HEAT AND MASS TRANSFER EFFECTS ON MHD NATURAL CONVECTION FLOW PAST AN INFINITE INCLINED PLATE WITH RAMPED TEMPERATURE

SIVA REDDY SHERI\textsuperscript{1†}, ANJAN KUMAR SURAM\textsuperscript{1}, AND PRASANTHI MODULGUA\textsuperscript{2}

\textsuperscript{1}DEPARTMENT OF MATHEMATICS, GITAM UNIVERSITY, HYDERABAD CAMPUS, TELANGANA, INDIA
E-mail address: sreddy7@yahoo.co.in

\textsuperscript{2}DEPARTMENT OF MATHEMATICS, CMRTC, KANDLAKOYA, MEDCHAL, TELANGANA, INDIA

\textbf{ABSTRACT.} This work is devoted to investigate heat and mass transfer effects on MHD natural convection flow past an inclined plate with ramped temperature numerically. The dimensionless governing equations for this investigation are solved by using finite element method. The effects of angle inclination, buoyancy ratio parameter, permeability parameter, magnetic parameter, Prandtl number, heat generation, thermal radiation, Eckert number, Schmidt number, chemical reaction parameter and time on velocity, temperature and concentration fields are studied and presented with the aid of figures. The effects of the pertinent parameters on skin friction, rate of heat transfer and mass transfer coefficients are presented in tabular form. The numerical results are compared graphically with previously published result as special case of the present investigation and results found to be in good agreement.

1. \textsc{Introduction}

Heat transfer is a study of the exchange of thermal energy through a body or between bodies which occurs when there is a temperature difference. Heat always transfers from hot to cold. Whereas mass transfer is the transport of constituent from a region of higher concentration to that of lower concentration. Heat and mass transfer is important in many engineering application such as food processing, nuclear reactors and polymer. A comprehensive discussion on heat and mass transfer was made by Abdallah and Zeghmati \cite{1} have investigated natural convection heat and mass transfer in the boundary layer along a vertical cylinder with opposing buoyancies. Olajuwon \cite{2} has presented convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion. Bisht et al. \cite{3} have analyzed the effects of variable thermal conductivity and chemical reaction on steady mixed convection boundary layer flow with heat and mass transfer inside a cone due to a point sink.

Received by the editors October 4 2016; Revised December 9 2016; Accepted in revised form December 9 2016; Published online December 20 2016.

2000 \textit{Mathematics Subject Classification.} 76R10, 76W105, 76D50, 76S05, 76D10.

\textit{Key words and phrases.} Heat and Mass transfer, MHD, Inclined Plate, Ramped temperature, FEM.

\footnote{Corresponding author.}
MHD is the science of motion of electrically conducting fluid in presence of magnetic field. The phenomenon of MHD flow with heat and mass transfer has been a subject of interest of many researchers because of its varied applications in science and technology. Such phenomena are observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies such as earth, etc. In light of these facts Eldabe et al. [4] have explained MHD free convection flow of visco-elastic fluid past an infinite vertical porous plate. Makinde [5] has studied heat and mass transfer by MHD mixed convection stagnation point flow toward a vertical plate embedded in a highly porous medium with radiation and internal heat generation. Mbeledogu et al. [6] have investigated unsteady MHD free convective flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer. Ishak et al. [7] have analyzed Magnetohydrodynamic (MHD) flow and heat transfer due to a stretching cylinder.

Dissipation is the process of converting mechanical energy of downward-flowing water into thermal and acoustical energy. Various devices are designed in streambeds to reduce the kinetic energy of flowing waters, reducing their erosive potential on banks and river bottoms viscous dissipation occurs in natural convection in various devices. Such dissipation effects may also be present in stronger gravitational fields and in process wherein the scale of the process is very large, e.g., on larger planets, in large masses of gas in space, and in geological processes in fluids internal to various bodies with viscous dissipative heat included in the energy equation. Amin et al. [8] have studied the effect of viscous dissipation on a power law fluid over plate embedded in porous medium. Siva Reddy and Srinivasa Raju [9] have explained Soret effect on unsteady MHD free convective flow past a semi infinite vertical plate in the presence of viscous dissipation. Raja Shekar and Hussain [10] have described the effect of viscous dissipation on MHD flow of a free convection power-law fluid with a pressure gradient.

Thermal radiation effects on hydromagnetic natural convection flow with heat and mass transfer play an important role in manufacturing processes taking place in industries for the design of fins, glass production, steel rolling, casting and levitation, furnace design, etc. Moreover, several engineering processes occur at very high temperatures where the knowledge of radiative heat transfer becomes indispensible for the design of pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. In this regard we may cite down the works done by Suneetha et al. [11] have discussed thermal radiation effects on MHD free convection flow past an impulsively started vertical plate with variable surface temperature and concentration. Cess [12] has explained the interaction of thermal radiation with free convection heat transfer. Howell et al. [13] have narrated thermal radiation heat transfer. Anjali Devi and Samuel Raj [14] have analyzed Thermo-diffusion effects on unsteady hydromagnetic free convection flow with heat and mass transfer past a moving vertical plate with time dependent suction and heat source in a slip flow regime. Venkateshwarlu et al. [15] have explained Thermal diffusion and radiation effects on unsteady MHD free convection heat and mass transfer flow past a linearly accelerated vertical porous plate with variable temperature and mass diffusion.

In certain porous media applications, the heat generation (source) or absorption (sink) effects are important. Representative studies dealing with these effects have been reported by
many authors such as Ibrahim and Shanker [16] have studied Unsteady MHD boundary-layer flow and heat transfer due to stretching sheet in the presence of heat source or sink. Ibrahim and Bhaskar Reddy [17] have investigated radiation and mass transfer effects on MHD free convection flow along a stretching surface with viscous dissipation and heat generation. Kesavaiah et al. [18] analyzed the effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Shivaiah and Anand Rao [19] have investigated Chemical reaction effect on an unsteady MHD free convection flow past a vertical porous plate in the presence of suction or injection. Venkateshwarlu et al. [20] have studied Radiation effects on MHD boundary layer flow of liquid metal over a porous stretching surface in porous medium with heat generation.

Several investigations were performed using both analytical and numerical methods under different thermal conditions which are continuous and well defined at the wall. Practical problems often involve wall conditions that are non-uniform or arbitrary. To understand such problems, it is useful to investigate problems subject to step change in wall temperature. For instance the fabrication of thin-film, nuclear heat transfer control, materials processing and turbine blade heat transfer. Chandran et al. [21] have studied Natural convection near a vertical plate with ramped wall temperature. Seth et al. [22] have explained MHD natural Convection flow with radiative heat transfer past an impulsively moving plate with ramped wall temperature. Ismail et al. [23] described MHD and radiation effects on natural convection flow in a porous medium past an infinite inclined plate with ramped wall temperature. Narahari et al. [24] considered Ramped temperature effect on unsteady MHD natural convection flow past an infinite inclined plate in the presence of radiation, heat source and chemical reaction. Siva Reddy et al. [25] have studied Transient approach to heat absorption and radiative heat transfer past an impulsively moving plate with ramped temperature.

Most of the research workers assumed vertical plate and they have neglected viscous dissipation. In the present work, we assumed inclined plate and at the same time effect of viscous dissipation is considered. The objective of this paper is to analyze the Heat and Mass transfer effects on MHD natural convection flow past an inclined plate with ramped temperature. A finite element scheme has been used to solve the governing equations. Variations in velocity, temperature and concentration with respect to various flow pertinent parameters are discussed graphically. The effects of the pertinent parameters on skin friction, rate of heat transfer and mass transfer coefficients are presented in tabular form. Finally our results are compared with results of Narahari et al. [24] and are found to be in excellent agreement.

2. Formulation of the Problem

Consider the unsteady laminar heat and mass transfer by natural convection along an infinite inclined plate through a porous medium. The physical model and Coordinate system are shown in Fig.1. The $x'$-axis is taken along the plate with the angle of inclination $\phi$ to vertical and the $y'$-axis is taken normal to the plate. Initially at $t' \leq 0$, the plate and the fluid are at the temperature $T'_\infty$ and concentration $C'_\infty$. At $u$ the plate temperature increases according to
the equation $T' = T'_{\infty} + (T'_{w} - T'_{\infty})t'/t_{0}$ and the chemical species concentration raised to $C'_{w}$. It is assumed that the plate is electrically non-conducting and a magnetic field of uniform strength $B_{0}$ is applied in the $y'$-direction. The fluid is assumed to be optically thin, constant property radiating gas except the density variation in the body force term of the balance of linear momentum equation and it is also assumed that the radiation heat flux in the $x'$-direction is negligible as compared to that in the $y'$-direction.

Since, the plate is assumed to be infinitely long in $x'$-axis direction so that all the dependent variables are functions of $y'$ and $t'$ only. Under the usual Boussinesq approximation, introducing heat due to viscous dissipation the governing equations for the momentum, energy and solute concentration can be written as follows:

\[ \frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta (T' - T'_{\infty}) \cos \phi + g\beta' (C' - C'_{\infty}) \cos \phi - \nu \frac{u'}{K'_{1}} - \frac{\sigma B_{0}^2}{\rho} u' \]  

(2.1)

\[ \frac{\rho c_{p}}{\partial T'_{w}} = \frac{\partial^2 T'}{\partial y'^2} - q_{c} (T' - T'_{\infty}) - \frac{\partial q_{r}}{\partial y'} + \frac{v}{\rho c_{p}} \left( \frac{\partial u'}{\partial y'} \right)^2 \]  

(2.2)

\[ \frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - \kappa (C' - C'_{\infty}) \]  

(2.3)

With the following initial and boundary conditions

\[
\begin{align*}
& t' \leq 0 : \quad u' = 0, \quad T' = T'_{\infty}, \quad C' = C'_{\infty} \quad \text{for all} \quad y' \geq 0, \\
& t' = 0 \quad u' = 0, \\
& T' = \begin{cases} 
T'_{\infty} + (T'_{w} - T'_{\infty}) \frac{t'}{t_{0}} & \text{for} \quad 0 < t' \leq t_{0} \ , \\
T'_{w} & \text{as} \quad y' \rightarrow \infty \\
\end{cases} \\
& C' = C'_{w} \\
& \text{as} \quad y' \rightarrow \infty \\
\end{align*}
\]  

(2.4)
For an optically thin constant property gas, the radiative heat flux $q_r$ satisfies the following nonlinear differential equation:

$$\frac{\partial q_r}{\partial y'} = 4\alpha\sigma^* (T'^4 - T_\infty'^4)$$  \hspace{1cm} (2.5)

Where $\alpha$ is the absorption coefficient and $\sigma^*$ is the Stefan-Boltzmann constant. It is assumed that the temperature differences within the flow are sufficiently small such that $T_\infty'^4$ may be expressed as a linear function of the fluid temperature $T'$ using the Taylor series about $T_\infty'$. After neglecting higher-order terms, gives

$$T'^4 \approx 4T_\infty'^3 T' - 3T_\infty'^4$$  \hspace{1cm} (2.6)

Using Equations (2.5) and (2.6), Equation (2.2) becomes

$$\rho c_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - q_c (T' - T_\infty') - 16\alpha\sigma^* T_\infty'^3 (T' - T_\infty') + \frac{\nu}{\rho c_p} \left( \frac{\partial u'}{\partial y'} \right)^2$$  \hspace{1cm} (2.7)

The following non-dimensional quantities are introduced:

$$y = \frac{y'}{\sqrt{\nu t_0}}, \quad t = \frac{t'}{t_0}, \quad t_0 = \frac{\nu^{1/3}}{g^{2/3}}, \quad u = \frac{u' t_0}{G_r \sqrt{\nu}}, \quad \theta = \frac{(T' - T_\infty')}{(T_w' - T_\infty')},
$$

$$G_m = \frac{g^3 (C_w' - C_\infty') t_0^{3/2}}{\sqrt{\nu}}, \quad N = \frac{G_m}{Gr}, \quad K = \frac{K_1'}{\nu t_0}, \quad M = \frac{\sigma^* B_0^2 t_0}{\rho},
$$

$$Q = \frac{q_c v t_0}{k}, \quad R = \frac{16\alpha\sigma^* v t_0 T_\infty'^3}{k}, \quad Sc = \frac{\nu}{D}, \quad \gamma = \kappa t_0, \quad Ec = \frac{U_0^2}{\gamma c_p (T_w' - T_\infty')},
$$

$$C = \frac{(C' - C_\infty')}{(C_w' - C_\infty')}, \quad Gr = \frac{g^3 (T_w' - T_\infty') t_0^{3/2}}{\sqrt{\nu}}, \quad Pr = \frac{\mu c_p}{k}$$

Where $u'$ is the fluid velocity in the $x'$-direction, $t'$ is the time, $\nu$ is the kinematic viscosity, $g$ is the acceleration due to gravity, $\beta$ is the volumetric coefficient of thermal expansion, $T'$ is the fluid temperature, $T_\infty'$ is the temperature of the fluid away from the plate, $\beta^*$ is the volumetric coefficient of concentration expansion, $C'$ is the species concentration away from the plate, $K_1'$ is the permeability of the porous medium, $\sigma$ is the electrical conductivity of the fluid, $\rho$ is the density, $C_p$ is the specific heat at constant pressure, $k$ is the thermal conductivity, $q_c$ is the volumetric heat generation or absorption, $q_r$ is the radiative heat flux in $y'$-direction, $D$ is the mass diffusivity, $\kappa$ is the chemical reaction parameter, $T_w'$ is the temperature of the plate, and $C_w'$ is the species concentration at the plate, $y$ is the dimensionless coordinate axis normal to the plate, $t$ is the dimensionless time, $u$ is the dimensionless velocity, $\theta$ is the dimensionless temperature, $C$ is the dimensionless species concentration, $Gr$ is the thermal Grashof number, $Gm$ is the mass Grashof number, $N$ is the mass to thermal buoyancy ratio parameter, $K$ is the dimensionless permeability parameter, $M$ is the magnetic field parameter (square of the Hartmann number), $Pr$ is the Prandtl number, $Q$ is the heat generation or absorption parameter, $R$ is the thermal radiation parameter, $Sc$ is the Schmidt number and $\gamma$ is the dimensionless chemical reaction parameter.
In view of equation (2.8), equations (2.1), (2.7), and (2.3) reduce to the following non-dimensional form, respectively

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta \cos \phi + CN \cos \phi - \left( \frac{1}{K + M} \right) u \tag{2.9}
\]

\[
Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - (Q + R) \theta + Ec \left( \frac{\partial u}{\partial y} \right)^2 \tag{2.10}
\]

\[
Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} - Sc \gamma C \tag{2.11}
\]

with the following initial and boundary conditions:

\[
t \leq 0 : \quad u = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad y \geq 0,
\]

\[
t > 0 : \left\{ \begin{array}{l}
\begin{array}{l}
\begin{array}{l}
t \leq 0 \quad \text{at} \quad y = 0, \\
t > 0 \quad \text{at} \quad y = 0,
\end{array}
\end{array}
\end{array}
\right.
\]

\[
C \rightarrow 1 \quad \text{as} \quad y \rightarrow \infty \tag{2.12}
\]

3. Method of Solution

The non-linear dimensionless partial differential equations (2.9)-(2.11) subject to boundary conditions (2.12) are solved by finite element method. This method consists of following five fundamental steps: discretization of the domain, derivation of element equations, assembly of element equations, imposition of boundary conditions and solution of the system of equations. An excellent description of these steps presented in the text books Bathe [26] and Reddy [27]. By using this procedure the whole domain is divided into a set of 60 intervals of equal length 0.1. At each node, three functions are to be evaluated. Hence, after assembly of the elements, we obtain a matrix of system of linear equations of order 60 \times 60. Consequently an iterative scheme is employed to solve the matrix system, which is solved by using the Gauss Seidel method. This process is repeated until the desired accuracy of \(10^{-6}\) is obtained. Hence, the finite element method is stable and convergent. The expressions for skin-friction, Nusselt number and Sherwood number are given by

The skin-friction coefficient is given by

\[
\tau = \left[ \frac{\partial u}{\partial y} \right]_{y=0} \tag{3.1}
\]

The Nusselt number is given by

\[
Nu = - \left[ \frac{\partial \theta}{\partial y} \right]_{y=0} \tag{3.2}
\]

The Nusselt number is given by

\[
Sh = - \left[ \frac{\partial C}{\partial y} \right]_{y=0} \tag{3.3}
\]
4. VALIDATION OF NUMERICAL RESULTS

In order to validate our numerical scheme we have presented a comparison in FIGURE 2 which displays contrast the velocity values for isothermal plate in the absence Eckert number with the values of the velocity obtained by Narahari et al. [24] for various values of $Q$. It is evident from FIGURE 2 that the numerical values of the velocity obtained through our numerical scheme are in very good agreement with the values of the velocity obtained by Narahari et al. [24]. This justifies the correctness of the results presented in the paper.

![Figure 2](image2.png)

**FIGURE 2.** Comparison of velocity profile for $Q$ (Isothermal)

We have also made a comparison in FIGURE 3 of the numerical values of velocity for ramped temperature obtained through our scheme with the solution presented by Narahari et al. [24] for various values of $Q$ taking $Ec = 0$ (i.e. in absence of Eckert number) in our model. It is observed that there is a very good agreement of the numerical solution with the solution.

![Figure 3](image3.png)

**FIGURE 3.** Comparison of velocity profile for $Q$ (Ramped)
5. Results and Discussions

In order to gain a clear insight of the physical problem Heat and Mass transfer effects on MHD natural convection flow past an inclined plate with ramped temperature, we have studied the effects of various parameters on velocity, temperature and concentration for both ramped temperature and isothermal plates and are presented graphically. In the present investigation we adopted the following default parameter values of finite element computations $\phi = 45^\circ$, $N = 0.2$, $K = 0.5$, $M = 1.0$, $Pr = 0.71$, $Q = -0.5$, $R = 5.0$, $Ec = 0.01$, $Sc = 0.6$, $\gamma = 1.0$, and $t = 0.6$.

5.1. Velocity field ($u$). The velocity of the fluid differs with the variation of the flow parameters such as angle of inclination, mass to thermal buoyancy ratio parameter, permeability parameter, magnetic field parameter (square of the Hartmann number), Prandtl number, heat generation or absorption parameter, radiation parameter, Eckert number, chemical reaction parameter, Schmidt number and time. The effects of these parameters on the flow field have been presented in FIGURES 4-14.

5.1.1. Effect of angle of inclination ($\phi$). FIGURE 4 portrays the effect of angle of inclination on dimensionless velocity $u$ for both ramped temperature and isothermal plates. From this FIGURE it is clear that velocity $u$ decreases on increasing angle of inclination. As $\phi$ increases the effect of the buoyancy force decreases since it is multiplied by $\cos \phi$, so the velocity profile decreases. Moreover, the momentum boundary layer is found to be thickened for increasing values of $\phi$. For $\phi = \pi/2$ the plate is horizontal and for $\phi = 0$ the plate assumes a vertical position. The gravitational effect is minimum for $\phi = \pi/2$ and maximum for $\phi = 0$.

![FIGURE 4. Effect of $\phi$ on dimensionless $u$](image)

The inclination parameter $\phi$ arises only in the buoyancy term $\cos \phi$ in the momentum equation (2.9). Thus, the fluid velocity is found to be maximized at the vertical position of the plate ($\phi = 0$) and minimized for the horizontal position of the plate ($\phi = \pi/2$).
5.1.2. *Effect of buoyancy ratio parameter ($N$).* FIGURE 5 displays the influence of buoyancy ratio parameter $N$ on the dimensionless velocity for both ramped and isothermal plates. Buoyancy ratio parameter is the ratio of thermal buoyancy force to mass buoyancy force. FIGURE 5 reveals that velocity gets accelerated on increasing buoyancy ratio parameter $N$ for aiding flow ($N > 0$). Since both thermal buoyancy force and mass buoyancy force move in same direction. On the other hand, velocity decreases on increasing buoyancy ratio parameter for opposing flow ($N < 0$). It is due to the reason that mass buoyancy force act conflicting to thermal buoyancy force.

5.1.3. *Effect of permeability parameter ($K$).* FIGURE 6 reveals the effect of permeability of porous medium $K$ on fluid flow in boundary layer region for both ramped temperature and isothermal plates. It is noticed from FIGURE 6 that, velocity $u$ increases on increasing $K$. An increase in permeability of medium implies that there is a decrease in the resistance of the porous medium which in turn accelerates fluid flow in boundary layer region for both ramped temperature and isothermal plates.
5.1.4. Effect of magnetic parameter ($M$). FIGURE 7 depicts the influence of magnetic field $M$ on fluid velocity for both ramped temperature and isothermal plates. It is evident from FIGURE 7 that velocity $u$ decreases in the region near the plate on increasing magnetic field parameter $M$ while it increases in the region away from the plate on increasing $M$. This implies that magnetic field tends to retard fluid velocity in the region near the plate whereas it has a reverse effect on fluid velocity in the region away from the plate. This shows that the Lorentz force (a resistive force developed due to the movement of an electrically conducting fluid in the presence of magnetic field) is dominant in the region near the plate and its effectiveness gets diminished by other forces in the region away from the plate.

![FIGURE 7. Effect of $M$ on dimensionless $u$](image)

5.1.5. Effect of Prandtl number ($Pr$). FIGURE 8 shows the influence of Prandtl number $Pr$ on the dimensionless velocity for both ramped and isothermal plates. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. From the FIGURE 8 it is clear that a rise in $Pr$ from 0.3 and 1.0 causes a decrease in velocity. Since an increase in the Prandtl number results a decrease of the thermal boundary layer thickness within the boundary layer.

![FIGURE 8. Effect of $Pr$ on dimensionless $u$](image)
5.1.6. *Effect of heat absorption or generation (Q).* FIGURE 9 displays the impact of heat source \((Q > 0)\) and heat sink \((Q < 0)\) on dimensionless velocity for both ramped temperature and isothermal plates. It is evident that velocity increases with the increase of heat source. Since in presence of a heat source the boundary layer generates energy which causes the temperature of the fluid to increase. This increase in temperature produces an increase in the flow field due to the buoyancy effect. On the other hand, the presence of a heat sink in the boundary layer absorbs energy which causes the temperature of the fluid to decrease. This decrease in the fluid temperature causes a reduction in the flow velocity in the boundary layer.

![Figure 9. Effect of Q on dimensionless u](image1)

5.1.7. *Effect of thermal radiation parameter (R).* FIGURE 10 exhibits the effect of thermal radiation \(R\) on dimensionless velocity for both ramped temperature and isothermal plates. From this FIGURE it is clear that velocity \(u\) decreases on increasing \(R\). This implies that radiation has a tendency to decelerate the fluid flow in the boundary layer region for both ramped temperature and isothermal plates. This is due to fact that fluid temperature is getting reduced due to thermal radiation and fluid flow in the boundary layer region is getting retorted.

![Figure 10. Effect of R on dimensionless u](image2)
5.1.8. Effect of Eckert number ($Ec$). FIGURE 11 shows the influence of Eckert number $Ec$ on dimensionless velocity for both ramped temperature and isothermal plates. Eckert number expresses the relationship between the kinetic energy in the flow and the enthalpy. From FIGURE 11 it is observed that, velocity experiences an enhancement on increasing Eckert number. This is due to the reason that Eckert number represents the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the velocity.

![Figure 11. Effect of $Ec$ on dimensionless $u$](image1)

5.1.9. Effect of Schmidt number ($Sc$). FIGURE 12 depicts the effects of Schmidt number $Sc$ on dimensionless velocity for both ramped temperature and isothermal plates. It is noticed from FIGURE 12 that, fluid velocity retard on increasing $Sc$. Schmidt number represents the ratio of momentum diffusivity to molecular (mass) diffusivity. This implies that mass diffusion tends to accelerate the fluid flow in the boundary layer region for both ramped temperature and isothermal plates.

![Figure 12. Effect of $Sc$ on dimensionless $u$](image2)
5.1.10. Effect of chemical reaction parameter (γ). FIGURE 13 demonstrates the effect of chemical reaction γ on dimensionless velocity for both ramped temperature and isothermal plates. It is noticed that fluid velocity decreases on increasing chemical reaction parameter γ. It should be mentioned that the studied case is for a destructive chemical reaction parameter γ. In fact, as γ increases, the considerable reduction in the velocity profiles is predicted, the presence of the peak indicates that the maximum value of the velocity occurs in the body of the fluid close to the surface but not at the surface.

![FIGURE 13. Effect of γ on dimensionless u](image)

5.1.11. Effect of time (t). FIGURE 14 portrays the effect of time t on dimensionless velocity for both ramped temperature and isothermal plates. From this FIGURE it is clear that velocity increases on increasing time t. This implies that fluid velocity tend to accelerate with the progress of time throughout the boundary layer region for both ramped temperature and isothermal plates.

![FIGURE 14. Effect of t on dimensionless u](image)
5.2. **Temperature field** ($\theta$). The Temperature of the fluid varies with the change in flow parameters like Prandtl number, heat generation or absorption parameter, radiation parameter, Eckert number and time. The influences of these parameters on the temperature field presented in FIGURES 15-19.

5.2.1. **Effect of Prandtl number** ($Pr$). FIGURE 15 displays the effect of Prandtl number $Pr$ on temperature for both ramped temperature and isothermal plates. It made clear that there is a significant decrease of the temperature as a result of an increase of the Prandtl number. Since Prandtl number is the ratio of thicknesses of the viscous and thermal boundary layers. Increasing the value of $Pr$ causes the fluid temperature and its boundary layer thickness to decrease significantly. This causes fluid temperature decreases from maximum at the boundary to a minimum value as far from the plate in both cases of ramped and isothermal plate.

![Figure 15. Effect of Pr on dimensionless $\theta$](image1)

5.2.2. **Effect of heat absorption parameter** ($Q$). FIGURE 16 represents the effects of heat source and heat absorption on the fluid temperature for both ramped temperature and isothermal plates. It portrays that fluid temperature decreases on increasing heat absorption. This may be attributed to the fact that the tendency of heat absorption (thermal sink) is to reduce the fluid temperature which causes the strength of thermal buoyancy force to decrease resulting in a net reduction in the fluid temperature.

![Figure 16. Effect of Q on dimensionless $\theta$](image2)
5.2.3. Effect of thermal radiation parameter \((R)\). FIGURE 17 symbolizes the effects thermal radiation \(R\) on temperature for both ramped temperature and isothermal plates. It says that there is a decrease in temperature with the increase of thermal radiation parameter. This result expresses that radiation parameter is to decrease the rate of energy transport to the fluid and thereby the temperature of the fluid decreases which causes the decrease in the fluid temperature.

\[\text{FIGURE 17. Effect of } R \text{ on dimensionless } \theta\]

5.2.4. Effect of Eckert number \((Ec)\). FIGURE 18 displays the influence of Eckert number \(Ec\) on dimensionless temperature for both ramped temperature and isothermal plates. It shows that the fluid temperature increases for increasing values of Eckert number \(Ec\) for buoyancy aided/opposed flows. The thermal boundary layer thickness decreases with increasing values of \(Ec\). The viscous dissipation, as a heat generation inside the fluid, increases the bulk fluid temperature. This can be attributed to the additional heating in the flow system due to viscous dissipation.

\[\text{FIGURE 18. Effect of } Ec \text{ on dimensionless } \theta\]
5.2.5. Effect of time \((t)\). FIGURE 19 displays the influence of time \(t\) on dimensionless temperature for both ramped temperature and isothermal plates. It expresses that the temperature increases with the increase of time \(t\). This implies that there is enrichment in field of temperature for both ramped temperature and isothermal plates.

![Figure 19](image1)

**FIGURE 19.** Effect of \(t\) on dimensionless \(\theta\)

5.3. Concentration field \((C)\). The variation of the concentration distribution of the flow field for various values of pertinent flow parameters like Schmidt number, Chemical reaction parameter and time are described from FIGURES 20-22.

5.3.1. Effect of Schmidt number \((Sc)\). FIGURE 20 exhibits the effect of Schmidt number \(Sc\) on dimensionless concentration for both ramped temperature and isothermal plates. It declares that the concentration distribution decreases at all points of the flow field with the increase of the Schmidt number. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases.

![Figure 20](image2)

**FIGURE 20.** Effect of \(Sc\) on dimensionless \(C\)
5.3.2. Effect of Chemical reaction parameter ($\gamma$). FIGURE 21 reveals the influence of chemical reaction parameter $\gamma$ on dimensionless concentration for both ramped temperature and isothermal plates. It displays that concentration decreases with the increase of chemical reaction parameter. Since chemical reaction parameter has a retarding influence on the concentration distribution of the flow field.

![Figure 21. Effect of $\gamma$ on dimensionless $C$](image)

5.3.3. Effect of time ($t$). FIGURE 22 exposes the effect time $t$ on dimensionless concentration for both ramped temperature and isothermal plates. It is evident that concentration distribution increases on increasing time. This implies that, for both ramped temperature plates there is improvement in concentration with the progress of time.

![Figure 22. Effect of $t$ on dimensionless $C$](image)

The numerical values of the skin friction, Nusselt number and Sherwood number are calculated for both ramped and isothermal plates by using equations (3.1)-(3.3) and are represented in the form of TABLES 1 to 3 for various values of $N$, $M$, $K$, $\phi$, $Pr$, $Q$, $R$, $Ec$, $Sc$, $\gamma$, and $t$.

TABLE 1 shows the values skin friction for both ramped and isothermal plates for different values of $N$, $M$, $K$ and $\phi$. The numerical values of skin friction are found by using the
expression (3.1). From TABLE 1 it is evident that there is an increase in Skin friction for increasing values of $N$, $K$ and $\phi$ whereas skin friction decreases for increasing values of $M$.

TABLE 1. Skin friction for ramped and isothermal plates

| $N$ | $M$ | $K$ | $\phi$ | Skin friction (Ramped) | Skin friction (Isothermal) |
|-----|-----|-----|------|----------------------|--------------------------|
| 0.2 | 0.5 | 1.0 | 30$^\circ$ | 0.054923 | 0.105677 |
| 0.6 | 0.5 | 1.0 | 30$^\circ$ | 0.124992 | 0.142456 |
| 0.2 | 1.0 | 1.0 | 30$^\circ$ | 0.032788 | 0.083927 |
| 0.2 | 0.5 | 1.5 | 30$^\circ$ | 0.093276 | 0.132048 |
| 0.2 | 0.5 | 1.0 | 60$^\circ$ | 0.107932 | 0.141680 |

TABLE 2 displays the values of Nusselt number for both ramped and isothermal plates for various values of $Pr$, $Q$, $R$ and $Ec$. Nusselt number values are obtained by using equation (3.2). From TABLE 2 it is clear that Nusselt number decreases with increasing values of $Pr$, $Q$ and $R$. On the other hand it increases with the increasing values of $Ec$.

TABLE 2. Nusselt number for both ramped and isothermal plates

| $Pr$ | $Q$ | $R$ | $Ec$ | Nusselt number (Ramped) | Nusselt number (Isothermal) |
|-----|-----|-----|-----|-------------------------|---------------------------|
| 0.3 | 0.5 | 1.0 | 0.01 | 0.503578 | 1.534721 |
| 0.7 | 0.5 | 1.0 | 0.01 | 0.757311 | 1.785213 |
| 0.3 | 1.0 | 1.0 | 0.01 | 0.481691 | 1.435678 |
| 0.3 | 0.5 | 5.0 | 0.01 | 0.34790 | 1.322670 |
| 0.3 | 0.5 | 1.0 | 0.02 | 0.69842 | 1.637805 |

TABLE 3 conveys the Sherwood values for both ramped and isothermal plates for diverse values of $Sc$, $\gamma$ and $t$. Values of Sherwood number are calculated by using equation (3.3). From TABLE 3 it is understood that Sherwood number increases with the progress of $t$ and it decreases with the increase of $Sc$ and $\gamma$.

TABLE 3. Sherwood number for both ramped and isothermal plates

| $Sc$ | $\gamma$ | $t$ | Sherwood number (Ramped) | Sherwood number(Isothermal) |
|-----|-------|----|-------------------------|---------------------------|
| 0.22 | 1.0 | 1.0 | 0.956791 | 1.4141678 |
| 0.6 | 1.0 | 1.0 | 0.864913 | 1.3678517 |
| 0.22 | 2.0 | 1.0 | 0.793463 | 1.3245690 |
| 0.22 | 1.0 | 1.4 | 1.000456 | 1.5254790 |

6. Conclusions

In this paper an attempt has made to investigate Heat and mass transfer effects on MHD natural convection flow past an inclined plate with ramped temperature. The governing partial
differential solved by using finite element method. Numerical results for velocity, temperature and concentration for both ramped temperature and isothermal plates are presented graphically for various relevant parameters. Based on the obtained graphical results, the following conclusions can be summarized.

- The fluid velocity increases with the increasing values of buoyancy ratio parameter when $N > 0$, permeability parameter $K$, heat generation ($Q < 0$), Eckert number $Ec$ and time $t$ and decreases with the increasing values of angle inclination $\phi$, buoyancy ratio parameter ($N < 0$), magnetic parameter $M$, Prandtl number $Pr$, heat absorption ($Q > 0$), thermal radiation $R$, Schmidt number $Sc$ and chemical reaction parameter $\gamma$.
- The fluid temperature increases with the increasing values of Eckert number $Ec$ and time $t$ and decreases with the increasing values of thermal radiation $R$, Prandtl number $Pr$ and heat absorption $Q$.
- The fluid concentration increases with the increasing values of time $t$ and decreases with the increasing values of Schmidt number $Sc$ and chemical reaction parameter $\gamma$.

ACKNOWLEDGEMENTS

The authors are thankful to the University Grants Commission, New Delhi, India for providing financial assistance to carry out this research work under UGC-Major Research Project [F. No. 42-22/2013 (SR)].

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