Two pion photoproduction on nucleons and nuclei in the $\rho$ and $\sigma$ regions

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In this talk we report on two physical processes, the photoproduction of $\rho$ mesons in nuclei and the $\pi^0\pi^0$ photoproduction in nuclei at low energies. In the first case the aim is to observe experimentally the theoretically predicted changes on the $\rho$ properties in the nuclear medium. In the second case one wishes to investigate the modifications of the $\pi\pi$ interaction in the nuclear medium in the region of the $\sigma$ meson. In the first case we observe that it is quite difficult to see large effects due to the $\rho$ modification in the medium because the detection of the $\rho$ is done through the two pions which are produced in the $\Delta$ region and are largely distorted in the medium. In the second case, when the $I=0$ part of the $\pi\pi$ interaction is substituted by the in-medium $\pi\pi$ amplitude, we observe a very large shift of strength in the invariant mass distribution to small values of the mass in $^{12}\text{C}$ and $^{208}\text{Pb}$ with respect to the distribution of the elementary reaction. This spectacular shift appears to be corroborated by recent experiments at Mainz reported in this same Workshop.

I. INDEX

In this talk I shall report on recent work concerning the photoproduction of two pions in nuclei. The structure of the talk is as follows:

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II. INTRODUCTION

The study of meson modification in a nuclear medium is a challenging problem which attracts permanent theoretical and experimental attention. Even more challenging is the modification of the meson meson scattering amplitudes inside the medium. Pioneering work in this direction was done in [1] where the modification of the scattering amplitude led to the appearance of peaks below the two pion threshold which were suggested as possible Cooper pairs of two pions. In related work this phenomenon has been interpreted as a drop of the $\sigma$ mass in the nuclear medium in a way that could be interpreted as partial restoration of chiral symmetry. The observation of an accumulation of strength at low invariant mass of two pions in the pion induced $\pi^+\pi^-$ production compared to the one in deuterium, gave support to that idea. More recently, in pion induced $\pi^0\pi^0$ production, the peaks seen in $\pi^+\pi^-$ production are not observed but there is also accumulation of strength at lower invariant masses compared to the free case. The most conclusive evidence could, however, come from photon induced two pion production experiments which are reported in the present Meeting.
The theoretical work of 1 stimulated many other works where more detailed calculations were done by 2-4, qualitatively confirming the findings of 1. More refined calculations imposing chiral constraints in the $\pi\pi$ amplitudes 1,2 softened the response somewhat but still there was an enhancement of strength at low invariant masses close to threshold compared to that in free $\pi\pi$ scattering.

The advent of $\chi PT$ as an approach to the QCD dynamics at low energies 13,14 has permitted to have a new look into the problem. Yet, with all its success at low energies, $\chi PT$ has a limited range of applicability and being perturbative in nature does not generate the poles in the t-matrix. An important step forward, extending the predictive power of $\chi PT$ to higher energies, has been done recently by reordering the chiral expansion and imposing unitarity in coupled channels. First steps in this direction were done in 15,16 for the meson baryon interaction using a combination of the Lippmann Schwinger equation and input from chiral Lagrangians, and in 17 in the meson meson interaction using the Bethe Salpeter equation and just the lowest order Lagrangian. Further refinements along this line have been done in 17,18. Another line of progress has been the use of the Inverse Amplitude Method in coupled channels which allows one to reproduce all the data of meson meson scattering up to 1.2 GeV 19,22. Similarly, another unitarization method 23 has been developed by means of the N/D method, together with the explicit exchange of genuine resonances which according to the work of 24 account for the second order Lagrangian of 13,14. All these methods, which are now known as unitarized chiral perturbation theory ($U\chi PT$), give rise basically to the same results and reproduce very well the data on meson meson scattering up to 1.2 GeV, leading to poles in the t-matrix for the different resonances appearing up to that energy. Among these resonances, the scalar ones, $\sigma(500)$, $f_0(980)$, $a_0(980)$, are generated dynamically, meaning that with the use of the strong interaction provided by the lowest order meson meson chiral Lagrangian, together with the unitarity constraints which generate multiple scattering of the mesons, leads automatically to these resonances without the need to introduce them explicitly in the formalism. This is not the case for the vector mesons which require the explicit inclusion in the N/D method of 24 or the implicit inclusion through the use of the higher order Lagrangians in 19,25. The $\sigma$ meson which is of relevance to the present work is thus a $\pi\pi$ scattering resonance, and not a genuine QCD state which would survive in the large $N_c$ limit, something already suggested in 26,27. Hence the study of the medium modification of the $\pi\pi$ interaction around the $\sigma$ region and below with the $U\chi PT$ approach seems most appropriate. This work was carried out in 27 and came to reconfirm the findings of the previous works where minimal chiral constraints were used 1,12. Yet, a detailed description of the pion induced two pion production in nuclei 29, using the information of 27, does not lead to the peaks found in the experiment 14. These results contrast with those in 30, where some approximations about the effective density met by the pions were done, but due to a strong pion absorption the effective densities are smaller than used in 30, as found in 29.

The $\rho$ renormalization in the medium has also been the subject of intense debate and we address the reader to the review article 11 for a discussion of the different approaches and results. For the purpose of the present work it is sufficient to recall that the most recent works, including 32, give a moderate shift of the $\rho$ mass to higher energies and the width is increased by about 50 percent at normal nuclear matter density. In heavy ion reactions, where bigger densities can be reached, experiments using dilepton detection show a qualitative evidence that the width of the $\rho$ is increased although the results could also be interpreted in terms of a dropping mass.

In the present paper we will try to investigate whether the predicted medium modifications of the $\rho$ could be observed in some other reaction, concretely in $\gamma N \rightarrow \pi\pi N$ photoproduction in nuclei. Recent advances in the two pion photoproduction have shown evidence of $\rho$ production 13 even at photon energies around 800 MeV, where only the tail of the resonance is seen. The $\rho$ comes from the decay of the $D_{13}(1520)$ resonance which is excited for photons around this energy. A recent theoretical study of two pion photoproduction 33, improving the early results of 17,34, includes the mechanisms of $\rho$ production and also effects of the $\Delta(1700)$ excitation and leads to invariant mass distributions compatible with the experimental ones found in 31. Of course the $\rho$ is better seen at higher energies where the peak can be seen clearly in the invariant mass of the two pions 35. So we have studied the photoproduction at these higher energies, but not too high as to make the $\rho$ be produced with such large momenta in the nucleus that it decays outside the nucleus, in spite of its large width, in which case we would only see the free $\rho$ decay.

### III. MODEL FOR $\gamma N \rightarrow \pi\pi N$

As commented in the introduction we shall be using the updated model for $\gamma N \rightarrow \pi\pi N$ of ref. 36. The model is meant to cover energies up to 800 MeV, Mainz energies, where most of the experiments are being performed. It contains tree level diagrams containing the coupling of photons and pions to nucleons and resonances. The resonances included are the $\Delta(1232)$, $N^*(1440)$, $N^*(1520)$ and $\Delta(1700)$. In addition two pions can come out as a $\rho$ meson from the decay of the $N^*(1520)$ and $\Delta(1700)$ resonances and from a $\gamma NN\rho$ contact term. The model is depicted in figure 1.
FIG. 1. Feynman diagrams for $\gamma N \rightarrow \pi\pi N$

FIG. 2. Total cross section for $\gamma p \rightarrow \pi^+\pi^- p$ and $\gamma p \rightarrow \pi^0\pi^0 p$
FIG. 3. Two pion invariant mass distribution for $\gamma p \rightarrow \pi^+ \pi^0 p$. Continuous line: results with $\rho$ meson and $\Delta(1700)$ terms. Dashed line: results of the model without those contributions. Experimental data from ref. [34].

Results for the cross sections and invariant mass distributions can be seen in [30]. We show in fig. 2 cross sections for $\pi^+ \pi^-$ and $\pi^0\pi^0$ production on the proton together with experimental results from several experiments. In fig. 3 we show the invariant mass distribution of the two pions in the case of $\pi^+ \pi^0$ production on the proton. As one can see there the effect of the $\rho$ production is important in shifting strength to higher invariant masses in agreement with experiment. The model is sufficiently accurate to be used to study the photoproduction of two pions in nuclei. We shall do that for two cases, the two pion production in a $\rho$ resonating state and the two pion production at low energies in the region of the $\sigma$ meson.

**IV. $\rho$ PHOTOPRODUCTION**

In fig. 3 we have seen how the $\rho$ production manifests itself in the two pion invariant mass distribution. Since our aim is to explore the possibility of finding medium effects through $\rho$ photoproduction, we have used the model for higher energies where there is sufficient phase space so that the $\rho$ is seen as a peak in the invariant mass distribution. We are aware that the model so far is only meant to work in the Mainz region but we find that the shape obtained is in good agreement with the experiment [36] although the absolute cross section is larger. Lack of unitarity corrections from $\rho N$ final state interaction could be responsible for this larger strength, which however would not modify the shape of the mass distribution. Hence, since we simply want to compare the results for photoproduction in nuclei and on the nucleon the extrapolation of the model can serve very well the purpose. With this caveat, in fig. 4 we show the results for the $\gamma p \rightarrow \pi^+ \pi^0 n$ reaction at an energy $E_\gamma = 1250 MeV$.

As we can see, the shape of the $\rho$ meson is clearly seen in the invariant mass distribution. The peak reflects both the position and the width of the free $\rho$ meson since the strength of the $\rho$ excitation is quite big compared to the background.
V. $\rho$ PHOTOPRODUCTION IN NUCLEI

In order to obtain the nuclear photoproduction cross section we evaluate the photon selfenergy in nuclear matter of density $\rho$ due to the diagram depicted in fig. 5.

![Diagram of Photon Selfenergy](image)

FIG. 5. Photon selfenergy to calculate the $\gamma N \rightarrow \pi\pi N$ cross section in nuclear matter

The imaginary part of this diagram is obtained when the intermediate states (a particle-hole and two pions) are placed on shell in the integrations over the momenta of the intermediate states. The nuclear cross section is then given by [39]:

$$\sigma = -\frac{1}{k} \int d^3r \text{Im}\Pi(k, \rho(r))$$

(1)

where $\Pi(k, \rho(r))$ is the photon selfenergy and $k$ the photon momentum. Equation (1) is making implicit use of the local density approximation, since the photon selfenergy is evaluated at a fixed density and then this density is replaced by the local nuclear density at the point $r'$ in the integral.

The photon selfenergy corresponding to the diagram of fig. 5 is given by
\[-i \Pi(k) = - \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \sum_{s_i, s_f \text{ pol}} (-i) T(-i) T^* \]

and thus, performing the integration of the energy variables and substituting in eq. 1, we find for the cross section

\[
\sigma = \frac{\pi}{k} \int d^3 r \int d^3 \tilde{q} \int \frac{d^3 \tilde{q}}{(2\pi)^3} \int \frac{d^3 q_1}{(2\pi)^3} \int \frac{d^3 q_2}{(2\pi)^3} \frac{1}{2 \omega(q_1)} \frac{1}{2 \omega(q_2)} \]

\[
\cdot \sum_{s_i, s_f \text{ pol}} | T |^2 \tilde{n}(\tilde{p})[1 - n(\tilde{k} + \tilde{p} - \tilde{q}_1 - \tilde{q}_2)]
\]

\[
\cdot \delta(k^0 + E(\tilde{p}) - \omega(q_1) - \omega(q_2) - E(\tilde{k} + \tilde{p} - \tilde{q}_1 - \tilde{q}_2))
\]

Eq. 3 incorporates explicitly the Pauli blocking factor and assumes that there is no distortion of the particles. This is actually not the case. The photon is not distorted but the pions can be absorbed in their way out of the nucleus from the point \( \vec{r} \) of production which is the variable of integration in eq. 3. In order to take into account the distortion of the pions we make an eikonal approximation and remove from the pion flux those pions which undergo absorption, which indeed disappear, and also those which undergo quasielastic collisions because, even if they do not disappear, they change the momentum and energy of the pions in a considerable amount such that the \( \rho \) meson shape in the mass distribution will be lost and the events will go into the background. The eikonal factors that we consider are given by eq. 4 and the \( \pi N \) cross sections, used for the distortion due to quasielastic steps, are taken from experiment, while the \( C_{\text{abs}} \) coefficients are calculated theoretically from studies of the \( \Delta \) selfenergy in a nuclear medium and tested against experimental pion absorption in nuclei.

\[
F_i(\vec{r}, \vec{q}_i) = \exp \left[ - \int \infty \vec{r}_i \left\{ \sigma_{\pi, \rho} \rho_\pi(\vec{r}_i) + \sigma_{\pi, n} \rho_n(\vec{r}_i) + \rho^2(\vec{r}_i) + \rho^3(\vec{r}_i) \right\} \right]
\]

\[
\cdot \delta(k^0 + E(\tilde{p}) - \omega(q_1) - \omega(q_2) - E(\tilde{k} + \tilde{p} - \tilde{q}_1 - \tilde{q}_2))
\]

The pions coming from \( \rho \) decay are mostly in the \( \Delta \) resonance region where the studies of \( \Delta \) are done. For the few events where pions are produced with higher energies we use the parameterization of the absorption coefficients given in \( \Delta \). The superindex \((2)\) and \((3)\) in the absorption coefficients indicate absorption on two or three nucleons, a topic thoroughly investigated experimentally in the meson factories.

The results obtained for the invariant mass distribution in the nucleus of \( ^{40}Ca \) for \( E_\gamma = 1250 \text{MeV} \) are shown in fig. \( \Delta \).
In the figure we show the results obtained using the free amplitude for $\gamma N \rightarrow \pi \pi N$ and those obtained by changing the width in the $\rho$ meson propagator by

$$\Gamma_\rho = \Gamma_{\rho\text{free}}(1 + \frac{\rho}{\rho_0})$$

(5)

This corresponds to doubling the $\rho$ width at nuclear matter density, which seems to be about double from the predicted in the latest theoretical approaches $[31]$, $[32]$. Even then, the results obtained are rather moderate. The changes in the width induce about a 20 percent decrease in the peak of the cross section and a corresponding increase of the width in about the same amount. The effects found in the cross section are much smaller than those assumed in the change of the in medium $\rho$ width. This disappointing result must be seen in the fact that the pions produced are mostly in the region of the $\Delta$ resonance where the probability that the pions undergo collisions or absorption is largest. There is also another factor that goes against finding large effects of $\rho$ modification. Unfortunately for this reaction, the $\rho$ is produced with a finite momentum and then before it decays it travels a certain distance inside the nucleus approaching the surface where the density dependent width resembles more the free width. We take this into account using in the integration the $\rho$ width at the point reached by the $\rho$ after its lifetime in the medium, which we take at the intermediate point between the point of production and the point of decay.

We have also checked the effects due to a hypothetical mass change of the $\rho$ and the results found, even for large changes in the mass like 150 MeV at normal nuclear matter density, are very moderate as can be seen in the figure. This finite traveling inside the nucleus is a handicap for observation of the medium effects, which is avoided in the dilepton production processes where one tries to maximize effects by looking to dileptons back to back, which would imply production of the $\rho$ at rest. One could try to minimize this effect by putting cuts in the $\rho$ momentum and concentrating only in events where the $\rho$ is produced with a small momentum, which would correspond to events where the $\rho$ is produced backwards in the $\gamma N$ CM frame and one takes further advantage of the Fermi motion. This procedure was successfully used in $[33]$ in order to find medium effects in $\phi$ photoproduction. Yet, one was looking there for slow kaons which were not so drastically distorted as the pions from $\rho$ decay. We have done similar tests here but the results are still very similar to those shown in fig. 3 only with smaller cross sections.

In view of the present results and discussion, the conclusion seems to be that trying to see medium effects of the $\rho$ from $\rho$ photoproduction in nuclei is quite a difficult task. The fact that the $\rho$ is produced with a finite momentum, and most important, that the pions are produced in the resonance region where the distortion of the pions is very large, has as a consequence that the process is very peripheral and tests very small densities where the medium modifications
are small.

VI. (γ, 2π) IN THE σ REGION

In section III we could see that the agreement with experiment of the γp → π0π0p reaction is overall good but in the region of the σ, Eγ around 400 to 500 MeV, our cross section is smaller than experiment. Actually, this is an interesting point, since as shown in [44], the consideration of chiral loops at threshold increased appreciably the π0π0 production rate. Here we will find similar results in the region of the σ, but in order to show this we need to introduce the elements of ππ scattering from the perspective of $U\chi PT$, which we review briefly in the next section.

VII. ππ INTERACTION IN UNITARY CHIRAL PERTURBATION THEORY

The starting point in this discussion is the chiral Lagrangian [14]

$$L_2 = \frac{f^2}{4} \langle \partial_\mu U^\dagger \partial^\mu U + M(U + U^\dagger) \rangle$$

(6)

where the symbol $\langle \rangle$ stands for the trace of the SU(3) matrices involved in the Lagrangian, $f$ is the pion decay constant, $M$ the mass matrix $\text{diag}(m_\pi^2, m_\rho^2, 2m_K^2 - m_N^2)$ and $U$ is the standard SU(3) matrix involving the meson fields $\{\pi, \rho, K, \bar{K}\}$. In addition there would be a second order Lagrangian involving the $L_i$ coefficients of Gasser and Leutwyler [14]. In $\chi PT$, omitting crossed loops, the lowest order Lagrangian provides the $O(p^2)$ contribution to the meson meson scattering matrix, $T_2$, while the $O(p^4)$ contribution comes from the loop in fig. 7 with the two vertices obtained from the lowest order Lagrangian, plus the polynomial contribution from the second order Lagrangian, $T_4^{(\text{pol})}$.

![Fig. 7. Contributions to de T matrix up to O(p^4)](image)

The $T$ matrix to order $O(p^4)$ is then given by

$$T = T_2 + T_4$$

(7)

The unitary extensions described in the introduction rely upon the implementation of exact unitarity in coupled channels

$$\text{Im}T = T\sigma T^*$$

(8)

$$\sigma \equiv \sigma_{\text{ll}} = -\frac{k_l}{8\pi\sqrt{s}} \theta(s - (m_{1l} + m_{2l})^2)$$

(9)

where $\sigma$ is a diagonal matrix( in the space of the coupled channels) which accounts for the phase space of the intermediate two mesons which we consider in our approach (ππ and $K\bar{K}$ in our case). In order to connect with $\chi PT$ we realize that $\sigma$ is the imaginary part of the loop function of two mesons in fig. 8, $G_{\text{ll}}$

$$G_{\text{ll}} = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_{1l}^2 + i\epsilon} \frac{1}{(P - q)^2 - m_{2l}^2 + i\epsilon}$$

$$\sigma_{\text{ll}} = \text{Im}G_{\text{ll}}$$

(10)

(11)

Hence we can write

$$\text{Im}T = T\text{Im}G^* \rightarrow \text{Im}G = -\text{Im}T^{-1}$$

(12)
\[ T^{-1} = ReT^{-1} - iImG \rightarrow T = (ReT^{-1} - iImG)^{-1} \]  

(13)

Although the different unitary approaches discussed at the beginning are technically different, the essence of all of them is that an expansion in powers of \( O(p^2) \) is done for \( ReT^{-1} \), not \( T \), and by virtue of this one succeeds in providing:

1. Faster convergence of the expansion
2. Larger convergence radius in the energy variable
3. The meson resonances up to 1.2 GeV

In the IAM method one gets now expanding \( ReT^{-1} \) up to \( O(p^4) \)

\[ T = T_2(T_2 - T_4)^{-1}T_2 \]

(14)

Now it is interesting to note that \( T_4 \) is fixed in chiral perturbation theory but neither the loop contribution not the \( T_4(\text{pol}) \) are defined since they both depend on the regularization scheme applied to renormalize the loops, only their sum is fixed. Now assume that we are able to choose a renormalization scheme (a cut off in \([16]\)) such that the \( T_4(\text{pol}) \) contribution is minimized. In such case \( T_4 \) is just \( T_2GT_2 \) and then eq. (13) becomes

\[ T = (1 - T_2G)^{-1}T_2 \rightarrow T - T_2GT = T_2 \]

(15)

which is nothing but the Bethe Salpeter equation with \( T_2 \) as a kernel (potential)

\[ T = T_2 + T_2GT \]

(16)

This procedure justifies why in \([16]\) one could get good results for the scalar sector in terms of only the lowest order Lagrangian. In particular one obtained the scalar resonances \( \sigma(500) \), \( f_0(980) \) \( a_0(980) \) and \( \kappa(900) \). The results obtained with this procedure are practically identical to those found in \([19,20]\) with the IAM and explicit use of the second order Lagrangian (see \([47]\) for updated results). However, the same procedure could not be used to generate the vector mesons and explicit use of the second order Lagrangian was needed in the IAM, or explicit exchange of vector meson resonances had to be assumed in \([23]\). This is why we associate these vector mesons to genuine QCD states while claim that the scalar mesons are dynamically generated.

VIII. THE \( \sigma \) AS A \( \pi\pi \) SCATTERING RESONANCE

The \( \sigma(500) \) is generated in this approach with the Bethe Salpeter equation and the lowest order chiral amplitude used as kernel and comes out as a broad resonance which is found as a pole in the second Riemann sheet and has a mass \( m_\sigma = 470+42 \text{ MeV} \) and a width around \( \Gamma_\sigma = 400+42 \text{ MeV} \). These results would be in good agreement with the recent experimental analysis of \([18]\), with \( m_\sigma = 478+24 \pm 17 \text{ MeV} \) and \( \Gamma_\sigma = 324+42 \pm 21 \).

IX. THE \( \pi\pi \) INTERACTION IN THE NUCLEAR MEDIUM

The \( \pi\pi \) interaction in a nuclear medium in the \( L=I=0 \) channel (\( \sigma \) channel) has stimulated much theoretical work lately. As commented in the Introduction, it was realized that the attractive P-wave interaction of the pions with the nucleus led to a shift of strength of the \( \pi\pi \) system to low energies and eventually produced a bound state of the two pions around \( 2m_\pi - 10 \text{ MeV} \) \([1]\). This state would behave like a \( \pi\pi \) Cooper pair in the medium, with repercussions in several observable magnitudes in nuclear reactions \([1]\). The possibility that such effects could have already been observed in some unexpected enhancement in the \( (\pi,2\pi) \) reaction in nuclei \([24]\) was also noticed there. More recent experiments where the enhancement is seen in the \( \pi^+\pi^- \) channel but not in the \( \pi^+\pi^0 \) channel \([2]\) have added more attraction to that conjecture (see also the talk of Starostin \([14]\) in this Workshop).

The advent of the chiral unitary methods has added new interest in the subject and has allowed one to focus on the implications of the chiral constraints which had been known to be relevant in this kind of studies \([1]\). In \([27]\) the \( \pi\pi \) interaction in a nuclear medium was studied following the lines of \([16]\), renormalizing the pion propagators in the medium and introducing vertex corrections for consistency. These vertex corrections play an interesting role since, as proved in \([25]\), they exactly cancel the off shell contribution of the two pion loops with renormalized pions. The diagrams considered are depicted in figs. \([3,10]\). The results for the imaginary part of the \( \pi\pi \) amplitude in \( L=I=0 \)
are shown in fig. 11. One can appreciate that there is an accumulation of strength in the region of small invariant masses of the two pion system, with qualitative results similar to those found in [1], but we do not get poles in that energy region. The accumulation of strength at these small invariant masses could raise hopes that the enhancement of strength at small invariant masses found in the $(\pi, 2\pi)$ reactions in nuclei in [3] could be explained. However, according to a recent study [29], the small nuclear densities involved in this reaction, which is rather peripheral, make the changes found in [27] insufficient to explain the experimental data.

The work of [27] includes only the $\pi\pi$ channel, since one is only concerned about the low energy region. This work has been generalized to coupled channels in [51] in order to make predictions for the modification of the $f_0(980)$ and $a_0(980)$ resonances in a nuclear medium. One finds there that both resonances become wider in the medium as the nuclear density increases, with the $a_0(980)$ eventually melting into a background for densities close to normal nuclear matter density. The $f_0(980)$ resonance, which in the free space is narrower than the $a_0(980)$, still would keep its identity at these high densities but with a width as large as 100 MeV or more. How to produce these resonances in a nucleus in order to check the predictions of these studies is a present experimental challenge.
FIG. 10. Diagram involving the three meson baryon contact terms of fig. 9 in each of the vertices

FIG. 11. Im $T_{22}$ for $\pi\pi \rightarrow \pi\pi$ scattering in $J = I = 0$ ($T_{00}$ in the figure) in the nuclear medium for different values of $k_F$ versus the CM energy of the pion pair. The labels correspond to the values of $k_F$ in MeV.

X. $(\gamma, 2\pi)$ IN NUCLEI IN THE $\sigma$ REGION

Unitarity is lost when one works with tree level Feynman diagrams. Yet, this is only partly true because in the Feynman diagrams one is usually including all possible resonances where one takes the width into account and then the most important part of the amplitude is included in a unitarized form. This would be the case when one has a delta in the final state, where the $\pi N$ channel would be unitarized. However, the final state of two pions is not unitarized. Once more this is only partly true because we explicitly account for $\rho$ production, which means that the unitarization of the two pion state in the $L=1, I=1$ channel is taken into account. But the unitarization in the $L=0, I=0$ is not done and we do not explicitly include a $\sigma$ production. As mentioned in section III, perturbative unitarity is taken into account in $[44]$ by means of loops at energies close to threshold. Here we wish to account for the final state interaction in $L=0$ for energies of the photon about 400-460 MeV, where Mainz experiments are being performed $[7]$. For that purpose we realize that at these energies the pions come mostly in S-wave. We actually carry out a projection of the amplitudes in this channel and find indeed that this is the case. However, we could have isospin $I=0$ or $I=2$. Since the interaction of pions at these energies in $I=2$ is very weak we implement the effects of final state interaction only in the $I=0$, for what we have to separate the pion production amplitude in two parts. This is done easily. We first write the isospin decomposition of the $\pi^0\pi^0$ state as

$$|\pi^0(1)\pi^0(2)\rangle = \frac{1}{3}|\pi^0(1)\pi^0(2) + \pi^+(1)\pi^-(2) + \pi^-(1)\pi^+(2)\rangle$$

(17)

I=0 part
Hence the $\gamma N \rightarrow N\pi^0\pi^0$ amplitude can be decomposed in two parts: the one that has as final state the combination of $I=0$, which requires to make the linear combination of amplitudes requested by the linear combination of $\pi\pi$ states in the $I=0$ term, first term of the RHS of eq. 17, and the $I=2$ combination, last two terms of eq. 17, which we leave as it is at the tree level.

To account for the pion final state interaction in $I=0$ we sum up the diagrammatic series in fig 12, which analytically can be expressed as

$$T_{(\gamma,\pi^0\pi^0)}(I_{\pi\pi} = 0) \rightarrow T_{(\gamma,\pi^0\pi^0)}(I_{\pi\pi} = 0) \left( 1 + G_{\pi\pi} t^{I=0}_{\pi\pi}(M_I) \right)$$

where $G_{\pi\pi}$ is the $G$ function for the loop function of the two pions, used before in the Bethe Salpeter equation, and $t^{I=0}_{\pi\pi}$ is the $\pi\pi$ scattering matrix in isospin $I=0$. By making use of the findings in [16] and [23], which tell us that in the two pion loop function the $T$ matrix can be factorized on shell, we assume this to be the case also for the tree level two pion production amplitude and this leads immediately to the equation written above. We have checked that this is a good approximation at low energies, but at the energies where we are working it has to be improved a bit and corrections of the order of 25 percent arise from these corrections. The main idea is that the dominant terms as we go to higher energies is the $\Delta$ Kroll Ruderman term, see diagram (i) of Fig (1). This term is present as it is at the tree level.

There is also a small technical detail. One of the terms in our approach contains the Roper excitation and its posterior decay into two pions in S-wave. The term is given by the Lagrangian

$$L_{N^*,\pi\pi} = -C_1^*m_\pi^2 \Sigma_N \bar{\phi}\phi \Sigma_N$$

where the constant $C_1^*$ is fitted to the partial decay width. Since this term is already projected in the $L=0$, $I=0$ channel, we substitute

$$C_1^* \rightarrow C_1^*/\left(1 + G(M_I) t^{I=0}_{\pi\pi}(M_I)\right)$$

where $M_I$ is an average $\pi\pi$ invariant mass in the decay of the Roper into a nucleon and two pions, such that when the renormalization due to the interaction is done explicitly in our approach in the region of energies where the Roper is excited, then we obtain the empirical term that leads to the observed partial decay width.

The cross section for the process is now given by the same equation 3, where now the distortion factors are

$$F_i(\vec{r}, \vec{q}_i) = exp \left[ \int_{\vec{r}}^{\infty} dl_i \frac{1}{q_i} Im \Pi(\vec{r}_i) \right]$$

where

$$\vec{r}_i = \vec{r} + l_i \frac{\vec{q}_i}{|\vec{q}_i|}$$

FIG. 12. Diagrammatic series for pion final state interaction in $I=0$
where $\Pi$ is the pion selfenergy. Here we are in the region of low energy pions and for the pion selfenergy we use the results of [53,54]. This potential has the advantage to have been tested against the different reaction cross sections, elastic, quasielastic and absorption. The imaginary part of the potential is split into a part that accounts for the probability of quasielastic collisions and another one which accounts for the pion absorption probability. As we shall see, the probability that there is loss of pion flux through pion absorption at low energies is larger than that where there are quasielastic collisions. One of the reasons is the Pauli blocking of the occupied states.

On the other hand, and this is one of the important points of the work, when we renormalize the $I=0$ amplitude to account for the pion pion final state interaction we change $G(s)$ and $t_{\pi\pi}(I=0)$ by their corresponding results in nuclear matter evaluated at the local density of the point $\vec{r}$ in the integral of eq. 3.

We have also used the $\Delta$ selfenergy from [40] to dress the $\Delta$ propagator. In addition to the proper real part of the selfenergy in [40] we add the effective contribution to the selfenergy $4/9 (f^*/\mu)^2 g' \rho$ coming from the iterated $\Delta h$ excitation driven by de Landau Migdal interaction [39, 40].

With all this introduction we pass now to present the results. In the first place, in fig. 13 we can see the results of the invariant mass distribution of the two pions for the $\gamma p \rightarrow \pi^0 \pi^0 p$ reaction.

![FIG. 13. Contributions of the different isospin channels to the $2\pi$ mass distribution.](image)

In the figure we can see the contribution of the $I=0$ part alone, the part of $I=2$ alone and the coherent sum of the two, both in the case when the $I=0$ amplitude is renormalized and when it is not. We can see that the renormalization of the $I=0$ amplitude has important effects nearly doubling the cross section. When then we sum coherently the $I=0$ and $I=2$ amplitudes we observe a curious shape of the distribution with a double hump, one at low invariant masses and the other one at the high mass part of the spectrum. This shape is corroborated by experiment as seen in the presentation in this Workshop by Volker Metag [7].

The integrated cross section compared with experiment can be seen in fig. 14 where we can appreciate that the inclusion of final state interaction has lead to an improvement of the cross section.
FIG. 14. Total cross section for $\gamma p \rightarrow \pi^0 \pi^0 p$ with and without pion final state interaction. Experimental data from ref. [56].

In fig. [15] we show the results of the invariant mass distribution for $\gamma p \rightarrow \pi^0 \pi^0 p$, $\gamma n \rightarrow \pi^0 \pi^0 n$ and $\gamma d \rightarrow \pi^0 \pi^0 pn$, the latest one obtained as a simple sum of the two other cross sections.

FIG. 15. Two pion invariant mass distribution for $2\pi^0$ photoproduction on proton, neutron and deuteron, (continuous, dashed and dashed-dotted line respectively).
Next we show the results of the invariant mass distribution in nuclei.

First we show in fig. 16 the results for $^{12}$C without including the pion pion final state interaction in nuclei, meaning we use the free $\gamma N \rightarrow \pi\pi N$ amplitude including the final state interaction of the pions in free space. However, we include the pion distortion. We can see that there is a reduction of about 40 percent in the cross section due to pion absorption. If we remove the pions which have undergone a quasielastic scattering this would reduce the cross section in an extra 20 percent. This effect is moderate. Part of these collisions would not change the charge of the pions, only their energy and momentum would be changed. In this case the two $\pi^0$ would still be there and their invariant mass would be somewhat changed. In other cases there could be change of charge and then we would not have two $\pi^0$ in the final state. However, this could also be compensated by having originally the $\pi^+\pi^0$ production followed by a collision of the $\pi^+$ with charge exchange. We shall show results eliminating only the pions absorbed but we can take the differences between the calculations removing only the pions absorbed, or the one where we remove all pions which are absorbed or undergo quasielastic collisions, as indicative of the theoretical uncertainties. The results for the case of removal of only the pions which are absorbed are obtained by putting in the imaginary part of the pion selfenergy, $\Pi$, the part which comes from the absorption and omitting the one that comes from quasielastic, which have been separated in [53]. In fig. 17 we can see the same results in $^{208}$Pb, which are qualitatively similar but where the amount of pions absorbed is much larger.
Now comes the interesting part when the medium effects are included in the pion pion final state interaction.

FIG. 18. Two pion invariant mass distribution for $2\pi^0$ photoproduction in $^{12}C$. Continuous line: Using the in medium final $\pi\pi$ interaction. Dashed line: using the final $\pi\pi$ interaction in free space.
In fig. 18 we can see the results for $^{12}C$. The only difference between the two curves has been the use of the in medium $\pi\pi$ scattering and $G(s)$ function instead of the free ones. As one can see in the figure there is an appreciable shift of strength to the low invariant mass region due to the in medium $\pi\pi$ interaction. The accumulation of strength found in all approaches close to the two pion threshold has its manifestation in this impressive shift of strength. This shift is remarkably similar to the one shown by Metag [7] in this Workshop.

In fig. 19 we can see the same results for $^{208}Pb$ which look qualitatively similar, but the shift to low invariant masses is further accused in this case, as it also happens in the experiment [7].

![Graph](image_url)

**FIG. 19.** Same as fig. 18 for $^{208}Pb$.

This is in our opinion the first clear manifestation of this modified pion pion interaction in the medium, given the fact that there are problems in the comprehension of the pion induced two pion production experiments and their theoretical description. The fact that the photons are not distorted has certainly an advantage and allows one to see inner parts of the nucleus. On the other hand there is an extra advantage because the $I=0$ amplitude for the case of $\pi^0\pi^0$ production was quite large since it involves the $\pi^+\pi^-$ production amplitudes which are much larger than the $\pi^0\pi^0$ one.

### XI. CONCLUSIONS

We touched here two issues looking for medium modifications of the mesons. The first one was about the modifications of the $\rho$ meson. Even when most authors agree about the important medium modifications of the $\rho$ meson properties, what we found here is that seeing these results in $\rho$ meson photoproduction in nuclei is very difficult. The fact that the pions are produced in the $\Delta$ resonance region distorts the pions considerably and only regions of low densities close to the surface can be tested where the medium effects are small.

On the other hand we could see that $\pi^0\pi^0$ photoproduction in nuclei at low energies is an excellent reaction to see the medium effects of the pion pion interaction close to the two pion threshold. This increased strength can be rightly attributed to the moving of the $\sigma$ resonance at low energies in the nuclear medium, and its decreased width, as suggested in [2] and found recently within the framework of $U\chi PT$ by M.J. Vicente Vacas and reported in this Workshop [55].
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$^{40}\text{Ca}$

$E_{\gamma}=1250$ Mev

No $\rho$ modification

$\rho$ width but no mass modification

$\rho$ width and mass+5% modification

$\rho$ width and mass+20% modification