A Sparse Linear Model and Significance Test for Individual Consumption Prediction

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Abstract—Accurate prediction of user consumption is a key part not also in understanding consumer flexibility and behavior patterns, but in the design of robust and efficient energy saving programs as well. Existing prediction methods usually have high relative errors that can be larger than 40%. In this paper, we propose a method to improve prediction accuracy of individual users by exploring sparsity in historical data and leveraging predictive relationship between different users. Sparsity is captured by popular least absolute shrinkage and selection (LASSO) estimator, while user selection is formulated as an optimal hypothesis testing problem and solved via a covariance test. We provide extensive simulation against well-known techniques such as support vector machine (SVM), principal component analysis (PCA) and random forest (RF) using real world data. Simulation results demonstrate that our proposed methods are operationally efficient, interpretable, and achieves optimal prediction performance.

Index Terms—Load forecasting, LASSO, sparse autoregressive model, significance test

I. INTRODUCTION

Electric load forecasting is an important problem in the power engineering industry and have received extensive attention from both industry and academia over the last century. Many different forecasting techniques have been developed during this time, and for an non-exhaustive list, see [1], [2], [3] and the references within. A common theme among many of the established methods is that they are used to forecast relative large loads, from substations serving megawatts to transmission networks serving more than gigawatts of power [4]. As the grid shifts to a more distributed system, the need of accurate forecasting for smaller sized loads are becoming increasing important.

Recent advances in technology such as smart meters, bidirectional communication capabilities and distributed energy resources have made individual households active participants in the power system. Many applications and programs based on these new technologies require estimating the future load of individual homes. For example, state estimation algorithms for distribution systems require pseudo-measurements [5], and these are provided by load forecasts. Another important class of application is demand response and dynamic pricing programs, where users’ future demand are needed to design appropriate incentives [6], [7], [8].

In contrast large aggregated loads, individual load forecasting is less developed. The current state of forecasting algorithms falls under three broad classes: simple averaging, statistical regression methods, and artificial intelligence (e.g. neural network). They are listed in increasing order of prediction accuracy and decreasing in order of interpretability. Simple averaging is intuitively pleasing since it is based on the mean of the previous similar days, but is often not very accurate. On the other end of the spectrum, artificial intelligence (AI) methods can be extremely accurate, but difficult to interpret. In this paper, we propose an algorithm that can achieve the performance of the state of the art AI methods, but retains the interpretability of regression methods.

The algorithm we propose is based on the well known LASSO algorithm in statistics and signal processing [9]. In particular, we formulate the load forecasting problem as learning parameters of a sparse autoregressive model. This algorithm has two main contributions. The first is that LASSO selects the best recurrent pattern in historical data automatically by shrinking irrelevant coefficients to zero. By selecting the correct features, the algorithm improves the interpretation of order selections in autoregressive models, and as we will show using real load data from PG&E, it also greatly improves the prediction accuracy of current regression models.

The second contribution of the proposed framework is that data from other users can be leveraged for forecasting. Intuitively, this means that knowing the past history of user $j$ improves forecasting of the user $i$. For a given user, we use a sequential hypothesis testing procedure to find the best other user’s historical data to include in the algorithm. This testing procedure differs significantly from standard clustering algorithms since it looks for the most “predictive” user, not necessarily the most similar user. We give a rigorous derivation of the hypothesis testing procedure and quantify the confidence of including other users. This allows us to show that the procedure is optimal, in the sense that given user $i$, it finds user $j$, whose historical data improves load forecasting for user $i$ the most among all users.

We derive rigorous theoretical justification for our methods as well as provide extensive simulation studies with respect to several well studied prediction methods. In particular, we compare against linear autoregression, support vector machine, principle component analysis, and random forest. Using a user’s own historical data, our proposed method and random forest both reduce prediction error by 30% compared to other predictors. Our proposed method is far more interpretable than random forest since the latter is a generic machine learning technique that relies on a random ensemble of decision
trees [10], [11]. Therefore, our method is useful to system operators in policy decisions without sacrificing prediction accuracy. By adding the historical data of another user, we can further improve the prediction accuracy.

The rest of the paper is organized as follows. Section I-A analyzes related work in short term load forecasting. Section II presents the autoregressive model for time series analysis. Section III introduces LASSO type linear regression model. Section IV proceeds with the significance test to pair users in order to improve prediction performance. It describes the significance test for LASSO, i.e., covariance test, to select the most significant user to form the pair with the current user. Section V introduces the evaluation criterion for prediction and details the simulation of the proposed methods compared to several other popular prediction methods. And finally Section VI concludes the paper and draws avenues for future work.

A. Literature review

There exists an extensive literature on short term load forecasting [12], [1]. In summary, research on energy consumption prediction can be divided into three groups [11], including simple averaging models, statistical models and AI models.

The simplest approach is to employ moving average [13]. Such models make predictions on mean of consumption data from previous similar days [14]. AI type methods (e.g. neural networks or random forests) yields high accuracy at the cost of complexity of the system, which may lead to overfitting [15]. Other drawbacks include difficult parametrization and non-obvious selection of variables, which are difficult to interpret. Statistical methods sits in between the previous methods in terms of complexity and accuracy, and includes regression models, probabilistic approach applied to regression models, and time series analysis such as autoregressive models.

In statistical methods, regression models combine several features to form normally a linear function. In [16], the authors build a regression tree model with weather data to predict consumption. Support vector machine is used in [17]. Gaussian process framework for prediction mitigating the uncertainty problem is proposed recently [18]. Besides these methods, times series analysis has also been widely applied to consumption data prediction [19]. An overview can be found in [20].

Another type models is the autoregressive integrated moving average (ARIMA) model [21][22]. Authors in [23] proposed a vector autoregressive model to include renewables, prices and loads together with sparsity recovery. In addition, to extend from linearity to nonlinearity, [24] addresses a mixed model combining ARIMA model to deal with the linear part and neural network with the nonlinear one. In our work, we recover the sparsity for univariate time series and multivariate time series under the framework of autoregressive models.

II. AUTOREGRESSIVE MODEL

Autoregressive models are widely used for prediction and inference of time series data. Here we adopt a linear autoregressive model of the hourly consumption of a single user, where future demands depends linearly on historical data plus random noise:

\[ y_t = \beta_0 + \sum_{i=1}^{I} \beta_i y_{t-i} + \epsilon_t. \]  

In this model, \( y_t \) denotes the demand of the user at time \( t \), \( \beta_i \) is the coefficient for order (lag order) \( i \) in the autoregressive model, and \( \epsilon_t \) is an additive random Gaussian noise. The time index \( t \) is measured in hours and the noise is identically and independently distributed at different hours. Note that in this paper we denote time series data by notation \( \{ \bullet \}_t \), where subscript \( t \) refers to the time slots in this time series data.

In addition, \( I \) is the number of orders that we include in the model. An autoregressive model with maximum order \( I \) is denoted by AR(\( I \)).

To use the model in (1) for prediction, the standard approach is to use ordinary least squares (OLS) to estimate the coefficients \( \beta_i, i \in I \). By convention, we write \( [y_{t-1}, y_{t-2}, \cdots, y_{t-I}]^T \) as a vector denoted by \( \mathbf{x}_t \). Using this notation, the model in (1) is written in a compact matrix form:

\[ \mathbf{y} = \mathbf{X}\mathbf{\beta} + \mathbf{\epsilon}, \]

where \( \mathbf{y} = [y_t, y_{t+1}, \cdots]^T \), \( \mathbf{X} \) is a matrix where \( t \)-th row is \([1 \ \mathbf{x}_t]^T\), \( \mathbf{\beta} = [\beta_0 \ \beta_1 \ \cdots]^T \), and \( \mathbf{\epsilon} = [\epsilon_t \ \epsilon_{t+1} \ \cdots]^T \). Vectors \( \mathbf{y}, \mathbf{\beta} \) and \( \mathbf{\epsilon} \) have dimension \( T \). Matrix \( \mathbf{X} \) has \( P \) columns, which we refer to as the dimension of \( \mathbf{X} \).

Applying standard OLS to (2), the estimate of \( \mathbf{\beta} \) is given by:

\[ \hat{\mathbf{\beta}}_{\text{OLS}} = \arg \min_{\mathbf{\beta}} \| (\mathbf{y} - \mathbf{X}\mathbf{\beta}) \|^2_2. \]  

Under some standard assumptions, the OLS estimator \( \hat{\mathbf{\beta}}_{\text{OLS}} \) is a consistent estimator for the true \( \mathbf{\beta} \), meaning that the expected difference between the estimator and the true value approaches zero when sample size becomes large [25]. This means that the bias going to zero as the sample size become big. The other asymptotic analysis on the OLS also applies in this case, such as asymptotic distribution for gaussianity of estimators and significance tests; however, in autoregressive models the estimators are typically not unbiased.

To learn and evaluate the estimator \( \hat{\mathbf{\beta}}_{\text{OLS}} \) from OLS, we separate the dataset into a training set and a test set. The estimator \( \hat{\mathbf{\beta}}_{\text{OLS}} \) is learned from the data from the training set and the estimation error is evaluated on the test set. Note that estimators may exhibit extremely good fits on the training set but poor estimation performance on the test set, as according to [26].

III. SPARSITY IN AUTOREGRESSIVE MODELS

Since the objective of OLS estimators is to minimize the sum of squared errors in the training set, OLS achieves optimal in-sample performance. This means that adding more regressors into \( \mathbf{\beta} \) can always decrease the sum of squared error and better fit the data within the training set. However, when we include too many irrelevant regressors, i.e. when we include too many lag orders from the historical data in (2), we are misled by the reduced in-sample bias. We will then
ignore the high variance introduced by the estimator which leads to model overfitting.

We use the PG&E dataset as an illustrative example. In this particular dataset, hourly consumption data for single households is recorded. If we use an AR(3) model for a particular household, it will result in an average in-sample sum of squared errors of 1.20, with an average out-of-sample error of 2.08, whereas AR(1) model has an average in-sample sum of squared error of 1.25, together with an average out-of-sample error of 1.91. Thus AR(1) gives better out-of-sample fitting results. If the potential lag orders are up to 10 days, i.e., 240, then an AR(240) model would produce large out-of-sample errors. Overall, we need to select the lag orders carefully to avoid model overfitting.

Determining the correct lag orders is not trivial because it is a combinatorial problem, which is NP-hard. To this purpose, we use LASSO, which is a convex relaxation of such combinatorial problems, to select relevant lag orders. The intuition for using LASSO is to get a sparse autoregressive models with high orders.

More formally, LASSO is a shrinkage and selection method for linear regression in \( \beta \). It shrinks parameter estimates towards zero in order to avoid overfitting as well as pick up the most relevant regressors. Operationally, it minimizes the usual sum of squared errors, with a bound on the sum of the absolute values of the coefficients:

\[
\hat{\beta}_{\text{LASSO}} = \arg\min_{\beta} \frac{1}{2} \| y - x\beta \|^2_2 + \lambda \| \beta \|_1 ,
\]

where \( \lambda \) is a tuning parameter to control the level of sparsity in the solution. The bigger \( \lambda \) is, the more sparse the solution \( \hat{\beta}_{\text{LASSO}} \) is. When \( \lambda = 0 \), the solution is the same as in (3). For practical purposes, we are using \( k \)-fold cross validation to determine the value of \( \lambda \) in our simulations, where \( k \) is either 5 or 10 [26].

LASSO has gained wide spread popularity in signal processing and statistical learning, see [27], [28], [29]. LASSO has also been applied to forecast electricity price [23],[30], but its application to load forecasting is still a new topic. In [31], LASSO has been applied to forecast short and middle term load. In our paper, we adopt LASSO to predict hour-ahead consumption and provide a comparison between LASSO and some other well-used prediction methods in literature. As discussed later in Section IV, LASSO is the most preferable model when considering model simplicity and prediction performances. Despite the fact that random forest (RF) achieves the best prediction performance with an average relative error less than 20%, it is highly non linear model and requires to tune many hyper-parameters. This is computationally expensive and not interpretable. On the other hand, LASSO achieves similar performance as RF and outperforms all other linear models considered in this paper, i.e., support vector machine (SVM) with a linear kernel, linear regression with principle component analysis (PCA), AR(1) and simple averaging method. It reduces the relative error by 30% compared to these models. Thus LASSO is considered the best in terms of simplicity and prediction accuracy based on our dataset.

Another advantage of using LASSO to recover sparsity is that it has some nice properties as to sign consistency [32],[33]. This means that \( \hat{\beta} \) has the same support as \( \beta \) and the sign of each element in this support is recovered correctly. Therefore LASSO recovers the exact sparsity of the underlying model. In our simulation, LASSO selects both the most recent lag orders and lag orders with intervals of roughly 24 hours, which performs as a combination of simple averaging and AR(\( \cdot \)). Furthermore, LASSO also gives a more interpretable results with respect to selected orders. In our simulation, for one electricity user as an example, LASSO selects lag orders as 1, 2, 5, 6, 16, 22, 23, 24, 48, 143, 144, 160, 191, 216, 238, 240. From these orders we can observe a clear behavior pattern of an interval of 24 hours. Some are multiples of 24 and some are not but close to multiples of 24. We thus observe that not every lag order that LASSO picks is a multiple of 24, otherwise we would directly employ simple averaging rather than LASSO, so LASSO is more adaptive and flexible than simple averaging or AR(\( \cdot \)). This implies that user behavior at current hour depends on similar hours happened in previous days. Unlike simple averaging which fix the lag orders at 24, 48, 72, etc., LASSO will automatically select these orders for each individual based on their respective historical data, instead of directly imposing fixed orders.

IV. USER PAIRING BY SIGNIFICANCE TEST

So far we have considered using historical data of an individual user for its own prediction. One way to leverage the fact that we have many users’ data is to improve the univariate autoregressive model by including other user’s historical data into the model. One popular way to perform this is to conduct vector autoregression (VAR), which extends the univariate autoregressive model to joint prediction for a vector of time series data.

To perform a complete VAR, we need to include all potentially relevant users into the autoregressive model, which will reduce the bias but increase the variance for estimators. This causes the same over fitting problem as occurred in univariate autoregressive model, when AR(3) yields a worse prediction on the test set but a better fit on the training set compared to AR(1). One possible way to overcome this problem is to first cluster similar users together and then perform VAR for each cluster. However, consider a scenario where two time series have the exact same values for each time slot. Then these two time series are clustered together since they are identical. In this case, clustering them together and performance multivariate autoregressive model does not help to enhance prediction because knowing the history of one time series would not help predict the future values of another time series. This problem distinguishes similarity from prediction performance, which is the focus of this section.

In this paper we focus on selecting the most relevant user to enhance prediction performance after doing univariate LASSO selection. To this end, we adopt LASSO significance test to select this most relevant user. In LASSO significance test, the inclusion of a particular user is based on how well this particular user’s history data explains the fitted residual after
performing LASSO to one user’s univariate autoregressive model. This is a hypothesis test of an exponential random variable [34]. Besides LASSO significance test, we also adopt the notion of Granger causality to achieve the same goal. However, it is less relevant to LASSO and is thus discussed in the appendices.

We will discuss more details of a LASSO solution to a regression model and the implementation of the LASSO significance test in the rest of this section. Overall from the simulation results presented in section V using significance test on top of univariate LASSO-type regression model improves the relative prediction error from 22.5% to 20.9%, which is almost as good as the prediction results from RF.

A. Linear Regression Model for the Fitted Residual and LASSO Path

In LASSO significance test, we want to test if the fitted residual from LASSO solution in (4) is indeed noise or if it can be better explained by other user’s historical data. Intuitively, we need to test inclusion of each user’s historical data. We therefore generate LASSO autoregressive coefficients for univariate time series in (1) and compute the residual \( \{e_t\} \) (true value minus fitted value) for each individual user. Then we use \( e_t \) as the response variable in a new linear regression model. To fit this new linear regression model, we include all users’ historical data at lag order one except for this user as the regressors. In this way we will have a high dimensional model. To fit this new linear regression model, we include all users’ historical data at lag order one except for this user as the regressors. In this way we will have a high dimensional model. In the following, we fix a regressor variable and test the significance of its entry in the active set.

We again apply LASSO for (5), to avoid overfitting by including too many irrelevant users. Building on the discussion in Section III, we define the LASSO path as the revolution of the estimator \( \hat{\alpha} \) in terms of a sequence of \( \lambda_k \)’s. The LASSO path \( \hat{\alpha}(\lambda_k) \) is given by:

\[
\hat{\alpha}(\lambda_k) = \arg\min_{\alpha} \frac{1}{2} \| (e - \xi \alpha) \|_2^2 + \lambda_k \| \alpha \|_1, \tag{6}
\]

where \( \lambda_k \) is called the knot along the LASSO path.

For different values of \( \lambda_k \), we obtain different solutions and sparsity at different levels. The active set at one particular value of \( \lambda_k \) is the set of all non zero coefficients estimated at that value, i.e., \( \mathcal{A}_p = \{ \hat{\alpha}_{p} \neq 0, \lambda = \lambda_k, p = 0, 1, \cdots, P \} \). The path \( \hat{\alpha}(\lambda_k) \) is continuous and piecewise linear with knots at these values \( \lambda_1 \geq \lambda_2 \geq \cdots \geq 0 \) [35]. With the formulation in (5), the goal is to test if an inclusion of one user’s historical data is helpful for prediction. Mathematically speaking, we want to test if the variables that sequentially enter the active set are statistically significant.

B. Covariance Test

Significance test applied to the LASSO path is fundamentally different from the standard hypothesis tests in linear regression. In standard testings with linear regression model, we fix a regressor variable and test the significance of its inclusion into the model. This can be done by \( t \)-test. However, the scenario is not the same in the case where we want to test the significance of variables along the LASSO path since the variables entering the active set are not known in advance. This means that we do not know which regressors to fix in order to conduct \( t \)-test. This problem is addressed by the authors in [34]. Instead of standard \( t \)-test, they propose a significance test for regressors progressively selected by the LASSO path, using the so called covariance statistic. They have shown that the covariance statistic asymptotically follows exponential distribution and can be used to test the significance of the entry of variables into the active set.

Using the covariance test defined in [34], we test whether it is significant to include one variable (one other user) into the active set against including no variables into the active set. Particularly in our case here, we want to analyze whether the residual, i.e., \( e \) is white noise, or it can be explained significantly by the lag one order historical data from other users.

The null hypothesis is thus that the true active set is an empty set, i.e., no other users can help better predict the current user:

\[
H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_P = 0. \tag{7}
\]

Another way to interpret this is:

\[
H_0 : e \sim N(0, \sigma^2 I). \tag{8}
\]

We focus on testing the significance of the first variable entering the active set and the two main reason for doing so are as follows. First, for the following variables entering the active set, the exponential distribution turns out to yield a slightly higher value than the true value of the test statistic, so the decision tends to be conservative. Second, If one
would like to include more significant regressors selected by LASSO, the Tailstop criterion \(^{[36]}\) for ordered selection can be used under the assumption that sequential p-values are independent. However, we restrain from looking at p-values at greater indexes because of the non-orthogonality of the columns of the regressor matrix \(\mathbf{X}\). From literature, interpreting p-values from sequential hypothesis and order selection under generic conditions is still an open question and is thus beyond the discussion of this paper.

To test whether the first variable entering the active set is significant, we first set \(\lambda = \infty\) and gradually reduce \(\lambda\) until one regressor variable has a non-zero \(\alpha\). Denote the value of this \(\lambda\) as \(\lambda_1\). Also denote \(\lambda_2\) as the value of \(\lambda\) when the second regressor variable enters the active set. For simplicity of representation, we scale the columns of the \(\mathbf{X}\) so that each column has unit norm.

Following this scaling strategy, the authors in \(^{[34]}\) define the covariance statistic for testing the entry of the first variable and it is written as:

\[
T_1 = \frac{\lambda_1(\lambda_1 - \lambda_2)}{\sigma^2}.
\]

From Theorem 2 in \(^{[34]}\), the authors have proved that \(\Pr(T_1 > t) \to e^{-t}\) as \(P \to \infty\) for all \(t \geq 0\). Thus we have that the asymptotic distribution of \(T_1\) obeys exponential distribution with parameter value as 1:

\[
T_1 \to \text{Exp}(1).
\]

In most cases, the value of \(\sigma^2\) is unknown. Provided that the dimension of \(\mathbf{X}\) is smaller than the sample size, i.e., \(P < N\), we can estimate \(\sigma^2\) by the residual sum of squared error using OLS:

\[
\sigma^2 = \frac{\|\mathbf{e} - \mathbf{X}\tilde{\alpha}_{OLS}\|^2}{N - P},
\]

where \(\tilde{\alpha}_{OLS}\) is obtained through:

\[
\tilde{\alpha}_{OLS} = \arg\min_{\alpha} \frac{1}{2} \|\mathbf{e} - \mathbf{X}\alpha\|^2.
\]

Plugging \((12)\) into \((9)\), we have a new statistic \(F_1\), which is asymptotically following \(F\) distribution (ratio of two independent \(\chi^2\) distribution) under the null \(^{[34]}\):

\[
F_1 = T_1 \sigma^2 = \frac{\lambda_1(\lambda_1 - \lambda_2)}{\sigma^2} \to F_{2,N-P}.
\]

In conclusion, to test whether another user’s historical data can explain the residual obtained by the univariate LASSO introduced in Section III we compute the value of \(\lambda_1\) and \(\lambda_2\), along with the full linear regression in \((12)\) which gives us \(\sigma^2\). Plugging these into \((13)\), we include the regressor variable (which represent one particular user) entering the active set at \(\lambda_1\) if \(F_1\) is greater than some threshold and reject the null hypothesis in \((8)\). Simulation results show that performing this significance test on top of univariate LASSO type regression model has improved the prediction by reducing relative test error by 38.3%, compared to AR(1) model.

V. SIMULATION RESULTS

We use the data from Pacific Gas and Electric Company. It contains hourly smart meter readings for residential users during a period of one year from 2010 to 2011. Temperature data is retrieved from an online database \(^{[33]}\) for the same period.

A. Data Preparation

Before proceeding with the prediction methods, we first decompose the consumption data into two parts: 24-hour periodic recurrence and the left-over non-periodic part is perceived as user consumption that reflects behavior. This goal is achieved by averaging for each time unit over all periods in the data \(^{[39]}\). The period is set to be 24 hours. The aim is to better predict the left over user behavior, which is the original consumption data minus the periodic recurrence. We also separate the weekday data and weekend data and we focus on weekday data in this paper.

The time series of decomposed consumption data is stationary. To verify this, we use augmented Dickey Fuller statistics to test the unit root for the autoregressive model. We also assume Gaussian noise with constant variance for the underlying true model and no perfect collinearity existed in the time series data, which follows the consistency assumptions to apply OLS to time series analysis.

To perform and evaluate the methods discussed in Section III and in Section VII we separate the weekday data into training and testing sets. The training set contains a sliding window with various lengths and we select the best length based on least MAPE with cross validation. The testing set contains one data point which is the next hour following this sliding window. As can be seen from Fig. 2, a sliding window with length as 1200 hours is the best for our dataset. We slide the window 240 times and evaluate the median MAPE based on the 240 obtained prediction errors.

B. Temperature Effect

According to the literature \(^{[40]}\), temperature plays an important role in predicting aggregated load. However, when it comes to individual consumption, the temperature effect is not that obvious due to the randomness of each individual. A linear regression of temperature and normalized load after removing the 24-hour recurrence is shown in Fig. 1. Since the slope is very small, with a value of 0.001741 against consumption data in a range between 0 and 1, in later analysis we exclude the temperature from regressors and the regressor matrix is made up with historical consumption data.

C. Prediction Evaluation Criterion

Before proceeding with the prediction methods, we first introduce the evaluation criterion for prediction. A naive way to evaluate prediction methods is to compare the sum of
squared error estimated within the testing set, but sum of squared error is not scale invariant. This will be misleading if we want to compare the sum of squared error for two datasets with significantly different scales and means. One way to solve this problem is to normalize the error or to compare the relative error, i.e., the prediction error with respect to the data scale. Here we use the Mean Absolute Percentage Error (MAPE), to capture the relative error:

$$\text{MAPE} = \frac{1}{n} \sum_{t=1}^{n} \frac{|y_t - \hat{y}_t|}{y_t}$$

where $y_t$ is the actual value, $\hat{y}_t$ is the predicted value, and $n$ is the number of fitted values.

In case of outliers, we adopt mean curtailing of 0.01 tail and head, or simply use the median to replace the mean value.

### D. Comparison with Different Prediction Methods

We compare our proposed methods with three other popular methods: 1) linear regression combined with principle component analysis (PCA), 2) support vector machines (SVM) and 3): random forest (RF). Input features are ten previous days’ consumption data which has a dimension of 240. These three methods are widely used machine learning techniques for prediction. Our simulations using real data show that our proposed method performs similar or better to these sophisticated algorithm while remaining easily interpretable.

For the extracted features obtained from PCA, we threshold the number of components with a tolerance $\zeta$ on how much the chosen components explain the covariance matrix of data $\xi$. Then we include those components as new features to the linear regression model. If we set up a tolerance $\zeta = 0.75$ and omit components if their standard deviations are smaller than $\zeta$ times the standard deviation of the first component, we obtain an average two principle components for each user based on our dataset. As to SVM, a linear kernel is trained as we are comparing SVM with several linear regression models. For RF, we grow 500 trees in total. At each split of the tree node we re-sample a third of the original features. Note that RF is a nonlinear model in terms of input features whereas the other models are linear in input features. For a more in-depth introduction to RF, please refer to [41].

The results of all the prediction methods that we consider in this paper are presented in Fig. 2 with 50 users in the training set. Result from PCA is outside the range of the results from the other prediction methods shown in Fig. 2 and is not presented. In addition, there is a constraint for the minimum number of samples in order to have a well behaved LASSO estimator. This lower bound is of order $O(s \log p)$, where $s$ is the cardinality of the true support and $p$ is the total number of regressors. However, since we do not know the level of sparsity $s$ in advance, the number of samples should be at least $O(s \log p)$, which is around 600. Nevertheless, since the columns in the regressor matrix are correlated, we resort to a training size larger than the theoretical bound. In the mean time, we cannot include too many samples into the training set to avoid losing stationarity. Here unit root test is applied to test stationarity. To these two ends, we experiment the training sizes of 720, 960 and 1200 samples and compare the respective performances by the proposed prediction methods.

From Fig. 2 we see that RF yields the smallest prediction error in terms of MAPE. This is because that RF is a non linear model with a much higher model complexity. When we increase the window size, LASSO is improved on a larger training set and its performance is as good as RF. For example, with a sliding window with length of 720 hours, RF has an average MAPE of 20.68% and LASSO has an average MAPE of 23.15%. When we increase the size of the sliding window to include 1200 hours’ consumption data, RF has an average MAPE of 16.85% and LASSO has an average MAPE as 17.2%. Besides these two prediction methods, SVM with either the original features or the principle components does not outperform the other prediction methods from our dataset, with AR(1) and simple averaging returning the medium performance with respect to MAPE.

We then compare the averaging computational time for the aforementioned methods. The simulations are tested on a MacBook with 2.7 GHz Intel Core i5 processor and 8GB 1867 MHz DDR3 memory. The empirical computational time for each prediction method is shown in Table I.

![Fig. 1: Consumption regressed on temperature, after decomposition and normalized.](image1)

![Fig. 2: MAPE for different prediction methods with various window size of the training set.](image2)
As can be seen from Table I, RF yields the worst computational time because it needs to be trained by the repeated bootstrapping samples. Linear regression with PCA is computationally efficient but does not provide desirable prediction accuracy. LASSO beats both SVM with either the original regressor matrix or the principle components in terms of time efficiency. Simple averaging and AR(1) are most time efficient but at the sacrifice of prediction accuracy. Combining the observations from Table I with Fig. 2, we can see that for univariate time series analysis, LASSO best trades off between prediction accuracy and time efficiency.

Based on the observations from Table 2 and Table I, the complexity grows as the prediction model varies from simple averaging method up to RF and consequently the computational time grows accordingly. It is even more time consuming when we try to tune the hyper-parameters in RF, for example, the number of features to bootstrap at each split of the tree, total number of trees, etc... What is more, although RF provides the most accuracy, it is the hardest to interpret in terms of the estimated parameters from the model, i.e., the decision boundary at each split of the tree node for each tree. Since throughout this paper the focus is on one hour ahead prediction, this complexity ruins the performance of the model and is not preferred. In contrast, LASSO yields a competing prediction performance with RF and has only one hyper-parameter ($\lambda$) to tune. The optimal value of this hyper-parameter can be determined from cross-validation and is easy to implement. Thus it is the most preferable method for univariate analysis based on our dataset.

Table II further consolidates this preference in terms of complexity of input data. Input dimension is defined as the number of input variables in the model. Simple averaging method has a dimension of 10 because it is a average of consumption data during the same hour for the past ten days. AR(1) also has a much smaller input space but its prediction performance is not as good as either LASSO or RF. For The prediction methods other than these two, the input data has a dimension of 240 which stands for the total number of hours in the past ten days. With the same input complexity, LASSO yield the most efficient computational time from Table I.

Based on the discussions from Fig. 2, Table I and Table II, LASSO is the best choice in terms of model efficiency and interpretability. Thus, in the following discussions, we restrict ourselves to compare the performances among LASSO, AR(1) and simple averaging for univariate time series analysis. The performance of RF is also shown to provide some comparisons with the proposed methods in this paper.

In addition, the average computational time for computing the optimal pairs using the covariance test is 3.51 seconds on average per each user. We then conclude that it is time efficient to compute the optimal pairs of users. What is more, since the optimal pair assignment is fixed once we compute it, this computational time can be amortized and is nearly negligible. Thus performing covariance test after the univariate LASSO selection does not have an impact on the total computational time.

$\textbf{TABLE I:}$ Average computational time per user per sliding training window for each prediction method.

| Prediction method | Time (in seconds) |
|-------------------|-------------------|
| Averaging         | less than 0.0001   |
| AR(1)             | 0.005             |
| LASSO             | 0.220             |
| PCA + linear regression | 0.175          |
| SVM               | 9.347             |
| SVM + PCA         | 9.522             |
| RF                | 21.562            |

$\textbf{TABLE II:}$ Input dimension for different prediction methods.

| Prediction method       | Input dimension          |
|-------------------------|--------------------------|
| Averaging               | 10                       |
| AR(1)                   | 1                        |
| LASSO                   | 240 (active regressors around 10-20) |
| PCA + linear regression | around 20                |
| SVM                     | 240                      |
| SVM + PCA               | around 20                |
| RF                      | 240 (split the tree 500 times) |

$\textbf{E. Results for Univariate Time Series Analysis}$

We perform detailed analysis on the simple averaging method, AR(1), RF and autoregressive model with LASSO. The results are shown in Fig. 3 and Table III for a random selection of 150 users from the dataset. As can be seen in Fig. 3, LASSO type regression reduces the variance of MAPE for these 150 users in the dataset. Its mean is also reduced. A comparison of the four methods is summarized in Table III.

What is more, besides the improved prediction performance compared to AR(1) and simple averaging method, LASSO also provides some straightforward interpretation of the model. Particularly, we include as much as 240 lag orders for LASSO, which includes all the historical data for the previous ten days. Taking user No.1 as an example, LASSO selects 16 non zero lag orders, according to 10-fold cross validation with a sequence of decreasing $\{k_i\}$. The lag orders that LASSO picks are 1, 2, 5, 6, 16, 22, 23, 24, 48, 143, 144, 146, 160, 191, 216, 238, 240. This pattern reflects that LASSO not only selects the most recent lag orders (which is similar to AR(1)), but the lag orders roughly at interval of one day, i.e., 24 hours as well (which is similar to simple averaging). The coefficients for the lag orders also have different scales. In the example of user No.1, the most recent lag orders given the largest coefficient ($\beta_1 = 0.259$). The second largest coefficient is given to lag order 24 ($\beta_{24} = 0.187$). The rest of the coefficients are scaled between 0.01 to 0.06.

Note that LASSO will return selected lag orders adaptively for different users. For user No.10, the lag orders that LASSO picks up for this user are 1, 23, 24, 120, 138, 144, 168, 239, 240. In this case, the three most recent lag orders are given the largest coefficients, where $\beta_1 = 0.579$, $\beta_{23} = 0.123$ and $\beta_{24} = 0.103$. The rest of the coefficients scale at around 0.02. So for this particular user, LASSO picks up less historical data at the same hour than user No. 1, at roughly one day ago, five days ago, six days ago, seven days ago and ten days ago. This is...
Fig. 3: Histogram comparison for univariate time series. Mean of MAPE for AR(1) is 0.339, for LASSO 0.225, for simple averaging method 0.359, and for RF 0.195. Standard deviation of MAPE for AR(1) is 0.151, for LASSO 0.094, for simple averaging method 0.237, and for RF 0.109.

different from simple averaging, since LASSO does not pick up the lag order 48 or 72, which are more recent lags than 120. Still, however different lag orders that LASSO picks for each user, we can observe a clear pattern of multiples of 24 hours, indicating user’s consumption behavior.

F. Results for Pairing the Users

As described in Section IV, we include historical consumption data from other users to fit the current user’s data. We compare the results of the two methods discussed as follows:

1) Reference method: LASSO selection in autoregressive model using one user’s own data.

2) Pairing by covariance test: the optimal pairing for a user is selected by the best user fitted to the residuals obtained from this user’ consumption data after LASSO in univariate autoregressive model. This best user corresponds to the first regressor in $\xi$ when its associated $p$-value is small. The lag orders for this user are selected by LASSO in the reference method (which is still up to 240 lags), but the lag order for its paired user is fixed to one.

The results for these two methods are in Fig. 4 and Table III.

The results are based on the same 150 users chosen from the dataset. It can be seen from Fig. 4 that with the covariance test that includes another useful user into the model, the prediction performance is enhanced. The mean of the MAPE on the test set is improved by 7% and the variance by 6%. Note that the selection of another user into the model is not mutual, for example, for user No.14, user No.137 is selected by covariance test whereas for user No.137, user No.50 is selected. This is essentially different from bivariate autoregressive model where pairs of users are included into the model simultaneously. Here, the pairing of the users is adaptive according to the covariance statistic defined in (9).

G. Aggregation Over Users

In the above analysis we discuss the univariate time series prediction according to each user’s consumption data. Alternatively, we can aggregate multiple users together and treat the aggregated data as one time series. The concept of aggregation of forecasting and baselines has been explored in [18] and [42]. Intuitively, the more users we aggregate, the less noise we will get. This is due to the fact that the aggregated noise tends to decay with the number of users being aggregated. Therefore, aggregating users will reduce the overall error and lead to better prediction, but at the cost of sacrificing individual consumption information. Fig. 5 shows user aggregation tradeoff between prediction performance based on MAPE and level of aggregation. The more we aggregate, the smaller MAPE is while losing some of the individual user’s information. The aggregation levels (number of users per aggregation) are set to be 1, 10, 50, 100, 200 and 250.

Note that the methods that we propose in this paper, i.e.,

| Method         | Averaging | AR(1) | LASSO | RF  | CovT  |
|----------------|-----------|-------|-------|-----|-------|
| mean (MAPE)    | 0.359     | 0.339 | 0.225 | 0.195 | 0.209 |
| sd (MAPE)      | 0.237     | 0.151 | 0.094 | 0.109 | 0.088 |

Fig. 4: Histogram comparison for multivariate time series, where covT stands for covariance test. Mean MAPE for LASSO is 0.225 and for performing covariance test after LASSO is 0.209. Standard deviation of MAPE has also reduced by 7% from LASSO to performing covariance test after LASSO.
ever, since multivariate time series analysis relies very much on users in the aggregation. The exact order that it chooses depends on user’s behavior. Therefore the optimal lag orders obtained from the aggregated time series data might not be directly applied to the individual user’s data. Take aggregation level at 50 users as an example. Our method treats the aggregated consumption data as an univariate time series and LASSO selects 22 non-zero lag orders, at 1, 6, 22, 23, 26, 50, 65, 67, 73, 83, 95, 138, 142, 147, 167, 172, 196, 198, 219, 239. This selection clearly differs from the selection either for user No.1 or for user No.10, but the selected lag orders become less variant when we aggregate more users together since the noise decays linearly with the number of users in the aggregation.

We also simulated aggregation for multivariate time series analysis, i.e., significance test with covariance statistic. However, since multivariate time series analysis relies very much on how we aggregate the users into groups, the result is not robust due to the limited size of dataset. Based on the dataset, for aggregation level at 500 users, we can at most generate 3 aggregated consumption data from our dataset through randomized grouping. Thus pairing for these consumption data is not as evident as in the individual consumption data analysis.

VI. Conclusion and future works

In this paper, we pose and analyze consumption prediction methods based on autoregressive models. For individual users, we adopt LASSO to recover the sparsity in linear regression models. LASSO selects the most recent lag orders and important lag orders with close multiples of 24 hours, which reveals user consumption pattern. Furthermore, we extend the idea from univariate time series analysis to multivariate analysis. We use the covariance statistic to test if inclusion of another user is significant to explain the fitted residual from univariate LASSO-type regression. We then compare several prediction methods and observe that LASSO best trades off between model complexity and prediction performance. Further simulation results show that LASSO with covariance test outperforms both simple averaging method and AR(1) for individual consumption prediction. The proposed prediction methods in this paper can help compare the user behavior with and without demand response incentives. In the future, we will use the results in this paper to investigate the demand response effect for users.

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Granger causality was proposed by Clive Granger, in 1969, to illustrate “causality” between two time series. Now suppose that we have two users’ consumption data, namely \( \{y_{1,t}\} \) and \( \{y_{2,t}\} \). In the context of Granger causality, the cause is prior to the effect, meaning that if a time series \( \{y_{1,t}\} \) is granger causing time series \( \{y_{2,t}\} \), then the past observations of \( \{y_{1,t}\} \) can help predict current status of \( \{y_{2,t}\} \). Obviously granger causality does not imply real causality, in the way that \( \{y_{1,t}\} \) and \( \{y_{2,t}\} \) can be both generated by some latent factors but \( \{y_{2,t}\} \) bares certain lag orders behind \( \{y_{1,t}\} \). Still, using historical data of \( \{y_{1,t}\} \) helps to predict \( \{y_{2,t}\} \).

To test whether \( \{y_{1,t}\} \) granger causes \( \{y_{2,t}\} \), we consider two autoregressive models to predict \( y_{2,t} \):

\[
\begin{align*}
y_{2,t} &= \beta_0 + \sum_{i=1}^I \beta_i y_{2,t-i} + \varepsilon_t \\
y_{2,t} &= \beta_0 + \sum_{i=1}^I \beta_i y_{1,t-i} + \sum_{j=1}^J \gamma_j y_{2,t-j} + u_t
\end{align*}
\]

where \( I \) and \( J \) are the lags to be included in the model, \( \varepsilon_t \) and \( u_t \) are the white noise. More specifically, we assume that \( \varepsilon_t \) and \( u_t \) follows normal distribution with zero mean and variance \( \sigma^2 \).

We say \( \{y_{1,t}\} \) granger causes \( \{y_{2,t}\} \) if (16) is statistically significantly better than (15). For our application, this definition of causality is justified even if it implies spurious causes in place of real causes. Our focus is on the improvement of prediction accuracy, not on real cause discovery.

To test granger causality, the null hypothesis is:

\[
H_0 : \gamma_1 = \gamma_2 = \ldots = \gamma_J = 0 \quad (17)
\]

The alternative hypothesis for (17) is that at least one \( \gamma \) is not zero.

To both models, we estimate the parameters by OLS. Recall that we assume Gaussian noise with variance \( \sigma^2 \) in unrestricted model presented in (16), the residual sum of squared error normalized by \( \sigma^2 \) follows the \( \chi^2_{J-I} \) distribution illustrated in (18), where \( \chi^2_{N-I-J-1} \) distribution is a sum of the squares of \( N-I-J-1 \) independent standard normal random variables. Detailed calculation and reasoning can be found in [44].

\[
\sum_{t=1}^N \hat{\varepsilon}_t^2 \sim \chi^2_{J-I} \quad (18)
\]

On the other hand, with some linear transformations of Gaussian variables, the difference between the residual sum of squared error of the restricted model and unrestricted model also follows \( \chi^2 \) distribution but with different degree of freedom:

\[
\frac{\sum_{t=1}^N \hat{\varepsilon}_t^2 - \sum_{t=1}^N \hat{\gamma}_t^2}{\sigma^2} \sim \chi^2_{J} \quad (19)
\]

Independence of \( \sum_{t=1}^N \hat{\varepsilon}_t^2 \) and \( \sum_{t=1}^N \hat{\gamma}_t^2 \) can also be shown using the independence between estimated parameters, i.e., \( \hat{\beta}_i \) and \( \hat{\gamma}_j \). Details can be found in [44]. Since the quantity in (18) and (19) are independent, we construct the F statistics as follows:

\[
F = \frac{(\sum_{t=1}^N \hat{\varepsilon}_t^2 - \sum_{t=1}^N \hat{\gamma}_t^2)/J}{\sum_{t=1}^N \hat{\gamma}_t^2/(N - (I + J + 1))} \quad (20)
\]

Under the null hypothesis, \( F \sim \chi^2_{J-I} \). If the value of \( F \) is larger than some thresholds or the \( p \)-value is less than 0.05, then we reject the null and claim that at least one \( \gamma \) is not zero. The \( p \)-value evaluates the probability of observing an extreme result under the null hypothesis, which can be used to quantify how likely the null hypothesis is to be true. We reject the null hypothesis if the \( p \)-value is smaller than some thresholds, normally taken as 0.05.

To test granger causality, we set \( I = J = p \), which equals to the maximum number of lag orders we wish to fit into the model. We can naturally include more historical data, i.e. at maximum lag order = \( p \) where \( p > 1 \). To test the best number of lag orders, we run a series of simulation with different \( p \) and it turns out that \( p = 1 \) has the best performance with regard to prediction accuracy as well as avoiding model overfitting. We also restrain from including more than two users’ historical data into the unrestricted model because granger causality tests possible relation between a pair of time series. We then test all possible pairs for each specific user using the F statistics defined in (20) and we reject the null hypothesis if its value is large, i.e., if the associated \( p \)-value is less than 0.05. We include the most relevant user into the final model by picking up the one user corresponding to the largest \( F \) statistics value.
APPENDIX B

SIMULATION RESULTS WITH GRANGER CAUSALITY

We compare the results in Fig. 4 with the results obtained by Granger causality test. The comparison is shown in Fig. 6. The results are based on the same dataset with 150 users.

From Fig. 6, we can see that both causality test and covariance test reduces MAPE. For causality test, MAPE is reduced from 0.225 to 0.218, and for covariance test, MAPE is reduced from 0.225 to 0.209. However, in our work we use the covariance test because it is closely related to univariate LASSO. It also provides an adaptive selection procedure, that can be extended to including more users into the model once the theory becomes more complete.