Weak Lensing By Nearby Structures

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Abstract

Weak gravitational lensing due to nearby structures, such as the Coma cluster, and the Local Supercluster can be expected to polarize images of distant galaxies by $\mathcal{O}(0.2\% \Omega)$ with coherence over scales of tens of square degrees. The Sloan Survey, which will image $\gtrsim 10^4$ galaxies deg$^{-2}$ over $\pi$ steradians, should be sensitive to polarizations of $\sim 0.1\% A^{-1/2}$, where $A$ is the area in square degrees. By measuring the polarization, one could determine $\Omega$ in local structures and compare this value to that derived from a variety of other techniques.

Subject Headings: gravitational lensing – large scale structure of the universe
1. Introduction

Weak lensing, the distortion of images by a gravitational field without the creation of multiple images, is a potentially powerful tool for studying the large-scale inhomogeneities of the universe (Kristian 1967; Gunn 1967). In essence, when rays from a distant galaxy pass by an overdensity in the matter distribution, the observed images will be elongated slightly tangentially with respect to the center of the perturbation (Lynds & Petrosian 1989; Soucail et al. 1987; Tyson, Valdes, & Wenk 1990; Fort et al. 1991). Independent perturbations along the line of sight add stochastically (Blandford et al. 1991; Miralda-Escudé 1991; Kaiser 1992).

Early searches for weak lensing outside of rich clusters (Kristian 1967; Valdes, Tyson & Jarvis 1983) met with negative results. However, Mould et al. (1994) have reported a tentative detection of a $2.8\%\pm0.4\%$ polarization in a random high-latitude field based on a very deep $10'$ square $r$ band CCD image. The reported mean polarization is well below the errors in the measurement of the ellipticities of individual galaxies on which the determination is based.

Previous weak lensing studies have generally focused on distant structures, either known clusters or field structures at redshifts of several tenths. There is a good reason for this. For fixed physical separation between a mass concentration and a given line of sight and for a source at an infinite distance, the strength of lensing scales as the distance to the lens. For fixed distance, the lensing declines with angular separation. Hence, lensing is easiest to observe by imaging structures that just fit on a single CCD image. To image an entire cluster (diameter $\sim 3h^{-1}$ Mpc) on a large CCD ($10'$), the cluster must be at $z \gtrsim 0.3$.

However, a great deal could be learned if it were possible to measure the weak lensing due to local structures, such as the Local Supercluster or the Coma cluster. Weak lensing is sensitive primarily to the total mass in a given structure. We have an enormous amount of information about the mass distribution of local structures that is not available for more distant structures. For example, we know the peculiar motions of many galaxies in the Local Supercluster, and even of galaxies and
clusters of galaxies at somewhat greater distances. We have much more detailed information about the gas distribution in the Coma cluster than we do of more distant clusters. Hence by measuring the weak lensing associated with these structures we can both gain new insight into the structures that we understand the best and also gain an external check for other methods of estimating masses and mass distributions.

In this Letter we show that the Sloan Survey (Gunn & Knapp 1993) is ideally suited to measure weak lensing induced by local structures in the north galactic cap. In § 2, we show that the weak lensing polarization can be measured with an accuracy $\sigma \sim (\Delta/N)^{1/2}$ where $\Delta$ is the accuracy of the measurement of the ellipticities of individual galaxies and $N$ is the number of galaxies measured. In § 3, we estimate that for a 1 square degree patch, the Sloan Survey can be used to measure polarizations to an accuracy $\sigma \sim 0.1\%$. In § 4, we show that local structures such as the Virgo cluster produce weak lensing of $\sim 0.2\%\Omega$ and that these structures are coherent over tens of square degrees. The Coma cluster produces much stronger lensing. In § 5, we indicate that the Sloan Survey will also provide useful information on lensing by more distant structures. In § 6, we discuss the calibration of systematic effects.

2. Detectability of Weak Lensing

The distortion of images in the weak lensing limit can be parameterized by a complex polarization $\xi$. This polarization results in a displacement in the complex ellipticity from the object, $\gamma$, to the complex ellipticity of the image, $\epsilon$. That is to lowest order

$$\xi = \epsilon - \gamma$$

(2.1)

where

$$\epsilon \equiv \frac{I_{xx} - I_{yy} - 2iI_{xy}}{I_{xx} + I_{yy}}; \quad \gamma \equiv \frac{O_{xx} - O_{yy} - 2iO_{xy}}{O_{xx} + O_{yy}},$$

(2.2)

and where $I_{xx}$, $I_{yy}$, and $I_{xy}$ are the three second moments of the image and $O_{xx}$,
$O_{yy}$ and $O_{xy}$ are the three second moments of the object.

Consider a set of $N$ objects with complex ellipticities, $\gamma_j$. First suppose that the $\gamma_j$ all have the same modulus $|\gamma|$, but have random phases. That is the $\gamma_j$ are random points on a circle centered at the origin. If all these images are subjected to the same lensing polarization, then the resulting ellipticities of the images $\epsilon_j$ will be points on a circle which is displaced from the origin by $\xi$. If the $\epsilon_j$ are measured with perfect accuracy, then real and imaginary parts of $\xi$ can be determined with an accuracy $\sigma = \left[ \langle Re(\gamma)^2 \rangle / N \right]^{1/2} = |\gamma|/(2N)^{1/2}$. If the real and imaginary parts of the $\epsilon_j$ can each be measured only to an accuracy $\Delta/2^{1/2}$, then $\sigma$ increases to

$$\sigma = \left( \frac{|\gamma|^2 + \Delta^2}{2N} \right)^{1/2}.$$  \hfill (2.3)

Now suppose that the moduli of the ellipticities are distributed as $f(|\gamma|)$ between 0 and 1, with $\int d|\gamma| f(|\gamma|) \equiv 1$. Then

$$\sigma = \left[ \int_0^1 d|\gamma| \frac{2Nf(|\gamma|)}{|\gamma|^2 + \Delta^2} \right]^{-1/2}. $$  \hfill (2.4)

For the special case $f(|\gamma|) = 1$ and $\Delta \ll 1$, equation (2.4) can be evaluated in closed form: $\sigma = [\Delta/(\pi N)]^{1/2}$. In general we may write

$$\sigma = \left( \frac{\Delta}{\zeta \pi N} \right)^{1/2}, $$  \hfill (2.5)

where $\zeta$ is a correction factor. Empirically, Mould et al. (1994) find that the distribution of ellipticities is more skewed toward low values than is a uniform distribution. That is $\zeta > 1$. Using their measured distribution, we estimate $\zeta \sim 2.2$. 

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Finally, we suppose that the ellipticities of $N$ images have been measured, each with accuracy $\Delta_j$. Then

$$\sigma = \left( \zeta_\pi \sum_{j=1}^{N} \Delta_j^{-1} \right)^{-1/2} = \left( \frac{\overline{\Delta}}{7N} \right)^{1/2}, \tag{2.6}$$

where $\overline{\Delta}$ is the harmonic mean of the $\Delta_j$ and where we have estimated $\zeta_\pi = 7$.

Note that $\sigma$, the accuracy of the measurement of the components of $\xi$, is proportional to the square root of $\Delta$, the accuracy of the measurements of the ellipticities of the observed images. This contrasts sharply with the usual situation where the error in the determination of a given parameter is directly proportional to the errors in the measured quantities.

3. Sensitivity of the Sloan Survey

The Sloan Survey is a digital survey of $\pi$ steradians about the north galactic pole. The survey will be performed on a 2.5m telescope in five bands by scanning the sky at approximately the sidereal rate. The nominal limit of the survey for a point source with signal-to-noise ratio of 5 is $r' = 23.1$ (D. Weinberg 1994, private communication, DW). Here we restrict consideration to galaxies with $r' < 21.5$. There are $\sim 10^4$ such galaxies per square degree and these have a median half-light diameter $\sim 2''5$ (DW). It is difficult to assess how well the ellipticities can be measured, but extrapolating from the experience of Mould et al. 1994, we conservatively estimate $\overline{\Delta} = 0.07$. We then estimate the sensitivity of the Sloan Survey from equation (2.6) to be

$$\sigma = 0.10\% \left( \frac{A}{\text{deg}^2} \right)^{-1/2}, \tag{3.1}$$

where $A$ is the angular area over which the mean value of $\xi$ is being measured.
4. Weak Lensing Signature of Nearby Structures

The Local Supercluster and other structures of the nearby universe produce weak lensing effects $|\xi| \sim 0.2\% \Omega$ where $\Omega$ is the density of the universe as a fraction of the critical density. The effects are coherent over tens of square degrees. This is apparent from Figures 1 and 2 which show the weak lensing patterns for $\Omega = 1$ and $\Omega = 0.3$ universes respectively. The orientation of the line segments indicates the direction in which the images are stretched. The length of the segment indicates the size of the effect: a 1 degree segment represents $|\xi| = 0.40\%$ in Figure 1 and $|\xi| = 0.12\%$ in Figure 2. To construct these figures, we computed the deflection of light in many directions due to all the galaxies in de Vaucouleurs et al. (1991 RC3) listed with redshifts and total blue magnitudes. We assumed that each galaxy has a total mass to $B$-light ratio of $2000\Omega h$ where $h$ is the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Binney & Tremaine 1987, assuming $\langle B - V \rangle_{\text{galaxies}} \sim 0.8$), and that the mass is distributed in a truncated isothermal sphere with radius $2.8\Omega \text{ Mpc}$. We then found the polarization at $0.2^\circ$ intervals from the traceless part (shear) of the magnification tensor, and finally averaged the results over the 25 positions within $1^\circ$ squares.

The accuracy of the lensing pattern shown in Figures 1 and 2 depends on the assumption that the RC3 is complete, and also on the correctness of the particular model we have chosen for the relation between mass and light. Both of these assumptions are likely to fail. However, as we show below, the incompleteness of the catalog will be rectified by the Sloan Survey itself, and the lensing pattern is mainly sensitive to $\Omega$ rather than to the details of the correlation between mass and light.
4.1. Completeness

For the Sloan Survey, the median redshift of the galaxies with redshifts will be \( z = 0.1 \). For galaxies with \( r' < 19.5 \) the median redshift will be \( z = 0.25 \) (DW). One can estimate the redshifts for these latter from their colors. The estimates should be fairly accurate at least in a statistical sense. Thus, it will be possible to make a good estimate of the distribution of galaxies brighter than \( L_* \) at least out to \( z = 0.25 \). Under the assumption that the \( L_* \) galaxies trace the distribution of all galactic light, one can then predict the lensing due to observed galaxies \( z = 0.25 \) for a given model relating mass to light. That is, it will be possible to “take out” the effect of lensing due to galaxies at intermediate redshift, leaving only the effects of the nearby structures and the galaxies with \( z > 0.25 \).

As we show explicitly in § 2.3, below, the effect of lensing due to galaxies with \( z > 0.25 \) is expected to be small on the \( \sim 5^\circ \) scales over which the effects of local structures are coherent. Hence, these distant galaxies can, to leading order, be ignored.

4.2. Mass Models

Figures 1 and 2 look very similar to the eye. The figures would look exactly the same except that in Figure 1, we assumed that the mass of galaxies is distributed to a radius of \( 2.8 h^{-1}\)Mpc, while in Figure 2, we assumed \( 0.84 h^{-1}\)Mpc. Figure 2 represents a universe with only \( 3/10 \) as much mass, but the line segments are \( 10/3 \) larger for the same amount of lensing. To the extent that Figures 1 and 2 look the same, one can measure \( \Omega \) directly from the amplitude of the observed lensing, without worrying about the details of the mass model. To the extent that they are different, one can use the differences to make inferences about the distribution of mass. Comparison of the two figures shows that it will be fairly easy to measure \( \Omega \) using weak lensing pattern averaged over many many square degrees. On the other hand, information about the mass distribution will come primarily from regions within a few degrees of clusters.
4.3. Polarization Due to Distant Galaxies

We can calculate the polarization field due to distant galaxies in a given cosmological model, e.g. Cold Dark Matter (CDM). To be specific, let us assume that there is a sheet of galaxies at redshift \( z_2 \) corresponding to a comoving angular diameter distance \( x_2 = 2(1 - (1 + z_2)^{-1/2}) \). The derivations below follow Blandford et al. (1991) and Mould et al. (1994). The derivations are formally valid only for \( \Omega = 1 \) but can be easily rescaled to other values of \( \Omega \). The polarization from all sources closer than \( x_1 \) is

\[
p(x_1, x_2) = 2 \int_0^{x_1} dy \frac{y(x_2 - y)}{x_2} F_0(y), \tag{4.1}
\]

\[
F_0 = \left( \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2} - 2i \frac{\partial^2}{\partial x \partial y} \right) \Phi_0, \tag{4.2}
\]

where the line of sight is the z-axis and \( \Phi_0 \) is the gravitational potential. This real space formulation is suitable for a known, or assumed, mass distribution, but for a cosmological model it is more useful to do a plane wave decomposition of the density field and then specify the power spectrum of density fluctuations \( P(k) = |\delta^2_0(k)| \). One can generate a map of the polarization field as a random realization with the appropriate power spectrum of polarization fluctuations \( Q(k) \) which can be calculated from \( P(k) \). Typically, the density field is assumed to be Gaussian which means that the phases of the waves are uncorrelated and random.

The field for a sheet of galaxies at distance \( x_2 \) from sources closer than \( x_1 \) is

\[
p(x_1, x_2) = -3 \int \frac{d^3k}{2\pi^3} \int_0^{x_1} dy y \left( \frac{x_2 - y}{x_2} \right) F_0(k) \exp \left( \frac{i k \cdot y}{2} \right) =
\]

\[
- \frac{3}{2} x_1^2 \int \frac{d^3k}{2\pi^3} F_0(k) \left[ \frac{x_1 j_1(\psi)}{x_2 \psi} + \left( 1 - \frac{x_1}{x_2} \right) (j_0(\psi) + ij_1(\psi)) \right] e^{i\psi}. \tag{4.3}
\]
where

$$\mathcal{F}_0(k) = \delta_0(k) \frac{(k_1 + ik_2)^2}{k_1^2 + k_2^2}; \quad \psi \equiv \frac{k \cdot x_1}{2}$$  \hspace{1cm} (4.4)$$

For \(x_1 = x_2\) this reduces to the standard result from Blandford et al. (1991). In the same way the polarization correlation \(C_{pp}(x_1, x_2, \theta)\) and the variance \(\sigma^2(x_1, x_2, \theta)\) can be calculated using the Fourier convolution theorem and the result

$$\int_{-\infty}^{\infty} da \exp(-iaq) \left[ \frac{x_1j_1(a)}{x_2} + \left(1 - \frac{x_1}{x_2}\right) (j_0(a) + ij_1(a)) \right]$$

$$= \pi \left[ \frac{x_1 (1 - q^2)}{2x_2} + \left(1 - \frac{x_1}{x_2}\right) (1 - q) \right] \Theta(1 + q) \Theta(1 - q).$$  \hspace{1cm} (4.5)$$

Then,

$$C_{pp}(x_1, x_2, \theta) = 36\pi^2 x_1^3 \int dk k^3 P(k) \int_0^1 ds \left[ \left( \frac{x_1}{x_2} \right)^2 s^2 (1 - s)^2 + \left(1 - \frac{x_1}{x_2}\right) s(1 - s) \right] J_0(kx_1 \theta s),$$  \hspace{1cm} (4.6)$$

and

$$\sigma^2(x_1, x_2, \theta) = 36\pi^2 x_1^3 \int dk k^3 P(k) \int_0^1 ds \left[ \left( \frac{x_1}{x_2} \right)^2 s^2 (1 - s)^2 + \left(1 - \frac{x_1}{x_2}\right) s(1 - s) \right] \left( \frac{2J_1(kx_1 \theta s)}{kx_1 \theta s} \right)^2.$$  \hspace{1cm} (4.7)$$

The variance is interpreted as the mean square polarization when the field is smoothed with a circular top hat weighting function of angle \(\theta\). The power spectrum of polarization fluctuations is the fourier transform of the polarization corre-
lation function,

\[ Q(k, x_1, x_2) = 18\pi x_1^3 \int_0^1 ds P\left( \frac{k}{s} \right) \left[ \left( \frac{x_1}{x_2} \right)^2 (1-s)^2 + \left( 1 - \frac{x_1}{x_2} \right) \frac{1-s}{s} \right]. \quad (4.8) \]

A random realization of the polarization field from sources between \( x_1 \) and \( x_2 \) can be generated by subtracting the field generated from \( Q(k, x_2, x_2) \) and \( Q(k, x_1, x_2) \) using the same set of random numbers. This is equivalent to specifying the same plane wave decomposition of the density field \( \delta_0(k) \) for the two polarization fields.

The polarization field predicted for CDM generated by sources between \( z_1 = 0.25 \) and \( z_2 = 0.4 \) is plotted in Figure 3 smoothed on a scale of 0.56°. This corresponds to a smoothing area of 1 square degree. It is plotted in the same way as the predicted polarization field in Figure 1. The rms polarization of the polarization field generated by all sources is 2.5% without smoothing and 1.1% with this 1 square degree smoothing. However, if we only look at sources more distant than \( z_1 = 0.25 \) then the rms polarization drops to 0.34% which is similar to what is predicted from the local galaxies in the RC3 catalogue with \( \Omega = 1 \). This strong reduction of the background signal comes about because the distant structures generate mostly small angular scale structure in the polarization field. Since the polarization field due to local structures is coherent over many square degrees, the polarization induced by distant galaxies should also be smoothed over a large area before comparing it with the locally induced structure. In Figure 4, we show the polarization due to distant galaxies smoothed over 25 square degrees. The rms polarization is 0.09% on this scale and therefore should not seriously interfere with measurement of the local structure.
5. More Distant Structures

As we discussed in § 1, it is particularly interesting to measure the lensing due to local structures since we have the most other information about them. However, there is also much to be gained by analyzing the lensing due to structures at intermediate redshifts $z < 0.25$. First, of course, by simultaneously fitting the lensing amplitude due to all the observed structures $z < 0.25$ rather than just the local structures, one could obtain a more accurate estimate of $\Omega$. Second, it is possible that a substantial part of the mass of the universe is correlated with the light only on scales of many Mpc, or perhaps tens of Mpc. To probe these large physical scales effectively, it is necessary to look at structures at larger distances.

6. Systematic Effects

Since weak lensing by local structures is extremely weak $O(0.2\%)$, one must be especially careful about small systematic effects. As discussed by Mould et al. (1994), these are basically of two types. First, trailing of the point spread function (PSF) and second, classical aberration. Mould et al. calibrated the trailing of the PSF primarily by measuring the trend of the polarization with inverse galaxy size, in effect extrapolating to galaxies of infinite size for which a trailing PSF would have no effect. They also checked for consistency with the trailing measured from stellar images. However, since there were $\sim 4000$ galaxies and only $\sim 80$ stars, the galaxies provided a somewhat more precise estimate than the stars.

In the Sloan Survey, by contrast, there will be $N = 10^4$ galaxies deg$^{-2}$ compared to $\sim 2000$ stars deg$^{-2}$. This means that the stars will provide much more information about the trailing of the PSF than the galaxies. It is easy to see that in fact the stars will provide adequate information to calibrate the trailing. The effective number of galaxies, that is those that enter with significant statistical weight, is $\zeta \pi \Delta N/2 \sim 2500$ deg$^{-2}$. The ellipticities of the stellar images can be measured somewhat better than those of the galactic images, but to be conservative we will assume equal accuracy. Then, if there were equal numbers of the stars...
and galaxies, the error in the galactic polarization induced by the uncertainty in
the measurement of stellar trailing would be $\sigma$ times the ratio of star-to-galaxy
areas, i.e., $\sim 16\%\sigma$ for a $1''$ seeing disk and median 2.5 diameter galaxies. Since
there are slightly more effective galaxies than stars, this fraction is raised by $\sqrt{1.25}$
to $\sim 18\%\sigma$. In other words, the problem of calibrating the trailing of the PSF
increases the error in the polarization measurement by a small fraction.

Mould et al. (1994) calibrated the classical aberration using photometry of
astrometric fields. We assume that calibration for the Sloan Survey can be carried
out in a similar manner. It is possible that flexure of the telescope will alter the
classical aberration as a function of the orientation of the telescope. If so, this
would complicate the calibration.

Finally, we note that the polarization measurements themselves will provide
an important check on how well the systematic effects have been calibrated. The
polarization will have significant power on scales of tens of square degrees due
to local structures, but not on scales of steradians. Hence, even a very small
systematic distortion induced by the instrumental setup should be recognizable.
Moreover, since the pattern of weak lensing due to local structures is approximately
known and relatively complicated, it should be possible to track down systematic
effects from unanticipated sources. That is, systematic effects from any source
would be unlikely to mimic this general pattern.

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FIGURE CAPTIONS

1) Weak lensing by nearby structures in an $\Omega = 1$ universe. Galaxies are assumed to be isothermal spheres truncated at 2.8 Mpc, with mass to blue light ratios of 2000. The lengths of the line segments in degrees are equal to magnitudes of the mean polarization in units of 0.40%. The directions of the line segments are the axes of elongation. The position (0,0) is the north galactic pole (NGP). The Coma cluster is $\sim 2^{\circ}$ from the NGP, the Virgo cluster is $\sim 15^{\circ}$ below it, and A1367 is $\sim 15^{\circ}$ below and to the right of the NGP. The heart of the Local Supercluster runs from the Virgo Southern Extension at roughly the lower-left corner through Virgo and out toward Ursa Major which lies beyond the upper right corner.

2) Same as Fig. 1 except for an $\Omega = 0.3$ universe with the truncation radii of galaxies set to 0.84 Mpc, the mass to blue light ratio set to 600, and with $1^{\circ}$ line segments representing 0.12% polarization.

3) Polarization field of galaxies at $z = 0.40$ due to a CDM ($h = 0.5$, $\Omega = 1$) mass distribution over the range $0.25 < z < 0.40$, smoothed over 1 square degree. As in Fig. 1, a line segment of length $1^{\circ}$ represents a polarization of 0.40%.

4) Same as Fig. 3, except smoothed over 25 square degrees. Direct comparison of this figure with Fig. 1, shows that the polarization pattern induced by distant galaxies will not seriously interfere with measurement of the field induced with local structures.