Influence of thermal conductivity of the partition wall on non-stationary conjugate natural convective heat exchange and temperature fields in the walls of a rectangular fuel tank

V S Berdnikov¹²* and K A Mitin¹

¹Kutateladze Institute of Thermophysics SB RAS, Novosibirsk, Russia
²Novosibirsk State Technical University, Novosibirsk, Russia

*E-mail: berdnikov@itp.nsc.ru

Abstract. The non-stationary conjugate heat transfer in a rectangular model of a fully filled thin-walled tank, divided by an internal vertical partition wall into two compartments of 1/3 and 2/3 of the tank volume is studied numerically in the conjugate formulation. The outer surfaces of the end walls and the upper wall of the tank are heat-insulated. The lower wall suddenly warms up under the influence of a uniform heat flux. The calculations are performed at the ratio of the thermal conductivity of solid walls to the thermal conductivity of fuel \( \lambda_s/\lambda_f = 1041.3 \) and \( \lambda_s/\lambda_f = 1 \) (low-heat partition wall) and thermal parameters of fuel T1 (Prandtl number \( \text{Pr} = 25.66 \)). The influence of internal vertical partitions wall on the spatial shape of convective flows and the regularities of non-stationary conjugate heat exchange is studied. The temperature fields in the liquid and in the tank walls are calculated.

1. Introduction

An actual problem in the operation of aviation equipment is thermal stresses in non-isothermal thin-walled elements of aircraft structures that appear during monotonous and cyclical temperature changes [1, 2]. Monotonous dependences on the time of thermal states of the aircraft structure are typical for takeoff and landing periods. However, at the initial stages of take-off and reaching cruising speed in the cooling modes of the outer walls, non-stationary thermogravitation convection develops in the fuel tanks. During supersonic flights, the aircraft covering warms up and thermogravitation convection develops in the fuel tanks. In the regimes of laminar-turbulent transitions and in the turbulent flow regimes, temperature fluctuations appear in the boundary layers of thin walls. In non-stationary conjugate convective heat exchange regime, heat waves propagate in the walls of fuel tanks and time-dependent alternating thermal stresses occur. In the presence of temperature gradients and uneven temperature deformation, additional stresses occur in the structural elements, including fuel tanks with reinforced elements and partition walls made of materials with different thermal conductivity [1]. When the temperature changes, the mechanical characteristics of the materials change. This influences the endurance and fatigue life of structures and subsonic and supersonic aircraft. The appearance of monotonically or cyclically changing temperature gradients and thermal stresses inside the tank walls depends on the regularities of local conjugate convective heat exchange. Non-stationary thermogravitation convection in the tanks affects the temperature conditions of the fuel and leads to uneven distributions of local heat fluxes. The spatial form of convective flows has a significant influence on the laws of local conjugate heat transfer. In turn, the shape of convective flows largely depends on the configuration of cavities and the location of the heated and cooled walls and their fragments [3-5].
For estimates and accurate calculation of thermal stresses, reliable knowledge of the laws of conjugated convective heat transfer in a design with non-stationary conditions on the external and internal surfaces of aircraft is required. Therefore, fundamental studies of the features of non-stationary thermogravitation convection in thin-walled structures are necessary [1, 2]. In view of the use of composite materials in modern aircraft structures, it is important to understand the features of the development of non-stationary convection in the tank model with low-heat-conducting walls. Similar problems are typical for many technical devices in the heating or cooling regimes on and off. In unevenly heated liquid volumes located in the gravity field, natural convective flows develop with liquid stratification by temperature, which significantly affects the development of boundary layers and heat exchange [3-6]. This work continues the series of works conducted at the S. S. Kutateladze Institute of Thermophysics SB RAS and aimed at studying the effect of conjugated natural convective heat transfer on temperature distribution in thin walls [3-5].

2. Model

Numerical simulations are performed in a dimensionless form in a two-dimensional conjugate formulation in Cartesian coordinates. The study domain is a two-dimensional rectangular cavity with an area width to height ratio of 6:1; it is filled with liquid and bounded on all sides by thin walls with a thickness equal to 0.0133 of the layer height. The area is divided by an internal vertical partition into two compartments of 1/3 and 2/3 of the total volume of the area. The outer surface of the upper wall and side (end) walls is adiabatic. The outer surface of the lower wall at the initial time begins to warm up due to a uniform heat flux.

Convective heat transfer in a liquid is described by a dimensionless system of thermogravitation convection equations in the Boussinesq approximation, written in terms of temperature, vortex, and stream function:

\[
\begin{align*}
\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} &= \frac{1}{\text{Pr}} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \\
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} &= -\omega, \\
\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} &= \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + \text{Gr} \frac{\partial T}{\partial x}.
\end{align*}
\]

here \( \text{Gr} = g \beta f H^3 \Delta T v^2 \) is the Grashof number, where \( g \) is the acceleration due to gravity, \( \beta \) is the coefficient of volume expansion of the fluid, \( \text{Pr} = v / \alpha_f \) is the Prandtl number, \( \alpha_f \) is the thermal diffusivity of the fluid, \( T \) is the dimensionless temperature, \( \omega \) is the dimensionless vortex, and \( \psi \) is the dimensionless stream function. The height of the layer is \( H \). The temperature scale is \( \Delta T = T_1 - T_2 \), where \( T_1 \) and \( T_2 \) are the temperatures on the outer surface of the lower and upper walls, respectively. The velocity scale is \( v / L \), where \( v \) is the kinematic viscosity of the liquid, and the time scale is \( H^2 / v \).

Conductive heat transfer in solid walls is described by the heat equation:

\[
\frac{\partial T}{\partial t} + \alpha_s \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0
\]

where \( \alpha_s \) is the thermal diffusivity of the solid wall material.

Numerical simulation is performed for the following parameters: \( \rho_f = 819 \text{ [kg/m}^3 \text{]} \) is the fluid density; \( \beta_f = 8.3 \cdot 10^{-4} \text{ [1/K]} \) is the coefficient of volume expansion of the liquid; \( \eta_f = 1.49 \cdot 10^{-3} \text{ [kg/(m·s)]} \) is the dynamic viscosity of the fluid; \( \lambda_f = 0.1162 \text{ [W/(m·K)]} \) is the fluid thermal conductivity; \( C_p = 2001.29 \text{ [J/(kg·K)]} \) is the heat capacity of the fluid; \( p_s = 2780 \text{ [kg/m}^3 \text{]} \) is the density of the material of the walls; \( \lambda_s = 121 \text{ [W/(m·K)]} \) is the solid thermal conductivity; and \( C_p = 921 \text{ [J/(kg·K)]} \) is the heat capacity of the
material of the walls. In the simulation with walls equal to the liquid thermal conductivity as the values of $\rho_s$ and $\lambda_s$, substituted values $\rho_f$ and $\lambda_f$, respectively. The thermophysical parameters correspond to the parameters of aviation fuel T1 and D16T.

The calculations were performed using the finite element method [7] on an irregular triangular grid with thickening to solid walls. Linear basis functions on triangles were used. A grid with 30,000 nodes was used.

3. Results and discussion

Calculations of non-stationary conjugate heat transfer in the regime of thermogravitation convection are performed in a rectangular model of a fully filled thin-walled tank divided by an internal vertical partition into two compartments 1/3 and 2/3 of the tank volume.

Figure 1. Temperature fields (top) and stream function isolines (bottom) at $\lambda_s/\lambda_f = 1041.3$ at times:

- a – t = 50;
- b – 60;
- c – 70;
- d – 80.

Figure 1 shows the evolution of temperature fields in time at $\lambda_s/\lambda_f = 1041.3$ at the initial time. It is noticeable that due to the higher thermal conductivity of thin walls, the vertical partition and side walls are rapidly heated. After that, by means of conductive heat transfer, the upper wall of the tank covering is heated.

Figure 2. Profiles of the vertical velocity component at $y = 0.5$ at $\lambda_s/\lambda_f = 1041.3$ at times:

- 1 – t = 50;
- 2 – 60;
- 3 – 70;
- 4 – 80.
In the left part of the tank, one ascending plume is formed, and a four-vortex flow is formed, due to two ascending flows on the heated vertical walls and one central ascending flow. In the right part, due to the greater length of the region, two ascending plumes and a system of six vortexes are formed. It is noticeable that in the right area in the bottom, the liquid is heated more intensively than in the left one, due to the larger area of the heated surface of the lower wall and, accordingly, smaller efficiency of heat removal through the vertical walls. There is a slight drift of the central updrafts to the side walls. Moreover, in the left part of the tank, the updraft drifts to the central partition, which is heated to a higher temperature than the left wall due to hot flows ascending along it from both sides (figure 2). Accordingly, the intensity of the upward convective flow is slightly higher and the thickness of the boundary layer decreases over time. Similarly, in the right part of the tank, the central updrafts diverge to the nearest vertical walls.

Figure 3 shows the further development of convective flows in the time of the temperature field at $\lambda_s/\lambda_f = 1041.3$. It is noticeable that a two-vortex stream forms in the left part of the tank. The central updraft alternately carries it to the right and to the left wall. That is, an oscillatory process is established. In the right part of the tank, a four-vortex flow is set in a similar way. Ascending jets originate in the central part of the compartment and drift to the nearest side wall. Figure 4 shows the time evolution of temperature fields at $\lambda_s/\lambda_f = 1$ at the initial time. It should be noted that convective heat exchange does not develop immediately. At the initial moment of time, the bottom area warms up and is in a state of mechanical equilibrium. This is only possible if there is no horizontal temperature gradient. In this case, the thermal conductivity of the liquid and the material of the tank walls coincide. Otherwise, a horizontal temperature gradient will inevitably occur, which initiates convective heat exchange.

![Figure 3. Temperature field (top) and stream function isolines (bottom) at $\lambda_s/\lambda_f = 1041.3$ at times:](image_url)

- a – $t = 640$
- b – $660$
- c – $680$
- d – $700$

Figure 4 shows the time evolution of temperature fields at $\lambda_s/\lambda_f = 1$ at the initial time. It should be noted that convective heat exchange does not develop immediately. At the initial moment of time, the bottom area warms up and is in a state of mechanical equilibrium. This is only possible if there is no horizontal temperature gradient. In this case, the thermal conductivity of the liquid and the material of the tank walls coincide. Otherwise, a horizontal temperature gradient will inevitably occur and initiate convective heat exchange.
Figure 4. Temperature field (top) and stream function isolines (bottom) at $\lambda_s/\lambda_f = 1$ at times: 

- a – $t = 700$; 
- b – 750; 
- C – 800; 
- g – 850.

In figures 4-5, it is noticeable that convective flows develop according to a different scenario compared to the case of high thermal conductivity of the tank wall material. In the left part, three ascending streams are formed: two at the vertical partitions and one in the center of the area. However, due to the fact that the vertical walls warm up too slowly, they form inverted downward flows that push the ascending jets to the center of the area. A similar pattern is observed in the right part of the tank, where 5 ascending jets are formed. Side jets are also pushed away from the side walls.

Figure 5. Vertical velocity component profiles at $y = 0.5$ at $\lambda_s/\lambda_f = 1$ at times:

- 1 – $t = 700$; 
- 2 – 750; 
- 3 – 800; 
- 4 – 850.

Figure 6 shows further development of convective flows over time of the temperature field at $\lambda_s/\lambda_f = 1$. As in the case of highly heat-conducting walls, the drift of ascending jets is noticeable. However, in the right area, the updrafts are attracted to the central stream. Neighboring ascending jets are attracted to the central stream, pouring into it and leading to an intense release of hot liquid masses.
upwards. At the same time, the updrafts at the side walls, which have already warmed up, do not change their position.

![Temperature field and stream function isolines](image)

**Figure 6.** Temperature field (top) and stream function isolines (bottom) at $\lambda_s/\lambda_f = 1$ at times: a – $t = 1100$; b – $1250$; c – $1400$; d – $1550$.

An interesting feature is observed in the left part of the tank. The central ascending stream is fed by an ascending stream from the left wall, as a result of which the central stream is shifted to the left wall, and subsequently attracts the nascent ascending streams on the left wall. The upstream flow on the central wall behaves stably because of the heating of the central wall due to the upstream flow in the right part of the tank.

![Temperature profiles](image)

**Figure 7.** Temperature profiles at $y = 0.01$ (inside the wall) at $\lambda_s/\lambda_f = 1041.3$ at times: 1 – $t = 640$; 2 – $660$; 3 – $680$; 4 – $700$.

Features of the time evolution of spatial forms of convective flows with different thermal conductivity of the walls affect the distribution of local heat fluxes and temperature in the walls. Local maxima on the temperature distributions along the longitudinal coordinate in figures 7 – 10 correspond to the positions of the heated liquid flows ascending from the wall.
Figure 8. Temperature profiles at y = 0.01 (inside the wall) at $\lambda_s/\lambda_f = 1041.3$ at times:
1 – $t = 700$; 2 – 725; 3 – 750; 4 – 775; 5 – 800.

Here, local heat fluxes from the wall are minimal. Local minima in temperature distributions are caused by the flow of cold liquid on the surface of the lower wall. The temperature values in the walls differ significantly. This is due to different rates of wall heating and very different temperature differences in the thickness of the walls.

Figure 9. Temperature profiles at y = 0.01 (inside the wall) at $\lambda_s/\lambda_f = 1$ at times:
1 – $t = 700$; 2 – 725; 3 – 750; 4 – 775; 5 – 800.

Figure 10. Temperature profiles at y = 0.01 (inside the wall) at $\lambda_s/\lambda_f = 1$ at times:
1 – $t = 1100$; 2 – 1250; 3 – 1400; 4 – 1550.

Conclusions
The non-stationary conjugate heat transfer in a rectangular model of a fully filled thin-walled tank divided by an internal vertical partition into two compartments of 1/3 and 2/3 of the tank volume has been studied numerically in the conjugate formulation. The outer surfaces of the end walls and the upper wall of the tank were heat-insulated. The lower covering was heated by a uniform heat flux. Calculations were performed for different thermal conductivity of the walls and internal partition of the tank and thermal parameters of fuel T1. The temperature fields in the liquid and in the tank walls were calculated. The evolution of convective flows and temperature fields after sudden heat supply under the base of the tank has been studied. The influence of internal vertical partitions on the spatial shape of convective flows and the regularities of non-stationary conjugate heat exchange have been studied. An inhomogeneous temperature field is shown to form inside solid walls of finite thermal conductivity. The thermal conductivity of internal partitions significantly affects the spatial shape of convective flows and the intensity of convective heat exchange. The calculations were performed for the Prandtl number $Pr =$
25.66, and the ratio of the thermal conductivity of solid walls to the thermal conductivity of fuel \( \lambda_s/\lambda_f = 1041.3 \) and \( \lambda_s/\lambda_f = 1 \) (low-heat partition wall).

It is shown that depending on the thermal conductivity of the tank wall material, the development scenarios and spatial shape of convective flows change significantly. In all cases, multi-vortex flows develop in both compartments of the tank and oscillatory processes are observed due to the demolition of vortexes. However, at high thermal conductivity, the vortexes are carried to the vertical walls, while at low thermal conductivity, the vortexes are carried to the central part of the tank compartments.

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References
[1] Belov V K, Belov V V 2011 Prochnost i ustojchivost raketychnykh i aviacionnykh konstrukcij pri termosilovom nagruzhenii (Novosibirsk: Izd-vo NGTU) p. 491 (In Russian)
[2] Zabrodin V S 1978 Temperaturnye polya v konstrukcii letatel'nykh apparatov (Metody rascheta) (M.: Mashinostroenie) p. 184 (In Russian)
[3] Berdnikov V S, Grishkov V A 2009 Sb. tr. Vserossiyskoy konferentsii po aerodinamike letatel'nykh apparatov i prochnosti aviacionnyh konstrukciy (SibNIA 17–19 iyunya 2008, Novosibirsk) p. 124–31 (In Russian)
[4] Berdnikov V S, Gaponov V A, Grishkov V A, Markov V A, Likhansky P M 2010 Thermophysics and Aeromechanics 17(2) 181–91
[5] Mitin K A, Kisliutsyn S A, Berdnikov V S 2019 Journal of Physics: Conference Series 1382 doi: 10.1088/1742-6596/1382/1/012199
[6] Geibhart B, Jaluria Y, Mahajan R L, Sammakia B 1988 Buoyency-induced flows and transport (Washington: Hemisphere Publishing Corporation) p. 613
[7] Solovejchik Yu G, Royak M E, Persova M G 2007 Metod konechnykh elementov dlya resheniya skal'nykh i vektornykh zadach (Novosibirsk: Izd-vo NGTU) p. 896 (In Russian)