Modulational instability of a Yukawa fluid excitation under the Quasi-localized charged approximation (QLCA) framework

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Keywords: quasi-localized charged approximation, modulational instability, Yukawa system, dusty plasma

Abstract

Collective response of a strongly coupled system departs from that in continuum phase upon transition to the quasi-crystalline phase, or a Wigner lattice. The nonlinearity driven modulational instability, for example, of a quasi-crystalline dusty plasma lattice wave, is predicted to inevitably grow macroscopic envelope structures at the expense of a mesoscopic carrier wave. The modulational instability in the dimensionally extended quasi-crystalline or amorphous phase of a strongly coupled system, uniquely accessed by the quasi-localized charge approximation (QLCA) formulation, is shown to offer conditional stability over the entire range of spectral scales by prescribing a narrower instability regime. In distinction from the excitations of linear one-dimensional chain of strongly coupled dust grains, the longitudinal modes of a quasi-crystalline phase incorporated by means of a pair correlation function in the present QLCA based treatment shows the lattice excitations to be stable for arbitrarily long wavelengths beyond a finite value of screening parameter \(\kappa = a/\lambda_D = 0.182\) at low enough temperature, where \(a\) is the inter dust separation and \(\lambda_D\) is the plasma Debye length. However, this unstable domain of the parameter space does grow with increase in the dust temperature which invokes the weak coupling-like effect. The present results show that in comparison to the one-dimensional chains, the dimensionally extended strongly coupled lattice are potentially stable with respect to the macroscopic amplitude modulations. Results offer a greater handle over the macroscopic structures growing from the mesoscopic fluctuations, a mechanism which underlies a variety of processes, ranging from the barrier formation in strongly coupled turbulence to the highly localized modification, induced by collective excitation, of the ultracold ions trapped in strong electromagnetic fields. The existence of the growth rate of instability as well as the maximum modulational growth rate of instability has been investigated for a wide range of values of the screening parameter.

1. Introduction

The dusty (complex) plasmas are observed in laboratory experiments [1, 2] and also in space plasmas (viz., the nucleus of a comet [3], Halley’s comet [4], planetary rings [5], the spokes of Saturn’s rings [6], solar wind [7] and etc [8, 9]). The dust particles have also been observed in the neighbourhood of the space station, Surveyor 5, 6 and 7 spacecraft, and the spacecraft of Apollo 17 mission [10, 11]. The presence of charged dust particles in plasma form a strongly coupled plasma system when the interacting potential energy of the dust particles surpasses the kinetic energy [2, 8, 9]. The strongly coupled plasma system exists in many astrophysical environments (viz., white dwarf [12], neutron star [13, 14]) and also in many laboratory environments (viz., resonant side-bands [15], ultracold neutral plasma [16], classical 2D electron liquid trapped on the surface of liquid helium [17], semiconductor electronic bilayers [18], polarized charged particles [19]). The dust particles are in the strongly coupled state when \(\Gamma_d = \frac{2z^2 e^2 \kappa}{\omega_D^2} = \Gamma e^{-\kappa} \gg 1\), where \(\Gamma = \frac{2z^2 e^2}{\omega_D^2}\) is the coupling parameter.
and \( \kappa = \frac{a}{\lambda_0} \) is the screening parameter with \( a \) is the average distance between the dust grains, and \( T_d \) is the dust temperature. For high coupling value, but below the crystallization limit, i.e., \( 1 \ll \Gamma \ll \Gamma_{\text{melting}} \), the dusty plasma remains in a liquid state. This paper addresses the modulational instability (MI) of DA waves in the liquid state, and in particular strongly coupled plasmas in a liquid state. Fundamental theories of the liquid are a general area of research that allows significant advance because in comparison to the much simpler case of a crystal (\( \Gamma_a \gg 1 \)) or gas (\( \Gamma_d \ll 1 \)), theories are complicated by the inherent disorder in the particle arrangement in liquid. The Quasi-localized charge approximation (QLCA) [20, 21] approach has proven to be a reliable predictor to linear collective excitations of such systems. Various instabilities such as ion–dust instability [19, 22], resonant & Buneman-type instability [22], dust–dust instabilities [23] and DA instability [23, 24] have also been investigated under the QLCA framework.

For small but finite amplitude of collective plasma excitations, the nonlinear self-interaction of the carrier waves grows an initial infinitesimal modulation of wave amplitude and generate waves of higher harmonics [25, 26]. The driven pump wave acts to grow the modulation and at large enough modulation amplitudes the pump may be cut off leading to the formation of solitary envelopes. This amplitude modulation of DA waves can be described as governed by a nonlinear Schrödinger equation (NLSE) under certain conditions [25, 26]. The NLSE can be derived by the reductive perturbation method (RPM) [25, 27]. For external (plane wave) perturbations of amplitude, the modulated envelope solitary wave may breakdown and results the modulational instability (this happens due to the second harmonic) [28]. Among the studies addressing the MI in strongly coupled regime of the dust, a limited number has accounted for an explicit localization by treating a linear, one-dimensional chain of the dust grains [29–32] whereas in other studies a more indirect inclusion of strong coupling is done by letting an effective dust temperature [32] represent the strong coupling effect [33–39]. Both these approaches treat intrinsically one-dimensional setups.

Relevant to many modern scientific applications discussed below, the present study shows that in comparison to the results from modulational instability (MI) in one-dimensional dust lattice excitations, the regime of instability can be highly restricted in an extended dust structure described by a pair–correlation function \( g(r) \) and its more general form. In particular, it is shown that for a wide range of values of screening parameter \( (\kappa = a/\lambda_0) \) explored, the unstable region is restricted up to a rather small value of \( \kappa \) for small dust temperatures. For strong coupling limit, the parameter space is explored by varying both the screening parameter \( (\kappa) \) and the dust temperature \( (\sigma_d = \frac{T_d}{Z_i T_i}) \), and it is shown that the temperature enhances the dimension of the unstable zone in the parameter space. The stabilization of amplitude modulation is verified for larger values of the wave number \( k \) by finding the maximum growth rate of the instability and showing that it indeed reduces to zero at the small value of \( \kappa = 0.183 \) for the dust temperature \( T_d = 3.33 \times 10^{-4} Z_d T_i \), where \( Z_d \) is the dust charge and \( T_i \) is the ion temperature.

Retaining dust temperature as a parameter extends the relevance of our analysis to systems where instability thresholds can be strong function of the temperature of the trapped species. Examples include the edge of the stability region of an RF/Laser ion trap [41] where the instabilities arising from collective excitation of lattice ions can facilitate ion manipulation as a result of the ratio between the collective interaction energy of ions (~0.1 eV) and depth of RF trapping field (> 100 eV) dropping significantly from its extreme bulk value of > 10^4. The similar collective interactions are involved in entrainment arising from Bragg scattering of unbound neutrons by the collective excitation of the Coulomb lattice in the inner crust of a neutron star [42, 43]. A number of other examples with either positive or negative consequences of the instability can also be cited from fusion plasmas [44], ultracold neutral plasmas [45, 46]. The effect of both coupling and the temperature remains important on the instability threshold and are incorporated in the present treatment showing that the instability threshold does reappear at smaller \( \kappa \) with the change of dust temperature while it stays stable over the entire scale spectrum at lower temperatures in contrast to the results for linear one-dimensional chain [29–32].

This paper is organized as follows. In section 2, the QLCA based analytical fluid model is considered, and consequently, the linear dispersion relation (DR) is derived. In section 3, using the reductive perturbation method (RPM) [25, 27], the spatiotemporal nonlinear Schrödinger equation (NLSE) is derived within the QLCA framework. In section 4, the nonlinear dispersion relation of modulated wave and the maximum modulational growth rate of instability are analytically derived. Results and discussions on instability analysis of modulated wave are presented in section 5. The summary and conclusions are presented in section 6.

2. Derivation of spatiotemporal equations with in the QLCA framework

We start with a more general expression described in recent analysis on strongly coupled rotating dusty plasma under the QLCA framework [47]
where the second and third terms in the right-hand side are the Coriolis force and centrifugal force, respectively. This equation can be reduced, in a non-rotating frame ($\Omega \rightarrow 0$), to a form given as

$$m_d \frac{\partial^2 r_{j\mu}}{\partial t^2} = \sum_j K_{ij\mu\nu} r_{j\nu} - 2m_d \left[ \Omega \times \left( \Omega \times r_{j\mu} \right) \right] - m_d [\Omega \times (\Omega \times r_{j\mu})] - \frac{\partial V}{\partial r_{j\mu}} = 0,$$

(1)

here $K_{ij\mu\nu}$ is the dynamical tensor, with the dimension indices $\mu, \nu = x, y, z (i, j$ enumerate particles), which represents the effect of the inter-particle interaction in a Yukawa system. In the non-retarded limit (velocity of electromagnetic waves tends to infinity), the dynamical tensor $K_{ij\mu\nu}$ is given by

$$K_{ij\mu\nu} = (1 - \delta_{ij}) \frac{\partial^2}{\partial r_{j\mu} \partial r_{i\nu}} \phi(|r_i - r_j|) - \delta_{ij} \left[ \sum_{l=1}^N (1 - \delta_{il}) \frac{\partial^2}{\partial r_{i\mu} \partial r_{l\nu}} \phi(|r_i - r_l|) \right]
+ \int d^3 r' \frac{\partial^2}{\partial r_{i\mu} \partial r'_{\nu}} \rho_b(r') \phi(|r_i - r'|),$$

(3)

where $\rho_b(r')$ and $\rho_0$ are the charge density of the background plasma and the unperturbed dust charge density respectively. And the potential $\phi$ is given by

$$\phi(|r_i - r_j|) = e^{-\kappa_b |r_i - r_j|} \frac{Z^2 e^2}{|r_i - r_j|},$$

(4)

with

$$\kappa_b = \sum_A 4\pi Z_A^2 e^2 n_A \beta_A.$$

(5)

The subscript $A$ represents the species in the background plasma, specifically electrons and ions in a typical dusty plasma. The Fourier transformed form of the potential $\phi(|r_i - r_j|)$ is given by

$$\phi(k) = \frac{4\pi Z^2 e^2}{k^2 + \kappa_b^2}.$$

(6)

The standard QLCA prescription [48] led to the linearized version of the above equation in their spectral space, given as,

$$[\omega^2 \delta_{ij} - C_{j\mu}(k, \omega)] \xi_{ki}(\omega) = 0,$$

where $C_{j\mu}$ contains the mean field and local field effects, produced by the random motion and the dust-dust correlation, respectively,

$$C_{j\mu}(k, \omega) = \omega_{pd}^2 \left\{ \frac{k_{j\mu} k_{\mu j}}{k^2 + \kappa_b^2} + D_{j\nu}(k, \omega) \right\},$$

(7)

with $\omega_{pd}$ being the dust acoustic frequency. The central quantity in the QLCA, the dynamical matrix in three dimensions, given as,

$$D_{k\tau}(k) = \omega_{pd}^2 \int_0^\infty dr e^{-\kappa r} \left[ g(r) - 1 \right] K_{k\tau}(kr, \kappa r),$$

(8)

includes dust-dust correlation effects, as this is a functional of the equilibrium pair correlation function (PCF).

In order to analyse the nonlinear effect within the QLCA framework, we require that the averages are done over spatiotemporal functions rather than their Fourier transformations. In the most approximate approach, we let the fluid conservation equations represent the evolution of these ensemble averages. This ensemble averaged (macroscopic) momentum equation is

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = eZ_d \frac{\partial \phi_d}{\partial x} - \frac{1}{m_d n_d} \frac{\partial P_{dk}}{\partial x} - \frac{1}{n_d u_d} \frac{\partial P_{d}}{\partial x}.$$

(9)

Similarly, the continuity equation of macroscopic particles obtained by ensemble averaging over the dust sites is

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x} (n_d u_d) = 0,$$

(10)
and the equation of state is

\[ P_{\text{d}} = P_{\text{i}0} n_{\text{d}}^2. \]  \hfill (11)

Where, \( n_{\text{d}}, u_{\text{d}}, \phi_{\text{d}} \) and \( P_{\text{d}} \) are the number density, velocity, the electrostatic potential and the pressure of dust particles, respectively. And also, \( m_{\text{d}} \) and \( Z_{\text{d}} \) are the mass of a dust particle and the average number of electrons residing on a dust particle, respectively. Local field effects are introduced via a correction \( P_{\text{d}} \) to the ideal-gas pressure term \([49, 50]\) and this correction \( P_{\text{d}} \) contains essential structural information. So, according to Hou et al. [49, 50], we consider \( \partial P_{\text{d}} / \partial \sigma = \left( \frac{\partial P_{\text{d}}}{\partial n_{\text{d}}} \right) \frac{1}{\sigma_{\text{d}}} \), and the dust layer compressibility \( \alpha \equiv \left( \frac{n_{\text{d}}}{n_{\text{e}} T_{\text{d}}} \right) \), is directly related to the dust-dust correlation energy of the Yukawa system [51, 52].

The system of basic equations (9)–(11) are closed by the following Poisson equation,

\[ \frac{\partial^2 \phi_{\text{d}}}{\partial x^2} = 4 \pi e (n_{\text{e}} - n_{\text{i}} + Z_{\text{d}} n_{\text{d}}), \]  \hfill (12)

where \( n_{\text{e}} \) and \( n_{\text{i}} \) are the ion and electron number density, respectively, follow the Boltzmann distribution give as,

\[ n_{\text{e}} = n_{\text{e}0} \exp \left[ - \frac{e \phi_{\text{d}}}{k_B T_{\text{e}}} \right], \]

\[ n_{\text{i}} = n_{\text{i}0} \exp \left[ - \frac{e \phi_{\text{d}}}{k_B T_{\text{i}}} \right], \]  \hfill (13)

\( T_{\text{i}} \) and \( T_{\text{e}} \) are the ion and electron temperature, respectively. The charge neutrality condition is given as \( n_{\text{e}0} = n_{\text{i}0} + Z_{\text{d}} \mu_{\text{i}0} = 0 \).

### 2.1. Linear theory

The linear dispersion relation \([9, 53, 54]\) of the DA wave is derived by linearising the equations (9)–(12) with respect to the dependent variables, and assuming the plane wave perturbation of the dependent variable to be of the form \( e^{i(kx - \omega t)} \).

\[ \omega^2(k) = \omega_{\text{d}}^2(k) + \frac{\gamma n_{\text{d}}}{2} k^2 + \alpha k^2, \]  \hfill (14)

where \( \alpha \) and \( k \) are the wave number frequency and wave number, respectively, and \( \omega_{\text{d}}^2(k) = \frac{\omega_{\text{d}0} k^2}{k^2 + \kappa^2}, \omega_{\text{d}}^2 = \frac{4 \pi e^2 n_{\text{d}} Z_{\text{d}}}{m_{\text{d}}}, \) \( \gamma = \frac{1}{2} \) are the ion coupling parameter and the medium parameter.

The last term of the equation (14), i.e., \( \alpha k^2 \) incorporates the essential QLCA (strong coupling) effect in the formulation since the dust layer compressibility (\( \alpha \)) can be approximated by the QLCA dynamical matrix \( D_{\text{L}} \)[50, 55], in a long wavelength limit, as \( \alpha = \lim_{k \to 0} D_{\text{L}}(k) / k^2 \)[50] where,

\[ D_{\text{L}}(k) = -\omega_{\text{d}}^2(k) + \omega_{\text{d}0}^2 k^2 \left( \frac{1}{1 + R \kappa} \times \left( \frac{1}{3} - \frac{2 \cos(Rk)}{Rk^2} + \frac{2 \sin(Rk)}{(Rk)^3} \right) - \frac{\kappa^2}{k^2} \left( \cos(Rk) + \frac{\kappa \sin(Rk)}{k} \right) \right), \]  \hfill (15)

\( R \approx 1 + \kappa / 10 \) is an excluded volume \([55]\) and \( \kappa = \frac{k}{\lambda_0} \) is the screening parameter. After substituting the parameter \( \alpha \) in eq.(14), the equation becomes,

\[ \omega^2(k) = \omega_{\text{d}}^2(k) + \frac{\gamma n_{\text{d}}}{2} k^2 + D_{\text{L}}(k). \]  \hfill (16)

The equation (14) represents the linear dispersion relation of dust acoustic wave in the strongly coupled Yukawa system.

### 2.2. Normalized basic equations

After normalizing the system of equations (9)–(12), we get the following normalized equations:

\[ \frac{\partial \tilde{n}_d}{\partial t} + \frac{\partial}{\partial x} (\tilde{n}_d \tilde{u}_d) = 0, \]  \hfill (17)

\[ \frac{\partial \tilde{u}_d}{\partial t} + \frac{\partial}{\partial x} \tilde{n}_d \frac{\partial \tilde{u}_d}{\partial x} = \mu \frac{\partial \tilde{\phi}_d}{\partial x} - \mu \gamma \sigma_{\text{d}} \tilde{n}_d^{-1} \frac{\partial \tilde{n}_d}{\partial x} - \frac{\alpha}{\tilde{n}_d} \frac{\partial \tilde{n}_d}{\partial x}, \]  \hfill (18)

\[ \frac{\partial^2 \tilde{\phi}_d}{\partial x^2} = \frac{1}{\mu} \left[ \left( \tilde{n}_d - 1 \right) + h_1 \tilde{\phi}_d + h_2 \tilde{\phi}_d^2 + h_3 \tilde{\phi}_d^3 \right]. \]  \hfill (19)

The space variable (\( x \)) and the time (\( t \)) are respectively normalized by \( a \) and \( \omega_{\text{pd}}^{-1} \).

The dust number density \( \tilde{n}_d \), the dust velocity \( \tilde{u}_d \) and dust electrostatic potential \( \tilde{\phi}_d \) are normalized by \( n_{\text{d}0}, a / \omega_{\text{pd}}^{-1} \) and \( k_B T_{\text{e}} / e \).
respectively. Other parameters involved in the calculation are given as, \( \sigma_d = \frac{Z_d}{Z_m T}, \mu = \frac{\epsilon_\gamma}{u_*^2} \) and \( \epsilon_\gamma = \sqrt{\frac{Z_a K_m T}{m_\gamma}} \).

The charge neutrality condition can be written as \( \mathcal{N}_e = \mathcal{N}_i + 1 = 0 \), where \( \mathcal{N}_e = \frac{n_{e0}}{Z_m m_e} \) and \( \mathcal{N}_i = \frac{n_{i0}}{Z_m m_i} \). The number densities (13) of both the electrons (\( \bar{n}_e \)) and ions (\( \bar{n}_i \)) can be written as

\[
\bar{n}_e = \mathcal{N}_e \exp [\sigma_e \bar{\phi}_d], \quad \bar{n}_i = \mathcal{N}_i \exp [-\bar{\phi}_d],
\]

where \( \sigma_e = \frac{Z_e}{T}, \bar{\nu}_1 = \mathcal{N}_e \sigma_{ie} + \mathcal{N}_i, \bar{\nu}_2 = \frac{1}{2}(\mathcal{N}_e \sigma_{ie}^2 - \mathcal{N}_i) \) and \( \bar{\nu}_3 = \frac{1}{6}(\mathcal{N}_e \sigma_{ie}^3 + \mathcal{N}_i) \).

In the next section 3, we have derived a spatiotemporal NLSE from the above system of nonlinear equations.

### 3. Derivation of spatiotemporal nonlinear equation

In many recent works in multi-species plasma, it has been shown that for small but finite amplitude plasma wave, the nonlinear terms give rise to the waves of higher harmonics [25]. If the amplitude varies slowly over the period of the oscillation, in certain cases the evolution of the wave amplitude can be described by a nonlinear Schrödinger equation [25, 26]. So, using the RPM [25, 27], a NLSE is derived to study the MI of DA waves in a strongly coupled Yukawa system within the QLCA framework. We consider the following stretching of space and time to separate the system into a rapidly varying part associated with the oscillation and the slowly varying amplitude [25, 56]

\[
\xi = \epsilon (x - V_g t), \quad \tau = \epsilon^2 t,
\]

and the perturbation expansions [25, 56] of the number density, velocity and electrostatic potential of the dust particles are

\[
\bar{f}_d = f^{001} + \sum_{l=1}^{\infty} \epsilon^l \sum_{a=-\infty}^{\infty} f^{a1}_a (\xi, \tau) \exp(ikx - \omega t),
\]

where \( k \) is carrier wave number, \( \omega \) is carrier wave frequency and \( \bar{f}_d = \bar{n}_d, \bar{u}_d, \bar{\phi}_d \) with \( f^{001} = [1 0 0]^T \).

Putting the perturbation expansions (22) for the field quantities \( \bar{n}_d, \bar{u}_d, \bar{\phi}_d \), into the set of Yukawa fluid equations (17)–(19), and sorting the distinct equations of distinct powers of \( \epsilon \), we get a sequence for distinct orders \( l = 1, 2, 3, \cdots \) and a sub-sequence for distinct harmonics \( a = 0, \pm 1, \pm 2, \cdots \).

#### 3.1. First order \( O(\epsilon = 1) \)

The set of zeroth harmonic (\( a = 0 \)) equations for the system of basic equations (17)–(19) are identically satisfied [56]. Solving the first harmonic (\( a = 1 \)) equations of the system of basic equations (17)–(19), we obtained the linear dispersion relation of DA waves for strongly coupled Yukawa system

\[
\omega^2 = \frac{k^2}{k^2 + \kappa^2} + \gamma \sigma_{ie} \kappa \mu + \alpha k^2,
\]

where \( \kappa^2 = \frac{h_1}{\mu} \). The essential QLCA effects are now incorporated via the compressibility parameter \( \alpha \) in the dispersion relation. The isothermal dust layer compressibility parameter \( \alpha \) can be calculated via the QLCA dynamical matrix \( D_{\perp}(k) \) which is a function of pair correlation function \( g(r) \). For a weakly coupled limit of the dusty plasma, i.e., \( R \rightarrow 0 \) then the QLCA dynamical matrix \( D_{\perp}(k) \rightarrow 0 \) as well as the isothermal dust layer compressibility \( \alpha \rightarrow 0 \), and consequently, the linear dispersion relation (23) of the strongly coupled dusty plasma reduces to the conventional DA wave linear dispersion relation [57] as follows

\[
\omega^2 = \frac{k^2}{k^2 + \kappa^2} + \gamma \sigma_{ie} \mu k^2.
\]

#### 3.2. Second order \( O(\epsilon = 2) \)

##### 3.2.1. First harmonics (\( a = 1 \))

Solving the set of first harmonic equations of the basic equation, we get the following compatibility condition which refers to the group velocity

\[
V_g = \frac{\omega^2 - W_d^2}{\omega k} = \frac{k}{\omega} \left[ \frac{h_1 \mu}{(k^2 \mu + h_1)^2} + \mu \gamma \sigma_{ie} + \alpha \right],
\]

where \( W_d^2 = \omega^2 - (\mu \gamma \sigma_{ie} + \alpha) k^2 \).

##### 3.2.2. Second harmonic (\( a = 2 \))

Solving the set of second harmonic modes which emerged from the nonlinear self-interaction of the carrier waves, we get
where \( g_1 = (\gamma - 2) \) and

\[
A_{\delta_1} = - \left[ \frac{b_2}{3k^2\mu} + \frac{k^2\omega^2\mu}{W_d^2} + (\mu\gamma\sigma_d g_1 - \alpha)k^2\mu \right],
\]
\[
A_{\alpha_1} = - [4(k^2\mu + h_1)A_{\delta_1} + h_2],
\]
\[
A_{\alpha_2} = \frac{\omega}{k} \left( A_{\alpha_3} - \frac{k^4}{W_d^2}\right).
\]

### 3.2.3. Zeroth harmonic (\( a = 0 \))

Zeroth harmonics also generates due to the nonlinear self-interaction of the carrier waves. So, solving the set of zeroth harmonic equations of the basic equations, we get

\[
(\phi_0^{(2)}, n_0^{(2)}, u_0^{(2)}) = (A_{\phi_0}, A_{n_0}, A_{u_0})|\phi_1^{(1)}|^2,
\]

where

\[
B_{\phi_0} = - \frac{1}{W_d^2}[h_1(V_g^2 - (\mu\gamma\sigma_d + \alpha)) - \mu] \times \left[ \frac{k^2}{W_d^2}(\mu\gamma\sigma_d g_1 - \alpha)k^4 + k^4\omega(2kV_g + \omega) \right.
\]
\[
+ 2h_2W_d^2(V_g^2 - (\mu\gamma\sigma_d + \alpha))]
\]
\[
B_{n_0} = - h_1B_{\phi_0} - 2h_2, \quad B_{u_0} = V_gB_{n_0} - \frac{2k^3\omega\mu}{W_d^2}.
\]

### 3.3. Third order (\( l = 3 \): first harmonic (\( a = 1 \))

Solving the set of first harmonic equations of the basic equations, we get the following NLSE

\[
i \frac{\partial \phi_1^{(1)}}{\partial \tau} + P_d \frac{\partial^2 \phi_1^{(1)}}{\partial \xi^2} + Q_d|\phi_1^{(1)}|^2\phi_1^{(1)} = 0,
\]

where

\[
P_d = \frac{W_d^2}{2k^2\omega} \left[ 1 - \frac{k^4}{W_d^2}(V_g - \frac{\omega}{k}) \left( 3V_g^2\omega^2 - 3(\mu\gamma\sigma_d + \alpha)\omega^3 \frac{k^3}{\omega} + 3(\mu\gamma\sigma_d + \alpha)V_g \right) \right],
\]
\[
Q_d = - \frac{W_d^2}{2k^2\omega} \left[ 2\frac{k^4\omega\mu}{W_d^2}(A_{\alpha_2} + B_{\alpha_2}) + k^2\mu \frac{\omega^3}{k^3} + (\mu\gamma\sigma_d g_1 - \alpha)k^2(A_{\alpha_2} + B_{\alpha_2}) \right.
\]
\[
+ (\mu\gamma\sigma_d g_1 + \alpha)k^4\mu \frac{\omega^3}{W_d^2} - 3h_3 - 2h_2(A_{\phi_0} + B_{\phi_0}) \right],
\]

where \( g_3 = (\gamma - 2) - \frac{3}{2} \). \( P_d \) and \( Q_d \) are the coefficients of the dispersive and nonlinear term in the NLSE (33), respectively. The QLCA based strong coupling effect via \( \alpha \) is appearing in the expressions of \( P_d \) and \( Q_d \), which are being explored in the next section.

### 4. Conditions for the modulational instability

In this section, we have studied the MI of DA waves in a strongly coupled (via the isothermal dust layer compressibility \( \alpha \)) dusty plasma within the QLCA framework. To find the instability conditions of the modulated DA waves from the above NLSE (33), it is necessary to consider that \( \phi_1^{(1)} = \phi_0 e^{iQ_d\phi_0^2\tau} \) is a steady state solution of the NLSE (33), where \( \phi_0 \) is a constant. To investigate the MI of the DA wave, we consider the perturbation expansion of \( \phi_1^{(1)} \) as

\[
\phi_1^{(1)} = (\phi_0 + \delta\phi) e^{iQ_d\phi_0^2\tau}, \quad \text{with } |\delta\phi| \ll |\phi_0|,
\]

and substituting the perturbation expansion (36) into the NLSE (33), and finally, linearizing the equation with respect to the perturbed quantity \( \delta\phi \), we get

\[
i \frac{\partial \delta\phi}{\partial \tau} + P_d \frac{\partial^2 \delta\phi}{\partial \xi^2} + Q_d|\phi_0|\phi_0^2(\delta\phi + \overline{\delta\phi}) = 0,
\]

where \( \overline{\delta\phi} \) is the complex conjugate of \( \delta\phi \). Substituting \( \delta\phi = U + iV \) into the above equation (37) and then separating into real and imaginary parts, we get
\[- \frac{\partial V}{\partial \tau} + P_{d} \frac{\partial^{2} U}{\partial \xi^{2}} + 2Q_{d}[U |\phi|^{2}] = 0, \tag{38}\]
\[\frac{\partial U}{\partial \tau} + P_{d} \frac{\partial^{2} V}{\partial \xi^{2}} = 0, \tag{39}\]

where $U$ and $V$ are real functions of $\xi$ and $\tau$. Finally, substituting

\[U = U_{0} \exp \{i(K\xi - \Omega\tau)\} + c.c., \tag{40}\]
\[V = V_{0} \exp \{i(K\xi - \Omega\tau)\} + c.c., \tag{41}\]

into the equations (38) and (39), we get

\[i\Omega V_{0} + (-P_{d}K^{2} + 2Q_{d}|\phi|^{2})U_{0} = 0, \tag{42}\]
\[i\Omega U_{0} + P_{d}K^{2}V_{0} = 0. \tag{43}\]

From the above coupled equations (42) and (43), we derive the following nonlinear dispersion relation \cite{56, 58, 59} of the modulated wave

\[\Omega^{2} = [P_{d}K^{2}]^{2}\left(1 - \frac{2Q_{d}|\phi|^{2}}{P_{d}K^{2}}\right), \tag{44}\]

where $\Omega$ and $K$ are the modulated wave frequency and modulated wave number respectively.

From the above nonlinear dispersion relation (44) of the modulated wave, we have derived the following instability conditions: (i) when $P_{d}Q_{d} < 0$ then $\Omega^{2} > 0$, so, the modulated DA wave is stable, (ii) when $P_{d}Q_{d} > 0$ and $K \gg K_{c}$ then $\Omega^{2} > 0$, so, the modulated DA wave is stable and (iii) when $P_{d}Q_{d} > 0$ and $K < K_{c}$ then $\Omega^{2} < 0$, so, the modulated DA wave is unstable, where $K_{c} = \sqrt{\frac{2Q_{d}|\phi|^{2}}{P_{d}K^{2}}}$.

Therefore, for $P_{d}Q_{d} > 0$ and $K < K_{c}$, the modulated DA wave is unstable and consequently the modulational growth rate of instability ($G$) in multi-species plasmas can be expressed as a function of the perturbed modulated wave number ($K$) \cite{58, 60, 59, 38, 32} is given by the following equation

\[G^{2} = (\Omega)^{2} = [P_{d}K^{2}]^{2}\left(\frac{2Q_{d}|\phi|^{2}}{P_{d}K^{2}} - 1\right), \tag{45}\]

where $i = \sqrt{(-1)}$. The growth rate of instability is an imaginary part of the perturbed modulated wave frequency ($\Omega$). In different conditions on the modulated wave number, this perturbed modulated wave frequency can be either purely real or purely imaginary function.

For fixed values of $P_{d}$ and $Q_{d}$, the growth rate of instability attains its maximum value $G_{\text{max}}$ at

\[K = \frac{k_{c}}{2} = \sqrt{\frac{Q_{d}|\phi|^{2}}{P_{d}K^{2}}}, \tag{46}\]

and consequently, the maximum modulational growth rate of instability $G_{\text{max}}$ is given by

\[G_{\text{max}} = |Q_{d}| |\phi|^{2}. \tag{46}\]

For different conditions, the envelope soliton solutions of a NLSE are possible. For $P_{d}Q_{d} > 0$ and $P_{d}Q_{d} < 0$, the soliton solution of the NLSE is known as bright and dark (black or gray) envelope soliton \cite{61–63} respectively. On the other hand, in the unstable region of the modulated DA wave, the lowest order rational solution of a NLSE is known as Akhmediev breathers \cite{64, 65} or MA solitons \cite{66} (in limiting case). And the higher order rational solution of the NLSE is known as rogue wave \cite{64}. Experimentally, Baijing et al \cite{67} observed the rogue waves in plasma physics. This Akhmediev breathers or rogue waves of waves has many applications in different fields, viz., plasma physics \cite{68}, nonlinear fibre optics \cite{69}, Bose–Einstein condensates \cite{70}, plasmonics \cite{71}, electromagnetic pulse propagation \cite{72}. Along with the existence of envelope solitons, rogue waves and breathers, there are also recently developed solutions of the NLSE. The soliton solution of a fractional order NLSE \cite{73}, one (two) soliton solution of a fifth-order NLSE \cite{74}, one (two)-soliton solutions of a coupled NLSE \cite{75}, the discrete envelope soliton solution of a time-fractional Ablowitz-Ladik model \cite{76}, envelope solitons of a three-component coupled nonlinear Schrödinger model using the neural network method \cite{77, 78}, the optical solitons of a coupled NLSE using the neural network method \cite{79}, the light bullet solitons of a (3 + 1)-D normalized nonlinear Schrödinger equation \cite{79} have been reported. Although, in our recent work, we are only restricted to study the modulational instability only.

5. Results and discussions

First, in order to compare our results with the $T_{\text{eff}}$ model, where a more indirect inclusion of strong coupling is done by letting an effective dust temperature represents the strong coupling effect, we have considered their set of parametric values (see figure 2 of [32]), and consequently, in the weakly coupled limit ($D_{c} = 0$), the linear dispersion relation is plotted in figure 1 which shows the trend $\omega \propto k$ at relatively higher $k$ value. The same trend
has also been recovered from [32] but in the strong coupling limit. It can be concluded that our weakly coupled ($D_L = 0$) linear dispersion relation shows a correspondence with the strongly coupled linear dispersion relation obtained in [32]. The linear dispersion relation for the set of parameters as given in table 1, with the QLCA effects ($D_L \neq 0$), is plotted in figure 2, which shows the signature of negative dispersion relation. While the negative dispersion is particular manifestation of the strong coupling effects, the $T_{\text{eff}}$ model does not predict this characteristic in the linear dispersion relation.

In figure 3, the dispersive coefficient $P_d$ and the nonlinear coefficient $Q_d$ are plotted against $k$ for the weakly coupled limit ($D_L = 0$) of dusty plasma for three set of parameters as given in table 1. The same qualitative nature of $P_d$ and $Q_d$ was also reported in strongly coupled limit of dusty plasma [32].

For the QLCA based strongly coupled ($D_L \neq 0$) dusty plasma, the dispersive coefficient $P_d$ and the nonlinear coefficient $Q_d$ are plotted against $k$ in figure 4. The red dashed, blue solid and green dash-dotted curves represent the coefficients $P_d$ and $Q_d$ for parameter $\kappa = 2.59$, 2.92 and 2.99, respectively, with relatively small values of $\sigma_d$.íf}

| Parameter | Set A | Set B | Set C |
|-----------|-------|-------|-------|
| $N_i$     | 1.1081| 2.9412| 1.9231|
| $N_e$     | 0.1081| 1.9412| 0.9231|
| $\sigma_i$| 0.01  | 0.02  | 0.0083|
| $\sigma_d$| $2.70 \times 10^{-4}$| $3.53 \times 10^{-4}$| $4.62 \times 10^{-4}$|
| $\kappa$  | 2.59  | 2.92  | 2.99  |
In both weak and strong coupling limit, the dispersive coefficient $P_d$ always remains negative, whereas in the weak coupling limit, the nonlinear coefficient $Q_d$ has both negative as well as positive values depending upon the value of $\kappa$. And in the strongly coupled limit it always remains positive at relatively higher value of $\kappa$. In order to explore the effect of relatively lower value of $\kappa$ on the coefficients $P_d$ and $Q_d$, for the strong coupling limit, we have plotted the $P_d$ and $Q_d$ against $k$ with different small values of $\kappa$ in figure 5. It has been identified, from figure 5, that the negative threshold values of the coefficient $Q_d$ arise at different position of wave-vector $k$ for different small values of $\kappa$. This negative threshold values are appeared at relatively higher value of the wave-vector $k$ by increasing the value of $\kappa$ up to $\kappa = 0.1825$, and beyond $\kappa = 0.1825$ the coefficient $Q_d$ again attains positive value.

Figure 3. $P_d$ and $Q_d$ are plotted against $k$ in (a) and (b) respectively for weakly coupled limit ($R \rightarrow 0$) of dusty plasma. Here, the red dashed curve correspond to parametric values $h_1 = 1.1092, h_2 = -0.5540, h_3 = 0.1847, \kappa = 2.59, \sigma_d = 0.58$, blue solid curve correspond to parametric values $h_1 = 1.9308, h_2 = -0.9615, h_3 = 0.3205, \kappa = 2.99, \sigma_d = 0.57$, and green curve dash dotted curve correspond to parametric values $h_1 = 2.9800, h_2 = -1.4702, h_3 = 0.4902, \kappa = 2.92, \sigma_d = 0.57$.

Figure 4. $P_d$ and $Q_d$ are plotted against $k$ in (a) and (b) respectively for strongly coupled limit of the dusty plasma within the QLCA framework. Here, the red dashed curve correspond to parametric values $h_1 = 1.1092, h_2 = -0.5540, h_3 = 0.1847, \kappa = 2.59, R = 1.259, \sigma_d = 0.00027$, green dash-dotted curve correspond to parametric values $h_1 = 1.9308, h_2 = -0.9615, h_3 = 0.3205, \kappa = 2.99, R = 1.299, \sigma_d = 0.000462$ and blue solid curve correspond to parametric values $h_1 = 2.9800, h_2 = -1.4702, h_3 = 0.4902, \kappa = 2.92, R = 1.292, \sigma_d = 0.000353$. 

In both weak and strong coupling limit, the dispersive coefficient $P_d$ always remains negative, whereas in the weak coupling limit, the nonlinear coefficient $Q_d$ has both negative as well as positive values depending upon the value of $\kappa$. And in the strongly coupled limit it always remains positive at relatively higher value of $\kappa$. In order to explore the effect of relatively lower value of $\kappa$ on the coefficients $P_d$ and $Q_d$, for the strong coupling limit, we have plotted the $P_d$ and $Q_d$ against $k$ with different small values of $\kappa$ in figure 5. It has been identified, from figure 5, that the negative threshold values of the coefficient $Q_d$ arise at different position of wave-vector $k$ for different small values of $\kappa$. This negative threshold values are appeared at relatively higher value of the wave-vector $k$ by increasing the value of $\kappa$ up to $\kappa = 0.1825$, and beyond $\kappa = 0.1825$ the coefficient $Q_d$ again attains positive value.
which stays positive for all values of $k$. To show this fact, the product $P_dQ_d$ against $k$ has been plotted in figure 6 with different smaller $\kappa$ values. It has been identified from figure 6 that the unstable region ($P_dQ_d > 0$), as presented with red and black curves, increases with an increment of $\kappa$ from 0.08 to 0.1. The threshold has been reached at $\kappa = 0.1825$, as represented by the blue line, beyond which the modulated wave is stable ($P_dQ_d < 0$) for all $k$ and $\kappa$.

In order to compare the stable and unstable regions in more detail, for weakly and strongly coupled limit, a plot with the contour $P_dQ_d = 0$ separating the stable and unstable regions on $k - \kappa$ plane has been presented in figure 7. In the weakly coupled limit, a relatively larger unstable region (pink) has been found, whereas in strongly coupled limit this unstable region has been reduced to a very small portion of the parameter-space. In comparison to analysis of modulational instability in a one dimensional chain [30], where it is predicted that an unstable region would exist for wide range of $\kappa$ value, the present QLCA based analysis, which incorporates isotropy of the dust structure and explicit localization of constituent particles, prescribes completely stable region (cyan color) arising beyond $\kappa = 0.183$ [see figures 7(b) and 6]. It can be seen from figure 7(b) that the modulated wave become stable above threshold value $\kappa = 0.183$ for all $k$.

The effect of dust temperature via parameter $\sigma_d$ on the modulated wave has been presented in figure 8, for both weak and strong coupling limit. For strong coupling limit, the contour plot of $P_dQ_d = 0$ in $k - \sigma_d$ space shows that the temperature enhances the unstable region (not presented in figure 7(b)). It can be seen in figure 8...
that the unstable region, represented by pink color region \((PdQd > 0)\), appears at relatively higher \(k\) and \(\sigma_d\) values. It can be concluded from these results that temperature competes with the localization (QLCA effects) and at higher temperatures the thermal effects dominate over the QLCA effects. The same trend has also been observed for weakly coupled limit of the present system.

For the strongly coupled limit, the maximum modulational growth rate of instability \((G_{\text{max}}/|\phi_0|^2)\) is plotted against \(k\) in figure 9 for different value of dust temperatures via \(\sigma_d\) within the QLCA framework. The blue, red, black and pink color curves correspond to \(\sigma_d = 0.00015, \sigma_d = 0.00025, \sigma_d = 0.00035\) and \(\sigma_d = 0.0004\), respectively. It has been shown that the region of existence of the maximum modulational growth rate of instability increases with increasing \(\sigma_d\). We can conclude that the dust temperature enhances the instability in the Yukawa system and this trend is also predicted from figure 8(b).

For strongly coupled limit, the maximum modulational growth rate of instability \((G_{\text{max}}/|\phi_0|^2)\) is plotted against \(k\) for different values of \(\kappa\) in figure 10. We have seen that there exist the critical values \(\kappa_1 = 0.1284\) and \(\kappa_2 = 0.1821\) of \(\kappa\) such that for \(\kappa < \kappa_1\) the maximum modulational growth rate of instability increases with the carrier wave number \(k\). Whereas for \(\kappa > \kappa_2\) the maximum value of the growth rate of instability increases with carrier wave number \(k\) lying within the range \(k_1 < k < k_2\). The peak value of maximum modulational growth rate of instability is increasing for \(\kappa > \kappa_2\) and is reducing for \(\kappa < \kappa_1\) this fact is also predicted from figure 7(b). Also, the maximum modulational growth rate of instability becomes zero at \(\kappa = 0.183\). It can therefore be concluded that the modulated wave will become stable after \(\kappa = 0.183\) for all values of \(k\) that is also predicted from figure 7(b).

In figure 11, we have shown the dependence of the growth rate of instability on the \(\kappa\) value. We have seen that the maximum value of the growth rate decreases with increasing \(\kappa\). This fact is also predicted from figure 10.
Before concluding the discussion, a relevance can be drawn between dusty plasma excitation treated here and, for example, with the observation in RF field trapping of the ultracold ions \[41\] where the motion of signaling ion species was found to be tunable at the edge of the stability region as a result of ions being quasi-localized and relatively stronger collective interaction becoming possible. The conclusion that collective ion interaction remains responsible for the observed delocalization in the boundary zone does indicate the role of constructively interacting collective ion excitation capable of producing a localized effect. The role of

\[\sigma_d < 0\] and \[\sigma_d > 0\]

\[P_d Q_d < 0\] and \[P_d Q_d > 0\]

\[P_d Q_d < 0\] and \[P_d Q_d > 0\]

\[h_1 = 2.9800, h_2 = -1.4702, h_3 = 0.4902, \kappa = 1.1, R = 1.11\text{ and } \gamma = 1.\]

\[\sigma_d = 0.00015, \sigma_d = 0.00025, \sigma_d = 0.00035, \sigma_d = 0.0004\]

\[h_1 = 2.9800, h_2 = -1.4702, h_3 = 0.4902, \kappa = 0.15, R = 1.015\text{ and } \gamma = 1.\]
temperature of trapped species in this case can indeed be expected to be marginal as for the Mathieu parameter $q \sim 1$ the mechanical motion of ions is entirely attributed to the collective (resonant) effect.

6. Conclusions

In this article, the QLCA based model has been adopted to study the MI of the DA waves in a strongly coupled Yukawa system consisting of negatively charged dust grains embedded in a polarizable plasma medium following the Boltzmann distribution. In order to study the modulated wave, we have derived the NLSE (33) using RPM [25, 27]. It has been seen from the linear analysis that the DA wave frequency is reduced when the strong coupling effects are incorporated via QLCA framework [32]. For the weak coupling case, it has been observed that the qualitative behaviour of the linear dispersion relation matches with the strongly coupled limit ($T_{\text{eff}}$ model) [32] in the dusty plasma. The MI of DA waves is numerically investigated for both the cases viz., for weakly and strongly coupled limits of the dusty plasma. It has been observed that in the weakly coupled limit a
relatively larger unstable region is recovered, whereas, in the strongly coupled limit this region is reduced to a very small zone of the parameter space. In comparison to analysis of the modulation instability in a one-dimensional chain [30], where existing studies have predicted an unstable region [30] for wide range of \( \kappa \) value, the present QLCA based analysis incorporating explicit and isotropic localization of constituent particles, has recovered unstable region upto a relatively smaller value of \( \kappa = 0.183 \) for a typical (small) dust temperature value \( T_d \sim 10^{-3}T \). For strong coupling limit, the contour plot of \( \kappa_dQ_d \) in \( \kappa - \sigma_d \) space shows that the larger dust temperature enhances the unstable region dimension in the parameter space. The peak value of maximum modulational growth rate of instability is reducing with \( \kappa \) for \( 0.13 \leq \kappa \leq 0.16 \) and that maximum modulational growth rate of instability become zero at \( \kappa = 0.183 \). The analysis on the instability criteria of a modulated wave, presented here, is largely applicable to quasi-crystalline state (amorphous solid) in which both free (slow, diffusive) motion as well as localization of the constituent particles coexist. The presented results are therefore expected to cover a wide range of natural systems where modulational instability is the prime mechanism for the weakly nonlinear (localized) collective effects. As a relevant example, the case of collective interaction driven delocalization of RF trapped ultracold ions is discussed which is observed at the stability boundary in a recent experiment where the background interference of the RF trapping field drops sharply, leaving the trapped ion species to be in a quasi-localized state. Within the QLCA framework, the nonlinear excitations of MI of DA waves in strongly coupled dusty plasma can be treated in the presence of a magnetic field as a future study. The investigation on existence of envelope solitary waves, analytically as well as numerically, in a strongly coupled Yukawa system within QLCA framework can be another area to be explored. Within the QLCA framework, the analytical obtained results can be numerically verified by considering a suitable numerical method [80] that one can consider this as a future work, at present this is beyond the scope of the present work.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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