Deconstructing the Low-Vol Anomaly

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The so-called low-volatility (or low-β) anomaly was noticed at least as early as 1970 by Black [1972]—who failed to convince Wells Fargo to launch a levered fund that would buy low-volatility stocks and sell high-volatility ones—and in 1972 by Robert Haugen—who equally failed to have his Haugen and Heins [1972] article published before his results contradicting the CAPM model were excised in Haugen and Heins [1975]. That low-volatility stocks should perform better than their high-volatility counterparts is indeed counterintuitive and in blatant contradiction with the idea, deeply rooted in economic theory, that risk should somehow be rewarded by some excess return (Sharpe [1964]).

Still, concurrent empirical evidence has accumulated since the early 1970s, and it broadly confirms that this low-volatility “puzzle” is a robust, universal stylized fact of stock markets (and, to a lesser extent, of bond markets and other asset classes as well); see Ang et al. [2006], Frazzini and Pedersen [2014], or Baltas et al. [2015]. The effect has indeed been persistent over time and is documented on a variety of stock markets throughout the world (developed countries and emerging markets alike); see, for example, Blitz and van Vliet [2007], Baker and Haugen [2012], Chen et al. [2012], and Iwasawa and Ushiwama [2013].

Such a striking departure from the efficient market lore begs for an explanation. Several plausible stories can in fact be found in the literature, as reviewed in Clemens [2012], Ramos and Hans [2013], and Hou and Loh [2014]. Some are based on behavioral biases (e.g., lottery ticket investing—see, e.g., Garrett and Russell [1999], Barberis and Huang [2008], Boyer et al. [2010], Iwasawa and Ushiwama [2013], and Hou and Loh [2014] and “glittering” stocks attracting the attention of investors, as in Barber and Odean [2008]). Some, such as Hsu et al. [2012], note that analysts are overly optimistic about high-volatility stocks; others highlight institutional constraints (high-volatility stocks as an alternative to leverage as in Black [1972]) or incentives (managers’ bonuses are in fact options on the performance of invested stocks, and thus more valuable for high-volatility stocks; see Baker et al. [2011] and Baker and Haugen [2012]). And others still are based on more “mechanistic” effects; see di Bartolomeo [2013] and Baltas et al. [2015].

The low-volatility (henceforth “low-vol”) anomaly is thus clearly relevant both from a theoretical and practical point of view: it challenges the pillars of modern academic finance and suggests interesting defensive stock strategies that would have significantly outperformed the market in the last 50 years. As such, it has attracted tremendous interest recently, with dozens of papers appearing in the academic and professional literature; see, among other papers, Ang et al. [2006],

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Clarke et al. [2006], Blitz and van Vliet [2007], Ang et al. [2009], Fu [2009], Baker et al. [2011], Baker and Haugen [2012], Iwasawa and Ushiwama [2013], Frazzini and Pedersen [2014], Li et al. [2014], Novy-Marx [2014], and Baltas et al. [2015].

Whereas all these papers confirm that the low-vol anomaly is strong and pervasive in stock markets, the origin of the effect is still debated. Novy-Marx, in particular, argues that the low-vol anomaly can actually be subsumed in more classic explanatory variables such as value or profitability (see Novy-Marx [2014]). We had ourselves undertaken an in-depth study of the low-vol effect when Novy-Marx’s paper came out, and our conclusions partly overlap with his. Still, some of our findings appear to be new, and we hope that they help shed light on these matters. We find, in particular, that a large proportion of the low-vol performance is in fact eked out from dividends. This is our central result, which follows from the strong negative correlation between volatility and dividend yields, which, oddly, does not seem to be clearly documented in the literature (but see Clemens [2012], where this correlation is implicitly discussed). However, the low-vol anomaly persists for ex-dividend returns. These are found to be roughly independent of the volatility level—that is, risk-adjusted ex-dividend returns are higher for low-vol stocks, which is an “anomaly” in itself.

Our overall practical conclusion is that although the low-vol (/low-β) effect is indeed compelling in equity markets, it is not a real diversifier in a factor-driven portfolio that already has exposure to value-type strategies. In a nutshell, the dividend yield factor explains (as expected) the dividend part of the low-vol performance, while the earnings-to-price factor explains the ex-dividend part. Furthermore, the strong dividend bias leads us to believe that the effect is probably not as convincing in other asset classes such as bonds (though see Frazzini and Pedersen [2014] and the discussion in Baltas et al. [2015] for a more in-depth discussion of this point).

METHODOLOGY

Validating the Anomaly on All Geographical Zones

To begin, we analyze the low-vol and low-β anomalies within the markets described in the appendix. In Exhibit 1, we indicate the starting dates of our simulations in every trading zone, as well as the maximum number of stocks in a given pool. We cover the main industrialized countries, as well as Brazil, Korea, and Hong Kong. Most pools start around 1996, but we used CRSP data for the U.S. zone to go back to 1966; the end date for all pools is July 16, 2015. The portfolio construction procedure, however, requires us to “burn” three years of data, so actual simulation results cover a slightly shorter period.

We define volatility as the 250-day standard deviation of the log of total daily returns (i.e., including dividends). To evaluate β, we stick to the procedure outlined in Frazzini and Pedersen [2014]: we compute the stock and index volatilities σ_i and σ_INDEX using the definition given above, while using overlapping three-day returns and a five-year period to compute stock index correlations ρ_i. We then write

\[ \beta_i = \rho_i \frac{\sigma_i}{\sigma_{\text{INDEX}}} \]  

Although not crucial, we then shrink our βs toward unity to reduce measurement noise:

\[ \beta_i \leftarrow 0.6 \times \beta_i + 0.4 \times 1 \]

We finally lag these measures of volatility and β by 20 days to make sure we do not capture any short-term
effects. This in particular excludes any interpretation of the low-vol anomaly in terms of short-term reversal of strong recent rallies, as argued in several recent papers—for example, Fu [2009] and Li et al. [2014]—because these reversal anomalies should not survive to such a lag. Besides, the well-known short-term reversal strategy shows a correlation of $< 0.1$ with the low-vol anomaly.

We then divide our stock universe into 10 decile portfolios, ranking the stocks with their past volatility or past $\beta$. In Exhibit 2, we show the information ratios of these portfolios on the 11 pools considered. We see quite clearly that the least volatile deciles have a better risk-adjusted return than the most volatile decile on all zones. Results would be very similar for $\beta$-constructed deciles. This shows the robustness of the low-vol anomaly across geographical zones. Also, the fact that it is quite significant for both the S&P 500 and U.S. small caps means that it is not a small-cap-only effect. Using different data, we have actually been able to confirm the low-vol anomaly as far back as 1927 on U.S. stocks, with a $t$-statistic beyond 8!

**Construction of the Factors**

We next want to create long–short portfolios that give exposure to this anomaly; namely, we want to be long low-vol stocks and short high-vol ones, while having zero (or close to zero) correlation with the market mode. The performance of such portfolios, and its statistical significance, is a test for accepting or rejecting the existence of these effects in the data. It is also crucial from an investor’s perspective.

Again, we will consider international pools of stocks, defined in the appendix. For the portfolio construction, we stick to what is proposed in Frazzini and Pedersen [2014], though we tested other approaches, including a Markowitz portfolio construction (see Exhibit 3). To control the cross-sectional distribution of our predictions over time, as well as to limit the amount of noise, we use the rank of the volatilities/$\beta$’s defined previously, rescaled between −1 and 1. In more mathematical terms,

$$s_i = \frac{2}{N} \text{rank} \left( \frac{1}{\sigma_i} \right) - 1$$  \hspace{1cm} (3)

where $s_i$ is the signal on stock $i = 1, \ldots, N$, $\sigma_i$ is its volatility (as defined), and $N$ is the number of stocks in the pool. We take the inverse of the volatility because we want to be long on low-vol stocks. We apply a similar procedure for the $\beta$ strategy.

However, a portfolio constructed blindly using this signal would end up having a net-short market bias, because by construction the long leg of the portfolio is less

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**EXHIBIT 2**

**Information Ratios of the “Total Return” Performance (i.e., including dividends) of Decile Portfolios Ranked by Past Volatility**

| Pool             | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   |
|------------------|------|------|------|------|------|------|------|------|------|------|
| Russell 3000     | 0.25 | 0.42 | 0.50 | 0.54 | 0.57 | 0.60 | 0.64 | 0.66 | 0.70 | 0.84 |
| S&P 500          | 0.49 | 0.59 | 0.71 | 0.73 | 0.79 | 0.78 | 0.88 | 0.86 | 0.90 | 1.01 |
| U.S. Small Caps  | 0.39 | 0.54 | 0.71 | 0.72 | 0.87 | 0.90 | 1.12 | 1.05 | 1.15 | 1.34 |
| Australia        | 0.18 | 0.18 | 0.51 | 0.51 | 0.65 | 0.69 | 0.73 | 0.79 | 0.88 | 0.90 |
| U.K.             | 0.13 | 0.26 | 0.21 | 0.29 | 0.56 | 0.52 | 0.27 | 0.47 | 0.94 | 0.64 |
| Europe (ex-U.K.) | 0.28 | 0.44 | 0.53 | 0.55 | 0.59 | 0.60 | 0.62 | 0.67 | 0.74 | 0.91 |
| Japan            | −0.16| 0.16 | 0.24 | 0.23 | 0.26 | 0.25 | 0.35 | 0.30 | 0.33 | 0.30 |
| Korea            | −0.35| −0.10| 0.28 | 0.55 | 0.67 | 0.73 | 0.57 | 0.75 | 0.65 | 0.67 |
| Hong Kong        | 0.49 | 0.67 | 0.75 | 0.66 | 0.71 | 0.70 | 0.73 | 0.98 | 1.03 | 1.15 |
| Brazil           | 0.22 | 0.25 | 0.23 | 0.41 | 0.49 | 0.47 | 0.43 | 0.41 | 0.76 | 0.59 |
| Canada           | 0.09 | 0.06 | 0.29 | 0.42 | 0.61 | 0.66 | 0.66 | 0.77 | 0.95 | 0.81 |
| Average          | 0.18 | 0.32 | 0.45 | 0.51 | 0.62 | 0.63 | 0.65 | 0.70 | 0.74 | 0.83 |

Notes: The first decile is the most volatile. The last line is a flat average over all zones/pools, which clearly reveals the progressive nature of the low-vol effect.
Deconstructing the Low-Vol Anomaly

volatile and has less market exposure than the short leg. To compensate for that effect, we releverage the long part of our portfolio by a factor \( \Sigma s_i \beta_i^{-1} = \beta_L^{-1} \), while we deleverage the short part by a factor \( \Sigma s_i \beta_i^{-1} = \beta_H^{-1} \). That way, both our short and long legs have the same market exposure. Though this portfolio already has a very low trading frequency, we rebalance our portfolio monthly instead of daily.

An interesting consequence of our portfolio construction is that the final dollar positions do not add up to zero, although the initial signal \( s_i \) did. Instead, the resulting net-dollar exposition is positive and roughly proportional to \( \beta_L^{-1} - \beta_H^{-1} \) (see online appendix at www.iijpm.com). In words, this is because of the releveraging of the (less volatile) longs that is needed to ensure the market neutrality. But this net-long dollar bias must be financed in order to get meaningful P&Ls. We therefore need a history of risk-free rate \( r_{RF} \), which we have listed in the last column of Exhibit 1. It is mostly the LIBOR three-months when available, but in Brazil for example, we had to rely on implied rates. We have quite long histories for these rates, so we are not limited when we perform our backtests. Therefore, we are able to simulate the effects we want to test over a wide variety of pools.

We obtain the final P&L of the low-vol/low-\( \beta \) strategies by summing over time and over stocks the (market neutral) positions for each stock, times the return of that stock minus the risk-free rate \( r_{RF} \), plus any dividend \( \delta_i \) distributed during that period that is taxed at rate \( \tau \):

\[
P&L = \sum_{i \leq 0} s_i \times (r_i + \delta_i - r_{RF}) \div \beta_H + \sum_{i > 0} s_i \times (r_i + (1 - \tau)\delta_i - r_{RF}) \div \beta_L
\]

(4)

The resulting time series is what we call the “performance” of the fully financed strategy. Note that we do not account for transaction costs (which depend on many extra assumptions, in particular on the size of the portfolio) or differences in the financing costs between the long and the short legs of the portfolio, and we set dividend taxes to zero—that is, \( \tau = 0 \), although we briefly discuss their impact below. These effects are

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**Exhibit 3**

Sharpe Ratio for Low-Vol, Low-\( \beta \) Anomalies for the Frazzini–Pedersen Portfolio Construction (FP), the Markowitz Portfolio Construction (M), and the Strictly Sector-Neutral (SN) Low-Vol Portfolio

| Pool              | Low-Vol FP | Low-\( \beta \)-FP | Low-Vol SN | Low-Vol M | Low-\( \beta \) M | \( p = 2 \) | \( p = 3 \) | Skew |
|------------------|------------|---------------------|------------|-----------|-----------------|-----------|-----------|------|
| Russell 3000     | 0.28       | 0.45                | 0.15       | 0.67      | 0.80            | 0.81      | 0.93      | 0.12 |
| S&P 500          | 0.07       | 0.21                | 0.02       | 0.24      | 0.14            | 0.90      | 0.81      | 0.07 |
| U.S. Small Caps  | 0.53       | 0.47                | 0.61       | 0.87      | 0.94            | 0.80      | 0.78      | 0.22 |
| Australia        | 0.77       | 0.98                | 0.76       | 0.64      | 0.70            | 0.85      | 0.79      | -0.19|
| U.K.             | 0.47       | 0.52                | 0.29       | 0.35      | -0.08           | 0.94      | 0.78      | -0.36|
| Europe (ex-U.K.) | 0.50       | 0.65                | 0.63       | 0.76      | 0.50            | 0.81      | 0.95      | 0.22 |
| Japan            | 0.57       | 0.43                | 0.57       | 0.45      | 0.48            | 0.87      | 0.96      | -0.03|
| Korea            | 0.84       | 0.90                | 0.86       | 0.86      | 0.56            | 0.64      | 0.94      | 0.59 |
| Hong Kong        | 0.36       | 0.25                | 0.20       | 1.00      | 1.01            | 0.69      | 0.94      | -0.10|
| Brazil           | 0.59       | 0.46                | 0.54       | 0.45      | 0.39            | 0.77      | 0.85      | -0.20|
| Canada           | 0.61       | 0.60                | 0.93       | 0.71      | 0.73            | 0.88      | 0.74      | -0.07|
| Average          | 0.54       | 0.65                | 0.63       | 0.56      | 0.57            | 0.85      | 0.85      | 0.08 |
| Global           | 0.63       | 0.65                | 0.86       | 0.74      | 0.66            | 0.88      | 0.86      | 0.41 |

Notes: Although not very significant at the single-index level (t-stats around 1.5–2), the performance is robust and consistent across geographical zones and does not rely very much on sectoral biases. We also give the correlations between low-vol and low-\( \beta \) (p 1–2) and between low-vol and low-vol-SN (p 1–3). Finally, the last column reports the skewness of the low-vol strategy (defined as mean minus median divided by rms). The last line refers to the results of the strategy applied with equal weight to all pools alive at any given date, also shown in Exhibit 4, with a t-stat of \( \approx 4.25 \) for Frazzini–Pedersen portfolios and above 5 for Markowitz portfolios. Note that the statistics are dominated by the U.S. market until 2000, which explains why the skewness of the global strategy can be quite different from the average skewness. The Sharpe ratio of the global low-vol strategy after 2002 actually rises to 1.41 (t-stat = 4.9).
usually not considered in the academic literature either (although see Li et al. [2014]).

For the sake of robustness, we also tested another portfolio construction by using a Markowitz procedure and neutralizing our signal with respect to the first 10 modes of the correlation matrix. The results are qualitatively similar (see Exhibit 3). This last procedure gives a better factor neutralization, but because it is less common in the literature and not immediate to reproduce, we stick to the first approach in the remainder of this article.

Analyzing the Low-Vol/Low-β Performance

We now present our simulation results, summarized in Exhibit 3, and discuss their significance and robustness. As already documented in Blitz and van Vliet [2007], Baker and Haugen [2012] and Chen et al. [2012], the performance of low-vol/low-β strategies appears to be very robust across all zones. In fact, there is little dispersion in the performance per pool. In particular, all pools return a positive contribution to both strategies, with no particular bias toward developed or emerging countries.

When aggregated in a global portfolio, these strategies yield a total performance plotted in Exhibit 4. Even if for each individual market, the statistical significance of low-vol and low-β is not impressive, the global Frazzini–Pedersen portfolios have Sharpe ratios of 0.63 and 0.65, respectively (see Exhibit 3), which give them a high statistical significance (t-stat) of 4.25. The results are even better for the Markowitz construction (Sharpe ratios are 0.86 and 0.74, respectively, with t-stats above 5). The performance of low-vol/low-β is furthermore only weakly correlated (∼0.1) between different geographical zones, and the mutual correlation between low-vol/low-β is as high as 0.88.

This leads us to conclude that these two anomalies are in fact very closely related, and we will not really distinguish them in the following discussion. A simple calculation explaining why low-vol/low-β are so strongly correlated is given in the online appendix. The reader must have noticed that we consider the total volatility of each stock, including that of the market. Some authors prefer to focus on the residual contribution only when defining low-vol and high-vol stocks. However, this leads to a strategy that is highly correlated to our definition of low-vol and to low-β; see again the online appendix for why this is the case.

Finally, we have computed the skewness of the performance for all zones (see Exhibit 3, last column). To reduce measurement noise, we have chosen to compute the skewness as the mean minus the median of daily returns, divided by the root mean square of the returns. This is a measure of tail risk that is decorrelated from volatility, and yet highly relevant in many contexts, as argued by Lempérière et al. [2017]. We find a small overall positive skewness for the global strategy, while the individual zones show skewnesses scattered around zero. As follows from our discussion of Lempérière et al. [2017], this confirms the intuitive conclusion that the excess return associated with low-vol stocks cannot be interpreted as a hidden (skewness-)risk premium and is probably of behavioral origin; see also, for example, Clemens [2012] and Hou and Loh [2014] and the following discussion.

THREE POSSIBLE BIASES

Dollar Bias and Financing Costs

As we discussed, the long leg of the portfolio must be releveraged to ensure that market neutrality leads to a net-long bias. Because the reason for this is not totally intuitive, we have provided a simple illustrative model...
that accounts for this effect in the online appendix. The financing cost was accounted for in the simulation results by systematically subtracting \( r_{RF} \) from the stock returns (see Equation 4). However, the difference between lending and borrowing rates was neglected in our study, as in most other academic studies.

We found no correlation between the financing rate level, and the unfinanced performance of the low-vol (\( \beta \)) strategy. Therefore, the net-long bias could stop these strategies from working if the risk-free rate became too high. Obviously, the 2015 situation is far from this case, but it could help us understand the relatively poor performance observed in the U.S. in the 1970s to the 1980s. (Note that the P&L displayed in Exhibit 4 is entirely coming from the U.S. stocks until 1996).

**Sector Biases**

There is considerable evidence (as well as strong intuitive reasons to believe) that the anomalies we study have persistent sectorial biases. In particular, we expect our low-vol portfolios to be long utilities/consumer noncyclical, and short technologies/consumer cyclical. We checked that this is indeed the case on all our zones.

Now, an interesting question arises: Are these anomalies merely picking up the sectors with the best risk-adjusted return? We address this issue directly by building sector-neutral portfolios—that is, we rank the volatility over every sector between \(-1\) and \(1\), and then apply our usual portfolio construction. By definition, these portfolios should not—and indeed do not—have any sectoral bias left.

We summarize the performance of this strategy for low vol in Exhibit 3 (results for low-\( \beta \) would lead to similar conclusions). As we can see, the Sharpe ratios are only slightly reduced and the correlation is still very high between the two implementations. This means that the sector component is not a dominant determinant of the low-vol/low-\( \beta \) anomaly—a conclusion that’s in line with Asness et al. [2014].

**Dividend Bias**

A rather striking observation (which, curiously, we have not seen clearly stated in the literature before) is that most of the gain of the low-vol strategy in fact comes from the dividend part, as can be seen from Exhibit 5. Because the portfolio is market neutral, its long, low-vol stocks must have received, on average, higher dividends than short, high-vol stocks. Indeed, we have found a significant negative causal correlation (\( \approx -0.2 \) for U.S. stocks) between past realized volatility and dividend yields, see Exhibit 6. This demonstrates that low-vol stocks are also, on average, high-dividend yielders.

Why is this so? One argument could be that high-vol “glittering” stocks are attractive because of all the biases mentioned in the introduction, so low-vol “boring” stocks must somehow compensate by offering larger dividends. A slight variation on this idea is that mature businesses pay dividends, whereas growing (risky) firms do not. However, the causality might be the other way round; not surprisingly, we find a similar negative correlation between earnings and volatility. One could thus argue that strong earnings and regular dividends make firms less risky and therefore less volatile.

In any case, this observation leads to the concern that part of the low-vol or low-\( \beta \) performance might eventually be eaten up by dividend taxes. The tax rate, however, is very much investor-dependent; our analysis suggests that the low-vol strategy can in fact withstand moderate dividend tax levels \( \tau \) (up to 50%) before becoming flat. Nonetheless, this is another concern for the viability of these strategies, on top of the level of interest rates to finance the leveraged positions. (Note that this tax applies only to the long part of our portfolio, so that neglecting the dividend component in the P&L or setting \( \tau \) to 100% are not equivalent.)

**Exhibit 5**

Contribution of the Dividend Gains (dvd) to the Performance of Low-Vol Strategies (total gain)

| Pool          | dvd/Total Gain |
|---------------|---------------|
| Russell 3000  | 88%           |
| S&P 500       | 3681%         |
| U.S. Small Caps | 73%         |
| Australia     | 49%           |
| U.K.          | 46%           |
| Europe (ex-U.K.) | 52%       |
| Japan         | 24%           |
| Korea         | 9%            |
| Hong Kong     | 57%           |
| Brazil        | 36%           |
| Canada        | 47%           |

*Note: Very similar figures are obtained for low-\( \beta \) as well.*
Incidentally, the above results suggest that simple dividend yield strategies need to be carefully risk controlled if one wants to avoid any market exposure. Indeed, because high-dividend-yield stocks are also low-vol/β, one would expect to end up with a short market exposure if longs are not releveraged. This is exactly what happens to the Fama–French portfolios based on the dividend yield (DY) factor, see Exhibit 7, where we have plotted the β of each of the 10 Fama–French decile portfolios created since 1950. As one can see, the highest ranking portfolios have a much smaller β compared to the others. This also means that going long the high-dividend portfolios and short the low ones in equal quantities results in a significantly negative correlation with the index of around −0.4. So it seems difficult to build a viable DY factor based on these portfolios.

**THE LOW-VOL STRATEGY: DECILES AND FACTORS**

**Deciles**

Because a large part (but not all) of the low-vol effect seems to come from a dividend bias, we reproduce Exhibit 2, but this time considering price returns only. In the following exhibits, we give the information ratio (i.e., the Sharpe ratio of the unfinanced strategy) of past volatility “decile” portfolios for all geographical zones for the ex-dividend strategy. One sees that the information ratio of low-vol portfolios is significantly higher than that of high-vol portfolios, even when dividends are left out (see the last line of Exhibit 8, in particular, which gives the average over all zones, and Exhibit 9). This means that there is a genuine low-vol effect here, on top of the strong dividend effect noted in the previous paragraph. We see that the anomaly is not localized on any of the 10 deciles, but is rather a smooth bias that builds up progressively as one moves from high-vol stocks to low-vol stocks. This remark is at odds with claims in the literature that the anomaly is chiefly due to penny stocks, or to extremely volatile stocks that plummet.

**Skewness**

We have also reported in Exhibit 10 the skewness of the different decile portfolios. One notices that all skewnesses are negative, but that the skewness of the high-vol portfolios is slightly less negative than that of the low-vol ones. This is somewhat unexpected because, as we have noted before, buying low-vol stocks and shorting high-vol stocks lead to a weakly positively...
Deconstructing The Low-Vol Anomaly fall skewed strategy (see last column of Exhibit 3). This of course is possible because skewnesses do not simply add up when returns are correlated.

Following up on this, one finds that while the average return of high-vol stocks is better than that of low-stocks on days in which the index goes up, a stronger opposite effect is observed on days when the index goes down. We have found that this is the case for all geographical zones. More precisely, the (negative) return differential between high-vol and low-vol stocks is ≈ 1.5 times larger when the index goes down—leading to strong positive gains that contribute to the positive overall skewness of the low-vol strategy. This is why these strategies are often called “defensive”: They allow investors to make profits in bear market environments, another argument against a risk premium interpretation.

Correlating with Standard Factors

It is now time to correlate the performance of low-vol strategies with other, more classical stock strategies. We include in these the usual Fama–French factors: UMD (momentum), SMB (size), and HML (value). We also included MKT (the market), because we eventually want to remove any residual correlation with the market in our analysis. Given the results of the previous section on the role of dividends, we thought it interesting to also study the correlation with other value/valuation-type metrics, so we consider earnings-to-price (E/P) and DY, in addition to book-to-price (i.e., HML).

We have shown that Fama–French portfolios are prone to market biases for both E/P and DY; furthermore, these portfolios are not readily available for all the zones we want to analyze. We have thus decided

| Exhibit 8 |
| Information Ratios of the Price Return (ex-dvd) Performance of Past Volatility Decile Portfolios |

| Pool          | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Russell 3000  | 0.25| 0.40| 0.48| 0.51| 0.53| 0.54| 0.56| 0.56| 0.56| 0.59|
| S&P 500       | 0.45| 0.53| 0.61| 0.61| 0.65| 0.61| 0.70| 0.65| 0.65| 0.63|
| U.S. Small Caps| 0.38| 0.53| 0.68| 0.67| 0.80| 0.80| 0.97| 0.84| 0.84| 0.84|
| Australia     | 0.12| 0.10| 0.36| 0.35| 0.44| 0.45| 0.49| 0.51| 0.55| 0.54|
| U.K.          | 0.08| 0.17| 0.11| 0.17| 0.42| 0.36| 0.12| 0.24| 0.72| 0.39|
| Europe (ex-U.K.)| 0.26| 0.39| 0.45| 0.46| 0.48| 0.48| 0.47| 0.53| 0.57| 0.71|
| Japan         | −0.18| 0.13| 0.20| 0.18| 0.21| 0.19| 0.28| 0.23| 0.25| 0.21|
| Korea         | −0.36| 0.11| 0.26| 0.52| 0.62| 0.69| 0.52| 0.70| 0.58| 0.58|
| Hong Kong     | 0.44| 0.61| 0.68| 0.58| 0.61| 0.58| 0.60| 0.84| 0.83| 0.83|
| Brazil        | 0.20| 0.22| 0.18| 0.36| 0.42| 0.39| 0.35| 0.31| 0.62| 0.43|
| Canada        | 0.08| 0.04| 0.24| 0.33| 0.49| 0.51| 0.52| 0.65| 0.59| 0.73|
| Average       | 0.16| 0.27| 0.39| 0.43| 0.52| 0.51| 0.52| 0.55| 0.63| 0.57|

Notes: The first decile is the most volatile. The last line is a flat average over all zones/pools.

| Exhibit 9 |
| Average over All Pools/Zones of the Information Ratios of the Total Return and Ex-Dividend Returns for the Decile Portfolios (from low vol to high vol) |

Notes: Total return is shown in black circles, and ex-dividend returns are shown in gray triangles. The average is computed as a flat average over the columns of Exhibits 2 and 8. We can clearly see that risk-adjusted, ex-dividend returns are slightly better for low-vol stocks than for high-vol stocks. The effect is amplified by dividends.
to recode all these factors using the same portfolio construction that we used for low-vol and low-β and that we outlined in Equation 4. We first rank the bare signal and then deleverage/releverage the long and short legs so that they have the same β. This allows us to have a uniform set of strategies across all our pools. We used standard signals to reconstruct these factors on all geographical zones, which we outline in Exhibit 11.

We are now in position to make a residual analysis, extracting the performance of low-vol and low-β performance with all these factors, computed using the corresponding global P&Ls (i.e., aggregating all geographical zones) and monthly data. As expected, we find a strong anticorrelation with SMB (small-cap stocks often have high volatilities), practically no correlation with HML, and strong positive correlation with either E/P or DY, again expected from the dividend bias previously documented. We also find some mild correlation between low-vol and UMD, which could come from some leverage effect on stocks. It should nonetheless be noted that this correlation vanishes with the Markowitz portfolio construction, so it may also be another artifact of the Frazzini–Pedersen approach.

We report in Exhibit 12 the β of the low-vol and low-β performance with standard factors, for the aggregated P&L over all geographical zones.

| Factor | Bare Signal |
|--------|-------------|
| HML    | Total equity of the company, normalized by its market capitalization |
| SMB    | Minus 100-day average of the market capitalization of the company |
| UMD    | One-year sum of the log-returns |
| E/P    | Annualized earnings of the company, normalized by its market capitalization |
| DY     | Annualized dividend of the company, normalized by its market capitalization |

Notes: We tried as much as possible to stick to the usual metrics. All quantities are lagged by one month.

### Exhibit 10
Skewnesses of the Total Return Performance of Volatility Decile Portfolios

| Pool            | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Russell 3000    | -0.59 | -0.49 | -0.29 | -0.28 | -0.36 | -0.41 | -0.38 | -0.39 | -0.48 | -0.39 |
| S&P 500         | -0.10 | -0.24 | -0.28 | -0.33 | -0.26 | -0.17 | -0.15 | -0.22 | -0.16 | -0.07 |
| U.S. Small Caps | -0.20 | -0.19 | -0.32 | -0.34 | -0.49 | -0.44 | -0.48 | -0.41 | -0.54 | -0.28 |
| Australia       | -0.11 | -0.61 | -0.56 | -0.57 | -0.44 | -0.38 | -0.58 | -0.43 | -0.68 | -0.43 |
| U.K.            | -0.16 | -0.13 | -0.41 | 0.33 | -0.24 | -0.29 | -0.22 | -0.63 | -0.20 | -0.58 |
| Europe (ex-U.K.)| -0.71 | -0.55 | -0.51 | -0.36 | -0.36 | -0.41 | -0.38 | -0.39 | -0.51 | -0.32 |
| Japan           | 0.03  | -0.15 | -0.04 | -0.03 | -0.22 | -0.06 | -0.09 | -0.25 | -0.07 | 0.10  |
| Korea           | -0.76 | -0.65 | -0.61 | -0.70 | -0.52 | -0.48 | -0.51 | -0.58 | -0.31 | -0.63 |
| Hong Kong       | -0.11 | 0.01  | -0.20 | -0.37 | -0.01 | -0.33 | -0.45 | -0.29 | -0.38 | -0.56 |
| Brazil          | 0.11  | -0.03 | -0.01 | -0.07 | -0.22 | -0.08 | -0.24 | -0.16 | -0.19 | -0.33 |
| Canada          | -0.37 | -0.31 | -0.50 | -0.77 | -0.68 | -0.70 | -0.71 | -0.90 | -0.72 | -0.67 |
| Average         | -0.27 | -0.30 | -0.34 | -0.32 | -0.35 | -0.34 | -0.38 | -0.42 | -0.38 | -0.38 |

Notes: The first decile is the most volatile. The last line is a flat average over all zones/pools, which helps us see a weak decrease in the skewness for lower volatility deciles.

### Exhibit 11
Definition of the Signals Used to Define the Reference Factors

| Factor | Bare Signal |
|--------|-------------|
| HML    | Total equity of the company, normalized by its market capitalization |
| SMB    | Minus 100-day average of the market capitalization of the company |
| UMD    | One-year sum of the log-returns |
| E/P    | Annualized earnings of the company, normalized by its market capitalization |
| DY     | Annualized dividend of the company, normalized by its market capitalization |

Notes: We tried as much as possible to stick to the usual metrics.
low vol is explained by traditional factors, there is still some additional unexplained performance, especially in the recent decades in which financing rates were lower. However, if we include E/P and DY as extra factors, the resulting residual P&L becomes essentially flat over the whole period, in agreement with Novy-Marx’s recent results (Novy-Marx [2014]). The breakdown by decade (for the aggregated strategy) is given in Exhibit 14; only the very last period seems to lead to a clearly positive residual performance—perhaps explaining the current interest in this strategy.

We have also done the same analysis using only backward looking β’s, computed over the previous 250 days, with the same conclusions. The ex ante residual performances are plotted in Exhibit 15 for the low-vol strategy and for the aggregated strategy, both within a four-factor and a six-factor model (the same results also hold for low-β).

As a good summary of the situation, we can say that the DY factor explains (as expected) the dividend part of the residual low-vol performance, while the E/P factor explains its ex-dividend part. The operational conclusion, however, is that low-vol is in fact another version of more standard valuation strategies that can be used as a diversifier in a quant-type portfolio, but is not expected to add much additional “alpha”.

### A COMPOUNDING EFFECT?

A simple idea that could account for the low-vol anomaly is the usual compounding effect (see, for example, di Bartolomeo [2013]): The geometrical mean is always smaller than the arithmetic mean.

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**Exhibit 13**

Ex Post Low-Vol Alpha with Respect to the Fama–French Three-Factor Model, Four-Factor Model (adding UMD), and Six-Factor Model (adding E/P and DY)

| Pool        | 3-Factors | 4-Factors | 6-Factors |
|-------------|-----------|-----------|-----------|
| Russell 3000| 0.35 (2.8) | 0.20 (2.2) | 0.13 (2.0) |
| S&P 500     | 0.13 (1.8) | 0.02 (0.5) | −0.11 (2.4) |
| Small Caps  | 0.05 (1.2) | −0.08 (2.2) | −0.07 (2.4) |
| Australia   | 0.07 (0.5) | 0.06 (0.4) | 0.03 (0.2) |
| U.K.        | 0.51 (3.2) | 0.44 (2.8) | 0.15 (1.3) |
| Europe      | 0.19 (2.1) | 0.13 (1.5) | 0.06 (0.6) |
| Japan       | 0.25 (2.3) | 0.08 (0.9) | −0.01 (0.1) |
| Korea       | 0.24 (2.0) | 0.24 (2.1) | 0.16 (1.4) |
| Hong Kong   | 0.24 (1.6) | 0.23 (1.5) | 0.03 (0.3) |
| Brazil      | 0.44 (2.8) | 0.29 (2.0) | 0.29 (2.2) |
| Canada      | 0.33 (2.0) | 0.2 (1.4)  | −0.04 (0.5) |
| Low-Vol (agg.) | 0.18 (5.9) | 0.09 (3.0) | −0.015 (0.6) |
| Low-β (agg.) | 0.22 (5.3) | 0.06 (1.06) | −0.07 (2.1) |

Notes: Ex post low-vol alpha is measured as the intercept of the regression of monthly returns. We indicate the t-stat of these alphas in parentheses. We also give the results at an aggregated level for both low-vol and low-β.

**Exhibit 14**

Residual Performance of the Low-Vol Strategy (aggregated over all zones) with Respect to the Six-Factor Model for Different Subperiods

| Decade     | 6-Factor Residual Sharpe Ratio | Low-Vol Sharpe Ratio |
|------------|-------------------------------|----------------------|
| 1970–1980  | −0.72                         | −0.63                |
| 1980–1990  | −0.31                         | 1.12                 |
| 1990–2000  | −0.79                         | −0.06                |
| 2000–2010  | 0.09                          | 0.46                 |
| 2010–2015  | 0.50                          | 1.24                 |

Notes: The second column gives the Sharpe ratio of the full low-vol strategy, again aggregated over all zones. Note that the last period is exceptionally good.

**Exhibit 15**

Residual P&Ls of Low Vol

Notes: The black line represents the ex ante residual of low vol once the four standard Fama–French factors (MKT, UMD, SMB and HML) are taken into account. The Sharpe ratio of the residual performance is ≈ 0.4. The gray line represents the ex ante residual of low vol once the above four standard Fama–French factors and the E/P and DY factors are taken into account. The performance now has an insignificant Sharpe ratio of 0.09.
mundane words, that −20% followed by +20% results in a drop of −4%. This implies that even if the average daily returns of all stocks (high vol or low vol) were exactly equal, the monthly or yearly average return of high-vol stocks would dip below that of low-vol stocks. Low-vol stocks might be “defensive” just because they avoid large drops from which it is hard to recover. This trivial mechanism could be a pervasive reason why one should expect volatile stocks (or any asset for that matter) to underperform in the long run.

To assess the relevance of this idea, we split our portfolio of stocks every day into 10 buckets of decreasing volatility (measured, as above, as a 250-days flat average). We then compute the average return (excluding dividends) for each of these deciles over \(n\) days, \(n = 1 \rightarrow 100\). We then plot as a function of \(n\) in Exhibit 16 the ratio of the average return of the first decile (high vol) to the average return of the last one (low vol). As we can see, ex-dividend daily returns are, on average, roughly the same for both deciles; depending on the geographical zone/market, their ratio is well scattered around unity with an average of \(\approx 1.2\). As the time scale over which the return is computed increases, the high-vol stocks outperform their defensive counterparts less and less. This is consistent with the compounding effect we outlined in the previous paragraphs.

However, the effect is not strong enough to explain on its own the performance of the low-vol anomaly (a similar conclusion can be drawn from the first two lines of Exhibit 1 in Baker et al. [2011, p. 48]). Even without the dividend bias, the mere fact that all deciles have roughly the same average return at one day is already at odds with the CAPM because one would expect high-vol stocks to compensate for risk. Within the CAPM, all stocks should have similar risk-adjusted returns, in contrast with empirical observation (see again Exhibits 8 and 9).

In conclusion, it seems that the compounding effect does indeed play a part in the low-vol anomaly, but it is only part of the explanation; without dividends, risk-adjusted daily returns of highly volatile stocks are already anomalously low.

**CONCLUSION**

In this article, we have dissected several aspects of the so-called low-vol and low-\(\beta\) strategies. We have established several “stylized” facts about these strategies; some are already well documented (such as the strength and universality of the effect over different geographical zones), whereas others have not been clearly discussed in the literature before (such as the strong dividend bias toward low-vol stocks).

Our most significant message is that the low-vol anomaly is the result of two quite independent structural effects, but of similar strength. One is the strong correlation between low-vol and high-dividend yields, showing that dividends do in fact contribute to a substantial part of the low-vol strategy’s performance. The second is the fact that ex-dividend returns themselves are, to a first approximation, independent of the volatility level on a daily time scale, leading to better risk-adjusted returns for low-vol stocks. This effect is further amplified by compounding.

We have also shown that the low-vol anomaly is not localized on extreme volatility deciles and does not come from sectoral biases, nor from short-term reversals or extreme movements. We furthermore find that the low-vol strategy has a slightly positive overall skewness, at variance with standard risk premia strategies characterized by negative skewnesses (see the extended discussion in Lempérière et al. [2017]). It would actually be very hard to explain intuitively why investing in low-vol, high-dividend stocks is carrying a specific risk factor that should be compensated for.
For practical purposes, the strong dividend bias and the resulting correlation with other valuation metrics such as E/P or HML does make the low-vol strategies to a large extent redundant, at least for stocks. This does not mean that such strategies should be excluded from the construction of “quant-factor” portfolios because they offer alternative implementations that increase diversification and reduce operational risk.

Finally, the underlying reasons for the low-vol anomaly to persist in equity markets are still, by and large, obscure. Although the behavioral/institutional stories that have been put forth are persuasive and compatible with the bias we observed in the holdings of mutual funds, there is no empirical smoking gun. We tend to believe in a universal “lottery ticket” or embedded option mechanism that equally affects institutional investors (perhaps through the bonus optionality) and private investors, leading them to be overly focused on potential spectacular upsides and forget much smaller but significant regular dividends.

**APPENDIX**

**POOLS OF STOCKS**

The pools considered here are mostly the composition of standard indices for every given day. More precisely, they are

- Australia: 200 stocks of the S&P/ASX 200.
- U.K.: 100 stocks of the FTSE 100.
- Europe: 600 most liquid (turnover in Euros) of the stocks mostly belonging to the SBF 250 (France), CDAX (Germany), OMX (Sweden), SMI (Switzerland), IBEX (Spain), AEX (the Netherlands), and FTSEMIB (Italy) indices, with a few stocks also coming from the Finnish, Norwegian, Belgian, and Danish indices.
- Japan: 500 most-liquid stocks of the all-shares TOPIX index.
- Korea: 200 stocks of the KOSPI.
- Hong Kong: around 450 stocks of the Hang Seng Composite Index.
- Canada: all stocks of the S&P/TSX index.
- Brazil: all stocks of the BOVESPA index (50 stocks in theory, but this number is actually variable in time—64 in 2015.)
- S&P 500 and Russell 3000: stocks belonging to these two indexes.
- U.S. Small Caps: U.S. stocks are ranked by their liquidity, and a pool is made with stocks ranking between 1501 and 2000.

Note that as a stock leaves the index, its position in the portfolio is liquidated at the next day’s close price.

**ENDNOTES**

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1 For well documented reviews on how poorly this idea fares in practice, one can refer to Fernandez [2015] or Thompson et al. [2006].

2 We therefore strongly disagree with the following statement in Li et al. [2014]: “In particular, we find that the excess return associated with forming the low risk zero-cost portfolios are short-lived as they are present only in month t + 1 and furthermore are largely subsumed by high transaction costs.” Note also that the analysis of Fu [2009] is affected by a look-ahead bias, as underlined in Guo et al. [2014].

3 Note that because low vol and value are correlated, this weak correlation is consistent with the decorrelation of the value anomaly over geographical zones reported in Asness et al. [2013].

4 We use the same definition of the volatility as above to rank the stock; we mean the past 250 days realized volatility, lagged by one month.

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