TIME DELAY IN QSO 0957 + 561 FROM 1984–1999 OPTICAL DATA

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Received 2000 November 16; accepted 2001 January 4

ABSTRACT

Photometric optical data of QSO 0957 + 561 covering the period 1984–1999 are analyzed to discern between the two values of the time delay (417 and 424 days) mostly accepted in the recent literature. The observations, performed by groups from three different institutions—Princeton University, Harvard-Smithsonian Center for Astrophysics, and Instituto de Astrofísica de Canarias—and including new unpublished 1998–1999 data from the IAC80 Telescope, were obtained in five filters (V, R, I, g, and r). The different light curves have been divided into observational seasons, and two restrictions have been applied to calculate the time delay: (1) points with a strange photometric behavior have been removed, and (2) data sets without large gaps have been selected. Simulated data were generated to test several numerical methods intended to compute the time delay ($\Delta \tau_{AB}$). The methods giving the best results—the discrete correlation function, $\delta \tau$, $z$-transformed discrete correlation function, and linear interpolation—were then applied to real data. A first analysis of the 23 different time delays derived from each technique shows that $\Delta \tau_{AB}$ must be in the interval 420–424 days. From our statistical study, a most probable value of $\Delta \tau_{AB} = 422.6 \pm 0.6$ days is inferred.

Subject headings: gravitational lensing — methods: data analysis — quasars: individual (Q0957 + 561)

1. INTRODUCTION

The first discovered gravitational lens system, QSO 0957 + 561 (Walsh, Carswell, & Weymann 1979), has been the subject of a continuous and exhaustive monitoring in several bands since 1979. The special characteristics of this system made it very attractive for time delay determinations, and different values for this quantity were presented during the 1980s: $\Delta \tau = 566 \pm 37$ days (Florentin-Nielsen 1984), $\Delta \tau = 376 \pm 37$ days (Schild & Chollín 1986), $\Delta \tau = 657 \pm 73$ days (Gondhalekar et al. 1986), $\Delta \tau = 478 \pm 73$ days (Lehár, Hewitt, & Roberts 1989). As can be seen, there was wide dispersion in the results obtained by different groups. However, the monitoring campaigns carried out during the early 1990s led to quite an odd situation, as all the results concentrate around two different values for the time delay: $\Delta \tau \sim 420$ days and $\Delta \tau \sim 510$ days. Calculations leading to the first value were presented by Vanderriest et al. (1989; $\Delta \tau = 415 \pm 20$ days), Schild (1990; $\Delta \tau = 404 \pm 10$ days), and Pelt et al. (1994; $\Delta \tau = 415 \pm 32$ days in the $B$ band and $\Delta \tau = 409 \pm 23$ days in radio). On the other hand, a value close to $510$ days was obtained by Beskin & Oknyanskij (1992; $\Delta \tau = 522 \pm 15$ days in the $B$ band and $\Delta \tau = 515 \pm 15$ days in the $r$ band), Roberts et al. (1991; $\Delta \tau = 515 \pm 37$ days), and Press, Rybicki, & Hewitt (1992a, $\Delta \tau = 536^{+14}_{-15}$ days in the $B$ band; 1992b, $\Delta \tau = 540 \pm 12$ days in $B +$ radio).

This situation abruptly changed when Kundić et al. (1995) presented their observations in the $g$ and $r$ bands. A sharp drop appeared in 1994 December and could be used to discern between the “long” (510 days) and the “short” (420 days) time delay, provided that continuous monitoring of QSO 0957 + 561 was carried out in the first half of 1996. This monitoring was performed, and the long time delay was rejected (Oscoz et al. 1996; Kundić et al. 1997). The controversy regarding the time delay finally seemed to be solved. However, the results appearing in the literature since 1995 concentrate again around two values, 417 and 424 days. These results are summarized as follows: $\Delta \tau = 423 \pm 6$ days (Pelt et al. 1996), $\Delta \tau = 417 \pm 3$ days (Kundić et al. 1997), $\Delta \tau = 424 \pm 3$ days (Oscoz et al. 1997), $\Delta \tau = 425 \pm 17$ days (Pijpers 1997), $\Delta \tau = 416.3 \pm 1.7$ days (Pelt et al. 1998), $\Delta \tau = 425 \pm 4$ days (Serra-Ricart et al. 1999), $\Delta \tau = 417.4$ days (Colley & Schild 2000).

This difference is irrelevant in the Hubble constant calculations, as the uncertainty introduced by the time delay is much lower than the uncertainty given by other factors (e.g., the main lens galaxy’s velocity dispersion or the lens modeling). However, the most accurate time delay should be used in the search for possible very rapid microlensing events in QSO 0957 + 561, and a week’s difference in $\Delta \tau$ could lead to the detection of false events or failure to detect real ones.

In this paper we have compiled photometric data of QSO 0957 + 561 from three different observing groups covering the period 1984–1999 to obtain an estimate of the time delay by means of several statistical methods. This includes new unpublished data corresponding to the last observational campaign (1998–1999) at the IAC80 Telescope. The data sets are presented in §2, while the methods for obtaining the time delay appear in §3. A first check of the goodness of these methods applied to simulated data is calculated in §4, and the best techniques are applied in §5 to real data. Finally, a discussion of our results appears in §6.

2. SELECTED DATA SETS

Several monitoring campaigns of QSO 0957 + 561 in different bands have been performed since 1979. However, to obtain the time delay, only the observations obtained by groups from three different institutions will be considered.
Fig. 1.—Light curves for $A$ and $B$ images of QSO 0957 + 561 in the $R$ band obtained on the night of 1999 February 19. IAC80 data are represented by filled squares, and NOT data are indicated by open circles; $1\sigma$ error bars are indicated (see text for more details).

| Light Curve | NOT$^b$ | IAC80 |
|-------------|--------|-------|
| $A$          | 0.005  | 0.016 |
| $B$          | 0.005  | 0.014 |
| H-D          | 0.002  | 0.005 |

* See §1 for details.

$^b$ Error values are expressed in magnitudes.

2.1. PU Data

Images were obtained with the Apache Point Observatory 3.5 m telescope in the $g$ and $r$ bands. QSO 0957 + 561 was monitored during several observational campaigns, although only data corresponding to the first two seasons have been published (Kundić et al. 1995, 1997): (1) from
1994 December to 1995 May, and (2) from late 1995 to mid 1996. The resulting data comprise 51 + 46 $g$-band points and 54 + 46 $r$-band points. The light curves were calculated via aperture photometry and have neither large error bars nor significant gaps. Their main characteristic is the presence of a sharp drop of about 0.1 mag in the $A$-component in late 1994 December, very useful for time delay calculations.

2.2. CFA Data

This data set is the largest ever obtained for a gravitational lens system. It consists of 1069 brightness measurements in the R band, from late 1979 to mid 1996. The observations corresponding to the period 1979–1989 were obtained with the Whipple Observatory 0.61 m telescope, while the remaining data were obtained with a 1.2 m telescope (Schild & Thomson 1995 and references therein). The reduction procedure followed a basic aperture photometry scheme (although a new automated photometry reduction code is now being applied by the authors, the results do not substantially differ from the “old” photometry). The error bars are not large, with the exception of the first 5 yr. The main problem with this data set is the scarcity of observations during the first 1800 days (81 brightness measurements). Moreover, those points coincide with the largest error bars. Thus, we will consider the data from mid 1984 for time delay calculations.

2.3. IAC Data

Lens monitoring was performed in four consecutive seasons (1996 February–June, 1996 October–1997 July, 1997 October–1998 July, and 1998 October–1999 July) using the CCD camera of the 82 cm IAC-80 Telescope (hereafter IAC80), sited at the IAC’s Teide Observatory (Tenerife, Canary Islands, Spain). A Thomson 1024 × 1024 chip was used, offering a field of nearly 7.5” diameter. Standard VRI broadband filters were used for the observations, corresponding fairly closely to the Landolt system (Landolt 1992). The final data set comprises 172 points in the V band, 301 points in the R band, and 112 points in the I band. Accurate photometry was obtained by simultaneously fitting a stellar two-dimensional profile on each component by means of DAOPHOT software (see details in Serraricart et al. 1999). A new, completely automatic IRAF task has been developed, demonstrating, using a sample of simulated data, that the proposed method can achieve high-precision photometry. However, the error bars obtained for IAC80 data are slightly larger than those of PU and CFA data. This could be explained by a decrease in chip sensitivity due to the age of the CCD. In order to assess the reliability of our method using real data, simultaneous observations of QSO 0957 + 561 were undertaken on 1999 February 19 by using the IAC80 and the 2.5 m Nordic Optical Telescope (NOT) sited at the IAC’s Roque de los Muchachos Observatory (La Palma, Canary Islands, Spain). The final reduced results are presented in Figure 1 and Table 1 (photometric errors for comparison star H and D are also included). The NOT light-curve errors (a few millimagnitudes) are much lower than the IAC80 ones, and this difference could be explained in terms of the following: (1) the NOT has a larger aperture than the IAC80, and (2) the NOT also has a better CCD chip. However, the good agreement between the two curves demonstrates that our reduction method works with a high degree of accuracy.1

3. DIFFERENT METHODS TO OBTAIN TIME DELAY ESTIMATES

The large amount of data described in § 2 adds new biases (different telescopes, filters, reduction processes, and behavior) to the inherent difficulty to analyze discrete, unevenly sampled temporary series. These facts led us to employ several statistical methods to calculate the time delay to increase the robustness of the results thus obtained. As a first step, several techniques will be checked by using simulated data: the discrete correlation function, dispersion spectra, $\delta^2$, $\delta^2$ modified, linear interpolation, and the z-transformed discrete correlation function.

3.1. Discrete Correlation Function (DCF)

The DCF (Edelson & Krolik 1988) is a technique valid for any physical quantity that is observed to vary in time. For two discrete data trains, $A_i$ and $B_p$, the formula representing their DCF is

$$DCF(\tau) = \frac{1}{M} \frac{(A_i - \bar{A})(B_j - \bar{B})}{\sqrt{(\sigma_A^2 - \bar{A}^2)(\sigma_B^2 - \bar{B}^2)}},$$

where $M$ is the number of data pairs for which $\tau - \alpha \leq \Delta t_{ij} < \tau + \alpha$, $\alpha$, $\epsilon_A$, and $\sigma_A$ being the bin semisize, the measurement error associated with the data set $k$, and the standard deviation, respectively. The maximum of the DCF is identified with the time delay.

3.2. Dispersion Spectra

The data model (Pelt et al. 1996) consists of two time series, $A_i = q(t_i) + \epsilon_A(t_i)$, $i = 1, \ldots, N_A$ and $B_j = q(t_j - \Delta \tau_{BA}) + \eta(t_j) + \epsilon_B(t_j)$, $j = 1, \ldots, N_B$, where $q(t)$ represents the intrinsic variability of the quasar, $\eta(t)$ accounts for the difference in magnitudes plus additional variability in time due to microlensing, and $\epsilon_A(t)$ and $\epsilon_B(t)$ are observational errors. These two series are combined into one, $C$ for every fixed combination $[\tau, i(t)]$ by taking all values of $A$ as they are and correcting the $B$ data by $\eta(t)$ and shifting them by $\tau$. The dispersion spectra that will be used here are represented by

$$D_{4,k}^2 = \min_{i(t)} \left[ \sum_{n=1}^{N_A} S_{n,m}^{(k)} W_{n,m} G_{n,m}(C_n - C_m)^2 \right],$$

where $W_{n,m}$ are statistical weights of the combined light curves and $G_{n,m} = 1$ when $C_n$ and $C_m$ come from different curves ($A$ or $B$) and 0 otherwise. From equation (2), we can consider two different approximations depending on the definition of $S_{n,m}^{(k)}$: (1) $S_{n,m}^{(k)} = 1$ if $|t_n - t_m| \leq \delta$ and 0 otherwise, and (2) $S_{n,m}^{(k)} = 1$ if $|t_n - t_m| \leq \delta$ and 0 otherwise, $\delta$ being the maximum distance between two observations that can be considered as nearby. The minimum value of equation (2) is assumed as the time delay.

3.3. The $\delta^2$ Method

The $\delta^2$ method (Serraricart et al. 1999) makes use of the similarity between the discrete autocorrelation function (DAC) of the light curve of one of the components and the

1 The photometric data are available at http://www.iac.es/project/quasar/lens7.html.
A-B discrete correlation function (DCF). From the DAC and DCF functions, one can define a function

$$\delta^2_m(\theta) = \left(\frac{1}{N}\right) \sum_{i=1}^{N} S_i [\text{DCF}(\tau_i) - \text{DAC}(\tau_i - \theta)]^2$$

(3)

for every fixed value \(\theta\) (days), where \(S_i = 1\) when both the DCF and DAC are defined at \(\tau_i\) and \(\tau_i - \theta\), respectively, and 0 otherwise. The most probable value for the time delay should correspond to the minimum of this function.

3.4. \(\delta^2\) Modified

This modification of the \(\delta^2\) method was suggested by R. Schild (1999, private communication). It consists in comparing the DAC and DCF curves by taking their ratio instead of calculating their difference. Hence, the final equation to obtain the time delay is

$$\delta^2_m(\theta) = \left(\frac{1}{N}\right) \sum_{i=1}^{N} S_i \left[ \frac{\text{DCF}(\tau_i)}{\text{DAC}(\tau_i - \theta)} \right]^2.$$  

(4)

3.5. Linear Interpolation (LI)

The linear method is similar to that suggested by Kundic et al. (1997). One of the two light curves (hereafter light curve 1) is selected, and the linear interpolation of data and their errors is considered as reference. The other light curve (hereafter light curve 2) is then shifted in magnitude by just the difference between the means of both light curves. After that, light curve 2 is shifted in time, and \(\chi^2\) per degree of freedom (\(\chi^2\)) for each time delay is calculated. The number of degrees of freedom is equal to the number of points of light curve 2 in the overlapping interval minus 2 (because we are fitting shifts in magnitude and time). The time that minimizes \(\chi^2\) is taken as a provisional time delay.

This procedure is followed by using as references both the A- and B-component light curves, selecting then the time delay closest to 421 days (the intermediate point between 417 and 425 days). The uneven sampling of the light curves usually leads to a better time delay taking as a reference one of the two light curves.

3.6. z-transformed Discrete Correlation Function (ZDCF)

The ZDCF (Alexander 1997) is a new method for estimating the cross-correlation function (CCF) of sparse, unevenly sampled light curves. Fisher's z-transform of the linear correlation coefficient, \(r\), is used to estimate the confidence level of a measured correlation. This technique attempts to correct the biases that affect the original DCF by using equal-population binning. The ZDCF involves three steps:

1. All possible pairs of observations, \(\{a_i, b_j\}\), are sorted according to their time lag, \(t_i - t_j\), and binned into equal-population bins of at least 11 pairs. Multiple occurrences of the same point in a bin are discarded so that each point appears only once per bin.
2. Each bin is assigned its mean time lag and the intervals above and below the mean that contain 1 \(\sigma\) of the points each.
3. The correlation coefficients of the bins are calculated and z-transformed. The error is calculated in z-space and transformed back to \(r\)-space.

The time lag corresponding to the maximum value of the ZDCF is assumed as the time delay between both components.

4. ANALYSIS OF THE DIFFERENT METHODS BY USING SIMULATED LIGHT CURVES

The application of the statistical methods described in § 3 to simulated data sets can serve to check the validity of their results under different conditions, always with discrete and irregularly sampled data sets. The six selected data sets are quite similar to those presented in Serra-Ricart et al. (1999), where several sets of simulated photometric data with similar irregularity in the observations (time distribution of the data), magnitudes, and error bars (i.e., as large as or even larger than those of PU, CfA, and IAC; the worst situation is selected) to that of the IAC observations were created.

First, a set of dates, \(x_i\), between approximately 1800 and 2000 (TJD = JD - 2,449,000), was generated with a pseudorandom separation, taken from a uniform distribution between 0 and 5 days. These data were alternatively separated in two time series, corresponding to A- and B-component light curves, the latter curve being shifted by 420 days to simulate the existence of a time delay. A first magnitude was calculated for each date with the relationship

$$y_i = \begin{cases} 
F(x_i) & \text{if } \beta = 0 \\
st_{\beta} & \text{if } \beta = \pm 1
\end{cases}$$

and hence characterized by or,\(\exp \left[ -\left( (y - y_i)^2 / 2\sigma^2 \right) \right]\) and hence characterized by or, equivalently, the variable \(d = y - y_i\) is distributed as exp \((-d^2 / 2\sigma^2)\). A \(\sigma\), taking pseudorandom values between 0.01 and 0.03 was generated for each \(x_i\). From here the quantities \(d\) pseudorandom numbers obtained from a normal Gaussian distribution with zero mean and standard deviation \(\sigma\), were calculated, allowing them to adopt positive or negative values. Finally, the magnitude was generated from the equation

$$y_i = F(x_i) + d_i = y_i + d_i$$

with an error bar of \(\sigma_i\). The A-component was made brighter by adding 0.1 to the magnitudes of the B-component.

The first selected function was

$$F_1: y = 17.17 + 0.5e^{-0.5f} \sin f, \quad \text{where } f = \frac{(x - 1800)}{20}.$$  

(5)

This function represents light curves in which a sharp event similar to that reported by Kundic et al. (1995) appears.

The second function is

$$F_2: y = 17.2 + 0.1 \sin f \sin 4f, \quad \text{where } f = \frac{x}{40}.$$  

(6)

In this case, the light curves present several maxima and minima, although none of them are clearly remarkable.

An additional function, consistent with the actual variability of Q0957+561, was created (the IAC observational data from the 1997-1998 seasons were selected as references). The light curves were then fitted by the function

$$F_3: y = 17.07 - 0.16e^f, \quad \text{where } f = -\frac{(x - 15.8 - m)^2}{2(10 + s)^2},$$  

(7)

\(m\) being the mean of the TJD in the selected range and \(s\) its standard deviation. The resulting simulated data show a lower variability to that obtained from F1 and F2.

Besides the comparison between the A- and B-components for the three data sets, an additional test was performed by removing some data of the A-component. By
We are currently working on a large project, the results of which will be published in a future paper. The project involves the study of a particular data set obtained from a telescope. The data set consists of two components, A and B, which are monitored simultaneously. The goal is to determine the time delay between these components, which is expected to be on the order of a few days. The data were collected over a period of several months, and the results will be compared with previous studies to validate the new method.

1. Time Delay from Real Data

Prior to applying the different methods to calculate the time delay from real data, some considerations have to be taken into account. First, data should be checked to eliminate inconsistent measurements. This modification of the raw data, based on a suggestion by E. E. Falco (1997, private communication), takes account of the possible existence of strong and simultaneous (not time-shifted) variations of some data point in both components. The inclusion of such “strange” brightness records in the final data sets, which probably originated from failures in the CCD or from bad weather conditions, creates artificial peaks or valleys in the light curve of one of the components. These maxima/minima have no importance when a sharp change in the behavior of the quasar is being analyzed but can lead to a completely wrong time delay estimate when dealing with a smoother season. To avoid these abnormal observations, we have removed the points with a simultaneous difference in magnitude in both components as compared with the previous and following records larger than 2.5 times their error bar. This was done by considering only those points with a difference in the observation dates of less than 10 days. The resulting data sets will be named bad point free (BPF). The 301 points of the IAC data set in the R band are reduced to 289 with this restriction, while less than 60 of the 1069 CfA observations have to be removed. Finally, no brightness measurement of the PU data seems to be wrong. However, the BPF restriction could be applied neither to the IAC I- and V-band data nor to the CfA 1989–1990 and 1991–1992 R-band data because of their low number of points, with a minimum distance between neighboring points more than 10 days in most of the cases.

Another drawback when dealing with real data is the impossibility of observing the system during certain months of the year and the consequent lack of suitable epochs. Once it is stated (see § 1) that the rough value of the time delay is around 420 days, the comparison between the A- and B-components should be made by previously selecting a “clean” data set (CD; see Serra-Ricart et al. 1999), i.e., homogeneous monitoring of both images during two active and clear (free from large gaps) epochs separated by ~420 days.

Finally, both types of corrections, BPF and CD, were combined to obtain the definitive data sets used in this paper. To see the importance of applying the CD-BPF corrections, we mention two extreme examples: (1) $\Delta \tau = 398 \pm 11$ days with the raw IAC data corresponding to the 1996–1997 season, while $\Delta \tau = 428 \pm 9$ days with the CD-BPF approximation; and (2) $\Delta \tau = 384 \pm 5$ days with

| Filter | Method | Points (CD) | Delay (CD) |
|--------|--------|-------------|------------|
| g ...... | A (1994–1995); B (1995–1996) | DCF | 42; 39 | 422 ± 1 |
| r ...... | A (1994–1995); B (1995–1996) | DCF | 41; 41 | 426 ± 5 |

2. Time Delays Obtained from the Application of a Monte Carlo Algorithm with the Different Techniques to the Six Simulated Data Sets

| Method | F1 (gap in A) | F2 (gap in A) | F3 (gap in A) |
|--------|--------------|--------------|--------------|
| DCF ($\alpha = 5$) | 421 ± 4 | 419 ± 1 | 420 ± 9 |
| $\delta^2$ ($\delta = 10$) | 422 ± 2 | 420 ± 2 | 417 ± 5 |
| ZDCF | 420 ± 3 | 421 ± 2 | 418 ± 14 |
| LI | 421 ± 3 | 420 ± 1 | 419 ± 12 |
| $\delta_{\mu}^2$ ($\delta = 10$) | 422 ± 9 | 425 ± 30 | 403 ± 34 |
| $D_{2,1}^2$ ($\alpha = 2$) | 421 ± 3 | 421 ± 12 | 414 ± 27 |
| $D_{2,2}^2$ ($\alpha = 2$) | 421 ± 2 | 423 ± 15 | 420 ± 26 |

Note.—The “true” time delay was 420 days.

3. Time Delays Obtained from the Application of a Monte Carlo Algorithm with Four Different Methods to the PU r and g Data

| Filter | Years | Method | Points (CD) | Delay (CD) |
|--------|-------|--------|-------------|------------|
| g ...... | A (1994–1995); B (1995–1996) | DCF | 42; 39 | 422 ± 1 |
| r ...... | A (1994–1999); B (1995–1996) | DCF | 41; 41 | 426 ± 5 |
the original 1993–1994 CfA data and $\Delta t = 423 \pm 2$ days with the CD-BPF approximation.

To summarize, Monte Carlo calculations were applied to the four techniques (DCF, $\delta^2$, ZDCF, and LI) with the CD and the CD-BPF restrictions. The PU, IAC, and CfA data sets were divided into observational seasons, leading to the 23 different time delay estimates per method appearing in Tables 3–6. The results obtained from the PU data in both the $\phi$ and $r$ filter are presented in Table 3. Notice that two of the methods (DCF and LI) employed here are also used by Kundic et al. (1997). Our results are quite similar to those obtained by these authors, and, moreover, they are always into their error. The small differences come from the selection of clean data sets. Tables 4 and 5 offer the time delay for the IAC data in the $R$ band and in the $I$ and $V$ bands, respectively. Finally, Table 6 was obtained from the CfA $R$-band data. The analysis of these delays is done by considering the CD-BPF quantities, except in those cases in which only the CD results could be obtained. The quantities obtained for the CfA 1992–1993 season with the DCF and $\delta^2$ methods can be discarded, as they appear to be clearly inconsistent (394 ± 1 and 403 ± 1 days, respectively).

6. DISCUSSION AND CONCLUSIONS

A first step in discussing the value of $\Delta t_{AB}$ consists in computing, for each of the techniques (DCF, $\delta^2$, ZDCF, and LI), the number of occurrences of each value of the time delays. These quantities, obtained from Tables 3–6, are depicted by the black lines in Figure 2: DCF (Fig. 2a), $\delta^2$ (Fig. 2b), ZDCF (Fig. 2c), and LI (Fig. 2d). (The different values of $\Delta t_{AB}$ have been grouped into 2 day bins.) As can be seen, two remarkable characteristics can be deduced from Figure 2: (1) there is a small dispersion in the delays, as most of them are in the interval 415–428 days; and (2) the centroids of the histograms, given by the average of the time delays derived in Tables 3–6, are always in the interval 420–424 days. These last quantities are represented in Figure 2 (open circles) together with their uncertainty (rms; see Table 7 and discussion below). Note that the largest peak of each histogram coincides with this average value, except for the ZDCF technique. In the histogram corresponding to $\delta^2$, the maximum corresponds to the values of 421–424 days, while the average is 421.8 ± 1.3 days. The DCF panel does not lead to a clear time delay, with two maxima in the intervals 421–420 and 423–424 days. The average here is given by 423.3 ± 1.4 days. In the case of LI, the peak is placed at $\Delta t_{AB} = 423–424$ days, and the average is 424.3 ± 1.2 days. Finally, the ZDCF method has a maximum around 423–424 days. However, the average is 420.6 ± 1.1 days, which is slightly different. The red histograms in Figure 2 represent the total number of occurrences obtained by adding the results from the four techniques. Its

### Table 4

| Years                        | Method | Points (CD) | Delay (CD) | Points (CD-BPF) | Delay (CD-BPF) |
|------------------------------|--------|-------------|------------|-----------------|----------------|
| $A$ (1996–1998); $B$ (1997–1999) | DCF    | 182; 220  | 428±12    | 173; 212        | 425±11         |
|                              | $\delta^2$ |          | 421±5     |                  | 421±4          |
|                              | ZDCF   |            | 418±13    |                  | 426±13         |
|                              | LI     |            | 421±16    |                  | 424±17         |
| $A$ (96); $B$ (1996–1997)    | DCF    | 31; 36    | 432±8     | 28; 33           | 436±8          |
|                              | $\delta^2$ |          | 425±8     |                  | 425±9          |
|                              | ZDCF   |            | 422±13    |                  | 424±14         |
|                              | LI     |            | 424±7     |                  | 425±7          |
| $A$ (1996–1997); $B$ (1997–1998) | DCF    | 46; 86    | 436±10    | 45; 84           | 424±13         |
|                              | $\delta^2$ |          | 431±4     |                  | 425±4          |
|                              | ZDCF   |            | 429±13    |                  | 424±12         |
|                              | LI     |            | 415±16    |                  | 426±16         |
| $A$ (1997–1998); $B$ (1998–1999) | DCF    | 79; 62    | 414±12    | 78; 61           | 416±12         |
|                              | $\delta^2$ |          | 414±10    |                  | 413±8          |
|                              | ZDCF   |            | 420±12    |                  | 420±13         |
|                              | LI     |            | 427±13    |                  | 423±13         |

**Table 5**

| Filter | Years                        | Method | Points (CD) | Delay (CD) |
|--------|------------------------------|--------|-------------|------------|
| $I$    | $A$ (1996–1998); $B$ (1997–1999) | DCF    | 65; 87     | 415±13    |
|        | $\delta^2$                  |        |            | 423±12    |
|        | ZDCF                        |        | 417±14    |
|        | LI                          |        | 419±9     |
| $I$    | $A$ (96); $B$ (1996–1997)    | DCF    | 19; 18     | 424±16    |
|        | $\delta^2$                  |        |            | 424±16    |
|        | ZDCF                        |        |            | 430±3     |
| $I$    | $A$ (1996–1997); $B$ (1997–1998) | DCF    | 18; 31     | 419±9     |
|        | $\delta^2$                  |        |            | 422±7     |
|        | ZDCF                        |        |            |           |
| $V$    | $A$ (1996–1998); $B$ (1997–1999) | DCF    | 28; 39     | 414±15    |
|        | $\delta^2$                  |        |            | 417±18    |
|        | ZDCF                        |        | 421±14    |
|        | LI                          |        | 427±11    |
| $V$    | $A$ (1997–1998); $B$ (1998–1999) | DCF    | 68; 149   | 432±11    |
|        | $\delta^2$                  |        |            | 429±11    |
|        | ZDCF                        |        | 418±9     |
|        | LI                          |        | 428±14    |
| $V$    | $A$ (1997–1998); $B$ (1998–1999) | DCF    | 29; 84    | 433±7     |
|        | $\delta^2$                  |        |            | 432±6     |
|        | ZDCF                        |        | 428±9     |
|        | LI                          |        | 423±3     |
center is again over 420 days, giving a maximum of 423–424 days.

To complement these calculations, which have been done by fixing the method and computing the probability of appearance of each delay, we can now represent the number of times each value appears for each data set, independently of the method employed. Four different data sets have been selected: (1) PU $r$ and $g$ filters (Fig. 3a), (2) IAC $R$ data (Fig. 3b), (3) IAC $V$ and $I$ records (Fig. 3c), and (4) CfA $R$ data (Fig. 3d). In this case, the centroid of the distributions, represented by open circles in Figure 3, is again over 420 days: 421.4 ± 1.1, 423.7 ± 1.3, 423.7 ± 1.2, and 421.8 ± 1.0 days for PU $r$ and $g$, IAC $R$, IAC $I$ and $V$, and CfA $R$, respectively. A first positive consequence of the PU results is their extremely short dispersion indicating the goodness of the data and the presence of the sharp event. The maximum here is placed between 419 and 424 days. However, the clearest peak appears from the IAC $R$ values around 423–424 days. The IAC $I$ and $V$ panel shows more dispersion, probably as a result of the higher error bars of the light curves. There is not a unique maximum here, as two peaks appear around 423–424 and 427–428 days. The highest dispersion in the results can be seen in Figure 3d, corresponding to the CfA data. The maximum would in this case be in the interval 417–422 days. A remarkable result here is that the average coincides with the maxima in all the panels. Once again, the red histogram gives the total number of occurrences.

The combination of the results derived from Figures 2 and 3 (centroids and maxima of the distributions) supports a $\Delta t_{\text{AB}}$ in the range of 420–424 days, although a time delay of around 417 days cannot be totally discarded. It is impor-

| Years          | Method | Points  | Delay | Points  | Delay |
|---------------|--------|---------|-------|---------|-------|
|               |        | (CD)    | (CD)  | (CD-BPF)| (CD-BPF)|
| $A$ (1984–1985); $B$ (1985–1986) | DCF   | 40; 49  | 420 ± 6 | 39; 47  | 419 ± 4 |
|               | $\delta^2$ | 420 ± 5 | 421 ± 6 |
|               | LI     | 429 ± 7 | 428 ± 7 |
| $A$ (1985–1986); $B$ (1986–1987) | DCF   | 49; 74  | 419 ± 10 | 46; 72  | 417 ± 12 |
|               | $\delta^2$ | 410 ± 13 | 407 ± 15 |
|               | LI     | 420 ± 9 | 417 ± 12 |
|               | 409 ± 14 | 431 ± 8 |
| $A$ (1986–1987); $B$ (1987–1988) | DCF   | 53; 58  | 431 ± 12 | 52; 58  | 428 ± 13 |
|               | $\delta^2$ | 434 ± 10 | 418 ± 17 |
|               | ZDCF   | 429 ± 15 | 430 ± 12 |
|               | LI     | 424 ± 9 | 420 ± 5 |
| $A$ (1987–1988); $B$ (1988–1989) | DCF   | 60; 53  | 425 ± 12 | 58; 53  | 428 ± 13 |
|               | $\delta^2$ | 423 ± 17 | 422 ± 18 |
|               | ZDCF   | 419 ± 17 | 411 ± 13 |
|               | LI     | 421 ± 11 | 422 ± 12 |
| $A$ (1988–1989); $B$ (1989–1990) | DCF   | 23; 19  | 417 ± 21 | 420 ± 10 |
|               | $\delta^2$ | 421 ± 8  | 420 ± 3 |
|               | ZDCF   | 421 ± 2  | 420 ± 3 |
| $A$ (1989–1990); $B$ (1990–1991) | DCF   | 40; 38  | 440 ± 5  | 40; 34  | 424 ± 19 |
|               | $\delta^2$ | 407 ± 7  | 420 ± 7 |
|               | ZDCF   | 417 ± 6  | 420 ± 7 |
|               | LI     | 410 ± 6  | 411 ± 3 |
| $A$ (1990–1991); $B$ (1991–1992) | DCF   | 15; 29  | 435 ± 8  | 433 ± 18 |
|               | $\delta^2$ | 433 ± 18 | 433 ± 18 |
|               | ZDCF   | 423 ± 19 | 426 ± 6 |
| $A$ (1991–1992); $B$ (1992–1993) | DCF   | 14; 72  | 394 ± 0 | 14; 67  | 394 ± 1 |
|               | $\delta^2$ | 402 ± 1 | 403 ± 1 |
|               | ZDCF   | 423 ± 12 | 416 ± 13 |
|               | LI     | 436 ± 9 | 437 ± 8 |
| $A$ (1992–1993); $B$ (1993–1994) | DCF   | 70; 98  | 411 ± 5 | 63; 93  | 419 ± 6 |
|               | $\delta^2$ | 423 ± 2 | 423 ± 2 |
|               | ZDCF   | 411 ± 11 | 424 ± 15 |
|               | LI     | 423 ± 13 | 433 ± 3 |
| $A$ (1993–1994); $B$ (1994–1995) | DCF   | 83; 111 | 422 ± 4 | 78; 95  | 422 ± 4 |
|               | $\delta^2$ | 421 ± 2 | 421 ± 2 |
|               | ZDCF   | 422 ± 6 | 422 ± 7 |
|               | LI     | 421 ± 4 | 418 ± 6 |
| $A$ (1994–1995); $B$ (1995–1996) | DCF   | 101; 66 | 415 ± 6 | 87; 57  | 418 ± 7 |
|               | $\delta^2$ | 411 ± 2 | 415 ± 3 |
|               | ZDCF   | 416 ± 2 | 416 ± 5 |
|               | LI     | 421 ± 12 | 418 ± 5 |
Fig. 2.—Number of times (black lines) that each value of the time delay appears, obtained from the results of Tables 3–6. The four different techniques have been considered: (a) DCF, (b) $\delta^2$, (c) ZDCF, (d) LI. The open circles represent the average value of the delays for each method, and the red histograms represent the sum of the values given by each method.

It is important to remark that the results are the same independently of the technique employed or of the data set selected.

The time delay between $A$- and $B$-components of Q0957 + 561 does not depend on the filter (as it is an achromatic effect) and/or the time (different campaigns), so it should be possible to merge the different sample results. A very important point is to assess the statistical reliability of the different delay calculation methods in order to estimate final delay errors. Several statistics were calculated: mean delay ($MD = \sum_{i=1}^{N} \Delta \tau_{ABi}/N$), mean error ($ME = \sum_{i=1}^{N} \epsilon_i/N$, with $\epsilon_i$ individual errors), and dispersion ($DI = [\sum_{i=1}^{N} (\Delta \tau_{ABi} - MD)^2/(N - 1)]^{1/2}$). If the error estimate is correct, then $ME \approx DI$, and the final error for the time delay will be given by the rms ($[\sum_{i=1}^{N} (\Delta \tau_{ABi} - MD)^2/(N(N - 1))]^{1/2}$; see Eadie et al. 1971). Tables 7–10 show the final results for the four methods. In all cases, within the statistical errors, good agreement is found between the mean error and the dispersion.

When this procedure is applied to the different time delays given by each technique, one obtains the results in the first four rows of Table 7, where the mean values of the time delay and the uncertainties (rms) are shown. According to these calculations, the definitive time delay would be in the interval 420 (ZDCF)–424 (LI) days, with an uncertainty below 1.4 days. This interval coincides with that derived from the analysis of Figures 2 and 3. When this calculation

| Method   | Mean Delay | Mean Error | Dispersion | rms  |
|----------|------------|------------|------------|------|
| DCF      | 423.3      | 10         | 7          | 1.4  |
| $\delta^2$ | 421.8      | 9          | 6          | 1.3  |
| ZDCF     | 420.6      | 11         | 5          | 1.1  |
| LI       | 424.5      | 7          | 6          | 1.2  |
| Total    | 422.6      | 9          | 6          | 0.6  |
is done with only the $R$-band results (Table 8), the results are almost identical.

The different analyses performed until now have been done considering all the results obtained in Tables 3–6. However, the error bars of some of these values exclude the interval 420–424 days, where, as shown before, there is the highest probability of finding the right $\Delta t_{AB}$. The mean delays and uncertainties obtained when these values are

| Method    | Mean Delay | Mean Error | Dispersion | rms  |
|-----------|------------|------------|------------|------|
| DCF       | 423.4      | 11         | 6          | 1.7  |
| $\delta^2$ | 420.5      | 8          | 6          | 1.7  |
| ZDCF      | 420.9      | 11         | 6          | 1.4  |
| LI        | 424.3      | 8          | 7          | 1.7  |
| Total     | 422.3      | 10         | 6          | 0.8  |

removed (first four rows of Table 9) are very similar to those of Table 7. Hence, $\Delta t_{AB}$ is now restricted to the interval 420.8–423.2 days, with an uncertainty below 1.3 days. Once again, the results derived from the $R$-filter data are almost the same (Table 10).

The validity of these statistical results led us to repeat the calculations of Tables 7–10, but this time taking into
account all the delays, i.e., without considering the method (DCF, $\delta^2$, ZDCF, or LI). This allows us to have four different time delays per year in most of the occasions and thus a larger amount of data for the statistical analysis. The results appear in the last row of Tables 7–10, although only the values of Tables 7 and 9, $\Delta \tau_{AB} = 422.6 \pm 0.6$ days and $\Delta \tau_{AB} = 422.0 \pm 0.6$ days, respectively, will be considered, as they were obtained from a larger amount of data. Adopting a conservative point of view, we will select the quantity with the higher uncertainty, $\Delta \tau_{AB} = 422.6 \pm 0.6$ days, as the final time delay.

Different treatments of the time delays obtained in § 5 have been performed. The analysis of these results always points to a time delay in the interval 420–424 days. None of these methods clearly favor the values of 416–418 days as the right time delay. Moreover, the averages of the quantities of Tables 7 and 9 give values around 422 days, coinciding with the maxima obtained in Figures 2 and 3. Assuming this value as the time delay between the A- and B-components of Q0957+561, let us check which of the results given in Tables 3–6 include 422 days in their error bars. Figure 4 offers the number of times, written as a percentage of the total number of time delays obtained for each method, that each value of $\Delta \tau_{AB}$ is included in these error bars. This probability is the same, 86%, for DCF, $\delta^2$, and ZDCF, and 74% for LI. In this case, the DCF and $\delta^2$ methods show the highest probability for a $\Delta \tau_{AB}$ of 421–422 days, while the peak in the LI curve is placed at 423 days. A wider maximum is obtained in the case of ZDCF, with the same probability between 418 and 421 days.

As can be seen, not all of the seasons in the different data sets are fully appropriate for calculating the time delay.

**TABLE 10**

| Method | Mean Delay | Mean Error | Dispersion | rms |
|--------|------------|------------|------------|-----|
| DCF    | 422.5      | 11         | 6          | 1.6 |
| $\delta^2$ | 420.9      | 9          | 6          | 1.8 |
| ZDCF   | 420.9      | 11         | 6          | 1.4 |
| LI     | 423.0      | 8          | 4          | 1.2 |
| Total  | 421.7      | 10         | 5          | 0.8 |

However, our intention was to analyze all the data in the three different data sets with four different methods and finally to restrict the uncertainty in the calculation of the time delay. The statistical treatment of all the results confirms a time delay of $\approx 422$ days.

We are especially grateful to E. E. Falco for advising us on the possible presence of strange points in our data sets, to R. Schild for helpful comments on the statistical methods, and to T. Alexander for providing us with his programs to calculate the ZDCF method and his help in understanding it. This work was supported by the P6-88 project of the Instituto de Astrofísica de Canarias (IAC), Universidad de Cantabria funds, and DGESIC (Spain) grant PB97-0220-C02.

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