Effect of plate amplified velocity on nanoparticle migration in free convection flow due to uniform heat flux

Nor Syafiqah, and Marneni Narahari

Fundamental and Applied Sciences Department, Universiti Teknologi PETRONAS, 32610 Seri Iskandar, Malaysia

nor_17007748@utp.edu.my, marneni@utp.edu.my

Abstract. The effects of the amplified velocity of the plate on nanoparticle migration and heat transfer characteristics in unsteady free convection flow have been studied with the uniform heat flux boundary condition. The governing set of partial differential equations for the non-homogeneous nanofluid model is employed. A numerically stable second-order accurate method is applied to solve the dimensionless governing equations. The numerical solutions are obtained for nanofluid velocity and temperature fields, nanoparticles displacement, skin-friction, and Nusselt number for different values of amplified velocity parameter, and shown in graphs. The results show that the amplified plate velocity decreases the nanoparticle migration slightly while it increases the average Nusselt number and skin-friction. Also, a good comparison is found between the present numerical results and available correlation results of steady-state local Nusselt number for the limiting case.

1. Introduction

A great improvement of thermal conductivity by dissolving solid particles into regular heat transfer fluids attracted the enthusiasm of many scientist and engineers. The dilute suspension of nanometer-size (<100 nm) particles in a base fluid (e.g. water, oil, ethylene glycol, etc.), known as a nanofluid, has outstanding potential as the commercial heat transfer fluid in the energy management and heat transfer applications [1, 2]. An impressive improvement in thermal conductivity of base fluids was found by many researchers with the addition of small volume fraction (<4%) of nanoparticles [3-5]. Gupta et al. [6] mentioned that nanofluids are widely used in various engineering applications such as electronic cooling, automobile engine cooling systems, drilling fluids, and cooling systems of nuclear reactors. Buongiorno [7] conducted a study on convective transport in nanofluids by focusing on various slip effects for the heat transfer enhancement and concluded that Brownian motion and thermophoresis are the important mechanisms causing the heat transfer enhancement in the nanofluid. He proposed a mathematical model for the convective transport of nanofluids with the effects of Brownian motion and thermophoresis. In certain engineering applications such as electronic components, boilers and chemical reactors involve a constant heat flux on the boundary of the surface. Dissipating high heat fluxes can cause high surface temperature that can destroy the engineering devices. Therefore, several researchers considered uniform heat flux boundary condition with the inclusion of Brownian motion and thermophoresis to gain a deeper understanding of the heat transfer characteristics of nanofluids.

Khan and Aziz [8] investigated the steady-state natural convection boundary layer flow of nanofluid along a vertical plate with uniform heat flux numerically. Later, Reddy [9] extended the work of Khan and Aziz [8] by counting the effect of thermal radiation. An investigation on hydromagnetic boundary
layer flow of nanofluid past a porous stretching sheet was performed by Jalilpour et al. [10] by taking into account the effects of heat generation/absorption with prescribed surface heat flux. Chamkha et al. [11] studied the effects of constant heat and concentration fluxes on non-Darcy free convection flow over a vertical cone in a non-Newtonian nanofluid saturated porous medium. Steady convective flow and heat transfer in a nanofluid past a permeable shrinking sheet were studied by Rahman et al. [12] in the presence of the convective surface condition. Later, Rahman et al. [13] extended their previous work [12] by considering the effects of second-order slip and zero normal flux of the nanoparticles at the boundary. Narahari et al. [14, 15] presented a second-order accurate finite-difference solution of the unsteady natural convection flow along a vertical plate for both uniform temperature and uniform heat flux conditions. The effects of uniform heat flux on mixed convection flow over a horizontal circular cylinder and an inclined wavy surface in a porous medium filled with nanofluid were analyzed, respectively, by Tham et al. [16] and Srinivasacharya et al. [17]. Hayat et al. [18] investigated the three-dimensional flow of magnetohydrodynamic nanofluid past a linearly stretching surface with uniform heat flux. However, existing literature indicates that the knowledge on the unsteady free convection flow of nanofluid past a moving vertical with uniform heat flux is scarce, and the effects of the amplified velocity of the plate on the convective transport of nanofluid have not been investigated yet in the literature.

The aim of this study is to investigate the effects of the amplified velocity of the plate on the migration of nanoparticles and heat transfer characteristics in the unsteady free convection flow of nanofluid with uniform heat flux boundary condition. A well tested, robust second-order accurate finite-difference method is used to find the solution for the governing system, and the numerical results are shown in graphs. Also, the results are validated by comparing the local Nusselt number with previous literature.

2. Mathematical analysis

The space coordinates \( x' \) and \( y' \) are taken parallel and normal to the vertical plate, respectively. Initially, the plate and the nanofluid are at rest with the ambient temperature, \( T'_\infty \) and nanoparticle concentration \( C'_\infty \). At time \( t'=0^+ \), the plate moves with velocity \( u' = \lambda u_0 \), where \( \lambda \) is the dimensionless amplified plate velocity parameter. Applying the Oberbeck-Boussinesq approximation, the non-dimensional form of the unsteady two-dimensional boundary layer equations for nanofluid with Brownian and thermophoresis diffusions along the plate are given in the usual notation [14] by

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \frac{\partial^2 u}{\partial y^2} + Gr\theta - Gr\eta C, \\
\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} &= \frac{1}{Pr} \left[ \frac{\partial^2 \theta}{\partial y^2} + Nb \frac{\partial C}{\partial y} \frac{\partial \theta}{\partial y} + Nt \left( \frac{\partial \theta}{\partial y} \right)^2 \right], \\
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= \frac{1}{Pr Le} \left[ \frac{\partial^2 C}{\partial y^2} + \frac{Nt}{Nb} \frac{\partial^2 \theta}{\partial y^2} \right].
\end{align*}
\]
The appropriate dimensionless initial and boundary conditions are
\[
\begin{align*}
    u &= 0, v = 0, \theta = 0 \quad \text{for all } x, y \geq 0 \text{ and } t \leq 0, \\
    u &= \lambda, v = 0, \frac{\partial \theta}{\partial y} = -1, Nb \frac{\partial C}{\partial y} + Nt \frac{\partial \theta}{\partial y} = 0 \text{ at } y = 0, x > 0, t > 0, \\
    u &= 0, v = 0, \theta = 0, C = 0 \quad \text{at } x = 0, t > 0, \\
    u &\rightarrow 0, v \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \quad \text{as } y \rightarrow \infty, t > 0.
\end{align*}
\] (5)

The dimensionless local and average skin-friction and Nusselt number expressions, respectively, are
\[
\begin{align*}
    \tau_x &= \frac{\tau'_x}{\rho u_0^2}, \\
    \tau &= \frac{\tau'_l}{\rho u_0^2} = \frac{1}{\alpha} \left( \frac{\partial u}{\partial y} \right)_{y=0} dx, \\
    Nu_x &= -x \left( \frac{\partial \theta}{\partial y} \right)_{y=0} / \left( \theta \right)_{y=0}, \\
    Nu &= \int_0^1 \left[ \left( \frac{\partial \theta}{\partial y} \right)_{y=0} / \left( \theta \right)_{y=0} \right] dx.
\end{align*}
\] (6) (7) (8) (9)

The dimensionless quantities used in the above equations are defined as:
\[
\begin{align*}
    x &= \frac{x' u_0}{v}, y &= \frac{y' u_0}{v}, t &= \frac{t' u_0^2}{v}, u = \frac{u'}{u_0}, v = \frac{v'}{u_0}, \theta = \frac{(T' - T'_\infty) k u_0}{q v}, \\
    C &= \frac{C' - C'_\infty}{C'_\infty}, Pr = \frac{\nu c_p}{k}, Gr = \frac{(1 - C'_\infty) g \beta q v^2}{k u_0^4}, Le = \frac{\alpha}{D_B}, \\
    Nr &= \frac{\rho_p - \rho_n}{\rho_n \beta q v (1 - C'_\infty)}, Nt = \frac{D_t q v (\rho C_p)\rho}{k u_0 \alpha T'_\infty \rho C_p}, Nb = \frac{D_B C'_\infty (\rho C_p)\rho}{\alpha p C_p}
\end{align*}
\] (10)

where \( u \) and \( v \) are the dimensionless fluid velocity along \( x \) and \( y \), respectively, \( \lambda \) is the dimensionless amplified plate velocity parameter, \( u_0 \) is the plate velocity, \( \alpha \), \( \theta \) and \( C \) are the dimensionless time, nanofluid temperature, and nanoparticles concentration, respectively, \( Pr \), \( Gr \) and \( Le \) are the Prandtl, Grashof, and Lewis numbers, respectively, \( Nr \), \( Nt \) and \( Nb \) are the buoyancy ratio, thermophoresis and Brownian motion parameters, respectively, \( \tau_x \), \( Nu_x \), \( \tau \) and \( Nu \) are the dimensionless local skin-friction, local Nusselt number, average skin-friction and average Nusselt number, respectively, \( u' \) and \( v' \) are the velocity of the fluid along \( x' \) and \( y' \), respectively, \( t' \) is the time, \( T' \) is the temperature of the fluid, \( T'_\infty \) is the ambient temperature of nanofluid, \( C' \) is the concentration of nanoparticles, \( C'_\infty \) is the ambient nanofluid concentration, \( \mu \) is the coefficient of nanofluid viscosity, \( \rho \) is the nanofluid density, \( \rho_p \) is the density of the nanoparticles, \( g \) is the gravitational acceleration, \( \beta \) is the volumetric thermal expansion coefficient of the nanofluid, \( C_p \) is the nanofluid heat capacity, \( k \) is the nanofluid thermal conductivity, \( (\rho C_p)\rho \) is the effective heat capacity of nanoparticles, \( D_B \) is the Brownian diffusion coefficient, \( D_t \) is the thermophoresis diffusion.
The partial differential equations (1) to (4) have been solved with the initial and boundary conditions (5) using the second-order accurate Crank-Nicolson finite-difference method. The computational region is selected with $0 \leq x \leq 1$ and $0 \leq y \leq 7$, where $y_{\text{max}} = 7$ reflects $y \to \infty$ which clearly laid outside the boundary layer for all the chosen parameters values. A grid size of $120 \times 280$ (i.e. $\Delta x = 0.0083$ and $\Delta y = 0.025$ meters) with a time step of $\Delta t = 0.01$ seconds is preferred after some trials and the results are unchanged up to four decimal places for all the selected parameter values. The results are computed by following procedure outlined in [14] with a stopping criterion of $10^{-6}$. The present numerical results for the local Nusselt number are validated with the clear fluid correlation results available in the literature [19]. The comparison in Table 1 shows a good agreement between the results indicating the correctness of the numerical algorithm.

| Pr   | Jiji [19] | Present | Difference (%) |
|------|-----------|---------|----------------|
| 1    | 0.8466    | 0.8492  | 0.31           |
| 5    | 1.2753    | 1.2782  | 0.23           |
| 12   | 1.5614    | 1.5649  | 0.22           |
| 25   | 1.8367    | 1.8411  | 0.24           |
| 75   | 2.3228    | 2.3292  | 0.28           |
| 150  | 2.6844    | 2.6920  | 0.28           |
| 300  | 3.0968    | 3.1058  | 0.29           |

3. Results and discussion
To obtain a clear understanding on the effect of amplified velocity on the flow field and heat transfer characteristics, numerical values of non-dimensionless velocity, temperature, nanoparticle concentration, local Nusselt number, average Nusselt number, local skin-friction, and average skin-friction have been computed at different values of amplified plate velocity. The graphical results are presented in Figure 1-7. The effects of varying amplified plate velocity ($\lambda$) on the velocity profile ($u$) is shown in Figure 1. It is clearly seen that by amplifying the plate velocity, the thickness of the momentum boundary layer increases. The fluid velocity is high at the vicinity of the plate and reduced gradually to the ambient velocity as the distance from the plate increases. This is because of the no-slip condition where the layer of the fluid in the immediate region of the boundary surface achieves the velocity of the boundary. Figure 2 depicts the impact of amplified velocity parameter ($\lambda$) on the temperature profile ($\theta$). The temperature of the fluid is observed to be slightly decreasing by amplifying the plate velocity parameter. Similar trend was observed by Jalilpour et al. [10] on the temperature profiles with the stretching parameter in the case of horizontal porous stretching sheet. It is noticed from the subplot of Figure 2, that when $\lambda = 0$ and $\lambda = 1$, the fluid temperature remain the same. An increment in the amplified velocity of the plate results in an increase in fluid velocity that leads to a decrement in viscous force. The loss in viscous force reduces the friction between the fluid and the plate surface, hence reduced the dissipation of heat within the boundary layer. Therefore, a slight decrease in fluid temperature occurs. However, the fluid temperature due to amplified plate velocity is negligibly decreasing. The effects of amplified velocity parameter ($\lambda$) on the nanoparticles concentration ($C$) are presented in Figure 3. Based on the results, it is noticed that the concentration of nanoparticles is
increased gradually near the plate by amplifying the plate velocity, thus the nanoparticle movement from hotter to colder region decreases with the amplified velocity of the plate. However for $\lambda = 0$ and $\lambda = 1$, there is no variation in the concentration profiles.

Figures 4 and 5 depict the impact of amplified velocity parameter on the local skin-friction ($\tau_x$) and local Nusselt number ($Nu_x$) variations along the axial coordinate $x$, respectively. It is found that the local skin-friction is reducing until a steady-state value of the friction is achieved when the velocity of the plate is amplified. When the velocity of the plate is zero ($\lambda = 0$), it is observed that the local skin-friction is increasing and reach a steady-state along the plate as the distance increases from the leading edge ($x = 0$). Amplifying the plate velocity reduces the wall shear stress along the length of the plate. It is also observed that as the amplified velocity increases, the skin-friction value is negative as the distance increases from the leading edge which indicates that the plate is exerting a drag force on the fluid.

**Figure 1.** The effects of amplified velocity ($\lambda$) on velocity distribution ($u$)

**Figure 2.** The effects of amplified velocity ($\lambda$) on temperatures distribution ($\theta$)

**Figure 3.** The effects of amplified velocity ($\lambda$) on concentration profiles ($C$)

**Figure 4.** The effects of amplified velocity ($\lambda$) on dimensionless local skin-friction ($\tau_x$)
From Figure 5, it is seen that by amplifying the plate velocity, it results in an increase of local Nusselt number. However, this increase is negligible near the trailing edge. It is also observed that the rate of heat transfer increases linearly along the plate when $\lambda = 0$. The amplified velocity of the plate increases the velocity of the fluid, thereby the heat transfer rate increases. The impacts of amplifying plate velocity ($\lambda$) on the average skin-friction ($\tau$) and average Nusselt number ($\overline{Nu}$) are presented in Figure 6 and 7, respectively. It is interesting to know from Figure 6 that the average skin-friction is increasing with the amplified velocity of the plate. In the case of $\lambda = 0$, the average skin-friction is increasing as the time progress. It is noticed that when the plate is moving ($\lambda > 0$), the average skin-friction decelerates with time and attain a steady-state as the time increases. As the plate velocity amplified, the momentum boundary layer thickness increases, thereby the wall shear stress increases which leads to the increment of average skin-friction. It is observed from Figure 7 that the average Nusselt number enhances by amplifying the plate velocity. When the plate is stationary ($\lambda = 0$), the average rate of heat transfer observed to be lower compared to the moving plate. As time escalate, the rate of heat transfer is decelerating to a steady state value. The amplified plate velocity leads to the increase of momentum boundary layer thickness and thereby enhances the rate of heat transfer.
4. Conclusions
The effects of the amplified velocity of the plate on nanoparticle migration and heat transfer characteristics in free convection flow of a nanofluid with constant heat flux boundary condition have been investigated by implementing Buongiorno’s model. The governing set of partial differential equations is solved numerically by using a second-order accurate finite-difference scheme. The numerical results are obtained for the velocity and temperature fields, and nanoparticle concentration for different values of amplified plate velocity parameter. It is found that by amplifying the plate velocity, the nanofluid velocity enhances but the temperature is slightly dropped. The nanoparticle movement from hotter to a colder region slightly decreased with the plate amplified velocity. The local skin-friction is seen to be decreased meanwhile the average skin-friction is increasing as the plate velocity amplified. The local and average heat transfer rates are increased with the amplified plate velocity.

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