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Multiphase flow in tight sandstone: an improved application for 3D intermingled fractal model

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Abstract

Multiphase displacement in tight sandstone is fundamental and critical for tight oil production. Micropores dominate the pore space in tight sandstone. Microscopic mineral components in micropores substantially influence the multiphase displacement behavior. Pore size distribution exhibits a self-similar or fractal property, similar to inorganic mineral grains. Utilizing fractal theory eases the rebuilding of the flowing space and calculation of relative permeability. As an improvement of previous work, we successfully extended the 3D intermingled fractal model (3D IFM). The extended 3D IFM can evaluate multiphase flow behavior in tight sandstone, considering fractal characteristics of pore size distribution and mineral grain distribution. The fractal information of pores and mineral properties are obtained by scalable scanning electron microscopy (SEM) and energy dispersive spectroscopy (EDS) images, respectively. This method is useful and reliable for calculating relative permeability and residual saturation. The method is also successfully used to evaluate the influence of oil-water viscosity ratio, mineral components and formation depth on multiphase displacement behavior, proving its convenience and potential in rapidly evaluating multiphase displacement behavior in tight sandstone.

Keywords  
Tight sandstone; Fractal; Mineral component; Multiphase displacement

1 Introduction

Most tight oils are contained in petroleum-bearing formations with tight matrix and low permeability. Tight matrices often contain an assemblage of rocks, reservoirs, and caprocks. Tight sandstone is a commonly discovered tight matrix. During economic production from tight sandstone formation, hydraulic fracturing is commonly used to enhance recovery. In hydraulic fracturing and flowback, multiphase displacement behavior between oil and fracturing liquid will considerably influence the
production. Tight sandstone features relatively low permeability and porosity compared with conventional sandstone; it commonly presents a gas permeability of less than 0.1 mD [1-2] and pore throat sizes ranging from approximately 0.03 μm to 2 μm [3]. Tight sandstones exhibit a complex and unique multiphase displacement mechanism. Non-Darcy effect, that is, the deviation from linear relation between flow velocity and pressure gradient, becomes notable as pore throat diameter decreases from the macro to micro scale. Wetting behavior, caused by the component and distribution of a variety of mineral grains, will also remarkably influence oil–water displacement behavior in micropores. The influence becomes perceptible in tight sandstone owing to the high capillary force in the matrix. Given the complex flow mechanism in tight sandstone, the multiphase displacement mechanism and rapid evaluation of relative permeability should be investigated for the analysis of oil production, hydraulic fracturing, and production forecasting, and may helpful for drilling and carbonate

Multiphase displacement, especially oil–water displacement, is a fundamental problem for tight sandstone reservoir development. Evaluating oil–water relative permeability curve is a key problem that requires research attention. However, the network of pore space should first be rebuilt precisely and effectively. The pore space of tight sandstone differs from that of conventional sandstone. Tight sandstone is relatively heterogeneous and features relatively low connectivity and complicated pore structure [4]. The characteristics of pore structure and pressure condition has significant influence on multi-phase displacement behavior, which has been studied for several years [5-10]. On the basis of effective pore network model, multiphase displacement behavior is calculated by introducing the capillary force, non-Darcy effect, and wettability behavior. Since Fatt [11] first combined the parallel tube bundle based on actual pore size distribution with the network model, various models have been developed over the years. Bryant et al. [12-13] presented a predictive calculation of two-phase relative permeability in equivalent network for granular porous media by considering stress deformation. Blunt [14] built a 3D equilibrium-based pore level network model to compute relative permeability and capillary pressure for drainage and imbibition cycles. Mani and Mohanty [15] extended the model into three-phase displacement. Dixit et al. [16] studied hysteresis phenomena and relative permeability curve by using a modified 3D rectangular network model. Related similar models are developed by Fishcher and Celia [17], Valvatne and Blunt [18], Yuan [19] and Lu et al. [20]. Bakke and Øren [21] developed a 3D random network that is more realistic than a regular network. Nguyen et al. [22] developed a dynamic network model that combines the complex interaction among displacement rate,
contact angle, aspect ratio, and pore and throat shapes on relative permeability. Recently, Ren et al. [23] developed an improved analytical oil–water displacement model to analyze nonlinear seepage characteristics in tight sandstone and verified the model with experiments.

Deterministic fractal is a geometric figure whose each and every part is similar to the whole, and that is repeated on different scales. Pores in tight sandstone possess fractal properties at a certain scale [24], similar to mineral grains[25]. The fractal theory conveniently represents complicated pore space and mineral grains by using self-similar properties[26-27]. Toledo et al [28] investigated the scaling laws between fractal dimension and capillary pressure and between fractal dimension and relative permeability of the wetting phase in sandstones. Cai et al. [29-31] used the fractal theory to calculate multiphase flowing behavior and applied the model in several applications. Xiao et al [32] and Xu et al[33] combined fractal and Monte Carlo method to predict the relative permeability of unsaturated porous media. Lei et al. [34] introduced fractal tortuosity into a tube bundle model to investigate stress-dependent relative permeability in sandstone. Balankin et al.[35-36] developed a fractal continuum model to calculate the flow process in random fractal porous media. Pia and Sanna[37] were the first to build an intermingled fractal unit (IFU) method for representing multi-fractal behavior of pore structure; the author also developed various applications in multiphase displacement process[38-40]. Cihan et al.[41-42] developed a new method, called the probabilistic capillary connectivity (PCC) method, to calculate permeability and relative permeability in 3D random fractal media. This method creatively represents the pore connectivity with connective probability. Recently, Li et al.[43] successfully combined the IFU model and PCC method to calculate shale permeability.

2 Basic theory of the fractal model and relative permeability calculation

2.1 Introduction of 3D IFU

First developed by Pia and Sanna[37], the IFU model represents random pores in porous media. The model is a combination of several randomized multi-fractal sub-units. These sub-units are revised from the basic Sierpinski carpet model. Pia and Sanna[37] have defined and explained in detail the IFU model in their work. Additionally, the PCC method, developed by Cihan et al.[41-42, 44], calculates the permeability of randomized Menger sponge. Previously, we have successfully developed a 3D intermingled fractal model (3D IFM) by combining revised IFU and PCC method and calculated the apparent permeability for
shale[43]. In the present work, we expand the previous model into multiphase calculation and introduce mineral components to calculate the displacement process in tight sandstone. Background information about model suggestions and basic parameter definitions can be found in the related references above. This information clarifies the 3D IFU model for comprehensibility. The general governing equation for unsteady state flow can be expressed as:

\[
\begin{align*}
\text{div}\left( \frac{K_{nw}}{\mu_{nw}} \text{grad}P_{nwr} \right) &= \phi \frac{\partial S_{nw}}{\partial t} \\
\text{div}\left( \frac{K_{w}}{\mu_{w}} \text{grad}P_{wr} \right) &= \phi \frac{\partial S_{w}}{\partial t}
\end{align*}
\] (1)

The general governing equation for steady state flow can be expressed as:

\[
\begin{align*}
\frac{K_{nw}}{\mu_{nw}} \text{grad}P_{nwr} &= \phi S_{nw} \bar{v}_{nw} \\
\frac{K_{w}}{\mu_{w}} \text{grad}P_{wr} &= \phi S_{w} \bar{v}_{w}
\end{align*}
\] (2)

where 'nw' and 'w' means non-wetting phase and wetting phase; \( K_{nw} \) and \( K_{w} \) are permeability of non-wetting phase and wetting phase; \( \mu_{nw} \) and \( \mu_{w} \) are viscosity; \( P_{nwr} \) and \( P_{wr} \) are pressure difference; \( S_{nw} \) and \( S_{w} \) are saturation; \( \bar{v}_{nw} \) and \( \bar{v}_{w} \) are average velocity, controlled by Poiseuille equation; \( \phi \) is porosity and \( t \) is time.

2.2 Multiphase probabilistic capillary connectivity (PCC) method

In this section, we extend the basic PCC method into relative permeability calculation in 3D IFU. The basic theory about PCC method is provided by Marshall et al.[45] and Cihan et al.[41]. In PCC method, connecting probabilities for pores in any cross section of sponge are assumed equal, that is, large pores will gain high connecting probability, thereby indicating high permeability.

In this work, our primary concern is to link PCC with relative permeability calculation and consider several mineral ingredients with different wetting behavior. To simplify the problem, we assume the following simplifications. a) Pores and bulks are squares/necks corrected with shape factors on cross section. b) Permeability is related to connecting probability in fractal porous media. c) Pores and bulks in 3D space feature random distributions. d) The number of pores is sufficient; thus, cross sections share the same fractal dimension and pore size distribution. e) Each mineral possesses its own wettability behavior, and mixed wetting is neglected. Fig.1 shows the schematic diagram for this displacement process.
Multiphase flow is more complicated than the single-phase one. As a result, multiphase introduces new problems, such as Jamin effect, dynamic meniscus behavior, and non-Darcy flow. To calculate the relative permeability of 3D IFU, we introduce two parameters: connected fraction $P_1$ and traverse probability $P_2$.

The connected fraction $P_{1,i}$ is the fraction of connecting pores (except dead ends) existing at $i$th iteration level and is a decimal between 0 and 1. For example, given $N_i$ pores at $i$th iteration level during the iterating process, $<N_i(1-P_{1,i})>$ pores are dead ends; only $<N_iP_{1,i}>$ pores will participate in the multiphase displacement process. The angle brackets '< >' are a symbol of nearest integer function, and they are used to keep the number of pores as integer. For real sandstone, $P_1$ at each iteration level can be easily derived from the capillary pressure–saturation ($P_c–Sw$) curve comparison between mercury injection experiment and CT or SEM data. The $P_c–Sw$ curve derived from mercury injection denotes the connected pores, whereas that observed and simulated from CT or SEM data comprises all pores. Specifically, we first obtain the observed pore size distribution information from CT or SEM data. Then, we use the IFU model to match the real size distribution and obtain simulated fractal distribution. Finally, we utilize simulated fractal pore size distribution to calculate the simulated $P_c–Sw$ curve. With the existence of dead ends, this $P_c–Sw$ curve will usually show no matching the mercury injection $P_c–Sw$ curve. Therefore, in each iteration level, we introduce and adjust the connected fraction $P_1$ to narrow down the differences in fractal and mercury $P_c–Sw$ curves. Finally, the introduced $P_1$ corrects the influence of dead ends in porous media.

The traverse probability $P_2$ is the probability of meniscus travel over the pore neck. $P_2$ is influenced by several forces, such as driving force, capillary force, and friction from walls. We set $P_{2,i→j}$ as the traverse probability from pore neck at iteration level $i$ to iteration level $j$. If the driving force is larger than the resistance, then the meniscus can travel over the pore neck. Thus, $P_{2,i→j} = 1$; otherwise, $P_{2,i→j} = 0$. To successfully complete a multiphase displacement at pore neck, two conditions must be met. First, the pores should be connected at pore neck (controlled by PCC). Second, the injected liquid should gain enough energy (driving force larger than resistance) to overcome resistance at pore neck.

We conduct three steps to calculate relative permeability on the basis PCC method, shown in Fig.2. First, we calculate the pore connective probability, followed by traverse probability and relative permeability. To calculate the pore connective probability, we build the probability calculation table according to Table 1. The table is rearranged in accordance with the
descending order of pore diameters. The largest pore is \( \lambda_1 \), where the number of pores is \( N_1 \). Only \( N_1 \) pores participate in the displacement process. Hence, the probability of \( \lambda_1 \) pores in one surface (face A) connecting to other \( \lambda_1 \) pores in other surfaces (face B) is \((\lambda_1^2 < N_1P_{1,1} > / A)^2\), which is called connecting probability. \( A \) refers to the cross-sectional area of porous media. To calculate traverse probability, driving forces and resistance are compared. If the driving force is large, then meniscus can travel over the pore neck. Therefore, \( P_{2,i \rightarrow j} = 1 \). Otherwise, meniscus will be arrested at pore neck, that is, \( P_{2,i \rightarrow j} = 0 \). Thus, the general connecting and successful displacement probability \( P_{CSDP} \) for iteration level 1 is as follows:

\[
P_{CSDP,1} = (\lambda_1^2 < N_1P_{1,1} > / A)^2 \times P_{2,1 \rightarrow 1,1}
\]

shown in Table 1. Similarly, at the second iteration level, the number of effective pores at level 2 is \( N_2 \), and number of dead ends is \( (N_2 - 1) \). At this level, according to PCC theory[41], the general probability is combined by the following three parts. First, the probability of \( \lambda_2 \) pores in face A connecting to \( \lambda_2 \) pores in face B is \((\lambda_2^2 < N_2P_{1,2} > / A)^2 \times P_{2,2 \rightarrow 2} \). Second, the probability of \( \lambda_1 \) pores in face A connecting to \( \lambda_2 \) pores in face B is \((\lambda_1^2 < N_1P_{1,1} > / A) \times (\lambda_2^2 < N_2P_{1,2} > / A) \times P_{2,1 \rightarrow 2} \). Third, the probability of \( \lambda_2 \) pores in face A connecting to \( \lambda_1 \) pores in face B is \((\lambda_2^2 < N_2P_{1,2} > / A) \times (\lambda_1^2 < N_1P_{1,1} > / A) \times P_{2,2 \rightarrow 1} \). In summary, the general connecting and successful displacement probability \( P_{CSDP} \) for iteration level 2 is computed by the following:

\[
P_{CSDP,2} = \frac{\lambda_1^2 < N_1P_{1,1} >}{A} \times \frac{\lambda_2^2 < N_2P_{1,2} >}{A} \times P_{2,1 \rightarrow 2} + \frac{\lambda_1^2 < N_1P_{1,1} >}{A} \times \frac{\lambda_2^2 < N_2P_{1,2} >}{A} \times P_{2,2 \rightarrow 1} + \frac{\lambda_2^2 < N_2P_{1,2} >}{A} \times \frac{\lambda_1^2 < N_1P_{1,1} >}{A} \times P_{2,2 \rightarrow 1}
\]

shown in Table 1. Similarly, we can obtain \( P_{CSDP} \) in the following iteration levels.

In accordance with the definition of \( P_{2,i \rightarrow j} \), we can give the detailed equation for \( P_{2,i \rightarrow j} \). By considering force displacement (non-wetting phase driving wetting phase) as an example, capillary force acts as a resistance. Hence, \( P_{2,i \rightarrow j} \) is calculated as follows:

\[
P_{2,i \rightarrow j} = \begin{cases} 1 & (\Delta P - P_c - (32\mu_w L_p / \lambda_1^2 + P_{\text{initial}}) - 32\mu_w (L - L_p) / \lambda_1^2 - P_G > 0) \\ 0 & (\Delta P - P_c - (32\mu_w L_p / \lambda_1^2 + P_{\text{initial}}) - 32\mu_w (L - L_p) / \lambda_1^2 - P_G < 0) \end{cases}
\]

In imbibition, capillary force acts as a driving force. Therefore, \( P_{2,i \rightarrow j} \) is represented by the following:

\[
P_{2,i \rightarrow j} = \begin{cases} 1 & (\Delta P + P_c - (32\mu_w L_p / \lambda_1^2 + P_{\text{initial}}) - 32\mu_w (L - L_p) / \lambda_1^2 - P_G > 0) \\ 0 & (\Delta P + P_c - (32\mu_w L_p / \lambda_1^2 + P_{\text{initial}}) - 32\mu_w (L - L_p) / \lambda_1^2 - P_G < 0) \end{cases}
\]

where \( \Delta P \) denotes the pressure difference between inlet and outlet of model; \( \mu_w \) and \( \mu_n \) refer to the viscosity of non-wetting
and wetting phases, respectively; $P_{initial}$ corresponds to the threshold pressure gradient, whose useful expression is provided by Tian et al[46]; $L$ identifies the edge length of model; setting cross section is perpendicular to injection direction; $L_p$ specifies the distance from cross section to pressure inlet and regarded as the average length of each iteration level in the invasion part; $P_G$ represents a projection of gravity on flow direction (when gravity is perpendicular to flow direction, gravity is neglected), and it can normally be neglected in tight porous media; $P_c$ is the capillary pressure, $P_c = \frac{\alpha C_n}{(\lambda_i/2)}$; $\sigma$ is the interfacial tension between two fluid phases; According to Ma et al.[47], $\tilde{C}_n$ is the curvature of a meniscus:

$$\tilde{C}_n = \begin{cases} \frac{2 \cos \theta_R}{\cos \theta_R + \sqrt{\tan \alpha}} \sin 2 \theta_R + \pi(1 - \frac{\alpha + \theta_R}{90}) & (90^\circ - \alpha < \theta_R < 90^\circ) \\ \cos \theta_R & (\theta_R < 90^\circ - \alpha) \end{cases} \tag{7}$$

where $\alpha$ is the half angle of the corner, that is, $\alpha = 45^\circ$ for a square cross section; $\theta_R$ denotes the dynamic contact angle, such as advancing and receding angle. The advancing and receding angle is controlled by capillary number, $C_a$,

$$C_a = \frac{\mu}{\sigma} \tag{8}$$

$C_a$ controlled by viscosity $\mu$, interfacial tension $\sigma$ and velocity. Technically, the dynamic contact angle shows a slight difference with the static contact angle. Hilpert [48-52] gives an expression about dynamic contact angle as:

$$\cos \theta_R - \cos \theta_{eq} = \tilde{\alpha C}_a \tilde{\beta} \tag{9}$$

where $\theta_{eq}$ is static contact angle. $\tilde{\alpha}, \tilde{\beta}$ are dimensionless parameters. Hilpert introduced the definition and common value of $\tilde{\alpha}, \tilde{\beta}$ in detail. On the basis of published experimental data and related conclusions [53-55], when capillary number $C_a > 10^{-5}$, the dynamic contact angle varies with velocity, and when $C_a < 10^{-3}$, $\tilde{\beta}$ approaches to 0, contact angle becomes a velocity-independent parameter. Given the relative slow displacement in tight media, we set dynamic contact angle as a velocity-independent parameter in this work.

Displacement efficiency is remarkably influenced by residual oil saturation. During and after the displacement process, certain residual phases will remain in corners. Several studies have provided residual saturation models[56]. We select the Ma’s residual saturation model[47], which provides the relationship between residual wetting phase $S_{wres}(\alpha, \theta_R)$ and shape of cross section:

$$S_{wres}(\alpha, \theta_R, i) = \frac{\tan \alpha}{C_n^2} \left[ \frac{\cos \theta_R \cos(\alpha + \theta_R) - \frac{\pi}{2} (1 - \frac{\alpha + \theta_R}{90})}{\sin \alpha} \right] \tag{10}$$
In summary, by considering the force injection process (non-wetting phase driving wetting phase in wetting-phase-saturated matrix, or primary drainage) as an example, we derive relative permeability in 3D IFU model as follows:

For iteration level \(i=1\), the following are utilized:

\[
k_{nw_{-sub}}(1) = C_1 \frac{\lambda_2^2 < N_1 P_{1,1} > \lambda_1^2 < N_1 P_{1,1} >}{A} \lambda_1^2 P_{2,1\rightarrow1} \tag{11}
\]

\[
k_{w_{-sub}}(1) = C_1 \frac{\lambda_1^2 < N_1 P_{1,1} > \lambda_1^2 < N_1 P_{1,1} > \lambda_1^2 (1 - P_{2,1\rightarrow1})}{A} \tag{12}
\]

where \(C_1\) refers to the shape factor\(^{29,56}\); \(k_{nw_{-sub}}(1)\) represents the non-wetting phase permeability at iteration level \(i=1\); \(k_{w_{-sub}}(1)\) corresponds to the wetting phase permeability at iteration level \(i=1\). Non-wetting phase residual saturation left in the first level pores is computed as follows:

\[
S_{nw_{-sub}}(1) = \frac{\lambda_2^2 < N_1 P_{1,1} > P_{2,1\rightarrow1} [1 - S_{wrez}(\alpha, \theta, 1)]}{\sum_{i=1}^{n} \lambda_i^2 P_{i,j}} \tag{13}
\]

For iteration level \(i=2\), the following are employed:

\[
k_{gw_{-sub}}(2) = C_2 \lambda_2^2 \left[ \frac{\lambda_2^2 < N_2 P_{2,1} > \lambda_1^2 < N_2 P_{2,1} > }{A} P_{2,2\rightarrow1} + \frac{\lambda_1^2 < N_1 P_{1,1} > \lambda_2^2 < N_2 P_{2,1} > }{A} P_{2,2\rightarrow1} \right] \tag{14}
\]

\[
k_{w_{-sub}}(2) = C_2 \lambda_2^2 \left[ \frac{\lambda_2^2 < N_2 P_{2,1} > \lambda_1^2 < N_2 P_{2,1} > (1 - P_{2,2\rightarrow1})}{A} + \frac{\lambda_1^2 < N_1 P_{1,1} > \lambda_2^2 < N_2 P_{2,1} > (1 - P_{2,2\rightarrow1})}{A} \right] \tag{15}
\]

\[
S_{nw_{-sub}}(2) = \frac{\lambda_2^2 < N_2 P_{2,1} > (P_{2,1\rightarrow2} + P_{2,2\rightarrow1} + 2P_{2,2\rightarrow2}) / 4 \times [1 - S_{wrez}(\alpha, \theta, 2)]}{\sum_{i=1}^{n} \lambda_i^2 P_{i,j}} \tag{16}
\]

For iteration level \(i=3\), the following equations are used:

\[
k_{gw_{-sub}}(3) = C_3 \lambda_3^2 \left[ \frac{\lambda_3^2 < N_3 P_{3,1} > \lambda_2^2 < N_3 P_{3,1} > }{A} P_{3,2\rightarrow1} + \frac{\lambda_2^2 < N_2 P_{2,1} > \lambda_3^2 < N_3 P_{3,1} > }{A} (P_{3,1\rightarrow3} + P_{3,2\rightarrow3}) \right] \tag{17}
\]

\[
k_{w_{-sub}}(3) = C_3 \lambda_3^2 \left[ \frac{\lambda_3^2 < N_3 P_{3,1} > \lambda_2^2 < N_3 P_{3,1} > }{A} (P_{3,2\rightarrow1} + P_{3,2\rightarrow2}) \right] \tag{18}
\]
\[ k_{w_{\text{sub}}}(3) = C_3 \lambda_i^3 \left( \frac{\lambda_i^3 < N_i P_{1,3} >}{A} - \frac{\lambda_i^3 < N_i P_{2,3} >}{A} \right) + \frac{\lambda_i^3 < N_i P_{1,3} >}{A} \frac{(1 - P_{2,3-s})}{(2 - P_{2,3-s} - P_{2,3-a})} \]  

\[ S_{w_{\text{sub}}}(3) = \frac{\lambda_i^3 < N_i P_{1,3} >}{A} \frac{(P_{2,1-s} + P_{2,3-s} + P_{2,2-s} + P_{2,3-a} + 2P_{2,3-s})}{6} \times \left( 1 - S_{\text{wres}}(\alpha, \theta, 3) \right) \sum_{j=1}^{n} \lambda_i^3 P_{1,i} \]  

Similarly, for iteration level \( i=n \), the following are utilized:

\[ k_{n_{\text{sub}}}(n) = C_n \lambda_n^2 \left[ \frac{\lambda_n^2 < N_n P_{1,n} >}{A} - \frac{\lambda_n^2 < N_n P_{2,n} >}{A} \right] + \frac{\lambda_n^2 < N_n P_{1,n} >}{A} \frac{(1 - P_{2,n-s})}{(2 - P_{2,n-s} - P_{2,n-a})} \]  

\[ S_{n_{\text{sub}}}(n) = \frac{\lambda_n^2 < N_n P_{1,n} >}{A} \frac{(P_{2,n-s} + P_{2,n-a})}{2n} \times \left[ 1 - S_{\text{wres}}(\alpha, \theta, n) \right] \sum_{i=1}^{n} \lambda_i^2 P_{1,i} \]  

To sum up, the permeabilities for non-wetting and wetting phases in 3D IFU are respectively calculated as follows:

\[ k_{n}(n) = \sum_{i=1}^{n} k_{n_{\text{sub}}}(i) \]  

\[ k_{w}(n) = \sum_{i=1}^{n} k_{w_{\text{sub}}}(i) \]  

Relative permeability denotes the ratio of apparent permeability to instinct permeability. The instinct permeability of 3D IFM is the case when all pores are filled with a single phase with normal Darcy flow. Based on Cihan et al.[41] and Li et al.[43], instinct Darcy permeability for 3D IFM is calculated as follows:
The relative permeability for non-wetting and wetting phases in 3D IFU model are respectively computed as follows:

\[
k_{r_{nw}}(n) = \frac{k_{nw}(n)}{k_{ins}(n)},
\]

\[
k_{r_{w}}(n) = \frac{k_{w}(n)}{k_{ins}(n)}.
\]  

The saturation for non-wetting and wetting phases in 3D IFU model are respectively given by the following:

\[
S_{nw}(n) = \sum_{i=1}^{n} S_{nw-iub}(i),
\]

\[
S_{w}(n) = 1 - S_{nw}(n).
\]

In general, the formula to describe multiphase displacement process under IFM method can be expressed as:

\[
k_{nw}(n) = \left[ \sum_{j=1}^{n} \left( \sum_{i=1}^{\xi_{j}} \frac{N_{i,j}}{A} \frac{N_{i,j}}{A} - C_{i} \left( 2 - P_{2,\rightarrow j} + P_{2,\rightarrow n} \right) - \left( \frac{N_{i,j}}{A} \right)^{2} C_{i} \left( 1 - P_{2,\rightarrow n} \right) \right) \right] C_{j} \hat{X}_{j}^{2}.
\]

\[
k_{w}(n) = \left[ \sum_{j=1}^{n} \left( \sum_{i=1}^{\xi_{j}} \frac{N_{i,j}}{A} \frac{N_{i,j}}{A} - C_{j} \left( P_{2,\rightarrow j} + P_{2,\rightarrow n} \right) - \left( \frac{N_{i,j}}{A} \right)^{2} C_{j} P_{2,\rightarrow j} \right) \right] C_{j} \hat{X}_{j}^{2}.
\]

### 3 Building the intermingled fractal model (IFM) for tight sandstone

#### 3.1 Basic information

In this section, we discuss how to build the 3D IFM for real sandstone sample and the IFM for real samples on the basis of experimental data. Tight sandstone exhibits a relatively complicated pore space. Various factors, such as pore structure, mineral component, wettability behavior, and non-Darcy effect, influence multiphase displacement behavior. In this study, we use a real tight sandstone sample to build a 3D IFM and calculate the relative permeability. Pore size distribution information is obtained by...
scalable scanning electron microscopy (SEM) images. Mineral components are determined by conducting energy dispersive spectroscopy (EDS) test. The information on connected pores and dead ends are obtained via mercury intrusion method. Combining all information together, we can establish the 3D IFM and calculate the relative permeability finally.

The sample is a fresh core sample from a tight-oil well in tight sandstone formation from Xinjiang. To obtain the pore distribution, we polish the sample using argon ion. Then, we subject the sample to scalable SEM test. Rock mineralogy of samples is analyzed in an area of 400 µm x 400 µm by using a Zeiss–Merlin rock mineralogy analysis machine with a maximal resolution of 4 nm. The results are shown in Fig.3. As observed through the SEM images, micropores are dominant in tight sandstone, featuring several pore types, such as interparticle pores, quartz dissolution pores and clay mineral-dominated pores. Fractal properties of these inorganic pores can be derived from image analysis and statistics. In addition, within the same analysis area of the sample, we conduct EDS test to derive the mineral component information. The analysis area of EDS is 400 µm x 400 µm with a resolution of approximately 1 µm.

3.2 Fractal representation for micropores.

Tight sandstone contains abundant interparticle and intraparticle pores. Fig.4. shows the detailed IFU modeling process. The building process is similar with IFU foundation processes in previous works ([40,43,58]). We list the steps briefly as follows:

In scalable SEM images, the first step is to calculate the representative elementary surface (RES) for pores by extending the method, as commonly observed in previous works. As the target subarea extends, the average grayscale in subarea is calculated simultaneously. When the average grayscale stabilizes, the subarea is regarded as RES. In this sandstone sample, the area of 400 µm x 400 µm reaches the RES. The next step is to adjust the pores from SEM images in scalable SEM images. After segmentation, the grayscale image becomes binarized. The pore cumulative distribution curve of the samples is then derived. Finally, within the binarized image, we adjust the fractal parameters to narrow the differences in cumulative curves between the basic IFU model and image calculation. Finally, certain parameters of the 3D IFU model are determined. The parameters are shown in Table 2. Also, CT images is an alternate fractal information source. The following building steps are similar with SEM images.
3.3 Fractal representation for minerals

Tight sandstone consists of several mineral grains, such as quartz, albite, dolomite, orthoclase, and illite. During displacement, the wettability of these grains substantially influences the relative permeability and residual distribution. Each mineral features its own wettability property. Displacement behavior is notably influenced by wettability. Therefore, rebuilding multi-fractal pores with unique contact angle is insufficient. Mineral wettability should be attached on the surface of 3D IFM. We use the EDS to gain mineral component information for tight sandstone. The EDS image shares the same target area with the SEM image. The resolution is 200 nm. The EDS results are shown in Fig.5.

In the EDS image, the distribution of mineral grains shows a self-similar or fractal property. The IFU model represents mineral grains well. Fig.6 shows the majority of minerals in this tight sandstone sample. The dominant minerals include quartz, albite, dolomite, orthoclase and illite, accounting for 92.36% of total mineral component. We select the top four minerals to rebuild the mineral space in this study. Similar with previous work, quartz is a base mineral that gains fractal parameters for other minerals separately. Then, we build a base mineral cube (quartz for this sample) and randomly scatter IFUs for albite, dolomite and orthoclase into the base cube. In this manner, IFU blocks will replace the original base mineral, and no overlapping will be observed with the scattered mineral blocks. Finally, we derive the 3D IFU model for minerals, shown in Fig.7.

3.4 Building 3D IFM.

Fig.8. show the detailed building process. Information on pore size distribution are obtained from scalable SEM images, whereas those on mineral content originate from the EDS images. We derive the fractal parameters for each mineral and build 3D IFU model for pores and minerals. As shown in Fig., the 3D IFU models for pores and for minerals overlap with each other. Thus, the pore walls will gain mineral properties. To simplify the fractal model, we assume that one pore wall gains only one mineral property, and one mineral gains only one wettability property (static contact angle). As a result, the minerals that gains the most proportion on one pore wall will give its mineral property on this pore wall. In this manner, each pore will receive one mineral characteristic. Finally, the 3D IFM can be successfully derived.

3.5 Relative permeability calculation

After the foundation of the 3D IFM, we extend Table 1 to contain mineral properties as Table 3. Within one iteration level,
the equivalent diameters are equal but share four different mineral properties. Therefore, this iteration level is extended into four rows. Each row includes only one mineral, and the pore number in this row denotes the number of pores containing this mineral on their walls. The sum pore number at this iteration level for all four minerals equals the pore number in the non-extended table. The basic 3D IFM is similarly extended in this manner. As shown in Table, for example, $N_2$ number of pores is given for pores with equivalent diameter $\lambda_2$; at the same time, $\lambda_2$ contains four kinds of mineral types with four different capillary pressures. The table is initially rearranged in accordance with the descending order of pore diameters. Then, we rearrange the rows that share the same diameter in accordance with the descending order of capillary pressure. The basic principle for relative permeability calculation is the same between PCC table and extended PCC table. By following the PCC rules described before, it is convenient to calculate residual oil saturation and relative permeability in the extended table for 3D IFM with mineral properties.

3.6 Model validation

The 3D IFM method for pore space representation was successfully verified in several works [43-44]. To verify the relative permeability calculation, we use the published experimental data [59-60]. The validation between experimental data and 3D-IFM calculation results are shown in Fig.9. The model results are generally close to the experimental data, which verify our 3D IFM model. Also, our 3D IFM model is relatively open and applicable. As described, in the process for fractal representation for micropores, SEM image is not the only information source for real pore size distribution. Other methods, like nuclear magnetic resonance and CT image, are acceptable because different method has different best performance pore size range. Similarly, in minerals representation process, when EDS test is not convenient to conduct, X-Ray powder diffraction method is also acceptable when the mineral grain distribution in space is not something to be focused on. For different types of non-Darcy fluids, 3D IFM model is also applicable, just slightly alter the base flowing behavior functions.

4 Application of 3D IFM: simple influencing factors of displacement

4.1 Pore size

In this section and the following section, we used the previous built 3D IFM for tight sandstone to investigate some important influencing factors in multi displacement process. The first is pore size.

Pore size is a critical parameter in multiphase flow in tight sandstone. During micro displacement, flow behavior is
synthetically influenced by several forces, such as capillary force, floating force, and driving force. The average weight of these forces will change with increasing pore size. To simplify the driving process, we consider three forces as an example: capillary force, floating force, and driving force. Then, we normalize these forces using the following equation:

\[ P_{cD} = \frac{P_c}{P_{sum}}, \quad P_{FD} = \frac{P_f}{P_{sum}}, \quad P_{jD} = \frac{P_j}{P_{sum}}, \]  

(32)

where \( P_c \) and \( P_{cD} \) denote the capillary force and normalized capillary force, respectively. \( P_f \) and \( P_{FD} \) represent the driving force and normalized driving force, respectively. \( P_j \) and \( P_{jD} \) refer to the floating force and normalized floating force, respectively. The sum of these forces is \( P_{sum} = P_c + P_f + P_j \). The normalized equivalent diameter is defined as follows:

\[ \lambda_{D} = \frac{\lambda}{L}, \]  

(33)

where \( \lambda \) and \( L \) are the equivalent diameter and length of the single pore, respectively. Shown in Fig.10, as the diameter of pore increases, the influence of capillary force decreases. In tight sandstone, the capillary force is a dominant force, whereas the influence of buoyant force increases with increasing pore diameter. According to calculations, in a single pore, the buoyant force is larger than the capillary force when normalized equivalent diameter surpass 10^{-6}, which is called the equilibrium point. In conventional sandstone, normalized equivalent diameter range is constantly at the right side of the equilibrium point, buoyant force has significant influence in multiphase flow. In tight sandstone, the influence of buoyant force is minimal compared with the other forces.

4.2 Oil–water viscosity ratio

Oil–water viscosity ratio is an important dimensionless parameter in tight oil development. A verity of parameters will influence oil–water viscosity ratio, like formation depth, temperature and asphalt content in oil. To focus on the ratio, we set the relative oil–water viscosity ratio \( \frac{\mu_r}{\mu_w} \) at 1, 5 and 10. Using the 3D IFM, we can calculate the relative permeability curve and displacement effect immediately. The result is shown in Fig.11. As the relative oil–water viscosity ratio increases, the isotonic point of relative permeability move to the left, relative permeability curve for oil also move to the left, indicating that the sandstone is more hydrophobic in general. More dense oil will resist in matrix after displacement, therefore, oil displacement efficiency \( Er_{o-w} \) will increase in primary drainage.
4.3 Mineral components

Mineral components will notably influence oil–water displacement, especially in tight sandstone. We simulate four types of mineral components in tight sandstone in the 3D IFM. Keeping the content of dolomite and orthoclase constant, we increase the percentage of quartz and decrease the percentage of albite, listed in Table 4. The results are shown in Fig.12. As the component of quartz increases, the isotonic point of relative permeability move to the right, relative permeability curve for water also move to the right. Based on several experiments, the hydrophilicity of quartz is relatively stronger than other minerals[61]. As the component of quartz increases, the simulated sandstone will become further hydrophilic, thereby increasing the irreducible water saturation. Finally, oil displacement efficiency $E_{r_{o-w}}$ will decrease.

5 Application of 3D-IFM: synthetic influencing factors of displacement

Multiphase flow in formation is a complicated process. Factors, such as pore compression, mineral component, wettability ageing and compound forces, will influence multiphase displacement process synthetically. Here, we make a simple model to simulate synthetic influencing factors on multiphase displacement at different formation depth. With the increase in reservoir depth, the temperature of formation will increase, thereby decreasing oil viscosity, threshold pressure and formation pressure, subsequently influencing the relative permeability curve and displacement effect synthetically. To simulate multi-flow in different depths and investigate synthetic influences on multi-flow, we define four depth increases from 1500 m to 6000 m. The viscosity-temperature curve comes from experimental data by Zhang et al.[59], geothermal gradient is 2.1°C/100 m according to the collected formation data from Xinjiang Basin. Using the 3D IFM, we can calculate the relative permeability curve and displacement effect immediately. The results are shown in Fig.13. With the increase in formation depth, the isotonic point of relative permeability move to the right, relative permeability curve for oil also move to the right. The reason is the viscosity of oil at formation will decrease as formation depth increase, oil are easier to flow, thereby maybe diminishing the fingering effect. Therefore, in deep formation, oil displacement efficiency $E_{r_{o-w}}$ will decrease.

6 Conclusions

A rapid relative permeability evaluation method for tight sandstone based on 3D IFM is introduced. Initially, a revised 3D IFM is built on the basis of high-resolution SEM and EDS images of tight sandstones with pores exhibiting mineral properties.
Then, we develop a PCC method to calculate the relative permeability and residual oil saturation of the 3D IFM. The accuracy of this model is verified by both experimental and simulated data, indicating that this method satisfactorily evaluates multiphase displacement in tight sandstone. Finally, as an application of 3D IFM method, the influences of several factors on multiphase displacement efficiency are discussed, including oil–water viscosity ratio, mineral components. Also, the synthetic influence by several factors. On displacement behavior is discussed.

The findings show that the modified 3D IFM is valid and useful. By using the self-similar properties, the expression process of pore size distribution and mineral properties is simplified, thereby providing a solid foundation for relative permeability calculation. From the simulation results, as oil-water viscosity ratio increase, the oil-water displacement efficiency will increase. And as the content of hydrophilic mineral decrease, the oil-water displacement efficiency will increase. In addition, the revised 3D IFM based on PCC method can rapidly evaluate the multiphase displacement process. This method provides another efficient way to research multiphase flow and shows a potential application for future use.

Nomenclature

\[ A = \text{cross-sectional area, m}^2 \]
\[ C = \text{shape factor} \]
\[ C_a = \text{capillary number} \]
\[ C_i = \text{shape factor for capillary at iteration level} \ i \]
\[ C_n = \text{curvature of a meniscus} \]
\[ E_{r_{o-w}} = \text{displacement efficiency for oil displace water} \]
\[ i = \text{iteration level in IFU model} \]
\[ k_{nw_{-iab}}(i) = \text{non-wetting phase permeability at iteration level} \ i \]
\[ k_{w_{-iab}}(i) = \text{wetting phase permeability at iteration level} \ i \]
\[ L = \text{the edge length of model, m} \]
\[ L_n = \text{distance from cross section to pressure inlet,m} \]
\[ N_i = \text{number of pores generated at iteration level} \ i \]
\[ N_{i-Mj} = \text{number of pores with} \ j \text{th mineral and Pore diameter} \ \lambda i \]
\[ P_1 = \text{fraction of connecting pores} \]
\[ P_{1,i} = \text{fraction of connecting pores (except dead ends) existing at} \ i \text{th iteration level} \]
\[ P_{1,j-Mi} = \text{fraction of connecting pores existing at} \ i \text{th iteration level with} \ j \text{th mineral} \]
\[ P_2 = \text{traverse probability} \]
\[ P_{2,i-j} = \text{traverse probability from pore neck at iteration level} \ i \text{to iteration level} \ j \]
\[ P_c = \text{capillary pressure, Pa} \]
\[ P_{CD} = \text{normalized capillary force} \]
\[ P_{CSDP} = \text{connecting and successful displacement probability} \]
\[ P_{CSDP,i} = \text{connecting and successful displacement probability for iteration level} \ i \]
\[ P_F = \text{driving force, Pa} \]
\[ P_{FD} = \text{normalized driving force} \]
\[ P_f = \text{floating force, Pa} \]
\[ P_{d1} = \text{normalized floating force} \]

\[ P_G = \text{projection of gravity on flow direction, Pa} \]

\[ P_{\text{initial}} = \text{the threshold pressure gradient, Pa} \]

\[ S_{\text{wres}}(\alpha, \theta_R, i) = \text{residual wetting phase at iteration level i} \]

\[ S_{\text{nw}_\text{sub}}(i) = \text{non-wetting phase residual saturation at iteration level i} \]

\[ \alpha = \text{half angle of the corner,}\degree \]

\[ \theta_{eq} = \text{static contact angle,}\degree \]

\[ \theta_R = \text{dynamic contact angle,}\degree \]

\[ \lambda = \text{equivalent tube diameter for capillary tube, m} \]

\[ \lambda_i = \text{equivalent tube diameter for IFU model at iteration level i, m} \]

\[ \lambda_D = \text{normalized equivalent diameter} \]

\[ \mu_{\text{nw}} = \text{viscosity of non-wetting phase, Pa.s} \]

\[ \mu_w = \text{viscosity of wetting phases, Pa.s} \]

\[ \sigma = \text{interfacial tension between two fluid phases} \]

\[ \phi_i = \text{percentage of mineral component i} \]

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Additional Information

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### Tables

#### Table 1

Connecting probability calculation process.

| Iteration level (i) | Pore diameter ($\lambda_i$) | Number of pores ($N_i$) | Connected fraction ($P_{1,i}$) | Connecting and successful displacement probability |
|---------------------|-----------------------------|-------------------------|---------------------------------|--------------------------------------------------|
| 1                   | $\lambda_1$                | $N_1$                   | $P_{1,1}$                       | $\frac{\lambda_i < N_i P_{1,1} > \lambda_{i+1}}{A} P_{2,1-i}$ |
| 2                   | $\lambda_2$                | $N_2$                   | $P_{1,2}$                       | $\frac{\lambda_i < N_i P_{1,2} > \lambda_{i+1}}{A} P_{2,2-i}$ + $\frac{\lambda_i < N_i P_{2,1} > \lambda_{i+1}}{A} P_{2,2-i}$ |
| ...                 | ...                         | ...                     | ...                             | ...                                               |
| n                   | $\lambda_n$                | $N_n$                   | $P_{1,n}$                       | $\sum_{i=1}^{n} \frac{\lambda_i < N_i P_{1,n} > \lambda_{i+1}}{A} P_{2,n-i}$ | $\frac{\lambda_i < N_i P_{1,n} > \lambda_{i+1}}{A} P_{2,n-i}$ (\(P_{2,i-n} + P_{2,n-i}\)) |

#### Table 2

IFU model input data for pores in sandstone.

| Unit | Df | n unit | $n_{pore(i=1)}$ | $\lambda_{\text{max}}$ (nm) | Iteration | $\lambda_{\text{min}}$ (nm) | Solid forever | total surface($\mu$m²) | $\varepsilon_{\text{SEM}}$ / $\varepsilon_{\text{IFU}}$ |
|------|----|--------|-----------------|-----------------------------|-----------|----------------------------|---------------|------------------------|-----------------------------|
| A    | 1.89 | 1      | 1               | 18420                       | 5         | 227.4                      | 0             | 512656                 | 0.082 / 0.082               |
| B    | 1.63 | 35     | 3               | 6500                        | 4         | 75.8                       | 0             |                        |                             |
| C    | 1.26 | 6000   | 5               | 780                         | 3         | 86.66                      | 0             |                        |                             |

#### Table 3

Extended connecting probability calculation table

| Pore diameter ($\lambda_i$) | Mineral type | Number of pores ($N_{i,M1}$) | Connected fraction ($P_{i,1-M1}$) | Capillary pressure ($P_{c,1,1-M1}$) |
|-----------------------------|--------------|--------------------------------|-----------------------------------|----------------------------------|
| $\lambda_1$                | quartz       | $N_{1,M1}$                     | $P_{1,1-M1}$                      | $P_{c,1,1-M1}$                  |
| $\lambda_2$                | quartz       | $N_{2,M1}$                     | $P_{1,2-M1}$                      | $P_{c,1,2-M1}$                  |
| $\lambda_2$                | albite       | $N_{2,M2}$                     | $P_{1,2-M2}$                      | $P_{c,1,2-M2}$                  |
| $\lambda_2$                | dolomite     | $N_{2,M3}$                     | $P_{1,2-M3}$                      | $P_{c,1,2-M3}$                  |
| $\lambda_2$                | orthoclase   | $N_{2,M4}$                     | $P_{1,2-M4}$                      | $P_{c,1,2-M4}$                  |
| $\lambda_3$                | quartz       | $N_{3,M1}$                     | $P_{1,3-M1}$                      | $P_{c,1,3-M1}$                  |
| ...                         | ...          | ...                            | ...                               | ...                             |
Table 4
Mineral components for the three cases

|        | \( \varphi_{\text{quartz}} \) | \( \varphi_{\text{albite}} \) | \( \varphi_{\text{dolomite}} \) | \( \varphi_{\text{orthoclase}} \) |
|--------|---------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Case1  | 40%                             | 57%                           | 2%                            | 1%                            |
| Case2  | 60%                             | 37%                           | 2%                            | 1%                            |
| Case3  | 80%                             | 17%                           | 2%                            | 1%                            |
Fig. 1. Connecting probability calculation model for multiphase displacement

Fig. 2. Schematic of pore connective calculation method

Fig. 3. SEM image of Xinjing tight sandstone
Fig. 4. Fractal representation for micropores

Fig. 5. EDS results for tight sandstone

Fig. 6. Statistics for minerals from EDS results
Fig. 7. Fractal representation for minerals

Fig. 8. IFM procedure for tight sandstone

Fig. 9. Model validation for 3D IFM
Fig. 10. The influence of pore size on several forces

Fig. 11. The influence of Oil–water viscosity ratio on displacement behavior

Fig. 12. The influence of mineral components on displacement behavior
Fig. 13. Synthetic influence on displacement behavior
Highlights

* An advanced 3D intermingled fractal method is used to multiphase flow in tight sandstone.
* Fractal characteristics of pore size distribution and mineral grain distribution are derived from real images from tight sandstone.
* This method is useful and reliable for calculating relative permeability, with main influencing factors included and fast evaluating process.