Wave energy localization near the rough surfaces of piezoelectric waveguide. Near-surface inhomogeneities as a resonator or filter

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Abstract. Propagation of high-frequency electro-elastic normal wave signal in a composite waveguide is investigated. The roughnesses on the two surfaces of the piezoelectric waveguide are filled with an ideal conductor and ideal dielectric respectively. The both of the influence of surface roughness and the influence non-homogeneity of materials near the surfaces on the process of high frequency electro-elastic normal wave signal propagation, is discussed. The behaviors the frequency characteristics of wave in the composite waveguide at the propagation of normal wave signals numerically are investigated. The problem by the method of virtual cross sections and input of Magneto Elastic Layered System (MELS) hypotheses is solved. It is shown that in a case of not filled roughness of the piezoelectric layer surfaces, only two short wave modes are arise. The filling of the surface roughness with a dielectric or a conductor respectively, leads to the appearance of up to four such wave modes, depending at the wave signal length. The dispersion dependencies for all possible characteristic modes of shear electroelastic waves are given. It is shown that in a case of the propagation of slow waves, on certain wave lengths the silence frequency zones are arise.

Introduction

The localization of wave energy near body surfaces is ordinary at the propagation of wave signals in mediums with geometric constraints. It is known that the reason of localization near boundary sections of medium is the interruption of homogeneity of physicomechanical characteristics of fields, which leads to loads on surface sections of medium. Often, based on the technical requirements, this phenomenon, as unnecessary, maybe eliminated by proper selection of geometry of structural elements or by material characteristics of medium. But, often it is possible to take the advantage of the presence of such phenomenon and select the structural elements in different devices with appropriate geometrical and physical characteristics.

Different types of localization of the wave energy are found in the sources about elastic surface waves [1–4]. More details about the conditions of wave energy localization near the boundary sections of medium, their varieties depending on the nature of the surface compounds and their applications in various devices can be found in [5–9], etc.

The reason of distortion of the propagating normal wave signal or localization of wave energy, together with the effective physicomechanical characteristics of the material can also be geometric surface heterogeneities, such as roughness and waviness of the surfaces of the waveguide [10–15].
Surface roughness and waviness of the waveguide formally form peculiarly efficient, geometrically thin heterogeneous layers in the near-border areas of the waveguide [16–18].

In the proposed work we consider the problem of possible localization of wave energy near to rough surfaces of homogeneous piezoelectric waveguide at different electro-mechanical boundary conditions. The electro-mechanical boundary conditions, different by nature, are obtained due to filling of surface heterogeneities with dielectric or conductive materials.

1. The modeling and formulation of the mathematical boundary value problem
Let we have the piezoelectric layer \( \Omega(x, y) \equiv \{ |x| < \infty; h_-(x) \leq y \leq h_+(x); |z| < \infty \} \) with rough surfaces \( y = h_{\pm}(x) \), in Cartesian coordinate system \( \{x; y\} \). In generally, the surface roughness is described by random functions \( y = H(x; z) \), which at high accuracy are described by functions \( y = h_{\pm}(x) \in L_2 \). Without losing the generality in the further considerations, the surface roughness can be sets respectively by functions of weak inhomogeneity

\[
    h_-(x) = -h_0[1 + \varepsilon_- \sin(k_- x) + \delta_- \cos(k_- x)], \quad h_+(x) = h_0[1 + \varepsilon_+ \sin(k_+ x) + \delta_+ \cos(k_+ x)].
\]

Here the coefficients \( \varepsilon_{\pm} \) and \( \delta_{\pm} \) characterize the amplitude and the initial phase of surface roughness. Moreover \( \gamma_{\pm} = \sqrt{\varepsilon_{\pm}^2 + \delta_{\pm}^2} \ll 1 \) which is the height of the profile roughness, and multipliers \( k_{\pm} = 2\pi/\lambda_{\pm} \), where \( \lambda_{\pm} \) is the step of the profile roughness, characterize surfaces waviness respectively.

Assume that the surface roughness \( y = h_+(x) \) up to the surface \( y = h_0(1 + \gamma_+) \) with a perfect dielectric material is filled. The surface roughness \( y = h_-(x) \) up to the surface \( y = -h_0(1 + \gamma_-) \) with a perfect conductor material is filled.

Then we obtain a composite waveguide, consisting of three layers of variable thickness:

1. the conductor layer \( \Omega^c_-(x, y) \equiv \{ |x| < \infty; h_0(1 - \gamma_-) \leq y \leq h_-(x); |z| < \infty \} \) with a variable thickness \( \xi_c(x) \equiv |h_0(1 + \gamma_-) + h_-(x)| \),
2. the piezoelectric layer \( \Omega(x, y) \equiv \{ |x| < \infty; h_-(x) \leq y \leq h_+(x); |z| < \infty \} \) with a variable thickness \( \xi_p(x) \equiv |h_+(x) - h_-(x)| \),
3. the dielectric layer \( \Omega^d_+(x, y) \equiv \{ |x| < \infty; h_+(x) \leq y \leq h_0(1 + \gamma_+); |z| < \infty \} \) with a variable thickness \( \xi_d(x) \equiv h_0(1 + \gamma_+) - h_+(x) \).

During processing of the basic piezoelectric layer, in addition to the surfaces roughness, material heterogeneity also occurs in the near-surface zones. It is important to take into account that near-surface zones especially in studies on the propagation of shortwave signals in the composite waveguide. For accounting these heterogeneities in the near-surface zones take virtual sections \( y = h_0(1 - \gamma_+) \) and \( y = -h_0(1 - \gamma_-) \). Instead of the waveguide base layer of variable thickness, we already will consider a three-layer piezoelectric waveguide consisting of a base homogeneous layer

\[
    \Omega_0(x, y) \equiv \{ |x| < \infty; h_0(1 - \gamma_-) \leq y \leq h_0(1 - \gamma_+); |z| < \infty \},
\]

and two inhomogeneous, through the thickness, near-surface thin layers of variable thickness

\[
    \Omega^c_0(x, y) \equiv \{ |x| < \infty; h_-(x) \leq y \leq -h_0(1 - \gamma_-); |z| < \infty \}, \quad \Omega^d_0(x, y) \equiv \{ |x| < \infty; h_0(1 - \gamma_+) \leq y \leq h_+(x); |z| < \infty \}.
\]

Thus, in the near-surface zone at the surface \( y = h_-(x) \) will have a composite layer \( \Omega_0(x, y) \) composed of laterally inhomogeneous piezoelectric and homogeneous, perfectly conducting materials \( \Omega_-(x, y) = \Omega^c_0(x, y) \cup \Omega^d_0(x, y) \). Also in the near-surface zone at the surface
$y = h_+(x)$ will have a composite layer $\Omega_0(x, y)$ composed of homogeneous dielectric and laterally inhomogeneous piezoelectric materials $\Omega_+(x, y) = \Omega^p_+(x, y) \cup \Omega^d_+(x, y)$. Thus, the homogeneous piezoelectric waveguide surface roughness of which are filled, is modeled as a multilayered waveguide made of different materials. We will investigate the localization of the shear elastic wave in the formed near-surface inhomogeneous thin layers $\Omega_-(x, y) = \Omega^p_-(x, y) \cup \Omega^d_-(x, y)$ and $\Omega_+(x, y) = \Omega^p_+(x, y) \cup \Omega^d_+(x, y)$. Let us assume high-frequency (shortwave) elastic shear (SH) wave signal, whose length is much less than the base layer thickness $\lambda_0 \ll 2h_0$, is propagating in the composite waveguide.

The material of the main piezoelectric layer $\Omega_0(x, y)$ belongs to the tetragonal class 4mm, or to the class of hexagonal symmetries 6mm, for which, when the axis $\bar{\alpha}x_3$ is parallel to the axis of symmetry of the fourth (or sixth) order piezoelectric crystal $\bar{p}$, electroactive shear deformation $\{0; 0; w(x, y, t); \varphi(x, y, t)\}$ is separated from the non-electroactive plane deformation $\{u(x, y, t); v(x, y, t); 0; 0\}$. Quasi-static equations of electroelasticity for these crystals, in the base layer of the composite waveguide, have the following forms:

$$\nabla^2 w(x, y, t) = c_{015}^{-2} \cdot \ddot{w}(x, y, t), \quad \nabla^2 \varphi(x, y, t) = \frac{e_{15}}{\varepsilon_{11}} \nabla^2 w(x, y, t). \quad (5)$$

Here $c_{015}^2 \equiv G/\rho$ is the speed of the shear electroelastic wave in the homogeneous piezoelectric, $G$ is the shear modulus, $\rho$ is the density, $e_{15}$ is the piezoelectric modulus and $\varepsilon_{11}$ is the dielectric coefficient of the medium.

The equations of electroelasticity of laterally inhomogeneous piezoelectric layer already will solve in virtually selected layers $\Omega^p_\pm(x, y)$ respectively:

$$G_\pm \frac{\partial^2 w_\pm(x, y, t)}{\partial x^2} + e_\pm(y) \frac{\partial^2 \varphi_\pm(x, y, t)}{\partial x^2} + \frac{\partial \sigma^\pm_{yz}(x, y, t)}{\partial y} = \rho_\pm(y) \ddot{w}_\pm(x, y, t), \quad (6)$$

$$e_\pm(y) \frac{\partial^2 w_\pm(x, y, t)}{\partial x^2} - e_\pm(y) \frac{\partial^2 \varphi_\pm(x, y, t)}{\partial x^2} + \frac{\partial D^\pm_y(x, y, t)}{\partial y} = 0, \quad (7)$$

where the material relations for the component of mechanical stress and induction of the electric field have the forms

$$\sigma^\pm_{yz}(x, y, t) = G_\pm \frac{\partial w_\pm(x, y, t)}{\partial y} + e_\pm(y) \frac{\partial \varphi_\pm(x, y, t)}{\partial y}, \quad (8)$$

$$D^\pm_y(x, y, t) = e_\pm(y) \frac{\partial w_\pm(x, y, t)}{\partial y} - e_\pm(y) \frac{\partial \varphi_\pm(x, y, t)}{\partial y}. \quad (9)$$

The motion equation for perfectly conducting layer $\Omega^c$ will have in the following form

$$G_c \frac{\partial^2 w^c(x, y, t)}{\partial x^2} + \frac{\partial \sigma^c_{yz}(x, y, t)}{\partial y} = \rho_c \ddot{w}^c(x, y, t), \quad (10)$$

where the relation for mechanical shear stress is the following

$$\sigma^c_{yz}(x, y, t) = G_c \frac{\partial w^c(x, y, t)}{\partial y}. \quad (11)$$

The equations of elastic shear motion and electrostatics in the dielectric layer $\Omega^d$ will have the following forms

$$G_d^+ \frac{\partial^2 w^d_+(x, y, t)}{\partial x^2} + \frac{\partial \sigma^d_{yz}(x, y, t)}{\partial y} = \rho_d^+ \ddot{w}^d_+(x, y, t), \quad (12)$$

$$- \varepsilon_d^+ \frac{\partial^2 \varphi^d_+(x, y, t)}{\partial x^2} + \frac{\partial D^d_y(x, y, t)}{\partial y} = 0. \quad (13)$$
where the material relations for the component of mechanical stress and induction of the electric field have the forms

\[
\sigma_{yz}^d(x, y, t) = G_+^d \frac{\partial u_z^d(x, y, t)}{\partial y}, \quad D_y^d(x, y, t) = -\varepsilon_+^d \frac{\partial \varphi^d(x, y, t)}{\partial y}. \tag{13}
\]

The continuity conditions of the electromechanical fields of piezoelectric and the continuity conditions of the perfect conductor are satisfied on the rough surface \(y = -h_0 \cdot (1 + \gamma_-)\) of the perfectly conducting thin layer

\[
\sigma_{yz}^c(x, -h_0 - \gamma_-, t) = G_c^e \frac{\partial w^c(x, y, t)}{\partial y} \bigg|_{y = -h_0 - \gamma_-} = 0. \tag{14}
\]

The continuity conditions of the electromechanical fields of piezoelectric and the continuity conditions of the perfect conductor are satisfied on the rough surface \(y = h_-(x)\)

\[
w_-(x, h_-(x), t) = w^c(x, h_-(x), t), \quad \varphi_-(x, h_-(x), t) = 0, \tag{15}
\]

\[h'_-(x)\sigma_{xx}^c(x, h_-(x), t) + \sigma_{zy}^c(x, h_-(x), t) = h'_-(x)\sigma_{xx}^c(x, h_-(x), t) + \sigma_{zy}^c(x, h_-(x), t). \tag{16}
\]

The continuity conditions of the electromechanical fields of homogeneous and heterogeneous piezoelectric layers are satisfied on the virtually selected surface \(y = -h_0(1 - \gamma_-)\)

\[
w_0(x, -h_0(1 - \gamma_-), t) = w_-(x, -h_0(1 - \gamma_-), t), \tag{17}
\]

\[
\varphi_0(x, -h_0(1 - \gamma_-), t) = \varphi_-(x, -h_0(1 - \gamma_-), t),
\]

\[
\sigma_{yz}^0(x, -h_0(1 - \gamma_-), t) = \sigma_{yz}^c(x, -h_0(1 - \gamma_-), t), \tag{18}
\]

\[D_0^y(x, -h_0(1 - \gamma_-), t) = D_y^c(x, -h_0(1 - \gamma_-), t). \tag{19}
\]

Similarly, taking into account the fact of the continuity conditions of the electromechanical fields, on the mechanically free surface \(y = h_0(1 + \gamma_+),\) for an electric field of the vacuum half-space outside of the dielectric layer are satisfied:

\[
\varphi_+^d(x, h_0(1 + \gamma_+), t) = \varphi^d(x, h_0(1 + \gamma_+), t), \tag{20}
\]

\[
\sigma_{yz}^d(x, h_0(1 + \gamma_+), t) = G_+^d \frac{\partial w_z^d(x, y, t)}{\partial y} \bigg|_{y = h_0(1 + \gamma_+)} = 0, \tag{21}
\]

\[D_y^d(x, h_0(1 + \gamma_+), t) = -\varepsilon_+^d \frac{\partial \varphi^d(x, y, t)}{\partial y} \bigg|_{y = h_0(1 + \gamma_+)}. \tag{22}
\]

The continuity conditions of electromechanical fields, considering surface roughness, are satisfied on the rough surface \(y = h_+(x)\) respectively

\[
w_+(x, h_+(x), t) = u_z^d(x, h_+(x), t), \varphi_+(x, h_+(x), t) = \varphi_+^d(x, h_+(x), t), \tag{23}
\]

\[h'_+(x)\sigma_{xx}^c(x, h_+(x), t) + \sigma_{zy}^c(x, h_+(x), t) = h'_+(x)\sigma_{xx}^c(x, h_+(x), t) + \sigma_{zy}^c(x, h_+(x), t),
\]

\[h'_+(x)D_x^d(x, h_+(x), t) + D_y^d(x, h_+(x), t) = h'_+(x)D_x^d(x, h_+(x), t) + D_y^d(x, h_+(x), t). \tag{24}
\]

On the virtually selected surface \(y = h_0(1 - \gamma_+),\) respectively are satisfied the continuity conditions of electromechanical fields

\[
w_0(x, h_0(1 - \gamma_+), t) = w_+(x, h_0(1 - \gamma_+), t), \quad \varphi_0(x, h_0(1 - \gamma_+), t) = \varphi_+(x, h_0(1 - \gamma_+), t), \tag{25}
\]

\[
\sigma_{yz}^0(x, h_0(1 - \gamma_+), t) = \sigma_{yz}^c(x, h_0(1 - \gamma_+), t), \quad D_0^y(x, h_0(1 - \gamma_+), t) = D_y^c(x, h_0(1 - \gamma_+), t). \tag{26}
\]
It is shown from the introduced boundary conditions, that tangential components of mechanical strain and induction of electric fields are participating in conditions (16) and (24) due to the rough surfaces $h_{±}(x)$ respectively. The tangential components of mechanical strain and the induction of electric fields have the following forms

\[
\begin{align*}
\sigma_{xx}^{\pm}(x, y, t) &= G_{\pm}(y) \frac{\partial w_{\pm}(x, y, t)}{\partial x} + e_{\pm}(y) \frac{\partial \varphi_{\pm}(x, y, t)}{\partial x}, \\
D_{xx}^{\pm}(x, y, t) &= e_{\pm}(y) \frac{\partial w_{\pm}(x, y, t)}{\partial x} - \varepsilon_{\pm}(y) \frac{\partial \varphi_{\pm}(x, y, t)}{\partial x}, \\
\sigma_{xx}^{d}(x, y, t) &= G_{d} \frac{\partial w_{d}(x, y, t)}{\partial x}, \\
D_{x}^{d}(x, y, t) &= -\varepsilon_{d} \frac{\partial \varphi_{d}(x, y, t)}{\partial x}, \\
\sigma_{xx}^{e}(x, y, t) &= G_{e} \frac{\partial w_{e}(x, y, t)}{\partial x}.
\end{align*}
\]

The values of potential and normal component of induction of the electric field of the vacuum half-space on the surface $y = h_{0}(1 + \gamma_{±})$ are involved in the boundary conditions (19) and (20) too. Quasi-static potential of the electric field $\varphi^{(e)}(x, y, t)$ is determined from the equation

\[
\nabla^{2} \varphi^{(e)}(x, y, t) = 0.
\]

Considering its decay at infinity $y \to \infty$, it will have the following form

\[
\varphi^{(e)}(x, y, t) = E_{0} e^{-ky} e^{i(kx - \omega t)}.
\]

Thus, the homogeneous piezoelectric waveguide with geometrically heterogeneous surfaces, smoothed by dielectric and perfectly conducting materials, is modeled as a multilayer waveguide of different materials.

**2. The problem solution: hypothetical approach on distributions of the electroelastic waves characteristics**

The obtained boundary-value problem from a mathematical point of view is complicated by the fact that the equations of electroelasticity (6) and (7) for laterally inhomogeneous piezoelectric, with variable coefficients, should be solved in virtually selected both layers of variable thicknesses $\Omega_{\pm}^{b}(x, y)$ and $\Omega_{\pm}^{e}(x, y)$. In addition, there are boundary conditions with variable coefficients on the rough surfaces $y = h_{-}(x)$ and $y = h_{+}(x)$.

To avoid from mathematical complexities, for building the solution of the mathematical boundary value problem apply a hypothetical approach.

The normal wave solution of the system of equations (5) in the base homogeneous piezoelectric layer $\Omega_{0}(x, y)$, will be written in the following form

\[
\begin{align*}
w_{0}(x, y, t) &= \left[A_{0} e^{\alpha_{0}ky} + B_{0} e^{-\alpha_{0}ky}\right] e^{i(kx - \omega_{0}t)}, \\
\varphi_{0}(x, y, t) &= \left[C_{0} e^{ky} + D_{0} e^{-ky} + \frac{\varepsilon_{15}}{\varepsilon_{11}} \left[A_{0} e^{\alpha_{0}ky} + B_{0} e^{-\alpha_{0}ky}\right]\right] e^{i(kx - \omega_{0}t)}.
\end{align*}
\]

Here $\alpha_{0} \triangleq \sqrt{1 - \eta_{0}^{2}}$ is the formation coefficient of elastic waves through the thickness of the base layer, and $\eta_{0} \triangleq (\omega/k)(\rho_{0}/G_{0})^{1/2}$ is the phase velocity of the normal wave in the baselayer $\Omega_{0}(x, y)$.

Considering the thinness of the other four boundaries virtually selected layers and the heterogeneity of $\Omega_{\pm}^{b}(x, y)$ and $\Omega_{\pm}^{e}(x, y)$ layers, through the thickness of each layer we input hypotheses MELS [16–18] upon to distributions of elastic shear and potential of electric field.
The elastic shear and electric field potential in the virtually selected heterogeneous piezoelectric layer $\Omega_+^P$ introduce in the following forms

$$w_+(x,y,t) = f_+\left( k h_0; \frac{h_+(x)}{h_0} \right) [w_+(x,h_+(x),t) - w_0(x,h_0(1-\gamma_+),t)] + w_0(x,h_0(1-\gamma_+),t), \quad (34)$$

$$\varphi_+(x,y,t) = f_+\left( k h_0; \frac{h_+(x)}{h_0} \right) [\varphi_+(x,h_+(x),t) - \varphi_0(x,h_0(1-\gamma_+),t)] + \varphi_0(x,h_0(1-\gamma_+),t). \quad (35)$$

Here $f_+(k h_0; h_+(x)/h_0) \triangleq \sinh\{\alpha_+ k[y-h_0(1-\gamma_+)\}]/\sinh\{\alpha_+ k[h_+(x)-h_0(1-\gamma_+)]\}$ is the distribution function (or formation) of electromechanical field in heterogeneous piezoelectric layer, corresponding to the electroelasticity equations (6) and (7). Obviously, here the formation function $f_+(k h_0; h_+(x))$ of the indefinite characteristics of the wave field is represented by the formation coefficient $\alpha_+(k) \triangleq [\rho_+ \omega_0^2/(k^2 G_+) - 1]^{1/2}$ and by the variable thickness $\xi_+(x) \triangleq h_+(x) - h_0(1-\gamma_+)$ of the layer.

Similarly, the elastic shear and potential of electric field in the homogeneous dielectric layer $\Omega_+^D$ introduce in forms

$$w_+(x,y,t) = f_+\left( k h_0; \frac{h_+(x)}{h_0} \right) [w_+(x,h_0(1+\gamma_+),t) - w_+(x,h_+(x),t)] + w_+(x,h_+(x),t), \quad (36)$$

$$\varphi_+(x,y,t) = f_+\left( k h_0; \frac{h_+(x)}{h_0} \right) [\varphi_+(x,h_0(1+\gamma_+),t) - \varphi_+(x,h_+(x),t)] + \varphi_+(x,h_+(x),t), \quad (37)$$

wherein the homogeneous dielectric layer the formation coefficient $f_+(k h_0; h_+(x)/h_0) \triangleq \sinh\{\alpha_+ k[y-h_+(x)]\}/\sinh\{\alpha_+ k[h_0(1+\gamma_+)-h_+(x)]\}$ already is presented by the appropriate parameters of the homogeneous layer $\alpha_+(k) \triangleq [\rho_+ \omega_0^2/(k^2 G_+) - 1]^{1/2}$ and $\xi_+(x) \triangleq h_0(1+\gamma_+)-h_+(x)$. In this case the representations (36) and (37) are automatically satisfied to the boundary conditions (23) and (25).

Analogically, the elastic shear and potential of electric field in the virtually selected heterogeneous piezoelectric layer $\Omega_-^P$ are introduced in the following forms

$$w_-(x,y,t) = f_-\left( k h_0; \frac{h_+(x)}{h_0} \right) [w_-(x,h_-(x),t) - w_0(x,-h_0(1-\gamma_-),t)] + w_0(x,-h_0(1-\gamma_-),t), \quad (38)$$

$$\varphi_-(x,y,t) = \left[ 1 - f_-\left( k h_0; \frac{h_+(x)}{h_0} \right) \right] \varphi_0(x,-h_0(1+\gamma_-),t), \quad (39)$$

wherein the inhomogeneous piezoelectric layer the formation function $f_-\left( k h_0; h_+(x)/h_0 \right) \triangleq \sinh\{\alpha_- k[y+h_0(1-\gamma_-)]\}/\sinh\{\alpha_- k[h_-(x)+h_0(1-\gamma_-)]\}$ is represented by new formation coefficient $\alpha_- (k) \triangleq [\rho_- \omega_0^2/(k^2 G_-) - 1]^{1/2}$ and variable thickness $\xi_-(x) \triangleq h_0(1-\gamma_-)-h_-(x)$ for the given layer.

The potential of the electric field is absent in the perfectly conducting layer $\Omega_-^C$, and for elastic shear will have the following representation

$$w_-(x,y,t) = f_-\left( k h_0; \frac{h_+(x)}{h_0} \right) [w_-(x,-h_0(1+\gamma_-),t) - w_-(x,h_-(x),t)] + w_-(x,h_-(x),t), \quad (40)$$

wherein the homogeneous perfectly conducting layer the formation function $f_-\left( k h_0; h_+(x)/h_0 \right) \triangleq \sinh\{\alpha_- k[y+h_-(x)]\}/\sinh\{\alpha_- k[-h_0(1+\gamma_-)-h_-(x)]\}$ is represented by formation coefficient $\alpha_- (k) \triangleq [\rho_- \omega_0^2/(k^2 G_-) - 1]^{1/2}$ and variable thickness $\xi_-(x) \triangleq h_-(x) - h_0(1+\gamma_-)$.

By the selection of formation functions and hypothetical representations (34)–(40), the boundary conditions (15), (17), (20), (23), and (25) for elastic shear and electric field potential are automatically satisfied.
In addition, the characteristic formation coefficients for each layer are involved in the distribution representations, as well as electromechanical field values on surfaces of adjacent layers are involved.

The elastic shear and electric field potential values on all layers are expressed by arbitrary amplitude constants of normal wave signal in the base piezoelectric waveguide and vacuum half-space \( \{A_0, B_0, C_0, D_0, E_0\} \).

Satisfying to the boundary conditions (16), (19), (22), and (24), we obtain a system of five homogeneous algebraic equations related to amplitude constants. The dispersion equation of the formed wave field is obtained from the condition of existence of nontrivial solutions in the following form

\[
 g_{35}(\alpha_d; \varepsilon^{(e)}; h_+(x); k h_0) \det \| g_{ij}(G_k; \rho_k; e_k; \varepsilon_k; h_\pm(x); \omega_0; k(x, \omega_0)) \|_{4 \times 4} = 0, \tag{41}
\]

where the variable coefficients \( \{g_{ij}(G_k; \rho_k; e_k; \varepsilon_k; h_\pm(x); \omega_0; k(x, \omega_0))\}_{4 \times 4} \) (tensor) of dispersion equation have bulky appearance. The coefficients of the fifth column of the tensor equal to zero \( g_{15} = g_{25} = g_{45} = g_{55} = 0 \). \( g_{35}(\alpha_d; \varepsilon^{(e)}/\varepsilon_d; h_+(x); k h_0) \geq 0 \) is positively definite and characterizes oscillations of the electric field in vacuum.

From the coefficient relations it is easy to see, that the amplitudes and the frequencies of wave forms of the propagating electroacoustic wave in the waveguide are depend on both of the physicomechanical constants of boundary materials and characteristic linear dimensions of the not-smoothness surface.

### 3. Numerical calculation and comparative analysis

The study on the propagation of high-frequency (shortwave \( k h_0 \gg 1 \)) wave signal in waveguides with rough surfaces, of course are due to the fact that the linear dimensions of these roughness are small compared to the thickness of the base layer \( \gamma_\pm = \sqrt{\varepsilon_\pm^2 + \delta_\pm^2} \ll 1 \). Also in paper [20], it is shown that the interaction of propagating waves and weak roughness hardly occurs at the propagation of long-wave signals. On the basis of numerical calculations are taken the numerical test data of material constants for appropriate layers, shown in table 1, as well as the geometric linear dimensions of the base layer and the surface roughness (\( h_0 = 1, \varepsilon_\pm = \delta_\pm = 1/100 \)).

**Frequency characteristic of propagating wave**

The dispersion equation (10) certainly does not have intuitive analytical solutions. It has already been said that in the case of long-wave approximation, when \( k h_0 \ll 1 \) the normal propagating wave signal does not interact with the surface roughness. In the case of propagation of short-wave (high-frequency) electro-elastic signal, when \( k h_0 \gg 1 \) the presence of a surface geometrical heterogeneity leads to the wave number dependence on the coordinates of the propagation \( \exp[k_n x - \omega t] \).

In this case, we can ignore the damped, from the surface up to the depth of base layer, wave forms of type \( \exp[-\alpha_i(\omega_0; k_n)y] \) and obtain two unrelated tasks of half-spaces with rough surfaces, which are filled with dielectric and conductor materials. Then we will lose the ability to accurately calculate the influence of surface roughness on the forming waves in the base layer of the waveguide.

Therefore, quantitatively small, but qualitatively important components are saved in the calculations.

For the comparative analysis, first we present the frequency characteristic of the propagating plane electro-elastic wave signal in piezoelectric homogeneous waveguide with mechanically free, rough surfaces, when one surface of the waveguide is electrically open and the other surface is
Table 1. Numerical test data of constants of composite waveguide materials.

| Material          | $G_i = c_{44}^{(i)}$ | $\rho_i$ | $\varepsilon_1 = \varepsilon_{11}^{(i)}$ | $\varepsilon_1 = \varepsilon_{15}^{(i)}$ |
|-------------------|----------------------|----------|------------------------------------------|------------------------------------------|
| Piezocrystal of class 1 | $1.49 \times 10^{10}$ N/m² | $4.82 \times 10^3$ kg/m³ | $7.99 \times 10^{-11}$ F/m | $-0.21$ |
| Piezocrystal of class 6mm (4mm) | $1.639 \times 10^{10}$ N/m² | $5.302 \times 10^3$ kg/m³ | $8.789 \times 10^{-11}$ F/m | $-0.231$ |
| Piezocrystal of class 6mm (4mm) ±10% | $1.341 \times 10^{10}$ N/m² | $4.338 \times 10^3$ kg/m³ | $7.191 \times 10^{-11}$ F/m | $-0.189$ |
| Dielectric        | $1.788 \times 10^{10}$ N/m² | $5.784 \times 10^3$ kg/m³ | $9.588 \times 10^{-11}$ F/m |             |
| Conductor         | $1.788 \times 10^{10}$ N/m² | $5.784 \times 10^3$ kg/m³ |                                 |             |
| Vacuum            | $1.192 \times 10^{10}$ N/m² | $3.856 \times 10^3$ kg/m³ | $6.392 \times 10^{-11}$ F/m     |             |

Figure 1. Dependence of wave number $k_n(\omega)$ on source frequency
It is also interesting that approximately when $k_n(\omega) \geq 350$, ultra short wave solutions do not exist. The investigation of emergent frequency images gives interesting results, when one of the waveguide surface roughness is filled with a perfect conductor and the other is filled with a good dielectric.

The calculations are show that the dependence of wave number $k(\omega)$ of the electroelastic normal waves with harmonic oscillations is almost identical to the previous case (figure 1 ↔ figure 3) at low frequency (long-wave) signals, up to some length $k_{0n}(\omega)$. This length by the physicomechanical constants of adjacent materials and by the ratio of geometrical linear dimensions of the base layer and surface roughness of the waveguide is determined. This means that surface weak heterogeneities, and very thin material layers on the surfaces of the layer of the waveguide do not have any effect on the low frequency, electro-elastic wave signal at its propagation.

The dispersion surface and the dependence of the wave number for waves with non-harmonic distribution $\sinh\{\alpha_0 k[y - h_\pm(x)]\}$ are shown in figures 2 and 4, respectively, from where it is obvious that the dispersion surface at high frequency (short wave) signal varies strongly. Here, as in the previous case, high frequencies lead to weak, quantitative change of the second wave with the wave number $k_{02}(\omega)$, which is well seen on figure 4. More interesting transformation occurs with a low-frequency form, with the corresponding wave number $k_{01}$. At relatively short wave signals $k_{01} \sim 25$ ($\lambda_{01} \sim 0.25\, \text{mm}$), wave number $k_{11}(\omega)$ strongly changes the direction, opening space for the emergence of new wave modes (figure 4). The wave number $k_{12}(\omega)$ of the newly emerged wave mode at first decreases, making the leap on the vertical $x_{01} = \text{const}$, and then increases up to the limit of the existence of high frequency oscillations.

According to the same scheme, two high frequency wave modes (figure 4) with changeable...
wave numbers $k_{31}(x)$ and $k_{41}(x)$ occur there. It is interesting that the existence limit of these ultrashort waves again is the same $k_{n1} \leq 350$.

It follows from figure 4, that at higher frequencies of the wave signal occurs branching of first low-frequency harmonic (figure 1) on four waves with different wave lengths $\lambda_{1n}(\omega) = 2\pi/k_{1n}(\omega)$ respectively. So, it means that the function $k_{n1}(\omega)$ has multiple branches which are not intersecting.

On some points the branches are becoming very closer to each other which is shown on figures 5 and 6.

Different orientations of the closer curves describing wave numbers, implies that there is fuss of new mode due to the surface roughness of the waveguide (figure 1), which is dissected on newly formed wave modes $k_{12}(\omega)$, $k_{13}(\omega)$, and $k_{14}(\omega)$ under wave interaction of the first main mode. Moreover, for all of the newly formed wave modes on the primary phase section $x_{01} = \text{const}$.

Such branching of course is a consequence of the wave signal dissipation on surface roughness.
Figure 7. Dependence of fast, long wave frequency $\omega(k)$ from wave number, when $k \in [0; 1.6]$

Figure 8. Dependence of fast, short wave frequency $\omega(k)$ from wave number, when $k \in [1.6; 350]$

and scattering of wave energy along selected layers of the waveguide. It also follows from figure 2, that the branching into different lengths (on different wave number) occurs at different wavelengths $\lambda_{1n}(\omega) = 2\pi/k_{1n}(\omega)$ (at different values of wave number $k_{1n}(\omega)$), which should lead to different dispersions. For fast waves when the phase speed is greater than values of shear body waves in the adjacent materials $V_{\phi}(k; \omega) \geq c_{nt}$, the dispersion of long waves, when $k \in [0; 1.6]$, happens in the interval $\omega(k) \in [0; 5000]$ (figure 2) and is close to the value $\omega_{01}(k) \approx 316000$. figure 3 shows that, the second frequency is induced at a certain value of wavelength. Also, for the fast, short wave the frequency is quite great $\omega(k) \in [17 \times 10^4; 8 \times 10^5]$. It is necessary to pay attention to the fact that, starting from some value of the wave length, a wave with a specific length can be propagated with three different frequencies.

In the case of slow wave signals, when the phase velocity is less than the values of shear body waves in the adjacent materials $V_{\phi}(k; \omega) < c_{nt}$, receive an interesting phase picture (figures 7 and 8).

It is seen from figure 9 that in contrast to the fast, long waves (figure 10), where to each wavelength corresponds two frequency values, here to each frequency value correspond two wave modes with different wavelengths. In this case the interval defining long $\lambda \in [2\pi/1.6; 2\pi/0.5]$ is larger than in the case of faster, longer waves. For slow, short waves, when $\lambda \in (\pi/175; \pi/90]$, to
Figure 9. Dependence of slow, long wave frequency $\omega(k)$ from wave number, when $k \in [0; 0.5]$

Figure 10. Dependence of slow, short wave frequency $\omega(k)$ from wave number, when $k \in [60; 350]$

each wave length corresponds two frequencies, and in the interval $\lambda \in (\pi/90; \pi/30]$ correspond already three oscillation frequencies. In the case of slow waves, it is noteworthy that there is a frequency zone of silence. For waves of the length $\lambda \in [\pi/30; 2\pi/0.5]$, frequencies does not exist $\omega(k) \in \emptyset$. Comparing figures 1–3 with figures 8–10 it is easy to see that, if long waves are propagating in the range of relatively low frequencies $\omega_{s,l} \in [0; 300]$ and $\omega_{q,l} \in [0; 5000]$, then short waves are propagating in the range of very high frequencies $\omega_{s,l} \in [170000; 800000]$ and $\omega_{q,l} \in [0; 600000]$.

Conclusion

A mathematical modeling of the problem on propagation of electro-elastic wave signal shear in a homogeneous piezoelectric waveguide with filled surface roughness is suggested. Using MELS hypotheses (hypothesis of Magneto Elastic Layered Systems), analytical distribution of the elastic shear and the electric potential in the base layer, as well as in each formed layer of the composite waveguide are built by inputting hypothesis of MELS. Numerically investigated the amplitude distribution and frequency characteristics of the wave field in the composite waveguide at the propagation of normal wave signal. It is shown that, in the case of non-filled surface roughness of the waveguide occur only one short-wave mode (figure 1), but the case of filled surface roughness leads to the appearance of up to four such wave modes, depending on the length of the wave signal (figure 2). The dispersion dependence $\omega(k)$ of all possible characteristic modes of shear elastic waves is shown (figures 7–10). It is shown that in the case of slow wave propagation $V_{\phi}(k; \omega) < c_{nl}$, occurs frequency zone of silence for waves with length $\lambda \in [\pi/30; 2\pi/0.5]$. 
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References
[1] Rayleigh L 1885 On waves propagated along the plane surface of an elastic solid Proc. London Math. Soc. 1-17 (1) 4–11
[2] Love A E H 1911 Some Problems of Geodynamics (Cambridge University Press)
[3] Lamb H 1917 On waves in an elastic plate Proc. Roy. Soc. London 93 (648) 114–28
[4] Bleustein J L 1968 A new surface wave in piezoelectric materials Appl. Phys. Lett. 13 (12) 412–3
[5] Avetisyan A S 2019 Wave Dynamics, Mechanics and Physics of Microstructured Metamaterials, Theoretical and Experimental Methods (Springer International Publishing)
[6] Achenbach J D 1984 Wave Propagation in Elastic Solids (New York: Elsevier)
[7] Biryukov S V, Gulyaev Y V, Krylov V, and Plessky V 1995 Surface Acoustic Waves in Inhomogeneous Media (Springer-Verlag Berlin Heidelberg)
[8] Royer D and Dieulesaint E 2000 Elastic Waves in Solids I: Free and Guided Propagation (Springer-Verlag Berlin Heidelberg)
[9] Brekhovskikh L 1980 Waves in Layered Media (Academic Press)
[10] Flannery C M and von Kiedrowski H 2002 Effects of surface roughness on surface acoustic wave propagation in semiconductor materials Ultrasonics 40 (1-8) 83–7
[11] Svetovoy V B and Palasantzas G 2015 Influence of surface roughness on dispersion forces Adv. Colloid Interface Sci. 216 1–19
[12] Singh S S 2011 Love wave at a layer medium bounded by irregular boundary surfaces J. Vibr. Control 17 (5) 789–95
[13] Potel C, Bruneau M, N'Djomo L C F, et al. 2015 Shear horizontal acoustic waves propagating along two isotropic solid plates bonded with a non-dissipative adhesive layer: Effects of the rough interfaces J. Appl. Phys. 118 224904
[14] Apostol B F 2014 The effect of surface inhomogeneities on the propagation of elastic waves J. Elasticity 114 (1) 85–99
[15] Valier-Brasier T, Potel C, Bruneau M, et al. 2011 Coupling of shear acoustic waves by gratings: Analytical and experimental analysis of spatial periodicity effect Acta Acustica United with Acustica 97 (5) 717–27
[16] Avetisyan A S 2015 On the formulation of the electro-elasticity theory boundary value problems for electromagneto-elastic composites with interface roughness Proc. NAS Armenia. Mech. 68 (2) 29–42
[17] Avetisyan A S 2016 The efficiency of application of virtual cross-sections method and hypotheses MELS in problems of wave signal propagation in elastic waveguides with rough surfaces J. Advances Phys. 11 (7) 3564–74
[18] Avetisyan A S 2015 The boundary problem modelling of rough surfaces continuous media with coupled physicomechanical fields Rep. NAS Armenia 115 (2) 119–31
[19] Avetisyan A S, Belubekyan M V, and Ghazaryan K B 2016 In Collection of Scientific Papers of the Int. Conf. “Modern Problems of Thermodynamics” pp. 142–44
[20] Hunanyan A A 2016 The instability of shear normal wave in elastic waveguide of weakly inhomogeneous material Proc. NAS Armenia. Mech. 69 (3) 16–27