Coherent Radio Emission from Pulsars

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Abstract. Generation of the pulsar radio emission from plasma waves excited by the two-stream instability is considered. Special attention is given to propagation effects.

1. Introduction

Strong coherent radio emissions from pulsars are believed to result from the development of plasma instabilities in the magnetosphere of the neutron star. The theory should explain a) what plasma instability excites collective plasma motions in the pulsar magnetosphere; b) how these collective motions generate radio waves and c) how the generated radio waves propagate through the magnetosphere. Such a complete theory has not available yet; moreover there is still no consensus on the basic emission mechanism. The two-stream instability, which readily excites strong plasma oscillations, is among the most widely discussed. Pulsar magnetospheres are believed to be filled with an electron-positron plasma generated in a cascade process from a primary particle beam accelerated in the rotationally induced electric field (see the recent work by Hibschman & Arons 2001 and references therein). This plasma streams along the open magnetic field lines, which extend beyond the light cylinder. Oscillations generated in this plasma may be a source of a powerful radio emission.

Conditions for the development of the two-stream instability in pulsar magnetospheres are not trivial. The plasma flow should involve streams with markedly different velocities. Usov (1987) pointed out that such a configuration arises if plasma production is extremely unsteady. Then the plasma flow is built up from separate clouds following each other along the magnetic field line and slower particles from a cloud would be overtaken by faster particles from the subsequent cloud giving rise to the two-stream instability (see also Uslov & Uslov 1988; Asseo & Melikidze 1998). A two-stream configuration arises also because the adjustment of electric current and charge density in the open field line tube requires a certain flow of particles to be directed downwards, against the main plasma flow (Lyubarskii 1992a,b; 1993a). At last conditions for the two-stream instability may arise as a result of interaction of the secondary pair plasma with the thermal emission from the neutron star surface (Lyubarskii & Petrova 2001). Discussion of these scenarios is out of the scope of my talk; it is enough to mention that in the plasma flow within the open magnetic line tube, the two-stream instability develops readily. Therefore let us assume that the instability does develop and consider the formation of pulsar radio emission from the excited plasma waves and the properties of the outgoing radiation.

2. Generation of radio emission from plasma oscillations in pulsar magnetospheres

2.1. Waves in the plasma embedded in a super strong magnetic field

Let us first outline qualitative properties of waves in the electron-positron plasma embedded in a super strong magnetic field. The pulsar radio beam is believed to originate from deep inside the magnetosphere, where the magnetic field may be considered as effectively infinite. Namely particles in such a field may be considered as beads on a wire; they move freely along the magnetic field lines and do not shift in the transverse direction. Properties of this plasma are relatively simple. They are especially simple if plasma is cold therefore below all estimates will be made in this limit. In the relativistically hot plasma properties of the waves are qualitatively the same (see, e.g., Volokitin et al. 1985; Arons & Barnard 1986; Lyubarskii 1996; Melrose & Gedalin 1999; Asseo & Riazuelo 2000).

Let us first consider waves in the plasma rest frame (all the quantities in this frame will be marked by prime). It is evident that a transverse wave propagating along the magnetic field does not interact with the plasma because the electric field in this wave is perpendicular to the background magnetic field and plasma particles are unable to move in this direction. Therefore transverse waves propagate along a super strong magnetic field just like in vacuum; for these waves $\omega = kc$. On another hand, there exist longitudinal, electrostatic waves propagating along the magnetic field and these waves are just the same as Langmuir waves in a nonmagnetized plasma because particles oscillating along the magnetic field do not "feel" this
Fig. 1. Dispersion curves in the proper plasma frame. Solid lines correspond to the waves propagating along the magnetic field and dashed, to those propagated obliquely. The two-stream instability excites waves in the shaded region.

\[
\frac{\omega'}{\omega_p} = \sqrt{\frac{4\pi e^2 n'}{m}},
\]

just as in a nonmagnetized plasma. Here \(n'\) is the plasma number density in the proper frame, \(e\) and \(m\) the electron charge and mass, respectively.

The two-stream instability excites waves in the shaded region.

Fig. 2. Dispersion curves in the pulsar frame.

In pulsars plasma streams with a Lorentz factor \(\gamma \sim 100\) along the open magnetic field lines. The dispersion curves in the pulsar frame may be obtained by Lorentz transformation of the curves shown in Fig. 1. The result is plotted in Fig. 2. The dispersion curve of the transverse wave propagating along the magnetic field, \(\omega = kc\), evidently remains unchanged whereas the horizontal line \(\omega' = \omega_p\), corresponding to the longitudinal waves, transforms into an inclined line such that the frequencies of the longitudinal plasma waves may vary significantly. The reason is that not only the frequency but also the wavelength enter the Lorentz transformation therefore the waves with different wavelengths and the same frequency in the proper plasma frame have different frequencies in the pulsar frame.

The wave \(\omega' = \omega_p, k' = 0\) have the frequency

\[
\omega_0 = \omega_p \sqrt{\gamma}
\]

in the pulsar frame. The waves with \(\omega < \omega_0\) are those propagating in the proper frame upstream the plasma flow. The dispersion curves of the transverse and longitudinal waves propagating along the magnetic field intersect at the point \(\omega = 2\omega_0, k = 2\omega_0/c\); above this point the longitudinal wave is subluminous. The dispersion curves for the oblique waves are also split into superluminous and subluminous branches. At \(\omega \gg \omega_p\) the superluminous wave becomes the vacuum transverse electromagnetic wave.

The waves resonate with particles if the phase velocity, \(\omega/k\), coincides with the particle velocity (more exactly the resonance condition reads as \(\omega = k \cdot v\)). Of course only subluminous waves participate in such an interaction. In the presence of a particle beam, such a
resonant interaction leads to the two-stream instability, which predominantly excites waves with the phase velocities close to the beam velocity. The region where waves are generated by a high velocity particle beam is shaded in Fig. 1. Waves with low enough phase velocities are heavily damped because they are in resonance with the thermal plasma particles (Landau damping).

Up to now only so called ordinary waves were considered. There is also an extraordinary wave; in this wave the electric field is perpendicular both to the background magnetic field and to the wave vector \( \mathbf{k} \). In the limit of infinitely strong magnetic field this wave evidently does not interact with the plasma and propagates like in vacuum.

### 2.2. Escape of waves

The crucial question is what waves can escape from the pulsar magnetosphere and form the observed radio beam. The answer is evident for the nonmagnetized plasma where the purely longitudinal as well as purely transverse waves exist. Namely the longitudinal wave, which is electrostatic, do not escape whereas the transverse wave escapes freely. There is no purely transverse waves in the strongly magnetized plasma and therefore one should ask what wave becomes the vacuum transverse electromagnetic wave when it propagates outward, in the plasma of decreasing density (Arons & Barnard 1986).

Let us consider propagation of a superluminous wave. In a steady state medium with smooth density gradients, the wave frequency remains constant while the wavelength is adjusted in order to satisfy the dispersion equation at any point. In the \((\omega, k)\) plane, the point representing a wave moves along the dispersion curve upward because plasma frequency decreases together with plasma density. If the wave propagates strictly along the magnetic field, it transits along the dispersion curve to the subluminous region. Eventually the wave phase velocity, \( \omega/k \), decreases such that the wave decays through the Landau damping. In the oblique propagation case, the wave remains superluminous and at \( \omega \gg \omega_0 \) transforms into the vacuum transverse electromagnetic wave, which escapes freely. Finally it should be noted that in a curved magnetic field, a wave becomes oblique, even if it was initially directed along the magnetic field (Barnard & Arons 1986, Lyubarskii & Petrova 1998). Therefore the superluminous longitudinal waves eventually escape from pulsar magnetospheres in the form of the vacuum transverse electromagnetic waves.

Applying the same consideration to the subluminous waves, one can see that such waves do not escape. When they propagate in the plasma of decreasing density, their phase velocities decrease and eventually the waves decay through the Landau damping. So in the strong magnetic field the superluminous wave escapes even if it was initially purely longitudinal (in the curved magnetic field) whereas the subluminous waves do not escape even though these waves are nearly transverse at \( \omega \ll \omega_0 \).

The two-stream instability generates predominantly longitudinal waves, which are in Cherenkov resonance with the particle beam, \( \omega = kv \). This waves are evidently subluminous, \( \omega/k < c \) (the region where the waves are generated is shaded in Figs. 1, 2). Such waves are unable to escape from plasma unless nonlinear processes redistribute the wave energy into the superluminous region. High brightness temperature of pulsar radio emission implies high wave energy density in pulsar magnetospheres and therefore nonlinear effects should be of paramount importance.

### 2.3. The second order processes

As the first step in studying of the nonlinear effects, one should consider the processes of the lowest order in the wave energy density, \( W \). These are the wave-wave interaction and induced scattering by the plasma particles (see, e.g., Tsytovich 1970; Melrose 1980). The wave-wave interactions involve, in the lowest order in \( W \), merging of two waves into one and decay of a wave into two waves. All three interacting waves should satisfy the dispersion equation, \( \omega = \omega(k) \), and also the energy and momentum conservation laws,

\[
\omega(k_1) + \omega(k_2) = \omega(k_3);
\]

\[
k_1 + k_2 = k_3;
\]

which place severe restrictions on the process. Therefore this process is rather ineffective in pulsar conditions (Bliokh & Lyubarskii 1997).

In contrast to wave-wave interactions, there is no restrictions on induced scattering and therefore this process is of crucial importance. At rather general conditions, the induced scattering redistributes the wave energy towards large wavelengths (in the plasma rest frame). One can easily see from Fig. 1 that when a longitudinal wave from the shaded region moves towards small \( k \) (larger wavelengths) it eventually becomes superluminous. However it was shown above that superluminous waves escape freely in the form of vacuum transverse waves if they propagate in the curved magnetic field in the plasma of decreasing density. So the observed radio emission may be generated

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1. Of course an infinitely small angle between the magnetic field and the propagation direction is insufficient for free escape of a longitudinal wave. If the angle is small enough, the superluminous branch approaches closely to the subluminous one near the point \( \omega = 2\omega_0, \quad k = 2\omega_0/c \) and the wave may jump, because of linear coupling, onto the subluminous branch. However in pulsar magnetospheres, this effect is of no importance because the corresponding critical angle is very small (Bliokh & Lyubarskii 1996).

2. There are more degrees of freedom for wave-wave interactions in a finite magnetic field (Machabeli 1983; Lyutikov 1999) however probabilities of these processes are rather small.
through the induced scattering of the longitudinal waves excited by the two-stream instability.

Redistribution of the wave energy is described by the kinetic equation; in the plasma in the infinite magnetic field it may be presented in the form

$$\frac{\partial W_k}{\partial t} + v_g \frac{\partial W_k}{\partial r} = \frac{k_z - k_{z1}}{\omega_1} \frac{\partial f}{\partial p} dp d\mathbf{k}_1,$$

where $W_k$ is the spectral energy density of waves, $f(p)$ the particle distribution function, $w_p(k, k_1)$ the probability of the scattering of the wave $k_1$ into the wave $k$, $v_g$ the group velocity of the wave; the field is directed along $z$ axis. The scattering probability was calculated by Lominadze et. al. (1979) for waves propagating along the magnetic field and by Lyubarskii (1993b) in the general case.

One can conveniently analyze the plasma processes in the proper plasma frame. In this frame (more exactly, in the frame of reference moving with the maximum of the particle distribution function) $\frac{\partial f}{\partial t} \leq 0$ and the right-hand side of Eq.(3) is positive if $k_z < k_{z1}$, so the wavelength increases in the scattering process. Kinetics of the induced scattering in the one-dimensional relativistic plasma was considered by Lyubarskii (1996). Of course in the presence of a particle beam, one more maximum appears in the distribution function and, provided the beam is strong enough, the above statement violates and the wave frequency may significantly increase (e.g., Weatherall 2001, Schopper et al. 2002). In this case the beam plays the role of the main plasma and, switching to the frame of the beam, we come to the same conclusion: the wavelength increases. In the frame of the beam this is accompanied by changing of the wave propagation direction and therefore the wave frequency grows in the laboratory frame. One should also note that the so called amplified linear acceleration emission (Melrose 1978; Rowe 1995) actually represents the induced scattering of a longitudinal, electrostatic wave into a (quasi) transverse one.

For a rough estimate, one can present the kinetic equation (3) in the simple form

$$\frac{dW}{dt} \sim \eta W,$$

where $W = \int W_k d\mathbf{k}$ is the total wave energy density. In the proper plasma frame, the redistribution rate may be roughly presented as

$$\eta \sim \frac{W'}{n'mc^2} \omega_p.$$

The structure of this simple estimate is common for any second order process: the corresponding redistribution rate is proportional to the ratio of the wave energy to the plasma energy density and some relevant frequency. In the case of interest, the characteristic plasma "temperature" is about of the electron rest energy and all frequencies are about the plasma frequency therefore the estimate is especially simple and may be in fact obtained from dimensional considerations.

2.4. Emission power

The mechanism under consideration generates electromagnetic waves with $k' < \omega/c$ (see Fig. 1); in the pulsar frame the frequencies of these wave are about $\omega_0$ (Eq.(2)). So the observed frequency is determined, via the plasma frequency, by the plasma density at the emission point. The electron-positron plasma is assumed to be generated at the base of the open field line tube; the generation rate is conveniently characterized by the multiplicity parameter, $\kappa$, such that the plasma density at the base of the open field line tube is normalized by the Goldreich-Julian (1969) density,

$$n = \frac{\kappa B}{\varepsilon c P},$$

where $B$ is the surface magnetic field, $P$ the pulsar period. Continuity of the plasma flow in the tube implies that the plasma density falls off as the tube cross section; for the dipole field, $n \propto 1/r^3$. Now the radiation frequency may be estimated as

$$\nu = \frac{\omega_0}{2\pi} = 1.4 \sqrt{\frac{\kappa_2 \gamma_2 B_{12}}{P}} \left(\frac{30r_5}{r}\right)^3 \text{GHz},$$

where $B_{12} \equiv 10^{12} B_{12}$ G is the surface magnetic field, $r_5$ the stellar radius, $\kappa_2 \equiv \kappa/10^2$, $\gamma_2 \equiv \gamma/10^2$. Observational data suggest that the radio emission is generated at altitudes about few hundreds km (e.g., von Hoensbroech & Xilouris 1997; Kijacki & Gil 1998; Kijacki 2001). At such altitudes, the width of the open field line tube is about the observed beam width, the last being estimated as $\delta = 0.1P^{-1/2}$ (Rankin 1993). The estimate (7) shows that the radio emission may be generated at the plasma frequency provided the multiplicity is about 100. Note that the presented radius-to-frequency mapping does not mean that the emission is narrow band. Plasma at a given radius radiates in the range $\Delta \nu \sim \nu$.

Because nonlinear redistribution rates depend on the wave energy density, one can easily estimate the radiation power comparing the redistribution rate with the escape rate. The two-stream instability generates Langmuir waves in the region shaded in Fig. 2. These waves propagate upward in the plasma of decreasing density; without nonlinear redistribution, the phase velocity of the waves would decrease and the waves would eventually decay. Phase velocity decreases with plasma density at the characteristic scale $r$; in the proper plasma frame, the corresponding time is about $r/(\gamma c)$. Emission of radio waves is possible if the induced scattering transforms the waves into superluminous ones for a lesser time, $\eta \geq r/(\gamma c)$. Making use of Eq. (5), one can estimate the minimal wave
energy density necessary for the emission as

$$W' = \frac{W}{n' mc^2} = \frac{W}{\gamma n mc^2} = 0.016 \frac{b^{5/3} P^{1/3}}{(v_0 B_{12})^{1/3}}.$$  \hspace{1cm} (8)

Here the radius-to-frequency mapping (7) was used to eliminate the radius in favor of the radiation frequency. If the plasma wave density is less than the above value, nonlinear processes are ineffective and the waves eventually decay. The pulsar radiates if the two-stream instability generates waves with the energy density exceeding the minimal value; then the generated waves are transformed into superluminous ones and may escape. The corresponding luminosity may be estimated multiplying the wave energy density by the speed of light and the cross section of the polar tube, $S = \pi r_0^2 \Omega/c$; for the two tubes one gets

$$L_{\text{min}} = 2 W_c S = 5 \cdot 10^{26} (k_2 B_{12})^{2/3} \frac{\gamma^{8/3}}{P_5^{5/3} \nu_9^{1/3}} \text{erg/s}. \hspace{1cm} (9)$$

The presented estimate roughly corresponds to the luminosity of weak pulsars. The radiation spectrum in this case may be estimated as $d\nu \propto \nu^{-4/3}$.

The pulsar luminosity is limited from above by the total energy of the plasma flow. So the maximal pulsar luminosity may be estimated as

$$L_{\text{max}} = 2 n mc^3 \gamma S = 2.5 \cdot 10^{28} \frac{k_2 B_{12} \gamma_2}{P_2} \text{erg/s}. \hspace{1cm} (10)$$

This estimate is compatible with the observed pulsar luminosities.

According to Eq. (7), the plasma multiplicity factor should be not large, $k \leq 100$, for radio emission to be emitted not too high in the magnetosphere. Originally the plasma production models predicted $k \sim 10^3 - 10^4$ and even more (Ruderman & Sutherland 1975; Daugherty & Harding 1982; Arons 1983). However recent calculations, which take into account the resonant scattering of the thermal radiation from the surface of the star, favor for smaller $k$ (Hibschman & Arons 2001), which are in better agreement with the assumption that the emission is generated at the plasma frequency. On the other hand, the observed pulsar luminosities require, according to Eq. (10), $k \geq 100$. Whilst these estimates are not incompatible, one would nevertheless fill more comfortable if the plasma could emit at frequencies well below the plasma frequency or if there was an additional energy source such that a low density plasma could provide the observed radiation power.

Radiation at $\nu \ll \nu_p$ is possible, in principle, in the subluminous mode (Arons & Barnard 1986; Lyutikov 2000; Melikidze et al. 2000). This wave propagates along the magnetic field preserving original direction of the wave vector. Being emitted at a level where the plasma density is high, the wave is ducted upwards along the magnetic field line and eventually decays when the phase velocity reaches the particle velocities (Barnard & Arons 1986).

The outgoing radiation may be generated if the wave is transformed into the superluminous wave before the decay takes place. One should expect that then the beam width will be determined by the width of the tube at the transformation level, which is in fact the level where $\nu \sim \nu_p$. So if $k$ is large, the beam should be too wide even if it was emitted at low altitudes. The beam will remain narrow only if the transformation process does not affect the original direction of $k$ however up to now nobody has proposed such a transformation process.

The possibility that the plasma in the open field line tube gains energy from an external source may not be ruled out because the electromagnetic energy in the pulsar magnetosphere greatly exceeds the plasma energy. True, plasma with $k > 1$ screens electric fields and therefore the huge electric potential generated in the open field line tube seems to remain beyond the reach of the plasma flow. Nevertheless adjustment of the electric charge density in the open field line tube requires some longitudinal electric field (Scharlemann 1974; Cheng & Ruderman 1977). It is possible that the plasma flow may gain some energy from this field; then the pulsar luminosity may exceed that of Eq. (10).

2.5. Strong turbulence

If the wave energy density is about Eq. (8), the induced scattering rate is comparable with the propagation time and therefore the waves are transformed into superluminous ones and escape. If the wave energy is larger, the induced scattering rate exceeds the escape time, which means that the effective optical depth becomes large. In this case the waves may be locked in the system therefore another nonlinear processes come into play.

The most important effect is the modulation instability. A spatially uniform distribution of plasma waves becomes unstable with respect to formation of regions with enhanced energy density of the waves (e.g., ter Haar & Tsidiovich 1981; Goldman 1984). In the nonrelativistic plasma, this instability develops already at rather small plasma wave energy densities. The threshold of the instability is

$$\frac{W}{W_{pl}} \sim (kD)^2,$$  \hspace{1cm} (11)

where $W_{pl}$ is the plasma energy density, $D$ the Debye length. The waves are excited by a particle beam with the velocity $v_b$ at the resonance $k = v_b/\omega_p$ therefore the right hand side of Eq. (11) may be presented as $(v_{th}/v_b)^2$, where $v_{th}$ is the electron thermal velocity. Taking into account that typically $v_b \gg v_{th}$, one can see that the threshold of the instability is rather low. In the relativistic plasma, $v_b \sim v_{th} \sim c$ and $W_{pl} \sim n' mc^2$ therefore the modulation instability develops only when the wave energy density becomes comparable with the plasma energy density. In this case the induced scattering rate (5) is comparable with the plasma frequency and therefore can not be neglected.
Interplay between the two effects should be considered, which has not been done yet. One can only anticipate that the induced scattering makes the development of the modulation instability easier because the wavelength increases.

It is still unclear how formation of cavities with the enhanced wave energy density influences the emission properties of the pulsar plasma. Evidently the effect results in a short term variability. However it is still unclear how outgoing waves are generated. Asseo et al. (1990), Asseo (1993), Melikidze et al. (2000) considered emission from the cavities, or solitons, assuming that they are stable. However Weatherall (1997, 1998) found numerically that the longwave Langmuir solitons collapse like in the non-magnetized plasma. He suggested that the waves escape from the collapsing cavities because longitudinal electrostatic waves evolve into oblique electromagnetic waves in the course of the collapse. However these waves may be locked within the cavity surrounded by a higher density plasma. More careful analysis is necessary, both numerical and analytical, to clarify the situation. Comparing analytical estimates with the results of numerical simulations, one will be able to find reliable scalings, which may be applied to real pulsars.

3. Propagation effects

3.1. Polarization limiting radius

If the radio emission is generated by the two-stream instability, the wave frequency should be of the order of the plasma frequency at the emission point. Then many observed characteristics of the outgoing radiation, first of all polarization, are dictated by the wave propagation in the magnetospheric plasma. Whatever the emission mechanism, radiation propagates in the plasma in the form of two orthogonally polarized normal waves. Deep inside the magnetosphere, normal waves are linearly polarized; the ordinary wave is polarized in the plane defined by the ambient magnetic field, \( \mathbf{B} \), and the wave vector, \( \mathbf{k} \), and the extraordinary wave in the \( \mathbf{k} \times \mathbf{B} \) direction. The polarization pattern follows, along the ray, the local orientation of the \( \mathbf{k} \times \mathbf{B} \) plane; Cheng & Ruderman (1979) used the term "adiabatic walking" to describe the evolution of the ray polarization in this regime. Ultimately the plasma density falls to an extent that the medium no longer affects wave propagation. The observed polarization is fixed in the transition region, at the so called polarization-limiting radius, which is determined by the condition

\[
kL \Delta n = 1, \tag{12}\]

where \( L \) is the characteristic scale length for changing polarization of the normal waves, \( \Delta n \) the difference in index of refraction between the normal waves. Because all characteristic scales in the pulsar magnetosphere are large as compared with the wavelength, \( kL \gg 1 \), \( \Delta n \) should be small at the polarization limiting radius, which typically means that both indexes are close to unity, i.e. the local plasma frequency is much lower then the wave frequency. So the polarization-limiting altitude should be significantly higher then the emission altitude.

In pulsar magnetospheres, the polarization-limiting radius was estimated by Cheng & Ruderman (1979), Melrose (1979), Barnard (1986). Unfortunately this value depends not only on the plasma density, which itself is rather uncertain, but also on the angle the wave vector makes with the magnetic field. Since this angle is small, even small deviation of the magnetosphere structure from the pure dipole is of crucial importance. Therefore one cannot firmly fix the polarization-limiting radius until the self-consistent model for the relativistic, rotating magnetosphere will be available.

Observations of the polarization position angle swing, i.e. of the smooth rotation of the plane of linear polarization through the pulse, place limits on the polarization limiting radius. The observed polarization position angle is determined by the projection of the magnetic field on the plane perpendicular to the ray direction at the polarization limiting radius. Deviations of the magnetosphere structure from the pure dipole at the polarization-limiting radius, as well as the aberration and retardation effects, should affect the position angle vs. pulse longitude curve. The success of the simple rotating vector model in description of the position angle swing (Radhakrishnan & Cooke 1969; Manchester & Tailor 1977) suggests that the polarization-limiting radius is small as compared with the light cylinder radius. In particular, the position angle sweep decreases with the altitude; this was demonstrated by Barnard (1986) for the Deutch magnetosphere but this effect is of general nature (Fig. 3). Large position angle sweeps, which are not uncommon in pulsars, favor for a small polarization limiting radius. Large sweeps may be obtained at large altitudes only if the magnetosphere sweepback caused by rotation is exactly compensated by the rotation itself, so the plasma motion is strictly radial and the magnetic field in the proper plasma frame is also radial. In this case the magnetic axis has exactly the form of the Archimedean spiral. Since the self-consistent model of the pulsar magnetosphere has not available yet, such a possibility cannot be excluded however below the general picture sketched in Fig. 3 is assumed.

The position angle sweep decreases with increasing ratio of the polarization-limiting radius to the light cylinder radius. Therefore the sweep should decrease, in average, with decreasing pulsar period (Barnard 1986). This conclusion agrees with shallow polarization swing observed in millisecond pulsars (Xilouris et al. 1998).

3.2. Circular polarization

The radiation from many pulsars contains detectable circular polarization, although the amounts are generally
much less than the degree of linear polarization (e.g., Radhakrishnan & Rankin 1990; Han et al. 1998). The circular polarization naturally arises at high altitudes where the cyclotron frequency is already not too large as compared with the radiation frequency. The normal waves are elliptically polarized in this region if the distribution functions of electrons and positrons are different. This is always the case because the Goldraich-Julian charge density should be maintained in pulsar magnetospheres. Therefore the circular polarization in the outgoing radiation may be attributed to the dispersive properties of the magnetospheric plasma provided the polarization-limiting radius is about the light cylinder radius (Melrose & Stoneham 1977; Cheng & Ruderman 1979; Melrose 1979; von Hoensbroech et al. 1998; von Hoensbroech & Lesch 1999; Gedalin et al. 2001). However large position angle sweeps imply a small polarization limiting radius, at least in significant fraction of pulsars. At such radii, the normal waves are linearly polarized and elliptical polarization may arise from the wave mode coupling in the polarization-limiting region (Cheng & Ruderman 1979).

Deep inside the polarization-limiting radius, where the plasma density is high enough, the normal waves propagate independently and their polarization plane is adjusted to the local orientation of the $\mathbf{k} \times \mathbf{B}$ plane. Outside this region, the plasma density is too low and the waves propagate as in vacuum. In the polarization limiting region, the normal waves are still influenced by the plasma but this influence is insufficient to make the wave polarization follow the orientation of the $\mathbf{k} \times \mathbf{B}$ plane. Provided the $\mathbf{k} \times \mathbf{B}$ plane turns along the ray path, wave mode coupling takes place resulting in the elliptical polarization of the outgoing waves. The $\mathbf{k} \times \mathbf{B}$ plane turns along the ray path due to rotation of the magnetosphere, or due to
sweepback of the magnetic field lines caused by the rotation or if the initial co-planarity between \( \mathbf{k} \) and the field line fails because of refraction.

The quantitative treatment of the polarization transfer in the rotating magnetosphere was held by Lyubarskii & Petrova (1999) and Petrova & Lyubarskii (2000). They found significant circular polarization of the outgoing radiation, in some cases with the sense reversal near the pulse center. The degree of circular polarization depends on the limiting polarization radius and reaches large values, \( V \sim 1 \), if this radius is about \( r_L \vartheta \), where \( \vartheta \) is the beam width, \( r_L \) the light cylinder radius.

Circular polarization is generally larger in the central part of the pulse profile, in the core beam according to Rankin’s (1983a) classification. This property may be naturally explained if the polarization limiting radius is small, \( < r_L \vartheta \), such that the magnetic axis remains within the beam (see Fig. 3). In this case the angle between the ray and the magnetic field decreases towards the center of the beam. The lesser this angle, the larger variation in the orientation of the \( \mathbf{k} \times \mathbf{B} \) plane may be caused by a small deviation from co-planarity between \( \mathbf{k} \) and the field line (Fig. 4). Therefore the circular polarization increases towards the center of the beam. Moreover violation of the regular position angle swing, which is commonly observed in the core beams (e.g., Rankin 1990), may be naturally explained by the same reason. One should also note that the angle between the magnetic field and the ray direction in the cone beam is also not large, therefore the difference between the polarization characteristics of the cone and core beams is not qualitative but only quantitative. This also explains why there is no abrupt transition from the cone to the core emission but merely a gradation of properties across the whole emission beam (Lyne & Manchester 1988; Han et al. 1998).

3.3. Orthogonal polarization modes and depolarization

The plasma in the pulsar magnetosphere is birefringent; the orthogonally polarized ordinary and extraordinary waves propagates independently producing the abrupt orthogonal transitions in polarization position angle that are commonly observed in studies of individual pulse polarization (Manchester et al. 1975; Stinebring et al. 1984a,b; Gil & Lyne 1995; Gangadhara 1997; McKinnon, Stinebring 1998, 2000). The observed depolarization of pulsar average profiles with increasing radio frequency (Manchester et al. 1973; Morris et al. 1981; Xilouris et al. 1996; von Hoensbroech et al. 1998; Kramer et al. 1999) may be attributed to merging of the ordinary and extraordinary beams at high frequency. McKinnon (1997) assumed, following Barnard & Arons (1986), that pulsars emit originally in both radiation modes and all the emission comes from a narrow range of heights above the stellar surface. The extraordinary mode propagates along straight ray paths, while the ordinary waves may be deflected from the original direction by refraction. At low frequencies, refraction separates the two modes resulting in a net polarization. At high frequencies (above the local plasma frequency), refraction is negligible and both modes merge producing a non-polarized beam.

It is still unclear if it is possible to generate the extraordinary waves deep inside the magnetosphere where the magnetic field is effectively infinite. In this case, the extraordinary wave, whose electric vector is orthogonal to the background magnetic field, does not interact with the plasma and cannot be emitted, at least in the homogeneous magnetic field. In principle, interaction is possible in a curved magnetic field, especially if one takes into account that the characteristic frequency of the curvature emission in pulsars is of the order of the emitted radio frequency. Unfortunately, nonlinear wave processes in a curved, super strong magnetic field have not been considered yet.

The extraordinary wave may appear in the outgoing radiation due to propagation effects even if initially only ordinary waves were emitted. Because the plasma production process in pulsars may be unsteady, the plasma flow in the open field line tube may be strongly inhomogeneous. One can naturally assume that the plasma is gathered in clouds separated by regions of the very low density such that the condition \( \Delta n kL \ll 1 \) is satisfied in the inter-cloud space and the waves propagate there preserving the po-

![Fig. 4. The turn of the \( \mathbf{k} \times \mathbf{B} \) plane caused by a variation of the magnetic field along the ray path. If the angle between the wave direction and the magnetic field is large (a), a turn of \( \mathbf{B} \) by a small angle \( \alpha \) result in a turn of the \( \mathbf{k} \times \mathbf{B} \) plane by a small angle \( \beta \). If the angle between \( \mathbf{k} \) and \( \mathbf{B} \) is small (b), a small turn of \( \mathbf{B} \) may result in a large turn of the \( \mathbf{k} \times \mathbf{B} \) plane.](image-url)
lарization position angle. Now let us consider an ordinary wave emitted within some cloud. Because the frequency of this wave is of the order of the local plasma frequency, refraction deviates the ray from the initial direction such that the ray may come to another plasma cloud where the orientation of the local magnetic field is different from that in the original cloud. Because adiabatic walking ceases in the inter-cloud space, the ray enters the second cloud not in a normal mode corresponding to the local orientation of the magnetic field. Therefore the wave is split into two independently propagating normal waves. This process may produce orthogonal polarization modes and may result in depolarization of the outgoing radiation.

The depolarization mechanism outlined above works if the refraction angle is comparable with the angular width of the open field line tube. Taking into account that the refraction angle is determined by the transverse density gradient, which decreases with the increasing of the tube width, one can state a general trend: the narrower the open field line tube the less degree of polarization. Then the radius-to-frequency mapping implies that the high frequency radiation, which is emitted at lower heights where the tube is narrow, should be less polarized, what is indeed observed. Another observational finding may be considered as a partial case of the above general trend. Namely because the width of the open field line tube decreases with the increasing pulsar period, depolarization should be more pronounced in slow pulsars. The observed high frequency polarization actually decreases with the pulsar period (Morris et al. 1981; von Hoensbroech et al. 1998).

Recently Petrova (2001b) considered another mechanism for the conversion of ordinary waves into extraordinary ones. Due to refraction, the ray may for a short while become nearly aligned with the local magnetic field. In this degenerate situation, refraction indexes of the two waves become equal and the conversion takes place. Calculations were made in the assumption that the plasma distribution in the open field line tube is axisymmetric, then the refracted ray remains in the plane of the magnetic field line. In the general case, co-planarity between the refracted beam and the field line fails and the ray may never become aligned with the magnetic field. Nevertheless this effect may reveal itself at some phases of the pulsar period.

3.4. Refraction

Refraction was already invoked in the above discussion of the depolarization. Of course the shape of pulses should be also strongly affected by refraction. The observed pulse widths typically broaden with decreasing frequency however at high frequencies (> 1 GHz) the beam width vs. frequency curve commonly flattens (Sieber et al. 1975; Rankin 1983b; Thorsett 1991). Barnard & Arons (1986) attributed this behavior to the refraction. They assumed that all the pulsar emission comes from the same height; then the frequency dependence of the beam width follows the frequency dependence of the refraction angle: at high frequencies refraction is negligibly small and the beam width is constant whereas at low frequencies the beam widens with decreasing frequency due to increasingly strong refraction.

In the model advocated here, radiation is emitted at about the local plasma frequency, which implies the standard radius-to-frequency mapping $\nu \propto r^{-3/2}$. In this case the refraction index, which depends on the ratio $\nu/\nu_p$, is the same for any ray at the emission point. However the emission height and the width of the open field line tube increases with decreasing frequency therefore the relative contribution of the refraction into the beam width decreases. Lyubarskii & Petrova (1978) considered the impact of refraction upon the beam width assuming the standard radius-to-frequency mapping and the axisymmetrical plasma distribution in the tube. The ray deviation occurs mainly on account of the density gradient across the tube. If the plasma density decreases towards the edge of the tube, the rays deviate outwards. Then at high frequencies the refraction angle exceeds the angular width of the open field tube and the beam width becomes independent of the frequency whereas at low frequencies the beam widens together with the open field line tube. If the plasma density increases towards the edge of the tube (the density is minimal at the magnetic axis), the ray deviate towards the magnetic axis and, provided the tube is narrow enough, may even reflect from the opposite side of the tube. As a result a trough arises at the beam width vs. frequency curve, which resembles the so called “absorption feature” observed in some pulsars (Rankin 1983b).

Assuming a hollow cone distribution of the plasma in the open field line tube (the plasma density is small at the axis, increases outwards and then decreases towards the edge) one can obtain pulse profiles with three components (Petrova & Lyubarskii 2000; Petrova 2000) resembling those observed in real pulsars. The rays deviating towards the axis form the core beam whereas the rays deviating outwards form the cone beam. Petrova (2001a) demonstrated that the refraction may also account for the unusual apparent structure of the emission region inferred from the observations of the interstellar scintillations (Wolszczan & Cordes 1987; Gupta et al. 1999).

The refraction may affect also the observed pulsar spectrum. Sieber (1997) demonstrated that many properties of the observed spectra may be attributed to the geometry of the beam. Petrova (2002) studied these effects in case the geometry of the beam is determined by refraction.

Of course the assumption about the hollow cone plasma distribution is rather restrictive. The plasma density at the axis should be small if the magnetic field is strictly dipolar down the stellar surface; then the plasma production rate is small at the magnetic axis. However it is naturally to believe that higher order multipoles con-
tribute to the field at the surface (see however Arons 1993) and then the plasma distribution across the tube is not so regular. Nevertheless one can anticipate that many qualitative features of the ray behavior remain in more general configurations. Let us assume, for example, that the instant plasma distribution in the tube is clumpy because the plasma production process is unsteady. Refraction in such a clumpy medium results in some scatter in the ray directions, which provides a minimal beam width even if the tube becomes very narrow. So saturation of the beam width vs. frequency curve at high frequencies (Sieber et al. 1975; Rankin 1983b; Thorsett 1991) seems to be a rather general feature.

4. Conclusion

It was demonstrated above that a large variety of the observed properties of the pulsar radio emission may be explained self-consistently, at least at the qualitative level, assuming that the emission is generated by the two-stream instability. Of course the theory falls short of being complete. Evidences for a large transverse size of the emission region inferred from the observations of the interstellar scintillations (Smirnova et al. 1996; Gwinn et al., 1997, 2000; Hirano & Gwinn 2001; Smirnova & Shishov 2001) make a challenge. A possible solution is the scattering of the beam on the plasma inhomogeneities in the upper pulsar magnetosphere (Lyutikov & Parikh 2000; Lyutikov 2001). Another difficult problem is posed by the recent finding that the Vela pulsar emits in the extraordinary mode (Lai et al., 2001). Plasma processes in an effectively infinite magnetic field are believed to emit predominantly in the ordinary mode, at least in the straight magnetic field. Possible emission of the extraordinary mode in the curved magnetic field should be considered.

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References

Arons J., 1983, ApJ 266, 215
Arons J., 1993, ApJ 408, 160
Arons J., Barnard J.J., 1986, ApJ, 302, 120
Asseo E., 1993, MNRAS 264, 940
Asseo E., Melikidze G. I., 1998, MNRAS 301, 59
Asseo E., Pelletier G., Sol H., 1990, MNRAS 247, 529
Asseo E., Riazuelo A., 2000, MNRAS 318, 983
Barnard J.J., 1986, ApJ 303, 280
Barnard J.J., Arons J., 1986, ApJ 302, 138
Bliokh K.Y., Lyubarskii Y.E., 1996a, Astron. Lett., 22, 482.
Bliokh K.Y., Lyubarskii Y.E., 1997, Plasma Phys.Rep. 23, 416
Cheng A.F., Ruderman M.A., 1977, ApJ 212, 800
Cheng A.F., Ruderman M.A., 1979, ApJ 229, 348
Daugherty J. K., Harding A. K., 1982, ApJ 252, 337
Gangadhara R. T., 1997, A&A 327, 155
Gedalin M., Grumen E., Melrose D. B., 2001, MNRAS 325, 715
Gil J., Lyne A.G., 1995, MNRAS 276, L55
Goldman M.N., 1984, Rev.Mod.Phys. 56, 709
Goldreich P., Julian W.H., 1969, ApJ 157, 869
Gupta Y., Bhat N. D. R., Rao A. P., 1999, ApJ 520, 173
Gwinn C. R., Ojeda M. J., Britton M. C., et al., 1997, ApJ 483, L53
Gwinn C. R., Britton M. C., Reynolds J. E., et al., 2000, ApJ 531, 902
Han J.L., Manchester R.N., Xu R.X., Qiao G.J., 1998, MNRAS 300, 373
Hibschman J.A., Arons J., 2001, ApJ 560, 871
Hirano C., Gwinn C. R., 2001, ApJ 553, 358
Kijak J., 2001, MNRAS 323, 557
Kijak J., Gil J., 1998, MNRAS 299, 855
Kramer M., Lange C., Lorimer D. R., et al. 1999, ApJ 526, 957
Lai D., Chernoff D. F., Cordes J.M. 2001, ApJ 549, 1111
Lominadze D.G., Mikhailovskii A.B. & Sagdeev R.Z., 1979., Sov. Phys.-JETP 50, 927 (1979)
Lyne A.G., Manchester R.N., 1988, MNRAS 234, 477
Lyubarskii Y.E., 1992a, A&A 261, 544
Lyubarskii Y.E., 1992b, A&A 265, L33
Lyubarskii Y.E., 1993a, Astron.Lett. 19, 14
Lyubarskii Y.E., 1993b, Astron.Lett. 19, 208
Lyubarskii Y.E., 1996, A&A, 308, 809
Lyubarskii Y.E., Petrova S.A., 1998, A&A 333, 181
Lyubarskii Y.E., Petrova S.A., 1999, Ap&SS 262, 379
Lyubarskii Y.E., Petrova S.A., 2000, A&A 355, 406
Lyutikov M., 1999, ApJ 525, L37
Lyutikov M., 2000, MNRAS 315, 31
Lyutikov M., 2001, Ap&SS 278, 81
Lyutikov M., Parikh A., 2000, ApJ 541, 1016
Machabeli G.Z., 1983, Soviet J Plasma Phys. 9, 1177
Manchester R.N., Tailor J.H., 1977, Pulsars. W.H.Freeman and Company, San Francisco
Manchester R.N., Tailor J.H., Huguenin G.R., 1973, ApJ 179, L7
Manchester R.N., Tailor J.H., Huguenin G.R., 1975, ApJ 196, 83
McKinnon M.M., 1997, ApJ 475, 763
McKinnon M.M., Stinebring D.R., 1998, ApJ 502, 883
McKinnon M.M., Stinebring D.R., 2000, ApJ 529, 435
Melikidze G.I., Gil J.A., Pataraya A.D., 2000, ApJ 544, 1081
Melrose D.B., 1978, ApJ 225, 557
Melrose D.B., 1979, Austr.J.Phys, 32, 61
Melrose D.B., 1980, Plasma Astrophysics. Gordon and Breach, NY
Melrose D.B., Gedalin M.E., 1999, ApJ 521, 351
Melrose D.B., Stoneham R.J., 1977, Proc.Astr.Soc.Austr. 3, 120
Morris D., Graham D.A., Sieber W., 1981, A&A 100, 107
Petrova S.A., 2000, A&A 360, 592
Petrova S.A., 2001a, MNRAS 324, 931
Petrova S.A., 2001b, A&A 378, 883
Petrova S.A., 2002, A&A 383, 120
Petrova S.A., Lyubarskii Y.E., 2000, A&A 355, 1168
Radhakrishnan V., Cooke D.J., 1969, Astrophys.Lett. 3, 225
Radhakrishnan V., Rankin J.M., 1990, ApJ 352, 258
Rankin J.M., 1983a, ApJ 274, 333
Rankin J.M., 1983b, ApJ 274, 359
Y.E. Lyubarsky: Coherent Radio Emission from Pulsars

Rankin J.M., 1990, ApJ 352, 247
Rankin J.M., 1993, ApJ 405, 285
Rowe E.T., 1995, A & A 296, 275
Ruderman M.A., Sutherland P.G., 1975, ApJ 196, 51
Scharlemann E. T., 1974, ApJ 193, 217
Schopper R., Kunzl T.A., Ruhl H., Lesch H., 2002, these proceedings
Sieber W., 1997, A&A 321, 519
Sieber W., Reinecke R., Wielebinski R., 1975, A&A 38, 169
Smirnova T. V., Shishov V. I., 2001, Ap&SS 278, 71
Smirnova T. V., Shishov V. I., Malofeev V. M., 1996, ApJ 462, 289
Stinebring D.R., Cordes J.M., Rankin J.M., Weisberg J.M., Boriakoff V., 1984a, ApJS 55, 247
Stinebring D.R., Cordes J.M., Rankin J.M., Weisberg J.M., Boriakoff V., 1984b, ApJS 55, 279
Toner H.A., Tsytovich V.N., 1981, Phys.Rep. 73, 175
Thorsett S. E., 1991, ApJ 377, 263
Tsytovich V.N., 1970, Non-linear Effects in Plasma. Plenum Press
Usov V.V. & Usov V.V., 1988, Ap&SS 140, 325
Usov V.V., 1987, ApJ 320, 333
Volokitin A.S., Krasnoselskikh V.V., Machabeli G.Z., 1985, Sov.J Plasma Phys. 11, 531
von Hoensbroech A., Kijak J., Krawczyk A., 1998, A & A 334, 571
von Hoensbroech A., Lesch H., 1999, A & A 342, L57
von Hoensbroech A., Lesch H., Kunzl T., 1998, A & A 336, 209
von Hoensbroech A., Xilouris K. M., 1997, A & A 324, 981
Weatherall J. C., 1997, ApJ 483, 402
Weatherall J. C., 1998, ApJ 506, 341
Weatherall J. C., 2001, ApJ 559, 196
Wolszczan A., Cordes J. M., 1987, ApJ 320, L35
Xilouris K. M., Kramer M., Jessner A., et al., 1996, A & A 309, 481
Xilouris K.M., Kramer M., Jessner A., et al., 1998, ApJ 501, 286