Klimontovich-Langevin Approach to the Fluctuation-Dissipation Theorem for a Nonlocal Plasma

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Abstract. Using the Klimontovich-Langevin approach and the multiscale technique, a kinetic theory of the time and space nonlocal fluctuations in the collisional plasma is constructed. In local equilibrium a generalized version of the Callen-Welton theorem is derived. It is shown that not only the dissipation but also the time and space derivatives of the dispersion determine the amplitude and the width of the spectrum lines of the electrostatic field fluctuations, as well as the form factor. There appear significant differences with respect to the non-uniform plasma. In the kinetic regime the form factor is more sensible to space gradient than the spectral function of the electrostatic field fluctuations. As a result of the inhomogeneity, these proprieties became asymmetric with respect to the inversion of the frequency sign. The differences in amplitude of peaks could become a new tool to diagnose slow space variations in the plasma.

1. Introduction
I have faith that the introduction of Langevin equation for the smoothed distribution function is the most significant achievement of Yu. L. Klimontovich. In contrast with the initial view on the concept, defined by B.B. Kadomtsev as a external casual force in the kinetic equation, Klimontovitch introduced the casual source under kinetic operator. As many other results of Klimontovich, this equation was founded on physical intuition. The present paper is devoted to the application of Klimontovich-Langevin equation to fluctuation-dissipation theorem. Klimontovich consecrated his last years on this issue. Fluctuations attract a great deal of attention. Besides being of interest from the fundamental point of view, there are situations where nonequilibrium fluctuations play an important role, namely in the neighborhood of bifurcations where the system has to choose a branch [1]. Moreover fluctuations determine the sensibility and the quality factor of the system. And, at last, fluctuations find an application in diagnostic procedures. Indeed, plasma parameters such as temperature, mean velocity, density and their respective profiles can be determined by incoherent (Thomson) scattering diagnostics [2], i.e. by the proper interpretation of data obtained from the scattering of a given electromagnetic field interacting with the system. The key point of interpreting them is the knowledge of the intensity of the dielectric function fluctuations or equally of the electron form factor \((\delta n_e \delta n_e)_{\omega, k}\). Here \(\omega\) and \(k\) are respectively the frequency and wavevector of the autocorrelations. Due to the Poisson equation the electron form factor in the spatially homogeneous system is directly linked to the electrostatic field fluctuations, which have been
temperatures and velocities per species (Θ state of the plasma is given by Maxwellian distributions characterized by plasma frequencies. The matter becomes more tricky in the non-equilibrium case. When the temperature Θ in energy units. The spectral function (1) has peaks, corresponding to proper
fluctuations): they are linked by the Poisson equation. But for the spatial inhomogeneous plasma this is not evident: the Poisson equation is a non-local relation. In reality, the non-local correction to the spectral function of the electrostatic field fluctuations is different from the correction to the form factor. For example, for the plasma mode we get:

\[
(\delta n_a \delta n_e)_{\omega k} = \frac{2n_e k^2}{\omega k_D^2} \left[ \frac{Im \varepsilon + \frac{\partial^2 Re \varepsilon}{\partial \omega \partial \mu} - \frac{1}{k^2} \frac{\partial}{\partial \mu_r} k_j \frac{\partial}{\partial \mu_r} k_j Re \varepsilon}{Re \varepsilon^2 + \left( Im \varepsilon + \frac{\partial^2 Re \varepsilon}{\partial \omega \partial \mu} - \frac{1}{k^2} \frac{\partial}{\partial \mu_r} k_j \frac{\partial}{\partial \mu_r} k_j Re \varepsilon \right)^2} \right].
\]
The inhomogeneous correction in (4) is greater than in (3) by a factor $1 + k_D^2/6k^2$. The structure of the formulae (3) and (4) is very interesting, because it contains both dissipative and dispersive contributions to the fluctuations. Moreover the non-homogeneous terms break the symmetry with respect to $\omega$. This leads to an asymmetric spectrum of the fluctuations: the position of the resonances remain the same with respect to the homogeneous case, but their intensity and broadening are different. The difference in the amplitudes of the peaks could be a diagnostic tool to measure local gradients in the plasma.

Note that for simple fluids a general theory of hydrodynamic fluctuations for nonequilibrium stationary inhomogeneous states has been developed in series of publications [7]. In particular, an asymmetry of the spectrum for the Brillouin scattering from a fluid in a shear flow or temperature gradient was obtained. The situation for the plasma problem we are considering is however rather different.

2. Electrostatic field fluctuations and form factor with respect to a inhomogeneous and nonstationary reference state.

To derive nonlocal expressions for the spectral function of the electrostatic field fluctuation and for the electron form factor we use the Klimontovich-Langevin approach to describe kinetic fluctuations [4]. The starting point of our procedure is the same as in [5]. A kinetic equation for the fluctuation $\delta f_a$ of the one-particle distribution function (DF) with respect to the reference state $f_a$ is considered. In the general case the reference state is a nonequilibrium DF which varies in space and time both on the kinetic scale (mean free path $l_e$ and interparticle collision time $\nu_a^{-1}$) and on the larger hydrodynamic scales. These scales are much larger than the characteristic fluctuation time $\omega^{-1}$. In the non-equilibrium case we can therefore introduce a small parameter $\mu = \nu_a/\omega$, which allows us to describe fluctuations on the basis of a multiple space and time scale analysis. Obviously, the fluctuations vary on both the "fast" $(r, t)$ and the "slow" $(\mu r, \mu t)$ time and space scales: $\delta f_a(x, t) = \delta f_a(x, \mu t, \mu r)$ and $f_a(x, t) = f_a(p, \mu t, \mu r)$. Here $x$ stands for the phase-space coordinates $(r, p)$. The Klimontovich-Langevin kinetic equation for $\delta f_a$ has the form [4]

$$\hat{L}_{\text{axt}}(\delta f_a(x, t) - \delta f_a^S(x, t)) = -e_a \delta \mathbf{E}(r, t) \cdot \frac{\partial f_a(x, t)}{\partial \mathbf{p}},$$

(5)

where

$$\hat{L}_{\text{axt}} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial r} + \hat{\Gamma}_a(t, r); \quad \hat{\Gamma}_a(t, r, p) = e_a \mathbf{E} \cdot \frac{\partial}{\partial p} - \delta \hat{I}_a;$$

$\delta \hat{I}_a$ is the linearized Balescu-Lenard collision operator.

The Klimontovich-Langevin source in eq. (5) is determined [5] by following equation:

$$\hat{L}_{\text{axt}} \delta f_a(x, t) \delta f_b(x', t')^S = \delta_{ab} \delta(t - t') \delta(x - x') f_a(x', t').$$

The solution of eq. (5) has the form

$$\delta f_a(x, t) = \delta f^S(x, t) - \sum b \int dx' \int dt' G_{ab}(x, t, x', t') e_b \delta \mathbf{E}(r', t') \cdot \frac{\partial f_b(x', t')}{\partial \mathbf{p}}.$$  

(6)

where the Green function $G_{ab}(x, t, x', t')$ of the operator $\hat{L}_{\text{axt}}$ is determined by

$$\hat{L}_{\text{axt}} G_{ab}(x, t, x', t') = \delta_{ab} \delta(x - x') \delta(t - t')$$

with the causality condition $G_{ab}(x, t, x', t') = 0$, when $t < t'$. Thus, $\delta f_a(x, t) \delta f_b(x', t')^S$ and $G_{ab}(x, t, x', t')$ are connected by the relation:

$$\delta f_a(x, t) \delta f_b(x', t')^S = G_{ab}(x, t, x', t') f_b(x', t').$$

For the stationary and nonstationary systems, when $\delta f_a$ and the operator $\hat{\Gamma}_a$ do not depend on time and space, $G_{ab}(x, t, x', t')$ can depend only on its time and space variables through
the difference \( t - t' \) and \( r - r' \). In the general case, when the one-particle DF \( f_a(p, \mu, \mu_t) \) and the operator \( \hat{\Gamma}_a \) slowly (in comparison with the correlation scales) vary in time and space, and when non-local effects are considered, the time and space dependence of \( G_{ab}(x,t,x',t') \) is more subtle.

\[
G_{ab}(x,t,x',t') = G_{ab}(p,p',r-r',t-t',\mu r',\mu t').
\]  

(7)

For the homogeneous case this non-trivial result was obtained for the first time in [8]. For inhomogeneous systems it has been generalized recently in [9].

The relationship (7) is directly linked with the constitutive relation between the electric displacement and the electric field:

\[
D_i(r,t) = \int dr' \int dt' \varepsilon_{ij}(r-r',t-t';\mu r' + \frac{r+r'}{2},\mu t' + \frac{t+t'}{2}) E_j(r',t').
\]  

(8)

Previously two kinds of constitutive relations were proposed phenomenologically for a weakly-inhomogeneous and slowly time-varying medium:

(i) the so-called symmetrized constitutive relation [10]:

\[
D_i(r,t) = \int dr' \int dt' \varepsilon_{ij}(r-r',t-t';\mu r,\mu t)) E_j(r',t').
\]  

(9)

(ii) the non-symmetrized constitutive relation [11]:

\[
D_i(r,t) = \int dr' \int dt' \varepsilon_{ij}(r-r',t-t';\mu r',\mu t')) E_j(r',t').
\]  

(10)

Both phenomenological formulations (i) and (ii) are unsatisfactory. The correct expression should be

\[
D_i(r,t) = \int dr' \int dt' \varepsilon_{ij}(r-r',t-t';\mu r',\mu t')) E_j(r',t').
\]  

(10)

Taking into account the first-order terms with respect to \( \mu \) from (6) and (7) we have

\[
\delta f_a(x,t) = \delta f_a^S(x,t) - \sum_b \int dp' d\rho \int_0^\infty d\tau
\]

\[
(1 - \mu \tau \frac{\partial}{\partial \mu_t} - \mu \rho \cdot \frac{\partial}{\partial \mu r}) e_b \delta E(r-\rho, t-\tau) G_{ab}(\rho, \tau, p, p', \mu, \mu_t, \mu r) \frac{\partial f_b(p', \mu, \mu r)}{\partial p'},
\]  

(11)

\((\rho = r-r', \tau = t-t')\).

From the Poisson equation

\[
\delta E(r, t) = -\frac{\partial}{\partial \tau} \sum_b e_b \int \frac{1}{|r-r'|} \delta f_b(x', t) d\mathbf{x}'
\]  

(12)

and performing the Fourier-Laplace transformation

\[
\delta E(k, \omega) = \int_0^\infty dt \int d\mathbf{r} \delta E(r, t) \exp(-\Delta t + i\omega t - i\mathbf{k} \cdot \mathbf{r}).
\]

from (11) we have
\[ \delta E(k, \omega, \mu, \mu r) = \delta E^*(k, \omega) + \sum_a 4\pi ie_a^2 \int dp [(1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial \mu}) \hat{L}^{-1}_{aw} \delta E(k, \omega, \mu r, \mu t) \cdot \frac{\partial f_a(p, \mu r, \mu t)}{\partial p}] \]

\[ -i \frac{\partial}{\partial \mu r_i} \delta E(k, \omega, \mu r, \mu t) \frac{\partial}{\partial k_i} \hat{L}^{-1}_{aw} \delta f_a(p, \mu r, \mu t), \]

(13)

Here and in the following for simplicity we omit \( \mu \), keeping in mind that derivatives over coordinates and time are taken with respect to the slowly varying variables. The resolvent in (13) is determined by the following relation:

\[ \int dp d\tau \exp(-\Delta \tau + i \omega \tau - i k \cdot \rho) G_{ab}(\rho, \tau, p, p', \mu r, \mu r) = \hat{L}^{-1}_{aw} \delta_{ab}(p - p'). \]

The approximation in which Eq. (13) was derived corresponds to the geometric optics approximation [12]. At first-order and after some manipulations, one obtains from Eq. (13) the transport equation in the geometric optics approximation, which is not considered in the present article, and the equation for the spectral function of the electrostatic field fluctuations:

\[ Re \tilde{\varepsilon}(\omega, k) \int (\delta E \delta E^*)_{\omega, k} = \frac{1}{|\tilde{\varepsilon}(\omega, k)|^2} (\delta E \delta E^*)_{\omega, k} = 0, \]

(14)

where we introduced

\[ \tilde{\varepsilon}(\omega, k) = 1 + \sum_a \tilde{\chi}_a(\omega, k); \varepsilon(\omega, k) = 1 + \sum_a \chi_a(\omega, k) \]

\[ \tilde{\chi}_a(\omega, k) = (1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} - i \frac{\partial}{\partial k} \cdot \frac{\partial}{\partial r}) \chi_a(\omega, k, t, r), \]

(15)

and where

\[ \chi_a(\omega, k, t, r) = -\frac{4\pi e^2}{e^2} \int dp \hat{L}^{-1}_{aw} k \cdot \frac{\partial}{\partial r} f_a(p, t, r) \]

is the susceptibility for a collisional plasma. In the same approximation the spectral function of the Klimontovich-Langevin source \((\delta E \delta E)_{\omega, k}^S\) takes the form

\[ (\delta E \delta E)_{\omega, k}^S = 32\pi^2 \sum_a e_a^2 Re \int dp [(1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} - i \frac{\partial}{\partial k} \cdot \frac{\partial}{\partial r}) \frac{1}{k^2} \hat{L}^{-1}_{aw} f_a(p, r, t)]. \]

(16)

If \( Re \tilde{\varepsilon}(\omega, k) \neq 0 \), it follows from Eqs. (14) and (16) that the spectral function of the nonequilibrium electrostatic field fluctuations is determined by the expression:

\[ (\delta E \delta E)_{\omega, k} = \frac{32\pi^2 \sum_a e_a^2 Re \int dp [(1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} - i \frac{\partial}{\partial k} \cdot \frac{\partial}{\partial r}) \frac{1}{k^2} \hat{L}^{-1}_{aw} f_a(p, r, t)]}{|\tilde{\varepsilon}(\omega, k)|^2}. \]

(17)

The effective dielectric function \( \tilde{\varepsilon}(\omega, k) \) in the denominator of Eq. (17) determines the spectral properties of the electrostatic field fluctuations and its imaginary part

\[ Im \tilde{\varepsilon}(\omega, k) = Re \varepsilon(\omega, k) + \frac{\partial}{\partial \omega} \frac{\partial}{\partial \mu} Re \varepsilon(\omega, k, t, r) - \frac{\partial}{\partial k} \cdot \frac{\partial}{\partial r} Re \varepsilon(\omega, k, t, r), \]

(18)

determines the width of the spectral lines near the resonance. Note that when expanding the Green function in Eq. (11) in terms of the small parameter \( \mu \), there appear additional terms at first order. It is important to note that the imaginary part of the dielectric susceptibility is now replaced by the real part, which is greater than imaginary part by the factor \( \mu^{-1} \). Therefore, the
second and third terms in Eq. (18) in the kinetic regime have an effect comparable to that of the first term. At second order in the expansion in \( \mu \) the corrections appear only in the imaginary part of the susceptibility, and they can reasonably be neglected. It is therefore sufficient to retain the first order corrections to solve the problem.

For the local equilibrium case where the reference state \( f_a \) is Maxwellian, we have the identity:

\[
\int dp (1 + i \frac{\partial}{\partial \omega} - i \frac{\partial}{\partial k} \varepsilon) \tilde{L}_{a\omega k} f_a (p, t, r) = \frac{1}{\omega_a} \int dp f_a (p, t, r) - \frac{\partial f_a}{\partial \omega} \tau_a (\omega, k) \quad (\omega_a = \omega - k V_a)
\]

and Eq.(17) takes the form

\[
(\delta \mathbf{E} \delta \mathbf{E})_{\omega, k} = \sum_a \frac{8 \pi}{\omega_a |\tilde{\varepsilon}(\omega, k)|^2} \text{Im} \tilde{\chi}_a (\omega, k).
\]

This expression is the effective damping decrement. For the case where the system parameters are homogeneous in space but vary in time, the correction is still symmetric with respect to the change of sign of \( \omega \), but the intensities and broadening are different, and the intensity integrated over the frequencies remains the same as in the stationary case. However, when the plasma parameters are space dependent this symmetry is lost. In the same manner as for simple fluids and gases [7] the spectral asymmetry is related to the appearance of space anisotropy in inhomogeneous systems.

\[
\text{Im} \varepsilon (\omega, k) = \sum_a \frac{8 \pi}{\omega_a |\tilde{\varepsilon}(\omega, k)|^2} \text{Im} \tilde{\chi}_a (\omega, k).
\]

To calculate explicitly \((\delta \mathbf{E} \delta \mathbf{E})_{\omega, k}\) we will restrict our analysis to the vicinity of the resonance, i.e. \( \omega = \pm \omega_0 \), where \( \text{Re} \varepsilon (\omega_0, k) = 0 \). We can develop \( \tilde{\varepsilon}(\omega, k) = (\omega - \omega_0 \text{sgn} \omega) \frac{\partial \text{Re} \varepsilon}{\partial \omega} |_{\omega = \omega_0 \text{sgn} \omega} + i \text{Im} \varepsilon + (\frac{\partial^2 \varepsilon}{\partial \omega \partial t} - \frac{\partial}{\partial k} \cdot \frac{\partial}{\partial r} \text{Re} \varepsilon) |_{\omega = \omega_0 \text{sgn} \omega} \). Thus \((\delta \mathbf{E} \delta \mathbf{E})_{\omega, k} = \frac{\tilde{\gamma}}{(\omega - \omega_0 \text{sgn} \omega) + \gamma^2 \omega |\text{Re} \varepsilon|/\text{Re} \varepsilon |_{\omega = \omega_0 \text{sgn} \omega}, where

\[
\tilde{\gamma} = \frac{\text{Im} \varepsilon + \frac{\partial^2 \varepsilon}{\partial \omega \partial t} \text{Re} \varepsilon - \frac{\partial}{\partial k} \cdot \frac{\partial}{\partial r} \text{Re} \varepsilon |_{\omega = \omega_0 \text{sgn} \omega}}{\text{Re} \varepsilon |_{\omega = \omega_0 \text{sgn} \omega}}
\]

is the effective damping decrement. For the case where the system parameters are homogeneous in space but vary in time, the correlation is still symmetric with respect to the change of sign of \( \omega \), but the intensities and broadening are different, and the intensity integrated over the frequencies remains the same as in the stationary case. However, when the plasma parameters are space dependent this symmetry is lost. In the same manner as for simple fluids and gases [7] the spectral asymmetry is related to the appearance of space anisotropy in inhomogeneous systems. The real part of the susceptibility \( \text{Re} \varepsilon \) is an even function of \( \omega \). This property implies that the contribution of the third term to the expression of the damping decrement (21) is an odd function of \( \omega \). Moreover this term gives rise to an anisotropy in \( k \) space.

Let us estimate this correction for the plasma mode \((\omega_0 = \omega_L)\) \( \text{Re} \varepsilon = 1 - \omega_L^2 / (1 + 3 \frac{k^2 \Theta}{m \omega^2}) \),

\[
\text{Im} \varepsilon = \frac{\omega_L^2}{\omega^2} \frac{\nu_\perp}{\omega}, \quad \omega_L^2 = \frac{4 \pi n e^2}{m} = \frac{\Theta k_D^2}{m}
\]

and

\[
\tilde{\gamma} = \frac{\nu_\perp + 2 \frac{\partial n}{\partial t} + 6 \frac{\omega_L}{nk_D^2} k \cdot \frac{\partial n}{\partial r} \text{sgn} \omega}{2}.
\]

For the spatially homogeneous case there is no difference between the spectral properties of the longitudinal electric field and of the electron density. They are connected by the Poisson equation. This statement is no longer valid when considering an inhomogeneous plasma. Indeed the longitudinal electric field is linked to the particle density by the nonlocal Poisson relation (12). In the latter case, an analysis similar to that made above can also be performed for the particle density. From Eq. (6) there follows

\[
\delta n_a (k, \omega, r, t) = \delta n_a^S (k, \omega, r, t) + \sum_b \frac{4 \pi i e_b e_a}{k^2} \int dp [(1 + i \frac{\partial}{\partial \omega} - i \frac{\partial}{\partial k} \varepsilon) \tilde{L}_{a\omega k} \delta n_b (k, \omega, r, t) \cdot \frac{\partial f_a (p, r, t)}{\partial p}
\]
expression for the electron form factor for a two-component (\(a = e, i\)) plasma:

\[-i \frac{\partial}{\partial r_j} \delta n_b(k, \omega, r, t) \frac{\partial}{\partial k_i} \tilde{I}_{-1}(\omega k) \frac{\partial f_a(p, r, t)}{\partial p}.\]  \quad (23)

At the first order approximation and after some manipulations, one obtains the following expression for the electron form factor for a two-component (\(a = e, i\)) plasma:

\[
(\delta n_e \delta n_e)_{\omega, k} = \frac{2n_e k^2}{\omega L} \left[ 1 + \frac{\chi_i(\omega, k)}{\chi_e(\omega, k)} \right] \frac{\partial}{\partial \omega} \frac{\partial R}{\partial \omega} \int \frac{k^2}{\chi_e(\omega, k)} \frac{\partial R}{\partial \omega} \bigg|_{\omega = \omega_L}.
\]

\[
+ \left[ \frac{\chi_e(\omega, k)}{\chi_i(\omega, k)} \right]^2 \frac{\partial}{\partial \omega} \frac{\partial R}{\partial \omega} \int \frac{k^2}{\chi_i(\omega, k)} \frac{\partial R}{\partial \omega} \bigg|_{\omega = \omega_L}.
\]  \quad (24)

where we used for local equilibrium the following expression for the "source" \((\delta n_a \delta n_b)_{\omega, k} = \delta a b \frac{k^2}{2 \pi e} \cos \omega \tilde{\chi}(\omega, k),\)

and \(\tilde{\chi}(\omega, k) = 1 + \sum \chi_a(\omega, k); \tilde{\chi}(\omega, k) = (1 + i \frac{\partial}{\partial \omega} \frac{\partial R}{\partial \omega}) \frac{\partial R}{\partial \omega} \bigg|_{\omega = \omega_L}.
\)

As above we can expand \(\tilde{\chi}(\omega, k)\) near the plasma resonance \(\omega = \omega_L.\) Thus, for the electron line,

\[
(\delta n_e \delta n_e)_{\omega, k} = \frac{2n_e k^2}{\omega L} \left[ 1 + \frac{\chi_i(\omega, k)}{\chi_e(\omega, k)} \right] \frac{\partial}{\partial \omega} \frac{\partial R}{\partial \omega} \int \frac{k^2}{\chi_e(\omega, k)} \frac{\partial R}{\partial \omega} \bigg|_{\omega = \omega_L}.
\]

we note that the damping decrement for the electrostatic field fluctuations (Eq. 21) and for the electron density fluctuations (Eq. 25) are not the same. The origin of this difference is that the Green function for electrostatic field fluctuation and density particle fluctuations are not the same. This property holds only in the inhomogeneous situation. An estimation for the plasma mode is then:

\[
\tilde{\chi} = \frac{1}{2} \left[ 1 + \frac{\partial n}{\partial \omega} \frac{\partial R}{\partial \omega} - \frac{1}{k^2} \frac{\partial}{\partial r_i} \frac{\partial}{\partial r_j} \frac{\partial}{\partial r_k} \frac{\partial R}{\partial \omega} \bigg|_{\omega = \omega_L} \right]\!
\]

From this equation we see that the inhomogeneous correction in Eq.26 is greater than the one in Eq. 22 by the factor \(1 + k^2/6k^2.\) For the same inhomogeneity; i.e., the same gradient of the density, we plot the form factor \((\delta n_e \delta n_e)_{\omega, k}\) together with the \((\delta \Theta \Theta \Theta)_{\omega, k}\) as functions of frequency.

This figure shows that the asymmetry of the spectral lines is present both for \((\delta n_e \delta n_e)_{\omega, k}\) and \((\delta \Theta \Theta \Theta)_{\omega, k}.\) However, this effect is more pronounced in \((\delta n_e \delta n_e)_{\omega, k}\) than in \((\delta \Theta \Theta \Theta)_{\omega, k}.\)

The fluctuation-dissipation dispersion theorem can be generalized to the arbitrary dissipative systems with slowly varying parameters. Using the momentum method \[13\] and the time multiscale technique, was derived \[14\], that a generalized Callen-Welton formula takes the form:

\[
(\delta A \delta B)_{\omega} = \text{Im} \alpha_{AB}(\omega) + \frac{\partial^2}{\partial t \partial \omega} \text{Re} \alpha_{AB}(\omega) \frac{2 \Theta}{\omega},
\]  \quad (27)

where \(\alpha_{AB}(\omega)\) is the response function. The width and the amplitude of the spectral lines of the fluctuations are determined not only by the dissipation but also by the derivatives of the dispersion. These two effects have a comparable influence for systems with a high quality factor. As an application a LC-circuit was considered. It is shown that the spectral function of the current depends not only on the real part of the impedance (dissipation) but also on the
3. Conclusion

We have shown that the amplitude and the width of the spectral lines of the electrostatic field fluctuations and form factor are affected by new non-local dispersive terms. They are not related to Joule dissipation and appear because of an additional phase shift between the vectors of induction and electric field. This phase shift results from the finite time needed to set the polarization in the plasma with dispersion. Such a phase shift in the plasma with space dispersion appears due to the medium inhomogeneity. These results are important for the understanding and the classification of the various phenomena that may be observed in applications; in particular, the asymmetry of lines can be used as a diagnostic tool to measure local gradients in the plasma.

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4. References

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