THE RATIO OF HELIUM- TO HYDROGEN-ATMOSPHERE WHITE DWARFS: DIRECT EVIDENCE FOR CONVECTIVE MIXING

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ABSTRACT

We determine the ratio of helium- to hydrogen-atmosphere white dwarf stars as a function of effective temperature from a model atmosphere analysis of the infrared photometric data from the Two Micron All Sky Survey combined with available visual magnitudes. Our study surpasses any previous analysis of this kind, both in terms of the accuracy of the \(T_{\text{eff}}\) determinations and the size of the sample. We observe that the ratio of helium- to hydrogen-atmosphere white dwarfs increases gradually from a constant value of \(<0.25\) between \(T_{\text{eff}} = 15,000\) and \(10,000\) K to a value twice as large in the range \(10,000 \text{ K} > T_{\text{eff}} > 8000\) K, suggesting that convective mixing, which occurs when the bottom of the hydrogen convection zone reaches the underlying convective helium envelope, is responsible for this gradual transition. The comparison of our results with an approximate model used to describe the outcome of this convective mixing process implies hydrogen mass layers in the range \(M_{\text{H}}/M_{\text{tot}} = 10^{-10} \text{ to } 10^{-8}\) for about 15% of the DA stars that survived the DA-to-DB transition near \(T_{\text{eff}} \sim 30,000\) K, the remainder having presumably more massive layers above \(M_{\text{H}}/M_{\text{tot}} \sim 10^{-6}\).

Subject headings: convection — stars: atmospheres — stars: interiors — white dwarfs

1. INTRODUCTION

The Two Micron All Sky Survey (2MASS) represents one of the largest sets of homogeneous infrared photometric data for all areas of the sky applicable to white dwarf stars of various spectral types. Indeed, the 2MASS survey contains \(JHK_s\) magnitudes for almost all point sources of the sky up to the magnitude thresholds of \(J \sim 16.8\), \(H \sim 16.5\), and \(K_s \sim 16.2\) for detections with any formal uncertainty (see, for instance, Fig. 1 of Tremblay & Bergeron 2007). This yields a nearly complete magnitude-limited survey of white dwarfs that allows us to trace a portrait of the local population of white dwarfs, assuming we can properly identify the white dwarfs in this survey. To that effect, the most complete compilation is that of the Villanova White Dwarf Catalog1 (WDC), which contains more than 5500 spectroscopically identified white dwarfs reported in the literature. The interest of such a sizeable sample of homogeneous infrared photometric observations is to reach an unprecedented level in the statistical analysis of white dwarf stars. In addition, Tremblay & Bergeron (2007) have demonstrated the overall reliability of the 2MASS Point-Source Catalog (PSC) photometry for the purpose of detailed comparisons with the predictions from white dwarf model atmospheres.

Studies of the spectral evolution of white dwarfs attempt to understand how the surface composition and the corresponding spectral signature evolve along the white dwarf cooling sequence (see Fontaine & Wesemael 1987 for a review). Because of the intense gravitational field present in these stars, hydrogen will float to the surface in the absence of competing mechanisms, while heavier elements will sink below the photospheric regions. Indeed, the majority of white dwarfs have optical spectra that are completely dominated by strong hydrogen Balmer lines—the DA stars—with hydrogen-dominated atmospheres. We also find that almost 25% of white dwarfs have helium-dominated atmospheres, mostly DO, DB, DQ, and DZ stars, and some of the DC stars as well. The fact that not all white dwarfs have hydrogen-rich atmospheres implies either that some stars are born with hydrogen-deficient atmospheres and remain that way throughout their evolution, or, alternatively, that several physical mechanisms, such as diffusion, accretion, convection, radiation pressure, and stellar winds, are competing with gravitational settling to determine their surface composition as they cool off. This particular issue can be resolved by analyzing the distribution of spectral types as a function of effective temperature (or any temperature index) for a large sample of white dwarf stars in a way similar to the analysis of Sion (1984) nearly twenty-five years ago.

Before discussing the physical processes that will affect the spectral evolution of hydrogen- and helium-atmosphere white dwarfs, it is important first to consider the constraints brought about by the studies of the hottest white dwarfs. The canonical mass fractions of light elements in white dwarfs considering the mass threshold for residual nuclear burning are \(M_{\text{He}}/M_{\text{tot}} \sim 10^{-2}\) and \(M_{\text{H}}/M_{\text{tot}} \sim 10^{-4}\). However, multiple excursions on the asymptotic giant branch (AGB) nuclear burning phase, during which very late helium flashes could remove essentially all of the hydrogen (Werner & Herwig 2006), could also produce white dwarfs with total hydrogen masses much lower than the canonical value of \(10^{-4}\). The first class would correspond to the progenitors of the hottest DA stars, while the second class would represent the progenitors of hot white dwarf stars with hydrogen-deficient atmospheres—the PG 1159 stars at very high effective temperatures, and the DO stars at slightly lower temperatures, whose spectra are dominated by He ii lines. It has traditionally been believed that by the time DO stars cool off to \(T_{\text{eff}} \sim 45,000\) K, residual amounts of hydrogen (\(M_{\text{H}}/M_{\text{tot}} \sim 10^{-16}\)), thoroughly mixed in their helium-rich envelope, would gradually accumulate at the surface, turning all helium-atmosphere white dwarfs into DA stars (Fontaine & Wesemael 1987, 1991). This scenario would account for the existence of the so-called DB gap, a range of temperatures between \(T_{\text{eff}} = 45,000\) and \(30,000\) K, at which all white dwarfs have a DA spectral type. However, the recent discovery in the Sloan Digital Sky Survey (SDSS) of several DB

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1 See http://www.astronomy.villanova.edu/WDCatalog/index.html.
stars within the gap (Eisenstein et al. 2006a) implies that the total mass of hydrogen left in the envelope of DO stars can be even smaller than previously believed. It is then generally admitted that the significant increase in the number of helium-atmosphere white dwarfs below \( T_{\text{eff}} \approx 30,000 \) K can be explained in terms of the convective dilution of the superficial hydrogen atmosphere by the underlying helium convective envelope, provided that the hydrogen layer is sufficiently thin \( (M_{\text{H}}/M_{\text{tot}} \approx 10^{-15}; \) MacDonald & Vennes 1991).

At even lower effective temperatures \( (T_{\text{eff}} \lesssim 12,000 \) K), hydrogen-atmosphere white dwarfs get a second opportunity to evolve into stars with helium-dominated atmospheres when the superficial hydrogen layer becomes convective over a significant fraction of its depth. This process is illustrated in Figure 1, where the extent of the hydrogen convection zone is displayed as a function of decreasing effective temperature for a 0.6 \( M_\odot \) DA white dwarf, based on evolutionary models with thick hydrogen layers similar to those described by Fontaine et al. (2001) and kindly provided to us by G. Fontaine and P. Brassard. These calculations show that if the hydrogen envelope is thin enough, the bottom of the hydrogen convection zone may eventually reach the underlying and more massive convective helium layer, resulting in a mixing of the hydrogen and helium layers (Strittmatter & Wickramasinghe 1971; Shipman 1972; Baglin & Vauclair 1973; Koester 1976; Vauclair & Reisse 1977). Figure 1 also indicates that the effective temperature at which this mixing occurs will depend on the thickness of the hydrogen envelope. The thicker the envelope, the lower the mixing temperature; if the hydrogen layer is more massive than \( M_{\text{H}}/M_{\text{tot}} \approx 10^{-6} \), mixing will never occur.

The simplest physical model that can be used to describe this convective mixing process is to assume that hydrogen and helium are homogeneously mixed. Since the helium convection zone is much more massive \( (M_{\text{He–conv}}/M_{\text{tot}} \approx 10^{-6}) \) than the hydrogen layer when mixing occurs, it is generally assumed that a DA star would be transformed into a helium-atmosphere white dwarf of type DB, DQ, DZ, or DC with only a trace abundance of hydrogen. More detailed calculations discussed by Fontaine & Wesemael (1991) confirm these predictions. If such a process takes place in cool white dwarfs, we then expect the ratio of helium- to hydrogen-atmosphere white dwarfs to increase at low effective temperatures. A comparison between the observed ratio as a function of \( T_{\text{eff}} \) and the theoretical expectations would make it possible to estimate the thickness of the hydrogen layers in DA white dwarfs. In turn, these determinations could be compared to independent measurements of the hydrogen layer mass inferred from ZZ Ceti asteroseismology.

The best approach to study the occurrence of this convective mixing process begins with the statistical analysis of large samples of cool white dwarfs for which we can determine the main atmospheric constituent and the effective temperature, or any analog temperature index such as the absolute \( V \) magnitude. Sion (1984) was the first to study this problem by estimating the non-DA–to–DA ratio at low effective temperatures using a proper-motion sample of 695 white dwarfs drawn from the catalog of spectroscopically identified white dwarfs available at that time. The results from Figure 1 of Sion (1984) are reproduced here in Figure 2 in terms of the non-DA–to–DA ratio (rather than the absolute number of DA and non-DA stars) as a function of the absolute visual magnitude \( M_V \). Looking at the results, we note a first increase in this ratio at \( M_V > 11.25 \), which corresponds to \( T_{\text{eff}} \approx 15,000 \) K for a 0.6 \( M_\odot \) white dwarf, a temperature much in excess of the theoretically predicted value for the convective mixing process. A second increase occurs at \( M_V \approx 12.5 \), or \( T_{\text{eff}} \approx 10,000 \) K, but the observed ratio drops suddenly afterward, and this increase is probably not significant. Then a third increase occurs at \( M_V > 13 \), or \( T_{\text{eff}} < 8000 \) K. Clearly, the evidence based on these results is not exactly convincing, and our current picture of the situation below 12,000 K is at best sketchy. The global portrait did not change significantly since the analysis of Sion (1984), and this remains the only available proof quoted in the literature of the transformation of some DA stars into non-DA stars (see also Greenstein 1986).

Our goal is to improve on the analysis of Sion (1984) by using the 2MASS photometric sample combined with the WDC database.
to identify white dwarfs and determine their atmospheric composition and, most importantly, to obtain more accurate temperature determinations using a full model atmosphere analysis of the 2MASS white dwarf sample used in our analysis. The determination of the atmospheric parameters is then presented in § 3, and the uncertainties related to our approach are discussed at length in § 4. The evolution of the ratio of helium- to hydrogen-atmosphere white dwarfs as a function of effective temperature is then determined in § 5. Our conclusions follow in § 6.

2. DEFINITION OF THE 2MASS WHITE DWARF SAMPLE

Our starting point is the Villanova White Dwarf Catalog (WDC) of spectroscopically identified white dwarfs. We took the most recent electronic version of the catalog (2006 August), which includes a total of 5557 white dwarfs. We made a first selection by taking only the objects with at least one published visual magnitude and with at least one measurement brighter than \( V = 18.5 \). Figure 3 shows the predicted \( V \) magnitude for which \( H_{\text{2MASS}} = 16.5 \) — a representative value for the 2MASS detection limit in this band—as a function of \( T_{\text{eff}} \) for our pure hydrogen and pure helium model atmospheres at log \( g = 8 \). Only white dwarfs located below these curves at a given effective temperature can be detected by 2MASS in the \( J \) and \( H \) bandpasses. For white dwarfs with \( T_{\text{eff}} > 5000 \) K, the \( V < 18.5 \) criterion is sufficient to find a maximum number of white dwarfs. Finally, since our study is aimed at tracing the spectral evolution of cool white dwarfs, we neglected all objects with early spectral classifications (e.g., PG 1159, DO, etc.) or for which the spectral classification was unavailable altogether. Thus, a total of 1473 remaining white dwarf stars define our initial WDC sample.

We then searched the 2MASS PSC for \( JHK \) photometric observations of all white dwarfs from this initial WDC sample.

We used the GATOR batch file tool and a 20′ search window centered on a set of improved coordinates measured by J. B. Holberg (2005, private communication). In most instances, multiple sources were found within the search window, and we unambiguously identified each object by comparing the 2MASS atlas with the finding charts available from the online version of the Villanova WDC. For several objects close to the detection limits, only one or two magnitudes are detected with formal uncertainties. We neglect all objects with only one detected magnitude, since in such cases, only 1 degree of freedom would be available in our minimization procedure described in § 3. We recovered 940 objects in the 2MASS PSC from the 1473 objects contained in our initial WDC sample. The remaining 533 objects are eliminated for the following reasons: 508 are too faint to be detected by 2MASS in at least two bands, 15 are bright enough in the 2MASS atlas but are not part of the 2MASS PSC, and 10 could not be unambiguously identified from the comparison of the 2MASS atlas and the published finding charts.

A spectral energy distribution is then built for each object by combining the 2MASS \( JHK \) photometry with the \( V \) magnitude (or Strömgren \( y \)) taken from the WDC. In addition, about 10% of our \( V \) magnitudes come from unpublished USNO photometry (C. C. Dahn et al. 2007, in preparation). When more than one visual magnitude is available, we take an unweighted average, unless we have reason to believe that a given measurement was less accurate than others. We also assume a typical uncertainty of 0.05 mag for the visual magnitudes.

3. ATMOSPHERIC PARAMETER DETERMINATIONS

We classify the white dwarfs in our sample into two broad categories, those with hydrogen atmospheres (DA stars, including DAB and DAZ stars) and those with helium atmospheres (DB, DZ, and DQ stars, as well as helium-atmosphere objects with weak hydrogen features, such as Ross 640 and L745-46A; the case of DC stars is discussed further below). When the spectral classification is ambiguous, we check in the literature to confirm the presence of hydrogen lines compatible with the estimated effective temperature. We then fit the \( VJHK \) (or \( VJH \)) photometry with our grids of pure hydrogen or pure helium model atmospheres described in Tremblay & Bergeron (2007) and references therein. Our fitting technique is similar to that outlined in Bergeron et al. (1997, 2001). Briefly, the magnitudes are converted into monochromatic fluxes using the zero points defined in Holberg & Bergeron (2006) for photon counting devices, but using the transmission function of Bessell (1990) for \( V \) and the 2MASS transmission functions of Cohen et al. (2003) for \( JHK \). The resulting fluxes are then compared with those predicted from hydrogen- and helium model atmospheres, properly averaged over the same bandpasses. The effective temperature and the solid angle \( \pi(R/D)^2 \) (where \( R \) is the radius of the star and \( D \) is the distance to Earth) are considered free parameters in the \( \chi^2 \) minimization procedure. We assume a value of log \( g = 8.0 \) throughout. The surface gravity has very little effect on the predicted \( V \) — \( J \) color indices of white dwarfs. The results shown in Figure 4 indicate that a difference of 1 dex in log \( g \) corresponds to a color difference smaller than 0.05 mag at all temperatures. This slight difference, which is of the order of our color uncertainties, allows us to fix the surface gravity at log \( g = 8.0 \) for all objects. Since the mass distributions of hydrogen- and helium-atmosphere white dwarfs have to a good approximation the same mean value (Bergeron et al. 1992; Beauchamp 1995; Liebert et al. 2005; Voss et al. 2007), this simplifying assumption should not affect the results of our analysis significantly.
Each case that the best fit is in excellent agreement with the white dwarfs in our sample are displayed in Figure 5. We note in photometric effective temperature determinations reported in the temperatures for these objects with independent spectroscopic or white dwarfs. We also compare in Table 1 the photometric temperatures for 18 DB stars with those determined spectroscopically from the ongoing spectroscopic survey of A. Gianninas (2007, private communication). We note in this case that both samples are generally in good agreement, but several discrepancies are also observed for which the spectroscopic temperatures are significantly larger than the photometric values ($\Delta T_{\text{eff}} > 0$), in particular at high temperatures. A first explanation for this discrepancy is that above $T_{\text{eff}} > 11,000$ K, the $V - J$ color index is a slowly varying function of effective temperature (see Fig. 6), and thus white dwarfs with large temperature uncertainties contaminate this particular region. This is confirmed by the fact that most white dwarfs showing a large discrepancy are near the 2MASS detection limit (objects not detected at $K_s$ in 2MASS are shown as open circles in Fig. 8). Therefore, we add the constraint that only white dwarfs detected in all three $JHK_s$ bands are allowed in our analysis for $T_{\text{eff}} > 11,000$ K. The second group of objects showing a positive $\Delta T_{\text{eff}}$ are unresolved double-degenerate systems, three of which are identified in Figure 8 by filled squares.

We attempt in this section to better quantify the uncertainties of our effective temperature determinations in order to determine the optimal bin widths for the histograms used in our study of convective mixing, presented in § 5. These uncertainties can be first quantified by looking at the rate of change of the $V - J$ color index as a function of $T_{\text{eff}}$. We show in Figure 6 the effective temperature variation, $\Delta T_{\text{eff}}$, for a corresponding change of the $V - J$ color index of $\Delta(V - J) = 0.08$, a value that defines the mean uncertainty of this color index in our sample. We note that the $T_{\text{eff}}$ uncertainties increase rapidly toward high effective temperatures, and for this reason we restrict our analysis to $T_{\text{eff}} < 15,000$ K.

We compare in Figure 7 our photometric $T_{\text{eff}}$ determinations for 133 cool white dwarfs in common with the analyses of Bergeron et al. (1997, 2001), which are based on independent $BVRIJHK_s$ photometric data and, in several cases, trigonometric parallax measurements from which surface gravities can be determined. To be internally consistent, we have revised their atmospheric parameters using our own fitting procedure based on the slightly different synthetic photometric calibration of Holberg & Bergeron (2006). As well, we consider only the stars that have been observed in at least the $J$ and $H$ bands. We can see that the agreement is excellent in this temperature range and that the $VJHK_s$ photometry is sufficient to constrain the $T_{\text{eff}}$ values with the precision sought in our analysis.

We present in Figure 8 a similar comparison of our photometric temperatures with independent values obtained from line profile fitting of DA and DB stars. The spectroscopic temperatures for the 198 DA stars in common with our sample are taken from the ongoing spectroscopic survey of A. Gianninas (2007, private communication). We note in this case that both samples are generally in good agreement, but several discrepancies are also observed for which the spectroscopic temperatures are significantly larger than the photometric values ($\Delta T_{\text{eff}} > 0$), in particular at high temperatures. A first explanation for this discrepancy is that above $T_{\text{eff}} > 11,000$ K, the $V - J$ color index is a slowly varying function of effective temperature (see Fig. 6), and thus white dwarfs with large temperature uncertainties contaminate this particular region. This is confirmed by the fact that most white dwarfs showing a large discrepancy are near the 2MASS detection limit (objects not detected at $K_s$ in 2MASS are shown as open circles in Fig. 8). Therefore, we add the constraint that only white dwarfs detected in all three $JHK_s$ bands are allowed in our analysis for $T_{\text{eff}} > 11,000$ K. The second group of objects showing a positive $\Delta T_{\text{eff}}$ are unresolved double-degenerate systems, three of which are identified in Figure 8 by filled squares. In all cases the photometric temperature is substantially lower than the spectroscopic value, and both spectroscopic and photometric fits are excellent. It is therefore nearly impossible to eliminate double white dwarf systems from our analysis. There remain 10 other objects with a $\Delta T_{\text{eff}}$ larger than 20% for which the photometric fit is good, and there is no clear explanation for a discrepancy. These could also correspond to unresolved degenerate binaries, although a closer inspection suggests rather that the $V$ magnitudes, which are old measurements, are probably in error.

In the bottom panel of Figure 8 we compare our photometric temperatures for 18 DB stars with those determined spectroscopically by Beaugrand (1995) and Beaugrand et al. (1996). We

For the featureless DC stars, we consider both hydrogen- and helium-atmosphere solutions. If the hydrogen solution is above $T_{\text{eff}} = 5000$ K, we adopt the helium solution since, in principle, Hα should have been detected spectroscopically, according to the photometric analyses of Bergeron et al. (1997, 2001). However, if the hydrogen solution is below this temperature, it is impossible to determine the main atmospheric constituent from the restricted available photometry, and therefore we set the limit of our analysis to $T_{\text{eff}} > 5000$ K.

There is also a significant fraction of binary white dwarfs in our sample. If the companion is a cool main-sequence star, the observed $JHK_s$ fluxes are substantially larger than those predicted by the models, and the $J - H$ color index takes a large positive value (Tremblay & Bergeron 2007), which typically yields a bad fit to the energy distribution. We eliminated 90 binary candidates satisfying at least two of the following criteria: 1) a photometric temperature below 6000 K incompatible with the spectral type, (2) an observed flux at $H$ larger than at $J$, and (3) a flux at $J$ larger than 10 times the flux at $V$. We have also eliminated nine binary systems that are partially resolved in the 2MASS atlas, but whose colors in the 2MASS PSC are likely to be contaminated. The large majority of our binary candidates have been previously identified as such in the literature.

Sample fits for eight bright ($11 < V < 13.5$) and well-studied white dwarfs in our sample are displayed in Figure 5. We note in each case that the best fit is in excellent agreement with the $VJHK_s$ photometry, for both hydrogen- or helium-atmosphere white dwarfs. We also compare in Table 1 the photometric temperatures for these objects with independent spectroscopic or photometric effective temperature determinations reported in the literature. This comparison indicates that our technique applied to cool white dwarfs of various spectral types yields $T_{\text{eff}}$ values with a precision of the order of 5% compared to other methods, provided we have accurate photometric data.

4. ATMOSPHERIC PARAMETER UNCERTAINTIES

We attempt in this section to better quantify the uncertainties of our effective temperature determinations in order to determine the optimal bin widths for the histograms used in our study of convective mixing, presented in § 5. These uncertainties can be first quantified by looking at the rate of change of the $V - J$ color index as a function of $T_{\text{eff}}$. We show in Figure 6 the effective temperature variation, $\Delta T_{\text{eff}}$, for a corresponding change of the $V - J$ color index of $\Delta(V - J) = 0.08$, a value that defines the mean uncertainty of this color index in our sample. We note that the $T_{\text{eff}}$ uncertainties increase rapidly toward high effective temperatures, and for this reason we restrict our analysis to $T_{\text{eff}} < 15,000$ K.

We compare in Figure 7 our photometric $T_{\text{eff}}$ determinations for 133 cool white dwarfs in common with the analyses of Bergeron et al. (1997, 2001), which are based on independent $BVRIJHK_s$ photometric data and, in several cases, trigonometric parallax measurements from which surface gravities can be determined. To be internally consistent, we have revised their atmospheric parameters using our own fitting procedure based on the slightly different synthetic photometric calibration of Holberg & Bergeron (2006). As well, we consider only the stars that have been observed in at least the $J$ and $H$ bands. We can see that the agreement is excellent in this temperature range and that the $VJHK_s$ photometry is sufficient to constrain the $T_{\text{eff}}$ values with the precision sought in our analysis.

We present in Figure 8 a similar comparison of our photometric temperatures with independent values obtained from line profile fitting of DA and DB stars. The spectroscopic temperatures for the 198 DA stars in common with our sample are taken from the ongoing spectroscopic survey of A. Gianninas (2007, private communication). We note in this case that both samples are generally in good agreement, but several discrepancies are also observed for which the spectroscopic temperatures are significantly larger than the photometric values ($\Delta T_{\text{eff}} > 0$), in particular at high temperatures. A first explanation for this discrepancy is that above $T_{\text{eff}} > 11,000$ K, the $V - J$ color index is a slowly varying function of effective temperature (see Fig. 6), and thus white dwarfs with large temperature uncertainties contaminate this particular region. This is confirmed by the fact that most white dwarfs showing a large discrepancy are near the 2MASS detection limit (objects not detected at $K_s$ in 2MASS are shown as open circles in Fig. 8). Therefore, we add the constraint that only white dwarfs detected in all three $JHK_s$ bands are allowed in our analysis for $T_{\text{eff}} > 11,000$ K. The second group of objects showing a positive $\Delta T_{\text{eff}}$ are unresolved double-degenerate systems, three of which are identified in Figure 8 by filled squares. In all cases the photometric temperature is substantially lower than the spectroscopic value, and both spectroscopic and photometric fits are excellent. It is therefore nearly impossible to eliminate double white dwarf systems from our analysis. There remain 10 other objects with a $\Delta T_{\text{eff}}$ larger than 20% for which the photometric fit is good, and there is no clear explanation for a discrepancy. These could also correspond to unresolved degenerate binaries, although a closer inspection suggests rather that the $V$ magnitudes, which are old measurements, are probably in error.

In the bottom panel of Figure 8 we compare our photometric temperatures for 18 DB stars with those determined spectroscopically by Beaugrand (1995) and Beaugrand et al. (1996). We

3 Of these, 16 did not exactly meet two criteria, but they have poor fits and atmospheric parameters that are incompatible with the published spectral type.
see that the agreement is excellent, although the spectroscopic temperatures are systematically larger than the photometric values by about 5%. This small shift could be due to the neglect of small traces of hydrogen or metals in both analyses or to inaccuracies in the treatment of the broadening theory for neutral helium lines at low effective temperatures, which will mainly affect the spectroscopic temperatures (see, e.g., Kepler et al. 2007). The effective temperature dispersions observed in the figures above indicate that histograms with bin widths of 1000 K below $T_{\text{eff}} = 11,000$ and of 2000 K above this temperature are acceptable. These values are reproduced in Figures 6–8.

5. RESULTS ON CONVECTIVE MIXING

We present in Table 2 the number of hydrogen- and helium-atmosphere white dwarfs in different effective temperature bins for our final 2MASS sample of 447 objects (after the temperature cutoffs at $T_{\text{eff}} = 15,000$ and 5000 K and the removal of 2MASS data with low accuracy above $T_{\text{eff}} = 11,000$ K have been applied).
This sample is nearly twice as large as that used in the analysis of Sion (1984) in the same range of effective temperature. We also display in Figure 9 the corresponding histogram of the number of hydrogen- and helium-atmosphere white dwarfs as a function of $T_{\mathrm{eff}}$. The total number of stars in each bin below $T_{\mathrm{eff}} = 11,000$ K is comparable, a result that can be explained by the competition between the increasing luminosity function and the decreasing detection probability of cooler, fainter objects. We further observe that hydrogen-atmosphere DA white dwarfs are always the dominant type. Consequently, convective mixing—if this process occurs at all—should affect only a small fraction of cool DA stars.

For a magnitude-limited sample—here the $H$ magnitude—we probe a limited volume of space. This volume is defined in terms of the distance at which a given white dwarf can be detected at the limiting magnitude of the survey. However, this distance is different for hydrogen- and helium-atmosphere white dwarfs because they have different continuum opacity sources and corresponding monochromatic fluxes. Figure 10 presents the ratio of the volume probed by hydrogen and helium white dwarfs as a function of effective temperature for a magnitude limit in the $H$ bandpass.

While the assumption of a fixed value of $\log g$ was appropriate for determining the effective temperatures, this may not be the case for the volume effect discussed here. Indeed, white dwarfs stars with large radii (or low masses) can be detected at larger distances. The differences in the mass distributions of DA and DB stars could therefore affect the ratio (see also Eisenstein et al. 2006a). While the results of Beauchamp et al. (1996) suggest that the mass distribution of DB stars is narrower than that of DA stars, the recent results of Voss et al. (2007) reveal instead that both distributions are comparable. Hence, the small residual differences in the low- and high-mass tails are not expected to affect our results significantly.

To take into account the volume effect, we henceforth correct the ratio of the number of helium- to hydrogen-atmosphere white dwarfs, $N_{\mathrm{He-atm}}/N_{\mathrm{H-atm}}$, by the corresponding factor given in Figure 11. This ratio corrected for the volume effect is reported in Table 2 and displayed in Figure 11 as a function of effective temperature. The error bars are statistical, and they are mainly governed

| White Dwarf  | Spectral Type | $T_{\mathrm{eff}}$ (K) |
|--------------|---------------|-----------------------|
| 0046+051     | DZ            | 6716                  |
| 0135–052     | DA            | 7353                  |
| 0310–688     | DA            | 15891                 |
| 1142–685     | DQ            | 8429                  |
| 1647+591     | DAV           | 12557                 |
| 1917+077     | DBQA          | 11488                 |
| 1940+374     | DB            | 15565                 |
| 2032+248     | DA            | 20287                 |

References.—(1) Dufour et al. 2007; (2) A. Gianninas 2007, private communication; (3) Dufour et al. 2005; (4) Oswalt et al. 1991; (5) Beauchamp et al. 1996.
by the limited number of helium-atmosphere objects in each bin. We also show in the same figure the ratio obtained by shifting the temperature bins in Figure 9 by 500 K.

We can immediately note one important trend in the observed ratio of helium- to hydrogen-atmosphere white dwarfs displayed in Figure 11. At high effective temperatures, this ratio takes a constant value of \( \frac{N_{\text{He-atm}}}{N_{\text{H-atm}}} \approx 0.25 \), and it shows a gradual increase from \( T_{\text{eff}} \approx 10,000 \) K to a fairly constant value twice as large (\( \frac{N_{\text{He-atm}}}{N_{\text{H-atm}}} \approx 0.5 \)) at \( 8000 \) K. The only physical mechanism that could account for this increase is the convective mixing of the thin hydrogen layer with the more massive underlying helium envelope. These results further imply that about 15% of all DA stars\(^4\) between \( T_{\text{eff}} \approx 15,000 \) and 10,000 K have turned into helium-atmosphere white dwarfs—either DQ, DZ (or DZA), or DC—by the time they reach 8000 K. Note that the results of our analysis become too uncertain above the temperature range displayed here (see Fig. 6).

We can gain a better understanding of our results by looking at the simple theoretical model of convective mixing discussed in § 1. As the white dwarf cools off, the bottom of the hydrogen convection zone will eventually reach the helium convection zone, provided the hydrogen layer is thin enough (see Fig. 1), with the mixing temperature depending on the thickness of the hydrogen layer. White dwarfs with thicker hydrogen layers will mix at lower effective temperatures. A comparison of the results shown in Figures 1 and 11 implies that there are very few DA stars between \( T_{\text{eff}} \approx 15,000 \) and 10,000 K—the region where ZZ Ceti stars are found—with hydrogen layer masses smaller than \( \frac{M_{\text{H}}}{M_{\text{tot}}} \approx 10^{-10} \). The gradual increase of the observed ratio from 10,000 K down to 8000 K further implies that only 15% of DA white dwarfs in the 10,000–15,000 K temperature range have superficial hydrogen layers with a mass in the range \( \frac{M_{\text{H}}}{M_{\text{tot}}} \approx 10^{-10} \). Note that this range of layer mass depends sensitively on the assumed convective efficiency (\( \frac{M_{\text{L}}}{M_{\text{H}}} \approx 0.6 \) in Fig. 1). Envelope models with a more efficient parameterization of the convective energy transport would imply even thicker hydrogen layers (see Fig. 9 of Tassoul et al. 1990). The lack of evidence for convective mixing at lower effective temperatures between \( T_{\text{eff}} \approx 8000 \) and 5000 K implies a discontinuity in the hydrogen layer mass distribution, with the large majority (\( \approx 85\% \)) of all DA stars between \( T_{\text{eff}} \approx 15,000 \) and 10,000 K having hydrogen layer masses...

\[^4\] The transformation implies the following change of ratio \( \frac{N_{\text{He-atm}}}{N_{\text{H-atm}}} \rightarrow \frac{(N_{\text{He-atm}} + N_{\text{H-transformed}})}{(N_{\text{H-atm}} - N_{\text{H-transformed}})}. \]

**TABLE 2**

| \( T_{\text{eff}} \) Range (10\(^3\) K) | \( N_{\text{He-atm}} \) | \( N_{\text{H-atm}} \) | Ratio\(^a\) |
|-----------------|-----------------|-----------------|-------------|
| 15–13           | 11              | 41              | 0.268       |
| 13–11           | 9               | 38              | 0.259       |
| 11–10           | 10              | 44              | 0.265       |
| 10–9            | 12              | 41              | 0.347       |
| 9–8             | 14              | 38              | 0.444       |
| 8–7             | 19              | 49              | 0.485       |
| 7–6             | 18              | 51              | 0.454       |
| 6–5             | 14              | 38              | 0.448       |

\(^a\) The ratio is corrected for the volume effect discussed in the text.

**Fig. 9.**—Histogram of the number of hydrogen-atmosphere (single-hatched region) and helium-atmosphere (double-hatched region) white dwarfs from the 2MASS sample as a function of \( T_{\text{eff}} \). The number of stars in each bin is per unit of \( 10^3 \) in \( T_{\text{eff}} \).

**Fig. 10.**—Ratio of the observed space volume for helium-atmosphere and hydrogen-atmosphere white dwarfs in an \( H \)-magnitude–limited survey as a function of \( T_{\text{eff}} \). \( T_{\text{eff}} \approx 15,000 \) and 10,000 K—the region where ZZ Ceti stars are found—with hydrogen layer masses smaller than \( \frac{M_{\text{H}}}{M_{\text{tot}}} \approx 10^{-10} \). The gradual increase of the observed ratio from 10,000 K down to 8000 K further implies that only 15% of DA white dwarfs in the 10,000–15,000 K temperature range have superficial hydrogen layers with a mass in the range \( \frac{M_{\text{H}}}{M_{\text{tot}}} \approx 10^{-10} \). Note that this range of layer mass depends sensitively on the assumed convective efficiency (\( \frac{M_{\text{L}}}{M_{\text{H}}} \approx 0.6 \) in Fig. 1). Envelope models with a more efficient parameterization of the convective energy transport would imply even thicker hydrogen layers (see Fig. 9 of Tassoul et al. 1990). The lack of evidence for convective mixing at lower effective temperatures between \( T_{\text{eff}} \approx 8000 \) and 5000 K implies a discontinuity in the hydrogen layer mass distribution, with the large majority (\( \approx 85\% \)) of all DA stars between \( T_{\text{eff}} \approx 15,000 \) and 10,000 K having hydrogen layer masses...

**Fig. 11.**—Ratio of the number of helium- to hydrogen-atmosphere white dwarfs as a function of effective temperature, corrected for the volume effect displayed in Fig. 10 and discussed in the text. The filled circles use the binning defined in Fig. 9, while the open circles present the results for the temperature bins shifted by 500 K. The error bars are statistical.
above $M_H/M_{\text{tot}} \sim 10^{-6}$ or so. The constant ratio of $\sim 0.25$ in this temperature range also implies that $\sim 20\%$ of all DA stars in the DB gap (or DB trough) between $T_{\text{eff}} = 45,000$ and $30,000 \text{K}$ have turned into DB stars by the time they reach $15,000 \text{K}$; we cannot tell from our results precisely at which effective temperature the DA-to-DB transition occurs.

The numbers given above could change slightly if one takes into account the presence of traces of metals (including carbon) and hydrogen in the model atmosphere calculations of DQ and DZ stars. Indeed, the atmospheric structure of cool helium-atmosphere white dwarfs is affected significantly by small traces of metals or hydrogen, which then provide most of the free electrons (Provencal et al. 2002; Dufour et al. 2005, 2007). These additional electrons increase the contribution of the He$^+$ free-free opacity, which in turn reduces the atmospheric pressure. The results of Dufour et al. (2005, 2007) reveal that photometric temperatures of DQ and DZ stars can be systematically reduced by $500 \text{K}$ on average when metals are included in the model calculations. Therefore, this could affect the precise temperature dependence of the change in the ratio of helium- to hydrogen-atmosphere white dwarfs observed in Figure 11.

We finally note that our results are compatible with the asteroseismological analyses of ZZ Ceti stars, which suggest that most ZZ Ceti stars have fairly thick hydrogen layers of the order of $M_H/M_{\text{tot}} \sim 10^{-6}$ and higher (see, e.g., Fontaine et al. 1992; Bergeron et al. 1993; Bradley 2001; Brassard & Fontaine 2006; Pech & Vauclair 2006). Only a small fraction (about $15\%$) of all ZZ Ceti stars should have thin layers according to our results. Furthermore, our results are also in qualitative agreement with the analysis of DZ stars by Dufour et al. (2007), who suggested that the presence of hydrogen in cool DZA white dwarfs could be explained better as the outcome of convectively mixed DA stars rather than due to accretion from the interstellar medium. This suggestion is based on estimates of the total mass of hydrogen present in the mixed hydrogen and helium envelope of DZ and DZA stars. Measurements of the hydrogen abundances in these stars suggest hydrogen layer masses in about the same range as that inferred here (see Fig. 13 of Dufour et al. 2007).

6. CONCLUSION

We have presented a detailed study of the spectral evolution of cool white dwarfs following the pioneering effort of Sion (1984). In particular, we have determined the ratio of helium- to hydrogen-atmosphere white dwarfs below $T_{\text{eff}} = 15,000 \text{K}$ as a function of effective temperature by increasing the size of the sample by a factor of 2 and by obtaining more precise measurements of the effective temperature of these stars through model atmosphere fits to Johnson $V$ (or Strömgren $y$) and 2MASS $JHK_s$ photometric observations. We have confirmed that in this temperature range the evolution of hydrogen- and helium-atmosphere white dwarfs is not independent, and we have revealed in greater detail the role of the convective mixing process responsible for the coupling between these two white dwarf types. We have found that about $15\%$ of cool hydrogen-atmosphere DA white dwarfs between $T_{\text{eff}} = 15,000$ and $10,000 \text{K}$ are transformed into helium-atmosphere non-DA white dwarfs at lower temperatures. Using a basic model for convective mixing, our results imply in turn that $15\%$ of the DA stars that have survived the DA-to-DB transition have hydrogen layer masses in the range $M_H/M_{\text{tot}} \sim 10^{-10}$ to $10^{-8}$, although the exact values depend on the assumed convective efficiency in the evolutionary model calculations. The remaining DA stars would presumably have more massive hydrogen layers, probably in excess of $M_H/M_{\text{tot}} \sim 10^{-6}$.

Further analysis of the convective mixing problem would require a significantly larger photometric sample, such as the ugriz photometric sample of the Sloan Digital Sky Survey (SDSS). This sample covers a few areas of the sky with a completeness of around $95\%$ up to $g = 20$. However, most spectroscopic follow-ups have relied on color-color diagrams to identify white dwarfs (Kleinman et al. 2004; Eisenstein et al. 2006b), and this method is strongly biased toward hot white dwarfs ($T_{\text{eff}} > 12,000 \text{K}$). Therefore, the SDSS white dwarfs identified so far cannot be used to study the convective mixing process at low effective temperatures. One way to proceed to identify cooler white dwarfs in the SDSS is through proper motion—color diagrams. Harris et al. (2006) have used this method to identify nearly 6000 white dwarfs, but a complete spectroscopic analysis similar to that of Kilic et al. (2006) has yet to be performed to take full advantage of this sample.

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