Emergence of classicality from quantum mechanics, a hotly debated topic, has had no satisfactory resolution so far. Various approaches including decoherence and gravitational interactions have been suggested. In the present work, the Schrödinger–Newton model is used to study the role of semi-classical self-gravity in the evolution of massive spin-1/2 particles in a Stern–Gerlach experiment. For small mass, evolution of the initial wavepacket in a spin superposition shows a splitting in the magnetic field gradient into two trajectories as in the standard Stern–Gerlach experiment. For larger mass, the deviations from the central path are less than in the standard Stern–Gerlach case, while for high enough mass, the wavepacket does not split, and instead follows the classical trajectory for a magnetic moment in inhomogeneous magnetic field. This indicates the emergence of classicality due to self-gravitational interaction when the mass is increased. In contrast, decoherence which is a strong contender for emergence of classicality, leads to a mixed state of two trajectories corresponding to the spin-up and spin-down states, and not the classically expected path. The classically expected path of the particle probably cannot be explained even in the many-worlds interpretation of quantum mechanics. Stern–Gerlach experiments in the macroscopic domain are needed to settle this question.

1. Introduction

Despite its enormous success, quantum theory still has certain unresolved problems, one of which is the emergence of classicality. Quantum theory characteristically involves superpositions of states, which are entirely absent in the classical world. If one believes that quantum theory is fundamental, it should lead to the classical world in some limit. Quantum theory in itself provides no mechanism for the emergence of classicality.

Consider the Stern–Gerlach experiment (see Figure 1), a prototype example of a quantum measurement and one that bears the distinction of demonstrating the fundamentally quantum nature of spin. A spin-1/2 particle subjected to a magnetic field gradient experiences a potential proportional to the component of the spin along the magnetic field direction.

A general quantum state of a spin-1/2 particle is a superposition $|\theta\rangle = \cos \frac{\theta}{2} |\uparrow\rangle + \sin \frac{\theta}{2} |\downarrow\rangle$ in the spin-$z$ basis. In a magnetic field gradient, the trajectory of this particle splits into two, and it always lands in one of two spots on the screen. This general state is also an eigenstate of $\hat{S}_z = \hat{S} \cdot \hat{n}$, the spin component along the direction $\hat{n}$ at an angle $\theta$ to the $z$-axis. Classically this would represent an angular momentum vector pointing along $\hat{n}$. The force on the classical magnetic moment in a field gradient is proportional to $\cos \theta$, so that the particle follows a single trajectory landing on the screen somewhere between the spots corresponding to the spin states $|\uparrow\rangle$ ($\theta = 0$) and $|\downarrow\rangle$ ($\theta = \pi$), with a continuous set of possibilities. The question then is whether this classical behavior can emerge from quantum dynamics if the particle is massive enough. To our knowledge a Stern–Gerlach experiment with macroscopic particles has not yet been performed, and whether it would lead to the classical result remains unverified.

A major contender among theories for the emergence of classicality is decoherence, which emphasizes the unavoidable role played by the environment in the evolution of a quantum system. Examining the experiment in the context of decoherence, the initial superposition of pure spin states evolves into a mixed state, which is an incoherent superposition of the two trajectories corresponding to those of $|\uparrow\rangle$ and $|\downarrow\rangle$. This predicts that the particle will still land in one of the two spots on the screen corresponding to $|\uparrow\rangle$ and $|\downarrow\rangle$. Decoherence does not lead to any intermediate trajectory corresponding to the classically expected behavior of a particle with magnetic moment passing through a Stern–Gerlach setup.

We wish to emphasize here that in various other experiments, such as a massive particle passing through a double-slit, decoherence leads to suppression of superpositions of the two alternate paths, and classically expected behavior emerges. In the Stern–Gerlach experiment, from classical dynamics one would expect a single outcome corresponding to the $z$-component of the magnetic moment. However, such an outcome is not explained by decoherence or by the collapse postulate, both of which lead to...
the emergence of one of the spin eigenvalues with probabilities predicted by the Born rule.

Following Roger Penrose’s proposal\[^5\] that gravity could play a role in the emergence of classicality in massive particles, several authors have investigated the effect of gravity on quantum systems in various ways.\[^6–10\] Here we explore the role of self-gravity of a massive particle in the emergence of classicality in a Stern–Gerlach experiment, using the Schrödinger–Newton (S–N) equation, earlier introduced by Diósi.\[^11\]

2. The Stern–Gerlach Experiment and Self-Gravity

2.1. The Schrödinger–Newton Equation

The seeds of the Schrödinger–Newton equation came from semi-classical gravity independently considered by Möller\[^12\] and Rosenfeld.\[^13\] In this approach, quantized matter is assumed to be coupled to the classical gravitational field,\[^14–16\] and the expectation value of the energy–momentum tensor with respect to the quantum state $|\Psi\rangle$ of matter is the source in the Einstein field equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \langle \Psi | \hat{T}_{\mu\nu} | \Psi \rangle$$ \hspace{1cm} (1)

In the Newtonian approximation, this naturally leads to the Schrödinger–Newton equation\[^16–19\]

$$\left( \frac{p^2}{2m} + V_c \right) \Psi(r, t) = i\hbar \frac{\partial \Psi(r, t)}{\partial t}$$ \hspace{1cm} (2)

where

$$V_c = -Gm^2 \int \frac{|\Psi'(r', t)|^2}{|r - r'|} d^3 r'$$ \hspace{1cm} (3)

is the potential due to gravitational self-interaction, derived from the spatial probability distribution of the massive particle. Since this potential depends on the wavefunction itself, the S–N equation is a non-linear modification of the Schrödinger equation. The non-linearity breaks the unitarity of Schrödinger evolution, and opens up the potentialities of effects that were precluded by the linearity of quantum dynamics, including perhaps a dynamical reduction of the wavefunction. Issues caused by the sacrosanct quantum-mechanical linearity being broken have been debated in the literature.\[^20\] However, the hope behind this approach is that the dynamics will be linear for all practical purposes at the scales at which quantum mechanics has been successfully tested, and the non-linearity will show up only when one approaches the classical limit. There have also been several other approaches in which non-linearity has been introduced in order to obtain wave-function collapse.\[^21–23\] The S–N equation, however, is elegant because no adjustable parameters are introduced, and it should naturally lead to the scale at which classicality might emerge.

2.2. Formulation of the Stern–Gerlach Problem

Consider a Stern–Gerlach experiment as shown in Figure 1, in which a spin-1/2 particle of mass $m$ travels along the x-axis, experiencing a magnetic field along the $z$-axis, with a constant gradient $B_0 \hat{z}$.\[^24\] If we assume that the spin-1/2 magnetic moment $\vec{\mu}$ is due to a single unpaired electron in the particle, then the potential experienced by the particle is given by

$$V_g(z) = -\vec{\mu} \cdot \vec{B} = -\mu_y B_0 z \sigma_z$$ \hspace{1cm} (4)

where $\mu_y$ is the Bohr magneton, and $\sigma_z$ is the Pauli spin-$z$ operator. This potential causes a spin-dependent deviation along the $z$-direction. Assuming an initial constant velocity $\vec{v}_0$, the dynamics of the particle along the x-axis is trivial, just translating the x-position of the particle by $x = vt$ in a given time $t$. So we do not consider the motion of the particle along the x-axis explicitly, and just focus on its dynamics along the $z$ axis. The quantum dynamics of the particle in the $z$ direction is given by the 1D Schrödinger equation under the influence of the potential $V_g(z)$ of Equation (4).

Imposing the additional self-gravitational potential given by Equation (3), the effective dynamics in the $z$ direction is governed by the S–N equation reduced to one dimension,\[^25\] along with $V_g(z)$

$$\left( \frac{p_z^2}{2m} + V_c(z) + V_g(z) \right) \Psi(z, t) = i\hbar \frac{\partial \Psi(z, t)}{\partial t}$$ \hspace{1cm} (5)

with $V_c(z) = -Gm^2 \int \frac{|\Psi'(r', t)|^2}{|r - r'|} d^3 r'$. The state of the particle in general is given by a spinor $|\Psi\rangle$, which in the position basis, is a two-component wave-function in a superposition of spin-up and spin-down states

$$\langle z | \Psi(t) \rangle = \chi_+(z, t) |\uparrow\rangle + \chi_-(z, t) |\downarrow\rangle$$ \hspace{1cm} (6)

The position–space probability distribution of the particle required to calculate the gravitational potential is given by

$$|\Psi(z, t)|^2 = |\langle z | \Psi(t) \rangle|^2 = |\chi_+(z, t)|^2 + |\chi_-(z, t)|^2$$ \hspace{1cm} (7)

where the cross terms between $\chi_+(z, t)$ vanish due to the orthogonality of $|\uparrow\rangle$ and $|\downarrow\rangle$. Using Equation (3), the self-gravitational

Figure 1. Schematic representation of a typical Stern–Gerlach setup: quantum spin-1/2 particles are deflected in one of two directions by an inhomogeneous magnetic field (blue lines) whereas classical magnetic moments are deflected by an amount proportional to the $z$-component of their spin (red).
potential becomes \( V_c(\Psi) = V_c(x_z) + V_c(x_{\perp}) \), where \( V_c(x_z) = -Gm^2 \int \frac{|x_z| \Psi^* (z') \Psi (z')}{|z-z'|} \, dz' \). The S–N equation for our system can now be expressed as

\[
\begin{bmatrix}
-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + V_c - \gamma z \\
0 \\
-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + V_c + \gamma z \\
\end{bmatrix}
\begin{bmatrix}
\tilde{\chi}_+ \\
0 \\
\tilde{\chi}_- \\
\end{bmatrix} = i\hbar \begin{bmatrix}
\dot{\tilde{\chi}}_+ \\
0 \\
\dot{\tilde{\chi}}_- \\
\end{bmatrix} \tag{8}
\]

This represents two equations for the time evolution of the two components \( \chi_z \), that are coupled nonlinear equations since the potential term \( V_c \) depends on both \( \chi_z ^2 \) and \( \chi_{\perp} \). Here \( \gamma = \mu_B B \) represents the magnetic force in our model. Introducing a length scale \( \sigma_r \) in terms of which time and mass scales are defined as

\[
t_z = \left( \frac{\sigma_r^5}{G\hbar} \right)^{\frac{1}{3}}, \quad m_z = \left( \frac{\hbar^2}{G \sigma_r} \right)^{\frac{1}{3}} \tag{9}
\]

we work in terms of the dimensionless variables

\[
\tilde{z} = \frac{z}{\sigma_z}, \quad \tilde{m} = \frac{m}{m_z}, \quad \tilde{t} = \frac{t}{t_z}
\tag{10}
\]

and rescaled wavefunctions \( \tilde{\psi} = \sqrt{\sigma_z} \psi, \quad \tilde{\chi}_\pm = \sqrt{\sigma_z} \chi_\pm \). The dimensionless magnetic force is

\[
\tilde{\gamma} = \frac{\gamma}{m_z \sigma_r / \tilde{t}_z} = \frac{\mu_B B}{m_z \sigma_r / \tilde{t}_z} \tag{11}
\]

Equation (8) can be rewritten in dimensionless terms as

\[
-\frac{1}{2\tilde{m}} \frac{\partial^2 \tilde{\chi}_\pm}{\partial \tilde{z}^2} \mp i \tilde{\gamma} \tilde{z} \tilde{\chi}_\pm - \frac{\tilde{m}^2 \tilde{\chi}_\pm}{\tilde{\sigma}_z} \int \frac{\tilde{\gamma}(\tilde{x}_\perp') \tilde{\chi}_\perp(\tilde{x}_\perp')}{|\tilde{z}-\tilde{z}'|} \, d\tilde{z}' = i \frac{\partial \tilde{\chi}_\pm}{\partial \tilde{t}} \tag{12}
\]

The S–N equation for a spin-1/2 particle in a Stern–Gerlach setup has been studied analytically by Großardt,[28] in the context of spin interference. We have carried out comprehensive numerical simulations of the problem to avoid being constrained by the limits of analytical approximations. We used the Crank–Nicolson method[27,28] to solve Equation (12), with spatial and temporal grid sizes chosen as \( \Delta \tilde{z} = 0.05 \) and \( \Delta \tilde{t} = 0.01 \) respectively, which satisfy the CFL (Courant–Friedrichs–Lewy) condition for stability of the solutions: \( \frac{\Delta \tilde{t}}{\Delta \tilde{z}} < 1 \). To tackle the singularity in the self-gravity potential, we regularize the integral in the potential using a small dimensionless \( \delta : -m^2 \int \frac{\tilde{\gamma}(\tilde{x}_\perp') \tilde{\chi}_\perp(\tilde{x}_\perp')}{|\tilde{z}-\tilde{z}'|} \, d\tilde{z}' \). \( \delta \) was fixed at 0.01. Fixing the spatial boundaries at \( \pm 100 \) avoided boundary effects.

The initial state of the particle was a Gaussian wave-packet of half width \( \epsilon \), localized at \( \tilde{z} = 0 \), with the spin state in a general superposition \( \cos \frac{\theta}{2}|\uparrow \rangle + \sin \frac{\theta}{2}|\downarrow \rangle \), so that

\[
\tilde{\chi}_\perp(\tilde{z},0) = \cos \frac{\theta}{2} e^{-\frac{\tilde{z}^2/2}{\epsilon^2}} \sqrt{\pi}, \quad \tilde{\chi}_\perp(\tilde{z},0) = \sin \frac{\theta}{2} e^{-\frac{\tilde{z}^2/2}{\epsilon^2}} \sqrt{\pi} \tag{13}
\]

3. Results and Analysis

3.1. Dynamics of Wavepackets

Figure 2 shows the time evolution of the probability distribution for a relatively small mass, \( \tilde{m} = 0.1 \), in an initially asymmetric spin superposition with \( \theta = \pi/3 \) and \( \epsilon = 4 \). The initial single spatial wave-packet splits into a superposition of two wave-packets traveling in different directions, consistent with the behavior expected in the typical Stern–Gerlach scenario.

A contour plot of \( |\tilde{\psi}(\tilde{z}, \tilde{t})|^2 \) as it evolves in time is shown in Figure 3. Comparing with the trajectories of the peaks of the spin-up and spin-down wavepackets under pure Schrödinger dynamics (shown overlaid in green), it is evident that for low enough mass, the presence of self-gravity does little to affect the pure quantum dynamics, and quantum superpositions remain unaffected.

As we increase the mass, the trajectories of the peaks of the wave-packets begin to deviate from those corresponding to pure Schrödinger dynamics. Figure 4 displays the contour plot of the probability density in the presence of self-gravity, together with the peak positions of the two wave-packets without self-gravity, for \( \tilde{m} = 0.5, \epsilon = 2 \). The split between the spin-up and spin-down parts of the wavefunction is less than that without gravity, which is expected due to the gravitational attraction between the two components. Interestingly, the deflection of the two trajectories is...
not symmetric about the $\tilde{z} = 0$ line when the spin superposition is unequal.

### 3.2. Emergence of Classicality

The most unexpected results are seen as we increase the mass further. For values of $\tilde{m}$ greater than about 0.6, the initial wave-packet does not separate into two, but is deflected along a single path (Figures 5 and 6) between the two paths of the pure Schrödinger case. This implies that in an experiment with an ensemble of massive particles in this spin state, the screen would show a single spot instead of two.

A classical particle with magnetic moment oriented at an angle $\theta$ with the $z$ axis would have an acceleration $a = (\tilde{\gamma} / \tilde{m})\cos \theta$ under the magnetic field gradient, and follow a classical trajectory.

Figure 6 shows the peak of the wavepacket for $\tilde{m} = 0.6$ closely following this trajectory (shown in purple). This seems to signal the emergence of classicality, and an apparent absence of quantization of the magnetic moment. It is interesting to note that this behavior appears as a consequence of self-gravity, despite the spin being fully quantized in the analysis.

We tested this effect for various values of $\theta$, changing the asymmetry in superposition of the spin states. The results summarized in Figure 7 show that for high mass ($\tilde{m} = 0.7$), dynamics under the S–N equation recovers the classically expected result in the Stern–Gerlach experiment. Another interesting feature is that the quantum expectation of the position also follows the classical trajectory reasonably well, indicating the emergence of classicality, driven by self-gravity.

Figure 7. Expectation value of position versus time for $\tilde{m} = 0.7$, for various values of $\theta$. In all cases the trajectory of the peak of the wave-packet follows the classical path reasonably well, indicating the emergence of classicality, driven by self-gravity.
4. Discussion

Our results indicate that self-gravity could play a role in the emergence of classicality in a spin measurement experiment. As the mass of the particle is increased, the expected deviations of the peaks of the spin-up and spin-down wave packets are less than for the pure quantum evolution, and for high enough mass the two wave packets do not separate, but instead travel as one wave packet along the classically expected path. This behavior does not emerge in the decoherence picture. Neither does it emerge in the many worlds interpretation. In the latter, the two wave packets with different spins, in the superposition state are believed to exist in two independent worlds. In each world, the particle is localized in position and has a distinct value of the z component of the spin. The classically expected path of the particle is nowhere in the picture in this interpretation. The S–N model thus appears to fare better than other popular candidates for emergence of classicality that do not invoke wave-function collapse. However, it should be mentioned that it is not clear if the S–N equation can explain the collapse of a superposition of two distinct states to one of the states, with the probability governed by Born rule. There have been suggestions that the S–N equation may need stochastic modifications to achieve that. It has also been pointed out that in the case of two spin-entangled particles, and one of them passing through a Stern–Gerlach setup, the S–N equation leads to a possibility, at least in principle, of faster-than-light signaling.

The question remains as to whether nature actually shows the classically expected path in a Stern–Gerlach experiment with massive particles. Magnetic experiments have been carried out with large atomic clusters, and nano-size particles. However, in such experiments the magnetic moment of the atomic cluster is also large, and the space quantization may not be apparent. The experiment has to be performed with spin-1/2 particles in the macroscopic domain. Coming up with large particles with small, stable magnetic moments may be challenging as spin relaxation is often observed in such situations.

Our results indicate that the effects of self-gravity in the Stern–Gerlach experiment are visible for \( \m = 0.3 \) onward, on evolution up to \( t \approx 10 \). Choice of \( \sigma_r \) is guided by Equation (9) and experimentally feasible masses. The smaller \( \sigma_r \) is, the larger the mass for which the effect of self-gravity will be noticeable over a relatively short time evolution. Equation (11) gives the required magnetic field gradient. For example, choosing \( \sigma_r = 0.371 \) nm leads us to \( m_r = 46.05 \times 10^9 \) u and \( t = 0.1 \) s. This implies that a particle of mass \( 27.63 \times 10^9 \) u (\( m_0 = 0.6 \)), passing through a magnetic field gradient of 28 mT m\(^{-1}\), will display the emergence classical behavior by time \( t = 1 \) s (\( t = 10 \)). These values appear well within the reach of the state-of-the-art technology. Magnetic field gradients up to 80 mT m\(^{-1}\) can already be achieved in clinical magnetic resonance imaging (MRI) scanners.

This experiment will be orders of magnitude less challenging than the Stern–Gerlach-like experiments proposed in the context of gravity induced entanglement, where masses of the order of \( 6 \times 10^{12} \) u and magnetic field gradients of the order of \( 10^8 \) T m\(^{-1}\) are needed. Stern–Gerlach experiments for testing the emergence of classicality have an advantage over interference experiments in that one need not worry about creating and maintaining coherent superpositions of massive particles. The only challenge, we believe, would be in coming up with a particle of such high mass, but having a spin-1/2. Our simulations show that masses lower than about \( 5 \times 10^9 \) u will not show any effect of self-gravity in a Stern–Gerlach experiment.

5. Conclusion

Including semiclassical gravitational self-interaction in the Schrödinger equation yields the interesting outcome of emergence of classicality for high masses in the Stern–Gerlach experiment. This is a feature that is amenable to testing in the laboratory, subject to the feasibility of creating spin-1/2 particles of high enough mass. The question which needs to be settled is whether nature shows the expected classical outcome for large mass particles, or does spin remain quantized all the way to the classical regime. It is not clear if the emergence of a single outcome corresponding to the average spin in a Stern–Gerlach experiment can be explained by any linear theory. Thus this experiment may turn out to be a crucial one in deciding the survival of nonlinear theories like the S–N equation. If the experiments show that for large masses the classically expected result is recovered, it would indicate that there is something lacking in the linear quantum mechanics, including the ideas of decoherence and the many worlds interpretation.

We believe this experimental test should be easier to implement compared to the interference experiments which are being attempted on the mesoscopic scale.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Keywords

Stern–Gerlach experiment, Schrödinger–Newton equation, self-gravitational interaction, semi-classical gravity, emergence of classicality

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