Uncertainty Analysis of Water Budget for Jinan Spring Basin Based on Numerical Model

Hanxi Ni¹, Longcang Shu¹, *, Hu Li², Portia Annabelle Opoku¹, Xiaobo Wang¹, Manqi Zhang¹ and Yafei Yu¹

¹School of Hohai University, Nanjing, China  
²Jinan Rail Transit Group Co., Ltd, Jinan, China

*Corresponding author: lcshu@hhu.edu.cn

Abstract. Groundwater numerical simulation is affected by many uncertain factors, which directly influence the precision and reliability of simulation results. Therefore, it is very important to analyze the influence of these factors. This paper studied the influence of the uncertainty of sources and sinks on the water budget and the effect on the constant spring flow. Jinan spring basin was chosen as a case study, and a groundwater numerical simulation model was established. In the model, precipitation, groundwater exploitation, and boundary conditions are selected for the local sensitivity analysis. The analysis showed that precipitation is the most important factor affecting the water budget in Jinan spring basin, whilst groundwater exploitation is the most sensitive factor for spring flow. Upon consideration of the impact of these two factors, six different scenarios were developed for the numerical simulation, and quantitative analysis was conducted of the impact of the uncertainties from the sources and sinks on Jinan spring. The results of this study can provide a scientific reference for the effective use of groundwater resources on the constant gushing of Jinan springs.

1. Introduction

Numerical simulation of groundwater has become one of the most important technical means of modern research on groundwater systems, and is widely used in domestic and international research on groundwater dynamics simulation, groundwater resource evaluation, groundwater development, and utilization. This is as a result of the improvement of the basic theory of groundwater dynamics, the continuous development of mathematical calculation methods, and the increasing update of computer technology [1]. However, limitations on technical and economic conditions make it difficult to have a comprehensive understanding of the hydrogeological conditions of a region, hence leading to various uncertainties in the predicted results of groundwater numerical simulations which directly affects the accuracy and reliability of groundwater simulations [2]. Uncertainty analysis is therefore very necessary to improve the numerical model.

In the past decades, the study of parameter uncertainty of groundwater models has gained widespread attention [3, 4, 5]. Beven et al. proposed equifinality for different parameters of hydrological models and developed the GLUE method to estimate the parameter uncertainty of hydrological models [6]. Vrugt et al. presented an MCMC (Markov Chain Monte Carlo) sampler to optimize and assess the uncertainty of hydrologic model parameters [7]. Hassan et al. also used the MCMC method to evaluate
the uncertainty of groundwater model parameters [8]. Srivastava et al. used fuzzy vertex alpha-cut techniques to analysis parameter uncertainty [9]. Zekri proposed a framework for estimating the optimal abstraction of groundwater under uncertainty [10]. Touhidul et al. presented a Bayesian approach to quantify the parameter uncertainty of fully distributed models [11]. Shu et al. combined uncertainty analysis with risk assessment for the determination of the safe yield of groundwater [12]. Lu et al. used the Morris method to do applicability evaluation and time series sensitivity analysis on hydraulic conductivity (K) and specific yield (μ) of the groundwater model [13].

However, many researchers pay more attention to the uncertainty of hydrogeological parameters when analyzing the uncertainty of the groundwater model, but often ignore the uncertainty of sources and sinks. In this paper, based on the groundwater numerical model, three factors, namely precipitation, exploitation, and boundary conditions, were selected for local sensitivity analysis to study the influence of sources and sinks on water budget and spring flow. Based on the results of the study, different scenarios were set up to analyze the impact of sources and sinks on Jinan springs, which can provide a scientific reference to ensure the continuous flow of Jinan springs.

2. Study Area
Jinan spring basin is a part of the city of Jinan. The range of Jinan spring basin starts from the Mashan Fracture in the west and extends to the Dongwu Fracture in the east, and is bounded by igneous rocks in the north and the surface watershed in the south (Fig. 1). Jinan is a world-famous "spring city", located in the north of China where carbonate karst highly occurred. There are many large karst springs in the urban area. There are more than 100 springs just within an area of 2.6 km$^2$ in the downtown area of Jinan, including the four famous spring groups: Baotu spring, Heihu spring, Wulongtan, and Zhenzhu spring.

Jinan City has a temperate monsoon climate, which is influenced by the monsoon and has an uneven spatial-temporal distribution of precipitation. Precipitation mainly falls from June to September accounting for about 70% of the whole year, while the overall spatial distribution is decreasing from south-east toward north-west. Jinan city is located on the low hills and alluvial plains of the central and northwest Shandong Province. The terrain generally shows the south high and the north low, and the plains gently slope down from southwest to northeast. Jinan City belongs to the monocline tectonic hydrogeological zone of north Mount Tai. Mount Tai was formed by metamorphism and is a watershed for regional surface water and groundwater, with the Cambrian and Sanshanzi Group being monocline which overlays on top of the metamorphic rocks, sloping to the north and buried under the quaternary system. There is a large area of Yanshanian igneous rocks in the urban area and the eastern and western suburbs. This specific topographical, geological and tectonic condition controls the spatial distribution of aquifers in the region, as well as the movement and circulation of groundwater [14, 15, 16]. The aquifers in the zone are mainly porous medium aquifers, carbonate fractured-karst aquifers, metamorphic and magmatic fractured aquifers.
3. Groundwater Numerical Model

3.1. The Conceptual Model

Based on the regional hydrogeological conditions, no-flow boundaries are considered at the south and north boundaries. For the eastern boundary, a constant flow is assigned at the north of Xujiazhuang (Fig.1) and a no-flow boundary is assumed at the south. Similarly, for the western boundary, a constant flow is assigned at the north of Sunzhuang (Fig.1) and a no-flow boundary is assumed at the south [17, 18, 19].

Based on the lithological description of boreholes and the hydrogeological conditions of the study area, the groundwater system of the study area was discretized into a three-dimensional unsteady flow with three layers of heterogeneous anisotropy media. The first layer was the porous Quaternary aquifer with a thickness of about 10-40 m; the second was the aquiclude made up of a clayey layer below the porous aquifer, and the thickness ranges from 5 to 40 meters; the third was the karst aquifer, which was the Cambrian Zhangxia Group, Fengshan Group, and Ordovician. The borehole data showed that the karst mainly developed within 600 m below the surface, so the depth of the model was 600 m.

The main recharge of groundwater in Jinan spring basin includes precipitation infiltration, irrigation return, river seepage, groundwater inflow, and artificial recharge. Discharge parameters are exploitation, groundwater outflow, and spring discharge. Evaporation discharge was not considered in this simulation because the buried depth of phreatic water in the study area was more than 4 m.

Table 1 shows the calculation results of water budget in Jinan spring basin from October 2012 to September 2013. It can be seen that the total amount of recharge in the basin was 217.0 million m$^3$/a, and the discharge was 173.1 million m$^3$/a. The total amount of recharge was greater than the total amount of discharge, which is a positive equilibrium.
Table 1. Calculation result of water budget in Jinan spring basin.

| Items                  | Values (million m³/a) | Percentage (%) |
|------------------------|-----------------------|----------------|
| Precipitation infiltration | 159.5                | 73.5           |
| River seepage           | 26.39                 | 12.2           |
| Irrigation return       | 4.160                 | 1.90           |
| Groundwater inflow      | 18.16                 | 8.40           |
| Artificial recharge     | 8.800                 | 4.10           |
| **Total**               | **217.0**             | **100**        |
| Groundwater outflow     | 9.960                 | 5.80           |
| Spring                  | 69.04                 | 39.9           |
| Exploitation            | 94.12                 | 54.4           |
| **Total**               | **173.1**             | **100**        |
| Difference              | 43.89                 |                |

3.2. The Mathematical Model

According to the method of reduced permeability coefficient, the four states of flow in the karst pipeline are represented by the unified flow rules, which can unify the flow in the karst pipeline with the flow in the pores and fractures [20]. Therefore, the equivalent porous media model was used to simulate the karst groundwater seepage in this study.

FEFLOW is a simulation software of groundwater flow and solute transport developed by WASY in Germany. The software uses a multi-dimensional finite element method (mesh generation) to solve the control equations of groundwater flow, mass, and heat migration in two-dimensional or three-dimensional complex pore-fracture media. It has many advantages such as a good GIS data interface, an optimized mesh generation technology and data analysis tools.

According to the above hydrogeological conceptual model, the corresponding mathematical model was established. The mathematical model of the confined aquifer is as follows:

\[
\begin{align*}
\begin{aligned}
\frac{\partial}{\partial x} \left( K_{xx} \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_{yy} \frac{\partial H}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_{zz} \frac{\partial H}{\partial z} \right) + W &= S_s \frac{\partial H}{\partial t} \quad (x,y,z) \in D \\
H(x,y,z,t) |_{r_1} &= H_1(x,y,z,t) \quad (x,y,z) \in r_1, t>0 \\
K_{xx} \frac{\partial H}{\partial n} \mid_{r_2} &= q(x,y,z,t) \\
H(x,y,z,t) |_{t=0} &= H_0(x,y,z) \quad (x,y,z) \in D
\end{aligned}
\end{align*}
\]

(1)

The model of the unconfined aquifer:

\[
\begin{align*}
\begin{aligned}
\frac{\partial}{\partial x} \left( K(H-B) \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y} \left( K(H-B) \frac{\partial H}{\partial y} \right) + \frac{\partial}{\partial z} \left( K(H-B) \frac{\partial H}{\partial z} \right) + W &= \mu \frac{\partial H}{\partial t} \quad (x,y,z) \in D \\
H(x,y,z,t) |_{r_1} &= H_1(x,y,z,t) \quad (x,y,z) \in r_1, t>0 \\
K(H - B) |_{r_2} &= q(x,y,z,t) \\
H(x,y,z,t) |_{t=0} &= H_0(x,y,z) \quad (x,y,z) \in D
\end{aligned}
\end{align*}
\]

(2)

Where, \( K \) is the hydraulic conductivity of the aquifer, \( K_{xx}, K_{yy} \) and \( K_{zz} \) are the components of the hydraulic conductivity in different directions, \( H \) is the water level, \( B \) is the elevation of the floor of the unconfined aquifer, \( W \) is sources and sinks, \( \mu \) is specific yield, \( S_s \) is the water storage rate, \( H_0(x,y,z) \) is the initial water level, \( r_1 \) is Dirichlet boundary, \( r_2 \) is Neumann boundary, \( n \) is the direction of outward normal of \( r_2 \).
The study area covers an area of 1795 km² and was discretized into 56964 cells in three layers. The period of October 1, 2012 to September 30, 2013 was chosen as the calibration period for the model, and the period of October 1, 2013 to September 30, 2014 was chosen as the verification period. The length of the stress period was a month, and the time step was set to 0.001 days.

The hydraulic properties of the aquifers were selected based on typical values for aquifer materials, borehole lithology, and pumping test data. The observation data of 9 observation wells within the study area were used for model calibration and verification.

### 3.3. Calibration and Verification

The calibration and verification of the model parameters were conducted using a trial-and-error method, which is one of the methods of reverse parameter calculation [21]. This was done by repeatedly adjusting selected parameters, fitting the dynamic process of the flow field and groundwater level at the same period to make the model closer to actual hydrogeological conditions of the study area as much as possible.

Fig.2 and Fig.3 show the groundwater level fitting of typical observation wells in the calibration and verification period. In general, the fitting error between the observed value and the calculated value of the groundwater level should be less than 10% of the variation of the water level in the calculation period. When the variation of water level is small (<5 m), the fitting error should be less than 0.5 m [22]. As can be seen from Figure 2 and Figure 3 below, the calculated values of the model at the observation point fit well with the observed values, and the simulation results are reasonable, which could reflect the real hydrogeological conditions of the study area. The calibrated and verified numerical model can be used to carry out groundwater dynamic prediction and groundwater resource evaluation under different scenarios.

**Figure 2.** Fitting curves of groundwater level for typical observation wells in the calibration period.
Figure 3. Fitting curves of groundwater level for typical observation wells in the verification period.

4. Uncertainty Analysis of Water Budget

4.1. Local Sensitivity Analysis

The main recharge of groundwater in the study area is precipitation infiltration, accounting for 70% of the total recharge amount. The main discharge is exploitation besides spring discharge. Therefore, we chose precipitation and exploitation as the factors for the uncertainty analysis of water budget, which can help to understand the recharge and discharge conditions of the groundwater system in the study area. Also, complex problems are often encountered when dealing with boundary conditions because the flow condition at the boundary is not only influenced by natural factors, but also influenced by human activities. Due to the coupling effect, the boundary conditions are highly uncertain in simulation and prediction, which also leads to the uncertainty of the water budget calculation. Therefore, this paper also analyzed the boundary conditions.

Local sensitivity analysis was used to study the influence of a single parameter change on the numerical simulation results. During the analysis, only the value of one selected parameter is changed, while other parameters remain unchanged. The sensitivity coefficient of a particular parameter $k$ can be approximated by the following formula [23, 24]:

$$X_k = \frac{\partial y}{\partial \alpha_k} \approx \frac{y(\alpha_k + \Delta \alpha_k) - y(\alpha_k)}{\Delta \alpha_k}$$

Based on the deterministic model, the selected sources and sinks (precipitation, exploitation, boundary conditions) were increased and decreased by 10% and 20%, respectively. The variation of spring flow was selected as the foundation of sensitivity analysis to observe the change of results of the model and compare the sensitivity of sources and sinks.

As can be seen from Fig. 4 (a), precipitation has the greatest influence on water budget, followed by exploitation, and the boundary conditions with the least influence. This also proves that groundwater in the study area is mainly recharged by precipitation. Fig. 4 (b) shows that the most significant factor influencing spring flow is exploitation, followed by precipitation, and the boundary conditions as the least. The reason for this result may be that there are many exploitation wells near the spring, so the groundwater exploitation has the most direct impact on the flow of the spring. Meanwhile, when the study area is large, the change of boundary conditions may have a far greater impact on the adjacent area than on the whole regional groundwater system.
4.2. Influence of Uncertainty from Water Budget

Since water budget and spring flow are mainly affected by precipitation and exploitation, different scenarios were set up to take into account the effects of both for model calculation and simulation. Based on the annual precipitation data from 1956 to 2016, the precipitation frequency was calculated, and precipitation at 25%, 50%, and 75% guaranteed rate were selected. Then precipitation of the corresponding typical years of 1996, 2008, and 1970 was selected as the precipitation of the prediction model. Different precipitation conditions were divided into present exploitation conditions (quantity: 257,900 m$^3$/d) and 30% reduced present exploitation conditions (quantity: 180,500 m$^3$/d). A total of six scenarios were set. The average annual precipitation for different years is as follows: wet years (813.1 mm), normal years (705.1 mm), and dry years (554.1 mm).

| Exploitation  | Precipitation | Spring flow (million m$^3$/a) | Spring water level (m) |
|---------------|---------------|-------------------------------|------------------------|
| 1             | Present exploitation | Wet year | 61.94 | 28.19 |
| 2             | Normal year | 58.55 | 28.12 |
| 3             | Dry year | 55.59 | 28.06 |
| 4             | Wet year | 71.25 | 28.75 |
| 5             | Normal year | 67.93 | 28.69 |
| 6             | Dry year | 64.20 | 28.60 |

As can be seen from Table 2, the spring flow varies significantly under different exploitation and precipitation conditions, while the spring water level is more affected by precipitation. Under the same exploitation condition, affected by precipitation, the variation of spring flow is within 3 and 4 million m$^3$/a, and the fluctuation of spring water level is within 0.06 and 0.09 m. Under the same precipitation condition, the spring flow under the reduced exploitation condition increased by about 15%, and the effect on the spring water level was greater, with the water level rising by 0.54~0.57 m. It shows that the spring flow and spring water level are significantly affected by exploitation, so the spring protection may not be effective only through groundwater recharge. It is necessary to adjust the exploitation layout, and the combination of reduced exploitation and groundwater recharge will be more effective action to ensure the continuous gushing of Jinan spring.
5. Conclusions
Based on the groundwater numerical model, the local sensitivity analysis of precipitation, exploitation, and boundary conditions shows that precipitation has the greatest impact on the water budget of Jinan spring basin, followed by exploitation, and boundary conditions the least. The most significant factor affecting spring flow is exploitation, followed by precipitation, and boundary conditions as the least influential.

In the meantime, considering the influence of precipitation and exploitation, six scenarios were set up under different precipitation and exploitation conditions, and a numerical model was used to simulate and predict spring flow and spring water level. Results showed that under the condition of reducing the current exploitation by 30%, spring flow increased by about 15%, and spring water level increased by 0.54 - 0.57 m. Spring flow was also influenced by precipitation, and the change was in the range of 3 - 4 million m$^3$/a. Recharge alone is not the best measure to preserve the spring, and the combination of reducing exploitation and groundwater recharge can be more effective in sustain Jinan springs.

This study has confirmed that parameter uncertainty is a part of the overall uncertainty of groundwater numerical model. Water budget determines the dynamic variation trend of groundwater in the whole region, so there is a need to be concerned about the uncertainty of water budget in a study area. In other words, how sources and sinks influence the groundwater flow system cannot be ignored. With the establishment of a groundwater numerical model, the uncertainty of sources and sinks can be properly captured. In lack of the uncertainty of sources and sinks in a single deterministic conceptual model, its prediction ability will be overestimated. Therefore, further studies shall focus more about the uncertainty analysis of the groundwater conceptual model.

Acknowledgments
This work was financially supported by the Major Innovation and Technology Projects of Shandong Province (No. 2019JZZY020105).

References
[1] Wang Y X, Lin X Y. The development strategy of China discipline: Groundwater science[M]. Beijing: Science Press, 2018:249-250.
[2] Wu J C, Lu L. Uncertainty analysis for groundwater modeling[J]. Journal of Nanjing University (Natural Sciences Edition), 2011, 47(3): 227-234. DOI: 10.13232/j.cnki.jnju.2011.03.009.
[3] Sreekanth, J., Moore, C. & Wolf, L. Estimation of Optimal Groundwater Substitution Volumes Using a Distributed Parameter Groundwater Model and Prediction Uncertainty Analysis. Water Resour Manage 29, 3663–3679 (2015). https://doi.org/10.1007/s11269-015-1022-y.
[4] Norouzi Khatiri, K., Niksokhan, M.H., Sarang, A. et al. Coupled Simulation-Optimization Model for the Management of Groundwater Resources by Considering Uncertainty and Conflict Resolution. Water Resour Manage 34, 3585–3608 (2020). https://doi.org/10.1007/s11269-020-02637-x.
[5] Shi X Q, Wu J C, Jiang B L, et al. Uncertainty analysis of groundwater models based on the Latin hypercube sampling technique[J]. Hydrogeology & Engineering Geology, 2009, 36(2): 1-6. DOI: 10.16030/j.cnki.issn.1000-3665.2009.02.008.
[6] Beven K, Binley A. The future of distributed models: Model calibration and uncertainty prediction[J]. Hydrological Processes, 1992, 6(3):279-298. DOI: 10.1002/hyp.3360060305.
[7] Vrugt J A, Gupta, H V, Bouten, W, et al. A Shuffled Complex Evolution Metropolis algorithm optimization and uncertainty assessment of hydrologic models parameters[J]. Water Resources Research, 2003, 39(8):1201. DOI: 10.1029/2002wr001642.
[8] Hassan A E, Bekhit H M, Chapman J B. Using Markov Chain Monte Carlo to quantify parameter uncertainty and its effect on predictions of a groundwater flow model[J]. Environmental Modelling & Software, 2009, 24(6):749-763. DOI: 10.1016/j.envsoft.2008.11.002.
[9] Srivastava, D., Singh, R.M. Groundwater System Modeling for Simultaneous Identification of Pollution Sources and Parameters with Uncertainty Characterization. Water Resour Manage 29, 4607–4627 (2015). https://doi.org/10.1007/s11269-015-1078-8.

[10] Zekri, S., Triki, C., Al-Maktoumi, A. et al. An Optimization-Simulation Approach for Groundwater Abstraction under Recharge Uncertainty. Water Resour Manage 29, 3681–3695 (2015). https://doi.org/10.1007/s11269-015-1023-x.

[11] Touhidul M S M, Jiri N, Gert G, et al. Estimation and Impact Assessment of Input and Parameter Uncertainty in Predicting Groundwater Flow with a Fully Distributed Model[J]. Water Resources Research, 2018, 54(9):6585-6608. DOI: 10.1029/2017WR021857.

[12] Shu L C, Zhu Y S, Sun Q Y, Peng X M. Risk analysis for the determination of the safe yield of groundwater[J]. Journal of Hydraulic Engineering, 2000(03):79-83. DOI:10.13243/j.cnki.slxb.2000.03.014.

[13] Lu C P, Shu L C, Liu L H, Liu P G, Dong G M. Adaptive evaluation of groundwater numerical simulation accuracy based on sensitivity analysis[J]. Journal of Hohai University(Natural Sciences), 2010, 38(01):26-30. DOI:10.3876/j.issn.1000-1980.2010.01.006.

[14] SUN B, PENG Y M. Boundary condition, water cycle and water environment changes in the Jinan spring region[J]. Carsologica Sinica, 2014, 33(3):272-279. DOI: 10.11932/zgyr20140302.

[15] XI D Y, SUN B, QIN P R. Research on Jinan springs[M]. Jinan: Jinan Press, 2017.

[16] Wang Q B, Duan X M, Gao Z D, et al. Groundwater numerical simulation in Jinan karst spring basin[J]. Hydrogeology & Engineering Geology, 2009, 036(005):53-60. DOI:10.16030/j.cnki.issn.1000-3665.2009.05.001.

[17] SUN B, PENG Y M, LI C S, et al. Division of karst water system and hydraulic connection of typical spring fields in Jinan city[J]. Shandong Land and Resources, 2016, 32(10):31-34+38. DOI:10.3969/j.issn.1672-6979.2016.10.007.

[18] SHANG G Y, WANG J J. Scientific protection of springs with a definite aim—an argument for boundary conditions of Jinan spring basin[J]. Groundwater, 2002: 191-194+223. DOI: 10.3969/j.issn.1004-1184.2002.04.001.

[19] DONG Y M, SU G X, LI Z H. To explore the west boundary of Jinan spring basin from Jixi pumping experiment[J]. Water Resources Protection, 2004:58-59. DOI: 10.3969/j.issn.1004-6933.2004.03.019.

[20] ZHAO J, LAI M, SHEN Z Z. Improved converting permeability coefficient method and variable permeability coefficient method for seepage calculation in karst region[J]. Chinese Journal of Rock Mechanics and Engineering, 2005: 1341-1347. DOI: CNKI:SUN:YSLX.0.2005-08-015.

[21] LI Q Y, REN Y, CHENG Z L. Identifying and verifying method and standard of groundwater numerical model[J]. South-to-North Water Transfers and Water Science & Technology, 2012, 10(A02):30-31.

[22] HAN Z S. Technical requirements for groundwater numerical method—Introduction to professional standard[J]. HYDROGEOLOGY AND ENGINEERING GEOLOGY, 1999:49-52. DOI:10.16030/j.cnki.issn.1000-3665.1999.04.013.

[23] Shu L C, Liu P G, Liu B, et al. Parameter sensitivity analysis of a mathematical model of a riverside well field——Based on riverside well field in Beipiao city[J]. Geotechnical Investigation & Surveying, 2006(08):29-31.

[24] Li S, Chen J J, Ye H H, et al. Sensitivity analysis of stochastic factors in groundwater numerical simulation[J]. Journal of Hydraulic Engineering, 2006, 037(008):977-984. DOI:10.13243/j.cnki.slxb.2006.08.014.