Panel Cointegration Testing in the Presence of Linear Time Trends∗

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Abstract

We consider a class of panel tests covering tests for the null hypothesis of no cointegration as well as cointegration. All tests under investigation rely on single-equations estimated by ordinary least squares, and they may be residual-based or not. We focus on test statistics computed from regressions with intercept only (i.e. without detrending) while at least one of the regressors (integrated of order 1) is dominated by a linear time trend. In such a setting, often encountered in practice, the limiting distributions and critical values provided for and applied with the situation “with intercept only” are not correct. It is demonstrated that their usage results in size distortions growing with the panel size $N$. Moreover, we show which are the appropriate distributions, and how correct critical values can be obtained from the literature.

Keywords Single-equations; large $N$ asymptotics; integrated series with drift

JEL C22; C23

1 Introduction

Most panel tests for the null hypothesis of (no) cointegration rely on single-equations, notable exceptions being Larsson, Lyhagen, and Löthgren (2001), Groen and Kleibergen (2003), Breitung (2005) and Karaman Örsal and Droge (2013) who proposed panel system approaches. Recent single-equation tests by Chang and Nguyen (2012) or Demetrescu,

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Hanck, and Tarcolea (2014) rely on nonlinear instrumental variable estimation, while the vast majority of such panel tests builds on (fully modified or dynamic) ordinary least squares [OLS]. Here, we study exactly this class of OLS based single-equation panel tests for the null of either cointegration or no cointegration.

We focus on the situation where the test statistics are computed from regressions with an intercept only (i.e. without detrending) while at least one of the integrated regressors displays a linear time trend on top of the stochastic trend. Such a constellation is often met in practical applications, see for instance Coe and Helpman (1995) and Westerlund (2005b) on R&D spillovers (total factor productivity and capital stock), Larsson, Lyhagen, and Löthgren (2001) on log. real consumption and income (per capita), or Hanck (2009) on prices and exchange rate series testing the weak purchasing power parity (PPP). The relevance of a linear trend in panel data has been addressed in Hansen and King (1998) when commenting on the link between health care expenditure and GDP, see McCoskey and Selden (1998); consequently, Blomqvist and Carter (1997), Gerdtham and Löthgren (2000) or Westerlund (2007) worked (partly) with detrended series, i.e. they included time as an explanatory variable in their panel regressions. Hansen (1992, p. 103), however, argues that “It seems reasonable that excess detrending will reduce the test’s power.” At the same time detrended relationships may not be economically meaningful. Think of the analysis of income ($x$) and consumption ($y$), where one is interested in checking for cointegration and $\beta = 1$ within $y_t = \bar{\alpha} + \bar{\beta}x_t + \bar{u}_t$, which amounts to stationary saving series. Here, a detrended regression, $y_t = \tilde{\alpha} + \tilde{\delta}t + \tilde{\beta}x_t + \tilde{u}_t$, does not make economic sense. The same holds true for the empirical application with PPP in this paper, see also the discussion in Hassler (1999). There are only few situations where economic interpretation can be attached to a linear time trend as explanatory variable, e.g. as technological progress in a production function. Therefore, we study the empirically relevant case where test statistics are computed from regressions with intercept only (i.e. without detrending) while at least one of the I(1) regressors displays a linear time trend, i.e. it is I(1) with drift.

Before becoming more technical, we want to explain our findings as a rule for empirical application. Let $Z^{(m)}$ denote a panel cointegration statistic computed from a regression with intercept only involving $m = k + 1$ I(1) variables. The regression may be static in levels,

$$y_{i,t} = \bar{\alpha}_i + \bar{\beta}_{i,1}x_{i,1,t} + \cdots + \bar{\beta}_{i,k}x_{i,k,t} + \bar{u}_{i,t}, \quad t = 1, \ldots, T, \ i = 1, \ldots, N,$$

or an error-correction equation,

$$\Delta y_{i,t} = \bar{\alpha}_i + \bar{\gamma}_iy_{i,t-1} + \bar{\beta}_{i,1}x_{i,1,t-1} + \cdots + \bar{\beta}_{i,k}x_{i,k,t-1} + \bar{\epsilon}_{i,t}, \quad t = 1, \ldots, T, \ i = 1, \ldots, N,$$
where additional lags of differences may be required to render $\varepsilon_{i,t}$ free of serial correlation. The test statistic may be constructed from pooling the data or from averaging individual statistics, see e.g. Pedroni (1999, 2004) or Westerlund (2007). Much of the nonstationary panel literature relies on sequential limit theory where $T \to \infty$ is followed by $N \to \infty$, such that limiting normality can be established under the assumption that none of the I(1) regressors follows a deterministic time trend:

$$\sqrt{N} \left( \bar{Z}^{(m)} - \bar{\mu}_m \right) / \bar{\sigma}_m \sim N(0,1).$$

The constants $\bar{\mu}_m$ and $\bar{\sigma}_m$ required for appropriate normalization are typically tabulated for a selected number of values of $m$. A different set of such moments $\tilde{\mu}_m$ and $\tilde{\sigma}_m$ is typically given for detrended regressions, too, where the test statistic $\tilde{Z}^{(m)}$ stems from regressions of the type $(m = k + 1)$

$$y_{i,t} = \tilde{\alpha}_i + \tilde{\delta}_t + \tilde{\beta}_{i,1} x_{i,1,t} + \cdots + \tilde{\beta}_{i,k} x_{i,k,t} + \tilde{u}_{i,t},$$

or

$$\Delta y_{i,t} = \tilde{\alpha}_i + \tilde{\delta} t + \tilde{\gamma}_i y_{i,t-1} + \tilde{\beta}_{i,1} x_{i,1,t-1} + \cdots + \tilde{\beta}_{i,k} x_{i,k,t-1} + \tilde{\varepsilon}_{i,t}.$$

Here, it holds irrespective of an eventual linear trend in the data that

$$\sqrt{N} \left( \tilde{Z}^{(m)} - \tilde{\mu}_m \right) / \tilde{\sigma}_m \sim N(0,1).$$

Our main contribution is twofold. First, it is shown that the normalization with $\bar{\mu}_m$ and $\bar{\sigma}_m$ and the resulting critical values for $\bar{Z}^{(m)}$ from the regression “with intercept only” are not correct in the presence of linear time trends in the data. It is analytically and numerically demonstrated that their usage results in size distortions growing with the panel size $N$. Second, we characterize the appropriate limiting distributions by showing that normalization of $\tilde{Z}^{(m)}$ with $\tilde{\mu}_{m-1}$ and $\tilde{\sigma}_{m-1}$ results in a standard normal limit, such that the size of the tests can be controlled. The general rule hence is (in non-technical terms): The limiting distribution arising from a regression on $k = m - 1$ I(1) variables with drift and on an intercept only amounts to the limiting distribution in the case of a regression on $k - 1 = m - 2$ I(1) variables and an intercept plus a linear time trend. Such a rule is known in a pure time series context for the special case of the residual-based Phillips-Ouliaris test for no cointegration since Hansen (1992, p. 103): “[...] deterministic trends in the data affect the limiting distribution of the test statistics whether or not we detrend the data”; see also the exposition in Hamilton (1994, p. 596, 597). It is more relevant in our panel framework since we illustrate numerically and analytically that the size distortions of an inappropriate normalization grow with the panel size $N$ (to zero or one, depending on the specific test).
The rest of the paper is organized as follows. The next section fixes some notation and assumptions. Section 3 establishes our asymptotic result and discusses its consequences for applied work. Section 4 considers the combination of p-values as an alternative approach to panel testing. Section 5 illustrates our asymptotic findings by means of Monte Carlo evidence. Section 6 contains the empirical application, and the last section summarizes. Mathematical proofs are relegated to the Appendix.

2 Notation and assumptions

Restricting our attention to the single-equation framework we partition the m-vector $z_{i,t}$ of observables into a scalar $y_{i,t}$ and a k-element vector $x'_{i,t}$, $z_{i,t} = (y_{i,t}, x'_{i,t})$, $m = k + 1$. As usual, the index $i$ stands for the cross-section, $i = 1, \ldots, N$, while $t$ denotes time, $t = 1, \ldots, T$. Each sequence $\{z_{i,t}\}$, $t = 1, \ldots, T$, is assumed to be integrated of order 1, I(1), where we allow for a non-zero drift, and assume for simplicity a negligible starting value, $z_{i,0} = 0$. While $\{z_{i,t}\}$ may be cointegrated or not, depending on the respective null hypothesis, we rule out cointegration among $\{x_{i,t}\}$. Technically, these assumptions translate as follows, where $W_{i,m}(\cdot)$ denotes an $m$-dimensional standard Wiener process, $\lfloor x \rfloor$ stands for the integer part of a number $x$, and $\Rightarrow$ is the symbol for weak convergence.

**Assumption 1** With obvious partitioning according to $(y_{i,t}, x'_{i,t})'$ we assume $(i = 1, \ldots, N)$

$$z_{i,t} = \mu_{i,t} + \sum_{j=1}^{t} e_{i,j} = \left( \begin{array}{c} \mu_{i,y} \\ \mu_{i,x} \end{array} \right) t + \sum_{j=1}^{t} \left( \begin{array}{c} e_{i,y,j} \\ e_{i,x,j} \end{array} \right), \quad t = 1, \ldots, T.$$  

The stochastic zero mean process $\{e_{i,t}\}$ is integrated of order 0 in that it satisfies

$$T^{-0.5} \sum_{t=1}^{rT} e_{i,t} \Rightarrow \Omega^{0.5}_{i} W_{i,m}(r) = \Omega^{0.5}_{i} \left( \begin{array}{c} W_{i,y}(r) \\ W_{i,x}(r) \end{array} \right), \quad r \in [0, 1],$$

with

$$\Omega_{i} = \left( \begin{array}{cc} \omega_{i,yy}^{2} & \omega_{i,xy}' \\ \omega_{i,xy} & \Omega_{i,xx} \end{array} \right)$$

where $\omega_{i,yy} > 0$ and $\Omega_{i,xx}$ is positive definite.

Let $\overline{S}^{(m)}_{t}$ and $\tilde{S}^{(m)}_{t}$ stand for test statistics computed from individual single-equation OLS regressions with “intercept only” and “intercept plus linear trend”, respectively. The superscript $(m) = (k + 1)$ stands for the dimension of the I(1) variables entering the equations.
One route to panel testing relies on so-called group statistics averaging individual statistics. We denote them as follows:

\[ \bar{G}^{(m)} = \frac{1}{N} \sum_{i=1}^{N} \bar{S}^{(m)}_i \quad \text{or} \quad \tilde{G}^{(m)} = \frac{1}{N} \sum_{i=1}^{N} \tilde{S}^{(m)}_i \]

Similarly, panel statistics rely on pooling the data across the within dimension, i.e. summing over terms showing up in the numerator and denominator separately,

\[ \bar{P}^{(m)} = g \left( \sum_{i=1}^{N} \bar{N}^{(m)}_{i,T}, \sum_{i=1}^{N} \bar{D}^{(m)}_{i,T} \right) \quad \text{or} \quad \tilde{P}^{(m)} = \tilde{g} \left( \sum_{i=1}^{N} \tilde{N}^{(m)}_{i,T}, \sum_{i=1}^{N} \tilde{D}^{(m)}_{i,T} \right) \]

Here it is assumed that \( \bar{N}^{(m)}_{i,T} \) and \( \bar{D}^{(m)}_{i,T} \) or \( \tilde{N}^{(m)}_{i,T} \) and \( \tilde{D}^{(m)}_{i,T} \) are computed from individually demeaned or detrended regressions, respectively. A typical example for the function \( g \) is \( g(x,y) = x/\sqrt{y} \) in the case of t-type statistics. We allow for group and panel statistics by introducing the generic notation \( \bar{Z}^{(m)} \) and \( \tilde{Z}^{(m)} \), and maintain for the panel the joint null hypothesis

\[ H_0 = \bigcap_{i=1}^{N} H_{i,0} \quad (1) \]

where a distinction between the individual null hypotheses \( H_{i,0} \) of cointegration or absence of cointegration is not required.

**Assumption 2** Consider linear single-equation OLS regressions (\( i = 1, \ldots, N, \ t = 1, \ldots, T \))

\[ y_{i,t} = \bar{\alpha}_i + \bar{\beta}_i x_{i,t} + \bar{\epsilon}_{i,t} \quad \text{and} \quad \tilde{y}_{i,t} = \tilde{\alpha}_i + \tilde{\beta}_i x_{i,t} + \tilde{\epsilon}_{i,t} \quad (2) \]

or

\[ \Delta y_{i,t} = \bar{\alpha}_i + \bar{\gamma}_i y_{i,t-1} + \bar{\beta}_i x_{i,t-1} + \bar{\epsilon}_{i,t} \quad \text{and} \quad \Delta y_{i,t} = \tilde{\alpha}_i + \tilde{\beta}_i x_{i,t-1} + \tilde{\epsilon}_{i,t} \quad (3) \]

where lags of \( \Delta z_{i,t-j} \) may be required as additional regressors in (12) to ensure residuals free of serial correlation. Let \( \bar{Z}^{(m)} \) and \( \tilde{Z}^{(m)} \) stand for group statistics or for panel statistics computed from regressions with “intercept only” and “intercept plus linear trend”, respectively. We then assume under the null hypothesis (1) that it holds

\[ \sqrt{N} \left( \bar{Z}^{(m)} - \bar{\mu}_m \right) \Rightarrow \mathcal{N}(0, \bar{\sigma}^2_m) \quad \text{if} \ \mu_{i,x} = 0, \]

\[ \sqrt{N} \left( \tilde{Z}^{(m)} - \tilde{\mu}_m \right) \Rightarrow \mathcal{N}(0, \tilde{\sigma}^2_m) \quad \text{for all} \ \mu_{i,x}, \]

as \( T \to \infty \) followed by \( N \to \infty \).
Tests by e.g. by Kao (1999), Pedroni (1999, 2004), Westerlund (2005a,b) or Westerlund (2007) meet Assumption 2 under different sets of restrictions, and they will be considered in the next section, see Remarks 1 through 3. In particular, these authors tabulate values of \((\bar{\mu}_m, \bar{\sigma}_m)\) and \((\tilde{\mu}_m, \tilde{\sigma}_m)\) for the different tests, \(m \geq 2\).

3 Results

The first paper allowing for linear time trends in a panel cointegration context was by Kao (1999). He considered a residual-based unit root test for the null hypothesis of no cointegration in the tradition of Phillips and Ouliaris (1990), see Remark 1 below. His test builds on pooling the data while allowing for a individual-specific intercept, which amounts to a least-squares dummy variable estimation. Kao (1999) did not consider regressions containing a linear time trend, but he does allow for a linear drift in the data when performing a regression with a fixed effect intercept. In the case of \(k = 1\) regressor (i.e. \(m = 2\), Kao (1999, eq. (15))) observed that the linear time trend dominates the I(1) component; hence, the limiting distribution amounts to that by Levin, Lin, and Chu (2002) upon detrending. To become precise: Let \(\bar{\mu}_1\) and \(\bar{\sigma}_1\) denote the normalizing constants provided by Levin et al. (2002) for detrended panel unit root tests; then one should use them for pooled residual-based panel cointegration testing in a bivariate regression if the regressor is I(1) with drift, see Kao (1999, Theo. 4):

\[
\sqrt{N} \left( \bar{P}^{(m)} - \bar{\mu}_1 \right) \Rightarrow N(0, \bar{\sigma}_1^2) \quad \text{for} \quad m = 2 \quad \text{under} \quad \mu_{i,x} \neq 0. 
\]

The claim that (4) continues to hold in the case of \(m > 2\) (see Kao (1999, Remark 12)), however, is not correct. Instead, we can prove Theorem 1 for panel or group statistics computed from regressions with intercept only in the presence of linear time trends.

**Theorem 1** Let Assumption 1 hold true for \(m \geq 2\) with \(\mu_{i,x} \neq 0\). Under Assumption 2 it holds under the null hypothesis that

\[
\sqrt{N} \left( \bar{Z}^{(m)} - \bar{\mu}_{m-1} \right) \Rightarrow N(0, \bar{\sigma}_{m-1}^2) 
\]

as \(N \to \infty\).

**Proof** See Appendix.

Related to Theorem 1 two research strategies are applied in practice when dealing with statistics resulting from regressions with intercept only. The first one simply ignores the possibility of potential linear time trends in the data and standardizes \(\bar{Z}^{(m)}\) with \(\bar{\mu}_m\) and
σ_m. A second strategy always accounts for an eventual drift according to Theorem 1, no matter whether the data display a linear time trend or not; in other words it always applies \( \tilde{Z}^{(m)} \) upon standardizing with \( \tilde{\mu}_{m-1} \) and \( \tilde{\sigma}_{m-1} \). We summarize as follows.

**Strategy S_I:** When \( \tilde{G}^{(m)} \) or \( \tilde{P}^{(m)} \) are computed from panel regressions without detrending, use the critical values relying on a standardization with \( \tilde{\mu}_m \) and \( \tilde{\sigma}_m \).

**Strategy S_A:** When \( \tilde{G}^{(m)} \) or \( \tilde{P}^{(m)} \) are computed from panel regressions without detrending, use the critical values relying on a standardization with \( \tilde{\mu}_{m-1} \) and \( \tilde{\sigma}_{m-1} \).

The situation analyzed in Theorem 1 has not been considered in the previous panel cointegration literature, with the notable exception of Kao (1999). Consequently, all applied papers we are aware of standardize \( \tilde{Z}^{(m)} \) with \( \tilde{\mu}_m \) and \( \tilde{\sigma}_m \) ignoring the effect of eventual trends in the series, which amounts to strategy S_I. The effect of both strategies, S_I and S_A, is discussed for growing \( N \) in the following proposition. In the absence of a linear trend, S_I performs without size distortion, while S_A provides correct inference under linear trends. Here we study the asymptotic consequences of inappropriate use of these strategies.

**Proposition 1** Let Assumptions 1 and 2 hold true for \( m \geq 2 \). Further assume

\[ \tilde{\mu}_{m-1} < \tilde{\mu}_m. \quad (5) \]

Under the null hypothesis one has the following.

a) For a test rejecting for too negative values, the probability to reject ...

\[ \text{increases with growing } N \text{ to } 1 \text{ under } S_I \text{ if } \mu_{i,x} \neq 0 \]
\[ \text{decreases with growing } N \text{ to } 0 \text{ under } S_A \text{ if } \mu_{i,x} = 0 \]

b) for a test rejecting for too large values, the probability to reject ...

\[ \text{decreases with growing } N \text{ to } 0 \text{ under } S_I \text{ if } \mu_{i,x} \neq 0 \]
\[ \text{increases with growing } N \text{ to } 1 \text{ under } S_A \text{ if } \mu_{i,x} = 0 \]

**Proof** See Appendix.

We now discuss a couple of panel tests satisfying Assumption 2 and (5), such that Theorem 1 and Proposition 1 apply.

**Remark 1** The residual-based unit root tests for the null hypothesis of no cointegration proposed by Pedroni (1999, 2004) build on static regressions. In particular, the suggested
group statistics are constructed from individual regression residuals:
\[ y_{i,t} = \bar{\alpha}_i + \bar{\beta}_i' x_{i,t} + \bar{u}_{i,t} \quad \text{or} \quad y_{i,t} = \tilde{\alpha}_i + \tilde{\delta}_i t + \tilde{\beta}_i' x_{i,t} + \tilde{u}_{i,t}. \]  

The null hypothesis (1) is rejected for too negative values. Expected values and standard deviations showing up in Assumption 2 are available from Pedroni (1999, Table 2) for \( m > 2 \) and from Pedroni (2004, Corollary 1) for \( m = 2 \). In order to apply Theorem 1 for \( m = 2 \), one requires \( \tilde{\mu}_1 \) and \( \tilde{\sigma}_1 \). These values stem from the detrended Dickey-Fuller distribution in the case of group statistics and have been tabulated by Nabey (1999, Table 4): \( \bar{\mu}_1 = -2.18136 \) and \( \bar{\sigma}_1 = 0.74991 \). Throughout we observe \( \bar{\mu}_{m-1} < \bar{\mu}_m < 0 \). Hence, Proposition 1 a) applies. If strategy \( S_I \) is employed under linear trends, then the probability to reject the true null hypothesis converges to one with growing panel size \( N \); if strategy \( S_A \) is employed in the absence of linear trends, then the probability to reject the null converges to zero with \( N \). Alternatively, Westerlund (2005a) suggested a group and a panel variance ratio type test along the lines of Breitung (2002). The null hypothesis of no cointegration is rejected again for too small values. To apply Theorem 1 with \( m = 2 \), we need \( \bar{\mu}_1 \) and \( \bar{\sigma}_1 \). For the detrended Breitung distribution we obtain by simulation \( \tilde{\mu}_1 = 0.0110 \) and \( \tilde{\sigma}_1 = 0.005197 \), which are value corresponding to the case of group statistics. Here, \( 0 < \tilde{\mu}_{m-1} < \tilde{\mu}_m \), so that (5) holds, see Westerlund (2005a, Table 1). Consequently, Proposition 1 a) applies, and the probability to reject the true null hypothesis under strategy \( S_I \) grows with \( N \) as long as there is a linear trend in the data. The other way round, strategy \( S_A \) results in increasingly conservative tests in the absence of linear trend, which will of course be accompanied by a loss in power.

**Remark 2** The error-correction test by Westerlund (2007) relies on regressions of the type
\[ \Delta y_{i,t+1} = \bar{\alpha}_i + \bar{\gamma}_i y_{i,t} + \bar{\beta}_i' x_{i,t} + \bar{\epsilon}_{i,t+1} \quad \text{or} \quad \Delta y_{i,t+1} = \tilde{\alpha}_i + \tilde{\delta}_i t + \tilde{\gamma}_i y_{i,t} + \tilde{\beta}_i' x_{i,t} + \tilde{\epsilon}_{i,t+1}, \]  

where lags of \( \Delta z_{i,t-j} \) may be required as additional regressors to ensure errors free of serial correlation. The null hypothesis of no cointegration is rejected for too negative t-values associated with \( \gamma \). In case of \( m = 1 \) (i.e. no \( x_{i,t} \) on the right-hand side), the limiting distributions are of the usual Dickey-Fuller type. Hence, \( \bar{\mu}_1 \) and \( \bar{\sigma}_1 \) for group statistics are again from detrended Dickey-Fuller-type distributions and given in Nabey (1999, Table 4), see above. From Westerlund (2007, Table 1) we hence have \( \bar{\mu}_{m-1} < \bar{\mu}_m < 0 \) meeting (5) again. Consequently, strategy \( S_I \) is increasingly liberal and the probability to reject the true null hypothesis approaches 1 in the limit as long as the series display a linear time trend.
Table 1: Approximate effective size of the group t-test by Pedroni (1999, 2004) at nominal level $\alpha$ under $S_I$ for $\mu_{i,x} \neq 0$

| $N$ | 10 | 20 | 30 | 40 | 50 |
|-----|----|----|----|----|----|
| $\alpha = 0.01$ | 0.030 | 0.053 | 0.079 | 0.107 | 0.137 |
| $k = 1$ | $\alpha = 0.05$ | 0.126 | 0.190 | 0.249 | 0.307 | 0.361 |
| | $\alpha = 0.10$ | 0.227 | 0.314 | 0.389 | 0.455 | 0.515 |
| $\alpha = 0.01$ | 0.017 | 0.024 | 0.030 | 0.036 | 0.043 |
| $k = 2$ | $\alpha = 0.05$ | 0.080 | 0.102 | 0.122 | 0.141 | 0.159 |
| | $\alpha = 0.10$ | 0.154 | 0.188 | 0.217 | 0.243 | 0.268 |
| $\alpha = 0.01$ | 0.014 | 0.017 | 0.020 | 0.022 | 0.025 |
| $k = 3$ | $\alpha = 0.05$ | 0.067 | 0.078 | 0.087 | 0.096 | 0.104 |
| | $\alpha = 0.10$ | 0.130 | 0.148 | 0.162 | 0.175 | 0.187 |

Remark 3 Westerlund (2005b) allowed to test the null hypothesis of cointegration. He proposed a CUSUM group test statistic for this null hypothesis. To apply Theorem 1 for $m = 1$, we provide as moments of the univariate, detrended distribution by simulation: $\tilde{\mu}_1 = 0.6367$ and $\tilde{\sigma}_1 = 0.14595$. The null hypothesis is rejected for too large values, and according to Westerlund (2005b, Table 1) $0 < \tilde{\mu}_{m-1} < \tilde{\mu}_m$. So, this time Proposition 1 b) comes in. Under strategy $S_I$ in the presence of linear trends, the test is increasingly undersized with growing $N$. This comes at the price of low power in the presence of linear time trends. Strategy $S_A$ without linear trends has the opposite, too liberal effect.

The statements obtained from Proposition 1 may be quantified more precisely by means of equations (19) through (22) given in the Appendix. The rejection probabilities apply approximately under the null hypothesis at nominal significance level $\alpha$. We report results for the group t-tests by Pedroni (1999, 2004) and by Westerlund (2007) in Tables 1 through 4, and for the group CUSUM test by Westerlund (2005a) in Table 5.

Generally, the size distortions in Tables 1 through 4 grow with $N$, while decreasing with $m = k + 1$ at the same time. The fact that $S_I$ is too liberal and $S_A$ too conservative is characteristic for these tests where we reject for too negative values. But it does not hold in general, as we see when reversing the null and alternative hypotheses. To quantify distortions for the CUSUM test we use equations (20) and (22) from the Appendix. When evaluating $S_I$ under $\mu_{i,x} \neq 0$, we observe rejection probabilities equal to zero up to three digits. For that reason we only report a Table 5 for strategy $S_A$ in the absence of linear time trends, where this strategy is very liberal under the null.

1 The univariate distribution is the supremum over the absolute value of a so-called second-level Brownian bridge, which shows up with the detrended KPSS test, too; see Kwiatkowski, Phillips, Schmidt, and Shin (1992).
Table 2: Approximate effective size of the group t-test by Pedroni (1999, 2004) at nominal level $\alpha$ under $S_A$ for $\mu_{i,x} = 0$

| $N$ = | 10 | 20 | 30 | 40 | 50 |
|------|----|----|----|----|----|
| $k = 1$ | $\alpha = 0.01$ | 0.003 | 0.001 | 0.001 | 0.000 | 0.000 |
|      | $\alpha = 0.05$ | 0.018 | 0.009 | 0.006 | 0.004 | 0.002 |
|      | $\alpha = 0.10$ | 0.038 | 0.022 | 0.014 | 0.009 | 0.006 |
| $k = 2$ | $\alpha = 0.01$ | 0.006 | 0.004 | 0.003 | 0.002 | 0.002 |
|      | $\alpha = 0.05$ | 0.030 | 0.023 | 0.018 | 0.014 | 0.012 |
|      | $\alpha = 0.10$ | 0.063 | 0.049 | 0.040 | 0.033 | 0.028 |
| $k = 3$ | $\alpha = 0.01$ | 0.007 | 0.006 | 0.005 | 0.004 | 0.004 |
|      | $\alpha = 0.05$ | 0.037 | 0.031 | 0.027 | 0.024 | 0.022 |
|      | $\alpha = 0.10$ | 0.076 | 0.065 | 0.058 | 0.053 | 0.048 |

Table 3: Approximate effective size of the group t-test by Westerlund (2007) at nominal level $\alpha$ under $S_I$ for $\mu_{i,x} \neq 0$

| $N$ = | 10 | 20 | 30 | 40 | 50 |
|------|----|----|----|----|----|
| $k = 1$ | $\alpha = 0.01$ | 0.139 | 0.352 | 0.564 | 0.732 | 0.846 |
|      | $\alpha = 0.05$ | 0.394 | 0.669 | 0.836 | 0.924 | 0.967 |
|      | $\alpha = 0.10$ | 0.566 | 0.808 | 0.921 | 0.969 | 0.988 |
| $k = 2$ | $\alpha = 0.01$ | 0.089 | 0.208 | 0.344 | 0.478 | 0.598 |
|      | $\alpha = 0.05$ | 0.283 | 0.484 | 0.644 | 0.763 | 0.846 |
|      | $\alpha = 0.10$ | 0.436 | 0.645 | 0.783 | 0.870 | 0.924 |
| $k = 3$ | $\alpha = 0.01$ | 0.067 | 0.150 | 0.247 | 0.349 | 0.450 |
|      | $\alpha = 0.05$ | 0.232 | 0.392 | 0.531 | 0.647 | 0.738 |
|      | $\alpha = 0.10$ | 0.372 | 0.553 | 0.687 | 0.783 | 0.852 |

Table 4: Approximate effective size of the group t-test by Westerlund (2007) at nominal level $\alpha$ under $S_A$ for $\mu_{i,x} = 0$

| $N$ = | 10 | 20 | 30 | 40 | 50 |
|------|----|----|----|----|----|
| $k = 1$ | $\alpha = 0.01$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
|      | $\alpha = 0.05$ | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 |
|      | $\alpha = 0.10$ | 0.006 | 0.001 | 0.000 | 0.000 | 0.000 |
| $k = 2$ | $\alpha = 0.01$ | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
|      | $\alpha = 0.05$ | 0.005 | 0.001 | 0.000 | 0.000 | 0.000 |
|      | $\alpha = 0.10$ | 0.012 | 0.003 | 0.001 | 0.000 | 0.000 |
| $k = 3$ | $\alpha = 0.01$ | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
|      | $\alpha = 0.05$ | 0.007 | 0.002 | 0.001 | 0.000 | 0.000 |
|      | $\alpha = 0.10$ | 0.016 | 0.005 | 0.002 | 0.001 | 0.000 |
Table 5: Approximate effective size of the group CUSUM test by Westerlund (2005b) at nominal level $\alpha$ under $S_A$ for $\mu_{i,x} = 0$

| $N$ | 10  | 20  | 30  | 40  | 50  |
|-----|-----|-----|-----|-----|-----|
| $\alpha = 0.01$ | 0.539 | 0.779 | 0.900 | 0.957 | 0.982 |
| $k = 1$ | $\alpha = 0.05$ | 0.706 | 0.887 | 0.958 | 0.985 | 0.994 |
|       | $\alpha = 0.10$ | 0.782 | 0.926 | 0.975 | 0.992 | 0.997 |
| $\alpha = 0.01$ | 0.456 | 0.697 | 0.841 | 0.920 | 0.961 |
| $k = 2$ | $\alpha = 0.05$ | 0.643 | 0.839 | 0.930 | 0.970 | 0.987 |
|       | $\alpha = 0.10$ | 0.732 | 0.893 | 0.958 | 0.983 | 0.994 |
| $\alpha = 0.01$ | 0.308 | 0.502 | 0.654 | 0.765 | 0.844 |
| $k = 3$ | $\alpha = 0.05$ | 0.502 | 0.695 | 0.816 | 0.891 | 0.936 |
|       | $\alpha = 0.10$ | 0.608 | 0.783 | 0.879 | 0.933 | 0.963 |

Much of the earlier panel (co)integration literature assumed independent units invoking a central limit theorem to establish Assumption 2. Independence, however, is not maintained in our Assumption 2. Westerlund (2005a, 2007) e.g. replaced $x_{i,t}$ and $y_{i,t}$ by the sectorally demeaned time series,$

$$
\bar{x}_t = \frac{1}{N} \sum_{i=1}^{N} x_{i,t}, \quad \bar{y}_t = \frac{1}{N} \sum_{i=1}^{N} y_{i,t},
$$

and established the limiting results from Assumption 2 when allowing for cross-correlation driven by a common factor. Consequently, Theorem 1 continues to hold.

Strategy $S_I$ is clearly predominant in the literature and applied with the tests mentioned in the remarks above. The performance of strategies $S_I$ or $S_A$ under the null hypothesis depends of course on the presence or absence of a linear time trend, and on the specific test. In view of Proposition 1, we find neither strategy $S_I$ nor strategy $S_A$ acceptable for group or panel statistics.

## 4 Combination of $p$-values

The idea to combine $p$-values from individual units to obtain panel significance on unit roots was put forward by Maddala and Wu (1999) and Choi (2001), see also Hanck (2009) for cointegration testing. With $p$-values $p_{i}, i = 1, \ldots, N$, Maddala and Wu (1999) proposed Fisher’s classical test statistic,

$$
F = -2 \sum_{i=1}^{N} \ln(p_{i}),
$$
which follows a $\chi^2(2N)$ distribution under the null hypothesis and for independent units. Under the same assumptions the so-called inverse normal method discussed by Choi (2001) relies on comparing

\[ INM = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Phi^{-1}(p_i) \]

with a standard normal distribution, where $\Phi^{-1}$ denotes the inverse of the standard normal distribution function. Clearly, this is a left-sided test rejecting the null hypothesis for too small values of $INM$. Hartung (1999) suggested to robustify the inverse normal method against certain forms of cross-correlation, see also the discussion in Demetrescu, Hassler, and Tarcolea (2006).

We now maintain the following assumption, where for notational simplicity the index $i$ is suppressed.

**Assumption 3** Consider linear single-equation OLS regressions $(t = 1, \ldots, T)$

\[ y_t = \alpha + \beta' x_t + u_t \quad \text{and} \quad y_t = \tilde{\alpha} + \tilde{\beta} t + \tilde{\beta}' x_t + \tilde{u}_t , \]

or

\[ \Delta y_t = \alpha + \gamma y_{t-1} + \beta' x_{t-1} + \tilde{\varepsilon} \quad \text{and} \quad \Delta y_t = \tilde{\alpha} + \tilde{\delta} t + \tilde{\gamma} y_{t-1} + \tilde{\beta}' x_{t-1} + \tilde{\varepsilon}_t , \]

where lags of $\Delta z_{i,t-j}$ may be required as additional regressors in the error-correction regressions to ensure residuals free of serial correlation. Let $\tilde{S}^{(m)}$ and $\tilde{S}^{(m)}$ stand for test statistics computed from regressions with “intercept only” and “intercept plus linear trend”, respectively. Let under $H_0$ hold as $T \to \infty$

\[ \tilde{S}^{(m)} \Rightarrow f \left( \tilde{Z}_m \right) =: \tilde{L}^{(m)} \quad \text{if } \mu_x = 0 , \quad (8) \]

\[ \tilde{S}^{(m)} \Rightarrow f \left( \tilde{Z}_m \right) =: \tilde{L}^{(m)} \quad \text{for all } \mu_x , \quad (9) \]

and

\[ \tilde{S}^{(m)} \Rightarrow \tilde{L}^{(m-1)} \quad \text{if } \mu_x \neq 0 , \quad (10) \]

where $f$ is a functional that maps $(r \in [0, 1])$

\[ \tilde{Z}'_m(r) = (W'_m(r), 1), \]

\[ \tilde{Z}'_m(r) = (W'_m(r), 1, r) , \]

such that these limiting distributions are free of nuisance parameters.

There are plenty of tests satisfying (8) and (9). We briefly mention three tests for which
(10) has been established as well, so that they meet Assumption 3. Note that (10) is the
time series analogue to our panel result from Theorem 1.

**PHILLIPS-OLIARI-S-HANSEN TEST** Building on the static regressions

\[ y_t = \bar{\alpha} + \bar{\beta}' x_t + \bar{u}_t \quad \text{or} \quad y_t = \tilde{\alpha} + \tilde{\delta} t + \tilde{\beta}' x_t + \tilde{u}_t, \]  

(11)

Phillips and Ouliaris (1990) suggested residual-based unit root test statistics for the null hypothesis of no cointegration. Under some additional assumptions the corresponding statistics satisfy Assumption (8) and (9). Hansen (1992, Theo. 7) proved (10) for the so-called \( Z \) statistics introduced by Phillips (1987) and Phillips and Perron (1988). For \( m = 2 \), the limit \( \tilde{L}^{(1)} \) is simply the detrended Dickey-Fuller distribution. Critical values for these tests are most often taken from MacKinnon (1991) or MacKinnon (1996), who also provided \( p \)-values.

**KPSS TEST** Under the null hypothesis of cointegration residual-based tests relying on the static regression (11) have been proposed. They rely on the KPSS statistic, see Kwiatkowski, Phillips, Schmidt, and Shin (1992). To circumvent the problem of endogeneity, Shin (1994) considered so-called efficient variants of least squares estimators, see also Harris and Inder (1994), and established (8) and (9). Independence of \( u_t \) and \( \Delta x_s \) is sufficient to render OLS efficient, too; hence, in such an environment (8) and (9) holds for OLS residuals, see also Leybourne and McCabe (1994). Hassler (2001) proved (10). Again, \( \tilde{L}^{(1)} \) is simply the detrended univariate distribution characterized in Kwiatkowski et al. (1992). To the best of our knowledge, however, \( p \)-values are not available for the KPSS cointegration test.

**ERROR-CORRECTION TEST** A test for the absence of an error correction mechanism has been proposed by Banerjee, Dolado, and Mestre (1998) suggesting the \( t \)-statistic for \( \gamma = 0 \) in

\[ \Delta y_{t+1} = \bar{\alpha} + \bar{\gamma} y_t + \bar{\beta}' x_t + \text{diff.s} + \bar{\varepsilon}_{t+1} \quad \text{or} \quad \Delta y_{t+1} = \tilde{\alpha} + \tilde{\delta} t + \tilde{\gamma} y_t + \tilde{\beta}' x_t + \text{diff.s} + \tilde{\varepsilon}_{t+1}. \]  

(12)

Under appropriate exogeneity restrictions (8) and (9) are verified. Critical values were given in Banerjee et al. (1998), and in greater detail in Ericsson and MacKinnon (2002), who also provide the means to compute \( p \)-values. Hassler (2000) established (10) for this error correction test. For \( m = 2 \), the limit \( \tilde{L}^{(1)} \) is again the detrended Dickey-Fuller distribution.

Strategies \( S_I \) and \( S_A \) become now:

**Strategy** \( S_I \): When \( \tilde{S}^{(m)} \) is computed from a regression without detrending, then ignore the possibility of a linear trend in the data and use the critical values from \( L^{(m)} \) provided for the case “intercept only”.

13
Strategy \( S_A \): When \( \bar{S}^{(m)} \) is computed from a regression without detrending, then account for the possibility of a linear trend in the data and use the critical values from \( \tilde{L}^{(m-1)} \) following (10).

Strategy \( S_I \) is predominant in the literature. An exception is the residual-based Phillips-Ouliaris test, because strategy \( S_A \) has been advocated by Hansen (1992) for this test. In fact, strategy \( S_A \) has been adopted for the Phillips-Ouliaris test in influential textbooks such as Stock and Watson (2003, Table 14.2). The reason for this is: \( S_A \) has correct size if \( \mu_x = 0 \), while it is only mildly conservative under the null hypothesis for \( \mu_x = 0 \). The latter statement is true because the tails of the distributions \( \tilde{L}^{(m-1)} \) and \( \bar{L}^{(m)} \) happen to be very close for residual-based Phillips-Ouliaris tests, with critical values from \( \tilde{L}^{(m-1)} \) being a bit more negative than those from \( \bar{L}^{(m)} \), see (13) below; the difference decreases with growing \( m \).

We now consider the strategies \( S_I \) and \( S_A \) with and without linear trends, respectively, which parallels Proposition 1. Let \( \bar{F}^{(m)} \) and \( \tilde{F}^{(m-1)} \) denote the distribution functions of \( \bar{L}^{(m)} \) and \( \tilde{L}^{(m-1)} \). We assume the following first-order stochastic dominance:

\[
\bar{F}^{(m)}(x) < \tilde{F}^{(m-1)}(x), \quad x \in \mathbb{R}.
\]  

(13)

**Proposition 2** Let Assumptions 1 and 3 hold true for \( m \geq 2 \). Further assume (13). Under the null hypothesis (1) one has the following.

a) If \( \text{INM} \) combines values from a test rejecting for too negative values, the probability that \( \text{INM} \) rejects ...

increases with growing \( N \) to 1 under \( S_I \) if \( \mu_{i,x} \neq 0 \)

... decreases with growing \( N \) to 0 under \( S_A \) if \( \mu_{i,x} = 0 \)

b) If \( \text{INM} \) combines values from a test rejecting for too large values, the probability that \( \text{INM} \) rejects ...

decreases with growing \( N \) to 0 under \( S_I \) if \( \mu_{i,x} \neq 0 \)

... increases with growing \( N \) to 1 under \( S_A \) if \( \mu_{i,x} = 0 \)

**Proof** See Appendix.

Using the codes accompanying MacKinnon (1996), we have verified (13) numerically for a fine grid covering the range of the two distributions with \( m = 2, \ldots, 9 \) for the Phillips-Ouliaris t-type test. Similarly, we did with the codes accompanying Ericsson and MacKinnon (2002). Hence, Proposition 2 a) applies for the Phillips-Ouliaris residual test and
the error correction test as well. We learn that strategy \( S_A \) recommended in a time series context by Hansen (1992) or Stock and Watson (2003) becomes increasingly conservative with growing \( N \) in the absence of linear trends just as \( S_I \) gets increasingly liberal under linear trends. For a numerical quantification, see the Monte Carlo section below.

Let \( \bar{c}^{(m)}(\alpha) \) and \( \tilde{c}^{(m-1)}(\alpha) \) denote critical values for the KPSS cointegration test at nominal level \( \alpha \) from the distributions \( \bar{L}^{(m)} \) and \( \tilde{L}^{(m-1)} \). From Shin (1994) and Kwiatkowski, Phillips, Schmidt, and Shin (1992) we observe \( \tilde{c}^{(m-1)}(\alpha) < \bar{c}^{(m)}(\alpha) \) for nominal level of 1%, 5% and 10%, which supports (13):

\[
\bar{F}^{(m)}(\tilde{c}^{(m-1)}(\alpha)) < \alpha = \tilde{F}^{(m-1)}(\tilde{c}^{(m-1)}(\alpha)).
\]

However, we must admit not to have the numerical means to verify (13) on a finer grid. Still, we conjecture that Proposition 2 b) holds for \( p \)-values from the KPSS cointegration test.

5 Monte Carlo evidence

5.1 Evidence on Proposition 2

The first set of experiments assesses the effect of the absence or presence of a linear time trend on the combination of independent \( p \)-values from Phillips-Ouliaris \( t \)-type tests for the null hypothesis \( H_0 \) of no cointegration. This illustrates Proposition 2 a). We now add the subscript \( PO \) to test statistics and related entities. Using the numerical distribution functions by MacKinnon (1996) we generate \( N \) pseudo random numbers from the limiting distribution \( \bar{L}_{PO}^{(1)} \) resulting from a bivariate regression with intercept under drifts, see (10).

With \( \bar{F}_{PO}^{(2)} \) being the distribution function of \( \bar{L}_{PO}^{(2)} \) we compute \( N \) \( p \)-values ignoring the linear time trend in the data, and feed them into the test statistic \( INM \). Then we perform a 5% test rejecting \( H_0 \) if \( INM < -1.6499 \). Repeating the experiment 10,000 times, we compute the rejection frequency, see Figure 1. For \( N = 1 \) it amounts to 6%. With increasing \( N \), however, the size distortion grows fast up to more than 20% for \( N = 50 \). Such distortions are easily avoided by accounting for the drift in the data, i.e. by applying (10). If \( \bar{F}_{PO}^{(1)} \) being the distribution function of \( \bar{L}_{PO}^{(1)} \) is used to determine the \( p \)-values, then the 5% level is maintained for all \( N \), see Figure 1.

We now extend the experiment to the conservative testing strategy, where \( \bar{S}_{PO}^{(2)} \) is always compared with critical values from \( \bar{L}_{PO}^{(1)} \). We thus generate \( N \) pseudo random numbers from the limiting distribution \( \bar{L}_{PO}^{(2)} \) without drift in the series. With \( \bar{F}_{PO}^{(1)} \) being the distribution function of \( \bar{L}_{PO}^{(1)} \) we compute \( N \) \( p \)-values to compute \( INM \). Again, we perform a 5% test building on combined \( p \)-values, see Figure 2. For \( N = 1 \) the experimental size amounts to roughly 4%. With increasing \( N \), however, the combination test gets very
Figure 1: Rejection of INM at nominal 5% level for Phillips-Ouliaris tests with $m = 2$ under $\mu_{i,x} \neq 0$ ($S_I$ vs. $S_A$)

Figure 2: Rejection of INM at nominal 5% level for Phillips-Ouliaris tests with $m = 2$ under $\mu_{i,x} = 0$ ($S_I$ vs. $S_A$)
conservative dropping below 1% for $N = 20$ already. In practice, this will come with a
power loss, of course. The distortions do not occur when working with the appropriate
limit in the absence of drifts: With $\tilde{F}_{PO}^{(2)}$ being the distribution function of $\tilde{L}_{PO}^{(2)}$ used to
determine the $p$-values, the 5% level is achieved for all $N$, see Figure 2.

The experiments show that the very mild distortions of the liberal or conservative strategies with the Phillips-Ouliaris tests are severely aggravated when combining significance of a panel with growing $N$.

5.2 Evidence on Proposition 1

The next set of experiments refers to the error-correction test by Westerlund (2007). As data-generating process (DGP) we consider

$$
y_{i,t} = x_{i,1,t} + x_{i,2,t} + \cdots + x_{i,k,t} + r_{i,0,t}, \quad t = 1, 2, ..., T = 250, \quad i = 1, 2, ..., N,
$$

(14)

$$
x_{i,j,t} = 1 + x_{i,j,t-1} + v_{i,j,t}, \quad j = 1, 2, ..., k,
$$

(15)

$$
r_{i,j,t} = r_{i,j,t-1} + v_{i,j,t}, \quad j = 0, 1, ..., k
$$

(16)

where $\{v_{i,j,t}\}$ are standard normal iid sequences independent of each other. Using the regression

$$
\Delta y_{i,t} = \alpha_i + \gamma_i y_{i,t-1} + \beta_i' x_{i,t-1} + b_i' \Delta x_{i,t} + e_{i,t},
$$

(17)

we computed the group t-statistic proposed by Westerlund (2007), and repeated this experiment 10,000 times. Tables 6 and 7 report the frequencies of rejection. In particular, Table 6 is for the same constellation as Table 3 containing approximate figures; the correspondence between the experimental sizes and the approximate sizes is rather close notwithstanding the fairly small panel dimension $N$. Table 7 illustrates how well the rule from (10) works: The experimental sizes are close to the nominal ones under strategy $S_A$ in the presence of linear trends. Of course, applying strategy $S_A$ in the case of no linear trends would yield to empirical size distortions analogous to those reported in Table 4.

5.3 Evidence on detrending

Since neither strategy $S_I$ nor strategy $S_A$ can be recommended when we are unsure about the presence or absence of a linear time trend in the data, one might always run detrended regressions. While this may not always be economically meaningful, it is a statistically

17
Table 6: Experimental size of the group t-test by Westerlund (2007) at nominal level \( \alpha \) under \( S_I \) for \( \mu_{i,x} \neq 0 \)

| \( N = \) | 10  | 20  | 30  | 40  | 50  |
|---------|-----|-----|-----|-----|-----|
| \( k = 1 \) | \( \alpha = 0.01 \) | 0.129 | 0.335 | 0.527 | 0.693 | 0.811 |
|         | \( \alpha = 0.05 \) | 0.367 | 0.635 | 0.804 | 0.905 | 0.951 |
|         | \( \alpha = 0.10 \) | 0.539 | 0.781 | 0.900 | 0.960 | 0.982 |
| \( k = 2 \) | \( \alpha = 0.01 \) | 0.082 | 0.180 | 0.301 | 0.415 | 0.534 |
|         | \( \alpha = 0.05 \) | 0.267 | 0.444 | 0.596 | 0.707 | 0.798 |
|         | \( \alpha = 0.10 \) | 0.412 | 0.605 | 0.747 | 0.830 | 0.896 |
| \( k = 3 \) | \( \alpha = 0.01 \) | 0.069 | 0.151 | 0.246 | 0.325 | 0.431 |
|         | \( \alpha = 0.05 \) | 0.227 | 0.382 | 0.519 | 0.614 | 0.709 |
|         | \( \alpha = 0.10 \) | 0.367 | 0.540 | 0.668 | 0.750 | 0.829 |

Table 7: Experimental size of the group t-test by Westerlund (2007) at nominal level \( \alpha \) under \( S_A \) according to Theorem 1 for \( \mu_{i,x} \neq 0 \)

| \( N = \) | 10  | 20  | 30  | 40  | 50  |
|---------|-----|-----|-----|-----|-----|
| \( k = 1 \) | \( \alpha = 0.01 \) | 0.010 | 0.009 | 0.008 | 0.008 | 0.008 |
|         | \( \alpha = 0.05 \) | 0.048 | 0.048 | 0.043 | 0.040 | 0.041 |
|         | \( \alpha = 0.10 \) | 0.093 | 0.095 | 0.087 | 0.080 | 0.083 |
| \( k = 2 \) | \( \alpha = 0.01 \) | 0.008 | 0.009 | 0.007 | 0.009 | 0.006 |
|         | \( \alpha = 0.05 \) | 0.046 | 0.040 | 0.039 | 0.040 | 0.034 |
|         | \( \alpha = 0.10 \) | 0.093 | 0.082 | 0.083 | 0.082 | 0.073 |
| \( k = 3 \) | \( \alpha = 0.01 \) | 0.010 | 0.010 | 0.012 | 0.010 | 0.011 |
|         | \( \alpha = 0.05 \) | 0.051 | 0.052 | 0.052 | 0.049 | 0.052 |
|         | \( \alpha = 0.10 \) | 0.101 | 0.101 | 0.102 | 0.098 | 0.098 |
Table 8: Experimental size of the group t-test by Westerlund (2007) at nominal level $\alpha$ under detrending

| $N$ = | 10 | 20 | 30 | 40 | 50 |
|-------|----|----|----|----|----|
| $\alpha = 0.01$ | 0.010 | 0.010 | 0.008 | 0.010 | 0.010 |
| $k = 1$ | $\alpha = 0.05$ | 0.047 | 0.047 | 0.045 | 0.048 | 0.044 |
| | $\alpha = 0.10$ | 0.098 | 0.095 | 0.091 | 0.091 | 0.087 |
| $\alpha = 0.01$ | 0.012 | 0.012 | 0.013 | 0.012 | 0.012 |
| $k = 2$ | $\alpha = 0.05$ | 0.057 | 0.057 | 0.056 | 0.059 | 0.057 |
| | $\alpha = 0.10$ | 0.112 | 0.108 | 0.106 | 0.108 | 0.111 |
| $\alpha = 0.01$ | 0.013 | 0.008 | 0.012 | 0.013 | 0.010 |
| $k = 3$ | $\alpha = 0.05$ | 0.055 | 0.050 | 0.050 | 0.055 | 0.049 |
| | $\alpha = 0.10$ | 0.104 | 0.990 | 0.101 | 0.098 | 0.099 |

sound procedure in that the probability of a type I error is controlled under the null hypothesis. However, the strategy of always detrending may come at a high price in terms of power, as we demonstrate again for the error correction test.

First, we generate again data under (14), and observe that the experimental size from detrended regressions is close to the nominal one under the null hypothesis of no cointegration, see Table 8. Second, the DGP under the alternative becomes:

$$
\Delta y_{i,t} = -0.02 (y_{i,t-1} - x_{i,1,t} - x_{i,2,t} - \cdots - x_{i,k,t}) + v_{i,0,t}
$$

where $x_{i,j,t}$ and $v_{i,0,t}$ are generated as before. We now report rejection frequencies under detrending and when applying strategy $S_A$, where the true DGP still contains linear time trends, see (15). The results from Table 9 are very clear: As a rule of thumb we observe that $S_A$ is twice as powerful as detrending in that the rejection frequencies are roughly twice as large (for $N \leq 30$).

### 6 Empirical application

We now turn to an analysis of the weak purchasing power parity (PPP) hypothesis. Let $p_{i,t}$ and $p_{US,t}$ denote the logarithms of domestic and US price levels (consumer price indices), respectively, and $s_{i,t}$ is the corresponding log exchange rate. According to weak PPP the relationship $p_{i,t} - \beta_1 p_{US,t} - \beta_2 s_{i,t}$ should be stationary, where the individual series are assumed to be I(1). With annual OECD data from 1973 until 2013 we run the following
Table 9: Experimental power of the group t-test at 5% by Westerlund (2007) under $\mu_{i,x} \neq 0$

| $N$ | 10   | 20   | 30   | 40   | 50   |
|-----|------|------|------|------|------|
| $k = 1$ | 0.486 | 0.727 | 0.859 | 0.933 | 0.965 |
| $k = 2$ | 0.284 | 0.433 | 0.559 | 0.672 | 0.751 |
| $k = 3$ | 0.155 | 0.234 | 0.288 | 0.336 | 0.387 |
| $S_A$: Theorem 1 | | | | | |

Detrended regression

| $k = 1$ | 0.235 | 0.379 | 0.497 | 0.615 | 0.707 |
| $k = 2$ | 0.163 | 0.247 | 0.331 | 0.399 | 0.452 |
| $k = 3$ | 0.087 | 0.118 | 0.133 | 0.156 | 0.173 |

error correction regression with vectors $\beta_i, b_i, a_i$ ($i = 1, \ldots, 11$),

$$\Delta p_{i,t} = \alpha_i + \gamma_i p_{i,t-1} + (p_{US,t-1}, s_{i,t-1})\beta_i + (\Delta p_{US,t}, \Delta s_{i,t})b_i + (\Delta p_{i,t-1}, \Delta p_{US,t-1}, \Delta s_{i,t-1})a_i + \varepsilon_{i,t},$$

(18)

for the $N = 11$ currencies reported in Table 10; Germany stands for the Euro. We deleted variables with coefficients not significantly different from zero (typically $\Delta s_{i,t}$, $\Delta p_{US,t-1}$ and $\Delta s_{i,t-1}$). Note that there are $m = 3 \mathbb{I}(1)$ variables where the mean growth of $p_{US,t-1}$ may be approximated by a linear time trend. Moreover, the 11 equations are clearly dependent. Sectorally demeaning to account for cross-sectional dependence as proposed by Westerlund (2007) is not feasible in our framework, since the identical US series enters each equation. Therefore, we combine $p$-values from individual regressions obtained from programmes accompanying Ericsson and MacKinnon (2002). These finite sample values computed according to strategies $S_I$ and $S_A$ are denoted by $\tilde{p}(3)$ and $\tilde{p}(2)$, respectively. The results are given in Table 10.

Clearly, there is mixed evidence with respect to cointegration (validity of weak PPP). For small $p$-values the difference between $S_I$ and $S_A$ is small in absolute terms. In relative terms, however, the difference is particularly large for smaller $p$-values as three examples illustrate:

- Canada: $\frac{0.4893 - 0.3562}{0.3562} = 37.37\%$
- Korea: $\frac{0.1937 - 0.1299}{0.1299} = 49.11\%$
- UK: $\frac{0.0478 - 0.0305}{0.0305} = 56.72\%$

We now combine the $p$-values from Table 10 by means of the inverse normal method.
Table 10: $p$-values from $t_\gamma$ when testing for $\gamma_i = 0$ according to strategies $S_I$ and $S_A$

| Country     | $t_\gamma$ from (18) | $S_I$: $\bar{p}(3)$ | $S_A$: $\tilde{p}(2)$ |
|-------------|-----------------------|----------------------|------------------------|
| Australia   | -3.0208               | 0.1408               | 0.2090                 |
| Canada      | -2.3409               | 0.3562               | 0.4893                 |
| Denmark     | -3.6772               | 0.0422               | 0.0656                 |
| Finland     | -4.0792               | 0.0177               | 0.0278                 |
| Germany     | -3.6465               | 0.0449               | 0.0697                 |
| Japan       | -3.5877               | 0.0503               | 0.0778                 |
| Korea       | -3.0703               | 0.1299               | 0.1937                 |
| Norway      | -2.5248               | 0.2871               | 0.4040                 |
| New Zealand | -2.7888               | 0.2011               | 0.2917                 |
| Switzerland | -3.2165               | 0.1015               | 0.1530                 |
| UK          | -3.8295               | 0.0305               | 0.0478                 |

\[ INM(\hat{\rho}) = \sum_{i=1}^{N} \Phi^{-1}(p_i) \sqrt{N + \hat{\rho}(N^2 - N)} \]

where consistent estimation $\hat{\rho}$ is discussed in Hartung (1999). This estimate is 0.6863 and 0.6299 under strategies $S_I$ and $S_A$. The resulting test statistic $INM(\hat{\rho})$ is listed in Table 10, too. The combined significance is given by the corresponding $p$-values computed from a one-sided standard normal distribution: 0.0623 and 0.0996 for $S_I$ and $S_A$, respectively. Hence, the conventional procedure $S_I$ results in a too optimistic view, rejecting the panel null hypothesis of no cointegration at a 6% level, while the procedure $S_A$ accounting for a linear time trend in the price series yields significance just at the 10% level.

7 Concluding remarks

In time series econometrics it has been known for a long time that “the deterministic trends in the data affect the limiting distributions of the test statistics whether or not we detrend the data.” (Hansen, 1992, p. 103). This has been shown for the residual-based Phillips-Ouliaris (or Engle-Granger) cointegration test by Hansen (1992), see also the exposition in Hamilton (1994, p. 596, 597). Analogous results have been given for other cointegration tests by Hassler (2000) and Hassler (2001). In this paper these findings are carried to the panel framework, and they are shown to continue to hold for
all single-equation tests relying on OLS, no matter whether the null hypothesis is absence or presence of cointegration. In a regression involving \( m \geq 2 \) variables, much of the panel cointegration theory relies on normalization with some \( \bar{\mu}_m \) and \( \bar{\sigma}_m \) and letting the panel dimension \( N \) to infinity to obtain a standard normal distribution. The numbers \( \bar{\mu}_m \) and \( \bar{\sigma}_m \) are typically tabulated for the case of regressions with intercept only; different figures \( \tilde{\mu}_m \) and \( \tilde{\sigma}_m \) are tabulated for regressions with intercept and linear time trend. We show the following: When statistics are computed from regressions with \( m \) integrated variables with intercept only, but when one of the integrated regressors is dominated by a linear time trend, then normalization with \( \tilde{\mu}_{m-1} \) and \( \tilde{\sigma}_{m-1} \) is required to achieve asymptotically valid inference under the null hypothesis. Normalization with \( \bar{\mu}_m \) and \( \bar{\sigma}_m \), however, which has been the conventional strategy so far, results in a loss of size control under the null hypothesis. In fact, employing \( \bar{\mu}_m \) and \( \bar{\sigma}_m \) in the presence of linear time trends gives rejection probabilities converging with \( N \) to 1 or 0, depending on whether the null hypothesis is no cointegration or cointegration, respectively (see Proposition 1). To avoid such size distortions one might always work with detrended regressions. Such a strategy, however, has two drawbacks: First, a regression with intercept only will be more powerful (see e.g. Hamilton, 1994, p. 598); second, a detrended regression may not be meaningful from an economic point of view.

A conservative strategy rejecting with a probability smaller than or equal to the nominal size irrespective of whether the data display a linear trend or not has been proposed by Hansen (1992) and adopted by e.g. Stock and Watson (2003, Ch. 14). It may be adequate for certain tests in a pure time series setting, but it is not acceptable in a panel framework where it becomes increasingly conservative as \( N \) grows. In panel applications it is hence crucial to recognize the presence of linear time trends in the data when testing from regressions with intercept only, in order to apply Theorem 1. In practice this requires pretesting for the presence of a linear time trend, which introduces the problems of multiple testing of course. Alternatively, one may consider the combination of the evidence from two cases: assuming a linear trend and assuming no linear trend. One could follow the lines by Harvey et al. (2009) or Harvey et al. (2012) who explored unit root testing under uncertainty over the presence of linear trends. The comparison of empirical strategies how to proceed under uncertainty about a linear time trend when testing panel cointegration is left for future research.
Appendix

Proof of Theorem 1

Since all panel statistics are computed from individually demeaned or detrended regressions, we suppress the index $i$ and consider the generic case, showing that a regression with intercept only in the presence of a linear time trend amounts to an appropriately modified regression including time as regressor.

Define the $k$-element vector $\lambda_1 := \mu_x / \sqrt{\mu_x' \mu_x}$ of unit length with

$$\tau_t := \lambda_1' x_t = \sqrt{\mu_x' \mu_x} t + \lambda_1 \sum_{j=1}^{t} e_{x,j}.$$ 

At the same time there exist $k - 1$ linearly independent $k$-element columns collected in the $k \times (k - 1)$-matrix $\Lambda_2$. Due to the Gram-Schmidt orthogonalization one may assume that the invertible matrix $\Lambda = (\lambda_1, \Lambda_2)$ is orthogonal: $\Lambda \Lambda' = I_k$. Consequently, all columns of $\Lambda_2$ eliminate the linear trend in $x_t$:

$$\xi_t := \Lambda_2' x_t = \Lambda_2' \sum_{j=1}^{t} e_{x,j}.$$ 

Hence, $\xi_t$ is a $(k - 1)$-vector integrated of order 1 without drift. Now, we are able to rewrite

$$\beta' x_t = \beta' \Lambda \Lambda' x_t
= \beta' \lambda_1 \tau_t + \theta' \xi_t, \quad \theta = \Lambda_2' \beta.$$ 

The deterministic term in $\tau_t$ dominates the I(1) component:

$$T^{-1} \tau_{[rT]} = \sqrt{\mu_x' \mu_x} \frac{|rT|}{T} + O_p(T^{-0.5}) \Rightarrow \sqrt{\mu_x' \mu_x} r.$$ 

To analyze residual statistics from (11) or $t$ statistics from (12), we further have to show that empirical moments involving $\tau_t$ equal those with $\sqrt{\mu_x' \mu_x} t$ up to $O_p(T^{-0.5})$. More precisely, we have from Park and Phillips (1988, Lemma 2.1) that the vector

$$\left( \frac{1}{T^2} \sum_{t=1}^{T} \tau_t, \frac{1}{T^3} \sum_{t=1}^{T} \tau_t^2, \frac{1}{T^{2.5}} \sum_{t=1}^{T} \tau_t \xi_t', \frac{1}{T^2} \sum_{t=1}^{T} \tau_t \Delta x_{t-j}' \right)$$ 

...
In the same way, one may analyze the effect of strategy $S_A$ in the absence of linear time trends:

$$
P \left( \sqrt{N} \frac{\bar{Z}^{(m)} - \mu_m}{\sigma_m} < -z_{1-\alpha} \right) = \Phi \left( \sqrt{N} \frac{\mu_m - \tilde{\mu}_{m-1}}{\tilde{\sigma}_{m-1}} - \frac{\sigma_m}{\tilde{\sigma}_{m-1}} z_{1-\alpha} \right).
$$

Hence, statistics computed from regressions including the term $c + \beta'x_t = c + \beta'\lambda_1 t + \theta'\xi_t$ amount to statistics from regressions with $c + \delta t + \theta'\xi_t$ asymptotically, where $\delta = \beta'\lambda_1 \sqrt{\mu''_x \mu_x}$. This completes the proof. □

**Proof of Proposition 1**

According to Theorem 1 the statistic $\bar{Z}^{(m)}$ requires under $\mu_{i,x} \neq 0$ normalization with $\bar{\mu}_{m-1}$ and $\bar{\sigma}_{m-1}$ in order to result in a standard normal distribution under $H_0$. Let $z_{1-\alpha}$ denote a quantile from the standard normal distribution. In the case where the panel tests reject for too negative values, the rejection probability of strategy $S_I$ under the null hypothesis becomes approximately for large $N$ (at nominal level $\alpha$):

$$
P \left( \sqrt{N} \frac{\bar{Z}^{(m)} - \mu_m}{\sigma_m} < -z_{1-\alpha} \right) = \Phi \left( \sqrt{N} \frac{\bar{\mu}_m - \tilde{\mu}_{m-1}}{\tilde{\sigma}_{m-1}} - \frac{\sigma_m}{\tilde{\sigma}_{m-1}} z_{1-\alpha} \right). \tag{19}
$$

Analogously, when rejecting for too large values, the rejection probability of strategy $S_I$ becomes under $\mu_{i,x} \neq 0$ according to Theorem 1 with growing $N$:

$$
P \left( \sqrt{N} \frac{\bar{Z}^{(m)} - \mu_m}{\sigma_m} > z_{1-\alpha} \right) = 1 - \Phi \left( \sqrt{N} \frac{\bar{\mu}_m - \tilde{\mu}_{m-1}}{\tilde{\sigma}_{m-1}} + \frac{\sigma_m}{\tilde{\sigma}_{m-1}} z_{1-\alpha} \right). \tag{20}
$$

In the same way, one may analyze the effect of strategy $S_A$ in the absence of linear time trends:

$$
P \left( \sqrt{N} \frac{\bar{Z}^{(m)} - \mu_{m-1}}{\sigma_{m-1}} < -z_{1-\alpha} \right) = \Phi \left( \sqrt{N} \frac{\bar{\mu}_{m-1} - \mu_m}{\bar{\sigma}_m} - \frac{\sigma_{m-1}}{\bar{\sigma}_m} z_{1-\alpha} \right), \tag{21}
$$

44
\[
P\left( \sqrt{N} \frac{\bar{Z}^{(m)} - \bar{\mu}_{m-1}}{\sigma_{m-1}} > z_{1-\alpha} \right) = 1 - \Phi\left( \sqrt{N} \frac{\bar{\mu}_{m-1} - \mu_m}{\sigma_m} + \frac{\bar{\sigma}_{m-1}}{\sigma_m} z_{1-\alpha} \right),
\]

(22)

For \( N \to \infty \) one gets the limits given in Proposition 1 from (19) through (22). ■

**Proof of Proposition 2**

We turn to the case of left-sided tests under a) first. By (13) it holds for \( p \)-values from the laws characterized in Assumption 3:

\[
\bar{p}^{(m)} < \tilde{p}^{(m-1)}.
\]

When employing strategy \( S_I \), one feeds \( \bar{p}^{(m)} \) into \( INM \), while \( \tilde{p}^{(m-1)} \) would be correct in the presence of linear time trends according to (10). Consequently for \( S_I \):

\[
\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Phi^{-1}(\bar{p}_i^{(m)}) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Phi^{-1}(\tilde{p}_i^{(m)}) - \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left( \Phi^{-1}(\tilde{p}_i^{(m-1)}) - \Phi^{-1}(\bar{p}_i^{(m)}) \right)
\]

\[
= Z - \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left( \Phi^{-1}(\tilde{p}_i^{(m-1)}) - \Phi^{-1}(\tilde{p}_i^{(m)}) \right)
\]

\[
< Z - \sqrt{N} \min_i \left( \Phi^{-1}(\tilde{p}_i^{(m-1)}) - \Phi^{-1}(\bar{p}_i^{(m)}) \right),
\]

where \( Z \) is a well defined random variable under the null hypothesis. Consequently, \( INM \) diverges to \(-\infty\). This establishes the first statement. The second one refers to \( S_A \) where \( INM \) is computed from \( \tilde{p}^{(m-1)} \), although \( \bar{p}^{(m)} \) would be correct:

\[
\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Phi^{-1}(\tilde{p}_i^{(m)}) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Phi^{-1}(\tilde{p}_i^{(m)}) - \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left( \Phi^{-1}(\bar{p}_i^{(m)}) - \Phi^{-1}(\tilde{p}_i^{(m-1)}) \right)
\]

\[
= Z - \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left( \Phi^{-1}(\bar{p}_i^{(m)}) - \Phi^{-1}(\tilde{p}_i^{(m-1)}) \right)
\]

\[
> Z + \sqrt{N} \min_i \left( \Phi^{-1}(\bar{p}_i^{(m-1)}) - \Phi^{-1}(\bar{p}_i^{(m)}) \right),
\]

which goes off to \( \infty \). This establishes a). Second, we turn to case b), where by (13)

\[
\bar{p}^{(m)} = 1 - \bar{F}^{(m)}(x) > 1 - \bar{F}^{(m-1)}(x) = \tilde{p}^{(m-1)}, \quad x \in \mathbb{R}.
\]

Repeating the above arguments, it is straightforward to complete the proof. ■
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