A Superfield Formalism of osp(1,2) Covariant Quantization

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We propose a superfield description of osp(1,2) covariant quantization by extending the set of admissibility conditions for the quantum action. We realize a superfield form of the generating equations, specify the vacuum functional and obtain the corresponding transformations of extended BRST symmetry.

1. Introduction

It is well-known that gauge field theory provides the universal setting for description of the fundamental interactions, while the manifestly covariant (Lagrangian) quantization of gauge theories in the path-integral approach is the most effective formalism for the study of their quantum properties. The main ingredient of covariant quantization is the concept of generating equations for the quantum action, expressed in terms of the corresponding generating operators and antibrackets (see, e.g., [1, 2, 3, 4]).

The general approach to covariant quantization is based on BRST symmetry [5], which is a global supersymmetry of the integrand in the vacuum functional. This symmetry was originally discovered in Yang–Mills theories (quantized according to the Faddeev–Popov rules) and afterwards generalized to extended BRST symmetry by adding so-called antiBRST transformations [6]. Extended BRST symmetry permitted Bonora, Pasti and Tonin [7] to discover a superfield description of quantum Yang–Mills theories, where this symmetry is realized as supertranslations along additional anticommuting coordinates.

The \( Sp(2) \) covariant quantization [2] realized extended BRST symmetry for arbitrary (general) gauge theories, i.e. theories of any stage of reducibility with a closed or open algebra of gauge transformations. This quantization scheme allows to describe the structure [1] of the complete configuration space \( \phi^A \) of a gauge theory in terms of irreducible representations of the group \( Sp(2) \), which leads to considerable simplifications of gauge-fixing compared to the well-known BV formalism [1] based on the standard BRST symmetry.

In [3], a superfield form of the \( Sp(2) \) covariant scheme [2] was discovered, which provides a superfield description of extended BRST symmetry for arbitrary gauge theories. This formalism allows to combine the entire set of variables [2], i.e. the fields and antifields \( (\phi^A, \phi^{*A}, \bar{\phi}_A) \), the Lagrangian multipliers \( (\pi^{Aa}, \lambda^A) \) and the sources \( J_A \) for the fields \( \phi^A \), into superfields \( \Phi^A(\theta) \) and supersources \( \Phi_A(\theta) \) defined in a superspace with two anticommuting coordinates \( \theta^a \). The quantum action is defined [3] as a functional of superfields and

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supersources which makes it possible to represent extended BRST symmetry as supertranslations (along the anticommuting coordinates) and transformations generated by the extended antibrackets, realized as superfield modifications of the extended antibrackets introduced in the $Sp(2)$ covariant approach [2].

In [4], an $osp(1,2)$ covariant scheme for general gauge theories was proposed to modify the $Sp(2)$ covariant formalism in a way allowing to ensure the symplectic invariance of the theory by means of subjecting the quantum action to a modified set of generating equations accompanied by analogous subsidiary conditions and special conditions of admissibility. As a consequence, apart form extended BRST symmetry related to the modified generating equations, the $osp(1,2)$ covariant scheme exhibits a new type of global symmetry related to the additional generating equations and admissibility conditions. This modification is achieved by extending the original set of variables [2] by auxiliary fields $\eta_A$ which allows to enlarge the set of operators encoding the generating equations so that the resulting set of operators satisfies relations isomorphic to the superalgebra $osp(1,2)$ belonging to the class of orthosymplectic superalgebras [8]. The superalgebra of the generating operators [4] contains explicit dependence on a mass parameter which enters these operators and is consequently inherited by the quantum action. It is expected [4] that the incorporated mass-dependence can be applied to achieve an $Sp(2)$ invariant renormalization of the theory. In the massless limit, the $osp(1,2)$ covariant formalism leads to the standard $Sp(2)$ covariant scheme considered in a special case of gauge-fixing and solutions to the generating equations.

In [9], a superfield description of $osp(1,2)$ covariant quantization was proposed, where the superalgebra $osp(1,2)$ is considered as a subalgebra of the superalgebra $sl(1,2)$, which can be regarded as the algebra of conformal generators in a superspace with two anticommuting coordinates. At the level of the vacuum functional, the supervariables of the formalism [2] are identical to those applied by the $Sp(2)$ covariant superfield scheme [4] with the components $\eta_A$ playing the role of sources to the fields $\phi^A$. The aim of the paper [4] was to generalize the transformations of extended BRST symmetry and those of the additional global symmetry to the full group of conformal transformations, containing the group of translations as a particular case.

Notice that the superfield description of the extended antibrackets used in [4] is different from that applied by the $Sp(2)$ covariant superfield approach [3]. Namely, the component form of these objects is identical with the extended antibrackets introduced in the original $Sp(2)$ covariant scheme [2]. This choice is due to the fact that the form of extended antibrackets [3] conflicts with the superalgebra $sl(1,2)$ satisfied by the symmetry generators [9]. In fact, this situation takes place also in the original formalism [4] based on the superalgebra $osp(1,2)$.

In the recent paper [10] it was demonstrated that the algebraic compatibility of the extended antibrackets with the symmetry generators is not sufficient for a consistent superfield quantization. It was shown [10] that without loss of generality the choice of superfield antibrackets in the form [3] is the only one compatible with extended BRST symmetry realized as supertranslations accompanied by transformations generated by the extended antibrackets. In particular, the form of extended antibrackets [2] proposed to describe extended BRST symmetry in terms of conformal transformations [3] is actually incompatible with this symmetry even in the particular case of supertranslations. This means that the problem of consistent $osp(1,2)$ covariant superfield quantization still remains open.
To advance in the solution of this problem, we observe, following [10], that the form of extended antibrackets [2] could be applied to a superfield description of extended BRST symmetry in the case of subjecting the quantum action to additional restrictions which cancel the non-invariance related to the particular choice of the extended antibrackets.

In this paper we propose a superfield approach to osp(1, 2) covariant quantization by means of extending the set of admissibility conditions for the quantum action [4, 9] on a manifestly superfield basis. This allows to set up a consistent superfield formalism which reproduces the original osp(1, 2) covariant scheme [4] as a particular case of gauge-fixing. The superfield representation of the extended antibrackets used in the present formalism was proposed in [9]. The construction of the vacuum functional is similar to the approach [3] with allowance for the necessary modifications.

The paper is organised as follows. In Section 2 we introduce the basic definitions and notations. In Section 3 we formulate the superfield rules of osp(1, 2) covariant quantization. In Section 4 we discuss the relation of the proposed formalism to the original osp(1, 2) covariant quantization. In Section 5 we summarise the results of the paper and make concluding remarks.

We use the conventions adopted in [3, 4]. Derivatives with respect to (super)sources and antifields are taken from the left, and those with respect to (super)fields, from the right. Left derivatives with respect to (super)fields are labelled by the subscript “l”.

2. Basic Definitions

Consider a superspace \( (x^\mu, \theta^a) \), where \( x^\mu \) are space-time coordinates, and \( \theta^a \) is an \( Sp(2) \) doublet of anticommuting coordinates. Notice that any function \( f(\theta) \) has a component representation,

\[
f(\theta) = f_0 + \theta^a f_a + \theta^2 f_3, \quad \theta^2 \equiv \frac{1}{2} \theta_a \theta^a,
\]

and an integral representation,

\[
f(\theta) = \int d^2 \theta' \delta(\theta' - \theta) f(\theta'), \quad \delta(\theta' - \theta) = (\theta' - \theta)^2,
\]

where raising and lowering the \( Sp(2) \) indices is performed by the rule \( \theta^a = \varepsilon^{ab} \theta_b, \theta_a = \varepsilon_{ab} \theta^b \), with \( \varepsilon^{ab} \) being a constant antisymmetric tensor, \( \varepsilon^{12} = 1 \), and integration over \( \theta^a \) is given by

\[
\int d^2 \theta = 0, \quad \int d^2 \theta \theta^a = 0, \quad \int d^2 \theta \theta^a \theta^b = \varepsilon^{ab}.
\]

In particular, for any function \( f(\theta) \) we have

\[
\int d^2 \theta \frac{\partial f(\theta)}{\partial \theta^a} = 0,
\]

which implies the property of integration by parts

\[
\int d^2 \theta \frac{\partial f(\theta)}{\partial \theta^a} g(\theta) = -\int d^2 \theta (-1)^{\varepsilon(f)} f(\theta) \frac{\partial g(\theta)}{\partial \theta^a}, \quad (2.1)
\]

where derivatives with respect to \( \theta^a \) are taken from the left.
We now introduce a set of superfields $\Phi^A(\theta), \varepsilon(\Phi^A) = \varepsilon_A$, with the boundary condition

$$\Phi^A(\theta)|_{\theta=0} = \phi^A,$$

and a set of supersources $\Phi_A(\theta)$ of the same Grassmann parity, $\varepsilon(\Phi_A) = \varepsilon_A$. The structure of the complete configuration space $\phi^A$ for a general gauge theory of $L$-stage reducibility is given by

$$\phi^A = (A^i, B^{\alpha}|a_1\ldots a_s}, C^{\alpha}|a_0\ldots a_s), \ s = 0, \ldots, L, \ (2.2)$$

where $A^i$ are initial classical fields, while $B^{\alpha}|a_1\ldots a_s}, C^{\alpha}|a_0\ldots a_s$ are pyramids of auxiliary and (anti)ghost fields, being completely symmetric $Sp(2)$ tensors of rank $s$ and $s + 1$, respectively.

For arbitrary functionals $F = F(\Phi, \Phi), G = G(\Phi, \Phi)$ we define the superbracket operations $(., .)^a, \{ . , . \}_a$

$$(F, G)^a = (-1)^{\varepsilon_A} \int d^2\theta \left\{ \frac{\partial^2}{\partial \theta^2} \left( \frac{\delta F}{\delta \Phi_A(\theta)} \right) \theta^a \frac{\delta G}{\delta \Phi_A(\theta)} - (F \leftrightarrow G)(-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)} \right\},$$

$${\{ F, G \}}_a = -\int d^2\theta \left\{ \frac{\partial^2}{\partial \theta^2} \left( \frac{\delta F}{\delta \Phi_A(\theta)} \right) \theta^2 \frac{\delta G}{\delta \Phi_B(\theta)} (\sigma_a)_B^A + (F \leftrightarrow G)(-1)^{(\varepsilon(F)+\varepsilon(G))} \right\}, \ (2.3)$$

where

$$\frac{\partial^2}{\partial \theta^2} \equiv \frac{1}{2} \varepsilon_{ab} \frac{\partial}{\partial \theta^b} \frac{\partial}{\partial \theta^a}.$$ 

Notice the properties of derivatives

$$\frac{\delta \Phi_A(\theta)}{\delta \Phi_B(\theta')} = \delta(\theta' - \theta)\delta^A_B = \frac{\delta \Phi_A(\theta)}{\delta \Phi_B(\theta')},$$

$$\frac{\delta \Phi_A(\theta)}{\delta \Phi_B(\theta')} = \delta(\theta' - \theta)\delta^A_B.$$

The elements of the matrix $(\sigma_A)_B^A \equiv -(\sigma_A)_B^A$ in eqs. (2.3) with the indices (2.2) are given by

$$(\sigma_A)_B^A \equiv \begin{cases} \delta_{\alpha_1 s}(s+1)(\sigma_A)_a^{\beta_1 \ldots \beta_s} S_{a_0 \ldots a_s}^{b_0 \ldots b_s} & A = \alpha_1 |a_1 \ldots a_s, B = \beta_1 |b_1 \ldots b_s, \\ \delta_{\alpha_1 s}(s+2)(\sigma_A)_a^{\beta_1 \ldots \beta_s} S_{a_0 \ldots a_s}^{b_0 \ldots b_s} & A = \alpha_1 |a_0 \ldots a_s, B = \beta_1 |b_0 \ldots b_s, \\ 0 & \text{otherwise}, \end{cases} \ (2.4)$$

where $S_{a_0 \ldots a_s}^{b_0 \ldots b_s}$ is a symmetrizer ($X^a$ being independent bosonic variables)

$$S_{a_0 \ldots a_s}^{b_0 \ldots b_s} \equiv \frac{1}{(s+2)!} \frac{\partial}{\partial X^{a_0}} \ldots \frac{\partial}{\partial X^{a_s}} X^a X^{b_0} \ldots X^{b_s},$$

with the properties

$$S_{a_0 \ldots a_s}^{b_0 \ldots b_s} = \frac{1}{s+2} \left( \sum_{r=0}^{s} \delta_{a_0}^{b_0} S_{a_1 \ldots a_r+1 \ldots b_s}^{b_0 \ldots b_s} + \frac{1}{s+1} \sum_{r=0}^{s} \delta_{a_0}^{a_1} S_{a_1 \ldots a_s}^{b_0 \ldots b_r+1 \ldots b_s} \right),$$

$$S_{a_0 \ldots a_s}^{b_0 \ldots b_s} = \frac{1}{s+1} \sum_{r=0}^{s} \delta_{a_0}^{b_0} S_{a_1 \ldots a_s}^{b_0 \ldots b_r+1 \ldots b_s}.$$
In eq. (2.4) we have

\[(\sigma_a)^{ab} = \varepsilon^{ac}(\sigma_c)^b = (\sigma_a)^a \varepsilon^{cb} = \varepsilon^{ac}(\sigma_c)_{ad} \varepsilon^{db}, \quad (\sigma_a)^b = -(\sigma_a)^b, \quad (\sigma_a)^{ab} = (\sigma_a)^{ba},\]

\[(\sigma_a)^a = (\sigma_a)^a = 0, \quad \varepsilon^{ad}\delta^b_c + \varepsilon^{bd}\delta^a_c = -(\sigma^a)^{ab}(\sigma_a)^c,\] (2.5)

where the matrices \(\sigma_a (\alpha = 0, +, -)\) form the algebra \(sl(2)\)

\[
\sigma_a \sigma_b = g_{ab} + \frac{1}{2} \varepsilon_{\alpha \beta \gamma} \sigma^\gamma, \quad \sigma^\alpha = g^{ab} \sigma_b, \quad \text{Tr}(\sigma_a \sigma_b) = 2g_{ab},
\]

\[g^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}, \quad g^{a\gamma} g_{\gamma b} = \delta^a_b,\]

with \(\varepsilon_{\alpha \beta \gamma}\) being an antisymmetric tensor, \(\epsilon_{0+} = 1\).

Let us introduce a set of first-order operators \(V_m^a, U_m^a\) (odd) and \(V_a, U_a\) (even),

\[
V_m^a = \int d^2 \theta \left( \frac{\partial \Phi_A(\theta)}{\partial \theta_a} \delta \Phi_A(\theta) + m^2 \left( (P_+)^{Ba} \theta^b \frac{\partial^2}{\partial \theta^2} (\theta^2 \Phi_B(\theta)) \right) \frac{\delta}{\delta \Phi_A(\theta)} \right)
\]

\[
U_m^a = \int d^2 \theta \left( \frac{\partial \Phi_A(\theta)}{\partial \theta_a} \delta \Phi_A(\theta) - m^2 \left( (P_+)^{Ba} \theta^b \frac{\partial^2}{\partial \theta^2} (\theta^2 \Phi_A(\theta)) \right) \frac{\delta}{\delta \Phi_B(\theta)} \right)
\]

\[
V_a = \int d^2 \theta \left( \Phi_B(\sigma_a)^B \Phi_A(\theta) \frac{\delta}{\delta \Phi_A(\theta)} - \frac{\partial^2}{\partial \theta^2} (\Phi_B(\theta) \theta^b) (\sigma_a)^b \theta^a \frac{\delta}{\delta \Phi_A(\theta)} \right)
\]

\[
U_a = \int d^2 \theta \left( \Phi_A(\sigma_a)^B \Phi_A(\theta) \frac{\delta}{\delta \Phi_A(\theta)} + \frac{\partial^2}{\partial \theta^2} (\Phi_A(\theta) \theta^a) (\sigma_a)^b \theta^b \frac{\delta}{\delta \Phi_A(\theta)} \right)
\] (2.6)

where \(m\) is a mass parameter. The matrices \((P_+)^{Ba}, (P_+)^{Ba}\) in eqs. (2.3) are given by

\[(P_+)^{Ba} = (P_+)^{Ba} - (P_+)^a_{Bb} \delta^a B + \delta^B \delta^a, \quad (P_+)^B = \delta^b (P_+)^{Ba}, \quad (\sigma_a)^B = (\sigma_a)^b (P_+)^{Ba},\]

with

\[
(P_+)^{Ba} = \begin{cases} 
\delta^a B & A = i, B = j, \\
\delta^a (s + 1) S_{a}^{b_{1 \cdots b_{s}}} & A = \alpha_{s} \alpha_{1 \cdots b_{s}}, B = \beta_{s} b_{1 \cdots b_{s}}, \\
\delta^a (s + 2) S_{a_{1 \cdots b_{s}}} & A = \alpha_{s} a_{1 \cdots b_{s}}, B = \beta_{s} b_{1 \cdots b_{s}}, \\
0 & \text{otherwise}
\end{cases}
\]

From the above definitions follow the relations [4]

\[\varepsilon^{ad}(P_+)^{Ba}_{Ad} + \varepsilon^{bd}(P_+)^{Ba}_{Ad} = -(\sigma^a)^{ab}(\sigma_a)^B,\]

\[\varepsilon^{ad}(P_+)^{Ba}_{Ac} + \varepsilon^{bd}(P_+)^{Ba}_{Ac} = -(\sigma^a)^{ab}(\sigma_a)^c (P_+)^{Ba}_{Ac} = -(\sigma^a)^{ab}(\sigma_a)^c \delta^B_A + \delta^d (\sigma_a)^B_A\]

and the property \((P_+)^{Ab}_{Cd} (P_+)^{Cd}_{Ba} = 0\).

The operators introduced in eqs. (2.6) obey the \(osp(1, 2)\) superalgebra [8], with the following non-trivial (anti)commutation relations:

\[V_a V_b = \varepsilon_{\alpha \beta} V_{\gamma}, \quad [V_a, V^b_m] = V^b_m (\sigma_a)^b, \quad \{V^a_m, V^b_m\} = -m^2 (\sigma^a)^{ab} V_a,\]

\[5\]
\[ [U_\alpha, U_\beta] = -\epsilon_{\alpha\beta} U_\gamma, \quad [U_\alpha, U^a_\beta] = -U^b_m (\sigma_\alpha)_b^a, \quad \{U^a_m, U^b_m\} = m^2 (\sigma^a)^{ab} U_a. \quad (2.7) \]

Let us introduce a set of second-order operators \( \Delta^a \) (odd) and \( \Delta_\alpha \) (even)

\[
\Delta^a = \int d^2 \theta \frac{\partial^2}{\partial \theta^2} \left( \frac{\delta l}{\delta \Phi_A(\theta)} \right) \frac{\delta}{\delta \Phi_A(\theta)} \theta^a \delta \bar{\Phi}_A(\theta), \\
\Delta_\alpha = (-1)^{\epsilon_\alpha+1} \int d^2 \theta \frac{\partial^2}{\partial \theta^2} \left( \frac{\delta l}{\delta \Phi_A(\theta)} \right) \theta^2 \delta \bar{\Phi}_B(\theta) (\sigma_\alpha)_B^A. \quad (2.8) \]

These operators possess the algebraic properties

\[
[\Delta_\alpha, \Delta_\beta] = 0, \quad \{\Delta^a, \Delta^b\} = 0, \quad [\Delta_\alpha, \Delta^a] = 0, \quad (2.9) \]

\[
[\Delta_\alpha, V_\beta] + [V_\alpha, \Delta_\beta] = \epsilon_{\alpha\beta} \gamma \Delta_\gamma, \\
\{\Delta_\alpha, V^a_\beta\} + \{V^a_\alpha, \Delta^b_\beta\} = -m^2 (\sigma^a)^{ab} \Delta_\alpha, \\
[\Delta_\alpha, V^a_\beta] + [V^a_\alpha, \Delta^b_\beta] = \Delta^b (\sigma_\alpha)_b^a. \quad (2.10) \]

From eqs. (2.8) it follows that the action of the operators \( \Delta^a \) and \( \Delta_\alpha \) on the product of two functionals defines the superbracket operations (2.3), namely

\[
\Delta_\alpha (FG) = (\Delta_\alpha F)G + F(\Delta_\alpha G) + \{F, G\}_\alpha, \\
\Delta^a (FG) = (\Delta^a F)G + F(\Delta^a G)(-1)^{\epsilon(F)} + (F, G)^a(1)^{\epsilon(F)}. \quad (2.11) \]

Using eqs. (2.9), (2.10), (2.11) one can derive the properties of the superbrackets at the algebraic level [4].

Let us introduce the operators

\[
\tilde{\Delta}^a_m \equiv \Delta^a + \frac{i}{\hbar} V^a_m, \quad \tilde{\Delta}_\alpha \equiv \Delta_\alpha + \frac{i}{\hbar} V_\alpha. \quad (2.12) \]

From eqs. (2.7), (2.9), (2.10) it follows that these operators obey the superalgebra

\[
[\tilde{\Delta}_\alpha, \tilde{\Delta}_\beta] = (i/\hbar) \epsilon_{\alpha\beta} \gamma \tilde{\Delta}_\gamma, \quad [\tilde{\Delta}_\alpha, \tilde{\Delta}^a_m] = (i/\hbar) \tilde{\Delta}^b_m (\sigma_\alpha)_b^a, \quad \{\tilde{\Delta}^a_m, \tilde{\Delta}^b_m\} = -(i/\hbar) m^2 (\sigma^a)^{ab} \tilde{\Delta}_\alpha, \]

isomorphic to \( osp(1, 2) \).

### 3. Superfield osp(1,2) Covariant Quantization

Let us consider a superfield analogue of the \( osp(1, 2) \) covariant formalism [4] constructed along the lines of the \( Sp(2) \) covariant superfield scheme [3]. Define the vacuum functional \( Z_m \) depending on the mass parameter \( m \) as the following path integral:

\[
Z_m = \int d\Phi d\bar{\Phi} \exp \left\{ \frac{i}{\hbar} \left[ W_m(\Phi, \bar{\Phi}) - \frac{1}{2} \epsilon_{\alpha\beta} U^a_m U^b_m F(\Phi) + m^2 F(\Phi) + \bar{\Phi} \Phi \right] \right\}, \quad (3.1) \]
where $W_m = W_m(\Phi, \bar{\Phi})$ is the $m$-extended quantum action that satisfies the generating equations

$$\Delta_m^a \exp \left\{ \frac{i}{\hbar} W_m \right\} = 0, \quad (3.2)$$

and the subsidiary conditions

$$\Delta_\alpha \exp \left\{ \frac{i}{\hbar} W_m \right\} = 0, \quad (3.3)$$

with $\Delta_m^a$ and $\Delta_\alpha$ given by eqs. (2.12). Eqs. (3.2) and (3.3) are equivalent to

$$\frac{1}{2} (W_m, W_m)^a + V_m^a W_m = i\hbar \Delta^a W_m, \quad (3.4)$$

$$\frac{1}{2} \{W_m, W_m\}_\alpha + V_{\alpha} W_m = i\hbar \Delta_\alpha W_m, \quad (3.5)$$

where the superbrackets $(\ , \ )^a$, $\{\ , \ \}_\alpha$ and the operators $V_m^a$, $V_\alpha$, $\Delta^a$, $\Delta_\alpha$ are defined by eqs. (2.3), (2.6), (2.8). The quantum action $W_m$ is assumed to be an admissible solution of eqs. (3.4) and (3.5), which implies the fulfillment of the restrictions

$$\int d^2\theta \theta^2 \left\{ \frac{\delta W_m}{\delta \Phi_A(\theta)} + \Phi^A(\theta) \right\} = 0, \quad (3.6)$$

$$\int d^2\theta \theta^2 \frac{\delta W_m}{\delta \Phi_A(\theta)} = 0, \quad (3.7)$$

$$\int d^2\theta \theta^a \frac{\delta W_m}{\delta \Phi_A(\theta)} = 0. \quad (3.8)$$

In eq. (3.1), $\Phi \Phi$ is a functional of the form

$$\Phi \Phi = \int d^2\theta \Phi_A(\theta) \Phi^A(\theta), \quad (3.9)$$

while $F = F(\Phi)$ is a gauge-fixing Boson restricted to the class of $Sp(2)$ scalars by the conditions

$$U_\alpha F(\Phi) = 0, \quad (3.10)$$

where $U_\alpha$ are the operators (2.6).

An important property of the integrand in eq. (3.1) is its invariance under the following transformations:

$$\delta \Phi^A(\theta) = \mu_a U_m^a \Phi^A(\theta), \quad \delta \bar{\Phi}_A(\theta) = \mu_a V_m^a \bar{\Phi}_A(\theta) + \mu_a (W_m, \bar{\Phi}_A(\theta))^a, \quad (3.11)$$

$$\delta \Phi^A(\theta) = \mu^a U_\alpha^a \Phi_A(\theta), \quad \delta \bar{\Phi}_A(\theta) = \mu^a V_\alpha^a \bar{\Phi}_A(\theta) + \mu^a \{W_m, \bar{\Phi}_A(\theta)\}_\alpha, \quad (3.12)$$

where $U_m^a$ are operators given by eqs. (2.6), while $\mu_a$ and $\mu^a$ are constant (anti)commuting parameters, $\varepsilon(\mu_a) = 1$, $\varepsilon(\mu^a) = 0$. Eqs. (3.11) realize the transformations of extended BRST symmetry and eqs. (3.12) express the symmetry related to the requirement of $Sp(2)$ invariance. The validity of the symmetry transformations (3.11), (3.12) follows from the generating equations (3.4), (3.5), the admissibility conditions (3.7), (3.8), the conditions (3.10) of $Sp(2)$ invariance for the gauge-fixing Boson, the algebraic properties (2.7) of the
operators \( U^a_m, U_a \), and the properties (2.5) of the matrices \( \sigma_a \). Besides, it is necessary to use integration by parts (2.1) as well as to take into account the operator representation (\( U^a_m \) are first-order operators)

\[
U^a_m U^b_m F(\Phi) = \{ U^a_m, [U^b_m, F(\Phi)] \}, \quad \varepsilon(F) = 0
\]

and the Jacobi identity

\[
[[\hat{F}, \hat{G}], \hat{H}](1) \varepsilon(\hat{F})\varepsilon(\hat{H}) + \text{cycl. perm.} (\hat{F}, \hat{G}, \hat{H}) \equiv 0
\]

for the supercommutator \([, ,]\).

Notice that the admissibility conditions (3.6) have not been used in the proof of invariance. These conditions serve to establish the relation of the present superfield formalism to the original \(osp(1,2)\) covariant scheme [4]. As will be explained in the next section, the conditions (3.6) are closely related to the absence in the path integral (3.1) of an additional integration weight used in the \(Sp(2)\) covariant superfield formalism [3].

### 4. Component Analysis

We now consider the component representation of the formalism proposed in the previous section in order to establish its relation to the original \(osp(1,2)\) covariant scheme [4].

The component form of superfields \( \Phi^A(\theta) \) and supersources \( \bar{\Phi}_A(\theta) \) reads

\[
\Phi^A(\theta) = \phi^A + \pi^A a \theta_a + \lambda^A \theta^2,
\]

\[
\bar{\Phi}_A(\theta) = \bar{\phi}_A - \theta^a \bar{\phi}^*_A - \theta^2 \bar{\eta}_A.
\]

The components \( (\phi^A, \pi^A a, \lambda^A, \bar{\phi}_A, \bar{\phi}^*_A, \bar{\eta}_A) \) are identical with the set of variables required for the construction of the vacuum functional in the \(osp(1,2)\) covariant formalism [4].

Denote \( F(\Phi, \bar{\Phi}) \equiv \bar{F}(\phi, \pi, \lambda, \bar{\phi}, \bar{\phi}^*, \eta) \). The superbrackets \( (, )^a \), \( \{ , \}_a \) in eqs. (2.3) have the following component representation:

\[
(F, G)^a = \delta F \frac{\delta G}{\delta \phi^A} \delta \phi^*_A - (\bar{F} \leftrightarrow \bar{G}) (1) \varepsilon(F)\varepsilon(G) + 1,
\]

\[
\{F, G\}_a = (\sigma_a)_B^A \delta F \frac{\delta G}{\delta \phi_A \delta \eta_B} + (\bar{F} \leftrightarrow \bar{G}) (-1)^{\varepsilon(F)\varepsilon(G)}.
\]

The component form of the second-order operators \( \Delta^a, \Delta_a \) given by eqs. (2.8) reads

\[
\Delta^a = (-1) \varepsilon^a \frac{\delta l^a}{\delta \phi^A \delta \phi^*_A},
\]

\[
\Delta_a = (-1) \varepsilon^a (\sigma_a)_B^A \frac{\delta l^a}{\delta \phi^A \delta \eta_B}.
\]

Eqs. (1.1), (4.2) coincide with the superbrackets and corresponding delta-operators used in the framework of the \(osp(1,2)\) covariant formalism [4].

The first-order operators \( V^a_m, V_a \) given by eqs. (2.6) have the following component representation:

\[
V^a_m = \varepsilon^{ab} \phi^*_a \frac{\delta}{\delta \phi^b_A} - \eta_A \frac{\delta}{\delta \phi^*_A} + m^2 (P_+)_{ab} \frac{\delta}{\delta \phi^*_a} \frac{\delta}{\delta \phi^*_b} - m^2 \varepsilon^{ab} (\bar{P} \rightarrow B)_{ac} \phi^*_c \frac{\delta}{\delta \phi^*_b},
\]

\[
V_a = \bar{\phi}_B (\sigma^B_A \frac{\delta}{\delta \phi^b_A} + \bar{\phi}^*_a (\sigma^B_A B) a \frac{\delta}{\delta \phi^*_A} + \eta_B (\sigma^B_A B) a \frac{\delta}{\delta \eta_A}.
\]
Eqs. (4.1)–(4.3) imply that the superfield generating equations (3.2), (3.3), or equivalently (3.4), (3.5), formally coincide with the generating equations for the quantum action in the framework of the \( \text{osp}(1,2) \) covariant approach [4].

The component form of the first-order operators \( U^a_m, U_\alpha \) given by eqs. (2.6) reads

\[
U^a_m = (-1)^{\varepsilon_a} \varepsilon^{ab} \lambda^A \frac{\delta l}{\delta \pi^{Ab}} - (-1)^{\varepsilon_a} \pi^{Aa} \frac{\delta l}{\delta \phi^A} + m^2 \varepsilon^{ab} (-1)^{\varepsilon_a} (P_+)^B_{Ab} \phi^A \frac{\delta l}{\delta \pi^{Bc}}
\]

\[-m^2 (-1)^{\varepsilon_a} (P_-)^B_{Ab} \phi^A \frac{\delta l}{\delta \lambda^B},
\]

\[
U_\alpha = \phi^B (\sigma_\alpha)_B^A \frac{\delta l}{\delta \phi^A} + (\pi^{Ab} (\sigma_\alpha)_B^a + \pi^{Ba} (\sigma_\alpha)_A^B) \frac{\delta l}{\delta \pi^{Aa}} + \lambda^B (\sigma_\alpha)_B^A \frac{\delta l}{\delta \lambda^A}.
\]  

The admissibility conditions (3.7), (3.8) for the quantum action \( \tilde{W}_m = \tilde{W}_m(\phi, \pi, \lambda, \bar{\phi}, \phi^*, \eta) \) can be represented as

\[
\frac{\delta \tilde{W}_m}{\delta \lambda^A} = \frac{\delta \tilde{W}_m}{\delta \pi^{Aa}} = 0.
\]  

These conditions have been introduced in order to compensate the non-invariance of the integrand in eq. (3.1) related to the specific choice of the superbrackets (1.1). At the same time, eqs. (4.3) restrict the variables of the functional \( \tilde{W}_m \) to the set \( (\phi^A, \bar{\phi}, \phi^*, \eta) \), which is the complete set of variables entering the quantum action in the original \( \text{osp}(1,2) \) covariant formalism [4]. On the hypersurface (4.5) the generating equations (3.5) become identical with the generating equations of the \( \text{osp}(1,2) \) covariant scheme.

The component form of the remaining admissibility condition (3.6)

\[
\frac{\delta \tilde{W}_m}{\delta \eta_A} = \phi^A
\]  

leads to an additional simplification of the quantum action:

\[
\tilde{W}_m = \mathcal{W}_m(\phi, \bar{\phi}, \phi^*) + \eta_A \phi^A.
\]  

From the viewpoint of the \( \text{osp}(1,2) \) covariant approach [4], this form of dependence on the sources \( \eta_A \), with allowance for eqs. (2.4), (2.5), gives the advantage of transforming the generating equations (3.4) into the simplified requirement of \( S^p(2) \) invariance [4]

\[
(\sigma_\alpha)_B^A \frac{\delta \tilde{W}_m}{\delta \phi^B} + V_\alpha \tilde{W}_m = 0.
\]

Let us restrict the gauge-fixing Boson in the vacuum functional (3.1) to the class of gauges used in the original \( \text{osp}(1,2) \) covariant formalism [4], i.e. gauges depending only on the fields: \( \tilde{F} = \tilde{F}(\phi) \). Then, with allowance for the component representation (1.4) of the operators \( U_\alpha \), the condition (3.10) of \( S^p(2) \) invariance imposed on the gauge-fixing Boson reduces to

\[
(\sigma_\alpha)_B^A \frac{\delta \tilde{F}}{\delta \phi^A} \frac{\delta \tilde{F}}{\delta \phi^B} = 0,
\]  

which, according to eq. (4.6), can be represented as

\[
(\sigma_\alpha)_B^A \frac{\delta \tilde{F}}{\delta \phi^A} \frac{\delta \tilde{W}_m}{\delta \eta_B} = 0.
\]
Eqs. (4.8) and (4.9) reproduce the whole set of additional restrictions imposed on the quantum action and gauge-fixing Boson in the $osp(1,2)$ covariant scheme [4]. Namely, eq. (4.8) is imposed to ensure the symplectic invariance of the gauge-fixing Boson [4], while the equation (4.9) emerges as a condition of admissibility for the quantum action, introduced to provide an $Sp(2)$ invariant gauge-fixing [4].

Notice that in the $osp(1,2)$ covariant formalism [4] the condition (4.6) is redundant, being in fact a particular solution of a more fundamental equation (4.9) imposed on the quantum action. On the contrary, in the present formalism the status of these two conditions is reversed, namely, eq. (4.9) emerges as a consequence of eq. (4.6), corresponding to a particular case of gauge-fixing.

The crucial role of the condition (4.6) for the present formalism is due to the following reasons. On the one hand, a superfield description of the $osp(1,2)$ covariant scheme [4] in terms of the variables $\Phi^A(\theta), \Phi^{\bar{A}}(\theta)$ requires a linear dependence of the quantum action on the auxiliary fields $\eta_A$. Indeed, if this dependence in the original $osp(1,2)$ covariant formalism is more than linear, then the vacuum functional must be parameterized by a set of variables [4] which is larger than the set of components of supervariables. On the other hand, the admissibility condition (4.6) allows to cancel the dependence on $\eta_A$ in the integrand (3.1) by observing that the functional $\Phi\Phi$ in eq. (3.9) has the component form

$$\Phi\Phi = \Phi_A^A + \Phi^{\bar{A}} \bar{A} - \eta_A \Phi^A.$$  (4.10)

This cancellation does not conflict with the original $osp(1,2)$ prescription [4], where the $\eta$-dependence is integrated out by imposing the delta-functional constraint $\delta(\eta)$, being, in fact, identical with the integration weight introduced within the $Sp(2)$ covariant superfield formalism [3]. At the same time, the presence of this integration weight in the case of a non-trivial dependence on $\eta_A$ violates the invariance of the integrand under the transformations (3.11), (3.12) because of the non-invariance of $\eta_A$, which can be observed from the component representation (4.3) of the symmetry generators $V^a_m, V_a$. Naturally, in the absence of $\eta$-dependence the integration weight $\delta(\eta)$ can be omitted, as in the case of the present approach (3.1), compared to the formalism [3].

Concluding, we demonstrate the relation of the vacuum functional (3.1), given in terms of $\tilde{W}_m = \tilde{W}_m(\phi, \bar{\phi}, \phi^*, \eta)$ and $\tilde{F} = \tilde{F}(\phi)$, to the vacuum functional of the $osp(1,2)$ covariant approach [4].

Notice that the integration measure in eq. (3.1) has the component representation

$$d\Phi d\bar{\Phi} = d\phi d\bar{\phi} d\pi d\bar{\pi} d\lambda d\bar{\lambda} d\eta.$$  

Using the component form of the operators $U^a_m$ given by eqs. (4.4) and integrating out the variables $\eta_A$ with allowance for eqs. (4.7), (4.10), we represent the vacuum functional (3.1) in the form

$$Z_m = \int d\phi d\phi^* d\pi d\bar{\pi} d\lambda d\bar{\lambda} \exp \left\{ \frac{i}{\hbar} (\mathcal{W}_m + \mathcal{X}_m + \bar{\phi}_A^A \lambda^A + \phi^*_{A} \bar{A} - \eta_A \Phi^A) \right\},$$  (4.11)

where the quantum action $\tilde{W}_m = \mathcal{W}_m + \eta_A \Phi^A$ satisfies eqs. (3.4), (3.5), (4.6), and the gauge-fixing term $\mathcal{X}_m$ is given by

$$\mathcal{X}_m = \frac{\delta \tilde{F}}{\delta \phi^A} \lambda^A - \frac{1}{2} m^2 (P^-)_B^A \delta \tilde{F} \phi^A - \frac{1}{2} \varepsilon_{ab} \bar{A} \bar{A} \delta \tilde{F} \bar{A} \bar{B} \bar{B} + m^2 \tilde{F},$$
with $\tilde{F}$ subject to eq. (4.8).

The vacuum functional of the $osp(1,2)$ covariant formalism [4] can be represented as

$$Z_m = \int d\phi \exp\{(i/\hbar)S_{m,\text{eff}}\}, \quad (4.12)$$

with

$$S_{m,\text{eff}}(\phi) = S_{m,\text{ext}}(\phi, \bar{\phi}, \phi^*, \eta)|_{\bar{\phi} = \phi^* = \eta = 0},$$

$$\exp\{(i/\hbar)S_{m,\text{ext}}\} = \hat{U}_m(Y) \exp\{(i/\hbar)S_m\},$$

where $S_m = S_m(\phi, \bar{\phi}, \phi^*, \eta)$ is the quantum action obeying the system of generating equations and subsidiary conditions (3.4), (3.5), (4.6) satisfied by $\tilde{W}_m = \tilde{W}_m(\phi, \bar{\phi}, \phi^*, \eta)$, and $\hat{U}_m(Y)$ is an operator of the form

$$\hat{U}_m(Y) = \exp\left\{\frac{\delta Y}{\delta \phi^A} \left(\frac{\delta}{\delta \bar{\phi}^A} - \frac{1}{2} m^2 (P^{-}_B) \frac{\delta}{\delta \eta_B} \right) - \frac{\hbar}{2i} \varepsilon_{ab} \frac{\delta}{\delta \phi_A^a} \frac{\delta^2 Y}{\delta \phi_B^b} \frac{\delta}{\delta \phi^*_{Bb}} + i \frac{\hbar}{m^2} Y\right\},$$

where $Y = Y(\phi)$ is a gauge-fixing $Sp(2)$ scalar restricted by the conditions (4.8) imposed on $\tilde{F} = \tilde{F}(\phi)$.

To establish the identity between the vacuum functionals (4.11) and (4.12), it is sufficient to set $S_m = \tilde{W}_m$, $Y = \tilde{F}$.

5. Concluding Remarks

In this paper we have proposed a superfield description of the $osp(1,2)$ covariant quantization formalism [4].

We have found superfield representations of the generating equations [4], constructed the superfield vacuum functional and found the corresponding transformations of extended BRST symmetry as well as the transformations of the additional global symmetry related to symplectic invariance. We have shown that the component representation of the formalism reduces to the original $osp(1,2)$ covariant quantization scheme [4] in a particular case of gauge-fixing.

On the one hand, the present approach is based on the component realization of extended antibrackets in the form introduced within the $Sp(2)$ covariant scheme [2], which provides the algebraic compatibility of the antibrackets with the generators of symmetry transformations obeying the superalgebra $osp(1,2)$. On the other hand, the formalism is based on extending the set of admissibility conditions [4, 9] for the quantum action, which allows to introduce the transformations of extended BRST symmetry in a manifestly superfield form by canceling the non-invariance related to the specific choice of the antibrackets.

In our opinion, the approach used in this paper to provide a superfield description of extended BRST symmetry on the basis of the $osp(1,2)$ superalgebra of symmetry generators should be considered as an intermediate step to the complete solution of the problem of $osp(1,2)$ covariant superfield quantization. Namely, it should be expected that a formalism providing such a solution must contain the $Sp(2)$ covariant superfield scheme [3] in the massless limit, which is suggested by the relation between the original $Sp(2)$ and $osp(1,2)$ covariant methods. The difficulty of the realization of such a program lies in setting up a formalism providing compatibility of the extended antibrackets used in the $Sp(2)$ covariant superfield scheme with the $osp(1,2)$ superalgebra of symmetry generators.
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