Defect Formation in First Order Phase Transitions with Damping

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Within the context of first order phase transitions in the early universe, we study the influence of a coupling between the (global U(1)) scalar driving the transition and the rest of the matter content of the theory. The effect of the coupling on the scalar is simulated by introducing a damping term in its equations of motion, as suggested by recent results in the electroweak phase transition. After a preceding paper, in which we studied the influence that this coupling has in the dynamics of bubble collisions and topological defect formation, we proceed in this paper to quantify the impact of this new effects on the probability of defect creation per nucleating bubble.

I. INTRODUCTION

According to the standard model and its extensions, symmetry breaking phase transitions are expected to occur in the early universe. The mechanism by which these transitions may take place could be either by the formation of bubbles of the new phase within the old one or by spinodal decomposition (i.e., either by a first order phase transition or by a second order one). In the particular case of the electroweak phase transition, for instance, common opinion inclines more towards the first of the two possibilities. As this scenario would have it, bubbles of the new phase nucleated within the old one (the nucleation process being described by instanton methods as far as the WKB approximation remains valid [1]), and subsequently expanded and collided with each other until they occupied all of the available volume at the time at which the transition was completed. In the process of bubble collision though, the possibility arises that regions of the old phase become trapped within the new one, giving birth to topologically stable localized energy concentrations known as topological defects.

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defects (for recent reviews see refs. [2]), much in the same way in which these structures are known to appear in condensed matter phase transitions. From a theoretical point of view, topological defects will appear whenever a symmetry group $G$ is spontaneously broken to a smaller group $H$ such that the resulting vacuum manifold $M = G/H$ has a non-trivial topology: cosmic strings for instance (vortices in two space dimensions) will form whenever the first homotopy group of $M$ is non-trivial, i.e., $\pi_1(M) \neq 1$.

To see how this could happen in detail, let's consider the Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi^* - V(\Phi)$$

for a complex scalar field $\Phi$. Let us assume that $V$ is of the type $V = \frac{\lambda}{4}(|\Phi|^2 - \eta^2)^2$, and that its parameters are functions of the temperature such that at high temperatures $\Phi = 0$ is the only minimum of $V$, while at zero temperature all the $|\Phi| = \eta$ states correspond to different degenerate minima. Then the structure of the vacuum manifold will be that of $S^1$. $\pi_1(S^1) \neq 1$ however, and thus we can form non-contractible loops in the vacuum manifold: the model admits cosmic string solutions.

The way in which these configurations would actually form during a phase transition is via the Kibble mechanism [3]. In the context of a first order transition the basic idea behind it is that bubbles are nucleated with random phases of the field, and that when two regions in which the phase takes different values encounter each other the phase should interpolate between this two regions following a geodesic path in the vacuum manifold—the so-called geodesic rule. A possible scenario for vortex formation in two space dimensions would then look like this: three bubbles with respective phases of 0, $2\pi/3$, and $4\pi/3$ collide simultaneously. Thus, if we walk from the first bubble to the second, then from the second to the third, and finally back from the third to the first one again, the phase will have wound up by $2\pi$ in our path, having traversed the whole of the vacuum manifold once along the way. Continuity of the field everywhere inside the region contained by our path demands then that the field be zero at some point inside of it, namely the vortex core. In the limit in which the bubbles extend to infinity, outwards from the center of collision, removal of this vortex would cost us an infinite amount of energy, since it would involve unwinding the field configuration over an infinite volume. The vortex is thus said to be topologically stable. In three space dimensions, the resulting object would obviously be a string, rather than a vortex.

Clearly, there are other ways in which strings could be formed. Collisions of more than three bubbles could also lead to string formation, or, for instance, two of the bubbles could hit each other first, with the third one hitting only at some later time while the phase is still equilibrating within the other two. This event in particular will be far more likely than a simultaneous three way (or higher order) collision, and it is probably the dominating process by which strings are formed (especially if nucleation probabilities are as low as required for WKB methods to be valid). When two bubbles collide, the phase will try to reach a homogeneous distribution within the single true vacuum cavity formed after the collision. However, in the absence of coupling between the scalar and the rest of the matter this process is never completed—essentially because the velocity at which the phase propagates inside the bubbles is the same as that with which the bubble walls expand—and thus this does not substantially modify the picture of defect formation described above.
Bubble collisions have been studied by a number of groups, most notably by Hawking et al. [4], Hindmarsh et al. [5], Srivastava [6], Melfo and Perivolaropoulos [7], and more recently by the author and Melfo [8]. Only in this last work however did the authors concern themselves with the interaction between the bubble field and the surrounding plasma however, despite of the fact that from current work in the electroweak phase transition we can expect that in many cases such interaction will not be negligible (e.g., see Ref. [9]). The basic underlying reason is relatively simple to understand: as the bubble wall sweeps through an specific point, the Higgs field $\Phi$ acquires an expectation value, and the fields coupled to it acquire a mass. Thus, particles with not enough energy to acquire their corresponding mass inside the bubble will bounce off the wall (thus imparting negative momentum to it), while the rest will get through. Obviously, the faster the wall propagates the stronger this effect will be, since the momentum transfer in each collision will be larger, and therefore a force proportional to the velocity with which the wall sweeps through the plasma should appear. In the overdamped regime then bubble walls will reach a terminal velocity in its expansion, which by most accounts will probably not be relativistic. For instance in [9] a value $v_{\text{ter}} \sim 0.1$ is predicted, although higher estimates do exist in the literature (e.g., see [10]). In this situation one could expect the process of phase equilibration after two bubbles collide—and thus of defect formation—to be different from that usually understood to take place when no dissipation is present and the walls acquire relativistic speeds. Indeed, in 1994 Kibble [11] suggested that these differences could be important enough to result in a lower value for the density of topological defects created during a full phase transition in the damped scenario. To investigate in detail these differences was the aim of ref. [8].

In it, new physical effects were found that modify the picture of how topological defects are formed when viscosity plays an important role in the motion of the bubble walls. The most salient feature of this new picture is perhaps the fact that, after they collide, the two bubbles behave as a resonant cavity where the spatial profile for the phase oscillates from its initial value to its inverse (that is, with the phases inverted with respect to their initial spatial distribution), then back to the initial profile, and so on. Therefore, an immediate consequence of the existence of this oscillating state is that it becomes possible, for the same set of three bubbles, to form a vortex, an antivortex or no defect at all depending on the precise timing of the last collision, in clear contrast to what occurs in the undamped scenario, where the initial phases of the bubbles determine the type of defect that will be formed. It was also seen there that these oscillations of the phase can potentially last for a long time (up to $25R$ for $v_{\text{ter}} \sim 0.1$, where $R$ is the radii of the bubbles at collision time). As a consequence, the two bubble system would have to remain in “isolation” for a very long time before acquiring a homogeneous phase, a very unlikely situation.

The preceding arguments would seem to suggest that there will not be a strong suppres-
sion of defect formation in these “slow” transitions, although it is still rather unclear exactly how much will these events be suppressed. The aim of this paper is thus to quantitatively investigate precisely this question: what will be final impact that this new dynamic of defect formation will have on the probability of forming defects?. In order to do this we have performed a series of simulations of full phase transitions for different values of the friction coefficient, $\gamma$, that phenomenologically models the coupling between the scalar driving the transition and the rest of the matter, thus finding the behavior of the density of nucleated defects per bubble as a function of $\gamma$. The paper is organized as follows: in section II we
give a brief account of the model and the results previously found in ref. [8], in section III we present the results of the simulations, finally, the conclusions are presented in section IV.

II. 2 AND 3 BUBBLE COLLISIONS IN A DAMPING ENVIRONMENT

Consider the Lagrangian (1) for a complex field $\Phi$. We will use the same form of potential that was used in [6,7], that is,

$$V = \lambda \left[ \frac{|\Phi|^2}{2} (|\Phi| - \eta)^2 - \frac{\epsilon}{3} \eta |\Phi|^3 \right].$$  \hspace{1cm} (2)

This is just a quartic potential with a minimum at $|\Phi| = 0$ (the false vacuum), and a set of minima connected by a $U(1)$ transformation (true vacuum) at $|\Phi| = \rho_{tv} \equiv \frac{9}{4} (3 + \epsilon + \sqrt{(3 + \epsilon)^2 - 8})$, towards which the false vacuum will decay via bubble nucleation. It is the dimensionless parameter $\epsilon$ that is responsible for breaking the degeneracy between the true and the false vacua.

The equations of motion for this system are then

$$\partial_\mu \partial^\mu \Phi = -\partial V/\partial \Phi.$$  \hspace{1cm} (3)

For the potential (2), approximate solutions of (3) exist for small values of $\epsilon$, the so-called thin wall regime [12], and are of the form

$$|\Phi| = \frac{\rho_{tv}}{2} \left[ 1 - \tanh \left( \frac{\sqrt{\lambda} \eta}{2} (\chi - R_0) \right) \right],$$  \hspace{1cm} (4)

where $R_0$ is the bubble radius at nucleation time and $\chi^2 = | \vec{x} |^2 - t^2$. The bubble then grows with increasingly fast speed and its walls quickly reach velocities of order 1. We are interested however in investigating a model with overdamped motion of the walls due to the interaction with the surrounding plasma. In order to model this effect, we will insert a frictional term for the modulus of the field in the equation of motion, namely

$$\partial_\mu \partial^\mu \Phi + \gamma \dot{|\Phi|} e^{i\theta} = -\partial V/\partial \Phi,$$  \hspace{1cm} (5)

where $|\dot{\Phi}| \equiv \partial |\Phi|/\partial t$, $\theta$ is the phase of the field, and $\gamma$ stands for the friction coefficient (which will as a matter of fact serve as parameter under which we will hide our lack of knowledge about the detailed interaction between the wall and the plasma). It can be shown (see [8]) that this equation does indeed posses a solution that shows the desired type of overdamped motion for the bubble walls. In the thin wall limit, this solution can be written as

$$\rho = \frac{\rho_{tv}}{2} \left[ 1 - \tanh \left( \frac{\sqrt{\lambda} \eta (r - v_{ter} t - R_0)}{2 \sqrt{1 - v_{ter}^2}} \right) \right],$$  \hspace{1cm} (6)

which is simply a Lorentz-contracted moving domain wall with a velocity $v_{ter}$ of the form $v_{ter} \sim \epsilon \delta_m/\gamma \rho_{tv}$, where $\delta_m$ is the bubble wall thickness.
In two bubble collisions, the phase will at first—before the collision actually takes place—interpolate between the values that it takes in each bubble by means of a phase wall situated at the midpoint in between the bubbles. This can be seen by taking the value of the modulus of the field $\rho$ far away from the bubbles to be given by the ansatz

$$\Phi(bubble 1 + bubble 2) \equiv \rho e^{i\theta} \simeq \Phi(bubble 1) + \Phi(bubble 2) \equiv \rho_1 e^{i\theta_1} + \rho_2 e^{i\theta_2},$$

(7)

together with the asymptotic forms for $\rho_1, \rho_2$ (in a 1 dimensional approximation where the bubble centers are situated at $\pm x_0$)

$$\rho_1 \simeq \rho_{tv} e^{(x+vt-x_0)/\delta_m},$$

$$\rho_2 \simeq \rho_{tv} e^{-(x-vt+x_0)/\delta_m},$$

(8)

and plugging the value thus obtained for $\rho$ into the equation of motion for the phase $\theta$. Note that we are assuming that (7) yields only a correct approximation for $\rho$, and not for $\theta$, since the motion of the two bubbles as they approach each other could in principle generate phase waves at the midpoint between them. The resulting equation for $\theta$ will depend on the phase difference between the two bubbles $\Delta \theta$. The most interesting value for this difference is however $\Delta \theta = \pi/2$, since for $\Delta \theta = 0$ there is no dynamics to the phase, and for $\Delta \theta = \pi$ the phase is undefined at the midpoint between the bubbles. A solution interpolating from say $\theta_1 = 0$ at $x \to -\infty$ to $\theta_2 = \pi/2$ at $x \to +\infty$ is

$$\theta = \frac{1}{2} \arcsin(\tanh(\frac{2x}{\delta_m})) + \frac{\pi}{4},$$

(9)

which clearly shows the structure of a phase wall placed at the origin and of width closely related to the bubble wall’s width $\delta_m$. Time dependent solutions to the equation for $\theta$ representing travelling waves do exist, but they die out in time scales of the order of $\delta_m/v_{ter}$.

At the moment of collision, and while the bubble walls merge with one another, this phase wall will thicken up to a value of at least twice the thickness of the bubble walls (since the time the walls take to complete the merging is $t_{merging} \sim \delta_m/v_{ter}$ and the thickening occurs at both sides of the phase wall). After the two bubbles have merged then, the phase is free to travel in the resulting true vacuum cavity since it is a Goldstone boson. The equation of motion for the phase is simply a wave equation, of which approximate solutions can be found by assuming SO(1,2) invariance in the $(t, x, y)$ subspace (the bubbles nucleate at $(0, 0, 0, \pm R)$). This symmetry is exact in the undamped case, but it still adequately describes the behavior of the phase in the damped scenario, especially in the bubble interior, far from the walls. With the initial conditions

$$\theta|_{\tau=0} = \theta_0 \varepsilon(z), \quad \partial_\tau \theta|_{\tau=0} = 0,$$

(10)

where $\tau^2 = t^2 - x^2 - y^2$, the solution to the equation of motion is

$$\theta = \begin{cases} 
\theta_0 & \text{for } z > \sqrt{t^2 - x^2 - y^2}, \\
\frac{\theta_0}{\sqrt{t^2 - x^2 - y^2}} & \text{for } |z| \leq \sqrt{t^2 - x^2 - y^2}, \\
-\theta_0 & \text{for } -z > \sqrt{t^2 - x^2 - y^2}.
\end{cases}$$

(11)

At some point after these waves start to propagate into each bubble, they will inevitably catch up with the bubble walls since they now move with a speed less than 1 due to the
viscosity. It is possible to perform an analysis of the asymptotic fate suffered by an incoming plane wave of frequency $\omega$ which hits the bubble wall; the results show that the reflection coefficient $R$ will be given by (see [8])

$$R = \begin{cases} 
1 & \text{for } \vert \omega \vert \delta_m < \beta (1 + v_{\text{ter}}), \\
\frac{\sinh^{2}(\pi (\omega \delta_m - \omega))}{\sinh^{2}(\pi (\omega \delta_m + \omega))} & \text{for } \vert \omega \vert \delta_m > \beta (1 + v_{\text{ter}}),
\end{cases}$$

where $\beta \equiv 1/\sqrt{1 - v_{\text{ter}}^2}$, and $\omega = \sqrt{\omega^2 \delta_m - 1}$. The appearance of the $\beta$ factor and the $v_{\text{ter}}$ summand in the conditions of (12) is obviously due to the fact that the incoming plane wave is colliding against a moving wall. In the rest frame of the wall the condition for $R = 1$ reads $\vert \omega' \vert \delta_m < 1$, where $\omega'$ is the frequency of the incoming wave in that frame. Thus, if the terminal velocity of the wall is non-relativistic (12) basically tells us that the incoming wave will be totally reflected by the wall if its wavelength is larger than the wall thickness, and partially reflected and partially transmitted if its wavelength is shorter than that—with the reflected part tending to zero as the wavelength diminishes. Remember however that at the moment of the bubble collision, during the merging of the walls, the thickness of the phase wall grew up to a value of the order of twice the bubble wall thickness, and more likely larger than that. It seems clear then that all of the Fourier components of the phase wave will have wavelengths that will fall into the total reflection regime, and thus the whole phase wave itself will simply be reflected by the wall. Let us imagine for the sake of clarity that, at collision time, bubble 1 had a phase $\theta_1$ and bubble two $\theta_2 > \theta_1$, with $\theta_2 - \theta_1 = \Delta \theta$. If the shape of the phase wall at the completion of the collision was $f(x)$ then, after it, we will have phase waves with shape $f(x - t)/2$, $f(x + t)/2$ propagating into each bubble, carrying a phase difference $-\Delta \theta/2$ into bubble 2 and $\Delta \theta/2$ into bubble 1. After these waves have bounced off the bubble walls and propagated back into the interior again, the phase of each bubble will however be, for bubble 2, $\theta_2 - 2\Delta \theta/2 = \theta_1$, and for bubble 1, $\theta_1 + 2\Delta \theta/2 = \theta_2$. The phases of the bubbles will thus have switched. The whole process is depicted in Figure 1, where the referred sequence has been plotted from a simulation. In Fig. 1a, the walls of the two bubbles are just about to finish their merging (continuous line), and the shape of the phase wall at that time is shown (dashed line). The bubble to the right plays the role of bubble 2 above, having $\theta_2 > \theta_1$. The following pictures show how the two phase waves propagate into the bubbles (Fig. 1b), and back after bouncing (Fig. 1c). As expected, after reflection each phase wave still carries a phase of $\pm \Delta \theta/2$, for bubbles 1 and 2 respectively. Finally, in Fig. 1d the two phase waves meet again. The phase polarity of the system has been completely inverted.

The next relevant question to ask is how long will it take until the phase equilibrates. The only mechanism through which the phase waves loose energy is the Doppler shifting of their frequency due to the fact that they bounce off a moving wall. A back of the envelope calculation shows that, in 1+1 dimensions and for a terminal velocity of the bubble walls of 0.1, the phase waves will reach a wavelength of the order of the radius of the single true vacuum cavity $R$ after completing 5 oscillations, that is, after an approximate time of $25R$, where $R$ is the radius of each bubble at collision time.

In 2+1 dimensions the process develops along the same qualitative lines, as we can see in Figs.2 and 3, where the contour lines of the phase (in units of $\pi$) are plotted in continuous
lines, while the bubble walls appear as dashed lines. In two spatial dimensions, at any point that the phase wave meets the wall, its propagation vector will have one component in the direction normal to the wall and another one tangential to it. Only the component in the normal direction will see the wall and bounce off it however, while the other one will continue to propagate freely. Thus, although in general the interaction between the wall and the phase wave will be a complicated superposition of these two processes, we can expect that after some time, in the region close the walls the phase will predominantly propagate tangentially to them, the rest of it having bounced towards the center. The developing of this process is depicted in Fig. 2, and its final stage is clearly visible in Fig. 3(a). At that point virtually all the phase propagating normal to the wall has bounced towards the center, and tangential propagation dominates close to the wall. Now we only have to wait for the central region of the phase, propagating along the $z$ axis, to get to the end of the bubble and bounce off the wall. In Figures 3(b), (c) and (d) we see how this takes place. While the area of the phase fronts propagating along the $z$ axis collides head on with the end of the bubbles and bounces as expected, the tangential components cross each other at that region. Note how while the bouncing and crossing is taking place the phase has a virtually homogeneous distribution inside the bubbles (Figs 3(b) and (c)). After this, the combination of these two phenomena brings about an inversion of the phase similar to that found in 1+1 dimensions, as can be clearly seen in Fig. 3(f). The main consequence that this process will have on the dynamics of vortex formation is now obvious: it will become possible, for the same set of three bubbles, to form a vortex, no defect at all, or an antivortex depending on the precise timing of the last collision (i.e., on the state of the resonant cavity at the moment at which the collision with the third bubble takes place).

The lifetime of the oscillating state is also very large in 2+1 dimensions, and we did observe some vortex nucleation events even after the phase had completed the 5 oscillations that we estimated for the relaxation time in 1+1 dimensions. Since this state is so long-lived, the two bubble system will not reach a homogeneous phase distribution until very late into the phase transition, if it ever does. Therefore, a full quantification of its effects on the probability of nucleating vortices needs to take into account multiple bubble collisions. This is the goal of the next section.

### III. MANY BUBBLE COLLISIONS: VORTEX FORMATION PROBABILITY AS A FUNCTION OF THE FRICTION COEFFICIENT

In order to be able to quantify the impact of the type of frictional coupling that we are considering on vortex formation probabilities we must then carry out simulations involving many bubble collisions. To explore the dependency of the vortex formation probabilities on the value of the friction coefficient (or what is the same, on the value of the terminal velocity for the bubble expansion), we have performed a series of simulations for $\gamma$ ranging from 0.25 (close to the undamped situation where $\gamma = 0$) up to a value of $\gamma = 100$ (corresponding to a terminal velocity for the bubble walls expansion of $v_{\text{ter}} \sim 10^{-2}$). Our simulations differ from the standard numerical simulations of defect production \cite{13} in some important aspects. In them, relative phases are assigned at random to sites on a lattice corresponding to the centres of causally disconnected regions of true vacuum (either bubbles in a first-order or domains in a second-order transition). Between these sites the phase is taken to vary along the shortest
path on the vacuum manifold—the geodesic rule. Defects are then formed wherever this
geodesic interpolation between sites generates a topologically nontrivial path in the vacuum
manifold. For a first-order transition this formalism corresponds to true vacuum bubbles
nucleating simultaneously, equidistant from all their nearest neighbors. Consequently all
collisions between neighboring bubbles occur simultaneously, and the associated phase dif-
fferences are simply given by the differences in the initial assigned phases. We on the other
hand have used a leap-frog method where the discretization of space and time was such
that the grid and time step sizes were several times smaller than the bubble wall thickness,
and included the ‘exact’ (discretized) field dynamics in the simulation. This of course was
needed in order to include the effects of phase bouncing and oscillations described in the
preceding section. To take full advantage of this approach then, bubbles were not nucleated
equidistant and simultaneously. We have followed [14] in nucleating the bubbles at random
points in space and time during the course of the simulation. The algorithm thus goes as
follows:

1. generate a population of time ordered bubble nucleation events distributed randomly
   within some finite volume of 2+1 dimensional space-time, assigning a random phase
to each bubble.

2. start with an initial state for which the field is in the false vacuum in all the simulation
   volume.

3. at the beginning of each time step check whether there are any bubbles to be nucleated
   at that time:

   (a) if there are any bubble nucleation events, nucleate only those bubbles that would
       fall entirely in a false vacuum region and discard the rest (i.e., avoid superimposing
       new bubbles on regions that are already in the true vacuum).

4. evolve the resulting field configuration to the next time step, following the field equa-
tion (5), using a leap-frog method.

In order to generate the bubble nucleation events, we first fix their number—multiplying
a sufficiently low but otherwise arbitrarily chosen nucleation rate by the simulation volume—,
then choose at random the space-time points at which they take place within the simulation
volume. Since we only nucleate those bubbles that fall within the false vacuum and discard
the rest, at later times into the transition it will become increasingly difficult for the new
bubbles to find themselves in the false vacuum, and consequently less and less bubbles will
be nucleated. Also, and although in a realistic situation one would expect the nucleation
rate to vary with the amount of dissipation present in the system, we kept the nucleation
rate constant throughout the simulation series. Since we will be concerned only about the
number of created vortices per nucleated bubble, \( n_d \), this will be of no consequence to
us. What is of concern to us in this case however is the number of bubbles used in the
simulations. We imposed periodic boundary conditions in our two dimensional box and
tried to keep the number of bubbles constant throughout simulations with different values
of the friction coefficient. For simulations with low \( \gamma \), an average around 35 bubbles per run
turned out to be sufficient. Figure 4 shows a particular phase transition for \( \gamma = 2.5 \) at four
different stages. Space and time are measured in units of the bubble wall thickness. Note how the vortex-antivortex pair that appears close to $x = 0, y = 50$ at $t = 550$ has annihilated by the end of the transition, at $t = 800$. This brings about the question of exactly what vortices are we counting, or rather, of when are we counting the vortices, since the number of vortices in the simulation volume is a function of time that in our case will tend to zero as $t \to \infty$. As it is only the final state of the system after the transition that we are interested in, we decided to count only those vortices surviving when the transition was about 95% completed. That is, in this case, the six defects that appear in Fig.4(d) —three vortices and three antivortices. However this rule of thumb has to be applied carefully—in a sense that is made clear in Figs.5 and 6. In Fig.5 we have four shots of a phase transition with $\gamma = 10$, for $t = 600, t = 900, t = 1200, and t = 1500$ —note the obvious difference in the time scale needed to complete the transition. By $t = 1500$ it would appear that we have two pockets of false vacuum left in the central region, but which do not produce any winding of the phase, then two vortices and perhaps an antivortex still closing at $x = 0, y = 80$. This would clearly violate the charge conservation that has to be fulfilled in our torus however. The solution to the problem comes from realizing that this last pocket of false vacuum actually has a winding number $-2$, and therefore it is bound to decay into two antivortices that will immediately start separating from each other. Figs. 6 (a) and (b) show precisely this process as it takes place, blowing out the corresponding region of simulation space. Note however that the process is completed only well after the transition has finished. A “correct” counting of the defects created in this case however would yield a value of 4. Note also how in Fig. 5 the phase has a much smoother structure than in Fig.4. This is easy to understand in physical terms: since we have periodic boundary conditions, in the absence of dissipation all the false vacuum energy would go into phase gradients and field oscillations. The higher the value that $\gamma$ takes however the more energy we dissipate and is not available to form gradients. In Fig.7 finally we show the last stage of a transition with a high value of $\gamma, \gamma = 75$. For such high values of $\gamma$ we had to resort to larger simulations to obtain a significant number of vortex creation events per run. Roughly, we multiplied by 4 the length of the box side, therefore multiplying by 16 both the area and the average number of nucleated bubbles —up to a total of about 570 bubbles per run. Note how, although gradients of the phase of course persist in larger scales, at the scale of Figs.4 and 5 the phase is even smoother now. We still have numerous small pockets of false vacuum left that will need some extra time to close completely. Most of them do not carry winding of the phase, but some of them do however, such as the vortex at $x = 100, y = 25$.

For simulations with relatively low $\gamma$, that is, for those with about 35 bubbles per run, we performed 50 runs for each value of $\gamma$ in order to get a good statistic for the average number of vortices created per nucleated bubble. We computed $n_d$ separately for each run dividing the total number of vortices existing at the end of the transition by the total number of bubbles nucleated in that run. For those values of $\gamma$ that required larger simulations however we could only perform 10 different simulations. The total computer time involved in the project is estimated to be around 500-600 hours in an Alpha 2100 DEC station. The results are shown in Fig.8., where $n_d$ is plotted versus $\gamma$ in a loglog plot together with a straight line fit. The first point corresponds to $\gamma = 0.25$, that is, a very nearly undamped regime. The value of $n_d, 0.24 \pm 0.01$, that we get for this case agrees with the value of the number of nucleated defects per bubble in the undamped case for 2 spatial dimensions quoted in the
literature [5], as could be expected. After all, if the terminal velocity of the bubble walls is only slightly smaller than 1, then the phase waves will need a large time interval in order to catch up with the bubble walls, maybe even larger than the total time needed to complete the transition, and so, we would still effectively be in the undamped scenario. After that we see a rather flat plateau up until the next value of \( \gamma, \gamma = 2.5 \), from which point on friction will start to have a noticeable effect. Starting at that point the data seem to suggest a soft power law decay of \( n_d \). We have plotted the least squares straight line that fits that data from \( \gamma = 2.5 \) to \( \gamma = 100 \) with a continuous line in Fig.8. This line has a slope of \(-0.58 \pm 0.05\), and thus we would have \( n_d \sim \gamma^{-0.58 \pm 0.05} \) with a 95% confidence margin, assuming of course that a simple regression is correct. The correlation coefficient for this regression is of \(-0.996\), which would seem to indicate a good fit. However, it is still unclear at this point whether this is the best we can do or whether the data admit a different interpretation. A detailed analysis and interpretation of the data will be left to a following paper in which the author is currently working jointly with J. Borrill. Notice that in any case the decay is rather soft, especially if we take into account that we are more likely to nucleate a larger number of bubbles in slower transitions.

**IV. CONCLUSIONS**

In first order phase transitions where a frictional coupling between the scalar driving the transition and the rest of the matter content is important, the mechanism for topological defects formation differs in some important features from the one usually understood to take place in undamped transitions. After having understood the detailed dynamics of two and three bubble collisions and defect formation in a previous paper, the aim of this paper was to try to quantify the effect that these differences have on the probability of defect formation per bubble, \( n_d \). In order to do that we have simulated phase transitions for a set of different values of \( \gamma \) ranging from the almost undamped case to \( \gamma = 100 \), thus finding the dependance of \( n_d \) on \( \gamma \) (i.e., on the terminal velocity for bubble wall expansion, since \( v_t \sim \gamma^{-1} \)). The total computer time involved in the project is estimated to be around 500-600 hours on an Alpha 2100 DEC station. The main result of the paper are Figs.8 and 9, where a soft power law decay of \( n_d \) with \( \gamma \) is shown.

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