Abstract

The encoding of cities into non-planar dual graphs reveals their complex structure. We investigate the statistics of the typical space syntax measures for the five different compact urban patterns. Universal statistical behavior of space syntax measures uncovers the universality of the city creation mechanism.

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1 Paradigm of a city

Although, nowadays the majority of people live in cities there is no one standard international definition of a city: the term may be used either for a town possessing city status; for an urban locality exceeding an arbitrary population size; for a town dominating other towns with particular regional economic or administrative significance. In most parts of the world, cities are generally substantial and nearly always have an urban core, but in the US many incorporated areas which have a very modest population, or a suburban or even mostly rural character, are designated as cities.
Cities have often been compared with biological entities [2]. The implication that social organizations and dynamics relating urbanization to economic development and knowledge production are extensions of biology, satisfying similar principles and constraints has a strong empirical ground. Almost all physiological characteristics of biological organisms scale with the mass of their bodies, $M$, as a power law. For example, it has been reported in [5] that the curve describing the power, $B$, required to sustain a living organism is a straight line in the logarithmic scale,

$$B \simeq M^{3/4}. \quad (1)$$

It is remarkable to note that all important demographic, socioeconomic, and behavioral urban indicators such as consumption of energy and resources, production of artifacts, information, and waste, are, on average, scaling functions of city size that appear to be very general to all cities, across urban systems [3].

The evolution of social and economic life in the city increases with its population size: wages, income, growth domestic product, bank deposits, as well as rates of invention, measured by new patents and employment in creative sectors scale super-linearly, over different years and nations with statistically consistent exponents [4].

The probable reason for such a similarity is that highly complex, self-sustaining structures, whether cells, organisms, or cities constitute of an immense numbers of units being organized in a form of self-similar hierarchical branching networks, which grow with the size of the organism [5]. A universal social dynamic underlying the scaling phenomena observed in cities implies that an increase in productive social opportunities, both in number and quality, leads to quantifiable changes in individual behavior of humans in a city integrating them into a complex dynamical network [6].

In the present paper, we consider the dual graph representation of urban texture. After a brief introduction into city space syntax (Sec. 2) we describe the typical space syntax measures (Sec. 3) and then investigate the statistics of their values for the five different compact urban patterns (Sec. 4). Universal statistical behavior of space syntax measures uncovers the universality of the creation mechanism responsible for the appearance of nodes of high centrality which acts over all cities independently of their backgrounds.
2 Graphs and space syntax of urban environments

Urban space is of rather large scale to be seen from a single viewpoint; maps provide us with its representations by means of abstract symbols facilitating our perceiving and understanding of a city. The middle scale and small scale maps are usually based on Euclidean geometry providing spatial objects with precise coordinates along their edges and outlines.

There is a long tradition of research articulating urban environment form using graph-theoretic principles originating from the paper of Leonard Euler (see [7]). Graphs have long been regarded as the basic structures for representing forms where topological relations are firmly embedded within Euclidean space. The widespread use of graph theoretic analysis in geographic science had been reviewed in [8] establishing it as central to spatial analysis of urban environments. In [9], the basic graph theory methods had been applied to the measurements of transportation networks.

Network analysis has long been a basic function of geographic information systems (GIS) for a variety of applications, in which computational modelling of an urban network is based on a graph view in which the intersections of linear features are regarded as nodes, and connections between pairs of nodes are represented as edges [10]. Similarly, urban forms are usually represented as the patterns of identifiable urban elements such as locations or areas (forming nodes in a graph) whose relationships to one another are often associated with linear transport routes such as streets within cities [11]. Such planar graph representations define locations or points in Euclidean plane as nodes or vertices \( \{i\}, \; i = 1, \ldots, N \), and the edges linking them together as \( i \sim j \), in which \( \{i, j\} = 1, 2, \ldots, N \). The value of a link can either be binary, with the value 1 as \( i \sim j \), and 0 otherwise, or be equal to actual physical distance between nodes, \( \text{dist}(i, j) \), or to some weight \( w_{ij} > 0 \) quantifying a certain characteristic property of the link. We shall call a planar graph representing the Euclidean space embedding of an urban network as its primary graph. Once a spatial system has been identified and represented by a graph in this way, it can be subjected to the graph theoretic analysis.

A spatial network of a city is a network of the spatial elements
of urban environments. They are derived from maps of open spaces (streets, places, and roundabouts). Open spaces may be broken down into components; most simply, these might be street segments, which can be linked into a network via their intersections and analyzed as a networks of movement choices. The study of spatial configuration is instrumental in predicting human behavior; for instance, pedestrian movements in urban environments [13]. A set of theories and techniques for the analysis of spatial configurations is called space syntax [14]. Space syntax is established on a quite sophisticated speculation that the evolution of built form can be explained in analogy to the way biological forms unravel [12]. It has been developed as a method for analyzing space in an urban environment capturing its quality as being comprehensible and easily navigable [13]. Although, in its initial form, space syntax was focused mainly on patterns of pedestrian movement in cities, later the various space syntax measures of urban configuration had been found to be correlated with the different aspects of social life, [15].

Decomposition of a space map into a complete set of intersecting axial lines, the fewest and longest lines of sight that pass through every open space comprising any system, produces an axial map or an overlapping convex map respectively. Axial lines and convex spaces may be treated as the spatial elements (nodes of a morphological graph), while either the junctions of axial lines or the overlaps of convex spaces may be considered as the edges linking spatial elements into a single graph unveiling the topological relationships between all open elements of the urban space. In what follows, we shall call this morphological representation of urban network as a dual graph.

The transition to a dual graph is a topologically non-trivial transformation of a planar primary graph into a non-planar one which encapsulates the hierarchy and structure of the urban area and also corresponds to perception of space that people experience when travelling along routes through the environment.

In Fig. 1, we have presented the glossary establishing a correspondence between several typical elements of urban environments and the certain subgraphs of dual graphs. The dual transformation replaces the 1D open segments (streets) by the zero-dimensional nodes, Fig. 1(1).

The sprawl like developments consisting of a number of blind
passes branching off a main route are changed to the star subgraphs having a hub and a number of client nodes, Fig. 1(2). Junctions and crossroads are replaced with edges connecting the corresponding nodes of the dual graph, Fig.1(3). Places and roundabouts are considered as the independent topological objects and acquire the individual IDs being nodes in the dual graph Fig. 1(4). Cycles are converted into cycles of the same lengths Fig. 1(5). A regular grid pattern is shown in Fig. 1(6). Its dual graph representation is called a complete bipartite graph, where the set of vertices can be divided into two disjoint subsets such that no edge has both end-points in the same subset, and every line joining the two subsets is present, [16]. These sets can be naturally interpreted as those of the vertical and horizontal edges in the primary graphs (streets and avenues). In bipartite graphs, all closed paths are of even length, [17].

It is the dual graph transformation which allows to separate the effects of order and of structure while analyzing a transport network on the morphological ground. It converts the repeating geometrical elements expressing the order in the urban developments into the
twins nodes, the pairs of nodes such that any other is adjacent either
to them both or to neither of them. Examples of twins nodes can
be found in Fig. 1(2,4,5,6).

3 Space syntax measures

A number of configurational measures have been introduced in so
far in quantitative representations of relationships between spaces
of urban areas and buildings. Below we give a brief introduction
into the measures commonly accepted in space syntax theory.

Although similar parameters quantifying connectivity and cen-
trality of nodes in a graph have been independently invented and
extensively studied during the last century in a varied range of disci-
plines including computer science, biology, economics, and sociology,
the syntactic measures are by no means just the new names for the
well known quantities. In space syntax, the spaces are understood as
voids between buildings restraining traffic that dramatically changes
their meanings and the interpretation of results.

Space adjacency is a basic rule to form axial maps: two axial
lines intersected are regarded as adjacency. Two spaces, i and j,
are held to be adjacent in the dual graph $G$ when it is possible to
move freely from one space to another, without passing through any
intervening.

The adjacency matrix $A_G$ of the dual graph $G$ is defined as fol-
lows:

$$
(A_G)_{ij} = \begin{cases} 
1, & i \sim j, \\
0, & \text{otherwise.} 
\end{cases}
$$

Let us note that rows and columns of $A_G$ corresponding to the twins
nodes are identical. Depth is a topological distance between nodes in
the dual graph $G$. Two open spaces, i and j, are said to be at depth
d_{ij} if the least number of syntactic steps needed to reach one node
from the other is $d_{ij}$. The concept of depth can be extended to
total depth, the sum of all depths from a given origin,

$$
D_i = \sum_{j=1}^{N} d_{ij},
$$
in which $N$ is the total number of nodes in $G$. The average number
of syntactic steps from a given node $i$ to any other node in the dual
graph $\mathcal{G}$ is called the mean depth,

$$\ell_i = \frac{D_i}{N - 1}.$$  

(4)

The mean depth (4) is used for quantifying the level of integration/segregation of the given node, [14].

**Connectivity** is defined in space syntax theory as the number of nodes that connect directly to a given node in the dual graph $\mathcal{G}$, [18]. In graph theory, the space syntax connectivity$^1$ of a node is called the node **degree**:

$$\text{Connectivity}(i) = \deg(i) = \sum_{j=1}^{N}(A_{ij}).$$  

(5)

The **accessibility** of a space is considered in space syntax as a key determinant of its spatial interaction and its analysis is based on an implicit graph-theoretic view of the dual graph.

**Integration** of a node is by definition expressed by a value that indicates the degree to which a node is integrated or segregated from a system as a whole (**global integration**), or from a partial system consisting of nodes a few steps away (**local integration**), [19]. It is measured by the **Real Relative Asymmetry** (RRA) [16],

$$\text{RRA}(i) = 2\frac{\ell_i - 1}{D_N (N - 2)}.$$  

(6)

in which the normalization parameter allowing to compare nodes belonging to the dual graphs of different sizes is

$$D_N = 2\frac{N \left( \log_2 \left( \frac{N+2}{3} \right) - 1 \right) + 1}{(N - 1)(N - 2)}.$$  

(7)

Another local measure used in space syntax theory is the **control value** (CV). It evaluates the degree to which a space controls access to its immediate neighbors taking into account the number of alternative connections that each of these neighbors has. The control value is determined according to the following formula, [14]:

$$\text{CV}(i) = \sum_{i\sim j} \frac{1}{\deg(j)} = \sum_{j=1}^{N^i} (A_{ij}D^{-1})_{ij}$$  

(8)

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$^1$In graph theory, connectivity of a node is defined as the **number of edges** connected to a vertex. Note that it is not necessarily equal to the degree of node, $\deg(i)$, since there may be more than one edge between any two vertices in the graph.
where the diagonal matrix is $D = \text{diag}(\text{deg}(1), \text{deg}(2), \ldots, \text{deg}(N))$.

A dynamic global measure of the flow through a space $i \in \mathcal{G}$ commonly accepted in space syntax theory is the global choice, \[.\] It captures how often a node may be used in journeys from all spaces to all others spaces in the city. Vertices that occur on many shortest paths between other vertices have higher betweenness than those that do not. Global choice can be estimated as the ratio between the number of shortest paths through the node $i$ and the total number of all shortest paths in $\mathcal{G}$,

$$\text{Choice}(i) = \frac{\#\text{shortest paths through } i}{\#\text{all shortest paths}}. \quad (9)$$

A space $i$ has a strong choice value when many of the shortest paths, connecting all spaces to all spaces of a system, passes through it. The Dijkstra’s classical algorithm \[21\] which visits all nodes that are closer to the source than the target before reaching the target can be implemented in order to compute the value $\text{Choice}(i)$.

The integration and the global choice index are the centrality measures which capture the relative structural importance of a node in a dual graph.

## 4 Space syntax as a complex network

The encoding of cities into non-planar dual graphs reveals their complex structure, \[22\].

In order to illustrate the applications of complex network theory methods to the dual graphs of urban environments, we have studied five different compact urban patterns.

Two of them are situated on islands: Manhattan (with an almost regular grid-like city plan) and the network of Venice canals (imprinting the joined effect of natural, political, and economical factors acting on the network during many centuries). In the old city center of Venice that stretches across 122 small islands in the marshy Venetian Lagoon along the Adriatic Sea in northeast Italy, the canals serve the function of roads.

We have also considered two organic cities founded shortly after the Crusades and developed within the medieval fortresses: Rothenburg ob der Tauber, the medieval Bavarian city preserving its original structure from the 13th century, and the downtown of Biele-
feld (Altstadt Bielefeld), an economic and cultural center of Eastern Westphalia.

To supplement the study of urban canal networks, we have investigated that one in the city of Amsterdam. Although it is not actually isolated from the national canal network, it is binding to the delta of the Amstel river, forming a dense canal web exhibiting a high degree of radial symmetry.

The scarcity of physical space is among the most important factors determining the structure of compact urban patterns. Some characteristics of studied dual city graphs are given in Tab. 1. There, \( N \) is the number of open spaces (streets/canals and places) in the urban pattern (the number of nodes in the dual graphs), \( M \) is the number of junctions (the number of edges in the dual graphs); the graph diameter, \( \text{diam}(\mathcal{G}) \), is the maximal depth (i.e., the graph-theoretical distance) between two vertices in a dual graph; the intelligibility parameter estimates navigability of the city, suitability for the passage through it.

In the framework of complex network theory, the focus of study is shifted away from the analysis of properties of individual vertices to consideration of the statistical properties of graphs, [23].

The degree distribution has become an important concept in complex network theory describing the topology of complex networks. It originates from the study of random graphs by Erdős and Rényi, [24]. The degree distribution is the probability that a node selected at random among all nodes of the graph has exactly \( k \) links,

\[
P(k) = \Pr \{ i \in \mathcal{G} \mid \deg(i) = k \}. \tag{10}
\]

The probability degree distributions for the dual graph representations of the five compact urban patterns mentioned in Tab. 1 has been studied by us in [25]. It is remarkable that the observed profiles are broad indicating that a street in a compact city can cross the different number of other streets, in contrast while in a regular grid. At the same time, the distributions usually have a clearly noticeable maximum corresponding to the most probable number of junctions an average street has in the city. The long right tail of the distribution which could decay even faster then a power law is due to just a few "broadways," embankments, and belt roads crossing many more streets than an average street in the city, [25]. It has been suggested in [22] that irregular shapes and faster decays in
the tails of degree statistics indicate that the connectivity distributions is \textit{scale-dependent}. As a possible reason for the non-universal behavior is that in the mapping and descriptive procedures, inadequate choices to determine the boundary of the maps or flaws in the aggregation process can damage the representation of very long lines. Being scale-sensitive in general, the degree statistics of dual city graphs can nevertheless be approximately universal within a particular range of scales.

4.1 Integration statistics of dual city graphs

The integration level of a node can be estimated by means of the Real Relative Asymmetry index (6). It is obvious that if an urban network has a clear hierarchical structure, just a few of its nodes is well integrated becoming a part of the city core. Most of other routes are less integrated growing like branches from the main routes and penetrating inside the quarters, but they do not penetrate too much inside the "flesh" of the city.

Figure 2: The histogram shows the distribution of nodes in the dual graph representing the morphology of the street grid in Manhattan over the Real Relative Asymmetry index (6) \( \text{RRA}(i) \in [0, 1] \).

The distribution of nodes over the integration values can be represented in the form of a histogram (Fig. 2). The height of each bar in the histogram is equal to the number of nodes in the dual
graph for which the integration values fall into the disjoint intervals (known as bins). There is no apriori the best number of bins for the data sample, and the histograms drawn with respect to the different sizes of bins can reveal different features of the data. We have used the Scott’s method [28] in order to calculate the bin width $h$,

$$h = 3.5\frac{\sigma}{N^{1/3}},$$

where $N$ is the graph size, and $\sigma$ is the standard deviation of the RRA data, $\text{RRA}(i), i \in \mathcal{G}$.

The integration histogram Fig. 2 has a sharp peak located at $\text{RRA}(i) = 0.17$. It indicates that the valuable fraction of streets in Manhattan belong to a class of secondary routes characterized by the relatively low RRA values. The sharp decay of the histogram at the utmost right cut represents a few nodes identified as an integration city core. A small number of streets that is at the utmost left position in Fig. 2 represent the segregated streets. It is worth to mention that in contrast to organic cities (which we discuss below), where the center of integration city core (the syntactic center of the city) usually matches the geographical city center, the essentially strongly integrated streets in the city of hierarchical structure are either the "broadways" crossing the city (the Broadway in Manhattan) or the belt routes encircling the city perimeter.

The mismatching of integration profiles can be used for the purpose of comparison between the different urban networks provided the correspondent integration histograms are normalized with respect to the network sizes.

Let us consider the probability distribution for the RRA index $P(i)$ such that a node $i$ selected at random among all nodes of the dual graph $\mathcal{G}$ has the value $\text{RRA}(i) = i$,

$$P(i) = \Pr [i \in \mathcal{G}|\text{RRA}(i) = i].$$

The continuous approximations for the empirical probability distributions $P(i)$ calculated for two German medieval cities subjected to the organic development unveil their structural difference.

The integration cores of the old organic cities usually form a compact, dense sub-structure. The empirical probability distributions for such cities are typically bell shaped (see the red dashed line representing the continuous approximation of $P(i)$ for the city
Figure 3: The continuous approximations of the empirical probability distributions $P(i)$ calculated for two German medieval cities: Rothenburg ob der Tauber, (the red dashed line) and the city of Bielefeld (the black solid line). While the city of Rothenburg preserves its original organic structure from the late 13th century, the Bielefeld downtown is composed of two different parts: the old section founded in the 13th century and another one subjected to partial urban redevelopment in modern times. The structural difference between two cities is clearly visible on their integration statistics profiles.

The major routes of such a city are well integrated becoming a part of the city core while the secondary routes branch from them penetrating inside the quarters, but not too much inside of the city 'bulk'.

The empirical probability distribution for the city of Bielefeld (see the black solid line in Fig. 3) is of essential interest since its downtown is comprised of two structurally different parts. The ancient part of the city downtown keeps its original organic structure from the late 13th century, while another part was subjected to the partial redevelopment twice, at the end of the 19th and in the 20th century.

The profiles of integration statistics clearly display structural differences between two urban patterns. The empirical integration distribution for the city of Bielefeld has two maxima that is an evidence in favor of two different most probable integration values. It appears due to the fact that the values of local integration are higher for streets belonging to one and the same city district, while it is lower
with respect to those streets from alternative city components. The only city route \((\text{Niederwall})\) being a boundary between the ancient and redeveloped parts of the city conjugates them both.

Another, probably more intuitive measure quantifying centrality of a node in a dual graph is the global choice \((9)\).

The global choice value \((9)\) of a given node counts the fraction of shortest paths between all origin/destination pairs passing through it, \(\text{Choice}(i) \in [0, 1]\). Vertices that occur on many shortest paths between other vertices have higher global choice index than those that do not. The global choice analysis is preferred in network analysis to mean shortest-path length, as it gives higher values to more central vertices. It has recently been implemented in order to study and compare the differences between urban centers \([26]\) and for angular analysis within space syntax \([27]\). The global choice is calculated by generating shortest paths between all nodes within the dual graph accordingly to the Dijkstra’s algorithm \([21]\).

We have investigated into the betweenness statistics of the dual graphs of compact urban patterns. The betweenness is a measure describing how shortest paths between the pairs of vertices are distributed over the network. The betweenness of vertex \(i\) is nothing else but the probability that a randomly chosen shortest paths in the graph passes through \(i\). Most of vertices in the dual graphs of the compact urban developments are characterized by the relatively low values of the global choice index (below, we call them as the ‘bulk’ nodes). At the same time just a few nodes, the main city itineraries, which play a key role in the overall city connectedness has the exceptionally high values of the global choice index.

Therefore a natural question arises on the distribution of the ‘bulk’ nodes in a dual graph over the global choice index (betweenness). Let us consider the probability that a ‘bulk’ node chosen randomly among all of them in the dual graph has the value of the global choice index \(s \in [0, 1]\):

\[
P(s) = \Pr [i \in \mathcal{G} | \text{Choice}(i) = s].
\]  

\((13)\)

In Fig. 4, we have presented the log-log plot for the continuous approximations for the probability distributions of the global choice index for five dual graphs of the compact urban patterns. The obvious discrepancy in the probability distribution profiles \(P(s) \)\((13)\) indicate the multiple structural differences between the dual graphs.
of the studied urban patterns. The probability that an arbitrary bulk node would have a very small global choice index is high for the organic cities (Bielefeld, Rothenburg, the canal networks of Venice and Amsterdam), while it is considerably low for the city planned in a regular grid (Manhattan).

The probability that a node has a strong global choice index decays very fast with the value of $s$ for the modern street grid in Manhattan and for the eternal Venetian canals. In both urban developments, the strongest choice is represented by just a few open spaces (Broadway, Franklin D Roosevelt Dr., West Side Hwy, and Henry Hudson Pkwy, in Manhattan; the Venetian Lagoon, the Giudecca Canal, and the Grand Canal, in Venice) while all other routes contributing to the statistics shown in Fig. 4 play merely the subsidiary role canalizing the local flows towards the main itineraries. In contrast to them, the probability distributions for the global choice index over the open spaces in the compact cities of organic development decay much slower, with the contributions from the main itineraries of the highest global choice indices being a part of the continuous distributions.
4.2 Control value statistics of dual city graphs

The CV($i$)-parameter quantifies the degree of choice the node $i \in \mathcal{G}$ represents for other nodes directly connected to it.

Figure 5: The log-log plot of the probability distribution that a node randomly selected among all nodes of the dual graph $\mathcal{G}$ will be populated with precisely $m$ random walkers in one step starting from the uniform distribution (one random walker at each node). The dashed line indicates the cubic hyperbola decay, $P(m) = m^{-3}$.

Provided random walks, in which a walker moves in one step to another node randomly chosen among all its nearest neighbors are defined on the graph $\mathcal{G}$, the parameter CV($i$) acquires a probabilistic interpretation. Namely, it specifies the expected number of walkers which is found in $i \in \mathcal{G}$ after one step if the random walks starts from a uniform configuration, in which all nodes in the graph have been uniformly populated by precisely one walker.

Then, a graph $\mathcal{G}$ can be characterized by the probability

$$P(m) = \Pr [i \in \mathcal{G} | \text{CV}(i) = m]$$

(14)

of that the control value of a node chosen uniformly at random
among all nodes of the graph $\mathcal{G}$ equals to $m > 0$.

The log-log plot of (14) is shown in Fig. 5. It is important to mention that the profile of the probability decay exhibits the approximate scaling well fitted by the cubic hyperbola, $P(m) \simeq m^{-3}$, universally for all five compact urban patterns we have studied.

Universal statistical behavior of the control values for the nodes representing a relatively strong choice for their nearest neighbors,

$$\Pr \{ i \in \mathcal{G} | CV(i) = m \} \simeq \frac{1}{m^3},$$

(15)

uncovers the universality of the creation mechanism responsible for the appearance of the ”strong choice” nodes which acts over all cities independently of their backgrounds. It is a common suggestion in space syntax theory that open spaces of strong choice are responsible for the public space processes driven largely by the universal social activities like trades and exchange which are common across different cultures and historical epochs and give cities a similar global structure of the ”deformed wheel” \[29\].

It has been shown long time ago by H. Simon \[30\] that the power law distributions always arise when the ”the rich get richer” principle works, i.e. when a quantity increases with its amount already present. In sociology this principle is known as the Matthew effect \[31\] (this reference appears in \[23\]) following the well-known biblical edict.

4.3 Intelligibility and Navigation

The kinds of human behavior that appear to be particularly structured by open spaces are pedestrian movement and navigation. It appears that the variance between volumes of pedestrian movement at different places in a city can be predicted reasonably accurately from investigations of spatial configurations alone. \[20\]. In space syntax, correlations between local property of a space (connectivity) and global configurational variables (integration) constitute a measure of the intelligibility, the global parameter quantifying the part-whole relationship within the spatial configuration. Intelligibility describes how far the depth of a space from the street layout as a whole can be inferred from the number of its direct connections \[18\] that is most important to way-finding and perception of environments \[32\] \[33\]. More integrated areas were also found to be
more “legible” by the residents who perceived their “neighborhood” to be of a greater size, [34].

In the traditional space syntax approach, the strong area definition and good intelligibility are identified in an *intelligibility scattergram* and then by means of the *Visibility Graph Analysis* (VGA) [35]. In statistics, a scatter plot is a useful summary of a set of two variables, usually drawn before working out a linear correlation coefficient or fitting a regression line. Each node of the dual graph contributes one point to the scatter plot. The resulting pattern indicates the type and strength of the relationship between the two variables, and aids the interpretation of the correlation coefficient [36].

The measure of spatial integration for each node $i \in \mathcal{G}$ is usually taken the mean depth $\ell_i$, however the analysis varies for specific case studies, and the precise measures of the graph are chosen to best correlate.

We prefer to use the global choice parameter (9) as a measure of spatial integration. The scatter plot for the downtown of Bielefeld (in the log-log scale) which shows the relationship between connectivity and global choice (centrality) is sketched on Fig. 6. The pattern of dots (representing the certain open spaces in the downtown of Bielefeld) slopes from lower left to upper right that suggests

![Figure 6](image-url)

*Figure 6:* The intelligibility scatter plot for the Bielefeld downtown. The slope of the regression line fitting the data by the method of least squares equals to 0.253.
a positive correlation between the connectivity and centrality variables being studied. A line of best fit computed using the method of linear regression exhibits the slope $0.253$. Let us note that the value of Pearson’s coefficient of linear correlations between the data samples of connectivity and global choice for Bielefeld equals to $0.681$.

The correlations between local and global properties within the spatial configurations of urban networks (intelligibility) can be quantified by means of different methods. The level of correlations can be reckoned by the slope of the regression line fitting the data of the scatter plot drawn in the logarithmic scale. Eventually, we can directly compute Pearson’s coefficient of linear correlations between the uniformly ordered connectivity and integration values. In order to show the compatibility of these methods, we collect the results of all intelligibility estimations for the five compact urban patterns that we studied in one diagram (see Fig. 7). It is clear

![Intelligibility of compact urban patterns](image)

Figure 7: The comparative diagram of intelligibility indices calculated for the five compact urban patterns by different methods.

from the diagram (7) that being an important characteristic related to a perception of place and navigation within that, intelligibility can be used in a purpose of comparison between urban networks. The obvious advantage of intelligibility is that it does not depend upon the network size.

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The intelligibility indices estimated by means of Pearson’s correlation coefficient have been given in Tab. 1.

5 Conclusion

We have investigated the complex networks of city space syntax observed in the several compact urban patterns. We have studied the local and various global measures characterizing a node in a dual city graph as well as the positive relationship between local and global properties known as intelligibility.

If cities were perfect grids in which all lines have the same length and number of junctions, they would be described by regular graphs exhibiting a high level of similarity no matter which part of urban texture is examined. This would create a highly accessible system that provides multiple routes between any pair of locations. It was believed that pure grid systems are easy to navigate due to this high accessibility and to the existence of multiple paths between any pair of locations [39, 38]. Although, the urban grid minimizes descriptions as long as possible in the ideal grid all routes are equally probable; morphology of the perfect grid does not differentiate main spaces and movement tend to be dispersed everywhere [22]. Alternatively, if cities were purely hierarchical systems (like trees), they would clearly have a main space (a hub, a single route between many pairs of locations) that connects all branches and controls movement between them. This would create a highly segregated, sprawl like system that would cause a tough social consequence, [22].

However, real cities are neither trees nor perfect grids, but a combination of these structures that emerge from the social and constructive processes [13]. They maintain enough differentiation to establish a clear hierarchy [40] resulting from a process of negotiation between the public processes (like trade and exchanges) and the residential process preserving their traditional structure. The emergent urban network is usually of a very complex structure which is therefore naturally subjected to the complex network theory analysis.
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Table 1: Some features of studied dual city graphs

| Urban pattern            | $N$ | $M$ | $\text{diam}(G)$ | Intelligibility |
|--------------------------|-----|-----|-------------------|-----------------|
| Rothenburg ob d.T.       | 50  | 115 | 5                 | 0.85            |
| Bielefeld (downtown)     | 50  | 142 | 6                 | 0.68            |
| Amsterdam (canals)       | 57  | 200 | 7                 | 0.91            |
| Venice (canals)          | 96  | 196 | 5                 | 0.97            |
| Manhattan                | 355 | 3543| 5                 | 0.51            |