Selection of displacement coefficients in external and internal involute gearing of planetary rotor hydraulic machine

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Abstract. Hydraulic and pneumatic volume displacement machines are commonly the most important and integral elements of modern mechanical systems. One of the known types of such machines is planetary rotary hydraulic machines (PRHM) with floating satellites in contact with central gearwheels. The numbers of the central gearwheels «waves» M and N may be different or the same. The article considers the case of the same number of «waves» when the central wheels are round. The algorithm of selection of the range of permissible values of the PRMM displacement coefficients, in which the central wheels of internal and external gearing have the same number of teeth, is proposed. Equal number of teeth is proposed to be obtained by application of maximum values of positive displacement of tool in processing of central gearwheel with internal gearing. The calculation was carried out according to formulas according to GOST 16532-70 and GOST 19274-73, which are based on the module "Shafts and mechanical transmissions" of the Kompas 3D software complex.

1. Introduction
In various technical systems, volume hydraulic machines are widely used: pumps and engines. Such machines include planetary rotary hydraulic machines PRHM. A typical PRGM is a planetary mechanism with central gears with one external and one internal gearing and enclosed floating satellites between them. The number of «waves» of the center gearwheel with the inner teeth N and the outer teeth M may vary or may coincide. In the case of the same number of «waves», when the center gearwheels are circular, it is possible to use standard evolute gearing of the gear links.

2. Theoretical bases for obtaining a planetary mechanism built in the base of the Planetary Rotor Hydraulic Machines with the same number of teeth
Planetary mechanism based on new PRHM [1] (figure 1) comprises two circular central gearwheels 1 and 2 with external gearing and internal gearing (M = N = 1). Two floating satellites 3, enclosed between central gearwheels, move along a central trajectory representing a circle. Axes of mutual rotation of gearwheels 1 and 2 are displaced relative to geometric axes of their centroids by equal distance e. The fundamental condition of the existence of such PRGM is equality of the numbers of teeth of the central gearwheels with external and internal gearing.

It should be noted that the equality of tooth numbers for the planetary mechanism laid down in the base of the PRGM is fundamentally new and has not been solved by anyone before. Some authors considered that such a condition is impossible [2]. Geometric parameters of such gearwheels can be...
obtained using standard evolvent external and internal gearing of gear links according to GOST 16532-70 [3] and GOST 19274-73 [4].

![Planetary Rotor Hydraulic Machines (PRHM) with round central gearwheels: 1 – solar gearwheel; 2 – epicyclic gearwheel; 3 – satellites.](image)

Equal number of teeth is proposed to be obtained by application of positive displacement $x_2$ of tool in processing of gear links of satellite, maximum positive displacement $x_3$ in processing of gear links of center gearwheel with internal gearing and negative displacement $x_1$ in processing of center gearwheel with external gearing. At the same time the range of permissible values of displacement coefficients of the initial contour is characterized by different quality factor.

3. Quality factors gearing

These factors include angles $\alpha_l$ and $\alpha_p$ characterizing the appearance of satellite gear tooth undercut fillet and permitted satellite gear tooth undercut; tooth thickness $s_{a2}$ around the satellite tip circumference; interference of transition surface of central gearwheel with tip of satellite tooth in external and internal gearing; overlap factor $\varepsilon_{2,3}$ in internal gearing of central gearwheel and satellite. The listed quality factors are calculated according to the method [5,6].

The boundary point determining the appearance of tooth undercut fillet of the corresponding toothed link cut by the rack tool is characterised by the angle $\alpha_l$ calculated by the formula:

$$tg\alpha_{lk} = tg\alpha - \frac{4 \cdot (h_l^* - h_a^* - x_k)}{Z_k \cdot \sin 2\alpha},$$

where $\alpha = 20^\circ$ is angle of initial contour profile; $h_l^* = 2$ is factor of adjacent point of initial contour tooth; $h_a^* = 1$ is factor of tooth point; $Z_k$ is the number of teeth of the corresponding gear link; $x_k$ is the coefficient of displacement of the initial contour of the tool during processing of the corresponding gear link; $k = 1$ for an externally gearing central gearwheel, $k = 2$ for a satellite, $k = 3$ for an internally gearing central gearwheel.

There is no undercut provided:

$$\alpha_{lk} > 0. \quad (2)$$

The bottom point of the active profile, which defines the undercut, is characterized by the angle $\alpha_p$, calculated by the formula:

at gearwheel $Z_1$:

$$tg\alpha_{p1} = tg\alpha_{ol-2} + \frac{Z_2}{Z_1} \cdot (tg\alpha_{a2} - tg\alpha_{ol-2}),$$

$$tg\alpha_{p2} = tg\alpha_{ol-3} + \frac{Z_3}{Z_2} \cdot (tg\alpha_{a3} - tg\alpha_{ol-3}),$$

$$tg\alpha_{p3} = tg\alpha_{ol-4} + \frac{Z_4}{Z_3} \cdot (tg\alpha_{a4} - tg\alpha_{ol-4}).$$
at gearwheel $Z_2$:

$$\tan \alpha_{p2} = \tan \alpha_{a2-3} + \frac{Z_3}{Z_2} \cdot \left( \tan \alpha_{a3} - \tan \alpha_{a2-3} \right),$$  \hspace{1cm} (4)

at gearwheel $Z_3$:

$$\tan \alpha_{p3} = \tan \alpha_{a2-3} + \frac{Z_2}{Z_3} \cdot \left( \tan \alpha_{a2} - \tan \alpha_{a2-3} \right),$$  \hspace{1cm} (5)

where $\alpha_{a2-3} = \arccos \left( m \cdot (Z_1 + Z_2) \cdot \cos \alpha / 2 \cdot \cos \beta \cdot a_{w1-2} \right)$ is angle of external gearing; $\alpha_{a2-3} = \arccos \left( m \cdot (Z_3 - Z_2) \cdot \cos \alpha / 2 \cdot \cos \beta \cdot a_{w1-2} \right)$ is angle of internal gearing; $a_{w1-2} = a_{w2-3}$ is center of distance; $\alpha_{a2} = \arccos \left( m \cdot Z_2 \cdot \cos \alpha / d_{a2} \right)$ is angle corresponding to point lying on the satellite tip circumference; $\alpha_{a3} = \arccos \left( m \cdot Z_3 \cdot \cos \alpha / d_{a3} \right)$ is angle corresponding to point lying on the central gearwheel tip circumference with internal gearing; $d_{a2}$ is diameter of tip circumference of satellite; $d_{a3}$ is diameter of tip circumference of central gearwheel with internal gearing; $\beta = 0$ is helix angle of tooth line.

Undercut becomes invalid if:

$$\alpha_{pk} > 0.$$  \hspace{1cm} (6)

The thickness of the teeth on the satellite tip circumference is as follows:

$$s_{a2} = d_{a2} \left( \frac{\pi}{2 \cdot z_2} + \frac{2 \cdot x_2 \cdot \tan \alpha}{z_2} + \text{inv} \alpha - \text{inv} \alpha_{a2} \right),$$  \hspace{1cm} (7)

where $\text{inv} \alpha_{a2} = \tan \alpha_{a2} - \alpha_{a2}$ is the evolvent angle corresponding to the profile angle.

Note here that equal number of central gearwheel teeth is ensured by correct selection of displacement coefficients of central gearwheel inner-gearing $x_1$, satellite $x_2$ and central gearwheel outer-gearing $x_1$. Preliminary selection of numerical values of displacement coefficients $x_1$, $x_2$, $x_3$ is made on the basis of solution of system of linear equations ensuring equality of center of distance of external and internal gearing:

$$\frac{x_2 + x_1}{Z_2 + Z_1} \cdot 2 \cdot \tan \alpha + \text{inv} \alpha - \text{inv} \alpha_{w1-2} = 0;$$

$$\frac{x_3 - x_2}{Z_3 - Z_2} \cdot 2 \cdot \tan \alpha + \text{inv} \alpha - \text{inv} \alpha_{w2-3} = 0;$$

$$\frac{(z_2 + z_1) \cdot \cos \alpha}{2 \cdot \cos \beta \cdot \cos \alpha_{w1-2}} - \frac{(z_3 - z_2) \cdot \cos \alpha}{2 \cdot \cos \beta \cdot \cos \alpha_{w2-3}} = 0,$$  \hspace{1cm} (8)

where $Z_1$, $Z_2$, $Z_3$ is the number of teeth of center gearwheel with external gearing, satellite and central gearwheel with internal gearing; $\text{inv} \alpha = \tan \alpha - \alpha$ is the evolvent angle corresponding to the profile angle.

Overlap factor in internal gearing is determined by formula:

$$\varepsilon_{2-3} = \frac{Z_3 \cdot (\tan \alpha_{a3} - \tan \alpha_{a2-3}) - Z_2 \cdot (\tan \alpha_{a2} - \tan \alpha_{a2-3})}{2 \cdot \pi}. $$  \hspace{1cm} (9)

Interference in external and internal gearing is absent provided:
4. Algorithm for selection of the range of permissible values of displacement coefficients in external and internal gearing of PRHM

The algorithm of selection of the range of permissible values of the PRHM displacement coefficients consists of the following main stages: 1) selection of the displacement coefficient of the initial contour of the instrument for satellite processing according to the conditions: absence of undercut of satellite tooth is formula (2) and (6); minimum thickness of satellite tooth is formula (7). 2) Preliminary selection of displacement coefficients of initial contour of tool for processing of central gear wheel with external and internal gearing according to condition of equality of center of distance is formula (8). 3) Selection of displacement coefficient of initial contour of tool for processing of central gear wheel with external and internal gearing according to conditions: absence of undercut of satellite tooth is formula (2) and (6); absence of gearwheel side surface interference and satellite tooth tip is formula (10). 3) Selection of displacement coefficients of initial contour of tool for processing of central gear wheel with internal gearing according to conditions: permissible value of overlap coefficient in internal gearing is formula (9); absence of interference between wheel side surface and satellite tooth tip is formula (10); absence of undercut of satellite tooth is formula (2) and (6).

The range of allowable values of the displacement coefficients of the initial contour for the planetary mechanism based on the PRHM can also be obtained using Mathcad computer mathematics software and Kompas-3D graphical modeling environment (built-in module «Shafts and mechanical transmissions»).

5. Research results

According to the proposed algorithm, areas of permissible values of displacement coefficients $x_2$ of the initial contour of the instrument were obtained during processin g of the satellite for $Z_2 = 8$ (figure 2) and diagrams of displacement coefficients according to the condition of equality of center of distance (figure 3). In figure 3 shows the range of permissible values of the displacement coefficients of the original contour during the processing of the PRHM gear links for external and internal gearing (figure 4), corresponding to the same number of central gearwheel teeth $Z_1 = Z_3 = 120$. The selection of geometric parameters of the planetary mechanism, which is based on the PRHM, is carried out using diagrams taking into account the range of permissible values of displacement coefficients. As a result of the calculations, the following rational parameters of the central gearwheels and satellites of the PRHM with the same number of teeth were selected: $Z_1 = Z_3 = 120; Z_2 = 8; x_1 = -0.8; x_2 = 0.6; x_3 = 11.315$.

$$\alpha_{lk} < \alpha_{pk}.$$  \hspace{1cm} (10)
Figure 3. Diagrams of displacement coefficients of PRHM gear links by condition of equality of center of distance

Figure 4. The range of permissible values of displacement coefficients of the initial contour during processing of PRHM gear links for external gearing and internal gearing: 1 is boundary of the satellite tooth undercut fillet; 2 is the interference of the satellite transition surface and the tip of the central gearwheel tooth; 3 is the line corresponding $\varepsilon = 1.1$; 4 is the line corresponding $\varepsilon = 1.05$.

Using Mathcad computer mathematics software and Kompas-3D graphical modeling environment, the movement of the PRHM links was visualized and an gearing pattern for the selected parameters was constructed (figure 5), confirming the correctness of the calculations.

For profiling of non-circular central gearwheels of hydraulic machines according to schemes $M = N = 2$, $M = N = 3$, etc., manufactured using 2D-technology, it is possible to use the procedure given in articles [7, 8]. In another method, the profiles of the central gearwheel rims are obtained by envelope the profile of the satellite performing the corresponding movement using CAD systems [9].
6. Conclusions
Thus, the algorithm proposed by the author, based on the formulas GOST 16532-70 and GOST 19274-73, arranged in a certain sequence using Kompas and Mathcad, allows to select the parameters of the central gearwheels of the planetary mechanism, built in the base of the PRHM with the same number of teeth, in case of evolvent gearing.

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