How far can Tarzan jump?

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Abstract

The tree-based rope swing is a popular recreational facility, often installed in outdoor areas. Hanging from a rope, users drop from a high platform and then swing at great speed like ‘Tarzan’, finally jumping ahead to land on the ground. The question naturally arises, how far can Tarzan jump using the swing? In this paper, I present an introductory analysis of the mechanics of the Tarzan swing, a large pendulum-like swing with Tarzan himself attached as weight. This enables determination of how much further forward Tarzan can jump using a given swing apparatus. The discussion is based on elementary mechanics and is, therefore, expected to provide rich opportunities for investigations using analytic and numerical methods.

(Some figures may appear in colour only in the online journal)

1. Introduction

An oft-repeated scene in Hollywood movies and adventure animations is that of someone jumping on to the end of a long rope suspended from above and, in pendulum motion, gallantly leaping over the various dangers below (enemies, wild animals, poisonous swamps, etc). Very often, a similar children’s play facility called the Tarzan swing [1] is set up in forest parks and on beaches. Holding on to the end of a rope and jumping down from a high altitude is a moment that would test anyone’s courage.

In terms of mechanics, the Tarzan swing can be defined as follows: it refers to a series of movements, starting from the forward jump to the actual landing, by using a half cycle of the pendulum’s full swing with one’s own bodyweight being the pendulum’s weight. After reading this sentence, a physics student may pose the following question: how far can one travel in the horizontal direction with this acrobatic jump? The deciding factor here is the moment when Tarzan releases the vigorously swinging rope. It would be intuitive that Tarzan can travel the maximum distance in the horizontal direction by releasing the rope at precisely the right time (neither too early or too late).

This argument brings us to the main theme of this paper. Let us recall that the rope hangs from a fulcrum and swings forward. The angle formed between the swinging rope and the
vertical line is expressed by a variable $\theta$. At which point, then, of the pendulum’s deflection angle $\theta$ should Tarzan take his hands off to travel the maximum distance in the horizontal direction?

If this problem is posed during a mechanics lecture, the students may respond—‘when $\theta = \pi/4$, the maximum distance travelled can be attained’. This answer is correct in the case of, for instance, firing a cannon from the earth’s surface. According to the simple ballistic model, a cannon fired at a launch angle of $\pi/4$ from the ground would travel the greatest distance (see the appendix). Even in the case of throwing a baseball, the ball would travel the greatest distance if thrown at an upward oblique angle of $\pi/4$, as anyone would probably know from experience. However, this answer is incorrect in the case of a Tarzan swing. As shown below, for Tarzan to travel the maximum distance it is essential for him to release the rope at a much smaller deflection angle than $\pi/4$. Interestingly, this optimum angle of deflection depends on the length of the rope used and the height of the platform, i.e. the starting position. To arrive at this conclusion, a knowledge of elementary mechanics and some knowledge of numerical calculation is sufficient. Furthermore, through a simple experiment using a pendulum, it is possible to compare the theory with the data measured and study the physical cause of disparity between them. Therefore, it can be said that the above problem is a suitable research topic that can be assigned to students in senior high school or in the first year of college to test their application of basic mechanics.¹

2. Establishment of the problem

For simplicity in the following discussion, consider Tarzan himself (who holds the rope) to be the weight with a mass of $m$. Assume that Tarzan moves in the vertical plane described by the $x$–$y$ axis and that air resistance is negligible due to his sufficiently large mass. We take no account of the mass and deflection of the suspended rope. Figure 1 shows: (i) a schematic diagram of the sequence of processes in the Tarzan swing from the take-off at a high platform until the landing and (ii) the definitions of constants and variables used in this paper. First, Tarzan jumps off the platform (point A of figure 1), reaches the pendulum swing nadir B

¹ A different recommended research theme is to study the case of Tarzan not releasing the rope due to fear or cowardice. In fact, since ‘swing mechanics’ is extremely useful for a first-year physics course, there have been many publications on extremely interesting topics in this area [3–5].
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(y = h) at time t = 0 and releases the rope (point C) at time t = t_r. We assume the pendulum’s deflection angle to be θ = θ_s and Tarzan’s velocity to be v = v_s at the moment he releases the rope. Then his position at t ≥ t_r, after he projects himself into the air, can be represented by the following equation.

\[ x(t) = r \sin \theta_s + v_s t \cos \theta_s, \quad (1) \]

\[ y(t) = h + r(1 - \cos \theta_s) + v_s t \sin \theta_s - \frac{g}{2} t^2. \quad (2) \]

Here, g is the acceleration due to gravity and r is the length of the rope. After diving through the air, Tarzan finally lands on the ground at t = t_f (point D).

Based on the above conditions, the problem that we need to solve can be summarized as ‘using the given constants, h, r, and θ_s, determine the swing deflection angle θ that maximizes the horizontal flight distance L, and evaluate the maximum distance’.

3. Formulation

Let us address the problem now. At t = t_f, Tarzan lands on the ground and thus we have x(t_f) = L and y(t_f) = 0. Substituting these equations into equations (1) and (2), we obtain

\[ L = r \sin \theta_s + v_s t_f \cos \theta_s, \quad (3) \]

\[ 0 = h + r(1 - \cos \theta_s) + v_s t_f \sin \theta_s - \frac{g}{2} t_f^2. \quad (4) \]

We eliminate t_f from equations (3) and (4) to obtain the expression of the flight distance L as

\[ L = r \sin \theta_s + \frac{v_s^2 \sin \theta_s \cos \theta_s}{g} \sqrt{\left(\frac{v_s^2 \sin \theta_s \cos \theta_s}{g}\right)^2 + \frac{2v_s^2 \cos^2 \theta_s [h + r(1 - \cos \theta_s)]}{g}}. \quad (5) \]

Note that the launch velocity v_s involved in equation (5) is dependent on θ_s. The θ_s-dependence of v_s originates from the energy conservation law represented by

\[ mg[h + r(1 - \cos \alpha)] = mg[h + r(1 - \cos \theta_s)] + \frac{m}{2} v_s^2. \quad (6) \]

It is reduced to a simpler expression:

\[ \frac{v_s^2}{g} = 2r(\cos \theta_s - \cos \alpha), \quad (7) \]

which clarifies the relationship between θ_s and v_s. The constraint given in equation (7) is the main reason that the optimal θ_s in the Tarzan swing becomes smaller than π/4. In the shell-firing case, in contrast, θ_s and v_s can be defined separately, as a result of which the optimal θ_s equals π/4 (see the appendix).

From equations (5) and (7), we attain the expression:

\[ \frac{L}{r} = \sin \theta_s + \Delta_2 \sin 2\theta_s + 2 \cos \theta_s \sqrt{\Delta_2 (\Delta_2 \sin^2 \theta_s + \Delta_1)}, \quad (8) \]

where

\[ \Delta_1 = \frac{h}{r} + (1 - \cos \theta_s), \quad \Delta_2 = \cos \theta_s - \cos \alpha. \quad (9) \]

In terms of physical meaning, Δ_1 quantifies the vertical height of the launching point, i.e. point C in figure 1) in units of r. Δ_2 characterizes the difference in height between the launching point and the starting point (point A in figure 1).
Equation (8) provides the flight distance $L$ covered by Tarzan for a given $\theta_s$. It is easy to derive from equation (8) that

$$\frac{L}{r} = \sin \alpha \ \text{at} \ \theta_s = \alpha, \quad \text{and} \quad \frac{L}{r} = 2 \sqrt{\frac{h}{r} (1 - \cos \alpha)} \ \text{at} \ \theta_s = 0. \ \ \ (10)$$

In addition, we can prove that

$$\alpha \frac{dL}{d\theta_s} \bigg|_{\theta_s=0} > L(\theta_s = \alpha) \quad (0 < \alpha < \frac{\pi}{2}). \ \ \ (11)$$

The inequality (11) guarantees the existence of at least one local maximum of $L$ within the range of $0 < \theta_s < \alpha$; the proof is left to readers as an exercise. Henceforth, we use $\tilde{\theta}_s$ to denote the optimal launching angle that yields the maximum flight distance $\tilde{L}$.

4. Results and discussion

Figure 2 illustrates the $\theta_s$-dependence of $L$ described by equation (8). Among the three parameters $\{h, r, \alpha\}$, the first two are set to have the relation $h/r = 0.3$ in figure 3(a) and $h/r = 10.0$ for (b). The value of $\alpha$ is tuned as presented in the plots. We observe that all $L$-curves are convex upward, possessing a maximum $\tilde{L}$ at $\tilde{\theta}_s$ less than $\pi/4$. In the two plots, the values of $\tilde{\theta}_s$ and $\tilde{L}$ for $\alpha = \pi/2$ are displayed as examples. As $\alpha$ reduces from $\pi/2$, both $\tilde{\theta}_s$ and $\tilde{L}$ decrease in a monotonic manner. These decreasing behaviours are universal regardless of the condition of $h/r$; this fact can be visually confirmed by figure 3, wherein the contour plots of $\tilde{\theta}_s$ and $\tilde{L}$ on the $(h/r)$-$\alpha$ plane are presented. From a physical viewpoint, the result in figure 3(b) is rather trivial; the larger (smaller) $\alpha$ yields the longer (shorter) flight distance $L$, because such $\alpha$ enhances (diminishes) the impetus of Tarzan at the launching point in accord with the energy conservation law (see equation (2)). Similarly, it is obvious that a larger (smaller) $h/r$ results in a larger (smaller) $L$, as it leads to a long (short) flight time duration in which Tarzan moves forward in the horizontal direction. On the other hand, the $\tilde{\theta}_s$-landscape shown in figure 3(a) is not so trivial and requires some consideration. Why is the
smaller $h/r$ associated with the larger $\tilde{\theta}_s$ and vice versa? This question is resolved in part by depicting the trajectories of Tarzan during the diving.

Figure 4(a) shows the trajectories of Tarzan for four different conditions of the paired parameter ($\alpha$ and $h/r$), which correspond to the positions of the four open circles depicted in figure 3. The two horizontal arrows in figure 4(a) symbolize the ropes spanned by the pendulum pivot (circle) and Tarzan’s hand for the case of $\alpha = \pi/2$. (A counterpart for $\alpha = \pi/10$ may be a slanted arrow with the same pivot, but this is omitted in the figure.) The figure makes it...
easy to grasp the two obvious facts already found in figure 3(b). First, for a fixed $\alpha$, the larger $h/r$ yields a larger $L$ owing to the long flight duration. Second, when $h/r$ is fixed, a large $\alpha$ yields a large $L$ owing to the large $x$-component of the launching velocity, denoted by $v_x$.

Moreover, figure 4(a) provides a clue to the answer to the previously raised question, i.e., the reason behind the smaller (larger) $h/r$ corresponding to a larger (smaller) $\tilde{\theta}_s$. A small $h/r$ implies a short flight duration; therefore, Tarzan prefers a large $\tilde{\theta}_s$ to obtain a certain degree of the $v_y^s$ component in the upward direction, because the upward $v_y^s$ component tends to prolong the flight duration. This situation corresponds to the thick bottom curve in figure 4(a), where Tarzan lifts off slightly upward like a flying ball to achieve $\tilde{L}$. In contrast, when $h/r$ is large enough, $\theta_s$-variation provides a minor contribution to the flight duration, since Tarzan’s vertical position $y(t)$ at $t \gg t_s$ is dominantly determined by the gravitational force effect, i.e. the term $-gt^2/2$ on the right side of equation (2). In other words, the flight duration is not significantly altered by acquiring the upward $v_y^s$ component. To reach maximum $L$, therefore, it is crucial to obtain a large $v_y^s$ component by setting a small $\theta_s$. However, one important aspect is to be kept in mind. Even though a small $\theta_s$ is desirable in principle, a very small $\theta_s$ is not appropriate to maximize $L$. This fact is schematically explained in figure 4(b) and 4(c), which demonstrate how the $\theta_s$-shift from the optimal value affects the Tarzan’s trajectory at $t \gg t_s$. If Tarzan jumps at a very small $\theta_s$, represented by the curve A, he necessarily lands before he successfully pulls out of the curve B, i.e. the optimal case in the horizontal direction. Much worse is the case of a very large $\theta_s$ (curve C), in which an insufficient $v_y^s$ component results in a shorter $L$ than for the other two cases.

5. Summary

We have developed a simplified model of the Tarzan swing. The optimal deflection angle $\tilde{\theta}_s$ to maximize the horizontal jump distance of Tarzan was considered; its dependence on the configuration parameters of the swing apparatus such as the height of the swing nadir ($h$), rope length ($r$) and the starting platform position ($\alpha$) was formulated in the realm of elementary mechanics. We found that $\tilde{\theta}_s$ is a monotonic decreasing function of $h/r$; the physical origin of the decreasing property was explained through the trajectories of Tarzan during his aerial diving.

6. Suggested problems

It is reasonable to propose the following questions for further work related to this topic.

- In figure 1, consider a case where $h = r$, $\alpha = \pi/2$ and $\theta_s = 0$. In this case, Tarzan travels through trajectories AB and BD, along each of which he falls down at the same distance of $h$. In addition, both the two initial velocities for the motions through AB and BD (i.e. velocities at points A and B, respectively) have zero velocity component in the vertical direction ($v_y^i = 0$). Now, determine the time required to travel from A to B (denoted by $t_{AB}$) and B to D ($t_{BD}$) and check whether they are equal. If they are not equal, give physical reasons for the inequality.
- In general, Tarzan at point B of figure 1 is subjected to a large centripetal force (upward) due to speed $v_0$. What is the maximum value of the sum of gravitational downward-pull and centripetal force at point B? If the force applied is extremely large, then we fear the rope breaking or Tarzan slipping off.
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- Carry out a study similar to that of this paper for a case where the rope expands and contracts like rubber. Find out the values that can be set for the rope’s elastic constants to maximize the horizontal distance covered by Tarzan.

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Appendix. The simplest ballistic curve

We suppose a flying ball launched upward from the ground at a slanted angle $\theta$ with respect to the horizontal line. For a given initial velocity $v$, the position of the ball at time $t$ is described by

$$x(t) = vt \cos \theta, \quad y(t) = vt \sin \theta - \frac{g}{2} t^2.$$  \hspace{1cm} (A.1)

At $t = t_f$, we obtain $x = L$ and $y = 0$; thus we have

$$L = vt_f \cos \theta, \quad 0 = vt_f \sin \theta - \frac{g}{2} t_f^2,$$  \hspace{1cm} (A.2)

which imply that

$$L = \frac{2v^2 \sin \theta \cos \theta}{g} = \frac{v^2 \sin 2\theta}{g}.$$  \hspace{1cm} (A.3)

Obviously, $L$ takes the maximum value at $\theta = \pi/4$.

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