A Generalized Approach to Estimating the Out-of-plane Buckling of Steel Sections with a Triangularly Corrugated Web

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Abstract. Building practice shows that I-sections consist of flanges and corrugated web could be quite effectively used for the construction of compressed structures. In the present work, an investigation into the behaviour and design of steel bending and compressed members with the triangularly corrugated web is carried out. Based on the result of the finite element analysis (FEA) conducted by the ABAQUS an attempt is made to generalize approaches in the evaluation of the stability of such structures. The necessity for this work is since, in the most building codes and studies, the issues of the global buckling of such structures are considered separately for beam structures and for structures at central and eccentric compression. Most of the building codes do not provide any recommendations and requirements for the design of compression members with corrugated webs at all and only consider the case of bending. It is shown that beam and column structures with triangularly corrugated web works almost identically in case of out-of-plane buckling. Finally, is proposed an expression allow to calculate the critical force in flanges of the cross-section in case of out-of-plane buckling caused by transverse bending, axial of eccentric compression uniformly, that could be useful for practical structural design.

1. Introduction

In recent decades steel sections with corrugated webs are found to be more and more widely applicable in construction and for this reason, they attract more interest among researchers and designers. Modern building codes make an afford to give the recommendations for the design of beam structures with corrugated web [1-4]. Currently, in the world practice, there are three main systems for corrugated webs: trapezoidal corrugations [5-8] (Middle East, USA, Japan, China, Europe), wavy corrugations [8-11] (Europe, Turkey, China), triangular corrugations [12-17] (Russia, Kazakhstan, Malaysia). The question of assessing the stability of thin-walled structures, which include any steel structures, and even more so thin-walled corrugated profiles, is often decisive in determining the possibility of their use and to determine their bearing capacity [18]. At the same time, for elements with corrugated webs, the flanges of which are also relatively thin-walled (with thickness from 6mm to 20mm, as a rule), several scenarios of buckling are distinguished, which should be taken into account and studied at the design stage. For steel I-section, the following are possible: overall buckling mode, that is topical for any steel structure of beam or column type, in which either the entire section
is in a compressed state (the case of axial compression), or just a part of the section is in tension, and another is compressed (the case of bending and eccentric compression).

The issues of local stability of compressed flange and corrugated web are left outside from the scope. This paper presents an attempt to formulate a generalizing approach to assessing the global buckling of steel sections with a triangularly corrugated web, as the most promising both from the point of industrial production and from the point of their application in the broadest conditions of practical construction.

The necessity for this work is since, in the most building codes and studies, the issues of the global buckling of such structures are considered separately for beam structures and for structures at central and eccentric compression. Most of the building codes do not provide any recommendations and requirements for the design of compression members with corrugated webs at all and only consider the case of bending [15]. However, building practice shows that such sections can be quite effectively used for the construction of compressed structures – see figure 1 and 2.

![Figure 1. Portal frame made of members with the triangularly corrugated web.](image1.jpg)

![Figure 2. The multi-storey framework of a residential building consists of compressed and bending structures with triangularly corrugated webs.](image2.jpg)

The present research deals with the global buckling effect of bending and axially compressed steel members with triangularly corrugated webs – see figure 3. The finite element (FEA) linear buckling analysis (LBA) by the software ABAQUS (2017) was carried out for various cases including different lengths of the structures and density of corrugation expressed as a ratio of height to length of half-wave of corrugation. During analysis were obtained the elastic critical moments and results were compared with estimations of the currently available design guidance [1-4, 12].

2. **Design rules for steel sections with triangular corrugated web**

Most of the modern design codes give recommendation only for steel members with trapezoidal or sinusoidal corrugated web [1-4]. Design features of triangularly corrugated web members are given in manual [12]. Since the corrugated web is not supposed to resist axial forces, what is frequently called “accordion effect”, fully or partially compressed members with corrugated web are designed considering that only the flanges resist the longitudinal axial force [5, 12-17]. Thus, for the structure stability, the following condition should be satisfied

\[ N_f \leq \varphi_f R_{yJ} b_f t_f \]  

(1)
in which $N_f$ is a force in a most compressed flange of the member, $b_f$ and $t_f$ are flange width and thickness respectively, $\varphi_f$ is reduction factor for out-of-plane buckling, $R_y$ is the material yielding strength, $\gamma_c = 1.0$ is working condition factor specified by [1]. It should be noticed that compressed force in a flange could be expressed as follows: $N_f = M/h_f$ for the bending member, $N_f = N/2$ for the axially compressed member, $N_f = M/h_f + N/2$ for the eccentrically compressed member.

Figure 3. (a) Considered design scheme for axial compressed member; (b) Triangularly corrugated cross-section; (c) Corrugated web parameters; (d) Considered design scheme for bending member.

According to design code [1, 2], the value of buckling reduction factor in case of a member with corrugated web could be adopted as for the section with a flat web that can be described by following expressions

\[
\varphi_f = \begin{cases} 
1 & \text{for } \bar{\lambda} < 0.6, \\
\pi^2 \gamma^\mu \cdot \bar{\lambda}^{-2} & \text{for } \bar{\lambda} > 4.4, \\
0.5 \left( \delta - \sqrt{\delta^2 - 39.48\bar{\lambda}^2} \right) / \bar{\lambda}^2 & \text{for other cases,}
\end{cases}
\]  

in which $\delta = 9.87 \left( 0.96 + 0.09\bar{\lambda} \right) + \bar{\lambda}^2$, $\bar{\lambda} = l / i_{\min} \sqrt{R_y/E}$, $l$ is a distance between anchorage points of the most compressed flange, $i_{\min}$ is a minimum value between the radius of gyration about strong $i_s = 0.5 (h_w + t_f)$ and weak $i_w = 0.289 b_f$ axis of corrugated web section, $\gamma^\mu = 1.3$ is a safety factor for elastic buckling.

3. Finite element analysis

While a numerical parametric study was considered two cases: axial compression and two-points bending of a member with corrugated web – see figure 3. All models are composed of two flanges 10mm x 200mm in size and 4mm x 600mm corrugated web. For the webs were assumed two different
corrugation densities $f/a = 50\text{mm}/100\text{mm}$ and $f/a = 70\text{mm}/200\text{mm}$. Such corrugation parameters represent practical geometries, which are commonly used for such structures in building practice [12-16]. A numerical study was considered four different lengths of compressed members $l$ equal to 4m, 6m, 8m and 10m and seven different spans of bending members $l$ equal to 3m, 4.5m, 5m, 6m, 7.5m, 9m and 12m. All compressed models were loaded by point axial force $P = 1\text{kN}$ located at one end of the structure at the cross-section centre line. All bending models were loaded by two-point loads $P = 1\text{kN}$ located in thirds of the span on the upper flange at the level of the structure centre line.

Boundary conditions were applied to either end of the models by restraining appropriate degrees of freedom to simulate the simply supported condition. An elastic material model was assumed with a yield strength value of 240MPa, modulus of elasticity $E = 206000$ MPa and Poisson’s ratio 0.3.

![Figure 4](image1.png)

*Figure 4.* The model adopted for the analysis of axially compressed member and its global elastic buckling mode.

![Figure 5](image2.png)

*Figure 5.* The model adopted for the analysis of the bending member and its global elastic buckling mode.

4. **Results of the numerical analysis**

All models accepted for present work were buckled under perfect conditions, wherein no initial imperfections and eccentric load are considered. The general view of the global buckling mode of compressed and bending members are shown in figure 4 and 5 respectively.

The critical buckling load was calculated using the load factor obtained from the eigenvalue’s analysis by multiplying it on the applied load. For example, $P$ accepted for this study is $1\text{kN}$, thus the critical buckling load $P_{cr} = 1\text{kN} \cdot \nu$, where $\nu$ is eigenvalue received from the buckling analysis. In the considering case, a critical moment at global buckling of the bending element could be obtained as $M_{cr} = P_{cr} l/3$.

In such an approach, the reduction factors were calculated as a ratio $M_{cr}/M_y$ for bending members and $N_{cr}/N_y$ for compressed members. Results of FEA are presented in table 1 and 2.

Analyzing the data, it could be noted that for the relatively slender members with slenderness parameter $\bar{K} > 4.4$ the minimum obtained values of reduction factors corresponds to the elastic
buckling predicting by Euler’s theory. For the relatively rigid members with \( \bar{\lambda} < 4.4 \), the minimum obtained values of reduction factors correspond to the inelastic buckling mode. In other words, members with corrugated webs behave like familiar structures with a flat web, but the first one has a greater load capacity as far as obtained reduction factors demonstrate the greater values for most cases up to 30% compared with the flat web structures of the same slenderness. As can be seen from the presented data the corrugation density weakly affects the buckling performance of the structure and values of the reduction factors for two types of corrugation looks almost similar. Moreover, it is distinctly that beam and column structures with triangularly corrugated web works almost identically in case of out-of-plane buckling. This fact once again confirms the “accordion effect” at which the corrugated web gets out from the work of the structure at bending (no matter longitudinal or transverse).

Generalizing the results, in figure 6 is given a graph shows the dependence of the values of reduction factors on the non-dimensional slenderness of member, that can be recommended for practical evaluation of load carrying capacity for steel triangularly corrugated web members.

Table 1. Results of linear buckling analysis (LBA) for bending members.

| Model     | \( \bar{\lambda} \) | \( M_{cr} \), kNm | \( M_y \), kNm | \( M_{cr}/M_y \) | \( \phi_f \) by Eq. (5)-(7) |
|-----------|---------------------|-------------------|----------------|-----------------|--------------------------|
| \( f/a = 50/100, l = 3m \) | 1.77                | 602.1             |                | 2.056           | 0.859                    |
| \( f/a = 50/100, l = 4.5m \) | 2.65                | 298.3             |                | 1.019           | 0.713                    |
| \( f/a = 50/100, l = 5m \) | 2.95                | 245.9             |                | 0.840           | 0.653                    |
| \( f/a = 50/100, l = 6m \) | 3.54                | 177.1             |                | 0.605           | 0.535                    |
| \( f/a = 50/100, l = 7.5m \) | 4.43                | 120.3             |                | 0.411           | 0.387                    |
| \( f/a = 50/100, l = 9m \) | 5.31                | 89.2              |                | 0.305           | 0.270                    |
| \( f/a = 50/100, l = 12m \) | 7.09                | 58.0              |                | 0.198           | 0.151                    |
| \( f/a = 70/200, l = 3m \) | 1.77                | 631.8             | 292.8          | 2.158           | 0.859                    |
| \( f/a = 70/200, l = 4.5m \) | 2.65                | 299.1             |                | 1.022           | 0.713                    |
| \( f/a = 70/200, l = 5m \) | 2.95                | 246.2             |                | 0.841           | 0.653                    |
| \( f/a = 70/200, l = 6m \) | 3.54                | 178.0             |                | 0.608           | 0.535                    |
| \( f/a = 70/200, l = 7.5m \) | 4.43                | 121.1             |                | 0.414           | 0.387                    |
| \( f/a = 70/200, l = 9m \) | 5.31                | 89.9              |                | 0.307           | 0.270                    |
| \( f/a = 70/200, l = 12m \) | 7.09                | 58.5              |                | 0.200           | 0.151                    |

Table 2. Results of linear buckling analysis (LBA) for axial compressed members

| Model     | \( \bar{\lambda} \) | \( N_{cr} \), kN | \( N_y \), kN | \( N_{cr}/N_y \) | \( \phi_f \) by Eq. (5)-(7) |
|-----------|---------------------|-----------------|--------------|-----------------|--------------------------|
| \( f/a = 50/100, l = 4m \) | 2.36              | 1670.2          |              | 1.740           | 0.767                    |
| \( f/a = 50/100, l = 6m \) | 3.54              | 745.38          |              | 0.776           | 0.535                    |
| \( f/a = 50/100, l = 8m \) | 4.72              | 420.67          |              | 0.438           | 0.341                    |
| \( f/a = 50/100, l = 10m \) | 5.91              | 269.23          |              | 0.280           | 0.218                    |
| \( f/a = 70/200, l = 4m \) | 2.36              | 1663.9          | 960          | 1.733           | 0.767                    |
| \( f/a = 70/200, l = 6m \) | 3.54              | 744.3           |              | 0.775           | 0.535                    |
| \( f/a = 70/200, l = 8m \) | 4.72              | 419.06          |              | 0.437           | 0.341                    |
| \( f/a = 70/200, l = 10m \) | 5.91              | 268.64          |              | 0.280           | 0.218                    |
5. Conclusions
In the present work, an investigation into the behaviour and design of steel triangularly corrugated web bending and compressed members is carried out. Based on the result of the FEA an attempt is made to generalize approaches in the evaluation of the stability of such structures. Proposed expression (1) allows to calculate the critical force in flanges of the cross-section in case of out of plane buckling caused by transverse bending, axial of eccentrical compression uniformly, that could be useful for practical structural design.

Surely the buckling performance of such structures requires further investigation considering the influence of various boundary conditions, imperfections in corrugated web and its alignment relative the flanges.

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