Superconducting Quantum Interference Device without Josephson Junctions

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A new type of a superconducting quantum interference device (SQUID) based on a single superconducting loop without Josephson junctions and with asymmetric contacts has been proposed. This SQUID offers advantages in simplicity of fabrication and a steeper dependence of measured quantities on the magnetic flux. To confirm the possibility of making this type of SQUID, the magnetic field dependence of the critical current in an aluminum ring with asymmetric contacts has been experimentally investigated.

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1. INTRODUCTION

A superconducting quantum interference device (SQUID) has provided a record-high sensitivity of magnetic flux and magnetic field measurements for several decades. It is based on a closed superconducting loop with (rf SQUID) one or (dc SQUID) two Josephson junctions [1]. Available SQUIDs implement [1] the relation $\Delta \phi + 2\pi \Phi/\Phi_0 = 2\pi n$ between the total phase difference $\Delta \phi$ at the Josephson junctions and the magnetic flux $\Phi$ through the loop. Here, $\Phi_0 = 2\pi \hbar/q$ is the flux quantum and $q = 2e$ is the charge of the electron pair. Since the supercurrent through a Josephson junction is $I = I_s \sin \Delta \phi$ [2], measurable quantities vary from minimum to maximum values on a scale of $\Phi_0/2$. In this work, we study the possibility of making a SQUID without Josephson junctions with a steeper dependence of measurable parameters on the magnetic flux. The idea is based on the effect discovered by V.A. Little and R.D. Parks [3]. The possibility of using a loop [4] or even a cylinder [5] without Josephson junctions as a quantum interferometer was studied both many years ago and recently [6]. A fundamentally new suggestion of this work, as compared to the previous ones, is the use of asymmetric contacts. Asymmetric connection to the interferometer with two Dayem bridges was implemented in recent work [7]. A new point as compared to [7] is the use of a sharp change in the average value of the critical current under a change in the quantum number near $\Phi = (n + 0.5)\Phi_0$.

2. POSSIBILITY OF A STEEPER MAGNETIC FLUX DEPENDENCE OF A PERSISTENT CURRENT

The idea of a new SQUID is based on a unique relation between the quantum number $n$, the magnetic flux through the loop without Josephson junctions, and the velocity of superconducting pairs

$$\oint dl = \frac{2\pi \hbar}{m} (n - \Phi/\Phi_0)$$

(1)

or the persistent current $I_p = qn\nu$. The magnitude of $I_p$ is determined by conditions of velocity quantization (1) and continuity of the current $I_c$ circulating in the loop with the cross section $s$ and the density $n_s$ of superconducting pairs: $I_p = [q \cdot 2\pi \hbar/ml(sn_s)^{-1}] |n - \Phi/\Phi_0| = I_{p,A} \cdot 2(n - \Phi/\Phi_0)$, where $(sn_s)^{-1} = l^{-1} \oint dl (sn_s)^{-1}$. If the quantity $sn_s$ is constant within the loop, $(sn_s)^{-1} = (sn_1)^{-1}$ and $I_{p,A} = sn_q \pi \hbar/ml$. Velocity (1), the persistent current, and the kinetic energy

$$E_n = \oint dl sn_s m \frac{\nu_a^2}{2} = I_p \oint dl m \frac{\nu_a^2}{2} = I_{p,A} \Phi_0 \left( n - \frac{\Phi}{\Phi_0} \right)^2$$

(2)

of superconducting pairs in the loop depend on the magnetic flux $\Phi$ and the quantum number $n$. The absolute value $|I_q| = I_{p,A}$ of the persistent current at $|n - \Phi/\Phi_0| = 0.5$, which determines the energy difference between allowed levels (2), increases with a decrease in temperature: $I_{p,A}(T) = I_{p,A}(0)(1 - T/T_c)$.
At the realistic currents $I_{p,A} = 1$, 10, and 100 $\mu$A [8], the energy differences $|E_{n+1} - E_{n}| \approx I_{p,A} \Phi_{0} \approx 2 \times 10^{-21}, 2 \times 10^{-20}$, and $2 \times 10^{-19}$ J at $\Phi \approx n \Phi_{0}$ correspond to the temperatures $T_{\text{dir}} = I_{p,A} \Phi_{0}/k_{B} \approx 150$, 1500, and 15000 K, respectively. At the transition of at least one segment to the normal state with $n = 0$, the persistent current becomes $I_{p,A} = q \pi \hbar / ml (sn_{v})^{-1}$ and energy (2) vanishes. At the reverse transition with the probability $P_{n} \propto \exp(-E_{n}/k_{B} T)$ given by the laws of statistical physics, the quantum number takes the value $n$. Numerous measurements of the critical current [8–10] and other quantities [11–13] confirm the overwhelming probability of low-energy state (2).

The interval $\Delta \Phi_{e}/\Phi_{0} = \Phi/\Phi_{0} - (n + 0.5)$, in which the probability of the state $n$ varies from $P_{n} \approx 1$ to $P_{n} \approx 0$, depends on the ratio $I_{p,A} \Phi_{0}/k_{B} T = T_{\text{dir}}/T$ because $E_{n+1} - E_{n} \approx I_{p,A} \Phi_{0} \cdot 0.5 \Delta \phi_{e}/\Phi_{0}$ in the vicinity of half of the flux quantum. The probability changes by a factor of 10 at $\Delta \phi_{e}/\Phi_{0} \approx \ln 10 k_{B} T/I_{p,A} \Phi_{0} = 2.3 T / T_{\text{dir}}$ and by a factor of 100 at $\Delta \phi_{e}/\Phi_{0} \approx \ln 100 k_{B} T/I_{p,A} \Phi_{0} = 4.6 T / T_{\text{dir}}$. At the temperature $T \approx 1$ K of measurements, the probability $P_{n}$ changes by a factor of 10 within the interval $\Delta \phi_{e} \approx 0.015 \Phi_{0}$, 0.0015$\Phi_{0}$, and 0.00015$\Phi_{0}$ in the loops with $I_{p,A} = 1$, 10, and 100 $\mu$A, respectively. These intervals are much smaller than the magnetic flux interval $\Delta \phi_{e} \approx 0.5 \Phi_{0}$ of the variation of the persistent current in rf and dc SQUIDs.

### 3. SUPERCONDUCTING LOOP WITH ASYMMETRIC CONTACTS

The distribution of the external current $I = q s_{\text{long}} n_{s, \text{long}} v_{\text{long}} - q s_{\text{sh}} n_{s, \text{sh}} v_{\text{sh}}$ (3) between the long ($l_{\text{long}}$ with the cross section $s_{\text{long}}$ and the pair density $n_{s, \text{long}}$) and short ($l_{\text{sh}}$ with the cross section $s_{\text{sh}}$ and the pair density $n_{s, \text{sh}}$) segments of the loop $I = l_{\text{long}} + l_{\text{sh}}$ (Fig. 1) is uniquely determined by quantization condition (1):

$$I_{\text{long}} v_{\text{long}} + I_{\text{sh}} v_{\text{sh}} = \frac{2 \pi \hbar}{m} (n - \Phi / \Phi_{0}).$$

(4)

At equal cross sections $s_{\text{long}} = s_{\text{sh}} = s$ and pair densities $n_{s, \text{long}} = n_{s, \text{sh}} = n_{s}$, the velocities

$$v_{\text{sh}} = \frac{l_{\text{long}} I_{\text{ext}}}{I q s_{v}} + \frac{2 \pi \hbar}{m l} (n - \Phi / \Phi_{0}),$$

(5a)

and

$$v_{\text{long}} = \frac{l_{\text{sh}} I_{\text{ext}}}{I q s_{v}} + \frac{2 \pi \hbar}{m l} (n - \Phi / \Phi_{0}),$$

(5b)

in the short and long segments, respectively, reach the critical value $|v_{\text{sh}}| = v_{c}$ at

$$I_{c} = \frac{l_{\text{long}}}{l_{\text{sh}}} \left[ q s_{v} v_{c} + q s_{v} \frac{2 \pi \hbar}{m l} (n - \Phi / \Phi_{0}) \right]$$

(6a)

and $|v_{\text{long}}| = v_{c}$ at

$$I_{c} = \frac{l_{\text{long}}}{l_{\text{sh}}} \left[ q s_{v} v_{c} - q s_{v} \frac{2 \pi \hbar}{m l} (n - \Phi / \Phi_{0}) \right]$$

(6b)

The positive direction is taken to be from left to right for $I$ and clockwise for all other quantities. In Fig. 1, the short segment of the loop is situated at the bottom.

The persistent current $I_{p} = I_{p,A} \cdot (2(n - \Phi / \Phi_{0})$ jumps from $I_{p} = -I_{p,A}$ to $I_{p} = +I_{p,A}$ with a change in the quantum number $n$ at $\Phi = (n + 0.5) \Phi_{0}$ (see Fig. 4.4 in [14]). Therefore, critical current (6) must also change abruptly by $(I_{\text{long}} - I_{\text{sh}}) / 2 I_{p,A}$ or by $(I_{\text{long}} - I_{\text{sh}}) / I_{p,A}$ at a higher $(I_{\text{long}} - I_{\text{sh}}) / I_{p,A}$ or lower $(I_{\text{long}} - I_{\text{sh}}) / I_{p,A}$ asymmetry, respectively. The jump vanishes at the symmetric connection of the contacts, $l_{\text{long}} = l_{\text{sh}} = l/2$.

### 4. MAGNETIC FLUX DEPENDENCE OF THE AVERAGE CRITICAL CURRENT AND VOLTAGE

In the vicinity of a half-integer number of flux quanta, at $|\Delta \phi_{e}| = |\Phi - (n + 0.5) \Phi_{0}| \ll \Phi_{0}$, critical current (6a) is $I_{c} \approx I_{0} - (l_{\text{long}}/l_{\text{sh}}) I_{p,A}$ and $I_{c} \approx I_{0} + (l_{\text{long}}/l_{\text{sh}}) I_{p,A}$ in the states $n$ and $n + 1$, respectively. At the
The average critical current \( I \approx I_{cl0} \), the voltage at the ring must be \( V = 0 \) and \( V = R_nI_{cl0} \) at \( n + 1 \) and \( n \), respectively. The average critical current \( I_c \approx I_{cl0} + (\mu_I I_{long})I_{p,A}(P_{n+1} - P_n) \) and voltage \( \bar{V} \approx R_nI_{cl0}P_n \) should vary between the minimum and maximum values within the same narrow interval \( \Delta \Phi_e \) of the magnetic flux as the probabilities \( P_n \) and \( P_{n+1} \) of the states \( n \) and \( n + 1 \). Since the resistance of the loop in the normal state is \( R_n = \rho l_{long}/s \) and \( I_{cl0} = s_l/l_{long} \), one has \( R_nI_{cl0} = \rho s_l/\rho_{cl0}. \) The change in the voltage is on the order of \( \partial V/\partial i_h \approx 10 \text{ mV} \) at \( i_h \approx 1 \mu \text{A} \) and typical values of the resistivity \( \rho \approx 10^{-5} \Omega \text{ cm} \) and critical current density \( j_c \approx 10^7 \text{ A/cm}^2 \) of known superconductors, e.g., niobium. This change is several orders of magnitude greater than the voltage change per flux quantum in a dc SQUID [1].

The sensitivity \( \partial V/\partial \Phi \) (an important parameter for the use of a dc SQUID as a measuring device [1]) can be additionally increased owing to the variation of the average voltage \( \bar{V} \approx R_nI_{cl0}P_n(\Delta \Phi_e) \) in a much smaller magnetic flux interval than the flux quantum (\( \Delta \Phi_e \ll \Phi_0 \)).

To measure the average voltage, the loop must be switched to the normal state for a short time. This can be accomplished by short (\( \Delta t \)) current pulses \( -\Delta I_{ext} > 2I_{ext} \), the sign of which is opposite to the dc excitation current \( (I_{cl} < I_{ext} < I_{c2}, \text{see Fig. 2}) \). The signs of \( I_{ext} \) and \( \Delta I_{ext} \) must be opposite owing to the hysteresis of the current–voltage characteristics (Fig. 2). The average voltage measured for a time much longer than the pulse period \( T \) should be \( \bar{V} \approx R_nI_{ext}P_1 \) at a short pulse length \( \Delta t/T \ll 1 \). The measured voltage should vary within the same narrow interval of \( \Phi \) values as the probability \( P_1 \) of the state with a lower critical current.

5. VOLTAGE–CURRENT CHARACTERISTICS AND MAGNETIC FLUX DEPENDENCE OF THE CRITICAL CURRENT IN AN ALUMINUM RING WITH ASYMMETRIC CONTACTS

To confirm the theoretical predictions, we performed the first measurements of the magnetic-field dependence of the critical current in the aluminum ring with the asymmetric connection of contacts. The structure shown in Fig. 1 was fabricated from an aluminum film with the thickness \( d = 20 \text{ nm} \) on a Si/SiO\(_2\) substrate by electron lithography with the use of liftoff technique. The electron exposure of the pattern (electron lithography) was performed on an EVO-50 scanning electron microscope equipped with a NanoMaker software/hardware system. The radius and width of the ring were \( r \approx 1 \mu \text{m} \) and \( w \approx 0.15 \mu \text{m} \), respectively. The resistance of the ring in the normal state was \( R_n \approx 60 \Omega \). The resistance ratio was \( R(300 \text{ K})/R(4.2 \text{ K}) \approx 1.6 \). The superconducting transition temperature was \( T_c \approx 1.52 \text{ K} \). The measurements were carried out by a four-terminal method in a glass helium cryostat. We used \(^4\text{He} \) as a coolant and pump-
The dependences $I_{c+}(B)$ and $I_{c-}(B)$ of the critical current on the magnetic field were found from the periodic (10 Hz) current–voltage characteristics (see Fig. 2) in a slowly (~0.01 Hz) varying magnetic field $B_{\text{sol}}$ according to the following algorithm: (i) the superconducting state of the structure was verified; (ii) after the threshold voltage (set above the pickup and noise level of the measuring circuit and determining the lowest measurable critical current) was exceeded, the magnetic field and critical current were measured with a delay of about 30 s. Thus, the critical current in the positive ($I_{c+}$) and negative ($I_{c-}$) directions of the excitation current $I$ were measured in sequence. Recording one $I_{c+}(B)$, $I_{c-}(B)$ curve of 1000 data points took 100 s. The magnetic field $B$ perpendicular to the sample plane was produced by a copper coil situated outside the cryostat. The measured quantities were recorded as functions of the current $I_{\text{sol}}$ in the coil. The magnitude of the magnetic field induced by the current in the coil was determined from the calibration $B_{\text{sol}} = k_{\text{sol}}I_{\text{sol}}$ with $k_{\text{sol}} \approx 129$ G/A found with the use of a Hall probe. To reduce the effect of the Earth’s magnetic field, the part of the cryostat where the sample was situated was screened by a permalloy cylinder. The measurement of the critical currents in opposite directions allowed us to determine the external magnetic field $B = B_{\text{sol}} + B_{\text{res}}$. Since simultaneous change of the direction of the total external magnetic field $B$ and the excitation current is equivalent to the rotation by $180^\circ$, one has $I_{c+}(B) = I_{c+}(B_{\text{sol}} + B_{\text{res}}) = I_{c-}(-B) = I_{c-}(-B_{\text{sol}} - B_{\text{res}})$. The residual magnetic field $B_{\text{res}} \approx 0.1$ G thus determined corresponds to the flux $SB_{\text{res}} = \pi r^2 \approx 0.02\Phi_0$ in the ring with the radius $r \approx 1$ μm.

When the current reaches the critical value $I_{c+}$ or $I_{c-}$, the ring switches abruptly to the normal state. In this case, the current–voltage characteristics exhibit hysteresis with a decrease in the excitation current (see Fig. 2). These features of the current–voltage characteristics typical of one-dimensional superconductors were explained in [8]. In this case, it is important that the ring can be switched to the normal state by a short pulse $-\Delta I_{\text{ext}} > 2I_{\text{ext}}$. After this pulse, the ring will switch back to the superconducting state with a lower ($I_{c1}$) or higher ($I_{c2}$) critical current. In the former and latter cases, the voltage at the excitation current $I_{\text{ext}} < I_{c1} < I_{c2}$ is $V = R_{\text{sh}}I_{\text{ext}}$ and $V = 0$, respectively. The voltage jump in Fig. 2 is $\Delta V = R_{\text{sh}} \approx 59 \Omega \cdot 14 \mu A \approx 0.8$ mV. The measured value is greater than the product $pI_{\text{sh}} \approx 0.2 \times 10^{-6}$ Ω cm, the critical current density $J_c \approx 10^6 A/cm^2$ at the temperature $T \approx 0.86T_c$, and the length $I_{\text{sh}} \approx 2$ μm of the short segment owing to the additional resistance of the wires with a length of 4 μm.

The measurements performed at various temperatures $T > 0.85T_c$ indicated that the dependences $I_{c+}(\Phi/\Phi_0)$ (Fig. 3) and $I_{c-}(\Phi/\Phi_0)$ agree with theory (6) near the integer number of flux quanta. Oscillations with the period $B_0 = \Phi_0/S \approx 5.66$ G corresponding to the area of the ring with the radius $r \approx 1.18$ μm were observed from $B = -15B_0$ to $B = +15B_0$. Owing to a finite width ($\approx 0.15$ μm) of the superconductor forming the ring, the critical current decreased smoothly with an increase in the magnetic field $B$ (Fig. 3). The values $I_{c0} \approx 11$ μA and $I_{p,A} \approx 1.1$ μA correspond to a higher asymmetry $(I_{\text{long}} - I_{\text{sh}})I_{\text{long}}/I^2 \approx 0.11 > I_{p,A}/I_{c0} \approx 0.10$ of the ring with $I_{\text{long}} \approx 1.18\pi r$ and $I_{\text{sh}} \approx 0.82\pi r$ (see Fig. 1). According to the above estimates, the probability $P_n$ of the state $n$ at the measured current $I_{p,A} \approx 1.1$ μA should change by a factor of 10 within the interval $\Delta \Phi = 0.014\Phi_0$ and the derivative $\partial V/\partial \Phi \approx 57$ mV/Φ₀ should be much higher than in a dc SQUID. Only the dependences $\bar{V}(\Phi/\Phi_0)$ and $\bar{I}_c(\Phi/\Phi_0)$ of the average values must be continuous. Individual measurements should show the discontinuity of $I_{c+}(\Phi/\Phi_0)$ and $I_{c-}(\Phi/\Phi_0)$ at $\Phi = (n + 0.5)\Phi_0$ (6a). The values between $I_{c0} \approx -0.82\pi r$ (at $n = I_{p,A} = 1.1$ μA) and $I_{c0} + (I_{\text{long}}/I_{p,A})$ (at $n + 1$) cannot occur. However, our measurements showed deviations from theory (6) in the intervals $(n - 0.33)\Phi_0 > \Phi > (n + 0.33)\Phi_0$ and the critical current (Fig. 3) contradicts quantization condition (1).

6. DISCUSSION

In conclusion, the measurements of the current–voltage characteristics have confirmed the possibility of obtaining a higher derivative $\partial V/\partial \Phi$ with the use of a superconducting loop with asymmetric contacts. The measurement of the magnetic-field dependence of the critical current has revealed disagreement with the theoretical predictions, which requires further investigation. We hope that a jump in the critical current associated with a change in the quantum number $n$ will be discovered in the investigations of other structures with asymmetric contacts probably made of other superconductors.

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