THE LENS GALAXY IN PG1115+080 IS AN ELLIPSE *

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ABSTRACT

We use the structure of the Einstein ring image of the quasar host galaxy in the four-image quasar lens PG1115+080 to determine the angular structure of the gravitational potential of the lens galaxy. We find that it is well described as an ellipsoid and that the best fit non-ellipsoidal models are consistent with the ellipsoidal model. We find upper limits on the standard parameters for the deviation from an ellipse of $|d^B|/a_0 < 0.035$ and $|d^A|/a_0 |< 0.064$. We also find that the position of the center of mass is consistent with the center of light, with an upper limit of $|\Delta x| < 0.0005$ on the offset between them. Neither the elliptical nor the non-ellipsoidal models can reproduce the observed image flux ratios while simultaneously maintaining a reasonable fit to the Einstein ring, so the anomalous flux ratio of the $A_1$ and $A_2$ quasar images must be due to substructure in the gravitational potential such as compact satellite galaxies or stellar microlenses rather than odd angular structure in the lens galaxy.

Subject headings: cosmology: observations — dark matter — gravitational lensing — quasars: individual (PG 1115+080)

1. INTRODUCTION

The scenario of hierarchical structure formation dominated by cold dark matter has been extremely successful in explaining various observations of large scale structure (see, e.g., York et al. 2000; Norberg et al. 2001; Spergel et al. 2003). In this scenario, small structures form first and merge to build progressively larger structure (Rees & Ostriker 1977; White & Rees 1978). One prediction of these theories, both analytically and in simulations, is the existence of far more low-mass halos than are observed either in the field or as satellites of the Milky Way. Detailed studies of small satellite halos show that many survive without being tidally disrupted, albeit with some mass loss, and remain as gravitationally bound substructures in the halo of the larger galaxy (Moore et al. 1999; Klypin et al. 1999). Most estimates lead to an abundance of satellites that are inconsistent with the observed abundances, suggesting that the substructure must consist of dark satellites in which star formation was suppressed by heating and ejecting the baryons (Bullock et al. 2000). Unfortunately, dark satellite halos are difficult to detect.

Fortunately, we have one probe of halo structure that can detect substructure purely through its gravitational effects - gravitational lensing (see, e.g., Blandford & Narayan 1992; Schneider et al. 1992; Kochanek et al. 2004). Substructure in the halo of a gravitational lens galaxy, whether dark satellites or simply stars, can modify the observed fluxes of the individual images relative to those in a smooth gravitational potential. These are most striking for the systems with anomalous flux ratios between images which should have similar flux ratios given any smooth potential centered on the primary lens galaxy (e.g., Schneider & Maj 1998; Zhao & Pronk 2001; Chiba 2002; Dalal & Kochanek 2002; Metcalf & Zhao 2002). For the case of microlensing by the stars in the lens galaxy, the existence of substructure is easily verified by the temporal variations of the flux ratio such as those observed in Q2237+0305 (Wozniak et al. 2000). Tests for satellites have been focused on lensed radio sources that are too large to be affected by microlensing and little affected by the interstellar medium of the lens galaxy. Kochanek & Dalal (2004) have shown statistically at least, that the flux ratios of lenses show the characteristic pattern of demagnified saddle point images expected for low optical depths of substructure embedded in a higher optical depth of smoothly distributed dark matter (Schechter & Wambsganss 2002; Keeton 2003). If these anomalies are interpreted as being due to substructure, the estimated mass fraction in the substructure (Dalal & Kochanek 2002) is consistent with theoretical expectations (Moore et al. 1999; Bullock et al. 2000). There is some debate over the contribution from small halos along the line of sight relative to those associated with the lens galaxy, with most studies favoring a dominant contribution from the lens galaxy (e.g., Metcalf 2002; Chen et al. 2003). However, from the point of view of understanding the halo mass function this is a moot point because these low mass halos are not observed as either satellites or in the field with an abundance approaching that predicted by the CDM scenario.

There have, however, been several explorations of whether the flux ratio anomalies can be explained by adding complex angular structures to the gravitational potential of the primary lens galaxy rather than by adding substructure (Evans & Witt 2003; Dalal & Kochanek 2002; Möller et al. 2002). In particular, Evans & Witt 2003 showed that for one particularly simple model with arbitrary angular structure the fit to the data can be reduced to a problem in linear algebra in which the lens constraints, including the flux ratios, can be modeled to arbitrary accuracy simply by including enough terms. The model parameters of this study were problematic (the quasar images needed only to be fitted to 0.05 over ten times the actual uncertainties), but the mathematical point that standard models fitting only the image positions fail to fit the image fluxes only because the gravitational potentials are assumed to be roughly ellipsoidal is correct. However, Kochanek & Dalal
considered models with some additional angular structure and compared models constrained by either only the positions of the quasar images or the positions of the images combined with any additional constraints available for the system. In each case, the large deviations from an ellipsoidal model that could improve the modeling of the flux ratios were ruled out by the additional constraints.

In this paper we explore whether the lens galaxy in the four-image quasar lens PG1115+080 (Weymann et al. 1980; Courbin et al. 1997; Keeton & Kochanek 1997; Impey et al. 1998; Treu & Koopmans 2002) has significant deviations from an ellipsoidal density distribution. PG1115+080 is an interesting case because the merging $A_1/A_2$ images have a flux ratio of $\approx 1.6$ rather than the ratio of $\approx 1.0$ predicted by smooth models. We will use the same models with arbitrary angular structure considered by Evans & Witt (2003, and earlier by Witt et al. 2000; Evans & Witt 2001; Kochanek et al. 2001; Zhao & Pronk 2001) but examine the constraints imposed by the Einstein ring image of the quasar host galaxy in detail. Kochanek et al. (2001) demonstrated that the shape of the Einstein ring completely encodes the angular structure of the potential, so we can quantitatively measure the deviations of the higher order multipoles of the lens gravity from a flat rotation curve, making them consistent with most estimates of the radial structure of lenses (e.g., Treu & Koopmans 2002; Rusin & Kochanek 2004). We expand the angular structure in multipole series,

$$F(\theta) = a_0 + \sum_{m=2}^{\infty} [a_m \cos m\theta + b_m \sin m\theta],$$

neglecting the dipole ($m=1$) terms which will be degenerate with a shift in source position. From Poisson’s equation, the angular structure of the convergence $\kappa$ is related to that of the potential by

$$f(\theta) = a_0 + \sum_{m=2}^{\infty} [a_m(1-m^2) \cos m\theta + b_m(1-m^2) \sin m\theta].$$

We extracted the ring curves as described in Kochanek et al. (2001). We first fit a photometric model to the image including the four lensed quasar images, the lens galaxy and the host galaxy. This fit provides estimates of the quasar image fluxes, and these are used to subtract the lensed quasar images from the images. We then select a point close to the center of the lens, and as a function of position angle $\theta$ we find the radius $r(\theta)$ of the peak surface brightness of the Einstein ring and its uncertainties. The total number of points extracted from an Einstein ring is $N_{\text{ring}} = 71$. Since the ring is well-detected even in a single image, we extracted rings from several subsets of the data to explore both statistical and systematic errors in the data. We will refer to extractions from the 1997 data as obs1 and extractions from the 2003 data as obs2. For obs1 we extracted the ring using three different choices of the point about which to extract: obs1-A was centered on the lens, obs1-B was offset by $0\arcsec 1$ in the $x$ direction of the image and obs1-C was offset by $0\arcsec 1$ in the $y$ direction. For the obs2 data we extracted the ring for each of the four individual orbits (obs2-A, obs2-B, obs2-C and obs2-D). For our final analysis we used a ring extracted from the combined image of the four individual orbits in obs2. Figure 1 shows $r(\theta)$ for these various extractions as compared to the final combined model and its uncertainties. For the image positions and fluxes, we use the HST/NICMOS observations described by Impey et al. (1998).

3. Lens Model of PG1115+080

In this section, we first describe our models for the lens galaxy of PG1115+080 and the nearby galaxy group as an external perturber. Then we describe our model for the Einstein ring and our approach to fitting the data.

3.1. Lens Potential

We model the lens galaxy as a scale-free potential with arbitrary angular structure,

$$\phi(r, \theta) = r F(\theta), \quad \kappa(r, \theta) = \frac{f(\theta)}{2r},$$

where $F(\theta)$ is an arbitrary function of angle $\theta$, and $f(\theta) = F + F''$ (Zhao & Pronk 2001; Evans & Witt 2001, 2003; Kochanek et al. 2001; Wucknitz 2002). These models have a flat rotation curve, making them consistent with most estimates of the radial structure of lenses (e.g., Treu & Koopmans 2002; Rusin & Kochanek 2004).
\[
= a_0 \left[ 1 + \sum_{m=2}^{\infty} \left( A_m \cos m\theta + B_m \sin m\theta \right) \right],
\]
where \( A_m = (1 - m^2) a_m / a_0 \) and \( B_m = (1 - m^2) b_m / a_0 \) for \( m \geq 2 \).

We are interested in the deviations of the potential from an ellipsoidal surface mass density,

\[
f(\theta) \sim \left[ 1 - (1 - q^2) \cos^2 \theta \right]^{-1/2} \times 1 + \sum_{m=1}^{\infty} a_{2m}^2 \cos 2m\theta,
\]
where \( q \) is the axis ratio of the ellipse with coefficients \( a_{2m}^2 \) to distinguish them from the coefficients \( a_{2m} \) of the general potential. We will look at deviations of the gravitational potential from an ellipsoid defined by the \( a_0, a_2, \) and \( b_2 \) coefficients. In this model, the quadrupole of the ellipsoid is defined by

\[
a_{20}^2 = \frac{3}{a_0} \left( a_2 \cos 2\theta + b_2 \sin 2\theta \right),
\]
for axis ratio \( q \) and major axis orientation \( \theta_L = 0.5\tan^{-1}(b_2 / a_2) \). The remainder of the coefficients \( a_{2m}^2 \) for \( m \geq 2 \) of the ellipsoidal model are defined by the standard multipole expansion of equation (3). We describe the rest of the model by the deviations \( \Delta a_m \) and \( \Delta b_m \) (\( m \geq 3 \)) from the ellipsoid. Thus, our overall model for the primary lens is

\[
F(\theta) = a_0 + a_2 \cos 2\theta + b_2 \sin 2\theta + \sum_{m=2}^{\infty} \frac{a_0}{1 - 4m^2} a_{2m}^2 \cos 2m\theta + \sum_{m=3}^{\infty} \left[ \Delta a_m \cos m\theta + \Delta b_m \sin m\theta \right],
\]
where \( a_0, a_2, b_2, \Delta a_m \) and \( \Delta b_m \) are model parameters. The deviations in the mass density from the ellipsoid are \( \Delta a_m = (1 - m^2) \Delta a_m / a_0 \) and \( \Delta b_m = (1 - m^2) \Delta b_m / a_0 \) for \( m \geq 3 \) where \( a_m \) and \( b_m \) are the standard coefficients that describe the deviations of the isophotes of elliptical galaxies from ellipsoids (e.g., Bender & Möllenhoff 1987; Bender & Möllenhoff 1989). Since the lens galaxy is fairly round \( q_L \approx 1 \), we expand the ellipsoid to order \( m = 5 \) and usually consider deviations for \( m = 3 \) to \( m = 5 \) beyond which the coefficients \( a_{2m} \) are negligible and the perturbations are well below the noise.

### 3.2. The External Perturber

When modeling gravitational external lens systems, it is important to include an independent external shear that can be misaligned with the light distribution (Keeton et al. 1997). In the case of PG1115+080, the dominant tidal perturbations are associated with the group to which the lens galaxy belongs (e.g., Impy et al. 1998). The group is composed of two luminous spirals and several smaller galaxies with a luminosity-weighted centroid \( \sim 20'' \) from the lens. Since the higher order perturbations of the group beyond an external shear are quantitatively important, we need to generalize the models of Kochanek et al. 2001 to include these higher order terms.

We model the group of galaxies as a singular isothermal sphere (SIS), \( \phi_{\text{ext}} = b_g |r - r_g| \), where \( b_g \) is the Einstein radius of the group and \( r_g \) is the position vector of the group relative to the lens center. If the group is far away from the lens in the sense that \( b_g / r_g \ll 1 \), then we need to include only the quadratic terms in an expansion of the potential,

\[
\phi_{\text{ext}}(r, \theta) \sim -\frac{1}{4} b_g r_g^2 - \frac{1}{4} b_g r_g^2 \cos 2(\theta - \theta_g) + O \left( \frac{b_g}{r_g} \right).
\]

The first term provides the mass sheet degeneracy and affects only the time delays (Falco et al. 1985; Gorenstein et al. 1988). The second term is the tidal shear, \( \gamma = b_g / 2r_g \approx 0.1 \) that is the dominant external perturbation (Schechter et al. 1997). However, in PG1115+080 the higher order terms are significant, so we must use a more complex model. We approximate the potential using only linear and quadratic terms in radial dependence,

\[
\phi_{\text{ext}}(r, \theta) \approx r \sum_{m=1}^{\infty} a_{2m}^0 \cos m(\theta - \theta_g) + r^2 \sum_{m=1}^{\infty} b_{2m}^0 \cos m(\theta - \theta_g)
\]

\[
\equiv r F_{\text{ext}}(\theta) + r^2 G_{\text{ext}}(\theta),
\]

because it has the advantage that the structure of the Einstein ring can be analytically calculated even when the potential has arbitrary angular structure. The quadratic term of the potential \( G_{\text{ext}}(\theta) \) is added to yield a better approximation to the radial deflections of a true SIS model. We determined the coefficients \( a_{2m}^0, b_{2m}^0 \) for the external perturber by minimizing the difference in the deflection of our approximation from that of a true SIS near the Einstein ring of PG1115+080. We need to expand \( \phi_{\text{ext}} \) only for \( m \leq 3 \), and the rms deflection difference between the resulting model and a true SIS model at \( r_g \approx 10'' \) from the lens is only 0.00005 over a 1'' annulus encompassing the ring. This is significantly less than our smallest astrometric errors (0.00055).

### 3.3. Einstein Ring Models

We model the Einstein ring using an extension of the theory developed by Kochanek et al. 2001 for our more general potential, \( \phi(r, \theta) = r \left[ F(\theta) + F_{\text{ext}}(\theta) \right] + r^2 G_{\text{ext}}(\theta) \). When the source is extended, the tangentially stretched images of the source form an Einstein ring. If we assume that the source has a monotonically decreasing surface brightness, the Einstein ring radius relative to the lens is simply

\[
r(\theta) = (F + F_{\text{ext}}) e_\theta - (F + F_{\text{ext}}) e_\theta + \sqrt{(F + F_{\text{ext}}) e_\theta - (F + F_{\text{ext}}) e_\theta - (M^{-1}) e_\theta},
\]

where \( e_\theta = (F + F_{\text{ext}}) e_\theta - (F + F_{\text{ext}}) e_\theta \), \( e_\theta = (1 - 2G_{\text{ext}}) e_\theta - M^{-1} e_\theta \), \( M^{-1} \) is the inverse magnification tensor, the source center is \( u_0 \), and \( S \) is the two-dimensional shape tensor of the source (see Kochanek et al. 2001 for the details). The only relevant parameters of the source are its axis ratio \( q_s \) and the position angle of its major axis \( \theta_s \).

### 3.4. Model Fitting

We use a simple \( \chi^2 \) statistic for the goodness of fit of a model described by parameters \( P \). The astrometric constraints consist of the observed Einstein ring radius \( r(\theta) \) and the four source positions \( u_i \). We divide the estimate into the goodness of fit to the astrometric constraints \( \chi^2_{\text{ast}} \) and the flux constraints \( \chi^2_{\text{flux}} \). The fit statistic for the astrometric constraints is

\[
\chi^2_{\text{ast}}(P) = \sum_{i=1}^{\text{Nimg}} \frac{[u_i - u_i(P)] M_i}{\sigma_i^2} + \sum_{i=1}^{\text{Nimg}} \frac{[r(\theta_i) - r(\theta_i, P)]^2}{\sigma_i^2},
\]

where \( M_i \) is the magnification of image \( i \), and the \( \sigma_i^2 \)’s are the uncertainties in either the image positions or the Einstein ring radii. The fit statistic for images is approximately correct because of the magnification weightings – the need to analyze models quickly in a large parameter space precluded using the exact image plane fit statistic. We can also constrain the
We ensure that we have found a gen-
parameter uncertainties using Levenberg-Marquardt method.
standard ring from obs1 extracted in three different positions
extracted from subsamples of the data as a check for systematic
both ellipsoidal and non-ellipsoidal models.

on a range of synthetic lenses with Einstein rings including
maximum with the downhill simplex method. We tested the code
or the parameters for the deviations from ellipsoidal symmetry
have estimated the goodness of fit for this model, we can add
potential
from the four individual orbits of obs2 (obs2-A,B,C,D). In each
case the lens was re-modeled as part of the extraction pro-
cedure. Unfortunately, the rotation of the PSF between the
two observations is negligible (see §2), so comparisons of the
data sets provide little constraint on systematic errors arising
from the PSF. Figure 2 compares the results for these seven
data sets for a range of variables using the full non-ellipsoidal
models. The main point to note is that the parameter estima-
tions are mutually consistent. Also note that the distributions
of the parameter estimates are consistent with the formal sta-
tistical uncertainties, except for the 5th order deviations where
they are smaller. For the remainder of the paper we consider
only the combined data.

4. RESULTS

In this section we address three issues. First, is the lens
consistent with an ellipsoid when we fit only the astrometric
constraints? Second, is the center of mass consistent with the
center of light? Third, are any of these conclusions changed
if we try to fit the flux ratios as well?

4.1. Is the Lens an Ellipsoid?

We start by fitting only the astrometric constraints, first
with a purely ellipsoidal model, and then adding deviations.
The results are presented in Table 1. An ellipsoidal model
with a very round lens ($q_L \sim 1$), a group located at a po-
sition angle of $\theta_s \sim -113^\circ$ and at a distance of $r_e \sim 11''$
provides a reasonable fit to our data and is consistent with
the earlier models using a singular isothermal ellipsoid for the
lens (Schechter et al. 1997; Keeton & Kochanek 1997).

model using the image fluxes, although fluxes can be contam-
inated by systematic errors, dust extinction, microlensing by
stars, or substructure. For parity-signed fluxes $f_i$, we add

$$\chi^2_{\mathrm{flux}}(\{p_i, f_i\}) = \sum_i^{N_{\mathrm{img}}} \frac{|f_i - M_i(p) f_i|^2}{\sigma^2_{\mathrm{flux}}},$$  \hspace{1cm} (11)

where the source flux $f_i$ is an additional free parameter of the
fit.

We start by fitting the astrometric constraints with a fiducial
ellipsoidal model that has 12 free parameters; the source po-
sition is $(u, v, u_0$, the axis ratio $q_L$ and major-axis position
angle $\theta_s$ of the source, the shear amplitude and the position
of the external perturber $\{\gamma, r_\theta, \theta_p\}$, the ellipsoidal lens po-
tential $\{a_0, a_2, b_2\}$, and the lens center is $(x_L, y_L)$. Once we
have estimated the goodness of fit for this model, we can add
the parameters for the deviations from ellipsoidal symmetry
or $\chi^2_{\mathrm{flux}}$, and examine the change in the goodness of fit and the
parameters.

We minimize the $\chi^2$ of a given model and estimate the
parameter uncertainties using Levenberg-Marquardt method
(e.g., Press et al. 1992). We ensure that we have found a genu-
ine $\chi^2$ minimum by repeating the minimization with a range
of initial parameters and by checking the existence of the min-
imum with the downhill simplex method. We tested the code
on a range of synthetic lenses with Einstein rings including
both ellipsoidal and non-ellipsoidal models.

As a last step before presenting our results, we fit rings ex-
tracted from subsamples of the data as a check for systematic
errors, and the level of our random errors. We consider the
standard ring from obs1 extracted in three different positions
relative to the lens galaxy (obs1-ABC) and the rings extracted
from the four individual orbits of obs2 (obs2-ABCD). In each

FIG. 2.— A check for systematic errors by comparing the results for the rings extracted from the data subsamples listed in the top left panel and discussed in §2. For illustration, we show the shear and PA of the group (top left), the lens galaxy position (top right), the quadrupole moment of the dens-
ty distribution (lower left) and the 50 moments (lower right). The formal uncertainties in parameters are only shown for the combined data.
variables significantly improves the models. In many cases, significantly improve the fit, and we find that none of the new non-ellipsoidal model in Figure 3, and we present their final constraints.

The total degrees of freedom means that we have somewhat overfit the potential are calculated in Cartesian coordinates in which $x$-direction is toward West. The errors are shown in parentheses.

We summarize the resulting $\chi^2$ of each model in Table 1. We use the F-test to estimate whether the new parameters significantly improve the fit, and we find that none of the new variables significantly improves the models. In many cases, adding the new variables even raises the $\chi^2$ per degree of freedom. All the best fit non-ellipsoidal models are consistent with the best fit ellipsoidal model. In short, the constraints on the angular structure from the Einstein ring force the lens of PG1115+080 to be consistent with an ellipsoidal density. The isophotal deviations of the best fit non-ellipsoidal models are $\sim 10^{-3}$ for both $a_m^n$ and $b_m^n$ ($m \leq 5$) somewhat smaller than the values of $a_0^0 / a_0 = 0.009$, $a_0^3 / a_0 = -0.004$ for $H$-band, and $a_1^3 / a_0 = -0.015$, $a_1^3 / a_0 = -0.004$ for $I$-band found by Evans & Witt (2003).

In our first models, we constrained the center of mass to agree with the observed lens position and its formal uncertainties of $0.003$. Evans & Witt (2003) were concerned that the center of light and mass may differ, and allowed the lens center enormous freedom to move relative to the observed position ($0.005$). For our next experiment, we drop the constraint on the lens position and re-optimize the models. We find a lens position of $(-0.001 \pm 0.002, -0.001 \pm 0.002)$ compared to the measured position of $(0.000 \pm 0.003, 0.000 \pm 0.003)$ or a net difference of $0.3\sigma$ for the RA and $0.4\sigma$ for the Dec. The center of mass of the lens is essentially identical to the center of light. The observed lens galaxy of PG1115+080 has an axis ratio $q \simeq 0.9$ and a position angle $\theta \simeq -70^\circ$ (Impey et al. 1998; Falco et al. 2000). The best-fit axis ratio $q_L$ of the mass is rounder than the light while the position angle $\theta_q$ is aligned with the light for both the elliptical and non-elliptical models.

### Table 1

| Parameter | Astrometry Only | Astrometry+Flux | Non-Ellipsoidal Model |
|-----------|----------------|----------------|-----------------------|
| $q_L$     | 0.69(02)       | 0.69(02)       | 0.70(02)              |
| $\theta_q$ (degrees) | -32.0(2.7) | -30.9(2.6) | -33.4(4.6) |
| $\gamma$  | 0.116(01)      | 0.116(01)      | 0.112(03)              |
| $r_e$ (arcsec) | 10.8(1.4)   | 12.4(1.8)     | 16.9(12.5)             |
| $\theta_3$ (degrees) | -113(00)     | -116(00)      | -113(01)               |
| $\chi_L$ (arcsec) | 0.001(02)    | 0.005(02)     | -0.003(03)             |
| $\chi_L$ (arcsec) | 0.001(01)    | 0.001(01)     | -0.002(03)             |
| $q_1$     | 0.97(02)       | 0.98(01)       | 0.96(03)               |
| $\theta_2$ (degrees) | -66.1(7.8)  | 68.4(9.0)     | -52.4(9.2)             |
| $a_0^0$   | 1.15(00)       | 1.15(00)       | 1.15(01)               |
| $10^3 a_2$ | 4.1(2.0)      | 3.2(0.9)      | 2.2(2.7)               |
| $10^3 b_2$ | 4.5(1.1)      | 3.0(0.6)      | 2.9(2.7)               |
| $10^3 \Delta a_3$ | 0       | 0           | -1.4(4.6)              |
| $10^3 \Delta b_3$ | 0       | 0           | 0.8(7.2)               |
| $10^3 \Delta a_4$ | 0       | 0           | -0.1(4.6)              |
| $10^3 \Delta b_4$ | 0       | 0           | 0.3(5.7)               |
| $10^3 \Delta a_5$ | 0       | 0           | -0.2(1.9)              |
| $10^3 \Delta b_5$ | 0       | 0           | 0.3(1.1)               |
| $\chi^2_{\text{ING}}$ | 35.6     | 38.1         | 29.5                  |
| $\chi^2_{\text{IMAGE}}$ | 0.2      | 5.4          | 0.1                   |
| $\chi^2_{\text{ENS}}$ | 0.1      | 2.8          | 1.6                   |
| $\chi^2_{\text{FLUX}}$ | -       | 45.4         | -                     |
| $\chi^2_{\text{EST}}$ | 36.0     | 91.7         | 31.1                  |

**Note.** The best fit ellipsoidal and non-ellipsoidal ($m = 5$) models of PG1115+080, fitting either only the astrometric constraints or the astrometric constraints and the observed flux ratios. All the angles are standard position angles while the lens position and the coefficients of the lens potential are calculated in Cartesian coordinates in which x-direction is toward West. The errors are shown in parentheses.

Courbin et al. 1997, Impey et al. 1998, Zhao & Pr"{o}nقط 2001, Kochanek & Dalal 2004. The total $\chi^2_{\text{ING}} \simeq 36$ for 69 degrees of freedom means that we have somewhat overfit the data and/or overestimated the uncertainties by 38%. Figure 4 shows the best fit ellipsoidal model using only the astrometric constraints.

Next, we fit the data using a sequence of non-ellipsoidal models to investigate how much the lens can deviate from ellipsoidal. We did this by sequentially including higher order poles starting with $m = 3 \ (\Delta a_3, \Delta b_3)$, then $m = 4$ and finally $m = 5$. For comparison with the ellipsoidal model, we show the critical line and the isodensity curve of the best fit $m = 5$ non-ellipsoidal model in Figure 5 and we present their final parameters in Table 1. It is particularly interesting to note that all the higher order deviations are consistent with zero.

We summarize the resulting $\chi^2$ of each model in Table 2. We use the F-test to estimate whether the new parameters significantly improve the fit, and we find that none of the new variables significantly improves the models. In many cases, adding the new variables even raises the $\chi^2$ per degree of freedom. All the best fit non-ellipsoidal models are consistent with the best fit ellipsoidal model. In short, the constraints on the angular structure from the Einstein ring force the lens of PG1115+080 to be consistent with an ellipsoidal density. The isophotal deviations of the best fit non-ellipsoidal models are $\sim 10^{-3}$ for both $a_m^n$ and $b_m^n$ ($m \leq 5$) somewhat smaller than the values of $a_0^0 / a_0 = 0.009$, $a_0^3 / a_0 = -0.004$ for $H$-band, and $a_1^3 / a_0 = -0.015$, $a_1^3 / a_0 = -0.004$ for $I$-band found by Evans & Witt (2003).

In our first models, we constrained the center of mass to agree with the observed lens position and its formal uncertainties of $0.003$. Evans & Witt (2003) were concerned that the center of light and mass may differ, and allowed the lens center enormous freedom to move relative to the observed position ($0.005$). For our next experiment, we drop the constraint on the lens position and re-optimize the models. We find a lens position of $(-0.001 \pm 0.002, -0.001 \pm 0.002)$ compared to the measured position of $(0.000 \pm 0.003, 0.000 \pm 0.003)$ or a net difference of $0.3\sigma$ for the RA and $0.4\sigma$ for the Dec. The center of mass of the lens is essentially identical to the center of light. The observed lens galaxy of PG1115+080 has an axis ratio $q \simeq 0.9$ and a position angle $\theta \simeq -70^\circ$ (Impey et al. 1998; Falco et al. 2000). The best-fit axis ratio $q_L$ of the mass is rounder than the light while the position angle $\theta_q$ is aligned with the light for both the elliptical and non-elliptical models.

### 4.2. The Flux Ratio Anomalies

Because we believe the anomalous flux ratios are due to substructure rather than a problem in the lens model, we have so far neglected the flux ratios as a model constraint. We now investigate the changes in the lens potential when the flux constraints are added. The best fit parameters of an ellipsoidal model with flux constraints are also shown in Table 1. There is little change from the parameters of the ellipsoidal model based only on the astrometric constraints. As expected, most of the contribution to the fit statistic $\chi^2_{\text{ensemble}}$ comes from the merging A1/A2 image pair showing the well-known flux anomaly. To reproduce the observed flux ratios, the critical line should be either distorted or moved closer to the bright-
We modeled the lensed quasar images and the Einstein ring formed from its host galaxy in the lens PG1115+080 using a scale-free potential with arbitrary angular structure. The best fit ellipsoidal model is consistent with previous results, and provides a reasonable fit to the astrometric constraints of both the Einstein ring and the images. Non-ellipsoidal models are constructed by adding higher-order deviations from the ellipsoidal model, but none of the additional degrees of freedom improves the models. The best-fit non-ellipsoidal models are still consistent with the best fit ellipsoidal model. We also find that the center of mass of the lens is consistent with the measured center of light even when this is not imposed as a constraint.

When we try to fit the fluxes as well, including the anomalous A1/A2 flux ratio, the best-fit non-ellipsoidal models still fail to match the observed flux ratios and are still consistent with the ellipsoidal model. We conclude that the suggestions that complex angular structure in the lens galaxy can explain the anomalous flux ratio in PG1115+080 are incorrect – the galaxy is indistinguishable from an ellipsoid. This result does not address the problem of whether the anomaly is due to microlensing or satellites and whether the source of substructure is in the lens or along the line of sight. Since Einstein rings are relatively common in H-band HST observations of lenses, we should be able to test the ellipsoidal hypothesis in many additional systems.

5. SUMMARY

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REFERENCES

Bender, R., & Möllenhoff, C. 1987, A&A, 177, 71
Bender, R., Surma, P., Döbereiner, S., Möllenhoff, C., & Madejsky, R. 1989, A&A, 217, 35
Blandford, R., & Narayan, R. 1992, ARAA, 30, 311
Bullock, J. S., Kravtsov, A. V., & Weinberg, D. H. 2000, ApJ, 539, 517
Chen, J., Kravtsov, A. V., & Keeton, C. R. 2003, ApJ, 592, 24
Chiba, M. 2002, ApJ, 565, 17
Courbin, F., Magain, P., Keeton, C. S., Koehl, C. S., Vanderriekt, C., Jaunsen, A. O., & Hjorth, J. 1997, A&A, 324, 1
Dalal, N., & Koehl, C. S. 2002, ApJ, 572, 25
Evans, N. W., & Witt, H. J. 2001, MNRAS, 327, 1260
Evans, N. W., & Witt, H. J. 2003, MNRAS, 345, 1351
Falco, E. E., Gorenstein, M. V., & Shapiro, I. I. 1985, ApJ, 289, L1
Falco, E. E., et al. 2000, in Gravitational Lensing: Recent Progress and Future Goals, ed. T. G. Brainerd & C. S. Koehl (San Francisco: ASP)
Gorenstein, M. V., Falco, E. E., & Shapiro, I. I. 1988, ApJ, 327, 693
Impy, C. D., Falco, E. E., Koehl, C. S., Lehár, J., McLeod, B. A., Rix, H.-W., Peng, C. Y., & Keeton, C. R. 1998, ApJ, 509, 551
Keeton, C. R. 2003, ApJ, 584, 664
Keeton, C. S., & Koehl, C. S. 1997, ApJ, 487, 42
Keeton, C. S., Koehl, C. S., & Seljak, U. 1997, ApJ, 482, 604
Klypin, A., Kravtsov, A. V., Valenzuela, O., & Prada, F. 1999, ApJ, 522, 82
Kochanek, C. S., & Dalal, N. 2004, ApJ, 610, 69
Kochanek, C. S., Keeton, C. R., & McLeod, B. A. 2001, ApJ, 547, 50
Kochanek, C. S., Schneider, P., & Wambsganss, J. 2004, Gravitational Lensing: Strong, Weak & Micro, Proceedings of the 33rd Saas-Fee Advanced Course, G. Meylan, P. Jetzer & P. North, eds. (Springer-Verlag: Berlin, astro-ph/0407232)
Lehár, J., et al. 2000, ApJ, 536, 584
McLeod, B. A. 1997, in 1997 HST Calibration Workshop, ed. S. Casertano et al., Space Telescope Institute, 281
Metcalfe, R. B. 2002, ApJ, 580, 696
Metcalfe, R. B., & Zhao H.-S. 2002, ApJ, 567, L5
Möller, O., Natarajan, P., Kneib, J.-P., & Blain, A. W. 2002, ApJ, 573, 562
Moore, B., Ghigna, S., Governato, F., Lake, G., Quinn, T., Stadel, J., & Tozzi, P. 1999, ApJ, 524, L19
Norberg, P., et al. 2001, MNRAS, 328, 64
Press, W. H., Flannery, B. P., Teukolsky, S. A., & Vetterling, W. T. 1992, Numerical Recipes (Cambridge: Cambridge Univ. Press)
Rees, M. J., & Ostriker, J. P. 1977, MNRAS, 179, 541
Rusin, D., & Kochanek, C. S. 2004, ApJ, submitted (astro-ph/0412001)
Schechter, P. L., et al. 1997, ApJ, 475, L85
Schechter, P. L., & Wambsganss, J. 2002, ApJ, 580, 685
Schneider, P., Ehlers, J., & Falco, E. E. 1992, Gravitational Lenses, (Berlin: Springer-Verlag)
Schneider, P., & Mao, S. 1998, MNRAS, 295, 587
Spergel, D. N., et al. 2003, ApJS, 148, 175
Treu, T., & Koopmans, L. V. E. 2002, ApJ, 575, 87
Weymann, R. J., et al. 1980, Nature, 285, 641
White, S. D. M., & Rees, M. J. 1978, MNRAS, 183, 341

TABLE 2

| Model (m) | N_par | dof | \(\chi^2_{df}\) | P(%) | N_par | dof | \(\chi^2_{df}\) | P(%) |
|----------|-------|-----|-------------|------|-------|-----|-------------|------|
| 0        | 12    | 69  | 0.522       | 100  | 13    | 72  | 1.273       | 100  |
| 3        | 14    | 67  | 0.474       | 69.1 | 15    | 70  | 1.310       | 90.5 |
| 4        | 16    | 65  | 0.488       | 78.7 | 17    | 68  | 1.301       | 92.7 |
| 5        | 18    | 63  | 0.493       | 82.0 | 19    | 66  | 1.285       | 96.6 |

Note: — Using astrometric constraints only, models with higher order deviations (Am) for order cos\(m\theta\) are compared to the fiducial ellipsoidal model (A0). When flux constraints are added, models (fm) for order cos\(m\theta\) are compared to F0. The dof column gives the degrees of freedom and the \(\chi^2_{df}\) column gives the \(\chi^2\) per degree of freedom. The probability P gives the F-test probability that a given non-ellipsoidal model is consistent with ellipsoidal. A non-ellipsoidal model with lower P implies a more significant improvement relative to the ellipsoidal model– none of the improvements is significant.

est image so that the flux ratio becomes higher than predicted by ellipsoidal models. In the Evans & Witt (2003) models, this was possible because the quasar, a lens positions could be shifted by far more than their actual uncertainties. However, with the correct QSO uncertainties and the Einstein ring constraints, especially on the angular structure of the lens potential, the best fit model with the flux constraints largely coincides with that using the astrometric constraints only.

We then added the higher order deviations from an ellipsoid and fit the data with non-ellipsoidal models. Since the magnitude, \(M \propto F'' \propto m^2 a_m\), is more sensitive to the higher order multipoles, the best-fit non-ellipsoidal models produce a better fit to the flux ratios, but it is still impossible to significantly improve flux ratios while simultaneously maintaining a good fit to the Einstein ring. The best fit model with flux constraints is still consistent with the ellipsoidal model. The fit statistics and the F-test results are shown again in Table 2.

Note that the major axis of the mass is misaligned with the light when we use the flux ratios as a model constraint.
Witt, H. J., Mao, S., & Keeton, C. R. 2000, ApJ, 544, 98
Woźniak, P. R, Alard, C., Udalski, A., Szymański, M., Kubiak, M., Pietrzyński, G., & Zebrun, K. 2000, ApJ, 529, 88
Wucknitz, O. 2002, MNRAS, 332, 951
York, D., et al. 2000, AJ, 120, 1579
Zhao, H. S., & Pronk, D. 2001, MNRAS, 320, 401