MSW mediated neutrino decay and the solar neutrino problem

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Abstract

We investigate the solar neutrino problem assuming simultaneous presence of MSW transitions in the sun and neutrino decay on the way from sun to earth. We do a global $\chi^2$-analysis of the data on total rates in Cl, Ga and Superkamiokande (SK) experiments and the SK day-night spectrum data and determine the changes in the allowed region in the $\Delta m^2 - \tan^2 \theta$ plane in presence of decay. We also discuss the implications for unstable neutrinos in the SNO experiment.
1 Introduction

The global analysis of the total rates measured in the Cl, Ga [1] and SK experiments and the day-night spectrum data of SK [2] indicate that the large mixing angle MSW solution gives the best description of the solar neutrino data [3]. However before a particular solution can be established one should rule out other possibilities. In this spirit people have considered various non-standard neutrino properties and their implication for the solar neutrino problem [4]. In this paper we consider a scenario where neutrinos are allowed to decay on their way from sun to earth after undergoing MSW transitions in the sun. Such a possibility was discussed earlier in [5].

We consider two flavor mixing between $\nu_e$ and $\nu_\mu/\nu_\tau$ with the mass eigenstates $\nu_1$ and $\nu_2$. We assume that the heavier mass state $\nu_2$ is unstable, while the lighter neutrino mass state $\nu_1$ has lifetime much greater than the sun-earth transit time and hence can be taken as stable. There are two possible non-radiative decay modes:

- **Model 1**: If neutrinos are Dirac particles one has the decay channel $\nu_2 \rightarrow \bar{\nu}_1 + \phi$, where $\bar{\nu}_1$ is a right handed singlet and $\phi$ is an iso-singlet scalar. Thus all the final state particles for this model are sterile and there is no distinct signature of this decay apart from in disappearance experiments. This model is discussed in [6]. In this model a light scalar boson $\phi$ with lepton number -2 and a singlet right handed neutrino is added to the standard model. The neutrino coupling to this scalar boson is given by $g_{21}^T \nu_R^T C^{-1} \nu_R$, $C$ being the charge conjugation operator.

- **Model 2**: If neutrinos are Majorana particles, the decay mode is $\nu_2 \rightarrow \bar{\nu}_1 + J$, where $J$ is a Majoron, produced as a result of spontaneous breaking of a global $U(1)_{L_e - L_\mu}$ symmetry [3]. In this model the neutrino masses are generated by extending the higgs sector of the standard model. Though the original triplet majoron model proposed by Gelmini and Roncadeli [9] is ruled out from the LEP data on Z decay to invisible modes [10], the model discussed in [4], which needs two additional triplet and one singlet scalar boson in the theory, can avoid the conflict with the LEP data, and at the same time predict fast enough neutrino decay necessary to solve the solar neutrino problem. In this model the $\bar{\nu}_1$ can be observed as a $\bar{\nu}_e$ with a probability $|U_{e1}|^2$ and as a $\bar{\nu}_\mu/\bar{\nu}_\tau$ with a probability $|U_{e2}|^2$.

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4Radiative decays are severely constrained [11].
In both the decay scenarios the rest frame lifetime of $\nu_2$ is given by 

$$\tau_0 = \frac{16\pi m_2 (1 + m_1/m_2)^{-2}}{g^2 \Delta m^2}$$

(1)

where $g$ is the coupling constant, $m_i$ is the $\nu_i$ mass and $\Delta m^2 = m_2^2 - m_1^2$. Assuming $m_2 >> m_1$ the equation (1) can be written as

$$g^2 \Delta m^2 \sim 16\pi \alpha$$

(2)

where $\alpha$ is the decay constant related to $\tau_0$ as $\alpha = m_2/\tau_0$. If we now incorporate the bound $g^2 < 4.5 \times 10^{-5}$ as obtained from K decay modes \cite{12} we get the bound $\Delta m^2 > 10^6 \alpha$. Since for a typical neutrino energy of 10 MeV, one starts getting decay effects over the sun-earth distance for $\alpha \sim 10^{-13}$ eV, the corresponding limit on $\Delta m^2$ is $\Delta m^2 \lesssim 10^{-7}$ eV$^2$. In an earlier paper we had considered high values of $\Delta m^2$ ($> 10^{-3}$ eV$^2$) so that the matter effects inside the sun can be neglected and one can have decay as well as $\Delta m^2$ independent average oscillations \cite{14}. In this paper we consider $10^{-6} \leq \Delta m^2 \leq 10^{-3}$ eV$^2$ such that the matter effects inside the sun are important. We incorporate the earth matter effects as well.

In section 2 we discuss the usual two flavor MSW solutions to the solar neutrino problem. We perform a $\chi^2$-analysis to the global data on rates and SK day-night spectrum and present the allowed regions in the $\Delta m^2 - \tan^2 \theta$ plane. In section 3 we introduce the possibility of having one of the neutrino states, $\nu_2$ to be unstable. We look for the effects of decay on the MSW solutions and present the allowed areas at various values of the decay constant. We show that for the majoron decay model (model 2) the SK data on $\bar{\nu}_e - p$ events restricts the allowed values of the decay constant to extremely low values. In section 4 we discuss the implications of non-zero neutrino decay for the SNO experiment. We finally present our conclusions in section 5.

2 MSW effect for stable neutrinos

As was first indicated in \cite{15}, interaction of electron neutrinos propagating through the sun modifies the vacuum mixing angle $\theta$ to matter mixing angle $\theta_M$ where,

$$\tan 2\theta_M = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2} G_F n_e E}.\quad (3)$$

\footnote{Since the $m_{ee}$ element of the mass matrix is zero in the majoron decay model considered, the $0\nu$-majoron $\beta\beta$ decay does not take place in this model \cite{8} and the more stringent bound on $g$ from majoron emission in $\beta\beta$ decay \cite{13} is not applicable.}
Here \( n_e \) is the ambient electron density, \( E \) the neutrino energy, and \( \Delta m^2 (= m_2^2 - m_1^2) \) the mass squared difference in vacuum. The vanishing of the denominator in eq. (3) defines the resonance condition, where the matter mixing angle is maximal.

The probability amplitude of survival for an electron neutrino is given by,

\[
A_{ee} = A_{e1}^{\odot} A_{1e}^{\odot} + A_{e2}^{\odot} A_{2e}^{\odot}
\]

where \( A_{ek}^{\odot} \) gives the probability amplitude of \( \nu_e \to \nu_k \) transition at the solar surface,

\[
A_{ek}^{\odot} = a_{ek} e^{-i\phi_k}
\]

\[
a_{e1}^{\odot} = P_J \sin^2 \theta_M + (1 - P_J) \cos^2 \theta_M = 1 - a_{e2}^{\odot} \tag{5}
\]

\( P_J \) is the non-adiabatic level jumping probability between the two mass eigenstates for which we use the standard expression from [16].

\[
P_J = \frac{\exp(-\gamma \sin^2 \theta) - \exp(-\gamma)}{1 - \exp(-\gamma)} \tag{7}
\]

\[
\gamma = \frac{\Delta m^2}{E} \left| \frac{d \ln n_e}{dr} \right|^{-1}_{r_r=r_{res}} \tag{8}
\]

\( A_{kk}^{\odot} \) is the transition amplitude from the solar surface to the earth surface,

\[
A_{kk}^{\odot} = e^{-iE_k(L-R_{\odot})} \tag{9}
\]

where \( L \) is the sun-earth distance and \( R_{\odot} \) the radius of the sun. \( A_{ke}^{\odot} \) denotes the \( \nu_k \to \nu_e \) transition amplitudes inside the earth. We evaluate these amplitudes by assuming the earth to consist of two constant density slabs. The \( \nu_e \) survival probability is given by

\[
P_{ee} = |A_{ee}|^2 = a_{e1}^{\odot} |A_{1e}^{\odot}|^2 + a_{e2}^{\odot} |A_{2e}^{\odot}|^2 + 2a_{e1}^{\odot} a_{e2}^{\odot} Re[A_{1e}^{\odot} A_{2e}^{\odot} e^{i(E_2-E_1)(L-R_{\odot})} e^{i(\phi_2-\phi_1,\odot)}] \tag{10}
\]

It can be shown that the square bracketed term containing the phases average out to zero in the range of \( \Delta m^2 \) in which we are interested [17].

Using eq. (10) as the probability we have done a \( \chi^2 \)-analysis of the current solar neutrino data on total rates from Cl, Ga\(^6\)[1] and SK and the 18+18 bins of data on\(^{6}\)We use the weighted average of the rates from Sage, Gallex and GNO.
the day-night electron energy spectrum from SK\cite{15}. The details of the code used can be found in \cite{19}. We use the 1117 days data of SK and incorporate the BBP00 solar model \cite{20}. The theory errors and their correlations in the analysis of total rates is included as in \cite{21}. In addition to the astrophysical uncertainties included in \cite{19} for the analysis of the total rates, we have included the uncertainty in the $S_0$-factor for the reaction $^{16}O(p, \gamma)^{17}F$ \cite{20}. For the day-night spectrum analysis we have included the correlation between the systematic errors of the day and the night bins. As in \cite{22} we vary the normalisation of the spectrum as a free parameter which avoids the overcounting of the rates and spectrum data for SK. Hence for the day-night spectrum analysis we have (36 - 1) degrees of freedom (d.o.f) while for the total rates we have 3, which makes a total of 38 d.o.f for the rates+spectrum analysis with no oscillation. For MSW analysis with two active neutrino flavors we present in Table 1 two sets of results

- with the $^8B$ normalisation factor $X_B$ in the total rates held at the SSM value.
- with the $^8B$ normalisation factor $X_B$ in the total rates kept as a free parameter.

| Nature of Solution | $\Delta m^2$ in eV$^2$ | $\tan^2 \theta$ | $\chi^2_{min}$ | Goodness of fit |
|--------------------|-------------------------|-----------------|----------------|----------------|
| $X_B$ fixed at SSM value | | | | |
| SMA | $5.48 \times 10^{-6}$ | $5.79 \times 10^{-4}$ | 43.22 | 19.01% |
| LMA | $4.17 \times 10^{-5}$ | 0.35 | 37.33 | 40.78% |
| LOW | $1.51 \times 10^{-4}$ | 0.64 | 39.54 | 31.48% |
| $X_B$ varying | | | | |
| SMA | $5.35 \times 10^{-6}$ | $4.34 \times 10^{-4}$ | 37.98 | 33.51% |
| LMA | $4.22 \times 10^{-5}$ | 0.25 | 34.20 | 50.67% |
| LOW | $1.51 \times 10^{-4}$ | 0.64 | 39.88 | 26.20% |

Table 1: The best-fit values of the parameters, $\chi^2_{min}$, and the goodness of fit from the global analysis of rates and the day-night spectrum data for MSW analysis involving two neutrino flavors.

Thus the best-fit comes in the LMA region. In fig. 1 we plot the 90\%, 95\% and 99\% C.L. allowed regions for the two-flavor MSW transition to an active neutrino. All the contours have been drawn with respect to the global minimum with the $X_B$ fixed at the SSM value. We have also done a $\chi^2$-analysis for $\nu_e - \nu_s$ MSW conversion, $\nu_s$ being a sterile neutrino, for both $X_B$ fixed at SSM and $X_B$ varying freely. The best-fit values of parameters, $\chi^2_{min}$ and the goodness of fit (g.o.f) are

- $\Delta m^2 = 3.74 \times 10^{-6}$ eV$^2$, $\tan^2 \theta = 5.2 \times 10^{-4}$, $\chi^2_{min} = 44.85$, g.o.f = 14.79\%, $X_B$ fixed at SSM value.
- $\Delta m^2 = 3.71 \times 10^{-6}$ eV$^2$, $\tan^2 \theta = 4.72 \times 10^{-4}$, $\chi^2_{min} = 43.42$, g.o.f = 15.53\%, $X_B$ free.
3 MSW effect and unstable neutrinos

If a neutrino of energy $E$ decays while traversing a distance $L$ then the decay term \( \exp(-\alpha L/E) \) gives the fraction of neutrinos that survive, where $\alpha$ is the decay constant discussed before. In fig. 2 we plot this \( \exp(-\alpha L/E) \) as a function of $\alpha$ for two different energy values. For $\alpha$ very small the exponential term is $\approx 1$ and there is no decay. As $\alpha$ increases one starts getting decay over the sun-earth distance only for $\alpha \sim 10^{-13}$ eV$^2$. As we have discussed in the introduction, since from K-decay $\Delta m^2 \geq 10^6 \alpha$, one can have simultaneous MSW and decay for $10^{-7} < \Delta m^2 < 10^{-3}$ eV$^2$. Below this $\Delta m^2$ the bound on the coupling constant coming from K-decay restricts the decay constant $\alpha$ to be small enough so that decay effects are negligible over the sun-earth distance. So henceforth we will be concerned with only the LMA and SMA solutions, the LOW region remaining unaffected due to decay. For $\Delta m^2 > 10^{-3}$ eV$^2$ there will not be any MSW effect.

The probability amplitude for $\nu_e$ survival in presence of neutrino decay is again given by eq.(4), with $A_{e\odot}^{\odot}$, $A_{e\odot}^{\oplus}$ and $A_{11}^{\text{vac}}$ as before, the only change being for $A_{22}^{\text{vac}}$ since the $\nu_2$ decays on its way from the sun to earth. $A_{22}^{\text{vac}}$ is given by

\[
A_{22}^{\text{vac}} = e^{-iE_2(L-R_{\odot})}e^{-\alpha(L-R_{\odot})/E_2}
\] (11)

Then, the $\nu_e$ survival probability is given by (ignoring the phase part),

\[
P_{ee} = a_{e1}^{\odot} |A_{e\odot}|^2 + a_{e2}^{\odot} |A_{e\odot}|^2 e^{-2\alpha(L-R_{\odot})/E_2}
\] (12)

The day-time probability (i.e. without the earth effect) is given by

\[
P_{ee}^{\text{day}} = \cos^2 \theta [P_J \sin^2 \theta_M + (1 - P_J) \cos^2 \theta_M]
+ \sin^2 \theta [(1 - P_J) \sin^2 \theta_M + \cos^2 \theta_M P_J] e^{-2\alpha(L-R_{\odot})/E_2}
\] (13)

From eq.(13) we note that the the decay term appears with a $\sin^2 \theta$ and is therefore appreciable only for large enough $\theta$. Thus we expect the effect of decay to be maximum in the LMA region. This can also be understood as follows. The $\nu_e$ are produced mostly as $\nu_2$ in the solar core. In the LMA region the neutrinos move adiabatically through the sun and emerge as $\nu_2$ which eventually decays. For the SMA region on the other hand $P_J$ is non-zero and $\nu_e$ produced as $\nu_2$ cross over to $\nu_1$ at the resonance and come out as a $\nu_1$ from the solar surface. Since $\nu_1$ is stable, decay does not affect this region. Including the earth effect the survival probability can be expressed as

\[
P_{ee} = P_{ee}^{\text{day}} + \frac{(\sin^2 \theta - P_{2e})(P_{ee}^{\text{day}} + e^{-2\alpha(L-R_{\odot})/E_2}(P_{ee}^{\text{day}} - 1))}{\cos^2 \theta - \sin^2 \theta e^{-2\alpha(L-R_{\odot})/E_2}}
\] (14)
$P_{2e}$ is the probability of the second mass eigenstate to get converted into the $\nu_e$ state at the detector.

In fig. 3 the solid lines show the survival probability versus neutrino energy for three different values of $\alpha$ with $\Delta m^2$ and $\tan^2 \theta$ fixed at the best fit value of the LMA solution. The introduction of decay reduces the survival probability, as the second term in eq. (12) reduces with $\alpha$. Decay also brings about a deviation from a flat probability towards the high energy end. The dotted line gives the probability for the same $\Delta m^2$ but a higher value of $\tan^2 \theta$ with $\alpha = 10^{-11}$ eV$^2$. We observe that there is a huge drop in the survival probability initially, followed by a sharp increase with energy. A comparison between the different curves shows that inclusion of decay results in the reduction of the survival probability as well as an energy distortion and both these effects increase with the mixing angle $\theta$.

We next perform a $\chi^2$-analysis of the total rates and the day-night spectrum data for non-zero decay. The best-fit comes for $\alpha = 0$, corresponding to no decay with the $\Delta m^2$ and $\tan^2 \theta$ as given in Table 1. However non-zero values of $\alpha$ giving finite decay probability also gives acceptable fit. For small values of $\theta$ as in the SMA region, the fraction of $\nu_2$ in the solar neutrino beam is very small. As a result very few neutrinos decay and all values of $\alpha$ may be allowed. On the other hand in the LMA region the $\theta$ is high and large number of neutrinos can decay producing a distortion in the energy spectrum. Therefore the data can put some restrictions on the allowed values of $\alpha$. In fig. 4 we plot $\Delta \chi^2 (= \chi^2 - \chi^2_{\text{min}})$ vs $\alpha$ keeping the other two parameters free (in the LMA regime). We also show the 99% C.L. limit for three parameters by the dotted line. From fig. 4 we see that in the LMA region, values of $\alpha$ up to $3.5 \times 10^{-11}$ eV$^2$ are allowed at 99% C.L. from the global analysis.

In fig. 5 we show the allowed regions in the $\Delta m^2 - \tan^2 \theta$ plane for various fixed values of $\alpha$. The first panel is for $\alpha = 0$ which is the case of no decay and the contours are the same as those presented in fig. 1, modulo the difference in the definition of the C.L. as now there are three parameters whereas in fig. 1 we had two. The other three panels are for different non-zero allowed values of $\alpha$ obtained from fig. 4.

As expected, we find that the SMA allowed regions remain mostly unaffected due to a non-zero $\alpha$. The LMA solution on the other hand is appreciably affected and the allowed area shrinks as $\alpha$ increases. This effect is seen to be more pronounced for larger $\tan^2 \theta$. The reason for this can be traced down to the fact that decay results in distortion of the high energy end of the neutrino spectrum and this distortion is more for higher $\tan^2 \theta$, a feature explicit in fig. 3. As a result higher values of $\tan^2 \theta$ are disfavored by the global data as $\alpha$ increases.

It is to be noted that although in the LOW region the $\sin^2 \theta$ is high, the $\Delta m^2$ is small and appreciable decay over the sun-earth distance is not obtained if one has to be consistent with the bounds on the coupling constant from K-decays. Because
of this the LOW region is not plotted in fig. 5.

The results presented in this section are in general valid for both the decay models though in model 2 the $\nu_2$ decays to a $\bar{\nu}_1$, which can interact with the electrons in SK detector as $\bar{\nu}_e$ and $\nu_x$. But since the $\bar{\nu}_1$ is degraded in energy [7], the effect of these additional ($\bar{\nu}_e - e$) and ($\nu_x - e$) scattering events in SK is not significant [14] and the final results remain the same. However additional constraint on model 2 come from the $\bar{\nu}_e p \rightarrow e^+ n$ events, which contribute to the background in SK. From the absence of a significant contribution above the background, the bound obtained on the total flux of $\bar{\nu}_e$ from $^8B$ neutrinos is $\Phi_{\bar{\nu}_e} (^8B) < 1.8 \times 10^5$ cm$^{-2}$ s$^{-1}$ which translates to a bound on the probability $P_{ee} < 3.5\%$ [23]. In fig. 6 we plot the conversion probability $P_{ee}$ vs $\alpha$. For each $\alpha$ we find the $\chi^2_{\text{min}}$ and plot $P_{ee}$ for the corresponding the best fit $\Delta m^2$ and $\tan^2 \theta$. We also show the allowed limit of $P_{ee} = 0.035$. So from this constraint for model 2 only $\alpha \leq 5.8 \times 10^{-13}$ eV$^2$ remain allowed.

4 Implications for SNO

The main detection processes in the heavy water of SNO are

$$\nu_e + d \rightarrow p + p + e^- $$  \hspace{1cm} (15)

$$\nu_x + d \rightarrow p + n + \nu_x$$  \hspace{1cm} (16)

The first is a charged current (CC) reaction with energy threshold of 1.44 MeV and the second one is a neutral current (NC) process with an energy threshold of 2.23 MeV. While the CC is sensitive to only $\nu_e$, the NC process is sensitive to neutrinos and antineutrinos of all flavors. The rate of ($\nu_e - d$) CC events recorded in the detector is given by

$$R_{\text{CC}} = \frac{\int dE_{\nu} \lambda_{\nu_e}(E_{\nu}) \sigma_{\text{CC}}(E_{\nu}) \langle P_{ee} \rangle}{\int dE_{\nu} \lambda_{\nu_e}(E_{\nu}) \sigma_{\text{CC}}(E_{\nu})}$$  \hspace{1cm} (17)

$$\sigma_{\text{CC}} = \int_{E_{\text{th}}}^{E_{\text{A}}} dE_A \int_0^{\infty} dE_T R(E_A, E_T) \int dE_{\nu} \frac{d\sigma_{\nu_e d}(E_T, E_{\nu})}{dE_T}$$  \hspace{1cm} (18)

where $\lambda_{\nu_e}$ is the normalised $^8B$ neutrino spectrum, $\langle P_{ee} \rangle$ is the time averaged $\nu_e$ survival probability, $d\sigma_{\nu_e d}/dE_T$ is the differential cross section of the ($\nu_e - d$) interaction [24], $E_T$ is the true and $E_A$ the apparent(measured) kinetic energy of
the recoil electrons, $E_{A_{th}}$ is the detector threshold energy which we take as 5 MeV and \( R(E_A, E_T) \) is the energy resolution function which is assumed to be Gaussian

\[
R(E_A, E_T) = \frac{1}{\sqrt{2\pi}(0.348\sqrt{\frac{E_T}{\text{MeV}}})} \exp \left( -\frac{(E_T - E_A)^2}{0.242E_T\text{MeV}} \right)
\]  

(19)

The NC event rate is given by

\[
R_{NC} = \frac{\int dE_\nu \lambda_{\nu_e}(E_\nu)\sigma_{NC}(E_\nu)\mathcal{P}}{\int dE_\nu \lambda_{\nu_e}(E_\nu)\sigma_{NC}(E_\nu)}
\]

(20)

where \( \mathcal{P} = \langle P_{ee} \rangle + \langle P_{e\mu} \rangle \) for two flavor \( \nu_e - \nu_\mu \) oscillation. The left hand panels in fig. 7 give the range of predicted values of \( R_{CC} \), \( R_{NC} \) and the double ratio \( R_{NC}/R_{CC} \) at 99% C.L., from the global MSW analysis of the solar neutrino data. The black dots in these figures represent the expected rates in SNO for the local best fit values of the parameters obtained. As the neutral current interaction is flavor blind, the \( R_{NC} \) remains 1 for oscillations to active neutrinos for which \( \mathcal{P} = 1 \). But for oscillations to sterile neutrinos \( \mathcal{P} = \langle P_{ee} \rangle \) and \( R_{NC} = 0.78 \) for the best fit case.

The right hand panels in fig. 7 show the the corresponding rates in SNO for the LMA region if the neutrinos are assumed to be unstable (the SMA solution remains largely unaffected as discussed in the previous section). We consider the decay model 1 and present the 99% C.L. predicted range of values of the rates for three different allowed values of the parameter \( \alpha \), along with the case for \( \alpha = 0 \), which is the same as the LMA region of global MSW analysis, modulo the difference in the definition of the C.L.. Also shown are the rates for the best fit value of \( \Delta m^2 \) and \( \tan^2 \theta \) for various fixed values of \( \alpha \). For each fixed value of \( \Delta m^2 \) and \( \tan^2 \theta \) one expects \( R_{CC} \) to fall sharply with alpha. But since the best fit of the global analysis for increasing \( \alpha \) corresponds in general to a higher \( \Delta m^2 \), that raises the value of \( R_{CC} \). Hence \( R_{CC} \) in SNO alone may not be able to distinguish the decay scenario from MSW conversion to active stable neutrinos. The value of \( R_{NC} \) suffers a marked decrease from 1 as the decay constant \( \alpha \) increases, since for the decay model 1 the \( \nu_e \) decays to sterile particles which cannot be detected in SNO.

From fig. 7 we see that if \( R_{NC} \) is below 1, one can have either MSW conversion to sterile neutrinos, or the possibility of unstable neutrinos. But the ambiguity between the last two cases can be lifted if one next looks at the value of \( R_{CC} \), which is much lower for the case of unstable neutrinos than what one would get for MSW conversion to sterile neutrinos. We also display the expected range of \( R_{NC}/R_{CC} \) for various fixed values of the decay constant. It is clear from the figure that from the value of \( R_{NC}/R_{CC} \) in SNO one can clearly distinguish the case of decay from the case of MSW transition to sterile neutrinos.
In the decay model 2 the $\nu_e$ decay to $\bar{\nu}_1$, which can in principle show up both in the NC events in SNO as well as in the charged current absorption reaction

$$\bar{\nu}_e + d \rightarrow n + n + e^+$$

But since the absence of $(\bar{\nu}_e - p)$ events in SK restrict $\alpha \leq 5.8 \times 10^{-13}$ eV$^2$, both the additional contribution to the NC event as well as the $(\bar{\nu}_e - d)$ absorption is negligible in SNO.

5 Conclusions

We have investigated the effect of neutrino decay on the MSW solution to the solar neutrino problem. There are two factors which control the effect of decay

- The mixing angle $\theta$ which determines the fraction of the unstable component in the $\nu_e$ beam
- The decay constant $\alpha$ which determines the decay rate

In order for the neutrinos to decay over the earth-sun distance $\alpha$ should be $\geq 10^{-13}$ eV$^2$. Then eq.(21) and the bounds on the coupling constant from K decays restricts $\Delta m^2 \gtrsim 10^{-7}$ eV$^2$ and decay does not take place in the LOW region. We therefore probe the SMA and the LMA regions. We find that even the SMA region is not affected much because the mixing angle being small very few neutrinos would decay. But the effect of decay on the LMA region is significant. This is borne out by the $\chi^2$-analysis of the global solar data on rates and SK day-night spectrum. We point out that although the data prefers no decay, it still cannot rule out the possibility of unstable neutrinos completely. We put limits on the allowed range of the decay constant $\alpha$ and present the allowed areas in the $\Delta m^2 - \tan^2 \theta$ parameter space for various allowed values of $\alpha$. The LMA allowed zone is seen to be severely constrained by decay.

From fig. 4, values of $\alpha \leq 3.5 \times 10^{-11}$ eV$^2$ are allowed, implying a neutrino rest frame lifetime $\tau_0 \geq 2 \times 10^{-5}$ sec. Thus one may encounter decay before neutrino decoupling in the early universe. This may result in increasing the number of light neutrinos $N_\nu$ to greater than 3. However, the upper limit on $N_\nu$ can be as large as 6 [25]. Hence our decay model is consistent with the bounds from early universe.

From the fact that the neutrinos from SN1987A have not decayed on their way one gets a lower bound on the electron neutrino lifetime as $\tau > 5.7 \times 10^5 (m_{\nu_e}/eV)$ sec. However if one includes neutrino mixing then shorter lifetimes are allowed provided $|U_{e2}| < 0.9$ [26]. This again is consistent with our analysis.

In this paper we assume one of the neutrino states to be unstable in vacuum and the values of the decay constant $\alpha$ are such that one has decay over earth-sun
distance. For these values of $\alpha$ decay inside the sun or the earth is negligible. However as was shown in [27] matter can induce neutrino decay with majoron emission even when neutrinos are stable in vacuum. It was shown later in [28] that in the presence of only standard interactions, the matter induced decay cannot provide a solution to the solar neutrino problem due to strong bounds on neutrino-majoron couplings. As we have discussed in this paper, decay models where the $\nu_2$ decays to a $\bar{\nu}_1$ and a majoron is independently disfavored from the non-observation of $(\bar{\nu}_e - p)$ events in SK.

We have looked at the implications of unstable neutrinos for the SNO detector. We first present the values of the CC and NC rates which SNO is expected to observe for MSW transitions with stable neutrinos. We have given the values for transitions to both active as well as sterile species. For unstable neutrinos the NC rate is less than 1 and comparable to $R_{NC}$ for standard MSW SMA transition to sterile neutrinos. But even though the value of $R_{NC}$ may be nearly same for the two scenarios, comparing the values of $R_{CC}$ or $R_{NC}/R_{CC}$ it may be possible in principle to distinguish the decay scenario from the MSW transition to sterile neutrinos.

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References

[1] B.T. Cleveland et al., *Nucl. Phys. B (Proc. Suppl.*) **38**, 47 (1995); Y. Fukuda et al., (The Kamiokande collaboration), *Phys. Rev. Lett.* **77**, 1683 (1996); W. Hampel et al., (The Gallex collaboration), *Phys. Lett.* **B388**, 384 (1996); J.N. Abdurashitov et al., (The SAGE collaboration), *Phys. Rev. Lett.* **77**, 4708 (1996); M. Altmann et al., (The GNO collaboration), hep-ex/0006034.

[2] The results presented at the nu2000 meeting at Sudbury, Canada, June 2000, can be found at http://nu2000.sno.laurentian.ca.

[3] M.C. Gonzalez-Garcia, C. Peña-Garay, hep-ph/0009041.

[4] O.G. Miranda, C. Peña-Garay, T.I. Rashba, V.B. Semikoz, J.W.F. Valle, hep-ph/0005259; D. Majumdar, A. Raychaudhuri and A. Sil, hep-ph/0009339; A.M. Gago, H. Nunokawa and R. Zukanovich Funchal, hep-ph/0012168; M.M. Guzzo, H. Nunokawa, P.C. de Holanda, O.L.G. Peres, hep-ph/0012089.

[5] R.S. Ragahavan, X. He and S. Pakvasa, Phys. Rev. **D38**, 1317 (1988).
[6] M. Fukugita, Phys. Rev. D36 (1987) 3817.
[7] A. Acker, S. Pakvasa and J. Pantaleone, Phys. Rev. D 43, (1991) R1754, D45 (1992) R1.
[8] A. Acker, A. Joshipura and S. Pakvasa, Phys. Lett. B285 (1992) 371.
[9] G.B. Gelmini and M. Roncadelli, Phys. Lett. B99, 411 (1981).
[10] LEPEWWG report, http://www.cern.ch.
[11] A. Acker and S. Pakvasa, Phys. Lett. B320 (1994) 320.
[12] V. Barger, W.Y. Keung and S. Pakvasa, Phys. Rev. D25, (1982) 907.
[13] D.O. Caldwell et al., Phys. Rev. Lett. 59, 419 (1987); P. Fisher et al., Phys. Lett. B192, 460 (1987).
[14] S. Choubey, S. Goswami, D. Majumdar, Phys. Lett. B484, 73 (2000).
[15] L. Wolfenstein, Phys. Rev. D34, 969 (1986); S.P. Mikheyev and A.Yu. Smirnov, Sov. J. Nucl. Phys. 42(6), 913 (1985); Nuovo Cimento 9c, 17 (1986).
[16] S.T. Petcov, Phys. Lett. B200, 373 (1988).
[17] A. S. Dighe, Q.Y. Liu and A.Yu. Smirnov, hep-ph/9903329 and references therein.
[18] Y. Suzuki of SK collaboration, private communication.
[19] S. Goswami, D. Majumdar, A. Raychaudhuri, Phys. Rev. D63, 013003, 2001.
[20] J.N. Bahcall, S. Basu, M. Pinsonneault, astro-ph/0010346.
[21] G.L. Fogli and E. Lisi, Astropart. Phys. 3, 185 (1995).
[22] M.C. Gonzalez-Garcia, P.C. de Holanda, C. Peña-Garay, and J.W.F. Valle, hep-ph/9906469, Nucl. Phys. B573, 3 (2000).
[23] P. Vogel, J.F. Beacom, Phys. Rev. D60 053003, (1999); E. Torrente-Lujan, hep-ph/9911458.
[24] S. Nakamura, T. Sato, V. Gudkov and K. Kubodera, nucl-th/0009012.
[25] K. Olive and D. Thomas, Astropart. Phys. 11 (1999) 403; E. Lisi, S. Sarkar and F. Villante, Phys. Rev. D59 (1999) 123520, S. Burles, K. Nollett, J. Truran and M.S. Turner, Phys. Rev. Lett. 82 (1999) 4176.
[26] J. Frieman, H. Haber and K. Freese, Phys. Lett. B200 (1988) 115.
[27] Z.G. Berezhiani and M.I. Vysotsky, Phys. Lett. B199, 281 (1987).
[28] Z.G. Berezhiani and A. Rossi, hep-ph/9306278.
Fig. 1: The 90, 95 and 99% C.L. allowed area from the global analysis of the total rates from Cl, Ga and SK detectors and the 1117 days SK recoil electron spectrum at day and night, assuming MSW conversions to stable sequential neutrinos.
Fig. 2: The decay term $\exp(\alpha L/E)$ vs $\alpha$ for two different neutrino energies. $L$ is taken as the earth-sun distance.
Fig. 3: The survival probability as a function of $\nu_e$ energy for different values of $\alpha$. The solid lines are for $\tan^2 \theta = 0.35$ while the dotted line is for $\tan^2 \theta = 0.85$. For all the curves $\Delta m^2 = 4.17 \times 10^{-5}$ eV$^2$. 
Fig. 4: The $\Delta \chi^2$ versus decay constant $\alpha$ for the global analysis of total rates and the SK day and night spectrum data. Also shown by the dotted line is the 99% C.L. limit for three parameters.
Fig. 5: The 90, 95 and 99% C.L. allowed area from the global analysis of the total rates and the 1117 days SK day-night spectrum data, for MSW conversion of unstable neutrinos at various values of the decay constant $\alpha$. The different fixed values of $\alpha$ in eV$^2$ are indicated.
Fig. 6: The conversion probability of $\nu_e$ to $\bar{\nu}_e$, $P_{e\bar{e}}$ versus decay constant $\alpha$. Also shown by the dotted line is the 99% C.L. value of $P_{e\bar{e}}$ allowed from the non-observance of $(\bar{\nu}_e - p)$ events in the SK detector.
**Fig. 7:** The predicted ranges of $R_{CC}$, $R_{NC}$ and $R_{NC}/R_{CC}$ in SNO for the standard MSW conversion to stable neutrinos (left hand panels) and for unstable neutrinos (right hand panels). The dots give the values of the rates for the local best fit.