Conductance microscopy of quantum dots weakly or strongly coupled to the conducting channel

K Kolasiński and B Szafran
AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, al. Mickiewicza 30, 30-059 Kraków, Poland

Received 3 March 2014, revised 9 April 2014
Accepted for publication 16 April 2014
Published 22 May 2014
New Journal of Physics 16 (2014) 053044
doi:10.1088/1367-2630/16/5/053044

Abstract
We consider scanning gate conductance microscopy of an open quantum dot that is connected to the conducting channel using the wave function description of the quantum transport and a finite difference approach. We discuss the information contained in conductance (\(G\)) maps. We demonstrate that the maps for a delta-like potential perturbation exactly reproduce the local density of states for a quantum dot that is weakly coupled to the channel, i.e. when the connection of the channel to the dot transmits a single transport mode only. We explain this finding in terms of the Lippmann–Schwinger perturbation theory. We demonstrate that the signature of the weak coupling conditions is the conductance, which for \(P\) subbands at the Fermi level varies between \(P - 1\) and \(P\) in units of \(2e^2/h\). For stronger coupling of the quantum dot to the channel, the \(G\) maps resolve the local density of states only for very specific work points, with the Fermi energy coinciding with quasi-bound energy levels.

Keywords: quantum transport, scanning gate microscopy, quantum dots

1. Introduction
Scanning gate microscopy (SGM) [1, 2] is an experimental technique that probes the reaction of a confined electron system to perturbation introduced by the charge of the atomic force microscope tip that sweeps above the sample area. The SGM technique has been applied to a
number of systems, including quantum point contacts [3–11], currents in the quantum Hall regime [12–14], quantum rings [1, 15–20], and quantum dots [2, 21–25]. The quantum dots that are studied with SGM include the closed ones [21], in which the transport is governed by the Coulomb blockade of the current. The potential of the tip changes the energy of the confined system to an extent that depends on the local charge density. The energy shift can be experimentally measured, which allows for read-out of the confined charge density distribution [21]. On the other hand, open quantum dots [2] are strongly coupled to the reservoirs, the flow of the current is never strictly blocked, and the tip potential only modifies the conductance. According to the Landauer–Büttiker theory the coherent conductance is determined by the properties of Fermi level wave functions [26]. In particular, the conductance maps of systems based on quantum point contacts are generally well correlated to the current flow at the Fermi level [3, 5, 7–10, 27–29]. For open quantum dots and quantum rings the SGM signal was attributed [1, 15–17, 22] to the electron density at the Fermi level, called the local density of states (LDOS). According to the semi-classical WKB approximation the perturbation—due to the tip for instance—changes the electron wavelength locally. The wavelength modulation should have the largest effect in the areas of large LDOS and thus the latter should be resolved in conductance maps.

Arguments against interpretation of conductance maps as images of the current flow or LDOS in the absence of the tip were raised along with formulation of the Lippmann–Schwinger theory of SGM [30, 31]. In terms of the perturbation theory [30, 31] the corrections to conductance due to the tip are non-local as expressed by the wave function, which is a global field. Interference effects introduced by the tip are observed in SGM experiments with quantum point contacts [4, 7, 32–34].

In this work we discuss the interpretation of the $G$ mapping for the quantum dot side-attached to a conducting channel. We develop the results of our previous study [35] for a cavity strongly coupled to the channel and a single mode transport, which indicated that the LDOS and $G$ maps are clearly correlated only at Fano resonances [36–40]. Here, we study a quantum dot with variable opening (coupling) to the channel. We focus on the limit of a weak point-like perturbation. In this way we determine the best possible spatial resolution of conductance maps and its possibly maximal correlation with the unperturbed LDOS. In quantum dots that are strongly coupled to the channel, the SGM map is usually far from the LDOS even for a weak point-like perturbation. Nevertheless, we find that for weakly coupled quantum dots the SGM conductance maps are highly correlated to the LDOS for any Fermi energy. We explain this finding and indicate the experimental conditions in which SGM measurements precisely probe the LDOS.

2. Model

We consider the system that is depicted in figure 1 with a square 300 nm × 300 nm quantum dot that opens to a 70 nm wide channel by a connection that is 130 nm long and has a width of $W$. The width of the opening $W$ that determines the strength of the coupling of the quantum dot to the channel is the central parameter discussed in this work. For $W < 300$ nm the dot is asymmetrically attached to the channel.

We apply a strictly two-dimensional model of the system and use the effective mass Hamiltonian
$H = -\frac{\hbar^2}{2m} \nabla^2 + V_c(x, y) + V_t(x, y, x_t, y_t)$, \hspace{1cm} (1)

where $m = 0.067m_0$ is the GaAs electron effective mass, $V_c$ is the confinement potential and $V_t$ is the potential of the tip. For $V_c$ we use a quantum well potential with $V_c = 0$ inside the quantum dot and the channel (gray area in figure 1) and $V_c \to \infty$ in the rest of the plane.

Figure 1. Schematic diagram of the considered system with 70 nm wide channels and a quantum dot of dimensions 300 nm x 300 nm that opens to the channel by a 130 nm long connection of width $W$. The computational box contains an $L = 700$ nm segment of the channel.

Figure 2. Dispersion relation for the channel that is 70 nm wide. The numbers of subbands are indicated. The horizontal green line shows the Fermi energy level for $P = 6$ subbands participating in the current flow.
In the channel far away from the scattering region the Hamiltonian eigenstates can be put in a separable form \( \Psi_n(x, y) = \exp(ikx)\psi_n(y) \), with the wave vector \( k \) and the subband quantum number \( n \). The dispersion relation for the channel is plotted in figure 2. In this work we consider up to \( P = 6 \) subbands participating in the current flow at the Fermi level. The conductance is evaluated using the Landauer formula

\[
G = \frac{2e^2}{h}T = \frac{2e^2}{h} \sum_{p=1}^{P} \sum_{q=1}^{P} T_{pq},
\]

where the factor of 2 accounts for the spin degeneracy, and \( T_{pq} \) is the electron transfer probability from the \( p \)th subband of the input lead to the \( q \)th subband of the output lead. The transfer probabilities are extracted from the Schrödinger equation, that is solved using the finite difference method with the gauge-invariant kinetic energy discretization [41]. The computational box covers a section of the channel of length \( L = 700 \text{ nm} \), which is large enough to neglect the evanescent modes appearing near the scattering region at the ends of the channel. The boundary conditions at these ends are then set by superposition of current carrying modes. For the electron incident from the left lead in subband \( p \) at the input channel one finds the reflected electron waves in all the subbands

\[
\Psi_in(x, y) = c_p \exp(ik_px)\psi_p(y) + \sum_{q=1}^{P} d_q \exp(-ik_q x)\psi_q^{-k_q}(y),
\]

where \( c_p \) and \( d_q \) are the amplitudes of the incident and reflected waves. At the right end of the box we find the wave function scattered to all the subbands

\[
\Psi_out(x, y) = \sum_{q=1}^{P} e_q \exp(ik_q x)\psi_q^{k_q}(y).
\]

The scattering amplitudes \( c_p, d_q, e_q \) are calculated using a self-consistent scheme of [18]. When the convergence of the scheme is reached [18], the transfer probability can be evaluated as

\[
T_{pq} = \left| \frac{e_q}{c_p} \frac{k_q}{k_p} \right|^2.
\]

The effective tip potential that results from the screening of the tip charge by the deformation of the two-dimensional electron gas was evaluated by Schrödinger–Poisson calculations [18, 19, 35]. The effective tip potential turns out to be close to a Lorentzian [18]. Here, we consider the point-like tip potential (see Introduction) using the Lorentz function

\[
V_t(x, y; x_t, y_t) = \frac{Ud^2}{(x - x_t)^2 + (y - y_t)^2 + d^2}
\]

with small values of both the tip height \( U = 0.05 \text{ meV} \) and the potential width \( d = 4 \text{ nm} \). The small value of the latter is of a more basic importance. In fact, all the results presented below—with the exception of the contrast of conductance maps—remain the same when \( U \) is significantly increased, by a factor of 20 for instance (see below figure 9). The value of \( d \) in the experiments is larger, of the order of the distance between the tip and the electron gas, as we established previously using the Schrödinger–Poisson calculations for the tip potential.
screening problem [18, 19, 35]. For larger $d$ the resolution of $G$ images is limited, as discussed in detail in [18]. In the present paper we study the limit of maximal resolution of $G$ map that is allowed by the quantum transport properties of the Fermi level electrons.

The LDOS [16, 22] is a feature of the unperturbed system that we want to extract using SGM. For a given Fermi energy the LDOS is defined as the sum of the scattering wave functions. Denoting by $\Psi_p^\pm$ the scattering wave function for the electron incident from subband $p$ from the left (+) or right (−) lead, the LDOS is evaluated as

$$\text{LDOS}(x, y) = \sum_{p=1}^{P} |\Psi_p^+(x, y)|^2 + \sum_{p=1}^{P} |\Psi_p^-(x, y)|^2.$$  

(7)

We compare $\text{LDOS}(x, y)$ with the $G(x, y)$ maps as functions of the tip position. For comparison we normalize both maps. For $G_{\text{max}}$ and $G_{\text{min}}$ standing for the maximal and minimal values of conductance as obtained when the system is scanned by the tip, the normalized conductance is defined by $N(G) = (G(x, y) - G_{\text{min}})/(G_{\text{max}} - G_{\text{min}})$. The correlation between the maps $a(x, y)$ and $b(x, y)$ is calculated as

$$r = \frac{\int_S (a(x, y) - \langle a \rangle)(b(x, y) - \langle b \rangle) \, dx \, dy}{S \sigma_a \sigma_b},$$  

(8)

where $\langle a \rangle = \frac{1}{S} \int_S a(x, y) \, dx \, dy$, $\sigma_a^2 = \frac{1}{S} \int_S (a(x, y) - \langle a \rangle)^2 \, dx \, dy$, and $S$ is the area where the comparison is made—here the square quantum dot area.

3. Results

3.1. Single-subband transport

Let us begin by characterization of the system as a scatterer for low Fermi energy with $P = 1$. Figure 3 (a) shows the electron transfer probability as a function of the connection width $W$ for the Fermi energy range that corresponds to the transport in the lowest subband. For small $W$ (see figure 1) the channel is only weakly coupled to the quantum dot and $T$ is generally close to 1. The backscattering of the incident electron involves Fano interference of the channel and quantum dot wave functions, which occurs only when the Fermi energy coincides with the energy of localized states within the dot. We determined the density of localized states using the Mandelshtam stabilization method [42]. The calculated localized states density have been plotted in figure 3(c). The pattern of localized energy levels has been additionally marked with the dashed lines in figures 3(a), (b). We can see that for small $W$ narrow valleys of low $T$ are formed near the quasi-bound energy levels. For larger $W$, as the dot–channel coupling becomes stronger, low $T$ areas also appear off the energy positions of the localized states (figure 3(a)).

Figure 3(b) shows the value of the correlation coefficient ($r$) between the LDOS and the conductance map. We notice that generally at the resonances the correlation coefficient $r$ changes its sign. The correlation between the $G$ map and LDOS is sensitive to the number of modes accessible for transport in the connection between the channel and the quantum dot (see figure 1). The connection of width $W$ becomes transparent for the Fermi energy which exceeds the energy of the lateral quantization. The number of transparent modes in the connection can
be evaluated from figure 3(d), where we have plotted the energy spectrum of the one-dimensional infinite quantum well of width $W$ as a function of its width $W (E_n = \frac{1}{2} m \left[ \frac{n \hbar}{W} \right]^2)$. The integers within the panel show the number of transparent modes of the connection. Red dots indicate the work points $(E$ and $W)$ which are discussed in the text.

Figure 3. Single subband transport. (a) Electron transfer probability as a function of the energy and the connection width $W$ (see figure 1). (b) Correlation coefficient $r$ of the LDOS (equation (8)) and conductance maps for the system perturbed by the tip potential (equation (6)) for $d = 4$ nm and $U = 0.05$ meV. (c) Density of localized states (in arbitrary units). The dashed lines in (a) and (b) show the positions of the most pronounced resonances of panel (c). (d) Energy levels of a 1D quantum well as a function of its width $W (E_n = \frac{1}{2} m \left[ \frac{n \hbar}{W} \right]^2)$. The integers within the panel show the number of transparent modes of the connection. Red dots indicate the work points $(E$ and $W)$ which are discussed in the text.

The top panel of figure 4 shows the cross section of figure 3(a) for $W = 80$ nm, with dips in $T$ (red line), which usually have asymmetric profiles—a characteristic of the Fano interference. The energy range marked 1 and 2 at the top of the figure corresponds to the number of transparent subbands of the channel–dot connection (see figure 3(d)). The dependence of $r$ on the energy is binary in range 1. For the second range $r$ becomes a continuous function of the energy. In this region we notice that $|r|$ tends to $1$ only at the conductance resonances.
Figure 4. Lowest subband transport $P = 1$ case. Top panel: transfer probability (red curve)—a cross section of figure 3(a) for $W = 80$ nm, $r$—correlation (black curve) of the LDOS to the conductance map as obtained with the Lorentz ansatz of equation (6)—a cross section of figure 3(b). Middle panel: $\text{sign}(r)$ (black curve) and $-0.9 \times \text{sign}(dT/dE)$ (red curve). Bottom panel: the gray signal shows $N(E)$—the cross section of figure 3(c), the red line shows the absolute value of $dT/dE$ (left axis), and the blue line the contrast of the $T$ map (right axis) for a given energy. Ranges 1 and 2 correspond to the number of subbands in the connection—see figure 3(d).

Figure 5. The same as figure 4 only for connection width $W = 200$ nm. The width of the channel is unchanged with $P = 1$, but the number of transparent modes in the connection reaches 5 at the end of the horizontal axis—see figure 3(d).
This feature is also preserved for a much wider dot–channel connection supporting a large number of transport modes—see the results for \( W = 200 \) nm in figure 5.

In figures 4 and 5 we notice that whenever \( T \) increases (decreases) in \( E \), the calculated correlation \( r \) between the \( G \) map and LDOS is negative (positive). This turns out to be a general rule observed also for larger energies and more subbands participating in the transport (see below). For illustration, in the central panels of figures 4 and 5 we plotted the sign of \( r \) and the inverted sign of the derivative \( dT/dE \). In subsection 3.3 we will provide an analytical argument for a perfect anticorrelation of \( r \) and \( dT/dE \) signs for strictly one-dimensional transport in the perturbation theory [30, 31]. This finding in semi-classical WKB terms can be understood as follows: when \( T \) grows with \( E \), a shorter electron wavelength is needed to tune the transfer probability to \( T = 1 \). The repulsive tip \( (V > 0) \) makes the wavelengths locally longer, \( \lambda (x, y) = 2\pi \hbar \sqrt{2 m \left( E - V(x, y; x_1, y_1) \right)} \), which for \( dT/dE > 0 \) lowers the value of \( T \).

Representative samples of \( G \) maps and LDOS were plotted in figures 6 (a), (b)) for \( W = 80 \) nm and in figures 6(c), (d) for \( W = 200 \) nm with energies 2.5 and 3.05 meV respectively (these work points are marked by dots in figure 3(d)). Both energies are taken off resonance and correspond to \( P = 1 \) with one and four channels of the dot to channel connection open.
3.2. Weak and strong coupling for $P > 1$

In order to verify the general character of the above findings we performed calculations for higher Fermi energy. The data will be discussed with respect to figure 7, which shows the number of modes open in the connection for a wider energy range than in figure 3(d).

Figure 8 shows $T$ and $r$ for $P = 2$ with a single ($W = 50\,\text{nm}$, figure 8(a)) or multiple ($W = 200\,\text{nm}$, figure 8(b)) transparent modes in the connection between the channel and the dot (see the gray belt for $P = 2$ in figure 7). For the smaller value of $W$, $r$ has a binary dependence on $E$. For the larger $W$ (see figure 8(b)) the variation of $r$ becomes smooth, and generally $|r|$ is far from unity, beside the resonances.

Results for six subbands at the Fermi level are given in figure 9. The binary dependence of $r$ is still obtained in spite of large $E$, but only for a very narrow opening with $W = 16\,\text{nm}$—see figure 9(a). This case corresponds to a single transparent mode of the connection—cf. the gray belt with $P = 6$ in figure 7. Similarly, as seen for $P = 2$ and $W = 50\,\text{nm}$ in figure 8(a) in the conditions of weak coupling, and the binary $r$ behavior, we find $T(E) \in [P - 1, P]$. For $W = 200\,\text{nm}$ at the resonances we obtain peaks of $|r|$ (figure 9(b)). Outside the resonances $|r|$ remains small.
The contrast of the $G$ map calculated for $U = 0.05$ meV is low outside the resonances (see figures 4 and 5). However, the contrast can be increased by application of a larger potential with a limited influence on the value of $r$. Figure 9(b) compares the correlation coefficient calculated for $U = 0.05$ meV (solid line) and for $U = 1$ meV (dashed line).

**Figure 8.** The energy range for $P = 2$. (a), (b) The transfer probability and LDOS to $G$ correlation coefficient for $W = 50$ nm (a) and $W = 200$ nm (b). (c) The sign of the correlation $r$ and $-dT/dE$ (the latter multiplied by 0.9 for clarity) for $W = 200$ nm. The blue line in (a) shows the contrast of the $G$ map (right axis).

**Figure 9.** Results for $P = 6$ subbands at the Fermi level for $W = 16$ nm (a) and $W = 200$ nm (b). For both (a) and (b) the red lines show the transfer probability (right axis), the blue lines the contrast, here calculated as $T_{\text{max}} - T_{\text{min}} - 1$ (left axis), and the black and grey curves the correlation $r$ (left axis). (a) The contrast and $r$ calculated for $U = 1$ meV. (b) $r$ calculated for $U=0.05$ meV and $U = 1$ meV (solid and dashed lines, respectively).
3.3. One-dimensional case

In the numerical results presented above we found that the sign of \( r \) is anticorrelated to the sign of the derivative \( \frac{dT}{dE} \). It is possible to explain this finding analytically for a one-dimensional system. Let us consider the quantum well of figure 10 (a). For the electron incident from the left, the scattering wave function has the form

\[
\phi_1 = \frac{1}{2} e^{ikL} e^{-ikL} \left( 1 + \frac{k}{k'} \right) e^{ikx} \left( 1 + e^{-2ik'x} e^{2ikL} k' - k \right) \]

for \( x \in (-L, L) \) and \( \phi_1 = t \exp (ikx) \) for \( x > L \), where \( k = \sqrt{2mE/\hbar} \) and \( k' = \sqrt{2m(E + U_0)/\hbar} \). For the symmetric quantum well a similar form of wave function is obtained for the electron incident from the right; in particular, inside the quantum well one has

\[
\phi_t = \frac{1}{2} e^{ikL} e^{-ikL} \left( 1 + \frac{k}{k'} \right) e^{-ikx} \left( 1 + e^{2ik'x} e^{2ikL} k' - k \right) .
\]

The LDOS within the quantum well is of the form

\[
\text{LDOS} = |\phi_1|^2 + |\phi_t|^2 = a' + b' \cos (2k'L) \cos (2k'x),
\]

where \( a' \) and \( b' \) are variables which do not depend on the spatial coordinate \( x \).
Let us now assume that the tip potential is of the delta form $V_t = U \delta (x - x_t)$. For small $U$ the conductance change induced by the tip can be evaluated in the first order of the perturbation analysis of [30, 31],

$$G(x_t) = G_0 - 4\pi i \{ c^* t V_{21}(x_t) \},$$

(10)

with $V_{21}(x_t) = U \phi_t(x_t)^* \phi_1(x_t)$. The product

$$c^* t = i \frac{4kk'(k' - k)(k' + k) \sin (2k'L)}{(k'^2 - k^2)^2 \cos (4k'L) - (k'^2 + k^2)^2 - 4k'^2k^2}$$

is purely imaginary. Therefore we need only the real part of the matrix element $V_{21}$, which reads

$$\Re \{ V_{21}(x_t) \} \propto 2\beta \cos (2k'L) + (\beta^2 + 1) \cos (2k'x_t),$$

where $\beta = \frac{k' - k}{k' + k}$. In consequence, the conductance as a function of the tip position $x_t$ has a form given by

$$G(x_t) = a + b \sin (2k'L) \cos (2k'x_t),$$

(11)

where $a$ and $b$ do not depend on spatial coordinate $x_t$. Thus the differences between the maps of $G(x_t)$ and LDOS $(x_t)$ disappear when the maps are normalized for calculation of the correlation $r$ (see equation (8)). For a perturbative delta-like tip one finds exactly $|r| = 1$ for any $E$ (see figure 10(b)). This is in perfect agreement with the conclusion reached in [16] for the discussion of LDOS—$G$ correlation in one dimension.

Now, let us consider the sign of the correlation $r$. The transfer probability is given by

$$T = \frac{8k'^2k^2}{\left( k'^2 + k^2 \right)^2 + 4k'^2k^2 - \left( k'^2 - k^2 \right)^2 \cos (4k'L)},$$

(12)

and reaches unity when $k' = \frac{m}{2W}$, for integer $n$. For these values of $k'$, for which $T = 1$, the product $c^* t$ changes sign. Hence, at the peaks of $T$ the correlation $r$ changes sign. The results for $T$ and $r$ are given in figure 10(b) with $r = -1 (+1)$ for the growing (decreasing) slope of $T$.

4. Discussion

The perfect correlation of LDOS with the $G$ map is due to the inversional symmetry of the considered scattering potential. The 2D system studied in this work is not inversionally invariant, yet a good correspondence is found for any $P$ provided that the channel to quantum dot connection is narrow. Let us explain why the LDOS is resolved by $G$ maps for any $E$ when the dot is weakly coupled to the channel. For this purpose it is convenient to return to the single-subband ($P = 1$) result of figure 3.

Figures 11 (a), (c) shows the scattering probability density for the electron incident from the left (figure 11) and right (figures 11(b), (d)) for $E = 4.1$ meV. We can see that for $W = 50$ nm —for a single channel within the connection (cf. Figure 3(d))—the densities inside the quantum dot are identical for the two transport directions (figures 11(a), (b)). For $W = 50$ nm there is only a single open transport channel inside the connection. Thus the electron wave function on its way to the quantum dot is bound to lose the information on the incidence direction. This is no longer the case for $W = 80$ nm (figures 11(c), (d)), for which two modes of the connection
participate in the transport (see figure 3(d) for $E = 4.1$ meV): the values of $r$ lose their binary dependence on $E$.

For a single open mode of the transport across the connection, not only is the probability density identical for both incidences but also the wave function inside the quantum dot becomes the same, $\phi_l = \phi_h$ (with precision to a phase). Thus the matrix element of the perturbation theory $V_{21}$ (see equation (10)) becomes proportional to the probability density and it perfectly matches the LDOS, which explains the central finding of the present work.

For $P > 1$ the conditions for the binary values of $r(E)$ can be recognized by the $T(E)$ dependence. We have $P$ subbands carrying the current in the main channel of the system (figure 1). In the discussed regime the connection to the quantum dot can only transmit the current in a single mode. Hence, the electron transfer probability summed over the incident subbands (equation (2)) should then vary between $(P - 1)$ and $P$. This is exactly what we found for binary $r(E)$ dependence with $P = 2$ (see figure 8(a)) and $P = 6$ (figure 10(b)) subbands at the Fermi level.

Let us comment on the experimental feasibility of the LDOS mapping for the weak coupling regime discussed in the present paper. The open quantum dots connected to electron reservoirs are routinely produced by gating [45], etching [25], or surface oxidation [46] techniques. The open dots [25, 45, 46] usually have sizes between 200 nm and 1 $\mu$m, i.e. of the order of the dimension discussed in the present paper. The dot needs to be connected to the reservoirs by a narrow channel. For low Fermi energy a channel of 80 nm is narrow enough. For higher Fermi energies we discussed channels of width as small as 16 nm. Such narrow channels can be defined with the surface oxidation technique that uses the AFM tip to deposit the oxide on specific locations of the sample. Formation of quantum wires of width as small as 4 nm by this technique has been reported [47]. For good resolution of the $G$ maps, the ratio of the size of the dot to the width of the tip should be the largest. The correlation of the $G$ maps to the LDOS depends only on the width of the connection and stays high when the quantum dot size is increased. Figure 12 shows the LDOS to $G$ map correlation factor for the quantum dot size increased to $600 \text{ nm} \times 600 \text{ nm}$ and $W = 80 \text{ nm}$. For $E$ below $\simeq 3.7$ meV the connection has a single conducting channel and $r$ exhibits binary dependence on the energy, as for the case of the

Figure 11. Single-subband transport $P = 1$, for $E = 4.1$ meV. The probability density for the electron incident from the left (a), (c) and right (b), (d) for $W = 50$ nm (a), (b) and $W = 80$ nm (c), (d).
smaller dot (see figure 4). On the other hand, the LDOS mapping can only be realized provided that the quantum dot size is smaller than the coherence length. The direct measurements [48] of the phase coherence length in the GaAs two-dimensional electron gas indicate that the length depends on the temperature $T$ as a $T^{-1/3}$ function. At $T = 100$ mK the length is of the order of 1 $\mu$m [48]. The present results show that the $G$ map to LDOS correlation factor is a strongly varying function of the energy. Besides the coherence length, the temperature also affects the spread of the Fermi level. For $T = 100$ mK, the spread $k_B T$ is of the order of 10 $\mu$eV. This value is of the order of the width of the resonances (see figure 4). However, outside the resonances for the weak coupling, i.e. in conditions where LDOS can be mapped for any $E$, variation of the transfer probability, the correlation factor and finally the LDOS with the Fermi energy is much slower, so the LDOS imaging by $G$ measurements should be within the reach of the experiment.

5. Summary and conclusions

We have studied the conductance response of an open quantum dot to a short range perturbation that scans the surface of the system in the context of mapping the LDOS. The study covered up to $P = 6$ subbands at the Fermi level and was supported by analysis of the density of states localized in the quantum dot on the energy scale. The latter turns out to determine the contrast of the SGM conductance maps and the sign of the correlation coefficient $r$ between the conductance maps and the LDOS: the sign of $r$ is always opposite to the sign of the derivative $dT/dE$.

We have found that for weak coupling of the quantum dot to the channel the correlation coefficient of LDOS to $G$ maps $r$ takes binary values, which are close to either $-1$ or $1$ for any $E$. For strongly coupled quantum dots (large values of $W$), the dependence of $r(E)$ is no longer binary, but $r(E)$ tends to $\pm 1$ at the resonances for energies corresponding to the dot localized quasi-bound states. We conclude that for weak coupling of the dot to the channel the tip does not seem to interfere with the scattering wave functions. In the conditions of weak coupling, the tip—when localized near the maximum of the LDOS—tunes the scattering conditions to or away from an extremal value of $T$. We have explained that these special conditions—when LDOS can be extracted from $G$ maps for an arbitrary Fermi energy—appear when the
connection of the channel to the quantum dot allows only a single transport mode to pass from the channel to the dot. Then, the wave functions inside the quantum dot for both incidence directions are the same up to a phase factor. This implies that LDOS and conductance maps become identical for a delta-like tip potential. We have indicated that an experimental signature of these conditions is the conductance changing between $P - 1$ and $P$ in units of $\frac{2e^2}{h}$ as the Fermi energy is changed.

Acknowledgments

This work was supported by the National Science Centre according to decision DEC-2012/05/B/ST3/03290, by the PL-Grid Infrastructure and by the Ministry of Science and Higher Education within statutory tasks of the faculty. Calculations were performed in ACK–CY-FRONET–AGH on the RackServer Zeus.

References

[1] Sellier H, Hackens B, Pala M G, Martins F, Baltazar S, Wallart X, Desplanque L, Bayot V and Huant S 2011 Sem. Sci. Tech. 26 064008
[2] Ferry D K, Burke A M, Akis R, Brunner R, Day T E, Meisels R, Kuchar F, Bird J P and Bennett B R 2011 Sem. Sci. Tech. 26 043001
[3] Crook R, Smith C G, Simmons M Y and Ritchie D A 2000 Phys. Rev. B 62 5174
[4] Topinka M A, LeRoy B J, Shaw S E J, Heller E J, Westervelt R M, Maranowski K D and Gossard A C 2000 Science 289 2323
[5] Topinka M A, LeRoy B J, Westervelt R M, Shaw S E J, Fleischmann R, Heller E J, Maranowski K D and Gossard A C 2001 Nature 410 183
[6] Aidala K E, Parott R E, Kramer T, Heller E J, Westervelt R M, Hanson M P and Gossard A C 2007 Nat. Phys. 3 464
[7] Jura M P, Topinka M A, Urban L, Yazdani A, Shtrikman H, Pfeiffer L N, West K W and Goldhaber-Gordon D 2007 Nat. Phys. 3 841
[8] Aoki N, Da Cunha C R, Akis R, Ferry D K and Ochiai Y 2005 Appl. Phys. Lett. 87 223501
[9] Pioda A, Kicin S, Brunner D, Ihn T, Sigrist M, Ensslin K, Reinwald M and Wegscheider W 2007 Phys. Rev. B 75 045433
[10] Schnez S, Rossler C, Ihn T, Ensslin K, Reichl C and Wegscheider W 2011 Phys. Rev. B 84 195322
[11] Kozikov A A, Weinmann D, Rössler C, Ihn T, Ensslin K, Reichl C and Wegscheider W 2013 New J. Phys. 15 083005
[12] Hackens B, Martins F, Faniel S, Dutu C A, Sellier H, Huant S, Pala M, Desplanque L, Wallart X and Bayot V 2010 Nat. Commun. 1 39
[13] Martins F, Faniel S, Rosenow B, Pala M G, Sellier H, Huant S, Desplanque L, Wallart X, Bayot V and Hackens B 2013 New J. Phys. 15 013049
[14] Ihn T, Rychen J, Vančura T, Ensslin K, Wegscheider W and Bichler M 2002 Physica E 13 671
[15] Aoki N, da Cunha C R, Akis R, Ferry D K and Ochiai Y 2005 Phys. Rev. B 72 155327
[16] Pala M G, Hackens B, Martins F, Sellier H, Bayot V, Huant S and Ouisse T 2008 Phys. Rev. B 77 125310
[17] Pala M G, Baltazar S, Martins F, Hackens B, Sellier H, Ouisse T, Bayot V and Huant S 2009 Nanotechnology 20 264021
[18] Szafran B 2011 Phys. Rev. B 84 075336
[19] Chwiej T and Szafran B 2013 Phys. Rev. B 87 085302
[20] Petrovic M D, Peeters F M, Chaves A and Farias G A 2013 J. Phys.: Condens. Matter 25 495301
[21] Fallahi P, Bleszynski A C, Westervelt R M, Huang J, Walls J D, Heller E J, Hanson M and Gossard A C 2005 Nano Lett. 5 223

Zhang L and Mand Fogler M M 2006 Nano Lett. 6 2206
Gildemeister A E, Ihn T, Sigrist M and Ensslin K 2007 Phys. Rev. B 75 195338
Bleszynski-Jayich A C, Fröberg L E, Björk M T, Trodahl H J, Samuelson L and Westervelt R W 2008 Phys. Rev. B 77 245327

Qian J, Halperin B I and Heller E J 2010 Phys. Rev. B 81 125323
Boyd E E and Westervelt R M 2011 Phys. Rev. B 84 205308
Boyd E E, Storm K, Samuelson L and Westervelt R M 2011 Nanotechnology 22 185201
Huefner M, Kueng B, Schnez S, Ensslin K, Ihn T, Reinwald M and Wegscheider W 2011 Phys. Rev. B 83 235326
Mantelli D, Cavaliere F and Sassetti M 2012 J. Phys. Condens. Matter 24 43202
Ziani N T, Cavaliere F and Sassetti M 2012 Phys. Rev. B 86 125451

[22] Mendoza M and Schulz P A 2003 Phys. Rev. B 68 205302
[23] Mendoza M and Schulz P A 2005 Phys. Rev. B 71 245303
[24] Ferry D K, Akis R and Bird J P 2004 Phys. Rev. Lett. 93 026803
[25] Burke A M, Akis R, Day T E, Speyer G, Ferry D K and Bennett B R 2010 Phys. Rev. Lett. 104 176801
[26] Datta S 1995 Electronic Transport in Mesoscopic Systems (Cambridge: Cambridge University Press)
[27] Kramer S 2013 Phys. Rev. B 88 125308
[28] Metalidis G and Bruno P 2005 Phys. Rev. B 72 235304
[29] Cresti A 2006 J. Appl. Phys. 100 053711
[30] Jalabert R A, Szewc W, Tomsovic S and Weinmann D 2010 Phys. Rev. Lett. 105 166802
[31] Gorini C, Jalabert R A, Szewc W, Tomsovic S and Weinmann D 2013 Phys. Rev. B 88 035406
[32] Abbout A, Lemarie G and Pichard J L 2011 Phys. Rev. Lett. 106 156810
[33] Jura M P, Topinka M A, Grobis M, Pfeiffer L N, West K W and Goldhaber-Gordon D 2009 Phys. Rev. B 80 041303(R)
[34] Kozikov A A, Rössler C, Ihn T, Ensslin K and Reich C 2013 New J. Phys. 15 013056
[35] Kolański K and Szafran B 2013 Phys. Rev. B 88 165306
[36] Mendoza M, Schulz P A, Vallejos R O and Lewenkopf C H 2008 Phys. Rev. B 77 155307
[37] Morfonios C, Buchholz D and Schmelcher P 2011 Phys. Rev. B 83 205316
[38] Clerk A A, Waintal X and Brouwer P W 2001 Phys. Rev. Lett. 86 4636
[39] Barnthaler A, Rotter S, Libisch F, Burgdörfer J, Gehler s, Kuhl U and Stockmann H-J 2010 Phys. Rev. Lett. 105 056801
[40] Miroshnichenko A E, Flach S and Kivshar Y S 2010 Rev. Mod. Phys. 82 2257
[41] Governale M and Ungarelli C 1998 Phys. Rev. B 58 7816
[42] Mandelshtam V A, Ravuri T R and Taylor H S 1993 Phys. Rev. Lett. 70 1932
[43] Akis R, Vasilopoulos P and Debray P 1997 Phys. Rev. B 56 9594
[44] Nöckel J U 1992 Appl. Phys. B 106 15348
[45] Bird J P, Akis R, Ferry D K, Vasileleska D, Cooper J, Aoyagi Y and Sugano T 1999 Phys. Rev. Lett. 82 4691
[46] Fuhrer A, Lüscher S, Ihn T, Heinzel T, Ensslin K, Wegscheider W and Bichler M 2001 Nature 413 822
[47] Martinez J, Martinez R V and Garcia R 2008 Nano Lett. 8 3636
[48] Ferrer M, Angers L, Rowe A C H, Guéron, Bouchiat H, Texier C, Montambaux G and Mailly D 2004 Phys. Rev. Lett. 93 246804