Crashes: symptoms, diagnoses and remedies

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Abstract. A brief historical perspective is first given concerning financial crashes, - from the 17th till the 20th century. In modern times, it seems that log periodic oscillations are found before crashes in several financial indices. The same is found in sand pile avalanches on Sierpinski gaskets. A discussion pertains to the after shock period with illustrations from the DAX index. The factual financial observations and the laboratory ones allow us some conjecture on symptoms and remedies for discussing financial crashes along econophysics lines.

Key words. Science and society; Econophysics; Crashes

1 Introduction

Because of its magnitude, the stock market crash of Oct. 19, 1987 outshines all downturns ever observed in the past. In one day, the Dow Jones lost 21.6\% and the worst decline reached 45.8\% in Hong Kong. By comparison, with the most famous crash of 1929, the crash was spread over 2 days: The Dow Jones sank 12.8\% on October 28 and 11.7\% the following day. This shows that a stock market index decline does not necessarily lead to a crash in one single day. The decline can be slow and last several days or even several months in what would be called not so drastic cases, but a long duration bear market. Notice that markets had been using breakers or periods of trading halts and limitations of daily variations in order to avoid anomalous drops. However, Lauterbach and Ben-Zion \(^4\) found that trading halts and price limits had no impact on the overall decline of October 1987.

Such financial factual observations should be turned into quantitative measures. That is why we looked at the similarities between index evolution in 87 and 97, as early as spring 97 \(^5\). Previous historically famous crashes served as a thinking basis, - see Sect. 2. In Sect. 3 it is recalled that log-periodic corrections are sometimes superimposed on an index trend before a crash. In Sect. 3 the types of fluctuations in a post-crash period are also touched upon. Sorting out distinct behaviors should be made \textit{a priori}, as here below for the DAX.

Fractal systems provide a good modelling of a wide class of media in particular those containing a hierarchical structure, like in financial markets \(^6\).
We conjecture that the Bak-Tang-Wiesenfeld (BTW) model \cite{5} of a sandpile, generalized for fractal bases contains several ingredients which could be translated \textit{mutatis mutandis} into a model for stock market index evolution because such a model contains log-periodic oscillations as found before crashes. Results on the BTW model \cite{5} on prefractal Regular Sierpinski Carpets (RSC) having various fractal dimensions and connectivity are reported in Appendix. Since \textit{log-periodic corrections} characterizing features in the distribution of sand avalanches depend on the ramification of the RSC, this allows us to propose reflection lines on remedies against crashes in Sect. 4.

2 Some Historical Notes

Although there might have been financial crises in previous times and locations all over the world, the most famous one in recent times is the Tulipomania and subsequent crash which occurred in the 17-th century in Holland \cite{6}. Everything started in 1559 when the first tulip bulb (TB) was brought to Holland. The flower was considered so rare that speculation ensued and the flower became wildly overvalued: in 1635, 1 TB was worth 4 tons of wheat + 4 oxen + 8 tons rye + 8 pigs + one bed + 12 sheep + clothes + 2 wine casks + 4 tons beer + 2 tons butter + 1000 pounds cheese + 1 silver drinking cup. In 1637, 1 TB was worth 550 NLG, i.e. a 117\% return/year. However within 1637, in a 6 week time the price of 1 TB went down 90\%. Nowadays a black tulip bulb costs about 0.5 EUR.

A set of similar financial crises is that made of the Compagnie du Mississippi \cite{7} and that of the South Sea Company \cite{8,9} bubbles. In 1715, John Law had persuaded Philippe, Regent of France, to consider a banking scheme that promised to improve the financial condition of the kingdom. In theory a private affair, the system was linked from the beginning with liquidating the national debt. When the monopoly of the Louisiana trade was surrendered in 1717, Law created a trading company known as the \textit{Compagnie d’Occident} (or \textit{Compagnie du Mississipi}) linked to the bank and in which government bills were accepted for the purchase of shares, Law gaining a monopoly on all French overseas trade. The result was a huge wave of speculation as the value of a share went from its initial 500 \textit{livre} value to 18 000 \textit{livres}. When the paper money was presented at the bank in exchange for gold, which was unavailable, panic ensued, and shares felt by a factor of 2 in a matter of days.

In England, the Whigs invented the Bank of England in 1694. The South Sea Company (SSC) was formed in 1711 by the Tory government to trade with Spanish America, and to offset the financial support which the Bank of England had provided for Whig governments. The Tories had in mind to establish a system like the Compagnie du Mississippi Monopoly \cite{7}, using trading privileges and monopolies granted to Britain after the Treaty of Utrecht. In 1720 a Parliament bill was passed enabling persons to whom the government owed portions of the national debt to exchange their claims for shares
in SSC stocks. On March 1 the SSC stocks were valued GBP 175. On June 1, the shares were valued 500 and more than 1 000 in August 1720. Speculators took advantage of investors to obtain subscriptions for patently impossible projects. By September 1720 however, the market had collapsed, and by December SSC shares were down to 124, dragging other, including government, stocks with them. Many investors were ruined including I. Newton.

In the years from 1925 to 1929 it was almost a craze to play the market. One could go to a broker and purchase stock on margin. That allowed the speculation bubble to grow unchecked. Many mini crashes and subsequent recoveries began as early as Monday March 25, 1929. The summer of 1929 was not too bad. However on Sept. 3, a bear market became firmly established, and on Thursday Oct. 24, 1929 a crash occurred. In fact such crashes, not mentioning the uncontrolled buying frenzy on IPOs stocks at the end of the 1990’s in companies for which owners do not have a coherent business plan, look all similar: euphoria and speculation.

Other recent summers (rather than octobers!) are in the memory of investors, e.g. the CAC 40 dropped every summer between 1990 and 1998, except for 1993 (Table 1). There were 11 declines on the S&P 500 since 1925. One of these were horrendous (ca. 43 % between Jan. 73 and Sept. 74). In so doing it can be emphasized that crashes can be very abrupt but a market drop of the same, and even bigger importance can also occur.

| Year | 1990 | 1991 | 1992 | 1994 | 1995 | 1996 | 1997 | 1998 |
|------|------|------|------|------|------|------|------|------|
| date | Aug.02 | Aug.18 | Jun.02 | Aug.30 | Aug.21 | Jun.30 | Jul.01 | Jul.20 |
| drop | 20 | 8 | 12 | 12 | 11 | 9 | 10 | 20 |
| $\Delta t$ | 3 w | 3 d | 2 m | 1 m | 1 m | 1 m | 2 m | 1 m |

3 Empirical Universality and Symptoms

Bates studied transactions prices of S&P 500 futures options a posteriori over 1985-1987 to find out expectations/precursors of a shock. He discovered that out-of-money puts were unusually expensive to out-of-money calls. Then, the use of a jump-diffusion model for daily options prices of 1987 led to the conclusion that an expected negative jump was predictable a year prior to the crash. Other techniques exist, like those looking at the probability distribution function of returns. The main difference with our studies and that of others, comes from the a priori analysis of the 1990-1997 scenario, trying to find out if a break point in
an index series becomes more and more probable, how?, and may be why?. These ideas are controversial [23,24].

3.1 Before: diagnoses

The application of statistical physics ideas to the forecasting of stock market behavior and crashes has been proposed earlier [15,17,18]. It was proposed that an economic index $y(t)$ could increase as a complex power law [17], i.e.

$$y = A + B \left( \frac{t_c - t}{t_c} \right)^{-m} \left[ 1 + C \sin \left( \omega \ln \left( \frac{t_c - t}{t_c} \right) + \phi \right) \right] \quad \text{for } t < t_c \quad (1)$$

where $t_c$ is the crash-time(day) or rupture point, $A$, $B$, $m$, $C$, $\omega$ and $\phi$ are free parameters, while the period of the oscillations converges to the rupture point at $t = t_c$. This law is similar to that of critical points at so-called second order phase transitions [25] but with a complex exponent $m + i\omega$, and generalizes the scaleless situation discrete scale invariance cases [26,27]. From the stock market point of view, the equation has been derived Canessa [28] along renormalization group lines.

The S&P500 data [16] for the period preceding the 1987 October crash were already fitted using Eq.(1). It has been stressed that a nonlinear parameter fit does not easily lead to robust values against small data perturbations [29], the more so when there are seven parameters. Various values of $m$, including negative ones, were in fact reported in the literature for various indices and events [15,16,31,32], and are summarized in Table 2.

It would be nice stipulating that $m$ could be universal by analogy with second order phase transitions, at least for the presently studied crashes, seemingly falling into financially similar classes, even though this is surely an unrealistic dream. A behavior which we considered was the logarithmic divergence

$$y = A + B \ln \left( \frac{t_c - t}{t_c} \right) \left[ 1 + C \sin \left( \omega \ln \left( \frac{t_c - t}{t_c} \right) + \phi \right) \right] \quad \text{for } t < t_c. \quad (2)$$

As in critical point data analysis the optimum test consists in separating the most diverging term from the others, after having eliminated the so called mean field trend and searching for the correction to scaling [33]. In fact, the fit can be made in two steps: (i) one looks for a $t_c^{\text{div}}$, i.e. for the logarithmic divergence; then (ii) for $t_c^{\text{osc}}$ for the oscillation convergence [16].

Due to the log-periodicity in Eq.(2), the relation

$$t_c^{\text{osc}} = \frac{t_n - t_{n+1}}{\lambda - 1} \quad (3)$$
Table 2. Values of the coefficients in Eq. (1) that result from fitting different financial indices to Eq. (1)

| Period | Index  | $m$   | $A$            | $B$            | $t^{div}_c$ | $R$     |
|--------|--------|-------|----------------|----------------|-------------|---------|
| 80-87  | Dow    | 0     | -499.4±16.1    | -532.9±5.6     | 87.85±0.02  | 0.951   |
| 80-87  | Dow    | 1/3   | -526.6±20.8    | 614.7±8.6      | 88.22±0.03  | 0.956   |
| 80-87  | Dow    | 1/2   | -5.7±15.2      | 257.8±4.1      | 88.46±0.04  | 0.956   |
| 80-87  | S&P500 | 0     | -57.4±2.5      | -68.9±0.9      | 87.89±0.03  | 0.947   |
| 80-87  | S&P500 | 1/3   | -80.3±3.7      | 88.2±1.6       | 88.45±0.04  | 0.949   |
| 80-87  | S&P500 | 1/2   | -11.6±2.8      | 38.8±0.8       | 88.78±0.05  | 0.949   |
| 80-87  | FTSE   | 0     | -563.5±31.9    | -512.9±9.8     | 87.85±0.03  | 0.960   |
| 80-87  | FTSE   | 1/3   | -449.9±41.4    | 549.9±15.1     | 88.21±0.05  | 0.958   |
| 80-87  | FTSE   | 1/2   | 59.1±31.5      | 222.3±7.1      | 88.41±0.06  | 0.956   |
| 90-97  | Dow    | 0     | -1919.6±38     | -1762±13.4     | 97.92±0.02  | 0.978   |
| 90-97  | Dow    | 1/3   | -2100.4±49     | 2081.8±20.3    | 98.39±0.03  | 0.982   |
| 90-97  | Dow    | 1/2   | -360.1±35.8    | 882±9.7        | 98.68±0.04  | 0.982   |
| 90-97  | S&P500 | 0     | -141.5±4.4     | -187±1.5       | 97.90±0.02  | 0.974   |
| 90-97  | S&P500 | 1/3   | -161.4±6.1     | 221.3±2.5      | 98.38±0.03  | 0.976   |
| 90-97  | S&P500 | 1/2   | 23.2±4.5       | 93.9±1.2       | 98.67±0.04  | 0.976   |
| 90-97  | FTSE   | 0     | -499.1±46.9    | -1109.9±19     | 98.44±0.04  | 0.951   |
| 90-97  | FTSE   | 1/3   | -1310±86       | 1633.3±40.9    | 99.51±0.08  | 0.948   |
| 90-97  | FTSE   | 1/2   | -189.8±66.3    | 770.6±22.3     | 80.0±0.10   | 0.948   |

holds true where $\lambda = \exp(\omega/2\pi)$ and $t_n$, $t_{n+1}$ are successive maxima or minima days (Table 3). The results readily show that the examined stock market indices well follow a logarithmic law divergence. It should be noted that $t^\text{osc}_c$ and $t^{div}_c$ are extremal dates since the index should necessarily fall down before it reaches infinity. Moreover for both 1980-87 and 1990-97 period cases, it is found that the value of $\lambda$ seems to be almost constant, (Table 3), corresponding to $\omega \approx 6$. An analysis along similar lines of thought, though emphasizing the no-divergence was discussed for the Nikkei and NASDAQ April 2000 crash.

### 3.2 During

First consider that a crash can occur under four different conditions, and be listed in four categories, i.e. PMP, PMM, MMP, MMM, where M and P indicate an index variation from one day to another. The middle variation represents the crash amplitude. This allows for mini and maxi crashes. For
Table 3. The $\lambda$ and $t^{osc}_c$ values obtained for three indices following the methodology explained in the text. The real rupture point of Oct. 19, 1987 is $t_c=87.79$, and that of Oct. 24, 97 is $t_c=97.81$.

| Period | Index | 80-87 | 90-97 |
|--------|-------|-------|-------|
|        | Dow   | S&P500 | FTSE  | Dow   | S&P500 | FTSE  |
| $\lambda$ | 2.382±0.123 | 2.528±0.127 | 2.365±0.137 | 2.278±0.045 | 2.549±0.163 | 2.375±0.054 |
| $t^{osc}_c$ | 87.91±0.10 | 87.88±0.07 | 87.87±0.10 | 97.89±0.06 | 97.85±0.08 | 97.85±0.05 |

This report considers the DAX variations between Oct. 1, 1959 and Dec. 30, 1998. In Figs. 1(a-d) we show the DAX partial distribution of fluctuations (pdf) resulting from distinguishing such categories. The pdf’s have fat tails far from a Gaussian distribution and scale as a power law with exponent $\mu$, $P(g(i)) \sim g(i)^{-\mu}$, where $g(i) = \log(y(i+1)/y(i))$ and $y(i)$ denotes the signal. Approximate values of the $\mu$ exponent are given in Table 4. The nine most drastic crashes in each category are shown in Table 5 according to the value of $g(i) = \log(y(i+1)) - \log(y(i))$, together with the corresponding day. Notice that the case studied by Lillo et al. [14], occurring on Aug. 31, 1998 is not included among these 36 crashes.

Table 4. The $\mu$ exponent of the pdf tail’s power law dependence, the mean spectral exponent $\beta$ and the corresponding mean fractal dimension $D$ for the DAX 600 day long recovery signals after the crash for each crash category, i.e. PMP, PMM, MMP and MMM of the DAX between Oct. 01, 1959 and Dec. 30, 1998.

| case  | $\mu$        | $< \beta >$ | $< D >$ |
|-------|--------------|-------------|--------|
| PMP   | 2.76±0.12    | 1.70±0.38   | 1.65±0.38 |
| PMM   | 2.76±0.19    | 1.79±0.30   | 1.60±0.30 |
| MMP   | 2.85±0.17    | 1.60±0.36   | 1.70±0.36 |
| MMM   | 2.83±0.23    | 1.68±0.35   | 1.66±0.35 |

3.3 After

To study the index evolution after the crash we construct an evolution signal (Fig. 2 (a-d)) that is the difference between the DAX value signal at each day $y(i)$ and the DAX value at the crash day $y_0$ for the 36 cases of interest reported in Table 5. For most of the crashes, i.e. all crashes that occur before Aug. 06, 1996, the evolution signal is 600 day long. However, for crashes that occur after Aug. 6, 96, e.g. less than 600 days before the last day of this study, the evolution signal is shorter, as for example the Oct. 01, 98 MMM case in Fig. 2d. There are 3 short cases in each category.
Notice that recovery can be slow. The PMP and PMM cases need of the order of thirty days before having a positive $y(i) - y_0$ value. The situation is more complicated for the MMP and MMM cases. To see if some periodic fluctuation occurs after the crash, the power spectrum of the DAX has been studied for the 600 day long signals, i.e. for 24 cases. The power spectrum corresponding to the major crash in each category is given in Figs. 3 (a-d). Note the high-frequency log-periodic oscillation regime of the power spectrum for the strongest MMM case that occurs on Oct. 19, 1987 on Fig. 3 d.

To estimate the behavior of the DAX index evolution signal post PMP, PMM, MMP, MMM crashes it is of interest to relate each spectral exponent $\beta$ to the fractal dimension $D$ of the signal through \[ D = E + \frac{3 - \beta}{2}, \] where $E$ is the Euclidian dimension. The values of the averaged $\beta$ and averaged fractal dimension $D$ are reported in Table 4.
Table 5. The nine strongest crashes in each PMP, PMM, MMP and MMM category listed in decreasing order strength measured by the value of \( g(i) = \log(y(i+1)) - \log(y(i)) \) for DAX between Oct. 1, 1959 and Dec. 30, 1998

|      | PMP       | PMM       | MMP       | MMM       |
|------|-----------|-----------|-----------|-----------|
| date | date      | date      | date      | date      |
|      | \( g(i) \) | \( g(i) \) | \( g(i) \) | \( g(i) \) |
| 1.   | Oct 26, 87 | -0.080    | Oct 28, 87 | -0.070    |
| 2.   | Oct 22, 87 | -0.069    | Aug 19, 91 | -0.099    |
| 3.   | Jan 04, 88 | -0.058    | Oct 22, 97 | -0.043    |
| 4.   | Mar 06, 61 | -0.056    | May 29, 62 | -0.075    |
| 5.   | Jul 07, 86 | -0.053    | Nov 10, 87 | -0.068    |
| 6.   | Oct 23, 97 | -0.048    | May 28, 62 | -0.065    |
| 7.   | Dec 06, 96 | -0.041    | Aug 21, 98 | -0.061    |
| 8.   | Apr 01, 97 | -0.040    | Aug 06, 90 | -0.056    |
| 9.   | Jan 21, 74 | -0.036    | Mar 13, 74 | -0.055    |

4 Predictability and remedies for a conclusion

Let us assume for the following arguments that one can discuss stock market crashes in terms of physical model considerations. Moreover, let us admit that signals can be treated as above, in terms of power laws, and oscillations. In so doing we use the framework which has been useful in analyzing the avalanche problem for sand piles in the Appendix. Let us wonder whether these considerations, and analogies can suggest remedies in order to control or even avoid crashes. It is easily argued that remedies can be either self-remedies or due to external fields. At thermodynamic phase transitions, impurities, or external fields can shift the critical temperature, and reduce the divergence of thermodynamic properties. Let us disregard here the case of external field, though several authors might consider that in some economies such fields are relevant, or more necessary than self-corrections.

Several variables, or parameters, are to be considered: (i) the time scale, or frequency \( \omega \), (ii) the amplitudes of the signal \( A_i \), (iii) the dimensionality of the system, (iv) the connectivity \( \lambda \) of the lattice. The amplitude is related to the ”amount of sand” or volume exchanged during transactions, while the connectivity is related to gradient of trades, which is somewhat similar to the sand ”angle of repose”.

First it is absolutely clear that if the sand flux is large, an avalanche will be very likely, and disordered. This can be seen to be analogous to the retention of orders on a market, and to the effect of breakers \[38\]. Contrary to reducing, stopping the avalanche effect, breakers are in fact accelerating the process.
Fig. 2. The evolution of the nine strongest DAX crashes in (a) PMP, (b) PMM, (c) MMP, and (d) MMM categories as listed in Table 5; \( y_0 \) denotes the index value at the “origin of recovery”, i.e. the value of the signal at the closure time of the crash day. The thick solid line corresponds to the average evolution of the recovery signal in each category.

One remedy is therefore to reduce the amount of sand, i.e. the number of orders should decrease with time, and some delay should be imposed between orders. This is similar to changing the angle of repose of the materials.

Another new remedy is hereby introduced, the connectivity. It has been seen in the Appendix that when the connectivity increases the avalanche distribution is more spread out, the log-periodicity feature is not so pronounced. Therefore it seems relevant that the number of actors on the market be increased at crash time, together with the decrease in exchanged volume. Furthermore the connectivity is a key ingredient in the spreading of information on a market \([31]\) and also leads to consider the effect of interacting agents and herding models \([40, 41]\). Notice that this effect is entirely contained in the oscillations which therefore smoothen the rate of divergence. It is of interest to observe that the connectivity, in the stock market, is a rather small number, ca. 6.0. To expect that the connectivity is a constant whatever the hierarchical stage is of course a utopia, but this can be taken as a first approximation.

Another consideration pertains to the question whether a model and its solution can be implemented, and if so if any crash can be avoided. Two
Fig. 3. The power spectrum of the 600 day DAX index evolution signal corresponding to the major crash in each category (a) PMP : Oct. 26, 1987, (b) PMM : Oct. 28, 1987, (c) MMP : Oct. 16, 1989, and (d) MMM : Oct. 19, 1987.

comments are in order. First, one has to bear with statistical physics that as long as hypotheses are fulfilled the class of transition is defined, and therefore a crash will occur if the conditions are fulfilled. Next economic and speculative considerations will always exist. Therefore crashes will always exist [42], except maybe under conditions/remedies outlined in the preceding section.

Whether there is an influence of a known theoretical model on a financial event like a crash is exactly similar to wondering whether the equilibrium market hypothesis holds true.

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Appendix

The Bak, Tang and Wiesenfeld (BTW) [5] accumulation-dissipation process model on regular lattices was extended to a sand pile version [43,44]. Recently, the BTW process was studied on a Sierpinski gasket of fractal dimension $D_f = \frac{\ln 3}{\ln 2}$ [45,46]. It has been shown that the avalanche dynamics is characterized by a power law with a complex scaling exponent $\tau + i\omega$ with $\omega = \frac{2\pi}{\ln 2}$. This was understood as the result of the underlying Discrete Scale Invariance (DSI) of the gasket, i.e. the lattice is translation invariant on a log-scale [26]. It is possible to extend the study of the BTW model on other (prefractal) Regular Sierpinski Carpets (RSC) by varying both the fractal dimension $D_f$ as well as the connectivity of the lattice. In so doing we have observed apparently connectivity-based corrections to power law scaling.

![Fig. 4](image-url)  
\[P(s)\]

**Fig. 4.** The size distribution of avalanches $P(s)$ for the BTW model on fractal RSC lattices having the same fractal dimension $D_f = \frac{\ln 6}{\ln 3} + i\frac{2\pi}{\ln 3}$. The generators of the Sierpinski carpets are indicated.

Four different RSC of generation $n = 2$ are illustrated in Fig. 4. They are characterized by the same complex fractal dimension $D_f = \frac{\ln 6}{\ln 3} + i\frac{2\pi}{\ln 3}$ but having different lacunarity, i.e. different measures of the heterogeneity [47]. The RSC’s of Fig. 5 are characterized by different fractal dimensions and connectivity, or ramification $R$. This quantity is defined as the minimum number of bonds which should be cut in order to remove a macroscopic part of the lattice. The RSC of Fig. 5a has an infinite ramification, though for all others in Figs. 4-5, the ramification $R$ is finite.
Let each site $j$ of a RSC be allowed to contain a finite number of states or entities $z_j = \{0, 1, 2, ..., z_j^c\}$, where $z_j^c$ is hereby taken equal to $R$.

At each step of the BTW-like dynamical process [5], one lattice site $j$ is chosen at random and its content is updated following:

$$ z_j \rightarrow z_j + 1, \quad (5) $$

i.e. accumulating entities on the site $j$. However if $z_j \geq z_j^c$, the $j$ site is assumed to become unstable (or “active”) and to relax (in other words an avalanche is initiated) according to the following rules:

$$ z_j \rightarrow z_j - z_j^c \quad (6) $$

$$ z_k \rightarrow z_k + 1 \quad (7) $$

where $k$ denotes the $z_j^c$ nearest neighboring sites of $j$. The dissipation rule is repeated $t$ times until the system reaches a stable state with all lattice sites $m$ implied in the avalanche having $z_m < z_m^c$. By definition, the size $s$ of the avalanche is the number of sites visited by the relaxation process after each perturbation. The duration of the avalanche is $t$. Another (or the same) site $j$ is next chosen and the (5)-(7) process repeated. One should remark that the borders of the square lattice on which the RSC is built play the role of absorbing sites for the dissipative process [6].

In Figs. 4-5, the distribution of avalanche size $P(s)$ is plotted for these different RSC lattices, the generators being indicated in the margin. Since
the generators have the same size $3 \times 3$, the imaginary part of $D_f$ is $\frac{2\pi}{\ln 3}$ for the illustrated lattices. About $10^6$ avalanches have been counted in each $P(s)$ distribution, rescaled by some arbitrary factor for clarity. Different types of behaviors can be observed ranging from jagged distributions with well defined peaks and valleys to "classic" smooth power law $P(s)$ distributions. For most distributions $P(s) \sim s^{-\tau}$, expressing the scale invariance of the dynamics. We have checked the power-law exponent ($\tau$) as a function of the fractal dimension of RSC lattices and have found that $\tau$ seems to be dependent of the real part of the fractal dimension $\Re\{D_f\}$. Notice that for $\Re\{D_f\} \rightarrow 2$, $\tau = 1.25 \pm 0.03$.

In order to estimate $\tau$, we have filtered the jagged curve $P(s)$ distribution obtained on lattices of size $n=3, 4, 5$ and 6. As done in [45], we have extrapolated the values of $\tau$ for $n \rightarrow \infty$ in order to minimize finite-size effects. Nevertheless, error bars are large (about 10%) due to the presence of the oscillations. We have observed significant deviations of $\tau$ from 1.25, i.e. the $d = 2$ value. It seems that the real part of the fractal dimension of the dissipative system is not the single parameter controlling the value of $\tau$. Usually when the dimension of the RSC lattice has a finite imaginary part $i\omega$, one can observe periodic structures in $P(s)s^\tau$ with a period $\frac{2\pi}{\omega}$. At the bottom of Fig. 5, there are huge peaks which are log-periodically spaced. These oscillations (peaks) can be thought to originate from the DSI of the RSC lattice as in [45], and to mimic those discussed in the main text for financial indices. We have noticed that a finite value of the ramification $R$ corresponds to the largest amplitude of the oscillations. One should remark that previous investigations [45,46] did not find huge oscillations nor sharp peaks. The authors considered a Sierpinski gasket having loops in the structure as well as a constant threshold $z_c^j = 4$ everywhere on the gasket. We emphasize that the connectivity of the lattice is one of the most relevant parameters. Notice that such log-periodic oscillations and linearly substructured peaks are observed in the time distribution of avalanche durations $P(t)$ as well.

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