GRB 110731A: EARLY AFTERGLOW IN STELLAR WIND POWERED BY A MAGNETIZED OUTFLOW

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ABSTRACT

One of the most energetic gamma-ray bursts, GRB 110731A, was observed from an optical to GeV energy range. Previous analysis of the prompt phase revealed similarities between the Large Area Telescope (LAT) bursts observed by Fermi: (1) a delayed onset of the high-energy emission (>100 MeV), (2) a short-lasting bright peak at later times, and (3) a temporarily extended component from this phase, lasting hundreds of seconds. Additionally to the prompt phase, multiwavelength observations over different epochs showed that the spectral energy distribution was better fitted by a wind afterglow model. We present a leptonic model based on an early afterglow that evolves in a stellar wind of its progenitor. We apply this model to interpret the temporally extended LAT emission and the brightest LAT peak exhibited by the prompt phase of GRB 110731A. Additionally, using the same set of parameters, we describe the multiwavelength afterglow observations. The origin of the temporally extended LAT, X-ray, and optical flux is explained through synchrotron radiation from the forward shock (FS) and the brightest LAT peak is described, evoking the synchrotron self-Compton emission from the reverse shock (RS). The bulk Lorentz factor required in this model ($\Gamma \approx 520$) lies in the range of values demanded for most LAT-detected GRBs. We show that the strength of the magnetic field in the RS region is ~50 times stronger than that in the FS region. This result suggests that, for GRB 110731A, the central engine is likely entrained with strong magnetic fields.

Key words: gamma-ray burst: individual (GRB 110731A) – radiation mechanisms: non-thermal

1. INTRODUCTION

In recent years, the detection of $\gamma$-rays and optical polarization in gamma-ray bursts (GRBs) has supported the idea that jets could be magnetized (Boggs & Coburn 2003; Coburn & Boggs 2003; Rutledge & Fox 2004). The jet evolution with magnetic content has been explored in several contexts. In these models, an electromagnetic component is introduced through the magnetization parameter ($\sigma$) and defined by the ratio of Poynting flux (electromagnetic component) and matter energy (internal + kinetic component; Wheeler et al. 2000; Blandford et al. 2002; Drenkhahn 2002; Lyutikov et al. 2002; Vlahakis & Königl 2003a, 2003b; Spruit et al. 2004; Zhang & Yan 2011; Fraija 2014).

The afterglow transition is one of the most interesting and least understood gamma-ray phases. During this phase, the relativistic ejecta interacts with the surrounding matter generating reverse and forward shocks (FSs). A strong short-lived reverse shock (RS) propagates into the ejecta whereas the long-lasting FS leads to a continuous softening of the afterglow spectrum (Nakar & Piran 2004; Panaitescu 2007). The dynamics of the RS in a wind and constant medium has been discussed by many authors (Mészáros & Rees 1997a; Sari & Piran 1999; Li & Chevalier 2003; Kobayashi & Zhang 2003; Wu et al. 2003; Fraija et al. 2012a, 2012b; Sacahui et al. 2012). The RS has been invoked to explain the early $\gamma$-ray, optical, and/or radio flares. After the peak, no new electrons are injected and the material cools adiabatically, although if the central engine emits slowly moving material the RS could survive from hours to days (Genet et al. 2007; Uhm & Beloborodov 2007). On the other hand, the origin of early flashes has also been discussed in the internal shock framework when they are nearly two orders of magnitude weaker than those produced by FSs (for the same total energy; Mészáros & Rees 1997a, 1999; Kumar & Piran 2000). Early observations of GRB afterglows would offer to clarify the question of whether the early emission takes place at internal or external shocks.

As has been pointed out in the literature, emission regions and radiative processes of high-energy (HE) photons with energies $\geq 100\text{ MeV}$ have been fully explored. On hadronic models, $\gamma$-ray components have been explained through photo-hadronic interactions between HE hadrons accelerated in the jet and internal synchrotron photons (Dermer et al. 2000; Asano et al. 2009), inelastic proton–neutron collisions (Mészáros & Rees 2000), and interactions of HE neutrons and photons out of the jet (Alvarez-Muñiz et al. 2004; Dermer & Atoyan 2004). On leptonic models, $\gamma$-ray fluxes have been explored with inverse Compton (IC), synchrotron self-Compton (SSC), and synchrotron processes at different regions of the jet. By considering electrons and photons at internal and external shocks, IC emissions have been discussed in detail in internal shocks (Papathomasiou & Meszaros 1996; Pilla & Loeb 1998; Panaitescu & Mészáros 2000; Gupta & Zhang 2007), FSs (Sari et al. 1996; Waxman 1997; Totani 1998; Panaitescu & Mészáros 1998a; Wei & Lu 1998; Chiang & Dermer 1999; Dermer et al. 2000; Panaitescu & Kumar 2000), and RSs (Wang et al. 2001a, 2001b; Pe’er & Wijers 2006). GeV photons generated from SSC emission in FS (Sari & Esin 2001; Wang et al. 2001a; Zhang & Mészáros 2001; Sacahui et al. 2012; Veres & Mészáros 2012) and RS (Granot & Guetta 2003; Wang et al. 2001a, 2001b) have been investigated separately and synchrotron radiation has only been examined in a few cases (Kumar & Barniol Duran 2009, 2010; Ghisellini et al. 2010; Piran & Nakar 2010; He et al. 2011; Liu & Wang 2011). Additionally, a particular lepto-hadronic model was developed to explain the HE emissions of GRB 090510 (Razzano 2010).

The bright and long GRB 110731A was detected by Fermi and Swift observatories, and by the MOA and Gamma-ray Burst Optical/Near-Infrared Detector (GROND) optical
The analysis of the prompt phase revealed the brightest peak in the Large Area Telescope (LAT) light curve (LC) starting at ~5.5 s (Ackermann et al. 2013) and a temporally extended LAT component described with a power law. In addition, temporal and spectral analysis in different wavelengths and epochs (just after the trigger time and extending for more than 800 s) favored a wind afterglow model. Recently, assuming that the long-lasting LAT component could be described as synchrotron radiation by relativistic electrons accelerated through FSs and requiring that the magnetic equipartition parameter varies as a function of time, $\epsilon_B \propto t^{-\alpha}$ with $0.5 \leq \alpha \leq 0.4$, Lemoine et al. (2013) determined that these GeV photons were likely produced in a region of strong $\epsilon_B$. They argued that the magnetization that permeates the blast wave of GRB 110731A can be described as partial decay of the micro-turbulence (Rossi & Rees 2003; Pe'er & Zhang 2006) as observed in particle-in-cell (PIC) simulations (Spitkovsky 2008; Martins et al. 2009; Haugbolle 2011; Sironi & Spitkovsky 2011; Sironi et al. 2013).

In this paper, we develop a leptonic model based on early afterglow with variable density (stellar wind, $s = 2$) to describe the temporally extended emission and the brightest peak present in the LAT LC of GRB 110731A. The paper is arranged as follows. In Section 2, we show a leptonic model based on external shocks (forward and reverse) that evolves adiabatically in a stellar wind. In Section 3, we apply this model to GRB 110731A as a particular case and, in Section 4, we give a brief summary.

2. EXTERNAL SHOCK MODEL

As the blast wave extends out into the stellar dense wind of the progenitor, it starts to be decelerated leading to FSs and RSs. The evolution of the afterglow will mainly depend on its mass and in some cases, the emission processes of internal shocks and RSs which could be simultaneously present in the LC. In the following subsections, we will develop the dynamics of the external shocks in stellar winds when Fermi-accelerated electrons are cooled down by synchrotron and Compton scattering emission at FSs and RSs. In addition, we will consider the RS in both the thick- and thin-shell cases. Hereafter, we use primes (unprimes) to define the quantities in a comoving (observer) frame and $c = h = 1$ in natural units. The subscripts $f$ and $r$ refer throughout this paper to the forward and RSs, respectively.

2.1. Forward Shocks

Afterglow hydrodynamics involves a relativistic blast wave expanding into the medium with density

$$\rho = A r^{-2} \quad \text{with} \quad A = \frac{M_\dot{m}}{4\pi V_\infty},$$

where $\dot{M}_\dot{m}$ is the mass-loss rate and $V_\infty$ is the wind velocity. For an ultra relativistic and adiabatic blast wave, the radius shock ($r$) spreading into this density can be written in the form

$$r = \frac{3\xi}{2\eta^{1/2}} (1 + z)^{-1/2} E^{1/2} \Gamma^{-1/2} A^{-1/2}.$$

Here the total energy ($E$) of the shock is constant and given by $E = 8\pi/9 A \Gamma^2 r$, $\Gamma$ is the bulk Lorentz factor, $\xi$ is a constant parameter (Panaitescu & Mészáros 1998b), $z$ is the redshift, and $t$ is the time in the observer’s frame (Sari 1997; Dai & Lu 1998; Panaitescu & Mészáros 1998b) which is given by

$$t = (1 + z) \frac{r}{4 \xi^2 \Gamma^2}.$$

From Equations (1) to (3), we get the scale of deceleration time

$$t_{\text{dec}} = \frac{9}{64\pi \xi^2 (1 + z) E \Gamma^{-1} A^{-4}}.$$

Synchrotron emission. Considering that electrons are accelerated to a power-law distribution $N(\gamma_e) \propto \gamma_e^{-\gamma} d\gamma$ with the electron spectral index $p > 2$, and the energy density ($U$) is equipartitioned to accelerate electrons ($U_e = \epsilon_{e,f} U = m_e \int_{\gamma_e} N(\gamma_e) d\gamma_e$) and to amplify the magnetic field $U_B,f = \epsilon_{B,f} U$ (with $U_B,f = B^2,f/8\pi$), the minimum electron Lorentz factor and the magnetic field can be written as

$$\gamma_{e,m,f} = \frac{(\rho - 2)m_p}{(\rho - 1)m_e} \epsilon_{e,f} \Gamma,$$

and

$$B^f_f \approx \frac{8\sqrt{2}}{3\xi} (1 + z)^{1/2} \epsilon_{B,f}^{1/2} \Gamma \eta^{-1/2} \eta^{-1/2} A,$$

respectively. Here, $\epsilon_{e,f}$ ($\epsilon_{B,f}$) is the electron (magnetic) equipartition parameter and $m_e$ ($m_p$) is the electron (proton) mass. When the expanding relativistic ejecta encounter the stellar wind, it starts to be decelerated, then electrons are first heated and then cooled down by synchrotron emission. Comparing both timescales, the deceleration time (Equation (4)), and the characteristic cooling time for synchrotron radiation $t_{\text{cool}} \approx 3m_e/16\sigma_T(1 + x_f)^{-1}(1 + z)\epsilon_{e,f}^{-1}\Gamma^{-1} \beta^{-1} \gamma^{-1}$, the characteristic electron Lorentz factor can be written in the form

$$\gamma_{e,c} = \frac{3m_e e^4}{\sigma_T} (1 + x_f)^{-1} (1 + z)^{-1} \epsilon_{B,f}^{-1} \Gamma^{-1} A^{-1} t.$$  

Here $\sigma_T$ is the Thomson cross section and the term $(1 + x_f)$ is introduced because a once-scattered synchrotron photon generally has energy larger than the electron mass in the rest frame of the second-scattering electrons. It is given by (Sari & Esin 2001)

$$1 + x_f = \begin{cases} 1 + \frac{\eta_{e,f}}{\epsilon_{B,f}}, & \text{if } \frac{\eta_{e,f}}{\epsilon_{B,f}} \ll 1, \\ 1 + \left(\frac{\eta_{e,f}}{\epsilon_{B,f}}\right)^{1/2}, & \text{if } \frac{\eta_{e,f}}{\epsilon_{B,f}} \gg 1, \end{cases}$$

where

$$\eta = \begin{cases} \gamma_{e,c}, & \text{for slow cooling,} \\ \gamma_{e,m,f}, & \text{for fast cooling.} \end{cases}$$

The maximum electron Lorentz factor can be calculated comparing the acceleration $t_{\text{acc}} \approx \frac{2m_e}{\dot{M}_\dot{m}} (1 + z) \Gamma^{-1} B^2,f$ and cooling ($t_{\text{cool}}$) timescales. Hence, the maximum electron
Lorentz factor is

\[ \gamma_{e,\text{max, } f} \approx \frac{9 \sqrt{2} q_e}{16 \pi \sigma T} e^{1/2} (1 + z)^{-1/4} \epsilon_{B, f}^{-1/4} \times \Gamma^{-1/2} E^{1/4} A^{-1/2} t^{1/4}, \]

where \( q_e \) is the elementary charge. From Equations (4), (5), (7), and (10), the synchrotron spectral breaks computed through the synchrotron emission \( E_{\gamma} = \frac{m_e}{q_e} (1 + z)^{-1} \Gamma B \gamma_{e, i, f} \) for \( i = m, c \) and max can be written as

\[ E_{\gamma, i, f} \approx \frac{2^{1/5} \pi^{1/5} q_e^{1/5} \epsilon_{e, f}^{1/5} (p - 2)^{3/5} (p - 2)^{1/5} \xi^{-6/5}}{3^{3/5} (3p + 2)(p - 2)^{1/5} m_p^{1/5} \epsilon_{e, f}^{1/2} E^{1/2} t^{1/2}} \times (1 + z)^{-2\xi} \xi_{e, f}^{1/5} E^{-2/5} A^{2/5} t^{-3/5}, \]

\[ E_{\gamma, m, f} \approx \frac{2^{1/3} \pi q_e m_p^{1/3} \epsilon_{e, f}^{1/3} (p - 2)^{1/3} (1 + z)^{2\xi} \xi_{e, f}^{1/3} E^{-3/4} A^{1/4} t^{-1/4}}{2^{1/3} \pi^{1/3} m_p^{1/3} \epsilon_{e, f}^{1/3} E^{3/4} A^{3/4} t^{1/4}} \times (1 + z)^{-3/2} \xi_{e, f}^{1/2} E^{-1/2} A^{-1/2} t^{-1/2}, \]

respectively. Here \( F_{\gamma, i, f} \) and \( F_{\gamma, m, f} \) are given by Equations (11) and (13), respectively. The transition time \( (t_{\text{syn}}^0) \) from the fast- to slow-cooling spectrum is

\[ t_{\text{syn}}^0 = \frac{\sigma_T m_p (p - 2)}{3^{1/3} \pi^{1/3} m_p^{1/3} (p - 1)^{1/3} \epsilon_{e, f} \epsilon_{B, f} A}. \]

Using the synchrotron spectral breaks radiated (Equation (11)) and synchrotron spectrum in the fast (Equation (14)) and slow (Equation (15)) cooling regime, one can obtain the LCs as a function of energy \( (E_{\gamma}) \). We get the flux for the fast-cooling regime

\[ F_{\gamma} = A_{\gamma} \gamma^{-3/4} \left( \frac{E_{\gamma}}{E_{\gamma, f}} \right)^{-3/4}, \]

where \( A_{\gamma} \) are given by

\[ A_{\gamma} \approx \frac{(2\pi)^{3/4} 27^{1/2} m_e^{3/2} \xi^{3/2}}{248^{1/2} m_p^{1/2} (p - 2)^{1/2}} \times (1 + x_f)^{-1} \left( \frac{1}{\epsilon_{e, f}} \right) \epsilon_{B, f}^{1/3} E^{3/4} D^{-2}, \]

and

\[ A_{\gamma} \approx \frac{2^{1/3} \pi q_e m_p^{1/3} (p - 2)^{1/3} (1 + z)^{2\xi} \xi_{e, f}^{1/3} E^{-3/4} A^{1/4} t^{-1/4}}{4^{1/3} \pi^{1/3} m_p^{1/3} (p - 2)^{1/3} \epsilon_{e, f}^{1/3} E^{3/4} A^{3/4} t^{1/4}} \times (1 + x_f)^{-1} \left( \frac{1}{\epsilon_{e, f}} \right) \epsilon_{B, f}^{1/3} E^{3/4} D^{-2}, \]

respectively. For the slow-cooling regime, we get

\[ F_{\gamma} = \frac{A_{\gamma} \gamma^{-3/4} \left( \frac{E_{\gamma}}{E_{\gamma, f}} \right)^{-3/4}}{\left( \frac{E_{\gamma}}{E_{\gamma, f}} \right)^{-3/4}} \times \left( \frac{E_{\gamma}}{E_{\gamma, f}} \right)^{-3/4}, \]

where \( A_{\gamma} \) are given by

\[ A_{\gamma} \approx \frac{(2\pi)^{3/4} 27^{1/2} m_e^{3/2} \xi^{3/2}}{248^{1/2} m_p^{1/2} (p - 2)^{1/2}} \times (1 + x_f)^{-1} \left( \frac{1}{\epsilon_{e, f}} \right) \epsilon_{B, f}^{1/3} E^{3/4} D^{-2}. \]
with $A_\text{syn}^m$ given by

$$A_\text{syn}^m = \left( \frac{\sqrt{2\pi} m_e \sigma_T}{24 q_e m_p \xi} \right) \left( \frac{3\sqrt{2} q_e m_p^2 (p - 2)^2}{8m_e^2 \xi^3 (p - 1)^2} \right)^{p-1} (1 + z)^{p+5} \frac{e_{p,1}^{-p+1}}{e_{p,2}^{-p+1}} A E^{p+1} D^{-2}. \quad (21)$$

**SSC emission.** Fermi-accelerated electrons in the FS may scatter synchrotron photons up to higher energies $E_{\gamma,f} \approx 2E_{\gamma,f}^\text{SYN}$. From the synchrotron spectral breaks (Equation (11)) and break electron Lorentz factors (Equations (5), (7), and (10)), the spectral breaks in the Compton regime are

$$E_{\gamma,\text{syn},f} \approx \frac{2^{1/2} \pi^{5/3} \sqrt{6} q_e m_p^2 \xi^5 (p + 2) \xi^{-11/5}}{5^{1/5} \Gamma(5/6)^{3/10} (p + 2)^{3/5} (p - 1)^{2/5}} \times (1 + z)^{1/10} \epsilon_{c,f}^{-1/5} A^{1/10} E^{1/10} \ t^{-11/10}$$

$$E_{\gamma,\text{syn},m} \approx \frac{729 \sqrt{2} q_e m_p^2 \xi^{12}}{64 \sigma_T^4} \left( 1 + z \right)^{-3/4} \left( 1 + x_f \right)^{-4} \epsilon_{B,f}^{-7/2} A^{-9/2} E^{-1/2}$$

$$E_{\gamma,\text{syn},m} \approx \frac{27 \sqrt{2} q_e m_p^2 \xi^{1/2}}{32 \pi^{3/2} \sigma_T^2 m_e} \left( 1 + z \right)^{-5/4} \epsilon_{B,f}^{-1} A^{-1/4}$$

with $x_f$ given by Equations (8) and (9), and $(E_{\gamma,F_{\gamma}})_{\text{syn},f} = E_{\gamma,F_{\gamma}}^\text{SYN}(E_{\gamma,F_{\gamma}})$ (Sari & Esin 2001; Granot & Guetta 2003). From the break photon energies (Equation (22)) and synchrotron spectrum in the fast (Equation (25)) and slow (Equation (26)) cooling regimes, one can obtain the LCs for the fast-cooling regime

$$F_{\nu} \sim \left[ \epsilon_{\nu} \left( E_{\gamma} \right) \right]^{1/2}, E_{\gamma,f} < E_{\gamma,s} < E_{\gamma,m,f},$$

and the slow-cooling regime

$$F_{\nu} \sim \left[ \epsilon_{\nu} \left( E_{\gamma} \right) \right]^{1/2}, E_{\gamma,f} < E_{\gamma,s} < E_{\gamma,m,f}.$$

2.2. **Reverse Shocks**

For the RS, a simple analytic solution can be derived taking two limiting cases, the thick- and thin-shell cases, (Sari & Piran 1995) by using a critical Lorentz factor ($\Gamma_c$), which is defined by

$$\Gamma_c = \frac{3}{8 \pi^{1/2} \xi} \left( 1 + z \right)^{1/4} E_{\gamma,F_{\gamma}}^{-1/4} A^{-1/4} T_{90}^{-1/4},$$

where $T_{90}$ is the duration of the GRB. For $\Gamma > \Gamma_c$ (thick shell), the shell is significantly decelerated by the RS\(^2\), otherwise, $\Gamma < \Gamma_c$ (thin shell), the RS cannot decelerate the shell effectively. Irrespective of the evolution of RS, the synchrotron spectral evolution between RS and FS is related by (Zhang et al. 2003; Fan et al. 2004a; Fan & Wei 2005; Shao & Dai 2005; Jin & Fan 2007; Kobayashi & Zhang

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\(^2\)Although bulk Lorenz factors (FS and RS) can be different at the shocked region, we have considered them to be similar.
\[ E_{\gamma,m,f}^{\text{syn}} \sim R_e^2 R_B^{1/2} R_M^{-2} E_{\gamma,m,f}^{\text{syn}} \]
\[ E_B^{\gamma,\tau,f} \sim R_B^{3/2} R_x^{-2} E_{\gamma,\tau,f}^{\text{syn}} \]
\[ F_{\gamma,max,f}^{\text{syn}} \sim R_B^{-1/2} R_M F_{\gamma,max,f}^{\text{syn}} \]  

where
\[ R_B = \frac{\epsilon_{B,f}}{\epsilon_{B,r}}, \quad R_x = \frac{1 + x_f}{1 + x_r + x_r^2} \]  
\[ R_M = \frac{\Gamma_d^2}{\Gamma} \]  

where \( \Gamma_d \) is the bulk Lorentz factor at the shock crossing time.  
The previous relations tell us that including the re-scaling there is a unified description between FSs and RSs, and the distinction between forward and reverse magnetic fields considers that in some central engine models (Usov 1992; Mészáros & Rees 1997b; Wheeler et al. 2000) the fireball could be endowed with “primordial” magnetic fields. Also, because the cooling Lorentz factor must be corrected, the \( R_x \) is introduced as a correction factor for the SSC cooling, where \( x_r \) is obtained by (Kobayashi & Zhang 2007)

\[ x_r = \begin{cases} \frac{\eta_{e,r}}{\eta_{e,r}}, & \text{if } \eta_{e,r} \ll 1, \\ \left(\frac{\eta_{e,r}}{\eta_{e,r}}\right)^{1/3}, & \text{if } \eta_{e,r} \gg 1. \end{cases} \]  

Here \( \eta = (\gamma_{e,r}/\gamma_{e,m})^{2-p} \) is given for slow-cooling and \( \eta = 1 \) for fast-cooling regime.

### 2.2.1. Thick-shell Case

In this case, the RS becomes relativistic during its propagation and the ejecta is significantly decelerated. The bulk Lorentz factor at the shock crossing time \( t_d \approx T_{90} \) is given by \( \Gamma_d \approx \Gamma_r \). Eventually, the shock crossing time could be shorter than \( T_{90} \) depending on the degree of magnetization of ejecta, defined as the ratio of Poynting flux to matter energy flux \( \sigma = L_p/L_k \approx \epsilon_B r \) (Fan et al. 2004b; Zhang & Kobayashi 2005; Kobayashi & Zhang 2007). In particular, numerical analysis performed by Fan et al. (2004b) revealed that for the value of the magnetization parameter \( \sigma \approx 1 \), the shock crossing time becomes \( t_d \approx T_{90}/6 \).

**Synchrotron emission.** Assuming that electrons are accelerated in the RS to a power-law distribution and the energy density is equipartitioned between electrons and the magnetic field, the minimum electron Lorentz factor and the magnetic field are

\[ \gamma_{e,m,r} = \epsilon_{e,r} \frac{(p - 2)}{(p - 1)} m_p \Gamma \]  
\[ \epsilon_{e,r} = \frac{\sqrt{8 \pi/3} m_p (p - 2)^{5/2}}{m_e (p - 1)} \]  
\[ \times \Gamma A^{1/4} E^{-1/4} T_{90}^{1/4} \]  

and

\[ B_r' \approx \frac{8 \sqrt{2} \pi}{3 \pi} (1 + z)^{1/2} \epsilon_{B,r} E^{-1/2} T_{90}^{-1/2} A. \]  

From the characteristic cooling time of synchrotron radiation and dynamical timescale, the characteristic electron Lorentz factor can be written as

\[ \gamma_{e,c,r} = \frac{27 m_e \epsilon_{e}}{64 \pi \sigma_T} \left(1 + x_r + x_r^2\right)^{-1} \epsilon_{B,r} E \Gamma^{-3} A^{-2}. \]  

By considering \( \gamma_{e,c,r} \approx \gamma_{e,m} \) (Sari & Esin 2001) and from Equation (31), we rescale the synchrotron self-absorption energy between FS and RS as \( E_{\gamma,max}^{\text{syn}} \sim R_B^{2} R_{90}^{-1/5} R_{M}^{-2} E_{\gamma,max}^{\text{syn}} \). Additionally, from Equations (11), (30), and (31), we get the synchrotron spectral breaks

\[ E_{\gamma,max}^{\text{syn}} \approx \frac{2^{40/5} \pi^{3/10} q_{e}^{8/5} (p + 2)^{3/5} (p - 1)^{8/5} \gamma^{2/5}}{5^{3/5} 3^{9/5} 5(5/6)^{3/5} (p + 2)^{3/5} (p - 2) m_{p}^{8/5}} \times \left(1 + z\right)^{-7/5} \epsilon_{e,r}^{-1} \epsilon_{B,r}^{1/2} E^{-7/5} T_{90}^{1/5} \]

\[ E_{\gamma,m}^{\text{syn}} \approx \frac{8 \pi \sqrt{2} q_{e} m_{p}^{4/5} (p - 2)^{2}}{3 m_{e}^{1/5} \left(p - 1\right)^{2}} \left(1 + z\right)^{-7/2} \epsilon_{e,r} \epsilon_{B,r} \Gamma^{-1/2} E^{-1/2} \times A T_{90}^{-1/2} \]

\[ E_{\gamma,r}^{\text{syn}} \approx \frac{27 \sqrt{2} \pi q_{e} m_{p}^{3/5}}{8 \pi} \left(1 + z\right)^{-3/2} \epsilon_{e,r}^{-3/2} \epsilon_{B,r}^{-2} A^{-2} \times E^{1/2} T_{90}^{-1/2} \]  

and the peak flux

\[ F_{\gamma,max}^{\text{syn}} \approx \frac{\sqrt{2} m_{e} \sigma_{T}}{64 m_{p} q_{e}^{5/2}} \left(1 + z\right)^{2} \times \epsilon_{B,r}^{1/2} \Gamma^{-1/2} A^{1/2} D^{-2} E^{-1/2} T_{90}^{-1}. \]  

**Synchrotron LCs.** Accelerated electrons can upscatter photons from low to high energies as

\[ F_{\gamma,f}^{\text{sc}} \sim 2 \gamma_{e,m,r}^{2} F_{\gamma,r}^{\text{sc}}, \quad F_{\gamma,m}^{\text{sc}} \sim 2 \gamma_{e,m}^{2} F_{\gamma,m}^{\text{sc}}, \quad F_{\gamma,max}^{\text{sc}} \sim k \tau F_{\gamma,max}^{\text{sc}}, \]

where \( k = 4(p - 1)/(p - 2) \) and \( \tau = \frac{N_{e}}{4 \pi c} \) is the optical depth of the shell. Here \( N_{e} \) is the number of radiating electrons. From Equations (33), (38), (44), and (47), we get the break
SSC energies

\[ E_{\text{SSC}} \approx \frac{2^{6/5} \pi^{36/10} q_e^8 \gamma_{\text{SSC}}^8 m_p^2 (p + 2)^{3/5} (p - 2)^{5/5}}{\sqrt{5} \pi^{24/5} \Gamma(5/6)^{33/5} (3p + 2)^{3/5} (p - 1)^{2/5} m_e^3} \times (1 + z)^{-1/10} \sigma_e \epsilon_{\text{SSC}}^4 E_{\gamma, I}^{-11/10} E_{\nu, I}^{4/5} T_{90}^{1/10} \]

\[ E_{\text{SSC}} \approx 128 \sqrt{2} \pi^{3/2} q_e^4 m_p^4 (p - 2)^4 \times (1 + z)^{-1} \epsilon_{\gamma, I}^4 \epsilon_{\nu, I}^{1/2} \Gamma^4 \]

\[ E_{\text{SSC}} \approx 0.85 m_p^2 q_e^2 \pi^{3/2} \sigma_e (p - 1) \times (1 + z)^{-3/2} (1 + x_r + x_e^{2})^4 \epsilon_{\nu, I}^{7/2} \]

\[ F_{\text{SSC}} \approx \frac{\sqrt{2} \sigma_e^2 m_e (p - 1)}{192 m_p^2 q_e \pi^{4/3} (p - 2)} \times D^{-2} E_{90}^2, \quad (39) \]

and the break energy at the KN regime is

\[ E_{\gamma, I}^{KN} \approx \frac{27 m_p^2 q_e^2}{64 \pi \sigma_e} (1 + z)^{-1} \left(1 + x_r + x_e^{2}\right)^{-1} \times \epsilon_{\gamma, I}^4 \Gamma E^{-2} A^2. \quad (40) \]

At the shock crossing time \( t_d \), the SSC flux reaches the peak

\[ F_{\gamma, \text{peak}, r} = \left( E_{\gamma, I}^{KN} / E_{\gamma, I}^{m, r} \right)^{1/2} F_{\gamma, \text{max}, r} \] (Kobayashi & Zhang 2003)

at

\[ F_{\gamma, \text{peak}, r} \approx \frac{0.13 m_p^2 q_e (p - 2) \xi^9}{\pi^{3/4} \sigma_e (p - 1)} \times (1 + z)^{-1/2} x_r \left(1 + x_r + x_e^{2}\right)^{-5} \times \epsilon_{\gamma, I}^4 \epsilon_{\nu, I}^{7/2} \Gamma^{-6} A^{-6} D^{-2} E_{90}^{1/2} \left| E_{\gamma, r}^{\text{syn}} \right|^{-1/2}. \quad (41) \]

SSC LCs can be analytically derived from Chevalier & Li (2000). For \( t < t_d \), we take into account that (1) the number of radiating electrons \( N_e \), and the spherical radius in the shocked shell region increase with time as \( N_e \sim t \) and \( r \sim t \), (2) the maximum flux of synchrotron is independent of time \( F_{\gamma, \text{max}, r} \sim t_0 \), and (3) the SSC cooling break energy \( E_{\gamma, r}^{\text{syn}} \sim t_0 \), increases as \( \sim t^3 \), then the SSC flux increases as \( F_{\gamma, r} \sim t_0^{3/2} F_{\gamma, r}^{\text{SSC}} \sim E_{\gamma, r}^{\text{SSC}} / t_0^{1/2} \sim t^{3/2} F_{\gamma, r}^{\text{SSC}} \). For \( t > t_d \), the SSC flux decreases as \( F_{\gamma, r} \sim t^{-3/2} F_{\gamma, r}^{\text{SSC}} \sim E_{\gamma, r}^{\text{SSC}} / t^{1/2} F_{\gamma, r}^{\text{SSC}} \), where \( E_{\gamma, r}^{\text{SSC}} \sim t^{-3/2} F_{\gamma, r}^{\text{SSC}} \) decreases as \( \sim t^{-1} \). It is valuable to note that the decay index of the emission for \( t > t_d \) might be higher than \( p - 1 \), due to the angular time delay effect (Kobayashi & Zhang 2003).

2.2.2. Thin-shell Case

In the thin-shell case, the RS cannot deaccelerate the shell effectively. The deceleration time and the minimum electron Lorentz factor are in the form

\[ t_{\text{dec}} = \frac{9}{64 \pi \xi^{2/3}} (1 + z) \Gamma A^{-1} \Gamma^{-4}, \quad (42) \]

and

\[ \gamma_{e, m, r} = \epsilon_{\gamma, r} \left( \frac{p - 2}{p - 1} \right) m_p / m_e, \quad (43) \]

respectively. Here the bulk Lorentz factor at the shock is \( \Gamma \approx \Gamma_d < \Gamma_t \).

**Synchrotron emission.** Performing a similar analysis of the thick-shell case, from Equations (11), (30), (31), and (43), we get that the synchrotron break energies and maximum flux can be written as

\[ E_{\gamma, I}^{\text{syn}} \approx \frac{2^{34/5} \pi^{27/10} q_e^8 \gamma_{\text{SSC}}^8 m_p^2 (p + 2)^{3/5} (p - 1)^{5/5}}{\sqrt{5} \pi^{24/5} \Gamma(5/6)^{33/5} (3p + 2)^{3/5} (p - 1)^{2/5} m_e^3} \times (1 + z)^{-1} \epsilon_{\gamma, r} \epsilon_{\nu, r}^{1/2} \Gamma^4 \]

\[ E_{\gamma, I}^{\text{syn}} \approx \frac{64 \sqrt{2} \pi^{3/2} q_e^4 m_p^4 (p - 2)^4}{9 m_p^2 q_e} \times (1 + z)^{-1} \epsilon_{\gamma, r} \epsilon_{\nu, r}^{1/2} \Gamma^4 A^{3/2} \]

\[ E_{\gamma, I}^{\text{syn}} \approx \frac{81 \sqrt{2} \sigma_e^2 m_e (p - 1)}{64 \pi^{4/3} \sigma_e^2} \times (1 + z)^{-1} \left(1 + x_r + x_e^{2}\right)^{-2} \epsilon_{\nu, r}^{7/2} E \times A^{-5/2} \Gamma^{-2}. \quad (44) \]

**SSC emission.** In a like manner to the thick-shell case, from Equations (33), (38), (44), and (47), we derive the break energies of SSC emission

\[ E_{\gamma, I}^{\text{SSC}} \approx \frac{2^{34/5} \pi^{27/10} q_e^8 \gamma_{\text{SSC}}^8 m_p^2 (p + 2)^{3/5} (p - 2)}{\sqrt{5} \pi^{24/5} \Gamma(5/6)^{33/5} (3p + 2)^{3/5} (p - 1)^{2/5} m_e^3} \times (1 + z)^{-1} \epsilon_{\gamma, r} \epsilon_{\nu, r}^{1/2} \Gamma^{-1} \]

\[ E_{\gamma, I}^{\text{SSC}} \approx \frac{128 \sqrt{2} \pi^{3/2} q_e^4 m_p^4 (p - 2)^4}{9 m_p^2 q_e} \times (1 + z)^{-1} \epsilon_{\gamma, r} \epsilon_{\nu, r}^{1/2} A^{3/2} \]

\[ E_{\gamma, I}^{\text{SSC}} \approx \frac{0.32 q_e m_e^3 \xi^{8/3}}{\pi^2 \sigma_e} \times A^{-13/2} \Gamma^{-8}. \quad (45) \]

The break energy at the KN regime is given by Equation (40).

3. APPLICATION: GRB 110731A

GRB 110731A was detected on 2011 July 31 by both instruments on board *Fermi*: the Gamma-Ray Burst Monitor (GBM) and LAT (Ackermann et al. 2013), the three instruments on board *Swift*: BAT, XRT, and UVOT (Oates et al. 2011) and ground-based observatories; the Microlensing Observations in Astrophysics (MOA) telescope (Tanvir et al. 2011) and GROND.

LAT localized GRB 110731A with coordinates R.A. = 18°51′00″ and decl. = −28°31′00″ (J2000), with a 68%
Swift/BAT immediately perceived this burst after the detection by both instruments of Fermi, whereas RXT and UVOT began observations 56 s after the BAT trigger. UVOT swiftly determined the afterglow position as R.A. = 11h42m00s99 and decl. = −28°32′13.8″ (J2000), with a 90% confidence. The lack of observation in the UV filters is consistent with the measured redshift $z = 2.83$ (Tanvir et al. 2011). MAO observations started 3.3 minutes after the Swift trigger for GRB 110731A. Using a 61 cm Boller & Chivens telescope at the Mt. John University Observatory in New Zealand, I- and V-band images were collected 105 minutes after the trigger (Tristram et al. 2011). Finally, GROND mounted on the 2.2 m MPG/ESO telescope at La Silla Observatory, Chile, imaged GRB 110731A for 2.74 days after the trigger (Greiner et al. 2008).

By considering the typical values of the stellar wind ($A = A_\star \times (5.0 \times 10^{11})$ gm cm$^{-2}$ with $A_\star = 0.1$; Chevalier & Li 2000), the parameter $\xi = 0.56$; Panaitescu & Mészáros 1998b) and those inferred by observations: redshift $z = 2.83$ (Tanvir et al. 2011), total energy $E \approx 10^{54}$ erg and duration of GRB $T_{90} = 7.3$ s, we will apply the leptonic model developed in this work to interpret the LAT LC observations. Taking into account the fact that the peak of the flux density was present at the end of the prompt phase; in the interval [5.47 s, 5.67 s], and that after the LAT flux decays smoothly during the whole temporally extended emission, we constrain the Lorentz factor $\Gamma \approx 520 > \Gamma_c$ so that the deceleration time takes place at

$$t_{\text{dec}} \approx 5.55 \left(\frac{1+z}{4}\right) E_{54} A_{*-1}^{-1} \Gamma_c^{-4/3} 2.72. \quad (46)$$

and the RS evolves in the thick-shell case with a critical Lorentz factor

$$\Gamma_c \approx 472.5 \left(\frac{1+z}{4}\right)^{1/4} A_{*-1}^{-1/4} E_{54}^{1/4} \left(\frac{T_{90}}{7.3 \text{ s}}\right)^{-1/4}. \quad (47)$$

We plot the synchrotron and SSC spectral breaks of the FS and RS as a function of equipartition parameters ($\epsilon_{B,fr}$ and $\epsilon_{e,fr}$), considering the typical values of the magnetic ($10^{-5} \leq \epsilon_{B,fr} \leq 1$) and electron ($\epsilon_{e,fr} = 0.5, 0.1$ and 0.05) parameters (Santana et al. 2014), as shown in Figure 1. We describe this figure as follows.

Figure 1. Break photon energies of synchrotron radiation from the forward (top left panel) and reverse (top right panel) shocks, and SSC emission from the RS (bottom panel) as a function of the magnetic equipartition parameter ($\epsilon_{B,fr}$) for $\epsilon_{e,fr} = 0.5, 0.1, 0.05$, and 0.01.
Synchrotron spectral breaks from FS (right panel). From this panel, we can see that the characteristic energy is an increasing function of $\epsilon_{B,f}$ and $\epsilon_{e,f}$, the cooling energy is a decreasing function of $\epsilon_{B,f}$ and the self-absorption energy is an increasing function of $\epsilon_{B,f}$ and a decreasing function of $\epsilon_{e,f}$. The characteristic energy lies in the ranges $5.3 \times 10^2$ eV $\leq E_{\gamma,m,f}^{\text{syn}}$ $\leq 2.8 \times 10^5$ eV, $3.5 \times 10^3$ eV $\leq E_{\gamma,m,f}^{\text{syn}}$ $\leq 6.8 \times 10^5$ eV, and $6.1 \times 10^3$ eV $\leq E_{\gamma,m,f}^{\text{syn}}$ $\leq 1.9 \times 10^7$ eV for $\epsilon_{e,f} = 0.05, 0.1$ and 0.5, respectively. Additionally, the cooling energy lies in the range $8.9 \times 10^{-6}$ eV $\leq E_{\gamma,c,f}^{\text{syn}}$ $\leq 2.1 \times 10^{-5}$ eV and the synchrotron self-absorption energy in the ranges $5.2 \times 10^{-3}$ eV $\leq E_{\gamma,a,f}^{\text{syn}}$ $\leq 4.6 \times 10^{-2}$ eV and $5.1 \times 10^{-4}$ eV $\leq E_{\gamma,a,f}^{\text{syn}}$ $\leq 6.3 \times 10^{-3}$ eV for $\epsilon_{e,f} = 0.05$ and 0.5, respectively. Furthermore, one can see that self-absorption energy is in the weak self-absorption regime ($E_{\gamma,a,f}^{\text{syn}} < E_{\gamma,c,f}^{\text{syn}}$) for $\epsilon_{B,f} < 0.003$ (0.007) and $\epsilon_{e,f} = 0.05$ (0.5), otherwise the synchrotron spectrum would be in the strong absorption regime.

Synchrotron spectral breaks from RS (left panel). From this panel, we can see that synchrotron spectral breaks from RS have the same behavior as FS, though at lower energy ranges. The characteristic synchrotron energy is in the ranges $0.01$ eV $\leq E_{\gamma,m,r}^{\text{syn}}$ $\leq 5.1$ eV, $0.08$ eV $\leq E_{\gamma,m,r}^{\text{syn}}$ $\leq 1.1 \times 10^2$ eV, and $1.2$ eV $\leq E_{\gamma,m,r}^{\text{syn}}$ $\leq 6.2 \times 10^2$ eV for $\epsilon_{e,r} = 0.05, 0.1$, and 0.5, respectively. The cooling energy is in the range $1.8 \times 10^{-6}$ eV $\leq E_{\gamma,c,r}^{\text{syn}}$ $\leq 46.2$ eV and the self-absorption energy in the ranges $3.8 \times 10^{-7}$ eV $\leq E_{\gamma,a,r}^{\text{syn}}$ $\leq 8.1 \times 10^{-7}$ eV and $7.3 \times 10^{-9}$ eV $\leq E_{\gamma,a,r}^{\text{syn}}$ $\leq 6.8 \times 10^{-8}$ eV for $\epsilon_{e,r} = 0.05$ and 0.5, respectively. It is important to note that synchrotron self-absorption is in the weak absorption regime ($E_{\gamma,a,r}^{\text{syn}} < E_{\gamma,c,r}^{\text{syn}}$) for any value of equipartition parameters considered here. However, as $E_{\gamma,a,r}^{\text{syn}} \propto A^{1/5}$ and $E_{\gamma,c,r}^{\text{syn}} \propto A^{-2}$ any significant increase of wind density, the absorption energy would become higher than the cooling energy ($E_{\gamma,a,r}^{\text{syn}} > E_{\gamma,c,r}^{\text{syn}}$), then the synchrotron spectrum could change to the strong absorption regime.
regime. In this case, the heating of low-energy electrons due to synchrotron absorption leads to the pile-up of electrons, and a thermal component besides the power-law spectrum appears (Gao et al. 2013).

SSC spectral breaks from RS (panel below). SSC spectral breaks have similar behavior as in the previous cases. The characteristic SSC energy lies in the range 52.3 eV \( \leq E_{\gamma}^{\text{syn,br}} \leq 1.12 \times 10^5 \) eV, 7.8 \( \times 10^3 \) eV \( \leq E_{\gamma}^{\text{sync}} \leq 2.3 \times 10^5 \) eV, and 6.2 \( \times 10^5 \) eV \( \leq E_{\gamma}^{\text{sync}} \leq 1.17 \times 10^8 \) eV for \( \epsilon_{e,r} = 0.05, 0.1, \) and 0.5, respectively, while the cooling energy lies in the range 8.2 \( \times 10^{-10} \) eV \( \leq E_{\gamma}^{\text{sync}} < 2.1 \times 10^{10} \) eV. To obtain the values of parameters \( \epsilon_{B,f(r)} \) and \( \epsilon_{e,f(r)} \) that reproduce the multiwavelength LC observations, we use the method of Chi-square (\( \chi^2 \)) minimization (Brun & Rademakers 1997).

We describe LAT flux observations by synchrotron radiation from FS and SSC emission from RS; the whole temporally extended emission using synchrotron LCs in the fast-cooling regime (Equation (17)) and the brightest peak at 5.5 s with SSC LCs in the fast-cooling regime (Equation (41)), for HE electrons radiating at \( \approx 100 \) MeV. Following the analysis shown in Ackermann et al. (2013), we fit the X-ray \( (t < 4.6 \) ks) flux with the synchrotron LC in the fast-cooling regime (Equation (17)) at \( t = 100 \) s for electrons radiating at \( E_{\gamma}^{\text{syn}} = 2 \) keV, and optical and X-ray \( (t > 4.6 \) ks) fluxes with the LC in slow-cooling regime (Equation (20)) at \( t = 600 \) s and 4000 s for \( E_{\gamma}^{\text{syn}} = 10 \) eV and 0.7 keV, respectively.

We plot the values of parameters \( \epsilon_{B,f(r)} \) and \( \epsilon_{e,f(r)} \) for \( p = 2.15, 2.2, 2.25, \) and 2.3 (see Figures 2 and 3) that reproduce the multiwavelength LC observations (see Figure 4). As shown in Figure 2, the areas in yellow (green) show the set of parameters that describes the temporarily extended LAT (optical) flux and in blue (red) ones show those parameters that describe the X-ray for \( t < 4.6 \) ks \( (t > 4.6 \) ks). The areas in brown show the set of parameters that reproduce more than one flux at the same time. For instance, as shown in the right panel below \( (p = 2.25) \), the equipartition parameters in the narrow strip between 0.38 \( < \epsilon_{e,f} < 0.52 \) and \( 10^{-4.5} < \epsilon_{B,f} < 10^{-4} \) would reproduce the temporally extended LAT, X-ray, and optical fluxes. From Figure 3, one can see the set of parameters that describe the brightest LAT peak. Also, it can be seen that as the electron spectral index increases the set of parameters is shifted to the right.

Given the values \( \epsilon_{B,f} = 10^{-4.15} \) and \( \epsilon_{e,f} = 0.40 \) for \( p = 2.25 \), from Equations (11) and (16) we determine that the synchrotron spectral breaks from FS are

\[
E_{\gamma}^{\text{syn,br}} \approx 5.56 \times 10^{-4} \text{ eV} \left( \frac{1 + z}{4} \right)^{-2/5} \epsilon_{e,f,0.4}^{1/5} \epsilon_{B,f,4.15}^{1/5} A_{\ast,1.1}^{-2/5} E_{54}^{-3/5} t_1^{-3/2}.
\]

\[
E_{\gamma}^{\text{sync}} \approx 77.45 \text{ keV} \left( \frac{1 + z}{4} \right)^{1/2} \epsilon_{e,f,0.4}^{1/2} \epsilon_{B,f,4.15}^{1/2} E_{54}^{1/2} t_1^{-3/2}.
\]

\[
E_{\gamma}^{\text{sync}} \approx 0.30 \text{ eV} \left( \frac{1 + z}{4} \right)^{-3/2} \left( \frac{1 + x_f}{11} \right)^{-2} \epsilon_{B,f,4.15}^{-3/2} A_{\ast,1.1}^{-2} E_{54}^{1/2} t_1^{1/2}.
\]

\[
E_{\gamma}^{\text{max,br}} \approx 36.94 \text{ GeV} \left( \frac{1 + z}{4} \right)^{-3/4} E_{54}^{1/4} A_{\ast,1.1}^{-1/4} t_1^{-1/4}.
\]

and the transition time from the fast- to slow-cooling regimes is

\[
t_0^{\text{syn}} = 123.46 s \left( \frac{1 + z}{4} \right) \epsilon_{e,f,0.4}^{1/2} \epsilon_{B,f,4.15} A_{\ast,1.1}^{-1}.
\]

It is important to highlight the fact that the maximum photon energy achieved by synchrotron radiation is \( E_{\gamma}^{\text{sync,max,br}} \approx 36.94 \text{ GeV} \) for \( t = 10 \) s and \( E_{\gamma}^{\text{syn,br}} \approx 20.77 \text{ GeV} \) for \( t = 100 \) s.

From Equation (26), we determine that the SSC scattering break energies are

\[
E_{\gamma}^{\text{sync}} \approx 11.66 \text{ TeV} \left( \frac{1 + z}{4} \right) \epsilon_{e,f,0.4}^{1/2} \epsilon_{B,f,4.15} \times A_{\ast,1/2}^{-1/2} E_{54}^{-1/2} t_1^{-2}.
\]

\[
E_{\gamma}^{\text{sync}} \approx 128.94 \text{ eV} \left( \frac{1 + z}{4} \right)^{-1/2} \epsilon_{e,f,0.4}^{1/2} \epsilon_{B,f,0.55} \Gamma_{2.72}^{1/2} A_{\ast,1/2}^{-1} E_{54}^{1/2} t_1^{-1/2}.
\]

\[
E_{\gamma}^{\text{sync}} \approx 0.93 \times 10^{-5} \text{ eV} \left( \frac{1 + z}{4} \right)^{-3/2} \left( \frac{1 + x_f + x_f^2}{3} \right)^{-2} \epsilon_{B,f,0.55}^{-1/2} \Gamma_{2.72}^{-1/2} A_{\ast,1/2}^{-2} E_{54}^{-1/2} t_1^{-1/2}.
\]

The break energy at the KN regime is \( E_{\gamma}^{\text{KN}} \approx 42.33 \text{ GeV} \). From the SSC LCs (Equations (27) and (28)), one can see that although the temporal power index of LC for \( E_{\gamma}^{\text{sync}} < E_{\gamma} < E_{\gamma}^{\text{sync}} \) would be in agreement with the temporally extended LAT flux, the energy range would not. The detection of the photon with energy 3.4 GeV at \( \approx 436 \) s could be explained by synchrotron radiation as well as Compton scattering emission. Considering \( \epsilon_{e,r} = \epsilon_{e,f} = 0.4 \), we obtain \( \epsilon_{B,f} = 0.28 \) for \( p = 2.25 \). From Equations (39) and (44), we determine that the synchrotron and SSC break energies are

\[
E_{\gamma}^{\text{KN}} \approx 4.28 \times 10^{-8} \text{ eV} \left( \frac{1 + z}{4} \right)^{-7/5} \epsilon_{e,r,0.4}^{1/5} \epsilon_{B,r,0.55} \Gamma_{2.72}^{1/2} A_{\ast,1/2}^{-1/2} E_{54}^{1/2} t_1^{-1/2}.
\]

\[
E_{\gamma}^{\text{sync}} \approx 0.41 \text{ Jy} \left( \frac{1 + z}{4} \right)^{1/2} \epsilon_{B,f,4.15}^{1/2} A_{\ast,1/2}^{-1/2} D_{28}^{-2} E_{54}. \]

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and

\[ E_{\gamma, m, r}^{\text{ssc}} \approx 103.55 \text{ MeV} \left( \frac{1 + z}{4} \right)^{-1} \epsilon_{\gamma, -0.4}^{4} \epsilon_{B, r, -0.55}^{1/2} \Gamma_{\gamma, 3}^{4} \times A_{s, 1}^{3/2} E_{m, 4}^{-1}, \]

\[ E_{\gamma, x, r}^{\text{ssc}} \approx 5.86 \times 10^{-3} \text{ eV} \left( \frac{1 + z}{4} \right)^{-3/2} \left( \frac{1 + x_{r} + x_{r}^{2}}{3} \right)^{-4} \times \epsilon_{B, r, -0.55}^{-7/2} \Gamma_{\gamma, 2}^{-6} A_{s, 1}^{-6} E_{m, 4}^{-5/2} \left( \frac{T_{90}}{7.38} \right)^{1/2}, \]

\[ F_{\gamma, \text{max}, r}^{\text{ssc}} \approx 1.42 \times 10^{2} \text{ Jy} \left( \frac{1 + z}{4} \right)^{3} \epsilon_{B, r, -0.55}^{1/2} \Gamma_{\gamma, 2}^{-2} A_{s, 1}^{3/2} \times D_{28}^{-2} E_{m, 4} \left( \frac{T_{90}}{7.38} \right)^{-2}, \]

respectively. The break energy at the KN regime is \( E_{\gamma, r}^{\text{KN}} = 52.71 \text{ GeV}. \)

Figure 3. Values of equipartition parameters (\( \epsilon_{B, r} \) and \( \epsilon_{e, r} \)) that describe the brightest LAT peak through SSC emission from RS.

Figure 4. Fits of the multiwavelength LCs of GRB 110731A observation with our model. We use the RS in the thick-shell regime to describe the brightest peak of the LAT flux (continuous line) and the FS to explain the temporally extended LAT, X-ray, and optical emissions (dashed lines).
4. CONCLUSIONS

We have presented a leptonic model based on the evolution of an early afterglow in the stellar wind. We apply this model to describe the temporally extended emission and the brightest peak present in the LAT LC of GRB 110731A, though additionally we fit the X-ray and optical LC afterglows.

We consider that the ejecta propagating in the stellar wind is decelerated early at ~5.5 s and the RS evolves in the thick shell regime. Taking into account the values of redshift $z = 2.83$ (Tanvir et al. 2011), energy $E \approx 10^{52}$ erg, duration of GRB $T_{90} = 7.3$ s (Oates et al. 2011; Tanvir et al. 2011; Tristram et al. 2011; Ackermann et al. 2013), and the stellar wind $A = 5.0 \times 10^{10}$ gm cm$^{-2}$ (Chevalier & Li 2000), we determine that the value of the Lorentz factor is $\Gamma \approx 520$.

We plot the SSC and synchrotron spectral breaks from FS and RS as a function of equipartition parameters, as shown in Figure 1. We found that the synchrotron self-absorption energies from FS and RS are in the weak self-absorption regime for $\epsilon_{B,f} < 0.003$ (0.007) and $\epsilon_{e,f} = 0.05$ (0.5) and for $10^{-5} < \epsilon_{B,r} < 1$ and $0.05 < \epsilon_{e,r} < 0.5$ (considered here), respectively. It is important to say that for $\epsilon_{e,r} \gg \epsilon_{e,m}$ (not considered in this work) the synchrotron spectrum would be in the strong absorption regime ($E_{\gamma,fs} > E_{\gamma,rs}$). In this case, a thermal peak due to pile-up of electrons would appear around $E_{\gamma,fs} \approx 6.23 \times 10^{-4} \times E_{r,s}^{7/4} \times E_{B,r}^{1/4} \times E_{e,r}^{-1/4}$ in the synchrotron spectrum of the RS, modifying the break power law spectrum (Kobayashi et al. 2004; Gao et al. 2013). To find the equipartition parameters $\epsilon_{B,(r,f)}$ and $\epsilon_{e,(r,f)}$ we fit the multiwavelength afterglow LCs (see Figure 4); the brightest LAT peak by SSC emission from RS and the extended temporally emissions (LAT, X-ray, and optical) by synchrotron radiation from the FS. We plot the set of values $\epsilon_{B,(r,f)}$ and $\epsilon_{e,(r,f)}$ that describes these observations, as shown in Figures 2 and 3. From the FS, we chose the values $\epsilon_{B,f} = 10^{-4.15}$ and $\epsilon_{e,f} = 0.4$ for $p = 2.25$, then we get the synchrotron spectral breaks: $E_{\gamma,fs} \approx 5.56 \times 10^{-4} \times E_{r,f}^{7/4} \times E_{B,f}^{1/4} \times E_{e,f}^{-1/4} \times (\frac{T_{90}}{7.3})^{11/14}$ in the synchrotron spectrum of the RS, modifying the break power law spectrum. Some authors have claimed that the GeV emission detected by LAT during the prompt phase could be described by SSC emission from FS and be candidates to be detected by TeV $\gamma$-ray observatories such as the High Altitude Water Cherenkov observatory (HAWC; Abeysekara et al. 2012, 2014).

In summary, we model not only the LAT LC (the long-lasting GeV emission extended up to 853 s by synchrotron emission from FS and the brightest peak by SSC emission from RS) but also explain the multiwavelength afterglow observations in GRB 110731A using the leptonic model based on an early afterglow that evolves in a stellar wind. The bulk Lorentz factor required in this model is $\Gamma \approx 520$ and the ejecta must be magnetized. In this model the onset of the HE emission is delayed because it is emitted from external shocks.

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