Features of the model of main functional failures of digital CMOS VLSIs under the action of ionizing radiation

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Annotation. The methods of functional-logical modeling of very large digital integrated circuits (VLSI) under the influence of ionizing radiation are considered. It is shown that the multiplicity of the node and the power of the spectrum are more accurately described when using the concept of fuzzy multiplicity. Methods are proposed for predicting LSI failures when exposed to ionizing radiation, which are based on the model of fuzzy digital and probabilistic reliability automata.

1. Introduction
As a rule, under the influence of radiation, the model of a digital automaton using the apparatus of probability theory is used. In the case when it is necessary to take into account the physical mechanisms of VLSI failure, the construction of a functional-logic model of this class assumes the use of a fuzzy digital Brauer automaton and a transition from the axiomatics of the Boolean lattice to the axiomatics of the vector lattice for each \( x \in X \) [1]. The real nature of VLSI radiation behavior is determined by the ratio of the radiation-sensitive parameters of its elements, taking into account the multiplicity of the node and the power of the VLSI spectrum. In this case, a more accurate estimate is possible when using the notion of fuzzy multiplicity. The relationship between the probability density distribution function and the criterial membership function (CFR) determines, in the final analysis, the appropriateness of using functional-logical models of VLSI radiation behavior for each specific case. Such a comparison is a necessary step in the analysis of the VLSI radiation resistance.

2. Topology tree of the modeling environment
In paper [1], the multiplicity of the node and the power of the spectrum of the VLSI tree were determined to estimate the power of the tree spectrum of the CMOS VLSI topology with the use of fuzzy numbers under the radiation effect. In this case, a more accurate assessment of the LSI stability is possible when using the notion of fuzzy multiplicity.

As a rule, with increasing the absorbed dose of ionizing radiation, the amplitude of the pulse of the logical element \( l \in [0,1] \) decreases [2], and the intrinsic probability of element switching also decreases. This decrease in probability can be interpreted as an increase in the threshold of operation \( l_H \in [0,1] \).

As an example, Fig. 1 shows the experimental results of the logical zero output voltage spread for CMOS VSLI chips of RAM 1617RU6 and their transistor structures that correspond to the normal
Since the dispersion $\sigma^2$ depends on the absorbed radiation dose, the maximum dose (6) can also be used to determine the maximum radiation dose.

**Figure 1.** Distributions of averaged voltages over the chip logical zero output of CMOS VSLI RAM 1617RU6 from the dose (1), threshold voltage of n-channel and p-channel MOS transistors (2, 3) obtained by an electron accelerator with $E_e = 130$ keV

Let $n$ branches leave the node in logical element connections. The node is selected with the maximum number of links (critical node). The multiplicity of the node $m = n$ is realized during normal operation. At the same time, if $l_H$ starts to approach $l$, an instability zone appears in which the elements switch randomly and the multiplicity of the node becomes undefined. Its magnitude, as is known [3], can be expressed by an indistinct number. To simplify the estimates, triangular $(L - R)$ fuzzy numbers are used in this paper.

At the same time, the thresholds $l_H$ are distributed according to some $f(x)$ law such that $f(x) \to 0$ (Fig. 2). Since the mathematical expectation $\xi = l_H$, the increase in the radiation dose $\xi \to l$ and the distribution is shifted to the right (dotted line in Fig. 2).
The fuzzy number mode obviously depends on the magnitude

\[ S = \int_0^l f(x) dx, \]

and the blurring is expressed by the relation [4]:

\[ \min \left\{ (S(\xi) - S(l)), \ (S_m(\xi) - S(l)) \right\}, \]

where \( S_m \) is the maximal value of \( S \).

In this case, we consider fuzzy numbers of the multiplicity of nodes for two distribution functions of \( I_{\mu} \) - Gauss and Laplace.

For the normal Gaussian distribution law, the following equality holds:

\[ S_G = \frac{1}{\sqrt{2\pi} \cdot \sigma} \int_0^l e^{-\frac{(x-\xi)^2}{2\sigma^2}} dx = \Phi\left( \frac{\xi}{\sigma} \right) + \Phi\left( \frac{l - \xi}{\sigma} \right), \quad (1) \]

\( \Phi(z) \) is the probability integral.

On the right in (1), the function is symmetric with respect to \( \xi = \frac{l}{2} \) and there is a weak maximum equal to \( \frac{2\Phi l}{2\sigma} \) while the ratio \( \frac{l}{\sigma} > 4 \).

This feature of the function makes it possible to use a simple trapezoidal approximation (Figure 3) [5], where

\[ \alpha = \frac{l}{\sigma}, \ x = \frac{\xi}{\sigma}, \ \alpha > 4, \ 0 \leq x \leq \alpha. \]
In this case, the fuzzy-number mode can be written in the form of the expression:
\[ M_G = [n \cdot \tilde{S}_G(x)], \]  \hfill (2)

The blur takes the following form:
\[ \Delta_G = 2[n \cdot \min\{\tilde{S}_G(x) - 0.5, (1 - \tilde{S}_G(x))\}], \]  \hfill (3)

where \( \alpha - \sqrt{\pi/2} < x < \alpha \).

In the expressions (2) and (3), \([Z]\) is the whole part of \(Z\).

Then from formulas (2) and (3) we obtain a \((L - R)\) fuzzy number, which is represented as an isosceles triangle with a base \(\Delta_G\) (Fig. 4).

If \(x < \alpha - \frac{1}{2} \sqrt{\frac{\pi}{2}}\), then the blurring does not include \(n\) and corresponding absorbed dose of ionizing radiation is considered catastrophic. In this case, the limiting absorbed dose is determined by the following equalities:
\[ x_{\text{max}} = \alpha - \frac{1}{2} \sqrt{\frac{\pi}{2}} \text{ или } \xi_{\text{max}} = l - \frac{\sqrt{\pi}}{2} \sqrt{\frac{\pi}{2}} \approx l - 0.063 \sigma, \]  \hfill (4)

The obtained expressions (4) are valid for one node.

For a real system, there are tree intersections, which greatly complicates the analysis. For the Laplace distribution we have:
\[ S_L = \frac{1}{2\beta} \int_0^{\frac{l-\xi}{\beta}} e^{\frac{-t}{\beta}} dt = 1 - \frac{e^{x-\alpha} + e^{-x}}{2}, \]  \hfill (5)
where \( x = \frac{\xi}{\beta}, \alpha = \frac{l}{\beta}, \alpha > x, \alpha > 4. \)

The Laplace distribution function (5) is analogous to the Gaussian distribution (1) and can also be approximated by a trapezoidal function \( S_L \) (Fig. 5).

![Figure 5. Trapezoidal approximation \( S_L \) for the Laplace distribution.](image)

In this case, the mode and the blurring of the fuzzy number are correspondingly equal to the relations [6]:

\[
M_L = [n \cdot S_L(x)], \\
\Delta_L = 2[n \cdot \min\left\{S_L(x) - 0.5, 1 - S_L(x)\right\}],
\]

where \( [Z] \) is the integer part of \( Z \), \( \alpha - 1 < x < \alpha \).

The use of the Laplace distribution is preferable, since it has a much smaller spread at the area of mathematical expectation.

For asymmetric distribution laws, formulas (2) - (7) remain valid, although with different limits of variation of the generalized parameter \( x \). The normalization of \( x \) is effected by a certain characteristic parameter of the distribution spreading [7].

To estimate the power of a tree, it is necessary to sum up the fuzzy numbers, however the mean values, as determined in [2], are calculated through the mean value of the modes and the blurrings.

In this case, the average value of the mode is:

\[
M_\xi = \frac{\sum_{i=1}^{K} M_i}{K},
\]

where \( K \) is the number of nodes in the tree, \( M_i \) is the mode of the fuzzy number of the node.

The average blur is determined from the ratio:

\[
\Delta_\xi = \frac{\sum_{i=1}^{K} \Delta_i}{K},
\]

where \( \Delta_i \) is the fuzzy number blur for the i-th node.

Thus, the power of the tree is also expressed by an indistinct number, though the mode and fuzziness of which, determined by formulas (8) and (9), are not necessarily integer. If the blur does not capture the maximum power values given in [2], then the corresponding absorbed radiation dose is considered catastrophic, and less than that which is had for one node [8].
However, this assessment remains overestimated, since it does not take into account the fact that for nodes of the intermediate level of the tree, if the element is not switched on, the multiplicity of the corresponding node is considered equal to 0.

3. Conclusion
The real nature of CMOS VLSI performance under ionizing radiation is determined by the specific spectrum of the VLSI topology tree and the ratio of radiation-sensitive parameters: the multiplicity of nodes, the power of the tree spectrum, and the fuzzy-number mode. In this case, the relationship between the distribution function of the spread and the fuzzy number mode determines, ultimately, the appropriateness of the use of functional-logical models of VLSI radiation behavior for each specific case. Providing such a comparison is a necessary stage in the general procedure of the VLSI radiation resistance analysis. It should be noted that in different ranges of levels or intensity of impact, the power assessment of the tree spectrum of the VLSI topology can be either fuzzy or probabilistic. In this case, fuzzy multiplicity can be specified in n-dimensional space with preservation of the basic model parameters. Radiation effects arising in VLSI at different irradiation levels are estimated in the model by changing the same parameter of the system. Such a structure of the model is probabilistic-fuzzy with operators of probability type, and the criterion membership function is a superposition of the statistical and deterministic criterion membership function. At the same time, the interdependence of fuzzy multiplicity and probabilistic logic is determined, which allows the most accurate estimation of the VLSI functioning under the influence of radiation.

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