Bottom and Charm quark masses from lattice NRQCD

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We present new values for the \( \overline{\mathrm{MS}} \) masses of \( b \) and \( c \) quarks based on lattice NRQCD simulations of the \( \Upsilon(b\bar{b}) \) and \( \psi(c\bar{c}) \) systems. These include three measurements of the \( b \) mass based on quenched simulations with lattice spacings ranging from 0.05fm to 0.15fm, which we find to be largely independent of lattice spacing. In addition, we find a consistent value from an unquenched simulation at 0.08fm.

1. Simulations

The \( \Upsilon(b\bar{b}) \) and \( \psi(c\bar{c}) \) systems possess several properties which permit accurate lattice simulations. They are physically small, allowing the use of small volumes. They are insensitive to the presence of light quarks and to uncertainties in their masses. They are well understood phenomenologically, which aids in the estimation of systematic errors. Their spin-averaged mass splittings are insensitive to the \( b \) or \( c \) masses, allowing independent tuning of the lattice spacing and bare masses. Their properties have been precisely measured. Finally, they may be efficiently simulated using a nonrelativistic effective action, NRQCD.

To compute the spin-averaged spectra we employed an NRQCD action for the heavy quarks, improved to include leading and next-to-leading order corrections for relativity and discretization errors. We used the average plaquette to tadpole improve the link operators. Several collaborations generously provided gauge field configurations for both zero and two light quark flavors, generated using the standard (unimproved) Wilson action for gauge fields and staggered fermions for the light quarks. We removed perturbatively the leading discretization errors in the spectrum due to use of this unimproved gluonic action. The details of our simulations and references for the gauge field configurations appear in Ref. [2].

2. Extracting \( \overline{M}_b \) and \( \overline{M}_c \)

Results from these simulations allowed us to obtain a value for the \( c \) quark \( \overline{\mathrm{MS}} \) mass \( \overline{M}_c \) and values for the \( b \) quark mass \( \overline{M}_b \) at several lattice spacings, as follows. We first measured the lattice spacing \( a \) for each simulation by comparing either the computed spin-averaged 1P-1S or 2S-1S splittings with data. Because these splittings are very insensitive to the bare mass \( aM_0^0 \), we were able to determine \( a \) without first finely tuning the mass. The results were accurate to within about 5% for the \( \Upsilon \) system, 15% for \( \psi \).

Using this \( a \), we tuned the bare mass \( aM_0^0 \) so
Table 1
Perturbative parameters connecting the bare mass $M^0$ to the $\overline{\text{MS}}$ mass $\overline{M}(\mu)$.

| $\beta$ | $n_\ell$ | $aM^0_\ell$ | $a\mu$ | $c^{(1)}(\mu)$ | $aq^*$ | $\alpha_\rho(q^*)$ |
|---------|----------|--------------|--------|----------------|--------|------------------|
| 5.7     | 0        | 3.15         | 5      | -1.18          | 3.03   | .1911            |
| 6.0     | 1.71     | 1.63         | 5.73   | .1315          |
| 6.2     | 1.22     | -1.79        | 9.80   | .1088          |
| 5.6     | 2        | 1.80         | -1.61  | 5.34           | .1570  |

$aM^0_\ell$

| 5.7 | 0 | 0.80 | 7 | -2.85 | 23.02 | .1086 |

that the kinetic mass $M_{\text{kin}}$, determined by fitting to the dispersion relation

$$E(p) = E(0) + \frac{p^2}{2M_{\text{kin}}} - \frac{p^4}{8M_{\text{kin}}^3},$$

agreed with experimental values for the $\Upsilon$ or $\eta_c$.

Once the fundamental parameters $aM^0$ and $a$ were fixed, we were able to convert quantities to physical units. By using the expression

$$M^0 = aM^0 \left( \frac{M_{\text{expt}}}{aM_{\text{kin}}} \right),$$

we minimized sensitivity to uncertainties in $aM^0$ and $a$. The bulk of $aM_{\text{kin}}$ is made up of $2aM^0$, with dynamics providing the remainder. As a result, the ratio $aM^0/aM_{\text{kin}}$ is very insensitive to errors in the dynamics from uncertainties in $aM^0$ and $a$; these represent only an error in an order $v^2$ correction to $1/2$. As an extreme illustration, computing $M^0_\ell$ using a value for $aM^0_\ell$ as far off as $aM^0_\ell$ gives a result off by only 25%. In contrast, using $a^{-1}$ directly in the form $M^0 = (aM^0)a^{-1}$ rather than Eq. 3 would unnecessarily promote errors in $a^{-1}$ by one power of the mass.

Refs. 3 and 4 give perturbative expressions relating the $\overline{\text{MS}}$ mass $\overline{M}$ and the bare mass $M^0$ to the pole mass, respectively. We combined these to obtain

$$\overline{M}(\mu) = M^0 \left[ 1 + c^{(1)}(\mu) \frac{\alpha_\rho(q^*)}{\pi} + \cdots \right],$$

with parameters listed in Table 1. We used the scheme in Ref. 3 to determine the scale $q^*$ for $\alpha_\rho$ in this relation. In particular, we chose to extract $\overline{M}(\mu)$ at a scale $\mu$ such that $q^*$ was well-defined. Values for $\alpha_\rho$ came from measurements of the plaquette in these same simulations.

Finally, we ran $\overline{M}(\mu)$ to its own scale using three-loop evolution. Our preliminary results are presented in Table 2. Insensitivity of the final result to the bare mass $aM^0_\ell$ is evident. The systematic errors quoted account for higher-order relativistic and discretization corrections, most of which were based on estimates from a potential model. They do not account for errors associated with using the wrong number of light quarks. Values for $\overline{M}_\ell$ at three lattice spacings and for zero and two light flavors $n_\ell$ also appear in Figure 1.

We also applied an alternate method to determine the $\Upsilon$ kinetic mass. Rather than fitting to the dispersion relation above, we determined $aM_{\text{kin}}$ from

$$aM_{\text{kin}} = 2 Z_m aM^0 + aE_{\text{NR}} - 2 aE_0.$$  

The first term on the right hand side gives twice the pole mass. The next two represent the energy for this state obtained in the simulation with the self energy of the quarks subtracted, which gives the binding energy. The quark mass renormalization $Z_m$ and self energy $aE_0$ were computed perturbatively to one loop in Ref. 4.

We used this kinetic mass to convert the bare mass to physical units and obtained the $\overline{\text{MS}}$ masses as above. Results were consistent with the first method, but with larger errors due to unknown higher-order terms in $Z_m$ and $aE_0$. 


Table 2

MS masses for $b$ and $c$ quarks (preliminary). The lattice spacing $a$ is determined from spin-averaged splittings between 1P and 1S states. Values for $M^0$ are from Eq. (2) rather than from $(aM^0)^{a(-1)}$, as discussed in the text. Errors are statistical and estimates of higher order relativistic and discretization corrections, respectively. Systematic errors are combined in $M^0$ and $\overline{M}(\overline{M})$. The final error in $\overline{M}(\overline{M})$ estimates neglected higher-order perturbative contributions. We do not include an error estimate to account for quenching.

| $\beta$ | $n_f$ | $aM^0_b$ | $a_{1P-1S}^{(-1)}$ (GeV) | $a_{2S-1S}^{(-1)}$ (GeV) | $M^0_b$ (GeV) | $\overline{M}_b(\overline{M}_b)$ (GeV) |
|---------|-------|-----------|-----------------|-----------------|-------------|-----------------|
| 5.7     | 0     | 3.15      | 1.41(4)(2)(5)   | 1.36(13)(2)(4)  | 4.22(5)(8)  | 4.15(5)(8)(3)   |
| 6.0     | 1.71  | 2.59(5)(3)(1) | 2.45(8)(3)(1)  | 4.11(3)(2)     | 4.28(3)(3)(3) |
|         | 1.80  |           |                 |                 | 4.16(3)(2)   |                 |
|         | 2.00  |           |                 |                 | 4.21(3)(2)   |                 |
|         | 3.00  |           |                 |                 | 4.32(3)(2)   |                 |
| 6.2     | 1.22  | 3.52(14)(4)(0) | 3.24(15)(4)(0) | 3.99(6)(2)     | 4.31(6)(3)(3) |
| 5.6     | 2     | 1.80      | 2.44(6)(3)(1)   | 2.38(10)(3)(1)  | 4.08(4)(3)  | 4.26(4)(3)(5)   |
|         |       |           |                 |                 |             |                 |
| 5.7     | 0     | 0.80      | 1.23(4)(3)(8)   | 1.20(20)(3)(8)  | .98(3)(9)   | 1.20(4)(11)(2)  |

errors: stat, rel, discr stat, sys stat, sys, pert

3. Conclusions

We presented new results for $\overline{M}_b$ and $\overline{M}_c$ from lattice NRQCD calculations of the $\Upsilon$ and $\psi$ spin-averaged spectra. Our results for $\overline{M}_b$ showed little dependence on $a$ or on the presence of light quarks. They are consistent with those presented at Lattice 97 and summarized in Ref. [6]: $\overline{M}_b = 4.15(5)(20)$ and $\overline{M}_c = 1.525(40)(125)$ from the APE group using HQET, and $\overline{M}_c = 1.33(8)$ from the FNAL group, using an action which interpolates between relativistic and nonrelativistic regions.

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