Measuring the neutron star compactness and binding energy with supernova neutrinos

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Abstract. We investigate the precision with which a neutron star gravitational binding energy can be measured through the supernova neutrino signal, without assuming any prior such as the energy equipartition hypothesis, mean energies hierarchy or constraints on the pinching parameters that characterize the neutrino spectra. We consider water Cherenkov detectors and prove that combining inverse beta decay with elastic scattering on electrons is sufficient to reach 11% precision on the neutron star gravitational binding energy already with Super-Kamiokande. The inclusion of neutral current events on oxygen in the analysis does not improve the precision significantly, due to theoretical uncertainties. We examine the possible impact on the conclusion of further theoretical input and of higher statistics. We discuss the implications of our findings on the properties of the newly formed neutron star, in particular concerning the assessment of the compactness or mass-radius relation.

Keywords: neutrino detectors, supernova neutrinos, neutron stars

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1 Introduction

The importance of neutrino astronomy and in particular of the observations of $\bar{\nu}_e$ from SN 1987A [1–5] is widely recognized.

A future observation of a core collapse supernova in the Milky Way, at a distance $3 \div 10$ times smaller than SN 1987A will have an impressive scientific potential and it is eagerly awaited by the scientific community. A special goal, here discussed, is the possibility of measuring directly the amount of gravitational binding (potential) energy $E_B$ of the newly formed neutron star, released in neutrinos, thanks to the neutrino telescopes. Besides those involved in SNEWS [6], new detectors such as DUNE [7], the large scale JUNO [8] and hopefully Hyper-Kamiokande [9] will also be operational.

The main sample of observed SN 1987A events consists of $\bar{\nu}_e$, seen through inverse beta decay (IBD), i.e. $\bar{\nu}_e + p \rightarrow e^+ + n$. Under the hypothesis that the energy is equally partitioned among the six neutrinos species, the gravitational energy was found to be $3 \times 10^{53}$ erg (at best-fit point and within errors) in agreement with expectations [10]. The other neutrino and antineutrino species are less easy to be seen. This requires in particular $\nu_e$ sensitive detectors such as those based on liquid argon, or on lead like HALO [11], including carbon as for the large scale scintillator detectors JUNO or oxygen in Super-Kamiokande [12] and the future Hyper-Kamiokande. The non-electron component of the neutrino fluxes could be extracted through elastic scattering on protons [13], and the important role of elastic scattering on electrons is discussed immediately below.

In principle, neutrino oscillations could come to the rescue if the spectrum of $\bar{\nu}_e$ probed by IBD would be composite, thereby offering a chance to observe the initial distributions of $\bar{\nu}_e$ and also of $\bar{\nu}_\mu$ and/or $\bar{\nu}_\tau$. This happens in the simplest case when the Mikheyev-Smirnov-Wolfenstein effect takes place [14]. Moreover, the neutrino spectra on Earth could also be composite due to the neutrino self-interactions, or reach equilibrium if fast conversion modes on short scales in the supernova occur [15].

On the other hand, astrophysical uncertainties might undermine the possibility to extract information on the emission spectra from the observations. Ref. [16] has pointed out that the misreading of the pinching parameter implies that the shape of the $\bar{\nu}_e$ observed spectrum cannot be used to learn about the emission spectra of $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$. Moreover, it was argued that
the measurement of $E_B$ is compromised by the uncertainties in the shape of the emitted neutrinos. This point was originally made by considering the IBD signal in Hyper-Kamiokande [9], but it applies also for Super-Kamiokande and JUNO, that have a much smaller mass.

Is it possible to evade this conclusion, namely, is it possible to measure the neutron star binding energy by supernova neutrino observations? Ref. [11] has argued that the range of pinching parameters can be significantly constrained by combining detection channels with different energy thresholds and using other types of detectors. In this work, we will focus on neutrino measurements in Super-Kamiokande and analyze the possibility to determine $E_B$ by exploiting other detection channels besides IBD in this detector.

We consider the fluences (i.e. the time integrated spectra) for three different types of (anti)neutrinos, namely, $\nu_e, \bar{\nu}_e, \nu_x$ — where $\nu_x$ means anyone of $\nu_\mu, \nu_\tau, \bar{\nu}_\mu$ and $\bar{\nu}_\tau$, supposed to have the same initial distribution. For each one of these three fluences, we introduce three parameters: 1) the emitted energy; 2) the average energy; 3) the parameter that accounts for deviations from a thermal distribution. Our work goes beyond existing analysis in many respects. First of all we vary these 9 parameters in wide ranges, to describe the effect of the astrophysical uncertainties, without making any assumption — for example on energy equipartition — or fixing the pinching parameter. Notice that energy equipartition is often employed in this kind of analysis, while it is not supported by current hydrodynamical simulations of supernova explosions, see e.g. [17]. We will give results under such hypothesis as well, to show how they compare with the available literature.

We consider the signal due to IBD events as in ref. [16] and combine it with elastic scattering $\nu + e^- \rightarrow \nu + e^-$ (ES) and neutral current events on oxygen (OS). We demonstrate that the inclusion of these signals, and in particular of ES, allows the measurement of $E_B$ with a precision of about 10%. Our results also show that a precision at a few percent level (and less) is an achievable experimental goal. Finally, we discuss the implications of our findings, in particular for extracting the compactness of the newly formed neutron star.

2 Method of analysis

2.1 Neutrino properties

We assume a supernova explosion at a distance $D = 10$ kpc — supposed to be known with a few percent precision from astronomical observations. We consider as true value of the total energy emitted in neutrinos $E_B^*$ the value $3 \times 10^{53}$ erg, equally distributed in fractions $E_i^*$ among the six species ($i = \nu_e, \bar{\nu}_e, \nu_x$). These quantities are the standard ones used in the literature, are not contradicted by astrophysical simulations and are consistent with SN 1987A data analyses [18–20]. However, before proceeding, it should be emphasized that, for the subsequent analysis, the equipartition ansatz is not implemented as a constraint: in other words, we do not assume to know which is the energy partition and we perform a model independent analysis of the simulated data.

We investigate the neutrino and antineutrino fluences, namely, their time-integrated fluxes. We parameterize the form of the fluence emitted at the source as suggested by [18, 21] and, again, in agreement with numerical simulations$^1$

$$F^0_i (E_\nu) = \frac{dF^0_i}{dE_\nu} = \frac{\mathcal{E}_i}{4\pi D^2} \frac{E_\nu^{\alpha_i} e^{-E_\nu/T_i}}{T_i^{\alpha_i+2} \Gamma (\alpha_i + 2)} \quad \text{with} \quad i = \nu_e, \bar{\nu}_e, \nu_x; \quad (2.1)$$

$^1$In the following we put $\hbar = c = k_B = 1.$
\[ \langle E^*_i \rangle [\text{MeV}] = 9.5 \in [5, 30], 12 \in [5, 30], 15.6 \in [5, 30] \]
\[ \alpha_i^* = 2.5 \in [1.5, 3.5], 2.5 \in [1.5, 3.5], 2.5 \in [1.5, 3.5] \]
\[ \kappa^* = 1 \in [0.8, 1.2] \]

Table 1. True parameter values assumed in the analysis and priors in which they can vary in the analysis. The quantities define the neutrino fluences — namely, the time-integrated fluxes — given by eq. (2.1). The first three rows describe the astrophysical parameters, while the fourth concerns the uncertainty in the OS cross section discussed later — see eq. (2.5).

where \( E_\nu \) is the neutrino energy, \( \Gamma(x) \) is the Euler gamma function and \( T_i = \langle E_i \rangle / (\alpha_i + 1) \). The true values of the parameters of the fluences (2.1), chosen for the simulation, are shown in table 1. The assumptions on the central values of the emitted energies \( E_i \) have been discussed previously. Other remarks on the values listed in this table follows:

1. The central values of the average energies \( \langle E_i \rangle \) do not contradict current theoretical simulations and display a moderate hierarchy of values, \( \langle E_{\bar{\nu}_e} \rangle / \langle E_{\nu_x} \rangle = 1 \). The mean energies come from [18] and are in agreement with what we know from SN 1987A so far [10].

2. The central values of the pinching parameters \( \alpha_i \) used for the true (simulated) spectra describe quasi-thermal distributions.\(^2\) The value \( \alpha^* = 2.5 \) is the midpoint of the conservative interval \([1.5, 3.5] \); even if its precise value is still unknown, for time integrated fluxes it is expected to be close to 2 [10], value that reproduces a Maxwell-Boltzmann distribution.

3. The most important information from the table, especially for the following, are the conservative ranges that we employ to analyze the simulated data, with the aim to reconstruct the (assumedly) true parameters. This statement applies for the 9 astrophysical parameters (emitted energies, average energies, pinching parameters for three species) and also for the oxygen cross section used for event detection (discussed below).

The neutrino oscillation mechanism is assumed to be described — as a first approximation — by the Mikheyev-Smirnov-Wolfenstein (MSW) effect [22, 23] under the hypothesis of normal hierarchy. The fluences for electron neutrinos and antineutrinos after oscillation become, in three flavors [14]

\[
\begin{align*}
\mathcal{F}_{\nu_e} &= \mathcal{F}_{\nu_e}^0 \\
\mathcal{F}_{\bar{\nu}_e} &= P_{e} \cdot \mathcal{F}_{\bar{\nu}_e}^0 + (1 - P_{e}) \cdot \mathcal{F}_{\nu_x}^0,
\end{align*}
\]

where the superscript 0 refers to the emitted fluences at the source and \( P_{e} = |U_{e1}|^2 \approx 0.70 \) [24]; notice that the measured \( \nu_{\bar{e}} \) correspond to the emitted \( \nu_x \).

\(^2\)This is a specific expected property of time-integrated spectra. By contrast, the time-dependent fluxes are expected to be strongly non-thermal especially at early times, and the corresponding pinching parameters can be much larger than \( \alpha_i = 2.5 \).
Table 2. Number of expected events in Super-Kamiokande, divided by the contribution given by each neutrino species. We use the notation $\nu_x = \nu_\mu + \bar{\nu}_\mu + \nu_\tau + \bar{\nu}_\tau$. The results correspond to the true parameters’ values used in the analysis, and reported in table 1. Remarks on NCR and OS events can be found in the text. The last column refers to the values extracted for the statistical analysis presented in this paper.

|       | $\nu_e$ | $\bar{\nu}_e$ | $\nu_x$ | sum | extracted |
|-------|---------|----------------|---------|-----|-----------|
| IBD   | —       | 2900           | 1672    | 4572| 4565      |
| ES    | 14.7    | 24.7           | 187     | 226 | 237       |
| NCR   | 11.6    | 345            | 204     | 561 | 554       |
| OS sig.| 0.53    | 2.04           | 43.0    | 45.5|           |
| IBD bkg.| —      | 324            | 77.8    | 401 |           |
| ES bkg.| 11.0    | 19.2           | 83.3    | 114 |           |

2.2 Expected events

As possible channels of neutrino detection in Super-Kamiokande we consider: 1) inverse beta decay (IBD); 2) elastic scattering on electrons (ES); 3) quasi-elastic neutral current scattering on $^{16}$O (OS). We recall that: 1) the first reaction is the one that gives the largest number of events, and the positrons are approximately distributed in an isotropic manner; 2) the second one provides us events in a narrow forward cone of about $20^\circ$ [25], and this allows its identification with sufficient precision; 3) the last reaction, finally, produces measurable $\gamma$-ray lines that will be treated simply as a contribution to the total number of events in a suitably chosen low energy region, discussed below.

In general, given a supernova explosion characterized by the parameters specified above, the number of expected events for the reaction $j$ generated by the neutrino species $i$ can be expressed as

$$N_{i,j} = N_{T,j} \int_{E_{\nu,\min}}^{\infty} dE_{\nu} \mathcal{F}_i(E_{\nu}) \sigma_{i,j}(E_{\nu}),$$

where $E_{\nu}$ is the neutrino energy, $N_{T,j}$ is the number of targets for the process $j$ and $\sigma_{i,j}(E_{\nu})$ is the energy dependent cross section for a given reaction and species. The integral goes from the minimum energy $E_{\nu,\min}$ to 300 MeV, which for all purposes is the same as infinity. For IBD and ES events in Super-Kamiokande we assume a threshold on the recoiling $e^\pm$ of 5 MeV [26, 27]. Concerning the OS signal, it is expected to be within a window of $4 \div 9$ MeV, obtained combining the window covered by the expected gamma lines ($\approx 5.3 \div 7.3$ MeV) [28] with the energy resolution of the detector ($\approx 1.1 \div 1.3$ MeV) [29]. In this region, it cannot be disentangled from the (many more) IBD and ES background events. Thus, we are bound to consider the sum of the three contributions. This constitutes the neutral current low energy region (NCR) that includes the OS signal. The number of expected events for each reaction is reported in table 2.

2.3 Statistical procedure

As likelihoods for IBD and ES events we use a standard binned form with the same bin widths as in [16]. Concerning the OS events we use a Gaussian function, with a caveat. For the OS cross section we assume the analytic form reported in [26]

$$\sigma_{\text{OS}}(E_{\nu}) \approx \sigma_0 (E_{\nu}/\text{MeV} - 15)^4 \text{ for } E_{\nu} \geq 15 \text{ MeV},$$

(2.4)
where $\sigma_0 = 4.21 \times 10^{-22}$ fm$^2$ for neutrinos and $\sigma_0 = 3.33 \times 10^{-22}$ fm$^2$ for antineutrinos.\(^3\)

In the literature, quasi-elastic neutral current cross section on oxygen can be found as computed by many models, e.g. \([28, 30–32]\). Comparing the calculations in the literature, the uncertainty seems to be of the order of several 10%. Therefore, we assume a optimistic but not unrealistic value of the uncertainty, namely 10%, that could be hopefully reached in the future.

Thus, we introduce a tenth parameter, $\kappa$, as a multiplicative constant for the whole cross section (identical for neutrinos and antineutrinos) in order to parametrize the systematic uncertainty. It varies according to a Gaussian of mean $\kappa^* = 1$ and standard deviation $\sigma_\kappa = 0.1$, in a prior [0.8, 1.2], see table 1. The NCR likelihood becomes

$$
\mathcal{L}_{\text{NCR}}(\text{param.}) \propto \exp \left[ -\frac{(n_{\text{NCR}} - N_{\text{NCR}})^2}{2N_{\text{NCR}}} - \frac{(\kappa - 1)^2}{2\sigma_\kappa^2} \right],
$$

where $N_{\text{NCR}}$ is the number of expected events as a function of the parameters and $n_{\text{NCR}}$ is the extracted one (see third row of table 2). It is useful to anticipate that our numerical analysis shows that, despite this optimistic assumption on $\kappa$, the inclusion of the NCR has only a minor impact on the conclusion. Moreover, it depends only weakly on the (supposed) value of $\kappa$. In fact, as shown in table 2, the amount of NCR events that can be affected by a modification in the cross section is really small if compared to the IBD+ES related background. For example, increasing $\kappa$ from 1.0 to 1.2 rises the number of OS events from 45.5 to 54.6, leading to an almost identical result.

In order to quantify the relevance of the various reactions, we perform three different analyses: 1) we start with IBD events alone; 2) then we add the ES information; 3) and eventually we include the NCR.

In the analyses we assume full tagging efficiency for the ES process. This assumption is justified as follows. The directional discrimination on the ES events allows us to reduce the IBD background to the 20° cone in which it overlaps to the ES signal — the 3% of the solid angle. Then, 20% of those IBD events can be rejected with the neutron tagging and a further 20% requiring a visible energy lower than 30 MeV \([33]\), already in the current configuration of Super-Kamiokande. Putting all together we end up with \(\sim 100\) IBD-beam related background events, whose statistical variation is \(\sim 10\), smaller than the one of the ES events. Moreover, gadolinium doping \([34]\) can provide an important improvement on neutron tagging \([35]\). According to current data and simulations, the IBD-beam related background events would lower to 20%; in fact, the efficiencies of neutron tagging is \(\sim 0.9\) and the neutron reconstruction efficiency is again \(\sim 0.9\) \([36]\).

The statistical procedure follows a Monte Carlo approach: \(n\)–dimensional random points are extracted in the region described by the priors listed in table 1. Then, each point $P$ is accepted within a certain confidence level (CL) if its likelihood satisfies the relation

$$
\log \mathcal{L}(P) \geq \log \mathcal{L}_{\text{max}} - \frac{A}{2} \quad \text{with} \quad \int_0^A \chi^2(N_{\text{dof}}; z) \, dz = \text{CL},
$$

where $\mathcal{L}_{\text{max}}$ is the likelihood maximum inside the prior and $A$ is defined with an integral of a chi-square distribution with $N_{\text{dof}}$ degrees of freedom. For instance, considering CL = 0.9973, namely 3$\sigma$, we find $A = 20.0621, 23.5746, 26.9011$ for $N_{\text{dof}} = 6, 8, 10$ respectively. This routine has been validated running the algorithm on the analysis \([16]\) and obtaining identical results.

\(^3\)Their values can be inferred from table I of \([28]\), from the branching ratios that explicitly have a $\gamma$-ray in the final state.
Figure 1. Reconstructed total energies $\mathcal{E}_B$ in the analysis without any prior (a) and with implemented energy equipartition among species (b). Different colors represent different amount of information. The prior gray distribution in (a) is obtained extracting random uniformly $\mathcal{E}_{\nu_e}, \mathcal{E}_{\bar{\nu}_e}, \mathcal{E}_{\nu_x}$ in $[0.2, 1] \times 10^{53}$ erg. Each distribution contains $30k$ extracted points and is normalized to $1$. Notice that the horizontal scales in the two cases are very different.

### 3 Results

For each extracted point $P$ the total energy can be reconstructed as

$$\mathcal{E}_{B,P} = \mathcal{E}_{\nu_e,P} + \mathcal{E}_{\bar{\nu}_e,P} + 4\mathcal{E}_{\nu_x,P}, \quad (3.1)$$

with a caveat for the analysis on IBD events alone: since $\mathcal{E}_{\nu_e,P}$ cannot be measured, it is taken randomly inside the prior. The results are gathered in a histogram and shown in figure 1a.

The distribution of the IBD-only analysis is quite similar to the prior distribution, and this implies that it is not possible to measure the total energy by using IBD events only. This negative conclusion fully agrees with the findings by [16]. When we add the ES events in the analysis, the distribution changes, as shown by the peak in figure 1a: the inclusion of the ES scattering events allows us to measure the total energy with Super-Kamiokande. This is a new result that changes qualitatively the conclusion concerning the relevance of water Cherenkov detectors.

This result can be understood looking at figure 2: two accepted points at $3\sigma$ CL, with very different $\mathcal{E}_{\bar{\nu}_e} + 4\mathcal{E}_{\nu_x}$, have similar IBD spectrum — the same within fluctuations — and can be distinguished through the contribution they give to the ES events.

Extracting mean and standard deviation from histogram(s) 1a we get

$$\frac{\mathcal{E}_B}{10^{53} \text{erg}} \xrightarrow{\text{IBD}} 3.4 \pm 0.9 \xrightarrow{+\text{ES}} 3.3 \pm 0.4 \xrightarrow{+\text{NCR}} 3.2 \pm 0.4. \quad (3.2)$$

We conclude that, for the combined three-channels analysis, the total energy can be reconstructed within an accuracy of about 10% (more precisely, $\approx 11\%$). This is the key result of our analysis.

**Remarks.** Often, analyses of simulated data that aim to forecast the physics reach of a future supernova assume equipartition hypothesis for granted. The results obtained if we make this assumption are shown in figure 1b: the three channel combined analysis gives
\[ \mathcal{E}_B = (3.02 \pm 0.09) \times 10^{53} \text{ erg} \] with an accuracy of \( \approx 3\% \). However, the equipartition hypothesis is expected to be reliable only within a factor of two [17, 37, 38]. In conclusion, the 10\% precision should be considered as reference value with current theoretical understanding, since this is model-independent.

The inclusion of the neutral current events in the NCR region yields a result consistent with the IBD+ES analysis, however it does not improve the determination significantly. If the number of beam-related background events due to IBD reaction decreases to 20\%, thanks to the tagging allowed by gadolinium doping, the accuracy for the combined three-channels analysis remains almost the same, namely changing from 11.0\% to 10.7\%. This conclusion does not depend on the uncertainty on the neutral current cross section. In fact, even assuming an ideal scenario in which the IBD background is 20\% and the OS cross section is known perfectly, the resolution becomes \( \approx 10.3\% \). In other words, the improvement due to the inclusion of the NCR is very small.

Therefore, we conclude that, in order to measure the neutron star binding energy with Super-Kamiokande or similar neutrino telescopes, it is very important to observe and analyze the events due elastic scattering of neutrinos and electron, even more than the neutral current events on oxygen. The fact that we include these events in our analysis explains the difference with the conclusion of ref. [16], that was based only on IBD events.

### 4 Discussion

The precise measurement of the gravitational binding energy in a future galactic supernova explosion is of interest for astrophysics, particle and nuclear physics. In supernova theory, the delayed neutrino-heating mechanism, elaborated by Wilson [39] and by Bethe and Wilson [40], is currently thought to be the mechanism producing the blow of most of supernovae type II and Ib/c. Close to giving successful explosions [41], three-dimensional simulations include realistic neutrino transport, nuclear networks, hydrodynamic instabilities, convection and turbulence. The precise measurement of the gravitational binding energy in a future galactic

![Figure 2. IBD spectrum (a) and contribution by $\bar{\nu}_e, \nu_x$ to the ES spectrum (b) for two set of points, characterized by very different values of $\mathcal{E}_\bar{\nu}_e + 4\mathcal{E}_\nu_x$. The first one is $P_1 = (\mathcal{E}_\bar{\nu}_e, \mathcal{E}_\nu_x, \langle \mathcal{E}_\bar{\nu}_e \rangle, \langle \mathcal{E}_\nu_x \rangle, \alpha_{\bar{\nu}_e}, \alpha_{\nu_x}) = (0.665 \times 10^{53} \text{ erg}, 0.2 \times 10^{53} \text{ erg}, 12.75 \text{ MeV}, 9.25 \text{ MeV}, 2.075, 2.155)$ and the other one is $P_2 = (0.294 \times 10^{53} \text{ erg}, 1 \times 10^{53} \text{ erg}, 13.45 \text{ MeV}, 11.9 \text{ MeV}, 2.075, 2.155)$. The sums $\mathcal{E}_\bar{\nu}_e + 4\mathcal{E}_\nu_x$ are 1.5 and $4.3 \times 10^{53} \text{ erg}$ respectively. Both points are accepted in 3\sigma CL IBD-only analysis and they can be discriminated only adding the ES reaction.](image-url)
explosion would provide key confirmation of the current paradigm of the delayed neutrino-heating mechanism. From the particle physics point of view, improved limits would be gathered on neutrino properties related to new physics, such as neutrino decay, the neutrino magnetic moment or the existence of sterile neutrinos.

Interestingly, the determination of $E_B$ would give the star compactness

$$\beta = \frac{GM}{R c^2}, \quad (4.1)$$

where $G$ is the gravitational constant, $M$ and $R$ are the gravitational mass and the radius of the newly formed neutron star. From the fit to the neutron star binding energies, for a large set of equations-of-state (EOS) that permit maximum masses larger than $1.65 M_\odot$ as a function of $\beta$, one has\(^4\) \[43\]

$$\frac{E_B}{M c^2} \approx \frac{(0.60 \pm 0.05) \beta}{1 - \beta/2}, \quad (4.2)$$

where $E_B = N_b m_b c^2 - M c^2 = M_{bc} c^2 - M c^2$ is the gravitational binding energy, $M_b$ is the baryonic mass corresponding to $N_b$ baryons of mass $m_b$. The latter can be taken as the mass of a neutron or a proton or of $^{56}\text{Fe}/56$ for a white-dwarf iron core.

The baryonic mass can be determined from simulations by considering the amount of mass inside the shock radius about 0.5 seconds after bounce, at the outer edge of the $^{56}\text{Ni}$ region and subtracting the amount of nickel in the supernova ejecta. After inserting the expression for $E_B$, the $\beta - E_B$ relation (4.2) reads

$$\beta = \frac{E_B}{0.6 M_{bc} c^2 - 0.1 E_B}, \quad (4.3)$$

For SN 1987A, using eq. (4.2) and considering that the $M_b = 1.733 M_\odot$ and $M = 1.53 M_\odot$ \[44\] one gets $\beta = 0.194$ and $R = 11.5$ km.

Relation (4.1) can also be used to get $R - M$ relation

$$R = \frac{0.6 G M^2}{E_B} + \frac{r_s}{4}, \quad (4.4)$$

where we have introduced the Schwarzschild radius $r_s = 2GM/c^2$. Alternatively, one can obtain the $M - R$ relation

$$M = \sqrt{\frac{E_B R}{0.6 G} \left[ \sqrt{1 + \epsilon^2} - \epsilon \right]} \quad \text{with} \quad \epsilon = \frac{1}{4} \sqrt{\frac{E_B G}{0.6 R c^2}}, \quad (4.5)$$

The main relativistic effect is given by the negative term $\propto \epsilon$, namely $0.42 E_B/c^2$.

Figure 3a shows the $\beta - E_B$ relation given by eq. (4.3) for different values of the baryonic mass known with 10% uncertainty. The binding energy $E_B$ is assumed to be determined more precisely than $M_b$ thanks to a future supernova galactic explosion. For definiteness we assume $\delta E_B \sim 1\%$. For a nominal value $E_B \approx 3.53 \times 10^{53}$ erg the neutron-star compactness would be of $\beta = 0.258 \pm 0.035, 0.223 \pm 0.030, 0.196 \pm 0.027$ for the representative values $M_b/M_\odot = 1.3, 1.5, 1.7$ respectively.

Figure 3b presents the gravitational mass-radius allowed region, based on relation (4.5) from neutron-star EOS. We show a blue band in the achievable case of 10% accuracy in

\(^4\)Notice that this relation, including some dependence on the radius, improves on a previous relation by Lattimer and Yahil, namely $E_B \approx 0.084 (M/M_\odot)^2 M_\odot c^2$ that is also accurate to $\sim 10\%$ \[42].
Figure 3. Neutron-star compactness $\beta$ as a function of the gravitational binding energy $\mathcal{E}_B$ (a), known with 1% accuracy, for different values of the baryonic mass $M_b$ supposed to be known with 10% precision from the observation of $^{56}$Ni and the pre-supernova model. Figure (b) presents the gravitational mass-radius allowed region, from eq. (4.5). The red point is a putative measurement of the neutron-star radius and the corresponding neutron star mass from a fit on the EOS given by relation (4.2).

In summary, we have shown that the gravitational binding energy of the next galactic supernova can be measured through its neutrinos in a detector such as Super-Kamiokande with a precision of $\sim 10\%$, without any priors and by combining inverse beta decay and elastic scattering events. Increasing volumes and number of detection channels is likely to reduce this error greatly; e.g., the statistical improvement due to the increased mass in Hyper-Kamiokande is a factor of $\approx 4$. We have shown here that this information can be used e.g. to determine the neutron star compactness with a precision of $\sim 10\%$; the limiting factor is the uncertainty on the baryon mass $M_b$ and not the one on $\mathcal{E}_B$. Future EOS investigations are likely to better constraint the $\beta - \mathcal{E}_B$ relation (4.2) allowing more precise inferences. Clearly, a precise measurement of $\mathcal{E}_B$, achievable in future observation of supernova neutrinos, would provide crucial observational constraints on the supernova mechanism, on neutrino properties and on the neutron star mass-radius relation.

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