Hadron loops effect on mass shifts of the charmed and charmed-strange spectra

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The hadron loop effect is conjectured to be important in understanding discrepancies between the observed states in experiments and the theoretical expectations of non-relativistic potential model. We present that, in an easily operable procedure, the hadron loop effect could shift the poles downwards to reduce the differences and provide better descriptions of both the masses and the total widths, at least, of the radial quantum number $n = 1$ charmed and charmed-strange states. The $1^3P_1 - 1^3P_1$ mixing phenomena could be naturally explained due to their couplings with common channels. The newly observed $D$ states are also addressed, but there are still some problems remaining unclear.

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I. INTRODUCTION

Discoveries of more and more charmed or charmed-strange states in experiments attract great interest on the theoretical side, because some members of them have unexpected properties. In the Particle Data Group (PDG) table[1], six lower charmed states, $D^0$, $D^*$ (2007)$^0$, $D_s^0$ (2460)$^0$, $D_s(2460)$, $D_s(2450)$, and their partners have already been established. Recently, some evidences of three new charmed states, $D(2550)_0$, $D(2610)_0$, and $D(2760)_0$ have been reported by the BABAR Collaboration [2], whose features lead to intense discussions and theoretical suggestions of the further experimental investigations [3–8]. There are also nine charmed-strange states quoted in the PDG table among which some states’ quantum numbers are undetermined. The mass spectra of these charmed and charmed-strange states are roughly depicted in the predictions of the non-relativistic potential model in the classic work by Godfrey and Isgur (referred as GI in the following) [9]. However, the observed masses are generally lower than the predicted ones. For example, the biggest discrepancies happening in both spectra are the $1^3P_0$ states. The $D_s^0(2318)$ is about 80MeV lower than the expectation, while the $D_s^*(2317)$ is about 160MeV lower. There are a body of theoretical efforts at solving this problem usually by changing the representation of the potential (see Ref. [10–14] and their references). Lattice calculation has also been made to explain the experiments [15–16]. However, the present systematic uncertainty of the Lattice calculations does not allow determinations of the charmed mesons with a precision less than several hundred MeV.

Another expectation to shed light on this problem is to take the coupled channel effects (or called hadron loop effects) into account, which plays an important role in understanding the enigmatic light scalar spectrum and their decays [17, 18]. In the light scalar spectrum, the strong attraction of opened or closed channels may dramatically shift the poles of the bare states to different Riemann sheets attached to the physical region and the poles on unphysical Riemann sheets appear as peaks or just humps of the modulus of scattering amplitudes in the experimental data. The mass shifts induced by the intermediate hadron loops have also been testified to present a better description of the charmonium states [19–22]. The coupled channel effects have already brought some insights into the nature of the charmed-strange $D_{sJ}(2317)$ and some other states [23–26]. However, although this effect could explain some of the observed charmed or charmed-strange states, there is still a concern that this effect may also exist in those states previously consistent with the theoretical expectation [27]. In this paper, we will address this point by considering the mass shifts, induced by hadron loops, of all the firmly established charmed and charmed-strange states. Here we propose an easily operable way, in which we use the imaginary part of the self-energy function calculated from the quark pair creation (QPC) model [28–30] in the dispersion relation to obtain the analytically continued inverse propagator and extract the physical mass and width parameters, and then apply it to the charmed and charmed-strange spectra to interpret their masses and total decay widths in a consistent way. It is found that the results of their masses and total widths are consistent with the experimental values, at least for the non-radially-excited states. This picture gives a natural explanation to the $1^3P_1 − 1^3P_0$ mixing by coupling with the same channels instead of using a phenomenological mixing angle. This scheme has some similarities to the methods used by Heikkila et al. [19] and Pennington et al. in their study of the charmonium and bottomonium states, but there are significant differences with them, as discussed in the text.

The paper is organized as follows: In Section II, the main scheme and how to model the decay channels are briefly introduced. The mixing mechanism is introduced in Section III. Numerical procedures and results are discussed in Section IV. Section V is devoted to our conclusions and further discussions.

II. THE SCHEME

We start by considering a simple model at the hadron level, in which the inverse meson propagator, $\mathcal{P}^{-1}(s)$, could be represented as [19, 20]

$$\mathcal{P}^{-1}(s) = m_0^2 - s + \Pi(s) = m_0^2 - s + \sum_n \Pi_n(s),$$  \hspace{0.5cm} (1)

where $m_0$ is the mass of the bare $q\bar{q}$ state and $\Pi_n(s)$ is the self-energy functions for the $n$-th decay channels. Here, the sum is over all the opened channels or including nearby unopened channels (“just virtual”). $\Pi_n(s)$ is an analytic function with only a right-hand cut starting from the $n$-th threshold $s_{th,n}$, and so one can write its real part and imaginary part through a dispersion relation

$$\text{Re} \Pi_n(s) = \frac{1}{\pi} \mathcal{P} \int_{s_{th,n}}^{\infty} dz \frac{\text{Im} \Pi_n(z)}{(z - s)},$$  \hspace{0.5cm} (2)

where $\mathcal{P} \int$ means the principal value integration. The pole of $\mathcal{P}(s)$ on the unphysical Riemann sheet attached to the physical region specifies its mass and total width of a meson by its position on the complex $s$ plane, usually defined...
as \( s_{\text{pole}} = (M_p - i\Gamma_p/2)^2 \).

One could recover a generalization of the familiar Breit-Wigner representation, usually used in experimental analyses, from Eq. (1), as

\[
\mathbb{P}^{-1}(s) = m(s)^2 - s + i m_{BW} \Gamma_{\text{tot}}(s),
\]

where \( m(s)^2 = m_0^2 + \text{Re}(s) \) is the “running squared mass” and \( \Gamma_{\text{tot}}(s) = \text{Im}(s)/m_{BW} \). \( m_{BW} \) is determined at the real axis where \( m(s)^2 - s = 0 \) is fulfilled. The mass and width parameters in these two definitions give similar results when one encounters a narrow resonance, but they differ when the resonance is broad or when there are several poles interacting with each other.

Based on the Cutkosky rule, the imaginary part of the self-energy function is expressed through the couplings between the bare state and the coupled channels. The relation could be pictorially expressed as Fig. 1.

\[
\text{FIG. 1. The imaginary part of the self-energy function. } \int d\Pi \text{ means the integration over the phase space.}
\]

Thus, one key ingredient of this scheme is to model the coupling vertices in the calculation of the imaginary part of the self-energy function. The QPC model, also known as the \( 3P_0 \) model in the literature, turns out to be applicable in explaining the Okubo-Zweig-Iizuka (OZI) allowed strong decays of a hadron into two other hadrons, which are expected to be the dominant decay modes of a meson if they are allowed. It is not only because this model has proved to be successful but also because it could provide analytical expressions for the vertex functions, which are convenient for extracting the shifted poles in our scheme. Furthermore, the vertex functions have exponential factors which give a natural cutoff to the dispersion relation and we need not to choose one by hand as in Ref. [20].

Here, we just make a brief review of the main results of the QPC model used in our calculation. (For a more complete review, see [31–33]) In the QPC model, the meson (with a quark \( q_1 \) and an anti-quark \( q_2 \)) decay occurs by producing a quark \( (q_3) \) and anti-quark \( (q_4) \) pair from the vacuum. In the non-relativistic limit, the transition operator can be represented as

\[
T = -3\gamma \sum_m \langle 1m1 - m|00 \rangle \int d^3\vec{p}_3 d^3\vec{p}_4 \delta^3(\vec{p}_3 + \vec{p}_4) \gamma^m_1 (\vec{p}_3 - \vec{p}_4) \chi^{34}_{1-m} \phi^{34}_0 \omega^{34}_0 b^4_1(\vec{p}_3) d^4_1(\vec{p}_4),
\]

where \( \gamma \) is a dimensionless model parameter and \( \gamma^m_1(\vec{p}) \equiv p^l Y^m_l(\theta_p, \phi_p) \) is a solid harmonic that gives the momentum-space distribution of the created pair. Here the spins and relative orbital angular momentum of the created quark and anti-quark (referred to by subscripts 3 and 4, respectively) are combined to give the pair the overall \( J^{PC} = 0^{++} \) quantum numbers. \( \phi^{34}_0 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3} \) and \( \omega^{34}_0 = \delta_{ij} \), where \( i \) and \( j \) are the SU(3)-color indices of the created quark and anti-quark. \( \chi^{14}_{1-m} \) is a triplet of spin.

Define the \( S \) matrix for the meson decay \( A \rightarrow BC \) as

\[
\langle BC|S|A \rangle = I - 2\pi i \delta(E_f - E_i)\langle BC|T|A \rangle,
\]

and then

\[
\langle BC|T|A \rangle = \delta^3(\vec{P}_f - \vec{P}_i) \mathcal{M}_A^{M_L_A M_M_A M_M_C}.
\]

The amplitude turns out to be

\[
\mathcal{M}_A^{M_L_A M_M_A M_M_C}(\vec{P}) = \gamma \sqrt{8E_A E_B E_C} \sum_{M_L_A, M_M_A, M_M_B, M_M_C, M_S_C, M_S_B} (L_A M_L_A S_A M_M_A |J_A M_L_A) \\
\times \langle L_B M_L_B S_B M_M_B |J_B M_M_B \rangle \langle L_C M_L_C S_C M_M_C |J_C M_M_C \rangle \langle 1m1 - m|00 \rangle \times \langle \chi^{32}_{S_C M_S_C} \chi^{14}_{S_B M_S_B} |\chi^{12}_{S_M_M_A} \chi^{34}_{S_A M_S_A} \rangle \langle \phi^{32}_{M_A} \phi^{34}_{M_B} \rangle \mathcal{M}_A^{M_L_A, M_M_A, M_M_C}(\vec{P}).
\]
The spatial integral $I_{MLA,MLC}^{MLA,MLC}(\vec{P})$ is given by

$$I_{MLA,MLC}^{MLA,MLC}(\vec{P}) = \int d^3k \bar{\psi}_{nC}^{*}(-\vec{k} + \frac{\mu_4}{\mu_1 + \mu_4} \vec{P}) \psi_{nC}^{*}(-\vec{k} + \frac{\mu_3}{\mu_2 + \mu_3} \vec{P}) \psi_{nA} \psi_{MLA}(-\vec{k} + \vec{P}) \chi_{I}^{m}(-\vec{k}),$$

where we have taken $\vec{P} = \vec{P}_{B} = -\vec{P}_{C}$ and $\mu_i$ is the mass of the $i$-th quark. $\psi_{nA} \psi_{MLA}(\vec{k})$ is the relative wave function of the quarks in meson $A$ in the momentum space.

The recoupling of the spin matrix element can be written, in terms of the Wigner’s 9-j symbol, as

$$\langle \lambda_{SC}\lambda_{SM} | \lambda_{A} \lambda_{A} \lambda_{A} \rangle = [3(2S_B + 1)(2S + 1)(2S_A + 1)]^{1/2} \times \sum_{S, M_S} \langle S_C | S_B S_M S_S | S_S \rangle \langle S_M | S_A M_{S_A} | 1, -m \rangle \begin{pmatrix} 1/2 & 1/2 & S_C \\ 1/2 & 1/2 & S_B \\ S_A & 1 & S \end{pmatrix}. \quad (9)$$

The flavor matrix element is

$$\langle \phi_{C}^{32}\phi_{B}^{14}\phi_{A}^{12}\phi_{0}^{34} \rangle = \sum_{I, I'} \langle I_{C}, I_{C}^{3}; I_{B} I_{B} I_{B}^{13} | I_{A} I_{A}^{13} \rangle [(2I_{B} + 1)(2I_{C} + 1)(2I_{A} + 1)]^{1/2} \begin{pmatrix} I_{2} & I_{3} & I_{C} \\ I_{1} & I_{4} & I_{B} \\ I_{A} & 0 & I_{A} \end{pmatrix}, \quad (10)$$

where $I_{i}(I_{1}, I_{2}, I_{3}, I_{4})$ is the isospin of the quark $q_{i}$.

The imaginary part of the self-energy function in the dispersion relation, Eq. (2), could be expressed as

$$Im\Pi_{A \rightarrow BC}(s) = -\frac{\pi^2}{2J_{A} + 1} \frac{|\bar{P}(s)|}{\sqrt{s}} \sum_{M_{J_A}, M_{J_B}, M_{J_C}} |\mathcal{M}^{M_{J_A}, M_{J_B}, M_{J_C}}(s)|^{2}, \quad (11)$$

where $|\bar{P}(s)|$ is the three momentum of $B$ and $C$ in their center of mass frame. So,

$$\frac{|\bar{P}(s)|}{\sqrt{s}} = \sqrt{(s - (m_B + m_C)^2)(s - (m_B - m_C)^2)}. \quad (12)$$

Care must be taken when Eq. (11) is continued to the complex $s$ plane. Since what is used in this model is only the tree level amplitude, there is no right hand cut for $\mathcal{M}^{M_{J_A}, M_{J_B}, M_{J_C}}(s)$. Thus, the analytical continuation of the amplitude obeys $\mathcal{M}(s + i\epsilon)^* = \mathcal{M}(s - i\epsilon) = \mathcal{M}(s + i\epsilon)$. The physical amplitude with loop contributions should have right hand cuts, and, in principle, the analytical continuation turns to be $\mathcal{M}(s + i\epsilon)^* = \mathcal{M}(s - i\epsilon) = \mathcal{M}^0(s + i\epsilon)$ by meeting the need of real analyticity. $\mathcal{M}(s + i\epsilon)$ means the amplitude on the physical Riemann sheet (the first sheet, in language of the analytic $S$ matrix theory), and $\mathcal{M}^0(s + i\epsilon)$ means the amplitude on the unphysical Riemann sheet (the $n$-th sheet) attached with the physical region.

With the analytical expression of the imaginary part of the coupled channel, one will be able to extract the poles or the Breit-Wigner parameters from the propagators by standard procedures. In principle, all hadronic channels should contribute to the meson mass, as considered by Heikkila et al. in studying the charmonium states. Even all the “virtual” channels will contribute to the real parts of $\Pi(s)$ and renormalize the “bare” mass. Pennington et al. proposed that a once-subtracted dispersion relation will suppress contributions of the faraway “virtual” channels and make the picture simpler. Since what we consider here is only the mass shifts, we could make a once-subtracted dispersion relation at some suitable point $s = s_0$. It is reasonably expected that the lowest charmed state, $D^0$, as a bound state, has the mass defined by the potential model, uninfluenced by the effect of the hadron loops. Its mass then essentially defines the mass scale and thus fixes the subtracted point. So, we set the subtracted point $s_0 = m_{D^0}^2$ or $s_0 = (m_c + m_u)^2$ in a practical manner. The inverse of the meson propagator turns out to be

$$\mathbb{P}^{-1}(s) = m_{pot}^2 - s + \sum_{n} \frac{s - s_0}{\pi} \int_{s_{th,n}}^{\infty} dz \frac{\text{Im}\Pi_n(z)}{(z - s_0)(z - s)}, \quad (13)$$

where $m_{pot}$ is the bare mass of a certain meson defined in the potential model.

### III. MIXING MECHANISM

In this scheme, all the states with the same spin-parity have interference effects and could mix with each other. For example, the two $J = 1$ states of the $P$-wave are usually regarded as linear combinations of $^1P_1$ and $^3P_1$ assignments. Here in considering the coupled channel effect, the mixing mechanism comes from the coupling via common channels.
It is also believed that the $2^3S_1$ and $1^3D_1$ states mix with each other, similar to the interpretation of the charmonium $\psi(3770)$ state [34].

The inverse of the propagator with two bare states mixing with each other reads

$$\mathbb{P}^{-1}(s) = \begin{pmatrix} M^2_{a,1}(s) & M^2_{a,2}(s) \\ M^2_{b,1}(s) & M^2_{b,2}(s) \end{pmatrix} - \delta_{a,b}s = \begin{pmatrix} m^2_{bare,1} - s + \Pi_{11}(s) & \Pi_{12}(s) \\ \Pi_{21}(s) & m^2_{bare,2} - s + \Pi_{22}(s) \end{pmatrix}, \quad (14)$$

where $M^2_{a,b}(s)$ is the mass matrix and $m_{bare,a}$ represents the mass parameter of the bare $a$ state. The off-diagonal terms of the self-energy function is represented by the $1PI$ diagram for the two mixed states. The physical states should be determined by the meson propagator matrix after diagonalization

$$M^2_{diag}(s) = \alpha(s)^{-1}M^2_{a,b}(s)\alpha(s), \quad (15)$$

where the mixing matrix $\alpha(s)$ satisfies $\alpha(s)^T\alpha(s) = I$, i.e., $\alpha(s)$ is a complex orthogonal matrix since $M^2_{a,b}(s)$ is symmetric. The $\alpha(s)$ matrix turns to be complex when the thresholds are open. The physical poles could be extracted, in an equivalent way, by finding the zero points of the determinant of the inverse propagator, that is to solve $det(\mathbb{P}^{-1}(s)) = 0$.

### IV. NUMERICAL ANALYSES

The bare masses of the related mesons are chosen at the values of the GI’s work [9], as also listed in Table I for comparison. As for the dimensionless parameter, $\gamma$, and the effective $\beta$ parameters in the QPC model to characterize the harmonic oscillator wave functions, we choose the same values as determined from the potential in GI’s work for self-consistency. The constituent quark masses are $M_c = 1.628$GeV, $M_s = 0.419$GeV, and $M_u = 0.22$GeV. $\gamma = 6.9$ and the values of $\beta$s are from Ref. [35, 36]. The physical masses concerned in the final states are the average values in the PDG table. The relative wave functions between the quarks in the mock-meson states are simple harmonic oscillator (SHO) wave functions usually used in the QPC model calculation, which brings some uncertainties into the calculation, as discussed later.

There are some further explanations for the effective $\beta$ parameters of $c\bar{u}$ states. Godfrey and Isgur have only presented their results of $n = 1$ $S$ and $P$-wave charmed states but not provided those of the $D$-wave and radial excited states which is needed in our discussion of the newly observed charmed states. We can only estimate the values by assuming that their ratios between the $\beta$ values of the $c\bar{u}$ states are similar to the ratios in the results from the other research groups. For example, we find the ratios of the $\beta$ values between different charmed states in Ref. [37] and Ref. [38] are almost same. Thus, the $\beta$ values of $c\bar{u}$ states used in our calculation, except those listed in Ref. [36], are $\beta(1^3D_1) = 0.44 \pm 0.02$GeV, $\beta(1^3S_1) = 0.47 \pm 0.02$GeV, and $\beta(1^3S_1) = 0.44 \pm 0.02$GeV, respectively.

The opened or nearby unopened two-body channels taken into account in our calculations are all listed in Table I. Those channels with the $\sigma$ meson are not considered, because $\sigma$ meson is not regarded as a conventional $qq$ state in the potential model [9]. It will quickly decay into two pions and the three-body decays usually present a minor contribution.

| Mode | channel | $1^-(1^3S_1)$ | $0^+(1^3P_0)$ | $1^+(1^3P_1)$ | $1^+(1^3P_2)$ | $2^+(1^3P_2)$ | $0^-(2^1S_0)$ | $1^-(2^1S_1)$ | $1^-(1^3D_1)$ | $3^-(1^3D_3)$ |
|------|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $0^- + 0^-$ | $D\pi$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| | $D\eta$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| | $D_s K$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| $1^- + 0^-$ | $D^*\pi$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| | $D^*\eta$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| | $D_s^* K$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| $0^- + 1^-$ | $D\rho$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| | $D\omega$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| $0^+ + 0^-$ | $D_0^0 \pi$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| $1^+(T) + 0^-$ | $D_1(2420) \pi$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| $1^+(S) + 0^-$ | $D_1(2430) \pi$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |
| $2^+ + 0^-$ | $D_2^*(2460) \pi$ | □ | □ | □ | □ | □ | □ | □ | □ | □ |

**TABLE I.** The channels of the charmed states considered in this paper.
The masses and widths are simultaneously determined, as listed in Table III for the charmed states, where we present the pole positions as well as the Breit-Wigner parameters for comparison. Remarkable improvements of the shifted masses of the already established charmed mesons could be found instantly. Furthermore, the total widths specified by twice of the imaginary part of the pole positions are also consistent in good quality with the values in the PDG table.

| $J^P(n_{2s+1}L_J)$ | Expt. mass | Expt. width | $m_{BW}$ | $\Gamma_{BW}$ | $\sqrt{s_{pole}} = M - i\Gamma/2$ | GI mass |
|---------------------|------------|-------------|----------|---------------|-------------------------------|---------|
| 0$^-$ ($1^1S_0$)   | 1867       |            |          |               |                               | 1880    |
| 1$^-$ ($1^3S_1$)   | 2007 ± 0.16| < 2.1      | 2016    | 0.02          | 2016 − 0.01i                 | 2040    |
| 0$^+$ ($1^3P_0$)   | 2318 ± 29  | 267 ± 40   | 2335    | 233           | 2275 − 125i                  | 2400    |
| “1$^+$ ($1^3P_1$)” | 2422 ± 0.6 | 20 ± 1.7   | 2420    | 16            | 2410 − 7i                    | 2440    |
| “1$^+$ ($1^3P_1$)” | 2427 ± 40  | 384$^{+130}_{-110}$ | 2409 | 163           | 2377 − 94i                   | 2490    |
| 2$^+$ ($1^3P_2$)   | 2462 ± 1   | 43 ± 3     | 2453    | 26            | 2452 − 12i                   | 2500    |
| 0$^-$ ($2^1S_0$)   | 2533 ± 14(?) | 128 ± 33(?) | 2534    | 25            | 2533 − 12i                   | 2580    |
| 1$^-$ ($2^3S_1$)   | 2608 ± 5(?) | 93 ± 19(?) | 2525    | 8             | 2523 − 5i                    | 2640    |
| 1$^-$ ($1^3D_1$)   | 2763 ± 5(?) | 61 ± 9(?)  | 2730    | 120           | 2686 − 66i                   | 2820    |
| 3$^-$ ($1^3D_3$)   | (?)        | (?)        | 2735    | 9             | 2735 − 4i                    | 2830    |

TABLE II. Compilation of the experimental masses and the total widths (the PDG average values) of the charmed states, the shifted pole positions and the mass spectrum in the GI’s model. The experimental values of $2^1S_0$, $2^3S_1$, and $1^3D_1$ are from Ref. 3. Here we only list the neutral charmed states. The unit is MeV.

The $D(1^1S_0)$ state is a long-lived particle in the strong interaction and there is no opened strong channel, so we regard it to be well described as a bound state in the potential model and choose its squared mass as the subtraction point of the dispersion relations.

When we calculate the mass shift of the $D(1^3S_1)$, the $D^0\pi^0$ and $D^+\pi^-$ threshold are both taken into account, because the $D^+\pi^-$ threshold is at about 2009MeV, just 2MeV higher than the observed $D^*$(2007)$^0$. It is a typical “just virtual” channel and in principle it will contribute a significant mass shift to the bare state. If the coupling to the $D^+\pi^-$ threshold is excluded, the pole mass will only be shifted to about 2031MeV using this set of parameters.

The pole of $D(1^3P_0)$ is significantly shifted down to 2275MeV, which is 125MeV down below the potential model prediction. The pole width is about 250MeV, which is in accordance with the experimental value within errorbar.

The shifted pole mass favors the BABAR and Belle results over the FOCUS result. The $D(1^1P_1)$ and $D(1^3P_1)$ states stay close to each other and they both have the same quantum numbers $J^P = 1^+$ and similar decay channels. The unmixed pole positions are at $\sqrt{s(1^1P_1)} = 2387 - i28$MeV and $\sqrt{s(1^3P_1)} = 2427 - i71$MeV, respectively. Both of the masses and the widths of the unmixed $1^3P_1$ state have large differences from the experimental values. It is the effect of their couplings with common channel $D^0\pi$ that significantly change their pole positions to one narrower and the other boarder. The poles determined by the zero points of the inverse propagator matrix are at $\sqrt{s(1^1P_1)} = 2410 - i7$MeV and $\sqrt{s(1^3P_1)} = 2377 - i94$MeV respectively, which characterize the two observed states quite well. Their related Breit-Wigner parameters agree with the experimental values better. It is interesting to mention that in this scheme the mixing ratio, as a function of $s$, are complex-valued and it is not easy to find its relation with the mixing angle commonly used in phenomenological analyses. Here, the approximate values of the mixing matrix at about 2410MeV is

$$\alpha_{s=(2.41 GeV)^2} = \begin{pmatrix} -0.57 + 0.26i & 0.87 + 0.17i \\ 0.87 + 0.17i & 0.57 - 0.26i \end{pmatrix},$$

whose imaginary parts are small compared with the real parts. If one just neglect the imaginary parts and define the mixing matrix as usual, one obtains

$$\begin{pmatrix} -\sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{pmatrix} \approx \begin{pmatrix} -0.57 & 0.87 \\ 0.87 & 0.57 \end{pmatrix},$$

which means the mixing angle $\theta \simeq (29^\circ \sim 35^\circ)$. It is in agreement with the value $\theta \simeq 35.3^\circ$ obtained by considering the heavy quark symmetry [31].

As for the other charmed states which are not quoted in the PDG table, the evidences of several new charmed states, $D(2550)$, $D(2610)$, and $D(2760)$, have been recently reported by the BABAR collaboration [2]. Several groups
have presented their tentative interpretations of the nature of these states \[3,7\], and we make a brief summary of their conclusions here. \(D(2550)\) is assigned to the \(2^1S_0\), but its large decay width could not be explained by the QPC model, the chiral quark model, and the relativistic quark model, so further experimental explorations were suggested. Although the potential model has predicted \(D(2^3S_1)\) to be located at about 2640MeV, the QPC model and the chiral quark model also favor \(D(2610)\) to be a mixed state of \(D(2^3S_1)\) and \(D(1^3D_1)\) to interpret its large width. There are conflicting opinions about the assignment of \(D(2760)\) as the heavier mixed state of \(D(2^3S_1)\) and \(D(1^3D_1)\), or as \(D(1^3D_3)\).

In our calculation, the mass shift induced by the intermediate states also reduces the pole masses of \(D(2^1S_0)\), down to 2533MeV, but its pole width is quite narrow compared with the experimental value, as shown in Table III. However, the mass of \(D(2^1S_1)\) is shifted too much down to about 2523MeV due to many intermediate channels opened. Actually, the pole is even shifted down about 100MeV below some thresholds, and its pole width is fairly small as well. It seems to become a quasi-bound state due to its strong coupling with the \(D_1\pi\) channel. Of course, there is still some parameter space for the effective \(\beta\) parameter and the dimensionless coupling strength parameter \(\gamma\) to be tuned to reduce the mass shift to fit the experimental signal, because these parameters have significant uncertainties.

In our opinion, one possible reason why the result seems to be inaccurate is the uncertainty of the SHO function used to estimate the coupling vertices, which might have a larger tail than the realistic one in the high \(s\) region, which will contribute to the mass shift through the dispersion relation. A more realistic wave function solved from the linear potential model could be more reliable to describe the meson property. However, usually, this kind of wave function does not have an analytical representation and it can not be easily continued into the complex \(s\) plane in our scheme. On the other hand, it is too early to get any firm conclusion, since these states still need further experimental confirmations. The pole of \(D(1^3D_1)\) is shifted down to \(2686 - 1666\)MeV as well, whose Breit-Wigner mass is about 2735MeV which is closer to the mass of \(D(2760)\). Unlike the \(1^3P_1 - 1^3P_1\) case, with this set of parameters, the mixing mechanism due to the coupling with their common channels does not change their positions much. The unmixed pole of \(D(1^3D_1)\) could also be estimated at about 2735 but its width is narrow.

| Mode channel | \(1^- (1^3S_1)\) | \(0^+(1^3P_0)\) | \(1^+(1^3P_1)\) | \(1^+(1^3P_1)\) | \(2^+ (1^3P_2)\) |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(0^- + 0^-\) | \(DK\)          | \(D^\ast K\)    | \(D^\ast K\)    | \(D^\ast K\)    | \(D^\ast K\)    |
| \(1^- + 0^-\) | \(DK\)          | \(D^\ast K\)    | \(D^\ast K\)    | \(D^\ast K\)    | \(D^\ast K\)    |

**TABLE III.** The opened and nearby closed channels of the charmed-strange states considered in this paper.

| \(J^P (n^{2s+1}L_J)\) | Expt. mass | Expt. width | \(\sqrt{s_{pole}} = M - i\Gamma/2\) | GI mass |
|----------------------|------------|-------------|----------------------------------|--------|
| \(0^- (1^1S_0)\)     | 1968       | < 1.9       | 2114 - 0i                         | 2130   |
| \(1^- (1^3S_1)\)     | 2112 ± 0.5 | < 3.8       | 2358 - 0i                         | 2480   |
| \(0^+(1^3P_0)\)      | 2317 ± 0.6 | < 3.5       | 2470 - 0i                         | 2530   |
| \("1^+ (1^3P_1)^\)\) | 2459 ± 0.6 | < 2.3       | 2508 - 1i                         | 2570   |
| \(2^+ (1^3P_2)\)     | 2573 ± 1   | 20 ± 5      | 2522 - 7i                         | 2590   |

**TABLE IV.** Compilation of the experimental masses and the total widths (the PDG average values \[1\]) of the charmed strange states. The unit is MeV.

The discrepancies that happen in the charmed-strange spectrum could be well addressed qualitatively, owing to their coupling with the opened thresholds and the nearby unopened OZI-allowed strong thresholds in Table III as the picture proposed by van Beveren and Rupp for explaining the \(D_{sJ}(2371)\) state \[23\]. Some of the thresholds are opened as a result of the isospin breaking effects, e.g., \(D_{sJ}(2371) \rightarrow D_s\pi^0, D_s^\ast\pi^0\), whose contributions are highly suppressed by a factor of about \((m_u - m_d)/(m_u + m_d)/2 \approx 1/38\), where the masses are the current quark masses. The coupling to such thresholds will contribute tiny imaginary parts of the self-energy functions, which hardly shift the mass of the state and only contribute to the decay widths with an order of KeV. So we completely neglect these thresholds with isospin breaking effects and those OZI suppressed. When we choose the value of dimensionless strength parameter \(\gamma = 6.9\) for the charmed-strange spectrum, the mass shifts will be a little larger. We change \(\gamma\) to be around 5.5 and obtained the shifted masses of the charmed-strange \(S\) and \(P\)-wave states, as listed in Table IV. One could regard this fine tuning procedure as a “fit”, because we only want to give a qualitative description for the charmed-strange spectrum. Indeed the \(\gamma\) parameter in the QPC model, determined by fitting to experimental decay processes, usually has an uncertainty of about 30\% \[6, 57\]. Here we only list the pole positions, because they do not differ much with the Breit-Wigner parameters in this case as quasi-bound states or narrow states.
If the isospin breaking and other weak interaction channels are unopened, $D_s^*(2112)$ and $D_{sJ}^*(2317)$ are the bound states when the bare $1^3S_0$ and $1^3S_1$ states are coupled to the "just virtual" $DK$ threshold. They show as the poles on the real axis of the physical Riemann sheet. It is the coupling of the bare states to the lower isospin breaking $D_0\pi^0$ thresholds and the other weak thresholds that shift the poles to the unphysical Riemann sheets when they are open in reality. When the mixing of the $1^3P_1$ and $1^3P_0$ states is not considered, they are both the bound states at 2478MeV and 2493MeV respectively below the $D_s^*K$ thresholds. The mixing owing to coupling with the common $D_s^*K$ thresholds shifts the $1^3P_1$ downwards along the real $s$ axis, and the $1^3P_0$ moves upwards and crosses $D_s^*K$ thresholds into the complex $s$ plane of unphysical Riemann sheet.

V. CONCLUSIONS

In this paper, we propose a simple procedure to extract the pole positions or determine the Breit-Wigner parameters of the charmed states based on the parameters in the non-relativistic potential model, by using the analytical representation of the QPC model to mimic the behaviors of the imaginary part of the self-energy function of the meson propagator. Overall improvements could be found between the pole positions or Breit-Wigner parameters of the newly observed states. Several charmed-strange states could be regarded as the quasi-bound states, due to the coupling with nearby unopened OZI-allowed thresholds. In this model, the $1^3P_1−1^3P_0$ mixing is explained by the coupling with common channels and these resultant pole masses and widths are consistent with the observed values. It is worth stressing that our calculation is the first one that systematically addresses such a broad spectrum and the decays of the members by considering the coupled channel effects, as far as we know. This calculation may help to improve our understanding the charmed and charmed-strange spectra.

There are still some differences between the shifted pole positions and the parameters of the newly observed states. Since at the present stage the statistics of the data is still not enough to make a firm determination, further experimental evidences are required for a confirmation of these mesons.

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