 1)(x^0)^{2n-1}(x^1)^2 + \ldots + (x^n)^2 + \ldots}. \tag{29}
\]

Thus, taking the limit \(x^0 \to 0\) we obtain the product of “delta functions”

\[
\lim_{x^0 \to 0} \frac{(x^0)^{n+1}}{(x^0)^{2n} + (x^1)^2 + (x^2)^2 + \ldots + (x^n)^2)^{n+1}} = \frac{\pi^n}{x^0} \delta(\sqrt{n+1} x^1) \delta(\sqrt{n+1} x^2) \ldots \delta(\sqrt{n+1} x^n)
\]

\[
= \frac{\pi^n}{x^0} \delta^n(\sqrt{n+1} \vec{x}), \tag{30}
\]

where \(\vec{x} = (x^1, x^2, \ldots, x^n)\). In the general case \(\epsilon^\beta/(\epsilon^2 + \vec{x}^2)^\alpha\) the above formula can be extended to

\[
\lim_{\epsilon \to 0} \frac{\epsilon^\beta}{(\epsilon^2 + \vec{x}^2)^\alpha} = \pi^n \delta^n(\sqrt{\alpha} \vec{x}) \tag{31}
\]

where \(\vec{x}\) is a vector in \(n\)-dimensions as in the previous formula and the powers \(\alpha\) and \(\beta\) are such that \(2\alpha - n = \beta > 0\).
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