Deformation of granular soil under combination of principal stress value and direction change

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ABSTRACT

Soil is often subjected to complex three-dimensional stress paths in geotechnical engineering. It is difficult to achieve such complex stress paths in laboratory tests. To this means, discrete element numerical simulations are carried out on cross-anisotropic granular material by using PFC3D in this study. The response of granular soil under three-dimensional seismic load is simulated and compared with results from corresponding pure principal stress magnitude change or direction change scenarios, illustrating the importance of principal stress value and direction combination on the mechanical behavior of soil. Simulations of three kinds of idealized stress paths are then conducted and discussed. In the idealized stress path simulations, the deformation and micro-scale fabric evolution is observed to be most significant under pure direction change.

Keywords: granular soil, deformation, principal stress value, principal stress direction, DEM simulation

1 INTRODUCTION

Naturally or artificially deposited soils are almost always anisotropic, and are often subjected to complex engineering loads, with a combination of change in both principal stress value and direction. However, due to mechanical limitations, experimental investigations into the response of soil under such loads are limited.

The effect of principal stress orientation change has been considered by many scholars, conducting undrain tests (Ishihara and Yamazaki, 1980; Ishihara and Tawhata, 1983; Ishihara and Yamazaki, 1984; Tawhata and Ishihara, 1985; Nakata et al., 1998) and drained tests (Miura et al., 1986; Miura et al., 1986; Tong et al., 2009; Yu et al., 2011; Tong et al., 2010) using the hollow cylinder torsional shear apparatus. In simple cases, combined principal stress value and direction change have also been considered (Ishihara and Yamazaki, 1980; Tawhata and Ishihara, 1985), proving that both types of changes influence soil deformation.

Numerical simulations have been conducted to study the effect of principal stress direction change using discrete element method (DEM, Cundall and Strack, 1979) (Li and Yu, 2010; Tong et al., 2014; Li et al., 2016; Xue et al., 2019; Wang et al., 2019).

The current study focuses on the deformation laws of soil under simultaneous change of principal stress value and direction, using DEM simulation via PFC3D (Li et al, 2016). The evolution of micro-scale fabric is discussed in relation to the macro deformation.

2 NUMERICAL METHOD

2.1 Test scheme and sample preparation

A total of 10 DEM simulations are carried out, as shown in Table 1. Shear stress $q$ and average spherical stress $p$ is computed using the following equations:

$$p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$ (1)

$$q = \sqrt{\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2}$$ (2)

$\sigma_1$ is the major principal stress, while $\sigma_2$ the middle principal stress, and $\sigma_3$ is the minor principal stress.

| Test ID | Stress path type | $q_{min}$ (kPa) | $q_{max}$ (kPa) | $p$ (kPa) |
|---------|------------------|-----------------|-----------------|-----------|
| L-0-30-F | Straight-line | 0 | 30 | 100 |
| S-30-30-R | Circular | 0 | 30 | 100 |
| D-0-30-R | “8”-shaped | 0 | 30 | 100 |
| D-F | Seismic load | - | - | - |
| D-R | Seismic load | - | - | - |
| D-M | Seismic load | - | - | - |

https://doi.org/10.3208/jgssp.v08.c40
In the test IDs, the first letter indicates the stress path type, where L represents a straight line in \( t_{xz} - (\sigma_z - \sigma_x)/2 \) plane, S represents circular path, B represents "8"-shaped path, and D represents seismic load stress path. The following two numbers represent the range of shear stress. The last letter represents whether rotation of principal stress is included: F - no rotation, R - continuous rotation, and M - both value and direction change. The three kinds of idealized paths are shown in Fig. 1. Under these three conditions, the mean effective stress is constant at 100 kPa, intermedia principal stress coefficient \( b = 0.5 \), and the principal stress is rotated around intermedia principal stress axis (y axis). The stress path of seismic load is designed to mimic the stress path of soil under a combination of seismic waves in various directions, and is discussed in further detail the following section.

Through such boundary conditions, complex stress paths can be achieved with ease by changing the force value on the lines and the orientation of lines.

The stress state be expressed as (Bagi, 1996):

\[
\sigma_{ij} = \frac{1}{V} \sum_{i=1}^{3} f_{i} f'_{j}
\]

\( \sigma_{ij} \) is the i, j stress component, \( c \) is the contact within calculation zone, whose volume is \( V \). \( f_{i} f'_{j} \) is contact force at i direction. \( f'_{j} \) is contact vector at j direction.

The strain is measured as (Drescher and De Jong, 1972):

\[
\text{d} \varepsilon_{ij} = \frac{1}{V} \int_{V} \text{d} \varepsilon_{ij} \, dV
\]

\( \text{d} \varepsilon_{ij} \) is the strain increment. \( \text{d} \varepsilon_{ij} \) is the average.

The deviatoric contact fabric tensor is used to represent material anisotropy using the following formula (Satake, 1978):

\[
F_{ij} = \frac{1}{N} \sum_{k=1}^{N} n_{i} n_{j} - \frac{1}{3} \delta_{ij}
\]

\( N \) is contacts number, \( k \) represents the \( k \)th contact, and \( n_{i} \) is the \( i \) component of unit contact vector.

2.3 Method verification

The efficacy of the numerical method in simulating granular material behavior under complex stress paths is first verified through simulations of pure principal stress axes rotation. Numerical simulation (S-30-30-R, \( p=100kPa \), \( b=0.5 \), \( q=30kPa \)) and laboratory tests by Tong et al. (\( b=0.5 \), \( p=100kPa \), \( q=86.6kPa \)) with pure principal stress axes rotation are compared qualitatively, without trying to calibrate the numerical simulation to be quantitatively matching to the test. Fig 2 shows the loading stress path achieved in S-30-30-R test. The relative error between measured and target stress is within 3%. Similar deformation patterns are observed qualitatively in numerical simulation and laboratory test.

2.2 Loading and measurement method

A force line method for the application of arbitrary stress boundaries on DEM samples developed by Fu et al (2012) is extended to 3D. For principal stress \( \sigma_{0} \) in a certain direction, force lines are uniformly generated. The distance between each force line is \( \Delta l' \), so the force represented by each line is \( \sigma_{0} \Delta l' \). The force is applied to the first and last particles of the sample that intersect the force line, in opposite directions. \( \Delta l' \) needs to be small enough to ensure accuracy.

![Fig. 1. Stress paths for simulation: (a) straight-line path; (b) circular path; (c) "8"-shaped path.](image)

![Fig. 2. Stress path achieved in S-30-30-R.](image)
Both principal stress value and direction can change in various geotechnical engineering scenarios. One typical case is the stress applied on soil under 3D seismic loading with shaking in both horizontal and vertical directions. The behavior of granular materials under such conditions is briefly investigated to highlight the importance of combining principal stress value change with direction change.

In a simplified scenario, the 3D seismic load to be simulated on a soil element is represented by the following equation:

\[
\begin{vmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_y & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_z
\end{vmatrix} =
\begin{bmatrix}
p_0 & 0 & \tau_0 \cos \theta \\
0 & p_0 & \tau_0 \sin \theta \\
\tau_0 \cos \theta & \tau_0 \sin \theta & p_0 + \tau_0 \cos \theta
\end{bmatrix}
\]  

where \( p_0 = 100\text{kPa} \), \( \tau_0 = 20\text{kPa} \), and \( \theta = 0\degree \text{ to } 360\degree \). The stress path is applied to the spherical sample, where \( \theta = 0\degree \) is set as the initial state.

For comparison purposes, three cases are simulated: the first one keeps the initial principal stress direction fixed with principal stress value variation amplitude the same as that in Eq. (6), namely D-F; the second one keeps the initial principal stress constant with continuous pure principal stress axes rotation also calculated from Eq. (6), i.e. D-R; the third one follows exactly Eq. (6), combining both principal stress value and direction change, i.e. D-M.

Fig. 4 shows the volume strain under the three stress conditions. In all cases, the sample experiences volume contraction accumulation. When only principal stress value changes, the contraction is insignificant. Under pure principal stress axes rotation, contraction becomes much more pronounced, where \( q = 40\text{kPa} \) and \( p = 107\text{kPa} \) kept constant. In the case of 3D seismic load, volumetric strain accumulation is the greatest. This is because under 3D seismic load, \( p \) is 93kPa at peak deviatoric stress \( q = 40\text{kPa} \), resulting greater deviatoric stress ratio compared with D-R. This indicates that the combined effect of change in principal stress value and direction is important to the deformation of soil.

4 IDEALIZED STRESS PATHS SIMULATION

Having highlighted the importance of the combined effects of principal stress value and direction changes, these two factors are investigated in further detail under idealized stress paths. As listed in Table 1, a straight-line stress path where only the principal stress value changes cyclically (L-0-30-F), a circular stress path where pure principal stress axes rotation occurs (S-30-30-R), and an “8” shaped stress path where both factors come into play (B-0-30-R) are adopted.

4.1 Volumetric strain

In all three cases, the mean effective stress \( p \) is kept constant at 100kPa, and the volumetric strain is caused solely by deviatoric stress increments. Fig. 5 shows volumetric strain for all three simulations. Continuous volume contraction is observed for all three cases, with oscillation during each cycle. Compared to strain under straight-line stress path, greater deformation occurs under “8”-shaped stress path even though the principal stress value variations are the same. The greatest volumetric strain is observed under circular path, as the maximum deviatoric stress amplitude is maintained.
under this stress path.

Fig. 5. Volumetric strain under three idealized stress paths.

4.2 Maximum shear strain

The maximum shear strain is defined as:

$$\gamma = \sqrt{\frac{2}{3} e : e} = \sqrt{\frac{2}{3} \sum_{i,j} e_{ij} e_{ij}}$$

(7)

e is the deviatoric strain tensor, calculated from the strain tensor:

$$e_{ij} = e_{ij} - \frac{1}{3} e_{kk} \delta_{ij}$$

(8)

Fig. 6. Maximum shear strain under three idealized stress paths.

4.3 Dilatancy ratio

Shear induced volumetric deformation, a unique characteristic of granular materials, can be described via the dilatancy ratio, defined as:

$$D = \frac{\Delta e_v}{\Delta \gamma}$$

(9)

As is shown in Fig. 7, for all three stress paths, positive dilatancy ratio is observed initially, indicating continuous volumetric contraction. After a few cycles, dilatancy begins to fluctuate around zero under the three types of cyclic loading. Compared to straight-line path, initial dilatancy ratio and its fluctuation are greater under circular and “8”-shaped stress paths, indicating more significant volumetric strain.

4.4 Influence of fabric evolution

For granular material, macro deformation is ultimately caused by micro-scale fabric evolution, which indicates rearrangement of soil particles. Fig. 8 shows the evolution of the angle between the major principal axis of the contact normal fabric and the z axis for all three idealized stress paths. Compared to the other two cases, the most significant angle variation is observed in circular path, implying the most significant particle rearrangement.

The relationship between maximum shear strain increment and deviatoric stress increment can be described as an equivalent shear modulus:

$$G = \left| \frac{ds}{de} \right| = \sqrt{\frac{\sum_{i,j} \sum_k \delta_{kj} \delta_{ij}}{\sum_{mn} \sum_{i,j} e_{mn} e_{ij}}}$$

(10)

s is the deviatoric stress tensor, computed from stress tensor, defined as:
This shear modulus can indicate how easily the granular soil can deform in shear under certain deviatoric stress increments. Fig. 9 shows the shear modulus for all three stress paths. Consistent with the evolution of contact fabric, the lowest shear modulus is observed under circular path, compared to that under the other two paths. As a result, the greatest maximum shear strain occurs under pure principal stress axis rotation, causing greatest volumetric contraction even though the dilatancy ratio under circular path and “8”-shaped paths are similar.

![Shear modules under three idealized stress paths.](image)

Fig. 9. Shear modules under three idealized stress paths.

5 CONCLUSIONS

Soil is often subjected to 3D complex stress paths in geotechnical engineering, with coupled change of principal stress value and direction. A 3D DEM loading method is verified applied in current study to investigate the response of soil under such condition. A simplified 3D seismic load with changes in both principal stress value and direction is simulated. The importance of the combined effects of principal stress value and direction changes is highlighted.

Three kinds of idealized stress paths are simulated to further investigate the influence of principal stress value and direction change. Compared to the straight-line path and the “8”-shaped path, the most significant contact fabric variation is observed under the circular path. The lowest shear modulus is observed under the circular stress path, while initial dilatancy ratio (in contraction) is similar for the circular and “8”-shaped stress paths, both greater than that under the straight-line path. These results in the greatest shear strain and volumetric contraction to be observed under the circular stress path.

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