Effects of Inelastic Scattering on Tunneling Time
in Generalized Nelson’s Quantum Mechanics

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We analyze the effects of inelastic scattering on the tunneling time theoretically, using generalized Nelson’s quantum mechanics. This generalization enables us to describe quantum system with optical potential and channel couplings in a real time stochastic approach, which seems to give us a new insight into quantum mechanics beyond Copenhagen interpretation.

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I. INTRODUCTION

An issue of the tunneling time, i.e., the time associated with the passage of a particle through a tunneling barrier, has been discussed in many theoretical studies [1–17] and is not settled yet. This difficulty arises mainly from the fact that time is not an observable represented by a self-adjoint operator but just a parameter in quantum mechanics.

In our previous paper [20], we proposed a new method to evaluate the tunneling time, using Nelson’s approach of quantum mechanics [18]. Our aim then was to treat tunneling effects in a detailed time-dependent and fully quantum mechanical way, as any theoretical expression of the tunneling time must be tested by experiments which are feasible at present and in near future.

As discussed in the reference [20], the Nelson’s approach of quantum mechanics has several advantages to study the tunneling time, a few of which are listed below.

First of all, this approach using the real time stochastic process enables us to describe individual experimental runs of a quantum system in terminology of “analogue” of classical mechanics. This is true even in the tunnel region where classical path is forbidden. From sample paths generated by the stochastic process we obtain information on the time parameter, in particular, the tunneling time.

As a matter of course, the whole ensemble of sample paths gives us all the same results as quantum mechanics in ordinary approach does, e.g., expectation values of observable, transmission and reflection probabilities in scattering problem and so on. It is important for us to note that in scattering phenomena (those without bound states) the transmission and reflection ensembles are defined unambiguously, namely each sample path is classified distinctively into either transmission ensemble or reflection one.

We need to accumulate a sufficient number of sample paths in numerical simulations. In thick or/and high potential case the transmission probability is low and consequently we have a difficulty that a number of sample paths belonging to the transmission ensemble is also low when each sample path is followed in forward time direction. However, in Nelson’s approach
there is not only the forward Langevin equation but also the backward Langevin equation (see (2) below), both being equivalent to each other in physical results. The difficulty above is avoided when the backward Langevin equation is employed.

Taking account of these advantages, we have developed a theoretical formulation of time-dependent description of tunneling phenomena based on the Nelson’s stochastic approach in [20]. Numerical simulations for a one-dimensional square well potential barrier model were demonstrated. An important result about the tunneling time then is that there are the three characteristic times, i.e., the passing time and the hesitating time, and their sum, the interacting time. The probability distribution of these three times were calculated numerically.

Our previous study treated only a quantum system of a single particle under a simple potential. But realistic experimental situations are much complicated. Naturally we are tempted to extend our previous formulation to more general scattering phenomena. In this paper we consider cases in which transition processes into other channels or absorptive processes takes place during scattering processes, and look into these effects on the tunneling time.

Processes of transition into other channels and absorption are described by channel coupling and optical potential (complex potential), respectively, in the ordinary quantum mechanics using the Schrödinger equation. So far it is known that the Nelson’s formulation is equivalent to the Schrödinger equation only for a one-body problem with a single channel and a real potential. The purpose of this paper is to generalize the Nelson’s stochastic quantization so that it can deal with multi-channel coupling and/or optical potential problems. As will be shown below, one can construct such generalized formulations of the Nelson’s approach with additional stochastic jumping processes. These theoretical formulations allow us to perform numerical simulations of stochastic processes as before [20]. This way we can investigate the effects of transition into other channels or absorption on the tunneling time.

The paper is organized as follows: In the next section the original Nelson’s quantum mechanics is reviewed briefly for later relevance. We propose a formulation of the Nelson’s approach, generalized to a quantum system with channeling-coupling in Sec. 3. The formu-
loration of Sec. 3 hints us how to develop a formulation for optical potential, which is shown in Sec. 4. In Sec. 5 numerical simulation for square well potential model, using the formulations in Sections 3 and 4, are demonstrated, and physical implications of these results are analyzed. The final section is devoted to summary and some comments.

II. BRIEF REVIEW OF NELSON’S QUANTUM MECHANICS

We start with a brief review of the original Nelson’s quantum mechanics which consists of two basic conditions, i.e., the kinematical condition and the dynamical one.

The kinematical condition is given by the Ito-type stochastic differential equation: There are two ways to express it, depending on forward or backward time direction. Explicitly we have for forward time evolution,

$$dx(t) = b(x(t), t)dt + dw(t),$$

and for backward time evolution,

$$dx(t) = b_*(x(t), t)dt + dw_*(t).$$

The $dw(t)$ is the Gaussian white noise (representing the quantum fluctuation) with the statistical properties of

$$<dw(t)> = 0 \quad \text{and} \quad <dw(t)dw(t)> = \frac{\hbar}{m} dt,$$

and the same properties for $dw_*(t)$ as in (3). Here $<\cdots>$ means a sample average. It is easy to show that for these two Langevin equations hold the following Fokker-Planck equations for the distribution function $P(x, t)$ of the random variables $x(t)$,

$$\frac{\partial P(x, t)}{\partial t} = \left[ -\frac{\partial}{\partial x} b(x, t) + \frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} \right] P(x, t) \quad (\text{forward in } t),$$

$$-\frac{\partial P(x, t)}{\partial t} = \left[ \frac{\partial}{\partial x} b_*(x, t) + \frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} \right] P(x, t) \quad (\text{backward in } t).$$

Thus a pair of equations (1) and (2) is mathematically equivalent to a pair of equations (4) and (5). We get an osmotic velocity, $u$ from the sum of Eqs. (4) and (5) as
\[ u = \frac{b - b_*}{2} = \frac{\hbar}{2mP} \frac{\partial P}{\partial x} \tag{6} \]

under the boundary condition of

\[ P(x \to \infty, t) \to 0. \tag{7} \]

The subtraction (5) from (4) gives

\[ \frac{\partial P}{\partial t} = -\frac{\partial}{\partial x}(vP) \tag{8} \]

where \( v \) is a current velocity

\[ v = \frac{b + b_*}{2}. \tag{9} \]

The elimination of \( P(x,t) \) from (6) and (8) leads to an equation called the kinematical equation,

\[ \frac{\partial u}{\partial t} = -\frac{\hbar}{2m} \frac{\partial^2 v}{\partial x^2} - \frac{\partial}{\partial x}(uv). \tag{10} \]

The dynamical condition is expressed through the “mean time derivatives” introduced as follows: The “mean forward time derivative” \( Df(t) \) is defined as

\[ Df(t) \equiv \lim_{\Delta t \to +0} \frac{f(t + \Delta t) - f(t)}{\Delta t} |f(s) (s \leq t) \text{ fixed}|, \tag{11} \]

and the “mean backward time derivative” \( D_*f(t) \) is defined as

\[ D_*f(t) \equiv \lim_{\Delta t \to +0} \frac{f(t) - f(t - \Delta t)}{\Delta t} |f(s) (s \geq t) \text{ fixed}|. \tag{12} \]

The “mean balanced acceleration” is introduced through the definitions of (11) and (12) as

\[ a(x(t), t) \equiv \frac{DD_* + D_*D}{2} x(t). \tag{13} \]

Note that this definition can be rewritten as

\[ a(x, t) = -\frac{\hbar}{2m} \frac{\partial^2 u}{\partial x^2} + \frac{1}{2} \frac{\partial}{\partial x}(v^2 - u^2) + \frac{\partial v}{\partial t}. \tag{14} \]
from (1) and (2) with (6) and (9). The dynamical condition is nothing but the classical Newton equation to this “mean balanced acceleration” \( a_x(t) \), that is,

\[
ma(x, t) = -\frac{\partial V}{\partial x},
\]

from which we derive the “Newton-Nelson equation”,

\[
\frac{\partial v}{\partial t} = \frac{\hbar}{2m} \frac{\partial^2 u}{\partial x^2} - v \frac{\partial v}{\partial x} + u \frac{\partial u}{\partial x} - \frac{1}{m} \frac{\partial V}{\partial x}
\]  

(16)

because of (14).

Summarize the mathematical structure of Nelson’s quantum mechanics. The two basic equations, (10) from the kinematical condition, and (16) from the dynamical condition, form a set of simultaneous equations for two unknown functions \( u(x, t) \) and \( v(x, t) \), or equivalently \( b(x, t) \) and \( b_*(x, t) \). Then we can determined the ensemble of sample paths or the distribution function \( P(x, t) \). Although it is practically very difficult to solve these equations directly due to their nonlinearity, one can easily show the equivalence between this approach and the ordinary approach of the Schrödinger equation. This fact tells us that one can solve the problem by means of the wave function much more easily. The equation

\[
\frac{\partial}{\partial x} \left[ i \frac{\hbar}{m} \frac{1}{\psi'} \frac{\partial \psi'}{\partial t} + \frac{1}{2} \left( \frac{\hbar}{m} \right)^2 \frac{1}{\psi'} \frac{\partial^2 \psi'}{\partial x^2} - \frac{1}{m} V \right] = 0
\]  

(17)

follows from the combination of (10)+i(16), where

\[
u + iv = \frac{\hbar}{m} \frac{1}{\psi'} \frac{\partial \psi'}{\partial x}.
\]  

(18)

Equation (17) clearly shows the relationship between \( \psi' \) and the wave function \( \psi \) as the solution of Schrödinger equation

\[
i\hbar \frac{\partial \psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V \right) \psi,
\]

(19)

namely

\[
\psi(x, t) = \psi'(x, t) \exp(-\frac{im}{\hbar} \int^t \eta(s) ds)
\]

(20)
with an arbitrary function of \( t, \eta(t) \), which has no physical relevance. It is easily seen from this proof of the equivalence that one has the expressions for \( b(x,t), b^*(x,t) \) and \( P(x,t) \) in terms of \( \psi(x,t) \),

\[
\begin{align*}
    b(x,t) &= \frac{\hbar}{m} (\text{Im} + \text{Re}) \frac{\partial}{\partial x} \ln \psi(x,t), \\
    b^*(x,t) &= \frac{\hbar}{m} (\text{Im} - \text{Re}) \frac{\partial}{\partial x} \ln \psi(x,t), \\
    P(x,t) &= |\psi(x,t)|^2.
\end{align*}
\]

(21)\( (22)\)\( (23)\)

### III. STOCHASTIC FORMULATION FOR QUANTUM SYSTEM WITH CHANNELING COUPLING

We now turn to generalize the above Nelson’s approach to a system with a channel coupling. For simplicity, consider the 2-channel Schrödinger equations (\( \{i,j\} = \{1,2\} \))

\[
\begin{align*}
    i\hbar \frac{\partial}{\partial t} \psi_i(x,t) &= \left( -\frac{\hbar^2}{2m_i} \frac{\partial^2}{\partial x^2} + V_{ii}(x,t) \right) \psi_i(x,t) + V_{ij}(x,t) \psi_j(x,t),
\end{align*}
\]

(24)

with

\[
V_{ij} = V_{ji}^*.
\]

(25)

Here and below the dummy index does not imply taking a sum. As will be seen, the generalization of the formulation in this section to \( N \)-channel case (\( N > 2 \)) is straightforward.

Consider the Fokker-Planck equations in the stochastic formulation, corresponding to (24). First we require a natural extension of (23) to the present case,

\[
\begin{align*}
    \frac{\partial P_i(x,t)}{\partial t} &= \left[ -\frac{\partial}{\partial x} b_i(x,t) + \frac{\hbar}{2m_i} \frac{\partial^2}{\partial x^2} - W(i \to j)(x,t) \right] P_i(x,t) \quad \text{(forward in time)},
\end{align*}
\]

(27)
\[-\frac{\partial P_i(x,t)}{\partial t} = \left[ \frac{\partial}{\partial x} b_{si}(x,t) + \frac{\hbar}{2m_i} \frac{\partial^2}{\partial x^2} + W_{(i\rightarrow j)}(x,t) \right] P_i(x,t) \quad \text{(backward in time), (28)}\]

as $P_i(x,t)$ increases or decreases, due to the potential $V_{ij}$ causing transitions between $i$ and $j$, at the rate of the absolute value of

$$W_{(i\rightarrow j)} P_i = -W_{(j\rightarrow i)} P_j = \frac{2}{\hbar} \text{Im} \psi_j^* V_{ji} \psi_i$$  \hspace{1cm} (29)

Although the sum of (27) and (28) leads to (6) with the index $i$,

$$u_i = \frac{b_i - b_{si}}{2} = \frac{\hbar}{2m_i} \frac{1}{P_i} \frac{\partial P_i}{\partial x},$$  \hspace{1cm} (30)

their difference provides us with

$$\frac{\partial P_i}{\partial t} = -\frac{\partial}{\partial x} (v_i P_i) - W_{(i\rightarrow j)} P_i$$  \hspace{1cm} (31)

instead of (8), where

$$v_i = \frac{b_i + b_{si}}{2}.$$  \hspace{1cm} (32)

As a result, eliminating $P_i(x,t)$ from (30) and (31), one derives the following kinematical equation

$$\frac{\partial u_i}{\partial t} = -\frac{\hbar}{2m_i} \frac{\partial^2 v_i}{\partial x^2} - \frac{\partial}{\partial x} (u_i v_i) - \frac{\hbar}{2m_i} \frac{\partial}{\partial x} W_{(i\rightarrow j)}$$  \hspace{1cm} (33)

instead of (10).

Here arises a natural question what are stochastic differential equations corresponding to the Fokker-Planck equations in (27) and (28), just as (1) and (2) correspond to (4) and (5). Apparently we need two random variables $x_i(t)$ ($i = 1, 2$), which are assumed to be subject to the stochastic differential equations, similar to (1) and (2),

$$dx_i(t) = b_i(x_i(t), t) dt + dw_i(t) \quad \text{(forward in time),} \hspace{1cm} (34)$$

$$dx_i(t) = b_{si}(x_i(t), t) dt + dw_{si}(t) \quad \text{(backward in time),} \hspace{1cm} (35)$$

with the properties for $dw_i(t)$ and $dw_{si}(t)$,
\[ \langle dw_i(t) \rangle = 0, \quad \langle dw_i(t)dw_j(t) \rangle = \frac{h}{m_i} \delta_{ij} dt \]
\[ \langle dw_{*i}(t) \rangle = 0, \quad \langle dw_{*i}(t)dw_{*j}(t) \rangle = \frac{h}{m_i} \delta_{ij} dt. \] (36)

As is easily seen, a naive interpretation of these independent stochastic differential equations leads only to the Fokker-Planck equations in (27) and (28) \textit{without the terms proportional to} \( W_{(i \to j)} \). An additional mechanism to take account of the quantum jump between \( i \) and \( j \) represented by the terms involving \( W_{(i \to j)} \) is necessary. For this purpose we supplement (34) and (35) with a stochastic jumping process between \( i \) and \( j \). Thus we attempt below the formulation for the two random variables \( x_i(t) \), subject to the stochastic differential equations (34) and (35) combined with a stochastic jumping process in the following way.

The “dynamical” rule to determine how each sample path \( x_i(t) \) changes its index (\( i = 1 \to 2 \) or vice versa) during passage of time is described by the following random jumping process (Fig. 1): At each time a dice is cast, \textit{independently of the stochastic equation (34)} and (35), and each sample path either keeps or changes its index at a certain rate. For forward time direction, we have the rule in case of \( W_{(i \to j)} > 0 \) (\( i \neq j \)),

\[
\begin{align*}
  x_i(t) & \rightarrow \begin{cases} 
x_j(t + dt) & \text{with the probability of } W_{(i \to j)}(x_i(t), t) dt, 
  x_i(t + dt) & \text{with the probability of } 1 - W_{(i \to j)}(x_i(t), t) dt \end{cases}, \\
  x_j(t) & \rightarrow x_j(t + dt) \quad \text{with the probability of } 1,
\end{align*}
\] (37)

and the rule in case of \( W_{(i \to j)} < 0 \)

\[
\begin{align*}
  x_j(t) & \rightarrow \begin{cases} 
x_i(t + dt) & \text{with the probability of } -W_{(i \to j)}(x_j(t), t) dt, 
  x_j(t + dt) & \text{with the probability of } 1 + W_{(i \to j)}(x_j(t), t) dt \end{cases}, \\
  x_i(t) & \rightarrow x_i(t + dt) \quad \text{with the probability of } 1.
\end{align*}
\] (38)

Likewise, the rules for backward time direction state that in case of \( W_{(i \to j)} > 0 \)

\[
\begin{align*}
  x_j(t) & \rightarrow \begin{cases} 
x_i(t - dt) & \text{with the probability of } W_{(i \to j)}(x_j(t), t) dt, 
  x_j(t - dt) & \text{with the probability of } 1 - W_{(i \to j)}(x_j(t), t) dt \end{cases}, \\
  x_i(t) & \rightarrow x_i(t - dt) \quad \text{with the probability of } 1,
\end{align*}
\] (39)
and in case of $W_{(i\rightarrow j)} < 0$

\[
\begin{align*}
  x_i(t) &\longrightarrow \begin{cases} 
    x_j(t - dt) & \text{with the probability of } -W_{(i\rightarrow j)}(x_i(t),t)dt, \\
    x_i(t - dt) & \text{with the probability of } 1 + W_{(i\rightarrow j)}(x_i(t),t)dt
  \end{cases}, \\
  x_j(t) &\longrightarrow x_j(t - dt) \text{ with the probability of 1.}
\end{align*}
\] (40)

According to the rules of the random jumping process above, the behavior of each sample path is illustrated as follows: For forward time direction, a sample path starts from $x_i(t_I)$, develops according to (34) with $i$ for a while, and when a chance comes, it changes its index from $i$ to $j$ and follows (34) with $j$ until the next jumping process takes place. The jumping process from $x_i$ to $x_j$ is allowed and the reverse process is forbidden where $W_{(i\rightarrow j)} > 0$, and vice versa where $W_{(i\rightarrow j)} < 0$. The jumping processes may be repeated or may not occur, depending on the sign and magnitude of $W_{(i\rightarrow j)}$. Sample paths show similar behavior for backward time direction.

It is remarked that $x_i(t)$ is generally a functional of both of $dw_1(s)$ and $dw_2(s)$ ($s < t$) (or $dw_{s1}(s)$ or $dw_{s2}(s)$ ($s > t$)) as it may repeat jumps between $i = 1$ and $i = 2$ in the past (in the future). Due to changes in the index for each sample path, there are several types of averages which are distinguished from each other carefully. It is convenient to introduce notations for conditional averages. The simple average $< \cdots >$ should be taken over both of $dw_1(s)$ and $dw_2(s)$ ($s < t$). To represent a physical average of the $i$-state at $t$, we introduce a notation of

\[
\ll f(x(t)) \gg_{\{x_i(t)\}} \equiv < f(x_i(t)) >
\] (41)

where the average on the left-handed side implies a conditional average only over sample paths, labeled by $i$ at $t$. This average should be expressed in terms of the probability distribution $P_i(x,t)$ as

\[
\ll f(x(t)) \gg_{\{x_i(t)\}} = \int dx \ f(x)\ P_i(x,t). \] (42)

The notation $\ll f(x(t)) \gg_{\{x_1(t)\}\cup\{x_2(t)\}}$ owns its trivial interpretations,
\[ \ll f(x(t)) \gg_{\{x_i(t)\} \cup \{x_j(t)\}} = \ll f(x(t)) \gg . \] (43)

Furthermore, conditional averages with different times such as \( \ll f(x(t)) \gg_{\{x_i(t+dt)\} \cap \{x_j(t)\}} \) can be introduced: This example represents the average only over the sample paths which have the index \( j \) at \( t \) and \( i \) at \( t+dt \).

Let us now evaluate the time derivative of the physical average \( \ll f(x(t)) \gg_{\{x_i(t)\}} \). For forward time direction, using appropriate conditional averages, we write down as

\[ \frac{d}{dt} \ll f(x(t)) \gg_{\{x_i(t)\}} = \frac{1}{dt} \left[ \ll f(x(t + dt)) \gg_{\{x_i(t+dt)\}} - \ll f(x(t)) \gg_{\{x_i(t)\}} \right] \]

\[ = \frac{1}{dt} \left[ \ll f(x(t + dt)) \gg_{\{x_i(t+dt)\} \cap \{x_i(t)\}} - \ll f(x(t)) \gg_{\{x_i(t+dt)\} \cap \{x_i(t)\}} - \ll f(x(t)) \gg_{\{x_j(t+dt)\} \cap \{x_i(t)\}} \right] + \ll f(x(t+dt)) \gg_{\{x_i(t+dt)\} \cap \{x_j(t)\}} \]

The three terms here are manipulated as

\[ \ll f(x(t + dt)) - f(x(t)) \gg_{\{x_i(t+dt)\} \cap \{x_i(t)\}} \]

\[ = \ll \frac{df(x)}{dx} \bigg|_{x=x(t)} dx(t) + \frac{1}{2} \frac{d^2 f(x)}{dx^2} \bigg|_{x=x(t)} (dx(t))^2 + o(dt^{3/2}) \gg_{\{x_i(t+dt)\} \cap \{x_i(t)\}} \]

\[ = \ll \frac{df(x)}{dx} b_i(x(t),t) dt + \frac{d^2 f(x)}{dx^2} \frac{h}{2m_i} dt \gg_{\{x_i(t+dt)\} \cap \{x_i(t)\}} \]

\[ = \ll \frac{df(x)}{dx} b_i(x(t),t) dt + \frac{d^2 f(x)}{dx^2} \frac{h}{2m_i} dt \gg_{\{x_i(t+dt)\} \cap \{x_j(t)\}} + o(dt^2) \]

\[ = dt \int dx \left( \frac{df(x)}{dx} b_i(x,t) + \frac{d^2 f(x)}{dx^2} \frac{h}{2m_i} \right) P_i(x,t) + o(dt^2) \]

\[ = dt \int dx f(x) \left( -\frac{\partial}{\partial x} b_i(x,t) + \frac{h}{2m_i} \frac{\partial^2}{\partial x^2} \right) P_i(x,t) + o(dt^2), \] (45)

\[ \ll f(x(t + dt)) \gg_{\{x_i(t+dt)\} \cap \{x_j(t)\}} = -dt \int dx f(x) W_{(i \rightarrow j)}(x,t) P_i(x,t) \theta(-W_{(i \rightarrow j)}(x,t)) + o(dt^2), \] (46)

and

\[ \ll f(x(t)) \gg_{\{x_j(t+dt)\} \cap \{x_i(t)\}} = dt \int dx f(x) W_{(i \rightarrow j)}(x,t) P_i(x,t) \theta(W_{(i \rightarrow j)}(x,t)) + o(dt^2), \] (47)
respectively, from (34), (36), (37), (38) and (42). Collecting (44) we obtain the correct
time evolution of (27). This shows the equivalence between (27) and the stochastic equation
(34) supplemented with the stochastic jumping process (37) and (38). Likewise one can
show the equivalence between (28) and the stochastic equation (35) supplemented with the
stochastic jumping process (39) and (40).

We need some careful treatment on the dynamical condition in the present case. For the
equivalence between the Nelson’s approach and the Schrödinger approach, the dynamical
condition is desired to have the form of

\[
\frac{\partial v_i}{\partial t} = \frac{\hbar}{2m_i} \frac{\partial^2 u_i}{\partial x^2} - v_i \frac{\partial v_i}{\partial x} + u_i \frac{\partial u_i}{\partial x} - \frac{1}{m_i} \frac{\partial \tilde{V}_{ii}}{\partial x}.
\]

(48)

Here we introduce a “quantum potential” \( \tilde{V}_{ii} \) which is to include the effect of channel coupling
as well as the usual potential \( V_{ii} \). The simplest way to achieve this equation is to define the
“mean balanced acceleration” \( a_i \) through the “mean (forward and backward) time derivative”
as usual but for the stochastic process without any jumping process. We simply consider a
stochastic process governed by (34) all the time, and denote \( X_i \) instead of \( x_i \) to distinguish
them from each other. There is no mixing of \( dw_i \) and \( dw_j \) in \( X_i \), contrary to \( x_i \). For each
\( X_i(t) \) we define the “mean balanced acceleration” \( a_i(X_i(t), t) \), and the “Newton” equations,

\[
m_i a_i(X_i(t), t) = -\frac{\partial \tilde{V}_{ii}}{\partial X_i},
\]

(49)

becomes (48).

The combination of the equations (33)+i(38) derives

\[
\frac{\partial}{\partial x}\left[ i \frac{\hbar}{m_i} \frac{1}{\psi_i'} \frac{\partial \psi_i'}{\partial t} + \frac{1}{2} \left( \frac{\hbar}{m_i} \right)^2 \frac{1}{\psi_i'} \frac{\partial^2 \psi_i'}{\partial x^2} - \frac{1}{m_i} \left\{ \tilde{V}_{ii} - \frac{i\hbar}{2} W_{i\rightarrow j} \right\} \right] = 0
\]

(50)

where the relation,

\[
u_i + iv_j = \frac{\hbar}{m_i} \frac{1}{\psi_i'} \frac{\partial \psi_i'}{\partial x},
\]

(51)

is used. If we shift the function \( \psi_i' \) to

\[
\psi_i(x, t) = \psi_i'(x, t) \exp(-im_i \int^t \eta(s) ds),
\]

(52)
choose the “quantum potential” as
\[
\tilde{V}_{ii} = V_{ii} + \text{Re} \frac{\psi_i^* V_{ij} \psi_j}{|\psi_i|^2},
\]
(53)
and use the relation
\[
W_{(i\rightarrow j)} = -\frac{2}{\hbar} \text{Im} \frac{\psi_i^* V_{ij} \psi_j}{|\psi_i|^2},
\]
(54)
we can reproduce the Schrödinger equations (24). By the use of the (51) and (52), the relations,
\[
b_i(x, t) = \frac{\hbar}{m_i} (\text{Im} + \text{Re}) \frac{\partial}{\partial x} \ln \psi_i(x, t)
\]
(55)
\[
b_{*i}(x, t) = \frac{\hbar}{m_i} (\text{Im} - \text{Re}) \frac{\partial}{\partial x} \ln \psi_i(x, t)
\]
(56)
and (26), are established again.

**IV. STOCHASTIC FORMULATION FOR QUANTUM SYSTEM OF OPTICAL POTENTIAL**

In this section, let us formulate the Nelson’s stochastic approach to a system of a single degree of freedom described by an optical potential. Then the Schrödinger equation with an imaginary part of potential, denoted by \(iU\) (a physically relevant situation, i.e., an absorptive process corresponds to \(U < 0\)), is written down as
\[
i\hbar \frac{\partial \psi(x, t)}{\partial t} = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) + iU(x, t) \right) \psi(x, t).
\]
(57)

The formulation in the previous section suggests a method to establish a stochastic formulation for this Schrödinger equation. The analogy between the channel coupling model and the present model becomes apparent when we attempt the Fokker-Planck equation corresponding to (57) in the form of
\[
\frac{\partial P(x, t)}{\partial t} = \left[ -\frac{\partial}{\partial x} b + \frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} + \frac{2U}{\hbar} \right] P(x, t) \quad \text{(forward in } t),
\]
(58)
\[
- \frac{\partial P(x, t)}{\partial t} = \left[ \frac{\partial}{\partial x} b_{*} + \frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} - \frac{2U}{\hbar} \right] P(x, t) \quad \text{(backward in } t).
\]
(59)
Equations (58) and (59) are compared with (27) and (28), then both are quite similar to each other with the correspondence between \( \frac{2U}{\bar{h}} \) and \( -W_{(i \rightarrow j)} \).

While the sum of (58) and (59) is given by (6), their difference leads to

\[
\frac{\partial P}{\partial t} = -\frac{\partial}{\partial x} (vP) + \frac{2U}{\bar{h}} P
\]  

instead of (8). From (3) and (60), follows the kinematical equation

\[
\frac{\partial u}{\partial t} = -\frac{\bar{h}}{2m} \frac{\partial^2 v}{\partial x^2} - \frac{\partial}{\partial x} (uv) + \frac{1}{m} \frac{\partial}{\partial x} U,
\]  

instead of (10).

The additional term in (58) simply describes production (absorption) effects for \( U > 0 \) (\( U < 0 \)), which one may put in such a way that the production (absorption) process is a transition from a “unphysical” sector to a “physical” one (from a “physical” sector to a “unphysical” one). At this point the analogy between the previous section and this section is helpful to find stochastic processes equivalent to the Fokker-Planck equations in (58) and (59): We consider the two random variables \( x_p(t) \) and \( x_u(t) \) for “physical” and “unphysical” sectors, respectively, and stochastic jumping between them occurs according to certain rules, which will be specified below. In contrast with the channel coupling case with the index \( i \), the stochastic differential equations for both of \( x_p(t) \) and \( x_u(t) \) can be common. Introducing a notation of a random variable \( x(t) \) standing for both of \( x_p(t) \) and \( x_u(t) \), we require the same form of stochastic differential equations for this \( x(t) \) as (1) and (2) all the time,

\[
\begin{align*}
\text{forward in time :} & \quad dx(t) = b(x(t), t)dt + dw(t) \\
\text{backward in time :} & \quad dx(t) = b_*(x(t), t)dt + dw_*(t)
\end{align*}
\]  

with the same properties for \( dw(t) \) as in(3) and so on. Each sample path is described by \( x(t) \) as a whole, but has to be classified into either \( x_p(t) \) or \( x_u(t) \) at each \( t \). Typically a sample path changes as, for example, \( x_p(t_1) \rightarrow x_u(t_2) \rightarrow \cdots \rightarrow x_p(t_n) \) as a result of repeated jumping processes. A sample path is said to be physically relevant at \( t \) if the sample is represented by \( x_p(t) \), while it is not so if it is represented by \( x_u(t) \). In other words, the
physical average at \( t \) is given by the average over ensemble of not all sample paths but only physically relevant sample paths at \( t \). The notation \( \ll f(x(t)) \gg_{\{x_p(t)\}} \) is introduced to represent this conditional average for \( f(x(t)) \). Similarly the notations of other conditional averages such as \( \ll f(x(t)) \gg_{\{x_u(t)\}} \) and \( \ll f(x(t)) \gg_{\{x_p(t)\} \cup \{x_u(t)\}} \) are clear, in particular

\[
\ll f(x(t)) \gg_{\{x_p(t)\} \cup \{x_u(t)\}} = < f(x(t) > .
\] (64)

Again conditional averages related to many times can be introduced, e.g., \( \ll f(x(t)) \gg_{\{x_p(t+dt)\} \cap \{x_u(t)\}} \) is supposed to represent the average over all the sample paths which are described by \( x_u \) at \( t \) and \( x_p \) at \( t + dt \).

Let us summarize the “dynamical” rule for stochastic jumping processes between \( p \) and \( u \). The rules are given as follows (Fig. 2): (i) For forward time direction, in case of \( U < 0 \),

\[
x_p(t) \rightarrow \begin{cases} 
  x_u(t+dt) & \text{with the probability of } -2U(x_p(t), t)/hdt, \\
  x_p(t+dt) & \text{with the probability of } 1 + 2U(x_p(t), t)/hdt 
\end{cases},
\]

\[
x_u(t) \rightarrow x_u(t+dt) \text{ with the probability of } 1,
\] (65)

and in case of \( U > 0 \),

\[
x_u(t) \rightarrow \begin{cases} 
  x_p(t+dt) & \text{with the probability of } 2U(x_u(t), t)/hdt, \\
  x_u(t) & \text{with the probability of } 1 - 2U(x_u(t), t)/hdt 
\end{cases},
\]

\[
x_p(t) \rightarrow x_p(t+dt) \text{ with the probability of } 1.
\] (66)

(ii) For backward time direction, in case of \( U < 0 \)

\[
x_u(t) \rightarrow \begin{cases} 
  x_p(t-dt) & \text{with the probability of } -2U(x_u(t), t)/hdt, \\
  x_u(t-dt) & \text{with the probability of } 1 + 2U(x_u(t), t)/hdt 
\end{cases},
\]

\[
x_p(t) \rightarrow x_p(t-dt) \text{ with the probability of } 1,
\] (67)

and in case of \( U > 0 \),

\[
x_p(t) \rightarrow \begin{cases} 
  x_u(t-dt) & \text{with the probability of } 2U(x_p(t), t)/hdt, \\
  x_p(t-dt) & \text{with the probability of } 1 - 2U(x_p(t), t)/hdt 
\end{cases},
\]

\[
x_u(t) \rightarrow x_u(t-dt) \text{ with the probability of } 1.
\] (68)
Note that for forward time direction the jumping process from $x_p$ to $x_u$ is allowed and the reverse process is forbidden where $U < 0$, and vice versa where $U > 0$, and that when $U$ is non-positive everywhere, the number of sample paths described $x_p(t)$ decreases and that in $x_u(t)$ increases as $t$ goes, the total number being conserved. Regardless of the indices of $p$ and $u$, each sample path is a stochastic process described by (62) (or (63)).

To prove the equivalence between the Fokker-Planck equation (58) and the stochastic differential equation (62) with the jumping rules (65) and (66), we calculate, for example,

$$
\frac{d}{dt} \ll f(x(t)) \gg \{x_p(t)\} = \frac{1}{dt} \left[ \ll f(x(t + dt)) \gg \{x_p(t+dt)\} - \ll f(x(t)) \gg \{x_p(t)\} \right]
$$

$$
= \frac{1}{dt} \left[ \ll f(x(t + dt)) - f(x(t)) \gg \{x_p(t+dt)\} \cap \{x_p(t)\} 
+ \ll f(x(t + dt)) \gg \{x_p(t+dt)\} \cap \{x_u(t)\} 
- \ll f(x(t)) \gg \{x_u(t+dt)\} \cap \{x_p(t)\} \right],
$$

with

$$
\ll f(x(t + dt)) - f(x(t)) \gg \{x_p(t+dt)\} \cap \{x_p(t)\} = dt \int dx f(x) \left( -\frac{\partial}{\partial x} b(x,t) + \frac{h}{2m} \frac{\partial^2}{\partial x^2} \right) P(x,t) + o(dt^2),
$$

$$
\ll f(x(t + dt)) \gg \{x_p(t+dt)\} \cap \{x_u(t)\} = dt \int dx f(x) \frac{2U(x,t)}{h} P(x,t) \theta(U(x,t)) + o(dt^2),
$$

and

$$
\ll f(x(t)) \gg \{x_u(t+dt)\} \cap \{x_p(t)\} = -dt \int dx f(x) \frac{2U(x,t)}{h} P(x,t) \theta(-U(x,t)) + o(dt^2).
$$

These equations (69)~(72) follows (58). The equivalence between the Fokker-Planck approach and the approach of the stochastic differential equation (62) with the stochastic jumping process (65) and (66) has been shown for forward direction. Similarly the equivalence between the two approaches can be proven for backward time direction.

As for the dynamical condition, we do not modify the original Nelson’s formulation. When the mean time derivatives $Df(t)$ and $D_* f(t)$ are concerned, there may be some ambiguity with respect to taking expectation. Here we will follow the argument given above.
We define the “mean balanced acceleration” through the “mean time derivatives” as usual but for the stochastic process without any jumping process. We simply consider a stochastic process governed by (62) and (63) all time. This leads to the “Newton-Nelson equation” in (16) in the present case.

The combination of the equations (61)+i(16) leads to

$$\frac{\partial}{\partial x} \left[ \frac{i}{m} \frac{1}{\psi'} \frac{\partial \psi'}{\partial t} \right] + \frac{1}{2} \left( \frac{\hbar}{m} \right)^2 \frac{1}{\psi'} \frac{\partial^2 \psi'}{\partial x^2} - \frac{1}{m} (V + iU) = 0$$

(73)

where the relation (18) is used. Again the relation between \( \psi' \) and the solution of (57) \( \psi \) is given as

$$\psi = \psi' \exp \left( -\frac{im}{\hbar} \int_{t}^{\eta} \eta(s) ds \right),$$

(74)

and

$$b(x, t) = \frac{\hbar}{m} (\text{Im} + \text{Re}) \frac{\partial}{\partial x} \ln \psi(x, t),$$

(75)

$$b_*(x, t) = \frac{\hbar}{m} (\text{Im} - \text{Re}) \frac{\partial}{\partial x} \ln \psi(x, t),$$

(76)

$$P(x, t) = |\psi(x, t)|^2.$$  

(77)

are satisfied.

V. NUMERICAL ANALYSIS

Now we can perform the numerical analysis of the effects of the optical potential and channel coupling on the tunneling time, using above generalized Nelson’s approach.

First, we discuss one-dimensional system with a static square well optical potential,

$$V(x) = \begin{cases} 0 & \text{in I} \quad (x < 0), \\ V_0 - iU_0 & \text{in II} \quad (0 < x < d), \\ 0 & \text{in III} \quad (d < x) \end{cases}$$

(78)

(Fig. 3). We set the solution of the Shrödinger equation
\[ i\hbar \frac{\partial}{\partial t} \psi(x,t) = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \psi(x,t) \]  

(79)

as

\[ \psi(x,t) = \int_{-\infty}^{\infty} A(k) \varphi_k(x) e^{-i\frac{E_k}{\hbar} t} dk \]  

(80)

with a coefficient function \( A(k) \) and \( E = \frac{\hbar^2 k^2}{2m} \). It is well-known that \( \varphi_k(x) \) is written as

\[ \varphi_k(x) = \begin{cases} 
    e^{ikx} + R_k e^{-ikx} & \text{in I,} \\
    C_k e^{\kappa x} + D_k e^{-\kappa x} & \text{in II,} \\
    T_k e^{ikx} & \text{in III,}
\end{cases} \]  

(81)

where

\[ \kappa = \sqrt{2m(V_0 - iU_0 - E)} \left/ \hbar \right. = \kappa_R - i\kappa_I \quad (\kappa_I > 0), \]  

(82)

\[ (\kappa_0 = \sqrt{2m(V_0 - iU_0 - E_0)} \left/ \hbar \right. = \kappa_{R_0} - i\kappa_{I_0} \quad (\kappa_{I_0} > 0), \]  

(83)

and \( R_k, T_k, C_k \) and \( D_k \) are given as

\[
\begin{bmatrix}
    R_k \\
    T_k \\
    C_k \\
    D_k
\end{bmatrix} = \mathcal{B}
\begin{bmatrix}
    -i(\kappa_0^2 + k^2) \sinh kd \\
    2\kappa_0 e^{-ikd} \\
    k(\kappa_0 + ik)e^{-\kappa_0 d} \\
    k(\kappa_0 - ik)e^{\kappa_0 d}
\end{bmatrix},
\]  

(84)

\[ \mathcal{B} = \frac{1}{2\kappa_0 \kappa_1 \cosh kd + i(\kappa_0^2 - k^2) \sinh kd}. \]  

(85)

We take a Gaussian form with its center at \( k = k_0 \), or

\[ A(k) = A_{k_0}(k) = C \exp \left\{ -\frac{(k_0 - k)^2}{4\sigma^2} \right\}, \]  

(86)

with a normalization constant \( C \). We put here \( \sigma = \frac{k_0}{100} \) and \( V_0 = 5E_0 = (\frac{\hbar k_0}{2m})^2 \). Using this solution, we calculate numerically \( (\mathbb{P}) \), \( (\mathbb{Q}) \) and \( (\mathbb{R}) \sim (\mathbb{S}) \).
Figure 4 shows the three typical sample paths calculated by (62) and (65) and (66). There is a sample path $x(t)$ which change it’s property from “physical” to “unphysical” in the tunnel region.

Figures 5 and 6 shows the parameter $U_0 E_0$ v.s. the average of passing time $\tau_p$, calculated by (63), (67) and (68). See the details of this “backward time evolution method” in our previous work [20]. Generally, $\tau_p$ decrease as the $U_0 E_0$ become larger. Let us estimate $\tau_p$ analytically on the W.K.B. like approximation. If we can write the wave function in the tunnel region II as

$$\psi(x,t) \sim \psi(x) \sim C' \exp(-\kappa_0 x) = \exp \left\{ -\left( \kappa_{R0} - i\kappa_{I0} \right)x \right\},$$

(87)

the drift of (63) becomes

$$b_* = \frac{\hbar}{m} (\kappa_{I0} + \kappa_{R0}) \sim \frac{h\kappa_0}{m} \left( 1 + \frac{\kappa_{I0}}{\kappa_{R0}} + o^2 \left( \frac{\kappa_{I0}}{\kappa_{R0}} \right) \right), \quad \kappa_0 = \frac{\sqrt{2m(V_0 - E_0)}}{\hbar},$$

(88)

from (76). In these cases, the “backward” time evolution of the distribution function $P_T(x,t)$, which has an “initial” distribution $\delta(x-d)$, is written as

$$\frac{\partial P_T(x,t)}{\partial t} = \left\{ \frac{\hbar}{m} \kappa_0 (1 + \frac{\kappa_{I0}}{\kappa_{R0}}) \frac{\partial}{\partial x} + \frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} - \frac{2}{h} U_0 \right\} P_T(x,t),$$

(89)

and we can get the solution of (89) easily

$$P_T(x,t) = \sqrt{\frac{2m\pi}{-\hbar t}} \exp \left\{ \frac{(x-d + \frac{\hbar\kappa_0(1 + \frac{\kappa_{I0}}{\kappa_{R0}})}{m}\frac{t}{2m} t)^2}{\frac{2m}{\hbar}t} - \frac{2U_0}{\hbar}t \right\}, \quad (t < 0).$$

(90)

There are two characteristic time intervals in this solution. One is the diffusion time $t_d \sim \frac{md}{\hbar}$ for which the distribution sizes up to the potential width $d$. The other is the current time

$$t_c \sim \frac{md}{h\kappa_0(1 + \frac{\kappa_{I0}}{\kappa_{R0}})} \sim \frac{md}{\hbar\kappa_0} \left( 1 - \frac{\kappa_{I0}}{\kappa_{R0}} \right),$$

(91)

for which the peak of the distribution moves from $x = d$ to $x = 0$. Of course, the approximation of (77) is justified when $\kappa_0 d$ is much larger than 1, and this leads us to the relation of

$$t_d \gg t_c,$$

(92)
and the time interval $t_c$ becomes the passing time in this extreme case. Note that this $t_c$ has tendency of decreasing as the $\frac{U}{E_0}$ become larger.

Second, we discuss a one-dimensional system with a static square well potential and 2-cannel coupling, or the case of the Shrödinger equation for this problem written down as

$$i \frac{\partial}{\partial t} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2m} \frac{\partial^2}{\partial x^2} + V & U \\ U & -\frac{1}{2m} \frac{\partial^2}{\partial x^2} + V \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}. \quad (93)$$

$V$ and $U$ are supposed to

$$V(x) = \begin{cases} 0 & \text{in I} \ (x < 0), \\ V_0 & \text{in II} \ (0 < x < d), \\ 0 & \text{in III} \ (d < x), \end{cases} \quad (94)$$

and

$$U(x) = \begin{cases} 0 & \text{in I} \ (x < 0), \\ U_0 & \text{in II} \ (0 < x < d), \\ 0 & \text{in III} \ (d < x). \end{cases} \quad (95)$$

Figure 7 shows the schematic illustration of our simulation. We can diagonalize (93) as

$$i \frac{\partial}{\partial t} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} = \begin{bmatrix} -\frac{1}{2m} \frac{\partial^2}{\partial x^2} + V + U & 0 \\ 0 & -\frac{1}{2m} \frac{\partial^2}{\partial x^2} + V - U \end{bmatrix} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix}. \quad (96)$$

where

$$\psi_+ = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2) \quad \text{and} \quad \psi_- = \frac{1}{\sqrt{2}}(\psi_1 - \psi_2), \quad (97)$$

and we can write down the time-dependent solution of $\psi_+$ and $\psi_-$ easily as same as (80) or

$$\psi_\pm(x,t) = \int_{-\infty}^{\infty} A(k)\varphi_\pm(x)e^{-i\frac{E}{\hbar}t}dk \quad (98)$$

with a Gaussian coefficient function $A(k)$ and $E = \frac{h^2k^2}{2m}$. $\varphi_\pm(x)$ is (81) substituted $\kappa$ for

$$\kappa_\pm = \sqrt{2m(V_0 \pm U_0 - E)} \quad (99)$$
Figure 8 shows the some typical sample paths calculated by (34), (37) and (38). There is a path which changes its index from 1 to 2 in the passage through the tunneling region. \( t_{i=1,2} \) Figs. 9 and 10 are the averages of the passing times over the sample paths which belong to \( \{x_i(t)\} \) at \( t \to \infty \). We can see in Fig. 10 that there is a critical parameter value of \( \frac{V_0 - U_0}{E_0} = 1 \) in the behavior of \( t_1 \) and \( t_2 \). This is understood as following; In the case of \( \frac{V_0 - U_0}{E_0} > 1 \), the “−” channel, which is dominant in the tunnel region II in comparison with “+” one, is not tunneling channel and it describes a particle which goes over the potential. Regardless of \( x_1(t) \) and \( x_2(t) \), the time spent in the “potential region” is expected to be agree with the one which is expected from classical mechanics or

\[
\frac{md}{\hbar k_{-0}}, \quad \text{where} \quad k_{-0} = \sqrt{\frac{2m(E_0 - V_0 + U_0)}{\hbar}}.
\]

(100)

This is seen in the Fig. 9 too. On the other hand, in the case of \( \frac{V_0 - U_0}{E_0} < 1 \), we can approximate the wave functions \( \psi_+ \) and \( \psi_- \) in the thick tunnel region to

\[
\begin{bmatrix}
\psi_+ \\
\psi_-
\end{bmatrix} \sim \begin{bmatrix}
0 \\
C_- \exp(-\kappa_{-0} x)
\end{bmatrix},
\]

(101)

and \( \psi_1 \) and \( \psi_2 \) to

\[
\begin{bmatrix}
\psi_1 \\
\psi_2
\end{bmatrix} \sim \frac{1}{\sqrt{2}} \begin{bmatrix}
C_- \exp(-\kappa_{-0} x) \\
-C_- \exp(-\kappa_{-0} x)
\end{bmatrix},
\]

(102)

where

\[
\kappa_{-0} = \sqrt{\frac{2m(V_0 - U_0 - E_0)}{\hbar}}.
\]

(103)

So we can estimate “passing time” of both channels (1 and 2) at

\[
\frac{md}{\hbar \kappa_{-0}}
\]

(104)

likewise (91).
VI. SUMMARY AND COMMENTS

In this paper, we have analyzed the effects of inelastic scattering on the tunneling time theoretically, using generalized Nelson's quantum mechanics. This generalization enables us to describe quantum systems with optical potentials and channel couplings in a real-time stochastic approach. In this formalism, the space-time development of dynamical variables, e.g., coordinate of particle, is described by a definite path determined stochastically. Each sample path has a definite form of trajectory in space-time diagram, while a physical quantity averaged over the ensemble of these sample paths recovers the effect of quantum coherence. This is true even in the Young's double slits interference experiment. Nelson's quantum mechanics gives each definite trajectory and the ensemble of it, but it does not predict which path is selected when one wants to measure the position of a particle. In this sense, this “real-time stochastic process approach” seems to give us a new insight into quantum mechanics beyond Copenhagen interpretation. On the other hand, the effects of more general cases (many-body systems, environments, temperatures, and so on) are subjects for future studies, and this work would be the first step to that study.

Recent experimental data of tunneling time using the neutron spin echo shift through magnetic films [19] seem to agree with the simulation based on our approach [21] and this study will be reported in near future.

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[1] E. P. Wigner, Phys. Rev. 98, 145 (1955).

[2] M. Büttiker and R. Landauer, Phys. Rev. Lett. 49, 1739 (1982).
[3] H. W. Lee and M. O. Scully, Found. Phys. 13, 61 (1983).

[4] M. Büttiker, Phys. Rev. B27, 6178 (1983).

[5] D. Sokolovski and L. M. Baskin, Phys. Rev. A 36, 4604 (1987).

[6] H. A. Fertig, Phys. Rev. Lett. 65, 2321 (1990).

[7] Ming-Quey Chen and M. S. Wang, Phys. Lett. A 149, 441 (1990).

[8] D. Sokolovski and J. N. L. Connor, Phys. Rev. A 47, 4677 (1993).

[9] V.S. Olkhovskiy and E. Recami, Phys. Rep. 214, 339 (1992).

[10] C. R. Leavens and G. C. Aers. In “Scanning Tunneling Microscopy III,” R. Wiesendanger and H.-J. Güntherodt, eds., Springer-Verlag, Berlin (1993).

[11] T. Martin and R. Landauer, Phys. Rev. A 47, 2023 (1993).

[12] C. R. Leavens and W. R. McKinnon, Phys. Lett. A 194, 12 (1994).

[13] S. Brouard, R. Sala, and J. G. Muga, Phys. Rev. A 49, 4312 (1994).

[14] A. M. Steinberg, Phys. Rev. Lett 74, 2405 (1995).

[15] S. Collins, D. Lowe, and J. R. Barker, J. Phys. C 20, 6213 (1987).

[16] E. H. Hauge and J. A. Støvneng, Rev. Mod. Phys. 61, 917 (1989).

[17] R. Landauer and Th. Martin, Rev. Mod. Phys. 66, 217 (1994).

[18] E. Nelson Phys. Rev. 150, 1079 (1966).

[19] M. Hino et.al “Traversal Time through Magnetic Thin Film Using larmor Precession” to be appeared in Supplement A to J. Phys. Soc. Jpn. Vol 65 (1996)

Proceedings of the International Symposium on Advance in Neutron Optics and Related Research Facilities (March 19-21, 1996 in Kumatori -NOK’96-)

[20] K. Imafuku, I. Ohba and Y. Yamanaka, Phys. Letters A 204, 329 (1995).
[21] I. Ohba, K. Imafuku and Y. Yamanaka, “Estimation of Tunneling Time Based on the Quantum Diffusion Process Approach and Neutron Scattering” to be appeared in Supplement A to J. Phys. Soc. Jpn. Vol 65 (1996)

Proceedings of the International Symposium on Advance in Neutron Optics and Related Research Facilities (March 19-21, 1996 in Kumatori -NOK’96-)
Figure caption

FIG. 1. Schematical illustration of the “dynamical” rule for stochastic jamping process between two cannels.

FIG. 2. Schematical illustration of “dynamical” rule for stochastic jamping process between physical and unphysical sector.

FIG. 3. Schematical illustration of one-dimensional optical barrier tunneling.

FIG. 4. The three typical sample paths in the optical potential case.

FIG. 5. The mean value of $\tau_p$ versus $\frac{U_0}{E_0}$ (thin potential cases).

FIG. 6. The mean value of $\tau_p$ versus $\frac{U_0}{E_0}$ (thick potential cases).

FIG. 7. Schematical illustration of one-dimensional scattering with channel coupling.

FIG. 8. The three typical sample paths with the the channel coupling.

FIG. 9. The mean values of $t_1$ and $t_2$ versus $\frac{U_0}{E_0}$ (thin potential cases).

FIG. 10. The mean values of $t_1$ and $t_2$ versus $\frac{U_0}{E_0}$ (thick potential cases).
$W_{1>2} > 0$

$W_{2>1} > 0$

$W_{1<2} < 0$

$W_{2<1} < 0$

Fig. 1
Fig. 2
FIG. 3

pre-tunnel region

$e^{ikx}$

$R_k e^{-ikx}$

reflection region

tunnel region

transmission region

$V_0 - iU_0$

$T_k e^{ikx}$

0  d
Fig. 4

- Tunnel region
- \( V_0 - iU_0 \)
- \( \tau_p \)
Fig. 5

\[ \text{time } (1/k_0^2) \]

- \( V_0/E_0 = 1.1 \)
- \( V_0/E_0 = 10 \)

path through

\[ d = 1/k_0 \]

\[ U_0/E_0 \]
Fig. 6

\[ d = \frac{10}{k_0} \]

Time \((1/k_0^2)\):

- \( V_0 / E_0 = 1.1 \)
- \( V_0 / E_0 = 10 \)

Path through

- \( U_0 / E_0 \)
Fig. 7
Fig. 8

Channel 1

Channel 2

Tunnel region

$\left( \begin{array}{cc} V & U \\ U & V \end{array} \right)$

Change from 1 to 2

$\tau_1$

$\tau_2$
Fig. 9

te\left(\frac{1}{k_0}\right)^2

\frac{md}{h k'}

\text{path through}

\frac{E_0}{V_0} = 5 \quad d = \frac{1}{k_0}

\frac{U_0}{E_0}
Fig. 10

\[ \frac{md}{h\kappa_0} \quad \frac{md}{hk'_0} \]

path through

\[ E_0/V_0 = 5 \quad d = 10/k_0 \]

\[ U_0/E_0 \]