Stiffness Simulation of Joints of Large Manipulators based on Cam Torsion Spring Mechanisms

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Abstract. There is elasticity in the joint of large manipulators, and the transfer torque is related to the relative rotation angle between input and output terminal. In order to simulate this stiffness characteristic in the test experiment, a cam torsion spring mechanism and its design method are proposed in this paper. The mechanism can simulate arbitrary angle-torque curve by directional design. After the necessary analysis and design, the actual characteristics of the mechanism are tested by experiments, which proves that the mechanism can simulate the stiffness characteristics of joints of large manipulator arm very well.

1. Introduction
There is a problem of joint flexibility in large manipulator arms, in other words, the joint is stiff [1-3]. The stiffness of flexible joints needs to be simulated in experimental testing, such as in the development of motor control. The stiffness of the joint shows that there is a functional relationship between the relative rotation angle of the input and output of the joint and the transmitted torque. Moreover, it is assumed that the function relationship is uncertain, the problem of simulating joint stiffness can be transformed into using a method to simulate arbitrary angle-torque curves. This paper considers the use of mechanical devices to achieve this purpose.

[4] proposed a spring mechanism based on pulley-cable systems. [5] have shown that it is possible to obtain perfect quadratic behavior with an expanding contour mechanism.[6] proposed a method for designing the 2nd order function of cam contour simulation. This paper puts forward a scheme of using cam torsion spring and gives a detailed design method and error analysis. Through the scheme proposed in this paper, arbitrary angle-torque curve can be simulated theoretically in a large angle range. Experiments show that the method is effective in simulating the elastic joints of large manipulators.

2. Analysis and Design of the Cam Torsion Spring Mechanism
2.1. Principle of Cam Torsion Spring Mechanism
The principle of cam torsion spring mechanism is shown in Fig. 2. The fine-tuning device is used to adjust the length of the free rope, which is fixed to the base with the pulley. The turntable (red part) can rotate freely relative to the base, and the cam sheet (green part) is fixed on the turntable. A high stiffness rope pulls two springs through two cam sheets. The device is completely centrosymmetrical and can be regarded as dynamic balance.
When the torsion spring rotates, the outline of the cam sheet extends by pulling the spring through the rope to generate the torque. So, by designing the profile of the cam and the length of the rope, the desired angle-torque function can be simulated.

![Fig. 1 Schematic diagram of cam torsion spring mechanism](image)

### 2.2. Analysis of Wrapping Cam Profile

According to the analysis proposed in [6], the design of cam profile can be analyzed as follow.

![Fig. 2 Principle Analysis of Cam Plate](image)

Because of the principle of relative motion, it can be considered that the turntable is not moving and the base is rotating.

F (u) represents the tension of the spring, G (α) is the torque produced by the torsional spring by single spring. \( u_0 \) represents initial length of spring. \( \alpha_0 \) represents initial angle of turntable. Conservation of energy,

\[
\int_{u_0}^{u} F(u) + u)du = \int_{\alpha_0}^{\alpha} G(\alpha + \alpha)d\alpha
\]

(1)

s represents the length of rope fitted to the cam profile, l the length of the rope suspended in the air. \( s_0 \) and \( l_0 \) are their initial values, respectively.
\[ u = s + l - r(\beta - \alpha) - s_0 - l_0 + r(\beta_0 - \alpha_0) \]  
(2)

\[ x + iy + (l + ir)e^{i\beta} = ae^{i\alpha} \]  
(3)

Solutions of simultaneous equations are available as follow.

\[ \beta = \sin^{-1}\left( \frac{1}{a} \frac{du}{d\alpha} - \frac{r}{\alpha} \right) + \alpha \]  
(4)

\[ \frac{d\beta}{d\alpha} = \frac{1}{a \cos(\beta - \alpha)} \frac{d^2u}{d\alpha^2} + 1 \]  
(5)

\[ l = a \cos(\beta - \alpha) \left( \frac{1}{a \cos(\beta - \alpha)} \frac{d^2u}{d\alpha^2} + 1 \right) \]  
(6)

It can be known.

\[ x = a \cos(\alpha) - l \cos(\beta) + r \sin(\beta) \]  
(7)

\[ y = a \sin(\alpha) - l \sin(\beta) - r \cos(\beta) \]  
(8)

\( x, y \) is the cam contour coordinates.

Formula (1) derivatives on both sides. \( k \) represents stiffness of spring.

\[ ku \frac{du}{d\alpha} = G(\alpha) \]  
(9)

The quadratic derivative of \( \alpha \) can be obtained.

\[ k \left( \frac{du}{d\alpha} \right)^2 + u \frac{d^2u}{d\alpha^2} = \frac{dG(\alpha)}{d\alpha} \]  
(10)

From the mechanism scheme, we can see that the contour continuity of the cam is very important, because the rope should be close to the contour of the cam. As can be seen from Eq (7) and Eq (8), in order to make the cam contour continuous, \( \alpha, \beta \) and \( l \) must be continuous. Because \( \alpha \) is continuous for cam contour, so in order to make \( \beta \) and \( l \) continuous, according to Eq (4) and Eq (6), \( \frac{du}{d\alpha} \) and \( \frac{d^2u}{d\alpha^2} \) must be continuous. Step by step recurrence according to the relation of equation, \( G(\alpha) \) and \( \frac{dG(\alpha)}{d\alpha} \) are continuous.

In conclusion, the angle-torque function and its derivatives must be continuous, otherwise the cam profile will not be smooth.
Eq (7) and Eq (8) calculates discrete points, so we need to fit the continuous curve through discrete points. Therefore, when the points are very dense, the straight line segment can be used instead of the curve.

2.3. Design method of cam torsion spring mechanism

The base can be made of rigid material such as hard aluminum, with a thickness of about 3 mm to 6 mm. So, in order to make the moment of inertia of the whole mechanism as small as possible, it is necessary to make the cross arm of the base as short as possible. So we need to give the limit condition of arm length.

As shown in the fig. 3, the red circle represents the turntable. The radius of the turntable is slightly larger than the outline of the cam sheet. The upper spring indicates that the spring has reached its maximum length, and the lower spring indicates that the spring is in its initial state. O is the center of the turntable, A is the center of the pulley, B is the center of the fine-tuning mechanism. It can also be considered that B is the center of the semicircle of the arm terminal. u is the elongation of the spring due to the rotation of the turntable. The calculation method of u is given below.

The spring used in the mechanism is a fixed stiffness spring. It is assumed that the stiffness of the spring will not change. Then it is obtained by integrating the left side of Eq (1).

\[ ku_0 u + \frac{1}{2} ku^2 = \int_0^\alpha G(\alpha + \alpha_0) d\alpha \]  

(11)

Solve the equation and discard negative root.

\[ u = \sqrt{u_0^2 + \frac{2}{k} \int_0^\alpha G(\alpha + \alpha_0) d\alpha - u_0} \]  

(12)

Spring elongation should not be greater than the maximum linear elongation rated \( u_{\text{max}} \).

\[ u + u_0 \leq u_{\text{max}} \]  

(13)

\( L_3 \) is B to the end of the spring. \( L_4 \) represents the original length of the spring. \( u_0 \) represents the initial elongation of the spring. \( L_5 \) represents the distance from the maximum elongation of the spring to A. The largest proportion is \( u \). \( u_{\text{max}} \) represents the maximum value of \( u \). Then we get the distance L.
\[ L = L_5 + u_{\text{max}} + u_0 + L_4 + L_3 \]  

(14)

In order to maximize the spring elongation without touching the turntable, the conditions can be calculated after connecting A and B.

\[ \sqrt{L_1^2 + L_2^2 - r^2 - 2L_1L_2 \cos \phi} \geq L \]  

(15)

In order let the spring don’t touch the turntable, the restrictive conditions can be obtained. The radius of the marked spring is \( r' \). The vertical distance from O point to spring is denoted as \( h \).

\[ h = \frac{L_1L_2 \sin(\phi)}{\sqrt{L_1^2 + L_2^2 - 2L_1L_2 \cos(\phi)}} > R + r' \]  

(16)

The design of the base not only satisfies the above formula, but also has two basic conditions.

As long as these conditions are satisfied, the minimum A and B can be found, thus reducing the inertia of the mechanism.

\[ \begin{cases} L_1 > R + r \\ L_2 > R \end{cases} \]  

(17)

2.4. Series connection of multi-mechanism

The analysis of single springs and cam sheets has been discussed in 2.2 and 2.3. However, as shown in Fig.1, a cam torsion spring mechanism consists of two sets of springs-cam sheet. They are connected by the same rope, so the torque is divided equally. If the angle required by the target curve is larger than that of a single mechanism, then multiple mechanisms need to be connected in series. The maximum angle of a single cam torsion spring is 130 to 140 degrees. Therefore, it is necessary to study how the angle-torque curve is evenly divided into a spring-cam sheet.

Assume that the number of mechanisms in series is \( N \). The objective function is \( T(\phi) \). Then the angle-torque image of a single spring \( G(\alpha) \) is equivalent to shortening \( T \) horizontally by \( N \) times and then lengthwise by 2 times.

\[ G(\alpha) = \frac{T(N\alpha)}{2} \]  

(18)

Or expressed as follows.

\[ T(\phi) = 2G\left(\frac{\phi}{N}\right) \]  

(19)

Many cam torsion spring mechanisms are connected by coupling. When the whole mechanism reaches the maximum angle, each cam torsion spring mechanism reaches the maximum angle at the same time.

2.5. Analysis of Torque Error Simulated

There are two kinds of errors, fitting error \( E_1 \) and realization error \( E_2 \). Then the total error \( E \) is:
\[ E = E_1 + E_2 \]  

Equation (20)

\[ E_1 = \max \left\{ |G_{i+1} - G_i| \right\} \]  

Equation (21)

\[ E_2 = E_{21} + E_{22} \]  

Equation (22)

\[ E_{21} = \Delta k \frac{\partial G(\alpha)}{\partial k} = \Delta k u \frac{du}{d\alpha} \]  

Equation (23)

\[ E_{22} = \Delta l \frac{\partial G(\alpha)}{\partial l} = \Delta l \frac{k u^2 a^2 \cos^2(\beta - \alpha)}{2l^2} \]  

Equation (24)

The observation error of the sensor is neglected. The influence of the inertia of the mechanism is neglected. The friction and machining error is neglected.

The single set of springs-cam sheet error of single cam torsion spring mechanism calculated above Eq (20)—Eq (24). The maximum error of the whole cam torsion spring mechanism \( E_T \) is theoretically as follows.

\[ E_T = 2NE \]  

Equation (25)

3. Specific realization and experimental results of the mechanism

3.1. Cam Torsional Spring Scheme

The joint stiffness of a large manipulator was measured. It is a piecewise function shown in Fig.4 without considering the rotation backlash.

\[ T = \begin{cases} 
  a\theta^2 & |\theta| < \theta_c \\
  K(\theta - \theta_c) + b & |\theta| \geq \theta_c 
\end{cases} \]

Fig. 4 Functional relation between relative angle of rotation and torque
Among Fig. 5, $K=0.314$, $\theta_c=1.558$, $T_{\text{max}}=2\text{N}\cdot\text{m}$. Then we can calculate the maximum rotation angle: $\theta_{c\text{max}} = 2.27541\pi = 409.5738^\circ$. We chose to connect three Cam torsion spring mechanisms in series. According to Eq (18), the unilateral objective function of each cam torsion spring mechanism can be obtained as follows, only positive angle is considered:

$$
G = \begin{cases} 
0.45345\theta^2 & 0 \leq \theta < 0.519 \\
0.471\theta - 0.1868 & 0.519 \leq \theta \leq 2.3823 
\end{cases}
$$

(26)

The selected spring parameters are: $k=401 \text{N}/\text{m}$, $\Delta k=0.03 \text{ N}/\text{m}$, $u_{\text{max}} = 85\text{mm}$, $r' = 3\text{mm}$. The maximum elongation of the rigid rope is 0.0005m at the maximum inside.

According Eq (20)—Eq (24), errors can be calculated. $E_1=3.216\text{mN}\cdot\text{m}$, $E_{21}=5.407 \text{ mN}\cdot\text{m}$, $E_{22}=5.254 \text{ mN}\cdot\text{m}$. Then the total error can be obtained. $E_T=83.262 \text{ mN}\cdot\text{m}$.

After repeated modifications and explorations according Eq (1)—Eq (10), it is obtained that the calculated cam profile curve, the position of the turntable and the pulley as shown in Fig. 5.

The initial elongation of the spring $u_0$ is 0.0249m. The fine-tuning device is responsible for fine-tuning the spring to its initial state.

![Fig. 5 Generated cam profile and base diagram](image)

Two cam torsion spring mechanisms in series are shown in Fig. 6. The white part is processed by 3D printing.

![Fig. 6 Physical drawing of cam torsion spring mechanism](image)
3.2. Experimental Results
The experiment of cam torsion spring mechanism is shown in Fig. 7. The left side is the control motor and the blue part is the torque sensor. Then the shaft is connected to three cam torsion spring mechanisms in series, and then the inertia disk is connected.

Let the angle change slowly and measure one value every 30 degrees. The direction of change is: 0 (deg) $\rightarrow$ 390 (deg) $\rightarrow$ -390 (deg) $\rightarrow$ 0 (deg).

The test results are shown in Fig. 8. The target curve is black, the actual measurement value is red, and the green curve is the difference between the two. The maximum error is 57 mN*m.

Conclusions
In order to simulate the joint stiffness of a large manipulator, the mechanical method is studied. This problem can be attributed to the use of mechanical devices to generate arbitrary angular-torque curves. This paper presents a scheme of cam torsion spring mechanism to achieve this purpose. Through the
analysis of the cam profile, the design condition of the mechanism, the error analysis of the mechanism and the series-parallel analysis of the mechanism, the application of the scheme in the simulation of the joint stiffness of the large manipulator is solved. Experiments show that the simulation error is within tens mN·m under the condition of large torque and low speed.

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