$\eta^{(\ell)}$ productions in semileptonic B decays

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Abstract

Inspired by the new measurements on $B^- \rightarrow \eta^{(\ell)}\ell\bar{\nu}_\ell$ from the BaBar Collaboration, we examine the constraint on the flavor-singlet mechanism, proposed to understand the large branching ratios for $B \rightarrow \eta'K$ decays. Based on the mechanism, we study the decays of $\bar{B}_{d,s} \rightarrow \eta^{(\ell)}\ell^+\ell^-$ and find that they are sensitive to the flavor-singlet effects. In particular, we show that the decay branching ratios of $\bar{B}_{d,s} \rightarrow \eta'\ell^+\ell^-$ can be as large as $O(10^{-8})$ and $O(10^{-6})$, respectively.

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Until now, the unexpected large branching ratios (BRs) for the decays $B \to \eta'K$ are still mysterious phenomena among the enormous measured exclusive $B$ decays at $B$ factories \cite{1, 2}. One of the most promising mechanisms to understand the anomaly is to introduce a flavor-singlet state, produced by the two-gluon emitted from the light quarks in $\eta^{(0)}$ \cite{3, 4}. In this mechanism, the form factors in the $B \to \eta^{(0)}$ transitions receive leading power corrections. Consequently, the authors in Ref. \cite{5} have studied the implication on the semileptonic decays of $\bar{B} \to P \ell \nu_\ell$ with $P = \eta^{(0)}$ and $\ell = e, \mu$. In particular, they find that the decay BRs of the $\eta'$ modes can be enhanced by one order of magnitude. Recently, the BaBar Collaboration \cite{6} has measured the semileptonic decays with the data as follows:

\begin{align}
\text{BR}(B^+ \to \eta \ell^+ \nu_\ell) &= (0.84 \pm 0.27 \pm 0.21) \times 10^{-4} < 1.4 \times 10^{-4} (90\% \text{ C.L.}) , \\
\text{BR}(B^+ \to \eta' \ell^+ \nu_\ell) &= (0.33 \pm 0.60 \pm 0.30) \times 10^{-4} < 1.3 \times 10^{-4} (90\% \text{ C.L.}) . \tag{1}
\end{align}

Although the measurements in Eq. (1) are only $2.55\sigma$ and $0.95\sigma$ significances, respectively, it is important to examine if they give some constraints on the form factors due to the flavor-singlet state in the decays of $\bar{B} \to \eta^{(0)} \ell \bar{\nu}_\ell$. It should be also interesting to investigate the implication of the measurements in Eq. (1) by concentrating on the flavor-singlet contributions on the flavor changing neutral current (FCNC) decays of $\bar{B}_{d,s} \to \eta^{(0)} \ell^+ \ell^-$. 

We start by writing the effective Hamiltonians for $B^- \to \eta^{(0)} \ell \nu_\ell$ and $\bar{B} \to \eta^{(0)} \ell^+ \ell^-$ at quark level in the SM as

\begin{align}
\mathcal{H}_I &= \frac{G_F V_{ub}}{\sqrt{2}} \bar{u}_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell , \\
\mathcal{H}_{II} &= \frac{G_F \alpha_{em} \lambda_{q'}^f}{\sqrt{2} \pi} \left[ H_{1\mu} L^\mu + H_{2\mu} L^{5\mu} \right] , \tag{2}
\end{align}

respectively, with

\begin{align}
H_{1\mu} &= C_{9}^{\text{eff}} (\mu) \lambda_{q'}^f \gamma^\mu P_L b - \frac{2m_b}{q^2} C_7 (\mu) \lambda_{q'}^f \sigma_{\mu\nu} q'^\nu P_R b , \\
H_{2\mu} &= C_{10} \lambda_{q'}^f \gamma^\mu P_L b , \\
L^\mu &= \bar{\ell} \gamma^\mu \ell , \quad L^{5\mu} = \bar{\ell} \gamma^\mu \gamma_5 \ell , \tag{3}
\end{align}

where $\alpha_{em}$ is the fine structure constant, $\lambda_{q'}^f = V_{tb} V_{tq'}^*$, $m_b$ is the current b-quark mass, $q$ is the momentum transfer, $P_{L(R)} = (1 \mp \gamma_5)/2$ and $C_i$ are the Wilson coefficients (WCs) with their explicit expressions given in Ref. \cite{7}. In particular, $C_{9}^{\text{eff}}$, which contains the

\begin{align}
&
\end{align}
contribution from the on-shell charm-loop, is given by \[7\]

\[
C_{9}^{\text{eff}}(\mu) = C_9(\mu) + (3C_1(\mu) + C_2(\mu)) h(z, s'),
\]

\[
h(z, s') = -\frac{8}{9} \ln \frac{m_b}{\mu} - \frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} x - \frac{2}{9} (2 + x) |1 - x|^{1/2}
\]

\[
\times \left\{ \begin{array}{ll}
\ln \left| \frac{x + 1}{\sqrt{1 - x}} \right| - i \pi, & \text{for } x \equiv 4z^2/s' < 1, \\
2 \arctan \frac{1}{\sqrt{x - 1}}, & \text{for } x \equiv 4z^2/s' > 1, \\
\end{array} \right.
\]

(5)

where \(h(z, s')\) describes the one-loop matrix elements of operators \(O_1 = \bar{s}_\alpha\gamma^\mu P_L b_\beta \bar{c}_\beta \gamma_\mu P_L c_\alpha\) and \(O_2 = \bar{s}_\alpha \gamma^\mu P_L b_\beta \bar{c}_\gamma \gamma_\mu P_L c_\alpha\) with \(z = m_c/m_b\) and \(s' = q^2/m_b^2\). Here, we have ignored the resonant contributions \[8, 9\] as they are irrelevant to our analysis. In Table I, we show the values of dominant WCs at \(\mu = 4.4\) GeV in the next-to-leading-logarithmic (NLL) order. We note that since the value of \(|h(z, s')|\) is less than 2, the influence of the charm-loop is much less than \(C_{9,10}\) which are dominated by the top-quark contributions.

| Table I: WCs at \(\mu = 4.4\) GeV in the NLL order. |
|--------|--------|--------|--------|--------|
| \(C_1\) | \(C_2\) | \(C_7\) | \(C_9\) | \(C_{10}\) |
| -0.226 | 1.096  | -0.305 | 4.344  | -4.599 |

To study the exclusive semileptonic decays, the hadronic QCD effects for the \(\bar{B} \to P\) transitions are parametrized by

\[
\langle P(p_P)|q'_\gamma \gamma^\mu b|\bar{B}(p_B)\rangle = f_+^P(q^2) \left( P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) + f_0^P(q^2) \frac{P \cdot q}{q^2} q_\mu, \]

\[
\langle P(p_P)|q'_\sigma \gamma_\mu q'' b|\bar{B}(p_B)\rangle = \frac{f_T^P(q^2)}{m_B + m_P} \left[ P \cdot q q_\mu - q^2 P_\mu \right],
\]

(6)

with \(P_\mu = (p_B + p_P)_\mu\) and \(q_\mu = (p_B - p_P)_\mu\). Consequently, the transition amplitudes associated with the interactions in Eqs. \[2\] and \[3\] can be written as

\[
\mathcal{M}_I = \frac{\sqrt{2} G_F V_{ub}}{\pi} f_+^P(q^2) \ell \not{p}_P \ell,
\]

\[
\mathcal{M}_{II} = \frac{G_F \alpha_{em} |q'|}{\sqrt{2} \pi} \left[ \tilde{m}_{g7} \not{p}_P + \tilde{m}_{10} \not{p}_P \gamma_5 \ell \right],
\]

(7)

(8)

for \(\bar{B} \to P \ell \bar{\nu}_\ell\) and \(\bar{B} \to P \ell^+ \ell^-\), respectively, with

\[
\tilde{m}_{g7} = C_{9}^{\text{eff}} f_+^P(q^2) + \frac{2m_b}{m_B + m_P} C_7 f_T^P(q^2), \quad \tilde{m}_{10} = C_{10} f_T^P(q^2).
\]

(9)
Since we concentrate on the productions of the light leptons, we have neglected the terms explicitly related to $m_\ell$. By choosing the coordinates for various particles:

\[
q = (\sqrt{q^2}, 0, 0, 0), \quad p_B = (E_B, 0, 0, |\vec{p}_{P}|),
\]
\[
p_P = (E_P, 0, 0, |\vec{p}_{P}|), \quad p_\ell = E_\ell(1, \sin \theta_\ell, 0, \cos \theta_\ell),
\]
where $E_P = (m_B^2 - q^2 - m_\ell^2)/(2\sqrt{q^2})$, $|\vec{p}_{P}| = \sqrt{E_P^2 - m_\ell^2}$ and $\theta_\ell$ is the polar angle, the differential decay rates for $B^- \rightarrow P\ell\bar{\nu}_\ell$ and $B_d \rightarrow P\ell^+\ell^-$ as functions of $q^2$ are given by

\[
\frac{d\Gamma_I}{dq^2} = \frac{2G_F^2|V_{ub}|^2m_B^3}{3\cdot2^6\pi^3}\sqrt{(1 - s + m_{\ell}^2)^2 - 4m_P^2}\left(f_+^P(q^2)\hat{P}_P\right)^2,
\]
\[
\frac{d\Gamma_{II}}{dq^2} = \frac{2G_F^2\alpha_{em}m_B^3}{3\cdot2^9\pi^5}|\lambda_1'|^2\sqrt{(1 - s + m_{\ell}^2)^2 - 4m_P^2}\hat{P}_P^2\left(|\bar{m}_{q_1}|^2 + |\bar{m}_{10}|^2\right),
\]
respectively, with $\hat{P}_P = 2\sqrt{s}|\vec{p}_{P}|/m_B = \sqrt{(1 - s - m_{\ell}^2)^2 - 4s\bar{m}_P^2}$, $\hat{P}_P = m_P/m_B$ and $s = q^2/m_B^2$.

To discuss the $P = \eta^{(T)}$ modes, we employ the quark-flavor scheme to describe the states $\eta$ and $\eta'$, expressed by $[10, 11]$

\[
\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}
\]

with $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$, $\eta_s = ss$ and $\phi = 39.3^\circ \pm 1.0^\circ$. Based on this scheme, it is found that the form factors in Eq. (9) at $q^2 = 0$ with the flavor-singlet contributions are given by $[4]$

\[
f_i^\eta(0) = \frac{\cos \phi}{\sqrt{2}} f_\pi^T(0) + \frac{1}{\sqrt{3}} \left(\sqrt{2}\cos \phi f_\pi^T - \sin \phi f_\pi s \right) f_i^{\text{sing}}(0),
\]
\[
f_i^{\eta'}(0) = \frac{\sin \phi}{\sqrt{2}} f_\pi^T(0) + \frac{1}{\sqrt{3}} \left(\sqrt{2}\sin \phi f_\pi^T + \cos \phi f_\pi s \right) f_i^{\text{sing}}(0),
\]
where $i = +, T$, $f_q = (1.07 \pm 0.02)f_\pi$, $f_s = (1.34 \pm 0.06)f_\pi$ $[11]$ and $f_i^{\text{sing}}(0)$ denote the unknown transition form factors in the flavor-singlet mechanism. We note that the flavor-singlet contributions to $B \rightarrow \eta^{(T)}$ have also been considered in the soft collinear effective theory $[12]$. For the $q^2$-dependence form factors $f_i^{\pi_{+T}}(q^2)$, we quote the results calculated by the light-cone sum rules (LCSR) $[13]$, given by

\[
f_i^{\pi_{+T}}(q^2) = \frac{f_i^{\pi_{+T}}(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha_{+T}(q^2/m_{B^*}^2))}.
\]
with $f^\tau_+(T)(0) = 0.27$, $\alpha_+(T) = 0.52(0.84)$ and $m_{B^*} = 5.32$ GeV. Since $f^\text{sing}_{+,T}(q^2)$ are unknown, as usual, we parametrize them to be

$$f^\text{sing}_{+,T}(q^2) = \frac{f^\text{sing}_{+,T}(0)}{(1 - q^2/m_{B^*}^2)(1 - \beta_{+,T} q^2/m_{B^*}^2)}$$

(16)

with $\beta_{+,T}$ being the free parameters. We will demonstrate that the BRs for the semileptonic decays are not sensitive to the values of $\beta_{+,T}$, but those of $f^\text{sing}_{+,T}(0)$. Moreover, based on the result of $f^\tau_+(0) \sim f_T^\tau(0)$ in the large energy effective theory (LEET) [14], we may relate the singlet form factors of $f^\text{sing}_{+,T}(0)$ and $f^\text{sing}_T(0)$. Explicitly, we assume that $f^\text{sing}_T(0) \sim f^\text{sing}_{+,T}(0)$. Note that this assumption will not make a large deviation from the real case since the effects of $f^\tau_T(0)$ on the dilepton decays are small due to $C_9 >> C_7$ in Eq. [14]. Hence, the value of $f^\text{sing}_{+,T}(0)$ could be constrained by the decays $\bar{B} \to \eta(0)\ell\bar{\nu}_\ell$.

Before studying the effects of $f^\text{sing}_{+,T}(q^2)$ on the BRs of semileptonic decays, we examine the $\beta_+$ dependence on the BRs. By taking $|V_{ub}| = 3.67 \times 10^{-3}$ and Eqs. (11) and (16), in Table II we present $BR(B^- \to \eta(0)\ell\bar{\nu}_\ell)$ and $BR(B_d \to \eta(0)\ell^+\ell^-)$ with $f^\text{sing}_{+,T}(0) = 0.2$ and various values of $\beta_+$. From the table, we see clearly that the errors of BRs induced by the errors of 60% in $\beta_+$ are less than 7% and 14% for the $\eta$ and $\eta'$ modes, respectively. Hence, it is a good approximation to take the $q^2$ dependence for $f^\text{sing}_{+,T}(q^2)$ to be the same as $f^\text{sing}_T(q^2)$. Consequently, the essential effect on the BRs for semileptonic decays is the value of $f^\text{sing}_{+,T}(0)$.

With $|V_{td}| = 8.1 \times 10^{-3}$ [15] and Eqs. (11) and (12), the decay BRs of $B^- \to \eta(0)\ell\bar{\nu}_\ell$ and $\bar{B}_d \to \eta(0)\ell^+\ell^-$ are shown in Fig. 1. In Table III we also explicitly display the BRs with $f^\text{sing}_{+,T}(0) = 0$, 0.1 and 0.2. From Table III we find that without the flavor-singlet effects, the result for $BR(B^- \to \eta(0)\ell\bar{\nu}_\ell)$ is a factor of 2 smaller than the central
value of the BaBar data in Eq. (1). Clearly, if the data shows a correct tendency, it indicates that there exist some mechanisms, such as the one with the flavor-singlet state, to enhance the decay of \( \bar{B} \to \eta \ell \bar{\nu}_\ell \) as illustrated in Table \( \text{III} \). Moreover, as shown in Fig. 1b and Table \( \text{III} \), the decays of \( B^- \to \eta' \ell \nu_\ell \) are very sensitive to \( f_{+}^{\text{sing}}(0) \). In particular, the current data has constrained that

\[
f_{+}^{\text{sing}}(0) \leq 0.2. \tag{17}
\]

It is interesting to note that for \( f_{+}^{\text{sing}}(0) = 0.2 \), \( BR(\bar{B}_d \to \eta' \ell^+ \ell^-) = 0.21 \times 10^{-7} \), which is as
large as $BR(B^- \to \pi^- \ell^+ \ell^-)$, while that of $B_d \to \eta \ell^+ \ell^-$ is slightly enhanced. In addition, it is easy to see that the flavor-singlet contributions could result in the BRs of the $\eta'$ modes to be over than those of the $\eta$ ones.

Our investigation of the flavor-singlet effects can be extended to the dileptonic decays of $\bar{B}_s \to \eta^{(')} \ell^+ \ell^-$. In the following, we use the notation with a tilde at the top to represent the form factors associated with $B_s$. Hence, similar to Eq. (14), we express the form factors for $\bar{B}_s \to \eta^{(')}$ with the flavor-singlet effects at $q^2 = 0$ to be

$$
\tilde{f}_+^\eta(0) = -\sin \phi \tilde{f}_+^m(0) + \frac{1}{\sqrt{3}} \left( \sqrt{2} \cos \phi \frac{f_q}{f_K} - \sin \phi \frac{f_s}{f_K} \right) \tilde{f}_+^\text{sing}(0),
$$

$$
\tilde{f}_+^{\eta'}(0) = \cos \phi \tilde{f}_+^{m,\eta'}(0) + \frac{1}{\sqrt{3}} \left( \sqrt{2} \sin \phi \frac{f_q}{f_K} + \cos \phi \frac{f_s}{f_K} \right) \tilde{f}_+^\text{sing}(0).
$$

For the $q^2$-dependence form factors of $f_+^m(0)$, we adopt the results calculated by the constituent quark model (CQM) [17], given by

$$
\tilde{f}_+^m(0) = \frac{\tilde{f}_+^{m,\eta}(0)}{1 - a_+^{\eta}(q^2/m_{B_s}^2 + b_+^{\eta}(q^2/m_{B_s}^2)^2)},
$$

with $\tilde{f}_+^{m}(0) = \tilde{f}_+^{m,\eta'}(0) = \tilde{f}_+^{m}(0) = 0.36$, $\tilde{f}_+^{m,\eta'}(0) = 0.39$, $a_+^{\eta} = a_+^{\eta'} = 0.60$, $b_+^{\eta} = b_+^{\eta'} = 0.20$, $a_T^{\eta} = a_T^{\eta'} = 0.58$ and $b_T^{\eta} = b_T^{\eta'} = 0.18$. By using $m_{B_s} = 5.37$ GeV and $V_{ts} = -0.04$ instead of $m_B$ and $V_{td}$ in Eq. (12), we present the BRs of $\bar{B}_s \to \eta^{(')} \ell^+ \ell^-$ in Table IV. We also display the BRs as functions of $\tilde{f}_+^\text{sing}(0)$ in Fig. 2. As seen from Table IV and Fig. 2, due to the flavor-singlet effects, the BRs of $\bar{B}_s \to \eta' \ell^+ \ell^-$ are enhanced and could be as large as $O(10^{-6})$ with around a factor of 3 enhancement, whereas those of $\bar{B}_s \to \eta \ell^+ \ell^-$ decrease as increasing $\tilde{f}_+^\text{sing}(0)$, which can be tested in future hadron colliders.

In summary, we have studied the effects of the flavor-singlet state on the $\eta^{(')}$ productions in the semileptonic B decays. In terms of the constraints from the current data of $B^- \to \eta^{(')} \ell \nu_\ell$,
FIG. 2: (a) [(b)] BRs (in units of $10^{-7}$) of $\bar{B}_s \rightarrow \eta[\eta']\ell^+\ell^-$ as functions of $f_+^{\text{sing}}(0)$.

we have found that the BRs of $\bar{B}_{d,s} \rightarrow \eta[\eta']\ell^+\ell^-$ could be enhanced to be $O(10^{-8})$ and $O(10^{-6})$, respectively. Finally, we remark that the flavor-singlet effects could result in the BRs of the $\bar{B}_{d,s} \rightarrow \eta'$ modes to be larger than those of $\bar{B} \rightarrow \eta$ and the statement is reversed if the effects are neglected.

Note added: After we presented the paper, Charng, Kurimoto and Li [18] calculated the flavor singlet contribution to the $B \rightarrow \eta[\eta']$ transition form factors from the gluonic content of $\eta[\eta']$ in the large recoil region by using the perturbative QCD (PQCD) approach. Here, we make some comparisons as follows:

1. While Ref. [18] gives a theoretical calculation on the flavor singlet contribution to the form factors in the PQCD, we consider the direct constraint from the experimental data. The conclusion that the singlet contribution is negligible (large) in the $B \rightarrow \eta[\eta']$ form factors in Ref. [18] is the same as ours. However, the overall ratios in Ref. [18] for the singlet contributions are about a factor 4 smaller than ours. On the other hand, as stressed in Ref. [18], they have used a small Gegenbauer coefficient, which corresponds smaller gluonic contributions. For a larger allowed value of the the Gegenbauer coefficient in Ref. [18], the overall ratios will be a few factors larger. In other words, the real gluonic contributions rely on future experimental measurements.

2. Although our assumption of $f_T^{\text{sing}}(0) \sim f_+^{\text{sing}}(0)$ seems to be somewhat different from the PQCD calculation as pointed out in Ref. [18] due to an additional term, the numerical values of $f_+^{\text{sing}}(0) = 0.042$ and $f_T^{\text{sing}}(0) = 0.035$ by the PQCD [18] do not change our assumption very much. After all, as stated in point 1 that there exist large uncertainties
for the wave functions in the PQCD. In addition, the difference actually is not important for our results as explained in the text.

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