Fuchs, Michael; Yu, Guan-Ru; Zhang, Louxin
On the asymptotic growth of the number of tree-child networks. (English) Zbl 1457.92126
Eur. J. Comb. 93, Article ID 103278, 21 p. (2021).

Summary: In a recent paper, C. McDiarmid et al. [Ann. Comb. 19, No. 1, 205–224 (2015; Zbl 1310.05120)] showed that the number of tree-child networks with \( n \) leaves has the factor \( n^{2n} \) in its main asymptotic growth term. In this paper, we improve this by completely identifying the main asymptotic growth term up to a constant. More precisely, we show that the number of tree-child networks with \( n \) leaves grows like

\[
\Theta\left(n^{-2/3} e^{a_1(3n)^{1/3}} \left(\frac{12}{e^2}\right)^n n^{2n}\right),
\]

where \( a_1 = -2.338107410 \cdots \) is the largest root of the Airy function of the first kind. For the proof, we bijectively map the underlying graph-theoretical problem onto a problem on words. For the latter, we can find a recurrence to which a recent powerful asymptotic method of A. Elvey Price et al. [J. Comb. Theory, Ser. A 177, Article ID 105306, 40 p. (2021; Zbl 1448.05034)] can be applied.

MSC:
92D15 Problems related to evolution
92D10 Genetics and epigenetics
92C42 Systems biology, networks

Keywords:
tree-child networks; asymptotic growth; phylogenetic networks

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