Ballot stuffing and participation privacy in pollsite voting

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Abstract. We study the problem of simultaneously addressing both ballot stuffing and participation privacy for pollsite voting systems. Ballot stuffing is the attack where fake ballots (not cast by any eligible voter) are inserted into the system. Participation privacy is about hiding which eligible voters have actually cast their vote. So far, the combination of ballot stuffing and participation privacy has been mostly studied for internet voting, where voters are assumed to own trusted computing devices. Such approaches are inapplicable to pollsite voting where voters typically vote bare-handed. We present an eligibility audit protocol to detect ballot stuffing in pollsite voting protocols. This is done while protecting participation privacy from a remote observer - one who does not physically observe voters during voting. Our protocol can be instantiated as an additional layer on top of most existing pollsite E2E-V voting protocols. To achieve our guarantees, we develop an efficient zero-knowledge proof (ZKP), that, given a value $v$ and a set $\Phi$ of commitments, proves $v$ is committed by some commitment in $\Phi$, without revealing which one. We call this a ZKP of reverse set membership because of its relationship to the popular ZKPs of set membership. This ZKP may be of independent interest.

1 Introduction

Conducting large, binding elections in a fair and free manner is hard. Many existing end-to-end verifiable (E2E-V) voting schemes address this problem by providing cryptographic guarantees to any voter that their vote was correctly counted as they intended [2,18,19,35,14,10]. However, these systems generally rely on polling officers and traditional processes to ensure that only eligible voters had voted. This poses a serious concern as an adversary controlling the polling booth could launch a ballot stuffing attack, by letting ineligible voters vote, letting eligible voters vote more than once, or injecting fake votes in lieu of voters who did not show up for voting. Ballot stuffing is considered one of the most serious attacks against pollsite voting systems [29,17,31]. As such attacks are often seen in situations of a booth capture or when one party dominates the polling booth, simple solutions such as deploying multiple polling agents to oversee the eligibility verification process may not always suffice.

While detection of ballot stuffing is essentially about verifying that only eligible voters had voted, a conflicting requirement is that of protecting participation privacy [30], i.e., which eligible voters had actually voted. Although local observers physically observing voters on voting day necessarily know who voted and who did not, a cryptographic voting protocol should not introduce new ways of leaking this information to a remote observer. Widely publishing the list of participating voters, as many existing E2E-V protocols do, makes large-scale systematic targeting for coercion and profiling much easier. Concretely, it runs the risk of large-scale forced abstention attacks [28], whereby voters are forced to abstain from voting, either under duress or in exchange for money, similar to how they may be forced to vote for a given candidate. Further, since voter identifiers could be linked to a host of other sensitive information, participation information can be used for profiling and selective targeting (e.g., by ignoring communities with poor voter turnout during policymaking). Thus, often the very act of voting or abstaining is considered an aspect of voter privacy and publishing voter participation information is often disallowed [24,1].

In this paper, we propose an eligibility audit protocol for a pollsite voting system that allows a public auditor to detect whether ballot stuffing has happened or not, while protecting participation privacy from a remote observer. Although this problem has been studied in the context of internet voting [28,5,36,23,20,6,30], these solutions are not applicable to pollsite voting because they assume that voters have access to trusted computing devices — for computing signatures, NIZK proofs, etc. In pollsite voting, voters typically vote bare-handed [35,18,19,2] and no devices are trusted either for correctness or secrecy of cast votes. Our goal is specific to such bare-handed pollsite voting protocols. In fact, we want to add the eligibility audit capability as a modular layer on top of any existing pollsite E2E-V voting protocol such as [35,18,19,2].
1.1 Our contributions

Towards this end, we make the following main contributions.

First, we propose a modular protocol structure for an eligibility audit protocol on top of any pollsite E2E-V voting protocol. We require the E2E-V protocol to be such that the vote casting process produces an encrypted vote (a voter receipt), encrypted votes are published on a public bulletin board and are verifiably decrypted to obtain the final tally. To add the eligibility audit capabilities, we introduce the notion of a token, which acts as an identifier for a real-world voter but hides the voter’s true identity. Tokens are issued to voters by a registrar during a registration step and are embedded in a voting access card, which voters need to present to a polling officer to cast their vote. Importantly, the polling officer cannot inject a vote without access to this information. After verifying the voter’s identity card, the polling officer allows them to cast a vote in a private booth and scans the voter receipt obtained from the underlying E2E-V protocol. The polling officer uploads the receipt (its encrypted vote) to a backend teller along with the voter’s token extracted from the voter’s access card. Post polling, the teller publishes the list of encrypted votes along with the corresponding tokens and engages with a public auditor in an audit protocol to prove in zero-knowledge that no ballot stuffing took place. The recorded encrypted votes are further processed as per the underlying E2E-V scheme’s tallying protocol.

Although the model requires voters to safekeep their voting access card until they cast their vote, the card design makes it robust against lapses in this safekeeping. First, losing the physical card does not disenfranchise the voter as the card’s information can readily be copied on auxiliary media and presented to the polling officer. Second, losing the information on the card to another voter does not transfer the right to vote to the other voter because the card’s information is cryptographically tied to the card owner’s identity. To bypass this, the other voter would also need to corrupt the identity verification process at the polling booth. In this way, we provide protection against ballot stuffing if at least one of the voter’s card and the polling booth is uncorrupted. Third, losing the information on the card does not reveal to a remote observer whether the card owner voted or not.

Second, we develop formal security definitions for ballot stuffing and participation privacy for pollsite voting. To the best of our knowledge, both these notions have not been formalised before in the context of pollsite voting. Our definition of ballot stuffing captures that the number of published encrypted votes should be at most the number of registered voters who had cast their vote plus the number of registered voters who leaked their voting access card. Ballot stuffing post decryption of the published encrypted votes already comes under the purview of verifiability definitions for E2E-V voting protocols [21]. For participation privacy, we adapt a standard definition by Bernhard et al. [13] to pollsite voting.

Third, we develop a novel cryptographic primitive called a ZKP of reverse set membership to instantiate a concrete eligibility audit protocol. Given a group $G$ of prime order $q$ with generators $g, h$, a set $\Phi$ of Pedersen commitments in group $G$ and a value $v \in \mathbb{Z}_q$, a ZKP of reverse set membership is a zero-knowledge proof of knowledge of an $r \in \mathbb{Z}_q$ such that $g^v h^r \in \Phi$. The interesting property is that when the proof is requested for $k$ values against the same set $\Phi$, it provides an amortised $O(|\Phi| + k)$ time complexity instead of $O(|\Phi|k)$, making it suitable for large elections. The ZKP of reverse set membership is named as such because of its relationship to the popular ZKPs of (forward) set membership, which, given a commitment $C \in G$ and a set $\phi$ of values in $\mathbb{Z}_q$, proves in zero-knowledge the knowledge of an opening $(v, r)$ such that $C = g^v h^r$ and $v \in \phi$. This ZKP may be of independent interest.

Fourth, we provide benchmarks for our ZKP of reverse set membership for $n = 10^6$, modelling an election of $10^6$ voters in the worst case when each voter participates.

2 Related work

Most pollsite E2E-V voting schemes [2,18,19,35,14,10] consider the question of ballot stuffing outside their scope and trust traditional polling processes for eligibility verification.

There exist many internet voting protocols that provide both eligibility verifiability and participation privacy [28,5,36,23,20,6,30]. In all these protocols, each voter has access to a trusted computing device that generates a ciphertext encrypting the voter’s vote and a non-interactive zero-knowledge proof establishing the eligibility of the sender. Adapting these protocols to the pollsite setting is non-trivial, because a) voters are bare-handed and b) delegating the computation of these ciphertexts/proofs to other devices/agents at the polling booth is problematic because it would leak the voters’ vote to them.
Akinyokun and Teague [4] proposed a pollsite protocol with similar goals as ours, of simultaneously preventing both ballot stuffing and forced abstention attacks. In their scheme, voters (who voted as well as who abstained) can verify for themselves whether their attendance was recorded correctly or not. However, as the authors point out, the scheme does not offer dispute resolution: the voter cannot prove to anyone else that a fake ballot was inserted against their identity. Dispute resolution is important to ensure that voters do not falsely claim ballot stuffing to discredit the election. In contrast, in our approach, the act of ballot stuffing can be publicly detected by anyone. See Appendix A for a more detailed description.

2.1 Prior work related to the ZKP of reverse set membership

Cramer et al. [22] gave a generic method to construct Σ-protocols for proving knowledge of a witness to one of many statements without revealing which one. Our ZKP can be cast in this framework (as PK{(v) : C_1 = g^{hv_1} \land \cdots \land C_n = g^{hv_n}}, where Φ := \{C_1, \ldots, C_n\}). However, it leads to an O(|Φ|k) complexity when the proof is requested for k values of v, even when the set of commitments Φ remains the same.

Groth and Kohlweiss [27] gave a scheme where given a list C_1, \ldots, C_n of commitments, the prover proves knowledge of an r such that one of the commitments opens to v = 0. The scheme improves the communication complexity over Cramer et al.’s generic construction to O(\log(n)). Our ZKP can be considered as a generalisation of the scheme for any v \in \mathbb{Z}_q with amortised linear complexity for multiple proofs against the same list of commitments.

In the blockchain literature, Zerocash [11] uses a primitive similar to the reverse set membership proof, but it is based on a general circuit-based NIZK construction, whereas ours is a discrete logarithm based interactive proof construction.

3 Formal security definitions

In this section, we describe our formal security definitions for ballot stuffing and participation privacy. We begin by formally describing the protocol structure of a pollsite eligibility audit protocol.

3.1 Protocol structure

Our eligibility audit protocol has the following participants: a registrar R, a set of voters V, a polling officer P, a teller T and an auditor A. We assume that each voter V_id \in V is identified by a unique real-world identifier id. The protocol structure is specified in terms of an underlying pollsite E2E-V voting protocol \Pi_{E2E-V}. We assume that \Pi_{E2E-V} provides the following sub-protocols: a) a vote casting protocol ev ← \Pi_{E2E-V}.Cast(v) that takes as input a voter’s intended vote v and outputs an encrypted voter receipt ev; and b) a tallying protocol \Pi_{E2E-V}.Tally that takes as input a list of published encrypted votes and outputs the final tally.

Formally, a pollsite eligibility audit protocol is defined by sub-protocols (Setup, Register, Cast, Publish, Audit), where (see Figure 1):

- pk_X, sk_X ← Setup(X(1^λ)) is a protocol run by each X \in \{R, P, T\} to generate its public key pk_X and secret key sk_X. All subsequent algorithms implicitly obtain pk_X even though we will sometimes omit it for brevity.
- t, va, pub ← Register(R(sk_R, id)) is an algorithm run by registrar R after manually verifying the identity and eligibility of voter V_id. The algorithm’s input is R’s secret key sk_R and the voter’s identifier id. Its output is a secret token t, some voting access information va (to be embedded as a QR-code in a voting access card) and given to V_id; and pub, containing a public identifier and a proof of registration of V_id, to be published to a public bulletin board BB_0 at index id. We call the voters whose identifiers are published on BB_0 the officially registered (or eligible) voters.
- cvr, ev ← Cast_{\Pi_{E2E-V}.Cast}(P(sk_P), V(va, v)) is a protocol between the polling officer P and a voter V \in V for vote casting. P’s private input is its secret key sk_P; V’s private input is its voting access information va (embedded in the access card) and its intended vote v. P verifies the voter’s identity and the presented card information and allows V to cast her vote in a private booth as per the underlying E2E-V voting scheme’s \Pi_{E2E-V}.Cast protocol. P scans the encrypted vote ev from the voter receipt generated during the \Pi_{E2E-V}.Cast protocol. With the information gathered so far, P obtains a cast vote record cvr that it sends to the teller T via an internal secure channel. V obtains the physical receipt ev.
- \((t_j, ev_j)_{j=1}^{CVR}, \text{SEC} \leftarrow \text{Publish}(T(\text{sk}_T, \text{CVR}))\) is an algorithm run by \(T\). The algorithm’s input is \(T\)’s secret key \(\text{sk}_T\) and a set \(\text{CVR}\) of cast vote records received from \(P\) during the Cast protocol. Its output is an array whose \(j\)th element contains a token \(t_j\) and an encrypted vote \(ev_j\), to be published to another bulletin board \(BB_1\) by \(T\), and a derived secret information \(\text{SEC}\), to be used by \(T\) in the Audit protocol later. \(ev_j\)’s can be matched against voter receipts and are further processed as per the underlying E2E-V scheme’s \(\Pi_{E2E-V}\) Tally protocol.

- \(\text{accept/reject} \leftarrow \text{Audit}(BB_0, BB_1, T(\text{SEC}), A())\) is a protocol between the teller \(T\) and the auditor \(A\). The common input is the information published to \(BB_0\) and \(BB_1\); \(T\)’s private input is the secret \(\text{SEC}\) obtained from the Publish algorithm. At the end of the protocol, \(A\) outputs accept or reject.

![Diagram](image)

Fig. 1: An overview of our eligibility audit protocol

### 3.2 Ballot stuffing

We now explain our formal definition of ballot stuffing. Intuitively, we say that ballot stuffing has happened in an election if the number of published votes is greater than the number of registered voters participating in vote casting.\(^6\) However, since the token embedded in a voter’s card acts as their voting access credential, if the voter’s card information is leaked, technically a vote can be published even for a voter who did not vote (provided the traditional identity and eligibility verification processes at the polling booth are also compromised). Further, the registrar may try to secretly generate tokens for some voters without officially registering them on \(BB_0\). Nevertheless, the adversary should be able to produce at most as many votes as the number of officially registered voters who participated in vote casting plus the number of officially registered voters who leaked their voter card. This is the notion our definition captures (see Definition 1).

As shown in experiment \text{Exp}_{PBS} in Figure 2a, the experimenter generates public/private keys for registrar \(R\), modelling that the registrar is (at least partially) trusted. A minimum level of trust on the registrar is inevitable because the mapping of a human voter to a digital artifact such as a token requires manual intervention. The adversary \(A\) controls \(P\) and \(T\) and supplies their public keys. After this, \(A\) is allowed to run the entire election and populate bulletin boards \(BB_0\) and \(BB_1\) using oracles \(O\text{Reg}\), \(O\text{Leak}\) and \(O\text{Cast}\), respectively for registering a voter, obtaining a voter’s access card information and casting a vote. We keep track of the tokens for all officially registered voters.

\(^6\) Note that we do not address the concern of an eligible voter’s vote being removed or changed during publication, because that is the concern of an E2E-V voting protocol.
registered voters in set RT and the registered tokens leaked to A, either during vote casting or otherwise, in set RLT. A wins if it makes the Audit protocol pass (playing the role of T) while having produced more number of entries on BB_1 than the total number of leaked tokens belonging to registered voters, i.e., the size of RLT.

In more detail, the OReg oracle allows A to register a voter with identifier id. It also expects A to supply a bit b pub indicating whether to officially publish id on BB_0 or not. If b pub = 1, the proof of registration pub_id is published to BB_0 and the generated token is added to set RT. Note that A only obtains the publicly posted information and not the voter’s card information va_id, modelling that honest registered voters must keep their voting cards secure.

The OCast oracle allows A to model the vote casting process. Since A controls both P and T, the voter’s card information necessarily gets leaked to the adversary during vote casting. In addition, A is also allowed to call the OLeak oracle, which models the leakage of voter card information outside of vote casting. The set of all leaked tokens are recorded in RLT. The size of set RLT thus represents the maximum number of encrypted votes A could report, without getting caught by the auditor.

Note that the size of RLT is updated only when the registered voters’ tokens are leaked, to discount leakages caused for unofficially registered voters. Further, it does not get updated if the OCast or OLeak oracles are called multiple times for the same id (the former modelling that each voter should be allowed to cast only one vote and the latter modelling that each leaked token is counted only once).

Fig. 2: Security experiments for ballot stuffing and participation privacy

Definition 1 (Prevention against ballot stuffing). We say that an eligibility audit protocol \( \Pi_{\text{elg}} := (\text{Setup}, \text{Register}, \text{Cast}, \text{Publish}, \text{Audit}) \) prevents ballot stuffing if for all PPT adversaries A and for all security parameters \( \lambda \in \mathbb{N} \), there exists a negligible function negl such that:

\[
\Pr[\text{Exp}_{\text{PBS}}^A(\lambda) = 1] \leq \text{negl}(\lambda)
\]

where \( \text{Exp}_{\text{PBS}}^A(\lambda) \) is as defined in Figure 2a.

3.3 Participation privacy

For the formal modelling of participation privacy, we adapt the definition given by Bernhard et al. [13] to polsitsite voting (the definition was given for an internet voting protocol called KTV-Helios [30]). The basic idea
of this definition is that participation privacy is maintained if an adversary that corrupts all voters except only two, say \( V_{id_0} \) and \( V_{id_1} \), cannot distinguish between the world where \( V_{id_0} \) votes and \( V_{id_1} \) abstains and the world where \( V_{id_1} \) votes and \( V_{id_0} \) abstains.

We capture this idea in Definition 2. As shown in experiment \( \text{Exp}_{pp}^A \) in Figure 2b, we consider a remote and external adversary \( A \) for participation privacy that does not corrupt any of \( R, P \) or \( T \). \( A \) uses the \( \text{OReg} \) oracle to register a voter, but obtains their voting access card. It can do so even for voters \( V_{id_0} \) and \( V_{id_1} \), modelling that even forcing a voter to surrender its card should not affect \( A \)'s belief about whether the voter voted or not (\( A \) must register both \( V_{id_0} \) and \( V_{id_1} \) for the experiment to proceed).

After this, \( A \) can use the \( \text{OCast} \) oracle to cast votes for corrupted voters by supplying their identifier, access card information and the vote value. At some point, \( A \) calls the \( \text{OCastChal} \) oracle once to ask the experimenter to cast a vote for either \( V_{id_0} \) or \( V_{id_1} \) (\( A \)'s \textit{challenge} would be to find out which one). For this call, \( A \) also supplies the vote that the challenge voter casts, to discount the case when the final tally itself reveals who voted. The experimenter keeps track of the cast vote records corresponding to both possible choices of the challenge voter and sends \( BB_1^b \) to \( A \), where bit \( b \) corresponds to the world in which \( V_{id_0} \) votes and \( V_{id_1} \) abstains. Finally, the experimenter engages with \( A \) in the \( \text{Audit} \) protocol, where \( A \) plays the role of the auditor. \( A \) wins if it can output a bit \( b' \) such that \( b' = b \) is the correct guess of \( b \).

Note that we do not need to reveal the decryption of \( BB_1^b \) to \( A \) (as revealed by the \( II_{E2E,V,Tally} \) protocol) because the tally in both the worlds would be exactly the same. Also note that in all of \( \text{OCast/OCastChal} \) oracle calls, \( A \) only obtains the voter receipt \( ev_{id} \) and not the internal cast vote record \( cvr_{id} \). This is consistent with our assumption of a remote and external adversary.

We believe that a remote and external adversary (that does not corrupt any of the polling officer, the teller or the registrar) is sufficient to model the threat of large-scale systematic targeting using participation information. Note that our definition does not assume voters to be honest and protects against malicious voters trying to prove to a remote coerer (e.g., by revealing their voter card information) that they did not vote. This is because the only way an abstaining voter interacts with the election system is when it obtains its access card, which the definition allows the coerer to obtain. Thus, the view of the coerer interacting with a voter who really abstained is identical to its view interacting with a voter that tries to evade coercion by casting its vote normally, hiding its receipts and claiming that it abstained.

**Definition 2 (Participation privacy).** We say that an eligibility audit protocol \( \Pi_{\text{alg}} := (\text{Setup}, \text{Register, Cast, Publish, Audit}) \) protects participation privacy if for all PPT adversaries \( \mathcal{A} \), for all security parameters \( \lambda \in \mathbb{N} \), and any two identifiers \( id_0, id_1 \), there exists a negligible function \( \text{negl} \) such that:

\[
| \Pr[\text{Exp}_{pp}^A(1^\lambda, id_0, id_1, 0) = 1] - \Pr[\text{Exp}_{pp}^A(1^\lambda, id_0, id_1, 1) = 1] | \leq \text{negl}(1^\lambda)
\]

where \( \text{Exp}_{pp}^A(1^\lambda, id_0, id_1, b) \) for \( b \in \{0, 1\} \) is as defined in Figure 2b.

### 4 Our eligibility audit protocol

Before we describe our proposed eligibility audit protocol, we define some notation and recall key cryptographic primitives we need.

#### 4.1 Preliminaries

**Notation.** Let \( \lambda \) be a security parameter; \( n \) be a constant denoting the number of eligible voters and \( q \) be a prime of length exponential in \( \lambda \). Let \( G_1, G_2, G_T \) denote cyclic groups of prime order \( q \) such that they admit an efficiently computable bilinear map \( e: G_1 \times G_2 \rightarrow G_T \), i.e., for all \( a, b \in \mathbb{Z}_q \) and generators \( g_1, g_2 \) of \( G_1 \) and \( G_2 \) respectively, \( e(g_1^a, g_2^b) = e(g_1, g_2)^{ab} \) and \( e(g_1, g_2) \neq 1_{G_T} \), where \( 1_{G_T} \) denotes the identity element of \( G_T \). We assume that the \( n \)-Strong Diffie Hellman assumption [15] holds in groups \( (G_1, G_2) \) and that the discrete logarithm problem is hard in group \( G_1 \). We let \( f_1, g_1, h_1 \in G_1, f_2, g_2 \in G_2 \) and \( f_T \in G_T \) denote randomly chosen generators of groups \( G_1, G_2 \) and \( G_T \) respectively. It is assumed that these generators are generated securely before the protocol starts so that nobody knows the mutual discrete logarithms of \( f_1, g_1, \) and \( h_1 \). This can be done by obtaining the generators from the output of a hash function, modelled as a random oracle, on some unpredictable input. We let \( \text{Perm}(n) \) denote the set of permutation functions with domain and range \( \{1, \ldots, n\} \).
Pedersen commitments. The quantity $g_1^v h_1^r$ is a Pedersen commitment [34] to a value $v \in \mathbb{Z}_q$ in group $\mathbb{G}_1$ under randomness $r \in \mathbb{Z}_q$. Pedersen commitments are \textit{computationally binding}: it is computationally hard to produce two pairs $(v, r)$ and $(v', r')$ such that $g_1^v h_1^r = g_1^{v'} h_1^{r'}$. Pedersen commitments are also \textit{perfectly hiding}: $g_1^v h_1^r$ reveals no information about $v$ if $r$ is chosen uniformly at random from $\mathbb{Z}_q$.

Public-key cryptosystem. We assume a public-key cryptosystem (PKC) for secure communication between the registrar, the polling officer and the teller (note that voters do not have to be involved in this PKC). We let $\Pi_{PKC} := \langle \text{Keygen}, \text{Enc}, \text{Dec}, \text{Sign}, \text{Ver} \rangle$ denote such a cryptosystem, where $\Pi_{PKE} := \langle \text{Keygen}, \text{Enc}, \text{Dec} \rangle$ forms an IND-CPA secure public-key encryption scheme and $\Pi_{PKS} := \langle \text{Sign}, \text{Ver} \rangle$ forms an EUF-CMA secure public-key digital signature scheme. We assume that the message space of $\Pi_{PKE}$ is $\mathbb{Z}_q$ and that of $\Pi_{PKS}$ is $\{0, 1\}^*$.

Public bulletin boards. A \textit{public bulletin board} represents an authenticated broadcast channel such that only the specified senders can successfully publish data to it and data once published cannot be changed.

BBS+ signatures. We also depend on BBS+ signatures [7] for our ZKP of reverse set membership (see Section 5). A BBS+ signature scheme is given by algorithms $(\text{BBS+Keygen}, \text{BBS+Sign}, \text{BBS+Ver})$ defined below (here, $x$ denotes the signer’s secret key, $y$ denotes its public key, and $m$ denotes a message to be signed):

- BBS+.Keygen: $x \xleftarrow{\$} \mathbb{Z}_q^*$, $y \leftarrow f_2^*$. 
- BBS+.Sign$(m, x)$: $c, r \xleftarrow{\$} \mathbb{Z}_q^*$; $S \leftarrow (f_1 g_1^m h_1^r)^{\frac{1}{e+\epsilon}}$. Output $\sigma \leftarrow (S, c, r)$.
- BBS+.Ver$(\sigma) = (S, c, r, m, y)$: If $e(S, y f_2^*) = e(f_1 g_1^m h_1^r, f_2)$, output 1; else output 0.

Obtaining signatures on committed values. An interesting property of BBS+ signatures is that given a Pedersen commitment $C = g_1^m h_1^r$, a \textit{signature on the committed value} $m$ can be obtained by sending to the signer only $C$ and a non-interactive zero-knowledge proof of knowledge (NIZKPoK) of its opening:

1. The committer sends $C = g_1^m h_1^r$ and a NIZKPoK $\Pi_{\text{NIZKPK}}\{(m, r) : C = g_1^m h_1^r\}$ to the signer. The signer verifies the NIZKPoK.
2. The signer computes a \textit{quasi signature} by choosing $c, r' \xleftarrow{\$} \mathbb{Z}_q$ and computing $S \leftarrow (f_1 C h_1^r)^{\frac{1}{e+\epsilon}}$. It sends $\sigma' := (S, c, r')$ to the committer.
3. The committer computes $\sigma \leftarrow (S, c, r' + r)$. Note that $\sigma = (\langle (f_1 g_1^m h_1^{r+r})^{\frac{1}{e+\epsilon}}, c, r' + r \rangle)$ is a valid BBS+ signature on message $m$ under the signer’s public key $y = f_2^*$.

Note that the signer learns nothing about the message because the message is committed.

4.2 The proposed protocol

At a high level, our protocol works as follows (see Figure 3). During the registration of a voter $V_{id}$, registrar $R$ generates a secret token $t_{id}$ uniformly sampled from $\mathbb{Z}_q$ and publishes a commitment $C_{id}$ to this token to a public bulletin board $BB_0$, along with a NIZK proof of knowledge of an opening to $C_{id}$. The token is also embedded inside $V_{id}$’s voting access card under an encryption against polling officer $P$’s public key. In addition, the card also contains an encryption of the commitment randomness $r_{id}$ against teller $T$’s public key (this will be used later by $T$ in the audit protocol). Assuming a voter’s access card was kept secure, $P$ obtains the voter’s token only when the voter participates in the Cast protocol.

If so, $P$ securely uploads the token to $T$ along with the voter-produced encrypted vote, passing along the encryption of the commitment randomness to $T$. Post polling, $T$ publishes tokens and encrypted votes of all participating voters to another public bulletin board $BB_1$, in a random order obtained by a secret permutation $\pi$. $T$ also decrypts the $r_{id}$’s to be used in the audit protocol. Specifically, $T$ acts as the prover in our ZKP of reverse set membership (see Section 5) that convinces the auditor that each token on $BB_1$ was committed by some commitment on $BB_0$, without revealing which one.

The ZKP implies that only those tokens can be published that were officially committed by $R$ in the registration phase. Further, since tokens are randomly selected from a large space and $P$ does not obtain a voter’s token unless the voter arrives to cast her vote, ballot stuffing cannot be done for absentee voters who keep their access cards secure.

We note that the registrar’s signature in the voting access card is only required to bind the identity of the voter to the issued token which facilitates the traditional identity verification process at the polling booth. Our cryptographic guarantee against ballot stuffing does not depend on the security of this signature scheme (see Theorem 2).
5 ZKP of reverse set membership

Before we present our ZKP of reverse set membership, we first present a ZKP of set membership protocol due to Camenisch et al. [16] to which our construction is closely related.

ZKP of set membership (Camenisch et al. [16]). Recall that a ZKP of set membership, denoted as $\text{PK}\{(v, r) : C = g_1^r h_i^v \land v \in \phi\}$, proves that a given commitment $C$ commits a value $v$ in some public set $\phi$. The main idea behind the Camenisch scheme is that the verifier sends to the prover Boneh-Boyen signatures [15] on elements of set $\phi$ under a fresh signing key. Then the prover can prove that $C$ commits a member of the set by proving that it knows a signature on the value committed by $C$. This can be done in zero knowledge by revealing only a blinded signature to the verifier and proving knowledge of appropriate blinding factors from which a valid signature can be obtained. If $C$ does not commit a member of $\phi$ then the proof fails because the prover does not obtain verifier’s signatures on non-members of the set.

The scheme is an honest-verifier ZKP of set membership if $|\phi|$-Strong Diffie Hellman assumption holds in $(G_1, G_2)$ [16]. A nice property of this scheme is that if proofs for $k$ commitments are requested against the same set $\phi$, the verifier’s signatures can be reused, resulting in only an $O(1)$ online overhead per commitment and thus overall $O(|\phi| + k)$ amortised complexity. In contrast, a scheme based on the generic OR construction ($\text{PK}\{(v, r) : C = g_1^r h_i^v \land (v = v_1 \lor \cdots \lor v = v_n)\}$), where $\phi = \{v_1, \ldots, v_n\}$, is $O(|\phi|k)$ when for proving set membership for $k$ commitments.

ZKP of reverse set membership. A ZKP of reverse set membership proves that a given value $v$ is committed by one of a set $\Phi$ of commitments. In the context of Pedersen commitments, this can be formalised as a proof of knowledge of an $r$ such that $g_1^r h_i^v \in \Phi$, i.e., $\text{PK}\{(r) : g_1^r h_i^v \in \Phi\}$.

Our protocol for ZKP of reverse set membership follows the Camenisch scheme’s theme of obtaining verifier’s signatures on elements of a set and proving knowledge of valid signatures. However, the Camenisch scheme cannot be adapted trivially to our setting because the Boneh-Boyen signatures require messages to be in group $\mathbb{Z}_q$ and cannot be directly used for signing Pedersen commitments, which are members of group $G_1$. Recall, however, from Section 4.1 that the BBS+ signature scheme [8] allows a committer to present a commitment to the signer and obtain a signature on the committed value.
Fig. 4: $\Pi_{ZKP-RSM}$: ZKP of reverse set membership for values $\{v_1, \ldots, v_k\}$ against set $\Phi := \{C_1, \ldots, C_n\}$. The prover is denoted by $\mathcal{P}$ and the verifier is denoted by $\mathcal{V}$. The common inputs to $\mathcal{P}$ and $\mathcal{V}$ are the following:

- $(C_1, \ldots, C_n)$, $(pr_1, \ldots, pr_n)$, $(v_1, \ldots, v_k)$ s.t. $(\forall i \in \{1, \ldots, n\} : C_i \in \mathbb{G}_1 \land pr_i = \text{NI-ZKP}\{(v, r) : C_i = \frac{v_i}{r_i}\})$ and $(v_1, \ldots, v_k) \in \mathbb{Z}_q^k, k \leq n$

The private inputs to $\mathcal{P}$ and $\mathcal{V}$ are as follows:

- Witness $(r_1, \ldots, r_k)$ s.t. $C_1 = g_1^{e_1} h_1^{r_1} \land \cdots \land C_n = g_1^{e_n} h_1^{r_k}$

Stage 1 (Obtaining BBS+ signatures on $v_1, \ldots, v_k$ from $\mathcal{V}$):

$\mathcal{P}$:

$\forall x \leftarrow \mathbb{Z}_q$

$y \leftarrow f(x)$

For each $i \in \{1, \ldots, n\}$:

- Verify $pr_i$ (abort if failed)

$e_i, r'_i \leftarrow \mathbb{Z}_q$

$S_i \leftarrow (f_i C_i h_i^{r'_i})_e$

$s_i' \leftarrow (S_i, e_i, c_i, r'_i)$

For each $j \in \{i_1, \ldots, i_k\}$:

$s_j := (S_j, c_j, r''_j) := s'_j$

$r''_j \leftarrow r'_j + r_j$

$s_j \leftarrow (S_j, c_j, r''_j)$

if BBS+Ver($s_j, v_j, y) \neq 1$: abort

Store $\langle s_{i_1}, \ldots, s_{i_k} \rangle$

Stage 2 (Proving knowledge of BBS+ signatures on $v_1, \ldots, v_k$ (as per [8])):

$\mathcal{P}$:

For each $j \in \{i_1, \ldots, i_k\}$:

- $\rho_1, \rho_2 \leftarrow \mathbb{Z}_q$

$B_1 := g_1^{\rho_1} h_1^{\rho_2}$

$B_2 := S_j g_1^{c_j}$

$\delta_1 := c_j \rho_1$

$\delta_2 := c_j \rho_2$

$\langle B_{i_1}, B_{2_{i_1}}, \ldots, B_{i_k}, B_{2_{i_k}} \rangle$

For each $j \in \{i_1, \ldots, i_k\}$:

$\Pi_{PK} := \text{PK}\{(\rho_1, \rho_2, c_j, r''_j, \delta_1, \delta_2) : B_1 = g_1^{\rho_1} h_1^{\rho_2} \land B_2 = g_1^{c_j} h_1^{\delta_2} \land e(B_2, y) = e(B_2, f_2)^{r''_j} e(g_1, y)^{\rho_2} e(g_1, f_2)^{\delta_2} e(h_1, f_2)^{r''_j} e(g_1, f_2)^{\delta_2}\}$
Thus, we let the reverse set membership verifier act as the signer that sends quasi-BBS+ signatures for each $C_i \in \Phi$ after verifying that $C_i$ is actually committed, via the NIZK proof $pr_i$ (see Figure 4 - stage 1). Thus, by the property of BBS+ signatures, the prover can obtain valid BBS+ signatures on values committed by each $C_i \in \Phi$. To prove that the given value $v$ is committed by some commitment in $\Phi$, the prover gives a ZKP of knowledge of a BBS+ signature on $v$. Since the prover only obtains signatures on values committed by commitments in $\Phi$, it cannot succeed if no $C \in \Phi$ committed $v$. For the proof of knowledge of a BBS+ signature in zero-knowledge, we use the technique proposed in [8] (stage 2). This protocol also enjoys an overall $O(|\Phi| + k)$ complexity for verifying reverse set membership for $k$ values, because signatures can be re-used.

6 Security analysis

Theorem 1 (ZKP of reverse set membership for $k$ values). If the computational binding assumption for Pedersen commitments holds in group $\mathbb{G}_1$, the $n$-Strong Diffie-Hellman ($n$-SDH) assumption holds in $(\mathbb{G}_1, \mathbb{G}_2)$, and for each $i \in \{1, \ldots, n\}$, then the protocol in Figure 4 is a ZKP of reverse set membership of values $(v_1, \ldots, v_k)$ against set $\Phi := \{C_1, \ldots, C_n\}$, i.e., PK$\{(r_{i1}, \ldots, r_{ik}) : g_{v_{i1}}^{r_{i1}} h_{i1}^{r_{i1}} \in \Phi \wedge \ldots \wedge g_{v_{ik}}^{r_{ik}} h_{ik}^{r_{ik}} \in \Phi\}$.

Proof. Completeness: For each $i \in \{1, \ldots, n\}$, the verifier passes the verification of $pr_i$ by the given condition.

Further, $\mathcal{P}$ obtains quasi-BBS+ signatures $\sigma_i = (S_i, c_i, r_i) = ((f_1 g_1^{v_i} h_i^{r_i})^{\frac{1}{x+c_i}}, c_i, r_i)$ from $\mathcal{V}$. From this, for each $j \in \{i_1, \ldots, i_k\}$, $\mathcal{P}$ obtains valid BBS+ signatures $\sigma_j = (S_j, c_{ij}, r_{ij} + r_j) = ((f_1 g_1^{v_j} h_1^{r_j + r_j})^{\frac{1}{x+c_j}}, c_{ij}, r_{ij} + r_j)$. Thus all BBS+ $\mathcal{V}$ checks pass.

In stage 2, for each $j \in \{i_1, \ldots, i_k\}$, $\mathcal{P}$ sends $B_{1j} = g_{1j}^{v_{1j}} h_{1j}^{r_{1j}}$ and $B_2 = g_{1j}^{v_{2j}} h_{1j}^{r_{2j}} g_{1j}^{r_{1j}}$ to $\mathcal{V}$ for any randomly chosen $r_{1j}, r_{2j} \in \mathbb{Z}_q$. We now show that these values pass the proof of knowledge $\Pi_{\text{PoK}}$.

First note that the first two conditions of $\Pi_{\text{PoK}}$ are trivially satisfied because $B_1 = g_{1j}^{v_{1j}} h_{1j}^{r_{1j}}$, $\delta_{1j} = c_j r_{1j}$ and $\delta_{2j} = c_j r_{2j}$. The RHS of the third condition of $\Pi_{\text{PoK}}$ simplifies to its LHS as follows:

\[
\frac{e(\delta_{1j}, f_2)}{e(B_2, f_2)} - e(g_{1j}, f_2)^{c_j} e(g_{1j}^{v_{1j}} h_{1j}^{r_{1j}}, f_2) = e((f_1 g_{1j}^{v_{1j}} h_{1j}^{r_{1j}})^{\frac{1}{x+c_j}}, c_j, r_j) = e(f_1, f_2).
\]

Special soundness: We show that if verifer $\mathcal{V}$ accepts then a PPT extractor $\mathcal{E}$ can extract $(r_{i1}, \ldots, r_{ik})$ such that $g_{v_{i1}}^{r_{i1}} h_{i1}^{r_{i1}} \in \Phi \wedge \ldots \wedge g_{v_{ik}}^{r_{ik}} h_{ik}^{r_{ik}} \in \Phi$. $\mathcal{E}$ simply runs the extractor $\mathcal{E}_{\text{pr}}$ for NIZKs $(pr_1, \ldots, pr_n)$ to obtain $((\hat{v}_{i1}, r_{i1}), \ldots, (\hat{v}_{in}, r_{in}))$ (by the special soundness of the NIZK proof of knowledge, $\mathcal{E}_{\text{pr}}$ exists and is PPT). Note that if $\{\hat{v}_{i1}, \ldots, \hat{v}_{in}\} \subseteq \{\hat{v}_1, \ldots, \hat{v}_n\}$ then $\mathcal{E}$ has been successful since $g_{v_{i1}}^{r_{i1}} h_{i1}^{r_{i1}} \in \Phi \wedge \ldots \wedge g_{v_{ik}}^{r_{ik}} h_{ik}^{r_{ik}} \in \Phi$.

Claim: $\{v_{i1}, \ldots, v_{ik}\} \subseteq \{\hat{v}_1, \ldots, \hat{v}_n\}$. Suppose for contradiction that there exists an $i_k \in \{i_1, \ldots, i_k\}$ such that $v_{i_k} \notin \{\hat{v}_1, \ldots, \hat{v}_n\}$. We show that if this happens then a forger $\mathcal{F}$ can be constructed against the BBS+ signature scheme. We show the construction of $\mathcal{F}$ in Figure 5.

Note that $\mathcal{F}$ produces a view for $\mathcal{P}$ that is indistinguishable from the view produced by the real verifier in Figure 4. In particular, $\mathcal{F}$ sends quasi-BBS+ signatures $\sigma_i' = ((f_1 g_1^{v_i} h_1^{r_i})^{\frac{1}{x+c_i}}, c_i, r_i - r_i)$ for randomly chosen $c_i$ and $r_i$ by $C$, which are identically distributed to quasi-BBS+ signatures sent by the real verifier in stage 1.
\( F((C_1, \ldots, C_n), (pr_1, \ldots, pr_n), (v_{i_1}, \ldots, v_{i_k}), 2) \):

\( C \rightarrow F: y \) (BBS+ signature public key)

\( F: \)

for \( i = 1, \ldots, n: \)

\( (\hat{v}_i, \hat{r}_i) \leftarrow E_{pr}(C, pr_i) \)

\( F \rightarrow C: (\hat{v}_1, \ldots, \hat{v}_n) \) (signature queries)

\( C \rightarrow F: (\sigma_1, \ldots, \sigma_n) \)

\( F: \)

for \( i = 1, \ldots, n: \)

\( (S_i, c_i, r_i) := \sigma_i \)

\( \sigma'_i := (S_i, c_i, r_i - \hat{r}_i) \)

\( F \rightarrow P: y, (\sigma'_1, \ldots, \sigma'_n) \)

\( P \rightarrow F: ((B_{1,i}, B_{2,i}), \ldots, (B_{k,i}, B_{k,i})) \)

\( F: \)

\( (\rho_1, \rho_2, c, r'', \delta_1, \delta_2) \leftarrow \mathcal{E}_{\text{PoK}}(y, v_{i_k}, B_{1,k}, B_{2,k}) \)

\( \hat{\sigma} := (B_{2,i}g_{1}^{\rho_2}, c, r'') \)

\( F: \) Output \((v_{i_k}, \hat{\sigma})\)

Fig. 5: Forger \( F \) breaking the BBS+ signature unforgeability game. \( C \) denotes the challenger of the unforgeability game and \( P \) denotes the prover of the reverse set membership protocol.

of Figure 4. Thus, \( F \) obtains \(((B_{1,i}, B_{2,i}), \ldots, (B_{k,i}, B_{k,i}))\) identically distributed to what the real verifier obtains in stage 2. By the soundness of \( \Pi_{\text{PoK}} \) in stage 2, values \( \rho_1, \rho_2, c, r'', \delta_1, \delta_2 \) extracted by its extractor \( \mathcal{E}_{\text{PoK}} \) satisfy the conditions of \( \Pi_{\text{PoK}} \). Thus, by Lemma 1, \( \hat{\sigma} = (B_{2,i}g_{1}^{\rho_2}, c, r'') \) is a valid BBS+ signature on message \( v_{i_k} \) under public key \( y \).

Since \( v_{i_k} \notin \{v_1, \ldots, v_n\}, \) \( F \) had not queried a signature on message \( v_{i_k} \) from \( C \). Thus, \((v_{i_k}, \hat{\sigma})\) is a BBS+ signature forgery. Since \( F \) queried \( n \) signatures in total, this is not possible under the \( n \)-Strong Diffie Hellman assumption, as shown in [7].

Lemma 1. If a PPT extractor can extract \((\rho_1, \rho_2, c, r'', \delta_1, \delta_2)\) that satisfy:

\[
B_1 = g_1^{\rho_1} h_1^{\rho_2} \tag{2}
\]

\[
B_1' = g_1^{\delta_1} h_1^{\delta_2} \tag{3}
\]

\[
e(B_2, y) \overline{e(f_1, f_2)} = e(B_2, f_2)^{-c} e(g_1, y)^{\rho_2} e(g_1, f_2)^{\delta_2} e(h_1, f_2)^{r''} e(g_1, f_2)^v \tag{4}
\]

then \((\hat{S}, \hat{c}, \hat{r}) = (B_2g_1^{-\rho_2}, c, r'')\) is a valid BBS+ signature on message \( v \) under public key \( y \), i.e., it satisfies the following BBS+ signature verification equation: \( e(\hat{S}, yf_2) = e(f_1g_1^v h_1^v, f_2) \).

Proof. From Equations 2 and 3, we get \( g_1^{\rho_1} h_1^{\rho_2} = g_1^{\delta_1} h_1^{\delta_2} \). Thus, it must be that \( \delta_1 = \rho_1 c \) and \( \delta_2 = \rho_2 c \), otherwise the extractor can be used to produce two different openings \((\rho_1 c, \rho_2 c)\) and \( (\delta_1, \delta_2) \) of the Pedersen commitment \( B_1' \). Substituting this in Equation 4, we get:

\[
e(B_2, y) \overline{e(f_1, f_2)} = e(B_2, f_2)^{-c} e(g_1, y)^{\rho_2} e(g_1, f_2)^{\rho_2 c} e(h_1, f_2)^{r''} e(g_1, f_2)^v \]

\[
\implies e(B_2, y) e(B_2, f_2^v) = e(g_1^{\rho_2}, y) e(g_1^{\rho_2}, f_2^v) e(f_1g_1^v h_1^{r''}, f_2) \]

\[
\implies e(B_2, y f_2) = e(g_1^{\rho_2}, yf_2) e(f_1g_1^v h_1^{r''}, f_2) \]

\[
\implies e(B_2g_1^{-\rho_2}, yf_2) = e(f_1g_1^v h_1^{r''}, f_2) \]

Honest-verifier zero-knowledge: Consider the following sequence of experiments:

- E0: Real protocol between \( P \) and \( V \).

- E1: Same as E0 except that for each \( i \in \{1, \ldots, n\} \), instead of supplying a real proof \( pr_i \), \( V \) is supplied a simulated proof. This is indistinguishable from E0 by the zero-knowledge property of NIZKs \((pr_1, \ldots, pr_n)\).

- E2: Same as E1 except that instead of running as per the real prover for \( \Pi_{\text{PoK}} \) in stage 2, simulator \( S_{\text{PoK}} \) is run instead. This is indistinguishable from E1 in the random oracle model by the zero-knowledge property of \( \Pi_{\text{PoK}} \).
E3: Same as E2 except that a) in stage 1, the prover’s steps are skipped; and b) in stage 2, for each 
\( j \in \{i_1, \ldots, i_k\}, B_{2j} \) is obtained as 
\( B_{2j} \leftarrow G_1 \) instead of as 
\( B_{2j} \leftarrow S_j g_1^{\rho_{2j}} \) and \( \delta_1, \delta_2 \) assignment steps are skipped. This is indistinguishable from E2 because 
\( S_j g_1^{\rho_{2j}} \) is indistinguishable from a uniformly random element of \( G_1 \) for uniformly chosen \( \rho_{2j} \in \mathbb{Z}_q \), which implies that the input statements for \( S_{\text{PoK}} \) in both the experiments are indistinguishable.

Since in E3, none of the prover’s private inputs are being used, the protocol is zero-knowledge.

General zero-knowledge: Note that \( \Pi_{\text{PoK}} \) in stage 2 can be directly converted from honest-verifier to general zero-knowledge in the random oracle model, using the standard Fiat-Shamir heuristic [26]. Further, the prover sends any message only if the verifier sends valid BBS+ quasi-signatures (because of the BBS+Ver signature verification step). Thus, the view of the verifier is identically distributed to its view in the honest-verifier case. Hence the claim holds.

**Theorem 2 (Prevention against ballot stuffing).** If the computational binding assumption for Pedersen commitments holds in group \( G_1 \), the n-Strong Diffie-Hellman (n-SDH) assumption holds in \( (G_1, G_2) \), the protocol presented in Figure 3 prevents ballot stuffing as per Definition 1.

**Proof.** Suppose for contradiction that there exists a PPT adversary \( A \) such that the probability that \( \text{Exp}_A^{\text{PKE}}(\lambda) \) outputs 1 is non-negligible. This means that with non-negligible probability, the Audit protocol passed but there are more entries in BB_1 than the size of the RLT set (see Figure 2a). Let PT denote the set of tokens published on BB_1. Because the Audit protocol verifies that all tokens published on BB_1 are distinct (see Figure 3), |PT| > |RLT|. Thus, there must exist at least one token \( t \in \) PT \setminus RLT. By the special soundness of the ZKP of reverse set membership (Theorem 1), there exists a PPT extractor that can extract an \( \hat{\tau} \) such that \( g_1^{\hat{\tau}} h_i^t \in \Phi \), where \( \Phi \) denotes the set of commitments published on BB_0. Let \( C_{\text{id}} \) be the commitment published on BB_0 such that \( C_{\text{id}} = g_1^{\hat{\tau}} h_i^t \). Let \( t_{\text{id}} \) denote the token committed by \( C_{\text{id}} \) during the Register protocol and \( r_{\text{id}} \) denote the corresponding randomness (see Figure 3).

**Case 1:** \( t \neq t_{\text{id}} \): If this case arises, then one can successfully produce two different openings \((t, \hat{\tau})\) and \((t_{\text{id}}, r_{\text{id}})\) for the commitment \( C_{\text{id}} \).

**Case 2:** \( t = t_{\text{id}} \): Since \( t \notin \text{RLT} \), this case leads to two sub-cases:

1. \( t \notin \text{RT} \): This case means that \( t \) was not an officially registered token. Note that \( \text{RT} = \{ t_{\text{id}} | t_{\text{id}} \text{ was generated during the OReg(id) call and id appears on BB}_0 \} \). Thus, this case is not possible because \( t = t_{\text{id}} \) and \( t_{\text{id}} \in \text{RT} \).
2. \( t \in \text{RT} \setminus \text{RLT} \): This case means that a commitment to \( t \) was appended on BB_0 against some identifier \( \text{id} \) but \( A \) does not obtain the voter card \( \text{va}_{\text{id}} \) from either the OLeak oracle or the OCast oracle. Since commitments are perfectly hiding, \( t \) is distributed identically to a uniform distribution in a large space \( \mathbb{Z}_q \) for \( A \) and the probability that \( A \) can guess it correctly is negligible.

**Theorem 3 (Participation privacy).** Assuming \( \Pi_{\text{PKE}} = (\text{Keygen}, \text{Enc}, \text{Dec}) \) is an IND-CPA secure encryption scheme, the scheme presented in Figure 3 protects participation privacy as per Definition 2 in the random oracle model.

**Proof.** For each \( b \in \{0, 1\} \), let \( E_b \) denote the experiment that is identical to \( \text{Exp}_b^{\text{PKE}}(\lambda, \text{id}_0, \text{id}_1, b) \) except that (I) instead of running the real prover for the ZKP of reverse set membership \( \Pi_{\text{ZKP-RSM}} \), it runs its simulator \( S_{\text{ZKP-RSM}} \); and (II) for \( \text{pr} \) generation, during the registration, it runs a simulator \( S_{\text{NIZKPK}} \) for NIZKPK. \( E_b \) is indistinguishable from \( \text{Exp}_0^{\text{PKE}}(\lambda, \text{id}_0, \text{id}_1, b) \) by the zero-knowledgeness property of both \( \Pi_{\text{ZKP-RSM}} \) (see Theorem 1) and NIZKPK.

We now show that if a PPT adversary \( A \) can distinguish between experiments \( E_{b=0} \) and \( E_{b=1} \) with non-negligible probability, then a PPT adversary \( B \) can break the IND-CPA security of encryption scheme \( \Pi_{\text{PKE}} \). We show the construction of \( B \) in Figure 6.

**Claim 1:** If \( C_{\text{IND-CPA}} \) selects \( b = 0 \), then \( A \)‘s view (while interacting with \( B \)) is identical to its view in experiment \( E_{b=0} \). \( A \) obtains from \( B \),

\(- \text{va}_{\text{id}_r} = (\text{id}_r, e_{\text{id}_r}, e_{\text{id}_r}, s_{\text{id}_r}) \) and \( \text{pub}_{\text{id}_r} = (C_{\text{id}_r}, \text{pr}_{\text{id}_r}) \) from the OReg(id_r) oracle call, \( r \in \{0, 1\} \), and
$C_{\text{IND-CPA}}(\lambda, b \in \{0, 1\})$ $B(\lambda, \text{id}_0, \text{id}_1)$ $\text{A}$

$pk_P, sk_P \leftarrow \text{Keygen}(\lambda)$

$pk_T, sk_T \leftarrow \text{Keygen}(\lambda)$

$pk_R, sk_R \leftarrow \text{Keygen}(\lambda)$

$\leftarrow pk_T, pk_P, pk_R$

for $\tau \in \{0, 1\}$:

$t_\tau \xleftarrow{\$} \mathbb{Z}_q$

$M_0 := (t_0, t_1)$

$M_1 := (t_1, t_0)$

$E_{b_0} \leftarrow \text{Enc}(pk_P, M_{b_0}[0])$

$E_{b_1} \leftarrow \text{Enc}(pk_P, M_{b_1}[1])$

$\xrightarrow{E_{b_0}, E_{b_1}}$

$et_0 := E_{b_0}; et_1 := E_{b_1}$

initialise $BB_0$

$\leftarrow \text{OReg}(\text{id}_\tau)$, $\text{OCast}(\text{id}, v, \text{SRegChal}(\text{id}))$

$t_{\text{id}_\tau} \leftarrow t_\tau$

$r_{\text{id}_\tau} \xleftarrow{\$} \mathbb{Z}_q$

$C_{\text{id}_\tau} \leftarrow g_{\text{id}_\tau}^{t_{\text{id}_\tau}}$

$\leftarrow et_{\text{id}_\tau}$

$\leftarrow \text{Enc}(pk_T, r_{\text{id}_\tau})$

$s_{\text{id}_\tau} \leftarrow \text{Sign}(sk_R, \text{id}, et_{\text{id}_\tau}, et_{\text{id}_\tau})$

$pr_{\text{id}_\tau} \leftarrow \text{SNIZKP}(C_{\text{id}_\tau})$

$va_{\text{id}_\tau} := (\text{id}, et_{\text{id}_\tau}, er_{\text{id}_\tau}, s_{\text{id}_\tau})$

$pub_{\text{id}_\tau} := (C_{\text{id}_\tau}, pr_{\text{id}_\tau})$

append $(\text{id}, pub_{\text{id}_\tau})$ to $BB_0$

$\leftarrow \text{OReg}(\text{id}_\tau)$, $\text{OCast}(\text{id}, v, \text{SRegChal}(\text{id}))$

$ev^*_v := \Pi_{\text{E2E-V.Cast}}(v^*)$

$\leftarrow \text{OReg}(\text{id}_\tau)$, $\text{OCast}(\text{id}, v, \text{SRegChal}(\text{id}))$

$BB'_1 = \begin{bmatrix}
\vdots \\
(t_0, ev^*_v) \\
\vdots 
\end{bmatrix}$

$\leftarrow \text{Audit}(BB_0, BB'_1, S_{\text{ZKP-Estb}}(\lambda), A(1^\lambda))$

$\downarrow$

Fig. 6: Adversary $B$ breaking the IND-CPA security of $\Pi_{\text{PKE}}$, given an adversary $A$ that distinguishes between experiments $E_{b=0}$ and $E_{b=1}$. $C_{\text{IND-CPA}}$ denotes the challenger of the IND-CPA game for $\Pi_{\text{PKE}}$. 

13
- $\langle t_0, ev_{b=0}^* \rangle$ when $BB_{b=0}^{\langle t_0 \rangle}$ is published, where $ev_{b=0}^* := II_{E2E-V}.Cast(v^*)$ for the value $v^*$ supplied by $A$ to the OCastChal oracle.

For $\tau \in \{0, 1\}$, the distribution of $[id_\tau, er_{id_\tau}, s_{id_\tau}, C_{id_\tau}]$ as part of $B$’s response is identical to that of $E_0$. For $et_{id_0}$, the encrypted token revealed in $va_{id_0}$ of $id_0$, we need to show that it decrypts to the same token which has been revealed in $BB_{b=0}^{\langle t_0 \rangle}$. In our reduction, the token revealed in $BB_{b=0}^{\langle t_0 \rangle}$ is $t_0$. We now see that $et_{id_0}$ also decrypts to $t_0$. Indeed, for $\tau \in \{0, 1\}$, $et_{id_\tau} = et_\tau = E_{b_0} = Enc(pk_\tau, M_{0}[\tau]) = Enc(pk_\tau, t_\tau)$. Therefore, $et_{id_0} = Enc(pk_\tau, t_0)$. Finally, it is easy to check that $B$ simulates $\text{OReg}(id)$ and $\text{OCast}(id, va, v)$ correctly.

Claim 2: If $C_{\text{IND-CPA}}$ selects $b = 1$, then $A$’s view (while interacting with $B$) is identical to its view in experiment $E_{b=1}$. $A$ obtains from $B$,

- $va_{id_\tau} = (id_\tau, et_{id_\tau}, er_{id_\tau}, s_{id_\tau})$ and $pub_{id_\tau} = (C_{id_\tau}, pr_{id_\tau})$ from the $\text{OReg}(id_\tau)$ oracle call, $\tau \in \{0, 1\}$, and
- $\langle t_0, ev_{b=1}^* \rangle$ when $BB_{b=1}^{\langle t_0 \rangle}$ is published, where $ev_{b=1}^* := II_{E2E-V}.Cast(v^*)$ for the value $v^*$ supplied by $A$ to the OCastChal oracle.

For $\tau \in \{0, 1\}$, the distribution of $[id_\tau, er_{id_\tau}, s_{id_\tau}]$ as part of $B$’s response is identical to that of $E_{b=1}$. In our reduction, the token revealed in $BB_{b=1}^{\langle t_0 \rangle}$ is also $t_0$. But we see that $et_{id_\tau}$ also decrypts to $t_0$. Indeed, for $\tau \in \{0, 1\}$, $et_{id_\tau} = et_\tau = E_{b_0} = Enc(pk_\tau, M_{0}[\tau]) = Enc(pk_\tau, t_{1-\tau})$. Therefore, $et_{id_\tau} = Enc(pk_\tau, t_{1-\tau}) = Enc(t_0)$. Moreover, for $\tau \in \{0, 1\}$, the distribution of $[et_{id_\tau}, C_{id_\tau}]$ as part of $B$’s response is identical to that of $E_{b=1}$. Indeed, since commitments are perfectly hiding, we have $[et_{id_\tau} = Enc(pk_\tau, t_{1-\tau}), C_{id_\tau} = g_1^{\tau h_{\tau}^*}] \approx [et_{id_\tau} = Enc(pk_\tau, t_{1-\tau}), g_1^{1-\tau h_{\tau}^*}]$. Finally, like $E_{b=0}$, $B$ simulates $\text{OReg}(id)$ and $\text{OCast}(id, va, v)$ in $E_{b=1}$ correctly.

Therefore, if $A$ has non-negligible advantage in distinguishing between experiments $E_{b=0}$ and $E_{b=1}$, then $|Pr[b' = 1 | b = 0] − Pr[b' = 1 | b = 1]|$ is non-negligible and thus $B$ breaks the IND-CPA security of $II_{\text{PKE}}$.

7 Performance

(1) Time to generate $n$ pr NIZKs (teller) 14 s  
(2) Time to verify $n$ pr NIZKs (auditor) 56 s  
(3) Time to generate $n$ BBS+ quasi-signatures (auditor) 51 s  
(4) Time to verify $n$ BBS+ quasi-signatures1 (teller) 11 s  
(5) Time to generate $n$ $II_{\text{PKE}}$ NIZKs (teller) 1725 s  
(6) Time to verify $n$ $II_{\text{PKE}}$ NIZKs (auditor) 4500 s

1 Optimised using the batch verification techniques suggested in [25].

| Experiment | Time |
|------------|------|
| Auditor [(2)+(3)+(6)] | 4607 s |
| Teller [(1)+(4)+(5)] | 1750 s |
| Size of $n$ BBS+ signatures | 90 MB |
| Size of $n$ pr NIZK proofs | 89 MB |
| Size of $n$ $II_{\text{PKE}}$ NIZK proofs | 572 MB |

Fig. 7: Benchmarks for $n = 10^6$ eligible voters when each voter had cast their vote. Run on a single machine with 100 cores.

We now briefly present some illustrative performance characteristics of our protocol, specifically, of our ZKP of reverse set membership. We implemented our ZKP using the Charm cryptographic library [3] with a PBC library [32] backend and chose the BN254 elliptic curve [9] to instantiate the pairing groups $(\mathcal{G}_1, \mathcal{G}_2)$. We ran our benchmarks on an Intel(R) Xeon(R) Silver 4210R CPU @ 2.40GHz with 46 GB of RAM and 104 cores. Figure 7 shows space/time benchmarks for a dummy election with $n = 10^6$ voters. We model the worst case where each voter had cast their vote so that the ZKP is to be given for $n$ tokens. Note from Figure 4 that each loop iteration is independent of any other iteration, which gives our ZKP an embarrassingly parallel character. We exploit this fact to distribute proof generation and verification among 100 cores. However, the signature-based nature of our ZKPs implies that they have to be given separately for each auditor. As the total verification takes about half an hour for the teller per auditor, it may be feasible to support independent audit only by some major stakeholders (such as civil society groups and the political parties). Nevertheless, against the signatures supplied by these trusted auditors in the preprocessing stage, anyone can verify individual tokens or a statistical sample of tokens rather efficiently (in a few seconds).

8 Conclusion and future work

We have proposed an eligible audit protocol to detect ballot stuffing for polls site voting while also protecting participation privacy from a remote coerced. Below we mention some ways in which our considered threat model could be strengthened in future work:
Making P unable to prove who voted. Although protecting participation privacy from P may be impossible, currently it can also prove participation of a voter to a remote coercer. To solve this problem, we can make use of designated-verifier signatures [33]. Specifically, R’s signature on the voter’s access card can be made a designated-verifier signature verifiable only by P. This allows P to verify that the card is authentic but does not enable it to prove its authenticity to any other party, even if it divulges all its secrets.

Multiple registrars and tellers. To prevent the registrar from leaking a voter’s token, this trust can be distributed to multiple registrars using standard threshold cryptography techniques. For distributing trust to multiple tellers, a distributed version of our ZKP has to be developed.

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We briefly discuss the individually and universally verifiable poll attendance scheme due to Akinyokun and Teague (see Figure 8). The scheme begins by a set of registrars jointly generating an El Gamal public key whose secret key is distributed among themselves. During registration, a designated registrar provides each voter a 1-bit secret $b$ (1 or -1) that they should keep confidential. The secret $b$ is provided in a receipt-free fashion, which means the voters do not hold any provable record of having received a given value for it. $b$ is then encrypted against the joint public key of the registrars and the resulting encryption $eb$ is uploaded to a bulletin board BB, indexed by the voter id. After the election, the polling officer sends to the registrars a list $\text{ID}_{\text{attended}}$ representing the voters who attended polling. For every registered voter on BB, say the $i$th voter, the registrars compute a bit $p_i = b_i$ if the voter attended and $p_i = -b_i$ if they did not attend. Additionally, the registrars compute an encryption $ea_i$ of the attendance bit $a_i$ (1 if attended; -1 if not attended) using information supplied by the polling officer. All voters can check whether their attendance bit has been correctly registered by checking if $p_i$ matches the bit $b$ given to them if they attended and $-b$ if they did not attend. Assuming a statistical number of voters perform this check, the number of voters who attended can be universally verified.

Although voters can check for themselves whether their attendance has been recorded correctly, they cannot prove this fact to anyone else because the bit $b$ is given to them in a receipt-free fashion. This makes dispute resolution hard and weakens the guarantee against ballot stuffing because voters could be falsely claiming ballot stuffing to discredit the election. Nevertheless, giving $b$ in a receipt-free fashion is required to prevent the forced
abstention attack (otherwise a coercer can derive whether the voter attended or not). In our scheme, we avoid this dichotomy by providing universal verification against ballot stuffing instead of providing guarantees to individual voters.

\[
\begin{align*}
\text{Setup}((R_j(1^n))_{j \in \{1, \ldots, k\}}) & : & \text{multiple registrars jointly generate El Gamal public/private keys} \\
g & \leftarrow G & \text{G is a group of prime order } q \\
x_j & \leftarrow Z_q & h_j & \leftarrow g^{x_j} \\
\text{Output } PK_R := h & \leftarrow \Pi_{j \in \{1, \ldots, k\}} h_j, SK_{R_j} := x_j
\end{align*}
\]

\[
\begin{align*}
\text{Register}(R_j(\langle PK_R, id \rangle)) & : & \text{registration of a voter id by the voter’s designated registrar } R_j \\
b & \leftarrow \{1, -1\}; r & \leftarrow Z_q \\
eb & \leftarrow \text{Enc}(PK_R, b) := (g^r, g^b \cdot h^r) \\
\text{Append } (id, eb) & \text{ to bulletin board } BB \text{ and embed in voter’s ID card.} \\
\text{Give } b & \text{ to the voter in a receipt-free fashion.}
\end{align*}
\]

\[
\begin{align*}
\text{PostProcess}(BB, ID_{\text{attended}}, (R_j(\langle SK_{R_j} \rangle))_{j \in \{1, \ldots, k\}}) : \\
\text{for each } i \in [n] : \text{total number of registered voters.} \\
(id_i, eb_i) & := BB_i \\
b_i & := \text{Dec}(eb_i, (R_j(\langle SK_{R_j} \rangle))_{j \in \{1, \ldots, k\}}) \text{ joint decryption of } eb_i \\
\text{if } id_i & \in ID_{\text{attended}} : p_i & \leftarrow b_i \\
\text{else : } p_i & \leftarrow -b_i \\
\text{append } (p_i, ea_i) & \text{ to } BB_i \\
\text{IndVerify}(V(\langle id, b \rangle)) : & \text{individual verification by the voter (who attended as well as who did not attend)} \\
\text{Look up } id & \text{ on } BB \\
\text{If attended, check } p_i & \leftarrow b \\
\text{Otherwise, check } p_i & \leftarrow -b
\end{align*}
\]

Fig. 8: Akinyokun and Teague’s scheme [4].