Fractionized Skyrmions in Dense Compact-Star Matter

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Summary

- The hadronic matter described as a skyrmion matter embedded in an FCC crystal is found to turn into a half-skyrmion matter with vanishing (in the chiral limit) quark condensate and non-vanishing pion decay constant $f_\pi$ at a density $n_{1/2}$ lower than or near the critical density $n_c$ at which hadronic matter changes over to a chiral symmetry restored phase with possibly deconfined quarks.

- When hidden local gauge fields and a dilaton scalar of spontaneously broken scale symmetry with decay constant $f_\chi$ are incorporated, half-skyrmion phase is characterized by $f_\chi \approx f_\pi \neq 0$ with the hidden gauge coupling $g \neq 0$ but $\ll 1$.

- While chiral symmetry is restored globally in this region in the sense that space-averaged, $\langle \bar{q}q \rangle$ vanishes, quarks are still confined in massive hadrons and massless pions.

- This phase is shown to play a crucial role in the model for a smooth transition from a soft EoS at low density to a stiffer EoS at high density, the changeover taking place at $n_{1/2}$. 
CONTENTS

I. Topology change

II. Skyrmion matter

III. Mapping $\frac{1}{2}$-skyrmions to “Bare” sHLS Lagrangian

IV. Applications to compact stars

V. Remarks
I. Topology Change
Skyrmion
A topology object in mesonic theory

\[ \mathcal{L} = \frac{f_\pi^2}{4} \text{Tr} (\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr} \left[ U^\dagger \partial_\mu U, U^\dagger \partial_\nu U \right]^2 \]

- \( f_\pi \): pion decay constant
- \( e \): Skyrme parameter

Topological soliton
wind number = baryon number

\[ B_\mu = \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \text{Tr} \left( U^\dagger \partial_\nuUU^\dagger \partial_\alphaUU^\dagger \partial_\beta U \right) \]

Although initiated by Skyrme in early 1960's for nuclear physics, is an approach barely explored and largely unfamiliar to most of the workers in the field.
Skyrmion makeover

Celebrating the treasures of topological twists.

After re-emerging from the depths of obscurity several times over, the spotlight is back on skyrmions. And a reader can only wonder what other neglected gems of mathematical ideas are tucked away in the literature, awaiting a creative scientist to recognize their value to the physical world?
Real-space observation of a two-dimensional skyrmion crystal

X. Z. Yu¹,², Y. Onose³,³, N. Kanazawa³, J. H. Park⁴, J. H. Han⁴, Y. Matsui¹, N. Nagaosa³,⁵ & Y. Tokura²,³,⁵

Figure 1 | Topological spin textures in the helical magnet Fe₀.₅Co₀.₅Si. 
- a, b, Helical (a) and skyrmion (b) structures predicted by Monte Carlo simulation. 
- c, Schematic of the spin configuration in a skyrmion. 
- d–f, The experimentally observed real-space images of the spin texture, represented by the lateral magnetization distribution as obtained by TIE analysis of the
Skyrmion matter from crystal

- Originally: I. Klebanov, Nucl. Phys. B262 (1985) 133-143. CC

- Kugler & Shtrikman, PLB208 (1988) 491; PRD40 (1989) 3421.

Widely used in condensed matter physics.
Order parameter:

\[ \langle \sigma \rangle = \frac{1}{(2L)^3} \int_0^{2L} d^3x \, \bar{q} q \]
Appearance of $\frac{1}{2}$-skyrmions is robust

B.Y. Park, V. Vento, MR et al since 1999

$$U = e^{2i\pi/f_\pi} \quad \rightarrow \quad \text{skyrmion}$$

$$U = \xi_L \xi_R^\dagger, \quad \xi_{L,R} \quad \rightarrow \quad \text{half-skyrmion}$$

$$\mathcal{L}_\xi = \frac{f_\pi^2}{2} \{ \text{Tr} \left[ |D_\mu \xi_L|^2 + |D_\mu \xi_R|^2 \right] \}$$
Topology change = Phase Change

Estimate: \( n_{1/2} \sim (1.3 - 2) n_0 \)
Meson spectrum: Going up $\infty$ tower

Nuclear force

- Vector meson:
  Essential for tensor force

- Light scalar meson:
  Attractive nucleon interaction

- Omega meson:
  Repulsive nucleon interaction

- At high density, short-distance interactions intervene, heavy-mass DoFs are needed.
Nuclear forces

\[ V(\rho) = \frac{f_2^2}{\rho} \frac{q^2}{q^2 + m_\pi^2} \left[ -\mathbf{\hat{\rho}} \cdot \mathbf{\hat{\rho}} - \mathbf{\hat{\rho}}_1 \cdot \mathbf{\hat{\rho}}_2 \right] \]

\[ V(\sigma) = \frac{g_\sigma^2}{q^2 + m_\sigma^2} \left[ -1 - \frac{\mathbf{\hat{L}} \cdot \mathbf{\hat{S}}}{2M^2} \right] \]

\[ V(\omega) = -\frac{g_\omega^2}{q^2 + m_\omega^2} \left[ +1 + \frac{3 \mathbf{\hat{L}} \cdot \mathbf{\hat{S}}}{2M^2} \right] \]

\[ V(\rho) = \frac{f_2^2}{12M^2} \frac{q^2}{q^2 + m_\rho^2} \left[ -2\mathbf{\hat{\rho}}_1 \cdot \mathbf{\hat{\rho}}_2 + \mathbf{\hat{\rho}}_1 \cdot \mathbf{\hat{\rho}}_2 \right] \]
The more vector mesons are included, the closer the static energy goes down and approaches the BPS bound. In other words, the higher tower of vector mesons drive the theory to a conformal theory.

\[ E^{(0)} = 1.235 \left( 8\pi^2 B \right) \]

\[ E^{(1)} = 1.071 \left( 8\pi^2 B \right) \]

\[ E^{(2)} = 1.048 \left( 8\pi^2 B \right) \]

The full tower of isovector mesons will bring this to the lower bound.

The high-lying vector mesons make the theory flow to a conformal theory.

Paul Sutcliffe, JHEP 1104 (2011) 045
Here we include the lowest-lying vector mesons $V = (\rho, \omega)$ and the scalar $\phi$ with the mass $\sim 600$ MeV.

The vectors will be incorporated as hidden local fields and the scalar as a dilaton.

For this we resort to scale-invariant hidden local symmetry (sHLS). We consider baryons generated from this sHLS Lagrangian as solitons.
Vector mesons as hidden local fields

\[ \mathcal{L}_\sigma = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) \quad \text{with} \quad U = \xi_L^\dagger \xi_R \]

Hidden Symmetry
\[ \xi_{L,R}(x) \to h(x)\xi_{L,R}(x), \quad h \in \text{SU}(2) \]
\[ V_\mu(x) \to ih(x)\partial_\mu h^\dagger(x) + h(x)V_\mu(x)h^\dagger(x) \]

Covariant derivative:
\[ D_\mu \xi_{L,R} = \partial_\mu \xi_{L,R} - iV_\mu \xi_{L,R} \]
\[ \hat{\alpha}_\mu = \frac{1}{2i} (D_\mu \xi_L \xi_L^\dagger + D_\mu \xi_R \xi_R^\dagger) \]
\[ \hat{\alpha}_\mu = \frac{1}{2i} (D_\mu \xi_L \xi_L^\dagger - D_\mu \xi_R \xi_R^\dagger) \]

Unitary gauge: \( \xi_L^\dagger = \xi_R = \xi \)

\[ \mathcal{L} = \mathcal{L}_A + a \mathcal{L}_V + \mathcal{L}_{\text{kin}} \]
\[ \mathcal{L}_A = f_\pi^2 \text{Tr}(\hat{\alpha}_\mu^2) = \mathcal{L}_\sigma \]
\[ \mathcal{L}_V = f_\pi^2 \text{Tr}(\hat{\alpha}_\mu^2) \]
\[ \mathcal{L}_{\text{kin}} = -\frac{1}{2g^2} \text{Tr}(F_{\mu\nu}^2) \]

\[ m_V^2 = ag^2 f_\pi^2 \]
\[ g_{\rho\pi\pi} = \frac{1}{2} ag \]

\( a = 2 \) gives KSRF relation and the universality of \( \rho \) coupling

M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. 164, 217 (1988)
Scalar meson as a hidden dilaton

A scalar meson of low mass ~600 MeV is an indispensable degree of freedom in nuclear physics.

It has figured for decades in phenomenological nucleon-nucleon potentials and in relativistic mean-field approaches for nuclear many body problems.

Although there are model-independent formalisms that establish its existence, now named $f_0(500)$ in PDG, its structure in the light of QCD is still more or less unknown.

We adopt the notion that the scalar meson needed in nuclear physics is a dilaton, a pseudo-Nambu-Goldstone boson, anchored on the conjecture that there is an IR fixed point in QCD that gives rise to a light-mass scalar, i.e., dilaton.

\[
\mathcal{L}_{dHLSI} = \mathcal{L}_{(2)}^{dHLSI} + \mathcal{L}_{(4)y}^{HLS} + \mathcal{L}_{(4)z}^{HLS} + \mathcal{L}_{\text{anom}}^{HLS} + \mathcal{L}_{\text{dilaton}},
\]  
\[
\partial^\mu D_\mu = \partial_\mu \frac{\beta(g_s)}{g_s} \text{Tr} G_{\mu\nu} G^{\mu\nu}
\]  
\[
\mathcal{L}_{(2)}^{dHLSI} = F_\pi^2 \left( \frac{\chi}{f_\chi} \right)^2 \text{Tr}[\hat{a}_\perp \hat{a}_\perp^\mu] + a F_\pi^2 \left( \frac{\chi}{f_\chi} \right)^2 \text{Tr}[\hat{a}_\parallel \hat{a}_\parallel^\mu] - \frac{1}{2 g^2} \text{Tr}[V_{\mu\nu} V^{\mu\nu}],
\]  
\[
\mathcal{L}_{\text{dilaton}} = \frac{1}{2} \partial_\mu \chi \partial^{\mu} \chi - \frac{m_\chi^2 f_\chi^2}{4} \left[ \left( \frac{\chi}{f_\chi} \right)^4 \left( \ln \left( \frac{\chi}{f_\chi} \right) - \frac{1}{4} \right) + \frac{1}{4} \right],
\]
II. Skyrmion Matter
The energy per particle of asymmetry nuclear matter is:

\[ E(n, \delta) = E(n, 0) + E_{\text{sym}} \delta^2 + O(\delta^4) \]
\[ \delta = (n_p - n_n)/(n_n + n_p) \]

To explore the symmetry energy from this model, we should collectively quantize the FCC multi-skyrmion system. Using the technique by Klebanov to collective-quantize the pure neutron system and for \( A \to \infty \),

\[ E_{\text{Nuclei}} = AE_{\text{Sol}} + \frac{1}{2A^2I} I^{\text{Tot}} (I^{\text{Tot}} + 1) \]

For a nuclei including both proton and neutron, defining \( \delta \equiv (N - P)/(N + P) \lesssim 1 \) we have

\[ I^{\text{Tot}} = \frac{1}{2} (N - P) = \frac{1}{2} (N + P) \frac{(N - P)}{(N + P)} = \frac{1}{2} A \delta. \]

So that the pre-nucleon energy can be expressed as

\[ E_{\text{Nucleon}} = \frac{E_{\text{Nuclei}}}{A} = E_{\text{Sol}} + \frac{1}{2A^2I} \frac{1}{2} A \delta \left( \frac{1}{2} A \delta + 1 \right) \approx E_0 + \frac{1}{8I} \delta^2. \]
Is the cusp real?

Answer: Yes

\[ E_{\text{sym}} = \frac{1}{8\lambda_I} \propto N_c^{-1} \]

B.Y. Park et al 2010

1/2-skyrmion
With the rich experimental data available, the symmetry energy is well determined up to nuclear matter density $n_0$.

At least up to $n_0$, Nature shows no clear indication for such a cusp structure seen in the skyrmion crystal; While $E_{sym}$ is unknown above $n_0$, the decrease toward $n_0$ is not visible in experiments.

The viability of the predicted feature? Unrealistic though it might appear, consistent with what is given by nuclear effective theory at the LO in many-body correlations.

Our main thesis: this topological structure gives a novelty to the nuclear tensor forces that govern the symmetry energy, give also a crucial clue to the structure of EoS at high density.
Another unexpected outcome from the transition from skyrmions to half-skyrmions is the behavior of the baryon mass as the density goes above $n_1/2$.

The mass of a single baryon estimated from the skyrmion crystal decreases smoothly as density approaches $n_1/2$ from below but at $n_1/2$ the quark condensate, while nonzero locally, vanishes globally and the baryon mass stays constant and non-zero.

The pion decay constant also remains non-zero.

Since hadron masses are gapped, the chiral symmetry remains broken.
One can write

\[ \frac{m_N^*}{m_N} \approx A + \Delta(\langle \bar{q}q \rangle) \]

- When the quark condensate averages to zero for \( n > n_{1/2} \), a large portion of the baryon mass remains nonzero.
- Although the bilinear quark condensate vanishes on average, it is non-zero locally and has a CDW structure. There is parity-doubling although pions are still present.
- At high temperatures, there are lattice indications for such an apparent chiral-invariant" mass.

L. Y. Glozman, et al, 2012
Inhomogeneous quark condensates

M. Rho, H. K. Lee, YM and M. Harada, PRD(2015).

1. Local chiral symmetry breaking (dense effect):

FIG. 1 (color online). The density effect on $\phi_0(x, y, 0)$ (first row) and $\phi_1(x, y, 0)$ (second row). The half-Skyrmion phase appears at $L = 1.45$ fm. (a) $L = 2.5$ fm. (b) $L = 1.5$ fm. (c) $L = 1.4$ fm. (d) $L = 0.9$ fm.
2. Local chiral symmetry breaking (resonance effect):

![Graphs showing chiral symmetry breaking](image)

**FIG. 2** (color online). The $\rho$, $\omega$, and $\chi$ meson effects on $\phi_0(x, y, 0)$ (first row) and $\phi_1(x, y, 0)$ (second row) in the Skyrmion phase at $L = 1.5$ fm. (a) HLS($\pi$). (b) HLS($\pi, \rho$). (c) HLS($\pi, \rho, \omega$). (d) dHLS-II($\pi, \rho, \omega, \chi$).
Mass spitting of the heavy-light mesons with chiral partner structure

- Chiral symmetry and chiral symmetry breaking have played predominant roles in hadron physics.

\[ m_q = 0 \]

\[ \downarrow \quad N_f \text{ flavor} \]

\[ U(N_f)_L \times U(N_f)_R \Rightarrow \text{The degeneracy of chiral partners} \]

Chiral Symmetry Breaking \(\Rightarrow\) chiral partner spectrum splitting

- The study of the chiral partner structure of hadrons can help us to reveal the magnitude of the chiral symmetry breaking, i.e., the order parameter.
Measurement of the heavy-light meson spectrum makes it possible to explore the chiral restoration in medium.

Chiral doubling for heavy-light mesons.

Doublets: \[
\begin{align*}
G &= D^{*}_{(s)0}(0^{+}), \quad D'_{(s)1}(1^{+}), \\
H &= D_{(s)0}(0^{-}), \quad D^{*}_{(s)1}(1^{-}),
\end{align*}
\]

Excited states; Ground states.

\[
\begin{aligned}
m_G - m_H &\approx 450 \text{ MeV}, \\
m_{D_{s0}(2317)} - m_{D_s} &\approx m_{D_{s1}(2460)} - m_{D^{*}_s} \approx 350 \text{ MeV},
\end{aligned}
\]

Chiral doubling seems to work.

M. A. Nowak, M. Rho and I. Zahed, Phys. Rev. D 48, 4370 (1993), Acta Physica Polonica (bf B35), 3277 (2004).
FIG. 1. (Color online) Medium modified heavy-light meson mass splitting as a function of the crystal size $L$. The vertical line indicates the critical density of the skyrmion-half-skyrmion phase transition.

D. Suenaga, B. R. He, YM, M. Harada, PRD 2015;
III. Mapping $\frac{1}{2}$-skyrmions to "Bare" sHLS Lagrangian
$E_{sym}$ is dominated by the tensor forces

G.E. Brown and R. Machleidt 1994 ... A. Carbone et al 2013

\[
V_{TM}^T(r) = S_M \left( \frac{f_{NM}^2}{4\pi} \right) \left( m_M \tau_1 \cdot \tau_2 S_{12} \biggl( \frac{1}{(m_M r)^3} + \frac{1}{(m_M r)^2} + \frac{1}{3m_M r} \biggr) e^{-m_M r} \right)
\]

$M = \pi, \rho$, $S_{\rho(\pi)} = +1(-1)$
Intrinsic density dependence of the Lagrangian parameters

The topology change at $n_{1/2}$ separates the density regime into two regions, Region-I for $n < n_{1/2}$ and Region-II for $n > n_{1/2}$. Combining experimental information available in R-I and what is inferred from presently available observables from compact stars for R-II, the IDD bare parameters of the Lagrangian can be obtained.
“Embed” in many-body correlations
To go above \( n_0 \)

If correct, signal for something new in hadron physics.
IV. Application to compact stars
Phenomena in R-I

C14 dating probes up to $n_0$

Can explain the long lifetime of carbon-14.
Phenomena in R-II

points: HLS $n_{1/2} = 2n_0$
A, B: Li constraints
C: Tsang constraint
Phenomena in R-II

(A): $n_{1/2} = 2.0n_0$, $M = 2.01M_{\odot}$, $R = 12.2\text{ km}$

(B): $n_{1/2} = 1.5n_0$, $M = 1.94M_{\odot}$, $R = 12.0\text{ km}$
A two-solar-mass neutron star measured using Shapiro delay. Here we present radio timing observations of the binary millisecond pulsar J1614-2230 that show a strong Shapiro delay signature. We calculate the pulsar mass to be $(1.97 \pm 0.04)M_\odot$, which rules out almost all currently proposed hyperon or boson condensate equations of state ($M_\odot$, solar mass). Quark matter can support a star this massive only if the quarks are strongly interacting and are therefore not ‘free’ quarks.

A Massive Pulsar in a Compact Relativistic Binary.

Results: We find that the white dwarf has a mass of $0.172 \pm 0.003 M_\odot$, which, combined with orbital velocity measurements, yields a pulsar mass of $2.01 \pm 0.04 M_\odot$. Additionally, over a span of 2 years,
V. Remarks
As skyrmions fractionize into half-skyrmions at $n = n_{1/2}$, there is an emergent parity doubling. The quark condensate vanishes on average, but locally non-zero, supporting CDW. There is pion with non-zero pion decay constant.

There also emerges an equally important scale symmetry, giving rise to “intrinisic density” dependence in the “bare” parameters of the effective Lagrangian.

The nucleon mass remains substantially non-melted, more or less density-independent within the range of density involved, with the bulk of the nucleon mass attributed to dilaton condensate and not to quark condensate.

Both the $\rho$ meson and the $\omega$ meson, play crucial roles for all range of densities. At low densities, the tensor force impacts strongly on nuclear structure.

At high densities, $n > 2n_0$, the VM of the $\rho$ meson controls the symmetry energy, making it stiffer at higher density.
Thank you for your attention.