Overcoming the speckle correlation limit to achieve a fiber wavemeter with attometer resolution

Graham D. Bruce, Laura O’Donnell, Mingzhou Chen, and Kishan Dholakia
SUPA, School of Physics and Astronomy, University of St Andrews, North Haugh, St Andrews KY16 9SS, UK
(Dated: February 8, 2019)

The measurement of the wavelength of light using speckle is a promising tool for the realization of compact and precise wavemeters and spectrometers. However, the resolution of these devices is limited by strong correlations between the speckle patterns produced by closely-spaced wavelengths. Here, we show how principal component analysis of speckle images provides a route to overcome this limit. Using this, we demonstrate a compact wavemeter which measures wavelength changes of a stabilized diode laser of 5.3 am, eight orders of magnitude below the speckle correlation limit.

The scattering of coherent light by a disordered medium converts the input field’s spatial profile into a granular speckle pattern. Much research is devoted to minimizing or removing this effect. However, the precise speckle pattern produced is strongly dependent on the properties of both the light field and the scattering medium. As a result, speckle has been widely used as a sensor for changes in the scattering medium [11]. Alternatively, as the coherence of the light field is preserved, the speckle pattern can be used to gather information about the incoming beam if the scatterer is fixed in time. This has been harnessed in focusing, micromanipulation and imaging through turbid media [4,9] and in measuring the polarization [7], wavelength [8–18] and modal character [19,20] of the light. Precision measurement of the wavelength of light is fundamental to many fields of science, including laser spectroscopy, optical sensing and telecommunications [21]. The most common spectrometers use diffraction to spatially separate the wavelength components of light in a one-to-one spectral to spatial mapping, with the resolution therefore depending on the separation of diffracting element and detector. This one-to-one mapping is, however, not a prerequisite for spectral recovery, and tracking wavelength via speckle translates this measurement into a two dimensional problem, allowing for both high resolution and large bandwidth in an intrinsically compact design [18].

A particularly appealing method for generating laser speckle is a multi-mode optical fiber (MMF), due to its low cost, ability to transport light over long distances with minimal losses and small footprint [22]. When coherent light traverses such a fiber, multiple-scattering and modal interference produce a speckle pattern. The predominant method to measure wavelength using speckle is via the transmission matrix method [18]. In brief, a transmission matrix \( T \) is first constructed by recording the speckle patterns produced for a set of known wavelengths, where each column of \( T \) contains the intensity pattern produced by a different wavelength. The MMF is stabilized so that \( T \) is time-invariant. Provided that the wavelengths used to construct the transmission matrix produced uncorrelated speckle patterns [10], \( T \) can subsequently be used as a reference from which one can extract the unknown wavelength that produced a measured speckle pattern. Using this method, compact spectrometers with an operating range of \( \sim 1 \mu m \) have been realized [12].

FIG. 1. The speckle patterns produced by an 18 cm length of multi-mode fiber vary with wavelength. Strong correlations are present for wavelength changes within the speckle correlation limit (the HWHM of the correlation curve), which is \( \sim 620 \mu m \) in our system. The white scale bar is common to all images and denotes a distance of 2 mm.

The limit to the wavelength resolution, and thus the accuracy, of this method is the requirement that the speckle patterns produced by two closely-spaced wavelengths (separated by \( \Delta \lambda \)) are sufficiently uncorrelated. The speckle correlation function

\[
C(\Delta \lambda) = \left\langle \frac{\langle I(\lambda, x) I(\lambda+\Delta\lambda, x) \rangle_\lambda}{\langle I(\lambda, x) \rangle_\lambda \langle I(\lambda+\Delta\lambda, x) \rangle_\lambda} - 1 \right\rangle_x,
\]

compares the intensity \( I(\lambda, x) \) of images with spatial coordinates \( x \) recorded at a range of wavelengths \( \lambda \), where \( \langle \ldots \rangle_\lambda \) denotes an average over wavelengths and \( \langle \ldots \rangle_x \) denotes an average over position. The values of \( C \) are normalized such that \( C(0) = 1 \) [9]. For clarity, we illustrate this correlation function in Fig. [1] using the speckle patterns produced by an 18 cm length of MMF injected...
by light from an external cavity diode laser tuned between $\lambda = 775\text{nm}$ and $785\text{nm}$ in steps of 0.5 nm. For large wavelength separations the speckle patterns are distinct and $C \approx 0$. However, for small wavelength changes the speckle patterns are strongly correlated, i.e. $C \approx 1$. The resolution limit of the transmission matrix method is called the speckle correlation limit and defined as the wavelength separation for which $C = 0.5$, i.e. the HWHM of the correlation curve $\Xi$. In the absence of strong mode coupling within the fiber, it scales inversely with the length of the optical fiber [12, 23–25]. In the example shown in Fig. 1, the speckle correlation limit is $\sim 620\text{pm}$.

Recent work investigating speckle in other, less widely-used, optical systems has demonstrated wavelength measurements which are well-resolved below the correlation limit [11, 17], via principal component analysis (PCA). These measurements may be either of absolute wavelength or of relative wavelength changes. The reason for this improvement has thus far not been fully explained nor placed in context. Here, we show that PCA circumvents the speckle correlation limit to the wavelength resolution because it is designed to identify the optimal basis in which to measure the changes of speckle patterns. We find that this offers not only an advantage in resolution, but also in separating wavelength-dependent changes of speckle patterns from those due to environmental changes. We demonstrate for the first time that the combination of a MMF and PCA can produce a compact, atometer-resolved wavemeter, thereby measuring wavelength changes eight orders of magnitude below the speckle correlation limit.

We investigate the attainable resolution of speckle-based wavelength measurement using a speckle wavemeter comprising an 18 cm-long step-index MMF and a fast CMOS camera (Fig. 2a). The MMF core diameter is 105 $\mu$m with NA = 0.22 (ThorLabs FG105LCA). After exiting the MMF, the light propagates for 5 cm and is captured by the camera (Mikrotron EoSens 4CXP). Laser light for testing the speckle wavemeter is generated by an external cavity diode laser (Toptica DL-100, LD-0785-P220), which is stabilized to the $^{87}\text{Rb} D_2$ line ($F = 2 \rightarrow F' = 2 \times 3$ crossover) using saturated absorption spectroscopy and current modulation. To vary the wavelength of the light, we employ an acousto-optic modulator (AOM) (Crystal Technologies 3110-120) in a cat-eye double-pass configuration, with a modulation range of 20 fm. The wavelength tuning is set by Labview and passed to an RF function generator as control voltage $V_{\text{RF}}$. In this letter, all wavelength measurements are quoted relative to $\lambda_0 = 780.212840\text{nm}$. The light is coupled into an angle-cleaved single-mode fiber (SMF) (ThorLabs P5-780PM-FC-10) to eliminate spatial variations of the beam. This is connected to the MMF via a standard mating sleeve (Thorlabs ADAFC1), delivering 300 $\mu$W of power to the camera.

We use PCA on the images recorded on the camera to extract the wavelength, as previously detailed in Ref. [17]. Briefly, a training set of independently-normalized speckle patterns at a range of known wavelengths is obtained. The principal components (PCs) of the data are given by the eigenbasis of the covariance matrix of the training set. There exists a linear relationship between the value of the first principal component PC1 (i.e. the eigenvector with largest eigenvalue) and wavelength, with the proportionality constant determined by the training set. For a speckle pattern produced by an unknown wavelength, the wavelength is extracted by projecting the speckle pattern into the PC-space of the training set.

For wavelength changes above a femtometer, we benchmarked the speckle wavemeter against a High-Finesse WS7 wavemeter (Fig. 2b). A sinusoidal wavelength modulation with 2 fm amplitude was applied at 2.5 Hz using the AOM. We recorded images of $1024 \times 1024$ pixels at 100 fps and 2 $\mu$s exposure. An estimation of the relative accuracy of the speckle wavemeter and the commercial wavemeter is provided by the standard deviation of the residuals between these two measurements, which was 0.28 fm over the 3 s measurement.

Compared to the other scattering media to which PCA has been applied, the MMF has a relatively low numerical aperture and supports relatively few transverse optical modes. This gives rise to a coarse-grained speckle pattern which is ideally suited to illustrate the underlying mechanisms allowing measurements below the speckle correla-
non-constant amplitude of oscillation and a time-
such as the SSI does capture the 10 mHz modulation, but
ing the changes in speckle through an image-based metric
the AOM at 10 mHz with an amplitude of 1 fm. Track-
ment, the wavelength was modulated by
6400 s, recorded at 10 fps and 1
0.97
5000 5700 6400
-0.4
0
0.4
PC1
5000 5700 6400
0.94
0.97
1
SSI
(a)
(b)
(c)
FIG. 3. The distribution of the pixel-weightings in PC1 (top) resembles a typical speckle pattern from the training set (middle). However, mapping the dissimilarity between these (bottom) shows that PCA gives increased weight to low-intensity pixels near the edges of the speckles.

FIG. 4. (a) Image-based recovery of the wavelength based on parameters like structural similarity index are susceptible to corruption due to coupling between environmental- and wavelength-dependent variations of the speckle. (b-c) However, principal component analysis, by identifying the optimal basis to recover the wavelength, decouples environmental factors from the wavelength-dependent variations in the speckle.
dependent mean value. The amplitude of the SSI oscillation at 10 mHz varies by one order of magnitude, from a minimum of 0.08% to a maximum of 0.6%, while the mean value of the SSI also varies by 5.3%. This implies a wavelength drift at least 8.8 times larger than the con-
trol modulation. In contrast, evaluating the same speckle images with PCA shows a separation of the wavelength modenation and slower drifts in PC1 and PC2 respectively. The amplitude of the oscillations in PC1 (Fig. 4(a)) varies by 16%, with a mean wavelength drift of a factor 1.1 times the modulation, consistent with observations using the reference wavemeter. The remaining changes in the speckle patterns are instead captured by higher order PCs. We highlight PC2 in Fig. 4.

To investigate the resolution of the device, we subsequently applied wavelength modulations below femtometer-level to the light using the AOM. The cal-
ibrated modulation signal from Labview $V_{FM}$ (highlighted in green in Fig.2) was $100 \pm 4 \lambda m$ at $6.97 \pm 0.03 \text{kHz}$. We acquired 5,000 frames of $8 \times 1024$ pix-
els, with 3\mu s exposure at 48,000 fps. The response of the speckle wavemeter, shown in Fig.4, clearly re-
solves this wavelength modulation, but with notable dev-
ations from the sinusoidal modulation signal. Each data point is a measurement of the instantaneous wave-
length, extracted from a single image. However, computing the amplitude spectrum of the complete image set allows us to illustrate why discrepancies occur between the control modulation and the response of the speckle wavemeter, and to test the accuracy and precision of the wavemeter. The amplitude spectrum (Fig. 5c) of the speckle wavemeter signal shows a dominant peak at 6.96 ± 0.01 kHz (which is caused by the control voltage) along with additional, smaller, modulation components. Using the dominant peak as a benchmark, we determined that the second peak in the spectrum is caused by an additional sinusoidal wavelength modulation at 10.42 ± 0.01 kHz with an amplitude of 20.7 ± 0.7 am. This modulation was caused by the external oscillator signal (dither) used to wavelength-stabilize the laser. We verified our wavelength measurements against the modulation voltage \( V_{\text{mod}} \) (shown in blue in Fig. 2a) applied to the laser diode current driver (Thorlabs LDC-202B). The wavelength dependence on \( V_{\text{mod}} \) was 53 ± 4 am mV\(^{-1}\), and the predicted wavelength modulation due to the dither was therefore 22 ± 3 am. For the benchmarking voltage measurements, the quoted uncertainties are the combination of 5% calibration error and the standard error of five repetitions. The relative accuracy of the speckle wavemeter is within this errorbar, and the precision of the speckle wavemeter is on attometer scale.

To further demonstrate attometer-resolved measurements with the speckle wavemeter, we recorded a time-series with no AOM modulation (Fig. 5b). The amplitude spectrum (Fig. 5d) of the recorded signal shows wavelength modulations occurring at three distinct frequencies. The largest of these was the dither, which was used to benchmark the additional modulation components. A single tone at a modulation frequency of 5.70 ± 0.02 kHz and a 0.4 kHz-broad, multipeak feature centered at 12.8 kHz were caused respectively by a noise component in the lock-in amplifier and the switch-mode power supply of the photodetector shown in Fig. 2. The lock-in amplifier circuitry produced a component to \( V_{\text{mod}} \) at 5.7 ± 0.3 kHz with \( V_{pk} = 96 ± 3 \mu \text{V} \), giving a wavelength modulation of 5.1 ± 0.5 am. This matches the 5.3 ± 0.4 am measurement obtained with the speckle wavemeter (Fig. 5e). This measurement is well-resolved, with a peak larger than 10\( \sigma \), where \( \sigma \) is the standard deviation of the noise floor. The voltage noise from the switch-mode supply was added to the photodetector signal, and thus also to \( V_{\text{mod}} \). The most prominent component of this feature had an amplitude of \( V_{pk} = 133 ± 4 \mu \text{V} \), i.e. a 7.1 ± 0.7 am wavelength modulation. The measurement from the speckle wavemeter was in agreement, at 6.5 ± 0.7 am.

In conclusion, we have demonstrated wavelength measurements which are well-resolved despite being eight orders of magnitude below the speckle correlation limit, through the use of PCA. This approach can be used in tandem with existing transmission matrix methods to maintain broadband performance. At present the resolution is limited by the white noise floor of our experiment, and intriguingly it is invariant for fiber lengths up to 50 m. A detailed analysis of the limits of PCA will be the subject of future work, but promisingly the length-invariance suggests that the approach may suitable for on-chip integrated photonics. The observation that environmental and wavelength changes to the speckle patterns are separated between principal components raises the possibility that, by implementation of an appropriate training process, the speckle wavemeter could be used as a multi-parameter sensor. Additionally, we will test whether the approach can be extended from measurements of narrow-linewidth, monochromatic laser light into a spectrometer capable of measuring multiple wavelengths or spectra.

**ACKNOWLEDGMENTS**

We acknowledge funding from Leverhulme Trust (RPG-2017-197) and EPSRC (EP/R004854/1, EP/R004854/1) and thank Donatella Cassettari, Philip Ireland and Paloma Rodríguez-Sevilla for technical assistance and useful discussions. Research data supporting this publication can be accessed at https://doi.org/10.17630/c567afee-de2a-4c8c-9413-0f0f1bb8df64.
[1] I. Yamaguchi, Journal of Physics E: Scientific Instruments 14, 1270 (1981).
[2] D. A. Boas and A. K. Dunn, Journal of Biomedical Optics 15, 011109 (2010).
[3] D. Briers, D. D. Duncan, E. R. Hirst, S. J. Kirkpatrick, M. Larsson, W. Steenbergen, T. Stromberg, and O. B. Thompson, Journal of Biomedical Optics 18, 066018 (2013).
[4] I. Vellekoop and A. Mosk, Optics Letters 32, 2309 (2007).
[5] I. M. Vellekoop, A. Lagendijk, and A. P. Mosk, Nature Photonics 4, 320 (2010).
[6] T. Čízmár, M. Mazilu, and K. Dholakia, Nature Photonics 4, 388 (2010).
[7] T. W. Kohlgraf-Owens and A. Dogariu, Optics Letters 35, 2236 (2010).
[8] B. Redding and H. Cao, Optics Letters 37, 3384 (2012).
[9] B. Redding, S. F. Liew, R. Sarma, and H. Cao, Nature Photonics 7, 746 (2013).
[10] B. Redding, S. M. Popoff, and H. Cao, Optics Express 21, 6584 (2013).
[11] M. Mazilu, T. Vettenburg, A. Di Falco, and K. Dholakia, Optics Letters 39, 96 (2014).
[12] B. Redding, M. Alam, M. Seifert, and H. Cao, Optica 1, 175 (2014).
[13] B. Redding, S. M. Popoff, Y. Bromberg, M. A. Choma, and H. Cao, Applied Optics 53, 410 (2014).
[14] M. Chakrabarti, M. L. Jakobsen, and S. G. Hanson, Optics Letters 40, 3264 (2015).
[15] N. H. Wan, F. Meng, T. Schröder, R.-J. Shiue, E. H. Chen, and D. Englund, Nature Communications 6, 7762 (2015).
[16] S. F. Liew, B. Redding, M. A. Choma, H. D. Tagare, and H. Cao, Optics Letters 41, 2029 (2016).
[17] N. K. Metzger, R. Spsytvstev, G. D. Bruce, B. Miller, G. T. Maker, G. Malcolm, M. Mazilu, and K. Dholakia, Nature Communications 8, 15610 (2017).
[18] H. Cao, Journal of Optics 19, 060402 (2017).
[19] M. Mazilu, A. Mourka, T. Vettenburg, E. M. Wright, and K. Dholakia, Applied Physics Letters 100, 231115 (2012).
[20] A. Mourka, M. Mazilu, E. Wright, and K. Dholakia, Scientific Reports 3, 1422 (2013).
[21] W. Demtröder, Laser spectroscopy: Basic concepts and instrumentation (Springer, 2013).
[22] J. Hecht, Understanding Fiber Optics, 5th ed. (Laser Light Press, 2015).
[23] E. G. Rawson, J. W. Goodman, and R. E. Norton, Journal of the Optical Society of America 70, 968 (1980).
[24] W. Freude, C. Frische, G. Grau, and L. Shan-Da, Journal of Lightwave Technology 4, 6472 (1986).
[25] P. Hubina, Journal of Modern Optics 41, 1001 (1994).
[26] W. Zhou, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli, IEEE Transactions on Image Processing 13, 600612 (2004).
[27] A. Davis, M. Rubinstein, M. Wadhwa, G. Mysore, F. Durand, and W. T. Freeman, ACM Transactions on Graphics (Proc. SIGGRAPH) 33, 79 (2014).