Teachers’ Content Knowledge (CK) Analysis and Students’ Performance on Fractional Materials: A Case Study in Al-Amien Prenduan Foundation School

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Abstract. Fractions is one of the richest and most complex mathematical topics in mathematics learning. This is caused fraction topics is being one of the difficult topics in learning mathematics. Therefore, this study aimed at analyzing Teachers’ Content Knowledge and Students’ Performance on fraction topics. This research is a qualitative descriptive study which is a case study of Teachers’ Content Knowledge and Students’ Performance on fractional materials at the Al-Amien Prenduan Foundation School, Sumenep, Madura, East Java, Indonesia. Data were taken from 29 students of 7th grade and 2 teachers who taught these students. The instrument used in this study was 3 fraction problems. Data collection techniques are carried out using tests and in-depth interviews to obtain information in detail. The results of this study are the similarity of the patterns found in students’ and teachers’ performance on fraction topic. Teachers deliver the concept of fractions by emphasizing on students’ performance that is procedural rather than students’ understanding. These methods result in students learn fractions by memorizing them so that misconceptions can occur.

1. Introduction
Fraction is one of the richest and the most complex topics of mathematics learning programs [1], and is considered to be one of the most difficult math topics to be taught, the most cognitively challenging, and most important for advanced mathematics [2], [3], especially for elementary school students [4]. This is corroborated by the research of a number of practitioners regarding the poor performance of students in working on algebra problems involving fractions [5], [6]. Refer to Kieren, states that ideas in fractions are different from ideas in natural numbers, causing students having difficulty in solving problems related to fractions[7]. Therefore, teachers can help students learn fractions by using representations that are precise and easy to understand. Examples of such representations are the use of certain models such as rectangles, circles, or number line.

Based on the reasons above, it is necessary to see teacher's Content Knowledge (CK) on fraction topic. The aim is to see how a teacher can properly convey fraction material so that it can be understood by students, or better known as Pedagogical Knowledge (PK). Furthermore, the integration of Content Knowledge (CK) and Pedagogical Knowledge (PK) is called Pedagogical Content Knowledge (PCK).
Shulman [8] classifies content knowledge (CK) in three categories: (1) Subject matter content knowledge, (2) Curricular knowledge, and (3) Pedagogical content knowledge. Of the three content knowledge categories, researchers focused more on content knowledge (CK). This is because CK is one of the distinguishing Body of knowledge (a term used to show professionalism in a work domain) in learning [8]–[10].

Izsaak [11] in his research focused on Mathematical Knowledge for Teaching (MKT) on numbers by developing Hill's research design [12]. Izsák's research explains the necessity for a learning model using image visualization to assist teachers in mastering knowledge about numbers. Izsák also mentioned that the conceptualization in his research helped disassemble the fine-grained of teacher knowledge. Fine-grained in question is the teacher's creativity in conceptualizing a fraction, illustrating a fraction operation in a number line, diagram, or a picture to facilitate students in understanding number operations, especially fraction.

Ekawati [13] focused her research on Mathematical Content Knowledge (MCK) and teachers' Mathematics Pedagogical Content Knowledge (MPCK) with the scope on Ratios and Comparisons. Through the cluster analysis approach, teachers are divided into three categories; high, medium and low. The results of the study mentioned that the teachers had difficulty in figural representation. In addition, compared to the results of the MCK, the results of teacher's performance on the MPCK were inconsistent. Based on these conditions, Ekawati [13] recommended establishing a professional development and teacher service program in Indonesia.

But in 2017, Tchoshanov conducted research on the relationship of teachers’ knowledge and students’ achievement using a mixed method approach, combining qualitative and quantitative research. Tchoshanov published research focusing on analyzing the relationship between teachers and students on specific topics knowledge. In depth, Tchoshanov examined the relationship between teachers’ and students’ knowledge and concluded the importance of teachers’ content knowledge on students’ performance. However, in that study, Tchoshanov only focused on the relationship between teachers’ content knowledge and students’ performance, it was not discussed the relationship of teachers’ pedagogical content knowledge to students’ performance, whereas teachers’ pedagogical knowledge affected classroom practice and students’ achievement [14].

Based on the description above, the purpose of this study is to analyze the relationship between teachers’ content knowledge and students’ performance on fraction topics. The relationship is seen by identifying patterns of students and teachers in solving fraction problems. The reason is the thought that a teacher will teach well if the content knowledge to be taught has been mastered first. As corroborated by a number of researchers, CK affects student’s performance results [13]–[15].

2. Research Methods

This research is descriptive qualitative, a case study of teachers’ content knowledge and students’ performance on fraction topics at the Al-Amien Prenduan Foundation School, Sumenep, Madura, East Java, Indonesia. Data were taken from 29 7th grade students of class VII A and 2 teachers who taught these students. The instrument used in this study was 3 fraction problems as shown in Figure 1, Figure 2, and Figure 3.

![Figure 1. Question 1](image-url)
Data collection techniques used were tests and in-depth interviews in detail. Furthermore, the data is processed using qualitative and descriptive analysis.

3. Result and Discussion

Based on the results of data analysis, the following results are obtained. For the first question as seen in Figure 1, all students answer correctly and the way they answer also looks similar, i.e. (1) changing the form \( \frac{2}{1} : \frac{3}{2} \) into ordinary fractions, (2) turning division into multiplication, and (3) on the last fraction, turning the denominator into a numerator. \( \frac{2}{1} : \frac{3}{2} = \frac{7}{3} : \frac{1}{2} = \frac{7}{3} \times \frac{2}{1} = \frac{14}{3} \). The teachers also answered exactly as the students answered. A temporary conclusion for the first question is the similarity of patterns between the performances of teachers and students.

The second question is as seen in Figure 2. The result showed that 24 of 29 students in class VII-A answered correctly (83%) and 5 others answered incorrectly (17%). Two teachers were also asked to answer the questions above, and both answered correctly. Based on the reasons written by students, a temporary conclusion is the students understand the rules of division, but there are 17% of students memorized the rules of division but do not understand them.

The teachers’ and students’ responses for question number 3 are as shown in the Figure 4 and Figure 5 below. Based on the results of students' answers, it was found 3 out of 29 students answered correctly (10%), but they were not able to give sufficient reasons to show their statements were true. Furthermore, one of the two teachers answered this question incorrectly, but both answered correctly in the first and second questions.
Based on the results above, there are similarities in patterns found in students’ and teachers’ performance that are shown in right and wrong answers. In the first question, even though all students and teachers answered correctly, it was found that similarities were found in the work of students and teachers. This was confirmed by interviews that had been conducted. The interview was, "Why can the division change into multiplication?". Some students answer, "That’s the formula, Sir" or other students say "That’s what the teacher says, Sir!". Meanwhile, one of the teachers when interviewed answered "The rules of distribution are like that, Sir". The second question is not commented on because it is only a multiple choice, where the teachers and students both answered correctly. While on the third question, 90% of students answered "surely wrong", and one of the two teachers also answered that way. It turns out that both the teachers and students giving the same reason about why the statement is wrong is because the rules of multiplication say $\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{ps}{qr}$. That’s why the answer is not $\frac{pr}{qs}$ but $\frac{ps}{qr}$. Therefore, the tentative conclusion is that there is a relationship between teachers’ mastery of the learning topics and students’ performance.

Then the information is obtained that one of the two teachers has a long teaching experience, which is 24 years, and is quite liked by students, but apparently the teacher made a mistake in working on the third question. These findings indicate that the long teaching history as well as good teaching skills are not enough to make students successfully master learning content without being supported by good teacher’s content knowledge. These results are in line with Olanoff's statement [16], fraction learning that emphasizes the application of procedures through rote procedure will be followed by a set of rules that become very ineffective. Thus, rote-based mathematics learning without being preceded by conceptual understanding will have an impact on the students' weak conceptual understanding, resulting in misconceptions.

The explanation of the teacher and student reflects that their understanding of fractions is still at the level of procedural understanding. Furthermore, teacher knowledge influences student learning and achievement [17], even being one of the most influential factors in student learning [18], making teachers expected to teach effectively. The teacher not only has to master the topics to be taught, but also be able to explain using a variety of ways so that students easily understand the topic.

### 4. Conclusion

Based on the discussion above, there are similar patterns found in students’ and teachers’ performance on fraction topic. The teachers teach the concept of fraction by emphasizing it on procedural performance instead of understanding. This results in students learning fractions by memorizing so that misconceptions can occur. The implication of this research is improving teachers’ mathematical content knowledge can be
one of effective ways to gain student’s performance. Specifically in fraction topic, learning that emphasizing concept is strongly recommended and learning that only focus on memorizing the concept is avoided so that the misconception can be minimalized.

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