Magnetic flux structures in various shaped composite structures with d- and s-wave superconductors (d-dots)

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Abstract. Utilizing phenomenological two-components Ginzburg-Landau equation, we investigate the vortex structures in nano-sized composite structures of d- and s-wave superconductors (d-dot), which shows spontaneous half-quantum magnetic fluxes. Especially, Effects of disorders in junctions between d- and s-wave superconductors are investigated. Holes in junction at the corners do not affect the magnetic flux patterns. Although the large hole at the corner changes the position of the flux. A wide disorder along the junction affects the magnetic structures much. At the edges of the non-disordered junction, there appear fractional fluxes.

1. Introduction
Nano-structured superconductors show much different property compared to bulk superconductors. Also, superconducting devices, especially superconducting qubit, have attracted much attention. Previously, we showed another candidate for such superconducting devices [1-17].

Our idea is based on the symmetry of high-Tc superconductors. Since the discovery of the high-Tc superconductors, the pairing symmetry, as well as its origin, has been controversial, but phase-sensitive experiment is crucial for determining the symmetry of the Cooper pairs. One such phase-sensitive experiment is a corner junction experiment [18, 19]. In this experiment, a square-shaped high-Tc superconductor is connected with a conventional s-wave superconductor with two Josephson junctions around a corner. Each junction is parallel to either the x- or the y-axis of the high-Tc superconductor. The critical current between the s- and the d-wave superconductors is zero under a zero external magnetic field because of the sign change of superconducting order parameter between Cooper pairs that go through the junction along the x direction and other Cooper pairs that go through...
another junction along the y direction. This is in apparent contrast with a corner junction between both s-wave superconductors for which the critical current is largest under zero external field. Therefore, from this experiment, the symmetry of the Cooper pairs in high-Tc superconductors was determined as d-wave, especially $d_{x^2-y^2}$. In addition, this experiment showed that the critical current became maximum when the total magnetic flux in the corner junction was $\Phi_0/2$. Here, $\Phi_0 = \hbar c/2e$ is the flux quantum. This means the state with a half-flux quantum magnetic flux is stable and that in the superconducting states of both superconductors, the stable states have a spontaneous magnetic flux at the corner without an external field. Experiments showing this property have been done. Especially, Hilgenkamp et al. showed in a zigzag junction between Nb and YBCO that a spontaneous magnetic flux appears at every corner under a zero external field [20]. This spontaneous magnetic flux is useful and can be used as a spin or a bit by itself.

In contrast to the previous approach for using the spontaneous half-flux quantum, we considered nano-sized d-wave superconductor embedded in a s-wave matrix \[1–17\]. We call it d-dot. We want to consider the whole d-dot system as one element, not as individual half-quantum fluxes. The single half-quantum flux is useful because there are two opposite directions that the flux can take. Also, our d-dot has two degenerate states if the spontaneous magnetic fluxes appear. This is because the state with spontaneous magnetic fluxes under zero external field breaks time-reversal symmetry. This property is independent of the shape, so a d-dot in any shape always has two degenerate stable states. Therefore, the d-dot as a whole can be considered as a single element with two level states. Experiments to establish these d-dots are now in progress \[21\].

In this study we investigate the magnetic flux structures for various shaped d-dot Especially we discuss the effect of disorders in the junctions. For this investigation, we use two-component Ginzburg-Landau equation and the finite element method, which are explained in next section.

2. Model

For analyzing the spontaneous half-quantum magnetic fluxes, we must take into account the anisotropy of d-wave superconductivity. Therefore we use following two-component Ginzburg-Landau free energy for d-wave superconductivity and s-wave superconductivity. For d-wave superconductors \[1,6,7,11,13,14,16,17\],

$$
F_d(\Delta_x, \Delta_y, A) = \int d\Omega \left[ \frac{3\alpha}{8} \lambda_d \left| \Delta_x \right|^2 + \frac{4}{3\alpha} \ln \frac{T_c}{T} \right] + \lambda_d \left| \Delta_y \right|^2 + \frac{2\alpha}{\alpha} \lambda_d \left| \Delta_y \right|^2 + \frac{1}{\alpha} \lambda_d \left| \Delta_y \right|^2 \\
+ \frac{1}{4} \alpha \lambda_d \nu_d^2 \left[ \Pi \Delta_y \right]^2 + 2 \left[ \Pi \Delta_y \right]^2 + \left( \Pi \Delta_y \Delta_y^* - \Pi \Delta_y \Delta_y^* + \Pi \Delta_y \Delta_y^* + h.c. \right] \\
+ \frac{1}{8\pi} \left[ \mathbf{H} - \mathbf{H}_c \right]^2 + \frac{1}{8\pi} \left( \text{div } A \right)^2 \right] d\Omega
$$

And for s-wave superconductors,
Here $\Delta_d$ and $\Delta_s$ are d- and s-wave superconducting order parameters, respectively. $\Pi = \frac{h}{i} \nabla - \frac{e}{c} A$ is the gauge-invariant derivative and $\alpha = \frac{7c(3)}{8(\pi T)}$. $\lambda_d$ and $\lambda_s$ are the strengths of the coupling constants for the d- and the s-wave interaction channels, respectively. We assume repulsive interaction for s-wave (d-wave) channel for d-wave superconductor (s-wave superconductor), respectively. $T_{cd}$ and $T_{cs}$ are transition temperatures for d- and s-wave superconductors, respectively. $H = \nabla \times A$ is a microscopic magnetic flux density and $\mathbf{H}$ is an external magnetic field.

In these free energies, an anisotropy of the d-wave superconductivity appears in the coupling term of the gradient of both order parameters, where the two terms that contain the gradients along the x and the y directions have different signs.

We also use alternative model, in which we consider the junction region explicitly. In the d-wave superconducting region, we only consider d-wave superconducting order parameter and in s-wave SC region, we only consider the s-wave SC order parameter and the free energy for each region is given as,

$$F_d(\Delta_d, A) = \int_\Omega \left( \frac{3}{8} \alpha_d \lambda_d \left[ |\Delta_d|^2 - \frac{4 \ln T_{cd}/T}{3 \alpha_d} \right] + \frac{3}{4} \alpha_d \lambda_d \left[ |\Pi \Delta_d|^2 \right] + \frac{1}{8 \pi} |h - H|^2 + \frac{1}{8 \pi} (\text{div } A)^2 \right) d\Omega,$$

(3)

$$F_s(\Delta_s, A) = \int_\Omega \left( \frac{3}{8} \alpha_s \lambda_s \left[ |\Delta_s|^2 - \frac{4 \ln T_{cs}/T}{3 \alpha_s} \right] + \frac{3}{4} \alpha_s \lambda_s \left[ |\Pi \Delta_s|^2 \right] + \frac{1}{8 \pi} |h - H|^2 + \frac{1}{8 \pi} (\text{div } A)^2 \right) d\Omega.$$

(4)

And in the junction region, we consider coupling between both order parameters as,

$$F_{js}(\Delta_d, \Delta_s, A) = \int_\Omega \left( \frac{3}{8} \lambda_d \lambda_s \left[ |\Delta_d|^2 - \frac{4 \ln T_{cd}/T}{3 \alpha_d} \right] + \frac{3}{4} \lambda_d \lambda_s \left[ |\Pi \Delta_d|^2 \right] + \frac{1}{8 \pi} |h - H|^2 + \frac{1}{8 \pi} (\text{div } A)^2 \right) d\Omega.$$

(5)

In order to minimize these free energies, we use the finite-element method [12–15] because we want to investigate variously shaped d-dots, especially disordered junction case in this study. We show an example of element partition in Fig. 1 for the alternative model. In this example the junction region is divided into smaller elements because the spatial variation of order parameters are rapid.

In the finite element method, we expand the order parameters and the vector potential in each element by using the area coordinates, which are defined for $e$-th element as

$$N_e^i(x, y) = \frac{1}{2S_e} \left( a_i^e x + b_i^e y + c_i^e \right) \quad (i = 1, 2, 3).$$

(6)
Here $a_i' = x^e_i y^e_j - x^e_j y^e_i$, $b_i' = y^e_i - y^e_j$, and $c_i' = x^e_i - x^e_j$, where $(i, j, k)$ is a cyclic permutation of $(1, 2, 3)$ and $(x^e_i, y^e_i)$ is the coordinate of $i$-th vertex of the $e$-th element. The order parameters and vector potential are given as,

$$\Delta_d (x, y) = \sum_{e=1}^{3} \Delta_{d_i}^e N^e_i (x, y),$$  \hspace{1cm} (7)

$$\Delta_s (x, y) = \sum_{e=1}^{3} \Delta_{s_i}^e N^e_i (x, y),$$  \hspace{1cm} (8)

$$A(x, y) = \sum_{e=1}^{3} A^e_i N^e_i (x, y),$$  \hspace{1cm} (9)

where $\Delta_{d_i}^e$ and $\Delta_{s_i}^e$ are the values of the d-wave and the s-wave order parameters at the $i$-th vertex in $e$-th element, respectively. And $A^e_i$ is the value of the vector potential at the $i$-th vertex in $e$-th element. Inserting these expression into previous free energy expression and minimizing it about those values, $\Delta_{d_i}^e$, $\Delta_{s_i}^e$ and $A^e_i$, we get the GL equations in the finite element method.

**Figure 1.** Example of finite elements partition.

3. Numerical Results

**Figure 2.** A square shaped d-dot. Configuration (a) and distribution of magnetic field (b) are shown.
First we show the magnetic field distribution for regular squared d-dot under zero external field in figure 1. There appear four spontaneous half-quantum magnetic fluxes at the corners of square-shaped d-wave superconducting region. They are aligned antiferromagnetically, because the repulsive interaction between magnetic fluxes. Also, in figure 2, we show the order parameter structures of s- and d- wave superconductivity. From this figure, interference of two order parameters in the junction region appears clearly.

![Figure 3](image)

**Figure 3.** A square shaped d-dot. Oder parameter structure for s-wave(a) and d-wave (b) are shown.

Next we consider the disorders in the junctions. In figure 4, we show the magnetic structures for d-dot with small disorder at the corners. As shown in figure 4 (a), the junction is broken at the corners. But the magnetic structures remain almost same as the perfect one (figure 2). This is because the spontaneous current between two superconductors flows around this hole. So small disorder at the corner does not affect the spontaneous current much.

![Figure 4](image)

**Figure 4.** A square shaped d-dot with disorder of junction at corners. Configuration (a) and distribution of magnetic field (b) are shown.

When the defect in the junction becomes large, then situation becomes different. As shown in figure 5, distribution of magnetic field is different from those in figures 2 and 4. In this case, the
magnetic fluxes appear inside of the d-wave superconducting region. In previous cases, the phase of the order parameter changes around a corner by $\pi$, because of the phase change of d-wave order parameter. But in this case, the phase of the s-wave order parameter fixed to be constant, and the phases of d-wave order parameter fixed at both edges around a corner to be continuously connected the s-wave order parameter. Then the phases of the d-wave order parameter at these edges are different by $\pi$ and in the d-wave region between these two edges the phase of d-wave order parameter varies gradually as shown in figure 6. So inside of d-wave SC region, current circulate around a point, which is apart from the corner. And half-quantum magnetic flux appears at this point. There is another solution where the phase change occurs around the corner, but it costs much kinetic energy because the current should flow large area around the large hole. Also because of this large hole, d-wave order parameter is free from the outside s-wave order parameter and its phase changes freely along this hole. In previous case, around the corner the phase of d-wave order parameter closely related to those of s-wave order parameter through the junction.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{A square shaped d-dot with large disorders of junction at corners. Configuration (a) and distribution of magnetic field (b) are shown.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Phase change of d-wave superconducting order parameter at the corner junction with a large hole.}
\end{figure}
Next we consider the case in which junction is broken in large area. In figure 7 we show an example where junction is broken over three corners. In this case, magnetic flux appears at the corner with good junction and the edges of the junction. There is no magnetic flux at the upper-right corner. Total magnetic flux is zero, because total phase change of order parameter around the d-wave region should be zero, otherwise the kinetic energy costs much. For other largely damaged junction cases, similar flux structures appear. Such magnetic flux structure were observed experimentally. Therefore avoiding disorder along the junction is crucial for forming clear magnetic flux structure.

\[\text{Figure 7. A square shaped d-dot with broken junction in large area. Configuration (a) and distribution of magnetic field (b) are shown.}\]

4. Summary and Conclusions

In this study we consider the magnetic flux structures in various shaped composite structures with d- and s-wave superconductors (d-dot). Especially we consider the effect of the broken junctions. Effect of disorders or holes at the corner junction depends on their size. For smaller holes, magnetic flux pattern remains as that of perfect case, but for larger holes, centers of spontaneous magnetic fluxes shift to inside of d-wave superconducting region. But overall structures remain same for both cases. Larger disorder along the junction is harmful to the magnetic flux structures.

Experimental realization of our d-dot system is a future problem.

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