Dynamically order-disorder transition in triangular lattice driven by a time dependent magnetic field

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We have elucidated the dynamic phase transition features and finite-size scaling analysis of the triangular lattice system under the presence of a square-wave magnetic field. It has been found that as the value of half-period of the external field reaches its critical value, whose location is estimated by means of Binder cumulant, the system presents a dynamic phase transition between dynamically ordered and disordered phases. Moreover, at the dynamic phase transition point, finite-size scaling of the Monte Carlo results for the dynamic order parameter and susceptibility give the critical exponents $\beta/\nu = 0.143 \pm 0.004$ and $\gamma/\nu = 1.766 \pm 0.036$, respectively. The obtained critical exponents show that present magnetic system belongs to the same universality class with the two-dimensional equilibrium Ising model.

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I. INTRODUCTION

The physical mechanism behind nonequilibrium phase transitions is less understood than that of equilibrium phase transitions for magnetic systems, and it deserves particular attention. Interacting spin systems under the existence of an oscillating magnetic field can display unusual and interesting magnetic behaviors, which can not be observed in their corresponding equilibrium parts. For the first time, the authors in Ref. [1] applied their mean field tools to characterize the kinetic nature of the Ising model being subjected to a time dependent magnetic field. From their analysis, it has been found that amplitude and period of the external field have an important role on the dynamic behavior of the studied system. For example, the system undergoes a dynamic phase transition (DPT) between dynamically ordered and disordered phase with increasing value of the applied field amplitude by keeping other system parameters fixed. Since then, many theoretical [2] [16] and several experimental [17] [21] studies have been performed to examine the DPTs and to understand in depth their origins observed in different types of magnetic systems. Note that in most of the theoretical studies mentioned above, DPT have been encountered by changing the applied field amplitude and temperature.

Some efforts were also taken to elucidate the influences of the period of the external magnetic field on the dynamic phase transition phenomena at constant applied field amplitude [23] [31]. Below its equilibrium critical temperature $T_c$, the kinetic Ising model undergoes a DPT between dynamically ordered and disordered phase when the period of the field reaches the critical period. For small period values of the field, the system does not have enough time to follow the external field instantaneously. Thereby, time dependent magnetization oscillates around a non-zero value indicating a dynamically ordered phase. However, magnetization can be capable of following the external field with a relatively small phase lag, which indicates the dynamically disordered phase. Some of previous published works indicate that there is a good consensus between DPTs and equilibrium phase transitions, especially for the determination of universality class of the spin system far from equilibrium. For instance, it has been found that the critical exponents of the two-dimensional (2D) kinetic Ising model subjected to a square-wave oscillatory magnetic field are consistent with the universality class of the corresponding 2D equilibrium Ising model [23] [26]. In another report, finite-size scaling analysis of Monte Carlo simulation supports these findings for the three dimensional kinetic Ising model [27]. These studies also show that the symmetry arguments reported in Ref. [32] is valid for the magnetic systems without surfaces [28] driven by a time dependent external field. We would like to mention that particular interests in works discussed above have been only dedicated to classify the universality classes of the square and simple cubic lattices in detail. We believe that much more work is required to have better understanding of the DPTs and classifying universality properties of the spin systems far from equilibrium in different geometries such as triangular, honeycomb, and kagome lattices.

In the present work, we consider the kinetic Ising model on a triangular lattice being subjected to a square-wave magnetic field, in order to contribute to the finite-size scaling properties and also universality properties of spin system far from equilibrium. Based on the finite-size scaling of the Monte Carlo results for the dynamic order parameter and susceptibility, it has been estimated the critical exponents. The obtained critical exponents demonstrate that present magnetic system belongs to the same universality class with the 2D equilibrium Ising model.

The outline of remainder parts of the paper is as follows: In section II we give details of model and simulation procedure. The results and discussion are presented

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in section III and finally section IV contains our conclusions.

II. MODEL AND SIMULATION DETAILS

We study the kinetic Ising model on a triangular lattice under presence of a time dependent magnetic field. The Hamiltonian of the present system can be written as follows:

$$H = -J \sum_{\langle ij \rangle} S_i S_j - h(t) \sum_i S_i$$

where $S_i = \pm 1$ is the Ising spin variable at the position $i$, and $J$ is the ferromagnetic ($J > 0$) spin-spin coupling between nearest neighbor (nn) spins in the system. The first summation in Eq. (1) is over the nn site pairs in the system while the second one is over all lattice sites in the 2D triangular lattice system. $h(t)$ describes the time dependent oscillating magnetic field. For the present study, we use a square-wave magnetic field source with amplitude $h_0$ and half-period $t_{1/2}$, following the references [22, 23, 27].

We use Monte Carlo simulation with local update Metropolis algorithm [33, 34] to understand and clarify the DPT characteristics and universality properties of the system on a $L \times L$ triangular lattice, where $L$ is the linear size of the system. Periodic boundary conditions are applied to the system in all directions. We consider the initial configuration where all spins are up, and spin configurations are generated by selecting the lattice site randomly through the triangular lattice. Here, we restrict ourselves to consider the values of the field amplitude $h_0/J = 0.3$ and of the temperature $T = 0.8T_c$, where $T_c = 3.60495J/k_B$ is the critical temperature of the 2D triangular lattice Ising model. After discarding the first 1000 period of the external field, numerical data were collected over next 200 000 periods of the field. We note that the time unit is one Monte Carlo step per site (MCSS).

In order to elucidate the critical properties of the dynamic phase transitions, one can consider the dynamic order parameter, which is the time averaged magnetization over a full cycle of the external magnetic field:

$$Q = \frac{1}{2t_{1/2}} \int M(t) dt,$$

where $M(t)$ is the instantaneous value of the magnetization per site, which can be obtained as follows:

$$M(t) = \frac{1}{L^2} \sum_{i=1}^{L^2} S_i.$$

We note that due to the symmetry of the system, the probability distribution of the dynamic order parameter is bimodal form in the dynamically ordered phase for the finite lattice sizes. Keeping this in mind, the order parameter is considered as $\langle Q \rangle$, namely average norm of $Q$.

In order to determine the dynamic critical point with a high precision, one of the suitable ways is to calculate Binder cumulant as a function of the system size:

$$U_L = 1 - \frac{\langle Q^4 \rangle_L}{3\langle Q^2 \rangle_L^2}.$$

Previous studies on the universality aspects of the kinetic Ising model suggest that the scaled variance of the dynamic order parameter can be regarded as susceptibility of the system, which can be defined as follows:

$$\chi^Q_L = L^2 \langle (Q^2) - \langle Q \rangle^2 \rangle.$$

In order to extract the critical exponents, one of the well-known methods is finite-size scaling method. In this method, the main tool is to determine the measured quantities as a function of the system size. Based on the finite-size scaling method for the system in thermal equilibrium [22, 33, 34], it is possible to write down the following scaling forms for the order parameter and susceptibility at the critical point:

$$\langle Q \rangle_L \propto L^{-\beta/\nu},$$

$$\chi^Q_L \propto L^{\gamma/\nu}.$$

Previous detailed investigations show that these scaling forms are also applicable to classify the universality classes of the magnetic systems driven by a time dependent oscillating magnetic field [23, 28].

III. RESULTS AND DISCUSSION

In Fig. I(a-c), we focus our attention on the time series of magnetization of the kinetic Ising model on a triangular lattice for a system size $L = 180$ at $T = 0.8T_c$ and $h_0/J = 0.3$. The time series are plotted at various values of the half-period of the external field: (a) $t_{1/2} = 50$ MCSS, (b) 150 MCSS and (c) 300 MCSS, respectively. As shown from the Fig. I(a), the magnetization of the system does not have enough time to follow the rapidly changing external field. Thereby, it oscillates around a non-zero value corresponding to the dynamically ordered phase ($Q \neq 0$). As the half-period of the external field is increased further, for example $t_{1/2} = 300$ MCSS, the system begins to reverse its magnetization corresponding to the dynamically disordered phase ($Q = 0$), as shown in Fig. I(c). It is clear that there exists a critical half-period value where a DPT takes place. Our Monte Carlo simulation results suggest that the critical half-period of the external field is $t_{1/2} = 142 \pm 1$ MCSS (which will be
discussed in the following) for the considered kinetic Ising model on a triangular lattice model. In Fig. 1(b), we give an example of the time series of the magnetization in the vicinity of the DPT of the system for value of half-period $t_{1/2} = 150$ MCSS.

In Fig. 2 we give period dependencies of the dynamic order parameter $Q$ of the kinetic Ising model on a triangular lattice, for the considered system parameters $L = 180$, $T = 0.8T_c$ and $h_0/J = 0.3$. The curves are obtained for three values of the half-period of the external field. $t_{1/2} = 50$ MCSS corresponds to the dynamically ordered phase where $Q$ oscillates around a finite value. Dynamic order parameter exhibits strongly fluctuating behavior at $t_{1/2} = 142$ MCSS, indicating the existence of a DPT. $Q$ oscillates around zero value for the value of $t_{1/2} = 300$ MCSS of the field, which is a signature of the dynamically disordered phase.

However, for $t_{1/2} = 300$ MCSS, period averaged $Q$ equals to zero indicating dynamically disordered phase. On the other hand, dynamic order parameter displays strongly fluctuating behavior at $t_1^{1/2} = 142$ MCSS. Large fluctuation behavior observed in the $Q$ as a function of the period of the external magnetic field is a clear evident of a DPT.

In Figs. 3 and 4 as an example of finite-size behavior, we show the data of the dynamic order parameters and their fluctuations for various values of the lattice sizes ranging from $L = 60$ to 210. The numerical data are collected by averaging over 200 000 periods of the magnetic field.

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FIG. 1. (Color online) Time dependent magnetization (blue solid lines) of the kinetic Ising model on a triangular lattice driven by a square-wave magnetic field (red dashed lines denote $h(t)/h_0$ where $h_0$ is amplitude of field) for three considered values of the half-period of the external field. (a) $t_{1/2} = 50$ MCSS and (c) $t_{1/2} = 300$ MCSS correspond to the dynamically ordered and disordered phases, respectively. (b) $t_{1/2} = 150$ MCSS, it is close to the dynamic phase transition point of the system. The numerical data were collected for a system size $L = 180$ at $T = 0.8T_c$ and for value of $h_0/J = 0.3$.

FIG. 2. (Color online) Period dependencies of the dynamic order parameter $Q$ of the kinetic Ising model on a triangular lattice, for the considered system parameters $L = 180$, $T = 0.8T_c$ and $h_0/J = 0.3$. The curves are obtained for three values of the half-period of the external field. $t_{1/2} = 50$ MCSS corresponds to the dynamically ordered phase where $Q$ oscillates around a finite value. Dynamic order parameter exhibits strongly fluctuating behavior at $t_{1/2} = 142$ MCSS, indicating the existence of a DPT. $Q$ oscillates around zero value for the value of $t_{1/2} = 300$ MCSS of the field, which is a signature of the dynamically disordered phase.

FIG. 3. (Color online) Half period dependency $t_{1/2}$ of the dynamic order parameter $\langle Q \rangle_L$ of the kinetic Ising model on a triangular lattice. The curves are obtained for varying values of lattice sizes ranging from $L = 60$ to 210. The numerical data are collected by averaging over 200 000 periods of the magnetic field.

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FIG. 4. (Color online) Half period dependency $t_{1/2}$ of the dynamic order parameter $\langle Q \rangle_L$ of the kinetic Ising model on a triangular lattice. The curves are obtained for varying values of lattice sizes ranging from $L = 60$ to 210. The numerical data are collected by averaging over 200 000 periods of the magnetic field.
field is increased starting from relatively lower values, dynamic order parameter begins to decrease for all studied values of lattice sizes. We also note that half-period dependency of $\langle |Q| \rangle$ tends to disappear with increasing system size. As displayed in Fig. 4 their corresponding susceptibility curves represent a behavior which tends to diverge as the lattice size of the system is increased, in the neighborhood of DPT.

In order to determine the critical half-period of the external field, we perform half-period dependency $t_{1/2}$ of the Binder cumulant $U_L$ at varying values of system size, as seen in Fig. 5 Our Monte Carlo simulation results indicate that the obtained Binder cumulants for varying lattice sizes cross at a special value of half-period of the external field $t_{1/2}^c = 142 \pm 1$ MCSS, where DPT takes place.

As we noted before, by means of Eq. [9] and [7] it is possible to determine the critical exponents of the kinetic Ising model on a triangular lattice. We give log-log plot of the dynamic order parameter $\langle |Q| \rangle_L$ as a function of the linear system size $L$ for the kinetic Ising model on a triangular lattice at $t_{1/2} = t_{1/2}^c$. We note that the filled symbols denote the numerical data obtained from MC simulation while the red line is the weighted least square fit. The numerical data are collected by averaging over 200 000 periods of the magnetic field.

As a final investigation, we obtain the critical exponent $\gamma/\nu$ by benefiting from the slopes of the log-log plot of the susceptibility $\chi_L^Q$ as a function of the system size. It has been found that the critical exponents are $\gamma/\nu = 1.766 \pm 0.036$ (using the data obtained at $t_{1/2}^c$) and $\gamma/\nu = 1.749 \pm 0.03$ (using the data at the peak location). It is interesting to note that our estimates on the critical exponents of the kinetic Ising model on a 2D triangular
lattice are very close to those of the 2D equilibrium Ising model, which are $\beta/\nu = 1/8 = 0.125$ and $\gamma/\nu = 7/4 = 1.75$. With the present study, it is possible to underline that the symmetry arguments reported in Ref. [32] is also valid for the 2D triangular lattice under presence of a square-wave magnetic field considered here, in addition to the previously published studies [23, 27].

IV. CONCLUDING REMARKS

In this study, we have investigated the magnetic response of the kinetic Ising model on a 2D triangular lattice to a square-wave magnetic field. We have performed Monte Carlo simulation with single site update Metropolis algorithm. Our numerical findings clearly indicate that the present system undergoes a DPT at the critical half-period of the external magnetic field $t^c_{1/2} = 142 \pm 1$ MCSS. It has been found that for large half-period of the magnetic field ($t_{1/2} \gg t^c_{1/2}$) time dependent magnetization can be capable of following the external field with a relatively small phase lag, which indicates the dynamically disordered phase. However, for small values of half-period of the external fields ($t_{1/2} \ll t^c_{1/2}$), magnetization does not have enough time to follow the external magnetic field, and it oscillates around a finite value corresponding to the dynamically ordered phase.

Moreover, we focus our attention on the finite-size scaling analysis and critical exponents of the present system, by changing the system size ranging from $L = 60$ to 210. Note that critical exponents with the statistical errors obtained in this study are found to be consistent with the universality class of the 2D equilibrium Ising model, as in the case of the previously published studies [23, 27]. It seems to be that kinetic spin models without surfaces may have the same critical exponents with the corresponding equilibrium Ising model. However, there exists a few systematic studies done in this direction. Hence, much more work is required to have better understanding of the DPTs and to classify universality properties of the spin system far from equilibrium.

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