Phase coherence, inelastic scattering, and interaction corrections in pumping through quantum dots

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Adiabatic quantum pumping in noninteracting, phase coherent quantum dots is elegantly described by Brouwer’s formula. Interactions within the dot, while suppressing phase coherence, make Brouwer’s formalism inapplicable. In this paper, we discuss the nature of the physical processes forcing a description of pumping beyond Brouwer’s formula, and develop, using a controlled adiabatic expansion, a useful formalism to study the effect of interactions within a generic perturbative scheme. The pumped current consists of a first contribution, analogous to Brouwer’s formula and accounting for the remanent coherence, and of interaction corrections describing inelastic scattering. We apply the formalism to study the effect of interaction with a bosonic bath on a resonant level pump and discuss the robustness of the quantization of the pumped charge in turnstile cycles.

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Introduction - Adiabatic quantum pumping in quantum dots (QD) is one of the simplest, yet non trivial settings where the interplay between phase coherence and non-equilibrium dynamics can be studied both theoretically and experimentally. In a quantum pump, a slow, cyclic variation of the system parameters in time generates a direct current through the system. For suitable cycles the pumped charge may be quantized, a fact that might have important implications for metrological purposes.

In noninteracting, phase coherent systems, this phenomenon is encoded elegantly in Brouwer’s formula, describing geometrically the charge pumped per cycle in terms of the instantaneous scattering matrices of the system. Pumping in interacting systems is much less understood. In the past years several ad hoc approaches have been developed to address specific interacting problems aimed at deriving generalized pumping formulas, analogous to Brouwer’s one, within different approximation schemes (the average time approximation, and linear response). The formulas so obtained, while in principle equivalent, are both hardly usable as a starting point to address generic pumping problems. In addition, the lack of a general understanding of the physics of interacting quantum pumps makes it difficult to develop systematic approximation schemes.

The purpose of this paper is twofold. First of all, we develop a systematic physical understanding of the new elements brought in by interaction in the physics of quantum pumps. Secondly, by making use of a controlled gradient expansion, we derive a handy formalism to describe the effect of interactions on pumping within a generic perturbative scheme. The current can be conveniently written as the sum of two terms. The first, analogous to Brouwer’s formula (but expressed in terms of the elastic S-matrices), accounts the remanent phase coherence. The second term express the corrections due purely to interaction and contains the new physics brought in by it: inelastic scattering. We demonstrate this by studying the effect of the coupling to a generic equilibrium environment on a resonant level quantum pump. Computing the dependence of the pumped charge on the bath temperature, we prove the robustness against interaction of the quantization of the pumped charge in turnstile cycles.

Inelastic scattering and pumping - We begin with a qualitative discussion of the effects of inelastic scattering on adiabatic quantum pumping. To illustrate our ideas, we consider a concrete example of interacting quantum pump: a single level quantum dot (QD) coupled to a generic bosonic environment in equilibrium at temperature T. The Hamiltonian is

\[ H(t) = \sum_{k,\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha} + \epsilon(t) d^\dagger d + \sum_q \omega_q a_q^\dagger a_q + \sum_{k,\alpha} [V_{\alpha}(t) d^\dagger c_{k\alpha} + h.c.] + d^\dagger d \sum_q \lambda_q (a_q^\dagger + a_{-q}) \] (1)

Here the d’s refer to the resonant level, the a_q’s describe the bosonic modes, while c_{k\alpha} annihilates an electron in the left/right leads. The chemical potential of the two leads is assumed equal and put to zero. The instantaneous strength of the coupling to the leads is characterized by the parameter \( \Gamma(t) = \sum_{\alpha} \Gamma_{\alpha} = \sum_{\alpha} 2 \pi \nu |V_{\alpha}(t)|^2 \), where \( \nu \) is the density of states in the leads at the Fermi level and the parameters \( V_{\alpha} \) (as well as \( \epsilon \)) are assumed to vary cyclically in time with frequency \( \Omega \). The environment is specified through its spectral density

\[ J(\omega) = \sum_q |\lambda_q^2| \delta(\omega - \omega_q) = C \theta(\omega) \omega^s e^{-\omega/\omega_c}, \] (2)

where \( C \) is a constant, \( \omega_c \) a high frequency cutoff, and \( s \ll 1 \) corresponds to a sub/super-ohmic bath. We assume a small relaxation rate \( \gamma \ll \Gamma \) for the bosonic modes.
In the adiabatic limit $\Omega \ll \min[\Gamma(t), \gamma]$ the pumping current $I_\alpha(t)$ from the lead $\alpha$ to the quantum dot can be cast as the sum of a first term, which we attribute to elastic processes and a second, purely interacting term accounting for inelasticity, $I_\alpha(t) = I^\text{el}_\alpha(t) + I^\text{in}_\alpha(t)$.

The first

$$I^\text{el}_\alpha(t) = \sum_\beta \int \frac{d\omega}{2\pi} (-f') \left[ S_{\alpha\beta} \partial_\omega S^{\dagger}_{\beta\alpha} \right], \quad (3)$$

is analogous to Brouwer’s formula [2], though written in terms of the instantaneous elastic S-matrix of the QD. Indeed, while here $f(\omega)$ is the Fermi function of the leads, $S(\omega, t)$ is defined as the Wigner transform $S(\omega, t) = \int d\epsilon e^{i\omega\tau} S(t + \tau/2, t - \tau/2)$ of the time dependent scattering matrix

$$S_{\alpha,\beta}(t, t') = \delta_{\alpha\beta} \delta(t - t') - i2\pi \nu V^a_\alpha(t) G^\text{r}(t, t') V^b_\beta(t'), \quad (4)$$

where $G^\text{r}$ is the full QD Green’s function [15]. When $\Omega \ll \gamma$, we may express using the Fisher-Lee relation $S_{\alpha,\beta}(\omega, t) = \delta_{\alpha\beta} - i2\pi \nu V^a_\alpha(t) G^\text{r}(\omega, t) V^b_\beta(t)$ in terms of the instantaneous QD Green’s function $G^\text{r}(\omega, t)$, solving the time independent problem with frozen parameters $\{\epsilon(t), V(t)\}$. Following Ref. [12], the latter is easily seen to be the expression of the instantaneous elastic S-matrix.

The second term $I^\text{in}_\alpha$, in its most general form given in Eq. (19)-(21), is associated to the effect of inelastic scattering on the pumped current. This statement is corroborated by the solution of the present problem at low temperatures ($T \ll \Gamma$), where second order perturbation theory in the QD-bath coupling applies. Indeed, assuming for simplicity real QD-lead couplings we obtain

$$I^\text{in}_\alpha(t) = 4 \sum_\beta \int \frac{d\omega}{2\pi} (-f') \frac{\Gamma^\text{in}(\omega, t)}{\Gamma(t)} \text{Im} \left[ T_{0,\alpha\beta}^{\text{r},a} \partial_\omega T_{\beta0}^{\text{r},a} \right], \quad (5)$$

where the noninteracting T-matrices $T_{0,\alpha\beta}^{\text{r},a}$ are defined by the Fisher-Lee relation as

$$T_{0,\alpha\beta}^{\text{r},a}(\omega, t) = 2\pi \nu V^a_\alpha(t) G^\text{r}_{0,\alpha}(\omega, t) V^b_\beta(t), \quad (6)$$

in terms of the instantaneous advanced/retarded QD Green’s function $G^\text{r}_{0,\alpha}(\omega, t) = (\omega - \epsilon(t) + \frac{i}{2} \Gamma(t))^{-1}$. Notice that $I^\text{in}$ contains explicitly the inelastic scattering rate associated to the coupling to the environment

$$\Gamma^\text{in} = \int \frac{d\epsilon}{2} \left[ J(\epsilon) - J(-\epsilon) \right] \text{Im} \left[ A(\omega + \epsilon, t) \{f(\omega + \epsilon) + N(\epsilon)\} \right], \quad (7)$$

where $A = -2 \text{Im}[G^\text{r}_{0,\alpha}]$, and $N(\epsilon)$ is the Bose distribution function. As a consequence, this term plays no role at $T = 0$ as a result of the identity $\Gamma^\text{in}(\omega = 0) = 0$. The limit of weak inelasticity/dephasing considered here amounts to $\Gamma^\text{in} \ll \Gamma$.

In order to illustrate the role of inelastic processes in a concrete pumping cycle, let us consider the turnstile cycle of Fig. 1, keeping the width of the level constant in time $\Gamma(t) = 2\Gamma$, the pumping parameters are chosen to be the energy level $\epsilon \in [-\epsilon_0, \epsilon_0]$ and $\delta \Gamma = \frac{1}{T} \epsilon \in [-\Gamma, \Gamma]$. This cycle consists in a periodic opening/closing of the coupling to the left/right leads, followed by an inversion of the position of the level $\epsilon$ when the dot is coupled to one lead. In the absence of interaction and at low temperatures $T \ll \Gamma$ the pumped charge is

$$Q_0 = \frac{2}{\pi} \left[ \text{Arctg}(x) + \frac{x}{x^2 + 1} \right] - \frac{8\pi T^2}{3\Gamma^2} \frac{x}{x^2 + 1}, \quad (8)$$

where $x = \epsilon_0/\Gamma$ and the temperature dependence is due to thermal broadening only. Notice that, for a noninteracting resonant level, $Q_0 \approx 1$ for $x \gg 1$.

Let us now account for interactions and clarify their effect on both the elastic and the inelastic channels by separating their contributions to the pumped charge. The pumped charge associated to Eq. (3) is $Q^{\text{el}} = Q_0 + \delta Q^{\text{el}}$ consists of the sum of the noninteracting result Eq. (8), plus a correction originating from the loss of unitarity of the elastic S-matrices. This second contribution, obtained by expanding Eq. (3) to second order in the QD-bath coupling, reads

$$\delta Q^{\text{el}} = \frac{-\pi \alpha_s C T^{s+1}}{6 \Gamma^2} \left[ \frac{4x(9 + 8x^2 + 3x^4)}{(1 + x^2)^2} + 12\text{Arctg}(x) \right]. \quad (9)$$

where we set for simplicity $\gamma \ll T \ll \Gamma$, and $\alpha_s = \int dy (3/8 \pi^2) \text{Sech}(\frac{y}{T})^2 \{-\Gamma(s + 1)(\text{Li}_{s+1}(-e^{-y}) + \text{Li}_{s+1}(e^{-y})) + (3/2\pi^2)\Gamma(1 + s)\text{Li}_{s+1}(1)\}$, $\text{Li}_s(z)$ being a polylogarithm and $\Gamma(z)$ the Gamma function. Notice that summing up both contributions we obtain

$$Q^{\text{el}} \simeq 1 - \frac{\pi^2 \alpha_s C T^{s+1}}{\Gamma^2} \quad \text{for } x \gg 1. \quad (10)$$

Intuitively, the growth of the inelastic component should be at the expense of the elastic one. In other words, $\delta Q^{\text{el}}$ and $Q^{\text{in}}$ should have opposite sign. The presence of inelastic channels, not accounted for in the
elastic S-matrix, is also behind the lack of quantization of $Q^\rl$ for $x \gg 1$. Nonetheless, a proper account of inelastic scattering processes restores quantization for the total pumped charge $Q = Q^\rl + Q^\ini$, where $Q^\ini = \int dt I^\ini$. Indeed, an computation gives

$$Q^\ini = \frac{\alpha}{2} \alpha \sigma_{i,2} \Gamma \left( \frac{x(x^2+3)}{3(x^2+1)^3} + 12 \arctan(x) \right).$$

Therefore, summing up all contributions we have

$$Q = Q_0 - \frac{8\pi}{3} \frac{x}{[x^2+1]^3} \frac{\alpha c \Gamma T^{x+1}}{\Gamma^2},$$

(11)

which is approximately one for $x \gg 1$. This statement is valid also in other temperature ranges (e.g. $T \ll \gamma, \Gamma$).

Physically, as long as the resonance in the dot can still be fully loaded and unloaded ($x \gg 1$) charge quantization persists [10], while for intermediate $x$ the inelastic broadening of the resonance has a net effect on the temperature dependence of $Q$.

**General formalism** - The importance of inelastic channels and of keeping track of unitarity pertain also to the general pumping formula from which Eq.(5) was obtained. Let us sketch its derivation. Maintaining full generality, we consider a generic multilevel, interacting quantum dot described by

$$\mathcal{H} = \sum_{k,\alpha,\beta} \epsilon_k c_{\alpha,k}^\dagger c_{\alpha,k} + \sum_i \epsilon_i(t) d_i^\dagger d_i + $$

$$+ \sum_{k,\alpha,\beta} \left[ V_{\alpha,i}(t) d_i^\dagger c_{\alpha,k} + h.c. \right] + \mathcal{H}_{\text{int}}[d_i, d_i^\dagger].$$

(12)

Here $\mathcal{H}_{\text{int}}$ does not involve electrons in the leads. Moreover, in order to be able to take the adiabatic limit, we assume the instantaneous state of the interacting system to be Fermi-liquid-like, characterized by an exponential decay of instantaneous dot Green’s functions with a typical rate $\tilde{\Gamma}$.

The average current flowing through the dot may be expressed in terms of Keldysh Green’s functions. Indeed, using

$$I_\alpha(t) = \frac{d}{dt} \langle N_\alpha(t) \rangle = 2 \Re \left[ i \sum_{k,l} V_{\alpha,i}(t) d_i^\dagger(t) c_{\alpha,k}(t) \right],$$

(13)

where $N_\alpha = \sum_k c_{\alpha,k}^\dagger c_{\alpha,k}$, and using standard techniques [13] it is possible to show that

$$I_\alpha(t) = i(f(t-t_1) \otimes T^a_{\alpha \alpha}(t_1, t) + T^a_{\alpha \alpha}(t, t_1) \otimes f(t_1-t)) - i T^\ini_{\alpha \alpha}(t, t).$$

(14)

Here $f(t)$ is the Fourier transform of the Fermi function of the leads and the symbol $\otimes$ stands for a convolution $A(t_1) \otimes B(t_1, t') = \int dt_1 A(t_1) B(t_1, t')$. The full T-matrices are defined as

$$T^r,^{<\gamma,\alpha}_{\alpha,\beta}(t, t') = 2\pi \nu V_{\alpha}(t) G^{r,^{<\gamma,\alpha}}(t, t') V_{\beta}(t')$$

(15)

where $G^{r,^{<\gamma,\alpha}}$ are the full Green’s functions of the QD levels [13], and the boldface notation implies summation over level indices. Information on the distribution function of the dot, and of its deviation from the equilibrium $f(\omega)$ is contained in $G^<$. A compact formula for the pumped current may be obtained expressing $T^<$ in terms of the QD self energy $\Sigma^< = \Sigma^\ini + \Sigma^\text{int}$, where $[\Sigma^\ini(t, t')]_{\alpha,\beta} = i \sum_{\alpha,k} V_{\alpha,i}(t) f(\epsilon_k) V_{\alpha,j}(t') \exp[-i \epsilon_k(t-t')]$ is the self energy associated to the coupling to the leads only, and $\Sigma^\text{int}$ is the one containing interaction (as well as tunneling) vertices. Using the relation $G^< = G^+ \otimes \Sigma^< \otimes G^-$ we may recast the current (14) in the form

$$I_\alpha(t) = i(f \otimes T^a_{\alpha \alpha} - T^r_{\alpha \alpha} \otimes f) + \sum_{\gamma} T^r_{\alpha \gamma} \otimes f \otimes T^a_{\gamma \beta}$$

$$- i 2 \pi \nu (V_{\alpha}^a G^+) \otimes (\Sigma^\ini \otimes (G^- V_{\alpha})),$$

(16)

where the time dependence of the r.h.s is analogous in structure to Eq.(13).

We are now in the position to take the adiabatic limit by performing a gradient expansion [14, 15]. To this end, we notice that we may write $I_\alpha(t) = \int d\omega/(2\pi) I(\omega, t)$, where $I(\omega, t)$ is the Wigner transform of the function on the r.h.s. of Eq.(16). In order to express it in terms of the instantaneous quantities we recall that Wigner transform of a convolution has the simple expansion [15]

$$[A \otimes B]_{\omega,\tau} = AB + \frac{1}{2i} (\partial_\tau A \partial_\omega B - \partial_\omega A\partial_\tau B) + .$$

(17)

In the adiabatic limit, the expansion may be truncated to lowest nonvanishing order. Indeed, each derivative produces a factor proportional to $\Omega/\tilde{\Gamma} \ll 1$.

It is crucial to note that, in order to expand Eq.(16) consistently one has to account for conservation of probability or unitarity. In equilibrium this is guaranteed by the optical theorem, which out of equilibrium generalizes to

$$T^\alpha_{\alpha \beta}(t, t') - T^r_{\alpha \beta}(t, t') = i \sum_{\gamma} T^r_{\alpha \gamma} \otimes T^a_{\gamma \beta} +$$

$$+ 2 \pi \nu (V_{\alpha}^a G^+) \otimes (\Sigma^\ini \otimes -\Sigma^\text{int}) \otimes (G^- V_{\alpha}),$$

(18)

as one can easily see using the Dyson equations. In the absence of interaction Eq.(18) reduces to the unitarity condition for the time dependent S-matrix, $\sum_{\gamma} S_{\alpha,\gamma} \otimes S_{\gamma,\beta} = \delta_{\alpha,\beta} \delta(t-t')$.

Expanding now Eq.(18) to first order in the gradients one obtains two terms: the first, proportional to the Fermi function $f(\omega)$, vanishes once we take into account consistently the optical theorem Eq.(18) expanded to the same order in the gradients. The remaining expression, which physically originates from the deviation of the dot distribution function from the equilibrium form, takes the form $I = I^\rl + I^\ini$ were the first term $I^\rl$ is given by
Eq. (3), and $I_{1}^{n} = I_{1}^{n} + I_{2}^{n}$, where

$$
I_{1}^{n} = i v \int d \omega \ f(\omega) \left[ \Sigma_{t}^{r}(\omega, t) - \Sigma_{t}^{l}(\omega, t) \right] \nonumber
$$

$$
\text{Im} \left[ \partial_{\omega} \left( V_{a}^{r}(t) G_{r}(\omega, t) \right) G_{r}^{*}(\omega, t) V_{a}(t) \right] \nonumber
$$

$$
I_{2}^{n} = i v \int d \omega V_{a}(t) G_{r}^{*}(\omega, t) \left[ \left( \Sigma_{t}^{r} \right)^{'}(\omega, t) \right] f(\omega) - \left( \Sigma_{t}^{l} \right)^{'}(\omega, t) G_{r}^{a}(\omega, t) \ V_{a}(t) \nonumber
$$

The equations above, together with Eq. (3), reduce the computation of the pumped current to that of the instantaneous interacting self energies/Green’s functions, $\Sigma_{t}^{r,a}$ and $G_{r}^{a}$, and of their first order expansion in gradients $\left( \Sigma_{t}^{r,a} \right)^{'}$. This may be computed using the standard tools of many body theory within a perturbative scheme. For example, the quantity $\left( \Sigma_{t}^{r,a} \right)^{'}$ may be easily obtained by i) deriving, e.g. diagrammatically, an expression of the full time dependent $\Sigma_{t}^{r,a} \ (t, t')$ in terms of the Green’s functions, ii) expanding the latter in gradients using Eq. (4), and iii) isolating the term containing a single time derivative.

To illustrate this in more detail, let us consider the calculation $\Sigma_{t}^{r}$ to second order in perturbation theory for the Hamiltonian Eq. (11). Neglecting the contribution of tadpole diagrams, which for $\Omega \ll \gamma$ can be incorporated from the beginning in the definition of $\epsilon(t)$, the self energy reduces to $\Sigma_{t}^{r} \ (t, t') = \sum_{a} \lambda_{a}^{2} G_{0}^{r}(t, t') D_{0}^{r}(t, t')$, where $G_{0}^{r}$ are the noninteracting Green’s functions of the dot, and $D_{0}^{r}(t, t') = -2\imath \pi (N(\omega_{q}) \exp(\pm i \omega_{q}(t - t')) + (1 + N(\omega_{q})) \exp(\pm i \omega_{q}(t - t'))$ are the bosonic propagators. In order to expand these self energies in gradients, one needs to first express $G_{0}^{r}(t, t') = G_{0}^{r}(t, t_{1}) \otimes \sum_{a} \Sigma_{a}^{r}(t_{1}, t_{2}) \otimes G_{0}^{r}(t_{2}, t_{1})$, where $\Sigma_{a}^{r}(t_{1}, t_{2}) = \pm i 2\pi \nu \sum_{a} V_{a}(t_{1}) f(\pm t_{1} - t_{2}) V_{a}(t_{2})$, and then take the Wigner transform of $\Sigma_{t}^{r} \ (t, t')$. Using the formula for the gradient expansion and substituting in Eq. (19)-(21) we find, for real $V$, Eq. (5).

Relation to previous works - Finally, we notice that a direct comparison shows that the formula obtained in Ref. [10] corresponds to the sum of $I_{1}^{n} + I_{1}^{n}$. Therefore, the additional term $I_{2}^{n}$ corresponds to the “irreducible vertex corrections” for the emissivity reported in Ref. [11] or to the corrections to the average time approximation in Ref. [12]. The formalism presented in this letter requires the extraction of the self energies with respect to a quadratic Hamiltonian. This does not constraint future applications to weakly interacting problems only: many interesting problems with strong correlations (e.g. Kondo effects) can be mapped into quadratic models plus deviations from them (e.g. by slave bosons, bosonization, etc.). An alternative approach studying pumping in a quantum dot in perturbation theory in the tunneling, but accounting exactly for the charging interaction, has been recently formulated in Ref. [13].

Conclusions - In this paper, we derived a useful formula for the pumped current through interacting quantum dots within a perturbative scheme and used it to demonstrate that inelastic scattering is the main effect forcing a description of pumping beyond Brouwer’s formula. Interestingly, even in the presence of inelasticity, pumping can be still described geometrically [8]. This statement draws a clear analogy between pumping and the dissipative Berry Phase problem [14], whose implications, in view of recent work connecting Brouwer’s formula to geometric phases [15], are worth of further investigations.

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1. D. J. Thouless, Phys. Rev. B 27, 6083 (1983).
2. P. W. Brouwer, Phys. Rev. B 58, R10135 (1998).
3. B. Spivak, F. Zhou, and M.T.B. Moned, Phys. Rev. B 51, 13226 (1995); F. Zhou, B. Spivak, and B. L. Altshuler, Phys. Rev. Lett. 82, 608 (1999); T. A. Shutenko, I. L. Aleiner, and B. L. Altshuler, Phys. Rev. B 61, 10366 (2000); J. E. Avron et al., Phys. Rev. Lett. 87, 236601 (2001); O. Entin-Wohlman, A. Aharony, and Y. Levinson, Phys. Rev. B 65, 195411 (2002).
4. M. G. Vavilov, V. Ambegaokar, and A. I. Aleiner, Phys. Rev. B 63, 195313 (2001).
5. I. A. Aleiner and A. V. Andreev, Phys. Rev. Lett. 81, 1286 (1998); R. Citro, N. Andrei, and Q. Niu, Phys. Rev. B 68, 165312 (2003); T. Aono, Phys. Rev. Lett. 93, 116601 (2004); P. Brouwer, A. Lamacraft, and K. Flensberg, Phys. Rev. B 72, 075316 (2005); E. Cota, R. Aguado, and G. Platero, Phys. Rev. Lett. 94, 107202 (2005); A. Schiller and A. Silva, condmat/07043014.
6. J. Splettstoesser, M. Governale, J. König, and R. Fazio Phys. Rev. Lett. 95, 246803 (2005).
7. J. Splettstoesser, M. Governale, J. König, F. Taddei, and R. Fazio Phys. Rev. B 75, 235302 (2007).
8. E. Sela and Y. Oreg, Phys. Rev. Lett. 96, 166802 (2006).
9. M. Smitkes et al., Science 283, 1905 (1995); S. K. Watson et al. Phys. Rev. Lett. 91, 258301 (2003).
10. V. Kashcheyevs, A. Aharony, and O. Entin-Wohlman, Phys. Rev. B 69, 195301 (2004) and references therein.
11. M. Moskalets and M. Böttiker, Phys. Rev. B 64, 201305 (2001); J. N. Cremers and P. W. Brouwer, Phys. Rev. B 65, 115333 (2002).
12. Liliana Arrachea and Michael Moskalets, Phys. Rev. B 74, 245322 (2006).
13. D. Langreth, Phys. Rev. 150, 516 (1966).
14. A. P. Jauho, N. S. Wingreen, and Y. Meir, Phys. Rev. B 50, 5528 (1994).
15. See, e.g., H. Haug and A.-P. Jauho, “Quantum Kinetics in Transport and Optics of Semiconductors” (Springer, Berlin, 1996), Chapter 6.
16. J. Splettstoesser et al., Phys. Rev. B 74, 085305 (2006).
[17] E. Sela and Y. Oreg, preprint, unpublished.
[18] R. S. Whitney et al., Phys. Rev. Lett. 94, 070407 (2005).
[19] H.-Q. Zhou et al., Phys. Rev. Lett. 91, 186803 (2003).