Estimation of radiation source distribution using machine learning with $\gamma$ ray energy spectra

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Abstract. A method is proposed for estimating the original radiation source distribution by machine learning using the dose and energy spectrum of $\gamma$ rays emitted from radiation sources placed at various positions. This method does not require complicated parameter settings and can be also applied when there is a shield like Pb between the radiation source and the measurement point. The estimation results displayed the original radiation source distribution with high accuracy. It is expected to be used for decontamination and decommissioning by developing this method.

Keywords: Machine learning, Monte Carlo method, Fukushima Daiichi Nuclear Power Station

1. Introduction

Large amounts of radiation sources ($^{134}$Cs and $^{137}$Cs) were released into the environment after the Fukushima Daiichi Nuclear Power Station (FDNPS) accident in 2011. The released cesium had been confirmed to deposit from the wide area of east Japan [1]. Therefore, it was necessary to know the distribution of radiation sources in order to recover the environment such as decontamination. In order to estimate the position of the radiocesium surrounding environment, an analysis method using an inverse problem solution has recently been proposed [2]. This method realizes radiation monitoring over a wide area of ground by theoretically converting the dose data measured using a helicopter into the air dose rate on the ground. However, this method has the problem that the more complicated the environmental conditions, the more difficult it is to set the parameters for the calculation. In addition, the inverse problem analysis is not adapted when there is a shield between detection points and the source distribution.

In this paper, a new estimation method using machine learning with dose data and $\gamma$-ray
energy spectra from radiation sources was presented to overcome these difficulties. In there are various types of machine learning, a forward-propagation neural network (FNN) using supervised learning algorithms were adopted among them. This FNN requires a lot of training data set consisting of the data input to the neural network and the answer data to compare with the output values of the FNN. A FNN for radiation source distribution estimation requires the source distribution as answer data and the dose and energy spectrum of γ-rays emitted from the source as input data. It is difficult to prepare much data for the FNN by actual measurement. Therefore, the training of the FNN was proceeded with a training dataset created by Monte Carlo (MC) simulation. The radiation source set for simulation was limited to $^{137}$Cs to simplify calculations. The $^{137}$Cs distribution was set on the plane as the answer data, and the dose and energy spectrum emitted from the $^{137}$Cs were simulated for use as input data. After the training, the versatility of FNN was verified by inputting dose data and energy spectra that were not used for training.

2. Methods

2.1. Network architecture

The FNN is formed from input, hidden, and output layers as shown in Fig. 1. In this study, FNN with one hidden layer was used for estimation. Those three layers are connected by synapses with weights and have $p,q$, and $r$ nodes, respectively.

![A typical forward-propagation neural network](image)

Figure 1: A typical forward-propagation neural network

When data $x$ are input to the input layer, the output value $y$ of the $i$-th node in the output layer and the output value $h$ of the $i$-th node in the hidden layer are described the following equations:

$$y_i = f\left(\sum_{k=1}^{q} h_k w_y(k,i) + b_y\right),$$

(1)
\[ h_i = f\left(\sum_{k=1}^{p} x_k w_{h}(k, i) + b_h\right), \]  

\[(2)\]

where \( f \) is the sigmoid function as below,

\[ f\left(\sum x w + b\right) = \frac{1}{1 + \exp\left(-\left(\sum x w + b\right)\right)}, \]  

\[(3)\]

Here, \( w_y(k, i), w_h(k, i) \) are weights of synapse connecting \( k \)-th and \( i \)-th node of Output layer and Hidden layer, respectively. In addition, \( b_y, b_h \) are biases of Output layer and Hidden layer, respectively. The process of propagating values from the input layer to the output layer is called forward propagation.

The training process of the FNN involves using the output-answer data to determine the weights and biases. These parameters were obtained by the backpropagation algorithm [3] using mean square error as below,

\[ E = \frac{1}{r} \sum_{k=1}^{r} (y_k - a_k)^2. \]  

\[(4)\]

Here, \( a_k \) is the answer data corresponding to the \( k \)-th node in the output layer. This method iteratively adjusts the weights and biases to achieve the least mean square error between the output of the FNN and the answer data.

### 2.2. MC simulation for radiation analysis

The \( \gamma \) rays from \( ^{137}\text{Cs} \) move while changing its position, direction and energy by scattering until it is absorbed as shown in Fig. 2. The \( ^{137}\text{Cs} \) emits \( \gamma \) rays with 662 keV, and if the \( \gamma \) rays pass through the shield, some \( \gamma \) rays are shifted to the low energy region due to Compton scattering. The energy of \( \gamma \) rays after Compton scattering \( E_{af} \) depends on the scattering angle \( \theta \) and the energy before scattering \( E_{be} \) as below,

\[ E_{af} = \frac{E_{be}}{1 + \frac{E_{be}}{0.511} (1 - \cos \theta)}. \]  

\[(5)\]

It is possible to obtain information like the number of particles or the absorbed energy by the MC simulation. The MC simulation was used to create a lot of training data for the training. Figure 3 shows the overview of the MC simulation for radiation analysis. The MC simulation simulates stochastic phenomena using random numbers which ranges from 0 to 1. These random numbers were used to determine the flight length, interaction, and scattering angle of \( \gamma \) rays.
The probability of each interaction occurring was calculated using a cross section depend energy of the γ ray. The cross section was obtained from the database “XCOM” [4] provided by the National Institute of Standards and Technology (NIST). The flight length $l$ is described using a random number $\eta_l$ ($0 < \eta_l < 1$) and material density of the shield $\rho$ by the following equation:

$$l = -\frac{1}{\rho(\sigma_{ab}(E) + \sigma_{sc}(E))} \ln \eta_l.$$  \hspace{1cm} (6)

Here, $\sigma_{ab}$, $\sigma_{sc}$ are cross section of absorption and scattering, respectively.

The absorption probability $P_{ab}$ of γ ray described as follows,

$$P_{ab} = \frac{\sigma_{ab}}{\sigma_{ab} + \sigma_{sc}}.$$  \hspace{1cm} (7)

The γ rays are absorbed when the following condition is satisfied,

$$\eta_p \leq P_{ab}.$$  \hspace{1cm} (8)

If the condition (8) is not satisfied, the γ rays scatter and the scattering angle cross section $\sigma_{an}$ is described by Klein-Nishina formula as follows,
\[
\frac{d\sigma_{\alpha n}}{d\Omega} = r_e \left( \frac{1}{2} \left( 1 + \alpha (1 - \cos \theta) \right)^2 \left( 1 + \cos^2 \theta + \frac{E}{0.511 (1 - \cos \theta)^2} \right) \right).
\] (9)

The cross section of the scattering angle is calculated every 0.1 degree, and the angle is determined using random numbers.

This calculation was accelerated by parallelization, and the motion and energy of many \( \gamma \) rays were analyzed. The parallel numerical simulations were performed on the Supercomputer system "AFI-NITY" at the Advanced Fluid Information Research Center, Institute of Fluid Science, Tohoku University.

2.3. Radiation source distribution estimation without shield

Figure 4 shows the MC simulation with no shield. Radiation sources randomly distributed on a 10 cm square plane separated by a 1 cm \times 1 cm mesh was set up for simulation to create a lot of dose distribution used as the training data for machine learning. The source distribution is displayed in red where there is a source and in blue no source for each mesh. If there is a source, it was set to emit 100,000 \( \gamma \) rays in random directions from the center of the mesh. The dose distribution indicates the number of \( \gamma \) rays that have passed through a detection surface which is a 10 cm square plane separated by a 1 cm \times 1 cm mesh as same as the source distribution plane. The detection surface was set 1 cm above the radiation source distribution plane.

The dose distribution obtained by the MC simulation without shield, as shown in Fig. 4, were input to the three-layer FNN as shown in Fig. 5. The training data sets were prepared 1000 patterns. The dose distribution were converted to one dimension and then input to Input layer. The number of nodes in the input layer is 100 since one dose data is determined per mesh of the detection surface. The number of nodes in the hidden layer was set to 100 as same as the input layer. Due to the source distribution is also divided by 100 meshes, the number of nodes in the output layer is 100. Using the set FNN and results of MC simulation without shield, forward propagation and back propagation were repeated 150,000 times.
Figure 5: Topology for the neural network using dose distribution

2.4. Radiation source distribution estimation with shield

A $\gamma$ ray emitted from the source was simulated by inserting a Pb block between the source distribution and the detection surface set in the previous estimation in 2.3 as shown in Fig. 6. The Pb block (dimensions: 5 cm×10 cm×1 cm height) was randomly placed along the Y axis. The simulation creates a lot of training data by changing the combination of source distribution and the Pb block position. The direction of $\gamma$ ray with energy decreased by scattering in Pb block was calculated by the MC method. As in the previous estimation, dose distribution was created by $\gamma$ rays through the detection surface. In addition, the energy spectrum was recorded for each mesh on the detection surface since the energy distribution of the detected $\gamma$-ray changes because it passes through the Pb block. Using the recorded energy spectrum, we calculated the ratio of sums of direct $\gamma$ ray count to sums of scattered $\gamma$ ray count ($R_{ds}$) as follows

$$R_{ds} = \frac{\sum C_{sc}}{\sum C_{di}}. \quad (10)$$

Here, $C_{sc}$ is the count of scattered $\gamma$ rays and $C_{di}$ is the count of direct $\gamma$ rays.

The dose distribution and $R_{ds}$ distribution obtained by the MC simulation with the Pb block, as show in Fig. 6, were input to the three-layer FNN as shown in Fig. 7. The calculation algorithm, the number of patterns in the training data set, the number of nodes in the output layer, and the number of trainings are the same as the previous estimation. The number of nodes in the hidden layer was set to 200 as same as the input layer. The number of nodes in the input layer is 200 since $R_{ds}$ distribution is added as input data. The dose and $R_{de}$ distribution are continuously input to the input layer.
3. Estimated results

Figure 8 (a) shows the dependence of the number of training for the mean square error calculated by Eq. (4) for training of FNN used for estimation of source distribution without shield described in 2.3. The mean square error between the output of the FNN and the answer data decreased with an increasing number of training. After training 150,000 times in 27 hours, the error was approximately $1.5 \times 10^{-3}$. In addition, a dose distribution was prepared 200 patterns as a verification data. The dose distribution as the verification data were calculated by the source distribution different from ones used to create the training data of 1000 patterns. The mean square error between the output for the validation data and the original source distribution was also calculated each of the training. This error also decreased with an increase...
ing the number of training and was $9.7 \times 10^{-3}$ after training 150,000 times.

\[ \text{Number of training} \times 10^{-3} \]

Figure 8: Dependence of the number of training for the mean square error for (a) the calculation without shield and (b) the calculation with shield.

The result of estimating the source distribution without the shield is shown in Fig. 9. The distribution with concentrated sources in the center was set for creating test data that was not used for training data as shown in (a) and (b) of Fig. 9. The dose distribution as test data calculated in the situation was input to the FNN with determined parameters by training as shown in Fig. 9 (c). The output value as shown in Fig. 9 (d) from the FNN was compared with the original source distribution. As a result, it was found that the output value was estimated with high accuracy the original source distribution. The mean square error between the original source and the estimated distribution shown in Fig. 9 (a) and (d) was approximately $1.0 \times 10^{-3}$.

The estimation accuracy was verified for other source distributions as shown in Fig. 10. The result of this estimation shows that the FNN is sufficiently versatile. The mean square errors between the original source and the estimated distribution were approximately $6.0 \times 10^{-3}$, $1.2 \times 10^{-3}$ and $1.4 \times 10^{-3}$ for Fig. 10(a), (b), and (c) respectively.

Figure 9: Schematic view of the set situation without shield, the test data calculated from the situations, and estimated result.
Figure 8 (b) shows the dependence of the number of training for the mean square error for estimation of source distribution with shield described in 2.4. The mean square error between the output of the FNN and the answer data decreased with an increasing the number of training. After training 150,000 times in 32 hours, the error was approximately $1.4 \times 10^{-3}$. As verification data, a dose distribution and $R_{ds}$ distribution were prepared 200 patterns. These the dose distribution and $R_{ds}$ distribution as the verification data were calculated by the source distribution different from used to create the training data of 1000 patterns. The mean square error between the output for the validation data and the original source distribution was also calculated each of the training. This error also decreased with an increasing the number of training and was $2.1 \times 10^{-2}$ after training 150,000 times.

Figure 11 shows the results of estimating the source distribution with a shield. The Pb block was placed as a shield at 2.5 cm to 7.5 cm on the Y axis of the original source distribution as shown in Fig. 11(a). The dose and $R_{ds}$ distribution as test data were input to the FNN as shown in Fig. 11 (b) and (c), respectively. As a result, it was found that the output value was estimated with high accuracy as shown in Fig. 11 (d). The mean square error between the original source and the estimated distribution shown in Fig. 10 (a) and (d) was approximately $6.8 \times 10^{-3}$. The results are promising in that the location of the sources hidden under the Pb block were estimated.

Figure 12 shows that other test data can also provide a good estimate of the source dis-
The position on the Y axis of the placed Pb block were 0.25 cm to 5.25 cm, 0.45 cm to 5.45 cm, and 3.60 cm to 8.60 cm for Fig. 12(a), (b), and (c), respectively. The mean square error between the original source and the estimated distribution were approximately $2.5 \times 10^{-2}$, $6.2 \times 10^{-3}$, and $1.1 \times 10^{-2}$ for Fig. 12(a), (b), and (c), respectively.

Figure 11: Schematic view of the set situations with shield, test data calculated from the situations, and estimated results

Figure 12: Estimation results of source distribution with shield
4. Conclusion

In this paper, a new method using machine learning was proposed for estimating radiation source distribution. As the result, it was found that the radiation source distribution was accurately estimated using the dose distribution and the $R_{ds}$ distribution as the teacher data for machine learning without complicated calculations even if the shield such as Pb covers the sources. It would be used for decontamination and the decommissioning furnace.

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