Dynamic behaviour of the steel footbridges with spatial pipe truss girders

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Abstract. The excessive vibrations of the steel footbridges can be an important problem during the verification of the requirements of the serviceability limit state of these structures. In the paper the issues of dynamic characteristics and dynamic behaviour of the steel footbridges on the basis of the results of the field tests and numerical analyses of the four single span steel footbridges with spatial pipe truss girders and span length from 30 to 50 m were presented. The dynamic properties of the structures (stiffness, mass, damping) were characterized and the dynamic response of the structures was studied. Results of the analyses and researches showing, in most cases, a very high dynamic susceptibility of the structures subjected to dynamic loads from slow running (jogging) users, mainly due to the very low damping value, which is important parameter affecting the vibration amplitudes.

1. Introduction

At the outset the information concerning primary rules of dynamics of the single degree of freedom system will be quoted to remind them for further analysis. Single degree of freedom system (SDOF), in the form of a mass suspended on a spring with added damping is described by three basic parameters $M$, $K$ and $C$ (mass, stiffness and damping respectively). It is well know that the natural vibration frequency of the SDOF system $f$ depends on its stiffness and mass. This dependence is described by the equation (1):

$$f = \frac{1}{2\pi} \sqrt{\frac{K}{M}}$$

Changing of the natural vibration frequency of the SDOF system is possible by changing of its stiffness $K$ [N/m] or mass $M$ [kg]. The natural vibration frequency of the SDOF system is directly proportional to the stiffness of the system $K$ and inversely proportional to its mass $M$. Modifications of the $M$ and $K$ parameters changes the natural frequency of the system. It is worth noting that natural vibration frequency of a SDOF system does not depend on damping value $C$. The equation (1) does not describe the effect of changes in $K$ and $M$ parameters on the amplitude of vibrations.

The influence of the $M$, $K$, $C$ parameters on the amplitude of forced vibrations of the system can be described by the dependencies (6), (8) and (10) resulting from the solution of the equation of motion of the SDOF system under harmonic excitation (2).

$$M \ddot{x}(t) + C \dot{x}(t) + K x(t) = P \sin(2\pi \cdot f_p \cdot t)$$

(2)
The solution of equation (2) can be written in the form (3) [1]:

\[ x(t) = A \sin(2\pi \cdot f_p \cdot t - \varphi) \]  

where:

\[ A = \frac{P}{\sqrt{(K - 4\pi^2 f_p^2 M)^2 + 4\pi^2 f_p^2 C^2}} \]  

\[ \tan \varphi = \frac{2\pi f_p C}{K - 4\pi^2 f_p^2 M} \]

\[ M, K, C \text{ – parameters of SDOF system (mass, stiffness, damping),} \]
\[ f_p \text{ – frequency of the harmonic excitation,} \]
\[ A \text{ – amplitude of forced vibration,} \]
\[ \varphi \text{ – phase shift.} \]

Using the equation (4), it is possible to analyse the influence of parameters \( M, K, C \) on the amplitude of forced vibrations \( A \) in three cases the relations between the frequency of the excitation \( f_p \) and the natural vibration frequency of the system \( f \).

Case No. 1: \( f_p << f \) – frequency of the excitation is significantly lower than the natural vibration frequency of the system. Assuming a small values of \( f_p \) and a small damping \( C \) (this assumption is correct for the majority of footbridges [2]) one obtains:

\[ A = \frac{P}{\sqrt{(K - 4\pi^2 f_p^2 M)^2 + 4\pi^2 f_p^2 C^2}} = \frac{P}{\sqrt{K^2}} = \frac{P}{K} \]  

Case No. 2: \( f_p >> f \) – frequency of the excitation is significantly greater than the natural vibration frequency of the system. Assuming a small values of \( f_p \) and a small damping \( C \) for:

\[ K = 4\pi^2 f_p^2 M \]  

one obtains:

\[ A = \frac{P}{\sqrt{(K - 4\pi^2 f_p^2 M)^2 + 4\pi^2 f_p^2 C^2}} = \frac{P}{\sqrt{(4\pi^2 f_p^2 M - 4\pi^2 f_p^2 M)^2 + 4\pi^2 f_p^2 C^2}} = \frac{P}{4\pi^2 f_p^2 M} \]

Case No. 3: \( f_p = f \) – frequency of the excitation is equal to the natural vibration frequency of the system (resonance effect) for:

\[ K = 4\pi^2 f_p^2 M = 4\pi^2 f_p^2 M \]  

one obtains:

\[ A = \frac{P}{\sqrt{(K - 4\pi^2 f_p^2 M)^2 + 4\pi^2 f_p^2 C^2}} = \frac{P}{\sqrt{(4\pi^2 f_p^2 M - 4\pi^2 f_p^2 M)^2 + 4\pi^2 f_p^2 C^2}} = \frac{P}{2\pi f_p C} \]

It can be seen that the first two cases are approximate solutions achieved with the adoption of simplified assumptions. Whereas, the case no. 3 is a strict solution applicable in the case of the resonance effect.

Equation (10) indicate that amplitude of forced vibration of the system in case of resonance is inversely proportional to the damping \( C \). Decreasing of the vibration amplitude in case of resonance
requires increasing the damping of the system. Changes in mass $M$ or stiffness $K$ of the system do not reduce the vibration amplitudes in case of resonant vibrations. According to equation (1) the correction of $M$ or $K$ parameters of the system can help to avoid the resonance effect by changing the vibrations frequency of the system. However, if the resonance effect can occur in a wide frequency range (e.g. in frequency range 1.40-3.40 Hz – vertical vibration or 0.70-1.20 Hz – horizontal vibration as it is in the case of vibration of footbridges caused by their users), changes in mass $M$ or stiffness $K$ of the structure may not lead to the desired effect. As can be seen from (10) changes in $M$ or $K$ do not helping to reduce vibration amplitudes in the case of resonant vibrations. Moreover, improper correction of $M$ and $K$ may result in increased dynamic susceptibility of the structure to other dynamic loads, in particular dynamic loads from wind action.

2. Characteristic of the footbridges

In order to examine the basic dynamic parameters and dynamic behaviour of steel footbridges with spatial pipe truss girders four exemplary structures presented in Fig. 1 were tested and analysed. The footbridges are single span structures with spans: 30.00 m, 44.77 m, 47.00 m, 50.05 m.

Footbridge in Kraków, span 30.0 m, is designed as a spatial truss with truss nodes spaced in intervals of 3.0 m and with orthotropic deck constructed with steel plate (thickness 8.0 mm) stiffened by longitudinal open ribs (thickness 10 mm) and supported on stiff transverse beams spaced in intervals of 1.50 m (beam web thickness 14.0 mm). Truss girder consist of 10 pyramidal parts forming the spatial truss. The details of the cross-section of the footbridge are presented in Fig. 2a.

Footbridge in Sławięcice, span 44.77 m, is designed as a spatial truss with truss nodes spaced in intervals of 4.07 m. Truss girder consist of 11 pyramidal parts forming the spatial truss. The deck of the footbridge is constructed using a system known as the “battle-deck-floor” (“b-d-f”) consisting of longitudinal wide flange beams (HEB 100) supported on transverse beams (HEB 140, arranged in intervals of 2.035 m) and covered by steel plate thickness 8.0 mm. The basic dimensions and construction details of the footbridge cross-section are presented in Fig 2b.

Footbridge in Biecz, span 47.0 m, is designed as a spatial truss with truss nodes spaced in intervals of 3.00 m in central part of a span ($L = 42.0$ m) and 2.50 m in initial part of the girder at both ends. Truss girder consist of 16 pyramidal parts forming the spatial truss. The upper chord of the truss girder is curvilinear with radius of 330.0 m. The distance between axis of the bottom chord and the axis of upper chord of the truss girder in the middle of the span equals 1.50 m and decreasing to 0.63 m at both ends of the girder near to the supports. The deck of the footbridge is constructed using a “battle-
deck-floor” system. The cross-section of the footbridge containing the basic dimensions is shown in Fig 2c.

Footbridge in Osjaków, span 50.05 m, is designed as a spatial truss with truss nodes spaced in intervals of 3.85 m. Truss girder consist of 13 pyramidal parts forming the spatial truss. The structural solutions used in a footbridge and its dimensions are similar to solutions applied in footbridge in Ślawińce. The details of the cross-section of the footbridge are presented in Fig. 2d.

![Cross-sections of the footbridges](image)

**Figure 2.** Cross-sections of the footbridges a) footbridge in Kraków, b) footbridge in Ślawińce [3], c) footbridge in Biecz, d) footbridge in Osjaków [4]

3. **Dynamic characteristics of the footbridges**

The dynamic characteristics of the footbridges were diagnosed during a series of field tests. In the time of the tests, the natural vibration frequency of the structures were identified. Acquired measurement signals allowed to determine the value of the maximum vibration acceleration generated during resonant excitation of vibrations by running and jumping or squatting people with a frequency adjusted to the frequency of the fundamental mode shape of the footbridge as well as to determine the vibration damping parameters in the form of the logarithmic decrement $\delta$ and corresponding value of the damping ratio $\zeta$. The results of the field tests are shown in Tab. 1, Fig. 3 and Tab. 2. In the case of vibrations excited by jumping/squatting people the values of vibration accelerations are presented after 10 s and 20 s of vibration excitation.
Table 1. Natural vibration frequencies of the footbridges (the results of the field tests)

| Mode shape | Frequency $f$ [Hz] | Kraków - 30.0 m | Sławięcie - 44.77 m | Biecz - 47.0 m | Osjaków - 50.0 m |
|------------|-------------------|-----------------|---------------------|---------------|-----------------|
| 1          | 1$^{st}$ vertical  | 4.21            | 1$^{st}$ vertical  | 2.50          | 1$^{st}$ vertical | 2.38          | 1$^{st}$ vertical |
| 2          | 1$^{st}$ horizontal | 6.23            | 1$^{st}$ horizontal | 3.23          | 1$^{st}$ horizontal | 4.71          | 1$^{st}$ horizontal |
| 3          | 1$^{st}$ torsional | 8.57            | 1$^{st}$ torsional | 6.26          | 2$^{nd}$ vertical  | 6.59          | 1$^{st}$ torsional  |
| 4          | 2$^{nd}$ vertical  | 12.85           | 2$^{nd}$ vertical  | 7.38          | 1$^{st}$ torsional  | 6.79          | 2$^{nd}$ vertical  |

Figure 3. Visualizations of the four mode shapes of the footbridges: a) 1$^{st}$ vertical mode shape, b) 1$^{st}$ lateral mode shape, c) 1$^{st}$ torsional mode shape, d) 2$^{nd}$ vertical mode shape

Table 2. Vibration damping and vibration accelerations for 1$^{st}$ vertical mode shape $f_1$ of the analyzed footbridges (the results of the field tests)

| Footbridge | Frequency $f_1$ [Hz] | Damping (mean value for 1$^{st}$ mode shape) | Maximum accelerations | One running person | One jumping person |
|------------|----------------------|---------------------------------------------|-----------------------|--------------------|-------------------|
| Kraków     | 4.21                 | $\delta_{test}$ = 0.0562, $\zeta_{test}$ = 0.0089 | 0.53                  | 0.83               | 0.97              |
| Sławięcie  | 2.50                 | $\delta_{test}$ = 0.0224, $\zeta_{test}$ = 0.0036 | 2.67                  | 2.37               | 3.44              |
| Biecz      | 2.38                 | $\delta_{test}$ = 0.0433, $\zeta_{test}$ = 0.0069 | 2.85                  | 2.12               | 3.32              |
| Osjaków    | 2.39                 | $\delta_{test}$ = 0.0084, $\zeta_{test}$ = 0.0013 | 3.27                  | 2.35               | 4.59              |

Analysis of the presented results allow to notice a relatively high fundamental vibration frequency of the footbridge in Kraków $f_1 = 4.21$ Hz > 3.0 Hz and much lower frequencies of the other footbridges. The 3.0 Hz value of vibration frequency is defined in some standards [5, 6] as a lowest allowable fundamental frequency of the footbridges. The structures with fundamental frequency in vertical mode less than 3.0 Hz (and 1.3 Hz in lateral mode) can experience excitations of resonance vibrations by pedestrians. If the fundamental frequency cannot satisfy these limitations, or if the second harmonic is a concern, an evaluation of the dynamic performance shall be made [5].

The fundamental frequencies of the footbridges in Sławięcie, Biecz and Osjaków are lower than 3.0 Hz, the dynamic analyses of the footbridges should be performed. However, it is worth noting that natural vibration frequencies of these footbridges are out of the range of the frequency of pacing during walking (1.4–2.4 Hz) and are in the range of frequency of running or jumping/squatting (1.90 – 3.40 Hz). This can be considered as a beneficial factor. Forces generated during running and jumping/squatting can be considered as a dynamic load cases which occur rarely on footbridges and for this reason resonance excitations are less likely. However, in the case of footbridges located along an important walking routes, connecting attractive and popular areas or footbridges located near important transport hubs (buss, railway or underground stations), footbridges located within sports and recreational areas and footbridges with long span/spans the loads generated by running people can be an important dynamic load case.

The case of a jumping/squatting people should be treated as an act of vandalism (vandal load [7, 8, 9]) which should be considered as an exceptional load case. This type of dynamic load usually leads to large vibration amplitudes. Analysis and prevention of vibrations generated by jumping or squatting people should be considered individually if it is important from point of view of comfort and safety of use of the structure.
In general all analyzed footbridges characterize a low damping (damping ratio $\zeta \approx 0.10–1.00\%$), especially footbridge in Osjaków with $\zeta \approx 0.13\%$. This low values of damping are insufficient to damp the vibrations forced by users of the footbridges. With regard to vibration damping, it can be generally stated that uncomplicated and rigid structures with small number of flexible connections have low damping. Various looseness, gaps and imperfections of connections increase the damping value. Steel structures with spatial truss girders made of pipes can be classified to first group of uncomplicated and rigid structures characterized by low damping. In the case of the footbridge in Kraków and Biecz the damping ratios have the relatively high values (generally two and half to six and half times higher than in the Sławińcice and Osjaków footbridges respectively). It may be a consequence of a more complex structural form of the footbridges in Kraków and Biecz and large number of welded joints applied in these footbridges (welded connections between deck, cross-beams and longitudinal ribs, additional decorative railings in Kraków; welded connections between longitudinal beams HEB100 and diagonal braces C80 placed under the deck in Biecz). This structural complexity may cause a slightly more efficient dissipation of energy from the vibrating structure.

The values of vibrations accelerations obtained during dynamic tests of the footbridges (during resonant vibrations caused by running and jumping/squatting people) indicate that the damping values in all analysed footbridges are insufficient to limit the amplitudes of forced vibrations of the footbridges (vibration acceleration) to an acceptable level. In almost all cases of resonant excitations the value of vibration acceleration (Tab. 2) significantly exceed the values of $a = 0.50 \text{ m/s}^2$ which was defined in [10] as the level of maximum comfort. The vibration accelerations also exceed values of $a = 0.70-1.00 \text{ m/s}^2$ which are generally acceptable but these vibrations can be slightly felt by walking people [11]. In almost all cases the vibration accelerations of the analysed footbridges are in the range of vibration accelerations causing significant walking disturbances i.e. vibration accelerations $a_{\text{max}} > 1.20 \text{ m/s}^2$ for the vibration frequency range $2.0 – 3.0 \text{ Hz}$ [12, 13] (Fig. 4).

![Figure 4. Vibration comfort criteria for footbridges in case of vertical vibrations [12, 13] and evaluation of comfort of use of the footbridges in Kraków, Sławińcice, Biecz and Osjaków during vibrations induced by running users:](image)

M1 – basic comfort curve, frequently occurring vibrations, vibrations are slightly felt or perceptible and do not disturb walking; M1.7 – rarely occurring vibrations, vibrations are clearly felt (fully perceptible), vibrations disturb walking; M10 – vandal intentional actions, comfort is strongly disturbed (free walking is impossible, standing or running is difficult and strongly disturbed). The M10 vibrations limits have been proposed to protect pedestrians against body injury (mainly legs) that can be caused by high-level vibration acceleration.

Only in the case of the footbridge in Kraków vibration accelerations induced by running or jumping/squatting people moving with frequency $f_{0.5} = \frac{1}{2} f_1 = 2.10 \text{ Hz}$ (movement with a frequency of $4.21 \text{ Hz}$ is very difficult to implement by people) are relatively small. The value of vibration acceleration $a = 0.97 \text{ m/s}^2$ in case of vibrations intentionally induced by vandals can be considered as a very low value of vibration. It can be regarded that the fundamental vibration frequency of the footbridge in Kraków $f_1 = 4.21 \text{ Hz}$ (resulting from the relatively high stiffness of the structure – see Tab. 3) effectively reduce the risk of pure resonant excitation. Nonetheless, during the several tests of vibration excitation by one jumping/squatting person with frequency $f_1 = 4.21 \text{ Hz}$ the vibration acceleration of the footbridge deck reaches the value of $2.5 \text{ m/s}^2$ after 10 s of vibration excitation.
4. Analyses of the dynamic parameters of the footbridges

In order to investigate the possibility of increasing the fundamental frequencies of the footbridges (to avoid the resonant vibrations of the structures) and the possibility of reduction of vibration amplitudes (vibration acceleration), the analyses of influence of the $M$ and $K$ changes on the fundamental frequency of the footbridges as well as influence of the changes of $C$ on the vibration amplitudes were carried out. Three footbridges with natural frequencies below 3.0 Hz were analyzed using the equivalent SDOF systems determined for the footbridges. In Tab. 3 the parameters of the equivalent SDOF systems are presented. SDOF for footbridge in Krakow was presented for comparative purposes.

The changes in mass $M$ of the structures were introduced by changing the construction system of the footbridge deck from “battle-deck-floor” into the orthotropic deck by replacing the longitudinal HEB100 beams by longitudinal ribs 100x10 mm, removing all diagonal braces and additionally changing the deck surface from asphalt to epoxy resin in the case of the footbridge in Sławięcice.

The changes in stiffness $K$ was introduced by changing the structural height of the spatial truss girder by increasing the initial height of the spatial truss girders $H$ (Fig. 2) by 10 cm, 20 cm and 50 cm.

Table 3. Equivalent parameters of the SDOF systems of analyzed footbridges

| Footbridge | Weight of the structure [kg/m] | Equivalent mass $M_e$ [t] | Equivalent stiffness $K_e$ [kN/mm] | Equivalent damping $C_e$ [kg/s] |
|------------|--------------------------------|--------------------------|-----------------------------------|--------------------------------|
| Kraków     | 696.40                         | 10.189                   | 7.129                             | 2156.17                        |
| Sławięcice | 1101.50                        | 19.677                   | 4.855                             | 2225.42                        |
| Biecz      | 717.40                         | 13.979                   | 3.126                             | 2884.77                        |
| Osjaków    | 833.80                         | 17.312                   | 3.904                             | 675.93                         |

Moreover, the analyses of the forced vibration of the footbridges were performed to investigate the impact of the changes of damping on the amplitudes of vibrations. During the analyses the dynamic load in form of squats performed by one person (body weight $- G = 850$ N) with frequency equal to the fundamental vibration frequency of the structure was used. The SDOF system was loaded by single harmonic equivalent force determined by the time-dependent force function presented in [8, 9] applicable to frequency range $f_{sq} > 1.60$ Hz. The results of the analyses are presented in Tab. 4 and Tab. 5. It can be seen that changes in mass of the structures slightly changes the fundamental natural vibration frequency of the footbridges, the resonant excitations are still possible. The changes in the height of the truss girder can help to increase the fundamental natural vibration frequency to the value of $f \geq 3.0$ Hz only in two cases of the footbridges in Sławięcice and Osjaków with constant height of the truss girder. In the case of the footbridge in Biecz with variable height of the truss girder (variable stiffness), in spite of increasing the truss girder height, the fundamental vibration frequency of the structure remained less than 3.0 Hz, further increase of the height of the girder is required.

Table 4. Dynamic parameters of the modified structures

| Footbridge | $M_{or}$ [t] | $K_{or}$ [kN/mm] | $f_{or}$ [Hz] | Equivalent stiffness $K_i$ [kN/mm] / fundamental frequency $f_i$ [Hz] |
|------------|--------------|-----------------|---------------|---------------------------------------------------------------|
| Sławięcice | 15.601       | 4.760           | 2.78          | 5.226 2.61 5.153 2.89 5.652 2.71 5.574 3.01 6.964 3.01 6.872 3.34 |
| Biecz     | 12.416       | 2.800           | 2.39          | 3.355 2.47 3.020 2.48 3.604 2.56 3.262 2.48 4.460 2.84 4.100 2.88 |
| Osjaków   | 15.720       | 3.817           | 2.48          | 4.248 2.49 4.155 2.59 4.600 2.59 4.500 2.69 5.685 2.88 5.567 3.00 |

$M_{or}, K_{or}, f_{or}$ – parameters of the structures with orthotropic deck,

$K_{bd}, f_{bd}$ – parameters of the structures with “battle-deck-floor” system (“b-d-f”).
Table 5. Vibration acceleration of the footbridges induced by one squatting person in a function of damping (results of numerical analyses)

| Footbridge   | Vibration acceleration $a_{\text{max}}$ [m/s$^2$] (duration of the squats $t_{sq}$ = 10 s) |
|--------------|------------------------------------------------------------------------------------------|
|              | $\zeta$ = 0.01 | $\zeta$ = 0.03 | $\zeta$ = 0.05 | $\zeta$ = 0.1 |
| Sławięcice   | 2.42 | 1.57 | 0.67 | 0.41 | 0.20 |
| Biecz       | 2.74 | 2.26 | 0.96 | 0.58 | 0.29 |
| Osiaków     | 3.27 | 1.82 | 0.77 | 0.47 | 0.24 |

The changes in the damping value allows to reduce the maximum vibration acceleration to the acceptable levels without changing the height of the girder. Attaining the damping ratio value of $\zeta \geq 0.05$ is required.

5. Summary
The issues presented in the paper relate to the subject of dynamic susceptibility of footbridges with steel spatial truss girders. It can be seen that analyzed structures have the proper dynamic characteristics in the range of the span length up to about 35.0 m. For longer spans the fundamental vibration frequency of the structures is less than 3.0 Hz and structures can experience resonant vibrations excited by users. The vibration acceleration can reach a large values (Tab. 2). Proper modifications of structural system of the truss girder, consisting in increasing the girder height, allow to increase the fundamental vibration frequency of the structures ($f \geq 3.0$ Hz) and minimize the risk of the resonant excitations. Increasing of the vibration damping level e.g. by installing TMD system, allows to effectively reduce the vibration acceleration of the structure to acceptable levels without changing the height of the girder if the damping ratio $\zeta \geq 0.05$ is achieved.

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