Elastic-plastic stress-strain state and strength of thick-walled pipe under the action of internal pressure

V N Barashkov and M Yu Shevchenko
Tomsk State University of Architecture and Building, 634003, Tomsk, Russia
E-mail: v.n.bar@mail.ru

Abstract. The task of calculating a plane axisymmetric elastic-plastic stress state of a thick-walled steel cylindrical pipe under the action of uniform internal pressure is considered. Thick-walled steel pipes are used in mechanical engineering, in heat engineering and heat-and-power engineering, in oil-producing and oil-refining industries, in the chemical industry. When loading in a pipe, two limiting states are realized: the elastic and plastic resistance of the material for two limiting values of pressure. For an intermediate pressure value, the cross-section of the pipe consists of two concentric annular zones – an inner elastic-plastic zone and an outer elastic zone. The exhaustion of the bearing capacity of the pipe occurs when the elastic-plastic deformation zone, spreading from the inner surface of the pipe, reaches its outer surface. Therefore, the presence of an elastic deformation zone at the outer surface of the pipe does not lead to the destruction of the structure. The aim of the work is to create an analytical technique for determining the calculated internal pressure, satisfying the strength condition under elastic-plastic deformation of steel cylindrical thick-walled pipes of different geometry, and leading to the most complete realization of the mechanical properties of the material. Recommendations on the effective search for the specified internal pressure are given.

1. Introduction

Thick-walled steel pipes are widely used in mechanical engineering for structures with high reliability requirements; in power and heat engineering for heating systems and boiler equipment; in the oil and oil refining industry in drilling operations and in the construction of oil pipelines; in the chemical industry, pipes are used to transport various aggressive media: solutions of alkalis, acids and salts, and operate under huge (up to 2-3 hundred atmospheres) loads. Welded structures of cylindrical shape are used for the manufacture of various chemical apparatus and tanks. In most cases, the pipes are operated under uniform internal pressure. The task of elastic-plastic deformation of pipes has been considered by many authors, for example, in [1-4]. The works [5, 6] are devoted to axisymmetric tasks of calculation of stress-strain state (hereinafter, SSS) of structures. Calculation of stresses and strains arising in pipes during operation is a mandatory stage of strength analysis in the design of various industrial facilities. In this case, the design result should be a structure that meets the strength condition, and the calculated value of the internal pressure should be as possible. The purpose of this work is to create an analytical technique for determining the calculated internal pressure, which satisfies the strength condition for elastic-plastic deformation of thick-walled steel cylindrical pipes of various geometries and leads to the most complete realization of the mechanical properties of the material.
2. Task statement

The article considers the solution of the task of flat elastic-plastic SSS and strength of thick-walled steel pipes of different geometry loaded with uniform pressure $p_a$ on the inner surface from the standpoint of the theory of elasticity and plasticity. The case of symmetric distribution of the SSS parameters relative to the longitudinal axis $z$ of the cylindrical coordinate system $r, \theta, z$ is of practical interest. Therefore, for the plane axisymmetric task of elasticity theory, the circumferential displacements, axial and angular deformations and shear stresses are zero. Radial displacements $u_r$, radial $\varepsilon_r$ and circumferential $\varepsilon_\theta$ deformations, radial $\sigma_r$ and circumferential $\sigma_\theta$ stresses depend only on the radius $r$ and do not depend on the polar angle $\theta$. The inner radius of the pipe is $r = a$, the outer is $-b$.

With increasing internal pressure $p_a$ from the inner surface to the outer thickness of the pipe begin to develop elastic-plastic deformation. Therefore, the pipe section consists of two concentric annular zones: elastic-plastic and elastic. The boundary of these zones along the length and thickness of the pipe is a cylindrical surface radius $r = c$ (Figure 1 reprinted from [7]).

![Figure 1. Deformation zones in the pipe](image)

Plastic deformations take place in the inner zone $a \leq r \leq c$ and remain elastic in the outer zone $c \leq r \leq b$ [7-9]. The exhaustion of the bearing capacity of the pipe occurs when the elastic-plastic deformation zone extends from the inner surface of the pipe to the outer surface $r = b$. Therefore, the presence of an elastic deformation zone at the outer surface of the pipe does not lead to the destruction of the structure.

When implementing the task, the conditions of the Huber–Mises plasticity and the incompressibility of the material are used. The pipe material is considered ideal elastic-plastic, for which the relationship between the intensities of stress and strain is described by the Prandtl diagram. The calculations were carried out using Microsoft Excel.

3. Analytical technique for solving the task

The paragraph consistently describes the stages of the analytical technique of solving the problem of determining the value of the calculated internal pressure that meets the condition of the strength of the pipe and the maximum implementation of the mechanical characteristics of the material. For the obtained value of the internal pressure at the outer surface of the pipe remains elastic deformation zone, which does not lead to the destruction of the structure.
3.1. Formulas for calculating stresses

In the elastic stage of the material operation, these normal stresses are determined by the formulas obtained from the solution of the Lame task in the absence of external pressure [7-9]

\[ \sigma_r = p_a \frac{a^2}{b^2 - a^2} \left( 1 - \frac{b^2}{r^2} \right), \quad \sigma_0 = p_a \frac{a^2}{b^2 - a^2} \left( 1 + \frac{b^2}{r^2} \right). \] (1)

If there are two deformation zones in the pipe, formulas (1) are used to calculate the stresses in the elastic zone, which, taking into account the size of this zone (a = c), are written as follows

\[ \sigma_r = q \frac{c^2}{b^2 - c^2} \left( 1 - \frac{b^2}{r^2} \right), \quad \sigma_0 = q \frac{c^2}{b^2 - c^2} \left( 1 + \frac{b^2}{r^2} \right), \] (2)

where \( q \) is the radial stress at the boundary between the zones, which is analogous to the external pressure.

In the elastic-plastic zone the stresses are determined by the formulas

\[ \sigma_r = -p_a + \frac{2}{\sqrt{3}} \sigma_T \ln \frac{r}{a}, \quad \sigma_0 = \sigma_r + \frac{2}{\sqrt{3}} \sigma_T, \] (3)

which follow from the joint solution of the differential equilibrium equation of the plane task and the Huber–Mises plasticity condition. To simplify the task, we assume that the pipe material is incompressible, i.e. we take the Poisson's ratio \( \nu = 0.5 \). Then, the use of generalized Hooke's law for deformation \( \varepsilon_r \) and equality \( \varepsilon_r = 0 \) in the plane deformation task leads to a formula for calculating the axial stresses in both deformation zones

\[ \sigma_z = 0.5(\sigma_r + \sigma_0). \] (4)

Determination of longitudinal stresses should be carried out using formulas (2) and (3) for elastic and elastic-plastic deformation zones of the pipe, respectively.

3.2. Formulas for determining the boundaries of the zones of elastic-plastic and elastic deformations through the thickness of the pipe.

From the condition of continuity of normal stresses \( \sigma_0 \) and \( \sigma_r \) on the boundary of the two zones \( r = c \) using the formulas (2) and (3) two equations for determining the parameters \( c \) and \( q \) are obtained:

\[ \ln \frac{c}{a} + \frac{1}{2} \left( 1 - \frac{c^2}{b^2} \right) = \frac{p_a \sqrt{3}}{2 \sigma_T}, \] (5)

\[ q = p_a - \frac{2}{\sqrt{3}} \sigma_T \ln \frac{c}{a}. \] (6)

The transcendental equation (5) is used to determine the radius \( c \) of the boundary surface. Further, the radial stress \( q \) acting on the boundary of the zones is calculated by the formula (6). Stresses in the elastic-plastic zone of the pipe are calculated by formulas (3) and (4), and in the elastic zone – by formulas (2) and (4).

3.3. Limiting states in the pipe

With increasing pressure in the pipe, two limiting states are realized: the limits of elastic and elastic-plastic resistance of the pipe. As an example, the selected pipe with dimensions: \( a = 0.16 \) m, \( b = 0.36 \) m, thickness \( h = 0.2 \) m; yield strength of the material \( \sigma_T = 230 \) MPa, Young's modulus \( E = 2 \cdot 10^5 \) MPa. The ratio of the radii is \( N_{ba} = b/a = 2.25. \)
3.3.1. The limit of elastic resistance pipe

The limit elastic state, in which the pipe material is deformed elastically throughout the thickness, corresponds to the radius value \( c = a \). Taking this radius value \( c \) into account in equation (5) leads to an expression for the elastic resistance limit pressure

\[
p_{ay} = \frac{\sigma_T}{\sqrt{3}} \left( 1 - \frac{a^2}{b^2} \right) = 0.463\sigma_T = 106.490 \text{ MPa}.
\]  

The substitution of the pressure expression \( p_{ay} \) in the ratio (1) for the stresses in the elastic zone leads to formulas for determining the stresses arising in the pipe at the limit of its elastic resistance,

\[
\sigma_r = \frac{\sigma_T}{\sqrt{3}} \frac{a^2}{b^2} \left( 1 - \frac{b^2}{r^2} \right), \quad \sigma_0 = \frac{\sigma_T}{\sqrt{3}} \frac{a^2}{b^2} \left( 1 + \frac{b^2}{r^2} \right).
\]  

Stress plots calculated by formulas (8) and (4) are shown in Figure 2.

![Figure 2. Plots of the normal stresses when reaching the limit of elastic resistance pipe](image)

Figure 2. Plots of the normal stresses when reaching the limit of elastic resistance pipe

It should be noted that the longitudinal stresses are smaller than the other two normal stresses. Analysis of the results allows us to conclude that the inner surface of the pipe is primarily deformed plastically, since it is in the most severe conditions. The intensity of stresses on the surface \( r = a \) is determined by the formula

\[
\sigma_i = \frac{\sqrt{2}}{2} \sqrt{(\sigma_r - \sigma_0)^2 + (\sigma_0 - \sigma_z)^2 + (\sigma_z - \sigma_i)^2}
\]

for stress are \( \sigma_r = 0.463\sigma_T, \ \sigma_0 = 0.691\sigma_T, \ \sigma_z = 0.114\sigma_T, \ \sigma_i = \sigma_T = 230 \text{ MPa} \).

Thus, the intensity of stresses on the pipe surface \( r = a \) is equal to the yield strength \( \sigma_T \), which follows when using the model of an ideal elastic-plastic body. When approaching the outer surface of the pipe value \( \sigma_i \) decreases, and on the surface of \( r = b \) intensity of stresses \( \sigma_i = 45.31 \text{ MPa} \). Therefore, with increasing internal pressure, the material of the outer surface of the pipe is deformed plastically last.

3.3.2. The limit of elastic-plastic resistance of pipe

The limit state of the plastic resistance of the pipe corresponds to the value of the radius \( c = b \). In this state the material of the pipe throughout the thickness of the deformed elastic-plastic. The formula for determining the limit pressure of this state follows from equation (5):
By substituting the expression of pressure $p_{a, pr}$ in formulas (3) for stresses in the elastic-plastic zone, formulas are obtained for determining the stresses arising in the pipe at the limit of its elastic-plastic resistance,

$$
\sigma_r = -p_{a, pr} + \frac{2}{\sqrt{3}} \sigma_T \ln \frac{r}{a}, \quad \sigma_0 = \sigma_r + \frac{2}{\sqrt{3}} \sigma_T.
$$

Using the formulas (9) and (4), it is possible to obtain the stress distribution over the pipe thickness. Stress plots are shown in Figure 3.

Analysis of the results allows us to conclude that in the elastic-plastic stage of deformation in the most severe conditions is the material on the outer surface of the pipe, where the circumferential stress $\sigma_\theta = 1.155 \sigma_T$. This result confirms the well-known fact that the destruction of the pipe begins with the formation of a longitudinal crack from the action of mainly tensile circumferential stresses $\sigma_\theta$ at zero values of radial stress $\sigma_r$.

The intensity of stresses in the entire volume of the pipe coincides with the value of the yield strength $\sigma_T$, which confirms the fulfillment of the Huber – Mises plasticity condition and is consistent with the Prandtl diagram adopted for the material.

### 3.4 Relations for deformations and radial displacements

It is known that radial $\varepsilon_r$ and circumferential $\varepsilon_\theta$ deformations are associated with radial displacements $u$ and radius $r$ relations

$$
\varepsilon_r = \frac{du}{dr}; \quad \varepsilon_\theta = \frac{u}{r},
$$

where does the expression for displacement

$$
u = \varepsilon_0 r.
$$

From the condition of incompressibility

$$
\varepsilon_r + \varepsilon_\theta + \varepsilon_z = 0,
$$

Figure 3. Plots of the normal stresses when reaching the limit of elastic-plastic resistance pipe
equal to zero axial deformation \( \varepsilon_z = 0 \) for the plane deformed state and the formula (10) is followed by a differential equation for radial displacement, valid in both elastic and elastic-plastic zones

\[
\frac{du}{dr} + \frac{u}{r} = 0.
\]  

Therefore, the dependence \( u(r) \) can be established by the formula (11) using Hooke's law and the values of stresses in the elastic zone:

\[
u = \frac{r}{E} \left[ \sigma_0 - \frac{1}{2} (\sigma_r + \sigma_z) \right].
\]

The substitution of stresses in this expression by formulas (8) and (4), taking into account that for the elastic zone the radius \( a = c \) leads to the dependence of the radial displacement on the radius, which is valid in both zones of the pipe deformation,

\[u = \frac{\sqrt{3} \sigma_T c^2}{2E} \frac{r^2}{r^2}.
\]

It should be noted that the value of the calculated internal pressure \( p_{calc} \) is implicitly present in the formula (14). This pressure is included in the transcendental equation (5), from which the radius \( c \) of the boundary surface is determined. Also in the same equation includes the geometric dimensions of the pipe – radii \( a \) and \( b \). For deformations according to formulas (10), we have the relations

\[\varepsilon_r = -0.866 \frac{\sigma_T c^2}{E r^2}, \quad \varepsilon_0 = 0.866 \frac{\sigma_T c^2}{E r^2},
\]

which satisfy the incompressibility condition (12) and the differential equation (13).

4. The results of the calculation of pipes of different geometry under the condition of strength

Consider the task of elastic-plastic deformation of the pipe for the calculated pressure \( p_{calc} = 0.696 \sigma_T = 160.08 \) MPa, which is about 75 % of the limit pressure \( p_{apr} \) and satisfying the inequality

\[p_{calc} < p_{apr} = 215.280 \text{ MPa},
\]

which is a strength condition for this task [9, 10].

From the solution of the transcendental equation (5) using the software Mathcad presented on the Internet, the radius of the zone boundary \( c = 0.2103 \) m is determined. In this case, only one of the two roots of equation (5) satisfies the condition \( a \leq c \leq b \). From the equation (6) for the known values of the parameters \( c \) and \( p_{calc} \), the value of the stress \( q \) acting on the boundary of the elastic and elastic-plastic zones is determined

\[q = p_{calc} - \frac{2}{\sqrt{3}} \sigma_T \ln \frac{c}{a} = 0.38 \sigma_T = 87.393 \text{ MPa}.
\]

Formulas (3) and (4) were used to calculate stresses arising in the internal elastic-plastic deformation zone \( a \leq r \leq c \). According to the results, the intensity of stresses throughout the thickness of the elastic-plastic zone of the pipe is equal to the yield strength \( \sigma_T \). Formulas (2) and (4) were used to calculate stresses in the external elastic zone \( c \leq r \leq b \). Judging by the stress intensity values, the entire outer zone of the thick-walled pipe is deformed elastically, and when approaching the surface \( r = b \), the value \( \sigma_r \) decreases. Figure 4 and Figure 5 shows plots of stresses, radial displacements and strains through the thickness of the pipe.
Figure 4. Plots of normal stresses in two zones of deformation pipe

Figure 5. Plots of displacements and deformations in two zones of deformation pipe

The data of Table 1 show the change in the maximum values of stresses and intensity of stresses on the outer surface of the pipe, radius $c$, stresses $q$ at the boundary of zones with increasing calculated pressure $P_{\text{calc}}$.

Table 1. The calculation results for pipe of the seven pressure values $P_{\text{calc}}$.

| $n$ | Case 1     | Case 2     | Case 3     | Case 4     | Case 5     | Case 6     | Case 7     |
|-----|------------|------------|------------|------------|------------|------------|------------|
|     | $P_{\text{calc}}$, MPa | 117.441    | 139.203    | 150.083    | 171.845    | 182.726    | 193.606    | 204.487    |
|     | $c$, m     | 0.168      | 0.188      | 0.199      | 0.226      | 0.242      | 0.262      | 0.290      |
|     | $q$, MPa   | 103.703    | 96.654     | 92.244     | 80.676     | 72.743     | 62.265     | 46.764     |
|     | $\sigma_r|_{r=c}$, MPa | -103.703   | -96.654    | -92.244    | -80.676    | -72.743    | -62.265    | -46.764    |
|     | $\sigma_0^{\text{max}}|_{r=b}$, MPa | 58.179     | 72.276     | 81.097     | 104.234    | 120.099    | 141.059    | 172.060    |
|     | $\sigma_z^{\text{max}}|_{r=b}$, MPa | 29.090     | 36.138     | 40.548     | 52.117     | 60.049     | 70.529     | 86.030     |
|     | $\sigma_i^{\text{max}}|_{r=b}$, MPa | 50.385     | 62.593     | 70.232     | 90.269     | 104.009    | 122.160    | 149.009    |
For the considered pipe, additional studies were carried out to determine the maximum possible value of the calculated pressure $p_{\text{calc}}^{\text{max}}$, at which there is still a solution of the transcendental equation (5) satisfying the condition $a < c < b$. The results showed that this value is $p_{\text{calc}}^{\text{max}} = 214.50$ MPa, which is 99.6% of the limit pressure $p_a^{\text{pr}}$. The value of the radius $c = 0.34$ m, i.e. the elastic-plastic deformation zone is 94.333% of the pipe thickness, and the width of the elastic zone is only 0.02 m. The stress at the boundary of two zones is $q = 14.608$ MPa. On the outer surface of the pipe circumferential stress is $\sigma_0 = 236.365$ MPa, axial stress is $\sigma_z = 118.1826$ MPa. The intensity of the stress is $\sigma_l = 204.698$ MPa, which is 89% of the yield strength. Thus, the purpose of determining the value of the internal pressure that best meets the strength condition (15) in the elastic-plastic deformation of a thick-walled pipe is achieved.

Using the equations of the theory of elasticity and plasticity, the deformation of the pipe, which can be attributed to the class of thin shells, is also considered. The solution of the problem with the help of the theory of shells allows us to obtain estimates of the parameters of the stress-strain state of the structure in the form of forces and moments, while the specific law of the stress distribution over the thickness is associated with the hypotheses of the theory of shells. Therefore, the calculation scheme based on the relations of the theory of shells can give only an approximate idea of the nature of the distribution of the parameters of the SSS on the thickness of the structure. At the same time, the spatial approach makes it possible to determine stresses and strains at any point of the structure [11].

For Figure 6 (reprinted from [10]) and in Table 2 the results of calculation of the stress state pipe of a large diameter are presented: $N_{ha} = 1.0526$, $h = 0.075$ m, $a = 1.425$ m, $b = 1.50$ m, $p_{ay} = 12.947$ MPa, $p_{a^{\text{pr}}} = 13.622$ MPa. Under the action of pressure $p_{\text{calc}}^{\text{max}} = 13.082$ MPa at the boundary of the two zones radius is $c = 1.433$ m, the stress is $q = 11.623$ MPa. The width of the elastic-plastic zone is only 0.008 m, i.e. 10.67% of the pipe thickness. Further increase of the calculated pressure $p_{\text{calc}}$ does not give solutions of the transcendental equation (5) satisfying the condition $a < c < b$.

![Figure 6. Stress plots in a large diameter pipe](image)

| $r$ | $a$ | $(a+c)/2$ | $c$ | $(b+c)/2$ | $b$ |
|-----|-----|-----------|-----|-----------|-----|
| $\sigma_r$, MPa | -13.0822 | -12.3517 | -11.6232 | -5.61202 | 0 |
| $\sigma_0$, MPa | 252.499 | 253.2295 | 253.958 | 247.949 | 242.337 |
| $\sigma_z$, MPa | 119.7084 | 120.4389 | 121.1685 | 121.1685 | 121.1685 |
| $\sigma_l$, MPa | 230.0 | 230.0 | 230.0 | 219.590 | 209.870 |

Table 2. The stress distribution through the thickness of the pipe
5. Conclusion

1. An analytical method for calculating the elastic-plastic stress-strain state and strength of steel cylindrical thick-walled pipes has been developed from the standpoint of the axisymmetric plane task of the theory of elasticity and plasticity. In solving the task, the condition of plasticity Huber-Mises is used. The model of incompressible, ideal elastic-plastic body for which the dependence between the stress and strain intensities is described by the Prandtl diagram is considered. For a wide range of geometric dimensions, the maximum possible values of the design pressure are obtained, which allow the most complete realization of the mechanical properties of the material in the calculation of the strength of pipes, and do not lead to the exhaustion of their bearing capacity due to the presence of an elastic deformation zone at the outer surface.

2. The reliability of the results obtained for pipes of different geometry is confirmed by a strict mathematical formulation of the task; by a results of test calculations; by fulfillment of the boundary conditions for radial stresses \( \sigma_r \); by the absence of stress \( \sigma_r, \sigma_\theta \) jumps and the equality of stresses \( \sigma_r \) and \( \sigma_q \) at the boundary of the zones, confirming the theoretical positions set out in [7, 9]; is confirmed by fulfillment of the conditions of plasticity Huber - Mises and Prandtl diagram in the elastic-plastic zone as well as the implementation of incompressibility conditions.

Nomenclature

| Symbol | Description |
|--------|-------------|
| \( r, \theta, z \) | cylindrical coordinates of the body point respectively radius (m), polar angle (°), axial coordinate (m) |
| \( u, v, w \) | respectively radial, circumferential and axial displacement (m) |
| \( \varepsilon_r, \varepsilon_\theta, \varepsilon_z \) | respectively radial, circumferential and axial deformation |
| \( \sigma_r, \sigma_\theta, \sigma_z \) | respectively radial, circumferential and axial stresses (MPa) |
| \( \sigma_i \) | stress intensity (MPa) |
| \( P_a \) | the internal pressure in the pipe (MPa) |
| \( P_{ay} \) | pressure corresponding to the elastic resistance limit of the pipe (MPa) |
| \( P_{apr} \) | pressure corresponding to the plastic resistance limit of the pipe (MPa) |
| \( P_{calc} \) | calculated internal pressure (MPa) |
| \( a \) | the inner radius of the pipe (m) |
| \( b \) | the outer radii of the pipe (m) |
| \( h \) | the thickness of the pipe (m) |
| \( N_{ba} \) | the ratio of the outer radius to the inner radius |
| \( c \) | boundary of elastic and elastic-plastic deformation zones (m) |
| \( \nu \) | Poisson ratio of pipe material |
| \( \sigma_f \) | pipe material yield strength (MPa) |
| \( E \) | Young's modulus of pipe material (MPa) |
| \( \sigma_{r, \theta, z}^{\text{max}} \) | respectively the maximum values of radial, circumferential, axial stresses and stress intensity on the outer surface of the pipe (MPa) |
The maximum possible value of the calculated internal pressure at which there is a solution of the transcendental equation (5) (MPa)

**Abbreviation**

SSS stress-strain state

**References**

[1] Alexandrov S E and Goldstein R V 2011 Deformation and destruction of materials 9 15–25
[2] Artemov M A, Larin I A and Potapov N S 2010 Vestnik Voronezhskogo gosudarstvennogo tekhnicheskogo universiteta. 6 9 117–119
[3] Kilikovskaya O A, Ovchinnikova N V and Pendyurina M N 2010 Vestnik Tul’skogo gosudarstvennogo universiteta. Matematika. Mekhanika. Informatika 16 1 72–87
[4] Mironova S N 1996 Vestnik Samarskogo gosudarstvennogo tekhnicheskogo universiteta Seriya Fiz.-mat. nauki. 4 85–92
[5] Orlov M Yu, Glazyrin V P and Orlov Yu N 2017 AIP Conference Proceedings 1893, 030133
[6] Orlov M Yu, Glazyrin V P, Orlov Yu N and Orlova Yu N EPJ Web of Conferences 183, 01049
[7] Samul V I 1970 Fundamentals of the theory of elasticity and plasticity Moscow: Vysshaya Shkola Publ. 288
[8] Malinin N N 1975 Applied theory of plasticity and creep Moscow: Mashinostroenie Publ. 400
[9] Barashkov V N 2015 Solution of axisymmetric plane task of the theory of elasticity and plasticity for bodies of rotation taking into account elastic and elastic-plastic deformations Tomsk: TSUAB Publ. 84
[10] Barashkov V N and Shevchenko M Yu 2018 Vestnik Tomskogo gosudarstvennogo arkhitekturno-stroitelnogo universiteta Journal of Construction and Architecture 20 3 100–111 DOI: 10.31675/1607-1859-2018-20-3-100-111
[11] Barashkov V N and Gerasimov A V 2016 Vestnik Tomskogo gosudarstvennogo arkhitekturno-stroitelnogo universiteta 5 102–109