Precision gaugino mass measurements as a probe of large trilinear soft terms at ILC

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Abstract

In supersymmetric models with radiatively-driven naturalness, higgsino-like electroweakinos (EW-inos) are expected to lie in a mass range 100–300 GeV, the lighter the more natural. Such states can be pair-produced at high rates at ILC where their masses are nearly equal to the value of the superpotential $\mu$ parameter while their mass splittings depend on the gaugino masses $M_1$ and $M_2$. The gaugino masses in turn depend on trilinear soft terms—the $A$ parameters, which are expected to lie in the multi-TeV range owing to the 125 GeV Higgs mass—via two-loop contributions to renormalization group running. We examine the extent to which ILC is sensitive to large $A$-terms via precision EW-ino mass measurement. Extraction of gaugino masses at the percent level should allow for interesting probes of large trilinear soft SUSY breaking terms under the assumption of unified gaugino masses.

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1 Introduction

The initial spate of results from LHC Run 1 at √s = 7–8 TeV and Run 2 with √s = 13 TeV have been delivered and have caused a paradigm shift in expected phenomenology of supersymmetric (SUSY) models. In pre-LHC years, a rather light spectrum of sparticle masses was generally expected on the basis of naturalness: that weak-scale SUSY should not lie too far beyond the weak scale as typified by the \(W, Z\) (and ultimately \(h\)) masses, where \(m_{\text{weak}} \simeq m(W, Z, h) \simeq 100\) GeV. These expectations were backed up by calculations of upper bounds on sparticle masses using the Barbieri–Giudice measure \([1–5]\)

\[
\Delta_{BG} \equiv \max_i \left| \frac{\partial \log m_Z^2}{\partial \log p_i} \right|
\]  

(1)

where \(m_Z\) is the \(Z\)-boson mass and the \(p_i\) label fundamental parameters of the theory, usually taken to be the unified soft SUSY breaking terms and the superpotential \(\mu\) parameter. The upper bounds turned out to be typically in the range of a few hundred GeV: for instance, in Ref. \([1]\), it was found that \(m_{\tilde{g}} \lesssim 350\) GeV for \(\Delta_{BG} < 30\).

Naive hopes for a rapid discovery of SUSY at LHC were dashed by the reality of data wherein lately the ATLAS and CMS collaborations have produced bounds on the gluino mass of \(m_{\tilde{g}} > 1.8\) TeV in simplified models assuming \(\tilde{g} \to b\bar{b}Z_1\) decays \([6, 7]\). The effect of direct sparticle mass limits was compounded by the rather high value of the Higgs-boson mass \(m_h \simeq 125\) GeV \([8]\) which was discovered. Such a high Higgs mass required top squarks in the 10–100 TeV range for small stop mixing and in the few TeV range for large stop mixing. While heavy stops were allowed by the \(\Delta_{BG}\) measure in the focus-point region \([9]\), they were seemingly dis-allowed by large logarithmic contributions to \(m_h\) as quantified by the high-scale large-log measure \(\Delta_{HS} [10, 11]\) which seemed to require three third generation squarks with mass \(\lesssim 500\) GeV \([12]\). Thus, the LHC data cast a pall on overall expectations for SUSY and indeed led to doubts as to whether weak scale SUSY did actually provide nature’s solution to the gauge hierarchy problem. In addition, lack of new physics at LHC cast doubt on the motivation for new accelerators such as the International Linear \(e^+e^-\) Collider (ILC) which would operate at \(\sqrt{s} = 0.5–1\) TeV \([13, 14]\): Would there be any prospect for detection of new particles, or would the role of such a machine be mainly to tabulate assorted precision measurements as a Higgs factory? Prospects for ILC detection of any SUSY particles \([15, 16]\) seemed dim, much less embarking on a program of precision SUSY particle measurements to determine underlying high scale Lagrangian parameters \([17]\).

An alternative approach was to scrutinize the validity of the theoretical naturalness calculations. The most conservative approach to naturalness arises from the weak scale link between the measured value of \(m_Z\) and the fundamental SUSY Lagrangian parameters \([18, 19]\). Minimization of the scalar potential in the minimal supersymmetric Standard Model (MSSM) leads to the well-known relation \([20]\)  

\[
\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 
\]

(2)

\[
\simeq - m_{H_u}^2 - \Sigma_u^u - \mu^2 
\]

(3)
where $\Sigma_u^u$ and $\Sigma_d^d$ denote the 1-loop corrections (expressions can be found in the Appendix of Ref. [19]) to the scalar potential, $m_{H_u}^2$ and $m_{H_d}^2$ are the Higgs soft masses at the weak scale, and $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$ is the ratio of the Higgs VEVs. The second line obtains for moderate to large values of $\tan \beta \gtrsim 5$ (as required by the Higgs mass calculation [21]). SUSY models requiring large cancellations between the various terms on the right-hand-side of Eq. (3) to reproduce the measured value of $m_Z^2$ are regarded as unnatural, or fine-tuned. In contrast, SUSY models which generate terms on the RHS of Eq. (3) which are all less than or comparable to $m_{\text{weak}}$ are regarded as natural. Thus, the electroweak naturalness measure $\Delta_{\text{EW}}$ is defined as [18,19]

$$\Delta_{\text{EW}} \equiv \max |\text{each additive term on RHS of Eq. (2)}| / (m_Z^2 / 2).$$

Including the various radiative corrections, over 40 terms contribute. Neglecting radiative corrections, and taking moderate-to-large $\tan \beta \gtrsim 5$, then $m_Z^2 / 2 \sim -m_{H_u}^2 - \mu^2$ so the main criterion for naturalness is that at the weak scale

- $m_{H_u}^2 \sim -m_Z^2$
- $\mu^2 \sim m_Z^2$ [22].

The value of $m_{H_d}^2$ (where $m_A \sim m_{H_d}(\text{weak})$ with $m_A$ being the mass of the CP-odd Higgs boson) can lie in the TeV range since its contribution to the RHS of Eq. (3) is suppressed by $1 / \tan^2 \beta$. The largest radiative corrections typically come from the top squark sector. Requiring highly mixed TeV-scale top squarks minimizes $\Sigma_{1,2}(\tilde{t}_1, \tilde{t}_2)$ whilst lifting the Higgs mass $m_h$ to $\sim 125$ GeV [19]. This framework is called the radiatively-driven natural SUSY (RNS) [18,19] scenario.

Using $\Delta_{\text{EW}} < 30$ or better than 3% fine-tuning[1] then instead of earlier upper bounds, it is found that

- $m_{\tilde{g}} \lesssim 3-4$ TeV,
- $m_{\tilde{t}_1} \lesssim 3$ TeV and
- $m_{\tilde{W}_1, \tilde{Z}_{1,2}} \lesssim 300$ GeV.

Thus, gluinos and squarks may easily lie beyond the current reach of LHC at little cost to naturalness while only the higgsino-like lighter charginos and neutralinos are required to lie near the weak scale. The lightest higgsino $\tilde{Z}_1$ comprises a portion of the dark matter and would escape detection at LHC. Owing to their compressed spectrum with mass gaps $m_{\tilde{W}_1} - m_{\tilde{Z}_1} \sim m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \sim 10-20$ GeV, the heavier higgsinos are difficult to see at LHC owing to the rather small visible energy released from their three body decays $\tilde{W}_1 \to f \bar{f} \tilde{Z}_1$ and $\tilde{Z}_2 \to f \bar{f} \tilde{Z}_1$ (where the $f$ stands for SM fermions). At the ILC, on the other hand, direct searches for higgsino-like neutralinos and charginos are more promising since the ILC background is much simpler. We will briefly review ILC direct searches for neutralinos and charginos in Sec. 2.

The apparent conflict between $\Delta_{\text{EW}}$ and $\Delta_{\text{BG}}$ was resolved when it was pointed out that $\Delta_{\text{BG}}$ was typically applied to low energy effective SUSY theories wherein the soft terms were

[1]For higher values of $\Delta_{\text{EW}}$, high fine-tuning sets in and is displayed visually in Fig. 2 of Ref. [23].
introduced as independent parameters to parametrize the unknown dynamics of hidden sector SUSY breaking. If instead the soft terms are all calculable e.g. as multiples of the gravitino mass $m_3/2$ as in gravity-mediation, then positive and negative high scale contributions to $m_Z^2$ cancel and the measure reduces to $\Delta_{EW}$ \cite{24,26}. Likewise, if one combines all dependent contributions in the large log measure $\Delta_{HS}$ then cancellations between $m_{H_u}^2(\Lambda)$ and $\delta m^2_{H_u}$ are possible so that $\Delta_{HS} \simeq \Delta_{EW}$: in this case the large negative correction $\delta m_{H_u}^2$ is used to drive the large GUT-scale soft term $m_{H_u}^2$ to a natural value at the weak scale.

A grand overview plot of the natural SUSY parameter space is shown in Fig. 1 where we show contours of $m_h = 123, 125$ and $127$ GeV (red, green and blue contours) in the $A_0$ vs. $m_0$ plane for $\mu = 150$ GeV, $m_{1/2} = 1$ TeV, $\tan \beta = 10$ and $m_A = 2$ TeV. We also show contours of electroweak naturalness $\Delta_{EW} = 30, 100$ and $400$ (black, orange and brown contours). While it is possible to be highly natural for small $m_0$ and small $A_0$ (as expected in pre-LHC days), this region generates a rather light Higgs mass of typically $m_h \sim 115-120$ GeV. However, it is also possible to be highly natural if one moves to multi-TeV values of $m_0$ and $A_0$ just above the gray excluded region (where CCB=charge or color breaking minima occur). In fact, in this large $m_0$ and $A_0$ region, we also see that the Higgs mass moves up to allowed values $\sim 125$ GeV. Thus, the intersection of these regions—highly natural with $m_h \sim 125$ GeV—requires multi-TeV values of $m_0$ and $A_0$. In fact, in a recent paper \cite{27}, it was suggested that in models with $\mu \ll m_{SUSY}$, the string landscape statistically favors as large as possible values of soft terms which are at the same time consistent with the anthropic requirement of $m_{\text{weak}} \sim 100$ GeV. In this region, the EW symmetry is barely broken leading to $m_{H_u}^2(\text{weak}) \sim -m_Z^2$. Then very large, multi-TeV values of $m_0$ and $A_0$ are to be expected.

Since $m_0$ sets the squark and slepton mass scale, we expect if we live in this region then the matter scalars will likely be out of LHC reach. Top squarks are also likely to be beyond both LHC and ILC reach so it will be difficult to ascertain whether the trilinear terms $A_0$ are indeed large. However, such a large $A_0$ modifies the gaugino masses via two loop terms in the renormalization group evolution. If the ILC measures gaugino masses at the percent level, it is possible to indirectly see the large $A_0$ which is required for natural SUSY. In this paper, we advocate precision measurements of the lightest chargino and neutralino masses as a means to test the requirement of multi-TeV trilinear soft breaking terms.

2 Review of radiatively-driven natural SUSY at ILC

While gluino masses may range up to 4 TeV for $\Delta_{EW} < 30$ \cite{19,23,28}, the $5\sigma$ reach of LHC14 for gluino pair production with $\sim 1000$ fb$^{-1}$ extends only to about 2 TeV while the 95\% CL exclusion for $\sim 3000$ fb$^{-1}$ extends to about 2.8 TeV. Other signal channels such as same sign diboson production \cite{29} from $pp \to W_2Z_4$ or $pp \to Z_1Z_2j \to \ell^+\ell^-j + E_T^{\text{miss}}$ \cite{30,32} should extend the LHC reach for natural SUSY spectrum \cite{33}. Meanwhile, construction of the ILC would be highly propitious for natural SUSY. If natural SUSY is correct, then ILC would likely be a higgsino factory where the reactions $e^+e^- \to W_1^+W_1^-$ and $Z_1Z_2$ would occur at high rates provided $\sqrt{s} > 2m(\text{higgsino})$ and the semi-compressed decays would be easily visible in the clean environment of $e^+e^-$ collisions\footnote{Even in the highly compressed (but less natural) case of sub-GeV EW-ino mass gaps, precision mass}.

\textsuperscript{2}
Figure 1: The $A_0$ vs. $m_0$ parameter plane for $\mu = 150$ GeV, $m_{1/2} = 1$ TeV, $\tan \beta = 10$ and $m_A = 2$ TeV. We show contours of $m_h = 123$, 125 and 127 GeV (red, green, blue). We also show contours of $\Delta_{EW} = 30$, 100 and 400. The intersection of $m_h \simeq 125$ GeV and low $\Delta_{EW}$ occurs along the edges of the allowed region at very large values of $A_0$.

A detailed phenomenological study of ILC measurements for RNS has been presented in Ref. [35]. More detailed studies are in progress using realistic detector simulations and more thorough background generation [37]. Briefly, the reaction $e^+e^- \rightarrow \tilde{W}_1^\pm \tilde{W}_1^-\nu_\ell$ followed by one $\tilde{W}_1 \rightarrow \ell \nu_\ell \tilde{Z}_1$ and the other $\tilde{W}_1 \rightarrow q\bar{q}'\tilde{Z}_1$ will allow for the $m_{\tilde{W}_1} - m_{\tilde{Z}_1}$ mass gap extraction via measurement of the dijet invariant mass. In addition, measurement of the kinematic lower and upper endpoints of the dijet energy $E(jj)$ distribution will allow for precision determination of $m_{\tilde{W}_1}$ and $m_{\tilde{Z}_1}$ [15,16,35]. Typically these endpoint measurements are expected to yield EW-ino masses at the percent level at ILC. In addition, using the variable beam polarization and the variable beam energy—for instance to do threshold scans—may allow sub-percent precision on these mass values [36].

Likewise, the reaction $e^+e^- \rightarrow \tilde{Z}_1\tilde{Z}_2$ followed by $\tilde{Z}_2 \rightarrow \ell^+\ell^- \tilde{Z}_1$ will allow the $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ mass gap to be extracted from the $m(\ell^+\ell^-)$ upper bound. The endpoints of the $E(\ell^+\ell^-)$ distribution should allow extraction of $m_{\tilde{Z}_2}$ and $m_{\tilde{Z}_1}$ to percent or better precision.

While the values of $m_{\tilde{W}_1}$, $m_{\tilde{Z}_2}$ and $m_{\tilde{Z}_1}$ are all expected to be $\sim \mu$, the mass gaps will depend on the bino and wino contamination of the mainly-higgsino eigenstates. Thus, it is expected that the weak scale bino $M_1$ and wino $M_2$ masses can be extracted from precision measurements. These virtual mass extractions will only be aided if measurements of $\tan \beta$ can be gained by studying deviations in the light Higgs $h$ decay modes [38,41] or if the heavy Higgs bosons $H$ or $A$ are accessible to LHC (or later $\sqrt{s} = 1$ TeV ILC) searches. In addition, other measurements are possible by using initial state photon radiation [34].
EW-ino reactions such as $e^+e^- \rightarrow \tilde{Z}_1\tilde{Z}_3$ may ultimately be accessible which would involve direct production of the mainly bino $\tilde{Z}_3$ state. At even higher energies, it may be possible to directly produce the mainly wino state $\tilde{Z}_4$ via $e^+e^- \rightarrow \tilde{Z}_1\tilde{Z}_4$. What is likely is that for RNS with light higgsinos, are very rich program of assorted SUSY measurements awaits the ILC program.

3 Testing multi-TeV trilinears via determination of gaugino masses

In this section, we present large $A$-term effects on gaugino masses via the renormalization group evolution from the high energy scale to the weak scale. The two-loop renormalization group equations (RGEs) for the $U(1)$, SU(2)$_L$ and SU(3)$_C$ gaugino masses in the DR scheme read $^{42}$:

$$\frac{dM_1}{dt} = \frac{2}{16\pi^2} \left[ \frac{33}{5} g_1^2 M_1 + \frac{2g_1^2}{(16\pi^2)^2} \left[ \frac{199}{25} g_1^2 (2M_1) + \frac{27}{5} g_2^2 (M_1 + M_2) + \frac{88}{5} g_3^2 (M_1 + M_3) \right. \right.$$  
$$+ \left. \frac{26}{5} f_t^2 (A_t - M_1) + \frac{14}{5} f_b^2 (A_b - M_1) + \frac{18}{5} f_{\tau}^2 (A_{\tau} - M_1) \right] ,$$  

(5)

$$\frac{dM_2}{dt} = \frac{2}{16\pi^2} \left[ \frac{3}{5} g_2^2 M_2 + \frac{2g_2^2}{(16\pi^2)^2} \left[ \frac{9}{5} g_1^2 (M_2 + M_1) + 25 g_2^2 (2M_2) + 24 g_3^2 (M_2 + M_3) \right. \right.$$  
$$+ \left. 6 f_t^2 (A_t - M_2) + 6 f_b^2 (A_b - M_2) + 2 f_{\tau}^2 (A_{\tau} - M_2) \right] ,$$  

(6)

$$\frac{dM_3}{dt} = \frac{2}{16\pi^2} \left[ -3 g_3^2 M_3 + \frac{2g_3^2}{(16\pi^2)^2} \left[ \frac{11}{5} g_1^2 (M_3 + M_1) + 9 g_2^2 (M_3 + M_2) + 14 g_3^2 (2M_3) \right. \right.$$  
$$+ \left. 4 f_t^2 (A_t - M_3) + 4 f_b^2 (A_b - M_3) \right] ,$$  

(7)

where $t \equiv \ln Q$ with $Q$ the renormalization scale; $g_1$, $g_2$, and $g_3$ are the $U(1)$, SU(2)$_L$ and SU(3)$_C$ gauge coupling constants, respectively (we use the SU(5) normalization for $g_1$); $f_t$, $f_b$ and $f_{\tau}$ are the $t$, $b$ and $\tau$ Yukawa couplings, respectively.$^3$ While generically the one-loop contributions to $dM_i/dt$ dominate, in the case where the soft trilinears are large, then the two loop contributions, which include the $A_i$ terms, can make a significant effect on the $M_i$ running $^{43}$.

In Fig. 2 we show the RG trajectories of the gaugino masses $M_i$ versus energy scale $Q$ starting from $Q = m_{\text{GUT}}$ down to the weak scale. We work in the 2-extra-parameter non-universal Higgs model (NUHM2) $^{44}$ where matter scalars have unified masses $m_0$ at $Q = m_{\text{GUT}}$ but where Higgs soft terms $m_{H_u}$ and $m_{H_d}$ are independent since they necessarily live in different GUT multiplets. For convenience, we trade the GUT scale parameters $m_{H_u}^2$ and $m_{H_d}^2$ for the weak-scale parameters $\mu$ and $m_A$ so that the model parameter space is given by

$$m_0, m_{1/2}, A, \tan \beta, \mu \text{ and } m_A \ (\text{NUHM2}),$$  

(8)

$^3$The third generation neutrino Yukawa coupling $f_\nu$ also contributes to the running of $M_1$ and $M_2$ above the right-handed neutrino mass scale since in simple SO(10) based grand unified theories (GUTs) we expect $f_\nu = f_i$ at the GUT scale. We however ignore its effects in the following analysis—if we assume the third generation neutrino mass to be $\sim 0.1$ eV, the corresponding right-handed neutrino mass should be $\sim 3 \times 10^{14}$ GeV. In this case, the inclusion of the neutrino Yukawa contribution changes $M_1$ and $M_2$ by $\lesssim 0.05\%$, which is totally negligible in the present discussion.
Figure 2: Evolution of gaugino masses in radiatively-driven natural SUSY for large positive and negative and small trilinear soft SUSY breaking terms.

We use the Isajet 7.85 code for sparticle mass spectrum generation in the NUHM2 model \[45\].

In Fig. 2, we adopt parameter choices $m_0 = 12$ TeV, $m_{1/2} = 0.8$ TeV, $\tan\beta = 10$, $\mu = 0.15$ TeV and $m_A = 2$ TeV. The solid lines show the $M_i$ $(i = 1, 2, 3)$ evolution for $A_0 = 0$. The dashed lines show the $M_i$ evolution for $A_0 = -1.8m_0$. In this case, the $A$-terms yield a negative contribution to the right-hand-sides of Eq’s \[5 \text{--} 7\] thus noticeably steepening the RG slope of the $M_3$ running and reducing the slopes of $M_1$ and $M_2$. The effect is especially noticeable for $M_2$ and $M_3$. Meanwhile, the dotted curves are plotted for $A_0 = +1.7m_0$. In this case, the two-loop contributions to the $M_i$ running are positive thus steepening the slopes for $M_1$ and $M_2$ while decreasing the slope for $M_3$. The extraction of $M_1$ and $M_2$ to high precision, provided that the gaugino masses are unified at the GUT scale, would be an excellent measure of large $A$-terms although we do not directly detect scalar quarks or scalar leptons.

In Fig. 3, we show the ratios of weak-scale gaugino masses versus $m_0$ which are generated for $m_{1/2} = 1$ TeV, $\tan\beta = 10$, $\mu = 0.15$ TeV and $m_A = 2$ TeV but where $A_0 = -1.6m_0$, $-1.8m_0$ and $-2m_0$ (solid curves colored blue, red and green respectively) and $A_0 = +1.6m_0$, $+1.8m_0$ and $+2m_0$ (dashed curves also colored blue, red and green). The black dotted regions correspond to where $124$ GeV < $m_h$ < $126$ GeV. The green curves end abruptly when the parameters conspire to give CCB minima in the scalar potential. Of direct relevance to ILC is the first frame, Fig. 3(a) which shows the ratio $M_2/M_1$ which can be extracted from precision measurements of EW-ino masses and mixings. The ratio varies by over $\sim 3\%$ on the range shown. Given sufficient accuracy in the ILC determination of $M_1$ and $M_2$, then it should be possible to determine the sign of $A_0$ and perhaps even gain information on the magnitude of $A_0$.

In the event that LHC also discovers gluinos via gluino pair production, then it is possible
the gluino mass can be extracted with order a few percent accuracy \cite{46,47} (depending on event rate, dominant gluino decay modes and backgrounds). If the value of $M_3$ can be extracted from the gluino pole mass, then also the ratios $M_3/M_1$ and $M_3/M_2$ should become relevant. These ratios are shown versus $m_0$ in frames Figs. 3(b) and 3(c). The ratio $M_3/M_2$ especially shows a significant disparity in values depending on whether the $A_0$ terms are positive or negative.

In Fig's 4 we show the weak-scale gaugino mass ratios versus variation in $A_0$ for $m_{1/2} = 1 \text{ TeV}$, $\tan \beta = 10$, $\mu = 150 \text{ GeV}$ and $m_A = 2 \text{ TeV}$ but where $m_0$ is scanned over the range $m_0: 0 - 15 \text{ TeV}$ and $-20 \text{ TeV} < A_0 < +20 \text{ TeV}$. The gray regions are disallowed by either CCB scalar potential minima or no EWSB. The green regions yield a light Higgs mass with $124 \text{ GeV} < m_h < 126 \text{ GeV}$ while the orange regions indicate $\Delta_{\text{EW}} < 50$. For Fig. 4(a), we see two intersecting regions: one for positive $A_0$ and one for negative $A_0$. The low fine-tuned regions which are consistent with the measured value of $m_h$ occur at large $|A_0|$ values indicating large mixing in the stop sector. The ratio $M_2/M_1 \gtrsim 1.81$ for the case of $A_0$ large.

\footnote{See Ref’s \cite{48,49} for the two-loop order calculations and Ref. \cite{50} for the leading three-loop contributions.}
negative while $M_2/M_1 \lesssim 1.81$ for $A_0$ large positive. We also notice that these two regions can be sufficiently separated from those which predict the correct Higgs mass but a large value of $\Delta_{EW}$. In particular, $A_0 \simeq 0$ points with $m_h \simeq 125$ GeV predict $M_2/M_1 \lesssim 1.78$, which can be distinguished from the above two regions if $M_2/M_1$ is measured with $\sim 1\%$ accuracy. Thus, assuming unified gaugino masses, it appears precision measurements of EW-ino masses and mixings will be sensitive to details of large trilinear soft terms.

In Fig. 4(b) we show the ratio $M_3/M_1$ for the case where LHC14 can discover the gluino and gain an estimate of $M_3$. Here, we see the intersecting orange/green regions occur for $M_3/M_1 \gtrsim 4.84$ for $A_0$ large positive while $M_3/M_1 \sim 4.75$–4.88 for $A_0$ large negative. While some range of $M_3/M_1$ overlaps between these two cases, only the large negative $A_0$ accesses the smaller range $M_3/M_1 \lesssim 4.85$. In Fig. 4(c) we plot the ratio $M_3/M_2$ versus $A_0$. Here we see that large positive values of $A_0$ yield $M_3/M_2 \sim 2.68$–2.7 while large negative $A_0$ yields instead $M_3/M_2 \sim 2.62$–2.65. The two cases appear distinguishable to combined ILC and LHC precision measurements.
Before concluding this section, we discuss some possible uncertainties in our calculations. In the calculation shown above, we assume the gaugino mass unification at the GUT scale. However, this can be spoiled by GUT-scale threshold corrections and by Planck-scale suppressed higher dimensional operators \cite{51,52}. These effects are expected to be $\lesssim 1\%$; however, they can be significant if the $A$- or $B$-terms for the GUT Higgs fields are much larger than gaugino masses, and/or if there are large representations of the GUT gauge group.

In the event that gaugino masses are not unified, then extrapolation of the measured values of $M_1$ and $M_2$ to high scales will intersect at some point other than $Q = m_{\text{GUT}}$ where the gauge couplings unify. This situation is illustrated in Fig. 5 where in frame (a) we show the running of gauge couplings while in frame (b) we show the running of gaugino masses from the compactified $M$-theory model of Ref. \cite{54}. If such a theory were discovered, then the precision measurements of $M_1$ and $M_2$ at LHC and/or ILC would be extrapolated in energy to meet at a unification point other than the energy scale where the gauge couplings unify. If $M_3$ is also measured, then the non-unification of all three gaugino masses would be apparent.

Precision gaugino mass measurements also play an important role in testing the split-SUSY type mass spectrum \cite{55}, where soft masses are taken to be $\mathcal{O}(100–1000)$ TeV and gaugino masses and $A$-terms are suppressed by loop factors, with which the 125 GeV Higgs mass can be obtained \cite{56}. In this case, gaugino masses are induced via anomaly mediation and proportional to the corresponding gauge coupling beta functions \cite{57}, which results in different gaugino mass

We however note that if top squarks are discovered in future collider experiments, we may infer $m_0$ and $A_0$ from the measurements of their masses, and in this case precision gaugino mass measurements in turn allow us to extract the soft parameters for the GUT Higgs fields via the GUT threshold corrections, just like the precise determination of the gauge couplings enables us to extract the mass spectrum of the GUT-scale fields via the GUT threshold corrections to the gauge couplings \cite{53}. In this sense, precision gaugino mass measurements are of importance even if the $A$-terms are directly measured.
ratios from those in the unified gaugino mass case. Deviations from the anomaly-mediation relation are caused by renormalization group effects below the soft mass scale and threshold corrections by higgsino–Higgs one-loop diagrams, which enable us to extract information on the SUSY scale and higgsino/heavy Higgs mass spectrum through gaugino mass measurements.

4 Conclusions

In the post LHC8 world with an improved understanding of electroweak naturalness in SUSY models, then we are directed to expect the existence of rather light higgsino-like \( \tilde{W}^\pm_1 \) and \( \tilde{Z}_{1,2} \) with mass \( \sim 100 - 250 \text{ GeV} \), the closer to \( m_h \) the better. Such light EW-inos are difficult to see at LHC (due to only soft tracks emerging from their decays) but which should be easily visible in the clean environment of an \( e^+e^- \) collider such as ILC with \( \sqrt{s} \geq 2m(\text{higgsino}) \). While it is expected that \( m_{\tilde{W}_1,\tilde{Z}_{1,2}} \sim |\mu| \), the mass splittings \( m_{\tilde{W}_1} - m_{\tilde{Z}_1} \) and \( m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \) are sensitive to the gaugino masses \( M_1 \) and \( M_2 \) via mixing effects.

From the rather high value of \( m_h \), we also expect large trilinear soft terms \( A_t \) which contribute to large mixing in the stop sector and an uplifting of \( m_h \). One way to test the presence of large trilinear soft terms is to look for their influence on the gaugino mass running which occurs at two-loop level. We point out that the sign of \( A_0 \) and its magnitude influence the expected ratios of gaugino masses which may be measured at ILC. Thus, this work motivates a dedicated program of precision measurements of EW-ino properties at a machine such as ILC which may test Lagrangian parameters well-removed from just those that directly determine the EW-ino masses.

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