Constraining churning and blurring in the Milky Way using large spectroscopic surveys – an exploratory study

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ABSTRACT

We have investigated the possibilities to quantify how much stars move in the Milky Way disc due to diffuse processes (blurring) and due to influences from spiral arms and the bar (churning). We assume that the formation radius of a star can be inferred if we know its elemental abundances and age and the metallicity profile of the interstellar medium at the time of the star’s formation. We use data for red giant branch stars from APOGEE DR14, parallaxes from Gaia, and stellar ages based on the C and N abundances. In our sample, we find that half of the stars have experienced some sort of radial migration, 10 per cent likely have suffered only from churning, and a modest 5–7 per cent have never experienced either churning or blurring making them ideal tracers of the original properties of the cool stellar disc. To arrive at these numbers, we imposed the requirement that the stars that are considered to be churned have highly circular orbits. If instead we require that the star has moved away from its formation position and at the same time that its Galactocentric radius at formation did not fall between the apo- and pericentre of its orbit today, we find that about half of the stars have undergone a radial migration. We have thus shown that it is possible to put up a framework to quantify churning and blurring. Future work includes investigations of how selection effects influence the results.

Key words: stars: kinematics and dynamics – ISM: abundances – Galaxy: evolution.

1 INTRODUCTION

It has long been understood that a star moving in the Galactic potential will be subject to transient gravitational interactions, such as passing close to another star or a giant molecular cloud or interacting with the density enhancement caused by a spiral arm (e.g. Wielen 1977). Many of these mechanisms have been studied in quite some detail and have allowed us to, for example, understand the structure in velocity space of the stars in the solar neighbourhood has invoked the idea that the Sun potentially came from a region interior to the current position of the Sun (Wielen, Fuchs & Dettbarn 1996). However, no reasonable set of physical interactions had been able to explain how the Sun could have undertaken this journey. Sellwood & Binney (2002) showed that another process, ‘churning’, can move a star from a more or less circular orbit to another circular orbit, thus erasing all memory of the past kinematic history of the star – this means that we cannot integrate the orbits of stars backwards to figure out where they came from (an example can be found in Martínez-Barbosa et al. 2016).

Since the seminal paper by Sellwood & Binney (2002), several studies have shown that the effects of radial migration (churning and blurring combined) can be substantial (Roškar et al. 2008; Loebman et al. 2011). However, although there is no doubt that the mechanisms resulting in churning and blurring are present, a proper quantification of their respective importance in the Milky Way stellar disc is still lacking. This is partially due to a lack of a large enough and reliable enough data set but also partially due to the fact that we are still exploring efficient ways to study these effects. Frankel et al. (2018) provide a model that aims to quantify the global efficiency of radial migration among stars in the Milky Way. Using data from APOGEE DR12 (Alam et al. 2015) they find that with a radial orbit migration efficiency of $3.6 \pm 0.1$ kpc. In this model, the Sun might have a formation radius of about $5.2$ kpc, a quite substantial distance away from its present position.

In this study, we explore a possible way to quantify the fraction of stars that have migrated radially in the Galaxy. We also attempt to derive the fractions of stars that have been blurred and churned, respectively. Our method borrows ideas from Minchev et al. (2018) and Grenon (1987) and is quite simple. What we do is to assume that there exists a model that describes how the radial metallicity gradient in the interstellar medium (ISM) in the Milky Way has evolved over time. Then, if we know the age and the metallicity of a star, it is straightforward to derive the Galactocentric radius at which the star formed. This then allows us to calculate how far the star has moved radially in our Galaxy from when it formed till today. Combining such information with orbital data allows to further study how stars with different kinematic properties have moved – enabling us to put up a method to constrain both churning and blurring.

In this way, we are able to identify the individual stars that have migrated and can, for example, study the properties of blurred stars and contrast them with the properties of the churned stars. This
allows, eventually, for a fine-grained understanding of the underlying stellar populations. It also allows a study of the properties of stars that have not moved from where they formed, providing further constraints on our understanding of how the stellar disc in the Milky Way formed.

This paper is organized as follows. Section 2 describes the data set used in this exploratory study, Section 3 explains the method and describes the different sets of ISM profiles we use. In Section 4, we derive upper limits on how many stars have radially migrated and how many have been churned or blurred in our sample. We also discuss limitations of the sample (selection effects) and take a look at how cosmological simulations could potentially be used to provide the evolution of the ISM. Section 4.5 provides a summarizing discussion of our results. Section 5 concludes the paper with a summary of our findings.

2 DATA

2.1 APOGEE and Gaia data

We use data from APOGEE [Sloan Digital Sky Survey IV (SDSS-IV) data release 14; Majewski et al. 2017] and Gaia data release 2 (DR2; Gaia Collaboration 2016, 2018a). From APOGEE DR14, we select all stars that fulfill the following four criteria:

(i) They are part of the main survey targets.
(ii) There were no failures in determining
   (a) the effective temperature ($T_{\text{eff}}$)
   (b) the surface gravity ($\log g$)
   (c) the rotation
   (d) the overall iron abundance ([Fe/H]$^1$).

(iii) Radial velocity could be determined for the star and the error in the radial velocity is $<0.5$ km s$^{-1}$.

(iv) The uncertainty in the determination of [Fe/H] is $<0.05$ dex.

Criterion (iii) ensures that we exclude potential binaries from the sample. The selected stars from APOGEE DR14 were cross-matched with Gaia DR2. We further require that the relative uncertainty in the parallax measured by Gaia is less than 10 per cent. This cut allows us to do robust and straightforward calculations of the stellar orbits (compare, e.g. discussion in Gaia Collaboration 2018b, about selecting the best stars for studying fine structure in the observed Herzspring–Russel diagram).

The combined sample drawn from APOGEE DR14 and Gaia DR2 fulfilling the criteria listed above amounts to about 85,000 stars. Fig. 1 shows [Mg/Fe] as a function of [Fe/H] for the sample.

2.2 Calculations of stellar orbits

Using the astrometric data from Gaia DR2 and radial velocities from APOGEE DR14, we used GALPY$^2$ (Bovy 2015) to calculate velocities and orbital parameters for the stars. We used the potential built into GALPY (MWPotential 2014; see Bovy 2015). The potential consists of a bulge modelled as a spherical potential with an exponential cut-off, a Miyamoto–Nagai potential modelling the disc, and a Navarro–Frenk–White dark matter halo (Miyamoto & Nagai 1975; Navarro, Frenk & White 1996). We use the code’s default values for the position and motion of the solar neighbourhood (position is 8 kpc from the Galactic centre and a circular velocity of 220 km s$^{-1}$). For further description of GALPY and its implementation, we refer the reader to Bovy (2015).

Each star is integrated in this potential from its current position forward 3 Gyr, which should be ample time for multiple, full Galactic orbits for any of the stars under consideration.

Our sample includes stars with orbits that reach deep into the stellar disc and therefore gets close to the region that potentially is much influenced by the bar. GALPY allows us to add a bar to the potential. We did so following the description in Dehnen (2000). We find that for the sample currently under investigation, the inclusion of a bar only marginally changes our derived orbital properties (see also Appendix B). We therefore choose to use the results without a bar.

Following Liu & van de Ven (2012), we also derive $L_c/L_z$ for a more robust estimate of the circularity of the orbits. Here, $L_c$ is the angular momentum in the $z$-direction (in cylindrical coordinates) while $L_z$ is the angular momentum in the $z$-direction the star would have were it on a circular orbit characterized by the same energy as the current orbit. Thus,

$$L_c = R v_c,$$

where $R_c$ follows from solving

$$E = \Phi + v_c^2/2,$$

where $E$ is the orbital energy and $v_c$ is the circular velocity, defined as

$$v_c = R_0 \Phi / \partial R$$

for $z = 0$.

The resulting distributions of $L_c/L_z$ are shown in Fig. 2(d). Fig. B1 shows a comparison of ecc. and $L_c/L_z$ for all stars in our sample. Overall, the two measures correlate well, but there are deviations. Following Liu & van de Ven (2012), we use the $L_c/L_z$ for the majority of our investigations; however, we apply a stricter definition of orbital circularity than they do (see Section 3.4).

2.3 Stellar ages

We need stellar ages for our investigation. However, our stellar sample consists entirely of giant stars (most of them being on or near the red clump) and determining ages is difficult for such stars. We make use of the investigation by Martig et al. (2016), who used carbon and nitrogen to infer the mass of the stars and hence provide the possibility to place the star on the right track in the HR-diagram.

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1. We use the standard notation where $[X/H] = \log(N_X/N_H)_{\text{sun}} - \log(N_0/N_1)_{\text{sun}}$, X being any element.
2. Astrophysics Source Code Library, record ascl:1411.008, https://galpy.readthedocs.io/en/v1.4.0/
and derive its age. These ages are good to about 40 per cent; this means that we cannot study ages of individual stars but the ensemble properties should be relatively robust.

2.4 Properties of the full sample

The final sample drawn from APOGEE DR14, Gaia DR2 (Gaia Collaboration 2018a), and Martig et al. (2016) with stellar orbits calculated using GALPY comprises some 18 000 stars. In this section, we describe the overall characteristics of the sample as well as validate that the ages provide a reasonable description of the stars in the stellar disc.

Fig. 2 shows the major characteristics of our sample. Figs 2(a)–(d) show how the full sample covers Galacticentric distances from about 6 to 11 kpc and how their Galacticentric distances relate to their other properties. The stars reach maximum distances above the plane ($z_{\text{max}}$) of several kpc, but the majority do not reach beyond 1.5 kpc. Metallicities range from $-0.5$ dex to supersolar and their distribution in Galacticentric distances does not depend on metallicity. Stellar ages range from 0.5 to about 10 Gyr. The overdensity of stars beyond the solar orbit has ages in the younger range while stars inside the solar orbit have somewhat larger ages. We will come back to these observations.

Fig. 3 provides a simple validation of the stellar ages. The stars have been divided into high- and low-\(\alpha\) stars, as indicated in Fig. 1. Fig. 3 then shows the sample divided into the high- and low-\(\alpha\) stars for four different ranges of $z_{\text{max}}$. For each sample, we also show the distributions of stellar ages for the high- and low-\(\alpha\) stars. We find that the sample of high-[Mg/Fe] stars on average is older than the low-[Mg/Fe] stars for the subsample closest to the plane (plots in the top row). This is expected from solar neighbourhood studies of dwarf stars (e.g. Bensby, Feltzing & Oey 2014; Fuhrmann et al. 2017) and thus confirms that our sample in all likelihood gives a reasonable description of the properties of Milky Way disc stars. The overall age of the low-\(\alpha\) stars changes as a function of height above the Galactic plane such that the stars get older as we move to higher heights. The high-\(\alpha\) stars, in contrast, have roughly the same median age and similar age spread at all $z_{\text{max}}$, thus resulting in an overall age gradient for the whole sample as we move away from the plane. This is an expected behaviour.

These simple considerations serve as a validation that our ages give a reasonable representation of the stellar populations and we can use them with confidence in our investigation.

We note that although the high-\(\alpha\) stars are subdominant at low $z_{\text{max}}$ as expected, the relative number of low-\(\alpha\) stars remains high also at large $z_{\text{max}}$. At 1 kpc, there are still more low-\(\alpha\) stars than high-\(\alpha\) stars. The difference is not very large and could be due to selection effects.

It is important to note that the method we use to infer formation radii for stars should only be applied to stars that are likely to have formed in a disc-like structure. It is very likely that so-called high-\(\alpha\) or thick disc stars have formed at a time when the ISM was highly turbulent and/or inhomogeneous (e.g. Bournaud, Elmegreen & Martig 2009). Only later did the ISM settle enough to allow for disc formation (Kassin et al. 2012) where it is meaningful to study the effects of churning and blurring (compare Frankel et al. 2018). Figs 2(e)–(h) show the properties of the sample when we restrict it to stars in the low-\(\alpha\) trend as defined in Fig. 1 and with $z_{\text{max}} < 0.5$ kpc.

3 ANALYSIS

Formation radii were derived in the same way as in Minchev et al. (2018) by assuming a model that describes how the radial metallicity gradient of the ISM changes over time and then simply referring
Figure 3. Stellar properties as a function of $z_{\text{max}}$. To the left, we show the [Mg/Fe] as a function of [Fe/H]. Colour coding as defined in Fig. 1. The middle and right-hand column show the age distributions of the high- and low-[Mg/Fe], respectively. The sample is divided into four bins in $z_{\text{max}}$: from $z_{\text{max}} < 250$ pc in the bottom row to $z_{\text{max}} > 1000$ pc in the top row. The number of stars in each subsample is indicated in the upper right-hand corner in each panel with age histograms.

We have selected four sets of radial profiles that describe how the ISM evolves. The profiles are shown in Fig. 4. Below, we briefly summarize the major characteristics of each set, but we refer the reader to the original publications for further details (Kubryk et al. 2015a; Kubryk, Prantzos & Athanassoula 2015b; Sanders & Binney 2015; Frankel et al. 2018; Minchev et al. 2018). We have also obtained radial metallicity profiles from an ongoing cosmological simulation (Agertz et al., in preparation). That analysis can be found in Section 4.4.

3.1 Choice of ISM profiles

For our analysis, we need a prescription of how the radial metallicity profile of the ISM varies over time. It is not our intention here to derive our own profiles, nor to test various profiles against observables (e.g. open clusters, O and B stars) but instead we are focusing on the method and exploring its strengths and weaknesses. Nevertheless, it is valuable to make use of a range of radial profiles to explore the method we develop. In the literature, it is possible to find a number of radial metallicity profiles that describe how the ISM evolves over time.

3.1.1 Minchev et al. (2018)

As described in their paper, Minchev et al. (2018) derived the ISM profiles by forcing a (small) solar neighbourhood sample of stars to reproduce the distributions of formation radii for stars with different ages from the models by Minchev, Chiappini & Martig (2013). Characteristic for these radial gradients is the steepening slope of the lines for older ages and the relatively narrow range of [Fe/H] at the smallest $R_{\text{gal}}$. The radial gradients implemented in our study are shown in Fig. 4(a).
Constraining radial migration in the Galaxy

3.1.2 Frankel et al. (2018)

Frankel et al. (2018) present a model that parametrizes the star formation history, the chemical enrichment history, and profiles that show where in the Galaxy a star forms given its age and metallicity. They assume that it is possible to describe the metallicity profile of the star-forming gas as a product of a radial profile and a term describing how the chemical enrichment evolves over time. They then combine these with a model for stellar migration modelled as diffusive processes. This results in a model that can quantify the global efficiency of radial migration.

For our purposes, we essentially invert their prescription of where a star forms given its age and metallicity (see Appendix A1). The result is radial gradients that define the evolution of the ISM with time suitable for our purposes. They are shown in Fig. 4(b). The gradients are straight lines that spread out more and more for the oldest ages. This is similar to one of the rejected tests by Minchev et al. (2018).

3.1.3 Sanders & Binney (2015)

Sanders & Binney (2015) set up a model of the Galaxy based on analytic distribution functions. Of interest to us is their function that describes the relation between age and metallicity for each radius in the Galaxy. The relevant function is derived by fitting to the model results from Schönrich & Binney (2009), which includes full chemical evolution as well as gas accretion and outflow.

Here, we invert their prescription to obtain a description of how the radial gradient of the ISM evolves with time (see Appendix A2). The result is shown in Fig. 4(c). We note that for young ages there is hardly any differentiation at all – all gas shares the same radial distribution at all times up and until 4 Gyr and for the oldest age (12 Gyr) the relation is flat at –1 dex since the model assumes that this is the metallicity of the ISM at the formation of the Galaxy (see table 3 in Sanders & Binney 2015).

3.1.4 Kubryk et al. (2015a)

Kubryk et al. (2015a) built a model to study the effects of radial migration on the chemical evolution of the Milky Way. Their model includes atomic and molecular gas, star formation that depends on the molecular gas and updated SN Ia rates and yields. They use parametrized time- and radius-dependent diffusion coefficients to describe the radial migration. The parametrization is based on N-body + SPH simulations. For further details, see their paper.

Although they do not provide a tabulated description of how the metallicity gradient in the ISM evolves with time, their fig. 4 (bottom middle panel) shows their results for three different ages. We reproduce these lines in Fig. 4(d) and use them for our study. Because this model explicitly takes into account the results from the N-body + SPH simulations, the gradients have less idealized shapes. We use the three available lines and interpolate between them to find the formation radii of the stars. This is a simplification but one we deem reasonable as the lines appear relatively well behaved.

3.2 Migratory distance (MD) – definition

In order to be able to quantitatively compare and analyse the results from the different model descriptions of how the radial metallicity gradient in the ISM has varied, we define the concept of migratory distance. This measure is simply the difference between the star’s current Galactocentric distance and the Galactocentric distance at its formation:

$$\text{MD} = (R_{\text{current}} - R_{\text{formation}}).$$

3.3 Error on inferred formation radii

The principle to infer the formation radius of a star is simple, but we also need to consider the uncertainties in the properties used to derive the formation radius. In particular, we need to consider uncertainties in [Fe/H] and age as well as in stellar coordinates, parallax, proper motions, and radial velocities, all of which transfer into the error of the current position of the star in the Galaxy.

For errors on [Fe/H], we use the individual errors as reported in APGEOE DR14 and for ages the error is 40 per cent according to Martig et al. (2016). For the Galactocentric radii, we find an error of 5 per cent being the maximum of our derived distances taking parallax errors into account. For most stars, the error is between 1 and 3 per cent.

We include these errors in our analysis in the following way. Iterating through each star in the full data set, we create a new star by assuming a normal error distribution and drawing new parameters for age, Galactic radius, and metallicity from a Gaussian where the mean is the original observed value and the standard deviation the associated uncertainty for each variable. We do this until we have generated 10 000 variants of each star and therefore 10 000 new and unique data sets.

We then proceed to analyse the data just as we would have done for the original sample but now with a sample where statistical uncertainties can be readily estimated.
Figure 5. Resulting distributions of $R_{\text{formation}}$ for stars with different ages. (a) Minchev et al. (2018), ages as indicated in the legend. (b) Frankel et al. (2018), ages as indicated in the legend. (c) Sanders & Binney (2015), ages as indicated in the legend. (d) Kubryk et al. (2015a), ages as indicated in the legend.

3.4 Results

Fig. 5 shows the resulting formation radii for stars of different ages for the four different sets of radial gradients. We note that Minchev et al. (2018), Frankel et al. (2018), and Kubryk et al. (2015a) show the same overall behaviour where older stars are on average formed further in towards the Galactic centre. This is largely in accordance with the criteria used in Minchev et al. (2018), i.e. an inside out formation of the disc where older stars form in the inner parts of the disc. Using the radial gradients from Sanders & Binney (2015) on the one hand results in that the majority of the stars in our sample form between roughly 5 and 8 kpc.

Fig. 6 shows the migratory distances as a function of age and formation radius of the stars. To create this figure, we have used the approach to include error estimates described in Section 3.3. Here, we show the results for Frankel et al. (2018) but all four sets of radial gradients qualitatively show the same results.

We also analyse a subsample of stars on highly circular orbits. After some experimentation, we find that $L_z/L_c > 0.99$ is the best definition of a highly circular orbit for our sample. This is a somewhat arbitrary choice but is supported by Fig. B1. Fig. 2 shows that such stars are present across the full range of Galactocentric distances and metallicities. We believe that the choice of this cut does not significantly bias our investigation.

For all four age bins, we find that stars on highly circular orbits ($L_z/L_c > 0.99$) have a smaller spread in migratory distance, on the order of 0.2 kpc. The median distance that a star has migrated depends on where it has formed – stars forming in the inner part of the disc have migrated significantly further distances than stars formed close to the Sun or further out in the disc.

4 ANALYSIS, INTERPRETATION, AND DISCUSSION

In this section, we show how combining the migratory distances with information about the stars’ orbits can be used to constrain how many of the stars in a sample have been churned. We discuss different ways of defining if a star has been churned or not and also look at stars that have not migrated at all. We also look at the formation radius of the Sun (Section 4.3) and investigate the possibility to obtain the radial metallicity gradients for the ISM from a cosmological simulation (Section 4.4).

4.1 Stars that have migrated

In this section, we will consider different ways to decide if a star or a stellar population has migrated in the Galaxy. We will provide some examples of how one can estimate the fractions of stars that have migrated and look at the fraction of stars that have only been churned.

Fig. 7 provides a summary of the results.

4.1.1 Estimating how many stars have radially migrated

The simplest definition of a star that has radially migrated in the Galaxy is a star for which its formation radius is not the Galactocentric radius the star is found at today. As can be inferred from Section 3.4, the fraction of stars that have moved radially in the Galaxy is large. Adding orbital information allows for a more interesting analysis.

Here, we consider a slightly more involved criterion for stars that have migrated in the Milky Way disc, namely stars that have a Galactocentric formation radius that is outside the Galactocentric radial range spanned by the apocentre and the pericentre of the current orbit of the star:

$$R_{\text{formation}} < R_{\text{peri, current orbit}} \quad \text{or}$$

$$R_{\text{formation}} > R_{\text{apo, current orbit}}.$$  (4)

Since there is no requirement set on the shape of the orbit, this implies that the sample of such stars must include both stars that
Figure 6. MDs as a function of age and formation radius (as indicated in the legend) using the model by Frankel et al. (2018). On the y-axis is shown the difference between the upper and lower quartile of the spread of the MDs. The solid line shows the result for the full sample, while the dashed line shows the results for stars on highly circular orbits, i.e. $L_z/L_c < 0.99$.

Figure 7. The fraction of stars on circular orbits, of stars that have been churned, and of stars that have not moved from their formation radius relative to the total sample of stars (Tables 1, B1, and 2). The colours identify the model used and the symbols different selections (see the legend). The results shown are for stars of all ages. Note that $R_{\text{apo}}$ and $R_{\text{peri}}$ refer to the apocentre and pericentre of the stars orbit today. If the formation radius ($R_f$) lies outside this orbit, the star is considered to have radially migrated (compare discussions in Section 4.1).

4.1.2 Estimating how many stars have been churned

In this section, we explore ways to estimate how many of our stars have experienced churning (churning being the radial migration that causes a star to move radially without losing the circularity of its orbit; Sellwood & Binney 2002, see also Section 4.5). For this, we need to define the subsample of stars that we think should have been subjected to just churning and not blurring. A star that has been blurred can also have been churned. This means that what we are trying to do here is to find a conservative lowest fraction of stars that have just been churned. We define a star to be a candidate as a churned star if it has a relatively circular orbit. We define such stars as those with $L_z/L_c > 0.99$.

For the other three models (Sanders & Binney 2015; Frankel et al. 2018; Agertz et al., in preparation) we obtain a higher fraction.
The results are shown in Fig. 7, where we also include the same estimates but with a more relaxed criterion on circularity (0.95 instead of 0.99). With the more relaxed criterion on circularity for the orbits, the minimum fraction of churned orbits increases to about 0.4. However, for reasons discussed below, we do not believe that this gives an indication of the fraction of churned stars.

4.1.3 Estimating the size of churning and blurring

It is also interesting to attempt to estimate the size of churning and blurring. An analysis as the one provided in Fig. 6 allows us to do this. It is readily seen from this figure that the spread in MD for all stars is about 0.2 kpc larger than for stars on highly circular orbits. If we regard the stars on the highly circular orbits as essentially uninfluenced by blurring then this gap is an indication of the size of the increased orbital spread due to blurring. We note that the difference between the full sample and the sample on highly circular orbits is the same for all formation radii for a given age. From our data, we cannot say if the difference between samples remains constant as age increases or not. As both churning and blurring can be modelled as diffusive processes, it is possible that the difference remains the same over time. All five models investigated in this study show the same patterns.

In our sample, we find that stars in our sample that formed inside the solar circle have migrated on average more than those that formed outside the solar circle, with a monotonic change in MD as a function of formation radius. We also note a trend with age where older stars have a larger MD for the same formation radius as compared to younger stars. This is consistent with radial migration being a diffusive process. Although the youngest stars have smaller MD they still show substantial radial migration, indicating that churning must be a process that acts quickly on a stellar sample. We know it cannot be blurring as blurring is a slow process and because even the circular sample shows significant migratory distances.

If we combine this finding with the finding that a minimum of about a tenth of the stars have suffered from such phenomena and hence its orbit will start making excursions away from the original circular orbit.

Fig. 8 shows the properties of the sample of stars that have not moved. We note that at all radii there is a large spread in metallicity as well as age. Metallicity and age appear to correlate well at each radius, such that younger ages are associated with higher metallicities. In a slightly circular argument this could be taken as proof that indeed at a given radius there is a tight age–metallicity relation. When we look at all stars in our sample this is not the case, indeed, many studies of stars in the solar neighbourhood have shown that there is an acute lack of such a relation (e.g. Edvardsson et al. 1993; Feltzing, Holmberg & Hurley 2001; Casagrande et al. 2011).

It should be noted for our method, that although per construction more metal-rich stars are younger at a given Galactocentric radius they need not be on orbits that have not moved via churning and/or blurring. It would have been entirely possible that there were only stars of one age or one metallicity at a given radius that were still on the same orbit they had when they formed. Thus, we believe that this does give observational support to the assumption that stars at a given radius in the Galaxy follow a tight age–metallicity relation.

4.3 The Galactocentric formation radius of the Sun

Returning to the question whether or not the Sun has formed at the solar radius, we find that using the radial metallicity gradients for the ISM from Frankel et al. (2018), Sanders & Binney (2015), and Kubryk et al. (2015a) the Sun formed at a distance from the Galactic centre of 5.7, 7.0, and 6.8 kpc, respectively. Clearly, there are uncertainties associated with these estimates; however, it is also clear that if we require an inside out-formation scenario in which the metallicity in the ISM is enriched over time in such a fashion as to produce flatter and flatter radial gradients, then the Sun most
Table 2. The fraction of stars that have not moved radially in the Galaxy since they formed (see Section 4.2).

| Model                  | Fraction of stars that have not moved |
|------------------------|----------------------------------------|
|                        | Age < 2      | 2 < Age < 4 | 4 < Age < 6 | 6 < Age < 8 | All ages          |
| Minchev et al. (2018)  | 0.100 ± 0.008 | 0.071 ± 0.005 | 0.056 ± 0.006 | 0.051 ± 0.010 | 0.073 ± 0.003 |
| Frankel et al. (2018) | 0.073 ± 0.006 | 0.054 ± 0.005 | 0.045 ± 0.006 | 0.039 ± 0.011 | 0.057 ± 0.003 |
| Sanders & Binney (2015)| 0.046 ± 0.005 | 0.055 ± 0.005 | 0.057 ± 0.007 | 0.051 ± 0.011 | 0.052 ± 0.003 |
| Kubryk et al. (2015a) | 0.124 ± 0.009 | 0.081 ± 0.006 | 0.047 ± 0.006 | 0.020 ± 0.007 | 0.076 ± 0.003 |

Figure 8. [Fe/H] as a function of Galactocentric radius for stars that have not moved (for definition, refer to Section 4.2) away from the radius they formed at. This example is for the model by Frankel et al. (2018). Age is colour coded according to the colour bar to the right.

likely formed between 5.5 and 7 kpc from the Galactic centre. This is largely consistent with other estimates of the Sun’s formation radius (Wielen et al. 1996; Minchev et al. 2013; Frankel et al. 2018).

Haywood et al. (2019) discuss the possibility that the Sun instead of coming from inside its current solar position in the Galaxy has in fact migrated in from the regions outside the Sun’s position. They find that this is supported by the numerical experiments carried out by Martínez-Barbosa et al. (2016), who use backwards integration in the Galactic potential to find out where a star comes from. As discussed earlier, with the inclusion of churning such integrations lose their ability to make such predictions, i.e. once a star has been churned, you cannot any more find out where it came from. As we have shown, not all stars have been churned; some may just be blurred while others are untouched by dynamical encounters. So, in theory the Sun may have an unchurned orbit and you could potentially retrace its orbit; however, we note that the fraction of stars in our sample fulfilling this criterion is small (compare Fig. 7).

4.4 Agertz et al. (in preparation)

Agertz et al. (in preparation) carried out a cosmological hydrodynamic + N-body zoom-in simulation of a Milky Way-mass galaxy ($M_* \sim 6 \times 10^{10} M_\odot$) forming in a dark matter halo with virial mass $M_{\text{vir}} = 1.3 \times 10^{12} M_\odot$ at $z = 0$. The simulation was carried out using the adaptive mesh refinement code RAMSES (Teyssier 2002), assuming a flat $\Lambda$-cold dark matter cosmology. We refer to Agertz et al. (in preparation) for details, as well as Pehlivan Rhodin et al. (2019), for an extensive description of the included physics and simulation settings. The simulation reaches the state-of-the-art resolution, with a mean resolution of $\sim 20$ pc in the cold ISM. For every simulation snapshot, we identify the most massive progenitor to the $z = 0$ Milky Way-mass galaxy. We identify the disc plane from the stellar and gaseous angular momentum vector, and compute radially averaged [Fe/H]-profiles from all neutral gas within a 2 kpc thick slab. The resulting radial ISM profiles are shown in Fig. 9. In panel (a), we show the gradients for all the snapshots from the model. As can be seen in some snapshots the gradient is very variable, this is in particular the case for the earlier times when there is still high activity of infalling material in the simulation. Around 6 Gyr the gradients become more stable and evolve steadily. In order for us to be able to calculate the formation radius of a star, we need the ISM gradients to be monotonically declining as a function of Galactocentric radius. We therefore have to smooth the gradients. In panel (b), we show the four selected snapshots for which we do this. We then, just as for example in the case of the gradients from Kubryk et al. (2015b), interpolate between the lines to find our final solution. Fig. 10 then
shows the resulting distribution of formation age as a function of time.

We note that insight drawn from abundance gradients in cosmological simulations of galaxy formation must be considered with care. While simulated galaxies can be selected to represent extended discs with global properties in line with the Milky Way’s, their detailed assembly histories will not necessarily be compatible with that of the Milky Way galaxy. However, for this explorative study, we find it informative to include these models as a part of our suite of diverse ISM models.

The radial gradients in Fig. 9(a) share many of the overall characteristics of those derived in Minchev et al. (2018) and those used in Frankel et al. (2018) in as much as they steepen towards the inner Galaxy and older profiles have lower iron content. Thus, we can expect that overall this description of the temporal evolution of the radial metallicity profile of the ISM should give rise to similar results as found for models explored previously. Indeed, that is also what we see, but with some modifications. Notably, Fig. 9(b) shows that although the median radius moves to smaller and smaller radii as age increases (i.e. an inside out-formation scenario as imposed in Minchev et al. 2018), there is significant spread at all ages.

Also, for this model, we have derived the fractions of stars fulfilling the different criteria discussed in Sections 4.1 and 4.2. We show these fractions in Fig. 7 together with the results from the other models. We note that the results from this model are largely the same as for the other models.

4.5 Discussion

One of the leading takeaways from our experimentation with this data set is the apparently rapid onset of radial migration. We find that the median migration distances of the stars in each of our radial bins remain very constant in time. This extends out to our oldest age bin, whose stars generally show identical median migration distances to our youngest age bin. This implies that the bulk of radial migration must take place relatively early in a star’s lifetime. We suggest that this could be attributed primarily to the fact that the period of the spiral bar pattern, dominantly responsible for churning, is significantly shorter than the width of our youngest age bin. A considerable number of attempts have been made to constrain the pattern speed of the spiral arms using hydrodynamic simulations (see for example, Bissantz, Englmaier & Gerhard 2003; Chakrabarty 2007) and observations of nearby velocity fields (Fernández, Figueras & Torra 2001). Most have placed the period of the spiral arms between 250 and 350 Myr (Gerhard 2011). Thus, a spiral arm crossing is well contained within the interval covered by our first age bin, and given that only a single transient interaction with a spiral arm is required to generate substantial changes to a stars angular momentum (Sellwood & Binney 2002), this should be sufficient to generate the displacements we observe.

This interpretation appears to be consistent with numerical N-body simulations that find substantial migration distances within 1 Gyr due to churning. Indeed, the time-scale for churning to displace the stars can be as short as 0.5 Gyr (Grand & Kawata 2016). It has also been observed in simulations that the distribution of stellar radii occupied for stars formed in the same location spreads dramatically in the first few Gyr, and slows down considerably at later times (Kubryk, Prantzos & Athanassoula 2013). Our data appear to provide empirical confirmation of these results.

Spiral galaxies show exponential disc profiles (van der Kruit & Freeman 2011). Although this appears to be a natural outcome of galaxy evolution, the exact mixture of physical processes that lead to these exponential profiles remains unclear; see Elmegreen & Struck (2013), Struck & Elmegreen (2018), and references therein. Our results seem to indicate that stars in the Milky Way are rapidly redistributed in the disc. Our sample is not constructed in such a way that we can readily test if there is a net inward migration of stars to allow for the build-up of the exponential disc or not. This would be an interesting aspect to include in future studies of Milky Way stars where the selection function of the sample can be better defined.

The swift onset of radial migration that we find, and subsequent invariability of median motions with time, indicates that churning is the dominant mechanism through which stars initially change their Galactic radii. It has long been understood that blurring alone is not sufficient to explain the distribution of stars observed in the local region and it is now understood that churning plays an integral part in explaining the locally observed distributions (examples can be found in Schönrich & Binney 2009; Minchev et al. 2013). However, the relative strengths of churning and blurring, that is, how much of a star’s displacement from the location it formed can be described by one process or the other, have not been much studied. This is due fundamentally to the fact that these processes act simultaneously, and are largely inseparable in observation.

It has been one of our aims in this work to investigate if it is possible to constrain the relative strengths of the two processes. Our ability to compare the migration distances of stars on circular orbits to the total sample enables us to observe how well the motions in our total sample are explained by a population that has not yet been blurred. In doing this, we see that churning immediately has a large effect on stellar radii, while blurring accounts for a comparatively small increase in spread at this time. This result is seen in the relatively small difference in spread between our total sample and the circular subset when compared to the overall spreads of these populations.

Blurring is traditionally modelled as a diffusive process, gradually increasing the spread of the radial distribution with time (Sellwood & Binney 2002; Schönrich & Binney 2009). As such, the deviations in radius are smaller and symmetric about the mean. In our data, this is represented by the equivalent median, though increased spread, in our total sample when compared to the population of stars on circular orbits. The difference in spread between these populations is presumably indicative of the added diffusion from blurring. Meanwhile, the expected changes to angular momentum caused by churning are quite large – several kpc from a single interaction with the spiral (Sellwood & Binney 2002) – when compared to the scales of the blurring found in our data: a spread only 0.2 kpc separating our
total sample and circular subset. This leads us to believe that churning is a much stronger force than blurring in terms of the magnitude of displacement. This is coupled with the observation that our data increases its spread with time, though only slightly from age bin to subsequent age bin. We have noted that this consistent dispersion in time is present in equal magnitude for both our total sample and our highly circular subsample. This implies that the aggregate spreading effect of churning and blurring in our total sample is on the same order as the effect purely from churning in our circular sample. Thus, beyond the initially large displacements caused by churning in the first hundreds of Myrs, churning begins to function indistinguishably from blurring, spreading only gradually with time. However, this is not to say that the distances covered by churning are less at later times. In fact, stars may be churned back and forth across the spiral patterns corotation radius many times in their lifetimes on so-called horseshoe orbits (Sellwood & Binney 2002). But this movement back and forth is not necessarily a purely random process. In fact, stars may be churned back and forth across the spiral patterns corotation radius many times in their lifetimes on so-called horseshoe orbits (Sellwood & Binney 2002). But this movement back and forth contributes little to any net migration when considering the average of the population (Halle et al. 2018), and serves only to broaden the distribution of present-day radii in a diffusive pattern reminiscent of blurring. Our results point to a net migration for a very large fraction of stars.

Previous attempts to constrain the strength of churning using observations from large spectroscopic surveys include Kordopatis et al. (2015), Hayden et al. (2018), and Hayden et al. (2020). These studies have mainly identified potentially migrated stars via the eccentricities of their orbits. Kordopatis et al. (2015) found that for stars with supersolar metallicities, about half of the stars in their RAVE sample had $\text{ecc} < 0.15$ (which they took to imply a circular orbit). That such stars are present at the solar radius is interpreted as evidence for churning as the ISM is at solar metallicity today and hence, more metal-rich stars need to come from somewhere else than the solar neighbourhood. Hayden et al. (2018) used stars from the Gaia-ESO Survey. They selected stars with [Fe/H] > 0.1 dex and found that about 20 per cent of the stars in their sample do not reach the Galactocentric radius at which they likely formed and can thus have been churned. Hayden et al. (2020) on the other hand, using a similar approach, find that for stars in the solar neighbourhood observed with GALAH and with $\text{ecc} < 0.2$ as many as 70 per cent have reached that location, thanks to churning/radial migration. Compared with our more modest total conservative estimate of about 15 per cent for our full sample, this seems to be a very large number. On the other hand, Hayden et al. (2020) include both high- and low-$\alpha$ stars, while we (in agreement with Frankel et al. 2018) argue that any attempts to constrain churning for $\alpha$-enhanced stars is likely to fail due to the complex nature and formation channel of that stellar component that includes both accretion and turbulent formation scenarios in the early Universe (some examples include Agerzt, Teyssier & Moore 2009; Bournaud et al. 2009). We note that our definition of a circular orbit is much more stringent than used in these studies. Referring to Fig. B1, we can see that by instead using a cut in $\text{ecc}$ we would indeed have many more stars, but importantly, say if we cut at $0.15$, then that would essentially include a very large portion of the stars with $0.95$. We note that if we relax the criterion for circularity to $0.95$ (from 0.99) then the difference between all stars and stars on circular orbits in Fig. 6 disappears. Meaning that then there is no difference between the two samples in terms of displacement, indeed it is not possible to distinguish between churning and blurring (see also discussion in Section 4). Hence, we conclude that although $\text{ecc}$ might appear as an easily understood measure of the orbital shape the more robust $L_s/L_c$ is a better indicator of the orbital shape.

At this point in the discussion it is important to reiterate two things: (i) our study is of an exploratory nature, we wish to see if we can find means to constrain churning and blurring the stellar populations in the cold stellar disc in the Milky Way, (ii) the sample we have used is far from perfect for the purposes. We believe that we have succeeded in showing that there are ways to effectively constrain the size and strength of churning using simple means to estimate the migratory distances for stars in the cold stellar disc. In this work we have made no attempt to account for the selection effects in our sample. Could some of our conclusions be influenced by this? Our main aim with this work is to establish ways to constrain the strength of churning and blurring. The results depend both on the models used (the radial metallicity gradients for the ISM) and the quality of stellar sample, including its physical distribution in the Galaxy. An inherent assumption is that there are no azimuthal changes in the stellar population in the Milky Way. This is likely incorrect, but current data do not allow to address this question. Future data releases from Gaia coupled with large spectroscopic surveys as well as dedicated follow-up of, e.g. Cepheids and A and B stars across the Milky Way disc, will illuminate this problem.

Figs 2(f) and (g) show that the stellar sample we use mainly is situated just outside the solar circle. Although there are stars between 5–11 kpc there is a concentration around 9 kpc. There is also a slight trend between [Fe/H] and present-day Galactocentric radius such that more metal-poor stars are found further out in the Galaxy. We know that the Milky Way stellar disc is well populated also inside the solar circle but we know less about the metallicities of those stars. Taking the data at face-value, we can thus conclude that it is likely that we are missing stars on smaller radii meaning that we will not have such a good view of the radial migration experienced by the stars that are currently inside the solar circle. We think, however, that for the stars beyond the solar circle and inside about 10.5 kpc we have a pretty good sampling of the stellar population as of today. Thus, our inferences about the strength of churning and blurring, as applied to these stars, should hold – churning is the stronger process and acts early on in the life of the stars. Later, both churning and blurring act as diffusive processes that grow slowly over time.

Future studies must still look into how the selection function of the stellar sample influences the results. We have seen that for our sample of cold disc stars drawn from a combination of Gaia and APOGEE, the results are largely model independent. That might not necessarily be the case with a sample defined in a different way and with a different selection function.

5 CONCLUSIONS

In this work, we set out to explore the possibility to quantify how many stars have radially migrated in the stellar disc and, eventually, will be able to put numbers on the strength and importance of the processes involved. We have taken some first steps on this path by utilizing a sample of red giant stars from APOGEE DR14, Gaia parallaxes and proper motions, and stellar ages derived from C and N abundances in the stars. This sample has allowed us to quantify how a large fraction of the stars in the sample have had their orbits changed from initially circular to non-circular, how many remain on circular orbits, and how many are on circular orbits that might have been “churned”.

We find that a conservative estimate is that about 10 per cent of the stars in the sample have been churned. This is in contrast to recent studies that have much higher numbers, however, we note that those studies essentially look at stars with supersolar metallicities while
we study stars of all metallicities. Furthermore, our definition of a highly circular orbit is deliberately conservative. If we instead select the stars have Galactocentric radii at formation that lays outside of its apo- as well as pericentre today, we find that about half of the stars have undergone some combination of churning and blurring. We estimate that a robust 5–7% of stars in our sample have not had their orbits blurred, nor have they been churned. These stars appear at all ages indicating that an individual star may escape these dynamical processes for quite a long time. Our study also provides tentative observational support to the assumption that stars at a given radius in the Galaxy follow a tight age–metallicity relation.

Looking towards the future, there are several ongoing and upcoming large spectroscopic surveys that would be able to provide data to further explore the relative importance and strengths of churning and blurring in our Galaxy (e.g. WEAVE, 4MOST; Balcells et al. 2010; de Jong et al. 2016, 2019; Jin et al., in preparation). From and blurring in our Galaxy (e.g. WEAVE, 4MOST; Balcells et al. to further explore the relative importance and strengths of churning and blurring in our Galaxy (e.g. WEAVE, 4MOST; Balcells et al. 2010; de Jong et al. 2016, 2019; Jin et al., in preparation). From Gaia, we will have the parallaxes and proper motions, which when combined with the radial velocities from the spectroscopic surveys will give the full 6D phase space information needed to calculate stellar orbits. The surveys will also provide the needed metallicity and elemental abundances. Stellar ages are difficult to derive. The best prospects are for turn-off stars (Sahlholdt et al. 2019) to reach large volumes of the Milky Way require us to use red giants. In this study, we have made use of stellar ages for red giants derived using elemental abundances of C and N and combinations thereof (Martig et al. 2016). It is essential that these and similar methods are further evaluated, developed, and validated such that they may be used, at least in a statistical sense, in studies like the one presented here.

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APPENDIX A: RADIAL METALLICITY GRADIENTS FOR THE ISM – EQUATIONS

A1 Frankel et al. (2018) radial metallicity gradients for the ISM

Fig. A1(a) shows the relation of $[\text{Fe/H}]$ as a function of age for different Galactocentric radii based on the prescriptions given in Frankel et al. (2018).

Manipulating the equations in Frankel et al. (2018), we arrive at the following equation that describes how the iron content in the ISM changes with radius ($R$) for a given age ($\tau$):

$$[\text{Fe/H}] (\tau) = -1 - (-1 - 0.07 \cdot 8.74) \cdot ((1 - \tau/12)^{0.32} - 0.07 \cdot R).$$

(A1)

Fig. A1(b) then shows the resulting radial gradients used in our work. For full references and discussion of the constants in equation (A1), we refer the reader to Frankel et al. (2018).

Figure A1. (a) $[\text{Fe/H}]$ as a function of age for different Galactocentric distances from Frankel et al. (2018) (compare their Figure 5). (b) $[\text{Fe/H}]$ as a function of Galactocentric radius for different ages, equation (A1).

A2 Sanders & Binney (2015) radial metallicity gradients for the ISM

Fig. A2(a) shows the relation of $[\text{Fe/H}]$ as a function of age for different Galactocentric radii based on the prescriptions given in Sanders & Binney (2015) (their fig. 1). Sanders & Binney (2015) obtain their functional form for $F(R)$ by fitting to the resulting output from Schönrich & Binney (2009), which has a steep present-day gradient $-0.082$ dex kpc$^{-1}$ as compared to $-0.07$ dex kpc$^{-1}$ in most other models (e.g. Frankel et al. 2018; Minchev et al. 2018)

Manipulating the equations in Sanders & Binney (2015), we arrive at the following equation that describes how the iron content in the ISM changes with radius ($R$) for a given age ($\tau$):

$$F(R) = -0.99 \cdot (1 - \exp(-0.064 \cdot (R - 7.37)/0.99)).$$

(A2)

$$[\text{Fe/H}] (\tau) = (F(R) + 0.99) \cdot \tanh((12 - \tau)/3.2) - 0.99.$$ 

(A3)

Fig. A2(b) then shows the resulting radial gradients used in our work. For full references and discussion of the constants in equations (A2) and (A3), we refer the reader to Sanders & Binney (2015). We note that as opposed to the other models used in our study, this one hardly has any evolution of the ISM radial gradient with time for the time span that is of interest for the formation of the stars in the cold stellar disc, i.e. the last 6 to 8 Gyrs.
APPENDIX B: ORBITAL DATA

As described in Section 2.2, orbital data for the stars in our sample were calculated using the GALPY package (Bovy 2015).

B1 Comparing $L_z/L_c$ and $e$.

In this work, following Liu & van de Ven (2012), we have chosen to use $L_z/L_c$ to characterize the circularity of the stellar orbits. Fig. B1 shows a comparison between $L_z/L_c$ and $e$. There is a clear correlation between the two properties. At a given $L_z/L_c$, there is a substantial spread in $e$. We note that our chosen cut for defining very circular orbits is 0.99 for $L_z/L_c$; this encompasses $e$ in the range 0–0.2.

B2 Including a bar in the Galactic potential

The GALPY package allows the user to include a bar when carrying out the orbital integrations (the reader is referred to Bovy 2015 for details about the potential). Fig. B2 shows the effect on $e$ when the bar is included in the potential. Although there are noticeable effects, we note that for stars that we consider to be on circular orbits (which all have $<0.2$, see Section B1), the effect is small and hence we have concluded that we did not need to include a bar in the Galactic potential for the purpose of this study.

B3 $L_z/L_c < 0.95$

For completeness, we include here the resulting fraction of stars on circular orbits when the constraint has been reduced to 0.95 instead of the 0.99 we use in the final analysis (see Table 1 for the 0.99 results). The data are given in Table B1 and are also shown in Fig. 7.
Table B1. Fractions of stars on different types of orbits for different age bins and for all stars (last column). See Section 4.1.2 for a description.

| Model                        | Age < 2 | 2 < Age < 4 | 4 < Age < 6 | 6 < Age < 8 | All ages |
|------------------------------|---------|-------------|-------------|-------------|----------|
|                              | Fraction of stars that have $L_z/L_c > 0.95$ | | | |
| Minchev et al. (2018)        | 0.747 ± 0.008 | 0.674 ± 0.008 | 0.624 ± 0.011 | 0.560 ± 0.020 | 0.670 ± 0.002 |
| Frankel et al. (2018)        | 0.755 ± 0.008 | 0.688 ± 0.008 | 0.649 ± 0.012 | 0.617 ± 0.024 | 0.695 ± 0.003 |
| Sanders & Binney (2015)      | 0.763 ± 0.008 | 0.684 ± 0.008 | 0.623 ± 0.012 | 0.569 ± 0.023 | 0.684 ± 0.002 |
| Kubryk et al. (2015a)        | 0.741 ± 0.009 | 0.671 ± 0.007 | 0.627 ± 0.010 | 0.604 ± 0.021 | 0.669 ± 0.003 |

|                              | Fraction of stars that have $L_z/L_c > 0.95$ and outside present-day orbit |
| Minchev et al. (2018)        | 0.339 ± 0.009 | 0.370 ± 0.008 | 0.385 ± 0.012 | 0.372 ± 0.020 | 0.366 ± 0.004 |
| Frankel et al. (2018)        | 0.414 ± 0.009 | 0.442 ± 0.009 | 0.457 ± 0.013 | 0.453 ± 0.025 | 0.453 ± 0.005 |
| Sanders & Binney (2015)      | 0.523 ± 0.009 | 0.436 ± 0.009 | 0.386 ± 0.013 | 0.373 ± 0.023 | 0.373 ± 0.003 |
| Kubryk et al. (2015a)        | 0.302 ± 0.010 | 0.332 ± 0.008 | 0.415 ± 0.011 | 0.498 ± 0.022 | 0.498 ± 0.005 |

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