Plasma potential around a single-dimensional dust particle chain placed in an external electric field

M V Salnikov, A V Fedoseev and G I Sukhinin

Institute of Thermophysics SB RAS, 1 Lavrentyev Ave., Novosibirsk, 630090, Russia

E-mail: salnikov@itp.nsc.ru

Abstract. In this paper a model that allows to self-consistently calculate the potential distribution and plasma space charge distribution near an infinite one-dimensional chain consisting of highly charged dust particles, which is influenced by an external electric field, is presented. The results show that with a gradual increase of the external field amplitude, unlike in the case of an isolated dust particle, there is no anisotropy in the distribution of the space charge; however, with a further increase of the electric field power, zones of local potential maxima appear between the dust grains.

1. Introduction

The problem of determining self-consistent charge and potential distributions in a system where only one dust particle drifts in plasma can be characterized as solved. In order to achieve this status, a number of theoretical models were developed and implemented by prominent figures in dust plasma science. These methods may be divided into three large groups: molecular dynamics methods (MD) [1], Particle-In-Cell methods (PIC) [2, 3], methods of linear response (LR) [4, 5].

To overcome the difficulties associated with computational laboriousness, in the majority of works devoted to the numerical studies of plasma dynamics around dust particles, the isolated dust particles are considered. In works where dust plasma is investigated by the experiments, the situation is reversed: most experimental studies relate to clusters of bounded dust grains [6–8]. In the vicinity of dust particles clusters, complex effects arise that are of interest both for theoretical studies and for experimental works.

The next step in the evolution of computational models is the calculation of self-consistent distributions of potentials and the plasma space charge around a multiple of dust particles. At the moment there are sophisticated models, created on the basis presented in the PIC method. These models with good accuracy describe the potential distribution for the case of dust grains chains [9]. However, in order to save computational resources (otherwise, such a task could never be counted), in these methods substantial simplifications are applied, for example, calculation of the self-consistent potential is carried out by finite difference schemes with first-order accuracy [9] or calculation of the simplified motion equations (leap frog calculation method) [2].

In the current paper a computational model in which, without significant simplifications, the self-consistent distributions of the space charge and the plasma potential around an infinite chain of dust particles are determined selfconsistently is considered.
2. Model

The model considered in the present paper is a strongly modified version of the model used in [10–12]. Therefore, only the main differences that made it possible to simulate plasma dynamics near an endless chain of dust particles will be addressed.

![Schematic representation of the simulated system of an infinite dust particle chain.](image)

The computational volume where the simulation takes place is chosen in the form of a regular quadrilateral prism (see figure 1). The length of the bigger bases edges is $L = 40 \lambda_i$, while bigger basis themselves lie in the XY plane. The length of the smaller edges is $D$, where $\lambda_i = (kT_i/4\pi e^2 n_\infty)^{1/2}$ is the ion Debye length, $n_\infty$ – is the unperturbed ion density, $T_i$ – is the temperature of ions, $D$ – is the distance between dust particles. In the center of the prism is a sphere of radius $r_0$.

The ion dynamics in the system, their contribution to temporal statistics, the method of data normalization, and the iterative calculation of the dust grain charge are coincide with the methods used in [10–12].

Since the computational volume is strongly flattened, segmentation of space according to spherical coordinates ($r$, $\theta$, $\phi$), as in [10–12], is not effective. Here the computational area is divided into elementary cylinders by coordinates ($r$, $\phi$, $z$). Due to the fact that the dust particles chain lies on the $z$ axis, the problem is cylindrical symmetric, and the spatial distributions are determined only by the coordinates ($r$, $z$). The volume of the prism is divided into segments ($i,j$). The volume of each such segment is $V_{ij} = 2\pi r_i \Delta r_i \Delta z_j$.

In the papers [10–12], as in this model, dimensionless quantities are implemented. In the dimensionless representation, the charge of the dust particle $\tilde{Q}$ and the external electric field $\tilde{E}$ are written as follows:

$$\tilde{Q} = \frac{e^2 Z_{d,i}}{\lambda_i kT_i}, \quad \tilde{E} = \frac{e \lambda_i E_z}{kT_i}.$$  \hspace{1cm} (1)

The initial potential distribution (zero iteration) is calculated as the superposition of the Debye-Hückel potentials of all dust particles and the external electric field:

$$U_0(r,\theta) = -\frac{\tilde{Q}}{r} e^{-r} - \sum_k \frac{\tilde{Q}_{r,k}}{r_{k,2}} e^{-\gamma_{k,2}} - \sum_k \frac{\tilde{Q}_{r,k}}{r_{k,3}} e^{-\gamma_{k,3}} - \tilde{E} z$$  \hspace{1cm} (2)
\[ r_{k,1}^2 = (D^2 + r^2 - 2kDz), \quad r_{k,2}^2 = (D^2 + r^2 + 2kDz), \quad (3) \]

where \( r_{k,1}, r_{k,2} \) is the distance from the side dust particles to the ion being monitored.

The calculation of the ions trajectories near the lateral dust grains in this model does not occur. The potential and space charge distributions are calculated in a volume bounded by a central straight prism, where one particle is simulated. The principle of the model is that larger faces of the same prisms are joined with larger edges of other regular prisms, where the spatial distributions are identical to those in the prism where the direct calculation of the trajectories takes place. That is, the boundary conditions in [10–12], where it was assumed that at the edges of the computational volume plasma is unperturbed, are replaced by the conditions of forces equality along the \( z \) axis

\[ \frac{\partial U(r, D)}{\partial z} = \frac{\partial U(r, -D)}{\partial z} = 0. \]

Since the computational domain segmentation is now performed by cylindrical coordinates, the calculation of the potential with the help of the space charge distribution expansion into the Legendre polynomials is not used here. However, the isotropic and anisotropic harmonics \( n(r) \) themselves are calculated for a limited region \( r < 0.5D \) in order to analyze the results obtained in more detail. In this article, the distribution of self-consistent potential is calculated by the formula:

\[ U(r, \theta) = -\frac{\bar{Q}}{r} \sum_k \frac{\bar{Q}}{r_{k,2}} \sum_k \frac{n(r', z')d^3r'}{|r' - r|} + \sum_k \frac{n(r', z')g^3r'}{|r_{k,1} - r'|} + \sum_k \frac{n(r', z')g^3r'}{|r_{k,2} - r'|} - \bar{E}z \]

\[ n(r, z) = \frac{n_i (r, z) - n_e (r, z)}{n_e} \]

where \( n_i (r, z), n_e (r, z), n(r, z) \) is dimensionless distributions of the ions, electrons, and volume charges, densities respectively. The sum of the integrals over \( k \) determines the surplus potential of dust particles in adjacent areas. If the effect of only two nearest neighboring particles is taken into account then \( k = 1 \).

The complete algorithm for calculating the self-consistent distributions in this problem was as follows. In the initial potential (2), the set of ion trajectories was calculated. After accumulation of the ion density statistics \( n_i (r, z) \) the potential \( U(r, z) \) is calculated (4). Then the calculation of the ion trajectories was repeated. Each iteration was followed by a correction of the dimensionless dust particle charge value \( \bar{Q} \), from the condition of equality of the flux of ions and electrons to the surface of the dust grain. The computational cycle was reproduced until the moment when the spatial distributions of the next step are not equal to the corresponding distributions obtained at the previous step.

3. Results

Similar to [10–12], here, when calculating ion trajectories, the parameter of the mean free path of the resonant charge exchange process \( l_i \) is used. For the data presented below, \( l_i = 5\lambda_i \).

As mentioned earlier, in this model, in order to analyze the obtained data in more detail, the model decomposes the space charge distribution \( n(r,z) \) into the Legendre polynomials. Figure 2 shows, isotropic, the expansion term \( n_0 (r) \) of this expansion, for different interparticle distances \( D \) for the case of the external electric field absence \( \bar{E} = 0 \).

The spatial distribution was decomposed over a sphere with a radius of 0.5 \( D \), and therefore did not affect the region \( z < 0.5D, r > 0.5D \). In addition to the data obtained for the task of the infinite dust particles chain, figure 2 shows data for the case of an isolated dust particle. As the interparticle distance \( D \) decreases, the discrepancy between the data obtained for an isolated dust particle and the dust particles chains increases. At the interparticle distance \( D = 4\lambda, \) the dust particles in the chain may
be considered as isolated, but as $D$ decreases further, the right edge of the function rises, suggesting that the neighboring dust particles having a significant effect on a space charge distribution.

In also can be judged by the dust particle charge $\tilde{Q}$, if the dust particle is isolated. The dependence of $\tilde{Q}$ on the interparticle distance is presented in figure 3. Figure 3 demonstrates charge behavior well known from numerical and experimental works. When the dust particles density increases, the charge of each individual particle decreases [13–15]. This effect is explained by the electron depletion in plasma. As a result of this depletion, less electrons fall on each individual particle. Therefore, from the condition of the ions and electrons fluxes equality to the surface of the dust grain, a smaller charge is produced on the grains.

![Figure 2](image1.png)

**Figure 2.** The first, isotropic, expansion term $n_0(r)$ of the space charge distribution $n(r,z)$ in the Legendre polynomials for different interparticle distances $D$.

![Figure 3](image2.png)

**Figure 3.** Dependence of the dimensionless dust particle charge $\tilde{Q}$ on the interparticle distance $D$.

According to the figures 2 and 3, it can be concluded that for the ion mean free path parameter $l_i = 5\lambda_i$ of the resonant charge exchange process, the dust particle may be considered isolated for $D > 6\lambda_i$.

Figure 4 shows the space charge distribution (a series of graphs in the upper half) and potential distribution (a series of graphs in the lower half) for different values of external electric field $\tilde{E}$.

From the upper part of figure 4, where the space charge distribution is shown, it can be seen that when an external field occurs, the clouds of ions merge, turning into one large screening layer. For the field value $\tilde{E} = 5$ one can distinguish some oscillations in this screening layer, but for the field value $\tilde{E} = 10$ the layer merges into a whole and does not represent much interest for the analysis.

The main interest here is the potential spatial distribution, where the contribution of the dust particles themselves is taken into account. For the external field $\tilde{E} = 0$, dust particles are practically isolated, screened relative to each other and the potential between them is $U(r, z) \approx -0.1kT/e$. In this case, the potential layer $U(r, z) \approx -0.05kT/e$ does not exceed the limits $\rho = 2\lambda_i$. With increasing external field, potential layers $U(r, z) \approx -0.5kT/e$, merge with each other, the screening length grows, which leads to the potential layer $U(r, z) \approx -0.05kT/e$ exceed $\rho = 4.3\lambda_i$ for $\tilde{E} = 20$. For the $\tilde{E} = 20$ local maxima appear between two dust particles. Similar to the case of an isolated particle, it is caused by the ion focusing behind the dust grains, which is described in numerous papers on computational methods [2, 3, 12]. In the case of an isolated particle, if such a large external field would be present in the system, a pseudo periodic oscillation structure called a wake would arise behind the dust particle.
Since in this model dust particles are placed directly behind each other, the wake is almost completely destroyed, leaving only the first local maximum. In addition, the occurrence of this wake is also delayed. For the case of an isolated dust particle for $l_i = 5\lambda_i$ the first local maximum in the potential appears at $\tilde{E} = 0.3$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{The space charge distribution (a series of graphs in the upper half) and potential distribution (a series of graphs in the lower half) for different values of external electric field $\tilde{E}$.}
\end{figure}

4. Conclusion
A numerical model has been developed that allows to obtain self-consistent potential and space charge distributions around an infinite chain of dust grains.

The dependence of the Legendre expansion first isotropic term of the space charge distribution on the interparticle distance is considered. The dependence of the dust particle charge on the interparticle distance is calculated. It obtained that the charge of dust grain decreases with the decrease of interparticle distance which coincide with the data obtained by numerous experimental and theoretical works. According to the data obtained, the condition is determined when the dust grain may be considered isolated.

The dependence of the self-consistent distributions of the potential and space charge on the value of the external electric field is shown. It has been demonstrated that, with an increase of the external electric field, the external layers of the ion density distribution become merged. The appearance of local maxima in the potential is shown. These maxima are part of the wakes forming as a result of ion focusing behind dust particles.

References
[1] Vladimirov S V, Maiorov S A and Cramer N F 2003 Phys. Rev. E 67 016407
[2] Ludwig P, Miloch W J, Kahlert H and Bonitz M 2012 New J. Phys. 14 053016
[3] Hutchinson I H and Haakonsen C 2013 Phys. Plasmas 20 083701
[4] Kompaneets R, Morfill G E and Ivlev A V 2016 Phys. Rev. Lett. 116 125001
[5] Dewar R L and Leykam D 2012 Plasma Phys. Control. Fusion 54 014002
[6] Truell H W, Kong J and Matthews L 2013 Physical review. E 87 053106
[7] Carstensen J, Greiner F, Block D, Schablinski J, Miloch W J and Piel A 2012 Physics of Plasmas 19 033702
[8] Forsyth B, Liu B Y H and Romay F J 2007 Particle charge distribution measurement for commonly generated laboratory aerosols Aerosol Science and Technology 28 489
[9] Schleede J, Lewerentz L, Bronold F X, Schneider R and Fehske H 2018 *Physics of Plasmas* **25** 043702
[10] Sukhinin G I, Salnikov M V, Fedoseev A V and A. Rostom A 2018 *IEEE Trans. Plasma Sci.* **46** 749
[11] Sukhinin G I, Fedoseev A V, Salnikov M V, Rostom A, Vasiliev M M and Petrov O F 2017 *Phys. Rev. E* **95** 063207
[12] Sukhinin G I, Fedoseev A V and Salnikov M V 2019 *Contributions to Plasma Physics* e201800152
[13] Sukhinin G I and Fedoseev A V 2010 *Physical review. E* **81** 016402
[14] Goertz I, Greiner F and Piel A 2011 *Phys. Plasmas* **18** 013703
[15] Picard R and Girshick S L 2016 *J. Phys. D: Appl. Phys.* **49** 095201