Structural and kinematic analysis of elbow rehabilitation equipment

G Vetrice and A Deaconescu
Department of Industrial Engineering and Management, Transilvania University of Brasov, Mihai Viteazul Street, no. 5, 500174, Brasov, Romania
E-mail: vetrice.georgiana@unitbv.ro

Abstract. The aim of this paper is to present the structural and kinematic analysis of a new constructive solution for the rehabilitation of patients suffering from posttraumatic elbow affections. The equipment with two degrees of freedom, one for flexion-extension movement and one for pronation-supination should be able to ensure the full recovery of the elbow joint. Each movement is achieved by means of a pair of pneumatic muscles. The structural analysis of the mechanism includes drawing the constructive, structural and block diagrams, identifying the exterior links (number of inputs and outputs), the mobility of the mechanism (the number of degrees of freedom), the analysis of the transmission of movements and forces and also the analysis of the hyperstaticity of the mechanism. The kinematic pairs composing the mechanisms are isostatic, i.e. with no potential links. The kinematic analysis involves equations that describe the kinematic behaviour of the rehabilitation equipment. Such behaviour can be described analytically or graphically. In the latter case, the equations will underlie the variation graphs of position, velocity and acceleration versus time plotted by means of SolidWorks software.

1. Introduction
Following studies carried out on the biomechanics of the elbow, the ranges of motion that resulted for each movement were: -90°...+50° (140° range of motion) for flexion-extension movement and ±90° for pronation-supination [1]. Figure 1 shows the kinematic diagram of the elbow biomechanics; this is the case of a mechanical pair with two degrees of freedom, the respective axes of rotation being perpendicular one to another. Thus, similarly, the rehabilitation equipment includes two modules of rotation, one for each movement. It was conceived to allow the full recovery of the joint, by applying constant motion to the joint, without contracting the patient’s muscles, a method so-called continuous passive motion.

The equipment must provide safety, comfort and easy utilization, since it can be used in both medical units and in patient homes. For a friendly human-machine interaction, the linkage of the equipment has to be similar to the linkage of the human upper limb, in order to perform exactly the elbow movements that need to be recovered.

Since the equipment developed so far are electrically driven and have a rigid structure, conceiving such equipment is necessary. Constructive simplicity, reduced recovery period, ergonomy and safety are characteristics that prove this equipment can be used efficiently.
2. Structural analysis of the elbow rehabilitation equipment
Taking into consideration that the equipment will perform the movements only separately, not simultaneously, they will be treated separately in the following.

According to [2], the structural diagram is a simplified representation of the mechanism, not scaled, including the basic element and the kinematic objects composing the mechanism, the kinematic pairs between the elements, according to their actual relative position, the inputs and outputs and the orthogonal trihedron as a frame of reference for the mechanism.

Figure 2 and figure 3 show the structural diagrams of the two mechanisms.

Figure 1. The kinematic diagram of elbow biomechanics.

Figure 2. Structural diagram of the mechanism for flexion-extension.

Figure 3. Structural diagram of the mechanism for pronation-supination.
In the first case, the mechanism consists of a fixed element (0), a mobile element (1) and a rotation pair A, which causes the movement of elements 2 and 2’ around the x axis.

In the second case, the mechanism consists of three fixed elements: (0), (1) and element (2’) that formerly was mobile and becomes fixed and the rotational pair B, which causes the rotation of element (2) around the y axis.

Each mechanism has a rotational kinematic pair, in order to perform the two rehabilitation motions. According to [3], a kinematic pair is a binary system characterized by two reciprocal linear spaces:
- the space of relative velocities allowed by links \{f\}, where the dimension of the space, \(f\), is called “degree of freedom” and the basis of space (f) represents the velocities (translation or rotation) allowed by the pair;
- the space of transferable forces through the link \{c\}, where the dimension of the space, \(c\), is called “degree of restrictiveness” and the basis of space (c) represents the forces and torques in the pair.

The equipment has a simple linkage, consisting of two open chains, each one having one lower pair, permitting a rotation.

Figure 4 presents the constructive diagram of the mechanism relating to flexion-extension and in figure 5 presents the constructive and structural diagrams of the rotation pair.

Figure 6 shows the constructive diagram of the mechanism relating to pronation-supination and figure 7 shows the constructive and structural diagrams of the rotation pair.

![Figure 4](image1.png)

**Figure 4.** Constructive diagram of the mechanism relating to flexion-extension.

![Figure 5](image2.png)

**Figure 5.** Constructive and structural diagram of the rotation pair relating to flexion-extension.

![Figure 6](image3.png)

**Figure 6.** Constructive diagram of the mechanism relating to pronation-supination.

![Figure 7](image4.png)

**Figure 7.** Constructive and structural diagram of the rotation pair relating to pronation-supination.
For the first mechanism (flexion-extension) the following quantities were determined:

- velocities space dimension \( f_{10} = f = 1 \);
- velocities space basis \((\omega_x)\);
- forces space dimension \( c_{10} = c = 6 - f = 5 \);
- space basis \((M_x, M_z, P_x, P_y, P_z)\).

For the second mechanism (pronation-supination) the following quantities were determined:

- velocities space dimension \( f_{21} = f = 1 \);
- velocities space basis \((\omega_x)\);
- forces space dimension \( c_{21} = c = 6 - f = 5 \);
- space basis \((M_x, M_z, P_x, P_y, P_z)\).

In figure 8 it can be noticed that the first mechanism (related to flexion-extension) has two exterior links, one input (including the position, velocity and acceleration parameters \(\theta_{10}, \dot{\theta}_{10}, \ddot{\theta}_{10}\)) and one output (including the position, velocity and acceleration parameters \(\theta_{21}, \dot{\theta}_{21}, \ddot{\theta}_{21}\)).

In figure 9 it can be noticed that the second mechanism (related to pronation-supination) has two exterior links as well, one input (including the position, velocity and acceleration parameters \(\theta_{21}, \dot{\theta}_{21}, \ddot{\theta}_{21}\)) and one output (including the position, velocity and acceleration parameters \(\theta_{22}, \dot{\theta}_{22}, \ddot{\theta}_{22}\)).

![Figure 8](image1.png)  
**Figure 8.** Structural diagram of the open chain for flexion-extension.

![Figure 9](image2.png)  
**Figure 9.** Structural diagram of the open chain for pronation-supination.

Each mechanism has also two exterior torques. For flexion-extension these are an input torque \(M_{10}\) (having the same direction as velocity \(\dot{\theta}_{10}\)) and an output torque \(M_{21}\) (in the opposite direction to velocity \(\dot{\theta}_{10}\)). For pronation-supination these are an input torque \(M_{21}\) (having the same direction as velocity \(\dot{\theta}_{21}\)) and an output torque \(M_{22}\) (in the opposite direction to velocity \(\dot{\theta}_{21}\)).

According to [3], the degree of freedom of an open kinematic mechanism is:

\[
M = \sum_{i=1}^{p} f_i \tag{1}
\]

where \(p\) is the number of kinematic pairs of the mechanism and \(f_i\) the degree of freedom of the kinematic pair.
Thus, 
- for flexion-extension: \( M = f_a = 1 \) (one rotational motion);  
- for pronation-supination: \( M = f_b = 1 \) (one rotational motion).

The degree of freedom of the equipment is \( M = f_a + f_b \).

According to [3], both mechanisms have the degree of hiperstaticity equal to 0, i.e. the mechanisms are isokinetic.

The number of inputs and outputs, calculated previously for each mechanism, is \( L = 2 \) (one input and one output), and the degree of freedom for each mechanism is \( M = 1 \) (one rotational motion).

As stated in [3], the values above have the following meaning:
- \( M = 1 \): one independent motion \( \theta_{10} \) (for flexion-extension) and \( \theta_{21} \) (for pronation-supination) and one torque transfer function \( M_{10} = M_{10}(\theta_{10}) \) for flexion-extension and \( M_{21} = M_{21}(\theta_{21}) \) for pronation-supination.
- \( L - M = 1 \): one independent torque \( M_{10} \) for flexion-extension and \( M_{21} \) for pronation-supination and one torque transfer function \( \theta_{21} = \theta_{21}(\theta_{10}) \) for flexion-extension and \( \theta_{22} = \theta_{22}(\theta_{21}) \) for pronation-supination.

Based on this information, the block diagrams for the two mechanisms are shown in figure 10 and figure 11.

**Figure 10.** Block diagram of mechanism for flexion-extension.  
**Figure 11.** Block diagram of mechanism for pronation-supination.

Both mechanisms consist of elementary kinematic pairs, thus they are isostatic (no potential links). The equations of the pair are based on the independent torques.

### 3. Kinematic analysis of the elbow rehabilitation equipment

The kinematic analysis entails writing the equations that describe the kinematic behaviour of the equipment. The equations of rotational motions are:

\[
\begin{align*}
\omega &= \omega_0 + \alpha t \quad (2) \\
\theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad (3) \\
\omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0) \quad (4)
\end{align*}
\]

where \( \theta \) is the angular displacement (\( \theta_0 \) - initial angular displacement), \( \omega \) is the angular velocity (\( \omega_0 \) - initial angular velocity), \( \alpha \) is the angular acceleration (\( \alpha_0 \) - initial angular acceleration) and \( t \) is the time necessary to perform a rotation.

Based on the equations (2...4) and the 3D model, further on the displacement, velocity and acceleration will be determined by means of the motion study module of SolidWorks software. Since the rotation of mechanism is generated by a pair of pneumatic muscles based on the antagonistic principle, two graphs are necessary for each quantity because the muscle is in relaxed state at different values of the angular displacement.

In the first case (flexion-extension) the mechanism must perform a rotation of 50° above the horizontal and of 90° below the horizontal. In order to obtain the 140° range of motion, the mechanism will be placed in an initial position once at +50° (maximum flexion) and then at -90° (maximum extension). According to the specifications of the elbow equipment manufacturer, one complete rotation is achieved in 30-45 seconds. In this case we considered 30 seconds.
In the second case (pronation-supination) the mechanism must perform a rotation of 90° to the right and of 90° to the left. In order to obtain the 180° range of motion, the mechanism will be placed in an initial position once at -90° (maximum pronation) and then at +90° (maximum supination). According to the specifications of the elbow equipment manufacturer, one complete rotation is achieved in 30-60 seconds. In this case we considered 30 seconds.

Figure 12 shows the graphs that describe angular displacement, angular velocity and angular acceleration for the pair of muscles performing flexion-extension.

Figure 12. Variation versus time of the angular displacement, angular velocity and angular acceleration for flexion-extension.

Figure 13 shows the graphs that describe angular displacement, angular velocity and angular acceleration for the pair of muscles performing pronation-supination.
4. Conclusions
The paper presents the structural and kinematic analysis of elbow rehabilitation equipment. In order to ensure the safety of the patient’s recovery process, particular attention should be paid to the design of this equipment, hence the importance of this analysis.

Since its linkage has to be compatible with the linkage of the human limb, the axes of the equipment will be identical to the ones of the human limb (perpendicular to one another). Even if from a constructive point of view the rotation modules are quite similar, the movements were discussed separately and the constructive, structural and block diagrams were represented separately as well.

Regarding kinematics, based on the equations of the rotation modules, graphs of the angular displacement, angular velocity and angular acceleration were presented for both pairs of muscles.

5. References
[1] Malagelada F, Dalmau-Pastor M, Vega J and Golanó P 2014 *Sports Injuries* In: Doral M, Karlsson J (eds.) Springer, Berlin, Heidelberg
[2] Săulescu R 2008 *Mechanisms – distance learning course* Publishing House of Transilvania University of Brașov, Brașov 6
[3] Dudiță F and Diaconescu D 1984 *Mechanisms course. Fascicule 1: Structure* Publishing House of Transilvania University of Brașov, Brașov 59
[4] Crețu S 2010 *Mechanisms. Structural analysis* Sitech Publishing House, Craiova 68