A Constructive GAN-based Approach to Exact Estimate Treatment Effect without Matching

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Abstract

Matching has become the mainstream in counterfactual inference, with which selection bias between sample groups can be significantly eliminated. However in practice, when estimating average treatment effect on the treated (ATT) via matching, no matter which method, the trade-off between estimation accuracy and information loss constantly exist. Attempting to completely replace the matching process, this paper proposes the GAN-ATT estimator that integrates generative adversarial network (GAN) into counterfactual inference framework. Through GAN machine learning, the probability density functions (PDFs) of samples in both treatment group and control group can be approximated. By differentiating conditional PDFs of the two groups with identical input condition, the conditional average treatment effect (CATE) can be estimated, and the ensemble average of corresponding CATEs over all treatment group samples is the estimate of ATT. Utilizing GAN-based infinite sample augmentations, problems in the case of insufficient samples or lack of common support domains can be easily solved. Theoretically, when GAN could perfectly learn the PDFs, our estimators can provide exact estimate of ATT.

To check the performance of the GAN-ATT estimator, three sets of data are used for ATT estimations: 1) A linear toy data set with 1-dimensional input and constant treatment effect is tested. The GAN-ATT estimate is 0.51% away from the preset ground truth. 2) A non-linear toy data set with 2-dimensional input and covariate-dependent treatment effect is tested. The GAN-ATT estimate is 1.70% away from the preset ground truth, which is better than traditional matching approaches including propensity score matching (PSM) and coarsened exact matching (CEM). 3) A real firm-level data set with high-dimensional input is tested and the applicability towards real data sets is evaluated by comparing the other two matching methods. Through the evidences obtained from the three tests, we believe that the GAN-ATT estimator has significant advantages over traditional matching methods in estimating ATT.

Key words: Counterfactual inference, Generative adversarial networks, Average treatment effect on the treated, Machine learning.

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1 Introduction

1.1 Aims and Motivations

Counterfactual inference is a causal inference research method that constructs hypotheses of no treatment happened in the past, making the effect that attribute to the treatment available to estimate. This method is extensively used in the evaluation of medical treatment, policy performance, and corporate decisions etc. Since counterfactuals did not really happen, one of the classic approaches for counterfactual inference is to search for non-treated samples that are as close as possible to the real samples that are treated, which can be considered as a matching process. In early studies, matching are usually proceed based on human experiences, in which selection bias may exist inevitably. In order to better eliminate selection bias, some advanced matching methods are further proposed, such as propensity score matching (PSM), coarsened exact matching (CEM).

In our previous study (You, Papps, 2022), both the two advanced matching methods are used to estimate the average treatment effect on the treated (ATT) of a firm-level data set. Evidence shows that the selection bias of covariates are significantly reduced. However, we also found that in high-dimensional sample matching processes, due to the existence of dimensionality reduction, sample dropping, or the lack of common support domains, the estimation of average treatment effect on the treated (ATT) may be inaccurate.

To find a solution, this paper proposes a constructive ATT estimator that integrates classic causal inference framework with a machine learning approach: generative adversarial networks (GAN), which can approximate the probability density function (PDF) of sample sets. By giving identical input condition into the PDFs of both treatment group and control group, the target conditional average treatment effect (CATE) can be estimated with no need of matching, and the ensemble average of the CATEs corresponding to all treated samples is the estimate of ATT. Using GAN-based infinite sample augmentations, problems caused by traditional matching can be easily solved. Theoretically, when GAN could perfectly learn the PDFs, our estimators can provide exact estimate of ATT.

Furthermore, in order to ensure the effectiveness of this approach, we constructed two sets of toy data with ATT ground truth to check the performance of the estimator. Finally, we apply the estimator to re-estimate the ATT of the previously-used firm-level data set.

1.2 Research Background and Literature Review

Generative Adversarial Networks (GAN) is systematically proposed by Goodfellow (2014), which is a latest deep learning approach involving dual neural network games. In early stages, GAN is mostly used for image processing. Through trained GAN models, images with certain given features can be generated arbitrarily. For example, with a few strokes on the screen, images of trees, rivers, or beaches can be randomly generated. With the popularity of GAN, this deep learning approach has gradually extended to other fields.

One of the innovative uses of GAN is for privacy protection. Huang et al. (2018) systematically proposed a generative network that can be used for privacy protection. Liu et al. (2018), Bae et al. (2019), Ponte (2020) use GAN for privacy control in the data of geolocation, medical information and marketing. Cai et al. (2021) systematically summarize all types of generative network used for privacy control in different perspectives.

While using GAN for privacy protection, scholars find that the function of GAN that generate identically distributed data sets can be further used for optimizing causal inference issues. Joint with counterfactual framework, Yoon & Van (2018) proposed GANITE to infer individualized treatment effect (ITE). With this method, Chu et al. (2019), Ge et al. (2020) estimate ITE based on medical data sets. Ghosh et al. (2021) use GAN to optimize the performance of propensity score matching.
However, current machine learning based counterfactual studies still focus on the idea of matching. The popular idea of Yoon & Van (2018) can be extended to the estimation of ATT, yet in their work GAN learning is used for inferring the counterfactual outcome of individual factual sample, which did not in fact jump out of the matching idea. In order to supplement the literature in this area, this paper proposes a constructive ATT estimator based on GAN deep learning approach: GAN-ATT, which is able to theoretically exact estimate ATT without matching. In details, GAN-ATT estimator includes four steps in estimation:

1) The two joint PDFs of samples in treatment group and control group are both learned from two GAN training;
2) Two synthesized data sets with ideally the same PDFs of real sample sets are generated based on the two trained models;
3) The conditional average treatment effect (CATE) is estimated by differentiating the two conditional PDFs with identical input condition;
4) ATT is estimated by the ensemble average of CATE over all treatment group samples.

There are several advantages of this estimator:

First, problems caused by insufficient samples or lack of common support domains can be completely solved by performing GAN-based sample augmentations, which can generate infinite samples of the two groups for PDF approximating. Second, the estimation of ATT can be theoretically proved 100% accurate in the ideal case that GAN generates samples with exact the same PDFs as the real data sets. Third, no matching performed in the estimator for ATT estimation. So the problem caused by matching, such as the information loss caused by dimensionality reduction in PSM (Marco & Sabine, 2008), or the sample loss caused by high-dimensional common support domain shortage in CEM (Iacus et al., 2012) can be bypassed. Fourth, incorporating machine learning approaches, the estimation results are more objective to the fact, the influence of human factors are reduced. Last, the privacy of sample providers can be protected if required.

In addition to proposing the estimator, we further construct two sets of toy data sets referring to the idea of Yoon & Van (2018), together with a real firm-level data set we previously used (You, Papps, 2022) to check the effectiveness of the GAN-ATT estimator. Evidence proves that the estimator is feasible and accurate.

2 Classic Counterfactual Inference Framework

2.1 Assumptions to Meet

To fit our estimator to the casual inference framework, according to (Rosenbaum and Rubin, 1983; Imbens and Rubin, 2015), the data set used for estimation should meet two assumptions: unconfoundedness (or conditional independence assumption) and overlap (or common support conditions).

**Assumption 1.** Unconfoundedness: $Y(d=0,1) \perp d \mid X$

In the assumption, $d$ is the treatment dummy, $Y(d = 0)$ is the outcome variable of the control group, $Y(d = 1)$ is the outcome variable of the treatment group, $X$ is covariate and $\perp$ denotes independence. This assumption can be understand as the difference between $Y(d = 1)$ and $Y(d = 0)$ under same $X$ should be attribute to the treatment (Caliendo & Kopeinig, 2008).
**Assumption 2. Overlap:** $0 < P(d = 1 \mid X) < 1$

This assumption indicates that for samples under any $X$, the probability of being treated or not being treated should not be equal to zero (Heckman et al., 1999). Otherwise, there will be a lack of common support domains for CATE calculations.

Note that the unconfoundedness assumption is relatively strong in practice. When focusing on estimating ATT only, the two assumptions can be weaken to 'unconfoundedness for controls' and 'weak overlap'. See Caliendo & Kopeinig (2008) for details.

### 2.2 Estimation of Average Treatment Effect on the Treated

Let \( \{y_d, x_d\} \) be the two sets of sample data sets that have been observed, where $y_d$ are outcome variables and $x_d$ are covariates; $d$ is the treatment dummy; $d = 1$ represents treatment group and $d = 0$ represents control group. Let the PDFs of the covariates $x_0$ and $x_1$ be $f_d(x)$, that is:

$$x_d \sim f_d(x), \ d = 0, 1$$

\hspace{1cm} (1)

Assumes that with given covariate $x_d$, the potential outcome variable $y_d$ conforms to the conditional probability density functions as follows:

$$y_d \mid x_d \sim g_d(y \mid x), \ d = 0, 1,$$

\hspace{1cm} (2)

where $y \mid x$ means the value of the potential outcome variable $y$ with given covariate $x$.

Note that when $f_0(x)$ and $f_1(x)$ share the same probability distribution, or share a common trend, the classic difference-in-difference (DID) method can be used for ATT estimation. However in practice, sample data sets are usually collected from observational experiments directly. The selection bias exists in covariates may lead to significant distribution differences between $f_0(x)$ and $f_1(x)$. In order to eliminate such selection bias, matching methods are usually required before ATT estimations.

Next, according to Heckman et al. (1997), ATT can be estimated as follows:

$$ATT = E_{x_1 \sim f_1(x)} \left\{ E_{y_1 \sim g_1(y \mid x)} \left\{ y_1^f \mid x_1 \right\} - E_{y_1 \sim g_1(y \mid x)} \left\{ y_1^c \mid x_1 \right\} \right\},$$

\hspace{1cm} (3)

where $y_1^f$ is the outcome variable of samples in the treatment group that are in real treated based on the fact, while $y_1^c$ is the counterfactual outcome variable of samples in the treatment group that is assumed not being treated.

Since the $y_1^f$ cannot be observed, to estimate the treatment effect, it is necessary to find samples with ideally the same features in the control group. That is to say, assuming no unobserved confounding covariates exist, $y_0^f \mid x$ and $y_1^c \mid x$ should have very similar statistical characteristics under same covariate $x$ (Heckman et al., 1997). This indicates that in the control group, $y_0^f \mid x_1$ with given covariate $x_1$ can be used as an estimate of $y_1^f \mid x_1$. In this case Equation (3) can be rewritten as:

$$ATT \approx E_{x_1 \sim f_1(x)} \left\{ E_{y_1 \sim g_1(y \mid x)} \left\{ y_1^f \mid x_1 \right\} - E_{y_0 \sim g_0(y \mid x)} \left\{ y_0^f \mid x_1 \right\} \right\}$$

\hspace{1cm} (4)

Note that in Equation (4), $E_{y_1 \sim g_1(y \mid x)} \left\{ y_1^f \mid x_1 \right\} - E_{y_0 \sim g_0(y \mid x)} \left\{ y_0^f \mid x_1 \right\}$ can be understand as the conditional average treatment effect (CATE) under condition $x_1$. Besides, the presumption of using Equation (4) is that the range of $x_1$ values is a subset of the range of $x_0$ values, otherwise it cannot
be guaranteed that any condition $x_1$ can be found in the control group for $y_0^f \mid x_1$, which means Assumption 2: Overlap cannot hold.

Classic matching-based ATT estimator such as PSM-DID and CEM-DID are all based on the idea of estimating $y_1^f \mid x_1$ via $y_0^f \mid x_1$. However, no matter which matching method is used, samples without common support domains and samples without matched pairs will be dropped, leading to an inevitable trade-off between estimation accuracy and information loss: the less samples are dropped, the greater the selection bias; the more samples are dropped, the more data information is lost. So, it is impossible for matching approaches to accurately estimate $y_1^f \mid x_1$ without information loss. To bypass this unsolvable problem, in the following section we propose a GAN-ATT estimator, which is able to estimate CATE in Equation (4) via GAN machine learning so that no matching process is required. For notation simplicity, in the following sections $y_0^f$ and $y_1^f$ are simplified to $y_0$ and $y_1$.

3 GAN-Counterfactual Inference Framework

3.1 Principles of GAN

Generative Adversarial Network (GAN) is first systematically proposed by Goodfellow et al. (2014). Generally, the training procedure of GAN can be summarized as the mutual iteration of two neural networks: generator $\mathcal{G}$ and discriminator $\mathcal{D}$. First, the generator $\mathcal{G}$ generates a set of data based on a white noise input. Next, with the comparison of the real data set, the discriminator $\mathcal{D}$ determines whether the synthesized data is real or fake (generated). As long as the value returned by discriminator $\mathcal{D}$ is fake, the generator $\mathcal{G}$ optimizes the synthesized data through a cost function, making it closer to the real. When the discriminator $\mathcal{D}$ cannot distinguish between real and fake, it can be considered that the training has converged. Based on this idea, Mirza & Osindero (2014) extend GAN into a conditional version, making inputs with conditions available to be trained. Referring to their work, conditional GAN will be used in this paper for CATE estimations. The specific algorithm is shown in the next section.

3.2 Algorithm of GAN Training

Figure 1(a) illustrates the detailed GAN training procedure. The two GAN training structures shown in Figure 1(a) are identical. Each training consists of two adversarial neural networks: generator $\mathcal{G}_d$ and discriminator $\mathcal{D}_d$, $d = 0, 1$, where $d = 1$ represents the GAN training for the treatment group and $d = 0$ represents GAN training for the control group. $\{y_d(i), x_d(i)\}$, $i = 1, \cdots, N_d$ is the real data set that is collected from sample pool. $\{\tilde{y}_d(i), \tilde{x}_d(i)\}$, $i = 1, \cdots, \tilde{N}_d$ is the synthesized data set that is generated from generator $\mathcal{G}_d$.

According to Goodfellow et al. (2014) and Mirza & Osindero (2014), generator $\mathcal{G}_d$ is a multi-layer feedforward neural network with adjustable parameters. The input of $\mathcal{G}_d$ is an independent and identically distributed Gaussian variable $z_d(i)$ together with a treatment dummy $d$. The output node of $\mathcal{G}_d$ is linear. The output is a synthesized data set $\{\tilde{y}_d(i), \tilde{x}_d(i)\}$ that will be next judged by the discriminator $\mathcal{D}_d$.

Discriminator $\mathcal{D}_d$ is also a multi-layer feedforward neural network. The input of $\mathcal{D}_d$ is $\{y_d(i), x_d(i)\}$ and $\{\tilde{y}_d(i), \tilde{x}_d(i)\}$. The output node of $\mathcal{D}_d$ is a sigmoidal function. The output value is $D_d(i)$, which gives a high score from input that is recognized from a real data set $\{y_d(i), x_d(i)\}$, or gives a low score from input that is recognized from a fake (synthesized) data set $\{\tilde{y}_d(i), \tilde{x}_d(i)\}$.
Next, with the output from discriminator $D_d$, generator $G_d$ reiterate the above procedure through a min-max game:

$$\min_{\mathcal{G}_d} \max_{\mathcal{D}_d} V(\mathcal{G}_d, \mathcal{D}_d) = \sum_{i \in [1, N_d]} \log [D_d(y_d(i), x_d(i))]+ \sum_{z_d(i) \sim \mathcal{N}(z_d)} \log [1 - D_d(\tilde{y}_d(i), \tilde{x}_d(i))],$$

(5)

where $\{y_d(i), x_d(i)\} \sim g_d^{\text{real}}(y, x)$ represents that $\{y_d(i), x_d(i)\}$ is collected from the real data set with the joint PDF of $g_d^{\text{real}}(y, x)$; $z_d(i) \sim \mathcal{N}(z_d)$ represents that $z_d(i)$ is collected from the sampling of a Gaussian distribution. It can be proved that when the real data set is sufficiently large, the output of the generator $\mathcal{G}$ $\{\tilde{y}_d(i), \tilde{x}_d(i)\} \sim g_d^{\text{syn}}(y, x)$ will approximate to the real data PDF $g_d^{\text{real}}(y, x)$ in terms of the Kullback-Leibler (KL) divergence (Goodfellow et al., 2014).

To sum up, GAN requires the input of $z_d(i)$ and $d$, and the output generated is an arbitrarily large synthesized data set $\{\tilde{y}_d(i), \tilde{x}_d(i)\}$ that approximately obeys the PDFs of the real data set $g_d^{\text{real}}(y, x)$. The content of Figure 1(b) will be explained in the next section.

### 3.3 Combining GAN with Counterfactual Inference

In order to integrate GAN into counterfactual inference framework, two separate GAN training procedures for the two data sets $\{y_d(i), x_d(i)\}$, $d = 0, 1$ are required to perform. After GAN training converges, the synthesized data sets $\{\tilde{y}_d(i), \tilde{x}_d(i)\} \sim g_d^{\text{syn}}(y, x)$ can be used for CATE estimations. According to Figure 1, the specific estimating procedure can divided into 4 steps:

1) Joint probability learning

As it is shown in Figure 1(a), two conditional GANs are first used to learn the joint PDFs of the treatment group $g_1^{\text{syn}}(y, x) \rightarrow g_1^{\text{real}}(y, x)$ and the control group $g_0^{\text{syn}}(y, x) \rightarrow g_0^{\text{real}}(y, x)$ based on their corresponding real data sets $\{y_1(i), x_1(i)\}$ and $\{y_0(i), x_0(i)\}$. Through the training process mentioned in section 3.2, when reaching convergence, two trained models which can reflect the corresponding PDFs of treatment group and control group can be saved for further calculations.
2) Synthesized data generating

As it is shown in Figure 1(b), based on the two obtained trained models, \( \mathcal{G}_1 \) and \( \mathcal{G}_0 \) can generate two synthesized data sets \( \{\tilde{y}_d(i), \tilde{x}_d(i)\} \sim g_{xd}^{syn}(y, x) \rightarrow g_{xd}^{cal}(y, x), \quad d = 0, 1 \). With joint PDFs approximated to the real, the synthesized data sets follow the conditional average \( E \{y_d | x_d\} \rightarrow E \{y_d | x_d\} \). Note that in this step, the sample size generated \( \tilde{N}_d \) can be arbitrarily large. The larger the sample size generated, the less common support domains between \( \{\tilde{y}_1(i), \tilde{x}_1(i)\} \) and \( \{\tilde{y}_0(i), \tilde{x}_0(i)\} \) are missing.

3) CATE approximating via histogram

After data sets are generated by \( \mathcal{G}_1 \) and \( \mathcal{G}_0 \), histogram method is used on the synthesized sample sets to approximate the conditional average \( E \{y_d | x_d\} \). Following the basic idea of Pearson (1894) and Iacus et al. (2012), the conditional probability of each covariate will be fitted by a combination of multiple bars so that a multidimensional distribution can be approximated.

In details, define the dimension of \( x_d \) in the data set be \( q \). Coarsen \( x_d \) into multiple small \( q \)-dimensional equal-sized cubes. Let \( \Delta | x_d \) denote such a cube that centered at \( x_d \). All samples of \( \tilde{y}_d(i) \) with their covariates \( x_d(i) \in \Delta | x_d \) will be averaged to obtain the conditional average \( \tilde{y}_d|x_d \rightarrow E \{y_d | x_d\} \). When the number of cubes is sufficient large, the target distribution can be approximated. Note that the smaller the cube size \( \Delta \) is chosen, the more accurate the approximation of \( E \{y_d | x_d\} \) can achieve. Yet with the decrease of cube size \( \Delta \), the sample size of the synthesized data set must be increased to ensure the existence of the common support domains.

It can be found that the \( E \{y_d | x_d\} \) estimation obtained is in fact a function of \( x_d \). So, define a function as \( \Delta y(x) = E \{y_1 | x\} - E \{y_0 | x\} \), making \( \Delta y(x) \) be a function of conditional average treatment effect (CATE).

4) ATT estimating

At last, bring \( \Delta y(x) \) into Equation (4), the estimation of ATT can be obtained by averaging over the treatment group data set\(^1\)

\[
ATT = E_{x_1 \sim g_1(y_1|x_1)} \{\Delta y(x_1)\} \approx \frac{1}{N_1} \sum_{j=1}^{N_1} \Delta y(x_1(j))
\]  

(6)

It can be found that by replacing traditional matching procedure, this estimate is theoretically proved to be able to accurately estimate ATT provided that the GAN is perfectly trained. In section 4.1, a standard linear benchmark with ground truth will be taken as an example to show that the proposed approach can exactly recover the preset ATT ground truth.

3.4 Extension to Conditional Tabular GAN

In practice, it is possible for covariates to be discontinuous, leading to failures that the generator of traditional GAN cannot deal with. To make our estimator more applicable to tabular economic data sets, in this paper we proceed the training through the conditional tabular generative adversarial network (CTGAN) proposed by Xu et al. (2019). By introducing conditional generator joint with training-by-sampling algorithm, CTGAN can effectively identify and learn distributions of data with both continuous and discrete inputs.

\(^1\)In fact, any data set that shares common support domains can be brought back. If there are no privacy or other concerns, it is an easiest way to use samples of the treatment group in real data set as input because they have very close common support domains with the synthesized data sets.
Referring to the Python code provided by CTGAN (Xu et al., 2019), the joint probability density functions of the real data sets in this paper are learned through a method equivalent to the algorithm described in section 3.2. Figure 2 shows the detailed training process. It can be found in the figure that the only difference from the original method is that two separate GANs are merged into one and are trained separately by distinguishing the one-hot input vector $\tilde{d}$ ($\tilde{d} = '10'$ for treatment group and $\tilde{d} = '01'$ for control group). The output is same to the previously-mentioned methods. The major reason of using this equivalent algorithm is to save computing resources. Detailed algorithm and neural network structure see Xu et al. (2019, section 4).

With the trained model, any number of synthesized tabular sample with the joint PDFs same to the real can be generated.

![Figure 2(a). Conditional GAN training process. A single combined GAN is used for learning both treatment group data and control group data under one-hot condition $\tilde{d} = 10$ or 01.](image)

![Figure 2(b). Conditional ATE estimation via histogram. A single combine generator is used for producing both synthesized treatment group data and control group data under one-hot condition $\tilde{d} = 10$ or 01.](image)

4 Standard Benchmark Establishment and Estimation Accuracy Tests

4.1 Linear Benchmark Establishment

In order to verify the accuracy of GAN-ATT estimator, a standard linear benchmark with preset ATT ground truth is established in this section.

First, assume that the data of both treatment group ($d = 1$) and control group ($d = 0$) satisfy the following linear model:

$$y_d = \alpha + \beta x_d + \gamma \cdot d + \varepsilon_d, \quad d = 0, 1$$

In this model, $\alpha$ is the fixed effect, $\beta$ is the parameter of covariates $x_d$, $\gamma$ is the treatment effect that need to be estimated, $\varepsilon_d$ is an Gaussian white noise.

In order to make the benchmark closer to the real, we artificially set the selection bias as follows:

$$x_d \sim \mathcal{N}(\mu_d, \sigma_{xd}), \quad d = 0, 1,$$

which means the covariates $x_0, x_1$ follows two different Gaussian distributions with mean $\mu_d$ and variance $\sigma_{xd}$.

2Detailed code introduction see https://sdv.dev/SDV/user_guides/single_table/ctgan.html
Subsequently, when $\varepsilon_d$ is set to be additive white Gaussian noise, combining with Equation (7), there is:

$$y_d \mid x_d \sim \mathcal{N}(\alpha + \beta x_d + \gamma d, \sigma_{\varepsilon_d}), \quad d = 0, 1$$  \hspace{1cm} (9)

Next, follow the idea in the section 2.2, combine Equation (8)-(9) with Equation (2), there is:

$$E_{y_1 \sim g_1(y \mid x)} \left\{ y_1 \mid x_1 \right\} = \alpha + \beta x_1 + \gamma$$  \hspace{1cm} (10)

$$E_{y_0 \sim g_0(y \mid x)} \left\{ y_0 \mid x_1 \right\} = \alpha + \beta x_1$$  \hspace{1cm} (11)

Bring Equation (10) and (11) back in Equation (4), there is $ATT = \gamma$.

The above derivation proves that no matter how much selection bias exists between $x_0$ and $x_1$, as long as the left sides of Equation (10) and Equation (11) can be calculated from sample sets, the ATT estimated by Equation (4) is theoretically the preset ground truth $\gamma$. Note that the ATT estimate of Equation (6) is an approximation of ATT estimate in Equation (4). That is to say, when the size of sample used for calculation is sufficiently large, the estimate of GAN-ATT is the preset ATT ground truth $\gamma$.

With the ATT ground truth preset, a linear benchmark is established. Referring to Equation (7), the model is set with $x_0 \sim \mathcal{N}(0, 1), x_1 \sim \mathcal{N}(1, 2); \varepsilon_0, \varepsilon_1 \sim \mathcal{N}(0, 0.1); \alpha = 0; \beta = 1.5; \gamma = 1$ is the ATT preset ground truth.

### 4.2 Estimator Performance Evaluation

After the linear benchmark is established, two toy data sets $\{y_d(i), x_d(i)\}$ (50,000 samples in each) are randomly generated from the given model. In order to avoid confusion, these two data sets will be called ‘real’ data sets in this section. Subsequently, two synthesized data sets (200,000 observations in each\(^3\)) with ideally the same distribution to the real data set is generated from GAN machine learning. Next, ATT will be estimated via CATE calculations, and the result will be compared to the preset ground truth $\gamma = 1$.

Figure 3 illustrates the $\Delta y(x)$ values (CATE) in all bar $\Delta$ in the real data set with common support domain; Figure 4 illustrates the $\Delta y(x)$ values in all bar $\Delta$ in the synthesized data set with common support domain. It can be found that there do exist some differences between the two figures, especially in the fringes of the bar axis. This is mainly caused by insufficient samples for CATE calculations on the fringe. That is, very few synthetic samples fall in these bars, making the estimation of $\Delta y(x)$ biased. However, in the case of estimating ATT, this difference has no major effect on the estimation accuracy. This is because similarly, when bringing the real treated samples back into $\Delta y(x)$, very few samples will fall into these biased bars, so the biased estimation will be few. Furthermore, by increasing the sample size of the synthesized data sets, the bias can be significantly reduced.

\(^3\)These samples are randomly generated by the trained model of GAN. These two synthesized data sets with arbitrarily large sample size will only be used for intermediate calculations of CATEs and does not affect the sample size for the final calculation of ATT.
Table 1 is the ATT estimation statistics of this benchmark, including real data sets and synthesized data sets of both treatment group and control group. In the last column, the mean of Estimated ATT = 0.9949 is the estimated average treatment effect of the treated (ATT) according to Equation (6). It can be observed that the estimated ATT is 0.51% away from the preset ATT.
ground truth $\gamma = 1$. So, it can be concluded that the GAN-ATT estimator has a good performance over this toy data set. Note that the standard error of $\text{Estimated}_\text{ATT}$ can also be given by our estimator. If required, the confidence interval of ATT estimate can be derived by the obtained mean and variance under Gaussian distributions.

Table 1 ATT estimate - Benchmark 1

| No. Obs | Real$_{y0}$ | Real$_{y1}$ | Synthetic$_{y0}$ | Synthetic$_{y1}$ | Estimated ATT |
|---------|-------------|-------------|------------------|------------------|---------------|
| 50,000  | 50,000      | 200,000     | 200,000          | 49,280           |
| Mean    | -0.0090     | 2.5152      | 0.0813           | 2.4337           | 0.9949        |
| Std. Err| 1.5012      | 2.9971      | 1.7132           | 2.4812           | 5.55e-4       |
| Inverted Kolmogorov-Smirnov Statistic | - | - | 0.9796 | 0.9651 | - |
| Continuous Kullback-Leibler Divergence | - | - | 0.6867 | 0.6247 | - |

4.3 Advanced Non-linear Benchmark Establishment

To further check the performance of the estimator proposed in this paper, in this section the standard linear benchmark is extended into a 2-dimensional non-linear data set with the treatment effect dependent on the covariates.

The benchmark is established as follows:

$$y_k^0 = \alpha \cdot \frac{1 - e^{-W \cdot X_0^i}}{1 + e^{-W \cdot X_0^i}} + \epsilon_k^0,$$

$$y_k^1 = \beta \cdot \frac{1 - e^{-W \cdot X_1^i}}{1 + e^{-W \cdot X_1^i}} + t_k + \epsilon_k^1,$$

$$t = \gamma \cdot e^{-\|X_1\|^2_{S^2}}$$

In this model, $t$ is the preset CATE. $W$ is a $2 \times 1$ random vector that satisfies a uniform distribution: $W \sim \mathcal{U}(-1, 1)^{2 \times 1}$; $X_0$ and $X_1$ are the vectors of covariates in control group and treatment group that satisfy $X_0^i \sim \mathcal{N}(\mu_0, \Sigma \cdot \Sigma^T)^{2 \times 1}$, $X_1^i \sim \mathcal{N}(\mu_1, \Sigma \cdot \Sigma^T)^{2 \times 1}$, $\Sigma \sim \mathcal{U}(-1, 1)^{2 \times 2}$, $\mu_0 = 0^{2 \times 1}$, $\mu_1 \sim \mathcal{U}(-1, 1)^{2 \times 1}$; $\alpha, \beta, \gamma, \sigma$ are parameters to control the value ranges of $y$ and $t$, here $\alpha = \beta = 5, \gamma = 1.5, \sigma = 4$; $\epsilon_0$ and $\epsilon_1$ are white noises that satisfy $\epsilon_0, \epsilon_1 \sim \mathcal{N}(0, 0.1)$. Note that in order to keep selection bias exist, $\mu_0 \neq \mu_1$. With these settings, the ATT preset ground truth is $0.6836$.

Similarly, it can be proved in the same way in section 4.1 that the ATT estimated by the GAN-ATT estimator is theoretically equal to the preset ATT ground truth.

4.4 Advanced Estimator Performance Evaluation

After the advanced benchmark is established, two toy data sets $\{y_d(i), x_d(i)\}$ (100,000 samples in each) are randomly generated from the given model. To avoid losing common support domains, the sample size is also increased as the covariate dimension increases. Same to section 3.1, two synthesized data sets (400,000 observations in each) with ideally the PDF same to the real data set is generated from trained GAN model, and the ATT is estimated and compared to the preset

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4Here, the value of the preset ATT ground truth is approximated by repeating Monte Carlo simulation since the integral calculation of $t$ is over-complicated.
ground truth. Note that here since the data dimension is upgraded to two-dimensional, the bars used for CATE estimations should be upgraded into two-dimensional cubes $\Delta$.

In Figure 5, the CATE values in all cube $\Delta$ over the real data set with common support domains are presented, while the CATE values in all cube $\Delta$ over the synthesized data set with common support domain are presented in Figure 6.

![Figure 5 CATE values of all cubes in real data set](image1)

![Figure 6 CATE estimates of all cubes in synthesized data set](image2)

Table 2 is the ATT estimation statistics of this benchmark. It can be observed that the mean of $\text{Estimated}_{\_\text{ATT}}=0.6720$ is the estimated average treatment effect of the treated (ATT) according to Equation (6), which is 1.70% away from the preset ATT ground truth $0.6836$. It can be concluded that the estimator has an acceptable performance over this toy data set.
Table 2 ATT estimate - Benchmark 2

|                      | Real\_y0 | Real\_y1 | Synthetic\_y0 | Synthetic\_y1 | Estimated\_ATT |
|----------------------|----------|----------|---------------|---------------|----------------|
| Mean                 | 0.0009   | 0.8094   | -0.0173       | 0.7905        | 0.6720         |
| No. Obs              | 100,000  | 100,000  | 400,000       | 400,000       | 99,166         |
| Std. Err.            | 2.0300   | 2.0472   | 2.2268        | 2.0317        | 0.0021         |
| Inverted Kolmogorov-Smirnov D Statistic | -        | -        | 0.9389        | 0.9570        | -              |
| Continuous Kullback-Leibler Divergence | -        | -        | 0.7483        | 0.7139        | -              |

Furthermore, in order to make our estimator better adapt to the practical cases of insufficient samples, we randomly select 1,000 treated samples in our benchmark 2 as a new treatment group, so without changing the preset ATT ground truth, the sample size of Real\_y1 is reduced from 100,000 to 1,000.

In the case when the training data is insufficient, problems of overfitting may exist. To solve problems of this kind, methods of data augmentations, which add noise to the inputs, are widely used in the field of machine learning. Referencing to Bishop (1995), samples of the treatment group are augmented to 100,000 with input white noise. From Table 3 it can be found that the ATT estimation obtained from our GAN-ATT estimator is 0.6796, which is 0.59% away from the preset ground truth. It can be concluded in the case of insufficient treated samples for machine learning, GAN-ATT can still provide relatively accurate estimations.

Table 3 ATT estimates comparing to matching approaches- Benchmark 2

|                | GAN-ATT | PSM-NN | PSM-Kernel | CEM |
|----------------|---------|--------|------------|-----|
| Estimated\_ATT| 0.6796  | 0.7038 | 0.7706     | 0.7064 |
| No. Obs (Treatment Group) | 996     | 999    | 999        | 971  |
| No. Obs (Control Group)    | -       | 2,918  | 99,583     | 86,722 |

Additionally, for comparison propose, two traditional matching approaches: PSM and CEM are also used to estimate ATT over this benchmark. From Table 3 it can be observed that the ATT estimation via PSM-Nearest Neighbour 1:3 is 0.7038, which is 2.95% away from the ground truth; estimation via PSM-Kernel is 0.7706, which is 12.73% away from the ground truth; estimation via CEM is 0.7064, which is 3.34% away from the ground truth.

From the table it can be observed that comparing to the other three matching methods, the estimate of GAN-ATT is more accurate. In details, the ATT estimate of PSM-NN is relatively accurate, yet loses too many samples in the control group. PSM-Kernel retains nearly full samples in both two groups, but the estimation is not accurate. This might be attributed to the inaccurate weightings, because the distance between propensity scores cannot precisely reflect the real distance before dimensionality reduction. CEM performs a better trade-off between information loss and estimation accuracy, yet still cannot get rid of the problem of sample dropping due to insufficient common support domains. Through the performance test of the benchmark in this section, it can be concluded that the GAN-ATT estimator has obvious advantages over traditional matching approaches.

4.5 Estimator Performance Evaluation via Firm-level Data

Referring to our previous study (You & Papps, 2022), we use the same data set to further test the performance of our estimator in a more complex case. The obtained results will also be compared to previous results estimated by PSM and CEM.
Table 4 Descriptive statistics - Firm-level data

|          | Profit | Size     | Age      | Asset    | Output    |
|----------|--------|----------|----------|----------|-----------|
| No. obs  | 1,401,594 | 1,411,470 | 1,411,472 | 1,411,467 | 1,411,472 |
| Mean     | 10,549.64 | 260.32   | 9.88     | 167,026.2| 200,378   |
| Std. Err.| 32,685.11 | 430.92   | 9.25     | 2,162,198| 4,208,789 |

Table 4 is the descriptive statistics of the original data set used in our previous study, which is a panel firm-level data set 2007-2013 extracted from the Chinese Industrial Enterprise Database. According to the previous discussions, 2007-2009 is the time period before the treatment, while 2011-2013 is the time period after the treatment. Samples are previously matched based on firm features of each year before the treatment to eliminate selection bias. Two ATT estimations are obtained via difference-in-difference (DID) after matching. The DID calculation used for ATT estimation is: 1) Calculate the subtractions of the three-year means of the outcome variables before and after the treatment; 2) Calculate the subtractions of samples matched between the treatment group and the control group.

In order to incorporate the GAN-ATT estimator into an equivalent estimation to the previous, step 1 remains unchanged, while step 2 is replaced by the calculation of the CATEs. In our previous study, each firm feature is divided into three covariates for matching purpose (e.g. firm profit between 2007-2009 is divided into profit2007, profit2008, profit2009). Firm size, which takes the average of 2011-2013, is selected as the outcome variable, and the rest four firm features are divided into covariates with 10 dimensions. Note that firm age is constant over time, for calculation simplicity, only one dimension is assigned to firm age. With this, the original data set is transformed into an equivalent data set with 10-dimensional covariates and 1-dimensional outcome variable. Besides, since the GAN training requires the integrity of data input, we eliminate samples with missing input and fill a small amount of data (less than 4%) through the mean over time. The final sample size is 1,250 for the treatment group and 241,289 for the control group. It can be observed that the sample size of the treatment group is too small for machine learning. In order to prevent overfitting, white noise is similarly added to the input according to Bishop (1995). By this method, the number of input sample for GAN training is extended into 125,000 (treatment group) and 241,289 (control group).

Table 5 ATT estimates comparing to matching approaches - Firm-level data

|                      | GAN-ATT | PSM-NN | PSM-Kernel | CEM  |
|----------------------|---------|--------|------------|------|
| Estimated_ATT - Firm Size | 6.258   | 7.269  | 2.029      | 10.009 |
| No. Obs (Treatment Group/Control Group) | 1,216/241,289 | 1,249/240,956 | 1,182/303,699 |
| Inverted Kolmogorov-Smirnov D Statistic (T/C) | 0.9795/0.9417 | - | - | - |
| Continuous Kullback-Leibler Divergence (T/C) | 0.9068/0.9733 | - | - | - |

Table 5 is the results of ATT estimates via our GAN-ATT estimator together with PSM (Nearest Neighbour 1:3), PSM (Kernel) and CEM. It can be observed that the estimation of GAN-ATT estimator is 6.258, in which CATEs are estimated by 2,000,000 synthesized samples in both treatment group and control group. It can be found that GAN-ATT still suffers a loss of 34 treated samples, which is caused by the limited personal computing power. To improve the results, simply increase the number of synthesized samples generated by GAN for CATE estimations. Furthermore, the results obtained previously by the three matching approaches are also presented for comparison purpose. It can be found that the GAN-ATT estimate is not far from the estimates of the other three methods. Short of ground truth, it is hard to say which method is better in this section, but it can be concluded that our estimator is competent to estimate ATT in the case of high-dimensional covariate inputs and insufficient treated samples, while providing theoretically more accurate estimations with less information loss.
5 Discussion

Based on the preliminary performance tests for the GAN-ATT estimator in section 4, there are several points we need to further discuss:

1) The aim of section 4.1-4.2 is to validate the usability of our estimator via the simplest benchmarked data set. In this regard, a standard linear benchmark with one-dimensional covariate and with selection bias preset is established. By estimating a preset constant ATT ground truth, the estimation accuracy of our estimator can be tentatively revealed. Compared to commonly-used semi-benchmarked data sets, such as TWINS, JOBS, and IHDP (Yao et al., 2021), tests based on our benchmark can more intuitively reflect the absolute accuracy of ATT estimates instead of relative accuracy of various estimation methods, especially methods based on linear regressions. Our evidence shows that the estimator proposed are within an accuracy loss of 1%. So it can be concluded that the GAN-ATT estimator is effective and accurate in estimating this benchmark.

2) In section 4.3-4.4, to make our estimators more broadly applicable to economic data sets, the standard linear benchmark is extended into an advanced non-linear benchmark, which has not been used in the existing literature. The selection bias is same preset, yet the treatment effect is set dependent to the covariates with a non-linear relationship. Furthermore, the dimension of the covariates input is increased to two. The goal of establishing this benchmark is to test the learning ability of neural networks towards the non-linearities implicit in the data set, so as to check the accuracy of estimating conditional average treatment effect (CATE) and ATT. Our evidence shows that the accuracy loss of our estimator is 1.70%. Compared to traditional matching approaches: propensity score matching (PSM) and coarsened exact matching (CEM), the GAN-ATT estimate is proved more accurate while having less information loss. Thus, it can be concluded that the estimator has a good performance in this benchmark.

3) In section 4.5, we implement our GAN-ATT estimator to a real firm-level data set, which is previous used in our study (You, Papps, 2022) for a matching-based counterfactual inference estimation. With proper data cleaning and augmentation, we find that the estimator is fully competent for ATT estimation in the case of high-dimensional covariate inputs and insufficient treated samples. Unfortunately, due to the need to privacy protections, the database stopped updating after 2013. However, if our GAN-ATT estimator could be used for casual inference proposes, privacy of the data providers would be secured. By officially providing the pre-trained GAN models, the usage of real data set with privacy concerns can be replaced by a synthesized data set under same PDF. Joint with the ATT estimation procedure proposed in this paper, sample privacy issues can be effectively resolved.

4) GAN is a relativity new deep learning technique, especially for tabular data sets. It must be admitted that there are still shortages of GAN performance in practice, such as the training validity problem existing in our estimator. Ideally, the accuracy of our estimator would approach 100% if GAN were able to learn the PDF of a data set with complete accuracy. However in practice, there is currently no systematic way to verify the performance of GANs in the case with multidimensional inputs. We found that in the cases when dimension of covariates is high, performance evaluation criteria provided by CTGAN (Xu et al., 2019), including KL distance, KS statistic, etc. may fail. In this paper we try to avoid this failure by repeating training process with adjusted hyperparameters and layer scales. The results with closest mean and variance of the real data set are selected for further calculations. Nevertheless, this is not sufficient to prove that the two sets of data are equally distributed. We believe that with the development of GAN training techniques, better testing methods can be found in our future research.

5) As it is mentioned in section 2.1, to use our estimator, the unconfoundedness premise must be satisfied. Yet this premise is usually difficult to check in the field of economics. In order to more effectively eliminate confounding factors, several methods have been developed in the early stage, such as learning representation (Bengio et al., 2013), T-Leaner, R-Learner (KÄŒenzel et al. 2019), etc.. To better solve this problem, we are considering a promising approach to fit all possible covariates using neural networks. If the R-squared is sufficiently close to 1, it can be considered that
there is no major unobserved covariates. In order to better use our GAN-ATT estimator, this issue awaits our further research.

6 Conclusion

In this paper we propose the GAN-ATT estimator that can better estimate average treatment effect on the treated (ATT) without matching. With the support of the latest machine learning technique: Generative Adversarial Network (GAN), the conditional probability density functions (PDFs) of samples in both treatment group and control group can be obtained. By differentiating conditional PDFs of the two groups under identical input condition, the conditional average treatment effect (CATE) can be estimated, and the ensemble average of corresponding CATE over treatment group samples is the estimate of ATT. In particular, when GAN could perfectly learn the conditional PDFs through training, the ATT estimate can be proved completely accurate.

Furthermore, we generate two benchmarks with preset ATT ground truth. Together with a real firm-level data set, the performance of GAN-ATT estimator is directly tested. Evidence shows that: 1) For the two benchmarks, whether the model is set linear or non-linear, and whether the treatment effect is set dependent to covariates or constant, the estimator can closely approximate the ATT ground truths. 2) A real firm-level data set with high-dimensional covariates and insufficient number of treated samples is further used for the test. With proper data augmentations, the estimator is considered competent for estimating ATT in such case.

Compared to matching approaches, the evidence shows that the GAN-ATT estimator does have advantages in estimating ATT. Without matching, there is no compromise between selection bias and information loss need to take into considerations in our estimator: With GAN-based sample augmentations, infinite samples can be generated based on the trained model, so that the problems of insufficient samples or common support domains are solved; With good GAN learning performance, ATT estimations can be very close to the fact; With machine learning technique, the estimates are less influenced by human factors; With providing per-trained GAN models instead of real samples, fewer privacy concerns will be taken into consideration.

However, we also found that the performance of GAN deep learning is away from our expectations. Same to other machine learning approaches, training results are influenced by the hyper-parameters. Yet in the case of GAN, with high-dimensional variable inputs, there is no systematic way so far we know to check training performance. Nevertheless, this paper has given a theoretical proof that the GAN-ATT estimate is exact under the premise that GAN performs perfectly. With future advancement of GAN techniques, we believe our estimator can perform completely accurate estimate of ATT.
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