Extracting $\gamma$ Through Flavour-Symmetry Strategies

ROBERT FLEISCHER

Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, D–22607 Hamburg, Germany

Abstract

A brief overview of flavour-symmetry strategies to extract the angle $\gamma$ of the unitarity triangle is given, focusing on $B \rightarrow \pi K$ modes and the $B_d \rightarrow \pi^+\pi^-$, $B_s \rightarrow K^+K^-$ system. We discuss also a variant of the latter approach for the $e^+e^- B$-factories, where $B_s \rightarrow K^+K^-$ is replaced by $B_d \rightarrow \pi^\pm K^\mp$.

Contribution to the Proceedings of the Workshop on the CKM Unitarity Triangle, CERN, Geneva, 13–16 February 2002
EXTRACTING $\gamma$ THROUGH FLAVOUR-SYMMETRY STRATEGIES

R. Fleischer
Deutsches Elektronen-Synchrotron DESY, Notkestraße 85, D–22607 Hamburg, Germany

Abstract
A brief overview of flavour-symmetry strategies to extract the angle $\gamma$ of the unitarity triangle is given, focusing on $B \to \pi K$ modes and the $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ system. We discuss also a variant of the latter approach for the $e^+e^-$ $B$-factories, where $B_s \to K^+K^-$ is replaced by $B_d \to \pi^+\pi^\pm$.

1 INTRODUCTION
An important element in the testing of the Kobayashi–Maskawa picture of CP violation is the direct determination of the angle $\gamma$ of the unitarity triangle of the CKM matrix. Here the goal is to overconstrain this angle as much as possible. In the presence of new physics, discrepancies may arise between different strategies, as well as with the “indirect” results for $\gamma$ that are provided by the usual fits of the unitarity triangle, yielding at present $\gamma \sim 60^\circ$ [1].

There are many approaches on the market to determine $\gamma$ (for a detailed review, see Ref. [2]). Here we shall focus on $B \to \pi K$ modes [3]–[12], which can be analysed through flavour-symmetry arguments and plausible dynamical assumptions, and the $U$-spin-related decays $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ [13]. The corresponding flavour-symmetry strategies allow the determination of $\gamma$ and valuable hadronic parameters with a “minimal” theoretical input. Alternative approaches, relying on a more extensive use of theory, are provided by the recently developed “QCD factorization” [14] and “PQCD” [15] approaches, which allow furthermore a reduction of the theoretical uncertainties of the flavour-symmetry strategies discussed here. Let us note that these approaches are also particularly promising from a practical point of view: BaBar, Belle and CLEO-III may probe $\gamma$ through $B \to \pi K$ modes, whereas the $U$-spin strategy, requiring also a measurement of the $B_s$-meson decay $B_s \to K^+K^-$, is already interesting for run II of the Tevatron [18], and can be fully exploited in the LHC era [19]. A variant for the $B$-factories [20], where $B_s \to K^+K^-$ is replaced by $B_d \to \pi^+\pi^\pm$, points already to an exciting picture [21].

2 $B \to \pi K$ DECAYS
Using the isospin flavour symmetry of strong interactions, relations between $B \to \pi K$ amplitudes can be derived, which suggest the following combinations to probe $\gamma$: the “mixed” $B^\pm \to \pi^\pm K$, $B_d \to \pi^\pm K^\pm$ system [3]–[7], the “charged” $B^\pm \to \pi^\pm K$, $B^\pm \to \pi^0K^\pm$ system [8]–[10], and the “neutral” $B_d \to \pi^0K$, $B_d \to \pi^\pm K^\pm$ system [10,11]. Interestingly, already CP-averaged $B \to \pi K$ branching ratios may lead to non-trivial constraints on $\gamma$ [5,8]. In order to determine this angle, also CP-violating rate differences have to be measured. To this end, we introduce the following observables [10]:

$$\begin{align*}
R & \equiv \frac{\text{BR}(B_d^0 \to \pi^-K^+) \pm \text{BR}(\bar{B}_d^0 \to \pi^+K^-)}{\text{BR}(B^+ \to \pi^+K^0) + \text{BR}(B^- \to \pi^-K^0)} \frac{\tau_{B^+}}{\tau_{B_d^0}} \\
A_0 & \equiv 2 \frac{\text{BR}(B^+ \to \pi^0K^+) \pm \text{BR}(B^- \to \pi^0K^-)}{\text{BR}(B^+ \to \pi^+K^0) + \text{BR}(B^- \to \pi^-K^0)} \\
R_n & \equiv \frac{\text{BR}(B_d^0 \to \pi^-K^+) \pm \text{BR}(\bar{B}_d^0 \to \pi^+K^-)}{2 \left( \text{BR}(B_d^0 \to \pi^0K^0) + \text{BR}(\bar{B}_d^0 \to \pi^0K^0) \right)} \\
A_0 & \equiv \frac{1}{2} \frac{\text{BR}(B_d^0 \to \pi^-K^+) \pm \text{BR}(\bar{B}_d^0 \to \pi^+K^-)}{2 \left( \text{BR}(B_d^0 \to \pi^0K^0) + \text{BR}(\bar{B}_d^0 \to \pi^0K^0) \right)}.
\end{align*}$$

If we employ the isospin flavour symmetry and make plausible dynamical assumptions, concerning mainly the smallness of certain rescattering processes, we obtain parametrizations of the following
structure [2, 10] (for alternative ones, see Ref. [9]):

\[ R_{(c,n)}, A^{(c,n)}_0 = \text{functions} \left( q_{(c,n)}, r_{(c,n)}, \delta_{(c,n)}, \gamma \right). \]  

(4)

Here \( q_{(c,n)} \) denotes the ratio of electroweak (EW) penguins to “trees”, \( r_{(c,n)} \) is the ratio of “trees” to QCD penguins, and \( \delta_{(c,n)} \) the strong phase between “trees” and QCD penguins. The EW penguin parameters \( q_{(c,n)} \) can be fixed through theoretical arguments: in the mixed system [3]–[6], we have \( q \approx 0 \), as EW penguins contribute only in colour-suppressed form; in the charged and neutral \( B \rightarrow \pi K \) systems, \( q_s \) and \( q_u \) can be fixed through the \( SU(3) \) flavour symmetry without dynamical assumptions [8]–[11]. The \( r_{(c,n)} \) can be determined with the help of additional experimental information: in the mixed system, \( r \) can be fixed through arguments based on factorization [3, 8] or \( U \)-spin [22], whereas \( r_c \) and \( r_n \) can be determined from the CP-averaged \( B^\pm \rightarrow \pi^\pm \pi^0 \) branching ratio by using only the \( SU(3) \) flavour symmetry [3, 8]. The uncertainties arising in this programme from \( SU(3) \)-breaking effects can be reduced through the QCD factorization approach [16], which is moreover in favour of small rescattering processes. For simplicity, we shall neglect such FSI effects in the discussion given below.

Since we are in a position to fix the parameters \( q_{(c,n)} \) and \( r_{(c,n)} \), we may determine \( \delta_{(c,n)} \) and \( \gamma \) from the observables given in (3). This can be done separately for the mixed, charged and neutral \( B \rightarrow \pi K \) systems. It should be emphasized that also CP-violating rate differences have to be measured to this end. Using just the CP-conserving observables \( R_{(c,n)} \), we may obtain interesting constraints on \( \gamma \). In contrast to \( q_{(c,n)} \) and \( r_{(c,n)} \), the strong phase \( \delta_{(c,n)} \) suffers from large hadronic uncertainties. However, we can get rid of \( \delta_{(c,n)} \) by keeping it as a “free” variable, yielding minimal and maximal values for \( R_{(c,n)} \):

\[ R^\text{ext}_{(c,n)}|_{\delta_{(c,n)}} = \text{function} \left( q_{(c,n)}, r_{(c,n)}, \gamma \right). \]

(5)

Keeping in addition \( r_{(c,n)} \) as a free variable, we obtain another – less restrictive – minimal value

\[ R^\text{min}_{(c,n)}|_{r_{(c,n)}} = \text{function} \left( q_{(c,n)}, \gamma \right) \sin^2 \gamma. \]

(6)

These extremal values of \( R_{(c,n)} \) imply constraints on \( \gamma \), since the cases corresponding to \( R^\text{min}_{(c,n)} > R^\text{max}_{(c,n)} \) and \( R^\text{exp}_{(c,n)} \) are excluded. Present experimental data seem to point towards values for \( \gamma \) that are larger than 90°, which would be in conflict with the CKM fits, favouring \( \gamma \sim 60^\circ \) [1]. Unfortunately, the present experimental uncertainties do not yet allow us to draw definite conclusions, but the picture should improve significantly in the future.

An efficient way to represent the situation in the \( B \rightarrow \pi K \) system is provided by allowed regions in the \( R_{(c,n)}^{-1}A^{(c,n)}_0 \) planes [2, 23], which can be derived within the Standard Model and allow a direct comparison with the experimental data. A complementary analysis in terms of \( \gamma \) and \( \delta_{(c,n)} \) was performed in Ref. [3]. Another recent \( B \rightarrow \pi K \) study can be found in Ref. [23], where the \( R_{(c)} \) were calculated for given values of \( A^{(c,n)}_0 \) as functions of \( \gamma \), and were compared with the \( B \)-factory data. In order to analyse \( B \rightarrow \pi K \) modes, also certain sum rules may be useful [23].

3 THE \( B_d \rightarrow \pi^+\pi^-, B_s \rightarrow K^+K^- \) SYSTEM

As can be seen from the corresponding Feynman diagrams, \( B_s \rightarrow K^+K^- \) is related to \( B_d \rightarrow \pi^+\pi^- \) through an interchange of all down and strange quarks. The decay amplitudes read as follows [15]:

\[ A_{B^0_d \rightarrow \pi^+\pi^-} \propto \left[ e^{i \gamma} - d e^{i \theta} \right], \quad A_{B^0_s \rightarrow K^+K^-} \propto \left[ e^{i \gamma} + \left( \frac{1-\lambda^2}{\lambda^2} \right) d' e^{i \theta} \right], \]

(7)

where the CP-conserving strong amplitudes \( d e^{i \theta} \) and \( d' e^{i \theta} \) measure, sloppily speaking, ratios of penguin to tree amplitudes in \( B^0_d \rightarrow \pi^+\pi^- \) and \( B^0_s \rightarrow K^+K^- \), respectively. Using these general parametrizations, we obtain expressions for the direct and mixing-induced CP asymmetries of the following kind:

\[ A^\text{dir}_{\text{CP}}(B_d \rightarrow \pi^+\pi^-) = \text{function}(d, \theta, \gamma), \quad A^\text{mix}_{\text{CP}}(B_d \rightarrow \pi^+\pi^-) = \text{function}(d, \theta, \gamma, \phi_d = 2\beta) \]

(8)
\[ \mathcal{A}_{CP}^{d_B}(B_s \to K^+K^-) = \text{function}(d', \theta', \gamma), \mathcal{A}_{mix}^{d_B}(B_s \to K^+K^-) = \text{function}(d', \theta, \gamma, \phi_s \approx 0). \]  

Consequently, we have four observables at our disposal, depending on six “unknowns”. However, since \( B_d \to \pi^+\pi^- \) and \( B_s \to K^+K^- \) are related to each other by interchanging all down and strange quarks, the \( U \)-spin flavour symmetry of strong interactions implies

\[ d'e^{i\theta'} = de^{i\theta}. \] 

Using this relation, the four observables in (9) depend on the four quantities \( d, \theta, \phi_d = 2\beta \) and \( \gamma \), which can hence be determined [13]. The theoretical accuracy is only limited by the \( U \)-spin symmetry, as no dynamical assumptions about rescattering processes have to be made. Theoretical considerations give us confidence into (8), as it does not receive \( U \)-spin-breaking corrections in factorization [13]. Moreover, we may also obtain experimental insights into \( U \)-spin breaking [13, 24].

The \( U \)-spin arguments can be minimized, if the \( B_d^0 - B_d^0 \) mixing phase \( \phi_d = 2\beta \), which can be fixed through \( B_d \to J/\psi K_S \), is used as an input. The observables \( \mathcal{A}_{CP}^{d_B}(B_d \to \pi^+\pi^-) \) and \( \mathcal{A}_{mix}^{d_B}(B_d \to \pi^+\pi^-) \) allow us then to eliminate the strong phase \( \theta \) and to determine \( d \) as a function of \( \gamma \). Analogously, \( \mathcal{A}_{CP}^{d_B}(B_s \to K^+K^-) \) and \( \mathcal{A}_{mix}^{d_B}(B_s \to K^+K^-) \) allow us to eliminate the strong phase \( \theta' \) and to determine \( d' \) as a function of \( \gamma \). The corresponding contours in the \( \gamma - d \) and \( \gamma - d' \) planes can be fixed in a theoretically clean way. Using now the \( U \)-spin relation \( d' = d \), these contours allow the determination both of the CKM angle \( \gamma \) and of the hadronic quantities \( d, \theta, \theta' \); for a detailed illustration, see Ref. [13]. This approach is very promising for run II of the Tevatron and the experiments of the LHC era, where experimental accuracies for \( \gamma \) of \( O(10^5) \) [18] and \( O(1^\circ) \) [19] may be achieved, respectively. It should be emphasized that not only \( \gamma \), but also the hadronic parameters \( d, \theta, \theta' \) are of particular interest, as they can be compared with theoretical predictions, thereby allowing valuable insights into hadron dynamics. For other recently developed \( U \)-spin strategies, the reader is referred to Refs. [23, 25].

4 THE \( B_d \to \pi^+\pi^- \), \( B_d \to \pi^\mp K^\pm \) SYSTEM AND IMPLICATIONS FOR \( B_s \to K^+K^- \)

A variant of the \( B_d \to \pi^+\pi^- \), \( B_s \to K^+K^- \) approach was developed for the \( e^+e^- \) \( B \)-factories [21], where \( B_s \to K^+K^- \) is not accessible: as \( B_s \to K^+K^- \) and \( B_d \to \pi^+\pi^- \) are related to each other through an interchange of the \( s \) and \( d \) spectator quarks, we may replace the \( B_s \) mode approximately through its \( B_d \) counterpart, which has already been observed by BaBar, Belle and CLEO. Following these lines and using experimental information on the CP-averaged \( B_d \to \pi^\mp K^\pm \) and \( B_d \to \pi^+\pi^- \) branching ratios, the relevant hadronic penguin parameters can be constrained, implying certain allowed regions in observable space [27]. An interesting situation arises now in view of the recent \( B \)-factor measurements of CP violation in \( B_d \to \pi^+\pi^- \), allowing us to obtain new constraints on \( \gamma \) as a function of the \( B_d^0 - B_d^{0*} \) mixing phase \( \phi_d \), which is fixed through \( \mathcal{A}_{mix}^{d_B}(B_d \to J/\psi K_S) \) up to a twofold ambiguity, \( \phi_d \sim 51^\circ \) or \( 129^\circ \). If we assume that \( \mathcal{A}_{mix}^{d_B}(B_d \to \pi^+\pi^-) \) is positive, as indicated by recent Belle data, and that \( \phi_d \) is in agreement with the “indirect” fits of the unitarity triangle, i.e. \( \phi_d \sim 51^\circ \), also the corresponding values for \( \gamma \) around \( 60^\circ \) can be accommodated. On the other hand, for the second solution \( \phi_d \sim 129^\circ \), we obtain a gap around \( \gamma \sim 60^\circ \), and could easily accommodate values for \( \gamma \) larger than \( 90^\circ \). Because of the connection between the two solutions for \( \phi_d \) and the resulting values for \( \gamma \), it is very desirable to resolve the twofold ambiguity in the extraction of \( \phi_d \) directly. As far as \( B_s \to K^+K^- \) is concerned, the data on the CP-averaged \( B_d \to \pi^+\pi^- \), \( B_d \to \pi^\mp K^\pm \) branching ratios imply a very constrained allowed region in the space of \( \mathcal{A}_{mix}^{d_B}(B_s \to K^+K^-) \) and \( \mathcal{A}_{mix}^{d_B}(B_s \to K^+K^-) \) within the Standard Model, thereby providing a narrow target range for run II of the Tevatron and the experiments of the LHC era [21]. Other recent studies related to \( B_d \to \pi^+\pi^- \) can be found in Refs. [14, 27].

ACKNOWLEDGEMENTS

I would like to thank Andrzej Buras, Thomas Mannel and Joaquim Matias for pleasant collaborations on the topics discussed above.
References

[1] See, for instance, A. J. Buras, F. Parodi and A. Stocchi, TUM-HEP-465-02 [hep-ph/0207101];
A. Höcker, H. Lacker, S. Laplace and F. Le Diberder, Eur. Phys. J. C 21 (2001) 225;
M. Ciuchini et al., JHEP 0107 (2001) 013.

[2] R. Fleischer, DESY-THESIS-2002-022 [hep-ph/0207108], to appear in Physics Reports.

[3] M. Gronau, J. L. Rosner and D. London, Phys. Rev. Lett. 73 (1994) 21.

[4] R. Fleischer, Phys. Lett. B 365 (1996) 399.

[5] R. Fleischer and T. Mannel, Phys. Rev. D 57 (1998) 2752.

[6] M. Gronau and J. L. Rosner, Phys. Rev. D 57 (1998) 6843.

[7] R. Fleischer, Eur. Phys. J. C 6 (1999) 451.

[8] M. Neubert and J. L. Rosner, Phys. Lett. B 441 (1998) 403;
Phys. Rev. Lett. 81 (1998) 5076.

[9] M. Neubert, JHEP 9902 (1999) 014.

[10] A. J. Buras and R. Fleischer, Eur. Phys. J. C 11 (1999) 93.

[11] A. J. Buras and R. Fleischer, Eur. Phys. J. C 16 (2000) 97.

[12] R. Fleischer and J. Matias, Phys. Rev. D 61 (2000) 074004.

[13] M. Bargiotti et al., Eur. Phys. J. C 24 (2002) 361.

[14] M. Gronau and J. L. Rosner, Phys. Rev. D65 (2002) 013004 [E: D65 (2002) 079901].

[15] R. Fleischer, Phys. Lett. B 459 (1999) 306.

[16] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914;
Nucl. Phys. B 606 (2001) 245.

[17] H.-n. Li and H. L. Yu, Phys. Rev. D 53 (1996) 2480;
Y. Y. Keum, H.-n. Li and A. I. Sanda, Phys. Lett. B 504 (2001) 6;
Y. Y. Keum and H.-n. Li, Phys. Rev. D 63 (2001) 074006.

[18] K. Anikeev et al., FERMILAB-Pub-01/197 [hep-ph/0201071].

[19] P. Ball et al., CERN-TH-2000-101 [hep-ph/0003238].

[20] R. Fleischer, Eur. Phys. J. C 16 (2000) 87.

[21] R. Fleischer and J. Matias, DESY-02-040 [hep-ph/0204101], to appear in Phys. Rev. D.

[22] M. Gronau and J. L. Rosner, Phys. Lett. B 482 (2000) 71.

[23] J. Matias, Phys. Lett. B 520 (2001) 131.

[24] M. Gronau, Phys. Lett. B 492 (2000) 297.

[25] R. Fleischer, Eur. Phys. J. C 10 (1999) 299, Phys. Rev. D 60 (1999) 073008;
P. Z. Skands, JHEP 0101 (2001) 008.

[26] M. Gronau and J. L. Rosner, Phys. Rev. D 65 (2002) 093012, 113008 and
TECHNION-PH-2002-21 [hep-ph/0205323];
C.-D. Lü and Z.-j. Xiao, BIHEP-TH-2002-22 [hep-ph/0205134].