Masses of the $Qar{Q}ar{Q}$ tetraquarks in the relativistic diquark–antidiquark picture

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Masses of the ground state teraquarks composed from heavy $c$ and $b$ quarks and antiquarks are calculated in the diquark-antidiquark picture in the framework of the relativistic quark model based on the quasipotential approach. The quasipotentials of the quark-quark and diquark-antidiquark interactions are constructed similarly to the previous consideration of mesons and baryons. It is assumed that the diquark and antidiquark interact in the tetraquark as a whole and the internal structure of the diquarks is taken into account. All such tetraquarks are found above the thresholds of decays to two heavy quarkonia. This is a result of the consideration of the diquark to be not a point-like object. Therefore such tetraquarks can be observed as broad structures decaying dominantly to quarkonia. The broad structure next to the di-$J/\psi$ mass threshold, recently observed by the LHCb Collaboration, can correspond to the ground $2^{++}$-state tetraquark consisting of four charm quarks.

I. INTRODUCTION

Theoretical and experimental investigations of the properties of exotic hadrons attract substantial interest especially in last two decades. This subject became a hot topic since the first explicit experimental evidence of the existence of hadrons with compositions different from usual $q\bar{q}$ for mesons and $qq$ for baryons became available (for recent reviews see [1–3] and references therein). Candidates both for the exotic tetraquark $qq\bar{q}\bar{q}$ and pentaquark $qqqq\bar{q}$ states were found. However, in the literature there is no consensus about the composition of these states [1–3]. For example, significantly different interpretations for the $qq\bar{q}\bar{q}$ candidates were proposed: molecules composed from two mesons loosely bound by the meson exchange, compact tetraquarks composed from a diquark and antidiquark bound by strong forces, hadroquarkonia composed from a heavy quarkonium embedded in a light meson, kinematic cusps etc. Discrimination between different approaches is a very complicated experimental task.

Investigation of exotic $QQ\bar{Q}$ states consisting from heavy quarks ($Q = c$ and/or $b$) is of a special interest, since their nature can be determined easier. They should be predominantly compact tetraquarks. Indeed, a molecular configuration is very unlikely. Only heavy $Q\bar{Q}$ meson can be exchanged between constituents in such a molecule and the arising Yukawa-type potential is not strong enough to provide binding. The hadroquarkonium picture is not applicable. Thus the diquark ($QQ$)-antidiquark ($\bar{Q}\bar{Q}$) configuration is preferable.

The CMS [4] and LHCb [5] Collaborations searched for the tetraquark states composed
only from bottom quarks in the $\Upsilon$-pair production. No evidence of such states was found. Very recently the LHCb Collaboration \[6\] reported results of the study of the $J/\psi$-pair invariant mass spectrum in proton-proton collision data at the center-of-mass energies of $\sqrt{s} = 7.8$ and 13 TeV. A narrow structure around 6.9 GeV and a broad structure just above twice the $J/\psi$ mass were observed. This discovery caused considerable theoretical activity interpreting these data (see \[7\]–\[10\] and references therein).

In this paper we calculate masses of the ground state $QQ\bar{Q}\bar{Q}$ tetraquarks in the framework of the relativistic quark model based on the quasipotential approach. It is assumed that such tetraquarks are composed from the doubly-heavy diquark ($QQ$) and antidiquark ($\bar{Q}\bar{Q}$). Such approximation significantly simplifies calculations, since instead of the very complicated relativistic four-body problem we need to solve more simple two relativistic two-body problems. First, masses and wave functions of diquarks (antidiquarks) are obtained by solving the relativistic quark-quark (antiquark-antiquark) quasipotential equation. Second, masses of tetraquarks are calculated by considering them to be the diquark-antidiquark bound states. The quasipotentials of the corresponding interactions are constructed using the same assumptions about their structure and parameters which were previously used for the investigation of the different properties of mesons and baryons \[11\]–\[15\]. The spin-independent and spin-dependent relativistic contributions to the quasipotentials of the $QQ$ interaction in a diquark $d$ and the $d\bar{d}$ interaction in a tetraquark are considered nonperturbatively. It is assumed that a diquark and antidiquark in a tetraquark interact as a whole, thus interactions between quarks from a diquark with antiquarks from an antidiquark are not considered. It is important to point out that diquarks and antidiquarks are not the point-like objects. Their short-distance interaction with gluons is smeared by the form factors which are calculated in terms of the overlap integrals of the diquark wave functions. Such an approach was previously applied for the calculation of the masses of heavy ($qQ\bar{q}\bar{Q}$, $QQ\bar{q}\bar{q}$, $Qq\bar{q}\bar{q}$) and light ($qq\bar{q}\bar{q}$) tetraquarks \[16\]–\[20\].

The paper is organized as follows. In Sec. II we describe our relativistic quark model. The quasipotentials of the $QQ$ and $d\bar{d}$ interactions are presented. The masses of the doubly-heavy diquarks and the form factors of their interaction with gluons are obtained. In Sec. III masses of the $QQ\bar{Q}\bar{Q}$ tetraquarks are calculated. They are confronted with lowest thresholds for the fall-apart decays to two heavy quarkonia. Detailed comparisons with previous theoretical predictions within different approaches are given. Finally, we present our conclusions and summary of the obtained results in Sec. IV.

II. RELATIVISTIC DIQUARK-ANTIDIQUARK MODEL

For the calculation of the masses of tetraquarks we use the relativistic quark model based on the quasipotential approach and the diquark-antidiquark picture of tetraquarks. First, we calculate the masses and wave functions ($\Psi_d$) of the doubly-heavy diquarks as the bound quark-quark states. Second, the masses of the tetraquarks and their wave functions ($\Psi_T$) are obtained for the bound diquark-antidiquark states. These wave functions are solutions of the Schrödinger-type quasipotential equations \[18\]

$$
\left(\frac{b^2(M)}{2\mu_R} - \frac{p^2}{2\mu_R}\right)\Psi_{d,T}(p) = \int \frac{d^3q}{(2\pi)^3} V(p, q; M)\Psi_{d,T}(q),
$$

(1)
with the on-mass-shell relative momentum squared given by
\[ b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}, \] (2)
and the relativistic reduced mass
\[ \mu_R = \frac{E_1E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}. \] (3)
The on-mass-shell energies \( E_1, E_2 \) are defined as follows
\[ E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}. \] (4)
The bound-state masses of a diquark or a tetraquark are \( M = E_1 + E_2 \), where \( m_{1,2} \) are the masses of quarks \( (Q_1 \text{ and } Q_2) \) which form the diquark or of the diquark \( (d) \) and antidiquark \( (\bar{d}') \) which form the heavy tetraquark \( (T) \), while \( \mathbf{p} \) is their relative momentum.

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The quasipotential operator \( V(\mathbf{p}, \mathbf{q}; M) \) in Eq. (1) is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. The quark-quark \( (Q\bar{Q}', Q') \) interaction quasipotential \(^1\) is considered to be \( 1/2 \) of the quark-antiquark \( (Q\bar{Q}) \) interaction and is given by
\[ V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)V(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q), \] (5)
with
\[ V(\mathbf{p}, \mathbf{q}; M) = \frac{1}{2} \left[ \frac{4}{3} \alpha_s D_{\mu\nu}(\mathbf{k}) \gamma^\mu \gamma_2 \gamma^\nu + V_{\text{conf}}^V(\mathbf{k}) \Gamma_1^\mu(\mathbf{k}) \Gamma_2;\mu(-\mathbf{k}) + V_{\text{conf}}^S(\mathbf{k}) \right]. \]
Here, \( D_{\mu\nu} \) is the gluon propagator in the Coulomb gauge, \( u(p) \) are the Dirac spinors and \( \alpha_s \) is the running QCD coupling constant with freezing
\[ \alpha_s(\mu^2) = \frac{4\pi}{(11 - \frac{2}{3}n_f) \ln \frac{\mu^2 + M_B^2}{\Lambda^2}}. \] (6)
where the scale \( \mu \) is chosen to be equal to \( 2m_1m_2/(m_1 + m_2) \), the background mass is \( M_B = 2.24\sqrt{A} = 0.95 \) GeV, and \( n_f \) is the number of flavours. The effective long-range vector vertex contains both Dirac and Pauli terms
\[ \Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m} \sigma_{\mu\nu} \tilde{k}^\nu, \quad \tilde{k} = (0, \mathbf{k}), \] (7)
where \( \kappa \) is the long-range anomalous chromomagnetic moment. In the nonrelativistic limit the vector and scalar confining potentials in configuration space have the form
\[ V_{\text{conf}}^V(r) = (1 - \varepsilon)(Ar + B), \quad V_{\text{conf}}^S(r) = \varepsilon(Ar + B), \]
\[ V_{\text{conf}}(r) = V_{\text{conf}}^V(r) + V_{\text{conf}}^S(r) = Ar + B, \] (8)

\(^1\) We consider diquarks in a tetraquark, as in a baryon, to be in the color triplet state, since in the color sextet there is a repulsion between two quarks.
where $\varepsilon$ is the mixing coefficient. Therefore in the nonrelativistic limit the $QQ'$ quasipotential reduces to

$$V^{\text{NR}}_{QQ}(r) = \frac{1}{2} V^{\text{NR}}_{QQ'}(r) = \frac{1}{2} \left(-\frac{4}{3} \frac{\alpha_s}{r} + Ar + B\right),$$

reproducing the usual Cornel potential. Thus our quasipotential can be viewed as its relativistic generalization. It contains both spin-independent and spin-dependent relativistic contributions.

Constructing the diquark-antidiquark ($d\bar{d}$) quasipotential we use the same assumptions about the structure of the short- and long-range interactions. Taking into account the integer spin of a diquark in the color triplet state, the quasipotential is given by $[17, 18]$

$$V(p, q; M) = \frac{\langle d(P)|J_{\mu}|d(Q)\rangle}{2\sqrt{E_dE_{\bar{d}}}} \frac{4}{3} \alpha_s D^{\mu\nu}(k) \frac{\langle d'(P')|J_{\mu}|d''(Q')\rangle}{2\sqrt{E_{d'}E_{\bar{d}'}}}$$

$$+ \psi_d^*(P) \psi_{d'}^*(P') \left[ J_{d\mu} J_{\bar{d'}}^{\mu} V^{\text{conf}}(k) + V^{\text{conf}}(k) \right] \psi(Q) \psi(Q'),$$

where $\psi_d(p)$ and $J_{d\mu}$ are the wave function and effective long-range vector vertex of the diquark, respectively. The vertex of the diquark-gluon interaction $\langle d(P)|J_{\mu}|d(Q)\rangle$ accounts for the internal structure of the diquark and leads to emergence of the form factor $F(r)$ smearing the one-gluon exchange potential.

All parameters of the model were fixed previously $[11–14]$ from the consideration of meson and baryon properties. They are the following. The constituent heavy quark masses: $m_b = 4.88$ GeV, $m_c = 1.55$ GeV. The parameters of the quasipotential: $A = 0.18$ GeV$^2$, $B = -0.3$ GeV, $\Lambda = 413$ MeV; the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$; the universal Pauli interaction constant $\kappa = -1$.

The resulting diquark-antidiquark potential for the tetraquark ground states (the orbital momentum $L = 0$), where quark energies $\epsilon_{1,2}(p)$ were replaced by the on-shell energies $E_{1,2}$ $[4]$ to remove the non-locality, is given by $[18]$

$$V(r) = \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}(r) + \frac{1}{E_1 E_2} \left\{ p \left[ \hat{V}_{\text{Coul}}(r) + V^V_{\text{conf}}(r) \right] p - \frac{1}{4} \Delta V^V_{\text{conf}}(r) \right\}$$

$$+ \frac{2}{3} \Delta \hat{V}_{\text{Coul}}(r) \mathbf{S}_1 \cdot \mathbf{S}_2 \right\}.$$  

(11)

Here

$$\hat{V}_{\text{Coul}}(r) = -\frac{4}{3} \frac{\alpha_s}{r} \frac{F_1(r) F_2(r)}{r}$$

is the Coulomb-like one-gluon exchange potential which takes into account the finite sizes of the diquark and antidiquark through corresponding form factors $F_{1,2}(r)$. $\mathbf{S}_{1,2}$ are the diquark and antidiquark spins. The numerical analysis shows that this form factor can be approximated with high accuracy by the expression

$$F(r) = 1 - e^{-\xi r - \zeta r^2}.$$  

(12)

Such form factor smears the one-gluon exchange potential and removes spurious singularities in the local relativistic quasipotential thus allowing one to use it nonperturbatively to find the numerical solution of the quasipotential equation. The masses and parameters of heavy diquarks are the same as in the doubly heavy baryons $[12]$ and are given in Table II.
TABLE I. Masses $M$ and form factor parameters of heavy $QQ'$ diquarks. $S$ and $A$ denote scalar and axial-vector diquarks, antisymmetric $[Q,Q']$ and symmetric $\{Q,Q'\}$ in flavour, respectively.

| Quark content $Q$ $\bar{Q}$ | Diquark type $Q_{\text{content}}$ | $M$ (MeV) | $\xi$ (GeV) | $\zeta$ (GeV$^2$) | $M$ (MeV) | $\xi$ (GeV) | $\zeta$ (GeV$^2$) |
|-----------------------------|----------------------------------|-----------|-------------|----------------|-----------|-------------|----------------|
| $[Q,c]$                     | $S$                              |           |             |               |           |             |               |
| $\{Q,c\}$                  | $A$                              | 3226      | 1.30        | 0.42          | 6526      | 1.50        | 0.59          |
| $\{Q,b\}$                  | $A$                              | 6526      | 1.50        | 0.59          | 9778      | 1.30        | 1.60          |

III. MASSES OF $QQ\bar{Q}\bar{Q}$ TETRAQUARKS

We substitute the quasipotential (11) in the quasipotential equation (1) and solve the resulting differential equation numerically. The calculated masses $M$ of the neutral $QQ'\bar{Q}\bar{Q}'$ tetraquarks composed from the heavy diquark ($QQ'$, $Q=b,c$), and heavy antidiquark ($\bar{Q}\bar{Q}'$) are given in Table III. The masses of the charged heavy $QQ'\bar{Q}\bar{Q}'$ tetraquarks are presented in Table IV. In these tables we give the values of the lowest thresholds $T$ for decays into two corresponding heavy mesons $[(QQ')]$, which were calculated using the measured masses of these mesons [21]. We also show values of the difference of the tetraquark and threshold masses $\Delta = M - T$. If this quantity is negative, then the tetraquark lies below the threshold of the fall-apart decay into two mesons and thus should be a narrow state. The states with small positive values of $\Delta$ could be also observed as resonances, since their decay rates will be suppressed by the phase space. All other states are expected to be broad and thus difficult to observe.

From these tables we see that the predicted masses of almost all $QQ\bar{Q}\bar{Q}$ tetraquarks lie significantly higher than the thresholds of the fall-apart decays to the lowest allowed two quarkonium states. All these states should be broad since they can decay to corresponding quarkonium states through quark and antiquark rearrangements and these decays are not suppressed either dynamically or kinematically. This conclusion is in accord with the current experimental data. Indeed, the CMS [4] and LHCb [3] Collaborations have not observed narrow beautiful tetraquarks in the $\Upsilon(1S)$-pair production. Note that the lattice nonrelativistic QCD [22] calculations also find the $bb\bar{b}\bar{b}$ tetraquarks above the corresponding two bottomonium thresholds. On the other hand the broad structure near the di-$J/\psi$ mass threshold very recently observed by the LHCb [6] can correspond to the $2^{++}$ state of the $c\bar{c}c\bar{c}$ tetraquark which mass is predicted to be 6367 MeV. The narrow structure, $X(6900)$, could be the orbital or radial excitation of this tetraquark. Such excited states can be narrow despite the large phase space since it will be necessary in the fall-apart process to overcome the suppression either due to the centrifugal barrier for the orbital excitations or due to the presence of the nodes in the wave function of the radially excited state.

In Tables IV-VII we compare our predictions for the masses of $QQ\bar{Q}\bar{Q}$ tetraquarks with the results of previous calculations [9, 23–41]. The nonrelativistic quark model and diquark-antidiquark structure of tetraquarks was employed in Ref. [23], while the authors of Refs. [9, 25] used for the calculations the string-junction picture and the constituent diquark-antidiquark model. References [24, 26, 27] present results obtained in different versions of the QCD sum rules. A simple constituent quark model with the color-magnetic interaction was applied in Ref. [28]. The relativized diquark-antidiquark model and variational method with harmonic oscillator trial wave functions were employed in Refs. [29, 31].
TABLE II. Masses $M$ of the neutral heavy diquark ($QQ'$)-antidiquark ($\bar{QQ}'$) states. $T$ is the threshold for the decays into two heavy-($QQ'$) mesons and $\Delta = M - T$. All values are given in MeV.

| Composition | $d\bar{d}$ | $J^{PC}$ | $M$ | $\eta_c(1S)\eta_c(1S)$ | $T$ | $\Delta$ |
|-------------|------------|----------|-----|-------------------------|-----|----------|
| $cc\bar{c}\bar{c}$ | $A\bar{A}$ | $0^{++}$ | 6190 | $\eta_c(1S)\eta_c(1S)$ | 5968 | 222 |
| | | | | $J/\psi(1S)J/\psi(1S)$ | 6194 | -4 |
| | | $1^{+-}$ | 6271 | $\eta_c(1S)J/\psi(1S)$ | 6081 | 190 |
| | | $2^{++}$ | 6367 | $J/\psi(1S)J/\psi(1S)$ | 6194 | 173 |
| $bb\bar{b}\bar{b}$ | $A\bar{A}$ | $0^{++}$ | 12813 | $\eta_c(1S)\eta_b(1S)$ | 12383 | 430 |
| | | | | $J/\psi(1S)Y(1S)$ | 12557 | 256 |
| | | | | $B_c^{\pm}B_c^{\mp}$ | 12550 | 263 |
| | | | | $B_{c^*}^{\pm}B_{c^*}^{\mp}$ | 12666 | 147 |
| $\frac{1}{\sqrt{2}}(A\bar{S} \pm S\bar{A})$ | $A\bar{A}$ | $1^{++}$ | 12826 | $\eta_c(1S)Y(1S)$ | 12444 | 382 |
| | | | | $J/\psi(1S)\eta_b(1S)$ | 12496 | 330 |
| | | | | $B_c^{\pm}B_{c^*}^{\mp}$ | 12608 | 218 |
| | | | | $B_{c^*}^{\pm}B_c^{\mp}$ | 12666 | 160 |
| $SS\bar{S}$ | $S\bar{S}$ | $0^{++}$ | 12824 | $\eta_c(1S)\eta_b(1S)$ | 12383 | 441 |
| | | | | $J/\psi(1S)Y(1S)$ | 12557 | 274 |
| | | | | $B_c^{\pm}B_{c^*}^{\mp}$ | 12550 | 274 |
| | | | | $B_{c^*}^{\pm}B_c^{\mp}$ | 12666 | 158 |

Note that we and authors of mass inequality relations among tetraquarks and heavy quarkonia were also obtained [29]. Different versions of the nonrelativistic quark model and diquark-antidiquark picture were used in Refs. [30, 33, 36]. The diffusion Monte Carlo method was applied to solve the nonrelativistic four-body problem for the $bb\bar{b}\bar{b}$ tetraquark in Ref. [35]. In Ref. [37] the multiquark color flux-tube model was employed. The meson-meson and diquark-antidiquark structures were considered: in the nonrelativistic chiral quark model using Gaussian expansion method in Refs. [38, 41]; in the nonrelativistic quark delocalization color screening model using resonating group method for bound states [39]; and extended relativized quark model using variational approach with Gaussian wave functions in Ref. [40].
TABLE III. Masses $M$ of the charged heavy diquark–antidiquark states. $T$ is the threshold for the decays into two heavy ($QQ'$) mesons and $\Delta = M - T$. All values are given in MeV.

| Composition | $dd$ | $J^P$ | $M$ | Threshold $T$ | $\Delta$ |
|-------------|------|-------|-----|---------------|--------|
| $cc\bar{c}b, cb\bar{c}$ | $A\bar{A}$ | $0^+$ | 9572 | $\eta_c(1S)B_c^\pm$ | 9259 | 313 |
| | | | | $J/\psi(1S)B_c^\pm$ | 9430 | 142 |
| | | | | $\eta_c(1S)B_c^\pm$ | 9317 | 285 |
| | | $1^+$ | 9602 | $J/\psi(1S)B_c^\pm$ | 9372 | 230 |
| | | | | $J/\psi(1S)B_c^\pm$ | 9430 | 172 |
| | | $2^+$ | 9647 | $J/\psi(1S)B_c^\pm$ | 9430 | 217 |
| $AS, SA$ | | $1^+$ | 9619 | $\eta_c(1S)B_c^\pm$ | 9317 | 302 |
| | | | | $J/\psi(1S)B_c^\pm$ | 9372 | 247 |
| | | | | $J/\psi(1S)B_c^\pm$ | 9430 | 189 |
| $cc\bar{b}b, bb\bar{c}$ | $A\bar{A}$ | $0^+$ | 12846 | $B_c^\pm B_c^\pm$ | 12550 | 296 |
| | | | | $B_c^\pm B_c^\pm$ | 12666 | 180 |
| | | $1^+$ | 12859 | $B_c^\pm B_c^\pm$ | 12608 | 251 |
| | | | | $B_c^\pm B_c^\pm$ | 12666 | 193 |
| | | $2^+$ | 12883 | $B_c^\pm B_c^\pm$ | 12666 | 217 |
| $cb\bar{b}b, bb\bar{c}$ | $A\bar{A}$ | $1^+$ | 16109 | $B_c^\pm \eta_b(1S)$ | 15674 | 435 |
| | | | | $B_c^\pm \Upsilon(1S)$ | 15793 | 316 |
| | | $2^+$ | 16117 | $B_c^\pm \Upsilon(1S)$ | 15735 | 382 |
| | | | | $B_c^\pm \eta_b(1S)$ | 15732 | 385 |
| | | | | $B_c^\pm \Upsilon(1S)$ | 15793 | 324 |
| | $SA, AS$ | $1^+$ | 16117 | $B_c^\pm \Upsilon(1S)$ | 15735 | 382 |
| | | | | $B_c^\pm \eta_b(1S)$ | 15732 | 385 |
| | | | | $B_c^\pm \Upsilon(1S)$ | 15793 | 324 |

Refs. [9, 23, 25, 29, 31, 34] consider diquarks and antidiquarks only in the color triplet and antitriplet color states, while the color sextet and antisextet configurations and their mixing is accounted in Refs. [28, 32, 33, 36–41]. In most of the previous calculations diquarks and antidiquarks were considered to be point-like. Our calculation shows that the account of the diquark structure (size) weakens the Coulomb-like one gluon-exchange potential, thus increasing tetraquark masses and reducing spin-spin splittings. We can see from Tables IV-VII that there are significant disagreements between different theoretical approaches. Indeed, Refs. [23, 26, 27, 30, 31, 34, 35] predict heavy tetraquark masses below or slightly above the thresholds of the decays to two quarkonia and, thus, stable or significantly suppressed against fall-apart decays with a very narrow decay width. On the other hand our model and other approaches predict such tetraquark masses significantly above these thresholds and, thus, they can be observed only as broad resonances.
TABLE IV. Comparison of theoretical predictions for the masses of the neutral \((QQ)(\bar{Q}\bar{Q})\) tetraquarks composed from the same flavour heavy quarks and antiquarks (in MeV).

| Reference | \(cc\bar{c}\bar{c}\) | \(0^{++}\) | \(1^{-+}\) | \(2^{++}\) | \(bb\bar{b}\bar{b}\) | \(0^{++}\) | \(1^{-+}\) | \(2^{++}\) |
|-----------|---------------------|-----|-----|-----|---------------------|-----|-----|-----|
| this paper | 6190                | 6271| 6367| 19314| 19320| 19330|
| [23]      | 5966                | 6051| 6223| 18754| 18808| 18916|
| [24]      | 6460-6470          | 6370-6510| 6370-6510| 18460-18490| 18320-18540| 18320-18530|
| [9, 25]   | 6192 ± 25           | 6429 ± 25| 18826 ± 25| 18956 ± 25|
| [26, 27]  | 5990 ± 80           | 6050 ± 80| 6090 ± 80| 18840 ± 90| 18840 ± 90| 18850 ± 90|
| [28]      | 6797                | 6899| 6956| 20155| 20212| 20243|
| [29]      | < 6140              |     |     | 18750|
| [30, 31]  | 5969                | 6021| 6115|     |     |     |
| [32, 33]  | 6487                | 6500| 6524| 19322| 19329| 19341|
| [34]      | 5883                | 6120| 6246| 18748| 18828| 18900|
| [35]      |                     |     | 18690 ± 30|     |     |     |
| [36]      | 6425                | 6425| 6432| 19247| 19247| 19249|
| [37]      | 6407                | 6463| 6486| 19329| 19373| 19387|
| [38]      |                     |     | 19178| 19226| 19236|
| [39]      | 6314                | 6375| 6407| 19237| 19264| 19279|
| [40]      | 6542                | 6515| 6543| 19255| 19251| 19262|

TABLE V. Comparison of theoretical predictions for the masses of the \((cb)(\bar{c}\bar{b})\) tetraquarks (in MeV).

| Reference | \(A\bar{A}\) | \(0^{++}\) | \(1^{-+}\) | \(2^{++}\) | \(\frac{1}{\sqrt{2}}(A\bar{S} \pm S\bar{A})\) | \(0^{++}\) | \(1^{-+}\) | \(2^{++}\) |
|-----------|-----------|-----|-----|-----|---------------------|-----|-----|-----|
| this paper | 12813     | 12826| 12849| 12831| 12831| 12824|
| [23]      | 12359     | 12424| 12566| 12485| 12488| 12471|
| [28]      | 13483     | 13520| 13590| 13510| 13592| 13553|
| [29]      | < 12620   |     |     |     |     |     |
| [32]      | 13035     | 13047| 13070| 13056| 13052| 13050|
| [34]      | 12374     | 12491| 12576| 12533| 12533| 12521|
| [37]      | 12829     | 12881| 12925|     |     |     |
| [41]      | 12746     | 12804| 12809|     |     |     |

IV. CONCLUSIONS

We calculated the masses of ground state tetraquarks composed only of heavy \((b\text{ or/and } c)\) quarks and antiquarks in the framework of the diquark-antidiquark picture and relativistic quark model based on the quasipotential approach. It was assumed that two heavy quarks and two heavy antiquarks form a doubly-heavy diquark and antidiquark, respectively. The dynamics of quarks in the diquark is governed by the relativistic \(QQ\) quasipotential which
TABLE VI. Comparison of theoretical predictions for the masses of the charged \((QQ)(\bar{Q}Q')\) tetraquarks (in MeV).

| Reference          | \(A\bar{A}\)       | \(A\bar{S},S\bar{A}\) |
|--------------------|--------------------|------------------------|
|                    | 0\(^+\)            | 1\(^+\)                | 2\(^+\)          |
| This paper         | 9572               | 9602                   | 9647             | 9619             |
| [28]               | 10144              | 10282                  | 10273            | 10174            |
| [29]               | < 9390             |                        |                  |                  |
| [32]               | 9740               | 9749                   | 9768             | 9746             |
| [37]               | 9670               | 9683                   | 9732             |                  |

\(cccc, cb\bar{c}\)

| Reference          | \(A\bar{A}\)       | \(A\bar{S},S\bar{A}\) |
|--------------------|--------------------|------------------------|
|                    | 0\(^+\)            | 1\(^+\)                | 2\(^+\)          |
| This paper         | 16109              | 16117                  | 16132            | 16117            |
| [28]               | 16823              | 16840                  | 16917            | 16915            |
| [29]               | < 15770            |                        |                  |                  |
| [32]               | 16158              | 16164                  | 16176            | 16157            |
| [37]               | 16126              | 16130                  | 16182            |                  |

TABLE VII. Comparison of theoretical predictions for the masses of the \(cc\bar{b}, b\bar{b}c\) tetraquarks (in MeV).

| Reference          | \(A\bar{A}\)       | \(cc\bar{b}, b\bar{b}c\) |
|--------------------|--------------------|--------------------------|
|                    | 0\(^+\)            | 1\(^+\)                  | 2\(^+\)          |
| This paper         | 12846              | 12859                    | 12883            |
| [28]               | 13496              | 13560                    | 13595            |
| [29]               | < 12580            |                          |                  |
| [32]               | 12953              | 12960                    | 12972            |
| [34]               | 12445              | 12536                    | 12614            |
| [36]               | 12866              | 12864                    | 12868            |
| [37]               | 12906              | 12945                    | 12960            |
| [41]               | 12892              | 12898                    | 12905            |

is one half of the \(Q\bar{Q}\) potential in the heavy quarkonium. Masses and wave functions of diquarks were calculated by the numerical solution of the quasipotential equation. The obtained diquark wave functions were used for the evaluation of the form factors of the diquark-gluon interaction \(F(r)\). Then the \(QQ\bar{Q}\bar{Q}\) tetraquark was considered as a bound diquark-antidiquark system. It was assumed that diquarks and antidiquarks interact as a whole. Constructing the quasipotential of the \(d - \bar{d}\) interaction the same assumptions about the structure of the long-range confining interaction were used with the correction to the integer spin of the diquark. In the potential of the one-gluon exchange between
the diquark and antidiquark the form factors $F(r)$ of the diquark-gluon interaction were introduced. They are expressed as the overlap integrals of the diquark wave functions and take into account the internal structure (finite size) of the diquarks and antidiquarks. These form factors significantly weaken the Coulomb-like potential, thus increasing masses of the tetraquarks and reducing spin splittings. Note that the approaches with a point-like diquark substantially underestimate the mass of the doubly charmed baryon $\Xi_{cc}$, while our model correctly predicted its mass \cite{12} long before its experimental discovery.

It was found that the predicted masses of all ground-state $QQ\bar{Q}\bar{Q}$ tetraquarks are above thresholds for decays into two heavy ($Q\bar{Q}$) mesons. Therefore they should rapidly fall-apart into two lowest allowed quarkonium states. Such decays proceed through quark rearrangements and are not suppressed both dynamically and kinematically. These states should be broad and, thus, difficult to observe experimentally. The states, which masses are predicted to be less than 200 MeV higher than the lowest allowed thresholds, are the $1^{+-}$ and $2^{++}$ states of the $c\bar{c}c\bar{c}$ tetraquark. They have the smallest phase space for the decay to two charmonium states. The former one decays mainly to $\eta_cJ/\psi$ while the latter one to $J/\psi J/\psi$. The $2^{++}$ $c\bar{c}c\bar{c}$ state with the predicted mass 6367 MeV can correspond to the broad structure recently observed by the LHCb Collaboration \cite{6} in the mass spectrum of $J/\psi$-pairs produced in proton-proton collisions. On the other hand all ground state $b\bar{b}b\bar{b}$ tetraquarks have masses significantly (400–500 MeV) higher than corresponding thresholds and thus should be very broad. This agrees well with the absence of the narrow beautiful tetraquarks in the $\Upsilon$-pair production reported by CMS \cite{4} and LHCb \cite{5}.

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