Two-loop Finiteness of Chern-Simons Theory in Background Field Method

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Abstract

We perform two-loop calculation of Chern-Simons in background field method using the hybrid regularization of higher-covariant derivative and dimensional regularization. It is explicitly shown that Chern-Simons field theory is finite at the two-loop level. This finiteness plays an important role in the relation of Chern-Simons theory with two-dimensional conformal field theory and the description of link invariant.

1. Introduction

The finiteness of 2 + 1-dimensional Chern-Simons field theory (CS) has played a crucial role in its connection with 2-dimensional conformal invariant field theory and in describing link invariant [1–3]. The one-loop perturbative finiteness has been explicitly shown in various regularization schemes despite the fact that there exists the dispute on the finite renormalization of the coupling constant [1]. In fact, from the general analysis of the analytic property of the one-loop amplitude by Speer [2], the perturbative finiteness of CS at one-loop is somehow trivial since it is defined in odd dimensional space-time. Only when one considers the quantum corrections at two-loop or higher-order, the UV divergences may appear. Thus it is necessary to work in a concrete regularization scheme and show whether the UV divergences really do or do not cancel. Although the UV finiteness of CS has been verified to every order in a regularization independent way [3], up to now there have appeared very few papers which deal with its two-loop or higher order perturbative behaviour.

In this letter, we shall explicitly show the two-loop perturbative finiteness of CS in the framework of background field method and by choosing the hybrid regularization scheme of higher-covariant derivative and dimensional regularization. The advantage of adopting the background field method is obvious: owing to explicit background gauge invariance [4], by considering only the two-point function of background field, one can determine the local quantum effective action. We shall choose the simplest higher-covariant derivative term, Yang-Mills action, which means that we start from the dimensional regulated topologically massive Yang-Mills theory (TMYM). From the viewpoint of regularization, the theory has
two regulators, one is the dimensional regulator, \( \epsilon = 3 - n \), and the other is the topological mass \( m \). Therefore, the two-loop finiteness is equivalent to the existence of the limits \( \epsilon \to 0 \) and \( m \to \infty \).

In Sect. II we briefly review the definition of dimensional continuation, which should receive a delicate consideration due to the antisymmetric tensor since it has been shown the naive dimensional regularization cannot make the theory well defined \([8]\). Then we give the Feynman rules in the Landau background covariant gauge. Sect. III contains the explicit calculations and the analysis of two-loop two-point function of background gauge field of topologically massive Yang-Mills theory. We show that the \( 1/\epsilon \) terms cancel exactly. The explicit background gauge invariance means that the three-point and other multi-point functions should also contain no pole terms. Further we make use of the cancellation theorem of IR divergence for Landau gauge given in Ref. \([9]\) to argue the existence of large topological mass limit and hence the two-loop finiteness of CS is verified. Sect. IV presents the conclusion and some discussions.

2. Feynman Rules of TMYM in Background Covariant Gauge

We start from the dimensionally regulated TMYM (in Euclidean space),

\[
S_m[A] = -i \operatorname{sgn}(k) \int d^3 x \epsilon^a_{\mu
u\rho} \left[ \frac{1}{2} A^a_\mu \partial_\nu A^a_\rho + \frac{1}{3!} g f^{abc} A^a_\mu A^b_\nu A^c_\rho \right] + \frac{1}{4m} \int d^n x F^a_\mu F^{\mu a}, \tag{1}
\]

which is in fact the hybrid regularization of dimensional and higher-covariant derivative regularization of CS theory. The \( n \)-dimensional continuation of \( \epsilon_{\mu\nu\sigma} \) adopted here is the one proposed by 't Hooft and Veltman \([10]\) and further by Breitenlohner and Maison \([11]\). For the Chern-Simons type theories, it was given explicitly in \([12]\),

\[
\epsilon_{\mu_1\mu_2\mu_3} \epsilon_{\nu_1\nu_2\nu_3} = \sum_{\pi \in \mathbb{P}_3} \operatorname{sgn}(\pi) \Pi_{i=1}^3 \tilde{g}_{\mu_i \nu_{\pi(i)}},
\]

\[
u_\mu = \tilde{u}_\mu \cup \hat{u}_\mu, \quad g_{\mu\nu} = \tilde{g}_{\mu\nu} \cup \hat{g}_{\mu\nu}, \quad \tilde{g}_{\mu\nu} \hat{u}^\nu = \hat{g}_{\mu\nu} \tilde{u}^\nu = 0,
\]

\[
\hat{g}_{\mu} = n - 3, \quad \tilde{g}_{\mu} = 3, \quad \mu, \nu = 1, 2, \ldots n,
\]

where \( g_{\mu\nu} = \delta_{\mu\nu} \) is the Euclidean metric on \( R^n \) and \( \tilde{g}_{\mu\nu} \) and \( \hat{g}_{\mu\nu} \) are its projections onto the subspaces \( R^3 \) and \( R^{n-3} \). Note that this definition of dimensional continuation actually makes the theory possess \( SO(3) \otimes SO(n - 3) \) invariance rather than \( SO(n) \) \([10, 12]\). To our knowledge, this prescription is the unique algebraically consistent definition for dimensional continuation. In the calculations, the integer \( n \) is promoted to a complex number. It should be stressed that the sequence of limits is first taking \( n \to 3 \) and then performing \( m \to \infty \). If the \( 1/\epsilon \) pole terms arise due to the UV divergences, they should be first removed by choosing a subtraction scheme and by introducing counterterms. Then the limit \( m \to \infty \) can be performed.

In the background field method we split the gauge field into two pieces \([7]\),

\[
A_\mu = A_\mu + Q_\mu \tag{3}
\]

with \( A_\mu \) being the background field and \( Q_\mu \) being the quantum one. Choosing the background covariant gauge condition \( D_\mu [A] Q^\mu = 0 \) and introducing the auxiliary field \( B \) to implement Landau type gauge, we obtain the gauge-fixed effective action,
\[ S_{\text{eff}} = S_m[A + Q] + \int d^nx B^a D_\mu^{ab}[A] Q^{\mu b} + \int d^nx \overline{\epsilon}^a (D_\mu[A] D^\mu[A + Q] c)^a, \] (4)

where \[ D_\mu^{ab}[A] = \partial_\mu \delta^{ab} + g \mu f^{abc} A_\mu c \] and \( \epsilon = (3 - n)/2. \)

Feynman rules can be derived in the standard way, which are listed as follows:

- **Quantum gauge field propagator**

\[ \Delta^{ab}_{\mu\nu}(\hat{p}, \hat{p}) = \langle Q^a_\mu(p)Q^b_\nu(-p) \rangle_{g=0} = \frac{\delta^{ab} m}{(p^2 + m^2 \hat{p}^2)} \left[ -\text{sgn}(k) m \epsilon_{\mu\rho\nu} \hat{p}^\rho + p^2 g_{\mu\nu} - p_\mu p_\nu \right] + \frac{m^2}{p^2} \left( \hat{p}_\mu \hat{p}_\nu - \hat{p}^2 \delta_{\mu\nu} - \hat{p}_\mu \hat{p}_\nu + \hat{p}_\mu \hat{p}_\nu \right) \] (5)

- **Ghost field propagator**

\[ G^{ab}(p) = \langle c^a(p)\overline{c}^b(-p) \rangle_{g=0} = \frac{1}{p^2} \delta^{ab}; \] (6)

- **QQQ and AQQ vertex**

\[ -ig f^{abc} \left\{ \text{sgn}(k) \epsilon_{\mu\rho\nu} + \frac{\mu}{m} \left[ (p - q)_\rho \delta_{\mu\nu} + (q - r)_\mu \delta_{\nu\rho} + (r - p)_\nu \delta_{\mu\rho} \right] \right\} \times (2\pi)^n \delta^{(n)}(p + q + r); \] (7)

- **QQQQ and AQQQ vertex**

\[ -\frac{g^2 \mu^{2e}}{M} \left[ f^{abe} f^{cde} (\delta_{\nu\rho} \delta_{\mu\lambda} - \delta_{\mu\rho} \delta_{\nu\lambda}) + f^{ace} f^{bde} (\delta_{\mu\nu} \delta_{\lambda\rho} - \delta_{\mu\rho} \delta_{\nu\lambda}) \right] + f^{dae} f^{bce} (\delta_{\nu\rho} \delta_{\mu\lambda} - \delta_{\mu\nu} \delta_{\rho\lambda}) \times (2\pi)^n \delta^{(n)}(p + q + r + k); \] (8)
• Acĉ vertex

\[
\begin{array}{c}
\pi_\mu, a \\
\mu \\
\nu, b
\end{array}
\]

\[- i g \mu^c f^{abc}(p - q)\mu(2\pi)^n\delta^{(n)}(p + q + r); \quad (9)\]

• Qcĉ vertex

\[
\begin{array}{c}
\pi_\mu, a \\
\mu \\
\nu, b
\end{array}
\]

\[- i g \mu^c f^{abc}p_\mu(2\pi)^n\delta^{(n)}(p + q + r); \quad (10)\]

• AAcĉ vertex

\[
\begin{array}{c}
\pi_\mu, a \\
\mu \\
\nu, b
\end{array}
\]

\[
g^2 \mu^2\delta_{\mu\nu}(f^{eac}f^{ebd} + f^{ebe}f^{ead})(2\pi)^n\delta^{(n)}(p + q + r + k); \quad (11)\]

• AQcĉ vertex

\[
\begin{array}{c}
\pi_\mu, a \\
\mu \\
\nu, b
\end{array}
\]

\[
g^2 \mu^2\delta_{\mu\nu}f^{eac}(2\pi)^n\delta^{(n)}(p + q + r + k). \quad (12)\]

Eq.(5) shows that the quantum gauge field propagator at the regularization level takes a very complicated form due to the dimensional continuation (4). However, it can be decomposed into two parts

\[
\Delta^{ab}_{\mu\nu} (\tilde{p}, \hat{p}) = D^{ab}_{\mu\nu} (p) + R^{ab}_{\mu\nu} (\tilde{p}, \hat{p}), \quad (13)
\]

where
\[ D^{ab\mu\nu}(p) = \frac{\delta^{ab}m}{p^2(p^2 + m^2)} \left[ -\text{sgn}(k) m\epsilon_{\mu\nu\rho\sigma} \hat{p}^\rho + p^2 g_{\mu\nu} - p_\mu p_\nu \right], \]

\[ R^{ab\mu\nu}(\hat{p}, \hat{p}) = \frac{\delta^{ab}m}{p^2[(p^2)^2 + m^2]} \left[ \frac{\hat{p}^2}{p^2 + m^2} (-\text{sgn}(k) m\epsilon_{\mu\nu\rho\sigma} \hat{p}^\rho + p^2 g_{\mu\nu} + \frac{m^2}{p^2} p_\mu p_\nu) \right. \]

\[ + \left. \hat{p}^2 \delta_{\mu\nu} + \hat{p}_\mu \hat{p}_\nu - p_\mu \hat{p}_\nu - \hat{p}_\mu p_\nu \right]. \]

A detailed analysis in Ref. [12] shows that \( D^{ab\mu\nu}(p) \) can be replaced \( \Delta^{ab\mu\nu}(\hat{p}, \hat{p}) \) as an effective propagator, since \( R^{ab\mu\nu}(\hat{p}, \hat{p}) \) is an evanescent quantity, which has good UV behavior and thus its contributions to the loop amplitudes vanish in the limit \( n \to 3. \)

### 3. Two-loop Perturbative Analysis and Explicit Calculation

The calculation of two-loop Feynman diagrams containing the propagators and vertices of gauge field is a very heavy work. However, since our aim is to verify the perturbative finiteness, so our attention is only paid on the UV divergent terms. Our strategy is first to show that the cancellation of the \( 1/\epsilon \) pole terms in TMYM theory indeed occurs and then to prove the existence of the large topological mass, and thus the two-loop finiteness of CS can be established.

#### A. Cancellation of \( 1/\epsilon \) Pole Terms in TMYM

There are several techniques to find out the pole terms:

First, at one-loop level TMYM is finite, despite that each Feynman diagram seems to be divergent from the superficial divergent degree, the one-loop UV divergences are actually cancelled. Thus there are no need for us to consider the subtraction of subdivergence. This differs from the case of four-dimensional field theory, where if the subdivergence cannot be well handled, the pole terms with non-polynomial coefficients will be produced and the unitarity of the theory will be violated. From Weinberg’s theorem [13], we can see that two-loop divergent behaviour of TMYM is similar to the one-loop behaviour of four-dimensional gauge theory. The only problem is whether one can compute the two-loop integral analytically.

Second, from the Feynman rules listed in Sect.2, one can see that the UV degree of the term involving the antisymmetric tensor in quantum effective propagator \( \epsilon^{\mu\nu\rho\sigma} \) is one unit less than that of the whole \( D^{ab\mu\nu}(p) \) (i.e. -1 and -2, respectively), so if \( \omega \) is the overall UV degree of a diagram at \( n = 3 \), any integral with \( N \) antisymmetric objects in the diagram amplitude will have an overall UV degree \( \omega - N \). It is easy to see that the actual superficial UV degree \( \omega \) is zero for every two-loop diagrams considered in the following. This fact implies that the integrals exhibiting singularities at \( n = 3 \) only come from pure Yang-Mills part, i.e. those obtained by formally setting every \( \epsilon^{\mu\nu\rho\sigma} \) to zero in the amplitude. Furthermore, the integral from Yang-Mills part with at least one external momentum in the numerator has a negative UV degree. Therefore, the only sources of UV singularities at \( n = 3 \) are two-loop integrals which contain neither \( \epsilon^{\mu\nu\rho\sigma} \) nor external momentum in their numerators. The one-loop finiteness means that there exists no subdivergence at \( n = 3 \). Hence the UV singularity is a simple pole at \( n = 3 \) independent of \( m \) and the external momentum \( p \). In addition, the dimensional analysis shows that the mass dimension of background field two-point function in momentum space is one and so the pole terms must take the following general form [12],

\[ A \frac{m}{3-n} g_{\mu\nu}, \]

\[ (15) \]
where \( A \) is a constant to be determined by explicit calculation.

Now let us come to the explicit calculation. All the two-loop diagrams contributing to background two-point function are listed in Fig. 2. We shall collect the divergent integrals from all of the two-loop diagrams and see whether they are cancelled.

We consider two diagrams as examples to illustrate the explicit calculation. The amplitude of Fig. 2a is read as follows:

\[
A_a = g^4 \mu^2 C_V^2 \delta^{ab} \left\{ \int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \frac{4m[k^2q^2 - (k.q)^2]}{q^4(q + k)^2(p + q)^2k^2(k^2 + m^2)} + \int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \frac{4m(k^2(p + q).p + q)[p\mu + 2q\mu]2\nu + 2q\nu)}{q^2(p + q + k)^2(p + q)^4k^2(k^2 + m^2)} \right\}.
\]

(16)

From the above analysis, we know that the divergent terms are independent of external momentum and thus to concentrate on the pole terms, we take external momentum \( p_\mu = 0 \). Rescaling the integration variables \( k \rightarrow km, q \rightarrow qm \), we obtain

\[
A_a^{(\text{div})} = g^4 \mu^2 C_V^2 \delta^{ab} 32m^{2n-5} \int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \frac{4m[k^2q^2 - (k.q)^2]}{q^6(q + k)^2k^2(k^2 + m^2)} q_\mu q_\nu \left\{ \frac{q_\mu q_\nu (k^2q^2 - (k.q)^2)}{q^6(q + k)^2k^2} - \frac{q_\mu q_\nu (k^2q^2 - (k.q)^2)}{q^6(q + k)^2(k^2 + 1)} \right\} \\
= g^4 \mu^2 C_V^2 \delta^{ab} 32m^{2n-5} \int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \left[ -\frac{(k.q)^2}{q^6(q + k)^2k^2} + \frac{1}{q^6(q + k)^2(k^2 + 1)} \right] q_\mu q_\nu.
\]

(17)

Using the formula collected in Appendix and the Feynman parameter integral,

\[
\frac{1}{A^m B^n} = \frac{\Gamma(m + n)}{\Gamma(m) \Gamma(n)} \int_0^1 dx \frac{x^m-1(1-x)^n-1}{(Ax + B(1-x))^{m+n}},
\]

\[
\lim_{\epsilon \rightarrow 0} \Gamma[-\epsilon] = \frac{1}{\epsilon} - \gamma,
\]

(18)

where \( \gamma \) is the Euler constant, it is easy to calculate and see that the first term in (17) vanishes, while the second term is

\[
- g^4 C_V^2 \delta^{ab} m \frac{1}{6\pi^2} \left( \frac{1}{\epsilon} + \gamma \right) \delta_{\mu\nu}
\]

(19)

and the third term is

\[
g^4 C_V^2 \delta^{ab} m \frac{1}{6\pi^2} \left( -\frac{1}{2} + \frac{1}{\epsilon} + \gamma \right) \delta_{\mu\nu}.
\]

(20)

Thus we have

\[
A_a^{(\text{div})} = 0.
\]

(21)
The divergent part is

\[ A_f = g^4 C_V \delta^{ab} \left\{ \int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \left[ \frac{m(p + 2k)_\nu(p + q + k)_\rho(k^2 \delta_{\mu \nu} - k_\mu k_\rho)}{q^2(p + q)^2(p + q + k)^2(k^2 + m^2)} \right] \right. \]

\[ + \frac{1}{2} \int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \left[ \frac{m q_\mu k^2 \delta_{\mu \nu} - k_\mu k_\nu}(p + 2k)_\nu}{(q + k)^2 k^2(k^2 + m^2)q^2(p + q)^2} \right] \]

\[ + \frac{1}{2} \int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \left[ \frac{m(2q - p)_\mu(2q + k - 2p)_\nu(k^2 \delta_{\nu \rho} - k_\nu k_\rho)}{(q - p)^2(q - p + k)^2k^2(k^2 + m^2)} \right] \]

\[ - \int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \left[ \frac{m(2q - p)_\mu(k^2 \delta_{\nu \rho} - k_\nu k_\rho)q_\rho}{(q - p)^2(q + k)^2(k^2 + m^2)q^2} \right] \]. \tag{22} \]

Its divergent part is

\[ A_f^{(\text{div})} = -5g^4 C_V \delta^{ab} m^{2n-5} \int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \frac{2k^2 q_\mu q_\nu - k_\mu q_\mu + k_\mu q_\nu}{q^4(q + k)^2(k^2 + 1)} \]

\[ = -5g^4 C_V \delta^{ab} m^{2n-5} \int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \left[ \frac{2q_\mu q_\nu}{q^4(q + k)^2(k^2 + 1)} + \frac{1}{2} \frac{k_\mu q_\nu}{q^2(q + k)^2(k^2 + 1)} \right] \]

\[ - \frac{1}{2} \frac{q_\mu k_\nu}{q^4(q + k)^2(k^2 + 1)} + \frac{1}{2} \frac{k_\mu q_\nu}{q^2(q + k)^2(k^2 + 1)} - \frac{1}{2} \frac{k_\mu q_\nu}{q^4(q + k)^2(k^2 + 1)} \]

\[ = -mg^4 C_V \delta^{ab} 5 \frac{1}{6(4\pi)^2} \left( \frac{1}{\epsilon} + \gamma \right). \tag{23} \]

The contributions from other diagrams are listed in Table [4]. We can see from Table [4] that the divergent pole terms of the two-loop background field two-point function cancel exactly. Further, recalling the relation between background field wave function renormalization constant and vertex renormalization constant [5],

\[ Z_g = Z_A^{-1/2}, \tag{24} \]

we can conclude that two-loop background field three-point function also has no 1/\epsilon pole terms and thus the limit \( \epsilon \to 0 \) exists.

**B. Existence of Large Topological Mass Limit**

In last subsection we have shown the cancellation of the 1/\epsilon pole terms by the explicit calculation in TMYM. However, to prove the two-loop finiteness of CS theory, we need to show the existence of the large topological mass limit, i.e. \( m \to \infty \). At first sight, it seems that we should calculate explicitly the complicated finite terms of TMYM with respect to the regulator \( \epsilon \) and then study the large \( m \)-limit. However, based on the IR finiteness of TMYM in Landau gauge, one can see that without needing any explicit calculation a simple analysis can show that the large \( m \)-limit indeed exists.

First, we use the fact that in Landau gauge the topological mass \( m \) is the only massive parameter at the two-loop level. Since the 1/\epsilon pole terms cancel, the artificial massive parameter \( \mu \) has disappeared after taking the \( \epsilon \to 0 \) limit. So the form factors of the two-loop background field two-point function are only the function of \( p^2/m^2 \), that is, the two-loop background field two-point function must take the following form,
\[\langle A_{\mu}(p)A_{\nu}(-p)\rangle^{(2)} = \epsilon_{\mu\nu\rho\sigma} p_{\rho} F_1 \left( \frac{p^2}{m^2} \right) + (p^2 \delta_{\mu\nu} - p_{\mu} p_{\nu}) F_2 \left( \frac{p^2}{m^2} \right), \]  

where we consider the explicit gauge symmetry of background field two-point function.

Second, we make use of a key property of TMYM in Landau gauge: TMYM is infrared finite to every order of perturbative theory. This means that the limit \( p^2 \rightarrow 0 \) exists, since the form factors \( F_1 \) and \( F_2 \) of background field two-point function can only be the functions of \( p^2/m^2 \), the limit \( p^2 \rightarrow 0 \) is equivalent to large-\( m \) limit. Of course, one may think that there could exist the terms with the following form factors:

\[
\frac{m^l}{(1 + p^2/m^2)^n}, \quad l > 0,
\]

for which the limit \( p^2 \rightarrow 0 \) is not equivalent to large-\( m \) limit. However, since the mass dimension of \( \langle A_{\mu}(p)A_{\nu}(-p)\rangle \) is one, we can see from Eq.(25) that this is not possible for \( l > 0 \), and hence the terms with this kind of form factors do not exist. Therefore, the large topological mass limit indeed exists.

4. Conclusion

Background field method is an elegant calculation technique in gauge field theories. The explicit gauge symmetry of background field is preserved if the background covariant gauge is chosen. The calculations are then reduced considerably. In this paper we have applied this method to the investigation of two-loop perturbative finiteness of CS theory in the concrete regularization scheme, namely the hybrid regularization of higher-covariant derivative and dimensional regularization. By explicit calculations we show that the pole terms with respect to the regulator \( \epsilon \) cancel exactly. Further, based on the gauge symmetry, the dimensional analysis and the IR finite property of TMYM, we show that the limit of large topological mass also exists. Thus the two-loop finiteness of CS theory has been explicitly proven. This has not only verified the reasonableness of non-perturbative analysis, but has also provided a strong support to the description of link polynomial in terms of CS theory and the Witten conjecture about the relation between CS and two-dimensional conformal field theories.

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APPENDIX A:

Some formulas used in the two-loop integrations are collected in this appendix:

\[
\int \frac{d^n q}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \frac{q_{\mu} k_{\nu}}{(q^2)^{\alpha} (k + q)^{\beta} (k^2 + 1)^{\gamma}} = -\frac{1}{(4\pi)^n} \frac{\delta_{\mu\nu} \Gamma(\alpha + \beta) \Gamma(n/2 - \alpha + 1) \Gamma(n/2 - \beta) \Gamma(\alpha + \beta + \gamma - 1 - n)}{\Gamma(\alpha) \Gamma(\beta) \Gamma(\gamma) \Gamma(n/2)}. \quad (A1)
\]
\[
\int \frac{d^n q \ d^n k}{(2\pi)^n (2\pi)^n (q^2 + 1)(k^2 + 1)^\beta [q + k]^2} k_\mu q_\nu = -\frac{1}{(4\pi)^n} \frac{\delta_{\mu\nu} \Gamma(\beta + 2\gamma - n/2)\Gamma(\beta + 2\gamma - n/2 - \beta)\Gamma(\alpha + \beta + \gamma - 1 - n)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)\Gamma(n/2)} \\
\times \Gamma(\alpha + \beta - n/2 - 1).
\]  
(A2)

\[
\int \frac{d^n q \ d^n k}{(2\pi)^n (2\pi)^n (q^2 + 1)(k^2 + 1)^\beta [q + k]^2} k_\mu q_\nu = -\frac{1}{(4\pi)^n} \frac{\delta_{\mu\nu} \Gamma(\beta + 2\gamma - n/2)\Gamma(\beta + 2\gamma - n/2 - \beta)\Gamma(\alpha + \beta + \gamma - 1 - n)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)\Gamma(n/2)} \\
\times \Gamma(\alpha + \beta + 2\gamma - n/2 - 1).
\]  
(A3)

\[
\int \frac{d^n q \ d^n k}{(2\pi)^n (2\pi)^n (q^2 + 1)(k^2 + 1)^\beta [q + k]^2} q_\mu q_\nu = \frac{1}{(4\pi)^n} \frac{\delta_{\mu\nu} \Gamma(n - \alpha - \beta + 1)\Gamma(\alpha + \beta + \gamma - n/2 - 1)}{\Gamma(n/2 + 1)\Gamma(n/2 - \alpha - 1)\Gamma(n/2 - \beta)\Gamma(n/2 - \gamma)\Gamma(n - \alpha - \beta + 2) \\
\times \Gamma(n - \alpha - \beta + 2)} \\
\times \left[ \frac{\Gamma(n/2 + 1)\Gamma(n/2 - \alpha - 1)\Gamma(n/2 - \beta + 1)\Gamma(n/2 - \gamma)}{\Gamma(n/2 - \beta)} \right].
\]  
(A4)

\[
\lim_{n \to 3} \int \frac{d^n q \ d^n k}{(2\pi)^n (2\pi)^n (q^2 + 1)(k^2 + 1)^\beta [q + k]^2} k_\mu q_\nu = \lim_{n \to 3} \int \frac{d^n q \ d^n k}{(2\pi)^n (2\pi)^n (q^2 + 1)(k^2 + 1)^\beta [q + k]^2} q_\mu k_\nu \\
= -\frac{1}{3} \left[ \frac{1}{16\pi^2} - \frac{1}{32\pi^2} \left( -\frac{1}{\epsilon} + 1 - \gamma + \ln\left(\frac{4\pi}{9}\right) \right) \right].
\]  
(A7)

\[
\lim_{n \to 3} \int \frac{d^n q \ d^n k}{(2\pi)^n (2\pi)^n (q^2 + 1)(k^2 + 1)^\beta [q + k]^2} q_\mu q_\nu = \frac{1}{3} \left[ -\frac{1}{16\pi^2} + \frac{1}{32\pi^2} \left( -\frac{1}{\epsilon} + 1 - \gamma + \ln\left(\frac{4\pi}{9}\right) \right) \right].
\]  
(A8)
REFERENCES

[1] E. Witten, Comm. Math. Phys. 121 (1989) 351.
[2] J.M.F. Labastida and A.V. Ramallo, Phys. Lett. 227 (1989) 92; 228 (1989) 214; S. Elitzur, G. Moore, A. Schwimmer and N. Seiberg, Nucl. Phys. B326 (1989) 108; E. Guadagnini, M. Martellini and M. Mintchev, Nucl. Phys. B336 (1990) 581; M. Bos and V.P. Nair, Int. J. Mod. Phys. A5 (1990) 959.
[3] E. Guadagnini, M. Martellini and M. Mintchev, Nucl. Phys. B330 (1990) 575; L. Rozansky and H. Saleur, Nucl. Phys. B376 (1992) 461; J.F.W.H. van de Weterring, Nucl. Phys. B379 (1992) 172; R.K. Kaul and T.R. Govindarajan, Nucl. Phys. B380 (1992) 293; M. Alvarez and J.M.F. Labastida, Nucl. Phys. B395 (1993) 198.
[4] E. Guadagnini, M. Martellini and M. Mintchev, Phys. Lett. B227 (1989) 111; L. Alvarez-Gaumé, J.M.F. Labastida and A.V. Ramallo, Nucl.Phys. B334 (1990) 103; W. Chen, G.W. Semenoff and Y.S. Wu, Mod. Phy. Lett. A5 (1990) 1833; Phys. Rev. D46 (1992) 5521; C.P. Martin, Phys. Lett. B241 (1990) 513; M. Asorey and F. Falceto, Phys. Lett. B241 (1990) 31; W.F. Chen and Z.Y. Zhu, J. Phys. A27 (1994) 1781; H.C. Kao, K. Lee and T. Lee, Phys. Lett. B373 (1996) 94; W.F. Chen, H.C. Lee and Z.Y. Zhu, Phys. Rev. D55 (1997) 3664.
[5] E.R. Speer, J. Math. Phy. 15 (1974) 1.
[6] A. Blasi and R. Collina, Nucl. Phys. B345 (1990) 472; F. Delduc, C. Lucchesi, O. Piguet and S. P. Sorrela, Nucl. Phys. B346 (1990) 313; D. Daniel and N. Dorey, Phys. Lett. B246 (1990) 82; N. Dorey, Phys. Lett. B 246 (1990) 87.
[7] L.F. Abbott. Nucl. Phys. B185 (1981) 189.
[8] M. Bos, Ann. Phys. (N.Y.) 181 (1988) 177; M. Chaichian and W.F. Chen, Inconsistency of Naive Dimensional Regularization and Quantum Correction of Non-Abelian Chern-Simons-Matter Theory (in preparation).
[9] R.D. Pisarski and S. Rao, Phys. Rev. D32 (1985) 2081.
[10] G. ’t. Hooft and M. Veltman, Nucl. Phys. 44 (1972) 189.
[11] Breitenlohner and Maison, Comm. Math. Phys. 52 (1977) 11.
[12] G. Giavarini, C.P. Martin and F. Ruiz Ruiz. Nucl. Phys. B381 (1992), 222.
[13] C. Itzykson and J.B. Zuber, Quantum Field Theory (McGraw-Hill, 1987).
TABLES

| a  | b    | c    | d    | e    | f    | g    | h    | i    | j    | k    |
|----|------|------|------|------|------|------|------|------|------|------|
| 0  | $\frac{191}{4\varepsilon}$ | $\frac{11}{3\varepsilon}$ | $\frac{71}{2\varepsilon}$ | 0    | $\frac{51}{6\varepsilon}$ | 0    | $\frac{81}{3\varepsilon}$ | $\frac{31}{4\varepsilon}$ | $\frac{11}{3\varepsilon}$ | $\frac{91}{2\varepsilon}$ |
| total | $a + b + c + d + e + f + g + h + i + j + k = 0$ | $1 + 1 + 1 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0$ |

TABLE I. Pole terms from two-loop diagrams (in the units of $1/(4\pi)^2 g^4 C_F^2$)
FIG. 1. Two-loop Feynman diagrams