Virtual Compton scattering off nuclei in the $\Delta$-resonance region

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Abstract

Virtual Compton scattering in the $\Delta$-resonance region is considered in the case of a target nucleus. The discussion involves generalized polarizabilities and is developed for zero-spin nuclei, focusing on the new information coming from virtual Compton scattering in comparison with real Compton scattering.

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Virtual Compton scattering (VCS) as a process where a space-like virtual photon is scattered into a real photon has recently attracted much interest in the case of a nucleon target [1]. In the hard-scattering regime it allows a stringent test of perturbative QCD [2], while below pion threshold it opens the possibility of measuring new electromagnetic observables which generalize the usual electric and magnetic polarizabilities [3]. These observables are clean probes of the non-perturbative structure of the nucleon and are complementary to the elastic form factors.

Compared to real Compton scattering, VCS experiments are much more difficult because of the necessary interplay between the Bethe-Heitler (BH) process and the full VCS process. The radiative corrections must be calculated with sufficient reliability in order to be able to extract the interesting information on the target structure. On the other hand, the increased number of available observables suggests that VCS could be a promising field of investigation also with nuclear targets.

In real Compton scattering on nuclei with polarized photons two structure functions contribute to the cross section [4, 5]. The first structure function, $W_T$, is the same quantity determined by scattering of unpolarized photons and is the incoherent sum of photon-helicity flip and non-flip contributions [6]. The second structure function, $W_{TT}$, contains interference contributions from helicity flip and non-flip amplitudes. Thus their separated determination is important for disentangling different reaction mechanisms. Such a separation has been achieved in a recent experiment on $^4$He in the $\Delta$-resonance region [7] adding further information to the precise data recently obtained at Mainz with tagged photons [8]. In this region $W_T$ and $W_{TT}$ are differently related to the resonant and background contributions. In a recent analysis of the data within the frame of the $\Delta$-hole model [9] a rather satisfactory agreement between theory and data is obtained under resonance conditions, while below the $\Delta$-resonance energy discrepancies are found at backward angles which can be ascribed to some lacking background mechanism.

The independent variation of energy and momentum transfer available in electron scattering makes VCS in the same energy region a suitable tool to investigate the relative importance of the longitudinal/transverse components of the current and the behaviour of background contributions as a function of the photon momentum transfer. In this letter we study the general nuclear VCS response and discuss the particular case of zero-spin nuclei, ignoring the
photon bremsstrahlung contribution of the electrons in the BH process.

Assuming the validity of the Born approximation, an incident electron with four-momentum $k^\mu = (E, \vec{k})$ is scattered to a final four-momentum $k'^\mu = (E', \vec{k}')$ exchanging with the nuclear target a virtual photon with four-momentum $q^\mu = k'^\mu - k^\mu = (\omega, \vec{q})$. The electroexcitation of the nucleus from the initial state $|J_i M_i\rangle$ is followed by emission of a photon $q'^\mu = (\omega' = \omega, \vec{q}')$ with the nuclear transition to the final state $|J_f M_f\rangle \equiv |J_i M_i\rangle$. The full VCS cross section involves the calculation of

$$M = \sum_{\lambda,\lambda',M_i,M_f} (T^{\nu\lambda}_{M_i M_f}) L_{\lambda\lambda'} (T^{\nu\lambda}_{M_i M_f})^{*},$$

where $L_{\lambda\lambda'}$ is the lepton tensor and $T^{\nu\lambda}_{M_i M_f}$ is the nuclear scattering amplitude of a virtual photon with polarization $\lambda = 0, \pm 1$ into a real photon with polarization $\lambda' = \pm 1$. The VCS amplitude $T^{\nu\lambda}_{M_i M_f}$ can be conveniently described in terms of nuclear polarizabilities as a direct generalization of the formalism developed in ref. [5] for real Compton scattering. Due to the nature of the virtual photon, in addition to the contribution of the $M^{\nu=1,|\lambda|=1} = M$ (magnetic) and $M^{\nu=2,|\lambda|=1} = E$ (electric) multipoles, the multipole expansion of the absorbed photon includes the $M^{\nu=0,|\lambda|=0} = C$ (Coulomb or longitudinal) multipole. As a consequence, the definition of the nuclear polarizabilities of eq. (1) of ref. [5] is generalized by

$$P_J(M^{\nu'\lambda'} L', M^{\nu\lambda} L; q'^\mu, q^\mu) =$$

$$= (-)^{L + L' - J_i} \hat{L}^2 \hat{L'}^2 \sum_{M_i M_f} \sum_{M'M} \sum_{\lambda,\lambda'} \lambda' \lambda' (1 + \delta_{\lambda,0})$$

$$\times (1 - \delta_{\nu,0} \delta_{|\lambda|,1}) (-)^{M_i} \left( \begin{array}{ccc} I_f & J & I_i \\ -M_i & M & M_i \end{array} \right) \left( \begin{array}{ccc} L & L' & J \\ M & M' & -m \end{array} \right)$$

$$\times \frac{1}{(8\pi^2)^2} \int dR \int dR' D_{M_i M_f}^{L*L'}(R') T^{\nu\lambda}_{M_i M_f} (q'^\mu, q^\mu) D_{M_M}^{L*L'}(R).$$

The total angular momentum $J$ transferred to the nucleus is limited by the Clebsch-Gordan coefficients as follows

$$|J_i - J_f| \leq J \leq J_i + J_f, \quad |L - L'| \leq J \leq L + L',$$
while parity conservation requires
\[ P_f(M^{\nu'\lambda'}L', M^{\nu\lambda}L; q'^\mu, q^\mu) = 0 \quad \text{if } (-1)^{L+\nu+L'+\nu'} \neq \pi_i\pi_f, \quad (4) \]
where the parities of the initial and final states of the nucleus have been denoted by \( \pi_i \) and \( \pi_f \), respectively.

In the case of unpolarized electrons, eq. (1) can be rewritten in terms of four response functions of the nucleus
\[ M = L_{00}W_L + L_{11}W_T + L_{01}W_{LT} \cos \alpha + L_{1-1}W_{TT} \cos 2\alpha, \quad (5) \]
where \( \alpha \) is the azimuthal angle of the emitted photon with respect to the electron scattering plane. In contrast to the case of real polarized photons, where only the pure transverse \( W_T \) and the transverse-transverse interference \( W_{TT} \) responses occur, now there are also the pure longitudinal \( W_L \) and the longitudinal-transverse interference \( W_{LT} \) structure functions.

Due to the selection rules (3) and (4), for a zero-spin target nucleus only the scalar polarizabilities \( P_0(EL, EL; q'^\mu, q^\mu) \), \( P_0(ML, ML; q'^\mu, q^\mu) \) and \( P_0(EL, CL; q'^\mu, q^\mu) \) survive. Then the decomposition of \( W_L \) and \( W_{LT} \) in terms of polarizabilities is given by
\[ W_L = \sum_{\lambda'=\pm 1} \left| \sqrt{2} \sum_{L} (-)^L \hat{L}^{-1} d_{0,\lambda}^{L}(\theta, \gamma) P_0(EL, CL; q'^\mu, q^\mu) \right|^2, \quad (6) \]
\[ W_{LT} = \sum_{L,L'} (-)^{L+L'} \hat{L}^{-1} \hat{L'}^{-1} \left( \sum_{\lambda,\lambda'=\pm 1} \lambda d_{0,\lambda}^{L}(\theta, \gamma) d_{\lambda',\lambda}^{L'}(\theta, \gamma) 2\text{Re} \left\{ P_0(EL, CL; q'^\mu, q^\mu) P_0(EL', CL; q'^\mu, q^\mu) \right\}^* \right) \quad (7) \]

For \( W_T \) and \( W_{TT} \) the same expressions in eqs. (10) and (11) of ref. [5] hold, where now \( D_{\pm 1, \pm 1}(R') \) are replaced by the reduced rotation matrices \( d_{\pm 1, \pm 1}(\theta, \gamma) \).

In principle, VCS allows us to access new information with respect to real Compton scattering through the scalar polarizability \( P_0(EL, CL; q'^\mu, q^\mu) \), which involves the coupling between the longitudinal virtual photon and the nuclear transition operator. This is achieved either by a Rosenbluth
separation of $W_L$ and $W_T$ in parallel conditions ($\theta_\gamma = 0$) or by looking at the left-right asymmetry ($\cos \alpha = \pm 1$) to extract $W_{LT}$.

In the $\Delta$-resonance region we are facing a situation similar to that occurring when studying the $E2/M1$ ratio for the $\Delta$ excitation in pion photoproduction, where the longitudinal-transverse $W_{LT}$ structure function emphasizes the role of the small $C2/E2$ transition through the amplifying factor given by the interfering $M1$ transition [10].

Within the frame of the modified version of the $\Delta$-hole model described in ref. [5], the nuclear transition operator is modelled by one-body interaction mechanisms between the incident photon and the single nucleon. As a consequence, the nuclear polarizabilities can be directly derived from the nucleon polarizability operators $P_\alpha^\nu(M^{\nu\lambda\nu'}, M^{\nu\lambda}; q^{\mu}, q^{\mu'})$, and in the particular case of longitudinal photons one obtains

$$P_0(EL, CL; q^{\mu}, q^{\mu'}) =$$

$$= \sqrt{\pi} \sum_{nm'} \sum_{ll'} \sum_{jm\lambda\alpha} \hat{n}^{n+n'} \hat{n'}^{2} \hat{L}^{l+l'} \hat{j}^{j} (-1)^{l'+L-m} \hat{n}(q'^{\mu}) \hat{n}(q^{\mu}) Y_{j, -m} (\hat{r}_{\alpha})$$

$$\times \left( \begin{array}{ccc} n & n' & j \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} n & l & L \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} n' & l' & L \\ 0 & -\lambda' & \lambda' \end{array} \right)$$

$$\times \left\{ \begin{array}{ccc} n & n' & j \\ l' & l & L \end{array} \right\} \left[ P_\alpha^\nu(EL', CL; q^{\mu}, q^{\mu'}) + \lambda' P_\alpha^\nu(Ml', Cl; q^{\mu}, q^{\mu'}) \right],$$

where the nucleon polarizabilities $P_\alpha^\nu$ satisfy the same selection rules of eqs. (4) and (5) applied to the nucleon quantum numbers.

Resonant multipoles for the $N\Delta$ transition are $M1$, $E2$, and $C2$. The corresponding transition currents can be cast in the form of, e.g., ref. [11]. Besides the obviously dominant pure $M1$ contribution, for zero-spin nuclei only the longitudinal $P_0^\alpha(E2, C2; q^{\mu}, q^{\mu})$ and transverse $P_0^\alpha(E2, E2; q^{\mu}, q^{\mu})$ are different from zero.

Background contributions come from the Kroll-Ruderman term and the seagull term in the two-photon amplitude. The latter is pure transverse. From the Kroll-Ruderman term one obtains the elementary polarizabilities $P_0^\alpha(E1, E1; q^{\mu}, q^{\mu})$ and $P_1^\alpha(E1, C1; q^{\mu}, q^{\mu})$. Specializing to zero-spin nuclei, it turns out that only the elementary scalar polarizability contributes. Then,
for the background one has $P_0(EL, CL; q^\mu, q^\mu) = 0$. As a result, in the above model only small contributions to $W_L$ and $W_{LT}$ are possible from the resonant $E2$ and $C2$ multipoles. In a more refined model other background terms could be included giving rise, e.g., also to possible contributions from $P_\alpha(1, 1; q^\mu, q^\mu)$. However, this terms are expected to be of even smaller size.

In fig. 1 the structure functions $W_T$, $W_{TT}$ and $W_{LT}$ are given for $^4$He across the $\Delta$-resonance region. The behaviour of $W_T$ and $W_{TT}$ is an extrapolation of what can be calculated and observed in real Compton scattering. The new structure function $W_{LT}$ is rather small and peaked in the forward hemisphere. Its separation requires high-precision experiments to get rid of the overwhelming photon bremsstrahlung contribution in the observed cross section.

References

[1] G. Audit et al., CEBAF proposal PR-93-050 (1993); MAMI proposal A1/1-95 (1995).

[2] G.R. Farrar and H. Zhang, Phys. Rev. D41 (1990) 3348.

[3] P.A.M. Guichon, G.O. Liu, and A.W. Thomas, Nucl. Phys. A591 (1995) 606.

[4] J. Vesper, D. Drechsel and N. Ohtsuka, Nucl. Phys. A466 (1987) 652.

[5] B. Pasquini and S. Boffi, Nucl. Phys. A598 (1996) 485.

[6] J.H. Koch, E.J. Moniz and N. Ohtsuka, Ann. Phys. (NY) 154 (1984) 99.

[7] C. Schaerf, in Conference on Perspectives in Nuclear Energy at Intermediate Energies, ed. S. Boffi, C. Ciofi degli Atti and M.M. Giannini (World Scientific, Singapore, 1996) p. 199.

[8] O. Selke et al., Phys. Lett. B369 (1996) 207.

[9] H. Arenhövel and D. Drechsel, Nucl. Phys. A233 (1974) 153.
[10] J.-M. Laget, in New Vistas in Electro-Nuclear Physics, ed. by E.L. Tomusiak, H.S. Caplan and E.T. Dressler (Plenum Publ. Co., New York, 1986) p. 361; Can. J. Phys. 62 (1984) 1046.

[11] J.-M. Laget, Nucl. Phys. A481 (1988) 765.
Figure captions

Fig. 1. Angular dependence of the structure functions \( W_{TT} \), \( W_{LT} \) and \( W_{T} \) for virtual Compton scattering off \(^4\text{He}\) at an excitation energy of 310 MeV. Dotted, dot-dashed, dashed and solid lines for an incident photon momentum of 330, 380, 430 and 480 MeV, respectively.