The effect of increased deformability of columns on the resistance to progressive collapse of buildings

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Abstract. The article discusses two constructive schemes of the building: two 20-storey buildings, in one model, columns with indirect reinforcement were used, in the other model, longitudinally reinforced columns were used. The calculation was carried out in the software complex Lira SAPR 2015. In the calculation, the finite element method was used in the form of the displacement method. The transition from the continual real model of structures to a discrete computational scheme was carried out by dividing the model into finite element grids with a step of no more than three element thicknesses, the number of degrees of freedom of FE is six. The limit state of reinforced concrete, which characterizes the work of structures in the stage of destruction, the “special” limit state, is considered. A particular limit state is characterized by partial destruction of the structure, but in which there is a resource of residual deformation, necessary for the redistribution of efforts arising from an emergency. Analyzing the calculations of the deformation and strength characteristics of the material with indirect reinforcement according to the normative document, it can be concluded that the characteristics of the material correspond to the operation of the structures when the “special” limit state is reached. The sufficient convergence of the data of the results of research of elements with indirect reinforcement and the theoretical results of calculation according to the normative document is established, which allowed in the modeling to make full use of the resources of elements with indirect reinforcement.

1. Introduction

Currently, the task of protecting buildings from progressive collapse becomes the most urgent, in view of the increasing pace of construction and design of more and more new and unique structures. Progressive collapse has major economic and social consequences.

The modern desire to understand the nature of progressive collapse, minimize damage, forces engineers to develop new methods for ensuring the sustainability of buildings: increasing the bearing capacity of elements when the purpose of the calculation is to increase the resistance of a specific element; increasing the degree of static indeterminacy; calculation methods that take into account the physical and geometric non-linearity of the structures when the “key element” is turned off; calculation methods taking into account the dynamic hardening of materials; increasing the plasticity of structures; the introduction of high strength materials; application of indirect reinforcement. The method of exclusion from the work of the “key element” is by far the most reasonable from the point of view of modeling progressive destruction. Engineers are facing another challenge – the economic feasibility. To solve this problem, it is necessary to adequately understand and analyze the stress-strain state of structures.

Analyzing the results of numerous studies, the loss of bearing capacity is associated with the achievement of ultimate strains in compressed concrete [1-6]. One of the most effective way to increase the bearing capacity and deformability of concrete is indirect reinforcement [7-11]. Rational
design of reinforced concrete elements with indirect reinforcement reduces the consumption of concrete and reinforcement. It is established that the transverse arrangement of reinforcement is very effective in compressing elements with random and small eccentricities. Indirect reinforcement of compressed reinforced concrete elements increases the resistance to transverse deformations due to the adhesion of grids with concrete [12]. When calculating for progressive destruction, an increase in the deformability of compressed elements allows to fully incorporate neighboring elements into the work and to maximally redistribute efforts from the element being excluded from operation.

The study of the work of elements with the use of indirect reinforcement in the protection of buildings from progressive collapse will allow a greater degree of use of reserves of structures and to avoid large damage.

2. Methodology

2.1. Analysis of the design scheme

The dimensions of the building in study were 50.4 x 18 m, height is 60 m. The building includes reinforced concrete stiffening cores mounted from walls 200 mm thick. The building is a frame. Vertical load-bearing elements – monolithic reinforced concrete columns and walls are rigidly connected with flat beamed slabs (Figure 1).

Staircases with monolithic marches and intermediate landings, of concrete class B30, W4, F100. The class of concrete for all above-ground load-bearing structures is not lower than B25, for underground bearing structures it is not lower than B30, the reinforcement class is A500C. Spatial rigidity, stability, and geometric immutability in both directions are ensured by the joint operation of columns rigidly clamped at the foundation level, with floor slabs and rigidity cores. The columns were modeled with core elements (elements N10). Slabs, walls were modeled with flat FE (elements N44).

The loads were set in accordance with the regulatory document. The calculation of the content of reinforcement in slabs, columns and walls was made by the DCF (design combinations of forces), based on the calculated values of the specified loads [13]. In this case, in the slabs, the orthogonal arrangement of the working reinforcement was taken.

According to the results of the static calculation, columns were selected with the section of 800 x 800 mm with reinforcement percentage of 2%.
Structural calculations have shown that the structural system of a building meets the requirements of regulatory documents for limiting maximum draft, relative difference in draft, horizontal movements of the building’s top, deflections of its floor slabs, as well as the requirement to ensure the stability of the building. According to the results of the static calculation, the most loaded element was determined. When modeling progressive collapse, this element will be excluded from the design scheme as a suddenly dismountable structure.

2.2. Determination of dynamic coefficient
Values were determined by the method of [14]:
\[ \gamma = \frac{K_{pl}}{K_{pl} - 0.5}, \]
where \( K_{pl} \) is the coefficient of plasticity, equal to the ratio of the total deflection of the element to the limiting elastic.

\[ K_{pl} = \frac{\varepsilon_{bud} \cdot \alpha_d \cdot E_s \cdot (0.78 - \varepsilon_d)}{(R_{sd} + 0.002 \cdot E_s) \cdot \varepsilon_d}, \]
where \( \varepsilon_{bud} \) – the relative deformation of concrete under central compression; \( \varepsilon_{bud} = 0.002 \); \( R_{sd} \) – concrete stress at dynamic loading. \( R_{sd} = R_{sd} \cdot \gamma_{sv}^{*} ; R_{sd} \) – the standard value of concrete resistance to compression; \( \gamma_{sv}^{*} \) – coefficient of dynamic hardening of concrete in compression. for reinforced concrete elements with a calculated reinforcement of compressed zone \( \gamma_{sv}^{*} = 1,1 \), \( R_{sd} = 22MPa \cdot 1,1 = 24,2MPa \); \( E_s \) – modulus of elasticity of reinforcement; \( E_s = 2 \cdot 10^5 MPa \); \( R_{sd} \) – the calculated value of the resistance of the reinforcement to tension under dynamic loading. \( R_{sd} = R_{sd} \cdot \gamma_{sy}^{*} \cdot R_{sd}, R_{sd} \) – the standard value of the resistance of the reinforcement to tension; \( \gamma_{sy}^{*} \) – coefficient of dynamic hardening of reinforcement under tension, \( \gamma_{sy}^{*} = 1,1 \) for reinforcement with \( R_{sd} = 400 \text{ MPa} \) and \( R_{sd} = 500 \text{ MPa} \); \( R_{sd} = 500 \text{ MPa} \cdot 1,1 = 550 \text{ MPa} \); \( \alpha_d \) – the coefficient of completeness of the stress profile of the compressed zone of concrete, \( \alpha_d = 0,85 - 0,006 \cdot R_{sd} = 0,7 \);

\( \varepsilon_{bud} \) – edge relative deformations of compressed concrete

\[ \varepsilon_{bud} = \frac{\varepsilon_{bud} \cdot 0.002}{\left(1 - \alpha_d \right) \frac{0.7}{1.1}} = 0.006; \]

\( \varepsilon_d = \frac{x_d}{h_0} \) – the relative height of the compressed zone of concrete under dynamic loading is determined by SNiP with dynamic resistance of reinforcement to tension \( R_{sd} \), compression \( R_{scd} \) and concrete \( R_{sd} \), calculated as a product of dynamic hardening coefficients and standard resistances of materials

\[ R_{scd} = \gamma_{sv}^{*} \cdot R_{sc}. \]

For reinforcement with \( R_{on} = 500 \text{ MPa}, R_{sc} = 450 \text{ MPa}, \gamma_{sv}^{*} = 1,0 \) \( \varepsilon_{bud} = 0,12 \).

\[ K_{pl} = \frac{0.006 \cdot 0.7 \cdot 2 \cdot 10^5 \cdot (0.78 - 0.12)}{(550 + 0.002 \cdot 2 \cdot 10^3) \cdot 0.12} = 4,54. \]

\[ \gamma = \frac{4,54}{4,54 - 0,5} = 1,12. \]
However, when using this method of calculation it is necessary to take into account empirical coefficients [15]. One of the requirements of its application is uniform reinforcement of slabs. This approach to determining the dynamic coefficient does not take into account the dynamic characteristics of the structure.

Also, a number of calculations were carried out, taking into account the dynamic factor $DIF = 2$, in accordance with Eurocode 3.02-108-2008 (Figure 2).

After a series of calculations, we obtained the value of the dynamic coefficient equal to 1,12. Analyzing Eurocode 3.02-108-2008, where the value of the dynamic factor is assumed to be 2, it is pre-proposed to perform a number of calculations with the dynamic factor of 1, and the dynamic factor equal to 2, as the worst possible events.

2.3. Accounting for indirect reinforcement based on non-linear deformation model

Mesh indirect reinforcement due to adhesion with concrete resist its transverse tensile deformations under the action of longitudinal compressive forces. In the rods of the meshes, tensile stresses, of course, arise, but they are caused by the forces of adhesion to concrete.

The force perceived by the mesh may be different depending on its thickness, determined by the surface of the clutch, which, in turn, depends on the diameter of the mesh rods and on the total length. The limiting resistance of rebar meshes to transverse deformations depends on the rebar type (smooth, periodic profile), the type of concrete, the technological features of its preparation, as well as the conditions for connecting the rods (for example, in welding).

Undoubtedly, in a reinforced concrete element working on axial compression, all meshes perceive the same efforts. The increase in the bearing capacity of elements due to indirect reinforcement is taken into account by means of the reduced prism strength of concrete $R_{b,red}$ instead $R_b$; the total cross-sectional area $A$ is replaced by the core area $A_{ef}$ (cross-sectional area in the axes of the outermost rods). The increase in bearing capacity occurs until the stress in the bars of the stress meshes reaches the standard resistance of the reinforcement $R_{sn}$.

Coefficient of volume reinforcement $R_{b,red} = R_b + \varphi \cdot \mu_{xy} \cdot R_{s,xy}$,

where $\mu_{xy} = \frac{n_x \cdot A_{st} \cdot I_x + n_y \cdot A_{sy} \cdot I_y}{A_{ef} \cdot s}$.

Coefficient of effectiveness of indirect reinforcement $\varphi = \frac{1}{0.23 + \alpha_{red} \cdot \varphi}$.

Coefficient of reduction $\alpha_{red} = \frac{\mu_{xy} \cdot R_{s,xy}}{R_b + 10}$,

where $R_{s,xy}$ – design resistance to tensile reinforcement meshes.
Limit deformations:
\[ \varepsilon_{b0,\text{red}} = \varepsilon_{b0} + 0.002 \cdot \alpha_{\text{red}}, \]
\[ \varepsilon_{b2,\text{red}} = \varepsilon_{b2} \cdot \frac{\varepsilon_{b0,\text{red}}}{\varepsilon_{b0}}. \]

The stiffness characteristics with indirect reinforcement in the normal section of elements for determining the deformations of concrete and reinforcement are determined by the formulas of the set of rules.

3. Results

3.1. Numerical research. Accounting for the convergence of empirical data with the normative document

Analysis of the results of empirical data of work [16]: in samples were tested with indirect reinforcement values equal to: \( \mu_{xy} = 0.054, \mu_{xy} = 0.031, \mu_{xy} = 0.02, \mu_{xy} = 0 \) on the actions of static and dynamic loads. The samples had a cross section of 20x20cm, height 80cm. The strength of concrete was adopted equivalent to the normative strength of concrete class B40. The indirect reinforcement meshes are made of A400 class reinforcement with a diameter of 8 mm with a yield strength of 410 MPa, the cell size is a = 43 cm.

According to [16], for these experimental results, the dependencies between stress and strain will be used:
\[ \sigma_b = \frac{N}{A_b} = \frac{\varepsilon_b}{E_b}, \]
\[ \sigma_{bx} = \frac{B_{bx} - V_{bp} \cdot (\mu_{xy} \cdot R_{xy} + \mu_{y} \cdot R_{y})}{V_{by} \cdot E_{bo}}, \]
where \( \sigma_{bx} = \frac{N}{A_b}, \sigma_{x0} = \mu_{xy} \cdot R_{xy} + \mu_{y} \cdot R_{y} = \mu_{xy} \cdot R_{xy} \) (when \( R_{xy} = R_{y} \)).

Stress versus Strains compressed concrete was written in the form:
\[ \sigma_v = \frac{\sigma_b}{E_b}, \]
\[ \frac{\sigma_b}{\varepsilon_b} = \frac{R_{b,\text{red}}}{\gamma_{b,\text{red}}} \cdot \gamma_{b,\text{red}} = \frac{R_{b,\text{red}}}{\gamma_{b,\text{red}}}, \]
where \( \gamma_{b,\text{red}} = \frac{R_{b,\text{red}}}{\gamma_{b,\text{red}}} \), \( \gamma_{b,\text{red}} = \frac{R_{b,\text{red}}}{\gamma_{b,\text{red}}}. \)

Values \( R_{b,\text{red}} \) and \( \gamma_{b,\text{red}} \) and are found by experimental dependences (2.3) and (2.4).

The dependences used allow us to determine the coefficient for concrete with indirect reinforcement meshes operating in the plastic stage.

For meshes operating in the elastic zone, other dependencies are valid; wherein
\[ |\varepsilon_{xy}| \leq \frac{410}{2 \cdot 10^6} = 2 \cdot 10^{-3}. \]

The sample section area \( A_b = 0.20^2 = 0.04 \text{ m}^2 \), taking into account the protective layer of concrete 20 mm, the section area of the concrete core is \( A_b = 0.025 \text{ m}^2 \).

Using the calculations [17, 18] the modulus of elasticity of concrete using indirect reinforcement grids can be determined at the site where the element is elastically deformed:
\[ \sigma_{bx} = \frac{N}{E_b} \cdot \varepsilon_{b}, \quad \varepsilon_{b} = 4 \cdot 10^{-3}, \quad \sigma_{bx} = \frac{400}{0.004} = 10000 \text{ kN/m}^2 = 10 \text{ MPa}, \quad \varepsilon_{b0} = \frac{10}{4 \cdot 10^{-3}} = 25 \cdot 10^3 \text{ MPa}. \]

Sample calculation with indirect reinforcement \( \mu_{xy} = 0.031 \).
For an element with such coefficient of indirect reinforcement we accept \( \nu_b = 0,25 \). To determine the initial parameter, we analyze the graph in figure 3.

![Figure 3](image-url)

**Figure 3.** Experimental data of the concrete compression diagram for samples with different content of mesh reinforcement:

1' – \( \mu_{\nu_0} = 0,054 \),

2' – \( \mu_{\nu_0} = 0,031 \),

3’ – \( \mu_{\nu_0} = 0,02 \),

4' – \( \mu_{\nu_0} = 0 \).

\[ N_{\text{max}} = 1470kN, \quad \sigma_{bx} = \frac{1470}{0,0256} = 57,42MPa, \quad \varepsilon_{b0} = 10,9 \times 10^{-3}, \quad \sigma_{d0} = 0,031 \cdot 410 = 12,71MPa. \]

Stresses in mesh reinforced concrete \( \bar{\sigma}_{bx} = 57,42 - 0,25 \cdot 12,71 = 54,24MPa. \)

Coefficient of elasticity at the end of the ascending branch

\[ \nu_{b0} = \frac{\bar{\sigma}_{bx}}{E_{b0} \cdot \varepsilon_{b0}} = \frac{54,24}{25 \times 10^3 \cdot 10,9 \times 10^{-3}} = 0,2. \]

Finding the strain value on the ascending branches at \( N_{\text{max}} = 1410kN. \)

\[ \sigma_{bx} = \frac{1410}{0,0256} = 55,08MPa, \quad \bar{\sigma}_{bx} = 55,08 - 0,25 \cdot 12,71 = 51,9MPa, \quad \eta = \frac{51,9}{55,08} = 0,942. \]

\[ \nu_0 = 1, \quad \omega_1 = 2 - 2,5 \nu_{b0} = 2 - 2,5 \cdot 0,2 = 1,5, \quad \omega_2 = -0,5, \]

\[ \nu_{b} = 0,2 + (1 - 0,2) \sqrt{1 - 1,5 \cdot 0,942 - 0,5 \cdot 0,942^2} = 0,194, \quad \varepsilon_{bx} = \frac{51,9}{25 \times 10^3 \cdot 0,194} = 10,70 \times 10^{-3}. \]

According to the experimental data, the deformations at \( N_{\text{max}} = 1410kN \) amounted to \( \varepsilon_{b} = 10,1 \times 10^{-3} \), the difference between the experimental data and the analytical calculation was 5,9%.

Finding the strain value on the downward branches at \( N_{\text{max}} = 1400kN. \)

\[ \sigma_{bx} = \frac{1400}{0,0256} = 54,7MPa, \quad \bar{\sigma}_{bx} = 54,7 - 0,25 \cdot 12,71 = 51,52MPa, \quad \eta = \frac{51,52}{54,7} = 0,942. \]

\[ \nu_0 = 2,05 \nu_{b0} = 0,413, \quad \omega_1 = 1,95 \nu_{b0} - 0,138 = 0,252, \quad \omega_2 = 0,748 \]

\[ \nu_{b} = 0,2 + (0,413 - 0,2) \sqrt{1 - 0,252 \cdot 0,942 - 0,748 \cdot 0,942^2} = 0,267, \quad \varepsilon_{bx} = \frac{51,52}{25 \times 10^3 \cdot 0,267} = 7,72 \times 10^{-3}. \]

According to the experimental data, the deformations at \( N_{\text{max}} = 1400kN \) amounted to \( \varepsilon_{b} = 8,65 \times 10^{-3} \), the difference between the experimental data and the analytical calculation was 10,7%. 
Finding the strain value on the downward branches at $N_{\text{max}} = 1330kN$.

$$\sigma_{bx} = \frac{1330}{0.0256} = 51.95\text{MPa}, \quad \bar{\sigma}_{bx} = 51.95 - 0.25 \cdot 12.71 = 48.77\text{MPa}, \quad \eta = \frac{48.77}{51.95} = 0.939.$$ 

$$\nu_0 = 2.05 \nu_{\beta 0} = 0.413, \quad \omega_1 = 1.95 \nu_{\beta 0} - 0.138 = 0.252, \quad \omega_2 = 0.748,$$

$$\nu_b = 0.2 + (0.413 - 0.2) \sqrt{1 - 0.252 \cdot 0.939 - 0.748 \cdot 0.939^2} = 0.232.$$ 

$$\varepsilon_{bx} = \frac{48.77}{25 \cdot 10^3 \cdot 0.232} = 8.41 \cdot 10^{-3}.$$ 

According to the experimental data, the deformations at $N_{\text{max}} = 1330kN$ amounted to $\varepsilon_{\beta} = 8.11 \cdot 10^{-3}$ the difference between the experimental data and the analytical calculation was 3.7%.

Analysis of the calculation of the characteristics of the element with indirect reinforcement according to normative document: $\mu_{xy} = 0.031$,

$$\alpha_{rod} = \frac{0.031 \cdot 410}{22 + 10} = 0.397,$$

$$\varphi = \frac{1}{0.23 + 0.397} = 1.595.$$ 

$$R_{b,\text{rod}} = 22 + 1.595 \cdot 0.031 \cdot 410 = 42.27\text{MPa}.$$ 

$$\varepsilon_{\beta 0,\text{rod}} = 0.002 + 0.002 \cdot 0.326 = 0.00852, \quad \varepsilon_{\beta 2,\text{rod}} = 0.0035 \cdot \frac{0.00852}{0.002} = 0.01491.$$ 

According to the results of the calculation for the normative document, a three-line diagram of concrete operation with indirect reinforcement was constructed (Figure 4).

3.2. Stress analysis

We find the stress value at the end of the ascending branch at $N_{\text{max}} = 1470kN$.

Stresses in reinforced concrete $\delta_{b,\text{max}} = 57.42 - 0.25 \cdot 12.71 = 54.24\text{MPa}.$

According to the calculations for the normative document, the limiting stresses are $R_{b,\text{rod}} = 51.75\text{MPa}.$

The results of numerical studies have shown sufficient convergence with the calculations according to the normative document.

4. Conclusions

1. The difference between the results of the calculation of deformations on the normative document and experimental data is valid.

2. Numerous test results of reinforced concrete elements with indirect reinforcement prove that indirect reinforcement meshes create a volumetric stress state in concrete, which increases the ultimate deformations and the carrier item ability. It is established that the destruction occurs due to the destruction of the protective layer of concrete, and the achievement of indirect reinforcement of the yield strength in the meshes.
3. When calculating the resistance to progressive collapse, it was proved that due to the increased deformability of the element with indirect reinforcement meshes, the element has the necessary residual deformation for complete redistribution effort. Due to the possibility of redistribution of forces, the elements in the zone of impact of the emergency load are not destroyed, which allows the rest of the circuit structures to remain in the design position.

4. The method of calculating columns with increased deformability is to use modern methods of calculation, which allow to take into account the non-linear operation of the structure, the correct modeling of the structure with indirect reinforcement; empirical comparison calculation, which allow to take into account the non-linear operation of the structure, the correct modeling of the structure with indirect reinforcement; empirical comparison data with theoretical studies led to the conclusion that in the method of calculation it is necessary to use the dependencies of the characteristics of the element with indirect reinforcement, described in the normative document.

References:
[1] Tamrazyan A and Avetisyan L 2016 MATEC Web of Conferences 86 01029.
[2] Tamrazyan A G and Avetisyan L A 2016 Procedia Engineering 153 pp 721-725
[3] Tamrazyan A and Avetisyan L 2014 Applied Mechanics and Materials 638 pp 62-65
[4] Tamrazyan A 2014 Applied Mechanics and Materials 475 pp 1563-1566
[5] Kabantsev O V and Tamrazian A G 2014 Magazine of Civil Engineering 49(5) pp 15-26
[6] Rastorguev B S and Vanus D S 2014 Construction and Renovation 6(56) pp 83-89
[7] Long Y and Cai J 2013 Journal of Constructional Steel Research 88 pp 1-14
[8] Attard M and Samani A K 2012 Eng. Struct. 41 pp 335-349
[9] Krishan A L, Troshkina E A, Rimshin V I, Rahmanov V A and Kurbatov V L 2016 Research Journal of Pharmaceutical, Biological and Chemical Sciences 7(3) pp 2518-2529
[10] Binici B 2005 Eng. Struct. 27 pp 1040-1051
[11] Sfer D, Gettu R and Etse G 2002 J. Eng. Mech. 128 p156-163
[12] Vanus D S 2011 The use of indirect mesh reinforcement to increase the rigidity and crack resistance of reinforced concrete elements. Dissertation of the candidate of technical sciences PhD Thesis (Moscow: Moscow State University of Civil Engineering) p 186
[13] Perelmuter A V, Kriskunov E Z and Mosina N V 2009 Magazine of Civil Engineering 2 pp13-18
[14] Popov N N, Rastorguev B S and Zabegaev A V 1992 Design calculation for dynamic special loads (Moscow: Vysshaya Shkola) p 319
[15] Trekin H H, Popov H H and Matkov N G 1986 Beton I Zhelezobeton [Concrete and Reinforced Concrete] 11 pp 33-34
[16] Vasiliev A P, Matkov N G and Filippov B N 1973 Beton I Zhelezobeton [Concrete and Reinforced Concrete] 4 pp 101-111
[17] Gvozdev A A 1949 Calculation of the bearing capacity of structures according to the method of limiting equilibrium (Moscow: Gosstroizdat) p 280
[18] Kolchunov V I and Savin S Yu 2018 Survivability criteria for reinforced concrete frame at loss of stability Magazine of Civil Engineering 80(4) pp 73-80

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