Adaptive blind equalization of fast time-varying channel with frequency estimation in impulsive noise environment

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Abstract
In this paper, a novel source signal recovery method for fast time-varying channels described by complex exponential-basis expansion model (CE-BEM) in the impulsive noise environment is proposed. This method consists of two phases. The first phase is the equalization of fast time-varying channels, in this phase, a novel algorithm FSE-FLOS-CMA is proposed. The convergence performance of this newly proposed algorithm is much better than that of existing fractionally spaced equalizer-constant modulus algorithm (FSE-CMA) in impulsive noise environment. In the second phase, a novel frequency estimation method is proposed. The estimated frequency in impulsive noise environment is obtained by calculating a specific p-order fractional low-order cyclic moment of the equalized signal. Simulation results show that the proposed FSE-FLOS-CMA and frequency estimation method can effectively estimate the source signals transmitted through the fast time-varying channels in impulsive noise environment.

1 | INTRODUCTION

In wireless communications, inter-symbol interference (ISI), as a form of signal distortion, is caused by multipath propagation and band limited propagation [1]. In real systems, the truncation of filter and the deviation of sampling time will both cause the overlap of the tail before and after the symbol, which will cause ISI. So, equalizers need to be designed to eliminate the influence of ISI. Including training sequence in the transmitted data will increase the transmission cost. In the fast time-varying (hereinafter referred to as TV) channel communication system, the time used to transmit the training sequence even takes up 50% of the total transmission capacity overhead [2, 3]. And in the measuring instrument, the transmitted signal is likely to be unknown. Therefore, the adaptive blind equalization is necessary to study. Communication signals often have the characteristic of constant envelope, that is, constant modulus. According to this characteristic, Godard proposed the famous constant modulus algorithm, CMA for short [4]. This algorithm is one of the best known and the simplest adaptive blind equalization algorithms.

CE-BEM model is widely used in the TV FIR channel [5–7]. The TV taps are expressed as a superposition of TV bases (e.g. complex exponentials when modelling Doppler effects) with time-invariant (hereinafter referred to as TI) coefficients. Time-variation offers diversity. Since the same input is modulated by different complex exponentials, some redundancy is introduced at the output which we call “channel diversity” [8, 9]. In order to deal with faster TV channels, a number of channel models have been developed to describe the TV SIMO-FIR channels [10, 8, 11, 12]. Just like the TI case, sampling faster than the symbol rate creates diversity that enables the problem to be cast into an SIMO framework. Multichannel diversity can also be achieved by using multiple antennas at the receiver [13–15]. The traditional adaptive blind equalization algorithms can only track slowly TV channels and many of them suffer from local...
minima. They fail when the speed of channel variations exceeds the convergence speed of the algorithms [10]. In practice, the CM algorithm has often been implemented using fractionally spaced equalizer (FSE) [16].

In many practical problems, the noise distribution encountered is more impulsive than Gaussian distribution. For examples, underwater noise, low frequency atmospheric noise, noise of semiconductor devices and many types of man-made noise are impulsive [17]. There exists a class of distributions called alpha-stable distributions that can be used to model impulsive noise [18]. In references [19, 20], a novel FLOS-CMA algorithm based on fractional low order constant modulus property of the measured signal is proposed. This algorithm can deal with impulsive noise and ISI in the data robustly.

The fourth-order statistics based estimation method [8] and higher-order statistics (HOS) based method both need to use a very large number of samples to achieve reasonable statistical performance and are expensive in computation [21]. Because the variance of second-order statistics (SOS) is usually lower than that of fourth-order and higher-order statistics, Tsatsanis et al. proposed an effective frequency estimation method combining second-order and fourth-order statistics [10], which has been widely used to solve the blind identification and equalization problems of TV channels [10, 8, 15, 22]. There is a special kind of non-stationary signal among communication, astronomy, ocean and other signals, their non-stationarity is characterized by cyclostationarity, that is the statistical characteristics change periodically or in many cycles (each cycle is incommensurable). The second-order cyclic moment is a powerful tool for utilizing cyclostationarity of signals [23].

In this paper, we propose a novel source signal recovery method which realize adaptive blind equalization and frequency estimation of fast TV channel in impulsive noise environment. The main contributions of this paper are summarized as follows. Firstly, we propose FSE-FLOS-CMA algorithm for the fast TV channel based on CE-BEM model. This algorithm can converge signals effectively in impulsive noise environment. Moreover, we propose a method to estimate the frequency of fast TV channel in impulsive noise environment, we calculate the fractional lower-order cyclic moment of the signal to obtain the frequency estimation.

The rest of the paper is organized as follows. In Section 2, the alpha-stable distribution is introduced as the model of impulsive noise. CE-BEM model applied to the fast TV channel and TV SIMO channel blind equalization architecture are both introduced in Section 3. In Section 4, we proposed a new channel equalization algorithm and a new frequency estimation method for the fast TV channel in impulsive noise environment. In Section 5, the simulation results are presented. Finally, the conclusion is shown in Section 6.

2 | ALPHA-STABLE DISTRIBUTION

The impulsive noise can be described by many models, such as Middleton Class A [24], Bernoulli-Gaussian [25–30] and symmetric alpha-stable distribution [18]. The parameters in model “Middleton Class A” are directly related to the physical channel, but not easy to deal with; models “Bernoulli-Gaussian” and “symmetric alpha-stable distribution” can both generate Gaussian and non-Gaussian noise. The alpha-stable distribution is the only distribution that satisfies the generalized central limit theorem, so we use alpha-stable distribution in this paper.

Alpha-stable distribution does not have a unified probability density function (pdf) expression, but it has a unified characteristic function expression.

\[ \varphi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ixt} \frac{1}{t^{\alpha}} \left(1 + \beta \operatorname{sgn}(t) \omega(\alpha, t)\right) dt \]  

where \( \omega(t, \alpha) = \begin{cases} \frac{\tan\left(\frac{\alpha \pi}{2}\right)}{\alpha}, & \alpha \neq 1 \\ \frac{2}{\pi} \log |t|, & \alpha = 1 \end{cases} \), \( \operatorname{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ \alpha, & t < 0 \end{cases} \)

is the characteristic exponent, which controls the level of impulse in the stable distribution process, the smaller the \( \alpha \), the stronger the impulse; \( \beta \) is the symmetry parameter, when \( \beta = 0 \), the distribution is symmetric, such distribution is called\( \alpha \)-stable distribution; \( \gamma \) is the dispersion parameter, which is similar to the variance of Gaussian distribution; \( \alpha \) is the location parameter, which corresponds to the mean value or mid-value of the stable distribution.

When \( \alpha = 2, \beta = 0 \), alpha-stable distribution corresponds to the Gaussian distribution, and it has explicit probability density function expression.

If \( 0 < \alpha < 2 \), there is no statistic higher than for the alpha-stable distribution. However, any order statistic exists for the Gaussian distribution (\( \alpha = 2 \)).

For any random variables \( X \) and \( Y \) which submit to the\( \alpha \)-stable distribution, their \( p \)-order fractional lower-order moment is defined as

\[ R_{XY}^{p} = \langle X, Y \rangle_{p} = E \left\{ X Y^{<p-1>} \right\}, \quad 1 \leq p \leq \alpha \]  

where \( < \cdot > \) represents: \( < x^{\alpha} > = x^{<\alpha> \alpha} \)

When \( p = 2 \), the \( p \)-order fractional moment is the general second-order moment.

Due to the characteristic of alpha-stable distribution, the conventional SNR becomes meaningless. Therefore, generalized SNR (GSNR) [31] is defined in impulsive noise environment, the expression is:

\[ \text{GSNR} = 10 \log_{10} \frac{\sigma_{i}^{2}}{\gamma} \]  

where \( \sigma_{i}^{2} \) is the variance of the input signal, \( \gamma \) is the dispersion parameter of alpha-stable distribution.

The Figure 1 below shows the impulse noise with GSNR is 25. Because we do not know the channel characteristics before processing the signal, we cannot process the channel noise in advance.
Suppose \( l \) is the order number of the channel, \( s(n) \) is the input source signal, \( r(n) \) is the channel noise. Therefore, as shown in Figure 2, the signal received by a channel is:

\[
x(n) = \sum_{q=1}^{Q} \sum_{l=0}^{L} b(q; l)s(n - l) + r(n)
\]  

Because the channel variations of the TV systems are too rapid, CE-BEM model uses spatial diversity, which is achieved by using multiple antennas at the receiver, to improve the quality of received signals. Therefore, TV channels usually use single-input multiple-output (SIMO) architecture [15].

The figure below shows the architecture of TV SIMO channel blind equalization.

As shown in Figure 3 above, there are \( M \) receiving antennas, which are expressed as \( x_1(n), x_2(n), \ldots, x_M(n) \) respectively. Suppose receive vector \( x(n) = [x_1(n), x_2(n), \ldots, x_M(n)]^T \), channel vector \( h(n; l) = [h_1(n; l), h_2(n; l), \ldots, h_M(n; l)]^T \), noise vector \( r(n) = [r_1(n), r_2(n), \ldots, r_M(n)]^T \). The number of equalizer taps is \( K + 1 \). Suppose \( \hat{x}(n) = [x^T(n), x^T(n - 1), \ldots, x^T(n - K)]^T \).

According to the analysis of the signals in CE-BEM model and TV SIMO channel blind equalization architecture, we can get:

\[
x(n) = \sum_{l=0}^{L} h(n; l)s(n - l) + v(n), \quad n = 0, 1, \ldots, N - 1
\]  

The expression of matrix \( H_j \) \((j = 0, 1, \ldots, L)\) is as follows. Its dimension is \( M \times Q \).

\[
H_j = \begin{bmatrix}
e^{j\alpha_1} h_{1,1}(n; l) & e^{j\alpha_2} h_{1,2}(n; l) & \cdots & e^{j\alpha_Q} h_{1,Q}(n; l) \\
e^{j\alpha_1} h_{2,1}(n; l) & e^{j\alpha_2} h_{2,2}(n; l) & \cdots & e^{j\alpha_Q} h_{2,Q}(n; l) \\
\vdots & \vdots & \ddots & \vdots \\
e^{j\alpha_1} h_{M,1}(n; l) & e^{j\alpha_2} h_{M,2}(n; l) & \cdots & e^{j\alpha_Q} h_{M,Q}(n; l)
\end{bmatrix}
\]  

The dimension of matrix \( H \) is \( M(K + 1) \times Q(P + 1) \). The expression of the matrix \( H \) is as follows.

\[
H = \begin{bmatrix}
H_0 & \cdots & H_L & \cdots & 0_{M \times Q} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0_{M \times Q} & \cdots & H_0 & \cdots & H_L
\end{bmatrix}
\]
The expressions of the matrix $C(n)$ is as follows, and its dimension is $Q(P+1) \times Q(P+1)$.

$$C(n) = \text{diag}(e^{j\omega_1 n}, ..., e^{j\omega_Q n}, ..., e^{j\omega_Q(n-P)}, ..., e^{j\omega_Q(n-P)})$$ (9)

Based on the above definitions of matrices and vectors, we can get the following equation in the noiseless case.

$$\tilde{x}(n) = HC(n)\tilde{s}(n)$$ (10)

where $\tilde{s}(n) = [s(n), ..., s(n-P), ..., s(n-P)]^T$.

It is known that $g_1, g_2, ..., g_M$ are the equalizers of the corresponding receivers. The length of each equalizer is $K + 1$.

### 4.1 FSE-FLOS-CM algorithm of channel equalization

Constant modulus algorithm (CMA) is a common adaptive blind equalization algorithm for constant modulus signals [4]. The cost function of CMA is:

$$J_{2,2}(w) = E\left\{\left|y(n)\right|^2 - R_2\right\}^2$$ (13)

where $R_2 = \frac{E\{\left|y(n)\right|^4\}}{E\{\left|y(n)\right|^2\}^2}$ is a constant determined by the statistical properties of the source signal.

Obviously, the cost function shown in the above formula contains the fourth-order moment. If the received signal is in impulsive noise environment, the second or higher order statistics do not exist, CMA is no longer applicable. Therefore, CMA is applicable only when the received signal is in Gaussian noise environment.

According to the characteristics of alpha-stable distribution, when the received signal is in the impulsive noise environment, only the statistics with order less than alpha are finite. We call this kind of statistics “fractional lower-order statistics (FLOS)”. The FLOS-CMA algorithm proposed in [19, 20] can be used in this environment. The cost function of FLOS-CMA shown below can suppress impulsive noise by using the fractional lower-order statistics.

$$J_{p,2}(w) = E\left\{\left|y(n)\right|^p - R_p\right\}^2$$ (14)

where $R_p = \frac{E\{\left|y(n)\right|^p\}}{E\{\left|y(n)\right|^2\}^p}, 2p < \alpha, i.e., p < \frac{\alpha}{2}$.

Using the stochastic gradient descent method, we can get:

$$\nabla J(w(n)) = \left(\left|y(n)\right|^p - R_p\right)\left|y(n)\right|^{p-2}y^*(n)x(n)$$ (15)

Then the iterative solution equation of weight vector is obtained:

$$w(n+1) = w(n) - \mu x(n)e(n), \quad p < \alpha/2$$ (16)

$$e(n) = \left(\left|y(n)\right|^p - R_p\right)\left|y(n)\right|^{p-2}y^*(n)$$ (17)

The above iterative solution equation of weight vector can achieve the signal convergence in impulsive noise environment. It can be seen, if $p = 2$, FLOS-CMA degenerates into CMA algorithm.

Enough diversity (at least $M > Q$) is required for zero-forcing FIR equalizers in the fast TV channels described by CE-BEM model. According to [15], the quadruplet $(M, L, Q, K)$ obeys

$$M(K + 1) > Q(L + K + 1)$$ (18)

In Figure 3, there are $M$ channels to transmit signal $s(n)$. In this situation, the existing CMA and FLOS-CMA are not applicable, because they are all used for symbol rate equalizers. Therefore, the existing FSE-CMA algorithm based on the idea of fractionally spaced equalization can equalize the fast TV channels in Gaussian noise environment.

Learned from FSE-CMA, we apply the idea of fractionally spaced blind equalization to FLOS-CMA and the fast TV channels in impulsive noise environment can be equalized effectively. We call this algorithm FSE-FLOS-CMA. For one channel, the order of the corresponding equalizer is $K + 1$, so the order
| TABLE 1 | The pseudo-code of the novel source signal recovery method |
|---|---|
| **Input:** | Signal $\tilde{x}(\sigma)$, which is received by $M$ receivers. |
| **Output:** | Signal $\hat{s}(\sigma)$, which is obtained by the equalization and the estimation of basis frequencies. |
| **First step:** | Calculate the dispersion constant $R_p$, which is only related to the statistical characteristics of the transmitted signal.  
$R_p = \frac{E[|s(n)|^p]}{E[|\tilde{x}(n)|^p]}$, $p < \alpha/2$ |
| **Second step:** | Set the initial value of equalizer tap, and the value of the middle two taps of $M \times (K + 1)$ taps is set to 1. |
| **Third step:** | for the index values of the received signal sequence  
calculate the current error value $e(\sigma)$  
calculate the current equalizer tap values  
end for |
| **Fourth step:** | Calculate the output signal $y(\sigma)$ of the equalizer.  
$y(\sigma) = y^T \cdot \tilde{x}(\sigma)$ |
| **Fifth step:** | The fractional lower-order cyclic moment is calculated to obtain $\hat{\omega}_q$, and $\hat{s}(\sigma)$ is obtained by calculating formula $\hat{s}(\sigma) = y(\sigma) \cdot e^{-j\hat{\omega}_q k}$. |

**FIGURE 4** The signals after equalization using different algorithms. (a) The equalization algorithm is FSE-CMA. (b) The equalization algorithm is FSE-FLOS-CMA. (c) The equalization algorithm is FLOS-CMA.
FIGURE 5  The waveforms after equalization using different equalizers. (a) \( K = 2 \) (b) \( K = 6 \)

FIGURE 6  The convergence performance of different blind equalization algorithms. (a) FSE-CMA algorithm, (b) FSE-FLOS-CMA algorithm
of the equalizer corresponding to $M$ channels is $M \times (K + 1)$. Therefore, signal $y(n)$ is obtained by the $M \times (K + 1)$-tap equalizer using FSE-FLOS-CMA.

For adaptive fractionally spaced blind equalization algorithm FSE-FLOS-CMA, the number of the basis frequencies (i.e. the value of $Q$) of the fast time-varying channels described by CE-BEM model can be up to 5. As mentioned in Section 3, $Q$ is set to 2–5 due to the high-speed time-varying multipath. Therefore, five basis frequencies can be applied to most channel cases using CE-BEM model. Ref. [15] proposes a blind equalization (non-adaptive) algorithm based on CE-BEM model, on which we can further study the adaptive blind equalization algorithm with $Q$ from 6 to 9.

### 4.2 Frequency estimation method

The fast time-varying channel model proposed in this paper is based on SIMO architecture, therefore, the received signals have cyclic frequencies. In the real system, the inconsistency of the frequencies and phases of the received and transmitted signals lead to the cyclic frequency offset [34, 35]. In this paper, it is assumed that the transmitted and received signals are completely synchronized, so there is no cyclic frequency offset.

In the Gaussian noise environment, the second-order statistics of signals can effectively suppress the stochastic noise, then can extract the useful signal information. The frequencies of complex exponentials in CE-BEM are calculated from the so-called cyclic moments, the Fourier series of the TV moments [36, 37].

However, when there is severe impulsive noise in the environment, the performance of the second-order statistics is degraded significantly. So, in the impulsive noise environment, the $p$-order fractional lower-order moment, which has been defined above, is needed to calculate the frequencies.

If signals $x(n)$ and $y(n)$ have at least one non-zero cyclic frequency, we expand their $p$-order fractional lower-order moment into Fourier series in order to get the $p$-order fractional lower-order cyclic moment. Therefore, the fractional lower-order cyclic moment function of signals $x(n)$ and $y(n)$ is:

$$ R_{xy}^{(p)}(\varepsilon) = \frac{1}{N} \sum_{n=0}^{N-1} x^{p-1} \cdot e^{j2\pi\varepsilon n} = \langle x^{p-1} \cdot e^{j2\pi\varepsilon n} \rangle, \quad 1 \leq p \leq \alpha \quad (19) $$

where $\varepsilon = k/N$ is the cyclic frequency.

It can be seen, when $p = 2$, $R_{xy}^{(2)}(\varepsilon) = \langle x^{2} \cdot e^{-j2\pi\varepsilon n} \rangle$, that is, fractional lower-order cyclic moment degenerates to second-order cyclic moment.
As can be seen above, the output after equalization is \(y(n)\), the received signal of one channel is \(x_m(n)\). From Equation (12), the \(p\)-order fractional lower-order cyclic moment is:

\[
\left< y(n) \cdot \sum_{l=m}^{K} x_{m}^{<p-1>}(n-\tau) \right>_a
= \left< \xi(n-\tau_0) e^{j \omega_y (n-\tau_0)} \sum_{l=m}^{K} x_{m}^{<p-1>}(n-\tau) \right>_a
\]

\(\xi\) is the basis frequency. Because 0-\(Q\) order fractional lower-order cyclic moment is:

\[
\langle y(n) \cdot \sum_{l=m}^{K} x_{m}^{<p-1>}(n-\tau) \rangle_a
= \left< \xi(n-\tau_0) e^{j \omega_y (n-\tau_0)} \sum_{l=m}^{K} \sum_{q=1}^{Q} \left| h_{m,q}(s) e^{j \omega_y (n-\tau_0)} y(n-\tau - l) \right|^{p-2} \sum_{q=1}^{Q} \left| h_{m,q}(s) e^{j \omega_y (n-\tau_0)} y(n-\tau - l) \right|^2 \right>_a
\]

(20)

From Equations (4), (5) and the definition of \(< \cdot, \cdot >\), the above formula can be further transformed into:

\[
\left< y(n) \cdot \sum_{l=m}^{K} x_{m}^{<p-1>}(n-\tau) \right>_a
= \left< \xi(n-\tau_0) e^{j \omega_y (n-\tau_0)} \sum_{l=m}^{K} \sum_{q=1}^{Q} \left| h_{m,q}(s) e^{j \omega_y (n-\tau_0)} y(n-\tau - l) \right|^{p-2} \sum_{q=1}^{Q} \left| h_{m,q}(s) e^{j \omega_y (n-\tau_0)} y(n-\tau - l) \right|^2 \right>_a
\]

(21)

By the Fourier series of Equation (21). The cyclic moment can be computed efficiently by taking the fast Fourier transform (FFT) of the variables product.

It can be seen from Equation (21) that the frequencies corresponding to the peak values of the waveform are the differences between \(\omega_y\) and \(\omega_y q = 1, 2, \ldots, Q\), the required frequency \(\omega_y\) is a basis frequency. Because 0 = \(\omega_1 < \omega_2 < \ldots < \omega_{Q}\), \(\omega_y - \omega_1\) corresponds to the maximum peak frequency, it is the value of \(\omega_y\).

4.3 The detailed process of the novel source signal recovery algorithm

Based on Sections 4.1 and 4.2, the following summarizes the steps of recovering the source signal from the received signal through a fast TV channel in impulsive noise environment.

Step 1: Use FSE-FLOS-CMA to equalize data through fast TV channels based on the architecture of linear TV SIMO channel blind equalization.

Step 2: Calculate \(p\)-order fractional lower-order cyclic moment to find the peak values of the waveform. The peak values represent the differences between \(\omega_y\) and \(\omega_y q = 1, 2, \ldots, Q\). Since \(\omega_1\) is equal to 0, the maximum frequency value corresponding to the peak values represents \(\omega_y q\).

Step 3: The equalized signal \(y(n)\) is obtained in step 1; the desired basis frequency \(\omega_y\) is obtained in step 2. According to Equation (12): \(\hat{y}(n) = y(n) \cdot e^{-j \omega_y n}\), the estimated value of the source signal can be calculated by \(\hat{y}(n)\) and \(\omega_y\)

Table 1 shows how to use the above three steps to realize the novel source signal recovery method in MATLAB.

5 SIMULATIONS

When the transmission system is full response system, the signal source used in the simulation is 4 quadrature-amplitude modulation (4-QAM) signal, which is a constant modulus signal. If the transmission system is partial response system, the modulation mode of 4-QAM signal is changed to 9 quadrature-partial response (9-QPR) signal, which is not a constant modulus signal [36]. In the partial response system, the inter-symbol interference (ISI) is introduced at the sampling time of some symbols, which can increase the utilization of frequency band and reduce the requirement of timing accuracy. However, the output of partial response system is changed from two-electrical level form to three-electrical level form, which means the anti-noise ability is reduced, so 4-QAM signal is still used in this paper.

Simulation results have illustrated the effective recovery performance of the proposed novel source signal recovery method in the impulsive noise environment.

The novel proposed algorithm is mainly used for the fast TV channels, but it can also be used for the TV channels. When used
5.1 The performance of FSE-FLOS-CMA

The signal is received through TV channel described by CE-BEM model and it is added by the impulsive noise. GSNR of the signal is 30 dB.

The values of variables in CE-BEM model are as follows: \( L = 1, M = 4, Q = 2, K = 4 \). Moreover, the characteristic exponent of the impulse noise is equal to 1.1. The figure below shows the signals after equalization according to the above settings.

As can be seen from Figure 4(a), FSE-CMA cannot effectively equalize the fast TV signals with obvious impulse noise; FLOS-CMA shown in Figure 4(c) cannot equalize the fast TV signals described by CE-BEM. However, as can be seen from Figure 4(b), the new proposed FSE-FLOS-CMA can effectively equalize this kind of signal.

When the parameters in CE-BEM model are assigned as \( L = 1, M = 4, Q = 2, K = 2 \), and the characteristic exponent of the impulse noise is equal to 1.1, it can be seen from the Figure 5(a,b) that the more the number of equalizer taps, the better the signal convergence.

5.2 The performance of ISI index

Based on BE-CEM model, the signal after channel equalization is expressed as:

\[
y(n) = g^{H} \hat{x}(n) = g^{H} H C(n) s(n) + v(n) = g^{H} (n) s(n) + v(n)
\]

(23)

where \( g(n) = (H C(n))^{H} g \) is the system impulse response. \( C(n) \) is the matrix of complex exponential bases of a TV channel.

Because of the channel diversity of BEM model, the matrix is related to the time point \( n \) of the input signal \( s(n) \).

Usually, ISI is used to measure the convergence performance of blind equalization. The definition is as follows:

\[
ISI = \frac{\sum_{n} |q(n)|^{2} - \max_{n} |q(n)|^{2}}{\max_{n} |q(n)|^{2}}
\]

(24)

where \( q(n) \) is defined in Equation (22).

The parameters in CE-BEM model are assigned as \( L = 1, M = 4, Q = 2, K = 2 \), and the characteristic exponent of the impulse noise is equal to 1.1.

As can be seen from Figure 6, if the received signal is equalized by FSE-CMA, the residual ISI does not converge; however, if the received signal is equalized by the newly proposed FSE-FLOS-CMA, the residual ISI can converge.

5.3 The performance of fractional lower-order cyclic moment

The parameters in CE-BEM model are assigned as \( L = 1, M = 4, Q = 3, K = 2 \), and the characteristic exponent of the impulse noise is equal to 1.1. The three basis frequencies in the TV channel are \( \omega_1 = 0, \omega_2 = 2\pi/60, \omega_3 = 2\pi/40 \).

When the received signal is equalized by FSE-CMA, the waveform of the second-order cyclic moment is shown in the lower plot of Figure 7(a). When the received signal is equalized by FSE-FLOS-CMA, the waveform of the \( p \)-order fractional lower-order cyclic moment is shown in the lower plot of Figure 7(b). Because FSE-CMA cannot equalize the signal in the impulsive noise environment, as shown in the upper plot of Figure 7(a), we cannot get the required frequency from the peaks of the waveform in the lower plot of Figure 7(a). However, the received signal can be equalized by FSE-FLOS-CMA, as shown in the upper plot of Figure 7(b), the required frequency.
estimation can be obtained from the peaks of the waveform in the lower plot of Figure 7(b).

As can be seen from Figure 6(b), the largest frequency of the peaks of the waveform is $2\pi/40$, so $\omega_q = \omega_q - \omega_1 = 2\pi/40$.

If the received signals are all equalized by FSE-FLOS-CMA, the waveform of the second-order cyclic moment is shown in the upper plot of Figure 8. The lower plot of Figure 8 shows the waveform of the $p$-order fractional lower-order cyclic moment. From these two plots, we can see that if the signal contains impulse noise, the noise of the second-order cyclic moment is too large to submerge useful signal; while the $p$-order fractional low-order cyclic moment can output all useful information of the signal.

FSE-FLOS-CMA algorithm can equalize the fast time-varying channel described by CE-BEM model with up to 5 basis frequencies in impulsive noise environment. Figure 9 below shows the equalization algorithm and frequency estimation algorithm for fast time-varying signals with 5 basis frequencies.

6 | CONCLUSION

In this paper, we propose a novel source signal recovery method for TV channels described by CE-BEM in impulsive noise environment. In channel equalization phase of the source signal estimation, we proposed FSE-FLOS-CM algorithm. Simulation results show that the convergence performance of FSE-FLOS-CMA is much better than that of FSE-CMA in impulsive noise environment. In frequency estimation phase of the source signal estimation, we obtain the estimated frequency in impulsive noise environment by calculating the $p$-order fractional low-order cyclic moments. Simulation results show that the correct estimated frequency can be obtained by calculating the fractional lower-order cyclic moment using equalized signal, which is obtained by FSE-FLOS-CMA. Otherwise, the correct estimated frequency can’t be obtained in the cases of equalizing signals by CMA or calculating the second-order cyclic moment of the equalized signals. Moreover, we use the ISI index to show that FSE-FLOS-CMA is effective for the signal convergence in impulsive noise environment. The ISI index is calculated based on CE-BEM and SIMO architecture.

The newly proposed adaptive equalization algorithms can equalize the fast time-varying channels described by CE-BEM with up to 5 basis frequencies in impulsive noise environment. For this deterministic model, $Q$ is the number of basis frequencies which is less than 10. So future work is required for extending the adaptive equalization algorithm in impulsive noise environment.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

1. Scarano, G., et al.: Second-order statistics driven LMS blind fractionally spaced channel equalization. IEEE Signal Process. Lett. 24(2), 161–165 (2017)
2. Boss, D., Kammeyer, K., Petermann, T.: Is blind channel estimation feasible in mobile communication systems? A study based on GSM. IEEE J. Sel. Areas Commun. 16(8), 1479–1492 (1998), https://doi.org/10.1109/49.730455
3. Sato, Y.: A method of self-recovering equalization for multilevel amplitude-modulation systems. IEEE Trans. Commun. 23, 679–682 (1975)
4. Godard, D.: Self-recovering equalization and carrier tracking in two-dimensional data communication systems. IEEE Trans. Commun. 28(11), 1867–1875 (1980), https://doi.org/10.1109/TCOM.1980.1094608
5. Coskun, A., Kale, I.: Blind multidimensional matched filtering techniques for single-input–multiple-output communications. IEEE Trans. Instrum. Meas. 59(5), 1056–1064 (2010)
6. Tugnait, J.K., Luo, W.: Blind space–time multiuser channel estimation in time-varying DS-CDMA systems. IEEE Trans. Veh. Technol. 55(1), 207–218 (2006)
7. Chen, F.-J., Kwong, S., Kok, C.-W.: Blind MMSE equalization of SISO 1R channels using oversampling and multichannel linear prediction. IEEE Trans. Veh. Technol. (2009)
8. Giannakis, G.B., Tepedelenlioglu, C.: Basis expansion models and diversity techniques for blind identification and equalization of time-varying channels. Proc. IEEE 86(10), 1969–1986 (1998)
9. Giannakis, G.B.: Blind equalization of time-varying channels: a deterministic multichannel approach. In: Proceedings of 8th Workshop on Statistical Signal and Array Processing, Corfu, Greece, pp. 180–183 (1996), https://doi.org/10.1109/SSAP.1996.534848
10. Tsatsanis, M.K., Giannakis, G.B.: Modeling and equalization of rapidly fading channels. Int. J. Adapt. Control Signal Process. 10(2/3), 159–176 (1996)
11. Borah, D.K., Hart, B.T.: Frequency-selective fading channel estimation with a polynomial time-varying channel model. IEEE Trans. Commun. 47, 862–873 (1999)
12. Martone, M.: Wavelet-based separating kernels for sequence estimation with unknown rapidly time-varying channels. IEEE Commun. Lett. 3, 78–80 (1999)
13. Giannakis, G.B., Tepedelenlioglu, C.: Direct blind equalizers of multiple FIR channels: A deterministic approach. IEEE Trans. Signal Process. 47(1), 62–74 (1999)
14. Xu, G., et al.: A least-squares approach to blind channel identification. IEEE Trans. Signal Process. 43, 2982–2993 (1995)
15. Liu, H., Giannakis, G.B.: Deterministic approaches for blind equalization of time-varying channels with antenna arrays. IEEE Trans. Signal Process. 46(11), 3003–3013 (1998)
16. Li, Y., Ding, Z.: Global convergence of fractionally spaced Godard (CMA) adaptive equalizers. IEEE Trans. Signal Process. 44(4), 818–826 (1996) https://doi.org/10.1109/78.492535
17. Nikias, C.L., Shao, M.: Signal Processing with Alpha-Stable Distribution and Application. John Wiley & Sons, New York (1995)
18. Shao, M., Nikias, C.L.: Signal processing with fractional lower order moment stable processes and their applications. Proc. IEEE 81, 986–1010 (1993)
19. Rupi, M., et al.: Constant modulus blind equalization based on fractional lower-order statistics. Signal Process. 84, 881–894 (2004)
20. Tang, H., Qiu, T., Li, T.: Capture properties of the generalized CMA in alpha-stable noise environment. Wireless Pers. Commun. 49, 107–122 (2009)
21. Bai, E., Ding, Z.: Blind decision feedback equalization of time-varying channels with DPSK inputs. IEEE Trans. Signal Process. 49(7), 1533–1542 (2001)
22. Tepedelenlioglu, C., Giannakis, G.B.: Transmitter redundancy for blind estimation and equalization of time- and frequency-selective channels. IEEE Trans. Signal Process. 48(7), 2029–2043 (2000)
23. Gardner, W.A., Napolitano, A., Paura, L.: Cyclostationarity: half a century of research. Elsevier Sci. Signal Process. 86(44), 639–697 (2006)
24. Middleton, D.: An Introduction to Statistical Communication Theory. vol. 960, McGraw-Hill, New York (1960)
25. Mestre, X., Payaró, M., Shrestha, D.: Maximum-likelihood detection of impulsive noise support for channel parameter estimation. In: 2019 27th European Signal Processing Conference (EUSIPCO). A Coruna, Spain, pp. 1–5 (2019), doi: 10.23919/EUSIPCO.2019.8902843
26. Chen, Y., Xu, S., Lu, S.: Cyclostationary impulsive noise mitigation algorithm for narrowband powerline communications. J. Franklin Inst. 357(1), 687–703 (2020)
27. Chen, C.-Y., Chiu, M.-C.: Parameter estimation of impulsive noise for channel coded communication systems. IET Commun. 15(3), 445–452 (2020)
28. Chen, Y.-R., Yu, H.-C.: Mitigating impulsive noise for wavelet-OFDM powerline communication. Energies 12, 1567 (2019)
29. Lopes, P.A.C., Gerald, J.A.B.: Iterative MMSE/MAP impulsive noise reduction for OFDM. Digital Signal Process. 69, 252–258 (2017)
30. Chen, Y.: Iterative channel estimation and impulsive noise mitigation algorithm for OFDM based receivers with application to powerline communications. IEEE Trans. Power Delivery 30(6), 2435–2442 (2015)
31. Sen, L., Tian, S., Daifeng, Z.: Adaptive blind equalization for MIMO systems using blind equalization for MIMO systems under α-stable noise environment. IEEE Commun. Lett. 13(8), 609–611 (2009), https://doi.org/10.1109/LCOMM.2009.081982
32. Jakes, W.C., Microwave Mobile Communications. IEEE Press, New York (1994)
33. Jeruchim, M.C., Balaban, P., Shanmugan, K.S.: Simulation of Communication Systems. Plenum Press, New York (1992)
34. Chopra, R., Ghosh, D., Mehra, D.K.: FRESH filter-based spectrum sensing in the presence of cyclic frequency offset. IEEE Wireless Commun. Lett. 5(2), 124–127 (2016)
35. Chien, Y.-R.; Lin, J.-L.; Tsao, H.-W.: Cyclostationary impulsive noise mitigation in the presence of cyclic frequency offset for narrowband powerline communication systems. Electronics 9, 988 (2020)
36. Giannakis, G.B.: Cyclostationary signal analysis. Digital Signal Processing Handbook. In: Madisetti V. K., Williams D. (eds.) CRC Press, Boca Raton, FL (1998)
37. Peng, D., et al.: CM-based blind equalization of time-varying SIMO-FIR channel with single pulsation estimation. IEEE Trans. Veh. Technol. 60(5), 2410–2415 (2011)
38. K. Vasudevan: Digital Communications and Signal Processing. 2nd ed. Universities Press (2010)