First identification of large electric monopole strength in well-deformed rare earth nuclei

K. Wimmer\textsuperscript{a}, V. Bildstein\textsuperscript{b}, K. Eppinger\textsuperscript{b}, R. Gernhäuser\textsuperscript{b}, D. Habs\textsuperscript{a}, Ch. Hinke\textsuperscript{b}, Th. Kröll\textsuperscript{b}, R. Krücken\textsuperscript{b}, R. Lutter\textsuperscript{a}, H.-J. Maier\textsuperscript{a}, P. Maierbeck\textsuperscript{b}, Th. Morgan\textsuperscript{a}, O. Schaile\textsuperscript{a}, W. Schwerdtfeger\textsuperscript{a}, S. Schwertel\textsuperscript{b} and P.G. Thirolf\textsuperscript{a}

\textsuperscript{a}Fakultät f. Physik, Ludwig-Maximilians-Universität München, 85748 Garching Germany
\textsuperscript{b}Physik Department E12, Technische Universität München, 85748 Garching Germany

Abstract

Excited states in the well-deformed rare earth isotopes $^{154}$Sm and $^{166}$Er were populated via “safe” Coulomb excitation at the Munich MLL Tandem accelerator. Conversion electrons were registered in a cooled Si(Li) detector in conjunction with a magnetic transport and filter system, the Mini-Orange spectrometer. For the first excited $0^+$ state in $^{154}$Sm at 1099 keV a large value of the monopole strength for the transition to the ground state of $\rho^2(E0; 0^+_2 \rightarrow 0^+_g) = 96(42) \cdot 10^{-3}$ could be extracted. This confirms the interpretation of the lowest excited $0^+$ state in $^{154}$Sm as the collective $\beta$-vibrational excitation of the ground state. In $^{166}$Er the measured large electric monopole strength of $\rho^2(E0; 0^+_4 \rightarrow 0^+_g) = 127(60) \cdot 10^{-3}$ clearly identifies the $0^+_4$ state at 1934 keV to be the $\beta$-vibrational excitation of the ground state.

1 Introduction

The structure of excited $0^+$ states in deformed even-even nuclei is still a matter of controversial discussion despite intensive investigation. Traditionally the first excited $0^+_2$ state has been interpreted as the $\beta$-vibrational excitation of the ground state. However, in many nuclei the $0^+_2$ state has only weak transitions to the ground-state band, while strong electric quadrupole transitions to the $\gamma$ band have been found \[1\]. This contradicts the traditional interpretation, since a transition from a $\beta$-vibrational state to the $\gamma$ band is suppressed due to the destruction of a $\beta$ phonon and, at the same time, the creation of a $\gamma$ phonon.
In this picture a $\beta$-vibrational state is characterized by a strong transition to the ground-state band, namely by a large $B(E2; 0^+_2 \rightarrow 2^+_g) \approx 10$ W.u. value and a strong E0 transition to the ground state with $\rho^2(E0) \approx 100 \cdot 10^{-3}$ [2]. Only in very few cases, such as $^{154}$Sm [3] and $^{166}$Er [4], it has been possible to identify candidates for a $\beta$-vibrational state by $\gamma$ spectroscopy. The unclear situation led to an intense debate about the structure of low-lying $0^+$ states. Based on calculations using the interacting boson approximation (IBA) [5,6], Casten and von Brentano [1] have proposed that the $0^+_2$ state in deformed nuclei should be interpreted as a second $\gamma$ phonon excitation built on the $\gamma$ vibration. Since in many cases the excitation energy of the $0^+_2$ state is located below the $\gamma$ band and $B(E2)$ values to the ground-state band as well as to the $\gamma$ band show large fluctuations, this interpretation has been challenged by Burke and Sood [7], Kumar [8] and Günther [9].

In the original work by Casten and von Brentano, it was assumed that the deformed nuclei are best described by a small area in the parameter space of the IBA, which led to the prediction of the character of the $0^+_2$ state in deformed nuclei as a two phonon $\gamma\gamma$ vibration. In the framework of the simplified ECQF formalism [11] nuclei are described by two parameters, $\zeta$ and $\chi$ and two scaling factors for energies and transition rates, respectively. Recent work by McCutchan et al. [10] mapped the position of the deformed nuclei for different isotopic chains of rare earth nuclei within the IBA symmetry triangle, revealing that the IBA parameters to describe the low-lying structure of these nuclei can differ significantly. The position within the symmetry triangle for well-deformed nuclei was later related to the underlying single-particle structure near the Fermi surface and the resulting quasi-particle structure of the $\gamma$-vibrational state [12].

It was also shown in recent years that the IBA consistently predicts that the E0 strength from the first or second excited $0^+$ state in deformed nuclei is large [13]. Near the $U(5) - SU(3)$ leg ($\chi = -\sqrt{7}/2$) the $0^+_2$ state carries the E0 strength, while near the $O(6)$ corner, the $0^+_3$ state exhibits large E0 strength. In an area in between the strength is shared among the $0^+_2$ and $0^+_3$ states. This IBA prediction for well-deformed nuclei is not confirmed experimentally, due to the lack of measured $\rho^2(E0)$ values of the E0 strength for excited $0^+$ states in these nuclei. For the few measured examples, such as $^{166}$Er and $^{172}$Yb, where a small E0 strength was observed for the $0^+_2$ states, it is not clear if these $0^+$ states correspond to those for which the IBA predicts large $\rho^2(E0)$ values. It is therefore important to obtain more experimental data on E0 strength in well-deformed nuclei, which may also lead to new insights in the nature of the low-lying $0^+$ states.

It is the purpose of this letter to report on the first observation of large E0 strength of excited $0^+$ states in the well-deformed nuclei $^{154}$Sm and $^{166}$Er, which confirm the interpretation of the $0^+_2$ and $0^+_3$ states, respectively, as $\beta$-
vibrational states. Before we describe the details of the performed experiments and their results in the next sections, we will briefly review the existing information on $0^+$ states in $^{154}$Sm and $^{166}$Er. Due to the existence of the systematic parameter studies [10] and the explicit predictions of the E0 strength in well-deformed nuclei [13] within the framework of the IBA, we will concentrate our discussion in the final section on a comparison with IBA calculations. Although a similar comparison could and should be done on the basis of collective models, such as the General Collective Model (GCM) [14], we are not aware of a systematic set of GCM calculations, including predictions for the E0 strength, for the nuclei in question.

The nature of excited $0^+$ states in $^{154}$Sm is particularly interesting, since $^{154}$Sm is the only rare earth nucleus with two excited $0^+$ states below the excitation energy of the band head of the $\gamma$ band at 1440 keV. The excitation energies of the two $0^+$ states are only 103 keV apart, however, they have very different properties. As the $0^+_3$ state is only weakly populated in Coulomb excitation a small transition strength to the ground-state band can be concluded [3]. The $0^+_2$ state at 1099 keV has very different properties, the measured lifetime of 1.3(3) ps results in a rather large transition probability of $B(E2; 0^+_2 \rightarrow 2^+_g) = 12(2)$ W.u.. This leads to the interpretation of the $0^+_2$ state as being the $\beta$ vibration built on the ground state, while the $0^+_3$ state cannot be interpreted as a collective excitation and also does not mix appreciably with the $0^+_2$ state. To confirm this interpretation, the electric monopole strength $\rho^2(E0; 0^+_2 \rightarrow 0^+_g)$ has to be determined.

In $^{166}$Er four excited $0^+$ states are known from two-neutron transfer experiments [15]. The $B(E2)$ values for the transitions from the first three excited $0^+$ states to the ground-state band and to the $\gamma$ band were obtained from lifetime measurements using the Doppler-shift attenuation method following inelastic neutron scattering [4]. The $0^+_2$ and $0^+_3$ states at 1460 keV and 1713 keV have small $B(E2)$ values to both the ground-state band as well as to the $\gamma$ band. This and the strong relative population in two-neutron transfer reactions [15] suggests that these states are mainly pair-type excitations. In contrast, the $0^+_4$ state at 1934 keV has a strong transition strength branch to the ground-state band ($B(E2; 0^+_4 \rightarrow 2^+_g) = 8.8(9)$ W.u.) and no observable decay to the $\gamma$ band. Thus the $0^+_4$ state is interpreted as a $\beta$-vibrational state. In addition, a fifth $0^+$ state was reported in Ref. [16] and interpreted as the $0^+$ member of the $\gamma\gamma$ phonon multiplet, due to its large $B(E2; 0^+_5 \rightarrow 2^+_g)$ value. As mentioned earlier, the electric monopole strength for the $0^+_2$ state was measured to $\rho^2(E0; 0^+_2 \rightarrow 0^+_g) = 2.2(8) \cdot 10^{-3}$ [17], hence a rather small value supporting the interpretation not to be the $\beta$ vibration.
Excited states in $^{154}\text{Sm}$ and $^{166}\text{Er}$ were populated via safe Coulomb excitation using isotopically enriched self-supporting targets (760 and 995 µg/cm$^2$, respectively) and an $^{16}\text{O}$ beam from the Tandem accelerator of the Maier-Leibnitz-Laboratory (MLL) in Munich ($E_{\text{lab}} = 55, 60$ and 65 MeV). Scattered particles were detected in a 64-fold segmented double-sided Silicon strip detector (DSSSD) in backward direction (covering angles from 152° to 170°). The electrons were registered in a cooled Si(Li) detector in conjunction with a Mini-Orange (MO) spectrometer. Simultaneously the $\gamma$ rays emitted by the excited nuclei were detected with a MINIBALL triple-cluster Germanium detector [18]. A sketch of the setup is shown in Fig. 1.

The electric monopole strength $\rho^2(E0)$ is used to characterize E0 transitions. It is given by

$$\rho(E0; i \rightarrow f) = \frac{\langle f|M(E0)|i\rangle eR^2}{eR^2}$$

where $R$ is the nuclear radius ($R \approx 1.2A^{1/3}$ fm) and $M(E0)$ is the monopole matrix element. The corresponding partial lifetime $\tau(E0)$ is given by the elec-
tric monopole strength \( \rho^2(E0) \) and the non-nuclear electronic factors \( \Omega \):

\[
\frac{1}{\tau(E0)} = \rho^2(E0) \cdot (\Omega_K + \Omega_L + \ldots + \Omega_{IP})
\]  

(2)

Experimentally the monopole strength is determined from the ratio of \( E0 \) and \( E2 \) K-conversion intensities \( q^2_K \) and the \( E2 \) transition rate \( W_\gamma(E2) \) [20].

\[
\rho^2(E0) = q^2_K(E0/E2) \cdot \frac{\alpha_K(E2)}{\Omega_K(E0)} \cdot W_\gamma = \frac{I_K(E0)}{I_K(E2)} \cdot \frac{\alpha_K(E2)}{\Omega_K(E0)} \cdot \frac{1}{\tau_\gamma}
\]  

(3)

The conversion coefficients \( \alpha_K \) and the electronic factors \( \Omega_K \) are tabulated [21], the lifetime of the excited \( 0^+ \) states of interest is known from previous experiments.

3 Results for \( ^{154}\text{Sm} \)

Fig. 2 shows the \( ^{154}\text{Sm} \) conversion electron singles spectrum for 60 MeV beam energy. The \( 0^+_2 \rightarrow 0^+_g \) and the \( 2^+_2 \rightarrow 2^+_g \) transitions in \( ^{154}\text{Sm} \) are only 3.5 keV apart and cannot be separated unambiguously in our experiment with a detector resolution of 4.6 keV and additional Doppler broadening. The binding energy for electrons in the K-shell amounts to 46.8 keV. Besides the K conversion peak at 1050 keV the L conversion can be seen at \( E_\gamma = 1091 \) keV (binding energy 7.7 keV).

Fig. 2. Singles energy spectrum of conversion electrons following Coulomb excitation of \( ^{154}\text{Sm} \).

Since the \( \rho^2(E0) \) value for the \( 2^+_2 \rightarrow 2^+_g \) transition is not known, the relative contributions of the two transitions (\( 0^+_2 \rightarrow 0^+_g \) and \( 2^+_2 \rightarrow 2^+_g \)) to the peak could not be determined. Therefore, the \( \rho^2(E0; 0^+_2 \rightarrow 0^+_g) \) value could not
be deduced from the singles spectrum. We performed Coulomb excitation calculations showing that the excitation probability for multiple excitations rises with increasing scattering angle. Since the $0^+$ states can only be excited in multiple-step processes, their excitation probability rises for large scattering angles, whereas the excitation probability of the $2^+_g$ state slightly drops with increasing angle. For particles that are scattered onto the particle detector, the excitation probability for the $0^+_g$ state is by a factor of 13 larger than for the $2^+_g$ state. Thus for electrons in coincidence with $^{16}$O ions hitting the DSSSD, the contribution from the $2^+_g \rightarrow 2^+_g$ transition can be neglected, even under the assumption that both E0 transitions have similar strength.

Fig. 3. Background-subtracted $^{154}$Sm conversion electron spectrum in coincidence with particles hitting the DSSSD.

Fig. 3 shows the conversion electron spectrum in coincidence with backscattered projectiles. The transitions $0^+_g \rightarrow 0^+_g$ at $E_e = 1053$ keV and $0^+_g \rightarrow 2^+_g$ at $E_e = 971$ keV from the first excited $0^+$ state are the strongest lines in the spectrum. The observed intensity in this spectrum is 23.4(48) counts in the E0 K-conversion transition line at $E_e = 1053$ keV and 10.4(32) counts in the E2 transition measured in 55 h beam time. The K conversion coefficient for the 1018 keV E2 transition in $^{154}$Sm is $\alpha_K(E2) = 2.045 \times 10^{-3}$ and the $\Omega$ factor is $\Omega_K(E0) = 3.688 \cdot 10^{10}$ s$^{-1}$ [21]. The lifetime of the first excited $0^+$ state has been measured to $\tau = 1.3(3)$ ps [3]. Thus a value of $\rho^2(E0; 0^+_g \rightarrow 0^+_g) = 96(42) \cdot 10^{-3}$ can be extracted.

With this value now also the electric monopole strength for the $2^+_g \rightarrow 2^+_g$ transition can also be determined from the number of counts in the peak in Fig. 2. The ratio $q_E^2(E0/E2)$ for the $2^+_g \rightarrow 2^+_g$ transition can be determined to be smaller than 0.97. The $\Omega$ factor is $\Omega_K(E0) = 3.65 \cdot 10^{10}$ s$^{-1}$ [21] and the lifetime of $2^+_g$ state has been measured in deuteron scattering ($\tau = 2.36(60)$ ps from $B(E2; 0^+_g \rightarrow 2^+_g) = 0.020(5) \cdot e^2b^2$ [22]) and Coulomb excitation ($\tau > 3.5$ ps [3]). Using $\tau = 2.36(60)$ ps (partial lifetime of the transition $2^+_g \rightarrow 2^+_g$):
5.8(15) ps) one obtains $\rho^2(E0; 2_2^+ \rightarrow 2_1^+) < 8.1 \cdot 10^{-3}$. However, this lifetime is not consistent with the present Coulomb excitation yield, which can only be reproduced with a $B(E2)$ value corresponding to a lifetime of 3.0(5) ps. This leads to an upper limit for the electric monopole strength of $\rho^2(E0; 2_2^+ \rightarrow 2_1^+) < 6.3 \cdot 10^{-3}$. This is a surprisingly low value and we will come back to this in the discussion.

4 Results for $^{166}$Er

Two separate experiments with the $^{166}$Er target have been performed at 55 and 65 MeV beam energy. The excitation of the $0_1^+$ state is clearly visible in the $\gamma$-ray energy spectra (not shown) by the observation of the 1854 keV $0_1^+ \rightarrow 2_2^+$ transition. For the determination of the electric monopole strength in $^{166}$Er only the conversion electron singles spectra could be used. The amount of conversion electron and particle detector coincidences attributed to Coulomb excitation reactions was less than 1 event/keV in 35 and 37.45 h run time respectively, for the two beam energies.

![Fig. 4. Singles spectrum of conversion electrons following the Coulomb excitation of $^{166}$Er at 55 MeV beam energy.](image)

Fig. 4 shows the singles energy spectrum of conversion electrons for 55 MeV beam energy. The binding energy of the K-shell amounts to 57.5 keV. The E0 and E2 transitions from the $0_1^+$ and the $0_2^+$ state could be identified in the spectrum, despite the poor statistics as the energies are known.

The combined statistics of the two experiments at both energies allowed to determine for the decay of the $0_2^+$ state an intensity ratio of $q_{K}(E0/E2) = 0.47(19)$, taking into account the ratio of the transmission of the Mini-Orange for the E0 and the E2 transition. The $\Omega_K(E0)$ factor for 1460 keV transition energy in $^{166}$Er amounts to $\Omega_K(E0) = 1.201 \cdot 10^{11} s^{-1}$, the conversion coefficient for K conversion is $\alpha_K(E2) = 1.505 \cdot 10^{-3}$ [21]. The lifetime of the $0_1^+$ state in $^{166}$Er has been measured to $\tau = 1.1(4)$ ps [4]. For the $0_2^+$ state an electric...
monopole strength of $\rho^2(E0; 0^+_2 \rightarrow 0^+_g) = 5.3(23) \cdot 10^{-3}$ (average value of our two measurements) could be determined which is slightly larger than the previously known value of $\rho^2(E0; 0^+_2 \rightarrow 0^+_g) = 2.2(8) \cdot 10^{-3}$ \cite{17} but not strictly contradicting. For the 1934 keV E0 transition from the $0^+_4$ state the electronic factor amounts to $\Omega_K(E0) = 1.702 \cdot 10^{11}$ s$^{-1}$ and $\alpha_K(E2) = 8.719 \cdot 10^{-4}$. The observed intensity ratio $q^2_K(E0/E2) = 1.94(91)$ and the lifetime of the $0^+_4$ state of $\tau = 78(8)$ fs \cite{4} result in a $\rho^2(E0; 0^+_4 \rightarrow 0^+_g) = 127(60) \cdot 10^{-3}$.

5 Discussion

The E0 measurements on $^{166}$Er and $^{154}$Sm presented here have revealed, despite the significant experimental uncertainties, large $\rho^2(E0)$ values from the states that have previously been associated with $\beta$-vibrational states in these well-deformed rare earth nuclei. The results also generally confirm for the first time the recent predictions by the IBA model \cite{13} of large $\rho^2(E0; 0^+ \rightarrow 0^+_g)$ values. However, as the following comparison to ECQF IBA calculations will show, the situation is not quite as straightforward and a number of open questions will remain.

This confirmation of large E0 strength from $\beta$-vibrational states is also supported by the results of a re-analysis of published conversion electron data for $^{240}$Pu. We found that in the superdeformed second minimum of the potential surface \cite{23} an average monopole strength of $\rho^2(E0; I^+_\beta \rightarrow I^+_g) = 55(24) \cdot 10^{-3}$ could be determined for the $\beta$-vibrational band members at 785.1 keV ($2^+_2$), 825.0 keV ($4^+_2$), 892.4 keV ($6^+_2$) and 986.8 keV ($8^+_2$).

$^{154}$Sm

$^{154}$Sm is the only nucleus in the rare earth region with two excited $0^+$ states below the excitation energy of the band head of the $\gamma$-vibrational band (1440 keV). Although these two states are only 103 keV apart, they have very different properties. The $0^+_2$ state has a strong collective transition to the ground-state band, $B(E20^+_2 \rightarrow 2^+_g) = 12(2)$ W.u. \cite{3}, while for the $0^+_3$ state only an upper limit $B(E20^+_3 \rightarrow 2^+_g) < 0.3$ W.u. is known. This is consistent with the interpretation that the $0^+_2$ state is the ground state of the $\beta$-vibrational band. Since $^{154}$Sm is only two neutrons away from the critical point nucleus $^{152}$Sm, the $0^+_3$ state may well be the spherical shape-coexisting $0^+$ state.

Fig. 5 shows the level scheme for the lowest positive parity states in $^{154}$Sm.
Fig. 5. $^{154}$Sm level scheme in comparison with IBA calculations. E0 transitions are marked with thick lines. Dashed lines indicate transitions which have not been observed. The inset shows the position of $^{154}$Sm within the IBA symmetry triangle.

together with the IBA prediction for the parameter pair $\chi = -\sqrt{7}/2$ and $\zeta = 0.68$ $^{25,26}$, thus positioning $^{154}$Sm directly on the $U(5) - SU(3)$ leg of the symmetry triangle. In this region of the IBA the $\gamma$ band is rather high in energy and the lowest excited $0^+$ state has a collective E2 transition to the ground-state band and a strong E0 transition to the ground state. For the IBA calculations the level energies are scaled to the experimental $E(2^+; 1^+ \rightarrow 0^+g) = 82.0$ keV, while the transition rates (indicated for each transition in Weisskopf units, W.u.) are scaled to the experimental $B(E2; 2^+ \rightarrow 0^+g) = 174$ W.u..

The experimental properties of the $0^+_2$ state are well described by the IBA calculations. For the $0^+_2$ state the large measured monopole strength of $\rho^2(E0; 0^+_2 \rightarrow 0^+_g) = 96(42) \cdot 10^{-3}$, measured within this work, and the collective E2 decay to the ground state band confirm the interpretation of the $0^+_2$ state as $\beta$-vibrational state. The experimental fact that the $0^+_3$ state is non-collective is reproduced in the IBA calculations which also show it to have a dominant contribution for the number of $d$ bosons $n_d = 0$, consistent with the interpretation as spherical shape-coexisting state (see Fig. 3 in Ref. $^{27}$).

While the situation for the $0^+_2$ states seems satisfactory, it is however not for the $2^+_2$ state, which is considered to be the $2^+$ member of the $\beta$-vibrational band. This state exhibits a transition strength to the ground state of $B(E2; 2^+_2 \rightarrow 0^+_g) = 0.64(12)$ W.u. consistent with the 0.76 W.u. predicted by the IBA, but at the same time a surprisingly small electric monopole strength of $\rho^2(E0; 2^+_2 \rightarrow$
$2^+_g) < 6.3 \cdot 10^{-3}$ was measured, which is almost 16 times smaller than the expectation that the E0 strength of the $2^+_2 \rightarrow 2^+_g$ transition should be about the same as that of the $0^+_2 \rightarrow 0^+_g$ monopole transition.

A possible explanation for this behavior may lie in a mixing of the $2^+_2$ state with other $2^+$ states. The obvious candidate would be the $2^+_3$ state at 1286 keV which belongs to the $K^\pi = 0^+$ band built upon the $0^+_3$ state. However, the $0^+_2$ and $0^+_3$ states exhibit at most a 4\% mixing \[3\], making it unlikely that the mixing of the $2^+_2$ and $2^+_3$ should be significantly larger. It may be more likely that there is significant mixing of the $\beta$ and $\gamma$ bands. However, the calculation of mixing amplitudes does not lead to consistent values, the branching ratios cannot be explained by a simple band mixing model. The mixing amplitude cannot be extracted quantitatively, however, the calculations reveal only a small mixing between $\beta$ band, $\gamma$ band and ground-state band. However, as shown in \[13\], in the IBA the total E0 strength depends on the subtle sum of many $n_d$ components, some with positive and some with negative sign. Thus even small admixtures of other states may lead to subtle but decisive changes of the $n_d$ distribution, possibly leading to the cancellation of the E0 strength.

166\textsuperscript{Er}

Fig. 6 shows part of the $^{166}$Er level scheme with its five $0^+$ states below 2 MeV in comparison with IBA calculations with the parameters obtained in Ref. \[10\], placing it near the O(6) corner of the symmetry triangle, although still being well-deformed with no significant $\gamma$ softness, as attested by the $R_{4/2} = E(4^+_1)/E(2^+_1)$ ratio of 3.29. For the IBA calculations the level energies are scaled to the experimental $E(2^+_1) = 80.6$ keV, while the transition rates (indicated for each transition in Weisskopf units, W.u.) are scaled to the experimental $B(E2; 2^+_1 \rightarrow 0^+_g) = 214$ W.u.. The $0^+_2$ state has no collective E2 transition to the ground-state band ($B(E2; 0^+_2 \rightarrow 2^+_g) = 2.7(10)$ W.u.) nor to the $\gamma$ band ($B(E2; 0^+_2 \rightarrow 2^+_\gamma) = 2.4(7)$ W.u.) \[4\] and only a small $\rho^2$(E0) to the ground state, which was confirmed in this experiment. For the $0^+_3$ state only upper limits for its decay to the ground-state band and the $\gamma$ band are known. This state has not been excited in this Coulomb excitation experiment and no E0 strength is known, but it is clear that this state is not a collective excitation of the ground state. Due to their non-collective behavior and the rather strong excitation via two-neutron transfer, both the $0^+_3$ state and the $0^+_2$ state have been interpreted as dominated by pair excitations \[4\]. Thus they are beyond the scope of the framework of the IBA. However, the energy of the $0^+_3$ state has been used in the fits of Ref. \[10\] to determine the IBA parameters, explicitly assuming that this state is collective in nature.
Fig. 6. \(^{166}\)Er level scheme in comparison with IBA calculations for the parameter pairs \((\zeta, \chi) = (0.91, -0.31)\) \(^{[10]}\) and \((\zeta, \chi) = (1.0, -0.35)\) in order to account for the high excitation energy of the \(0^+\gamma\) state. E0 transitions are marked with thick lines. Dashed lines indicate transitions which have not been observed. The inset indicates the position of the two IBA parameter sets within the IBA symmetry triangle.

However, the IBA parameters do not change dramatically, if the \(0^+_3\), \(0^+_4\) or even \(0^+_5\) states are used, as is apparent from Fig. 3 of Ref. \(^{[24]}\) because the ratio \(R_{0\gamma} = [E(0^+_2) - E(2^+_\gamma)]/E(2^+_\gamma)\) changes from 8.3 to 14.2 and \(^{166}\)Er is located slightly closer to the \(O(6) - SU(3)\) leg of the symmetry triangle. The excitation energy of the \(0^+\gamma\) state at 1943 keV can be approximately reproduced in the ECQF using IBA parameters \((\zeta, \chi) = (1.0, -0.35)\).

In this region of the IBA symmetry triangle, the \(0^+_2\) state shows a very collective decay to the \(2^+_\gamma\) state and only weak transitions to the ground state band, being consistent with the interpretation of this state being the \(0^+\) member of the \(\gamma\gamma\)-phonon multiplet. These decay properties are most consistent with that of the experimental \(0^+_5\) state \(^{[16]}\). At the same time the \(0^+_3\) state in the IBA calculations shows a large E0 strength to the ground state band, which would be consistent with the expectation for a \(\beta\)-vibrational state. However, for this region of the IBA symmetry triangle, no excited \(0^+\) state exhibits a collective E2 decay to the ground state band, which would be a prerequisite for this interpretation. Thus it seems that in the IBA there exists no state in this parameter range that is consistent with the traditional concept of a \(\beta\) vibration (namely large \(\rho^2(E0)\) and large \(B(E2; 0^+ \rightarrow 2^+\))).

However, the experimental situation in \(^{166}\)Er is not consistent with this IBA picture, since the \(0^+_4\) state does exhibit a strong collective transition to the ground-state band \((B(E2 0^+_4 \rightarrow 2^+\gamma) = 8.8(9)\) W.u. \(^{[4]}\) and no observable transition to the \(\gamma\) band. Moreover, two-nucleon transfer reactions showed
that the (p,t) cross section is low, which led to the interpretation that the $0^+_1$ state is the band head of the $\beta$-vibrational band [4]. The large value of $\rho^2(0;0^+_1 \rightarrow 0^+_g) = 127(60) \cdot 10^{-3}$ to the ground state obtained in this work is consistent with this interpretation.

Thus, even by considering the experimental $0^+_2$ and $0^+_3$ state as non-collective and therefore not within the framework of the IBA the transition properties of the $0^+$ states cannot be reproduced.

6 Conclusion

Excited states in the well-deformed rare earth isotopes $^{166}$Er and $^{154}$Sm were populated via Coulomb excitation at the MLL Tandem accelerator. Conversion electrons were registered in a cooled Si(Li) detector in conjunction with a magnetic transport and filter system, the Mini-Orange.

For the first excited $0^+$ state in $^{154}$Sm at 1099 keV a large value of the monopole strength for the transition to the ground state of $\rho^2(0;0^+_2 \rightarrow 0^+_g) = 96(42) \cdot 10^{-3}$ was extracted. This confirms the interpretation of the lowest excited $0^+$ state in $^{154}$Sm as the collective $\beta$-vibrational excitation of the ground state.

In $^{166}$Er we observed E0 transitions from the $0^+_2$ as well as from the $0^+_4$ state. For the $0^+_2$ state we obtained a value of $\rho^2(0;0^+_2 \rightarrow 0^+_g) = 5.3(23) \cdot 10^{-3}$ in agreement with the known value of $2.2(8) \cdot 10^{-3}$ [17]. The newly measured large electric monopole strength of $\rho^2(0;0^+_4 \rightarrow 0^+_g) = 127(60) \cdot 10^{-3}$ is consistent with the previous assignment [4] of the $0^+_4$ state at 1934 keV to be the $\beta$-vibrational excitation of the ground state.

In a re-analysis of published conversion electron data for $^{240}$Pu in the superdeformed second minimum of the potential surface [23] an average monopole strength of $\rho^2(0;I^+_\beta \rightarrow I^+_g) = 55(24) \cdot 10^{-3}$ could be determined for the $\beta$-vibrational band members up to the $8^+_2$ state.

The observed large monopole strength in all three deformed nuclei for the first time experimentally confirms the theoretical predictions [13] that the lowest excited $0^+$ states in deformed nuclei exhibit strong monopole transitions to the ground state.

A more detailed comparison of the level schemes of the two rare earth nuclei with ECQF IBA calculations reveals that not all experimental features are reproduced by the IBA. In the region of the IBA symmetry triangle where the $\gamma$-vibrational band is at relatively low energy and the first excited $0^+$ state is well above the $2^+_1$ state no excited $0^+$ state shows collective E2 strength.
to the ground state band while the $0^+_2$ or $0^+_3$ states have large E0 strength to the ground state. In this region of IBA parameters the $0^+_2$ state has the characteristics of a double-$\gamma$ vibration but no $0^+$ state with the characteristics of a traditional $\beta$ vibration exists. The case of $^{166}$Er, where a $\beta$-vibrational state has been clearly observed, seems to be in contradiction to that feature of the IBA calculations. The appearance of the low-lying non-collective $0^+_2$ and $0^+_3$ states and the fact that the $0^+_5$ double-$\gamma$ vibrational state is almost degenerate with the $\beta$-vibrational $0^+_4$ state make this a quite unusual case.

Near the $U(5) - SU(3)$ leg of the IBA symmetry triangle the $0^+_2$ state in deformed nuclei lies below the $2^+_\gamma$ state and exhibits all characteristics of a $\beta$-vibrational excitation. $^{154}$Sm seems to be a very good example of this situation. However, the properties of the $2^+_2$ state are not in agreement with the IBA predictions probably due to a mixing with other $2^+$ states.

We conclude that the two nuclei $^{154}$Sm and $^{166}$Er are in general representative for two regions in the IBA triangle, one with low lying $\beta$ vibration near the $U(5) - SU(3)$ leg and one closer to the $O(6)$ corner (but still with $R_{4/2} \geq 3.1$) with the $0^+_2$ state being the two phonon $\gamma\gamma$ vibration but without a $0^+$ state with the characteristics of a $\beta$ vibration (namely large $\rho^2(E0)$ and large $B(E2; 0^+ \rightarrow 2^+_g)$). However, significant discrepancies in some details are observed and it is an interesting question if other collective models, such as the GCM, may be able to obtain better agreement with the experimental data. However, no systematic studies exist at this point.

From the current investigation, we draw the conclusion, that it is very important to obtain as much detailed experimental information on all low-lying $0^+$ states as possible, including data on transfer strength as well as electromagnetic decay properties. In many cases, where only partial information on excited $0^+$ states is available, it is not clear that these states are indeed the ones described in the framework of the collective models. In addition, mixing of different structures can lead to significant modifications of the properties, leading to large deviations from the simple expectations, which should, if at all, just be used as guiding principles.

Acknowledgement
We acknowledge fruitful discussions with Prof. K. Heyde, N.V. Zamfir and R.F. Casten. This work was supported by the DFG Cluster of Excellence “Origin and Structure of the Universe”.

References

[1] R.F. Casten and P. von Brentano, Phys. Rev. C 50 (1994) R1280.
[2] P.E. Garrett, J. Phys. G 27 (2001) R1.
[3] R. Krücken et al., Phys. Lett. B 454 (1999) 15.
[4] P.E. Garrett et al., Phys. Lett. B 400 (1997) 250.
[5] A. Arima and F. Iachello, Phys. Rev. Lett. 35 (1975) 1070.
[6] A. Arima and F. Iachello, Ann. Phys. 111 (1978) 201.
[7] D.G. Burke and P.C. Sood, Phys. Rev. C 51 (1995) 3525.
[8] K. Kumar, Phys. Rev. C 51 (1995) 3524.
[9] C. Günther et al., Phys. Rev. C 54 (1996) 679.
[10] E.A. McCutchan et al., Phys. Rev. C 69 (2004) 064306.
[11] P. O. Lipas, P. Toivonen and D. D. Warner, Phys. Lett B 155 (1985) 295.
[12] C. Hinke, R. Krücken, R.F. Casten, V. Werner and N.V. Zamfir, Eur. Phys. J. A 30 (2006) 357.
[13] P. von Brentano et al., Phys. Rev. Lett 93 (2004) 152502.
[14] G. Gneuss and W. Greiner, Nucl. Phys. A 171 (1971) 440; D. Troltenier, J.A. Maruhn and P.O. Hess in Computational Nuclear Physics, edited by K. Langanke, J.A. Maruhn and S.E. Koonin (Springer Verlag, Berlin, 1991) p. 105.
[15] D.G. Burke and P.E. Garrett, Nucl. Phys. A 550 (1992) 21.
[16] P.E. Garrett et al., Phys. Rev. Lett 78 (1997) 4545.
[17] J.L. Wood, E.F. Zganja, C. de Coster and K. Heyde, Nucl. Phys. A 651 (1999) 323.
[18] J. Eberth, G. Pascovici, H.G. Thomas, N. Warr et al., Prog. Part. Nucl. Phys. 46 (2001) 389.
[19] J. van Klinken and K. Wisshak, Nucl. Instr. Meth. 98 (1972) 1.
[20] T. Kibédi and R.H. Spear, ADNDT 89 (2005) 77.
[21] J. Kantele, Handbook of nuclear spectrometry (Academic Press. London, 1995).
[22] E. Veje, B. Elbek, B. Herskind and C. Olesen, Nucl. Phys. A 109 (1969) 489.
[23] D. Gaßmann et al., Phys. Lett. B 497 (2001) 181.
[24] E.A. McCutchan et al., Phys. Rev. C 74 (2006) 057302.
[25] O. Scholten, F. Iachello and A. Arima, Ann. Phys. 115 (1978) 235.
[26] C. Hinke, Diploma Thesis, TU München (2005), unpublished.
[27] F. Iachello, N.V. Zamfir and R.F. Casten, Phys. Rev. Lett 81 (1998) 1191.