Detectability of orbital motion in stellar binary and planetary microlenses

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ABSTRACT

A standard binary microlensing event light curve allows just two parameters of the lensing system to be measured: the mass ratio of the companion to its host and the projected separation of the components in units of the Einstein radius. However, other exotic effects can provide more information about the lensing system. Orbital motion in the lens is one such effect, which, if detected, can be used to constrain the physical properties of the lens. To determine the fraction of binary-lens light curves affected by orbital motion (the detection efficiency), we simulate light curves of orbiting binary star and star–planet (planetary) lenses and simulate the continuous, high-cadence photometric monitoring that will be conducted by the next generation of microlensing surveys that are beginning to enter operation. The effect of orbital motion is measured by fitting simulated light-curve data with standard static binary microlensing models; light curves that are poorly fitted by these models are considered to be detections of orbital motion. We correct for systematic false positive detections by also fitting the light curves of static binary lenses. For a continuous monitoring survey without intensive follow-up of high-magnification events, we find the orbital motion detection efficiency for planetary events with caustic crossings to be 0.061 ± 0.010, consistent with observational results, and 0.0130 ± 0.0055 for events without caustic crossings (smooth events). Similarly, for stellar binaries, the orbital motion detection efficiency is 0.098 ± 0.011 for events with caustic crossings and is 0.048 ± 0.006 for smooth events. These result in combined (caustic-crossing and smooth) orbital motion detection efficiencies of 0.029 ± 0.005 for planetary lenses and 0.070 ± 0.006 for stellar binary lenses. We also investigate how various microlensing parameters affect the orbital motion detectability. We find that the orbital motion detection efficiency increases as the binary mass ratio and event time-scale increase, and as the impact parameter and lens distance decrease. For planetary caustic-crossing events, the detection efficiency is highest at relatively large values of semimajor axis ~4 au, due to the large size of the resonant caustic at this orbital separation. Effects due to the orbital inclination are small and appear to significantly affect only smooth stellar binary events. We find that, as suggested by Gaudi, many of the events that show orbital motion can be classified into one of the following two classes. The first class, separational events, typically show large effects due to subtle changes in resonant caustics, caused by changes in the projected binary separation. The second class, rotational events, typically show much smaller effects, which are due to the magnification patterns of close lenses exhibiting large changes in angular orientation over the course of an event; these changes typically cause only subtle changes to the light curve.

Key words: gravitational lensing: strong – gravitational lensing: micro – planets and satellites: general – Galaxy: general.

1 INTRODUCTION

The current gravitational microlensing surveys, OGLE (Udalski 2003) and MOA (Hearnshaw et al. 2005), discover ~700 unique microlensing events per year, of which, of the order of 10 per cent.
show signatures of lens binarity. A small fraction of these, those with a high probability of planet detection, are followed up by a number of follow-up teams, which intensively monitor the events for the signatures of planets. In the coming years, this strategy will be augmented and extended by a strategy of continuous, high-cadence surveys performed by a global network of wide field telescopes. Such a network will monitor all the microlensing events it discovers with a cadence similar to that achieved by the follow-up networks for a handful of events today.

The light curve of a standard static binary lens, in which the lens components are fixed and the source follows a straight path, can be described by a minimum of seven parameters. Only three of these parameters contain physical information about the lens system. Two are dimensionless parameters: the mass ratio \( q \) and the projected separation of the lens components \( d \), measured in units of the Einstein radius. The third, the Einstein time-scale of the event, \( t_E \), is the time taken for the source to cross one Einstein radius:

\[
    t_E = \frac{r_E}{v_s},
\]

where \( v_s \) is the relative projected lens-source velocity and \( r_E \) is the Einstein radius. This is defined as

\[
    r_E = \left( \frac{4G}{c^2} \right) x (1 - x) D_s D_l M,
\]

where \( x = D_l/D_s \) is the ratio of the lens distance \( D_l \) to the source distance \( D_s \) and \( M \) is the total mass of the binary. Of the other four parameters, three are purely geometrical and the final parameter is the unlensed source flux.

The mass ratio and separation are closely related to the most interesting properties of the binary, the component masses and the semimajor axis of the orbit. They can be measured very accurately from a light curve, but only describe the binary’s properties in terms of ratios relative to the typical physical scales of the system. The Einstein crossing time-scale, \( t_E \), contains information on these scales, but this information is wrapped up in a three-fold degeneracy (the so-called microlensing degeneracy) between the total binary mass, the lens distance and the source velocity. It is also dependent on the source distance, but this is usually well constrained by measurements of the baseline flux. To gain any more knowledge of the lens system requires that this degeneracy be broken, either by the detection of higher order effects in the event light curve or by the detection of the lens flux and proper motion as the lens and source separate.¹ These detections yield measurements of the lens distance and source velocity, respectively, allowing the lens mass to be solved for (Gould & Loeb 1992; Bennett et al. 2006). Higher order effects, such as finite source effects (Gould 1994; Nemicoff & Wickramasinghe 1994; Witt & Mao 1994; Alcock et al. 1997) and microlensing parallax (Refsdal 1966; Gould 1992; Alcock et al. 1995), allow the microlensing degeneracy to be broken or reduced through measurement or constraining some of the parameters that are combined in \( t_E \). For example, detections of finite source effects and microlensing parallax in the same event yield two independent measurements of the angular Einstein radius \( \theta_E = r_E/D_s \), which allow the source velocity and lens distance to be eliminated, and the lens mass determined (e.g. An & Gould 2001).

Orbital motion of the binary lens is another such higher order effect. If the binary-lens components are gravitationally bound, then they will orbit each other and their projected orientation will change as a microlensing event progresses. As the magnification pattern produced by a binary lens is not rotationally symmetric, the change in orientation may be detectable in the light curve of the event. If the orbit is inclined relative to the line of sight, then the projected separation of the lens components will also evolve, causing changes in the structure of the magnification pattern, which again may be detectable. In a small fraction of binary microlensing events, we can expect to see the effects of this orbital motion in their light curves, though this is the first work that attempts to quantify this fraction. If orbital motion can be detected in a microlens, then it can provide constraints on the mass of the lens and information about the binary orbit.

To date, six binary microlensing events have shown strong evidence of orbital motion in the lens system. The first, MACHO-97-BLG-41, was a stellar mass binary. Modelling of the event was only able to measure the change in the projected angle and separation of the binary in the time between two caustic encounters, but was unable to constrain the orbital parameters (Albrow et al. 2000). The second event, EROS-BLG-2000-5, had a very good light-curve coverage, which allowed the measurement of the rates of change of the binary’s projected separation and angle; these measurements were then used to obtain a lower limit of the orbit’s semimajor axis and an upper limit on the combined effect of inclination and eccentricity (An et al. 2002). The third and fourth examples, OGLE-2003-BLG-267 and OGLE-2003-BLG-291, both seem to show orbital motion effects (Jaroszynski et al. 2005). However, only OGLE survey data were used in their analysis, without follow-up measurements, so the light-curve coverage was not ideal. Combined with parallax measurement, the masses of both binary lenses were constrained, but no constraints could be placed on the orbits (Jaroszynski et al. 2005).

In each of these four cases, the ratio of the component masses is large (near unity), indicative of the lens systems being binary stars; however, orbital motion has recently been measured in two events involving planetary mass secondaries. After this paper was submitted, two further events have been shown to display orbital motion effects: OGLE-2005-BLG-153 (Hwang et al. 2010) and OGLE-2009-BLG-092/MOA-2009-BLG-137 (Ryu et al. 2010).

OGLE-2006-BLG-109 was an event involving a triple lens, with analogues of Jupiter and Saturn orbiting an \( \sim 0.5 M_\odot \) star (Gaudi et al. 2008). The light curve of the event had extremely good coverage and showed multiple features, allowing the orbital motion of the Saturn analogue to be detected. The detection was so strong that the semimajor axes of both planets could be strongly constrained (Gaudi et al. 2008). A more complete analysis of the event, incorporating measurements of the lens flux and orbital stability constraints, carried out by Bennett et al. (2010), tightly constrained four out of six Keplerian orbital parameters of the Saturn analogue and weakly constrained a fifth. The planet OGLE-2005-BLG-071Lb is an \( \sim 4 M_{\text{Jupiter}} \) planet orbiting an \( \sim 0.5 M_\odot \) star (Udalski et al. 2005). Measurements of the orbital motion in this event have allowed some constraints to be placed on the planet’s orbit (Dong et al. 2009). In all six events, other higher order effects have also been detected, most notably microlens parallax and finite source effects, which have allowed the measurement of the lens mass.

Despite these detections, there has been relatively little theoretical work on orbital motion in microlensing, likely due to the traditional assumption that the effects of orbital motion on a binary microlens light curve will be small and in most cases negligible (e.g. Mao & Paczyński 1991). The problem was first considered in detail by Dominik (1998) who concluded that in most microlensing events, the effects of lens orbital motion were likely to be small, though in some cases, light curves could be dramatically different. Dominik

¹ Throughout we will use the terms lens motion and source motion interchangeably.
(1998) points out that the effect is most likely to be seen in long-duration binary microlensing events with small projected binary separations. Ioka, Nishi & Kan-Ya (1999) also studied the problem and pointed out that the effect of binary-lens rotation is likely to be important in self-lensing events in the Magellanic Clouds. Rattenbury et al. (2002) showed that orbital motion could affect the planetary signatures seen in high-magnification events.

The six microlensing events that display orbital motion make up a significant fraction of the few tens of binary microlensing events that have been modelled (e.g. Alcock et al. 2000; Jaroszynski 2002; Jaroszynski et al. 2004, 2006; Skowron et al. 2007), which begins to shed doubt on the previous conclusion that lens orbital motion is likely to be unimportant in most binary events. The two planetary events constitute approximately 15 per cent of the entire published microlensing planet population. These observations motivate us to revisit the question: how likely are we to see lens orbital motion in a microlensing event? This question is made especially pertinent in the context of the next generation of high-cadence microlensing surveys, which will make the exquisite light-curve coverage of EROS-BLG-2000-5 and OGLE-2006-BLG-109 the norm rather than the exception. To gain a better understanding of how frequently orbital motion affects microlensing light curves, we simulate a large number of microlensing events caused by orbiting binary lenses. We also investigate the factors that affect this frequency.

The structure of this work is as follows. In Section 2, we will review the basic theory of binary microlensing and the effects of orbital motion on such lensing systems. Section 3 describes our simulations of microlensing events and Section 4 describes how we measure the effects of orbital motion. In Section 5, we present the results of the simulations. We discuss the results in Section 6 and conclude in Section 7.

2 MICROLENSING WITH ORBITING BINARIES

2.1 Binary microlensing

The lens equation of a binary point-mass gravitational lens describes the mapping of light rays from the source plane to the image plane and can be written in complex form (Witt 1990) as

\[ z_s = z - \frac{1}{1 + q} \left( \frac{1}{z - z_1} + \frac{q}{z - z_2} \right) \]

where \( z_1 = x_1 + iy_1 \) is the complex coordinate in the source plane, \( z = x + iy \) is the complex coordinate in the image plane, \( q \leq 1 \) is the mass ratio of the secondary mass to the primary, \( z_1 = x_1 + iy_1 \) and \( z_2 = x_2 + iy_2 \) are the complex coordinates of the primary and secondary lenses, respectively, and bars represent complex conjugation. All lengths have been normalized to the Einstein radius of the total lensing mass. The positions of the images \( z \) for a given source position are found by solving this equation, which can be rearranged into a fifth-order complex polynomial in \( z \). The total magnification of the images is given by the ratio of image areas to the source area. This information is contained in the Jacobian of the lens mapping \( J \), and the magnification is given by

\[ A = \frac{1}{J}. \]

The symbol \( x \) used here should not be confused with that representing the ratio of lens to source distances. Its meaning should be clear from the context in which it is used and it will not be used again in this context without a subscript.

\[ det J = \frac{1 + q}{(z - z_1)^2 + \frac{q}{(z - z_2)^2}} \]

where \( det J \) is the determinant of the Jacobian and is given by

\[ A = \frac{1}{det J}, \]

\[ \phi = \frac{1}{2q} \left( 1 - \frac{1}{d_c} \right)^3, \]

which separates regions of close and resonant topology, and

\[ d_w^2 = \frac{1 + q}{1 + q}, \]

which separates regions of resonant and wide topology.

Figure 1. Plot showing the \( d-q \) plane separated into three regions, where the caustics take on close, resonant and wide topologies, with increasing \( d \); the dashed lines, \( d_c(q) \) and \( d_w(q) \), separate the regions of different topologies. The solid lines are examples of each of the caustic topologies, drawn to the same scale, for a binary lens with \( q = 0.1 \), and \( d = 0.7 \) (close), 1.05 (resonant) and 1.75 (wide). The filled circles show the positions of the lenses for each topology, with the primary (more massive) lens being positioned leftmost in all cases. After Cassan (2008); figure 1.
2.2 Orbital motion in a binary microlens

The light curve of a microlensing event can be considered as a one-dimensional probe, by the source, of the two-dimensional magnification pattern produced by the lens. The magnification pattern of a single lens is rotationally symmetric about the position of the lens, but the magnification pattern of a binary lens is more complicated, containing strong caustic structures that exhibit a reflectional symmetry about the binary axis, the axis connecting the lens components. However, far away from the caustics, the magnification pattern can resemble that of a single lens.

As the lens components orbit each other, their position angle and their projected separation can change. These changes cause changes in the orientation and structure of the magnification pattern, respectively. It is clear, however, that only if the source traverses regions of the magnification pattern that differ significantly from that of a single lens, will it be possible to detect these effects of orbital motion. For the effects to be measurable, the light curve of the event must be affected in a significant way that is not reproducible by a static binary-lens model. It is also possible to detect the effect of orbital motion by showing that a static model is less physically plausible than an orbiting model, but this will usually require further information about the event, such as an independent constraint on the lens mass.

The effects of orbital motion on a light curve can also be mimicked by other higher order effects, especially parallax and xallarap. Parallax effects are caused by the motion of the earth about the sun and cause the source to take an apparently curved path through the magnification pattern (e.g. Smith, Mao & Paczyński 2003). In the case of xallarap, the source travels along a curved path through the magnification pattern as a result of binary orbital motion in the source system (Griest & Hu 1992; Paczyński 1997; Dominik 1998; Rahvar & Dominik 2009). These curved paths can look very similar to those taken by the source in the rotating binary-lens centre-of-mass frame and hence it can sometimes be difficult to identify the true cause of the effect.

3 SIMULATING A HIGH-CADENCE MICROLENSING SURVEY

The primary aim of this study is two-fold: first, to determine the fraction of microlensing events that will be affected by orbital motion, as seen by the next-generation microlensing surveys; and secondly, to investigate the factors that affect the detectability of orbital motion, to aid in the targeting of such events without resorting to exhaustive modelling efforts. To achieve the first goal, various factors that go into the observation of a microlensing event should be simulated, accurately modelling the observing setup, the distribution of planetary and binary star–lens systems, and the distribution of the sources and lenses throughout the Galaxy. To achieve the second goal, we must simplify the parameter space, we investigate, as far as possible, without removing essential elements from the model, so as to allow a clear interpretation of the results.

To balance these somewhat contradictory requirements, we choose to accurately simulate ideal photometry and use a semiréalistic model of the Galaxy, while investigating a logarithmic distribution of companion masses and separations. This allows us to use our simulations to gain a good order of magnitude estimate of the results expected from future surveys, whilst simultaneously investigating the factors that have the largest impact on the detection of orbital motion over a relatively uniform parameter space.
stars in the field. The amount of blending can be quantified by a blending fraction $f_b$, which we define to be the fraction of the total flux of the observed blend that the source contributes when unmagnified, such that the time-dependent magnitude of the blend is

$$I(t) = I_b - 2.5 \log[f_b A(t) + (1 - f_b)],$$

(13)

where $I_b$ is the baseline magnitude of the observed blend when the source is unmagnified and $A(t)$ is the magnification caused by the lens.

The distribution of baseline magnitudes and blending fractions is drawn from simulations of blending effects by Smith et al. (2007) who perform photometry on mock images of typical Galactic bulge fields with high stellar density. Specifically, we calculate the blending fraction and baseline magnitude for each event from the input and output magnitudes of source stars drawn from their simulation with 1.05 arcsec seeing and input stellar density of 133.1 stars arcmin$^{-2}$, before any detection efficiency cuts are made to the catalogue. As the phenomenon of negative blending, the source apparently contributing a fraction $f_b > 1$ to the total flux of the blend (Park et al. 2004; Smith et al. 2007), is poorly understood, we only include sources with moderate negative blending, requiring that $f_b < 1.2$.

The mock images are produced by Smith et al. (2007) using the method of Sumi et al. (2006), drawing stars from the Hubble Space Telescope I-band luminosity function of Holtzman et al. (1998), adjusted to account for denser fields and brighter stars using OGLE data. Extinction was accounted for using the extinction maps of Sumi (2004) and the baseline magnitudes were measured using the standard OGLE pipeline based on $I_{phot}$ (Schechter, Mateo & Saha 1993) (for further details, see section 3 of Smith et al. 2007, and references therein).

The lens systems are composed of a primary of mass $M_1$ and secondary of mass $M_2$. The primary’s mass is drawn from a broken power-law distribution

$$\frac{dn}{dM_1} \propto M_1^{\alpha+0.5},$$

$$\alpha = \begin{cases} -1.3 & M_1 \leq M_{break} \\ -2.0 & M_1 > M_{break} \end{cases},$$

with lower and upper limits of 0.05 and 1.2 $M_\odot$, respectively, and where $M_{break} = 0.5 M_\odot$. The addition of 0.5 to the power-law index is to account for the dependence of the microlensing event rate on the mass of the lens. We do not include a population of stellar remnant lenses, such as white dwarfs, neutron stars and black holes. The mass ratio $q$ of the secondary to the primary is drawn from a logarithmic distribution, with limits $10^{-2} \leq q < 1$ for stellar binary lenses and $10^{-5} \leq q < 10^{-2}$ for planetary lenses. Note that for lower mass primaries, the distribution of stellar binary mass ratios does include secondaries with masses as low as $\sim 5 M_{Jupiter}$, well into the planetary mass regime, and the lower limit of the planetary mass ratio distribution implies a secondary of $\sim 1 M_\oplus$ for a 0.3 $M_\odot$ primary.

The components of the lens orbit their combined centre of mass in Keplerian orbits, of semimajor axis $a$, distributed logarithmically over the range $0.1 \leq a \leq 20$ au. These orbits are inclined to the line of sight, with inclination angles distributed uniformly over a sphere. For binary stars, we performed two sets of simulations, one with zero eccentricity $e$ and another with bound, eccentric orbits with eccentricities distributed uniformly over $0 \leq e < 1$.

The source trajectories were parametrized by the angle of the source trajectory relative to the binary axis $\theta_0$, at the time of closest approach $t_0$, and the impact parameter $u_0$, the projected source–lens separation in units of Einstein radii at $t_0$. We set $t_0 = 0$, for simplicity, and $\theta_0$ and $u_0$ were distributed uniformly over the ranges $0 \leq \theta_0 < 2\pi$ and $-1.5 \leq u_0 < 1.5$, respectively.

### 3.3 Simulation of photometry

In the hunt for planets, the proposed next generation of microlensing surveys will consist of a (potentially homogeneous) network of telescopes located throughout the Southern hemisphere, such that the target fields in the Galactic bulge can be monitored continuously during the times when the bulge is observable. The telescopes will have diameters between 1.3–2.0 m and fields of view between 1.4–4.0 deg$^2$. They will operate at a cadence of approximately 10 min and are expected to discover several thousand microlensing events per year. An example is KMTNet, a network of three identical 1.6-m telescopes due to enter operation in 2014 (Kim et al. 2010). Such surveys can operate effectively without the need for intensive follow-up observations due to their high cadence and continuous coverage. However, it is likely that the survey/follow-up observing paradigm will persist, with low-cadence surveys monitoring far larger areas of sky.

Unfortunately, the effects of the weather, amongst other things, make completely continuous, high-cadence observations unachievable in reality. Rather than including complicated models of these effects, as well as other effects, such as the lunar cycle and their effects on the photometry, we instead choose a simpler prescription. Each event is monitored with continuous photometry at a reduced cadence of 30 min. These observations are performed by telescopes with 1.3 m effective diameter observing in the $I$ band. For each exposure of 120 s, the seeing is chosen from a lognormal distribution with mean 1.2 arcsec and standard deviation 0.25 arcsec, and a background flux distributed as

$$F = 8500 \text{LN}(1.5, 0.4) \text{ photon arcsec}^{-2},$$

(15)

which is integrated over a seeing disc, where LN($\mu, \sigma$) is a lognormal distribution with mean $\mu$ and standard deviation $\sigma$. New values of seeing and background flux are chosen for each observation. A lower limit on the photometric accuracy is imposed by adding a Gaussian noise component, with dispersion 0.3 per cent, to the photon counts, which are calculated by adding 10 photon m$^{-2}$s$^{-1}$ reaches the observer from an $I = 22$ source.

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To ensure that all the features of a light curve are covered, and that there is a good balance between the baseline, peak and features of the light curve when fitting (see the next section), the light curve is monitored continuously over the times $-5t_b \leq t - t_0 < 5t_b$, and over $10.5t_b \leq |t - t_0| < 9.5t_b$ to sample the baseline. To ensure that all features are covered, if the magnification of the source rises above $A \geq A_{\text{thresh}} = 1.0062$, the coverage is extended so as to be continuous within one Einstein time-scale of the feature and continuous between the feature and $t = t_0$. Fig. 3 shows an example of a light curve where coverage had to be extended.

4 MEASURING ORBITAL MOTION

Ultimately, we are interested in finding the fraction of binary microlensing events, which show signs of orbital motion. To do this, we must classify the events, we simulate, into those binary events that do show orbital motion, those that do not and events that do not show binary signatures. To do this, we fit each event first with a single-lens model and then we fit those events, which are poorly fitted with this model, with a static binary-lens model. To evaluate the effectiveness of each stage of the fitting process, in addition to the fitting of the light curves simulated with orbiting binary lenses, we must also fit light curves simulated with point-mass lenses and static binary lenses.

4.1 Light-curve modelling

The single-lens model has five parameters: the time of closest approach $t_0^s$, the event time-scale $t_b^s$, the impact parameter $u_0^s$, the baseline magnitude $I_b^s$ and the blending fraction $f_b^s$. We performed a $\chi^2$ minimization using the MINUIT routine from CERNLIB (James & Roos 1975), with all parameters free; all parameters were unconstrained, except for $f_b^s$, which was constrained to be within $0.0 < f_b^s < 1.2$. For each event, seven single-lens fits were performed, with different initial blending fractions, $f_b^s$ = 0.05, 0.2, 0.4, 0.6, 0.8, 1.0 and 1.2. For each fit, the initial guesses for each parameter were: $t_0^s = 0$, the time-scale was the true time-scale, the baseline magnitude was taken to be the magnitude of the first data point on the light curve, and the impact parameter was chosen such that at $t = t_0^s$ the magnitude of the event would be that of the brightest data point. This prescription works well for events which are well modelled by a single-lens model, but not so for events with strong binary features or events which are heavily blended and barely rise above the baseline. It is therefore useful to eliminate events falling into the latter category before performing the fitting, such that the only events that the single-lens model fails to fit are ones that show genuine signs of lens binary. This cut will be described in the next section.

To fit the binary-lens light curves, we found it necessary to split the events into caustic-crossing events and non-caustic-crossing events, and to fit each category using a different parametrisation. The non-caustic-crossing events were fitted with a standard parametrisation, with a reference frame centred on the primary lens. The parameters are the time of closest approach to the lens primary $t_0^b$, the event time-scale $t_b^b$, the impact parameter between the lens primary and the source $u_0^b$, the angle of the source trajectory to the binary axis $\theta_0^b$, the logarithm of the projected binary separation $\log d^b$, the logarithm of the normalized secondary mass $\log m_2^b$, the baseline magnitude $I_b^b$ and the blending fraction $f_b^b$. For brevity, we introduce the vector notation

$$p^s = (t_0^s, t_b^s, u_0^s, \theta_0^s, \log d^b, \log m_2^b, t_b^s, f_b^s),$$

(16)

to represent the parameter set of the standard binary parametrisation.

For the number of light curves necessary to obtain a good statistical sample, a full search of the full binary-lens parameter space is not computationally feasible, so we perform just one minimization per light curve. We must therefore pay special attention to the choice of initial guesses we use, first, so as to maximize the chance of finding a good minimum, and secondly, so as to treat the fitting of the static binary events comparably to the orbiting binary events. The static binary simulations are drawn from the same distributions as the orbiting binary simulations, the only difference being that the lens is frozen in the state it would be in at $t = t_0$.

As we have simulated the microlensing events, we already have a perfect knowledge of the systems and we can use this knowledge to obtain a good set of initial guesses. We note that at a given time, the state of an orbiting binary lens can be described by a static binary model. We can therefore describe our lens at time $t$ using the time-dependent parameter set

$$p(t) = (t_0, t_b, u_0, \theta_0(t), d(t), q, \theta_0, f_b),$$

(17)

where we have used the same definitions and centre-of-mass reference frame as in the previous section. Note that only two of the parameters are time-dependent and so we can use the true values of the constant parameters as initial guesses, having applied the appropriate coordinate transformations. However, we are still left with the problem of choosing the guesses of $\theta_0^b$ and $d^b$. We could choose $\theta_0(t_0)$ and $d(t_0)$, but this would bias the fitting-success probability unfairly towards static binary events: the initial guess would be the
actual model used to simulate the data. Instead, we choose to use \( d(t_f) \) and \( \theta_0 (t_f) \), where \( t_f \) is the time of a feature in the light curve. We define a feature simply as any maximum in the light curve, or a maximum or minimum in the Paczyński residual (the residual of the true light curve with respect to the best-fitting single-lens model) with \( I - I_{0\text{pk}} > 0.1 \), where \( I \) is the I-band magnitude of the true model and \( I_{0\text{pk}} \) is the I-band magnitude of the best-fitting Paczyński model. As there is in general more than one feature, we choose the feature that gives the best \( \chi^2(p(t_f)) \). If the initial guesses for fits to static binary light curves are chosen in the same way, as if the binary were orbiting, then the initial guesses for static lenses should be worse than for orbiting lenses, as at the time of the chosen feature, the true orbiting lens magnification will exactly match the magnification of the initial guess static model. In reality, for \( t_i \approx t_0 \), there will likely be a bias in favour of static lenses and for \( t_i \neq t_0 \), there will be a bias in favour of orbiting lenses, but we do not believe this will affect results significantly. To fit the events, we again use the MINUIT minimizer, allowing all parameters to vary. All parameters are unconstrained, except for \( f_C^2 \), which is constrained to the range \( 0 < f_C^2 < 1.2 \).

While this method was suitable for events which showed smooth binary features, it is not always suitable for those events which exhibit caustic crossings. For these events, in addition to fitting with the standard parametrization, we also used the alternative parametrization of Cassan (2008). This replaces the parameters specifying the source trajectory \( (t_{\text{en}}^C, t_{\text{ex}}^C, u_{\text{en}}^C, u_{\text{ex}}^C, \theta_{\text{en}}^C, \theta_{\text{ex}}^C) \) with parameters that better reflect the sharp caustic-crossing features of the light curve \( (t_{\text{en}}^C, t_{\text{ex}}^C, s_{\text{en}}^C, s_{\text{ex}}^C) \), the times of a caustic entry and exit, and the positions of the entry and exit on the caustic, respectively, where \( s_{\text{en}}^C \) and \( s_{\text{ex}}^C \) are defined to be the chord length along the caustic, normalized such that \( 0 \leq s_{\text{en}}^C < 2 \) and \( 0 \leq s_{\text{ex}}^C < 2 \). Full details of the parametrization can be found in Cassan (2008). The parameter set we use for caustic-crossing events is therefore

\[
p^C = (t_{\text{en}}^C, t_{\text{ex}}^C, s_{\text{en}}^C, s_{\text{ex}}^C, \log d^C, \log q^C, t^C_0, f^C_0),
\]

where the parameter \( \log m^2 \) has been replaced by \( \log q^C \) as a matter of preference; the two parameters are related by \( m^2 = q^C/(1+q^C) \).

The accurate calculation of the \( s_{\text{en}}^C \) and \( s_{\text{ex}}^C \) parameters is quite computationally expensive, compared to the calculation of a light curve, and needs to be repeated each time \( d \) or \( q \) changes. Also, despite the improved parametrization, the \( \chi^2 \) surface is still very complicated, especially in the \( s_{\text{en}}^C-s_{\text{ex}}^C \) plane, containing many local minima. For these reasons, we pursue a three-stage minimization process. We begin by conducting a grid search over the entire \( s_{\text{en}}^C-s_{\text{ex}}^C \) plane, with \( 128 \times 128 \) points spaced evenly in \( s_{\text{en}}^C, s_{\text{ex}}^C \) and with all other parameters, including the caustic-crossing times, fixed at their true values, except for \( \log d^C \). \( \log d^C \) is fixed at a random value chosen in from the range \( \Delta \log d^C = 1.5[\log (d_{\text{en}}) - \log (d_{\text{ex}})]\) or \( \Delta \log d^C = 0.015 \), whichever is greater, centred on the midpoint of \( \log d \) between the caustic crossings, where \( d_{\text{en}} \) and \( d_{\text{ex}} \) are the projected separations at the caustic entry and exit times, respectively. The range of \( \Delta \log d^C \) is truncated, if necessary, to ensure that it only covers the caustic topologies at the time of the crossings. For the static lenses, \( \log d^C \) is chosen from a uniform distribution with the same range as if the lens were orbiting. The grid search is then refined by performing a second \( 128 \times 128 \) grid search over a box of side length 1/32 about the grid point with the lowest \( \chi^2 \). Six \( 2 \times 128 \times 128 \) grid searches are performed with different random values of \( \log d^C \). In cases where there are multiple caustic crossings, different pairs of caustic crossings are used to define \( (t_{\text{en}}^C, t_{\text{ex}}^C, s_{\text{en}}^C, s_{\text{ex}}^C) \) for each grid search. Fig. 4 shows an example light curve where the first caustic exit defines \( (t_{\text{en}}^C, s_{\text{en}}^C) \) and the second caustic entry defines \( (t_{\text{ex}}^C, s_{\text{ex}}^C) \).

The second stage of the fitting simply polishes the result of the grid search by performing a minimization over \( s_{\text{en}}^C, s_{\text{ex}}^C \), \( t_{\text{en}}^C \), \( t_{\text{ex}}^C \), \( f^C_0 \), and all other parameters fixed, using MINUIT. In the final stage of the fitting, all parameters, except for \( f^C_0 \), are allowed to vary in a further minimization. Again, all parameters were unconstrained, except for \( f^C_0 \), which was constrained to the range \( 0 < f^C_0 < 1.2 \).

We found that, at all stages of the minimization for caustic-crossing events, the minimization is performed better when the first and last data points inside the caustic crossing were not considered in the fit. This is because, with the high-cadence observations that we simulate, the point source is typically very close to the inside of the fold caustic, and hence is magnified by many orders of magnitude. This leads to unrealistic photometry in two ways: first, in a real detector, saturation would become a problem, and secondly, a real, finite, source would not be magnified in such an extreme way.

4.2 Classification of events

With the modelling procedures in place, we now describe the classification of the events. The classification is performed by a series of cuts based on the \( \chi^2 \) results of the fitting described in the last section. The first cut, the variability cut, removes events, which do not show significant variability from the analysis. This is done, without fitting, by comparing the \( \chi^2 \) values of the simulated data relative to the true model, \( \chi_{\text{OM}}^2 \), and relative to a constant light curve with no variability at the true baseline magnitude, \( \chi_b^2 \). We exclude events that do not satisfy

\[
\frac{\Delta \chi_b^2}{\chi_b^2} = \frac{\chi_{\text{OM}}^2 - \chi_{\text{OM}}^2}{\chi_b^2} > 0.3,
\]

where \( n_{\text{obs}} \) is the number of observations.

The second cut is used to classify events into single-lens-like events and binary-lens-like events or events that do not and do exhibit binary-lens features in their light curves. Using the results of the single-lens modelling, \( \chi_{\text{PC}}^2 \), \( \chi^2 \) of the simulated data with respect to the single-lens model, we define events that satisfy

\[
\Delta \chi_{\text{PC}}^2 \equiv \chi_{\text{PC}}^2 - \chi_{\text{OM}}^2 > 200
\]
to be binary events and those that do not be single events. Binary events can then be split into caustic-crossing binary events and smoothly varying events or caustic-crossing and smooth events, respectively. We define a caustic-crossing event as one where at least one data point is measured when the source is inside a caustic.\footnote{The removal of data points in the fitting process does not affect the classification.}

The final cut is based on the result of light-curve fitting with binary models. Events that satisfy

$$\Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{OM}} > 200$$  \hspace{1cm} (21)

are classified as events that exhibit orbital motion (orbital motion events) and those that do not are classified as static events, where $\chi^2$ is taken to be the $\chi^2$ of the best-fitting static binary model. For smooth events, this is the $\chi^2$ of the best-fitting standard binary model, and for caustic-crossing events, it is the $\chi^2$ of the better fitting of the Cassan (2008) caustic-crossing model or the standard binary model. In the case of the caustic-crossing fits, the data points removed from the light curve do not contribute to $\chi^2_{\text{OM}}$.

With these classifications in place, we can now define the binary detection efficiency and the orbital motion detection efficiency. The binary detection efficiency is the fraction of detectable microlensing events that show binary signatures

$$\epsilon_{\text{BS}} \equiv \frac{N_{\text{BS}}}{N_{\text{ml}}},$$  \hspace{1cm} (22)

where $N_{\text{ml}}$ is the number of events satisfying $\Delta \chi^2_{\text{BS}}/\Delta \chi^2_{\text{OM}} > 0.3$ and $N_{\text{BS}}$ is the number of events satisfying $\Delta \chi^2_{\text{BS}} > 200$. The orbital motion detection efficiency is the fraction of binary events that show orbital motion signatures:

$$\epsilon_{\text{OM}} \equiv \frac{N_{\text{OM}}}{N_{\text{BS}}}$$  \hspace{1cm} (23)

where $N_{\text{OM}}$ is the number of events satisfying $\Delta \chi^2_{\text{OM}} > 200$.

To be confident of our results, we must quantify the effectiveness of the modelling prescriptions we use. We can do this by measuring the rate of false positives in our samples. To measure these rates, we simulate both single-lens events and static binary-lens events, drawn from the same distributions as the orbiting lens events. These events then go through the same fitting procedure as the orbiting lens events and are subject to the same cuts. The binary-lens false positive rate $\epsilon_{\text{BS}}^{\text{single}}$ is therefore the fraction of detectable single-lens microlensing events that survive the $\Delta \chi^2_{\text{BS}} > 200$ cut and the orbital motion false positive rate $\epsilon_{\text{OM}}^{\text{static}}$ is the fraction of static binary-lens events that survive the $\Delta \chi^2_{\text{OM}} > 200$ cut.

5 RESULTS

5.1 What fraction of events show orbital motion?

We begin by presenting and analysing the results of the simulations as a whole, calculating the fraction of microlensing events in which we expect to see orbital motion events. Tables 1 and 2 summarize the results of the cuts described in the previous section, for planetary and stellar binary events, respectively. It should be noted that in a small number of caustic-crossing events, the fitting procedure failed and so these events have been excluded from the analysis of the orbital motion detection efficiency, but not from the analysis of the binary detection efficiency. These events are included in the Binary and Caustic rows of Tables 1 and 2, but not in the others. Fig. 5 shows some light curves, which were slightly below the threshold for each cut.

Table 1. Summary of the results for planetary lenses.

| Orbit       | Static | Circular |
|-------------|--------|----------|
| Single      | 48511  | 49226    |
| Binary      | 1364   | 1366     |
| Caustic     | 410    | 449      |
| Caustic static | 397 | 414      |
| Caustic orbital motion | 7   | 35       |
| Smooth      | 954    | 917      |
| Smooth static | 931 | 883      |
| Smooth orbital motion | 23  | 34       |

Table 2. Summary of the results for stellar binary lenses.

| Orbit       | Static | Circular | Eccentric |
|-------------|--------|----------|-----------|
| Single      | 4151   | 4046     | 4153      |
| Binary      | 1413   | 1424     | 1385      |
| Caustic     | 641    | 635      | 613       |
| Caustic static | 608 | 538      | 550       |
| Caustic orbital motion | 25  | 86       | 61        |
| Smooth      | 772    | 789      | 772       |
| Smooth static | 764 | 743      | 729       |
| Smooth orbital motion | 8   | 46       | 43        |

While in many cases we may not be able to say that a light curve in our simulations definitively shows orbital motion signatures, due to relatively high rates of false positive detections, there is a clear excess of detections in the circular and eccentric orbit simulations relative to the static ones, though the detection of this excess is only marginal in smooth planetary events. Interestingly, there appears to be a discrepancy in the orbital motion detection efficiencies for

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stellar binary caustic-crossing events. The same static orbit simulation results were used to calculate the corrected orbital motion efficiencies for both circular and eccentric orbits, which means that the measurements are not independent. Also, the eccentricity of the orbits allows the projected separation to take a wider range of values than the circular orbits, which means the false positive rate measured with the same distribution for circular orbits is likely an overestimate for eccentric orbits; for caustic-crossing events, the majority of false positives are caused by events with resonant caustic topology (see Fig. 19 later in this section). We therefore believe the discrepancy to be caused largely due to a combination of a relatively large statistical fluctuation in the number of eccentric orbit events that do show orbital motion and an overestimate of the false positive rate for eccentric orbits.

5.2 What affects the detectability of orbital motion?

We now investigate the effects that various system parameters have on the detectability of orbital motion. We look at the dependence of the orbital motion detectability on both the standard microlensing parameters and the physical orbital parameters, and compare them where appropriate. While we conducted two sets of simulations, one with circular orbits and the other with eccentric orbits, we only present the results for those with circular orbits here, as both sets are in good agreement.

We begin by looking at the dependence on the impact parameter, the sole parameter that determines the maximum magnification of a single-lens microlensing event $\Lambda_{\text{max}} = (u_0^2 + 2)/\sqrt{u_0^2 + 4}$ (Paczyński 1986). For all binary lenses, except wide stellar binaries, the central caustic is located near the centre of mass and so $u_0$ determines whether or not the source will encounter this caustic. Fig. 6 plots the orbital motion detection efficiency as a fraction of caustic-crossing or smooth binary events (top panels) and the total number of orbital motion detections (bottom panels), against the impact parameter for both planets (left-hand panels) and binary stars (right-hand panels). In the plots, red lines represent data for caustic-crossing events and blue lines for smooth events. In the top panel, the orbital motion detection efficiency has been corrected for systematic false detections as described in the previous section, whereas the bottom panel shows the number of detections for both orbiting (solid lines, filled points) and static lenses (dashed lines, open points). Note that the orbital motion detection efficiency can be negative, due to statistical fluctuations, but if it is, then the measurement should be consistent with zero. The events have been binned into bins of constant width, on the scale that they are plotted. It should also be noted that the number of planetary events simulated was a factor of 9 larger than the number of stellar binary events.

The plots of orbital motion detection efficiency (from here on detection efficiency) against $|u_0|$ for caustic-crossing events show much the same trends for both planetary and stellar binary lenses, with significant detection efficiencies for high-magnification (low $|u_0|$) events only, with no caustic-crossing planetary detections for $|u_0| \gtrsim 0.6$ and only a few for stellar binaries. This is due to the location of central and resonant caustics close to the centre of mass, which the source can only cross in events with small $|u_0|$. Consequently, for the events with larger $u_0$, the source can only cross weaker secondary caustics, which in the case of wide binaries will typically move slowly, and in the case of close binaries are typically very small and are rarely crossed. The secondary caustics

Table 3. Binary and orbital motion detection efficiencies.

| Orbit    | Circular | Eccentric |
|----------|----------|-----------|
| $q < 0.01$ | $\epsilon_{BS}$ | 0.0772 ± 0.0014 | – |
| $q < 0.01$ (caustic) | $\epsilon_{OM}$ | 0.061 ± 0.010 | – |
| $q < 0.01$ (smooth) | $\epsilon_{OM}$ | 0.0130 ± 0.0055 | – |
| $q < 0.01$ (all) | $\epsilon_{OM}$ | 0.029 ± 0.005 | – |
| $q \geq 0.01$ | $\epsilon_{BS}$ | 0.260 ± 0.004 | 0.251 ± 0.004 |
| $q \geq 0.01$ (caustic) | $\epsilon_{OM}$ | 0.098 ± 0.011 | 0.060 ± 0.010 |
| $q \geq 0.01$ (smooth) | $\epsilon_{OM}$ | 0.048 ± 0.006 | 0.045 ± 0.006 |
| $q \geq 0.01$ (all) | $\epsilon_{OM}$ | 0.070 ± 0.006 | 0.052 ± 0.006 |
Table 4. Microlensing parameters of example light curves in this paper.

| Figure | Orbita | $u_0$ | $\theta_0/\circ$ | $d_0$ | $q$ | $t_{\text{E}}/d$ | $t_0$ | $f_s$ |
|--------|--------|-------|-----------------|-------|-----|----------------|-------|-------|
| 3      | C      | 0.48  | 307             | 8.64  | 0.22| 14.9           | 17.9  | 1.04  |
| 4      | S      | −0.091| 186             | 0.95  | 0.054| 14.7           | 19.2  | 0.59  |
| 5 (top panel) | C | 1.43  | 315             | 5.23  | 0.030| 7.5            | 18.8  | 0.41  |
| 5 (middle panel) | C | −0.16 | 155             | 0.61  | 0.14 | 12.6           | 19.3  | 0.082 |
| 5 (bottom panel) | C | 0.37  | 255             | 2.92  | 0.21 | 6.9            | 14.5  | 0.93  |
| 21a    | C      | −0.011| 255             | 1.06  | 0.0016| 26.2           | 17.1  | 0.19  |
| 21b    | C      | −0.024| 285             | 1.31  | 0.0076| 132.2          | 18.7  | 0.067 |
| 21c    | C      | −0.071| 81              | 1.04  | 0.0015| 12.2           | 19.6  | 0.71  |
| 21d    | C      | 0.22  | 265             | 0.87  | 0.00045| 65.7           | 18.0  | 0.38  |
| 21e    | C      | 0.16  | 169             | 0.94  | 0.0038| 26.3           | 17.3  | 0.15  |
| 21f    | E      | −0.20 | 16              | 0.55  | 0.49 | 14.8           | 17.3  | 0.073 |
| 22a    | C      | 0.15  | 52              | 0.57  | 0.33 | 54.6           | 18.6  | 0.67  |
| 22b    | C      | 0.033 | 69              | 0.45  | 0.56 | 88.3           | 18.2  | 0.72  |
| 22c    | C      | −0.56 | 353             | 0.18  | 0.30 | 49.3           | 16.0  | 1.04  |
| 22d    | C      | −0.076| 245             | 2.38  | 0.0059| 9.0            | 20.0  | 1.04  |
| 22e    | E      | −0.33 | 163             | 0.34  | 0.29 | 82.4           | 15.3  | 0.96  |
| 22f    | E      | 0.21  | 77              | 0.79  | 0.29 | 24.3           | 18.7  | 0.20  |

Table 5. Physical parameters of example light curves in this paper.

| Figure | Orbit | $M_1/M_\odot$ | $M_2$ | $a/au$ | $T/\text{d}$ | $e$ | $i/\circ$ | $v_t/\text{km s}^{-1}$ | $D_t/\text{kpc}$ |
|--------|-------|---------------|-------|--------|--------------|-----|-----------|------------------------|-----------------|
| 3      | C     | 0.084         | 0.018 M_\odot | 10.7 | 39799       | 0   | 214       | 134.8                  | 5.75            |
| 4      | S     | 0.70          | 0.038 M_\odot | 1.88 | 1090        | 0   | 300       | 215.7                  | 7.40            |
| 5 (top panel) | C | 0.058 | 0.0018 M_\odot | 4.46 | 14047      | 0   | 173       | 196.3                  | 6.04            |
| 5 (middle panel) | C | 0.13   | 0.017 M_\odot | 1.22 | 1298       | 0   | 311       | 183.8                  | 5.95            |
| 5 (bottom panel) | C | 0.10    | 0.021 M_\odot | 3.52 | 6852       | 0   | 112       | 282.8                  | 6.43            |
| 21a    | C     | 0.55         | 0.89 M_{Jupiter} | 5.82 | 6924       | 0   | 93        | 167.3                  | 6.12            |
| 21b    | C     | 0.75         | 6.0 M_{Jupiter} | 4.32 | 3767       | 0   | 115       | 39.8                   | 6.01            |
| 21c    | C     | 0.27         | 0.43 M_{Jupiter} | 0.51 | 256        | 0   | 243       | 63.2                   | 7.91            |
| 21d    | C     | 0.89         | 0.42 M_{Jupiter} | 3.83 | 2899       | 0   | 136       | 88.8                   | 2.13            |
| 21e    | C     | 1.17         | 4.7 M_{Jupiter} | 3.42 | 2130       | 0   | 56        | 173.5                  | 7.19            |
| 21f    | E     | 0.21         | 0.10 M_\odot | 0.61 | 306        | 0.92 | 102,216   | 183.0                  | 6.90            |
| 22a    | C     | 0.56         | 0.18 M_\odot | 1.88 | 1098       | 0   | 16        | 101.2                  | 2.44            |
| 22b    | C     | 0.38         | 0.21 M_\odot | 1.69 | 1044       | 0   | 40        | 57.4                   | 2.69            |
| 22c    | C     | 0.68         | 0.20 M_\odot | 0.65 | 205        | 0   | 30        | 115.8                  | 5.97            |
| 22d    | C     | 0.65         | 4.0 M_{Jupiter} | 2.70 | 2005       | 0   | 2        | 218.3                  | 7.75            |
| 22e    | E     | 0.59         | 0.17 M_\odot | 1.35 | 656        | 0.77 | 303,213   | 68.2                   | 5.56            |
| 22f    | E     | 0.39         | 0.11 M_\odot | 2.14 | 1609       | 0.18 | 2,143     | 187.0                  | 5.64            |

For events with eccentric orbits, two values of the inclination are quoted, representing inclinations about two orthogonal axes on the sky. The effect of this second inclination is absorbed into the source trajectory for circular orbits and to first order can be reduced to the range $0^\circ \leq i \leq 90^\circ$.

of close stellar binaries are significantly larger and stronger than those of planetary lenses and so the chances of the source crossing them is higher, and the caustic has a longer time in which to change due to orbital motion as the source crosses it, leading to the small positive efficiency for $|u_0| \gtrsim 0.6$. For smooth events, the planetary and stellar binary lenses show weak but opposing trends, with the efficiency increasing slightly as $|u_0|$ increases for planetary events and decreasing slightly as $|u_0|$ increases for stellar binary events, indicating that the impact parameter only plays a small role in orbital motion detectability for smooth light curves. Note, however, that for both smooth and caustic-crossing events, the number of orbital motion detections, as opposed to the detection efficiency, is a strong function of $|u_0|$, peaking at small values, due to the dependence of the binary detection efficiency on the impact parameter.

Fig. 7 plots the detection efficiency against the event time-scale $t_{\text{E}}$. All classes of binary events (planetary or binary, smooth or caustic crossing) show a strong detection efficiency dependence on the event time-scale. The reason for this dependence is simply because a longer time-scale allows the lens to complete a larger fraction of its orbit and hence causes a larger change in the magnification pattern, during the time in which the source probes regions of the magnification pattern that deviate from that of a single lens. In the case of planetary lenses, it seems that a time-scale of greater than $\sim 10$ d is necessary for caustic-crossing events and slightly longer for smooth events. Caustic-crossing events show larger detection efficiency than smooth events, even at shorter time-scales. This is likely due to the high accuracy with which caustic-crossing times and the light-curve shape around caustic crossings can be measured. In the case of OGLE-2006-BLG-109, this has allowed the orbital motion of the lens to be measured from data covering just $\sim 0.2$ per cent of the orbit (Gaudi et al. 2008; Bennett et al. 2010). Smooth events, in contrast, require a much larger fraction of the orbit to cause significantly detectable changes in the light curve and hence require a longer time-scale to achieve the same detection efficiency. However, typically, it is possible for smooth features to
time-scale means that the slope of the high-$t_E$ tail of the distribution of orbital motion events is much shallower than $t_E^{-3}$.

The plots of detection efficiency against the projected separation $d_0$ (Fig. 8) and semimajor axis $a$ (Fig. 9) tell largely the same story. The detection efficiency in binary stars has a significant inverse dependence on both $d_0$ and $a$, as would be expected from the dependence of the orbital velocity of the semimajor axis. However, the behaviour for planetary lenses is less intuitive: for caustic-crossing events, there is a significant peak in the detection efficiency at $a \sim 4$ au and a peak/shoulder at $d_0 \sim 2$. There is a second peak in $\epsilon_{OM}$ with $d_0$. The two peaks occur at values of $d_0$ where the boundaries between caustic topologies occur for the highest mass ratio planets. It is at these boundaries that, for a small change in projected separation $d(\log d)$, the largest changes in the caustics occur. The peak in $\epsilon_{OM}$ against $a$ at $a \sim 4$ au for caustic-crossing planetary events is accompanied by a hint of a peak at small values of $a$. The peak at $a \sim 4$ au can be explained by considering the typical scale of the Einstein ring and by considering the trend of $\epsilon_{OM}$ with the event time-scale. The typical size of the Einstein ring for a microlensing event is $2-3$ au, but as seen in Fig. 7, orbital motion effects typically occur in events with larger time-scales. As the time-scale is correlated with the Einstein ring size, and caustic-crossing events typically occur in systems with $d_0 \sim 1$, the peak orbital motion detection efficiency occurs at a semimajor axis slightly above the typical Einstein ring size, at $a \sim 4$ au. The increase in orbital velocity as $a$ decreases likely causes the second weaker peak in $\epsilon_{OM}$ at smaller $a$. Little can be said about the trend of $\epsilon_{OM}$ with $a$, due to the small numbers of events and the distribution of Einstein radii that serves to smear out any obvious trends. However, when plotted against $d_0$, $\epsilon_{OM}$ does increase towards smaller values of $d_0$, as would be expected from orbital velocity considerations.

Returning to the caustic-crossing stellar binary events, $\epsilon_{OM}$ flattens off as $a$ increases to $\sim 4$ au, before dropping to zero. This flattening likely has the same cause as the peak for planetary caustic-crossing events. We see the more intuitive inverse trend in stellar binaries because of the stronger and larger magnification pattern features that they exhibit, and the larger range of $d$ over which the caustics have a significant size. This results in a distribution of events over $a$ and $d_0$, which is broader and somewhat less peaked than for planetary events (see the lower panels of Figs 8 and 9). This allows the inverse relationship between the orbital velocity and semimajor axis to have a greater influence on the trend in the orbital motion detection efficiency. We note that the reason we see behind such a complicated relationship between $\epsilon_{OM}$ and $a$ and $d_0$, but not, for example, between $\epsilon_{OM}$ and $t_E$, is that the orbital separation affects the orbital velocity in a relatively simple way, and the caustic size and strength in a complicated way, whereas $t_E$ only affects or more accurately is the result of a fairly simple dependence on a single factor in the detection of orbital motion, the source speed.

Fig. 10 plots the detection efficiency against the mass ratio $q$. For planetary lenses, orbital motion features can be detected effectively over almost the entire range of $q$ that we simulated, though with a low efficiency for small mass ratios. For events with $q \lesssim 0.01$, the detection efficiency reaches $\sim 20$ per cent for smooth events and $\sim 10$ per cent for caustic-crossing events. The detection efficiencies are similar for planetary events. The majority of planetary and binary events showing orbital motion have time-scales of around $\sim 10-40$ d, with few events at larger time-scales $t_E$ due to the steep $t_E^{-3}$ distribution at large time-scales (Mao & Paczyński 1996). However, the strong dependence of $\epsilon_{OM}$ on $q$ and $t_E$ means that the slope of the high-$t_E$ tail of the distribution of orbital motion events is much shallower than $t_E^{-3}$.

For stellar binary lenses, orbital motion features can be detected effectively over almost the entire range of time-scales that we simulated, though with a low efficiency for time-scales below $\sim 40$ d for smooth events and $\sim 10$ d for caustic-crossing events. For events with time-scales over $\sim 100$ d, the detection efficiency reaches $\sim 20$ per cent for smooth events and $\sim 40$ per cent for caustic-crossing events. The detection efficiencies are similar for planetary events. The majority of planetary and binary events showing orbital motion have time-scales of around $\sim 10-40$ d, with few events at larger $t_E$ due to the steep $t_E^{-3}$ distribution at large time-scales (Mao & Paczyński 1996). However, the strong dependence of $\epsilon_{OM}$ on $q$ and $t_E$ means that the slope of the high-$t_E$ tail of the distribution of orbital motion events is much shallower than $t_E^{-3}$.

The plots of detection efficiency against the projected separation $d_0$ (Fig. 8) and semimajor axis $a$ (Fig. 9) tell largely the same story. The detection efficiency in binary stars has a significant inverse dependence on both $d_0$ and $a$, as would be expected from the dependence of the orbital velocity of the semimajor axis. However, the behaviour for planetary lenses is less intuitive: for caustic-crossing events, there is a significant peak in the detection efficiency at $a \sim 4$ au and a peak/shoulder at $d_0 \sim 2$. There is a second peak in $\epsilon_{OM}$ with $d_0$. The two peaks occur at values of $d_0$ where the boundaries between caustic topologies occur for the highest mass ratio planets. It is at these boundaries that, for a small change in projected separation $d(\log d)$, the largest changes in the caustics occur. The peak in $\epsilon_{OM}$ against $a$ at $a \sim 4$ au for caustic-crossing planetary events is accompanied by a hint of a peak at small values of $a$. The peak at $a \sim 4$ au can be explained by considering the typical scale of the Einstein ring and by considering the trend of $\epsilon_{OM}$ with the event time-scale. The typical size of the Einstein ring for a microlensing event is $2-3$ au, but as seen in Fig. 7, orbital motion effects typically occur in events with larger time-scales. As the time-scale is correlated with the Einstein ring size, and caustic-crossing events typically occur in systems with $d_0 \sim 1$, the peak orbital motion detection efficiency occurs at a semimajor axis slightly above the typical Einstein ring size, at $a \sim 4$ au. The increase in orbital velocity as $a$ decreases likely causes the second weaker peak in $\epsilon_{OM}$ at smaller $a$. Little can be said about the trend of $\epsilon_{OM}$ with $a$, due to the small numbers of events and the distribution of Einstein radii that serves to smear out any obvious trends. However, when plotted against $d_0$, $\epsilon_{OM}$ does increase towards smaller values of $d_0$, as would be expected from orbital velocity considerations.

Returning to the caustic-crossing stellar binary events, $\epsilon_{OM}$ flattens off as $a$ increases to $\sim 4$ au, before dropping to zero. This flattening likely has the same cause as the peak for planetary caustic-crossing events. We see the more intuitive inverse trend in stellar binaries because of the stronger and larger magnification pattern features that they exhibit, and the larger range of $d$ over which the caustics have a significant size. This results in a distribution of events over $a$ and $d_0$, which is broader and somewhat less peaked than for planetary events (see the lower panels of Figs 8 and 9). This allows the inverse relationship between the orbital velocity and semimajor axis to have a greater influence on the trend in the orbital motion detection efficiency. We note that the reason we see behind such a complicated relationship between $\epsilon_{OM}$ and $a$ and $d_0$, but not, for example, between $\epsilon_{OM}$ and $t_E$, is that the orbital separation affects the orbital velocity in a relatively simple way, and the caustic size and strength in a complicated way, whereas $t_E$ only affects or more accurately is the result of a fairly simple dependence on a single factor in the detection of orbital motion, the source speed.

Fig. 10 plots the detection efficiency against the mass ratio $q$. For planetary lenses, orbital motion features can be detected effectively over almost the entire range of $q$ that we simulated, though with a low efficiency for small mass ratios. For events with $q \lesssim 0.01$, the detection efficiency reaches $\sim 20$ per cent for smooth events and $\sim 40$ per cent for caustic-crossing events. The detection efficiencies are similar for planetary events. The majority of planetary and binary events showing orbital motion have time-scales of around $\sim 10-40$ d, with few events at larger $t_E$ due to the steep $t_E^{-3}$ distribution at large time-scales (Mao & Paczyński 1996). However, the strong dependence of $\epsilon_{OM}$ on $q$ and $t_E$ means that the slope of the high-$t_E$ tail of the distribution of orbital motion events is much shallower than $t_E^{-3}$.

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Figure 6. Plot of the orbital motion detection efficiency, corrected for systematic false positives (top panels), and the absolute number of orbital motion detections in the simulations (lower panels), against the impact parameter $u_0$. Results are shown for lenses with planetary mass ratios (left-hand panel) and binary star mass ratios (right-hand panel). Red/grey lines with filled squares show the results for caustic-crossing events and blue/black lines with filled circles show the results for smooth events. In the upper panels, a line marks zero orbital motion detection efficiency. All events had circular orbits, and in the lower panels, results are shown for events where the lens components were in orbit (solid lines, filled points) and where they were held static for the calculation of the false positive rate (dashed lines, open points). Events have been binned into bins of equal width and points plotted at the centre of the bin. Note that in the lower panels the scales are different. Colour version available online.

Figure 7. Same as Fig. 6, but plotted against the event time-scale $t_E$. Covered a much larger fraction of the light curve time than caustic-crossing features, lessening the effect of this discrepancy.

For stellar binary lenses, orbital motion features can be detected effectively over almost the entire range of time-scales that we simulated, though with a low efficiency for time-scales below $\sim 40$ d for smooth events and $\sim 10$ d for caustic-crossing events. For events with time-scales over $\sim 100$ d, the detection efficiency reaches $\sim 20$ per cent for smooth events and $\sim 40$ per cent for caustic-crossing events. The detection efficiencies are similar for planetary events. The majority of planetary and binary events showing orbital motion have time-scales of around $\sim 10-40$ d, with few events at larger $t_E$ due to the steep $t_E^{-3}$ distribution at large time-scales (Mao & Paczyński 1996). However, the strong dependence of $\epsilon_{OM}$ on $q$ and $t_E$ means that the slope of the high-$t_E$ tail of the distribution of orbital motion events is much shallower than $t_E^{-3}$.
through by this dependence (unlike the curves of the number of orbital motion detections, which show a strong dependence on \( q \)), to leave a very shallow orbital motion detection efficiency curve. The other effect that \( q \) has on the light-curve features is to make them stronger as \( q \) increases. In caustic-crossing events, the caustic features are usually strong, independent of the value of \( q \), and hence the caustic-crossing event curve is shallower than the curve for smooth events, for which the dependence of the feature strength on \( q \) is much more important.

Fig. 11 shows the detection efficiency plotted against the primary lens mass. The dependence is as expected for both mass ratio regimes and for both types of binary events, increasing as the mass of the primary increases. The trend is strongest in smooth, stellar binary events.

Fig. 12 plots the detection efficiency against the lens distance. In all cases, a trend of increasing detection efficiency with decreasing lens distance is seen, though caustic-crossing events suffer from small number statistics at low values of \( D_l/D_s \). Note, however, that the frequency distribution (plotted in the lower panels of Fig. 12) of orbital motion events, once false positives have been approximately accounted for, is different, being peaked at \( D_l/D_s \sim 0.7 \).

Fig. 13 shows the detection efficiency plotted against the orbital period. Both types of stellar binary events show a significant inverse trend. Planetary caustic-crossing events show a peak, and stellar caustic-crossing events a flattening, at large periods. These features correspond directly to similar features in the curves of \( \epsilon_{\text{OM}} \) with \( a \) and will have the same cause.

Figs 14 and 15 plot the detection efficiency against the baseline magnitude \( I_b \) and blending fraction \( f_b \), respectively. For our purposes, the primary effect of both parameters is to affect the accuracy with which microlensing variations can be measured in the light curve. For a fixed observing setup, the baseline magnitude determines the photometric accuracy, which should lead to a trend of increasing detection efficiency with decreasing magnitude. This is seen to a certain extent in all cases, but brighter events may suffer significantly from blending, due to faint source stars falling entirely within the large point spread function of a much brighter star. Blending determines the relative strength of features in the light curve and as such has a much more significant effect on the detection of smooth binary features, which have a continuous range of shapes and sizes, compared to the effect on caustic crossings, which are typically sharp and very strong, at least when finite sources are not considered. It is no surprise therefore that smooth stellar binary events show a significant increase in the orbital motion detection efficiency with blending fraction. This is less obvious in planetary lenses, likely because the smooth light-curve features of planetary lenses are often very weak and difficult to detect even without the hindrance of the blending, and would not permit the measurement of higher order effects for any value of blending fraction. It is more surprising, perhaps, that caustic-crossing events show a significant dependence on blending, as in the simulations all caustic-crossing events were detected as binaries, regardless of blending. This
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implies that, at least in some orbital motion detections in caustic-crossing events, the additional smooth features in the light curve, such as peaks and shoulders due to cusp approaches outside the caustic, and features due to fold caustic approaches within the caustic, play an important role in the detection of orbital motion [e.g. light curves (a) and (e) in Fig. 21].

Fig. 16 plots the detection efficiency against inclination. There is little evidence for any significant dependence on inclination for all caustic-crossing events, and for smooth microlenses. There is, however, a stronger trend for smooth stellar binary events, the detection efficiency decreasing as the inclination increases. This would be expected in systems where \( a/r_c \ll d_s \), the boundary between close and resonant caustic topologies, where a reduction in the projected separation due to inclination would reduce the size of the caustics and reduce the detectability of both binary features and orbital motion signatures. Unfortunately, due to the similar effects of inclination and eccentricity on the projected orbit, the data from the eccentric orbit simulations did not show any dependence of \( \epsilon_{om} \) on eccentricity. This, however, implies that the effects of eccentricity on the orbital motion detection efficiency are not likely to be significantly stronger than those of inclination.

It is important not just to consider the system parameters in isolation, but also their combined effects on the orbital motion detection efficiency. For example, Dominik (1998) introduced two dimensionless ratios to describe the magnitude of orbital motion effects on a binary lens:

\[
R_t = \frac{I_0}{T},
\]

the ratio of time-scales, and

\[
R_v = \frac{v_{circ}}{v_t},
\]

the ratio of velocities, where \( v_{circ} = a/2\pi T \) is the circular velocity of the orbit. These ratios attempt to encapsulate the most important factors that determine if an event will show orbital motion features. Figs 17 and 18 plot the detection efficiency against \( R_t \) and \( R_v \), respectively. Both ratios prove to be good descriptors of the orbital motion detection efficiency, with \( \epsilon_{om} \) showing strong increasing trends as \( R_t \) and \( R_v \) increase, across all mass ratios and light-curve types, though with a lower significance in planetary events. It would even seem that, in the case of smooth events, there exists a threshold value of the ratios, below which the orbital motion detection efficiency is negligible. For the ratio of time-scales, the threshold is \( \log R_t \approx -2 \) for both planetary and stellar binary lenses, while for the ratio of velocities, the value appears to be more dependent on the mass ratio, taking values of \( \log R_v \approx -2.5 \) for planetary lenses and \( \log R_v \approx -2.75 \) for stellar binary lenses. There may be similar thresholds for caustic-crossing events, but at smaller values of \( R_t \) and \( R_v \).

Fig. 17 plots the detection efficiency against inclination. There is little evidence for any significant dependence on inclination for all caustic-crossing events, and for smooth microlenses. There is, however, a stronger trend for smooth stellar binary events, the detection efficiency decreasing as the inclination increases. This would be expected in systems where \( a/r_c \ll d_s \), the boundary between close and resonant caustic topologies, where a reduction in the projected separation due to inclination would reduce the size of the caustics and reduce the detectability of both binary features and orbital motion signatures. Unfortunately, due to the similar effects of inclination and eccentricity on the projected orbit, the data from the eccentric orbit simulations did not show any dependence of \( \epsilon_{om} \) on eccentricity. This, however, implies that the effects of eccentricity on the orbital motion detection efficiency are not likely to be significantly stronger than those of inclination.

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Fig. 15. Same as Fig. 6, but plotted against the fraction of baseline flux associated with the source \( f_s \).
5.3 Are there two classes of the orbital motion event?

Gaudi (2009) has suggested that orbital motion can affect the light curves of microlensing events in two ways. In the first scenario, the orbital motion effects are dominated by rotation in the lens, as the orientation of the binary axis changes during the time between two widely separated light-curve features. The second type of effect is due to changes in the projected separation over the course of a single light-curve feature, such as a resonant caustic crossing. In this section, we will describe the typical features of each type of event before investigating to what extent orbital motion events can be classified in such a way.

Gaudi (2009) describes the separational class of events as typically occurring in archetypal binary microlenses with resonant caustic crossings. If the binary’s orbit is inclined, the projected separation of the lenses changes, causing a stretching or compression of the resonant caustic. If the projected separation is close to a boundary between caustic topologies, \( d \sim d_c \) or \( d \sim d_w \), the changes in the caustic structure can be very rapid. If the microlensing event occurs while the changes are happening, and the source crosses, or passes close to, the caustics, then there is a very good chance of detecting the orbital motion. As a whole, though the changes in caustic structure during the caustic-crossing time-scale will be fairly small, for example, the difference in the caustic-crossing time between the static lens and the orbiting lens may be of the order of minutes to hours (cf. the orbital period of several years). It is only the extremely good accuracy with which caustic crossings can be measured and timed that facilitates the high orbital motion detection probability. These changes to the caustic shape will often be more significant than the changes in orientation of the caustic due to rotation and so we class them as separational orbital motion effects.

Gaudi (2009) described the rotational class of events as occurring when a source encounters two disjoint caustics of a typically close topology lens. In the time between the two caustic encounters, which are separated by a time \( \Delta t \sim t_c \), the lens components have time to rotate and show detectable signatures of orbital motion. We extend the class by considering the important effect to be the long baseline over which binary-lensing features can be detected. If binary-lens features are detectable across a significant fraction of the light curve, then a significant amount of rotation can occur in the lens while the features are detectable. Up to now, our discussion has focused mainly on caustic features, whether the source crosses them or not, but, in stellar binary lenses especially, the magnification pattern of the lens can differ significantly, if subtly, from the single-lens form over large parts of the pattern, and well away from caustics. For example, in close binary lenses, there is a region of excess magnification that can stretch the entire distance between the facing cusps of the central and secondary caustics. In stellar binary lenses, this can extend for distances larger than an Einstein radius. In planetary lenses, the magnification excesses are weaker, but there tends to be a large region of demagnification between the two planetary caustics. If lenses with such features rotate rapidly, then the source may encounter them in such a way that a static lens interpretation of the light-curve features is not possible and lens rotation must be invoked.

We begin by looking for evidence of two classes of events in the locations of the orbital motion events in the \( d_0-q \) plane. Fig. 19 plots \( q \) against \( d_0 \) for all binary events; events which do not show orbital motion signatures are plotted with small, open points with light colours, whereas those that do are plotted with large, filled points with darker colours. Caustic-crossing events are plotted with red squares and smooth events with blue circles. The upper panels show stellar binary lenses and lower panels show planetary lenses, while the left-hand panels show orbiting lenses and the right-hand panels show static lenses. The black lines show the boundaries between the caustic topologies (equations 7 and 8). It is immediately clear that caustic-crossing and smooth orbital motion events reside in different regions of the \( d_0-q \) plane, with virtually all events within the intermediate topology regime being caustic crossing. Almost

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**Figure 16.** Same as Fig. 6, but plotted against the orbital inclination \( i \).

**Figure 17.** Same as Fig. 6, but plotted against the ratio of microlensing to orbital time-scales \( R_T = t_c/T \).

**Figure 18.** Same as Fig. 6, but plotted against the ratio of orbital and source velocities \( R_v = v_{\text{circ}}/v_s \).
all smooth orbital motion events are located in the close topology region. This broadly reflects the underlying pattern for all binary events and is not in itself evidence of two classes of orbital motion events, but is instead a result of different caustic sizes in the different caustic topologies.

Another feature of the plot is the clustering of caustic-crossing orbital motion events near the boundary of the close and intermediate topologies. It is close to the topology boundaries that the changes in projected separation cause the largest changes in the caustics. It is, however, difficult to attribute this clustering to faster caustic motions due to separational changes, as orbital velocity is inversely correlated with \( d_0 \), and so there should be more orbital motion events at smaller values of \( d_0 \) in any case. In support of the existence of a separational class, there is a hint of clustering against the resonant-wide boundary. However, the caustic size peaks at both topology boundaries, as the single resonant caustic stretches before splitting apart into central and secondary caustics, possibly meaning that simply the increased size of the caustics causes the increased density of detections.

Fig. 20 plots the impact parameter against \( d_0 \) and is very useful in separating different kinds of binary events, especially for planetary lenses. The events follow a distinctive pattern, with a large clump of events centred at \( |u_0| \sim 0 \) and \( \log d_0 \sim 0 \), which consists of high-magnification events that encounter the central or resonant caustic. At very small \( |u_0| \), this clump extends over a significant range in \( d_0 \), but narrows as \( |u_0| \) increases, to its narrowest point at \( |u_0| \sim 0.3 \), corresponding to the maximum size of the region affected by resonant caustics (or at larger \( |u_0| \) for stellar binaries). As \( |u_0| \) increases, the plot shows a distinctive ‘V’ shape, with no binary signatures being detected for events with \( d_0 \sim 0 \). This ‘V’ shape arises as, in events with larger \( |u_0| \), the source passes through regions of the magnification pattern that can only contain secondary caustics and does not enter the regions containing central or resonant caustics, that is, the binary features in lenses with \( d_0 \sim 1 \) only occur in regions of the magnification pattern that the sources with large \( |u_0| \) do not probe.

The events which occur on the branch with large \( |u_0| \) and large \( d_0 \) are caused by wild topography lenses and therefore involve only a single secondary caustic encounter. The rotation of these lenses is typically very slow and over the short duration of the binary features (typically of the order of a day) the lens completes only a very small fraction of its orbit. This points towards separational changes being the dominant effect in the detection of orbital motion features in events on this branch, even with the enhancement of rotational velocity due to the solid body ‘lever arm’.

The events that occur on the branch with large \( |u_0| \) and small \( d_0 \) are largely smooth events, with the occasional caustic-crossing event. The smooth events are likely caused by the source crossing the large cusp extensions that occur in close binary lenses, suggesting that they belong to the rotational class of events.

Unfortunately, it is difficult to attribute the cause of any one grouping of orbital motion events in Figs 19 and 20 to either the rotational or the separational class, partly because both types of motion will affect each event to some extent. Despite this, it is possible to classify many individual events as either a separational or a rotational event. Figs 21 and 22 show example light curves of both classes of orbital motion events, rotational and separational, respectively. The plots show the light curve in the upper left-hand panels, with simulated data in red, the true model in blue, the best-fitting static binary model in green and the best-fitting single-lens model in black. Also shown are the residuals from the single-lens model and the static binary model in the middle and lower left-hand panels, respectively. Shown in the right-hand panel is a plot of the source trajectory, shown in black, and snapshots of the caustics at various times during the event, shown in different colours. The coloured points on the time axis of the light curve show the time at which the caustic snapshots occurred and the coloured points on the source trajectory show the position of the source at these times. The source trajectory and caustics are shown in the frame of reference that rotates with the binary axis, with its origin at the centre of mass. In this frame, rotation of the lens causes the source trajectory to appear curved and changes in the lens separation cause the caustics to change its shape and move. Note that in event (f) in Fig. 21, and events (e) and (f) in Fig. 22, the lens orbits are eccentric, so that the source does not travel along the shown trajectory at a constant rate.

Fig. 21 shows examples of separational events. In each example, the source trajectory appears relatively straight, indicating that the lens rotates little; however, in each case, the caustics move significantly. Events (a), (b), (c) and (e) all involve resonant caustic crossings and conform well to the picture described by Gaudi (2009). Event (d) could be described as the encounter of two disjoint caustics, similar to the original description of the rotational class of events by Gaudi (2009), but other than the close topology, the event

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**Figure 19.** Scatter plot of \( q \) against \( d_0 \) for microlensing events with detectable binary signatures. Caustic-crossing events are plotted with red squares and smooth events with blue circles. Events classified as orbital motion events are plotted with larger, darker filled points and those classified as static with smaller, lighter, open points. The black lines show the positions of the caustic topology boundaries.

**Figure 20.** Same as Fig. 19, but showing \( |u_0| \) plotted against \( d_0 \).
Figure 21. Example light curves of simulated events affected by separational type orbital motion effects. In each subfigure, the left-hand panels show the light curve, its residual with respect to the best-fitting Paczyński model and its residual with respect to the best-fitting static binary model, from the top to bottom, respectively. Simulated data are shown in red, the Paczyński model is shown in black, the static binary model is shown in green and the true model is shown in blue. The source trajectory is plotted in black and the caustics are colour-coded according to the time. Coloured points on the light-curve panel show the time at which the caustic was in the state shown and the coloured points on the source trajectory show the position of the source at this time. The parameters of the microlensing events can be found in Tables 4 and 5.
Figure 22. Same as Fig. 21, but showing example light curves of simulated events affected by rotational type orbital motion effects.
is remarkably similar to event (c); the source trajectory is slightly curved, but it is clear that separational effects are dominant. At first glance, event (f) would clearly fit into the picture of disjoint caustic encounters, but the source trajectory reveals that rotation plays only a minor role. In this event, a static fit to just the features about \( t = t_0 \) would suggest a close encounter with a large secondary caustic at \( t \approx 1.5 t_{\text{E}} \), but instead changes in the binary’s separation cause the source to not only encounter, but also cross a now much smaller secondary caustic at \( t \approx 2 t_{\text{E}} \).

In contrast to Fig. 21, the source trajectories in Fig. 22 show significant curvature. Event (a) fits the description of rotational events by Gaudi (2009) exactly. The source first encounters a secondary caustic, but the rotation of the lens causes the source to pass the opposite side of the central caustic. Rotation also prevents the source from crossing the magnification excess between central caustic and the other secondary caustic. During the entire event, separational changes cause only slight changes in the caustics. In event (c), the rotation is more extreme, but the caustics smaller. The binary features are therefore more subtle, being caused by small magnification excesses between the caustics, the secondary caustics being located at \( \sim (\pm 3, \pm 4) \) and the central caustic at \( \sim (0, 0) \). The rotation of the lens causes the source to cross each excess more than once, and there are several minor deviations visible in the residual between the static and true model of the event. Event (d), while being caused by a wide lens, expected to rotate slowly, is clearly caused by rotation. During the event, there are virtually no separational changes, but the precision with which the secondary caustic-crossing and cusp approach features constrain the source trajectory means that the very slight rotation, which brings the source closer to the central caustic, is detectable. Events (b) and (e) both show strong signs of rotation in their source trajectories, but separational changes are also important. While we assign them to the rotational class of events, in reality, they may better fit into a third, hybrid class. Event (f) also shows signs of both rotational and separational orbital motion effects, but we assign it to the rotational class, because without rotation the second caustic crossing would be significantly shorter.

6 DISCUSSION

6.1 Limitations of the study

The questions that we wanted to answer in this work were: what fraction of microlensing events observed by the next-generation surveys will be affected by orbital motion and what type of events are the effects likely to be seen? While we do not claim to have fully answered these questions, we do feel that this work represents an important step in that direction. The simulation of the photometry is slightly optimistic and does not include the effects of weather and the systematic differences in the site conditions and observing systems, distributed across the globe, that would make up the network of telescopes needed for a continuous monitoring microlensing survey. The observing setup we simulated is in some respects more like a space-based microlensing telescope than a ground-based network. However, the photometric accuracy that we simulated is not too optimistic, and the differences between the static and orbiting simulations show that orbital motion plays a significant role in a significant fraction of microlensing events.

As discussed in Section 3, our choice of models will not fully answer the question of how many microlensing events with orbital motion effects will be seen; however, they do provide a good order of magnitude estimate. The binary detection efficiencies we find assume that all stars have a companion and must be adjusted accordingly to account for this. For example, current estimates suggest that only \( \sim 33 \) per cent of stellar systems are binaries (e.g. Lada 2006), so assuming that a next-generation microlensing survey detects \( \sim 2000 \) events per year, we can expect to see \( \sim 30 \) binary microlensing events showing orbital motion signatures per year. However, the true rate may be higher as the mass ratio distribution that we use for stellar binaries is not realistic; the real distribution is likely to be peaked in the range \( 0.1 \leq g \leq 1 \) (e.g. Duquennoy & Mayor 1991). A similar calculation for planetary lenses, assuming the fraction of stars hosting planets is \( \sim 0.5 \), yields a detection rate of \( \sim 1.5 \) caustic-crossing orbital motion events per year. Again, this estimate is affected significantly by our assumptions. Our mass ratio distribution is optimistic, with current microlensing results suggesting an inverse relation between the planet frequency and mass ratio in the regions microlensing is sensitive to (Gould et al. 2010; Sumi et al. 2010). This implies our estimate is optimistic, but we have also assumed that there is only one planet per system. Many multiplanet systems have been discovered to date (e.g. Fischer et al. 2008; Gaudi et al. 2008) and they are thought to be common. The microlensing planet detection efficiency in multiplanet systems is increased, as the planets are spread over a range of semimajor axes. This will somewhat compensate for the overestimate due to the incorrect mass ratio distribution.

The major limitation of this work is that finite source effects are not considered. The finite size of the source acts to smooth out the extreme magnification peaks as a source crosses a caustic, limiting the precision with which magnifications can be measured, and caustic crossings timed, and thus plays an important role in orbital motion detection. However, in most cases, the caustic entry times can still be timed accurately if the caustic crossing is monitored with high enough cadence. In some cases, the effect may increase the detectability of orbital motion as the source will probe more of the magnification pattern, especially when a source travels approximately parallel to and very close to the inside of a fold caustic, producing additional peaks between the caustic crossings. We cannot quantitatively estimate the effects that the finite source size has on the orbital motion detection efficiency, but we do not believe it will significantly affect our order of magnitude estimates. Unfortunately, including finite source sizes in the modelling of a microlensing event increases the required computation time by several orders of magnitude, so the effect could not easily be included in the simulations without significantly reducing the sample size.

6.2 Comparison with observations

While our simulations are more representative of future microlensing surveys, it is possible for us to compare the results of our simulations with the results of the current microlensing observations. Current microlensing planet searches using the survey/follow-up strategy routinely achieve a cadence similar or better than that expected for future high-cadence surveys for a small number of microlensing events per year (e.g. Dong et al. 2009). We can therefore compare the detection efficiency of orbital motion in the events where planets are detected. At the time of writing, there were 10 published detections of planets by microlensing (Bond et al. 2004; Udalski et al. 2005; Beaulieu et al. 2006; Gould et al. 2006; Bennett et al. 2008; Gaudi et al. 2008; Dong et al. 2009; Janczak et al. 2010; Sumi et al. 2010) and of these, seven had high-cadence coverage of a significant proportion of the light curve. In two of these events, the orbital motion of the planet was detected (Gaudi et al. 2008;
Dong et al. 2009), leading us to estimate an orbital motion detection efficiency of \( \sim 0.29^{+0.13}_{-0.10} \) per cent. This efficiency is larger than we find in our simulations. However, the orbital motion effects in the OGLE-2005-BLG-71 event are very subtle and improve the fit by \( \Delta \chi^2 \lesssim 200 \) (Udalski et al. 2005; Dong et al. 2009), meaning that it would not be classed as a detection in our simulations; this reduces the comparable detection efficiency estimate to \( 0.14^{+0.11}_{-0.07} \), with which our estimates of the detection efficiency for planetary caustic-crossing events are consistent. It should be noted that this figure could be biased as events showing orbital motion signatures will take significantly longer to analyse. Unfortunately, a similar estimate for binary star lenses is not so simple as they are usually not followed up to the same degree as that planetary events are, either in terms of observations or in terms of modelling.

We have identified two different classes of orbital motion events, so it is natural to try to classify the orbital motion events that have already been seen. The orbital motion detected in OGLE-2006-BLG-109 (Gaudi et al. 2008; Bennett et al. 2010) was detected due to deformation of a resonant caustic and the event can easily be assigned to the class with separational changes. OGLE-2005-BLG-71 (Udalski et al. 2005; Dong et al. 2009) is harder to classify, as the orbital motion effects observed were very subtle. The event suffers from the well-known close–wide degeneracy (Griest & Safizadeh 1998; Dominik 1999) and rather strangely, for the close (\( d < 1 \)) solution, separational changes are more prominent than rotational and vice versa for the wide (\( d > 1 \)) solution, where we might normally expect the opposite. We therefore do not assign the event to either class. Of the stellar binary lenses, MACHO-97-BLG-41 (Albrow et al. 2000) was mainly influenced by rotation and was detected by two disjoint caustic crossings, so is classed as a rotational event. EROS-2000-BLG-5 (An et al. 2002) undoubtedly belongs to the separational class; the caustic structure was resonant with \( d \) close to \( d_e \) and changes in the separation were measured with high significance, while rotational changes were consistent with zero. The final events, OGLE-2003-BLG-267 and OGLE-2003-BLG-291 (Jaroszynski et al. 2005), are not very well constrained, so we do not attempt to classify them.

We finally suggest that the event OGLE-2002-BLG-069 (Kubas et al. 2005) is a strong candidate for showing rotational type orbital motion effects. The event was modelled successfully by Kubas et al. (2005) without including orbital motion, with a close binary solution favoured physically and by the modelling. The event had a time-scale \( t_N \approx 105 \) d and binary parameters \( d = 0.46 \) and \( q = 0.58 \).

The light curve was very similar to event (b) shown in Fig. 22, having a long, well-covered central caustic crossing, with measurements of both caustic entry and exit. The physical lens parameters obtained from the modelling suggest lens masses of \( M_1 = 0.51 M_\odot \) and \( M_2 = 0.30 M_\odot \), and a projected separation of \( \sim 1.7 \) au, with a corresponding minimum period of \( T \gtrsim 900 \) d. The baseline is relatively bright, at \( I_b \sim 16.2 \), and so subtle magnification deviations could probably be constrained by the data, if they have been covered.

### 6.3 Future prospects

Interestingly, our results show that the orbital motion detection efficiency depends only weakly on the mass ratio. In the case of planetary events, caustic-crossing orbital motion detections occur preferentially in high to moderate magnification events (\( A \gtrsim 5 \)), while smooth orbital motion detections occur in all but high-magnification events. Our results therefore suggest that the strategy of targeting high-magnification events (Griest & Safizadeh 1998; Han & Kim 2001) should allow caustic-crossing orbital motion events to be detected efficiently. However, the strong dependence of orbital motion detection efficiency on the event time-scale suggests that long time-scale events should also be routinely followed up. While follow-up of these events requires a significant investment of resources from the follow-up teams, like high-magnification events, they are relatively rare. For a given cadence, these events allow a better signal-to-noise ratio detection of planetary deviations and also allow more time for the prediction of future features. Long time-scale events are also more likely to show parallax features, allowing constraints to be placed on the lens mass.

High-cadence, continuous monitoring microlensing surveys will begin operating in the next few years. Already, the MOA-II survey (Hearnshaw et al. 2005; Sako et al. 2008) has been surveying a fraction of its total survey area with a cadence of \( \sim 10 \) min for some time and the OGLE-IV survey (Udalski 2009) has begun operations this year and should provide significant increases in cadence over OGLE-III. KMTnet, a uniform network of telescopes with near continuous coverage, and operating at a cadence of \( \sim 10 \) min, should begin operating around 2014; this promises an almost order of magnitude increase in the detection rate of microlensing events and a similar, if not bigger, increase in the detection rate of planets by microlensing. The uniform nature of the survey network will also make statistical analysis of the planets detected easier, greatly enhancing the work already done in this direction (Gould et al. 2010; Sumi et al. 2010). The work we have presented shows that a significant fraction of the events will show signs of orbital motion, which will significantly complicate the interpretation of future planet detections. However, these complications can be used to provide valuable additional constraints on the lens.

Often overlooked are binary star microlensing events. The next-generation surveys will detect many more binary star events than planetary events. A large number of these lenses will be located in the Galactic bulge and be composed of low-mass stars, providing an opportunity to study the properties of the bulge binary star population. Our results show that a significant fraction of these events will show orbital motion signatures and it is likely that in a significant number of these events, it will be possible to measure the masses of the system. It should therefore be possible to measure the statistics of a population that is difficult to reach by current spectroscopic and astrometric methods due to their low brightness and long periods.

### 7 SUMMARY

We have simulated the light curves of \( \sim 100 \) 000 microlensing events caused by stars orbited by a companion star or planet. By fitting simulated data with single-lens and static binary-lens models, we have determined the fraction of these events where the bigness of the lens is detected and we also estimate the fraction of these events where orbital motion is detected. For an observational setup that resembles a near future microlensing survey conducted by a global network of telescopes without intensive follow-up...
observations, we found that orbital motion was detected in $\sim 5$–10 per cent of simulated binary star microlensing events depending on the characteristics of the event. Similarly, the rate of detection of orbital motion in simulated microlensing events where a planet is detected was $\sim 1$–5 per cent.

We investigated the effects of various event parameters on the fraction of events showing orbital motion. Orbital motion detection efficiency as a fraction of binary detections was found to depend only weakly on the mass ratio of the binary, but strongly on the event time-scale. We found that a significant number of microlensing events showing orbital motion can be classified into one of two classes: those where the dominant cause of orbital motion effects is either the separational motion of the binary either due to inclination or due to eccentricity, or the rotational motion of the binary.

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