Application of Graph Theory in Electrical Network

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Abstract: Graph theory is helpful in various practical problems solving in circuit or network analysis and data structure. It leads to graph practically not possible to analyze without the aid of computer. In electrical engineering the word is used for edge, node for vertex and loop for circuit. An electrical network is the set of electronic components i.e. resistors, inductors and capacitors etc. Electric network analysis and synthesis are the study of network topology. Electric network problem can be represented by drawing graphs. In this paper, we present a circuit network in the concept of graph theory application and how to apply graph theory to model the circuit network.

Keywords: Graph theory, adjacency matrix, electrical circuit and analysis

1. Introduction

A connected graph without closed path i.e. tree was implemented by G.Kirchhoff in 1847 and he employed graph theoretical concept in the calculation of currents in network or circuits and was improved upon J.C.Maxwell in 1892.[4] Ever since, graph theory has been applied in electrical network analysis. An electrical network is a collection of components and device interconnected electrically. The network components are idealized of physical device and system, in order to for them to represent several properties, they must obey the Kirchhoff’s law of currents and voltage.[1] A graph representation of electrical network in terms of line segments or arc called edges or branches and points called vertices or terminals.

2. Basic Definition of Graph Theory

Graphs are amenable for pictorial representation of a system using two basic components vertex and edges. A vertex is represented by a dot and an edge is represented by line segment connecting the dots associated with the edge. If the edges of a graph direct one vertex to the other vertex, then the graph is called as a directed graph. Otherwise graph is called an undirected graph. [2] Formally, a graph G= (V, E) contains a finite set V= (v1, v2,…..vn) of elements called vertices and a finite set (e1, e2,…..en) of elements called edges. In an undirected graph G= (V, E), the edges are unordered pairs, and each edge e1 in E is associated with two vertices v1 and v2, and it is written as either e1= (v1,v2) or e1= (v2,v1).But, In a directed graph, each edge e1 in E is associated with an ordered pairs of vertices (v1, v2) and it is denoted the directed edge e1 from v1 to v2. Two vertices v1 and v2 of a graph are adjacent, if there is an edge, v1v2 connecting them, then vertices are them considered incident to the edge v1v2. [5]

3. Analysis of Electrical Circuit

Ohm’s law states that for an edge ‘e’, the current flowing across that edge Ie is given by Ie = \frac{PC}{RC} = PC.CE

We see that this means that i{uv} = -i{vu} and the negative current as positive currents flowing the different way. The weight of an edge as the conductance of that edge, which denote Ce for a given edge e. The resistance of an edge re is defined as re = \frac{1}{CE}. [3]

Both the resistance and conductance are independent of edge such as r(vu) = r(uv) and c(uv) = c(vu).

4. Kirchhoff’s Circuit Law

Kirchhoff’s voltage law states that for a closed loop SV=0 or SV rise is equal to SV drops.[1] The total resistance of ‘n’ resistors in series is RT= R1+R2+R3+……..+Rn and the total power are PT= P1+P2+P3+……..Pn

In series, So that the same current flows through all the components but a different potential voltage can exist across every one. In parallel, so that the same potential difference exists across every components but each component may carry a different current.

Representation of circuit and its graph:
A graph model is used to represented circuit network inn graph by tracing the nodes of the circuit and edges contain in circuit.

Here is the graph of the circuit,
A circuit is a path which ends at the vertex it begins. An electric circuit is a closed loop formed by source, wires, load, and a switch, when switch is turned on the electrical circuit is complete and current flows from negative terminals of the power source. An electrical circuit is categories into three type namely series, parallel and series and parallel circuit. The representation of graph in circuit network are one of the type of representation of graph in which the current flows in circuit and present the linking of connection between resistors series and parallel connection are determined in the circuit.[4] The representation is

The schematic figure of the electric circuit is as follows,

The electrical features of individual network components can be representing suitably in the form of primitive network matrix that describe the performance of interconnected network.

5. Graph Representation of Matrix

A graph can actually be represented using matrices method the two of the most widely used matrices for graph representation is adjacency and incidence matrices. An adjacency matrix is a square matrix in which each row and column is represented by a vertex [5].

Consider figure 4, as an example it has three vertices V={R1, R2, R3} this mean that the square matrix must be 3x3 let each row and column is represented by each of the six vertices in V.

|   | R1 | R2 | R3 |
|---|----|----|----|
| R1| 0  | 1  | 0  |
| R2| 1  | 0  | 1  |
| R3| 0  | 1  | 0  |
| R4| 0  | 0  | 1  |
| R5| 0  | 0  | 0  |
| R6| 0  | 1  | 1  |

The adjacency matrix of 3×3 matrix square matrix represented as follows

\[
G = \begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 0
\end{bmatrix}
\]

6. Conclusion

In this research we focus on the application of graph theory to electrical network analysis and matrix approach as an electrical network analysis. Graph theory is a very interesting topic in mathematics due to numerous applications in various fields especially in computer and electrical engineering. We use the graph theory concept and techniques that we have developed to study electrical networks. Thus, graph theory has more practical application particularly in solving electric network.

References

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