Hole dynamics in generalized spin backgrounds in infinite dimensions

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Abstract

We calculate the dynamical behaviour of a hole in various spin backgrounds in infinite dimensions, where it can be determined exactly. We consider hypercubic lattices with two different types of spin backgrounds. On one hand we study an ensemble of spin configurations with an arbitrary spin probability on each sublattice. This model corresponds to a thermal average over all spin configurations in the presence of staggered or uniform magnetic fields. On the other hand we consider a definite spin state characterized by the angle between the spins on different sublattices, i.e. a classical spin system in an external magnetic field. When spin fluctuations are considered, this model
describes the physics of unpaired particles in strong coupling superconductors.
I. INTRODUCTION

It is well known that the discovery of high Tc superconductors has triggered an extensive study of highly correlated systems. The Hubbard and $t - J$ models have been prototype models towards the understanding of most of the features of those materials. For low doping (near half filling) the system consists of dilute mobile holes in a spin background.

A lot of effort has been devoted to the understanding of the dynamics of holes in spin backgrounds. Brinkman and Rice (BR) [2] considered several configurations of the spin background (ferromagnetic (FM), Néel and random) and studied the density of hole-states and dc conductivity. Their calculation was based on Nagaoka’s expansion of expectation values in terms of background-conserving hole paths [3]. Of these, they only considered retraceable paths, i.e., no loops were taken into account. They obtained very accurate results for single-particle Green’s functions of a hole in the Néel background, being exact in one dimension (where all paths are retraceable). This retraceable paths approximation (rpa) was used to study many other quantities such as dynamical conductivity [4], electrical resistivity, thermal conductivity, thermopower and specific heat [5].

Using different approximations, many other analytical approaches have been reported for the ground state and excited properties of a single hole [6,7,8]. There are also numerous studies using exact diagonalization techniques in low dimensions [9,10,11,12]. These results show a well defined quasiparticle peak for $J \simeq t$ whereas for small enough $J/t$ an incoherent spectrum carries most of the spectral weight.

Most of the studies have been concentrated on the dynamics of a hole in an antiferromagnetic (AF) background. The case of the polarized $t - J$ model has been studied in connection with strong coupling superconductors [13]. The negative $U$ Hubbard model with $n$ particles is equivalent to a positive-$U$ case with one particle per site and a net magnetization given by $S_z = \frac{(1-n)}{2}$ [14]. So the dynamics of a single unpaired particle moving in a background of strongly bounded paired particles (strong negative $U$ limit) is described by the $t - J$ model.
In the present work we study the single-particle Green’s functions in certain backgrounds, extending the results of Metzner et al. \cite{1} to generalized spin backgrounds and in particular to one that describes the physics of holes in strong coupling superconductors.

The calculations are performed for infinite dimensions where the results are exact. The limit of high lattice dimensions, $d \to \infty$, has been used to study correlated fermions \cite{15} and helped to clarify the validity of several approximations and construct new ones \cite{16}.

A self-consistent approximation for finite dimensions can be performed in a similar way as in Ref. \cite{1}. More realistic calculations applicable to high Tc superconductors, for example, should also include interactions between holes and spin fluctuations. We neglect spin fluctuations since they disappear in infinite dimensions, and concentrate on the corrections due to the inclusion of loops to the BR retraceable path approximation. This latter approach doesn’t distinguish between different spin backgrounds since paths without loops are always background conserving. While in the Néel background the BR approximation is correct up to order $1/d^4$, where $d$ is the dimension, the contribution of loops becomes important whenever there are clusters of aligned spins. In particular, in a FM background, any hole path leaves the background unchanged, leading to Nagaoka’s theorem \cite{3}.

The paper is organized as follows: in Sec. II we calculate the dynamics of a hole in an ensemble of spin configurations in a hypercubic lattice, considering arbitrary spin averages or probabilities in each sublattice; in Sec. III we consider a definite spin state characterized by the angle between the spins on different sublattices and we obtain expressions for the local and $k$—dependent propagator. We summarize in Sec. IV.

II. ONE HOLE IN A GENERALIZED SPIN BACKGROUND

We consider the $t – J$ model for infinite dimensions with $J = 0$, which with standard notation reads:

$$H = -t \sum_{<ij>,\sigma} (1 - n_{i-\sigma}) c_{i\sigma}^\dagger c_{j\sigma} (1 - n_{j-\sigma})$$

(1)
To keep the average kinetic energy finite in $d \to \infty$, one must scale the hopping amplitude $t$ as

$$t = \frac{t^*}{\sqrt{Z}}$$

$t^*$ fixed, where $Z$ is the number of nearest neighbours ($Z = 2d$ in a hypercubic lattice).

The Green’s function of a hole in an arbitrary spin background $S$ is:

$$G_{ij}^S(z) = \sum_{\sigma} G_{ij\sigma}^S(z)$$

$$G_{ij\sigma}^S(z) = \langle c_{i\sigma}^\dagger z - H c_{j\sigma} \rangle^S$$

where $\langle \cdots \rangle^S = \sum_{S'} w_{S'} (S' | \cdots | S')$ and $w_{S'}$ is a normalized distribution of spin configurations $S'$. The spins of different sites are statistically independent. We consider a hypercubic lattice and characterize the spin ensembles by assigning a probability for spin $\sigma$ in the sublattice $X$: $p_{X\sigma}$; $X = (A, B)$ and $\sigma = (\uparrow, \downarrow)$. The random case considered in Ref. [1] corresponds to $p_{A\sigma} = p_{B\sigma}$ and interpolates between the FM ($p_{A\sigma} = 1$) and the unpolarized random ($p_{A\sigma} = 1/2$). We generalize the calculation to interpolate also between the random and the Néel background ($p_{A\sigma} = p_{B-\sigma} = 1$). The unpolarized random case corresponds to a thermal average over all spin configurations which contribute with equal weight. The other polarized cases correspond to thermal averages in presence of uniform or staggered magnetic fields.

We follow closely Ref. [1] for the notations. The calculations are based principally on Nagaoka’s expansion for the Green’s functions in powers of $t/z$ [3]. For our hypercubic lattice we have to distinguish two local Green’s functions:

$$G_{AA}(z) = \frac{1}{z[1 - S_A(z)]} \quad \text{and} \quad G_{BB}(z) = \frac{1}{z[1 - S_B(z)]}$$

where $S_X$ ($X = A, B$) is given by the sum over all graphs for which the hole returns to the starting point only once. Due to the simple topology of the loop trees, $S_X$ can be written as a sum over loops with dressed vertices $C_X$. This dressed vertex is given by the bare one plus all possible $S_X$ insertions, so we have:

$$C_{X\sigma}(z) = p_{X\sigma} \sum_{l=0}^{\infty} [S_X(z)]^l = \frac{p_{X\sigma}}{1 - S_X(z)}$$
and

\[ S_A(z) = \sum_{\sigma} \sum_{n=1}^{\infty} u_{2n} [C_{B\sigma}^n C_{A\sigma}^{n-1}] \left( \frac{t^*}{z} \right)^{2n} \]  
\[ S_B(z) = \sum_{\sigma} \sum_{n=1}^{\infty} u_{2n} [C_{A\sigma}^n C_{B\sigma}^{n-1}] \left( \frac{t^*}{z} \right)^{2n} \]  

(6a) \hspace{1cm} (6b)

where \( u_n \) is the number of self-avoiding return paths of length \( n \).

Defining a generating function for \( u_n \)

\[ M(\xi) = 1 + \sum_{n=2}^{\infty} u_n \xi^n \]  

(7)

equation (6) can be written as

\[ S_A(z) = \sum_{\sigma} \frac{M[t^* \sqrt{C_{A\sigma} C_{B\sigma}}] - 1}{C_{A\sigma}} \]  
\[ S_B(z) = \sum_{\sigma} \frac{M[t^* \sqrt{C_{A\sigma} C_{B\sigma}}] - 1}{C_{B\sigma}} \]  

(8a) \hspace{1cm} (8b)

From Eqs. (4), (5) and (8) and defining \( G^0 \) as the free particle Green’s function and \( G_{-1}^0 \) its inverse, the following relations hold in the FM limit \( (p_{A\uparrow} = p_{B\uparrow} \rightarrow 0) \)

\[ M(\xi) = 1 + \mathcal{O}(\xi^2) \text{ for small } \xi \implies M(tG^0) = zG^0 \]  

(9)

This implies

\[ M(\xi) = \frac{\xi}{t^*} G^0_{-1}(\xi/t^*) \]  

(10)

Using this relation in a similar way as in \( [1] \) we find:

\[ 1 + p_{A\uparrow} p_{A\downarrow} (z G_{AA} - 1) = \sum_{\sigma} p_{A-\sigma} \sqrt{p_{A\sigma} p_{B\sigma} G_{AA} G_{BB} G_{-1}^0} \left[ \sqrt{p_{A\sigma} p_{B\sigma} G_{AA} G_{BB} G_{-1}^0} \right] \]  
\[ 1 + p_{B\uparrow} p_{B\downarrow} (z G_{BB} - 1) = \sum_{\sigma} p_{B-\sigma} \sqrt{p_{A\sigma} p_{B\sigma} G_{AA} G_{BB} G_{-1}^0} \left[ \sqrt{p_{A\sigma} p_{B\sigma} G_{AA} G_{BB} G_{-1}^0} \right] \]  

(11a) \hspace{1cm} (11b)

A more compact expression can be obtained in the symmetric case where \( p_{A\sigma} = p_{B-\sigma} \).

This implies \( G_{AA} = G_{BB} = G \) and Eqs. (11) simplify to

\[ \gamma G = G^0 \left[ 1 + \gamma^2 (z G - 1) \right] \]  

(12)
with $\gamma = \sqrt{p_{\alpha\sigma}p_{A-\sigma}}$ ranging from Néel to uniform random backgrounds ($0 \leq \gamma \leq 1/2$). The density of states $D(\omega) = -\frac{1}{\pi} \text{Im} G(\omega + i0^+)$ for this symmetric case coincides exactly with the density of states of the tilted configuration with the correspondence $\gamma = \alpha$ for $\alpha \leq 1/2$ (see Eq. (26) and Fig. 2).

As for $|\omega| \gg t^* \Rightarrow G(\omega) \sim 1/\omega$, Eq. (12) implies $G(\omega) \sim 1/\omega$ and $\gamma G \sim G_0(\omega/\gamma)$. Using the density of states of a free particle at large $|\omega|$ we have:

$$D(\omega) \sim \frac{1}{\sqrt{2\pi\gamma t^*}} e^{-\omega^2/(2\gamma t^*)}$$

The density of states has exponential tails for large $|\omega|$ whenever $\gamma \neq 0$. These come from the presence of FM clusters in the spin configuration.

In the Néel limit ($p_{A\downarrow} \to 0 \Rightarrow \gamma \to 0$), to first order in $p_{A\downarrow}$, Eq. (11) leads to

$$zG - 1 = t^*G^2 \Rightarrow G = \frac{z - \sqrt{z^2 - 4t^*2}}{2t^*}$$

This coincides with the results of the rpa [2]. The corresponding density of states

$$D(\omega) = \frac{1}{2\pi t^*} \sqrt{4t^*2 - \omega^2}; \quad |\omega| \leq 2t^*$$

has a semielliptic shape with band edges at $\pm 2t^*$ with a square root singularity.

We also calculated the self-energy for this symmetric case. It is $k$-independent for $d \to \infty$ and from symmetry $\Sigma_{A\sigma} = \Sigma_{B-\sigma}$. It is not necessary to use the Nagaoka expansion for the off-diagonal propagator in the site representation as we did for the calculation of the local propagator [1]. Instead, we use the Dyson equation together with the locality of the self energy.

In a matrix representation for the $k$-dependent Green’s function in the reduced Brillouin zone ($\epsilon_k = -2t(\cos k_1 + \cdots + \cos k_d, < 0$) we have

$$G_{k\sigma}^{-1}(z) = \begin{pmatrix} z - \Sigma_{A\sigma} & \epsilon_k \\ \epsilon_k & z - \Sigma_{B\sigma} \end{pmatrix}$$

This leads to
\[ G_{k\sigma}(z) = \frac{1}{D} \begin{pmatrix} z - \Sigma_{B\sigma} & -\epsilon_k \\ -\epsilon_k & z - \Sigma_{A\sigma} \end{pmatrix} \]  

(17)

\[ D = \left( \sqrt{(z - \Sigma_{A\sigma})(z - \Sigma_{B\sigma}) - \epsilon_k} \right) \left( \sqrt{(z - \Sigma_{A\sigma})(z - \Sigma_{B\sigma}) + \epsilon_k} \right) \]  

(18)

After some algebra and considering the fact that

\[ \langle 1 + \epsilon_k \rangle_{k} = \int_{-\infty}^{0} D^{0}(\epsilon) a + \epsilon = \int_{0}^{\infty} D^{0}(\epsilon) a - \epsilon = \langle 1 - \epsilon_k \rangle_{k} \]  

(19)

and that \( G_{X\sigma} = \langle G_{k\sigma} \rangle_{k} = p_{X\sigma} G \) we obtain:

\[ \sqrt{z - \Sigma_{A\sigma}} G^{0} \left[ \sqrt{(z - \Sigma_{A\sigma})(z - \Sigma_{B\sigma})} \right] = p_{A\sigma} G \]  

(20a)

\[ \sqrt{z - \Sigma_{B\sigma}} G^{0} \left[ \sqrt{(z - \Sigma_{A\sigma})(z - \Sigma_{B\sigma})} \right] = p_{B\sigma} G \]  

(20b)

From Eqs. (12) and (20) we obtain:

\[ \Sigma_{A\sigma} = z(1 - p_{B\sigma}) - \frac{1 - \gamma^{2}}{p_{A\sigma} G} \]  

(21a)

\[ \Sigma_{B\sigma} = z(1 - p_{A\sigma}) - \frac{1 - \gamma^{2}}{p_{B\sigma} G} \]  

(21b)

These expressions lead to the known results for the Néel limit \( (\gamma = 0) \) and the random case \( (\gamma = 1/2) \) (see Ref. [1]). Here \( Im \Sigma_{X\sigma}(\omega - i0^{+}) > 0 \) \( \forall \omega \) and there are no quasiparticles in the system. This is expected since this system doesn’t have a Fermi surface.

### III. ONE HOLE IN A TILTED SPIN BACKGROUND

A more compact and straightforward calculation can be done by considering a magnetic field acting on the system polarizing the spins in a way sketched in Fig. 1. As in the previous section, we are considering a hypercubic lattice in a pure spin configuration with spins \( \sigma_A \) and \( \sigma_B \) lying on the \( yz \) plane, forming an angle \( \phi \) between them. In this way we can continuously connect the Néel \( (\phi = \pi) \) to the FM \( (\phi = 0) \) background.
As we mentioned in the Introduction, this model also describes the dynamics of an unpaired particle in a strong coupling superconductor. The spin variables up and down correspond to an empty and a doubly occupied site, respectively. The external magnetic field plays the role of a chemical potential since the total magnetization is given by the total number of particles \( S_z = (1 - n) / 2 \). The magnetization in the \( x - y \) plane corresponds to the superfluid order parameter.

By moving along a loop, a hole will replace \( A \)-spins with \( B \)-spins and the overlap between two states at any given site is

\[
\alpha = \langle \uparrow | e^{i\phi \sigma_x / 2} | \uparrow \rangle = \cos(\frac{\phi}{2})
\]  

For a \( 2n \)-loop, \( 2n - 2 \) sites are changed, therefore

\[
S(z) = \sum_{n=1}^{\infty} u_{2n} C^{2n-1} \alpha^{2n-2} \left( \frac{t^*}{z} \right)^{2n} = \frac{M [\alpha C t^*/z] - 1}{\alpha^2 C}
\]

where \( M \) is given by Eq. (7).

Here again

\[
G(z) = \frac{1}{z(1 - S(z))} \quad \text{and} \quad C(z) = \frac{1}{1 - S(z)}
\]

From Eqs. (10), (23) and (24) we obtain

\[
\alpha G(z) = G^0 \left[ \alpha z + \frac{1 - \alpha^2}{\alpha G(z)} \right]
\]

This expression is equal to Eq. (12) for the symmetric random model with the correspondence

\[
\alpha = \cos(\phi/2) = \gamma = \sqrt{p_{A\sigma} p_{A-\sigma}}
\]

for \( \alpha \leq 1/2 \). But Eq. (25) is more general because \( \alpha \) can assume any value between 0 and 1: \( \alpha = 1 \) corresponds to the FM case and \( \alpha = 0 \) to the Néel case. The hole dynamics for \( \alpha = 1/2 \) \((\Rightarrow \phi = 2\pi/3)\) happens to coincide exactly with that for the unpolarized random background.
In Fig. 2 we show the density of states corresponding to the one-particle Green function of Eq. (23) for several values of $\phi$. $G$ has been calculated self-consistently using the Néel Green function (14) as a seed. The only case without tails is the Néel configuration ($\phi = \pi$) that presents square root singularities at the band edges. The states in the tails of the other configurations correspond to low energy (say $\omega \rightarrow -\infty$) states due to FM clusters. It also seems that for all values of $\alpha$ there is an energy $\omega_0$ such that $D(\omega_0)$ is independent of $\alpha$ (this crossing also happens for the real part of the propagator at another energy). But this coincidence is only approximate, being, nevertheless, a curious feature.

We calculated the full propagator in the tilted background. For the disposition of axes chosen, the spin on the A-sites can be written as:

$$e^{i\sigma_x \phi/4} | \uparrow \rangle = \cos(\phi/4) | \uparrow \rangle - \sin(\phi/4) | \downarrow \rangle$$

and on the B-sites by changing $\phi$ by $-\phi$. So the Green function can be written in a matrix way in the spin representation

$$G_{AA\sigma\sigma'} = G \begin{pmatrix} \cos^2(\phi/4) & -\sin(\phi/4) \cos(\phi/4) \\ -\sin(\phi/4) \cos(\phi/4) & \sin^2(\phi/4) \end{pmatrix}$$

(28a)

$$G_{BB\sigma\sigma'} = G(\phi \leftrightarrow -\phi)$$

(28b)

For this type of configurations there is a generalized Bloch theorem. The elements of the symmetry group that leave the configuration $|X\rangle$ and the Hamiltonian invariant are $g_l = T_l e^{i\pi \sigma_z/2}$, where $l = \sum_{n=1}^{d} l_n$; i.e. translations coupled with rotations in $\pi$ about the $z$ axis. So $c_{k\sigma}^\dagger = \frac{1}{\sqrt{V}} \sum_{l} c_{l\sigma}^\dagger e^{ikl}$ is an eigenoperator of $G_l$ with eigenvalue $q$ such that $e^{iq1} = (\sigma i)^l e^{ik1} = e^{i(k+\sigma Q/2)\cdot l}$ and $Q = (\pi, \cdots, \pi)$. This implies that $c_{k\sigma}^\dagger$ can couple only to $c_{k+Q-\sigma}^\dagger$.

We define a propagator in the spin representation

$$G_{k\sigma\sigma'} = \begin{pmatrix} \langle \langle c^\dagger_{k\uparrow} | c_{k\uparrow} \rangle \rangle & \langle \langle c^\dagger_{k\uparrow} | c_{k+Q\downarrow} \rangle \rangle \\ \langle \langle c^\dagger_{k+Q\uparrow} | c_{k\uparrow} \rangle \rangle & \langle \langle c^\dagger_{k+Q\uparrow} | c_{k+Q\downarrow} \rangle \rangle \end{pmatrix}$$

(29)

Then
\[
\langle G_{k\sigma'} \rangle_k = G'_{AA\sigma'} \sigma' G_{BB\sigma'}
\]  
(30)

Considering the locality of the self energy we can write the inverse of Eq. (29) as:

\[
G_{k\sigma' \sigma}^{-1}(z) = \begin{pmatrix}
  z - \epsilon_k - \Sigma_{\uparrow \uparrow} & -\Sigma_{\uparrow \downarrow} \\
  -\Sigma_{\downarrow \uparrow} & z + \epsilon_k - \Sigma_{\downarrow \downarrow}
\end{pmatrix}
\]  
(31)

so

\[
G_{k\sigma' \sigma}(z) = \frac{1}{D} \begin{pmatrix}
  z + \epsilon_k - \Sigma_{\downarrow \uparrow} & \Sigma_{\downarrow \uparrow} \\
  \Sigma_{\uparrow \downarrow} & z - \epsilon_k - \Sigma_{\uparrow \uparrow}
\end{pmatrix}
\]  
(32)

\[
D = (z - \epsilon_k - \Sigma_{\uparrow \uparrow})(z + \epsilon_k - \Sigma_{\downarrow \uparrow}) - \Sigma_{\downarrow \uparrow} \Sigma_{\uparrow \downarrow} = - (\epsilon_k - E_+)(\epsilon_k - E_-)  
\]  
(33)

where

\[
E_{\pm} = \frac{1}{2}(\Sigma_{\downarrow \downarrow} - \Sigma_{\uparrow \uparrow}) \pm \frac{1}{2}\sqrt{W}
\]  
(34)

This leads to

\[
G_{k\sigma' \sigma}(z) = \frac{1}{E_+ - E_-} \begin{pmatrix}
  -z + E_+ - \Sigma_{\downarrow \uparrow} + \frac{z + E_+ - \Sigma_{\downarrow \uparrow}}{E_+ - \epsilon_k} & \Sigma_{\downarrow \uparrow} \\
  \Sigma_{\uparrow \downarrow} & -z - E_- + \Sigma_{\uparrow \uparrow} + \frac{z - E_- + \Sigma_{\uparrow \uparrow}}{E_- - \epsilon_k}
\end{pmatrix}
\]  
(35)

Using Eqs. (19), (30) and (34) we obtain

\[
G'_{AA\sigma'} = - \frac{1}{\sqrt{W}} \begin{pmatrix}
  [z - \frac{1}{2}(\Sigma_{\uparrow \uparrow} + \Sigma_{\downarrow \downarrow})](G^0_{\downarrow \downarrow} - G^0_{\uparrow \uparrow}) - \frac{1}{2}\sqrt{W}(G^0_{\downarrow \downarrow} + G^0_{\uparrow \uparrow}) & \Sigma_{\downarrow \uparrow}(G^0_{\downarrow \downarrow} - G^0_{\uparrow \uparrow}) \\
  \Sigma_{\uparrow \downarrow}(G^0_{\downarrow \downarrow} - G^0_{\uparrow \uparrow}) & [z - \frac{1}{2}(\Sigma_{\uparrow \uparrow} + \Sigma_{\downarrow \downarrow})](G^0_{\downarrow \downarrow} - G^0_{\uparrow \uparrow}) + \frac{1}{2}\sqrt{W}(G^0_{\downarrow \downarrow} + G^0_{\uparrow \uparrow})
\end{pmatrix}
\]  
(36)

where

\[
W = (\Sigma_{\downarrow \downarrow} - \Sigma_{\uparrow \uparrow})^2 - 4\Sigma_{\uparrow \downarrow} \Sigma_{\downarrow \uparrow} + 4(z - \Sigma_{\uparrow \uparrow})(z - \Sigma_{\downarrow \downarrow})
\]  
(37)

and we have defined $G^0(E_{\pm}) = G^0_{\pm}$.

Eq. (35) implies $\Sigma_{\uparrow \downarrow}(z) = \Sigma_{\downarrow \uparrow}(z)$. Together with Eq. (28) it also gives a set of three equations for $\Sigma_{\uparrow \uparrow}$, $\Sigma_{\downarrow \downarrow}$ and $\Sigma_{\uparrow \downarrow}$. 

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\[(2z - \Sigma_{\uparrow\downarrow} - \Sigma_{\downarrow\uparrow})(G_+^0 - G_-^0) = G\sqrt{W}\]  
(38a)

\[G_+^0 + G_-^0 = G\cos\left(\frac{\phi}{2}\right)\]  
(38b)

\[2\Sigma_{\uparrow\downarrow}(G_+^0 - G_-^0) = -G\sin\left(\frac{\phi}{2}\right)\sqrt{W}\]  
(38c)

From these equations we obtain \(G_-^0 = 0\) which implies \(E_- \to \infty\) as well as the three self energies. From \(G^0(E_+) = G\cos\left(\frac{\phi}{2}\right)\) and Eq. (25) it follows that

\[E_+ = z\cos\left(\frac{\phi}{2}\right) + \frac{\sin^2\left(\frac{\phi}{2}\right)}{G\cos\left(\frac{\phi}{2}\right)}\]  
(39)

We cannot obtain separate expressions for the self energies but \(G_{k\sigma\sigma'}\) can be calculated by knowing the ratios \(\Sigma_{\uparrow\downarrow}/E_-, \Sigma_{\downarrow\uparrow}/E_-\) and \(\Sigma_{\uparrow\downarrow}/E_-\). From Eq. (38) and the definition of \(E_-\) (34), we have, for \(E_- \to \infty\)

\[\Sigma_{\downarrow\uparrow}/E_- \to \left(1 - \tan^2\left(\frac{\phi}{4}\right)\right)^{-1}\]  
(40a)

\[\Sigma_{\uparrow\downarrow}/E_- \to \left(\cot^2\left(\frac{\phi}{4}\right) - 1\right)^{-1}\]  
(40b)

\[\Sigma_{\uparrow\downarrow}/E_- \to \left(2\cot\left(\frac{\phi}{2}\right)\right)^{-1}\]  
(40c)

Finally the full propagator becomes:

\[G_{k\sigma\sigma'} = \frac{\tan\left(\frac{\phi}{2}\right)}{2(E_+ - \epsilon_k)} \begin{pmatrix} \cot(\phi/4) & -1 \\ -1 & \tan(\phi/4) \end{pmatrix}\]  
(41)

where \(E_+\) is given by Eq. (34). From Eq. (28) we see that the spin-flipped propagators are obtained by replacing \(k\) by \(k + Q\), \(i.e.\ \epsilon_k\) by \(-\epsilon_k\).

In Fig. 3 we show the spectral density \(\rho_k(\omega)\) obtained from these equations for several values of \(\phi\) and \(k = Q\). The Néel case is equal to the local density of states since it is \(k\)-independent. In particular, no quasiparticle peak is present since the system has no Fermi surface. Only the FM background presents a quasiparticle peak that corresponds to the dynamics of a free particle.
IV. SUMMARY AND CONCLUSIONS

We have obtained general expressions for the single particle Green’s functions describing the dynamics of a hole in a generalized spin background in infinite dimensions. Our calculations are a generalization of those performed in Ref. [1].

We characterized the ensemble of spin configurations in a hypercubic lattice by the average value of the spin in each sublattice. Closed expressions were found for the symmetric case \( p_{A\sigma} = p_{B-\sigma} \), interpolating between Néel order and the unpolarized random case.

We also performed the calculations considering a magnetic field acting on the system that tilts the spins in such a way that spins of different sublattices form an angle \( \phi \) between them. In this case we obtain expressions for the local and \( k \)-dependent propagator as a function of the angle \( \phi \), so the results interpolate between the FM and Néel configurations.

The local density of states gets narrower when departing from the Néel order and acquires exponential tails that come from the contribution of FM clusters to the hole motion. The spectral function \( \rho_k(\omega) \) also shows this behaviour. A quasiparticle peak is seen only in the FM case and it corresponds to the movement of a free particle.

A self-consistent calculation for finite dimensions can also be carried out in a similar way as in Ref. [1], considering the exact expression for \( G^0 \) in \( d \)-dimensions and scaling the hopping as in Eq. (2). Nevertheless, one has to take into account that in this calculation only loop trees have been considered. Paths with loops on which a hole walks around more than once or with multiply connected loops, are suppressed with respect to the loop tree [7] by some integer power of \( 1/d \). Also, in the Néel case, at the band edges, where the density of states is small, corrections become very large. Another important feature that is neglected in these calculations is the presence of spin fluctuations that become important at low dimensions. Nevertheless, this approximation can always be corrected by properly taking into account all the relevant hole paths (for the 2D Néel case see Ref. [18]) or by allowing background restoring spin flips along the path.
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FIGURES

FIG. 1. Schematic representation of the spin configuration in the tilted background, in the presence of a magnetic field $h$.

FIG. 2. Density of states for the tilted background for different values of $\alpha = \cos(\phi/2)$: $\alpha = 1$ (FM case, dashed-dotted line), $\alpha = .8$ (dashed), $\alpha = .5$ (dotted), $\alpha = .3$ (full), $\alpha = 0$ (Néel, long-dashed line).

FIG. 3. Spectral density for the tilted background for $k = Q$ and different values of $\alpha = \cos(\phi/2)$: The symbols correspond to those of Fig. 2. The FM case presents a delta function at $\omega = 0$.
Fig. 1
Fig. 2
Fig. 3

Plot showing the dependence of $\rho_Q(\omega)$ on $\omega/t^*$.