Sky coverage for Layer Oriented MCAO: a detailed analytical and numerical study

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ABSTRACT

One of the key-points for the future developments of the multiconjugate adaptive optics for astronomy is the availability of the correction for a large fraction of the sky. The sky coverage represents one of the limits of the existing single reference adaptive optics systems. Multiconjugate adaptive optics allows to overcome the limitations due to the small corrected field of view and the Layer Oriented approach, in particular by its Multiple Field of View version, increases the number of possible references using also very faint stars to guide the adaptive systems. In this paper we study the sky coverage problem in the Layer Oriented case, using both numerical and analytical approaches. Taking into account a star catalogue and a star luminosity distribution function we run a lot of numerical simulation sequences using the Layer Oriented Simulation Tool (LOST). Moreover we perform for several cases a detailed optimization procedure and a relative full simulation in order to achieve better performance for the considered system in those particular conditions. In this way we can retrieve a distribution of numerically simulated cases that allows computing the sky coverage with respect to a performance parameter as the Strehl Ratio and to the scientific field size.

Keywords: Multi-Conjugate Adaptive Optics, Layer oriented MCAO, Wavefront Sensors, Sky Coverage

1. INTRODUCTION

The theoretical and technological developments of multiconjugate adaptive optics\textsuperscript{1–3} (MCAO) give room to design a new generation instruments for 8-10 meters class telescopes. Moreover budget considerations induce to think more attractive ground based solutions than space ones, at least for the optical and near infrared portions of the electromagnetic spectrum. Actually large, very large and extremely large telescopes\textsuperscript{4} projects take into account the MCAO option in order to achieve the theoretical diffraction limit resolution. However is still pending the way to approach to the MCAO correction. One of the biggest drivers for the selection of which MCAO technique should be used is the effective feasibility of the adaptive correction. For all natural guide stars techniques this becomes the probability to find (or not) a suitable stars asterism around the object, target of the scientific exposure. The probability to find these reference stars is what we call sky coverage. Laser guide stars techniques are quite solving this problem even if, to date, more technological efforts to achieve stability and high power for the laser-beam\textsuperscript{5} are needed. Sky coverage usually is analysed in statistical way: assuming a star luminosity density function\textsuperscript{6} (for example the Bahcall and Soneira function) for the Galaxy the value of the coverage for different galactic latitudes and longitudes are then retrieved.

In this paper we will discuss the sky coverage problem focusing on the Layer-Oriented\textsuperscript{7} (LO) MCAO technique and in particular on the Multiple Field of View\textsuperscript{8} (MFoV) version. This approach is very attractive because of the possibility to add optically the light of many reference stars sensed simultaneously on the same detector. In this
way the signal to noise ratio (SNR) on the wave-front measurements does not depend on the brightness of the single reference star, but by the overall integrated magnitude of the asterism. This capability increases the sky coverage by including also faint stars in the list of the possible references. In LO each couple of deformable mirror (DM) and Wave-Front Sensor (WFS) is conjugated to a turbulent layer in an independent closed loop (from the hardware point of view). So, in principle, it is possible to drive a MCAO system using different reference stars for each detector-DM loop.

The MFoV approach goes in the direction of increasing the sky coverage using for the ground layer correction a large field of view (FoV) of 6 arcmin or more, instead of the typical 1 or 2 arcmin (with a factor 16 gain in terms of sky area) where to look for the reference stars. The ground layer usually is the strongest contributor to the atmospheric seeing and if it is subtracted then the residual phase to be corrected by the other (or others) loop is considerably smaller. High layers have usually Fried parameter ($r_0$) larger than the ground one and a bigger coherence time ($\tau_0$). One the advantages of the LO is the possibility to tune the characteristics of the loops as the integration time and the dimension of the sub-apertures, that compose the measured wave-front, to the statistical properties of the turbulent layers. In this case the MFoV systems, or more generally the LO ones, take advantage of this characteristic driving the high layer with a frequency usually different than the ground (because the turbulence is usually lower in the high layers than in the ground one and in the MFoV case also the reference stars are different between the two conjugation altitudes) optimising in this way the signal to noise ratio for two WFSs. In the system we are going to analyse in this study the reference stars for the ground layer correction are chosen in a ring of 6 arcmin external diameter excluding the inner corrected FoV of two arcmin, that is the field used for the selection of the guide stars for the high WFS.

In 2002 we started the development of a numerical simulation code, then called LOST - Layer Oriented Simulation Tool\(^9\), to analyse the performance of the MCAO system using the LO approach. It is very close to being an end-to-end simulation code: the main difference is in the way the WFSs are simulated. In LOST the phase noise introduced by each detector is computed through the relation derived for the Shack-Hartmann WFS and then it is added to the noise-free measurements.

In this paper we will describe the USNO catalogue\(^{10}\), the definition of sky coverage and the adaptive system we considered in this study. Finally the data are discussed.

2. SKY COVERAGE ESTIMATION USING STARS CATALOGUE

2.1 The USNO B catalogue

Every sky-coverage study is strictly dependent on the initial parameter taken into account for the adaptive system being considered, and dramatically on the way to select the reference stars. We analysed three 1 square degree real sky-fields, reading the stars data in the USNO-B catalogue (version 1.0). In this catalogue are listed the objects identified by scanning plates taken in the last 50 years by different observatories. These data cover all the galactic latitudes and longitudes and the catalogue gives information about positions, proper motions, magnitudes in various optical pass bands (B, R and I), and a star/galaxy estimator for more than 1 billion objects. This catalogue has an accuracy of about 0.2 arcsec for astrometry, 0.3 arcsec in photometry and 85 percent in star/galaxy classification. It is supposed to be completed until the magnitude 21 in the V band.

However the study we are presenting refers to the R band only, and we checked that the catalogue presents lack of data for stars fainter than 19\(^{th}\) magnitude in this band (see Figure\(^{[1]}\)).

2.2 Sky Coverage

The Sky Coverage is defined as the fraction of sky where the MCAO correction is feasible. However this concept suffers of a lot of ambiguities. These can be generated by the definition itself. In fact to assess that MCAO correction it is possible one can refer to simple closure of the adaptive loop with, for example, a small gain in terms of SR or Full Width Half Maximum of the PSF, or he can refer to the level of SR that an astronomer define as good for his purposes, or taking into account encircled energy or other details. Of course this definition depends on the target of MCAO correction and on the system characteristics (for example the performance requested for a ground layer corrector are different than a Multi-conjugate system).
Someone\textsuperscript{7,11} defines sky-coverage as the percentage of cases where the SR achieve at least the 50\% of the infinite Signal to Noise Ratio (SNR) case. But this definition is not regarding at all the absolute performance of the system, and in several cases the obtained results could not be enough high to give the requested correction (because of very low SR or big un-uniformity in the correction) but included in the list of the sky-coverage as good.

We propose to relate the sky-coverage definition to the astrophysical target to be study without speaking of “absolute sky coverage”. This means that we have to set in some way a condition parameter to distinguish the region of the sky where there is (or not) the coverage. Looking to the system we are considering (MCAO on a 8 meter telescope) a reasonable parameter can be the SR, and the lower limit could be 5 or, better 10 percent (at the K band), useful for many astrophysical studies. Here we considered different classes of objects according to their angular dimensions so we defined different coverage for 3 different FoVs, averaging the SR data over the interested region. In this way we take into account also un-uniformities of the correction that were not considered if we only take as representative the maximum SR over the field taken into account.

The sky-coverage problem regarding the LO-MFoV technique has been studied\textsuperscript{11} using a statistical approach based on the Bahcall and Soneira star density function and an analytical model for the Pyramid WFS\textsuperscript{12,13}. Here the limiting integrated magnitude to have a degradation of 0.5 in SR with respect to the case with infinite bright reference stars are 14.5 for the ground loop and 16.5 for the high loop. There were found out sky coverage between 8\% and 20\% for the galactic poles and between 46\% and 96\% for low galactic latitudes.

Here we consider the definition of coverage we proposed few lines above and this method: first of all we consider real 1 square degree fields, from the USNO-B catalogue, around the Galactic poles, North (NGP) and South (SGP) and the Galactic anti-Centre (GAC) and then we define on them a square grid of 32 × 32 points with a step of 101 arcsec. Every grid point represents the on-axis direction where the FoV circles of 2 and 6 arcmin are centred.

But in the USNO-B catalogue are listed also extended non-stellar objects that cannot be references for MCAO: we discard these objects from the data by considering the star/galaxy estimator of the catalogue. In this way we have the “raw” data for the analysis of the field stars distribution and characteristics.

Of course the sky-coverage is limited by the presence of the stars in the field of view but also by other very important parameters. The reference stars are selected (if any) simultaneously in the two FoVs of 2 and 6 arcmin and according to the technical constrains summarized in Table\textsuperscript{11}. The constrain on the maximum range of variation for the references brightness usually tends to decrease the probability to find very bright star (brighter than 12-13 mag) in the central 2 arcmin FoV because the other stars in the Field are “statistically” fainter than
3 magnitudes. Another limit to the sky-coverage is the minimum distance that separates two close references. This is due to the physical dimension of the mounting of the pyramid (also called star enlargers). These values depend on the system, in particular we used numbers taken from an existing project\(^\text{14}\); the minimum separations adopted are 20 arcsec and 30 arcsec respectively for the high and for the ground WFSs.

The stars are selected in order to retrieve the brightest as possible asterism considering the limits given and trying to maximize the separation between the references.

| Limit integrated magnitude for the asterism | Min Separation GL | Min Separation HL | Max magnitude range | Min NGS number GL and HL | Max NGS number GL | Max NGS number HL |
|---------------------------------------------|-------------------|-------------------|---------------------|--------------------------|-----------------|-----------------|
| R < 19                                      | 30 arcsec         | 20 arcsec         | 3 mag               | 3                        | 12              | 8               |

Table 1. In this table are listed the main constrain used for the selection of the Natural Guide Stars. All this condition must be verified simultaneously in order to select the stars asterism for a simulation run. If no asterism verifies these conditions then in that region we say that there is not sky coverage.

A simulation run is performed for each grid-point where a suitable asterism has been found, retrieving in this way a grid of SR values on the 2 arcmin central FoV (see Figure 2). Finally the simulations outputs are the data used for the sky-coverage analysis.

2.3 The system taken into account

In order to compare the results obtained on different asterisms of the same 1 degree field the characteristics of the LO system and of the atmosphere must be fixed. We consider an 8-meter telescope and the median turbulence profile typical of the Paranal (see Table 4), characterized by an average seeing value at the V band of 0.73 arcsec equivalent to an overall \( r_0 \) of 0.14 meters (in the V band). In both cases the results are computed in the K band, 2.2\( \mu m \), with a correspondent \( r_{0,K} = 0.83 \) meters. We consider a MFoV system with 2 DMs conjugated to 0 and 8.5 km. The spatial geometry of the system has been taken fixed to a sampling 8\( \times \)8 for the ground and 7\( \times \)7 for the high in order to be deep in term of limiting magnitude but not optimising for the maximum achievable SR. The other mains MCAO system parameters are listed in Table 2. Each simulation covers an elapsed time of 0.5 seconds, but only the last 0.4 are taken into account for the computation of the long exposure SR.

| Overall efficiency | Sensing wavelength | Scientific wavelength | Bandwidth | Conjugation altitude | RON e/frame | Dark Current | Binning | Max # Zernike modes |
|--------------------|--------------------|-----------------------|-----------|----------------------|-------------|--------------|---------|---------------------|
| 0.2                | 0.7\( \mu m \)     | 2.2\( \mu m \)       | 0.4\( \mu m \) | 0 km                 | 3.0         | 200 e\(^-\)/sec | 2\(\times\)2 | 59                  |
|                    | 8.5 km             | 3.0                   | 200 e\(^-\)/sec | 4\(\times\)4 | 45            |

Table 2. In this table are presented the main characteristic of the MCAO system considered. We simulate an MFoV system with corrected field of 2 arcmin and a technical FoV of 6 arcmin relative to the high and ground WFSs respectively.

The integration times applied to the two WFSs are tuned to the integrated magnitude on the 6arcmin ring and on the 2arcmin FoVs for the ground and the high respectively. As in the case of the spatial sampling, we choose the frame rates of the two WFSs to retrieve a high limiting magnitude instead of a higher SR. The values of the integrated times are listed in Table 3 below.

2.4 Optimization test

The integration times used for both loops were set according to the integrated magnitude of the asterism in the 6 arcmin annular FoV and in the central 2 arcmin FoV, respectively for the Ground and the High WFS (see Table 3). This solution to set this important couple of parameters is correct only for a first order approach. In fact, for example, it neglects the effect of the different illumination of the sub-apertures in the High WFS due to the references position and different brightness. A smarter analysis should take into account a fine-tuning of
| Ground Loop | $R_{int} < 11$ | $11 < R_{int} < 13$ | $13 < R_{int} < 15$ | $15 < R_{int} < 16.5$ | $R > 16.5$ |
|-------------|---------------|------------------|------------------|------------------|---------|
|             | 2 msec        | 4 msec           | 10 msec          | 20 msec          | 40 msec |

| High Loop   | $R_{int} < 10$ | $10 < R_{int} < 12$ | $12 < R_{int} < 14$ | $14 < R_{int} < 16.5$ | $R > 16.5$ |
|-------------|----------------|------------------|------------------|------------------|---------|
|             | 2 msec         | 4 msec           | 10 msec          | 20 msec          | 40 msec |

Table 3. In this table are listed the integration times used for the two WFS with respect to the integrated magnitude of the both references asterism. The values are tuned to the statistical characteristics of the conjugated planes.

| Layer ID | Layers Altitude [m] | Cn2 fraction | Wind [m/s] |
|----------|---------------------|--------------|------------|
| 1        | 0                   | 0.65         | 6.6        |
| 2        | 1800                | 0.08         | 12.4       |
| 3        | 3200                | 0.12         | 8.0        |
| 4        | 5800                | 0.03         | 33.7       |
| 5        | 7400                | 0.03         | 23.2       |
| 6        | 13100               | 0.08         | 22.2       |
| 7        | 15800               | 0.01         | 8.0        |

Table 4. Here are listed the atmospheric parameters used in the simulations. For each layer an outer-scale of 20 m has been considered. The isoplanatic angle for the overall atmosphere is about 15 arcsec at the 2.2$\mu$m pass band. This model is not the most recent one where there is a bit more turbulence power in the ground layer (67% instead of 65%). In this study a 0”.73 seeing in V Band and 0”.66 seeing in R band were considered.

the frame rates for the two WFSs. But an optimization procedure to be applied to all the asterisms considered it’s not feasible because in this case the overall number of simulations performed will increase too much. In other cases described so far$^{15,16}$ we considered a grid of possible values for the two frame rates in order to taking into account the different combinations of the two integration times. Here we considered a small $3 \times 3$ grid with values around to the ones specified in Table 3 (these depending on the integrated magnitude) ranging from 25% less and 25% more the two values taking into account for the simulation performed yet. We optimized the integration time only for a small set of the asterisms (20) used in the three $1 \times 1$ square degree fields. The results of this optimization compared to the non-optimized data allow extrapolating the optimized SR values for all the simulated cases.

3. DATA ANALYSIS

3.1 CPU time and Workstation

We found in the catalogue 40000 useful stars over the 3 sky-fields considered. We analysed 3072 regions, running 2000 simulations on the found asterisms. Each simulation took 3-6 hours of CPU time according to the CPU clock (at most we used 2000 MHz).

The overall CPU time used was of 330days, the most spent on the Arcetri Beowulf cluster with 16 nodes, each one equipped with two CPUs. The LOST code is not paralleled yet, but we ran “in parallel” different simulations on different nodes at the same time.

We developed an IDL procedure to manage the different steps described in the section above. Moreover this found the best asterism for each couple of 2arcmin and 6arcmin fields; the best integration times for the two loops and it computed an analytical SR over the $31 \times 1$ deg$^2$ fields considering the asterism and integration times found before.

3.2 Data analysis

For each simulation LOST computed the SR values over the 2arcmin FoV. A $6 \times 6$ grid of SR evolution data was retrieved for each of the 32$\times$32 positions in the three 1-degree field (see Figure 2). Using these data the averages SR were computed on the 1arcmin circle and the 2-arcmin FoV. Moreover the on axis direction SR was taken into account averaging the 4 “probe” stars close to the centre of the field (see Figure 2).
Figure 2. In this plot the “+” sign defines the direction where the SR is computed in all the simulated cases, the step of this square grid is 24arcsec. The two circles represent the 1arcmin field and the 2arcmin. The correction was applied in the 2 arcmin FoV. In addition to these 6 × 6 directions the SR data were computed also in the Natural Guide Stars positions.

Figure 3. This picture shows a map of the analytical SR found for the Galactic Plane case. Each square covers 101×101arcsec region. If there is a lack of data or no good asterisms to drive the adaptive system this square is black. The color bar on the right indicates the value of the SR. For example: on the left of this map there is a very bright star that saturates all the plates taken into account for the catalogue preparation: in fact a circular hole without stars appears in our analysis. The analytical SR gives only an idea of the possible SR achievable because it does not take into account neither the distribution of the stars in the 6 and 2 arcmin FoV nor the turbulence profile.
Figure 4. This picture shows the sky coverage results for the three galactic latitude cases taken into account and relative to the on axis direction only. The functions plotted here represent the percentage of the simulated case where at least the SR showed in abscissa was achieved. Dotted line represents the North Galactic Pole; the dashed one refers to the South Galactic Pole and the solid to the Galactic Anti-centre. The percentage is relative to the 32×32 directions considered for each galactic field.

We optimize the MCAO system parameters looking for the loops closure and the robustness of the correction and we do not optimize the system in order to achieve high SR (more than 60%). In fact considering only the effect of the spatial sampling of the metapupils for the ground (8×8) and for the high loop (7×7) the best SR achievable is between 55%-60% with the atmospheric parameters here considered.

Each simulation performed has an iteration step of 2 msec for a total of 250 iterations that gives an overall time of 0.5 seconds. The length of these simulations is not enough to estimate a representative long exposure SR because of the effect of the bootstrap, so we assumed the maximum SR achieved during the run as reference for our analysis (the SR values we consider take into account the tip-tilt residual also). Analyzing the SR data of the 3 different 1 square degree fields considered, we drew different SR maps according to the 3 FoV sizes we assumed. These cases can be seen as representative of different instruments:

- A camera with few arcsec FoV;
- An instrument with 1 arcmin FoV;
- An instrument, or more instruments mounted on the same system covering the corrected 2arcmin field.

### 3.2.1 Few arcsec FoV Case

We assumed mounted at the focus of the telescope a camera with a small FoV of few arcsec centered in the optical axis direction. In this case the SR must be uniform because the FoV is smaller than the isoplanatic patch size. So we considered representative for this case the value of the SR obtained for the 4 “probe” stars more close to the centre of the FoV (see Figure 2). Figure 4 presents the results in terms of sky coverage VS threshold SR:

In the 98% of the cases taken into account the SR on axis was higher than 10%, while percentages of 48% and 25% were retrieved respectively for the North and the South Galactic poles. For the low Galactic latitudes in half of the cases considered the SR on axis was higher than 40%.
3.2.2 1 arcmin FoV case

Now we describe the sky coverage analysis for an instrument with 1arcmin FoV centered in the axis direction (the same axis relative to the correction applied by the adaptive system). For this case the representative SR is the average SR over the central one arcmin computed by the simulations. The Figure 5 shows the results relative to this field size: for the low galactic latitude (the Galactic Anticentre) in the 98% of the directions considered the average SR was higher than 10%, while the same values for the North and South Poles were 38% and 17% respectively.

3.2.3 2 arcmin FoV case

In this last case we supposed several instruments (or a unique big camera) observing the whole region corrected by the MCAO system (we considered a corrected FoV of 2 arcmin). As in the 1arcmin case we took into account the average SR, but now over the 2arcmin FoV.

In the the results are presented: considering a 10% threshold for the SR as condition to define the coverage we found sky coverage of 99% for Galactic plane, 25% and 13% respectively for the North and South Galactic poles.

We want to stress that the sky-coverage values within the 3 FoV sizes changes of a factor $\sim 2$ for the galactic poles while it is un-changed for the Galactic anticentre (see Figure 7). This different behaviour depends on the different stars density of the two galactic regions. The poles are poor of stars with respect to the low galactic latitudes: this translates in a less number of reference stars for the galactic poles and so a lower uniformity for the correction with respect to the galactic plane where it is quite easy to cover homogenously the corrected 2arcmin FoV with natural guide stars.

3.2.4 Dealing to a different definition

Now we want to analyse the results presented in the previous sections (3.2.1, 3.2.2 and 3.2.3) according to a different definition of sky coverage, using, for example, that one given in the reference$^{11}$, and that we discussed also above (section 2.2). We assume as reference for the infinite SNR SR the maximum SR achieved in each of the 3 cases of Field of View considered that are: $\sim 0.6$ for the few arcsec field of view case, $\sim 0.5$ for the 1 arcmin case and $\sim 0.4$ for the 2 arcmin FoV. Using these values we found the SR$_{50}$ thresholds (50% of the SR relative to the infinite SNR case) for each of these cases: 0.3, 0.25 and 0.2 respectively. Applying these thresholds instead of the 10% one, we found coverage of 90% for the low galactic latitudes case and between 7%and 15%
Figure 6. This picture shows the results in the overall corrected 2 arcmin FoV and for the three galactic latitude cases analyzed. The functions plotted here are representing the percentages where at least the SR showed in abscissa was achieved. Dotted line represents the North Galactic Pole; the dashed one refers to the South Galactic Pole and the solid to the Galactic Anti-centre. The SR here considered is average SR over the corrected field of 2 arcmin.

Figure 7. This figure shows the percentage of sky-coverage with respect to a threshold SR. Dashed line represents the coverage with respect to the on axis direction SR; the dotted one refers to the average SR over 1 arcmin FoV and the solid line to the SR averaged over the 2arcmin corrected field. All the 3 curves refer to the analysis performed on the 1×1 square degree field centered in the North Galactic Pole.

for the galactic poles. Even if referring to different wavelengths these values agree with the ones given in the references\textsuperscript{8,11} (here we considered correction in the K band while it was the R band in the reference\textsuperscript{11}).

4. EXAMPLE OF SCIENCE-COVERAGE: CLUSTER OF GALAXIES AT HIGH RED-SHIFT

What we said and stressed about different sky-coverage for different FoV becomes important when we consider the possible astronomical applications. For example we took as possible astronomical target the cluster of galaxies. As everybody knows their apparent dimension changes with their distance, but because of the structure and evolution of the universe this depend also by cosmological parameters.
The angular size of clusters is related to the red-shift ($z$) then for these objects the sky-coverage is a function of the $z$. We plotted the angular size taken from the reference with respect to the redshift\textsuperscript{17} in Figure 9.

This plot says that high-$z$ clusters have higher sky-coverage. If the cluster has dimension bigger than 2 arcmin then more contiguous asterisms are needed to cover its entire dimension and, in this case sky coverage decreases.

5. CONCLUSIONS

We analyse the sky coverage problem in the case of a specific Layer Oriented Multiple Field of View system. We showed basic relationship between sky coverage and field dimension to be studied and presented a scientific case. We analyse the definition of sky coverage and we stressed that it must be related to the performance requested to the adaptive system and the class of objects to be studied with the scientific instrument to be used. In particular we set a reasonable threshold to the 10% in SR as condition to define if there is sky coverage or not. We showed that for low galactic latitudes the correction is feasible about everywhere while at the galactic poles the coverage decreases, but down to reasonable values (20%-40%) to justify the use of this natural guide stars technique also for high galactic latitudes targets.

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REFERENCES

[1] Beckers, J. M., “Increasing the size of the isoplanatic patch with multiconjugate adaptive optics.,” in [ESO Conference on Very Large Telescopes and their Instrumentation], 2, 693–703 (1988).
Figure 9. This figure shows the angular size with respect to the red-shift according to the data in the reference\textsuperscript{17}. We plotted a dashed line to a radius of 1 arcmin, corresponding to the 2arcmin FoV case we considered in the sky-coverage analysis. Following our results the clusters at \( z \sim 0.9 \) have sky coverage of 25% (at North Galactic Pole).

\[2\] Beckers, J. M., “Detailed compensation of atmospheric seeing using multiconjugate adaptive optics,” in \textit{Active Telescope Systems}, \textit{Proc. SPIE}, 215–217 (1989).

\[3\] Ellerbroek, B. L., “First-order performance evaluation of adaptive-optics systems for atmospheric-turbulence compensation in extended-field-of-view astronomical telescopes,” \textit{Journal of the Optical Society of America A} \textbf{11}, 783–805 (1994).

\[4\] Dierickx, P., Beckers, J., Brunetto, E., Conan, R., Fedrigo, E., Gilmozzi, R., Hubin, N. N., Koch, F., Le Louarn, M., Marchetti, E., Monnet, G. J., Noethe, L., Quatti, M., Sarazin, M. S., Spyromilio, J., and Yaitskova, N., “The eye of the beholder: designing the OWL,” in \textit{Adaptive Optical System Technologies II}, J. R. P. Angel & R. Gilmozzi, ed., \textit{Proc. SPIE} \textbf{4840}, 151–170 (2003).

\[5\] Rabien, S., Davies, R. I., Ott, T., Li, J., Hippler, S., and Neumann, U., “Design of PARSEC the VLT laser,” in \textit{Adaptive Optical System Technologies II}, P. L. Wizinowich & D. Bonaccini, ed., \textit{Proc. SPIE} \textbf{4839}, 393–401 (2003).

\[6\] Bahcall, J. N. and Soneira, R. M., “The universe at faint magnitudes. I - Models for the galaxy and the predicted star counts,” \textit{ApJS} \textbf{44}, 73–110 (1980).

\[7\] Ragazzoni, R., Farinato, J., and Marchetti, E., “Adaptive optics for 100-m-class telescopes: new challenges require new solutions,” in \textit{Adaptive Optical Systems Technology}, Wizinowich, P. L., ed., \textit{Proc. SPIE} \textbf{4007}, 1076–1087 (2000).

\[8\] Ragazzoni, R., Diolaiti, E., Farinato, J., Fedrigo, E., Marchetti, E., Tordi, M., and Kirkman, D., “Multiple field of view layer-oriented adaptive optics. Nearly whole sky coverage on 8 m class telescopes and beyond,” \textit{A&A} \textbf{396}, 731–744 (2002).

\[9\] Arcidiacono, C., Diolaiti, E., Tordi, M., Ragazzoni, R., Farinato, J., Vernet, E., and Marchetti, E., “Layer-Oriented Simulation Tool,” \textit{Appl. Opt.} \textbf{43}, 4288–4302 (2004).

\[10\] Monet, D. G., Levine, S. E., Canzian, B., Ables, H. D., Bird, A. R., Dahn, C. C., Guetter, H. H., Harris, H. C., Henden, A. A., Leggett, S. K., Levison, H. F., Lugnabuhl, C. B., Martini, J., Monet, A. K. B., Munn, J. A., Pier, J. R., Rhodes, A. R., Riepe, B., Sell, S., Stone, R. C., Vrba, F. J., Walker, R. L., Westerhout, G., Brucato, R. J., Reid, I. N., Schoening, W., Hartley, M., Read, M. A., and Tritton, S. B., “The usno-b catalog,” \textit{AJ} \textbf{125}, 984–993 (2003).

\[11\] Marchetti, E., Ragazzoni, R., and Diolaiti, E., “Which range of magnitudes for layer oriented mcao?,” in \textit{Adaptive Optical System Technologies II}, P. L. Wizinowich & D. Bonaccini, ed., \textit{Proc. SPIE} \textbf{4839}, 566–577 (2003).
[12] Ragazzoni, R., “Pupil plane wavefront sensing with an oscillating prism,” *Journal of Modern Optics* **43**, 289–293 (1996).

[13] Ragazzoni, R. and Farinato, J., “Sensitivity of a pyramidal Wave Front sensor in closed loop Adaptive Optics,” *A&A* **350**, L23–L26 (1999).

[14] Ragazzoni, R., Herbst, T. M., Gaessler, W., Andersen, D., Arcidiacono, C., Baruffolo, A., Baumeister, H., Bizenberger, P., Diolaiti, E., Esposito, S., Farinato, J., Rix, H. W., Rohloff, R.-R., Riccardi, A., Salinari, P., Soci, R., Vernet-Viard, E., and Xu, W., “A visible MCAO channel for NIRVANA at the LBT,” in *Adaptive Optical System Technologies II*, Wizinowich, P. L. and Bonaccini, D., eds., *Proc. SPIE* **4839**, 536–543 (2003).

[15] Arcidiacono, C., Diolaiti, E., Ragazzoni, R., Viard, E., and Farinato, J., *LINC-NIRVANA Preliminary Design Review document*, 267, Max Planck Institute Für Astronomie (2003).

[16] Arcidiacono, C., Diolaiti, E., Ragazzoni, R., Viard, E., Farinato, J., and Baruffolo, A., *Layer Oriented Wavefront Sensor for MAD Final Design Review: System Design Analysis*, ESO (2003).

[17] Ettori, S., Tozzi, P., Borgani, S., and Rosati, P., “Scaling laws in x-ray galaxy clusters at redshift between 0.4 and 1.3,” *A&A* **417**, 13–27 (2004).