A Unified Power Control Algorithm for Multiuser Detectors in Large Systems: Convergence and Performance*

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Abstract

A unified approach to energy-efficient power control, applicable to a large family of receivers including the matched filter, the decorrelator, the (linear) minimum-mean-square-error detector (MMSE), and the individually and jointly optimal multiuser detectors, has recently been proposed for code-division-multiple-access (CDMA) networks. This unified power control (UPC) algorithm exploits the linear relationship that has been shown to exist between the transmit power and the output signal-to-interference-plus-noise ratio (SIR) in large systems. Based on this principle and by computing the multiuser efficiency, the UPC algorithm updates the users’ transmit powers in an iterative way to achieve the desired target SIR. In this paper, the convergence of the UPC algorithm is proved for the matched filter, the decorrelator, and the MMSE detector. In addition, the performance of the algorithm in finite-size systems is studied and compared with that of existing power control schemes. The UPC algorithm is particularly suitable for systems with randomly generated long spreading sequences (i.e., sequences whose period is longer than one symbol duration).

1 Introduction

Power control is used for interference management and resource allocation in both the downlink and the uplink of code-division-multiple-access (CDMA) networks. Power control for CDMA systems has been studied extensively over the past decade (see for example [1, 2, 3, 4, 5, 6, 7]). In the uplink, the purpose of power control is for each user to transmit just enough power to achieve the required quality of service (QoS) without causing unnecessary interference.

Multiuser receivers are expected to be deployed in future wireless systems, especially in the uplink, because of their superior performance to the conventional matched filter [8]. Because of this, power control for multiuser detectors has attracted attention in recent years. In particular, power control algorithms for the linear minimum-mean-square-error (MMSE) detector and successive interference cancellation (SIC) receiver have been proposed in [4] and [7], respectively. In the proposed schemes, the output signal-to-interference-plus-noise ratio (SIR) is measured for each user and then the user’s transmit power is adjusted to achieve the desired target SIR.

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Almost all of the power control schemes proposed so far have a specific receiver in mind. Reference [9], however, proposes a unified power control (UPC) algorithm which is applicable to a large family of multiuser detectors. This algorithm is based on the large-system results in [10] where a linear relationship between the input power and the output SIR has been shown to exist for a family of multiuser detectors. Members of this family include many well-known receivers such as the matched filter (MF), the decorrelator (DE), and the MMSE detector as well as the individually optimal (IO) and jointly optimal (JO) multiuser detectors [8]. This linear relationship, which is characterized by the multiuser efficiency, is exploited in obtaining the proposed power control algorithm. In [9], the convergence of the UPC algorithm is not proved and is only demonstrated through simulation. In this paper, we prove the convergence of the UPC algorithm for the matched filter, the decorrelator and the MMSE detector. In addition, we study the performance of the UPC algorithm in finite-size systems and compare it with that of the existing power control algorithms. Since the UPC algorithm is based on large-system results, it does not depend on instantaneous spreading sequences. Therefore, it is particularly useful in CDMA systems with long spreading sequences (e.g., the uplink of cdma2000). In systems with long spreading sequences (i.e., sequences whose period is longer than one symbol duration), other power control algorithms may need to adjust the users’ transmit powers symbol-by-symbol even when all the users’ channels stay fixed. The UPC algorithm, on the other hand, does not need to update the powers if the channel gains stay the same. Since the true SIR does depend on the spreading sequences, as the spreading sequence changes from one symbol to the next, the SIR achieved by the UPC algorithm deviates from the target SIR. However, we show that if the processing gain is reasonably large, the SIR achieved by the UPC algorithm stays close to the target SIR most of the time.

The organization of this paper is as follows. In Section 2, we provide the system model and some relevant background on multiuser detection in large systems. In Section 3, the unified power control algorithm is presented and its convergence for linear receivers is proved. The performance of the UPC algorithm in finite-size systems is studied in Section 4 and simulation results are presented in Section 5. Finally, conclusions are given in Section 6.

2 Multiuser Detection in Large Systems

We consider the uplink of a synchronous DS-CDMA system with $K$ users and processing gain $N$. Let $p_k$, $h_k$, and $\gamma_k$ represent the transmit power, channel gain and output SIR, respectively, for user $k$. Also, define

$$\Gamma_k = \frac{p_k h_k}{\sigma^2}$$

(1)

as the received signal-to-noise ratio (SNR) for user $k$ where $\sigma^2$ here is the background noise power (including other cell interference). The received signal (after chip-matched filtering) sampled at the chip rate over one symbol duration can be represented as

$$Y = \sum_{k=1}^{K} \sqrt{\Gamma_k} X_k s_k + W,$$

(2)

where $s_k$ and $X_k$ are the spreading sequence and transmitted symbol of user $k$, respectively. We assume random spreading sequences for all users, i.e., $s_k = \frac{1}{\sqrt{N}} [v_1 \ldots v_N]^T$, where the $v_i$’s are independent and identically distributed (i.i.d.) random variables taking values $\{-1, +1\}$ with equal probabilities. The $X_k$’s are assumed to be i.i.d. with probability density $p_X$. In (2), $W$ is the normalized noise vector consisting of independent standard Gaussian entries.

It is shown in [10] that in large systems with random spreading sequences, the multiuser channel combined with the uplink detector can be decoupled into equivalent parallel single-user Gaussian channels with some degradation in the SNR due to multiple-access interference as
shown in Fig. 1. By a large system, we refer to the limit in which the number of users and the spreading factor in a CDMA system both tend to infinity but with a fixed ratio, i.e.,

$$\lim_{K,N \to \infty} \frac{K}{N} = \alpha .$$  \hspace{1cm} (3)

The degradation parameter, known as the multiuser efficiency, completely characterizes the performance of each individual user and can be determined by solving some fixed-point equations, which we shall discuss shortly. This “decoupling principle” holds for a large family of detectors, called posterior mean estimators (PMEs), which includes the matched filter, the decorrelator, and the MMSE detector, as well as the individually and jointly optimal multiuser detectors. This decoupling result implies that in large systems there is a linear relationship between a user’s transmit power and its output SIR characterized by the multiuser efficiency, $\eta_k$:

$$\gamma_k = \eta_k \Gamma_k .$$  \hspace{1cm} (4)

This relationship is mainly due to the fact that in a large system, as the user’s transmit power changes, the interference seen by that user essentially stays the same as long as the overall distribution of the received powers remains the same. This result generalizes to multirate and multicarrier systems as well [11,12]. In general, the multiuser efficiency depends on the received SNRs, the spreading sequences as well as the type of detector. However, in the asymptotic case of large systems, the dependence on the spreading sequences disappears and the received SNRs affect $\eta$ only through their distribution\(^1\). Note that although (4) is true only in the large-system limit, it is a very good approximation for most finite-size systems of practical interest. In this paper, we focus on the implication of this result in designing a unified power control scheme for multiuser detectors. The linear dependence of the user’s SIR on its transmit power can also be used to generalize the results of [13] to nonlinear detectors such as the individually and jointly optimal multiuser detectors (see [9]). In [13], it has been shown that if we model power control as a non-cooperative game with a utility function that measures the energy efficiency of the network, then, for all linear receivers, this game has a unique Nash equilibrium that is SIR-balanced.

It is shown in [10] that for a particular class of PME, which contains many popular receivers such as the conventional matched filter, the decorrelator, and the MMSE detector as well as the individually and jointly optimal detectors, the large-system multiuser efficiency is obtained by solving the following joint equations:

$$\eta^{-1} = 1 + \alpha \mathbb{E}\{\Gamma \cdot \mathcal{E}(\Gamma; \eta, \xi)\},$$  \hspace{1cm} (5)

$$\xi^{-1} = \varrho^2 + \alpha \mathbb{E}\{\Gamma \cdot \mathcal{V}(\Gamma; \eta, \xi)\},$$  \hspace{1cm} (6)

where $\alpha = \lim_{K,N \to \infty} \frac{K}{N}$, and $\varrho$ is a parameter determined by the receiver type. In [5] and [6], $\mathcal{E}$ and $\mathcal{V}$ are functions that can be easily computed given the distribution of the transmitted

\(^1\)Since different users may have different receiver types, the multiuser efficiency, in general, may be different for different users.
symbols and the type of receiver; and the expectations are taken with respect to the received SNR distribution, \( P \). For some popular detectors, the multiuser efficiency is given as the solution of the following equations (see [10]):

\[
\eta_{MF} = \frac{1}{1 + \alpha E\{\Gamma\}}, \tag{7}
\]

\[
\eta_{DE} = 1 - \alpha \quad \text{for} \quad \alpha < 1, \tag{8}
\]

\[
\frac{1}{\eta_{MMSE}} = 1 + \alpha E\left(\frac{\Gamma}{1 + \eta_{MMSE} \Gamma}\right), \tag{9}
\]

\[
\frac{1}{\eta_{IO}} = 1 + \alpha E\left\{ \Gamma \left[ 1 - \int_{-\infty}^{+\infty} e^{-z^2/2} \frac{\tanh(\eta_{IO} \Gamma - z \sqrt{\eta_{IO} \Gamma})}{\sqrt{2\pi}} \, dz \right] \right\}. \tag{10}
\]

Except for (10), which assumes binary inputs (i.e., BPSK modulation), the rest are valid for all input distributions.

3 The Unified Power Control Algorithm

Recall from Section 2 that in large systems, the output SIR of user \( k \) is given by

\[
\gamma_k = \eta_k \Gamma_k = \eta_k \frac{p_k h_k}{\sigma^2} \quad \text{for} \quad k = 1, \cdots, K. \tag{11}
\]

The objective of the UPC algorithm is for each user to iteratively adjust its transmit power in order to reach an output SIR equal to \( \gamma^* \). While here we have assumed that the target SIR is the same for all users, the UPC algorithm is general in the sense that it can be applied to the case of unequal target SIRs. The algorithm is also applicable to multirate systems. The description of the UPC algorithm is as follows.

**Algorithm 1**: The Unified Power Control (UPC) Algorithm:

1. \( n=0 \), start with initial powers \( p_1(0), \cdots, p_K(0) \).
2. Based on the power profile, compute the multiuser efficiency, \( \eta_k(n) \), using (5) and (6).
3. Update the powers using \( p_k(n+1) = \frac{1}{\eta_k(n)} \frac{\gamma^* \sigma^2}{\eta_k} \) for \( k = 1, \cdots, K \).
4. \( n=n+1 \), stop if convergence; otherwise, go to Step 2.

In Step 2, while finding an analytical expression for \( \eta_k \) is difficult for most multiuser detectors, \( \eta_k \) can be easily obtained using numerical methods. Note that (9) and (10) need to be solved only once per iteration for each user. The uplink receiver (e.g., base station) can, for example, compute \( \eta_k \) and feed it back to the user terminal. Also, if all the users have the same type of receiver, \( \eta_k \) will be independent of \( k \) and, hence, we need to solve for \( \eta \) only once per iteration which greatly reduces the computational complexity of the algorithm. The above algorithm is applicable to a large family of receivers which includes many popular receivers such as the matched filter, the decorrelator, and the MMSE detector as well as individually and jointly optimal multiuser detectors. In actual implementation of the algorithm, the expectations in (9) and (10) can be replaced by summations over all users’ received SNRs (or their estimates). For example, (9) can be expressed as

\[
\eta_{MMSE} = 1 + \frac{1}{K} \sum_{k=1}^{K} \frac{1}{\eta_k} \frac{\Gamma_k}{1 + \eta_{MMSE} \Gamma_k},
\]

We now prove the convergence of the UPC algorithm for the matched filter, the decorrelator, and the MMSE detector\(^2\). To prove the convergence, let \( \Gamma = [\Gamma_1, \cdots, \Gamma_K] \) and let us define an interference function, \( I(\Gamma) = [I_1(\Gamma), \cdots, I_K(\Gamma)] \), where

\[
I_k(\Gamma) = \frac{\gamma^*}{\eta_k(\Gamma)}. \tag{12}
\]

\(^2\)The convergence proof for a general receiver remains an open problem. To prove the convergence in the general case, one has to deal directly with (5) and (6) which are difficult to work with.
Here, we have explicitly shown the dependence of $\eta$ on $\Gamma$. Also, when we write $\Gamma' \geq \Gamma$, we mean that $\Gamma'_k \geq \Gamma_k$ for $k = 1, \cdots, K$. Now, recall that $\Gamma_k = \frac{P_k b_k}{\sigma^2}$. Hence, based on (12), the UPC algorithm can be expressed as

$$\Gamma(n + 1) = I(\Gamma(n)). \quad (13)$$

**Proposition 1** For the matched filter, the decorrelator, and the MMSE detector, if there exists a $\tilde{\Gamma}$ such that $\tilde{\Gamma} \geq I(\Gamma)$, then for every initial vector $\Gamma(0)$, the sequence $\Gamma(n + 1) = I(\Gamma(n))$ converges to the unique fixed point solution of $\Gamma^* = I(\Gamma^*)$. Furthermore, $\Gamma^* \leq \tilde{\Gamma}$ for all $\tilde{\Gamma} \geq I(\Gamma)$.

**Proof:** The condition that there exists a $\tilde{\Gamma} \geq I(\Gamma)$ states that a feasible SNR vector exists for achieving $\gamma^*$. To prove the proposition, it is sufficient to show that $I(\Gamma)$ is a standard interference function (see [3]), i.e., for all $\Gamma \geq 0$, the following three properties are satisfied.

1) **Positivity:** $I(\Gamma) > 0$; 2) **Monotonicity:** If $\Gamma' \geq \Gamma$, then $I(\Gamma') \geq I(\Gamma)$; 3) **Scalability:** For all $\theta > 1$, $\theta I(\Gamma) > I(\theta \Gamma)$.

The dependence of the multiuser efficiency on $k$ is due to the fact that different users may have different receivers. However, we can assume, without loss of generality, that all users have the same receiver type (and hence the same multiuser efficiency). Therefore, to prove the proposition, it suffices to show that for each receiver type, the three properties (i.e., positivity, monotonicity, and scalability) are satisfied for $\hat{I}(\Gamma) = \frac{1}{\eta + \eta'}$.

Positivity of $\hat{I}(\Gamma)$ is trivial by [5] for all receivers since $\eta \in [0, 1]$.

For the matched filter, the multiuser efficiency is given by [7], i.e., $\eta = \frac{1}{1 + \alpha E\{\Gamma\}}$. Now, if $\Gamma' \geq \Gamma$, then $E\{\Gamma'\} \geq E\{\Gamma\}$. Therefore, $\hat{I}(\Gamma') \geq \hat{I}(\Gamma)$. To prove the third property, note that, for $\theta > 1$, $\hat{I}(\theta \Gamma) = 1 + \alpha E\{\theta \Gamma\} = \theta \hat{I}(\Gamma)$.

For the decorrelating detector, the multiuser efficiency is given by [8], i.e., $\eta = 1 - \alpha$ for $\alpha < 1$. Since in this case, $\eta$ is independent of $\Gamma$, proving properties 2 and 3 is straightforward.

For the MMSE detector, the multiuser efficiency is the solution to [9], or equivalently, the solution of $\eta + \alpha E\left\{\frac{1}{\eta + \eta'}\right\} = 1$. Note that the left-hand side increases if both $\eta$ and $\Gamma$ increase. Thus if $\Gamma' \geq \Gamma$, we must have $\eta(\Gamma') \leq \eta(\Gamma)$ to maintain the equality. Hence, $\hat{I}(\Gamma') \geq \hat{I}(\Gamma)$. To prove the third property, let us define $\eta' = \eta(\theta \Gamma)$ and $\eta'' = \theta \eta'$, where $\theta > 1$.

Therefore, we have $\eta' + \alpha E\left\{\frac{1}{\eta' + 1}\right\} = 1$, or equivalently, $\eta'' + \alpha \theta E\left\{\frac{1}{\eta'' + 1}\right\} = \theta$. Showing $\theta \hat{I}(\Gamma) > \hat{I}(\theta \Gamma)$ is equivalent to showing $\eta'' > \eta$. Since $\eta'' + \alpha E\left\{\frac{1}{\eta'' + 1}\right\} = 1 + (1 - \frac{1}{\theta}) \eta'' > 1$ and $\eta + \alpha E\left\{\frac{1}{\eta + 1}\right\} = 1$, and because $\eta + \alpha E\left\{\frac{1}{\eta + 1}\right\}$ is increasing in $\eta$, we must have $\eta'' > \eta$.

Therefore, $\theta \hat{I}(\Gamma) > \hat{I}(\theta \Gamma)$.

This completes the proof. \hfill \Box

In the following section, we study the performance of the UPC algorithm for finite-size systems and compare it with that of existing power control algorithms. A more detailed study of the UPC algorithm can be found in [14].

### 4 Performance Evaluation and Discussion

The existing SIR-based power control algorithms such as the ones proposed in [1] and [4], update the transmit powers of the users according to

$$p_k(n + 1) = \frac{\gamma^*}{\gamma_k(n)} p_k(n), \quad (14)$$
where $\gamma_k$ is the output SIR of user $k$. For the matched filter, the decorrelator, and the MMSE detector, $\gamma_k$ is expressed as

$$
\gamma_k^{MF} = \frac{p_k h_k}{\sigma^2 + \sum_{j \neq k} p_j h_j (s_k^T s_j)^2},
$$

(15)

$$
\gamma_k^{DE} = \frac{p_k h_k}{\sigma^2 \left[ (S^T S)^{-1} \right]_{kk}},
$$

(16)

$$
\gamma_k^{MMSE} = p_k h_k (s_k^T A_k^{-1} s_k),
$$

(17)

where $S = [s_1, s_2, \ldots, s_K]$, $A_k = \sum_{j \neq k} p_j h_j s_j^T + \sigma^2 I$, and $\left[ (S^T S)^{-1} \right]_{kk}$ is the $(k, k)$ entry of the matrix $(S^T S)^{-1}$.

The SIR-based power control algorithm in (14) cannot be easily applied to the optimal multiuser receivers since finding the output SIR for these receivers is not straightforward. In addition, since the expressions for the output SIR are all dependent on the spreading sequences of the users, in systems with long spreading sequences, the SIR-based algorithm in (14) has to continuously adjust the users’ transmit powers as the spreading sequences change from symbol to symbol even if the channel gains stay unchanged. The UPC algorithm, on the other hand, is a large-system approach and is, hence, independent of the users’ spreading sequences. Therefore, after convergence, the users’ transmit powers need not be updated as long as the channel gains stay the same. Obviously, the true SIR does depend on the spreading sequences (as shown in (15)–(17)). A question of interest is: if we use the UPC algorithm, how close will the resulting SIRs be to the target SIR? To answer this question, we focus on the decorrelating and MMSE detectors.

### 4.1 Decorrelating Detector

We proved via Proposition 1 that the UPC algorithm converges to the fixed point solution of $\Gamma^* = I(\Gamma^*)$. For the decorrelating detector with $\alpha < 1$, the multiuser efficiency is given by $\eta^{DE} = 1 - \alpha$. As a result, we have $\Gamma_k^* = \frac{\gamma}{1-\alpha}$, for $k = 1, \ldots, K$. Therefore, based on (16), the true output SIR for the decorrelating detector, in this case, is given by

$$
\gamma_k = \left(\frac{\gamma}{1-\alpha}\right) \left[ (S^T S)^{-1} \right]_{kk}.
$$

(18)

It can be shown that in systems with large processing gains, the distribution of $\left[ (S^T S)^{-1} \right]_{kk}$ can be approximated by a beta distribution with parameters $(N - K + 1, K - 1)$ [15]. As a result, for the decorrelator, the probability density function (PDF) of $\gamma_k$ is given approximately by

$$
f_{\gamma_k}(z) = \left(\frac{1}{\Gamma_{DE}^*}\right)^{N-1} z^{N-K} (\Gamma_{DE}^* - z)^{K-1} B(N-K+1, K-1) \text{ with } z \leq \Gamma_{DE}^*,
$$

(19)

where $B(a, b) = \int_0^1 t^{a-1} (1 - t)^{b-1} dt$ and $\Gamma_{DE}^* = \frac{\gamma}{1-\alpha}$. Therefore, as the spreading sequences change from symbol to symbol, the probability that $\gamma_k$ stays within $\Delta$ dB of $\gamma^*$ is given by

$$
P_{\Delta,DE} \equiv \Pr \{ |\gamma_{DE}(dB) - \gamma^*(dB)| \leq \Delta \} = \int_{\gamma_L}^{\gamma_H} f_{\gamma_k}(z) dz,
$$

(20)

where $\gamma_L = 10^{-\Delta \gamma^*} \gamma^*$ and $\gamma_H = 10^{\Delta \gamma^*} \gamma^*$.

### 4.2 MMSE Detector

If all users have the same target SIR, $\gamma^*$, the steady-state SNRs will be identical after the UPC algorithm converges (i.e., $\Gamma_1^* = \cdots = \Gamma_K^* = \Gamma^*$). The multiuser efficiency in this case will be given by
\[ \eta_{\text{MMSE}} = \frac{1 - \alpha}{2} - \frac{1}{2\Gamma_{\text{MMSE}}^*} + \sqrt{\left(1 - \frac{\alpha}{2}\right)^2 + \frac{1}{2\Gamma_{\text{MMSE}}^*} + \left(\frac{1}{2\Gamma_{\text{MMSE}}^*}\right)^2}, \tag{21} \]

with \(\Gamma_{\text{MMSE}}^* = \frac{\gamma_{\text{MMSE}}^*}{1 - \alpha(1 + \frac{1}{\gamma_{\text{MMSE}}^*})}\) assuming that \(\alpha < 1 + \frac{1}{\gamma_{\text{MMSE}}^*}\). It can be shown that for the MMSE detector, the fluctuation of the true SIR around \(\gamma_{\text{MMSE}}^*\) is approximately Gaussian with variance \(\frac{1}{2\gamma_{\text{MMSE}}^*}\), i.e., \(\gamma_{\text{MMSE}} \sim \mathcal{N}(\gamma_{\text{MMSE}}^*, \frac{1}{2\gamma_{\text{MMSE}}^*})\). Hence, the probability that \(\gamma_k\) stays within \(\Delta\) dB of \(\gamma_{\text{MMSE}}^*\) is given approximately by

\[ P_{\Delta,\text{MMSE}} \equiv \Pr\{\left|\gamma_{\text{MMSE}}(\text{dB}) - \gamma_{\text{MMSE}}^*(\text{dB})\right| \leq \Delta\} \simeq \Phi\left(\sqrt{\frac{N}{c}} (\gamma_H - \gamma_{\text{MMSE}}^*)\right) - \Phi\left(\sqrt{\frac{N}{c}} (\gamma_L - \gamma_{\text{MMSE}}^*)\right), \tag{22} \]

where \(\Phi(\cdot)\) is the cumulative distribution function of the standard normal distribution.

It is seen that for both the decorrelator and the MMSE detector, the variance of fluctuations of SIR decreases as \(1/N\). In the following section, we demonstrate the performance of the UPC algorithm using simulations and also investigate the accuracy of the theoretical approximations discussed above.

## 5 Numerical Results

We consider the uplink of a DS-CDMA system with \(K\) users and processing gain \(N\), with random (long) spreading sequences. The background noise power, \(\sigma^2\), is assumed to be \(1.6 \times 10^{-13}\) Watts and the target SIR, \(\gamma^*\), is equal to 6.4 (i.e., 8.1 dB). Our choice for the target SIR comes from the results in [13].

We first demonstrate the convergence of the UPC algorithm by considering a system with 8 users and spreading factor 32 (i.e., \(K = 8\) and \(N = 32\)). The channel gain for user \(k\) is given by \(h_k = 0.1d_k^{-4}\) where \(d_k\) is the distance of user \(k\) from the uplink receiver (e.g., base station) and is assumed to be given by \(d_k = 100 + 10k\) in meters. We implement the UPC algorithm for the decorrelator and the MMSE detector as well as the maximum likelihood (ML) detector (which is equivalent to the jointly optimal detector). In Fig. 2 we show the transmit powers for users 1, 4, and 8 at the end of each iteration. It is seen that for all three receiver types, the UPC algorithm converges very quickly to steady-state values. The results are similar when the initial power values and/or \(K\) and \(N\) are changed. It is also observed that the steady-state transmit powers for the decorrelator and the MMSE detector are close to those of the ML detector (in this case, the difference is less than 22%). This means that in terms of energy efficiency\(^3\), the decorrelator and the MMSE detector are almost as good as the ML detector.

We now compare the performance of the UPC algorithm with that of the SIR-based algorithm of [14]. Fig. 3 shows the SIR and bit-error-rate (BER) of user 1 for the UPC and SIR-based algorithms for the MMSE detector. It is seen that the SIR-based algorithm achieves the target SIR, \(\gamma^*\), at all time whereas the output SIR for the UPC algorithm fluctuates around the target SIR as the spreading sequences change. It should be noted that the BER values in this plot fluctuate around \(Q(\sqrt{\gamma^*}) = 1 - \Phi(\sqrt{\gamma^*}) = 0.006\) which is the BER corresponding to \(\gamma^* = 6.4\) (assuming additive Gaussian noise/interference).

To evaluate the accuracy of the theoretical approximations given in Section 4 we have plotted the cumulative probability distribution functions (CDFs) of \(\gamma\) for the decorrelating and MMSE detectors for different processing gains with both low and high system loads in Fig. 4. In this figure, we have plotted the CDFs obtained from simulation (based on 100,000 realizations) as well as those predicted by the theoretical approximations. It is seen from the figure that the theoretical approximations become more accurate as the processing gain increases. Also,

\(^3\)We define energy efficiency as the utility (in bits per joule) achieved by the users in the network at the Nash equilibrium. The utility function of a user is the ratio of the user’s throughput to its transmit power (see [14]).
Figure 2: Users’ transmit powers for the ML, MMSE, and decorrelating detectors, using the UPC algorithm ($N = 32$ and $K = 8$).

Figure 3: User 1 output SIR and BER for the UPC algorithm and SIR-based algorithm with the MMSE detector ($N = 32$ and $K = 8$).
in general, the approximations are more accurate when the system load is low. This figure suggests that the UPC algorithm is more useful when the processing gain is high and/or the system load is low.

To quantify the discrepancies between the simulation results and the theoretical approximations, we have computed $P_{\Delta,DE}^{\delta}$ and $P_{\Delta,MMSE}^{\delta}$ using the CDFs obtained from simulation as well as those predicted by theory (see (20) and (22)). Table 1 shows the results for different processing gains and system loads for $\Delta = 1$ dB. The numbers in the table represent the probability that $\gamma$ is within 1 dB of $\gamma^*$. The probabilities obtained by simulation suggest that the UPC algorithm performs better for the MMSE detector than for the decorrelator. It is also seen from the table that when the processing gain is small, the fluctuation in the output SIR is considerable, especially when the system load is high. The performance improves as the processing gain increases. For example, for the MMSE detector, when $N = 256$ and $\alpha = 0.75$, the SIR stays within 1 dB of the target SIR 98% of the time. It is also observed that the theoretical approximations are optimistic for the decorrelator and very pessimistic for the MMSE detector.
6 Conclusions

A unified power control (UPC) algorithm which is applicable to a large family of detectors including many of the most widely studied multiuser detectors has recently been proposed. In this work, we have studied the convergence and performance of the UPC algorithm. In particular, we have proved the convergence of the algorithm for the matched filter, the decorrelator, and the (linear) MMSE detector. In addition, the performance of the algorithm in finite-size systems with long spreading sequences has been studied and compared with that of the existing power control schemes. Theoretical approximations for predicting the performance of the UPC algorithm have been presented and their accuracies have been investigated using simulations. We have shown that in systems where achieving the target SIR is crucial to the performance, the UPC algorithm is useful primarily when the processing gain is large and/or the system load is small.

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