Isoscalar \(NN\) spin-orbit potential from a Skyrme model with scalar mesons

Abdellatif Abada

Theoretical Physics Group, Department of Physics and Astronomy,
University of Manchester, Manchester M13 9PL, England
(March 26, 2022)

As a first step toward circumventing the difficulty to obtain an attractive isospin-independent \(NN\) spin-orbit force from Skyrme-type models involving only pions, we investigate an improved Skyrme Lagrangian that incorporates the scalar-isoscalar meson \(\varepsilon\) which can be viewed as the cause behind the enhancement of the \(\pi\pi\) \(S\)-wave. We find that at large distances, the main contribution to the spin-orbit potential comes from the scalar Lagrangian and it is found to be attractive. We briefly discuss how to pursue this work to finally obtain a medium-range attractive interaction.

PACS numbers(s): 11.10 Lm, 12.39 Dc, 13.75 Cs.

Report-no: MC/TH 96/17

I. INTRODUCTION

Ten years before the advent of QCD, Skyrme proposed a model [1] describing hadronic physics which involves only pion fields and where baryons emerge as topological solitons. This model is recognized as the simplest chiral realization of QCD at low energies and large \(N_c\) [2]. The corresponding Lagrangian contains, in addition to the well known nonlinear \(\sigma\) model, an antisymmetric term of fourth order in powers of the derivatives of the pion field (the so-called Skyrme term). The latter has been added by Skyrme in order to avoid soliton collapse. Despite its relative success in describing some properties of baryons (cf. Ref. [3] for a review), the Skyrme model presents several shortcomings such as its prediction of a very small value for the axial coupling constant [4] and an almost zero nucleon mass when the Casimir effects are taken into account [5]. But probably the most important crisis of the Skyrme model is that it does not allow for the formation of nuclei. Indeed, it predicts a repulsive \(NN\) central potential [6].

Because of these drawbacks, several extensions of the standard Skyrme model have been proposed in order to improve its prediction power. The most famous one consists in adding to the Skyrme Lagrangian a term of order six in powers of the derivatives of the pion field, proportional to the square of the baryon current as first proposed in [7]. The extended Skyrme model thus obtained contains a second, fourth and sixth-order term, but still is an effective theory of pions. It has been shown recently [8] that this extended model is much more realistic when describing low-energy hadronic physics than the standard Skyrme model.

In contrast with the velocity-independent part of the \(NN\) potential, the kinetic part and especially the \(NN\) spin-orbit force has not been studied extensively within the framework of Skyrme-type models. However, all the authors

*Supported by the EPSRC, UK.
†Address after Feb. 1st, 1997: BP Finance, BP International Ltd, Britanic House, 1 Finsbury Circus, London EC2M 7BA.
who worked on this subject, e.g. \[9,10\], arrived at the disappointing result that the standard Skyrme model predicts a repulsive isospin-independent spin-orbit force while, according to the phenomenology \[11\], it should be attractive. A glimpse of hope has emerged from Ref. \[12\] where it was claimed that when considering the sixth-order term, an attractive isoscalar spin-orbit interaction could be obtained. However, we have shown recently \[13\] that this is not the case: the sixth-order term contributes with a positive sign as is the case for the Skyrme term. Indeed, the authors of Ref. \[12\] considered in their calculations only one piece of the force due to the sixth-order term and omitted the piece that stems from the baryon exchange current, which turns out to be not only repulsive, but the dominant one too \[13\]. Thus, despite its success in improving the predictions of 1-baryon data, even an extended Skyrme model including higher-order terms in powers of the derivatives of the pion field fails to predict an attractive $NN$ isoscalar spin-orbit interaction as it fails to predict the correct $NN$ central potential \[14\]. Therefore, one has to consider more realistic low-energy models to describe the $NN$ interaction.

The Skyrme term as well as the sixth-order term can both be regarded as an infinite mass limit, the so-called local approximation, of a model with $\rho$-meson \[15\] and $\omega$-meson \[7\] exchanges respectively. These findings have provided the ground for improving Skyrme-type models by constructing effective Lagrangians which include, in addition to the pions, the other low-lying mesons. The authors of Ref. \[16\] have investigated the $NN$ force by considering such an effective model incorporating the mesons $\pi, \rho, \omega, A_1$ and the scalar meson $\epsilon$. Unlike previous works, they have introduced the vector meson fields by gauging the linear $\sigma$-model instead of the nonlinear one. They have found that the different channels of the $NN$ interaction arising from the potential piece of the Lagrangian are very well described, in particular the central potential for which an attraction has been found in a quantitative agreement with the phenomenology \[11\]. These results lead us to think that, similarly to the central potential, it may be that the right way to circumvent the difficulty of finding an attractive isoscalar spin-orbit force with Skyrme-type models is to consider effective theories which incorporate low-mass mesons with finite masses. In order to check these claims, one has to investigate the kinetic energy of a two-nucleon system within an effective Lagrangian such as the one in \[16\]. This is of course not an easy task and goes beyond the scope of this short paper. As a first step, we rather focus here only on the scalar meson contribution to the isoscalar spin-orbit force. Indeed, it has been pointed out in Ref. \[16\] that the presence of scalar degrees of freedom plays the crucial role of recovering the missed central attraction, so one may think that this will also be the case for the isoscalar spin-orbit force.

This idea to account for a scalar field in the spin-orbit channel has been investigated in Ref. \[17\]. There, a dilaton field is coupled to Skyrmions in order to mimic the scale breaking of QCD. We will not discuss here if a dilaton field is suitable or not to provide a good description of low-energy hadron physics; we refer the interested reader to Ref. \[18\] for a detailed discussion on this subject. The fact is that a combination of the sixth-order term and dilaton coupling yields an attractive isoscalar spin-orbit force, as it has been claimed in \[17\]. Note though that this result remains questionable since these authors, like those of Ref. \[12\], ignored the baryon exchange current contribution in their calculations \[13\]. In any case, here we will follow a more transparent way in order to incorporate scalar degrees of freedom \[16\]. It consists in considering the linear $\sigma$ model in which the presence of the scalar field is intrinsic. In Sec.II, the model with the scalar-isoscalar $\epsilon$-meson is presented. Sec.III contains the derivation of the isoscalar spin-orbit force. Our results and discussions are presented in Sec.IV.
II. THE MODEL WITH SCALAR MESONS

The infinite-mass limit of the effective Lagrangian with \( \pi, \rho, \omega \) and \( \epsilon \) mesons \([10]\) reads:

\[
\mathcal{L} = \mathcal{L}_{\text{NL}} + \mathcal{L}_4 + \mathcal{L}_6 + \mathcal{L}_{\text{SB}} = \frac{f_\pi^2}{4} \text{Tr} \left( \partial_\mu U \partial^\mu U^+ \right) + \frac{1}{32\pi^2} \text{Tr} \left[ (\partial_\mu U U^+) (\partial_\nu U U^+) \right]^2 - e_0^2 B_\mu B^\mu + \frac{f_\pi^2 m_\pi^2}{2} \text{Tr} (U - 1) \tag{1}
\]

where \( B^\mu = \epsilon^{\mu \nu \alpha \beta} \text{Tr} \left( (\partial_\nu U U^+) (\partial_\alpha U U^+) (\partial_\beta U U^+) \right) / 24\pi^2 \) is the baryon current \([3]\), and \( U \) an SU(2) matrix which characterizes the pion field. The first term in Eq. (1) corresponds to the nonlinear \( \sigma \) model, \( f_\pi \) being the pion decay constant. The second term, parameterized by the coupling constant \( e_0 \), is the so-called Skyrme term. The third term is of order six in the derivatives of the pion field and corresponds to \( \omega \)-meson exchange in the case of an infinite \( \omega \)-meson mass \([4]\). The coupling constant \( e_0^2 \) is a parameter related to the \( \omega \rightarrow \pi\gamma \) width. The last term in Eq. (1) which is proportional to the square of the pion mass \( m_\pi \) (139 MeV) implements a small explicit breaking of chiral symmetry.

As compared to the standard Skyrme model, the extended model \([1]\), when used with realistic parameters, provides a more accurate description of the 1-baryon properties (e.g., the nucleon quantum mass, the \( \Delta-N \) mass splitting, the breathing mode energy the axial-vector coupling constant \( g_A, \ldots \) \( [5,8] \)). However, as mentioned in the introduction, it does not describe properly neither the \( NN \) isospin-independent spin-orbit force \([13]\) nor the central potential \([14]\).

The scalar-isoscalar meson \( \epsilon \) is viewed as the responsible for the \( S \)-wave attraction in the \( \pi\pi \) interaction and, therefore, should be considered as an essential ingredient in the low-energy hadronic phenomenology. It is incorporated in the Lagrangian \([1]\) by replacing the nonlinear \( \sigma \) model \( \mathcal{L}_{\text{NL}} \) with the linear one. Namely,

\[
\mathcal{L}_{\text{NL}} \rightarrow \mathcal{L}_\sigma = \frac{1}{2} \partial_\mu \xi \partial^\mu \xi + \frac{\xi^2}{4} \text{Tr} \left( \partial_\mu U \partial^\mu U^+ \right) - \lambda (\xi^2 - f_\pi^2)^2 \tag{2}
\]

where \( \xi \) is the isoscalar content of the quaternion field \( \sigma + i\tau \cdot \pi, \sigma \) being the scalar chiral partner of the pion field. The scalar field \( \epsilon \) is defined as \( \epsilon(\mathbf{r}) = f_\pi - \xi(\mathbf{r}) \). The coupling constant \( \lambda \) is related to the \( \epsilon \)-meson mass through \( m_\epsilon^2 = 8\lambda f_\pi^2 \). It is straightforward to check that by taking \( m_\epsilon \rightarrow \infty \) in Eq. (2), one recovers the nonlinear \( \sigma \) model \( \mathcal{L}_{\text{NL}} \). Indeed, in that limit, \( \lambda \) becomes infinite and so the \( \xi \)-field has to be frozen to its asymptotic value \( f_\pi \) in order to keep the potential energy in Eq. (2) finite.

Therefore, for a realistic finite \( \epsilon \)-meson mass, the Lagrangian density \([1]\) is then replaced by the following one:

\[
\mathcal{L}' = \mathcal{L}_\sigma + \mathcal{L}_4 + \mathcal{L}_6 + \mathcal{L}_{\text{SB}} \tag{3}
\]

The one-soliton system is commonly solved by assuming the hedgehog ansatz for the pion field:

\[
U(\mathbf{r}) \equiv U_H(\mathbf{r}) = \exp \left( i \tau \cdot \mathbf{F}(\mathbf{r}) \right) \tag{4}
\]

where the \( \tau_a \)'s are the Pauli matrices and the notation \( \hat{\mathbf{r}} \) means \( \mathbf{r}/r \). The static Euler-Lagrange equations for the chiral field \( \mathbf{F} \) and the \( \xi \)-field corresponding to the Lagrangian (3) read

\[
\left( \frac{r^2 \xi^2}{c^2} \frac{2}{c^2} \sin^2(F) + \frac{\xi^2}{2\pi^4 r^2} \sin^4(F) \right) \mathbf{F}' = \left( \xi^2 + \frac{1}{c^2 r^2} \sin^2(F) \right) \sin(2F) + \left( \frac{1}{c^2} + \frac{\xi^2}{2\pi^4 r^2} \sin^2(F) \right) \sin(2F) \mathbf{F}' + f_\pi^2 m_\pi^2 \mathbf{F} \sin(\xi^2) \tag{5}
\]

\[
(r\xi)' = \left( \frac{r^2 \xi^2}{c^2} \frac{2}{c^2} \sin^2(F) + 4\lambda (\xi^2 - f_\pi^2) \right) r\xi
\]
where primes indicate radial-coordinate differentiation. In order to ensure a winding number one the chiral angle $F$ obeys the usual boundary conditions while the $\xi$-field fulfills the conditions $\xi'(0) = 0$ and $\xi(\infty) = f_\pi$. Let us observe that the above Euler-Lagrange equations are solved with a set of parameters whose values have been fixed by fitting to the mesonic sector \cite{5,8,16}. Namely, $f_\pi = 93$ MeV, $e = 7.2$, $e_6 = 1.66$ fm, $m_\pi = 650$ MeV. These parameters yield, e.g., a value of the nucleon mass of $\sim 1$ GeV after subtracting the Casimir energy \cite{5}, and a $\Delta N$ mass splitting of $\sim 267$ MeV. We have plotted in Fig.1 the chiral function $F$ and the $\xi$-field, solutions to the equations of motion \cite{3} obtained with the set of parameters given above. We will use below these two functions as an input in the numerical computation of the spin-orbit force. For reference, we have also shown in Fig.1 the chiral solution $F$ in the case of the model \cite{1}, i.e. $\xi = f_\pi$, \cite{8}.

III. THE SPIN-ORBIT FORCE

So far, all the calculations on the $NN$ spin-orbit force within Skyrme-type models have been carried out in the framework of the product ansatz as suggested by Skyrme \cite{19} (cf. Ref. \cite{9,10} and references therein). It is worthwhile noticing that the latter is only a simple approximation to the two-baryon configuration and not a self-consistent solution. We will not discuss here the degree of validity of this approximation. It is commonly chosen because of its relative simplicity as compared to other two-baryon field configurations which can be found in the literature \cite{20}. Furthermore, it becomes exact for large $NN$ separation\footnote{The region of validity of the product ansatz corresponds to a relative distance $r$ much larger than 1 fm.}. Thus, following common practice, we use it here to describe a system of two interacting solitons. Furthermore, in order to obtain the appropriate spin and isospin structure, we also introduce rotational dynamics \cite{4}. Hence, the field configuration of the two-nucleon system separated by a vector $r$ reads:

$$U(A_1, A_2, x, r) = U_1 U_2 = A_1 U_H(r_1) A_1^+ A_2 U_H(r_2) A_2^+, \tag{6}$$

where $A_1$ and $A_2$ are $SU(2)$ matrices, and $U_H$ the hedgehog single soliton \cite{3}. For the scalar field, we use the configuration suggested in Ref. \cite{16}

$$\xi_P(x, r) = \frac{1}{f_\pi} \xi(r_1) \xi(r_2) \tag{7}$$

which obviously is the form the most compatible with the product-ansatz approximation \cite{1}. To carry out a simultaneous quantization of the relative motion of the two nucleons and the rotational motion, we need to treat $r, A_1$ and $A_2$ as collective coordinates. Therefore, we make all these parameters ($r, A_1, A_2$) time-dependent.

The spin-orbit potential will emerge from a coupling between the relative motion and the spins of the two nucleons so that we have to calculate the kinetic energy corresponding to (2). As reported in Ref. \cite{11,13}, generally, one has to treat with care the conversion from velocities to canonical momenta before identifying and extracting the spin-orbit potential. Indeed, one has to start from a Lagrangian formalism, take its “classical” kinetic energy and extract from it the mass matrix and then invert it properly in order to move to a Hamiltonian formalism. However, for a large relative
distance \( r \), the region of validity of the product ansatz, this procedure \[^{10}\] is equivalent to that of Refs. \[^{4,13}\] which we will use here. In this latter one starts, in the case \[^{4}\], directly from

\[
K_{L\sigma}(A_1, A_2, r) = (-1) \int d^3x \left( \frac{1}{4} \xi^2 \text{Tr}(\partial_0 U \partial_0 U^+) + \frac{1}{2} (\partial_0 \xi r)^2 \right)
\]  

(8)

where the minus sign in front of the integral is put in explicitly so that account is taken of the change in sign of the off-diagonal terms of the \( 2 \times 2 \) mass matrix under inversion \[^{14}\]. Afterwards, one makes the usual identifications \[^{4,21}\] :

\[
\dot{r}_n \to \frac{p^{(n)}}{M} , \quad \omega_n = -i \frac{1}{2} \text{Tr}(\tau A_n^+ \dot{A}_n) \to \frac{s^{(n)}}{2\Lambda} , \quad n = 1, 2 , \quad (9)
\]

where \( p^{(n)} \) and \( s^{(n)} \) are respectively the radial momentum and the spin of the \( n \)-th nucleon while \( M \) and \( \Lambda \) are respectively the mass and the moment of inertia of the single soliton. Inserting the product ansätz (10,11) in the expression (8) gives

\[
K_{L\sigma} = -\frac{1}{2f^2} \int d^3x \xi^2(r_1)\xi^2(r_2) \left( R_{0a}(U_1) + L_{0a}(U_2) \right)^2 + \cdots
\]  

(10)

where we have omitted to write the second term in Eq. (8) since it does not contain angular velocities and thus will not contribute to the spin-orbit force. In Eq. (10), \( R_{0a} \) and \( L_{0a} \) are the time components of the right and left currents respectively:

\[
R_{0a}(U_1) = -i \frac{1}{2} \text{Tr}(\tau_0 U_1^+ \partial_0 U_1) = D_{ab}(A_1) \left( T_{cb}^{(1)} \frac{s_c^{(1)}}{2\Lambda} + \frac{p_i^{(1)}}{M} R_{ib}^{(1)} \right) ,
\]

\[
L_{0a}(U_2) = -i \frac{1}{2} \text{Tr}(\tau_0 \partial_0 U_2 U_2^+) = D_{ab}(A_2) \left( -T_{bc}^{(2)} \frac{s_c^{(2)}}{2\Lambda} + \frac{p_i^{(2)}}{M} R_{bi}^{(2)} \right) .
\]  

(11)

In the above equations, the sum from 1 to 3 on repeated indices is understood. The tensors \( T_{ab}^{(n)} \) and \( R_{ab}^{(n)} \) depend only on the position \( r_n \) of the \( n \)-th nucleon, and their explicit expressions can be found in Eq. (11) of Ref. \[^{13}\]. The \( D_{ab} \)'s are the matrix elements of the \( 3 \times 3 \) rotation matrix in the adjoint representation: \( D_{ab}(A) = \text{Tr}(\tau_a A \tau_b A^+) / 2 \). To obtain the right-hand sides of expressions (10), we have used the quantization scheme (9). The next step now is to expand the square in the expression of \( K_{L\sigma} \) (10) after inserting expressions (11) and making use of the \( D \)-matrix properties and the definitions of the tensors \( T_{ab}^{(n)} \) and \( R_{ab}^{(n)} \) \[^{13}\]. Thus, by considering the isoscalar part\[^{2}\] and keeping only terms proportional to \( \mathbf{L} \cdot \mathbf{S} \), where \( \mathbf{S} = s^{(1)} + s^{(2)} \) is the total spin and \( \mathbf{L} = \mathbf{r} \times \mathbf{p} \) the angular momentum (\( \mathbf{p} \) being the relative momentum, i.e., \( \mathbf{p} = p^{(2)} = -p^{(1)} \)), we obtain:

\[
K_{L\sigma} \to \frac{1}{M \Lambda} V_{L\sigma}(r) \mathbf{L} \cdot \mathbf{S}
\]

\[
V_{L\sigma}(r) = -\frac{1}{f^2} \int d^3x \frac{\xi^2(r_1)\xi^2(r_2)}{r_1} \frac{1}{r_1} \sin^2(F(r_1)) \dot{r}_1 \mathbf{r}_1 .
\]  

(12)

The total isoscalar spin-orbit potential of the model \[^{3}\] is then:

\[
V_{SO} = \frac{1}{M \Lambda} (V_{L\sigma}(r) + V_4(r) + V_6(r)) \mathbf{L} \cdot \mathbf{S}
\]  

(13)

where the linear \( \sigma \)-model contribution \( V_{L\sigma}(r) \) is given in Eq. (12). The rather lengthy expressions of the Skyrme-term contribution \( V_4 \) and the sixth-order term contribution \( V_6 \) can be found explicitly in Refs. \[^{3,10,17}\] and \[^{3}\],

\[^{2}\] Due to the projection theorem, the terms proportional to \( D(A_1^+ A_2) \) contribute to the isospin dependent force.
respectively, and do not need to be rewritten here. Note that the analytical expressions of \( V_4 \) and \( V_6 \) are not affected by the presence of the scalar field and still depend explicitly only on the chiral field \( F \). This is because the \( \xi \)-field does not couple to \( \mathcal{L}_4 \) nor \( \mathcal{L}_6 \), as it can be seen from Eq. (8). But obviously their numerical values will change due to the dependence of \( F \) on the scalar degrees of freedom.

**IV. RESULTS AND DISCUSSION**

Before displaying our numerical results concerning \( V_{L,\sigma} \), let us first discuss the analytical properties of the terms \( V_4(r) \) and \( V_6(r) \) at large distances [cf. Eq. (13)]. As already shown in Ref. [13], these potentials decrease as \( e^{-2m_\pi r} \) a large \( r \). Indeed, one can see from Eq. (13) that at large distances the chiral angle \( F(r) \) falls off as \( e^{-m_\pi r}/r \) (note that at large \( r \), \( \xi \to f_\pi \)). Then, following the approximation of Ref. [21] which consists of treating at large distances the field of one soliton as constant in the presence of the other, one obtains from [9,10,17] that \( V_4 \) decreases as \( e^{-2m_\pi r} \) over a power of \( r \). For the potential \( V_6 \) we have shown in Ref. [13] that it is a sum of two terms. The first term (the one which has been considered in [12,17]) decreases as \( e^{-3m_\pi r} \). This behaviour is expected since the sixth-order term \( \mathcal{L}_6 \) in Eq. (1) can be regarded as the local approximation of an effective model of \( \omega \) meson coupled to three pions [4]. However the asymptotic expression of the second part of \( V_6 \), which arises from the baryon exchange current [13], contains both \( e^{-2m_\pi r} \) and \( e^{-3m_\pi r} \) terms so that it behaves at large \( r \) as \( e^{-2m_\pi r} \). This means that the Lagrangian \( \mathcal{L}_6 \) generates, in addition to the expected three-pion exchange piece, a two-pion exchange one (coming from the exchange current) similar to that of the the fourth-order Skyrme term \( \mathcal{L}_4 \). Thus we see that for large distance both potentials \( V_4 \) and \( V_6 \) are of Yukawa type with range \( 2m_\pi \). In fact, the main motivation of including a scalar-isoscalar meson field by hand in Skyrme-type models [10,17] is to mimic the two-pion exchange since the latter is not well addressed in these models with simple zero-mode quantization.

A straightforward manner to calculate numerically the integrals giving the different contributions to the spin-orbit force consists in making the change in variable \( r' = x - r/2 \) and taking the \( NN \) separation vector \( r \) along the \( z' \) axis. E.g., with these changes, the expression of the linear \( \sigma \) model potential \( V_{L,\sigma} \) in Eq. (12) becomes:

\[
V_{L,\sigma}(r) = -\frac{2\pi}{f_\pi^2} \int_0^\infty \! dr' \int_{-1}^1 \! du \, u^2 \left( \sqrt{r'^2 + u^2} + 2ru \right) \sin^2(F(r')) \xi^2(r') \int_0^1 \! r^2 \, dr' \sin^2(F(r')) \xi^2(r')
\]

(14)

where \( u = \hat{r} \cdot \hat{r} = \frac{z'}{r'} \). In Fig.2, we plot \( V_{L,\sigma}, V_4, V_6 \) together with the total sum \( V_{L,\sigma} + V_4 + V_6 \) as functions of the relative distance \( r \) between the two nucleons. As it was the case for the extended Skyrme model [6], \( V_4 \) and \( V_6 \) are still repulsive in the case of the model (8). On the other hand, while the nonlinear \( \sigma \) model \( \mathcal{L}_{NL,\sigma} \) has a zero contribution to the isoscalar spin-orbit force the linear \( \sigma \) model contributes and with a *negative* sign as it can be seen from the behaviour of \( V_{L,\sigma} \) in Fig.2. This result is in agreement with the phenomenology [11]. The total potential (full line in Fig.2) is found to be repulsive for intermediate distances (\( r \leq 2\text{fm} \)) and attractive for large distances (\( r > 2\text{fm} \)). For short distances (not shown on Fig.2), neither the product ansatz nor the effective model should be trusted since this region corresponds to processes involving perturbative QCD. The total potential is attractive at large distances because in this region the scalar meson contribution [12] is the dominant one as compared to \( V_4 \) and \( V_6 \). This can

---

\(^3\) Indeed, \( \mathcal{L}_{NL,\sigma} \) contributes only to the isospin-dependent force [22].
be checked straightforwardly by comparing the asymptotic behaviour of each expression in Eq (13). We have also displayed in Fig.2 the potential related to the linear $\sigma$ model and the sixth-order term only, $V_{L\sigma} + V_6$. We observe from that curve that the attraction starts at about $1.7$ fm instead of the $2$ fm in the case of the total potential discussed above. This result is in some way in agreement with the phenomenological isoscalar spin-orbit potential for which it is well known that scalar-isoscalar and $\omega$ mesons are the mesons which play the most important role [11].

Similar results have already been found in Ref. [17] where a different way has been used to introduce a scalar-isoscalar degrees of freedom. In Ref. [17], the scalar-isoscalar meson field is a dilaton, and it explicitly couples to the sixth-order term. However the contribution to the spin-orbit force coming from the sixth-order term was not complete in Ref. [17]. Indeed, these authors ignored the baryon exchange current contribution to the spin-orbit force in their calculations, and we have shown in Ref. [13] that this contribution is significant, with respect to the direct term [12, 17], and should be taken into consideration. It is then legitimate to question whether the qualitative results of Ref. [17] remain valid if the baryon exchange current contribution is taken into account. In this work, by using a different way of including the scalar-isoscalar meson in the Skyrme Lagrangian [14, 16], and by considering the entire contribution of the baryon current to the spin-orbit channel, we show that the main result of Ref. [17] (i.e., an attractive $N-N$ spin-orbit force at large distance from Skyrmions with scalar mesons) is still valid. In this sense, my results can be viewed as a confirmation of those of Kälbermann and Eisenberg [17].

Even though we have obtained an attractive force at large distances in the isoscalar spin-orbit channel, attraction is still missing at intermediate distances, and thus the problem remains unsolved within Skyrme-type models. This is not surprising since we have considered here only a finite-mass scalar field without the other vector mesons. However, the result obtained here, namely, the change from a zero contribution to the isospin-independent spin-orbit interaction to an attractive one when replacing the frozen $\epsilon$-meson field (nonlinear $\sigma$ model) with a realistic one (linear $\sigma$ model), is very encouraging. Indeed, it suggests that the right way to obtain an attractive isospin-independent $NN$ spin-orbit force within Skyrme-type models in the framework of the product ansatz is to replace the pion theory [1] with a realistic effective model including, in addition to the scalar-isoscalar $\epsilon$ field, all low-lying vector mesons and taking into account the finiteness of their masses. In a sense, the model considered here can be seen as a minimal and modest improvement of the pion Skyrme theory. Similarly, it has been shown in Ref. [14] that the model [3] gives rise to attraction in the $NN$ central potential but the attraction occurs at distances larger than required by phenomenology. The problem has been finally solved when the other vectors mesons were included [14, 16]. For this reason, we believe that in order to cure the problem of the isoscalar spin-orbit force, one has to consider effective models which incorporate the first mesonic resonances with finite masses. For instance, when considering a finite-mass $\omega$-meson model, the $\omega$-field couples directly to the nucleon via the baryon current (defined after Eq. (1)) and generates the common three-pion exchange diagram, contrary to the case of the local approximation $L_6$ (cf. Eq. (1)). Indeed, in the latter, an unexpected two-pion exchange piece coming from the baryon exchange current arises in addition to the three-pion piece and contributes with a positive sign to the isoscalar spin-orbit force yielding a repulsive interaction [12]. This problem with the baryon exchange current is obviously avoided when the sixth order term $L_6$ is replaced with a more realistic $\omega$-meson model.

Finally, in addition to considering finite-mass mesons Lagrangians, we would like to mention a further way that might lead to the desired attractive isoscalar spin-orbit force within the framework of Skyrme-type models. It is concerned with the approximation of the product ansatz configuration [1]. In the latter, the two nucleons are
supposed to keep their spherical shape without deformation even after overlapping. This is certainly not true in reality. Thus, while still using the product ansatz so that we keep benefiting from its simplicity and being able to perform analytical calculations, we may improve on the approximation \(^{(1)}\) by allowing the shape of the nucleon to deform when approaching the other nucleon (cf. Ref. \(^{(23)}\) and references therein). Calculations of the spin-orbit force within this approach are under way \(^{(24)}\).

[1] T. H. R. Skyrme, Proc. Roy. Soc. A260, 127 (1961).
[2] E. Witten, Nucl. Phys. B 223, 422 and 433 (1983).
[3] I. Zahed and G. E. Brown, Phys. Rep. 142, 1 (1986).
[4] G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. B 228, 552 (1983).
[5] B. Moussallam, Ann. Phys. (N.Y.) 225, 264 (1993).
[6] A. Jackson, A. D. Jackson, and V. Pasquier, Nucl. Phys. A 432, 567 (1985).
[7] A. Jackson et al, Phys. Lett. B 154, 101 (1985).
[8] A. Abada and H. Merabet, Phys. Rev. D 48, 2337 (1993).
[9] E. M. Nyman and D. O. Riska, Phys. Lett. B 175, 392 (1986);
   D. O. Riska and K. Dannbom, Phys. Scr. 37, 7 (1988);
   T. Otofuji et al, Phys. Lett. B 205, 145 (1988).
[10] R. D. Amado et al, Phys. Lett. B 314, 159 (1993); Phys. Lett. B 324, 467 (1994);
    B. Shao et al, Phys. Rev. C 48, 2498 (1993); Phys. Rev. C 49, 3360 (1994);
    A. Abada, “The isospin independent spin-orbit force in the extended Skyrme model”, hep-ph/9401341, unpublished.
[11] M. Lacombe et al, Phys. Rev. C21, 861 (1980);
    R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. 149, 1 (1987).
[12] D. O. Riska and B. Schwesinger, Phys. Lett. B 229, 339 (1989).
[13] A. Abada, J. Phys. G 22, L57 (1996).
[14] D. Kalafatis, hep-ph/9406411 thesis, Univ. Paris XI.
[15] K. Iketani, Kyushu University preprint 84-HE-2 (1984), unpublished.
[16] D. Kalafatis and R. Vinh Mau, Phys. Rev. D 46, 3903 (1992) and references therein.
[17] G. Kälbermann and J. M. Eisenberg, Phys. Lett. B 349, 416 (1995).
[18] M. C. Birse, J. Phys. G 20, 1287 (1994).
[19] T. H. R. Skyrme, Nucl. Phys. 31, 556 (1962).

[20] N. R. Walet, Nucl. Phys. A 586, 649 (1995) and references therein.

[21] E. M. Nyman, Phys. Lett. B 162, 244 (1985).

[22] D. O. Riska and E. M. Nyman, Phys. Lett. B 183, 7 (1987).

[23] A. Rahimov et al, Phys. Lett. B 378, 12 (1996).

[24] A. Abada, work in progress.
FIG. 1. The chiral function $F$ (full line) and the $\xi$-field (dashed line) in fm$^{-1}$ solutions of Eq. (5). The dotted line corresponds to the chiral field $F$ in the nonlinear model ($\xi = f_\pi$).

FIG. 2. The potentials $V_{L\sigma}$ (dashed line), $V_4$ (dotted line), $V_6$ (dashed-dotted line), $V_{L\sigma} + V_6$ (starry line) and $V_{L\sigma} + V_4 + V_6$ (full line) with respect to the relative $NN$ distance $r$. 