Study on the Correlations between the Twin Kilohertz Quasi-periodic Oscillations in Low-mass X-ray Binaries

H. X. Yin\textsuperscript{1*}, Y. H. Zhao\textsuperscript{1}

\textsuperscript{1} National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China
Email:yhx@lamost.org

Abstract

The recently updated data of the twin kilohertz quasi-periodic oscillations (kHz QPOs) in the neutron star low-mass X-ray binaries are analyzed. The power-law fitting $\nu_1 = a(\nu_2/1000)^b$ and linear fitting $\nu_2 = A\nu_1 + B$ are applied, individually, to the data points of four Z sources (GX 17+2, GX 340+0, GX 5-1 and Sco X-1) and four Atoll sources (4U 0614+09, 4U 1608-52, 4U 1636-53 and 4U 1728-34). The $\chi^2$-tests show that the power-law correlation and linear correlation both can fit data well. Moreover, the comparisons between the data and the theoretical models for kHz QPOs are discussed.

Key words: QPOs; Accretion disks; Neutron stars; X-rays binaries.

1 Introduction

The kilohertz quasi-periodic oscillations (kHz QPOs) were firstly discovered in Sco X-1, a luminous Z source in neutron star (NS) low-mass X-ray binaries (LMXBs) (e.g. van der Klis et al. 1996), and now they have been detected in twenty more sources (e.g. van der Klis 2000, 2006, for reviews). Usually, these kHz QPOs appear in pairs, the upper kHz QPO frequency ($\nu_2$, hereafter the upper-frequency) and the lower kHz QPO frequency ($\nu_1$, hereafter the lower-frequency), which are discovered in three classes of sources, i.e. accretion powered millisecond pulsars, bright Z sources and less luminous Atoll sources (e.g., Hasinger & van der Klis 1989).

The kHz QPO peak separation, $\Delta\nu = \nu_2 - \nu_1$, in a given source generally decreases with frequency, except the recently detected kHz QPOs in Cir X-1, in which the peak separation increases with frequency (Boutloukos et al.)
In addition, the variable peak separations are not equal to the NS spin frequencies. However, the averaged peak separation is found to be either close to the spin frequency or to half of it (e.g., van der Klis 2006; Linares et al. 2005).

The above observations offer strong evidence against the simple beat-frequency model, in which the lower-frequency is the beat between the upper-frequency $\nu_2$ and the NS spin frequency $\nu_s$ (e.g. Strohmayer et al. 1996; Zhang et al. 1997; Miller et al. 1998), i.e. $\nu_1 = \nu_2 - \nu_s$. Furthermore, with the discovery of pairs of 30–450 Hz QPOs from a few black-hole candidates with the frequency ratios 3:2 (e.g., van der Klis 2006), Abramowicz et al. (2003) reported that the ratios of twin kHz QPOs in Sco X-1 tend to cluster around a value about 3:2, and they argued this fact to be a promising link with the black hole high-frequency QPOs (e.g. van der Klis 2006).

For the all Z and Atoll sources, the data plots of the upper-frequency versus the lower-frequency can be fitted by a power law function (e.g., Zhang et al. 2006a), and also roughly fitted by a linear function (Belloni et al. 2005). However, for the individual kHz QPO source, for instance Sco X-1, its kHz QPOs can be well fitted by a power law function (e.g. Psaltis et al. 1998; Yin et al. 2005).

In this paper, to investigate the twin kHz QPO correlation for the individual Z or Atoll source, we fitted the data with a power-law and a linear function for four typical Z sources and four typical Atoll sources, and a comparison of both fittings by $\chi^2$-tests is discussed in section 2, where comparisons with the models are discussed. The conclusions and consequences are given in section 3.

2 Correlations between twin kHz QPOs

Until now, twin kHz QPOs have been detected in 21 LMXBs, including 2 accretion powered millisecond X-ray pulsars, 8 Z sources and 11 Atoll sources, as listed in Tab. 1. In Fig. 1 and Fig. 2, we plotted twin kHz QPO data for the Z sources and Atoll sources, showing the correlations of $\nu_1$ vs. $\nu_2$, $\Delta \nu$ vs. $\nu_2$ and $\nu_2/\nu_1$ vs. $\nu_2$, where the power-law and linear fitting lines for the eight Z and Atoll sources are presented. The results of the fittings and $\chi^2$-tests are listed in Tab. 2.
2.1 A power law fitting

The power-law function is chosen as

\[ \nu_1 = a \left( \frac{\nu_2}{1000 \text{ Hz}} \right)^b \text{ Hz} \]  \tag{1} 

to fit twin kHz QPO data points of all Atoll (Z) sources, as well as 4 individual Atoll (Z) sources, separately. It is noted that a same function was applied to the fitting of kHz QPOs of Sco X-1 by Psaltis et al. (1998) with a smaller set of kHz QPO data points. The fitting results of the normalization coefficient \( a \), the power-law index \( b \) and \( \chi^2/d.o.f. \) for various cases are listed in Tab. 2, which correspond to the fitting curves as presented in Fig. 1. We find that the power-law index for the fitting of all Z sources (see Tab. 2) is 1.87, obviously bigger than that of the fitting for all Atoll sources (1.61). Then, for the individual case, the power-law index for Z source is generally bigger than that in Atoll source, except GX 17+2.

2.2 A linear fitting

For the same data sets, the linear fitting function is chosen as,

\[ \nu_2 = A \nu_1 + B \text{ Hz} \]  \tag{2} 

which was exploited by Belloni et al. (2005) to discuss the kHz QPO fitting in Sco X-1, 4U 1608-52, 4U 1636-53, 4U 1728-34 and 4U 1820-30. By means of the \( \chi^2 \)-tests, as shown in Tab. 2, we find that the linear fitting concordes with the data well in some cases, and there is no much systematic difference between the linear slope parameters of the Atoll sources and those of Z sources.

2.3 Comparison between the power-law and the linear correlation

As a comparison between models and the data, it is remarked that the relativistic precession model (e.g. Stella & Vietri 1999) and the Alfvén wave oscillation model (e.g. Zhang 2004; Li & Zhang 2005) both can lead to power-law relations approximately, and then the beat-frequency model (e.g. Miller et al. 1998) and the 3:2 resonance model (e.g. Abramowicz et al. 2003, this model is successfully applied to black hole candidates) predicted the linear relations between twin kHz QPO frequencies in the lowest approximation (Abramowicz et al. 2005). In Tab. 2, we can see that the \( \chi^2/d.o.f. \) of the power-law relation
is usually less than the linear one for the same source, except the two Atoll sources 4U 0614+09 and 4U 1636-53. And a linear function cannot give a firstly increasing and then decreasing tendency of all Z data as shown in Fig. 2b. But a power-law one would fit it well as shown in Fig. 1b. So, these maybe mean that a power-law correlation is better than a linear one.

2.4 Testing the constant peak separation $\Delta \nu = 300$ Hz

Since the discovery of kHz QPOs, it is known that the peak separation for Sco X-1 (van der Klis et al 1997; Méndez & van der Klis 2000) is a not constant, and the same is true for the other Z sources, e.g., GX 17+2 (Homan et al. 2002) and Cir X-1 (Boutloukos 2006). As for the Atoll sources, the peak separation of 4U 1728–34 (Migliari, van der Klis, & Fender 2003; Méndez & van der Klis 1999) is always significantly lower than the burst oscillation frequency, and the peak separation of 4U 1636–53 (Jonker, Méndez, & van der Klis 2002b; Méndez, van der Klis, & van Paradijs 1998) is varying between being lower and higher than half the spin frequency. In addition, 4U 1608-52 (Méndez et al. 1998) and 4U 1735-44 (Ford et al. 1998) are found to share the varied peak separations.

In Fig. 1b or Fig. 2b, we show that the peak separations in all Z sources decrease (increase) systematically with the upper frequency if the upper frequency is larger (less) than $\sim 700$ Hz (e.g. van der Klis 2000, 2006; Boutloukos et al. 2006). But this firstly increasing and then decreasing with frequency is not clearly found for the kHz QPO data of all Atoll sources, as shown in Fig. 1e and Fig. 2e, which perhaps is on account of the less amount of data in the low kHz QPO frequencies in Atoll sources. From Fig. 1 and Fig. 2, we find that the peak separations are scattered in a wide range of frequency for each source. Therefore, a constant peak separation, i.e. $\Delta \nu = 300$ Hz, cannot fit for these data.

Fig. 3 shows the results of $\chi^2$-tests against a general constant peak separation of the twin kHz QPOs in the 8 individual sources. The minimum $\chi^2$/d.o.f. are all with values $>> 1$, which means that any constant peak separation model cannot fit for these data anymore.

2.5 Testing the constant peak ratio $\nu_2/\nu_1 = 3/2$

From Fig. 1c (Fig. 1f) or Fig. 2c (Fig. 2f), we find that twin frequency ratios distribute in a wide range from 1.2 to 4.2, with the averaged value 1.73 (1.50) for all known Z (Atoll) data. Obviously, a constant ratio $\nu_2/\nu_1 = 3/2$, which can be applicable to some black hole QPO sources, is not consistent with
the observed NS/LMXB data. In the $\nu_2/\nu_1$ vs. $\nu_2$ plots of Fig. 1 and Fig. 2, the frequency ratios systematically decrease with the upper-frequency for both Z and Atoll sources. In detail, the incompatible 3:2 ratio peak distribution has been also studied by Belloni et al. (2005) in several sources, who showed that the distribution of QPO frequencies in Sco X-1, 4U 1608–52, 4U 1636–53, 4U 1728–34, and 4U 1820–30 is multi-peaked, with the peaks occurring at the different $\nu_2/\nu_1$ ratios.

3 Conclusions

In this paper, the updated data sets of twin kHz QPO frequencies simultaneously detected in NS LMXBs are analyzed, and the power-law and linear fittings are studied for the individual Z/Atoll and all Z/Atoll sources, respectively. Our main conclusions are presented as follows. (1) In Fig. 1 and Fig. 2, we can notice that a simple constant peak separation model, such as the beat-frequency model (e.g. Strohmayer et al. 1996; Zhang et al. 1997; Miller et al. 1998), or a constant peak ratio assumption, as in a naive extrapolation of the observed resonant frequency ratio from the black hole sources to the neutron stars (e.g. Abramowicz et al. 2003), cannot fit the observed data. Namely, any simple constant peak separation and constant peak ratio models are generally inconsistent with the data. The peak separations in all Z sources tend to increase (decrease) with the upper-frequency if the upper-frequency is less (larger) than $\sim 700$ Hz. But this tendency does not appear in all Atoll sources because of less amount of data at low kHz QPO frequency. Statistically, the twin frequency ratios tend to decrease with the upper-frequency in both Z and Atoll sources. (2) Our results show that the index of the fitted power-law relation of Z source is generally bigger than that of Atoll source, except GX 17+2. On the consideration of model, this different index value in Z and Atoll sources might be related to the diversity in their luminosity or magnetic field. However the linear correlations do not show any systematic differences between Z and Atoll sources. (3) The power-law fitting is somewhat better than the linear one for most of the sources, because the $\chi^2/d.o.f.$ value of the power-law correlation is generally less than that of linear one, and a linear correlation cannot give the firstly increasing and then decreasing tendency of peak separations in all Z data. As a comparison with model’s prediction, we mention the Relativistic Precession model (e.g. Stella & Vietri 1999) and the Alfvén Wave Oscillation model (Zhang 2004), since both models can give an approximated power-law correlation between the twin kHz QPOs, however none of them can distinguish the influences of the luminosity of Z and Atoll sources.

As a summary, if the future data still support the conclusions obtained in the paper, they will pose the meaningful constraints on the models for
explaining kHz QPOs.

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Fig. 1. Plots of a and d $\nu_1$ vs. $\nu_2$, b and e $\Delta \nu$ vs. $\nu_2$ and c and f $\nu_2/\nu_1$ vs. $\nu_2$ for Z sources and Atoll sources. Power-law fitting lines and the two reference lines ($\nu_2/\nu_1 = 3/2$, and $\Delta \nu = 300$ Hz) are presented also.
Fig. 2. Plots of a and d $\nu_1$ vs. $\nu_2$, b and e $\Delta \nu$ vs. $\nu_2$ and c and f $\nu_2/\nu_1$ vs. $\nu_2$ for Z sources and Atoll sources. Linear fitting lines and the two reference lines ($\nu_2/\nu_1 = 3/2$, and $\Delta \nu = 300$ Hz) are presented also.
Fig. 3. $\chi^2$–tests for a constant peak separation of the 8 individual sources list in Tab. 2.
| Sources | $\nu_1$ | $\nu_2$ | $\Delta\nu$ | $\nu_2/\nu_1$ | References |
|---------|---------|---------|------------|---------------|------------|
| Millisecond pulsar (2) |
| XTE J1807-294 | 127-360 | 353-587 | 179-247 | 1.51-2.78 | 1,2, |
| SAX J1808.4-3658 | 499 | 694 | 195 | 1.39 | 3 |
| Z source (8) |
| Cir X-1 | 56-226 | 229-505 | 173-340 | 2.23-4.19 | 4 |
| Sco X-1 | 544-852 | 844-1086 | 223-312 | 1.26-1.57 | B,M,K |
| GX 340+0 | 197-565 | 535-840 | 275-413 | 1.49-2.72 | B,K,P,5 |
| XTE J1701-462 | 620 | 909 | 289 | 1.47 | 6 |
| GX 349+2 | 712-715 | 978-985 | 266-270 | 1.37-1.38 | B,K,7 |
| GX 5-1 | 156-634 | 478-880 | 232-363 | 1.38-3.06 | B,K,P,8 |
| GX 17+2 | 475-830 | 759-1078 | 233-308 | 1.28-1.60 | B,K,P,9 |
| Cyg X-2 | 532 | 856.6 | 324 | 1.61 | B,K,P |
| Atoll source (11) |
| 4U 0614+09 | 153-823 | 449-1162 | 238-382 | 1.38-2.93 | B,K,P,10,11 |
| 4U 1608-52 | 476-876 | 802-1099 | 224-327 | 1.26-1.69 | M,B,K,12 |
| 4U 1636-53 | 644-921 | 971-1192 | 217-329 | 1.24-1.51 | B,K,P,13,14 |
| 4U 1702-43 | 722 | 1055 | 333 | 1.46 | K,P,15 |
| 4U 1705-44 | 776 | 1074 | 298 | 1.38 | B,K,P |
| 4U 1728-34 | 308-894 | 582-1183 | 271-359 | 1.31-1.89 | B,K,P,11,16 |
| KS 1731-260 | 903 | 1169 | 266 | 1.29 | B,K,P |
| 4U 1735-44 | 640-728 | 982-1026 | 296-341 | 1.41-1.53 | B,K,P |
| 4U 1820-30 | 790 | 1064 | 273 | 1.35 | B,K,P |
| 4U 1915-05 | 224-707 | 514-1055 | 290-353 | 1.49-2.3 | B,K,P |
| XTEJ2123-058 | 849-871 | 1110-1140 | 261-270 | 1.31-1.31 | B,K,P |

(1): the range of $\nu_1$; (2): the range of $\nu_2$; (3): the range of $\Delta\nu$; (4): the range of $\nu_2/\nu_1$. K: van der Klis 2000, van der Klis 2006; M: Méndez et al. 1998ab, Méndez & van der Klis 1999, 2000; B: Belloni et al. 2002, Belloni et al. 2005; P: Psaltis et al. 1999ab. 1: Linares 2005; 2: Zhang et al. 2006b; 3: Wijnands et al. 2003; 4: Boutloukos et al. 2006; 5: Jonker et al. 2000; 6: Homan 2006 (personal communication); 7: O’Neill et al. 2002; 8: Jonker et al. 2002a; 9: Homan et al. 2002; 10: van Straaten et al. 2002; 11: van Straaten et al. 2000; 12: van Straaten et al. 2003; 13: Di Salvo et al. 2003; 14: Jonker et al. 2002b; 15: Markwardt et al. 1999; 16: Migliari et al. 2003.
Table 2
List of the results of fittings and $\chi^2$-tests.

$$\nu_1 = a(\nu_2/(1000 \text{ Hz}))^b \text{ Hz}$$

$$\nu_2 = A\nu_1 + B \text{ Hz}$$

| Source* | $\nu_1$ | $\nu_2$ | $\chi^2$/d.o.f. | $\chi^2$/d.o.f. |
|---------|---------|---------|-----------------|-----------------|
| Sco X-1 | 721.95 ± 0.69 | 1.85 ± 0.01 | 33.9/87 | 0.765 ± 0.007 | 445.84 ± 4.73 | 54.8/87 |
| GX 340+0 | 763.85 ±38.03 | 2.12 ± 0.15 | 21.3/17 | 0.884 ± 0.067 | 374.96 ±24.41 | 27.2/17 |
| GX 5-1 | 833.02 ±26.08 | 2.26 ± 0.10 | 29.7/27 | 0.828 ± 0.039 | 379.54 ±15.10 | 43.7/27 |
| GX 17+2 | 723.40 ± 5.53 | 1.56 ± 0.06 | 11.2/19 | 0.906 ± 0.038 | 341.13 ±24.27 | 12.0/19 |
| All Z | 725.38 ± 2.50 | 1.87 ± 0.02 | 205.8/169 | 0.924 ± 0.012 | 336.22 ± 6.81 | 380.1/169 |

**Atoll source**

| Source* | $\nu_1$ | $\nu_2$ | $\chi^2$/d.o.f. | $\chi^2$/d.o.f. |
|---------|---------|---------|-----------------|-----------------|
| 4U 0614+09 | 673.74 ± 4.77 | 1.49 ± 0.06 | 80.1/40 | 1.002 ± 0.033 | 320.75 ±20.37 | 68.4/40 |
| 4U 1608-52 | 717.36 ± 4.89 | 1.81 ± 0.08 | 7.8/16 | 0.781 ± 0.036 | 436.60 ±25.12 | 9.1/16 |
| 4U 1636-53 | 685.16 ±15.44 | 1.72 ± 0.16 | 10.7/11 | 0.737 ± 0.064 | 505.18 ±53.51 | 10.1/11 |
| 4U 1728-34 | 667.86 ± 5.59 | 1.51 ± 0.07 | 20.9/23 | 0.997 ± 0.046 | 329.33 ±32.09 | 25.5/23 |
| All Atoll | 683.48 ± 3.01 | 1.61 ± 0.04 | 293.3/109 | 0.912 ± 0.020 | 371.08 ±13.91 | 308.7/109 |