Developing cross entropy genetic algorithm for solving Two-Dimensional Loading Heterogeneous Fleet Vehicle Routing Problem (2L-HFVRP)

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Abstract. Two-dimensional Loading Heterogeneous Fleet Vehicle Routing Problem (2L-HFVRP) is a combination of Heterogeneous Fleet VRP and a packing problem well-known as Two-Dimensional Bin Packing Problem (BPP). 2L-HFVRP is a Heterogeneous Fleet VRP in which these costumer demands are formed by a set of two-dimensional rectangular weighted item. These demands must be served by a heterogeneous fleet of vehicles with a fix and variable cost from the depot. The objective function 2L-HFVRP is to minimize the total transportation cost. All formed routes must be consistent with the capacity and loading process of the vehicle. Sequential and unrestricted scenarios are considered in this paper. We propose a metaheuristic which is a combination of the Genetic Algorithm (GA) and the Cross Entropy (CE) named Cross Entropy Genetic Algorithm (CEGA) to solve the 2L-HFVRP. The mutation concept on GA is used to speed up the algorithm CE to find the optimal solution. The mutation mechanism was based on local improvement (2-opt, 1-1 Exchange, and 1-0 Exchange). The probability transition matrix mechanism on CE is used to avoid getting stuck in the local optimum. The effectiveness of CEGA was tested on benchmark instance based 2L-HFVRP. The result of experiments shows a competitive result compared with the other algorithm.

1. Introduction

Packing and distribution processes are among important activities in a manufacturing company. Packing and distribution processes in the combinatorial optimization field have been studied intensively but separately. This combination has been reviewed by a number of researchers in the recent years. In this paper, we address the combination of packing and distribution problem by adding some constraints to bring the model closer to the reality.

There are several types of distribution problems which are popular as the object of study by researchers. Vehicle Routing Problem (VRP) is a popular distribution problem as a research object. A great amount of research about VRP have been discussed since 1959 when Danzig and Ramser have described the Traveling Salesman Problem (TSP) [1]. In contrast with to VRP, in the TSP there are no customer demand and capacity. In this paper, we discuss the variant of Two-dimensional Loading Capacitated Vehicle Routing Problem (2L-CVRP). 2L-CVRP is a generalization of the Capacitated Vehicle Routing Problem, in which customer demand is formed by a set of two dimensional, rectangular, weighted items [2].
This paper addresses a variant of 2L-CVRP, called Two-dimensional Loading Heterogeneous Fleet Vehicle Routing Problem (2L-HFVRP). 2L-HFVRP is a combination of Heterogeneous Fleet Vehicle Routing Problem (HFVRP) and 2L-CVRP. HFVRP is a VRP with heterogeneous fleet of vehicle which have various capacities. Since most enterprises own a heterogeneous fleet of vehicle or hire different type of vehicles to serve their customers, it is therefore crucial to study VRP with a fleet of heterogeneous vehicles [3]. As the NP-hard problem, this paper used metaheuristic approach as the solver. Cross Entropy Genetic Algorithm (CEGA) is a metaheuristic that is in this paper. CEGA algorithm was first used by Santosa et al. [4] and successfully solved a scheduling problem.

2. Problem description
The 2L-HFVRP is defined on an undirected connected graph \( G = (V, E) \), where \( V = \{0, 1, \ldots, n\} \) is a vertex set corresponding to the depot (vertex 0) and the customers (vertices \( 1, 2, \ldots, n \)) and \( E = \{e_{ij}; i, j \in V\} \) is an edge set. For each \( e_{ij} \in E \) is associated with distance \( d_{ij} \) and \( d_{il} = 0 \). A fleet of \( P \) different type of vehicles is located at the depot, and the number of vehicles is unlimited for each type. Every type of vehicle \( t \) \((t = 1, 2, \ldots, P)\) is associated with capacity \( Q_t \), fix cost \( F_t \), variable cost \( V_t \), length \( L_t \), and width \( W_t \). The loading surface of vehicle of type \( t \) is \( A_t = L_t \times W_t \). On the basis that a vehicle with larger capacity usually has higher cost and greater fuel consumption, we assume that \( Q_1 \leq Q_2 \leq \ldots \leq Q_P \), \( F_1 \leq F_2 \leq \cdots \leq F_P \), and \( V_1 \leq V_2 \leq \cdots \leq V_P \). The traveling cost of each edge \( e_{ij} \in E \) by a vehicle type \( t \) is \( c_{ij} = V_t \times d_{ij} \). The transportation cost of route for vehicle type \( t \) is \( C_R = F_t + \sum_{i=1}^{R} V_t \times d_{R(i),R(i+1)} \), where \( R \) is the route which starts and ends in the same point (depot). 2L-HFVRP has an objective function to minimize total transportation cost that will be calculated by the sum of \( C_R \) for all route \( R \).

For each customer \( i \) \((i = 1, 2, \ldots, n)\) demands a set of \( m_i \) rectangular items, denoted as \( IT_i \), and the total weight of \( IT_i \) equal to \( D_i \). Each item \( IT_i \) \((r = 1, 2, \ldots, m_i)\) has a specific length \( l_{ir} \) and width \( w_{ir} \). Total area of the items of customers \( i \) denote as \( a_i = \sum_{r=1}^{m_i} w_{ir} \times l_{ir} \). In 2L-HFVRP, a feasible loading must satisfy the following constraints:

- All items of given customer must be loaded on the same vehicle and split deliveries are not allowed.
- All items can be rotated 90° and must be loaded with their sides parallel to the sides of the loading surface.
- Each vehicle must start and finish at the depot.
- Each customer can only be served once.
- The capacity, length and width of the vehicle cannot be exceeded.
- No two items can overlap in the same route.

3. Packing heuristic
Packing heuristic is composed by 5-heuristics. 5-heuristics are used to solve packing problem and generating a feasible loading position of an inserted item. 5-heuristics are based on the research by Zachariadis et al. [2]. 5-heuristics are used in sequence, and start from heur1. If heur1 fails to produce a feasible solution, then algorithm will be called next heur1 \((i = 1, 2, \ldots, 5)\). Heur2 is more complex than heur1 and heur5 is the most complex heuristic. If a feasible solution is found, then the packing heuristic stops and the solution is stored.

PosList is a list of possible loading positions for the items. At the beginning, the only available loading position is \( \text{PosList} = \{(0, 0)\} \). \( \text{PosList} = \{(0, 0)\} \) is at front left corner of the container. Whenever an item is inserted, a selected loading position will be removed from the PosList. New loading positions are generated and added into the PosList whenever an item inserted.

The loading process the item to the container divided into two scenarios. Both the sequential and the unrestricted scenarios are considered on this paper. The sequential scenario is a scenario that deals with loading and unloading item into vehicle. The sequence of the loading process is affected by the sequence of the route. Whereas the unrestricted scenario is a scenario that only deals with feasible loading of item.
into the vehicle. In this paper, the loading process is allowed to 90° rotation position to maximize the utility of container surface.

4. Theoretical basis

4.1. Cross entropy
Cross Entropy was originally used to estimate the probability of rare events with the implementation of adaptive algorithms in complex stochastic events, by minimizing the variation [5]. Basically, on CE algorithm, a number of samples are generated randomly. In the next iteration, a number of sample are generated randomly using the parameters that generated from the previous iteration. CE algorithm is executed by the iterative procedure, where each iteration is divided into two parts, as follows:
- Generate a number of sample randomly using a specific mechanism
- Update the parameter. This parameter will be used to generate the new samples in the next iteration. The updated parameter is based on the elite sample in the previous iteration. Elite sample is a percentage of sample that will be used to update the parameter.

4.2. Genetic algorithm
GA algorithm is an algorithm that is based on the principles of genetics and natural selection. The elements that exist in genetics and natural selection adopted in GA to solve optimization problems. The elements are cross over, mutation and elitism. Besides being able to solve the problems of continuous optimization, GA is also widely used to solve combinatorial problems such as TSP, VRP, and crew scheduling for the airline. The procedure of finding solutions in GA, is based only on the value of the objective function, not gradient or calculus techniques [5].

4.3. Local improvement
2-opt, 1-1 exchange and 1-0 exchange procedure is a mutation mechanism that is used to solve VRP. These procedures were based on Ai and Kachitvichyanukul [6]. These procedures are used to improve the quality of VRP solution. The 2-opt procedure works in single route by systematically exchanging the route direction between two pairs of consecutive customers in the route and evaluating whether the routing cost of route is improved or not. The 1-1 exchange procedure systematically interchanges one customer from the first route with another customer from the second route. The 1-0 exchange procedure systematically moves one customer from the first route to the second route [6].

5. Proposed algorithm
CEGA used in this study is based on research by Santosa et al. [4] with some modifications. Crossover mechanism is not used in this study because it would be difficult to generate the feasible solution of 2L-HFVRP. The crossover mechanism was replaced with transition probability matrix mechanism. This mechanism is only performed when the sample did not change over three iterations. This mechanism aims to escape from the local optimum. Transition probability matrix mechanism has a similar concept with crossover mechanism because they both generate solutions based on elite sample (parent).

There are two criteria for termination of iterations. They are the maximum number of iterations and terminating criterion $\varepsilon$. Terminating criterion $\varepsilon$ is the difference of mutation rate on the current iteration $p_m(it)$ with the previous iteration $p_m(it-1)$. The value of $\varepsilon$ is usually set near 0. This means that the objective function of the elite sample has the same value between iterations because they indicated already reached optimum. If one of the terminating criterion is met, then stop the iterations. The steps above are described in more detail in algorithm 1.

Algorithm 1: CEGA Algorithm
1. Generate a population of $N$
2. Calculate the objective function of the population in actual iteration.
3. Set a number of the elite sample $\rho \times N = N_{\text{elite}}$
4. If there are a number of samples that have not changed in three iterations, then generate a new sample to replace them. It uses the transition probability matrix mechanism. Transition probability matrix \( P_M \) will be calculated for each iteration by formula 1.1.

\[
P_M(\text{it}) = \alpha \omega + (1 - \alpha) P_M(\text{it-1})
\] (1.1)

5. Replace the sample that have not changed in three iterations by the new sample from step 4.

6. Run the mechanism of mutation for each sample
   a. Generate random numbers \( r \)
   b. If \( r \leq 0.33 \) then select 2-opt procedure
   c. If \( 0.33 < r \leq 0.66 \) then select 1-0 Exchange procedure
   d. If \( r > 0.66 \) then select 1-1 Exchange procedure
   e. Repeat the step 6a until all samples finished mutated.

7. Update \( G_{\text{best}} \), if the best sample in the actual iteration has a better value of objective function than the previous \( G_{\text{best}} \).

8. Update the mutation rate \( p_m \) using elite sample with equation (1.2).

\[
p_m(\text{it}) = \frac{A_{(\text{it})}}{2}
\] (1.2)

With the value of \( A_{(\text{it})} \) is obtained from equation (1.3) and equation (1.4)

\[
A_{(\text{it})} = (1 - \alpha)u + A_{(\text{it-1})}\alpha
\] (1.3)

\[
u = \frac{z_e}{z_{\text{best}}^2}
\] (1.4)

Repeat the step 2 if the maximum number of iterations or termination criterion \( \varepsilon \) unmet

\[
\varepsilon = |p_m(\text{it}) - p_m(\text{t-1})|
\] (1.5)

If the maximum iteration or the value of \( \varepsilon \) is met, then the iteration will stop and the best solution in the last iteration will be recorded as output of the algorithm CEGA.

Note:
\( z_e \) : The average objective function of elite sample.
\( z_{\text{best}} \) : The best solution for all iterations.

To determine the type of vehicle, the algorithm used a mechanism as described in figure 1. The first step is calculating the total area, weight and maximum length of the items that has not entered into the route. After that, compare them with total area, capacity (weight) and the length of container.

**Figure 1.** Pseudocode of determining the types of vehicles.

```plaintext
Allnode = {1,2,...,n}; Route \subseteq Allnode; nonRoute \subseteq Allnode \setminus \{\text{Route}\}
Vehicle = \{P,P-1,.....,1\}; VehicleP > VehicleP-1 > .... > Vehicle1
TotArea = \text{sum}(\text{Vol(areaRoute)})
TotWeight = \text{sum}(\text{Weight(nonRoute)})
MaxDim = \text{max}(\text{Dimension(nonRoute)})

for i = P : 1 step -1
    if AND(TotVol > AreaVehicle P, TotWeight > CapVehicle P, MaxDim > LVehicle P)
        Select = Vehicle P
    elseif AND(TotArea > AreaVehicle (i), TotWeight > CapVehicle (i), MaxDim > LVehicle (i))
        Select = Vehicle (i+1)
    elseif AND(TotArea < AreaVehicle 1, TotWeight < CapVehicle 1, MaxDim < LVehicle 1)
        Select = Vehicle 1
    end
end
```
6. Experiment and analysis
Experiments were performed with three different cases which are CVRP, 2L-CVRP and 2L-HFVRP. Each case of 2L-CVRP and 2L-HFVRP is divided into two versions which are the sequential and the unrestricted version. CEGA algorithms were run on MATLAB program. All experiments were run on a CPU core i3 5015U 2.1GHz with 4 GB ram in the operating system Windows 10.

6.1. Data set
This paper uses the data sets as used in Leung et al. [3]. This data set was originally from Zachariadis et al. [2], with heterogeneous fleet data added onto it. Out of 36 instances only instances 1-15 are used in the data set. Each instance consists of 5 classes. Class 1 is for pure CVRP and class 2-5 is for 2L-HFVRP. On the 2L-HFVRP case the number of vehicles is unlimited for of each type. There are 4 types of vehicles A, B, C, and D. Each vehicle has the capacity ($Q_t$), the length of the container ($L_t$), the width of the container ($W_t$), variable cost ($V_t$), and Fix cost ($F_t$). $i = A, B, C, D$.

6.2. Parameter setting
CEGA algorithm in this study uses a number of parameters that must be determined. These parameters include:
- $N$: The number of samples in population (Population size); $N$=5
- $\alpha$: Smoothing coefficient.
- $\rho$: Elite sample ratio. Elite sample rate ($\rho$) generally in range between 1% -10% [4], so in this paper the value of $\rho = 2\%$.
- $\varepsilon$: Terminating criterion
- $\text{maxiter}$: Maximum number of iteration.

The value of $\alpha$, $\varepsilon$, and $\text{maxiter}$ are determined by performing several experiments with certain combinations. Each combination performed 5 times to find the solutions. Convergence and Divergence are the indicators in determining the value of $\alpha$, $\varepsilon$, and $\text{maxiter}$. Convergence is a condition that illustrates the ability of the algorithm to produce a near optimal solution. Convergence is obtained by dividing the difference between the average solution and the optimal solution with the optimal solution. The divergence is a condition that illustrates the deviation between the solutions from the algorithm compared to the optimal solution. The divergence is calculated by subtracting the worst solution in five replications with the optimal solution.

Based on the experiment, the combination of parameter $\alpha = 0.8$, $\varepsilon = 0.00001$ and $\text{maxiter} = 30$ is the best of out 18 combinations for 2L-HFVRP. It is the best combination because it has the smallest value of the convergence and divergence. The combination of parameter $\alpha = 0.2$, $\varepsilon = 0.00001$ and $\text{maxiter} = 60$ is selected to be the best combination for 2L-CVRP.

6.3. Computational result
In this study, the CEGA algorithm is compared with SA_HSL [3] algorithm. This is because not many studies that discuss about 2L-HFVRP. The comparison of the two algorithms for sequential and unrestricted scenario can be seen in table 1.

The sequential version of 2L-HFVRP produce $\%\text{gap} = -4.78\%$. That means the average result of CEGA algorithm is 4.78% higher than SA_HSL algorithm. The unrestricted version of 2L-HFVRP produce $\%\text{gap} = -5.68\%$. That means the average result of CEGA algorithm is 5.68% higher than SA_HSL algorithm. Instances 7,8,10,11, and 13 in the sequential and the unrestricted scenario have higher minus value of $\%\text{gap}$ than others. $\text{Sec tot}$ is the time when the last best solution was updated and $\text{Sec h}$ is the overall time to run the algorithm.

6.3.1. Applied to the pure CVRP
As for the pure CVRP, the results of CEGA are compared with results of previous studies which were using the SA_HLS [3], GTS [2], ACO [7], and GRAPS x ELS [8] algorithm. Table 2 displays the results comparison from each algorithm for pure CVRP. Based on the resulting solution, CEGA algorithm is
able to compete with the other comparator algorithms. On the Instance 1,2,4,5 and 6, CEGA algorithm is able to produce the optimal solution. For all comparator algorithms, CEGA has an average %gap = -1. That means the average result of CEGA algorithm is 1% higher than compared to other algorithms.

Table 1. The average computational result of 2L-HFVRP in class 2-5.

| Inst | Average Class 2-5 | Sequential | Unrestricted |
|------|-----------------|------------|-------------|
|      | CEGA | SA_HSL | CEGA | SA_HSL | Sec h | CEGA | SA_HSL | Sec h | %gap | CEGA | SA_HSL | CEGA | SA_HSL | Sec h | CEGA | SA_HSL | Sec h | %gap |
| 1    | 607.786 | 603.15 | 75.8 | 5.73 | 121.05 | 31.04 | -0.77% | 1 | 607.49 | 600.77 | 79.68 | 4.52 | 118.33 | 29.89 | -1.12% |
| 2    | 719.525 | 705.03 | 52.27 | 6.09 | 104.53 | 31.28 | -2.06% | 2 | 711.71 | 699.21 | 78.09 | 5.2 | 113.6 | 32.88 | -1.79% |
| 3    | 777.527 | 771.81 | 189.33 | 10.35 | 232.03 | 36.61 | -0.74% | 3 | 768.79 | 770.12 | 159.69 | 10.37 | 274.36 | 33.84 | 0.17% |
| 4    | 710.593 | 704.87 | 198.91 | 8.71 | 339.1 | 35.19 | -0.81% | 4 | 706.37 | 698.19 | 201.65 | 7.73 | 344.5 | 30.04 | -1.17% |
| 5    | 799.536 | 802.56 | 410.03 | 9.35 | 546.85 | 27.62 | 0.38% | 5 | 789.95 | 786.84 | 504.02 | 7.82 | 563.99 | 27.68 | -0.39% |
| 6    | 841.512 | 834.76 | 311.91 | 9.92 | 393.03 | 43.9 | -0.81% | 6 | 831.9 | 831.32 | 329.04 | 8.9 | 429.64 | 42.78 | -0.07% |

Table 2. Average comparison results for the pure CVRP.

| Inst | CEGA | SA_HLS | GTS | ACO | G x E | SA_HLS | GTS | ACO | G x E |
|------|------|-------|-----|-----|-------|-------|-----|-----|-------|
|      |      |       |     |     |       | %gap  | %gap| %gap| %gap |
| 1    | 278.73 | 278.73 | 278.73 | 278.73 | 278.73 | 0.0% | 0.0% | 0.0% | 0.0% |
| 2    | 334.96 | 334.96 | 334.96 | 334.96 | 334.96 | 0.0% | 0.0% | 0.0% | 0.0% |
| 3    | 360.09 | 358.4 | 358.4 | 358.4 | 358.4 | -0.5% | -0.5% | -0.5% | -0.5% |
| 4    | 430.88 | 430.89 | 430.89 | 430.89 | 430.89 | 0.0% | 0.0% | 0.0% | 0.0% |
| 5    | 375.28 | 375.28 | 375.28 | 375.28 | 375.28 | 0.0% | 0.0% | 0.0% | 0.0% |
| 6    | 495.85 | 495.85 | 495.85 | 495.85 | 495.85 | 0.0% | 0.0% | 0.0% | 0.0% |
| 7    | 570.15 | 568.56 | 568.56 | 568.56 | 568.56 | -0.3% | -0.3% | -0.3% | -0.3% |
| 8    | 574.95 | 568.56 | 568.56 | 568.56 | 568.56 | -1.1% | -1.1% | -1.1% | -1.1% |
| 9    | 608.89 | 607.65 | 607.65 | 607.65 | 607.65 | -0.2% | -0.2% | -0.2% | -0.2% |
| 10   | 536.86 | 535.8 | 535.8 | 535.8 | 535.8 | -0.2% | -0.2% | -0.2% | -0.2% |
| 11   | 516.24 | 505.01 | 505.01 | 505.01 | 505.01 | -2.2% | -2.2% | -2.2% | -2.2% |
| 12   | 618.71 | 610 | 610 | 610 | 610 | -1.4% | -1.4% | -1.4% | -1.4% |
| 13   | 2081.20 | 2006.34 | 2006.34 | 2006.34 | 2006.34 | -3.7% | -3.7% | -3.7% | -3.7% |
| 14   | 846.73 | 837.67 | 837.67 | 837.67 | 837.67 | -1.1% | -1.1% | -1.1% | -1.1% |
| 15   | 867.59 | 837.67 | 837.67 | 837.67 | 837.67 | -3.6% | -3.6% | -3.6% | -3.6% |

6.3.2. Applied to the 2L-CVRP

As for the sequential scenario of 2L-CVRP, the results of CEGA are compared with results of previous studies which were using the SA_HLS [3], GTS [2], and ACO [7] algorithm. For unrestricted scenario, the algorithm GRAPS x ELS [8] was added.

Table 3 displays the results comparison from each algorithm for 2L-CVRP. In table 3, the average %gap of sequential 2L-CVRP generated by SA_HLS, GTS, and the ACO algorithm are -0.22%; 0.55%; and -2.04%, respectively. As for sequential 2L-CVRP, based on the average of %gap, the solutions
produced by CEGA has a better average than the GTS algorithm. Meanwhile in instance 6, 7, 9, and 10, CEGA algorithm is able to generate a better average solution than the SA_HLS algorithm. However, CEGA algorithm outperforms ACO algorithm only in instance 6. Whereas, the average %gap of unrestricted 2L-CVRP generated by SA_HLS, GTS, ACO and the G x E algorithm are -1.95%, -1.01%, -3.02%, and -3.87% respectively. Meanwhile in instance 1, 3, 4, 8 and 9, CEGA algorithm is able to generate a better average solution than the GTS.

| Inst | Average Class 2-5 | % gap | Inst | Average Class 2-5 | % gap |
|------|-------------------|-------|------|-------------------|-------|
|      | CEGA | SA_HLS | GTS | ACO | SA_HLS | GTS | ACO | CEGA | SA_HLS | GTS | ACO | G x E | SA_HLS | GTS | ACO | G x E |
| 1    | 299.05 | 298.34 | 304.22 | 294.48 | -0.24% | 1.70% | -1.55% | 1 | 291.84 | 290.37 | 295.74 | 285.77 | 282.65 | -0.49% | 1.33% | -2.11% | -3.24% |
| 2    | 347.71 | 345.23 | 346.71 | 345.24 | -0.72% | -0.29% | -0.72% | 2 | 344.49 | 341.35 | 341.89 | 341.02 | 339.26 | -0.92% | -0.76% | -1.02% | -1.54% |
| 3    | 392.23 | 390.78 | 393.35 | 381.4 | -0.37% | 0.28% | -2.84% | 3 | 380.02 | 377.37 | 384.49 | 376.32 | 376.32 | -0.75% | -1.11% | -1.03% | -1.03% |
| 4    | 444.28 | 444.21 | 444.62 | 441.11 | -0.02% | 0.08% | -0.72% | 4 | 439.31 | 435.01 | 441.45 | 438.65 | 435.01 | -0.99% | 0.49% | -0.15% | -0.99% |
| 5    | 389.73 | 387.59 | 396.36 | 382.39 | -0.55% | 1.67% | -1.92% | 5 | 386.45 | 379.49 | 382.22 | 379.03 | 379.03 | -1.83% | -1.11% | -1.96% | -1.96% |
| 6    | 498.86 | 503.66 | 505.04 | 499.49 | 0.95% | 1.22% | 0.13% | 6 | 502.53 | 501.02 | 499.47 | 497.27 | 497.04 | -0.30% | -0.61% | -1.06% | -1.11% |
| 7    | 712.71 | 719.95 | 723.83 | 702.27 | -1.01% | 1.54% | -1.49% | 7 | 705.74 | 698.65 | 703.49 | 696.91 | 691.11 | -1.02% | -0.32% | -1.27% | -2.12% |
| 8    | 723.21 | 716.62 | 715.72 | 711.65 | -0.92% | -1.05% | -1.62% | 8 | 696.77 | 703.13 | 705.6 | 691.44 | 678.84 | 0.90% | 1.25% | -0.82% | -2.64% |
| 9    | 618.26 | 621.23 | 622.2 | 614.54 | 0.48% | 0.63% | -0.60% | 9 | 613.17 | 612.02 | 615.65 | 612.02 | 612.01 | -0.19% | 0.40% | -0.19% | -0.19% |
| 10   | 718.42 | 727.73 | 727.86 | 697.2 | 1.28% | 1.30% | -3.04% | 10 | 716.68 | 701.61 | 713 | 682.53 | 675.79 | -2.15% | -0.52% | -5.00% | -6.05% |
| 11   | 765.38 | 761.25 | 768.15 | 728.61 | -0.54% | 0.36% | -5.05% | 11 | 765.83 | 736.3 | 740.04 | 721.82 | 705.95 | -0.41% | -3.49% | -6.10% | -8.48% |
| 12   | 623.68 | 618.98 | 628.62 | 615.76 | -0.76% | 0.79% | -1.29% | 12 | 636.95 | 613.94 | 616.83 | 611.26 | 611.26 | -3.75% | -3.26% | -4.20% | -4.20% |
| 13   | 2664.35 | 2641.36 | 2679.34 | 2591.77 | -0.87% | 0.56% | -2.80% | 13 | 2667.53 | 2561.4 | 2599.4 | 2520.73 | 2490.62 | -4.14% | -2.62% | -5.82% | -7.10% |
| 14   | 1093.85 | 1079.47 | 1092.78 | 1042.33 | -1.33% | -0.10% | -4.94% | 14 | 1081.9 | 1015.81 | 1036.77 | 991.26 | 984.42 | -6.51% | -4.35% | -9.14% | -9.90% |
| 15   | 1239.35 | 1230.28 | 1234.21 | 1212.93 | -0.74% | -0.42% | -2.18% | 15 | 1230.38 | 1193.97 | 1197.83 | 1167.28 | 1144.69 | -3.05% | -2.72% | -5.41% | -7.49% |

Table 3. The average computational result of 2L-CVRP in class 2-5.

7. Analysis

CEGA algorithm used on this paper in finding the solutions is affected by two factors: the value of \( \alpha \) and the number of iterations. The number of iterations is limited by maxiter and the value of \( \varepsilon \). The value of \( \varepsilon \) will make the iterations stop when its value has reached its limit even if a solution is not optimal. In general, the smaller value of \( \alpha \) will produce a better solution but needs more iterations. However, extending the iteration becomes ineffective when a higher value of \( \alpha \) is used. This is because the algorithm has a tendency to get stuck in a local optimum when using a high value of alpha. Thus, if uses a high value of \( \alpha \), then the algorithm is possible to get near optimal solution. This option also has a possibility to stuck in a local optimum. The ability to find a solution would be better to use the small value of \( \alpha \), but this requires a longer number of iterations.

Based on the average % gap, it can be seen that 2L-HFVRP have higher average % gap compared to other algorithms. However, CEGA was able to resolve the case CVRP and 2L-CVRP properly. 2L-HFVRP is a variant of 2L-CVRP by adding variations such as different types of vehicles. In 2L-HFVRP, there is a mechanism to determine the type and the number of vehicles used. This mechanism was observed as the factor affecting the high value of %gap in 2L-HFVRP. On the other hand, in the context of 2L-CVRP where this mechanism does not apply, the value of %gap is relatively small.

However, it should be noted that not all examples of the 2L-HFVRP produce a solution with a high value of % gap. Instance 7, 8, 10, 11, and 13 produce a high value of %gap. On the instance 7, 8, 10, 11,
and 13 there are a number of nodes in which the item’s weight are higher than the average item’s weight of the other nodes. Some of the items are almost filling into one type of vehicle with the largest capacity. These types of problems require a various type of vehicles. Whereas, the mechanism of selecting the type of vehicle used in this paper tended to choose the highest capacity in such a way that for some instances in 2L-HFVRP high values of %gaps are produced.

8. Discussion and conclusion

Based on the experimental results and analysis, it can be concluded that the CEGA successfully developed to solve Two-Dimensional Constraint Loading Heterogeneous Fleet Vehicle Routing Problem (2L-HFVRP). CEGA algorithms are generally able to complete CVRP and-CVRP 2L and 2L-HFVRP with some notes. Therefore, we can conclude that CEGA algorithm is able to compete with the algorithms in previous research. CEGA algorithm requires many iterations when using a small value of α while it has a risk of getting stuck in a local optimum if using a high value of α. The mechanisms of determining the type of vehicles tend to choose a vehicle with a highest capacity. This makes the search of possible solutions is limited. Local improvement is proved to have a good ability in improving the solution, but the terms of mutation make this mechanism is limited in searching solutions.

9. References

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