Tests of Complete Positivity in Fiber Optics

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**Abstract.** We consider the propagation of polarized photons in optical fibers under the action of randomly generated noise. In such situation, the change in time of the photon polarization can be described by a quantum dynamical semigroup. We show that the hierarchy among the decay constants of the polarization density matrix elements as prescribed by complete positivity can be experimentally probed using standard laboratory set-ups.

1. Introduction

A quantum system immersed in a large environment represents the paradigm of a physical situation that is often encountered in quantum optics, quantum chemistry and atomic physics. Since it is not isolated, such system is called open: it exchanges energy and entropy with the external environment, whose dynamics is typically unaffected by the presence of the small subsystem. The evolution of such open quantum systems, obtained by tracing over the (infinite) degrees of freedom of the environment, is in general very complicated, exhibiting non linearities and memory effects.

However, when the coupling between the two systems is weak, a very common situation in many practical applications, the reduced time evolution can be described by a one-parameter (\(\gamma_t\)) family of linear maps, a so-called quantum dynamical semigroup \([1]-[5]\). These linear transformations map states of the subsystem, described by density matrices, into states, while possessing many desirable physical properties. In particular, these maps exhibit irreversibility, encoded in the semigroup property (i.e. in the forward in time composition law), decoherence and dissipation, the typical noisy effects induced by the environment. Further, these generalized time evolutions turn out to be completely positive, a property that guarantees the physical consistency of the dynamics in all physical situations.

Nevertheless, in many phenomenological treatments of the dynamics of open quantum systems, this last property is often dismissed as irrelevant, nothing more
than a mathematical artifact [6]-[10]. The argument supporting this point of view is based on the following considerations. In order to represent a physical state of the subsystem S, a density matrix ρ must be a positive operator, since its eigenvalues have the meaning of probabilities; this is at the root of the statistical interpretation of quantum mechanics. The time evolution \(\rho(0) \mapsto \rho(t) = \gamma_t[\rho(0)]\) must then preserve this fundamental property, and therefore map a positive initial \(\rho(0)\) into a positive final \(\rho(t)\). Such a property of the linear transformation \(\gamma_t\) is called positivity, it is apparently sufficient to assure the physical consistency of the dynamics.

On the other hand, complete positivity is a more restrictive requirement than simple positivity: it guarantees the positivity not only of \(\gamma_t\) but also of the dynamics of a larger system built with the system S statistically coupled to a second ancillary system \(S'\), which however remains inert under the time evolution. The dynamics of this enlarged system is then described by \(\Gamma_t = \gamma_t \otimes \text{id}\), where \(\text{id}\) represents the identity operation. Positivity of \(\Gamma_t\) for any ancillary system \(S'\) means complete positivity of the map \(\gamma_t\). It is worth noting that this property is intimately related to entanglement, i.e. to the possibility that S may have interacted in the past with \(S'\) and become quantum correlated with it [4, 11].

Admittedly, although logically stringent, the above motivation of why an open system dynamics should be completely positive is not very appealing from the physical point of view; indeed, the ancilla system \(S'\) is remote from S and fixed. Nevertheless, a more concrete scenario can be offered, that of two equal, mutually non interacting systems \(S\), immersed in the same external bath; such a situation is not uncommon and it is for instance encountered in high energy particle physics in the study of correlated neutral mesons [12, 13, 14]. In this case the two systems evolve with the product map \(\Gamma_t = \gamma_t \otimes \gamma_t\), and one can show that the positivity of \(\Gamma_t\) is equivalent to the complete positivity of \(\gamma_t\) [15].

Another important advantage of having a completely positive open system dynamics is that this property fully characterizes the form of the map \(\gamma_t\) [16, 17]. In particular, when S is a two-level systems, as discussed below, the decay times of the diagonal \(T_1\) and off-diagonal \(T_2\) elements of the corresponding density matrix are seen to satisfy a characteristic order relation: \(2 T_1 \geq T_2\) [11, 15]. It is precisely the presence of such hierarchy that is often questioned in the phenomenological literature on open quantum systems.

The aim of the present investigation is to analyze in detail the possibility of a direct test of the above inequality in the laboratory, using polarized photons. The basic idea is to study the change in their polarization while the photons travel across a weakly coupled, noisy environment. A possible practical realization of this scenario involve photons with a given polarization injected inside a high quality, polarization preserving optical fiber, which is subjected to random noise generated by external high frequency sound waves; as we shall see, the random noise will play
a role similar of that of an external environment. By making the photons traverse the fiber twice with the insertion of a Faraday mirror at one end, it is possible to measure the ratio between the diagonal and off-diagonal elements of the $2 \times 2$ density matrix representing the final photon polarization state, and thus determine the combination $2T_1 - T_2$.

All the necessary techniques needed for the realization of the apparatus and the realization of the measure are standard and available in modern quantum optical laboratories. We hope that our results will stimulate the actual realization of the experiment, thus clarifying in a controlled setting the role of complete positivity in open quantum dynamics.

2. Master equation

In describing the evolution of polarized photons in an optical fiber we shall adopt the standard effective description in terms of a two-dimensional Hilbert space, the space of helicity states [19]-[23]. Any vector in this space represents a given polarization and can be identified by two angles $\theta$ and $\varphi$:

$$|\theta, \varphi\rangle = \cos \theta |+\rangle + e^{i\varphi} \sin \theta |-\rangle ,$$

where $|+\rangle$ and $|-\rangle$ are two orthonormal basis vectors, representing linearly polarized states. Another convenient basis in this space is given by the circularly polarized states:

$$|R\rangle = \frac{1}{\sqrt{2}} (|+\rangle + i|-\rangle) , \quad |L\rangle = \frac{1}{\sqrt{2}} (|+\rangle - i|-\rangle) .$$

With respect to this basis, any (partially) polarized photon state can be represented by a $2 \times 2$ density matrix $\Sigma$; as already pointed out before, this is a hermitian, positive operator, i.e. with positive eigenvalues, normalized to have unit trace.

The time evolution of the photons inside the fiber, while subjected to random noise, can be cast in a standard Liouville - von Neumann form:

$$\frac{\partial \Sigma(t)}{\partial t} = -i[H_0, \Sigma(t)] + L_t[\Sigma(t)] .$$

The first piece on the r.h.s. describes the propagation of the photons in absence of noise; in the chosen basis, the effective hamiltonian $H_0$ can be cast in the general form:

$$H_0 = \frac{\omega_0}{2} \vec{n} \cdot \vec{\sigma} ,$$

where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the vector of Pauli matrices. For sake of generality, we have kept $\omega_0$ nonvanishing, in order to take into account possible birefringence effects due to the fiber mechanical bending.
The fiber is further subjected to random noise, generated by sound waves of frequency much higher than the inverse flight time of the photons in the fiber. As a consequence, the travelling photon will propagate in wave guide that is randomly changing; it is as if it was effectively moving into a rapidly fluctuating medium, which can be described by classical stochastic fields. The action of this fields on the travelling photons can then be expressed via the commutator with a time-dependent hermitian matrix $F(t)$,

$$L_t[\Sigma(t)] = -i[F(t), \Sigma(t)], \quad F(t) = \vec{F}(t) \cdot \vec{\sigma}, \quad (5)$$

whose components $F_1(t)$, $F_2(t)$, $F_3(t)$ form a real, stationary Gaussian stochastic field $\vec{F}(t)$; in general, they have nonzero constant means and translationally invariant correlations:

$$G_{ij}(t-s) \equiv \langle F_i(t) F_j(s) \rangle - \langle F_i(t) \rangle \langle F_j(s) \rangle, \quad i, j = 1, 2, 3. \quad (6)$$

Since the generalized hamiltonian $F(t)$ in (5) involves stochastic variables, the density matrix $\Sigma(t)$, solution of the equation of motion (3), is also stochastic. Instead, we are interested in the behaviour of the reduced density matrix $\rho(t) \equiv \langle \Sigma(t) \rangle$ which is obtained by averaging over the noise; it is $\rho(t)$ that describes the effective evolution of the photons in the randomly behaving fiber. By making the additional assumption that photons and noise be decoupled at $t=0$, so that the initial state is $\rho(0) \equiv \langle \Sigma(0) \rangle = \Sigma(0)$, a condition very well satisfied in practice, an effective master equation for $\rho(t)$ can be derived by going to the interaction representation, where we set:

$$\bar{\Sigma}(t) = e^{-itH_0} \Sigma(t) e^{-itH_0}, \quad \bar{\sigma}(t) = e^{-itH_0} \vec{\sigma} e^{-itH_0}, \quad \bar{L}_t[\cdot] = -i[\vec{F}(t) \cdot \bar{\sigma}(t)], \quad (7)$$

The time evolution of the reduced density matrix in this representation, $\bar{\rho}(t) \equiv \langle \bar{\Sigma}(t) \rangle$, can then be expressed as a series expansion involving multiple correlations of the operator $\bar{L}_t$:

$$\bar{\rho}(t) = N_t[\bar{\rho}(0)] \equiv \sum_{k=0}^{\infty} \int_0^t ds_1 \int_0^{s_1} ds_2 \cdots \int_0^{s_{k-1}} ds_k \langle \bar{L}_{s_1} \bar{L}_{s_2} \cdots \bar{L}_{s_k} \rangle [\bar{\rho}(0)]. \quad (8)$$

The sum $N_t[\bar{\rho}] \equiv \sum_{k=0}^{\infty} N_k[\bar{\rho}]$ in (8) with $N_0[\bar{\rho}] \equiv \bar{\rho}$ can be formally inverted, and a suitable resummation gives (a dot represents time derivative) [24]:

$$\frac{\partial \bar{\rho}(t)}{\partial t} = \dot{N}_t N_t^{-1}[\bar{\rho}(t)] = \left\{ \dot{N}_1 + (\dot{N}_2 - \dot{N}_1 N_1) + \ldots \right\}[\bar{\rho}(t)]. \quad (9)$$

Since the interaction of the travelling photons with the stochastic field is weak, one can focus the attention on the dominant terms of the previous expansion, neglecting all contributions higher than the second-order ones. Further, since the
characteristic fluctuation time of the random noise is by assumption much smaller than the typical photon time of flight across the fiber, the memory effects implicit in (9) should not be physically relevant and the use of the Markovian approximation justified. This is implemented in practice by extending to infinity the upper limit of the integrals appearing in $\hat{N}^{(2)}$ and $N^{(1)}$ [1]-[3].

By returning to the Schrödinger representation, one finally obtains [4, 25]

$$\frac{\partial \rho(t)}{\partial t} = -i[H, \rho(t)] + L[\rho(t)],$$

(10)

where

$$H = H_0 + H_1 + H_2 \equiv \vec{\omega} \cdot \vec{\sigma},$$

(11)

and

$$L[\rho] = \frac{1}{2} \sum_{i,j=1}^{3} C_{ij} \left[ 2\sigma_j \rho \sigma_i - \{\sigma_i, \sigma_j, \rho\} \right].$$

(12)

The effective Hamiltonian $H$ differs from the one in absence of noise $H_0$ by first order terms (coming from the piece $\hat{N}^{(1)}$ in (9)) depending on the noise mean values:

$$H_1 = \langle \vec{F}(t) \rangle \cdot \vec{\sigma},$$

(13)

and by second-order contributions (coming from the second-order terms in (9)),

$$H_2 = \sum_{i,j,k=1}^{3} \epsilon_{ijk} C_{ij} \sigma_k,$$

(14)

involving the noise correlations through the time-independent combinations:

$$C_{ij} = \sum_{k=1}^{3} \int_0^\infty dt \, G_{ik}(t) \, U_{kj}(-t),$$

(15)

where the $3 \times 3$ orthogonal matrix $U(t)$ is defined by the following transformation rule: $e^{itH_0} \sigma_i e^{-itH_0} = \sum_{j=1}^{3} U_{ij}(t) \, \sigma_j$. On the other hand, the contribution $L[\rho]$ in (12) is a time-independent, trace-preserving linear map involving the symmetric coefficient matrix $C_{ij} \equiv C_{ij} + C_{ji}$, which needs to be positive, $C_{ij} \geq 0$, in order to generate a completely positive dynamics [1]-[4]. It introduces irreversibility, inducing in general dissipation and loss of quantum coherence.

Altogether, equation (10)-(12) generates a semigroup of linear maps, $\Gamma_t : \rho(0) \mapsto \rho(t) \equiv \Gamma_t[\rho(0)]$, for which composition is defined only forward in time: $\Gamma_t \circ \Gamma_s = \Gamma_{t+s}$, with $t, s \geq 0$; it is usually referred to as a quantum dynamical semigroup [1]-[4].
It is convenient to expand the photon density matrix in terms of the Pauli matrices and the identity $\sigma_0$,

$$\rho = \frac{1}{2} \left[ \sigma_0 + \rho \cdot \vec{\sigma} \right].$$

(16)

Then, the linear equation (10) reduces to a diffusion equation for the components $\rho_1$, $\rho_2$, $\rho_3$ of the (Bloch) vector $\vec{\rho}$:

$$\frac{\partial \vec{\rho}(t)}{\partial t} = -2\mathcal{H} \vec{\rho}(t) ;$$

(17)

the entries of the $3 \times 3$ matrix $\mathcal{H}$ can be expressed in terms of the coefficients $\omega_i$ and $C_{ij}$ appearing in the hamiltonian and noise contribution in $[1, 4]$:

$$\mathcal{H} = \begin{bmatrix} a & b + \omega_3 & c - \omega_2 \\ b - \omega_3 & \alpha & \beta + \omega_1 \\ c + \omega_2 & \beta - \omega_1 & \gamma \end{bmatrix},$$

(18)

with $a = C_{22} + C_{33}$, $\alpha = C_{11} + C_{33}$, $\gamma = C_{11} + C_{22}$, $b = -C_{12}$, $c = -C_{13}$, $\beta = -C_{23}$.

The condition of complete positivity, $C_{ij} \geq 0$, can then be expressed more explicitly through the following inequalities:

$$2R \equiv \alpha + \gamma - a \geq 0 , \quad RS \geq b^2 ,$$

$$2S \equiv a + \gamma - \alpha \geq 0 , \quad RT \geq c^2 ,$$

$$2T \equiv a + \alpha - \gamma \geq 0 , \quad ST \geq \beta^2 ,$$

$$RST \geq 2bc\beta + R\beta^2 + Sc^2 + Tb^2 .$$

(19)

The solution of (17) involves the exponentiation of the matrix $\mathcal{H}$,

$$\vec{\rho}(t) = \mathcal{M}(t) \vec{\rho}(0) , \quad \mathcal{M}(t) = e^{-2\mathcal{H}t} .$$

(20)

From this relation, one immediately sees that the $3 \times 3$ matrix $\mathcal{M}(t)$ represents the (reduced) Mueller matrix connecting the initial Stokes vector $\vec{\rho}(0)$ with the evolved one $\vec{\rho}(t)$ at time $t$ $[21, 22, 23, 26]$.

3. Complete positivity

As shown in the previous section, the entries of the matrix $\mathcal{H}$ in (18) are directly related to the stochastic field correlations in (10). For a generic noise, one expects all parameters in $\mathcal{H}$ to be nonvanishing; only physical considerations may allow a simplification and therefore a manageable, explicit expression for $\vec{\rho}(t)$.

For a typical random noise, one can generically assume the correlation functions to have an exponentially damped form. Further, the off-diagonal correlations are
usually much suppressed with respect to the diagonal ones, so that, without much loss of generality, one can write:

\[ G_{ij}(t) = G_i e^{-\lambda_i |t|} \delta_{ij}, \quad G_i, \lambda_i \geq 0. \]  

(21)

Using the expression (1) for the hamiltonian \( H_0 \), one easily finds the form of the matrix \( U_{ij}(t) \) giving the time evolution of the Pauli matrices,

\[ U_{ij}(t) = n_i n_j + (\delta_{ij} - n_i n_j) \cos \omega_0 t - \sum_{k=1}^{3} \varepsilon_{ijk} n_k \sin \omega_0 t \]  

(22)

and then through the definition (15), one finally obtains:

\[ C_{ij} = \lambda_i \Lambda_i \left[ \delta_{ij} + \frac{\omega_0^2}{\lambda_i} n_i n_j + \frac{\omega_0}{\lambda_i} \sum_{k=1}^{3} \varepsilon_{ijk} n_k \right], \]  

(23)

where

\[ \Lambda_i = \frac{G_i}{\lambda_i^2 + \omega_0^2}. \]  

(24)

The symmetric and antisymmetric pieces of \( C_{ij} \) give the noise contributions to the dissipative (12) and hamiltonian (14) parts of the evolution equation (10).

In order to further simplify the treatment, we shall assume the unit vector \( \vec{n} \) defining the starting hamiltonian \( H_0 \) to be directed along the third axis and the stochastic field \( \vec{F}(t) \) to have zero mean. In this case, the effective hamiltonian in (11) remains proportional to \( \sigma_3 \), \( H = \omega \sigma_3 \), with a new frequency \( \omega \equiv \omega_3 \) explicitly containing zero and second order pieces: \( \omega = \omega_0/2 + \omega_0 (\Lambda_1 + \Lambda_2) \). Similarly, also the dissipative contributions simplifies, since one finds \( c = \beta = 0 \), while

\[ a = 2\lambda_2 \Lambda_2 + 2 \frac{G_3}{\lambda_3}, \quad b = \omega_0(\Lambda_2 - \Lambda_1), \]  

\[ \alpha = 2\lambda_1 \Lambda_1 + 2 \frac{G_3}{\lambda_3}, \quad \gamma = 2\lambda_1 \Lambda_1 + 2\lambda_2 \Lambda_2. \]  

(25)

Notice that the conditions assuring the complete positivity of the time evolution given by the inequalities in (19) are not all automatically satisfied by the assignments in (25). The following relation needs to be imposed, \( \omega_0^2(\Lambda_2 - \Lambda_1)^2 < 4 \lambda_1 \lambda_3 \Lambda_1 \Lambda_2 \): it can always be fulfilled by a suitable choice of the noise parameters in (21).

This situation is not exceptional: the derivation of a physically consistent, Markovian reduced dynamics starting from the exact Liouville-von Neumann equation in (3) is in general very involved and can be treated with the necessary mathematical rigor only in special cases [1]-[3]. In general, as briefly discussed
in Section 2, one instead resorts to various approximations, justified by physical considerations. However, this naive procedure does not always lead to physically consistent reduced dynamics [4]: further conditions, like that of complete positivity, need to be imposed at the end. This explains why this property is often dismissed as irrelevant in phenomenological applications, and therefore why it is so important to verify its fulfillment in an experimentally controlled setting.

With the above assignments, the exponentiation of $H$ can be easily evaluated in closed form, and for the Mueller matrix one explicitly finds:

$$M(t) = \begin{bmatrix}
  e^{-(a+\alpha)t} A_+(t) & e^{-(a+\alpha)t} B_+(t) & 0 \\
  e^{-(a+\alpha)t} B_-(t) & e^{-(a+\alpha)t} A_-(t) & 0 \\
  0 & 0 & e^{-2\gamma t}
\end{bmatrix}, \quad (26)$$

where

$$A_{\pm}(t) = \cos(2\Omega t) \pm \frac{\alpha - a}{2\Omega} \sin(2\Omega t),$$

$$B_{\pm}(t) = -b \pm \omega \Omega \sin(2\Omega t), \quad (27)$$

with $\Omega = \left[ \omega^2 - b^2 - (a - \alpha)^2/4 \right]^{1/2}$. The evolution matrix in (26) contains both oscillating and damping terms, and can be considered as a generalization of a rotator [21]-[23]. Notice, however, that the oscillator behaviour depends on the magnitude of effective frequency $\omega$ with respect to the dissipative parameters $a$, $b$ and $\alpha$; when $\omega < [b^2 + (a - \alpha)^2/4]^{1/2}$, the frequency $\Omega$ becomes purely imaginary and $M(t)$ contains only exponential terms.

In any case, for large times, i.e. for infinitely long fibers, the damping terms always dominate [27] and the Mueller matrix becomes that of a total depolarizer; in other words, as result of the action of the stochastic noise, the photon density matrix $\rho(t)$ will become asymptotically proportional to the unit matrix (i.e. $\rho(t) \sim 0$), independently from the initial state $\rho(0)$.

Through the definition (16), the evolution matrix (26) gives the time behaviour of the entries of the photon density matrix $\rho(t)$, which in turn can be experimentally determined using suitable tomographic procedures. Therefore, at least in principle, thanks to the different time-dependence of the entries in (26), one can measure the magnitude of all the dissipative parameters $a$, $b$, $\alpha$ and $\gamma$ and thus check the conditions of complete positivity, that in particular requires: $a + \alpha - \gamma \geq 0$.

Unfortunately, this can not be done by a single passage of the photons in the noisy fiber, since in this case the time $t$ appearing in (26) is fixed, being the time of flight along the fiber. In order to isolate the exponential damping terms from the oscillating ones, $t$ needs to be varied. This can be achieved by placing at the end of the fiber a Faraday mirror [21], which inverts the polarization of
the photons, while reflecting them back into the fiber. In this way, travelling backward, the rotation induced by the standard Hamiltonian $H_0$ on the photon polarization is “undone”, while the dissipative effects due to the stochastic noise further accumulates. Indeed, the backward evolution is given by a Mueller matrix $\tilde{M}(t)$ still of the form (25), but with the frequency $\omega_0$ replaced by $-\omega_0$, which in turn gives $\omega \rightarrow -\omega$ and $b \rightarrow -b$, while leaving the remaining dissipative parameters unchanged.

Thanks to the semigroup property of the dynamics, the complete evolution for the double passage of the photons in the fiber is obtained by composing the two Mueller matrices, and therefore, after having travelled for a time $2t$ inside the noisy fiber, the Stokes parameters representing the photon polarization state become at the end:

$$\tilde{\rho}(2t) = \tilde{M}(t) \cdot M(t) \tilde{\rho}(0). \quad (28)$$

Let us now consider an initially linearly polarized photon, so that $\tilde{\rho}(0) \equiv \tilde{\rho}(^{+})(0) = (1, 0, 0)$. After having passed once through the noisy fiber, this Stokes vector becomes $\tilde{\rho}(^{+})(t)$, as in (20), while further evolves to $\tilde{\rho}(^{+})(2t)$, as in (28), after having travelled backwards to the beginning of the fiber. Similar results hold for a circularly polarized initial photon, for which: $\tilde{\rho}(0) \equiv \tilde{\rho}(^{R})(0) = (0, 0, 1)$. One can then form the following combination of components of the above Stokes vectors:

$$R(t) = \frac{1}{\rho_3^{^{(R)}}(t)} \left[ \rho_1^{^{(R)}}(2t) + \rho_2^{^{(+)}}(2t) \frac{\rho_1^{^{(R)}}(t)}{\rho_2^{^{(R)}}(t)} \right]. \quad (29)$$

Using the explicit expressions for these Stokes components as obtained from (20), one easily finds: $R(t) = \exp[-2(a + \alpha - \gamma)t]$. Recall that complete positivity requires the combination $a + \alpha - \gamma$ to be nonnegative. Therefore, by measuring the components of the Stokes vectors appearing in (29), one can determine the quantity $R(t)$ and thus check whether or not

$$R(t) \leq 1. \quad (30)$$

Although this condition is in general only necessary for complete positivity, it becomes also sufficient for the phenomenologically relevant situation for which $\Lambda_1 = \Lambda_2$. Recalling (25), in this case one has $b = 0$ and further $a = \alpha$, so that $2\alpha \geq \gamma$ is the only surviving inequality of those listed in (19).

The two nonvanishing dissipative parameters $\alpha$ and $\gamma$ have now the meaning of inverse relaxation times for the off-diagonal and diagonal entries of the photon density matrix $\rho(t)$ [1]. They are usually called $1/T_2$ and $1/T_1$, respectively. The condition that assures the complete positivity of the open dynamics for photons

\footnote{Note that this inequality determines the sign of the combination $a + \alpha - \gamma$ for any $t$, i.e. irrespective of the length of the fiber, provided it behaves in a stochastic way. This fact might help reduce the systematic uncertainties in the actual experimental test of (29).}
travelling along the noisy fiber is therefore precisely $2T_1 \geq T_2$, as was mentioned in the introductory remarks.

In summary, we have shown that the condition assuring the complete positivity of the dynamics of polarized photons in a noisy fiber can be cast in the form (30). By measuring the combination $\mathcal{R}$ in (24), it can be tested using set-ups and techniques that are routinely used in optical laboratories. We find this possibility very intriguing and hope will trigger the interest of the vast community of specialists working in quantum optics experiments.

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