THE IMPACT OF SMALL-CELL BANDWIDTH REQUIREMENTS ON STRATEGIC OPERATORS

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5G Trends

- Heterogeneous networks
  - Cells (Macro/Small)

- Heterogeneous services
  - Mobility, Quality of Experience

How does policy influence the strategic behavior of the service providers?
- Pricing
- Resource allocation (macro vs. micro)
5G Trends

- Heterogeneous networks
  - Cells (Macro/Small)
- Heterogeneous services
  - Mobility, Quality of Experience

How does policy influence the strategic behavior of the service providers?
- Licensed vs. unlicensed
- Regulatory constraints (sharing rules)
Spectrum Sharing

- 100 MHz
- Shared with naval radar
- Three-tier sharing rules
  - Incumbents
  - Priority Access Licenses
  - General Access
- Low power
  - small cells

3.5 GHz
THE INNOVATION BAND

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Spectrum Sharing

- 100 MHz
- Shared with naval radar
- Three-tier sharing rules
- Low power ➔ small cells

How will the low power / small-cell requirement affect prices, bandwidth allocation, and social welfare?

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Assumptions

- SPs manage two networks:
  - Macro-cell / Small-cell

- Two types of users: mobile / fixed
  - Mobile users **must** connect to macro-cell network
  - Fixed users can connect to macro- or small-cell network

- Utility is a function of the rate received
  - Shared spectrum
    - bandwidth (rate) is split evenly among users
Assumptions

- SPs manage two networks:
  - Macro-cell / Small-cell
- Two types of users: mobile / fixed
  - Mobile users must connect to macro-cell network
  - Fixed users can connect to macro- or small-cell network
- Utility is a function of the rate received
- Each SP must provide a minimum amount of bandwidth for small cells.
Related Work

- Chen et al:
  - Workshop on Smart Data Pricing, 2015
    Model for competing service providers
  - Infocom, 2016
    Licensed and unlicensed spectrum

- Differences from other related work:
  - Two classes of users (mobile/fixed)
  - Providers set prices and optimize bandwidth
  - Constraint on minimum small-cell bandwidth
Model

Supply

Mobile user

Demand

Fixed user

Users select service, rate, pay service fee.

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Service Competition

Prices (per unit rate)

Bandwidth Allocation

Macrocell

Small-cell

\[ B_{iM} \]

\[ p_{iM} \]

\[ B_{iS} \]

\[ p_{iS} \]
Model

Bandwidth Allocation

Model

Supply

Demand

Users select service, rate, pay service fee.

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How do the small cell constraints affect bandwidth and prices?
An equilibrium always exists and is unique.

Adding the constraints can only decrease social welfare (α-fair utilities).
Adding Small-Cell Bandwidth

- SPs have exclusive-use bands $B_1$ and $B_2$, which can be split between macro and small cells.
- Add bandwidth $B$ designated for small cells.
Social Welfare: Large $B$

$B_1^0 = 1$, $B_2^0 = 1.2$, $B = 10$

Maximum SW without constraint

Maximum SW with constraint

SW for equilibrium with constraint
Social Welfare: Smaller $B$

$B_1^0 = 1, B_2^0 = 1.2, B = 6$

- Maximum SW without constraint
- $= \text{Maximum SW with constraint}$
- SW for equilibrium with constraint
Main Results (2)

- An equilibrium always exists and is unique.
- Possible effect of adding constraint on equilibrium:

Diagram:

Small-cell BW

No constraint

With constraint

$B_1,S$

$B_2,S$

$B_1,S$

$B_2,S$
Main Results (2)

- An equilibrium always exists and is unique.
- Possible effect of adding constraint on equilibrium:

```
| Small-cell BW | No constraint | With constraint |
|---------------|--------------|-----------------|
|               | $B_{1,S}$    | $B_{1,S}$       |
|               | $B_{2,S}$    | $B_{2,S}$       |
```
Effect of Constraint on Equilibrium

- Required bandwidth for SP 1 small cells ($B_{1,S}^0$, $B_{2,S} = B_{2,S}^0$).
- No change in equilibrium.
- SP 2 violates constraint: $B_{1,S} > B_{1,S}^0$, $B_{2,S} = B_{2,S}^0$.
- SPs 1 and 2 violate constraint: $B_{1,S} = B_{1,S}^0$, $B_{2,S} = B_{2,S}^0$.
- SP 1 violates constraint: $B_{1,S} = B_{1,S}^0$, $B_{2,S} > B_{2,S}^0$.
- $B_1 = 2$, $B_2 = 1$.
Utility

- Utility for each user is a function of the rate $r$.
- Total rate (capacity) depends on spectral efficiency $R_0$.

- Macro-cell capacity for SP $i$: $C_{i,M} = B_{i,M}R_0$
- Small-cell capacity for SP $i$: $C_{i,S} = \lambda_s B_{i,S}R_0$

$\lambda_s > 1$ accounts for higher density and/or spectral efficiency of small-cell network.
Utility

- Utility for each user is a function of the rate $r$.
- Total rate (capacity) depends on spectral efficiency $R_0$.
  - Macro-cell capacity for SP $i$: $C_{i,M} = B_{i,M}R_0$
  - Small-cell capacity for SP $i$: $C_{i,S} = \lambda S B_{i,S}R_0$
- Will assume the class of $\alpha$-fair utility functions:
  $$u(r) = \frac{r^{1-\alpha}}{1-\alpha}$$
  - $\alpha \to 0$, $u(r)$ becomes linear
  - $\alpha \to 1$, $u(r)$ becomes logarithmic
Sequential (Two-Stage) Game

1. SPs set bandwidths \( B_{i,M} \) \( B_{i,S} \)
2. SPs set prices \( p_{i,M} \) \( p_{i,S} \)

Fixed users choose network to maximize surplus (utility minus cost): \( S(r) = u(r) - p \cdot r \)

rate \( r^* = \arg \max S(r) = D(p) \) (demand function)

We will characterize sub-game perfect Nash equilibria:

1. Price equilibrium / user association given bandwidth allocation.
2. Bandwidth allocation given that prices are set according to 1.
Revenue Maximization

\[
\max S_i = K_{i,M} p_{i,M} D(p_{i,M}) + K_{i,S} p_{i,S} D(p_{i,S})
\]

subject to

\[
K_{i,M} D(p_{i,M}) \leq C_{i,M}
\]

\[
K_{i,S} D(p_{i,S}) \leq C_{i,S}
\]

\[
B_{i,M} + B_{i,S} \leq B_i
\]

\[
0 \leq p_{i,M}, p_{i,S} < \infty
\]

\[
B_{i,M} \geq 0, \quad B_{i,S} \geq B_i^0
\]

fraction of users in macro-/small-cell networks

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Social Welfare (Utility) Objective

With $\alpha$-fair utility functions the equilibrium maximizes $SW$ without small-cell bandwidth constraints.

$$SW = \sum_{i=1}^{N} K_{i,M} u(r_{i,M}) + K_{i,S} u(r_{i,S})$$
Social Welfare Loss

- **SW loss occurs when**

\[
\frac{N_f \lambda_S^{1/\alpha-1}}{N_f \lambda_S^{1/\alpha-1} + N_m} \sum_{i \in \mathcal{N}} B_i < \sum_{i \in \mathcal{N}} B_{i,S}^0
\]

- **The loss satisfies:**

\[
\frac{SW_{w}^{NE}}{SW_{w_o}^{*}} \geq \left( \frac{N_f \lambda_S^{1/\alpha-1}}{N_m + N_f \lambda_S^{1/\alpha-1}} \right)^\alpha
\]

- **Equality holds when** \( B_{i,S}^0 = B_i \) **for every SP** \( i \).
Given new bandwidth $B$, there exists a threshold $T$ such that if $B > T$, constraining $B$ for small cells reduces SW.

$$T = \frac{(B_1^0 + B_2^0) N_f \lambda_{S}^{1/\alpha - 1}}{N_m},$$

If $B < T$, $B$ can be split between SPs 1 and 2 so that the competitive equilibrium achieves the maximum SW.
Social Welfare: Smaller $B$

$B_1^0 = 1, B_2^0 = 1.2, B = 6$

Maximum SW without constraint

= Maximum SW with constraint

SW for equilibrium with constraint

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Conclusions

- Adding constraints on small-cell bandwidth can change competitive equilibrium and lead to a loss in SW.
- The constraint may cause an SP to reduce its small-cell bandwidth, although the total allocation cannot decrease.
- Constraining new bandwidth $B$ leads to inefficient allocations when $B$ exceeds a threshold.