A REMARK ON THE GEOGRAPHY PROBLEM IN HEEGAARD FLOER HOMOLOGY

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Abstract. We give new obstructions to the module structures arising in Heegaard Floer homology. As a corollary, we characterize the possible modules arising as the Heegaard Floer homology of an integer homology sphere with one-dimensional reduced Floer homology. Up to absolute grading shifts, there are only two.

1. Introduction

Heegaard Floer homology is a collection of three-manifold invariants defined by Ozsváth and Szabó which were inspired by the Seiberg–Witten equations in gauge theory [17]. The most refined of these invariants is $HF^+$, which is a graded module over $\mathbb{F}[U]$, where $U$ is an endomorphism of degree $-2$, and $\mathbb{F}$ denotes the field $\mathbb{Z}/2\mathbb{Z}$. The simplest example is the Heegaard Floer homology of the 3-sphere for which $HF^+(S^3) = \mathcal{T}^+_0$, where $\mathcal{T}^+_0$ denotes $\mathbb{F}[U,U^{-1}]/U : \mathbb{F}[U]$ with $gr(1) = d$. More interesting examples include $HF^+ (\pm \Sigma(2,3,5)) = \mathcal{T}^+_{(\pm2)}$ and $HF^+ (\Sigma(2,3,7)) = \mathcal{T}^+_0 \oplus \mathbb{F}(0)$, while $HF^+ (\Sigma(2,3,7)) = \mathcal{T}^+_0 \oplus \mathbb{F}_{(-1)}$ where a positive orientation refers to the orientation induced on the boundary of a positive-definite plumbing. In fact, for every Spin$^c$ rational homology sphere $(Y,\mathfrak{s})$, we have a (non-canonical) splitting $HF^+(Y,\mathfrak{s}) = \mathcal{T}^+_d \oplus HF_{\text{red}}(Y,\mathfrak{s})$, where $HF_{\text{red}}(Y,\mathfrak{s})$ is a finitely generated torsion module and $d \in \mathbb{Q}$. If $Y$ is an integer homology sphere, then there is a unique Spin$^c$ structure.

The $d$-invariant is an invariant of Spin$^c$ rational homology cobordism, and has become pervasive in applications to singularity theory, knot concordance, and unknotted numbers of knots (see for instance [1, 15, 18]). On the other hand, if $HF_{\text{red}}(Y) = \bigoplus_{\mathfrak{s}} HF_{\text{red}}(Y,\mathfrak{s}) = 0$, then $Y$ cannot admit a co-orientable taut foliation [16]. The interplay between the $d$-invariants and the reduced Floer homology is also quite powerful; this was used to prove the Dehn surgery characterization of the unknot in $S^3$ [6] (see also [16, 5]).

In this note, we give new restrictions on the module structure of the Heegaard Floer homology of rational homology spheres.

Theorem 1. Let $Y$ be a rational homology sphere and $\mathfrak{s}$ a self-conjugate Spin$^c$ structure. If $HF_{\text{red}}(Y,\mathfrak{s})$ is supported only in degrees strictly greater than $d(Y,\mathfrak{s})$, then $\dim_{\mathbb{F}} HF_{\text{red}}(Y,\mathfrak{s})$
is even. The same statement holds if $HF_{\text{red}}(Y, s)$ is supported in degrees strictly less than $d(Y, s) - 1$.

Note that every three-manifold admits at least one self-conjugate Spin$^c$ structure and the unique Spin$^c$ structure on an integer homology sphere is tautologically self-conjugate. Theorem 1 immediately yields a characterization of the modules with one-dimensional reduced Floer homology.

**Corollary 2.** Let $Y$ be a rational homology sphere equipped with a self-conjugate Spin$^c$ structure $s$. If $\dim_F HF_{\text{red}}(Y, s) = 1$ then $HF^+(Y, s) = T(d) \oplus F(d)$ or $HF^+(Y, s) = T(d) \oplus F(d-1)$.

By the computations of $HF^+(\pm \Sigma(2,3,7))$ stated above, we see that the two possible relatively graded modules with $\dim_F HF_{\text{red}} = 1$ are realized.

The argument will be a result of the isomorphisms with monopole Floer homology (see Theorem 3) and its relationship, via the Gysin sequence of Lin [13], with the Pin(2)-monopole Floer homology. For the reader with a distaste for gauge theory, we point out that the arguments only use the formal properties of these theories. We briefly review these properties in Section 2, and provide a proof of Theorem 1 in Section 3. We hope that this note encourages further work utilizing the strengths of both Heegaard Floer homology and Seiberg-Witten theory in conjunction.

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2. **Background**

In order to prove Theorem 1, we use the Pin(2)-symmetry of solutions of the Seiberg-Witten equations to rule out certain graded module structures in Heegaard Floer homology. To be more precise, we use the Pin(2)-monopole Floer homology as defined by Lin in [13]. The latter is a Morse–Bott version of Kronheimer and Mrowka’s monopole Floer homology (see [7]). In this article, we will not need the definitions of either the monopole Floer homology or the Pin(2)-monopole Floer homology. It suffices to work with their formal properties, which we review next.

First, to make connection with the Seiberg-Witten equations, we appeal to the isomorphism between Heegaard Floer homology and monopole Floer homology as is proved in [8, 9, 10, 11, 12]. The part of the isomorphism between Heegaard Floer homology and monopole Floer homology relevant to this article also follows from work of Taubes [19, 20, 21, 22, 23] and Colin-Ghiggini-Honda [2, 3, 4].
Theorem 3 (Main Theorem in [8]). Let $Y$ be a closed oriented three-manifold and $\mathfrak{s}$ be a Spin$^c$ structure on $Y$. Then, $HF^+(Y, \mathfrak{s})$ (respectively, $HF^-(Y, \mathfrak{s})$ and $HF^\infty(Y, \mathfrak{s})$) and $\widetilde{HM}_*(Y, \mathfrak{s}, c_b)$ (respectively, $\widetilde{HM}_*(Y, \mathfrak{s}, c_b)$ and $\widetilde{HM}_*(Y, \mathfrak{s}, c_b)$) are isomorphic as relatively graded $\mathbb{F}[U]$-modules.

Here, $\widetilde{HM}(Y, \mathfrak{s}, c_b)$ denotes the monopole Floer homology of $(Y, \mathfrak{s})$ with a balanced perturbation. In the case of a torsion Spin$^c$ structure, this is the same as the standard monopole Floer homology (see [7, §30]). Otherwise, one would work with completions of these modules, denoted $\widetilde{HM}_*(Y, \mathfrak{s}, c_b)$, with respect to the variable $U$, making them modules over $\mathbb{F}[[U]]$. According to [7, Theorem 31.1.1], we have the following isomorphisms:

$$
\widetilde{HM}_*(Y, \mathfrak{s}, c_b) \cong \widetilde{HM}_*(Y, \mathfrak{s}), \quad \widetilde{HM}_*(Y, \mathfrak{s}, c_b) \cong \widetilde{HM}_*(Y, \mathfrak{s}), \quad \widetilde{HM}_*(Y, \mathfrak{s}, c_b) \cong \widetilde{HM}_*(Y, \mathfrak{s}).
$$

In any case, completion does not affect the chain complex that defines $\widetilde{HM}$ since every element of $\widetilde{HM}$ is annihilated by some finite power of $U$.

Like in Heegaard Floer and monopole Floer homologies, the Pin(2)-monopole Floer homology comes equipped with a more interesting module structure; this is the same structure which enables the more refined invariants leading to Manolescu’s disproof of the Triangulation Conjecture in dimensions $\geq 5$ [14] (see also [13]). For a closed oriented three-manifold $Y$ equipped with a self-conjugate Spin$^c$ structure $\mathfrak{s}$, the invariants $\widetilde{HS}_*(Y, \mathfrak{s}), \widetilde{HS}_*(Y, \mathfrak{s}), \widetilde{HS}_*(Y, \mathfrak{s})$ take the form of $\mathbb{Q}$-graded modules over $\mathcal{R} = \mathbb{F}[[V]](Q)/Q^3$, where $V$ and $Q$ are endomorphisms of degrees $-4$ and $-1$, respectively. Note that we can also naturally equip any $\mathbb{F}[[U]]$-module with an $\mathcal{R}$-module structure, by having $Q$ act by $0$ and $V$ by $U^2$.

The following proposition displays the clear analogy between the flavors of the monopole Floer and Pin(2)-monopole Floer homologies.

Proposition 4 (Proposition 4.6 in Chapter 4 of [13]). Let $\mathfrak{s}$ be a self-conjugate Spin$^c$ structure on a rational homology sphere $Y$. Then, up to an absolute grading shift, $\overline{HS}_*(Y, \mathfrak{s}) \cong \mathbb{F}[[V, V^{-1}]](Q)/Q^3$.

This is analogous to the fact that $\overline{HM}_*(Y, \mathfrak{s}) \cong \mathbb{F}[[U, U^{-1}]]$ for any Spin$^c$ rational homology sphere. Recall that this implies, using the long exact sequence relating $\widetilde{HM}_*, \widetilde{HM}_*$, and $\overline{HM}_*$, that in sufficiently large gradings, the dimension of $\widetilde{HM}_*(Y, \mathfrak{s})$ alternates between one and zero. Likewise, Proposition 4 has the following consequence with regard to the rank of $\widetilde{HS}$ of rational homology spheres in sufficiently large gradings.

Lemma 5. Let $\mathfrak{s}$ be a self-conjugate Spin$^c$ structure on a rational homology sphere $Y$. Then $\dim_{\mathbb{F}} \widetilde{HS}_k(Y, \mathfrak{s}) \leq 1$ for $k \gg 0$.
Proof. This follows readily from the definition of the groups $\hat{HS}_\bullet$, $\hat{HS}_\bullet$, $\overline{HS}_\bullet$, the long exact sequence relating them, and Proposition 4. To be more explicit, by definition $\hat{HS}_k(Y, s)$ is zero for all sufficiently large $k \gg 0$. Then the long exact sequence,  

$$ \cdots \to \hat{HS}_{k+1}(Y, s) \xrightarrow{j_*} \overline{HS}_k(Y, s) \xrightarrow{p_*} \hat{HS}_k(Y, s) \xrightarrow{i_*} \overline{HS}_k(Y, s) \xrightarrow{j_*} \hat{HS}_{k-1}(Y, s) \to \cdots, $$

implies that $\overline{HS}_k(Y, s) \cong \hat{HS}_k(Y, s)$ as vector spaces over $\mathbb{F}$ for all sufficiently large $k \gg 0$. On the other hand, Proposition 4 implies that $\overline{HS}_k(Y, s)$ has rank at most 1 for any $k \in \mathbb{Z}$. This gives the desired result.

The key fact which allows us to transport information from $\hat{HS}_\bullet$ to $\hat{HM}_\bullet$ is the following Gysin sequence.

**Proposition 6** (Proposition 3.10 in Chapter 4 of [13]). Let $Y$ be a closed oriented three-manifold equipped with a self-conjugate $\text{Spin}^c$ structure $s$. Then there exists a long exact sequence:

$$ \cdots \to \hat{HS}_{k+1}(Y, s) \xrightarrow{c_{k+1}} \hat{HS}_k(Y, s) \xrightarrow{\iota_*} \hat{HM}(Y, s) \xrightarrow{\pi_*} \hat{HS}_k(Y, s) \xrightarrow{c_k} \hat{HS}_{k-1}(Y, s) \to \cdots $$

Further, the maps in this long exact sequence respect the $\mathcal{R}$-module structures.

With the preceding understood, we are ready to prove Theorem 1.

### 3. Proof of Theorem 1

In order to prove Theorem 1, we will simply show that an $\mathbb{F}[[U]]$-module of the form $\mathcal{T}_{(d)}^+ \oplus N$ where $N$ is an $r$-dimensional torsion module supported in degrees greater than $d$ with $r$ an odd integer cannot fit into the Gysin sequence with an $\mathcal{R}$-module satisfying Lemma 5. As explained momentarily, a duality argument rules out the case where $N$ is supported in degrees less than $d - 1$. This will imply that such an $\mathbb{F}[[U]]$-module cannot occur as the monopole Floer homology of a rational homology sphere with a self-conjugate $\text{Spin}^c$ structure. The $\mathcal{R}$-module structure will be key.

Note that the isomorphisms of Theorem 3 are only relatively graded. However, from the proof, it will be clear that the absolute grading does not play a role. We therefore assume for notational simplicity that $d = 0$ throughout.

Meanwhile, by [7, Proposition 28.3.4], $\hat{HM}_\bullet(Y, s) \cong \hat{HM}^\bullet(-Y, s)$ via an isomorphism sending elements in grading $k$ to elements in grading $-(k + 1)$. Working with coefficients in the field $\mathbb{F}$, we also have $\hat{HM}^\bullet(-Y, s) \cong \hat{HM}_\bullet(-Y, s)$. Hence, if $\hat{HM}_\bullet(Y, s) \cong \mathcal{T}_{(0)}^+ \oplus HM_{\text{red}}(Y, s)$ where $HM_{\text{red}}(Y, s)$ is $r$-dimensional and is supported in degrees less than $-1$ with $r$ an odd integer, then $\hat{HM}_\bullet(-Y, s) \cong \mathcal{T}_{(0)}^+ \oplus HM_{\text{red}}(-Y, s)$ where $HM_{\text{red}}(-Y, s)$ is $r$-dimensional and is supported in degrees greater than 0. Therefore, it suffices to prove the
non-realizability of $T^+_{(0)} \oplus HM_{red}$ where $HM_{red}$ is $r$-dimensional and is supported in degrees greater than 0 with $r$ an odd integer.

With the preceding understood, suppose that $\widetilde{HM}_\bullet(Y, s) \cong T^+_{(0)} \oplus HM_{red}(Y, s)$ with $HM_{red}(Y, s)$ supported only in positive degree.

**Lemma 7.** $\widetilde{HS}_k(Y, s) = 0$ for $k < 0$ and $\widetilde{HS}_0(Y, s) = F$.

**Proof.** Since $\widetilde{HM}_k(Y, s) = 0$ for $k < 0$, the Gysin sequence in Proposition 6 gives isomorphisms $\widetilde{HS}_k(Y, s) \cong \widetilde{HS}_{k-1}(Y, s)$ for all $k < 0$. Thus $\widetilde{HS}_k(Y, s)$ is isomorphic for all $k < 0$. But $\widetilde{HS}_k(Y, s) = 0$ for sufficiently negative $k$ by definition, so we must have that $\widetilde{HS}_k(Y, s) = 0$ for all $k < 0$. Finally, $\widetilde{HM}_0(Y, s) = F$ (since $HM_{red}$ is 0 in degree 0), and the exactness of

$$\cdots \rightarrow \widetilde{HS}_0(Y, s) \xrightarrow{i} \widetilde{HM}_0(Y, s) \xrightarrow{\pi_0} \widetilde{HS}_0(Y, s) \rightarrow 0$$

(1)

implies that $\pi_0$ is an isomorphism and $\widetilde{HS}_0(Y, s) = F$. \hfill $\Box$

Given $k \geq 0$ even, let $\widetilde{\pi}_k$ denote the restriction of the map $\pi_k : \widetilde{HM}_k(Y, s) \rightarrow \widetilde{HS}_k(Y, s)$ in the Gysin sequence in Proposition 6 to the part of the tower $T^+_{(0)}$ with grading $k$.

Although the splitting of $\widetilde{HM}(Y, s) \cong T^+_{(0)} \oplus HM_{red}(Y, s)$ is non-canonical, we can identify $T^+_{(0)}$ canonically as a submodule of $\widetilde{HM}(Y, s)$ by considering the image of $U^\ell$ for $\ell \gg 0$. Thus, the restriction of $\pi_k$ to $T^+_{(0)}$ is well-defined.

**Lemma 8.** For each $i \geq 0$, $\widetilde{\pi}_{4i}$ is nontrivial and $\widetilde{\pi}_{4i+2}$ is trivial.

**Proof.** Suppose that $\widetilde{\pi}_i$ is nontrivial for some even $i$. We deduce that $\widetilde{\pi}_{i+4}$ is also nontrivial from the fact that the Gysin sequence respects the module structures on $\widetilde{HM}_\bullet$ and $\widetilde{HS}_\bullet$. In particular $\widetilde{\pi}_i \circ U^2 = V \circ \widetilde{\pi}_{i+4}$. Since $U^2$ gives a nontrivial map between (the restrictions to $T^+_{(0)}$ of) $\widetilde{HM}_{i+4}(Y, s)$ and $\widetilde{HM}_i(Y, s)$ and $\widetilde{\pi}_i$ is nontrivial, we must have that $\widetilde{\pi}_{i+4}$ is nontrivial. That $\widetilde{\pi}_0$ is nontrivial follows from Equation (1) (note that since $HM_{red}$ is trivial in degree 0, we have $\widetilde{\pi}_0 = \pi_0$). By induction, it follows that $\widetilde{\pi}_{4i}$ is nontrivial for all $i \geq 0$.

Now suppose that $\widetilde{\pi}_{4i+2}$ is also nontrivial for some $i \geq 0$. Then by the argument above $\widetilde{\pi}_{4j+2}$ is nontrivial for all $j \geq i$, and so $\widetilde{\pi}_{2k}$ is nontrivial for all $k \geq 2i$. For sufficiently high degrees (in particular, higher than the support of $HM_{red}$), the Gysin sequence breaks into pieces of the form

$$0 \rightarrow \widetilde{HS}_{2k+1}(Y, s) \rightarrow \widetilde{HS}_{2k}(Y, s) \rightarrow \widetilde{HM}_{2k}(Y, s) \rightarrow \widetilde{HS}_{2k-1}(Y, s) \rightarrow 0.$$

Thus we have isomorphisms $\widetilde{HS}_{2k+1}(Y, s) \cong \widetilde{HS}_{2k}(Y, s)$ and $\dim \widetilde{HS}_{2k}(Y, s)$ is strictly larger than $\dim \widetilde{HS}_{2k-1}(Y, s)$. It follows that $\dim \widetilde{HS}_{k}(Y, s)$ grows without bound for sufficiently large $k$, violating Lemma 5. Thus $\widetilde{\pi}_{4i+2}$ must be trivial for all $i \geq 0$. \hfill $\Box$
With these lemmas, we are ready to complete the proof of Theorem 1. For sufficiently large $k$, $\widehat{HM}_{4k+1}(Y, \mathfrak{s})$ and $\widehat{HM}_{4k+3}(Y, \mathfrak{s})$ are zero. Consider the portions of the Gysin sequence centered around $\widehat{HM}_{4k+2}(Y, \mathfrak{s})$ and $\widehat{HM}_{4k}(Y, \mathfrak{s})$:

$$
0 \to \widehat{HS}_{4k+3}(Y, \mathfrak{s}) \to \widehat{HS}_{4k+2}(Y, \mathfrak{s}) \to \mathbb{F} \xrightarrow{0} \widehat{HS}_{4k+1}(Y, \mathfrak{s}) \to \widehat{HS}_{4k}(Y, \mathfrak{s}) \to 0,
$$

$$
0 \to \widehat{HS}_{4k+1}(Y, \mathfrak{s}) \to \widehat{HS}_{4k}(Y, \mathfrak{s}) \xrightarrow{0} \mathbb{F} \xrightarrow{\text{id}} \widehat{HS}_{4k-1}(Y, \mathfrak{s}) \to 0.
$$

Note that the non-triviality (respectively triviality) of the map $\widehat{\pi}_{4k} : \mathbb{F} \to \widehat{HS}_{4k}(Y, \mathfrak{s})$ (respectively $\widehat{\pi}_{4k+2} : \mathbb{F} \to \widehat{HS}_{4k+2}(Y, \mathfrak{s})$) is determined by Lemma 8. For $k \gg 0$, this gives the following isomorphisms:

$$
\widehat{HS}_{4k+1}(Y, \mathfrak{s}) \cong \widehat{HS}_{4k}(Y, \mathfrak{s}),
$$

$$
\widehat{HS}_{4k+2}(Y, \mathfrak{s}) \cong \widehat{HS}_{4k+1}(Y, \mathfrak{s}),
$$

$$
\widehat{HS}_{4k+3}(Y, \mathfrak{s}) \oplus \mathbb{F} \cong \widehat{HS}_{4k+2}(Y, \mathfrak{s}),
$$

$$
\widehat{HS}_{4k+4}(Y, \mathfrak{s}) \cong \widehat{HS}_{4k+3}(Y, \mathfrak{s}) \oplus \mathbb{F}.
$$

In particular, since $\widehat{HS}_{4k+i}(Y, \mathfrak{s}) \cong \widehat{HS}_{4k+3}(Y, \mathfrak{s}) \oplus \mathbb{F}$ and by Lemma 5 we have that $\dim_{\mathbb{F}} \widehat{HS}_{4k+4}(Y, \mathfrak{s}) \leq 1$, it follows that $\widehat{HS}_{4k+3}(Y, \mathfrak{s}) = 0$ for all sufficiently large $k$.

Fix some large $k$ such that $4k + 3$ is larger than the maximum degree in the support of $HM_{\text{red}}(Y, \mathfrak{s})$ and $\widehat{HS}_{4k+3}(Y, \mathfrak{s}) = 0$. Consider the Gysin sequence between $\widehat{HM}_{4k+3}(Y, \mathfrak{s}) = 0$ and $\widehat{HM}_{-1}(Y, \mathfrak{s}) = 0$. By exactness, the sum of dimensions of each group in this sequence must be even. The groups that appear in this sequence are $\widehat{HM}_i(Y, \mathfrak{s})$ for each $0 \leq i \leq 4k+2$, $\widehat{HS}_{4k+3}(Y, \mathfrak{s}) = 0$, $\widehat{HS}_{-1}(Y, \mathfrak{s}) = 0$, and two copies of $\widehat{HS}_i(Y, \mathfrak{s})$ for each $0 \leq i \leq 4k + 2$. It follows that

$$
\sum_{i=0}^{4k+2} \dim_{\mathbb{F}} \widehat{HM}_i(Y, \mathfrak{s}) = 2(k + 1) + \sum_{i=0}^{4k+2} \dim_{\mathbb{F}} HM_{\text{red},i}(Y, \mathfrak{s})
$$

is even, where the first term on the right is the contribution to the tower from degrees zero to $4k+2$, and the second term is simply $\dim_{\mathbb{F}} HM_{\text{red}}(Y, \mathfrak{s})$ since by assumption $HM_{\text{red}}(Y, \mathfrak{s})$ is supported in positive degrees at most $4k + 2$. Therefore, $\dim_{\mathbb{F}} HM_{\text{red}}(Y, \mathfrak{s})$ must be even. This completes the proof of Theorem 1.

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