A “Lorentz-Poincaré”–Type Interpretation of Relativistic Gravitation

Jan (B.) Broekaert

November 1, 2021

Abstract

The nature of time, space and reality are to large extent dependent on our interpretation of Special (SRT) and General Relativity Theory (GRT). In SRT essentially two distinct interpretations exist; the “geometrical” interpretation by Einstein based on the Principle of Relativity and the Invariance of the velocity of light and, the "physical" Lorentz-Poincaré interpretation with underpinning by rod contractions, clock slowing and light synchronization, see e.g. Bohm (1965); Bell (1987). It can be questioned whether the “Lorentz-Poincaré”-interpretation of SRT can be continued into GRT. We have shown that till first Post-Newtonian order this is indeed possible (Broekaert, 2004). This requires the introduction of gravitationally modified Lorentz transformations, with an intrinsically spatially-variable speed of light $c(r)$, a scalar scaling field $\Phi$ and induced velocity field $w$. Still the invariance of the locally observed velocity of light is maintained (Broekaert, 2005). The Hamiltonian description of particles and photons recovers the 1-PN approximation of GRT. At present we show the model does obey the Weak Equivalence Principle from a fixed perspective, and that the implied acceleration transformations are equivalent with those of GRT.

1 Introduction

We believe that a Lorentz-Poincaré (L-P) interpretation of relativistic gravitation has the merit of retaining a classical ontology of gravitational fields, and Hamiltonian mechanics in a flat-metric space. However, proper to an L-P interpretation of gravity, the physically observed measurements of space and time still result in a curved space-time. Basically this is the idea of Poincaré’s geometric conventionalism (Poincaré, 1902): it is formally indistinguishable to have free geodesic motion in curved space-time, or to have and adjusted gravitational dynamics —which also affects electromagnetism in rods and clocks— in a flat space and cosmic time. The present scalar–vector model is accordingly based on isotropic scaling, i.e. contraction and dilation, of physical quantities depending on position in the gravitation field and directional scaling due to velocity relative to gravitational sources.

Two levels of description must be discerned in a L-P type model (see also Cavalleri, Spinelli (1980); Thirring [1961]; Dicke [1957]; Wilson [1921]): gravitationally affected observations (or scaled) versus gravitationally unaffected observation (or unscaled). Our model implements the gravitational effects on rods and clocks through appropriate gravitationally modified Lorentz Transformations (GMLT’s) for space and time. These relate both, affected and unaffected observers, but also by combination of the previous; distinctly affected observers. Moreover the GMLT’s for energy and momentum provide the hamiltonian expressions which cover adequately the gravitational phenomenology of GRT (Broekaert, 2005). While the latter has been verified till 1-PN, some aspects of the model remain to be verified.

A critical requirement of a viable formulation of gravitation is the sufficient fulfillment of the Equivalence Principle (EP) even more the Weak Equivalence Principle (WEP) (Will [1993] (see however also Damour [2001])). The WEP purports the local indistinguishability of acceleration and gravitation, or the equivalence of inertial and gravitational mass. The EP on the other hand requires all physical laws in local free-falling frames to be equivalent and, equivalent with unaccelerated frames in a homogeneous (zero) gravity field. Similarly the WEP can be stated as the principle of universality of free-fall; and the free-falling observer locally observes gravitation to be eliminated. The WEP should be apparent to a fixed affected observer as well. In that case the acceleration of the particle should invariantly be independent of its rest mass and energy. This configuration can be easily covered by the acceleration transformations derived from the space-time GMLT’s. We will detail in Section 3, that in this configuration there
is a residual relative acceleration which is dependent on the velocity of the particle. It will be shown that the GRT expression of this acceleration is the same as in our L-P type model (Section 4). With the independence of the acceleration on the mass of the free-falling system (Section 2), this proves the validity of the WEP for fixed observers in our Lorentz-Poincaré model in the same manner as in General Relativity Theory.

2 The Lorentz-Poincaré type interpretation

A physical observer at coordinate position \( \mathbf{r} \) will locally measure affected quantities \((dx', dt')\). The explicit structure of the Lorentz-transformation adapted to gravitation straightforwardly reveals the effects of gravitational rod contraction, clock slowing and synchronization at the given location,

\[
\begin{align*}
    dx' &= \left( (dx_{o\|} - u_o dt_o) \gamma (u_o) + dx_{o\perp} \right) \Phi (r)^{-1} \tag{1} \\
    dt' &= \left( dt_o - u_o dx_o c_o (r)^{-2} \right) \gamma (u_o) \Phi (r) \tag{2}
\end{align*}
\]

which relates them to the unaffected space and time intervals of coordinate space. These are to be obtained taking into account the induced velocity field due to source movement in a Galilean relation in coordinate space:

\[
    dx = dx_o + w dt_o, \quad dt = dt_o \tag{3}
\]

The frame velocity \( \mathbf{u} \) of the affected observer relative to coordinate space, is then related to \( u_o \) by \( u_o = u - w \), while the velocity of light is given by \( c_0 = |c - w| \). The specific form of the relation \((1,2)\) is based on \( u_o \) and \( \gamma (u_o) \) appearing as an effective dynamical velocity and relativistic kinematical factor in the associated Hamiltonian mechanics and, concomitantly, the Lorentz-transformation-form which constrains the velocities to comply to the invariance of the locally observed velocity of light.

The associated velocity transformation is straightforward:

\[
    \mathbf{v}' = \frac{\mathbf{v}_o \perp - \gamma^{-1} \mathbf{v}_{o\perp} c_o^{-2}}{1 - \mathbf{u}_o \cdot \mathbf{v}_o c_o^{-2}} \frac{1}{\Phi^2} \tag{4}
\]

The gravitational scaling and induced velocity fields \( \{ \Phi, w \} \) are given by the equations:

\[
\begin{align*}
    \Delta \Phi &= 4 \pi G' \rho(r) \Phi + \frac{(\nabla \Phi)^2}{\Phi} \tag{5} \\
    \Delta w &= -\frac{16 \pi G'}{c^2} \mathbf{v}_p(x, t) \tag{6}
\end{align*}
\]

in no-retardation approximation. For example in the static source configuration we find the well known solution:

\[
    \Phi \equiv \exp(\varphi), \quad \varphi = \frac{G'}{c^2} \int_S \frac{\rho(r^*)}{|r - r^*|} d^3r^* \tag{7}
\]

The required hamiltonians are derived from associated energy-momentum GMLT’s which expose gravitational affecting different from the space-time GMLT’s, and consistent with Newtonian fitting (Broekaert 2004):

\[
    H = m c^2 + \mathbf{p} \cdot \mathbf{w}, \quad m = m_o \gamma (p) \Phi^{-3} \tag{8}
\]

We have shown in previous work that the L-P model gives the correct 1-PN equations of motion for particles (and photons) in harmonic coordinate space:

\[
    \mathbf{a} \approx -c^2 \nabla (\varphi + 2 \varphi^2) - v^2 \nabla \varphi + 4 \mathbf{v} \cdot \nabla \varphi - \mathbf{v} \times (\nabla \times \mathbf{w}) + 3 \mathbf{v} \partial_t \varphi + \partial_t \mathbf{w} \tag{9}
\]

In order to implement Poincaré’s Principle of Relativity, and obtain the calibration of the \( \mathbf{w} \)-equation, an acceleration transformation has been obtained; reproduced here as Eq. (10). Since the unaffected free-fall acceleration, Eq. (9), is independent of rest-mass or energy, this procedure —to obtain the acceleration in affected perspective by transformation from the unaffected perspective— trivially shows that the free-fall acceleration in the affected perspective as well does not depend on the falling particle rest-mass or energy. This independence shows then that the L-P model abides the basic premiss of the Weak Equivalence Principle. However it remains to be verified whether whether this transformation correctly exposes the velocity-dependent terms in the free-fall acceleration in the affected perspective of a fixed observer.
3  Acceleration transformations in the Lorentz-Poincaré type model

The acceleration transformation in the L-P model is obtained by taking the standard time derivative of the velocity transformation \( \Phi \). In the case of a general kinematic source, \( \omega \neq 0 \) and \( \Phi \neq 0 \), the acceleration observed by the affected observer is given by:

\[
a' = \left(1 - \mathbf{u}_o \cdot \mathbf{v}_o c_o^{-2}\right)^{-1/2} \left[\gamma_o^{-1} \Phi^{-3} \left\{ \mathbf{a}_o - \mathbf{o}_o + \left(\gamma_o^{-1} - 1\right) \mathbf{a}_o \right\} + \mathbf{v}' \Phi^2 \mathbf{a}_o \cdot \mathbf{v}_o c_o^{-2} \right.
\]
\[
- \left(\gamma_o^{-1} - 1\right) u_o^2 \left(\mathbf{o}_o \cdot \mathbf{v}_o + \mathbf{u}_o \cdot \mathbf{v}_o - 2 \mathbf{o}_o \cdot \mathbf{v}_o\right)
\]
\[
- v_o \gamma_o \mathbf{a}_o \cdot \mathbf{v}_o c_o^{-2} + \mathbf{v}' \Phi^2 \mathbf{a}_o \cdot \mathbf{v}_o c_o^{-2}
\]
\[
- 2 \mathbf{v}' \Phi^2 \left(1 + \mathbf{u}_o \cdot \mathbf{v}_o c_o^{-2}\right) \mathbf{a}_o + 2 \mathbf{v}_o \gamma_o u_o^2 c_o^{-2} \mathbf{a}_o \right\}
\]

with \( \mathbf{o} = \mathbf{u} \) the observer-frame acceleration, \( \mathbf{a}' = \mathbf{v}' \) the test-particle acceleration, both in terms of coordinate space, and \( \mathbf{a}' = \mathbf{v}' \) the test-particle acceleration in the perspective of the affected observer. Effectively the nature of the affected observer is completely defined by its frame particulars \( \{ \mathbf{u}, \mathbf{u}' \} \). The acceleration transformation can therefore be adapted to the affected observer being fixed (\( \mathbf{u} = 0, \mathbf{u}' = 0 \)) or dragged (\( \mathbf{u} \neq 0, \mathbf{u}' = 0 \)). We compare the first case for the L-P model and GRT, while other cases, and the LIF-case in particular, will need to be elaborated in future work (see however \[\text{Broekaert} (2005)\]) for a specific dragged configuration.

3.1 Acceleration relative to a fixed observer in the L-P model

The relative acceleration with respect to a fixed observer, i.e. *frame* acceleration and velocity \( \mathbf{o} = 0 \) and \( \mathbf{u} = 0 \), at 1-PN is obtained from Eq. \( \text{[10]} \) by approximation:

\[
a' = \Phi^{-3} \left\{ \mathbf{a} - 2 \mathbf{v}' \Phi^3 \mathbf{a}' \right\}
\]

where we have made use of the contravariant space-time GMLT, \( S' \) to \( S \) (the unaffected observer) for gradient operators:

\[
\nabla = \nabla_s = \Phi^{-1} \left( \gamma_s (\nabla_{||s} + \mathbf{u}' c_s^{-2} \partial_{t'}) + \nabla_{\perp s} \right)
\]

\[
\partial_t + \mathbf{w} \cdot \nabla = \partial_{t'} = \gamma \Phi \left( \partial_t + \mathbf{u}' \cdot \nabla' \right)
\]

Rendering all expressions of Eq. \( \text{[9]} \) explicitly in terms of \( S' \), gives for \( \mathbf{a}' \):

\[
a' = -(c^2 + v^2) \nabla' \mathbf{a}' + 2 \mathbf{v}' \cdot \nabla' \mathbf{a}' + \mathbf{v}' \mathbf{a}' - \mathbf{v}' \times (\nabla' \times \mathbf{w}') + \partial_{t'} \mathbf{w}'
\]

where \( \nabla = \Phi^{-1} \nabla' \) and \( \partial_{t'} = \Phi \partial_{t'} \) has been used for the fixed observer. We must consider next the same configuration in the framework of GRT.

4  GRT and relative acceleration

We first observe that the expression of the 3-acceleration of particles or photons in the affected perspective, *i.e.* in the curved space-time of the observer, is not commonly used in GRT due to its explicit dependence on the chosen metric. The description of particle-motion is obtained by setting zero the second covariant proper-time derivative of LIF-coordinates; the (null) geodesic equation. Appropriate coordinate transformations then lead to acceleration expressions in the physical coordinates of an observer.

A number of specific acceleration *transformation* laws were described in GRT by \[\text{Rindler, Mishra (1993); Mishra (1994)}\] and, in generic (and Fermi–) coordinates \[\text{Misner et al. (1973); Bins et al. (1993); Bunchaft, Carneiro (1998)}\):

\[
a = g - g \cdot \mathbf{v} \cdot \mathbf{v}
\]

where \( \mathbf{a} \) is the local 3-proper-acceleration of a relativistic particle in a static gravitational field, relative to a stationary observer, and \( g \) is this same acceleration but with the “physical” relative velocity \( \mathbf{v} = 0 \).

In order to have correspondence with the L-P model we compare to the expressions in Schwarzschild coordinates by \[\text{McGruder (1982)}\]. For the affected, physically observed, radial acceleration in GRT, McGruder gives:

\[
a_t = g \left( v_R^2 - v_I^2 - 1 \right) + O(r^{-3})
\]

3
with \( v_R \) the affected radial velocity and \( v_t \) the affected transversal velocity (here \( g = c^2 \kappa / r^2 \)). The comparison to the L-P model requires however that we work with its associated 1PN metric:

\[
c^2 dt_o^2 - dx_o^2 = \Phi^2 c^2 dt^2 (1 - \omega^2 / c^4) - \Phi^{-2} (dx - w dt)^2
\]

then for \( x_o^{\mu} = x^{\mu}_o (x) \):

\[
g_{\mu\nu} = \begin{pmatrix} \Phi^2 (1 - \omega^2 c^{-2}) & 0 \\ 0 & -\Phi^{-2} \end{pmatrix}
\]

which are defined up to a Lorentz boost \( \Lambda_{\mu}^\nu (u') \) between LIF’s. However, in the fixed observer case a Lorentz boost is not present and the observer’s coordinates are obtained directly through:

\[
dx^\lambda_o = b^\lambda_\mu dx^\mu
\]

which are found in the last step the expressions have been cast in terms of \( x_o^{\mu} \) and \( x^{\mu}_o \) are taken in coordinate space, we find till 1PN:

\[
\Phi^2 a^{i} + 2 \Phi^{1} \phi \phi^{i}
\]

\[
\Phi^3 a^{i} + 2 \Phi^{1} \phi \phi^{i}
\]

\[
\Phi^2 b^{i} + 2 \Phi^{1} \phi \phi^{i} + \omega \phi^{i}
\]

\[
\Phi^2 b^{i} + 2 \Phi^{1} \phi \phi^{i} + \omega \phi^{i}
\]
5  The WEP in the Lorentz-Poincaré Model, conclusions

We have shown —under restricted conditions of observation— the validity of the Weak Equivalence Principle in the L-P model and, detailed aspects of its formulation.

In particular we have shown that the free-fall acceleration observed by a fixed observer in a general gravitation field, according the L-P model and in GRT, coincides at first Post-Newtonian order. The WEP should of course be verified in more general conditions; the kinematics of the the observer should be generalized. However, due to the fact that the L-P model uses an acceleration transformation in order to obtain the free-fall acceleration in physical perspective, the independence of the rest mass is trivial, since the coordinate-space expression of the acceleration itself is independent of the rest mass. The successful implementation of the WEP in the L-P model therefor hinges on the specific aspects of an acceleration transformation of the type Eq. (10). It remains to be studied whether or not adaptations to it are required to cover LIF observations in concordance with the Weak Equivalence Principle.

References

Bell J. S., *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, 1987

Bini D., Carini P and Jantzen R.T., Relative observer kinematics in general relativity, *Classical and Quantum Gravity*, 12, 2549-2563, 1995

Bohm D., *The Special Theory of Relativity*, W.A. Benjamin Inc, 1965

Broekaert J., A Spatially-VSL gravity model with 1-PN limit of GRT, 2004, gr-qc/0405015 (submitted)

Broekaert J., A Modified-Lorentz-Transformation based Gravity Model Confirming Basic GRT-Experiments, *Foundations of Physics*, 35, 839-864, 2005

Bunchaft F., Carneiro S., The static spacetime relative acceleration for the general free fall and its possible experimental test, *Classical and Quantum Gravity*, 15, 1557, 1998

Cavalleri G., Spinelli G., Field-theoretic approach to gravity in flat space-time, *La Rivista del Nuovo Cimento*, 3, 8, 1980

Damour T., Questioning the Equivalence Principle, 2001, gr-qc/0109063

Dicke R.H., Gravitation without a Principle of Equivalence, *Reviews of Modern Physics*, 29, 363-376, 1957

McGruder III, Ch. H., Gravitational Repulsion in the Schwarzschild field, *Physical Review D*, 25, 3191-3194, 1982

Mishra L., The relativistic acceleration addition theorem, *Classical and Quantum Gravity*, 11, L97 -L102, 1994

Misner C.W, Thorne K.S., Wheeler J.A., *Gravitation*, W.H. Freeman, San Francisco,1973

Ni W.T., Theoretical Frameworks For Testing Relativistic Gravity. IV. A Compendium of Metric Theories of Gravity and their Post-Newtonian Limits, *Astrophysical Journal*, 176, 769-796, 1972

Poincaré H., *La Science et l’Hypothèse*, Edition Flammarion, Paris, 1902: 1968

Rindler W., Mishra L., The nonreciprocity of relative acceleration in relativity, *Physics Letters A*, 173, 105-108, 1993

Thirring T.E., An Alternative Approach to the Theory of Gravitation, *Annals of Physics*, 16, 96-117, 1961

Weinberg S., *Gravitation and Cosmology. Principles and applications of the General Theory of Relativity*, Wiley, London, 1972.

Will C., *Theory and Experiment in Gravitational Physics*, Cambridge University Press,1995

Wilson H.A., An Electromagnetic Theory of Gravitation, *Physical Review*, 17, 54-59, 1921