Analytical reconstruction of isotropic turbulence spectra based on the Gaussian transform

A. Wohlbrandt*, N. Hu, S. Guérin, R. Ewert

*Institute of Propulsion Technology, Engine Acoustics Department
German Aerospace Center (DLR), Müller-Breslau-Str. 8, 10623 Berlin, Germany
Department of Aerodynamics and Flow Technology, Technical Acoustics
German Aerospace Center (DLR), Lilienthalplatz 7, 38108 Braunschweig, Germany

Abstract

The Random Particle Mesh (RPM) method used to simulate turbulence-induced broadband noise in several aeroacoustic applications is extended to realise isotropic turbulence spectra. With this method turbulent fluctuations are synthesised by filtering white noise with a Gaussian filter kernel that in turn gives a Gaussian spectrum. The Gaussian function is smooth and its derivatives and integrals are again Gaussian functions. The Gaussian filter is efficient and finds wide-spread applications in stochastic signal processing. However in many applications Gaussian spectra do not correspond to real turbulence spectra. Thus in turbo-machines the von Kármán, Liepmann, and modified von Kármán spectra are more realistic model spectra. In this note we analytically derive weighting functions to realise arbitrary isotropic solenoidal spectra using a superposition of weighted Gaussian spectra of different length scales. The analytic weighting functions for the von Kármán, the Liepmann, and the modified von Kármán spectra are derived subsequently. Finally a method is proposed to discretise the problem using a limited number of Gaussian spectra. The effectiveness of this approach is demonstrated by realising a von Kármán velocity spectrum using the RPM method.

Keywords:
Synthetic, Isotropic Turbulence, Broadband Noise Simulation, Gaussian filter, Gaussian transform, von Karman and Liepmann spectra, Fast Random-Particle-Mesh Method

1. Introduction

A stochastic noise signal of a certain spectral shape can be generated by convolution of a white noise signal by a filter kernel of an appropriate shape [1].

One of the most common filter kernels is the Gaussian filter kernel that realises a Gaussian spectrum. The Gaussian filter is very simple and time efficient as it has beneficial characteristics: its derivatives and integrals are again Gaussian shape; the filtering decouples in multi-dimensional space and fast filter methods are available, such as Purser [2] and Young & Van-Vliet [3] filters.

But Gaussian spectra seldom represent the physics of turbulence. Here more elaborate spectra are needed, such as Kolmogorov, von Kármán or Liepmann spectra. For these spectra the filter kernels are very complicated to use and they are fully coupled in space, as shown by Dieste and Gabard [4].

Siefert et al. [5] succeeded in realising the Kolmogorov spectrum using the superposition of Gaussian spectra weighted manually. Others have adopted this method for other kinds of spectra, e.g. just recently Gea-Aguilera et al. [6] and Kim et al. [7] published their findings. Note that the herein presented method has already been presented by the authors on conferences, but not derived in detail [8,9].

The objective of this note is to provide a theoretical background for determining the appropriate analytical weighting function by means of Gaussian transform [10]. The analytical weighting function is derived for the von Kármán, the Liepmann and the modified von Kármán spectra. Furthermore, an efficient method is proposed to discretise the weighting function with a limited number of Gaussian spectra. Suggestions are made to choose the number of filters and their length scales. As illustration, the realised velocity spectrum using the Random Particle Mesh (RPM) method [11] is compared to the analytically derived velocity spectrum.

2. Method - Gaussian Transformation

2.1. Turbulence spectra

The most popular models for isotropic turbulence are the von Kármán, Liepmann, and modified von Kármán models [11].

2.1.1. The von Kármán Spectrum

The von Kármán spectrum is commonly used to represent homogeneous isotropic turbulence. It satisfies the energy law distribution of $k^4$ for the large eddies which contain most of the energy and reproduces the $-5/3$ - law in the inertial subrange. The energy spectrum is given by

$$E_k(\hat{k}) = \frac{55}{5\pi^2} u_t^2 \Lambda \frac{k^4}{(1 + \hat{k}^2)^{5/6}}$$

(1)
with the mean turbulent velocity \( u_t \), related to the turbulent intensity \( T_u \) and the mean flow velocity \( u_0 \) by \( u_t^2 = (T_u - u_0)^2 \), the integral length scale \( \Lambda \), and the reduced wavenumber \( k \) defined by \( \hat{k} = k' / k_e \), where \( k' = k \Lambda \), and \( k_e = \sqrt[4]{\pi / (1/3)} \).

### 2.1.2. The Liepmann Spectrum

Liepmann determined turbulence longitudinal correlation coefficients from measurements and found that they can be approximated by an exponential law \( f(x) = \exp \left( \frac{-x}{\epsilon} \right) \). The resulting model spectrum is given as [11][12]:

\[
E_L(k') = \frac{8u_t^2 \Lambda}{\pi} k'^4 \left( 1 + k'^2 \right)^{-3/2}.
\]

(2)

The Liepmann spectrum is comparable in shape to the von Kármán spectrum.

### 2.1.3. The Modified von Kármán Spectrum

According to Bechara [13] the von Kármán spectrum can be modified to be representative over the entire wavenumber range including the dissipation subrange:

\[
E_M(\hat{k}) = E_L(\hat{k}) \exp \left( -2 \frac{k_e^2}{k'^2} \right)
\]

(3)

with the Kolmogorov wavenumber \( k_e = \left( \frac{\nu}{\epsilon} \right)^{1/4} \), where \( \epsilon \) is the specific dissipation rate and \( \nu \) is the eddy viscosity.

### 2.2. Weighting function

According to Ewert et al. [11] filtering of a white noise field with a Gaussian filter kernel of a specific length scale realises a Gaussian spectrum of the form

\[
E_G(k) = \frac{4\pi^2 \Lambda}{\nu^3} k'^4 e^{-k'^2/\nu^2}.
\]

(4)

For convenience we introduce a new spectrum \( e_G \) such that its integral over the wavenumber range is one, i.e.

\[
\int_0^\infty e_G(k) dk = 1 \quad \Rightarrow \quad E_G(k) = \frac{3}{2} k'^2 e_G(k).
\]

(5)

We are looking for a weighting function \( f(l, \Lambda) \) to realise an arbitrary spectrum \( e(k) \) of integral length scale \( \Lambda \) by means of a superposition of Gaussian spectra \( e_G \) of length scales \( l \):

\[
e(k, \Lambda) = \int_0^\infty f(l, \Lambda) e_G(k, l) dl.
\]

Using Eq. (4) and (5) yields the following solution:

\[
e(k, \Lambda) = \int_0^\infty f(l, \Lambda) \frac{8 \sigma^2 \Lambda^4}{3\pi^2} k'^4 \exp \left( -\frac{k'^2 \sigma^2}{\pi} \right) dl.
\]

(6)

Note that only the weighting function \( f(l, \Lambda) \) depends on the integral length scale \( \Lambda \). In the following we drop \( \Lambda \) in the expression of the weighting function \( f \) and write \( f(l, \Lambda) = f(l) \).

A parameter \( \sigma \) is introduced to write Equation (6) in a suitable manner for Gaussian transform as defined by Alecu et al. [10]. This parameter verifies the two following relationships:

\[
\hat{l}^2 = \frac{\pi}{2\sigma^2} \quad \text{and} \quad \frac{dl}{d\sigma^2} = -\frac{\sqrt{\pi}}{2\sqrt{2}\sigma^3}.
\]

(7)

Equation (6) rewrites:

\[
e(k) = \frac{k^4}{p(k)} \int_0^\infty f \left( \sqrt{\frac{\pi}{2\sigma}} \right) - \frac{1}{\sqrt{2\pi}\sigma^2} \exp \left( -\frac{k^2}{2\sigma^2} \right) d\sigma^2.
\]

(8)

According to Alecu et al., \( p(k) \) is a zero-mean generic symmetric distribution, \( G(\sigma^2) \) is the mixture function and \( \mathcal{N}(k\sigma^2) \) is the zero-mean Gaussian distribution. They define the Gaussian transform \( \mathcal{G} \) as the operator which transforms \( p(x) \) into \( G(\sigma^2) \). The inverse Gaussian transform \( \mathcal{G}^{-1}(G(\sigma^2)) = p(x) \) is simply given by Equation (7).

From Eq. (7) the weighting function \( f(l) \) is given as

\[
f(l) = \frac{\pi^{1/2}}{2\sigma^2} = \frac{3\sqrt{2\pi}\sigma^2}{\sqrt{\pi}} G(\sigma^2).
\]

(9)

### 2.2.1. Von Kármán weighting function

With the von Kármán spectrum given in Eq. (1) the left-hand side of Equation (7) becomes

\[
p(k) = \frac{110}{27\pi} k_e^5 \frac{1}{(k_e^2 + k^2\Lambda^2)^{17/6}}.
\]

(10)

The direct Gaussian Transform is given by Alecu et al. [10 Eq.(4)]:

\[
\mathcal{G}(p(\sigma)) = \frac{1}{\sigma^2} \sqrt{\frac{\pi}{2\sigma^2}} \left( \mathcal{L}^{-1} \left( \mathcal{G}^{-1}(G(\sigma^2)) \right) \right)_{p(\sigma)}.
\]

(11)

where \( \mathcal{L}^{-1} \) is the inverse Laplace transform. Using the relation

\[
\mathcal{L}^{-1} \left( \frac{1}{(p - \alpha)^n} \right)(t) = \frac{e^{\alpha t} t^{n-1}}{\Gamma(n)},
\]

(12)

where \( \Gamma(n) = (n-1)! \) is the gamma function, we find for Eq. (12)

\[
\mathcal{G}(p(\sigma)) = \frac{55}{54\sqrt{\pi}(17/6)} \frac{k_e^5}{2\sigma^2\Lambda^2} \exp \left( -\frac{k_e^2}{2\sigma^2} \right)
\]

(13)

and the weighting function for the von Kármán spectrum is given by

\[
f_k(l) = \frac{55}{18\Gamma(17/6)\sqrt{\pi^3}} \frac{k_e^5}{\pi\Lambda^2} \exp \left( -\frac{k_e^2}{\pi\Lambda^2} \right).
\]
2.2.2. Liepmann weighting function

With the Liepmann spectrum given in Eq. (2) the left-hand side of Equation (7) becomes

\[ p(k) = \frac{16 \Lambda^4}{3\pi} \left( \frac{\pi}{\Lambda k} + k^2 \right)^{5/2} \]  

This is of the form of the generalised Cauchy distribution shown in appendix of Ref. [10] with \( \nu = 2.5 \) and \( b = \frac{1}{\Lambda} \). We identify

\[ p(x) = \frac{16 \Lambda^4}{3\pi} \sqrt{\pi} (2.5)^{5/2} \frac{p_c(x|\nu, b)}{2} \]  

The Gaussian transform of \( p_c(x|\nu, b) \) is given in [10] as

\[ G_{\chi|\nu, b}(\sigma^2) = \frac{b^2}{\sigma^2 \sqrt{2 \pi} \Gamma(2.5)} \left( \mathrm{e}^{-\left( b^2 + i\omega \right)^2} \right) \text{erf} \frac{\omega}{2} \]  

So the Gaussian transform of \( p(x) \) is

\[ G(\sigma^2) = \frac{2}{3\Lambda \sigma^6 \sqrt{2\pi} \sigma^2} \mathrm{e}^{-\frac{\sigma^2}{3\Lambda}} \]  

Finally, this yields the weighting function for a Liepmann spectrum as

\[ f_L(l) = \frac{2}{\pi \Lambda} \mathrm{e}^{-\frac{l^2}{3\Lambda}} \]  

This function is plotted in Fig. 1.

2.2.3. Modified von Kármán weighting function

With the modified von Kármán spectrum given in Eq. (5) the left-hand side of Equation (7) becomes

\[ p(k) = \frac{2E_m(k)}{k_d^3} \]  

\[ = \frac{110}{27\pi} \Lambda^3 \frac{k_d^{5/3}}{(k_d^2 + \Lambda^2)^{1/3}} \exp \left( -\frac{2k_d^2}{k_d^2} \right) \]  

The direct Gaussian Transform is again given by Eq. (10) and we write

\[ G(p(\sigma)) = \frac{1}{\sigma^2} \sqrt{\frac{\pi}{2\sigma^2}} \left( \Lambda^3 \frac{k_d^{5/3}}{(k_d^2 + \Lambda^2)^{1/3}} \right)^{-1} \exp \left( \frac{-2k_d^2}{k_d^2} \right) \]  

This can be solved analytically, with the partial fraction expansion [14, p.783]. The image function is defined by

\[ F(p) = H(p)/J(p), \]  

where \( H(p) \) is the inverse Laplace function of Gaussian spectra of various length scales \( l \). The analytical weighting functions for the von Kármán (solid), the Liepmann (dashed), and the modified von Kármán (dash-dot) spectra are shown.

So using the convolution theorem

\[ \Lambda^{-1}(1/J(p)) = \Lambda^{-1}(1/J(p)) \ast \Lambda^{-1}(H(p)) \]  

We determine the inverse Laplace function to \( H(p) \) and \( 1/J(p) \). We get the inverse Laplace function by applying the convolution theorem afterwards.

The inverse Laplace function of \( 1/J(p) \) is given by Eq. (11) as

\[ \Lambda^{-1}(1/J(p)) = \Lambda^{-1}(1/J(p)) \ast \Lambda^{-1}(H(p)) \]  

Equation (20) simplifies to

\[ G(p(\sigma)) = \begin{cases} \frac{55\pi}{18\Gamma(17/6)} \sqrt{\frac{k_d^2}{\Lambda^2}} \left( \frac{\pi}{\sqrt{2\pi}} - \frac{2k_d}{k_d^2} \right)^{-\frac{1}{2}} \exp \left[ \frac{-k_d^2}{\Lambda^2} \left( \frac{\pi}{\sqrt{2\pi}} - \frac{2k_d}{k_d^2} \right) \right] & \text{if } l \geq \frac{-2k_d}{k_d^2} \\ \frac{1}{\sqrt{2\pi \Gamma(17/6)}} \left( \frac{\pi}{\sqrt{2\pi}} - \frac{2k_d}{k_d^2} \right)^{1/2} \exp \left[ -\frac{k_d^2}{\Lambda^2} \left( \frac{\pi}{\sqrt{2\pi}} - \frac{2k_d}{k_d^2} \right) \right] & \text{else.} \end{cases} \]  

This function is plotted in Fig. 1. From the plot we see that the modified von Kármán and the von Kármán weighting functions...
are identical in a region of length scales above the Kolmogorov scale. From the modified von Kármán weighting function a cut-off condition for the smallest relevant length scales can be derived as

\[ k_d \geq \frac{\sqrt{2\pi}}{l}. \]  

(28)

2.3. Weighting of velocity spectra

Following Ref. [13], in isotropic turbulence the one-dimensional velocity spectra \( E_\omega(k_1) \) are determined by the energy-spectrum function \( E(k) \):

\[ E_\omega(k_1) = \int_{-\infty}^{\infty} \frac{E(k)}{2\pi k^2} \left(1 - \frac{k^2}{k_i^2}\right) dk_2 dk_3. \]  

(29)

As these integrals are independent of the integral length scale and the weighting function \( f(l) \) is independent of the wavenumbers it is obvious that the realisation of arbitrary isotropic velocity spectra by superposition of Gaussian velocity spectra is straightforward given by combining Eq. (6) and (29) to

\[ E_\omega(k_1, \Lambda) = \int_{0}^{\infty} f(l, \Lambda) E_G^\Lambda(k_1, l) dl. \]  

(30)

2.4. Two-dimensional turbulence

The derivation has been performed in three-dimensional space. The application to two-dimensional turbulence is analogous. As the axial velocity spectrum \( E_{11}(k_1) \) is identical in 2D and 3D space, the weighting functions \( f(l) \) apply without modifications to 2D turbulence, resulting in a different one-dimensional transverse velocity spectrum \( E_{22}(k_1) \) and energy-spectrum function \( E(k) \) for 2D turbulence.

3. Discretisation

The main idea of the weighting function is to realise an arbitrary spectrum \( E(k) \) of length scale \( \Lambda \) with the superposition of a limited number of Gaussian spectra \( E_G \) of length scales \( l_m \) with \( 0 \leq m \leq M \). For this we discretise the integral in Eq. (6) to

\[ E(k) = \int_{0}^{\infty} f(l) E_G(k, l) dl \]  

(31)

\[ \approx \sum_{m=0}^{M} f(l_m) E_G(k, l_m) \Delta l_m \]  

(32)

with the spacing \( \Delta l_m \).

For an efficient realisation we want \( M \) to be as small as possible. As many orders of the wavelength \( k \) have to be covered an exponential distribution of the length scales \( l_m \) seems natural. We propose to discretise \( l_m \) by

\[ l_m = \ell_0 q^m, \quad q = \left(\frac{l_M}{l_0}\right)^{\frac{1}{M}}, \quad m = 0 \ldots M \]  

(33)

with the minimum and maximum length scales \( l_0 \) and \( l_M \), respectively. To define the spacing \( \Delta l_m \), the trapezoidal rule is applied:

\[ (\Delta l)_0 = \frac{l_1 - l_0}{2}, \quad (\Delta l)_p = \frac{l_{p+1} - l_p}{2}, \quad (\Delta l)_M = \frac{l_M - l_{M-1}}{2}. \]  

(34)

Analytical parameter variations show that a realisation with five filters per order of \( k \)-variation is already sufficient for to get a spectrum that is visually smooth. An example result is shown in Fig. 2a. The green curve indicates the analytical von Kármán spectrum and the black curve is the realisation with 10 weighted Gaussian spectra for two orders of \( k \)-variation. The set of blue curves show the \( M = 10 \) Gaussian spectra of length scales \( l_m \) weighted with the analytical weighting function \( f_k(l_m) \) of Eq. (13). In comparison a realisation with only 5 weighted Gaussian spectra is plotted with the orange curve. The pink curve shows the original Gaussian spectrum of Eq. (4) of the integral length scale \( \Lambda \).

Note that the smallest used length scale \( l_1 \) is chosen to resolve the highest wavenumbers of interest, but the length scale does not have to be smaller than the Kolmogorov length scale \( \ell_G = \frac{2\pi}{k_G} \). For low wavenumbers no additional Gaussian spectra are needed as this region is efficiently covered if the largest
length scale verifies $l_M \geq 4A$. This is due to the fact that all investigated model spectra, including the Gaussian spectrum, follow the power law $E(k) \sim k^4$ in the low wavenumber region.

As long as the 1D model spectra can be assumed valid, the findings are problem independent. Nevertheless depending on the wanted accuracy another discretisation or distribution density might be more appropriate and a quantification of the error is needed.

4. Realisation in RPM

Now we implement the analytical weighting functions in the RPM method of Ewert et al. [1]. The RPM method delivers unsteady fluctuating streamfunctions $\psi$ and the turbulent velocity fluctuations $v$ by convolution of mutually uncorrelated spatio-temporal white-noise $\dot{\psi}$ in a Lagrangian frame [16] with the Gaussian filter kernel $G(t)$

$$v = \nabla \times \psi = \nabla \times \int \hat{A} G(x-x') \dot{\psi}(x,t) dx', \quad (35)$$

realising a Gaussian velocity spectrum. The dimension of the problem is indicated by $d$ (2 for two-dimensional turbulence and 3 for three-dimensional turbulence). The variance of the fluctuations is inferred by Ref. [11] as

$$\hat{R} = \psi(x,t) \dot{\psi}(x,t) = \frac{\hat{A}(x)^2 \Lambda^d}{\rho_0}, \quad (36)$$

where $\rho_0$ is the local mean flow density. In order to realise a target kinetic energy $k_t = \frac{1}{2} \dot{u}_m^2$ with length scale $\Lambda$ the source variance is defined as

$$\hat{R} = \frac{2^{4-d} \Lambda^2 k_t}{3\pi}, \quad (37)$$

From Eq. (36) the amplitude infers as

$$\hat{A} = \sqrt{\frac{\rho_0 \hat{R}}{\Lambda^d}}. \quad (38)$$

Now, to realise the discretised model spectra from Equations (13), (18) or (27) the Gaussian energy spectra $E_G(k, l_m)$ are weighted with $f(k_m) \Delta l_m$ and superposed as seen in Eq. (32). Hence using Eq. (32), (35) and (37) the amplitude for each discrete weighted Gaussian energy spectrum $E_G(k, l_m)$ must be

$$\hat{A}_m = \sqrt{\frac{\rho_0 \Delta l_m}{l_m^2}} \frac{2^{4-d} k_t}{3\pi}. \quad (39)$$

Note that the correlations of the velocity fluctuations given in Equation (35) perfectly match the complete correlation tensor of isotropic turbulence given in [16] Equation (9). Therefore the model spectra derived in this paper also realise isotropic solenoidal turbulence as they are a superposition of Gaussian spectra.

5. Application

The analytical weighting has been successfully applied by the authors to reproduce the measured inflow turbulence spectrum used for the fundamental test case 1 of the AIAA benchmark workshop [13] with the RPM method. The energy spectrum discretisation shown in Fig. 2a is the one used for the benchmark. Turbulent fluctuations of an integral length scale of $\Lambda = 8\text{mm}$ had to be resolved in the frequency range $100\text{Hz} \leq f \leq 10\text{kHz}$. For a resolved von Kármán spectrum in that range we needed $M = 10$ Gaussian spectra reaching from length scales $l_0 = \Lambda/5$ to $l_M = 4\Lambda$. In Fig. 2b the resulting measured and synthesised turbulent spectra of the axial velocity component are compared; the results show close agreement with the target spectrum.

As each filtering process, generating the Gaussian spectra of length scales $l_m$, is fully decoupled from the others, the method can be implemented thread-parallel. In this way the overhead for realising arbitrary spectra is easily handled by using additional computational power resulting in negligible penalty in efficiency compared to a computation realising a Gaussian spectrum.

6. Conclusion

Analytical weighting functions have been derived to realise von Kármán, Liepmann, and modified von Kármán spectra of integral length scale $\Lambda$ by integrating weighted Gaussian spectra of different length scales.

A discretisation of this integral is necessary in real applications. Using an exponential distribution it was shown that very few Gaussian realisations per order of frequency are enough for a smooth resulting spectrum. The discretisation is limited in the lower frequency range by $\frac{L}{\Lambda} \leq 4\Lambda$ and the upper limit is either given by grid resolution or in case of the modified von Kármán spectrum by the Kolmogorov length scale $l_d = 2\pi/k_d$. The method has been validated by generating a von Kármán spectrum with the RPM method by a superposition of 10 Gaussian realisations to resolve 2 orders of magnitude of the frequency range.

7. Acknowledgement

The authors wish to acknowledge the financial support of the European Commission, provided in the framework of the FP7 Collaborative Project IDEALVENT (Grant Agreement no 314066).

References

[1] R. Ewert, J. Dierke, J. Siebert, A. Neifeld, C. Appel, M. Siefert, O. Körnow, CAA broadband noise prediction for aeroacoustic design. Journal of Sound and Vibration 330 (17) (2011) 4139–4160. doi:10.1016/j.jsv.2011.04.014.
[2] R. J. Purser, W.-S. Wu, D. F. Parrish, N. M. Roberts, Numerical aspects of the application of recursive filters to variational statistical analysis. part II: Spatially inhomogeneous and anisotropic general covariances, Monthly Weather Review 131 (8) (2003) 1536–1548. doi:10.1175/2563.1.
[3] I. T. Young, L. J. van Vliet, Recursive implementation of the gaussian filter, Signal Processing 44 (2) (1995) 139–151. doi:10.1016/0165-1684(95)00020-E

[4] M. Dieste, G. Gabard, Random particle methods applied to broadband fan interaction noise, Journal of Computational Physics 231 (24) (2012) 8133–8151. doi:10.1016/j.jcp.2012.07.048

[5] M. Siefert, R. Ewert, Sweeping sound generation in jets realized with a random particle-mesh method, American Institute of Aeronautics and Astronautics, Miami, Florida, 2009, pp. 3369–3382.

[6] F. Gea-Aguilera, X. Zhang, X. Chen, J. R. Gill, T. Nodé-Langlois, Synthetic Turbulence Methods for Leading Edge Noise Predictions, American Institute of Aeronautics and Astronautics, 2015. doi:10.2514/6.2015-2670.

[7] J. W. Kim, S. Haeri, An advanced synthetic eddy method for the computation of aerofoil–turbulence interaction noise, Journal of Computational Physics 287 (2015) 1–17. doi:10.1016/j.jcp.2015.01.039.

[8] E. Envia, J. Coupland, [AA-39 panel session: Fan broadband noise prediction] in: 20th AIAA/CEAS Aeroacoustics Conference, Atlanta, Georgia, 2014. URL www.oai.org/aeroacoustics/FBNWorkshop

[9] C. Rautmann, J. Dierke, R. Ewert, N. Hu, J. Dells, Generic airfoil trailing-edge noise prediction using stochastic sound sources from synthetic turbulence, in: 20th AIAA/CEAS Aeroacoustics Conference, AIAA Aviation, American Institute of Aeronautics and Astronautics, 2014. doi:10.2514/6.2014-3298.

[10] T. I. Alecu, S. Voloshynovskiy, T. Pun, The gaussian transform, in: EU-SIPCO2005, 13th European Signal Processing Conference, 2005.

[11] H. M. Atassi, M. M. Logue, Effect of turbulence structure on broadband fan noise, in: 29th AIAA Aeroacoustics Conference, Vancouver, Canada, 2008.

[12] J. O. Hinze, Turbulence, 2nd Edition, McGraw-Hill series in mechanical engineering, McGraw-Hill, New York, 1975.

[13] W. Bechara, C. Bailly, S. M. Candel, P. Lafon, Stochastic approach to noise modeling for free turbulent flows, AIAA Journal 32 (3) (1994) 455–463. doi:10.2514/3.12008

[14] I. N. Bronstein, K. A. Semendjaev, G. Musiol, H. Mühlig, Taschenbuch der Mathematik, 7th Edition, Deutsch, Frankfurt am Main, 2008.

[15] S. B. Pope, Turbulent Flows, 8th Edition, Cambridge University Press, 2000.

[16] R. Ewert, Broadband slat noise prediction based on CAA and stochastic sound sources from a fast random particle-mesh (RPM) method, Computers & Fluids 37 (4) (2008) 369–387. doi:10.1016/j.compfluid.2007.02.003