Damage process of a fiber bundle with a strain gradient

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We study the damage process of fiber bundles in a wedge-shape geometry which ensures a constant strain gradient. To obtain the wedge geometry we consider the three-point bending of a bar, which is modelled as two rigid blocks glued together by a thin elastic interface. The interface is discretized by parallel fibers with random failure thresholds, which get elongated when the bar is bent. Analyzing the progressive damage of the system we show that the strain gradient results in a rich spectrum of novel behavior of fiber bundles. We find that for weak disorder an interface crack is formed as a continuous region of failed fibers. Ahead the crack a process zone develops which proved to shrink with increasing deformation making the crack tip sharper as the crack advances. For strong disorder, failure of the system occurs as a spatially random sequence of breakings. Damage of the fiber bundle proceeds in bursts whose size distribution shows a power law behavior with a crossover from an exponent 2.5 to 2.0 as the disorder is weakened. The size of the largest burst increases as a power law of the strength of disorder with an exponent 2/3 and saturates for strongly disordered bundles.

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I. INTRODUCTION

The damage and fracture of disordered materials is an important scientific and technological problem which has attracted an intensive research during the past years [1, 2, 3]. Theoretical and experimental studies have revealed that at the beginning of the loading process of highly disordered materials, first micro-cracks nucleate randomly, covering the entire volume of the specimen without any spatial correlations [1, 2, 3, 4]. Approaching the critical load, localization occurs resulting in a single growing crack along which the specimen fails apart. In a composite system of two solid blocks glued together along an interface, the damage usually concentrates along the weak plane of the glue [5, 6, 7, 8]. Loading such composites, interface crack propagation occurs which has also been found to be a complex sequence of crack growth and arrest with interesting spatial and temporal fluctuations [5, 6, 7, 9, 10, 11, 12]. The crackling noise accompanying the failure of disordered systems (bulk or interface cracking) can be recorded in the form of a complicated trail of signals whose analysis provides important information about the microscopic dynamics of damaging [11, 13, 14, 15, 16, 17, 18].

Fiber bundle models (FBM) are one of the most important theoretical approaches to the progressive damage of disordered materials. During the last decade FBMs have provided a deep insight into the collective nature of the microscopic dynamics and statistical properties of degradation phenomena. Recently, FBMs have also been applied to study interfacial failure of glued solid blocks under shear loading [5, 8] and wear [6, 7, 19]. Interesting novel results have been obtained on the temporal and spatial fluctuations of local breakings which precede macroscopic failure, and on the analogy of fracture with phase transitions and critical phenomena [2, 6, 7, 8, 11]. In this paper we study the damage process of a fiber bundle in a wedge-shape geometry, which provides a constant strain gradient of fibers. To obtain a simple representation of the geometry and loading conditions, we consider a bar subject to three-point bending. The bar is modelled as two rigid blocks coupled together by an elastic interface which is then discretized by a bundle of parallel fibers. Deformation and damage of the bar is concentrated in the interface resulting in a linear deformation profile of fibers, while the two blocks remain intact. Besides interfacial failure, the model provides the mean field limit of the failure of disordered materials under three-point bending. Varying the amount of disorder of fibers, we can control the strength of non-linearity before macroscopic failure, and hence, the type of fracture (brittle-quasi-brittle) of the bundle [15, 20, 21]. We focus on the progressive damage of the fiber bundle analyzing the damage profile, crack formation, and bursts of local breakings. We find that for weak disorder an interface crack is formed by a continuous region of failed fibers. Ahead the interface crack, a process zone develops which proved to shrink with increasing deformation making the crack tip sharper. For strong disorder the failure of the bundle occurs due to a spatially random sequence of local breakings. Very interestingly we find that the size distribution of bursts is a power law whose exponent shows a crossover from exponent 2.5 to 2.0 when the strength of disorder is lowered.

We demonstrate that the novel results of our model calculations are the consequence of the strain gradient; in the homogeneous case of zero gradient our model recovers all recent results of FBMs with varying threshold disorder [13, 15, 20, 21, 27, 28]. Our results imply that interfacial fracture problems can lead to novel universality classes of breakdown phenomena.

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of fibers \( l_i \) can be expressed as a function of \( \delta \) as
\[
    l_i = l_1 + 2\delta a \frac{i - 1}{b(N - 1)}, \quad i = 1, \ldots, N
\]
where \( l_1 = l_0 + 2(b - \sqrt{b^2 - \Delta^2}) \) is the length of fiber index \( i = 1 \) at the top of the bar. It follows that also the elongation \( \Delta l_i \) and longitudinal strain \( \varepsilon_i \) of fibers increase linearly as a function of their position \( i \)
\[
    \Delta l_i = 2b - 2\sqrt{b^2 - \Delta^2} + 2\delta a \frac{i - 1}{b(N - 1)},
\]
\[
    \varepsilon_i = \frac{\Delta l_i}{l_0}.
\]
Equilibrium of the system is obtained when the total momentum of forces with respect to the clamping points is zero. During the deformation process, those fibers which exceed their threshold value break, i.e. they are removed from the interface. Since \( 1 - P(\varepsilon_i(\delta)) \) is the probability that the interface element of index \( i \) remained intact under the externally imposed deformation \( \delta \), based on the equilibrium condition, the constitutive equation \( \sigma(\delta) \) of the deflected bar can be cast in the form
\[
    \sigma(\delta) = \frac{1}{NL} \sum_{i=1}^{N} \left[ \delta + \sqrt{b^2 - \Delta^2} \frac{a(i - 1)}{b(N - 1)} \right] \times [1 - P(\varepsilon_i(\delta))] E \varepsilon_i(\delta),
\]
where \( L = 2b + l_0 \) is the overall length of the bar and the sum goes over all the fibers. On the right hand side \( \varepsilon_i(\delta) \) should be substituted from Eqs. (2, 3).

The amount of disorder of the failure thresholds \( \varepsilon_i^c \) has a substantial effect on the macroscopic response of the fiber bundle \( \sigma(\delta) \). In the limiting case of zero disorder, i.e. when all the fibers have the same breaking threshold \( \varepsilon_i^c = \varepsilon^c_0 \), the failure of the bundle starts at the bottom of the interface where the stretching deformation is the highest and proceeds upwards as \( \delta \) is increased. It can be seen in Fig. 2 that the corresponding constitutive curve is sharply peaked. The critical deformation \( \delta_c \) defined by the peak position corresponds to the instant of the first fiber breaking \( \varepsilon^c_0 = \varepsilon_N(\delta_c) \). Beyond the peak stress, \( \sigma \) rapidly decreases due to the gradual breaking of fibers as \( \delta \) increases. In order to study how the behavior of the system changes when the amount of disorder of fibers is varied, we consider a uniform distribution for the breaking thresholds.

\[
    P(\varepsilon) = \frac{\varepsilon^c - (\varepsilon^c_0 - \Delta)}{2\Delta}, \quad \varepsilon^c_0 - \Delta < \varepsilon^c < \varepsilon^c_0 + \Delta.
\]

The strength of disorder of the breaking thresholds is characterized by \( \Delta \), while \( \varepsilon^c_0 \) sets the scale of fiber
FIG. 2: The constitutive curve $\sigma(\delta)$ of the interface composed of $N = 10000$ fibers varying the width of the threshold distribution $\Delta$ at the fixed value of $\varepsilon_0 = 0.01$. For $\Delta = 0$ the constitutive curve $\sigma(\delta)$ has a sharp peak which gets rounded and develops into a quadratic maximum when $\Delta$ is increased.

strength. The width $\Delta$ can be varied over the interval $[0, \varepsilon_0]$ where the limits $\Delta = 0$ and $\Delta = \varepsilon_0$ corresponds to zero disorder and the strongest disorder, respectively. Fig. 2 shows that increasing $\Delta$, the peak of the constitutive curve gets more and more rounded and develops into a quadratic maximum. The maximum of $\sigma(\delta)$ is preceded by a longer and longer non-linear regime due to the breaking of fibers, so that for $\Delta \to \varepsilon_0$ the linear behavior prevails only for small deformations $\delta \to 0$. On the microlevel this process is accompanied by the randomization of the breaking sequence of fibers along the interface, i.e. for $\Delta \neq 0$ fibers do not simply break in the decreasing order of their index $i$ (from bottom to top of the bar).

### III. SIMULATION TECHNIQUES

The complete constitutive curve of the system presented in Fig. 2 can only be recovered by deformation controlled loading. When $\delta$ is controlled externally, the local load on the fibers is solely determined by the externally imposed deformation, so that there is no load redistribution after fiber breaking. Under stress controlled conditions, the breaking of fibers is followed by the redistribution of load over the intact ones. Due to the wedge shape of the deformed interface, at a given external load $\sigma$, the load on the fibers linearly increases from top to bottom. It has the consequence that in spite of the rigidity of the two solid blocks, the load redistribution following fiber failure differs from the usual equal load sharing (ELS) approximation commonly used for the study of parallel bundles of fibers [2, 11, 20, 22, 23]. The rigid surfaces, however, ensure that the load is redistributed globally in such a way that the excess load received by an intact fiber depends on its position along the interface but not on its distance from the failed one. This implies that no stress enhancement arises in the vicinity of the failed fibers as in the case of local load sharing approximation (LLS) of fiber bundles [2, 21, 24]. Our fiber bundle model provides the mean field limit of the damage and fracture of disordered materials under three-point bending conditions and also represents an interesting interface rupture problem.

In order to analyze the microscopic damage mechanism of FBMx with a constant strain gradient, we worked out an efficient simulation technique for a sample where the interface is composed of $N$ fibers with breaking thresholds $\varepsilon_i^c$, $i = 1, \ldots, N$ sampled from the probability distribution $\delta^c_i = \delta(c, i) = \delta(c, i)$.

Performing stress controlled experiments, after the breaking of the $(i-1)$th and the $i$th fibers, the constitutive equation of the system takes the form

$$\sigma \leftarrow \frac{E}{20L} \left\{ 2\delta \left[ b - \sqrt{b^2 - \delta^2} \right] N_{\text{intact}} ight. \right.$$

$$+ \left. \left[ \frac{\delta^2}{2a} - 2ab + 2a\sqrt{b^2 - \delta^2} \right] \frac{1}{N-1} \sum_{j=1}^{N} (j-1) \right\},$$

where the prime indicates that the summation is restricted to indices of intact fibers (which are not necessarily consecutive integers). Note that in Eq. (6) the value of $\delta$ falls in the range $\delta_i^c = \delta(c, i)$.

Performing stress controlled experiments, after the breaking of a fiber the deformation of the specimen can freely change resulting in a redistribution of load over the intact fibers. The excess load taken up by the intact fibers can give rise to further fiber failures which may trigger an entire avalanche of breakings. This avalanche either stops and the bar becomes stable under the externally imposed load, or it spans the entire interface and
the specimen breaks into two pieces (the entire bundle ruptures). In order to study numerically this microscopic breaking process, in the simulations first we increase the deformation \( \delta \) such that a single fiber breaks, \( \text{i.e.} \ \delta = \delta_1 \) with index \( i_1 \). Then the load needed to maintain this deformation \( \sigma \) is calculated from Eq. (6) for \( N_{\text{intact}} = N \) fibers. After the breaking of fiber \( i_1 \) its load has to be redistributed over the remaining \( N - 1 \) fibers. In order to determine the load of intact fibers after the removal of the broken one, we remove fiber \( i_1 \) on the right hand side of Eq. (6) and invert the equation for \( \delta(\sigma) \) keeping the load \( \sigma \) fixed. The fibers with threshold values \( \delta^c_i < \delta(\sigma) \) break as a consequence of load redistribution. This iteration has to be repeated under a fixed external load \( \sigma \) until the breaking sequence stops or all the fibers break resulting in a macroscopic failure of the system.

**IV. SPATIAL EVOLUTION OF DAMAGE**

In a bundle of fibers loaded between two parallel rigid plates, due to the equal load sharing after fiber breaking, the failure of a fiber is solely determined by its breaking threshold \( \varepsilon^c_i \). Hence, fibers break in a completely random sequence without any spatial correlations. In our system, however, during the loading process the fibers break in the increasing order of their critical macroscopic deformation \( \delta^c(\varepsilon^c_i, i) \) which depends both on the local breaking thresholds \( \varepsilon^c_i \) and on the spatial position \( i \) of fibers. In the limiting case of zero disorder, \( \text{i.e.} \ \Delta = 0 \) and \( \varepsilon^c_i = \varepsilon^c_0 \) the critical deformation \( \delta^c(\varepsilon^c_i, i) \) is a monotonically decreasing function of \( i \), which implies that the fibers break one-by-one starting from the bottom. This breaking sequence can be conceived as a crack which penetrates the interface upward such that below the crack tip all the fibers are broken while above it the fibers are intact. Under strain controlled loading stable crack propagation is obtained gradually breaking the fibers by the strain increments. Controlling the external load, however, the onset of crack propagation occurs in an unstable manner resulting in immediate catastrophic failure when the maximum of \( \sigma(\delta) \) is reached (see Fig. 2).

Increasing the strength of disorder \( \Delta \), the breaking sequence of fibers determined by \( \delta^c_i \) becomes spatially randomized. At a given deformation \( \delta \), the fibers with \( \delta^c_i < \delta \) have already failed. If the disorder is not too strong, an interesting spatial distribution of these broken fibers emerges: starting from the bottom of the interface a continuous region of failed fibers develops forming a crack. On the opposite side, starting from the top of the interface a continuous region of intact, elongated fibers can be observed. The two regimes are separated by a process zone which, is a sparse sequence of intact and broken elements. To illustrate this feature, in Fig. 3 we show the probability \( P_b(i) \) that the fibers are broken along the interface for several different values of \( \Delta \) using the threshold distribution Eq. (5). The probability \( P_b(i) \) that fiber \( i \) is broken at the deflection \( \delta \) can be obtained directly from the threshold distribution \( P_b(i) = P(\varepsilon_i(\delta)) \). The process zone is defined as the regime where for the probability \( P_b \) of fiber breaking \( 0 < P_b < 1 \) holds. It can be observed in Fig. 3 that the process zone sharpens, \( \text{i.e.} \) its width decreases as the deformation \( \delta \) increases which makes the crack tip sharper as the crack advances. It can be obtained analytically that at a given deformation \( \delta \) the width \( W \) of the process zone depends both on the strength of disorder \( \Delta \) and on the geometrical extensions \( a, b \) of the specimen. For the explicit calculations, it is
worth considering separately the following three regimes of the damage profile: (A) for the breaking probability at the bottom of the interface it holds that $P_{b}(i = N) < 1$, i.e. the crack has not yet developed ($\delta_{1}$ in Fig. 3); (B) the process zone is completely contained by the interface $P_{b}(i = 1) = 0$ and $P_{b}(i = N) = 1$ ($\delta_{2}, \delta_{3}, \delta_{4}$ in Fig. 3), and (C) there is no intact region $P_{b}(i = 1) > 0$ ($\delta_{5}$ in Fig. 3). The width $W$ of the process zone can be obtained analytically as a function of $\delta$ for the three cases

\[ W = N \left[ 1 - \frac{\varepsilon_{0}^{2}}{2\delta^{2} - \delta^{2} b^{2} a^{2}} \right], \]  
(7)

\[ W = \frac{\Delta b}{\delta a} (N - 1), \]  
(8)

\[ W = \frac{\varepsilon_{0}^{2} + \Delta - 2b + 2\sqrt{b^{2} - \delta^{2} b^{2} a^{2}}}{2\delta}, \]  
(9)

We also determined the width of the process zone $W$ numerically for a system of $10^6$ fibers, which is presented in Fig. 4 together with the corresponding analytic results. In Fig. 4 the curves of $W(\delta)$ are composed of three distinct parts corresponding to the regimes (A), (B), and (C) of Eqs. (7) (9). It can be seen that for smaller values of $\Delta/\varepsilon_{0}$ first $W$ increases and reaches a maximum where the crack occurs. As the crack advances, the width of the process zone decreases according to Eq. (8) and finally, as the tip of the process zone reaches the top of the interface, $W$ rapidly decreases as given by Eq. (9). It is interesting to note that for large $\Delta$ values no crack can be identified, i.e. for the parameter set used in Fig. 4 the damage profile spans the entire interface max[$W/N]$ = 1 when $\Delta/\varepsilon_{0}$ exceeds 0.46. It follows from the above arguments that the disorder of the interface can be considered strong if the damage proceeds as a spatially random sequence of local breakings without the formation of a propagating crack. No crack can develop if at the deformation $\delta$ where the top of the interface may already be damaged $\varepsilon_{1}(\delta) > \varepsilon_{0} - \Delta$ the bottom of the interface may still be intact $\varepsilon_{N}(\delta) < \varepsilon_{0} + \Delta$. Making use of Eq. (2) and assuming $b \gg \varepsilon_{0} - \Delta$, the condition of strong disorder can be formulated as

\[ \frac{b}{a^{2}} \Delta^{2} + \Delta > \varepsilon_{0}^{2}, \]  
(10)

which implies that the average fiber strength $\varepsilon_{0}$, the width of the distribution $\Delta$ and the geometrical layout $a,b$ of the specimen together determine the relevance of disorder. At a given value of $a, b$, and $\varepsilon_{0}$, the crossover point $\Delta^{*}$ between weak and strong disorder can be obtained as

\[ \Delta^{*} \approx \frac{a^{2}}{2b} \left[ \sqrt{1 + \frac{4b\varepsilon_{0}^{2}}{a^{2} b^{2}} - 1} \right], \]  
(11)

so that for $\Delta > \Delta^{*}$ no crack is formed, while for $\Delta < \Delta^{*}$ crack propagation occurs with a shrinking process zone ahead the crack tip. For the parameter values of Fig. 3 the crossover point $\Delta^{*} = 0.4633$ was obtained from Eq. (11) in an excellent agreement with the numerical results.

V. BURSTS OF FIBER BREAKINGS

In order to characterize the damage process of the fiber bundle under stress controlled conditions, we determined the distribution of burst sizes $s$ of fiber breakings varying the width $\Delta$ of the disorder distribution. Simulations were carried out by increasing the external load to break a single fiber and following the cascading fiber breakings with the algorithm discussed in Sec. III. The avalanche size distributions $D(s)$ are presented in Fig. 5 varying the amount of disorder in a broad range. It can be observed in Fig. 5 that $D(s)$ has a power law form

\[ D(s) \sim s^{-\tau}, \]  
(12)

at any finite value of $\Delta$ with an exponential cutoff at large avalanches. Simulations revealed an interesting change of the value of the exponent $\tau$ as the strength of disorder is varied: for strong disorder the value of $\tau$ coincides with the mean field (equal load sharing) exponent of the classical parallel bundle of fibers $\tau \approx 2.5$ [13, 20]. However, as the disorder is weakened, the distribution exhibits a crossover to another power law with a lower exponent $\tau \approx 2.0$. To better illustrate this effect, in Fig. 5 burst distributions are shown separately for the limiting case of very weak disorder. The numerical results are well described by a power law with an exponent 2.0.

It is interesting to note that the size of the largest burst $s_{max}$ has a strong dependence on the value of $\Delta$ (see Fig. 5(b)), namely, as the strength of disorder is reduced the largest avalanche decreases. To obtain a quantitative characterization of this effect, Fig. 5 presents the average size of the largest bursts $\langle s_{max} \rangle$ as a function of $\Delta$, where a power law dependence is evidenced

\[ \langle s_{max} \rangle \sim \Delta^{\alpha}, \]  
(13)

for the case of weak disorder $\Delta < \Delta^{*}$. As $\Delta$ exceeds $\Delta^{*}$, the largest avalanche $s_{max}$ reaches a maximum and levels
off (see Fig. 6). Computer simulations revealed that the exponent $\alpha$ has a universal value $\alpha = 2/3$ indicated by the straight line drawn in Fig. 6 to guide the eye. The value of $\Delta$ where $\langle s_{\text{max}} \rangle$ saturates is in a good agreement with the corresponding value of $\Delta^*$ estimated from Eq. (11).

It is important to emphasize that in our system no critical disorder distribution can be identified in the sense defined in Refs. [15, 21, 27, 28, 29, 30]. For equal load sharing fiber bundles the threshold distribution is considered to be critical if the breaking of the weakest fiber gives rise to an immediate macroscopic failure of the system. For uniformly distributed threshold values in the interval $[x_0, 1]$ the distribution becomes critical for $x_0 \rightarrow 0.5$ (21, 27, 28, 29, 30). It has been pointed out that approaching the critical disorder, the macroscopic response of the system becomes perfectly brittle and on the micro-level the size distribution of bursts exhibits a crossover from a power law of exponent 2.5 to a significantly lower value (21, 27, 28, 29) such that for $\Delta < \Delta_c$ a single fiber breaking triggers a catastrophic avalanche. The corresponding numerical results are presented in Fig. 7(a), where it can be seen that approaching $\Delta_c$ from above, the burst size distribution exhibits a crossover from the exponent $\tau = 5/2$ to a lower value $\tau = 3/2$ in agreement with the predictions of Refs. [15, 27, 28]. The deviation of $\Delta_c$ from $\Delta_c^*/3$ arises due to the non-linear terms of $\delta$ in Eqs. (21, 27, 28). It can also be observed in the figure that in the homogeneous case the largest burst $s_{\text{max}}$ increases as criticality is approached in agreement with Ref. [21].

However, at any finite value of $a$, i.e. in the presence of a strain gradient, the picture drastically changes: criticality occurs solely in the limit $\Delta \rightarrow 0$ so that catastrophic failure is always preceded by avalanches with a power law distribution but the cutoff avalanche size goes to zero as a power law of $\Delta$ when $\Delta$ is decreased. Fig. 7(b) presents simulation results obtained with $10^6$ fibers at the value $a = 0.01$, which results in a much lower strain gradient than in Fig. 5. Comparing the burst size distributions of different values of $\Delta$ for the homogeneous (Fig. 7(a)) and inhomogeneous (Fig. 7(b)) cases, a clear difference can be observed. The higher value of the crossover exponent $\tau = 2$ compared to $\tau = 3/2$ of the homogeneous case shows that in the absence of a finite critical disorder, the large avalanches are less dominating in the distributions $D(s)$.

VI. DISCUSSION

We presented a detailed study of the progressive damage and fracture of fiber bundles in a wedge-shape geometry which provides a linear deformation profile for fibers. For a simple representation of the geometrical and loading conditions of the system, we considered a bar sub-
ject to three point bending. The bar is composed of two rigid blocks coupled by an elastic interface which is then discretized in terms of parallel fibers. We showed that in the limit of zero disorder of fibers’ strength the bundle has a perfectly brittle macroscopic response, i.e., under stress controlled loading global failure occurs as a sudden collapse of the system without any precursory activity, furthermore, fibers break in a completely ordered sequence from bottom to top of the wedge creating an unstable crack with a sharp tip. The relevance of disorder is determined together by the parameters (mean and width) of the strength distribution of fibers and by the geometrical layout of the wedge. We demonstrated that a propagating interface crack can only be defined for weak disorder. Ahead the crack a process zone is formed whose width decreases with increasing deformation making the crack tip sharper as the crack advances. For strong disorder a spatially random sequence of local breakings occur along the entire bundle.

The breaking of single fibers can trigger cascades of breaking events. The size distribution of these bursts is found to be a power law with an interesting crossover effect as the strength of disorder is varied: for strong disorder the mean field exponent \( \tau = 5/2 \) of equal load sharing fiber bundles is recovered indicating the complete randomness of the failure process. However, for weak disorder where a propagating crack with a process zone develops, a lower exponent \( \tau = 2.0 \) is obtained. In the weak disorder regime the largest burst increases as a power law of the width of the disorder distribution with an exponent \( \alpha = 2/3 \). We showed that in the limit of zero strain gradient our calculations reproduce the crossover of burst exponents from 5/2 to 3/2 predicted recently. The novel features of our system originate from the finite strain gradient, i.e., from the geometrical constraints of fibers which naturally occur, for instance, at interfacial fracture problems. Our model provides also the mean field limit of the damage and fracture of disordered materials under three-point bending. The results imply that the statistical physics of interfacial rupture can reveal novel universality classes of breakdown phenomena.

Recently, it was found experimentally that the cracking of a bar under three point bending proceeds in bursts which are characterized by power law distributions \[31\]. The experiments showed that the exponents of the amplitude, area and energy distribution of magnetic emission signals recorded during the fracture process of ferromagnetic materials are sensitive to the type of fracture, i.e., the noise spectra of ductile materials are characterized by higher exponents than the brittle ones. The boundary and loading conditions ensured in the experiments that the damage localizes to a relatively thin layer of the specimen giving rise to a single growing crack so that the crackling noise measured during the loading process characterizes the crack propagation. Note that in our model the crossover to a lower exponent of burst sizes with decreasing disorder is accompanied on the macro-level by an increasing degree of brittleness showing that this simple mean field approach can qualitatively account for the changing properties of crackling noise observed experimentally \[31\].

Acknowledgments

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