Geometric Resonance in Modulated Quantum Hall Systems Near
\[ \nu = 1/2 \]

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Abstract

We propose a theory for the new effects recently observed by Willett et al [1] in the magnetoresistance of a weakly modulated two dimensional electron gas near filling factor 1/2. Minima in transverse magnetoresistance and maxima in longitudinal magnetoresistance at the same magnetic field producing the new resonance structure are reported. The structure occurs due to geometric resonance of the composite fermion cyclotron orbits with the modulation period of the effective magnetic field \( B_{\text{eff}} \) due to the applied density modulation. The transverse minimum occurs due to the inhomogeneity in the field \( B_{\text{eff}} \) in the presence of density modulations, whereas the longitudinal maximum can arise due to a shape-effect (distortion) of the composite fermion Fermi surface (CF–FS). Thus the minima and maxima reflect different physical mechanisms.

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The present work is motivated by the new experimental results of dc transport experiments of Willett et al [1] in a two-dimensional electron gas (2DEG) in the fractional quantum Hall regime near half filling of the lowest Landau level ($\nu = 1/2$). The 2DEG was modulated with a density modulation of small period applied in one direction. A new resonance structure produced by the modulation was observed in the magnetoresistance of the 2DEG. The resonance structure was superimposed on a minimum which occurs at about $\nu = 1/2$ in the magnetic field dependence of that dc resistivity component for a current driven across the modulation lines ($\rho_\perp$), and on a maximum in the resistivity corresponding to a current driven along the modulation lines ($\rho_\parallel$). Such a maximum in the magnetic field dependence of the dc resistivity of a modulated 2DEG was never observed before.

Here we develop a semiquantitative theory of these results [1]. Our work is based on the theory of the quantum Hall system at and near $\nu = 1/2$ proposed by Halperin, Lee and Read (HLR) [2,3], which corresponds to the physical picture of the electrons decorated by attached quantum flux tubes. These are the relevant quasiparticles of the system – so called composite fermions (CF). The CFs are charged spinless fermionic quasiparticles which move in the reduced effective magnetic field $B_{eff} = B - 4\pi hcn/e$ ($n$ is the electron density) [4]. At $\nu = 1/2$ the CFs form a Fermi sea and exhibit a FS. Within the HLR theory the CF–FS can be taken as a circle in quasimomentum space. Its radius $p_F$ equals $\sqrt{4\pi n\hbar^2}$.

The density modulation influences the CF system in two ways: through the direct effect of the modulating potential which can deform the CF–FS [5], and also through the effect of an additional inhomogeneous magnetic field $\Delta B(r)$ proportional to the density modulation $\Delta n(r)$ [6]. To analyse the effect systematically we have to solve the Boltzmann transport equation for the CF distribution function in the presence of a spatially inhomogeneous disturbance due to the density modulation in a similar fashion to the work of Ref.[7] for the 2DEG in a low magnetic field. When, however, the CFs mean free path $l$ is larger than the radius of their cyclotron orbit at the effective magnetic field $R$ and the period of modulation $\lambda$, we can obtain the desired response functions using simplifications based on the work of Beenakker [8] and Gerhardts [9]. The result of our work is that the deformation of the CF–FS affects principally the longitudinal response, while the additional field $\Delta B(r)$ principally affects the transverse response.

We start from the Lorentz force equations describing the CF motion along the orbit:

$$\frac{dp_x}{dt} = -\frac{|e|}{c} B(r)v_y; \quad \frac{dp_y}{dt} = \frac{|e|}{c} B(r)v_x;$$  \hspace{1cm} (1)

where $p_x$ and $v_x$ are the components of the CF quasimomentum and velocity;

$$B(r) = B_{eff} + \Delta B(r) \equiv B_{eff} - 4\pi hc\Delta n(r)/e.$$

We will consider first a single-harmonic sinusoidal density modulation of period $\lambda = 2\pi/g$ along the "$y"$ direction: $\Delta n(r) \equiv \Delta n(y) = \Delta n \sin(gy)$. We assume that the correction term $\Delta B(r)$ is small compared to $B_{eff}$. Under this assumption we can write the CF velocity $v$ in the form $v = v_0 + \delta v$, where $v_0$ is the uniform-field velocity and the correction $\delta v$ arises due to the inhomogeneity of the magnetic field. For a circular CF–FS we have: $v_{x0} = v_F \cos \Omega t; v_{y0} = v_F \sin \Omega t; \Delta n(y) \approx \Delta n \sin(gY - gR \cos \Omega t)$, where $v_F$ is the CF’s Fermi velocity, and $Y$ is the "$y"$ coordinate of the guiding center. Substituting these expressions for $v$ and $\Delta n(y)$ into Eqs.(1) and keeping only first-order terms we obtain:
\[
\frac{d(\delta v_x)}{dt} = -\Omega \delta v_y - \frac{\Delta B}{B_{\text{eff}}} \Omega v_F \sin \Omega t \sin(gY - gR \cos \Omega t);
\]
\[
\frac{d(\delta v_y)}{dt} = \Omega \delta v_x + \frac{\Delta B}{B_{\text{eff}}} \Omega v_F \cos \Omega t \sin(gY - gR \cos \Omega t).
\]

We remark here that in the presence of the density modulation the CF’s Fermi velocity gets a correction due to the modulation. To evaluate this correction we calculate the average of \(\Delta n(y)\) over the cyclotron orbit. Expanding the functions \(\cos(gR \cos \Omega t)\) and \(\sin(gY - gR \cos \Omega t)\) in Bessel functions we arrive at the following result for the averaged correction to the inhomogeneous density modulation:

\[
< \Delta n(y) > = \frac{\Delta n}{2\pi} \int_0^{2\pi} \sin(gY - gR \cos \psi) d\psi = \Delta n \sin(gY) J_0(gR).
\]

Here \(\psi = \Omega t\).

The result (3) gives a spatially inhomogeneous correction to the chemical potential of the CFs and to their Fermi velocity: \(\tilde{v}_F = v_F \sqrt{1 + \frac{\Delta n}{n} \sin(gY) J_0(gR)}\). However we can neglect the difference between \(v_F\) and \(\tilde{v}_F\) in the equations (2) because it gives corrections which are an order of magnitude smaller than those which are kept. It is natural to suppose that to the first order in the modulating field the corrections \(\delta v_x\) and \(\delta v_y\) are periodic over the unperturbed cyclotron orbit. This assumption is equivalent to that used in Ref.[9]. Under this assumption we can calculate averages of Eqs.(2) over the cyclotron orbit. This gives us the following expressions for the components of the velocity of the guiding center drift \(V_x\) and \(V_y\) defined below:

\[
V_x(Y) = < \delta v_x > = -\frac{v_F}{2\pi B_{\text{eff}}} \frac{\Delta B}{B_{\text{eff}}} \int_0^{2\pi} \cos \psi \sin(gY - gR \cos \psi) d\psi = \frac{\Delta B}{B_{\text{eff}}} \cos gY J_1(gR);
\]
\[
V_y(Y) = < \delta v_y > = -\frac{v_F}{2\pi B_{\text{eff}}} \frac{\Delta B}{B_{\text{eff}}} \int_0^{2\pi} \sin \psi \sin(gY - gR \cos \psi) d\psi = 0.
\]

To evaluate semiquantitatively the CF conductivity we assume that the \(x\) component of the CF velocity can be written in the form \(v_x(Y) = v_{x0} + V_x(Y)\). We also assume that the cyclotron frequency \(\Omega\) can be replaced by the quantity \(\Omega(Y) = \Omega + < \Delta \Omega(y) >\) where \(< \Delta \Omega(y) >\) is the correction to the cyclotron frequency arising due to the inhomogeneity of the effective magnetic field averaged over the cyclotron orbit:

\[
\Omega(Y) = \Omega \left\{ 1 + \frac{\Delta B}{B_{\text{eff}}} \frac{1}{2\pi} \int_0^{2\pi} \sin(gY - gR \cos \psi) d\psi \right\} = \Omega \left\{ 1 + \frac{\Delta B}{B_{\text{eff}}} \sin(gY) J_0(gR) \right\}.
\]
We showed before [5] that in the semiquantitative analysis of the magneto-transport in the modulated 2DEG we can use the following approximation for the CF conductivity:

$$\sigma_{\alpha\beta}^{cf} \approx \frac{g}{2\pi} \int_{-\pi/g}^{\pi/g} \sigma_{\alpha\beta}^{cf}(Y) dY;$$  \hspace{0.5cm} (6)

where

$$\sigma_{\alpha\beta}^{cf}(Y) = \frac{e^2 m_c \tau}{2\pi \hbar^2} \sum_k \frac{v_{k\beta}(Y) v_{-k\beta}(Y)}{1 + i k \Omega(Y) \tau}. \hspace{0.5cm} (7)$$

Here $m_c$ is the CF cyclotron mass; $\tau$ is the relaxation time; $v_{k\beta}(Y)$ are the Fourier transforms for the CF velocity components: $v_{kx} = \tilde{v}_F (\delta_{k,1} + \delta_{k,-1}) + V_x(Y) \delta_{k,0}$; $v_{ky} = \frac{i\tilde{v}_F}{2} (\delta_{k,1} - \delta_{k,-1})$.

Keeping terms of the order of $(\Delta B/B_{eff})^2$ or larger we obtain the following approximations for the CF conductivity components:

$$\sigma_{xx}^{cf} \approx \sigma_0 \left[ 1 + \frac{\Delta n}{n} \frac{p_F}{\hbar} \frac{1}{l_F} \right] \left[ 1 + 3 \left( \frac{\Delta n}{n} \frac{p_F}{\hbar} \frac{1}{l_F} \right)^2 \frac{J_0^2(gR)}{1 + (gR)^2} \right]; \hspace{0.5cm} (8)$$

$$\sigma_{yy}^{cf} \approx \sigma_0 \left[ 1 + \frac{\Delta n}{n} \frac{p_F}{\hbar} \frac{1}{l_F} \right] \left[ 1 + 3 \left( \frac{\Delta n}{n} \frac{p_F}{\hbar} \frac{1}{l_F} \right)^2 \frac{J_0^2(gR)}{1 + (gR)^2} \right]; \hspace{0.5cm} (9)$$

$$\sigma_{xy}^{cf} = -\sigma_{yx}^{cf} \approx \sigma_0 \frac{\Omega \tau}{1 + (\Omega \tau)^2} \left[ 1 + \frac{1}{2} \left( \frac{\Delta n}{n} \frac{p_F}{\hbar} \frac{1}{l_F} \right)^2 \frac{J_0^2(gR)}{1 + (gR)^2} \right]. \hspace{0.5cm} (10)$$

where $\sigma_0 = ne^2l/p_F$ is the CF conductivity in a homogeneous magnetic field.

The last term in the expression for $\sigma_{xx}^{cf}$ describes the contribution from CFs diffusing along the "x" direction which arises due to the guiding center drift. To show it we can calculate the corresponding contribution to the diffusion coefficient $\delta D$. Following [8,9] we write:

$$\delta D = \tau \frac{g}{2\pi} \int_{-\pi/g}^{\pi/g} V_x^2(Y) dY. \hspace{0.5cm} (11)$$

This term $\delta D$ gives the additional contribution to the "x" component of the diffusion tensor $D$. The latter is connected with the CF conductivity through the Einstein relation $\sigma_{\alpha\beta}^{cf} = Ne^2 D_{\alpha\beta}$ ($N$ is the CF density of states). Substituting Eq.(11) into this relation we obtain the expression for this diffusion correction to $\sigma_{xx}^{cf}$ which coincides with the last term in Eq.(8).

According to the HLR theory, the 2DEG resistivity tensor $\rho$ equals: $\rho = \rho^{cf} + \rho^{cs}$ where $\rho^{cf}$ is the CF resistivity tensor ($\rho^{cf} = (\sigma^{cf})^{-1}$) and the contribution $\rho^{cs}$ arises due to a fictitious magnetic field which originates from the Chern-Simons formulation of the theory. The latter has only off diagonal elements. Hence the diagonal components of the 2 DEG
resistivity tensor coincide with the corresponding components of the CF resistivity tensor \( \rho_{cf} \). After straightforward calculations we arrive at the result:

\[
\rho_{||} \approx \frac{1}{\sigma_0} \left\{ 1 + \left( \frac{\Delta B}{B_{eff}} \right)^2 \chi_1(gR) \right\}^{-1}; \quad (12)
\]

\[
\rho_{\perp} \approx \frac{1}{\sigma_0} \left\{ 1 + \left( \frac{\Delta n}{n} k_{F} l \right)^2 \frac{\chi_2(gR)}{1 + (\Delta B/B_{eff})^2 \chi_3(gR)} \right\}; \quad (13)
\]

Here \( \chi_i(gR) = \alpha_i J_0^2(gR) + J_1^2(gR) \) \((i = 1, 2, 3)\) and the coefficients \( \alpha_i \) are given by the expressions:

\[
\alpha_2 = 0; \quad \alpha_1 = -\frac{1}{2} \frac{(\Omega \tau)^2}{1 + (\Omega \tau)^2}; \quad \alpha_3 = \frac{(\Omega \tau)^2}{1 + (\Omega \tau)^2}. \quad (14)
\]

When the density modulation is very weak \((\Delta n/k_{F} l \ll 1)\) the corrections to the magnetoresistivity are small and we can neglect them. Under this condition the inhomogeneity of the effective magnetic field does not significantly affect dc transport. For stronger modulation \((\Delta n/k_{F} l \sim 1)\) the resistivity component \( \rho_{||} \), as previously, depends weakly on modulations but \( \rho_{\perp} \) is significantly changed.

We can easily extend our consideration to include higher harmonics of the periodic density modulation. Suppose that the correction \( \Delta n(r) \) has the form:

\[
\Delta n(r) = \sum_{s=1}^{\infty} \Delta n_s \sin(sgy) \quad (15)
\]

In this case we have:

\[
B(r) = B_{eff} \left\{ 1 + \sum_{s=1}^{\infty} \beta_s \sin[sgY - s \cos(\Omega \tau)] \right\} \quad (16)
\]

where \( \beta_s = \Delta B_s/B_{eff} \) and \( \Delta B_s \) is the Fourier transform of the correction \( \Delta B(r) \). For weak modulations we can assume \( \beta_s \ll 1 \). Proceeding in a similar way as before we arrive at the following results for the correction \( \Delta v_x \) and the cyclotron frequency \( \Omega \) averaged over the CF cyclotron orbit:

\[
V_x(Y) = v_F \sum_{s=1}^{\infty} \beta_s \cos sgy J_1(sgy) \quad (17)
\]

\[
\Omega(Y) = \Omega \left[ 1 + \sum_{s=1}^{\infty} \beta_s \sin(sgy) J_0(sgy) \right] \quad (18)
\]

As before \( V_y(Y) = 0 \).

Using these expressions we can obtain the following results for the desired resistivity components.
\[ \rho_\parallel = \rho_{xx} \approx \frac{1}{\sigma_0} \left[ 1 + \sum_{s=1}^{\infty} \beta_s^2 \chi_1(sgR) \right]^{-1}; \quad (19) \]

\[ \rho_\perp = \rho_{yy} \approx \frac{1}{\sigma_0} \left[ 1 + \sum_{s=1}^{\infty} \beta_s^2 \chi_2(sgR) \right]^{-1} \left( 1 + \sum_{s=1}^{\infty} \beta_s^2 \chi_3(sgR) \right). \quad (20) \]

For a nearly harmonic density modulation we can neglect all coefficients \( \beta_s \) (\( s \neq 1 \)) and suppose that \( \beta_1 \approx \Delta B/B_{\text{eff}} \). As a result Eqs.(19),(20) turn into (12),(13). Now we turn to consider different behavior of the components \( \rho_\perp \) and \( \rho_\parallel \) as functions of magnetic field. First we note that all the considerations up to this point are based on the assumption that the CF–FS is a circle.

The magnetic field dependence of the magnetoresistivity is determined by the parameter \( gR \). For \( gR \sim 1 \) the functions \( \chi_i(sgR) \) at \( s = 1 \) take values of the order of unity and can increase upon increase of \( B_{\text{eff}} \). For \( \frac{\Delta n}{n} k_F l \sim 1, \frac{\Delta B}{B_{\text{eff}}} << 1 \) the features of dependence of magnetoresistivity \( \rho_\perp \) on the magnetic field is determined by the function \( \chi_2(gR) \) (\( s = 1 \)) in the numerator of Eq.(20). The increase of \( \chi_2(gR) \) upon increase of the effective magnetic field corresponds to a minimum in the magnetic field dependence of \( \rho_\perp \) around \( \nu = 1/2 \). Such a minimum was observed in the experiments [1,10]. The magnetic field dependence of the resistivity component \( \rho_\parallel \) is determined by the first term of the sum \( \sum_{s=1}^{\infty} \beta_s^2 \chi_1(sgR) \) in the denominator of Eq.(19). Within a certain range of magnitude of the field \( B_{\text{eff}} \) this term may increase on increase of \( B_{\text{eff}} \) and thus give a maximum in the dependence of this resistivity component on the effective magnetic field at about \( \nu = 1/2 \). The maximum can be developed when the CF mean free path and the modulation period are large enough to obey the inequalities \( l > R \) and \( gR \sim 1 \) for sufficiently small \( B_{\text{eff}} \). However this maximum is too small in magnitude (because of the small factor \( \beta^2 \)) and narrow compared with a maximum in the magnetic field dependence actually observed in experiments [1]. We conclude therefore that the maximum in the magnetic field dependence of the resistivity \( \rho_\parallel \) cannot be successfully described within the framework of the same approach which gives rather good results for the component \( \rho_\perp \).

We conjecture, that this discrepancy mainly originates from a noncircular shape of the CF-FS which was not taken into account in the original HLR theory [2,3], and also not considered here in deriving the formulas (19),(20) for the resistivity components. It follows from these expressions (19),(20) that in the absence of density modulations both resistivity components \( \rho_\parallel \) and \( \rho_\perp \) do not depend on the magnetic field. This is correct only for a circular CF–FS. However the CF–FS may have a noncircular shape due to the effect of the crystalline field of adjacent layers of GaAs/AlGaAs. For a modulated 2DEG the CF–FS can undergo an extra distortion because of the modulation itself [5]. For a noncircular geometry of the CF–FS the resistivity components do depend on \( B_{\text{eff}} \) even for unmodulated 2DEG, as we now show.

Consider an unmodulated CF system with a closed FS of arbitrary shape. It follows from Eq.(7) that at \( \nu = 1/2 \) \( (B_{\text{eff}} = 0) \) we have \( \rho_{xx}(\nu = 1/2) = 1/\sigma_0 a_{xx}, \rho_{yy}(\nu = 1/2) = 1/\sigma_0 a_{yy} \). The dimensionless coefficients \( a_{\alpha\beta} \) equal:
Away from \( \nu = 1/2 \) when \( B_{\text{eff}} \) is large enough to satisfy an inequality \( \Omega \tau >> 1 \) we obtain: \( \sigma_{xx}^{\text{eff}} = \sigma_{0a_{xx}}/((\Omega \tau)^2); \ \sigma_{yy}^{\text{eff}} = \sigma_{0a_{yy}}/((\Omega \tau)^2); \ \sigma_{xy}^{\text{eff}} = -\sigma_{xy}^{\text{eff}} = \sigma_{0a_{xy}}/\Omega \tau. \) This gives the following approximations for resistivity components: 
\[
\rho_{xx} = \rho_{xx}(\nu = 1/2)/b; \\
\rho_{yy} = \rho_{yy}(\nu = 1/2)/b \text{ where } b = a_{xy}^2/a_{xx}a_{yy}.
\]

The value of this parameter \( b \) is determined from the geometry of the CF–FS. For a circular CF–FS we have \( a_{xx}a_{yy} = a_{xy}^2 \) (\( b = 1 \)). Then it can be shown that the asymptotics for resistivity components at the limits of zero and strong effective magnetic field coincide with each other (resistivity components \( \rho_{xx} \) and \( \rho_{yy} \) do not depend on \( B_{\text{eff}} \)). In the general case, however, \( a_{xx}a_{yy} \neq a_{xy}^2 \) (\( b \neq 1 \)) and asymptotically the resistivity components at the limits of zero and strong \( B_{\text{eff}} \) can differ; i.e. they depend on the magnetic field. When \( b < 1 \) the resistivity components at strong effective magnetic field take on values greater than at \( B_{\text{eff}} = 0 \) (\( \nu = 1/2 \)), which agrees with the assumption that magnetoresistivity increases upon increase of the field \( B_{\text{eff}} \). For \( b > 1 \) \( \rho_{xx} \) and \( \rho_{yy} \) in the limit of strong magnetic field \( B_{\text{eff}} \) are smaller than at \( \nu = 1/2 \) which can correspond to a maximum around \( \nu = 1/2 \) in the dependence of both resistivity components on \( B_{\text{eff}} \).

It follows from our results (19),(20) that the inhomogeneous magnetic field arising due to the density modulations influences the resistivity \( \rho_\perp \) significantly more than the resistivity \( \rho_\parallel \). The effect of the inhomogeneous magnetic field can predominate for \( \rho_\perp \) whereas the magnetic field dependence of \( \rho_\parallel \) is determined predominantly by the CF–FS geometry. As a result: a minimum of the resistivity \( \rho_\perp \) at \( \nu = 1/2 \) arising due to the modulations of the effective magnetic field can match a maximum of the resistivity \( \rho_\parallel \) originating from the geometry of the CF–FS. We suppose that such a maximum was observed in the experiments [1] in the magnetic field dependence of the resistivity \( \rho_\parallel \).

We also remark that the resistivity \( \rho_\parallel \) can be strongly influenced by the mobility modulations which occur due to the presence of the modulating field whereas \( \rho_\perp \) is nearly independent of them [7]. Therefore a systematic study of an effect of the CF–FS shape on the magnetic field dependence of the 2DEG resistivity near \( \nu = 1/2 \) for \( \rho_\parallel \) requires that we go beyond the relaxation time approximation used in this work. We believe, however, that our semiquantitative estimations capture the essential physics of the effect.

The functions \( \chi_s sgR \) for \( s > 1 \) near \( \nu = 1/2 \) describe geometrical oscillations of a special kind. The oscillations appear due to the commensurability of the CF cyclotron orbit with the periodic magnetic field induced by the density modulation which periodically arises upon change (increase or decrease) of \( B_{\text{eff}} \). The corresponding oscillating structures have to be symmetrically arranged around \( \nu = 1/2 \).

Similar geometric resonance (so called Weiss oscillations) were observed in dc transport experiments in both electrostatic and magnetic field modulated 2DEG in low magnetic fields [11–14]. A theory of these oscillations in modulated 2DEG systems was first developed within the framework of a quantum mechanical approach [15–17]. An equivalent semiclassical approach to the analysis of these phenomena was first proposed by Beenakker [8] who pointed that Weiss oscillations could be explained by means of the guiding center drift of cyclotron orbits of the electrons in the presence of the modulating electric field. The most complete semiclassical consideration of magneto-transport in a modulated 2DEG is presented in Ref.[7]
(See also Ref.[9]).

The magnetic field dependence of the resistivity component $\rho_\perp$ is shown in Fig.1. The theoretical curve in Fig.1 is described by Eq.(20) where terms corresponding to $s < 4$ are kept in the sums over $s$. For simplicity it is assumed that the coefficients $\beta_s$ in the retained terms are equal to $\Delta B/B_{\text{eff}}$. The shape of the curve is in qualitative agreement with the new experimental data for $\rho_\perp$ of Willett et al [1]. Two local minima symmetrically arranged about $\nu = 1/2$ correspond to the geometrical Weiss oscillations of the CFs at the presence of modulating magnetic field.

Our simplified formula (20) cannot be applied to the region immediately adjacent to $\nu = 1/2$ where the condition $\Delta B/B_{\text{eff}} << 1$ is not satisfied. So we cannot analyze the effect of channeled orbits of CFs which occur this region of $\nu$ where $B_{\text{eff}}$ is of the same order of magnitude as the oscillating correction $\Delta B$. To analyze the dc response of the modulated 2DEG for $\Delta B \approx B_{\text{eff}}$ we have to solve the CF transport problem as treated in Ref.[7]. Nevertheless our semiquantitative approach gives simple analytical results applicable for the comparison with experimental data.

In summary, we show that within the single relaxation time approximation, assuming the CF’s have a circular FS we obtain good semiquantitative agreement with experimental observation of a minimum in transverse resistivity $\rho_\perp$. Assuming the Fermi circle is spontaneously distorted we can obtain the observed maximum. It is significant that the different mechanisms: magnetic field modulation, and Fermi circle distortion can predominate in transverse and longitudinal geometry. Additional experiments can help identify these mechanisms.

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Figure Caption

Fig.1. dc magnetoresistivity versus the effective magnetic field. Dashed line – experiment of Ref.[1], solid line – theory for the parameters $n = 1.2 \times 10^{11} \text{cm}^{-1}$, $\Delta n/n = 0.01$ , $\lambda = 0.7 \mu m$, $l = 10^{-4} \text{cm}$. The dotted part of the theoretical curve corresponds to the region of the values of $B_{eff}$ where Eq.(20) cannot be applied.
FIGURES

FIG. 1.