Introduction:

Neutrino physics is currently poised at an interesting juncture. Among the parameters of the neutrino mass matrix, oscillation experiments have measured the mass squared differences \( \Delta_{21}, |\Delta_{31}| (\Delta_{ij} = m_i^2 - m_j^2) \) and the mixing angles \( (\theta_{12}, \theta_{23}, \theta_{13}) \) with considerable precision. The remaining unknown oscillation parameters are (i) the mass hierarchy: normal hierarchy (NH, \( m_3 > m_2 > m_1 \)) or inverted hierarchy (IH, \( m_3 < m_2 \approx m_1 \)), (ii) octant of \( \theta_{23} \): \( \theta_{23} < 45^\circ \) (lower octant, LO) or \( \theta_{23} > 45^\circ \) (higher octant, HO) and (iii) the CP-violating phase \( \delta_{CP} \).

The global analysis of current oscillation data gives no indication of the mass hierarchy. There is also no clear indication of the octant of \( \theta_{23} \). For inverted hierarchy higher octant is preferred while for normal hierarchy the lower octant is preferred [1]. However these indications are still fragile. Recently, an indication for \( \delta_{CP} \sim -90^\circ \) has been obtained, driven mainly by a combination of T2K and reactor data [2]. This hint comes from T2K running in the neutrino mode with 8% of the expected total flux of T2K \((7.8 \times 10^{21} \text{ protons on target (pot))} [2] \).

In this paper we consider the possibility of determination of the above unknowns in future runs of T2K. The relevant channel is the \( \nu_\mu \to \nu_e \) oscillation probability \( P_{\mu e} \). This is sensitive to all the three unmeasured parameters described above. However the lack of knowledge of \( \delta_{CP} \) can give rise to three additional spurious solutions corresponding to wrong hierarchy-right octant, right hierarchy-wrong octant and wrong hierarchy-wrong octant in addition to the correct solution. Thus there is a 4-fold degeneracy which is a subset of the 8-fold degeneracy discussed in the literature [3]. These degenerate solutions can occur for different values of \( \delta_{CP} \) other than the true value making its determination difficult. In this study, we work with the assumption that the current hint of \( \delta_{CP} = -90^\circ \) coming from T2K data will be further confirmed by T2K running in its full beam power in only neutrino mode. In that case, we argue that the data will also indicate the true hierarchy as NH and true octant as HO. This is because, for the other combinations of true hierarchy and octant in nature, degenerate solutions occur for wrong values of \( \delta_{CP} \), preventing a distinct signature for \( \delta_{CP} = -90^\circ \). This provides an indirect indication of the hierarchy and octant from T2K.

Thus, for the first time we show that the T2K experiment by itself can give a hint of all three unknowns by running only in neutrino mode. In this context we also elucidate on the role played by antineutrinos in improving CP sensitivity, and identify the cases for which antineutrino runs can be helpful. An early hint of the unknown parameters is useful from the point of view of planning neutrino facilities in future. In addition, knowledge of these parameters provides an important test for neutrino mass models and will therefore significantly influence our search for models of new physics beyond the Standard Model.

Degeneracies in \( P_{\mu e} \):

The T2K experiment uses the neutrino beam from J-PARC and has a baseline of 295 km. The probability relevant for the measurement of CP violation is [4],

\[
P_{\mu e} = 4s_{13}^2 s_{23}^2 \sin^2 (\hat{A} - 1) \Delta \\
+ 2s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos (\Delta + \delta_{CP}) \\
\frac{\sin A \Delta \sin (\hat{A} - 1) \Delta}{\hat{A}} + O(\alpha^2). \quad (1)
\]
Here $s_{ij}(c_{ij}) \equiv \sin \theta_{ij}(\cos \theta_{ij})$, $\Delta = \Delta_{31} L/E$ where $L$ is the distance travelled and $E$ is the energy of the neutrino. $\bar{A} = 2EV/\Delta_{31}$, where $V = \sqrt{2}G_F n_e$ is Wolfenstein’s matter potential in terms of the electron density $n_e$. The lack of definite information about hierarchy, octant and $\delta_{CP}$ gives rise to two types of degeneracies. They are (i)Hierarchy-$\delta_{CP}$ degeneracy: $P_{\mu e}(\delta_{CP}, \Delta) = P_{\mu e}(\delta_{CP}, -\Delta)$ i.e. the probability for NH can be mimicked by IH and a different $\delta_{CP}$ value giving rise to wrong hierarchy-wrong $\delta_{CP}$ solutions \cite{3, 4} and (ii) Octant-$\delta_{CP}$ degeneracy: $P(\theta_{23}^{LO}, \delta_{CP}) = P(\theta_{23}^{HO}, \delta_{CP})$, i.e. the probability in the right octant can be the same as that for wrong octant and a different $\delta_{CP}$ giving wrong octant-wrong $\delta_{CP}$ solutions \cite{5, 6}. Hierarchy determination is facilitated if nature has chosen favourable combinations of hierarchy and $\delta_{CP} - \{\delta_{CP} \in [-180^\circ, 0^\circ], \text{NH}\} \text{ and } \{\delta_{CP} \in [0^\circ, 180^\circ], \text{IH}\}$, which hold for both neutrinos and antineutrinos \cite{8}. The situation is different for octant determination. For true LO, the degenerate solutions arise for $\{\delta_{CP} \in [-180^\circ, 0^\circ]\}$ and for true HO they occur for $\{\delta_{CP} \in [0^\circ, 180^\circ]\}$ in the neutrino mode; for antineutrino mode the behaviour is opposite \cite{9}. This feature in the oscillation probability can be understood from the following simple arguments. For neutrinos the values of $P_{\mu e}$ are higher for NH and lower for IH, and it is opposite for anti-neutrinos. But there is also a flip in the relative sign of $\delta_{CP}$ between neutrinos and antineutrinos. That causes the hierarchy-$\delta_{CP}$ degeneracy to appear in the same region for both neutrinos and antineutrinos. On the other hand the value of $P_{\mu e}$ is lower for LO and higher for HO for neutrinos as well as antineutrinos. Therefore the octant-$\delta_{CP}$ degeneracy behaves differently with neutrinos and antineutrinos. This implies that combination of neutrino and antineutrino channel is helpful for removal of octant-$\delta_{CP}$ degeneracy but it does not help in removal of hierarchy-$\delta_{CP}$ degeneracy.

**Results:**

We simulate the T2K experiment using the GLoBES package \cite{10} along with its auxiliary files \cite{11}. We consider T2K running a total of $8 \times 10^{21}$ pot. Event rates have been simulated for various combinations of hierarchy (NH or IH) and octant (LO or HO). Here LO corresponds to $\theta_{23} = 39^\circ$ and HO corresponds to $\theta_{23} = 51^\circ$, while NH/IH correspond to $\Delta_{31} = \pm 2.4 \times 10^{-3}eV^2$. 

![FIG. 1: $\delta_{CP}$ sensitivity of T2K neutrino run](image1)

![FIG. 2: $\delta_{CP}$ sensitivity of T2K for equal neutrino+antineutrino run (Total pot = $8 \times 10^{21}$)](image2)
(a) **Hint for hierarchy and octant:** Fig. 1 shows the sensitivity of T2K to $\delta_{CP}$, for each of the four true combinations in nature – NH-LO, NH-HO, IH-LO and IH-HO. The true value of $\delta_{CP}$ in nature is taken to be $-90^\circ$, since we are working with the assumption that the current T2K hint will be confirmed in future.

In the first panel, NH-LO is taken as the true combination of hierarchy and octant. The various curves show fits to $\delta_{CP}$ for all of the four test combinations, of which one is correct and the other three wrong. Expectedly, the correct combination (NH-LO) gives a best-fit (minimum $\chi^2$) at the true value of $\delta_{CP} = -90^\circ$. With NH-HO as the test combination, we get a best-fit around $135^\circ$ while with IH-HO as the test combination, we have a best-fit close to $0^\circ$. The combination IH-LO is seen to be excluded with minimum $\chi^2 \approx 3$.

Therefore, on marginalizing over the hierarchy and octant to find the overall best-fit value of $\delta_{CP}$, one would see allowed values of $\delta_{CP}$ around $-90^\circ$, $0^\circ$ and $135^\circ$. In other words, there would not be a strong indication for any single value of $\delta_{CP}$ from the data. Similar conclusions can be drawn if the true combinations in nature are IH-LO or IH-HO, as evidenced from the last two panels. The true combination NH-LO is the only one, for which one can see an unambiguous signal at $-90^\circ$, as the second panel shows. This is easy to explain, since neutrino probabilities for NH are higher than for IH, and those for HO are higher than those for LO. Therefore, it is not possible for any other combination of parameters to match the high event rates of NH-HO and create a degeneracy when $\delta_{CP} = -90^\circ$. Thus, a hint for $\delta_{CP} = -90^\circ$ would also signify normal mass hierarchy and higher octant of $\theta_{23}$ by elimination of the other options.

(b) **Impact of antineutrino run:** Fig. 2 is obtained using $4 \times 10^{21}$ pot exposure each in the neutrino and antineutrino modes. This figure demonstrates the actual role of antineutrinos in improving CP sensitivity of T2K. As before, the true value of $\delta_{CP}$ is assumed to be $-90^\circ$. The first panel for true NH-LO shows that with equal neutrino-antineutrino data the degenerate solutions with wrong octant – NH-HO and IH-HO are excluded at $>2\sigma$ confidence level. Similarly for true IH-LO (third panel) the degenerate solutions corresponding to IH-HO get disfavoured with the antineutrino run. But since the antineutrinos do not help in solving hierarchy degeneracy, the wrong hierarchy solutions corresponding to NH-LO are still allowed. For true IH-HO case (last panel) the degenerate solutions occurred for NH-HO and antineutrino run does not help in solving these. Rather the precision for $\delta_{CP} = -90^\circ$ i.e the statistical significance with which other $\delta_{CP}$ values can be disfavoured, reduces as compared to the full neutrino run because of less statistics. The plot in the second panel is for true NH-HO. Since there is no degeneracy, in this case also the CP sensitivity for $\delta_{CP} = -90^\circ$ becomes worse with antineutrino run.

(c) **Discovery of CP violation:** Discovering CP violation implies differentiating between a true value of $\delta_{CP}$ from the CP conserving values $0^\circ$ or $180^\circ$. In the previous sections, we have seen that except for true NH-HO, in all other cases the correct fit $\delta_{CP} = -90^\circ$ is accompanied by degenerate solutions around $0^\circ$ or $180^\circ$ using only the neutrino data. This affects the CP discovery potential of T2K. In this section we examine how far antineutrino run is useful in disfavoring these solutions thereby improving the chance of discovering $\delta_{CP}$ using various combinations of hierarchy and octant. For this purpose the simulated event spectrum is generated for true values of $\delta_{CP}$ spanning the range $[-180^\circ, 180^\circ]$. This is compared with the test event spectrum, with $\delta_{CP}$ = $0^\circ$ or $180^\circ$. In Fig. 3 we plot the CPV discovery potential of T2K for different combinations of true hierarchy and octant – NH-LO, NH-HO, IH-LO and IH-HO. In the test events, the hierarchy and octant are marginalized over. We present the results for different combinations of neutrino+antineutrino exposures – 8+0, 7+1, 6+2, 5+3 and 4+4 in units of $10^{21}$ pot.

Let us consider the behaviour of the $\chi^2$ for true $\delta_{CP} = -90^\circ$. From the first panel, we observe that for true...
NH-LO no hint is possible at $\delta_{CP} = -90^\circ$ with only neutrino mode. This is because the $\chi^2_{\text{min}}$ occurs for test $\delta_{CP} = 0^\circ, 180^\circ$ for the test IH-HO case, as we have already seen in the top left panel of Fig. 1. Thus $\delta_{CP} = -90^\circ$ cannot be distinguished from $\delta_{CP} = 0^\circ, 180^\circ$ since the minima occurs at these values for the wrong hierarchy-wrong octant solutions. These solutions can be disfavoured by antineutrino runs as can be seen from Fig. 2 improving the CP discovery $\chi^2$ as is seen from the top-left panel of Fig 3. However we find that equal neutrino and antineutrino run is not always the best option. The optimal antineutrino run is determined by the amount of antineutrino data that is required to disfavour the wrong $\delta_{CP}$ solutions occurring with the wrong octant. Once this is accomplished, further antineutrino data is superfluous. In fact, since the antineutrino cross-sections are smaller, such runs suffer from lesser statistics, worsening the sensitivity. Similarly as seen from the third panel of Fig. 3 for the true IH-LO case, a lower value of $\chi^2_{\text{min}}$ occurs because of the degenerate solutions at $\delta_{CP}^{\text{test}} = 0^\circ, 180^\circ$ for NH-LO. Here, the CP discovery signal gets masked by the wrong-hierarchy solution. For true IH-HO the degenerate case NH-HO can give lower $\chi^2_{\text{min}}$ at $\delta_{CP}^{\text{test}} = 0^\circ, 180^\circ$ and hence once again, no signal for CP discovery is obtained. In both the above cases the CP discovery potential is compromised because of wrong hierarchy solutions. Since hierarchy degeneracy does not get resolved by antineutrino data, including antineutrino run does not improve the $\chi^2$ in these cases for true $\delta_{CP} = -90^\circ$. For true NH-HO, there are no degenerate wrong octant solutions. Therefore, adding the antineutrino data is not useful for this case as well. If T2K were to run in both neutrino and antineutrino mode with equal exposure, then a hint at $\delta_{CP} = -90^\circ$ could be an indication for either NH-LO or NH-HO, with a slight preference for the lower octant. Note that similar conclusions can also be drawn if true $\delta_{CP} = +90^\circ$ − in which case the indication would be for IH-LO from only neutrino run.

Conclusions:

In this paper we have shown that if future T2K data confirms the current hint of $\delta_{CP} \sim -90^\circ$ by running only in the neutrino mode then that would indicate the hierarchy to be normal and $\theta_{23}$ octant to be higher. For the other hierarchy-octant combinations, parameter degeneracies forbid a clear hint for $\delta_{CP} = -90^\circ$ in only neutrino mode. The main role of the antineutrino data in enhancing $\delta_{CP}$ sensitivity is to remove the octant degeneracy. If this degeneracy is not present then the decrease in statistics associated with antineutrino run worsens the CP sensitivity. Even when this degeneracy is present, equal neutrino and antineutrino run do not always give the best results. Once the degenerate solution is eliminated further addition of antineutrino data worsens the CP sensitivity because of reduced statistics.

If the current indication for $\delta_{CP}$ is true, then the T2K experiment has the unique opportunity to give hints on all the unknown oscillation parameters by running only in the neutrino mode. The results also underscore the importance of optimizing antineutrino run, and can significantly impact the planning of neutrino facilities in future.

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