Growth of Black Holes in the interior of Rotating Neutron Stars

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Mini-black holes made of dark matter that can potentially form in the interior of neutron stars have been always thought to grow by accreting the matter of the core of the star via a spherical Bondi accretion. However, neutron stars have sometimes significant angular velocities that can in principle stall the spherical accretion and potentially change the conclusions derived about the time it takes for black holes to destroy a star. We study the effect of the star rotation on the growth of such black holes and the evolution of the black hole spin. Assuming no mechanisms of angular momentum evacuation, we find that even moderate rotation rates can in fact destroy spherical accretion at the early stages of the black hole growth. However, we demonstrate that the viscosity of nuclear matter can alleviate the effect of rotation, making it possible for the black hole to maintain spherical accretion while impeding the black hole from becoming maximally rotating.

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I. INTRODUCTION

A lot of effort has been devoted by theoretical and experimental physicists in order to unveil the mystery of dark matter. One possible way to indirectly observe the existence or constrain the properties of dark matter is by looking on possible effects dark matter might have on stars. This includes for example constraints on dark matter through asteroseismology \cite{1,2}, modification of the transport properties of the star \cite{3,4}, exotic new effects \cite{5,6,7}, and hybrid dark matter rich compact stars \cite{8,9}. For compact stars such as neutron stars and white dwarfs there are additional ways to constrain dark matter either by studying the effect of Weakly Interacting Massive Particles (WIMP) annihilation in the core of the star \cite{10,11,12}, or — in the case of asymmetric dark matter — by studying the possibility of forming a mini-black hole out of WIMPs that eventually destroys the host compact star. This is the possibility that we entertain in this paper.

Under certain circumstances, the compact star might accumulate a sufficient number of WIMPs which, after losing their initial kinetic energy, thermalize with the surrounding nuclear matter in the core of the star. Depending on the nature of the particle (i.e., boson or fermion), the existence or not of self-interactions, and the mass, the WIMPs might become self-gravitating and collapse forming a black hole in the center of the star. Once the black hole is formed, its fate is determined by the relative strength of two competing processes: the accretion of matter from the surrounding nuclear matter onto the black hole, and the Hawking evaporation. The initial mass of the black hole dictates the result of the competition. Since the accretion rate increases with increasing black hole mass while the Hawking radiation decreases, the black holes that are lighter than a certain critical mass evaporate, while those which are heavier grow.

The outcome in the two cases is completely different. The evaporation of a black hole can possibly heat up the star slightly, which is difficult to observe, if possible at all. On the contrary, the growth of the black hole leads to the eventual star destruction. Based on this catastrophic scenario first studied in \cite{14}, severe constraints have been imposed on the mass and WIMP-nucleon cross section for the bosonic asymmetric dark matter \cite{15,16,17,18}, and for asymmetric fermionic dark matter with attractive interactions \cite{19}. Based on similar scenarios, constraints on primordial black holes as dark matter have been imposed in \cite{20,21} as well as constraints on the spin-dependent WIMP-nucleon cross section \cite{22} (in the latter case, white dwarfs have been considered instead of neutron stars).

However, in order for these constraints to be valid, one must make sure that every single step in this chain of events — the WIMP capture, thermalization, collapse, and growth of resulting black hole — takes place. Failure in any of these steps might lead to the complete invalidation of the constraints. In view of this, several issues in the thermalization that can potentially affect the constraints on the light WIMPs were examined in \cite{23}, as well as some issues regarding the number of modes emitted in Hawking radiation \cite{24}. In addition, it has been pointed out that for non-self-interacting asymmetric bosonic WIMPs, no constraints can be imposed on heavy WIMPs with masses in the TeV range or higher because the collapse of the WIMPs in this case would lead to the creation of successive small black holes that evaporate instead of a single large one that would grow \cite{25,26}.

Finally, similar arguments have been used in the safety assessment report for LHC \cite{27} in the context of the hy-
Theoretical possibility of black hole formation in particle collisions at the LHC.

There is a common element in all the constraints that have been derived so far. It has been assumed that once the black hole forms, the accretion of matter onto the black hole proceeds via the so-called Bondi accretion which gives a large accretion rate scaling as $M^2$ with the black hole mass. However, the Bondi solution is based upon the assumption of spherical accretion. This means that matter falls into the black hole isotropically. If the falling matter carries angular momentum, it may change the picture completely. It is a well known fact in astrophysics that if the infalling matter possesses angular momentum, it forms a disc around the black hole rather than falling in isotropically. This can significantly diminish the accretion rate and therefore could change or even invalidate the derived constraints in two ways: the Hawking radiation might dominate for heavier black holes, and/or the accretion rate may be too slow to lead to the star destruction in the lifetime.

In reality, the ideal conditions for Bondi accretion are never met since all compact stars, and in particular neutron stars always rotate, sometimes with high angular velocities. The rotating in-falling matter not only can stall the accretion due to angular momentum, but it can also spin up the black hole to maximum rotation rates, thus changing the accretion rate as compared to the (non-rotating) Bondi case. So, the effect of rotation requires a detailed study, which is the purpose of this paper. In fact, as we will show, in realistic cases rotation has an important effect, and the conditions for the Bondi accretion are not met automatically. We will demonstrate that only if one considers the effect of the viscosity of nuclear matter in the core of the neutron star, one can recover the conditions for Bondi accretion and thus save the imposed constraints.

### II. THE EFFECT OF ROTATION

Two main issues have to be addressed in order to make sure that the neutron star rotation does not change substantially the estimates for the accretion rate. First, one has to check whether the Bondi accretion regime is valid through the entire star consumption process. Second, it has to be checked that the black hole itself does not become maximally rotating and the Schwarzschild solution remains a good approximation.

#### A. Validity of the Bondi regime

In the Bondi regime, the accreted matter is characterized by an $r$-dependent energy density, pressure and velocity \[^{30}\]. This description holds down to the Bondi radius $r_s$,

$$ r_s = \frac{GM}{c_s^2}, \quad (1) $$

where $M$ is the black hole mass and $c_s = 0.17$ is the sound speed of matter in the core of a neutron star far away from the black hole. At $r < r_s$ the flow becomes supersonic. Note that in the case of a black hole inside a neutron star the sound speed is a finite fraction of the speed of light, and the Bondi radius is only a few times larger than the black hole horizon size.

Ideally, the Bondi accretion is spherically symmetric and assumes zero vorticity of matter collapsing into the black hole. However, all stars rotate to some extend, and in particular neutron stars may have rotation periods as short as milliseconds. Rotation of matter falling into the black hole can destroy the conditions for the Bondi accretion. Due to conservation of angular momentum an accretion disc can be formed reducing significantly the accretion rate. This may invalidate the constraints that are based on the star destruction as an observable effect, because there may not be enough time for a black hole to consume the whole star. We show below that, although the rotation cannot be ignored, the conditions for the Bondi accretion are still valid.

The rotation cannot break the Bondi accretion regime if the accreted matter reaches the innermost stable orbit with the angular momentum much smaller than the keplerian angular momentum it would have at that orbit. The keplerian specific angular momentum at the innermost stable orbit is

$$ l_{\text{iso}} = 2 \sqrt{3} \psi G M, \quad (2) $$

where $\psi$ is 1 for a nonrotating Schwarzschild black hole, and 1/3 for an extreme Kerr black hole. Considering the worst case of matter near the equatorial plane, the specific angular momentum of a piece of matter at a distance $r_0$ from the center of the star rotating with an angular velocity $\omega_0$ is $l = \omega_0 r_0^2$. At the time this piece of matter reaches the innermost stable orbit, all the matter at smaller radii has been already accreted, so that the mass of the black hole is $M = 4/3 \pi \rho_0 r_0^3$, where we have neglected the initial mass of the black hole (which was simply the mass of the collapsed WIMP population that triggered the formation of the black hole). The condition $l < l_{\text{iso}}$ translates into the condition $M > M_{\text{crit}}$ for the mass $M$ of the black hole. In other words, once the black hole mass grows beyond the critical value

$$ M_{\text{crit}} = \frac{1}{12 \sqrt{3}} \left( \frac{3}{4 \pi \rho_c} \right)^2 \left( \frac{\omega_0}{G} \right)^3 \frac{1}{\psi^3}, \quad (3) $$

the angular momentum of the accreting matter cannot stall the accretion and can be safely ignored. Using a typical value $\rho_c = 5 \times 10^{38} \text{ GeV/cm}^3$, we find

$$ M_{\text{crit}} = 2.2 \times 10^{46} P_{-3}^{-1} \text{ GeV}, \quad (4) $$

where $P_1$ is the star rotation period $P$ measured in seconds. In practice, we are interested in constraints from nearby old neutron stars (e.g. J0437+4715 and J2124-3358) that have periods of $P \sim 5 \text{ ms}$. For such a period one finds $M_{\text{crit}} = 1.7 \times 10^{53} \text{ GeV} \sim 10^{-4} M_\odot$. 

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\[^{30}\] See L. I. Schiff, *Relativistic Quantum Mechanics*, McGraw-Hill, New York, 1968.
Consider now the growth of the black hole mass from the initial value to $M_{\text{crit}}$. It is clear from the above discussion that rotating infalling matter can and will stall the accretion as long as there is no mechanism of getting rid of the extra angular momentum. There are several mechanisms that can potentially achieve this. We need to demonstrate that at least one can reduce efficiently the angular momentum of infalling matter. Viscosity can in fact play this role. It was pointed out in \[36\], that in the case of a black hole accreting matter via Bondi accretion inside a star, viscosity can enforce an essentially rigid rotation of matter (with $\omega = \omega_0$) for radii larger than 

$$r_\nu = \frac{G^2 M^2}{c_s^2 \nu}.$$  

(5)

where $\nu$ is the kinematic viscosity of the star’s matter and $M$ is the mass of the black hole. For radii smaller than $r_\nu$, the viscosity cannot brake efficiently the rotation, so that the angular velocity grows with the decreasing radius as follows from angular momentum conservation, $\omega = \omega_0 r^2/r^2$. The kinematic viscosity (assuming neutron superfluidity) for a typical neutron star is \[37\]

$$\nu = 2 \times 10^{11} T_5^{-2} \text{ cm}^2/\text{s},$$

where $T_5 = T/(10^5 \text{K})$, $T$ being the temperature of the star.

Because of the viscosity, a given piece of nuclear matter falls into the black hole rotating with a constant angular velocity $\omega_0$ as long as its distance to the black hole is larger than both $r_\nu$ and $r_s$. If any of these conditions is violated, we will assume (conservatively) that the angular momentum is conserved.

One has to distinguish two cases: $r_\nu < r_s$ and the opposite one. As it can be seen from Eq. \[4\], $r_\nu$ grows with $M$. By comparing $r_\nu$ and $r_s$ one can verify that the initial black hole (right after the collapse of the WIMP sphere) falls into the first case for all WIMP masses in the range between keV to tens of GeV. The nuclear matter that is consumed in this regime has specific angular momentum $\sim \omega_0 r_\nu^2$. The condition $l < l_{\text{iso}}$ even for a fast rotating $\sim 5$ ms star is satisfied for $M < 8 \times 10^{57}$ GeV, which is always true. So, in the first regime the Bondi accretion is not in danger.

However, as the mass of the black grows, so does $r_\nu$. The transition to the regime $r_\nu > r_s$ takes place when the black hole mass becomes $M = M_{\nu}$ with

$$M_{\nu} \simeq 2.1 \times 10^{51} T_5^{-2} \text{ GeV}. \quad (6)$$

Once the black hole gets heavier than $M_{\nu}$, the specific angular momentum of the consumed matter becomes $l = \omega_0 r_\nu^2$. Once again, the Bondi accretion holds as long as $l < l_{\text{iso}}$. This leads to the condition

$$M < M_B$$

Here we have set $\psi = 1$ assuming a slowly rotating black hole. For a 5 ms pulsar, this condition becomes $M < 3.4 \times 10^{53} T_5^{-4/3} \text{ GeV}$. Above this value of the black hole mass the viscosity is not efficient in evacuating the angular momentum and the Bondi accretion may not be valid.

To summarize, from the collapse of the WIMP sphere up to the mass $M_B$ of Eq. \[7\], the black hole grows in the Bondi regime because the angular momentum is efficiently evacuated by the viscosity. On the other hand, for black holes heavier than $M_{\text{crit}}$ of Eq. \[41\], the angular momentum of infalling matter is not sufficient to stall the Bondi accretion. So, the Bondi regime can only be violated for black hole masses in the range $M_B < M < M_{\text{crit}}$.

At temperatures of $T = 10^5 \text{K}$ characteristic of old neutron stars, one actually has $M_{\text{crit}} < M_B$, so accretion proceeds always in the Bondi regime. But for a rapidly rotating warmer star with the period $P \sim 5$ ms and the temperature $10^7 \text{K}$, one has $M_{\text{crit}} > M_B$ and the viscosity stops being effective when the black hole mass satisfies $M_B < M < M_{\text{crit}}$. At this stage of the growth, and for this neutron star temperature the Bondi accretion does not hold anymore. However note that this is the last and the shortest stage of the star destruction: were the Bondi regime still valid, the rest of the star would have been consumed in about a second. So, it is clear that the star destruction cannot be substantially postponed even if the Bondi regime is not valid.

Although it is difficult to calculate the exact accretion rate at this stage, one can easily convince oneself that the star destruction is eminent and the accretion cannot be stalled for a long time. To make the argument, recall that so far we have based our estimates on the worst possible case of the accretion of matter from the equatorial plane. In Eq. \[7\] we estimated the black hole mass $M_B$ above which the specific momentum at the equator satisfies $l > l_{\text{iso}}$. However, matter along the rotation axis carries smaller specific angular momentum. Keeping in mind that for $r > r_\nu$ matter rotates with angular velocity $\omega_0$, the specific angular momentum of matter at $r = r_\nu$ is $l = \omega_0 r_\nu^2 \sin^2 \theta$ where $\theta$ is the polar angle i.e. the angle formed between the rotation axis and the line that connects the center of the star with the particular piece of matter in consideration. For the sake of the argument let us assume that although the Bondi accretion might have ceased for matter around the equator, matter at higher latitude that satisfies $l < l_{\text{iso}}$ can still be accreted with a Bondi rate. Under this assumption, the overall accretion rate is suppressed, as compared to the Bondi case, by the ratio of the solid angle of the polar cups where the condition $l < l_{\text{iso}}$ holds to $4\pi$. It is easy to show that $l < l_{\text{iso}}$ as long as $\sin^2 \theta < (M_B/M)^3$. The rate therefore can be written as follows,

$$\frac{dM}{dt} = \frac{4\pi \lambda \rho G^2}{c_s^2} M^2 f(M), \quad (7)$$

where $\lambda$ is a parameter equal to 0.707 for the polytropic
neutron star equation of state with an index $\Gamma = 4/3$, and the suppression factor $f(M)$ is
\[ f(M) = 1 - \sqrt{1 - M_B^3/M^3}. \]  
(9)
The Bondi rate is recovered by setting $f(M) = 1$.

From Eq. (8), the growth time $t$ of the black hole from $M = M_B$ to some $M > M_B$ is given by the equation
\[ t = \tau \int_{M/M_B}^M \frac{d\xi}{\xi^2 \left( 1 - \sqrt{1 - 1/\xi^8} \right)}, \]  
(10)
where
\[ \tau = \frac{c_s^3}{4\pi\rho_c G^2 M_B} \]  
(11)
is the time scale of the process. For the worst case of $P = 5$ ms and $T = 10^7$ K one has $M_B \sim 7.4 \times 10^{50}$ GeV and $\tau \simeq 0.7$ s. In the Bondi case one has to omit the factor in the brackets in denominator of Eq. (10). In this case one would have $t \sim \tau$, as stated above. With the reduced accretion rate, the growth to some $M \gg M_B$ takes time
\[ t \sim \left( \frac{M}{M_B} \right)^2 \tau. \]  
(12)
As explained above, given that the black hole has to grow in this regime only by $2 - 3$ orders of magnitude until it reaches $M_{\text{crit}}$, and that $\tau \lesssim 1$ s, the total growth time remains short (of the order of hours), and the neutron star destruction is eminent.

B. Evolution of the black hole spin

Let us now show that the accreting matter does not make the black hole maximally rotating. The parameter that characterizes how fast the black hole rotates is
\[ a = J/J_{\text{crit}}, \]
where $J_{\text{crit}} = GM^2$. Maximally rotating black hole corresponds to $a = 1$.

We first observe that black holes are formed from a BEC state with a very small angular momentum, $a \ll 1$. Approximating for simplicity the WIMP sphere in the condensed state as a rigid homogeneous sphere with a moment of inertia $I = 2/5M r_c^2$, $r_c$ (the radius of the condensed state) is [15]
\[ r_c = \left( \frac{8\pi}{3} G \rho_c m^2 \right)^{-1/4} \simeq 1.6 \times 10^{-4} \left( \frac{\text{GeV}}{m} \right)^{1/2} \text{ cm}. \]  
(13)
The angular momentum is therefore $J = I \omega_0$, where $\omega_0$ is the neutron star angular velocity. The mass of the black hole at birth is given by [15]
\[ M = \frac{2}{\pi} \frac{M_{\text{pl}}^2}{m}, \]  
(14)
where $m$ is the WIMP mass. Assuming the neutron star has a period of rotation of 5 ms one finds $a \simeq 0.035 < 1$.

In the Bondi regime, the growth of the black hole is governed by the equation [24]
\[ \frac{dM}{dt} = \frac{4\pi\rho_c G^2}{c_s^3} M^2 - \frac{f(a)}{M^2}, \]  
(15)
where the first term corresponds to the Bondi accretion rate (cf. Eq. (8) with $f = 1$) and the second to the Hawking radiation. As it was pointed out in [34], the coefficient of the Hawking radiation depends not only on the number of different particle species that the black hole can emit, but also on the black hole rotation: the larger the value of $a$, the higher is the emission rate.

A remark is in order at this point concerning the establishment of the Bondi accretion. If Bondi accretion is not established fast, Hawking radiation might win over the weak initial accretion and lead to evaporation of the black hole. This might shift to higher values the critical mass that the black hole needs in order to survive. However, this is not an important effect since the hydrodynamic limit establishes very fast. Making use of Eqs. (13) and (14), the dynamical time scale is given by
\[ t_{\text{dyn}} = \sqrt{\frac{r_c^3}{GM}} \simeq 6 \times 10^{-10} \left( \frac{\text{GeV}}{m} \right)^{1/4}, \]  
(16)
which is many orders of magnitude smaller than the evaporation times of black holes that are formed from WIMPs of masses in the keV – GeV range.

Consider now the evolution of the black hole spin during the accretion. It is easy to see that if one ignores viscosity and assumes angular momentum conservation, the black hole can become maximally rotating. Indeed, if a spherical region of a rotating neutron star of a mass $M$ collapses, the angular momentum of the resulting black hole scales like $M^{5/3}$, while the corresponding critical angular momentum is proportional to $M^2$. Therefore, for sufficiently large $M$ the resulting black hole is subcritical. The boundary value of the mass $M$ at which an exactly critical black hole would be formed parametrically coincides with $M_{\text{crit}}$ of Eq. (4) and equals
\[ M_{\text{max}} = 5.8 \times 10^{46} P_1^{-3} \text{ GeV}. \]  
(17)
For the neutron star rotation period of 5 ms this value equals $4.6 \times 10^{53}$ GeV, which is much larger than the initial mass of the black hole formed from WIMPs. Thus, if the angular momentum were conserved, the black hole would become critical as soon as it accretes a mass much larger than the initial one, and would stay close to critical until the black hole reaches the mass $M \sim M_{\text{max}}$.

The presence of viscosity changes this picture. To see this, consider the evolution of the parameter $a$. Since $a = J/GM^2$, the rate of its change is
\[ \frac{1}{a} \frac{da}{dt} = \frac{1}{J} \frac{dJ}{dt} - 2 \frac{1}{M} \frac{dM}{dt}. \]  
(18)
The rate of the change of the black hole mass is given by Eq. (15), whereas the rate of change of the momentum is given by

$$\frac{dJ}{dt} = F_J - \frac{g(a)J}{G^2 M^3},$$

where $F_J$ is the angular momentum accretion rate, and $g(a)$ is a dimensionless number that characterizes the strength of the angular momentum loss rate due to Hawking radiation.

In the presence of viscosity, the matter is accreted with the specific angular momentum given by either $l = \omega_0 r_s^2$ or $l = \omega_0 \theta_0^2$, whichever is bigger. As we have seen above, at the very first stages of the black hole growth when the critical one. So, this term can be safely ignored.

At $M > M_\ast$, Eq. (21) is modified. The angular momentum accretion rate now becomes $F_J = \omega_0 r_s^2 dM/dt$ with $r_\ast$ given by Eq. (6). Thus, the new equation is obtained from Eq. (21) by the replacement $r_s \rightarrow r_\ast$. It can be rewritten in a form analogous to Eq. (22),

$$\frac{1}{a} \frac{da}{dt} = \left( \frac{1}{a} \frac{M^3}{M_\ast^3} - 1 \right) \frac{2}{M} \frac{dM}{dt} - \frac{g(a)}{G^2 M^3},$$

but with a different parameter $M_\ast = (2v/a_0)^{1/3} \times c_s^2/G \simeq 1.6 \times 10^{54} P_1^{1/3} T_5^{-4/3}$ GeV. For $P = 5$ ms one has $M_\ast \simeq 2.8 \times 10^{-5} T_5^{-1/3}$ GeV. If the last term is neglected, this equation can also be solved exactly. Denoting again $M/M_\ast \equiv \mu$, the solution is

$$a(\mu) = \frac{\mu^2}{\mu^2 a_0 + \frac{2}{3} \mu^3 - \mu_0^3},$$

where $a_0 \approx 1.7 \times 10^{-4} / P_1$ and $\mu_0 = 9.8 \times 10^{-23} / (m P_1)$ are the initial values. The behavior of this solution is quite simple. Initially, the first term dominates and $a$ increases. When $\mu$ becomes much larger than $\mu_0$ (i.e. when the black hole mass becomes much larger than its initial value), the second term eventually starts to dominate and then

$$a(\mu) \simeq \frac{2}{3} \mu = \frac{2M}{3M_\ast}.$$  

This behavior continues as long as Eq. (22) holds, i.e. until $M = M_\ast$. When the mass reaches the value $M_{\text{max}} = 5.8 \times 10^{46} P_1^{-1}$ GeV given by Eq. (17), beyond which the total angular momentum of accreted matter is not sufficient to make the black hole maximally rotating, the angular momentum starts decreasing. Note that, although now we are not assuming conservation of angular momentum, the latter is always transported from inner to outer layers of the star, and thus black holes with $M > M_{\text{max}}$ are subcritical even in the presence of viscosity. Thus, the parameter $a$ can become of order 1 only at $M < M_{\text{max}}$. As $M$ grows to $M_{\text{max}}$, the parameter $a$ grows to its maximum value

$$a_{\text{max}} = 2 \times 10^{-23} T_5^{-1} F_1^{10}.$$  

As one can see, while generically the parameter $a$ remains small, there is a strong dependence of its maximum value on the temperature and the period of the star, and for small periods and large temperatures $a$ may actually reach the value $a \sim 1$.

It is clear from Eq. (27) that the problem arises for the values of masses $M \gtrsim M_\ast$. Comparing the definition of
\( \dot{M} \) with \( M_B \) of Eq. (17) we see that they are parametrically the same. On the other hand, the mass at which \( a \) reaches the maximum value parametrically coincides with \( M_{\text{crit}} \) of Eq. (1). Therefore, the black hole spin could reach the values \( a \sim 1 \) (and thus the accretion rate could be modified) for the range of masses \( M_B \lesssim M \lesssim M_{\text{crit}} \), which is the same range where the Bondi accretion does not hold (see Sect. II A). As it has been discussed there, at these last stages of the black hole growth, the modifications of the accretion rate can no longer prevent the star destruction.

III. TEMPERATURE CONSIDERATIONS

It is clear from the above discussion that because the viscosity depends strongly on the temperature \( \sim T^{-2} \), the estimates presented above may not be valid if the accretion onto the black hole increases the temperature of the surrounding nuclear matter. Protons, neutrons and particularly electrons that accelerate towards the black hole radiate due to thermal Bremsstrahlung or free-free emission. This happens dominantly close to the event horizon. If a sufficiently large amount of radiation is produced, two things can happen: a) the heat might increase the temperature of nuclear matter, making viscosity significantly smaller and therefore invalidating the mechanism of the evacuation of the angular momentum, and b) the radiation pressure may become significant, so that the accretion rate could slow down and become much smaller than the Bondi rate.

The Bremsstrahlung radiation is produced dominantly close to the event horizon. For Bondi accretion with \( \Gamma = 4/3 \), the density and temperature at the event horizon are respectively [30]

\[
\frac{n_n}{n_{\infty}} \simeq \frac{\lambda}{4} \left( \frac{c}{c_s} \right)^3 = 36, \tag{29}
\]

\[
\frac{T_h}{T_{\infty}} \simeq \left( \frac{\lambda}{4} \right)^{1/3} \left( \frac{c}{c_s} \right) = 3.3, \tag{30}
\]

where we used again \( c_s = 0.17 \) and \( \lambda = 0.707 \). For neutral nuclear matter of total density \( \rho \) in \( \beta \)-equilibrium, one can estimate the Fermi momenta and number densities of the neutrons, protons and electrons as follows (see page 310 of [30])

\[
n_n = 5.4 \times 10^{38} \left( \frac{\rho}{\rho_0} \right) \text{ cm}^{-3},
\]

\[
n_e = n_p = 9.7 \times 10^{36} \left( \frac{\rho}{\rho_0} \right)^2 \text{ cm}^{-3},
\]

\[
p_{F_n} = 0.5 \left( \frac{\rho}{\rho_0} \right)^{1/3} \text{ GeV},
\]

\[
p_{F_e} = p_{F_p} = 0.13 \left( \frac{\rho}{\rho_0} \right)^{2/3} \text{ GeV}, \tag{31}
\]

where \( \rho_0 = 5 \times 10^{38} \text{ GeV/cm}^3 \) is the standard value of the neutron star density we have used throughout this paper.

Let us first notice that as particles flow towards the event horizon, it is expected that densities will scale by the same factor 36 for all components (i.e. protons, electrons and neutrons) as given by Eq. (29). Indeed, the cross section for weak equilibration is \( \sim G_F^2 p_F T \) where \( G_F \) is the Fermi constant, \( p_F \) the Fermi momentum and \( T \) the temperature. The time scale for weak equilibration is \( 1/(G_F^2 p_F T n) \) where \( n \) is the density of the nucleons that are not Pauli blocked. This time scale is much longer than the dynamic time scale of Eq. (10) and therefore we can safely assume that all components scale by the same factor since the weak interactions are not fast enough to convert species to one another.

Let us now estimate the mean free path \( d \) of a photon produced close to the event horizon. It is given by \( d = p_{F_e}/(3T_h n_p \sigma_T) \), where \( \sigma_T = 6.6 \times 10^{-25} \text{ cm}^2 \) is the Thomson cross section, \( n_p = 3.5 \times 10^{38} \text{ cm}^{-3} \) is the electron density at the horizon, and \( 3T_h/p_{F_e} \) is the suppression factor due to Pauli blocking of electrons close to the event horizon. It is understood that \( p_{F_e} \) is also evaluated at the event horizon, which gives 0.43 GeV. Since for relativistic degenerate matter the number density scales as \( p_{F_e}^3 \), \( p_{F_{eh}} \) is 36\(^{1/3} \) larger than the asymptotic value of Eq. (31). Making use of these numbers and Eq. (30), we get \( d = 2.2 \times 10^{-8}/T_5 \text{ cm} \). This has to be compared to the Bondi radius. If the mean free path \( d \) is much smaller than the Bondi radius, photons are re-absorbed very quickly and no heat transport from the horizon region to the surrounding nuclear matter takes place. A simple comparison shows that this happens for a black hole of mass \( M > 2 \times 10^{37}/T_5 \text{ GeV} \). Therefore, from this black hole mass to the total destruction of the star, the Bondi radius is much larger than the mean free path of photons and no heating of nuclear matter by the accreted particles has taken place, beyond the hydrodynamic factor 3.3.

However, this is not sufficient. As explained above, black holes with masses as low as \( \sim 10^{36} \text{ GeV} \) can be formed out of WIMPs. Moreover, this case of low-mass black holes is of particular interest as it corresponds, via Eq. (14), to the highest WIMP mass \( m \) of order a few GeV that is considered in many recently proposed WIMP models. If the black hole has a mass \( M < 2 \times 10^{43}/T_5 \text{ GeV} \), photons produced via thermal Bremsstrahlung of (mostly) electrons can heat the surrounding nuclear matter and consequently might reduce its viscosity, which may make it impossible to have a Bondi accretion because the angular momentum is not effectively subtracted. More importantly, they might slow down the accretion significantly since the photon pressure can counterbalance the gravitational attraction from the black hole.

We consider now this case in more detail. The Bremsstrahlung radiation can be due to either thermal collisions of protons with electrons, protons with protons...
and electrons with electrons. We are going to look at the electron-electron Bremsstrahlung keeping in mind that the other two combinations are of the same order. The Bremsstrahlung radiation in the relativistic case has a cross section

$$\frac{d\sigma}{d\omega} \approx \frac{3\alpha\sigma_T}{2\pi E_0^2\omega} \left( E_0^2 + E^2 - \frac{2}{3} EE_0 \right) \left( \ln \frac{2E_0E}{m_e\omega} - \frac{1}{2} \right),$$

(32)

where $\alpha = 1/137$ is the fine structure constant, $\omega$ is the energy of the emitted photon, $E_0$ is the initial energy of the electron and $E = E_0 - \omega$ is the final energy. Since the electrons are degenerate, they must have momentum around the Fermi surface $\sim p_{Feh}$ where the index $h$ is a remainder that the Fermi momentum must be evaluated close to the event horizon. For $\omega << p_{Feh}$, one has

$$\frac{d\tau}{d\omega} \approx \frac{2\alpha\sigma_T}{\pi\omega} \ln \frac{2p_{Feh}^2}{m_e\omega}. \tag{33}$$

The total emissivity i.e. energy per volume per time is

$$\Lambda_{ee} = n_{eh}^2 \int_0^{p_{Feh}} \omega \frac{d\sigma}{d\omega} d\omega,$$

(34)

where $n_{eh}$ is the electron density close to the event horizon, including the suppression factor $3T_h/p_{Feh}$ due to the Pauli blocking since only the electrons that lie within $\sim T_h$ of the Fermi surface can interact with each other in order to emit a photon. This is also the reason why the integration stops at $\omega = p_{Feh}$. Inserting Eq. (33) into Eq. (34), and performing the integration we get

$$\Lambda_{ee} = \frac{18\alpha}{\pi} \sigma_T n_{eh}^2 T_h^2 p_{Feh} \ln \frac{2p_{Feh}}{m_e}. \tag{35}$$

As before $T_h \approx 3.3T$, $n_{eh} \approx 36n_e$ and $p_{Feh} = 36^{1/3}p_{Feh}$, where the quantities without subscript $h$ refer to the values of Eqs. (31) far away from the horizon.

The total emitted luminosity is

$$L_{ee} = \int_{r_h}^{R} \Lambda_{ee} 4\pi r^2 dr \approx \Lambda_{ee} 4 \frac{4}{3} \pi r_h^3. \tag{36}$$

Because the densities increase as $r$ decreases, the integral is dominated by the lower bound at $r = r_h$. One can now estimate the efficiency of radiation, i.e. the ratio of emitted energy over the total accreted energy,

$$\epsilon = \frac{L_{ee}}{dM/dt} \approx 5 \times 10^{-12} T_5 \left( \frac{M}{M_0} \right),$$

(37)

where $M_0 = 2 \times 10^{43}/T_5$ GeV is the black hole mass above which the mean free path of the photons becomes smaller than the Bondi radius and the hydrodynamic approximation holds. Since the infalling particles are already semi-relativistic and the efficiency is so small, radiation pressure from Bremsstrahlung cannot impede the accretion. The out-flowing momentum carried by the radiation is negligible compared to the in-flowing momentum, and therefore the infalling particles are not affected by it.

Consider now the increase of the temperature of the nuclear matter due to the radiation from the horizon region. A constant flow of energy $\Lambda_{ee}$ from the center of the star increases the temperature of the material at a distance $r$ as [27]

$$\delta T = \frac{L_{ee}}{4\pi kr} \approx 458 \left( \frac{M}{M_0} \right)^2 \left( \frac{T_5}{r} \right) K, \tag{38}$$

where $k \approx 10^3 GeV^2$ is the thermal conductivity of nuclear matter [27]. Even at the Bondi radius the increase in the temperature is much smaller than its asymptotic value of $10^3 - 10^7 K$. Therefore, we conclude that the Bremsstrahlung process cannot change significantly the temperature and does not affect the viscosity mechanism of the angular momentum evacuation.

Notice that this result is independent of the asymptotic temperature for realistic values of the latter. We should also mention here that by simple inspection of the relevant equations, Bremsstrahlung from electron-proton collisions gives identical result whereas the proton-proton one gives a somewhat smaller contribution (due to the mass dependence in Eq. (35) and the fact that $m_p > m_e$). Therefore, the radiation efficiency is always small and cannot impede the accretion neither by radiation pressure nor by changing the temperature and reducing the viscosity of the surrounding nuclear matter.

### IV. CONCLUSIONS

Stringent constraints have been imposed recently on asymmetric dark matter candidates based on neutron star destruction by black holes formed out of collapsing dark matter in the cores of the stars. For these constraints to be valid, one should make sure that once formed, the black hole consumes the star in a reasonably short time. This is indeed the case if spherical Bondi accretion is assumed. In this paper we have studied the effect of the host star rotation on the growth of the black hole in its interior.

If conservation of angular momentum is assumed for the accreted matter, we found that even moderate rotations can disrupt the spherical accretion at all except the very last stages of the black hole growth. In this case the black hole growth would proceed via a disc-type accretion with possibly significantly lower rate. This is important for two reasons: firstly because the lifetime of the star with the black hole inside might then be larger than a few billions years, in which case no constraints can be placed. The second reason is that the slow accretion rate may not be enough to win over Hawking evaporation at the initial stages of the black hole growth. Since the accretion rate increases, while the Hawking evaporation rate decreases with the mass of the black hole, the fate of the latter is determined by the competition of the two
rates right after its birth. A slow accretion rate means that the black hole would have to be more massive from the start in order to survive. Thus, knowing exactly the accretion rate at the early stages of the formation of the black hole is crucial for determining the range of parameters where the derived constraints are valid.

In this paper we demonstrated that the viscosity of the nuclear matter plays a crucial role in alleviating the effect of the star rotation, which enables the black hole to grow via a spherical Bondi-type accretion. More precisely, the evacuation of the angular momentum due to viscosity is enough to preserve the Bondi accretion regime from the black hole birth until the moment it reaches the mass $M = M_B$ of Eq. (7). Thus, rotation has no effect neither on the balance between the accretion and the Hawking radiation rates, nor on the longest initial stage of the black hole growth.

For a large part of the parameter space the growth beyond $M_B$ also proceeds in the Bondi regime. But for rapidly rotating and hot neutron stars (e.g. $P = 5$ ms and $T = 10^7$ K), in the range of masses $M_B < M < M_{\text{crit}}$, where $M_{\text{crit}}$ is given by Eq. (1), the Bondi accretion does not hold. However, this does not prevent the rapid star destruction. The reason is that the Bondi regime breaks at the very last and short stage of the black hole growth: were the Bondi regime still valid, the destruction of the star would take less than a second. We have estimated, by considering accretion from the polar regions where the angular momentum is small and is not an obstacle for the accretion, that the time until destruction cannot be longer than a few hours. The star destruction is thus eminent.

We also demonstrated that the spin of the black hole initially decreases, but later on it starts growing and may become close to the maximum one in the mass range $M_{\text{crit}} < M < M_B$, i.e. when the spherical Bondi accretion is not valid anyway. In view of the argument of the previous paragraph, this has no important consequences for the star destruction time.

Finally, we have investigated whether the Bremsstrahlung radiation of the infalling matter can impede the accretion or increase the temperature of the nuclear matter around the black hole in such a way as to modify significantly its viscosity. We found that the radiation efficiency (i.e. the power of the emitted Bremsstrahlung radiation over the accretion rate) is insignificant and cannot slow down the accretion, while the rise in the temperature can be at most $\sim 500$ K, which is negligible compared to the internal temperatures of even old neutron stars.

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