Global Asymptotic Stability for a New Class of Neutral Neural Networks

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Abstract: In the present world, due to the complicated dynamic properties of neural cells, many dynamic neural networks are described by neutral functional differential equations including neutral delay differential equations. These neural networks are called neutral neural networks or neural networks of neural-type. The differential expression not only defines the derivative term of the current state but also explains the derivative term of the past state. In this paper, global asymptotic stability of a neutral-type neural networks, with time-varying delays, are presented and analyzed. The neural network is made up of parts that include: linear, non-linear, non-linear delayed, time delays in time derivative states, as well as a part of activation function with the derivative. Different from prior references, as part of the considered networks, the last part involves an activation function with the derivative rather than multiple delays; that is a new class of neutral neural networks. This paper assumes that the activation functions satisfy the Lipschitz conditions so that the considered system has a unique equilibrium point. By constructing a Lyapunov-Krasovskii-type function and by using a linear matrix inequality analysis technique, a sufficient condition for global asymptotic stability of this neural network has been obtained. Finally, we present a numerical example to show the effectiveness and applicability of the proposed approach.

Keywords: Global Asymptotic Stability, Neutral Neural Network, Time-Varying Delay, Sufficient Condition

1. Introduction

Since Hopfield proposed a neural network model which was named after him in 1984, the Hopfield neural network has been applied to various fields, such as combinatorial optimization [1–4], image processing [5, 6], pattern recognition [7], signal processing [8], and communication [9]. As for one of the recurrent neural networks, the Hopfield neural network has been continuously investigated in the past decades [10-14]. In fact, due to the finite speeds of the switching and transmission of signals in a network, time delays exist in a working network and thus should be incorporated into the model equation of the network. Neural networks in the presence of time delays have received a great deal of attention in recent literature [15-22].

Due to the complexity of nerve cells in the real world, people have found that many existing neural network models cannot easily or accurately describe the characteristics of the neural response process. So, the different information of the past states should be incorporated to describe such a complex neural reactive dynamic system. Since a time derivative of the state is a function of time, delay parameters need to be introduced into the time derivatives of states of the system. The neural network model containing time delays in the time derivatives of states is called a delayed neutral-type neural network. Neutral-type neural networks are a special type of time-delayed neural networks, in which the information relating to derivatives of the past states is introduced to describe the system dynamics [23-27]. Further neural networks and applications can be referred to other studies [28-30]. Neutral-type neural networks are usually described by the following ordinary differential equations:
\[
\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t - \tau(t))) + E\dot{x}(t - \tau(t)) + u
\]  

(1)

In 2008, C. Bai investigated system (1) and has given sufficient conditions ensuring the existence and global exponential stability of the continuously differentiable, almost periodic, solution by using a fixed point theorem and differential inequality technique [25]. In 2012, Orman has obtained sufficient conditions for the existence, uniqueness and global asymptotic stability of the equilibrium point for the class of neutral-type systems (1) [26]. In 2013, S. Lakshmanan et al. have studied the following neutral delay Hopfield neural network model:

\[
\begin{aligned}
\dot{y}(t) &= -Ay(t) + Bg(y(t)) + Cg(y(t - \tau(t))) \\
&\quad + D\int_{t-\tau}^{t} g(y(s))ds + E\dot{y}(t - d(t)) + I
\end{aligned}
\]  

(2)

S. Lakshmanan and his group developed several LMI-based criteria to guarantee the asymptotic stability of neural networks. They considered the delay partitioning on both discrete and distributed delays and triple integral term for the neutral delay for the development of the derived results, and by employing a new type of Lyapunov–Krasovskii functionals, new delay-dependent stability criteria were derived [27].

Motivated by the idea and work of S. Lakshmanen et al., in this paper, the following neutral delay neural network model will be studied:

\[
\begin{aligned}
\dot{y}(t) &= A_1y(t) + A_2g(y(t)) + A_3g(y(t - \tau_1(t))) \\
&\quad + A_4\dot{y}(t - \tau_2(t)) + A_5\dot{g}(y(t)) + I
\end{aligned}
\]  

(3)

where \(y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T\) denotes the state variable, and \(g(y(t)) = (g_1(y_1(t)), g_2(y_2(t)), \ldots, g_n(y_n(t)))^T\) denotes the activation function. \(A_1 = \text{diag}\{a_1, a_2, \ldots, a_n\}, A_2 = (b_{ij})_{n \times n}, A_3 = (c_{ij})_{n \times n}, A_4 = (d_{ij})_{n \times n}, A_5 = (e_{ij})_{n \times n}\) and the constants \(a_{ij}, b_{ij}, c_{ij}, d_{ij}, e_{ij}\) denote, respectively, the connection weights of the \(i\)th neuron on the \(j\)th neuron. If the output of the \(j\)th neuron activates the \(i\)th neuron (or inhibits it), then \(b_{ij} > 0\) (or \(b_{ij} < 0\)), \(c_{ij}, d_{ij}, e_{ij}\) are similar. \(\tau_1(t)\) and \(\tau_2(t)\) denote the transmission delays of corresponding terms, and satisfies \(0 \leq \tau_1(t) \leq \bar{\tau}_1 < \infty, \quad 0 \leq \tau_2(t) \leq \bar{\tau}_2 < 1\), where \(\dot{\tau}_1(t)\) is the first derivative of \(\tau_1(t)\), and \(\dot{\tau}_2(t)\) are positive constants \((i = 1, 2)\). \(I = (I_1, I_2, \ldots, I_n)^T\), in which \(I_i\) is the external bias on the \(i\)th neuron.

Let the activation functions \(g_i(\cdot)\) satisfy the following conditions:

(H) The activation function is bounded and satisfies, \(g_i(x_i) \leq M_i, i = 1, 2, \ldots, n\), \(0 \leq g_i(z_1) - g_i(z_2) \leq L_i, \forall z_1, z_2 \in \mathbb{R}\), and \(z_1 \neq z_2\), where \(M_i\) and \(L_i\) are positive constants.

Because the activation functions satisfies (H), by using the Brouwer fixed-point theorem, it would be inferred that the neural network model (3) has an unique equilibrium point \(y^*\) for each \(i\); it is similar to the proof provided by the literature [25].

The equilibrium point \(y^*\) in (3) is shifted to the origin by letting \(x(t) = y(t) - y^*\), and then the system (3) can be transformed into the following form

\[
\begin{aligned}
\dot{x}(t) &= A_1x(t) + A_2f(x(t)) + A_3f(x(t - \tau_1(t))) \\
&\quad + A_4\dot{x}(t - \tau_2(t)) + A_5f(x(t))
\end{aligned}
\]  

(4)

where \(x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T\) is the state vector of the transformed system, and \(f(x(t)) = f(x(t) + y^*) - f(y^*)\), obviously, \(f(0) = 0\).

From the assumption (H), the transformed neuron activation function satisfies \(f_j(y_i) \leq M_i, i = 1, 2, \ldots, n\), \(0 \leq f_j(z_1) - f_j(z_2) \leq L_i, \forall z_1, z_2 \in \mathbb{R}\), and \(z_1 \neq z_2\), where \(M_i\) and \(L_i\) are positive constants.

2. Main Results

In this section, the sufficient conditions for global asymptotic stability of system (4) would be established. Throughout this paper, the following notations will be made use of:

i. \(\eta_1 = x(t), \quad \eta_2 = f(x(t)), \quad \eta_3 = f(x(t - \tau_1(t))), \quad \eta_4 = \dot{x}(t - \tau_2(t)), \quad \eta_5 = \dot{f}(x(t))\); 
ii. \(L_M = \max\{L_1, L_2, \ldots, L_n\}\), \(a_M = \max\{a_1, a_2, \ldots, a_n\}, \quad \lambda = \frac{a_M}{L_M}\); 
iii. The superscript \(T\) denotes the transpose of the matrix and the notation \(X \geq Y\) (respectively, \(X > Y\)), where \(X\) and \(Y\) are symmetric matrices, means that \(X - Y\) is positive semi-definite (respectively, positive definite). \(\bar{\lambda}_m(P)\) and \(\lambda_m(P)\) denote the minimum eigenvalue of \(P\) and the maximum eigenvalue of \(P\), respectively. \(\text{Diag}\{\cdots\}\) denotes the block diagonal matrix. \(I\) denotes identity matrix.

By using notations (I), system (4) can be rewritten as:

\[
\begin{aligned}
\dot{x}(t) &= A_1\eta_1 + A_2\eta_2 + A_3\eta_3 + A_4\eta_4 + A_5\eta_5
\end{aligned}
\]  

(5)

Next, a theorem, which provides sufficient conditions for global asymptotic stability of the system (4), will be established.

Theorem 1 The zero solution of system (4) is globally asymptotically stable if there exist symmetric positive definite matrices \(P, Q\), such that
\[
\begin{align*}
\Omega &= \left\{ 2A_t + \Sigma_{11} + A_x + \Sigma_{12} + A_j + \Sigma_{13} \right\} \\
&\left\{ (2A_t + \Sigma_{12}) + 2A_t + P + \Sigma_{22} + A_j + \Sigma_{23} \right\} \\
&\left\{ 2A_t + \Sigma_{13} + A_j + \Sigma_{24} \right\} \\
&\left\{ A_j + \Sigma_{14} + A_t + \Sigma_{15} + A_t + \Sigma_{25} + A_j + \Sigma_{26} \right\}
\end{align*}
\]

where \( \Sigma_{ij} = A_i^T A_j \), \( i, j = 1, 2, 3, 4, 5 \).

**Proof.** Define the following Lyapunov-Krasovskii functional for system (4),

\[
V(x(t)) = x^T(t)x(t) + 2 \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \left( f_i(s) + \dot{f}_i(s) \right) ds
\]

\[
+ \int_{t_{i-1}}^{t_i} f_i^T(t+s)P f_i(x(t+s)) ds
\]

\[
+ \int_{t_{i-1}}^{t_i} \dot{x}^T(t+s)Q \dot{x}(t+s) ds
\]

By using assumption (H), we derived

\[
0 \leq f_i(x_i(t)) \leq L_i \leq L_M, \quad (6)
\]

\[
0 \leq \dot{f}_i(s) \leq L_i \leq L_M, \quad \left| f_i(x_i(t)) \right| \leq L_i \mid x_i(t) \right|, \quad \left| \dot{f}_i(x_i(t)) \right| \leq L_i \mid x_i(t) \right|
\]

From (6), it is easy to understand that the signs of \( f_i(x_i(t)) \) and \( x_i(t) \) are the same (both positive, or both negative) if \( f_i(x_i(t)) \neq 0 \), therefore

\[
0 \leq f_i(x_i(t)) f_i(x_i(t)) \leq L_i x_i(t) f_i(x_i(t)) \leq L_M x_i(t) f_i(x_i(t))
\]

\[
f_i(x_i(t)) x_i(t) \geq \frac{1}{L_m} f_i(x_i(t)) f_i(x_i(t)),
\]

according to \( a_i < 0, a_M = \max\{a_1, a_2, \ldots, a_n\} \), and \( \hat{\lambda} = \frac{a_M}{L_M} \), the following is true

\[
f_i(x_i(t)) a_i x_i(t) \leq \frac{1}{L_M} f_i(x_i(t)) a_i f_i(x_i(t))
\]

\[
f(x(t)) A_i x(t) = \sum_{i=1}^{n} f_i(x_i(t)) a_i x_i(t)
\]

\[
\leq \frac{1}{L_M} \sum_{i=1}^{n} f_i(x_i(t)) a_i f_i(x_i(t))
\]

\[
\leq a_M \hat{\lambda} f^T(x(t)) f(x(t))
\]

that is

\[
f(x(t)) A_i x(t) \leq \hat{\lambda} f^T(x(t)) f(x(t)) \quad (7)
\]

By using the notations in (1), (7) can be rewritten as

\[
\eta_2 A_i \eta_1 \leq \hat{\lambda} \eta_1^T \eta_2
\]

By calculating the derivative of \( V(x(t)) \) along the trajectories of system (4), then

\[
\frac{dV(x(t))}{dt} = 2 \dot{x}^T(t) \dot{x}(t) + 2 f^T(x(t)) \dot{x}(t)
\]

\[
+ 2 \dot{f}^T(x(t)) \dot{x}(t) + f^T(x(t)) P f(x(t))
\]

\[
+ f^T(t) Q \dot{x}(t) - (1 - \tau_1) f^T(t-t_1(t))(P f(x(t-t_1(t)) + (A_i \eta_1 + A_2 \eta_2 + A_j \eta_3 + A_4 \eta_4 + A_5 \eta_5)
\]

\[
+ \eta_2^T P \eta_2 - (1 - \tau_2) \eta_1^T \eta_3 Q \eta_4
\]

\[
(A_i \eta_1 + A_2 \eta_2 + A_j \eta_3 + A_4 \eta_4 + A_5 \eta_5)
\]

\[
+ \eta_2^T P \eta_2 - (1 - \tau_2) \eta_1^T \eta_3 Q \eta_4
\]

\[
= 2 \eta_i^T A_i \eta_1 + 2 \eta_j^T A_j \eta_3 + 2 \eta_4^T A_4 \eta_4 + 2 \eta_5^T A_5 \eta_5
\]

\[
\leq 2 \eta_i^T A_i \eta_1 + 2 \eta_4^T A_4 \eta_4 + 2 \eta_5^T A_5 \eta_5
\]

\[
\leq 2 \eta_i^T A_i \eta_1 + 2 \eta_4^T A_4 \eta_4 + 2 \eta_5^T A_5 \eta_5
\]
\[ +2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i \]

\[ +2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i \]

\[ +\eta^T_i P \eta_i - (1 - \tau_i(t))\eta^T_i P \eta_i \]

\[ +\{A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\}^T Q \]

\[ \{A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\} - (1 - \tau_i(t))\eta^T_i Q \eta_i \]

\[ +2\lambda \eta^T_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i \]

\[ +\eta^T_i P \eta_i - (1 - \tau_i(t))\eta^T_i P \eta_i + \{A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\} \]

\[ +A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\}^T Q \{A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\} \]

\[ - (1 - \tau_i(t))\eta^T_i Q \eta_i \]

\[ = (2\eta^T_i A_i \eta_i + (\eta^T_i A_i \eta_i + \eta^T_i A_i \eta_i)) + (\eta^T_i A_i \eta_i + \eta^T_i A_i \eta_i) + \{A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\} \]

\[ +2\lambda \eta^T_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i \]

\[ +\eta^T_i P \eta_i - (1 - \tau_i(t))\eta^T_i P \eta_i + \{A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\} \]

\[ +A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\}^T Q \{A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\} \]

\[ - (1 - \tau_i(t))\eta^T_i Q \eta_i \]

\[ = \eta^T_i (A_i + A_i Q A_j) \eta_i + \{A_i + A_i Q A_j\} + \eta^T_i (A_i + A_i Q A_j) \eta_i + \eta^T_i (A_i + A_i Q A_j) \eta_i \]

\[ +2\lambda \eta^T_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i + 2\eta^T_i A_i \eta_i \]

\[ +\eta^T_i P \eta_i - (1 - \tau_i(t))\eta^T_i P \eta_i + \{A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\} \]

\[ +A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\}^T Q \{A_i \eta_i + A_i \eta_i + A_i \eta_i + A_i \eta_i\} \]

\[ - (1 - \tau_i(t))\eta^T_i Q \eta_i \]

\[ \Omega = \begin{bmatrix} 2A_i + \xi_{11} & A_2 + \xi_{12} & A_3 + \xi_{13} \\ A_2^T + \xi_{22} & 2\lambda \eta + 2A_2 + P + \xi_{22} & A_3 + \xi_{23} \\ A_3^T + \xi_{33} & \xi_{23} + \xi_{33} & (1 - \tau_i(t))P + \xi_{33} \end{bmatrix} \]

\[ \begin{bmatrix} A_4 + \xi_{44} \\ A_4 + \xi_{44} \\ A_4 + \xi_{44} \end{bmatrix} \]

\[ \begin{bmatrix} A_4 + \xi_{44} \\ A_4 + \xi_{44} \\ A_4 + \xi_{44} \end{bmatrix} \]

\[ \biggl( \begin{bmatrix} 1 - e^{-x} \\ 1 + e^{-x} \end{bmatrix} \biggr) \]

\[ A_4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, A_5 = \begin{bmatrix} -1 & 0.15 \\ 0.15 & -1.7 \end{bmatrix} \]

\[ A_4 = \begin{bmatrix} 4 & 0 \\ 0 & 0.34 \end{bmatrix} \]

\[ A_4 = \begin{bmatrix} 0.1 & 0 \\ 0 & 1.1 \end{bmatrix}, A_5 = \begin{bmatrix} 4 & 0 \\ 0 & 0.34 \end{bmatrix} \]

where

\[ \| Q \|_f < \lambda \]

According to the known conditions, \( \Omega \) is negative definite, so \( \dot{V}(x(t)) \) is negative definite, thus system (4) is globally asymptotically stable. This completes the proof.

3. A numerical Example

In this section, an example to test the validity of results would be presented. Consider the following model with two neurons:

\[ \begin{align*}
\dot{x}_1(t) &= A_1 x_1(t) + A_2 f(x_1(t)) + A_3 \left( f'(x_1(t)) \right) \\
\dot{x}_2(t) &= A_4 x_2(t) + A_5 f(x_2(t)) + A_6 \left( f'(x_2(t)) \right)
\end{align*} \]
Obviously, the functions \( f(x(t)) \) satisfies condition (H), and
\[
L_M=0.5, \quad a_M=-1, \quad \lambda = \frac{d_M}{L_M} = -2, \quad \tau_1^* = 0.05, \quad \tau_2^* = 0.03. \quad \text{Let}
\]
\[
P = \begin{bmatrix} 0.5 & 0 \\ 0 & 5 \end{bmatrix}, \quad Q = \begin{bmatrix} 1.1 & 0 \\ 0 & 1.2 \end{bmatrix}
\]

The matrix \( \Omega \) is negative definite. According to the result of Theorem 1, system (8) is globally asymptotically stable at its equilibrium point.

4. Conclusion

This paper investigated the problem of global asymptotic stability of delayed neutral neural networks. These neutral delayed neural networks are formulated by neutral delay differential equations. They not only include the derivative term of the current state but also the derivative term of the past state.

A new Lyapunov-Krasovskii-type technique is employed to develop sufficient conditions of global asymptotic stability of the neural networks. All these stability criteria are expressed in the form of linear matrix inequalities, which are solvable by the use of the Matlab software.

Finally, a numerical example is presented to illustrate the effectiveness and usefulness of the obtained results. Our results are suited to investigate the neural networks containing activation functions with both the state derivatives and multiple delays.

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References

[1] J. J. Hopfield. Neurons with graded response have collective computational properties like those of two-state neurons. Proc. Acad. Sci. USA, 1984, 81: 3088-3092.

[2] S. Abe, J. Kawakami, K. Hirasawa. Solving inequality constrained combinatorial optimization problems by the Hopfield neural networks. Neural Networks, 1992, 5: 663-670.

[3] H. Tamura, Z. Zhang, X. S. Xu, M. Ishii, Z. Tang. Lagrangian object relaxation neural network for combinatorial optimization problems, Neurocomputing, 2005, 68: 297-305.

[4] R. L. Wang, Z. Tang, Q. P. Cao. A learning method in Hopfield neural network for combinatorial optimization problem. Neurocomputing, 2002, 48: 1021-1024.

[5] S. Rout, Seethalakshmy, P. Srivastava, J. Majumdar. Multi-modal image segmentation using a modified Hopfield neural network Original Research Article. Pattern Recognition, 1998, 31: 743-750.

[6] R. Sammouda, N. Adgaba, A. Touir, A. Al-Ghamdi. Agriculture satellite image segmentation using a modified artificial Hopfield neural network. Computers in Human Behavior, 2014, 30: 436-441.

[7] P. Suganthan, E. Teoh, D. Mital. Pattern recognition by homomorphic graph matching using Hopfield neural networks Original Research Article. Image and Vision Computing, 1995, 13: 45-60.

[8] N. Laskaris, S. Fotopoulos, P. Papathanasopoulos. A. Bezerianos. Robust moving averages, with Hopfield neural network implementation, for monitoring evoked potential signals Original Research Article. Electroencephalography and Clinical Neurophysiology/Evoked Potentials Section, 1997, 104: 151-156.

[9] D. Calabuig, J. F. Monserrat, D. Gmez-Barquero, O. Lzano. An efficient dynamic resource allocation algorithm for packet-switched communication networks based on Hopfield neural excitation method. Neurocomputing, 2008, 71: 3439-3446.

[10] W. Zhang. A weak condition of globally asymptotic stability for neural networks. Applied Mathematics Letters, 2006, 19: 1210–1215.

[11] X. Li, Z. Chen. Stability properties for Hopfield neural networks with delays and impulsive perturbations. Nonlinear Analysis: Real World Applications, 2009, 10: 3253-3265.

[12] L. Wang, Y. Gao. Global exponential robust stability of reaction–diffusion interval neural networks with time-varying delays. Physics Letters A, 2006, 350: 342-348.

[13] X. Lou, Q. Ye, B. Cui. Parameter-dependent robust stability of uncertain neural networks with time-varying delay. Journal of the Franklin Institute, 2012, 349: 1891-1903.

[14] C. Marcus, R. Westervelt. Stability of analog neural networks with delay. Physical Review A, 1989, 39: 347-359.

[15] J. Wu. Symmetric functional-differential equations and neural networks with memory. Transactions of the American Mathematical Society, 1998, 350: 4799-4838.

[16] J. Wu, X. Zou. Patterns of sustained oscillations in neural networks with time delayed interactions. Applied Mathematics and Computation, 1995, 73: 55-75.

[17] K. Gopalsamy, X. He. Stability in asymmetric Hopfield nets with transmission delays. Physica D, 1994, 76: 1344-1358.

[18] P. van den Driessche, X. Zou. Global attractivity in delayed Hopfield neural network models. SIAM Journal on Applied Mathematics, 1998, 58: 1878-1890.

[19] H. Zhao. Global asymptotic stability of Hopfield neural network involving distributed delays. Neural Networks, 2004, 17: 47–53.
[20] R. Rakkiyappan, P. Balasubramaniam. Delay-dependent asymptotic stability for stochastic delayed recurrent neural networks with time varying delays. Applied Mathematics and Computation, 2008, 198: 526–533.

[21] H. Xu, Y. Chen, K. L. Teo. Global exponential stability of impulsive discrete-time neural networks with time-varying delays. Applied Mathematics and Computation, 2010, 217: 537-544.

[22] R. Luo, H. Xu, W. Wang, J. Sun, W. Xu. A weak condition for global stability of delayed neural networks. Journal of Industrial and Management Optimization, 2016, 12: 505-514.

[23] R. Samli, S. Arik. New results for global stability of a class of neutral-type neural systems with time delays. Applied Mathematics and Computation, 2009, 210: 564–570.

[24] X. Zeng, Z. Xiong, C. Wang. Hopf bifurcation for neutral-type neural network model with two delays. Applied Mathematics and Computation, 2016, 282:17–31.

[25] C. Bai. Global stability of almost periodic solutions of Hopfield neural networks with neutral time-varying delays. Applied Mathematics and Computation, 2008, 203: 72–79.

[26] Z. Orman. New sufficient conditions for global stability of neutral-type neural networks with time delays. Neurocomputing, 2012, 97: 141-148.

[27] S. Lakshmanan, J. H. Park, H. Y. Jung, O. M. Kwon, R. Rakkiyappan. A delay partitioning approach to delay-dependent stability analysis for neutral type neural networks with discrete and distributed delays. Neurocomputing, 2013, 111: 81-89.

[28] J. Ye, H. Xu, E. Feng, Z. Xiu. Optimization of a fed-batch bioreactor for 1, 3-propanediol production using hybrid nonlinear optimal control. Journal of Process Control. 2014, 24: 1556-69.

[29] R. Luo, H. Xu, W. Wang, X. Wang. A new stability criterion of neutral neural networks with time-varying delays. Pacific Journal of Optimization, 2016, 12: 487-96.

[30] H. Zeng, K. L. Teo, Y. He, H. Xu, W. Wang. Sampled-data synchronization control for chaotic neural networks subject to actuator saturation. Neurocomputing, 2017, 260: 25-31.