Vibration analysis of five layer composite plate at different boundary conditions

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Abstract

Generally plates are subjected to the load conditions which may change by fixing different edges of the plate, that cause deflections transverse to the plate. In this paper a Vibration Analysis of a five layer composite plate is presented at different boundary condition. Vibration is the most influencing parameter of life & performance of particular machine element or engineering structures, and invariably, damping is used to reduce that. Various types of damping mechanisms have been developed over time to control the undesired vibration of structures. A composite plate having five layers is modeled in ANSYS parametric Design Language (APDL) platform & Finite element procedure is followed. Then, a set of results are presented to show the applicability of the present problem to various types of boundary conditions under free vibration conditions. Subsequently the results are compared with isotropic plate.

Keywords: Composite Plate; Vibration Analysis; Finite Element Methods; Isotropic Plate; Boundary Conditions Etc.

1. Introduction

Plates are straight, flat and non-curved surface structures whose thickness is slight compared to their other dimensions. Generally plates are subjected to load conditions that cause deflections transverse to the plate. Geometrically they are bound either by straight or curved lines. Plates have free, simply supported or fixed boundary conditions. The static or dynamic loads carried by plates are predominantly perpendicular to the plate surface. The load carrying action of plates resembles that of beams or cables to a certain extent. Hence plates can be approximated by a grid work of beams or by a network of cables, depending on the flexural rigidity of the structures [1]. Plates are of wide use in engineering industry like ships, containers, etc. In Aeronautics require complete enclosure of plates without use of additional covering, for which composite plates have been, used which consequently saves the material and labor.

The analysis of plates first started in the 1800s. Euler [2] was responsible for solving free vibrations of a flat plate using a mathematical approach for the first time. Then it was the German physicist Chladni [3] who discovered the various modes of free vibrations. Then later on the theory of elasticity was formulated. Navier [4] can be considered as the originator of the modern theory of elasticity. Navier’s numerous scientific activities included the solution of various plate problems. He was also responsible for deriving the exact differential equation for rectangular plates with flexural resistance. For the solution to certain boundary value problems Navier introduced exact methods which transformed differential equations to algebraic equations. Poisson in 1829 [5] extended the use of governing plate equation to lateral vibration of circular plates. Later, the theory of elasticity was extended as there were many researchers working on the plate and the extended plate theory was formulated. Kirchhoff [6] is considered as the one who formulated the extended plate theory. In the late 1900s, the theory of finite elements was evolved which is the basis for all the analysis on complex structures. However the analyses using finite elements are now being carried out using comprehensive
software which requires high CPU resources to compute the results. Another method for analysis of plates statically and
dynamically was later developed for arbitrary shapes using advanced finite elements. Actually there was a method
called the weighted residual method which was used in analysis of plate even before the finite element method of
analysing the plate was formulated.
Composite material refers to material that is created by the synthetic assembly of two or more organic or inorganic
materials in order to obtain specific material properties such as high strength and high stiffness to weight ratio,
corrosion resistance, thermal properties, fatigue life and wear resistance and increased tolerance to damage [7].

2. Objective of present work

The main objective of present work is to find the frequencies of vibration of five layer composite plate under different
boundary conditions. Following are the parameters which have been varied to conduct a comprehensive parametric
study on the effects of:
1) Boundary Condition
2) Width-length ratio
3) Thickness ratio

3. Five layer composite plate

In this study, a five layer composite plate is modeled & finite element analysis is conducted using ANSYS parametric
Design Language (APDL) platform. An 8 node shell element, (specified as SHELL 281 in ANSYS) is used throughout
the study. The element has eight nodes with six degrees of freedom at each node: translations in the x, y, and z axes, and
rotations about the x, y, and z-axes (when using the membrane option, the element has translational degrees of freedom
only). Thus each element has 48 degree of freedom in total. SHELL281 is well-suited for linear, large rotation, and
large strain nonlinear applications [8].

Fig. 1: Layup of 5 Ply Composite Plates

The material properties used are: E1 = 280 GPa, E2 = 7 GPa, G12 = G13 =4.2 GPa, G23 = 3.5 GPa and v12 = v13 =
0.25. v23 = 0.1. Fiber orientation is -45/45/-45/45/-45/.

4. Results

4.1. Five layer composite plate

In Table 1 natural frequencies of the five layer composite plate at different boundary conditions are shown. It shows
very good convergence in the values of obtained results by the present study and the open literature. Ratio of Width (A)
& Thickness (h) of the plate is taken as another parameter.
For the study of effect of boundary condition on the natural frequency of plate a 5 ply composite rectangular plate of
h/A = 0.01 and B/A = 1.25 is modeled in ANSYS. The material properties are: E1 = 280 GPa, E2 = 7 GPa, G12 = G13 =
4.2 GPa, G23 = 3.5 GPa and v12 = v13 = 0.25, v23 = 0.1. Fiber orientation is -45/45/-45/45/-45./
Table 1: Natural Frequencies of 5 Ply Rectangular Plate

| Boundary condition | Thickness Ratio A/h | Mode | Present Study | Kalita [10] |
|--------------------|----------------------|------|---------------|-------------|
|                    |                      | 1    |               |             |
| 5                  |                      | 12  | 938.2986     | 1126.767    |
| 10                 |                      | 2   | 916.5363     | 1120.226    |
| 5                  |                      | 3   | 938.4175     | 1481.05     |
| 10                 |                      | 1   | 917.448      | 1460.04     |
| 5                  |                      | 2   | 938.4473     | 1486.005    |
| 10                 |                      | 3   | 917.9138     | 1461.031    |
| 5                  |                      | 1   | 940.0329     | 1486.698    |
| 10                 |                      | 2   | 923.5823     | 1463.212    |

Table 2: Natural Frequencies of 5 Ply Composite Plates with Different Boundary Conditions.

| Mode Number | SSSS | SSSC | SSCC | SCCC | CCCC | CFFF |
|-------------|------|------|------|------|------|------|
| 1           | 140.87 | 163.77 | 174.67 | 201.15 | 212.15 | 12.365 |
| 2           | 271.98 | 288.48 | 311.13 | 327.72 | 352.54 | 56.43 |
| 3           | 349.47 | 389.61 | 396.47 | 439.04 | 444.83 | 69.681 |
| 4           | 443.77 | 458.41 | 486.8 | 502.31 | 532.77 | 145.44 |
| 5           | 522.84 | 546.44 | 565.03 | 588.44 | 606.46 | 153.19 |
| 6           | 553.41 | 662.98 | 687.42 | 707.61 | 737.29 | 173.21 |
| 7           | 634    | 684.37 | 691.02 | 738.21 | 740.81 | 243.54 |

Fig. 3: Natural Frequencies at Different Boundary Conditions for 5 Ply Composite Plates.

For the study of effect of aspect ratio on the natural frequency of plate a 5 ply composite rectangular plate of h/A = 0.01 is modelled in ANSYS with boundary condition SSSS. The material properties are: $E_1 = 280$ GPa, $E_2 = 7$ GPa, $G_{12} = G_{13} = 4.2$ GPa, $G_{23} = 3.5$ GPa and $\nu_{12} = \nu_{13} = 0.25$, $\nu_{23} = 0.1$. Fiber orientation is $-45/45/-45/45/-45$.

Table 3: Natural Frequencies of 5 Ply Composite Plates with Different Aspect Ratio.

| Mode Number | Aspect ratio (B/A) |
|-------------|--------------------|
|             | 0.5 | 1.0 | 1.25 | 1.5 | 2.0 | 2.5 |
| 1           | 377.01 | 172.11 | 140.87 | 122.05 | 101.24 | 90.617 |
| 2           | 603.4 | 341.94 | 271.98 | 223.73 | 167.94 | 138.03 |
| 3           | 878.32 | 407.37 | 349.47 | 322.68 | 254.48 | 199.95 |
| 4           | 1004.7 | 517.24 | 443.77 | 356.99 | 296.83 | 272.08 |
| 5           | 1034.6 | 558.99 | 522.84 | 450.86 | 360.86 | 288.37 |
| 6           | 1194 | 649.44 | 553.41 | 520.87 | 375.78 | 335.85 |
| 7           | 1223 | 703.88 | 634 | 607.97 | 480.39 | 364.78 |
| 8           | 1484.1 | 806.06 | 648.26 | 610.41 | 484.92 | 410.9 |
For the study of effect of aspect ratio on the natural frequency of plate a 5 ply composite rectangular plate of $B/A = 1.25$ is modelled in ANSYS with boundary condition SSSS. The material properties are: $E_1 = 280 \text{ GPa}$, $E_2 = 7 \text{ GPa}$, $G_{12} = G_{13} = 4.2 \text{ GPa}$, $G_{23} = 3.5 \text{ GPa}$ and $\nu_{12} = \nu_{13} = 0.25$, $\nu_{23} = 0.1$. Fiber orientation is $-45/45/-45/45/-45$.

## Table 4: Natural Frequencies of 5 Ply Composite Plates with Different Thickness Ratio.

| Mode Number | $h/A$ ratio | 0.001 | 0.01  | 0.05  | 0.1   | 0.25  |
|-------------|-------------|-------|-------|-------|-------|-------|
| 1           |             | 14.64 | 140.87| 434.72| 514.35| 547.28|
| 2           |             | 29.009| 271.98| 553.71| 554.64| 561.08|
| 3           |             | 38.483| 349.47| 699.96| 774.62| 664.68|
| 4           |             | 48.884| 443.77| 803.26| 885.89| 807.09|
| 5           |             | 59.849| 522.84| 1000.5 | 1078.9 | 925.78 |
| 6           |             | 74.082| 553.41| 1024.8 | 1088.1 | 1008.8 |
| 7           |             | 74.948| 634  | 1096.3 | 1098.3 | 1097.7 |

### 4.2. Contour plot for 5 ply composite plate

The various displacement in translational X, Y and Z direction as well as displacement in rotational X, Y and Z direction is shown in this section for 5 ply composite SSSS Plate with $B/A=1.25$, $h/A=0.01$. 

![Fig. 6: X Component of Translational Displacement in 5 Ply Composite Plates.](image)
5. Conclusion

The problem of five layer composite plate is related uniform thickness rectangular plates having different thickness ratios, different aspect ratios and different boundary conditions. The problem is simulated in ANSYS v14.5. Meshing is done by taking a grid of 40X40 shell 281 elements. The variations of the first eight natural frequencies with respect to the length to thickness ratio for different aspect ratio and boundary condition are presented. The natural frequency decreases with increase in thickness ratio. Frequency is found to be increasing with increase aspect ratio. The present analysis is useful for the design of composites plates for dynamic response.

On the basis of present study following conclusions are drawn:

1) In the 5 ply composite plate the natural frequency increases with increase in mode number.
2) In the 5 ply composite plate the natural frequency increases with increase in constraints.
3) In the 5 ply composite plate of all the tested boundary conditions natural frequency is lowest for a cantilever plate (CFFF).
4) In the 5 ply composite plate of all the tested boundary conditions natural frequency is highest at all sides clamped plate (CCCC).
5) In the 5 ply composite plate the natural frequency increases with decrease in B/A ratio. It means when by keeping A as constant and B is kept on increasing natural frequency of the plate decreases.
6) In the 5 ply composite plate the natural frequency increases with increase in h/a ratio. It means when by keeping A as constant and h is kept on increasing natural frequency of the plate increases.
7) In the 5 ply composite plate thicker the plate more the natural frequency.
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