DEGREE TO WHICH CVC IS ESTABLISHED THROUGH 
BETA DECAY ALONE 

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Precision studies of $0^+ \rightarrow 0^+$ superallowed $\beta$ decays provide the necessary data to test stringently the CVC hypothesis. They will also provide a value for the weak vector coupling constant and the $V_{ud}$ quark mixing element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The determination of this element is crucial for the test of the unitarity of the CKM matrix, and the search for physics beyond the Standard Model. This paper reviews the current status of $0^+ \rightarrow 0^+$ $\beta$-decay data and their implications, as well as some relevant data from the $\beta$ decay of $^{19}$Ne and the neutron.

1 Introduction 

Studies of $0^+ \rightarrow 0^+$ superallowed $\beta$ decays are compelling because of their simplicity. The axial-vector decay strength is zero for such decays, so the measured $ft$ values are directly related to the weak vector coupling constant through the following equation:

$$ft = \frac{K}{G'^2 \langle M_F \rangle^2},$$

where $K$ is a known constant, $G'_V$ is the effective vector coupling constant and $\langle M_F \rangle$ is the Fermi matrix element between analogue states. Eq. (1) is only accurate at the few-percent level since it omits calculated correction terms. Radiative corrections, $\delta_R$, modify the decay rate by about 1.5% and charge-dependent corrections, $\delta_C$, modify the “pure” Fermi matrix element by about 0.5%. Thus, Eq. (1) is transformed into the equation:

$$F t = ft(1 + \delta_R)(1 - \delta_C) = \frac{K}{G'^2 \langle M_F \rangle^2}.$$
CKM matrix. Any violation of unitarity would signal the need for physics beyond the Standard Model, such as extra $Z$ bosons or the existence of right-handed currents.

2 The experiments

The required experimental data on $0^+ \to 0^+$ beta decays fall into three categories: (i) $Q_{EC}$ values, (ii) half-lives and, (iii) branching ratios. In order to be useful, they need to be determined to an accuracy of about 40 parts per million (ppm) for $Q_{EC}$ and 200 ppm for the other two, a challenge that requires ingenuity and rigorous procedures. At present, nine superallowed beta emitters meet this requirement. Fig. 1 shows the degree to which the necessary experimental data is known in these nine cases. Specific examples of precise $Q_{EC}$-value, half-life and branching-ratio measurements, with their associated problems and techniques, are given below.

2.1 $Q_{EC}$ values

Most precision $Q_{EC}$-value measurements employ $(p, n)$ or $(^3He, t)$ reactions, the inverse beta-decay process, which provide a simple and direct relation to the beta-decay energy. Such reaction $Q$ values can be determined for the nine well-known superallowed beta emitters since they all have stable daughter nuclides that will form the target material in the reaction studies. In three cases, $^{26m}$Al, $^{34}$Cl and $^{42}$Sc, the

![Figure 1. Contributions to the $\mathcal{F}$-value uncertainty from the uncertainties in the $Q_{EC}$-values, half-lives and branching ratios of the nine precisely known $0^+ \to 0^+$ superallowed beta emitters. The arrows indicate values that exceed the scale of the plot.](image-url)
nuclide with one less proton than the $\beta$-decay parent is stable (or long lived) as well. Measurements of both the $(p, \gamma)$ and $(n, \gamma)$ reaction $Q$-values with targets comprised of these stable nuclides also provide a direct relation to the $\beta$-decay $Q_{EC}$ value.

In all these measurements the main difficulty lies in calibrating the projectile and/or ejectile energies. The Auckland group has frequently used the $(p, n)$ reaction and eliminated the need for an ejectile calibration by measuring the sensitive and rapid onset of neutron emission just at the reaction threshold. They calibrate their proton energy by passing the beam through a magnetic spectrometer, with narrow entrance and exit slits, before it impinges on the target. The spectrometer is calibrated before and after runs with beams of surface ionized K or Cs from an ion source that is inserted before the spectrometer at a point traversed by the proton beam. The source extraction voltage is adjusted until the alkali beam follows the same path through the spectrometer as the proton beam and then the energy of the latter can easily be deduced from the applied extraction voltage. The spectrometer is NMR stabilized and never changed between or during runs and calibrations. The small final adjustment of the proton beam energy to map out the $(p, n)$ reaction threshold is done by applying a bias on the target. The threshold measurements and the calibrations thus all revert to precise readout of voltages.

At Chalk River the complications of precise, absolute $Q_{EC}$-value measurements have been avoided by measuring instead the differences in $Q_{EC}$ values between superallowed beta emitters. The target material is a mixture of two beta-decay daughter nuclides, the ($^{3}_{}He, t$) reaction is used and the outgoing tritons are analysed by a Q3D spectrometer. Triton groups originating from both types of target nuclei are therefore present in the spectrometer focal plane. As in the Auckland measurements the target can be biased, in this case by up to 150 kV. When a bias of $X$ volts is applied to the target the incoming, doubly-charged $^{3}_{}He$ projectiles are retarded by $2X$ eV whereas the outgoing, singly-charged tritons are reaccelerated by $X$ eV. The net effect of the target bias is a shift of the triton group by $X$ eV along the focal plane. In these types of $Q_{EC}$-value difference measurements the target bias is adjusted until the shifted position of a triton peak from one target component exactly coincides with the unshifted position of a triton group from the other component. When matched, the trajectories of both triton groups through the spectrometer are identical and a detailed knowledge of the spectrometer optics is not required. The $Q_{EC}$-value difference determination is then reduced to measuring the target bias. If the two selected, matched triton groups do not correspond to the final product nuclei being in their ground states, then a knowledge of the excitation energies of the reaction products is also required. The $Q_{EC}$-value difference measurements nicely complement the absolute $Q_{EC}$-value measurements and together they result in a precise $Q_{EC}$-value grid with more than one piece of information for each superallowed $\beta$ emitter, a situation that is especially valuable when probing data for possible systematic biases.
2.2 Half-lives

Measurements of half-lives appear deceptively simple but when high precision is required they are fraught with possible biases. Precise half-life measurements of short-lived ($\sim 1 \text{ s}$) nuclides pose interesting and unique difficulties. Many samples need to be counted, with high initial rates, in order to obtain adequate statistics. The purity of the radioactive samples must be ensured, so they need to be prepared with on-line isotope separators. This is a major technical challenge because of the low volatility and short half-life of many of the superallowed beta emitters. The detector must be capable of high-rate counting and have a high efficiency. It must also have a very low-energy threshold so as to minimize its sensitivity to possible, minor gain changes and pile up. At high rates the necessary correction for deadtime becomes the most worrisome aspect of the counting electronics. At Chalk River, the samples are prepared with an isotope separator equipped with a high temperature He-jet ion source. The detector used is a $4\pi$ gas counter with a 92% efficiency for positrons. An instantly retriggerable gate generator is used to provide a well-defined, non-extendable pulse width, which introduces a dominant known deadtime at least five times longer than any preceding electronics deadtime.

An accurate result is not guaranteed even if the problems with sample preparation, detector and electronics have all been addressed because of the possible bias introduced by the analysis procedure, a source of potential problems that is often overlooked. The procedure employed is based on Poisson statistics, but the counting conditions do not strictly satisfy the Poisson criteria in that the sample size is relatively small, often less than $10^4$ atoms, and is counted with high efficiency until only background is visible. Furthermore, the variance for each data point is not the Poisson variance because of the perturbations introduced by dead time, nor is it easily calculable even though the dead-time losses themselves are properly accounted for. Numerous samples are counted in a typical experiment. Normally, one might expect to obtain the final result by averaging the individual half-lives obtained from a decay analysis of each sample or by adding together the data from all samples and then performing a decay analysis, but neither method is strictly correct.

The analysis procedure employed by the Chalk River group uses modified Poisson variances and is based on a simultaneous fit of up to 500 decay curves with a single common half-life, but with individual intensity parameters for each decay curve. Because an exact treatment is not possible, we have evaluated the bias introduced by our analysis simplifications by using hypothetical data generated to simulate closely the experimental counting conditions. The analysis of the hypothetical data should return the same half-life from which they were generated. Our analysis has been proven to be correct at the 0.01% level. Our tests with the event generator have also shown that different analysis procedures, based on reasonable assumptions, may bias the outcome by more than 0.1%, well outside the accuracy required for a $0^+ \rightarrow 0^+$
2.3 Branching ratios

The last experimental quantity required is the precise magnitude of the superallowed branch for each $0^+ \rightarrow 0^+$ emitter. For eight of the nine well-known cases (excluding $^{10}$C) the superallowed branch is the dominant one by far and other branches, if known, are well below the 1% level. The non-superallowed branches seen so far have either been allowed Gamow-Teller transitions to $1^+$ states in the daughter or been non-analogue transitions to excited $0^+$ states in the daughter. The latter transitions, although very weak, are of special interest because their magnitudes are related to the size of one of the necessary calculated charge-dependent corrections. Studies of such non-analogue transitions are the only way so far to test the predictive power of those calculations.

For the eight well known $0^+ \rightarrow 0^+$ cases where the superallowed decay branch is dominating, a precise measurement of its intensity is achieved by a sensitive, but less precise, measurement of the weak, competing branches. The difficulty in observing these branches stems from the intense background generated by the prolific and energetic positrons from the superallowed branch. At Chalk River a sensitive counting technique has been developed where events in an HPGe detector, used to observe the $\gamma$ rays resulting from non-superallowed beta transitions to excited states in the daughter, are tagged by the direction of coincident positrons seen in plastic scintillators. The majority of unwanted events in the HPGe detector originate from positrons heading towards the detector since they may interact with it directly or through bremsstrahlung and annihilation-in-flight processes. Such events are removed by a condition that HPGe events must be coincident with positrons heading away from that detector. This condition leads to a dramatic reduction of the background produced by positrons from the dominant, superallowed ground-state branch, which is not accompanied by $\gamma$ rays, whereas events from the excited-state branches, which are accompanied by subsequent $\gamma$ rays, are still efficiently recorded. The decays of six superallowed beta emitters have been investigated with this technique, the results for four of them have been published so far. Gamow-Teller transitions were observed in three cases and non-analogue transitions in two cases. Very stringent upper limits for similar branches were determined for the cases where none was observed.

3 The theoretical corrections

The charged particles involved in a nuclear beta decay interact electromagnetically with the field from the nucleus as well as with each other. These interactions modify the decay when compared to a case where only the weak force is involved and they thus need to be accounted for by theoretical corrections. For positron decay the
electromagnetic interaction between the proton involved and the nuclear field, an
effect absent in a bare nucleon decay, results in a charge-dependent correction, \( \delta_C \),
to the Fermi matrix element. The similar interaction between the emitted positron
and the nuclear field is already accounted for in the calculation of the statistical rate
function, \( f \). The interactions between the charged particles themselves and their
associated bremsstrahlung emissions leads to a series of radiative corrections to the
bare beta-decay process. It has been found advantageous to group the radiative
corrections into two classes, those that are nuclear-structure dependent, denoted \( \delta_R \),
and those that are not, denoted \( \Delta_R \).

The charge-dependent correction, \( \delta_C \), arises from the fact that both Coulomb and
charge-dependent nuclear forces act to destroy the isospin symmetry between the ini-
tial and final states in superallowed beta decay. The odd proton in the initial state is
less bound than the odd neutron in the final state so their radial wavefunctions differ
and the wavefunction overlap is not perfect. Furthermore, the degree of configuration
mixing between the final state and other, excited \( 0^+ \) states in the daughter is slightly
different from the similar configuration mixing in the parent, again resulting in an
imperfect overlap. As was mentioned in Sec. 2.3 the configuration-mixing predic-
tions have been tested against data on non-analogue \( 0^+ \rightarrow 0^+ \) transitions. There is
good agreement between the data and the most recent calculations. The radial
wavefunction difference correction has been calculated with Woods-Saxon wavefunc-
tions and the shell model, the Hartree-Fock method and the shell model, and, most
recently, the Hartree-Fock method and the Random Phase Approximation. In gen-
eral, the three types of calculations exhibit similar trends in the predicted values as
a function of the mass of the emitter, but the absolute values of \( \delta_C \) from ref. differ
on average from that of ref. by 0.07%.

The nuclear-structure dependent radiative correction, \( \delta_R \), depends on the energy
released in the beta decay and consists of a series of terms of order \( Z^n \alpha^n \) (with \( m < n \))
where \( Z \) is the proton number of the daughter nucleus and \( \alpha \) is the fine-structure
constant. The first three terms of this converging series have been calculated. To
them is added a nuclear-structure dependent, order \( \alpha \), axial-vector term, denoted by
\( (\alpha/\pi)C_{NS} \) in ref. to form the total correction, \( \delta_R \).

The nuclear-structure independent radiative correction, \( \Delta_R \), is dominated by its
leading logarithm, \( (2\alpha/\pi)\ln(m_Z/m_p) \), where \( m_p \) and \( m_Z \) are the masses of the proton
and \( Z \)-boson. It also incorporates an axial-vector term, \( (\alpha/2\pi)[\ln(m_p/m_A)+2C_{Born}] \),
whose principal uncertainty is the value assigned to the low-energy cut-off mass,
\( m_A \). We adopt a range \( m_A/2 < m_A < 2m_A \) with the central value given by the
\( A_1 \)-resonance mass, \( m_A = 1260 \) MeV. The resulting nucleus-independent radiative
correction, \( \Delta_R \), then becomes \( (2.40 \pm 0.08)\% \).
Table 1. Experimental results ($Q_{EC}$, $t_{1/2}$ and branching ratio, $R$) and calculated corrections ($\delta_C$ and $\delta_R$) for $0^+ \rightarrow 0^+$ transitions.

| Element | $Q_{EC}$ (keV) | $t_{1/2}$ (ms) | $R$ (%) | $\delta_C$ (%) | $\delta_R$ (%) | $F_t$ (s) |
|---------|----------------|----------------|---------|---------------|---------------|-----------|
| $^{10}$C | 1907.77(9)     | 19290(12)      | 1.4638(22) | 0.16(3)       | 1.30(4)       | 3074.4(54) |
| $^{14}$O | 2830.51(22)    | 70603(18)      | 99.336(10) | 0.22(3)       | 1.26(5)       | 3069.7(26) |
| $^{26}$Al | 4232.42(35)    | 6344.9(19)     | $\geq 99.97$ | 0.31(3)       | 1.45(2)       | 3070.0(21) |
| $^{34}$Cl | 5491.71(22)    | 1525.76(88)    | $\geq 99.988$ | 0.61(3)       | 1.33(3)       | 3070.1(24) |
| $^{38m}$K | 6043.76(56)    | 923.95(64)     | $\geq 99.998$ | 0.62(3)       | 1.33(4)       | 3069.4(31) |
| $^{42}$Sc | 6425.58(28)    | 680.72(26)     | 99.9941(14) | 0.41(3)       | 1.47(5)       | 3077.3(24) |
| $^{46}$V | 7050.63(69)    | 422.51(11)     | 99.9848(13) | 0.41(3)       | 1.40(6)       | 3074.4(27) |
| $^{50}$Mn | 7632.39(28)    | 283.25(14)     | 99.942(3)   | 0.41(3)       | 1.40(7)       | 3073.8(27) |
| $^{54}$Co | 8242.56(28)    | 193.270(63)    | 99.9955(6)  | 0.52(3)       | 1.39(7)       | 3072.2(27) |

Figure 2. $F_t$ values for the nine well-known cases and the best least-squares one-parameter fit.

4 Results

The measured data on $Q_{EC}$-values, half-lives and branching ratios for the nine precisely known $0^+ \rightarrow 0^+$ emitters as well as the calculated charge-dependent and radiative corrections are given in Table 1. The deduced $F_t$ values for the nine cases are also shown in Fig. 2.

It is evident that the nine separate cases are in good agreement, as is expected from CVC. The average $F_t$ value is 3072.3 ± 1.0 s, with a reduced chi-square of 1.20. The constancy of the $F_t$ values from the nine individual cases establishes that the CVC hypothesis, as tested through nuclear beta decay, is accurate at the $4 \times 10^{-4}$ level.

The weak vector coupling constant $G'_V = G_V (1 + \Delta_R)^{1/2} = (K/2F_t)^{1/2}$, deduced
from superallowed decay is $G'_V/(\hbar c)^3 = (1.1496 \pm 0.0004) \times 10^{-5}$ GeV$^{-2}$, where the error on the average $F_t$ value has been doubled to include an estimate of the systematic uncertainties in the calculated correction, $\delta_C$. The $V_{ud}$ quark mixing element of the CKM matrix is defined as $V_{ud} = G_V/G_\mu$, where $G_\mu$ is the coupling constant deduced from the purely leptonic muon decay. We arrive at $V_{ud} = 0.9740 \pm 0.0005$. With values of the other two elements of the first row of the CKM matrix taken from ref. the unitarity test produces the following result

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9972 \pm 0.0013.$$ (3)

The discrepancy with unity, shown in Fig. 3, is more than two standard deviations.

Precision measurements of non $0^+ \rightarrow 0^+$ superallowed beta decays can also yield $F_t$ and $V_{ud}$ values. The decays of $^{19}$Ne and the neutron have been studied extensively but because their superallowed decay branches contain Gamow-Teller components, beta-asymmetry measurements are required to separate them from the Fermi components. Consequently the precision achieved so far in these two cases is less than that achieved in the $0^+ \rightarrow 0^+$ decay measurements. The first row unitarity test with a $V_{ud}$ value based on the most current $\beta$-decay asymmetry measurement of $^{19}$Ne is also shown in Fig. 3. It is in good agreement with the $0^+ \rightarrow 0^+$ data, which further supports the CVC hypothesis but still leaves a unitarity problem. The $^{19}$Ne data also yields a result for $G_A(M_{GT})$, which can be used to deduce a value for the axial-vector
coupling constant, $G_A$. However, unlike the case of the neutron, the Gamow-Teller matrix element, $\langle M_{GT} \rangle$, is not explicitly given by theory but needs to be calculated, for example, with the shell model. Consequently a very high precision is not attainable for $G_A$ from $^{19}$Ne decay studies and only the $G_A$ value is shown in Fig. 3. The results for $G_V$ and $G_A$ from the neutron decay studies are also shown in Fig. 3. With $V_{ud}$ based on the neutron studies the unitarity test also fails, but now the sum of the matrix elements is too large.

The current status is thus far from ideal. All three types of data, nuclear superallowed beta decay, $0^+ \rightarrow 0^+$ and non $0^+ \rightarrow 0^+$, and the decay of the neutron, result in a failure to meet the unitarity condition. Only two of the three types of data agree among themselves. However, it is worth pointing out that the different types of data have their own particular strengths and weaknesses. The strength of the superallowed decay studies is the large number of cases, which dilute the effect of any possible, erroneous measurement, and their consistency. The weakness is the number, magnitude and complexity of the necessary, calculated corrections. It is unlikely to expect a large change in the $\mathcal{F}t$ value deduced from superallowed beta emitters from further experimental or theoretical work.

In contrast, the strength of the neutron decay studies is the simplicity of the calculated corrections. The weakness is that it is a single case with fewer measurements and, consequently a greater susceptibility to one possible erroneous measurement. (In fact, the two most recent $\beta$-asymmetry measurements disagree.) The neutron case thus appears to have greater potential but it is also the one most likely to see its $\mathcal{F}t$ value move substantially from its present location.

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