KINEMATICAL vs DYNAMICAL RELATIVISTIC EFFECTS IN \( A(\vec{e}, e'\vec{p})B \)

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The relativistic mean field approach is applied to the description of coincidence \( A(e, e'p)B \) reactions where both the incident electron beam and knockout proton are polarized. Effects introduced by the dynamical enhancement of the lower components of the bound nucleon wave function are analyzed within RPWIA for the polarized responses and transferred polarizations. Results obtained by projecting out the negative-energy components are also compared with various nonrelativistic reductions.

1. INTRODUCTION

As is well known, over the years quasielastic coincidence \((e, e'p)\) reactions have provided one of the most powerful tools to study nuclear structure. In particular, single-particle properties such as momentum distributions and spectroscopic factors corresponding to nucleon shells in the vicinity of the Fermi level have been extracted from the analysis of these processes. Moreover, \((e, e'p)\) reactions have also clearly proved the limits of the single-particle approach for nuclei on which the mean field approximation is based. When polarization degrees of freedom are involved, a much richer variety of polarization observables becomes accessible. These contain in general interferences between the various amplitudes and consequently a complete decomposition into the electromagnetic matrix elements can in principle be achieved. In the case of final-state nucleon polarization measurements, i.e., \(A(\vec{e}, e'\vec{p})B\) processes, the differential cross section can be written as

\[
\frac{d\sigma}{de_e d\Omega_e d\Omega_N} = \sigma_0 [1 + \vec{P} \cdot \vec{\sigma} + h(A + \vec{P}' \cdot \vec{\sigma})],
\]

where \(\sigma_0\) is the unpolarized cross section, \(h\) is the incident electron helicity, \(A\) denotes the electron analyzing power, and \(\vec{P} \, (\vec{P}')\) represents the induced (transferred) polarization. Note that \(\vec{P}\) only depends on the outgoing nucleon polarization, whereas \(\vec{P}'\) becomes accessible when the outgoing proton and electron beam polarizations are both measured. The conventional three perpendicular directions chosen to specify the recoil nucleon polarization are given by \(\vec{l}\) (parallel to the momentum \(\vec{p}_N\) of the outgoing nucleon), \(\vec{n}\) (perpendicular to the plane containing \(\vec{p}_N\) and the transfer momentum \(\vec{q}\)), and \(\vec{s}\) (determined by \(\vec{n} \times \vec{l}\)). In coplanar kinematics, the only surviving components are \(P_n\), \(P'_l\) and \(P'_s\). Moreover, the induced polarization \(P_n\) is zero when final state interactions (FSI) between the outgoing nucleon and the residual nuclear system are neglected.

The transfer polarization components \(P'_l\) and \(P'_s\) may provide valuable information on the nucleon form factors. In the case of electron-nucleon scattering one gets a close relationship between the nucleon form factors and the polarization transfer components

\[
\frac{P'_l}{P'_l} = \frac{G_E}{G_M} \left[ \frac{1}{2} \left( 1 + (1 + \tau) \tan^2 \frac{\theta_e}{2} \right) \right]^\frac{1}{2},
\]

with \(\tau = 4Q^2 / M_N^2\), being \(Q^2\) the transfer four-momentum, and \(\theta_e\) the electron scattering angle. It is important to remark that the above relationship is only strictly correct for electron scattering from a free nucleon. In the case of bound nucleons the polarization ratio should be evaluated within the scheme of a particular nuclear model, and thus eq. (2) only holds approximately. In spite of this, the ‘polarization’ technique to determine the nucleon form factors, presents clear advantages compared with the usual Rosenbluth separation method. It does not require one to vary the beam energy and/or the spectrometer angle, thus eliminating the systematic uncertainties that make it so difficult to extract \(G_E\) at high \(Q^2\) within the Rosenbluth method.
Furthermore, recoil polarization calculations for low/medium missing momenta have also proven to be relatively insensitive to different ingredients in the description of the reaction mechanism, namely off-shell ambiguities and optical potentials used to describe FSI \[11\]. These results allow one to consider the polarization technique to be a promising probe in studying the behaviour of the nucleon form factors.

In recent years there has been a concerted experimental effort to shed some light on the issue of the possible modification of the form factors of the nucleons inside the nuclear medium. High precision polarization transfer measurements on complex nuclei have recently been presented by Malov et al. in $^{16}\text{O}(e,e')^{15}\text{N}$ \[1\] and by Dieterich et al. in $^4\text{He}(e,e')^3\text{H}$ \[2\]. Although the general conclusions in both experiments are not free from ambiguities due to experimental uncertainties, the authors in \[1\] show that standard nonrelativistic calculations are in clear disagreement with the experimental data. This result constitutes a strong indication of the necessity for a fully relativistic calculation in order to describe the spin transfer observables.

The relativistic mean field approach has been used recently to evaluate several electron scattering observables. These have been successfully compared with experimental data for transferred and induced polarizations \[3\], as well as for unpolarized observables: the interference transverse-longitudinal response $R_{TL}^f$, left-right asymmetry $A_{TL}^f$, single-particle momentum distributions \[10\] and spectroscopic factors \[11\]. In all the cases, the fully relativistic analysis shows a clear improvement in describing the experimental data compared with standard nonrelativistic treatments.

Although a treatment of FSI is necessary to describe experimental data, various studies have appeared in recent years dealing with the relativistic plane-wave impulse approximation (RPWIA). This simplified approach has proved to be very useful in order to disentangle relativistic effects from distortion effects. The unpolarized responses in $A(e,e'p)$ reactions within RPWIA were already presented in \[12\]. There important modifications with respect to the standard plane-wave impulse approximation (PWIA) calculation were found due to the presence of negative-energy components in the relativistic bound nucleon wave function. The interference $TL$ response and asymmetry, $A_{TL}$, were shown to be very sensitive to dynamical effects of relativity affecting the lower components. These results persist in more realistic calculations including FSI. In fact, data on $R_{TL}^f$ and $A_{TL}^f$ are a strong indication of the crucial role played by dynamical relativistic effects \[9\].

Let us recall that the differences between the fully relativistic approach and the standard nonrelativistic one can be divided into kinematical and dynamical effects. The first are due to the 4-vector current operator, compared with the nonrelativistic one that usually involves $\vec{p}/M_N$ expansions. The latter come from the difference between the nucleon (bound and ejected) wave functions involved. Within these dynamical relativistic effects one may distinguish effects associated with the so-called Darwin term, that mainly affect the determination of spectroscopic factors at low missing momenta, and the ones due to the dynamical enhancement of the lower components of the relativistic wave functions, which are expected to be more relevant at high missing momenta, although they might produce noticeable effects for some particular observables even at low/medium $p_m$ values.

Our main aim in this paper is to study, within RPWIA, the new response functions that enter in the analysis of $A(e,e'p)B$ processes due to the presence of spin-dependent degrees of freedom. All the results shown are for the reaction $^{16}\text{O}(e,e'p)^{15}\text{N}$. Following the arguments presented for the unpolarized case in \[12\], here we extend the analysis to the polarized situation and for the polarized responses try to identify clear signatures due solely to the dynamical relativistic effects coming from the negative-energy projections (NEP) of the relativistic bound nucleon wave function. Moreover, the role played by the NEP on the transfer polarization components, $P_T^f$, $P_T^s$, is also analyzed in detail. These issues are presented in Section 2. In Section 3 we also estimate the kinematical relativistic effects introduced by using various possible non-relativistic reductions of the nuclear current operator. Finally in Section 4 we summarize our main conclusions.

## 2. DYNAMICAL RELATIVISTIC EFFECTS

As mentioned in the introduction, in this work we restrict our attention to the RPWIA. Hence the ejected proton is described as a plane wave, i.e., FSI are neglected. Within this scheme, kinematical relativistic effects are included via the use of the fully relativistic CC1 and/or CC2 current operators \[14\]. Here we are mainly interested in the dynamical relativistic effects coming from the presence of the negative-energy projections of the bound nucleon wave function that gives rise to the dynamical enhancement of the lower components.
From the total of eighteen response functions that enter in the analysis of \( A(e, e'p)B \) reactions \(^1\), only nine survive within RPWIA. Dynamical relativistic effects affecting the four unpolarized responses (\( R^L, R^T, R^{TL} \) and \( R^{TT} \)) were already studied in detail in \(^2\). Here we present results for the polarized response functions and transfer polarization components corresponding to two different selected kinematics:

1. \((q - \omega)\) constant kinematics with \( q = 500 \text{ MeV/c} \) and \( \omega = 131.56 \text{ MeV} \). The value of the transfer energy \( \omega \) corresponds to the quasielastic peak value.

2. Parallel kinematics. The outgoing nucleon kinetic energy is fixed at 120.2 MeV, and the angle \( \theta \) between the missing momentum \( \vec{p}_m \) and the transfer momentum \( \vec{q} \) is fixed at 0\(^\circ\) (\( p_m > 0 \)) and/or 180\(^\circ\) (\( p_m < 0 \)).

In both cases coplanar kinematics (\( \phi = 0^\circ \)) has been selected and we focus on the proton knockout from the 1\( p_{1/2} \) shell in \(^{16}\)O. The bound state wave function has been computed within the Walecka relativistic model. It corresponds to a Dirac-Hartree solution from a phenomenological relativistic Lagrangian with scalar and vector meson potentials. The parameters of the set HS \(^3\) and the TIMORA code \(^4\) have been used. Other possible parameterizations of the bound state wave function have been also checked. The results obtained show similar trends to the ones presented in the figures that follow. For all the applications discussed below the Coulomb gauge has been chosen.

Let us first discuss the results for the polarized responses. From the five polarized responses surviving within RPWIA, one of them, \( R^{TL} \), only enters for out-of-plane kinematics. Therefore, in coplanar kinematics, the analysis is reduced to four polarized response functions: \( R^T_1, R^{TL}_1, R^T_4 \) and \( R^{TL}_4 \). The subindex refers to the three directions of the final-state polarization defined in the Introduction. The presence of the negative-energy components of the bound nucleon wave function breaks the factorization property that holds when only positive-energy components are taken into account. Within RPWIA each response function can be decomposed in the form

\[
R^K = R^K_P N_P(p) + R^K_C N_C(p) + R^K_N N_N(p). \tag{3}
\]

The first term is proportional to the square of the positive-energy projection, and is analogous to the standard result that appears in a nonrelativistic calculation. The remaining two terms are proportional (quadratically and linearly) to the negative-energy projection \( (R^K_N N_N(p)) \) and \( R^K_C N_C(p) \).

In Fig. 1 we represent the results obtained for the polarized response functions corresponding to \((q - \omega)\) constant kinematics and CC1 current operator. We show the fully relativistic result (solid) versus its three contributions as given in (3): the positive-energy (dotted), the crossed (short-dashed) and the negative-energy (long-dashed) contributions. Thus the dynamical relativistic effects are easily appreciated by just comparing the negative-energy and crossed terms with the total response. It is clearly seen that in two responses, \( R^T_1 \) and \( R^{TL}_4 \), the contribution of NEP is almost negligible, that is, dynamical relativistic effects from the bound nucleon wave function do not affect these responses. On the contrary, the two remaining polarized responses are very sensitive to these effects. In both cases, although the negative-energy term does not contribute significantly to the total result, the crossed term plays an important role, especially for the \( R^{TL}_4 \) response where its contribution is similar to the one coming from the positive-energy projection. This result resembles what appeared for the unpolarized interference \( TL \) response. Hence there exists a strong discrepancy between RPWIA results and those corresponding to the standard PWIA (we must recall that although the positive-energy term in (3) is not identical to the PWIA result, for which we must take the nonrelativistic momentum distribution \( N_{mr}(p) \), the difference is very small provided that \( N_{mr}(p) \sim N_P(p) \)).

In Fig. 2 we show the polarized responses obtained using the CC2 form of the nucleon current operator. As observed, the general trend is similar to the one discussed for the CC1 case, except for the magnitude of the relativistic effects. Although the role of the negative-energy and crossed terms is significantly reduced for the CC2 current (Fig. 2), their effects are still quite sizeable on \( R^T_4 \) and \( R^{TL}_4 \). This behaviour is similar to the one already stated for the unpolarized responses in \(^1\). The use of the CC1 operator maximizes the role of the negative-energy projections. This can be traced back to the fact that the CC2 form of the current is obtained (for free nucleons) by
simply imposing general constrains over the more general form of the current. On the contrary, the CC1 operator is obtained from the CC2 one by applying the Gordon decomposition, only valid for free $u$ Dirac spinors. Note that in RPWIA one should also take into account couplings to $v$ Dirac spinors for which the Gordon decomposition is not valid. This explains why the difference between CC1 and CC2 results is much more important for the negative-energy and crossed terms than for the strictly positive-energy contribution (standard PWIA calculations).

In what follows we analyze the behaviour of the transfer polarization asymmetries $P'_l$ and $P'_s$. Note that the ratio $P'_s/P'_l$ is related to the nucleon electric/magnetic form factors. In Fig. 3 we show the results corresponding to forward ($\theta_e = 30^\circ$) and backward ($\theta_e = 150^\circ$) electron scattering angles. In the former case the electron beam energy is given by $\varepsilon_e = 1$ GeV and in the latter $\varepsilon_e = 324$ MeV. We compare the fully relativistic results (dashed lines) corresponding to the CC1 and CC2 current operators with their positive-energy projection contributions (dotted lines). The difference between the relativistic and projected results observed for very small missing momentum values is directly connected to the quantum number $\ell = 0$ of the lower component in the $1p_{1/2}$ state (see ref. [12] for details). Apart from this behaviour for very small missing momenta, it is important to note that fully relativistic and positive-energy projected results do not differ appreciably (especially for backward angles) for $p_m$-values up to $\sim 300$ MeV/c. For $p_m > 300$ MeV/c both (relativistic vs projected) results start to deviate from each other. This general behaviour is what one should expect because of the clear dominance of the positive-energy projection component of the momentum distribution in the region $p_m \leq 300$ MeV/c [12]. On the contrary, in the region of high missing momentum, $p_m > 300$ MeV/c, the crossed and negative-energy components, $N_C(p), N_N(p)$, are similar to or even larger than that of $N_P(p)$, hence the effects of the dynamical enhancement of the lower components in the bound relativistic wave function are clearly visible in the transfer polarization asymmetries.

Finally, it is also clear from the results shown in Fig. 3 that the dynamical effects are maximized...
in the forward electron scattering situation. Here the differences between fully relativistic and projected results are important even for low/medium $p_{m}$ values, in particular for the sideways transfer polarization, $P'_{s}$. In Table I we present the various electron kinematical factors that enter in the polarized cross section. We display the values corresponding to forward ($\theta_{e} = 30^\circ$) and backward ($\theta_{e} = 150^\circ$) kinematical situations. As noted, the purely transverse responses dominate at backward angles; hence the most relevant contributions to the polarization asymmetries in this case come from the transverse polarized responses $R_{l}^{T'}$ and/or $R_{s}^{T'}$ in the numerator, and from the unpolarized $R^{T}$ response in the denominator. From these three responses, only the small $R_{s}^{T'}$ is particularly sensitive to the effect of the negative-energy components. On the contrary, at a forward angle ($\theta_{e} = 30^\circ$) all the kinematical factors (table I) are of similar order, and hence the contribution of the responses that are more sensitive to dynamical relativistic effects is clearly maximized. Therefore, the important role played by the negative-energy components of the bound relativistic wave function is much better appreciated for the transfer polarization asymmetries measured at forward angles.

In parallel kinematics only two polarized responses survive: $R_{l}^{T'}$ and $R_{s}^{TTL'}$. In Fig. 3 we present

| $\theta^{(\circ)}$ | $v_L$  | $v_T$  | $v_{TL}$ | $v_{TT}$ | $v_{T'}$ | $v_{TL'}$ |
|------------------|-------|-------|----------|----------|----------|----------|
| 30               | 0.866 | 0.537 | -0.659   | -0.465   | 0.268    | -0.176   |
| 150              | 0.866 | 14.39 | -2.537   | -0.465   | 14.386   | -2.456   |

Table I: Electron kinematical coefficients $v_k$ for perpendicular kinematics and two values of the electron scattering angle.
3. KINEMATICAL RELATIVISTIC EFFECTS

For a long time the standard procedure to treat \( (e, e'p) \) reactions has been based on a nonrelativistic description of the hadronic current. Bound nucleon wave functions have been described as solutions of the Schroedinger equation and a nonrelativistic treatment of FSI has been considered in describing the ejected nucleon wave function. The main reason for this analysis is directly connected with the fact that most nuclear models have been derived within a nonrelativistic framework. Hence, in order to be consistent with such description of the nucleus, one is also forced to perform a nonrelativistic reduction of the relativistic electromagnetic current.

In the previous section we have analyzed the dynamical relativistic effects coming from the bound state. In doing that we have compared the fully relativistic results obtained within RPWIA with the positive-energy projected calculations. In this section we focus on the kinematical relativistic effects. In order to disentangle clearly both types of relativistic effects and simplify the discussion we only retain the positive-energy component of the responses. Thus by comparing these projected
results with those evaluated by performing a nonrelativistic reduction one would be able to estimate the magnitude of the relativistic kinematical effects.

The standard nonrelativistic procedure has been based on expansions in all dimensionless momenta, i.e., nonrelativistic expansions of the current were made in powers of the transferred momentum, \( \frac{q}{M_N} \), transferred energy, \( \frac{\omega}{M_N} \), and momenta of the initial-state struck nucleons, \( \frac{p_m}{M_N} \). These approximations are not justified in present experiments, since values of \( q \) can be even higher than the nucleon mass. Here, for comparison with experimental data, we treat the problem exactly for the transferred energy and momentum, considering only expansions in powers of \( \frac{p_m}{M_N} \). Analyses of the nonrelativistic reductions along this line can be found in the literature \[17, 18\]. In this work, instead of making use of existing nonrelativistic expressions for the single-particle current matrix elements, we directly expand the single-nucleon responses in powers of \( \frac{p_m}{M_N} \) up to first order. We compare the results so obtained to the fully relativistic PWIA calculation. Our main aim is to establish how precise the expansion in powers of \( \frac{p_m}{M_N} \) is, and under which conditions and/or for which observables it does or does not work. In what follows we only analyze the transfer polarization asymmetries which, within PWIA, do not depend on the nuclear model, since factorization is fulfilled.

In Fig. 5 we represent two different nonrelativistic results (dotted and short-dashed lines) corresponding to the \((q - \omega)\) constant kinematics (see ref. [19] for details). Fully relativistic (solid line) and positive-energy projected (long-dashed line) calculations are also shown for comparison. The kinematical effects are clearly visible by comparing the nonrelativistic curves to the projected ones. We observe that kinematical effects are almost negligible in the low \( p_m \) region (\( p \leq 300 \text{ MeV} \)), starting to show up for higher missing momenta. Note that this was also the case for the dynamical effects. Moreover, kinematical relativistic effects are also maximized at forward electron scattering angle (bottom panels). Finally, it is important to remark that the use of nonrelativistic reductions may even produce unphysical results: the sideways transferred polarization, \( P'_s \), for \( \theta_e = 30^\circ \) gets bigger than 1 (in absolute value) for high \( p_m \) which means negative cross section. Therefore, one should be very careful in doing non-relativistic expansions to use in the analysis of present-day experiments.

4. SUMMARY

Dynamical relativistic effects associated with the bound nucleon wave function have been analyzed within RPWIA for polarized hadronic responses and transferred polarization observables. Two different kinematical situations have been considered: i) \((q - \omega)\) constant kinematics and ii) parallel kinematics. We have found that the four polarized responses that enter in the analysis of coincidence electron scattering reactions may be quite sensitive to the presence of negative-energy projections in the relativistic bound state. In particular,
FIG. 5: Longitudinal and sideways transferred polarizations for the $1p_{1/2}$ shell in $^{16}\text{O}$ in $(q - \omega)$ constant kinematics, using the Coulomb gauge and the CC1 current. Top panels correspond to $\theta_e = 150^\circ$ and bottom panels to $\theta_e = 30^\circ$. The fully relativistic results are represented with solid lines, the projected results are given by long-dashed lines and the other two curves (dotted and shot-dashed) correspond to two different nonrelativistic reductions.

- $R^T_L$ and $R^{TL}_L$ show important deviations from the positive-energy projected result in $(q - \omega)$ constant kinematics, whereas the role of NEP on the two remaining polarized responses is almost negligible.

- $R^T_s$ and $R^{TL}_s$ are, however, particularly sensitive to NEP in the case of parallel kinematics.

Concerning the transfer polarizations, it should be pointed out that they can be strongly affected by the negative-energy contributions, mainly at high $p_m$-values. In particular, at a forward electron scattering angle ($\theta_e = 30^\circ$) the dynamical enhancement of the lower components in the bound nucleon wave function may modify completely the structure of the polarization asymmetries. Kinematical effects can also be very important under the same conditions, and we must keep in mind their limits of validity at high missing momenta. This region is particularly interesting if one wishes to investigate short-range correlations.

Finally, although being aware of the important modifications that FSI may introduce in the analysis, we are rather confident that the high sensitivity of polarization-related observables to negative-energy projections already shown within RPWIA, will be probably also maintained within more elaborated relativistic distorted-wave impulse approximation (RDWIA) calculations. Work along this line is presently in progress.

ACKNOWLEDGMENTS

M.C.M. acknowledges a grant from the Fundación Cámara of the Universidad de Sevilla.

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