Notes on D-Branes

Joseph Polchinski, Shyamoli Chaudhuri, Clifford V. Johnson

Institute For Theoretical Physics
University of California
Santa Barbara, CA 93106-4030 USA
email: joep,sc,cvj@itp.ucsb.edu

Abstract
This is a series of remedial lectures on open and unoriented strings for the heterotic string generation. The particular focus is on the interesting features that arise under $T$-duality—D-branes and orientifolds. The final lecture discusses the application to string duality. There will be no puns. Lectures presented by J. P. at the ITP from Nov. 16 to Dec. 5, 1995. References updated through Jan. 25, 1996.
Contents

1. Lecture I: *Open and Unoriented Bosonic Strings*
   
   1.1. Open Strings
   
   1.2. Chan-Paton Factors
   
   1.3. Unoriented Strings

2. Lecture II: *T-Duality*
   
   2.1. Self Duality of Closed Strings
   
   2.2. Open Strings and Dirichlet-Branes
   
   2.3. Chan-Paton Factors and Multiple D-Branes
   
   2.4. D-Brane Dynamics
   
   2.5. D-Brane Tension
   
   2.6. Unoriented Strings and Orientifolds.

3. Lecture III: *Superstrings and T-Duality*
   
   3.1. Open Superstrings
   
   3.2. Closed Superstrings
   
   3.3. *T*-Duality of Type II Superstrings
   
   3.4. *T*-Duality of Type I Superstrings

4. Lecture IV: *D-Branes Galore*
   
   4.1. Discussion
   
   4.2. Multiple Branes and Broken Supersymmetries
   
   4.3. D-strings
   
   4.4. Five-Branes
   
   4.5. A Brief Survey
   
   4.6. Conclusion
String theory today is a bit like particle physics in the good old days: we have a great deal of ‘data’ coming in, and are looking for the theory that explains it. Of course the data is not experimental but theoretical. For the most part, it consists of evidence that all the different string theories and string backgrounds that have been found are different states in a single theory. For many years students were told that it was sufficient to study closed oriented strings, the heterotic string in particular, but now various strongly coupled limits of the heterotic string theory are weakly coupled open or unoriented theories. Moreover, certain solitonic states, required by string duality, turn out to have a simple interpretation in terms of open strings with exotic boundary conditions.

These lectures are thus intended as a remedial course in open and unoriented strings and the exotic things that happen to them under $T$-duality, in particular the appearance of orientifolds and D-branes. We will start with the bosonic string, as many of the interesting features already appear there, but some of the essential structure will arise only in the supersymmetric case. The presentation is largely an expanded version of refs. [1,2,3], with a few of the more recent developments. In addition to references at appropriate points in the text, we will include at the end a short survey of the literature on this subject, both pre- and post-string duality. A general familiarity with string theory is assumed.

1. Lecture I: Open and Unoriented Bosonic Strings

1.1. Open Strings

To parameterize the open string’s world sheet, let the ‘spatial’ coordinate run $0 \leq \sigma^1 \leq \pi$ as in the figure:

\[ \frac{1}{4\pi\alpha'} \int_{\mathcal{M}} d^2\sigma \left( \partial_a X^\mu \partial_a X_\mu \right) \quad (1.1) \]
Under a variation, after integrating by parts,
\[ \delta S = -\frac{1}{2\pi\alpha'} \int_{\mathcal{M}} d^2 \sigma \left( \delta X^\mu \partial^2 X_\mu \right) + \frac{1}{2\pi\alpha'} \int_{\partial\mathcal{M}} d\sigma \left( \delta X^\mu \partial_n X_\mu \right) \] (1.2)
where \( \partial_n \) is the derivative normal to the boundary. The only Poincaré invariant boundary condition is the Neumann condition \( \partial_n X_\mu = 0 \). The Dirichlet condition \( X^\mu = \text{constant} \) is also consistent with the equation of motion, and we might study it for its own sake. However, we will follow history and begin with the Neumann condition, finding that Dirichlet condition is forced upon us later. From the first term in (1.2), we have to simply solve Laplace’s equation (or the wave equation, if we have Minkowski signature on the worldsheet.)

The general solution to Laplace’s equation with Neumann boundary conditions is
\[ X^\mu(z, \bar{z}) = x^\mu - i\alpha' p^\mu \ln(z\bar{z}) + i \sqrt{\alpha'} \sum_{m \neq 0} \frac{\alpha_m^\mu}{m} (z^{-m} + \bar{z}^{-m}), \] (1.3)
where \( x^\mu \) and \( p^\mu \) are the position and momentum of the center of mass. As is conventional in conformal field theory, this has been written in terms of the coordinate \( z = e^{\sigma^2 + i\sigma^1} \), so that time runs radially:

After the usual canonical quantization:
\[ [x^\mu, p^\nu] = i\eta^{\mu\nu}; \]
\[ [\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n} \eta^{\mu\nu}, \] (1.4)
and we get the mass spectrum
\[ M^2 = -p^\mu p_\mu = \frac{1}{\alpha'} \left( \sum_{m=1}^{\infty} mN_m - 1 \right). \] (1.5)
Here $N_m$ is the number of excited oscillators in the $m^{th}$ mode, and the $-1$ is the zero point energy of the physical bosons. So for example we have the following two particle states:

\[
\text{tachyon} : \quad |k\rangle, \quad M^2 = -\frac{1}{\alpha'}, \quad V = \exp(i k \cdot X); \\
\text{photon} : \quad \alpha_{-1}^\mu |k\rangle, \quad M^2 = 0, \quad V^\mu = \partial_\mu X^\mu \exp(i k \cdot X),
\]

where $V$ is the particle state’s vertex operator. Here $\partial_t$ is the derivative tangent to the string’s world sheet boundary.

### 1.2. Chan-Paton Factors

It is consistent with spacetime Poincaré invariance and world-sheet conformal invariance to add non-dynamical degrees of freedom at the ends. Their Hamiltonian vanishes so these degrees of freedom have no dynamics—an end of the string prepared in one of these states will remain in one of these states. So in addition to the Fock space label for the string, we could label each end $i$ or $j$ where the labels run from 1 to $N$:

\[
\begin{array}{c}
\text{i} \\
\downarrow \\
\text{j}
\end{array}
\]

The $n \times n$ matrix $\lambda^{a}_{ij}$ forms a basis into which to decompose a string wavefunction $|k, a\rangle = \sum_{i,j}^N |k, ij\rangle \lambda^{a}_{ij}$. These are the Chan-Paton factors. Each vertex operator carries such a factor. String fields satisfy a reality condition (e.g. the graviton must be real), so for the Chan-Paton factors $\lambda_{ij} = \lambda^{*}_{ji}$—they are Hermitian. Later we shall see there are extra conditions on the $\lambda$ arising from requiring certain factorization properties of amplitudes, and one-loop consistency of the theory. The tree diagram for four oriented strings is:

---

1 A boson with periodic boundary conditions has zero point energy $-\frac{1}{24}$, and with antiperiodic boundary conditions it is $\frac{1}{48}$. For fermions, there is an extra minus sign. For the bosonic string in 26 dimensions, there are 24 transverse (physical) degrees of freedom.
Since the Chan-Paton degrees of freedom are non-dynamical, the right end of string 1 must have the same index as the left end of string 2, and so on, and so the diagram comes with a factor

\[ \lambda^1_{ij} \lambda^2_{jk} \lambda^3_{kl} \lambda^4_{li} = \text{Tr}(\lambda^1 \lambda^2 \lambda^3 \lambda^4). \]  

(1.7)

So in general, we multiply by such factors in order to account for the Chan-Paton degrees of freedom in amplitudes. The massless vertex operator \( V^{a \mu} = \lambda^a_{ij} \partial_t X^\mu \exp(ik \cdot X) \) transforms as the adjoint under the \( U(N) \) symmetry of the Chan-Paton degrees of freedom (under which the endpoints transform as an \( N \) and \( \overline{N} \) respectively), so this is a gauge symmetry in spacetime.

The massless bosonic background fields of the open string are the graviton \( G_{\mu \nu} \), dilaton \( \phi \), and antisymmetric tensor \( B_{\mu \nu} \). The closed string coupling is related to the expectation value of the dilaton field \( \phi_0 \) and is given by \( g = e^{\phi_0} \). In the low energy limit, the massless fields satisfy equations of motion which may be obtained by varying the following action:

\[
S = \int d^{10} X \left\{ \frac{1}{2} e^{-2\phi} \left( R + 4(\nabla \phi)^2 - \frac{1}{12} H_{\mu \nu \kappa} H^{\mu \nu \kappa} \right) - \frac{c}{4} e^{-\phi} \text{Tr} F_{\mu \nu} F^{\mu \nu} + O(\alpha') \right\} 
\]  

(1.8)

This action arises from tree level in string perturbation theory—the closed string kinetic terms are accompanied by \( g^{-2} \), from the sphere, and the open string kinetic terms by \( g^{-1} \), from the disk. Each further order in \( \alpha' \) brings two extra derivatives and terms such as \( \alpha' e^{-2\phi} R^2 \) and \( \alpha' e^{-\phi} \text{Tr}[F_{\mu \nu} F_{\phi \lambda}^\alpha F^{\mu \nu}] \) appear. Some of these terms vanish in the supersymmetric case. The normalization of the open string action will be discussed later.

### 1.3. Unoriented Strings

Let us begin with the open string sector. World sheet parity acts as the \( z \leftrightarrow -\overline{z} \), reflecting right-moving modes into left moving modes. In terms of the mode expansion, \( X^\mu(z, \overline{z}) \rightarrow \)

\[ \text{In string perturbation theory world-sheets contribute a factor } g^{2h-2+b+c}, \text{ where } h, b \text{ and } c \text{ (which completely characterize the topology of a two-dimensional manifold) are the number of handles, boundaries and crosscaps, respectively.} \]
\( X^\mu (-\mathfrak{z}, -z) \) takes

\[
\begin{align*}
X^\mu & \rightarrow X^\mu \\
\pi^\mu & \rightarrow \pi^\mu \\
\alpha^\mu_m & \rightarrow (-1)^m \alpha^\mu_m .
\end{align*}
\] (1.9)

This is a global symmetry of the open string theory above, but we can also consider the theory in which it is gauged. When a discrete symmetry is gauged, only invariant states are left in the spectrum.\(^3\) The open string tachyon is even and survives the projection, while the photon does not, as

\[
\begin{align*}
\Omega |k\rangle &= +|k\rangle ; \\
\Omega \alpha^\mu_{-1} |k\rangle &= -\alpha^\mu_{-1} |k\rangle .
\end{align*}
\] (1.10)

We have made an assumption here about the overall sign of \( \Omega \). This sign is fixed by the requirement that \( \Omega \) be conserved in string interactions, which is to say that it is a symmetry of the operator product expansion (OPE). The assignment (1.9) matches the symmetries of the vertex operators (1.6): the minus sign is from the orientation reversal on the tangent derivative \( \partial_t \).

World-sheet parity reverses the Chan-Paton factors on the two ends of the string, but more generally it may have some additional action on each endpoint,

\[
\Omega \lambda_{ij} |k, ij\rangle \rightarrow \lambda'_{ij} |k, ij\rangle
\] (1.11)

\[
\lambda' = M \lambda^T N .
\]

Further it must be that \( M = N^{-1} \) in order that this be a symmetry of general amplitudes such as (1.7).

Acting twice with \( \Omega \) squares to the identity on the fields, leaving only the action on the Chan-Paton degrees of freedom. States are thus invariant under

\[
\lambda \rightarrow M M^{-T} \lambda M^T M^{-1} .
\] (1.12)

The \( \lambda \) must span a complete set of \( N \times N \) matrices: if strings \( ik \) and \( jl \) are in the spectrum for any values of \( k \) and \( l \), then so is the state \( ij \). First, \( jl \) implies \( lj \) by CPT, and a splitting-joining interaction in the middle gives \( ik + lj \rightarrow ij + lk \). But now Schur’s lemma requires \( M M^{-T} \) to be proportional to the identity, so \( M \) is either symmetric or antisymmetric. This gives two cases, up to choice of basis:\(^5\)

\textbf{a.} \( M = M^T = I_N \). (Here, \( I_N \) is the \( N \times N \) unit matrix.) In this case, for the photon \( \lambda_{ij} \alpha^\mu_{-1} |k\rangle \) to be even under \( \Omega \) and therefore survive the projection, we must have \( \lambda = -\lambda^T \), which means that our gauge group is \( SO(N) \).

\(^3\) The familiar example of this is the orbifold construction, in which some global world-sheet symmetry, usually a discrete symmetry of spacetime, is gauged.
b. \( M = -M^T = i \begin{pmatrix} 0 & I_{N/2} \\ -I_{N/2} & 0 \end{pmatrix} \). In this case, \( \lambda = -M \lambda^T M \), which defines the gauge group \( USp(N) \).

Now consider the closed string sector. For closed strings, we have the familiar mode expansion \( X^\mu(z, \bar{z}) = X^\mu(z) + X^\mu(\bar{z}) \) with:

\[
X^\mu(z) = x^\mu + i \sqrt{\alpha'} \frac{1}{2} \left( -\alpha_0^\mu \ln z + \sum_{m \neq 0} \frac{\alpha_m^\mu}{mz^m} \right),
\]

\[
X^\mu(\bar{z}) = \tilde{x}^\mu + i \sqrt{\alpha'} \frac{1}{2} \left( -\tilde{\alpha}_0^\mu \ln \bar{z} + \sum_{m \neq 0} \frac{\tilde{\alpha}_m^\mu}{m\bar{z}^m} \right). \tag{1.13}
\]

The theory is invariant under a world-sheet parity symmetry \( \sigma^1 \to -\sigma^1 \). For a closed string, the action of \( \Omega \) is to reverse the right- and left-moving oscillators:

\[
\Omega : \quad \alpha_m^\mu \leftrightarrow \tilde{\alpha}_m^\mu. \tag{1.14}
\]

For convenience, parity is here taken to be \( z \to \bar{z} \), differing by a \( \sigma^1 \)-translation from \( z \to -\bar{z} \). This is a global symmetry, but again we can gauge it. We have \( \Omega |k\rangle = |k\rangle \), and so the tachyon remains in the spectrum. However

\[
\Omega \alpha_{-1}^\mu \tilde{\alpha}_{-1}^{\nu} |k\rangle = \tilde{\alpha}_{-1}^{\mu} \alpha_{-1}^{\nu} |k\rangle, \tag{1.15}
\]

so only states symmetric under \( \mu \leftrightarrow \nu \) survive from this multiplet, i.e. the graviton and dilaton. The antisymmetric tensor is projected out.

When a world-sheet symmetry is gauged, a string carried around a closed curve on the world-sheet need only come back to itself up to a gauge transformation. Gauging world-sheet parity thus implies the inclusion of unoriented world-sheets. The oriented one-loop closed string amplitude comes only from the torus, while insertion of the projector \( \frac{1}{2} \text{Tr}(1 + \Omega) \) into a closed string one-loop amplitude will give the amplitude on the torus and Klein bottle respectively:

\[\begin{array}{c}
\includegraphics[width=0.2\textwidth]{torus}
\quad \equiv \quad \begin{array}{c}
\includegraphics[width=0.2\textwidth]{klein_bottle}
\end{array}
\end{array}\]

\[\begin{array}{c}
\includegraphics[width=0.2\textwidth]{orient_string}
\quad \equiv \quad \begin{array}{c}
\includegraphics[width=0.2\textwidth]{unorient_string}
\end{array}
\end{array}\]

\[\begin{array}{c}
\includegraphics[width=0.2\textwidth]{klein_bottle_insert}
\quad \equiv \quad \begin{array}{c}
\includegraphics[width=0.2\textwidth]{torus_insert}
\end{array}
\end{array}\]

In the notation where \( USp(2) \equiv SU(2) \).
Similarly, the unoriented one-loop open string amplitude comes from the annulus and Möbius strip. The lowest order unoriented amplitude is the projective plane, which is a disk with opposite points identified:

A circular hole with opposite points identified is a crosscap. The Klein bottle can be represented as a cylinder with a crosscap at each end, as shown in the figure above. This representation will be useful and will be explained further in section 2.6.

Gauging world-sheet parity is similar to the usual orbifold construction, gauging an internal symmetry of the world-sheet theory \[6\]. One difference is that there is no direct analog of the twisted states, because the Klein bottle does not have the modular transformation \( \tau \to -1/\tau \). Perturbative unitarity of an orbifold theory requires the twisted states, but the unoriented theory is perturbatively unitary without additional states. There are however some senses in which open strings can be regarded as twisted states under world-sheet parity \[6\]; we will return to this later.

2. Lecture II: T-Duality

2.1. Self Duality of Closed Strings

For closed strings, let us first study the zero modes. We have

\[
X^\mu(z, \bar{z}) \sim -i \sqrt{\alpha'} \left( \alpha^\mu_0 + \tilde{\alpha}^\mu_0 \right) \sigma^2 + \sqrt{\alpha'} \left( \alpha^\mu_0 - \tilde{\alpha}^\mu_0 \right) \sigma^1 + \cdots. \tag{2.1}
\]

Noether’s theorem gives the spacetime momentum of a string as

\[
p^\mu = \frac{1}{\sqrt{2\alpha'}} (\alpha^\mu_0 + \tilde{\alpha}^\mu_0), \tag{2.2}
\]

while under \( \sigma^1 \sim \sigma^1 + 2\pi \), \( X^\mu(z, \bar{z}) \) changes by \( 2\pi \sqrt{(\alpha'/2)(\alpha^\mu_0 - \tilde{\alpha}^\mu_0)} \). For a non-compact spatial direction \( \mu \), \( X^\mu(z, \bar{z}) \) is single-valued, and so

\[
\alpha^\mu_0 = \tilde{\alpha}^\mu_0 = \frac{\alpha'}{2} p^\mu. \tag{2.3}
\]
Since vertex operators must leave the space (2.3) invariant, only the sum \( x^\mu + \bar{x}^\mu \) may appear.

For a compact direction of radius \( R \), say \( \mu = 25 \), \( X^{25} \) has period \( 2\pi R \). The momentum \( p^{25} \) can take the values \( n/R \). Now, under \( \sigma^1 \sim \sigma^1 + 2\pi \), \( X^{25}(z, \bar{z}) \) can change by \( 2\pi wR \), which means that

\[
\alpha_0^{25} + \tilde{\alpha}_0^{25} = \frac{2n}{R} \sqrt{\frac{\alpha'}{2}} \tag{2.4}
\]

and so

\[
\alpha_0^{25} = \left( \frac{n}{R} + \frac{wR}{\alpha'} \right) \sqrt{\frac{\alpha'}{2}}
\]

(2.5)

Turning to the mass spectrum, we have

\[
M^2 = -p^\mu p_\mu = \frac{2}{\alpha'} (\alpha_0^{25})^2 + \frac{4}{\alpha'} (L - 1)
= \frac{2}{\alpha'} (\tilde{\alpha}_0^{25})^2 + \frac{4}{\alpha'} (\bar{L} - 1) \tag{2.6}
\]

Here \( \mu \) runs only over the non-compact dimensions, \( L \) is the total level of the left-moving excitations, and \( \bar{L} \) the total level of the right-moving excitations. The mass spectra of the theories at radius \( R \) and \( \alpha'/R \) are identical with the winding and Kaluza-Klein modes interchanged \( n \leftrightarrow w \) \[7\], which takes

\[
\alpha_0^{25} \to \tilde{\alpha}_0^{25}
\]

(2.7)

The interactions are identical as well \[8\]. Write the radius-\( R \) theory in terms of

\[
X'^{25}(z, \bar{z}) = X^{25}(z) - X^{25}(\bar{z}) \tag{2.8}
\]

The energy-momentum tensor and OPE and therefore all of the correlation functions are invariant under this rewriting. The only change is that the zero mode spectrum in the new variable is that of the \( \alpha'/R \) theory.

The \( T \)-duality is therefore an exact symmetry of perturbative closed string theory. Note that it can be regarded as a spacetime parity transformation acting only on the right-moving degrees of freedom. We will denote this transformation as \( T_{25} \), where \( T_{\mu_1 \ldots \mu_k} \) refers to the corresponding transformation on \( X^{\mu_1 \ldots \mu_k} \).
This duality transformation is in fact an exact symmetry of closed string theory. To see why, recall the appearance of an $SU(2)_L \times SU(2)_R$ extended gauge symmetry at the self-dual radius. Additional left- and right-moving currents are present at this radius in the massless spectrum, $\partial X^{25}(z)$, $\exp(\pm 2iX^{25}(z)/\sqrt{\alpha'})$ for $SU(2)_L$, and $\partial X^{25}(\bar{z})$, $\exp(\pm 2iX^{25}(\bar{z})/\sqrt{\alpha'})$ for $SU(2)_R$. The marginal operator for the change of radius, $\partial X^{25} \partial X^{25}$, transforms as a $(3, 3)$, so a rotation by $\pi$ in one of the $SU(2)$’s transforms it into minus itself. The transformation $R \rightarrow \alpha'/R$ is therefore a $\mathbb{Z}_2$ subgroup of the $SU(2) \times SU(2)$. We may not know what non-perturbative string theory is, but it is a fairly safe bet that it does not violate spacetime gauge symmetries explicitly, else the theory could not be consistent. Note that the $\mathbb{Z}_2$ is already spontaneously broken, away from the self-dual radius.

It is important to note that $T$-duality acts nontrivially on the dilaton. By the usual dimensional reduction, the effective 25-dimensional coupling is $e^{\phi} R^{-1/2}$. Duality requires this to be equal to $e^{\phi'} R'^{-1/2}$, hence

$$e^{\phi'} = e^{\phi} R^{-1/2}$$  \hspace{1cm} (2.9)

2.2. Open Strings and Dirichlet-Branes

Let us rewrite the open string mode expansion for the compact direction as follows:

$$X^{25}(z) = \frac{x^{25}}{2} + C - i\alpha' p^{25} \ln z + i \sqrt{\frac{\alpha'}{2}} \sum_{m \neq 0} \frac{\alpha_m^\mu}{m z^m},$$

$$X^{25}(\bar{z}) = \frac{x^{25}}{2} - C - i\alpha' p^{25} \ln \bar{z} + i \sqrt{\frac{\alpha'}{2}} \sum_{m \neq 0} \frac{\alpha_m^\mu}{m \bar{z}^m}. \hspace{1cm} (2.10)$$

Then $X^{25}(z, \bar{z}) = X^{25}(z) + X^{25}(\bar{z})$ is the usual open string coordinate. Again rewrite the theory in terms of

$$X'^{25}(z, \bar{z}) = X^{25}(z) - X^{25}(\bar{z}) = 2C - i\alpha' p^{25} \ln z + (oscillators)$$

$$= 2C + 2\alpha' p^{25} \sigma^1 + (oscillators)$$

$$= 2C + 2\alpha' n R \sigma^1 + (oscillators). \hspace{1cm} (2.11)$$

The oscillator terms vanish at the endpoints $\sigma^1 = 0, \pi$. Notice that there is no dependence on $\sigma^2$ in the zero modes. Therefore the endpoints of the string do not move in the $X^{25}$ direction. We could also see this directly, from the boundary condition $0 = \partial_n X^{25} = \partial_t X'^{25} \hspace{1cm} (11)$. At the ends,

$$\sigma^1 = 0 : \quad X'^{25} = 2C;$$

$$\sigma^1 = \pi : \quad X'^{25} = 2C + 2\pi \alpha' p^{25}$$

$$= 2C + 2\pi n R'. \hspace{1cm} (2.12)$$
This means that in the dual theory (with radius $R' = \alpha'/R$) the ends of the open strings are located for all time at position $X'^{25} = 2C$. They can wind $n$ times around the spacetime circle, and they are free to move in the other directions:

In the above diagram, the vertical direction represents the other 24 spatial directions. The string endpoints are constrained to lie on the 24-dimensional hypersurface $X'^{25} = 2C$; in the present case we could define $X'^{25}$ such that $C \to 0$, but later $C$ will be necessary. We will see that this hypersurface is a dynamical object, a membrane, hence called a Dirichlet-brane (D-brane).

### 2.3. Chan-Paton Factors and Multiple D-Branes

Now we study the effect of Chan-Paton factors [4]. Consider the case of $U(N)$, the oriented open string. In compactifying the $X^{25}$ direction, we can include a Wilson line $A^{25} = \text{diag}\{\theta_1, \theta_2, \ldots, \theta_N\}/2\pi R = \partial_{25} \Lambda$, generically breaking $U(N) \to U(1)^N$. Locally this is pure gauge, $\Lambda = (X^{25}/2\pi R) \text{diag}\{\theta_1, \theta_2, \ldots, \theta_N\}$, but because $X^{25}$ is periodic, $A^{25}$ has non-trivial holonomy as we go around the circle:

$$\Lambda(2\pi R) = \Lambda(0) + \text{diag}\{\theta_1, \theta_2, \ldots, \theta_N\}. \tag{2.13}$$

We can make a gauge transformation to remove this, but states which are charged under $U(N)$ pick up a phase in going around the compact dimension, $|ij\rangle$ being multiplied by $\exp(i[\theta_j - \theta_i])$. So

$$p^{25} = \frac{n}{R} + \frac{\theta_j - \theta_i}{2\pi R} \tag{2.14}$$

giving

$$X'^{25}(z, \bar{z}) = 2C + 2\alpha' p^{25} \sigma^1$$

$$= 2C + 2\alpha' \left( \frac{n}{R} + \frac{\theta_j - \theta_i}{2\pi R} \right) \sigma^1. \tag{2.15}$$
Now we can deduce the value of $C$. The position of the left endpoint should depend only on its Chan-Paton degree of freedom $i$ and similarly for the right endpoint $j$ (else an interaction at one end would have an instantaneous effect at the other). If we set $2C = \theta_i R'$ then indeed

\[
\begin{align*}
\sigma^1 &= 0 : \quad X'^{25} = \theta_i R' \\
\sigma^1 &= \pi : \quad X'^{25} = 2\pi n R' + \theta_j R'.
\end{align*}
\]  

(2.16)

That is, an endpoint in state $i$ is located on a D-brane at $\theta_i R'$, modulo the periodicity of the dual spacetime.

Illustrated above are three D-branes with various strings wound between them.

2.4. D-Brane Dynamics

Let us first note that this whole picture goes through if several coordinates $X^m = \{X^{25}, X^{24}, \ldots, X^{p+1}\}$ are periodic, and we rewrite the periodic dimensions in terms of the dual coordinate. The open string endpoints are then confined to $N (p+1)$-dimensional hyperplanes. The Neumann conditions on the world sheet, $\partial_\sigma X^m(\sigma^1, \sigma^2) = 0$ have become Dirichlet conditions $\partial_\tau X^m(\sigma^1, \sigma^2) = 0$ for the dual coordinates. The $(p+1)$-dimensional hypersurface is the world-volume of a $p$-dimensional extended object called a ‘Dirichlet $p$-brane’, or ‘D-brane’ for short.

It is natural to expect that the D-brane is dynamical. Closed strings can interact with the D-branes (indirectly via open strings), and so the D-branes feel the effects of gravity in the closed string massless sector. We would therefore expect that they can fluctuate in shape and position as dynamical objects. We can see this by looking at the massless spectrum of our theory, interpreted in the dual coordinates.

\[\text{In this terminology, the original Type I theory contains } N \text{ 25-branes. A 25-brane fills space, so the string endpoint can be anywhere: it just corresponds to an ordinary Chan-Paton factor.}\]
Returning to our illustration with a Dirichlet 24-brane, let us look at the mass spectrum. We have
\[ M^2 = (p^{25})^2 + \frac{1}{\alpha'}(L - 1) \] which gives, for multiple branes:
\[ M^2 = \left\{ \frac{[2\pi n + (\theta_i - \theta_j)]R'}{2\pi \alpha'} \right\}^2 + \frac{1}{\alpha'}(L - 1). \] (2.17)

Note that \( [2\pi n + (\theta_i - \theta_j)]R' \) is the minimum length of the string. We see that the massless states arise for non-winding open strings whose end points are on the same D-brane, as the string tension contributes an energy to a stretched string. We have therefore the massless states:

- \( \alpha^{\mu}_{-1}[k, ii] \), vertex operator \( \propto \partial_t X^\mu \). This is the gauge field in the directions transverse to the D-brane, with \( p + 1 \) components.
- \( \alpha^{25}_{-1}[k, ii] \), vertex operator \( \propto \partial_t X^{25} = \partial_n X^{25} \). This is the gauge field in the compact direction of the original theory, which became the position of the D-brane in the dual theory. We considered a classical background for this field, but the string quanta in this state, which are built into string perturbation theory, correspond to transverse fluctuations of the D-brane shape. The relation here is that same as that between the classical background metric and the graviton states of the string. The world-brane theory thus consists of a \( U(1) \) vector field plus \( 25 - p \) world-brane scalars describing the fluctuations.

It is interesting to look at the \( U(N) \) symmetry breaking in the dual picture. When no D-branes coincide, there is just one massless vector each, or \( U(1)^N \) in all, the generic unbroken group. If \( m \) D-branes coincide, there are new massless states because (non-winding) strings which are stretched between these branes can achieve vanishing length. Thus, there are \( m^2 \) vectors, forming the adjoint of a \( U(m) \) gauge group. This coincident position corresponds to \( \theta_1 = \theta_2 = \cdots = \theta_m \) for some subset of the original \( \{\theta\} \), so in the dual theory the Wilson line left a \( U(m) \) subgroup unbroken. At the same time, there appears the set of \( m^2 \) massless scalars: the \( m \) positions are promoted to a matrix. This is curious and hard to visualize, but has proven to play an important role in the dynamics of D-branes [11,12]. As one consequence, consider the figure, which shows two singly wound strings and one doubly wound string on a compact dimension.

For fundamental strings these are distinct. For D-branes, however, the integral over the D-brane \( U(2) \) group will include an integral over the holonomy in going around the compact dimension. Since this acts on the two D-branes, the two parts of the figure just represent...
holonomies

\[
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}, \quad \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]  

(2.18)
respectively, and are continuously connected through configurations that cannot be drawn.

Note that if all \( N \) branes are coincident, we recover the \( U(N) \) gauge symmetry.

This picture seems a bit exotic, and will become more so in the unoriented theory. But all we have done is to rewrite the original open string theory in terms of variables which are more natural in the limit \( R \ll \sqrt{\alpha'} \). Various puzzling features of the small-radius limit become clear in the \( T \)-dual picture.

2.5. D-Brane Tension

One can use nonlinear sigma model methods to find the conditions for conformal invariance of the D-brane CFT. The field equations lead to an effective action for a D-brane moving in a closed string background \[13,1\]

\[
S = -T_p \int d^{p+1}\sigma \left\{ e^{-\phi} \text{Tr} \det^{\frac{1}{2}} \left( \tilde{G}_{\mu\nu} + \tilde{B}_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} \right) \right\},
\]

(2.19)

where \( \tilde{G}_{\mu\nu} \) and \( \tilde{B}_{\mu\nu} \) are the induced fields on the world brane, \( F_{\mu\nu} \) is the open string \( U(1) \) gauge field strength, and the trace is over the Chan-Paton degrees of freedom. The tension is of order \( e^{-\phi} = g^{-1} \), because the D-brane is invisible to amplitudes on the sphere and contributes first at disk order. The action for the field strength can be understood from \( T \)-duality. Start with the original Type I theory but now with a background \( A^{25}(X) \) depending on \( X^\mu \) for \( \mu \neq 25 \). By the earlier construction, the dual is now a curved D-brane, \( X'^{25}(X) = 2\pi\alpha' A^{25} \). The Dirac-Born-Infeld action then gives the area of this D-brane. Because \( B_{\mu\nu} \) appears, the antisymmetric tensor gauge invariance must act on the photon as well,

\[
\tilde{B}_{\mu\nu} \rightarrow \tilde{B}_{\mu\nu} + \partial_\mu \chi_\nu - \partial_\nu \chi_\mu \\
A_\mu \rightarrow A_\mu - \chi_\mu.
\]

(2.20)

At the world-sheet level this occurs because a surface term from the variation of \( B_{\mu\nu} \) must be canceled by variation of the open string vector.

It is instructive to compute the D-brane tension \( T_p \). As noted above, this is proportional to \( g^{-1} \). It arises from considering the disk tadpole, where in this case the disk is trapped with its boundary on a D-brane:

\[\text{M. Douglas (seminar at ITP) has made the interesting observation that one can think of this as an enlargement of the usual statistics group on } m \text{ particles from } S_m \text{ to } U(m)!\]
One could obtain the tension by calculating the disk with a graviton vertex operator, but it is easier to proceed as follows. Consider two parallel Dirichlet $p$-branes at positions $X'_{\mu} = 0$ and $X'_{\mu} = Y_{\mu}$. These two objects can feel each other’s presence by exchanging closed strings,

This string graph is an annulus, with no vertex operators. It is therefore easily calculated. The poles from graviton and dilaton exchange then give the coupling of closed string states to the D-brane, that is, $T_p$.

Parameterize the world-sheet as $(\sigma^1, \sigma^2)$ where $\sigma^1$ runs from 0 to $\pi$, and $\sigma^2$ is a periodic coordinate running from 0 to $2\pi t$. This vacuum graph has the single modulus $t$, running from 0 to $\infty$. If we time-slice horizontally, so that $\sigma^2$ is world-sheet time, we see two open strings appearing out of the vacuum and then disappearing, giving the open string loop channel. Time-slicing vertically instead, so that $\sigma^1$ is time, we see a single closed
string propagating in the tree channel. The world-line of the open string boundary can be regarded as a vertex connecting the vacuum to the single closed string, i.e., a one-point closed string vertex. The two channels are related by duality \[14,15\].

Consider the limit $t \to 0$ of the loop amplitude. This is the ultra-violet limit for the open string channel, but unlike the torus, there is no modular group acting to cut off the range of integration. However, because of duality, this limit is correctly interpreted as an infrared limit. Time-slicing in the vertical direction shows that the $t \to 0$ limit is dominated by the lowest lying modes in the closed string spectrum. In keeping with string folklore, there are no “ultraviolet limits” of the moduli space which could give rise to high energy divergences. All divergences in loop amplitudes come from pinching handles and are controlled by the lightest states, or the long distance physics.

One loop vacuum amplitudes are given by the Coleman-Weinberg formula, which amounts to summing the zero point energies of all the modes \[16\]:

$$A = \int_0^\infty \frac{dt}{2t} \sum_{i,k} e^{-2\pi\alpha' t (k^2 + M_i^2)}. \quad (2.21)$$

Here the sum $i$ is over the physical spectrum of the string, equivalent to the transverse spectrum, and the momentum $k$ is in the $p + 1$ extended directions of the D-brane worldsheet. The mass spectrum is given by

$$M^2 = \frac{1}{\alpha'} \left( \sum_{n=1}^\infty n\alpha_n^i \alpha_n^i - 1 \right) + \frac{Y \cdot Y}{4\pi^2\alpha'^2}. \quad (2.22)$$

where $Y^m = x_1^m - x_2^m$ is the separation of the D-branes. The sums over $N_n^i \equiv \alpha_n^i \alpha_n^i$ are as usual geometric, giving

$$A = 2V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi^2\alpha' t)^{-\frac{(p+1)}{2}} e^{-Y \cdot Y t / 2\pi\alpha'} q^{-2} \prod_{n=1}^\infty (1 - q^{2n})^{-24} \quad (2.23)$$

where $q = e^{-\pi t}$ and the overall factor of 2 is from exchanging the two ends of the string. We need the asymptotics as $t \to 0$. More generally, define

$$f_1(q) = q^{1/12} \prod_{n=1}^\infty (1 - q^{2n})$$

$$f_2(q) = \sqrt{2} q^{1/12} \prod_{n=1}^\infty (1 + q^{2n})$$

$$f_3(q) = q^{-1/24} \prod_{n=1}^\infty (1 + q^{2n-1})$$

$$f_4(q) = q^{-1/24} \prod_{n=1}^\infty (1 - q^{2n-1}) \quad (2.24)$$
The asymptotics as $t \to \infty$ are manifest. The asymptotics as $t \to 0$ are then obtained from the modular transformations
\[
f_1(e^{-\pi/s}) = \sqrt{s}f_1(e^{-\pi s}), \quad f_3(e^{-\pi/s}) = f_3(e^{-\pi s}), \quad f_2(e^{-\pi/s}) = f_4(e^{-\pi s}). \tag{2.25}
\]
In the present case
\[
A = 2V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi^2 \alpha')^{(p+1)/2} e^{-Y \cdot Y/2t} t^{12} \left( e^{2\pi i/t} + 24 + \ldots \right). \tag{2.26}
\]
The leading divergence is from the tachyon and is an uninteresting bosonic artifact. The massless pole, from the second term, is
\[
A \sim V_{p+1} \frac{24}{2^{12}} (4\pi^2 \alpha')^{11-p} \pi^{(p-23)/2} \Gamma((23-p)/2)|Y|^{p-23}
= V_{p+1} \frac{3\pi}{2^7} (4\pi^2 \alpha')^{11-p} G_{25-p}(Y^2) \tag{2.27}
\]
where $G_D(Y^2)$ is the massless scalar Green’s function in $D$ dimensions.

This can be compared with a field theory calculation, the exchange of graviton plus dilaton between a pair of D-branes, with the bulk action (1.8) and the coupling (2.19) to the D-brane. This is a bit of effort because the graviton and dilaton mix, but in the end one finds
\[
A \sim \frac{D-2}{4} V_{p+1} T_p^2 G_{25-p}(Y^2) \tag{2.28}
\]
so
\[
T_p = \frac{\sqrt{\pi}}{16} (4\pi^2 \alpha')^{(11-p)/2}. \tag{2.29}
\]
The units are obscured because we are working with dimensionless $\kappa = e^\phi$. The physical tension is $\tau_p = e^{-\phi}T_p = T_p/\kappa$, which is dimensionally correct.

As one application, consider $N$ 25-branes, which is just an ordinary $N$-valued Chan-Paton factor. Expanding the 25-brane Lagrangian (2.19) to second order in the gauge field gives
\[
T_{25}^2 (2\pi \alpha')^2 e^{-\phi} \text{Tr} F_{\mu\nu} F^{\mu\nu}, \tag{2.30}
\]
with the trace in the fundamental representation of $U(N)$. This gives the precise numerical relation between the open and closed string couplings [17].

The asymptotics (2.26) have an obvious interpretation in terms of a sum over closed string states exchanged between the two D-branes. One can write the cylinder path integral in Hilbert space formalism treating $\sigma_1$ rather than $\sigma_2$ as time. It then has the form
\[
\langle B | e^{-(L_0 + L_0)\pi/t} | B \rangle \tag{2.31}
\]
where the boundary state $|B\rangle$ is the closed string state created by the boundary loop. We will not have time to develop this formalism but it is useful in finding the couplings between closed and open strings [14,15].
2.6. Unoriented Strings and Orientifolds.

For closed strings, the original coordinate is \( X^m(z, \bar{z}) = X^m(z) + X^m(\bar{z}) \) and the dual is \( X^m(z, \bar{z}) = X^m(z) - X^m(\bar{z}) \). The action of world sheet parity reversal is to exchange \( X^\mu(z) \) and \( X^\mu(\bar{z}) \). In terms of the dual coordinate this is

\[
X^m(z) \leftrightarrow -X^m(\bar{z}),
\]

which is the product of a world-sheet and spacetime parity operation. In the unoriented theory, strings are invariant under the action of \( \Omega \). Separate the string wavefunction into its internal part and its dependence on the center of mass \( x^m \), and take the internal wavefunction to be an eigenstate of \( \Omega \). The projection then determines the wavefunction at \(-x^m\) to be the same as at \( x^m \), up to a sign. This is the same as the orbifold construction, the only difference being that the internal part includes a world-sheet parity reversal; thus we will call it an orientifold \[6,1\]. The compact spacetime is effectively the orbifold \( T^{25-p}/Z_2 \). For the case of a single compact dimension, for example, spacetime is the line segment \( 0 \leq x^{25} \leq \pi R' \), with orientifold fixed planes at the ends. It should be noted that orientifold planes are not dynamical. Unlike the case of D-branes, there are no string modes tied to the orientifold plane to represent fluctuations in its shape.

In the case of open strings, the situation is similar. Let us focus for convenience on a single compact dimension. Again there is one orientifold fixed plane at 0 and another at \( \pi R' \). Introducing \( SO(N) \) Chan-Paton factors, a Wilson line can be brought to the form

\[
\begin{pmatrix} 0 & i\theta_1 & 0 & 0 & \cdots \\ -i\theta_1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & i\theta_2 & \cdots \\ 0 & 0 & -i\theta_2 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}
\]

or equivalently

\[
\text{diag}\{\theta_1, -\theta_1, \theta_2, -\theta_2, \ldots, \theta_{N/2}, -\theta_{N/2}\}.
\]

Thus in the dual picture there are \( \frac{1}{2} N \) D-branes on the line segment \( 0 \leq X'^{25} < \pi R' \), and \( \frac{1}{2} N \) at their image points under the orientifold identification.
Strings can stretch between D-branes and their images as shown. The generic gauge group is again $U(1)^{N/2}$. As in the oriented case, if $m$ D-branes are coincident there is a $U(m)$ gauge group. Now if the $m$ D-branes in addition lie at one of the fixed planes, then strings stretching between one of these branes and one of the image branes also become massless and we have the right spectrum of additional states to fill out $SO(2m)$. The maximal $SO(N)$ is restored if all of the branes are coincident at a single orientifold plane. Note that this maximally symmetric case is asymmetric among the fixed planes, a fact that will play an important role later. Similar considerations apply to $USp(N)$.

The orientifold plane, like the D-brane, couples to the dilaton and metric. The amplitude is the same as in the previous section, but with $RP^2$ in place of the disk; that is, a crosscap replaces the boundary loop. The orientifold identifies $X^m$ with $-X^m$ at the opposite point on the crosscap, so the crosscap is localized near one of the orientifold fixed planes. Again the easiest way to calculate this is via vacuum graphs, the cylinder with one or both boundary loops replaced by crosscaps. These give the Möbius strip and Klein bottle, respectively:

![Diagram of Möbius strip and Klein bottle](image)

To understand this, consider the figure below, which shows two copies of the fundamental region for the Möbius strip:

![Diagram of fundamental region](image)

The lower half is identified with the reflection of the upper, and the edges $\sigma^1 = 0, \pi$ are boundaries. Taking the lower half as the fundamental region gives the familiar representation of the Möbius strip as a strip of length $2\pi t$, with ends twisted and glued. Taking
instead the left half of the figure, the line $\sigma^1 = 0$ is a boundary loop while the line $\sigma^1 = \pi/2$ is identified with itself under a shift $\sigma^2 \rightarrow \sigma^2 + 2\pi t$ plus reflection of $\sigma^1$: it is a crosscap. The same construction applies to the Klein bottle, with the right and left edges now identified.

The M"obius strip is now given by the vacuum amplitude

$$ A_M = \int_0^\infty \frac{dt}{2t} \sum_{i,k} \frac{\Omega_i}{2} e^{-2\pi \alpha' t (k^2 + M_i^2)} , \quad (2.35) $$

where $\Omega_i$ is the $\Omega$ eigenvalue of state $i$. The oscillator contribution to $\Omega_i$ is $(-1)^L$ from eq. (1.9). For the $SO(N)$ open string the Chan-Paton factors have $\frac{1}{2}N(N+1)$ even states and $\frac{1}{2}N(N-1)$ odd for a net of $+N$. For $USp(N)$ these numbers are reversed, for a net of $-N$. Focus on a D-brane and its image, which correspondingly contribute $\pm 2$. The diagonal elements, which contribute to the trace, are those where one end is on the D-brane and one on its image. The total separation is then $Y^m = 2x^m + 2\pi n^m R'$, corresponding to a fixed plane (or its period image) at $x^m = \pi n^m R'$. Taking into account these factors, the $n^m = 0$ term is

$$ A_M = \pm V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi^2 \alpha' t)^{-\frac{p+1}{2}} e^{-2\pi \alpha' t x^m / \pi \alpha'} \left[ q^{-2} \prod_{k=1}^{\infty} (1 + q^{4k-2})^{-24} (1 - q^{4k})^{-24} \right] $$

The factor in braces $[ ]$ is

$$ f_3(q^2)^{-24} f_1(q^2)^{-24} = (2t)^{12} f_3(e^{-\pi/2t})^{-24} f_1(e^{-\pi/2t})^{-24} = (2t)^{12} \left( e^{\pi/2t} - 24 + \ldots \right) . \quad (2.37) $$

One thus finds a pole

$$ \mp 2^{p-12} V_{p+1} \frac{3\pi}{2^7} (4\pi^2 \alpha')^{11-p} G_{25-p}(X^2) . \quad (2.38) $$

This is to be compared with $\frac{1}{2} (D - 2) T_p T'_p G_{25-p}(Y^2)$, where $T'_p$ is the fixed-plane tension; a factor of 2 as compared to the earlier field theory calculation (2.28) comes from the spacetime boundary. Thus the fixed-plane and D-brane tensions are related

$$ T'_p = \mp 2^{p-13} T_p . \quad (2.39) $$

A similar calculation with the Klein bottle gives a result proportional to $T'^2_p$.

---

7 In the compact directions there are two additional signs that cancel: the world-sheet parity contributes an extra minus sign in the directions with Dirichlet boundary conditions, and the spacetime part an additional sign.
Noting that there are $2^{25-p}$ fixed planes, the total fixed-plane source is $\mp 2^{12} T_p$. The fixed-plane and D-brane sources cancel for the group $SO(2^{13}) = SO(D/2)$ \cite{[18]} For this group the dilaton and graviton tadpoles cancel at order $g^{-1}$. This has no special significance in the bosonic string, as the one loop $g^0$ tadpoles are nonzero and imaginary due to the tachyon instability, but similar boundary combinatorics will give a restriction on anomaly free Chan Paton gauge groups in the superstring.

The Möbius strip and Klein bottle, like the cylinder, can be written in terms of the closed string Hilbert space \cite{[14][15]}. Like a boundary loop, the crosscap can be thought of as creating a closed string in a state $|C\rangle$. The two amplitudes are then

$$\langle B|e^{-(L_0 + \tilde{L}_0)\pi/4t}|C\rangle, \quad \langle C|e^{-(L_0 + \tilde{L}_0)\pi/2t}|C\rangle$$

where the different $t$ dependences in eqs. (2.31), (2.40) follow from the mapping between the two ways of drawing each surface.

This concludes our survey of bosonic open and unoriented strings and their $T$-dualities. The final picture is rather exotic, but remember that this is just the original string theory, rewritten in terms of variables which display most clearly the physics of the $R \to 0$, $R' \to \infty$, limit.

3. Lecture III: Superstrings and $T$-Duality

3.1. Open Superstrings

All of the exotic phenomena that we found in the bosonic string will appear in the superstring as well, together with some important new ingredients. We first review open and unoriented superstrings.

The superstring world-sheet action is

$$S = \frac{1}{4\pi \alpha'} \int_{\mathcal{M}} d^2\sigma \left\{ \alpha'^{-1} \partial X^\mu \partial X_\mu + \bar{\psi}^\mu \partial \psi_\mu + \tilde{\psi}_\mu \partial \tilde{\psi}^\mu \right\}$$

where the open string world-sheet is the strip $0 < \sigma^1 < \pi$, $-\infty < \sigma^2 < \infty$. The condition that the surface term in the equation of motion vanishes allows two possible Lorentz invariant boundary conditions on world-sheet fermions:

$$\text{R} \quad \psi^\mu(0,\sigma^2) = \tilde{\psi}^\mu(0,\sigma^2) \quad \psi^\mu(\pi,\sigma^2) = \tilde{\psi}^\mu(\pi,\sigma^2)$$

$$\text{NS} \quad \psi^\mu(0,\sigma^2) = -\tilde{\psi}^\mu(0,\sigma^2) \quad \psi^\mu(\pi,\sigma^2) = \tilde{\psi}^\mu(\pi,\sigma^2)$$

This corresponds to $2^{12}$ D-branes—it would be overcounting to include also their images. Incidentally, we use the D-brane tension of the oriented theory, because the local physics away from the fixed planes is oriented; in the unoriented theory the tension is smaller by $\sqrt{2}$. 

21
We can always take the boundary condition at one end, say $\sigma^1 = \pi$, to have a $+$ sign by redefinition of $\bar{\psi}$. The boundary conditions and equations of motion are conveniently summarized by the doubling trick, taking just left-moving (analytic) fields $\bar{\psi}^\mu$ on the range 0 to $2\pi$ and defining $\bar{\psi}^\mu(\sigma^1, \sigma^2)$ to be $\psi^\mu(2\pi - \sigma^1, \sigma^2)$. These left-moving fields are periodic in the Ramond (R) sector and antiperiodic in the Neveu-Schwarz (NS).

In the NS sector the fermionic oscillators are half-integer moded, giving a ground state energy of

$$\left(-\frac{8}{24}\right) + \left(-\frac{8}{48}\right) = -\frac{1}{2}$$

from the eight transverse coordinates and eight transverse fermions. The ground state is a Lorentz singlet and has odd fermion number, $(-1)^F = -1$. This assignment is necessary in order for $(-1)^F$ to be multiplicatively conserved.\footnote{In the ‘$-\frac{1}{2}$ picture’ \cite{ref} the matter part of the ground state vertex operator is the identity but the ghost part has odd fermion number. In the ‘0 picture’ this is reversed.} The GSO projection, onto states with even fermion number, removes the open string tachyon from the superstring spectrum. Massless particle states in ten dimensions are classified by their $SO(8)$ representation under Lorentz rotations which leave the momentum invariant. The lowest lying states in the NS sector are the eight transverse polarizations of the massless open string photon, $A^\mu$,

$$\psi_{-1/2}^\mu |k\rangle, \quad M^2 = \frac{1}{\alpha'}(L - \frac{1}{2})$$

forming the vector of $SO(8)$.

The fermionic oscillators in the Ramond sector are integer-moded. In the R sector the ground state energy always vanishes because the world-sheet bosons and their superconformal partners have the same moding.\footnote{This will remain true later when some bosons are integer moded and some half-integer. Note the the R and NS sectors are always identified by the periodicity properties of the world-sheet supercurrent.} The Ramond vacuum is degenerate, since the $\psi_0^\mu$ take ground states into ground states, so the latter form a representation of the ten-dimensional Dirac matrix algebra

$$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$$

The following basis for this representation is often convenient. Form the combinations

$$d_+^i = \frac{1}{\sqrt{2}} \left(\psi_0^{2i} \pm i\psi_0^{2i+1}\right) \quad i = 1, \cdots, 4$$

$$d_0^\pm = \frac{1}{\sqrt{2}} \left(\psi_0^1 \mp \psi_0^0\right)$$

$$\left(-\frac{8}{24}\right) + \left(-\frac{8}{48}\right) = -\frac{1}{2}$$

(3.3)
In this basis, the Clifford algebra takes the form
\[ \{d_i^+, d_j^-\} = \delta_{ij} \] (3.7)
The \(d_i^+, i = 0, \cdots, 4\) act as raising and lowering operators, generating the 32 Ramond ground states. Denote these states
\[ |s_0, s_1, s_2, s_3, s_4\rangle = |\mathbf{s}\rangle \] (3.8)
where each of the \(s_i\) is \(\pm \frac{1}{2}\), and where
\[ d_i^- | -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\rangle = 0 \] (3.9)
while \(d_i^+\) raises \(s_i\) from \(-\frac{1}{2}\) to \(\frac{1}{2}\). The significance of this notation is as follows. The fermionic part of the ten-dimensional Lorentz generators is
\[ S_{\mu\nu} = -\frac{i}{2} \sum_{r \in \mathbb{Z} + \kappa} \psi_{-r}^\mu | \psi_r^\nu \rangle \] (3.10)
where \(\kappa = 0(\frac{1}{2})\) in the R(NS) sector. The states above are eigenstates of \(S_0 = iS_{01}, S_i = S_{2i,2i+1}\), with \(s_i\) the corresponding eigenvalues. Since the Lorentz generators always flip an even number of \(s_i\), the Dirac representation 32 decomposes into a 16 with an even number of \(-\frac{1}{2}\)'s and 16' with an odd number.

Physical states are annihilated by the zero mode of the supersymmetry generator, which on the ground states reduces to \(G_0 = p_\mu \psi_0^\mu\). In the frame \(p^0 = p^1\) this becomes \(s_0 = \frac{1}{2}\), giving a sixteen-fold degeneracy for the \textit{physical} Ramond vacuum. This is a representation of \(SO(8)\) which again decomposes into \(8_\mathbf{a}\) with an even number of \(-\frac{1}{2}\)'s and \(8_\mathbf{c}\) with an odd number.

The GSO projection keeps one irreducible representation; the two choices, 16 or 16', are physically equivalent, differing only by a spacetime parity redefinition. It is useful to think of the GSO projection in terms of locality of the OPE with the gravitino vertex operator. Suppose we take a projection which includes the operator \(e^{-\varphi/2 + i(H_0 + H_1 + H_2 + H_3 + H_4)/2}\), where the \(H_i\) are the bosonization of \(\psi^\mu\). In the NS sector this has a branch cut with the ground state vertex operator \(e^{-\varphi}\), accounting for the sign discussed above. In the R sector the ghost plus longitudinal part is local, so we have
\[ \sum_{i=1}^{4} s_i = 0 \pmod{2}, \] (3.11)
picking out the \(8_\mathbf{a}\).

The ground state spectrum is then \(8_\mathbf{a} \oplus 8_\mathbf{s}\), a vector multiplet of \(D = 10, N = 1\) spacetime supersymmetry. Including Chan-Paton factors gives again a \(U(N)\) gauge theory in the oriented theory and \(SO(N)\) or \(USp(N)\) in the unoriented.
3.2. Closed Superstrings

The closed string spectrum is the product of two copies of the open string spectrum, with right- and left-moving levels matched. In the open string the two choices for the GSO projection were equivalent, but in the closed string there are two inequivalent choices, taking the same (IIb) or opposite (IIa) projections on the two sides. These lead to the massless sectors

- Type IIa: \((8_{\text{v}} \oplus 8_{\text{s}}) \otimes (8_{\text{v}} \oplus 8_{\text{c}})\)
- Type IIb: \((8_{\text{v}} \oplus 8_{\text{s}}) \otimes (8_{\text{v}} \oplus 8_{\text{s}})\)

of \(SO(8)\).

The various products are as follows. In the NS-NS sector, this is

\[
8_{\text{v}} \otimes 8_{\text{v}} = \phi \oplus B_{\mu \nu} \oplus G_{\mu \nu} = 1 \oplus 28 \oplus 35.
\]

(3.13)

In the R-R sector, the IIa and IIb spectra are respectively

- IIa: \(8_{\text{s}} \otimes 8_{\text{c}} = [1] \oplus [3] = 8_{\text{v}} \oplus 56_{\text{c}}\)
- IIb: \(8_{\text{s}} \otimes 8_{\text{s}} = [0] \oplus [2] \oplus [4]_{+} = 1 \oplus 28 \oplus 35_{+}\)

(3.14)

Here \([n]\) denotes the \(n\)-times antisymmetrized representation of \(SO(8)\), with \([4]_{+}\) being self-dual. Note that the representations \([n]\) and \([8-n]\) are the same, being related by contraction with the 8-dimensional \(\epsilon\)-tensor. The NS-NS and R-R spectra together form the bosonic components of \(D = 10\) IIa (nonchiral) and IIb (chiral) supergravity respectively. In the NS-R and R-NS sectors are the products

\[
8_{\text{v}} \otimes 8_{\text{c}} = 8_{\text{s}} \oplus 56_{\text{c}}
\]

\[
8_{\text{v}} \otimes 8_{\text{s}} = 8_{\text{c}} \oplus 56_{\text{s}}.
\]

(3.15)

The \(56_{\text{s},\text{c}}\) are gravitinos, their vertex operators having one vector and one spinor index. They must couple to conserved spacetime supercurrents. In the IIa theory the two gravitinos (and supercharges) have opposite chirality, and in the IIb the same.

Let us develop further the vertex operators for the R-R states. This will involve a product of spin fields \([19]\): \(e^{-\frac{\phi}{2} - \frac{\tilde{\phi}}{2}} S_\alpha \tilde{S}_\beta\). These again decompose into antisymmetric tensors, now of \(SO(9,1)\):

\[
V = e^{-\frac{\phi}{2} - \frac{\tilde{\phi}}{2}} S_\alpha \tilde{S}_\beta (\Gamma^{[\mu_1} \ldots \Gamma^{\mu_n]} C)_{\alpha\beta} H_{\mu_1 \ldots \mu_n} (X)
\]

(3.16)

with \(C\) the charge conjugation matrix. In the IIa theory the product is \(16 \otimes 16\)' giving even \(n\) (with \(n \cong 10 - n\)) and in the IIb theory it is \(16 \otimes 16\) giving odd \(n\). As is usual, the classical equations of motion follow from the physical state conditions, which at the massless level reduce to \(G_0 \cdot V = \tilde{G}_0 \cdot V = 0\). The relevant part of \(G_0\) is just \(p_\mu \psi_0^\mu\) and
similarly for $\tilde{G}_0$. The $p_\mu$ acts by differentiation on $H$, while $\psi_0^\mu$ acts on the spin fields as it does on the corresponding ground states: as multiplication by $\Gamma^\mu$. Noting the identity

$$\Gamma^\nu \Gamma^{[\mu_1 \ldots \Gamma^{\mu_n}] = \Gamma^{[\nu \ldots \Gamma^{\mu_n}]} + \left( \delta^{\nu \mu_1} \Gamma^{\mu_2 \ldots \Gamma^{\mu_n}} + \text{perms} \right)$$

(3.17)

and similarly for right multiplication, the physical state conditions become

$$dH = 0 \quad d^* H = 0.$$  

(3.18)

These are the Bianchi identity and field equation for an antisymmetric tensor field strength. This is in accord with the representations found: in the IIA theory we have odd-rank tensors of $SO(8)$ but even-rank tensors of $SO(9,1)$ (and reversed in the IIB), the extra index being contracted with the momentum to form the field strength. It also follows that R-R amplitudes involving elementary strings vanish at zero momentum, so strings do not carry R-R charges.

As an aside, when the dilaton background is nontrivial, the Ramond generators have a term $\partial_\mu \phi \psi^\mu$, and the Bianchi identity and field strength pick up terms proportional to $d \phi \wedge H$ and $d \phi \wedge * H$. The Bianchi identity is nonstandard, so $H$ is not of the form $dB$. Defining $H' = e^{-\phi} H$ removes the extra term from both the Bianchi identity and field strength. In terms of the action, the fields $H$ in the vertex operators appear with the usual closed string $e^{-2\phi}$ but with non-standard dilaton gradient terms. The fields we are calling $H'$, which in fact are the usual fields used in the literature, are decoupled from the dilaton. This fact has played an important role in recent discussions of string solitons and duality.

The IIB theory is invariant under world-sheet parity, so we can again form an unoriented theory by gauging. Projecting onto $\Omega = +1$ interchanges left-moving and right-moving oscillators and so one linear combination of the R-NS and NS-R gravitinos survives, leaving $D = 10, N = 1$ supergravity. In the NS-NS sector, the dilaton and graviton are symmetric under $\Omega$ and survive, while the antisymmetric tensor is odd and is projected out. In the R-R sector, it is clear by counting that the $1$ and $35_+$ are in the symmetric product of $8_s \otimes 8_s$ while the $28$ is in the antisymmetric. The R-R vertex operator is the product of right- and left-moving fermions, so there is an extra minus in the exchange and it is the $28$ that survives. The bosonic massless sector is thus $1 \oplus 28 \oplus 35$, the $D = 10, N = 1$ supergravity multiplet. This is the same multiplet as in the heterotic string, but now the antisymmetric tensor is from the R-R sector.

The open superstring has only $N = 1$ supersymmetry; in order that the closed strings couple consistently they must also have $N = 1$ supergravity and so the theory must be unoriented. In fact, spacetime anomaly cancelation implies that the only consistent $N = 1$ superstring is the $SO(32)$ open plus closed string theory. Now, as a general principle any such inconsistency in the low energy should be related to some stringy inconsistency. This is the case, but it will be more convenient to discuss this later after some discussion of $T$-duality.

25
3.3. T-Duality of Type II Superstrings

Even in the closed oriented Type II theories $T$-duality has an interesting effect \[ \text{[3.1]} \]. Consider compactifying a single coordinate $X^9$. In the $R \to \infty$ limit the momenta are $p_R^9 = p_L^9$, while in the $R \to 0$ limit $p_R^9 = -p_L^9$. Both theories are $SO(9, 1)$ invariant but under different $SO(9, 1)$’s. Duality reverses the sign of the right-moving $X^9(\bar{z})$; therefore by superconformal invariance it does so on $\tilde{\psi}^9(\bar{z})$. Separate the Lorentz generators into their left-and right-moving parts $M_{\mu\nu} + \tilde{M}_{\mu\nu}$. Duality reverses all terms in $\tilde{M}^{\mu 9}$, so the $\mu 9$ Lorentz generators of the $R \to 0$ limit are $M^{\mu 9} - \tilde{M}^{\mu 9}$. In particular this reverses the sign of the helicity $\tilde{s}_4$ and so switches the chirality on the right-moving side. If one starts in the IIa theory, with opposite chiralities, the $R \to 0$ theory has the same chirality on both sides and is the IIb theory, and vice versa. More simply put, duality is a one-sided spacetime parity operation, and reverses the relative chiralities of the right- and left-moving ground states. The same is true if one dualizes on any odd number of dimensions, while dualizing on an even number returns the original Type II theory.

Since the IIa and IIb theories have different R-R fields, $T_9$ duality must transform one set into the other. The action of duality on the spin fields is of the form

\[
S_\alpha(z) \rightarrow S_\alpha(z) \\
\tilde{S}_\alpha(\bar{z}) \rightarrow \rho_9 \tilde{S}_\alpha(\bar{z})
\]

for some matrix $\rho_9$. In order for this to be consistent with the action $\tilde{\psi}^9 \rightarrow -\tilde{\psi}^9$, $\rho_9$ must anticommute with $\Gamma^9$ and commute with the remaining $\Gamma^\mu$. Thus $\rho = \Gamma^9 \Gamma^{11}$ (the phase of $\rho_9$ is determined, up to sign, by hermiticity of the spin field). Other $\rho_m$ are similarly defined. Now consider the effect on the R-R vertex operators (3.16). The $\Gamma^{11}$ just contributes a sign, because the spin fields have definite chirality. Then by the $\Gamma$-matrix identity (3.17), the effect is to add a 9-index to $H$ if none is present, or to remove one if it is; the effect on the potential $B$ ($H = dB$) is the same. Take as an example the Type IIa vector $B_\mu$. The component $B_9$ maps to the IIb scalar $B$, while the $\mu \neq 9$ components map to $B_{\mu 9}$. The remaining components of $B_{\mu\nu}$ come from $B_{\mu\nu 9}$, and so on.

3.4. T-Duality of Type I Superstrings

The action of $T$-duality in the open and unoriented Type I theory produces D-branes and orientifold planes, just as in the bosonic string. Let us focus here on a single D-brane, taking a limit in which the other D-branes and the orientifold planes are distant and can be ignored. Off the D-brane, only closed strings propagate. The local physics is that of the Type II theory, with two gravitinos. This is true even if though we began with the unoriented Type I theory which has only a single gravitino. The point is that the closed string begins with two gravitinos, one with the spacetime supersymmetry on the right-moving side of the world-sheet and one on the left. The orientation projection of the Type
I theory leaves one linear combination of these. But in the $T$-dual theory, the orientation projection does not constrain the local state of the string, but relates it to the state of the (distant) image gravitino. There are two independent gravitinos, with equal chiralities if an even number of dimensions have been dualized and opposite if an odd number.

However, the open string boundary conditions are invariant under only one supersymmetry. In the original Type I theory, the left-moving world-sheet current for spacetime supersymmetry $j_{\alpha}(z)$ flows into the boundary and the right-moving current $\tilde{j}_{\alpha}(\bar{\tau})$ flows out, so only the total charge $Q_{\alpha} + \tilde{Q}_{\alpha}$ of the left- and right-movers is conserved. Under $T$-duality this becomes $Q_{\alpha} + \prod_{m} \rho_{m} \tilde{Q}_{\alpha}$, the product running over all the dualized dimensions. Closed strings couple to open, so the general amplitude has only one linearly realized supersymmetry. That is, the vacuum without D-branes is invariant under $N = 2$ supersymmetry, but the state containing the D-brane is invariant under only $N = 1$: it is a BPS state $^3$.

BPS states must carry conserved charges. In the present case there is only one set of charges with the correct Lorentz properties, namely the antisymmetric R-R charges. The world volume of a $p$-brane naturally couples to a $(p+1)$-form potential $A_{p+1}$, which has a $(p+2)$-form field strength $F_{p+2}$. This identification can also be made from the $g^{-1}$ behavior of the D-brane tension: this is the behavior of an R-R soliton $^{20,21,22,23}$. The IIa theory has $p = 0, 2, 4, 6,$ and $8$-branes. The vertex operators (3.16) describe field strengths of all even ranks. By a $\Gamma$-matrix identity the $n$-form and $(10-n)$-form field strengths are Hodge dual to one another, so a $p$-brane and $(6-p)$-brane are sources for the same field, but one magnetic and one electric. The field equation for the 10-form field strength allows no propagating states, but the field can still have a physically significant energy density $^{3,24}$. Curiously, the 0-form field strength should couple to a $(-2)$-brane, but it is not clear how to interpret this.

The IIb theory has $p = -1, 1, 3, 5, 7,$ and $9$-branes. The vertex operators (3.16) describe field strengths of all odd ranks, appropriate to couple to all but the 9-brane. A $(-1)$-brane is a Dirichlet instanton, defined by Dirichlet conditions in the time direction as well as all spatial directions. The 9-brane does couple to a nontrivial potential, as we will see below.

The action for a $(p+1)$-form potential takes the form

$$S = \frac{1}{2} \int F_{p+2} F_{p+2} + i \mu_{p} \int_{p-\text{branes}} A_{p+1},$$

(3.20)

where the $(p+1)$-form charge is $\mu_{p}$. In addition the coupling of the D-brane to NS-NS and open string states has the same form (2.19) as the bosonic D-brane theory.

It is interesting to consider the effect of $T$-duality. Consider a $p$-brane, which couples to the R-R potential with $p + 1$ indices tangent to the brane world-sheet. Take the $T$-dual

---

$^{11}$ This is not correct for $p = 3$, for which the field strength is self-dual. There is no covariant action in this case.
in a direction \( \mu \) perpendicular to the D-brane. The Dirichlet condition in this direction becomes Neumann, so in the dual theory there is a \((p+1)\)-brane. At the same time, as discussed at the end of section 3.3, the R-R potential acquires an extra \( \mu \) index, as needed to couple to the \((p+1)\)-brane. Similarly, if we take the T-dual in a direction \( m \) along the brane, it becomes a \((p-1)\)-brane and the R-R potential loses its \( m \) index.

The D-brane, unlike the fundamental string, carries R-R charge. It is interesting to see how this is consistent with our earlier discussion of string vertex operators (this argument was first given by Bianchi, Pradisi, and Sagnotti [25]). The R-R vertex operator (3.10) is in the \((-\frac{1}{2}, -\frac{1}{2})\) picture, which can be used in almost all processes. In the disk, however, the total right+left ghost number must be \(-2\). With two or more R-R vertex operators, all can be in the \((-\frac{1}{2}, -\frac{1}{2})\) picture (with picture changing operators included as well), but a single vertex operator must be in either the \((-\frac{3}{2}, -\frac{1}{2})\) or the \((-\frac{1}{2}, -\frac{3}{2})\) picture. The \((-\frac{1}{2}, -\frac{1}{2})\) vertex operator is essentially \( e^{-\phi}G_0 \) times the \((-\frac{3}{2}, -\frac{1}{2})\) operator, so besides the shift in the ghost number the latter has one less power of momentum and one less \( \Gamma \)-matrix. The missing factor of momentum turns \( H \) into \( A \), and the missing \( \Gamma \)-matrix gives the correct Lorentz representations for the potential rather than the field strength.

For open string gauge fields, the \(-1\) picture involves the potential and the \( 0 \) picture the field strength. In interactions involving one R-R field and \( k \) open string gauge fields, in order for the pictures to add to \(-2\), exactly one vector or else the R-R field must appear as a potential, and the rest as field strengths. Thus these interactions are of Chern-Simons form [15]. Their detailed form has been discussed recently [28,26]. All interactions

\[
\int A_k F^l \tag{3.21}
\]

having the correct rank to be integrated over the \( p \)-brane world-sheet (that is, \( k+2l = p+1 \)) appear. These have played an important role in various recent discussions of D-brane dynamics [11,12,27].

To obtain the D-brane tension and R-R charge, one can consider the same vacuum cylinder as in the bosonic string [13]. Carrying out the traces over the open superstring spectrum gives

\[
A = 2V_{p+1} \int \frac{dt}{2t} \left( 8\pi^2 \alpha' t \right)^{-(p+1)/2} e^{-t} \frac{Y^2}{2\pi \alpha'} \prod_{n=1}^{\infty} \left( 1 - q^{2n} \right)^{-8} \tag{3.22}
\]

\[
\frac{1}{2} \left\{ -f_2(q)^8 + f_3(q)^8 - f_4(q)^8 \right\},
\]

where again \( q = e^{-\pi t} \). The three terms in the braces come from the open string R sector with \( \frac{1}{2} \) in the trace, from the NS sector with \( \frac{1}{2} \) in the trace, and the NS sector with \( \frac{1}{2}(-1)^F \) in the trace; the R sector with \( \frac{1}{2}(-1)^F \) gives no net contribution. These three terms sum to zero by the ‘abstruse identity,’ because the open string spectrum is supersymmetric. In terms of the closed string exchange, this reflects the fact that D-branes are BPS states,
the net forces from NS-NS and R-R exchanges canceling. The separate exchanges can be identified as follows. In the terms with \((-1)^F\), the world-sheet fermions are periodic around the cylinder corresponding to R-R exchange, while the terms without \((-1)^F\) have antiperiodic fermions and come from NS-NS exchange. Obtaining the \(t \to 0\) behavior as before gives

\[
A \sim \frac{1}{2}(1 - 1)V_{p+1} \int \frac{dt}{t} (2\pi t)^{-(p+1)/2} (t/2\pi\alpha')^4 e^{-t\frac{Y^2}{8\pi^2\alpha'^2}} = (1 - 1)V_{p+1} 2\pi(4\pi^2\alpha')^{3-p} G_{9-p}(Y^2).
\]

The \((1 - 1)\) is from the NS-NS and R-R exchanges respectively. Comparing with field theory calculations gives \[3\]

\[
\mu_p^2 = 2T_p^2 = 2\pi(4\pi^2\alpha')^{3-p}.
\]

Just as a constant Wilson line is dual to a translation of the D-brane, a constant field strength is dual to a rotation or boost \[29\]. D-branes which are not parallel feel a net force because the cancelation is no longer exact. In the extreme case, where one of the D-branes is rotated by \(\pi\), the coupling to the dilaton and graviton is unchanged but the coupling to the R-R tensor is reversed in sign, and the two terms in the cylinder amplitude add. In fact, a well-known divergence of Dirichlet boundary conditions sets in for non-parallel branes: the \(t\)-integration diverges at zero. This is similar to the Hagedorn divergence, and represents an instability of the D-branes when brought too close \[30\].

The orientifold planes also break half the supersymmetry and are R-R and NS-NS sources. In the original Type I theory the orientation projection keeps only the linear combination \(Q_\alpha + \tilde{Q}_\alpha\); in the dualized theory this becomes \(Q_\alpha + \prod_m \rho_m \tilde{Q}_\alpha\) just as for the D-branes. The force between an orientifold plane and a D-brane can be obtained from the Möbius strip as in the bosonic case; again the total is zero and can be separated into NS-NS and R-R exchanges. The result is similar to the bosonic result (2.39),

\[
\mu'_p = \mp 2^{p-5}\mu_p, \quad T'_p = \mp 2^{p-5}T_p
\]

Since there are \(2^{9-p}\) orientifold planes, the total fixed-plane charge is \(\mp 16\mu_p\), and the total fixed-plane tension is \(\mp 16T_p\).

A nonzero total tension represents a source for the graviton and dilaton, so that at order \(g\) these fields become time dependent as in the Fischler-Susskind mechanism \[31\]. A nonzero total R-R source is more serious: the field equations are inconsistent, because R-R flux lines have no place to go in the compact space.\[12\] So we need exactly 16 D-branes with the \(SO\) projection, giving the \(T\)-dual of \(SO(32)\). So we find that the spacetime

\[12\] The Chern-Simons coupling \[3.21\] implies that the open string field strengths are also R-R sources, so there will be more general consistent solutions with nonzero values for these.
anomalies for $G \neq SO(32)$ are accompanied by a divergence \[32\]; this also leads to a world-sheet conformal anomaly that cannot be canceled because of the inconsistency of the field equations. All this can be discussed in the original $D = 10$ Type I theory \[15\]. The Neumann open strings correspond to 9-branes, since the endpoints can be anywhere. The Dirichlet and orientifold 9-branes couple to an R-R 10-form,

$$i(32 \mp N) \frac{\mu_{10}}{2} \int A_{10}, \tag{3.26}$$

and the field equation from varying $A_{10}$ is $G = SO(32)$ \[13\]!

4. Lecture IV: D-Branes Galore

4.1. Discussion

We have seen that $T$-duality of the Type I string leads to a theory with precisely 16 Dirichlet $p$-branes on a $T_{9-p}/Z_2$ orientifold, for any given value of $p$. We now understand that the restriction to 16 comes from conservation of R-R charge. It follows that in a non-compact space, where the flux lines could run to infinity, we could have a consistent theory with any number and configuration of $p$-branes, with all $p$ being even in the IIa theory or odd in the IIb. Indeed, cluster decomposition plus $T$-duality forces this upon us. The $T$-dual of a flat torus gives flat D-branes, but because they are dynamical this is continuously connected to configurations where the D-branes fold back and forth, and in this way one can reach a configuration which in any local region has an arbitrary set of $p$-branes. Moreover, while $T$-duality gives at first only $p$-branes for a single value of $p$, we can then deform to a configuration with perpendicular $p$-branes. A further $T$-duality along a direction which is parallel to one $p$-brane and perpendicular to another interchanges Neumann and Dirichlet conditions along that direction, and so produces a $(p+1)$-brane and a $(p-1)$-brane. In this way we reach a general configuration.

Thus it is natural to consider all these configurations as different states in a single theory, with the usual Type I and II strings being perturbative expansions around particular states (the latter being the no-brane state). There is an important consistency check here. The field strengths to which a $p$-brane and $(6-p)$-brane couple are dual to one another, $H_{p+2} = \ast H_{10-p}$. This implies a Dirac quantization condition, as generalized by Teitelboim and Nepomechie \[13\]. Integrating the field strength $\ast H_{p+2}$ on an $(8-p)$-sphere surrounding a $p$-brane, the action \[3.20\] gives a total flux $\Phi = \mu_p$. We can write $\ast H_{p+2} = H_{8-p} = dB_{7-p}$ everywhere except on a Dirac ‘string’. Then

$$\Phi = \int_{S_{8-p}} \ast F_{p+2} = \int_{S_{7-p}} B_{7-p}, \tag{4.1}$$

30
where we perform the last integral on a small sphere surrounding the Dirac string. A 
(6−p)-brane passing circling the string picks up a phase $e^{i\mu_6-p\Phi}$. The condition that the 
string be invisible is

$$\mu_6-p\Phi = \mu_6-p\mu_p = 2\pi n. \quad (4.2)$$

The D-branes charges (3.24) satisfy this with the minimum quantum $n = 1$.\[13\]

This calculation has the look of a ‘string miracle.’ It is not at all obvious why the one-loop 
open string calculation should have given just this result. Had the R-R charges not satisfied 
the quantization condition, one could likely use the argument from the first paragraph of 
this section to show that the Type I theory has some sort of non-perturbative anomaly. 
Perhaps this can be used to find a more direct topological calculation of the D-brane 
charge.

Thus far weak/strong coupling string duality has not entered. All we have done is to 
follow T-duality to its logical conclusions, though not all of these were noticed until string 
duality focused attention on the important issues. Now, the key point is that string 
duality relates ordinary strings (which carry electric NS-NS charges), as well as string 
solitons carrying magnetic NS-NS charges, to R-R charged states [21,22]. The D-brane 
description of the R-R charged states has allowed many new and successful tests of string 
duality. Most fundamentally, string duality makes a specific prediction for the quantum 
of R-R charge [34,35], which is precisely the value (3.24) carried by the D-brane [3]. In 
the remainder of this lecture a few additional consequences will be derived.

Before the observation that D-branes carry R-R charge, the R-R charged states required 
by string duality were assumed to be black holes. One can always find such black hole 
solutions [20]. What is the relation between these descriptions? My understanding is as 
follows. Because the dilaton scales out of the R-R action, the R-R solitons are small, their 
size being given by the Planck scale [36]. For weak string coupling this is smaller than 
the string length, so the nonlinear part of the black hole solution is just not relevant, and 
the D-brane is actually a small perturbation on the geometry. This is consistent with the 
discussion of single-fermion tunneling in the matrix model [37], which is also an effect of 
order $g^{-1}$, and so is a small disturbance as compared to a normal field theory tunneling 
event. A rather opposite interpretation is that an open string ending on a D-brane is 
actually a closed string, half of which is stuck behind the horizon of a black hole! This is 
curiously similar to the picture of the black hole entropy in ref. [38].

\[13\] This argument does not apply directly to the case $p = 3$, as the self-dual 5-form field strength 
has no covariant action. However, using T-duality to relate this to $p = 2$ shows that the $p = 3$ 
quantum is minimal also.

\[14\] To be precise, there remains a factor of two discrepancy in the literature, which can plausibly be 
attributed to the problem of defining the action for the chiral bosons of the string soliton [34].

\[15\] Suggested by E. Witten and A. Strominger.

31
Taking this issue further, for sufficiently many coincident D-branes string perturbation theory will break down, the expansion parameter being $gN$. However, for large enough R-R charge, the description in terms of low energy field theory becomes valid because the black hole is large. In some cases it is possible to continue between these regimes by varying parameters, and to follow the BPS states. Very recently, this has led to the counting of the BPS states of a black hole, and the number is indeed that given by the Bekenstein-Hawking entropy. This is the first time, after two decades of attempts, that the black hole entropy has been related to a counting of states in a controlled way.

4.2. Multiple Branes and Broken Supersymmetries

$T$-duality of the Type I string lead to parallel D-branes with given $p$. This configuration has the same supersymmetry as the original Type I theory. For convenience we will in this section use $D = 4$ units, so this is $N = 4$ SUSY, broken from the $N = 8$ of the Type II theory.

Now we are considering more general configurations of D-branes, and so will determine the unbroken supersymmetry of such configurations (there is some discussion of this in ref. [41]). For simplicity we will analyze only the case that all D-branes are oriented along some set of coordinate axes, so each can be defined by taking Dirichlet boundary conditions on some subset $S_i \subset \{0, 1, \ldots, 9\}$ of the coordinates ($i$ labeling the D-brane) and Neumann conditions on the remaining coordinates $\overline{S}_i$. We can use $T$-duality to simplify the discussion. By dualizing on each of the axes in $S_1$ we can take the first D-brane to be a nine-brane, with fully Neumann conditions. Now consider a second D-brane. From earlier discussion, we know that the supersymmetries left unbroken by the two D-branes are respectively

$$Q_\alpha + \tilde{Q}_\alpha, \quad Q_\alpha + \rho_{(2)\alpha\beta} \tilde{Q}_\beta.$$ (4.3)

The unbroken supersymmetries are the intersections of these two sets, and therefore are in one-to-one correspondence with the +1 eigenvalues of $\rho_{(2)} = \prod_{m \in S_2} (\Gamma^m \Gamma^{11})$. Note that $S_2$ has an even number of elements, because we must now be in the IIB theory. For the case of two elements, meaning that the second D-brane is a seven-brane, there are no unbroken supersymmetries: we have $(\Gamma^m \Gamma^{11})^2 = -1$ so the eigenvalues of $\rho_{(2)}$ are $\pm i$. For four elements, $\rho_{(2)}^2 = +1$; half the eigenvalues are +1 and so the unbroken supersymmetry is $N = 2$. Similarly, six elements break all the supersymmetry, and eight break half.

We can state the above result in a $T$-duality invariant way. Consider open strings with one end on one D-brane and one on the other. Some coordinates will have Neumann conditions on both ends (NN), some Dirichlet (DD), and some mixed (ND). Unbroken supersymmetry

16 The BPS strategy was applied to Neveu-Schwarz black holes by Larsen and Wilczek [39]. In this case the black hole continue to look like a black hole no matter how weak the coupling becomes, and so one does not have an explicit understanding of the space of states even at weak coupling.
requires the number of ND directions to be a multiple of 4. We can also see this as follows. The open string mode expansion is
\[
X(0, \sigma_1) \sim i \sqrt{\frac{\alpha'}{2}} \sum_r \frac{\alpha_r}{r} (e^{ir\sigma_1} \pm e^{-ir\sigma_1}).
\] (4.4)

Here \( r \) is integer for NN and DD coordinates, with the upper and lower sign respectively (the \( r = 0 \) terms have not been written explicitly), while \( r \) is half-integer for ND and DN coordinates. The fermions have the same moding in the R sector and opposite in the NS sector. Let \( \nu \) be the number of ND coordinates. The string zero point energy is 0 in the R sector as always, and
\[
(8 - \nu) \left( -\frac{1}{24} - \frac{1}{48} \right) + \nu \left( \frac{1}{24} + \frac{1}{48} \right) = -\frac{1}{2} + \frac{\nu}{8}
\] (4.5)
in the NS sector. Only for \( \nu \) a multiple of 4 is degeneracy between the R and NS sectors possible.

Similarly, one can show that a nine-brane plus two five-branes, with \( S_2 = \{6, 7, 8, 9\} \) and \( S_3 = \{4, 5, 8, 9\} \), break the supersymmetry down to \( N = 1 \). This is \( T \)-dual to three five-branes (with \( S_1 = \{4, 5, 6, 7\} \), \( S_2 = \{4, 5, 8, 9\} \) and \( S_3 = \{6, 7, 8, 9\} \)); also to three seven-branes, and so on.

The story above cannot be complete. A \( p \)-brane and \( (p + 2) \)-brane, when separated, do indeed break all supersymmetries. However string duality requires that they have a bound state where the \( p \)-brane is fully contained in the \( (p + 2) \)-brane, with as much supersymmetry as the \( (p + 2) \)-brane has by itself. We can see a hint of this by considering open strings with one end on each D-brane. The zero point energy found above is negative for these. When the \( p \)- and \( (p + 2) \)-branes are well separated, the energy of stretching makes the open strings massive, but for sufficiently small separation there are tachyonic open strings, not all of which are removed by the GSO projection. This is like the Hagedorn instability, and cannot be quantitatively treated. However, in this case one can see in another description that there is a stable BPS state to decay to. The \( (p + 2) \)-brane has a world-sheet gauge field. Consider a constant background for its field-strength \( F_{mn} \). The Chern-Simons coupling (3.21),
\[
\int A_{p+1} F
\] (4.6)
implies that the \( (p + 2) \)-brane now couples to the \( (p + 1) \)-form potential as well as the usual \( (p + 3) \)-form. That is, it has the total R-R quantum numbers of the \( (p + 2) \)-brane and the \( p \)-brane. In effect the \( p \)-brane has dissolved in the \( (p + 2) \)-brane!

\[17\] The remainder of this section is a result of discussions between J. Harvey, G. Moore, J. Polchinski, and A. Strominger.
This discussion is exactly parallel to the discussion of the binding of fundamental and D-strings in ref. [11]. In fact it is dual to it. By $T$-duality we may consider a D three-brane and D one-brane in Type IIB theory. This theory has an $SL(2,\mathbb{Z})$ self-duality which takes the three-brane into itself and the D one-brane into a fundamental string. Now $T$-dualize in the two directions parallel to the three-brane and perpendicular to the string. The result is parallel Dirichlet and fundamental strings, the case considered in ref. [11].

4.3. D-strings

The only supersymmetric objects in the Type I theory, besides the nine-branes, will be one- and five-branes. This is consistent with the fact that the only R-R field strengths are the three-form and its Hodge-dual seven-form. To close these lectures we will study these objects in the Type IIb and Type I theories.

Start with the Dirichlet one-brane, or D-string, first in the Type IIb theory and then in the Type I theory. In the Type II string, a $p$-brane is just the $T$-dual of the Type I nine-brane. In particular, its massless sector is just the dimensional reduction of the $D = 10, N = 1$ gauge multiplet to $p + 1$ dimensions. Thus we have the bosonic states

$$\psi^{-1/2}_{-\mu}(k,ij), \quad \mu = 0, \ldots, p \quad \psi^{m}_{m}(k,ij), \quad m = p + 1, \ldots, 9 \quad (4.7)$$

where $k^\mu$ is a $(p + 1)$-dimensional momentum vector. The spacetime spinor of $SO(8)$ is projected along Dirichlet and Neumann directions, under an $SO(9-p) \times SO(p+1)$ decomposition.

For the Type IIb D-string, $p = 1$, the gauge field has no local dynamics, so the only bosonic excitations are the transverse fluctuations. Applying the GSO projection (e.g. via locality with the gravitino vertex operator), the right-moving spinors on the D-string are in the $\bar{8}_s$ of $SO(8)$, and the left-moving spinors in the $8_c$. This is the same as the world-sheet theory of a macroscopic fundamental IIb string [11]. This is as required by weak/strong self-duality of the IIb string [21,22]. The fundamental and D-strings couple respectively to NS-NS and R-R two-form potentials, which are interchanged by weak/strong duality. Their tensions are respectively $O(1)$ and $O(e^{-\phi})$ in the string metric, which become $O(e^{\phi/2})$ and $O(e^{-\phi/2})$ in the Einstein metric, and so are interchanged under $\phi \to -\phi$. Indeed, given the argument that D-branes must appear in the Type II spectrum, the BPS bound implies that at strong coupling they are the lightest degrees of freedom. This strongly suggests that the physics in this limit is given by an effective theory of D-strings: string duality.

---

18 The binding of $p$ and $p+2$ is therefore not relevant here.

19 It does not imply that duality holds to all energies, but this is the simplest possibility. That is, given that physics below the Planck energy is described by some specific string theory, it seems likely that there is a unique extension to higher energies.
The full duality group of the $D = 10$ Type IIb theory is believed to be $SL(2,\mathbb{Z})$ \cite{21,22}. This relates the fundamental string not only to the R-R string but to a whole set of strings $(m,n)$ for $m$ and $n$ relatively prime \cite{42}. Here $m$ and $n$ are respectively the NS-NS and R-R tensor charges of the string. Witten has used the D-brane picture to show that these strings exist as bound states of $n$ D-branes and $m$ fundamental strings \cite{11}. The non-dynamical $U(n)$ gauge symmetry of $n$ coincident one-branes plays an essential role here. This method has been generalized successfully to a number of other counting problems \cite{11,12}, most notably the black hole entropy \cite{40} discussed above.

Now let us move on to the one-brane of the Type I theory \cite{13}. There are two modifications. The first is the projection onto oriented states. The $U(1)$ gauge field, with vertex operator $\partial_t X^\mu$, is removed just as the vector of the Type I string spectrum. The collective coordinates, with vertex operators $\partial_n X^\mu$, remain in the spectrum because the normal derivative is even under $\Omega$. That is, in terms of its action on the $X$ oscillators $\Omega$ has an additional $-1$ for the $m = 2, \ldots, 9$ directions, as compared to the action on the usual NN strings. By superconformal symmetry this must extend to the fermions, so that on the ground states $\Omega$ is no longer the identity but acts as $R = e^{i\pi(S_1+S_2+S_3+S_4)}$. This removes the left-moving $8_c$ and leaves the right-moving $8_s$ (or vice versa: we have made an arbitrary choice in defining $R$).

The second modification is the inclusion of 1-9 strings, strings with one end on the one-brane and one on a nine-brane, the latter corresponding to the usual $SO(32)$ Chan-Paton factor.

\begin{equation}
|\pm; i\rangle = (\psi_0^0 \pm \psi_0^1)|i\rangle,
\end{equation}

where $i$ is a Chan-Paton index for the nine-brane end. One of the two states $|\pm; i\rangle$ is removed by the GSO projection, and the $G_0$ physical state condition then implies that these massless fermions are chiral on the one-brane. Spacetime supersymmetry can only be satisfied if the GSO projection is such that they move oppositely to the 1-1 fermions; at the world-sheet level this would have to follow from a careful analysis of the OPE of the gravitino. The 1-9 strings, with one Chan-Paton index, are vectors of $SO(32)$.
Thus the world-sheet theory of the Type I one-brane is precisely that of the heterotic string, with the spacetime supersymmetry realized in Green-Schwarz form and the current algebra in fermionic form. This is again strong evidence for string duality, here between the $SO(32)$ Type I and heterotic strings. Curiously, the same conclusion follows from the black one-brane description [44], even though the details are quite different.

The fermionic $SO(32)$ current algebra requires a GSO projection. It is interesting to see how this arises in the D-string. Consider a closed D-string. The $\Omega$ projection removed the $U(1)$ gauge field, but is consistent with a discrete gauge symmetry, a holonomy $\pm 1$ around the D-brane. This discrete gauge symmetry is the GSO projection, and evidently the rules of D-branes require us to sum over all consistent possibilities in this way.

We can now see how D-strings account for the spinor representation of $SO(32)$ in the Type I theory. In the R sector of the discrete D-brane gauge theory, the 1-9 strings are periodic. The zero modes of the fields $\Psi^i$, representing the massless 1-9 strings, satisfy the Clifford algebra
\[ \{ \Psi^i_0, \Psi^j_0 \} = \delta^{ij}, \quad i, j = 1, \cdots, 16. \quad (4.9) \]

The quantization now proceeds just as for the fundamental heterotic string, giving spinors $\mathbf{2}^{15} + \mathbf{2}^{15}$. We can follow this further, looking for the $E(8) \times E(8)$ Type I string (much of the following is based on ref. [43] and discussions with E. Witten). Let us start with a single $E(8)$. Compactify the $SO(32)$ heterotic string on a circle, with $U(1)^{16} \subset SO(32)$ Wilson line
\[ \left( \frac{1}{7}, 0^9 \right). \quad (4.10) \]

This breaks the $SO(32)$ to $SO(14) \times SO(18)$. As the radius is reduced, massless winding states appear at $R^2 = \frac{1}{8} \alpha'$. Winding numbers $\pm 1$ contribute spinors $(64, +1)$ and $(64, -1)$ of $SO(14) \times U(1)$, the $U(1)$ being the left-moving Kaluza-Klein momentum. Winding numbers $\pm 2$ contribute $(14, \pm 2)$. These add up to the adjoint of $E(8)$.

In the Type I theory the winding states map to D one-branes, and we should be able to find all these states. But there is a paradox [43]. In the Type I theory, the $E(8)/SO(14) \times U(1)$ gauge bosons are D-branes, not perturbative string states. With all the recent work on supersymmetric gauge theories and string theories, we have gotten used to something that once seemed unlikely: nonperturbative states can become massless at special values of the parameters. But in all known examples, this happens only when perturbation theory breaks down. This is consistent with the idea that perturbation theory is in some sense asymptotic (at fixed energy)—light nonperturbative states would violate this. In field theory one can probably prove it. In string theory we have no nonperturbative formulation on which to base a proof, but it seems likely and is consistent with all examples. Now apply this to the present case. Hold the heterotic string radius fixed at the symmetry point, $R_h = \sqrt{\alpha'/8}$ and take the coupling $g_h$ large. According to the Type I–heterotic
duality, the corresponding Type I theory has
\[ R_I = R_h g_h^{-1/2}, \quad g_I = g_h^{-1}. \] (4.11)

In the limit of interest \( R_I \) is becoming small, so the physics will be clearer in the \( T \)-dual theory, which we will denote \( I' \). Then, taking into account the transformation (2.9) of the dilaton,
\[ R_{I'} = R_h^{-1} g_h^{1/2}, \quad g_{I'} = g_h^{-1/2} R_h^{-1}. \] (4.12)

The limit \( g_h \to \infty, R_h \) fixed then corresponds to a weakly coupled Type I' theory on a large \( S_1/Z_2 \), where one would not expect to see a massless soliton.

The resolution involves the special features of the Type I' theory. We know that the Wilson line (4.10) puts seven D-branes at one fixed plane and nine at the other. The R-R charge of the fixed planes is canceled globally but not locally (the latter, by symmetry, would require eight at each end). So the spacetime is an R-R capacitor, with a net source at one end and net sink at the other. But by the BPS property there are also local dilaton sources, so there is a dilaton gradient. This gradient is of order \( g_I' \), but \( R_{I'} \) is of order \( g_I'^{-1} \) so the effect is of order 1. The precise dilaton dependence is obtained by solving the field equations of the effective supergravity theory. This is the IIa supergravity theory, since we have dualized one dimension. The R-R background is a nine-form potential, which is non-dynamical in \( D = 10 \) but contributes an effective cosmological constant. This is the supergravity theory found by Romans [45]. The solution was found in ref. [43]. It has the following property. As \( R_h \) is decreased toward the \( E(8) \) radius, \( R_{I'} \) increases and so does the effect of the dilaton gradient. Precisely at the critical radius, the dilaton diverges at the end with seven D-branes. This is so even though the effective nine-dimensional coupling, involving some average of the dilaton, remains weak.

The paradox is thus evaded, and the precise point of breakdown gives further evidence for Type I–heterotic duality. We can go further and find the \( E(8)/SO(14) \times U(1) \) gauge bosons in the Type I' D-brane spectrum. In the heterotic theory these are winding states, so one-branes in the Type I theory and zero-branes in the Type I'. The winding number one states map into single zero-branes, which by symmetry must be at one fixed plane. From the relation (2.16),
\[ X'^9(\pi, \sigma^2) - X'^9(0, \sigma^2) = 2\pi\alpha'p^9, \] (4.13)

one can deduce that the one–branes from the current algebra R sector map to zero–branes at the end with nine eight–branes, and those from the current algebra NS sector to the end with seven. It is the latter that are of interest. These have a mass of order \( e^{-\phi} \), which does indeed go to zero at this end when the radius becomes critical. Of course, we cannot follow the state all the way to strong coupling, but in the range where the coupling is still weak this is a BPS state and the supergravity solution for \( \phi \) gives the mass required by duality. The reader can work out the one–brane spectrum just as done for the zero–brane
above, and it is as expected. It is free to move on the fixed plane but not away from it, and the 0-8 strings give rise to a spinor representation of $SO(14)$. The $U(1)$ is the R-R gauge field which couples to zero-branes. The remaining gauge bosons have winding number two and so map to a pair of zero-branes. The necessary bound states can be studied as in refs. [11][12], but we have not carried this out in detail.

For the $E(8) \times E(8)$ theory the Wilson line is

\[
\left( \frac{1}{2}, \frac{1}{2} - \lambda, \lambda, 0^7 \right)
\]

with critical radius $R^2_h = \alpha' \lambda \left( \frac{1}{2} - \lambda \right)$. This maps into a configuration with seven D-branes at each fixed point and the other two placed symmetrically. The dilaton behaves as shown.

![Diagram](image)

In the strong-coupling limit, the eighth D-brane moves toward each end. In between, we have the IIa theory, in which an eleventh dimension is supposed to decompactify at strong coupling. The strong coupling limit is then M-theory on $S_1/Z_2 \times S_1$, which is the same as Horava and Witten’s description of the $E(8) \times E(8)$ M-theory [47]. Letting the $S_1/Z_2$ have size $r_1$ and the $S_1$ size $r_2$, the dualities give

\[
R_h = r_2 r_1^{1/2}, \quad g_h = r_1^{3/2}
\]

\[
R_I' = r_1 r_2^{1/2}, \quad g_I' = r_2^{3/2}.
\]

These agree with the mapping (4.12) between the Type $I'$ and the $T$-dual of the heterotic theory.

### 4.4. Five-Branes

For the Type IIb five-brane we obtain again a world-brane $U(1)$ gauge field plus the scalar transverse fluctuations, and their superpartners. In the Type I theory there is an interesting subtlety. Consider multiple five-branes, so the Chan-Paton index $i$ runs over
both nine- and five-branes. The calculation (1.12) appears to imply that a common $SO$ or $Sp$ projection must be taken on both types; we know from the nine-brane that this must be $SO$. However, in eq. (1.12) it was assumed that $\Omega^2$ on the fields was simply the identity. A more careful analysis, explained in detail in section 2 of ref. [48], shows that $\Omega^2$ is $-1$ in the 5-9 sector of the open string Hilbert space. To cancel this we must take the opposite projections on the five- and nine-branes, so the former have a symplectic world-sheet group. In particular this implies that they must appear in even numbers.

Let us therefore study two coincident five-branes. The massless states are

$$\lambda_{ij}\alpha_{-1}^\mu|k,ij\rangle, \quad \lambda'_{ij}\alpha_{-1}^m|k,ij\rangle$$

with $i,j$ running over $1,2$. The orientation projection then implies that

$$\epsilon\lambda = -\lambda^T\epsilon, \quad \epsilon\lambda' = \lambda'^T\epsilon.$$  

This implies that $\lambda$ is one of the Pauli matrices, giving five-brane gauge group $SU(2) = USp(2) = Sp(1)$. The collective coordinate wavefunction $\lambda'$ is the identity, so the two five-branes move as a unit; physically it is a single five-brane with a two-valued Chan-Paton factor. This can also be seen in another way [49]. In the Type I theory the force between 5-branes, and between 1-branes, is half of what we found earlier, because of the orientation projection on the sum over states. The product of the charges of a single one-brane and single five-brane would then be only half a Dirac-Teitelboim-Nepomechie unit; but since the five-branes are always paired the quantization condition is respected.

This result, a symplectic gauge group on the five-brane, is required by string duality [50]. The Type I five-brane is dual to the instanton five-brane of the heterotic theory. The symplectic gauge group is needed to give the correct moduli space of instantons.

In the Type I theory there will also be 5-9 strings transforming as a $(2,32)$ under the five-brane and nine-brane gauge groups. The R sector and NS sectors both have vanishing zero point energies, and in each there are four periodic transverse fermions. The zero modes thus generate four states, reduced to two by the GSO projection. In terms of the $D = 6 N = 1$ supersymmetry of this configuration (equivalent to $D = 4, N = 2$), this is the content of half a hypermultiplet. This is allowed because the representation $(2,32)$ is pseudoreal.

4.5. A Brief Survey

Dirichlet boundary conditions were a subject of frequent fascination even before the relevance to string duality was realized, and were interpreted in several different ways. In

\[\text{This argument also implies that Dirichlet three- and seven-branes are inconsistent in Type I theory, as expected from the absence of an appropriate R-R field.}\]
this final section we will briefly survey the pre- and post-duality literature on D-branes, omitting some papers discussed elsewhere in these notes.

Boundaries with Dirichlet conditions on all coordinates (D-instantons or D (−1)-branes in the current terminology) were first considered as off-shell probes of the theory [51]. It was then proposed that introducing a gas of such boundaries would produce the partonic behavior needed in a string theory of QCD [52]. In ref. [2] it was proposed that these D-instantons were actually an essential part of string theory, based on the $e^{-1/g}$ behavior noted in ref. [51].

Boundaries with Dirichlet conditions on some coordinates and Neumann on others were first suggested to represent a form of compactification [53], since the open strings move in a space of reduced dimension; however, the closed strings still move in the critical dimension. Such boundaries also arose in an attempt to put the Type I string on a K3 orbifold [54]. The interpretation in terms of a dynamical object was made in ref. [1]. There were many studies of compactification of open superstrings with orientifold projections and Dirichlet boundaries, with spacetime anomaly and/or divergence cancellation imposed [54,5,55]. Recent systematic studies of two examples can be found in refs. [48,56].

Many other interesting duality properties of D-branes and orientifolds have recently been discussed [11,12,27,57]. There is no perturbative string theory in eleven dimensions, so our knowledge of M theory is limited for now to some understanding of its compactifications and duality symmetries. The D-brane description of R-R charges has been useful in unraveling some of this [58].

As mentioned above, the original interest in D-instantons was their hard behavior at short distance. Now that we are interpreting these as an essential part of string theory we have to rethink this [2]: are D-branes a sign of degrees of freedom at distances less than the string scale [50]? There have been several studies of D-brane–D-brane and string–D-brane scattering. The picture that emerges is not entirely clear. String scattering from a p-brane for $p \geq 0$ has structure on the string scale [59], unlike D-instanton corrections to scattering. D-brane–D-brane scattering shows some sign of shorter distance structure [29]. The string scale structure has been interpreted as a ‘string halo’ that hides the shorter distance physics [50]. In fact, there is one other kind of scattering that seems to cut through the string halo and see pointlike structure. Consider a macroscopic string ending on a D-brane. We can send ripples down the string and watch them bounce off the end. This is easy: to lowest order in string perturbation theory, the Dirichlet boundary condition just gives an energy-independent phase shift, indicating a pointlike structure. This holds up to arbitrarily high energies, so is cut off only where string perturbation theory breaks down. It will be interesting to pursue this further.

---

21 Dirichlet boundary conditions are not superconformally invariant in heterotic string theory, a fact which caused some discomfort in ref. [4]. D-branes have no known analog in the heterotic string, there being no analog of the R-R fields, and so no understanding of the $e^{-1/g}$ there.
4.6. Conclusion

So where does this leave us? The goal is to answer the question “What is string theory?” We have learned that string theory contains a new kind of object, the D-brane, which is a sort of topological defect where strings can end. This has clarified many of the connections between dual theories, and turned string duality into a much tighter structure. However, it also points up, even more strongly, that our current understanding of string theory is only effective, provisional. It is hard to imagine that string theory will be defined in a precise way as some sort of sum over string and D-brane world-sheets. Rather, the perturbative string description is valid only up to some scale, and the sum over D-brane histories makes no sense at shorter distances.

We are in a position similar to that of Wilson [60], when he was trying to answer the question “What is field theory?” He began to make progress when he found a model, the pion-nucleon static model, which was simple enough to be understood yet rich enough to display the essence of field theory. We have finally found models, namely string backgrounds with extended supersymmetry, which are simple enough that we can make progress, but rich enough to display a great deal of new and surprising dynamics. But the model was only a stepping-stone to the principle, which was to think about field theory scale-by-scale. That principle made possible both a precise definition of field theory and an understanding of the dynamics and phase structure. In string theory we are still looking for the underlying principle, and there is good reason to expect that it will be similarly beautiful and powerful.
References

[1] J. Dai, R. G. Leigh and J. Polchinski, Mod. Phys. Lett. A4 (1989) 2073; R. G. Leigh, Mod. Phys. Lett. A4 (1989) 2767.

[2] J. Polchinski, Phys. Rev. D50 (1994) 6041.

[3] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724.

[4] J. Paton and Chan Hong-Mo, Nucl. Phys. B10 (1969) 519.

[5] J. H. Schwarz, in Florence 1982, Proceedings, Lattice Gauge Theory, Supersymmetry and Grand Unification, 233; Phys. Rept. 89 (1982) 223; N. Marcus and A. Sagnotti, Phys. Lett. 119B (1982) 97.

[6] A. Sagnotti, in Non-Perturbative Quantum Field Theory, eds. G. Mack et. al. (Pergamon Press, 1988) 521; G. Pradisi and A. Sagnotti, Phys. Lett. B216 (1989) 59; M. Bianchi and A. Sagnotti, Phys. Lett. 247B (1990) 517; Nucl. Phys. B361 (1991) 519; P. Horava, Nucl. Phys. B327 (1989) 461; Phys. Lett. B289 (1992) 293; Nucl. Phys. B418 (1994) 571; Open Strings from Three Dimensions: Chern-Simons-Witten Theory an Orbifolds, Prague preprint PRA-HEP-90/3, to appear in J. Geom. Phys.

[7] K. Kikkawa and M. Yamanaka, Phys. Lett. B149 (1984) 357; N. Sakai and I. Senda, Prog. Theor. Phys. 75 (1986) 692.

[8] V.P. Nair, A. Shapere, A. Strominger, and F. Wilczek, Nucl. Phys. B287 (1987) 402.

[9] M. Dine, P. Huet, and N. Seiberg, Nucl. Phys. B322 (1989) 301.

[10] P. Horava, Phys. Lett. B231 (1989) 251; M. B. Green, Phys. Lett. B266 (1991) 325.

[11] E. Witten, Bound States of Strings and p-Branes, preprint IASSNS-HEP-95-83, hep-th/9510133.

[12] A. Sen, A Note on Marginally Stable Bound States in Type II String Theory, preprint MRI-PHY-23-95, hep-th/9510229; U Duality and Intersecting D-Branes, preprint MRI-PHY-27-95, hep-th/9511026; C. Vafa, Gas of D-Branes and Hagedorn Density of BPS States, preprint HUTP-95-A042, hep-th/9511088; Instantons on D-Branes, preprint HUTP-95-A049, hep-th/9512078.

[13] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. B163 (1985) 123.
[14] C. Lovelace, Phys. Lett. B34 (1971) 500;  
L. Clavelli and J. Shapiro, Nucl. Phys. B57 (1973) 490;  
M. Ademollo, R. D’ Auria, F. Gliozzi, E. Napolitano, S. Sciuto, and P. di Vecchia,  
Nucl. Phys. B94 (1975) 221;  
C. G. Callan, C. Lovelace, C. R. Nappi, and S. A. Yost, Nucl. Phys. B293 (1987) 83.  

[15] J. Polchinski and Y. Cai, Nucl. Phys. B296 (1988) 91;  
C. G. Callan, C. Lovelace, C. R. Nappi and S.A. Yost, Nucl. Phys. B308 (1988) 221.  

[16] S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888;  
J. Polchinski, Comm. Math. Phys. 104 (1986) 37.  

[17] J. A. Shapiro and C. B. Thorn, Phys. Rev. D36 (1987) 432;  
J. Dai and J. Polchinski, Phys. Lett. B220 (1989) 387.  

[18] M. Douglas and B. Grinstein, Phys. Lett. B183 (1987) 552; (E) 187 (1987) 442. S.  
Weinberg, Phys. Lett. B187 (1987) 278;  
N. Marcus and A. Sagnotti, Phys. Lett. B188 (1987) 58.  

[19] D. Friedan, E. Martinec, and S. Shenker, Nucl. Phys. B271 (1986) 93.  

[20] G. T. Horowitz and A. Strominger, Nucl. Phys. B360 (1991) 197.  

[21] C. M. Hull and P.K. Townsend, Nucl. Phys. B438 (1995) 109.  

[22] E. Witten, Nucl. Phys. B443 (1995) 85.  

[23] A. Strominger, Nucl. Phys. B451 (1995) 96.  

[24] J. Polchinski and A. Strominger, New Vacua for Type II String Theory, preprint  
UCSBTH-95-30, NSF-ITP-95-136, hep-th/9510227 (1995).  

[25] M. Bianchi, G. Pradisi, and A. Sagnotti, Nucl. Phys. B376 (1992) 365.  

[26] M. R. Douglas, Branes within Branes, preprint RU-95-92, hep-th/9512077.  

[27] M. Bershadsky, C. Vafa, and V. Sadov, D-Branes and Topological Field Theories,  
preprint HUTP-95-A047, hep-th/9511222;  
A. Strominger, Open p-Branes, preprint hep-th/9512059.  

[28] M. Li, Boundary States of D-Branes and Dy-Branes, preprint BROWN-HET-1020,  
hep-th/9510161.  

[29] C. Bachas, D-Brane Dynamics, preprint NSF-ITP-95-144, hep-th/9511043.  

[30] T. Banks and L. Susskind, Brane - Anti-Brane Forces, preprint RU-95-87, hep-  
th/9511194.  

[31] W. Fischler and L. Susskind, Phys. Lett. B171 (1986) 383; 173 (1986) 262.  

43
[32] M. B. Green and J. H. Schwarz, Phys. Lett. B149 (1984) 117; B151B (1985) 21.

[33] R. I. Nepomechie, Phys. Rev. D31 (1985) 1921; C. Teitelboim, Phys. Lett. B167 (1986) 63, 69.

[34] J. A. Harvey and A. Strominger, Nucl. Phys. B449 (1995) 535.

[35] C. Vafa and E. Witten, Nucl. Phys. B447 (1995) 261.

[36] S. H. Shenker, Another Length Scale in String Theory? preprint RU-95-53, [hep-th/9509132] and seminar at ITP, Jan. 1996.

[37] S. H. Shenker, in Cargese 1990, Proceedings: Random Surfaces and Quantum Gravity (1990) 191.

[38] L. Susskind and J. Uglum, Phys. Rev. D50 (1994) 2700.

[39] F. Larsen and F. Wilczek, Phys. Lett. B375 (1996) 37.

[40] A. Strominger and C. Vafa, Microscopic Origin of the Bekenstein-Hawking Entropy, preprint HUTP-96-A002, [hep-th/9601029].

[41] M. Bershadsky, C. Vafa, and V. Sadov, D-Strings on D-Manifolds, preprint HUTP-95-A035, [hep-th/9510225].

[42] J. H. Schwarz, Phys. Lett. B360 (1995) 13; (E) B364 (1995) 252.

[43] J. Polchinski and E. Witten Evidence for Heterotic - Type I Duality, preprint IASSNS-HEP-95-81, [hep-th/9510169].

[44] A. Dabholkar, Phys. Lett. B357 (1995) 307; C. M. Hull, Phys. Lett. B357 (1995) 545.

[45] L. J. Romans, Phys. Lett. B169 (1986), 374.

[46] P. Ginsparg, Phys. Rev. D35 (1987) 648.

[47] P. Horava and E. Witten, Heterotic and Type I String Dynamics from Eleven Dimensions, preprint IASSNS-HEP-95-86, [hep-th/9510209].

[48] E. Gimon and J. Polchinski, Consistency Conditions for Orientifolds and D-Manifolds, preprint NSF-ITP-96-01, [hep-th/9601038].

[49] E. Witten, private communication.

[50] E. Witten, Small Instantons in String Theory, preprint IASSNS-HEP-95-87, [hep-th/9511030].

[51] J. H. Schwarz, Nucl. Phys. B65 (1973), 131; E. F. Corrigan and D. B. Fairlie, Nucl. Phys. B91 (1975) 527;
M. B. Green, Nucl. Phys. B103 (1976) 333;
A. Cohen, G. Moore, P. Nelson, and J. Polchinski, Nucl. Phys. B267, 143 (1986); B281, 127 (1987).

[52] M. B. Green, Phys. Lett. B69 (1977) 89; B201 (1988) 42; B282 (1992) 380; B329 (1994) 435.

[53] W. Siegel, Nucl. Phys. B109 (1976) 244.

[54] J. A. Harvey and J. A. Minahan, Phys. Lett. 188B (1987) 44.

[55] N. Ishibashi and T. Onogi, Nucl. Phys. B318 (1989) 239;
Z. Bern, and D. C. Dunbar, Phys. Lett. B242 (1990) 175; Phys. Rev. Lett. 64 (1990) 827; Nucl. Phys. B319 (1989) 104; Phys. Lett. 203B (1988) 109;
M. Bianchi and A. Sagnotti, Phys. Lett. B247 (1990) 517; Nucl. Phys. B361 (1991) 519;
A. Sagnotti, Phys. Lett. B294 (1992) 196; Some Properties of Open-String Theories, preprint ROM2F-95/18, hep-th/9509080.

[56] A. Dabholkar and J. Park, An Orientifold of Type IIB Theory on K3, preprint CALT-68-2038, hep-th/9602030.

[57] C. Vafa and E. Witten, Dual String Pairs with N = 1 and N = 2 Supersymmetry in Four Dimensions, HUTP-95-A023, hep-th/9507050 (1995);
C. G. Callan, Jr. and I. R. Klebanov, D-Brane Boundary State Dynamics, preprint P UPT-1578, hep-th/9511173;
S.-T. Yau and E. Zaslow, BPS States, String Duality, and Nodal Curves on K3, preprint hep-th/9512121;
R. G. Leigh, Anomalies, D-Flatness, and Small Instantons, preprint RU-95-94, hep-th/9512191.

[58] J. Maharana, M Theory p-Branes, preprint IASSNS-HEP-95-98, hep-th/9511159;
P. K. Townsend, D-Branes from M-Branes, preprint DAMTP-R-95-59, hep-th/9512062;
A. Sen, T-Duality of p-Branes, preprint MRI-PHY-28-95, hep-th/9512203;
E. Witten, Five-Branes and M Theory on an Orbifold, preprint IASSNS-HEP-96-01, hep-th/9512219;
C. Schmidhuber, D-Brane Actions, preprint PUPT-1585, hep-th/9601003;
J. H. Schwarz, M Theory Extensions of T-Duality, preprint CALT-68-2034, hep-th/9601077.

[59] I. R. Klebanov and Larus Thorlacius, The Size of p-Branes, preprint PUPT-1574, hep-th/9510200;
S. S. Gubser, A. Hashimoto, I. R. Klebanov, and J.M. Maldacena, Gravitational Lensing by p-Branes, preprint PUPT-1586, hep-th/9601057.

45
J. L. F. Barbon, *D-Brane Form-Factors at High Energy*, preprint PUPT-1590, hep-th/9601098.

[60] K. Wilson, Rev. Mod. Phys. **55** (1983) 583.