ANALYSIS OF THE REACTION \( np \rightarrow n p \pi^+ \pi^- \) FROM THE POINT OF VIEW OF OPER-MODEL

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Abstract

The reaction \( np \rightarrow n p \pi^+ \pi^- \) was studied at the various momenta of incident neutrons. It was shown that the characteristics of the reaction at the momenta above 3 GeV/c could be described by the model of reggeized \( \pi \) exchange (OPER). At the momenta below 3 GeV/c, it was necessary to use additionally the mechanism of one baryon exchange (OBE).

1 Introduction: study of inelastic np interactions at accelerator facility of LHEP JINR

The data about inelastic np interactions were obtained due to irradiation of 1m hydrogen bubble chamber (4\( \pi \) geometry) by quasimonochromatic neutron beam (\( \delta P < 2.5\% \)) at the following incident momenta:

\[ P_0 = 1.25, 1.43, 1.73, 2.23, 3.10, 3.83, 4.10 \text{ and } 5.20 \text{ GeV/c} \]

The unique of fullness and precision data are obtained [1]. It permits to carry out the detailed study of inelastic np interactions in a wide region of energies.

![Cross-sections of some inelastic np interactions](image)

Figure 1: Cross-sections of some inelastic np interactions (black squares - our data)
2 The reaction $np \rightarrow np\pi^+\pi^-$ at $P_0 > 3$ GeV/c

This reaction is characterized by:

- plentiful production of the $\Delta$-resonance (see Fig.2),
- large peripherality of the secondary nucleons.

![Figure 2: The distributions of $M_{p\pi^+}$ and $M_{n\pi^-}$ from the reaction $np \rightarrow np\pi^+\pi^-$ at $P_0 = 3$ GeV/c](image)

Various modifications of the one pion exchange models (OPE) are used to describe the experimental data of the inelastic $NN$, $N\bar{N}$, $\pi N$-interactions. At that parameters of these models are different for various processes and even for concrete reactions at various energies. Various models differ also in respect of the reggeization of $\pi$-meson: at times an exchange by elementary $\pi$-meson is used [2] at other times - by reggeized $\pi$-meson [3].

The models of Regge pole exchange[4, 5] are based on the method of complex momenta and consider an exchange in t-channel by a virtual state R that has quantum numbers of particle (resonances) with variable spin and is on some trajectory $\alpha_R(t)$ named Regge trajectory. According to this model, the amplitude of binary and quasi-bynary processes such as $a + b \rightarrow c + d$

![Figure 3: Diagram of the process $a + b \rightarrow c + d$](image)
is written in the following form [3]:

\[
T_R(s, t) = i8\pi s_0 \ g_R^{ac}(t) \ \eta_R(t) \left( \frac{ss_0}{m_c^2 m_d^2} \right)^{\alpha_R(t)} \ g_R^{bd}(t)
\]  

(1)

where \( s_0 \) - energy scale factor,
\( g_R^{ac}(t) \) and \( g_R^{bd}(t) \) - vertex functions,
\( \eta_R(t) \) - signature factor, that is determined in the following form:

\[
\eta_R(t) = -\sigma + \exp(-i\pi \alpha_R(t)) \sin[\pi \alpha_R(t)]
\]

Signature \( \sigma \) is the quantum number characterizing particles (resonances) and correspondingly Regge pole trajectory passing through them. It is determined by a parity of the particle (resonance):

\[
\sigma = (-1)^{l_R} \text{ for integer } l_R \text{ (bosons)},
\sigma = (-1)^{l_R+1/2} \text{ for semi-integer } l_R \text{ (fermions)}.
\]

The most developed and detailed model of reggeized \( \pi \)-meson exchange is the model suggested in ITEP [3]. The advantages of this model are:

- small number of free parameters (3 in our case),
- wide region of the described energies (2 \( \div \) 200 GeV),
- calculated values are automatically normalized to the reaction cross-section.

Within the framework of this model the diagrams of the following form give main contribution into the reaction \( np \rightarrow np\pi^+\pi^- \):

\[
\begin{align*}
\text{Figure 4: Main diagram for the reaction } NN \rightarrow NN\pi\pi
\end{align*}
\]

Lets introduce the following notations:

\[
\begin{align*}
&\ s = (Q_1 + Q_2)^2, \ s_1 = (q_1 + k_1)^2, \ s_2 = (q_2 + k_2)^2, \\
&\ t_1 = (Q_1 - q_1)^2, \ t_2 = (Q_2 - q_2)^2, \ t = (Q_1 - q_1 - k_1)^2 = (Q_2 - q_2 - k_2)^2,
\end{align*}
\]

In that case the matrix element for this diagrams is written in the following form:

\[
M_{22} = \frac{T_{\pi N \rightarrow \pi N}^{up}(s, t, s_1, s_2...)}{t - m^2_c} \cdot F_{22}(s, t, s_1, s_2...) \ T_{\pi N \rightarrow \pi N}^{down}(t - m^2_d)
\]  

(2)
\[ T_{\pi N \to \pi N} - \text{amplitudes of elastic } \pi N\text{-scattering off mass shell} \]

\[ F_{22}(s, t, s_1, s_2...) - \text{form-factor:} \]

\[ F_{22} = e^{R_2^2(t-m_{\pi}^2)} \left\{ \frac{s \, \kappa_1^2 \, \kappa_2^2}{s_0 \, s_1 \, s_2} \right\}^{\alpha_\pi(t)}, \]

where \( \kappa_i^2 = k_{i\perp}^2 + m_{\pi}^2 - c(t - m_{\pi}^2) \),

Usually energy scale factor is determined as \( s_0 = 1 \text{ GeV}^2 \) and Regge trajectory of \( \pi \)-meson as linear one \( \alpha_\pi(t) = \alpha'_\pi(t - m_{\pi}^2) \). In that way the used version of OPER-model has 3 free parameters: \( \alpha'_\pi, R_2^2, c \).

The following main diagrams correspond to the reaction \( np \to np\pi^+\pi^- \) within the framework of OPER model:

![Diagrams](image)

Figure 5: OPER diagrams 2×2 for the reaction \( np \to np\pi^+\pi^- \)

The study has shown that the contribution of the "hanged" diagrams (d) and (e) into the reaction cross-sections at \( P_0 < 10 \text{ GeV}/c \) is negligible. Interference between diagrams (a), (b) and (c) is small and do not exceed some \% at \( P_0 < 10 \text{ GeV}/c \).

The use of some specific kinematic cuts similar to used in [7] permits to select the kinematic region of the reaction \( np \to np\pi^+\pi^- \) in which the contribution of the diagram (a) is dominating. This approach permits to determine the parameters of the model more precisely.

The slope of the \( \pi \)-meson trajectory was taken equal \( \alpha'_\pi = 0.7 \) (as in [6]). However some modifications of the model were made to describe the experimental characteristics of the reaction \( np \to np\pi^+\pi^- \). In particular it was determined that it is better to replace parameter \( c = 0.08 \) in expression for \( \kappa_2^2 \) by \( c = \frac{2m_\pi}{\sqrt{s - 2m_N}} \). Moreover the amplitude of elastic \( \pi N \) scattering off mass shell should be written in the form...
The value of the amplitude of the elastic $\pi N$-scattering on mass shell is calculated using the data of partial wave analysis (PWA)\cite{9}. Parameter $R_2^2$ was taken equal $3.3 \text{ GeV}^{-2}$.

The results of the calculations using OPER-model with such set of the parameters are shown in Fig.6 for the selected kinematic region of the reaction at $P_0 = 5.2 \text{ GeV/c}$.

Figure 6: Distributions for the reaction $np \rightarrow np\pi^+\pi^-$ at $P_0 = 5.20 \text{ GeV/c}$ obtained due to specific cuts.

One can see a good agreement between the experimental distributions and theoretical calculations.

However it is insufficient the diagrams (5a), (5b) and (5c) to describe the reaction $np \rightarrow np\pi^+\pi^-$ in full kinematic region. It is necessary to take into account the diagrams that include the vertex of inelastic $\pi N \rightarrow \pi\pi N$-scattering shown in Fig.7.

The matrix element of these diagrams is written in the form similar to the diagrams of the reaction $NN \rightarrow NN\pi$ in \cite{6}:

$$M_{13} = G\bar{u}(q_N)\gamma_5 u(Q_N) \frac{F_{13}}{(t - m_\pi^2)} T_{\pi\pi N \rightarrow \pi N}$$

where $T_{\pi\pi N \rightarrow \pi N}$ - amplitudes of inelastic $\pi N$-scattering off mass shell

$$T_{\pi N \rightarrow \pi\pi N}^{off} = \sqrt{\frac{Q(s, t)}{Q(s, t, m_\pi^2)}} T_{\pi\pi N \rightarrow \pi N};$$

$G\bar{u}(q_N)\gamma_5 u(Q_N)$ - $(N\pi N)$-vertex ($G^2/4\pi = 14.6$);

$F_{13} = c R_2^2(t - m_\pi^2) \left[1 + s^2\right]^{\alpha_+(t)}$ - form factor.
The calculation of the amplitudes of $\pi N \to \pi\pi N$ reactions are described in [12]. It is significant to mention a detail in the determination of value $\kappa^2$ in the formfactor $F_{13}$. The value $\kappa^2 = k_{\pi \perp}^2 + m_\pi^2 - c(t - m_\pi^2)$ for the reaction $NN \to NN\pi\pi$. But the parameterization of the reaction $\pi N \to \pi\pi N$ assumes that it is in fact the sum of separate 2-particle channels (see Appendix in [12]):

- $\pi N \to N^*(\Delta^*) \to \Delta\pi$,
- $\pi N \to N^*(\Delta^*) \to N\rho$,
- $\pi N \to N^*(\Delta^*) \to N\epsilon$,
- $\pi N \to N^*(\Delta^*) \to N^{*1440}_\pi$.

Then in accordance with [6] there are 4 formfactors:

- $F_{13\Delta}$ for $\pi N \to \Delta\pi$ with $\kappa^2 = k_{\pi \perp}^2 + m_\pi^2 - c(t - m_\pi^2)$ and $c = \frac{m_\pi}{\sqrt{s - 2m_N}}$;
- $F_{13\rho}$ for $\pi N \to N\rho$ with $\kappa^2 = k_{\rho \perp}^2 + m_\rho^2 - c(t - m_\rho^2)$ and $c = \frac{m_\rho}{\sqrt{s - 2m_N}}$;
- $F_{13\epsilon}$ for $\pi N \to N\epsilon$ with $\kappa^2 = k_{\epsilon \perp}^2 + m_\epsilon^2 - c(t - m_\epsilon^2)$ and $c = \frac{m_\epsilon}{\sqrt{s - 2m_N}}$;
- $F_{13N^*}$ for $\pi N \to N^{*1440}_\pi$ with $\kappa^2 = k_{\pi \perp}^2 + m_\pi^2 - c(t - m_\pi^2)$ and $c = \frac{m_\pi}{\sqrt{s - 2m_N}}$.

This choice of the formfactor provide an explanation for the absence of the clear signal of the $\rho$-meson production in the effective masses of $\pi\pi$-combinations from $NN \to NN\pi\pi$ reactions. The channels of the production both $\rho$-meson and $\epsilon$-meson are suppressed in comparison with the channels of the $\Delta$ and $N^*$ production due to a considerably larger values of $\kappa^2$ in formfactor $F_{13}$.

It was shown in [10] that the processes of diffractive production of $N^{*1440}_{1440}$- and $N^{*1680}_{1680}$-resonances make also sizeable contribution into the reaction $n p \to p p \pi^\pm \pi^\mp$. Therefore it is necessary to take into account such processes for the reaction $np \to np\pi^+\pi^-$ that are described by the diagrams similar to diagrams in Fig.7 with the replacement of $\pi$-meson exchange by the exchange of vacuum pole (pomeron). The matrix element for the diagrams of pomeron exchange.
exchange is written in the following form:

\[ T_{N^*}(s, t) = i8\pi s_0 \, g^P_N(t) \, F_{13} \, T_{\pi N \rightarrow N^* \rightarrow \pi\pi N} \]  \hspace{1cm} (4)

where \( g^P_N(t) = g^P_N(0) \, e^{-R^2_N|t|} \) - vertex function,

\[ \alpha_P(t) = \alpha_P(0) + \alpha'_P \, t \]  

Regge trajectory of pomeron.

The values \( g^P_N(0), R^2_N, \alpha_P(0) \) and \( \alpha'_P \) were taken from [4].

The results of the description of the reaction \( n p \rightarrow p p \pi^- \) by diagrams (5), (7) and pomeron exchange at \( P_0 = 5.2 \) GeV/c are presented in Fig. 8:

![Graphs showing distributions for the reaction np → npπ⁺π⁻ at P₀ = 5.20 GeV/c](image)

Figure 8: Distributions for the reaction \( np \rightarrow np\pi^+\pi^- \) at \( P_0 = 5.20 \) GeV/c

One can see a good agreement between the experimental distributions and theoretical calculations.
3 The reaction \( np \rightarrow np\pi^+\pi^- \) at \( P_0 < 3 \text{ GeV/c} \)

The study of effective mass spectra of \( np \)-combinations at \( P_0 = 1.73 \) and \( 2.23 \text{ GeV/c} \) (Fig.9) shows the clear peak close the threshold (\( M_{np} = m_n + m_p \)) that can not be described within the framework of OPER-model using the diagrams from Fig.5 and Fig.7.

The model of Regge poles with baryon exchange and nonlinear trajectories, suggested in [13] was used to describe these features. The following diagrams of one baryon exchange (OBE) were taken into account within the framework of this model:

![OBE Diagrams](image)

Figure 10: OBE diagrams for the reaction \( np \rightarrow np\pi^+\pi^- \)

The vertex function of elastic \( np \rightarrow np \) scattering was calculated using the data from [14].
distribution from the reaction $np \rightarrow np\pi^+\pi^-$ at $P_0 = 1.73$ and 2.23 GeV/c (Fig.9 and Fig.11).

Figure 11: Distributions for the reaction $np \rightarrow np\pi^+\pi^-$ at $P_0=1.73$ GeV/c
4 OPER model and other reactions

The other reactions of np interactions are scheduled to study by means of OPER model:

\[ np \rightarrow pp\pi^- \quad \text{vertex functions } 1 \times 2 \]
\[ np \rightarrow pp\pi^0 \quad \text{vertex functions } 2 \times 2 \text{ and } 1 \times 3 \]
\[ np \rightarrow pp\pi^+\pi^- \quad \text{vertex functions } 2 \times 3 \]
\[ np \rightarrow pp\pi^+\pi^-\pi^0 \quad \text{vertex functions } 3 \times 3 \]
\[ np \rightarrow np\pi^+\pi^- \quad \text{vertex functions } 3 \times 3 \]

Similar reactions of \(pp, \bar{p}p\) and \(\pi N\) interactions also can be described by OPER model. The following reactions were simulated for HADES experiment [16]:

\[ pp \rightarrow pp\pi^+\pi^- \text{ at } T_{kin}=3.5 \text{ GeV} \]
\[ np \rightarrow np\pi^+\pi^- \text{ at } T_{kin}=1.25 \text{ GeV} \text{ (see Fig[12]} \]
\[ np \rightarrow np\pi^+\pi^-e^- \text{ at } T_{kin}=1.25 \text{ GeV with vertex function of } \gamma N \rightarrow Ne + e^- \]

Since the \(\pi N \rightarrow \pi N\) and \(\pi N \rightarrow \pi\pi N\) vertex functions are taken in helicity representation it seems to be perspective to use OPER model for description of the reaction with polarized particles.

Figure 12: Disdributions for the reaction \(np \rightarrow np\pi^+\pi^- \text{ at } T_{kin}=1.25 \text{ GeV}, \) calculated for HADES set-up. Dashed area - results of HADES acceptance.
5 Conclusion

Reaction $np \rightarrow np\pi^+\pi^-$ is characterized by the plentiful production of the $\Delta$ resonance and the large peripherality of the secondary particles. The experimental data are successfully described by the further development of OPER model.

However at $P_0 < 3$ GeV/c it is necessary to take into account another mechanism of the reaction (such as OBE).

OPER model permits to describe another $N(\bar{N}) - N$ reactions with the production of some $\pi$-mesons. The further development of OPER-model can be very promising to describe the production of $e^+e^-$-pairs in hadronic interactions.

OPER model can be used as an effective tool to simulate various reactions of hadronic interactions.
References

[1] C. Besliu et al. YaF, 43:888-892, 1986.

[2] G. Wolf. PR182, 1969, p.1538.

[3] E.L. Berger. PRL21, 1968, p.701.

[4] Yu.P.Nikitin and I.L.Rozental. Nuclear Physics of High Energies. Atomizdat, Moscow, 1980. (in russian)

[5] P.D.B Collins P.D.B. An Introduction to Regge Theory and High Energy Physics. Cambridge University Press, 1977.

[6] L.Ponomarev. Part. and Nucl., v.7(1), pp. 186-248, 1976, JINR, Dubna (in russian).

[7] G.W. van Apeldoorn et al. NP B156, 1979, p.111.

[8] A.P.Jerusalimov et al. JINR Rapid Comm., v.35(2) pp.21-26, 1989, JINR, Dubna (in russian).

[9] R.A.Arndt et al. IJMP A18(3), 2003, p.449.

[10] K.G.Boreskov et al. Yad.Fiz.15:557-565,1972. (in russian)

[11] D.J.Herndon and P.Söding. PR D11, 3165 (1975); D.J.Herndon et al. PR D11, 3183 (1975).

[12] A.P.Jerusalimov et al. Study of the Reaction $np \rightarrow np\pi^+\pi^-$ at Intermediate Energies. [http://arxiv.org/pdf/1102.1574.pdf](http://arxiv.org/pdf/1102.1574.pdf)

[13] A.B. Kaydalov and A.F. Nilov. YaF, v.41(3), pp. 768-776, 1985 ; YaF, v.52(6), pp. 1683-1696, 1990.

[14] NN and ND interactions - a compilation. UCRL-20000 NN, August 1970.

[15] V.Barashenkov and B.Kostenko. JINR Comm. 4-84-761, 1984, JINR, Dubna.

[16] P.Salabura et al. HADES Collaboration. NP A749, 150, 2005.