New mechanism for Type-II seesaw dominance in SO(10) with TeV scale $Z'$ and other verifiable predictions

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Abstract

Dominance of type-II seesaw mechanism for neutrino masses has attracted considerable attention because of a number of advantages. We show a novel approach to achieve Type-II seesaw dominance in non-supersymmetric $SO(10)$ grand unification where a low mass $Z'$ boson and specific patterns of right-handed neutrino masses are predicted within the accessible energy range of the Large Hadron Collider. In spite of the high value of the seesaw scale, $M_{\Delta_L} \simeq 10^8 - 10^9$ GeV, the model predicts new dominant contributions to neutrino-less double beta decay in the $W_L - W_L$ channel close to the current experimental limits via exchanges of heavier singlet fermions used as essential ingredients of this model even when the light active neutrino masses are normally hierarchical or invertedly hierarchical. We obtain upper bounds on the lightest sterile neutrino mass $m_s \lesssim 3.0$ GeV, 2.0 GeV, and 0.7 GeV for normally hierarchical, invertedly hierarchical and quasi-degenerate patterns of light neutrino masses, respectively. The underlying non-unitarity effects lead to lepton flavor violating decay branching ratios within the reach of ongoing or planned experiments and the leptonic CP-violation parameter nearly two order larger than the quark sector. Some of the predicted values on proton lifetime for $p \to e^+\pi^0$ are found to be within the currently accessible search limits.

Keywords: Beyond standard model, neutrino masses and mixing, lepton number and lepton flavor violations, grand unified theory
1 INTRODUCTION

Experimental evidences on tiny neutrino masses and their large mixings have attracted considerable attention as physics beyond the standard model (SM) leading to different mechanisms for neutrino mass generation. Most of these models are based upon the underlying assumption that neutrinos are Majorana fermions that may manifest in the detection of events in neutrino-less double beta decay ($0\nu\beta\beta$) decay experiments on which a number of investigations are in progress [1, 2, 3].

Theories of neutrino masses and mixings are placed on a much stronger footing if they originate from left-right symmetric (LRS) [8, 9] grand unified theories such as $SO(10)$ where, besides grand unification of three forces of nature, P (=parity) and CP-violations have spontaneous-breaking origins, the fermion masses of all the three generations are adequately fitted [10], all the 15 fermions plus the right-handed neutrino ($N$) are unified into a single spinorial representation $16$ and the canonical ($\equiv$ type-I) seesaw formula for neutrino masses is predicted by the theory. Although type-I seesaw formula was also proposed by using extensions of the SM (SM) [5, 6], it is well known that this was advanced even much before the atmospheric neutrino oscillation data [7] and it is interesting to note that Gell-Mann, Ramond and Slansky had used the left-right symmetric $S(10)$ theory and its Higgs representations $10_H, 126_H$ to derive it. A special feature of left-right (LR) gauge theories and $SO(10)$ grand unification is that the canonical seesaw formula for neutrino masses is always accompanied by type-II seesaw formula for Majorana neutrino mass matrix

$$M_\nu = m_{\nu}^{II} + m_{\nu}^I,$$

$$m_{\nu}^I = -M_D^{-1}M_N^{-T},$$

$$m_{\nu}^{II} = f v_L$$

where $M_D(M_N)$ is Dirac (RH-Majorana) neutrino mass, $v_L$ is the induced vacuum expectation value (VEV) of the left-handed (LH) triplet $\Delta_L$, and $f$ is the Yukawa coupling of the triplet. Normally, because of the underlying quark-lepton symmetry in $SO(10)$, $M_D$ is of the same order as $M_u$, the up-quark mass matrix, that drives the seesaw scale to be large, $M_N \geq 10^{11}$ GeV.
Similarly the type-II seesaw scale is also large. With such high seesaw scales, these two mechanisms in SO(10) cannot be directly verified at low energies or by the Large hadron collider except for the indirect signature through the light active neutrino mediated $0\nu\beta\beta$-decay and possibly leptogenesis.

It is well known that the theoretical predictions of branching ratios for LFV decays such as $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, and $\tau \rightarrow e\gamma$ and $\mu \rightarrow e\bar{e}e$ closer to their experimental limits are generic features of SUSY GUTs even with high seesaw scales but, in non-SUSY models with such seesaw scales, they are far below the experimental limits. Recently they have been also predicted to be experimentally accessible along with low-mass $W_R, Z_R$ bosons through TeV scale gauged inverse seesaw mechanism [14] in SUSY SO(10). In the absence of any evidence of supersymmetry so far, alternative non-SUSY SO(10) models have been found with predictions of substantial LFV decays and TeV scale $Z'$ bosons with inverse seesaw dominance [16] or with the predictions of low-mass $W_R, Z_R$ bosons, LFV decays, observable neutron oscillations, and dominant LNV decay in the $W_L - W_L$ channel via extended seesaw mechanism [17]. Another feature of non-SUSY SO(10) is rare kaon decay and neutron-antineutron oscillation which is discussed in a recent work [19] with inverse seesaw mechanism of light neutrino masses. The viability of the model of ref.[14] depends on discovery of TeV scale SUSY, TeV scale $W_R, Z_R$ bosons, and TeV scale pseudo-Dirac neutrinos which are almost degenerate in masses. The viability of the non-SUSY model of ref.[16] depends on the discovery TeV scale low-mass $Z_R$ boson and partially degenerate pseudo Dirac neutrinos in the range 100 – 1200 GeV. Both types of model predict proton lifetime within the Super-K search limit. The falsifiability of the non-SUSY model of ref.[19] depends upon any one of the following predicted observables: TeV scale $Z_R$ boson, dominant neutrino-less double beta decay, heavy Majorana type sterile and right-handed neutrinos, neutron oscillation, and rare kaon decays. Whereas the neutrino mass generation mechanism in all these models is through gauged inverse seesaw mechanism, our main thrust in the present work is type-II seesaw.

A key ansatz to resolve the issue of large mixing in the neutrino sector and small mixing in the quark sector has been suggested to be through type-II seesaw dominance [20] via renormalisation group evolution of quasi-degenerate

* Possibility of LHC accessible low-mass $Z'$ has been also investigated recently in the context of heterotic string models [18]
neutrino masses that holds in supersymmetric quark-lepton unified theories \[8\] or \(SO(10)\) and for large values of \(\tan \beta\) which represents the ratio of vacuum expectation values (VEVs) of up-type and down type doublets. In an interesting approach to understand neutrino mixing in SUSY theories, it has been shown \[21\] that the maximality of atmospheric neutrino mixing is an automatic consequence of type-II seesaw dominance and \(b - \tau\) unification that does not require quasi-degeneracy of the associated neutrino masses. A number of consequences of this approach have been explored to explain all the fermion masses and mixings including type-II seesaw, or a combination of both type-I seesaw and type-II \[22, 23\] through SUSY \(SO(10)\). As a further interesting property of type-II seesaw dominance, it has been recently shown \[24\] without using any flavor symmetry that the well known tri-bimaximal mixing pattern for neutrino mixings is simply a consequence of rotation in the flavor space. Although several models of Type-II seesaw dominance in SUSY \(SO(10)\) have been investigated, precision gauge coupling unification is distorted in most cases because of the following reason.\[†\]: fermion mass fitting in the conventional one-step breaking of SUSY GUTs, including fits to the neutrino oscillation data require the left-handed triplet to be lighter than the type-I seesaw scale. The gauge coupling evolutions being sensitive to the quantum numbers of the LH triplet \(\Delta_L(3, -2, 1)\) under SM gauge group, act to misalign the precision unification in the minimal scenario achieved without the triplet being lighter.

Two kinds of \(SO(10)\) models have been suggested for ensuring precision gauge coupling unification in the presence of type-II seesaw dominance. In the first type of SUSY model \[25\], \(SO(10)\) breaks at a very high scale \(M_{10} \geq 10^{17} \text{GeV}\) to SUSY \(SU(5)\) which further breaks to MSSM at the usual SUSY GUT scale \(M_U \sim 2 \times 10^{16} \text{GeV}\). Type-II seesaw dominance is achieved by fine tuning the mass of the full \(SU(5)\) multiplet \(15_H\) containing the \(\Delta_L(3, -2, 1)\) to remain at the desired type-II scale \(M_{\Delta_L} = 10^{11} - 10^{13} \text{GeV}\). Since the full multiplet \(15_H\) is at the intermediate scale, although the paths of the three gauge couplings of the MSSM gauge group deflect from their original paths for \(\mu > M_{\Delta_L}\), they converge exactly at the same scale \(M_U\) as the MSSM unification scale but with a slightly larger value of the GUT cou-

\[†\] A brief review of different SUSY \(SO(10)\) models requiring type-II seesaw or an admixture of type-I and type-II for fitting fermion masses is given in ref. \[24\] and a brief review of distortion occurring to precision gauge coupling unification is given in ref. \[26\]
pling leading to a marginal reduction of proton-lifetime prediction compared to SUSY $SU(5)$. In the second class of models applicable to a non-SUSY or a split-SUSY case, the grand unification group $SO(10)$ breaks directly to the SM gauge symmetry at the GUT-scale and by tuning the full $SU(5)$ scalar multiplet $15_H$ to have degenerate masses at $M_{\Delta L} = 10^{11} - 10^{13}$ GeV, type-II seesaw dominance is achieved. The question of precision unification is answered in this model by pulling out all the super-partner scalar components of the MSSM but by keeping all the fermionic superpartners and the two Higgs doublets near the TeV scale. In the non-SUSY case the TeV scale fermions can be equivalently replaced by complex scalars carrying the same quantum numbers. The proton lifetime prediction $\tau_P(p \rightarrow e + \pi^0) \simeq 10^{35}$ Yrs. in this model.

In the context of LR gauge theory, type-II seesaw mechanism was originally proposed with manifest left right symmetric gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times D (g_{2L} = g_{2R}) (\equiv G_{2213D})$ where both the left- and the right-handed triplets are allowed to have the same mass scale as the LR symmetry breaking (or the Parity breaking) scale $[13]$. With the emergence of D-Parity and its breaking leading to decoupling of Parity and $SU(2)_R$ breakings $[27]$, a new class of asymmetric LR gauge group emerged: $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C (g_{2L} \neq g_{2R}) (\equiv G_{2213})$ where the left-handed triplet acquired larger mass than the RH triplet leading to the type-I seesaw dominance and suppression of type-II seesaw in $SO(10)$ $[29]$. It is possible to accommodate both types of intermediate symmetries in non-SUSY $SO(10)$, but these models make negligible predictions for branching ratios of charged LFV processes and they leave no other experimental signatures to be verifiable at low or LHC energies.

The purpose of this work is to show that in a class of models descending from non-SUSY $SO(10)$ or from Pati-Salam gauge symmetry, type-II seesaw dominance at intermediate scales ($M_{\Delta} \simeq 10^8 - 10^9$ GeV) and $M_N \sim O(1)$ TeV can be realised by cancellation of the type-I seesaw contribution but with a TeV scale $Z'$ boson accessible to the large Hadron Collider (LHC) where $U(1)_R \times U(1)_{B-L}$ breaks spontaneously to $U(1)_Y$ through the vacuum expectation value (VEV) of the RH triplet component of Higgs scalar contained in $126_H$ that carries $B - L = -2$. We also discuss how the type-II seesaw contribution dominates over the linear seesaw formula. Whereas in all previous Type-II seesaw dominance models in $SO(10)$, the RH Majorana
neutrino masses have been very large and inaccessible for accelerator energies, the present model predicts these masses in the LHC accessible range. In spite of large values of the $W_R$ boson and the doubly charged Higgs boson $\Delta^{++}_L, \Delta^{++}_R$ masses, it is quite interesting to note that the model predicts a new observable contribution to $0\nu\beta\beta$ decay in the $W_L - W_L$ channel. The key ingredients to achieve type-II seesaw dominance by complete suppression of type-I seesaw contribution are addition of one $SO(10)$ singlet fermion per generation ($S_i, i = 1, 2, 3$) and utilization of the additional Higgs representation $16_H$ to generate the $N - S$ mixing term in the Lagrangian through Higgs-Yukawa interaction. The underlying leptonic non-unitarity effects lead to substantial LFV decay branching ratios and leptonic CP-violation accessible to ongoing search experiments. We derive a new formula for the half-life of $0\nu\beta\beta$ decay as a function of the fermion singlet masses and extract lower bound on the lightest sterile neutrino mass from the existing lower bounds on half-life of different experimental groups. For certain regions of parameter space of the model, we also find the proton lifetime for $p \rightarrow e^+\pi^0$ to be accessible to ongoing or planned experiments.

This paper is organized as follows. In Sec.2, we give an outline of the model and discuss gauge coupling unification where we also discuss proton lifetime predictions. In Sec.3 we derive type-II seesaw dominance formula and show how the model predicts RH neutrino masses from fits to the neutrino oscillation data. In Sec.4 we discuss the derivation of Dirac neutrino mass matrix from the GUT scale fit to fermion masses. In Sec.5, we discuss prediction on lepton flavour violation and leptonic CP violation due to the underlying non-unitarity effects. In Sec.6 we discuss analytic derivation of amplitudes on lepton number violation. In Sec.7 we discuss predictions on effective mass parameters and half life for $0\nu\beta\beta$ where we also obtain the singlet fermion mass bounds. We summarize and conclude our results in Sec.8.

2 UNIFICATION WITH TeV SCALE Z’

In this section we devise two symmetry breaking chains of non-SUSY $SO(10)$ theory, one with LR symmetric gauge theory with unbroken D-Parity and another without D-Parity at the intermediate scale. In the subsequent sections we will compare the ability of the two models to accommodate type-II seesaw dominance to distinguish one from the other. As necessary require-
ments, we introduce one $SO(10)$-singlet per generation ($S_i, i = 1, 2, 3$) and Higgs representations $126_H$ and $16_H$ in both the models.

2.1 Models from $SO(10)$ symmetry breaking

Different steps of symmetry breaking is given below for two models.

Model-I

\[
SO(10) \xrightarrow{(M_U = M_P)} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \ [G_{2213}]
\]

\[
\xrightarrow{(M_R^+)} SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \ [G_{2113}]
\]

\[
\xrightarrow{(M_R^0)} SU(2)_L \times U(1)_Y \times SU(3)_C \ [SM]
\]

\[
\xrightarrow{(M_Z)} SU(3)_C \times U(1)_Q
\]

Model-II

\[
SO(10) \xrightarrow{(M_U)} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \ [G_{2213D}]
\]

\[
\xrightarrow{(M_R^+ = M_P)} SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \ [G_{2113}]
\]

\[
\xrightarrow{(M_R^0)} SU(2)_L \times U(1)_Y \times SU(3)_C \ [SM]
\]

\[
\xrightarrow{(M_Z)} SU(3)_C \times U(1)_Q
\]

In the Model-II, $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times D \ [\equiv G_{2213D}](g_{2L} = g_{2R})$ is obtained by breaking the GUT-symmetry and by giving vacuum expectation value (VEV) to the D-Parity even singlet $(1, 1, 0, 1) \subset (1, 1, 15) \subset 210_H$ \cite{27, 28} where the first, second, and the third set of quantum numbers of the scalar components are under $G_{2213P}$, the Pati-Salam symmetry $G_{224}$, and $SO(10)$, respectively. As a result, the Higgs sector is symmetric below $\mu = M_U$ leading to equality between the gauge couplings $g_{2L}(M_R^+)$ and $g_{2R}(M_R^+)$ . In this case the LR discrete symmetry (\equiv Parity) survives down to the intermediate scale $M_{R+} = M_P$. The second step of symmetry breaking is implemented by assigning VEV to the neutral component of the right-handed (RH) Higgs triplet $\sigma_R(1, 3, 0, 1) \subset 45_H$ that carries $B - L = 0$. The third step of breaking to SM is carried out by assigning the order $5 - 10$ TeV VEV to the $G_{2113}$ component $\Delta_R^0(1, 1, -2, 1)$ contained in the RH triplet $\Delta_R(1, 3, -2, 1) \subset 126_H$ carrying $B - L = -2$. This is responsible for RH Majorana neutrino mass generation $M_N = fv_R$ where $v_R = \langle \Delta_R^0 \rangle$ and $f$
is the Yukawa coupling of $126^\dagger$ with $SO(10)$ spinorial fermionic representation :$f_{16.16.126}^\dagger_H$. We introduce $SO(10)$ invariant $N - S$ mixing mass via $y_\chi_{16.16.126}^\dagger$ and obtain the mixing mass $M = y_\chi V_\chi$ where $V_\chi = \langle \chi_R^0 \rangle$ by noting that under $G_{2113}$ the submultiplet $\chi_R^0 (1, 1/2, -1, 1)$ is contained in the $G_{2213}$ doublet $\chi_R (1, 2, -1, 1) \subset 16_H$. The symmetry breaking in the last step is implemented through the SM Higgs doublet contained in the bidoublet $\phi(2, 2, 0, 1) \subset 10_H$ of $SO(10)$. This is the minimal Higgs structure of the model, although we will utilise two different Higgs doublets $\phi_u \subset 10_H$ and $\phi_d \subset 10_H$ for fermion mass fits.

In Model-I, the GUT symmetry breaks to LR gauge symmetry $G_{2213}(g_2L \neq g_2R)$ in such a way that the D-parity breaks at the GUT scale and is decoupled from $SU(2)_R$ breaking that occurs at the intermediate scale. This is achieved by giving GUT scale VEV to the D-parity odd singlet scalar component in $(1, 1, 0, 1)_H \subset (1, 1, 15)_H \subset 45_H$ where the first, second, and third submultiplets are under $G_{2213}$, the Pati-Salam symmetry $G_{224}$, and $SO(10)$, respectively. In this case by adopting the D-Parity breaking mechanism [27] in $SO(10)$ normally the LH triplet component $\Delta_L (3, 1, -2, 1) \subset 126_H$ and the LH doublet component $\chi_L (2, 1, -1, 1) \subset 16_H$ acquire masses at the GUT scale while the RH triplet and RH doublet components, $\Delta_R (1, 3, -2, 1) \subset 126_H$ $\chi_R (1, 2, -1, 1) \subset 16_H$, can be made much lighter. We have noted that in the presence of color octet at lower scales, found to be necessary in this Model-I as well as in Model-II, precision gauge coupling is achieved even if the the parameters of the Higgs potential are tuned so as to have the LH triplet mass at intermediate scale, $M_{\Delta_L} \simeq 10^8 - 10^9$ GeV. The presence of $\Delta_L (3, 1, -2, 1)$ at the intermediate scale plays a crucial role in achieving Type-II seesaw dominance as would be explained in the following section. The necessary presence of lighter LH triplets in GUTs with or without vanishing $B-L$ value for physically appealing predictions was pointed out earlier in achieving observable matter anti-matter oscillations [30], in the context of low-scale leptogenesis [31], and type-II seesaw dominance in SUSY, non-SUSY and split-SUSY models [25, 26], and also for TeV scale LR gauge theory originating from SUSY $SO(10)$ grand unification [14].

2.2 Renormalisation group solutions to mass scales

In this section while safeguarding precise unification of gauge couplings at the GUT scale, we discuss allowed solutions of renormalisation group equations for the mass scales $M_U, M_{R^+}$, and $M_{R^0}$ as a function of the mass $M_C$.
of the lighter color octet $C_8(1,1,0,8) \subset 45_H$. The renormalisation group coefficients for the minimal cases have been given in Appendix A to which those due to the color octet scalar in both models and the LH triplet $\Delta_L$ in Model-I in their suitable ranges of the running scale have been added.

**Model-I:**

As shown in Table 1 for Model-I, with $M_{\Delta_L} = 10^8$ GeV the $G_{2213}$ symmetry is found to survive down to $M_{R^+} = (10^8 - 10^{10})$ GeV with larger or smaller unification scale depending upon the color octet mass. In particular we note one set of solutions,

\[ M_{R^0} = 10 \text{ TeV}, \quad M_{R^+} = 10^{9.7}\text{GeV}, \quad M_U = 10^{15.62}\text{GeV}, \]
\[ M_{\Delta_L} = 10^8 \text{ GeV}, \quad M_C = 10^{10.9}\text{GeV}. \]  

As explained in the following sections, this set of solutions are found to be attractive both from the prospects of achieving type-II seesaw dominance and detecting proton decay at Hyper-Kamiokande. With $M_U = 6.5 \times 10^{15}$ GeV when the color octet mass is at $M_C \sim 10^{11}$ GeV. As discussed below the proton lifetime in this case is closer to the current experimental limit. With allowed values of $M_{R^0} = (5 - 10) \text{ TeV}$, this model also predicts $M_{Z'} \simeq (1.2 - 3.5) \text{ TeV}$ in the accessible range of the Large Hadron Collider. As discussed in the following Sec.3 because of the low mass of the $Z'$ boson associated with TeV scale VEV of $V_R$, the type-II seesaw mechanism predicts TeV scale RH neutrino masses with known mixings among them. These RH neutrinos can be testified at the LHC or future high energy accelerators.

| $M_{R^0}$ (TeV) | $M_C$ (GeV) | $M_{R^+}$ (GeV) | $M_G$ (GeV) | $\alpha_G^{-1}$ | $\tau_p$ (yrs.) |
|----------------|-----------|-----------------|-------------|----------------|----------------|
| 10             | $10^{4.5}$| $10^9$          | $10^{16.91}$| 41.14          | $5.4 \times 10^{39}$ |
| 10             | $10^{9}$  | $10^{8.9}$      | $10^{16.74}$| 41.41          | $1.12 \times 10^{39}$ |
| 10             | $10^9$    | $10^{9}$        | $10^{16.73}$| 41.73          | $8.38 \times 10^{37}$ |
| 10             | $10^{10.9}$| $10^{9.7}$      | $10^{15.63}$| 41.93          | $3.2 \times 10^{34}$ |
| 5              | $10^{7.8}$| $10^{8.8}$      | $10^{16.4}$ | 41.54          | $9 \times 10^{37}$  |

Table 1: Allowed mass scales as solutions of renormalisation group equations for gauge couplings for Model-I with fixed value of the LH triplet mass $M_{\Delta} = 10^8$ GeV,

The RG evolution of gauge couplings for the set of mass scales given in eq. (4) is presented in Fig. showing clearly the unification of the four gauge couplings of the $G_{2213}$ intermediate gauge symmetry.
Figure 1: Two loop gauge coupling unification in the $SO(10)$ symmetry breaking chain with $M_U = 10^{15.62}$ GeV and $M_R^+ = 10^{9.7}$, $M_{ΔL} = 10^8$GeV with a low mass $Z'$ boson at $M_{R}^0 = 10$ TeV for Model-I.
Model-II:
As shown in Table 2 for Model-II, the $G_{213D}$ symmetry is found to survive down to $M_{R^+} = M_P = 10^{8.2}$ GeV with $M_U = 6.5 \times 10^{15}$ GeV when the color octet mass is at $M_C = 10^8$ GeV. As discussed below the proton lifetime in this case is closer to the current experimental limit.

| $M_R^0$ (TeV) | $M_C$ (GeV) | $M_R^+$ (GeV) | $M_G$ (GeV) | $\alpha_G^{-1}$ | $\tau_p$ (yrs.) |
|---------------|-------------|---------------|-------------|-----------------|----------------|
| 10            | $10^{4.5}$  | $10^{1.886}$  | $10^{16.15}$| 40.25           | $1.6 \times 10^{36}$|
| 10            | $10^{6.5}$  | $10^{8.5}$    | $10^{16.04}$| 40.64           | $1.6 \times 10^{34}$|
| 10            | $10^8$      | $10^{8.789}$  | $10^{15.62}$| 41.49           | $3.9 \times 10^{34}$|
| 10            | $10^{8.5}$  | $10^{8.8}$    | $10^{15.5}$ | 41.69           | $1.2 \times 10^{34}$|
| 5             | $10^{9.8}$  | $10^{7.2}$    | $10^{15.83}$| 41.15           | $2.3 \times 10^{34}$|

Table 2: Allowed mass scales as solutions of renormalisation group equations for Model-II as described in the text.

One example of RG evolution of gauge couplings is shown in Fig. 2 for $M_{R^0} = 10$ GeV, $M_{R^+} = 10^{8.7}$ GeV, $M_C = 10^8$ GeV, and $M_U = 6.5 \times 10^{15}$ GeV. Clearly the figure shows precise unification of the three gauge couplings of the intermediate gauge symmetry $G_{2213P}$ at the GUT scale. For all other solutions given in Table-I the RG evolutions and unification of gauge couplings are similar.

In both the models, with allowed values of $M_{R^+} \gg M_{R^0} = 5 - 10$ TeV, the numerical values of gauge couplings $g_{2L}, g_{1R}$ and $g_{B-L}$ predict $\left< M_{Z'} \right> = (1.2 - 3.5) \text{TeV}$. 

### 2.3 Proton lifetime prediction

In this section we discuss predictions on proton lifetimes in the two models and compare them with the current Super-Kamiokande limit and reachable limits by future experiments such as Hyper-Kamiokande $^{33}$. Currently, the Superkamiokande detector has reached the search limit

$$ (\tau_p)_{\text{expt.}} (p \rightarrow e^+\pi^0) \geq 1.4 \times 10^{34} \text{ yrs}, $$

The proposed 5.6 Megaton years Cherenkov water detector at Hyper-Kamiokane, Japan is expected to probe into lifetime $^{33}$,

$$ (\tau_p)_{\text{Hyper-K}} (p \rightarrow e^+\pi^0) \geq 1.3 \times 10^{35} \text{ yrs}, $$
Figure 2: Two loop gauge coupling unification in the $SO(10)$ symmetry breaking chain with $M_U = 10^{15.62}$ GeV and $M_{R^+} = 10^{8.7}$ GeV with a low mass $Z'$ boson at $M_{R^0} = 10$ TeV for Model-II.
The width of the proton decay for $p \to e^+\pi^0$ is expressed as

$$\Gamma(p \to e^+\pi^0) = \frac{m_p}{64\pi f_\pi^2} \left(\frac{g G^4}{M_U^4}\right) |A_L|^2 |\bar{\alpha}_H|^2 (1 + D + F)^2 \times R. \quad (8)$$

where $R = [(A_{SR}^2 + A_{SL}^2)(1 + |V_{ud}|^2)^2]$ for $SO(10)$, $V_{ud} = 0.974$ is the $(1,1)$ element of $V_{CKM}$ for quark mixings, $A_{SL}(A_{SR})$ is the short-distance renormalization factor in the left (right) sectors and $A_L = 1.25$ is long distance renormalization factor. $M_U$ is the degenerate mass of 24 superheavy gauge bosons in $SO(10)$, $|\bar{\alpha}_H|$ is the hadronic matrix element, $m_p$ is the proton mass = 938.3 MeV, $f_\pi$ is the pion decay constant = 139 MeV, and the chiral Lagrangian parameters are $D = 0.81$ and $F = 0.47$. With $\alpha_H = \bar{\alpha}_H(1 + D + F) = 0.012$ GeV$^3$ obtained from lattice gauge theory computations, we get $A_R \simeq A_L A_{SL} \simeq A_L A_{SR} \simeq 2.726$ for both the models. The expression for the inverse decay rates for the models is expressed as,

$$\tau_p = \frac{1}{\Gamma(p \to e^+\pi^0)} = \frac{64\pi f_\pi^2}{m_p} \left(\frac{M_U^4}{g G^4}\right) \frac{1}{|A_L|^2 |\bar{\alpha}_H|^2 (1 + D + F)^2 \times R}. \quad (9)$$

where the factor $F_q = 2(1 + |V_{ud}|^2)^2 \simeq 7.6$ for $SO(10)$. Now using the given values of the model parameters the predictions on proton lifetimes for both the models are given in Table 1 and Table 2. We find that for proton lifetime predictions accessible to Hyper-Kamiokande detector [33], it is necessary to have an intermediate value of the color octet mass $M_C \geq 10^{8.6}$ GeV in Model-II and $M_C \geq 10^{10.8}$ GeV in Model-I. The predicted proton lifetime as a function of the color octet mass is shown in Fig. 3 both for Model-I and for Model-II.

These analyses suggest that low color octet mass in the TeV scale and observable proton lifetime within Hyper-Kamiokande limit are mutually exclusive. If LHC discovers color octet within its achievable energy range, proton decay searches would need far bigger detector than the Hyper-K detector. On the other hand the absence of color octet at the LHC would still retain the possibility of observing proton decay within the Hyper-K limit.

3 TYPE-II SEESAW DOMINANCE

In this section we discuss prospects of having a type-II seesaw dominated neutrino mass formula in the two $SO(10)$ based models discussed in Sec 2.
3.1 Derivation of type-II seesaw formula

We have added to the usual spinorial representations $16_F$ ($i = 1, 2, 3$) for fermion representations in $SO(10)$, one fermion singlet per generation $S_i$ ($i = 1, 2, 3$). The $G_{2213}$ symmetric Yukawa Lagrangian descending from $SO(10)$ symmetry can be written as

$$
\mathcal{L}_{\text{Yuk}} = \sum_{i=1,2} Y_i^\dagger \overline{\psi}_L \psi_i \Phi_i + f (\psi_R^c \psi_R \Delta_R + \psi_L^c \psi_L \Delta_L) + y_\chi (\overline{\psi}_R S \chi_R + \overline{\psi}_L S \chi_L) + \text{h.c.},
$$

where $\Phi_1, 2 \subset 10_{H_1, H_2}$ are two bidoublets, $(\Delta_L, \Delta_R) \subset 126_F$ and $(\chi_L, \chi_R) \subset 16_H$. As discussed in sec.2, the spontaneous breaking of $G_{2213} \rightarrow G_{2113}$, takes place by the VEV of the RH triplet $\sigma_R(1, 3, 0, 1) \subset 45_H$ carrying $B - L = 0$ which does not generate any fermion mass term. As we discuss below when the Higgs scalar $\Phi_i$, $\Delta_R$ and $\chi_R$ acquire VEV’s spontaneous symmetry breaking $G_{2113} \rightarrow SM \rightarrow U(1)_{em} \times SU(3)_C$, occurs and generate $N - S$ mixing mass term $M = y_\chi \langle \chi_R^0 \rangle$ by the induced VEVs. In addition $v_{\chi L} = \langle \chi_L^0 \rangle$ and $v_L = \langle \Delta_L^0 \rangle$ are automatically generated even though the LH doublet $\chi_L$ and the RH triplet $\Delta_L$ are assigned vanishing VEVs directly.

In models with inverse seesaw [35] or extended seesaw [16, 17, 36, 37]...
mechanisms, a bare mass term of the singlet fermions $\mu_S S^T S$ occurs in the Lagrangian. Being unrestricted as a gauge singlet mass term in the Lagrangian, determination of its value has been left to phenomenological analyses in neutrino physics. Larger values of the parameter near the GUT-Planck scale [38] or at the intermediate scale [39] have been exploited. On the other hand, fits to the neutrino oscillation data through inverse seesaw formula by a number of authors have shown to require much smaller values of $\mu_S$ [17, 36, 14, 16, 42]. Even phenomenological implications of its vanishing value have been investigated recently in the presence of other non-standard and non-vanishing fermion masses [40, 41] in the $9 \times 9$ mass matrix. Very small values of $\mu_S$ is justified on the basis of ’t Hooft’s naturalness criteria representing a mild breaking of global lepton number symmetry of the SM [43]. While we consider the implication of this term later in this section, at first we discuss the emerging neutrino mass matrix by neglecting it.

In addition to the VEVs discussed in Sec.2 for gauge symmetry breaking at different stages, we assign the VEV to the neutral component of RH Higgs doublet of $16_H$ with \(<\chi_R(1,1/2,-1/2,1) = V_\chi\) in order to generate $N - S$ mixing mass term $M_{NS}$ between the RH neutrino and the sterile fermion where the $3 \times 3$ matrix $M = y_\chi V_\chi$. We define the other $3 \times 3$ mass matrices $M_D = Y^{(1)} v_u$ and $M_N = f v_R$. We also include induced small contributions to the vacuum expectation values of the LH Higgs triplet $v_L = <\Delta_L(3,0,-2,1)>$ and the LH Higgs doublet $v_{\chi L} = <\chi_L(2,0,-2,1)>$ leading to the possibilities $\nu - S$ mixing with $M_L = y_\chi v_{\chi L}$ and the induced type-II seesaw contribution to LH neutrino masses $m_{\nu}^{II} = f v_L$ given in eq.(20). The induced VEVs are shown in the left and right panels of Fig.4. We have also derived them by actual potential minimisation which agree with the diagramatic contribution. Including the induced VEV contributions the mass term due to Yukawa Lagrangian can be written as

$$\mathcal{L}_{\text{mass}} = (M_D N + \frac{1}{2} M_N N^T \bar{N} + M_N N^T S + M_L \bar{N} S + h.c) + m_{\nu}^{II} \nu^T \nu \quad (11)$$

In the $(\nu, S, N^C)$ basis the generalised form of the $9 \times 9$ neutral fermion mass matrix after electroweak symmetry breaking can be written as

$$\mathcal{M} = \begin{pmatrix} m_{\nu}^{II} & M_L & M_D \\ M_L^T & 0 & M^T \\ M_D^T & M & M_N \end{pmatrix} \quad (12)$$
where $M_D = Y \langle \Phi \rangle$, $M_N = f v_R$, $M = y_\chi \langle \chi_R^0 \rangle$, $M_L = y_\chi \langle \chi_L^0 \rangle$ and we have used $\mu_s = 0$. In this model the symmetry breaking mechanism and the VEVs are such that $M_N > M \gg M_D$. The RH neutrino mass being the heaviest fermion mass scale in the Lagrangian, this fermion is at first integrated out leading to the effective Lagrangian at lower scales \cite{44, 31, 45},

$$- \mathcal{L}_{\text{eff}} = \left( m_{\nu}^{I} + M_D \frac{1}{M_N} M_D^T \right)_{\alpha \beta} \nu_\alpha^T \nu_\beta + \left( M_L + M_D \frac{1}{M_N} M_D^T \right)_{\alpha m} \left( \nu_\alpha S_m + \overline{S}_m \nu_\alpha \right),$$

where the heaviest RH neutrino mass matrix $M_N$ separates out trivially, the other two $3 \times 3$ mass matrices $\mathcal{M}_\nu$, and $m_S$ are extracted through various steps of block diagonalisation \cite{17}. The details of various steps are given in Appendix B and the results are

$$\mathcal{M}_\nu = m_{\nu}^{I} + (M_D M_N^{-1} M_D^T) - (M_D M_N^{-1} M_D^T) + M_L (M^T M_N^{-1} M)^{-1} M_L^T - M_L (M^T M_N^{-1} M)^{-1} (M^T M_N^{-1} M_D^T) - (M_D M_N^{-1} M) (M^T M_N^{-1} M)^{-1} M_L^T,$$

$$m_s = -M M_N^{-1} M^T + ....,$$

$$m_N = M_N. \quad (14)$$

From the first of the above three equations, it is clear that the type-I seesaw term cancels out \cite{44, 31, 45} with another of opposite sign resulting from block diagonalisation. Then the generalised form of the light neutrino mass matrix turns out to be

$$\mathcal{M}_\nu = f v_L + M_L M^{-1} M_N (M^T)^{-1} M_L^T - [M_L M_D^T M^{-1} + M_D M_L^T M^{T^{-1}}]. \quad (15)$$

With $M_L = y_\chi v_\chi L$ that induces $\nu - S$ mixing, the second term in this equation is double seesaw formula and the third term is the linear seesaw formula which are similar to those derived earlier \cite{39}.

From the Feynman diagrams, the analytic expressions for the induced VEVs are

$$v_L \sim \frac{V_R}{M^2_{\Delta L}} \left( \lambda_1 v_1^2 + \lambda_2 v_2^2 \right),$$

$$16$$
Figure 4: Feynman diagrams for induced contributions to VEVs of the LH triplet (diagram (a)) and the LH doublet (diagram (b)) in Model-I and Model-II.

\[ v_{\chi_L} \sim \frac{V_{\chi}}{M_{\chi L}^2} (\lambda_1' M_1' v_1 + \lambda_2' M_2' v_2), \]
\[ = C_{\chi} \frac{V_{\chi} M_{R^+} v_{wk}}{M_{\chi L}^2}, \]  
(17)

where \( v_{wk} \sim 100 \text{ GeV} \), and

\[ C_{\chi} = \frac{(\lambda_1' M_1' v_1 + \lambda_2' M_2' v_2)}{(M_{R^+} v_{wk})}. \]  
(18)

In eq.(17), \( v_i (i = 1, 2) \) are the VEVs of two electroweak doublets each originating from separate \( 10_H \subset SO(10) \) as explained in the following section, and \( M_1', M_2' \) are Higgs trilinear coupling masses which are normally expected to be of order \( M_{R^+} \). In both the models \( V_R = 5-10 \) TeV and \( V_\chi \sim 300-1000 \) GeV. Similar expressions as in eq.(17) are also obtained by minimisation of the scalar potential.

### 3.2 Suppression of linear seesaw and dominance of type-II seesaw

Now we discuss how linear seesaw term is suppressed without fine tuning of certain parameters in Model-I but with fine tuning of the same parameters.
in Model-II. The expression for neutrino mass is given in eq. (15) where the first, second, and the third terms are type-II seesaw, double seesaw, and linear seesaw formulas for the light neutrino masses. Out of these for all parameters allowed in both the models, (Model-I and Model-II) the double seesaw term will be found to be far more suppressed compared to the other two terms. Therefore we now discuss how the linear seesaw term is suppressed compared to type-II seesaw term allowing the dominance of the latter.

In Model-I, gauge coupling unification has been achieved such that
\[ M_P = M_{\chi L} \sim M_U \geq 10^{15.6} \text{ GeV}, \]
\[ M_{\Delta L} = 10^8 \text{ GeV} \]
where \( M'_1 \sim M'_2 \sim M_{R^+} \sim 10^9 \text{ GeV} \). Using these masses in eq. (15), we find that even with \( C_{\chi} \sim 0.1 - 1.0 \)
\[ v_{\chi L} \sim 10^{-18} \text{ eV} - 10^{-17} \text{ eV}, \]
\[ v_L \sim 0.1 \text{ eV} - 0.5 \text{ eV}, \]
(19)

Such induced VEVs in the Model-I suppress the second and the third terms in eq. (15) making the model quite suitable for type-II seesaw dominance although the Model-II needs finetuning in the induced contributions to the level of \( C_{\chi} \leq 10^{-5} \) as discussed below.

In Model-II, \( M_{\Delta L} \sim M_{\chi L} \sim M_P \sim 10^9 \text{ GeV} \), and without any fine tuning of the parameters in eq. (16), we obtain \( v_L \sim 10^{-10} \text{ GeV} \). From eq. (17), we get \( v_{\chi L} \sim C_{\chi} \times 10^{-6} \text{ GeV} \sim 10^{-7}\text{GeV} \) for \( C_{\chi} \sim 0.1 \). With \( (M_D)_{(3,3)} \leq 100 \text{ GeV} \) and \( \frac{M_P}{M_D} \sim 0.1 - 1, \) the most dominant third term in eq. (15) gives \( M_{\nu} \geq 10^{-8} \text{ GeV} \). This shows that finetuning is needed in the parameters occurring to reduce \( C_{\chi} \leq 10^{-5} \) to suppress linear seesaw and permit type-II seesaw dominance in Model-II whereas the type-II seesaw dominance is achieved in Model-I with \( c_{\chi} \sim 0.1 - 1.0 \) without requiring any such fine tuning.

In what follows we will utilise the type-II seesaw dominated neutrino mass formula to study neutrino physics, neutrino-less double beta decay and lepton flavor violations in the context of Model-I, although they are similar in Model-II subject to the finetuning needed for \( C_{\chi} \).

*Following the similar block diagonalisation procedure given in Appendix B, but in the presence of \( \mu_S S^T S \) in the Yukawa Lagrangian with mass ordering \( M_N > M >> M_D, \mu_S \) results in the appearance of the inverse seesaw part of the full neutrino mass matrix,
\[ \mathcal{M}_p = f v_L + (\frac{M_D}{M_P}) \mu_S (\frac{M_D}{M_P})^T. \]
Although we plan to investigate the implications of this formula in a future work, for the present purpose we assume \( \mu_S \simeq 0 \) such that type-II seesaw dominance prevails.

18
$$M_\nu \simeq f v_L.$$  

(20)

## 3.3 Right-handed neutrino mass prediction

Global fits to the experimental data \[46\] on neutrino oscillations have determined the mass squared differences and mixing angles at 3$\sigma$ level

$$\sin^2 \theta_{12} = 0.320, \quad \sin^2 \theta_{23} = 0.427,$$

$$\sin^2 \theta_{13} = 0.0246, \quad \delta_{CP} = 0.8 \pi,$$

$$\Delta m^2_{\text{sol}} = 7.58 \times 10^{-5} \text{eV}^2,$$

$$|\Delta m^2_{\text{atm}}| = 2.35 \times 10^{-3} \text{eV}^2.$$  

(21)

For normally hierarchical (NH), inverted hierarchical (IH), and quasi-degenerate (QD) patterns, the experimental values of mass squared differences can be fitted by the following values of light neutrino masses

$$\hat{m}_\nu = (0.00127, 0.008838, 0.04978) \text{eV} \quad \text{(NH)}$$

$$= (0.04901, 0.04978, 0.00127) \text{eV} \quad \text{(IH)}$$

$$= (0.2065, 0.2058, 0.2) \text{eV} \quad \text{(QD)}$$  

(22)

We use the diagonalising Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$U_{\text{PMNS}} = \begin{pmatrix}
   c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
   -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & s_{23}c_{13} \\
   s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23}
\end{pmatrix}.$$  

(23)

and determine it using mixing angle and the leptonic Dirac phase from eq. \[21\]

$$U_{\text{PMNS}} = \begin{pmatrix}
   0.81437 & 0.5585 & -0.1267 - 0.092i \\
   -0.359 - 0.049i & 0.6710 - 0.0341i & 0.645298 \\
   0.4486 - 0.0576i & -0.484 - 0.0395i & 0.7476
\end{pmatrix}.$$  

(24)

Now inverting the relation $\hat{m}_\nu = U_{\text{PMNS}}^\dagger M_\nu U_{\text{PMNS}}^*$ where $\hat{m}_\nu$ is the diagonalised neutrino mass matrix, we determine $M_\nu$ for three different cases and
further determine the corresponding values of the $f$ matrix using $f = M_\nu/v_L$ where we use the predicted value of $v_L = 0.1$ eV. Noting that

\[ \hat{M}_N = fV_R = M_\nu V_R/v_L, \]

we have also derived eigen values of the RH neutrino mass matrix $\hat{M}_N$, as the positive square root of the $i^{th}$ eigen value of the Hermitian matrix $M_N^\dagger M_N$.

**NH**

\[
f = \begin{pmatrix}
0.1176 + 0.02242i & -0.12431 - 0.0031i & 0.1443 + 0.0251i \\
-0.12431 - 0.0031i & 0.1588 - 0.0148i & -0.14168 + 0.0172i \\
0.1443 + 0.0251i & -0.1416 + 0.0172i & 0.313 - 0.00029i
\end{pmatrix}, \tag{25}
\]

\[ |\hat{M}_N| = \text{diag}(160, 894, 4870) \text{ GeV.} \tag{26} \]

**IH**

\[
f = \begin{pmatrix}
0.3904 - 0.01711i & 0.0993 + 0.0103i & -0.1616 + 0.0533i \\
0.0993 + 0.0103i & 0.379 + 0.0222i & 0.1762 + 0.0363i \\
-0.1616 + 0.0533i & 0.1762 + 0.0363i & 0.218 - 0.0114i
\end{pmatrix}, \tag{27}
\]

\[ |\hat{M}_N| = \text{diag}(4880, 4910, 131) \text{ GeV.} \tag{28} \]

**QD**

\[
f = \begin{pmatrix}
2.0256 + 0.029912i & 0.00118776 + 0.0231615i & -0.01905 + 0.3072i \\
0.00118776 + 0.0231615i & 2.03411 + 0.01721i & 0.02129 + 0.2106i \\
-0.01905 + 0.3072i & 0.02129 + 0.2106i & 1.9904 - 0.0482i
\end{pmatrix} \tag{29}
\]

For $v_L = 0.1$ eV, we have

\[ |\hat{M}_N| = \text{diag}(21.46, 20.34, 18.87) \text{ TeV,} \tag{30} \]

but for $v_L = 0.5$ eV, we obtain

\[ |\hat{M}_N| = \text{diag}(4.3, 4.08, 3.77) \text{ TeV.} \tag{31} \]

These RH neutrino masses predicted with $v_L = 0.1$ eV for NH and IH cases and with $v_L = 0.5$ eV for the QD case are clearly verifiable by the LHC.
4 THE DIRAC NEUTRINO MASS MATRIX

The Dirac neutrino mass matrix which has its quark-lepton symmetric origin \cite{8} plays a crucial role in the predictions of lepton flavor violations \cite{14,16} as well as lepton number violations as pointed out very recently \cite{17,36}. The determination of the Dirac neutrino mass matrix \( M_D(M_{\text{Pl}}) \) at the TeV seesaw scale is done which was discussed in \cite{16,47}.

4.1 Extrapolation to the GUT scale

The RG extrapolated values at the GUT scale are,

\[
\begin{align*}
\mu &= \text{M}_{\text{GUT}}: \\
m_e^0 &= 0.00048 \text{ GeV}, m_{\mu}^0 = 0.0875 \text{ GeV}, m_{\tau}^0 = 1.8739 \text{ GeV}, \\
m_d^0 &= 0.0027 \text{ GeV}, m_s^0 = 0.0325 \text{ GeV}, m_b^0 = 1.3373 \text{ GeV}, \\
m_u^0 &= 0.001 \text{ GeV}, m_c^0 = 0.229 \text{ GeV}, m_t^0 = 78.74 \text{ GeV}, \\
m_{\bar{d}}^0 &= 0.0027 \text{ GeV}, m_{\bar{s}}^0 = 0.0325 \text{ GeV}, m_{\bar{b}}^0 = 1.3373 \text{ GeV}, \\
m_{\bar{u}}^0 &= 0.001 \text{ GeV}, m_{\bar{c}}^0 = 0.229 \text{ GeV}, m_{\bar{t}}^0 = 78.74 \text{ GeV}, \tag{32}
\end{align*}
\]

The \( V^0_{\text{CKM}} \) matrix at the GUT scale is given by

\[
V^0_{\text{CKM}}(\text{M}_{\text{GUT}}) = \begin{pmatrix}
0.9748 & 0.2229 & -0.0003 - 0.0034i \\
-0.2227 - 0.0001i & 0.9742 & 0.0364 \\
0.0084 - 0.0033i & -0.0354 + 0.0008i & 0.9993
\end{pmatrix}. \tag{33}
\]

For fitting the charged fermion masses at the GUT scale, in addition to the two complex 10_{H,2} representations with their respective Yukawa couplings \( Y_{1,2} \), we also use the higher dimensional operator \cite{14,16}

\[
\frac{\kappa_{ij}{16}_i{16}_j{10}_H{45}_H{45}_H}}{M^2_G}. \tag{34}
\]

In the above equation the product of three Higgs scalars acts as an effective 126_{H} operator \cite{14}. With \( M_G \approx M_{\text{Pl}} \) or \( M \approx M_{\text{string}} \), this is suppressed by \( (M_G/M_U)^2 \approx 10^{-3} - 10^{-5} \) for GUT-scale VEV of 45_{H}. Then the formulas for different charged fermion mass matrices are

\[
\begin{align*}
M_u &= G_u + F, & M_d &= G_d + F, \\
M_e &= G_d - 3F, & M_D &= G_u - 3F. \tag{35}
\end{align*}
\]
Following the procedure given in [16], the Dirac neutrino mass matrix at the GUT scale is found to be

\[
M_D(M_{R0}) = \begin{pmatrix}
0.0145 & 0.0456 - 0.0119i & 0.1039 - 0.303i \\
0.0456 + 0.0119i & 0.3511 & 2.66 + 0.0007i \\
0.1039 + 0.303i & 2.66 - 0.0007i & 79.20
\end{pmatrix} \text{ GeV.} \quad (36)
\]

5 LEPTON FLAVOR VIOLATION

In the present non-SUSY SO(10) model, even though neutrino masses are governed by high scale type-II seesaw formula, the essential presence of singlet fermions that implement the type-II seesaw dominance by cancelling out the type-I seesaw contribution give rise to experimentally observable LFV decay branching ratios through their loop mediation. The heavier RH neutrinos in this model being in the range of \( \sim 1 - 10 \) TeV mass range also contribute, but less significantly than the singlet fermions.

5.1 Estimation of non-unitarity matrix

In order to study non-unitarity effects and lepton flavor violations, we assume the \( N-S \) mixing matrix M to be diagonal for the sake of simplicity and also for exercising economy of parameters,

\[
M = \text{diag} (M_1, M_2, M_3), \quad (37)
\]

The non-unitarity deviation \( \eta \) is defined as

\[
\eta = \frac{1}{2} X.X^\dagger = M_D M^{-2} M_D^\dagger, \\
\eta_{\alpha\beta} = \frac{1}{2} \sum_{k=1,2,3} \frac{M_{D\alpha k} M_{D\beta k}^*}{M_k^2}. \quad (38)
\]

which for the degenerate case, \( M_i = M_{Deg}(i = 1, 2, 3) \), gives,

\[
\eta = \frac{1\text{GeV}^2}{M_{Deg}^2} \begin{pmatrix}
0.03947 & 0.1461 - 0.4032i & 4.1732 - 11.990i \\
0.1461 + 0.4032i & 3.6024 & 105.805 - 0.0022i \\
4.1732 + 11.990i & 105.805 + 0.0022i & 3139.82
\end{pmatrix}. \quad (39)
\]
For the general non-degenerate case of M, we saturate the upper bound $|\eta_{\tau\tau}| < 2.7 \times 10^{-3}$ \[49\] to derive

$$\frac{1}{2} \left[ \frac{0.1026}{M_1^2} + \frac{7.0756}{M_2^2} + \frac{676.4}{M_3^2} \right] = 2.7 \times 10^{-3}, \quad (40)$$

By inspection this equation gives the lower bounds

$$M_1 > 4.35 \text{ GeV}, \quad M_2 > 36.2 \text{ GeV}, \quad M_3 > 1120 \text{ GeV},$$

and for the degenerate case $M_{\text{Deg}} = 1213 \text{ GeV}$. For the partial degenerate case of $M_1 = M_2 \neq M_3$ the solutions can be similarly derived as in ref[16] and one example is $M(100, 100, 1319.67) \text{ GeV}$.

### 5.2 Branching ratio and CP Violation

One of the most important outcome of non-unitarity effects is expected to manifest through ongoing experimental searches for LFV decays such as $\tau \to e\gamma$, $\tau \to \mu\gamma$, $\mu \to e\gamma$. In these models the RH neutrinos and the singlet fermions contribute to the branching ratios $[14, 16, 15]$.

Because of the condition $M_N >> M$, neglecting the RH neutrino exchange contribution compared to the sterile fermion singlet contributions, our estimations for different cases of M values are presented in Table 43. These values are many orders larger than the standard non-SUSY contributions and are accessible to ongoing or planned searches $[4]$. For the degenerate case

$$\Delta \mathcal{J}_{12}^{\mu\tau} = -2.1 \times 10^{-6},$$
$$\Delta \mathcal{J}_{23}^{\mu\tau} = -2.4 \times 10^{-6},$$
$$\Delta \mathcal{J}_{31}^{\mu\tau} = 1.4 \times 10^{-4},$$
$$\Delta \mathcal{J}_{31}^{\mu\tau} = 1.2 \times 10^{-4}, \quad (41)$$

we have the branching ratio

$$BR(\mu \to e\gamma) = 6.43 \times 10^{-17},$$
$$BR(\tau \to e\gamma) = 8.0 \times 10^{-16},$$
$$BR(\tau \to \mu\gamma) = 2.41 \times 10^{-12}. \quad (42)$$
Because of the presence of non-unitarity effects in the present model, the leptonic CP-violation can be written as [16, 49, 50]. The moduli and phase of non-unitarity and CP-violating parameter for the degenerate case is given in (43).

\[
\begin{align*}
|\eta_{e\mu}| &= 2.73 \times 10^{-8}, \\
\delta_{e\mu} &= 1.920, \\
|\eta_{e\tau}| &= 4.54 \times 10^{-7}, \\
\delta_{e\tau} &= 1.78, \\
|\eta_{\mu\tau}| &= 2.31 \times 10^{-5}, \\
\delta_{\mu\tau} &= 2.39 \times 10^{-7}.
\end{align*}
\]

(43)

Our estimation presented in (43) shows that in a wider range of the parameter space, the leptonic CP violation parameter could be nearly two orders larger than the CKM-CP violation parameter for quarks.

6 NEUTRINO-LESS DOUBLE BETA DECAY

Even with the vanishing bare mass term \(\mu_S = 0\) in the Yukawa Lagrangian of eq.(10), the singlet fermions \(S_i(i = 1, 2, 3)\) acquire Majorana masses over a wide range of values and, in the leading order, the corresponding mass matrix given in eq.(14) is \(m_S = -M_{MN}M^T\). As far as light neutrino mass matrix is concerned, it is given by the type-II seesaw formula of eq.(20) which is independent of the Majorana mass matrix \(m_S\) of singlet fermions. But the combined effect of substantial mixing between the light neutrinos and the singlet neutrinos, and the Majorana neutrino mass insertion \(m_S\) due to the singlet fermions in the Feynman diagram gives rise to new contributions to the amplitude and the effective mass parameter for \(0\nu\beta\beta\) even in the \(W_L-W_L\) channel. This may be contrasted with conventional type-II seesaw dominated non-SUSY \(SO(10)\) models with only three generations of standard fermions in \(16_i(i = 1, 2, 3)\) where there is no such contributions to \(0\nu\beta\beta\) decay.

The charged current interaction Lagrangian for leptons in this model in the flavor basis is

\[
\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \sum_{\alpha = e, \mu, \tau} \left[ \bar{\ell}_{\alpha L} \gamma_\mu \nu_{\alpha L} W^\mu_L + \bar{\ell}_{\alpha R} \gamma_\mu N_{\alpha R} W^\mu_R \right] + \text{h.c.} \quad (44)
\]
In the present model the $W_R^\pm$ bosons and the doubly charged Higgs scalars, both left-handed and the right handed, are quite heavy with $M_{W_R} \sim M_\Delta \approx 10^8 - 10^9$ GeV that makes negligible contributions due to the RH current effects and Higgs exchange effects for the $0\nu\beta\beta$ decay amplitude. The most popular standard and conventional contribution in the $W_L^- - W_L^-$ channel is due to light neutrino exchanges. But one major point in this work is that even in the $W_L^- - W_L^-$ channel, the singlet fermion exchange allowed within the type-II seesaw dominance mechanism, can yield much more dominant contribution to $0\nu\beta\beta$ decay rate. For the exchange of singlet fermions ($S_j$), the Feynman diagram is shown in the Fig 5. For the exchange of heavier RH Majorana neutrinos ($\tilde{N}_k$), the diagram is same as the right-panel of this figure but with the replacement of the mixing matrix and masses by $\mathcal{V}_{\nu S} \rightarrow \mathcal{V}_{\nu N}$ and $S_i \rightarrow N_i$. The heavier RH neutrino exchange contributions are found to be negligible compared to the singlet fermion exchange contributions.

In the mass basis, the contributions to the decay amplitudes by $\nu$ and $S$ exchanges are estimated as

\begin{align}
A_{\nu LL}^L &\propto \frac{1}{M_{WL}^4} \sum_{i=1,2,3} \left(\frac{\mathcal{V}_{\nu e}^{\nu e_i}}{p^2} \right)^2 m_{\nu_i} \tag{45} \\
A_{S LL}^L &\propto \frac{1}{M_{WL}^4} \sum_{j=1,2,3} \left(\frac{\mathcal{V}_{\nu S}^{\nu S_j}}{m_{S_j}} \right)^2 m_{S_j} \tag{46} \\
A_{N LL}^L &\propto \frac{1}{M_{WL}^4} \sum_{j=1,2,3} \left(\frac{\mathcal{V}_{\nu N}^{\nu N_j}}{m_{N_j}} \right)^2 m_{N_j}, \tag{47}
\end{align}

where $|p| \approx 190$ MeV represents the magnitude of neutrino virtuality momentum [51, 53]. Using uncertainties in the nuclear matrix elements [52, 53] we have found it to take values in the range $|p| = 120$ MeV – 200 MeV.

In order to understand physically how the singlet fermion Majorana mass insertion terms as a new source of lepton number violation contributes to $0\nu\beta\beta$ process, we draw the Feynman diagram Fig 5 with mass insertion. In this model, the Majorana mass matrix for the singlet fermion after block
diagonalisation is $m_S = -M M_N^{-1} M^T$. Then exchanges of such singlets generate dominant contribution through their mixings to active neutrinos and this mixing is proportional to the Dirac neutrino mass $M_D$ derived in [4]. It is clear from the Fig. 5 that the the singlet fermion exchange amplitudes are derived [17] to have the same form as in eq.(46).

7 EFFECTIVE MASS PARAMETER AND HALF LIFE

Adding together the $0\nu\beta\beta$ decay amplitudes arising out of light-neutrino exchanges, singlet fermion exchanges, and the heavy RH neutrino exchanges in the $W_L - W_L$ channel from eq.(46) and using suitable normalisations [52, 53], we express the inverse half life

$$[T_{1/2}^{0\nu}]^{-1} \simeq G_{01}^{0\nu} |M_\nu^{0\nu}|^2 \left| (m_{\nu}^{ee} + m_{S}^{ee} + m_{N}^{ee}) \right|^2,$$

$$= K_{0\nu} \left| (m_{\nu}^{ee} + m_{S}^{ee} + m_{N}^{ee}) \right|^2,$$

$$= \left| m_{\nu}^{ee} \right|^2 K_{0\nu} \left| m_{\nu}^{ee} \right|^2$$

(48)

In the above equation $G_{01}^{0\nu} = 0.686 \times 10^{-14} \text{yrs}^{-1}$, $M_\nu^{0\nu} = 2.58 - 6.64$, $K_{0\nu} = 1.57 \times 10^{-25} \text{yrs}^{-1} \text{eV}^{-2}$, and the three effective mass parameters for light-
neutrino, singlet fermion, and heavy RH neutrino exchanges are

\[ m_{ee} = \sum_i (\nu_{ei}^\nu)^2 m_{\nu_i} \]  \hspace{1cm} (49)

\[ m_S^{ee} = \sum_i (\nu_{ei}^S)^2 \frac{|p|^2}{m_{S_i}} \]  \hspace{1cm} (50)

\[ m_N^{ee} = \sum_i (\nu_{ei}^N)^2 \frac{|p|^2}{m_{N_i}} \]  \hspace{1cm} (51)

with

\[ m_{\text{eff}} = m_{ee} + m_S^{ee} + m_N^{ee} \]  \hspace{1cm} (52)

Here \( m_{S_i} \) is the eigen value of the \( S- \) fermion mass matrix \( m_S \), and the magnitude of neutrino virtuality momentum \( |p| = 120 \text{ MeV–200 MeV} \). As the predicted values of the RH neutrino masses carried out in Sec.3 have been found to be large which make their contribution to the \( 0\nu\beta\beta \) decay amplitude negligible, we retain only contributions due to light neutrino and singlet fermion exchanges. The estimated values of the effective mass parameters due to the \( S- \) fermion exchanges and light neutrino exchanges are shown separately in Fig. 6 where the magnitudes of corresponding mass eigen values used have been indicated.

### 7.1 Numerical estimations of effective mass parameters

Using the equations of normalized mass parameters [17], we estimate numerically the nearly standard contribution due to light neutrino exchange and the dominant non-standard contributions due to singlet fermion exchanges.

**A. Nearly standard contribution**

In this model the new mixing matrix \( \mathcal{N} \equiv \mathcal{V}^{\nu\nu} = (1 - \eta) U_\nu \) contains additional non-unitarity effect due to non-vanishing \( \eta \) [17].

Using \( M_{\text{Deg}} = 1213 \text{ GeV} \) in the degenerate case, we estimate

\[ \mathcal{N}_{ei} = (0.81437, 0.54858, 0.1267 + 0.0922i). \]  \hspace{1cm} (53)
Since all the $\eta-$ parameters are constrained by $|\eta_{\alpha\beta}| < 10^{-3}$, it is expected that $|\mathcal{N}_{\alpha i}| \simeq |U_{\alpha i}|$ for any other choice of $M$. In the leading approximation by neglecting the $\eta_{ai}$ contributions, the effective mass parameter in the the $W_L - W_L$ channel with light neutrino exchanges is expressed as

$$m_{\nu}^{ee} = \sum_i N_{ei}^2 \hat{m}_i$$

$$\simeq (c_{12}c_{13})^2 \hat{m}_1 e^{i\alpha_1} + (s_{12}c_{13})^2 \hat{m}_2 e^{i\alpha_2} + s_{13}^2 e^{i\delta} \hat{m}_3,$$

(54)

where we have introduced two Majorana phases $\alpha_1$ and $\alpha_2$. As discussed subsequently in this section, they play crucial roles in preventing cancellation between two different effective mass parameters.

Using $\alpha_1 = \alpha_2 = 0$ and the experimental values of light neutrino masses and the Dirac phase $\delta = 0.8\pi$ from eq.(21), the light neutrino exchanges have their well known values,

$$|m_{\nu}^{ee}| = \begin{cases} 0.0039 \text{ eV} & \text{NH}, \\ 0.04805 \text{ eV} & \text{IH}, \\ 0.23 \text{ eV} & \text{QD}. \end{cases}$$

(55)

B. Dominant non-standard contributions

The $(ei)$ element of the $\nu - S$ mixing matrix is [17]

$$\mathcal{V}_{ei}^{\nu S} = \left( \frac{M_D}{M} \right)_{ei}.$$  

(56)

where the Dirac neutrino mass matrix $M_D$ has been given in eq.(36) and the diagonal elements are estimated using the non-unitarity equation as discussed in the previous section. We derive the relevant elements of the mixing matrix $\mathcal{V}_{ei}^{\nu S}$ using the structures of the the Dirac neutrino mass matrix $M_D$ given in eq.(36) and values of the diagonal elements of $M = (M_1, M_2, M_3)$ satisfying the non-unitarity constraint in eq.(40). The eigen values of the $S-$ fermion mass matrix $m_S$ are estimated for different cases using the structures of the RH Majorana neutrino mass matrices given in eq.(26), eq.(28), and eq.(30) in the formula $m_S = -M \frac{1}{M_N} M^T$. It is clear that in the effective mass parameter the non-standard contribution due to sterile fermion exchange has a sign opposite to that due to light neutrino exchange and also its magnitude is
inversely proportional to the sterile fermion mass eigen values. In the NH case the estimated effective mass parameters are shown in Fig.6 where the values of diagonal elements of $M$ and the eigen values of $m_s$ have been specified. For comparison the effective mass parameters in the standard case without singlet fermions have been also given. It is clear that for allowed masses of the model, the non-standard contributions to effective mass parameters can be much more dominant compared to the standard values irrespective of the mass patterns of light neutrino masses, NH, IH or QD.

![Figure 6: Variation of the effective mass parameters with lightest LH neutrino mass. The dominant non-standard contributions due to fermion singlet contributions are shown by three horizontal lines with corresponding mass values in GeV units. The subdominant effective mass parameters due to NH and IH cases shown are similar to the standard values.](image)

### 7.2 Cancellation between effective mass parameters

When plotted as a function of singlet fermion mass eigen value $m_{S_1}$, the resultant effective mass parameter shows cancellation for certain region of the parameter space, the cancellation being prominent in the QD case.
Like the light neutrino masses, the singlet fermion masses $m_{S_i}$ are also expected to have two Majorana phases. When all Majorana phases are absent, both in the light active neutrino as well as in the singlet fermion sectors, it is clear that in the sum of the two effective mass parameter there will be cancellation between light active neutrino and the singlet fermion contributions because of the inherent negative sign of the non-standard contribution. Our estimations for NH, IH, and QD patterns of light neutrino mass hierarchies are discussed separately.

A. Effective mass parameter for NH and IH active neutrino masses

In Fig. 7, we have shown the variation of the resultant effective mass parameter with $m_{S_1}$ for NH and IH patterns of active light neutrino masses in the left-panel and the right panel, respectively. It is clear that for lower values of $m_{S_1}$, the singlet fermion exchange term continues to dominate. For larger values of $m_{S_1}$, the resultant effective mass parameter tends to be identical to the light neutrino mass contribution due to the vanishing non-standard contribution. We note that the values $|m_{\text{eff}}| = 0.5 - 0.1$ eV can be easily realised for $|m_{S_1}| = 3 - 5$ GeV in the NH case but for $|m_{S_1}| = 1 - 2$ GeV in the IH case.

![Figure 7: Variation of effective mass of $0\nu\beta\beta$ decay with the mass of the lightest singlet fermion for the value of $|p| = 190$ MeV.](image)

B. Effective mass parameter for QD active neutrinos

The variation of effective mass with $m_{S_1}$ for the QD case with one experimen-
tally determined Dirac phase $\delta = 0.8\pi$ and assumed values of two unknown Majorana phases is given in Fig. 8. The left-panel of Fig. 8 shows the variation with $\alpha_1 = \alpha_2 = 0$ for different choices of the common light neutrino mass $m_0 = 0.5$ eV, 0.3 eV, and 0.2 eV for the upper, middle, and the lower curves, respectively, where cancellations are clearly displayed in the regions of $m_{s_1} = 0.4 - 1.5$ GeV. However, before such cancellation occurs, the dominance of the singlet exchange contribution has been clearly shown to occur in the regions of lower values of $m_{s_1}$. For larger values of $m_{S_1} > 5$ GeV, the singlet exchange contribution tends to be negligible and the light QD neutrino contribution to $m_{\text{eff}}$ is recovered.

In the right panel of Fig. 8 the upper curve corresponds to $\alpha_1 = \pi, \alpha_2 = \pi$ at $m_0 = 0.2 eV$. The middle line corresponds to $\alpha_1 = \pi, \alpha_2 = 0$ at $m_0 = 0.5 eV$. The lower line corresponds to $\alpha_1 = 0, \alpha_2 = \pi$ at $m_0 = 0.3 eV$. We find that because of introduction of appropriate Majorana phases the dips in two curves have disappeared.

![Figure 8: Variation of effective mass of $0\nu\beta\beta$ decay with the mass of the lightest singlet fermion for QD light neutrinos with one Dirac phase (left panel) and with one Dirac phase and two Majorana phases (right panel).](image)

**7.3 Half-life as a function of singlet fermion masses**

In order to arrive at a plot of half-life against the lightest singlet fermion mass in different cases, at first we estimate the mass eigen values of the three
singlet fermions for different allowed combinations of the $N - S$ mixing matrix elements satisfying the non-unitarity constraint of eq.(40) and by using the RH neutrino mass matrices predicted for NH, IH, and QD cases from eq.(26), eq.(28), eq.(30), and eq.(31). These solutions are shown in Table 3.

We then derive expressions for half-life taking into account the contributions of the two different amplitudes or effective mass parameters arising out of the light neutrino and the singlet fermion exchanges leading to

\[
\left[T_{1/2}^{0\nu}\right] = \frac{m_{s_1}^2}{K_{0\nu}|p|^4(M_D/M)_{e1}} \left[|1 + X + Y|\right]^{-2}, \tag{57}
\]

where

\[
X = \frac{(M_D/M)_{e2}^2 m_{s_1}}{(M_D/M)_{e1}^2 m_{s_2}} + \frac{(M_D/M)_{e3}^2 m_{s_1}}{(M_D/M)_{e1}^2 m_{s_3}}, \tag{58}
\]

\[
Y = \frac{m_{e\nu}^e m_{s_1}}{p^2(M_D/M)_{e1}^2}. \tag{59}
\]

Here we have used the expression for $m_{e\nu}^e$ given in eq.(19). In eq.(57), $Y = 0$ gives complete dominance of the singlet fermion exchange term. However this formula of half-life is completely different from the one obtained using inverse seesaw dominance in SO(10) [19]. In this model half-life depends directly to the square of the lightest singlet fermion mass and it is independent of the right-handed neutrino mass which is non-diagonal. But in [19], the half-life of neutrino less double beta decay is directly proportional to the fourth power of the lightest singlet fermion mass and square of the lightest right handed neutrino mass leading into a different result.

A. Half-life in the NH and IH cases

We have computed the half-life for NH and IH patterns of active neutrino masses, taking the contributions of singlet fermion as well as light active neutrino exchanges. This is shown in the left-panel for NH case and in the right panel for IH case in Fig.9.

Taking both $X$ term and $Y$ term in eq.(57), we find that for smaller value of $m_{s_1}$, the contribution due to sterile neutrino is dominated for both NH and IH. But with the increase in the value of $m_{s_1}$, the half-life increases showing its decreasing strength. The predicted half-life curve saturates the
Table 3: Eigen values for singlet fermion mass matrix for different allowed $N - S$ mixing matrix elements for NH, IH, and QD patterns of light neutrino masses

| $M$ (GeV) | $\tilde{m}_s(NH)$ (GeV) | $M$ (GeV) | $\tilde{m}_s(IH)$ (GeV) |
|----------|-------------------------|----------|-------------------------|
| (40,400,1180) | (1.2,502,883) | (40,450,1280) | (0.4,54.32,7702) |
| (100,400,1180) | (7.65,515,909) | (60,450,1280) | (0.9,54.4,7705) |
| (150,400,1180) | (16,533,951) | (70,450,1280) | (1.2,54.4,7706) |
| (200,400,1180) | (25,55,1011) | (100,450,1280) | (2.5,55,7715) |
| (250,400,1180) | (35,588,1093) | (300,450,1280) | (22.56,7831) |
| (300,400,1180) | (43,622,1200) | (400,450,1280) | (36.2,59,7933) |
| (350,400,1180) | (50,659,1331) | (450,450,1280) | (42,64,7996) |

| $M$ (GeV) | $\tilde{m}_s(QD)$ (GeV) |
|----------|-------------------------|
| (100,600,1500) | (0.5,17.7,109) |
| (130,600,1500) | (0.8,17.7,109) |
| (200,600,1500) | (1.97,17.7,109) |
| (300,600,1500) | (4.4,17.7,109) |
| (350,600,1500) | (6.05,17.7,109) |
| (400,600,1500) | (8,17.7,109) |
| (500,600,1500) | (12.3,17.7,109) |
| (600,600,1500) | (17.7,17.7,109) |
experimental data at $m_{S_1} \simeq 3\,\text{GeV}$ and $m_{S_1} \simeq 2\,\text{GeV}$, for NH and the IH cases, respectively. The interesting predictions are that if the lightest sterile neutrino mass satisfies the bound $m_{S_1} \leq 3\,\text{GeV}$, then the $0\nu\beta\beta$ decay should be detected with half-life close to the current experimental bound even if the light neutrino masses have NH pattern of masses. Similarly the corresponding bound for the IH case is $m_{S_1} \leq 2\,\text{GeV}$. But in one of the paper [19] which is inverse seesaw dominant, the corresponding bound for the NH and IH case is $m_{S_1} \leq 14\,\text{GeV}$.

![Figure 9: Variation of half-life of $0\nu\beta\beta$ decay with the sterile neutrino mass for NH(left-panel) and IH(right-panel) of patterns of light active neutrino masses for $|p| = 190\,\text{MeV}$](image)

**B. Lifetime prediction with QD neutrino masses.**

For QD masses of light active neutrinos we considered the $X$ term and $Y$ term of eq.(57) i.e including both the sterile neutrino exchange and light neutrino exchange contributions. For the light-neutrino effective mass parameter occurring in $Y$, we have considered three different cases with common light-neutrino mass values $m_0 = 0.2\,\text{eV}, 0.3\,\text{eV}$, and $0.5\,\text{eV}$ resulting in three different curves shown in the left- and the right- panels of Fig. [10]. In the left-panel only the experimentally determined Dirac phase $\delta = 0.8\pi$ has been included in the PMNS mixing matrix for light QD neutrinos while ignoring the two Majorana phases($\alpha_1 = \alpha_2 = 0$). In the right panel while keeping $\delta = 0.8\pi$ for all the three curves, the Majorana phases have been chosen as indicated against each of them.

As the sterile neutrino exchange amplitude given in eq.(49) is inversely proportional to the eigen value of the corresponding sterile neutrino mass $m_{S_i}$, even in the quasi-degenerate case this contribution is expected to dominate for allowed small values of $m_{S_i}$. This fact is reflected in both the figures.
given in Fig[10]. When Majorana phases are ignored, this dominance gives half-life less than the current bounds for $m_{S_1} < 0.5$ GeV when $m_0 = 0.5$ eV, but for $m_{S_1} < 0.7$ GeV when $m_0 = 0.2 - 0.3$ eV. When Majorana phases are included preventing cancellation between the two contributions, these crossing points are changed to $m_{S_1} < 0.7$ GeV when $m_0 = 0.3$ eV, but $m_{S_1} < 1.0$ GeV when $m_0 = 0.2 - 0.5$ eV. In one of the paper [19] which is inverse seesaw dominant, the corresponding bound for the QD case is $m_{S_1} \leq 12.5$ GeV. The peaks in the half-life prediction in the curves appear because of cancellation between the two effective parameters. Inclusion of Majorana phases annuls cancellation resulting in constructive addition of the two effective mass parameters and reduced values of half-life accessible to ongoing searches. For larger values of $m_{S_1} >> 20$ GeV, the sterile neutrino contribution to $0
u\beta\beta$ amplitude becomes negligible and the usual contributions due to light quasi-degenerate neutrinos are recovered.

8 Summary and conclusion

In this work we have investigated the prospect of having a type-II seesaw dominated neutrino mass generation mechanism in non-SUSY $SO(10)$ GUT by a novel procedure by introducing one additional singlet fermion per generation. Following the popular view that the only meaningful fermion masses in the Lagrangian must have dynamical origins, and taking the non-dynamical singlet fermion mass $\mu_S$ to be negligible, one of the models (Model-I) di-
cussed is found to exhibit type-II seesaw dominance and predicts TeV scale $Z'$ boson accessible to LHC. The would be dominant type-I seesaw contribution to neutrino masses cancels out. The induced contribution to the $\nu - S$ mixing mass term $M_L$ in the model is shown to be damped out because of GUT-scale mass of the LH doublet in $16_H$ that renders the linear seesaw contribution to light neutrino masses negligible in this model. In spite of the high values of the seesaw scale $M_{\Delta L} \simeq 10^8 - 10^9$ GeV $>> M_Z$, the model predicts new dominant contributions to $0\nu\beta\beta$ decay in the $W_L - W_L$ channel mediated by sterile neutrinos which acquire Majorana masses. The predicted LFV decay branching ratios for $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, and $\tau \rightarrow e\gamma$, are found to be accessible to ongoing and planned experiments. We discuss the impact on the resultant effective mass parameter and $0\nu\beta\beta$ half-life showing cancellation between light-neutrino exchange and sterile neutrino exchange contributions. The cancellation occurs because of the opposite signatures of the two effective mass parameters due to light neutrino exchange and the sterile neutrino exchange.

We derive an analytic formula for the half-life of $0\nu\beta\beta$ decay as a function of singlet fermion masses which predicts a lower bound on the lightest sterile neutrino mass eigen value from the current experimental data. We find that the half-life close to the current lower bound or even lower can be easily accommodated even with NH or IH patterns of light neutrino masses. We find that the QD nature of light neutrino masses is not a necessary criteria to satisfy existing lower bounds on the half life estimated by different experimental groups. Even if the light active neutrino masses are NH or IH, a half-life prediction $T_{1/2} \simeq (2 - 5) \times 10^{25}$ yrs is realizable if the lightest sterile neutrino mass $m_{S_1} \simeq 2 - 3$ GeV. Depending upon the common mass of the light QD neutrinos, the model also predicts lifetime $T_{1/2} \leq 2 \times 10^{25}$ yrs for $m_{S_1} \leq (0.5 - 1.0)$ GeV. Large cancellation between the two contributions is found to occur in the quasi-degenerate case of active neutrinos in the regions of sterile neutrino mass $m_{S_1} \simeq 2 - 8$ GeV. The bounds obtained in the sterile neutrino mass in this type-II seesaw dominance model is smaller than that of the bounded obtained in the inverse seesaw model [19].

As the sterile neutrino contribution to the $0\nu2\beta$ decay is inversely proportional to the corresponding mass eigen values, the smallness of the lightest mass eigen values causes dominant contributions compared to those by light neutrinos in NH, IH, and QD cases. For the same region the new contributions are damped out for large sterile neutrino mass eigen values.

Because of the underlying type-II seesaw formula for neutrino masses,
the model predicts different and distinct spectra of RH neutrino masses with specified mixings corresponding to NH, IH and the QD patterns of light neutrino masses which can be also testified at the LHC and future high energy accelerators. The proton lifetime predictions for $p \to e^+\pi^0$ for some regions of the parameter space are also accessible to ongoing experimental searches especially for intermediate mass values of the color octet scalar which has been found to be necessary for gauge coupling unification.

**Acknowledgment**

M. K. P. thanks the Department of Science and Technology, Govt. of India for a research project, SB/S2/HEP-011/2013. B. N. thanks Siksha 'O' Anusandhan University for research fellowship.
8.1 Appendix A

Beta function coefficients for RG evolution of gauge couplings

| Symmetry | $a_i$ | $b_{ij}$ (GeV) |
|----------|------|----------------|
| $G_{213}$ | $(-19/6, 41/10, -7)$ | $(199/50, 27/10, 44/5)$  
|          |      | $9/10, 35/6, 12$ |
|          |      | $11/10, 9/2, -26$ |
| $G_{2113}$ | $(-3, 57/12, 37/8, -7)$ | $(8, 1, 3/2, 12)$  
|           |      | $3/2, 33/57, 63/8, 12$ |
|           |      | $9/2, 63/8, 209/16, 4$ |
|           |      | $9/2, 3/2, 1/2, 26$ |
| $G_{2213}$ | $(-2, -3/2, 29/4, -7)$ | $(31, 6, 39/2, 12)$  
|           |      | $6, 115/6, 3/2, 12$ |
|           |      | $81/2, 6, 181/8, 4$ |
|           |      | $9/2, 9/2, 1/2, -26$ |
| $G_{2213D}$ | $(-3/2, -3/2, 15/2, -7)$ | $(319/6, 6, 57/4, 12)$  
|           |      | $6, 319/6, 57/4, 12$ |
|           |      | $171/4, 171/4, 239/4, 4$ |
|           |      | $9/2, 9/2, 1/2, -26$ |

Table 4: One-loop and two-loop beta function coefficients for gauge coupling evolutions described in the text taking the second Higgs doublet mass at 1 TeV

8.2 Appendix B

Block diagonalization and determination of $\mathcal{M}_\nu$

In this section we discuss the various steps of block diagonalization in order to calculate the light neutrino mass, sterile neutrino mass and right-handed neutrino mass and their mixings. The complete $9 \times 9$ mass matrix in the flavor basis $\{
u_L, S_L, N_R^C\}$ is

$$
\mathcal{M} = \begin{pmatrix}
  m_{\nu}^{II} & M_L & M_D \\
  M_L^T & 0 & M \\
  M_D^T & M^T & M_N
\end{pmatrix}
$$

(60)
where $M_L = y_L v^{}_{X_L}$, $M = y_R v^{}_{X_R}$, $M_N = f^{}_{V_R}$ and $M_D$ is the Dirac neutrino mass matrix as discussed in Sec.4.

Assuming a generalized unitary transformation from mass basis to flavor basis, gives

$$|\psi\rangle_{\text{flavor}} = V |\psi\rangle_{\text{mass}} \quad (61)$$

or

$$\begin{pmatrix} \nu^{}_{\alpha} \\ S^{}_{\beta} \\ N^{}_{\gamma} \end{pmatrix} = \begin{pmatrix} \nu^{}_{\alpha i} & \nu^{}_{\alpha j} & \nu^{}_{\alpha k} \\ \nu^{}_{\beta i} & \nu^{}_{\beta j} & \nu^{}_{\beta k} \\ \nu^{}_{\gamma i} & \nu^{}_{\gamma j} & \nu^{}_{\gamma k} \end{pmatrix} \begin{pmatrix} \hat{\nu}^{}_{i} \\ \hat{S}^{}_{j} \\ \hat{N}^{}_{k} \end{pmatrix} \quad (62)$$

with

$$V^\dagger M^{}_{\nu} = \hat{M} = \text{diag} \left( \hat{M}^{}_{\nu^{}_{\alpha i}}; \hat{M}^{}_{S^{}_{j}}; \hat{M}^{}_{N^{}_{k}} \right) \quad (63)$$

Here $M^{}_{\nu}$ is the $9 \times 9$ neutral fermion mass matrix in flavor basis with $\alpha, \beta, \gamma$ running over three generations of light-neutrinos, sterile-neutrinos and right handed heavy-neutrinos in their respective flavor states and $\hat{M}^{}_{\nu}$ is the diagonal mass matrix with $(i, j, k = 1, 2, 3)$ running over corresponding mass states.

In the first step of block diagonalization, the full neutrino mass matrix is reduced to a block diagonal form $M^{}_{\text{BD}}$ and in the second step we further block diagonalize to obtain the three matrices as three different block diagonal elements, $M^{}_{\text{BD}}= \text{diag}(M^{}_{\nu^{}_{\alpha i}}, m^{}_{S^{}_{j}}, m^{}_{N^{}_{k}})$ whose each diagonal element is a $3 \times 3$ matrix. In our estimation, we have used the mass hierarchy $M^{}_{N} > M \gg M^{}_{D^{}}, M^{}_{L^{}}, f^{}_{v^{}_{L}}$. Finally in the third step we discuss complete diagonalization to arrive at the physical masses and their mixings.

### 8.2.1 Determination of $M^{}_{\text{BD}}$

With two unitary matrix transformations $Q^{}_{1}$ and $Q^{}_{2}$,

$$Q^\dagger M^{}_{\nu} Q^* = \hat{M}^{}_{\text{BD}}, \quad (64)$$

39
where

$$Q = Q_1 Q_2$$  \hspace{1cm} (65)$$

i.e the product matrix $Q = Q_1 Q_2$ directly give $M_{BD}$ from $M_\nu$. Here $\hat{M}_{BD}$, and $M_{BD}$ are the intermediate block-diagonal, and full block-diagonal mass matrices, respectively,

$$\hat{M}_{BD} = \begin{pmatrix} M_{\text{eff}} & 0 \\ 0 & m_N \end{pmatrix}$$ \hspace{1cm} (66)$$

and

$$M_{BD} = \begin{pmatrix} M_\nu & 0 \\ 0 & m_S \\ 0 & 0 & m_N \end{pmatrix}$$ \hspace{1cm} (67)$$

### 8.2.2 Determination of $Q_1$

In the leading order parametrization the standard form of $Q_1$ is

$$Q_1 = \begin{pmatrix} 1 - \frac{1}{2} R^* R^T \\ -R^T \end{pmatrix} \begin{pmatrix} R^* \\ 1 - \frac{1}{2} R^T R^* \end{pmatrix}, \hspace{1cm} (68)$$

where $R$ is a $6 \times 3$ dimensional matrix.

$$R^\dagger = M_N^{-1} (M_D^T, M^T) = (K^T, J^T)$$ \hspace{1cm} (69)$$

$$J = MM_N^{-1} K = M_D M_N^{-1} I = K J^{-1} = M_D M^{-1}$$ \hspace{1cm} (70)$$

Therefore, the transformation matrix $Q_1$ can be written purely in terms of dimensionless parameters $J$ and $K$

$$Q_1 = \begin{pmatrix} 1 - \frac{1}{2} K K^\dagger & -\frac{1}{2} K J^\dagger & K \\ -\frac{1}{2} J K^\dagger & 1 - \frac{1}{2} J J^\dagger & J \\ -K^\dagger & -J^\dagger & 1 - \frac{1}{2} (K^\dagger K + J^\dagger J) \end{pmatrix}$$ \hspace{1cm} (71)$$

while the light and heavy mass matrices are

$$M_{\text{eff}} = \begin{pmatrix} f v_L M_L \\ M_L^T \end{pmatrix} - \begin{pmatrix} M_D M_N^{-1} M_D^T & M_D M_N^{-1} M \\ M_D^T M_N M_D^T & M_D^T M_N^{-1} M \end{pmatrix}$$ \hspace{1cm} (72)$$

$$m_N = M_N + ..$$ \hspace{1cm} (73)
Denoting

\[ M_{\text{eff}} = \begin{pmatrix} Z & B \\ C & D \end{pmatrix} \]  

\[ Z = f v_L - M_D M_N^{-1} M_D^T, \]
\[ B = M_L - M_D M_N^{-1} M, \]
\[ C = M_L^T - M_T M_N^{-1} M_D^T, \]
\[ D = M_T M_N^{-1} M, \]

8.2.3 Determination of \( Q_2 \)

The remaining mass matrix \( M_{\text{eff}} \) can be further block diagonalized using another transformation matrix

\[ S^\dagger M_{\text{eff}} S^* = \begin{pmatrix} M_\nu & 0 \\ 0 & m_s \end{pmatrix} \]  

such that in eq.(8.2.1)

\[ Q_2 = \begin{pmatrix} S & 0 \\ 0 & 1 \end{pmatrix} \]

\[ S = \begin{pmatrix} 1 - \frac{1}{2} P^* P^T \\ -P^T \\ 1 - \frac{1}{2} P^T P^* \end{pmatrix} \]  

Using eq.(81) in eq.(79), we get through eq.(74)-eq.(78),

\[ P^\dagger = (M_T M_N^{-1} M)\left( M_T M_N^{-1} M_D - M_T^T \right) = M^{-1} M_D^T - M^{-1} M_N M^{-1} M_L \]  

where we have used \( y_\chi \) to be symmetric, leading to

\[ M_\nu = m_\nu^H + (M_D M_N^{-1} M_D^T) - (M_D M_N^{-1} M_D^T) + M_L (M_T M_N^{-1} M)\left( M_T M_N^{-1} M_D^T \right) - M_L (M_T M_N^{-1} M)\left( M_T M_N^{-1} M_D^T \right) - (M_D M_N^{-1} M) (M_T M_N^{-1} M)\left( M_T M_N^{-1} M_D^T \right), \]
\[ m_s = -M M_N^{-1} M^T + ..., \]
The $3 \times 3$ block diagonal mixing matrix $Q_2$ has the following form

$$Q_2 = \begin{pmatrix} S & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2} I I^\dagger & 1 - \frac{1}{2} I I^\dagger \\ -I^\dagger & 0 \end{pmatrix}$$

where we have used eq. (70) to define $I = KJ^{-1} = M_D M^{-1}$.

**Complete diagonalization and physical neutrino masses**

The $3 \times 3$ block diagonal matrices $M_\nu$, $m_S$ and $m_N$ can further be diagonalized to give physical masses for all neutral leptons by a $9 \times 9$ unitary matrix $U$ as

$$U = \begin{pmatrix} U_\nu & 0 & 0 \\ 0 & U_S & 0 \\ 0 & 0 & U_N \end{pmatrix}.$$  

where the $3 \times 3$ unitary matrices $U_\nu$, $U_S$ and $U_N$ satisfy

\[
U_\nu^\dagger M_\nu U_\nu^* = \hat{M}_\nu = \text{diag} (M_{\nu 1}, M_{\nu 2}, M_{\nu 3}),
\]

\[
U_S^\dagger m_S U_S^* = \hat{m}_S = \text{diag} (m_{S 1}, m_{S 2}, m_{S 3}),
\]

\[
U_N^\dagger m_N U_N^* = \hat{m}_N = \text{diag} (m_{N 1}, m_{N 2}, m_{N 3}).
\]

With this discussion, the complete mixing matrix is

$$V = Q \cdot U = Q_1 \cdot Q_2 \cdot U = \begin{pmatrix} 1 - \frac{1}{2} K K^\dagger & -\frac{1}{2} K J^\dagger & K \\ -\frac{1}{2} J K^\dagger & 1 - \frac{1}{2} J J^\dagger & J \\ -K^\dagger & -J^\dagger & 1 - \frac{1}{2} (K^\dagger K + J J^\dagger) \end{pmatrix} \begin{pmatrix} 1 - \frac{1}{2} I I^\dagger & I & 0 \\ -I^\dagger & 1 - \frac{1}{2} I I^\dagger & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} U_\nu & 0 & 0 \\ 0 & U_S & 0 \\ 0 & 0 & U_N \end{pmatrix}$$

\[
= \begin{pmatrix} 1 - \frac{1}{2} I I^\dagger & I - \frac{1}{2} K J^\dagger & K \\ -I^\dagger & 1 - \frac{1}{2} (I I^\dagger + J J^\dagger) & J - \frac{1}{2} I I^\dagger \\ 0 & -J^\dagger & 1 - \frac{1}{2} J J^\dagger \end{pmatrix} \cdot \begin{pmatrix} U_\nu & 0 & 0 \\ 0 & U_S & 0 \\ 0 & 0 & U_N \end{pmatrix} \]

\[ (87) \]

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