Supernova Kicks and Misaligned Microquasars

Rebecca G. Martin, Christopher A. Tout and J. E. Pringle
University of Cambridge, Institute of Astronomy, The Observatories, Madingley Road, Cambridge CB3 0HA

ABSTRACT

The low-mass X-ray binary microquasar GRO J1655–40 is observed to have a misalignment between the jets and the binary orbital plane. Since the current black hole spin axis is likely to be parallel to the jets, this implies a misalignment between the spin axis of the black hole and the binary orbital plane. It is likely the black holes formed with an asymmetric supernova which caused the orbital axis to misalign with the spin of the stars. We ask whether the null hypothesis that the supernova explosion did not affect the spin axis of the black hole can be ruled out by what can be deduced about the properties of the explosion from the known system parameters. We find that this null hypothesis cannot be disproved but we find that the most likely requirements to form the system include a small natal black hole kick (of a few tens of km s$^{-1}$) and a relatively wide pre-supernova binary. In such cases the observed close binary system could have formed by tidal circularisation without a common envelope phase.

Key words: X-rays: binaries; supernovae supernovae

1 INTRODUCTION

It has long been known that neutron stars have much greater space velocities than their likely progenitors (Gunn & Ostriker 1970) and it is now widely accepted that that is because they acquire large velocity kicks in the supernova explosions in which they form (Shklovskii 1970; Sutantyo 1978). The reasons for these kicks is still a matter for debate with the leading candidates being asymmetric neutrino emission and/or asymmetric mass release during the supernova explosion and core collapse (Brandt & Podsiadlowski 1995; Podsiadlowski et al. 2002).

We address here the question of the extent to which similar kicks may be present when black holes form. Because the material forming the black holes passes through the event horizon it is quite possible that few neutrinos can escape (Gourgoulhon & Haensel 1993) so that the black hole could form with little or no natal kick. On the other hand, asymmetric collapse to form a black hole might lead to copious emission of gravitational waves (Bonnell & Pringle 1992; Kobayashi & Mészáros 2003). In addition, Lyne & Lorimer (1994) note that in a binary system a black hole can form by accretion of matter on to a neutron star. In that case the kick given to the original neutron star would appear as a kick given to the current black hole. Evidence that black holes are indeed kicked comes from the work of Jonker & Nelemans (2004). They looked at the out-of-plane distributions of low mass X-ray binaries and neutron stars. They found no significant difference between the two, leading to the conclusion that black holes are subject to similar kicks at their formation.

In this paper we accept the evidence that stellar black holes do indeed acquire a velocity kick when they form, but we inquire further into the nature of the kick. In particular we ask whether the mechanism which gives rise to the kick might also give rise to a misalignment of the black hole spin axis with the original spin axis of the star from which it formed. The black hole progenitor star most likely had its spin aligned with the binary orbit before the supernova. A simple spherical collapse would preserve the spin axis, but a more complicated collapse might not. We consider here the simple null hypothesis that the spin axis remains unaltered by the kick process and test the extent to which this might be contradicted by the evidence.

We focus on the microquasar GRO J1655-40. As we discuss in Section 2 there is considerable information for this system, about the size of the natal velocity kick and also on the misalignment between the current black hole spin and the binary orbital axis. In Section 3 we outline the dynamics of natal kicks and their implications for spin/orbital misalignment and in Section 4 we apply these results to GRO J1655-40. We discuss our results in Section 5.

2 THE MICROQUASAR GRO J1655-40

Microquasars are black-hole X-ray binaries with relativistic radio jets (Mirabel & Rodríguez 1999). GRO 1655-40 is a binary system consisting of a black hole of mass $M_2 = 6.3 \pm$
and a lobefilling companion star with mass $M_2 = 2.4 \pm 0.4M_\odot$ (Greene, Bailyn & Orosz 2001). The binary system has a large systemic radial velocity with respect to the Sun of $V_R = -142.4 \pm 1.5 \text{ km s}^{-1}$ (Orosz & Bailyn 1997; Shahbaz et al. 1999) and this together with the observed proper motion led Mirabel et al. (2002) to deduce that the system has a current space velocity of $112 \pm 18 \text{ km s}^{-1}$. This space velocity can be the result of a combination of two physical causes, instantaneous mass loss during the supernova explosion and an additional kick owing to asymmetry in the explosion itself. The first depends on the mass lost during the explosion as well as on the pre-supernova orbital velocity. That is, it depends on the nature of the binary just prior to the explosion. Further, if the amount of mass lost is too large, then an additional, carefully directed kick may be required to keep the system bound. Given the current position and space velocity of the orbit, computation of the likely post-explosion orbit coupled with analysis of the likely nature of the pre-explosion system have led Willems et al. (2005) to conclude that immediately after the formation of the black hole the system had a space velocity in the range $45 \text{ km s}^{-1} < v_{\text{sys}} < 115 \text{ km s}^{-1}$. We make use of this constraint in Section 3. They conclude that although a symmetric supernova explosion has no intrinsic black hole kick cannot be ruled out, the constraints can be satisfied more comfortably if the black hole did indeed have an intrinsic natal kick of a few tens of $\text{km s}^{-1}$. They set an upper limit of $210 \text{ km s}^{-1}$ to the intrinsic kick of $210 \text{ km s}^{-1}$.

In this paper we consider the additional constraint that the axis of the spin of the black hole, (measured by the direction of the relativistic jets), is misaligned with the orbital angular velocity, along with the null hypothesis that any intrinsic kick imparted to the black hole does not change its direction of spin. This hypothesis implies that the kick imparts linear but not angular momentum to the black hole. The angle the jets make to the line of sight was measured by Hjellming & Rupen (1992) to be $\theta_{\text{jet}} = 85\degree \pm 2\degree$. The inclination of the binary rotation axis to the line of sight is $i_{\text{orb}} = 70\degree.2 \pm 1\degree.9$ (Greene, Bailyn & Orosz 2001). We take the angle $i$ between the black hole spin axis and the binary orbital axis to lie in the range $15\degree.5 \leq i \leq 165\degree.5$ (Martin, Tout & Pringle 2008). Martin, Tout & Pringle (2008) also show that if the black hole is spinning, interaction between the hole and the accretion disc tends to reduce $i$. The only rigorous constraint that we can put on the misalignment angle $i$ immediately post-supernova is that $i > 10\degree$ though it could be much greater than this.

The result of the mass loss coupled with any intrinsic kick would have left the the binary system in an eccentric orbit (Brandt & Podsiadlowski 1995) but of course the orbit of GRO J1655-40 is now circular. To achieve this it is necessary that the post-explosion orbit be such that it could be circularised by tides. Using the formulae given by Harley, Tout & Pols (2002) we can estimate the circularisation timescale of the system in its current state to be about $2 \times 10^8$ yr. This is sufficiently shorter than the evolution time, of at least $3 \times 10^9$ yr since mass transfer began (Martin, Tout & Pringle 2008) so the circular orbit now is not an unreasonable expectation.

### 3 MODEL

In the absence of any other information on the kick it still makes sense to see what a kick distribution like that used for neutron stars might imply about inclinations in microquasars. We assume that prior to the supernova the binary is in a circular orbit with two stars of masses $M_1$ and $M_2$ with relative orbital velocity $v_{\text{orb}}$. We then suppose that star 2 then has an asymmetric supernova explosion in which it loses mass $M_2 - M_1$ and gains it an intrinsic kick with velocity magnitude $0 < v_k < \infty$. The direction of the kick is parameterised by the angle out of the binary plane, $-\pi/2 < \phi < \pi/2$, and the angle between the direction to the instantaneous velocity of the star and the projection of the velocity kick on to the binary orbital plane of $0 < \omega < 2\pi$ (see fig. 1 of Martin, Tout & Pringle 2008 for a diagram showing these angles). We denote the angle between the angular momentum axes of the old and new orbits as $i$ and, according to our null hypothesis, assume that this is the misalignment angle between the spin of the newly formed black hole and the new binary orbital axis. That is we assume that no angular momentum kick is imparted to the remnant. This means that on average material ejected in the explosion simply carries away its specific angular momentum (Podsiadlowski, Rappaport & Pfahl 2002).

Martin, Tout & Pringle (2009) find the angles to be related by

$$\cos \omega = \frac{v_{\text{orb}}}{v_k} \frac{1}{\cos \phi} - \frac{|\tan \phi|}{\tan i}. \quad (1)$$

For a given misalignment angle, $i$, we must have $\cos \omega$ real and so the velocity kick must lie between the locus of $\cos \omega = 1$ ($\omega = 0\degree$), which corresponds to a velocity

$$v_+ = \frac{v_{\text{orb}}}{\cos \phi} \left(1 + \frac{|\tan \phi|}{\tan i}\right)^{-1}, \quad (2)$$

and the locus of $\cos \omega = -1$ ($\omega = 180\degree$), which corresponds to a velocity

$$v_- = \frac{v_{\text{orb}}}{\cos \phi} \left(-1 + \frac{|\tan \phi|}{\tan i}\right)^{-1}. \quad (3)$$

We consider this further in Section 4 once we have examined the other constraints on the kick.

#### 3.1 Bound Systems

The system must remain bound after the supernova. This implies that the velocity kick must be less than

$$v_{\text{bound}} = -\frac{v_{\text{orb}} |\sin \phi|}{\tan i} + \sqrt{\left(3 - 2f\right)v_{\text{orb}}^2 + v_{\text{orb}}^2 \sin^2 \phi \tan^2 i}. \quad (4)$$

(Martin, Tout & Pringle 2008). Here the fraction of mass lost in the supernova from the system $f$ is given by is

$$f = 1 - \frac{M'}{M}, \quad (5)$$

where the total system mass is $M = M_1 + M_2$ before the supernova and $M' = M_1 + M_2$ after it.

#### 3.2 System Velocity

We now calculate the system velocity that results from mass loss coupled with the intrinsic supernova kick. We work in
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3.3 Misalignment Probability

We are interested in the misalignment angle of the system, \( \theta \), after the supernova kick. This represents the angle between the old and new angular momenta of the orbit as shown in fig. 2 of Martin, Tout & Pringle (2004). If \( 0 \leq \theta < \pi / 2 \) then the system is closer to alignment than counter-alignment and if \( \pi / 2 < \theta \leq \pi \) then the system is closer to counter-alignment.

We assume that the intrinsic kick is independent of the geometry of the pre-supernova system. Thus the kick direction is taken to be uniformly distributed over a sphere. We also assume that the velocity distribution of the kick is a Maxwellian. Then, Martin, Tout & Pringle (2004) find the misalignment angle probability distribution for a Maxwellian kick distribution with velocity dispersion \( \sigma_k \) to be

\[
P(\phi) \, d\phi = \int_0^\pi \int_0^\infty I \, d\phi \, d\sigma_k \, d\phi,
\]

where

\[
I = \frac{2}{\sigma_k^3} \sqrt{\frac{3}{\pi}} \frac{\sin^2 \phi}{\sin \omega} \sin \omega
\]

and \( \omega = \omega(\phi, i, \theta_k) \) is defined by equation (1). The region \( R \) in their figure is the region in the \( \phi - \theta_k \) plane for which a given misalignment angle \( \theta \) can be produced. We integrate this using Mathematica and Monte-Carlo methods. We consider this further below (Section 3) where we apply these particular ideas to the system GRO J1655–40.

4 MISALIGNMENT OF GRO J1655–40

In order to calculate misalignment probabilities we need to know the properties of the system just before the supernova. In particular the masses before and after the supernova are not known but varying them does not significantly affect our conclusions. So in order to have something specific to work with we start with the state of GRO J1655–40 after the supernova as discussed by Martin, Tout & Pringle (2003). We found two very similar models and we use the second which has \( M_1 = 2.8 \, M_\odot \) and \( M_2' = 5.08 \, M_\odot \) and the circular period just after the supernova is 1.481 days as an example. Then we assume that the progenitor was a naked helium star of mass \( M_2 = 10 \, M_\odot \). This fits with the work of Podsiadlowski et al. (2002), Eldridge & Tout (2004) and Willems et al. (2003). We expect the system to be circular before the supernova and we need to know the relative orbital velocity just before the explosion, \( v_{\text{orb}} \). The maximum pre-supernova circular velocity that can be achieved corresponds to the closest binary separation that can accommodate the main-sequence star without it filling its Roche lobe. The companion naked helium star would be sufficiently compact to fit well inside its own lobe. For a lobe-filling main-sequence star of mass \( M_1 = 2.8 \, M_\odot \) and companion of mass \( M_2 = 10 \, M_\odot \), this maximum speed is \( v_{\text{orb}} = 500 \, \text{km} \, \text{s}^{-1} \). In practice, the probability of forming the current system becomes very low at high values of \( v_{\text{orb}} \). We shall consider a range \( 50 \leq v_{\text{orb}} / \text{km} \, \text{s}^{-1} \leq 590 \).

4.1 Possible Systems

In Figs. 1 and 2 we show, for a given pre-explosion orbital velocity, the possible kick parameters in terms of \( \theta_k \) and \( \phi \) that can produce a particular misalignment angle \( \theta \). In Fig. 1 we consider systems with pre-supernova orbital velocity \( v_{\text{orb}} = 400 \, \text{km} \, \text{s}^{-1} \) and post-supernova values of \( i = 10^\circ \) and \( i = 20^\circ \). In Fig. 2 we consider a lower value of \( v_{\text{orb}} = 150 \, \text{km} \, \text{s}^{-1} \) and values of \( i = 10^\circ \) and \( i = 40^\circ \). In the \( \phi - \theta_k \) plane we require that the values of \( \phi \) and \( \theta_k \) lie in the region between the \( v_\phi \) (equation 2 solid lines) and \( v_\theta \) (equation 3 dashed lines) contours. In this region we have real values of cos \( \omega \). This allowed region is further limited by two more factors. First the system must remain bound after the supernova. The maximum velocity kick for a system to remain bound (equation 4 is shown by the dotted line. Above this line the system is unbound but below remains bound. Secondly the velocity of the system after the supernova further limits the allowed region in the \( \phi - \theta_k \) plane that can produce a given inclination \( \theta \). We plot the lines where


\[ v_{\text{sys}} = 45 \text{ and } 115 \text{ km s}^{-1} \] (Willems et al. 2003) and require that the system must lie between these lines (equation 10).

For the high value of the pre-supernova orbital velocity \( v_{\text{orb}} = 400 \text{ km s}^{-1} \) the allowed regions are very small. From the upper panels of Fig. 1 we can see that the range of allowable values of \( v_\kappa \) and \( \phi \) is highly restricted. In fact there are no possible combinations of \( \phi \) and \( v_\kappa \) that can lead to a bound system with a misalignment of \( i > 23^\circ \) with the required system velocity. For \( v_{\text{orb}} = 590 \text{ km s}^{-1} \) the space is further reduced and we cannot produce a system with \( i > 17^\circ \) (see also Section 4.3). A high value of \( v_{\text{orb}} \) implies that the pre-supernova system was tightly bound. Without a kick, mass loss in the explosion results in a very unbound system with each star having its pre-supernova velocity vector unchanged. To keep such a system bound the kick must reduce the post-supernova relative velocity. Such a kick must lie close to the orbital plane and so cannot give rise to a large post-explosion misalignment angle \( i \).

For the lower relative orbital velocity, that is for a wider pre-supernova system, \( v_{\text{orb}} = 150 \text{ km s}^{-1} \), we see from Fig. 2 that the permitted regions of parameter space are noticeably larger although still somewhat restricted. Because the kick no longer has to be aimed quite so accurately to cancel out the post-explosion relative velocity it is possible to acquire larger post-explosion misalignment angles \( i \).

### 4.2 Probability Distribution

We have shown in Section 4.1 that the range of values of \( v_\kappa \) and \( \phi \) needed to give rise to the required systems is rather restricted. And of course each individual supernova explosion has no means of aiming for any particular restricted set. Thus, if we assume that the kick velocities acquired by the black hole have a certain distribution, and also that the directions of the kicks are randomly oriented in space, we can compute a probability distribution for the resulting misalignment angles \( i \).

As a distribution of intrinsic kick velocities we use the standard (Hobbs et al. 2005) Maxwellian distribution of supernova kicks. We consider two cases, kicks with a high velocity peak (\( \sigma_\kappa = 265 \text{ km s}^{-1} \)) and a low velocity peak (\( \sigma_\kappa = 26.5 \text{ km s}^{-1} \)). For neutron stars there is some evidence that a combination of two such velocity distributions is required to fit the observational data (Arzoumanian, Chernoff & Cordes 2002). We integrate equation (11) over the yellow regions illustrated in Figs. 1 and 2 for bound systems with a system velocity in the required range. We integrate the probability over this area using the Monte-Carlo integrator within MATHEMATICA. In Fig. 3 we plot the probability distribution for two different values of the pre-supernova orbital velocity \( v_{\text{orb}} \) for these two values of \( \sigma_\kappa \). The high-velocity distribution of kicks (left panel) has a much lower probability of producing the observed system than the low-velocity distribution (right panel). Also, as we discussed above, lower values of \( v_{\text{orb}} \) are more able to give rise to larger values of \( i \).

In Table 1 we list in Column 2 the probability \( P_i \) of getting a bound system with the post-supernova system velocity in the right range for different combinations of \( \sigma_\kappa \) and \( v_{\text{orb}} \). In the remaining columns we show the probabilities that, given the conditions for \( P_i \) are satisfied, the misalignment angle is greater than a particular value. As is evident from Fig. 3 we see that the probability of getting a bound system is much smaller with the higher \( \sigma_\kappa \). However, if we produce a bound system, then we are more likely to get higher inclinations with the higher \( \sigma_\kappa \). We discuss this further in the Section 5.

### 4.3 Velocity Kick Range

For a given initial orbital velocity, \( v_{\text{orb}} \), and misalignment angle, \( i \), there is a range of permissible velocity kicks, \( v_\kappa \). These ranges are illustrated for particular values of \( v_{\text{orb}} \) and \( i \) with the highest and lowest velocity kick in the permitted regions described in Figs. 1 and 2. We plot the possible ranges of \( v_\kappa \) as functions of \( i \) for three particular values of \( v_{\text{orb}} \) in Fig. 4. We can deduce that the lower the pre-supernova orbital velocity the larger the maximum possible velocity kick. To see this, we consider the range of velocity kicks that can produce misalignments of \( i = 10^\circ, 20^\circ \) and \( 40^\circ \). For \( v_{\text{orb}} = 590 \text{ km s}^{-1} \) we find that we need a velocity such that \( v_\kappa / v_{\text{orb}} \approx 0.33 \) or \( v_\kappa \approx 198 \text{ km s}^{-1} \) to produce an inclination of \( i = 10^\circ \). This is shown in Table 2. This is the largest orbital velocity that can produce the system so this is the largest possible velocity kick. The ranges of permitted velocity kicks for other values of \( v_{\text{orb}} \) and \( i \) are shown in Table 2.

### 5 CONCLUSIONS

We have considered what can be learned about the natal kick acquired by a black hole in a supernova by considering the system GRO 1655–40. In line with the analysis of...
Figure 1. The region of the $v_k$–$\phi$ plane for which the combination can produce a system with a misalignment of $i = 10^\circ$ (top left and top right for a more detailed look at the interesting region) and $i = 20^\circ$ (bottom left and bottom right for more detail). Here $v_k$ is scaled by the pre-supernova orbital velocity $v_{\text{orb}} = 400 \text{ km s}^{-1}$. The solid lines are the curves of $v_\perp$ (equation 2) where $\omega = 0$. Below the solid line $\omega$ is not real. The dashed lines are $v_\perp$ (equation 3) where $\omega = 180^\circ$. Above this line $\omega$ is not real valued. Thus allowed values of $v_k$ and $\phi$ must lie between these two curves. Below the dotted line the systems are bound and above they are unbound (equation 4). Thus permitted values must also lie below the dotted line. In each panel the green dot-dashed line with smaller $\phi$ values is for $v_{\text{sys}} = 115 \text{ km s}^{-1}$ and the other magenta one is for $v_{\text{sys}} = 45 \text{ km s}^{-1}$ (equation 10). For the post-supernova system velocity to lie in the required range the permitted parameters must lie between these two lines. The fully constrained regions are yellow. For $i = 10^\circ$ the permitted range is a small region of the total parameter space and is even smaller for $i = 20^\circ$.

We make use of kinematic data, but we add the additional constraint that the current black hole spin is not aligned with the orbital rotation. Of course, if the supernova explosion and collapse process in which the hole is formed gives not just a linear impulse, but also angular momentum, to the hole, then the current misalignment is just a measure of the added angular momentum.

The task we set ourselves is to ask if it is possible, given the various constraints, to rule out the possibility that the natal kick (if any) added just linear, and not angular, momentum. To simplify the analysis we have fixed the masses of the stars before and after the supernova. To be fully consistent we could allow these masses to vary too and assign probabilities based on the likelihood of a given set of param-
Figure 2. As Fig. 1 but for $v_{\text{orb}} = 150 \text{ km s}^{-1}$ with misalignments $i = 10^\circ$ (left) and $i = 40^\circ$ (right). For $i = 10^\circ$ the yellow permitted region lies above the solid line, below the dotted line, to keep the system bound and below the two dot-dashed lines, to satisfy system velocity constraints. For $i = 40^\circ$ (right panel) the permitted region lies above the solid line, below the dotted line (for a bound system) and below the dot-dashed line to satisfy $v_{\text{sys}} \leq 115 \text{ km s}^{-1}$. The right panel does not have a line for $v_{\text{sys}} = 45 \text{ km s}^{-1}$ because for this no combination of $v_k$ and $\phi$ can produce such a high inclination with such a low system velocity. At this lower value of $v_{\text{orb}}$ the allowed regions are larger than when $v_{\text{orb}} = 400 \text{ km s}^{-1}$.

Figure 3. Probability distributions for the misalignment angle $i$ between the spin of the black hole and the orbital axis for the microquasar GRO J1655–40 with $v_{\text{orb}} = 150 \text{ km s}^{-1}$ (dashed lines) and $v_{\text{orb}} = 400 \text{ km s}^{-1}$ (solid lines). The left panel has $\sigma_k = 265 \text{ km s}^{-1}$ and the right has $\sigma_k = 26.5 \text{ km s}^{-1}$.

Parameters. However one set of masses proved to be sufficient to not rule out the null hypothesis.

Our main results are summarised in Table 1. We have considered randomly oriented natal kicks with Maxwellian velocity distributions of 265 and 26.5 km s$^{-1}$ for various separations of the pre-supernova binary, parametrised in terms of its relative orbital velocity $v_{\text{orb}}$. Column 2 gives the probability $P_1$ that a bound system with the appropriate properties can be formed, ignoring any information about misalignment. It is immediately evident, that for low-velocity kicks...
The probability of forming the system is high, being typically around a few tens of percent (compare \cite{Willems et al. 2005}). The probabilities are lower for higher-velocity kicks, being typically around a percent. That is only around one in a hundred such pre-supernova systems would end up looking like GRO 1655–40. This is still probably not unreasonable.

The remaining columns in Table 1 show the probabilities, given \( P_1 \) that the additional constraint of the misalignment angle \( i \) can be satisfied, given our null hypothesis. It is evident that we are not able to rule out this null hypothesis, and so it is quite possible that the black hole formation process does not affect the angular as well as the linear momentum of the resulting black hole.

It is worth noting, however, that even without the alignment information, it is easier to form the observed system with a low-velocity kick. Moreover, if the formation process did just impart linear momentum, then, with account for the fact that the likely alignment timescale is comparable to the age of the system since the formation of the hole \cite{Martin, Tout & Pringle 2008} so that the initial misalignment angle \( i \) would have been in the range \( 20^\circ - 40^\circ \), then it is evident that there is a strong preference for the pre-supernova system to have been fairly wide. Note that \( v_{\text{orb}} = 50 \text{ km s}^{-1} \) corresponds to a binary separation of around 4.6 AU before the supernova. It is interesting that such a wide system could have avoided the traditional common envelope required to shrink the orbit \cite{Verbunt 1996}.

We also note, that if the current misalignment angle were large (say, \( i > 40^\circ \)) then it would become increasingly difficult to satisfy the constraints for GRO 1655–40. In this respect we note that V4641 Sgr is a microquasar similar in many respects to GRO 1655–40 and it has a current misalignment angle of around 4.5\(^\circ\). This is probably still not unreasonable.

Table 1. For each combination of \( \sigma_k \) and \( v_{\text{orb}} \) we show the probability that the kick produces a bound system, \( P_1 \), with the required system velocity in Column 2. In Column 3 we show the probability that given the conditions attached to \( P_1 \) are satisfied, it also produces a system with a misalignment greater than \( 10^\circ \) given that the system is bound and has the correct system velocity. Column 4 is the same for misalignments \( i > 15^\circ \), column 5 for \( i > 20^\circ \) and column 6 for \( i > 40^\circ \). To satisfy the constraints of post-supernova system velocity and misalignment angle, the relevant probabilities need to be multiplied together.

\begin{table}[h]
\centering
\begin{tabular}{cccccc}
\hline
\( v_{\text{orb}}/\text{km s}^{-1} \) & \( P_1 \) & \( P(i > 10^\circ) \) & \( P(i > 15^\circ) \) & \( P(i > 20^\circ) \) & \( P(i > 40^\circ) \) \\
\hline
\( \sigma_k = 265 \text{ km s}^{-1} \) & & & & & \\
50 & 0.00155 & 0.99355 & 0.99355 & 0.98710 & 0.97419 \\
100 & 0.00967 & 0.92968 & 0.89142 & 0.85212 & 0.65460 \\
150 & 0.01179 & 0.81033 & 0.70993 & 0.60431 & 0.19161 \\
200 & 0.01260 & 0.70259 & 0.54751 & 0.39109 & 0.00088 \\
300 & 0.01174 & 0.38446 & 0.12884 & 0.00000 & 0.00000 \\
500 & 0.01136 & 0.28383 & 0.04257 & 0.00000 & 0.00000 \\
\hline
\( \sigma_k = 26.5 \text{ km s}^{-1} \) & & & & & \\
50 & 0.06070 & 0.98575 & 0.97831 & 0.97049 & 0.93358 \\
100 & 0.18639 & 0.73864 & 0.60720 & 0.47644 & 0.07337 \\
150 & 0.28588 & 0.48675 & 0.27915 & 0.12740 & 0.00098 \\
200 & 0.36393 & 0.28225 & 0.09411 & 0.01917 & 0.00000 \\
300 & 0.46989 & 0.06416 & 0.00419 & 0.00008 & 0.00000 \\
400 & 0.51754 & 0.08087 & 0.00004 & 0.00000 & 0.00000 \\
500 & 0.50958 & 0.00051 & 0.00000 & 0.00000 & 0.00000 \\
\hline
\end{tabular}
\caption{The range of permissible velocity kicks \( v_{\text{kmin}} \leq v_k \leq v_{\text{kmax}} \) for a given pre-supernova circular velocity \( v_{\text{orb}} \) that can produce the current system with misalignment angle \( i \). These are independent of \( \sigma_k \).}
\end{table}
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