An inventory model with self-generation of priorities and retrial of customers

Jomy Punalal, S. Babu
Department of Mathematics, University College, Kerala, India
E-mail: jomypunalal@gmail.com, drbabu.s@gmail.com

Abstract. This paper mainly focuses on a single server retrial inventory system. The arrival of customers follows the Poisson process, and when the inventory level depletes to the reorder level $s$ due to service, replenishment order placed according to $(s,Q)$ policy. The lead-time governed by an exponential distribution with parameter $\beta$. A customer who finds the server busy or inventory dry moves to an orbit of infinite capacity with probability $\sigma$. Customers in orbit independently tries to access the server at a rate of $\theta$, which depends on the number of customers in orbit. A retrial customer returns to the orbit with probability $\delta$ if the server is busy or inventory is dry. Each customer in orbit generates priority according to an exponential distribution with rate $\gamma$. If the server is free, a priority generated customer will get immediate service. Else such a customer have to wait in a space $A_1$ of capacity one, which is reserved only for priority generated customers. The service times follows exponential distribution with different rates for ordinary and priority generated customers. The model turned out to be a level-dependent quasi birth-death (LDQBD) process, and we use Matrix-Analytic Method as a tool for obtaining a solution.

Keywords. Self-generation of priorities, Inventory with service time, Positive lead time, Retrial queue of customers, Matrix Analytic Method.

1. Introduction

There are abundance of probability models on priority queues in literature (Gross and Harris [1] chapter 3, Jaiswal [2] chapters 3 and 7, and Takagi [3] chapter 3). Studies on priority queues found many applications in health care systems Brahimi [4], Taylor [5]. Some external priority rules used in all of these models. In many applied areas, this approach is inadequate. The concept of self-generation of priorities of customers in queues is coined by Gomez-Corral, Krishnamoorthy, and Viswanath [6, 7]. The preliminary works on self-generation of priorities are done by Krishnamoorthy, S. Babu, and Viswanath [8, 9]. The idea of retrial queues are explored extensively in a survey by Yang [10] and Falin [11]. A book by Falin and Templeton [12] discussed all concepts of retrial queues revealed at that time. Along with the concept of self-generation of priorities, S. Babu discussed some models with the retrial of customers [13, 14]. Jomy Punalal and S. Babu extend this work with customer induced interruption in [15]. Inventory is a physical stock of items to meet future demands. A voluminous number of deterministic inventory systems can found in Churchman [16], Hadley and Whitin [17], Naddor [18] and Sahin [19]. An inventory model with positive service time can found in the survey [20], a discussion [21] and a comparison of three such models [22] are innovative works of Krishnamoorthy and K.P. Jose.
2. Model Description

In this paper, we discuss a retrial inventory model with special emphasize on self-generation of priorities. The first section of the paper contains an accurate description of the model. Mathematical formulation, matrix transitions and system stability discussed in succeeding sections. Some essential system characteristics derived, and we define an expected total profit function with the help of these measures. Different system parameters in profit function and its effect in various performance measures discussed and optimum \((s, S)\) pair also calculated. Results are shown graphically in the last section.

The arrival of customers to a retrial inventory follows a Poisson process with parameter \(\lambda\). When the level of inventory depletes to the reorder level \(s\) due to service of the single server, a replenishment order placed according to \((s, Q)\) policy. The order quantity is \(Q = S - s\). The lead-time ruled by an exponential distribution with parameter \(\beta\). The customer will move to an orbit of infinite capacity with probability \(\sigma\) if the customer finds the busy server or dry inventory. Otherwise, the customer is lost with probability \(1 - \sigma\). Customers in orbit independently tries to access the server with rate \(\theta\) in such a fashion that the rate of a retrial is \(i\theta\) provided there are \(i\) customers in orbit. A retrial customer returns to the orbit with probability \(\delta(< 1)\) if the server busy or inventory dry. Otherwise, such a customer will be lost forever with probability \(1 - \delta\). Each customer in orbit generates priority according to an exponential distribution with rate \(\gamma\). If the server is free, the priority generated customer will get immediate service. Else, such customer have to wait in a waiting space \(A_1\) of capacity one, which reserved for priority generated customers. The duration of service times of ordinary customer follows an exponential distribution with parameter \(\mu_1\), and that of priority generated customer follows an exponential distribution with parameter \(\mu_2\). Pictorial representation of the model shown in Figure 1.

Figure 1. Pictorial representation of the model
3. Mathematical formulation

This mathematical model gives a level dependent quasi birth-death (LDQBD) process and for further analysis, we use the following notations:

\[ N_1(t) = \text{Number of customers in the orbit at time } t, \]

\[ C(t) = \begin{cases} 
0, & \text{server is idle at time } t, \\
1, & \text{server busy with ordinary customer at time } t, \\
2, & \text{server busy with priority generated customer at time } t,
\end{cases} \]

\[ N_2(t) = \text{Number of priority generated customers waiting for service at time } t, \]

\[ I(t) = \text{Inventory level at time } t. \]

Here \( \{\chi(t) : t \geq 0\} \) where \( \chi(t) = \{N_1(t), C(t), N_2(t), I(t)\} \) form a continuous time Markov chain on the state space \( \Omega = L_1(i) \cup L_2(i) \), where

\[ L_1(i) = \{(i, 0, j, k) : i \geq 0; j = 0, 1; k = 0, 1, 2, \cdots, S\}, \]

\[ L_2(i) = \{(i, c, j, k) : i \geq 0, c = 1, 2; j = 0, 1; k = 0, 1, 2, \cdots, S\}. \]

We partition the state space according to the levels concerning the number of customers in orbit, the infinitesimal generator of the above Markov process takes the form:

\[
Q = \begin{bmatrix}
A_{10} & A_0 & \cdot & \cdot & \cdot & \cdot \\
A_{21} & A_{11} & A_0 & \cdot & \cdot & \cdot \\
& A_{22} & A_{12} & A_0 & \cdot & \cdot \\
& & & A_{2N} & A_{1N} & A_0 \\
& & & & A_{2N+1} & A_{1N+1} \\
& & & & & \cdot \\
& & & & & \cdot \\
\end{bmatrix}
\]

where \( A_0, A_{10}, A_{21}, A_{11}, \) \( i = 1, 2, 3, \cdots \) are matrices of order \( (6S + 2) \times (6S + 2) \). The details of the matrix transitions can be summarized as follows:

3.1. Matrix entries of \( A_{10} \) are the transition rates within level 0.

Let \( c = 1, 2; j = 0, 1; k_1 = 1, 2, 3, \cdots, S; k_2 = 0, 1, 2, 3, \cdots, S; k_3 = 1, 2, 3, \cdots, s; k_4 = 0, 1, 2, 3, \cdots, s; k_5 = s + 1, s + 2, \cdots, S; \)

- \( (0, 0, j, k_1) \xrightarrow{\lambda} (0, 1, j, k_1) \)
- \( (0, 0, j, s - k_4) \xrightarrow{\beta} (0, 0, j, S - k_4) \)
- \( (0, c, j, s - k_3) \xrightarrow{\beta} (0, c, j, S - k_3) \)
- \( (0, 0, j, k_3) \xrightarrow{-\lambda - \beta} (0, 0, j, k_3) \)
- \( (0, 0, j, k_5) \xrightarrow{-\lambda} (0, 0, j, k_5) \)
- \( (0, c, j, k_4) \xrightarrow{-\lambda \sigma - \beta} (0, c, j, k_4), c = 0, 1, 2 \)
- \( (0, c, j, k_5) \xrightarrow{-\lambda \sigma} (0, c, j, k_5) \)

3.2. Matrix entries of \( A_0 \) are the transition rates from level \( i \) to \( (i + 1) \).

Let \( j = 0, 1; c = 1, 2; k_1 = 1, 2, 3, \cdots, S \)

- \( (i, 0, j, 0) \xrightarrow{\lambda \sigma} (i + 1, 0, j, 0) \)
3.3. Matrix entries of $A_{2i}$ are the transition rates from level $i$ to $(i - 1)$.
Let $c = 1, 2; j = 0, 1; k_1 = 1, 2, 3, \cdots, S; k_2 = 0, 1, 2, 3, \cdots, S; k_3 = 1, 2, 3, \cdots, s; k_4 = 0, 1, 2, 3, \cdots; s; k_5 = s + 1, s + 2, \cdots, S$.

- $(i, c, j, k_1) \xrightarrow{\lambda\sigma} (i + 1, c, j, k_1)$

3.4. Matrix entries of $A_{1i}$ are transitions within level $i$.
Let $c = 1, 2; j = 0, 1; k_1 = 1, 2, 3, \cdots, S; k_2 = 0, 1, 2, 3, \cdots, S; k_3 = 1, 2, 3, \cdots, s; k_4 = 0, 1, 2, 3, \cdots; s; k_5 = s + 1, s + 2, \cdots, S$.

- $(i, c, 0, k_1) \xrightarrow{\mu_c} (i, c, 0, k_1 - 1)$
- $(i, c, 1, k_1) \xrightarrow{\mu_c} (i, 2, 0, k_1 - 1)$
- $(i, 0, j, k_1) \xrightarrow{\lambda} (i, 1, j, k_1)$
- $(i, 0, j, s - k_4) \xrightarrow{\beta} (i, 0, j, S - k_4)$
- $(i, c, j, s - k_3) \xrightarrow{\beta} (i, c, j, S - k_3)$
- $(i, 0, j, 0) \xrightarrow{-\lambda\sigma - \beta - i\theta(1 - \delta) - i\gamma} (i, 0, j, 0)$
- $(i, 0, j, k_3) \xrightarrow{-\lambda - \beta - i\theta - i\gamma} (i, 0, j, k_3)$
- $(i, 0, j, k_5) \xrightarrow{-\lambda - i\theta - i\gamma} (i, 0, j, k_5)$
- $(0, c, j, k_3) \xrightarrow{-\lambda\sigma - \mu_c - \beta - i\theta(1 - \delta) - i\gamma} (0, c, j, k_3)$
- $(0, c, j, k_5) \xrightarrow{-\lambda\sigma - \mu_c - i\theta(1 - \delta) - i\gamma} (0, c, j, k_5)$

4. Stability of System

**Theorem:** The above described system is stable.

**Proof:** To prove the stability, we consider Lyapunov test function defined by $\phi(r) = i$ where $r$ is a state in level $i$. For a state $r$ in level $i$, the mean drift

$$y_r = \sum_{p \neq r} [\phi(p) - \phi(r)] q_{rp},$$

$$= \sum_{r'} [\phi(r') - \phi(r)] q_{rr'} + \sum_{r''} [\phi(r'') - \phi(r)] q_{rr''} + \sum_{r'''} [\phi(r''') - \phi(r)] q_{rr'''},$$
where \( r', r'', r''' \) vary over states belonging to levels \( i - 1, \ i \) and \( i + 1 \) respectively. Then 
\[
\phi(r) = i, \ \phi(r') = i - 1, \ \phi(r'') = i, \ \phi(r''') = i + 1,
\]
\[
y_r = -\sum_{r'} q_{rr'} + \sum_{r''} q_{rr''},
\]
\[
y_r = \begin{cases} -i\gamma - i\theta, & \text{when server is idle and inventory not dry}, \\ -i\gamma - i\theta(1 - \delta) + \lambda\sigma, & \text{otherwise}. \end{cases}
\]

As \( (1 - \delta) > 0 \), for any \( \epsilon > 0 \), we can find \( K^* \) large enough that \( y_r < -\epsilon \) for any \( r \) belonging to level \( i \geq K^* \). Thus the theorem follows from Tweedie’s result [23].

5. Stationary Probability vectors

Assuming \( Q \) is irreducible and the steady state probability vector of the process \( \chi \) be
\[
\xi = (\xi_0, \xi_1, \xi_2, \cdots). \]
Now by a theorem from Neuts[24] \( \xi \) satisfies the relation \( \xi_{N+k} = \xi_{N-1}R^{k+1} \), \( k \geq 0 \) where \( R \) is the required solution of the matrix quadratic equation \( A_0 + RA_1 + R^2A_2 = 0 \) with \( A_1 = A_{1N}, \ A_2 = A_{2N} \). When the number of retrying customers exceeds a large number, the most of the retrials become unsuccessful and the retrials greater than a large number \( N \) have no effect on the system. The truncation level \( N \) is calculated by using Neuts-Rao Truncation [25]. Now, \( \xi_{N-i} = \xi_{N-i-1}R_{N-i}, \ k \geq 0, \ 1 \leq i \leq N - 1 \) where \( R_{N-i} = -A_0(A_{1N-i} + R_{N-i}A_{2,N-i+1})^{-1} \) and \( R_1 = -A_0(A_{11} + R_{22})^{-1} \). Now from \( \xi_0A_{10} + \xi_1A_{21} \) we find the steady state distribution \( \xi_0 \) of the finite state Markov chain with generator \( A_{10} + R_{1}A_{21} \). Finally each \( \xi_i \) is divided with \( \sum_{i=0}^{\infty} \xi_i e \) to compute \( \xi \).

6. Some important system characteristics

For the computation of system performance measures we partition each \( \xi_i, \ i \geq 0 \) as
\[
\xi_i = (x_i, y_i, z_i) = (x_i(1), \ldots, x_i(2S + 2), y_i(1), \ldots, y_i(2S), z_i(1), \ldots, z_i(2S)).
\]
Some standard probabilities and other measures are calculated as follows:

- Prob(the server is idle) = \( P_{idle} = \sum_{i=0}^{\infty} x_i e \).
- Prob(the server is busy with ordinary customer) = \( P_{sb} = \sum_{i=0}^{\infty} y_i e \).
- Prob(the server is busy with priority generated customer) = \( P_{sbpr} = \sum_{i=0}^{\infty} z_i e \).
- Prob(the server is idle with customers in the orbit) = \( P_{sid} = \sum_{i=0}^{\infty} x_i e - x_i e \).
- Expected number of customers in the orbit, \( E_{or} = \sum_{i=1}^{\infty} i\xi_i e \).
- Expected number of customers in the orbit when server is idle, \( E_{sid} = \sum_{i=1}^{\infty} ix_i e \).
- Successful retri rate, \( S_{rr} = \theta \sum_{i=1}^{\infty} ix_i e \).
- Overall retri rate, \( O_{rr} = \theta \sum_{i=1}^{\infty} ix_i e \).

Let \( \xi_i = (\xi(c, j, k) \) where \( \xi(c, j, k) \) is a row vector with respect to \( C(t) = c, \ N_2(t) = j \) and \( I(t) = k, \ c = 0, 1, 2; \ k = 0, 1, 2, \cdots S; \) then

- Expected ordinary customers in the orbit, \( E_{orc} = \sum_{i=1}^{\infty} \sum_{j=0}^{1} \sum_{k=0}^{S} i\xi_i(0, j, k) + \sum_{i=1}^{\infty} \sum_{j=0}^{1} \sum_{k=0}^{S} i\xi_i(1, j, k) + \sum_{i=1}^{\infty} \sum_{j=0}^{1} \sum_{k=0}^{S} (i - 1)\xi_i(2, j, k) \).
- Expected number of priority generated customers in the orbit, \( E_{prc} = \sum_{i=0}^{\infty} \sum_{j=0}^{S} \xi_i(0, 1, k) + \sum_{i=1}^{\infty} \sum_{j=0}^{S} \xi_i(1, 1, k) + \sum_{i=0}^{\infty} \sum_{j=0}^{S} \xi_i(2, 0, k) + \sum_{i=0}^{\infty} \sum_{j=0}^{S} 2\xi_i(2, 1, k) \).
- Expected inventory level, \( E_{inv} = \sum_{i=0}^{\infty} \sum_{j=0}^{2} \sum_{k=0}^{S} k\xi_i(0, j, k) + \sum_{i=1}^{\infty} \sum_{j=0}^{2} \sum_{k=0}^{S} \xi_i(1, j, k) + \sum_{i=0}^{\infty} \sum_{j=0}^{2} \sum_{k=0}^{S} \xi_i(2, j, k) \).
- Expected reorder rate, \( E_{rr} = \mu_1 \sum_{i=0}^{\infty} \sum_{j=0}^{1} \xi_i(1, j, s + 1) + \mu_2 \sum_{i=0}^{\infty} \sum_{j=0}^{1} \xi_i(2, j, s + 1) \).
• Expected number of departure of ordinary customers after completing service,
  \[E_{do} = \mu_1 \sum_{i=0}^{\infty} \sum_{j=0}^{1} \sum_{k=1}^{S} \zeta_i(1, j, k)\].
• Expected number of departure of priority generated customers after completing service,
  \[E_{depr} = \mu_2 \sum_{i=0}^{\infty} \sum_{j=0}^{1} \sum_{k=1}^{S} \zeta_i(2, j, k)\].
• Expected number of customers lost before entering the orbit per unit time,
  \[E_{clo} = \lambda(1 - \sigma) \sum_{i=0}^{\infty} \sum_{j=0}^{1} \zeta_i(0, j, 0) + \lambda(1 - \sigma) \sum_{i=0}^{\infty} \sum_{c=1}^{2} \sum_{j=0}^{1} \sum_{k=1}^{S} \zeta_i(c, j, k)\].
• Expected number of customers lost after retrials per unit time,
  \[E_{deor} = \theta(1 - \delta) \sum_{i=0}^{\infty} \sum_{j=0}^{1} i \zeta_i(0, j, 0) + \theta(1 - \delta) \sum_{i=0}^{\infty} \sum_{c=1}^{2} \sum_{j=0}^{1} \sum_{k=1}^{S} i \zeta_i(c, j, k)\].
• Expected number of priority generated customers lost per unit time,
  \[E_{prl} = \gamma \theta(1 - \delta) \sum_{i=1}^{\infty} \sum_{k=1}^{S} i \zeta_i(0, 1, k) + \gamma \theta(1 - \delta) \sum_{i=1}^{\infty} \sum_{c=1}^{2} \sum_{k=1}^{S} i \zeta_i(c, 1, k)\].

7. Cost Analysis
In this section, we propose an optimization problem and explaining with a numerical example. Let us consider the cost and revenue variables as follows:
• A revenue of \(r_1\) monetary units for each ordinary customer getting service and leaving the system.
• A revenue of \(r_2\) monetary units for each priority generated customer getting service and leaving the system.
• A holding cost of \(c_1\) monetary units for each unit of time that an ordinary customer has to wait in orbit.
• A holding cost of \(c_2\) monetary units for each unit of time that a priority generated customer has to wait in orbit.
• A cost of \(c_3\) monetary units for each customer lost before entering the orbit.
• A cost of \(c_4\) monetary units for each customer lost after the retrial.
• A cost of \(c_5\) monetary units for each priority generated customer lost due to no space in waiting space \(A_1\) at the time of priority generation occurs.
• Procurement cost of \(c_6\) monetary units for each unit.
• A holding cost of \(c_7\) monetary units for each unit in the inventory.
• A miscellaneous fixed cost \(c_{\text{fixed}}\).

Define a revenue(profit) function as:
\[\mathcal{ETP} = r_1 E_{do} + r_2 E_{depr} - c_1 E_{orcl} - c_2 E_{prc} - c_3 E_{do} - c_4 E_{de} - c_5 E_{pr} - c_6 (S - s) E_{rr} - c_7 E_{inlev} - c_{\text{fixed}}\].

Our goal is to find an optimum \((s, S)\) pair with all other parameters fixed that maximizes the expected total profit \(\mathcal{ETP}\). We provide a numerical example in the following section.

8. Numerical illustration
By variation of underlying parameters, we calculate the system characteristics numerically in this section.

8.1. \(\mathcal{ETP}\) and optimum \((s, S)\) pair
We fix \(\gamma = 1.2, \mu_1 = 1.2, \mu_2 = 2.4, \theta = 10.2, \sigma = 0.7, \delta = 0.6, \beta = 10.5,\)
\(r_1 = 500, r_2 = 1600, c_1 = 5, c_2 = 1, c_3 = 3, c_4 = 4, c_5 = 3, c_6 = 1, c_7 = 1, \) \(c_{\text{fixed}} = 1\) and compute \(\mathcal{ETP}\) for selected \((s, S)\) entries with \(\lambda = 10.0\). Results are tabulated in Table 1 and 2. For another set of selected \((s, S)\) entries with \(\lambda = 20.0\) is tabulated in Table 3 and 4. Optimum \((s, S)\) pair and corresponding \(\mathcal{ETP}\) are shown in Tables 1, 2, 3 and 4.
8.2. Effect of $\beta$ in $ETP$

We fix $\lambda = 15.0$, $\gamma = 10.2$, $\mu_1 = 5.2$, $\mu_2 = 2.4$, $\theta = 1.2$, $\sigma = 0.7$, $\delta = 0.2$, $r_1 = 200$, $r_2 = 100$, $c_1 = 5$, $c_2 = 1$, $c_3 = 3$, $c_4 = 4$, $c_5 = 3$, $c_6 = 5$, $c_7 = 1$, $c_{fixed} = 1$ and compute $ETP$ for different $\beta$ and selected $(s, S)$ pairs. Results are plotted in Figure 2 and it shows that when $\beta$ increases $ETP$ also increases for all selected $(s, S)$ pairs.

8.3. Effect of $\lambda$ in $ETP$

We fix $\gamma = 10.2$, $\beta = 1.5$, $\mu_1 = 5.2$, $\mu_2 = 2.4$, $\theta = 1.2$, $\sigma = 0.7$, $\delta = 0.2$, $r_1 = 200$, $r_2 = 100$, $c_1 = 5$, $c_2 = 1$, $c_3 = 3$, $c_4 = 4$, $c_5 = 3$, $c_6 = 5$, $c_7 = 1$, $c_{fixed} = 1$ and compute $ETP$ for different $\lambda$ and selected $(s, S)$ pairs. Results are plotted in Figure 3 and it shows that when $\lambda$ increases $ETP$ also increases for selected $(s, S)$ pairs.

8.4. Effect of $\theta$ in $ETP$

We fix $\lambda = 5.0$, $\gamma = 10.2$, $\beta = 1.5$, $\mu_1 = 1.2$, $\mu_2 = 1.4$, $\sigma = 0.7$, $\delta = 0.2$, $r_1 = 1200$, $r_2 = 1100$, $c_1 = 5$, $c_2 = 1$, $c_3 = 3$, $c_4 = 4$, $c_5 = 3$, $c_6 = 5$, $c_{fixed} = 1$ and compute $ETP$ for different $\theta$ and selected $(s, S)$ pairs. Results are plotted in Figure 4 and it shows that when $\theta$

Table 1. $ETP$ with $s$ and selected $S$ for $\lambda = 10.0$

| $s$ | S=30 | S=31 | S=32 | S=33 | S=34 | S=35 | S=36 | S=37 | S=38 |
|-----|------|------|------|------|------|------|------|------|------|
| 4   | 745.41 | 745.57 | 745.66 | **745.66** | 745.56 | 745.36 | 745.02 | 744.54 | 743.89 |
| 5   | 744.93 | 745.12 | 745.24 | **745.27** | 745.21 | 745.05 | 744.77 | 744.34 | 743.76 |
| 6   | 744.45 | 744.66 | 744.81 | **744.87** | 744.85 | 744.73 | 744.49 | 744.12 | 743.60 |
| 7   | 743.96 | 744.19 | 744.36 | 744.46 | **744.48** | 744.40 | 744.20 | 743.89 | 743.43 |
| 8   | 743.45 | 743.71 | 743.91 | 744.04 | **744.09** | 744.04 | 743.90 | 743.63 | 743.22 |
| 9   | 742.94 | 743.22 | 743.45 | 743.60 | **743.69** | 743.68 | 743.57 | 743.35 | 743.00 |
| 10  | 742.43 | 742.73 | 742.97 | 743.16 | 743.27 | **743.30** | 743.23 | 743.06 | 742.76 |
| 11  | 741.90 | 742.22 | 742.49 | 742.70 | 742.84 | **742.91** | 742.88 | 742.75 | 742.50 |
| 12  | 741.37 | 741.71 | 742.00 | 742.24 | 742.41 | 742.50 | **742.51** | 742.42 | 742.22 |
| 13  | 740.83 | 741.19 | 741.50 | 741.76 | 741.96 | 742.08 | **742.13** | 742.08 | 741.92 |

Table 2. $ETP$ with $s$ and selected $S$ for $\lambda = 10.0$

| $s$ | S=50 | S=51 | S=52 | S=53 | S=54 | S=55 | S=56 | S=57 | S=58 |
|-----|------|------|------|------|------|------|------|------|------|
| 14  | **737.87** | 737.86 | 737.75 | 737.52 | 737.15 | 736.63 | 735.92 | 735.01 | 733.85 |
| 15  | 737.47 | **737.50** | 737.43 | 737.25 | 736.94 | 736.48 | 735.85 | 735.02 | 733.97 |
| 16  | 737.05 | **737.11** | 737.09 | 736.96 | 736.70 | 736.30 | 735.74 | 735.00 | 734.04 |
| 17  | 736.61 | 736.72 | **736.73** | 736.64 | 736.44 | 736.10 | 735.61 | 734.94 | 734.07 |
| 18  | 736.17 | 736.30 | **736.35** | 736.31 | 736.16 | 735.88 | 735.45 | 734.86 | 734.07 |
| 19  | 735.71 | 735.88 | 735.96 | **735.96** | 735.96 | 735.86 | 735.63 | 735.27 | 734.74 |
| 20  | 735.24 | 735.44 | 735.56 | **735.60** | 735.53 | 735.36 | 735.05 | 734.60 | 733.97 |
| 21  | 734.76 | 734.99 | 735.14 | **735.21** | 735.19 | 735.07 | 734.82 | 734.43 | 733.87 |
| 22  | 734.27 | 734.52 | 734.71 | 734.81 | **734.84** | 734.76 | 734.56 | 734.23 | 733.74 |
| 23  | 733.78 | 734.05 | 734.26 | 734.40 | **734.46** | 734.42 | 734.28 | 734.01 | 733.59 |
Table 3. \( \varepsilon TP \) with \( s \) and selected \( S \) for \( \lambda = 20.0 \)

| s | S=31 | S=32 | S=33 | S=34 | S=35 | S=36 | S=37 | S=38 | S=39 |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 775.45 | 775.86 | 776.15 | 776.28 | **776.22** | 775.92 | 775.31 | 774.32 | 772.84 |
| 5 | 774.74 | 775.20 | 775.54 | 775.74 | **775.76** | 775.57 | 775.09 | 774.28 | 773.04 |
| 6 | 774.02 | 774.51 | 774.90 | 775.17 | **775.27** | 775.17 | 774.81 | 774.13 | 773.05 |
| 7 | 773.28 | 773.81 | 774.25 | 774.57 | **774.74** | 774.73 | 774.49 | 773.95 | 773.06 |
| 8 | 772.53 | 773.09 | 773.57 | 773.90 | **774.17** | 774.17 | 773.81 | 773.05 | 772.95 |
| 9 | 771.76 | 772.36 | 772.88 | 773.30 | **773.74** | 773.69 | 773.40 | 772.81 | 772.81 |
| 10 | 770.99 | 771.61 | 772.16 | 772.63 | **772.98** | 772.93 | 772.30 | 772.33 | 772.58 |
| 11 | 770.21 | 770.85 | 771.44 | 772.14 | **772.35** | 772.36 | 772.30 | 772.30 | 772.31 |
| 12 | 769.41 | 770.08 | 770.70 | 771.24 | **771.21** | 771.21 | 771.21 | 771.21 | 771.99 |
| 13 | 768.61 | 769.30 | 769.94 | 770.52 | **770.61** | 770.71 | 770.65 | 770.65 | 771.61 |

Table 4. \( \varepsilon TP \) with \( s \) and selected \( S \) for \( \lambda = 20.0 \)

| s | S=50 | S=51 | S=52 | S=53 | S=54 | S=55 | S=56 | S=57 | S=58 |
|---|---|---|---|---|---|---|---|---|---|
| 14 | 765.67 | 766.03 | 766.25 | **766.28** | 766.08 | 765.59 | 764.72 | 763.38 | 761.43 |
| 15 | 764.99 | 765.40 | 765.68 | **765.80** | 765.71 | 765.35 | 764.66 | 763.55 | 761.90 |
| 16 | 764.29 | 764.75 | 765.09 | 765.28 | **765.29** | 765.06 | 764.52 | 763.59 | 762.17 |
| 17 | 763.57 | 764.07 | 764.47 | 764.73 | **764.83** | 764.70 | 764.31 | 763.56 | 762.38 |
| 18 | 762.83 | 763.37 | 763.82 | 764.14 | **764.32** | 764.30 | 764.03 | 763.45 | 762.46 |
| 19 | 762.08 | 762.65 | 763.15 | 763.53 | **763.78** | 763.78 | 763.78 | 763.78 | 762.47 |
| 20 | 761.31 | 761.92 | 762.45 | 762.89 | **763.36** | 763.31 | 763.00 | 762.38 | 762.38 |
| 21 | 760.54 | 761.17 | 761.74 | 762.22 | **762.98** | 762.82 | 762.82 | 762.82 | 762.47 |
| 22 | 759.75 | 760.41 | 761.01 | 761.54 | **762.39** | 762.39 | 762.39 | 762.39 | 761.98 |
| 23 | 758.95 | 759.64 | 760.27 | 760.83 | **761.89** | 761.89 | 761.89 | 761.89 | 761.68 |

increases \( \varepsilon TP \) decreases for all selected \((s, S)\) pairs.

8.5. Effect of \( \mu_1 \) in \( \varepsilon TP \)
We fix \( \lambda = 15.0, \; \gamma = 10.2, \; \beta = 1.5, \; \theta = 1.2, \; \mu_2 = 1.4, \; \sigma = 0.7, \; \delta = 0.2, \; r_1 = 200, \; r_2 = 100, \; c_1 = 5, \; c_2 = 1, \; c_3 = 3, \; c_4 = 4, \; c_5 = 3, \; c_6 = 5, \; c_7 = 1, \; c_{fixed} = 1 \) and compute \( \varepsilon TP \) for different \( \mu_1 \) values and selected \((s, S)\) pairs. Results are plotted in Figure 5 and it shows that when \( \mu_1 \) increases \( \varepsilon TP \) increases for all selected \((s, S)\) pairs.

8.6. Effect of \( \mu_2 \) in \( \varepsilon TP \)
We fix \( \lambda = 5.0, \; \gamma = 10.2, \; \beta = 1.5, \; \theta = 1.2, \; \mu_1 = 1.2, \; \sigma = 0.7, \; \delta = 0.2, \; r_1 = 1200, \; r_2 = 1100, \; c_1 = 5, \; c_2 = 1, \; c_3 = 3, \; c_4 = 4, \; c_5 = 3, \; c_6 = 5, \; c_7 = 5, \; c_{fixed} = 1 \) and compute \( \varepsilon TP \) for different \( \mu_2 \) values and selected \((s, S)\) pairs. Results are plotted in Figure 5 and it shows that when \( \mu_2 \) increases \( \varepsilon TP \) increases for all selected \((s, S)\) pairs.
8.7. Effect of $\delta$ in $\mathcal{ETP}$
We fix $\lambda = 5.0$, $\gamma = 10.2$, $\beta = 1.5$, $\theta = 1.2$, $\mu_1 = 1.2$, $\sigma = 0.7$, $r_1 = 1200$, $r_2 = 1100$, $c_1 = 5$, $c_2 = 1$, $c_3 = 3$, $c_4 = 4$, $c_5 = 3$, $c_6 = 5$, $c_7 = 5$, $c_{fixed} = 1$ and compute $\mathcal{ETP}$ for different $\delta$ values and selected $(s,S)$ pairs. Results are plotted in Figure 6 and it shows that when $\delta$ increases $\mathcal{ETP}$ also increases.

8.8. Effect of $\sigma$ in $\mathcal{ETP}$
We fix $\gamma = 10.2$, $\beta = 1.5$, $\lambda = 5.0$, $\mu_1 = 2.2$, $\mu_2 = 2.4$, $\theta = 1.2$, $\delta = 0.2$, $r_1 = 200$, $r_2 = 100$, $c_1 = 5$, $c_2 = 1$, $c_3 = 3$, $c_4 = 4$, $c_5 = 3$, $c_6 = 5$, $c_{fixed} = 1$ and compute $\mathcal{ETP}$ for different $\sigma$ values and selected $(s,S)$ pairs. Results are plotted in Figure 6 and it shows that when $\sigma$ increases $\mathcal{ETP}$ also increases.

9. Conclusion
An inventory system with special focus on self-generation of priorities studied in this paper. The arrival follows the Poisson process, and the single server provides service with different exponential distribution. Here we consider a retrial inventory and give a probability rule for returning to orbit after an unsuccessful retrial. Important system characteristics calculated for a suitable system designing and numerically illustrated with examples. An optimization problem is also discussed by introducing a profit function and calculated optimum $(s,S)$ pairs.
Figure 4. $\mathcal{ETP}$ vs. retrial rate $\theta$ for selected $(s, S)$ pairs

Figure 5. $\mathcal{ETP}$ vs. service rate $\mu_1$, $\mu_2$ for selected $(s, S)$ pairs

Figure 6. $\mathcal{ETP}$ vs. $\delta$, $\sigma$ for selected $(s, S)$ pairs

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