Nonfactorization in B and D decays

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ABSTRACT

We discuss the role of nonfactorized contributions in $B \to \psi + K^*$ and charmed meson decays. We demonstrate, using $D_S^+ \to \phi \pi$ as a model, how nonfactorization, annihilation and final state interactions can be built into effective and unitarized $a_1$ and $a_2$.

1. Introduction

Factorization approximation, where the matrix element of a product of two color-singlet weak currents is approximated by the product of the matrix elements of the individual currents, is the most commonly used scheme to calculate the amplitudes for two-body hadronic decays of B and D mesons. It was shown recently that this approximation failed to reproduce the longitudinal polarization observed in $B \to \psi + K^*$ decay in all the commonly used models of form factors. Subsequently it was shown how a small nonfactorization contribution helps in understanding the longitudinal polarization in $B \to \psi + K^*$ decay in all the commonly used models of form factors.

In the following I will discuss three topics: (I) Nonfactorization and polarization in color-suppressed $B \to \psi + K^*$ decay, (II) Nonfactorization in Cabibbo-favored D decays and (III) Nature of $a_1$ and $a_2$.

2. $B \to \psi + K^*$ decay

The effective weak Hamiltonian for decays of kind $b \to sc\bar{c}$ is:

$$H_{W}^{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left\{ C_1 (\bar{c}b)(\bar{s}c) + C_2 (\bar{c}c)(\bar{s}b) \right\}$$

(1)

where $(\bar{c}b)$ etc. represents color-singlet (V-A) current, $V_{cb}$ and $V_{cs}$ are the relevant Cabibbo-Kobayashi-Maskawa (CKM) mixing parameters and $C_1$ and $C_2$ are the QCD coefficients for which we use the values:

$$C_1 = 1.12 \pm 0.01, \quad C_2 = -0.27 \pm 0.03$$

(2)
If we Fierz-transform in color space, we can write

\[
(\bar{c}c)(\bar{s}b) = \frac{1}{N_c}(\bar{c}b)(\bar{s}c) + \frac{1}{2} \sum_a (\bar{c}\lambda^a b)(\bar{s}\lambda^a c) \tag{3a}
\]

and

\[
(\bar{c}b)(\bar{s}c) = \frac{1}{N_c}(\bar{c}c)(\bar{s}b) + \frac{1}{2} \sum_a (\bar{c}\lambda^a c)(\bar{s}\lambda^a b) \tag{3b}
\]

where \(\lambda^a\) are the Gell-Mann matrices and \(N_c = 3\) is the number of colors. It is convenient to define two parameters \(a_1\) and \(a_2\) as follows:

\[
a_1 = C_1 + \frac{1}{N_c}C_2 = 1.03 \pm 0.01 \tag{4a}
\]

\[
a_2 = C_2 + \frac{1}{N_c}C_1 = 0.10 \pm 0.03 \tag{4b}
\]

The decay amplitude for \(B \to \psi + K^*\) can be written in the following form:

\[
A(B \to \psi + K^*) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left\{ a_2 < K^*\psi \mid (\bar{c}c)(\bar{s}b) \mid B > + C_1 < K^*\psi \mid \tilde{H}_W^8 \mid B > \right\} \tag{5}
\]

where

\[
\tilde{H}_W^8 = \frac{1}{2} \sum_a (\bar{c}\lambda^a c)(\bar{s}\lambda^a b) \tag{6}
\]

is a product of color-octet currents. In the factorization approximation, only the first term in Eq.(5) is kept.

Confining ourselves to the factorization approximation, we can write the following expressions for the decay amplitude in terms of the relevant form factors \(\tilde{G}_F\):

\[
A(B \to \psi + K^*) = \tilde{G}_F a_2 m_\psi f_\psi \left\{ (m_B + m_{K^*}) A_1^{BK^*}(m_\psi^2) - \frac{2}{(m_B + m_{K^*})} \epsilon_1 \cdot p_B \epsilon_2 \cdot p_B A_2^{BK^*}(m_\psi^2) + \frac{2i}{(m_B + m_{K^*})} \epsilon_{\mu
u
\rho
\sigma} \epsilon_1^\mu \epsilon_2^\nu p_{K^*}^\rho p_B^\sigma V^{BK^*}(m_\psi^2) \right\} \tag{7}
\]

where \(\tilde{G}_F = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^*\) and \(\epsilon_1\) and \(\epsilon_2\) are the polarization vector of the \(\psi\) and \(K^*\) respectively. From Eq.(7) one obtains:

\[
B(B \to \psi + K^*) = 2.84 a_2^2 \left| A_1^{BK^*}(m_\psi^2) \right|^2 (\Sigma_{LL} + \Sigma_{TT}) \% \tag{8}
\]

where for longitudinal and transverse states of polarization,

\[
\Sigma_{LL} = (a - bx)^2, \quad \Sigma_{TT} = 2(1 + c^2 y^2) \tag{9}
\]
Figure 1: The domain of $x$ and $y$ allowed by the polarization data Eq.(12). Points A,B,... represent predicted values of $x$ and $y$ in various models. See Ref. [2] for details.

with

$$x \equiv \frac{A_2(m_\psi^2)}{A_1(m_\psi^2)}, \quad y \equiv \frac{V(m_\psi^2)}{A_1(m_\psi^2)},$$

(10a)

and

$$a = 3.147, \quad b = 1.297, \quad c = 0.434$$

(10b)

From Eq.(9), we obtain the longitudinal polarization as follows:

$$P_L = \frac{\sum_{LL}}{\sum_{LL} + \sum_{TT}} = \frac{(a - bx)^2}{(a - bx)^2 + 2(1 + c^2y^2)}$$

(11)

Experimentally

$$P_L = 0.78 \pm 0.07.$$  

(12)

In Figure 1 we have shown the allowed domain in (x-y) plane to one standard deviation of the central value of $P_L$ data. We have also shown the value of x and y predicated by six different models. For details the reader is referred to Ref. [2].

It is clear that the commonly used models of form factors generate points in (x-y) plane that are several standard deviations removed from those required by polarization data.

Assume now that there are nonfactorized contributions to the decay amplitude of Eq. (5). These could arise from two sources: (i) nonfactorized contribution to the first term comprised of color-singlet currents and (ii) contribution from $\bar{H}_w^{(8)}$ which is a product of two color-octet currents. This latter contribution is enhanced relative to
the former by the fact that $|C_1| \approx 10 |a_2|$. If for simplicity we assign nonfactorized contribution to the Lorentz structure belonging to $A_1$ only, then we need to modify our formalism with the replacement

$$A_1^{BK^*}(m_{\psi}^2) \rightarrow A_1^{BK^*}(m_{\psi}^2) + A_1^{(1)nf} + \frac{C_1}{a_2}A_1^{(8)nf}$$

where $A_1^{(1)nf}$ and $A_1^{(8)nf}$ are the two nonfactorized contributions referred to above.

The resulting formula for $P_L$ becomes

$$P_L = \frac{(a\xi - bx)^2}{(a\xi - bx)^2 + 2(\xi^2 + c^2y^2)}$$

where

$$\xi = 1 + \frac{C_1}{a_2}\chi \quad (15a)$$

and

$$\chi = \left(\frac{A_1^{(8)nf} + \frac{a_2}{C_1}A_1^{(1)nf}}{A_1^{BK^*}(m_{\psi}^2)}\right)$$

In Figure 2 we have plotted $P_L$ for different values of $\chi$ which is a measure of nonfactorized contribution.

We note that for $\chi \approx 0.1$ all models of form factors accommodate data. We remark that if nonfactorization were put in, say $A_2$, we would have needed much larger values of the parameter analogous to $\chi$. Thus even as little as a 10% nonfactorized contribution provides an understanding of the longitudinal polarization in all commonly used
3. Nonfactorization in Cabibbo-favored $D$ decays.

In the following we will discuss the role of nonfactorization in a selected few Cabibbo-favored $D$ decays: $D \to \pi \bar{K}, \pi \bar{K}^*, \rho \bar{K}$.

The relevant effective Hamiltonian is

$$H_W^{eff} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \{ C_1(\bar{u}d)(\bar{s}c) + C_2(\bar{u}c)(\bar{s}d) \}$$  \hspace{1cm} (16)$$

where for $C_1$ and $C_2$ at charm mass scale we adopt the following values \[6,7, C_1 = 1.26 \pm 0.04, \quad C_2 = -0.51 \pm 0.05\] so that with $N_c = 3$, we obtain

$$a_1 = 1.09 \pm 0.04, \quad a_2 = -0.09 \pm 0.05$$  \hspace{1cm} (17)$$

Fierz transformation of the current products in color-space leads to

$$(\bar{uc})(\bar{sd}) = \frac{1}{N_c} (\bar{ud})(\bar{sc}) + \frac{1}{2} \sum_a (\bar{u}\lambda^a d)(\bar{s}\lambda^a c)$$  \hspace{1cm} (19a)$$

and

$$(\bar{ud})(\bar{sc}) = \frac{1}{N_c} (\bar{uc})(\bar{sd}) + \frac{1}{2} \sum_a (\bar{u}\lambda^a c)(\bar{s}\lambda^a d)$$  \hspace{1cm} (19b)$$

Define

$$H_W^{(8)} = \frac{1}{2} \sum_a (\bar{u}\lambda^a d)(\bar{s}\lambda^a c)$$  \hspace{1cm} (20a)$$

and

$$\tilde{H}_W^{(8)} = \frac{1}{2} \sum_a (\bar{u}\lambda^a c)(\bar{s}\lambda^a d)$$  \hspace{1cm} (20b)$$

Using the above equations we calculate the decay amplitude for $D \to \bar{K}\pi, \bar{K}^*\pi$ and $\bar{K}\rho$ decays. As an example,

$$A(D^0 \to K^-\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* f_\pi (m_D^2 - m_K^2) a_1^{eff} F_{0}^{DK}(m_\pi^2)$$  \hspace{1cm} (21a)$$

and

$$A(D^0 \to \bar{K}^0\pi^0) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* f_\pi (m_D^2 - m_\pi^2) a_2^{eff} F_{0}^{D\pi}(m_\pi^2)$$  \hspace{1cm} (21b)$$

where

$$a_1^{eff} = a_1 \left\{ 1 + \frac{F_{0}^{(1)nf}}{F_{0}^{DK}(m_\pi^2)} + \frac{C_2}{a_1 F_{0}^{DK}(m_\pi^2)} \right\}$$  \hspace{1cm} (22a)$$
and
\[ a_2^{\text{eff}} = a_2 \left\{ 1 + \frac{\tilde{F}_0^{(1)nf}}{F_0^{D\pi}(m_K^2)} + \frac{C_1}{a_2} \frac{\tilde{F}_0^{(8)nf}}{a_2 F_0^{D\pi}(m_K^2)} \right\} \]  
\[ (22b) \]

\[ F_0^{(1)nf} \] and \[ \tilde{F}_0^{(1)nf} \] are the nonfactorized contributions from the product of color-singlet currents and \[ F_0^{(8)nf} \] and \[ \tilde{F}_0^{(8)nf} \] arise from \[ H_W^{(8)} \] and \[ \tilde{H}_W^{(8)} \] of Eq. (20), respectively.

>From Eq. (21) we calculate the isospin amplitudes \[ A_1^{\frac{3}{2}}, A_2^{\frac{3}{2}}, A_3^{\frac{3}{2}}, \] by setting the phases equal to zero in the following isospin decomposition,

\[ A(D^0 \rightarrow K^-\pi^+) = \frac{1}{\sqrt{3}} \left\{ A_2^{\frac{3}{2}} e^{i\delta_2^{\frac{3}{2}}} + \sqrt{2} A_1^{\frac{3}{2}} e^{i\delta_1^{\frac{3}{2}}} \right\} \]  
\[ (23a) \]

\[ A(D^0 \rightarrow \bar{K}^0\pi^0) = \frac{1}{\sqrt{3}} \left\{ \sqrt{2} A_2^{\frac{3}{2}} e^{i\delta_2^{\frac{3}{2}}} - A_1^{\frac{3}{2}} e^{i\delta_1^{\frac{3}{2}}} \right\} \]  
\[ (23b) \]

Having thus determined \( A_1^{\frac{3}{2}} \) and \( A_2^{\frac{3}{2}} \), we reinstated the phases with \( \delta_2^{\frac{3}{2}} - \delta_1^{\frac{3}{2}} = (86 \pm 8)^\circ \) and calculate the branching ratio. Fitting the branching ratio data determined \( a_1^{\text{eff}} \) and \( a_2^{\text{eff}} \) to lie in the following ranges,

\[ 1.13 \leq a_1^{\text{eff}} \leq 1.17, \quad -0.46 \leq a_2^{\text{eff}} \leq -0.42 \]  
\[ (24) \]

We repeated this procedure for \( D \rightarrow \bar{K}^*\pi \) and \( \bar{K}\rho \) decays where the difference of isospin phases is known and obtained,

\[ \bar{K}^*\pi : \quad 1.74 \leq a_1^{\text{eff}} \leq 1.96, \quad -0.53 \leq a_2^{\text{eff}} \leq -0.43 \]  
\[ (25a) \]

and

\[ \bar{K}\rho : \quad 1.17 \leq a_1^{\text{eff}} \leq 1.32, \quad -1.00 \leq a_2^{\text{eff}} \leq -0.75 \]  
\[ (25b) \]

In the above calculation we used the measured form factors as much as possible; \( F_0^{DK}(0), A_1^{DK^*}(0), A_2^{DK^*}(0) \) and \( V^{DK^*}(0) \) from Ref. [10], \( F_0^{D\pi}(0) \) from Ref. [8] and the as-yet-unmeasured \( A_0^{DK}(0) \) from the model of Ref. [1]. From Eq. (24) we note that the effective \( a_1 \) and \( a_2 \) are very close to the values advocated in Ref. [1] which gave rise to the belief that \( N_C \rightarrow \infty \) limit was relevant to \( D \) decays. However the value of \( a_1^{\text{eff}} \) and \( a_2^{\text{eff}} \) determined from \( \bar{K}^*\pi \) and \( \bar{K}\rho \) decays belie that belief.

4. Nature of \( a_1^{\text{eff}} \) and \( a_2^{\text{eff}} \)

Up to this stage our discussion has assumed that the effective \( a_1^{\text{eff}} \) and \( a_2^{\text{eff}} \) are real. Here we discuss how final-state interactions can be introduced and how they render \( a_1^{\text{eff}} \) and \( a_2^{\text{eff}} \) complex.
To illustrate the ideas, we look at the decay $D_s^+ \to \phi\pi^+$. It involves a single isospin amplitude, $I = 1$, in the final state. Before one introduces inelastic final state interactions, the decay amplitude for $D_s^+ \to \phi\pi^+$ is given by

$$A(D_s^+ \to \phi\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1^{\text{eff}}(2m_\phi) f_\pi \epsilon \cdot p_{D_s} A_0^{D_S\phi}(m_\pi)^2$$  \hspace{1cm} (26)

where

$$a_1^{\text{eff}} = a_1 \left\{ 1 + \frac{A_0^{(1)nf}}{A_0^{D_S\phi}(m_\pi^2)} + \frac{C_2}{A_0} \frac{A_0^{(8)nf}}{A_0^{D_S\phi}(m_\pi^2)} + \frac{f_{D_S}}{f_\pi} \frac{A_0^{\text{ann}}}{A_0^{D_S\phi}(m_\pi^2)} \right\}$$  \hspace{1cm} (27)

where apart from the nonfactorized contributions we have also absorbed a real annihilation term in the definition of $a_1^{\text{eff}}$.

We next unitarize the amplitude through final-state interactions. This renders the amplitude complex and one can define a complex, unitarized $a_1^{\text{eff}}$ as follows (the superscript $U$ stands for 'unitarized'):

$$A^U(D_s^+ \to \phi\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1^{U,\text{eff}}(2m_\phi) f_\pi \epsilon \cdot p_{D_s} A_0^{D_S\phi}(m_\pi^2)$$  \hspace{1cm} (28)

For the sake of illustration, we consider coupling of the $\phi\pi^+$ channel to the G-even $K^*K$ channel given by the symmetric state

$$| K^*K >_S = \frac{1}{\sqrt{2}} \left\{ | K^{*+}\bar{K}^0 > + | K^{*0}\bar{K}^+ > \right\}$$  \hspace{1cm} (29)

In Figure 3 we have shown schematically how the channels are coupled.
We adopt the K-matrix formalism \cite{3} to unitarize the decay amplitude. The unitarized decay amplitude, are given in terms of the following coupled equations,

\begin{equation}
\left( \begin{array}{c} A_{U,0}^{\phi} \\ A_{U,0}^{K^*K} \end{array} \right) = \left( \begin{array}{c} 1 - ik^3 \end{array} \right) \left( \begin{array}{c} \hat{A}_{\phi}^{\pi} \\ \hat{A}_{\phi}^{K^*K} \end{array} \right)
\end{equation}

where $K$ is a real, symmetric $2 \times 2$ matrix with dimensions $(GeV)^{-3}$, $k^3$ is a diagonal matrix with entries $k_1^3$ and $k_2^3$ as appropriate P-wave threshold factors for the two channels and $A_{\phi}^{\pi}$ and $A_{\phi}^{K^*K}$ are the un-unitarized amplitudes. The latter is constructed out of the following two amplitudes,

\begin{equation}
A(D_s^+ \rightarrow \bar{K}^0 K^{*+}) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_{2}^{eff} f_{K}(2m_{K^*}) \epsilon \cdot p_{Ds} A_{0}^{DsK^*}(m_{K^*}^2)
\end{equation}

with

\begin{equation}
a_{2}^{eff} = a_{2} \left\{ 1 + \frac{B_{0}^{(1)nf}}{A_{0}^{DsK^*}(m_{K^*}^2)} + \frac{C_{1}}{a_{2} A_{0}^{DsK^*}(m_{K^*}^2)} + \frac{a_{1} f_{Ds} B_{0}^{ann}}{a_{2} f_{K}^{*} A_{0}^{DsK^*}(m_{K^*}^2)} \right\}
\end{equation}

and

\begin{equation}
A(D_s^+ \rightarrow \bar{K}^0 K^{*+}) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_{2}^{eff} f_{K^*}(2m_{K^*}) \epsilon \cdot p_{Ds} F_{1}^{DsK}(m_{K^*}^2)
\end{equation}

with

\begin{equation}
\hat{a}_{2}^{eff} = a_{2} \left\{ 1 + \frac{\hat{B}_{0}^{(1)nf}}{\hat{F}_{1}^{DsK}(m_{K^*}^2)} + \frac{\hat{B}_{0}^{(8)nf}}{\hat{F}_{1}^{DsK}(m_{K^*}^2)} + \frac{a_{1} f_{Ds} \hat{B}_{0}^{ann}}{a_{2} f_{K^*} \hat{F}_{1}^{DsK}(m_{K^*}^2)} \right\}
\end{equation}

$B_{0}^{(1)nf}, B_{0}^{(8)nf}$ and $B_{0}^{ann}$ are the analogue of $A_{0}^{(1)nf}, A_{0}^{(8)nf}$ and $A_{0}^{ann}$ of Eq. (27). The hatted quantities refer to the channel $\bar{K}^0 K^+$. Using the above equations the amplitude $A_{\phi}^{K^*K}$ is written as,

\begin{equation}
A_{\phi}^{K^*K} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* (2m_{K^*}) \sqrt{2} \left\{ a_{2}^{eff} f_{K} A_{0}^{DsK^*}(m_{K^*}^2) + \hat{a}_{2}^{eff} f_{K^*} \hat{F}_{1}^{DsK}(m_{K^*}^2) \right\}
\end{equation}

The $K$-matrix being real and symmetric is parametrized as follows,

\begin{equation}
K = \left( \begin{array}{cc} a & b \\ b & c \end{array} \right)
\end{equation}

On carrying out the unitarization through Eq.(30) one obtains,

\begin{equation}
A_{\phi}^{U}(D_s^+ \rightarrow \phi\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_{1}^{eff} f_{\pi\epsilon} \epsilon \cdot p_{Ds} (2m_{\phi}) A_{0}^{Ds\phi}(m_{\phi}^2)
\end{equation}

with

\begin{equation}
a_{1}^{eff} = \frac{a_{1}^{eff}}{\Delta} \left\{ 1 - ik_2^3 c + i \frac{m_{K^*}}{2m_{\phi}} k_2^3 b F \right\}
\end{equation}
where

\[ F = \frac{a_2^{\text{eff}} f_K A_0^{D_S K^*}(m_K^2)}{a_1^{\text{eff}} f_\pi A_0^{D_S \phi}(m_\pi^2)} + \frac{\hat{a}_2^{\text{eff}} f_{K^*} F_1^{D_S K}(m_{K^*}^2)}{a_1^{\text{eff}} f_\pi A_0^{D_S \phi}(m_\pi^2)} \]

and \( \Delta = \det (1 - i k^3 \mathbf{K}) \).

If the final-state interactions were elastic, \( b = c = 0 \) and \( \Delta = 1 - i k_1^3 a \), we would have obtained

\[ a_1^{U,\text{eff}} = \frac{a_1^{\text{eff}} e^{i\delta}}{\sqrt{1 + k_1^6 a^2}} \]

with \( \delta = \tan^{-1}(a k_1^3) \), the elastic P-wave \( \phi \pi \) scattering phase.

In summary, one can always define an effective, and complex \( a_1 \) through the expression in Eq.(37). What we have shown is how effects such as nonfactorized contributions, annihilation and final-state interactions are built into it. A corollary of our point of view is that claims of test of factorization by comparing two-body hadronic rates to semileptonic rates are really nothing more than determinations of \( |a_1^{U,\text{eff}}| \).

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