Unified Model for Inflation and the Dark Side of the Universe

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Abstract.
We present a model with a complex and a real scalar fields and a potential whose symmetry is explicitly broken by Planck-scale physics. For exponentially small breaking, the model accounts for the period of inflation in the early universe and for the period of acceleration of the late universe or for the dark matter, depending on the smallness of the explicit breaking.

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INTRODUCTION

The Standard Model (SM) of particle physics based on the gauge group $SU(3) \times SU(2) \times U(1)$ is considered to be a successful model, able to accommodate all existing empirical data with high accuracy. Nevertheless, there are many deep questions for which the SM is unable to give the right answer, such that many physicists believe that it is not the ultimate theory of nature. In any extension of the SM, the idea of supposing new additional symmetries is quite justified, taking into account that there are known symmetries that at low energies are broken, but at higher energies are restored. If we assume that global symmetries are valid at high energies, we should expect that they are only approximate, since Planck-scale physics breaks them explicitly [1, 2]. Even with an extremely small breaking, very interesting effects may appear. As discussed in [3, 4], when a global symmetry is spontaneously broken and in addition there is a small explicit breaking, the corresponding pseudo-Golstone boson (PGB) can play a role in cosmology. The focus in [3] was to show that the PGB could be a dark matter constituent candidate, whereas in [4] it might play the role of a quintessence field responsible for the present acceleration of the universe.

In the present contribution we will relate the period of very early acceleration of the universe (inflation) either with the present period of acceleration, or with the mysterious dark matter, depending on the smallness of the effects of Planck-scale physics in breaking global symmetries. Direct or indirect observational evidence for the existence of dark energy and dark matter together with the need for inflation come mainly from supernovae of type Ia as standard candles [5], cosmic microwave background anisotropies [6], galaxy counts [7] and others [8]. The physics behind inflation, dark matter or dark energy may be completely unrelated, but it is an appealing possibility that they have a common origin. An idea for this kind of unification is "quintessential inflation", that has
been forwarded by Frieman and Rosenfeld [9]. Their framework is an axion field model where there is a global $U(1)_{PQ}$ symmetry, which is spontaneously broken at a high scale and explicitly broken by instanton effects at the low energy QCD scale. The real part of the field is able to inflate in the early universe while the axion boson could be responsible for the dark energy period. The authors of [9] compare their model of quintessential inflation with other models of inflation and/or dark energy. Here, in the framework of a global symmetry with Planck-scale explicit breaking, we offer an explicit scenario of quintessential inflation. As an alternative, we also consider the possibility that, in the same framework, the axion boson is a dark matter constituent. We may have one alternative or the other depending on the magnitude of the explicit symmetry breaking.

**THE MODEL**

In our model, we have a complex field $\Psi$ that is charged under a certain global $U(1)$ symmetry and a potential that contains the following $U(1)$-symmetric term

$$V_1(\Psi) = \frac{1}{4} \lambda \left( |\Psi|^2 - v^2 \right)^2 \quad (1)$$

where $\lambda$ is a coupling constant and $v$ is the energy scale of the spontaneous symmetry breaking (SSB).

Without knowing the details of how Planck-scale physics breaks our $U(1)$ symmetry, we introduce the most simple effective $U(1)$-breaking term

$$V_{\text{non-sym}}(\Psi) = -g \frac{1}{M_P^{n-3}} |\Psi|^n \left( \Psi e^{-i\delta} + \Psi^* e^{i\delta} \right) \quad (2)$$

with an integer $n > 3$. We base our model on the idea that the coupling $g$ is expected to be very small [10]. If $g$ is of order $10^{-30}$ then we will see that the resulting PGB is a dark matter candidate, while for $g$-values of order $10^{-119}$ it will be a quintessence field.

The complex scalar field $\Psi$ may be written in the form

$$\Psi = \phi e^{i\theta / v}. \quad (3)$$

Our basic idea is that the radial part $\phi$ of the field $\Psi$ is responsible for inflation, whereas the angular part $\theta$ can play either the role of the present dominating dark energy of the universe, or of the dark matter, depending on the values of $g$ parameter that appears in (2).

In order for $\phi$ to inflate, one has to introduce a new real field $\chi$ that assists $\phi$ to inflate. The $\chi$ field is supposed to be massive and neutral under $U(1)$. In the process of SSB at temperatures $T \sim v$ in the early universe, the scalar field $\phi$ develops in time, starting from $\phi = 0$ and going to values different from zero, as in inverted hybrid inflation [11, 12] models. We shall follow ref.[12] and couple $\chi$ to $\Psi$ with a $-\Psi^* \Psi \chi^2$ term. More specifically we introduce the following contribution to the potential

$$V_2(\Psi, \chi) = \frac{1}{2} m_\chi^2 \chi^2 + \left( \lambda^2 - \frac{\alpha^2 |\Psi|^2 \chi^2}{4\Lambda^2} \right)^2 \quad (4)$$
where $\alpha$ is a coupling and $\Lambda$ and $m_\chi$ are mass scales. The interaction between the two fields will give the needed behavior of the real part of $\Psi$ to give inflation. Such models of inflation are realized in supersymmetry, using a globally supersymmetric scalar potential [12].

To summarize, our model has a complex field $\Psi$ and a real field $\chi$ with a total potential

$$V(\Psi, \chi) = V_{sym}(\Psi, \chi) + V_{non-sym}(\Psi) + C$$

where $C$ is a constant that sets the minimum of the effective potential to zero. The non-symmetric part is given by (2), whereas the symmetric part is the sum of (1) and (4),

$$V_{sym}(\Psi, \chi) = V_1(\Psi) + V_2(\Psi, \chi)$$

### Inflation

Let us study, firstly, the conditions to be imposed on our model to describe the inflationary stage of expansion of the primordial Universe. In order to do this, we will only work with the symmetric part of the effective potential, which dominates over the non-symmetric part at early times, and after making the replacement (3) we obtain

$$V_{sym}(\phi, \chi) = \Lambda^4 + \frac{1}{2} \left( m_\chi^2 - \alpha^2 \phi^2 \right) \chi^2 + \frac{\alpha^4 \phi^4 \chi^4}{16 \Lambda^4} + \frac{1}{4} \lambda (\phi^2 - v^2)^2,$$

Here, $\phi$ is the inflaton field and $\chi$ is the field that plays the role of an auxiliary field, which ends the inflationary regime through a "waterfall" mechanism. We note that the $\phi^4 \chi^4$ term in Eq.(7) does not play an important role during inflation, but only after it ends, and it sets the position of the global minimum of $V_{sym}(\phi, \chi)$.

From (7) we notice that the field $\chi$ has an effective mass given by $M_\chi^2 = m_\chi^2 - \alpha^2 \phi^2$, so that for $\phi < \phi_c = \frac{m_\chi}{\alpha}$, the only minimum of $V_{sym}(\phi, \chi)$ is at $\chi = 0$. The curvature of the effective potential in the $\chi$ direction is positive, while in the $\phi$ direction is negative. Because we expect that after the SSB, $\phi$ is close to the origin and displaced from it due to quantum fluctuations, it will roll down away from the origin, while $\chi$ will stay at its minimum $\chi = 0$ until the curvature in $\chi$ direction changes sign. That happens when $\phi > \phi_c$ and $\chi$ becomes unstable and starts to roll down its potential.

The conditions to be imposed on our model are the following:

- The vacuum energy term in (7) should dominate over the others: $\Lambda^4 > \frac{1}{4} \lambda v^4$
- The absolute mass squared of the inflaton should be much less than the $\chi$-mass squared, $|m_\phi^2| = \lambda v^2 \ll m_\chi^2$, which fixes the initial conditions for the fields: $\chi$ is initially constrained at the stable minimum $\chi = 0$, and $\phi$ may slowly roll from its initial position $\phi \simeq 0$
- Slow-roll conditions in $\phi$-direction, which are given by the following requirements:
  $$\epsilon \equiv \frac{M_P^2}{16 \pi} \left( \frac{V_{sym}}{V_{sym}} \right)^2 \ll 1, \quad |\eta| \equiv \left| \frac{M_P^2 V_{sym}'}{8 \pi V_{sym}} \right| \ll 1,$$
  where a prime means derivative with respect to $\phi$
• Sufficient number of e-folds of inflation:
\[ N(\phi) = \int_{t_{\text{end}}}^{t_f} H(t) \, dt = \frac{8\pi}{M_p} \int_{\phi_{\text{end}}}^{\phi} \frac{V_{\text{sym}}}{V_{\text{sym}}} \, d\phi \]
where \( \phi_{\text{end}} \equiv \phi(t_{\text{end}}) = \phi_c \) marks the end of slow-roll inflation

• Fast roll of \( \chi \) field at the end of inflation: \( |\Delta M^2_\xi| \gg H^2 \), where \( |\Delta M^2_\xi| \) is the absolute variation of the \( \chi \)-mass squared in a Hubble time \( H \), around the point where \( \phi \simeq \phi_c \)

• Fast roll of \( \phi \) after \( \chi \) settles down to the minimum. This is possible because the potential has a non-vanishing first derivative at that point which forces \( \phi \) to oscillate around the minimum of the potential, with a frequency \( \omega \) which we want to be greater than the Hubble parameter \( H: \omega > H \).

From the last condition we obtain an upper limit for the SSB scale \( v \)
\[ v < M_P. \quad (8) \]

**Dark matter**

As stated above, our idea is that the PGB \( \theta \) that appears after the SSB of \( U(1) \) can play the role of quintessence or of dark matter, depending on the values of \( g \)-parameter. Let us start investigating the case where \( \theta \) describes dark matter. For a detailed study we send the reader to our work [3]. Here, we will just highlight the main features and conclusions of our study in [3].

Due to the small explicit breaking of the \( U(1) \) symmetry, \( \theta \) gets a mass
\[ m^2_\theta = 2g \left( \frac{v}{M_P} \right)^{n-1} M_P^2 \quad (9) \]
which depends on the two free parameters \( v \) and \( g \). In what follows, we fix the value of \( n = 4 \) except if explicitly mentioned.

For \( \theta \) to be a dark matter candidate, it should satisfy the following astrophysical and cosmological constraints:

• It should be stable, with a lifetime \( \tau_\theta > t_0 \), where \( t_0 \) is the lifetime of the universe
• Its density should be comparable to the dark matter density \( \Omega_\theta \sim \Omega_{DM} \sim 0.25 \)
• Because it can be produced in stars, it should not allow for too much energy loss and rapid cooling of stars
• Even if it is stable, \( \theta \) can be decaying in the present and thus contribute to the diffuse photon background of the universe, which is bounded experimentally.

In order to calculate the density of produced \( \theta \)-particles we took into account the different production mechanisms: **thermal production** in the hot plasma, and **non-thermal production** by \( \theta \)-field oscillations and from the decay of cosmic strings produced in the SSB. A detailed study [3] showed that for \( v < 7.2 \times 10^{12} \text{ GeV} \), there is thermal production of \( \theta \) particles, and the number density produced is given by \( n_{\text{th}} \simeq 0.12 T^3 \). The number density produced by the misalignment mechanism is \( n_{\text{osc}} \simeq \frac{1}{4} m_\theta v^2 \) and by cosmic strings decay is \( n_{\text{str}} \simeq v^2/t_{\text{str}} \). Also, we have to take into account that non-thermal produced \( \theta \) may finally thermalize, depending on the values of \( g \) and \( v \). Astrophysical
constraints place a limit on \( v \), but not on \( g \)

\[
v > 3.3 \times 10^9 \text{GeV}.
\] (10)

The combinations of astrophysical and cosmological constraints lead to the following values for \( v \) and \( g \) for \( \theta \) to be a dark matter candidate

\[
v \sim 10^{11} \text{GeV}, \quad g \sim 10^{-30}.
\] (11)

As a final comment, we mention that one could obtain values of order the electric charge for \( g \), if one puts \( n = 7 \), with all \( n < 7 \) prohibited for some unknown reason.

**Dark energy**

Let us find now the values for \( v \) and \( g \) in order for \( \theta \) to be a quintessence field responsible for the present acceleration of the universe. There are two conditions it should satisfy:

- The field \( \theta \) should be displaced from the minimum of the potential \( V_{\text{non-sym}}(\theta) \), and we suppose that its value is of order \( v \); it will only start to fall towards the minimum in the future

\[
m_{\theta} < 3H_0
\] (12)

- The energy density of the \( \theta \) field, \( \rho_0 \), should be comparable to the present critical density \( \rho_{c0} \), if we want \( \theta \) to explain all of the dark energy content of the universe.

\[
\rho_\theta \sim \rho_{c0}
\] (13)

In the above equation (12), \( H_0 \) is the Hubble constant. Taking into account the expression for the mass of \( \theta \), Eq. (9), \( m_{\theta} = \sqrt{2g \left( \frac{v}{M_p} \right)^{\frac{n-1}{2}} M_p} \), condition (12) becomes

\[
g \left( \frac{v}{M_p} \right)^{n-1} < \frac{9H_0^2}{2M_p^2}.
\] (14)

The energy density of the \( \theta \) field is given by the value of the non-symmetric part of the effective potential, \( V_{\text{non-sym}}(\phi, \theta) \), with the assumption that the present values of both fields are of order \( v \)

\[
\rho_\theta \simeq V_{\text{non-sym}}(v, v) = g \left( \frac{v}{M_p} \right)^{n-1} M_p^2 v^2.
\] (15)

Introducing (15) into (13) and remembering that the present critical energy density \( \rho_{c0} = \frac{3H_0^2 M_p^2}{8\pi} \), we have that

\[
g \left( \frac{v}{M_p} \right)^{n-1} \simeq \frac{3H_0^2}{8\pi v^2}.
\] (16)
Combining (14) and (16) we obtain a constraint on \( v \)

\[
\nu > \frac{1}{6}M_P. \tag{17}
\]

This is the restriction to be imposed on \( v \) in order for \( \theta \) to be the field describing dark energy. Notice that it is independent of \( n \). It is also interesting to obtain the restriction on the coupling \( g \), which can be done if we introduce (17) into (16) giving

\[
g < \frac{3 \times 6^{n+1} H_0^2}{8\pi M_P^2}. \tag{18}
\]

Replacing the value for \( H_0 \sim 10^{-42} \) GeV and taking the smallest value \( n = 4 \), we obtain the limit

\[
g < 10^{-119}. \tag{19}
\]

CONCLUSIONS

We have presented a model that is able to explain inflation and dark energy, or inflation and dark matter. Although it is possible that there is no connection between them, the idea of unifying such important ingredients of cosmology into the same model is exciting.

Our model contains two scalar field: one, \( \Psi \), which is complex and charged under a certain global \( U(1) \) symmetry, and another one, \( \chi \), which is real and neutral under \( U(1) \). The real part of \( \Psi \) is supposed to give inflation by coupling to the real field \( \chi \). The imaginary part of \( \Psi \) can be either a dark matter candidate, or a quintessence field responsible for the recent acceleration of the universe. We suppose that we have a \( U(1) \)-symmetric potential to which we add a small term which explicitly breaks the symmetry due to Planck-scale physics. Our conclusion is that the explicit breaking has to be exponentially suppressed. In fact, this is suggested by quantitative studies on the breaking of global symmetries by gravitational effects [10]. If the suppression parameter \( g \) is of order \( 10^{-30} \) and \( v \sim 10^{11} \) GeV, the PGB that appears after the SSB of \( U(1) \) is a dark matter candidate. For a much stronger suppression \( g \sim 10^{-119} \) and a higher SSB scale \( v \sim M_P \), the PGB is a candidate to the dark energy of the universe.

Previous work on explicit breaking of global symmetries can also be found in [13], and related to Planck-scale breaking, in [14]. Cosmological consequences of some classes of PGBs are discussed in [15].

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