Contribution of interband effects caused by long-wavelength transverse optical phonons to electron-phonon coupling in doped polar insulators

Aleksandr Pishtsiev

Institute of Physics, University of Tartu, Ria 142, 51014 Tartu, Estonia

We estimate the contribution of the long-wavelength el–TO–ph interaction and discuss the effect it has on electron pairing in doped polar systems like SrTiO$_3$ and PbTe. The analytical and numerical results presented in the study indicate that the el–TO–ph interaction tends to contribute little to the total strength of electron-phonon coupling in these and related materials. To explain this fact we consider possible reasons why the effect of the polar long-wavelength transverse optical phonons on the coupling constant $\lambda$ is far less than one might suppose.

PACS numbers: 63.20.kd; 74.20.Fg; 74.70.-b; 77.84.Bw

I. INTRODUCTION

A classical representative of polar crystals SrTiO$_3$, which is the prototypical among insulating ABO$_3$ perovskite oxides, is a well-known material with exceptional dielectric properties that made its attractive for a number of electronic applications. The ion-covalent bonds, the long-range dipole forces and extremely close proximity to the ferroelectric state determine SrTiO$_3$ to be an example of a low carrier density system with a metallic-like conductivity. Doping into insulating state of SrTiO$_3$ is performed by electron donating ions like La$^{3+}$ or Nb$^{5+}$ for Ti$^{4+}$ which transfer the doped electrons into the originally empty d$^0$ configuration [1]. Experimentally observable superconductivity in the electron-doped SrTiO$_3$ has been assumed to be described by the plasmon-polar optic phonon mechanism [2].

The polar-coupling electron–phonon system in SrTiO$_3$ has attracted a substantial amount of interest [2] [4]. The most important reason for this is that in such materials there exists a strong coupling between electronic and lattice degrees of freedom. Correspondingly, the properties of conductivity electrons are strongly affected by local lattice properties. The main motivation for the present study is related to previous work, in which we showed that in polar crystals like SrTiO$_3$ the strength of the interactions between valence electrons and transverse optical (TO) phonon modes is especially large [7] [8]. Application of these results to the case of doping will allow us to have a better understanding of how electron dynamics is affected by the strong el–TO–ph interaction. Note also that a good knowledge about the roles of the coexisting interactions responsible for various physical properties is valuable to further advances in polar and related systems. Therefore, it would be desirable to examine in detail whether and under which conditions does the el–TO–ph interaction contribute to the total electron-phonon coupling constant. The accomplishment of this task is the scope of the present communication.

II. MODEL

Consider a linearly coupled system of electrons and polar (infrared-active) TO lattice vibrations. Assume that the band structure is represented by the set of one-particle Bloch wave functions $|\sigma k\rangle$ and energies $E_{\sigma}(k)$ which can be determined within the framework of DFT-LDA methods with the correct accounting for the quasiparticle effects. The relevant Hamiltonian, which describes the dynamic mixing of electronic states of the valence ($\sigma = 1$) and conduction band ($\sigma = 2$) caused by the TO phonons, reads

$$H_{el-ph} = N^{-1/2} \sum_{\sigma, \sigma', j} \sum_{q, k} g_{\sigma\sigma'}(qj) a_\sigma^k a_{\sigma'}^{k-qj} u_{qj} \ (1)$$

where the quantities $g_{\sigma\sigma'}(qj)$ describe the el–TO–ph interaction, $a_\sigma^k$ ($a_\sigma^*$) are the creation (annihilation) operators for the states $|\sigma k\rangle$, $N$ is the number of unit cells; the TO phonon modes are characterized by the normal coordinate $u_{qj}$, the wave vector $q$ and the vibration branch $j$.

The consistent microscopic analysis of the model (1) is presented in [8]. Some relevant characteristics of this model are summarized as follows: (1) Eq. (1) is mostly related to polar crystals in which the difference in long-range fields given by longitudinal and transverse optical phonon modes is essential [9]. (2) For $q = 0$, the interaction of band electrons and TO lattice vibrations has completely the interband nature (the intraband ($\sigma = \sigma'$) terms in Eq. (1) vanish due to inversion symmetry [3] [8]). (3) The model represents the Fröhlich-type interaction which may give rise to an asymmetric charge distribution [3]. (4) At the macroscopic level, the strength of the el–TO–ph interaction is the direct effect of coupling between the dipole polarization associated with the polar TO mode [10] and the resulting shift of the electron density [8]. Note also that the electronic and structural significance of the el–TO–ph interaction is incorporated within the vibronic model (e.g., [8]).

---

*E-mail: ap@eeter.fi.tartu.ee*
can be a family of the ferroelectric ABO$_3$ perovskite oxides, in which hybridization involves strong dynamic mixing between the O 2p and the d$^0$ (Ti$^{4+}$, Nb$^{5+}$, Ta$^{5+}$, etc.) states caused by the TO $F_{1u}$ soft vibrations. Estimates of the el–TO–ph interaction constants for a wide number of polar compounds are reported in [5].

III. THEORETICAL ANALYSIS IN THE LONG-WAVELENGTH LIMIT

A widely-used approach for estimating the electron-phonon coupling strength is formulated in terms of the quasiparticle mass enhancement, $m^*/m$, due to the electron-phonon renormalization factor $1 + \lambda$ (e.g., [13, 14]):

$$m^*/m = 1 + \lambda.$$  \hspace{1cm} (2)

Here $\lambda$ is a dimensionless effective constant which characterizes the total strength of electron-phonon coupling. As well-known [13, 14], transition temperatures in conventional superconductors depend directly on $\lambda$. According to common definitions (the representation of $\lambda$ as a sum over mode coupling constants) one can define the partial coupling constant $\lambda_{\text{TO}}$ which is related to the corresponding contribution of the el–TO–ph interaction to $\lambda$.

Being based on the model [11], our goal is to determine which characteristic values of $\lambda_{\text{TO}}(j)$ for the $j$th TO mode correspond to the el–TO–ph interactions (which are most strong in a polar crystal). The simplest and practical method of obtaining $\lambda_{\text{TO}}(j)$ is to calculate the corresponding corrections to the single-particle energies. In the long-wavelength limit, upon comparing different physical processes and energy scales operative in the system, we can conceptually isolate, in specific properties, those effects arising from interactions between electrons and phonons from those caused by the other factors. This suggests $\lambda_{\text{TO}}$ to be composed of three distinct elements, i.e. under the above assumptions one has in the low-doping regime:

$$\lambda_{\text{TO}}(j) = (4N)^{-1} \sum_{\text{q}} V(j, q) \times A(j, q) \times K(q).$$ \hspace{1cm} (3)

where

$$V(j, q) = \frac{4|g(qj)|^2}{M_j \omega_{qj}^2 (\Delta(k, q) - \hbar \omega_{qj})} \bigg|_{k=0}, \hspace{1cm} (4)$$

$$A(j, q) = \frac{\hbar \omega_{qj}}{\Delta(k, q) - \hbar \omega_{qj}} \bigg|_{k=0}, \hspace{1cm} (5)$$

$$K(q) = \frac{m_2}{\hbar^2} \left[ \frac{\partial^2 \Delta(k, q)}{\partial |k|^2} - 2 \left( \frac{\partial \Delta(k, q)}{\partial |k|} \right) \frac{\partial \Delta(k, q)}{\partial |k|} \right] \bigg|_{k=0}, \hspace{1cm} (6)$$

$\Delta(k, q) = E_2(k) - E_1(k + q)$, $g(qj) \equiv g_{12}(qj)$, and $m_2$ denotes the band mass in the absence of the el–TO–ph interaction. The square of the TO phonon frequency $\omega_{qj}^2$ is represented as a sum of the unperturbed term $\omega_{qj}^2$ and the additional term $\Delta \omega_{qj}^2$, which accounts for the el–TO–ph interaction: $\omega_{qj}^2 = \omega_{qj}^2 + \Delta \omega_{qj}^2$. We expect the “normal” part $\omega_{qj}$, which may be regarded as the bare TO phonon frequency with respect to the el–TO–ph interaction, to be free of softening anomalies. The proposed analytical decomposition, Eq. (3), allows us, firstly, to distinguish various physical factors contributed to $\lambda_{\text{TO}}(j)$, secondly, to treat them separately in order to understand how the interplay of these elements affects the magnitude of $\lambda_{\text{TO}}(j)$, and, thirdly, by using available experimental data, to obtain reliable upper bounds for $\lambda_{\text{TO}}(j)$.

The first element $V(j, q)$ is the main player of the decomposition since involves the effects of the hybridization of the electronic states by the TO phonon modes. It can be analyzed by using our previous results [7, 8] as follows. First of all, note that the quantity $V(j, q)$ supplies a measure of the zone-center TO phonon softening arising from the interband scatterings of the band electrons. That is, $V(j, q)$ is proportional to a ratio of the electronic contribution $|g(qj)|^2$ to the TO mode $\omega_{qj}$, determined by the electron-ion potential and the band structure, and the lattice force constant $M_j \omega_{qj}^2$ associated with the bare TO vibrational mode. The momentum dependence of the quantity $g(qj)$ is seen from the first-principles analysis [8]: first, in a polar crystal, the maximum of $g(qj)$ as a function of $q$ is strictly related to the center point $q = 0$; second, the function $g(qj)$ is decreasing with increasing $q$. The last key aspect, which helps us to evaluate $V(j, q)$, is the delicate balance of long- and short-range forces peculiar to ferroelectrics (e.g., [13, 14]). By using the connection between long-range forces and the el–TO–ph interaction [7, 8], one can relate the quantity $g(qj)$ at $q \to 0$ to the relevant material parameters. As a result, in the long-wavelength limit, an upper-bound estimate for $V(j, q)$ can be described by the expression:

$$V(j, q) \leq \tau_j, \hspace{1cm} \tau_j = \frac{\bar{S}(j)}{S(j) + \epsilon_{\infty} + 2}. \hspace{1cm} (7)$$

Here, $\bar{S}(j) = S(j)/\Omega_{j0}^2$ and $S(j)$ is a dipole oscillator strength associated with the $j$th zone-centre TO vibrational mode; $\epsilon_{\infty}$ is the core (electronic high-frequency) contribution to the dielectric function. In the derivation of Eq. (7) we supposed that contributions of the zone-centre TO phonons to the dielectric function $\epsilon(\omega)$ of the far-IR spectral range are represented by a classical oscillator model [16, 17]:

$$\epsilon(\omega) = \epsilon_{\infty} + \sum_j \frac{S(j)}{\Omega_{j0}^2 - \omega^2}. \hspace{1cm} (8)$$

It is seen that the parameter $\tau_j$ always less than unity so that the inequality $V(j, q) < 1$ can be viewed as a reliable qualitative estimate. Note that, in a macroscopic sense, the difference between 1 and $\tau_j$ can be considered as a measure of the polarity in such a manner that in polar materials, such as ferroelectrics, $1 - \tau_j$ is much closer to zero than in compounds of lower polar nature.
Following \[7, 8\], we note that the second element $A(j, q)$ serves as an adiabatic parameter which relates two different energy scales – one is the electronic scale determined by magnitude of an insulating gap $E_g$, the other is the conventional phonon frequency scale $\hbar \omega_0$. In view of the balance of long- and short-range forces as well as weak dispersion of the long-wavelength limit, this gives the estimate: $A(j, q) \approx \hbar \Omega_{j0} / (E_g \sqrt{1 - \tau_j - \hbar \Omega_{j0} / E_g})$.

The last element $K(q)$ associated with the derivate of $\Delta(k, q)$ measures at $q = 0$ the effective curvature of the conductivity and valence bands. In contrast to $V(j, q)$ and $A(j, q)$, the role of the quantity $K(q)$ is not so important for two reasons. First, it is not directly related to the lattice properties. Second, based on electronic structure calculations in polar materials like ABO$_3$ perovskite oxides, it is reasonable to assume that peculiarities of the relevant band structure close to the Fermi level do not critically influence the quantity $K(q)$. Therefore, for the sake of simplicity, assuming that the bands are parabolic around $k = 0$, we restrict here to the case $K(q) \approx (1 + m_2/m_1)$ where $m_1$ corresponds to the bare effective mass in the valence band. (Note that in more general case the relevant estimate of $K$ may include the summation over $q$.) Of course, it is not guaranteed that such an approximation of $K(q)$ would be almost the same for all the ABO$_3$ systems of interest. However, by above considerations, the assumption of the functional non-importance of $K(q)$ is quite evident in the present case.

With these remarks in mind, the relatively simple expression that can be used to find upper-bound estimates for $\lambda_{TO}(j)$ is obtained by gathering all three observed estimates together. The quantity $\lambda_{TO}$ is then estimated in terms of observable parameters:

$$\lambda_{TO}(j) \lesssim \frac{1}{4} \left(1 + \frac{m_2}{m_1}\right) \frac{\sqrt{1 - \tau_j - \hbar \Omega_{j0} / E_g}}{\hbar \Omega_{j0} / E_g}.$$  \hspace{1cm} (9)

In Eq. (9), the adiabatic ratio $\hbar \Omega_{j0} / E_g$ serves as a restricting factor which minimizes significantly the magnitude of $\lambda_{TO}(j)$. This implies that there is no need to find more sharper estimates since the ratio of two parameters, $\hbar \Omega_{j0}$ and $E_g$, is sufficiently small for systems under consideration. Thus, in order to ensure smallness of $\lambda_{TO}(j)$ for any polar compound of ABO$_3$ perovskite oxides, we need only the corresponding values of material parameters such as $\epsilon_\infty$, $S(j)$, $\Omega_{j0}$, and $E_g$ as well as the prefactor $(1 + m_2/m_1)$ (which can be deduced from electronic-band-structure calculations and/or from experimental data). Upper-bound estimates for the parameter $\lambda_{TO}(j)$ calculated with the help of Eq. (9) for representative polar compounds belonging to typical perovskites are presented in Table 1. Note that these estimates can be referred to as first principles because Eq. (9) contains no adjustable parameters.

| Compound   | $\epsilon_\infty$ | $E_g$ (eV) | $\Omega_{j0}$ | $S(j)$ | $\tau_j$ | $\lambda_{TO}$ | $\lambda_{TO} = \sum_{j} \lambda_{TO}(j)$ |
|------------|-------------------|------------|---------------|--------|----------|----------------|-----------------------------------|
| SrTiO$_3$  | 5.2               | 3.3        | 1             | 88     | 0.977    | 0.016         | $\leq 0.02$                       |
| BaTiO$_3$  | 5.2               | 3.3        | 1             | 42     | 1.250    | 0.016         | $\leq 0.02$                       |
| KNbO$_3$   | 4.7               | 3.1        | 1             | 96     | 0.972    | 0.017         | $\leq 0.03$                       |
| PbTe       | 32.8              | 0.31       | 32            | 632    | 0.948    | 0.031         | $\leq 0.03$                       |
| SnTe       | 37.0              | 0.35       | 45            | 330    | 0.894    | 0.025         | $\leq 0.05$                       |
| PbSe       | 22.9              | 0.28       | 39            | 289    | 0.921    | 0.033         |                                  |
| PbS        | 17.2              | 0.42       | 67            | 155    | 0.890    | 0.031         |                                  |

Based on the comparative analysis of the obtained results, we can draw several general conclusions and characterize the corresponding roles of the el–TO–ph interaction as follows:

(i) First of all, regarding effects associated with the el–TO–ph interaction, we stress a key distinction: On the one hand, it has been established that the large magnitude of the el–TO–ph interaction ($\sim 5$ eV/$\AA$ in the wide-gap compounds like ABO$_3$ perovskite oxides and $\sim 0.6$ eV/$\AA$ in the $A^IVB^VI$ narrow-gap semiconductors) is an important inherent property of polar crystals.\[7, 8\]. According to the vibronic theory (e.g., \[3\]), this provides a significant softening of the zone-center TO vibrations, and may lead to a structural instability of a crystal lattice. On the other hand, the contribution of the el–TO–ph interaction to $\lambda$ was found to be suppressed by the ratio $\hbar \Omega_{j0} / E_g$. For example, the data in Table 1 indicate that in the ABO$_3$ materials the highest values of $\lambda_{TO}(j)$ are in the range of $0.016–0.017$ for the low-lying TO modes ($j = 1$). The high-lying modes have the smaller $\lambda_{TO}(j)$ ($j = 2, 3$) because the relevant el–TO–ph interactions are smaller. In addition to these numerical estimates, we have presented in Table 1 the corresponding results for $A^IVB^VI$ narrow-gap semiconductors. It is easy to see that similar estimates are also obtained for these materials.

(ii) The suppression of the contribution of the el–TO–ph interaction to the total electron-phonon coupling $\lambda$ is needed to be clarified. From a physical point of view, one may distinguish two factors why this is so. Firstly, note that one of the main features of the el–TO–ph interaction is its direct connection with the relevant changes in the electron density distribution.\[8\]. The resulting shift of the electron density depends on the interband transitions (hybridization) which exhibit an energy scale
pared with the magnitude of $E_g$. In parallel, the carrier scattering effects associated with the long-wavelength TO phonon modes have the typical energy scale of the order of $\hbar \Omega_{j0}$. Because the corresponding processes proceed in different ways, the smallness of the contribution of the el–TO–ph interaction to $\lambda$ can be explained by the great disproportion between these scales: $\hbar \Omega_{j0} \ll E_g$. Secondly, note that the el–TO–ph interaction directly depends on the electronic screening [3]: the factor $(\epsilon_{\infty}+2)$ in Eq. (7) accounts for local-fields effects-induced partial screening of the bare el–TO–ph interaction. This implies that when the values of $\hat{S}(j)$ and $\epsilon_{\infty}$ are comparable, the local-fields effects are not to be ignored.

(iii) The strength of the Fröhlich interaction of electrons with the polar long-wavelength longitudinal optical (LO) vibrations can be measured by the coupling constant $\alpha_{eff}$ represented as a sum of partial constants [3]:

$$\alpha_{eff} = \sum_i \alpha(i) \text{ where the index } i \text{ belongs to the relevant vibrational branches. A similar summation can also be introduced for the TO phonons as } \lambda_{\text{TO}} = \sum_j \lambda_{\text{TO}}(j).$$

Since the effective contribution of the LO phonons to $\lambda$ is $\sim \alpha_{eff}/6$ (for weak coupling), the ratio $\lambda_{\text{TO}}/(\alpha_{eff}/6)$ gives then a qualitative comparison of the corresponding contributions associated with the TO and LO phonons. Using the polaron coupling constant $\alpha_{eff} \sim 2$ reported in [7] and the data of Table 1, one can readily see that for SrTiO$_3$ the ratio $\lambda_{\text{TO}}/(\alpha_{eff}/6) \leq 0.06$. As it is apparent from this comparison, the polar LO phonons tend to provide much stronger couplings of electrons than the long-wavelength TO phonons. This implies that the properties of the conduction electrons are weakly affected by the the long-wavelength el–TO–ph interaction, no matter how great the strength of this interaction is.

(iv) In the context of superconductivity in SrTiO$_3$, it is of interest to note that, although the el–TO–ph interaction produces little influence on $\lambda$, the relative contribution of the el–TO–ph interaction might be capable of playing some role in the doping phase diagram. (Here, we have followed the arguments presented within a two-band scenario of superconductivity [19, 20].)

(v) Note that the derivation of Eq. (9) assumed an adequate choice of simplifications to recognize the relevant estimates for $\lambda_{\text{TO}}$. This allows us to test the applicability range of Eq. (9) to other systems. Let us demonstrate the values of $\bar{g}(j)$ for both narrow-gap compounds appear to be close to each other. In materials where the el–TO–ph interaction is operative, it can be verified by inspection of the magnitude of the zone-center polar TO vibrational mode effective charges $|Z^*(j)|$ [7, 8]. This point is well illustrated in Table 3, which presents the estimated values of the transverse effective charges, $|Z^*(j)|$, both for wide-gap (FeO) and narrow-gap (FeSi, PbTe) compounds. Thus, the estimates presented in Table 3 suggest that the interband channels of electronic scatterings associated with the zone-center TO phonons play a certain role in the lattice-dynamical properties of FeSi.

### IV. CONCLUSION

In the present paper, motivated by the need to extend the results of the previous works [7, 8], we have analyzed the influence of the interaction of electrons with infrared-active long-wavelength TO lattice vibrations on the total electron-phonon coupling constant $\lambda$. To examine in detail in what extent el–TO–ph interactions are important for the total strength of electron-phonon coupling we considered the generic two-band model involving the interband hybridization by the TO vibrational modes. The main result of the analysis points out the marginal role

| TO mode ($j$) | 1 | 2 | 3 | 4 |
|---------------|---|---|---|---|
| $\Omega_{j0}$ (cm$^{-1}$) | 198 | 318 | 338 | 445 |
| $\hat{S}(j)$ | 8.0 | 8.5 | 2.8 | 1.3 |
| $\tau_j$ | 0.44 | 0.46 | 0.22 | 0.11 |
| $\lambda_{\text{TO}} = \lambda_{\text{TO}}(j)$ | 0.068 | 0.142 | 0.056 | 0.041 |
| Ref. [21] | 0.063 | 0.080 | 0.096 | 0.036 |

| Ref. [21] | 0.270 | 1.65 | 4.17 | 2.19 |

| $\bar{g}(j)$ (eV/Å) | 0.27 | 0.45 | 0.28 | 0.25 | 1.9 | 0.65 |
| $|Z^*(j)|$ | 1.63 | 2.70 | 1.65 | 1.47 | 2.19 | 8.37 |
played by the el–TO–ph interaction on electron pairing in doped polar insulating systems: the presented analytical and numerical arguments show that the partial contribution from the el–TO–ph interaction to $\lambda$ is much smaller than the relevant contribution from the interaction with the polar long-wavelength LO phonons. The corresponding explanation is that the important physical properties of the el–TO–ph interaction are described by the matrix elements which mix the electron states of the valence and conduction bands separated by relatively large energy gap. For given intra- and interband interactions coexisting in a doped insulator, due to distinct energy scales, the amount of the contribution of the el–TO–ph interaction is ruled by the adiabatic ratio $\hbar\Omega_0/E_g$. In most systems of interest, the quantities, $E_g$ and $\hbar\Omega_0$, are of a different order of magnitude so that the real impact of the el–TO–ph interaction on $\lambda$ turns out to be strongly suppressed. This conclusion has a clear physical meaning: upon completion of the intraband scatterings, a conduction electron becomes already nearly “undressed”. Our studies suggest that the el–TO–ph interaction (via valence electrons) is of great concern for the lattice and polarizability properties, the changes in the electron density distribution, and the renormalization (soft behavior) of the zone-center TO vibrational modes. At the same time, in the context of superconductivity in doped polar insulating compounds, the el-TO-ph interaction due to a difference in energy scales of underlying interactions tends to contribute little to the total effective coupling constant.

Acknowledgments

The work was supported by the ETF grant No. 6918.

[1] W. Wunderlich, H. Ohta, K. Koumoto, Physica B 404 (2009) 2202.
[2] Y. Takada, J. Phys. Soc. Jpn. 49 (1980) 1267.
[3] N. Kristofel, P. Konsin, Phys. stat.sol. (b) 149 (1988) 11.
[4] J. L. M. van Mechelen, D. van der Marel, C. Grimaldi, A. B. Kuzmenko, N. P. Armitage, N. Reyren, H. Hagemann, I. I. Mazin, Phys. Rev. Lett. 100 (2008) 226403.
[5] J. T. Devreese, S. N. Klimin, J. L. M. van Mechelen, D. van der Marel, Phys. Rev. B 81 (2010) 125119.
[6] W. Meerasana, X. J. Zhou, B. Moritz, C-C. Chen, R. H. He, S-I. Fujimori, D. H. Lu, S-K. Mo, R. G. Moore, F. Baumberger, T. P. Devereaux, D. van der Marel, N. Nagaosa, J. Zaanen, Z-X. Shen, New J. Phys. 12 (2010) 023004.
[7] A. Pishtshev, Physica B 405 (2010) 4128.
[8] A. Pishtshev, arXiv:1002.0294
[9] W. Zhong, R. D. King-Smith, D. Vanderbilt, Phys. Rev. Lett. 72 (1994) 3618.
[10] M. Born, K. Huang, Dynamical Theory of Crystal Lattices, Oxford Univ. Press, Oxford, 1954.
[11] P. Konsin, N. Kristoffel, Ferroelectrics 226 (1999) 95.
[12] P. Konsin, B. Sorkin, Ferroelectrics, 353 (2007) 63.
[13] G. Rickayzen, Green’s functions and condensed matter, Academic Press, London, 1980.
[14] V. L. Ginzburg and D. A. Kirzhnits (Eds.), High-Temperature Superconductivity, Consultants Bureau, New York, 1982.
[15] R. Blinc, B. Žekš, Soft Modes in Ferroelectrics and Antiferroelectrics, Noth-Holland Publishing Company/American Elsevier, Amsterdam Oxford New York, 1974.
[16] W. Cochran, R. A. Cowley, J. Phys. Chem. Solids 23 (1962) 447.
[17] J. Petzelt, S. Kamba, Spectrosc. Prop. Inorg. Organomet. Compd. 40 (2009) 49.
[18] V. I. Kaidanov, Yu. I. Ravich, Sov. Phys. Usp. 28 (1985) 31.
[19] P. Konsin, N. Kristoffel, T. Öz, Annalen der Physik 2 (1993) 279.
[20] A. Bussmann-Holder, R. Micnas, A. R. Bishop, Eur. Phys. J. B 37 (2004) 345.
[21] A. Damascelli, K. Schulte, D. van der Marel, Phys. Rev. B 55 (1997) R4863.
[22] S. Paschen, E. Felder, M. A. Chernikov, L. Degiorgi, H. Schwer, H. R. Ott, D. P. Young, J. L. Sarrao, Z. Fisk, Phys. Rev. B 56 (1997) 12916.
[23] S. Bocelli, F. Marabelli, R. Spolenak, E. Bauer, Mat. Res. Soc. Symp. Proc. 402 (1996) 361.