I. INTRODUCTION

One of advantages of studying gravitino cosmology lies in the fact that a gravitino mass ($m_{3/2}$) is directly related to a supersymmetry (SUSY)-breaking scale (\sqrt{F}), i.e. $m_{3/2} \approx F/(\sqrt{3}M_P)$ where $M_P \approx 2.4 \times 10^{18}$GeV. For that reason, understanding effects that gravitinos can potentially have on experimental data can be of great help in studying a SUSY-breaking scale in the Universe. Then, is there any inevitable physical effect induced by gravitinos so that its absence implies exclusion of a certain SUSY-breaking scale?

Inspired by this question, in this work, we give our special attention to how to probe SUSY-breaking scenarios when a gravitino mass is so light that gravitinos are bound to serve as the lightest SUSY particle (LSP). Particularly, we focus on the gravitino mass range $100\text{eV} \lesssim m_{3/2} \lesssim 1\text{keV}$. In this case, gravitinos are expected to exist today as the form of warm dark matter (WDM) with a free-streaming length amounting to $O(0.1)\text{Mpc}$.

Very interestingly, for the sub-keV gravitino, the relic abundance ($\Omega_{3/2} h^2$) is insensitive to the reheating temperature ($T_{\text{RH}}$) \cite{1}, but sensitive to $m_{3/2}$ and the decoupling temperature ($T_{\text{3/2,dec}}$). This means that once the sparticle mass spectrum is fixed, there is a definite prediction for the relic abundance of the gravitino WDM. Given the null observation of any sparticle thus far, it is fair to state that the gravitino decoupling temperature is at least $O(1)\text{TeV}$. Therefore, for $100\text{eV} \lesssim m_{3/2} \lesssim 1\text{keV}$, there exist solid lower bounds on $\Omega_{3/2} h^2 \equiv \omega_{3/2}$ for each $m_{3/2}$ as far as $T_{\text{RH}}$ is larger than sparticle masses.

On the other hand, Lyman-\alpha forest observation and redshifted 21cm signals in EDGES observation give the stringent lower bounds on WDM mass in the case where the whole of DM population consists only of a WDM particle. Constraints from each experiment read $m_{\text{wdm}} > 5.3\text{keV}$ \cite{2} and $m_{\text{wdm}} > 6.1\text{keV}$ \cite{3, 4} respectively. Hence, as is well-known, sub-keV gravitino WDM cannot be responsible for 100% DM population today if the Universe went through PeV scale SUSY-breaking. Then, following this understanding, there arises an immediate and natural question which is “what is a upper bound on $\omega_{3/2}$ for a given $m_{3/2}$ in $[100\text{eV}, 1\text{keV}]$?”

In this work, we address this question by making an estimate for the expected number of dwarf satellite galaxies ($N_{\text{sat}}$) in the Milky Way resulting from the Universe with the mixture of cold dark matter (CDM) and gravitino WDM. Defining the fraction of DM population contributed by the gravitino WDM to be $f_{3/2} \equiv \omega_{3/2}/\omega_{\text{DM}}$, we compute $N_{\text{sat}}$ based on the matter power spectrum resulting from $100(1-f_{3/2})\%$ CDM and $100 f_{3/2}\%$ gravitino WDM. Then by requiring $N_{\text{sat}} \gtrsim 63$, we obtain the maximally allowed $f_{3/2,\text{max}}$ for each $m_{3/2}$.

In accordance with our observation that $f_{3/2,\text{max}} < f_{3/2}$, we argue that the entropy production is requisite for $m_{3/2} \in [100\text{eV}, 1\text{keV}]$. We derive the necessary amount of entropy production for each $m_{3/2}$, and then argue that this entropy production is inevitably imprinted on the primordial (inflationary) gravitational wave (pGW) spectrum $\Omega_{\text{GW}}$ \cite{5-9}. Since these features in the pGW spectrum depend on $m_{3/2} \in [100\text{eV}, 1\text{keV}]$, our study can be useful as a smoking gun for the PeV scale SUSY-breaking scenarios. Even if future surveys detect only a pGW spectrum without any suppression in the frequency range $O(10^{-10})\text{Hz} \lesssim f_{\text{pGW}} \lesssim O(10^{-5})\text{Hz}$, our study contributes to ruling out all the PeV scale SUSY-breaking scenarios.

The outline of this paper is as follows. In Sec. II, we

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Probe PeV scale SUSY-breaking with Satellite Galaxies and Primordial Gravitational Waves

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We study an inevitable cosmological consequence in PeV scale SUSY-breaking scenarios. We focus on the SUSY-breaking scale corresponding to the gravitino mass $m_{3/2} = 100\text{eV} - 1\text{keV}$. We argue that the presence of an early matter-dominated era and the resulting entropy production are requisite for the Universe with this gravitino mass. We infer the model-independent minimum amount of the entropy production $\Delta$ by requiring that the number of dwarf satellite galaxies $N_{\text{sat}}$ in the Milky Way exceed the currently observed value, i.e. $N_{\text{sat}} \gtrsim 63$. This entropy production is inevitably imprinted on the primordial gravitational waves (pGWs) produced during the inflationary era. We study how the information on the value of $\Delta$ and the time of entropy production are encoded in the pGW spectrum $\Omega_{\text{GW}}(f_{\text{GW}})$. If the future GW surveys observe suppression in the pGW spectrum for the frequency range $O(10^{-15})\text{Hz} \lesssim f_{\text{GW}} \lesssim O(10^{-5})\text{Hz}$, it works as a smoking gun for PeV SUSY-breaking scenarios. Even if they do not, our study can be used to rule out all such scenarios.

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raise questions we address in this work by making a brief review for $m_{3/2} = \mathcal{O}(100)\text{eV}$ gravitino cosmology. Then in Sec. III, we discuss how to obtain the maximally allowed $f_{3/2,\text{max}}$ for each $m_{3/2}$ and discuss the result of the analysis. In Sec. IV, we discuss inevitable signatures on pGW in the Universe with PeV scale SUSY-breaking. We study what sort of deviation of pGW from its usual form in the standard Universe is induced by the presence of an early matter dominated (EMD) era followed by the entropy production via the decay of a heavy degree of freedom. Finally our conclusion is made in Sec. V.

II. PeV SCALE SUSY BREAKING SCENARIO

In a local SUSY model, a SUSY-breaking scale ($M_{\text{SUSY}} = \sqrt{|F|}$) is directly connected to a gravitino mass ($m_{3/2}$) via

$$m_{3/2} = \frac{F}{\sqrt{3}M_{P}} \quad (1).$$

Especially for the low SUSY-breaking scale amounting to $F \simeq 10^{11} - 10^{12}\text{GeV}^{2}$ ($M_{\text{SUSY}} = \mathcal{O}(1)\text{PeV}$), based on Eq. (1), the corresponding gravitino mass is found to be $m_{3/2} \in [100\text{eV}, 1\text{keV}]$.

This gravitino mass range is of particular interest in that the relic abundance of the gravitinos is given almost in a model-independent way. On the spontaneous SUSY-breaking, gravitinos become massive by absorbing Goldstino field. As far as particles are present in the MSSM thermal bath, gravitinos keep thermal equilibrium with the thermal bath. Afterward, the thermal bath continues to be cooled until its temperature ($T$) reaches the following gravitino decoupling temperature $T_{3/2,\text{dec}} [1]^{1}$

$$\text{Max} \left[ m_{\tilde{g}}, 10\text{GeV} \left( \frac{g_{*p}(T_{3/2,\text{dec}})}{230} \right)^{\frac{1}{2}} \left( \frac{m_{3/2}}{1\text{keV}} \right)^{2} \left( \frac{1\text{TeV}}{m_{\tilde{g}}} \right)^{2} \right].$$

where $g_{*p}(T)$ is the effective number of degrees of freedom for the energy density in the MSSM thermal bath at a temperature $T$ and $m_{\tilde{g}}$ is a gluino mass.

For $F \simeq 10^{11} - 10^{12}\text{GeV}^{2}$, a low scale gauge-mediated SUSY-breaking (GMSB) can explain soft masses of sfermions and gauginos. The gluino mass ($m_{\tilde{g}}$) is dominantly generated by the loop correction contributed by colored messengers and it reads

$$m_{\tilde{g}} \simeq N_{\text{mess}} \frac{g_{c}^{2}}{(4\pi)^{2}} \frac{yF}{M_{\text{mess}}}, \quad (3)$$

where $N_{\text{mess}}$ is the number of messengers, $g_{c}$ is the gauge coupling of the MSSM $SU(3)_{c}$ color gauge group, $y$ is the coupling constant for the interaction between messenger fields and SUSY-breaking field, and $M_{\text{mess}}$ is the mass of the messenger.

For a perturbative gauge mediation model, in order for a SUSY-breaking vacuum to be stable to date, the condition $M_{\text{mess}}^{2} \gg kF$ is required to be satisfied [10]. This implies that $m_{\tilde{g}}$ can be at most $\mathcal{O}(10)\text{TeV}$ for $m_{3/2} \in [100\text{eV}, 1\text{keV}]$. Thus, we see that $T_{3/2,\text{dec}} \simeq m_{\tilde{g}}$ holds true in Eq. (2). For a given $m_{3/2}$, $T_{3/2,\text{dec}}$ so obtained leads to the following estimate for the relic abundance of gravitinos

$$\omega_{3/2} \equiv \Omega_{3/2}h^{2} = \left( \frac{T_{3/2,0}}{T_{\nu,0}} \right)^{3} \left( \frac{m_{3/2}}{94\text{eV}} \right) = \left( \frac{10.75}{g_{*s}(T_{3/2,0})} \right) \left( \frac{m_{3/2}}{94\text{eV}} \right), \quad (4)$$

where $\Omega_{3/2}$ is the ratio of the present gravitino energy density to the critical energy, $h$ parametrizes the Hubble expansion rate via $H_{0} = 100h\text{km}/\text{Mpc}/\text{sec}$, $g_{*s}(T)$ is the effective number of degrees of freedom for the entropy density in the MSSM thermal bath at a temperature $T$, and $T_{3/2,0}$ and $T_{\nu,0}$ are the temperature of gravitinos and neutrinos at present respectively.

Now that $g_{*s}(T_{3/2,\text{dec}})$ depends on a model-dependent mass spectrum, one-to-one correspondence between $\omega_{3/2}$ and $m_{3/2}$ is not precisely determined via Eq. (4). However, because $g_{*s}$ in the MSSM is at most $g_{*s} \simeq 230^{2}$, it is fair to state that for each $m_{3/2}$ the minimum inevitable value of $\Omega_{3/2}h^{2}$ is given by

$$\omega_{3/2} \gtrsim \omega_{3/2,\text{min}} = \left( \frac{10.75}{230} \right) \left( \frac{m_{3/2}}{94\text{eV}} \right). \quad (5)$$

Given $\omega_{3/2,\text{min}}$ in Eq. (5), we notice that for the Universe with $m_{3/2} \in [100\text{eV}, 1\text{keV}]$, as the lightest SUSY particle (LSP), gravitinos must exist today in the form of DM. Depending on its mass, gravitino’s relic abundance can explain a fraction of DM today or even exceed the DM relic abundance in the absence of any density dilution. Making the estimate of the free-streaming length ($\lambda_{\text{FS}}$) of gravitinos [11],

$$\lambda_{\text{FS}} \sim \frac{2\pi}{5} \left( \frac{m_{3/2}}{1\text{keV}} \right)^{-1} \left( \frac{10.75}{g_{*s}(T_{3/2,\text{dec}})} \right)^{\frac{1}{2}} \text{Mpc}, \quad (6)$$

it is realized that sub-keV gravitinos with $g_{*s} \simeq 230$ serve as WDM owing to $\lambda_{\text{FS}} \sim \mathcal{O}(0.1) - \mathcal{O}(1)\text{Mpc}$.

Therefore, if our Universe went through the spontaneous SUSY-breaking at PeV scale, it becomes inevitable today for DM population to be composed of a CDM candidate and gravitino WDM (mixed DM scenario) or fully of gravitino WDM with a certain mechanism for diluting the relic abundance. If so, then one critical question

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1 Max[A,B] means the larger one among A and B.

2 Assuming GMSB scenario, $g_{*s}$ can be slightly greater than $g_{*s} \simeq 230$ due to additional contribution made by messengers.
to be addressed could concern whether the presence of gravitino WDM is consistent with various cosmological and astrophysical observables or not.

Regarding this question, for the later possibility, already the mass constraints on WDM from both of Lyman-α forest observation $m_{\text{wdm}}^{\text{thermal}} > 5.3$ keV [2] and the redshifted 21cm signals in EDGES observations $m_{\text{wdm}}^{\text{thermal}} > 6.1$ keV [3, 4] require the energy density dilution. This implies that for $m_{3/2} \in [100$ eV, $1$ keV], gravitinos can reside in the present Universe only as a fraction of DM population.

Along this line of reasoning, we ask two key questions to which answers are the direct cosmological consequences of PeV SUSY-breaking scenarios and thus can be used as a powerful cosmological probe of PeV SUSY-breaking scenario. These questions include

1. For each $m_{3/2}$, what is the minimum amount of the entropy production for consistency with experimental observation?

2. How to probe and confirm such an entropy production?

In the coming Sec. III and Sec. IV, we will answer these questions by invoking the estimate of the number of satellite galaxies in the Milky Way (MW) and pGW. In Sec. III, we quantify the amount of the entropy production by $\Delta \equiv s/\bar{s}$ where $s$ is the entropy newly produced by the decay of a heavy particle $X$ prior to Big Bang Nucleosynthesis (BBN) era and $\bar{s}$ is the existing entropy of the MSSM thermal bath before the decay took place.\(^3\) To this end, for each $m_{3/2}$, we apply today’s observed number of the satellite galaxies ($N_{\text{sat}}$) in MW to constrain a DM fraction responsible for gravitinos. This shall eventually provide us with the minimum required dilution factor $\Delta_{\text{min}}$. In Sec. IV, we study imprints on pGW left by the early matter-dominated (EMD) era and the entropy production. Regarding testability, we discuss how the different future GW detection experiments could be employed to either confirm or rule-out PeV-scale SUSY-breaking scenario.

III. $\Delta_{\text{min}}$ FROM $N_{\text{sat}}$

The mixed DM model (MDM) where the present DM population consists of both CDM and WDM is parametrized by two parameters: a mass of WDM and a fraction of DM relic abundance today attributable to WDM. For our case with gravitinos being WDM, these are denoted by $(m_{3/2}, f_{3/2})$. For a given set of $(m_{3/2}, f_{3/2})$, the matter power spectrum in MDM scenario ($P_{\text{MDM}}(k)$) is given by

$$P_{\text{MDM}}(k) = \sqrt{T(k, m_{3/2}, f_{3/2})} P_{\text{CDM}}(k),$$

where $T(k, m_{3/2}, f_{3/2})$ is a transfer function relating the matter power spectrum in CDM case ($P_{\text{CDM}}(k)$) to that in MDM case. We compute $P_{\text{MDM}}(k)$ for a given set of $(m_{3/2}, f_{3/2})$ by using the Boltzmann solver CLASS [15].

As one of ways to constrain $(m_{3/2}, f_{3/2})$, we attend to the estimate of the number of dwarf satellite galaxies in the Milky Way. The approach we adopt in this paper is based on the one given in Ref. [16] (see also Refs. [17–25]): Fifteen satellite galaxies were observed by SDSS (Sloan Digital Sky Survey) with the sky coverage $f_{\text{sky}} \approx 0.28$. When this limited sky coverage and eleven classically known satellites are taken into account together, $N_{\text{sat}} \approx 63$ is inferred as the total number of satellites of the Milky Way. Considering the possibility that more satellites are found in the future surveys, we take $N_{\text{sat}} \approx 63$ as the lower bound of the number of satellite galaxies that a DM model should satisfy. This set-up shall give us an upper bound on $m_{3/2}$ for each $f_{3/2}$ below which the presence of gravitinos is consistent with the number of satellite galaxies of the Milky Way.

Given $P_{\text{MDM}}(k)$, one can make an estimate of the expected number of dwarf satellite galaxies residing in a host halo. Our estimate closely follows Refs. [21, 22], which are based on the extended Press-Schechter approach [26, 27] with the conditional mass function [28]. Adopting a sharp-$k$ filter for the window function, it is calculated as [21, 22]

$$N_{\text{sat}} = \int_{M_{\text{min}}}^{M_h} dM_s \frac{1}{C_n} \frac{1}{6\pi^2 R_i^3} \left( \frac{M_h}{M_i^2} \right) \frac{P_{\text{MDM}}(1/R_i)}{2\pi (S_i - S_h)},$$

where $M_i$, $R_i$ and $S_i$ are the mass, the filter scale and the variance of a satellite galaxy for $i = s$ (a host halo for $i = h$) respectively. While the relation between the mass $M_i$ and the filter scale $R_i$ is in principle unconstrained in the sharp-$k$ modeling, we follow Ref. [21] and adopt $M_i = (4\pi/3) \times (c R_i)^3 \times \Omega_m \times \rho_{c,0}$ with $c = 2.5$, which matches best with observations. Here $\rho_{c,0}$ is the critical density. The overall normalization is taken to be $C_n = 45$ to reproduce the N-body simulation result [29]. In addition, we take $M_{\text{min}} = 10^8 h^{-1} M_\odot$ as the minimum mass of the dwarf satellite galaxies [30] and $M_h = 1.77 \times 10^{12} h^{-1} M_\odot$ as the Milky Way mass based on Ref. [31]. The variance $S_i$ is the function of $R_i$ and given by

$$S_i = \frac{1}{2\pi^2} \int_0^{1/R_i} dk \ k^2 P_{\text{MDM}}(k).$$

In the left panel of Fig. 1, using Eq. (5), for each $m_{3/2}$ we show (i) the minimum fraction of DM contributed by

\(^3\) For avoiding to spoil BBN by causing mismatch in the light element amount, we assume that the entropy production took place before the temperature of the Universe becomes 10 MeV. In addition, as a way to achieve the entropy production, we consider the decay of a heavy particle $X$ whose decay rate determines the decay time (temperature) via $\Gamma_X \approx H$ where $\Gamma_X$ is the decay rate of $X$ and $H$ is the Hubble expansion rate. As a candidate of $X$, one may consider $X$ in a messenger sector in GMSB scenario [12], $X$ in the SUSY-breaking sector [13] or lightest right-handed sneutrino [14].
the gravitino WDM which is unavoidable in PeV SUSY-breaking scenarios in the absence of the energy dilution (red line) and (ii) the fraction of the gravitino satisfying \( N_{\text{sat}} = 63 \) (blue line). Below the blue line, the number of the satellite galaxies are larger than 63. As one can see from the gap between the two lines, even the minimum predicted amount of gravitino relic abundance exceeds the observationally allowed one. This implies that our Milky Way should be left with too few satellites if there is no any history of dilution of the gravitino energy density. We take this point, therefore, as a strong hint for the presence of the era when the entropy production is made via, for example, a heavy particle decay.

In the right panel of Fig. 1, we show the minimum amount of the entropy production necessary for consistency with the observed number of the satellite galaxies of the Milky Way. For each \( m_{3/2} \), \( \Delta_{\text{min}} = s/\bar{s} \) was obtained by dividing \( f_{3/2, \text{min}} \) (red line) by \( f_{3/2} \) associated with \( N_{\text{sat}} = 63 \) (blue line) in the left panel of Fig. 1. Note that when the precise \( g_{s,\ast}(T_{3/2, \text{dec}}) \) is taken into account, a larger \( \Delta \) would be required.

Before leaving Fig. 1, one may wonder whether the Lyman-\( \alpha \) forest observation can offer a stronger constraint on \( f_{3/2} \) than shown in the blue line in the left panel of Fig. 1. Indeed in Ref. [32], it was pointed out that the MDM is very difficult to accommodate Lyman-\( \alpha \) data unless \( m_{3/2} \) is very tiny. Provided \( f_{3/2} \) is more severely constrained by Lyman-\( \alpha \) forest data, then apparently more entropy production is required than the green line in the right panel of Fig. 1 for each \( m_{3/2} \). In any case, the green line remains as the most conservative lower bound of the necessary amount of \( \Delta \) to be consistent with a variety of cosmological observations.

Given this inevitable consequence (requirement for the entropy production) that any PeV SUSY-breaking scenarios confront with, the next important question could be about how one could confirm the presence of the EMD era and the time when the sudden increase in the radiation energy density occurred. In the next section, we try to address this question by studying pGW spectrum with the assumption of the inflationary phase of the early Universe. Non-vanishing tensor modes produced by the quantum fluctuation during the inflation forming pGW, those non-standard events (EMD era and the entropy production) can be seen by the modes re-entering the horizon before the entropy production.

We envision the scenario where a heavy particle \( X \) is present in the first radiation-dominated (RD1) era. \( X \) is assumed to be out-of-equilibrium with the thermal bath and thus its energy density scales as \( \rho_X \propto a^{-3} \). When \( \rho_X \) becomes comparable to the energy density of the existing thermal bath, EMD era starts and continues until \( X \) decays to produce the additional entropy. After the decay, the second radiation-dominated (RD2) era gets started and continues until the familiar matter-radiation equality at \( z_{\text{eq}} \approx 3300 \) is reached.

### IV. PRIMORDIAL GRAVITATIONAL WAVES

The gravitational waves \( h_{ij}(t,x) \) in the expanding background are tensor perturbations to the Friedmann-Robertson-Walker (FRW) metric, i.e.

\[
ds^2 = a(t)^2[-dt^2 + \delta_{ij}dx^i dx^j], \quad (i,j = 1, 2, 3),
\]

\[
(10)
\]
where \( \tau \) is the conformal time defined via \( dt = a d\tau \). Here \( h_{ij} \) is traceless and transverse, i.e. \( h_{ii} = \partial_i h_{ij} = 0 \). In the Fourier space, \( h_{ij}(t, \mathbf{x}) \) can be written as

\[
h_{ij}(t, \mathbf{x}) = \sum_{\lambda = +, \times} \int \frac{d^3k}{(2\pi)^3} h_{ij}(t, \mathbf{k}) e^{i \mathbf{k} \cdot \mathbf{x}},
\]

\[
h_{ij}(t, \mathbf{k}) = h^\lambda_k(t) \epsilon^\lambda_{ij}(\mathbf{k}),
\]

where \( \lambda = +, \times \) are the GW polarizations, \( \mathbf{k} \) is the unit vector along the three momentum \( \mathbf{k} \), and the polarization tensors \( \epsilon^\lambda_{ij} \) are traceless and transverse as with \( h_{ij}(t, \mathbf{x}) \). The polarization tensors are normalized via \( \epsilon^\lambda_{ij} (\epsilon^\lambda_{ij})^\ast = 2 \delta^{\lambda\lambda'} \). The Fourier components \( h^+_k \) and \( h^\times_k \) commonly obey the following time evolution equation in the absence of an anisotropic stress

\[
\ddot{h}^\lambda_k + 3H \dot{h}^\lambda_k + \frac{k^2}{a^2} h^\lambda_k = 0,
\]

where the dot denotes the time derivative.

After the relevant modes get sufficiently sub-horizon, the energy density of pGW is given as

\[
\rho_{GW} = \frac{1}{32\pi G} \left\langle (\partial h_{ij}/\partial \tau)^2 \right\rangle / a^2,
\]

where \( G \equiv (8\pi M_F^2)^{-1} \) is the Newtonian constant and \( \langle \ldots \rangle \) denotes the oscillation average. In this paper we assume the homogeneity and isotropy of the pGWs. As long as we average over sufficiently large number of oscillations, we may add the spatial average to the definition of \( \langle \ldots \rangle \). Using the Fourier transform of \( h_{ij} \) in Eq. (11),

\[
\left\langle (\partial h^\lambda_k / \partial \tau)^2 \right\rangle \approx k^2 \left\langle h^\lambda_k(\tau)^2 \right\rangle
\]

for the sub-horizon \( k \)-modes and translating the spatial average into the ensemble average, one obtains the pGW energy density per logarithmic wavenumber

\[
\Omega_{GW}(k, \tau) \equiv \frac{1}{\rho_{tot}} \frac{d\rho_{GW}}{d \ln k} = \frac{1}{12} \left( \frac{k}{aH} \right)^2 P_T(k, \tau),
\]

where we define \( k = \sqrt{k^2 - \mathbf{k} \cdot \mathbf{k}} \). The label \( \tau \) is understood as the time coordinate after taking oscillation average around it. The tensor power spectrum \( P_T(k, \tau) \) is defined via the ensemble average as

\[
\left\langle h_{ij}(\tau, \mathbf{k})h_{ij}(\tau, \mathbf{k}') \right\rangle = \delta^3(\mathbf{k} - \mathbf{k}') P_T(k, \tau),
\]

which can be decomposed into the primordial part and the transfer function encoding the time evolution

\[
P_T(k, \tau) = P_T^{\text{prim}}(k) T_T^2(k, \tau),
\]

The transfer function is defined through \( h^\lambda_k(\tau) = h^\lambda_k^{\text{prim}} T_T(k, \tau) \). For the primordial part, \( P_T^{\text{prim}}(k) \) is written in terms of the CMB pivot scale \( k_{\text{CMB}} = 0.002 \text{Mpc}^{-1} \) as

\[
P_T^{\text{prim}}(k) = r P_T^{\text{prim}}(k_{\text{CMB}}) \left( \frac{k}{k_{\text{CMB}}} \right)^{n_T},
\]

where \( P_T^{\text{prim}}(k_{\text{CMB}}) \approx 2.1 \times 10^{-9} \) is the amplitude of the scalar perturbation spectrum, \( r \equiv P_T^{\text{prim}}(k_{\text{CMB}})/P_T^{\text{prim}}(k_{\text{CMB}}) \) is the tensor-to-scalar ratio and \( n_T \) is a tensor spectral index. Note that the most recent bound on the tensor-to-scalar ratio is \( r \lesssim 0.056 \) [34] which gives \( P_T^{\text{prim}}(k_{\text{CMB}}) \lesssim 1.197 \times 10^{-10} \).

While \( P_T^{\text{prim}}(k_{\text{CMB}}) \) is determined by the initial conditions set during the inflationary era as in Eq. (18), \( T_T^2(k, \tau) \) reflects physical processes experienced by the \( k \)-mode after its horizon re-entry. Even within the Standard Model, the pGW spectrum carries rich information on the particle content in/out of the thermal bath [35–38]. In BSM scenarios we expect much richer structure [39–42]. Similarly in the present scenario, the cosmological effects of the PeV scale SUSY-breaking is encoded in \( T_T^2(k, \tau) \) through the entropy production [43–46]. The decay of pGW amplitude inversely proportional to the scale factor after horizon re-entry implies \( T_T^2(k, \tau) \propto a^{-2} \).

For the modes re-entering the horizon during deep in either RD1 or RD2 era, we can write it as \( [47]^5 \)

\[
T_T^2(k, \tau) = \frac{1}{2} \left( \frac{a k}{a} \right)^2,
\]

where the subscript \( k \) denotes the time of re-entry \( k = a H k \), and the factor \( 1/2 \) arises from the oscillation average deep inside the horizon. Thus we obtain the following pGW spectrum expression that holds for these modes

\[
\Omega_{GW}(k, \tau) = \frac{1}{24} \left( \frac{a k}{a} \right)^4 \left( \frac{H_k}{H} \right)^2 P_T^{\text{prim}}(k).
\]

We further use the Friedmann equation \( 3M^2 T^2 = \rho_{rad} = g_{s,p,k} (\pi^2 / 30) T_4^4 \) at the time of horizon re-entry together with the entropy relation \( s_{s,k} a^3 T_k^3 = \Delta^{-1} \times g_{s,s-0} a^3 T_0^3 \) (for the modes re-entering during RD1) or \( g_{s,s-0} a^3 T_0^3 \) (RD2) to obtain the present pGW spectrum

\[
\Omega_{GW,0}(k) = \frac{\Omega_{rad,0}}{24} \left( \frac{g_{s,p,k}}{g_{s,p,0}} \right)^{\Delta^{-1}} \left( \frac{g_{s,s-0}}{g_{s,s,0}} \right)^{4} P_T^{\text{prim}}(k)
\]

\[
\times \begin{cases} \Delta^{-\frac{3}{2}} & \text{RD1}, \\ 1 & \text{RD2}. \end{cases}
\]

Here \( \Omega_{rad,0} \equiv \rho_{rad,0}/\rho_{crit,0} = 4.2 \times 10^{-5} h^{-2} \) is the radiation energy fraction today, and \( g_{s,p} \) and \( g_s \) account for the relativistic degrees of freedom for the energy and entropy density, respectively.\(^6\) We use \( g_{s,p,0} = 3.838 \) and

\(^5\) Depending on the equation of state at the time of horizon re-entry, an extra factor appears in Eq. (19) due to the non-zero “thickness” of the horizon [47]. For the modes re-entering during a RD era, this extra factor takes unity and Eq. (19) holds true.

\(^6\) Though we do not focus much on the change in the number of relativistic degrees of freedom, it leaves an important imprint of the high energy theory on the pGW spectrum. See e.g. (to be written).
$g_{s*0} = 3.931$ for their present values. While Eq. (21) already tells us the rough behavior of the pGW spectrum at low and high wavenumbers, we parameterize it as

$$\Omega_{GW,0}(k) = \frac{\Omega_{rad,0}}{24} \left( \frac{g_{*p,k}}{g_{*p,0}} \right) \left( \frac{g_{s*,0}}{g_{s*,k}} \right)^{\frac{4}{3}} P_{T}^{prim}(k) C_{\Delta}(k),$$

in order to account for the wavenumbers re-entering the horizon during the entropy production.

To obtain $C_{\Delta}(k)$, we explicitly calculate the time evolution of the pGW field. The final spectral shape depends on two parameters: the temperature at which the entropy production occurs ($T_\Delta$) and the dilution factor ($\Delta = s/\dot{s}$). Through $H(T_\Delta) = \Gamma_X$, the first parameter is exchangeable with the decay rate of a heavy particle ($\Gamma_X$), which sources the extra entropy. We obtain the value of the second parameter once the value of $m_{3/2}$ is specified based on Fig. 1. Different combinations of the two parameters ($T_\Delta$, $\Delta$) will give rise to different $C_{\Delta}(k)$s. This difference in $C_{\Delta}(k)$ enables us to probe the early Universe history through $\Omega_{GW}(f_{GW})$ with $f_{GW} = k/(2\pi a_0)$.

Before the heavy particle decays, the energy density of the Universe is mainly contributed from the radiation ($\rho_{rad}$) and a heavy particle $X$ ($\rho_X$). Thus the time evolution of the Hubble expansion rate in Eq. (13) is affected by that of $\rho_{rad}$ and $\rho_X$, i.e.

$$\dot{\rho}_{rad} + 4H\rho_{rad} = \Gamma_X \rho_X$$

$$\dot{\rho}_X + 3H\rho_X = -\Gamma_X \rho_X$$

$$H^2 = \frac{\rho_{rad} + \rho_X}{3M_p^2}.$$  \hspace{1cm} (23)

When solved together with Eq. (23), Eq. (13) yields $h^+_{\Delta}(\tau)$ and $h^\times_{\Delta}(\tau)$ which in turn produce the transfer function.

In the left panel of Fig. 2, we show $C_{\Delta}(k)$ obtained by numerically solving Eq. (13) for different choices of the dilution factor $\Delta$. Each different color corresponds to the specified dilution factor. The suppression in red, blue and purple spectra is due to the specified dilution factor. The green dashed line is the sensitivity curve of SKA.

$\Omega_{GW,0}(k) = \frac{\Omega_{rad,0}}{24} \left( \frac{g_{*p,k}}{g_{*p,0}} \right) \left( \frac{g_{s*,0}}{g_{s*,k}} \right)^{\frac{4}{3}} P_{T}^{prim}(k) C_{\Delta}(k),$

In the right panel of Fig. 2, we show the spectra of pGWs in the Universe with $T_\Delta = 100\text{MeV}$. The upper ($n_\tau = 0.4$) and lower spectra ($n_\tau = -0.007$) result from the different choice of $n_\tau$. Suppression in red, blue and purple spectra is due to the specified dilution factor. The green dashed line is the sensitivity curve of SKA.

$\Omega_{GW,0}(k) = \frac{\Omega_{rad,0}}{24} \left( \frac{g_{*p,k}}{g_{*p,0}} \right) \left( \frac{g_{s*,0}}{g_{s*,k}} \right)^{\frac{4}{3}} P_{T}^{prim}(k) C_{\Delta}(k),$
frequency of pGW and the temperature \( T_{hc} \) of the Universe when the relevant mode re-entered the horizon is related via

\[
f_{GW} = \frac{k}{2\pi a_0} \simeq 2.65 \text{Hz} \left( \frac{g_{*s,k}}{106.75} \right)^{\frac{1}{3}} \left( \frac{g_{*s,k}}{106.75} \right)^{-\frac{1}{4}} \left( \frac{T_{hc}}{10^8 \text{GeV}} \right). \tag{26}
\]

To see \( \Omega_{GW} \)'s dependence on an inflation model, we assumed both of the standard single-field slow-roll inflation with \( n_T = -r_{\text{max}}/8 = -0.007 \) with \( r_{\text{max}} \) the maximum allowed tensor-to-scalar ratio and a non-minimal inflation model with an assumed both of the standard single-field slow-roll inflation during RD era. The larger \( \Delta \) induces the greater suppression in the spectrum for the modes \( k \gtrsim k_{\text{dec}} \). Although only \( T_\Delta = 100 \text{MeV} \) case was shown in the right panel of Fig. 2, for other choice of \( T_\Delta \), we expect to see the same shape of the spectrum with suppression occurring at the relevant \( f_{GW,\Delta} \).

It should be noted that in the range \( 10 \text{MeV} \lesssim T_\Delta \lesssim O(1) \text{TeV} \), the lower bound is for ensuring the successful BBN. On the other hand, as discussed in Sec. II, \( T_{3/2,\text{dec}} \) can be at most \( O(10) \text{TeV} \) since \( m_\tilde{g} \lesssim O(10) \text{TeV} \). Since the entropy production should happen after gravitinos decouple from the thermal bath, we conclude that \( T_\Delta \lesssim O(1) \text{TeV} \) should be the case. Hence, for PeV SUSY-breaking scenario, we can make quite solid argument that the suppression of \( \Omega_{GW} \) due to \( \Delta \neq 0 \) should occur at \( f_{GW,\Delta} = O(10^{-10}) \text{Hz} \)–\( O(10^{-5}) \text{Hz} \), irrespective of an assumed inflation model.

In the right panel of Fig. 2, given the green shaded region as the parameter space that can be potentially probed by SKA [56–58], we notice that when \( n_T = 0.4 \), SKA may have a chance to see the suppression directly for \( T_\Delta = O(100) \text{MeV} \). However, for other cases with either of different \( T_\Delta \) or \( n_T = -r/8 \), direct observation of the suppression in \( \Omega_{GW} \) based on SKA seems not possible.

In this regard, if accomplished, detection of B-mode polarization of the CMB in future is expected to play a critical role for our study even if its relevant frequency regime is too low for direct probe of \( f_{GW,A} \) of our interest. As the information for \( r \) obtained from the detection in principle can provide us with the spectrum of \( \Omega_{GW} \) for \( O(10^{-10}) \text{Hz} \lesssim f_{GW} < 1 \text{Hz} \) via extrapolation, the indirect investigation for the presence of the suppression in \( \Omega_{GW} \) can be conducted with the aid of space interferometers such as LISA [59, 60], DECIGO [61, 62], Taiji [63], TianQin [64, 65], and BBO [66–68]. Although this way of indirect investigation has indeed the caveat that it cannot pin down \( T_\Delta \), since the projected sensitivity of these future space-based interferometers can cover the suppressed \( \Omega_{GW} \) as small as \( \sim O(10^{-18}) \), at least \( \Delta \) can be measured. Therefore, the investigation supported by the combination of the CMB B-mode polarization survey and future space-based interferometers (DECIGO and BBO) still renders PeV-SUSY-breaking scenario testable provided the future pulsar timing arrays (IPITA [69–71] and SKA) are not sensitive enough to probe the entropy production at \( 10 \text{MeV} \lesssim T_\Delta \lesssim O(1) \text{TeV} \).

We end this section by commenting on effects that sub-GeV gravitinos can have on the spectrum of pGW as the free-streaming dark radiation (DR) [72–74]. Gravitinos decouple from the primordial MSSM thermal bath at \( T_{3/2,\text{dec}} = O(1) \text{TeV} \), they start to behave as DR. For the usual case without a significant late time entropy production after the decoupling, \( \Delta \mathcal{N}_{\text{eff}} \) contributed by gravitinos (say \( N_{3/2} \)) is given by

\[
N_{3/2} = \left( \frac{T_{3/2}}{T_\nu} \right)^4 \left( \frac{g_{*s}(T_{\nu,\text{dec}})}{g_{*s}(T_{3/2,\text{dec}})} \right)^{\frac{3}{2}} \lesssim 0.017 \left( \frac{g_{*s}(T_{3/2,\text{dec}})}{230} \right)^{-\frac{1}{2}}, \tag{27}
\]

where \( T_{\nu,\text{dec}} \) is the neutrino decoupling temperature and \( g_{*s}(T_{\nu,\text{dec}}) = 10.75 \) is the effective number of degrees of freedom for the entropy density evaluated at \( T_{\nu,\text{dec}} \).

We notice that for the case where a DR makes a significant contribution to \( \Delta \mathcal{N}_{\text{eff}} \neq 0 \), there can be an enhancement of the normalization of the pGW spectrum which is the net effect resulting from the enhancement due to \( \Delta \mathcal{N}_{\text{eff}} \neq 0 \) and the suppression due to the non-zero anisotropic stress induced by the free-streaming of a DR [47, 75–77]. However, for our case, since \( N_{3/2} \) is too small, we see that the amount of the enhancement due to \( N_{3/2} \) in Eq. (27) is too small to be visible in the right panel of Fig. 2.

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7 As for the choice of \( n_T = 0.4 \), we refer to Ref. [43] where \( n_T = 0.4 \) was chosen to assess the maximal reach of future GW experiments and the explanation for consistency with CMB observation [34], constraints from LIGO and Virgo [54, 55], and BBN, LIGO, and pulsars [46] was made.

8 Note that at the GW frequency range relevant to DECIGO and BBO, some or all of MSSM particles are relativistic. Thus, when the suppression of \( \Omega_{GW} \) due to \( \Delta \neq 0 \) is probed using DECIGO and BBO, the additional suppression caused by the larger \( g_{*s}(T_{\text{ini}}) \) and \( g_{*s}(T_{\text{ini}}) \) in Eq. (22) should be taken into account.
V. CONCLUSION

In this work, we demonstrated the necessity of the late time entropy production prior to BBN era for the Universe with PeV scale SUSY-breaking. Our argument is based on the observation that the theoretically predicted relic abundance of gravitino WDM with \(m_{3/2} \in [100\text{eV}, 1\text{keV}]\) is too large to be consistent with the observed number of satellite galaxies \((N_{\text{sat}})\) residing in the Milky Way provided there is no any energy density dilution mechanism. By demanding that \(N_{\text{sat}} \gtrsim 63\) be satisfied in the current Universe in which sub-keV gravitino WDM is responsible for a fraction of DM population today, we quantified the minimum amount of the dilution factor \((\Delta_{\text{min}})\) necessary for observational consistency.

We also proposed to use the spectrum of pGW \(\Omega_{GW}(f_{GW})\) to investigate whether there were once EMD era and the entropy production prior to BBN era. The degree and location \((f_{GW}, \Delta)\) of the suppression in \(\Omega_{GW}(f_{GW})\) can give us information for \((\Delta, T_{\Delta})\) once observed by the future pulsar timing array such as SKA. The suppression is expected to occur at \(\mathcal{O}(10^{-10})\text{Hz} \lesssim f_{GW} \lesssim \mathcal{O}(10^{-5})\text{Hz}\) in PeV scale SUSY-breaking scenarios. In case where the magnitude of \(\Omega_{GW}(f_{GW})\) is not large enough to be probed by SKA, coordination between the future CMB B-mode polarization detection and the space-based interferometers can alternatively probe the suppression in \(\Omega_{GW}(f_{GW})\) induced by \(\Delta \neq 0\). Particularly when the two regimes of \(f_{GW} < \mathcal{O}(10^{-10})\text{Hz}\) and \(f_{GW} > \mathcal{O}(10^{-5})\text{Hz}\) are compared, if \(\Omega_{GW}(f_{GW})\) does not show any suppression attributable to \(\Delta \neq 0\), our study can be used to rule-out PeV scale SUSY-breaking scenarios.

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