Large CP violation is an interesting phenomenon both theoretically and experimentally. Last year, LHCb Collaboration found in some three-body decays of bottom mesons that large CP violations appear in regions of the Dalitz plots that are not dominated by contributions from narrow resonances. In this paper, we present a mechanism which can induce such kind of large CP violations. In this mechanism, large localized CP asymmetries in phase space can be induced by the interference of two intermediate resonances with different spins. We also apply this mechanism to the decay channel $B^\pm \to K^\pm \pi^+ \pi^-$. 

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1. INTRODUCTION

Charge-Parity (CP) violation is one of the most fundamental and important properties of weak interactions. It was first discovered in $K^0 - \bar{K}^0$ systems in 1964 [1]. In Standard Model (SM), CP violation is originated from the weak phase in Cabibbo-Kobayashi-Maskawa (CKM) matrix which describes the mixing of different generations of quarks [2, 3]. Besides the weak phase, in order to have a CP asymmetry that is large enough to detect, a large strong phase is needed. Usually, this large phase is provided by QCD loop corrections.

It was suggested long time ago that large CP violation should be observed in $B$ meson systems [4, 5]. Last year, LHCb Collaboration found clear evidence for CP violation in some three-body decay channels of $B$ mesons [6–8]. Intriguingly, large direct CP asymmetries were found in some localized phase spaces of the decay channel $B^\pm \to \pi^\pm \pi^+ \pi^-$, which dose not clearly correspond to any resonance [7, 8]. The observed large localized CP asymmetry lies in the region $m^2_{\pi^+ \pi^- \text{ low}} < 0.4 \text{ GeV}^2$ and $m^2_{\pi^+ \pi^- \text{ high}} > 15 \text{ GeV}^2$, and takes the value:

$$A_{CP} = +0.622 \pm 0.075 \pm 0.032 \pm 0.007,$$

while in the region $m^2_{\pi^+ \pi^- \text{ low}} < 0.4 \text{ GeV}^2$ and $m^2_{\pi^+ \pi^- \text{ high}} < 15 \text{ GeV}^2$, no large CP asymmetry is observed.

In our previous paper [9], we proposed a mechanism which can generate large localized CP asymmetries in phase space of three-body decay by the interference of two intermediate resonances with different spins. With this mechanism, we showed that the large CP asymmetry difference between the aforementioned two regions can be interpreted as the interference of amplitudes which correspond to two intermediate resonances, $\rho^0(770)$ and $f_0(500)$, respectively.

In fact, similar CP asymmetry behavior was also observed in $B^\pm \to K^\pm \pi^+ \pi^-$. When the invariant mass of the $\pi^+ \pi^-$ pair is around the vicinity of $f_0(500)$, a CP asymmetry larger than about 30% was observed for smaller invariant mass of the $K^\mp \pi^\pm$ system, while a CP asymmetry that is slightly smaller than 0 (about 0 to -10%) was observed for larger invariant mass of the $K^\mp \pi^\pm$ system. In this paper, we will first give a more general analysis of the aforementioned mechanism, and then apply it to the channel $B^\pm \to K^\pm \pi^+ \pi^-$.  

\footnote{For the decay channel $B^- \to \pi^- \pi^+ \pi^-$, there are two identical pions with negative charge. When combining the momentum of each $\pi^-$ meson with that of the $\pi^+$ meson, we will have two Lorentz invariant masses squared which are usually different in values and are denoted as $m^2_{\pi^+ \pi^- \text{ low}}$ and $m^2_{\pi^+ \pi^- \text{ high}}$ in Ref. [8], respectively.}
The remainder of this paper is organized as follows. In Sec. II we first present a detailed analysis of the aforementioned mechanism which can be generate large localized $CP$ asymmetries in three-body decays of bottom mesons. In Sec. III we apply the mechanism to the decay channel $B^\pm \to K^\pm \pi^+ \pi^-$. In Sec. IV we present our conclusions.

II. GENERAL CONSIDERATION ON THE INTERFERENCE OF TWO NEARBY RESONANCES WITH DIFFERENT SPINS

For a cascade decay process, $B \to XM_3$, $X \to M_1M_2$, with all the initial and final particles being spin-0 ones, the transition amplitude is proportional to $P_{J_X} (g_{s_{12}}(s_{13}))$ [10], where $P_{J_X}$ is the $(J_X + 1)$-th Legendre Polynomial, $s_{ij} (i, j = 1, 2, 3)$ is the invariant mass squared of $M_i$ and $M_j$, $J_X$ is the spin of $X$, and

$$g_{s_{12}}(s_{13}) = \frac{\hat{s}_{13} - s_{13}}{\Delta_{13}},$$

with $\hat{s}_{13} = (s_{13,\text{max}} + s_{13,\text{min}})/2$, $\Delta_{13} = (s_{13,\text{max}} - s_{13,\text{min}})/2$, and $s_{13,\text{max(min)}}$ being the maximum (minimum) value of $s_{13}$ for fixed $s_{12}$.

Inspired by the above statement, we can expand the transition amplitude of the decay process, $B \to M_1M_2M_3$, in terms of Legendre polynomials for fixed $s_{12}$:

$$M(s_{12}, s_{13}) = \sum_l a_l P_l (g_{s_{12}}(s_{13})).$$

Note that $a_l$, $\Delta_{13}$, and $\hat{s}_{13}$ may depend on $s_{12}$, but all of them are independent of $s_{13}$. For certain value of $s_{12}$ (denoted by $\bar{s}_{12}$) when $a_J$ is much larger than other $a_l$’s, the transition amplitude $M$ will be dominated by the $(J + 1)$-th Legendre Polynomial:

$$M(\bar{s}_{12}, s_{13}) \approx a_J P_J (g_{\bar{s}_{12}}(s_{13})).$$

One would observe a spin-$J$ resonance lying around $s_{12} = \bar{s}_{12}$, which is in fact responsible for the aforementioned cascade decay.

Another interesting situation arises when two different Legendre Polynomials with $l = J_1$ and $l = J_2$ are dominant for fixed $s_{12} = \bar{s}_{12}$. The decay amplitude $M$ will take the form

$$M(\bar{s}_{12}, s_{13}) \approx a_{J_1} P_{J_1} (g_{\bar{s}_{12}}(s_{13})) + a_{J_2} P_{J_2} (g_{\bar{s}_{12}}(s_{13})).$$

If this decay process is a weak one, $a_l$’s may take a general form

$$a_l = \left[ T_l + P_l e^{i(\alpha_l + \phi)} \right] e^{i\delta_l},$$
where $\phi$ is the weak phase, while $\delta_i$ and $\alpha_i$ are strong phases, $T_i$ and $P_i$ represent tree and penguin amplitudes, respectively. The strong phases $\delta_i$ and $\alpha_i$ can be properly chosen so that both $T_i$ and $P_i$ are real. The differential CP violation parameter, which is defined as

$$A_{CP} = \frac{|M|^2 - |\overline{M}|^2}{|M|^2 + |\overline{M}|^2};$$

(7)
can then be expressed as $A_{CP} = D/F$, where

$$D = -2 \sin \phi \left\{ P_{j_1} P_{j_2} T_{j_1} P_{j_2} \sin(\delta_{j_2} - \delta_{j_1} + \alpha_{j_2}) + P_{j_1}^2 T_{j_1} P_{j_1} \sin \alpha_{j_1} \right\} + \{J_1 \leftrightarrow J_2\},$$

(8)

$$F = \left\{ P_{j_1}^2 (T_{j_1}^2 + P_{j_1}^2) + P_{j_1} P_{j_2} \left[ T_{j_1} T_{j_2} \cos(\delta_{j_1} - \delta_{j_2}) + P_{j_1} P_{j_2} \cos(\delta_{j_1} - \delta_{j_2} + \alpha_{j_1} - \alpha_{j_2}) \right] \right. \left. + 2 \cos \phi \left[ P_{j_1} P_{j_2} T_{j_1} P_{j_2} \cos(\delta_{j_2} - \delta_{j_1} + \alpha_{j_2}) + P_{j_1}^2 T_{j_1} P_{j_1} \cos \alpha_{j_1} \right] \right\} + \{J_1 \leftrightarrow J_2\},$$

(9)

with $P_{j_i} (i = 1, 2)$ being the abbreviation for $P_{j_i}(g_{s_{13}}(s_{13}))$. One can see that the CP asymmetry depends on $s_{13}$ through $P_{j_1}$ and $P_{j_2}$. This is a very interesting behavior. On the other hand, no $s_{13}$-dependence of the CP asymmetry appears if only one Legendre Polynomial dominates because the common factor $P_{j_1}^2$ will be cancelled between $D$ and $F$. When $\alpha_1$ and $\alpha_2$ equal zero, only one strong phase $\delta \equiv (\delta_{j_1} - \delta_{j_2})$ contributes to CP violation, and $D$ and $F$ reduce to

$$D = -2 \sin \phi \left\{ P_{j_1} P_{j_2} T_{j_1} P_{j_2} \sin(\delta_{j_2} - \delta_{j_1}) \right\} + \{J_1 \leftrightarrow J_2\},$$

(10)

$$F = \left\{ P_{j_1}^2 (T_{j_1}^2 + P_{j_1}^2) + P_{j_1} P_{j_2} \left[ T_{j_1} T_{j_2} + P_{j_1} P_{j_2} + 2 T_{j_1} P_{j_2} \cos \phi \right] \cos(\delta_{j_1} - \delta_{j_2}) \right\} + \{J_1 \leftrightarrow J_2\}.$$

(11)

In the following of this section, we will focus on the situation when $J_1 = 0$ and $J_2 = 1$. Since the zero point for $P_1(g_{s_{13}}(s_{13}))$ lies at $s_{13} = \hat{s}_{13}$, this allows us to divide the allowed region of $s_{13}$ into two parts: $\Omega$ and $\bar{\Omega}$, where in $\Omega$ $s_{13} > \hat{s}_{13}$ and in $\bar{\Omega}$ $s_{13} < \hat{s}_{13}$. The CP asymmetries in the regions $\Omega$ and $\bar{\Omega}$, after integration over $s_{13}$, are found to be

$$A_{\Omega}^{\Omega} = \frac{\hat{S}_{\Omega}^\Omega + \hat{A}_{\Omega}^\Omega}{S_{\Omega}^\Omega + A_{\Omega}^\Omega}, \quad A_{\bar{\Omega}}^{\Omega} = \frac{\hat{S}_{\bar{\Omega}}^\Omega + \hat{A}_{\bar{\Omega}}^\Omega}{S_{\bar{\Omega}}^\Omega + A_{\bar{\Omega}}^\Omega},$$

(12)
where

\[ \hat{S}_\Omega^- = -2 \sin \phi \left[ T_0 P_0 \sin \alpha_0 + \frac{1}{3} T_1 P_1 \sin \alpha_1 \right], \quad (13) \]

\[ \hat{S}_\Omega^+ = \left[ T_0^2 + P_0^2 + 2 T_0 P_0 \cos \alpha_0 \cos \phi + \frac{1}{3} \left( T_1^2 + P_1^2 + 2 T_1 P_1 \cos \alpha_1 \cos \phi \right) \right], \quad (14) \]

\[ \hat{A}_\Omega^- = \sin \phi \left[ T_0 P_1 \sin(\alpha_1 + \delta_1 - \delta_0) + T_1 P_0 \sin(\alpha_0 + \delta_0 - \delta_1) \right], \quad (15) \]

\[ \hat{A}_\Omega^+ = -\left\{ T_0 T_1 \cos(\delta_0 - \delta_1) + P_0 P_1 \cos(\alpha_0 - \alpha_1 + \delta_0 - \delta_1) \right. \]

\[ + \cos \phi \left[ T_0 P_1 \cos(\alpha_1 + \delta_1 - \delta_0) + T_1 P_0 \cos(\alpha_0 + \delta_0 - \delta_1) \right] \}. \quad (16) \]

From the above expressions, one can check that under the interchange of \( \Omega \) and \( \bar{\Omega} \), \( \hat{S}_\pm \) are symmetric while \( \hat{A}_\pm \) are antisymmetric, i.e.,

\[ \hat{S}_\pm = \hat{S}_\pm, \quad \hat{A}_\pm = -\hat{A}_\pm. \quad (17) \]

Because of the presence of the antisymmetric terms, \( CP \) asymmetries in the two regions can be very different.

In practice, the two regions \( \Omega \) and \( \bar{\Omega} \) are not defined for fixed \( s_{12} \). Instead, \( s_{12} \) lies in a small interval where both of the two resonances are dominant, for example, \( \tilde{s}_{12} - \lambda_1 < s_{12} < \tilde{s}_{12} + \lambda_2 \) (\( \lambda_1 \) and \( \lambda_2 \) are small). Then the localized \( CP \) asymmetry in the region \( \omega \) (\( \omega = \Omega, \bar{\Omega} \)) takes the form

\[ A_{CP}^{\omega} = \frac{\int_{\tilde{s}_{12} - \lambda_1}^{\tilde{s}_{12} + \lambda_2} ds_{12} (\hat{S}_\omega + \hat{A}_\omega)}{\int_{\tilde{s}_{12} - \lambda_1}^{\tilde{s}_{12} + \lambda_2} ds_{12} (\hat{S}_\omega + \hat{A}_\omega)}, \quad (18) \]

which is exactly the case in Ref. [9].

Besides the \( CP \) asymmetry, other quantities may also have interesting behaviors. For example, one can check that the quality \( R_+ \), which is defined as

\[ R_+ = \frac{\int_{\Omega} ds_{13} (|M|^2 + |\overline{M}|^2) - \int_{\bar{\Omega}} ds_{13} (|M|^2 + |\overline{M}|^2)}{\int_{\Omega} ds_{13} (|M|^2 + |\overline{M}|^2) + \int_{\bar{\Omega}} ds_{13} (|M|^2 + |\overline{M}|^2)} \]

equals to \( A_{+}^{\Omega}/S_{+}^{\Omega} \) and is nonzero. Even if the decay process \( B \to M_1 M_2 M_3 \) is not a weak one, the interference of spin-0 and spin-1 resonances also leads to interesting phenomenology. The quantity \( R_+ \) is again nonzero. The nonzero value of \( R_+ \) is originated from the interference of the spin-0 and spin-1 resonances. If one (no matter which one) of the resonances dominates over the other one, \( R_+ \) will equal to zero.
III. APPLICATION TO $B^\pm \to K^{\pm}\pi^+\pi^-$

In this section, we will apply the mechanism which was considered in last section to the decay $B^\pm \to K^{\pm}\pi^+\pi^-$. We will show that the interference of the two resonances, $f_0(500)$ and $\rho^0(770)$, which are spin-0 and spin-1, respectively, can lead to large localized $CP$ asymmetry difference around the vicinity of $f_0(500)$ in the phase space. The corresponding effective Hamiltonian can be expressed as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{uq}^*(C_1 O_1^u + C_2 O_2^u) + V_{cb} V_{cq}^*(C_1 O_1^c + C_2 O_2^c) - V_{tb} V_{tg}^{*10} \sum_{i=3}^{10} C_i O_i \right] + \text{h.c.}, \quad (20)$$

where $G_F$ is the Fermi constant, $V_{qq'}$ is the CKM matrix element, $C_i(\mu)$ ($i = 1, \ldots, 10$) are the Wilson coefficients, $O_i(\mu)$ are the operators from Operator Product Expansion, $\mu$ is the typical energy scale for the decay process. The local four quark operators $O_i$ can be written as

$$O_1^q = \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) q_\beta \gamma_\mu (1 - \gamma_5) b_\alpha, \quad O_2^q = \bar{q}_\beta \gamma_\mu (1 - \gamma_5) q_\alpha \gamma_\mu (1 - \gamma_5) b_\beta,$$

$$O_3 = \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q'}_\beta \gamma_\mu (1 - \gamma_5) q', \quad O_4 = \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q'}_\beta \gamma_\mu (1 - \gamma_5) q_\alpha,$$

$$O_5 = \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q'}_\beta \gamma_\mu (1 + \gamma_5) q', \quad O_6 = \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q'}_\beta \gamma_\mu (1 + \gamma_5) q_\alpha,$$

$$O_7 = \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q'}_\beta \gamma_\mu (1 + \gamma_5) q', \quad O_8 = \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q'}_\beta \gamma_\mu (1 + \gamma_5) q_\alpha,$$

$$O_9 = \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q'}_\beta \gamma_\mu (1 - \gamma_5) q', \quad O_{10} = \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q'}_\beta \gamma_\mu (1 - \gamma_5) q_\alpha,$$

where $\alpha$ and $\beta$ represent color indices, $e_{q'}$ is the charge of the quark $q'$ in unit of the absolute electron charge. With the effective Hamiltonian at hand, we can derive the matrix element for $B^- \to \rho^0\pi^-$ and $B^- \to f_0(500)\pi^-$ via the factorization approach.

We also need the effective Hamiltonian for $\rho^0 \to \pi^+\pi^-$ and $f_0(500) \to \pi^+\pi^-$, which can be formally expressed as

$$\mathcal{H}_{\rho^0\pi\pi} = -i g_{\rho\pi\pi} \rho^0_{\mu} \rho^0_{\nu} \gamma_\mu \gamma_\nu \partial_\pi^- \to \pi^-, \quad (21)$$

$$\mathcal{H}_{f_0\pi\pi} = g_{f_0\pi\pi} f_0(2\pi^+\pi^- + \rho^0\pi^0), \quad (22)$$

where $\rho^0_{\mu}$, $f_0$ and $\pi^\pm$ are the field operators for $\rho^0$, $f_0(500)$ and $\pi$ mesons, $g_{\rho\pi\pi}$ and $g_{f_0\pi\pi}$ are the effective coupling constants, which should be in principle determined by the underlying
theory, i.e., QCD. The effective coupling constants can be expressed in terms of the decay constants:

\[ g_{\rho\pi\pi}^2 = \frac{48\pi}{(1 - \frac{4m_\rho^2}{m_\rho^2})^{3/2}} \cdot \frac{\Gamma_{\rho \to \pi^+\pi^-}}{m_\rho}, \tag{23} \]

\[ g_{f_0\pi\pi}^2 = \frac{4\pi m_{f_0} \Gamma_{f_0 \to \pi^+\pi^-}}{(1 - \frac{4m_\rho^2}{m_{f_0}^2})^{1/2}}. \tag{24} \]

Both \( f_0(500) \) and \( \rho^0(770) \) decay into one pion pair dominantly. One can easily check that \( \Gamma_{\rho^0} \simeq \frac{3}{2} \Gamma_{f_0 \to \pi^+\pi^-} \).

The vector meson \( \rho^0(770) \) are usually the dominant resonance for \( B \) meson decay channels with one \( \pi^+\pi^- \) pair in the final state, while \( f_0(500) \) is not. This makes both the two resonances, \( f_0(500) \) and \( \rho^0(770) \), are dominant when the invariant mass of the \( \pi^+\pi^- \) pair is around the mass of \( f_0(500) \). As a result, the decay amplitude for \( B^- \to K^-\pi^+\pi^- \) can be expressed as

\[ \mathcal{M}_{B^- \to K^-\pi^+\pi^-} = \mathcal{M}_{f_0} + \mathcal{M}_{\rho^0} e^{i\hat{\delta}}, \tag{25} \]

when the invariant mass of the \( \pi^+\pi^- \) pair is around the vicinity of \( f_0(500) \), where \( \mathcal{M}_{f_0(\rho^0)} \) is the transition amplitude for the cascade decay \( B^- \to K^-f_0(\rho^0), f_0(\rho^0) \to \pi^+\pi^- \), \( \hat{\delta} \) is the relative strong phase between \( \mathcal{M}_{\rho^0} \) and \( \mathcal{M}_{f_0} \).

With the effective Hamiltonians at hand, one can in principle calculate the transition amplitude via the QCD factorization approach \[12\] or perturbative QCD approach \[13\], etc. These approaches will generate complex phases in the effective Wilson coefficients. However, these strong phases usually result in a small net strong phase between the penguin and tree parts of the amplitude. Besides, since we are working in the vicinity of \( f_0(500) \), any factorization approach seems not to be accurate for \( B^\pm \to K^\pm \rho^0 \) when \( \rho^0 \) is off shell.

In view of this, we will use a naive factorization approach for both \( B^\pm \to K^\pm \rho^0 \) and \( B^\pm \to K^\pm f_0(500) \). As a result, the amplitudes take the form\footnote{Just as the case of \( B^\pm \to \pi^\pm \pi^+\pi^- \) in Ref. \[9\], for the decay \( B^\pm \to K^\pm \pi^+\pi^- \), there are also annihilation terms which are also chiral enhancement terms in the meantime. However, these terms are about four times smaller for \( B^\pm \to K^\pm \pi^+\pi^- \) than \( B^\pm \to \pi^\pm \pi^+\pi^- \). Because of this, we simply neglect these terms here.}
\[ M_{\rho} = \frac{2m_{\rho}g_{\rho\pi\pi}(s_{K^{-}}\pi^{-} - s_{K^{-}}\pi^{+})}{s - m_{\rho}^2 + i m_{\rho} \Gamma_{\rho}} \cdot \left\{ V_{ub}V_{us}^* \left[ \frac{1}{\sqrt{2}} a_1 f_{\rho} F_1 + a_2 f_{K} A_0 \right] ight\} \]

\[ = -V_{tb}V_{ts}^* \left[ \frac{3}{2\sqrt{2}} (a_7 + a_9) f_{\rho} F_1 + (a_4 + a_{10} - \frac{2(a_6 + a_8)m_{K}^2}{(m_s + m_u)(m_b + m_u)}) f_{K} A_0 \right] \right\}, \quad (26) \]

\[ M_{f_0} = \frac{2g_{f_0\pi\pi}}{s - m_{f_0}^2 + i m_{f_0} \Gamma_{f_0}} f_{\pi} m_B^2 F_{f_0}^{B\to f_0}(m_{K}^2) \]

\[ \cdot \left\{ V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \left[ (a_4 + a_{10} - \frac{2(a_6 + a_8)m_{K}^2}{(m_s + m_u)(m_b + m_u)}) \right] \right\}, \quad (27) \]

where \( F_1 \) and \( A_0 \) are short for the form factors \( F_1^{(B\to K)}(m_{\rho}^2) \) and \( A_0^{(B\to \rho)}(m_{K}^2) \), respectively, all the \( a_i \)'s are built up from the Wilson coefficients \( C_i \)'s, and take the form \( a_i = C_i + C_{i+1}/N_c \) for odd \( i \) and \( a_i = C_i + C_{i-1}/N_c \) for even \( i \). In deriving the above expression for the amplitudes, we have assumed that both \( f_0(500) \) and \( \rho^0(770) \) do not have the \( s\bar{s} \) component (or at least negligible). This is a rough estimation, especially for \( f_0(500) \), because the structure of \( f_0(500) \) is still unclear \(^3\).

We use a set of Wilson coefficients from Ref. [11]:

\[
C_1 = -0.185, \quad C_2 = 1.082, \quad C_3 = 0.014, \quad C_4 = -0.035, \\
C_5 = 0.009, \quad C_6 = -0.041, \quad C_7 = -0.002\alpha, \\
C_8 = 0.054\alpha, \quad C_9 = -1.292\alpha, \quad C_{10} = 0.263\alpha,
\]

where \( \alpha \) is the fine structure constant and all the Wilson coefficients are taken in the naive dimensional regularization scheme for \( \mu = \overline{m}_b(m_b) = 4.40 \text{ GeV}, m_t = 170 \text{ GeV}, \Lambda_{(5)}^{MS} = 225 \text{ MeV} \). We also need three form factors, \( F_1^{(B\to K^-)} \), \( A_0^{(B\to \rho^0)} \) and \( F_0^{(B\to f_0)} \). In our numerical calculation, we use [15]

\[
F_1^{(B\to K)}(0) = 0.35, \quad (28) \\
A_0^{(B\to \rho)}(0) = 0.28. \quad (29)
\]

Since most of the models indicate that the \( B \) meson to a light meson form factor at zero recoil lies around 0.3, we simply set

\[
F_0^{(B\to f_0)}(0) = 0.3. \quad (30)
\]

\(^3\) Theoretical analysis shows that it has a large \( qq\bar{q}q \) component [14].
One of the commonly used approximations for these form factors is the monopole approximation:

\[ f(s) = \frac{f(0)}{1 - \frac{s}{m_p^2}}, \tag{31} \]

where \( f = F_1^{(B \to K)}, A_0^{(B \to \rho)}, \) or \( F_0^{(B \to f_0)}, \) \( m_p \) is the pole mass. The pole mass should be different for different form factors (around 5 to 6 GeV). However, since \( s_L \) and \( m_\pi^2 \) are small compared with the pole mass squared, we will simply replace \( f(s) \) or \( f(m_K^2) \) by \( f(0) \).

We confront with two resonances, \( \rho^0(770) \) and \( f_0(500) \). The masses and total decay widths of these two resonances in our numerical calculation are (in GeV) [16]

\[
m_{\rho^0(770)} = 0.775, \quad \Gamma_{\rho^0(770)} = 0.149, \\
m_{f_0(500)} = 0.500, \quad \Gamma_{f_0(500)} = 0.500.
\]

With all the above considerations, one can see that we have only one free parameter, which is the strong phase \( \tilde{\delta} \). The latest experimental \( CP \) asymmetry data for the decay channel \( B^\pm \to K^\pm \pi^+\pi^- \) is from LHCb Collaboration [6]. Their experimental results indicate that when the invariant mass of \( \pi^+\pi^- \) is around the vicinity of \( f_0(500) \), the \( CP \) asymmetry can be larger than about 30% for small invariant mass of the \( K^\mp\pi^\pm \) pair, and lies between 0 to -10% for large invariant mass of the \( K^\mp\pi^\pm \) pair. These experimental constraints imply that the strong phase \( \tilde{\delta} \) should be between 200° and 249°. In FIG. 11, we show the differential \( CP \) asymmetry as a function of \( g_s(s_{K^\mp\pi^\pm}) \) when \( s = m_{f_0}^2 \) for \( \tilde{\delta} = 200°, 220°, \) and 240°, respectively. One can see that when \( g \) is smaller than 0 (corresponding to \( s_{K^\mp\pi^\pm} < s_{K^\mp\pi^\pm,s_{K^\mp\pi^\pm,max}} \), which is just the region \( \Omega \)), the \( CP \) asymmetry is very small, while when \( g \) is larger than about 0.5 (corresponding to \( s_{K^\mp\pi^\pm,min} < s_{K^\mp\pi^\pm} < s_{K^\mp\pi^\pm} - 0.5\Delta_{K^\mp\pi^\pm} \)), the \( CP \) asymmetry becomes very large. This is exactly what the LHCb experimental results showed.

IV. CONCLUSION

In this paper, we presented a general analysis of the mechanism that induces \( CP \) violation by the interference of two resonances with different spins. We applied this mechanism to the decay process \( B^\pm \to K^\pm\pi^+\pi^- \). When the invariant mass of the \( \pi^+\pi^- \) pair is around the vicinity of \( f_0(500) \), we found that a large \( CP \) asymmetry difference may exist between large and small invariant masses of the \( K^\pm\pi^- \) system. A key observation of this large \( CP \)
FIG. 1. The differential $CP$ asymmetry (curved lines) as a function of $g = g_{m_{K}^{0}}(s_{K^{+}\pi^{\pm}})$. We also show the localized $CP$ asymmetries (straight lines) averaged over the regions $\Omega$ and $\bar{\Omega}$, respectively. Dash-dotted lines, solid lines, and dashed lines are for $\tilde{\delta} = 200^\circ$, $220^\circ$ and $240^\circ$, respectively.

Asymmetry difference is that it can be interpreted as the interference of the amplitudes induced by $\rho^0$ and $f_0(500)$ as the intermediate resonances, respectively.

Unlike the $B^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ case where there are up to five free parameters, we have only one parameter for the case $B^{\pm} \rightarrow K^{\pm}\pi^{+}\pi^{-}$, which is the relative strong phase $\tilde{\delta}$. This makes our analysis of the decay channel $B^{\pm} \rightarrow K^{\pm}\pi^{+}\pi^{-}$ much more simplified. We found that when the relative strong phase $\tilde{\delta}$ lies between $200^\circ$ and $249^\circ$, theoretical analysis is consistent with the data.

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