Research Article

An Improved Data Generalization Model for Real-Time Data Analysis

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1. Introduction

Weibull distribution is a well-known distribution. It helps in observing the failures of various phenomena and components. Over the past few decades, various investigators concentrate on distribution. Alizadeh et al. [1] provide an extensive analysis of offering a number of historical facts, the various forms of this distribution considered by the practitioners, and probable errors and confusions that arise owing to the nonuniqueness. Bagheri et al. [2] perform a comprehensive analysis of distribution. It is the distribution that attains maximal attention. Moreover, Weibull distribution is applied to biological, medical, and earth sciences. Dattner et al. [3] discuss the effectual Weibull-based shape parameter estimation which relies on modified profile likelihood. The reliability analysis with an additive Weibull model based on failure rate function is considered by various approaches. Some consistent approaches are considered for the evaluation of various WD parameters [4, 5]. Assume that X is a random variable with WD with shape parameter $\alpha$ and scale parameter $\lambda$, and the probability distribution function (PDF) and cumulative distribution function (CDF) is expressed as follows:

$$f(x) = \frac{\alpha}{\lambda} x^{\alpha-1} e^{-\frac{x^\alpha}{\lambda}}, x > 0, \alpha > 0, \lambda > 0,$$

(1)

$$F(x) = 1 - e^{-\frac{x^\alpha}{\lambda}}, x > 0, \alpha > 0, \lambda > 0.$$  

(2)

It is essential to examine the productive estimation of CDF and PDF of Weibull distribution due to its significance over various Weibull distribution applications. There are various estimation approaches like maximal likelihood estimation, uniformly minimal variance unbiased estimation, least square estimation, percentile estimation, and weighted least square estimation [6]. Various investigations are noted that specifically concentrate on other distributions, for
instance, the derive estimator of CDF and PDF of three
generalized parameters with Poisson and Exponential dis-
trIBUTION; however, its shape parameter is known to be
assumed [7]. Some derived estimator of CDF and PDF of
generalized parameter (Rayleigh) distribution when all par-
parameters except shape parameter are considered to be
known. Nagatsuka et al. [8] evaluate CDF and PDF of the
Weibull extension model when all, however, the shape
parameters are considered to be known. Some recent works,
including exponential Weibull distribution, generalized
exponential distribution, and exponential Gumbel distri-
bution, are considered and analyzed [9]. The extension of
this work includes the likelihood of the Weibull distribution
that is unknown. Some works with CDF and PDF are
evaluated and unknown parameters [10].

2. Maximal Likelihood-Weibull
Estimator Model

Consider $X_1, \ldots, X_n$ as the random sample acquired for
Evaluating the Weibull estimator, and the estimator $\lambda$
deﬁnes $\lambda = \sum_{i=1}^{n} x_i^\alpha$. Thus, this work attains
the maximal likelihood of CDF and PDF using
\begin{equation}
\tilde{f}(x) = \frac{\alpha}{\lambda} x^{\alpha-1} e^{-x/\lambda},
\end{equation}
\begin{equation}
\tilde{F}(x) = 1 - e^{-x/\lambda}.
\end{equation}

The probability density of $T = \sum_{i=1}^{n} x_i^\alpha$ is expressed as
\begin{equation}
h^*(t) = \frac{t^{n-1} e^{-t/\lambda}}{\Gamma(n)\lambda^n}.
\end{equation}

For $t > 0$ where some elementary algebra, this work attains the PDF
of $w = \lambda$ as follows:
\begin{equation}
g(w) = n^n w^{n-1} e^{-w/\lambda}. \frac{\Gamma(n)\lambda^n}{\Gamma(n)}.
\end{equation}

For $w > 0$, this work computes $E(\tilde{f}(x)^r)$ and computes
$E(F(x)^r)$.

3. Parametric Distribution of
Weibull Distribution

The novelty behind Weibull distribution is its competency to
examine the failure trends and failure prediction based on
the provided dataset. The major advantage relies on its
versatility, and it can be applied over smaller dataset
samples. The Weibull distribution is shown in various forms
[11]. The distribution functions are expressed as follows:
\begin{align}
F(t) &= 1 - \exp\left[-\left(\frac{t-T}{\alpha}\right)\right], t \geq 0, \tag{7}
\end{align}
\begin{align}
F(t) &= 1 - \exp\left(-x^\beta\right), t \geq 0. \tag{8}
\end{align}

The preliminary properties of Weibull distribution are discussed as follows:
\begin{align}
f(t) &= \beta \alpha x^{\alpha-1} \exp\left[-\left(\frac{t-T}{\alpha}\right)\right], \tag{9}
\end{align}
\begin{align}
f(t) &= \beta \lambda (t-r)^{\beta-1} \exp\left[-(t-r)^\beta\right], t \geq 0. \tag{10}
\end{align}

The mode of MLWD is denoted as
$t = \alpha (\beta - 1) \beta + r \tau_b > 1$ and $r = 0 < \beta \leq 1$.
The medium of MLWD is described at $\alpha (\log 2)^{1/\beta} + \tau$.
Consider $T$ speciﬁes the random variable based on the
WD parameters [12]. The transformed variables
$T' = (T - \tau)/\alpha$ are measured as a standard form. The density
function is expressed as follows:
\begin{align}
f(t) &= \beta \alpha x^{\alpha-1} \exp\left[-x^\beta\right], x > 0, \beta > 0. \tag{11}
\end{align}

The moment value of $T$ is expressed as follows:
\begin{align}
\mu_r^T = E\left(T'^r\right) = \Gamma \left(\frac{r + 1}{\beta}\right). \tag{12}
\end{align}

\begin{align}
Var(T) = \Gamma \left(\frac{2}{\beta} + 1\right) - \left[\Gamma \left(\frac{1}{\beta} + 1\right)\right]^2. \tag{13}
\end{align}

4. Entropy

It is an uncertainty measure that helps in various applications
like physics, statistical measure, hydrology, and en-
geering [13]. Generally, entropy facilitates us to provide
proper statements and perform certain evaluations and
factors related to pressing issues. It is generally a measure of
uncertainty. Here, two diverse entropies are depicted as
follows:
\begin{align}
I_r(y) &= \frac{1}{1-y} \log \left(\int g^y(x) \, dx\right), y \neq 0, 1. \tag{14}
\end{align}

Here, substitute the PDF of the WD in

\begin{align}
I_r(y) &= \frac{1}{1-y} \left[ y \log \left(\alpha \log \frac{\alpha}{\alpha + 1}\right) + (y-1) \log \frac{1}{\lambda} - \frac{1}{\lambda} \log \beta + \log \left(\sum_{k=0}^{\infty} \frac{y(1-1/\lambda + 1/\gamma)}{k! (y+k)^{(1-1/\lambda + 1/\gamma)}}\right)\right]. \tag{15}
\end{align}

\begin{align}
H_x = -E_x [\log f(x)]. \tag{16}
\end{align}

With PDF, the expression attained is
\[
H_s = 1 - \frac{\alpha \log \alpha}{\alpha - 1} - \log \left( \frac{\beta \log \alpha}{\alpha - 1} \right) + \frac{\alpha}{\alpha - 1} \sum_{k=1}^{\infty} \left( -\log \frac{\alpha}{\alpha - 1} \right)^k + 1 \left( \frac{1}{1 + k} - \frac{\lambda - 1}{\lambda} \right) (\emptyset(1) - \log \beta - \log(1 + k)) \right).
\]

(17)

When the distribution with provided with maximal entropy distribution when the mean and standard deviation is known for the provided real-time dataset, this makes the sense that the people use the distribution often as it is easier to evaluate the mean and SD of any provided dataset.

### 5. Maximal Likelihood Evaluator

This section attains the maximal likelihood estimator of various parameter distributions. Consider \( \mathbf{x} = \{x_1, x_2, \ldots, x_n\} \) is a random sample with size \( n \) from the provided distribution with PDF [14]. The likelihood function is expressed as

\[
l(\theta|\mathbf{x}, c) = \prod_{i=1}^{n} f_X(x_i) = \left( \frac{\log \alpha}{\alpha - 1} \right)^n (\lambda\beta)^n e^{-\beta \sum_{i=1}^{n} x_i^\lambda} \sum_{i=1}^{n} (1 - e^{-\beta x_i^\lambda}) \prod_{i=1}^{n} x_i^{\lambda-1}.
\]

(18)

The log-likelihood function is expressed as

\[
\log \ell = n \log \left( \frac{\log \alpha}{\alpha - 1} \right) + n \log \beta + n \log \lambda + (\lambda - 1) \sum_{i=1}^{n} \log x_i - \beta \sum_{i=1}^{n} x_i^\lambda + \log \alpha \sum_{i=1}^{n} (1 - e^{-\beta x_i^\lambda}).
\]

(19)

The log-likelihood estimator equations are expressed as

\[
\frac{\delta \log l}{\delta \alpha} = \frac{n}{\alpha} \log \alpha - \frac{n}{\alpha - 1} + \frac{\sum_{i=1}^{n} (1 - e^{-\beta x_i^\lambda})}{\alpha} = 0,
\]

(20)

\[
\frac{\delta \log l}{\delta \beta} = \frac{n}{\beta} - \sum_{i=1}^{n} x_i^\lambda + \log \alpha \sum_{i=1}^{n} x_i^\lambda e^{-\beta x_i^\lambda} = 0,
\]

(21)

\[
\frac{\delta \log l}{\delta \lambda} = \frac{n}{\lambda} + \sum_{i=1}^{n} \log(x_i) - \beta \sum_{i=1}^{n} x_i^\lambda \log x_i + \beta \log \alpha \sum_{i=1}^{n} x_i^\lambda \log x_i e^{-\beta x_i^\lambda} = 0,
\]

(22)

where the maximal likelihood of \( \alpha, \beta, \lambda \) is specified by \( \hat{\alpha}, \hat{\beta}, \) and \( \hat{\lambda} \), respectively, and attained by resolving the previous equations.
6. Applications of Maximal Likelihood Distribution

This section provides three diverse parameters for maximal likelihood distribution towards the real-time data for illustration purposes. It shows the feasibility of the new distribution while modelling the positive data [15]. The empirical hazard function of the provided dataset relies on the total time on the test. The distribution and the scaled transform are expressed as

\[
H^{-1}(u) = \int_0^{F^{-1}(u)} S(y) \, dy, \quad 0 < u < 1 \text{ is expressed as } g(u) = H^{-1}\left(\frac{u}{H}\right). \quad (23)
\]

The empirical version of total time on the test is expressed as

\[
g_n\left(\frac{r}{n}\right) = H_n^{-1}\left(\frac{r}{n}\right) = \left[ \frac{\sum_{i=1}^{r} x_{i:n} - (n - r) x_{r:n}}{\sum_{i=1}^{n} x_{i:n}} \right]. \quad (24)
\]

where \( r = 1, 2, \ldots, n \) and \( x_{i:n}, i = 1, 2, \ldots, n \), specifies the order statistics of various samples. It is convex, and the rate is increasing or decreasing. The maximal WD is considered the best choice for the provided datasets [16]. However, the distribution fits the prevailing parameters of Weibull distributions. The following are the corresponding distributions of Weibull and expressed as in equations (25) to (29).

Generally, the WD model is expressed as

\[
F(x) = 1 - \exp(-\beta x^\lambda); \quad \beta, \lambda > 0. \quad (25)
\]

The modified maximal WD model is expressed as

\[
F(x) = 1 - \exp(-\beta x^\lambda e^{\alpha x}); \quad \beta, \lambda, \alpha > 0. \quad (26)
\]

The modified WD model based on CDF is expressed as

\[
\% \ F(x) = 1 - \exp(-\beta x^\lambda - \alpha x), \quad \beta > 0, \lambda > 0, \alpha \geq 0 \text{ with } \beta + \alpha > 0. \quad (27)
\]

The exponential WD model based on CDF is expressed as

\[
F(x) = 1 - \exp(-\beta x^\lambda); \quad \beta > 0, \lambda > 0, \alpha \geq 0 \text{ with } \beta + \alpha > 0. \quad (28)
\]

The extended generalized gamma distribution is expressed as

\[
f(x) = \frac{\lambda^\alpha \beta^\lambda}{\Gamma(\alpha)} x^{\lambda \alpha - 1} \exp(-\alpha (\beta x)^\lambda); \quad \beta, \lambda > 0, \alpha > 0. \quad (29)
\]

Here, the formal fitness test is performed to verify the distribution, superior to the real-time dataset. However, it is not fulfilled that the anticipated model offers a superior fit to the other approaches. However, it can offer a better fit [17]. Thus, the likelihood distribution model is utilized as a substitute for the existing well-known distribution such as modified, exponential, and gamma [18]. Here, an online available UCI Machine Learning repository for disease prediction and emotion recognition datasets is considered [19, 20]. The dataset is taken from the online resource known as the Kaggle dataset for disease prediction. The standard error and \( p \) value computations are expressed in Tables 1 and 2. Figures 1 and 2 show the standard error comparison for datasets 1 and 2.

7. Weighted Least Square Analysis

Various estimators recommend evaluating the beta parameter distribution. Here, a regression-based estimator of certain unknown parameters is derived. It is used for other distributions [21, 22]. Assume that \( X_1, X_2, \ldots, X_n \) is a random sample with \( n \) size from the CDF with function \( F(.) \) and \( X_{(i)}, i = 1, 2, \ldots, n \) specify the sample order in ascending order. The sample size \( n \) is expressed as

\[
E[F(X_{(i)})] = \frac{j}{n + 1}. \quad (30)
\]

\[
\var[F(X_{(i)})] = \frac{j(n - j + 1)}{(n + 1)^2 (n + 2)}. \quad (31)
\]

With the variance and expectation, two diverse variants of the least squares methods are given as follows.

7.1. Least Square Estimator. The minimal estimator is attained as follows:

\[
\sum_{j=1}^{n} \left( F(X_{(j)}) - \frac{j}{n+1} \right)^2. \quad (32)
\]

It is provided based on unknown parameters [23]. Thus, Weibull distribution with least square estimators of \( \lambda \) and \( \lambda_{ls} \) and attained with minimized as

\[
\sum_{j=1}^{n} \left( 1 - \exp(-\alpha (\beta x)^\lambda) - \frac{j}{n+1} \right)^2. \quad (33)
\]
To attain the CDF and PDF, we use the method for ML estimator. Thus,

\[ \tilde{f}_{ls}(x) = \frac{\alpha}{f_{ls}} x^{\alpha-1} e^{-x/\lambda_{ls}}, \]  

(34)

\[ \tilde{F}_{ls}(x) = 1 - e^{-x/\lambda_{ls}}. \]  

(35)

It is complex to predict the expectation and error rate of the estimators with mathematical modelling.

7.2. Weight-Based Least Square Estimators. The minimal weighted least square estimator is expressed as
Based on all these analyses and unknown parameter estimation, \( w_j = 1/\text{Var}(F(X_{(j)})) = (n+1)^2(n+2)/j(n-j+1) \). Thus, Weibull distribution for \( \lambda \) weighted square estimator is attained based on

\[
\sum_{j=1}^{n} w_j \left( F(X_{(j)}) - \frac{j}{n+1} \right)^2. \tag{36}
\]

With the weighted least estimator with CDF and PDF, the estimator is expressed as

\[
\tilde{F}_{\text{we}}(x) = \frac{\lambda \alpha}{\lambda} x^{\alpha-1} e^{-x^{\alpha}/\lambda_{\alpha}}, \tag{38}
\]

\[
F_{\text{we}}(x) = 1 - e^{-x^{\alpha}/\lambda_{\alpha}}. \tag{39}
\]

It is difficult to predict these weighted estimators’ error rates and expectations with the mathematical modelling in Table 3 and also the weighted estimation comparisons are depicted in Figure 3.

8. Failure Data Analysis

There are two diverse kinds of mathematical modelling. The data failure is categorized into two diverse types: incomplete and complete. Concerning complete data, the actual data values realized are well-known for every observation, while in the case of incomplete data, the actual values are not known for certain observations [24]. The negative entropy value represents that something turns to be less disordered and huge energy is used. It may not occur spontaneously, and it reduces the randomness:

(1) Theory-based modelling relies on the well-established component failures, also called physics- or white-box-based models

(2) Empirical modelling: the available data relies on the model construction. It is also known as a black-box and data-dependent model

With empirical modelling, the mathematical formulation is provided based on the preliminary analysis of available data. When the data analysis specifies a higher degree of variability, the model needs to capture the variability. It needs a stochastic and probabilistic way to express the provided dataset. The black-box modelling includes three diverse steps: (1) model assortment; (2) parameter evaluation; and (3) model validation.

To choose various models with the probable model, the investigators need to understand various properties. It is often known as a trial and error model. The goodness fitting test is the key process for choosing the statistical distribution of the observed data’s best fits. The larger amount of the Weibull distribution model is known from various literature. The selection of an appropriate model from the WD is based on the probability plots. It also provides the crude evaluation of model parameters. There are various statistical testing processes for model validation.

9. Hypothesis Testing Analysis:

**Goodness Measure**

There is another goodness fit-based model for the empirical distribution function, for instance, the Cramer von Mises test, KS test, chi-square test, and AD test. The goodness measure of the WD model is provided as \( H_0 \), and the population is well-suited for WD vs. \( H_1 \). There
are various extensions like modification and generalization adopted by the WD model. It leads to the rise of model features of the provided empirical dataset, which is not determined by the three parameters of the WD model [25]. The monotonic properties of the Weibull are not capable of capturing the dataset’s nature. It leads to failure rate as in Table 4.

Product reliability relies on the development, manufacturing, and design decisions made before the product launch. It influences the failures when the product is provided for launch. The prelaunching stage includes various phases. Some investigations indicate the target value of product reliability [26]. The product reliability is evaluated during the design phase based on the component reliability. Moreover, the enhancement is identified in the upper limit. When the target value is below the provided limit, the design with the components attains the desired target values. The program to enhance the reliability via the fix-test-cycle is done in the development phase [27]. The prototype is tested when the failure is noted, and the cause is examined. Based on these metrics, the design variations are done to handle the predicted failure causes. This process is continued until reliability is attained [28]. The item reliability during manufacturing intends to change from the manufacturing process. The quality control and proper process during the manufacturing phase and the variations are managed.

During the postlaunch stage, the item reliability is reduced due to the deterioration and outcomes from the utilization. The deterioration is influenced by various factors, including the operating condition, environment, and maintenance. The deterioration rate is managed via the prevention maintenance phase. The reliability outcomes with higher maintenance costs for the buyer. Also, it leads to higher warranty costs and outcomes from the rectifying cost within the warranty period. Some products are composed of various components due to the component’s failure. The WD model is used to deal with the failure of various components, and the literature is wider. During the development phase, it is essential to use acceleration testing to hasten the process of component failures. The enhancement in reliability during the development phase is modelled in various ways. The Weibull intensity model helps design the failure rate improvements as a development time function. The breakthroughs lead to enhancements as random points. During the manufacturing phase, the nonconforming fractions are lesser when the process is in-control, enhancing the process goes out of control. The likelihood-Weibull distribution is utilized to design the control duration in the control charts design to predict the

|   | $\alpha$ estimation | $\lambda$ estimation | Log-likelihood |
|---|-----------------|-----------------|----------------|
| MALE | 5.7            | 16.74           | -15.2          |
| PCE  | 5.4            | 14.28           | -15.4          |
| LSE  | 7.4            | 39.64           | -18.9          |
| WLSE | 7.016          | 32.139          | -17.3          |
failure data analysis is done with theoretical and empirical various metrics like standard errors from this dataset are explained with the analysis is done with dataset 1 and dataset 2. The proposed model can be used in various real-time applications. The maximal likelihood estimator are considered to measure the variance parameters. The parameter distribution and maximal likelihood estimation in a reliable environment, and it is a versatile distribution that considers the characteristics of other distributions based on the provided scale and shape parameters. The parameter distribution and maximal likelihood estimator are considered to measure the variance $\text{Var}(T)$. The uncertainty measure assists in statistical analysis with certain criteria. The maximal likelihood estimator is measured with the PDF and CDF values. The proposed model can be used in various real-time applications where the analysis is done with dataset 1 and dataset 2. The standard errors from this dataset are explained with various metrics like $p$ value. The weighted least square and failure data analysis is done with theoretical and empirical modelling, where the hypothesis for measuring the testing analysis is done with goodness measure. The model gives better outcomes than others, and some other distribution models will be examined in the future.

### Table 4: Failure rate measure.

| Failure Rate | Dataset 1 | Dataset 2 |
|-------------|-----------|-----------|
| 0.13        | 0.45      | 0.93      |
| 6.80        | 7.65      | 8.43      |
| 18.67       | 19.25     | 24.40     |
| 32.65       | 35        | 40.71     |
| 49.45       | 52.18     | 58.63     |
|             | 72.33     | 72.22     |

### Table 5: Advantages of various applications.

| References     | Advantages                                |
|----------------|-------------------------------------------|
| Navid et al. [21] | Yield steel strength and fatigue of steel life |
| Rana et al. [22]  | Fracture strength of glass                 |
| Shahbaz et al. [23] | Pitting pipe corrosion                      |
| Singh et al. [24]  | Adhesive wearable in metals                |
| Tahir et al. [25]  | Carbon-fibre composite failure             |
| Tomer et al. [26]  | Coating failures                           |
| Yousof et al. [28] | Brittle material failure                   |
| Zografos et al. [29] | Composite material failure                 |
| Yuri et al. [30]   | Concrete components                        |

### 10. Likelihood Maximal Weibull Distribution and Other Applications

To perform statistical data analysis, computer software is needed. It can be extensively utilized in various other applications to examine various issues. There are enormous works that provide nominal outcomes to measure the distribution rate. Additionally, statistical software like SPSS, MiniTab, SAS, etc., is considered, and spreadsheet-like Excel evaluates the success and failure rate. The advantage of likelihood maximal Weibull distribution is its ability to solve the problem related to density evaluation. It performs maximization of likelihood function to predict the probability distribution and parameters of the finest observed data.

### 11. Conclusion

This research extensively analyses the maximal likelihood-Weibull distribution model for data generalization. The Weibull distribution is widely adopted for lifetime distribution evaluation in a reliable environment, and it is a versatile distribution that considers the characteristics of other distributions based on the provided scale and shape parameters. The parameter distribution and maximal likelihood estimator are considered to measure the variance $\text{Var}(T)$. The uncertainty measure assists in statistical analysis with certain criteria. The maximal likelihood estimator is measured with the PDF and CDF values. The proposed model can be used in various real-time applications where the analysis is done with dataset 1 and dataset 2. The standard errors from this dataset are explained with various metrics like $p$ value. The weighted least square and failure data analysis is done with theoretical and empirical variations from in-to out-control. When the failure rate is identified, it is prone to earlier failure. For the product that exhibits a failure rate, burn-in is utilized to eliminate these failures and enhance the product reliability before the release for sale (see Table 5).

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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