Innovative method for state of energy estimation based on improved Cubature Kalman filter

Xinan Zhou1, Sida Zhou1, Qiangwei Li1, Zhiwei Fan2, Yang hua1, Shichun Yang1* and Yulong Shao3

1 School of Transportation Science and engineering, Beihang University, Beijing, Beijing, 100191, China
2 Evergrande New Energy Automotive R&D Institute Global Headquarters, Guangzhou, Guangdong, 510080, China
3 Zhengzhou Yutong Bus Co., Ltd, Zhengzhou, Henan, 450016, China
*Corresponding author’s e-mail: yangshichun@buaa.edu.cn

Abstract. As the increasing concern of driving ranges for electric vehicles, the precision of state of energy (SOE) estimation contributes to the better performance of battery management. In this article, an innovative method based on 5th-order simplex square-radius cubature Kalman filter is developed to achieve the precise and robust estimation for SOE on electric vehicles. Based on second-order equivalent circuit model and particle-swarm-optimization algorithm, the implementation helps the validation of SOE estimation. The max error of SOE estimation under stable condition is less than 3%, and that for dynamic stress test condition is under 4%. Moreover, the robustness is investigated based on diverse deviations on initialization, delivering the future potential applications on embedded system on cloud-controlling.

1. Introduction
As the increasing concerning of cleaner energy, electric vehicles (EVs) are recognized as representative productions for substituting the traditional fuel vehicles[1]. Moreover, EVs contribute to the reduction of imported petroleum resources, leading to the widely promotion in China. Considered to be the main energy storage system for EVs, lithium-ion batteries have been globally promoted compared against lead-acid batteries, sodium-ion batteries or fuel cells according to the comprehensively better performances[2]. The long lifespan and satisfactory energy/power density help the popularization of lithium batteries[3]-[4]. However, there still remains the challenges for in-depth applications, especially for precise battery state estimation.

Generally, the essential states of lithium battery can be divided into: state of charge (SOC), state of health, state of power and state of energy (SOE). State of health indicates the general degradation of battery, and state of power demonstrates the performances on permitted charging/discharging power. SOC and SOE describe the useful capacity or energy for batteries, which determine the safe boundary for battery operation. Both SOC and SOE contribute to the safe management for serial batteries, but SOE is rather essential for evaluating the driving range of vehicles, which is associated with experiences for drivers. Unfortunately, the current estimation still encounters challenges on and further precise calculating methods still need deep research[5].

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Considering the general methods for estimating SOE, model-based methods are of crucial importance for embedded system on EVs compared against data-driven methods or electrochemical models[6]. Equivalent-circuit-model is one of the most promoted models for describing the stable/dynamic performances according to serial resistances and capacitances, and satisfactory performances can be achieved coupled with parameter identification algorithms[7]-[8]. Due to the rigorous mathematical description of models, the state-space model can be established and further applications on filtering algorithms are applicable. Moreover, the Kalman filter family has been widely promoted for SOE estimations, owing to the associativity on the state-space equations based on equivalent-circuit-model[9]-[10].

Herein, an innovative 5th-order simplex spherical-radius cubature Kalman filter (5th-SSRCKF) is developed and accurate SOE estimation method is delivered based on second-order equivalent circuit model (SECM). Furthermore, the particle-swarm-optimization (PSO) method is introduced to achieve parameterization of models. Both stable and dynamic conditions are applied for validating the precision of proposed methods, and performances of robustness are also discussed based on simulation results.

2. Modeling of Lithium-ion battery

2.1. Second-order equivalent circuit model

Although diverse kinds of equivalent models can be applied for SOE estimation [7], SECM is widely promoted based on sufficient precision with limited parameters[11]-[12]. Generally, SECM consists of an ideal power to simulate the open circuit characteristics, serial resistance to represent DC impedance, and two RC links to describe dynamic performances. The profile of SECM is presented as figure 1, and system equations can be established based on Kirchhoff’s voltage principle. Demonstrated as equation (1), the model description can be transferred into state-space model.

\[
\begin{align*}
U_{\text{dis}} &= I_t R_o \\
\dot{U}_{p1} &= \frac{I_t}{C_{p1}} - \frac{U_{p1}}{R_{p1} C_{p1}} \\
\dot{U}_{p2} &= \frac{I_t}{C_{p2}} - \frac{U_{p1}}{R_{p2} C_{p2}} \\
U_t &= U_{\text{ocv}} - U_{dp1} - U_{p2} - U_{\text{dis}} \\
SOE_k &= \int_{t_{k-1}}^{t_k} U_t I_k - I_k^2 R dt
\end{align*}
\]

Figure 1. Second-order RC equivalent circuit model.
\[
\begin{bmatrix}
U_{p,k+1} \\
U_{d,k+1}
\end{bmatrix} =
\begin{bmatrix}
e^{-\frac{1}{R_p C_p}} & 0 \\
0 & e^{-\frac{1}{R_d C_d}}
\end{bmatrix}
\begin{bmatrix}
U_{p,k} \\
U_{d,k}
\end{bmatrix} +
\begin{bmatrix}
(1-e^{-\frac{1}{R_p C_p}})R_p \\
(1-e^{-\frac{1}{R_d C_d}})R_d
\end{bmatrix} I_{k,k}
\]

(2)

Where: \(R_{p1}\) and \(C_{p1}\) are the activation polarization resistance and capacitance respectively; \(R_{p2}\) and \(C_{p2}\) are the concentration polarization resistance and capacitance respectively. \(U_L\) is the terminal voltage, \(I_k\) is the current (assumed positive for charge, negative for discharge). \(U_{p1}\) and \(U_{p2}\) describe the diffusion voltage over the RC network.

2.2. Particle-swarm-optimization

Considering the state-space model of SECM, there are six parameters still needed to be parameterized, including \(U_{ocv}\), \(R_o\), \(R_{p1}\), \(R_{p2}\), \(C_{p1}\), \(C_{p2}\). In this article, open-circuit-voltage is identified based on pulsing discharging experiments, where the terminal voltage of battery cell can be abstract as open-circuit-voltage with sufficient resting time. Herein, 1 hour is applied after the cell discharging for achieving relaxation. Moreover, \(R_o\) can be also parameterized, according to the voltage drop when the current vanishes suddenly. Thus, only four polarization parameters should be identified based on parameterization algorithm.

The parameter identification method adopted is the interpolation curve-fitting method, which identifies parameters by fitting constant current pulse charging and discharging experimental data with the circuit equation[13]. Generally, multiple methods can be applied especially for intelligent algorithm [14], such as simulated annealing algorithm, PSO, ant colony optimization algorithm and sunflower algorithm[15]-[16]. Considering the comprehensive performances on simplicity, precision and calculational cost, PSO method is adopted for solving parameter identification in this article[17]. The governing equations are presented as equation (2) and (3), and the state \(x\) is referred to the parameter vector, presented as equation (4).

\[
v_{k+1} = \omega v_k + c_1 rand_1[p_{opt,k} - x_k] + c_2 rand_2[g_{opt,k} - x_k]
\]

(3)

\[
x_{k+1} = x_k + v_k
\]

(4)

\[
x = [R_{p1}; R_{p2}; C_{p1}; C_{p2}]
\]

(5)

Where: \(x\) is current state and \(v\) is the optimizing velocity; \(c_1\) and \(c_2\) are learning factors from particle and swarm; \(rand_1\) and \(rand_2\) are random numbers between 0 and 1; \(p\) is historical best solution for particle swarm and \(g\) is historical best solution for global swarm.

Considering the random search in each iteration, the optimized solutions may not be the same for different optimization. Thus, all solutions can be received which can decrease the general deviation between simulated results and experimental dataset. The identification experimental conditions is selected as follows. The specific pulse power discharge process and experimental result are shown as figure 2. Moreover, the parameters identified by PSO are partly presented as figure 3. As the open-circuit-voltage and internal resistance can be obtained from pulsing discharging test, the associated parameters are identified directly based on experiments.

1#: 1C discharge at ambient temperature of 25 °C for 6 minutes
2#: rest for 1 h
3#: cycle 10 times between 1# and 2# until the terminal voltage is limited to least one.
3. 5th-SSRCKF and SOE estimation

Diverse kinds of filtering algorithms can be applied for battery state estimation, such as classical Kalman filter, extended Kalman filter and unscented Kalman filters[18]-[20]. Although algorithms can be adopted for calculating SOE of batteries, an innovative algorithm with rigorous mathematical proof still can be appreciated for applications. Herein, a novel 5th-order SSRCKF method is introduced and associated principles are presented for understanding. And the related method with SOE estimation is presented.

Generally, the system equations can be abstract as non-linear probabilities. As presented in equation (6), the updating of system state has reliance within historical times, and observation state is associated with current state. Considering the prediction at k time for Kalman filter theories, a non-linear integral equation can be presented as equation (7), which has the similarity with Hammerstein equation. Considering the operational applications, the predigestion of complex equations is of crucial importance and Gaussian method is widely promoted for the simplification, which is developed as equation (8).

Considering the selection of sampling points and linked weights, an innovative 5th order simplex spherical-radius cubature principle is introduced. Demonstrated as equation (9) to (12), the sampling points and weights are presented for each iteration, and the equations can be implemented into Kalman filter profile. Moreover, the associated 5th SSRCKF algorithm is presented as table 1, where the general procedure can be divided into initialization, prediction, correction and updating.

Although adaptive correction can be achieved based on Kalman gain, the system performances is still influenced by initial state. Compared against other filters, the prediction period is rather important for Kalman filter profile, and the predicted state contributes to the simulation of non-linear equations, which delivers the evolution tendency for current system state. Moreover, the correction period
develops the simultaneous correcting of state based on the deviation between simulated terminal voltage and experimental samples.

\[
x_k = f(x_{k-1}, w_k) \\
z_k = h(x_k, v_k)
\]

\[
x_{k|k-1} = \int f(x_{k-1}, w_k)N(x_{k-1}, P_{x,k-1})dx_{k-1}
\]

\[
\int f(x)p(x)dx = \sum_{i=1}^{L} \omega_i f(\xi_i)
\]

\[
\xi_1 = \overline{x}, i = 1
\]

\[
\xi_{1+i} = \overline{x} + \sqrt{(n+2)P_\alpha a,}
\]

\[
\xi_{i+2} = \overline{x} - \sqrt{(n+2)P_\alpha b,}
\]

\[
\xi_{n^2/2+5+i+i+3} = \overline{x} - \sqrt{(n+2)P_b b}, i = 2n+2, 2n+5, \ldots n^2 + 2n + 3
\]

\[
w_i = \frac{2}{n+2}, i = 1
\]

\[
w_i = \frac{(7-n)n^2}{2(n+1)^2(n+2)^2}, i = 2, 3, \ldots 2n + 3
\]

\[
w_i = \frac{2(n-1)^2}{(n+1)^2(n+2)^2}, i = 2n+2, 2n+5, \ldots n^2 + 2n + 3
\]

\[
a_{i,j} = \begin{cases} 
-\left(\frac{n+1}{n(n-i+2)(n-i+1)}\right), & i < j \\
\left(\frac{(n+1)(n-j+1)}{n(n-i+2)}\right), & i < j \\
0, & 0, i < j 
\end{cases}
\]

\[
\{b_j\} = \left\{\frac{n}{2(n-1)}(a_i + a_j) : i \leq l, l = 1, 2, \ldots n+1\right\}
\]

Table 1. The profile of 5th SSRCKF method.

| Initialization | State prediction |
|----------------|------------------|
| \(\hat{x}_0 = E(x_0)\) | \(P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]\) |

Sample according to equation (9) to (12) 
\(\xi_{i,k-1}, i = 1, 2, \ldots, 2n\)

Sample state prediction 
\(\xi_{i,k-1} = f(\xi_{i,k-1}, w_{k-1})\)

Prior state estimate 
\(\hat{x}_{k|k-1} = \sum_{i=1}^{L} \omega_i \xi_{i,k-1}\)

Prior covariance matrix estimate 
\(P_{k|k-1} = \sum_{i=1}^{L} \omega_i (\xi_{i,k-1} - \hat{x}_{k|k-1})(\xi_{i,k-1} - \hat{x}_{k|k-1})^T + Q_{k-1}\)
Correction

Sample according to equation (9) to (12)
\[ \bar{\xi}_{ik}\|_{k-1}, i = 1, 2, \ldots, 2n \]

Sample measurement
\[ z_{ik}\|_{k-1} = h(\bar{\xi}_{ik}\|_{k-1}, v_{k-1}) \]

Measurement estimate
\[ \hat{z}_{ik}\|_{k-1} = \sum_{i=1}^{2n} \omega_i z_{ik}\|_{k-1} \]

Measurement error
\[ e_k = z_k - \hat{z}_{ik}\|_{k-1} \]

Error estimate
\[ P_{zz} = \sum_{i=1}^{2n} \omega_i (z_{ik}\|_{k-1} - \hat{z}_{ik}\|_{k-1}) (z_{ik}\|_{k-1} - \hat{z}_{ik}\|_{k-1})^T + R_{k-1} \]

Correct gain
\[ G_k = P_{zz}^{-1} P_{zk}\|_{k-1} \]

Update

State update
\[ \hat{x}_k = \hat{x}_{ik}\|_{k-1} + G_k (z_k - \hat{z}_{ik}\|_{k-1}) \]

Covariant matrix update
\[ P_k = P_{ik}\|_{k-1} - G_k P_{zz} G_k^T \]

4. Experimental validation and discussion

4.1. Experiment setup

For providing the parameterization and algorithm validation, sufficient experiments are of necessary. Presented as figure 4, the workbench consists of diverse equipment, including battery cycler (managing the current for charging/discharging, and sampling), PC (storing the dataset), environmental chamber (managing the temperature and humidity) and battery pack (a battery module with serial connection). It is worthy to mention that the experimented battery pack is a battery module with 6 cells under serial connection, and is recognized as a large battery cell without the consideration of internal inconsistency. The simplicity of the battery pack leads to a greater promotion for real-time calculation of SOE, where the embedded system cannot afford the computational cost for dealing with each cell.

Figure 4. Workbench design.
The tested battery pack is prismatic lithium-ion battery of 180AH and the cathode material is LiNi_{x}Co_{y}Mn_{z}O_{2}. The energy density of battery pack is about 180 Wh/kg. Generally, the maximum charging voltage is 4.2V, and the minimum discharging voltage is 2.7. When the 6 cells are serial connected, the operational voltage is between 16.2V – 25.2V.

4.2. Precision and robustness analysis
Considering the operational applications on SOE estimation, different conditions should be validated under stable/dynamic conditions. Normally, stable conditions are used for simulating constant current discharging/charging, and dynamic conditions aims at the simulation of transient current conversion for electric vehicle driving. Herein, a constant current discharging condition and dynamic stress test condition are applied for investigating the performances on battery pack. The results are presented in figure 5 and figure 6.

A maximum absolute error of 2.4% can be observed from figure 5, where figure 5 (a) presented the simulated SOE curve compared against experiments, and figure 5 (b) demonstrates the error analysis. It is worthy to mention that the error is relatively less in the medium SOE ranges compared with high/low ranges. The potential causes may be attributed to the failure of SECM, where the second-order equivalent model cannot simulate the output performances according to the limited parameters and violent electrochemical reactions.

Similarly, a maximum absolute error of 3.89% under dynamic stress test condition validates the effectiveness of proposed method for calculating SOE. The fitting curve and error analysis are presented as figure 6. Compared against stable conditions, the intense fluctuation can be observed according to figure 6 (b). The changes of error are associated with sudden changes of the current, where the jitter voltage results in the delay of SOE correction. Moreover, the immediate response of second-order equivalent circuit still cannot meet the demand of real battery sampling intervals, which enhances the fluctuation of error curves.

Additionally, the robustness analysis is carried out and the results are demonstrated as figure 7. Different deviation of initialization for SOE is applied to the simulation, where: 80% SOE, 60% SOE, 40% SOE, 20% SOE are adopted for investigating robustness. The simulated results deliver the result that the proposed method contributes to the convergence of deviated initialization, and the system can be corrected to the actual one. Moreover, the convergence time is about 200 – 300s, demonstrating the sufficient efficiency for real-time estimation.

![Figure 5. SOE estimation under stable discharging condition.](image)
5. Conclusion
In this article, an innovative method based on 5th-order simple square-radius cubature Kalman filter is developed to achieve the precise and robust estimation for SOE on electric vehicles. The implementation of second-order equivalent circuit model and offline parameterization based on particle swarm optimization are delivered for combining the battery model and filtering algorithm. The maximum error of SOE estimation under stable is less than 3%, and that for dynamic stress test condition is under 4%.

More scenarios of application can be extended based on the proposed modeling framework, and the methods can be further applied to estimate the SOC or SOH of battery pack under more complex driving cycles. In the future, more improvement for algorithms and online state of heath estimation will also be considered in modeling to promote the promising methods.

Acknowledgments
This work is supported by the National Key R&D Program of China (2018YFB0104100).

References
[1] D. Wang et al. (2020) A lithium-ion battery electrochemical–thermal model for a wide temperature range applications. Electrochimica Acta, vol. 362.
[2] S. Wang, C. Fernandez, C. Yu, Y. Fan, W. Cao, and D.-I. Stroe. (2020) A novel charged state prediction method of the lithium ion battery packs based on the composite equivalent modeling and improved splice Kalman filtering algorithm. Journal of Power Sources, vol. 471.

[3] A. Carnovale and X. Li. (2020) A modeling and experimental study of capacity fade for lithium-ion batteries. Energy and AI, vol. 2.

[4] J. Li et al. (2020) Aging modes analysis and physical parameter identification based on a simplified electrochemical model for lithium-ion batteries. Journal of Energy Storage, vol. 31.

[5] G. Dong, X. Zhang, C. Zhang, and Z. Chen. (2015) A method for state of energy estimation of lithium-ion batteries based on neural network model. Energy, vol. 90, pp. 879-888.

[6] G. Dong, Z. Chen, J. Wei, C. Zhang, and P. Wang. (2016) An online model-based method for state of energy estimation of lithium-ion batteries using dual filters. Journal of Power Sources, vol. 301, pp. 277-286.

[7] M. Hu, Y. Li, S. Li, C. Fu, D. Qin, and Z. Li. (2018) Lithium-ion battery modeling and parameter identification based on fractional theory. Energy, vol. 165, pp. 153-163.

[8] M.-K. Tran, A. Mevawala, S. Panchal, K. Raahemifar, M. Fowler, and R. Fraser. (2020) Effect of integrating the hysteresis component to the equivalent circuit model of Lithium-ion battery for dynamic and non-dynamic applications. Journal of Energy Storage, vol. 32.

[9] X. Li et al. (2017) A physics-based fractional order model and state of energy estimation for lithium ion batteries. Part I: Model development and observability analysis. Journal of Power Sources, vol. 367, pp. 187-201.

[10] X. Li, J. Xu, J. Hong, J. Tian, and Y. Tian. (2021) State of energy estimation for a series-connected lithium-ion battery pack based on an adaptive weighted strategy. Energy, vol. 214.

[11] Y. Li, M. Vilathgamuwa, T. Farrell, S. S. Choi, N. T. Tran, and J. Teague. (2019) A physics-based distributed-parameter equivalent circuit model for lithium-ion batteries. Electrochimica Acta, vol. 299, pp. 451-469.

[12] J. Yang, Y. Cai, C. Pan, and C. Mi. (2019) A novel resistor-inductor network-based equivalent circuit model of lithium-ion batteries under constant-voltage charging condition. Applied Energy, vol. 254.

[13] M. Kim et al. (2019) Data-efficient parameter identification of electrochemical lithium-ion battery model using deep Bayesian harmony search. Applied Energy, vol. 254.

[14] J. Xu, T. Wang, L. Pei, S. Mao, and C. Zhu. (2020) Parameter identification of electrolyte decomposition state in lithium-ion batteries based on a reduced pseudo two-dimensional model with Padé approximation. Journal of Power Sources, vol. 460.

[15] R. A. El-Sehiemy, M. A. Hamida, and T. Mesbahi. (2020) Parameter identification and state-of-charge estimation for lithium-polymer battery cells using enhanced sunflower optimization algorithm. International Journal of Hydrogen Energy, vol. 45, no. 15, pp. 8833-8842.

[16] X. Lai, S. Wang, S. Ma, J. Xie, and Y. Zheng. (2020) Parameter sensitivity analysis and simplification of equivalent circuit model for the state of charge of lithium-ion batteries. Electrochimica Acta, vol. 330.

[17] S. Zhou, X. Liu, Y. Hua, X. Zhou, and S. Yang. (2021) Adaptive model parameter identification for lithium-ion batteries based on improved coupling hybrid adaptive particle swarm optimization- simulated annealing method. Journal of Power Sources, vol. 482.

[18] Y. Wang, C. Zhang, and Z. Chen. (2016) Model-based State-of-energy Estimation of Lithium-ion Batteries in Electric Vehicles. Energy Procedia, vol. 88, pp. 998-1004.

[19] G. Zhai, S. Liu, Z. Wang, W. Zhang, and Z. Ma. (2017) State of Energy Estimation of Lithium Titane Battery for Rail Transit Application. Energy Procedia, vol. 105, pp. 3146-3151.

[20] X. Zhang, Y. Wang, J. Wu, and Z. Chen. (2018) A novel method for lithium-ion battery state of energy and state of power estimation based on multi-time-scale filter. Applied Energy, vol. 216, pp. 442-451.