Gravitational form factors of $\rho$ meson with a light-cone constituent quark model

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The $\rho$ meson gravitational form factors are studied based on a light-front constituent quark model which has been successfully employed to calculate its generalized parton distributions and some low-energy observables. The distributions of energy, spin, pressures, and shear forces inside the $\rho$ meson are explicitly given.

I. INTRODUCTION

We know that the gravitational form factors (GFFs) are defined through the matrix element of the energy-momentum tensor (EMT) [1]. Since the GFFs relate to the mass, spin, shear forces, and $D$-term of the particles [2, 3], they (or EMT form factors) involve a large range of physics, such as the gravitation physics and the physics in hard scattering processes [4]. It is a promising way to extract more information about the mechanical properties of a hadron (especially in the non-perturbative region) from the study of GFFs. Those tasks mainly try to answer some fundamental questions, like how the hadron mass and spin are carried out by quarks and gluons or what the mechanism that the trace anomaly contributes to hadron mass is, and how the strong force distributes inside the hadron, etc.. Besides, the Fourier transforms of the EMT matrix elements define the static EMT which can further tell the distributions of pressure and shear forces [4].

In the 1960s, the total GFFs were already introduced for both spin-0 and spin-1/2 hadrons [1]. The most natural but also the least practical way to probe GFFs is the scattering off gravitons. However, it’s more practical to extract GFFs through their connections to the generalized parton distributions (GPDs). The relations between GFFs and GPDs were discussed in detail in Refs. [3, 4]. As the soft part of the hard-exclusive reactions, GPDs have been received many theoretical and experimental investigations [5-12]. As a reflection of the broken scale invariance of QCD, the matrix element of the trace anomaly part of EMT naturally connects with the hadron mass [13, 14]. Especially the gluonic operator is believed to contribute to the majority part. This may give another possible way to probe the GFFs via the exclusive production of heavy quarkonium states, such as near-threshold $J/\psi$ and $\Upsilon$ photoproduction processes at JLab. and RHIC etc. [15-17].

At present, it is still not clear what the specific relations among the strong force, pressure and shear forces are. Nevertheless, one may get some hints from the phenomenological studies of the static EMT of particles with different spins. For instance, the GFFs of pion (spin-0) were evaluated in chiral quark models in Refs. [18, 19]. Ref. [20] applies the Q-ball model to spin-0 particles as well, where the $D$-term, energy density, pressure, and shear forces were investigated in detail. There are also Lattice QCD calculations related to the pion GFFs [21]. For the spin-1/2 hadrons, there are model calculations from the AdS/QCD approach [22], and the chiral quark soliton model [23], etc.. More can be found in a review article (see Ref. [4]). As for a spin-1 hadron, the formalism of its GFFs and force

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distributions are discussed very recently by Refs. [24, 25]. In the literature, the model calculations for the spin-1
particles include the AdS/QCD approach [26] and the Nambu-Jona-Lasinio (NJL) model [27].

It is shown that the light-cone quark model (LCCQM) for the ρ meson employed in our previous works can describe
the ρ meson well in the low energy region, such as its electromagnetic form factors, GPDs, etc. [28–30]. In this work,
we’ll apply our LCCQM and the previous results of GPDs to study the ρ meson GFFs and its mechanical properties,
and try to get the information about those fundamental questions.

This paper is organized as follows. In Section II the definitions of GFFs, pressure, and shear forces for a spin-1
particle are briefly presented. Moreover, the LCCQM for the ρ meson applied in our previous works is also shortly
reviewed in this section. Section III gives our numerical results for the ρ meson GFFs, pressure, and shear forces, etc.
In this part, we introduce a phenomenological three-dimensional (3D) Gaussian form wave package, when calculating
the static EMT, since our obtained GFFs do not drop fast enough. We also display our model-dependent
D-term of the ρ meson, which is not affected by the Gaussian form wave package. Finally, section IV is devoted for a summary.

II. GFFS OF SPIN ONE PARTICLES AND OUR MODEL

A. GFFs of spin one particles

The formalism of GFFs of a spin one particle and its other mechanical properties have been discussed and given
explicitly in the recent works [24, 25]. Here, we briefly summarize them as follows. In this paper, we use the
covariant normalisation \( \langle p', \sigma' | p, \sigma \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(p' - p) \delta_{\sigma \sigma'} \) for the system, and introduce the kinematic variables
\( P = \frac{1}{2}(p' + p), \Delta = p' - p, t = \Delta^2 \). Then, the EMT form factors of a spin-1 particle in QCD are defined as,

\[
\langle p', \sigma' | \hat{T}^a_{\mu \nu}(x) | p, \sigma \rangle = \left[ 2P_{\mu} P_{\nu} \left( -\epsilon^a \cdot \epsilon A_0^a(t) + \frac{\epsilon^a \cdot P \epsilon \cdot P}{m^2} A_1^a(t) \right) + 2 \left[ P_{\mu} (\epsilon_{\nu}^a \cdot P + \epsilon_{\nu} \epsilon^a \cdot P) + P_{\nu} (\epsilon_{\mu}^a \cdot P + \epsilon_{\mu} \epsilon^a \cdot P) \right] J^a(t) \right.
\]

\[
+ \frac{1}{2} (\Delta_{\mu} \Delta_{\nu} - g_{\mu \nu} \Delta^2) \left( \epsilon^a \cdot P D_0^a(t) + \frac{\epsilon^a \cdot P \epsilon \cdot P}{m^2} D_1^a(t) \right)
\]

\[
+ \frac{1}{2} (\epsilon_{\mu} \epsilon_{\nu}^a + \epsilon_{\nu} \epsilon_{\mu}^a) \Delta^2 - (\epsilon_{\mu}^a \Delta_{\nu} + \epsilon_{\nu}^a \Delta_{\mu}) \epsilon \cdot P
\]

\[
+ (\epsilon_{\mu} \Delta_{\nu} + \epsilon_{\nu} \Delta_{\mu}) \epsilon^a \cdot P - 4g_{\mu \nu} \epsilon^a \cdot P \epsilon \cdot P \right] E^a(t)
\]

\[
+ \left( \epsilon_{\mu} \epsilon_{\nu}^a + \epsilon_{\nu} \epsilon_{\mu}^a - \frac{\epsilon^a \cdot \epsilon}{2} g_{\mu \nu} \right) m^2 f^a(t)
\]

\[
+ g_{\mu \nu} \left( \epsilon^a \cdot m^2 c_0^a(t) + \epsilon^a \cdot P \epsilon \cdot P c_1^a(t) \right) \right] e^{ip'(p + p)\tau},
\]

where \( a = g, u, d, \ldots \), which represent the contributions of gluon and all flavors of quarks, and the polarization vectors
\( \epsilon_{\mu} = \epsilon_{\mu}(p', \sigma') \), \( \epsilon_{\mu} = \epsilon_{\mu}(p, \sigma) \) with \( \sigma = x, y, z \), respectively. The 6 quark and gluon GFFs \( A_{0,1}^a, D_{0,1}^a, J^a \) and \( E^a(t) \) are
individually momentum-energy conserving, and the other 3 GFFs, \( f^a \) and \( c_{0,1}^a \), are not.

As shown in our previous works, in the Breit frame, the above expression can be re-organized according to the power
of the quadrupole operator of the spin one particles. The static EMT \( T_{\mu \nu}'(\vec{r}, \sigma', \sigma) \) of the spin-1 system is defined by
the Fourier transform of the EMT with respect to \( \vec{A} \) as

\[
T_{\alpha}^{\mu \nu}(\vec{r}, \sigma', \sigma) = \int \frac{d^3 \vec{A}}{2E(2\pi)^3} e^{-i\vec{A} \cdot \vec{r}} \langle p', \sigma' | \hat{T}^{\mu \nu}_{\alpha}(0) | p, \sigma \rangle.
\]
Eq. 2 contains the energy densities, the distributions of spin, pressure and shear forces with different power of quadrupole operator. For the energy distributions, we have

$$T^{00}(\vec{r}, \sigma', \sigma) = \int \frac{d^3 \Delta}{2E(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} \langle p', \sigma' | \tilde{T}^{00}(0) | p, \sigma \rangle$$

(3)

$$= \varepsilon_0(r) \delta_{\sigma'\sigma} + \varepsilon_2(r) \hat{Q}^{ij} Y_2^{ij},$$

(4)

where $r = |\vec{r}|$, $Y_2^{ij} = r^i r^j / r^2 - \delta^{ij} / 3$, $\hat{Q}^{ij} = (\tilde{Q}^{ij})_{\sigma'\sigma}$, and

$$\varepsilon_0(r) = 2m^2 \hat{\varepsilon}_0(r), \quad \varepsilon_2(r) = -r \frac{d}{dr} \left( \frac{d}{dr} \right) \hat{\varepsilon}_2(r),$$

(5)

(6)

with

$$\hat{\varepsilon}_{0,2}(r) = \int \frac{d^3 \Delta}{2E(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} \varepsilon_{0,2}(t)$$

(7)

where $\varepsilon_{0,2}(t) = \sum_a \varepsilon_{a,0,2}(t)$.

For the spin distribution, the 0j component is

$$T^{0j}(\vec{r}, \sigma', \sigma) = \int \frac{d^3 \Delta}{2E(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} \langle p', \sigma' | \tilde{T}^{0j}(0) | p, \sigma \rangle.$$  

(8)

The individual contributions of quarks and gluons to the spin of the particle is

$$J^i_a(\vec{r}, \sigma', \sigma) = \epsilon^{ijk} r^k T^{0k}(\vec{r}, \sigma', \sigma),$$

(9)

$$= \hat{S}^i_{\sigma'\sigma} \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} \left[ \left( \tilde{J}^i(t) + \frac{2}{3} \frac{d}{dt} \tilde{J}^i(t) \right) \delta^{ij} + \left( \Delta^i \Delta^j - \frac{1}{3} \delta^{ij} \right) \frac{d \tilde{J}^i(t)}{dt} \right],$$

(10)

with $\tilde{J}^i(t) = \frac{\pi}{2m} J^i(t)$.

In the spirit of Ref. 31, the stress tensor, defined by the ij-components of EMT in Eq. (2), can be written generically to the quadrupole order as:

$$T^{ij}(\vec{r}, \sigma', \sigma) = \int \frac{d^3 \Delta}{2E(2\pi)^3} e^{i\vec{\Delta} \cdot \vec{r}} \langle p', \sigma' | \tilde{T}^{ij}(0) | p, \sigma \rangle$$

$$= p_0(r) \delta^{ij} \delta_{\sigma'\sigma} + s_0(r) Y_2^{ij} \delta_{\sigma'\sigma} + p_2(r) \hat{Q}^{ij} + 2s_2(r) \left[ \hat{Q}^{ip} Y_2^{pj} + \hat{Q}^{jp} Y_2^{pi} - \delta^{ij} \hat{Q}^{pq} Y_2^{pq} \right]$$

$$- \frac{1}{m^2} \hat{Q}^{ik} \hat{Q}^{jk} \left[ p_3(r) \delta^{ij} + s_3(r) Y_2^{ij} \right],$$

(11)

where the (quadrupole) pressure $p_n(r)$ and shear forces functions $s_n(r)$ are

$$p_n(r) = \frac{1}{3} \partial^2 \tilde{D}_n(r), \quad s_n(r) = -\frac{1}{2} \frac{d}{dr} \frac{d}{dr} \tilde{D}_n(r)$$

(12)

with $\partial^2 = \frac{1}{r^2} \frac{d}{dr} \frac{d}{dr}$ being the radial part of 3D Laplace operator and

$$\tilde{D}_n(r) = \int \frac{d^3 \Delta}{2E(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r}} D_n(t), \quad n = 0, 2, 3$$

(13)

with $D_n(t) = \sum_a D^a_n(t)$. The detailed definitions for the form factors $\varepsilon_{a,0,2}(t)$, $\tilde{J}^a(t)$ and $D^a_{0,2,3}(t)$ are shown in the appendix.
B. Phenomenological light-front constituent quark

We know that the GFFs can be obtained from GPDs via the sum rules. Fig. 1 illustrates the process we are considering for GPDs in our phenomenological model. The notations are

\[ t = \Delta^2 = (p' - p)^2 = (q - q')^2, \quad Q^2 = -q^2, \]
\[ \xi = -\frac{\Delta \cdot n}{2P \cdot n} = -\frac{\Delta^+}{2P^+}, \quad |\xi| = \frac{\Delta^+}{2P^+}, \quad (|\xi| \leq 1), \]
\[ x = \frac{k \cdot n}{P \cdot n} = \frac{k^+}{P^+}, \quad (-1 \leq x \leq 1), \]

(14)

where \( n \) is a light-like 4-vector with \( n^2 = 0 \). Here \( q \) is the virtual photon momentum, and \( q' \) is treated as a real one.

In a numerical calculation, we employ the phenomenological light-front quark model to describe the interaction between the spin-1 \( \rho \) meson and its two constitutes \( u \) and \( d \) [28, 32]. It is based on an effective interaction Lagrangian for the \( \rho \rightarrow \bar{q}q \) vertex,

\[ L_I = -\frac{iM}{f_\rho} \bar{q} \Gamma^\mu \tau q \cdot \rho \mu \]
\[ = -\frac{i\sqrt{2}M}{f_\rho} \left[ \bar{u} \Gamma^\mu u - \bar{d} \Gamma^\mu d + \bar{u} \Gamma^\mu d \rho_\mu^+ + \bar{d} \Gamma^\mu u \rho_\mu^- \right], \]

where \( \rho_\mu \) is the \( \rho \) meson field, \( f_\rho \) is the \( \rho \) decay constant, and \( \Gamma^\mu \) is a Bethe-Salpeter amplitude (BSA) describing the interaction between the meson and the quark-antiquark pair,

\[ \Gamma^\mu = N \frac{\gamma^\mu - (k_q + k_\bar{q})^\mu/(M_{u,d} + 2m)}{[k_q^2 - m_R^2 + \imath\epsilon][k_\bar{q}^2 - m_R^2 + \imath\epsilon]}, \]

(16)

where, for the \( u \) quark contribution, the struck \( u \) quark momentum is \( k_u = k - \Delta/2 \) and the spectator constituent momentum is \( k_s = k_{\bar{d}} = k - P \). \( N \) is the normalization constant, \( m \) and \( m_R \) are the masses of the constituent quark and the regulator, respectively, and \( M_{u,d} \) are the kinematic invariant masses. Details are referred to Refs. [28, 32].

III. NUMERICAL RESULTS

Following our previous work on the \( \rho \) meson GPDs [28, 30], we take the two model parameters, the constituent mass \( m = 0.403 \) GeV and regulator mass \( m_R = 1.61 \) GeV. In our LCCQM, the gluon contributions are assumed to be absorbed into the constituent quark mass. After summing over all the contributions from the quark flavors, we get
the total GFFs, where the 3 energy-momentum non-conserving terms are canceled, and only 6 conserving terms are considered.

Our results for GFFs are shown in Fig. 2. We know that the GFFs $A_0(t)$ and $J(t)$ are related to the generators of the Poincare group for the mass and spin of the particle (here is the $\rho$ meson) which give the constraints at zero-momentum transfer $A_0(0) = 1$ and $J(0) = 1$. Except for the cases of free particles and Goldstone bosons, there is no any other general principles or constraints for $D$-term. The results for the $D$-terms of the proton and pion have been given by different phenomenological analyses based on the experimental data of leading order deep virtual Compton scattering (DVCS) process [33]. In spin-1 case, the $D$-term is related to the GFFs $D_0(t)$ and $E(t)$. It is expected, with a similar approach, one can also obtain an estimation for the mechanical properties for other spin-1 particles, in particular for deuteron, which may be measured in the future JLab experiment [34].

In the literature, there are some other model calculations for the spin-1 GFFs before this present work. Ref. [26] gives the results for $A_0(t)$ and $J(t)$ by applying the AdS/QCD approach, and Ref. [27] shows the six non-zero GFFs in the NJL model. The relations among the different notations have been explicitly discussed in Ref. [24]. Besides the common constraints from the total mass ($A_0(0) = 1$) and spin ($J(0) = 1$), the present calculation and the mentioned two other approaches, however, show different decreasing $t-$ dependent behaviors of the two GFFs $A_0(t)$ and $J(t)$. For the rest four non-zero GFFs $A_1$, $D_0$, $D_1$ and $E$, our results are also quite different from those of Ref. [27]. One could find, in the common region of momentum transfer, our $A_1(t)$ ($\sim 1$ at $t = 0$) is much larger than the results ($\sim 0.4$ at $t = 0$) in Ref. [27] and our $E(t)$ ($\sim 0.15$ at $t = 0$) is smaller.
than those ($\sim 0.5$ at $t = 0$) in Ref. [27]. For $D_0(t)$ and $D_1(t)$, we get opposite signs comparing with those from Ref. [27].

The present model result for $\rho$ meson D-term is

\[ D = D_0(0) = -D_0(0) + \frac{4}{3}E(0) = 0 + \frac{4}{3}(-0.161) = -0.21 < 0 . \]  

(17)

It should be stressed that the negative value of the $D$-term satisfies the requirement for the mechanical stability [4], i.e.,

\[ \frac{2}{3}s(r) + p(r) > 0 . \]  

(18)

Although the $t$-dependent behaviors of GFFs obtained in different approaches are quite different, we find the $D$-term value obtained from the GFFs of Ref. [27] (although it’s not given explicitly in the paper) is around $-0.33$, which is close to ours $-0.21$.

Moreover, our result for the gravitational radius

\[ \langle r^2 \rangle_{\text{grav}} = \frac{\int d^3r r^2 T^{00}(\vec{r})}{\int d^3r T^{00}(\vec{r})} = \frac{1}{m} \int d^3r r^2 T^{00}(\vec{r}) = -\frac{6}{m} \left. \frac{dA_0(t)}{dt} \right|_{t \to 0} , \]  

(19)

is $\sqrt{\langle r^2 \rangle_{\text{grav}}} = 0.55$ fm, which is smaller than the charge radius (0.72 fm) from our previous calculation. This feature is reasonable and consistent with the nucleon case [23, 35]. Besides, we find both of the radius and the $D$-term are sensitive to the constituent quark mass $m_q$. As binding energy is approaching zero ($m_q$ is approaching to the half of total $\rho$ meson mass), the radius increases rapidly and the absolute value of $D$-term decreases close to zero. It means the bound system is getting looser or even falls apart in the view of the constituent quark model.

In principle, one can calculate the static EMT $T^{\mu\nu}(r)$ (also the energy density and pressure) straightforwardly from the obtained GFFs with Fourier transformation. However, the integrals may not converge if the GFFs drops slowly w.r.t. the momentum transfer square $t$. According to the analyses of pQCD and AdS/CFT, at the large momentum transfer ($-t = Q^2 \to \infty$), the six GFFs decease roughly with the following power respectively [26],

\[ (A_0 , D_0 , J , E) \sim 1/t^2 , \quad (A_1 , D_1) \sim 1/t^3 . \]  

(20)

In the nucleon case, Ref. [36] adopts an assumption that its GFFs behave $d_1(t) \sim t^{-3}$, and the converged results are obtained.

Because of the limited capability of our LCCQM, especially at the large momentum transfer region, we believe that a modification of our model results in the large momentum transfer region is needed. To simulate the $t$-dependent behaviors of the obtained GFFs, we consider the forms like

\[ a \left(1 - \frac{t}{b}\right)^c \]  

(21)

to present our numerical results at a momentum transfer region, $0 < -t < 10$ GeV$^2$, and we find five of the six GFFs
are approximately described by

\[ A_0(t) = (1 - 0.996 t)^{-1.28} , \]  
\[ A_1(t) = -1.20 \left( 1 - \frac{t}{0.73} \right)^{-1.38} , \]  
\[ D_1(t) = 0.814 \left( 1 - \frac{t}{1.32} \right)^{-1.64} , \]  
\[ J(t) = 0.965 \left( 1 - \frac{t}{0.68} \right)^{-0.877} , \]  
\[ E(t) = -0.161 \left( 1 - \frac{t}{4.2} \right)^{-0.909} . \]

Due to the limit \(|\xi| \leq 1/\sqrt{1 - 4M^2/t}\), there are some small oscillation in the numerical result of the GFF \(D_0(t)\) at \(0 < -t < 3 \text{ GeV}^2\). After a carefully check of its \(t\)-dependent behavior, we find it is oscillating around a curve that can be simulated by

\[ \left( 1 - \frac{t}{b} \right)^c \left( 1 - \frac{t}{d} \right)^e . \]

Thus, for \(D_0(t)\), we get

\[ D_0(t) = \left( 1 - \frac{0.97}{t} \right)^{-1.97} \left( 1 - \frac{t}{0.14} \right)^{-0.86} . \]

It turns out, unlike the nucleon case in Ref. \[36\], neither our GFF results in Eqs. \[22\] and \[24\] nor the pQCD predictions in Eq. \[20\] drop fast enough to give converging results for \(T^{\mu\nu}(r)\) (see Eq. \[2\]). This issue has already been pointed out and discussed for the pion meson case. For the spin-0 case, Ref. \[20\] estimated the relativistic corrections by a way of smearing out the point-particles (a delta function is replaced by a Gaussian function). It is believed that the relativistic corrections \((\delta_{\text{rel}} = 1/(2m^2R_h^2))\) is negligible when \(mR_h \gg 1\) where \(m\) is the hadron mass and \(R_h\) is the hadron size. For the light meson pion, its \(\delta_{\text{rel}} = 220\% \) \[20\]. Although the \(\rho\) meson is spin-1 hadron, we may simply "borrow" the argument for pion to roughly estimate how large the relativistic corrections are for the case of the \(\rho\) meson. With our model estimation of the radius \((\sqrt{\langle r^2 \rangle} \sim 0.55 \text{ fm})\), one gets \(\delta_{\text{rel}} \sim 11\%\) which is not important and we believe that the concept of the 3D densities is applicable for the \(\rho\) meson. So far, there is no experimental data for the \(\rho\) meson radius, the future experimental information about its size would be essential for our estimate.

To proceed with a modification of our phenomenological model calculation, particularly in the large \(t\) region, we introduce a Gaussian form wave package, as we did in previous work \[29\], to suppress the contribution from the high energy region. It’s reasonable since only limited values of \(t\) can be measured in the experiments. Choosing the Gaussian form wave package originates from the observation that a hadron is an extended object and is smeared out in space \[37\]. It should be stressed that the value of \(D\)-term is not affected by this modification and by the consideration of the relativistic corrections since it’s defined by the value of GFFs at the zero momentum transfer \[1\] \[20\].

In our previous study for the \(\rho\)-meson transverse distributions in the two-dimension (2D) impact parameter space \[29\], we introduced a 2D Gaussian form wave packet in both of the incoming and outgoing states in order to avoid the similar divergences, and we found the appropriate value for the wave packet width being around \(\sigma = 1 \sim 2\text{GeV}^{-1}\). This treatment was also pointed out and employed in other calculations of nucleon GPDs in the impact parameter space \[37\]. Here, a similar 3D Gaussian form wave packet,

\[ e^{-\Delta^2 \sigma_0^2/4} , \]
is adopted (with width $\sigma_0 \sim 2 \text{GeV}^{-1} = 0.39 \text{ fm}$) to carry out the calculation for the spatial distributions. As a result, the expressions for the energy densities are modified to be

$$
\varepsilon_0^{(\sigma_0)}(r) = 2m^2 \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r} - \vec{\Delta}^2\sigma_0^2/4} \mathcal{E}_0(t),
$$

$$
\varepsilon_2^{(\sigma_0)}(r) = -i \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r} - \vec{\Delta}^2\sigma_0^2/4} \mathcal{E}_2(t).
$$

Then, the spin distribution is modified to be

$$
J_n^{(\sigma_0)}(r, \sigma', \sigma) = J_n^{(\sigma_0)}(\vec{r}, \sigma', \sigma)
$$

$$= \hat{S}_{\sigma'\sigma} \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r} - \vec{\Delta}^2\sigma_0^2/4} \left[ \left( \vec{J}^a(t) + \frac{2}{3} \frac{d\vec{J}^a(t)}{dt} \right) \delta^{ij} + \left( \vec{\Delta}^i \Delta^j - \frac{1}{3} \vec{\Delta}^2 \delta^{ij} \right) \frac{d\vec{J}^a(t)}{dt} \right],
$$

where $\vec{J}^a(t) = \frac{m}{E} \mathcal{J}^a(t)$ and the first step in the above equation is based on the observation that $J_n^{(\sigma_0)}$ depends on $\vec{r}$ only through $r = |\vec{r}|$. Moreover, the function $\vec{D}_n(r)$ in Eq. (13), which defines the distributions of pressure and shear force, is modified as

$$
\vec{D}_n^{(\sigma_0)}(r) = \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\vec{\Delta} \cdot \vec{r} - \vec{\Delta}^2\sigma_0^2/4} \mathcal{D}_n(t), (n = 0, 2, 3).
$$

Correspondingly, $p_n(r) \to p_n^{(\sigma_0)}(r)$ and $s_n(r) \to s_n^{(\sigma_0)}(r)$.

After summing over all the partons (only quarks here) in Eq. (27), one has

$$
J_\sigma^{(\sigma_0)}(\vec{r}) = \sum_a J_a^{(\sigma_0)}(\vec{r}, \sigma', \sigma), \quad i, \sigma', \sigma = x, y, z.
$$

In the Cartesian basis, average over all polarizations and spatial directions, one further has

$$
J^{(\sigma_0)}(r) \equiv \frac{1}{\text{Tr}[\hat{S}^2]} \sum_{\sigma' \sigma} \hat{S}_{\sigma'\sigma} J_a^{(\sigma_0)}(\vec{r}) = iJ_y^{(\sigma_0)}(r),
$$

which is a real quantity and is the 3D spatial spin distribution of the $\rho$ meson.

With those preparations, the energy densities calculated from the GFFs in Eqs. (22) and (24) are shown in Fig. 9. In Fig. 3(b), the normalization is changed to the form of $4\pi r^2 \varepsilon_0^{(\sigma_0)}(r)/M$, which gives 1 after averaging over the polarizations and integrating over the whole radial space. The higher-order term $\varepsilon_2^{(\sigma_0)}(r)$ doesn’t contribute to the energy distributions in the unpolarized case. Its negative value indicates that the mass or energy distribution would deviate from the center because of the polarization effect. In the polarized case, for example, $\sigma = \sigma' = x$, the contour plane of the 3D energy distribution would be a long ellipsoid with $x$ being the long axis. The result for the spin distribution is shown in Fig. 4. In our previous work with LCCQM, we obtain the fraction of spin carried by the constituent quark and antiquark in the $\rho$ meson is 86% 60. The rest part is believed to come from the orbital angular momentum (no gluon in our model) 58.

Finally, our results for pressures $p_n^{(\sigma_0)}$ and shear forces $s_n^{(\sigma_0)}$ are shown in Fig. 5. To the best of our knowledge, there are some model calculations for spin-0 and spin-1/2 nucleon, but none for the spin-1 particle before. In the unpolarized case, only the first order pressure and shear force contribute to $T^{ij}$ in Eq. (11). So far in almost all model studies for different spin cases, it is found that the (unpolarized) pressure is positive in the inner region and negative...
in the outer region. Under the present convention, we know that the positive sign means repulsion towards outside and the negative sign means attraction towards inside. However, the specific relationship between these values and the strong force remains obscure. As one can see, $p_0^{(σ_0)}(r)$ changes its sign for the first time at around $r ≈ 0.5$ fm, which is roughly the gravitational radius. Physically, the sign changing means the forces change from "stretching" to "squeezing". This phenomenon is the same as in the nucleon case [4]. At the region, $r ≥ 1$ fm, the pressures and shear forces are all quickly approaching to zero with small oscillations w.r.t. the center region values. There always exist oscillations in this approach and the oscillations depend on what the value of $σ_0$ is used in our numerical calculation (larger $σ_0$ generates stronger suppression on the amplitudes of the oscillations). There is still no constraint on how many times the changing would happen and no explanation for the meaning of those numbers. It is of great interest to have further study and to answer these intriguing questions. Nevertheless, here we present the first model estimation for the pressures and shear forces for the spin-1 $ρ$ meson. Similar distributions may indicate some common properties of the strong force in forming the hadron systems.
Figure 5: The pressure and shear forces functions with $\sigma_0 = 2 \text{ GeV}^{-1}$.

IV. SUMMARY

In this work, we extend our previous approach on the $\rho$ meson GPDs with the phenomenological light-front constituent quark model to its GFFs and further to the distributions of pressure and shear forces. For the GFFs $A_0$ and $J$ which are related to the mass and spin, our model estimations are consistent with the result of other approaches, such as the NJL model and AdS/CFT etc. For the rest four GFFs, there are no specific constraints as the mass and spin cases, and the results from different approaches have large discrepancies even with opposite signs. Moreover, the $D$-term is given through our calculated GFFs, and it is estimated to be $-0.21$. The negative value satisfies the stability condition. We also calculate the distributions of energy, pressure, spin, and shear forces. Since the LCCQM works well mainly within the low momentum transfer regions, we consider a Gaussian wave package during the Fourier transforms to suppress the contributions from large momentum transfer region. Thus, the present results should be considered as a qualitative estimation. We expect that our results may provide some hints for the understanding of the mechanical properties, especially in the case of spin-1 hadron.
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Appendix A: Definition for the form factors

The definitions for the form factors used in this work are

$$\mathcal{E}^a_0(t) = A^a_0(t) + \frac{1}{4} \bar{f}^a(t) - \frac{1}{2} \bar{c}^a_0(t)$$

$$+ \frac{t}{12m^2} \left[ -5A^a_0(t) + 3D^a_0(t) + 4J^a(t) - 2E^a(t) + A^a_1(t) + \frac{1}{2} \bar{f}^a(t) + \bar{c}^a_0(t) + \frac{1}{2} \bar{c}^a_1(t) \right]$$

$$- \frac{t^2}{24m^4} \left[ -A^a_0(t) + D^a_0(t) + 2J^a(t) - 2E^a(t) + A^a_1(t) + \frac{1}{2} D^a_1(t) + \frac{1}{4} \bar{c}^a_1(t) \right] + \frac{t^3}{192m^6} \left[ A^a_0(t) + D^a_1(t) \right],$$

(A1)

$$\mathcal{E}^a_2(t) = -A^a_0(t) + 2J^a(t) - E^a(t) + \frac{1}{4} \bar{f}^a(t) + \frac{1}{2} \bar{c}^a_0(t) + \frac{1}{2} \bar{c}^a_1(t)$$

$$- \frac{t}{4m^2} \left[ -A^a_0(t) + D^a_0(t) + 2J^a(t) - 2E^a(t) + A^a_1(t) + \frac{1}{2} D^a_1(t) + \frac{1}{4} \bar{c}^a_1(t) \right]$$

$$+ \frac{t^2}{32m^4} \left[ A^a_0(t) + D^a_1(t) \right],$$

(A2)

$$\mathcal{J}^a(t) = J^a(t) + \frac{1}{2} \bar{f}^a(t) - \frac{t}{4m^2} \left( J^a(t) - E^a(t) \right).$$

(A3)

$$\mathcal{D}^a_0(t) = -D^a_0(t) + \frac{4}{3} E^a(t) + \frac{t}{12m^2} \left[ 2D^a_0(t) - 2E^a(t) + D^a_1(t) \right] - \frac{t^2}{48m^4} D^a_1(t),$$

(A4)

$$\mathcal{D}^a_2(t) = -E^a(t),$$

(A5)

$$\mathcal{D}^a_3(t) = \frac{1}{4} \left[ 2D^a_0(t) - 2E^a(t) + D^a_1(t) \right] - \frac{t}{16m^2} D^a_1(t).$$

(A6)

When sum over all partons, the momentum-energy non-conserving terms, $\bar{f}^a$ and $\bar{c}^a_{0,1}$, will drop and they have no contribution.

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