Evolution of density perturbations in double exponential quintessence models

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Abstract

In this work we investigate the evolution of matter density perturbations for quintessence models with a self-interaction potential that is a combination of exponentials. One of the models is based on the Einstein theory of gravity, while the other is based on the Brans-Dicke scalar tensor theory. We constrain the parameter space of the models using the determinations for the growth rate of perturbations derived from data of the 2-degree Field Galaxy Redshift Survey.

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I. INTRODUCTION

In the past few years, it has become apparent that the energy budget of our universe is dominated by an unknown component called "dark energy". The WMAP table of "best" cosmological parameters [1], for instance, gives a 0.73 ± 0.04 abundance for it, and a value of its equation of state $w < -0.78$. In order to relieve some problems of the popular $ΛCDM$ model (like the fine tuning issue), a dynamical $Λ$-term has been proposed as representative of the dark energy. It’s more popular version is a slowly rolling scalar field named quintessence. Many alternative cosmological models have been proposed, and indeed it is a challenge the work of ruling out all the "incorrect ones" on observational grounds. For instance many different potentials for these self interacting scalar fields (quintessence) have been proposed. However, it is obvious the importance of the observational exploration of the proposed cosmological models, and in this paper we give a further step in this direction.

A variety of quintessence self-interaction potentials have been studied. Among them, a single exponential is the simplest case. This last model has two possible late-time attractors in the presence of a barotropic fluid: a scaling regime where the scalar field mimics the dynamics of the background fluid with a constant ratio between both energy densities, or an attractor solution dominated by the scalar field. Some of these models has been studied in references [2, 3, 4, 5]. Given that single exponential potentials can lead to one of the above scaling solutions, then it should follow that a combination of exponentials should allow for a scenario where the universe can evolve through a radiation-matter regime (attractor 1) and, at some recent epoch, evolve into the scalar field dominated regime (attractor 2). For this reason the combination of exponentials represents an interesting alternative. Minimally coupled models with double exponential potentials are studied in [4, 5], meanwhile, Brans-Dicke (BD) models of quintessence with this kind of potential have been studied, for instance, in [6].

The aim of this short paper is to (observationally) check models of the universe with potentials that are combination of exponentials: $V = V_1 \exp(-\alpha \phi) + V_2 \exp(-\beta \phi)$ (both in minimally coupled (Einstein) and BD theories), by considering another aspect of structure formation: the galaxy motions and clustering, i. e., the evolution of density perturbations in the Universe. In the past few years, observations of the large scale structure of the Universe have improved greatly. The development of fiber-fed spectrographs that can simultaneously measure spectra of hundreds of galaxies has provided large redshift surveys such as the 2-degree Field Galaxy Redshift Survey (2dFGRS) and the Sloan Digital Sky Survey (SDSS). In particular, the Anglo-Australian Telescope of the 2dFGRS has obtained the redshift of a quarter million galaxies. This collaboration has produced abundant data and technical papers [6] about galaxy motions and clustering, and we will refer to some of this, in particular their velocity to density comparisons.

The paper is organized as follows: in section II we outline the main characteristics of the models, in section III the main aspects of velocity to density comparisons are exposed and the equation for the growth of perturbations is solved, in section IV the observational check is presented and interpreted, while in section V conclusions are drawn.

II. THE MODELS

In this section we supply the main equations characterizing the dynamics of the models with double exponential potential of the form:

$$V = V_1 e^{-\alpha \phi} + V_2 e^{-\beta \phi},$$  (1)

where $V_1$, $V_2$, $\alpha$ and $\beta$ are free constant parameters. We study separately models with minimal coupling (basically Einstein gravity) and BD models written in Einstein frame variables. We adopt also throughout the paper units with $8\pi G = c = 1$.
A. Einstein’s Theory

We analyze flat Friedmann-Robertson-Walker (FRW) solutions to Einstein’s theory with two fluids: a background fluid of ordinary matter and a self-interacting scalar fluid that accounts for the dark energy component.

If we set the following relationship between the barotropic index of the background (γ) and the free parameters of the above equation (1):

\[ \beta = \frac{3\gamma}{\alpha} \]  (2)

then it can be shown that

\[ a(\tau) = \left\{ \sqrt{\frac{1}{2 - 3\varepsilon}} \sinh[\mu_E(\tau + \tau_0)] \right\}^{\frac{2}{3\gamma - \alpha^2}} . \]  (3)

is an exact solution of the Einstein’s field equations for the above potential.\[5\] Where

\[ \mu_E = (3\gamma - \alpha^2) \sqrt{\frac{\gamma(2 - 3\varepsilon)}{3\gamma - \alpha^2}} \]  (4)

and \( \tau_0 \) is a constant of integration and the following time coordinate \( \tau \) has been introduced instead of the cosmic time \( t \): \( dt = a^{-\frac{3\gamma - \alpha^2}{3\gamma}} \mathrm{d}\tau \).

This solution is very interesting since, the aforementioned relationship between \( \alpha \) and \( \beta \) leads always to a transition from a matter-dominated phase of the cosmic evolution at high redshift, into a (late time) dark energy dominated phase. This is, precisely, the kind of feature observational data suggests the cosmic evolution should share. Other parameters of observational interest are the Hubble expansion parameter:

\[ H(\tau) = \sqrt{\frac{\gamma(2 - 3\varepsilon)}{3\gamma - \alpha^2} a(\tau)^{-\frac{\alpha^2}{3\gamma}} \coth[\mu_E(\tau - \tau_0)]} \]  (5)

the matter density parameter:

\[ \Omega_m(\tau) = (1 - \varepsilon)[\cosh[\mu_E(\tau - \tau_0)]]^{-2} \]  (6)

and the equation of state (EOS) parameter:

\[ \omega_\phi(\tau) = -1 + \frac{\alpha^2}{3(1 - \Omega_m(\tau))} . \]  (7)

This solution depends only on three free parameters (\( \gamma, \alpha, \varepsilon \)), where \( \varepsilon \) is the density \( \Omega_\phi(z) \) of dark energy in the early stages of the evolution (high redshift \( z \gg 1 \)). Nevertheless we will led with a reduce parameters space(\( \alpha, \varepsilon \)) because of we fix \( \gamma = 1 \), meaning cold dark matter dominance at present. Using CMB\[7\], nucleosynthesis\[8\] and galaxy formation\[9\] observations, the parameters space can be constrained to be: 0 < \( \alpha < 1 \) & 0 ≤ \( \varepsilon \) ≤ 0.045\[5\].\[1\]

B. Brans-Dicke gravity

Now we study flat FRW exact solutions to BD theory with two fluids: a background of ordinary matter and a self-interacting BD scalar field fluid accounting for the dark energy in the universe. In this case we are faced with two relevant frames (the Jordan frame and the Einstein frame), in which BD theory can be formulated. There has been discussion on whether these two frames are equivalent\[18\]. It is not our aim to participate in this controversy and for practical reasons (simplicity of mathematical handling) we chose the Einstein frame.

In this frame, if one assumes the following relationship\[2\] between the free parameters \( \alpha \) and \( \beta \) in (1) and \( \gamma \):

\[ \beta = \frac{3\gamma}{\alpha} + \frac{4 - 3\gamma}{2\sqrt{\omega + \frac{3}{2}}} \]  (8)

where \( \omega \) is the BD coupling parameter; then

\[ \bar{a}(r) = \left\{ \sqrt{\frac{1}{2 - 3\varepsilon}} \sinh[\mu_B(r + r_0)] \right\}^{\frac{2}{3\gamma - \alpha^2}} \]  (9)

is an exact solution with:

\[ \mu_B = \frac{\alpha(\beta - \alpha)}{2} \sqrt{\frac{2 - 3\varepsilon}{3\alpha^2(1 - \varepsilon)}} \]  (10)

and

\[ n = \frac{2\sqrt{\omega + \frac{3}{2}} - \alpha}{2\sqrt{\omega + \frac{3}{2}}} \]  (11)

The time coordinate \( r \) and the cosmic time in the Einstein frame \( \bar{t} \) are related by the following expression: \( d\bar{t} = a^{\frac{\alpha^2}{3\gamma}} \mathrm{d}r \). The bar notation means we are working in the Einstein frame of BD theory. The other interesting cosmological parameters are the Hubble expansion parameter:

\[ \bar{H}(r) = \sqrt{\frac{2 - 3\varepsilon}{3\alpha^2(1 - \varepsilon)} \bar{a}(r)^{-\frac{\alpha^2}{3\gamma}} \coth[\mu_B(r + r_0)]} \]  (12)

the matter density parameter

\[ \bar{\Omega}_m(r) = n^2(1 - \varepsilon)[\cosh[\mu_B(r + r_0)]]^{-2} \]  (13)

and the equation of state parameter

\[ \bar{\omega}_\phi(r) = -1 + \frac{\alpha^2}{3(1 - \bar{\Omega}_m(r))} \]  (14)

\( \alpha, \) see \[5\]. Indeed, it is well known the controversy about the degeneracy of supernovae observations\[12, 13, 14, 15, 16, 17\].

\[2\] Recall that this relationship leads to a very desirable pattern for the cosmic evolution. See similar discussion under subsection A.
This cosmological model also depends on three free parameters \((\gamma, \alpha, \varepsilon)\). Like in the former case we fix the value of \(\gamma = 1\) and executing a similar analysis with CMB, nucleosynthesis and galaxy formation observations, the parameter space can be constrained: \(0 < \alpha < 0.37\) & \(0 \leq \varepsilon \leq 0.045\).\(^3\)

### III. PERTURBATION GROWTH

We will perform here a preliminary study of the evolution of the mass density contrast \((\delta = \delta \rho / \rho)\) in the mass distribution, modelled as a pressureless fluid, in linear perturbation theory. The evolution of quintessence density contrast in not considered in this paper for simplicity. We understand, however, that these perturbations might influence perturbations of background matter\(^1\). This method is based on Newtonian mechanics, that is better suited to the study of the development of structure such as galaxies and clusters of galaxies. This computation requires that we be able to isolate a region small enough for the Newtonian gravitational potential energy and the relative velocities within the region to be small (non relativistic)\(^1\). The equation of time evolution of mass density contrast is

\[
\ddot{\delta}_m + 2H \dot{\delta}_m - 4\pi G \rho_m \delta_m = 0, \tag{15}
\]

where the dot means derivative with respect to the comoving time. In this equation the relative contribution of dark energy to the energy budget enters into the expansion rate \(H\). We shall consider this equation in the matter dominated era, when the radiation contribution is really negligible. The linear theory relates the peculiar velocity field \(v\) and the density contrast by \(^11\)

\[
v(x) = H_0 \frac{f}{4\pi} \int \delta_m(y) \frac{x - y}{|x - y|^3} d^3y, \tag{16}
\]

where the growth index \(f\) is defined as

\[
f \equiv \frac{d \ln \delta_m}{d \ln a}. \tag{17}
\]

To solve the equation \((15)\) for the evolution or perturbations, it is useful to rewrite it in terms of suitable variables, allowing some simplification

\[
X^2(1 + X^2) \frac{d^2 \delta_m}{dX^2} + X \frac{d \delta_m}{dX} (X^2 c + d) - e \delta_m = 0, \tag{18}
\]

where the new variable \(X\), the parameters \(c, d\) and \(e\) are characteristic of model as follow:

| TABLE I: This table show the characteristic parameters for each model |
|----------------|------------------|------------------|------------------|
| parameters     | Einstein’s Theory | BD gravity       |
| \(X\)           | \(\sinh[\mu_E(\tau + \tau_0)]\) | \(\sinh[\mu_B(\tau + r_0)]\) |
| \(c\)           | \(0.5 - 3\alpha^2 / (2(3 - \alpha))\) | \(\alpha(d - 2\alpha + 4) / (\alpha(\beta - \alpha)^2)\) |
| \(d\)           | \(X^2 / (2(3 - \alpha)^2)\) | \(4\alpha^2 / (\alpha(\beta - \alpha)^2)\) |
| \(e\)           | \(1 / 2(3 - \alpha)^2\) | \(6\alpha^2 / (\alpha(\beta - \alpha)^2)\) |

Equation \((18)\) has two linearly independent solutions, the growing mode \(\delta_{m+}\) and the decreasing mode \(\delta_{m-}\), which can be expressed in terms of hypergeometric functions of second type \( _2F_1\). We get

\[
\delta_{m+} \propto X^{1/2}(1 - D + g), \quad _2F_1 \left[ \frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, g, \frac{1}{4}, \frac{1}{2},\right]
\]

and

\[
\delta_{m-} \propto X^{1/2}(1 - D - g), \quad _2F_1 \left[ \frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, g, \frac{1}{4}, \frac{1}{2},\right]
\]

where

\[
g = \sqrt{(d - 1)^2 + 4\varepsilon}. \tag{19}
\]

For \(\tau \ll 1\) and \(r \ll 1\) we can write

\[
\delta_{m+} \propto X^{1/2}(1 - D + g), \tag{20}
\]

\[
\delta_{m-} \propto X^{1/2}(1 - D - g). \tag{21}
\]

For determining the growth index of the perturbations we use the growing mode \(\delta_{m+}\) \((20)\) and substitute into \((17)\). It is well known the biasing effect in galaxy formation, i.e.; the relative perturbations in the galaxy field and the matter field, on a point-by-point basis, are not equal:

\[
\frac{\delta n}{n} (x) = b \frac{\delta \rho}{\rho} (x), \tag{22}
\]

where \(n\) refers to the galaxy number density and \(b\) is the bias parameter. The parameter \(\beta = \frac{f}{b}\) relates the growth rate \(f\) of the perturbations (and hence the velocity field of the galaxy motions) with the density bias \(b\). In this sense astrophysicists speak on a velocity/density comparison. Indeed, a compelling agreement is seen to exist between

\(^3\) Note again that SN Ia observations are not useful to constrain the parameter space.
the velocity and density fields, which offers one possible test for the gravitational instability picture for the origin of structure [22].

The 2dFGRS has measured the position and the redshift of a quarter million galaxies and from the analysis of the correlation function, determined the redshift distortion parameter $\beta$ with the bias parameter $b$ quantifying the difference between the galaxies and the dark haloes distribution. Using the estimated $\beta$ and the method employed by [14, 20, 21] to determine the bias $b$, one may estimate $f = 0.51 \pm 0.11$ at the survey effective depth $z = 0.15$. As we can see the growing mode depend of the free parameters $(\alpha, \varepsilon)$ as the growth index $f$ to. Now we use this fact to additionally constrain the above mentioned parameter space.

IV. OBSERVATIONAL CHECK

Applying equation (17), in the flat FRW solutions to Einstein’s theory, does not give further constrain on $\varepsilon$, but it does on $\alpha$, as Figure 1 show.

Assuming a flat universe, the parameter $\varepsilon = \Omega_\phi(\infty)$, is the amount of dark energy in the very early universe ($z \approx \infty$). It is known that an appreciable amount of dark energy at that epoch would imply an expansion fast enough to prevent the formation of structure at $z \sim 3$, but this parameter had been already constrained in Ref. 5, so it is not problematic that now it has not been additionally constrained. In this case, the velocity/density comparison allows to locate $\alpha$ in a rather narrow region, thus acting as a stringent selector of attractor-like solutions of the kind $\mathbb{R}$, $\mathbb{R}$, $\mathbb{R}$, $\mathbb{R}$. Two things are to be said. Firstly, Figure 1 shows the joint region $(\alpha, \varepsilon)$ satisfying the velocity/density comparision, for $\alpha$ alone we might have a wider variation. Secondly, though narrow the interval, still we can speak of $\alpha$ as a selector of a class of solutions.

Figure 2 shows the parameter space that was obtained for the model based on Brans-Dicke gravity. Taking account this result and the former obtained in [5] we construct the final parameter space which obey all constrains considered so far.

As we can see in Figure 3 the parameters $(\alpha, \varepsilon)$ were already considerably constrained from the original observations.

V. CONCLUSIONS

In the models presented here we found that the preliminary study of perturbation growth is a good tool to constraint the parameter space in quintessence models with a self-interaction potential that is a combination...
of exponentials. This kind of potential often appears in fundamental theories like the string or supersymmetry. Besides, it can accommodate a pattern of cosmic evolution that is characterized by a late time, dark energy dominated attractor.

Research on the origins and evolution of the large-scale of the universe is one of the hottest topics in cosmology. In this work, we have used the relation between the peculiar velocity field of the galaxies, the growth rate of perturbations and the density bias in galaxy formation to make another step in the observational check of two quintessence models, resulting in a further constrain on the parameter space of the models. We consider that above discussion adds another argument in favor of the use of exponential potentials in quintessence cosmology. We plan in the oncoming future proceed this works using another cosmological probes, like CMB, for instance. It could be desirable to consider perturbations of the quintessence field also.

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