HOW TO SHARE PARTIAL INFORMATION WITH COMPETITIVE MANUFACTURERS

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(Communicated by Shuhua Zhang)

Abstract. This paper investigates the partial information sharing of a supply chain including one retailer and two competitive manufacturers. The retailer has information about the uncertain market demand, and can share partial information with neither, one, or both of the manufacturers. We formulate three pricing decision models, and explore how some parameters (e.g., the amount of shared information, competition intensity, etc.) affect pricing and profits. Moreover, we give the sufficient conditions that each member benefits from the retailer’s partial information sharing.

1. Introduction. Effective supply chain management is to enhance the profits of all supply chain members by undertaking some appropriate coordination and collaboration mechanisms [12]. With the increasing complexity of business environment and increasing competition intensity, coordinations and collaborations between supply chain members are becoming more complicated and complex, and many problems still need to be addressed, such as the issue of information sharing. Information sharing can promote firms’ collaborations and optimize the supply chain’s performance because strategic and operational information may be available to other members of supply chain in sharing process [29]. With the rapid development of information technology, information sharing has received more attention from business practice. Especially, it is more common for downstream retailers to share some

2020 Mathematics Subject Classification. Primary: 90B50; Secondary: 91A27.
Key words and phrases. Supply chain management, uncertain demand, competitive manufacturers, partial information sharing, demand signal.

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information with upstream firms. According to the Chinese bank’s report, 61% of the surveyed firms said that it is essential and important to share information for their business success [1]. The annual surveys by Bearing Point reported that the downstream retailers in the United States had a rapid growth not only in the frequency but also in the scope of communication with their upstream suppliers [9]. A majority of the surveyed retailers communicated with their suppliers once a week, and thus could coordinate their supply chains well. As one of the most brilliant examples, Walmart shares its weekly sales information with its suppliers.

In general, the more the shared information, the more the business is successful. However, complete/full information sharing among supply chain members is very difficult due to the decentralization, competitiveness, and globalization of current supply chains. Double marginalization may occur if information between supply chain members becomes totally transparent, which damages the sharer and benefits the receiver [13]. Moreover, it is also very common that some supply chain members do not share their information. Shang et al. [24], using empirical study, show that only 27% of downstream retailers shared point of sales data with other supply chain members. So, partial information sharing is more prevalent in practice. According to prior studies, partial information sharing in supply chains usually takes place when the information is delayed, partially revealed, inaccurate (either intentionally or unintentionally), asymmetrically shared among supply chain members, or shared only between some supply chain members [4]. To our best knowledge, the research on information sharing concentrates almost exclusively on complete information sharing, while there are very few discussions on partial information sharing, which is one of our research motivations.

On the other hand, the existing research results show that, in different supply chain structures, information sharing assumptions, conditions, and mechanisms may be different, although information sharing can reduce uncertainty, smooth operations, and increase the whole supply chain’s profit [29]. Moreover, existing research on information sharing has been performed mainly in the one-to-one, or one-to-many, or two competitive supply chains, while there are few researchers who consider the information sharing in many-to-one supply chains. The many-to-one structure (i.e., monopoly common retailer channel) is one of the most common structure types, which characterizes numerous markets including those with department stores, supermarkets, specialty stores etc. [34]. Although relatively fruitful research results have been achieved about the effects of competition between downstream firms or between supply chains on information sharing, there are few studies on the effect of competition between upstream firms on information sharing besides Shang et al. [24]. This is the other motivation of our research.

Up to now, as we know, the current practice and problems haven’t been adequately explored and explained, so this work tries to fill in this gap by examining partial information sharing in a two-echelon supply chain including two upstream manufacturers and one downstream retailer. A distinguished feature of the paper is that the common retailer possesses some private information on uncertain market demand, and can share partial information selectively with two competitive manufacturers. Thus, three information sharing patterns may exist in the two-manufacturer one-retailer supply chain, i.e., sharing partial information with neither, one, or both of manufacturers. The following problems are explored: i) How do the supply chain members make their wholesale/retail pricing decisions
when facing different information sharing patterns? ii) How does the partial information shared by the retailer affect the supply chain members’ optimal pricing decisions and maximal profits? iii) In which condition, all supply chain members benefit from partial information sharing? iv) How do some important parameters (e.g., the amount of shared information, the competition intensity between two manufacturers, the retailer’s signal precision and the demand variation) affect the supply chain members’ optimal pricing decisions and maximal profits? This work aims to give enterprise managers and decision-makers some useful guidelines to make information sharing strategies facing competition between upstream enterprises, and provide them with a better understanding about the effects of the amount of shared information, the retailer’s signal precision, the competition intensity between two manufacturers, and the demand variation on the optimal pricing decisions and maximal profits.

The remainder of this article is organized as follows. In the next section, we review the related research literature. This is followed by the problem description and notations of models in Section 3. In Section 4, we give the main results under three information sharing patterns. Then, we compare and analyse the maximal profits and the expected profits in Section 5. Numerical study and parameter sensitivity analysis are implemented in Section 6. Finally, Section 7 concludes the paper with a summary and provides some potential directions for future research. All the proofs of Propositions are given in Appendix A.

2. Literature review. Our research is closely related to the following two research topics, i.e., complete information sharing and partial information sharing.

2.1. Complete information sharing. In economics field, earlier researchers mainly study demand information sharing in horizontal markets, and explore the incentive issue of information sharing between a firm and its competitors, such as Vives [28], Gal-Or [6], Li [17] and Raith [22]. These researches show that the product attributes (complement or substitute) and competitive patterns (Cournot or Bertrand) affect a firm’s decision on disclosing its private information.

In marketing management and operational research, researchers have studied the vertical information sharing in supply chains extensively. According to different supply chain structures, the related studies can be classified into four types. The first stream of research studies complete information sharing in one-to-one supply chain, such as Yue and Liu [31], Chu and Lee [3], Li and Zhang [19], Yan and Pei [30], Zhang et al. [33] and Cai et al. [2] etc. Yue and Liu [31] investigate the issue of sharing information on demand forecast in a dual-channel supply chain with one manufacturer and one retailer. Chu and Lee [3] consider voluntary information sharing between a downstream retailer and an upstream vendor, and show that the ways of facilitating information sharing are increasing the profit margin and reducing the cost of sharing information. Li and Zhang [19] study a retailer who shares demand information with a make-to-stock manufacturer, and show that the retailer has incentive to share information voluntarily if the magnitude of demand uncertainty is intermediate, which is different from the prior studies where the retailer is not willing to share information when the manufacturer adopts a make-to-order strategy. Yan and Pei [30] study the effects of information sharing on the firms’ performances in various market structures, and indicate that information sharing is always beneficial to the firms in the Stackelberg mode, but is beneficial to firms in the Bertrand mode under certain conditions. Cai et al. [2], considering the
demand uncertainty and demand forecast, explore the information sharing strategy under manufacturer warranty and supplier warranty, and show that under some conditions, the manufacturer has an incentive to share the demand information under both warranty policies.

The second stream of literature considers firms’ incentives to share complete information in a supply chain including many downstream firms (retailers) and an upstream firm (a manufacturer). Li [18] examines two effects of information sharing: “direct effect” and “leakage effect”, and shows that the leakage effect encourages the retailers to share their cost information with the manufacturer, while discourages them from sharing their demand information; the direct effect always discourages the retailers from sharing their information. Furthermore, the conditions are identified under which information can be traded when voluntary information sharing is not possible. Zhang [32] shows that, if the leakage effect is more beneficial to the retailers, or if the retailers’ information is statistically less accurate, complete information sharing will be achieved by using a side payment. On these basis, Li and Zhang [20] investigate the confidentiality issue of information exchange, and obtain the following results: 1) without confidentiality, information sharing benefits the manufacturer and hurts the retailers; 2) if all retailers share information confidentially, the supply chain profit will get the maximum; 3) with confidentiality, all firms are willing to share information when retail competition is intense. Lei et al. [16] investigate the issue of vertical and horizontal information sharing in a dual-channel supply chain, and show that vertical information sharing is beneficial to the manufacturer, and horizontal information sharing does not affect the manufacturer’s expected profit, while the retailers are willing to share information horizontally but not vertically. Tai et al. [27] investigate the value of information sharing in a two-level supply chain composed of one manufacturer and two retailers under promotional competition, and show that sharing information with a nondisclosure agreement is always valuable.

The third stream of research examines the issue of sharing complete information among members of the many-to-one supply chain. Zhao et al. [35] consider a client’s outsourcing problem where two suppliers face asymmetric information on service cost and compete for the client’s service contract. Shi et al. (2014) analyse two suppliers’ collusion and information-sharing problem, and show that cooperation and information sharing are beneficial to two suppliers, while detrimental to the manufacturer. Huang et al. [12] show that the reduction of inventory level and total costs of the suppliers is seriously affected by information sharing when the demand of successive periods is more correlated. Shang et al. [24] show that the main factors affecting the retailer’s incentive to share information include the nonlinear production cost, competition intensity between two manufacturers, and a payment contract for the information.

The fourth stream of research considers the supply chains’ competition. Ha and Tong [10] and Ha et al. [11] show that sharing information in one supply chain can coordinate the supply chain and effectively manage the threats from the competitive supply chain. Shamir and Shin [23] examine the effects of public disclosure of forecast information on supply chain members’ operational decisions. Guo et al. [9] consider the strategic disclosure after obtaining the demand signal, and study the strategic information sharing of two competitive channels. Bian et al. [1] consider the bilateral information sharing of two competitive supply chains,
and find whether information sharing benefits the supply chains depends on forecast error and competition intensity.

The aforementioned literatures show that different information sharing assumptions, conditions, and mechanisms are required in different supply chain structures. Different from many prior studies on complete information sharing with competition between downstream firms or between supply chains, this paper focuses on partial information sharing with competition between upstream firms. The most related literature is Shang et al. [24] which also considers a supply chain including competitive manufacturers and one retailer. However, there are two main differences between Shang et al. (2016) and this paper. First, Shang et al. [24] examines a scenario where a retailer only shares complete information with one, both, or neither of the competitive manufacturers, while our work considers that the retailer can choose to share partial information with two competitive manufacturers. From the perspective of mathematical modelling, this paper extends and complements the prior work of Shang et al. [24]. Second, Shang et al. [24] emphasize the issue of information contracting under production economy/diseconomy, while we focus on the issue of decision-making under different information sharing patterns, and further analyse the effects of the shared information and competition intensity between upstream manufacturers, which have not yet been studied in prior articles.

2.2. Partial information sharing. Compared with complete information sharing, there are relatively few researches on partial information sharing, which mainly focus on sharing partial information among supply chain members or sharing only among some supply chain members. For example, Lau et al. [15] consider the effect of different levels of sharing information on inventory replenishment in three-stage distribution supply chains. Shnaiderman and Ouardighi [26] consider several levels of partial sharing on demand information in a supply chain with one manufacturer and one retailer, and show that each firm informs the other not of the exact value of demand, but of an interval. Ganesh et al. [7, 8] extend the incentive issue of information sharing to a multilevel supply chain through considering two types of partial information sharing, i.e. upstream information sharing and downstream information sharing. Huang et al. [12] consider the information sharing in a two-echelon supply chain including multiple suppliers and one retailer, and show that partial information sharing may generate the bullwhip effect problem. Dominguez et al. [4] study the effect of using different strategies to implement partial information sharing among heterogeneous retailers on supply chain performance. The main difference between the aforementioned literatures and this paper is in the modeling assumption on market demand. Most of papers are based on the same assumption: the demand follows an AR (1) process, while this paper assumes that the demand is an arbitrary random variable, and the retailer has a sample about random demand with \( n \) observations and takes the sample mean as its private information.

This work is most related to Zhou et al. [36]. The two papers both consider the problem of partial information sharing and sharing information with partial supply chain members. However, the main difference of the two papers is in supply chain structures. Zhou et al. [36] consider a one-to-two supply chain where two competitive manufacturers (downstream firms) have some private information on the uncertain demand, and can choose to share a part with one group purchasing organization (upstream firm), while this paper focuses on a supply chain including two competitive manufacturers (upstream firms) and one retailer (downstream firm), i.e., a two-to-one supply chain. Moreover, Zhou et al. [36] show that it is optimal
3. Problem description and assumptions. This paper considers a two-echelon supply chain including two competitive manufacturers (manufacturer 1 and manufacturer 2, we use the pronoun “he” to refer to one of two manufacturers) and one retailer (she). Manufacturer $i$ wholesales product $i$ to the retailer at wholesale price $p_i$, $i = 1, 2$. Products 1 and 2 are substitutable to each other. Without loss of generality, both manufacturers’ and retailer’s marginal costs are normalized to zero [21, 14]. In the process of defining the consumer demand function, following Vives [28], Li and Zhang [20], and Shang et al. [24], we use a linear form which has been used extensively in the literature. The demand function of product $i$ is given as

$$d_i = a + \varepsilon - p_i + \gamma p_j, \ i = 1, 2, j = 3 - i.$$  

Parameter $\gamma \in (0, 1)$ shows two products’ substitutability degree, and also represents the two manufacturers’ competition intensity. Larger $\gamma$ indicates that the competition between both manufacturers is more intensive. The demand intercept $a + \varepsilon$ is the market base of product $i$, where parameter $a$ is a constant and $\varepsilon$ is a random variable with mean zero and variance $\sigma^2$. We assume $a >> \sigma$ to make the probability of negative demand intercept negligible [36].

The random variable $\varepsilon$ shows the market demand uncertainty, which is not observable by both manufacturers upstream. Due to directly contacting with consumers, the downstream retailer can acquire some private information about $\varepsilon$. After $n$ observations, the retailer will get a sample $M = \{\varepsilon + u_k | k = 1, 2, \ldots, n\}$ about random variable $\varepsilon$, where the $u_1, u_2, \ldots, u_n$ are independent and identically distributed, with mean zero and variance $\sigma^2_u$, and which are independent of $\varepsilon$. Set $Y = \varepsilon + \sum_{k=1}^{n} u_k / n$, which is the sample mean of $M$ and also is the retailer’s private information. One can easily check that $E[Y] = \varepsilon$ and $E[Var(Y|\varepsilon)] = \sigma^2_u / n$, which means $Y$ is an unbiased estimator of random variable $\varepsilon$. Set $t = \sigma^2 / E[Var(Y|\varepsilon)]$, which is regarded as the expected precision of the sample mean $Y$ relative to the precision of random variable $\varepsilon$, and which also measures the amount of information received by the retailer. $t \to \infty$ indicates that the demand is fully observable, and $t \to 0$ means that the demand is completely invisible.

We assume two manufacturers play a Nash game, and they play a Stackelberg game with the retailer (Stackelberg follower). Furthermore, we describe the corresponding information sharing patterns and the order of events of the multistage game as follows.

1. The retailer first chooses whether to share information with manufacturer $i$. Denote two manufacturers’ information sharing patterns as $X_i, X_j$, where $i = 1, 2$, $j = 3 - i$. Then, $X_i (X_j)$ may be $I$ or $N$, which means manufacturer $i$ (manufacturer $j$) is informed or uninformed. If the retailer decides to share information, she selects $n_0$ observations from the private sample $M$, denoted by $K_0$, to share with two manufacturers or one of them. Take $y (y = \varepsilon + \sum_{k=1}^{n_0} u_k / n_0)$, the sample mean of $K_0$, as the informed manufacturer’s signal. Here $y$ is also an unbiased estimator of $\varepsilon$, $E[y] = \varepsilon$, and $E[Var(y|\varepsilon)] = \sigma^2_u / n_0$. Let $\tau = \sigma^2 / E[Var(y|\varepsilon)]$ represent the amount of shared information. $\tau = t$ means complete information
sharing, \( \tau = 0 \) implies that there is no information sharing, and \( \tau \in (0, t) \) shows the partial information sharing.

2. Two manufacturers then determine their own wholesale prices simultaneously to optimize their own expected profits. The informed manufacturer \( i \)'s expected profit is defined as

\[
E[\pi_m(w_i)|y] = E[w_id_i], \quad i = 1, 2, \quad (2)
\]

and the uninformed manufacturer \( j \)'s expected profit is

\[
E[\pi_m(w_j)] = E[w_jd_j], \quad j = 3 - i. \quad (3)
\]

3. For given wholesale prices, the retailer then determines retail prices \( p_1 \) and \( p_2 \) to maximize her expected profit by using the information \( Y \). The retailer’s expected profit is

\[
E[\pi_r(p_1, p_2)|Y] = E\left[ \sum_{i=1}^{2} (p_i - w_i)d_i | Y \right]. \quad (4)
\]

4. Finally, after the consumers’ demands arrive, the retailer then wholesales two products from two manufacturers in order to meet the demands.

The information structure in our model is based on the following assumptions:

(a) \( E[Y|x] = E[y|x] = x \cdot \beta \) \( E[x|X] \) is an affine in \( X \) for any subset \( X \subseteq \{Y, y\} \); (c) \( Y \) and \( y \) are independent conditional on \( \varepsilon \).

Referring to the Lemma 1 of Li [17], and the above assumptions imply that

\[
E[\varepsilon|Y] = \frac{t}{1 + t} Y, \quad E[\varepsilon|y] = \frac{\tau}{1 + \tau} y, \quad E[Y|y] = \frac{\tau(1 + t)}{t(1 + \tau)} y. \quad (5)
\]

In addition, one can verify that

\[
E[y^2] = (1 + \frac{1}{\tau})\sigma^2, \quad E[Y^2] = E[YY] = (1 + \frac{1}{t})\sigma^2. \quad (6)
\]

Table 1 summarizes the notations adopted in this paper, and Figure 1 shows the information sharing patterns discussed in this paper.

**Table 1. Notations and Descriptions**

| Notation | Description |
|----------|-------------|
| \( p_i \) | retail price of product \( i \), decided by retailer |
| \( w_i \) | wholesale price of product \( i \), decided by manufacturer \( i \) |
| \( d_i \) | demand of product \( i \) |
| \( a \) | determined market base |
| \( \varepsilon \) | market demand uncertainty, a random variable with mean zero and variance \( \sigma^2 \) |
| \( \gamma \) | competition between both manufacturers, which denotes the substitutability degree between products 1 and 2 |
| \( Y \) | retailer’s private demand information |
| \( y \) | informed manufacturer’s demand signal |
| \( t \) | retailer’s signal precision |
| \( \tau \) | the amount of shared information |
| \( \pi_m \) | profit of manufacturer \( i \) |
| \( \pi_r \) | profit of retailer \( i \) |
| \( \Pi_m \) | expected profit of manufacturer \( i \) |
| \( \Pi_r \) | expected profit of retailer |
| \( \nu_m \) | value of manufacturer \( i \)'s information sharing |
| \( \nu_r \) | value of retailer information sharing |
| \( V_m \) | expected value of manufacturer \( i \)'s information sharing |
| \( V_r \) | expected value of retailer information sharing |
4. Model formulation and analysis. To explore the effect of the retailer’s different information sharing patterns on channel members’ optimal decisions and maximum profits, in this section, we first formulate the pricing game models under different information sharing patterns, and then solve them by backward induction. The optimal equilibrium solutions are obtained.

4.1. Sharing information with two manufacturers. We first consider the setting where the retailer shares information with both manufacturers, i.e., Figure 1(a) \( X_iX_j = \text{II} \). The pricing decision model is formulated as follows

\[
\begin{align*}
\max_{w_i} \mathbb{E}[\pi_m(w_i, p_1(w_1, w_2), p_2(w_1, w_2)) | y] \\
\max_{w_j} \mathbb{E}[\pi_m(w_j, p_1(w_1, w_2), p_2(w_1, w_2)) | y] \\
\max_{(p_1, p_2)} \mathbb{E}[\pi_r(p_1, p_2) | Y]
\end{align*}
\]

where \( p_1(w_1, w_2), p_2(w_1, w_2) \) are derived from solving the following problem

\[
\max_{(p_1, p_2)} \mathbb{E}[\pi_r(p_1, p_2) | Y]
\]

We need to first solve the retailer’s response functions for given both manufacturers’ wholesale price decisions. After obtaining the retailer’s response functions, both manufacturers then give their wholesale prices, simultaneously, to maximize their own expected profits. Using the backward induction, we can obtain the corresponding results as in Propositions 1-2.

**Proposition 1.** Given two manufacturers’ earlier decisions \( w_1 \) and \( w_2 \), the retailer’s response functions are

\[
p_i(w_i, w_j) = \frac{w_i}{2} + \frac{a}{2(1-\gamma)} + \frac{1}{2(1-\gamma)} \mathbb{E}[\epsilon | Y], \quad i = 1, 2, \quad j = 3 - i.
\]

We put the proofs of Proposition 1 and other Propositions in Appendix A.

**Proposition 2.** If the retailer chooses to share information with two manufacturers, the optimal wholesale price \( w_i^{II*} \) and the optimal retail price \( p_i^{II*} \) (\( i = 1, 2 \)) are

\[
w_i^{II*} = \bar{w} + \frac{A^{II} y}{2}, \quad p_i^{II*} = \bar{p} + \frac{A^{II} y}{2} + BY,
\]

where \( \bar{w} = \frac{a}{2-\gamma} \), \( \bar{p} = \frac{(3-2\gamma)a}{2(1-\gamma)(2-\gamma)} \), \( A^{II} = \frac{\gamma}{2(1-\gamma)(1+\gamma)} \), \( B = \frac{t}{2(1-\gamma)(1+\gamma)} \).

Eq. (9) shows that the informed manufacturer adopts a linear strategy to adjust the wholesale price in response to the shared demand signal \( y \); the retailer also adopts a linear strategy to adjust the retail prices in response to her private signal \( Y \) and the shared demand signal \( y \). The coefficient \( A^{II} \) captures the responsiveness
of the optimal prices to \( y \), and \( B \) captures the responsiveness of the optimal retail prices to \( Y \).

It is easy to see that \( A_{II} > 0 \), which means the shared signal \( y \) has a positive impact on the optimal prices. This implies that the informed manufacturer \( i \) adjusts his wholesale price \( w_{i}^{II} \) to respond positively to the shared demand signal \( y \) so that he can benefit from both the stronger double marginalization and the lower order uncertainty. Moreover, it follows from \( \frac{\partial B_{II}}{\partial \gamma} > 0 \) and \( \frac{\partial B_{II}}{\partial \tau} > 0 \) that the impact of the shared signal \( y \) is stronger with more intense competition and more amount of shared information.

Similarly, we can also obtain that \( B > 0 \), \( \frac{\partial B}{\partial \gamma} > 0 \), and \( \frac{\partial B}{\partial \tau} > 0 \), which mean the private signal \( Y \) has a positive impact on the optimal retail prices, and its impact is stronger with more intense competition and more accurate signal \( Y \).

**Corollary 1.** When the retailer shares complete information with two manufacturers, i.e., \( \tau = t \), the optimal prices of product \( i \) (\( i = 1, 2 \)) are \( w_{i}^{II} = \bar{w} + A_{II}^i Y \), \( \hat{p}_{i}^{II} = \bar{p} + B_{II}^i Y \), where \( A_{II}^1 = \frac{t}{(2-\gamma)(1+t)} \), \( B_{II}^1 = \frac{(3-2\gamma)}{2(1-\gamma)(2-\gamma)(1+t)} \).

In addition, it is possible for the retailer to share information with both manufacturers, but the amount of information shared is not the same. Thus, we further consider the extended case wherein the retailer shares the amount of information \( \tau_i = \sigma^2/E[\text{Var}(y_i|\varepsilon)] \) with manufacturer \( i \) and shares the amount of information \( \tau_j = \sigma^2/E[\text{Var}(y_j|\varepsilon)] \) with manufacturer \( j \) (\( \tau_i \neq \tau_j \)), and \( y_i, y_j \) are independent and are both unbiased estimators of \( \varepsilon \). By backward induction, we obtain the corresponding results as follows.

**Corollary 2.** When the retailer shares different amount of information with both manufacturers, i.e., manufacturer \( i \)’s amount of information is \( \tau_i \) and manufacturer \( j \)’s amount of information is \( \tau_j \), the optimal prices are \( w_{i}^{II} = \bar{w} + A_{II}^i y_i \), \( \hat{p}_{i}^{II} = \bar{p} + \frac{1}{2}A_{II}^i y_i + BY \), where \( A_{II}^i = \frac{\tau_i[2+(2-\gamma)(1+t)]}{(3-2\gamma)} \).

### 4.2. Sharing information with one manufacturer

In this section, we consider the setting where the retailer chooses to share information only with one of the both manufacturers, without loss of generality, assume the manufacturer \( i \) (\( i = 1, 2 \)), which is illustrated in Figure 1(b). The following pricing decision model is established

\[
\max_{w_i} E[\pi_{m}(w_i, p_1(w_1, w_2), p_2(w_1, w_2))|y] \\
\max_{w_j} E[\pi_{m}(w_j, p_1(w_1, w_2), p_2(w_1, w_2))|y], \quad i = 1, 2, \quad j = 3 - i \\
\max_{(p_1, p_2)} E[\pi_r(p_1, p_2)|Y] \\
p_1(w_1, w_2), p_2(w_1, w_2) \text{ are derived from solving the following problem } \tag{10}
\]

By backward induction, it is easy to derive the optimal equilibrium strategies under this case, which are summarized in Proposition 3.

**Proposition 3.** If the retailer chooses to share information only with manufacturer \( i \), i.e., the sharing information pattern \( X_iX_j = IN \), the optimal wholesale prices and the optimal retail prices are

\[
w_{i}^{IN} = \bar{w} + A_{IN}^i y_i, \quad p_{i}^{IN} = \bar{p} + \frac{A_{IN}^i}{2} y_i + BY, \quad i = 1, 2, \tag{11}
\]

\[
w_{j}^{IN} = \bar{w}, \quad p_{j}^{IN} = \bar{p} + BY, \quad j = 3 - i, \tag{12}
\]
where \( \bar{w} = \frac{a}{2-\gamma} \), \( \bar{p} = \frac{(3-2\gamma)a}{2(1-\gamma)(2-\gamma)} \), \( A^N = \frac{2\tau}{(4-\gamma)(1+\gamma)} \), \( B = \frac{t}{2(1-\gamma)(1+t)} \).

It follows from Eq. (11) that \( w_i^{NN}\) responds positively to the shared demand signal \( y \) because \( A^N > 0 \). This makes three effects: the double marginalization of \( w_i^{NN} \) stronger, the order uncertainty of \( d_i \) lower and the order uncertainty of \( d_j \) higher. The first two effects benefit the manufacturer \( i \), while the second and the third hurt the retailer and manufacturer \( j \), respectively. It follows from Eq. (12) that the uninformed manufacturer \( j \) will not respond to the demand signal, and the retail price of product \( j \) is only responsive to the retailer’s private signal. It is obvious that the scenario of sharing information only with manufacturer \( i \) is not beneficial to manufacturer \( j \).

It’s worth noting that \( A^N < A^I \), which means that the optimal prices of product \( i \) are less responsive to the shared demand signal due to the uninformed manufacturer \( j \). In other words, the informed manufacturer \( i \) benefits more when manufacturer \( j \) is also informed.

**Corollary 3.** When the retailer shares complete information only with manufacturer \( i \) (i.e., \( \tau = t \)), the optimal prices of product \( i \) are \( w_i^{NN} = \bar{w} + A^{NY} \) and \( p_i^{NN} = \bar{p} + B^{NY} \), while the optimal prices of product \( j \) remain unchanged, where \( A^N = \frac{2\tau}{(4-\gamma)(1+\gamma)} \), \( B^N = \frac{(6-2\gamma-\gamma^2)t}{2(1-\gamma)(4-\gamma)(1+t)} \), \( i = 1, 2 \), \( j = 3 - i \).

**4.3. No sharing information.** In this section, we consider the scenario where the retailer chooses to share no information with both manufacturers, i.e., Figure 1(c) \( X_iX_j = NN \). The pricing decision model is

\[
\begin{align*}
\max_{w_i} & E[\pi_{m_i}(w_i, p_1(w_1, w_2), p_2(w_1, w_2))] \\
\max_{w_j} & E[\pi_{m_j}(w_j, p_1(w_1, w_2), p_2(w_1, w_2))] \\
\text{subject to} & \quad p_1(w_1, w_2), p_2(w_1, w_2) \text{ are derived from solving the following problem} \\
\max_{(p_1, p_2)} & E[\pi_e(p_1, p_2)|Y]
\end{align*}
\]  

(13)

In fact, the optimal equilibriums of the above model can be easily obtained by setting the amount of information \( \tau \) shared by the retailer to zero in Proposition 2 or Proposition 3. The following Proposition provides the corresponding results.

**Proposition 4.** When the retailer shares no information with both manufacturers, the optimal wholesale price \( w_i^{NN} \) and the optimal retail price \( p_i^{NN} \) (\( i = 1, 2 \)) are

\[
w_i^{NN} = \bar{w}, \quad p_i^{NN} = \bar{p} + BY,
\]

(14)

where \( \bar{w} = \frac{a}{2-\gamma} \), \( \bar{p} = \frac{(3-2\gamma)a}{2(1-\gamma)(2-\gamma)} \), \( B = \frac{t}{2(1-\gamma)(1+t)} \).

**Remark 1.** The results in Corollaries 1 and 2, and Proposition 4 are consistent with those of the Lemma 1 in Shang et al. [24], which show that the retailer and the informed manufacturer both use a linear strategy to adjust the retail prices and the wholesale price in response to the demand signal, respectively. The uninformed manufacturer will not respond to the demand signal \( Y \). So, this paper extends and complements the model in Shang et al. [24] from the aspect of the quantity of shared information. The result of Lemma 1 in Shang et al. [24] is a special case when the retailer shares complete information in this paper.
5. **Comparison and analysis.** Based on the optimal pricing strategies in Propositions 2-4, we can derive the maximal profits and expected profits of two manufacturers and the retailer under different information sharing patterns. Table 2 summarizes the corresponding results.

Firstly, we examine the supply chain members’ maximal profits. If the retailer shares no information, denote the maximal profits of two manufacturers and the retailer as $\Phi_m$ and $\Phi_r$, respectively, which reflect the effect of the demand signal $Y$.

If the retailer chooses to share information with one or both of two manufacturers, the maximal profits include two parts. The first part is the effect of the demand signal $Y$ (i.e., $\Phi_m$ or $\Phi_r$), and the other shows the effect of information sharing, i.e., the value of information sharing, which is $v_k^I$, $k = m_i, m_j$, or $r$.

Under information sharing pattern $X_iX_j = NN$, the quantity sold by the retailer is $d_{NN}^N = a + E|Y| - p_i^N + \gamma p_j^N$. To ensure $d_{NN}^N$ is positive, the demand signal $Y$ should satisfy the condition $Y > \frac{(1+\tau)a}{(2-\gamma)t}$. Under the condition, it is easy to prove that the maximal profits $\Phi_m$ and $\Phi_r$ are positive.

| Table 2. The maximal profits and the expected profits |
|-----------------------------------------------------|
| $X_iX_j = II$                                      |
| $X_iX_j = 1N$                                      |
| $X_iX_j = NN$                                      |
| **The maximal profits**                            |
| $\pi_{m_i} III = \Phi_m + v_{m_i}^I$               |
| $\pi_{m_j} III = \Phi_m + v_{m_j}^I$               |
| $\pi_r III = \Phi_r + v_r^I$                      |
| $\pi_{m_i} NN = \Phi_m + v_{m_i}^N$               |
| $\pi_{m_j} NN = \Phi_m + v_{m_j}^N$               |
| $\pi_r NN = \Phi_r + v_r^N$                      |
| **The expected profits**                           |
| $\Pi_{m_i} III = \Pi_{m_i} + V_{m_i}^I$           |
| $\Pi_{m_j} III = \Pi_{m_j} + V_{m_j}^I$           |
| $\Pi_{m} III = \Pi_{m_i} + \Pi_{m_j} + F_i^I + F_j^I$ |
| $\Pi_{m_i} NN = \Pi_{m_i} + V_{m_i}^N$           |
| $\Pi_{m_j} NN = \Pi_{m_j} + V_{m_j}^N$           |
| $\Pi_{m} NN = \Pi_{m_i} + \Pi_{m_j} + F_i^N + F_j^N$ |
| $\Pi_{m} II = \Pi_{m_i} + V_{m_i}$               |
| $\Pi_{m} 1N = \Pi_{m_j} + V_{m_j}$               |
| $\Pi_{m} NN = \Pi_{m_i} + \Pi_{m_j} + F_i^N + F_j^N$ |

where $\Phi_m = \frac{a + (2-\gamma)E|Y|}{2(2-\gamma)t}$, $\Phi_r = \frac{a + (2-\gamma)E|Y|}{2(1-\gamma)(2-\gamma)}$, $v_{m_i}^I = \frac{E|y|[(3-\gamma)E|y| - (2-\gamma)E|y| + (4-\gamma)a]}{2(2-\gamma)^2}$, $v_{m_j}^N = \frac{\gamma a E|y|}{2(2-\gamma)^2}$, $v_r^I = \frac{E|y|[(3-\gamma)E|y| - (2-\gamma)E|y| + (4-\gamma)a]}{2(2-\gamma)^2}$, $\Pi_{m_i} = \frac{a^2}{2(2-\gamma)^2}$, $\Pi_{m_j} = \frac{a^2}{2(2-\gamma)^2}$, $\Pi_{m} = \frac{a^2}{2(2-\gamma)^2}$, $\Pi_{m} = \frac{a^2}{2(2-\gamma)^2}$, $V_{m_i}^I = \frac{(2-\gamma)^2\sigma^2}{(1+\tau)^2}$, $V_{m_j}^N = \frac{(2-\gamma)^2\sigma^2}{(1+\tau)^2}$, $V_{m}^N = \frac{(2-\gamma)^2\sigma^2}{(1+\tau)^2}$, $V_{m}^I = \frac{(2-\gamma)^2\sigma^2}{(1+\tau)^2}$.

**Proposition 5.**

(i) $v_{m_i}^I > 0$, if $y > \max\{0, \frac{1+\tau}{(3-\gamma)^\tau} \frac{(2-\gamma)^t}{1+t} Y - (4-\gamma)a\}$ or $y < \min\{0, \frac{1+\tau}{(3-\gamma)^\tau} \frac{(2-\gamma)^t}{1+t} Y - (4-\gamma)a\}$;

(ii) $v_{m_j}^N > 0$, if $y > \max\{0, \frac{2(1+\tau)}{(1-\gamma)^\tau} \frac{(2-\gamma)^t}{1+t} Y + a\}$ or $y < \min\{0, \frac{2(1+\tau)}{(1-\gamma)^\tau} \frac{(2-\gamma)^t}{1+t} Y + a\}$;
(ii) \( v^{IN}_{m_i} > 0 \), if \( y > \max \{0, \frac{(4 - \gamma^2)(1 + \tau)}{2(3 - \gamma^2)\tau} \left[ \frac{t}{1 + t} Y - (2 + \gamma)a \right] \} \) or 
\[ y < \min \{0, \frac{(4 - \gamma^2)(1 + \tau)}{2(3 - \gamma^2)\tau} \left[ \frac{t}{1 + t} Y - (2 + \gamma)a \right] \} \]
\[ v^{NI}_{m_j} > 0, \text{if } y > 0; \]
\[ v^{IN}_{r} > 0, \text{if } y > \max \{0, \frac{(2 + \gamma)(1 + \tau)}{\tau} \left[ \frac{(2 - \gamma)t}{1 + t} Y + a \right] \} \) or 
\[ y < \min \{0, \frac{(2 + \gamma)(1 + \tau)}{\tau} \left[ \frac{(2 - \gamma)t}{1 + t} Y + a \right] \}. \]

Proposition 5 gives the sufficient conditions to guarantee nonnegative values of information sharing under two information sharing patterns. Based on Proposition 5, the retailer can strategically use the condition to maximize the value of information sharing so that each member benefits from information sharing.

In Figure 2, we illustrate the feasible region of information sharing where the retailer’s information sharing is beneficial to supply chain members. One can easily see that the feasible region satisfied by \( v^{X_iX_j}_{m_i} > 0 \) is smaller than that satisfied by \( v^{X_iX_j}_{m_j} > 0 \). Figure 2(a) shows that the left upper shaded area and the lower right shaded area can ensure all supply chain members benefit under the pattern \( X_iX_j = II \), while Figure 2(b) shows only the left upper shaded area can ensure all supply chain members benefit under the pattern \( X_iX_j = IN \). So, the feasible region of information sharing under the pattern \( X_iX_j = II \) is obviously larger than that under the pattern \( X_iX_j = IN \).

![Figure 2. Feasible region of information sharing](image)

Secondly, we examine the expected profits shown in Table 1. We find that, if the retailer shares information, her expected profit can be divided into three parts: the deterministic profit \( \Pi_r \), the expected value of information sharing(EVIS) for retailer \( r \) or \( V^{II}_r \) or \( V^{IN}_r \), and the forecast profit \( F_r \); the expected profit of the informed manufacturer \( i \) can be divided into two parts: the deterministic profit \( \Pi_{m_i} \), and EVIS for manufacturer \( i \), \( V^{II}_{m_i} \) or \( V^{IN}_{m_i} \). If there is no information sharing, the corresponding EVIS is zero.

In the following, we aim to analyze the effects of information sharing from the perspective of EVIS.
Proposition 6.

(i) $V_{m_i}^{II} > V_{m_i}^{IN} > 0$; (ii) $V_{r}^{II} < V_{r}^{IN} < 0$; (iii) $V_{r}^{II} + V_{m_1}^{II} + V_{m_2}^{II} < 0$,
and $V_{r}^{IN} + V_{m_i}^{IN} < 0$.

From Proposition 6, we can see that the retailer’s information sharing is beneficial to the informed manufacturer, but hurts the retailer and the whole supply chain; the more informed manufacturers, the more damage received by the retailer, and the better for the informed manufacturer.

Remark 2. From the perspective of EVIS, we make a comparison with some prior studies on the effects of information sharing, and summarize in Table 3. The results show that no matter under which scenarios, information sharing always benefits the manufacturer while hurts the retailers. However, it has different impacts on the whole supply chain.

Table 3. Comparison on the effects of information sharing

| paper         | SC structure | information | effects of IS |
|---------------|--------------|-------------|---------------|
| Zhang [32]    | 1-2          | ×           | B H BC        |
| Li [18]       | 1-n          | ×           | B H BC        |
| Mishra et al. [21] | 1-1      | √           | *B H H        |
| This paper    | 2-1          | ×           | B H BC        |
|               |              |              | †B/N H H      |

Note: M: manufacturer R: retailer SC: supply chain IS: information sharing B: beneficial H: harmful BC: beneficial under some conditions N: no effect *: make-to order scenario -: make-to-stock scenario †: share with both manufacturers †: share with one of manufacturers B/N: information sharing benefits the informed manufacturer, and has no effect on the uninformed manufacturer

Thirdly, we examine the effect of parameter $t$ on the forecast profit $F_r$, the effect of parameter $\tau$ on EVISs and the effect of parameter $\sigma$ on the expected profits. The corresponding results are given in Proposition 7.

Proposition 7.

(i) $\frac{\partial F_r}{\partial t} > 0$; (ii) $\frac{\partial V_{r}^{X_iX_j}}{\partial t} > 0$ and $\frac{\partial V_{r}^{X_iX_j}}{\partial \tau} < 0$;

(iii) $\frac{\partial V_{m_1}^{II}}{\partial \gamma} > 0$, $\frac{\partial V_{m_1}^{II}}{\partial \gamma} < 0$, $\frac{\partial V_{m_1}^{IN}}{\partial \gamma} < 0$, and $\frac{\partial V_{m_1}^{IN}}{\partial \gamma} < 0$;

(iv) $\frac{\partial \Pi_{m_1}^{**}}{\partial \sigma} = 0$, $\frac{\partial \Pi_{m_1}^{**}}{\partial \sigma} > 0$ and $\frac{\partial \Pi_{m_1}^{**}}{\partial \sigma} > 0$, where $X_iX_j = II$ or $IN$.

It follows from Proposition 7 (i) that as the precision of demand signal $Y$ increases, the retailer’s forecast profit increases, and thus the retailer’s profit increases. Result (ii) means the more information shared by the retailer, the better for the informed manufacturer, and the worse for the retailer. From (iii), we see that, as the competition intensity between two manufacturers increases, two manufacturers’ EVISs increase, while the retailer’s EVIS decreases if the retailer shares partial
information with two manufacturers. Surprisingly, if the retailer chooses to share partial information only with one manufacturer, the increase of competition intensity between two manufacturers hurts not only the retailer but also the informed manufacturer. Result (iv) shows that demand variation $\sigma$ affects the expected profits of the retailer and the informed manufacturer positively, while it has no effect on the uninformed manufacturer’s expected profit.

6. Numerical study. In this section, we implement the sensitivity analysis of several main parameters (i.e., $\tau$, $t$, $\sigma$, $\gamma$) on the expected profits under different information sharing patterns through numerical approach, because it is impossible to analyze them analytically. We perform many groups of parameter values to carry out the numerical studies, and find some interesting results as follows. We show some results in Figures 3-6, where the default values of parameters are $a = 10$, $\tau = 3$, $t = 5$, $\sigma = 5$ and $\gamma = 0.6$.

6.1. Impact of the amount of shared information on the expected profits. In this section, we study the impact of the amount of shared information $\tau$ on the expected profits of the whole supply chain and its members. We assume the parameter $\tau \in [0, 5]$, the other parameters take their default values, and show the corresponding results in Figure 3. Figure 3 tells us that under information sharing patterns $X_i X_j = II$ and $X_i X_j = IN$, with the increase in the amount of shared information $\tau$, (i) the informed manufacturer’s expected profit increases; (ii) the retailer’s expected profit decreases; (iii) the uninformed manufacturer’s expected profit has no change; (iv) the total expected profit decreases slightly.

![Figure 3. The effect of $\tau$ on the expected profits](image1)

Result (i) means that the increase of $\tau$ makes the informed manufacturer adopt a higher wholesale price strategy, and the retailer, in turn, makes a higher retail price, which directly lowers the demand of the informed manufacturer’s product. Because the revenue from increased wholesale price offsets the loss of decreased product’s demand, so the informed manufacturer’s expected profit increases. The explanation for observation (ii) is as follows: with Eqs. (9) and (11), we can see that with the increase of $\tau$, the increase in retail price is half of the increase in wholesale price. Thus, for the retailer, the product’s demand and profit margin (i.e., $d_i^{X_i X_j \ast}$ and $p_i^{X_i X_j \ast} - w_i^{X_i X_j \ast}$, where $i = 1, 2$; $X_i X_j = II, IN$) both decrease, which leads to her expected profit decreases. The two observations are consistent with the result (ii) of Proposition 7.
It follows from $\bar{\Pi}_{m_j}$ in Table 1 that parameter $\tau$ has no effect on the uninformed manufacturer’s expected profit. Eq. (12) shows that the optimal prices of the uninformed manufacturer’s product are not affected by parameter $\tau$, and the expected demand for his product is a linear function of $y$. Therefore, the amount of shared information does not affect the uninformed manufacturer’s expected profit. The expected profit of the whole supply chain decreases slightly, because the decrease of the retailer’s expected profit slightly exceeds the increase of the informed manufacturer’s expected profit.

Additionally, if the retailer shares no information with two manufacturers, the amount of shared information does not affect the expected profits of the whole supply chain and its members. This observation is intuitive from the expected profits under the pattern $X_iX_j = NN$ shown in Table 1.

6.2. Impact of the retailer’s signal precision on the expected profits. In this section, we investigate the impact of the retailer’s signal precision $t$ on the expected profits, and show the corresponding results in Figure 4, where $t \in [0, 10]$ and other parameters take their default values. Figure 4 shows that regardless of the pattern of information sharing, as $t$ increases, the retailer’s expected profit increases, while two manufacturers’ expected profits do not change, which is easy to understand according to the expected profits in Table 1 and the result $\frac{\partial F_r}{\partial t} > 0$ in Proposition 7. Furthermore, it is obvious that with the increase in the retailer’s signal precision, the expected profit of the whole supply chain increases at the same rate as the retailer’s expected profit.

6.3. Impact of demand variation on the expected profits. We explore the effect of the demand variation $\sigma$ on the expected profits, which is shown in Figure 5, where the parameter $\sigma$ changes from 0 to 25 and the others take the default values. Figure 5 shows that under three patterns of information sharing, as parameter $\sigma$ increases, the expected profits all increase except that of the uninformed manufacturer. This verifies the result (iv) of Proposition 7. This observation is consistent with that of Mishra et al. [21], which consider bilateral information sharing in a supply chain including one manufacturer and one retailer.

Moreover, the impact of parameter $\sigma$ on the retailer’s expected profit is larger than that on the informed manufacturer’s expected profit, which is consistent with the analysis of the expected profits in Table 1. This observation is easily understandable because the downstream retailer is closer to the market demand than the informed manufacturer, which results in her expected profit is more affected by the demand variation.

6.4. Impact of competition intensity between two manufacturers on the expected profits. We examine the impact of competition intensity $\gamma$ between two manufacturers on the expected profits and show the corresponding results in Figure 6, where parameter $\gamma$ changes from 0 to 0.9 and the other parameters take their default values. Figure 6 shows that no matter which pattern of information sharing is adopted, as the competition intensity $\gamma$ increases, the expected profits of supply chain and its members all increase; the positive impacts of $\gamma$ on the expected profits of the whole supply chain and the retailer are higher than those on two manufacturers’ expected profits.

From the theoretical analysis on the expected profits in Table 1, it is easy to see that the competition intensity has positive effects on the deterministic profits and the forecast profit. Furthermore, together with this observation, we can infer that
its positive effects exceed the negative effects on some EVISs, which is shown in the \((iii)\) of Proposition 7. This observation is different from the results of Zhang [32]. Zhang [32] shows that with the increase in competition intensity between two retailers, the expected profits of all supply chain members decrease, where two competitive downstream retailers share their private information with the upstream manufacturer.

6.5. **Comparison of expected profits under three patterns.** In this section, we compare the expected profits of each firm and the whole supply chain under three different information sharing patterns. From Figure 3-6, the following observations can be obtained.

\((i)\) \(\Pi_{II}^{*} > \Pi_{IN}^{*} > \Pi_{NN}^{*}\); \(\Pi_{m_{j}}^{II} > \Pi_{m_{i}}^{IN} > \Pi_{m_{i}}^{NN}\). These results verify the correction of Results (\(i\)) and (\(ii\)) in Proposition 6, and further indicate that information sharing is harmful for the retailer, and is beneficial to the informed manufacturer. Moreover, the more informed manufacturers, the more damage received by the retailer, and the better for the informed manufacturer. The above observations are consistent with the result of Lemma 3 in Shang et al. [24].

\((ii)\) \(\Pi_{m_j}^{II} > \Pi_{m_j}^{NI} = \Pi_{m_j}^{NN}\), which means that manufacturer \(j\) is obviously beneficial if he is informed of partial demand information. However, if the retailer only shares information with manufacturer \(i\), the shared information has no effect on the uninformed manufacturer \(j\), which is different from the result of Proposition 1(a) in Shang et al. [24] where the uninformed manufacturer is damaged under the production diseconomy.

\((iii)\) The whole supply chain’s expected profit slightly increases under no information sharing, which verifies the correction of result (\(iii\)) in Proposition 6. Moreover, the total expected profit under sharing with one manufacturer is a little higher than that under sharing with both manufacturers.

7. **Conclusion.** This paper studies the issue of partial information sharing in a supply chain including two competitive manufacturers who wholesale products to one common retailer. The retailer possesses private information about uncertain market demand, and can choose to share partial information with neither, one, or both of the two manufacturers. We formulate three pricing decision models under different information sharing patterns and derive the corresponding equilibrium solutions. This work enriches the literature by investigating the effect of partial information sharing in a two-to-one supply chain.
Specifically, through the comparison and analysis of our results, we show that the shared information affects the optimal prices positively, and the positive effect when the retailer chooses to share partial information with two manufacturers is stronger than that when the retailer chooses to share partial information only with one manufacturer. Our results also show that the values of information sharing of supply chain members depend on the shared demand signal, the retailer’s private signal and the system parameters, and two manufacturers are more likely to get positive values of information sharing. Moreover, all supply chain members are more likely to obtain positive values of information sharing when the retailer chooses to share information with two manufacturers. From the perspective of expected value of information sharing, partial information sharing always benefits the informed manufacturer, but it hurts both the retailer and the whole supply chain. Furthermore, numerical analysis shows that the amount of shared information affects the informed manufacturer’s expected profit positively, and affects the expected profits of the retailer and the whole supply chain negatively, moreover, the amount of shared information does not affect the uninformed manufacturer’s expected profit; increasing competition intensity between two manufacturers will increase the expected profits of the supply chain and its members.

To sum up, our work enriches the current research in a few important aspects. Specifically, this paper considers a two-to-one supply chain, which is similar to Shang et al. [24]. However, this paper considers the problem of partial information sharing and sharing information with partial supply chain members, which extends and complements the prior work of Shang et al. [24]. In addition, although both our paper and Zhou et al. [36] consider partial information sharing, this paper considers partial information sharing under upstream competition, while Zhou et al. [36] consider partial information sharing under downstream competition. More importantly, our paper show that sharing information with one manufacturer is more beneficial to the whole supply chain than with both manufacturers, which is different from Zhou et al. [36].

Although this work enriches the literature in some aspects, the present paper still has some limitations, and some possible extensions of our results could be addressed in future research. In this paper, we assume only the retailer possesses demand forecast information. In practice, with the development of information technology, many manufacturers can also acquire imperfect demand information. Thus, for future research, it is worthwhile to consider bilateral information sharing among two manufacturers and a retailer, including vertical and horizontal information sharing. The other potential topic for future research is to analyse the effect of different game scenarios among supply chain members on information sharing, such as Stackelberg game between two manufacturers, or Nash game between two manufacturers and the retailer, which exist popularly in real life.

**Acknowledgment.** We would like to thank the editors and reviewers for their constructive suggestions and corrections to enhance the clarity of the present article.

**Appendix A.**

**Proof of Proposition 1.** Through substituting Eq. (1) into Eq. (4) and calculating the first and second orders partial derivatives to $p_i$ ($i = 1, 2$), the following
results can be obtained:
\[
\frac{\partial E[\pi_r(p_1, p_2)]}{\partial p_i} = -2p_i + 2\gamma p_j + w_i - \gamma w_j + a + E[\varepsilon\vert Y], \tag{A.1}
\]
\[
\frac{\partial^2 E[\pi_r(p_1, p_2)]}{\partial p_i^2} = -2, \quad \frac{\partial^2 E[\pi_r(p_1, p_2)]}{\partial p_i \partial p_j} = 2\gamma, \quad j = 3 - i. \tag{A.2}
\]

It follows from Eq. (A.2) that the Hessian matrix of \(E[\pi_r(p_1, p_2)\vert Y]\) is negative definite, so \(E[\pi_r(p_1, p_2)\vert Y]\) is jointly concave in \(p_1\) and \(p_2\). Setting the first-order partial derivatives be zero and solving them simultaneously, the retailer’s response functions can be obtained, i.e., Eq. (8). Proposition 1 holds.

**Proof of Proposition 2.** Through substituting Eq. (8) into Eq. (2), and calculating first and second order derivatives to \(w_i\) \((i = 1, 2)\), we have
\[
\frac{dE[\pi_m(w_i)]}{dw_i} = -w_i + \frac{\gamma}{2} w_j + \frac{a}{2} + \frac{1}{2} E[\varepsilon\vert Y], \quad j = 3 - i, \tag{A.3}
\]
\[
\frac{d^2 E[\pi_m(w_i)]}{dw_i^2} = -1 < 0. \tag{A.4}
\]

Eq. (A.4) shows that the expected profit function \(E[\pi_m(w_i)]\) is concave in \(w_i\). Setting the first-order derivatives to \(w_1\) and \(w_2\) be zero, and solving for \(w_1\) and \(w_2\) simultaneously, we have the optimal wholesale price \(w_{1}^{I*}\). Substituting \(w_{1}^{I*}\) into Eq. (8), the optimal retail price \(p_{1}^{I*}\) is derived. Thus, Proposition 2 holds.

**Proof of Proposition 3.** Substituting Eq. (8) into Eqs. (2)-(3), and calculating first and second orders derivatives to \(w_i\) and \(w_j\) respectively, we have
\[
\frac{dE[\pi_m(w_i)]}{dw_i} = -w_i + \frac{\gamma}{2} w_j + \frac{a}{2} + \frac{1}{2} E[\varepsilon\vert Y], \tag{A.5}
\]
\[
\frac{dE[\pi_m(w_j)]}{dw_j} = -w_j + \frac{\gamma}{2} E[w_i] + \frac{a}{2}, \tag{A.6}
\]
\[
\frac{d^2 E[\pi_m(w_i)]}{dw_i^2} = \frac{d^2 E[\pi_m(w_j)]}{dw_j^2} = -1 < 0. \tag{A.7}
\]

Eq. (A.7) shows that the expected profit functions \(E[\pi_m(w_i)]\) and \(E[\pi_m(w_j)]\) are concave in \(w_i\) and \(w_j\), respectively. Setting Eq. (A.5) to zero and solving for \(w_i\), we have \(w_i(w_j)\). Then, substituting \(w_i(w_j)\) into Eq. (A.6) and setting it to zero, \(w_{j}^{I-N*}\) is obtained. Substituting \(w_{j}^{I-N*}\) into \(w_i(w_j)\), \(w_{i}^{I-N*}\) is derived. Finally, substituting \(w_{i}^{I-N*}\) and \(w_{j}^{I-N*}\) into Eq. (8), we have the optimal retail prices \(p_{i}^{I-N*}\) and \(p_{j}^{I-N*}\). Thus, Proposition 3 is valid.

**Proof of Proposition 5.**

(i) \(v_{m_i}^{II} > 0 \iff E[\varepsilon\vert Y](3 - \gamma)E[\varepsilon\vert Y] - (2 - \gamma)E[\varepsilon\vert Y] + (4 - \gamma)a > 0\)
\[
\Leftrightarrow \begin{cases} 
E[\varepsilon\vert Y] > 0 \\
(3 - \gamma)E[\varepsilon\vert Y] - (2 - \gamma)E[\varepsilon\vert Y] + (4 - \gamma)a > 0
\end{cases}
\]

or 
\[
\begin{cases} 
E[\varepsilon\vert Y] < 0 \\
(3 - \gamma)E[\varepsilon\vert Y] - (2 - \gamma)E[\varepsilon\vert Y] + (4 - \gamma)a < 0
\end{cases}
\]
\( \iff \begin{cases} 
  y > 0 \\
  y > \frac{1 + \tau}{3 - \gamma \tau} \left( \frac{(2 - \gamma)t}{1 + t} Y - (4 - \gamma)a \right) 
\end{cases} \)

or

\( \iff \begin{cases} 
  y < 0 \\
  y < \frac{1 + \tau}{3 - \gamma \tau} \left( \frac{(2 - \gamma)t}{1 + t} Y - (4 - \gamma)a \right) 
\end{cases} \)

\(\iff y > \max \{0, \frac{1 + \tau}{3 - \gamma \tau} \left[ \frac{(2 - \gamma)t}{1 + t} Y - (4 - \gamma)a \right] \} \)

or

\( y < \min \{0, \frac{1 + \tau}{3 - \gamma \tau} \left[ \frac{(2 - \gamma)t}{1 + t} Y - (4 - \gamma)a \right] \}. \)

Therefore, \( v^{II}_r > 0 \) and \( v^{II}_m > 0 \) hold.

(ii) \( v^{II}_m > 0 \iff E[\varepsilon|y] [2(3 - \gamma^2)E[\varepsilon|y] - (4 - \gamma^2)E[\varepsilon|Y] + (2 + \gamma)(2 - \gamma)^2a] > 0 \)

\( \iff \begin{cases} 
  E[\varepsilon|y] > 0 \\
  2(3 - \gamma^2)E[\varepsilon|y] - (4 - \gamma^2)E[\varepsilon|Y] + (2 + \gamma)(4 - \gamma^2)a > 0 
\end{cases} \)

or

\( \iff \begin{cases} 
  E[\varepsilon|y] < 0 \\
  2(3 - \gamma^2)E[\varepsilon|y] - (4 - \gamma^2)E[\varepsilon|Y] + (2 + \gamma)(4 - \gamma^2)a < 0 
\end{cases} \)

\( \iff y > \max \{0, \frac{2(1 + \tau)}{(1 - \gamma)\tau} \left[ \frac{(2 - \gamma)t}{1 + t} Y + a \right] \} \)

or

\( y < \min \{0, \frac{2(1 + \tau)}{(1 - \gamma)\tau} \left[ \frac{(2 - \gamma)t}{1 + t} Y + a \right] \}. \)
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\[
\left\{ \begin{array}{l}
y < 0 \\
y < \frac{(4-\gamma^2)(1+\tau)}{2(4-\gamma^2)(1+\tau)} t Y - (2-\gamma)a \\
y > \max\{0, \frac{(4-\gamma^2)(1+\tau)}{2(4-\gamma^2)(1+\tau)} t Y - (2-\gamma)a\} \\
or \\
y < \min\{0, \frac{(4-\gamma^2)(1+\tau)}{2(4-\gamma^2)(1+\tau)} t Y - (2-\gamma)a\} \\
or
\end{array} \right.
\]

\[\begin{align*}
\gamma a \mathbb{E}[\varepsilon|y] > 0 &\iff y > 0 \\
v_{m_i}^{NI} > 0 &\iff \mathbb{E}[\varepsilon|y] > 0 \\
v_r^{IN} > 0 &\iff \mathbb{E}[\varepsilon|y] - (4-\gamma^2)\mathbb{E}[\varepsilon|Y] - (2+\gamma)a > 0 \\
or \\
\mathbb{E}[\varepsilon|y] - (4-\gamma^2)\mathbb{E}[\varepsilon|Y] - (2+\gamma)a > 0 &\iff y > 0 \\
\mathbb{E}[\varepsilon|y] - (4-\gamma^2)\mathbb{E}[\varepsilon|Y] - (2+\gamma)a &< 0 \\
or \\
\mathbb{E}[\varepsilon|y] - (4-\gamma^2)\mathbb{E}[\varepsilon|Y] - (2+\gamma)a &> 0 \\
\mathbb{E}[\varepsilon|y] - (4-\gamma^2)\mathbb{E}[\varepsilon|Y] - (2+\gamma)a &< 0 \\
or \\
y > \max\{0, \frac{(2+\gamma)(1+\tau)}{\tau} \frac{(2-\gamma)t}{1+t} Y + a\} \\
or \\
y < \min\{0, \frac{(2+\gamma)(1+\tau)}{\tau} \frac{(2-\gamma)t}{1+t} Y + a\}.
\end{align*}\]

Thus, \(v_{m_i}^{NI} > 0, v_{m_j}^{NI} > 0) and \(v_r^{IN} > 0) can be derived.

**Proof of Proposition 6.** From the Table 1, we have

\[
\begin{align*}
V_{m_i}^{II} - V_{m_i}^{IN} &= \gamma(4+3\gamma)\tau \left(\frac{2}{4-\gamma^2} \frac{\sigma^2}{(1+\tau)}\right) > 0, \\
V_r^{II} - V_r^{IN} &= \frac{(1+\gamma)(\gamma^2-2\gamma^2-6)\tau}{2(4-\gamma^2)(1+\tau)} \sigma^2 < 0, \\
V_{m_i}^{II} + V_{m_2}^{II} + V_r^{II} &= -\frac{(1-\gamma)\tau}{2(2-\gamma^2)(1+\tau)} \sigma^2 < 0, \\
V_{m_i}^{IN} + V_r^{IN} &= \frac{\tau}{(4-\gamma^2)(1+\tau)} \sigma^2 < 0.
\end{align*}
\]

**Proof of Proposition 7.** From the Table 1, we have

\[
\begin{align*}
1) \frac{\partial F_r}{\partial t} &= \frac{1}{2(1-\gamma)(1+t)^2} \sigma^2 > 0;
\end{align*}
\]

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\[ ii) \frac{\partial V_{m}^{II}}{\partial \tau} = \frac{1}{2(2-\gamma)^2(1+\tau)^2} \sigma^2 > 0, \quad \frac{\partial V_{m}^{IN}}{\partial \tau} = \frac{(2-\gamma^2)}{(4-\gamma^2)^2(1+\tau)^2} \sigma^2 > 0, \]
\[ \frac{\partial V_{r}^{II}}{\partial \tau} = \frac{-(3-\gamma)}{2(2-\gamma)^2(1+\tau)^2} \sigma^2 < 0, \quad \frac{\partial V_{r}^{IN}}{\partial \tau} = \frac{(3-\gamma^2)}{(4-\gamma^2)^2(1+\tau)^2} \sigma^2 < 0; \]
\[ iii) \frac{\partial V_{m}^{II}}{\partial \gamma} = \frac{\tau}{(2-\gamma)^3(1+\tau)} \sigma^2 > 0, \quad \frac{\partial V_{m}^{IN}}{\partial \gamma} = \frac{(4-\gamma^3(1+\tau))}{(4-\gamma^2)^3(1+\tau)} \sigma^2 < 0, \]
\[ \frac{\partial V_{r}^{II}}{\partial \gamma} = \frac{-(4-\gamma)\tau}{2(2-\gamma)^3(1+\tau)} \sigma^2 < 0, \quad \frac{\partial V_{r}^{IN}}{\partial \gamma} = \frac{2\gamma(2-\gamma^2)\tau}{(4-\gamma^2)^3(1+\tau)} \sigma^2 < 0; \]
\[ iv) \frac{\partial \Pi_{m}^{II^s}}{\partial \sigma} = 0, \quad \frac{\partial \Pi_{m}^{II^s}}{\partial \sigma} = \frac{\tau}{(2-\gamma)^2(1+\tau)} \sigma > 0, \quad \frac{\partial \Pi_{m}^{IN^s}}{\partial \sigma} = \frac{2(2-\gamma^2)\tau}{(4-\gamma^2)^2(1+\tau)} \sigma > 0, \]
\[ \frac{\partial \Pi_{r}^{II^s}}{\partial \sigma} = \frac{t}{(1-\gamma)(1+t)} - \frac{(3-\gamma)\tau}{(2-\gamma)^2(1+\tau)} \sigma > 0, \quad \frac{\partial \Pi_{r}^{IN^s}}{\partial \sigma} = \frac{t}{(1-\gamma)(1+t)} - \frac{2(3-\gamma^2)\tau}{(4-\gamma^2)^2(1+\tau)} \sigma > 0. \]

\[ \square \]

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Received August 2021; 1st revision November 2021; 2nd revision December 2021; early access January 2022.

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