Valley-Hall-like photonic topological insulators with vanishing Berry curvature

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Valley-Hall-like photonic topological insulators are designed in kagome-lattice photonic crystals with \( C_{3v} \) point-group symmetry. The photonic crystal consists of circular air holes in pure dielectric materials. Different from conventional valley-Hall photonic topological insulators with nonvanishing Berry curvature and valley Chern numbers, the proposed insulators have vanishing Berry curvature and their topological invariants are described by a quantized electric polarization. Topological transition can be realized by tuning the structural size and topological edge states appear at the interface between photonic crystals with different topological phases. Numerical analyses show the proposed insulators preserve important features of valley-Hall photonic insulators such as valley transport with little backscattering.

I. INTRODUCTION

Topological photonics provide attractive approaches to reduce photon losses in waveguides or cavities thus gain great developments in recent years. Photonic topological insulators (PTIs), as the extension of topological insulators from electronics to optics, have been realized using different mechanism such as photonic quantum anomalous Hall (QAH) [1-8], quantum spin Hall (QSH) [9-16] or quantum valley Hall (QVH) effects [17-22]. Due to small magnetic response of magneto-optical materials in optical band, great efforts have been made in exploring PTIs which do not need external magnetic field to break the time reversal symmetry. A powerful solution is discovered by applying the concept of valley degree of freedom from valleytronics [23-25] to photonic crystals (PCs). By introducing structural asymmetry in the photonic lattice, the Berry curvature: the “magnetic field” in momentum space, can be nonvanishing and opposite signed at the time-reversal valleys (\( K \) and \( K' \)), resulting in nonzero valley Chern number exhibiting nontrivial topological phase [17-19]. Because of zero coupling between forward-moving \( K \)-valley and backward-moving \( K' \)-valley fields, robust topological waveguiding with reflection suppressed through sharp turns is possible, which is newly experimentally demonstrated at telecommunication band [22].

In recent publications, PTIs in the absence of Berry curvatures are reported in a simple square lattice [26,27]. Their topological invariants are characterized by two-dimensional (2D) electric polarization related to 2D Zak’s phase [28,29]. Although this kind of PTIs can not avoid backscattering at turns, they reveal a non-vanishing Berry curvature may not be necessary for topologically nontrivial systems. Supplementary understandings of the classical QSH PTIs in hexagonal-lattice PCs with \( C_{6v} \) point-group symmetry (PGS) [13] are explained using this theory in their earlier reports [30]. Those square-lattice PTIs with zero Berry curvature, which can be considered as 2D photonic generalization of the Su-Schrieffer-Heeger (SSH) model, are also used to construct higher-order topological insulators (HOTIs) [31]. In the second-order PTIs, besides one-dimensional edge states there are important zero-dimensional corner states.

In this work, we propose a valley-Hall-like photonic topological insulator that owns many similar properties with conventional valley-Hall PTIs but has vanishing Berry curvature.
designated PTI is made of periodic circular air hole array in pure dielectrics, forming a kagome-lattice photonic crystal with $C_{3v}$ point-group symmetry. We theoretically demonstrated its topology can transit by shrunk or expend the $C_{3v}$ PGS unit cell. The topological nontrivial phase of its photonic band gap is quantized by a fractional electric polarization $\left(\begin{array}{c}
\frac{1}{3} \\
\frac{1}{3}
\end{array}\right)$ connected to 2D Zak’s phase.

Numerical simulations illustrate topological edge states can emerge at the interface between two PCs with different topological phases. Unlike those square-lattice HOTIs with non-negligible reflection at turns, the proposed PTI preserves the key features of valley transport with little reflection at turns along the same valley. The principle can also be used to construct rod-type PTIs with similar features in other electromagnetic bands through proper structural designs.

II. QUICK REVIEW ON PHOTONIC VALLEY-HALL PHASE

At the beginning, we present a quick review of a valley hole-type photonic crystal (PC). The unit cell consists of two air holes embedded in pure dielectrics with the hole diameters of $d_1$ and $d_2$ respectively in hexagonal lattice, as shown the hexagon in Fig.1(a). The permittivity of the dielectric is $\varepsilon=13$. We fix the size $d_1=0.4a$ where $a$ is the lattice constant and change the size $d_2$ of the center hole of the unit cell. The photonic transverse-electric (TE) mode band structure of the lowest three bands are shown in Fig.1(b),(c),(d) under three different $d_2$. When $d_2=0.2a<d_1$, there is a band gap opening between the lowest two bands. When $d_2=0.4a=d_1$, the lowest gap closes and reduces to a degeneracy featuring a Dirac cone at $K(K')$ point in the momentum space, as pointed out with a red dash circle in Fig.1(c). When $d_2=0.6a>d_1$, the lowest gap opens again.

The $n$th-band Berry curvature is defined as $F^{(n)}_k = \nabla_k \times A^{(n)}_k$ with $A^{(n)}_k$ denoting the Berry connection of the $n$th band energy $\epsilon_n$ [32]. By using an efficient algorithm [33], we did a fast calculation on the Berry curvature of the first band over a roughly discretized portion of the Brillouin zone (BZ) under different situations, as shown in Fig. The asymmetry between the sizes of the two holes ($d_1\neq d_2$) within the unit cell breaks the inversion symmetry, leading to non-zero Berry curvature with opposite signs at the $K$ and $K'$ points. This causes non-zero valley Chern numbers thus both the cases for $d_2=0.2a$ and $d_2=0.6a$ show topologically nontrivial valley-Hall phase. Above valley-hall physics have already been discussed in many publications related to the valley Hall PTIs [17-22]. It is worth to mention an exchange process of the Berry curvature with respect to the band inversion. If $d_2$ starts increasing from $d_2< d_1$, a band inversion between the 1st and 2nd bands will take place at the band degeneracy when $d_2= d_1$. As compared the fields in Fig.1(e), bands 1, 2 and 3 in $d_2=0.2a$ case correspond to band 3, 1 and 2 in $d_2=0.6a$ case respectively, clearly showing the inversion. As arrowed the power flux, the helicity of the lowest mode is usually used to explain the backward suppression of valley transports because zero coupling is verified between the field distribution of left-hand circular polarization (LCP) and right-hand circular polarization (RCP) modes [17]. As shown in Fig.1(f), the Berry curvature at $K(K')$ point will have an exchange between lowest two bands after the inversion, resulting in a reversion of sign of the Berry curvature near $K(K')$ point. The valley Chern number will then change its sign from $C_{KK}=\mp 0.5$ to $C_{KK}=\mp 0.5$. In addition, simply rotating the original unit cell by 180° will also reverse $C_{KK}$ from $\pm 0.5$ to $\mp 0.5$. Topological edge states can emerge at the edge between two PCs with different valley Chern numbers.
FIG. 1. (a) Typical structure of a valley hole-type photonic crystal; (b-d) Band structures of the photonic crystal with the diameter of center air hole in the cell $d_2=0.2a$, 0.4$a$, 0.6$a$ respectively; (e) Normalized field distribution of the lowest three bands at K point under $d_2=0.2a$ and $d_2=0.6a$ cases, showing the band inversion. The white arrows present the direction of power flux; (f) Berry curvature of the first band under $d_2=0.2a$ and $d_2=0.6a$ cases.

III. VALLEY-HALL-LIKE PHOTONIC TOPOLOGICAL INSULATOR

Here we consider a kagome-lattice photonic crystal, the unit cell of which is composed of three identical air holes with $C_3$, PGS embedded in pure dielectrics, as shown in Fig. 2(a). The permittivity of the dielectric is $c=13$. We set the diameter of the holes $d=0.36a$ where $a$ is the lattice constant. We explore the topological phase transition by a method of shrinking or expanding the air-hole array within the unit cell, similar to the method in the publications [13]. According to the 2D Su-Schrieffer-Heeger (SSH) model, the two parameters of the intracellular hopping $\gamma$ and the intercellular hopping $\gamma'$ determine the topology of the system. Thus the dominant thing in our system is the same distance $W$ between the center of each air hole and the center of the unit cell. One can easily find $W=a/(2\sqrt{3})$ as a critical value when the intracellular hopping equals to the intercellular hopping ($|\gamma'}/|\gamma|=1$). With this specific structural size of the photonic crystal, the photonic band structure of the lowest three bands are shown in Fig. 2(c). Here, we only focus on the
TE modes propagating in the x–y plane. Obviously there are Dirac cones formed by the lowest two bands as marked with a red circle at $K(K')$ point in the first BZ. We then vary the distance $W$ with respect to $a$ to get the three holes shrunken ($W < a / (2\sqrt{3})$) or expended ($W > a / (2\sqrt{3})$). The corresponding band structures for $W = a/4$ and $W = a/3$ are shown in Fig.2(b) and (d) respectively. The shrunken and expended unit cells are shown in Fig.2(e). Comparing Fig.2 with Fig.1, the transformation of the bands has many things in common with that of the valley-hall PTI in Sec.II. During the expending process of $W$ from $a/4$ to $a/3$, band inversions take place between the 1st and 2nd bands at $K(K')$ degeneracy when $W = a / (2\sqrt{3})$. As compared the fields in Fig.2(f), bands 1, 2 and 3 in $W = a/4$ case correspond to bands 3, 2 and 1 in $W = a/3$ case respectively, showing the inversion. The process makes the original existing lowest band gap start from open to closed and then reopened.

![Diagram](image)

**FIG.2.** (a) The structure of a hole-type photonic crystal with three identical air holes in the unit cell forming kagome lattice; (b–d) Band structures of the photonic crystal with the distance between the center of each air hole and the center of the unit cell $W = a/4, a/(2\sqrt{3}), a/3$ respectively; (e) The schematics of a shrunken unit cell with $W = a/4$ and a expended unit cell with $W = a/3$; (f) Normalized field distribution of the lowest three bands at K point under $W = a/4$ and $W = a/3$ cases, showing the band inversion. The white arrows present the direction of power flux; The vanishing Berry curvature of the first band is not shown here.
However, through computation it is found that the Berry curvature of the first band vanishes everywhere in the BZ no matter $W$ is beyond the critical value or not, thus we do not show the zero Berry curvature here. This is quite different with the case of valley-Hall PTI in Sec.II. The band inversions will not cause visible exchange of the curvature at $K(K')$ points. As a result, the topology of such a system can not be describe by the valley Chern numbers. We demonstrate a 2D electric polarization as a complimentary quantum number to characterized the topological transition in our PCs. The 2D polarization $\mathbf{P} = (p_1, p_2)$ is given by the integral of the Berry connection over the 2D Brillouin zone and expressed as \[ p_i = -\frac{1}{(2\pi)^2} \int_{BZ} dk_1 dk_2 \text{Tr}[A_i(k)], \] with $i$ indicating the component of $\mathbf{P}$ along the reciprocal lattice vector $\mathbf{b}_i$ ($i=1,2$). Here \[ [A_i(k)]^{mn} = -i \langle u^m(k) | \partial k_i | u^n(k) \rangle \] is the Berry connection matrix where $m$ and $n$ run over occupied energy bands. $| u^n(k) \rangle$ is the periodic Bloch function for the $n$th band. $\mathbf{k} = k_1 \mathbf{b}_1 + k_2 \mathbf{b}_2$ is the wave vector. Via a direct integration with Eq.(1), it is found the 2D polarization $\mathbf{P}$ of the proposed PC along $\mathbf{b}_{1,2}$ (as illustrated in Fig.2(a)) equals $\left(\frac{1}{3}, \frac{1}{3}\right)$ for $W > a/(2\sqrt{3})$ showing topological nontrivial phase, while it equals $(0,0)$ for $W < a/(2\sqrt{3})$ showing topological trivial phase. The calculation is conducted using the Wilson-loop approach following Refs.[34,35]. Its mathematical details are presented in the Supplemental Material [36]. This polarization can also be obtained by making use of $C_3$ PGS. Under the specific PGS, the 2D polarization can be written as \[ \exp(2\pi p_{1,2}) = \prod_{n\in\text{occ}} \beta_n(K), \] where $\beta_n(k)$ is the eigenvalue of the operator $R_3$ at $C_3$-invariant $\mathbf{k}$ point on the $n$th band. Its derivation is given in the Supplemental material [36]. The polarization is related to the 2D Zak phase through $\phi_{1,2} = 2\pi p_{1,2}$ [28,29]. So the topological nontrivial phase would be $\frac{2\pi}{3}$ in our case.

Next, we investigate the topological edge modes between two PCs with different 2D polarization. Four types of edges supporting topological edge modes are considered with their supercell structure and projected band diagrams along the edge direction (x-axis). The first one in Fig.3(a) is formed by an expanded PC ($W=a/3$) with $\mathbf{p} = (\frac{1}{3}, \frac{1}{3})$ and a shrunken PC ($W=a/4$) with $\mathbf{P} = (0,0)$. A topological edge mode proved to be confined at the edge through the normalized field distribution is presented by red line inside the lowest topological photonic band gap. Its time-reversal partners respectively at positive and minus $k_x$ region are in y-axial symmetry but always with opposite group velocity. The edge mode does not connect the upper and lower bulk bands, similar to that of those square-lattice PTIs with zero Berry curvature [27,31]. The second edge in Fig.3(b) is formed by simply rotating the expended PC of the first edge structure by 60 degree,
leading to the topological nontrivial polarization becoming \( p = (-\frac{1}{3}, -\frac{1}{3}) \). The unique topological edge mode also appears but the field seems less confined at the interface comparing to the first edge. The third edge in Fig.3(c) is constructed by substituting the upper shrunken PC of the first edge structure with ordinary triangular array of air holes (also topologically trivial) with the diameter of 0.6\( a \), so the properties of the edge mode have no much difference with the first edge. The last edge in Fig.3(d) consists of two expanded PCs. The lower region is still the expanded PC with \( p = (\frac{1}{3}, \frac{1}{3}) \), while the upper is the expanded PC with 60°-rotated unit cell possessing \( p = (-\frac{1}{3}, -\frac{1}{3}) \). Two topological edge modes in the gap is supported by such an edge structure, which can not be constructed in square-lattice PTIs with zero Berry curvature.

![Projected band diagrams and topologically protected states of the edge between two PCs with different 2D polarization.](image)

**FIG.3.** Projected band diagrams and topologically protected states of the edge between two PCs with different 2D polarization. (a) edge between a shrunken PC and an expanded PC; (b) edge between a shrunken PC and an expanded PC with the cell rotating 60°; (c) edge between an ordinary hole-type PC and an expanded PC; (d) edge between an expanded PC and an expanded PC with the cell rotating 60°.

At last, we discuss the wave propagation of the topological edge modes. The time-averaged power flow is computed by using Comsol Multiphysics and the source excites a harmonic wave with the frequency of 0.23(c/a) locating inside the band gap. The first situation is a stable straight waveguiding between two expanded PCs with different unit-cell orientations, as shown in Fig.4(a). Such a waveguide structure is quite similar to that of a valley-hall PTI interface between two PCs with different orientations, which is not possible in square-lattice PTIs. The second waveguiding between an expanded PC and a shrunken PC through a sharp oblique turn (the same \( K \) direction) further demonstrates that the proposed PTI preserve key features of the conventional valley-hall PTI. As shown in Fig.4(b), the topological transport is robust through the turn with reflection suppressed, due to zero field coupling between the forward and the backward modes. The mechanism lies on that the lowest band of the expanded nontrivial PC owns helical field properties, as shown the white arrowed power flux at \( K \) point for \( W = a/3 \) case in Fig.2(f). Obviously the corresponding field
properties at $K'$ point will show opposite helicity. So ideally there is no field coupling between the forward wave along $K$ direction and backward wave along $K'$ direction, which is the same as in the valley-Hall PTIs [17]. The corresponding transmission spectrum illustrated in Fig. 4(d) clearly reveals the transmission band with a nearly unitary transmittance. This is also the main reason that we call the proposed PC a valley-hall-like PTI. However in Fig.4(c), when edge waves propagate though a sharp orthogonal turn, larger reflection takes place with the transmittance though the turn reducing to 76%. This is because at orthogonal corner the forward mode can flip to another backward mode with opposite group velocity and helicity. To show the influence brought by disorders, a missing lattice is introduced on the edge between an expanded PC and an ordinary triangular-lattice air-hole PC. As shown in Fig.4(e), the edge waves can go around the defect to some degree but the existing backscattering causes a reduced transmittance of about 90% through the disorder. Above results demonstrate valley-like edge transports of the expanded PCs.

![Image](image_url)

**FIG.4.** Edge propagation towards different directions. (a) Straight edge guiding between an expanded PC and an expanded PC with the cell rotating 60°. (b) Edge guiding through a sharp oblique turn between a shrunken PC and an expanded PC. (c) Edge guiding through an orthogonal turn between a shrunken PC and an expanded PC. (d) The transmittance over the sharp oblique turn in (b). (e) Straight edge guiding through a disorder formed by removing a unit cell close to the edge.

**IV. Conclusion**

We have studied kagome-lattice photonic crystals with three air holes in the unit cell obeying $C_{3v}$ point-group symmetry. They show valley-hall-like topological phases but have vanishing Berry curvature. Their topological properties are protected by 2D electric polarization related to 2D Zak's phase. They possess many similar characteristics to conventional valley-Hall photonic crystals, such as the edge states in the system here can turn along the same valley through a sharp corner with suppressed backscattering. Such features are unable to exist in square-lattice photonic crystal system with zero Berry curvature and meanwhile prove non-zero valley Chern numbers are not necessary for valley-like transports. The proposed structures of air holes in pure dielectrics are easy to fabricate and can also extend to rod-type photonic crystals with the same point-group symmetry. The last important thing is, hexagonal-lattice higher-order topological insulators with oblique optical corner...
modes can be realized in the proposed photonic crystals and then may be used to fabricate optical cavities, which needs a further investigation. Our works prove the understanding of using fractional electric polarization as a quantum number to describe topology in photonic honeycomb system, and add new sights on valley waveguiding in photonic topological insulators.

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