Measuring $|V_{ub}|$ at future $B$–Factories

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ABSTRACT

We calculate the so–called Fermi motion parameter $p_F$ of ACCMM model using the variational method in a potential model approach. We also propose hadronic invariant mass distribution as an alternative experimental observable to measure $V_{ub}$ at future asymmetric $B$ factories.

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1. INTRODUCTION

In the standard $SU(2) \times U(1)$ gauge theory of Glashow, Salam and Weinberg the fermion masses and hadronic flavor changing weak transitions have a somewhat less secure role, since they require a prior knowledge of the mass generation mechanism. The simplest possibility to give mass to the fermions in the theory makes use of Yukawa interactions involving the doublet Higgs field. These interactions give rise to the Cabibbo–Kobayashi–Maskawa (CKM) matrix: Quarks of different flavor are mixed in the charged weak currents by means of an unitary matrix $V$. However, both the electromagnetic current and the weak neutral current remain flavor diagonal. Second order weak processes such as mixing and CP–violation are even less secure theoretically, since they can be affected by both beyond the Standard Model virtual contributions, as well as new physics direct contributions. Our present understanding of CP–violation is based on the three–family Kobayashi–Maskawa model [1] of quarks, some of whose charged–current couplings have phases. Over the past decade, new data have allowed one to refine our knowledge about parameters of this matrix $V$.

The CKM matrix element $V_{ub}$ (or $|V_{ub}|/|V_{cb}|$) is measured through $B$–meson weak decay, but the determination of the value is highly model dependent. One method is to investigate the end point region of the lepton energy distribution in the inclusive semileptonic $B$–meson decay: $B \rightarrow X_u l \nu$. In Section 2, we assume the Gaussian ansatz of the so called ACCMM model [2] and determine the parameter $p_{F}$ using the variational method in a potential model approach. In Section 3, we point out that directly measuring the invariant mass of final hadrons in $B$–meson semileptonic decays offers an alternative way to select $b \rightarrow u$ transitions [3] that is in principle more efficient than selecting the upper end of the lepton energy spectrum.

2. ACCMM MODEL AND THE PARAMETER $p_{F}$

The simplest model for the semileptonic $B$–decay is the spectator model which describes the decaying $b$–quark of the $B$–meson as a free particle. The decay width with phase space and radiative corrections can be written as

$$\Gamma(b \rightarrow X_u l \nu) = |V_{bq}|^2 \left( \frac{G_F^2 m_b^5}{192 \pi^3} \right) f \left( \frac{m_q}{m_b} \right) \left[ 1 - \frac{2}{3} \frac{\alpha_s}{\pi} g \left( \frac{m_q}{m_b} \right) \right]. \quad (1)$$

QCD correction can be approximated [4] within 0.2%,

$$g(\epsilon = \frac{m_q}{m_b}) \simeq (\pi^2 - 31/4)(1 - \epsilon)^2 + 3/2. \quad (2)$$

Because of the $m_b^5$ factor in Eq. (1), a small error on the $b$-quark mass $m_b$ would be significantly amplified in the width. To take into account the bound state effect and thus
circumvent the $m_b^5$ factor problem, a model (called the ACCMM model [2]) had been proposed.

The ACCMM model incorporates the bound state effect by treating the $b$-quark as a virtual state particle, thus giving momentum dependence to the $b$-quark mass. The momentum dependence of the virtual state $b$-quark mass is given by

\[m_{b^*}^2(\vec{p}) = m_B^2 + m_{sp}^2 - 2m_B\sqrt{\vec{p}^2 + m_{sp}^2}\]  

(3)

in the $B$-meson rest frame, where $m_{sp}$ is the spectator quark mass and $m_B$ the $B$-meson mass. In this way the bound effect is introduced through the spectator quark contribution to the decay width.

For the momentum distribution of the quark’s Fermi motion inside the $B$-meson, the ACCMM model simply assumes a Gaussian distribution,

\[
\phi(\vec{p}) = \frac{4}{\sqrt{\pi p_F^3}} e^{-|\vec{p}|^2/p_F^2}
\]  

(4)

with a free parameter $p_F$. Thus the decay width is modified from the spectator model to

\[
\frac{d\Gamma_B}{dE_l}(p_F, m_{sp}, m_q) = \int_0^{p_{max}} dp \, p^2 \phi(\vec{p}) \frac{d\Gamma_b}{dE_l}(m_q, m_b)
\]  

(5)

in the ACCMM model, where $\Gamma_b$ is given by eq.(1).

The model, therefore, introduces a new parameter, $p_F$, for the momentum measure of the Gaussian distribution, instead of the $b$-quark mass of the spectator model. The new parameter $p_F$ is regarded as free parameter, and the value 0.3 is widely used in the literature without theoretical justification. We assume here the Gaussian ansatz of the ACCMM model and determine the parameter $p_F$ using the variational method in a potential model approach. By doing that we give some definite values to $p_F$ in terms of the $b$-quark mass, $m_b$, and potential model parameters. The Gaussian probability distribution of the momentum, Eq. (4), is interpreted in our approach as the absolute square of the momentum space wave function of the bound $B$-meson, i.e.

\[
\phi(\vec{p}) = 4\pi |\chi(\vec{p})|^2
\]  

with \[\chi(\vec{p}) = \frac{1}{(\sqrt{\pi p_F})^{3/2}} e^{-\vec{p}^2/2p_F^2} .\]  

(6)

The Fourier transform of the momentum space wave function $\chi(\vec{p})$ is regarded as the
position-space wave function \( \psi(\vec{r}) \), which is itself Gaussian,

\[
\psi(\vec{r}) = \left( \frac{p_F}{\sqrt{\pi}} \right)^{3/2} e^{-r^2 p_F^2/2}.
\] (7)

We will use the variational method with the Hamiltonian operator,

\[
H = \sqrt{\vec{p}^2 + m_{sp}^2} + \sqrt{\vec{p}'^2 + m_b^2} + V(r),
\] (8)

and a trial function,

\[
\psi(\vec{r}) = \frac{1}{(\sqrt{\pi} \mu)^{3/2}} e^{-\mu^2 r^2/2},
\] (9)

where \( \mu \) is a variational parameter. The ground state is given by minimizing the expectation value of \( H \),

\[
\langle H \rangle = \langle \psi | H | \psi \rangle = E(\mu)
\]

and \( \frac{d}{d\mu} E(\mu) = 0 \) at \( \mu = \bar{\mu} \). (10)

Then we approximate \( m_B \sim E(\bar{\mu}) \) with \( \bar{\mu} = p_F \). The \( p_F \) or \( \mu \) of the Gaussian wave function corresponds to a measure of the radius of the two-body bound state as can be seen from the expectation values of \( r \),

\[
\langle r \rangle = \frac{2}{\sqrt{\pi} \mu}, \quad \langle r^2 \rangle^{1/2} = \frac{3}{2} \frac{1}{\mu}
\] (11)

or the most probable \( r = 1/\bar{\mu} \).

For the potential in the variational method, we use the linear plus coulomb potential,

\[
V(r) = -\frac{\alpha_c}{r} + Kr.
\] (12)

The best fit for the quark masses and the potential parameters, \( \alpha_c = \frac{4}{3} \alpha_s \) and \( K \), had been determined by Hagiwara et. al. [5],

\[
\alpha_c = 0.47 \quad (\alpha_s = 0.35), \quad K = 0.19 \text{ GeV}^2,
\]

\[
m_b = 4.75 \text{ GeV}
\] (13)

for \((c\bar{c})\) and \((b\bar{b})\) bound states. We will use these values in our analysis and also the value \( \alpha_c = 0.32 \quad (\alpha_s = 0.24 = \alpha_s (m_b^2)) \) for comparison.
We use the relativistic kinematics only for the light $u$ or $d$ quark, thus the Hamiltonian is written as

$$H \simeq M + \frac{\vec{p}^2}{2M} + \sqrt{\vec{p}^2 + m^2 + V(r)} , \quad (14)$$

where $M = m_b$ and $m = m_{sp}$. Trying to solve the eigenvalue equation of the differential operator (14) may be faced with difficulty because of the square-root operator in $H$. However, in our variational method, the expectation value of the $H$ can be calculated with either positron-space wave functions or momentum-space wave functions,

$$\langle H \rangle = \langle \psi(\vec{r}) | H | \psi(\vec{r}) \rangle = \langle \chi(\vec{r}) | H | \chi(\vec{r}) \rangle . \quad (15)$$

Fortunately, our trial wave function is Gaussian both in position space and in momentum space. Also the Gaussian function is a smooth function, and the derivatives of any order are square integrable, that is, defined on Hilbert space. Thus any power of the Laplacian operator $\nabla^2$ is a hermitian operator at least under Gaussian functions. For more details, see Ref. [6].

With the input value of $m = 0.15$ GeV (as in the literature), the calculated values are

$$\bar{\mu} = 0.54, \quad \bar{E} = 5.54 \quad \text{for } \alpha_s = 0.35 , \quad (16)$$

and

$$\bar{\mu} = 0.49, \quad \bar{E} = 5.63 \quad \text{for } \alpha_s = 0.24 .$$

For comparison, we calculated $\langle H \rangle$ for the case of $m = 0$ in which the integral of the square root operator is exact,

$$\bar{\mu} = 0.53, \quad \bar{E} = 5.52 \quad \text{for } \alpha_s = 0.35 , \quad (17)$$

and

$$\bar{\mu} = 0.48, \quad \bar{E} = 5.60 \quad \text{for } \alpha_s = 0.24 .$$

The calculated values of the $B$-meson mass, $\bar{E}$, are much larger than the measured value of 5.28. The large values for the mass is originated mainly from the Hamiltonian (14), which is flavor-degenerate for $B$ and $B^*$ (vector meson). The difference between the pseudoscalar meson and the vector meson is the chromomagnetic hyperfine splitting, which is given by Fermi-Breit as

$$V_s = \frac{2}{3Mm} \vec{s}_1 \cdot \vec{s}_2 \nabla^2 \left( -\frac{\alpha_c}{r} \right) . \quad (18)$$

And the expectation value of $V_s$ is given by

$$\langle V_s \rangle = -\frac{2}{\sqrt{\pi}} \frac{\alpha_c \mu^3}{Mm} \quad \text{for } B , \quad (19)$$

$$= \frac{2}{3\sqrt{\pi}} \frac{\alpha_c \mu^3}{Mm} \quad \text{for } B^* .$$

Since $\langle V_s \rangle$ is proportional to $-\mu^3$ for $B$, $E(\mu)$ has no minimum in $\mu$. Hence we can treat
\langle V_s \rangle \) only as a perturbation for \( B \)-meson, resulting in

\[ p_F = 0.54, \quad E_B = 5.52 \quad \text{for } \alpha_s = 0.35, \]

and

\[ p_F = 0.49, \quad E_B = 5.56 \quad \text{for } \alpha_s = 0.24. \] (20)

The perturbative result for \( B^* \) is shown

\[ p_F = 0.54, \quad E_{B^*} = 5.58 \quad \text{for } \alpha_s = 0.35, \]

and

\[ p_F = 0.49, \quad E_{B^*} = 5.65 \quad \text{for } \alpha_s = 0.24. \] (21)

The calculated values of the \( B \)-meson mass, 5.42 (\( \alpha_s = 0.35 \)) and 5.56 (\( \alpha_s = 0.24 \)) are in reasonable range compared to the experimental value 5.28; the relative errors are 2.7\% and 5.3\%, respectively. But for \( p_F \), the calculated values, 0.54 (\( \alpha_s = 0.35 \)) and 0.49 (\( \alpha_s = 0.24 \)), are much larger than the value 0.3 widely used in the literature. For more details, see Ref. [6].

3. HADRONIC INVARIANT MASS DISTRIBUTION ON SEMILEPTONIC \( B \)-DECAY

The CKM matrix element \( V_{ub} \) characterizing \( b \to u \) quark transitions plays an important role in the description of CP violation within the three-family Standard Model, but is still not accurately known. The most direct way to determine this parameter is through the study of \( B \) meson semileptonic decays; recent results from the CLEO [7] and ARGUS [8] data on the end-point region of the lepton spectrum have established that \( V_{ub} \) is indeed non-zero and have given an approximate value for its modulus. The central problem in the extraction of \( V_{ub} \) is the separation of \( b \to u \) events from the dominant \( b \to c \) events. In semi-leptonic \( B \)-meson decays, the usual approach is to study the upper end of the charged lepton spectrum, since the end-point region

\[ E_\ell > \frac{m_B^2 - m_D^2 + m_\ell^2}{2m_B} \] (22)

in the CM frame is inaccessible to \( b \to c \) transitions and therefore selects purely \( b \to u \). However, only about 20\% of \( b \to u \) transitions actually lie in the region of Eq. (22); it is therefore not a very efficient way to select them. In situations of physical interest, the situation is even somewhat worse. For example, in \( \Upsilon(4S) \to BB \) decay, each \( B \) meson has a small velocity in the \( \Upsilon \) rest frame; the magnitude \( \beta \) of this velocity is known, but its direction is not. In this frame, which is the laboratory frame when \( \Upsilon \) is produced at a symmetric \( e^+e^- \) collider, the \( b \to u \) selection region based on Eq. (22) becomes

\[ E_\ell > \gamma(1 + \beta)(m_B^2 - m_D^2)/(2m_B), \] (23)

for the cases \( \ell = e \) or \( \mu \). Here \( \gamma = (1 - \beta^2)^{-1/2} = m_\Upsilon/(2m_B) \) and we neglect the lepton mass. (At an asymmetric collider, where \( e^+ \) and \( e^- \) beams have different energies, it will be
necessary to boost lepton momenta from the laboratory frame to the Υ rest frame before applying this cut.) Equation (23) accepts an even smaller percentage of $b \rightarrow u$ decays than Eq. (22), about 10% in fact.

The essential physical idea behind Eqs. (22)-(23) is that $b \rightarrow c$ transitions leave at least one charm quark in the final state; hence for a general semileptonic decay $B \rightarrow \ell + \nu + X$ the invariant mass $m_X$ of the final hadrons exceeds $m_D$ and this implies a kinematic bound on $E_\ell$. In Ref. 3, we gave this old idea a new twist. We first observe that there is no unique connection between $m_X$ and $E_\ell$, due to the presence of the neutrino, so the bound on $E_\ell$ is not an efficient way of exploiting the bound on $m_X$. We then observe a more efficient way to exploit the latter bound is to measure $m_X$ itself and to select $b \rightarrow u$ transitions by requiring

$$m_X < m_D \quad (24)$$

instead of Eqs. (22)-(23). This condition is of course frame-independent.

The final hadronic invariant mass distribution depends both on the $c$-quark energy distribution $d\Gamma(b \rightarrow c\ell\nu)/dE_c$ and on the Fermi momentum distribution $\phi(p)$ which is normalized to $\int_0^\infty dp \phi(p) = 1$. The lowest-order contribution to the $c$-quark energy distribution is given by

$$\frac{d\Gamma^0(b \rightarrow c\ell\nu)}{dx_c} = \frac{G_F^2 m_b^5}{96 \pi^3} |V_{cb}|^2 (x_c^2 - 4\epsilon^2)^{1/2} \left[ 3x_c(3 - 2x_c) + \epsilon^2(3x_c - 4) \right], \quad (25)$$

where $x_c = 2(c \cdot b)/m_b^2 = 2E_c/m_b$ in the $b$ rest-frame, with kinematical range $2\epsilon \leq x_c \leq 1 + \epsilon^2$. When QCD radiative corrections are included, the real and virtual gluon contributions must be subject to resolution smearing so that their singular parts will cancel; this we approximate by absorbing real soft gluons into the effective final $c$-quark and correcting $d\Gamma^0/dx_c$ by the factor $g(\epsilon)$:

$$\frac{d\Gamma(b \rightarrow c\ell\nu)}{dx_c} \simeq \frac{d\Gamma^0(b \rightarrow c\ell\nu)}{dx_c} \left[ 1 - \frac{2\alpha_s}{3\pi} g(\epsilon) \right]. \quad (26)$$

For each value of the Fermi momentum $p$ we calculate $d\Gamma/dE_c$ in the $b$ rest-frame (isotropic angular distribution here) and fold it with the spectator energy-momentum vector to form the distribution $d\Gamma/dm_X$ with respect to the invariant mass $m_X$ of the final charmed hadronic system,

$$m_X^2 = (E_c + E_{sp})^2 - (p_c + p_{sp})^2. \quad (27)$$

The spectator energy and momentum in the $b$ rest-frame are

$$E_{sp} = \left[ (p^2 + m_b^2)^{1/2} (p^2 + m_{sp}^2)^{1/2} + p^2 \right]/m_b, \quad (28)$$

$$p_{sp} = \left[ (p^2 + m_b^2)^{1/2} + (p^2 + m_{sp}^2)^{1/2} \right] p/m_b,$$
and $m_b$ is everywhere defined by Eq. (3). The maximum and minimum values of $m_X^2$ for given $p$ are

$$m_X^2(\text{max}) = m_c^2 + m_{sp}^2 + m_b(E_{sp} + p_{sp}) + m_c^2(E_{sp} - p_{sp})/m_b,$$

$$m_X^2(\text{min}) = \begin{cases} (m_c + m_{sp})^2, & \text{if } (m_b^2 - m_c^2)E_{sp} \geq (m_b^2 + m_c^2)p_{sp}, \\ m_c^2 + m_{sp}^2 + m_b(E_{sp} - p_{sp}) + m_c^2(E_{sp} + p_{sp})/m_b, & \text{otherwise.} \end{cases} \quad (29)$$

These relations show explicitly that small $p$ results in $m_X$ values close to $(m_c + m_{sp})$. The upper limit on $p$ for the decay to be possible (from Eq. (3) with $m_b > m_c$) is

$$p < \lambda^{1/2}(m_B^2, m_c^2, m_{sp}^2)/(2m_B), \quad (30)$$

where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$.

For $b \to u$ transitions the effect of individual resonances in $X$ quickly disappears above the $\pi$ and $\rho$ region, and multiparticle jet-like continuum final states should give the dominant contributions [9]; this makes it reliable to calculate the $m_X$ distribution using the modified spectator decay model. Figure 1 gives the hadronic invariant mass distribution from $B \to \ell\nu X$ semileptonic decays, showing that more than 90% of $b \to u$ decays lie within the region selected by Eq. (24). In this illustration we use $m_B = 5.273$ GeV, $E_B = m_X/2 = 5.29$ GeV, $m_c = 1.6$ GeV, $m_u = 0.1$ GeV, $p_F = 0.3$ GeV, including QCD corrections up to order $\alpha\alpha_s$ according to Ref. [10]. The figure is normalized for simplicity to the case $|V_{ub}/V_{cb}| = 1$. For more details, see Refs. [3,11].

In order to exploit Eq. (24) instead, it is desirable to isolate uniquely the products of a single $B$ meson decay; to achieve this it is generally necessary to reconstruct both $B$ decays in a given event. One of these decays can be semileptonic, since kinematic constraints can often determine the missing neutrino four-momentum well enough to reconstruct a peak at zero in the invariant square of this four-momentum. Double-semileptonic decay events will not generally reconstruct uniquely, however. To study semileptonic channels, we are therefore concerned with those events (about 30% of the total) where one $B$ decays hadronically, one semileptonically with $\ell = e$ or $\mu$. Of order 1% of these have $b \to u\ell\nu$ semileptonic transitions (because of the very small ratio $|V_{ub}/V_{cb}| \approx 0.1$). About 10% of the latter satisfy the criterion $E_\ell > 2.5$ GeV of Eq. (23). With present data, it appears to be possible to reconstruct a few percent of such events, but perhaps only about one percent without ambiguity.

*Figure 1.* Hadronic invariant mass distribution in $B \to \ell\nu X$ semileptonic decays.
We note that there is a question of bias. Some classes of final states (e.g. those with low multiplicity, few neutrals) may be more susceptible to a full and unambiguous reconstruction. Hence an analysis that requires this reconstruction may be biased. However the use of topological information from microvertex detectors should tend to reduce the bias, since vertex resolvability depends largely on the proper time of the decay and its orientation relative to the initial momentum (that are independent of the decay mode). Also such a bias can be allowed for in the analysis, via suitable modeling. Finally there may be a background from continuum events that accidentally fake the $\Upsilon$ events of interest. This can be measured directly at energies close to the resonance.

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