Weak Decays of Triply Heavy Baryons

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After the experimental establishment of doubly heavy baryons, baryons with three quarks are the last missing pieces of the lowest-lying baryon multiplets in quark model. In this work we study semileptonic and nonleptonic weak decays of triply heavy baryons, $\Omega^{++}_{ccc}$, $\Omega^+_{cbb}$, $\Omega^0_{cbb}$, $\Omega^-_{bbb}$. Decay amplitudes for various channels are parametrized in terms of a few SU(3) irreducible amplitudes. We point out that branching fractions for Cabibbo allowed processes, $\Omega^{++}_{ccc} \to (\Xi^{++}_{cc}K^0, \Xi^{++}_{cc}K^−\pi^+, \Omega^+_{cc}\pi^+, \Xi^+_cD^+, \Xi'_cD^+, \Lambda_cD^+\overline{K}^0, \Xi^+_cD^0\pi^+, \Xi^0_cD^+\pi^+)$ may reach a few percents. We suggest our experimental colleagues to perform a search at hadron colliders and the electron and positron collisions in future, which will presumably lead to discoveries of triply heavy baryons and complete the baryon multiplets. Using the expanded amplitudes, we derive a number of relations for the partial widths which can be examined in future.

I. INTRODUCTION

In the past decades, hadron spectroscopy has experienced a rapid renaissance, predominantly propelled by discoveries of a number of hadron resonances that defy the standard quark model interpretations. These resonant states are generically classified as hadron exotics, and for reviews on recent progresses, please see Refs. [1–4]. Among all exotic hadrons, the $X(3872)$ plays a most important role due to its special properties. Aside from these unexpected discoveries, there are also gradual progresses in the traditional sector of the charmonium and bottomonium spectroscopy. One of the highlights in recent years is the discovery of $\Xi^{\pm\pm}_{cc}$ by the LHCb collaboration [5]:

$$m_{\Xi^{\pm\pm}_{cc}} = (3621.40 \pm 0.72 \pm 0.27 \pm 0.14)\text{MeV}. \quad (1)$$

This observed state fills well in the quark model as the lowest-lying $ccu$ baryon [6]. After the tentative establishment of $\Xi^{\pm\pm}_{cc}$, it is plausible to fill the baryon family with the last missing members, namely, baryons made of three heavy quarks. Charm and bottom quarks are much heavier than the $u, d, s$, thus baryons with three heavy quarks will refrain from light quark contaminations and the study of triply heavy baryons can help us to better understand the dynamics of strong interactions.

Previous studies of triply heavy baryons concentrated on three facets: spectroscopy, production and decays. Most theoretical investigations in the literature, especially in recent years, have focused...
on the masses and magnetic moments\textsuperscript{7,20}, while less attentions have been paid to the production and decay properties. The only available estimate of the production is conducted in Refs.\textsuperscript{21,22}, where the cross sections at the LHC with $\sqrt{s} = 7$ TeV are found to reach the 0.1 nb level depending on different kinematics cuts. In the $b \to c$ transitions among triply heavy baryons, Ref.\textsuperscript{23} has discussed the implications of heavy quark spin symmetry. Some decay modes of the $\Omega_{ccc}$ are analyzed recently in Ref.\textsuperscript{24}.

The main focus of this paper is to provide a systematic analysis of weak decays of the lowest-lying triply heavy baryons, $\Omega_{ccc,ccb,cbb,bbb}$. The $\Omega_{ccc}$ and $\Omega_{bbb}$ have spin $3/2$, while the $\Omega_{ccb}$ and $\Omega_{cbb}$ can be the $J^P = 1/2^+$ or $J^P = 3/2^+$ state. As we will show, various types of weak decays of triply heavy baryons occur, but unfortunately, a universal dynamical (factorization) approach can not be established yet. This gives a barrier for us to predict their decay widths. Instead we will use an optional theoretical tools to analyze heavy quark decays, the flavor SU(3) symmetry\textsuperscript{24–54}. One advantage of the SU(3) analysis is that it is independent of the factorization details. In this work, we consider semileptonic decay channels with one or two hadrons in the final state, while for nonleptonic decays, the two-body and three-body modes will be analyzed.

The rest of this paper is organized as follows. In Sec.\textsuperscript{II} we will collect the representation matrices for various particle multiplets in the SU(3) group. In Sec.\textsuperscript{III} semileptonic decay modes with one or two hadrons in the final state are analyzed. In Sec.\textsuperscript{IV}, Sec.\textsuperscript{V}, and Sec.\textsuperscript{VI} nonleptonic decays of $\Omega_{ccc}$, $\Omega_{bbb}$, $\Omega_{ccb}$ and $\Omega_{cbb}$ will be studied in order. In Sec.\textsuperscript{VII} we shall present a collection of golden modes which are most likely to discover the triply heavy baryons. A short summary is given in the last section.

\section{II. PARTICLE MULTIPLETS}

In this section, we will collect the representations for hadron multiplets under the flavor SU(3) group. We start with the baryon sector. Light baryons made of three light quarks can group into an SU(3) octet and a decuplet. The light baryon decuplet is symmetric in SU(3) flavor space and is characterized by the following matrix

\begin{align}
(T_{10})^{111} &= \Delta^{++}, \quad (T_{10})^{112} = (T_{10})^{121} = (T_{10})^{211} = \frac{1}{\sqrt{3}} \Delta^+, \\
(T_{10})^{222} &= \Delta^-, \quad (T_{10})^{122} = (T_{10})^{212} = (T_{10})^{221} = \frac{1}{\sqrt{3}} \Delta^0, \\
(T_{10})^{113} &= (T_{10})^{131} = (T_{10})^{311} = \frac{1}{\sqrt{3}} \Sigma'^{+}, \quad (T_{10})^{223} = (T_{10})^{232} = (T_{10})^{322} = \frac{1}{\sqrt{3}} \Sigma'^{-}, \\
(T_{10})^{123} &= (T_{10})^{132} = (T_{10})^{213} = (T_{10})^{231} = (T_{10})^{312} = (T_{10})^{321} = \frac{1}{\sqrt{6}} \Sigma^0, \\
(T_{10})^{133} &= (T_{10})^{313} = (T_{10})^{331} = \frac{1}{\sqrt{3}} \Xi'^{0}, \quad (T_{10})^{233} = (T_{10})^{323} = (T_{10})^{332} = \frac{1}{\sqrt{3}} \Xi'^{-}, \\
(T_{10})^{333} &= \Omega^-.
\end{align}
The octet has the expression:

\[
T_8 = \left( \begin{array}{ccc}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & \Sigma^+ & p \\
-\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & \Xi^- & n \\
\Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}} \Lambda^0
\end{array} \right).
\] (3)

The triply heavy baryons form an SU(3) singlet, while doubly heavy baryons are an SU(3) triplet:

\[
T_{cc} = \begin{pmatrix} \Xi_{cc}^{++}(ccu) \\ \Xi_{cc}^+(ccd) \\ \Omega_{cc}^{cc}(ccs) \end{pmatrix}, \quad T_{bc} = \begin{pmatrix} \Xi_{bc}^+(bcu) \\ \Xi_{bc}^0(bcd) \\ \Omega_{bc}^0(bcs) \end{pmatrix}, \quad T_{bb} = \begin{pmatrix} \Xi_{bb}^0(bbu) \\ \Xi_{bb}^0(bbd) \\ \Omega_{bb}^0(bbs) \end{pmatrix}.
\] (4)

Singly charmed and bottom baryons with two light quarks can form an anti-triplet or sextet. For the anti-triplet and sextet, we have the matrix expression:

\[
T_{c3} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}, \quad T_{c6} = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}} \Sigma_c^+ & \frac{1}{\sqrt{2}} \Xi_c^+ \\ \frac{1}{\sqrt{2}} \Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}} \Xi_c^0 \\ \frac{1}{\sqrt{2}} \Xi_c^+ & \frac{1}{\sqrt{2}} \Xi_c^0 & \Omega_c^0 \end{pmatrix}.
\] (5)

In the meson sector, a light pseudo-scalar meson is formed by a light quark and one light antiquark. It forms an octet:

\[
M_8 = \begin{pmatrix} \pi^0 \sqrt{\frac{1}{2}} + \eta \sqrt{\frac{1}{6}} \\ \pi^- \sqrt{\frac{1}{2}} + \eta \sqrt{\frac{1}{6}} \\ K^0 \sqrt{\frac{1}{2}} - 2\eta \sqrt{\frac{1}{6}} \end{pmatrix},
\] (6)

while the singlet \( \eta_1 \) is not considered. Our analysis is also applicable to light vector mesons and other light mesons. The charmed meson forms an SU(3) anti-triplet:

\[
D_i = \begin{pmatrix} D^0, D^+, D_s^+ \end{pmatrix},
\] (7)

and the anti-charmed meson forms an SU(3) triplet:

\[
\overline{D}^i = \begin{pmatrix} \overline{D}^0, D^-, D_s^- \end{pmatrix}.
\] (8)

The above two SU(3) triplets are also applicable to the bottom mesons.

In the following we will construct the hadron-level effective Hamiltonian for various decay modes. It is necessary to point out that a hadron in the final state must be created by its anti-particle field. For instance, in order to produce a \( \Xi_{ccu}^{++} \), one must use the \( \Xi_{ccu}^{--} \) in the Hamiltonian, and the doubly heavy baryon anti-triplet is abbreviated as \( \overline{T}_{cc} \).
TABLE I: Amplitudes for semileptonic $\Omega_{ccc}$ decays derived from Eq. (10). The light meson in the final state can be replaced by its vector counterpart. For instance, the $K^0$ can be replaced by a $K^{*0}$ decaying into $K^-\pi^+$. 

| channel $\Omega_{ccc}$ | amplitude | channel $\Omega_{ccc}$ | amplitude |
|------------------------|-----------|------------------------|-----------|
| $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \ell^+ \nu_\ell$ | $a_1 V_{cd}^*$ | $\Omega_{ccc}^{++} \rightarrow \Lambda_\ell^+ D^0 \ell^+ \nu_\ell$ | $a_3 V_{cd}^*$ |
| $\Omega_{ccc}^{++} \rightarrow \Omega_{ccc}^{++*} \ell^+ \nu_\ell$ | $a_1 V_{cs}^*$ | $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{+} D^0 \ell^+ \nu_\ell$ | $a_3 V_{cs}^*$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \nu_\ell \ell^+ \nu_\ell$ | $a_2 V_{cs}^*$ | $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{0} D^0 \ell^+ \nu_\ell$ | $a_3 V_{cs}^*$ |
| $\Omega_{ccc}^{++} \rightarrow K^- \ell^+ \nu_\ell$ | $a_2 V_{cs}^*$ | $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{+} D^0 \ell^+ \nu_\ell$ | $-a_3 V_{cs}^*$ |
| $\Omega_{ccc}^{++} \rightarrow K^0 \ell^+ \nu_\ell$ | $\frac{a_2 V_{cs}^*}{\sqrt{2}}$ | $\Omega_{ccc}^{++} \rightarrow \Sigma^0 D^+ \ell^+ \nu_\ell$ | $\frac{a_3 V_{cs}^*}{\sqrt{2}}$ |
| $\Omega_{ccc}^{++} \rightarrow K^0 \ell^+ \nu_\ell$ | $a_2 V_{cs}^*$ | $\Omega_{ccc}^{++} \rightarrow \Sigma^0 D^+ \ell^+ \nu_\ell$ | $a_3 V_{cs}^*$ |

III. SEMI-LEPTONIC DECAY CHANNELS

A. Semileptonic $\Omega_{ccc}$ decays

The $c \rightarrow q \ell \nu$ transition is induced by the effective electro-weak Hamiltonian:

$$H_{e.w.} = \frac{G_F}{\sqrt{2}} \left[ V_{cq}^* q^\mu (1 - \gamma_5) c \nu_\ell \bar{\gamma}_\mu (1 - \gamma_5) \ell \right] + \text{h.c.}, \quad (9)$$

where $q = d, s$ and $\ell = e, \mu$. The $V_{cd}$ and $V_{cs}$ are CKM matrix elements. Heavy-to-light quark operators are an SU(3) triplet, denoted as $H_3$ with the nonzero components $(H_3)^2 = V_{cd}^*$, $(H_3)^3 = V_{cs}^*$. At hadron level, the effective Hamiltonian for three-body and four-body semileptonic $\Omega_{ccc}$ decays can be constructed as:

$$H_{\text{eff}} = a_1 \Omega_{ccc} (T_{cc})_i (H_3)^i \bar{\nu}_\ell \ell + a_2 \Omega_{ccc} (T_{cc})_i (M_8)_j (H_3)^j \bar{\nu}_\ell \ell + a_3 \Omega_{ccc} (T_{cc})_i (M_8)_j (H_3)^j \bar{\nu}_\ell \ell + a_4 \Omega_{ccc} (T_{cc})_i (M_8)_j (H_3)^j \bar{\nu}_\ell \ell.$$ \hspace{1cm} (10)

where the $a_i$s are SU(3) irreducible amplitudes. Decay amplitudes for different channels can be deduced from the Hamiltonian in Eq. (10), and collected in Tab. I. A few remarks are given in order.

- In this table and following ones, the light pseudoscalar mesons can be replaced by their light counterparts. For instance the $K^0$ can be replaced by a $K^{*0}$, which is reconstructed by the $K^-\pi^+$ final state. The $\eta$ meson is difficult to reconstruct at hadron colliders, while the vector $\phi$ meson can be reconstructed in the $K^+K^-$ final state with a high efficiency.

- The $c \rightarrow s$ transition is proportional to the $V_{cs} \sim 1$, and the $c \rightarrow d$ transition has a smaller CKM matrix element $V_{cd} \sim 0.2$. Inspired by the experimental data on $D$ meson decays [6],
we can infer that branching fractions for the $c \to s$ channels are about a few percents, and the ones for the $c \to d$ transitions are at the order $10^{-3}$.

- A number of relations for decay widths can be easily read off from Tab. II. For instance for the $c \to s$ decays into a doubly charmed baryons, we have

$$\Gamma(\Omega_{ccc}^{++} \to \Xi_{cc}^{++} K^- \ell^+ \nu_\ell) = \Gamma(\Omega_{ccc}^{++} \to \Xi_{cc}^{++} K^0 \ell^+ \nu_\ell) = \frac{3}{2} \Gamma(\Omega_{ccc}^{++} \to \Omega_{ccc}^{++} \eta \ell^+ \nu_\ell).$$

However it is necessary to point out that the above relations will be modified due to the different masses of the final hadrons. Once the mass of $\Omega_{ccc}$ is experimentally measured in future, kinematical corrections can be included, and these relations can be refined.

**B. Semileptonic $\Omega_{b\bar{b}b}$ decays**

The $b$ quark decay is controlled by the electro-weak Hamiltonian

$$\mathcal{H}_{e.e.} = \frac{G_F}{\sqrt{2}} \left[ V_{q'q} \gamma^\mu (1 - \gamma_5) b \ell \gamma_\mu (1 - \gamma_5) \nu_\ell \right] + h.c.,$$

with $q' = u, c$, and here $\ell = e, \mu, \tau$. The $b \to c$ transition is an SU(3) singlet and thus the transition is simply a singlet. The $b \to u$ transition forms an SU(3) triplet $H_u^3$ with $(H_u^3)^1 = 1$ and $(H_u^3)^2,3 = 0$. The hadron level Hamiltonian is given as

$$\mathcal{H}_{eff} = b_1 \Omega_{b\bar{b}b} \Sigma_{\ell\bar{\ell}} \bar{\nu}_\ell \ell \nu_\ell + b_2 \Omega_{b\bar{b}b}(\mathcal{T}_{bb})_{ij} \mathcal{T}_{bb}^{ij} \bar{\nu}_\ell \ell \nu_\ell + b_3 \Omega_{b\bar{b}b}(\mathcal{T}_{bc})_i \mathcal{T}_{bb}^{ij} \bar{\nu}_\ell \ell \nu_\ell + b_4 \Omega_{b\bar{b}b}(\mathcal{T}_{bb})_i (H_u^3)^i \bar{\nu}_\ell \ell \nu_\ell + b_5 \Omega_{b\bar{b}b}(\mathcal{T}_{bb})_{ij} (H_u^3)^{ij} \bar{\nu}_\ell \ell \nu_\ell + b_6 \Omega_{b\bar{b}b}(\mathcal{T}_{bb})_{ij} (H_u^3)^{ij} \bar{\nu}_\ell \ell \nu_\ell.$$

The $b_i$s are the SU(3) independent amplitudes like the $a_i$s in Eq. 10. The decay amplitudes can be deduced from this Hamiltonian, and the results are given in Tab. III.

A few remarks are given in order.

- The $\Omega_{cbb}$ in the final state can be $1/2^+$ or $3/2^+$. It is similar for the $T_{bb}, T_{bc}$ and others.
The $b \to c$ transition has a larger CKM matrix element $V_{cb} \sim 0.04$, and the typical branching fractions might reach the order $10^{-3}$ to $10^{-2}$. However such decay modes still contain a triply heavy baryon which must be detected through its subsequent weak decays.

The $b \to u$ transition is suppressed due to $V_{ub} \sim 10^{-3}$. Typical branching fractions are at the order $10^{-4}$.

### C. Semileptonic $\Omega_{ccb}$ decays

Both charm quark and bottom quark in $\Omega_{ccb}$ can weakly decay. Thus the hadron level Hamiltonian for semileptonic $\Omega_{ccb}$ decays is given as

$$\mathcal{H}_{\text{eff}} = c_1 \Omega_{ccb} \langle T_{bc} \rangle_i (H_3)^j \bar{\nu}_e \ell + c_2 \Omega_{ccb} \langle T_{bc} \rangle_i (M_8)^j (H_3)^j \bar{\nu}_e \ell$$

$$+ c_3 \Omega_{ccb} \langle T_{bc} \rangle_{ij} \langle T_{bc} \rangle_{jk} (H_3)^j \bar{\nu}_e \ell + c_4 \Omega_{ccb} \langle T_{bc} \rangle_{ij} \langle T_{bc} \rangle_{jk} (H_3)^j \bar{\nu}_e \ell$$
Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. III.

D. Semileptonic $\Omega_{cbb}$ decays

Similarly the hadron level Hamiltonian for semileptonic $\Omega_{cbb}$ decay is given as

$$\mathcal{H}_{\text{eff}} = d_1 \Omega_{ccb}(T_{bb})_{(ij)} \bar{D}^j (H_3^j) \bar{\ell} \nu_\ell + d_2 \Omega_{ccbb}(T_{cc})_{(ij)} \bar{B} (H_3^j) \bar{\ell} \nu_\ell + d_3 \Omega_{ccbb}(T_{bc}) \bar{D}^j \bar{\ell} \nu_\ell$$

$$+ d_4 \Omega_{ccbb}(T_{bc}) (H_3^j) \bar{\ell} \nu_\ell + d_5 \Omega_{ccbb}(T_{bc}) (M_3^j) (H_3^j) \bar{\ell} \nu_\ell$$

$$+ d_6 \Omega_{ccbb}(T_{bb}) (H_3^j) \bar{\ell} \nu_\ell + d_7 \Omega_{ccbb}(T_{bc}) (H_3^j) \bar{\ell} \nu_\ell + d_8 \Omega_{ccbb}(T_{bc}) (H_3^j) \bar{\ell} \nu_\ell.$$  

(14)
FIG. 1: Feynman diagrams for two-body decay modes induced Cabibbo-allowed transitions. In the first two panels, the final state contains a doubly charmed baryon and a light meson, described by Eq. (20). The last panel corresponds to decays into a charmed baryon and a charmed meson. The first panel is color-allowed, while the last two panels are suppressed by $1/N_c$.

\[ +d_8 \Omega_{cab}(T_{bb})_{ij} \overline{D}(H'_3)^j \ell \bar{\nu}_\ell + d_9 \Omega_{cab}(T_{cb})_{ij} \overline{B}'(H'_3)^j \ell \bar{\nu}_\ell \\
+ d_{10} \Omega_{cab}(T_{bb})_i (H_3)^j \bar{\nu}_\ell \ell + d_{11} \Omega_{cab}(T_{bb})_i M_{ij}^1 (H_3)^j \bar{\nu}_\ell \ell \\
+ d_{12} \Omega_{cab}(T_{bb})_{ij} \overline{B}'(H_3)^j \bar{\nu}_\ell \ell + d_{13} \Omega_{cab}(T_{bb})_{ij} \overline{B}'(H_3)^j \bar{\nu}_\ell \ell. \tag{15} \]

Expanding the above equations, we will obtain the decay amplitudes given in Tab. IV.

IV. NON-LEPTONIC $\Omega_{ccc}$ DECAYS

Nonleptonic charm quark decays into light quarks are classified into three groups:

\[ c \rightarrow s\bar{d}u, \quad c \rightarrow u\bar{d}d/\bar{s}s, \quad c \rightarrow d\bar{s}u. \tag{16} \]

Feynman diagrams for two-body decays induced by the $c \rightarrow s\bar{d}u$ are given in Fig. I. In the first two panels, the final state contains a doubly charmed baryon and a light meson, while the last panel corresponds to decays into a charmed baryon and a charmed meson. The first panel is color-allowed, while the last two panels are suppressed by $1/N_c$.

Penguin contributions in charm quark decays are highly suppressed, and thus are neglected in our analysis. Tree operators transform under the flavor SU(3) symmetry as $3 \otimes \overline{3} \otimes 3 = 3 \oplus 3 \oplus \overline{6} \oplus 15$. For charm quark decays, the vector representation $H_3$ will vanishes as an approximation. For the $c \rightarrow s\bar{d}u$ transition, we have

\[ (H_3')_{12} = -(H_3')_{23} = 1, \quad (H_3')_{13} = (H_3')_{23} = 1, \tag{17} \]

while for the doubly Cabibbo suppressed $c \rightarrow d\bar{u}s$ transition, we have

\[ (H_3')_{31} = -(H_3')_{12} = \sin^2 \theta_C, \quad (H_3')_{23} = (H_3')_{31} = \sin^2 \theta_C. \tag{18} \]

CKM matrix elements for $c \rightarrow \bar{u}\bar{d}d$ and $c \rightarrow u\bar{s}s$ transitions are approximately equal in magnitude but different in sign. With both contributions, one has the nonzero components:

\[ (H_3')_{31} = -(H_3')_{12} = (H_3')_{12} = -(H_3')_{23} = \sin(\theta_C), \]
In the following we will use the Hamiltonian:

\[ \mathcal{H}_{eff} = a_1 \Omega_{ccc} (T_{cc})_i (M_8)^j (H_{ij})^k + a_2 \Omega_{ccc} (T_{cc})_i (M_8)^j (H_{ij})^k, \]

whose Feynman diagrams are given in Fig. 11. It is necessary to stress that the above SU(3) independent amplitudes \( a_s \) are different with the ones in Eq. (10).

A. Decays into a doubly-charmed baryon and one (two) light meson(s)

For decays into a doubly-charmed baryon and a light meson, one may derive the effective Hamiltonian:

\[ \mathcal{H}_{eff} = a_1 \Omega_{ccc} (T_{cc})_i (M_8)^j (H_{ij})^k + a_2 \Omega_{ccc} (T_{cc})_i (M_8)^j (H_{ij})^k, \]

whose Feynman diagrams are given in Fig. 11. It is necessary to stress that the above SU(3) independent amplitudes \( a_s \) are different with the ones in Eq. (10).

In the following we will use \( s_C \) to abbreviate the sine of Cabibbo angle \( \theta \).

\[
(H_{15})^{31}_3 = (H_{15})^{13}_3 = -(H_{15})^{12}_2 = -(H_{15})^{21}_2 = \sin(\theta_C). \quad (19)
\]

**TABLE V:** Amplitudes for \( \Omega_{ccc} \) decays into doubly-charmed baryon and a light meson. Cabibbo-allowed, singly Cabibbo-suppressed and doubly Cabibbo-suppressed decay channels are included in this table and the following ones. The amplitudes \( a_s \) are different with the ones in Tab. II.

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^0 \) | \( a_2 - a_1 \) | \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} \pi^0 \) | \( (a_2 - a_1)s_C \) |
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} \pi^+ \) | \( a_1 + a_2 \) | \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} \eta \) | \( \sqrt{\frac{3}{2}} (a_1 - a_2)s_C \) |
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^0 (a_1 - a_2)(s_C) \) | | \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} \pi^+ (a_1 + a_2)(-s_C) \) | |
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^0 (a_1 + a_2)s_C^2 \) | | \( \Omega^{++}_{ccc} \rightarrow \Omega^{++}_{ccc} K^0 (a_1 + a_2)s_C \) | |

**TABLE VI:** Amplitudes for \( \Omega_{ccc} \) decays into doubly-charmed baryon and two light mesons. The amplitude \( b_2 \) defined in Eq. (22) is not shown since it always accompanies with \( b_1 \) in the form \( b_1 - b_2 \).

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^0 \pi^0 \) | \( \frac{b_1 - b_1 + b_2}{\sqrt{2}} \) | \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} \pi^0 \pi^0 \) | \( (b_1 - b_3 + b_4)s_C \) |
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^0 \pi^+ \) | \( -b_1 + b_3 + b_4 \) | \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} \pi^0 \pi^+ \) | \( (b_1 - b_3 - b_4)s_C \) |
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^0 \eta \) | \( \frac{b_1 - b_1 + b_2}{\sqrt{3}} \) | \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} \eta \pi^+ \) | \( -\sqrt{\frac{2}{3}} (b_1 - b_3 + b_4)s_C \) |
| \( \Omega^{++}_{ccc} \rightarrow \Omega^{++}_{ccc} K^+ \pi^0 \) | \( b_1 + b_3 + b_4 \) | \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} \pi^0 \pi^+ \) | \( \sqrt{2b_4}s_C \) |
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^0 \pi^+ \) | \( \frac{(b_1 - b_3 + b_4)s_C}{\sqrt{6}} \) | \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^0 \pi^0 \) | \( (b_1 - b_3 - b_4)(s_C) \) |
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^0 \eta \) | \( \frac{(b_1 - b_3 + b_4)s_C}{\sqrt{3}} \) | \( \Omega^{++}_{ccc} \rightarrow \Omega^{++}_{ccc} K^0 \eta \) | \( (b_1 + b_3 - b_4)s_C \) |
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^+ \pi^0 \) | \( \frac{(b_1 - b_3 + b_4)s_C}{\sqrt{3}} \) | \( \Omega^{++}_{ccc} \rightarrow \Omega^{++}_{ccc} K^+ \pi^0 \) | \( (b_1 + b_3 + b_4)s_C \) |
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^+ \eta \) | \( \frac{(b_1 - b_3 + b_4)s_C}{\sqrt{5}} \) | \( \Omega^{++}_{ccc} \rightarrow \Omega^{++}_{ccc} K^+ \eta \) | \( -\frac{(b_1 - b_3 + b_4)s_C}{\sqrt{5}} \) |
| \( \Omega^{++}_{ccc} \rightarrow \Xi^{++}_{ccc} K^0 \pi^+ \) | \( b_1 + b_3 + b_4 \) | \( \Omega^{++}_{ccc} \rightarrow \Omega^{++}_{ccc} K^0 \pi^+ \) | \( (b_1 + b_3 - b_4)s_C \) |
| \( \Omega^{++}_{ccc} \rightarrow \Omega^{++}_{ccc} K^0 \pi^0 \) | \( 2b_4s_C \) |
Expanding the above equations, we will obtain the decay amplitudes given in Tab. VII. From Eq. \((20)\) and Tab. VI one can see that there are two SU(3) independent amplitudes, and thus there exists a few relations for decay widths. These relations can be directly read off from Tab. VII and can be examined by future experiments.

For the reactions with one additional light meson in the final state, one has the Hamiltonian:

\[
H_{\text{eff}} = b_1\Omega_{ccc}(T_{cc})_i(M_S)_j^i(H_P)_k^i + b_2\Omega_{ccc}(T_{cc})_i(M_S)_j^i(H_P)_k^i + b_3\Omega_{ccc}(T_{cc})_i(M_S)_j^i(H_{15})_k^i + b_4\Omega_{ccc}(T_{cc})_i(M_S)_j^i(H_{15})_k^i. \tag{21}
\]

Expanding the above equation, we will obtain the decay amplitudes given in Tab. VII. The following remarks are in order.

- From the expanded Hamiltonian, one can find that the amplitudes \(b_1\) and \(b_2\) always appear in the combination \(b_1 - b_2\). Thus we have removed the amplitude \(b_2\) in Tab. VII.

- For channels with two identical particles, there is a factor 1/2 in the decay width.

### B. Decays into a charmed baryon and a charmed meson

For the two-body decays into a charmed baryon and a charmed meson, the effective Hamiltonian for the decays of \(\Omega_{ccc}\) into a singly charmed baryon and a charmed meson is given as:

\[
H_{\text{eff}} = c_1\Omega_{ccc}(T_{c8})_{ij}\overline{D}^k(H_6)_k^{ij} + c_2\Omega_{ccc}(T_{c6})_{ij}\overline{D}^k(H_{15})_k^{ij}. \tag{22}
\]

The Feynman diagram is shown in the last panel of Fig. II and decay amplitudes are collected in Tab. VII.

The three-body decays of \(\Xi_{ccc}^{++}\) can involve an additional light meson in the final state. For the modes with an anti-triplet baryon, we have

\[
H_{\text{eff}} = d_1\Omega_{ccc}(T_{c8})_{ij}\overline{D}^i(M_S)_k^i(H_6)_l^k + d_2\Omega_{ccc}(T_{c8})_{ij}\overline{D}^i(M_S)_k^i(H_6)_l^k + d_3\Omega_{ccc}(T_{c8})_{ij}\overline{D}^i(M_S)_k^i(H_{15})_l^k + d_4\Omega_{ccc}(T_{c8})_{ij}\overline{D}^i(M_S)_k^i(H_{15})_l^k. \tag{23}
\]

| channel amplitude | channel amplitude |
|-------------------|-------------------|
| \(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} D^+\) | \(-2c_1\) | \(\Omega_{ccc}^{++} \rightarrow \Lambda_{ccc}^{++} D^+\) | \(2c_1 s_C\) |
| \(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} D^+\) | \(\sqrt{2c_2}\) | \(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} D^+\) | \(-2c_1 s_C\) |
| \(\Omega_{ccc}^{++} \rightarrow \Sigma_{ccc}^{++} D^+\) | \(\sqrt{2c_2 s_C}\) | \(\Omega_{ccc}^{++} \rightarrow \Sigma_{ccc}^{++} D^+\) | \(-\sqrt{2c_2 s_C}\) |
### TABLE VIII: Amplitudes for three-body $\Omega_{ccc}$ decays into a singly-charmed baryon (anti-triplet), D meson and a light meson.

| Channel | Amplitude |
|---------|-----------|
| $\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^+ K^0$ | $-d_2 + d_3 + d_4 - d_5$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^0 \pi^+$ | $-2d_1 - d_2 - d_4$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ \pi^0$ | $2d_1 + d_2 + d_3$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ \eta$ | $-2d_1 + d_2 - 3d_3$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ \bar{K}^0$ | $-2d_1 - d_2 + d_4$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ K^0$ | $-d_2 + d_3 - d_4 + d_5$ |
| $\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^0 K^+$ | $(2d_1 + d_2 - d_4 - d_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ \pi^0$ | $(2d_1 - d_2 - d_4 - d_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ \eta$ | $(2d_1 + d_2 - d_4 - d_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ \bar{K}^0$ | $(2d_1 - d_2 - d_4 - d_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ K^0$ | $(2d_1 - d_2 - d_4 - d_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D_s^+ K^0$ | $(d_2 - d_3 - d_4 + d_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D_s^+ K^0$ | $(d_2 - d_3 - d_4 - d_5)$ |

### TABLE IX: Amplitudes for three-body $\Omega_{ccc}$ decays into a singly-charmed baryon (sextet), D meson and a light meson.

| Channel | Amplitude |
|---------|-----------|
| $\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^0 K^0$ | $e_2 - e_4$ |
| $\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^+ \pi^0$ | $(e_2 - e_4)$ |
| $\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^+ \eta$ | $-\sqrt{2}(e_2 - e_4)$ |
| $\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^+ \bar{K}^0$ | $e_2 + e_3 - e_4 - e_5$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^{++} D^0 \pi^0$ | $\frac{1}{2}(-2e_1 + e_3 + e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^{++} D^+ \eta$ | $\frac{1}{2}(2e_1 + e_3 - e_4 - e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^{++} D^+ \bar{K}^0$ | $e_2 + e_3 + e_4 - e_5$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^{++} D^+ K^0$ | $e_2 + e_3 + e_4 + e_5$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^{++} D_s^+ K^0$ | $(e_2 + e_3 + e_4 + e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^{++} D_s^+ \pi^+$ | $(e_2 + e_3 + e_4 + e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^{++} D_s^+ \eta$ | $\sqrt{2}(e_2 + e_3 + e_4 + e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Omega_{ccc}^{0+} D_s^+ K^0$ | $(2e_2 + e_3 + e_4 + e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Omega_{ccc}^{0+} D_s^+ \pi^+$ | $(2e_2 + e_3 + e_4 + e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Omega_{ccc}^{0+} D_s^+ \eta$ | $\sqrt{2}(2e_2 + e_3 + e_4 + e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Omega_{ccc}^{0+} D_s^+ \bar{K}^0$ | $(e_2 + e_3 - e_4 - e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^{++} D_s^+ K^0$ | $(e_2 + e_3 + e_4 + e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^{++} D_s^+ \pi^+$ | $(e_2 + e_3 + e_4 + e_5)$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_c^{++} D_s^+ \eta$ | $\sqrt{2}(e_2 + e_3 + e_4 + e_5)$ |
TABLE X: Amplitudes for $\Omega_{b\bar{b}b}$ decays into a $J/\psi$ and a doubly bottom baryon or two bottom hadrons.

| channel amplitude | channel amplitude |
|-------------------|-------------------|
| $\Omega_{b\bar{b}b} \rightarrow \Xi_{bb}^* J/\psi$ | $\Xi_{bb}^* J/\psi$ |
| $\Omega_{b\bar{b}b} \rightarrow \Xi_{bb}^* J/\psi$ | $f_1 V_{cd}^*$ |
| $\Omega_{b\bar{b}b} \rightarrow \Xi_{bb}^* J/\psi$ | $f_3 V_{cs}^*$ |
| $\Omega_{b\bar{b}b} \rightarrow \Xi_{bb}^* J/\psi$ | $f_1 V_{cd}^*$ |
| $\Omega_{b\bar{b}b} \rightarrow \Xi_{bb}^* J/\psi$ | $f_3 V_{cs}^*$ |
| $\Omega_{b\bar{b}b} \rightarrow \Xi_{bb}^* J/\psi$ | $f_1 V_{cd}^*$ |
| $\Omega_{b\bar{b}b} \rightarrow \Xi_{bb}^* J/\psi$ | $f_3 V_{cs}^*$ |
| $\Omega_{b\bar{b}b} \rightarrow \Xi_{bb}^* J/\psi$ | $f_1 V_{cd}^*$ |
| $\Omega_{b\bar{b}b} \rightarrow \Xi_{bb}^* J/\psi$ | $f_3 V_{cs}^*$ |

while the effective Hamiltonian for a sextet baryon is constructed as:

$$H_{\text{eff}} = e_1 \Omega_{ccc}(\bar{c}6\{ij\})^j_i (M_8)_{ij} (H_{15})_{ij}^k + e_2 \Omega_{ccc}(\bar{c}6\{ij\})^j_i (M_8)_{ij} (H_{15})_{ij}^k$$

$$+ e_3 \Omega_{ccc}(\bar{c}6\{ij\})^j_i (M_8)_{ij} (H_{15})_{ij}^k + e_4 \Omega_{ccc}(\bar{c}6\{ij\})^j_i (M_8)_{ij} (H_{15})_{ij}^k$$

$$+ e_5 \Omega_{ccc}(\bar{c}6\{ij\})^j_i (M_8)_{ij} (H_{15})_{ij}^k.$$  \hspace{1cm} (24)

Expanding the above equations, we will obtain the decay amplitudes given in Tab. VIII for anti-triplet baryon and in Tab. IX for sextet baryon.

V. NON-LEPTONIC $\Omega_{b\bar{b}b}$ DECAYS

For the bottom quark decay, there are generically 4 kinds of quark-level transitions:

$$b \rightarrow c\bar{c}d/s, \quad b \rightarrow c\bar{c}d/s, \quad b \rightarrow u\bar{c}d/s, \quad b \rightarrow q\bar{q}q,$$  \hspace{1cm} (25)

which will be studied in order.

A. $b \rightarrow c\bar{c}d/s$

1. Decays into a $J/\psi$

Such decays will have the same topology with the $b \rightarrow s\ell^+\ell^-$ decays. The transition operator $b \rightarrow c\bar{c}d/s$ can form an SU(3) triplet:

$$H_{\text{eff}} = f_1 \Omega_{b\bar{b}b}(\bar{c}6\{ij\})(H_{3})_{ij}^j J/\psi + f_2 \Omega_{b\bar{b}b}(\bar{c}6\{ij\})(M_8)_{ij}^j J/\psi$$

$$+ f_3 \Omega_{b\bar{b}b}(\bar{c}6\{ij\})(M_8)_{ij}^j J/\psi + f_4 \Omega_{b\bar{b}b}(\bar{c}6\{ij\})(M_8)_{ij}^j J/\psi,$$  \hspace{1cm} (26)

with $(H_{3})_2 = V_{cd}^*$ and $(H_{3})_3 = V_{cs}^*$. Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. IX.
Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XI.

### Tab. XI: Amplitudes for $\Omega_{uub}$ decays into $\Omega_{ub}$ and a anti-charmed meson.

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| $\Omega_{uub} \rightarrow \Omega_{ub}^0 D^-$ | $g_1 V_{u}^*$ | $\Omega_{uub} \rightarrow \Omega_{ub}^0 D_s^-$ | $g_1 V_{cs}^*$ |
| $\Omega_{uub} \rightarrow \Omega_{ub}^0 \tilde{D}^- \pi$ | $g_2 V_{c}^*$ | $\Omega_{uub} \rightarrow \Omega_{ub}^0 \tilde{D}^- K^-$ | $g_2 V_{cs}^*$ |
| $\Omega_{uub} \rightarrow \Omega_{ub}^0 \tilde{D}^- \pi^0$ | $-\frac{g_2 V_{c}^*}{\sqrt{2}}$ | $\Omega_{uub} \rightarrow \Omega_{ub}^0 \tilde{D}^- K^0$ | $g_2 V_{cs}^*$ |
| $\Omega_{uub}^0 \rightarrow \Omega_{ub}^0 D_s^- \eta$ | $\frac{g_2 V_{c}^*}{\sqrt{6}}$ | $\Omega_{uub}^0 \rightarrow \Omega_{ub}^0 D_s^0 K^0$ | $g_2 V_{cs}^*$ |

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| $\Omega_{uub} \rightarrow \Omega_{ub}^0 \tilde{D}^+ \eta$ | $h_1 V_{ub} V_{ud}^*$ | $\Omega_{uub} \rightarrow \Omega_{ub}^0 \tilde{D}^- \pi^0$ | $\frac{h_2 V_{ub} V_{ud}^*}{\sqrt{2}}$ |
| $\Omega_{uub} \rightarrow \Omega_{ub}^0 \tilde{D}^- \pi^0 \tilde{K}$ | $h_2 V_{ub} V_{ud}^*$ | $\Omega_{uub} \rightarrow \Omega_{ub}^0 \tilde{D}^- \pi^0 \tilde{K}^*$ | $h_2 V_{ub} V_{ud}^*$ |
| $\Omega_{uub} \rightarrow \Omega_{ub}^0 \tilde{D}^- \pi^0 \tilde{K}$ | $h_2 V_{ub} V_{us}^*$ | $\Omega_{uub} \rightarrow \Omega_{ub}^0 \tilde{D}^- \pi^0 \tilde{K}^*$ | $h_2 V_{ub} V_{us}^*$ |

2. **Decays into a triply heavy baryon $cbb$ plus a anti-charmed meson**

The $b \to c\bar{c}d/s$ transition can lead to another type of effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = g_1 \Omega_{uub} \overline{\eta}_{cbb} D_i (H_3)^i + g_2 \Omega_{uub} \overline{\eta}_{cbb} D_j (M_8)^j (H_3)^i.$$  \hspace{1cm} \text{(27)}

which corresponds to the decays into doubly heavy baryon $bcq$ plus a anti-charmed meson. Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XI.

### Tab. XII: Amplitudes for $\Omega_{uub}$ decays into $\Omega_{ucb}$ and light meson(s).

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| $\Omega_{uub} \rightarrow \Omega_{uc}^0 \pi^-$ | $h_1 V_{cb} V_{ud}^*$ | $\Omega_{uub} \rightarrow \Omega_{ucb}^0 K^-$ | $h_1 V_{cb} V_{us}^*$ |
| $\Omega_{uub} \rightarrow \Omega_{ucb}^0 \tilde{K}^-$ | $\frac{h_2 V_{cb} V_{ud}^*}{\sqrt{2}}$ | $\Omega_{uub} \rightarrow \Omega_{ucb}^0 \tilde{K}^- \pi^0$ | $\frac{h_2 V_{cb} V_{us}^*}{\sqrt{2}}$ |
| $\Omega_{uub} \rightarrow \Omega_{ucb}^0 \tilde{K}^- \pi^0 \tilde{K}$ | $h_2 V_{cb} V_{ud}^*$ | $\Omega_{uub} \rightarrow \Omega_{ucb}^0 \tilde{K}^- \pi^0 \tilde{K}^*$ | $h_2 V_{cb} V_{us}^*$ |

B. **$b \to c\bar{d}/s$ transition**

1. **Decays into a triply heavy baryon $cbb$ plus light mesons**

The operator to produce a charm quark from the $b$-quark decay, $\bar{c}bq$, is given by

$$\mathcal{H}_{\text{e.w.}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{au}^* \left[ C_1 O_1^{cu} + C_2 O_2^{cu} \right] + h.c.$$  \hspace{1cm} \text{(28)}

The light quarks in this effective Hamiltonian form an octet with the nonzero entry

$$\langle H_8 \rangle_1^2 = V_{cb} V_{ud}^*.$$  \hspace{1cm} \text{(29)}

for the $b \to c\bar{d}$ transition, and $\langle H_8 \rangle_3^3 = V_{cb} V_{us}^*$ for the $b \to c\bar{u}s$ transition.

$$\mathcal{H}_{\text{eff}} = h_1 \Omega_{uub} \overline{\eta}_{cbb} M_i^j (H_8)^i_j + h_2 \Omega_{uub} \overline{\eta}_{cbb} M_k^j M_l^i (H_8)^i_j.$$  \hspace{1cm} \text{(30)}

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XII.
TABLE XIII: Amplitudes for nonleptonic \( \Omega_{bb}^- \) decays into a charmed meson plus a doubly bottom baryon or two bottom hadrons.

| channel                  | amplitude          | channel                  | amplitude          |
|--------------------------|--------------------|--------------------------|--------------------|
| \( \Omega_{bb} \rightarrow \Xi_{bb}^0 D^0 \) | \( j_1 V_{ub} V_{us}^* \) | \( \Omega_{bb} \rightarrow \Lambda_b^0 B^- D^0 \) | \( -(j_5 + j_6) V_{ub} V_{us}^* \) |
| \( \Omega_{bb} \rightarrow \Xi_{bb}^- D^0 \) | \( j_1 V_{ub} V_{us}^* \) | \( \Omega_{bb} \rightarrow \Xi_{bb}^- B^- D^0 \) | \( -(j_5 + j_6) V_{ub} V_{us}^* \) |
| \( \Omega_{bb} \rightarrow \Xi_{bb}^0 D^0 \pi^- \) | \( (j_1 + j_2) V_{ub} V_{us}^* \) | \( \Omega_{bb} \rightarrow \Xi_{bb}^- B^- D^0 \) | \( -(j_5 + j_6) V_{ub} V_{us}^* \) |
| \( \Omega_{bb} \rightarrow \Xi_{bb}^0 D^0 K^- \) | \( (j_1 + j_2) V_{ub} V_{us}^* \) | \( \Omega_{bb} \rightarrow \Xi_{bb}^- B^- D^0 \) | \( -(j_5 + j_6) V_{ub} V_{us}^* \) |
| \( \Omega_{bb} \rightarrow \Xi_{bb}^0 D^0 \pi^0 \) | \( \frac{i j_5 (\sqrt{2} V_{ub} + V_{us})}{\sqrt{2}} \) | \( \Omega_{bb} \rightarrow \Xi_{bb}^- B^- D^0 \) | \( -(j_5 + j_6) V_{ub} V_{us}^* \) |
| \( \Omega_{bb} \rightarrow \Xi_{bb}^0 D^0 K^0 \) | \( \lambda V_{ub} V_{us}^* \) | \( \Omega_{bb} \rightarrow \Xi_{bb}^- B^- D^0 \) | \( -(j_5 + j_6) V_{ub} V_{us}^* \) |
| \( \Omega_{bb} \rightarrow \Omega_{bb} D^0 D^- \) | \( \frac{i j_5 (j_1 + j_2) V_{ub} V_{us}^*}{\sqrt{2}} \) | \( \Omega_{bb} \rightarrow \Xi_{bb}^- B^- D^0 \) | \( -(j_5 + j_6) V_{ub} V_{us}^* \) |
| \( \Omega_{bb} \rightarrow \Omega_{bb} D^0 D^- \) | \( \frac{i j_5 (j_1 + j_2) V_{ub} V_{us}^*}{\sqrt{2}} \) | \( \Omega_{bb} \rightarrow \Xi_{bb}^- B^- D^0 \) | \( -(j_5 + j_6) V_{ub} V_{us}^* \) |
| \( \Omega_{bb} \rightarrow \Omega_{bb} D^0 D^- \) | \( \frac{i j_5 (j_1 + j_2) V_{ub} V_{us}^*}{\sqrt{2}} \) | \( \Omega_{bb} \rightarrow \Xi_{bb}^- B^- D^0 \) | \( -(j_5 + j_6) V_{ub} V_{us}^* \) |

2. Decays into a charmed meson plus a doubly bottom baryon or two bottom hadrons.

If the \( c \bar{c} \) are separated, then the final state could be a doubly bottom baryon and a charmed meson. The three-body modes can also include decays into a bottom baryon, a bottom meson and a charmed meson. Thus one can have the effective Hamiltonian as:

\[
\mathcal{H}_{\text{eff}} = j_1 \Omega_{bbb} (\bar{T}_{bb}) (\bar{T}_{bb}) (H_8) + j_2 \Omega_{bbb} (\bar{T}_{bb}) (\bar{T}_{bb}) (M_8) (H_8) + j_3 \Omega_{bbb} (\bar{T}_{bb}) (\bar{T}_{bb}) (M_8) (H_8) + j_5 \Omega_{bbb} (\bar{T}_{bb}) (\bar{T}_{bb}) (H_8) + j_6 \Omega_{bbb} (\bar{T}_{bb}) (\bar{T}_{bb}) (H_8) + j_7 \Omega_{bbb} (\bar{T}_{bb}) (\bar{T}_{bb}) (H_8) + j_8 \Omega_{bbb} (\bar{T}_{bb}) (\bar{T}_{bb}) (H_8). \tag{31}
\]

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. \( \text{T XIII} \).

C. The CKM suppressed \( b \rightarrow u c / s \) transition

For the anti-charm production, the operator having the quark contents \((\bar{u}b)(\bar{q}c)\) is given by

\[
\mathcal{H}_{\text{e.w.}} = \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^* \left[ C_1 \bar{O}_1^u + C_2 \bar{O}_2^u \right] + \text{h.c.} \tag{32}
\]
The two light anti-quarks form the 3 and 6 representations. The anti-symmetric tensor $H''_3$ and the symmetric tensor $H_6$ have nonzero components

\[(H''_3)^{13} = -(H''_3)^{31} = V_{ub}V_{cs}^*, \quad (H_6)^{13} = (H_6)^{31} = V_{ub}V_{cs}^*, \]

for the $b \to ucs$ transition. For the transition $b \to ucd$ one requests the interchange of 2 $\leftrightarrow$ 3 in the subscripts, and $V_{cs}$ replaced by $V_{cd}$.

The effective Hamiltonian is derived as:

\[
\mathcal{H}_{\text{eff}} = k_1 \Omega_{bb\bar{b}} \langle T_{bb} \rangle D_J(H_3)^{ij} + k_2 \Omega_{bb\bar{b}} \langle T_{bb} \rangle D_J(H_6)^{ij} \\
+ k_3 \Omega_{bb\bar{b}} \langle T_{bb} \rangle D_J(M_3)^{ij} (H_3)^{jk} + k_4 \Omega_{bb\bar{b}} \langle T_{bb} \rangle D_J(M_6)^{ij} (H_3)^{jk} \\
+ k_5 \Omega_{bb\bar{b}} \langle T_{bb} \rangle D_J(M_3)^{ij} (H_6)^{jk} + k_6 \Omega_{bb\bar{b}} \langle T_{bb} \rangle D_J(M_6)^{ij} (H_6)^{jk} \\
+ k_7 \Omega_{bb\bar{b}} \langle T_{bb} \rangle D_J(M_3)^{ij} (H_3)^{jk} + k_8 \Omega_{bb\bar{b}} \langle T_{bb} \rangle D_J(M_6)^{ij} (H_3)^{jk}
\]
TABLE XV: Amplitudes for $\Omega_{bb\bar{b}}$ decays into a doubly bottom baryon and a light meson. The $b \to d$ transitions are given in the left columns, and the $b \to s$ ones are shown in the right columns.

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| $\Omega_{bb\bar{b}} \to \Xi_{bb}^0 \pi^- l_1 + l_2 + 3l_3$ | $\Omega_{bb\bar{b}} \to \Xi_{bb}^0 K^-$ | $l_1' + l_2' + 3l_3'$ |
| $\Omega_{bb\bar{b}} \to \Xi_{bb}^0 \eta^0 - \frac{t_1 + 7t_2 + 5t_3}{\sqrt{2}}$ | $\Omega_{bb\bar{b}} \to \Xi_{bb}^0 \eta$ | $l_1' - l_2' - l_3'$ |
| $\Omega_{bb\bar{b}} \to \Xi_{bb}^- \eta$ | $\Omega_{bb\bar{b}} \to \Xi_{bb}^0 \eta$ | $-\sqrt{2}(l_1' - 2l_3')$ |
| $\Omega_{bb\bar{b}} \to \Xi_{bb}^0 K^0 l_1 - l_2 - l_3$ | $\Omega_{bb\bar{b}} \to \Xi_{bb}^0 K^0$ | $-\sqrt{2}(l_1' - 3l_3')$ |

$$
+k_9 \Omega_{bb\bar{b}} (T_{b3})_{ik} B^k D_j (H_6)^{ij} + k_{10} \Omega_{bb\bar{b}} (T_{b6})_{ik} B^k D_j (H_3)^{ij}$$

$$
+k_{11} \Omega_{bb\bar{b}} (T_{b6})_{ij} B^k D_k (H_6)^{ij} + k_{12} \Omega_{bb\bar{b}} (T_{b6})_{ik} B^k D_j (H_6)^{ij}. \quad (34)
$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XIV.

D. Charmless $b \to q_1 q_2 q_3$ Decays

1. Decays into a doubly bottom baryon $bb\bar{b}$ and a light meson

The charmless $b \to q$ ($q = d, s$) transition is controlled by the weak Hamiltonian $\mathcal{H}_{\text{eff}}$:

$$
\mathcal{H}_{\text{e.w.}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V^{*}_{us} \left[ C_1 O_1^{uu} + C_2 O_2^{uu} \right] - V_{tb} V^{*}_{ts} \left[ \sum_{i=3}^{10} C_i O_i \right] \right\} + \text{h.c.}, \quad (35)
$$

where $O_i$ is a four-quark operator or a moment type operator. In the SU(3) group, penguin operators behave as the 3 representation while tree operators can be decomposed in terms of a vector $H_3$, a traceless tensor antisymmetric in upper indices, $H_{\overline{3}}$, and a traceless tensor symmetric in upper indices, $H_{15}$.

For the $\Delta S = 0 (b \to d)$ decays, the non-zero components of the effective Hamiltonian are:

$$
(H_3)^2 = 1, \quad (H_{\overline{3}})^2 = -(H_{\overline{3}})^2 = -(H_{\overline{3}})^2 = -(H_{\overline{3}})^2 = 1,
$$

$$
2(H_{15})^2 = (H_{15})^2 = -(H_{15})^2 = -(H_{15})^2 = -(H_{15})^2 = 6. \quad (36)
$$

and all other remaining entries are zero. For the $\Delta S = 1 (b \to s)$ decays the nonzero entries in the $H_3$, $H_{\overline{3}}$, $H_{15}$ are obtained from Eq. (36) with the exchange $2 \leftrightarrow 3$.

If the final state contains one light meson, the effective Hamiltonian is given as:

$$
\mathcal{H}_{\text{eff}} = l_1 \Omega_{bb\bar{b}} (T_{bb})_j (M_8)_j^i H_3^i + l_2 \Omega_{bb\bar{b}} (T_{bb})_j (M_8)_j^k H_6^k + l_3 \Omega_{bb\bar{b}} (T_{bb})_j (M_8)_j^k H_6^k. \quad (37)
$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XV.
TABLE XVI: Amplitudes for $\Omega_{bb}^-$ decays into a doubly bottom baryon and two light mesons. The $b \to d$ transitions are given in the left columns, and the $b \to s$ ones are shown in the right columns. The amplitude $l_7$ is neglected since it correlates with $l_6$.

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| $\Omega_{bb}^- \to \Xi_{bb}^0 \pi^- \pi^0$ | $4\sqrt{2}l_9$ | $\Omega_{bb}^- \to \Xi_{bb}^0 K^0$ | $l_4 + l_6 + 3l_8 - l_9$ |
| $\Omega_{bb}^- \to \Xi_{bb}^0 K^- K^0$ | $\sqrt{2}(l_4 + l_6 + 3l_8 + 3l_9)$ | $\Omega_{bb}^- \to \Xi_{bb}^0 K^- \eta$ | $-l_4' + l_6' + 3l_8' - 9l_9'$ |
| $\Omega_{bb}^- \to \Xi_{bb}^- \pi^0 \pi^0$ | $l_4 + 2l_5 - l_6 + l_8 - 5l_9$ | $\Omega_{bb}^- \to \Xi_{bb}^- K^- K^0$ | $l_4' - l_6' - l_8' + 3l_9'$ |
| $\Omega_{bb}^- \to \Xi_{bb}^- \pi^- \pi^+$ | $l_4 + 2l_5 - l_6 + l_8 + 3l_9$ | $\Omega_{bb}^- \to \Xi_{bb}^- \pi^- \pi^+$ | $-l_4 + l_6 + 3l_8 + 5l_9$ |
| $\Omega_{bb}^- \to \Xi_{bb}^- \eta \pi^0$ | $-l_4 + l_6 + 3l_8 + 5l_9$ | $\Omega_{bb}^- \to \Xi_{bb}^- \eta \pi^0$ | $2(l_5 + l_6)$ |
| $\Omega_{bb}^- \to \Xi_{bb}^- K^- K^+$ | $2(l_5 + l_6)$ | $\Omega_{bb}^- \to \Xi_{bb}^- K^- K^+$ | $2l_5 + 2l_6$ |
| $\Omega_{bb}^- \to \Xi_{bb}^- \eta \pi^0$ | $-l_4 + l_6 + 3l_8 + 5l_9$ | $\Omega_{bb}^- \to \Xi_{bb}^- \eta \pi^0$ | $-2l_6 + 2l_8 + 2l_9$ |
| $\Omega_{bb}^- \to \Omega_{bb}^- K^0$ | $l_4 + 3(2l_5 + l_6 + l_8 + l_9)$ | $\Omega_{bb}^- \to \Omega_{bb}^- K^0$ | $l_4' + 2l_5' - l_6' + l_8' + 3l_9'$ |
| $\Omega_{bb}^- \to \Omega_{bb}^- K^0$ | $-l_4 + l_6 + 3l_8 + 5l_9$ | $\Omega_{bb}^- \to \Omega_{bb}^- K^0$ | $l_4' + 2l_5' + l_6' - 3l_8' - l_9'$ |
| $\Omega_{bb}^- \to \Omega_{bb}^- K^0$ | $l_4 - l_6 - l_8 + 3l_9$ | $\Omega_{bb}^- \to \Omega_{bb}^- K^0$ | $2(l_5 + l_6)$ |

With one additional light meson, we have

$$
\mathcal{H}_{\text{eff}} = l_4 \Omega_{bb}^-(\overline{T}_{bb})_i j_i(M_S)_j^2(M_S)_k^2(H_3)^k + l_5 \Omega_{bb}^-(\overline{T}_{bb})_i j_i(M_S)_j^2(M_S)_k^2(H_3)^k + l_6 \Omega_{bb}^-(\overline{T}_{bb})_i j_i(M_S)_j^2(M_S)_k^2(H_3)^k + l_7 \Omega_{bb}^-(\overline{T}_{bb})_i j_i(M_S)_j^2(M_S)_k^2(H_3)^k + l_8 \Omega_{bb}^-(\overline{T}_{bb})_i j_i(M_S)_j^2(M_S)_k^2(H_3)^k + l_9 \Omega_{bb}^-(\overline{T}_{bb})_i j_i(M_S)_j^2(M_S)_k^2(H_3)^k.
$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XVI. A few remarks are given in order.

- In Tab. XVI and Tab. XVI both $b \to d$ and $b \to s$ channels are included. Since the CKM matrix elements are different, the SU(3) irreducible amplitudes for the $b \to s$ transition are primed.

- Expanding Eq. (35), one can find the amplitudes $l_6$ and $l_7$ are not independent and they always appear in the product $l_6 - l_7$. So in the two tables, we did not show the $l_7$.

- Inspired from the $B$ meson decay data, we can infer that the typical branching fractions are at the order $10^{-6}$. Thus these channels are rare decays, and can be studied with a large amount of data. However, the direct CP asymmetries in these channels are typically sizable.

- For the $b \to q_1 \bar{q}_2 q_3$ decays, there are two amplitudes with different CKM factors. One can consider the U-spin connected decays with the decay amplitudes

$$
A(\Delta S = 0) = r (V_{ub} V_{ud} A^T + V_{tb} V_{td} A^T),
$$
Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XVII.

\[ A(\Delta S = 1) = V_{ub}V_{ts}^*A_T + V_{tb}V_{ts}^*A_P. \] (39)

where \( r \) is a constant factor, and \( A_T \) and \( A_P \) are the amplitudes without the CKM factors. Such channel pairs include \([\Omega_{bb}^+ \rightarrow \Xi_b^0 B^+, \Omega_{bb}^0 \rightarrow \Xi_b^0 B^0]\), \([\Omega_{bb}^- \rightarrow \Xi_b^- B^+, \Omega_{bb}^- \rightarrow \Xi_b^- B^0]\), and etc. As pointed out in Refs. [29, 32, 53], there exists a relation for the CP violating quantity \( \Delta = \gamma - \bar{\gamma} \):

\[ \frac{A_{CP}(\Delta S = 0)}{A_{CP}(\Delta S = 1)} = -r^2 \frac{\Gamma(\Delta S = 1)}{\Gamma(\Delta S = 0)}. \] (40)

The future experimental data will be valuable to test flavor SU(3) symmetry and the CKM mechanism for CP violation.

2. Decays into a bottom meson and a bottom baryon \( bqq \)

If the bottom baryon is an anti-triplet, we have the effective Hamiltonian for two-body and three-body decays:

\[
\mathcal{H}_{\text{eff}} = m_1 \Omega_{bb} (\mathcal{T}_{b3})_{ij} \mathcal{B}^j (H_3)^i + m_2 \Omega_{bb} (\mathcal{T}_{b3})_{ij} \mathcal{B}^k (H_6)^i_j + m_3 \Omega_{bb} (\mathcal{T}_{b3})_{ij} \mathcal{B}^j (M_8)^j_i \]
\[+ m_4 \Omega_{bb} (\mathcal{T}_{b3})_{ij} \mathcal{B}^j (M_8)^j_i (H_3)^i + m_4 \Omega_{bb} (\mathcal{T}_{b3})_{ij} \mathcal{B}^j (M_8)^j_i \]
\[+ m_5 \Omega_{bb} (\mathcal{T}_{b3})_{ij} \mathcal{B}^j (M_8)^j_i (H_3)^i + m_6 \Omega_{bb} (\mathcal{T}_{b3})_{ij} \mathcal{B}^j (M_8)^j_i \]
\[+ m_7 \Omega_{bb} (\mathcal{T}_{b3})_{ij} \mathcal{B}^j (M_8)^j_i (H_3)^i + m_8 \Omega_{bb} (\mathcal{T}_{b3})_{ij} \mathcal{B}^j (M_8)^j_i (H_15)^i + m_9 \Omega_{bb} (\mathcal{T}_{b3})_{ij} \mathcal{B}^j (M_8)^j_i (H_15)^i. \] (41)

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XVII.
TABLE XVIII: Amplitudes for $\Omega_{\text{b}}$ decays into a bottom baryon (sextet)

| Channel | Amplitude |
|---------|-----------|
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}$ | $n_{0}+6n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}^{-}B'$ | $n_{1}-2n_{2}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}^{-}B'$ | $n_{1}-2n_{2}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{4}+3n_{6}+3n_{7}+n_{8}+n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}^{0}B^{-}$ | $n_{4}+3n_{6}^{'}+3n_{7}^{'}+n_{8}^{'}+n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{3}-n_{4}+6n_{5}+5n_{6}-n_{8}-2n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}^{-}B^{-}$ | $n_{4}-n_{5}^{'}-3n_{6}^{'}+n_{7}^{'}+n_{8}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}^{-}B'$ | $n_{1}^{'}+3n_{6}^{'}-n_{7}^{'}-n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{3}-2n_{5}+3n_{7}-n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}+6n_{9}^{'}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{3}-2n_{5}-n_{7}+n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}-2n_{5}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{3}-2n_{5}+3n_{7}-n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}+6n_{9}^{'}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{3}-2n_{5}-n_{7}+n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}-2n_{5}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{3}-2n_{5}+3n_{7}-n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}+6n_{9}^{'}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{3}-2n_{5}-n_{7}+n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}-2n_{5}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{3}-2n_{5}+3n_{7}-n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}+6n_{9}^{'}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{3}-2n_{5}-n_{7}+n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}-2n_{5}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{3}-2n_{5}+3n_{7}-n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}+6n_{9}^{'}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{3}-2n_{5}-n_{7}+n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}-2n_{5}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{3}-2n_{5}+3n_{7}-n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}+6n_{9}^{'}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{3}-2n_{5}-n_{7}+n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}-2n_{5}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Sigma_{0}B^{-}n_{3}-2n_{5}+3n_{7}-n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}+6n_{9}^{'}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{3}-2n_{5}-n_{7}+n_{9}$ |
| $\Omega_{\text{b}} \rightarrow \Xi_{0}B^{-}n_{1}^{'}-2n_{5}+3n_{6}^{'}+3n_{7}^{'}-2n_{8}^{'}-n_{9}^{'}$ |

In the case of a sextet, we have

$$H_{\text{eff}} = n_{1} \Omega_{\text{b}} \overline{b}_{6}(T_{6})_{(ij)}B(H_{3})^{ij} + n_{2} \Omega_{\text{b}} \overline{b}_{6}(T_{6})_{(ij)}B(H_{3})^{ij}$$

$$+ n_{3} \Omega_{\text{b}} \overline{b}_{6}(T_{6})_{(ij)}B(M_{5})^{ij}k + n_{4} \Omega_{\text{b}} \overline{b}_{6}(T_{6})_{(ij)}B(M_{5})^{ij}k$$

$$+ n_{5} \Omega_{\text{b}} \overline{b}_{6}(T_{6})_{(ij)}B(M_{5})^{ij}k + n_{6} \Omega_{\text{b}} \overline{b}_{6}(T_{6})_{(ij)}B(M_{5})^{ij}k$$

$$+ n_{7} \Omega_{\text{b}} \overline{b}_{6}(T_{6})_{(ij)}B(M_{5})^{ij}k + n_{8} \Omega_{\text{b}} \overline{b}_{6}(T_{6})_{(ij)}B(M_{5})^{ij}k$$

$$+ n_{9} \Omega_{\text{b}} \overline{b}_{6}(T_{6})_{(ij)}B(M_{5})^{ij}k.$$ (42)

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XVIII.

VI. NON-LEPTONIC $\Omega_{\text{c}}$ AND $\Omega_{\text{b}}$ DECAYS

For the mixedtriply heavy baryons, $\Omega_{\text{c}}$ and $\Omega_{\text{b}}$, most of their weak decays can be obtained from the $\Omega_{\text{ccc}}$ and $\Omega_{\text{b}}$ decay channels with some replacements. For instance, decays of $\Omega_{\text{c}}$ induced by the charm quark can be obtained from the ones of $\Omega_{\text{ccc}}$ by replacing one charmed meson by...
TABLE XIX: Amplitudes for $\Omega_{bb}$ decays into a singly bottom baryon (triplet)

| Channel | Amplitude |
|---------|-----------|
| $\Omega_{cb}^0 \rightarrow \Lambda_b^0 \pi^0$ | $-\sqrt{2}a_5 V_{cd}$ |
| $\Omega_{cb}^0 \rightarrow \Lambda_b^0 K$ | $(a_1 + a_5) V_{cs}$ |
| $\Omega_{cb}^0 \rightarrow \Lambda_b^0 \eta$ | $\frac{\sqrt{2}}{3} a_1 V_{cd}$ |
| $\Omega_{cb}^0 \rightarrow \Xi_b^0 K^0$ | $(a_1 - a_5) V_{cs}$ |
| $\Omega_{cb}^0 \rightarrow \Xi_b^0 \eta$ | $\frac{(a_1 - a_5)^2 V_{cd}}{\sqrt{3}}$ |

| Channel | Amplitude |
|---------|-----------|
| $\Omega_{cb}^0 \rightarrow \Lambda_b^0 K^0$ | $(a_1 + a_5) V_{cd}$ |
| $\Omega_{cb}^0 \rightarrow \Xi_b^0 \eta$ | $\frac{1}{\sqrt{6}} (a_1 - 2a_4 + a_6) V_{cd}$ |
| $\Omega_{cb}^0 \rightarrow \Xi_b^0 K^+$ | $(a_1 - 2a_4 + a_6) V_{cs}$ |
| $\Omega_{cb}^0 \rightarrow \Xi_b^0 \eta K^0$ | $\frac{(a_1 - 2a_4 + a_6)^2 V_{cd}}{\sqrt{6}}$ |

FIG. 2: Feynman diagrams for W-exchange. The spectator is a bottom or a charm quark. If the final $u$ quark is replaced by a charm quark, the W-exchange contribution is an SU(3) triplet, and this triplet contribution has been incorporated in the $b \rightarrow q_1 \bar{q}_2 q_3$.

the corresponding bottom meson, or a charmed baryon by the corresponding bottom baryon, or a doubly charmed baryon $H_{cc}$ by its counterpart $H_{bc}$.

In addition, there are new decay modes, which are induced by W-exchange transition shown in Fig. 2, $bc \rightarrow ud$ or $bc \rightarrow us$, with two heavy quarks annihilating into two light quarks. The spectator quark is a bottom or charm quark. These diagrams are dynamically suppressed by factors of $1/m_{b,c}$. The electroweak Hamiltonian is similar with Eq. (33). Taking $\Omega_{cb}^0$ as the example, one

---

1 If the final $u$ quark is replaced by a charm quark, the W-exchange contribution is an SU(3) triplet, and this triplet contribution has been incorporated in the $b \rightarrow q_1 \bar{q}_2 q_3$. In this case, the CKM matrix element is $V_{cb} V_{cd}/\sqrt{3}$. 

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TABLE XX: Amplitudes for $\Omega_{\text{bb}}$ decays into a singly bottom baryon (sextet)

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| $\Omega_{\text{cbb}}^{0} \rightarrow \Sigma_{b}^{0} \pi^{-}$ | $(b_{1} + b_{4}) V^{a}_{cd}$ | $\Omega_{\text{cbb}}^{0} \rightarrow \Xi_{b}^{0} \pi^{0}$ | $\frac{1}{2} (b_{4} - b_{1}) V^{a}_{cs}$ |
| $\Omega_{\text{cbb}}^{0} \rightarrow \Sigma_{b}^{+} K^{-}$ | $(b_{1} + b_{4}) V^{a}_{cs}$ | $\Omega_{\text{cbb}}^{0} \rightarrow \Xi_{b}^{0} K^{0}$ | $\frac{1}{\sqrt{2}} (b_{4} + b_{1}) V^{a}_{cs}$ |
| $\Omega_{\text{cbb}}^{0} \rightarrow \Sigma_{b}^{0} \eta^{0}$ | $-b_{1} V^{a}_{cd}$ | $\Omega_{\text{cbb}}^{0} \rightarrow \Xi_{b}^{0} \eta$ | $-(b_{4} + b_{1}) V^{a}_{cs}$ |
| $\Omega_{\text{cbb}}^{0} \rightarrow \Sigma_{b}^{+} \pi^{0}$ | $(b_{1} + b_{4}) V^{a}_{cs}$ | $\Omega_{\text{cbb}}^{0} \rightarrow \Xi_{b}^{-} \pi^{+}$ | $\frac{1}{\sqrt{2}} (b_{4} + b_{1}) V^{a}_{cs}$ |
| $\Omega_{\text{cbb}}^{0} \rightarrow \Sigma_{b}^{0} K^{0}$ | $b_{4} - b_{1} V^{a}_{cs}$ | $\Omega_{\text{cbb}}^{0} \rightarrow \Xi_{b}^{-} K^{+}$ | $\frac{1}{\sqrt{2}} (b_{4} - b_{1}) V^{a}_{cs}$ |
| $\Omega_{\text{cbb}}^{0} \rightarrow \Sigma_{b}^{+} \pi^{0}$ | $b_{4} - b_{1} V^{a}_{cs}$ | $\Omega_{\text{cbb}}^{0} \rightarrow \Xi_{b}^{-} K^{+}$ | $(b_{4} - b_{1}) V^{a}_{cs}$ |

should notice that the final state contains only one heavy bottom quark. Thus at hadron level, the final state can be a bottom baryon (antitriplet) which has:

$$
\mathcal{H}_{\text{eff}} = a_{1} \Omega_{\text{cbb}}(\mathcal{T}_{\text{bb}}) [i | j] (M_{8})^{k}_{j} (H_{6}^{i})^{[ij]} + a_{2} \Omega_{\text{cbb}}(\mathcal{T}_{\text{bb}}) [i | j] (M_{8})^{k}_{j} (H_{6}^{i})^{[ij]} + a_{3} \Omega_{\text{cbb}}(\mathcal{T}_{\text{bb}}) [i | j] (M_{8})^{k}_{j} (H_{6}^{i})^{[ij]} + a_{4} \Omega_{\text{cbb}}(\mathcal{T}_{\text{bb}}) [i | j] (M_{8})^{k}_{j} (H_{6}^{i})^{[ij]} + a_{5} \Omega_{\text{cbb}}(\mathcal{T}_{\text{bb}}) [i | j] (M_{8})^{k}_{j} (H_{6}^{i})^{[ij]} + a_{6} \Omega_{\text{cbb}}(\mathcal{T}_{\text{bb}}) [i | j] (M_{8})^{k}_{j} (H_{6}^{i})^{[ij]} .
$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XIIX

The final state can be a bottom baryon (sextet) which has:

$$
\mathcal{H}_{\text{eff}} = b_{1} \Omega_{\text{cbb}}(\mathcal{T}_{\text{bb}}) [i | j] (M_{8})^{k}_{j} (H_{6}^{i})^{[ij]} + b_{2} \Omega_{\text{cbb}}(\mathcal{T}_{\text{bb}}) [i | j] (M_{8})^{k}_{j} (H_{6}^{i})^{[ij]}
$$
### TABLE XXI: Amplitudes for Ω_{cbb} decays into a bottom meson and a light baryon (octet)

| Channel | Amplitude | Channel | Amplitude |
|---------|-----------|---------|-----------|
| Ω_{cbb} \rightarrow B^{-} \Sigma^{+} | \frac{-2c_1 + c_2}{\sqrt{2}} V_{bb} V_{bb} | Ω_{cbb} \rightarrow \Lambda^{0} | \sqrt{2} (-2c_1 + c_2) V_{bb} V_{bb} |
| Ω_{cbb} \rightarrow \overline{B}^{0} | \frac{-2c_1 + 3c_2}{\sqrt{2}} V_{bb} V_{bb} | Ω_{cbb} \rightarrow \Xi^{-} | -\sqrt{2} c_2 V_{bb} V_{bb} |
| Ω_{cbb} \rightarrow \Xi^{-} | \frac{2c_1 - 3c_2}{\sqrt{2}} V_{bb} V_{bb} | Ω_{cbb} \rightarrow \Xi^{0} | (-2c_1 + c_2) V_{bb} V_{bb} |
| Ω_{cbb} \rightarrow B^{-} \Lambda^{0}_{n} | \frac{2c_1 - c_2}{\sqrt{2}} V_{bb} V_{bb} | Ω_{cbb} \rightarrow B^{-} \Sigma^{+}_{n} | \frac{2c_1 + c_2}{\sqrt{2}} V_{bb} V_{bb} |

\[ + b_{v3} \Omega_{cbb}(T_{6b})_{\{ik\}}(M_{8}^{\frac{1}{2}})(H_{0}^{\prime})^{ij} \]
\[ + b_{v3} \Omega_{cbb}(T_{6b})_{\{ij\}}(M_{8}^{\prime})^{ij} + b_{v3} \Omega_{cbb}(T_{6b})_{\{ik\}}(M_{8})_{ij}^{\prime}+(H_{0}^{\prime})^{ij} \]
\[ + b_{v3} \Omega_{cbb}(T_{6b})_{\{ik\}}(M_{8})_{ij}^{\prime}(H_{0}^{\prime})^{ij}. \quad (44) \]

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XX

If the final state contains a bottom meson and a light baryon (octet), we have

\[ H_{\text{eff}} = c_{1} \Omega_{cbb} B \epsilon_{ij} (T_{8})_{k}^{ij} (H_{0}^{\prime})^{[ij]} + c_{2} \Omega_{cbb} \overline{B} \epsilon_{ij} (T_{8})_{k}^{ij} (H_{0}^{\prime})^{[ij]} \]
\[ + c_{3} \Omega_{cbb} B m_{ij} (T_{8})_{k}^{ij} (H_{0}^{\prime})^{[ij]} + c_{4} \Omega_{cbb} \overline{B} m_{ij} (T_{8})_{k}^{ij} (H_{0}^{\prime})^{[ij]} \]
\[ + c_{5} \Omega_{cbb} B m_{ij} (H_{0}^{\prime})^{[ij]} + c_{6} \Omega_{cbb} \overline{B} m_{ij} (H_{0}^{\prime})^{[ij]} \]
\[ + c_{7} \Omega_{cbb} B m_{ij} (H_{0}^{\prime})^{[ij]} + c_{8} \Omega_{cbb} \overline{B} m_{ij} (H_{0}^{\prime})^{[ij]} \]
\[ + c_{9} \Omega_{cbb} B m_{ij} (H_{0}^{\prime})^{[ij]} + c_{10} \Omega_{cbb} \overline{B} m_{ij} (H_{0}^{\prime})^{[ij]} \]
\[ + c_{11} \Omega_{cbb} B m_{ij} (H_{0}^{\prime})^{[ij]} + c_{12} \Omega_{cbb} \overline{B} m_{ij} (H_{0}^{\prime})^{[ij]} \]. \quad (45) \]

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XXI. One can find the amplitudes $c_1$ and $c_2$ are not independent, they always appear in the product $2c_1 - c_2$. So in Tab. XXI we did not show $c_2$. 

TABLE XXII: Amplitudes for $\Omega_{cbb}$ decays into a bottom meson and a light baryon (decuplet)

| channel | amplitude |
|---------|-----------|
| $\Omega_{cbb}^0 \rightarrow B^- \Delta^+$ | $\sqrt{3} d_4 V_{ub} V_{cd}^*$ |
| $\Omega_{cbb}^0 \rightarrow B^- \Sigma^+$ | $\sqrt{3} d_4 V_{ub} V_{cd}^*$ |
| $\Omega_{cbb}^0 \rightarrow B^0 \Delta^+$ | $\sqrt{3} d_4 V_{ub} V_{cd}^*$ |
| $\Omega_{cbb}^0 \rightarrow B^- \Sigma^+$ | $\sqrt{3} d_4 V_{ub} V_{cd}^*$ |
| $\Omega_{cbb}^0 \rightarrow B^- \Omega^+$ | $\sqrt{3} d_4 V_{ub} V_{cd}^*$ |
| $\Omega_{cbb}^0 \rightarrow B^- \Xi^+$ | $\sqrt{3} d_4 V_{ub} V_{cd}^*$ |
| $\Omega_{cbb}^0 \rightarrow B^- \Omega^+$ | $\sqrt{3} d_4 V_{ub} V_{cd}^*$ |
| $\Omega_{cbb}^0 \rightarrow B^- \Xi^+$ | $\sqrt{3} d_4 V_{ub} V_{cd}^*$ |

If the final state contains a bottom meson and a light baryon (decuplet), we have

$$H_{\text{eff}} = d_1 \Omega_{cbb} \overline{B}^j (T_{10})_{ijk} M_{m}^k (H_3^{m})^{[im]} + d_2 \Omega_{cbb} \overline{B}^j (T_{10})_{ijk} M_{m}^k (H_3^{m})^{[im]}$$

$$+ d_3 \Omega_{cbb} \overline{B}^j (T_{10})_{ijk} M_{m}^k (H_3^{m})^{[km]} + d_4 \Omega_{cbb} \overline{B}^j (T_{10})_{ijk} M_{m}^k (H_3^{m})^{[ij]}$$

$$+ d_5 \Omega_{cbb} \overline{B}^j (T_{10})_{ijk} (H_6^{m})^{[ij]} + d_6 \Omega_{cbb} \overline{B}^j (T_{10})_{ijk} M_{m}^k (H_6^{m})^{[ij]}$$

$$+ d_7 \Omega_{cbb} \overline{B}^j (T_{10})_{ijk} M_{m}^k (H_6^{m})^{[im]} + d_8 \Omega_{cbb} \overline{B}^j (T_{10})_{ijk} M_{m}^k (H_6^{m})^{[jm]}$$

$$+ d_9 \Omega_{cbb} \overline{B}^j (T_{10})_{ijk} M_{m}^k (H_6^{m})^{[jm]} + d_10 \Omega_{cbb} \overline{B}^j (T_{10})_{ijk} M_{m}^k (H_6^{m})^{[km]}.$$  \hspace{1cm} (46)

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Tab. XXII. It is interesting to notice that the above amplitudes $d_s$ are not all independent. The products $d_1 + d_2 + d_3, d_4 + d_5, d_6 + d_9, d_7 + d_8 + d_{10}$, appear in the expansion, thus we have removed the $d_2, d_3$ and $d_5$ and $d_6$ and $d_8, d_{10}$ in Tab. XXII.

VII. GOLDEN CHANNELS

Based on the above analysis, we give a collection of the CKM allowed decay channels for the $\Omega^{++}_{cbb}$ in Tab. XXIII and for the $\Omega_{bb}$ in Tab. XXIV. The ones for $\Omega_{cbb}$ and $\Omega_{cbb}$ can be obtained by the replacements as discussed in the above section.
TABLE XXIII: Cabibbo allowed decays of $\Omega_{ccc}$ with typical branching fractions at a few percent level. A light meson $K^0$ can be replaced by a $\bar{K}^0$.

| channel | channel | channel | channel |
|---------|---------|---------|---------|
| $\Omega_{ccc}^{++} \rightarrow \Omega_{ccc}^{+} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{+} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{+} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{+} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ | $\Omega_{ccc}^{+} \rightarrow \Xi_{cc}^{0} + \nu_\ell$ |

TABLE XXIV: CKM allowed decay channels of $\Omega_{bbb}^-$.

| channel | channel | channel | channel |
|---------|---------|---------|---------|
| $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ |
| $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ |
| $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ |
| $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ | $\Omega_{bbb} \rightarrow \Xi_{bb}^{0} + \nu_\ell$ |

- The light pseudoscalar meson in these two tables can be replaced by its vector counterpart. For instance a $K^0$ can be replaced by a $\bar{K}^0$ decaying into $K^-\pi^+$.  

- Branching fractions for semileptonic $\Omega_{cc}$ decay channels in Tab. XXIII can reach a few percent, but there is a neutrino in the final state, reducing somewhat experimental efficiency.

- Nonleptonic $\Omega_{ccc}$ such as $\Omega_{ccc}^{+++} \rightarrow \Xi_{cc}^{++} + \nu_\ell$ might be used to search for $\Omega_{ccc}$ especially at LHC, since their branching fractions are sizable, and the final state can be easily identified. This will make use of the doubly heavy baryon $\Xi_{cc}^{++}$ which has been just discovered by LHCb.

- For nonleptonic decays of $\Omega_{bbb}^-$, the largest branching fraction might reach $10^{-3}$. Taking into account its daughter decays, we expect the branching fraction for $\Omega_{bbb}^-$ decaying into charmless final state is at most $10^{-9}$. Thus the triply bottom baryon can be only observed with a large amount of data in future, such as the high luminosity LHC.
VIII. CONCLUSIONS

Up to date, quark model is a most successful theoretical tool to describe the hadron spectrum especially the lowest lying hadrons. Since the charm and bottom quarks are much heavier than the lighter ones, hadrons with a different number of heavy quarks will have distinct dynamics. On experimental side, light hadrons with no heavy quark, singly heavy baryons, and doubly heavy baryons have been established, but triply heavy baryons are still missing. Thus it deserves more theoretical and experimental efforts to study various properties of triply heavy baryons from both theoretical and experimental sides.

In this work, we have systematically analyzed weak decays of triply heavy baryons for the first time in the literature. Decay amplitudes for various transitions have been parametrized in terms of the SU(3) independent amplitudes. Using these results, we find a number of relations for the partial decay widths. We also give a list of decay channels with sizable branching fractions. We suggest our experimental colleagues to perform a search at hadron colliders and the electron and positron collisions in future.

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