Numerical analysis of meshing heating of involute spur gear

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Abstract. Involute spur gears generate heat due to tooth surface meshing friction. Excessive temperature rising affects transmission accuracy and reduces work reliability. By establishing the normalized coordinates of the meshing curve and based on the frictional heat generation theory, the mathematical analysis model of the meshing surface heating is studied, the factors affecting the average heat flux density of meshing are explored, and the distribution law of these factors along the normalized coordinate of the meshing is analysed. The analysis shows that the tangential velocity of the meshing point and the half-bandwidth of the time domain contact have the greatest influence on the average heat flow density; the average heat flow density distribution of the driving wheel and the driven wheel are similar. The heat flow density of the driving wheel is greater than that of the driven wheel. Tooth shape modification minimizes tooth surface meshing contact stress, reduces meshing heat generation, controls temperature rise and improves transmission reliability.

1. Introduction
The volume and size are restricted in the high-speed and heavy-duty transmission system, due to its working conditions. The involute spur gear generates a lot of frictional heat during the meshing transmission process. The heat convection with internal lubricating oil. Excessive temperature rise of the gear will cause the thermal-elastic coupling deformation of the gear, the load distribution between the teeth generated by the alternate meshing with the gear, the impact of meshing, and the manufacturing error of the gear teeth. The meshing characteristics of the gear transmission system bring about serious consequences of transmission system vibration, the noise was increased, and even the transmission failure. Therefore, it is important to control the temperature rise during gear meshing.

This article will establish a mathematical analysis model of the planetary transmission gear body temperature based on the theory of frictional heat generation. Taking the mesh of a pair of involute spur gears as an example to explore the factors that affect the average heat flux density of meshing, and analyse these factors to be unified along the meshing According to the distribution law of chemical coordinates, the main factors of gear tooth meshing heat are determined, and the target solutions are proposed.

2. Normalization of gear meshing coordinates
According to the transmission characteristics of the involute spur gear meshing studied in this paper, the transmission process can be decomposed into: $N_1, N_2$ is the theoretical meshing line, $A$ is the meshing
point, $B$ is the single-tooth area meshing-in point, and $C$ is the single-tooth area meshing-out point and $D$ is the biting out point. Since the meshing of single and double teeth are produced alternately, the load on each gear fluctuates [1]. The contact pressure of the gear teeth in the double-tooth meshing area is related to the load distribution coefficient between the teeth and the comprehensive radius of curvature of the contact points of the driving and driven gears. Therefore, the normal load borne by the gear teeth changes with the operation of the gear. It produces a non-uniform contact pressure distribution on the meshing tooth surface of the gear teeth [2].

Figure 1. Schematic diagram of gear meshing.

In order to facilitate the accurate study of the meshing transmission gear, any points above $N_1, N_2$ are selected as the research object, the meshing line is taken as the coordinate axis. A dimensionless function $\Gamma$ is introduced to describe the meshing line coordinates, and the normalized coordinate of this point named $K$ is specified, which is: the distance from any point to the node is greater than the distance from the upper node to the starting point of the relative coordinate, and the edge of any point is the positive direction, otherwise it is the negative direction. According to the geometric relationship shown in the figure, the coordinate formula of any point $K$ on the meshing line axis can be derived:

$$\Gamma_K = \frac{PK}{N_1P} = \frac{\tan \alpha_{K1}}{\tan \alpha'} - 1$$

(1)

Where: $\alpha'$ is the pressure angle of the gear pitch circle, that is, the actual meshing angle. $\alpha_{K1}$ is the pressure angle of the corresponding driving wheel at the meshing point, which is calculated by the formula:

$$\alpha_{K1} = \arccos \left( \frac{r_{K1}}{r_{K1}} \right)$$

(2)

Where: $r_{K1}$, $r_{K1}$ are respectively the radius of the driving wheel corresponding to $N_1$, $K$ point.

From the above formula, the normalized coordinates of each key point on the theoretical meshing line $N_1, N_2$ can be calculated separately.

Theoretical initial meshing point, $\Gamma_{N_1} = -1$. Theoretical meshing end point, $\Gamma_{N_2} = \pm i$. The intersection point of the pitch circle of the driving wheel $O_1$ and the driven wheel $O_2$, $\Gamma_p = 0$. Other points are calculated as follows:
3. Analysis of gear meshing heat transfer

Thermal analysis is based on the heat balance equation of the principle of conservation of energy, which calculates the temperature distribution of each node of an object and its thermophysical parameters under given thermal boundary conditions [2].

During the gear transmission meshing process, due to the temperature difference between the gear surface, air, lubricating oil and the transmission box, heat exchange will be caused mainly through conduction and convection for heat dissipation. In the thermal equilibrium state, the working conditions of each gear tooth of the transmission gear want to go through both the flow generated by friction and heat conduction and heat dissipation. According to the conservation of energy and Fourier's law, the three-dimensional temperature of the single tooth in the Cartesian coordinate system can be derived. The thermal balance differential equation of the field, as shown in the following formula:

\[
\rho c \frac{\partial \theta}{\partial t} = \lambda \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + q_v \tag{4}
\]

Where: \( \theta \) is temperature, \( t \) is time, \( \rho \) is density, \( c \) is specific heat capacity, \( q_v \) is the intensity of the external heat source. For the body temperature field of the meshing gear, the intensity of the external heat source, \( q_v = 0 \).
The body temperature field under thermal steady-state equilibrium, its temperature does not change with time, and is a constant function of time, \( \frac{\partial \theta}{\partial t} = 0 \). At this time, the thermal balance differential equation of the three-dimensional state temperature field of a single tooth can be simplified as:

\[
\lambda \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) = 0
\]

(5)

The thermal boundary conditions of the body temperature field calculation include: the heat flux density of the tooth surface and the convective heat transfer of the gear. Taking a single tooth of a meshing transmission gear for analysis, each surface has different types of boundary conditions due to different meshing states. The meshing gear pair studied in this paper is well lubricated, and L-CKD320 industrial closed gear oil is selected. Its basic parameters are shown in Table 2.

| Lubricant model | Density \( \rho_0 \) (kg/m\(^3\)) | Kinematic viscosity \( \nu_0 \) (cst) | Specific heat capacity \( c_0 \) (J/(kg·K)) | Thermal conductivity \( \lambda_0 \) (W/(m·K)) |
|----------------|-------------------------------|---------------------------------|---------------------------------|---------------------------------|
| L-CKD320       | 870                           | 320(40℃)                       | 2000                            | 0.14                            |
|                | 38.5(100℃)                   |                                 |                                 |                                 |

4. Numerical analysis of gear meshing heating
A pair of meshing gears generate heat by friction at the meshing point, and the frictional heat flow is the heat generated per unit time per unit area, that is, the average heat flow density. The calculation formula of average heat flux is [3]:

\[
q_f = \frac{\pi \psi \gamma \sigma_{\text{fr max}} \theta_n V_c}{2 T_o}
\]

(6)

There are many design parameters for the calculation of average heat flux, and the calculation is complicated, and the parameters need to be discussed and analysed.

4.1. Gear meshing period \( T_o \)
The calculated average heat flux density is the average value of heat generated by frictional meshing in one cycle, so the driving wheel rotates for one circle \( T_o = 60/n_1 \).

4.2. Heat flux distribution coefficient between teeth \( \psi \)
It is because the dimensions of the main and driven wheels are inconsistent, and the tangential speed at \( K \) point is inconsistent, the heat flux generated by meshing friction is not evenly distributed. The distribution coefficient \( \psi \) is related to the speed \( V_t \) of the tangent direction of the meshing point, the radius of curvature \( \rho \), the thermal conductivity of the material \( \lambda \), and the specific heat capacity \( c \).

The tangential velocity \( V_n \) at the meshing point and the heat flux distribution coefficient \( \psi \) between teeth are calculated as follows:
\[
\begin{align*}
V_{i1} &= \rho_1 \omega_1 = \frac{\pi n_1}{30} (1 + \Gamma) a' \sin \alpha' \\
V_{i2} &= \rho_2 \omega_2 = \frac{\pi n_2}{30} (i + \Gamma) a' \sin \alpha' \\
\psi_1 &= \frac{\sqrt{\lambda_1 \rho_1 c V_{i1}}}{\sqrt{\lambda_1 \rho_1 c V_{i1}} + \sqrt{\lambda_2 \rho_2 c V_{i2}}} \\
\psi_2 &= \frac{\sqrt{\lambda_2 \rho_2 c V_{i2}}}{\sqrt{\lambda_1 \rho_1 c V_{i1}} + \sqrt{\lambda_2 \rho_2 c V_{i2}}}
\end{align*}
\]

(7)

4.3. Maximum Hertz contact stress \( \sigma_{H_{\text{max}}} \)

According to the Hertz contact formula: \( \sigma_{H_{\text{max}}} = \frac{4}{\pi} \sigma_{E} = \frac{4}{\pi} \sqrt{\frac{F_{Z}}{2\pi L}} \cdot \frac{E}{\rho_{E}}, \) the solar wheels and the planetary wheels are the same material, so the elastic modulus is the same, which is \( E. \) The number of teeth is different, so the comprehensive equivalent radius of curvature at the meshing point is different. The calculation is as follows:

\[
\begin{align*}
\rho_1 &= \frac{(1 + \Gamma)}{(i \pm 1)} \cdot a' \sin \alpha' \\
\rho_2 &= \frac{(i + \Gamma)}{(i \pm 1)} \cdot a' \sin \alpha' \\
\rho_E &= \frac{\rho_1 \rho_2}{\rho_2 \pm \rho_1}
\end{align*}
\]

(8)

4.4. Thermal energy conversion coefficient \( \gamma \)

Generally, \( \gamma \) is in the range of 0.9–0.95[4]. The involute spur gear studied in this paper is used in high-power thin seam shearsers. The underground working environment is harsh and the heat dissipation conditions are poor. Therefore, the thermal energy conversion coefficient is taken in the calculation. The maximum value is 0.95.

4.5. Time domain contact half bandwidth \( t_0 \)

\[
t_0 = a/V_{i1} = \frac{8F_{Z}}{\pi L} \cdot \frac{\rho_E}{E} / V_{i1}
\]

(9)

4.6. Coefficient of friction \( f \)

The determination of tooth surface friction coefficient has always been relatively complicated. The real-time friction coefficient during the meshing process should change with the change of the meshing point. So far, there is no accurate theory that can accurately calculate the real-time friction coefficient under complex conditions. Considering the influence of load on the friction coefficient, the American gear standard AGMA217-01 design code is selected to solve the friction coefficient under load. The empirical formula of AGMA in literature [5] is:

\[
f = 0.12 \left( \frac{w R_a}{\eta V_c \rho_E} \right)^{0.25}
\]

(10)
Where: \( w_t \) is unit tangential load, the calculation formula is:
\[
w_t = \frac{F_x}{L} \cos \alpha'.
\]
\( \bar{R}_a \) is the average roughness of the tooth surface of the meshing gear is 1.6. \( \eta \) is the dynamic viscosity coefficient of lubricating oil, its calculation formula is:
\[
\eta = \rho e^{\frac{21.54 - 3.54\ln(2.718286 - 0.6 - 6 \times 10^{-6})}{c}}.
\]
\( \bar{\rho} \) is the density of the lubricating oil, the specific value is shown in Table 2, \( \bar{\theta}_c \) is the average temperature of the gear, and the empirical value 70℃ is selected. \( V_c \) is the sum of the tangential velocity at the meshing point is calculated as follows:
\[
V_c = |V_{1} - V_{2}| = \frac{\pi \eta}{30} \cdot \alpha' \sin \alpha' \cdot \left| \frac{1}{i} \cdot |\Gamma| \right|.
\]  
(11)

The calculated value is in the range of 0.02–0.05. In engineering practice, the average friction coefficient of the tooth surface measured by experiments is generally 0.04–0.06. In general calculations, the constant value of the friction coefficient is taken for the tooth surface with fluid friction and critical friction at the same time [6].

According to the above calculation formula, it is concluded that the main factors affecting the average heat flux density of a pair of meshing gear pairs are as follows. In order to comprehensively study the distribution of the average heat flux along the normalized coordinates of the meshing curve, the distribution curves of the above-mentioned influencing factors were drawn respectively in Figure 2. For the meshing spur gear pairs in Table 1 of this paper, the plot of average heat flux distribution of tooth surface friction along the normalized coordinates as shown in Figure 3.

![Figure 2](image)

(a) The sum of the tangential velocity  
(b) Maximum Hertz contact stress  
(c) Coefficient of friction  
(d) Time domain contact half bandwidth  
(e) Average heat flux density of driving wheel \( O_i \)  
(f) Average heat flux density of driven wheel \( O_2 \)

Figure 2. The distribution of the average heat flux density of the wheels along normalized coordinate
5. Conclusion

In this paper, an involute straight-tooth cylindrical wheel set is taken as an example to carry out a numerical analysis of tooth surface meshing heating. The calculation parameters that affect the average heat flux between teeth are calculated in detail, and the distribution of the parameters along the normalized meshing coordinates is drawn. Through the analysis of the average heat flux density and influencing factors of meshing transmission gears, it can be known that:

1. From an overall point of view, the rate of change of the distribution of various factors along the meshing line is different. The two that have the greatest influence on the average heat flux density between teeth are: the maximum contact stress on the tooth surface $\sigma_{H_{\text{max}}}$ and the tangential velocity $V_C$ of the meshing point.

2. The average heat flux density is distributed along the normalized coordinates in a V shape. The driving wheel obtains the maximum value at the starting point $\Gamma_{A}$ of the double-tooth meshing area at the engagement end, and the driven wheel obtains the maximum value at the engagement end point $\Gamma_{D}$. Both the master and the driven wheels obtain the minimum value of 0 at the node $P$. Because the gears are purely rolling at the pitch circle, the sum of the tangential speeds at the meshing point $V_C = 0$.

3. The average heat flux density generated by tooth surface friction during the entire meshing process of the driving wheel $\Omega_1$ is greater than the value of the driven wheel $\Omega_2$. It is because that the number of teeth of the driving gear $z_1 = 15$, and the number of the driven gears $z_2 = 27$. There is more meshing number while the driving wheel $\Omega_1$ turns more in the same meshing cycle, and the average heat flux density of tooth surface friction is greater.

In summary, there are many factors that affect the temperature field of gears, such as the geometric parameters of the gears, speed, load, lubrication conditions, and heat dissipation devices. Under the condition that the geometrical parameters of the gear pair are unchanged, the heat dissipation are fixed and the working conditions are determined, the gear modification design method can be used to reduce the contact stress of the tooth surface, effectively control the temperature rise of the meshing tooth surface, and ensure the reliability of the gear transmission.

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