Preliminary study on internal flow simulation of centrifugal dredge pump by SPH algorithm

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Abstract. In order to simulate the internal flow of centrifugal dredge pump accurately and efficiently, a meshfree simulation method is proposed, which is a method of smooth particle hydrodynamics (SPH). The fluid domain is discretized into Lagrangian particles, the blades, the pump body are discretized as solid particles, the governing equations for fluid, and solid mechanics are transformed into corresponding particle force. When the initial state is started, the particles move and continue to evolve under the action of these forces. The motion parameters of the flow field and solid can be obtained by interpolating the kernel function with the information of particle. The correctness of the control equations and the boundary conditions is verified by simulating the standard examples. The preliminary analysis of using this method to simulate the movement of the two-dimensional centrifugal dredge pump shows that the simulation results are reasonable. It lays theoretical basis for 3-dimensional simulation of internal flow and transition process of centrifugal dredge pump by SPH algorithm. It will be very conducive to reveal the internal transient flow mechanism of flow system transition process in centrifugal dredge pump, which is of great theoretical significance to ensure the reliable and stable operation of dredge pump system.

1. Introduction

The implement of China’s coastal port infrastructure construction, deeper water channel dredging, reclamation of islands and reefs in the South China Sea, land reclamation and other projects, to bring development opportunity for china’s dredging industry. So that China has become a veritable dredger construction powers. The work efficiency of TianJing dredger which is used for reclamation of islands and reefs in the South China Sea is up to 4500 square sand per hour, the centrifugal dredge pump is the key equipment of the dredger. The structure of dredge pump is easy to be destroyed due to poor operating condition and severe wear. It seriously affects reliability and stability of the operation, which will lead to the decrease of dredge pump’s head and efficiency, increase of vibration, short of the flow part’s service life, frequent replacement and maintenance. To ensure the reliability and stability of the dredge pump system is of great importance to the rapid and efficient development of national economy, national defence, dredging and reclamation.
As China's dredging project develop to large-capacity, ultra-long-distance transport direction, dredge pump’s reliability and stability issues are more and more. Therefore, it is very important and urgent to study the stability of the large dredge pump unit. The transition process of dredge pump unit is an important part in the operation of dredge pump and runs through the entire dredge pump operation process. Its characteristics have an important influence on the safety, reliability, stable operation and service life of the dredge pump unit. Therefore, it is undoubtedly the best basis for improving the reliability and stability of the dredge pump to master the internal flow characteristics of the dredge pump during the transition process.

Compare with the finite volume method or finite element method which is mesh method, the meshfree SPH algorithm has the same number of particles in the course of moving and it strictly follows the conservation of mass. The particles are used to represent the simulated dielectric system to avoid deformation which leads to mesh distortion, so it is suitable for simulating deformation problem. The Lagrange method is also used to describe the historical process of the medium motion. Convection and transport terms are avoided and particles are arranged at specific locations, the movement of matter is determined by tracking the motion of the particles, and complex calculations are not required to track motion characteristics such as moving boundaries and motion interfaces. These characteristics will obviously improve the calculation accuracy of fluid mechanical transition process. Therefore, using SPH algorithm to achieve the transition process calculation of centrifugal dredge pump will be conducive to reveal the internal transient flow mechanism of centrifugal dredge pump flow system transition process.

2. The basic theory of smoothed particle dynamics method

Smoothed Particle Hydrodynamics (SPH) is a meshfree method developed step by step in recent years. The basic idea of this method is to describe the continuous fluid with the interaction mass point group. Each material point carries a variety of physical quantities, including mass, velocity, density and so on. The mechanical behaviour of the whole system is obtained by solving the dynamic equations of mass point group and tracking the trajectory of each particle. It is similar to the particle cloud simulation in physics. In principle, as long as the number of particle is enough, the mechanical process can be described accurately. The basic idea of SPH algorithm is to think of continuous fluid as interacting particles. These particles interact with each other to form a complex fluid motion, and it also follows the most basic Newton’s second law for each individual fluid particle.

At present, SPH algorithm is widely used in the problem of fluid mechanical related fields, such as dam break study[1], porous media flow[2], interstitial flow[3], free surface flow[4], surface tension[5], multiphase flow[6], the infiltration of water on dry particles[7] and so on. The expansion of the application field of SPH shows its strong adaptability and solving ability. Figure.1 shows an example of the SPH algorithm application.

The earliest applications of SPH algorithm in hydrodynamics field include elastic flow, magnetic fluid mechanical, multiphase flow, gravity flow, flow through porous media, quasi-incompressible flow, heat conduction, heat transfer, mass transfer and so on. In 1983, Monaghan[8] simulated the shock tube in one dimension by comparing the equations of fluid mechanics and the calculation method of artificial viscosity, and introduced the artificial viscosity term applied to the smoothed particle method into the momentum equation of hydrodynamics, which improves the traditional viscosity coefficient and effectively controls the oscillation characteristics of the results. In 1996, Johnson and Cook[9] simulate rupture growth process with SPH algorithm. In 1998, Ricard Gutfraind et.al[10] simulated the flow of sea-ice by SPH algorithm. In the numerical simulation, the flow of sea-ice is set to viscoplastic flow, and the flow of fractured sea-ice in wedge-shaped channel is numerically simulated by solving the relevant transport equation. The SPH algorithm, as an effective method for solving differential equations, can solve a specific problem by adding an effect term. Libersky[11] introduce the effect of material strength into the numerical simulation of elastic fluid phenomenon and obtain satisfactory results. Bonet et.al[12] simulated the metal forming process by smoothed particle hydrodynamics algorithm. Before 2002, the application of SPH algorithm in fluid
mechanics focused mainly on incompressible fluids. After that, Monaghan \[13\] proposed a smooth hydrodynamic method for compressible turbulent fluid and make the SPH algorithm further extended to compressible fluid. In the same year, Shao et al.\[14\] combined LES with SPH algorithm to simulate the climbing and propagation phenomenon of solitary waves, and compared experimental results with results simulated by SPH algorithm. The simulation results are in good agreement with the experimental results. As mentioned earlier, the SPH algorithm has inherent advantages in dealing with large deformations and even extreme deformations. Liu\[15\]-\[17\] simulated the explosion process of high energy explosive under water by SPH algorithm, and successfully simulated the underwater shock waves caused by the explosion and the buffering process of the water flow during the impact process.

The successful use of smoothed particle dynamics in various fluid mechanics problems and solid mechanics problems has also contributed to the development and improvement of the algorithm. In the process of improvement, scholars also put forward some problems of SPH algorithm in the calculation\[18\]. For example, during research process, Swegle et al. proposed the phenomenon that there is tension instability in SPH algorithm when simulating the problem of material strength\[19\]-\[20\], and put forward advice to improve. Since the SPH algorithm was first used in astrophysics, there would be boundary distortion in computation and so on. In order to improve the computational accuracy at the boundary, Chen\[21\]-\[22\] proposed corrective smoothed particle method (CSPM) by improving SPH algorithm. Dilts \(2000\) \[23\]-\[24\] proposed moving least square particle method (MLSPH).

Since the 1990s, Chinese scholars have gradually focused on the smoothed particle hydrodynamics method. SPH algorithm were respectively applied to the free liquid surface, high-energy explosives underwater explosion and ultra-high-speed collision. Li Meie\[25\] of Xi'an Jiao Tong University presented a new simulation method of the free surface on the basis of the SPH algorithm, and applied this algorithm to simulate incompressible free face. In the simulation, the information of the particle nodes in the calculation domain is solved by the kernel function, and the time integral is carried out by the fractional step method. So the instability problem in the numerical simulation of the incompressible fluid free liquid surface is solved. Xu Zhihong\[26\] comes from National University of Defense Technology has improved the SPH algorithm based on the Taylor expansion of Riemann solution. After joining thermal engineering, Ji Shunying\[27\] from Dalian University of Technology simulated the process of sea-ice drift in Liaodong on the basis of Hibler viscous sea-ice constitutive equation by SPH algorithm. Zong zhi\[28\] from Dalian University of Technology proved a smooth length optimization algorithm based on SPH algorithm by simulating underwater explosion. Yan xiaojun\[29\] of Beihang university modelled by combining the finite element grid with the particle node of SPH algorithm, and simulated the ultra-high-speed collision process of the space debris. Cheng Lin\[30\] applies the SPH algorithm to the two-dimensional hydraulic turbine simulation of an open system, and it has not yet achieved three-dimensional simulation.

Above all, the application of SPH algorithm in China is rarely related to the field of fluid machinery, and it still has a long way to go to apply the SPH algorithm to the field of fluid machinery.

SPH algorithm uses the particle groups to represent the flow field, and each particle carries the information of its position, such as velocity, pressure, temperature, density and other flow field information. The algorithm uses the smooth kernel approximation and the particle approximation to discrete the Navier-Stokes equations, and the partial differential equations are transformed into ordinary differential equations, and then the discrete algebraic equations are obtained. The information of each particle point can get by solving the equations. There is no problem of distortion and reconstruction of the mesh due to the algorithm is completely meshfree and particle-based method, and it is suitable for the fluid mechanical transient process and dynamic stress simulation.

The process of approximating partial differential equations by SPH algorithm consists of two steps: kernel approximation and particle approximation. The kernel approximation of the function is achieved by integrating the function and the weight function. The particle approximation is the weighted summation of all the particles in a region. For the traditional SPH algorithm, on a particle i, the kernel approximation.
(\langle f(x) \rangle)$ of any function $f(x)$ can be defined as:

$$\langle f(x) \rangle = \int_{\Omega} f(x') \delta(x-x') dx'$$  \hfill (1)

The function $\delta(x-x')$ is the Dirac function, and replaces the delta function with a weight function $W(x-x', h)$, then the integral expression of $f(x)$ can be written as:

$$\langle f(x) \rangle = \int_{\Omega} f(x') W(x-x', h) dx'$$ \hfill (2)

$$\langle f(x_i) \rangle = \sum_{j=1}^{N} f(x_j) W(x_j - x_i, h) \frac{m_j}{\rho_j}$$ \hfill (3)

$W$ is often called a smooth function or a kernel function. $h$ defines the influence area of the smooth function $W$ and is called the smooth length. Equation (2) is the integral representation of the arbitrary field function $f(x)$, that is the kernel approximation. The particle approximation of the discrete form of $f(x)$ is the weighted sum of all the particles in the support domain of the associated particle $i$. $W$ is smooth function, $kh$ is support domain, $S$ is the surface of the calculation domain $\Omega$.

Figure 1. SPH algorithm application.

Figure 2. Approximate illustration of SPH particles in two-dimensional space.
The Navier-Stokes equations are discretized by smooth kernel approximation and particle approximation. It is described as formula 4–6 after discretization, they are the conservation equations of mass, the conservation equations of momentum and the energy conservation equations:

\[
\frac{D\rho_i}{Dt} = \sum_{j=1}^{N} m_j v_{ij} \frac{\partial W_{ij}}{\partial x_{i}^j}
\]  
(4)

\[
\frac{Dv_{ij}}{Dt} = \sum_{j=1}^{N} \frac{\sigma_{ij} + \alpha_{ij} \rho_j}{\rho_i} \frac{\partial W_{ij}}{\partial x_{i}^j} + \mu_i \frac{\varepsilon_{ij}}{2\rho_i} \varepsilon_{ij}
\]  
(5)

\[
\frac{De_i}{Dt} = \frac{1}{2} \sum_{j=1}^{N} m_j \left( \frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) v_{ij} \frac{\partial W_{ij}}{\partial x_{i}^j} + \mu_i \varepsilon_{ij} \varepsilon_{ij}
\]  
(6)

The SPH algorithm uses the integral representation of smooth functions. Smooth kernel functions play a key role in the whole algorithm. This function not only defines the dimensions of the particles in the support domain, but also determines the consistency of the algorithm results and the accuracy of the results obtained by the approximation of the kernel and the approximation of the particles. The study of the Gaussian smooth function, B-spline smooth function and quasi-spline function will be respectively carried out.

3. Simulation of centrifugal dredge pump internal flow by smooth particle dynamics method

The whole process of the SPH algorithm not only includes the kernel function approximation process but also the particle calculation process, so its program structure has its corresponding particularity. These special processing works of the SPH algorithm are included in the time cycle of each section. Figure 4 shows the flow chart for the SPH algorithm.

![Flow chart of the SPH algorithm](image)

Figure 3. Flow chart of the SPH algorithm.

In order to verify the correctness of the program that simulates the viscous incompressible fluid, a program simulates two-dimensional Couette and Poiseuille flow with analytical solutions will be written, and then used to simulate the flow inside the two-dimensional centrifugal pump. In figure 4, the pump inlet and outlet are equipped with particle buffer, the blade consists of particles and as the moving boundary, rotating with a given speed. The pump body is composed of solid particles and keep still.
Figure 4. Approximate illustration of SPH Particles in Two-dimensional space.

The specific algorithm steps are as follows: Debugging development program of good SPH numerical model; Complete the numerical model to calculate the interface data exchange; According to the motion equation of centrifugal dredge pump impeller and outlet valve, the velocity of valve particles and impeller field particles is obtained. And update the particle properties in time; After 2 to 3 steps, call the corresponding numerical model to calculate step by step. Output related values after calculation and keep all fluid field data fluent; Deal with the data and end calculation.

4. Example analysis

4.1. Couette and Poiseuille flow

The two-dimensional Couette and Poiseuille flows are simulated by the SPH method. The calculation model is shown in figure 5. The distance between two parallel infinite planes is 0.001m, and the direction of the calculated region x is 0.002m, the liquid region is discretized into 50×100 particles, the upper and lower plates are dispersed into 4×100 solid wall particles. In order to simulate the infinite computational region in the x direction, the left and right boundaries adopt periodic boundaries.

Figure 5. Calculation area.

4.1.1. Couette flow. The liquid is filled between two infinitely large plates and the lower plate remains stationary. The upper plate moves at constant speed along the x direction. Equation 7 is the series solution of the velocity distribution of the flow field in the y direction. It is given as:

\[ u_y(y,t) = \frac{v_0}{L} y + \sum_{n=1}^{\infty} \frac{2v_0}{n\pi} (-1)^n \sin \left( \frac{n\pi}{L} y \right) \exp \left( -v \frac{n^2\pi^2}{L^2} t \right) \] (7)
In equation (7): \[ v = 10^{-6} \text{m}^2/\text{s}, \quad \rho = 10^3 \text{kg/m}^3, \quad L = 2 \times 10^{-3} \text{m}, \quad v_0 = 1.0 \times 10^{-5} \text{m/s} \]

In the simulation, the top 4 layers of SPH solid wall particles keep velocity \( v_0 \) moving uniformly towards the right, and the bottom 4 layers of SPH solid wall particles remain still. Figure 6 shows the comparison of the velocity distribution in the \( y \)-direction calculated by the different time program and the theoretical velocity distribution obtained by the equation (7). The maximum error between the calculated and theoretical solutions is less than 0.8%.

![Figure 6](image)

Figure 6. Comparison of couette flow velocity of theoretical solution and SPH calculation result.

4.1.2. Poiseuille flow. Two infinite plates remain stationary, and the liquid starts to move in the \( x \) direction under the action of the volume force \( F \). The series solution of the velocity distribution in the \( y \) direction is:

\[
 u_y(y, t) = \frac{F}{2v} y (L - y) - \sum_{n=0}^{\infty} \frac{4FL^2}{v \pi^3 (2n + 1)^3} \sin \left( \frac{\pi y}{L} (2n + 1) \right) \exp \left( -\frac{(2n + 1)^2 \pi^2 v}{L^2} t \right) \tag{8}
\]

In equation (8): \[ v = 10^{-6} \text{m}^2/\text{s}, \quad \rho = 10^3 \text{kg/m}^3, \quad L = 2 \times 10^{-3} \text{m}, \quad F = 10^{-4} \text{m/s}^2 \]

Figure 7 shows the comparison of the velocity distribution in the \( y \)-direction calculated by the different time program and the theoretical velocity distribution obtained by the equation (8). The maximum error between the calculated and theoretical solutions is 0.8%.

![Figure 7](image)

Figure 7. Comparison of poiseuille flow velocity of theoretical solution and SPH calculation result.
The results of the above two examples show that the control equations and calculation procedures used by the SPH method are correct when simulating viscous incompressible flow.

4.2. Two-dimensional dredge pump internal flow

The research focus is based on the SPH method and make the two-dimensional dredge pump as the research object. Create a physical model as shown in figure 5, the liquid region is discretized into fluid particles; the pump body area is discretized as solid wall particles and remains stationary, the blade area is discretized as solid wall particles and rotates at a fixed rotational speed of 300 r/min. The velocity vector of the flow field at different times is shown in figure 8.

![Figure 8](image)

**Figure 8.** The velocity vector of the flow field at different times.

5. Conclusions

It is verified the correctness of the governing equations and boundary handling methods in the simulation of viscous incompressible flow with SPH method. At present, the calculation process is limited to two dimensions, and it is convenient to simulate and calculate the transition process of two-dimensional centrifugal dredge pump. But the program needs to be optimized to simulate the three-dimensional model more effectively. The next step is to study the 3D SPH algorithm suitable for the simulation of centrifugal dredge pump, reveal the mechanism of SPH algorithm, develop and improve the three-dimensional transition process model of centrifugal dredge pumps and its numerical solution strategy and establish a mathematical model to accurately describe the transition process flow.
parameters of centrifugal dredge pump full flow system.

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