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**A–T Volume Integral Formulations for Solving Electromagnetic Problems in the Frequency Domain**

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A volume integral formulation for solving electromagnetic problems in the frequency domain is proposed. First, it is based on a magnetic flux and current density face element interpolation for representing the electromagnetic problem through an equivalent circuit. Second, magnetic vector potentials and electric vector potentials \( T \) are considered, thanks to the use of finite element mesh connectivity matrices. The formulation is particularly well adapted to solving electromagnetic problems with large air domains, in the presence of thin electric regions and magnetic materials.

*Index Terms*— Electromagnetism, equivalent circuit, face and edge elements, vector potential, volume integral formulation.

I. INTRODUCTION

DIFFERENT works have shown the interest of using volume integral method (VIM) for 3-D magnetic field analysis [15], the main advantage being that the air region does not need to be meshed. In particular, the VIM Partial Element Equivalent Circuit Method has shown great ability to handle really complex devices in the presence of conductors and complex electrical circuits [10], such as power electronic devices, which is the type of problem we want to address. Moreover, in recent years, there was a regain of interest for solving Maxwell’s equations by Green’s function integrals triggered by the development of matrix compression algorithms, which greatly improves storage and resolution of full matrix systems.

On the other hand, Whitney face interpolation for current density \( J \) and magnetic flux density \( B \) is well suited for representing an electromagnetic problem through an equivalent circuit [2]. This approach provides a solution that ensures the conservation of flux and current through the facets of the mesh [3]. In this work, we propose an alternative vector potential approach derived from the previous.

II. EQUIVALENT CIRCUIT APPROACH THROUGH FACET ELEMENT AND VOLUME INTEGRAL FORMULATION

Let us consider a linear magnetodynamic problem with magnetic regions \( \Omega_m \) (presence of magnetization \( M \)), conducting electrical regions \( \Omega_e \) (presence of current density \( J \)), and coils (imposed current density \( J_0 \)). The two constitutive laws linking the current density \( J \) to the electric field \( E \) (in \( \Omega_e \)) and the flux density \( B \) to the magnetic field \( H \) (in \( \Omega_m \)) are expressed as follows:

\[
J = \sigma E \quad M = \chi H = (v_0 - v)B.
\]

We can express the electric field \( E \) and the magnetic field \( H \) through vector potentials (magnetic \( A \) and electric \( T \)) and scalar potentials (electric \( V \) and magnetic \( \phi \)) such as

\[
E = -j\omega A - \nabla V \quad H = T - \nabla \phi
\]

\[
A(P) = \frac{\mu_0}{4\pi} \left( \int_{\Omega_e} \frac{J(Q)}{r} d\Omega + \int_{\Omega_e} \frac{J_0(Q)}{r} d\Omega + \int_{\Omega_m} \frac{M(Q)}{r^2} d\Omega \right)
\]

\[
T(P) = \frac{1}{4\pi} \left( \int_{\Omega_e} \frac{J(Q)}{r} d\Omega - \int_{\Omega_m} \frac{\nabla \phi}{r^2} d\Omega \right)
\]

\[
\phi(P) = \frac{1}{4\pi} \left( \int_{\Omega_m} \frac{M(Q) \cdot r}{r^3} d\Omega \right)
\]

where \( \omega \) is the angular frequency and \( r \) is the distance between point \( P \) and integration point \( Q \) in \( \Omega_e \) and \( \Omega_m \). Meunier et al. [3] proposed to solve electromagnetic problems through a volume integral formulation based on the first-order face elements discretization for \( B \) and \( J \) (the mesh is limited to \( \Omega_m \) for \( B \) and to \( \Omega_e \) for \( J \)).

\[
J = \sum_j w_j J_j \quad B = \sum_g w_g \Psi_g
\]

where \( w_j \) and \( w_g \) are the face shape functions, and \( J_j \) and \( \Psi_g \) the electric and magnetic fluxes, respectively. In practice, integral volume formulation consists in matching electric and magnetic fields obtained by (1) with electric and magnetic fields expressed by local laws (2). It can be achieved by applying two Galerkin procedures, respectively, associated with \( \Omega_e \) and \( \Omega_m \) and by using previous face shape functions \( w_j \) and \( w_g \) for projection.

\[
\int_{\Omega_e} w_j \left( \frac{J}{\sigma} - E \right) d\Omega = 0 \quad \int_{\Omega_m} w_j \left( \frac{M}{\chi} - H \right) d\Omega = 0.
\]

Equation (4) allows us to build two equivalent circuits for magnetic (Fig. 1) and electric regions, whose graphs are dual meshes of the primal meshes used for \( \Omega_m \) and \( \Omega_e \). Inserting (2) and (3) in (4) provides

\[
\left[ \begin{array}{ccc}
R + j\omega L & Y & I \\
X & K + M & J
\end{array} \right] \left[ \begin{array}{c}
\Delta V \\
\Delta \Phi
\end{array} \right] = \left[ \begin{array}{c}
U \\
\Omega
\end{array} \right]
\]

(5)
which allow to express magnetic and electric potential differences \( \{ \Delta \Phi \} \) and \( \{ \Delta V \} \) on the branches of the equivalent circuits [3]. Matrix coefficients are \( \{ \text{fluxes such as } J \} \) between edges and nodes, \( \{ \text{strongly imposing the solenoidality of the magnetic flux } B \} \) are the branch-fundamental independent loop matrices, allow differences \( \{ \text{which allow to express magnetic and electric potential } \} \) and \( \{ \text{vector potentials } A \} \) by introducing an equivalent surface representation [12]. In Fig. 1. Circuit representation for magnetic regions.

\[
R_{ij} = \int_{\Omega_k} w_i \cdot \frac{w_j}{\sigma} d\Omega \quad L_{ij} = \frac{\mu_0}{4\pi} \int_{\Omega} w_i \cdot \int_{\Omega} \frac{w_j}{r} d\Omega d\Omega
\]

\[
Y_{fg} = \int_{\Omega_m} w_f \cdot \int_{\Omega_m} (v_0 - v) \frac{w_g \wedge r}{r^3} d\Omega d\Omega
\]

\[
K_{fg} = \int_{\Omega_m} w_f \cdot \int_{\Omega_m} \frac{w_g}{r} d\Omega d\Omega
\]

\[
M_{fg} = \int_{\Omega_m} \int_{\Omega_m} (v - v_0) \frac{w_g \wedge r}{r^3} d\Omega d\Omega
\]

\[
X_{ij} = -\frac{\mu_0}{4\pi} \int_{\Omega_k} \frac{1}{S_f \Omega_m} \int_{\Omega_m} \left( \int_{\Omega_m} \left( \frac{\mathbf{j}_{nk}}{r} \right) \right) I_k d\Omega d\Omega
\]

\[
Q_f = \frac{1}{4\pi} \int_{\Omega_m} w_f \cdot \sum_k \left( \int_{\Omega_m} \left( \frac{\mathbf{j}_{nk} \wedge r}{r} \right) \right) I_k d\Omega d\Omega
\]

where \( \mathbf{j}_{nk} \) is the source vector current density, which produces a current of 1 A in coil \( k \) and \( I_k \) is the current flowing through coil \( k \). System (5) can be solved by the use of a circuit solver by using mesh analysis. Circuit equations \( M_m \{ \Delta \Phi \} = 0 \) and \( M_c \{ \Delta V \} = 0 \) [3], where \( M_c \) and \( M_m \) are the branch-fundamental independent loop matrices, allow strongly imposing the solenoidality of the magnetic flux \( B \) and current density \( \mathbf{J} \). The unknowns are mesh currents and fluxes such as \( \{ I \} = M_c \{ I_m \} \) and \( \{ \Psi \} = M_m \{ \Psi_M \} \). The formulation can be easily extended to the case of thin regions by introducing an equivalent surface representation [12]. In Section III, we propose to solve equations (5) by an alternative vector potentials \( A-T \) formulation.

### III. SIMPLY CONNECTED \( A-T \) FORMULATION

Let us consider usual connectivity matrices of primal magnetic and electric finite element meshes: \( G_m \) and \( G_c \) between edges and nodes, \( C_m \) and \( C_c \) between faces and edges, and \( D_m \) and \( D_c \) between volumes and faces. Matrices of the dual complex on \( \Omega_m \) are then defined by \( \tilde{G}_m, \tilde{C}_m \) and \( \tilde{D}_m \), with \( \tilde{G}_m = D_m^T, \tilde{C}_m = C_m, \tilde{D}_m = -G_m^T \) \[4\], \[5\]. Similar relations are used for \( \tilde{G}_c, \tilde{C}_c, \tilde{D}_c \) on \( \Omega_c \).

On the primal meshes, on which \( \mathbf{B} \) and \( \mathbf{J} \) are interpolated, the connectivity matrices \( C_m \) and \( C_c \) link the integrals of magnetic and electric vector potential (denoted \( \mathbf{A} \) and \( \mathbf{T} \)) along the edges \( e \) of each facet \( f \), with \( \Psi \) and \( I \) being the magnetic fluxes and the currents through faces

\[
\{ \Psi \} = C_m \{ A \}
\]

\[
\{ I \} = C_c \{ T \}
\]

On the dual meshes, which define the equivalent circuits, the connectivity gradient matrices \( \tilde{G}_m \) and \( \tilde{G}_c \) link edges and nodes such as

\[
\{ \Delta \Phi \} = \tilde{G}_m \{ \Phi \} \{ \Delta V \} = \tilde{G}_c \{ V \}.
\]

By considering that \( \tilde{G} = D^T \) and \( D.C = 0 \) \[4\], we have

\[
C_m \{ \Delta \Phi \} = C_m^T D_m^T \{ \Phi \} = 0, \quad C_c \{ \Delta V \} = C_c^T D_c^T \{ V \} = 0.
\]

By letting (8) and (9) in (5), we obtain a novel dense and non-symmetric system (11) to be solved

\[
\begin{bmatrix}
C_m^T Z_c C_c & C_m^T Y_c C_m \\
C_m^T X C_c & C_m^T Z_m C_m
\end{bmatrix}
\begin{bmatrix}
T \\
A
\end{bmatrix} =
\begin{bmatrix}
C_c U \\
C_m^T Q
\end{bmatrix}
\]

with \( Z_c = R + j \omega L \) and \( Z_m = K + M \).

### IV. NUMERICAL RESULTS

We have tested the \( A - T \) formulation on the problem proposed in \[3\] (Fig. 2). Integrals of Green’s kernels of \( L, M, X, \) and \( Y \) matrices are computed with the use of analytical integrations \[7\], \[8\]. No boundary condition is imposed on \( \mathbf{A} \), and \( T = 0 \) is imposed on the boundary of conducting regions. Note that the volume integral matrix of magnetic interactions \( M \) can be transformed to a surface integration when the problem is linear \[13\]. This is not the case in the presence of volume electric regions, since we have to compute full interaction matrices \( L, X, \) and \( Y \). This can be a limitation of VIM formulation in the presence of strong skin effect. A reference solution is obtained with a \( T - \Phi \) finite element method (FEM) from 0 to 1000 Hz. A mesh of 3700 elements is used for the volume integral \( A - T \) formulation, which provides
A convergence is observed with a solution without gauge condition [6], and a better system of equations is compatible, an iterative solver providing the determination of independent loops. Moreover, since the discrepancy from the FEM is about 1.5% of eddy current losses.

\[
\begin{align*}
\text{Without} & \quad \text{ILU factorization} & \quad \text{LU factorization} \\
\text{A-T} & \quad 113 / 117 & \quad 29 / 45 & \quad 26 / 36 \\
\text{IM-} & \quad 2064 / 2306 & \quad 86 / 100 & \quad 83 / 99
\end{align*}
\]

Table I: Number of GMRES iterations for A–T and IM–ΨM formulations with and without using a preconditioner (100 Hz/1000 Hz)

A simple solution consists in substituting the connectivity matrix \( C_c \) by a fundamental branch independent loop matrix \( M_c \), which can be given by a circuit solver [14]. System (11) becomes

\[
\begin{bmatrix}
M'_{t} Z_{t} M_{t} & M'_{t} Y_{m} C_{m} \\
C_{m}^t X_{m} C_{m} & C_{m}^t Z_{m} C_{m}
\end{bmatrix}
\begin{bmatrix}
I_{t} \\
C_{m}^t Q
\end{bmatrix} = \begin{bmatrix}
M'_{t} U \\
C_{m}^t Q
\end{bmatrix}.
\] (12)

This formulation is then general and allows solving any type of magnetodynamic problem in the presence of magnetic and electric volume regions, including external circuits.

In order to efficiently get \( M_c \), we propose a specific algorithm, which combines the use of connectivity matrix \( C_c \) and independent loops search. The \( C_c \) matrix is first easily obtained from the finite element mesh. Then, a specific algorithm dedicated to circuit solver [14] analyzes this matrix on the dual finite element mesh circuit. In the presence of non-simply connected regions, the incidence matrix \( C_c \) is then completed to consider missing loops.

With thin regions, i.e., using a surface mesh, the initial \( C_c \) matrix leads directly to an independent branch loop matrix in the case of simply connected regions. For a non-simply connected region, the number of supplementary loops is then determined by circuit analysis. For instance, in the case of a torus, two supplementary loops are found corresponding to the eddy currents, which can circulate along the torus and the section. In the 3-D volume regions, the \( C_c \) matrix represents a branch loop matrix that is not independent in the general case. The circuit analysis allows eliminating superfluous loops (if desired) and adding the loops due to the presence of holes.

We tested our formulation on a strong magnetic–electric coupling problem. In order to validate on a reliable reference solution, we solved in 3-D an axisymmetric problem composed of a volume magnetic region (average radius 4 mm, thickness 2 mm, height 10 mm, relative permeability \( = 100 \)), a thin surface copper region (radius 5.5 mm, thickness 0.1 mm, height 10 mm, conductivity 55 e+6 S/m), surrounded by a thin surface coil (radius 6 mm, thickness 0.1 mm, height 10 mm) (Fig. 4). The reference solution is an axisymmetric finite element solution associated with a very fine mesh. The 3-D mesh of the VIM method is composed of 572 quadrangles for the conducting region and 1728 tetrahedras for the magnetic region.

Fig. 3. Comparison of eddy current losses computed by FEM and VIM.

Fig. 4. Test geometry and current density on the conducting region.
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