A Novel Stochastic Model of the Photo-Thermoelasticity Theory of the Non-Local Excited Semiconductor Medium

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Received: 29 April 2022 / Accepted: 6 July 2022 / Published online: 25 July 2022 © Springer Nature B.V. 2022

Abstract
A novel technique under the effect of stochastic heating due to the thermal effect of the photothermal theory is investigated. Realistically, stochastic processes are taken on the boundary of the non-local semiconductor medium. The interactions between optical, thermal, and mechanical waves in a half-space of the medium are studied according to the photo-thermoelasticity theory. The governing equations are described in one dimension (1D) according to the elastic-electronic deformation. Laplace transforms with short-time approximation are used to analyze the main physical fields in linearity form. To study the problem more realistically, some conditions are taken as random with white noise on the free surface of the elastic medium. The deterministic physical quantities are obtained with a stochastic calculus when a numerical inversion of the Laplace transform is applied. The silicon material is utilized to make the stochastic simulation. The comparisons are carried out between the distributions of deterministic and stochastic (statistically, the mean and variance) of the main physical quantities along different sample paths graphically and discussed for the non-local silicon semiconductor material.

Keywords Stochastic process · Photo-thermoelasticity theory · White noise · Semiconductor · Sample path · Non-local parameter

1 Introduction
The stochastic models in recent years have many applications in a wide range of modern prominent science. The stochastic processes are used to describe the new mathematical and statistical models of biology, microbiology, biochemistry, epidemiology (the science of random spread of diseases like Covid-19), modern physics, mechanics, and microelectronics. The view of modern mathematical models that describe many problems in nature has differed depending on the surrounding random changes. So when studying these previous models, when adding randomness to the phenomenon will make the phenomenon more realistic. In this case, the stochastic technique (random process) is used to describe some deterministic processes in nature. On the other hand, such deterministic models represent ideal situations, which is unrealistic in nature, so such models are often improved by including random effects. In fact, stochastic techniques are mathematical-statistical tools that help us deal with the random nature of the new models. Many reasons require that we replace the deterministic models with random (stochastic) ones [1, 2]. The first reason is that the systems under study are not completely isolated, so it is added noise. The second reason, not all the variables found in the phenomenon under study can be taken into account when making a mathematical model for the phenomenon, so these variables lead to additional noise. The third reason, the devices used in the measurements are not 100% accurate. According to these previous reasons, stochastic simulation
The photothermal diffusion process is used with the photoacoustic spectroscopy to investigate the impact of laser light sources on the semiconductor medium that falls on it [12]. After was used to investigate the effect of laser light sources on the semiconductor medium, high temperature and its gradient resulting from nuclear reactions cause high thermal pressures on their structures, which calls for their construction from special materials. In the second half of the 20th century, Biot [3] introduced the coupled thermoelasticity theory (CD theory) which it can be obtained from the heat Fourier law by resolving the contradiction in the classical uncoupled theory (CUCT). The CUCT has a physical paradox, in which it assumed a parabolic behavior of the thermal wave with infinite speed. To solve this contradiction, a single relaxation time was inserted in the heat equation [4]. On the other hand, Green and Lindsay (G-L) [5] developed the heat equation when inserting into it a double relaxation time and introduced the generalized thermoelasticity theory. Many authors [5–8] used the generalized thermoelasticity theory widely to describe many physical problems. Recently, a thermal shock problem is used in the context of the generalized thermoelasticity theory during the two temperature theory in two dimensions with many external fields [9, 10].

On the other hand, the great importance of semiconductor materials has been taken into account, especially in modern industries, especially electrical circuits that are used in electronic industries. Therefore, models that describe semiconductors should be studied, especially when they are exposed to external influences such as light that has a thermal effect on those materials. As a result of the internal excitation processes, some changes in the displacements of the particles result in what is known as thermoelastic (TE) deformation. Also, as a result of optical (photothermal transport processes) excitation, a transfer of surface electrons (carrier density appears with plasma wave) occurs in a process called electronic deformation (ED) (photo-excitation). Gordon et al. [11] was one of the first to study the properties of semiconductors by studying the acoustic waves due to electromagnetic radiation. A sensitive analysis of photoacoustic spectroscopy was used to investigate the effect of laser light sources on a semiconductor medium that falls on it [12]. After that many researchers used the photothermal theory and photoacoustic spectroscopy to investigate the impact of laser beams falling on a sample of solid and fluid [13–16]. The photothermal diffusion process is used with the gradient in temperature of semiconductor samples to study the variable thermal conductivity [17]. The photothermal theory is used to measure theoretically some physical and chemical properties of the solid semiconductor media. The photothermal theory is used to explain the interaction between the plasma and thermal-elastic waves for semiconductors [18]. Lotfy and Tantawi [19] investigated the interaction which occurs between the thermoelasticity theory and photothermal theory in a non-homogeneous nano-composite semiconductor, called a photo-thermoelasticity theory. Recently, various scientists [20–23] studied the behavior of plasma-thermal-elastic waves in semiconductors when the magneto-thermoelasticity theory in general form is considered when intricate with other new theories as to the two-temperature theory and the hyperbolic two-temperature.

A differential equations model which it has one or more stochastic terms is called a stochastic differential equation (SDE), and this model has a resulting solution is itself a random process. A stochastic differential equation can be used to formulate a physical system subject to thermal fluctuations. Recently, the techniques of stochastic simulation are used for describing the thermoelastic problem during the heat conduction transport process. Sherief et al. [24] developed a stochastic thermal shock problem according to the generalized thermoelasticity theory. Allam et al. [25, 26] studied a stochastic half-space problem under the effect of an internal heat source in the theory of generalized diffusion–thermoelasticity theory in an infinitely long annular cylinder. Gupta and Mukhopadhyay [27] investigated the stochastic thermoelastic interaction with random temperature distribution at the free surface of an elastic solid medium under a dual phase-lag model. On the other hand, Kant and Mukhopadhyay [28] used the stochastic temperature distribution at the boundary to describe an elastic half-space problem. Many authors [29–33] modified modern physical and biological models according to the fractional time derivative due to the impact of laser pulses for dipolar bodies of microstretch elastic materials.

According to a previous review of literature, the stochastic simulation process of the semiconductor material with the photo-thermoelasticity theory is not studied before. In this work, the optical properties of elastic semiconductor medium are considered when stochastic temperature distributions are taken at the boundary. A novel theoretical technique is studied when the thermal shock problem in the ideal case is deterministic together with the realistic case during some noise exists in the context of the photo-excitation processes. The governing equation is described in one-dimensional (1D) deformation under the impact of random heating on the waves behavior of the semiconductor medium. The analytical solutions of temperature (thermal wave), displacement, stress (mechanical wave), and carrier density (plasma wave) are obtained using the 1D Laplace transform. The inverse of Laplace transforms

\[ \mathcal{L}^{-1}\{F(s)\} = f(t) \]
with some numerical approximation is used to obtain the complete solutions. Statistically, the mean and the variance of the main physical fields are derived. Some comparisons are represented graphically and discussed.

2 Mathematical Formulation of the Problem

The main variables according to the semiconductor medium are the temperature distribution (thermal wave) $T(\vec{r}, t)$, the carrier density distribution (plasma wave) $N(\vec{r}, t)$, the displacement distribution $u(\vec{r}, t)$ (elastic wave). The governing equations which describe the coupled system in semiconductor are [33–39].

\[
\frac{\partial N(\vec{r}, t)}{\partial t} = D_E \nabla^2 N(\vec{r}, t) - \frac{N(\vec{r}, t)}{\tau} + \kappa T(\vec{r}, t) \tag{1}
\]

\[
\rho C_v \frac{\partial T(\vec{r}, t)}{\partial t} = k \nabla^2 T(\vec{r}, t) - \frac{E_g}{\tau} N(\vec{r}, t) + \gamma T_0 \nabla \cdot \frac{\partial u(\vec{r}, t)}{\partial t} \tag{2}
\]

\[
\rho (1 - \xi_1^2 \nabla^2) \frac{\partial^2 f(\vec{r}, t)}{\partial t^2} = \mu \nabla^2 N(\vec{r}, t) + (\mu + \delta \nabla (\nabla \cdot N(\vec{r}, t))) - \gamma VT(\vec{r}, t) - \delta_u \nabla N(\vec{r}, t) \tag{3}
\]

Where $\kappa = \frac{\partial N_0}{\partial \tau}$ is a parameter describes the non-zero activation coupling at the equilibrium carrier concentration $N_0$ and $\xi_1$ is non-local scale parameter. The thermal-optical-elastic physical constants are $D_E$, $\tau$, $E_g$, $\mu$, $\lambda$, $\rho$, $k$ and $T_0$ which they represent the carrier diffusion coefficient, the life-time relaxation time, the gap energy, the Lame’s constants for elastic medium, the density, the thermal conductivity and the reference temperature respectively. The volume thermal expansion is $\gamma = (3\lambda + 2\mu)\alpha_T$, $\alpha_T$ is the linear thermal expansion parameter, $C_v$ is specific heat coefficient, $\delta_u = (2\mu + 3\lambda) d_n$, is the difference between valence band and conductive potential, $d_n$ is the coefficient of electronic deformation.

On the other hand, the non-local equation in non-local case which is described by the stress components is [40, 41]:

\[
\left(1 - \xi_1^2 \nabla^2\right) \sigma_{ij}' = \sigma_{ij}' = \mu(\delta_{ij} + w_{ij}) + (\lambda w_{kk} - \gamma_\theta - \gamma_\theta N) \delta_{ij}' \tag{4}
\]

For more simplicity, the problem is taken in 1D deformation. The displacement vector can be represented as $\vec{u} = (u(x, t), 0, 0)$. On the other hand, the following non-dimensional variables in 1D are introduced as:

\[
(x', \ u', \ \xi_1') = \left(\frac{x - u(t)}{C_T T^*}, \ u(t'), \ N^* \right) = \left(\frac{T(t)}{\sqrt{2 \mu + \lambda}}, \ \sigma' = \frac{\sigma}{\mu} \right) \tag{5}
\]

The dashed is dropped and taken all variables in 1D (in the direction of $x$-axis). Therefore, the above system of Eqs. (1)–(4) with helping Eq. (14), yields:

\[
\left(\frac{\partial^2}{\partial x^2} - A_2 - A_1 \frac{\partial}{\partial t}\right) N + \varepsilon_3 T = 0, \tag{6}
\]

\[
\left(\frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial t}\right) T - \alpha_2 N + \alpha_3 \frac{\partial^2 u}{\partial x \partial t} = 0, \tag{7}
\]

\[
\left(\frac{\partial^2}{\partial x^2} - \left(1 - \xi_1^2 \frac{\partial^2}{\partial t^2}\right) \frac{\partial^2}{\partial x^2}\right) u - \frac{\partial N}{\partial x} - \frac{\partial T}{\partial x} = 0, \tag{8}
\]

\[
\sigma = \sigma_{xx} = \frac{2\mu + \lambda}{\mu} \left(\frac{\partial u}{\partial x} - (T + N)\right). \tag{9}
\]

Where:

\[
\alpha_2 = \frac{\alpha_T E_i E_j^*}{\alpha_T E_i E_j^*}, \alpha_3 = \frac{\gamma T_c^* c_j}{\mu}, A_1 = \frac{k}{D_T^*}, A_2 = \frac{k^*}{D_T^*}, \varepsilon_3
\]

\[
\frac{k_{\text{ed}}^* c_j}{\alpha T_c^*}, C_f = \sqrt{\frac{2\mu + \lambda}{\mu}}, \rho = \frac{k_{\text{ed}}^*}{\mu C_f c_j} \tag{10}
\]

3 The Solution of the Problem

Using the Laplace transform to convert the partial differential equations (PDE) into ordinary differential equations (ODE), Laplace transform for any function $\zeta(x, t)$ taken the following form:

\[
L[\zeta(x, t)] = \overline{\zeta}(x, s) = \int_0^\infty \zeta(x, t) e^{-st} \, dt \tag{11}
\]

The homogenous initial conditions can be considered in this problem as:

\[
\left\{\begin{array}{l}
\left. u(x, t) \right|_{t=0} = \left. \frac{du}{dt} \right|_{t=0} = 0, \quad T(x, t)|_{t=0} = \left. \frac{dT}{dt} \right|_{t=0} = 0, \\
\left. \sigma(x, t) \right|_{t=0} = \left. \frac{d\sigma}{dt} \right|_{t=0} = 0, \quad N(x, t)|_{t=0} = \left. \frac{dN}{dt} \right|_{t=0} = 0.
\end{array}\right\} \tag{12}
\]

Applying the Laplace transform Eq. (10) for the system of main 1D Eqs. (6)–(9), yields:

\[
\left(\frac{\partial^2}{\partial x^2} - A_1 \frac{\partial}{\partial t}\right) N + \varepsilon_3 T = 0 \tag{13}
\]

\[
\left(\frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial t}\right) T - \alpha_2 N + \alpha_3 \frac{\partial^2 u}{\partial x \partial t} = 0 \tag{14}
\]

\[
\left(\frac{\partial^2}{\partial x^2} - \left(1 - \xi_1^2 \frac{\partial^2}{\partial t^2}\right) \frac{\partial^2}{\partial x^2}\right) u - \frac{\partial N}{\partial x} - \frac{\partial T}{\partial x} = 0, \tag{15}
\]

Where $A_1 = A_2 + sA_3, A_4 = \frac{(2\mu + \lambda)}{\mu}$, $R = \frac{x^2}{(1 + \xi_1^2 x^2)^2}, \frac{1}{Z} = \frac{1}{(1 + \xi_1^2 x^2)}$.
Eliminating $\overline{T}, \overline{n}$ and $\overline{N}$ in the system of ODE Eqs. (12), (14) and (Mathematica program is used to check that), yield:

$$D^6 - \prod_i D^4 + \prod_j D^2 - \prod_k = 0$$

(16)

Where the coefficients of Eq. (16) can be obtained as:

$$\prod_i = (-s - R - a_1 + Z \alpha_2),$$

$$\prod_j = (\alpha R + s \alpha_1 + 2 \alpha_1 - Z \alpha_1 \alpha_2 + Z \alpha_2 \alpha_3 - \epsilon_2 \epsilon_3),$$

$$\prod_k = (-s \alpha R + 2 \alpha_1).$$

(17)

The solution of the thermal distribution according to the linearity case can be written as:

$$\overline{T}(x, s) = \sum_{i=1}^{3} M_i(s) e^{-k_i x}$$

(18)

The other solutions of the main fields can be represented as:

$$\overline{n}(x, s) = \sum_{i=1}^{3} M'_i(s) e^{-k_i x} = \sum_{i=1}^{3} M_i(s) e^{-k_i x}$$

(19)

$$\overline{N}(x, s) = \sum_{i=1}^{3} M''_i(s) e^{-k_i x} = \sum_{i=1}^{3} M_i(s) e^{-k_i x}$$

(20)

$$\overline{\sigma}(x, s) = \sum_{i=1}^{3} M''' e^{-k_i x} = \sum_{i=1}^{3} h_{3i} M_i(s) e^{-k_i x}$$

(21)

Where $M_i, M'_i, M''_i$ and $M'''_i (i = 1, 2, 3)$ represent the unknown parameters which they depend on the Laplace parameter $s$ only, and

$$h_{1i} = \frac{-k_i^2 s + h_{3i} \alpha_2}{k_i^2 \alpha_2},$$

$$h_{2i} = -\frac{k_i \alpha_1}{k_i - \alpha_1},$$

$$h_{3i} = -(1 + h_{2i} - h_{1i} k_i) \alpha_4.$$

(22)

### 4 Boundary Conditions

To obtain the unknown parameters $M_i$, some conditions can be applied on the surface of the stochastic semiconductor medium [45].

(I) The system is exposed to thermal shock (Danilovskaya’s problem; $T(t) = T_0(t) = T^\ast h(t)$) at the outer surface ($x=0$) as:

$$\overline{T}(0, s) = \overline{T_0}(s) = T^\ast R(s) \Rightarrow \sum_{i=1}^{3} M_i(s) = \overline{T_0}(s) = \frac{T^\ast}{s}$$

(23)

(II) The traction is free at the outer surface ($x=0$) which it leads to:

$$\overline{\sigma}(0, s) = 0 \Rightarrow \sum_{i=1}^{3} M_i h_{3i} = 0$$

(24)

(III) The recombination processes during the plasma distribution at the free surface ($x=0$), which leads to

$$\overline{N}(0, s) = 0 \Rightarrow \sum_{i=1}^{3} h_{3i} M_i(s, x) = 0$$

(25)

Where, $T^\ast = T_0$ is a constant reference temperature. From the above system of boundary conditions, the parameters $M_i$ be determined. In this case, the temperature distribution can be rewritten as:

$$\overline{T}(x, s) = M_i e^{-k_i x} + M_i e^{-k_i x} + M_i e^{-k_i x}$$

(26)

### 5 Invers of the Laplace Transforms

The complete deterministic solutions the basic fields is obtained in the domain of space-time coordinates when the Riemann-sum approximation technique is used numerically by inversion Laplace transform [41]. The main fields in case of deterministic distributions according to the short time approximation is taken into consideration for the large value of $s$. The inverse of any function $\overline{\zeta}(x, s)$ can be obtained as:

$$\zeta(x, t') = L^{-1}\left\{ \overline{\zeta}(x, s) \right\} = \frac{1}{2\pi i} \int_{-\infty}^{\infty} e^{st'} \overline{\zeta}(x, s) ds$$

(27)

Where $s = n + iM$ ($n, M \in R$), the solution of Eq. (26) taken the form:

$$\zeta(x, t') = \frac{\exp(nt)}{2\pi} \int_{-\infty}^{\infty} \exp(nt) \overline{\zeta}(n + i\beta) d\beta$$

(28)

Using the numerical Fourier series expand for Eq. (27) in the closed interval[0, $2t'$], yields:

$$\zeta(x, t') = \frac{\exp(nt)}{t'} \left[ \frac{1}{2} \zeta(x, n) + \text{Re} \sum_{k=1}^{N} \overline{\zeta}(x, n + \frac{ik\pi}{t'}) (-1)^n \right]$$

(29)

Where $i$ represents the imaginary unit, $N$ can be chosen free as a large integer and $Re$ refers to the real part and the notation $nt \approx 4.7$ [46].
6 Stochastic Main Physical Fields

6.1 Stochastic Temperature (Thermal Wave)

Now, a stochastic distribution of the temperature (thermal) at the boundary can be defined on the following form [29, 30]:

\[ T_0(t) = T + \varphi_0(t) \]  

(30)

Where \( T(t) = \frac{T_0(t)}{h(t)} \), \( T^* \) is a constant temperature, \( h(t) \) is the Heaviside unit function and a stochastic process is \( \varphi_0(t) \) which based on the time-parameter \( t \) and satisfies:

\[ E[\varphi_0(t)] = 0 \]  

(31)

The stochastic process \( \varphi_0(t) \) can be chosen from the white noise type (the most common type), the stochastic process for the function \( x(t) \) satisfies the following relation:

\[ E[L[x(t)]] = L[E[x(t)]] \]  

(32)

On the other hand, the physical fields involve a boundary condition during a stochastic process, in this case, the physical fields have a stochastic process mainly due to the random function \( \varphi_0(t) \). Therefore, from Eqs. (26) and (30), yield:

\[ E[\overline{T}(x,s)] = L[E[T(x,t)]] = \overline{T}(x,s) \]  

(33)

Accordingly, the mean of all the sample paths of the temperature field, \( E[T(x,t)] \) is similar to the solution for the deterministic case.

But in the case of placing the temperature distribution on the following form:

\[ \overline{T}(x,s) = \overline{\vartheta}(x,s) + \overline{\phi}(x,s)\overline{T}_0(s) \quad \text{or} \quad \overline{T}(x,s) = \left( A_1 + A_2\overline{T}_0(s) \right) \exp(-k_1x) + \left( B_1 + B_2\overline{T}_0(s) \right) \exp(-k_2x) + \left( C_1 + C_2\overline{T}_0(s) \right) \exp(-k_3x) \]  

(34)

According to Eq. (34), yields:

\[ \overline{\vartheta}(x,s) = A_1 \exp(-k_1x) + B_1 \exp(-k_2x) + C_1 \exp(-k_3x) \]  

(35)

\[ \overline{\phi}(x,s) = A_2 \exp(-k_1x) + B_2 \exp(-k_2x) + C_2 \exp(-k_3x) \]  

(36)

On the other hand, when using Eqs. (30) and (34) can be rewritten as [29, 30]:

\[ \overline{T}(x,s) = \overline{\vartheta}(x,s) + \overline{\phi}(x,s)\overline{T}_0(s) \]  

(37)

Applying the inverse of Laplace transform technique of Eq. (37), in this case Eq. (37) can be written as:

\[ \overline{T}(x,s) = \left\{ \overline{\vartheta}(x,s) + \overline{\phi}(x,s)\overline{T}(s) \right\} + \overline{\phi}(x,s)\overline{\varphi_0}(s), \]  

(38)

\[ \overline{T}(x,s) = \overline{T}_0(s) + \overline{\phi}(x,s)\overline{\varphi_0}(s), \]  

(39)

where \( \overline{T}_0(s) = \overline{\vartheta}(x,s) + \overline{\phi}(x,s)\overline{T}(s) \). According to the inverse of Laplace transform method Eq. (39) takes the form:

\[ T(x,t) = T^*(x,t) + \int_0^t \phi(x,t-u)\varphi(u)du \]  

(40)

where \( T^*(x,t) \) expresses the deterministic temperature distribution and \( \phi(x,t) \) is the Laplace inverse of Eq. (36). However, Eq. (40) can be reduced as:

\[ T(x,t) = T^*(x,t) + \int_0^t \phi(x,t-u)W(u) \]  

(41)

where \( W(u) \) is the Wiener process.

To complete the stochastic properties, the variance of the problem should be obtained, in this case, squaring Eq. (40), yields [31]:

\[ |T(x,t)|^2 = |T^*(x,t)|^2 + \int_0^t \left( \int_0^1 \phi(x,t-u)\varphi(x,t-u) \right)du \]  

(42)

Applying the expectation process to both sides of Eq. (42), yields:

\[ E[\varphi(u)] = 0, \quad E[\varphi(u_1)\varphi(u_2)] = \delta(u_1 - u_2). \]  

(44)

The variance can be written in the following form:

\[ \text{Var}[T(x,t)] = \int_0^t \int_0^t \phi(x,t-u_1)\phi(x,t-u_2)\delta(u_2 - u_1)du_1du_2. \]  

(45)

Using the following relation:

\[ \int_a^b f(x)f(x-x_0)dx = f(x_0), a < x_0 < b. \]  

(46)

In this case, the variance can be rewritten as:

\[ \text{Var}[T(x,t)] = \int_0^t \phi(x,t-u_1)\phi(x,t-u_2)du_1. \]  

(47)

During the path \( u_1 = u_2 \), yields:
\[ Var[T(x, t)] = \int_0^t \left[ \phi(x, t - u_1) \right]^2 du_1. \]  

(48)

Substituting by \( t - u_1 = \theta \), then the variance of temperature field can be written as:

\[ Var[T(x, t)] = -\int_t^\infty (\phi(x, \theta))^2 d\theta = \int_0^t (\phi(x, \theta))^2 d\theta. \]  

(49)

### 6.2 Deterministic Stress Distribution

Using the same technique which is described in section 6.1, under the deterministic boundary condition Eqs. (24) and (21), the solution of the deterministic stress field which it can be described as [29, 30]:

\[ \tau_{ss}(x, s) = h_1 M_1 \exp(-k_1 x) + h_2 M_2 \exp(-k_2 x) + h_3 M_3 \exp(-k_3 x). \]  

(50)

### 6.3 Stochastic Stress Distribution

Using the same form of Eq. (30) and proceeding in the same manner as in section 6.1, the stochastic stress distribution can be obtained as:

\[ E[\sigma_{ss}(x, s)] = L[E\{\sigma_{ss}(x, s)\}] = \sigma_{ss}(x, s). \]  

(51)

Therefore, the mean of all the sample paths of the stress field distribution, \( E[\sigma_{ss}(x, s)] \) can be taken in a similar manner of Eq. (30) for the deterministic case which can be put as:

\[ \pi_{ss}(x, s) = \bar{\pi}_{ss}(x, s) + \bar{\pi}_{ss}(x, s) \overline{T_0}, \text{ or} \]

\[ \pi_{ss}(x, s) = \left( D_1 + D_2 \overline{T_0}(s) \right) \exp(-k_1 x) + \left( E_1 + E_2 \overline{T_0}(s) \right) \exp(-k_2 x) + \right. \]

\[ \left( F_1 + F_2 \overline{T_0}(s) \right) \exp(-k_3 x) \].  

\[ \text{(52)} \]

On the other hand, from Eq. (52), \( \Omega(x, s) \) can be rewritten as:

\[ \Omega(x, s) = D_1 \exp(-k_1 x) + E_1 \exp(-k_2 x) + F_1 \exp(-k_3 x) \]  

(53)

and \( \overline{\omega}(x, s) \) can be written as:

\[ \overline{\omega}(x, s) = D_2 \exp(-k_1 x) + E_2 \exp(-k_2 x) + F_2 \exp(-k_3 x) \]  

(54)

Therefore, using Eq. (30), yields:

\[ \overline{\sigma}_{ss}(x, s) = \Omega(x, s) + \overline{\omega}(x, s) \left[ \overline{T} + \overline{\varphi}_0(t) \right] \]  

(55)

Using the Laplace transform inverse of the above Eq. (55) under the influence of the convolution property of Laplace inverse, then:

\[ \sigma_{ss}(x, t) = \sigma^1(x, t) + \int_0^t \omega(x, t - u) \varphi(u) du \]  

(56)

where, \( \sigma^1(x, t) \) is the stress distribution in the deterministic case and \( \omega(x, t) \) is the Laplace inverse of \( \overline{\sigma}(x, s) \). By the same way which it is used in section 6.1, the variance for the stress distribution can be written as:

\[ Var[\sigma_{ss}(x, t)] = \int_0^t \omega^2(x, \theta) d\theta \]  

(57)

### 6.4 Deterministic Displacement Distribution

In view of the deterministic boundary conditions (19), where the displacement distribution is a part of stress distribution, however by the same technique which described in section 4, in this case the solution of the displacement field can be written in the following form

\[ \bar{u}(x, s) = h_1 M_1 \exp(-k_1 x) + h_2 M_2 \exp(-k_2 x) + h_3 M_3 \exp(-k_3 x) \]  

(58)

### 6.5 Stochastic Displacement Distribution

Using the stochastic boundary condition as obtained in Eq. (30) by the same technique which is used in section 6.1, the mean displacement distribution can be written as [29, 30]:

\[ E[\bar{u}(x, s)] = L[E\{u(x, s)\}] = \bar{u}(x, s) \]  

(59)

Where the mean \( E\{u(x, s)\} \) of all the displacement field sample paths, which has the similar solution of the deterministic case.

Considering,

\[ \bar{u}(x, s) = \bar{u}_1(x, s) + \bar{u}_2(x, s) \overline{T_0}, \text{ or} \]

\[ \bar{u}(x, s) = \left( G_1 + G_2 \overline{T_0}(s) \right) \exp(-k_1 x) + \left( H_1 + H_2 \overline{T_0}(s) \right) \exp(-k_2 x) + \left( K_1 + K_2 \overline{T_0}(s) \right) \exp(-k_3 x) \]  

(60)

On the other hand, the values of \( \bar{\Gamma}(x, s) \) and \( \bar{U}(x, s) \) according to Eq. (60), can be written as:

\[ \bar{\Gamma}(x, s) = G_1 \exp(-k_1 x) + H_1 \exp(-k_2 x) + G_1 \exp(-k_3 x) \]  

(61)

\[ \bar{U}(x, s) = G_2 \exp(-k_1 x) + H_2 \exp(-k_2 x) + G_2 \exp(-k_3 x) \]  

(62)

Using Eq. (30), Eq. (60) can be rewritten as:
Fig. 1 The deterministic temperature, normal stress, carrier density and displacement against the distance for different values of non-local parameters in the context of the photo-thermoelasticity theory
\( \overline{u}(x, s) = \overline{T}(x, s) + \overline{U}(x, s)\left[\overline{T}(s) + \overline{\varphi}_0(s)\right] \) \hspace{1cm} (63)

Under the influence of the inverse of Laplace transform, Eq. (63) can be reduced as:

\[ u(x, t) = u^1(x, t) + \int_0^t U(x, t - u)\varphi(u)du \] \hspace{1cm} (64)

where \( u^1(x, t) \) is the deterministic displacement field and \( U(x, t) \) is the Laplace inverse of the function \( \overline{U}(x, s) \).

By the same way, the variance for the displacement distribution can be obtained as in section 6, variance for the in this case, the variance for the displacement distribution can be written as:

\[ \text{Var}[\sigma_{xx}(x, t)] = \int_0^t U^2(x, \theta)d\theta \] \hspace{1cm} (65)

### 6.6 Deterministic Carrier Density Distribution

Using the stochastic boundary condition as obtained in Eq. (30) by the same technique which is used in the above deterministic sections, the deterministic carrier density field solution is obtained as:

\[ N(x, s) = M_1h_1\exp(-k_1x) + M_2h_2\exp(-k_2x) + M_3h_3\exp(-k_3x) \] \hspace{1cm} (66)

### 6.7 Stochastic Carrier Density Distribution

Using the stochastic boundary condition as obtained in Eq. (30) by the same technique which is used in the above stochastic sections, the mean displacement distribution can be written as [29, 30]:

\[ E[N(x, s)] = L[E[N(x, s)]] = \overline{N}(x, s) \] \hspace{1cm} (67)

where the mean \( E[N(x, s)] \) taken of all carrier density field sample paths.

Considering the stochastic carrier density distribution can be written as:

\[
\overline{N}(x, s) = \overline{m}(x, s) + \overline{m}(x, s)\overline{T}_0, \quad \text{or}
\]

\[
\overline{N}(x, s) = \left(a_1 + a_2\overline{T}_0(s)\right)\exp\left(-k_1x\right) + \left(b_1 + b_2\overline{T}_0(s)\right)\exp\left(-k_2x\right) + \left(c_1 + c_2\overline{T}_0(s)\right)\exp\left(-k_3x\right)
\] \hspace{1cm} (68)

Where the values of \( \overline{m}(x, s) \) and \( \overline{n}(x, s) \) can be taken the form:

\[ \overline{m}(x, s) = a_1\exp\left(-k_1x\right) + b_1\exp\left(-k_2x\right) + c_1\exp\left(-k_3x\right) \] \hspace{1cm} (69)

\[ \overline{n}(x, s) = a_2\exp\left(-k_1x\right) + b_2\exp\left(-k_2x\right) + c_2\exp\left(-k_3x\right) \] \hspace{1cm} (70)

Under the effect of stochastic term which defined by Eq. (30), stochastic carrier density distribution can be rewritten as:

\[ \overline{N}(x, s) = \overline{m}(x, s) + \overline{m}(x, s)\overline{T}_0 + \varphi_0(s) \] \hspace{1cm} (71)

Using the inverse of Laplace transform, yields:

\[ N(x, t) = N^1(x, t) + \int_0^t n(x, t - u)\varphi(u)du \] \hspace{1cm} (72)

where \( N^1(x, t) \) is the deterministic carrier density field and \( n(x, t) \) is the Laplace inverse of the function \( \overline{m}(x, s) \).

The variance for the carrier density field can be obtained as:

\[ \text{Var}[N(x, t)] = \int_0^t n^2(x, \theta)d\theta \] \hspace{1cm} (73)

### 7 Numerical Results

This section analyses the obtained results by considering the input parameters (physical constants) of non-local silicon semiconductor material. The numerical simulation of the waves propagation accompanying the basic physical quantities can be done and represented graphically under

---

**Table 1** The Si input physical parameters in SI units

| Unit          | Symbol | Si        |
|---------------|--------|-----------|
| \( \text{N/m}^2 \) | \( \lambda \) | \( 6.4 \times 10^{10} \) |
| \( \text{kg/m}^3 \) | \( \mu \) | \( 6.5 \times 10^{10} \) |
| mg/L          | \( K \) | 2330      |
| sec(s)        | \( T_0 \) | 800       |
| m²/s          | \( D_e \) | \( 2.5 \times 10^{-3} \) |
| m³            | \( d_n \) | \( -9 \times 10^{-31} \) |
| eV            | \( E_g \) | 1.11      |
| \( \text{K}^{-1} \) | \( \alpha_t \) | \( 4.14 \times 10^{-6} \) |
| Wm⁻¹K⁻¹       | \( k \) | 150       |
| \( \text{J/(kg K)} \) | \( C_e \) | 695       |
| m/s           | \( \bar{\gamma} \) | 2         |
the impact of the noise intensity. Accordingly, the physical constants of silicon are shown in Table 1 [37, 39–41].

Figure 1 ((1-a) to (1-d)) shows the variation of the main deterministic physical variables (temperature, carrier density, normal stress and displacement) against the horizontal distance according to the different values of three dimensionless non-local parameters namely \(\xi_1 = 0.0, \xi_1 = 0.5\) and \(\xi_1 = 1.5\) for small time \(t = 0.05\). The behavior of waves propagation for different values of non-local parameters (\(\xi_1 = 0.0, \xi_1 = 0.5\) and \(\xi_1 = 1.5\)) takes the same behavior in two cases when \(\xi_1 = 0.5\) and \(\xi_1 = 1.5\) with a difference in magnitude and different in local case when \(\xi_1 = 0.0\). In Fig. 1a the deterministic distribution of temperature (thermal wave) starts from a positive point that fulfills the thermal condition and continues to increases continuously to reach the maximum value when \(\xi_1 = 0.5\) and \(\xi_1 = 1.5\) in the first range, and in the second range decrease with taking the exponential behavior without any jumps until reaching the minimum before it coincides with the zero line. But in local case when \(\xi_1 = 0.0\), the thermal wave decreases sharply in the first range to coincide with the zero line. In Fig. 1b the deterministic stress begins at zero satisfying the mechanical condition which decreases sharply in local case when \(\xi_1 = 0.0\) near the surface until reaches the peak minimum point and then increases in the negative region until coincide with the zero line. Due to the increasing of the thermal effect on the surface, in the non-local cases when \(\xi_1 = 0.5\) and \(\xi_1 = 1.5\), the stress distribution begins to increase when \(\xi_1 = 1.5\)and decrease when \(\xi_1 = 0.5\) in a continuous and smooth manner until it reaches a state of stability by going deeper into the material and this is shown by matching with the zero line. In Fig. 1c, the carrier density in the deterministic case (plasma wave) in three cases of non-local parameters (\(\xi_1 = 0.0, \xi_1 = 0.5\) and \(\xi_1 = 1.5\)) starts from a minimum point at rest (steady-state) that meets the state of plasma recombination at the beginning on the non-local surface and then begins to increase continuously according to the thermal effect of light in the smooth state without any jumps until reaches the maximum values near the surface. In the second range, the plasma waves decrease with exponential behavior until converge with zero line when the distance increases to reach the steady-state. Finally, in Fig. 1d, the deterministic (elastic wave) displacement distribution on the surface starts from an extreme point in two non-local cases when \(\xi_1 = 0.5\) and \(\xi_1 = 1.5\) with taken the exponential behavior until reach to the steady-state case. But in local case when \(\xi_1 = 0.0, \xi_1 = 0.5\) and \(\xi_1 = 1.5\) shows no effect on wave propagation.

Figure 2 ((2-a) – (2-d)) displays the dispersion of the distributions of the fundamental physical quantities around their mean which is described by variance. The numerical simulation was carried out at different values of the dimensionless non-local parameters (\(\xi_1 = 0.0, \xi_1 = 0.5\) and \(\xi_1 = 1.5\)) to obtain the behavior of all distributions (the variance of the main fields), due to a white noise in stochastic case. It can be seen that the variance travels like a wave (wave behavior or exponential behavior) and ends with a diffuse portion for both the stress and the carrier density distributions. Moreover, the variance on the free surface of non-local semiconductor is zero except for the temperature and displacement distributions. Where all physical properties are invariant on the boundary except for temperature and displacement. The dispersion maximum for all properties shifts to the right except for the two displacement and temperature distributions, where the maximum occurs at the free surface. In general, occurs at the free surface. In general, the physical field distributions vanish after a finite distance (\(x = 1.3\)) for the stochastic as well as deterministic.

Figure 3 ((3-a) to (3-d)) illustrates the comparison between stochastic and deterministic temperature, normal thermal stress, carrier density and displacement distributions according to three different sample paths at \(\xi_1 = 0.5\). Figure 3a displays a combined plot of the temperature distribution in a deterministic case and a stochastic case for three sample paths. From this figure, it becomes clear a large variation between the stochastic temperature (thermal) distribution and the deterministic temperature distribution along three different sample paths which clearly visible near the boundary. On the other hand, the stochastic distribution of temperature coincides with the deterministic distribution with increasing distance to enter the depth of the semiconductor material. Figure 3b shows the deterministic normal stress distribution (mean) with stochastic normal stress distribution according to three different random sample paths. From this figure, a significant variation between the stochastic normal stress distribution and deterministic normal stress distribution is obtained at the beginning near the surface boundary. This difference continues with the increase in the distance until entering the depth of the material as a result of the thermal stresses on the semiconductor material, then matching occurs until the state of stability. Figure 3c shows the comparison between the deterministic carrier density (plasma) distribution with the stochastic plasma distribution. From this figure, it is clear that there is a discrepancy between the distributions, but in general the mean of all three sample paths of the stochastic plasma distribution coincides with the deterministic plasma distribution with increasing distance. Finally, Fig. 3d displays the deterministic displacement (elastic) distribution which is in its comparison with a displacement (elastic) distribution. Similarly in Fig. 1, there is a large variation near the boundary surface and then there is a correspondence with the increase in the distance by entering into the depth of the material.
8 Conclusion

Present work is based on the interaction between the photothermal theory and thermoelasticity theory when a non-local semiconductor material is used. The main concept of this problem depends on two types of analyses: deterministic and stochastic types. The used stochastic type is defined when the Gaussian white noise stochastic process is added to the deterministic conditions. The numerical deterministic solutions of temperature, normal stress, carrier density, and displacement distributions show no significant differences (negligible differences) between the stochastic and the deterministic distributions. All physical distributions are obtained to be continuous according to the physical domain. This physical-mathematical model can be used to improve product quality and production efficiency of semiconductor materials. The analysis and obtained results in this work will be very much significant in studying the uses of semiconductors such as diodes, triodes, and modern electronic devices.

Appendix

The coefficients of Eq. (26) are:

\[ A_1 = -\frac{-\lambda(h_{31} - h_{33})}{sD_x(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})} \]  

(74)

\[ A_2 = -\frac{-h_{32}h_{33}}{h_{22}h_{31} - h_{23}h_{31} + h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33}} \]  

(75)

The coefficients of Eq. (50) are:

\[ D_1 = -\frac{\lambda h_{31}(h_{32} - h_{33})}{sD_x(-h_{22}h_{31} + h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})} \]  

(80)

\[ E_1 = -\frac{\lambda h_{32}(-h_{31} + h_{33})}{sD_x(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})} \]  

(81)

\[ F_1 = \frac{\lambda h_{33}(h_{31} - h_{32})}{sD_x(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})} \]  

(82)

\[ D_2 = \frac{h_{31}(-h_{23}h_{32} + h_{22}h_{33}) t_0}{(h_{22}h_{31} - h_{23}h_{31} + h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})} \]  

(83)

\[ E_2 = -\frac{h_{32}(-h_{23}h_{31} + h_{22}h_{33}) t_0}{sD_x(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})} \]  

(84)

\[ F_2 = -\frac{h_{33}(-h_{22}h_{31} + h_{21}h_{32}) t_0}{(h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})} \]  

(85)

The coefficients of Eq. (58) are:

\[ G_1 = -\frac{\lambda h_{11}(h_{32} - h_{33})}{sD_x(-h_{22}h_{31} - h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})} \]  

(86)

\[ H_1 = -\frac{\lambda h_{12}(-h_{31} + h_{33})}{sD_x(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})} \]  

(87)

\[ K_1 = -\frac{\lambda h_{13}(h_{31} - h_{32})}{sD_x(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})} \]  

(88)
The Deterministic and stochastic distributions of temperature, normal stress, carrier density and displacement against the distance when $\xi = 0.5$ in the context of the generalized photo-thermoelasticity theory

$$G_2 = \frac{h_{11}(h_{23}h_{32} + h_{22}h_{33})}{(h_{22}h_{31} - h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})}$$  
(86)

$$H_2 = \frac{h_{12}(h_{23}h_{31} - h_{21}h_{33})}{(h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})}$$  
(87)

$$K_2 = \frac{h_{13}(h_{22}h_{31} + h_{21}h_{32})}{(h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})}$$  
(88)

The coefficients of Eq. (66) are:

$$a_1 = -\frac{\lambda h_{23}(h_{32} - h_{33})}{sD_e(h_{22}h_{31} - h_{23}h_{31} - h_{21}h_{32} + h_{23}h_{32} + h_{21}h_{33} - h_{22}h_{33})}$$  
(89)

$$b_1 = \frac{\lambda h_{32}(-h_{11} + h_{13})}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})}$$  
(90)

$$c_1 = \frac{\lambda h_{23}(h_{31} - h_{32})}{sD_e(-h_{22}h_{31} + h_{23}h_{31} + h_{21}h_{32} - h_{23}h_{32} - h_{21}h_{33} + h_{22}h_{33})}$$  
(91)

$$a_2 = -\frac{h_{21}(h_{32}h_{31} - h_{33}h_{32})}{(h_{22} - h_{23})(h_{31} - h_{21} - h_{23}h_{32} + (h_{21} - h_{22})h_{33})}$$  
(92)

$$b_2 = \frac{h_{22}(h_{23}h_{31} - h_{21}h_{33})}{((-h_{22} + h_{23})h_{31} + (h_{21} - h_{23})h_{32} - (h_{21} - h_{22})h_{33})}$$  
(93)

$$c_2 = \frac{h_{23}(h_{22}h_{31} - h_{23}h_{32})}{((-h_{22} + h_{23})h_{31} + (h_{21} - h_{23})h_{32} - (h_{21} + h_{22})h_{33})}$$  
(94)

Authors Contributions Kh. Lotfy: Conceptualization, Methodology, Supervision, Software, Data curation, Validation. A. Ahmed: Writing-Original draft preparation. A. El-Bary: Visualization, Investigation, Software. Ramdan. S. Tantawi: Writing- Reviewing and Editing.

Data Availability The information applied in this research is ready from the authors at request.

Declarations This study and all procedures performed involving human participants were in accordance with the ethical standards.

Competing Interests The authors have declared that no Competing Interests exist.

Consent to Participate All authors consent to participate to this publication.

Consent for Publication All authors consent to the publication of the manuscript in SILICON, should the article be accepted by the Editor-in-chief upon completion of the refereeing process.

Ethics Approval Not applicable.

Disclosure Statement No potential conflict of interest was reported by the author.
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