The application of displaced trajectories in tasks of aircraft navigation in time

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Abstract. Modern requirements in accuracy of aircraft flight trajectory following is done with specifications to the systems of zone navigation in the international air space RNAV (Area Navigation) and RNP (Required Navigation Performance) may apply tight restrictions for the possibilities to correct for the aircraft arrival time to the points of route. Applying the flight speed varying is often insufficient to be on the time chart because of restricted range of available speeds. Application of standard waiting zones in route and in the vicinity of the airport to eliminate the time excess is restricted to the value of minimal flight time which is around 4 minutes or more. As an alternative for arrival time excess elimination to the necessary route point you may use the method of parallel displaced trajectories which is supported functionally with RNP systems. This method is done in the on-board navigation system navigation in time. Depending on the value of the time arrival excess to the necessary route point the navigation coil in time creates the recommendations for the crew how to use displayed trajectories to form the necessary point of exiting to the displaced trajectory to define the side displacement and to evaluate the arrival time correction.

1. Introduction

Steering an aircraft with parallel displaced trajectories is foreseen as special functions being realized with RNP systems. In its definition, an RNP system is a navigation system which helps to stay with necessary accuracy the necessary 3D flight trajectory [1, 2]. Additionally, to 3D trajectories the flights with displacement is known to be applied for tactical tasks of air traffic control when the related services are sanctioned to control the air space. In particular, when two aircrafts hold the same air road [3, 4].

The displaced trajectory mode functionality in modern systems of flight control requires to choose the value of side displacement from the current flight route with increment in 1 sea mile, to start moving to the parallel route in accordance with land services of control and then keep following the route of that displaced value [5, 6]. If you need to cancel the flight of displaced trajectory and to return to the primary route is done normally automatically if it is not possible to keep that route or manually by the crew command. In this case the requirements of RNP specifications is done by holding the fixed value of side displacement from the primary route [7, 8].

Having implemented the concept of 4th flight trajectory which requires not only holding the necessary 3D trajectory, route time flight evaluation and control it may expand the functionality of displaced trajectory application [9, 10].

It is more than clear the application of displaced trajectory to correct the arrival time to the route
points by elimination of time excess. This article describes the method of displaced trajectory application to solve the task of the necessary arrival time control. This task is solved by navigation in time of the on-board navigation system.

The navigation in time coil is done automatizedly from the primary coil of aircraft steering and may prognosticate the arrival time to the points of route on the primary route and also may form and display the recommendations of flight plan change and its altitude and speed parameters in route to hold the aircraft on the necessary time chart.

The received recommendations can be used by aircraft crew to hold the aircraft on the primary route in time or by correction of the flight plan, or by changing of the flight parameters. And also to coordinate and verify the aircraft position with the help of land services of air traffic control.

To eliminate the arrival time excess by using a displaced trajectory in the coil of navigation in time they form the recommended point of exiting to the displaced trajectory by defining the side displacement and evaluation of corrected arrival time to the necessary point of route.

2. Materials and methods

The parameters of recommended displayed trajectory being applied in the navigation in time to eliminate the time excess are given in figure 1.

![Figure 1. Parallel displaced trajectory being applied for time excess elimination (B, L – coordinates of the point where displacement begins, Z – side displacement, S – distance of displaced trajectory application, t - the flight time in displaced trajectory).](image)

Those parameters include:

- the position of displacement initiation point on the primary trajectory;
- side displacement value;
- the application distance of displaced trajectory;
- flight time in displaced trajectory.

So the side displacement value is equal to 1 sea mile and cannot exceed 20 sea mile. Transition to the displaced trajectory and return to the primary route is done with the aircraft line change for $45^\circ$ or less if the side displacement can be reached when the course is changed less than $45^\circ$.

The application method of parallel displaced trajectories to eliminate the time excess include:

- to choose a recommended side displacement with a step of 1 sea mile to provide the necessary elimination of time excess;
- value calculation of the time excess being eliminated and minimal length of the displaced trajectory;
• to form recommendations how to use the displacement to eliminate the time excess and to display that data.

To satisfy the necessary accuracy in time calculation using the displaced trajectory the displaced trajectory flight is studied with particular aircraft specifications and the current parameters of wind.

To choose the recommended side displacement is done with preliminary calculation on the condition of arrival time excess elimination provision to the route point with time control. Time excess value is calculated in navigation in time by comparison of the estimated time of arrival to the necessary one. The estimated time of arrival is calculated after the results of flight modelling on the primary route.

Generally, the arrival time excess ($\Delta t$) is calculated as:

$$\Delta t = ETA - RT,$$

where $RTA$ – necessary time of arrival to the route point with time control; $ETA$ – estimated time of arrival.

The necessary side displacement ($Z$) is calculated with the following approximate dependence:

$$Z = 4R + (0.5 \Delta t V^2 - 2R \alpha_0) \frac{\sin \alpha_0}{1 - \cos \alpha_0},$$

where $R$ – calculated radius of turn with maximum permitted careen; $V$ – true air speed; $\alpha_0$ – way line angle of the transition between the primary and the displaced trajectories.

Standard value of the transition way line angle is $45^\circ$. But with small values of displacement this angle could not be reached. The conditions when it is not possible to reach the transition way line angle of the value $45^\circ$.

$$\Delta t < \frac{4R}{V} (\alpha_0 - \sin \alpha_0).$$

In this case the value of transition way line angle with sufficient degree of accuracy can be accepted as:

$$\alpha_0 = \cos^{-1} \left(1 - \frac{(1 - \cos^{2} \alpha_0) \Delta t}{4R(\frac{\pi}{4} - \sin^{2} \alpha_0)}\right).$$

The received value of side displacement ($Z$) is rounded up to the lower value with equality to 1 sea mile. The value of side displacement cannot be more than 20 sea miles and subsequently can be restricted by the crew up to the flight plan, air space characteristics, requirements of air traffic land control services.

The calculation of time excess being eliminated value and minimal length of the displaced trajectory can be done by division of the displaced trajectory in particular the entry point and return to the primary route in separate distances. This how the necessary level of flight accuracy is granted.

Type scheme of the displaced trajectory division of the coordinate system of the primary line route including the entry point and return to the primary route is given in figure 2.

Figure 2. Type scheme of the displaced trajectory division into sections.
Transition sections between the displaced and the primary trajectories include: entry section to the turn on the transition way line (position 1), section of turn on the transition way line (position 2), entry point on the transition way line (position 3), section of flight in the transition way line (position \( L \)), entry point on the turn to the displaced or primary trajectories (position 4), turn section on the trajectory (position 5), entry point to the trajectory (position 6), the section of flight in the displaced trajectory (position \( D \)).

Entry point to the turn is defined by the entry time of the aircraft careen \( t_0 \). And of course the exit time to the turn \((t_{1,4})\):

\[
t_{1,4} = t_0
\]

Keeping in mind:

\[
\frac{d\psi}{dt} = \frac{g \tan \gamma}{V}, \\
\frac{d\gamma}{dt} = \frac{\gamma_{\text{max}}}{t_0}
\]

if the careen angle is changed in the range until \( 30^\circ \) the value of course change in this section \( (\Delta \psi_1) \) can be represented as:

\[
\Delta \psi_1 = 0.5 \frac{V t_0}{R}.
\]

In the calculated radius of turn:

\[
R = \frac{V^2}{g \tan \gamma_{\text{max}}},
\]

where \( \psi \) - the angle of course; \( \gamma \) - the angle of careen; \( \gamma_{\text{max}} \) – the maximum angle of careen; \( g \) – the acceleration of free fall; \( V \) – the true air speed; \( t \)– the current time.

The exit point to the way line is defined with the accepted law of the careen angle control being done as the aircraft control as [4]:

\[
\gamma = -k_\gamma \frac{dZ}{dt} - k_z Z_x
\]

where \( Z \) – side displacement from the way line, \( k_\gamma \) and \( k_z \) – transmitting coefficients of speed and displacement.

This section is characterized with small aircraft displacement from the necessary way line and small dissonance value of way angle, careen angle control law \((\gamma)\) can be represented as:

\[
\gamma = -k \tan (\psi + \psi_0),
\]

where \( k \) – transmitting coefficients in course, \( \psi \) - current course calculated from the necessary way line, \( \psi_0 \) – slide angle on the primary trajectory.

Transmitting coefficients value can be obtained from the accepted law of the careen angle control. If we accept the careen angle restriction in the range until \( 30^\circ \) course change value can be represented as:

\[
\Delta \psi_2 = \frac{V^2}{R g k}.
\]

Exiting time to the way line \((t_{3,6})\) can be found with the expression:

\[
\Delta \psi_2 = \frac{0.5 V t_{3,6}}{R}.
\]
The turn section is characterized with maximum careen angle for the current flight mode. While the turn the course value changes in the range between the course received at the end of the turn section and exiting course to the way line.

Flight time in the turn section \( t_{2.5} \):

\[
t_{2.5} = R \left( \alpha - \Delta \psi_2 - \Delta \psi_1 \pm \psi_0 \right).
\]

The sign of the slide angle is defined with the turn direction relatively to the direction of wind: in turn with the wind they use the sign «plus».

The transition way line is characterized with keeping the course angle value in this line of way line: to the necessary way line in the correspondent section, \( U_z \) – side component of the wind speed.

Formula of side displacement calculation in the correspondent sections and section projections to the necessary way line (\( S \)) is given in table 1, where \( U_z \) – the value of the wind speed longitudinal component which is positive in the back wind and negative in the face wind.

The sign of the slide angle and side component of the wind speed is defined with the direction of the side displacement relatively to the way line: if the side displacement is done with the wind direction they use the sign «plus»; if it is against the wind – «minus».

**Table 1.** Formula of side displacement calculation in the correspondent sections and section projections to the necessary way line.

| \( Z \) | \( S \) |
|-------|-------|
| 1     | \( 0 \) | \( V \cos \psi_{f0} + U_z t_{1.4} \) |
| 2     | \( \frac{V^2}{gk} (2 \sin \alpha - \Delta \psi_2 \cos \alpha) \pm U_z t_{1.3.6} \) | \( \frac{V^2}{gk} (2 \cos \alpha + \Delta \psi_2 \sin \alpha) \pm U_z t_{1.3.6} \) |
| 3     | \( V_{1.4} \cos \alpha \pm U_z t_{1.4} \) | \( V_{1.4} \cos \alpha \pm U_z t_{1.4} \) |
| 4     | \( \frac{V^2}{gk} \Delta \psi_2 \pm U_z t_{1.3.6} \) | \( \frac{V^2}{gk} + U_z t_{1.3.6} \) |
| 5     | \( (V \sin \alpha \pm U_z) t_L \) | \( (V \cos \alpha + U_z) t_L \) |

Course angle of the transition way line (\( \alpha \)) can be found after the solution of given equation system. With sufficient degree of accuracy, the calculated value of the transition way line course angle relatively to the current way line can be defined as:

\[
\alpha = \cos \frac{\sqrt{2 \cos \psi_{f0} + \psi_0 (\psi_1 + \psi_2) Z_0}}{2} \left( \psi_1 + \psi_2 \right),
\]

where

\[
Z_0 = R (\pm \psi_0 + \frac{\pm \psi_0 (2 (\psi_1 + \psi_2) - (\pm \psi_0)) + \frac{Z}{R}}{2}).
\]

The angle value should not be higher than:
\[ \alpha = 45^\circ \pm \psi_0. \]

So having chosen the recommended value of side displacement they should find the course angles of
the transition way line from the primary route to the displaced trajectory and back. Then they should
find the time of flight in the transition sections and summary time spent for transition from the primary
route to the displaced trajectory and back \( (t_x) \). They find the minimal distance of displaced trajectory
utilizing \( (S) \) as sum of projections to the necessary way line all stages of transition.

The value of time excess being eliminated when they take the chosen displaced trajectory should be:

\[ \Delta t = \frac{S}{V} (\cos \psi_0 + \frac{U_s}{V}) t_x. \]

If the calculated value of the time excess being eliminated exceeds the given value, they correct the
value of recommended side displacement.

As recommendations how to use side displacement to eliminate the time excess which are given to
the aircraft crew are accepted:

- the position of calculated point of exiting to the displaced trajectory;
- the value of side displacement from the necessary way line;
- to evaluate the time of arrival to the route point with given arrival time when they use the
displaced trajectory.

The calculated point of exiting to the displaced trajectory is established from the minimal distance
from the current way point provided that the return to the primary flight route is before a change of the
current point. The distance is calculated as calculated minimal distance of displaced trajectory using. If
it is not possible to return to the primary flight route before a change of the current way point
displacement trajectory calculation is done for the following flight route sections and of course for the
next way point.

The calculated value of side displacement is displayed and can be corrected if necessary. After its
correction the value of the time excess being eliminated must be re-calculated and minimal distance of
the trajectory displacement.

Arrival time evaluation to the route point with given arrival time is done by subtracting from the
calculated arrival time value and the time excess being eliminated on the displaced trajectory. Obtained
so the correctable arrival time is displayed to make a decision if to take the displaced trajectory or not.

With the displayed information the crew makes a decision if they take the displaced trajectory and
also in which direction to exit on that trajectory and if necessary that procedure must be sanctioned with
the air traffic land control services and without passing by the calculated point they initiate the exit to
the parallel route. Having reached the necessary value of side displacement and they initiate the cancel
of flight on the displaced trajectory and the aircraft returns to the primary route.

3. Evaluation results and realization

The evaluation of utilizing the parallel displaced trajectories to eliminate the arrival time excess is done
by aircraft flight modelling on the primary route with different given values of true air speed and
different values of wind speed components.

Dependence of side displacement value \( (Z) \) and minimal distance of the displaced trajectory \( (S) \) from
the value of the excess time being eliminated for some values of air speed is given in figure 3. The
maximum careen angle value being used – 25°. Ordinary lines show the results obtained if there is no
wind. Line and dot lines – the results obtained if the side wind is 180 km/h.
Display option of recommendations to take the displacement to eliminate the time excess in navigation view of a multi-functional indicator in the crew compartment is given in figure 4.

Figure 3. Dependence of the displaced trajectory minimal distance value \(S\) (a) and side displacement \(Z\) (b) from the value of the time excess being eliminated \(\Delta t\) (ordinary lines – if there is no wind, line and dot lines – if the side wind is 180 km/h).

Figure 4. Display option of recommendations to take parallel displaced trajectory to eliminate the time excess in navigation view of a multifunctional indicator.

In this option of informative support when the given arrival time \((RTA)\) to the route point \(UNESCO\) 11:25:10 and the estimated arrival time \((ETA)\) 11:24:14 they propose the crew to take a parallel displaced trajectory with the value of side displacement 9 sea miles and where the exit point is in the distance of 56 km from the current point of route. The prognosticated time of arrival \((CTA)\) to the route point \(UNESCO\) when that displaced trajectory is used 11:25:05. In the flight plan the point of
recommended exit to the displaced trajectory is marked as index «↔»; and information line shows the current distance and estimated time of arrival to that point.

4. Conclusion
The flights with 4th trajectories and parallel displaced trajectories which is supported as a function by the RNP system may help to eliminate the arrival time excess to the necessary route point and also keeping the necessary navigation characteristics and realization of a little predictability. And the value of the time excess being eliminated by that option can be 4 minutes of flight.

Creation of recommendations to take the displaced trajectory is proposed as navigation in time of the aircraft steering system. The given recommendations can be used by the aircraft crew to keep the flight time chart in route and to coordinate and concord the aircraft position in cooperation with traffic control land services.

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