The Relationship Between Time and Space

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Abstract. The relationship between time and space has been studied by people for a long time. Although it is still in a development stage, it is necessary to be researched. This paper aims to introduce the space-time relationship from classical physics to modern physics. Subsequently, the author analyses Galilean transformation and its limitations to lead naturally to Lorentz transformation. After that, the special relativity is shown by giving some examples. Special relativity is based on two postulates, including the Principle of Relativity and the Principle of Invariant Light Speed. Mickelson-Morley experiment and mass-energy equivalence can explain the Principle of Invariant Light Speed well. The paper then introduces intuitively Minkowski spacetime, which can help readers understand four-dimensional spacetime as well. In addition, the distinctions of different dimensional spaces will be mentioned. The essay also discusses the general relativity that time is also affected by gravity and the principle of equivalence, but it’s really only a quick cheat-sheet summary. Finally, this paper points out the possibility of reaching the speed of light, and if it can be achieved, spacetime will be changed so that lots of unbelievable consequences will come one after another. The article draws a conclusion that time and space are like a whole, one change makes all change.

1. Introduction
The relationship between time and space is a significant question about modern physics, which can help people understand the theory of relativity and solve many problems. Compared with the scope of exploring the universe, human have begun to think about what is time, what is space, and the relationship between these two as early as two thousand years ago. The space humans live in is a physical space. Since matter and motion are inseparable, space and motion are inseparable. Any point is the center of space at the present moment. Therefore, it is necessary to introduce the conception of time to describe space. Only by endowing space with the concept of time can one form a systematic and scientific understanding of the past, present and future of space movement. In the 16th century, Isaac Newton put forward the concept of absolute spacetime, which maintains that the motion of an object does not affect time and space, so time and space are the same for any objects moving in any situation [1]. With the development of modern physics, Albert Einstein predicted different views in 1905, when he introduced the concept of relative time as part of special relativity, which proved to be contrary Newton’s idea of absolute spacetime. In 1916, Einstein published his theory of general relativity, which further explained four-dimensional spacetime [2]. Studying the relationship between time and space is the foundation of learning modern physics and theory of relativity.

2. Absolute Spacetime under Galilean Transformation
The reference frame that satisfies Newton’s first law is called the inertia frame [3]. If there are two inertial reference frames called S and S’, and S’ moves in an uniform motion in a straight line relative
to S, the laws of motion relative to the two reference frames are exactly the same. From this fact a prediction can be inferred that all mechanical relation are equivalent to these two reference frames. For example, a girl moves with a constant velocity relative to a boy. If the boy sees a body moving with constant velocity, the girl will see the body moves with constant velocity. On the contrary, if the body’s motion is variable motion, they will see the body moves with different velocities because of girl’s motion relative to boy’s; and so one comes to the point that there is not just one inertial observer, but a whole set of inertial observers. Therefore, there is more than one, but a number of inertial reference frames, which move in uniform motion in a straight line with each other. In these reference frames, the properties of time and space are the same so that the mechanical laws of each other are the same. This conclusion is called the relativity principle of mechanics.

\[
\begin{align*}
&x' = x - vt \\
y' = y \\
z' = z
\end{align*}
\]  

(1)

The assumption of absolute time is one of the foundations of the classical mechanics, so there is same time between these two frames.

\[ t = t' \]  

(2)

The formula 1 and 2 are called Galilean transformation.

However, when the speed close to is close to the speed of light, Galilean transformation is impossible. At this time, Lorentz transformation should be considered.

3. Lorentz Transformation

People have been solving physics problems by Newton’s calculus and classical mechanics for a long time. Until the end of 19th century, Maxwell’s equation was built, which marked the establishment of classical electromagnetic theory. However, there is a series of contradictions between Newton’s mechanics and electromagnetic theory. At this time, the appearance of Lorentz transformation adjusted the contradictions, and then it became the basic formulas in special relativity.

Lorentz transformation could be considered as a dictionary that observers residing on the two frames can make their meaning clear to every other. It is used for studying a coordinate transformation between two inertial frames. Unlike Galilean transformation, it gives more specific formulations when the velocity approaches the speed of light and adds a research of time.
As figure 2 shows, the transformation relationship between different inertial reference systems is consistent with the Lorentz transformation in mathematical repressions [5].

\[
\begin{align*}
    x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\
    y' &= y \\
    z' &= z \\
    t' &= \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}
\end{align*}
\]  

(3)

In this case, \(x, y, z, t\) are the coordinates and time in the inertial coordinate system \(S\) respectively, and \(x', y', z', t'\) are the coordinates and time in the inertial coordinate system \(S'\) respectively, \(v\) is the movement speed of the coordinate system \(S'\) relative to the coordinate system \(S\), and the direction is along the axis \(X\).

In the above equation set, changing \(v\) into \(-v\), and \(x', y', z', t'\) are exchanged with \(x, y, z, t\) so the inverse Lorentz transformation can be obtained.

\[
\begin{align*}
    x &= \frac{x' - vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \\
    y &= y' \\
    z &= z' \\
    t &= \frac{t' + \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}
\end{align*}
\]  

(4)

When \(\frac{v}{c^2} = \frac{1}{c} \frac{v}{c} \rightarrow 0\),

then \(v << c\), or \(\frac{v}{c} \rightarrow 0\),

Figure 2. Lorentz Transformation
and Lorentz transformation is same as Galilean transformation [5].

\[
\begin{align*}
    x' &= x - vt \\
    y' &= y \\
    z' &= z \\
    t' &= t
\end{align*}
\]

Lorentz transformation is the basic equation set in special relativity. It can be seen that the special relativity is not contradictory with classical mechanics. The special relativity extends classical mechanics to the general situation of macroscopic objects at any speed. And classical mechanics is just an approximation of relativity theory at low speed (v is far less than c). Therefore, it is necessary to consider different situations to choose Galilean transformation or Lorentz transformation.

4. Special Relativity

With the development of electromagnetic theory, scientists found that many electromagnetic phenomena do not confirm with Newton’s classical mechanics [3]. In 1905, Albert Einstein originally proposed the theory of special relativity in *On the Electrodynamics of Moving Bodies*, where many problems cannot be explained by Newton’s theories are interpreted.

4.1. Two Basic Hypotheses of Special Relativity

Special relativity is based on two postulates, which are the Principle of Relativity and the Principle of Invariant Light Speed.

The first hypothesis of Special Relativity is The Principle of Relativity. The laws of physics are invariant in all inertial frames of reference, which means all inertial observers are equivalent [6]. These were the same inertial observers that Newton introduced. They occur in the same ways in Einstein’s theory. Another interpretation of this principle is the mathematical forms of physical laws were not changed under Lorentz transformation [3].

\[\text{Figure 3. Galilean Transformation}\]

As figure 3 shows, frame S’ (x’, y’, z’) moving uniformly without spinning relative to the inertial frame S (x, y, z), then the frame S’ is also inertial reference frame according to Galilean transformation. The evolution of natural phenomena relative to frame S’ will follow the same physical laws as the frame S, which is called the Principle of Relativity.

The second hypothesis of Special Relativity is the Principle of Invariant Light Speed. The propagation speed of light in vacuum is always a constant no matter it is observed in what frames. There are a lot of experiments that have proved this principle. The most famous experiment is Mickelson-Morley experiment.
The precondition of experiment is the mediums of light is ether (later proved to be wrong). As shown in figure 4, the device moves to the right in the ether at a speed \(v\), and the distance from the practically silver-plated glass sheet to the two mirrors is \(L\). Then the speed of light \(1\) is \((c - v)\), and the time it takes is \(t_1 = \frac{L}{c - v}\), the speed when returning is \((c + v)\), and the time \(t_2 = \frac{L}{c + v}\). So the total time is

\[
T = t_1 + t_2 = \frac{2Lc}{c^2 - v^2} = \frac{2L}{c(1 - \frac{v^2}{c^2})} \quad (5)
\]

For the light \(2\), suppose the time it takes to reach the mirror is \(t_3\). During this time, the mirror moves \(vt_3\) to the right, so the distance traveled by the light is a hypotenuse, then the return time is same as this.

\[
L^2 = (ct)^2 - (vt)^2 = (c^2 - v^2)t_3^2 \quad (6)
\]

\[
t_3 = \frac{L}{\sqrt{c^2 - v^2}} \quad (7)
\]

\[
2t_3 = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c(1 - \frac{v^2}{c^2})} = t_1 + t_2 \quad (8)
\]

The Principle of Invariant Light Speed can be explained in a simple example. As shown in figure 5, there are two men at the same point carrying a bow and arrow who shoot simultaneously at 200 km/hr but one of them stand in a train traveling at 100 km/hr and another stand on the ground. Obviously, the arrow of the man standing in the train will reach the target at first according to Galilean transformation. However, if they emit a beam of light in the same situation and the train travels at the speed of light, the consequence will be quite different – two beams will reach the target at the same time. Therefore, the speed of light is always a constant value, which stays as \(3 \times 10^8\) m/s in any frames.
4.2. Relativistic Mechanics
As mentioned earlier, the principle of velocity superposition in classical mechanics is no longer valid when the velocity approaches the speed of light. Relativistic mechanics proposed more specific formulations about mass-velocity and mass-energy relation [7].

Before introducing relativistic mechanics, people should know what is mass. In relativistic mechanics, masses of objects are divided into rest mass \( m_0 \) and relativistic mass \( m \). By contrast, in the tradition of Newton, these “masses” are seen as different indirect ways of measuring the same unique quantity, mass, and not as three distinct ‘masses’. The relationship between \( m_0 \) and \( m \) is called mass-velocity relation [8]:

\[
m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}
\]  

(9)

Einstein made another highly important contribution to a more dynamic concept of mass, by stating that mass is ‘a reservoir of energy’, and by insisting on the ‘equivalence’ of mass and energy. The relationship between energy \( E \) and mass is referred to as mass-energy relation [2]:

\[
E = mc^2
\]  

(10)

According to mass-velocity relation and mass-energy relation, the principle of invariant light speed is proved again. If an object whose \( m \) is unequal to 0 moves at speed of light, its energy will be infinitely great. In other words, the energy of accelerating such an object to the speed of light is infinite, so the speed of light is a limited speed, which will not be changed.

5. Four-dimensional Spacetime - Minkowski Space
Human can only observe three-dimensional space. Describing space in mathematical models, zero-dimension is a point without units; one-dimension is a straight line or curve expressed with the length units like meter; two-dimension is a flat or curve surface and the unit is square meter or other area units; three-dimension is solid figure, whose unit is cubic meter or other volume units. In quantum mechanics, string theory believes that the world is eleven-dimensional.

Minkowski presented his view on relativity in 1908 [6]. To understand the special relativity, one must understand Minkowski space (figure 6), which describes four-dimensional space (figure 7) according to special relativity and Lorentz transformation.
There are two kinds of invariance arising in association with the equations of Newton’s mechanics. One is the invariance connected with an arbitrary change of position and another connected with uniform translation. Compared traditional space (three-dimensional space, according to Newton’s mechanics and Galilean translation), Minkowski space exists in a fictitious four-dimensional space, which includes time axis and space axis [6]. Minkowski explained the first fact that if the x, y, z axes for t = 0 are related around the origin of coordinates, then the expression remains invariant:

\[ x^2 + y^2 + z^2 \]

In addition, he introduced the second fact that the laws of mechanics remain unchanged under some new transformations that send x, y, z, t to x – αt, y – βt, z – γt, t, with constant values α, β, γ. Under these transformations, the axis t can be given whatever upward direction, which represents past and future.

6. The Relationship Between Time and Gravity
The relationship between time and gravity was mentioned in general relativity by Einstein. In general relativity, time should appear to run slower when it is near a massive body such as the earth [9].

If there is a man in an elevator in empty space without gravity so there is not “up” or “down”, elevator starts to move with constant acceleration while he is floating freely. Subsequently, he must feel gravity that a pull toward one end of the elevator, which suddenly seems like he is on the floor. He has already been in a gravity field. If he throw a ball, it will drop to the floor. In fact, it is led by the principle of equivalence. Both inertial mass, which is in Newton’s second law that determines how much you accelerate in response to a force, and gravitational mass in Newton’s law of gravity that determines how much gravitational force people feel are the same. Therefore, all objects in a gravitational field will fall at the same rate, no matter what their mass.
7. Conclusion
From Galilean transformation to general relativity then to some new theories in modern physics, humans have been unremittingly exploring the relationship between time and space. People are also constantly trying to change spacetime and select this themes to make films. The spacetime cannot be changed at present stage, because the precondition of it is reaching the speed of light to break the line and point of spacetime. It is impossible with our current technology. However, people have known the relationship between time and space in different situations depends on velocity, and the research will be also developed from this aspect in the future.

In reality, no matter what speed people are, people are in different time and space every moment because time is flowing and the earth is turning. Time changes with space changing, and the space is also changing as time goes by. Time and space are like a whole, even one change makes all change.

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