µm-class Control of Relative Position and Attitude for a Formation Flying Synthetic Aperture Telescope with Micro-satellites

Ryo SUZUMOTO, Satoshi IKARI, Norihide MIYAMURA, and Shinichi NAKASUKA

1) Department of Aeronautics and Astronautics, The University of Tokyo, Tokyo 113–8656, Japan
2) Department of Interdisciplinary Science and Engineering, Meisei University, Tokyo 191–8506, Japan

Earth remote sensing from geostationary orbit (GEO) can realize high temporal resolution; however, the spatial resolution is commonly worse than observation from low Earth orbit. In order to achieve high-frequency and high-resolution GEO remote sensing, a “Formation Flying Synthetic Aperture Telescope (FFSAT)” with multiple micro-satellites has been proposed. The FFSAT greatly improves the spatial resolution using a synthetic aperture technique. Therefore the relative positions and attitudes between the optical units of each satellite must be controlled with an accuracy better than 1/10 of the observation wavelength. However, even mm-class accuracy control has not been demonstrated on orbit. As a first practical application of the FFSAT, a forest fire monitoring mission using infrared rays is being considered, in which control accuracy requirement is relaxed as its wavelength is longer than visible light. We proposed a point spread function optimization method for controlling formation flying with an accuracy of approximately 1–1,000 times the wavelength (1 µm–1 mm) in the absence of sensors, which can measure absolute distance with µm-accuracy. The effectiveness of the method was demonstrated through simulations in which the satellites’ system and the optical system are coupled. The simulation results show that the method can control the formation within the wavelength order.

Key Words: Formation Flying, Synthetic Aperture Telescope, Micro-satellites, Remote Sensing, Control Method

Nomenclature

- \( P \): pupil function
- \( \mathcal{P}(x, y) \): complex pupil function
- \( \mathcal{F}\{.\}_f \): Fourier transform by frequency \( f \)
- \( \mathcal{L} \): linear interpolation
- \( N_p \): number of pixels on one side of image sensor
- \( N_s \): wavelength division number
- \( \mathcal{O}(\cdot) \): amount of calculation
- \( \mathcal{P}(x, y) \): pupil function
- \( \mathcal{P}(x, y) \): complex pupil function
- \( p \): pixel pitch of image sensor
- \( (p/v)_{c_d} \): peak-to-valley ratio of \( s_d(u) \)
- \( \tilde{Q} \): number of sub-apertures
- \( r \): distance
- \( S(\lambda) \): spectral intensity distribution
- \( \mathcal{R} \): Strehl ratio
- \( s(u, v) \): point spread function (PSF)
- \( s_d(u) \): cross-sectional profile of PSF
- \( u = (u, v) \): pupil plane coordinate
- \( W(x, y) \): wavefront aberration
- \( w \): width of LSF
- \( x = (x, y) \): image plane coordinate
- \( z_{f} \): focal length
- \( z_{m}(x, y) \): shape of mirror surface
- \( \Delta W \): max \( W(x) - \min W(x) \)
- \( \Delta \delta \): control amount
- \( \Delta \lambda \): effective bandwidth
- \( \delta \): misalignment of relative position and attitude
- \( \{ \delta_d \} \): \( \delta_1, \ldots, \delta_Q \)
- \( \delta x, \delta y \): shift misalignment
- \( \delta z \): piston misalignment in optical axis
- \( \delta \varphi, \delta \theta \): tilt misalignment
- \( \delta \psi \): rotation misalignment around optical axis
- \( \lambda \): wavelength, observation wavelength
- \( \xi = (\xi, \eta) \): object plane coordinate

Subscripts
- \( C \): center of aperture
- \( \max \): maximum
- \( q \): \( q \)th sub-aperture
- \( \text{syn} \): synthetic aperture
- \( \lambda \): value at the wavelength

1. Introduction

Disaster monitoring using Earth observation satellites is a valuable information source to grasp wide-area ground situations at one time. In addition, spatial resolution and observation frequency (called “time resolution”) are also important.\(^1\) Geostationary orbit (GEO) remote sensing, which utilizes Earth observation satellites positioned in GEO, can realize high-frequency observation at several minute interval. However, because of its high altitude, it has the disadvantage that it is difficult to obtain high resolution compared to observation from low Earth orbit (LEO). In order to improve the resolution, there is no other way to increase the aperture of the
optical system. However due to restrictions such as the size of a launch vehicle or development cost of such large satellites, it would be very difficult to put a huge optical system in GEO. As an alternative way to improve the resolution without using a huge structure, “a synthetic aperture telescope” has been studied.\(^2\) The method obtains a virtual large aperture by precisely arranging small sub-apertures that are interferometrically combined to achieve a resolution comparable to a primary aperture equivalent to the entire array size. Therefore, a research group at The University of Tokyo has proposed a “Formation Flying Synthetic Aperture Telescope (FFSAT)” comprised of multiple micro-satellites (Fig. 1) that attempts to actualize the synthetic aperture telescope by satellite formation flying. The FFSAT can virtually realize a large aperture surpassing that of a single satellite and can attain a high resolution which could not be achieved until now.

The research group has proposed an application of this FFSAT for GEO remote sensing as a method to realize both of high-resolution and high-frequency observation, especially focusing on monitoring Australian forest fires. In previous projects, satellites Terra and Aqua equipped with NASA’s visible/infrared radiometer MODIS, which achieved 1 km ground sample distance (GSD) from the altitude of 705 km, with time resolution of one to two days at the same point.\(^3\) By contrast, the FFSAT proposed here is expected to achieve both high-resolution observation of 30 m GSD and high-frequency observation on the order of 10 min. However, in order to realize the FFSAT, it is necessary to control the satellite relative positions and attitudes with an accuracy better than \(\lambda/10\) (e.g., in the case of infrared rays, on the order of 0.1 \(\mu\)m),\(^2\) which is extremely high accuracy as compared with the past studies on formation flying.\(^4\) In order to achieve such extremely high-accuracy formation flying, high-precision sensors and actuators are essential. Although the details of the existing devices will be described later in Section 2.3.4 and Fig. 3, the biggest problem is that there is no absolute distance meter which can achieve measurement accuracy from mm order to \(\mu\)m order. In other words, absolute distance meters such as laser rangefinders can measure absolute distances with mm-class accuracy, and when optical compensations such as phase diversity\(^5\) is used, wavelength order misalignment (e.g., in case of infrared rays, on the order of 1 \(\mu\)m) can be estimated. However, there is no absolute distance meter to improve the control accuracy from mm order to \(\mu\)m order. Therefore, in this research, we developed a control method called “a point spread function (PSF) optimization method” to overcome the gap of the accuracy range of absolute distance meters and realize extremely high-accuracy formation flying of the FFSAT.

Even in the single-aperture telescope or in the segmented-mirror telescope, some methods for removing misalignments in the optical system based on the PSF have been proposed as previous research.\(^6\) On the other hand, the following things are unique in a synthetic aperture telescope such as the FFSAT: since each sub-aperture is composed of formation flying satellites, the sub-apertures are physically separated; since each sub-aperture is physically separated, misalignments significantly larger than the wavelength must be removed; and since the aperture is discrete, the PSF has many side lobes. In addition, in order to set the preconditions and simulation parameters for examining the proposed PSF optimization method, the conceptual design of a control system and the control sequence of the FFSAT with multiple micro-satellites for forest fire monitoring was carried out. Additionally, relative position and attitude determination using highly accurate sensing and its control were discussed.

In Section 2 of this paper, overview of the forest fire monitoring mission, key elements necessary for the FFSAT, and previous research are described. Section 3 shows the overview of the control system and the control sequence of the FFSAT. To develop a control method for formation flying with an accuracy of approximately 1–1,000 times the wavelength, a simulator in which the satellites’ system and the optical system are coupled is produced in Section 4, and the proposed PSF optimization method to improve the accuracy of relative positions and attitudes of formation flying satellites from mm order to \(\mu\)m order is explained in Section 5. Finally, Section 6 summarizes the results and future research directions.

2. Mission Overview, Key Elements, and Previous Research

This section summarizes the mission overview and concept, the comparison with other methods, and key elements necessary for realizing the mission.

2.1. Mission overview

The first attempt of using the FFSAT, which can realize both of high-resolution observation and high-frequency observation, is its application to the early detection of forest fires, “bushfire,” frequently occurring in Australia. Australia suffers from frequent bushfires. Bushfires that occurred in Victoria in 2009 killed 173 people and caused a total economic loss exceeding four billion dollars.\(^7\) We hope the FFSAT can make initial fire fighting possible and minimize damage by detecting the occurrence positions of bushfires at early stages and transmitting the information to
Table 1. Comparison between methods to realize both of high-resolution and high-frequency observation.

|                        | FFSAT in GEO          | Single aperture in GEO | Constellation in LEO |
|------------------------|-----------------------|------------------------|----------------------|
| Aperture size          | Sub-aperture diameter may be small | Must be large         | May be small         |
| Upper limit of         | Depends on the sensor range | Depends on the rocket size | Depends on the rocket size |
| aperture size          | Fixed point observation is possible | Fixed point observation is possible | Once a day even with 50 satellites |
| Observation frequency  | Large synthetic aperture size is necessary because of high altitude | Large aperture size is necessary because of high altitude | Aperture can be small because the altitude is low |
| Spatial resolution     | Always able to communicate | Always able to communicate | For limited time only |
| Communication with      | The possibility that the spatial frequency becomes discrete | A normal image with continuous spatial frequency | A normal image with continuous spatial frequency |
| ground station         | In addition to high pointing accuracy, relative position and attitude with an accuracy of $\lambda/10$ is also essential | High pointing accuracy is required | Required control accuracy is low |
| Captured image         | Low cost due to small aperture size and small number of satellites | High cost due to large structure | A huge number of satellites are required, and a cost increase |
| Cost                   | While there are robustness and operation flexibility by formation flying, there is a danger of collision | Airbus is planning a satellite called GO3S | AXELSPACE is carrying out a plan called AxelGlobe |

Fig. 2. Burnt area in 1997–2003 and observation area (southeast in summer, north in winter).

ground stations without delay. Research in Australia indicated that the place where bushfires occur is seasonal; in southeast Australia, bushfires tend to be most common during summer and autumn (December to March), and in northern Australia, they usually occur during the dry season (April to September). We set the primary observation target as a 1,000 km square in the southeast of Australia from the summer to the autumn, because this is the most densely populated area, and from the winter to the spring, a 2,000 km square in the north is the main observation area (Fig. 2). To change the observation areas, changing the focal length of the FFSAT is proposed (Fig. 4(b)). Although this is also mentioned in Section 2.3.3, the method is feasible because the satellites are physically separated from each other in the FFSAT.

2.2. Comparison with other missions

Three methods for compatibility between high-resolution observation and high-frequency observation are compared in Table 1. In the LEO constellation, high-frequency observation within minute order interval cannot be achieved. In order to arrange a gigantic single-aperture satellite in GEO, the project called GO3S is planned. The cost of GO3S is estimated to be billions of dollars, and it may cost much more than the FFSAT. As to the FFSAT in GEO, there are issues that very-high control accuracy is required, the spatial frequency of the captured image can be discrete, the aperture area tends to be small and the amount of light can be insufficient, and it is difficult to change the pointing of the telescope because it is necessary to rotate the entire formation. Nevertheless, we think that the FFSAT is a realistic system.

2.3. Key elements and previous studies

The concept of the FFSAT is to arrange a synthetic aperture telescope realized by micro-satellites formation flying, which is capable of high-resolution observation, on GEO suitable for high-frequency observation and to perform earth remote sensing. In this section, the key elements of the FFSAT are summarized.

2.3.1. GEO remote sensing

Remote sensing from GEO represented by meteorological satellites is suitable for high-frequency observation required for disaster monitoring because it can always observe a specific point and the obtained data can be downlinked to the ground without delay. On the other hand, because of its high orbital altitude, the spatial resolution is greatly deteriorated compared to observation from LEO.

2.3.2. Synthetic aperture telescope

A synthetic aperture is a technique that dramatically improves the spatial resolution, which is an issue of GEO remote sensing. The upper limit of the resolution of a optical system is determined by the diffraction limit, and it is given by $\lambda/D$. Therefore, in order to improve the spatial resolution, a large-aperture telescope is required. A synthetic aperture can virtually configure a large telescope by precisely arranging small sub-apertures that are interferometrically combined to achieve a resolution comparable to a primary aperture.
that it is necessary to align each optical unit with an accuracy equivalent to the entire array size. The technological issue is to achieve a relative position and attitude control accuracy on the order of 0.1 μm. In the radio-wave region, because the wavelength is sufficiently long, the phase of the observation wave is observable, which makes it possible to configure an interferometer later using a computer. Therefore, a synthetic aperture telescope in radio-waveband is already realized such as Very-long-baseline interferometry (VLBI). However, since the wavelength is much shorter in the near-infrared and visible light regions, it has not yet been realized as a space telescope.

### 2.3.3. Formation flying with micro- or nano-satellites

Formation flying with micro- or nano-satellites is superior in cost, flexibility, and robustness compared to a single large satellite. In the FFSAT of this mission, a synthetic aperture telescope consisting of micro-satellites formation flying is constituted. Furthermore, since the satellites are physically separated from each other, it is possible to flexibly change the specifications of the telescope, such as the focal length as shown in Fig. 4(b). There are only a few missions planned for the formation flying with four or more micro- or nano-satellites. The best relative position determination accuracy, which is currently achieved with micro- or nano-satellite formation flying, is 25 cm of the CanX-4 and 5. Therefore, the relative position and attitude control accuracy on the order of λ/10 (on the order of 0.1 μm) cannot be achieved using the conventional method, and a new control method is needed.

#### 2.3.4. Current available sensors and actuators

Figure 3 summarizes the accuracy and range of sensors and actuators available for micro-satellites formation flying.

1. **Sensors and sensing methods**

   In the phase where the distance between the satellites is sufficiently far, their positions are determined using GPS. Although GPS is a positioning system for the ground and LEO, demonstrations using it in GEO have also been conducted in recent years. In addition, highly accurate relative position and attitude estimation among satellites can be achieved by precisely measuring the distance between multiple points between satellites. However, there is no absolute distance meter in the range marked “Gap” in Fig. 3. In order to fill this gap, the PSF optimization method has been developed. The details of the method are explained in Sections 4 and 5.

2. **Actuators and control methods**

   Electrospay thruster, cold gas thruster, and so on are known as thrusters with extremely accurate thrust control capability and low thrust. A deformable mirror used in the field of optical compensation can drive a mirror surface with an accuracy of the wavelength order or less. In addition, piezo actuators can perform positioning with extremely high accuracy and wide stroke. The FFSAT can be realized through control using these actuators.

#### 3. Control System and Control Sequence

In this section, the control system and sequences of the FFSAT for the early detection of Australian bushfires are discussed.

##### 3.1. System overview of the FFSAT

From the characteristics of spatial frequency of the image to be captured, a synthetic aperture telescope with six sub-apertures was designed as shown in Fig. 4. The FFSAT is realized by combining seven 50 kg-class micro-satellites (i.e., one imaging satellite and six mirror satellites). The details of the preliminary design for the entire FFSAT system are discussed in Refs. 13 and 14.

##### 3.2. Mission requirements

The mission requirements for the early detection of bushfires are as follows: (1) An observation frequency that can scan the whole observation area within 10 min; (2) the ability to instantaneously transmit the information of detected bushfires to a ground station; (3) a design life of 10 years or more; (4) requirements on optical system necessary for early detection of bushfires from GEO; and (5) relative position and attitude control accuracy that satisfy the optical requirements. The optical requirements of (4) and the required control accuracy of (5) are summarized in Table 2. The GSD is determined from the ground resolution required to detect bushfires before they spread, the field of view (FOV) is determined from the observation areas (Fig. 2), and the observation wavelength λ is determined from the spectrum of bushfires and the atmospheric transmission of electromagnetic waves. Finally, other specifications like $D_{syn}, z, p$ and so on are
Table 2. Optical requirements and required control accuracy of the FFSAT.

|                | Summer | Winter |
|----------------|--------|--------|
| GSD            | m      | 30     | 60     |
| FOV            | km × km| 1,000 × 1,000 | 2,000 × 2,000 |
| λ              | μm     | 4 (infrared) | 4 (infrared) |
| p              | μm     | 18     | 18     |
| zt             | m      | 21.5   | 10.7   |
| Dsyn           | m      | 5.82   | 2.91   |
| Piston control accuracy | nm | 400 | 400 |
| Tilt control accuracy | μrad | 0.42 | 0.84 |

also determined by GSD, λ, the FOV and imaging sensor size.

3.3. Control system for the FFSAT

For control from rough formation construction to 100 nm-class (λ/10 order class) formation construction and keeping, sensors and actuators with a wide range of accuracy are required. Therefore, the multiple sensors and actuators shown in Fig. 3 are installed to enable observation and control of the relative position and attitude of each satellite in a wide dynamic range. The next section describes the mission sequences each of which utilizes different sensors and actuators.

3.4. Mission sequence and control sequence

(1) Launch and transition to GEO

The one imaging satellite and the six mirror satellites are mounted on one boost stage and altogether launched into GTO via a rocket. Then, the boost stage delivers all the satellites to GEO, where seven satellites are separated.

(2) Formation construction

This is a phase to make the formation and construct a synthetic aperture telescope. In order to achieve extremely accurate relative position and attitude control of 400 nm, which is 1/10 of the observation wavelength, different kinds of sensors and actuators are used for each sub-phase as follows.

(2-1) Rough formation construction or in case of anomaly

GPS receivers, star trackers (STT), mN-class thrusters are used in this sub-phase, where the distance between satellites is several tens of meters or more, such as during initial formation construction phase or at the time of an anomaly. When the distance is less than several tens of meters, formation control at cm-class accuracy is executed using ultra-wide-band devices (UWB) that can achieve cm-class positioning.

(2-2) mm-class formation construction

By measuring the absolute distance with laser rangefinders from the imaging satellite to the optical unit of the mirror satellites, it is possible to observe the piston misalignment of relative position in the optical axis direction δz with an accuracy of 1.0 mm and the tilt misalignment of relative attitude δp, δθ with an accuracy of 0.2°. Furthermore, by photographing the mirror satellites from the imaging satellite using the optical navigation camera, it is possible to observe the shift misalignment of relative position δx, δy with an accuracy of 1.5 mm and the rotation misalignment around the optical axis with an accuracy of 0.4°. In this sub-phase, μN thrusters are used as actuators.

(2-3) μm-class (λ-order class) formation construction

There is no compact sensor that can directly measure distance between satellites at the level of the required accuracy. Therefore, the PSF optimization method explained in Sections 4 and 5 is used in this sub-phase. The fine movement of the optical units is driven by piezo actuators, and when the stroke is saturated, the thrusters are used to move the satellites.

(2-4) 100 nm-class (λ/10-order class) formation construction

During final relative position and attitude adjustment phase, a wavefront aberration is estimated using a technique of adaptive optics such as phase diversity, and the estimated aberration is compensated for using the piezo actuators and deformable mirrors.

(3) Formation keeping and bushfire monitoring

The laser displacement meter can estimate the change of the relative position and attitude between the optical units of the satellites with an accuracy of 100 nm or less. Since the absolute distance cannot be measured with this accuracy, once the above mentioned highly accurate alignment is achieved, this formation is kept using this displacement sensor and the actuators. The method for changing the telescope orientation to switch the shooting location is discussed in Ref. 16).

4. Model and Simulator of FFSAT in Which the Satellites’ System and the Optical System Are Coupled

As mentioned in Section 2.3.4, since existing absolute distance meters cannot measure absolute distance with an accuracy in the mm to μm range, a control method without absolute distance meters is needed. In order to develop such kind of control method, the FFSAT was modeled and a simulator in which the satellites’ system and the optical system are coupled was made. For a single-pupil mirror or segment mirrors, modeling methods of wavefront aberration in the aperture with several wavelengths have been proposed in previous studies; however, for the multiple sub-apertures, which are physically separated and dynamically moving, any modeling method that can express wavefront aberrations hundreds of times larger than the wavelength has not yet been proposed. Therefore, a simulator that has the state of the relative positions and attitudes of each satellite and calculates the captured image when observing the point light source (e.g., a star) with the relative arrangement of the FFSAT was developed in this study. The coordinate systems and the definition of misalignments are shown in Figs. 5 and 6, and a schematic diagram of the simulator is shown in Fig. 7. As illustrated in Fig. 7, the synthetic aperture telescope part of the simulator simulates the dynamics of each satellite, and the image of the point light source is simulated in the optical simulator part based on information of each satellite’s position and attitude. The feature quantities are extracted from the captured image by the feedback algorithm part, and each mirror satellite is controlled in the actuator part. In this case, any disturbances (e.g., orbital perturbation
disturbances, torque disturbances, control noises, and sensor noises) are ignored.

4.1. Optical simulator

The point light source image when the relative position and attitude of the optical unit of the mirror satellite \( q \) has a misalignment of \( \delta_q \) is simulated, and the image (pixel information) obtained by the imaging sensor is calculated in this simulator. The wavefront aberration \( W_q \) of each sub-aperture is calculated from \( \delta_q \). When \( \delta_q \) is given, mirror surface position of the mirror satellite can be calculated, and the optical path length at each point on the mirror surface can be calculated geometrically. The difference in optical path length with and without misalignment is \( W_q \). The PSF \( s \), which is the point light source image, is given by

\[
s(u) = \int S(\lambda) s_j(u, \lambda) d\lambda \propto \int S(\lambda) |F^* [P_q(x, \lambda)] f_{x_0} f_{y_0} \frac{i}{\lambda} |^2 d\lambda, \tag{1}
\]

where

\[
P_q(x, \lambda) = \sum_q P_q \cdot \exp \left[ i \cdot \frac{2\pi}{\lambda} W_q(x, \delta_q) \right]. \tag{2}
\]

Here, \( s \) is normalized by the maximum value of the non-aberration PSF, thereby the maximum value \( s_{\text{max}} \) corresponds to the Strehl ratio \( SR \).

4.2. Feature quantities extraction

Feedback control using the entire image is difficult because of excessive data, so some feature quantities extracted from the obtained image by the FFSAT are used for feedback control. The feature quantities for the feedback algorithm described in Section 5 are defined. There are several types of feature quantities, and they are used properly for different purposes in the control method.

(1) Line spread function (LSF): \( a \)

\[
a_u(u) = \text{Lerp} \left( \frac{\sum_v s(u, v)}{\max_v s(u, v)} \right) \tag{3}
\]

\[
a_v(v) = \text{Lerp} \left( \frac{\sum_u s(u, v)}{\max_u s(u, v)} \right) \tag{4}
\]

where \( a_u \) and \( a_v \) are normalized by the maximum value, and linearly interpolated since the image pixels are discretized.

Figure 8 shows examples of PSF and its LSF.

is calculated from \( \delta_q \). When \( \delta_q \) is given, mirror surface position of the mirror satellite can be calculated, and the optical path length at each point on the mirror surface can be calculated geometrically. The difference in optical path length with and without misalignment is \( W_q \). The PSF \( s \), which is the point light source image, is given by

\[
s(u) = \int S(\lambda) s_j(u, \lambda) d\lambda
\]

where

\[
P_q(x, \lambda) = \sum_q P_q \cdot \exp \left[ i \cdot \frac{2\pi}{\lambda} W_q(x, \delta_q) \right].
\]

Here, \( s \) is normalized by the maximum value of the non-aberration PSF, thereby the maximum value \( s_{\text{max}} \) corresponds to the Strehl ratio \( SR \).

4.2. Feature quantities extraction

Feedback control using the entire image is difficult because of excessive data, so some feature quantities extracted from the obtained image by the FFSAT are used for feedback control. The feature quantities for the feedback algorithm described in Section 5 are defined. There are several types of feature quantities, and they are used properly for different purposes in the control method.

(1) Line spread function (LSF): \( a \)

\[
a_u(u) = \text{Lerp} \left( \frac{\sum_v s(u, v)}{\max_v s(u, v)} \right) \tag{3}
\]

\[
a_v(v) = \text{Lerp} \left( \frac{\sum_u s(u, v)}{\max_u s(u, v)} \right) \tag{4}
\]

where \( a_u \) and \( a_v \) are normalized by the maximum value, and linearly interpolated since the image pixels are discretized.

Figure 8 shows examples of PSF and its LSF.
5. Relative Position and Attitude Control Method Using Mission Image Feedback

As mentioned in Sections 2.3.4 and 3.4, the accuracy of the existing absolute distance meters is, at most, approximately on the order of mm. Therefore, in this section, a method for improving the accuracy of controlling the relative positions and attitudes of the satellites’ optical units from the mm order (i.e., several hundred times of the wavelength order) to μm order (i.e., the wavelength order) is considered. For this control, we propose a PSF optimization method. The proposed control method uses the image feedback algorithm. In this feedback algorithm, images are taken by using the synthetic aperture telescope of the FFSAT to estimate the misalignments. In order to take an ideal image, satellites are controlled with high accuracy based on information of the estimated misalignments.

5.2. Control algorithm of the PSF optimization method

5.2.1. Overall flow

First, the overall flow of the proposed control algorithm is explained here as follows:

(1) In the first step, the images of each sub-aperture are scattered (Fig. 11).

(2) The images of each sub-aperture are roughly collected near the center of the imaging sensor (Fig. 12).

(3) The image of a single sub-aperture is optimized. At this step, the other five sub-apertures are tilted by a specified amount so that they will not disturb the optimization of the selected sub-aperture, and only the point light of the intended sub-aperture is imaged to the center of the imaging sensor, and then the shape of the image is optimized (Fig. 13).
(4) By interfering the images of the two sub-apertures sequentially, the \( z \) directional aberrations are removed. At this time as well, only two of the light sources reflected from the intended sub-apertures are imaged to the center of the imaging sensor (Fig. 14).

(5) The images of all of the sub-apertures are collected to the center of the imaging sensor (Fig. 15).

(6) The whole image is optimized by superimposing all of the sub-apertures’ images (Fig. 16).

5.2.2. Details of each step

Each algorithm step outlined above is described in detail here.

(1) Initial state

In the initial state, the relative positions and attitudes on the six axes of the mirror satellites relative to the imaging satellite have misalignments due to the limited accuracy of the absolute distance meters and attitudes sensors. Therefore, the images of each sub-aperture are scattered on the imaging sensor.

(2) The arrangement of each sub-aperture’s image at the center of the imaging sensor

The images of each sub-aperture are roughly collected near the center of the imaging sensor. When the optical unit of a certain mirror satellite \( q \) is moved, the image from the sub-aperture moves on the imaging sensor accordingly. By utilizing this fact, each image corresponds to a specific sub-aperture, and the respective images are gathered in the vicinity of the center of the imaging sensor. With the objective function \( J \) of a mirror satellite \( q \) as

\[
J_q = -w_{aq} \tag{12}
\]

the gradient method defined in Section 5.1 is carried out for each mirror satellite by controlling the position and the attitude of each satellite.

(3) Optimization of the single sub-aperture’s image

The image of the single sub-aperture is optimized. The other five sub-apertures are rotated by a specified amount around the \( x \) axis, and only the point light of the intended sub-aperture is imaged to the center of the imaging sensor. Here, the gradient method is repeatedly used with several objective functions. First, in order to eliminate image distortion, the gradient method is performed with the objective function \( J \) as

\[
J_q = -w_{aq} \tag{13}
\]

Next, with \( J \) as

\[
J_q = -r_{max_q} \tag{14}
\]

the image is aligned to the center of the imaging sensor, and by setting

\[
J_q = s_{max_q} \tag{15}
\]

the \( SR \) of the single sub-aperture is maximized. Then, by repeating the gradient method by setting \( J \) again as

\[
J_q = -w_{aq} \tag{16}
\]

\[
J_q = -r_{max_q} \tag{17}
\]

the image shifted in the process of maximizing the \( SR \) is aligned again to the center of the imaging sensor.

(4) Removing the \( z \) directional aberration

By interfering with the images of the two sub-apertures in this step, the aberrations in the \( z \) direction are removed. The gradient method usually falls into a local optimum, and it is difficult to remove the aberrations or misalignments in the \( z \)
direction. Therefore, in order to find the global optimum state, a method to sweep the mirror satellites in the $z$ direction was devised. However, the system has six mirror satellites, and it is difficult to find the global optimum in the search space of six dimensions. Taking this issue into account, by fixing one satellite and sweeping another mirror satellite to the $z$ direction, the $z$ directional aberration between the two satellites is removed, and those of the other satellites are removed as well. Note that only the images of the two sub-apertures to be interfered are imaged to the center of the imaging sensor so that the other images do not disturb the optimization process as well as in Step 3.

Figure 17 shows $(p/v)_{\delta z}$ when the mirror satellite 1 is fixed and the mirror satellite 4 is swept in the $z$ direction at the situation where the optimization up to Step 3 is completed. The $(p/v)_{\delta z}$ drops greatly only around the state of $\delta z_1 - \delta z_4 = 0$, and this means that there is almost no aberration between the sub-apertures of mirror satellites 1 and 4 (aberration in the aperture might slightly remain). Using this feature, although the sign and the amount of aberration cannot be estimated, it is possible to find whether or not aberration exists with precision of a few times the wavelength.

In addition, since the farther the distance between the two sub-apertures is, the clearer the interference fringes (as shown in Fig. 9) become, the sub-apertures to be subjected to interference are chosen from those other than the adjacent ones. The order of interference is:

1. Fix the mirror satellite 1, sweep the mirror satellite 3
2. Fix the mirror satellite 1, sweep the mirror satellite 4
3. Fix the mirror satellite 1, sweep the mirror satellite 5
4. Fix the mirror satellite 4, sweep the mirror satellite 2
5. Fix the mirror satellite 4, sweep the mirror satellite 6

This optimization method finally controls the $z$ directional aberrations among all six sub-apertures to be a few times the wavelength or less. It should be noted that when it cannot be detected where $(p/v)_{\delta z}$ drops substantially, the sweep range is expanded and this step is executed again. Furthermore, when the centers of two images are gradually shifted and cannot be made to interfere with cleanly by sweeping the sub-aperture, the images are aligned to the center of the imaging sensor again.

(5) Superimposing all sub-apertures’ images at the center of the imaging sensor

All of the images which have been moved out of the center of the imaging sensor are gathered to the center and superimposed. Using high-precision displacement meters and piezo actuators, it becomes possible to restore each sub-aperture to the proper relative position and attitude.

(6) Optimization with all sub-apertures’ images superimposed

With the images of all the sub-apertures superimposed, the gradient method is performed to optimize the whole image with the objective function $J$ as

$$J_{\text{syn}} = \mathcal{SR} = s_{\text{max}}.$$  

In this step, all of 6 DOF of six mirror satellites are controlled in order to take an ideal image.

### 5.3. Simulation results and discussion of the control method

The proposed PSF optimization method was verified using the Monte Carlo simulation of 3,000 cases with the simulator introduced in Section 4. Initial misalignments for all mirror satellites $[\delta_\theta]$ were given as a random value having a position standard deviation of 0.2 mm and an attitude standard deviation of 0.05 mrad. The other specifications of simulation are summarized in Table 3. The specifications nearly conform to the FFSAT for Australian forest fire monitoring discussed in Section 3, but because the PSF was used for control, the pixel pitch $p$ of the imaging sensor was finer in order to capture clearer PSF.

The simulation results for the summer case (30 m GSD) and the winter case (60 m GSD) are shown in Figs. 18 and 19, respectively. The horizontal axis shows the maximum wavefront aberration $\Delta W$ before and after the execution of the PSF optimization method. Note that the axis is normalized by the observation wavelength $\lambda$. The figures show that the proposed PSF optimization method successfully converges $\Delta W$ to within 3.5 times in all cases. In other words, when converted $\Delta W$ to misalignments of satellites, $\Delta W$ converged to within approximately 7 $\mu$m (approximately 7 $\mu$m). The results show that the formation of the FFSAT with the relative position and attitude errors that cannot be removed by the absolute distance meters and attitude sensors can be controlled.
up to the state where the wavefront aberration becomes a few times the wavelength by the proposed control method. Furthermore, it should be noted that despite the fact that the images targeting the ideal conditions for optimization are largely different in the summer case and the winter case as shown in Fig. 10, the FFSAT can be controlled by the same control method. This indicates the high versatility of the proposed PSF optimization method.

Finally, onboard processing of this control method should be considered, as the onboard computational resource is not so high level as those on the ground. In orbit, the calculation result of the optical simulator part in Fig. 7 is obtained using the imaging sensor of the imaging satellite. Only the feedback algorithm part in Fig. 7 needs to be calculated by the onboard processors of the satellites. It took about 0.5 s to run one loop of the process in Fig. 7 using a standard laptop computer. However, most of the calculation time was spent on calculating Eq. (1) in the optical simulator part, the amount of calculation of which is \( O(N_t N_p^2 \log^2 N_p) \), and the calculation in the feedback algorithm part is only \( O(N_p^2) \) and small. Therefore, it is inferred that the proposed control method can be conducted within the usually limited onboard computational resource.

6. Conclusion

GEO remote sensing has strong merits of continuous monitoring of the same place, high observation frequency (time resolution), and almost real-time downlink of the obtained data, which are very suitable features for disaster monitoring missions. The drawback of GEO remote sensing is low spatial resolution as the distance is farther than LEO remote sensing, and therefore a synthetic aperture telescope called FFSAT is proposed here to dramatically improve the spatial resolution. As one example, a forest fire monitoring mission in Australia is proposed in this paper. The sequence of the mission was decided, requirements of the FFSAT were identified, components satisfying the requirements were considered, and a control system was discussed. The most important technological issue is how to fill the gap between the accuracy range of absolute distance meters such as laser rangefinders (i.e., several hundred times the wavelength, or mm order) and that of optical compensations such as phase diversity (i.e., less than a few times the wavelength, or \( \mu m \) order), “a PSF optimization method” was proposed to solve this issue. This method enables \( \mu m \)-class control of formation flying in the absence of sensors that can measure absolute distance with \( \mu m \)-accuracy. In the method, to remove misalignments and aberrations in that range, the PSF from the image taken by the synthetic aperture telescope of the FFSAT is optimized. In order to verify the method, a simulator that coupled the satellites’ system and the optical system of the FFSAT was developed, and the Monte Carlo simulation was performed. The results show that under the assumption that there are no optical disturbances, orbital disturbances, or control disturbances, the relative position and attitude of the optical units of the mirror satellites could be controlled, and the wavefront aberrations were within 3.5\( \lambda \) (approximately 7\( \mu m \) in terms of misalignments) even in the worst case. The achieved control accuracy was far beyond the \( \mu m \) order accuracy that could be observed using absolute distance meters. This is enough level of accuracy in the sense that from this accuracy level, phase diversity can make the final correction to realize the FFSAT at the target accuracy. The more practical simulation study is yet to be conducted including disturbances and measurement or control errors, but it can be said that the proof of concept has been obtained in this study up till now.

The relative position and attitude control on the order of 1/10 of the wavelength is essential to realize a FFSAT. There-
fore, a seamless control method with a wide control range, from final control accuracy of several 100 nm for observation phase to that of the m order or more for the initial formation construction phase, is required. The results of this research prove that the greatest barrier in constructing a seamless control method has been removed and the possibility of realizing a FFSAT has been greatly improved.

The FFSAT has the possibility to realize much larger aperture diameter and to achieve much higher spatial resolution compared with conventional space telescopes. By applying the FFSAT to an astronomical observation telescope, unprecedented observations will be possible. For that purpose as well, it is necessary to continue studying toward the realization of a FFSAT in the future.

References

1) Nakamura, T.: A Study on Total Serviceability Evaluation for Disaster Monitoring by Satellites (1st Report), J. Ips. Soc. Aeronaut. Space Sci., 63, 4 (2015), pp. 143–149 (in Japanese).
2) Rousset, G., Mugnier, G. M., Cassaing, F., and Sorrente, B.: Imaging with Multi-aperture Optical Telescopes and an Application, C. R. Acad. Sci. Paris, serie IV, 2, 1 (2001), pp. 17–25.
3) Hutchison, K. D.: Applications of MODIS Satellite Data and Products for Monitoring Air Quality in the State of Texas, Atmospheric Environment, 37 (2003), pp. 2403–2412.
4) Bandyopadhyay, S., Fouest, R., Subramanian, G. P., Chung, S. J., and Hadaegh, F. Y.: Review of Formation Flying and Constellation Missions Using Nanosatellites, J. Spacecraft Rockets, 53 (2016), pp. 567–578.
5) Paxman, R. G., Schulz, T. S., and Fienup, J. R.: Joint Estimation of Object and Aberrations by Using Phase Diversity, J. Optical Society America A, 9 (1992), pp. 1072–1085.
6) Acton, D. S., Towell, T., Schwenker, J., Shields, D., Sabatke, E., Contos, A. R., Hansen, K., Shi, F., Dean, B., and Smith, S.: End-to-end Commissioning Demonstration of the James Webb Space Telescope, Optical Engineering + Applications 2007, San Diego, United States, Proc. of SPIE, 668706, 2007.
7) The 2009 Victorian Bushfires Royal Commission, 2009 VBRC–Final Report–Summary–Interactive Version, http://royalcommission.vic.gov.au/Commission-Reports/Final-Report/Summary/Interactive-Version.html (accessed January 19, 2019).
8) Savanna Explorer–Discover the People and Landscapes of Northern Australia, Fire in Australia’s Tropical Savanna, http://www.savanna.org.au/all/fire.html (accessed January 10, 2018).
9) AIRBUS: GO3S–Permanent Surveillance from Space, http://www.airbus.com/defence.html#earth-observation-satellites/go3s/ (accessed January 10, 2018).
10) AXELSPACE: AXELGLOBE, https://www.axelspace.com/en/axelglobe/ (accessed January 19, 2019).
11) Nakajima, Y., Yamamoto, T., Sekiguchi, T., Nishijo, K., Kumagai, S., Kawakami, S., Harada, R., and Kasahara, M.: Development of A GPS Receiver for Geosynchronous Satellites toward Autonomous Operation, 62nd Space Sciences and Technology Conference, Kurume, Japan, 2G04, 2018 (in Japanese).
12) Funaki, I., Nakayama, Y., Horisawa, H., and Ando, M.: Micro-thruster Options for the Japanese Space Gravitational Wave Observatory Missions, 32nd International Electric Propulsion Conference, Wiesbaden, Germany, IEPC-2011-308, 2011.
13) Suzumoto, R.: Accurate Control of Relative Position and Attitude for Formation Flying Synthetic Aperture Telescope with Multiple Micro-satellites, 32nd International Symposium on Space Technology and Science, Tokushima, Japan, 2019-n-41s, 2019.
14) Suzumoto, R., Mori, D., Ikari, S., Miyamura, N., and Nakasuka, S.: Preliminary Design of Formation Flying Synthetic Aperture Telescope with Multi-Micro-satellites, 62nd Space Sciences and Technology Conference, Kurume, Japan, P51, 2018 (in Japanese).
15) Ikura, M., Ikari, S., Tomoki, A., Funase, R., and Nakasuka, S.: Estimation Algorithm of Relative Position and Attitude during Proximity Rendezvous and Docking Using Multiple Ultra-Wide-Band Devices, Trans. JSASS Aerospace Technology Japan, 17 (2019), pp. 43–50.
16) Miyamura, N., Suzumoto, R., Funabiki, N., Matsushita, S., Kawabata, Y., Ikari, S., and Nakasuka, S.: Optical Design of the Synthetic Aperture Telescope with Small Satellites, 62nd Space Sciences and Technology Conference, Kurume, Japan, 3N13, 2018 (in Japanese).
17) Knight, J. S., Lightsey, P., Barto, A., and Acton, D. S.: Image Quality Verification Analysis of the JWST, Astronomical Telescopes + Instrumentation 2010, San Diego, United States, Proc. of SPIE, 77381Z, 2010.

Christie Maddock
Associate Editor

©2021 JSASS