Recoilless resonant absorption of a monochromatic neutrino beam for measuring $\Delta m_{31}^2$ and $\theta_{13}$

Hisakazu Minakata$^{1,2,3}$ and Shoichi Uchinami$^1$

$^1$ Department of Physics, Tokyo Metropolitan University, 1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, Japan
$^2$ Abdus Salam International Center for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy
E-mail: minakata@phys.metro-u.ac.jp and uchinami@phys.metro-u.ac.jp

New Journal of Physics 8 (2006) 143
Received 18 April 2006
Published 24 August 2006
Online at http://www.njp.org/
doi:10.1088/1367-2630/8/8/143

Abstract. We discuss, in the context of precision measurement of $\Delta m_{31}^2$ and $\theta_{13}$, physics capabilities enabled by the recoilless resonant absorption of a monochromatic antineutrino beam enhanced by the Mössbauer effect recently proposed by Raghavan. Under the assumption of a small relative systematic error of the level of a few tenths of a per cent between measurements at different detector locations, we give analytical and numerical estimates of the sensitivities to $\Delta m_{31}^2$ and $\sin^2 2\theta_{13}$. The accuracies of their determination are enormous; the fractional uncertainty in $\Delta m_{31}^2$ achievable by ten point measurement is 0.6% (2.4%) for $\sin^2 2\theta_{13} = 0.05$, and the uncertainty of $\sin^2 2\theta_{13}$ is 0.002 (0.008) both at 1σ confidence level (CL) with the optimistic (pessimistic) assumption of systematic error of 0.2% (1%). The former opens a new possibility of determining the neutrino mass hierarchy by comparing the measured value of $\Delta m_{31}^2$ with that from accelerator experiments, while the latter will help to resolve the $\theta_{23}$ octant degeneracy.

3 Author to whom any correspondence should be addressed.
1. Introduction

Recently, the intriguing possibility was suggested by Raghavan [1, 2] that the resonant absorption reaction

\[ \bar{\nu}_e + ^3\text{He} + \text{orbital } e^- \rightarrow ^3\text{H} \]  

(1)

with simultaneous capture of an atomic orbital electron can be dramatically enhanced. The key idea is to use a monochromatic \( \bar{\nu}_e \) beam with the energy 18.6 keV from the inverse reaction \( ^3\text{H} \rightarrow \bar{\nu}_e + ^3\text{He} + \text{orbital } e^- \), by which the resonance condition is automatically satisfied. (See [4, 5] for earlier suggestions.) He then suggested an experiment to measure \( \theta_{13} \) by utilizing the ultra low-energy monochromatic \( \bar{\nu}_e \) beam. Though similar to the reactor \( \theta_{13} \) experiments [6]–[8], the typical baseline length is of order 10 m due to the much lower energy of the beam by a factor of \( \simeq 150 \), making it achievable in the laboratories. The mechanism, in principle, would work with a more generic setting in which \( ^3\text{H} \) and \( ^3\text{He} \) in (1) are replaced by nuclei \( A(Z) \) and \( A(Z + 1) \).

The author of [1, 2] then went on to an even bolder proposal of enhancement by a factor of \( \sim 10^{11} \) by embedding both \( ^3\text{H} \) and \( ^3\text{He} \) into solids [4, 5] by which the broadening of the beam due to nuclear recoil is severely suppressed by a mechanism similar to the Mössbauer effect [9]. Then, the event rate of the \( \theta_{13} \) experiment is enhanced by the same factor, allowing an extremely high counting rate. Thanks to the ultimate energy resolution of \( \Delta E_\nu/E_\nu \simeq 2 \times 10^{-17} \) enabled by the recoilless mechanism, he was able to propose a table top experiment to measure the gravitational red shift of neutrinos, a neutrino analogue of the Pound–Rebk experiment for photons [10].
In this paper, we examine possible physics potentials of the $\theta_{13}$ experiment proposed in [1, 2]. The characteristic feature of the experiment, which clearly marks the difference from the reactor $\theta_{13}$ measurement, is the use of a monochromatic beam apart from the shorter baseline by a factor of $\simeq 150$. Then, the most interesting question is how accurately $\Delta m^2_{31}$ can be determined. Note that even without recoilless setting, the beam energy width is of the order of $\Delta E_\nu/E_\nu \sim 10^{-5}$, and it can be ignored for all practical purposes. It is also interesting to explore the accuracy of the $\theta_{13}$ measurement. In addition to possible extremely high statistics, the baseline as short as $\sim 10$ m should allow us to utilize the setting of a continuously movable detector, which was once proposed in a reactor $\theta_{13}$ experiment [11] but that did not survive in the (semi-) final proposal. The method will greatly help to reduce the experimental systematic uncertainties of the measurement.

We will show in our analysis that the accuracies one can achieve for $\Delta m^2_{31}$ and $\theta_{13}$ determination by recoilless resonant absorption are enormous. At $\sin^2 2\theta_{13} = 0.05$, for example, the fractional uncertainty in $\Delta m^2_{31}$ determination is 0.6% (2.4%) and the uncertainty of $\sin^2 2\theta_{13}$ is 0.002 (0.008) both at 1$\sigma$ CL under an optimistic (pessimistic) assumption of systematic error of 0.2% (1%).

What is the scientific merit of such precision measurement of $\Delta m^2_{31}$ and $\theta_{13}$? With a 1% level precision of $\Delta m^2_{31}$, the method for determining neutrino mass hierarchy by comparing between the two effective $\Delta m^2$ measured in reactor and accelerator (or atmospheric) disappearance measurements [12, 13] would work, opening another door for determining the neutrino mass hierarchy. It is also proposed [7] that the $\theta_{23}$ octant degeneracy can be resolved by combining reactor measurements of $\theta_{13}$ with accelerator disappearance (appearance) measurements of $\sin^2 2\theta_{23} (s_{23}^2 \sin^2 2\theta_{13})$.4 (See [16, 19] for earlier qualitative suggestions.) The results of the recent quantitative analysis [20], however, indicate that the resolving power of the method is limited at small $\theta_{13}$ primarily because of the uncertainties in reactor measurement of $\theta_{13}$. Therefore, the highly accurate measurement of $\Delta m^2_{31}$ and $\theta_{13}$ which is enabled by using the resonant absorption reaction should help resolve the mass hierarchy and the $\theta_{23}$ degeneracies.

In section 2, we discuss ‘conceptual design’ of the possible experiments. In section 3, we define the statistical method for our analysis. In section 4, we present a numerical analysis of the sensitivities of $\theta_{13}$ and $\Delta m^2_{31}$ measurement. In section 5, we complement the numerical estimate in section 4 by giving an analytic estimate of the sensitivities. In section 6, we give some remarks on the implications of our results. In the appendix, we give a general formula for the inverse of the error matrix.

2. Which kind of $\theta_{13}$ experiment?

We give preliminary discussions on which kind of setting is likely to be the best one for an experiment to measure $\Delta m^2_{31}$ and $\theta_{13}$ with use of an ultra-low-energy monochromatic $\bar{\nu}_e$ beam. In this section, we rely on the one-mass scale dominant [21] (or the two-flavour) approximation of the neutrino oscillation probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, though we use the full three-flavour expression

4 Here is a short summary for the parameter degeneracy. It is the phenomenon that there exist multiple solutions for mixing parameters, $\theta_{13}, \theta_{23}$ and $\delta$, for a given set of measurement of $\nu_\mu$ disappearance, $\nu_e$ and $\bar{\nu}_e$ appearance probabilities, and the octant ambiguity of $\theta_{23}$ is among them. The nature of the degeneracy may be characterized as the intrinsic degeneracy of $\theta_{13}$ and $\delta$ [14], which is duplicated by the unknown sign of $\Delta m^2_{31}$ [15] and the octant ambiguity of $\theta_{23}$ for a given $\sin 2\theta_{23}$ [16]. For an overview, see e.g., [17, 18].
In our quantitative analysis performed in section 4. It reads

\[ P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m^2_{31} L}{4E} \right), \]  

(2)

where the neutrino mass squared difference is defined as \( \Delta m^2_{ji} \equiv m_j^2 - m_i^2 \) with neutrino masses\(^5\) \( m_i \) \((i = 1–3)\) and \( L \) is the distance from source to detector. With \( E_\nu = 18.6 \text{ keV} \), the first oscillation maximum (minimum in \( P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \)) is reached at the baseline distance

\[ L_{OM} = 9.2 \left( \frac{\Delta m^2_{31}}{2.5 \times 10^{-3} \text{ eV}^2} \right)^{-1} \text{ m.} \]  

(3)

While the current value of \( \Delta m^2_{31} \) which comes from the atmospheric [22] and the accelerator [23] measurement has large uncertainties, it should be possible to narrow down the value, thanks to the ongoing and the forthcoming disappearance measurement by MINOS [24] and T2K [25] experiments. Furthermore, the experiment considered in this paper is powerful enough to determine both quantities accurately at the same time, if detector locations are appropriately chosen.

The whole discussion of the \( \theta_{13} \) experiment must be preceded by the test measurement at \( \sim 10 \text{ cm} \) or so to verify the principle, namely to demonstrate that the mechanism of resonant enhancement proposed in [1, 2] is indeed at work. At the same time, the flux times cross-section must be measured to check the consistency of the Monte Carlo estimate. Then, one can go on to the measurement of \( \theta_{13} \) and \( \Delta m^2_{31} \), and possibly other quantities. Because of the expected high statistics of the experiment it is natural to think about using spectrum information. In the case of a monochromatic beam, this amounts to considering measurements at several different detector locations.

Let us estimate the event rate. Although the precise rate is hard to estimate, the numbers displayed below will give the reader a feeling on what would be the timescale for the experiment. The \( \bar{\nu}_e \) flux from \(^3\)H source with strength \( S \text{ MCI} \) due to bound state beta decay is given by

\[ f_{\bar{\nu}_e} = 1.4 \times 10^{7} \left( \frac{S}{1 \text{ MCI}} \right) \left( \frac{L}{10 \text{ m}} \right)^{-2} \text{ cm}^{-2} \text{ s}^{-1}, \]  

(4)

where the ratio of bound state beta decay to free space decay is taken to be \( 4.7 \times 10^{-3} \) based on [26]. The rate of the resonant absorption reaction can be computed by using cross-section \( \sigma_{\text{res}} \) and number of target atoms \( N_T \) as \( R = N_T f_{\bar{\nu}_e} \sigma_{\text{res}} \). Without the Mössbauer enhancement, the cross-section is estimated to be \( \sigma_{\text{res}} \simeq 10^{-42} \text{ cm}^2 \) [1, 2] based on [3]. Then, the rate with target mass \( M_T \) without neutrino oscillation is given by

\[ R = 2.4 \times 10^{-4} \left( \frac{SM_T}{1 \text{ MCI kg}} \right) \left( \frac{L}{10 \text{ m}} \right)^{-2} \text{ day}^{-1}. \]  

(5)

\(^5\) When we speak about discriminating the neutrino mass hierarchy by comparing the two ‘large’ \( \Delta m^2 \) measured in \( \bar{\nu}_e \) disappearance and \( \nu_\mu \) disappearance channels, one has to be careful about the definition of \( \Delta m^2 \) which enters into the survival probabilities [12]. While keeping this point in mind, we do not try to elaborate the expressions of \( \Delta m^2 \) this paper by just writing it as \( \Delta m^2_{31} \) in the \( \bar{\nu}_e \) disappearance channel which may be interpreted as \( \Delta m^2_{\text{eff}1} \) in [12].
An improved estimate in [2] entailed a factor of $\simeq 10^{11}$ enhancement of the cross-section by the Mössbauer effect after the source and the target are embedded into solids. Assuming the enhancement factor, $\sigma_{\text{res}} \simeq 5 \times 10^{-32} \text{cm}^2$ and the rate becomes

$$R_{\text{enhanced}} = 1.2 \times 10^4 \left( \frac{SM_T}{1 \text{MCi g}} \right) \left( \frac{L}{10 \text{m}} \right)^{-2} \text{day}^{-1}. \quad (6)$$

Therefore, one obtains about $1.2 \times 10^6$ events per day for a 1 MCI source and 100 g $^3\text{He}$ target at a baseline distance $L = 10$ m. If the enhancement factor is not reached, the running time for collecting the same number of events becomes longer accordingly.

Thus, once the $^3\text{He}$ (and much easier $^3\text{H}$) implementation into solid is achieved, the event rate is sufficient. The real issue for high sensitivity measurement of $\Delta m_{31}^2$ and $\sin^2 2\theta_{13}$ is whether the produced $^3\text{H}$ can be counted directly without waiting for decay back to $^3\text{He}$ by emitting an electron. This is because the long lifetime of 12.33 years [27] of $^3\text{H}$ makes it impossible to identify in which period the decayed $^3\text{H}$ was produced, resulting in errors of the event rate at each detector location. Possibilities of real-time counting and direct counting by extracting $^3\text{H}$ atoms are mentioned in [2]. In this paper, we assume that at least one such method works, and it offers the opportunity of direct counting of events. Note that the detection efficiency need not be high because of huge number of events. What is important is the time-stable counting rate which allows relative systematic errors between measurements at different detector locations small enough.

3. Statistical method for analysis

In this section, we define the statistical procedure for our analysis to estimate the sensitivities of $\Delta m_{31}^2$ and $\sin^2 2\theta_{13}$ to be carried out in the following sections. We aim at illuminating general properties of the $\chi^2$ under the assumption of the small uncorrelated systematic errors compared to the correlated ones.

3.1. Definition of $\chi^2$ and characteristic properties of errors

We consider measurement at $n$ different distances $L = L_i$ ($i = 1, 2, \ldots, n$) from the source. Then, the appropriate form of $\Delta \chi^2$ which is suited for analytic study [28] and is simply denoted as $\chi^2$ hereafter, is as follows:

$$\chi^2 = \sum_{i=1}^{n} \left[ \frac{N_{\text{obs}} - (1 + \alpha)N_{\text{exp}}}{N_{\text{exp}}^2} \right]^2 + \left( \frac{\alpha}{\sigma_c} \right)^2$$

where $N_{\text{obs}}$ is the number of events computed with the values of parameters given by nature, and $N_{\text{exp}}$ is the one computed with a certain trial set of parameters. $\sigma_c$ is the systematic error common to measurement at $n$ different distances, the correlated error, whereas $\sigma_{\text{sys},i}$ indicate errors that cannot be attributed to $\sigma_c$, the uncorrelated errors. Examples of the former and latter errors are as follows:

1. $\sigma_c$ (correlated error): uncertainties in number of target $^3\text{He}$ atoms, errors in counting the number of produced tritium nuclei, errors in calculating resonant absorption cross-section, errors in estimating the efficiency of counting tritium nuclei.
2. $\sigma_{\text{usys},i}$ (uncorrelated error): possible time dependences of number of decaying tritium nuclei and detection efficiency of events.

Since we consider a moving detector setting the list of the possible uncorrelated systematic errors is quite limited. If a near detector with an identical structure to a movable far detector exists the error can, in principle, be vanishingly small. One may think of errors of the order of 0.1–0.3%. This is because the flux times cross-section can be monitored in real time by a near detector. In fact, similar values for uncorrelated systematic error are adopted in sensitivity estimates of some of the reactor $\theta_{13}$ experiments such as the Braidwood, the Daya Bay, and the Angra projects [29]–[31]. In near future experiments, somewhat larger values are taken, 0.6% in the Double–Chooz project [32] and 0.35% in KASKA [33].

On the other hand, it may not be so easy to control the correlated systematic error $\sigma_c$. The number of $^3$H nuclei may be measured when they are implemented into solid. The number of target nuclei times the resonant absorption cross-section may be measured in a research and development stage with a near detector. Therefore, we suspect that the largest error may come from uncertainty in the counting rate of the produced $^3$H nuclei. Of course, a reliable estimate of systematic errors $\sigma_c$ and $\sigma_{\text{usys}}$ requires specification of the site to estimate the background caused by $^n$H reaction etc. But it can be experimentally measured by the source on and off procedure, as pointed out in [1]. Lacking definitive numbers for $\sigma_c$ at the moment, we use a tentative value $\sigma_c = 10\%$ throughout our analysis. We have checked that the results barely change even if we use the more conservative number $\sigma_c = 20\%$.

If direct counting of $^3$H atoms does not work, we may have to expect much larger systematic errors, because one has to extract the event rate at each detector location only by fitting the decay curve. In this case, determination of baseline-dependent event rates would be more and more difficult for a larger number of detector locations. Probably a better strategy without direct counting would be to place multiple identical detectors (or of the same structure) at appropriate baseline distances. Even in this case, it is quite possible that the uncorrelated systematic error $\sigma_{\text{usys}}$ can be controlled to 1% level, as expected in a variety of reactor $\theta_{13}$ experiments [8].

### 3.2. Approximate form of $\chi^2$ with hierarchy in errors

By eliminating $\alpha$ through minimization the $\chi^2$ can be written as

$$
\Delta \chi^2 = \bar{x}^T V^{-1} \bar{x},
$$

where $\bar{x}$ is defined as

$$
\bar{x}^T = \left[ \frac{N_1^{\text{obs}} - N_1^{\text{exp}}}{N_1^{\text{exp}}}, \frac{N_2^{\text{obs}} - N_2^{\text{exp}}}{N_2^{\text{exp}}}, \ldots, \frac{N_n^{\text{obs}} - N_n^{\text{exp}}}{N_n^{\text{exp}}} \right].
$$

Using the general formula given in the appendix, $V^{-1}$ is given by

$$
(V^{-1})_{ij} = \frac{\delta_{ij}}{\sigma_{\alpha i}^2} - \frac{1}{\sigma_{\alpha i}^2 \sigma_{\alpha j}^2} \left[ 1 + \left( \sum_{k=1}^{n} \frac{1}{\sigma_{\alpha k}^2} \right) \sigma_c^2 \right].
$$
where $\sigma_{ui}^2 \equiv \sigma_{usys,i}^2 + 1/N_i^{exp}$. By construction, the $\chi^2$ depends upon $\sigma_{usys,i}$ and $N_i^{exp}$ only through this combination. Therefore, the particular case that will be taken in the next section, in fact, includes many cases with different event number but with the same $\sigma_u$.

Under the approximation $\sigma_{ui}^2 \ll \sigma_c^2$, $V^{-1}$ simplifies;

$$
(V^{-1})_{ij} = \delta_{ij} \frac{1}{\sigma_{ui}^2} - \frac{1}{\sigma_{ui}^2 \sigma_{aj}^2 \left( \sum_{k=1}^{n} \frac{1}{\sigma_{uk}^2} \right)}. \quad (11)
$$

The remarkable feature of (11) is the ‘scaling behaviour’ in which $\chi^2$ is independent of the correlated error $\sigma_c$, and the sensitivity to $\sin^2 2\theta_{13}$ and $\Delta m_{31}^2$ can be made higher as the uncorrelated systematic errors as well as the statistical error become smaller. It may be counter-intuitive because the leading term of the error matrix $V$ is of order $\sigma_c^2$. (See the appendix.) It is due to the singular nature of the leading order matrix, as noted in [34].

4. Estimation of sensitivities of $\Delta m_{31}^2$ and $\theta_{13}$

We now examine the sensitivities of $\Delta m_{31}^2$ and $\sin^2 2\theta_{13}$ achievable by the recoilless resonant absorption of monochromatic $\bar{\nu}_e$ enhanced by the Mössbauer effect. The numerical estimate of the sensitivities in this section will be followed by that by the analytic method in section 5.

The setting of the movable detector and the expected high statistics of the experiment make it possible to consider the situation that an equal number of events are taken at each detector location. Of course, the farther the detector from the source, the longer an exposure will take.

In the following analysis, the number of events is assumed to be $10^6$ at each detector location. Of course, the farther the detector from the source, the longer an exposure will take. Given the rate in (6), and assuming that direct counting works, it is obtainable in $\sim 10$ days for a $100 \text{g} \ ^3\text{He}$ target even if the detector is located at the second oscillation maximum, $L = 3L_{OM}$. On the other hand, the number of events $10^6$ is sufficient for our purpose because it is unlikely that the uncorrelated systematic errors can be made much smaller than 0.1%.

We take a ‘common-sense approach’ to determine the locations of the detectors and postpone the discussion of the optimization problem. We examine the following four types of Run: Run I, IIA, IIB, and III, for the measurement.

1. Run I: measurement at five detector positions, $L = \frac{1}{2}L_{OM}, \frac{7}{2}L_{OM}, L_{OM}, \frac{9}{2}L_{OM}$, and $\frac{5}{2}L_{OM}$ are considered so that $\Delta \equiv \Delta m_{31}^2 L/4E = \pi/10, 3\pi/10, \pi/2, 7\pi/10, 9\pi/10$ and $10$ are covered.

2. Run IIA: a setting for precision determination of $\Delta m_{31}^2$ by measurement at ten detector positions: $L = L_i$ ($i = 1, \ldots, 10$) where $L_{i+1} = L_i + \frac{1}{3}L_{OM}$ and $L_1 = \frac{1}{3}L_{OM}$ so that the range $\Delta = 0$ to $\pi$ is covered.

3. Run IIB: a setting for precision determination of $\Delta m_{31}^2$ by measurement at ten detector positions: $L = L_i$ ($i = 1, \ldots, 10$) where $L_{i+1} = L_i + \frac{2}{3}L_{OM}$ and $L_1 = \frac{1}{3}L_{OM}$ so that the entire period, $\Delta = 0$ to $2\pi$, is covered.

4. Run III: a setting of 20 detector positions: $L = L_i$ ($i = 1, \ldots, 20$) where $L_{i+1} = L_i + \frac{1}{2}L_{OM}$ and $L_1 = \frac{1}{2}L_{OM}$ (\(\Delta = 0 - 2\pi\)). This is to check the scaling behaviour of the sensitivity with respect to errors.

In the following two subsections 4.1 and 4.2, we examine the cases of the optimistic ($\sigma_{usys} = 0.2\%$) and the pessimistic ($\sigma_{usys} = 1\%$) systematic errors. We stress here that the analyses
we will present there contain much more general cases. For example, because of the scaling
behaviour discussed in the previous section, the case with \( N = 10^6 \) and \( \sigma_{\text{usys}} = 0.2\% \) is equivalent
to \( N = 2 \times 10^5 \) and \( \sigma_{\text{usys}} = 0.0\% \). Similarly, the case with \( N = 10^6 \) and \( \sigma_{\text{usys}} = 1\% \) is equi-
tivalent to \( N = 1.33 \times 10^4 \) and \( \sigma_{\text{usys}} = 0.5\% \). In subsection 4.3, we give an estimate of the
sensitivities using a tentative setting which may be possible without direct counting of \(^3\text{H}\) atoms.

### 4.1. Case of optimistic systematic error

We focus in this subsection on the case of optimistic systematic error, from which one may
obtain some feeling for the ultimate sensitivities achievable by the present method with the
four Run options described above. As we mentioned earlier, the correlated systematic error \( \sigma_c \) is taken to be a tentative value of 10% throughout our analysis. The uncorrelated systematic error \( \sigma_{\text{usys}} \), which is assumed to be equal for all detector locations, is taken to be 0.2% in this
subsection.

In figure 1, we show in \( \sin^2 2\theta_{13} - \Delta m^2_{31} \) plane the expected allowed region by Run I, II, A, II B, and III with number of events \( 10^6 \) in each location. Throughout the analysis, the true value of \( \Delta m^2_{31} \) is assumed to be \( \Delta m^2_{31} = 2.5 \times 10^{-3} \text{eV}^2 \). The input values of \( \sin^2 2\theta_{13} \) are taken
as 0.1 and 0.01 in the left- and the right-hand panels in figure 1, respectively. Throughout the
numerical analyses in this paper, the other oscillation parameters are taken as: \( \Delta m^2_{12} \) as 0.1 and 0.01 in the left- and the right-hand panels in figure 1, respectively. Throughout the analysis, the true
value of \( \Delta m^2_{31} \) is roughly satisfied, as indicated in table 1. (See section 5 for more detailed discussions.) For a small value of \( \theta_{13} \), \( \sin^2 2\theta_{13} = 0.01 \), the sensitivities to \( \sin^2 2\theta_{13} \) are much worse, as shown in
table 1. They are about 6% in Run IIA, and 3% in Run IIB both at 1\( \sigma \). If Run III is carried
out it can go down to 2%.

In table 2, the expected sensitivities to \( \sin^2 2\theta_{13} \) at 1\( \sigma \) and 3\( \sigma_{\text{CL}} \) (the latter in parentheses) for 1 DOF are given. The sensitivities to \( \sin^2 2\theta_{13} \) can be better characterized by \( \delta(\sin^2 2\theta_{13}) \), not its
fraction to \( \sin^2 2\theta_{13} \), as will be understood in our analytic treatment in section 5. By Run I
one can already achieve an accuracy of \( \delta(\sin^2 2\theta_{13}) \approx 0.003 \), and Run IIA or IIB reach
\( \delta(\sin^2 2\theta_{13}) \approx 0.002 \). The effect of measurement at multiple detector locations on improvement of the sensitivity is relatively minor in the case of sensitivities to \( \sin^2 2\theta_{13} \). This is in sharp contrast to the case of \( \Delta m^2_{31} \).

To show the sensitivity limit on \( \theta_{13} \) achievable by the present method, we present in figure 2
the excluded regions in \( \sin^2 2\theta_{13} - \Delta m^2_{31} \) space, assuming the case of no depletion of \( \bar{\nu}_e \) flux.
The four panels in figure 2 correspond to Run I, IIA, IIB, and III. In each panel, the red-solid,
the green-dashed, and the blue-dotted lines are for 1\( \sigma \) (68.27%), 2\( \sigma \) (95.45%) and 3\( \sigma \) (99.73%) CL for 1 DOF, respectively.
Figure 1. The expected allowed region by Run I, IIA, IIB, and III with number of events $10^6$ in each location are depicted. The red-solid, the green-dashed, and the blue-dotted lines are for $1\sigma$ (68.27%), $2\sigma$ (95.45%) and $3\sigma$ (99.73%) CL for 2 degrees of freedom (DOF), respectively. The input values of the mixing parameters are marked by asterisks and they are as follows: $\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV$^2$, $\sin^2 2\theta_{13} = 0.1$ and 0.01 in the left- and the right-hand panels.
Table 1. The expected fractional uncertainty $\delta(\Delta m^2)/\Delta m^2_{31}(0)$ in % for the optimistic systematic error of $\sigma_{\text{sys}} = 0.2\%$ reachable by Runs I–III defined in the text. The uncertainties are given at $1\sigma$ (68.27%) CL for 1 DOF, and the numbers in parentheses are the ones at $3\sigma$ (99.73%) CL for 1 DOF. In the left, middle, and right columns, the input value of $\theta_{13}$ are taken as $\sin^2 2\theta_{13} = 0.1$, 0.05, and 0.01, respectively.

| $\sigma_{\text{sys}} = 0.2\%$ Run type | $\sin^2 2\theta_{13} = 0.1$ | $\sin^2 2\theta_{13} = 0.05$ | $\sin^2 2\theta_{13} = 0.01$ |
|--------------------------------------|-----------------|-----------------|-----------------|
| Run I (5 locations) | 0.84 (2.5) | 1.7 (5.0) | 9.6 ($^{+1.1}_{-0.9}$) |
| Run IIA (10 locations) | 0.56 (1.7) | 1.2 (3.5) | 6.0 ($^{+0.8}_{-1.0}$) |
| Run IIB (10 locations) | 0.28 (0.8) | 0.56 (1.6) | 2.8 ($^{+0.6}_{-0.9}$) |
| Run III (20 locations) | 0.20 (0.56) | 0.40 (1.2) | 2.0 ($^{+0.5}_{-0.6}$) |

Table 2. The expected sensitivity to the $\sin^2 2\theta_{13}$ for the optimistic systematic error of $\sigma_{\text{sys}} = 0.2\%$ reachable by Runs I–III defined in the text. The uncertainties $\delta(\sin^2 2\theta_{13})$ are given at $1\sigma$ (68.27%) CL for 1 DOF, and the numbers in parentheses are the ones at $3\sigma$ (99.73%) CL for 1 DOF. In the left, middle, and right columns, the input value of $\theta_{13}$ are taken as $\sin^2 2\theta_{13} = 0.1$, 0.05 and 0.01, respectively.

| $\sigma_{\text{sys}} = 0.2\%$ Run type | $\sin^2 2\theta_{13} = 0.1$ | $\sin^2 2\theta_{13} = 0.05$ | $\sin^2 2\theta_{13} = 0.01$ |
|--------------------------------------|-----------------|-----------------|-----------------|
| Run I (5 locations) | $0.1 \pm 0.0026$ (0.0078) | $0.05 \pm 0.0027$ (0.0081) | $0.01 \pm 0.0028$ (0.0085) |
| Run IIA (10 locations) | $0.1 \pm 0.0019$ (0.0058) | $0.05 \pm 0.0020$ (0.0061) | $0.01 \pm 0.0021$ (0.0064) |
| Run IIB (10 locations) | $0.1 \pm 0.0017$ (0.0050) | $0.05 \pm 0.0018$ (0.0053) | $0.01 \pm 0.0018$ (0.0055) |
| Run III (20 locations) | $0.1 \pm 0.0013$ (0.0038) | $0.05 \pm 0.0014$ (0.0041) | $0.01 \pm 0.0014$ (0.0042) |

4.2. Case of pessimistic systematic error

It might be possible that we end up with the error of $\sim 1\%$ due to, e. g., time dependence of the source even though the method of movable detector with direct counting of $^3$H works. In figure 3, we present the similar allowed region in $\sin^2 2\theta_{13} - \Delta m^2_{31}$ space obtained by the same Run I, IIA, IIB, and III with the same number of events of $10^6$ in each location but with a pessimistic systematic error of $\sigma_{\text{sys}} = 1\%$. At large $\theta_{13}$, $\sin^2 2\theta_{13} = 0.1$, we still have reasonable sensitivities to $\Delta m^2_{31}$. For Run IIB and III, for example, the sensitivities are about 1–2% level for 2 DOF. At $\sin^2 2\theta_{13} = 0.01$, however, the sensitivity to $\Delta m^2_{31}$ is lost except for the one at $1\sigma$CL in Run III. It indicates that the value of $\theta_{13}$ is close to the sensitivity limit, and hence we do not include the figure for it though we did for the case of optimistic error, figure 2.

For more detailed information on sensitivities with the pessimistic systematic error of $\sigma_{\text{sys}} = 1\%$, we give in tables 3 and 4 the sensitivities at $1\sigma$ and $3\sigma$ CL (the latter in parentheses) for 1 DOF to $\Delta m^2_{31}$ and $\sin^2 2\theta_{13}$, respectively. The column without number represents that no limit is obtained, in a similar way as seen in the right-hand panels of figure 3. At relatively large $\theta_{13}$, $\sin^2 2\theta_{13} = 0.1$ and 0.05, the sensitivities to $\Delta m^2_{31}$ remain good, 1.2 and 2.4% at $1\sigma$ CL for
The sensitivity to $\sin^2 2\theta_{13}$ measurement of $^3$H in the target is not possible. Then we may have to take the case without direct detection of $^3$H. In this case, most probably, we have to accept a pessimistic value of the uncorrelated systematic error of $\Delta m_{31}^2$ at small $\theta_{13}$.

**4.3. Case without direct detection of $^3$H**

Suppose that the direct detection of $^3$H in the target is not possible. Then we may have to take the option of multiple detectors with the same structure, giving up the idea of the movable detector. In this case, most probably, we have to accept a pessimistic value of the uncorrelated systematic error of 1–3%. This will cause two important changes in designing the experiment. (i) Number of events that can be accumulated in a reasonable timescale would be smaller by a factor of $\sim 10$ than the case of direct detection. (ii) Number of detectors that can be prepared by keeping their identity to suppress the uncorrelated systematic errors may be limited. Therefore, three detector settings, for example, (in addition to a near detector which monitors the flux) would be more practical.

To understand the performance of such a reduced setting with larger errors, we have carried out a similar $\chi^2$ analysis as done in the previous subsections. We take three detector settings with practical.
Figure 3. The same as in figure 1 but with the pessimistic systematic error of σ_{sys} 1%. The input values of sin² 2θ_{13} is taken as 0.1 and 0.01 in the left- and the right-hand panels.

tentatively determined baselines L = \frac{1}{3} L_{OM}, L_{OM} and 3L_{OM}, and assume 10^5 events in each detector. We call the setting Run 0. The three cases of uncorrelated systematic errors, 1, 2 and 3%, are examined. In table 5, the expected fractional uncertainty δ(Δm^2)/Δm^2_0(0) and sin² 2θ_{13} for Run 0 are presented. With 1% of the uncorrelated systematic errors, while a sensitivity comparable to Run I is reached for sin² 2θ_{13}, uncertainty of Δm^2_{31} is larger by a factor of ≃2
We complement our numerical analysis of the sensitivities in the previous section by presenting
and no numbers are obtained for uncertainties at 3
efficient extraction, of the produced 3H is mandatory to make the type of experiments under
not from an order of magnitude smaller number of events.
The results obtained above indicate that direct counting, either real-time counting or an
efficient extraction, of the produced 3H is mandatory to make the type of experiments under discussion useful.

5. Analytic estimation of the sensitivities

We complement our numerical analysis of the sensitivities in the previous section by presenting
analytical treatment of the uncertainties in the \( \Delta m^2_{31} \) and \( \theta_{13} \) determination. In particular,
Again the independence of the $\chi^2_{Hn}$ with respect to the uncorrelated systematic error. Then, the ratio of small uncorrelated systematic error compared to correlated one, $\sigma_{u}/\sigma_{c}$, we have argued, assuming feasible direct counting of $^3$H atoms, that the hierarchy of errors to the case that an equal number of events are taken in each baseline, $N_{i}^{\text{obs}} = N_{i}^{\text{exp}}$. We also assume, for simplicity, the case of equal uncorrelated systematic error in each detector location, $\sigma_{ui} = \sigma_{uys,i}^{2} + 1/N_{i}^{\text{exp}} = \sigma_{u}^{2}$. Under these assumptions, $V^{-1}$ has a simple form

$$V^{-1} = \frac{1}{\sigma_{u}^{2}} \left[ I - \frac{1}{n} H_{n \times n} \right] = \frac{1}{\sigma_{u}^{2}} \left( \begin{array}{cccc} 1 - \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \\
-\frac{1}{n} & 1 - \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \\
-\frac{1}{n} & -\frac{1}{n} & 1 - \frac{1}{n} & \cdots & \frac{1}{n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-\frac{1}{n} & -\frac{1}{n} & -\frac{1}{n} & \cdots & 1 - \frac{1}{n} \end{array} \right),$$

(12)

where $H_{n \times n}$ is an $n \times n$ matrix whose elements are all unity, $H_{ij} = 1$ for any $i$ and $j$. It indicates again the independence of the $\chi^2$ on the correlated error $\sigma_{c}^{2}$ and the ‘scaling behaviour’ with respect to the uncorrelated systematic error. Then, the $\chi^2$ simplifies:

$$\chi^2 = \frac{1}{n\sigma_{u}^{2}} \sum_{i,j=1}^{n} \left( \frac{N_{i}^{\text{obs}}}{N_{i}^{\text{exp}}} - \frac{N_{j}^{\text{obs}}}{N_{j}^{\text{exp}}} \right)^2.$$

(13)

Table 5. The expected fractional uncertainty $\delta(\Delta m^2_{31})/(\Delta m^2_{31}(0))$ in % and $\delta(\sin^2 \theta_{13})$ for the pessimistic systematic error of $\sigma_{uys} = 1, 2$ and 3% reachable by Run 0 with three detectors as defined in the text. The uncertainties are given at 1$\sigma$ (68.27%) CL for 1 DOF, and the numbers in parentheses are the ones at 3$\sigma$ (99.73%) CL for 1 DOF. In the left, middle, and right columns, the input value of $\theta_{13}$ are taken as $\sin^2 \theta_{13} = 0.1, 0.05$ and 0.01, respectively. The column without number represents that no limit is obtained.

| Run 0 | $\sin^2 \theta_{13} = 0.1$ | $\sin^2 \theta_{13} = 0.05$ | $\sin^2 \theta_{13} = 0.01$ |
|-------|----------------|----------------|----------------|
| $\sigma_{uys}$ | $(^{+0.06}_{-0.07})$ | $(^{+0.06}_{-0.07})$ | $(^{+0.06}_{-0.07})$ |
| 1% | 0.1 $\pm$ 0.01 (0.035) | 0.05 $\pm$ 0.012 (0.037) | 0.01 $^{+0.013}_{-0.037}$ $(0.038)$ |
| 2% | 0.1 $\pm$ 0.022 $(^{+0.06}_{-0.07})$ | 0.05 $\pm$ 0.024 $(^{+0.06}_{-0.07})$ | 0.01 $^{+0.024}_{-0.037}$ $(0.069)$ |
| 3% | 0.1 $\pm$ 0.033 $(^{+0.088}_{-0.090})$ | 0.05 $\pm$ 0.035 $(^{+0.094}_{-0.098})$ | 0.01 $^{+0.035}_{-0.098}$ |

we derive analytic formulae for the sensitivities of $\Delta m^2_{31}$ and $\sin^2 \theta_{13}$ under the approximation of small uncorrelated systematic error compared to correlated one, $\sigma_{u}^{2} \ll \sigma_{c}^{2}$. In section 3, we have argued, assuming feasible direct counting of $^3$H atoms, that the hierarchy of errors is very likely to hold.

We restrict ourselves, consistent with the numerical analysis done in the previous section, to the case that an equal number of events are taken in each baseline, $N_{i}^{\text{obs}} = N_{i}^{\text{exp}}$, which may be translated into $N_{i}^{\text{exp}} = N_{i}^{\text{exp}}$. We also assume, for simplicity, the case of equal uncorrelated systematic error in each detector location, $\sigma_{ui}^{2} = \sigma_{uys,i}^{2} + 1/N_{i}^{\text{exp}} = \sigma_{u}^{2}$. Under these assumptions, $V^{-1}$ has a simple form
5.1. Optimal baselines and sensitivities for two detector locations

Let us start by examining sensitivities for the case of two detector locations. Because of a simple setting with monochromatic $\bar{\nu}_e$ beam, we can give explicit expression of $\chi^2$ in terms of small deviation of the parameters from the true (nature’s) values. For this purpose, we note that the number of events is given by

$$ N^{\text{exp}}(N^{\text{obs}}) = f_{\bar{\nu}_e} \sigma_{\text{res}} N_T TP_{\text{ee}}(\theta_{13}, \Delta m^2_{31}, L), $$

(14)

where $f_{\bar{\nu}_e}$ denotes the neutrino flux, $\sigma_{\text{res}}$ the absorption cross-section, $N_T$ the number of target nuclei, $T$ the running time, and $P_{\text{ee}}$ is a short-hand notation for $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$. In the setting in this section, $T$ is adjusted such that an equal number of events is collected at each location of the detector. We recall that $N^{\text{obs}}$ denotes the event number computed with the true value of the parameters, $\Delta m^2_{31} = \Delta m^2(0)$ and $\sin^2 2\theta_{13} = \sin^2 2\theta_{13}(0)$, whereas $N^{\text{exp}}$ denotes the event number computed with possible small deviations $\delta(\Delta m^2)$ and $\delta(\sin^2 2\theta_{13})$ from the true values of the parameters. Then, $i$th component of $\vec{x}$ vector in (9) is given to first order in the deviation as

$$ \frac{N_i^{\text{obs}}}{N_i^{\text{exp}}} - 1 = \frac{1}{P_{\text{ee}}^{(0)}(L_i)} \left[ \sin^2 \left( \frac{\Delta m^2_{31}(0)L_i}{4E} \right) \delta(\sin^2 2\theta_{13}) \right. $$

$$ + \left. \frac{1}{2} \sin^2 2\theta_{13}(0) \sin \left( \frac{\Delta m^2_{31}(0)L_i}{2E} \right) \times \left( \frac{\delta(\Delta m^2)L_i}{2E} \right) \right], $$

(15)

where $P_{\text{ee}}^{(0)}(L_i) \equiv P_{\text{ee}}(\theta_{13}(0), \Delta m^2_{31}(0), L_i)$ for which we have used the two-flavour expression (2).

For simplicity we restrict our discussion in this section to the analysis with single DOF. This is a natural setting for estimating ultimate sensitivities; When we discuss sensitivity of $\Delta m^2_{31}$ we optimize $\chi^2$ in terms of $\sin^2 2\theta_{13}$, and vice versa. Or, one can think of the situation that, in determination of $\Delta m^2_{31}$, $\sin^2 2\theta_{13}$ is accurately determined by other ways, e.g., long-baseline accelerator experiments. Under the approximation $\sigma^2_{\bar{\nu}_e} \ll \sigma^2_\nu$ and using $V^{-1}$ in (12), $\chi^2$ is given for small deviations of $\theta_{13}$ and $\Delta m^2_{31}$ as follows:

$$ \chi^2_\theta = \frac{\left( \delta(\sin^2 2\theta_{13}) \right)^2}{2\sigma^2_\theta} \left[ \sin^2 \left( \frac{\Delta m^2_{31}(0)L_1}{4E} \right) \frac{\sin^2 \left( \frac{\Delta m^2_{31}(0)L_2}{4E} \right)}{P_{\text{ee}}^{(0)}(L_1)} - \frac{\sin^2 \left( \frac{\Delta m^2_{31}(0)L_2}{4E} \right)}{P_{\text{ee}}^{(0)}(L_2)} \right]^2, $$

(16)

$$ \chi^2_{\Delta m^2} = \frac{\sin^2 2\theta_{13}(0)}{8\sigma^2_{\Delta m^2}} \left[ \frac{\delta(\Delta m^2)}{\Delta m^2_{31}(0)} \right]^2 \times \left[ \frac{\left( \frac{\Delta m^2_{31}(0)L_1}{2E} \right) \sin \left( \frac{\Delta m^2_{31}(0)L_1}{2E} \right) - \frac{\sin \left( \frac{\Delta m^2_{31}(0)L_2}{2E} \right)}{P_{\text{ee}}^{(0)}(L_1)} \right] \right]^2. $$

(17)

Now, we can address the problem of optimal baseline and estimate the sensitivities of $\sin^2 2\theta_{13}$ and $\Delta m^2_{31}$ under the approximations stated above. Since $\sin^2 2\theta_{13} \lesssim 0.1$ [35], it may be a reasonable approximation to set $P_{\text{ee}}^{(0)}(L_i) = 1$ in the denominator, as we do in the rest of the section.
5.1.1. Optimal setting and sensitivity to $\sin^2 2\theta_{13}$. To maximize (16) one should take $L_1$ as short as possible, and $L_2$ at the oscillation maximum, the well-known feature in the reactor $\theta_{13}$ experiments. Thus, we take $L_1 = 0$ and $L_2 = L_{OM}$ which makes the square parenthesis in (16) unity. Then, one can obtain the sensitivity at $N_{CL}\sigma_{CL}$ for two detector locations as

$$
\delta(\sin^2 2\theta_{13}) = 2N_{CL} \frac{\sigma_u}{\sqrt{2}} = 2N_{CL} \sqrt{\frac{\sigma^2_{\text{sys}} + \frac{1}{N}}{2}}. 
$$

(18)

For $\sigma_u = 0.2\%$, $\delta(\sin^2 2\theta_{13}) = 2.8 \times 10^{-3}$ (8.5 $\times$ 10$^{-3}$) at 1$\sigma$ (3$\sigma$) CL. For $\sigma_u = 1\%$, $\delta(\sin^2 2\theta_{13}) = 0.014$ (0.043) at 1$\sigma$ (3$\sigma$) CL, which is not so far from the sensivities quoted in the literatures of the reactor $\theta_{13}$ experiments.

5.1.2. Optimal setting and sensitivity to $\Delta m^2_{31}$. The optimal baseline setting is quite different for $\Delta m^2_{31}$. We first recall a property of the function $x \sin x$: it has the first maximum $x \sin x = 1.82$ at $x = 2.02$ ($L = 0.64L_{OM}$), has the first minimum $x \sin x = -4.81$ at $x = 4.91$ ($L = 1.56L_{OM}$), and then the second maximum $x \sin x = 7.92$ at $x = 7.98$ ($L = 2.54L_{OM}$), and so on. For simplicity, we restrict ourselves to $x \leq 3\pi$, which means $L \leq 3L_{OM}$ so that the running time does not blow up. Then, the optimal setting is $L_1 = 0.64L_{OM}$ and $L_2 = 1.56L_{OM}$ if we restrict to $L \leq 2L_{OM}$, and $L_1 = 1.56L_{OM}$ and $L_2 = 2.54L_{OM}$ if we allow baseline until $L \leq 3L_{OM}$. Despite the factor of \approx 4 different baseline lengths, we still assume equal numbers of events at $L = 0.64L_{OM}$ and $L = 2.54L_{OM}$, which implies \approx 16 times longer exposure time at the latter distance.

The $\chi^2$ is given approximately by

$$
\chi^2_{\Delta m^2} = \frac{c}{\sigma_u^2} \sin^4 2\theta_{13}(0) \left( \frac{\delta(\Delta m^2_{31})}{\Delta m^2_{31}(0)} \right)^2. 
$$

(19)

The coefficient $c$ is 5.5 for $L \leq 2L_{OM}$ and 20.3 for $L \leq 3L_{OM}$. Then, we obtain the sensitivity at $N_{CL}\sigma_{CL}$ for two detector locations as

$$
\frac{\delta(\Delta m^2_{31})}{\Delta m^2_{31}(0)} = \frac{1}{\sin^2 2\theta_{13}(0)} N_{CL} \frac{\sigma_u}{\sqrt{c}}. 
$$

(20)

Hence, the sensitivity to $\Delta m^2_{31}$ depends very sensitively on $\sin^2 2\theta_{13}$.

With $\sigma_u = 0.2\%$, $\delta(\Delta m^2_{31})/\Delta m^2_{31}(0) = 8.5 \times 10^{-3}$ at 1$\sigma$ CL for $\sin^2 2\theta_{13} = 0.1$, if we restrict to $L \leq 2L_{OM}$. If we allow $L \leq 3L_{OM}$, the sensitivity becomes better, $\delta(\Delta m^2_{31})/\Delta m^2_{31}(0) = 4.4 \times 10^{-3}$ at the same CL. If the systematic error is worse, $\sigma_u = 1\%$, the sensitivity at $\sin^2 2\theta_{13} = 0.1$ becomes to $\delta(\Delta m^2_{31})/\Delta m^2_{31}(0) = 4.3 \times 10^{-2}$ and $\delta(\Delta m^2_{31})/\Delta m^2_{31}(0) = 2.2 \times 10^{-2}$ at 1$\sigma$ CL for $L \leq 2L_{OM}$ and $L \leq 3L_{OM}$ cases, respectively.

5.2. The problem of $n$ detector locations reduces to the two-location case

We first show that the problem of optimal setting of $n$ detector locations reduces to the case of two locations under the assumption of equal number of events in each location. To indicate the essential point let us first consider a simplified $\chi^2$ of the form $\chi^2 = \sum_{i,j=1}^{n} (x_i - x_j)^2$ and $0 \leq x_i \leq 1$, which is the essential part of $\chi^2$ for $\sin^2 2\theta_{13}$, the equation (16). In the case of
Thus, the problem of optimal setting with equal number of events at each detector location is maximized when

\[ \chi^2(n = 2) = 1 \]

for \( n \) locations the configuration which maximizes \( \chi^2(n) \) is

1. Even \( n : n = 2M; x = 0 \) appears \( M \) times, and \( x = 1 \) appears \( M \) times, \( \chi^2(n)_{\text{max}} = M^2 \).
2. Odd \( n : n = 2M + 1; x = 0 \) appears \( M+1 \) times, and \( x = 1 \) appears \( M \) times, or vice versa, \( \chi^2(n)_{\text{max}} = M(M + 1) \).

Thus, the problem of optimal setting with equal number of events at each detector location is reduced to the two-location case.

For \( \chi^2 \) for \( \Delta m_{31}^2 \) in (17), the situation is slightly different because the function \( |x \sin x| \) increases without limit as \( x \) becomes large. Therefore, mathematically speaking, one can obtain better and better accuracies as one goes to longer and longer distances in our setting of equal number of events at any detector location.\(^6\) But, since we want to remain to a reasonable running time, we have restricted our discussions to baselines limited by \( L \leq 2L_{OM} \) or \( L \leq 3L_{OM} \) in the two-location case, the restriction which is kept throughout this section. Then, one can show that the same result follows for \( \chi^2 \) for \( \Delta m_{31}^2 \), (17). Namely, in the case of \( n \) locations, the highest sensitivity is achieved at the same baselines \( L_1 \) and \( L_2 \) of the two location case; \( L_1 \) in \( [\frac{2}{3}] \) \((\lfloor \frac{2}{3} \rfloor + 1 \) for odd \( n \) times, and \( L_2 \) in \( [\frac{2}{3}] \) times, where \([\cdot]\) implies Gauss’ symbol.

The maximal value of \( \chi^2 \) is, therefore, given by

\[
\chi^2(n)_{\text{max}} = \left( \frac{n}{2} \right) \chi^2(n = 2)_{\text{max}}; \quad (\text{even } n),
\]

\[
\chi^2(n)_{\text{max}} = \left( \frac{n^2 - 1}{2n} \right) \chi^2(n = 2)_{\text{max}}; \quad (\text{odd } n),
\]

(21)

It can be translated into the uncertainties at \( N_{CL} \) as

\[
\delta(\sin^2 2\theta_{13})(n) = \sqrt{\frac{2}{n}} \delta(\sin^2 2\theta_{13})(n = 2),
\]

\[
\left( \frac{\delta(\Delta m_{31}^2)}{\Delta m_{31}^2(0)} \right)(n) = \sqrt{\frac{2}{n}} \left( \frac{\delta(\Delta m_{31}^2)}{\Delta m_{31}^2(0)} \right)(n = 2),
\]

(22)

for even \( n \). For odd \( n, n \) in (22) must be replaced by \((n^2 - 1)/n, \delta(\sin^2 2\theta_{13})(n = 2)\) and \((\delta(\Delta m_{31}^2)/\Delta m_{31}^2(0))(n = 2)\) are given respectively by (18) and (20). Therefore, the sensitivity gradually improves as number of runs becomes larger.\(^7\)

At the end of this subsection, we want to note the following: the reason why we did not take these sets of optimal distances for \( \theta_{13} \) and \( \Delta m_{31}^2 \) obtained in this subsection in the numerical analyses in section 4 is that the sensitivity to \( \Delta m_{31}^2 \) is lost if we tune the setting optimal for \( \sin^2 2\theta_{13} \), and vice versa. The reason for this is easy to understand; at baselines \( L_1 \) and \( L_2 \) which maximizes \( \chi_0^2(\chi_{\Delta m_{21}^2}, \chi_{\Delta m_{31}^2}(\chi_{\Delta m_{31}^2}^2) \) vanishes (approximately vanishes), because \( x_1 = \sin x_2 = 0 \) at \( x_1 = 0 \) and \( x_2 = \pi (\sin^2 \frac{3\pi}{2} \approx \sin^2 \frac{3\pi}{2} \approx \frac{1}{2} \) at \( x_1 \approx \frac{\pi}{2} \) and \( x_2 \approx \frac{3\pi}{2} \). Nonetheless, we will see,

\(^6\) This explains at least partly the reason why the sensitivities to \( \Delta m_{31}^2 \) and \( \sin^2 2\theta_{13} \) differ in dependence on distance from the source to a detector, as indicated in figure 3 in [36].

\(^7\) It is a well-known feature in the multiple detector setting in the reactor \( \theta_{13} \) experiments in which one obtains better sensitivity as in (18) if the two identical detectors, the near and the far, are each divided into small detectors in the same way, if the uncorrelated systematic error \( \sigma_{\text{sys}} \) is made to be equal with that of the original large detector and if the statistical errors are negligible even for divided detectors. This point was emphasized by Yasuda [37].
We examine the cases of restriction

\( L \leq 2L_{OM} \) for the optimistic and the pessimistic systematic errors of

in our numerical analysis in section 4. Then, an approximation of small deviation from the best fit.

They compare well with the numbers in table 2 though the latter are obtained with not-

reasonably well with each other.

in the following subsections, that the sensitivities analytically estimated with optimal baseline distances and the numerically calculated ones with baselines taken by ‘common sense’ agree reasonably well with each other.

5.3. Analytic estimation of the sensitivities; \( \sin^2 2\theta_{13} \)

Let us examine the case of \( \sigma_{\text{usys}} = 0.2\% \) and the number of events \( N = 10^6 \) which was considered in our numerical analysis in section 4. Then, \( \sigma_u = 0.22\% \). In the case of five locations, \( n = 5 \),

\( \delta(\sin^2 2\theta_{13}) = 2.0 \times 10^{-3} \) (6.1 \( \times 10^{-3} \)) at 1\( \sigma \) (3\( \sigma \)) CL. Similarly for ten and 20 locations

\( \delta(\sin^2 2\theta_{13}) = 1.4 \times 10^{-3} \) (4.2 \( \times 10^{-3} \)) and 0.99 \( \times 10^{-3} \) (3.0 \( \times 10^{-3} \)), respectively, at 1\( \sigma \) (3\( \sigma \)) CL. They compare well with the numbers in table 2 though the latter are obtained with not-

so-tuned baseline settings. Notice that \( \delta(\sin^2 2\theta_{13}) \) is independent of \( \theta_{13} \) under the present approximation of small deviation from the best fit.

With \( \sigma_{\text{usys}} = 1\% \) the corresponding sensitivities are \( \delta(\sin^2 2\theta_{13}) = 9.2 \times 10^{-3} \) (2.8 \( \times 10^{-2} \)),

6.4 \( \times 10^{-3} \) (1.9 \( \times 10^{-2} \)), and 4.5 \( \times 10^{-3} \) (1.3 \( \times 10^{-2} \)) for five, ten and 20 locations, respectively, at 1\( \sigma \) (3\( \sigma \)) CL. They are again roughly consistent with the ones in table 4.

5.4. Analytic estimation of the sensitivities; \( \Delta m_{31}^2 \)

We examine the cases of restriction \( L \leq 2L_{OM} \) and \( L \leq 3L_{OM} \). In the first case, the maximum of the function \( x \sin x \) is at \( x = 2.02 \) (\( L = 0.64L_{OM} \)) where \( x \sin x = 1.82 \), and the minimum at

\( x = 4.91 \) (\( L = 1.56L_{OM} \)) where \( x \sin x = -4.81 \). In the case of milder restriction \( L \leq 3L_{OM} \), the maximum of the function \( x \sin x \) is at \( x = 7.98 \) (\( L = 2.54L_{OM} \)) where \( x \sin x = 7.92 \), and the minimum at \( x = 4.91 \) (\( L = 1.56L_{OM} \)) as above.

In table 6, we give the fractional uncertainties of \( \Delta m_{31}^2 \), \( \delta(\Delta m^2)/\Delta m_{31}^2(0) \) in % at 1\( \sigma \) CL for the optimistic and the pessimistic systematic errors of \( \sigma_{\text{usys}} = 0.2 \) and 1\( \% \) (the latter in

Table 6. The analytically estimated fractional uncertainties of \( \Delta m_{31}^2 \), \( \delta(\Delta m^2)/\Delta m_{31}^2(0) \) in % are given for the optimistic systematic error of \( \sigma_{\text{usys}} = 0.2\% \) and for the pessimistic one (in parentheses) of \( \sigma_{\text{usys}} = 1\% \). The uncertainties are given at 1\( \sigma \) (68.27%) CL for 1 DOF for the cases of 5, 10, and 20 detector locations. The upper three rows are for the case of restricted baselines, \( L \leq 2L_{OM} \), whereas the lower three rows are for the cases of somewhat relaxed baseline setting, \( L \leq 3L_{OM} \). In the left, middle, and right columns, the input value of \( \theta_{13} \) are taken as \( \sin^2 2\theta_{13} = 0.1, 0.05, \) and 0.01, respectively.

| Number of locations | \( \sigma_{\text{usys}} = 0.2\% \) (\( \sigma_{\text{usys}} = 1\% \)) | \( \delta(\Delta m^2)/\Delta m_{31}^2(0) \) (in %) at 1\( \sigma \) CL |
|---------------------|--------------------------------|--------------------------------|
| \( L \leq 2L_{OM} \) | \( \sin^2 2\theta_{13} = 0.1 \) | \( \sin^2 2\theta_{13} = 0.05 \) | \( \sin^2 2\theta_{13} = 0.01 \) |
| 5 locations         | 0.61 (2.8)                     | 1.2 (5.5)                      | 6.1 (28)                      |
| 10 locations        | 0.42 (1.9)                     | 0.85 (3.8)                     | 4.2 (19)                      |
| 20 locations        | 0.30 (1.3)                     | 0.6 (2.7)                      | 3.0 (13)                      |
| \( L \leq 3L_{OM} \) | 0.32 (1.5)                     | 0.62 (2.9)                     | 3.2 (15)                      |
| 5 locations         | 0.22 (0.99)                    | 0.44 (2.0)                     | 2.2 (9.9)                     |
| 10 locations        | 0.15 (0.68)                    | 0.31 (1.4)                     | 1.5 (6.8)                     |

Reference:

New Journal of Physics 8 (2006) 143 (http://www.njp.org/)
parenthesis), respectively, obtained by using the equations (20) and (22). We do not show errors at 3σ CL because it is obtained simply by multiplying 3. Overall, the analytically estimated uncertainties are in reasonable agreement with those obtained by the numerical analysis in section 4. Notice that one has to compare the sensitivities of Run I and IIA with the case of severer restriction \( L \leq 2L_{OM} \), and the ones of Run IIB and III with the case of milder restriction \( L \leq 3L_{OM} \), because distances beyond 2\( L_{OM} \) are involved in the latter runs. The fact that our analytical estimates of the errors are smaller than the numerical ones by \( \sim 30\% \) or so, apart from approximations involved, is consistent with that the latter are based on non-optimal baseline distances. It also implies that the baseline setting chosen by the ‘common sense’ used in the numerical analysis in section 4 is not so far from the optimal one, indicating that the sensitivities are rather stable against changes of baseline setting.

Based on the numerical and the analytical estimate of the uncertainties of \( \Delta m^2_{31} \) determination, we conclude that sensitivities of less than 1% at 90% CL required for resolution of mass hierarchy proposed in [12, 13] are in reach if the uncorrelated systematic error of \( \sigma_{usys} = 0.2\% \) is realized, and if \( \theta_{13} \) is relatively large, \( \sin^2 2\theta_{13} \gtrsim 0.05 \) in Run IIB.

We have confined ourselves to the problem of optimal setting of distances under the constraint of equal number of events at each location. To minimize running time for a given sensitivity, we have to address the problem of optimal detector locations and exposure times for a given total running time. This is left for a future study.

6. Concluding remarks

In this paper, we have explored the potential of high sensitivity measurement of \( \Delta m^2_{31} \) and \( \theta_{13} \) which is enabled by using the resonant absorption of a monochromatic \( \bar{\nu}_e \) beam enhanced by the Mössbauer effect. With baseline distances of \( \sim 10 \) m, the movable detector setting is certainly possible. Assuming that the direct detection of produced \(^3\text{H} \) atom either by real-time counting or extraction of \(^3\text{H} \) atoms works, we have argued that the uncorrelated systematic error can be as small as 0.1–0.3%, if the near detector has the same structure as the far one. This will allow us to determine \( \Delta m^2_{31} \) to the accuracies \( \Delta m^2_{31} \) of \( \simeq 0.3(\sin^2 2\theta_{13}/0.1)^{-1}\% \) at 1σ CL (Run IIB, \( \sigma_{usys} = 0.2\% \). The error of \( \sin^2 2\theta_{13} \) is also small, \( \sin^2 2\theta_{13} \sim 1.8 \times 10^{-3} \) almost independently of \( \theta_{13} \) with the same setting. The accuracy of the \( \theta_{13} \) measurement even in Run I, if the systematic error of 0.2% is reached, is comparable with that of the next generation accelerator \( \nu_e \) appearance experiments [25, 38]. It may exceed the accelerator sensitivity if Run IIB is performed. If the systematic error is of 1% level, the sensitivity to \( \theta_{13} \) is comparable to the first-stage reactor \( \theta_{13} \) experiments even in Run IIB.

What is the scientific merit of the precision measurement of \( \Delta m^2_{31} \) and \( \theta_{13} \)? As we have already mentioned in section 1, the precision measurement of \( \Delta m^2_{31} \) and \( \theta_{13} \) will have a great impact, at least, on two of the unknowns in lepton flavour mixing: the neutrino mass hierarchy and resolving the \( \theta_{23} \) octant degeneracy. It would be very interesting to carry out quantitative analyses of such possibilities. We emphasize that such physics capabilities can only be made possible by direct counting of the produced \(^3\text{H} \) atoms which makes a movable detector feasible. We hope that these exciting possibilities will stimulate further development of the experimental technology towards that goal.

8 Such an analysis on mass hierarchy resolution has recently been carried out; see [39].
What are the additional capabilities of the resonant absorption of the monochromatic $\bar{\nu}_e$ beam? With 18.6 keV of neutrino energy, the solar oscillation maximum would be reached at $L_{\text{solarOM}} = 290(\Delta m^2_{21}/8 \times 10^{-5} \text{ eV}^2)^{-1}$ m. Then, it would be worthwhile to explore the possibility of precision measurement of $\theta_{12}$ and $\Delta m^2_{21}$, as was done for the reactor experiments [36]. But in the present case, neither geo-neutrinos nor $\bar{\nu}_e$ flux from nearby reactors (if any) contaminate the measurement. In particular, a possible movable or multiple baseline set up should allow improvement of accuracy of $\Delta m^2_{21}$ determination.

Detection of CP violating effect due to $\delta$ in the $\bar{\nu}_e$ disappearance measurement requires going down by a factor of $O(10^{-6})$ compared to CP conserving terms [40]. Unfortunately, this would not be within reach despite great potential sensitivities achievable by the resonant absorption reaction.

Finally, the setup of multiple baseline lengths of $\sim 10$ m allows a precision test of the pure vacuum oscillation hypothesis by observing a sine curve slightly modified by the solar $\Delta m^2_{21}$ oscillation. It will constrain various possible sub-leading effects such as de-coherence or new neutrino interactions to great precision.

Acknowledgments

We thank Hiro Sugiyama and Osamu Yasuda for informative correspondences and helpful discussions on statistical procedure for our analysis. The encouraging comments from Raju Raghavan on the first version of the manuscript are gratefully acknowledged. HM thanks Abdus Salam International Center for Theoretical Physics for hospitality where this work was completed. This work was supported in part by the grant-in-aid for scientific research, no. 16340078, Japan Society for the Promotion of Science.

Appendix. General formula for $\chi^2$ for $n$-uncorrelated and $\ell$-correlated errors

We consider a general setting with $n$-uncorrelated and $\ell$-correlated errors. $\chi^2$ is given in a form as

$$\chi^2 = \tilde{x}^T V^{-1} \tilde{x}$$  \hspace{1cm} (A.1)

where $\tilde{x}^T = (x_1, x_2, \ldots, x_n)$ and $x_i = N_i^{\text{obs}}/N_i^{\text{exp}} - 1$. We also introduce a vector notation for parameters $\alpha_p (p = 1, \ldots, \ell)$ for correlated error as $\tilde{\alpha}^T = (\alpha_1, \alpha_2, \ldots, \alpha_\ell)$ for so called the ‘pull type’ $\chi^2$ [41, 42]. Then, $\chi^2$ can be written [28]

$$\chi^2 = \min_{\tilde{\alpha}}[(\tilde{x} - H\tilde{\alpha})^T D_u^{-1}(\tilde{x} - H\tilde{\alpha}) + \tilde{\alpha}^T D_c^{-1} \tilde{\alpha}]$$

$$= \min_{\tilde{\alpha}}[(\tilde{\alpha} - A^{-1} H^T D_u^{-1} \tilde{x})^T A(\tilde{\alpha} - A^{-1} H^T D_u^{-1} \tilde{x})]$$

$$+ \tilde{x}^T (D_u^{-1} - D_u^{-1} H A^{-1} H^T D_u^{-1}) \tilde{x}).$$  \hspace{1cm} (A.2)

After minimization by $\alpha_j$, $V^{-1}$ can be written as

$$V^{-1} = D_u^{-1} - D_u^{-1} H A^{-1} H^T D_u^{-1}.$$  \hspace{1cm} (A.3)

In the above equations, $D_u$ is an $n \times n$ matrix $D_u \equiv \text{diag}(\sigma_{u1}^2, \ldots, \sigma_{un}^2)$, $D_c$ is an $\ell \times \ell$ matrix $D_c \equiv \text{diag}(\sigma_{c1}^2, \ldots, \sigma_{c\ell}^2)$, and $H$ is an $n \times \ell$ matrix whose elements are all unity, $H_{p,i} = 1$ for
any \( p = 1, \ldots, \ell \) and \( i = 1, \ldots, n \). Finally, the \( \ell \times \ell \) matrix \( A \) is defined by

\[
A \equiv D_c^{-1} + H^T D_u^{-1} H. \tag{A.4}
\]

Note that the statistical errors are incorporated in the uncorrelated errors as \( \sigma_{ui}^2 = \sigma_{\text{sys},ui}^2 + 1/N_i^{\text{exp}} \).

A simple ‘path-integral’ proof is given by [28] that \( V = D_u + H D_c H^T \).

Here, we are interested in obtaining the explicit form of \( V^{-1} \) in (A.3). We first compute \( A^{-1} \). We note that the matrix \( A \) given in (A.4) can be written explicitly as

\[
A = \left( \sum_{i=1}^{n} \frac{1}{\sigma_{ui}^2} \right) T, \tag{A.5}
\]

\[
T = \begin{pmatrix}
1 + \epsilon_1 & 1 & 1 & \ldots & 1 \\
1 & 1 + \epsilon_2 & 1 & \ldots & 1 \\
1 & 1 & 1 + \epsilon_3 & \ldots & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 1 & 1 & \ldots & 1 + \epsilon_\ell
\end{pmatrix}, \tag{A.6}
\]

where \( \epsilon_i \) in (A.6) is given by

\[
\epsilon_p = \frac{1}{\sigma_{cp}^2} \left( \sum_{i=1}^{n} \frac{1}{\sigma_{pi}^2} \right)^{-1} \quad (p = 1, \ldots, \ell). \tag{A.7}
\]

It is easy to show that \( T^{-1} \) is given as

\[
(T^{-1})_{pp} = \frac{1}{\epsilon_p} \left[ 1 + \sum_{r=1}^{\ell} \frac{1}{\epsilon_r} \right] = \frac{1}{\epsilon_p} - \frac{1}{\epsilon_p^2} \left[ 1 + \sum_{r=1}^{\ell} \frac{1}{\epsilon_r} \right], \tag{A.8}
\]

\[
(T^{-1})_{pq} = -\frac{1}{\epsilon_p \epsilon_q} \left[ 1 + \sum_{r=1}^{\ell} \frac{1}{\epsilon_r} \right] \quad (p \neq q). \tag{A.9}
\]

Then, the second term of \( V^{-1} \) in (A.3) is given by

\[
(V^{-1}_{(2nd \term)})_{ij} = \frac{1}{\sigma_{ui}^2 \sigma_{uj}^2} \left[ \sum_p H_{ip} (A^{-1})_{pp} H_{pj} + \sum_{p \neq q} H_{ip} (A^{-1})_{pq} H_{qj} \right],
\]

\[
= \frac{1}{\sigma_{ui}^2 \sigma_{uj}^2} \left[ \sum_p (A^{-1})_{pp} + \sum_{p \neq q} (A^{-1})_{pq} \right]. \tag{A.10}
\]

We thus obtain \( V^{-1} \) as

\[
(V^{-1})_{ij} = \frac{\delta_{ij}}{\sigma_{ui}^2} - \frac{1}{\sigma_{ui}^2 \sigma_{uj}^2} \left[ 1 + \left( \sum_{k=1}^{n} \frac{1}{\sigma_{uk}^2} \right) \left( \sum_{p=1}^{\ell} \frac{\sigma_{cp}^2}{\sigma_{ui}^2} \right) \right]. \tag{A.11}
\]
References

[1] Raghavan R S 2005 Preprint hep-ph/0511191
[2] Raghavan R S 2006 Preprint hep-ph/0601079
[3] Mikaelyan L A, Tsinoe v B G and Borovoi A A 1967 Yad. Fiz. 6 349
  Mikaelyan L A, Tsinoe v B G and Borovoi A A 1968 Sov. J. Nucl. Phys. 6 254 (Engl. Transl.)
[4] Visscher W M 1959 Phys. Rev. 116 1581
[5] Kells W P and Schiffer J P 1983 Phys. Rev. C 28 2162
[6] Kozlov Y, Mikaelyan L and Sinev V 2003 Yad. Fiz. 66 497 (Preprint hep-ph/0109277)
  Kozlov Y, Mikaelyan L and Sinev V 2003 Phys. At. Nucl. 66 469 (Engl. Transl.)
[7] Minakata H, Sugiyama H, Yasuda O, Inoue K and Suekane F 2003 Phys. Rev. D 68 033017
  Minakata H, Sugiyama H, Yasuda O, Inoue K and Suekane F 2004 Phys. Rev. D 70 059901 (erratum)
  (Preprint hep-ph/0211111)
[8] Anderson K et al 2004 White Paper Report on Using Nuclear Reactors to Search for a Value of $\theta_{13}$
  Preprint hep-ex/0402041
[9] Mössbauer R L 1958 Z. Phys. 151 124
  Frauenfelder H 1962 The Mössbauer Effect (New York: Benjamin)
[10] Pound R V and Rebka G A 1960 Phys. Rev. Lett. 4 337
  Pound R V and Snider J L 1964 Phys. Rev. Lett. 13 539
[11] Shaevitz M H and Link J M 2004 Proc. 4th Workshop on Neutrino Oscillations and Their Origin (NOON2003)
  eds Y Suzuki, M Nakahata, Y Itow, M Shiozawa and Y Obayashi (Singapore: World Scientific) p 171
  Preprint hepph/0306031
[12] Minakata H, Parke S and Funchal R Z 2005 Phys. Rev. D 68 013009 (Preprint hep-ph/0503283)
[13] de Gouvea A, Jenkins J and Kayser B 2005 Phys. Rev. D 71 113009 (Preprint hep-ph/0503079)
[14] Barenboim G and de Gouvea A 2002 Preprint hep-ph/0209117
[15] Minakata H and Nunokawa H 2001 J. High Energy Phys. JHEP08(2001)001 (Preprint hep-ph/0108085)
  Minakata H and Nunokawa H 2002 Nucl. Phys. Proc. Suppl. 110 404 (Preprint hep-ph/0111131)
[16] Fogli G and Lisi E 1996 Phys. Rev. D 54 3667 (Preprint hep-ph/9604415)
[17] Barger V, Marfatia D and Whisnant K 2002 Phys. Rev. D 65 073023 (Preprint hep-ph/0112119)
[18] Minakata H, Nunokawa H and Parke S 2002 Phys. Rev. D 66 093012 (Preprint hep-ph/0208163)
[19] Barenboim G and de Gouvea A 2002 Preprint hep-ph/0209117
[20] Hiraike K, Minakata H, Nakaya T, Nunokawa H, Sugiyama H, Teves W J C and Funchal R Z 2006
  Phys. Rev. D 73 093008 (Preprint hep-ph/0601258)
[21] Minakata H 1995 Phys. Rev. D 52 6630 (Preprint hep-ph/9503417)
  Minakata H 1995 Phys. Lett. B 356 61 (Preprint hep-ph/9504222)
  Bilenky S M, Bottino A, Giunti C and Kim C W 1995 Phys. Lett. B 356 273 (Preprint hep-ph/9504405)
  Babu K S, Pati J C and Wilczek F 1995 Phys. Lett. B 359 351 (Preprint hep-ph/9505334)
  Fogli G L, Lisi E and Scioscia G 1995 Phys. Rev. D 52 5334 (Preprint hep-ph/9506350)
[22] Fukuda Y et al (Super-Kamiokande Collaboration) 1998 Phys. Rev. Lett. 81 156 (Preprint hep-ex/9807003)
  Ashie Y et al (Super-Kamiokande Collaboration) 2004 Phys. Rev. Lett. 93 101801 (Preprint hep-ex/0404034)
[23] Ahn M H et al (K2K Collaboration) 2003 Phys. Rev. Lett. 90 041801 (Preprint hep-ex/0212007)
[24] Tagger N (MINOS Collaboration) 2006 Preprint hep-ex/0605058
[25] Itow Y et al 2006 Preprint hep-ex/0106019 online at http://neutrino.kek.jp/jhfnu/loi/loi.v2.030528.pdf
[26] Bahcall J N 1961 Phys. Rev. 124 495
[27] LBNL Isotopes Project—LUANDS Universitet Nuclear Data Dissemination Home Page online at
  http://ie.lbl.gov/toi.html

New Journal of Physics 8 (2006) 143 (http://www.njp.org/)
[28] Sugiyama H, Yasuda O, Suekane F and Horton-Smith G A 2006 Phys. Rev. D 73 053008 (Preprint hep-ph/0409109)

[29] Bolton T (Braidwood Collaboration) 2005 Nucl. Phys. Proc. Suppl. 149 166 Braidwood Project Description online at http://braidwood.uchicago.edu/

[30] Cao J (Daya Bay Collaboration) 2005 Preprint hep-ex/0509041

[31] Anjos J C et al (Angra Collaboration) 2005 Preprint hep-ex/0511059

[32] Ardellier F et al (Double–Chooz Collaboration) 2004 Preprint hep-ex/0405032

[33] Suekane F (KASKA Collaboration) 2004 Preprint hep-ex/0407016

[34] Minakata H and Sugiyama H 2004 Phys. Lett. B 580 216 (Preprint hep-ph/0309323)

[35] Apollonio M et al (CHOOZ Collaboration) 1998 Phys. Lett. B 420 397 (Preprint hep-ex/9711002)

[36] Minakata H, Nunokawa H, Teves W J C and Funchal R Z 2005 Phys. Rev. D 71 013005 (Preprint hep-ph/0407326)

[37] Yasuda O 2004 Preprint hep-ph/0403162

[38] Ayres D S et al (NOvA Collaboration) 2005 Preprint hep-ex/0503053

[39] Minakata H, Nunokawa H, Parke S J and Funchal R Z 2006 Preprint hep-ph/0607284

[40] Minakata H and Watanabe S 1999 Phys. Lett. B 468 256 (Preprint hep-ph/9906530)

[41] Fogli G L, Lisi E, Marrone A, Montanino D and Palazzo A 2002 Phys. Rev. D 66 053010 (Preprint hep-ph/0206162)

[42] Stump D et al 2002 Phys. Rev. D 65 014012 (Preprint hep-ph/0101051)

Botje M 2002 J. Phys. G: Nucl. Part. Phys. 28 779 (Preprint hep-ph/0110123)

Pumplin J, Stump D R, Huston J, Lai H L, Nadolsky P and Tung W K 2002 J. High Energy Phys. JHEP08(2002)012 (Preprint hep-ph/0201195)