Quantum divisibility test and its application in mesoscopic physics

G.B. Lesovik\textsuperscript{a,b}, M.V. Suslov\textsuperscript{c}, and G. Blatter\textsuperscript{b}

\textsuperscript{a}L.D. Landau Institute for Theoretical Physics RAS, 117940 Moscow, Russia
\textsuperscript{b}Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland and
\textsuperscript{c}Moscow Institute of Physics and Technology, Institutskii per. 9, 141700 Dolgoprudny, Moscow District, Russia

(Dated: June 26, 2009)

We present a quantum algorithm to transform the cardinality of a set of charged particles flowing along a quantum wire into a binary number. The setup performing this task (for at most $N$ particles) involves $\sim \log_2 N$ quantum bits serving as counters and a sequential readout. Applications include a divisibility check to experimentally test the size of a finite train of particles in a quantum wire with a one-shot measurement and a scheme allowing to entangle multi-particle wave functions and generating Bell states, Greenberger-Horne-Zeilinger states, or Dicke states in a Mach-Zehnder interferometer.

PACS numbers: 03.67.Ac 03.67.Bg 73.23.-b

INTRODUCTION

Quantum mechanics offers novel algorithms allowing to speed up the solution of specific computational tasks, some modestly, such as sorting a list \cite{1,2}, while others, such as prime factorization \cite{1,3}, are accelerated exponentially. While applications in quantum cryptography \cite{1,4} are close to commercial realization \cite{5}, the endeavor of building a universal quantum computer with thousands of quantum bits lies in the distant future, if ever realized. In this situation, it is interesting to consider special tasks which are less demanding in their requirement with regard to the number of qubits and the complexity of its network. An example of such an application is the use of a qubit as a measuring device in the realization of full counting statistics \cite{6}. In a similar spirit, an iterative phase estimation algorithm has been proposed as a testbed application for a limited amount of qubits, in particular, a two-qubit benchmark \cite{7}. Here, we discuss other applications where a few qubits serve as active or passive detectors. The core element on the algorithmic side is a specific physical setup with $K$ qubits allowing to perform a (non-demolition) count of the elements $n < N = 2^K$ in a stream of particles flowing in a quantum wire, i.e., determining its cardinality. This algorithm resembles the phase estimation problem \cite{8,9} in inverted form: Rather then determining a phase $\phi$ with $N$ gate operations, here, the phase $\phi$ is known and we seek to find the number $N$ of operations associated with the passage of the particles. Our measurement scheme, involves a sequential readout, where the $j$-th reading depends on the results of the previous $j-1$ measurements, reminding about binary search trees \cite{10}. A simpler, simultaneous (rather then conditional) readout of the $K$ qubits provides a divisibility check (by $2^K$) of the cardinality. Combining the counter with a Mach-Zehnder interferometer in a ‘which path’ setup \cite{11}, we study interference effects in the particle flow across the device and show how to make use of the counter in the fabrica-

tion of entangled many-particle wave functions of various kinds. With this program, we position ourselves at the interface between information theory and its application in mesoscopic physics; rather than universal, our quantum counting scheme is a special purpose algorithm, admitting a relatively simple implementation while offering practical applications.

The use of two-level systems as clocks or counters has a long history: using the Larmor precession of a spin as a clock attached to the particle itself, Baz’ \cite{12} and Rybachenko \cite{13} proposed to measure the time it takes a particle to traverse a barrier in a tunneling problem. In the context of full counting statistics in mesoscopic physics, Levitov and Lesovik \cite{14} introduced the idea to use an independent stationary spin as a measurement device to count the electrons flowing in a nearby quantum wire. In quantum optics, Brune et al. \cite{15} proposed to make use of atoms excited to Rydberg states as atomic clocks to count photons in a cavity, a proposal that has been experimentally realized recently \cite{16}; in this case, the flying atoms measure the number of localized photons in the cavity. Our algorithm can be used to count photons as well; in our dual setup the counters are fixed and (microwave) photon pulses propagate in a transmission line.

COUNTING ALGORITHMS

Classical algorithm

We start out with the counting problem, the transformation of the magnitude of a set of (charged) particles (i.e., its cardinality $n$) into a binary number. First consider the obvious classical algorithm and assume that each particle passing a classical counter generates a pulse; to simplify the discussion, we can assume taking the particle ‘•’ itself from the string. Consider a register with $K$ ($n < 2^K$) empty slots $\{0,0,\ldots,0,0,0\}$, then the first par-
particle is placed in the right most position $[0, 0, \ldots, 0, 0, \bullet]$, the second induces a shift of the first to the next register $([0, 0, \ldots, 0, 0, \bullet] \rightarrow [0, 0, \ldots, 0, \bullet, 0])$, the third particle fills the first slot again $([0, 0, \ldots, 0, 0, \bullet, \bullet])$, the fourth particle induces two shifts $([0, 0, \ldots, 0, 0, \bullet, \bullet, 0] \rightarrow [0, 0, \ldots, 0, \bullet, 0, 0])$, etc. The transformation of the set’s magnitude $n < N = 2^K$ to a binary number then involves $\sim n \log_2 n$ steps.

Quantum measurement

The simplest scheme using spin counters to determine the number of particles flowing across a wire requires $\propto n^2$ measurements and thus is even more demanding: to fix ideas, we assume transport of charged particles along $x$ and $N_m$ spins initially polarized along the positive $y$-axis. Upon passage of a charge, the induced $B$-field, locally directed along the $z$-axis, rotates the spins in the $x$-$y$ plane by a fixed angle $\phi < \pi/N$. In a real experiment, the spins could be replaced by suitable qubits and below, we will use the terms ‘spin’ and ‘qubit’ synonymously. The use of $N_m$ spins is equivalent to an $N_m$-fold repetition of the same experiment with one spin, allowing us to transfer the $n < N$ particles once and perform $N_m$ measurements on equally prepared spins—this procedure corresponds to a single shot measurement of $N_m$ spins (note, that the no-cloning theorem prevents us from using one spin and then clone it after the passage of the $n$ particles). Measuring the spin along the $y$-axis, the (theoretical) probability to find it pointing upwards is given by $P^\uparrow = \langle m^\uparrow \rangle_k/N_m = \cos^2(n\phi/2)$, where $\langle m^\uparrow \rangle_k$ denotes the average of finding $m^\uparrow$ of the $N_m$ spins pointing up in a sequence of $k \rightarrow \infty$ realizations of the entire experiment. On the other hand, the one-time measurement $m^\uparrow_k$ provides the experimental result $P^\uparrow_m = m^\uparrow_m/N_m$, from which we can find the number $n = (2/\phi) \arccos([P^\uparrow_m]^{1/2})$. A variant of this scheme is proposed in Ref. [12], where a sequence of Rydberg atoms brought into a quantum superposition through the interaction with cavity photons is used to project the cavity-field onto a photon number state.

The above procedure is a statistical one and we have to determine how many spins (measurements) $N_m$ are needed to predict the particle number $n$ with certainty. The difference (we assume $N > n \gg 1$) $\delta P^\uparrow = [P^\uparrow(n+1) - P^\uparrow(n)] \approx \partial_n P^\uparrow = (\phi/2) \sin(n\phi)$ has to be much larger than the uncertainty $[\langle(\delta m^\uparrow)^2 \rangle_k]^{1/2} \equiv [\langle m^\uparrow - \langle m^\uparrow \rangle_k \rangle^2]^{1/2}$ in the measurement, $\delta P^\uparrow \gg [\langle(\delta m^\uparrow)^2 \rangle_k]^{1/2}/N_m$. Given the binomial statistics of the measurement process (the values $\uparrow$ and $\downarrow$ are measured with probabilities $P^\uparrow$ and $(1 - P^\uparrow)$), we obtain $\langle(\delta m^\uparrow)^2 \rangle_k = P^\uparrow(1 - P^\uparrow) N_m$ and combining these results, we find that $N_m \gg 1/\phi^2 > N^2/\pi^2 \gg 1$ spins are needed in order to accurately measure the particle number $n < N$.

Quantum algorithm

We now consider a more sophisticated measurement scheme, assuming the role of a quantum algorithm, where we need only a number $K \sim \log_2 N$ of spins to encode the magnitude of a set with $n < N$ particles into a binary number. The $K$ spins are all pointing up initially, see Fig. 1(a). Upon passage of a particle, the $j$-th spin is rotated (clockwise) by $\phi_j = 2\pi/2^j$ upon passage of one particle. After the passage of all particles, the first spin is measured along the $y$-axis and provides the number’s parity. Depending on the parity, the second spin is measured along the $y$-axis (even parity) or $x$-axis (odd parity); a measurement along (opposite to) the axis is encoded with a 0 (1). The further iteration is straightforward: depending on the previous outcomes of the $(j-1)$ measurements, the $j$-th spin is measured along one of the angles $l\phi_j$ with $l \in \{0, \ldots, j-1\}$ and the $j$-th position in the binary register assumes values 0 or 1 depending on the measurement result. The figure shows the reading after passage of 5 particles with $K = 4$. (b) Divisibility check: qubit states after passage of $n = 0, \ldots, 8$ electrons for $K = 3$; for $n = 1, \ldots, 7$ there is exactly one qubit ending up in the $|1\rangle$-state (shaded), signalling that the cardinality $n$ of the sequence is not divisible by $2^3 = 8$.

FIG. 1: Illustration of the quantum algorithm to transform the cardinality $n$ of a set of particles into a binary number. (a) Initially, all spins point into the $+y$ direction. The $j$-th spin is rotated (clockwise) by $\phi_j = 2\pi/2^j$ upon passage of one particle. After the passage of all particles, the first spin is measured along the $y$-axis and provides the number’s parity. Depending on the parity, the second spin is measured along the $y$-axis (even parity) or $x$-axis (odd parity); a measurement along (opposite to) the axis is encoded with a 0 (1). The further iteration is straightforward: depending on the previous outcomes of the $(j-1)$ measurements, the $j$-th spin is measured along one of the angles $l\phi_j$ with $l \in \{0, \ldots, j-1\}$ and the $j$-th position in the binary register assumes values 0 or 1 depending on the measurement result. The figure shows the reading after passage of 5 particles with $K = 4$. (b) Divisibility check: qubit states after passage of $n = 0, \ldots, 8$ electrons for $K = 3$; for $n = 1, \ldots, 7$ there is exactly one qubit ending up in the $|1\rangle$-state (shaded), signalling that the cardinality $n$ of the sequence is not divisible by $2^3 = 8$. 

Quantum algorithm

We now consider a more sophisticated measurement scheme, assuming the role of a quantum algorithm, where we need only a number $K \sim \log_2 N$ of spins to encode the magnitude of a set with $n < N$ particles into a binary number. The $K$ spins are all pointing up initially, see Fig. 1(a). Upon passage of a particle, the $j$-th spin is rotated (clockwise) by the amount $\phi_j = 2\pi/2^j$ (rotation by $U_z(-\phi_j)$); such different rotation angles are implemented through different coupling strengths of the spins/qubits to the wire, cf. Ref. [12]. The passage of $n$ particles rotates the $j$-th spin by the amount $n\phi_j$. In particular, the first spin is rotated by the angle $\pi r_7$ and either points upward if the number’s parity is even (we store a ‘0’ in the first position of the binary number) or downward (we store a ‘1’ in the first position of the binary number) if the parity is odd. Hence the measurement of the first spin along the $y$-axis provides already the parity of the number (note that we had to perform $n \log_2 n$ operations in the classical algorithm to find the parity). In addition, the first spin will determine the axis in the measurement of the second spin: for an even $n$, we measure $\phi_2$ along the $y$-axis and store a 0 if the spin is pointing up and a 1 if the spin is pointing down, while for an odd-parity $n$, we measure $\phi_2$ along the $x$-axis. More specifically,
if \( n = 4l_2 \), with \( l_2 \) the number of full rotations of the spin number 2, the state of the first spin signals even parity and the second spin, measured along the \( y \) direction, points into the direction \( +y \), hence we store a ‘0’ in the second position of the binary number. Similarly, for \( n = 4l_2 + 1 \), the spin 2 is directed along \( +x \); spin 1 signals odd parity, the measurement is done along the \( x \)-axis, and we store a ‘0’. For \( n = 4l_2 + 2 \), spin 2 is directed along \( -y \) (even parity, measurement along \( y \), store ‘1’), and for \( n = 4l_2 + 3 \) the spin 2 is directed along \( -x \) (odd parity, measurement along \( x \), store ‘1’).

The iteration of the algorithm is straightforward: the \( j \)-th spin is measured along the angle \( m_j - 1 \phi_j \) with the integer \( m_j - 1 \) corresponding to the binary number encoded in the \( j - 1 \) previous measurements. The \( j \)-th position in the binary register then assumes a value 0 or 1 depending on the measurement result, 0 for a spin pointing along the axis and 1 for a spin pointing opposite. The entire algorithm requires \( \lfloor \log_2(n+1) \rfloor \) steps \( (i_r = \lfloor r \rfloor \) is the closest integer \( i_r > r \)), the same number as bits required to store the number \( n \) in binary form, and provides an exponential speedup compared with the classical algorithm [41].

SINGLE SHOT DIVISIBILITY CHECK

A variant of the above counting algorithm is a test for divisibility by powers of two: given a finite train of electrons propagating in a wire, we wish to check whether the number of electrons in the train (its cardinality) is divisible by \( 2^K \). Obviously, the information on the divisibility of the train’s cardinality by \( 2^K \) is reduced as compared to the information on its cardinality; correspondingly, we expect a reduced effort to achieve this task. Indeed, using the above setup, the divisibility check involves \( K \) qubits and their simultaneous measurement along the \( y \)-axis at the end of the train’s passage (rather than the conditional measurement above). The train’s cardinality is divisible by \( 2^K \), if all spins are pointing up, i.e., along the positive \( y \)-axis, cf. Fig. 1(b); the non-divisibility is signalled by the ‘opposite’ outcome, i.e., there is at least one spin pointing down.

The above statement relies on the fact, that after the passage of \( n = 2^K \) particles all counters end up in the spin-up state, while for \( n \neq 2^K \) there is exactly one spin residing in a spin-down state, cf. Fig. 1(b) (note the difference in having counters in up/down states with well defined measurement outcomes and statistical results of up/down measurements for counters pointing away from the \( y \) direction). We provide a formal proof of this statement: starting in the initial state (with the quantization axis along \( z \)) \( |in\rangle = |+y\rangle = (|\uparrow\rangle + i |\downarrow\rangle)/\sqrt{2} \), after passage of \( n \) particles, the \( j \)-th spin ends up in the final state \( |f\rangle = [e^{i\pi n/2j}|\uparrow\rangle + i e^{-i\pi n/2j}|\downarrow\rangle]/\sqrt{2} \). The probability to measure this spin along the \( +y \)-direction is \( |(+y|f\rangle|^2 = \cos^2(\pi n/2j) \), \( j = 1, \ldots, K \). There is exactly one spin \( 1 \leq j^* \leq K \), for which this probability vanishes: this follows from the statement, that any number \( 0 < n < 2^K \) can be represented in the form \( 2mI \) with \( 0 \leq m < K \) and \( I \) an odd integer. Then, for the spin \( j^* = m + 1 \) (and only for this spin) the phase \( \pi n/2j^* = \pi I/2 \) is an odd multiple of \( \pi/2 \) and hence the probability \( |\cos(\pi I/2)|^2 \) to find it pointing along \( +y \) vanishes, i.e., the spin is pointing down. For all other spins \( j \neq m + 1 \), the phase is a multiple of \( \pi \) (for \( j < m + 1 \), the spin is pointing up) or a fraction \( I/2^{m-1} \) of \( \pi/2 \) (for \( j > m + 1 \), the spin is not pointing down).

IMPLEMENTATION

The spins required in the above counting- and divisibility-check algorithms can be implemented using various types of qubits; note that, while the special nature of our algorithm avoids the large number of qubits and huge network complexity of a general-purpose quantum computer, we do require individual qubits with high performance. Most qubits naturally couple to the electrons in the quantum wire, either via the gauge field (current) or via the scalar potential (charge). The coupling to charge is strong, with typical rotation angles \( \phi \) of order \( (e^2/k_BT_0) \ln(L/d) \), with \( L \) the wire’s length, \( d \) its distance from the qubit, and \( v_F \) the Fermi velocity, thus allowing for a \( \pi \)-phase rotation upon passage of one unit of charge. Transverse coupling via the current is weak, usually requiring enhancement with a flux transformer, cf. the discussion in Ref. [21].

To fix ideas, below we discuss an implementation with charge qubits in the form of double quantum dots (DQD) as one attractive possibility which can be manipulated via electronic gates and offers various modes of operation. DQDs have been implemented in GaAs/AlGaAs heterostructures [19, 20] or as an isolated (leadless) version in Si technology [21], the former with typical oscillation frequencies in the few GHz regime and nano-second phase decoherence times, resulting in quality factors of order 1 to 10; characteristic tunneling couplings/decoherence times are a factor 100 smaller/larger in the isolated qubit [21]. Alternatively, one may consider superconducting charge qubits, e.g., the ‘Quantronium’ [22], with a decoherence time reaching nearly a \( \mu s \); this value, measured at the ‘sweet spot’, will be reduced, however, when choosing a working point which is suitably sensitive to charge. At present, the resolution in the competition between a suitable charge sensitivity to achieve rotation angles of order \( \pi \) and the decoherence due to fluctuating charges in the environment remains a technological challenge. On the other hand, todays best solid state qubits (with a decoherence time above \( 2 \mu s \)), the transmon [23, 24], could be used as photon counters in the microwave regime [25].

The above qubit characteristics have to be compared
with the typical time scale of electronic transport in the wire. While under dc bias conditions, subsequent electrons are separated by the voltage time \( \tau = h/eV \), single-electron wave packets can be generated by unit-flux voltage pulses of Lorentzian shape. Recently, an alternative scheme has been used by Feve et al., who have injecting individual electrons from a quantum dot into an edge channel formed in the quantum Hall regime. Typical time scales \( \tau = h/T \delta \) of single-electron pulses in their experiment range between 0.1 and 10 nano-seconds, where \( T \) and \( \delta \) denote the tunneling probability and the level separation between states in the dot feeding the quantum wire. We conclude that today’s charge qubits are at the border of becoming useful for the proposed electron counting experiments.

We assume the two dots aligned perpendicular to the wire, such that they couple differently to the electron charge in the wire and model the double dot as a two-well potential with quasi-classical states \( |T \rangle \equiv |↑ \rangle \) (top well, see Fig. 2 we use spin language in our analysis below) and \( |B \rangle \equiv |↓ \rangle \) (bottom well) and ground/excited states \( |± \rangle = (|↑ \rangle \pm |↓ \rangle)/\sqrt{2} \) separated by the gap \( \Delta \). We first consider a ‘phase mode operation’ of the counter. Assuming a large barrier separating the quasi-classical states, the tunneling amplitude \( \Delta \) is exponentially small. In order to prepare the qubits in the state \( |+y \rangle = (|↑ \rangle + i|↓ \rangle)/\sqrt{2} \), we measure their states and subsequently rotate them around \( x \) by an angle \(-\pi/2 \) (\( \pi/2 \)) if the state \(| \rangle \) (|\rangle \rangle) was measured. The rotation around \( x \) involves a lowering of the barrier separating the quasi-classical states, allowing for an amplitude shift between them, cf. Fig. 2 the opening of a finite gap \( \Delta \) during the time \( t \) adds an additional phase evolution \( e^{-i\Delta t/\hbar} \) to the excited state \(-\rangle \) and thus corresponds to a rotation of the spin around the \( x \)-axis by the angle \( \Delta t/\hbar \); choosing a time \( t = \hbar \pi/2\Delta \) generates a rotation of the state \(|\rangle \rangle \) to the state \(|+y\rangle \). Alternatively, the qubits are relaxed to the ground state \(|+\rangle \) (corresponding to a spin pointing \(+x\) and subsequently rotated by \( \pi/2 \) around the \( z \)-axis via a suitable bias pulse applied to the double-dot, adding the relative phase \( \pi/2 \) to the quasi-classical state \(|\rangle \rangle \), cf. Fig. 2.

The passage of electrons in the wire generates a final state \(|b \rangle = (|↑ \rangle + ie^{-i\phi} |↓ \rangle)/\sqrt{2} \), where \( \phi_j = 2\pi/2^j \) is the properly tuned phase difference picked up by the quasi-classical states upon passage of one electron. The readout step for the divisibility check involves a rotation around the \( x \)-axis by an angle \( \pi/2 \). The divisibility check then tests for the presence of all dot electrons in the state \(|\rangle \rangle \); if the answer is positive, the number \( n \) of particles passing the \( K \) double-dots is divisible by \( 2^K \). In order to find the exact value of the cardinality \( n \), another rotation around the \( z \)-axis by an angle \( m_{j-1}\phi_j \) has to be performed before rotating around \( x \), where the integer \( m_{j-1} \) corresponds to the binary number encoded in the first \( j-1 \) measurement outcomes; e.g., for the third qubit \( j = 3 \), after passage of 7 electrons, the measurement of the first two qubits provides the binary number \((1,1)\), hence \( m_2 = 3 \), and a rotation by \( 3\pi/4 \) around \( z \) makes the third spin point along the \(-y\) direction; storing a 1 as the third digit of the binary number we obtain \( m_3 = 7 \), cf. Fig. 1(b).

Using the above phase mode operation, the double dot does not act back on the passing electrons in the wire, since the charge distribution remains unchanged during all of the detection phase. Another version of the divisibility check makes use of the back action of the double dot on the wire and tests for the divisibility without explicit measurement of any of the final qubit states. This gain in performance has to be traded against two disadvantages: first, the back action has to be properly controlled, and second, the particle train has to be properly sequenced in time, following a prescribed time separation between two consecutive particles.

The setup then involves a quantum point contact (QPC) that can be manipulated through an external gate \( V_{\text{ext}} \), see Fig. 3(a)). This time, the double dot qubits are prepared with asymmetric states (with energy difference \( \varepsilon \)) in the unbiased situation. Initially, each qubit is in the (high energy) \(|B\rangle \rangle \) state with the electron further away from the wire. The passage of an electron in the wire brings the two states \(|T\rangle \rangle \) and \(|B\rangle \rangle \rangle \) into degeneracy and a fraction of the wave function tunnels from \(|B\rangle \rangle \) to \(|T\rangle \rangle \). The role of the angle \( \phi \) is now played by the phase \( \phi \approx \Delta \delta_{\text{deg}}/\hbar \), with \( \Delta \) the tunneling gap and \( \delta_{\text{deg}} \) the degeneracy time (in reality, the time evolution of the electric potential due to the passing electron has to be properly accounted for). In order to assure proper evolution of the DQD’s wavefunction due to the passage of subsequent electrons, the trivial phase evolution in between \(|T\rangle \rangle \longleftrightarrow |B\rangle \rangle \rangle \) tunneling events has to be an integer.

**FIG. 2:** Implementation of the counting algorithm with double-dot charge qubits modelled as double-well systems. Initial state \(|+y\rangle \rangle = (|↑ \rangle + i|↓ \rangle)/\sqrt{2} \). Particles are counted via their associated voltage pulses generating a phase shift between the states \(|T\rangle \rangle \) and \(|B\rangle \rangle \) (rotation around \( z \)). The initialization and readout involve manipulations of phase (disbalancing the levels \(|T\rangle \rangle \) and \(|B\rangle \rangle \), rotation around \( z \)) and of amplitude (lowering the barrier between \(|T\rangle \rangle \) and \(|B\rangle \rangle \), rotation around \( x \)).
The electrons in the qubits act back on the quantum wire through a capacitive coupling and can block the channel. We define the ‘critical’ \( V_{\text{ext}}^c \) and the ‘open’ \( V_o^c \) bias settings of the external gate in the following way (see Fig. 3(b)): With all qubits in the \(|\text{T}\rangle\) state, we tune the QPC to one transmitting channel barely open such that the shift of the electron in one of the qubits to the \(|\text{T}\rangle\) state suffices to block the channel—this defines \( V_{\text{ext}}^c \). On the other hand, setting the bias to \( V_o^c \) widely opens the transmitting channel such that the electrons move with appreciable velocity through the channel. Given these two settings, the divisibility check is easily implemented: We apply a bias \( V_o^c \) to the external gate and let the particle train pass the QPC. Subsequently, we switch the external gate to its critical value \( V_{\text{ext}}^c \) and send one more (test) electron through the QPC. If the cardinality of the train is divisible by \( 2^K \), then all qubits have returned back to the (bottom) state, the (test) electron can pass the QPC and is detected on the other side, e.g., via a single electron transistor. On the other hand, if the cardinality of the train is not divisible by \( 2^K \), then exactly one of the \( K \) qubits is in the top state, see Fig. 3(c), and definitively blocks the channel (while other qubits may reflect particles only indeterministically). This scheme provides a single shot test for the divisibility by \( 2^K \) of the particle train’s cardinality.

![FIG. 3: (a) Setup for divisibility check with ‘self-measurement’. The disbalanced states \(|\text{T}\rangle\) and \(|\text{B}\rangle\) of the two-level charge qubits are brought into degeneracy when electrons pass the QPC and the time evolution moves a fraction of the qubit’s wave function between \(|\text{T}\rangle\) and \(|\text{B}\rangle\). In turn, the qubits’ charges act back on the QPC and modify its conductance \( G \) via narrowing (wave function at the top) and widening (wave function in the bottom) the constriction. \( \bullet \) External bias set to \( V_{\text{ext}}^c \) with the channel barely open and to \( V_o^c \) defining a wide open channel.](image)

Note that the channel has to be wide open during the transmission of the electrons in order to prevent their entanglement with the counter-qubits through back action: the scalar interaction \( V \) between the qubit and the electron in the wire decelerates the latter. This deceleration generates a time delay \( t_{\text{del}} = \int dx \{1/v|V(x)| - 1/v|V = 0|\} \) which depends on the charge state of the qubit, hence the two qubit states \(|\text{T}\rangle\) and \(|\text{B}\rangle\) are entangled with portions of the particle’s wave packet which are delayed in time. We then require the electron to move fast through the channel and thus demand that the QPC be biased away from criticality, implying a weak backaction and hence a negligible time delay.

**Self-organized bunching**

A modified setup of the qubit-controlled quantum point contact can be used to generate self-organized bunching, however, this setup involves strong backaction and is difficult to control (see Ref. 29 for a recent study where a strongly coupled qubit modifies the transport through a QPC). The basic idea is, that while a simple quantum scatterer (a tunneling barrier) transforms a regular stream of particles into a perfectly random sequence with binomial statistics (transmission versus reflection), our qubit-controlled QPC will generate a non-trivial, tunable, and non-Markovian random sequence. We consider the simplest case with one qubit controlling the QPC, cf. Fig. 4 and start with an initial state where the qubit electron resides in the high-energy left state \(|\text{L}\rangle\) and the quantum point contact is barely closed through a critical tuning of the QPC with the external gate \( V_{\text{ext}}^c \). The passage of one electron brings the right state \(|\text{R}\rangle\) into resonance with \(|\text{L}\rangle\) and the qubit electron tunnels to the state \(|\text{R}\rangle\) away from the QPC (we assume a phase angle \( \phi = \pi \)). The QPC then is open and as the next electron flows down the channel, the qubit electron tunnels back to \(|\text{L}\rangle\), thus closing the channel again (we assume a proper time sequenced flow). Adding a second control qubit (e.g., on the other side of the QPC, with the channel blocked when both qubit electrons reside in the \(|\text{L}\rangle\) state) with phase angle \( \phi = \pi/2 \), bunched electron trains with four particles can be formed. The train can be initiated randomly through tunneling of the initial electron or in a controlled way via a voltage pulse. The complexity

![FIG. 4: Setup for bunching. The passage of an electron in the wire brings the states \(|\text{L}\rangle\) and \(|\text{R}\rangle\) into degeneracy and the time evolution shifts a fraction \( \phi/\pi \) of the wave function to the other level. The qubit’s charge acts back on the QPC and changes its conductance \( G \), blocking the channel when residing in the \(|\text{L}\rangle\) state and opening the channel after tunneling to the \(|\text{R}\rangle\) state. Note that in the present geometry, reflected particles do not bring the qubit into resonance and hence do not modify the relative weights in the qubit’s wave function.](image)
of the system’s evolution is already appreciated for the case with only one qubit controlling the QPC (we denote the left (right) state by $|\uparrow\rangle$ ($|\downarrow\rangle$). The incoming electron $|\text{in}\rangle = |\Psi_0\rangle$ is transmitted across ($|\text{t}\rangle$) or reflected by ($|\text{r}\rangle$) the QPC, depending on the state of the qubit,

$$|\text{in}\rangle|\sigma\rangle \to |\text{t}\rangle \sum_{\sigma'} t_{\sigma\sigma'} |\sigma'\rangle + |\text{r}\rangle \sum_{\sigma'} r_{\sigma\sigma'} |\sigma'\rangle.$$ (1)

Hence, after scattering of one electron, the system’s initial state $|\text{in}\rangle (a_{\uparrow} |\uparrow\rangle + b_{\downarrow} |\downarrow\rangle)$ evolves to the final state $a_f |\text{t}\rangle |\downarrow\rangle + b_f |\text{t}\rangle |\downarrow\rangle + d_f |\text{r}\rangle |\downarrow\rangle$ with

$$a_f = a_{t\uparrow} + b_{t\downarrow}, \quad b_f = a_{t\downarrow} + b_{t\uparrow}, \quad c_f = a_{r\uparrow} + b_{r\downarrow}, \quad d_f = a_{r\downarrow} + b_{r\uparrow}.$$ For an ideal setup, we have $|r_{\uparrow\downarrow}| = |t_{\uparrow\downarrow}| = 1$ and all other coefficients vanish, hence the final state assumes the simple form $a_{t\uparrow} |\text{t}\rangle |\downarrow\rangle + b_{t\downarrow} |\text{t}\rangle |\downarrow\rangle$. Further extensions beyond two qubits are more difficult to realize, as the qubits have to act jointly on the QPC; this will introduce uncontrollable interaction effects among the qubits, perturbing their proper ‘rotation’. Also, keeping the QPC close to criticality, the velocity of the transmitted electrons depends on the qubits’ states, which thus get entangled with correspondingly time-delayed portions of the wave function. The above idealized bunching will then give way to some self-organized bunching (a non-Markovian process) which might be interesting in itself, though not perfectly controlled.

So far, we have discussed the setup of the ‘quantum cardinality-counter’ and its application to the manipulation of classical information; below we use these ideas to control and modify quantum information.

**MANIPULATION OF WAVE FUNCTIONS**

Next, we discuss the manipulation of one- and two-body wavefunctions by a spin/qubit counter. Consider a particle entering the Mach-Zehnder interferometer, see Fig. 5, from the lower-left lead and exiting the loop through the upper-right lead $u$ where it is measured. We denote the initial state injected into the interferometer by $|\text{in}\rangle = |\psi_0\rangle$. The wave function can propagate along two trajectories, the upper arm $U$ where the particle picks up a phase $\varphi_U$ and the spin counter is flipped (we choose a rotation angle $\varphi = \pi$), or the lower arm $D$ accumulating a phase $\varphi_D$ and leaving the spin unchanged. Assuming symmetric splitters with transmission $t$ ($|\text{t}\rangle^2 = 1/2$) and reflection $r = \pm it$, the projection of the wave function in the upper outgoing channel $u$ reads

$$\Psi_u = tr e^{i\varphi_U} |\text{in}\rangle |\uparrow\rangle + tr e^{i\varphi_D} |\text{in}\rangle |\downarrow\rangle$$ (2)

$$= (\pm i) \{e^{i\varphi_U} |\text{in}\rangle |\uparrow\rangle + e^{i\varphi_D} |\text{in}\rangle |\downarrow\rangle\}/2.$$ A partial summation over the spin states provides us with the particle’s density matrix $\rho_u = |\text{in}\rangle\langle\text{in}|/2$. The result tells us that all interference effects are gone due to the decoherence by the spin counter: the visibility $V \equiv \max P_u - \min P_u/\max P_u + \min P_u = \cos(\varphi/2)$ of the oscillations in the probability $P_u = |\Psi_u|^2$ vanishes, while the spin carries the full information ($I$) on the particle’s path $[30]$. Choosing another rotation angle $\varphi$ for the spin counter, the visibility can be tuned to any value between zero and unity, with the conjugate behavior of the information gain by the counter, $V^2(\varphi) = P^2(\varphi) = 1$, cf. Ref. [11] where a similar behavior has been observed in a ‘which path’ experiment.

![FIG. 5: Mach-Zehnder interferometer with spin counter. Particles enter the interferometer through the left leads (here the bottom lead) and are measured on the right. The spin counter in the upper arm $U$ detects the passage of particles via a rotation by the angle $\varphi$. The magnetic flux $\Phi$ through the loop allows to tune the phase difference when propagating along different arms.](image)

Next, we send two particles into the Mach-Zehnder (MZ) loop with the spin-counter flipping by $\varphi = \pi$ upon passage of one particle in the upper arm. We assume the two wave functions describing the initial state $|\text{in}\rangle = |\psi_{10}\psi_{02}\rangle$ to be well separated in space, allowing us to ignore exchange effects in our (MZ) geometry. The part of the wave function with two particles measured in the top-right arm then reads

$$\Psi_{2u} = [-1]|\text{t}\rangle^2 r_2 e^{i2\varphi_U} |\text{in}\rangle |\uparrow\rangle + t_2 r_2 e^{i\varphi_U} e^{i\varphi_D} |\text{in}\rangle |\downarrow\rangle$$ (3)

$$+ r_2^2 t_2^2 e^{i2\varphi_D} |\text{in}\rangle |\uparrow\rangle + r_2 t_2 e^{i2\varphi_D} |\text{in}\rangle |\downarrow\rangle$$

$$= (-1)\{[-1]e^{i2\varphi_U} |\text{in}\rangle |\uparrow\rangle + e^{i2\varphi_D} |\text{in}\rangle |\downarrow\rangle$$

$$+ 2e^{i(\varphi_U + \varphi_D)} |\text{in}\rangle |\downarrow\rangle\}/4.$$ The factor $[-1]$ accounts for the phase $\pi$ picked up in the rotation of the spin-state by $2\pi$ (for a qubit, this phase can be tuned and assumes the value $[-1]$ if the coupling shifts the qubit levels symmetrically up and down), while the factor $(−1)$ accounts for the additional scattering phases ($\pm i$) in the reflection process. This time, the interference partly survives; the summation over the spin states provides us with the probability $P_{2u}$ to detect both particles in the upper arm,

$$P_{2u} = P_{2u\uparrow} + P_{2u\downarrow} = [-1]e^{i2\varphi_U} + e^{i2\varphi_D}/16$$

$$+ 2e^{i(\varphi_U + \varphi_D)}/16 \equiv (1 + [-1] \cos[2(\varphi_U - \varphi_D)])/8 + 1/4.$$ (4)
The two interference terms behave quite differently: in the first one (associated with the unrotated spin \( |\uparrow\rangle \)), the phases accumulated by the two particles when both travel along the upper/lower arms add up and we observe a two-particle interference pattern (the occurrence of two-particle interference in a Hanbury-Brown Twiss interferometer has been proposed by Yurke and Stoler \[31\] and by Samuelsson et al. \[32\] and observed in an experiment by Neder et al. \[33\]). The second term (associated with the flipped spin \( |\downarrow\rangle \)) describes particles travelling in different arms and the phases picked up along the upper and lower arms cancel. Nonetheless, we find that we have constructive interference with a doubled total probability 1/4 (the maximal value of the first term), but no Aharonov-Bohm oscillations show up.

When calculating the average number of particles detected in the upper lead, we have to add the probability resulting from those trajectories where only one particle leaves the device through the lead \( u \). Keeping track of the out-terminals with the index \( u \) or \( d \), we obtain the corresponding part of the wave function

\[
\Psi_{1u} = r t \{ 1 \} e^{2i\varphi_U} \left[ \text{in, } u, d, \uparrow + \text{in, } d, u, \uparrow \right]
+ r^2 t^2 e^{i(\varphi_U + r\rho)} \left[ \text{in, } u, d, \downarrow + \text{in, } d, u, \downarrow \right]
+ r^2 e^{2i\varphi_D} \left[ \text{in, } u, d, \uparrow + \text{in, } d, u, \uparrow \right] \}.
\]

Extracting the component associated with the up state of the spin-counter, we find the probability

\[
P_{1u\uparrow} = 2 \left[ 1 \right] e^{2i\varphi_U} + (-1) e^{2i\varphi_D} \left| 16 \right|
= (1 + [-1] \cos(2(\varphi_U - \varphi_D))) / 4.
\]

The total particle number \( N_u \) measured in the upper lead and associated with the spin-up state is given by \( N_u = 2 P_{2u\uparrow} + P_{1u\uparrow} = 1/2 \) and the interference term cancels out.

**PROJECTIVE MULTI-QUBIT ENTANGLEMENT**

The standard way to entangle quantum degrees of freedom makes use of interaction between the constituents. An alternative is provided by a projection technique, where a measurement selects the desired entangled state. In some cases, the projection makes use of the entangled state but simultaneously implies its destruction—more useful for quantum information processing are those schemes which entangle qubits for further use after the projection. Examples for the latter have been proposed using various arrangements of double-dot charge qubits combined with a quantum point contact serving as a quadratic detector \[34, 35\] or (free) flying spin-qubits tracked via a charge detector, where the charge provides an additional non-entangled degree of freedom associated with the entangled spins \[36, 37\]. Here, we generate multi-qubit orbital entanglement of flying qubits via their entanglement with our spin counters serving as ancillas; after reading of the counter states, the entangled multi-qubit state can be further used.

The setup in figure 5 conveniently lends itself for the generation of entanglement. The simplest example is provided by the two-particle propagation analyzed above: Evaluating the wave function at the position \( A \) before mixing in the second splitter, we find the expression (cf. Eq. 4): we use scattering coefficients for a symmetric beam splitter, e.g., \( t^2 = 1/2 \) and \( r^2 = -(1/2) \) and \( r t = (\pm i)/2 \) for the two particles injected from the bottom left:

\[
\Psi_{2A} = \{ 1 \} e^{2i\varphi_U} \left( |\uparrow, \uparrow\rangle + (-1) e^{2i\varphi_D} |\downarrow, \downarrow\rangle \right) \otimes |1\rangle
+ (\pm i) e^{i(\varphi_U + r\rho)} \left( |\downarrow, \downarrow\rangle + |\uparrow, \uparrow\rangle \right) \otimes |1\rangle \} / 2,
\]

where we have introduced a pseudo-spin notation to describe the propagation of the two particles along the two arms: a spin \( \uparrow \) (\( \downarrow \)) refers to propagation in the upper (lower) arm. Choosing \( \varphi_U = \varphi_D \), the measurement of the spin-counter in the \( \uparrow \)-state projects the particle wave function to the Bell state \( |\uparrow, \uparrow\rangle + |\downarrow, \downarrow\rangle \), while the measurement of the \( \downarrow \)-state generates the state \( |\downarrow, \downarrow\rangle + |\uparrow, \downarrow\rangle \).

The remaining two Bell states can be obtained by injecting the two particles through the different leads on the left: The state \( |\uparrow, \uparrow\rangle (|\downarrow, \downarrow\rangle) \) then involves the coefficient \( r t \) \( (t r) \) rather than \( t^2 \) \( (r^2) \) and hence we obtain a minus sign in the combination \( |\uparrow, \uparrow\rangle - |\downarrow, \downarrow\rangle \) (and similar for \( |\downarrow, \downarrow\rangle - |\uparrow, \downarrow\rangle \)), which now involves the coefficients \( t^2 \) \( (r^2) \) rather than the factor \( r t \) before). Alternatively, one may thread a flux \( \Phi \) through the loop in order to manipulate the relative phase \( \varphi_D - \varphi_U = 2\pi \Phi/\Phi_0 \), with \( \Phi_0 = h e / c \) the unit flux, in the state \( |\uparrow, \uparrow\rangle + \exp(2\pi i \Phi/\Phi_0)|\downarrow, \downarrow\rangle \). Note that the indistinguishability of particles exploited in the above entanglement process is an ‘artificial’ one defined by the qubit detector, rather than the ‘fundamental’ one of identical particles.

The above scheme for entangling two particles with one spin-counter is easily extended to \( 2^K \) particles and an array of \( K \) spin-counters measuring the cardinality of the particle set flowing through the upper arm. As an illustration we consider the case \( K = 2 \), four particles and two spin-counters. We use the shorthand \( |j\rangle \), \( j = 0, 1, 2, 3, 4 \) with the identification \( |0\rangle = |\uparrow\rangle = |\uparrow, \uparrow\rangle \) for the four different counter states, assume again a symmetric splitter, \( \varphi_U = \varphi_D \), and injection from the bottom left; then

\[
\Psi_{4A} = \{ 1 \} e^{2i\varphi_U} \left( |\uparrow, \uparrow\rangle + |\downarrow, \downarrow\rangle \right) \otimes |0\rangle
+ (\pm i) \left[ \left| \uparrow, \uparrow\rangle + \left| \downarrow, \downarrow\rangle + \ldots \right| \right] \otimes |3\rangle
+ [-1] (-1) \left[ \left| \uparrow, \downarrow\rangle + \left| \downarrow, \uparrow\rangle + \ldots \right| \right] \otimes |2\rangle
+ (\mp i) \left[ \left| \uparrow, \downarrow\rangle + \left| \downarrow, \uparrow\rangle + \ldots \right| \right] \otimes |1\rangle \} / 4
\]

and proper projection provides us with specific entangled states with all pseudo-spins aligned (Greenberger-Horne-Zeilinger states \[38\]), or specific superpositions with ex-
actly one-, two-, and three pseudospins pointing downward, among them the Dicke states \[39\] with an equal number of pseudospins pointing upward and downward. The generalization to other values of \(K\) is straightforward, including also cases with \(n > 2^K\) producing a reduced but finite entanglement.

Letting the particles propagate beyond the line \(A\), the many-particle wavefunction undergoes mixing in the second beamsplitter, cf. Fig. 5. By properly choosing the transmission \((t = \cos \theta)\) and reflection \((r = i \sin \theta)\) coefficients of the second splitter, the pseudospins can be rotated into any direction, though all of the pseudo-spins are rotated equally. Different rotations of the pseudospins can be implemented by changing the characteristics of the splitter in time—the time separation of the particle wavepackets can be enlarged, while compromising between leaving sufficient time for the manipulation of the splitter and keeping the system coherent.

For two particles, the Bell test for the pseudo-spin 'singlet' state \(|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle\) is particularly simple, as the four polarization angles \(\theta_{1,2,1',2'}\) can all be chosen to reside in the first quadrant, thus keeping the manipulation of the beam splitter simple: the maximum violation is obtained for the usual \[40\] angles \(\theta_{12} = \theta_{1} - \theta_{2} = \theta_{1'2'} = \pi/8\) and \(\theta_{12'} = 3\pi/8\) (the indices 1(2) refer to the first (second) particle). The analysis of the pseudo-spin triplet states involves angles in the second quadrant as well (as we have to replace \(\theta_2 \rightarrow -\theta_2\)) and their experimental analysis is more demanding.

The action of the spin counter entangling two particles in a pseudo-spin 'singlet' state \(|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle\) can be observed in a simple experiment involving a finite Aharonov-Bohm flux \(\Phi\) through the interferometer. We inject the two (time delayed, to avoid exchange effects) particles through the two different leads on the left and measure the cross-correlator \(\langle N_u N_d \rangle\) on the right \((N_{u,d} \in \{0, 1, 2\})\) denote the number of particles observed in the leads \(u\) and \(d\). Without the counter, the product state generates the result \(\langle N_u N_d \rangle = P_{u,1} P_{d,2} + P_{d,1} P_{u,2}\), where \(P_{1x,i}\) denotes the probability to find the particle \(i\) in the lead \(x\). With \(P_{u,1} = |t^2 + r^2 \exp(2\pi i \Phi/\Phi_0)|^2\) and \(P_{u,2} = |r t [1 + \exp(2\pi i \Phi/\Phi_0)]|^2\) and assuming symmetric splitters with \(t^2 = 1/2, r^2 = -1/2\), we find that \(\langle N_u N_d \rangle = |1 + \cos^2(2\pi \Phi/\Phi_0)|/2\). On the other hand, with the spin-counter selecting the singlet state, only paths where the combined trajectories encircle the loop survive and the correlator is independent of \(\Phi\), \(\langle N_u N_d \rangle = 2P_{u,2d} = |t^4 + r^4 + 2t^2r^2|^2 = 1/2\) for symmetric splitters. Hence post-selecting the spin-flipped events entangles the particles and quenches the Aharonov-Bohm oscillations in the cross correlator \(\langle N_u N_d \rangle\). Such an analysis, although not as rigorous as the classic Bell-inequality test but much simpler to implement, may nevertheless serve as a preliminary indicator for the presence of entanglement.

**CONCLUSION**

In conclusion, the simple spin counter in the ‘Gedanken Experiment’ of full counting statistics proves itself a fruitful idea—not only can it be implemented as a real counting device with the help of quantum bits, its generalization to many qubits combined with a non-trivial measurement protocol allows for the fabrication of a ‘quantum cardinality counter’, where a ‘primitive’ physical information (rather than a binary one) can be transformed into a binary form or directly used in the control and manipulation of other—in particular quantum—information within a mesoscopic setting. On a more general level, we propose that restricting ambitions to the design and implementation of special purpose devices may open up new directions in quantum information processing which appear to be much simpler to realize than the universal quantum computer, while still allowing for interesting applications.

We thank Fabian Hassler, Andrey Lebedev, Renato Renner, Alexandre Blais, Denis Vion, John Martinis, Patrice Bertet, and Fabien Portier for discussions and acknowledge financial support by the CTS-ETHZ and the Russian Foundation for Basic Research under grant No. 08-02-00767-a.

[1] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).
[2] L.K. Grover, Proceedings of the 28th Annual ACM Symposium on the Theory of Computing, (May 1996) p. 212.
[3] P. Shor, Proceedings of the 35th Annual Symposium on Foundations of Computer Science, Santa Fe, NM (1994) and SIAM, J. Sci. Statist. Comput. 26 1484 (1997).
[4] C.H. Bennett and G. Brassard, Proceedings IEEE Int. Conf. on Computers, Systems and Signal Processing, Bangalore, India (IEEE, New York, 1984), p. 175.
[5] D. Stucki, N. Gisin, O. Guinnard, G. Ribordy, and H. Zbinden, New Journal of Physics 4, 41.1 (2002); see also www.idquantique.com.
[6] G.B. Lesovik, F. Hassler, and G. Blatter, Phys. Rev. Lett. 96, 106801 (2006).
[7] M. Dobsicek, G. Johansson, V. Shumeiko, and G. Wendin, Phys. Rev. A 76, 030306 (2007).
[8] A.Y. Kitaev, Russ. Math. Surv. 52, 1191 (1997).
[9] R. Cleve, A. Ekert, C. Macchiavello, and M. Mosca, Proc. R. Soc. Lond. A 454, 339 (1998).
[10] E. Andersson and D.K.L.Oi, Phys. Rev. A 77, 052104 (2008).
[11] E. Buks, R. Schuster, M. Heiblum, D. Mahalu, and V. Umansky, Nature 391, 871 (1998).
[12] A.I. Baz’, Sov. J. Nucl. Phys. 4, 182 (1967).
[13] V.F. Rybchenko, Sov. J. Nucl. Phys. 5, 635 (1967).
[14] L.S. Levitov and G.B. Lesovik, cond-mat/9401004 (1994); L.S. Levitov, H.W. Lee, and G.B. Lesovik, J. Math. Phys. 37, 4845 (1996).
[15] M. Brune, S. Haroche, V. Lefevre, J.-M. Raimond, and
N. Zagury, Phys. Rev. Lett. 65, 976 (1990).

[16] C. Guerlin, J. Bernu, S. Deléglise, C. Sayrin, S. Gleyzes, S. Kuhr, M. Brune, J.-M. Raimond, and S. Haroche, Nature 448, 889 (2007).

[17] W.K. Wootters and W.H. Zurek, Nature 299, 802 (1982).

[18] D. Dieks, Physics Letters A 92, 271 (1982).

[19] T. Hayashi, T. Fujisawa, H.D. Cheong, Y.H. Jeong, and Y. Hirayama, Phys. Rev. Lett. 91, 226804 (2003).

[20] J.R. Petta, A.C. Johnson, C.M. Marcus, M.P. Hanson, and A.C. Gossard, Phys. Rev. Lett. 93, 186802 (2004).

[21] J. Gorman, D.G. Hasko, and D.A. Williams, Phys. Rev. Lett. 95, 090502 (2005); see also the comment/reply in ibid 97, 208901 (2006).

[22] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, M.H. Devoret, Science 296, 886 (2002).

[23] J. Koch, T.M. Yu, J. Gambetta, A.A. Houck, D.I. Schuster, J. Majer, A. Blais, M.H. Devoret, S.M. Girvin, and R.J. Schoelkopf, Phys. Rev. A 76, 042319 (2007).

[24] J.A. Schreier, A.A. Houck, J. Koch, D.I. Schuster, B.R. Johnson, J.M. Chow, J.M. Gambetta, J. Majer, L. Frunzio, M.H. Devoret, S.M. Girvin, and R.J. Schoelkopf, Phys. Rev. B 77, 180502 (2008).

[25] Alexandre Blais, private communication.

[26] L.S. Levitov, H.W. Lee, and G.B. Lesovik, J. Math. Phys. 37, 4845 (1996).

[27] J. Keeling, I. Klich, and L.S. Levitov, Phys. Rev. Lett. 97, 116403 (2006).

[28] G. Fève, A. Mahé, J.-M. Berroir, T. Kontos, B. Plaçais, D.C. Glattli, A. Cavanna, B. Etienne, and Y. Jin, Science 316, 1160 (2007).

[29] I. Neder, N. Ofek, Y. Chung, M. Heiblum, D. Mahalu, and V. Umansky, Nature 448, 333 (2007).

[30] R. Ruskov and A.N. Korotkov, Phys. Rev. B 67, 241305 (2003).

[31] B. Trauzettel, A.N. Jordan, C.W.J. Beenakker, and M. Büttiker, Phys. Rev. B 73, 235331 (2006).

[32] S. Bose and D. Hume, Phys. Rev. Lett. 88, 050401 (2002).

[33] C.W.J. Beenakker, D.P. DiVincenzo, C. Emary, and M. Kindermann, Phys. Rev. Lett. 93, 020501 (2004).

[34] D.M. Greenberger, M.A. Horne, and A. Zeilinger, in Bell’s Theorem, Quantum Theory, and Conceptions of the Universe, M. Kafatos (Ed.) (Kluwer, Dordrecht, 1989), 69-72; see also arXiv:0712.1922v1.

[35] R.H. Dicke, Phys. Rev. 93, 99 (1954).

[36] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 49, 91 (1982).

[37] Strictly speaking, this statement is valid if the unit time step in the algorithm is larger than the time separation between particles.