Soft Leptogenesis

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Abstract

We study “soft leptogenesis”, a new mechanism of leptogenesis which does not require flavour mixing among the right-handed neutrinos. Supersymmetry soft-breaking terms give a small mass splitting between the CP-even and CP-odd right-handed sneutrino states of a single generation and provide a CP-violating phase sufficient to generate a lepton asymmetry. The mechanism is successful if the lepton-violating soft bilinear coupling is unconventionally (but not unnaturally) small. The values of the right-handed neutrino masses predicted by soft leptogenesis can be low enough to evade the cosmological gravitino problem.
1 Introduction

After the experimental confirmation of neutrino oscillations, leptogenesis [1] has become the most economical and attractive scenario to explain the cosmic baryon asymmetry. Within a range of neutrino mass and mixing parameters compatible with experimental data, it successfully reproduces the value $n_B/s = (0.87 \pm 0.04) \times 10^{-10}$ derived from nucleosynthesis and CMB measurements. The see-saw mechanism [2] employed in leptogenesis requires the existence of right-handed neutrinos with masses close to the GUT scale. Since both the stability of the GUT mass hierarchy and gauge coupling unification strongly suggest low-energy supersymmetry, leptogenesis is more natural in a supersymmetric framework. Once supersymmetry is introduced, sneutrino decays offer a new channel for generating an asymmetry.

In this paper we want to discuss how the sneutrino decay channel is fundamentally different than the neutrino channel. Supersymmetry-breaking terms remove the mass degeneracy between the two real sneutrino states belonging to the supermultiplet of a single neutrino generation [3]. They also provide a source of CP violation, and the mixing between the two sneutrino states can generate a CP asymmetry in the decay. Although the scale of supersymmetry-breaking is much smaller than the right-handed neutrino mass, the asymmetry can be sizable because of the resonant effect [4, 5] of the two nearly-degenerate states. Contrary to leptogenesis from neutrino decay, where at least two generations of right-handed neutrinos are required, a single-generation right-handed sneutrino decay is sufficient to generate the CP asymmetry. The soft terms, and not flavour physics, provide the necessary mass splitting and CP-violating phase. This new mechanism of leptogenesis, which we will call “soft leptogenesis” can then be an alternative or an addition to the traditional scenario of mixing between different flavour states.

This paper is organized as follows. In sect. 2 we describe the one-generation see-saw model in presence of supersymmetry-breaking effects and compute the relevant CP asymmetry. In sect. 3 we rederive the asymmetry following a different field-theoretical approach, and comment on the effect of the initial-state coherence. The baryon-asymmetry efficiency factor is computed in sect. 4 by integrating the complete Boltzmann equations. Finally our results for the baryon asymmetry are presented and discussed in sect. 5.

As we were completing this work, a paper has appeared [6] presenting the same idea.

2 The CP Asymmetry

The supersymmetric see-saw model is described by the superpotential

$$W = Y_{ij} N_i L_j H + \frac{1}{2} M_{ij} N_i N_j,$$

(1)

where $i, j = 1, 2, 3$ are flavour indices and $N_i, L_i, H$ are the chiral superfields for the right-handed neutrinos, the left-handed lepton doublets and the Higgs, respectively. The
supersymmetry-breaking terms involving the right-handed sneutrinos \( \tilde{N}_i \) are

\[
- \mathcal{L}_{soft} = \tilde{m}_{ij}^2 \tilde{N}_i^\dagger \tilde{N}_j + \left( A_{ij} Y_{ij} \tilde{N}_i \ell_j H + \frac{1}{2} B_{ij} M_{ij} \tilde{N}_i \tilde{N}_j + \text{h.c.} \right),
\]

(2)

with standard notations.

We will consider a single generation of \( N \) because, as explained in the introduction, our effect survives even in this limiting case. For simplicity, we will also assume proportionality of soft trilinear terms and drop the flavour index for the coefficient \( A \). Under these conditions, a CP-violating phase is still present. Indeed, with a superfield rotation we can eliminate all phases from the superpotential parameters \( Y_{1i} \) and \( M \) (\( \equiv M_{11} \)), and with an \( R \)-rotation we can eliminate the relative phase between \( A \) and \( B \). However, the remaining phase is physical.

The right-handed neutrino \( N \) has a mass \( M \), while sneutrino and antisneutrino states mix in the mass matrix. Their mass eigenvectors

\[
\tilde{N}_+ = \frac{1}{\sqrt{2}} \left( e^{i\Phi/2} \tilde{N} + e^{-i\Phi/2} \tilde{N}^\dagger \right),
\]

\[
\tilde{N}_- = \frac{-i}{\sqrt{2}} \left( e^{i\Phi/2} \tilde{N} - e^{-i\Phi/2} \tilde{N}^\dagger \right),
\]

(3)

with \( \Phi \equiv \arg(BM) \), have mass eigenvalues

\[
M^2_\pm = M^2 + \tilde{m}^2 \pm |BM|.
\]

The sneutrino interaction Lagrangian in the basis of flavour \((\tilde{N}, \tilde{N}^\dagger)\) and mass \((\tilde{N}_+, \tilde{N}_-)\) eigenstates is, respectively,

\[
- \mathcal{L}_{int} = \tilde{N} \left( Y_{1i} \tilde{H} \ell_i^\dagger L + MY_{1i}^* \tilde{\ell}_i H^* + AY_{1i} \tilde{\ell}_i H \right) + \text{h.c.}
\]

\[
= \frac{Y_{1i}}{\sqrt{2}} \tilde{N}_+ \left[ \tilde{H} \ell_i^\dagger L + (A + M) \tilde{\ell}_i H \right] + i \frac{Y_{1i}}{\sqrt{2}} \tilde{N}_- \left[ \tilde{H} \ell_i^\dagger L + (A - M) \tilde{\ell}_i H \right] + \text{h.c.}
\]

(5)

Here, for simplicity, we have set \( \Phi = 0 \) choosing, from now on, a basis where \( A \) is the only complex parameter.

The system of \( \tilde{N} - \tilde{N}^\dagger \) is completely analogous to the \( K^0 - \bar{K}^0 \) or \( B^0 - \bar{B}^0 \) system, and in this section we will treat it with the same formalism (see e.g. ref. [7]). Its evolution is determined (in the non-relativistic limit) by the Hamiltonian \( H = \hat{M} - i\hat{\Gamma}/2 \) where, at leading order in the soft terms,

\[
\hat{M} = M \begin{pmatrix} 1 & B \frac{B}{2M} \\ B \frac{B}{2M} & 1 \end{pmatrix},
\]

\[
\hat{\Gamma} = \Gamma \begin{pmatrix} 1 & A^* \frac{A}{M} \\ A^* \frac{A}{M} & 1 \end{pmatrix}.
\]

(7)

(8)
Here $\Gamma$ is the total $\tilde{N}$ decay width

$$\Gamma = \frac{(YY^\dagger)^{11}}{4\pi} M \equiv \frac{G_F}{\sqrt{2}} m M^2. \quad (9)$$

With this (standard) definition, $m = (YY^\dagger)^{11}(H)^2/M$ sets the scale for the physical (mainly left-handed) neutrino masses $m_\nu$, since $m = \sum_i |r_i|^2 m_i^\nu$, under the condition $\sum_i r_i^2 = 1$.

The eigenvectors of the Hamiltonian $H$ are

$$\tilde{N}_L = p\tilde{N} + q\tilde{N}^\dagger$$

$$\tilde{N}_H = p\tilde{N} - q\tilde{N}^\dagger,$$

$$(\frac{q}{p})^2 = \frac{\hat{M}_{12}^* - i\hat{\Gamma}_{12}^*}{\hat{M}_{12} + i\hat{\Gamma}_{12}}. \quad (10)$$

We consider an initial state at $t = 0$ with equal densities of $\tilde{N}$ and $\tilde{N}^\dagger$. At time $t$, the state has evolved into

$$\tilde{N}(t) = g_+(t)\tilde{N}(0) + \frac{q}{p} g_-(t)\tilde{N}^\dagger(0)$$

$$\tilde{N}^\dagger(t) = \frac{p}{q} g_-(t)\tilde{N}(0) + g_+(t)\tilde{N}^\dagger(0), \quad (12)$$

$$g_+(t) = e^{-iMt} e^{-\Gamma t/2} \cos(\Delta M t/2)$$

$$g_-(t) = ie^{-iMt} e^{-\Gamma t/2} \sin(\Delta M t/2). \quad (13)$$

Here $\Delta M \equiv M_+ - M_- = |B|$ and we have neglected $\Delta \Gamma$ with respect to $\Delta M$.

We can now compute the total integrated lepton asymmetry, defined by

$$\epsilon = \frac{\sum_f \int_0^\infty dt \left[ \Gamma(\tilde{N}(t) \rightarrow f) + \Gamma(\tilde{N}(t)^\dagger \rightarrow f) - \Gamma(\tilde{N}(t) \rightarrow \bar{f}) - \Gamma(\tilde{N}(t)^\dagger \rightarrow \bar{f}) \right]}{\sum_f \int_0^\infty dt \left[ \Gamma(\tilde{N}(t) \rightarrow f) + \Gamma(\tilde{N}(t)^\dagger \rightarrow f) + \Gamma(\tilde{N}(t) \rightarrow \bar{f}) + \Gamma(\tilde{N}(t)^\dagger \rightarrow \bar{f}) \right]} \cdot (14)$$

Here $f$ is a final state with lepton number equal to 1 and $\bar{f}$ is its conjugate. Since we want to exploit the enhancement due to the resonance [4, 5], we will disregard any other subleading effects. In particular, we will neglect direct CP violation in the decay (vertex diagrams) and include only the effect of the $\tilde{N} - \tilde{N}^\dagger$ mixing (wave-function diagrams). This means that the decay amplitudes of the flavour sneutrino eigenstates can be immediately derived from the interaction Lagrangian in eq. (5), setting $A = 0$. We will include the factors $c_F$ and $c_B$ to parametrize the phase space of the fermionic ($f = \tilde{H}\ell$) and bosonic ($f = H\ell$) final states. Taking into account the time dependence described by eq. (12), the CP asymmetry is given by

$$\epsilon = \frac{1}{2} \left( \left| \frac{q}{p} \right|^2 - \left| \frac{p}{q} \right|^2 \right) \left( \frac{c_B - c_F}{c_F + c_B} \right) \int_0^\infty dt \left| g_- \right|^2 \int_0^\infty dt \left( |g_+|^2 + |g_-|^2 \right). \quad (15)$$
Figure 1: $\Delta_{BF}$, defined in eq. (14), as a function of $z = M/T$.

Evaluating eq. (11) in the limit $\hat{\Gamma}_{12} \ll \hat{M}_{12}$, we find

$$\left| \frac{g}{p} \right|^2 \simeq 1 - \text{Im} \frac{\hat{\Gamma}_{12}}{\hat{M}_{12}} = 1 + \frac{2\Gamma \text{ Im} A}{BM},$$

(16)

Performing the time integral

$$\frac{\int_0^\infty dt |g^-|^2}{\int_0^\infty dt (|g^+|^2 + g^-|^2)} = \frac{\Delta M^2}{2 (\Gamma^2 + \Delta M^2)},$$

(17)

we obtain the final expression for the CP asymmetry

$$\epsilon = \frac{\Gamma B \text{ Im} A}{\Gamma^2 + B^2 M} \Delta_{BF},$$

(18)

$$\Delta_{BF} = \frac{c_B - c_F}{c_F + c_B}.$$

(19)

It is easy to understand the origin of the different terms present in eq. (18). The factor $AB$ signals the presence of supersymmetry breaking and the violation of lepton number; $(B/M)\text{ Im} A$ signals CP violation. The resonance effect is described by $\Gamma B/(\Gamma^2 + B^2)$, which is maximal when $\Gamma \sim |B|$. As we move away from the resonance condition, $\epsilon$ suffers an extra power suppression.

An exact cancellation occurs between the asymmetry in the fermionic and bosonic channels, if $c_F = c_B$. Thermal effects, which break supersymmetry, remove this degeneracy. This happens both because of final-state Fermi blocking and Bose stimulation [8], and because of the effective masses acquired by particle excitations inside the plasma (for a full discussion of the thermal effects in leptogenesis, see ref. [9]). We find

$$c_F = (1 - x_{\ell} - x_H) \lambda(1, x_{\ell}, x_H) [1 - n_F(E_{\ell})] [1 - n_F(E_H)],$$

(20)

$$c_B = \lambda(1, x_H, x_{\ell}) [1 + n_B(E_H)] [1 + n_B(E_{\ell})],$$

(21)

$$E_{\ell,H} = \frac{M}{2} (1 + x_{\ell,H} - x_{\ell,H}), \quad E_{H,\ell} = \frac{M}{2} (1 + x_{H,\ell} - x_{H,\ell}),$$

(22)

$$\lambda(1, x, y) = \sqrt{(1 + x - y)^2 - 4x}, \quad x_a \equiv \frac{m_a (T)^2}{M^2},$$

(23)

$$n_F(E) = \frac{1}{e^{E/T} - 1}, \quad n_B(E) = \frac{1}{e^{E/T} + 1},$$

(24)
where the thermal masses for the relevant supersymmetric degrees of freedom are
\begin{align}
m^2_H(T) &= 2m^2_H(T) = \frac{3}{8}g_2^2 + \frac{1}{8}g_Y^2 + \frac{3}{4}\lambda_t^2, \\
m^2_{\tilde{\ell}}(T) &= 2m^2_{\tilde{\ell}}(T) = \frac{3}{8}g_2^2 + \frac{1}{8}g_Y^2.
\end{align}

Here \( g_2 \) and \( g_Y \) are gauge couplings and \( \lambda_t \) is the top Yukawa, renormalized at the appropriate high-energy scale. The value of \( \Delta_{BF} \) as a function of \( z = M/T \) is plotted in fig. 1. Because of Bose stimulation, \( \Delta_{BF} \) is positive and grows with temperature. However, for \( z < 1.2 \), the sum of Higgs and slepton thermal masses becomes larger than \( M \), and the bosonic channel is kinematically closed. Eventually, for \( z < 0.8 \), also the fermionic channel becomes unaccessible. This explains the abrupt changes of \( \Delta_{BF} \) shown in fig. 1.

### 3 Field-Theoretical Approach

In this section we want to study the CP asymmetry using a different procedure. We use an effective field-theory approach of resummed propagators for unstable (mass eigenstate) particles, as described in ref. [5]. The decay amplitude \( \hat{f} \) of the unstable external state \( \tilde{N}_- \) into a final state \( f \) is described by a superposition of amplitudes with stable external states \( f_\pm \). Adding the contributions shown in fig. 2, we obtain
\begin{align}
\hat{f}_-(\tilde{N}_- \to f) &= f_+ - f_- + \frac{i\Pi_{+-}}{M_-^2 - M_+^2 + i\Pi_{++}}, \\
\hat{\bar{f}}_-(\tilde{N}_- \to \bar{f}) &= f_-^* - f_+^* + \frac{i\Pi_{+-}}{M_-^2 - M_+^2 + i\Pi_{++}}.
\end{align}

Squaring the amplitudes and multiplying by the phase-space factors \( c_F \) and \( c_B \), we obtain the asymmetry
\begin{align}
\epsilon_- &= \frac{\sum_f \left[ \Gamma(\tilde{N}_- \to f) - \Gamma(\tilde{N}_- \to \bar{f}) \right]}{\sum_f \left[ \Gamma(\tilde{N}_- \to f) + \Gamma(\tilde{N}_- \to \bar{f}) \right]} \\
&= \frac{2(M_-^2 - M_+^2)}{\sum_f |f_-|^2 (M_-^2 - M_+^2)^2 + |f_- \Pi_{+-} - f_+ \Pi_{++}|^2} \frac{\sum_f \text{Im} (f_-^* f_+) \Pi_{+-} c_f}{\sum_f \text{Im} (f_-^* f_+) \Pi_{+-} c_f}.
\end{align}

The corresponding results for \( \tilde{N}_+ \) are obtained by interchanging the indices + and −.
Figure 2: Interfering $\tilde{N}_-$ decay amplitudes for the fermionic final states. Analogous diagrams exist for bosonic final states. The two-point function $\Pi_{+\text{}}$ denoted by a blob contains a sum of all possible intermediate states.

Neglecting supersymmetry-breaking in vertices, from the interaction Lagrangian in eq. (6) we obtain, up to an overall normalization, $f_+ = 1$, $f_- = -i$ for the scalar-channel final state (Higgs and slepton) and $f_+ = 1$, $f_- = i$ for the fermionic channel (higgsino and lepton). Inserting these values in eq. (31) and combining the asymmetries from $\tilde{N}_-$ and $\tilde{N}_+$, we obtain the final expression for the total CP asymmetry

$$\epsilon = \frac{4\Gamma B}{4\Gamma^2 + \Gamma^2} \frac{\text{Im}A}{M} \Delta_{BF}. $$

(32)

This result agrees with eq. (18) in the limit $\Gamma \ll \Delta M$. When $\Gamma \gg \Delta M$, the two states are not well-separated particles. Therefore, the result for the asymmetry depends on how the initial state is prepared. If sneutrinos (like $K$ and $B$) are produced in current eigenstates and evolve freely (e.g. if produced in inflaton decay out of equilibrium), the formalism followed in sect. 2 gives the correct answer, taking into account the coherence of the initial state. On the other hand, if $\tilde{N}$ are in a thermal bath with a thermalization time $\Gamma^{-1}$ shorter than the oscillation time $\Delta M^{-1}$, coherence is lost and eq. (32) gives a more appropriate description. Therefore in principle we are sensitive to the details of the initial state. In practice, the difference is inessential since we can just recast eq. (18) into eq. (32) with a redefinition of the unknown soft parameters, $A \to 2A$, $B \to 2B$. In the following, we will use eq. (32) in our discussion.

4 Solutions to the Boltzmann Equations

The baryon asymmetry is given by

$$\frac{n_B}{s} = -\left(\frac{24 + 4n_H}{66 + 13n_H}\right) \frac{\epsilon}{\Delta_{BF}} \eta \frac{Y_{\tilde{N}}^{eq}}{\Delta_{BF}}. $$

(33)

The first factor takes into account the reprocessing of the $B-L$ asymmetry by sphaleron transitions, with the number of Higgs doublets $n_H$ equal to 2. $Y_{\tilde{N}}^{eq} = 45\zeta(3)/\pi^4 g_*$ is the sneutrino equilibrium density in units of entropy density, for temperatures much larger than $M$. For the minimal supersymmetric model with one generation of right-handed neutrinos,
the number of effective degrees of freedom is $g_* = 225$. Then, we obtain

$$\frac{n_B}{s} = -8.6 \times 10^{-4} \frac{\epsilon}{\Delta_{BF}} \eta. \tag{34}$$

The efficiency factor $\eta$ describes the effects caused by: $i)$ the sneutrino density being smaller than the equilibrium density, $ii)$ the wash-out from the lack of perfect out-of-equilibrium decay, $iii)$ the temperature-dependence of $\epsilon$ through $\Delta_{BF}$. It is obtained by integrating the relevant Boltzmann equations. We have numerically solved the set of differential equations describing decay, inverse decay, and scattering processes for all supersymmetric particles, including thermal masses for the particles involved [9]. With our definition of $\eta$, the temperature-dependent part $\Delta_{BF}$ has been factored out from $\epsilon$, see eq. (34). We have included in $\Delta_{BF}$ thermal masses and final-state statistical factors, as described by eqs. (20)–(21), but we have neglected thermal corrections to the loop diagram generating the asymmetry (for complete expressions of the thermal corrections, see ref. [9]).

In fig. 3 (left) we plot the absolute value of the efficiency $\eta$ as a function of $m$ for fixed $M = 10^{10}$ GeV. We consider two different initial conditions for $Y_{\tilde{N}}$, the sneutrino density in units of the entropy density. In the first case, we assume that the $\tilde{N}$ population is created by their Yukawa interactions with the thermal plasma, and set $Y_{\tilde{N}}(z \to 0) = 0$. The second case corresponds to an initial $\tilde{N}$ abundance equal to the thermal one, $Y_{\tilde{N}}(z \to 0) = Y^\text{eq}_{\tilde{N}}(z \to 0)$. Here we are assuming that some unspecified high-energy interaction (e.g. GUT couplings) is responsible for bringing the sneutrinos into an equilibrium density at $T \gg M$. In fig. 3 (right) we present isocurves of $|\eta| = 10^i$, $i = -2, -3, -4$ on the $(m, M)$ plane, for the initial condition $Y_{\tilde{N}}(z \to 0) = 0$. This demonstrates that the efficiency is almost independent of $M$, in the range of $M$ that is relevant for us.

Figure 3: Left: Efficiency $|\eta|$ as a function of $m$ for $M = 10^{10}$ GeV and for two different initial conditions: (i) vanishing initial $\tilde{N}$ abundance (solid red curve); (ii) thermal initial $\tilde{N}$ abundance, $Y^\text{eq}_{\tilde{N}}(z \to 0)$ (short-dashed blue curve). Right: isocurves of $|\eta| = 10^i$, $i = -2, -3, -4$ on the $(m, M)$ plane for the case (i).
Figure 4: Evolution of the absolute values of the abundances $|Y_X|$ with $z = M/T$ for $M = 10^{10}$ GeV, $m = 10^{-4}$ eV (left) and $m = 10^{-3}$ eV (right). $Y^\text{eq}_N(z)$ is denoted with short-dashed black line, $Y_N(z)$ by the solid green line, while the red long-dashed line denotes the lepton asymmetry $Y_L(z)/\epsilon_{\text{const}}$ with $\epsilon_{\text{const}} = 10^{-6}$.

The results in Fig. 3 indicate that, because of $\Delta_{\text{BF}}$, there is an extra suppression of the soft-leptogenesis efficiency compared to the standard leptogenesis case. Notice that this suppression occurs also if $Y_N(z \to 0) = Y^\text{eq}_N(z \to 0)$ (dashed line). The smaller $m$, the stronger the suppression, because the out-of-equilibrium decay occurs at lower $T$, where $\Delta_{\text{BF}}$ is smaller, see fig. 1.

In the case $Y_N(z \to 0) = 0$ (solid line) we observe a double-peak structure in $|\eta|$. To understand this behaviour we plot in Fig. 4 the evolution of the abundances with $z$ for $M = 10^{10}$ GeV and $m = 10^{-4}$ eV (left), $m = 10^{-3}$ eV (right). The solid green lines denote $Y_N(z)$ and the red long-dashed lines denote the lepton asymmetries $Y_L(z)/\epsilon_{\text{const}}$, for a fixed arbitrary value $\epsilon_{\text{const}} = 10^{-6}$. For reference, we also plot the equilibrium density $Y^\text{eq}_N(z)$ with the short-dashed black line.

For $z < 1.2$ the fermionic channel of sneutrino (inverse) decay creates an asymmetry. As soon as the bosonic channel is open ($z > 1.2$, see fig. 1), it dominates and the asymmetry flips sign. This is illustrated by the dip of the dashed lines at $z = 1.2$ in fig. 4 (both left and right), but this effect is inconsequential for the final asymmetry. During the $\tilde{N}$-production phase, a wrong-sign asymmetry is generated compared to the right-sign asymmetry produced in $\tilde{N}$ decays. For small $m$ (fig. 4 left) the Yukawa interactions are weak and the decay occurs at late time (small $T$) when $\Delta_{\text{BF}}$ is small. Therefore the generation of the right-sign asymmetry cannot overcome the wrong-sign asymmetry. For larger $m$ (fig. 4 right) the washout of the initial wrong-sign asymmetry is more efficient, and at late time an asymmetry with the right sign is created. This is observed in the right plot of fig. 4 as the additional sign-flip of $Y_L$ (or dip of the long-dashed curve). At an intermediate value of $m$ the two effects perfectly compensate each other, and the final asymmetry vanishes, as shown in fig. 3 (solid line). In the case of an initial thermal $\tilde{N}$ distribution (dashed line in fig. 4) this cancellation never occurs, since the production phase is irrelevant.
5 Discussion of the Results

We now have all the ingredients to discuss the results of the baryon asymmetry generated by the proposed mechanism of soft leptogenesis. The CP asymmetry is maximal when the parameters lie on the resonance condition, \( \Gamma = 2|B| \), where eq. (32) becomes

\[
\frac{\epsilon}{\Delta_{BF}} = \frac{\text{Im} A}{M}.
\] (35)

From eq. (34) and from the results shown in fig. 3, we obtain that the presently observed baryon asymmetry requires

\[
M < \frac{\text{Im} A}{10^{8-9}} \text{ GeV}.
\] (36)

The resonance condition \( \Gamma = 2|B| \) occurs when

\[
M = \left( \frac{10^{-3} \text{ eV}}{m} \right)^{1/2} \left( \frac{B}{100 \text{ GeV}} \right)^{1/2} 10^{10} \text{ GeV}.
\] (37)

For typical values of \( B \) around the electroweak scale, the value of \( M \) in eq. (37) is larger than what is required by eq. (36), and \( n_B/s \) is predicted to be too small. Soft leptogenesis can give a significant contribution to the baryon asymmetry only for very small values of \( B \).

Very low values of \( B \) require that the lepton-violating bilinear soft term should not be generated at the leading order in supersymmetry breaking, but only by some higher-dimensional operators. Let us consider the supersymmetry-breaking spurion superfield \( X = \theta \tilde{m} M_{\text{Pl}} \). Our assumption is that the leading contribution to \( B \), coming from the operator \( \int d^2 \theta X M N^2 / M_{\text{Pl}} \), vanishes. In a general supergravity scenario, this is not the case. One can however envisage dynamical relaxation mechanisms (see e.g. ref. [11]) which set \( B = 0 \) at leading order. Then, \( B \) is determined by the operator in the Kähler potential \( \int d^4 \theta X \tilde{X} N^2 / M_{\text{Pl}}^2 \), which gives a value \( B \sim \tilde{m}^2/M \). The resonance condition \( \Gamma = 2|B| \) in terms of

\[
B_M \equiv \sqrt{BM},
\] (38)

1The proportionality of the trilinear soft terms, assumed here, is certainly a questionable hypothesis and should not be strictly applied. However, the stability of the electroweak vacuum implies a bound on the size of \( A \). Let us consider the see-saw one-generation model along a \( D \)-flat and \( F \)-flat \( (\partial W/\partial N = 0) \) direction

\[
\tilde{t} = \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad H = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad \tilde{N} = -\frac{\lambda \phi^2}{M}.
\]

The scalar potential becomes

\[
V = \frac{2M}{Y^2} \left[ x^3 + \left( \frac{B}{2} - A \right) x^2 + \tilde{m}^2 x \left( 1 + \frac{x}{2M} \right) \right],
\]

where \( x \equiv Y^2 \phi^2/M \) and, for simplicity, we have taken equal soft masses \( \tilde{m} \) for all scalar fields. Minima of the potential occur at \( x = [A - B/2 \pm \sqrt{(A - B/2)^2 - 3\tilde{m}^2}] \). The request that the potential is positive at these minima (to avoid instabilities of the electroweak vacuum) leads to the condition \( |A - B/2| < 2\tilde{m} \). This shows that a departure from proportionality cannot significantly enhance the CP asymmetry, unless we accept to live on metastable vacua.
Figure 5: Regions of \((m,M)\) plane where soft leptogenesis predicts \(n_B/s > 0.83 \times 10^{-10}\) for \(\text{Im}A < \text{TeV}\) and \(B_M = 100 \text{ GeV}\) (dashed line) and 1 TeV (solid line). Soft leptogenesis is successful inside the contours. We have assumed a vanishing (left) or thermal (right) initial sneutrino density.

According to our previous hypothesis, \(B_M\) is the parameter to be taken of the order of the electroweak scale. In this case, the value of \(M\) in eq. (39) is in agreement with eq. (36).

Our hypothesis of a small value of \(B\) is not technically unnatural. Indeed, radiative corrections to the lepton-violating bilinear term are of the form

\[
\delta B \sim (YY^\dagger)_{11} A \ln\left(\Lambda^2/M^2\right)/(16\pi^2) \sim \Gamma A/M,
\]

where \(\Lambda\) is some ultraviolet cutoff scale. Thus, \(\delta B\) is much smaller than the assumed tree-level value \((B \sim \Gamma)\). On the other hand, we stress that it would have been unnatural to choose a very small trilinear coefficient, since \(A\) receives gauge radiative corrections.

In fig. 5 we quantify our results by showing the regions of parameters in the \((m,M)\) plane where soft leptogenesis can predict \(n_B/s = (0.87 \pm 0.04) \times 10^{-10}\) for \(\text{Im}A < \text{TeV}\) and \(B_M\) between 100 GeV (dashed line) and 1 TeV (solid line). Soft leptogenesis is successful in the \((m,M)\) region inside the contours. The two plots (left and right) correspond to vanishing and thermal initial sneutrino density, respectively. There is no overlap between the region of \((m,M)\) parameters favourable for soft leptogenesis with the one suggested by conventional leptogenesis.

The values of \(M\) required by soft leptogenesis (see fig. 5) are smaller than the usual see-saw expectation, and imply very small Yukawa couplings, \(Y < 10^{-4}(M/10^7 \text{ GeV})^{1/2}\). It should be said that soft leptogenesis is more natural in presence of a large mass hierarchy.
of right-handed neutrinos, since one is working in the one-generation limit. Therefore it is not inconsistent to predict that one generation of $N$ lies at a mass scale significant lower than the GUT scale.

This result has interesting consequences for the gravitino problem. In traditional leptogenesis, the mass of the right-handed neutrino is bounded from below $M > 2.4(0.4) \times 10^9$ GeV for vanishing (thermal) initial neutrino densities [12]. Such values of $M$ are often uncomfortably large when compared with the upper bounds on the reheat temperature after inflation $T_{RH} < 10^8-10$, obtained by the requirement that relic gravitinos do not upset the successful predictions of nucleosynthesis [13]. On the other hand, soft leptogenesis needs values of $M$ in the range $10^6-8$ GeV, well within the limits imposed by the gravitino cosmological problem.

In conclusion, we have discussed how soft leptogenesis provides an interesting interplay between lepton-number violating interactions at high energy and low-energy supersymmetry-breaking terms. We have found that soft leptogenesis can explain the observed baryon asymmetry within the range of parameters shown in fig. 5. This requires i) an unconventional (but not unnatural) choice of the lepton-violating bilinear soft parameter, such that $B_M$ in eq. (38) of the order of the electroweak scale; ii) values of $M$ in the range $10^6-8$ GeV, which is favourable to evade the gravitino problem.

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References

[1] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.

[2] M. Gell-Mann, P. Ramond and R. Slansky, Proceedings of the Supergravity Stony Brook Workshop, New York, 1979, eds. P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam); T. Yanagida, Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan 1979 (eds. A. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba); R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[3] Y. Grossman and H. E. Haber, Phys. Rev. Lett. 78 (1997) 3438 [arXiv:hep-ph/9702421].

[4] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384 (1996) 169; [arXiv:hep-ph/9605319]; M. Flanz, E. A. Paschos and U. Sarkar, Phys. Lett. B 345 (1995) 248 [Erratum-ibid. B 382 (1996) 447] [arXiv:hep-ph/9411366].
[5] A. Pilaftsis, Phys. Rev. D 56 (1997) 5431, arXiv:hep-ph/9707235.

[6] Y. Grossman, T. Kashti, Y. Nir and E. Roulet, arXiv:hep-ph/0307081.

[7] Y. Nir, SLAC-PUB-5874 Lectures given at 20th Annual SLAC Summer Institute on Particle Physics: The Third Family and the Physics of Flavor, Stanford, CA, 13-24 Jul 1992.

[8] L. Covi, N. Rius, E. Roulet and F. Vissani, Phys. Rev. D 57 (1998) 93 arXiv:hep-ph/9704366.

[9] G.F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, to appear.

[10] J. A. Harvey and M. S. Turner, Phys. Rev. D 42 (1990) 3344.

[11] M. Yamaguchi and K. Yoshioka, Phys. Lett. B 543 (2002) 189 arXiv:hep-ph/0204293.

[12] S. Davidson and A. Ibarra, Phys. Lett. B 535 (2002) 25 arXiv:hep-ph/0202239.

[13] W. Buchmuller, P. Di Bari and M. Plumacher, Nucl. Phys. B 643 (2002) 367 arXiv:hep-ph/0205349; J. R. Ellis and M. Raidal, Nucl. Phys. B 643 (2002) 229 arXiv:hep-ph/0206174.

[14] For a review, see S. Sarkar, Rept. Prog. Phys. 59 (1996) 1493 arXiv:hep-ph/9602260.