Mass spectrum of orbitally and radially excited heavy-light mesons in the relativistic quark model

D. Ebert and V. O. Galkin*
Institut für Physik, Humboldt–Universität zu Berlin, Invalidenstr.110, D-10115 Berlin, Germany

R. N. Faustov
Russian Academy of Sciences, Scientific Council for Cybernetics, Vavilov Street 40, Moscow 117333, Russia
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Abstract

The mass spectrum of orbitally and radially excited states of $B$ and $D$ mesons is calculated in the framework of the relativistic quark model. The expansion in inverse powers of the heavy quark mass is carried out up to the first order, while the light quark is treated without expansion. We find that the relativistic treatment of the light quark plays an important role. Different patterns of $P$ level inversion are discussed. The obtained masses of orbitally and radially excited states are in accord with available experimental data and heavy quark symmetry relations.

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*On leave of absence from Russian Academy of Sciences, Scientific Council for Cybernetics, Vavilov Street 40, Moscow 117333, Russia.
I. INTRODUCTION

The investigation of the properties of excited heavy-light mesons represents an interesting and important problem. The experimental data on the orbitally excited $B$ and $D$ meson states are becoming available now. The $B$ and $c - \tau$ factories will provide more accurate and comprehensive data for these states. On the other hand, the presence of the heavy quark in these systems considerably simplifies the theoretical description of heavy-light mesons. The heavy quark symmetry, arising in the limit of infinitely heavy quark mass $m_Q$, imposes strict constraints on the mass spectrum of heavy-light mesons [3]. In this limit the heavy quark mass and spin decouples and all the meson properties are determined by light degrees of freedom alone. As a result the heavy quark spin degeneracy of the levels emerges. The light quark’s spin couples with its orbital momentum, resulting for $S$-wave mesons in two degenerate $j = 1/2$ states, corresponding to $1^-$ and $0^-$. For $P$-wave mesons two degenerate $j = 3/2$ states, $(1^+, 2^+)$, and two degenerate $j = 1/2$ states, $(0^+, 1^+)$, arise. The heavy quark symmetry constrains also the strong decays of these states. The decay rates of the $P$-wave states in degenerate doublets should be the same. The states with $j = 1/2$ are expected to be wide, because they decay in $S$-wave, while $j = 3/2$ mesons are narrow, because they decay in $D$-wave. Since the real $b$ and $c$ quarks are not infinitely heavy, the corrections in inverse powers of the heavy quark mass turn out to be important. These corrections break down the degeneracy of the levels. The heavy quark effective theory (HQET) (see e.g. [3] and references therein) provides a systematic method for treating $1/m_Q$ corrections. However, in order to obtain quantitative predictions it is necessary to combine it with some dynamical nonperturbative approaches.

Many different approaches have been used for the calculation of orbital and radial excitations of heavy-light mesons [3-4]. However, almost in all of them the expansion in inverse powers not only of the heavy quark mass ($m_Q$) but also in inverse powers of the light quark mass ($m_q$) is carried out. The estimates of the light quark velocity in these mesons show that the light quark is highly relativistic ($v/c \sim 0.7 \div 0.8$). Thus the nonrelativistic approximation is not adequate for the light quark and one cannot guarantee the numerical accuracy of the expansion in inverse powers of the light quark mass. In this paper we use the relativistic quark model [10, 12] for the calculation of the masses of orbitally and radially excited $B$ and $D$ mesons without employing the expansion in $1/m_Q$. Thus the light quark is treated fully relativistically. Concerning the heavy quark we apply the expansion in $1/m_Q$ up to the first order. Our relativistic quark model is based on the quasipotential approach in quantum field theory. It has been used for the calculation of heavy quarkonia mass spectrum [10] and electroweak decays of heavy mesons [11, 12]. Recent nonperturbative investigations indicate that the confining potential cannot be simply scalar. This is in agreement with our model assumptions on the Lorentz structure of quark confinement. The comparison of our model results with model independent constraints of HQET is given in [13].

The paper is organized as follows. In Sec. [I] we describe our relativistic quark model giving special emphasize on the role of the Lorentz-structure of quark confinement. In Sec. [II]
we construct the quasipotential of the interaction of a light quark with a heavy antiquark. We use the heavy quark $1/m_Q$ expansion to simplify the construction. The light quark is treated relativistically. First we consider the $m_Q \to \infty$ limit and then the corrections of the first order in $1/m_Q$. We present the predictions of our model for orbitally and radially excited states of $D$, $D_s$, $B$ and $B_s$ mesons. In Sec. V we compare our results with the heavy quark symmetry and other quark model predictions as well as available experimental data. Section V contains our conclusions.

II. RELATIVISTIC QUARK MODEL

In the quasipotential approach a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation [14] of the Schrödinger type [15]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{p^2}{2\mu_R}\right)\Psi_M(p) = \int \frac{d^3q}{(2\pi)^3} V(p, q; M)\Psi_M(q), \quad (1)$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_qE_Q}{E_q + E_Q} = \frac{M^4 - (m_q^2 - m_Q^2)^2}{4M^3}, \quad (2)$$

and $E_q, E_Q$ are given by

$$E_q = \frac{M^2 - m_Q^2 + m_q^2}{2M}, \quad E_Q = \frac{M^2 - m_q^2 + m_Q^2}{2M}, \quad (3)$$

here $M = E_q + E_Q$ is the meson mass, $m_{q,Q}$ are the masses of light and heavy quarks, and $p$ is their relative momentum. In the centre of mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_q + m_Q)^2][M^2 - (m_q - m_Q)^2]}{4M^2}. \quad (4)$$

The kernel $V(p, q; M)$ in Eq. (1) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. An important role in this construction is played by the Lorentz-structure of the confining quark-antiquark interaction in the meson. In the literature there is no consent on this item. For a long time the scalar confining kernel has been considered to be the most appropriate one [16]. The main argument in favour of this choice is based on the nature of the heavy quark spin-orbit potential. The scalar potential gives a vanishing long-range magnetic contribution, which is in agreement with the picture of quark confinement of Ref. [17], and allows to get the fine structure for heavy quarkonia in accord with experimental data. However, the calculations of electroweak decay rates of heavy mesons with a scalar confining potential alone yield results which are in worse agreement with data than for a vector potential [18,19]. The radiative $M1$-transitions in quarkonia such as e. g. $J/\psi \to \eta_c\gamma$ are the most sensitive for the Lorentz-structure of the confining potential. The relativistic corrections for these decays arising from vector and
scalar potentials have different signs \[18,19\]. In particular, as it has been shown in Ref. \[19\], agreement with experiments for these decays can be achieved only for a mixture of vector and scalar potentials. In this context, it is worth remarking, that the recent study of the $q\bar{q}$ interaction in the Wilson loop approach \[20\] indicates that it cannot be considered as simply a scalar. Moreover, the found structure of spin-independent relativistic corrections is not compatible with a scalar potential. A similar conclusion has been obtained in Ref. \[21\] on the basis of a Foldy-Wouthuysen reduction of the full Coulomb gauge Hamiltonian of QCD. There, the Lorentz-structure of confinement has been found to be of vector nature. The scalar character of spin splittings in heavy quarkonia in this approach is dynamically generated through the interaction with collective gluonic degrees of freedom.

All these new results are in agreement with the assumptions of the relativistic quark model approach \[19,10\]. Indeed, in constructing the quasipotential of quark-antiquark interaction we have assumed that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli interaction. The quasipotential is then defined by \[10\]

$$V(p, q; M) = \bar{u}_q(p)\gamma_\mu D_{\mu\nu}(k)\gamma^\nu u_Q(-p)\left\{\frac{4}{3}\alpha_S D_{\mu\nu}(k)\gamma_\mu \gamma_\nu + V^V_{\text{conf}}(k)\Gamma_{\mu Q\nu} + V^S_{\text{conf}}(k)\right\}u_q(q)u_Q(-q), \quad (5)$$

where $\alpha_S$ is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge

$$D^{00}(k) = -\frac{4\pi}{k^2}, \quad D^{ij}(k) = -\frac{4\pi}{k^2}\left(\delta^{ij} - \frac{k^i k^j}{k^2}\right), \quad D^{i0} = D^{0i} = 0, \quad (6)$$

and $k = p - q$; $\gamma_\mu$ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}}\left(\frac{1}{\epsilon(p) + m}\right)^{\frac{1}{2}}\epsilon(p), \quad (7)$$

with $\epsilon(p) = \sqrt{p^2 + m^2}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(k) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \quad (8)$$

where $\kappa$ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V^V_{\text{conf}}(r) = (1 - \varepsilon)(Ar + B),$$
$$V^S_{\text{conf}}(r) = \varepsilon(Ar + B), \quad (9)$$

reproducing

$$V_{\text{conf}}(r) = V^S_{\text{conf}}(r) + V^V_{\text{conf}}(r) = Ar + B, \quad (10)$$
where \( \varepsilon \) is the mixing coefficient.

The expression for the quasipotential for the heavy quarkonia, expanded in \( v^2/c^2 \), can be found in Ref. [10]. The structure of the spin-dependent interaction is in an agreement with the parameterization of Eichten and Feinberg [22]. All the parameters of our model like quark masses, parameters of linear confining potential \( A \) and \( B \), mixing coefficient \( \varepsilon \) and anomalous chromomagnetic quark moment \( \kappa \) were fixed from the analysis of heavy quarkonia masses [10] and radiative decays [19]. The quark masses \( m_b = 4.88 \text{ GeV}, m_c = 1.55 \text{ GeV}, m_s = 0.50 \text{ GeV}, m_{u,d} = 0.33 \text{ GeV} \) and the parameters of the linear potential \( A = 0.18 \text{ GeV}^2 \) and \( B = -0.30 \text{ GeV} \) have standard values of quark models. The value of the mixing coefficient of vector and scalar confining potentials \( \varepsilon = -1 \) has been determined from the consideration of the heavy quark expansion [13] and meson radiative decays [13]. Finally, the universal Pauli interaction constant \( \kappa = -1 \) has been fixed from the analysis of the fine splitting of heavy quarkonia \( ^3P_J \)-states [10]. Note that the long-range magnetic contribution to the potential in our model is proportional to \( 1 + \kappa \) and thus vanishes for the chosen value of \( \kappa = -1 \).

### III. QUASIPOTENTIAL OF THE INTERACTION OF A LIGHT QUARK WITH A HEAVY ANTIQUARK

The expression for the quasipotential (5) can, in principle, be used for arbitrary quark masses. The substitution of the Dirac spinors (7) into (5) results in an extremely nonlocal potential in the configuration space. Clearly, it is very hard to deal with such potentials without any simplifying expansion. Fortunately, in the case of heavy-light mesons, one can carry out (following HQET) the expansion in inverse powers of the heavy quark mass \( m_Q \). The leading terms then follow in the limit \( m_Q \to \infty \).

#### A. Infinitely heavy quark limit

In the limit \( m_Q \to \infty \) the heavy quark Dirac spinor has only an upper component. Thus only the zeroth component of Dirac matrices in (5) contributes, and we get

\[
V(p, q; M) = \bar{u}_q(p) \left\{ -\frac{4}{3} \alpha_s \frac{4\pi}{k^2} \gamma_0 \right. \\
+ V_{\text{conf}}^V(k) \left[ \gamma_0 + \frac{\kappa}{2m_q} \gamma_0 (\gamma k) \right] + V_{\text{conf}}^S(k) \right\} u_q(q). \tag{11}
\]

The resulting quasipotential is still nonlocal in configuration space. However, taking into account that \( B \) and \( D \) mesons are weakly bound [2] we can replace \( \varepsilon_q(p) \to E_q \) in the Dirac spinors (7). Such simplifying substitution is widely used in quantum electrodynamics [23–25] and introduces only minor corrections of order of the ratio of the binding energy \( \langle V \rangle \) to \( E_q \). This substitution makes the Fourier transformation of the potential (11) local and works well

\(^2\)The sum of constituent quark masses \( m_Q + m_q \) is very close to the ground state meson mass \( M \).
for the confining part of the potential. However, it leads to a fictitious singularity $1/r^3$ at the origin arising from the one-gluon exchange part, which is absent in the initial potential. Note that this singularity is not important if the perturbation theory in $1/m_q$ is used. As we are not going to expand our potential in $1/m_q$, additional analysis is necessary. The explicit one-gluon contribution to (11) is given by

$$\bar{u}_q(p) \left( -\frac{4}{3}\alpha_s \frac{4\pi}{k^2} r_0 \right) u_q(q) = \left( -\frac{4}{3}\alpha_s \frac{4\pi}{k^2} \right) \left[ \frac{\epsilon_q(p) + m_q}{2\epsilon_q(p)} - \frac{\epsilon_q(q) + m_q}{2\epsilon_q(q)} \right] \times \left( 1 + \frac{pq + i\sigma_q[pq]}{[\epsilon_q(p) + m_q][\epsilon_q(q) + m_q]} \right).$$

The most interesting is the last term in this expression, which comes from the lower components of the spinors $u_q$. In configuration space, it has an $1/r$ asymptotics for $r \to 0$ and decreases as $1/r^3$ for $r \to \infty$. If we simply replace $\epsilon_q \to E_q$, then we get the asymptotics $1/r^3$, which is correct at infinity but too singular at the origin. To cure this asymptotics, let us notice that if binding effects are taken into account, it is necessary to replace $\epsilon_q \to E_q - V$, where $V$ is the quark interaction potential. If we consider larger distances of order of the hadron radius, the contributions of $V$ are of order of the (small) binding energy and can be neglected. But for $r \to 0$ the Coulomb singularity in $V$ becomes important. Thus, we keep this contribution (up to the first order in the binding energy) in the last term in (12). As a result we find that the Coulomb-like one-gluon potential $V_{Coul}(r) = -\frac{4}{3}\alpha_s E_q$ in this term should be replaced by

$$\bar{V}_{Coul}(r) = V_{Coul}(r) \frac{1}{1 + \frac{4}{3}\frac{\alpha_s}{E_q} r} \left( 1 + \frac{4}{3}\frac{\alpha_s}{E_q + m_q} \frac{1}{r} \right),$$

leading to the correct asymptotics both at the origin and infinity.

The resulting local quark-antiquark potential for $m_Q \to \infty$ can be presented in configuration space in the following form

$$V_{mQ\to\infty}(r) = \frac{E_q + m_q}{2E_q} \left[ V_{Coul}(r) + V_{conf}(r) + \frac{1}{(E_q + m_q)^2} \left\{ p[\bar{V}_{Coul}(r) + V_{conf}(r)] - \frac{E_q + m_q}{2m_q} \Delta V_{conf}(r) [1 - (1 + \kappa)] \right\} + \frac{2}{r} \left( V_{Coul}(r) - V_{conf}(r) \right) \left[ \frac{E_q}{m_q} - 2(1 + \kappa) \frac{E_q + m_q}{2m_q} \right] \textbf{LS}_q \right\}.$$

Here the prime denotes differentiation with respect to $r$, $\textbf{L}$ is the orbital momentum, and $\textbf{S}_q$ is the spin operator of the light quark. Note that the last term in (14) is of the same order as the first two terms and thus cannot be treated perturbatively.

In the infinitely heavy quark limit the quasipotential equation (11) in configuration space becomes

$$\left( \frac{E_q^2 - m_q^2}{2E_q} - \frac{P^2}{2E_q} \right) \Psi_M(r) = V_{mQ\to\infty}(r) \Psi_M(r),$$

and the mass of the meson is given by $M = m_Q + E_q$. 

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Solving (15) numerically \(^26\) we get the eigenvalues \(E_q\) and the wave functions \(\Psi_M\). The obtained results are presented in Table I. We see that the heavy quark spin decouples in the limit \(m_Q \to \infty\), and thus we get the number of degenerated states in accord with the heavy quark symmetry prediction.

B. \(1/m_Q\) corrections

The heavy quark symmetry degeneracy of states is broken by \(1/m_Q\) corrections. The corrections of order \(1/m_Q\) to the potential (14) arise from the lower components of heavy quark Dirac spinors in (5). The matrix elements of the spatial components of Dirac matrices are not zero now. Thus the contributions at first order in \(1/m_Q\) come from the one-gluon-exchange potential and the vector confining potential, while the scalar potential gives no contribution at first order. The resulting \(1/m_Q\) correction to the heavy quark potential (14) is given by the following expression

\[
\delta V_{1/m_Q}(r) = \frac{1}{E_q m_Q} \left\{ p \left[ V_{\text{Coul}}(r) + V_{\text{conf}}'(r) \right] p + V_{\text{Coul}}'(r) \frac{L^2}{2r} \right. \\
- \frac{1}{4} \Delta V_{\text{conf}}'(r) + \left. \left[ \frac{1}{r} V_{\text{Coul}}'(r) + \frac{(1 + \kappa)}{r} V_{\text{conf}}'(r) \right] LS \right.
\]

\[
+ \gamma \left[ \left[ \frac{1}{r} V_{\text{Coul}}'(r) - V_{\text{conf}}'(r) + (1 + \kappa)^2 \left[ \frac{1}{r} V_{\text{conf}}'(r) - V_{\text{conf}}'(r) \right] \right] \right.
\]

\[
\left. \times \left[ -S_q S_Q + \frac{3}{r^2}(S_q r)(S_Q r) \right] \right)
\]

\[
+ \frac{2}{3} \left[ \Delta V_{\text{Coul}}(r) + (1 + \kappa)^2 \Delta V_{\text{conf}}'(r) \right] S_Q S_q \right\},
\]

where \(S = S_q + S_Q\) is the total spin. The first three terms in (16) represent spin-independent corrections, the fourth term is responsible for the spin-orbit interaction, the fifth one is the tensor interaction and the last one is the spin-spin interaction. It is necessary to note that the confining vector interaction gives a contribution to the spin-dependent part which is proportional to \((1 + \kappa)\) or \((1 + \kappa)^2\). Thus it vanishes for the chosen value of \(\kappa = -1\), while the confining vector contribution to the spin-independent part is non zero.

The quasipotential at \(1/m_Q\) order is given by the sum of \(V_{m_Q \to \infty}(r)\) from (14) and \(\delta V_{1/m_Q}(r)\) from (16). By substituting it in the quasipotential equation (1) and treating the \(1/m_Q\) correction term \(\delta V_{1/m_Q}(r)\) using perturbation theory, we are now able to calculate the mass spectrum of \(D, D_s, B\) and \(B_s\) mesons with the account of \(1/m_Q\) corrections. The results of our calculations are presented in Tables II-V.

IV. RESULTS AND DISCUSSION

Let us compare the obtained results with model independent predictions of HQET. In HQET the heavy-light meson mass \(M\) of ground-state pseudoscalar and vector mesons is given by \(^3\)

\[
M = \mu_Q + \bar{\Lambda} - \frac{\lambda_1 - 2 \left[ J(J + 1) - \frac{3}{2} \right] \lambda_2}{2\mu_Q} + O(1/\mu_Q^2),
\]

(17)
where the parameter $\bar{\Lambda}$ represents contributions arising from terms in the HQET Lagrangian which are independent of the heavy quark mass. The terms $\lambda_1$ and $\lambda_2$ parametrize contributions from the kinetic energy and chromo-magnetic interaction.

Note that the HQET heavy quark mass $\mu_Q$ is in principle different from our mass $m_Q$ by the simple reason, that HQET uses the pole heavy quark mass, while quark models use the constituent quark mass. Obviously, in the heavy quark symmetry limit this difference between constituent and pole heavy quark masses, however, disappears. In particular, we find the relation $E_q = \bar{\Lambda} + O(1/m_Q)$. Thus, the values of $E_q$ listed in Table I are just the values of the HQET parameters $\bar{\Lambda}$.

The structure of $1/m_Q$ corrections in our model is consistent with (17). However, we find that the parameter corresponding to $\lambda_1$ in our model contains not only the kinetic energy part but also other terms originating from spin-independent corrections in Eq. (16). A similar result has been found in the relativistic quark model in Ref. [27]. The value of this parameter turns out to be very sensitive to the value of the heavy quark mass. Indeed, assuming that the heavy quark pole mass $\mu_Q$ and constituent quark mass $m_Q$ are related by $[27]$

$$m_Q = \mu_Q - \frac{\lambda^Q}{2\mu_Q} + O(1/\mu^2_Q),$$  

we see that only $\lambda_1$ is influenced, and it is connected to the quark model value $\lambda_1^{QM}$ by

$$\lambda_1 = \lambda_1^{QM} + \lambda^Q.$$  

In our model, for constituent quark masses $m_b = 4.88$ GeV and $m_c = 1.55$ GeV, we find $\lambda_1^{QM} \approx 0.85$ GeV$^2$ for $B$ mesons and $\lambda_1^{QM} \approx 0.26$ GeV$^2$ for $D$ mesons. If we require the parameter $\lambda_1$ to be equal to the mean value of $-\langle p^2 \rangle$ (in our model $\langle p^2 \rangle \approx 0.25$ GeV$^2$ for $B$ and $D$ mesons), then from (18) and (19) we get the heavy quark masses $\mu_b = 4.75$ GeV and $\mu_c = 1.40$ GeV. These values agree with values of the heavy quark pole masses used in HQET.

The values of the parameter $\lambda_2$, which determines the hyperfine splitting, coincide in HQET and quark models. We find $\lambda_2 \approx 0.112$ GeV$^2$ for $B$ mesons and $\lambda_2 \approx 0.125$ GeV$^2$ for $D$ mesons.

Heavy quark symmetry provides relations between excited states of $B$ and $D$ mesons, such as

$$\bar{M}_{B_1} - \bar{M}_{D_1} = \bar{M}_{B_{1s}} - \bar{M}_{D_{1s}} = \bar{M}_B - \bar{M}_D = \bar{M}_{B_s} - \bar{M}_{D_s} = m_b - m_c,$$

where $\bar{M}_{B_1} = (3M_{B_1} + 5M_{B_s})/8$, $\bar{M}_B = (M_B + 3M_{B^*})/4$ are appropriate spin averaged $P$- and $S$-wave states. We get from Tables II-V the following values of mass splittings

$$\bar{M}_{B_1} - \bar{M}_{D_1} = 3.29 \text{ GeV}$$

3Note that, if we had used also an expansion in inverse powers of the light quark mass, then, in the static limit, we would get $\lambda_1 = -\langle p^2 \rangle$. 

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\[ M_{B_{s1}} - M_{D_{s1}} = 3.30 \text{ GeV} \]
\[ M_B - M_D = 3.34 \text{ GeV} \]
\[ M_{B_s} - M_{D_s} = 3.33 \text{ GeV}, \]

in agreement with (20). There arise also the following relations between hyperfine splittings of levels

\[ \Delta M_B \equiv M_{B_2} - M_{B_1} = \frac{m_c}{m_b} \Delta M_D \equiv \frac{m_c}{m_b} (M_{D_2} - M_{D_1}), \]

and the same for \( B_s \) and \( D_s \) mesons as well as for \( P_1 - P_0 \) states. Our model predictions for these splittings are displayed in Table VI.

In Tables II-V we compare our relativistic quark model results for heavy-light meson masses with the predictions of other quark models of Godfrey and Isgur [4], Isgur [5], Eichten, Hill and Quigg [7] and experimental data [28,1,2]. All these quark models use the expansion in inverse powers both of the heavy \( m_Q \) and light \( m_q \) quark masses for the \( Q\bar{q} \) interaction potential. In Ref. [6] some relativization of the potential has been put in by hand, such as relativistic smearing of coordinates and replacing the factors \( 1/m_q \) by \( 1/\epsilon_q(p) \). However, the resulting potential in this approach accounts only for some of the relativistic effects, while the others, which are of the same order of magnitude, are missing. The considerations of Refs. [8,7] are closely related. The heavy quark expansion is extended to light (\( u,d,s \)) quarks and the experimental data on \( P \) wave masses of \( K \) mesons are used to obtain predictions for \( B \) and \( D \) mesons.

In the paper [8] it is argued that the heavy quark spin \( P \)-wave multiplets with \( j = 1/2 \) (\( 0^+,1^+ \)) and \( j = 3/2 \) (\( 1^+,2^+ \)) in \( B \) and \( D \) mesons are inverted [29]. The \( 2^+ \) and \( 1^+ \) states lie about 150 MeV below the \( 1^+ \) and \( 0^+ \) states. In the limit \( m_Q \to \infty \), we find the same inversion of these multiplets in our model, but the gap between \( j = 1/2 \) and \( j = 3/2 \) states is smaller (\( \sim 90 \text{ MeV for } B \) and \( D \) mesons and \( \sim 70 \text{ MeV for } B_s \) and \( D_s \) mesons), and \( 1/m_Q \) corrections reduce this gap further. However, the hyperfine splittings among the states in these multiplets turn out to be larger than in [8]. As a result, the states from the multiplets for \( D, D_s \) and \( B_s \) mesons overlap in our model, however the heavy quark spin averaged centres are still inverted (see Figs. 1-4). We obtain the following ordering of \( P \) states (with masses increasing from left to right): \( B \) meson — \( 1P_1(\frac{3}{2}), 1P_2, 1P_0, 1P_1(\frac{1}{2}) \); \( D_s \) meson — \( 1P_0, 1P_1(\frac{3}{2}), 1P_2, 1P_1(\frac{1}{2}) \); \( D \) and \( B_s \) mesons — \( 1P_1(\frac{3}{2}), 1P_0, 1P_2, 1P_1(\frac{1}{2}) \). Thus only for \( B \) meson we get the purely inverted pattern. Note that the model [6] predicts the ordinary ordering of levels.

The results of our model agree well with available experimental data. The experimental values in Tables II-V for ground state and \( P \)-wave masses are taken from Refs. [28,1]. For the radially excited states we use the preliminary data from DELPHI [2].

V. CONCLUSIONS

In this paper we have presented the calculation of the mass spectra of orbitally and radially excited states of heavy-light mesons in the framework of the relativistic quark model. The main advantage of the proposed approach consists in the relativistic treatment of the light quarks (\( u,d,s \)). We apply only the expansion in inverse powers of the heavy quark
(b, c) mass, which considerably simplifies calculations. The infinitely heavy quark limit as well as the first order $1/m_Q$ corrections are considered. Our model respects the constraints imposed by heavy quark symmetry on the number of levels and different splittings.

We find that the heavy quark spin multiplets with $j = 1/2 \ (0^+, 1^+)$ and $j = 3/2 \ (1^+, 2^+)$ are inverted in the $m_Q \rightarrow \infty$ limit. This inversion is caused by the following reason: The confining potential contribution to the spin-orbit term in (14) exceeds the one-gluon exchange contribution. Thus the sign before the spin-orbit term is negative, and the level inversion emerges. However, $1/m_Q$ corrections, which produce the hyperfine splittings of these multiplets, are substantial. As a result the purely inverted pattern of $P$ levels occurs only for the $B$ meson. For $D$ and $B_s$ mesons the levels from these multiplets begin to overlap. This effect is more pronounced in the $D_s$ meson, where the ordinary ordering is restoring. Thus we see that the $D_s$ meson occupies an intermediate position between heavy-light mesons and heavy-heavy mesons (quarkonia, $B_c$ meson). It is necessary to note, that both the presence of the relativistic light quark and the light-to-heavy quark mass ratio play an important role in the formation of level ordering patterns. The light quark determines the meson radius, while the $m_q/m_Q$ ratio indicates the validity of the application of the heavy quark symmetry limit.

The found mass values of orbitally and radially excited heavy-light mesons are in good agreement with available experimental data. At present only narrow $P$-wave $2^+, 1^+ \ (j = 3/2)$ levels of $D, D_s$ and $B$ mesons have been measured. It will be very interesting to observe also $0^+, 1^+ \ (j = 1/2)$ levels, which is more complicated, because these states are expected to be broad. This will allow to determine the ordering of $P$ levels and to test quark dynamics in a heavy-light meson. We plan to apply the found meson wave functions for the calculation of semileptonic and nonleptonic decays of $B$ mesons into $P$-wave $D$ and $D_s$ mesons, which are important for the experimental observation of these states.

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TABLES

TABLE I. The values of $E_q$ and meson masses $M$ in the limit $m_Q \to \infty$ (in GeV). We use the notation $(n^J L^j)$ for meson states, where $n$ is the radial quantum number, and $j$ is the total angular momentum of the light quark.

| State | $E_{u,d}$ | $M$ | $E_s$ | $M$ | $E_{u,d}$ | $M$ | $E_s$ | $M$ |
|-------|-----------|-----|-------|-----|-----------|-----|-------|-----|
| $1^\frac{3}{2}S$ | 0.497 | 2.047 | 0.607 | 2.157 | 0.514 | 5.394 | 0.628 | 5.508 |
| $1^\frac{3}{2}P$ | 0.800 | 2.350 | 0.913 | 2.463 | 0.800 | 5.680 | 0.927 | 5.807 |
| $1^\frac{1}{2}P$ | 0.886 | 2.436 | 0.985 | 2.535 | 0.898 | 5.778 | 0.998 | 5.878 |
| $2^\frac{3}{2}S$ | 0.940 | 2.490 | 1.047 | 2.597 | 0.955 | 5.835 | 1.063 | 5.943 |

TABLE II. Mass spectrum of $D$ mesons with the account of $1/m_Q$ corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV. We use the notation $(nL^J)$ for meson states, where $J$ is the total angular momentum of the meson.

| State | our | $[6]$ | $[8]$ | experiment $[28,1,2]$ |
|-------|-----|-------|-------|------------------------|
| $1S_0$ | 1.875 | 1.88 | | 1.8645(5) |
| $1S_1$ | 2.009 | 2.04 | | 2.0067(5) |
| $1P_2$ | 2.459 | 2.50 | 2.460 | 2.4589(20) |
| $1P_1$ | 2.414 | 2.47 | 2.415 | 2.4222(18) |
| $1P_0$ | 2.501 | 2.46 | 2.585 | |
| $1P_0$ | 2.438 | 2.40 | 2.565 | |
| $2S_0$ | 2.579 | 2.58 | | |
| $2S_1$ | 2.629 | 2.64 | | 2.637(9) |

TABLE III. Mass spectrum of $D_s$ mesons with the account of $1/m_Q$ corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV. The same notation as in Table II is used for meson states.

| State | our | $[6]$ | $[7]$ | experiment $[28,1,2]$ |
|-------|-----|-------|-------|------------------------|
| $1S_0$ | 1.981 | 1.98 | | 1.9685(6) |
| $1S_1$ | 2.111 | 2.13 | | 2.1124(7) |
| $1P_2$ | 2.560 | 2.59 | 2.561 | 2.5735(17) |
| $1P_1$ | 2.515 | 2.56 | 2.526 | 2.535(4) |
| $1P_0$ | 2.569 | 2.55 | | |
| $1P_0$ | 2.508 | 2.48 | | |
| $2S_0$ | 2.670 | 2.67 | | |
| $2S_1$ | 2.716 | 2.73 | | |
TABLE IV. Mass spectrum of $B$ mesons with the account of $1/m_Q$ corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV. The same notation as in Table II is used for meson states.

| State | our    | 6 | 8 | 7 | experiment [28,1,2] |
|-------|--------|---|---|---|---------------------|
| $1S_0$| 5.285  | 5.31 |      | 5.2792(18)         |
| $1S_1$| 5.324  | 5.37 |      | 5.3248(18)         |
| $1P_2$| 5.733  | 5.80 | 5.715| 5.771 5.759        |
| $1P_1$| 5.719  | 5.78 | 5.700| 5.759  |
| $1P_1$| 5.757  | 5.78 | 5.875|  |
| $1P_0$| 5.738  | 5.76 | 5.870|  |
| $2S_0$| 5.883  | 5.90 |      | 5.90(2)      |
| $2S_1$| 5.898  | 5.93 |      |         |

TABLE V. Mass spectrum of $B_s$ mesons with the account of $1/m_Q$ corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV. The same notation as in Table II is used for meson states.

| State | our    | 6 | 8 | 7 | experiment [28,1,2] |
|-------|--------|---|---|---|---------------------|
| $1S_0$| 5.375  | 5.39 |      | 5.3693(20)         |
| $1S_1$| 5.412  | 5.45 |      | 5.416(4) ?         |
| $1P_2$| 5.844  | 5.88 | 5.861| 5.853(15) ?        |
| $1P_1$| 5.831  | 5.86 | 5.849|  |
| $1P_1$| 5.859  | 5.86 |      |  |
| $1P_0$| 5.841  | 5.83 |      |  |
| $2S_0$| 5.971  | 6.27 |      |  |
| $2S_1$| 5.984  | 6.34 |      |         |

TABLE VI. Hyperfine splittings of $P$ levels. All values are given in MeV.

| States  | $\Delta M_D$ | $\frac{m_c}{m_b} \Delta M_D$ | $\Delta M_B$ | $\Delta M_{D_s}$ | $\frac{m_c}{m_b} \Delta M_{D_s}$ | $\Delta M_{B_s}$ |
|---------|--------------|-------------------------------|-------------|-----------------|--------------------------------|-----------------|
| $1P_2 - 1P_1$ | 45           | 14                       | 14          | 45              | 14                           | 13              |
| $1P_1 - 1P_0$ | 63           | 20                       | 19          | 61              | 19                           | 18              |
FIG. 1. The ordering pattern of $D$ meson states. The mass scale is in GeV.
FIG. 2. The ordering pattern of $D_s$ meson states. The mass scale is in GeV.
FIG. 3. The ordering pattern of $B$ meson states. The mass scale is in GeV.
FIG. 4. The ordering pattern of $B_s$ meson states. The mass scale is in GeV.