Students’ Error on Proof of The Group with "Satisfy Axioms Proof" based on Newman Error Analysis

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Abstract. This study aims to analyze students’ errors in solving proof problems on group theory, focus on proofs with satisfying axioms proof. The analysis used refers to the Newman Error Analysis, namely: reading, understanding, transformation, process skills, and coding. The participants in this study consisted of students at the Mathematics Department that enrolled in the group theory course during the odd semester of the 2021/2022 academic year in Universitas Negeri Makassar. Research data was obtained through tests, followed by interviews based on student answers from the test. Based on the results of the error analysis conducted in this study, it can be concluded that: (1) There were no reading errors; (2) Comprehension error was incorrectly write down the meaning of what is known from the problem in symbolic form; (3) Transformation error was error determining the type of proof, mistake write down a formula to show an axiom in the group; (4) Process skill error was an error using arithmetic operations for the validation of an axiom; and (5) Encoding error was an error in writing the final answer, wrong evaluation to conclude.

1. Introduction

These The group theory as a part of modern algebra is a subject with a strict axiomatic deductive structure. As Birkhoff [1] states “the most striking characteristics of modern algebra is a deduction of theoretical properties of such formal systems as groups, ring, fields, and vector spaces.” Thus, group theory is full of definitions and theorems so that students learn the skills to prove theorems, and can take advantage of existing theorem definitions in solving problems that are generally used for proof. Such as the textbooks are written by Birkhoff [1], Fraleigh [2]; Herstein [3]; Suradi [4] in general, the solving problems in the textbooks include proof tasks.

One of the goals of group theory courses for students is that they have conceptual abilities and basic skills related to group concepts, and can carry out the process of proving group problems using definitions or using theorems. Thus, students often have difficulties in learning it. To overcome this, an understanding is needed to find out the types of errors experienced by students in solving proof questions.

There is a need to analyze the errors that have been made by students in working on proof questions in group theory. By knowing the types of errors made by students, it is possible to make improvements so that the errors will not happen again. Various methods can be used to analyze the types of errors made by students. Among others, using the Newman Error Analysis (NEA) procedure. Newman (in Bulu [5]) states that there are five procedures found by Anne Newman, namely reading, comprehension, transformation, process skills, and encoding. Thus, this study investigates students’ errors in solving group proofs using the "Satisfy Axioms Proof" classification based on Newman Error Analysis.
The group theory course introduces the concept of abstract algebra which emphasizes the ability to think logically and to do mathematical reasoning systematically in solving problems. The topics discussed in this course are based on axioms, which are needed by students in solving proof problems. According to Soedjadi [6] axiom is a base statement in the mathematical structure that is useful to avoid circling in proof. While the term system is defined as a collection of elements or elements that are related to each other that contain or note a hierarchical relationship. Soedjadi further stated that a set of axioms can be a system if it satisfies (1) consistent: the axioms do not accept any contradictions; (2) independent: it cannot be proven from the other axioms; and (3) complete: every statement derived from the system is capable of being proven true or false. Particularly, the collection of four axioms in groups, namely closed, associative, identity, and inverse forms a system of axioms, which is called the group axiom system or commonly called the group structure.

Based on the description above, to solve proof problems in group theory, a deep understanding of group structure, defined concepts, and various theorems is required. One way to prove mathematical problems or problems in group theory is to understand the relationship between structures in group theory. According to Polya [7], problems in mathematics are grouped into two types, namely problems to find and problems to prove. Thus, the purpose of the proof is to show that a statement is true or false, not both. We must answer the question is it right or wrong?

The process of proving mathematics according to Suradi ([8], [9]) can use definitions, theorems, or statements that have been proven previously. Therefore, in establishing confidence in the evidence that has been obtained, every step used in the evidence must always be questioned "why" and "what is the reason". Likewise, in proving questions on group theory, every step taken must always be questioned for its validity. For this reason, mastery of concepts is the main requirement in solving proof problems.

According to Suradi [9], many of the questions that contained in: (1) algebraic structure textbooks, and (2) questions that are often raised in mid-semester exams, quizzes, and end-semester exams, generally revolve around several problems as the following:

- Proving based on known axioms or based on theorems, whether a set and its defined operations are a group or not.
- Proving whether a group is abelian or not based on the given conditions.
- Proving whether or not a given non-empty subset of a group is a subgroup.
- Proving whether or not a given subgroup is a normal subgroup.
- Proving whether a given function of a group is a homomorphism, an epimorphism, or an isomorphism.

Furthermore, Hart (in Asikin, [10]) classifies several types of proof questions in group theory as follows, namely:

- Satisfy axioms proof, where one has to prove that something is a group.
- Set-definition proof, where one has to prove that particular subset, given by a defining property, is subgroup/subring.
- Uniqueness proof, where one has to prove the existence of a unique ‘idempotent’ element.
- Syntactic proof, where one uses a syntactic i. e. a procedural ‘symbol pushing to prove that given is a group abelian.
- Non-routine proof, where one has to prove that a group with every number of elements has an element that squares to the identity.

Based on findings from the previous studies of Suradi’s research [11], several causes of errors made by students during the lecture process, including: (1) conceptual errors; (2) lack of student understanding of the problem; and (3) cannot use the theorem in carrying out the proof.

The errors made by students in solving problems related to description questions can be traced using the Newman Error Analysis (NEA) procedure. The NEA procedure was first introduced in 1977 by Anne Newman, a mathematics teacher in Australia. According to Prakitipong [12], The Newman Procedure is a method that analyzes errors in sentence problems. Thus, the Newman procedure is a method for analyzing errors in description problems. The stages proposed by Newman in analyzing errors made by students can be traced from their activities in (1) reading problems (reading); (2)
understand the problem (comprehension); (3) transformation of the problem (transformation); (4) process skills (process skills); and (5) writing the final answer (encoding).

Based on the description above, the research question address the purpose of this study is what are the types of student errors in solving problems related to proof in group theory courses, especially proofs using "satisfy axioms proof" based on five NEA procedures.

Students need to explore mathematical thinking and reasoning when solving a proof problem, especially proof by satisfying axioms proof on groups, understanding the meaning of an axiom in the problem. Moreover, students are required to be able to relate the axioms, concepts, or statements contained in the problems they are facing correctly. In addition, wrong in understanding a concept cause them to be wrong in solving the problems. Furthermore, in solving the problem of proof, it needs to be done sequentially or systematically.

According to Layn [13], common problem-solving mistakes are related to procedural errors. To avoid errors, it is necessary to do a lot of practice so that students will be more skilled and understand in working on the relevant questions. Further, in the research conducted by Layn, et al, other possible students’ errors are misunderstanding the instruction; although the student used a correct procedure, they did not finish it; incorrect answers caused by technical errors such as an error in calculation, and incorrectly understand what the question asked about.

2. Research method

Problem The main problem presented in this article is the mistakes made by students in solving proof problems using NEA. The research subjects were students at the Mathematics Department of UNM that passed in group theory and were temporarily taking the ring theory course of the academic year 2021/2022. The number of students involved in this study was 37 student.

Data were obtained by using tests and followed by interviews. The test used is a matter of proving the group according to Hart's classification (in Asikin, 1997), namely "Satisfy axioms proof". The problem of proof: "Suppose \( G \) is a set of positive rational numbers, and the operation \(*\) in \( G \) is defined by \( a * b = \frac{ab}{2}, \forall a, b \in G \). Prove that \( \langle G, * \rangle \) is a group."

3. Result and discussion

The results of descriptive data analysis using the NEA procedure obtained the following results.

- For reading error according to Newman's analysis, no error was recorded (0.0%), meaning that all students can read the questions properly and correctly. However, the difficulty they experienced was not being able to interpret the sentences they read properly. At this stage, students understanding the context of the question but they did not understand the meaning correctly.

- In comprehension errors, students had problems with understanding the meaning of the problem of proof. It was found that there were only 2 students (5.4%) who did not write down information that is known to prove the problem. However, there were still 14 students (37.8%) who did not understand the problem. The error at this stage is that students can read all the words in the problem, but cannot understand all the meanings of the words contained in the problem symbolically. For example, students cannot write down the meaning of \( G \) is a set of positive rational numbers in symbolic form, namely \( G = \{ x \in \mathbb{Q} : x > 0 \} \) as the information of the problem.

- Transformation error occurs when students were unable to write or mention a formula to show the validity of an axiom in the group. The percentage of students who could not show the "closed axiom" in the proof problem was about 100%. Students just write: if \( a, b \in G \) arbitrary, then \( a * b = \frac{ab}{2} \in G \), so that it fulfills the closed nature. It just simply repeats what is known from the problem, which should be shown that \( a * b = \frac{ab}{2} > 0 \).
Moreover, it cannot plot a solution to show the validity of an axiom. There were 5 students (13.5%) who forgot the concept of associative axioms by writing $a \ast b = b \ast a$ as an associative proof. In addition, there were still 8 students (21.6%) who misinterpret the concept of identity for the operation "$\ast$" such as writing "1" as an identity element.

- **Process skills errors** occur when students could not perform arithmetic operations or calculation steps correctly. However, errors in process skills also occur due to errors in determining the formula at the problem transformation stage. A total of 35 students (94.6%) could not operate "$\ast$" to determine the inverse element of the problem. An example, they got the inverse of $a \in G$ is $a^{-1} = \frac{b}{4}$ (which should be $a^{-1} = \frac{4}{a}$).

- **Encoding errors** occur when students were not careful (not double-checked) to conclude the inverse of an element of a group by using the operation "$\ast$". Therefore, 100% of the students wrote the final answer that $(G, \ast)$ is a group, but it is not correct.

Based on the above results, it can be concluded that several errors were made by the research subjects to prove the group on the problem "Suppose $G$ is a set of positive rational numbers, and the operation $\ast$ in $G$ is defined by $a \ast b = \frac{ab}{2}$, $\forall a, b \in G$. Prove that $(G, \ast)$ is a group" of which: (1) the student did not write down the membership of the set $G$ that is known from the problem, (2) the student did not write down what will be shown to fulfill the closed axiom in $G$. (3) the student has a conceptual error to show the associative axiom by writing $a \ast b = b \ast a$, (3) students have misconceptions about the axiom of identity by writing that the identity element of the problem is 1, and (4) students made a procedural error in determining the inverse element of the membership in $G$.

Based on the classification of NEA, various causes revealed by students' errors in solving proof problems are (1) comprehension error, students could write down what is known from the problem but could not write down the meaning contained in what is known (especially writing it in the form of symbol); (2) transformation error, failed to determine the formula that will be shown in the axioms that will be proven in the group; (3) process skills error, including being less thorough, unable to perform operations correctly, lack of practice working on proof questions; (4) encoding error, indicates that students did not re-check the final answer for each stage in proving the validity of the axioms in the group, especially checking related to the validity of the identity axiom, and the inverse axiom.

Relevantly, according to [14] that most of the students are capable in performing the first stage of Newman’s Analysis (Read and Recode) however they faced difficulties in performing the second to five stage of Newman’s Model.

4. Conclusion

The results showed that the highest number of errors made by students especially those related to the proof of the "satisfy axioms proof" of a group, based on Newman Error Analysis was in transformation and encoding (100%), incorrectly writes or mentions the formula to show the validity of an axiom in the group and wrong evaluation to conclude; followed by process skills (94.6%), wrong in performing arithmetic operations for the validity of an axiom; and comprehension (43.2%), incorrectly writing down the meaning of what is known from the problem in symbolic form.

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