Analysis of trajectory adjusting system using a magnetic lens for the superconducting IFE target

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Abstract. In order to minimize the deviation of the position of spherical superconducting inertial fusion energy (IFE) target from the laser shot point, a target trajectory adjusting system using a magnetic lens was analysed. The system consists of magnets or coils placed symmetrically along the line between the target injection point and the laser shot point. An approximation to the magnetic charge, which forms the magnetic lens, is obtained for general configuration. The magnitude of the magnetic charge limits allowable target displacement at the magnetic lens. Sensitivity analysis of the target deviation is reported to design an active adjusting system which compensates for the perturbation of the parameters of an individually injected target.

1. Introduction
In a laser fusion reactor, where the fuel target is delivered to the center of the reaction chamber, it is necessary to either adjust the laser shot point to the target position or to adjust the target position to the fixed laser shot point. Traditional design study takes the former approach and requires that the final mirrors are steered based on the measurement of the target position in flight and on the real-time calculation of the target trajectory. There is a technically difficult problem that more than a hundred final mirrors of large diameter (~1m) must be simultaneously controlled in a few milliseconds. It is clear that the latter approach can eliminate the need for beam steering.

Recently, the latter approach, the magnetostatic method [1] and the electrostatic method [2], were proposed to adjust the target position to the fixed laser shot point. Both methods make use of the force on the target in the magnetic or electric field.

In this paper, in order to develop a design study of the target trajectory adjusting system using a magnetic lens, an analysis of the system is reported. The principle of the system is presented. System operation in a gravitational field and allowable target displacement at the magnetic lens are discussed. Sensitivity analysis of the target deviation from the fixed laser shot point is carried out to design a dynamic trajectory adjusting system.

2. Principle of the trajectory adjusting system using a magnetic lens
The principle of the trajectory adjusting system consists of three points [1]. (1) A spherical fuel target is coated with Pb thicker than 0.04 µm and cooled below 7.2 K. The target becomes a spherical superconductor. (2) A symmetrical magnetic field, or magnetic lens, is formed in the section of the target injection path. (3) If the actual target trajectory is displaced from an ideal one, the target
receives a repulsion force, which is proportional to the displacement at the magnetic lens, due to the Meissner effect. Thus, the target is accurately delivered to the laser shot point as shown in figure 1.

3. Analyses and results

3.1. Magnetic charge of the magnetic lens

A symmetrical magnetic field, which works as a magnetic lens, is produced by magnets (magnetic charges) or electrical currents. Its quantity must be determined to exactly adjust the target trajectory. Using a model of the magnetic charge and an average force in the effective interaction region, we obtained an approximation to the magnetic charge $q$ in the case of the magnetic lens was placed at the center of the injection path [1].

Here, we extend our previous results to a general configuration. Let us suppose that the magnetic lens is composed of two equal magnetic point charges $q_A (=q)$ and $q_B (=q)$ placed at $A (0, -a)$ and $B (0, a)$ as shown in figure 2. The spherical fuel target is injected at (-L1, 0) with the velocity ($V_x$, $V_y$), and the laser shot point is set at ($L_2$, 0). Consider the total force $F_y (\text{total})$ exerted on the spherical superconducting target in the effective interaction region (-a/2 < x < a/2). The induced magnetic field occurs outside the target as if there were magnetic charges inside the target. These charges are composed of image point charges ($q_{A'}$ and $q_{B'}$) and image line charges ($\lambda_{A''}$ and $\lambda_{B''}$) as shown in figure 3 [3]. Calculating the force between the real magnetic charges ($q_A$ and $q_B$) and the image magnetic charges ($q_{A'}$, $q_{B'}$, $\lambda_{A''}$, $\lambda_{B''}$), we obtain the total force $F_y (\text{total})$ exerted on the target as

$$F_y (\text{total}) = F_y(AA') + F_y(AA'') + F_y(BB') + F_y(BB'') + F_y(AB') + F_y(AB'') + F_y(AB) + F_y(BA'),$$

(1)

where the subscript in parenthesis denotes a pair of interacting charges. If we set the target center $O (x, y)$ on the y-axis ($x=0$) and expand the total force $F_y (\text{total})$ in the Taylor series around $Y (= y/a)$ = 0, the force $F_y (\text{total})$ is represented as the series of odd powers of $Y$

$$F_y (\text{total}) = \frac{q^2 a r}{4\pi \mu_0}[C_1 Y + C_3 Y^3 + O(Y^5) + \cdots],$$

(2)

Figure 1. Gas gun type target injector and trajectory adjusting system using a magnetic lens.

Figure 2. Geometry of the analysis.

Figure 3. Image charges ($q_{A'}$, $q_{B'}$, $\lambda_{A''}$, $\lambda_{B''}$) induced by real charges ($q_A$, $q_B$).
where \( r \) is the radius of the target. If \( Y \) is small enough, then the force \( F_y(\text{total}) \) is proportional to the target displacement. And, this force causes lens action. Coefficients \( C_1 \) and \( C_3 \) at \( x = 0 \) are written as

\[
C_1 = \frac{-8a^2r^2(a^2 + 3r^2)}{(a^2 - r^2)^2} < 0, \quad C_3 = \frac{-32a^4r^2(2a^6 + 17a^4r^2 + 4a^2r^4 + 3r^6)}{(a^2 - r^2)^2} < 0.
\]

(3)

Since the assumption \( r < a \), the coefficients are negative. Assuming that the force \( F_y(\text{total}) \) at \( x = 0 \) equals the average force \( <F_y> \) exerted on the target in the effective interaction region and calculating the momentum transfer in this region, we obtain the magnetic charge

\[
q = \pm \left(\frac{m\pi\mu_0}{2a^3r^3(a^2 + 3r^2)} \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\right)^{1/2} V_x, \quad (4)
\]

where \( m \) is the mass of the target. In the case of \( L_1 = 4 \text{ m}, L_2 = 6 \text{ m}, V_x = 100 \text{ m/s}, V_y = 0.01 \text{ m/s}, a = 10 \text{ mm}, m = 5 \text{ mg} \) and \( r = 2 \text{ mm} \), the magnetic charge \( q (= 2.09571 \times 10^{-5} \text{ Wb}) \) obtained from equation (4) agrees well with the magnetic charge \( q_{\text{sim}} (= 2.06120 \times 10^{-5} \text{ Wb}) \) obtained from the numerical target trajectory simulation.

Additionally, applying the method described above to the case that the magnetic lens is composed of two opposite magnetic point charges \( q_A (= -q) \) and \( q_B (= q) \), we obtain the result that the target trajectory is adjusted and converges at the laser shot point.

### 3.2. System operation in a gravitational field

The trajectory adjusting system using a magnetic lens can be applied in a gravitational field by shifting the injection point and the laser shot point.

Let us suppose that the target is injected at \((-5, 0.01226)\) and the laser shot point is \((5, -0.03678)\) as shown in figure 4. If the velocity \( V_y = 0 \text{ m/s} \), the target displacement on the y-axis is zero. Thus, we obtain the same result of zero gravity for the target deviation \( \Delta y \) at the laser shot point. If the velocity \( V_y \) changes in the range of \( 0 \sim 0.02 \text{ m/s} \), the target deviation \( \Delta y \) is smaller than \( 43.6 \mu \text{m} \) for \( a = \pm 0.01 \text{ m} \) and \( q = 1.97104 \times 10^{-5} \text{ Wb} \), and the target deviation \( \Delta y \) is smaller than \( 10.4 \mu \text{m} \) for \( a = \pm 0.02 \text{ m} \) and \( q = 1.19168 \times 10^{-4} \text{ Wb} \). If there is no magnetic lens, the target deviation \( \Delta y = 1000 \mu \text{m} \) is caused by the velocity \( V_y = 0.01 \text{ m/s} \).

![Figure 4. Target trajectory in a gravitational field.](image)

**Figure 4.** Target trajectory in a gravitational field. \( g = 9.80665 \text{ m/s}^2 \), \( m = 5 \text{ mg} \), \( r = 2 \text{ mm} \), \( V_x = 100 \text{ m/s} \) and \( L_1 = 5 \text{ m} \). Solid line : \( V_y > 0 \text{ m/s} \). Broken line : \( V_y = 0 \text{ m/s} \).

### 3.3. Allowable target displacement at the magnetic lens

If the magnetic field intensity on the target exceeds a critical value, the superconducting phenomenon disappears. For \( \text{Pb} \), the critical magnetic field intensity is estimated to be \( 0.0532 \text{ T} \) for \( 4.2 \text{ K} \) by the parabolic law [4]. Not to exceed the critical field intensity, we need to know allowable target displacement \( y_{\text{max}} \) at \( x = 0 \), because the magnetic field intensity on the target surface has a maximum value when the target crosses the y-axis (\( x = 0 \)).

Let us suppose that the target is placed at \((0, y)\) between two magnetic point charges \( q_A (0, -0.02) \) and \( q_B (0, 0.02) \). These charges are set at \( q = 1.19773 \times 10^{-4} \text{ Wb} \) to cancel the deviation at the laser shot point in the case of \( L_1 = L_2 = 5 \text{ m} \), \( V_x = 100 \text{ m/s} \), \( V_y = 0.01 \text{ m/s} \), \( m = 5 \text{ mg} \) and \( r = 2 \text{ mm} \).
As a result, allowable target displacement $y_{\text{max}}$ on the y-axis is 5 mm for target temperature 4.2 K. From equation (4), the increase of $V_x$ causes the increase of $q$, and it also causes the decrease or disappearance of allowable target displacement at the magnetic lens. In this case, by dividing a single stage magnetic lens into multiple stages, we can decrease the maximum field intensity on the target.

### 3.4. Sensitivity analysis for design parameters

The deviation $\Delta y$ of the target position at the laser shot point is a function of design parameters, such as the radius $r$ and the magnetic charge $q$. To design a practical target trajectory adjusting system, a sensitivity analysis for design parameters is carried out by differentiating $\Delta y$ as

$$
\Delta y = \frac{\partial (\Delta y)}{\partial r} \Delta r + \frac{\partial (\Delta y)}{\partial m} \Delta m + \frac{\partial (\Delta y)}{\partial V_x} \Delta V_x + \frac{\partial (\Delta y)}{\partial V_y} \Delta V_y + \frac{\partial (\Delta y)}{\partial a} \Delta a + \frac{\partial (\Delta y)}{\partial q} \Delta q + \frac{\partial (\Delta y)}{\partial L_1} \Delta L_1 .
$$

(5)

Here, $a$ is the distance from the center of the magnetic lens to the magnetic charge, and $L_1$ ($L_1 + L_2 = \text{const.} = 10$ m) is the distance to the injection point.

As shown in figure 5, the target deviation $\Delta y$ at the laser shot point calculated by equation (5) agrees well with those by the numerical simulation of the target trajectory. Using equation (5), we can compensate for the perturbation of the parameters of an individually injected target ($\Delta r$, $\Delta m$, $\Delta V_x$) by adjusting the parameters of the magnetic lens ($\Delta a$, $\Delta q$). This enables us to design a dynamic trajectory adjusting system using a magnetic lens.

![Figure 5. Deviation at the laser shot point for the error of the parameters. $r = 2$ mm, $m = 5$ mg, $V_x = 100$ m/s, $V_y = 0.01$ m/s, $a = \pm 20$ mm, $q = 1.19773 \times 10^{-4}$ Wb and $L_1 = 5$ m. Lines: Results from equation (5). Points: Results of numerical simulation.](image)

### 4. Summary

The principle of the target trajectory adjusting system using a magnetic lens was presented. An approximation to the magnetic charge $q$, which forms a magnetic lens, was obtained for general configuration. The trajectory adjusting system can be applied in a gravitational field by shifting the target injection point and the laser shot point. The magnitude of the magnetic charge limits allowable target displacement at the magnetic lens. Sensitivity analysis of the target deviation was carried out for the perturbation of the parameters of individually injected target. By adjusting the magnetic lens parameters, we can compensate for the perturbation of the parameters of individually injected target.

### References

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