Anomalous exciton Hall effect

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It is well known that electrically neutral excitons can still be affected by crossed electric and magnetic fields that make them move in a direction perpendicular to both fields. This effect is at the origin of the magnetic Stark effect for excitons and is described in terms of a synthetic gauge field. We show that a similar effect appears in the absence of external electric fields, in the case of scattering of an exciton flow by charge impurities in the presence of the external magnetic field. As a result, the exciton flow changes the direction of its propagation that may be described in terms of the Hall conductivity for excitons. We develop a theory of this effect, which we refer to as the anomalous exciton Hall effect, to distinguish it from the exciton Hall effect that arises due to the valley selective exciton transport in transition metal dichalcogenides. According to our estimations, the effect is relatively weak for optically active or bright excitons in conventional GaAs quantum wells, but it becomes significant for optically inactive or dark excitons, because of the difference of the lifetimes. This makes the proposed effect a convenient tool for spatial separation of dark and bright excitons.

Introduction. Thomas and Hopfield [1] were the first to point out that excitons propagating in the presence of an external magnetic field orthogonal to their velocity acquire stationary dipole polarisation perpendicular to both the magnetic field and their propagation direction. This is the manifestation of the magnetic Stark effect for excitons that was experimentally evidenced in a variety of semiconductor systems [2, 3]. This effect is caused by the Lorentz force that pulls an electron and a hole apart if an exciton as a whole particle moves in the presence of a magnetic field. Imamoglu [4, 5] pointed out that once an exciton is placed in crossed electric and magnetic fields, it starts moving as a whole in the direction perpendicular to the directions of both fields, that leads to the renormalization of the excitonic dispersion in quantum wells or two-dimensional semiconductor crystals. Onga et al. [6] have recently reported the experimental observation of an exciton Hall effect in atomically thin layers of MoS2 that manifests itself in the appearance of an off-diagonal exciton conductivity in the presence of a magnetic field. The effect is caused by the strong spin-valley locking in monolayer transition metal dichalcogenides (TMDs).

Here we propose one further step with respect to the three effects described above. We show that in conventional GaAs quantum wells containing charged impurities, an exciton flow may be reoriented in the real space due to the combined effect of the local electrostatic potential created by charged impurities and the magnetic field applied in normal to the plane direction. Conceptually, this effect is similar to the cross-field effect proposed by Imamoglu [4, 5] and it manifests itself in a very similar phenomenology to the exciton Hall effect studied by Onga et al. [6], however, it is different from both above mentioned effects as (i) an external electric field and (ii) spin-valley locking are not needed. To distinguish from the previous studies we refer to the effect we study as an anomalous exciton Hall effect.

We argue that the effect may have a significant magnitude in fluids of optically inactive, dark excitons due to their long lifetimes, and correspondingly is much weaker for short-living bright excitons in conventional GaAs-based quantum wells. This makes the anomalous exciton Hall effect a powerful tool for spatial separation of dark and bright excitons.

Synthetic gauge fields. In condensed matter physics gauge fields are ubiquitous. The best known example is a magnetic field B, which can be introduced into a single particle Hamiltonian by substitution $p_i \rightarrow p_i - q A_i$, with $q$ being the electric charge of the particle, $A_i$ being components of the vector potential, $B = \nabla \times A$. The presence of a magnetic field dramatically modifies the properties of the system, and leads to such fundamental phenomena as the quantum Hall and Aharonov-Bohm effects.

For neutral particles with $q = 0$, such as photons, cold atoms, or excitons, a magnetic field does not affect the orbital degree of freedom directly, and thus can not be considered as a real gauge field. However, if a particle possesses internal degrees of freedom, such as spin, polarization, or internal set of energy levels, creation of so called synthetic gauge fields becomes feasible. In particular, for cold atoms both Abelian and non Abelian gauge potentials can be engineered by resonant drive of the system with spatially inhomogeneous laser beams [8].
For photons, the methods to create synthetic gauge fields include dynamic modulation \[9\], use of coupled optical resonators \[10\], engineering lattices with strain \[11\], or reciprocal metamaterials \[12\].

In condensed matter physics, the typical example of an electrically neutral particle is an exciton. Excitons can be created by optical excitation. They govern the optical response of semiconductor materials at low temperatures. In systems of the reduced dimensionality, such as TMDs, excitonic features are highly pronounced and may be robust up to the room temperature \[13\]. The recent study of the electric field effect on the gauge fields for exciton strongly coupled to light (exciton-polaritons) \[8\] showed the high potentiality of the gauge field approach to the description of exciton and polariton dynamics in the presence of a magnetic field. In this Letter we demonstrate how the combination of the magnetic Stark effect \[1\] with excitonic scattering by an impurity potential, leads to the anomalous exciton Hall effect due to the appearance of an effective U(1) gauge field acting on the motion of the exciton center-of-mass.

**Phenomenological model.** To reveal the nature of the proposed effect, we start with a simplified phenomenological model, assuming that the motion of the exciton center-of-mass is decoupled from the internal dynamics of the electron-hole pair forming it. We consider a 2D exciton confined in the xy-plane and subject to the external magnetic field directed along the z-axis. If the center of mass of the exciton is characterised by a non-zero momentum \(\mathbf{k} \neq 0\), the magnetic field acting on the electron and hole would dipole-polarize the exciton in the direction perpendicular to \(\mathbf{k}\), so that the electric dipole moment of the exciton reads \(\langle \mathbf{d} \rangle = f(B)\mathbf{e}_z \times \mathbf{k}\), where \(f(B)\) is a function of the magnetic field, which depends linearly on \(B\) at weak fields, but becomes inversely proportional to \(B\) at the large fields in the magneto-exciton regime \[14\,16\]. At small magnetic fields one can find \(f(B)\) by passing to the center-of-mass reference frame, where the magnetic field is absent, but an electric field \(\mathbf{E}' = \frac{\hbar \mathbf{k} \times \mathbf{B}}{M}\) appears, here \(M\) is the exciton mass. If this electric field is weak, then \(\langle \mathbf{d} \rangle = -e\mathbf{r} = \alpha \mathbf{E}' = \alpha [\hbar \mathbf{k} \times \mathbf{B}] / M\), where \(\alpha = 21a_0^34\pi\varepsilon_0/128\) is the 2D exciton polarizability \[22\], thus \(f(B) = -\alpha \hbar B / M\). At strong magnetic fields, in the magneto-exciton regime, the dipole moment is given by \[16\] \(\langle \mathbf{d} \rangle = -e\mathbf{r} = -e[\mathbf{e}_z \times \mathbf{k}]l_B^2\), where \(l_B = \sqrt{\hbar / eB}\) is the magnetic length, thus \(f(B) = -\hbar / B\).

The presence of impurities and fluctuations of the doped quantum well width leads to the appearance of the scattering potential \(U_{sc}(\mathbf{R})\) for excitons, here \(\mathbf{R}\) denotes the position of the exciton center-of-mass. Moreover, it generates some non-zero in-plane distribution of the electric field \(\mathbf{E}(\mathbf{R})\), which can affect the exciton dipole moment (see Fig.\[1\]).

![FIG. 1: The sketch of the system under consideration. Excitons created by the optical pump, travel in the plane of a disordered quantum well in the presence of a uniform orthogonal magnetic field \(\mathbf{B}\). The magnetic field induces an in-plane dipole moment of the excitons which leads to the asymmetric scattering of the excitons by impurities. This problem can be mapped to the scattering of a charged particle by an impurity in the presence of the position-dependent synthetic magnetic field \(\mathbf{B}_{\text{eff}}(\mathbf{R})\), depicted by the grey domain around the scatterer in the figure.](image)

Using these assumptions we write down the model Hamiltonian of the system as:

\[
\hat{H}^{\text{dip}} = \frac{\hat{p}^2}{2M} + \hat{V}^{\text{dip}} = \frac{\hat{p}^2}{2M} + U_{sc}(\mathbf{R}) - \frac{1}{2}(\mathbf{d} \cdot \mathbf{E} + \mathbf{E} \cdot \mathbf{d}),
\]

where \(\mathbf{d} = f(B)\mathbf{e}_z \times \mathbf{k}\), \(M\) is the exciton mass, and \(\hat{p} = \hbar \mathbf{k}\) is the exciton center-of-mass momentum operator. Note, that as an exciton is a composite particle, there is no simple straightforward relation between the scattering potential and the electric field \(\mathbf{E} = \mathbf{E}(\mathbf{R})\), produced by the scatterer. After some simple algebra this can be cast in form \(\hat{H}^{\text{dip}} = (\hat{p} - e\mathbf{A}_{\text{eff}}(\mathbf{R}))^2 / (2M) + U_{\text{eff}}(\mathbf{R})\), where \(\mathbf{A}_{\text{eff}}(\mathbf{R}) = M f(B)(\hbar e)^{-1}(\mathbf{e}_z E_y(\mathbf{R}) - \mathbf{e}_y E_z(\mathbf{R}))\) is a synthetic vector potential, corresponding to the magnetic field \(\mathbf{B}_{\text{eff}}(\mathbf{R}) = -\mathbf{e}_z M f(B)(\hbar e)^{-1} \text{div} \mathbf{E}(\mathbf{R})\). \(U_{\text{eff}}(\mathbf{R}) = U_{sc}(\mathbf{R}) - M f^2(B)(2\hbar)^{-1} \mathbf{E}(\mathbf{R})^2\) is an effective scalar potential. One can see that in order to have a non trivial synthetic gauge field, two conditions need to be fulfilled: (a) \(B \neq 0\) and (b) the presence of an inhomogeneous electric field.

We consider first the scattering of an exciton by a single radially symmetrical impurity with \(U_{sc}(\mathbf{R}) = U_{sc}(\mathbf{R})\) and \(\mathbf{E}(\mathbf{R}) = E(\mathbf{R})\mathbf{R}/R\). Then, for the elastic scattering we obtain the following scattering matrix elements between the exciton states with the center-of-mass momenta \(\mathbf{k}\) and \(\mathbf{k}'\) (\(|\mathbf{k}| = |\mathbf{k}'|\))

\[
V_{\mathbf{k},\mathbf{k}'}^{\text{dip}} = U_{sc}(\Delta \mathbf{k}) + \frac{i\hbar f(k' \times \mathbf{k})|z|}{4\pi|\Delta \mathbf{k}|} \int_0^\infty dR^2 EJ_1(|\Delta \mathbf{k}|R).
\]

Here we performed the integration over the polar angle, which yielded the Bessel functions, \(\Delta \mathbf{k} = \mathbf{k}' - \mathbf{k}\) is
the transferred momentum, the arguments of \( f(B) \) and \( E(R) \) are omitted for brevity. The first term \( U_{\text{sc}}(\Delta k) \approx (2\pi)^{-1} \int \text{d}R U_{\text{sc}}(R) J_0(|\Delta k| R) \) is real and it describes the normal symmetric scattering of an exciton by an impurity, while the second imaginary term accounts for the breaking of the time reversal symmetry by the magnetic field (as \( V_{k,k'} \neq V_{-k,-k'}^{\text{dip}} \)). This term gives rise to the asymmetric scattering (analogous to the skew scattering) of excitons by the impurities and thus leads to the excitonic analogue of the anomalous Hall Effect.

Microscopic theory. In the presence of a magnetic field the center-of-mass motion and the internal motion are coupled, which has been neglected in the simplified qualitative model presented above. Now we proceed with a full microscopic model of exciton-impurity scattering in the presence of a magnetic field.

The Hamiltonian \( \hat{H}_{\text{rel}} \) of relative motion of an \( e-h \) pair characterized by the center-of-mass momentum \( \hbar \mathbf{k} \) in the presence of a perpendicular magnetic field \( \mathbf{B} = (0, 0, B) \) has the form \[14, 15\]

\[
\hat{H}_{\text{rel}} = -\frac{\hbar^2}{2M} \nabla^2 - \frac{i e B}{2} \left( \frac{1}{m_e} - \frac{1}{m_h} \right) |\mathbf{r} \times \nabla| + \frac{e^2 B^2}{8\mu} r^2 + \frac{e B}{M} (\mathbf{r} \cdot \mathbf{k}) \cdot \nabla \frac{r^2}{|\mathbf{r}|} - \frac{\hbar^2 k^2}{2M} \tag{3}
\]

where \( \mathbf{r} = \mathbf{r}_e - \mathbf{r}_h \) is the relative \( e-h \) coordinate, and \( \mu^{-1} = m_e^{-1} + m_h^{-1} \). Writing this expression we have taken advantage of the existence of an exact integral of motion, namely the magnetic center-of-mass momentum \[15\], defined by the operator \( \hbar \mathbf{k} = -i \hbar \nabla \mathbf{R} - e \mathbf{A}(\mathbf{r}) \), where \( \mathbf{R} = (m_e \mathbf{r}_e + m_h \mathbf{r}_h) / M \) is the center-of-mass coordinate, \( M = m_e + m_h \), and the vector potential is taken in the symmetrical gauge \( \mathbf{A}(\mathbf{r}) = \mathbf{B} \times \mathbf{r} / 2 \).

The exciton wave function has the form \( \Psi_{\mathbf{k}}(\mathbf{R}, \mathbf{r}) = \exp \{ i \mathbf{R} \cdot [\hbar \mathbf{k} + e \mathbf{A}(\mathbf{r})] \} \Phi_{\mathbf{k}}(\mathbf{r}) \), where \( \Phi_{\mathbf{k}}(\mathbf{r}) \) is the corresponding eigenstate of the Hamiltonian above. An important point is that the wave function \( \Phi_{\mathbf{k}} \) of the relative motion depends on the center-of-mass momentum \( \hbar \mathbf{k} \) i.e., the relative motion and the center-of-mass motion are coupled. The scattering matrix elements \( V_{\mathbf{k},\mathbf{k}'} \) between exciton states with the center-of-mass momenta \( \mathbf{k} \) and \( \mathbf{k}' \) in the external potential \( \mathcal{V} = V_e(\mathbf{r}_e) + V_h(\mathbf{r}_h) \) are given below.

Weak magnetic fields. In the weak-field limit, \( l_B \gg a_B \), the problem can be treated analytically, here \( a_B = 4\pi\epsilon_0\hbar^2/(\mu e^2) \) is the Bohr radius (the 1s exciton radius is \( a_B/2 \)). We shall neglect excitonic transitions to the excited states of internal \( e-h \) motion, i.e. the center-of-mass momentum \( |\mathbf{k}| \ll a_B^{-1} \). The ground-state wave function \( \Phi_{\mathbf{k}}(\mathbf{r}) \) is calculated in a magnetic field using the perturbation theory. The corresponding scattering matrix elements \( V_{\mathbf{k},\mathbf{k}'} \) are obtained in Ref. \[17\] and read as follows

\[
V_{\mathbf{k},\mathbf{k}'} = \tilde{V}_e(\Delta \mathbf{k}) \mathcal{F}_e(\Delta \mathbf{k}) + \tilde{V}_h(\Delta \mathbf{k}) \mathcal{F}_h(\Delta \mathbf{k}) + i |\mathbf{k}' \times \mathbf{k}| a_B^2 \left( \frac{a_B}{l_B} \right)^2 \left( \tilde{V}_e(\Delta \mathbf{k}) \alpha_e - \tilde{V}_h(\Delta \mathbf{k}) \alpha_h \right) . \tag{4}
\]

Here \( \tilde{V}_j(\mathbf{k}) \) is the two-dimensional Fourier transform of the potentials \( V_j(\mathbf{r}) \) (\( j = e, h \)), \( \mathcal{F}_e(\Delta \mathbf{k}) = \left[ 1 - 3a_B^2 m_e^2(\Delta \mathbf{k})^2 / (32M^2) \right] + \beta_e(\Delta \mathbf{k})^2 a_B^2 (a_B/l_B)^4 \), and \( \Delta \mathbf{k} = \mathbf{k}' - \mathbf{k} \) is the transferred momentum. In the derivation above only the essential terms of up to the second order in \( B \) and the lowest order in \( |\mathbf{k}| a_B \) are taken into account. The expressions for the dimensionless constants \( \alpha_{e(h)} \) and \( \beta_{e(h)} \) for a 2D Wannier-Mott exciton can be found in \[17\] (we corrected typos in the original formulas): \( \beta_{e(h)} = 4^{-6} M^{-2} (105m_e^2 - 159\mu^2/2) \) and \( \alpha_{e(h)} = -2m_e(\kappa_k) / M, \kappa = -21/165M \). Note that \( \beta_e, \beta_h > 0 \), therefore, the exciton scattering cross-section increases with \( B \) when \( l_B \gg a_B \). An important point is that the time-reversal symmetry is broken \( V_{k,k'} \neq V_{-k,-k'} \), and the structure of \( V_{k,k'} \) resembles its counterpart \[24\] from the simplified phenomenological model. The ground-state exciton energy spectrum is given by

\[
\epsilon(\mathbf{k}) = \frac{\hbar^2 k^2}{2M} \left[ 1 - 2|\kappa| \left( \frac{a_B}{l_B} \right)^4 \right] - \epsilon_0 \left[ 1 - \left( \frac{l_2}{l_B} \right)^4 \right] , \tag{5}
\]

where the parameter \( l_2 = (3/128)^{1/4} a_B \) determines the diamagnetic shift. The first term stands for the center-of-mass kinetic energy, whereas the second term represents the binding energy.

Magneto-exciton regime. In the opposite regime, when the magnetic field is strong (i.e., \( a_B \gg l_B \)), one can neglect the Coulomb interaction term in the Hamiltonian \[23\] and the exciton dynamics is governed by the magnetic field solely. In this regime, which is often referred to as the magneto-exciton regime, one can calculate the dispersion of the ground state of the system, treating the \( e-h \) Coulomb potential as a perturbation \[24,25\]

\[
\epsilon(\mathbf{k}) = \frac{\hbar^2}{2} \omega_C - \sqrt{\frac{\pi}{4 M^2 \epsilon_0 l_B}} e^2 \frac{\epsilon_0^2}{4 \pi^3} \left( \frac{a_B^2 l_B}{M} \right)^{1/2} \int_0^{\infty} \text{d}r \left( \frac{k^2 l_B^2}{4} \right) . \tag{6}
\]

The corresponding impurity scattering matrix elements are given by

\[
V_{\mathbf{k},\mathbf{k}'} = \tilde{V}_e(\Delta \mathbf{k}) \exp \left( -\frac{i}{2} |\mathbf{k}' \times \mathbf{k}| l_B^2 - \frac{\Delta \mathbf{k}^2 l_B^2}{4} \right) + \tilde{V}_h(\Delta \mathbf{k}) \exp \left( +\frac{i}{2} |\mathbf{k}' \times \mathbf{k}| l_B^2 - \frac{\Delta \mathbf{k}^2 l_B^2}{4} \right) , \tag{7}
\]

here \( \omega_C = eB/\mu \) is the cyclotron frequency.

Scattering rates. The impurity scattering matrix elements, discussed above for the two regimes, is the starting
ially found eigenstates of Eq. (3), whereas the dashed lines are obtained from the numerical solution of the plot are typical of GaAs quantum wells: the effective eigenvalue corresponding to $|\psi_k\rangle$ under consideration, respectively, and $\theta = \varphi - \varphi'$ is the scattering angle, $\varphi, \varphi'$ are the polar angles of $k, k'$, and $n_0 = M/(2\pi \hbar^2)$ is the 2D density of states of free particles with parabolic dispersion, which is introduced here to write the scattering matrices in dimensionless form. The density of states is defined as $\nu(k) = |\partial \epsilon(k)/\partial k|^{-1} k/(2\pi)$. Here we restrict ourselves to the case of elastic scattering $|\mathbf{k}| = |\mathbf{k}'|$. It is the asymmetric part $\mathcal{J}_{k,k'}$ of exciton scattering by impurities that gives rise to the Hall current. The properties of $\mathcal{J}_{k,k'}$ are discussed in details in Ref. [19–21].

The elastic scattering rate $W_{k,k'}$ from $k'$ to $k$ state is expressed with use of the Fermi golden rule $W_{k,k'} = 2\pi \hbar^{-1} n_{\text{imp}} |\langle T_{k,k'} \rangle|^2 \delta (\epsilon_k - \epsilon_{k'})$, where $n_{\text{imp}}$ is the surface density of impurities. The presence of the magnetic field breaks the time inversion symmetry in the problem and leads to the fact that scattering rates $W_{k,k'}$ and $W_{k',k}$ become non-equivalent and thus a non-zero Hall contribution $\mathcal{J}_{k,k'}$ emerges.

Let us assume that the impurity potential is described by the Coulomb potential $V_{\text{Coul}}(r) = -V_0(r) = -e q_{\text{imp}}/(4\pi \varepsilon_0 r)$. The Lippmann-Schwinger equation can not be treated perturbatively in our system, which is why we solve this integral equation numerically.

Having solved the Lippmann-Schwinger equation numerically in the two regimes under consideration, one may proceeds with calculating the Hall current appearing in the system. We study the classical transport regime $|\mathbf{k}|l \gg 1$, where $l$ is the exciton mean free path. We use the semiclassical Boltzmann equation for the occupation numbers of exciton states $n_k$ having wave vectors $k$.

$$\frac{dn_k}{dt} = P_k - \Gamma n_k + \int \frac{d^2 k'}{(2\pi)^2} (W_{k,k'} n_{k'} - W_{k',k} n_k),$$

where $P_k$ is the coherent pump term, $\Gamma = 1/\tau_0$ is the particle decay rate. When the equilibrium is reached, $n_k$ doesn’t depend on time, and thus the time derivative term can be omitted. Then, performing the integration over the absolute value of $k'$, the Boltzmann kinetic equation can be rewritten as

$$0 = P_k - \Gamma n_k + \int d\varphi' (w_{k,k'} n_{k'} - w_{k',k} n_k),$$

where $w_{k,k'} = n_{\text{imp}} q_0/(\hbar v(k')^2) (g_{k,k'} + j_{k,k'})$ and the exciton density of states $\nu(k) = |\partial \epsilon(k)/\partial k|^{-1} k/(2\pi)$. Let us assume the in-plane wave vector $\mathbf{k}_0$ of the pump is pointing along the $x$-axis, i.e. $\mathbf{k}_0 = (k_0, 0)^T$, which implies $P_k = P_0 \delta(k-k_0) = (P_0/k) \delta(k-k_0) \delta(\varphi)$ and we note that this function is even with respect to $\varphi$. Since $w_{k,k'}$ depends on the difference between the momentum angles, to perform the integration over $\varphi'$ in the collision integral, the particle density $n_k$ and the scattering rate $w_{k,k'}$ should be expanded in cylindrical harmonics $\cos m\varphi, \sin m\varphi$, where $m \geq 0$ is integer. Assuming that only the dipole type of anistropy of the momentum-space
distribution function is significant, we represent \( n_k = n_0(k) + \delta n(k) \), where \( \delta n(k) = n_+(k) \cos \varphi + n_-(k) \sin \varphi \), \( n_0(k) \) is the isotropic part of the distribution function which depends only on energy. The coefficients \( n_{\pm}(k) \) are to be found. Substituting this decomposition into the kinetic equation (10) and integrating over \( \varphi' \) in the collision term, we obtain

\[
0 = \frac{P_0}{k} \delta (k - k_0) \delta(\varphi) \cos \varphi \left( \Omega(k) n_-(k) - \frac{n_+(k)}{\tau(k)} \right) - \frac{n_k}{\tau_0},
\]

where \( \tau(k) \) is given by the symmetric scattering term \( \tau(k)^{-1} = n_{\text{imp}}n_0/(\hbar v(k)^2) \int_0^{2\pi} \mathcal{G}_{k,k'}(1 - \cos \theta) d\theta \), here \( \theta = \varphi - \varphi' \) is the scattering angle. The factor \( \Omega(k) \) is determined by the asymmetric scattering term \( \Omega(k) = -n_{\text{imp}}n_0/(\hbar v(k)^2) \int_0^{2\pi} \mathcal{G}_{k,k'} \sin \theta d\theta \), and it mixes the even \( n_+(k) \) and odd \( n_-(k) \) contributions to the density distribution, yielding a Hall current in the transverse \( y \)-direction. Using the orthogonality of \( \sin \varphi \) and \( \cos \varphi \), the kinetic equation is readily solved, yielding \( n_-(k) = -\Omega(k) \tau_{\text{tot}}(k) n_+(k) \). Here we have used the total relaxation time \( \tau_{\text{tot}}(k) = (\tau_0^{-1} + \tau(k)^{-1})^{-1} \). The Hall angle, defined as the ratio of the Hall current \( j_y \) and the longitudinal current \( j_x \), is

\[
\frac{j_y}{j_x} = -\frac{\Omega(k_0) \tau_{\text{tot}}(k_0)}{\tau_0} \approx -0.08%.
\]

At a strong magnetic field \( B = 18 \text{T} \) we solve the Lippmann-Schwinger equation (11) numerically for the distribution function \( \mathcal{G}_{k,k'} \), which yields \( j_y/j_x = -\Omega(k_0) \tau_{\text{tot}}(k_0) \approx -1.8\% \).

Distribution functions in dark and bright excitons. One may expect, by looking at Fig. 2(a), that at intermediate magnetic field strengths (about \( B \approx 10 \text{T} \)) the Hall angle would be significantly larger, as the exciton dipole moment reaches its largest value in this non-perturbative regime, while still not switching to the magneto-exciton regime. We also note, that the range of magnetic fields corresponding to the largest dipole moment is easily achievable experimentally, which makes the observation of the predicted phenomenon realistic. The dependence of the Hall angle on the exciton lifetime is shown in Fig. 2(b). Clearly, for dark excitons whose lifetime is significantly larger than the lifetime of bright excitons, the Hall angle is notably larger. This shows that the anomalous exciton Hall effect may be used as a tool for spatial separation of dark and bright excitons.

Conclusions. In conclusion, we demonstrated that the magnetic Stark effect for 2D excitons may lead to the emergence of an effective U(1) gauge field. This field can result in the excitonic analogue of the anomalous Hall effects. For the later we presented a detailed microscopic description of the scattering mechanism and analyzed the transport properties, showing that the effect can be observed experimentally in conventional GaAs quantum wells and that it is much stronger for dark than bright excitons.

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