Calculation of deception probability of netted radar based on non-central chi-square distribution

Yue Yuan¹, Gang-yi Tu², Ben Wang¹ and Ling-ling Wang¹

Abstract
Aiming at the problems of complex factors affecting the rate of deception probability of networked radar nets, the large amount of calculation by Monte Carlo simulation and the inability to quantitatively analyze the influence of various factors on the deception probability of networked, a calculation method of deception probability of networked is proposed. First, according to the homology measurement method based on the Mahalanobis distance, the probability density model of the deception probability of networked is calculated. Its probability density model obeys the non-central chi-square distribution. Then, a hypothesis test model is established to calculate the deception probability of networked mathematical expression. The simulation results show that the error between the calculation method of the deception probability of networked and the calculation result of 1000 times Monte Carlo is less than 2%. The method in this article can analyze the quantitative effect of false target position, interference distance interval, radar position, true target position, and other factors on the deception probability of networked, instead of Monte Carlo simulation, to provide a trade-off between the true target recognition rate and the deception probability of networked theoretical basis.

Keywords
Netted radar, deception false target, non-central chi-square distribution, deception probability of networked

Date received: 31 January 2021; accepted: 5 June 2021
Handling Editor: Lyudmila Mihaylova

Introduction
With the rapid development of modern electronic warfare, active deceptive jamming has great advantages in pertinence, cost-effectiveness, combat flexibility, and so on, so it is widely used. In particular, the emergence of Digital Radio Frequency Memory (DRFM) enables the active deceptive jamming system¹,² to quickly copy the received radar signal, generating multiple false targets distributed around the real target, consuming radar system resources, cause the radar system to be overloaded, and even make the radar misjudge the interference as the real target,³ so the identification of deceptive false targets is of great significance.⁴–⁸

A large number of research achievements have been made in the field of anti-spoofing of range false targets. Multiple input multiple output (MIMO) radar of the Frequency Diversified Array (FDA)⁹,¹⁰ can resist main-lobe jamming because it provides controllable degrees of freedom in both the range and angle domains. Netted radars have the advantages of different systems and different frequency bands in clustering. Usually, the information of the network radar station will not be fully known by the enemy, so it is difficult for the forward jammer to generate active false target jamming

¹No. 8 Research Academy of CSSC, Nanjing, China
²Nanjing University of Information Science and Technology, Nanjing, China

Corresponding author:
Gang-yi Tu, Nanjing University of Information Science and Technology, Nanjing 210044, China.
Email: 1572467004@qq.com

Creative Commons CC BY: This article is distributed under the terms of the Creative Commons Attribution 4.0 License (https://creativecommons.org/licenses/by/4.0/) which permits any use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).
with consistent position information and speed information for each radar node, thus forming collaborative deception. In other words, the false target has no spatial correlation, while the true target is relatively concentrated after the unified coordinates in all radar measurements, that is, it has spatial correlation. Based on the above physical phenomena, a false target elimination method based on homologous measurement and inspection fusion was proposed in the literature\(^1\)\(^-\)\(^10\) to reduce the probability of being spoofed by netted radar. The homology detection in the above literatures all adopts central chi-square test, which has a good detection performance in the detection of linear and non-linear signals in Gaussian white noise.\(^17\),\(^18\) Existing literatures have established mathematical models of the accuracy of true target recognition. However, the probability of netted radar being spoofed is obtained through Monte Carlo experiment statistics. Monte Carlo simulation calculation is heavy, and it is impossible to quantitatively describe the influence of different factors on the probability of netted radar being spoofed. The mathematical model of spoofing probability of netted radar is not given in the existing literature.

In response to the above problems, this paper proposes a method for calculating the probability of being deceived by a deceptive networked radar based on a non-central chi-square distribution, and establishes a probability density model for the deception probability of a non-central chi-square distribution of a networked radar, and derives the deception probability. The mathematical expression of the probability of deception. Through simulation verification, it can be concluded that the calculation method proposed in this article is consistent with the Monte Carlo simulation results and can replace Monte Carlo simulation.

The organizational structure of this article is as follows. In section “System model,” the system model of netted radar against spoofing is established. In section “Probability of being deceived by networked radar,” a method for calculating the spoofing probability of netted radar based on non-central chi-square distribution is proposed. In section “Simulation analysis,” the validity of the method is verified by numerical simulation. Finally, section “Concluding remarks” concludes this article.

**System model**

For the convenience of discussion, suppose that the networked radar is composed of two three-coordinate radars, and the geographic coordinates of the \(i\)th node radar are \((L_i, B_i, H_i)\) \((i = 1, 2)\), where \(L_i, B_i, H_i,\)

respectively, indicate the longitude, latitude, and altitude of radar \(i\). Geocentric coordinates of radar \(i\) \((X_{Ri}, Y_{Ri}, Z_{Ri})\) for

\[
\begin{align*}
X_{Ri} &= (N_R + H_i) \cos L_i \cos B_i \\
Y_{Ri} &= (N_R + H_i) \cos L_i \sin B_i \\
Z_{Ri} &= [N_R(1 - e_i^2) + H_i] \sin L_i
\end{align*}
\]

(1)

Among them, \(e_i^2 = (a^2 - b^2)/a^2\) is the first eccentricity; \(N_R = a/\sqrt{1 - e_i^2}\sin^2 B_i\) is the radius of curvature of the circle; \(a = 6,378,137\) m is the semi-major axis of the ellipsoid; and \(b = 6,356,752\) m is the semi-minor axis of the ellipsoid.

Suppose the coordinates of the target in the radar polar coordinate system are \((\rho_i, \phi_i, \theta_i)\). The coordinates in the east-north-up (ENU) coordinate system from radar \(i\) to \((x_i, y_i, z_i)\). Get

\[
\begin{align*}
x_i &= \lambda_{\phi}^{-1} \lambda_{\theta}^{-1} \rho_i \sin(\phi_i) \cos(\theta_i) \\
y_i &= \lambda_{\phi}^{-1} \lambda_{\theta}^{-1} \rho_i \sin(\phi_i) \cos(\theta_i) \\
z_i(k) &= \lambda_{\theta}^{-1} \rho_i \sin(\theta_i)
\end{align*}
\]

(2)

where \(\lambda_{\phi} = e^{-\sigma_{\phi}^2/2}\), \(\lambda_{\theta}^\prime = e^{-2\sigma_{\theta}^2}\), \(\lambda_{\theta} = e^{-\sigma_{\theta}^2/2}\), and \(\lambda_{\theta}^\prime = e^{-2\sigma_{\theta}^2}\).

In the earth-centered, earth-fixed (ECEF) coordinate system, the coordinates of the target \(Z_{ECEF} = (x_{ECEF}, y_{ECEF}, z_{ECEF})\)

\[
\begin{bmatrix}
x_{ECEF} \\
y_{ECEF} \\
z_{ECEF}
\end{bmatrix} = R_g \begin{bmatrix}
x_i \\
y_i \\
z_i
\end{bmatrix}
\]

(3)

\[
R_g = \begin{bmatrix}
-\sin L_i & -\sin B_i \cos L_i & \cos B_i \cos L_i \\
\cos L_i & -\sin B_i \sin L_i & \cos B_i \sin L_i \\
0 & \cos B_i & \sin B_i
\end{bmatrix}
\]

(4)

The measured noise covariance under the ECEF coordinate system is

\[
R = R_g \cdot \begin{bmatrix}
r_{11} & r_{12} & r_{13} \\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{bmatrix} \cdot R_g^T
\]

(5)

Among them

\[
r_{11} = \left(\lambda_{\phi} \lambda_{\theta}\right)^{-2} - 2 \rho^2 \cos^2 \phi \cos^2 \theta + \frac{1}{4} \left(\rho^2 + \sigma_{\rho}^2\right) \left(1 + \lambda_{\phi}^\prime \cos 2\phi\right) \left(1 + \lambda_{\theta}^\prime \cos 2\theta\right)
\]

(6)

\[
r_{22} = \left(\lambda_{\phi} \lambda_{\theta}\right)^{-2} - 2 \rho^2 \sin^2 \phi \cos^2 \theta + \frac{1}{4} \left(\rho^2 + \sigma_{\rho}^2\right) \left(1 - \lambda_{\phi}^\prime \cos 2\phi\right) \left(1 + \lambda_{\theta}^\prime \cos 2\theta\right)
\]

(7)
\[ r_{33} = (\lambda_\phi^{-2} - 2) \rho^2 \sin^2 \epsilon + \frac{1}{2} \left( \rho^2 + \sigma_\rho^2 \right) (1 - \lambda_\theta \cos 2\theta) \]  
(8)

\[ r_{12} = \left[ \left( \lambda_\phi \lambda_\theta \right)^{-2} - 2 \right] \rho^2 \sin \phi \cos \phi \cos^2 \theta + \frac{1}{4} \left( \rho^2 + \sigma_\rho^2 \right) \lambda_\phi \sin 2\phi (1 + \lambda_\theta \cos 2\theta) \]  
(9)

\[ r_{13} = (\lambda_\phi^{-1} \lambda_\theta^{-1} - \lambda_\phi^{-1} - \lambda_\phi) \rho^2 \cos \phi \sin \phi \cos \theta \]  
\[ + \frac{1}{2} \left( \rho^2 + \sigma_\rho^2 \right) \lambda_\phi \lambda_\theta \cos \phi \sin 2\theta \]  
(10)

\[ r_{23} = (\lambda_\phi^{-1} \lambda_\theta^{-2} - \lambda_\phi^{-1} - \lambda_\theta) \rho^2 \sin \phi \sin \phi \cos \theta \]  
\[ + \frac{1}{2} \left( \rho^2 + \sigma_\rho^2 \right) \lambda_\phi \lambda_\theta \sin \phi \sin 2\theta \]  
(11)

The distance multi-false target jamming is in the same straight line as the radar and the jammer, so the space position of the false target is related to the geographic coordinates of the radar station. When the radar station is different, the position of the false target is also different, and the real target and the radar station are mutual. Independent, the location of the real target has nothing to do with the geographic location of the radar station. Therefore, when processing data in the radar network information processing center, false targets can be identified based on the location of different radar stations.

After coordinate transformation and unbiased processing, target measurement in ECEF coordinate system is \( Z_{ECEF}^1 \) with \( Z_{ECEF}^2 \). In order to realize the tracking of the target by different radars, it is necessary to judge whether the measurement of the target by different radars can be correlated. If the two measurements can be correlated, based on the above results, the two measurements must come from the same real target. If the measurements from different radars cannot be correlated on other radars, the radar measurement is a false target. The equation is as follows:

\[ V_{ECEF}(k) = Z_{ECEF}^1(k) - Z_{ECEF}^2(k) \]  
(12)

Because \( Z_{ECEF}^1 \) with \( Z_{ECEF}^2 \). Independent, so

\[ R_{ECEF}^V = R_{ECEF}^1 + R_{ECEF}^2 \]  
(13)

In formula (13), \( R_{ECEF}^1 \) and \( R_{ECEF}^2 \) are the measurement covariance matrices of \( Z_{ECEF}^1 \) with \( Z_{ECEF}^2 \), respectively.

In order to study the degree of difference in radar measurements, the easiest way is to construct the distance between the three measurements. Although Euclidean distance is useful, its obvious disadvantage is that it does not consider the covariance of the measured coordinates. For this reason, the Mahalanobis distance of the measurement point is selected as the statistic, which overcomes the adverse effect of the covariance of the measurement error on the statistic. Set statistics

\[ \eta = V_{ECEF} \cdot (R_{ECEF}^1 + R_{ECEF}^2)^{-1} \cdot V_{ECEF} \]  
(14)

This statistic obeys the chi-square distribution with 3 degrees of freedom (DOFs).

Therefore, according to the radar network data association criterion based on statistical judgment, the data association problem from different radars can be analyzed and judged with the following assumptions:

H0: if \( \eta \leq \psi \), it is judged that the measurements of radar 1 and radar 2 come from the same real target, that is, the true-true situation.

H1: if \( \eta > \lambda \), then it is judged that the measurement of radar 1 or radar 2 contains false targets, that is, true false, false false false, and false true.

Among them, \( \psi = \chi^2_\beta(n) \) is the statistical decision threshold, \( \beta N = 3 \) is the degree of freedom of the chi-square distribution. The significance level table of chi-square test for different degrees of freedom is shown in Table 1. Considering that the recognition probability of netted radar is 99%, the detection threshold can be set according to the significance level table with degree of freedom equal to 3 \( \psi = 11.343 \); considering that the recognition probability of real target is 90%, the test threshold is set \( \psi = 6.251 \). \( \psi \) is the chi-square distribution test threshold. It is only related to the true target recognition probability and does not change with the

| \( n \) | 0.01 | 0.03 | 0.05 | 0.1 | 0.25 | 0.5 |
|-------|------|------|------|-----|------|-----|
| 1     | 6.347| 16.013| 5.024| 3.841| 1.323| 0.455|
| 2     | 9.210| 11.345| 7.378| 5.991| 2.773| 1.386|
| 3     | 15.086| 13.277| 11.433| 9.348| 4.108| 2.366|
| 4     | 16.812| 14.618| 11.644| 9.488| 5.285| 3.357|
| 5     | 18.475| 16.013| 12.833| 11.070| 6.626| 4.351|
| 6     | 16.812| 15.086| 12.833| 11.644| 10.645| 5.348|
| 7     | 18.475| 16.013| 12.833| 11.644| 12.017| 6.347|
target position and the radar measurement error of the node, which is due to the statistics obtained from the above transformation $\eta$. After normalization, the scale problem of Euclidean distance is corrected, which is not affected by the dimension, so the variance of each degree of freedom is 1. When the two plots involved in the test are real target measurements, after unbiased correction, the mean value is 0. Therefore, $\eta$ obeys the standard chi-square distribution with 3 degrees of freedom.

It should be noted that the premise of the above test is that the detection probability of the two radars to the true target is 1. For the case that only one radar detects the true target, and there is no corresponding true target plot matching with it in the other radar, the true target probability will be eliminated as false target.

For the network of $N$ radars, the measured values of $N$ radars are correlated $C_2$. A total of $M$ correlation measurement sequences were formed $z_i$. Each sequence consists of measurements of different serial numbers from $n$ different radars

$$
\mathbf{z}_i = (Z_{i1}, Z_{i2}, \ldots, Z_{iN}), \quad i = 1, 2, \ldots, m;
$$

in which $M_{\text{max}}$ is the maximum number of measuring points in $n$ radars, and $Z_{im}$ indicates the $n$th trace from the $i$th radar for correlation measurement sequence $z_i$. Two pairs of chi-square test are carried out, and then the intersection is taken. The plot which cannot satisfy the two chi-square tests at the same time will be discarded.

### Probability of being deceived by networked radar

According to the previous analysis, when $Z_{ECEF}^1$ and $Z_{ECEF}^2$ all come from real targets, statistics $\eta$ obey the standard chi-square distribution. When the measurement of radar 1 comes from a real target and the measurement of radar 2 comes from a false target, that is, a true-false situation, The mean of $Z_{ECEF}^1$ is the true value of the target position $Z_{ECEF}^1$. The mean of $Z_{ECEF}^2$ is the true value of the false target $Z_{ECEF}^2$, so the mean value of the random variables $V_{ECEF} = Z_{ECEF}^1 - Z_{ECEF}^2$ is not 0, but the mean value is $Z_{ECEF}^1 - Z_{ECEF}^2$. When the measurements of radar 1 and radar 2 come from false targets, that is, false-false situation, The mean value of $Z_{ECEF}^1$ is the true value of the false target received by radar ( $Z_{ECEF}^1$). The mean value of $Z_{ECEF}^2$ is the true value of radar b receiving false targets $Z_{ECEF}^2$. The mean value of the random variable $V_{ECEF} = Z_{ECEF}^1 - Z_{ECEF}^2$ is also not zero, and its mean value is $Z_{ECEF}^1 - Z_{ECEF}^2$. Due to the normalization of the covariance matrix of the Mahalanobis distance, the test statistic at this time is $\eta$. The variance of the Gaussian component of each degree of reason is still 1, but the mean value is not 0, and the test statistic becomes a non-central chi-square distribution, and the influence on the probability of being deceived by the networked radar becomes the threshold of the standard chi-square distribution to test the non-central chi-square distribution. The random variable

$$
Q_i \sim N(\mu_i, \sigma^2) \quad (16)
$$

$$
E = \sum_{i=1}^{n} Q_i^2 \quad (17)
$$

Among them $\sigma^2 = 1$. And make $\lambda = \sum \mu_i^2$. It obeys the non-central chi-square distribution with $n$ degrees of freedom, and its probability dense function is

$$
p_{E(\eta)} = \frac{1}{2\sigma^2} \left( \frac{\eta}{\lambda} \right)^{\frac{n-2}{2}} \exp \left( -\frac{\eta + \lambda}{2\sigma^2} \right) I_{n-2} \left( \frac{\sqrt{\eta\lambda}}{\sigma^2} \right) \quad (18)
$$

Its mean value and variance are, respectively

$$
E(\eta) = n\sigma^2 + \lambda \quad (19)
$$

$$
Var(\eta) = 2n\sigma^2 + 4\sigma^2 \lambda \quad (20)
$$

The $xyz$ axis of VECEF is not independent, so it is impossible to directly calculate the mean formula of each degree, according to the mean and covariance relationship formula

$$
E(AB) = \text{cov}(A, B) + E(A)E(B) \quad (21)
$$

$$
\eta = V_{ECEF}^T \cdot \left( R_{ECEF}^1 + R_{ECEF}^2 \right)^{-1} \cdot V_{ECEF} \quad (22)
$$

Then

$$
E(\eta) = w_{11}[\text{cov}(\Delta x, \Delta x) + E(\Delta x)E(\Delta x)] + w_{21}[\text{cov}(\Delta y, \Delta x) + E(\Delta y)E(\Delta x)] + w_{31}[\text{cov}(\Delta z, \Delta x) + E(\Delta z)E(\Delta x)] + w_{12}[\text{cov}(\Delta x, \Delta y) + E(\Delta x)E(\Delta y)] + w_{22}[\text{cov}(\Delta y, \Delta y) + E(\Delta y)E(\Delta y)] + w_{32}[\text{cov}(\Delta z, \Delta y) + E(\Delta z)E(\Delta y)] + w_{13}[\text{cov}(\Delta x, \Delta z) + E(\Delta x)E(\Delta z)] + w_{23}[\text{cov}(\Delta y, \Delta z) + E(\Delta y)E(\Delta z)] + w_{33}[\text{cov}(\Delta z, \Delta z) + E(\Delta z)E(\Delta z)] \quad (23)
$$
Substituting into the covariance matrix, we get
\[
E(\eta) = w_{11}[v_{11} + E(\Delta x)E(\Delta x)] + w_{21}[v_{21} + E(\Delta y)E(\Delta y)] + w_{31}[v_{31} + E(\Delta z)E(\Delta z)]
\]
\[
+ w_{12}[v_{12} + E(\Delta x)E(\Delta y)] + w_{22}[v_{22} + E(\Delta y)E(\Delta y)] + w_{32}[v_{32} + E(\Delta z)E(\Delta y)]
\]
\[
+ w_{13}[v_{13} + E(\Delta x)E(\Delta z)] + w_{23}[v_{23} + E(\Delta y)E(\Delta z)] + w_{33}[v_{33} + E(\Delta z)E(\Delta z)]
\]
(24)

Get
\[
\lambda = E(\eta) - n\sigma^2
\]  
(25)

Substituting formula (24) and \(\sigma^2 = 1\) into formula (17), the probability density function of the non-central chi-square test statistic is obtained, and the probability density function is integrated to obtain the distribution function of the non-central chi-square test statistic.

\[
\phi_E(x) = \int_{-\infty}^{\infty} \rho_E(\eta) d\eta
\]
\[
= \frac{1}{\sqrt{2\pi} (\lambda)} \exp\left(-\frac{\eta + \lambda}{2\sigma^2}\right) I_{\frac{\lambda-1}{2\sigma^2}}\left(\frac{\sqrt{\pi\lambda}}{\sigma^2}\right) d\eta
\]
(26)

where \(\phi_E(\psi)\) is the probability of false target being misjudged as true target under chi-square test threshold. \(\psi\) is a chi-square test threshold determined by the true target recognition rate.

Equation (26) is the deception probability value of node radars 1 and 2 taking one measurement value, respectively. Considering multiple real targets and multiple false targets, the comprehensive deception probability is calculated. It is assumed that there are NT true targets in the observed airspace, NT true targets and NF1 false targets in radar 1 measurement plot, and NT true targets and NF2 false targets in radar 2 measurement point trace. The (NT + NF1) and (NT + NF2) measurement pairs were formed by pairwise combination. There are NT groups for true true measurement pairs, ntnf2 pairs for true false measurement pairs, nftnt pairs for false true measurement pairs, and nftnf2 pairs for false false measurement pairs. If one pair of measurement pairs is taken each time, the points in the measurement pairs passing the chi-square test will not participate in the chi-square test. The deception probability of false target comes from the probability of true false true and false false measurement pairs passing chi-square test. The total deception probability is the weighted average of the above three cases. In the case of \(N\) radar networks, the measurement values in the correlation measurement sequence are combined in pairs, and the two measurements in each combination are carried out by chi-square test, only if all combinations pass the hypothesis test, the measurement sequence is considered to be corresponding to the true target, and the associated measurement sequence that fails the hypothesis test is eliminated.

**Simulation analysis**

In order to verify the validity of the calculation of the false target deception probability model, the probability model simulation and 1000 Monte Carlo simulations in this article are carried out, respectively. The information of the two-node radar is shown in Table 2.

It is assumed that the number of real targets in the air is three (including one jammer), and the jammer is self-defense jamming. The remaining two true targets are around the jammer. The jammer generates 10 equally spaced distance deception false targets on its connection with node radars 1 and 2, among which five are in front of the jammer and five are behind the jammer. Let the longitude and latitude of jammer change within a certain range, with latitude coordinates from 30.6° to 31.5° and longitude coordinates from 25.8° to 27.4°. The significant level is \(\alpha = 0.1\), and the detection threshold is \(\psi = 6.251\). The distance deception interval is set at 300 and 500 m, respectively, and the Monte Carlo simulation results of 1000 times are compared with the calculation results of this method.

Figure 1 is a graph showing the variation of radar deception probability of radar 1 and radar 2 networks when the distance deception value is 300 m. It can be seen from Figure 1(a) and (b) that for targets in different regions, the calculated results in this article are close to the Monte Carlo simulation results of 1000 times, and the statistical error between the two calculated results is less than 2%, the average error is 0.00026983, and the standard deviation of error is 0.0042.

Figure 2 is a graph showing the variation of radar deception probability of radar 1 and radar 2 networks when the distance deception value is 500 m. It can be seen from Figure 2(a) and (b) that for targets in

| Nodal radar number | GPS coordinates   | Ranging accuracy/m | Bearing accuracy/rad | Elevation accuracy/rad |
|-------------------|-------------------|--------------------|----------------------|------------------------|
| 1                 | [30.5,26.5,300]   | 5                  | 0.001                | 0.001                  |
| 2                 | [30.5,26.7,300]   | 5                  | 0.001                | 0.001                  |
different regions, the calculated results in this article are close to the Monte Carlo simulation results of 1000 times, and the statistical error between the two calculated results is less than 2%, with an average error of 0.00019628 and an error standard deviation of 0.0044.

It can be seen from Figures 1 and 2 that the farther the false target is from the node radar, the greater the probability that the networked radar will be deceived. Comparing Figure 1 with Figure 2, it can be seen that the smaller the distance deception interval, that is, the denser the false targets, the greater the probability of deception of netted radar.

Figure 3(a) shows the probability density of the test statistics composed of the false targets released by the jammer in the area far from the node radar and the false targets released by radar 2. The probability density of the true target test statistic is a chi-square distribution with 3 DOFs. Figure 3(b) shows the probability density of the test statistics composed of the false targets released by the jammer in the area close to the node radar and the false targets released by radar B. The detection threshold is \( \psi = 6.251 \). Comparing Figure 3(a) and (b), it can be obtained that when the jammer is close to the nodal radar, the probability density distribution of false target inspection statistics is smoother. When the jammer is far away from the nodal radar, the probability density of the true target test statistic and the false target test statistic in Figure 3(a) is close, and it is difficult to distinguish them by setting a threshold.

The true target recognition probability is the chi-square distribution in \( [0, \psi] \). The probability of being deceived by the networked radar is the non-central chi-square distribution in \( [0, \psi] \). It can be seen that the probability of being deceived by the networked radar in Figure 3(b) is smaller than that in Figure 3(a). When the detection threshold is increased, the true target recognition rate will increase, and the probability of being deceived by the networked radar will also increase. If the detection threshold is lowered, the true target recognition rate will be consumed in exchange for a lower probability of being deceived by the networked radar. According to Figure 3, the chi-square distribution of the true target test statistic and the non-central chi-square distribution of the false target test statistic have an intersection. When the detection threshold is less than this intersection, when the threshold is lowered,

Figure 1. Deception probability of networked radar with distance deception interval of 300 m: (a) 1000 Monte Carlo experiments, (b) the calculation results of this article, and (c) the difference between the two calculation results.
the loss of the true target recognition rate is greater than the network. The benefit of the probability of radar being deceived: when the detection threshold is greater than the intersection point, the detection threshold is reduced, and the loss of the true target recognition probability is less than the benefit of the networked radar being deceived. When the distance is far enough, the non-central chi-square distribution will degenerate to the central chi-

Figure 2. Deception probability of networked radar with distance deception interval of 500 m: (a) 1000 Monte Carlo experiments, (b) the calculation results of this article, and (c) the difference between the two calculation results.

Figure 3. Probability density of false target and true target test statistics under different positions of the jammer: (a) far away from the nodal radar and (b) close to the node radar.
square distribution, and the probability density of the test statistics of the false target and the true target coincides. When the threshold is set for detection, the true target recognition rate is equal to the probability of being deceived by the networked radar. The calculation method of deception probability of netted radar based on non-central chi-square distribution proposed in this article still needs some improvement. First, the calculation method in this article does not consider the detection probability of node radars, and the calculation premise is that the detection probability of each node radar to the real target is 1. Second, this article gives the probability density model of netted radar with non-central chi-square distribution. According to the probability model, the test threshold is not set adaptively to improve the overall anti-jamming performance of networked radar. The above work will be carried out in the follow-up study.

**Concluding remarks**

This article presents a method for calculating the probability of being deceived by a netted radar. Based on the homologous measurement method of the Mahalanobis distance, a probability density model of the probability of being deceived by the networked radar is established. Through theoretical analysis and simulation experiments, it can be seen that the mathematical model of networked radar deception probability established in this article is consistent with the 1000 Monte Carlo simulation calculation results, and the error of the two calculation results is less than 2%. It provides theoretical value for weighing the true target recognition rate and the probability of being deceived by the networked radar. How to adaptively set the test threshold according to the non-central chi-square distribution probability density model is the focus of the next research.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) received no financial support for the research, authorship, and/or publication of this article.

**ORCID iD**

Yue Yuan https://orcid.org/0000-0001-9330-4502

**References**

1. He Y, Xiu J, Zhang J, et al. Radar data processing and application. 3rd ed. Beijing, China: Beijing Electronics Industry Press, 2013, pp.87–97.

2. Stavroulakis P, Farsaris N and Xenos TD. Anti-jamming transmitter independent radar networks. In: International conference on signal processing, communications and networking, Chennai, India, 4–6 January 2008, pp.269–273. Piscataway, NJ: IEEE.

3. Li N and Zhang Y. A survey of radar ECM and ECCM. IEEE T Aero Elec Sys 1995; 31(3): 1110–1120.

4. Akhtar J. Orthogonal block coded ECCM schemes against repeat radar jammer. IEEE T Aero Elec Sys 2009; 45(3): 1218–1226.

5. Rao B, Xiao S, Wang X, et al. Maximum likelihood approach to the estimation and discrimination of exoatmospheric active phantom tracks using motion features. IEEE T Aero Elec Sys 2012; 48(1): 794–818.

6. Huang C, Chen Z and Duan R. Novel discrimination algorithm for deceptive jamming in polarimetric radar. In: Lu W, Cai G, Liu W, et al. (eds) Proceedings of the 2012 international conference on information technology and software engineering. Berlin: Springer-Verlag, 2013, pp.369–365.

7. Greco M, Gini F and Farina A. Radar detection and classification of jamming signals belonging to a cone class. IEEE T Signal Proces 2008; 56(5): 1984–1993.

8. Bandiera F, Farina A, Orlando D, et al. Detection algorithms to discriminate between radar targets and ECM signals. IEEE T Signal Proces 2010; 58(12): 5984–5993.

9. Li S, Zhang L, Liu N, et al. Adaptive detection with conic rejection to suppress deceptive jamming for frequency diverse MIMO radar. Digit Signal Process 2017; 69: 32–40.

10. Ciuonzo D, Aubry A and Carotenuto V. Rician MIMO channel-and jamming-aware decision fusion. IEEE T Signal Proces 2017; 65(15): 3866–3880.

11. Zhao Y, Chen Y, Meng J, et al. Distributed netted radar anti-multiple false target deception jamming processing method l-J3. Electro-Optics Contr 2011; 18(3): 25–30.

12. Zhao Y, Wang X, Wang G, et al. Tracking technology of netted radar under multiple false target deception jamming. Chinese J Electron 2007; 35(3): 454–458.

13. Zhao S, Zhang L, Zhou Y, et al. Using the difference of spatial scattering characteristics to identify active false targets. J Xidian Univ (Natur Sci Ed) 2015; 42(2): 20–27.

14. Zhao S, Zhang L, Zhou Y, et al. Anti-false target jamming method based on networked radar spot information fusion. J Univ Electron Sci Technol China 2014; 43(2): 207–211.

15. Wang G and Ji Z. Range multi-jamming target identification of radar net based on multiple discrimination. Syst Eng Electron Technol 2017; 39(1): 40–48.

16. Liu J, Zhang L, Zhao S, et al. A method of anti-deception false target considering site error. J Xi'an Jiaotong Univ 2017; 51(6): 54–58.

17. Cugnon F and Kay S. Alternative approaches to data compression for distributed detection. In: Radar conference, 2016. Piscataway, NJ: IEEE.

18. Ciuonzo D. On time-reversal imaging by statistical testing. IEEE Signal Proc Let 2017; 24: 1024–1028.