Abstract

The objective of this study is to propose prediction equations for performance parameters of Glass Fibre Reinforced polymer (GFRP) strengthened High Strength Concrete (HSC) beams. To examine the effectiveness of GFRP on high strength concrete beams, a total of fourteen beams of size 150 x 250 x 3000mm were cast and tested. All the test specimens were subjected to static and cyclic loading up to failure, in a loading frame of capacity 250kN. Necessary measurements were taken for each load increment. The proposed equation shows higher accuracy for predicting the performance parameters of high strength concrete beams strengthened with GFRP laminates.

Keywords: Cyclic, GFRP, Laminate, Regression, Static, Strengthening

Introduction

A worldwide use of high strength concrete during the last decade and an expansion in the material technology has made it possible to design concrete having superior mechanical properties, material properties and structural behavior. Whilst use of high strength concrete has accelerated, the progress in development of revised design standards have not progressed at the same rate. Acceleration is due to its enhanced mechanical properties and better structural performance as compared to Normal Strength Concrete. Also High Strength Concrete offers economy and superior performance in terms of strength and long-term behavior.

The application of Fibre Reinforced Polymer (FRP) for strengthening and rehabilitation of reinforced concrete structures has become a promising technique. FRP may be applied as externally bonded reinforcement as well as internal reinforcing rods. The specific advantage of FRP is the high strength to weight ratio and easy installation of the system. Strengthening RC elements using fibre reinforced polymers (FRP) has proved as a potential technique to tackle strength lack troubles.

FRP can be used for a wide range of strengthening works, like strengthening beams in shear and flexure, strengthening of slabs using FRP strips, strengthening of columns using FRP tubes and adhesive bonded FRP wraps, strengthening of masonry walls against lateral forces. Strengthening of beams in flexure is typically carried out by the application of FRP strips bonded to the soffit of the beam or by U-shaped wrapping which covers two sides and soffit portion of the beam. Improvement in shear strength of beams can be achieved by the application of U-shaped strips bonded at regular intervals.

The present research work has been undertaken for evaluating the effectiveness of glass fibre reinforced polymer (GFRP) laminated high strength concrete (HSC) beams. Under static and cyclic loading Emphasis has been given to the strength and deformation properties of GFRP strengthened HSC beams. Non-linear finite element analysis has been carried out to understand static response of the strengthened beams. Comparison has been made between the un-strengthened and strengthened beams and appropriate conclusions are drawn based on the results of investigation carried out.

Review of Literature

1Proposed energy dissipation index to enumerate the RC beams subjected to cyclic loading. The authors considered
strength of concrete, reinforcement and cyclic load data to evaluate the energy dissipation index and gave recommendation for designing the RC beams under cyclic loading. \textsuperscript{3} Proposed regression equations to predict the performance parameters of GFRP strengthened reinforced concrete columns. The corrosion level, GFRP type and GFRP thickness were the independent variables considered in this study. The authors reported that the proposed regression equation enabled the predictions to be made with reasonable accuracy. \textsuperscript{4} Proposed regression equation to evaluate the ultimate shear strength of reinforced concrete beams with micro fibre reinforcements without stirrups. The proposed equation predict results with close agreement with those of eighty nine beams failed in shear mode. The authors also identify the factors which influences the shear strength of fiber reinforced concrete beams. The authors concluded that shear span to - depth ratio, percentage steel ratio, concrete strength, fiber volume, size and fibre type were the identified factors which influences the shear strength of fiber reinforced concrete beams. The proposed equation for predicting cracking and ultimate shear strength agree well with those experimental results. \textsuperscript{5} Evaluated the flexural strengthening of high strength concrete (HSC) beams by external bonding of FRP sheets. Six concrete beam specimens with dimensions of $150 \times 250 \times 3000$ mm were cast and tested under two point loading. The principal variables included in their study were dissimilar layouts of CFRP sheets and percentage steel reinforcement ratio. The authors concluded that increase in percentage steel reinforcement ratio resulted in reduction of strength due to externally bonded CFRP. The proposed finite element model showed good agreement with those of experimental results. \textsuperscript{6} Conducted an experimental investigation to evaluate the effectiveness of CFRP laminates in shear and in flexure on structurally damaged full-scale RC beams. The percentage steel ratio, retrofitting position and CFRP length were the principal variables considered in this study. The authors concluded that beams strengthened with CFRP laminate showed increase in stiffness when compared to that of control beam. RC beams strengthened with CFRP laminate enhance the ultimate strength substantially\textsuperscript{a} proposed an regression equation to predict the flexural strength of SFRC beams under partially and fully prestressed condition. The predicted equation showed good correlation with those of experimental results in finding the flexural strength of beams. \textsuperscript{7} Proposed regression equation to predict the shear strength of rectangular reinforced concrete beams subjected to concentrated load. According to different behaviour of strength the beam test data were separated and regression equations are predicted for each category.

Regression Analysis

Regression analysis is a statistical tool for the examination of relations between independent and dependent variables. Regression analysis having single explanatory variable is said to be simple regression. Likewise multiple regression will have more than one independent variable. It is much important to quantify the impact of several simultaneous influences ahead a single dependent variable. Regression analysis is a procedure for relating known input variables and output parameter by means of statistical philosophy. The general regression technique is to assume a form of relationship for the input parameters and the results with unknown coefficients. The unknown coefficients are found out using the data available from experiments or other sources by means of Legendre's standard of least squared errors.

Legendre's standard of least squared errors, is a general purpose curve fitting technique which helps to choose the values of unknown coefficients, also called the regression coefficient. Some of the terms related to regression analysis are defined in the following subsections.

Terminology used in Regression Analysis

The mathematical technique used for fitting curves, whether linear or non-linear of the predetermined shape. The purpose of regression is to evaluate the unknown coefficients in an equation. The form of the equation is assumed a priori in such a way that it might best suite the anticipated relationship between the input and the output.

Regression Co-efficient

The unknown factor introduced into a regression equation. This unknown factor suitably changes the input variables and makes the regression predictions nearer to the experiential results.

Sum of Squared Errors

The squared of error value after adding to signify the total arithmetical error level coupled with the predictions made by the regression equation.
Mean Squared Error

Mean Squared Error (MSE) = \frac{\text{Sum of squared errors}}{\text{Number of samples}}

MSE is higher to Sum of Squared Errors because MSE indicates the squared error per sample while SSE simply gives the sum, irrespective of the number of samples concerned.

Root Mean Squared Error

Root Mean Squared Error (RMSE) = \sqrt{\text{Mean Squared Error}}

RMSE represents the numerical value by which the prediction may diverge from the observed value. The deviation may be positive or negative, hence making the predictions bit lower or higher than the observed values.

Fitness of Function

The fitness of a function is calculated as the ratio between variance for predicted values and observed values. The fitness value will lie in between 0 to 1.

Multivariate Linear Type Regression

Multivariate linear type regression helps to construct first order equations involving more than one independent data. The basic formulation for multivariate linear type regression is,

\[
\begin{align*}
\frac{\partial}{\partial a_0} \sum_{i=1}^{K} \left( P_i - (a_0 + a_1 x_{i1} + a_2 x_{i2} x_{i3} x_{i4} + \cdots + a_n x_{in}) \right) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\end{align*}
\]

where, \(a_0, \ldots, a_n\) are unknown coefficients to be resolved, \(x_{i1}, x_{i2}, x_{i3}, x_{i4}, \ldots, x_{in}\) denotes independent variables, \(P\) denotes dependent variable for the set of \(i\)th input data and \(K\) is the number data sets available for regression. By performing the partial derivative operators, the above basic equation reduces to,

By adding up both the independent and dependent variables value with necessary operations the above equation can be solved. The logic offered in above equation was developed by OriginPro software to derive the unknown coefficients.

Modeling of Regression Equations

The research study is intended to evaluate the effectiveness of Glass Fibre Reinforced Polymer (GFRP) laminates on High Strength Concrete (HSC) beams subjected to static and cyclic loading. A regression based models have also been developed for estimating the performance parameters of GFRP strengthened high strength concrete beams. The required data to carry out regression modeling is presented in Table 1.

Results of Regression Equation

In this study, multiple linear regression analysis was carried out to determine the relations between the dependent variable of various performance parameters of GFRP strengthened HSC beams and the independent variables, i.e., thickness of GFRP wrap and tensile strength of GFRP laminate. The results of regression equation for various parameters associated with GFRP strengthened high strength concrete (HSC) beams are given in Table 2. The proposed regression equations enabled the predictions to be made with reasonable accuracy. The predicted results obtained through regression modeling were compared with those of the experimental values and are presented in Table 3.

Summary on Regression Equations

Regression equation can estimate the prediction results with realistic accuracy level for yield load, deflection at yield load, ultimate load, deflection at ultimate load, crack width, number of cracks, average spacing of cracks, deflection ductility, deflection ductility ratio, energy ductility and energy ductility ratio.
**Table 1.** Data Considered for the Regression Analysis

| Beam Designation | Thickness of Fibre Tk | Tensile strength of Fibre ffu | First Crack Load in kN | Deflection at First Crack Load in mm | Yield Load in kN | Ultimate Load Deflection in mm | Crack Width in mm | Number of Cracks | Average spacing of cracks in mm | DD | DDR | ED | EDR |
|------------------|-----------------------|------------------------------|------------------------|-------------------------------------|----------------|--------------------------------|------------------|----------------|--------------------------------|----|-----|----|-----|
| CBHSC-S          | 0                     | 0                            | 9.5                    | 0.91                                | 35             | 2.34                           | 7.03             | 1.21           | 12                             | 126 | 2.9 | 1  | 2.26|
| C3HSC-S          | 3                     | 126.2                        | 12.5                   | 1.01                                | 40             | 3.09                           | 8.95             | 0.97           | 17                             | 93  | 3.01| 1.038 | 2.67|
| C5HSC-S          | 5                     | 156                          | 14                     | 1.18                                | 45             | 3.78                           | 90               | 0.78           | 20                             | 81  | 3.17| 1.093 | 4.62|
| W3HSC-S          | 3                     | 147.4                        | 15.5                   | 1.23                                | 45             | 3.91                           | 94.4             | 0.84           | 28                             | 79  | 2.41| 0.831 | 2.2 |
| W5HSC-S          | 5                     | 178.09                       | 17.5                   | 1.31                                | 50             | 4.17                           | 122              | 0.74           | 31                             | 70  | 3.07| 1.059 | 4.77|
| U3HSC-S          | 3                     | 446.9                        | 21                     | 1.37                                | 68             | 4.32                           | 120              | 0.79           | 34                             | 64  | 3.73| 1.286 | 3.98|
| U5HSC-S          | 5                     | 451.5                        | 24.5                   | 1.46                                | 76             | 4.46                           | 144              | 0.7            | 37                             | 59  | 4.36| 1.503 | 5.4 |

Note: DD-Deflection Ductility; DDR - Deflection Ductility ratio; ED-Energy Ductility; EDR-Energy Ductility ratio

**Table 2.** Regression Equations

| Sl.No | Prediction Parameter                              | Regression Equation                                                      | Adj. R² Value |
|-------|--------------------------------------------------|-------------------------------------------------------------------------|---------------|
| 1     | First Crack Load in kN                           | 8.96+0.5314Tk+0.0258ffu                                                 | 0.9341        |
| 2     | Deflection at First Crack Load in mm             | 0.8992 + 0.0408Tk+0.0007ffu                                              | 0.8801        |
| 3     | Yield load in kN                                 | 32.16+0.1777Tk+0.0860ffu                                                 | 0.9615        |
| 4     | Yield Load Deflection in mm                      | 2.43+0.2194Tk+0.0024ffu                                                  | 0.8779        |
| 5     | Ultimate Load kN                                 | 44.06+4.37Tk+0.1537ffu                                                   | 0.9161        |
| 6     | Ultimate Load Deflection in mm                   | 5.77+0.5562Tk+0.0212ffu                                                  | 0.9398        |
| 7     | Crack Width in mm                                | 1.18-0.0736Tk-0.00035ffu                                                 | 0.9420        |
| 8     | Number of Cracks                                 | 12.34+1.28Tk+0.0410ffu                                                   | 0.8027        |
| 9     | Average spacing of cracks in mm                  | 121.13-6.75Tk-0.0755ffu                                                  | 0.9252        |
| 10    | Deflection Ductility                             | 2.57-0.0057Tk+0.00314ffu                                                 | 0.7020        |
| 11    | Deflection Ductility Ratio                       | 0.889-0.00194Tk+0.0010ffu                                                 | 0.7018        |
| 12    | Energy Ductility                                 | 1.52+0.4754Tk+0.0025ffu                                                  | 0.7505        |
| 13    | Energy Ductility Ratio                           | 0.6764+0.2103Tk+0.0011ffu                                                 | 0.7503        |

Note: Tk – Thickness of Fibre; ffu – Tensile Strength of Fibre
Linear regressions are naturally restricted in their ability to model very complex sets of data, since first order regression parameters try to fit a monotonically varying linear relationship curvature for the prediction parameter.

Validation of Test Results

For the purpose of validation, the data relating to this topic of research has been taken from several published literature.

Towards accomplishing the above objective, data pertaining to the proposed regression model such as breadth of beam (b), overall depth of beam (D), percentage of steel (%Ast), compressive strength (fck) and thickness of FRP (tk) have been taken from the research papers published by Darwin (2001), Scribner and Wright (2010), Wight and Sozen (2011), Thandavamoorthy (2002) and Gopinathan (2016). Source data considered for regression analysis towards validation are given in Table 4. OriginPro software has been utilized for the formation of proposed regression equation. The proposed equation has been validated against the test data reported in the published literature by some authors who have carried out investigation of this nature. The proposed regression equation is presented in Table 5. The validation of results is presented in Table 6.

(Continued)
Table 4. Continued

| Authors name | Beam Designation | b in mm | D in mm | % Steel | fck in N/mm² | FRP tk in mm | No. of Cycles | Exp | Any. |
|--------------|-----------------|---------|---------|---------|-------------|--------------|--------------|-----|------|
| Nmai and Darwin (2001) | F-1 | 190.5 | 387.35 | 1.03 | 22.1 | 0 | 5 | 6.17 |
| | F-2 | 190.5 | 390.652 | 1.02 | 20.1 | 0 | 2 | 5.88 |
| | F-3 | 190.5 | 387.35 | 1.02 | 23.2 | 3 | 6 | 7.40 |
| | F-4 | 190.5 | 390.652 | 0.69 | 24.2 | 0 | 9 | 6.48 |
| | F-5 | 190.5 | 387.35 | 0.69 | 23.8 | 3 | 8 | 7.60 |
| Scribner and Wright (2010) | 1 | 203.2 | 218.4 | 1.27 | 19.2 | 0 | 12 | 9.51 |
| | 2 | 203.2 | 218.4 | 1.27 | 19.2 | 3 | 12 | 10.62 |
| | 3 | 203.2 | 256.4 | 1.63 | 19.2 | 0 | 7 | 8.67 |
| | 4 | 203.2 | 256.4 | 1.63 | 19.2 | 3 | 8 | 9.78 |
| | 5 | 254 | 307.34 | 2.62 | 19.2 | 0 | 10 | 9.92 |
| | 6 | 254 | 307.34 | 2.62 | 19.2 | 3 | 12 | 11.03 |
| Wight and Sozen (2011) | 1 | 149.86 | 256.5 | 1.47 | 26.2 | 0 | 7 | 6.82 |
| | 2 | 152.4 | 256.5 | 1.47 | 26.2 | 3 | 7 | 8.06 |
| | 3 | 157.48 | 256.5 | 1.47 | 26.2 | 0 | 9 | 7.21 |
| | 4 | 154.94 | 256.5 | 1.47 | 26.2 | 3 | 9 | 8.19 |
| Thandavamoorthy (2002) | FB1-1 | 150 | 400 | 0.565 | 57.493 | 0 | 8 | 8.08 |
| | FB1-2 | 150 | 400 | 0.565 | 57.48 | 0 | 12 | 8.08 |
| | FB2-1 | 150 | 400 | 0.392 | 51.27 | 0 | 6 | 7.44 |
| | FB2-2 | 150 | 400 | 0.392 | 52.79 | 0 | 9 | 7.62 |
| Gopinathan (2016) | CBHSC-C | 150 | 250 | 0.603 | 67 | 0 | 9 | 11.93 |
| | C3HSC-C | 150 | 250 | 0.603 | 67 | 3 | 11 | 13.03 |
| | C5HSC-C | 150 | 250 | 0.603 | 67 | 5 | 13 | 13.77 |
| | W3HSC-C | 150 | 250 | 0.603 | 67 | 3 | 12 | 13.03 |
| | W5HSC-C | 150 | 250 | 0.603 | 67 | 5 | 14 | 13.77 |
| | U3HSC-S | 150 | 250 | 0.603 | 67 | 3 | 14 | 13.03 |
| | U5HSC-S | 150 | 250 | 0.603 | 67 | 5 | 16 | 13.77 |

Table 5. Proposed regression Equations for Number of Cycles

| Sl.No | Prediction Parameter | Regression Equation | Fitness Value |
|-------|----------------------|---------------------|---------------|
| 1     | Number of Cycles     | 1.56 + 0.0507b – 0.0185D – 0.3861%S + 0.1137fck + 0.3681tk | 0.818         |

Note: b-breadth of beam; D-overall depth of beam; %S-percentage of steel; fck-compressive strength; tk- thickness of FRP.
Conclusions

The regression equations proposed in the present study closely predict the study parameters of yield load, deflection at yield load, ultimate load, deflection at ultimate load, crack width, number of cracks, average spacing of cracks, deflection ductility, deflection ductility ratio, energy ductility, energy ductility ratio and number of cycles of GFRP laminated high strength concrete beams.

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