Consistency between 11D and U-duality*

Itzhak Bars
Department of Physics and Astronomy
University of Southern California
Los Angeles, CA 90089-0484

U-duality transformations must act on a basis of states that form complete multiplets of the U group, at any coupling, even though the states may not be mass degenerate, as for a broken symmetry. Similarly, if superstring theory is related to a non-perturbative 11D M-theory, then an 11D supermultiplet structure is expected, even though the multiplet may contain states of different masses. We analyse the consistency between these two multiplet schemes at the higher excited string levels for various compactifications of the type IIA superstring. While we find complete consistency for a number of compactifications, there remain some unanswered questions in others. The relation to D-branes also needs further clarification.

1. Introduction

Circumstantial evidence for U-duality in string theory has been accumulating [1]-[10]. Also, starting with 11D supergravity in the form of the low energy limit, there seems to be hints [2] for an 11 dimensional extended structure lurking behind the non-perturbative aspects of string theory [3][11][12]. Further study of duality and the possible connections with 11-dimensions is bound to reveal more non-perturbative properties of the underlying theory. In this lecture I will concentrate on the assumed 11D theory and will try to discuss parts of its stringy spectrum and its consistency with U-duality. In particular, I will examine the toroidally compactified type-II superstring on \(R^d \times T^c\) (with \(d + c = 10\)) and analyse its spectrum for various values of \((d, c)\), looking for connections to 11D and U-duality. This is a summary of the work in [3][13] and more recent observations along the same lines regarding BPS saturated stringy states which are related to D-branes.

Beginning with the uncompactified type IIA string theory \((d=10, c=0)\), in [3] it was suggested that, if the type IIA string theory on \(R^{10}\) is related to a non-perturbative 11D structure, then its stringy spectrum (at any strength of the coupling) must display a certain pattern consistent with 11D supersymmetry and 11D Lorentz group. The patterns were displayed up to excitation level \(l = 5\) as follows.

1.1. Hidden 11D in 10D superstring

In the Green-Schwarz formalism, the perturbative states of type IIA superstring in 10D, at level \(l\)

\[
\text{(Bose} \oplus \text{Fermions)}^{(0)} | \text{vac}, \ p^\mu >
\]

(1)

start out as SO(8) supermultiplets in the light-cone gauge. Here \(\text{vac}\) stands for the \(2^7_B + 2^7_F\) dimensional Clifford vacuum of 8-left and 8-right fermionic zero modes. Because of Lorentz invariance in the critical dimension, one expects that massless states must form SO(8) supermultiplets, while massive states must form SO(9) supermultiplets. However, by detailed examination of the oscillator states one finds that there are higher structures that begin to give more hints of 11-dimensions. The oldest of these hints is that the massless sector comes as SO(9) multiplets \((l = 0 : 2^7_B + 2^7_F\), identical to the massless states of 11D supergravity), while the first massive level
comes as SO(10) supermultiplets \[ l = 1 : \quad 2 \frac{15}{B} + 2 \frac{15}{F}. \] (2)

In fact, these correspond to the short and long multiplets of 11D supersymmetry and therefore are expected to occur in any supersymmetric theory that has 11D supersymmetry. The fact that they arise in the 10D string theory is already of striking significance. This should be viewed partly as a consequence of the fact that the 32 components of the two 10D supercharges of type IIA reassemble to a single supercharge in 11D, and partly due to the oscillator content of the theory.

The states above are purely perturbative. In \[ \text{3} \] it was suggested that the non-perturbative BPS saturated “black hole” solutions of \[ \text{3} \] can be interpreted as Kaluza-Klein excitations of 11D supergravity, thus elevating the \( l = 0 \), \( 2 \frac{7}{B} + \frac{7}{F} \), multiplet from being 10-dimensional fields \( \phi (x^\mu) \) to 11-dimensional fields \( \phi (x^\mu, x^{11}) \). From the point of view of the vacuum state this corresponds to having a quantized 11th momentum

\[ |\text{vac}, p^\mu, p^{11} > \] which is the central extension in the type IIA 10D supersymmetry algebra, that occurs non-perturbatively in the 11D superalgebra. It is then natural to expect that, in the theory on \( R^{10} \) (or in its assumed non-perturbative 11D structure), there are non-perturbative string excitations with non-trivial values of the central extension at all excitation levels, at any coupling

\[ (\text{Bose} \oplus \text{Fermi oscill.})^{(l)}|\text{vac}, p^\mu, p^{11} > . \] (4)

Thus, the \( 2 \frac{15}{B} + 2 \frac{15}{F} \) multiplet at the first massive level \( l = 1 \) is also elevated to 11-dimensional fields \( \phi (x^\mu, x^{11}) \). Then the \( l = 0, 1 \) fields know about 11 dimensions in two ways: (i) the indices, (ii) the momentum (or space) dependence. We will refer to these two spaces as “index space” and “base space” respectively.

Since the \( l = 0 \) states are BPS saturated one can give their mass in the form \[ \text{3} \]

\[ l = 0 : \quad M_{10} = |p_{11}|. \] (5)

This is equivalent to saying that the 11-dimensional mass is zero for all the non-perturbative or perturbative states

\[ l = 0 : \quad M_{11} = \sqrt{M_{10}^2 - p_{11}^2} = 0. \] (6)

In \[ \text{2} \] it was pointed out that \( M_{10} = |p_{11}| = cn/\lambda \) become massless at infinite coupling \( \lambda \rightarrow \infty \). In \[ \text{3} \] it was emphasized that \( M_{11} = 0 \) at all values of the coupling. Stated in this way all the states satisfying the condition \( M_{11} = 0 \) are viewed as a single multiplet of degenerate states in the 11-dimensional theory, at any coupling. From the point of view of 10D, the eleven dimensional symmetry is broken, but its 11D multiplet structure is preserved. For the \( l = 1 \) states we cannot give a non-perturbative mass formula, although we may still define \( M_{11} = \sqrt{M_{10}^2 - p_{11}^2} \neq 0 \). It is evident that we still have an 11D multiplet structure for all \( l = 1 \) states, including all those that have non-perturbative values of \( p_{11} \).

The perturbative index space at higher levels is obtained by examining the content of the oscillators in \[ \text{4} \]. For the theory in 10D this is done explicitly in \[ \text{3} \] with the following result

\[ \text{indices} \Rightarrow \left( 2 \frac{15}{B} + 2 \frac{15}{F} \right) \times R^{(l)}. \] (7)

The factor \( 2 \frac{15}{B} + 2 \frac{15}{F} \), although obtained from a combination of the oscillators and \( \text{vac} \), can be reinterpreted as the action of 32 supercharges on a set of \( SO(9) \) representations \( R^{(l)} \) at oscillator level \( l \), where \( SO(9) \) is the spin group in 10-dimensions for massive states. The factor \( R^{(l)} \) is of the form of direct products of \( SO(9) \) representations coming from left/right movers

\[ R^{(l)} = \left( \sum_i r_i^{(l)} \right)_L \times \left( \sum_i r_i^{(l)} \right)_R \] (8)

such that the left-factor is identical to the right-factor, and is given by the collection of \( SO(9)_{L,R} \) representations listed in Table 1, where the subscripts \( B/F \) stand for boson/fermion respectively.

This structure shows that the factor \( R^{(l)} \) really has the index structure classified by the larger group

\[ SO(9)_L \otimes SO(9)_R. \] (9)

2The left/right excitation levels are the same for the 10D theory, \( l = l_L = l_R \).
Table 1
L/R oscillator states in 10D superstring.

| Level | SO(9),R reps \( \sum_i r_i^{(l_{L,R})} \)_{L,R} |
|-------|----------------------------------|
| \( l_{L,R} = 1 \) | 1B |
| \( l_{L,R} = 2 \) | 9B |
| \( l_{L,R} = 3 \) | 44B + 16F |
| \( l_{L,R} = 4 \) | \( \begin{align*} & (1 + 36 + 44 + 84)B \\ & + 231 + 450 \\ & + [16 + 128 + 576]_B \end{align*} \) |
| \( l_{L,R} = 5 \) | \( \begin{align*} & (1 + 36 + 44 + 84)B \\ & + 231 + 450 \\ & + [16 + 128 + 576]_B \end{align*} \) |

Furthermore, the supercharge factor \( 2_{15}^B + 2_{15}^R \) has an even larger classification group

\( SO(32) \)

with \( 2_{15}^B + 2_{15}^R \) corresponding to its two spinor representations\(^3\). The diagonal \( SO(9) \) subgroup of all these factors is the familiar rotation group in the 10D Lorentz group \( SO(9,1) \).

If there is an underlying non-perturbative 11D structure one must expect to find \( SO(10) \) supermultiplets for both the index and base spaces at all levels \( l \). However, the perturbative type IIA string given above can guarantee only \( SO(9) \) multiplets for the index space. Despite the larger classification schemes of indices displayed above one cannot find a common \( SO(10) \) subgroup, and hence, except for level \( l = 1 \), the 11D structure is absent at excited levels if only the perturbative indices are taken into account.

One possible conclusion is that there is no 11D, and that the \( l = 0, 1 \) index structures are just accidents. However, if one thinks of the 10D theory as a perturbative starting point to describe the hidden 11D theory, it must be that the 11D symmetry is broken spontaneously. This is the hint provided by the \( l = 0, 1 \) levels discussed above. Thus, consider the scenario in which the weak coupling type IIA string theory happens to describe a self consistent 10D part of the 11D multiplet, while the complete non-perturbative theory has additional states, such that when combined with the perturbative string states, they make up complete 11D multiplets. Under such an assumption the minimal extension of the index space of table-I that give 11D multiplets is as follows\(^3\): At level \( l_{L,R} = 2 \) the \( 9_B \) must be extended to \( 10_B = 9_B + 1_B \) for both left/right. At level \( l_{L,R} = 3 \) the \( 44_B \) must be extended to \( 54_B = 44_B + 9_B + 1_B \), while the \( 16_B \) is already a complete \( SO(10) \) representation, and so on for higher levels. The additional states are non-perturbative states from the point of view of string theory, but are naturally expected as part of a multiplet if there is an underlying 11D non-perturbative theory that is spontaneously broken.

It is interesting that this minimal extension of the indices has a definite pattern. Namely, in order to get complete \( SO(10) \) representations at level \( l_{L,R} \), we need to add new states whose indices are isomorphic to the indices of all previous levels. Then the following set of indices form complete \( SO(10) \), \( l_{L,R} \) multiplets separately for L/R movers

\[
|l_{L,R} >_p \oplus |l_{L,R} - 1 >_n p \oplus |l_{L,R} - 2 >_n p \oplus \cdots \oplus |l_{L,R} = 2 >_n p \oplus |l_{L,R} = 1 >_n p |
\]

as can be verified by examining Table 1. The subscript \( p \) describes the perturbative states listed in Table 1, while the subscript \( n \) describes the non-perturbative states that need to be added, but whose indices are isomorphic to those in Table 1. It is not clear what this pattern indicates, although some suggestions appear in\(^3\). Although verified explicitly up to \( l = 5 \), this pattern is conjectured to be true at all levels.

The additional new states were conjectured on the basis of 11D. However, in recent work with S. Yankielowicz\(^3\) we found out that they are required on the basis of U-duality in the compactified theory, as explained below.

There may be additional, purely non-perturbative, complete 11D multiplets, whose states are not connected by Lorentz transformations to the perturbative string states in Table 1. Some such possibilities are mentioned in\(^3\) but there may be others as well. By analysing the compactified string theory and using U-duality we may find U-duality transformations that connect different 11D multiplets to each other. Indeed there are signs of

\(^3\)Note that \( SO(32) \) contains successively \( SO(9)_L \otimes SO(9)_R, SO(10)_L \otimes SO(10)_R, SO(16)_L \otimes SO(16)_R \).
such phenomena in our recent work \[13\].

2. Compactifications, U-duality and 11D

2.1. The states

In the toroidally compactified type II string on \(R^d \otimes T^c\), with \(d+c = 10\), the perturbative vacuum state has Kaluza-Klein (KK) and winding numbers that label the "perturbative state". Hence the perturbative states are identified by their equal excitation levels \(l_{L,R}\) for left/right movers. The perturbative states are

\[
(\text{Bose} \oplus \text{Fermi oscillators})^{(l_L)}_{L,R}
\times (\text{Bose} \oplus \text{Fermi oscillators})^{(l_R)}_{R}
\times |\text{vac}, p^i; \vec{m}, \vec{n} >
\] (12)

where the \(c\)-dimensional vectors \((\vec{m}, \vec{n})\) are the KK and winding numbers that label the "perturbative base". These quantum numbers satisfy the relations

\[
l_L + \frac{1}{2} p^2_L = l_R + \frac{1}{2} p^2_R = M^2_d
\]

\[
\vec{p}^2_L - \vec{p}^2_R = \vec{m} \cdot \vec{n} = l_L - l_R
\]

where \(\vec{p}_{L,R}\) depend as usual \[21\] on \((\vec{m}, \vec{n})\) and \((G_{ij}, B_{ij})\) that parametrize the torus \(T^c\), while \(M_d\) is the mass in \(d\)-dimensions \(M^2_d = \vec{p}^2_R\). By using the same methods as \[8\] we can identify the following supermultiplet "perturbative index" structure for the string states \[12\] at levels \((l_L, l_R)\):

\[
(0,0) : (2^7_B + 2^7_F) \otimes 1_L \otimes 1_R
\]

\[
(0,l_R) : (2^{11}_B + 2^{11}_F) \otimes 1_L \otimes \sum_i r^{(l_R)}_{iR}
\]

\[
(l_L,0) : (2^{11}_B + 2^{11}_F) \otimes \sum_i r^{(l_L)}_{iL} \otimes 1_R
\]

\[
(l_L, l_R) : (2^{15}_B + 2^{15}_F) \otimes \sum_i r^{(l_L)}_{iL} \otimes \sum_i r^{(l_R)}_{iR}
\]

The \(2^{11}_B + 2^{11}_F\) corresponds to the intermediate supermultiplet of 11D supersymmetry. The structures \(\sum_i r^{(l_{L,R})}_{iL,R}\) are the same ones listed in Table 1, but with the \(SO(9)_{L,R}\) representations reduced to representations of \(SO(d-1)_{L,R} \otimes SO(c)_{L,R}\). So, a general perturbative string state is identified by "index space" and "base space" in the form

\[
\phi^{(l_{L,R})}_{\text{indices}} (\text{base})
\]

where both the base and the indices are given through \[13\] and Table 1.

The spectrum of the non-perturbative states is much richer in the compactified theory. There are many central charges in the supersymmetry algebra and those provide sources that couple to the NS-NS as well as R-R gauge potentials. Therefore one finds a bewildering variety of non-perturbative solutions of the low energy field equations as examples of non-perturbative states that carry the non-perturbative charges. A complete classification of all these charges, including p-brane charges will be given elsewhere \[13\]. Here we concentrate on 0-branes. The base quantum numbers are now extended to

\[
|\text{vac}, p^i; \vec{m}, \vec{n}, z^I >
\]

where \(z^I\) includes the \(p_{11}\) of the previous section as well as many other quantum numbers related to 0-branes. These correspond to the bosonic scalar central operators in the SUSY algebra that can be simultaneously diagonalized. This is the non-perturbative base.

There are two types of new non-perturbative states: those obtained by applying oscillators on the non-perturbative base and those that cannot be obtained in this way, but which are required to be present to form a basis for U-duality transformations. The latter are generalizations (at fixed \(l_{L,R}\)) in the spirit of the extra states listed in \[14\], but not necessarily identical (see below). So, a general state in the theory is identified at each \(l_{L,R}\) as in \[14\]. Both the base and the indices have non-perturbative extensions as explained in \[13\]. The full set of states turns out to form a basis for U-duality transformations at each fixed value of \(l_{L,R}\). These states are not degenerate in mass, hence the idea of a multiplet is analogous to the multiplets in a theory with broken symmetry.

The BPS saturated states are those with either \(l_L = 0\) or \(l_R = 0\). Even for BPS saturated states there are the two types of non-perturbative states. In particular the non-perturbative indices occur
for $l_L \geq 2$, $l_R = 0$ (or interchange $L, R$) similar to (11).

For the BPS saturated states one can derive an exact non-perturbative formula for the mass by using the supersymmetry algebra with central charges. For example for a state with non-trivial quantum numbers $(\vec{m}, \vec{n}, p_{11})$ and $l_R = 0, l_L = \vec{m} \cdot \vec{n}$, the mass is \[M^2_4 = p_{11}^2 + \frac{1}{2}p_R^2.\] (17)

The presence of the non-perturbative $p_{11}$, as in (10), modifies the mass formula (13). Further generalizations involving other non-trivial $z^I$ will be given elsewhere [13]. For non BPS saturated states we cannot give an exact mass formula.

2.2. Dualities

In this paper I will give some examples of how U-duality acts on the non-perturbative states (including oscillators) to connect them to perturbative states, and how from these transformation properties one can obtain the content of the quantum numbers for both the non-perturbative base and the non-perturbative indices.\footnote{After these lectures were delivered last summer, it was later understood that some of the non-perturbative states discussed here and in [13] are related to D-branes [16], as explained in [14]. See further remarks at the end of section 2.3.}

Details of these ideas [13] and their extensions to include p-branes appear elsewhere [14]. I must emphasize that in this way we can consider both BPS saturated as well as BPS non-saturated states.

In [13] the emphasis was on BPS non-saturated states whereas here I will discuss BPS saturated ones.

The $T$-duality group is $T = O(c, c; Z)$ in all cases [20]. The non-compact $U$-groups, their maximal compact subgroups $K \subset U$, and the maximal compact subgroup $k$ of the $T$-group,

\[k = O(c)_L \times O(c)_R\] (18)

are listed for various dimensions in Table 2 [1].

Since $T \subset U$ then $k \subset K$. It is understood that these groups are continuous in supergravity but only their discrete version can hold in string theory.

The string states involved in the T-duality transformations are not all degenerate in mass. Therefore, T-duality must be regarded as the analog of a spontaneously broken symmetry, and the string states must come in complete multiplets despite the broken nature of the symmetry. It is well known that $T = O(c, c; Z)$ acts linearly on the the $2c$ dimensional vector $(\vec{m}, \vec{n})$ [21]. However it is important to realize that it also acts on the indices in definite representations.

The action of $T$ on the indices is an induced $k$-transformation that depends not only on all the parameters in $T$ but also on the background $c \times c$ matrices $(G_{ij}, B_{ij})$ that define the tori $T^c$ [13].

Since the states in the previous section are all in $k = O(c)_L \times O(c)_R$ multiplets, the $T$-duality transformations do not mix perturbative states with non-perturbative states.

A U-multiplet contains both perturbative as well as non-perturbative T-multiplets. Like the $T$-duality transformations, the U-duality transformations act separately on the base and the indices of the states described by (15) without mixing index and base spaces. The action on the base quantum numbers $(\vec{m}, \vec{n}, z_I)$ is in linear representations (the representations are explicitly known, see e.g. [13]). The action on index space is an induced field-dependent gauge transformation in the maximal compact subgroup $K$, whose only free parameters are the global parameters in $U$. This $(U, K)$ structure extends the situation with the $(T, k)$ structure of the T-duality transformations described in the previous paragraph. The logical/mathematical basis for this structure is more fully explained in [13]. The bottom line is that in order to have U-duality multiplets, in addition to the non-perturbative base, the “indices” on the fields in (17) must be extended to form complete $K$-multiplets.

By knowing the structure of a U-multiplet we can therefore predict algebraically the quantum numbers of the non-perturbative states by extending the quantum numbers of the known perturbative states given in (14). The prediction of these non-perturbative quantum numbers is one of the immediate outcomes of our approach. In addition, our formulation sheds some light and raises some questions on other non-perturbative
2.3. An example

It is very easy to analyze the case \((d, c) = (6, 4)\) so we present it here as an illustration. In this case the spin group is \(SO(5)\) and there are 4 internal dimensions. The duality groups and index spaces follow from Tables 1, 2 and \([14]\). The relevant information is summarized by

\[
\begin{align*}
U & = SO(5, 5), \quad K = SO(5) \otimes SO(5) \\
T & = SO(4, 4), \quad k = SO(4)_L \otimes SO(4)_R \\
L, R & = 1: \quad \left( \sum_i r_i^{(L, R)} \right)_{L, R} = 1_{L, R} \\
L, R & = 2: \quad \left( \sum_i r_i^{(L, R)} \right)_{L, R} = 9_{L, R} \\
L, R & = 3: \quad \text{ etc.}
\end{align*}
\]

where the indices \(9_{L, R}\) have been reclassified according to their space and internal components. The reclassification is done also for the short \((2^7_B + 2^7_F)\), intermediate \((2^{11}_B + 2^{11}_F)\) and long \((2^{15}_B + 2^{15}_F)\) supermultiplet factors (see footnote \#2). It is clear from this form that the \(k = SO(4)_L \otimes SO(4)_R\) structure follows directly from the separate left/right internal components, while the spin of the state is to be obtained by combining left and right content of the space part.

Here I will discuss an example involving BPS states which is very similar to another discussion on non-BPS states given in \([13]\). Let us consider the BPS saturated states \((l_L \neq 0, l_R = 0)\). The base quantum numbers in \(\phi_{indices}^{(l_L, 0)}(\text{base})\) form the 16 dimensional spinor representation of \(U = SO(5, 5)\)

\[
\text{base} = (\vec{m}, \vec{n}, z^I) = 16 \quad \text{of} \quad SO(5, 5)
\]

Among these the eight quantum numbers \((\vec{m}, \vec{n})\) are perturbative, while the remaining eight \(z^I\) are non-perturbative. 0-branes that carry these quantum numbers provide the sources for the field equations of the 8 massless NS-NS vectors and the 8 R-R vectors respectively. The representation content of the indices in \(\phi_{indices}^{(l_L, 0)}(\text{base})\) is

\[
\text{indices} = (2^{11}_B + 2^{11}_F) \times \\
\times \left[ \left( \sum_i r_i^{(l, I)} \right)_L \\
\quad + \text{non-perturbative} \right]
\]

where \((2^{11}_B + 2^{11}_F)\) is interpreted as the SUSY factor. The full set of indices must form complete

\[
K = SO(5)_L \otimes SO(5)_R
\]

multiplets for consistency with the general U-duality transformation. It can be shown generally that the SUSY factor does satisfy this requirement because the supercharges themselves are representations of \(SO(5)_{\text{spin}} \times K\) \([13]\). Therefore, the remaining factor in brackets must be required to be complete \(SO(5)_{\text{spin}} \times K\) multiplets.

At level \(l_L = 1\) the piece \(\sum_i r_i^{(1)} = 1\) is a singlet, as seen in Table 1. Hence no additional non-perturbative indices are needed at this level. At level \(l_L = 2\) the piece \(\sum_i r_i^{(2)} = 9_L = 5_{\text{space}} \otimes 4_{\text{internal}}\) is classified under \(SO(5)_{\text{spin}} \times SO(4)_L \otimes SO(4)_R\) as

\[
(5, (0, 0)) + (0, (4, 0))
\]

Obviously, this is not a complete \(SO(5)_{\text{spin}} \times SO(5)_L \otimes SO(5)_R\) multiplet. Therefore, non-perturbative indices must be added just in such

| \(d/c\) | \(U\) | \(K\) | \(k\) |
|-------|-------|-------|-------|
| 9/1   | \(SL(2) \otimes SO(1, 1)\) | \(U(1)\) | \(Z_2\) |
| 8/2   | \(SL(3) \otimes SL(2)\) | \(SO(3) \otimes U(1)\) | \(U(1) \otimes U(1)\) |
| 7/3   | \(SL(5)\) | \(SO(5)\) | \(SO(3) \otimes SO(3)\)
| 6/4   | \(SO(5, 5)\) | \(SO(5) \otimes SO(5)\) | \(SO(4) \otimes SO(4)\) |
| 5/5   | \(E_{6, 6}\) | \(USp(8)\) | \(Sp(4) \otimes Sp(4)\) |
| 4/6   | \(E_{7, 7}\) | \(SU(8)\) | \(SU(4) \otimes SU(4)\) |
| 3/7   | \(E_{8, 8}\) | \(SO(16)\) | \(SO(7) \otimes SO(7)\) |
a way as to extend the $(4,0)$ of $k = SO(4)_L \otimes SO(4)_R$ into the $(5,0)$ of $K = SO(5)_L \otimes SO(5)_R$. That is

$$(4_{int})_L \rightarrow (5_{int})_L.$$  \hfill (23)

This extension determines the required non-perturbative indices for this case. Note that this amounts to extending the $9_L$ into a $10_L$, and similarly for right-movers

$$9_{L,R} \rightarrow 10_{L,R}. \hfill (24)$$

This is precisely what was needed in section-1 in order to obtain consistency with an underlying 11D theory \[1\].

At all higher levels $l_{L,R}$ the requirement for complete $K$-multiplets coincides precisely with the requirement of an underlying 11D theory. Therefore the full set of indices are the same as those given in eq.\[11\]. The story is the same with the non-BPS-saturated states at arbitrary $l_{L,R}$. This result was found in \[3\] by assuming the presence of hidden 11-dimensional structure in the non-perturbative type-IIA superstring theory in 10D. In ref.\[3\] a justification for \[24\] could not be given. However, in \[13\] and in the present analysis $U$-duality demands \[23\] and therefore justifies \[24\], and similarly for all higher levels.

Therefore for this particular compactification on $R^6 \otimes T^4$, U-duality and 11D Lorentz representations imply each other.

When similar results were reported last summer in these lectures, D-branes had not yet entered the duality picture. However, later a consistency check between U-duality and D-branes was reported in \[19\]. It is of interest to compare that analysis to ours at the time of writing these lectures. We find complete agreement at level $l_L = 1$. But at higher levels $l \geq 2$ our scheme requires more states than the D-brane degeneracy computed in \[19\]. There the states corresponding to the non-perturbative indices were not considered, seemingly because the special U-duality transformation considered (interchanging the two 8’s in the 16 of \[24\]) has a trivial transformation on our index space (does not go outside of the $4_{int}$). We have seen that under more general U-transformations the extra indices are needed both for U-duality multiplets as well as for 11D interpretation. Thus, the D-brane or other interpretation of these extra states is currently unknown.

For $(d,c) = (10,0), (9,1), (8,2), (6,4)$ the analysis for $l_{L,R} = 2, 3, 4, 5$ produces exactly the same conclusion as the 11D analysis. That is, $U$-duality demands that the $SO(9)_L \otimes SO(9)_R$ multiplets $\sum_i r_i^{(l_{L,R})}$ should be completed to $SO(10)_L \otimes SO(10)_R$ multiplets. The minimal completion \[12\] is sufficient in this case. Hence, in these compactifications $U$-duality is consistent with a hidden 11D structure, and in fact they imply each other.

On the other hand for the other values $(d,c) = (7,3), (5,5), (4,6), (3,7)$ the story is more complicated. At various low levels we found that the minimal index structure required to satisfy $U$-duality is different than the minimal structure of 11-dimensional supersymmetry multiplets \[12\]. If both U-duality and 11D are true then there must exist an even larger set of states such that they can be regrouped either as 11D multiplets or as U-duality multiplets. Exposing one structure may hide the other one. In fact we have shown how this works explicitly in an example in the case $(7,3)$ at low levels $l_{L,R}$ \[13\]. However, it is quite difficult to see if the required set of states can be found at all levels.

3. Summary

We have found that at levels $l_{L,R} = 0, 1$ the existing index structure for perturbative states is all that is needed to define complete $U$-multiplets in the form $\phi^{(l_{L,R})}_{(\text{indices})} (\text{base})$ for all values of $(d,c)$, and that this result directly follows from the simplest short, intermediate and long multiplet structure of 11D space-time supersymmetry. This is easily seen since in table-I the first entry $1_{L,R}$ is just a singlet.

At levels $l_{L,R} = 0, 1$ all non-perturbative aspects appear in the base $= (\tilde{m}, \tilde{n}, z')$. The base quantum numbers are the central charges of the 11D SUSY algebra and these correspond to the 0-brane sources that couple to the massless vector particles in supergravity (generalizations including p-brane central charges in the SUSY algebra are found in \[13\]). $U$ acts as a linear transfor-
formation on the base in a representation that is identical to the one applied to the massless vector fields in compactified 11D supergravity. Furthermore the indices correspond to complete representations of $K$ and they mix with a transformation induced by $U$. Hence, for $l_{L,R} = 0,1$ both index space and base space of $U$-multiplets have firm connections to 11D, for all compactifications.

To have $U$-duality at higher levels $l_{L,R} \geq 2$ additional non-perturbative states are needed to complete the index structure. If these additional states are absent in the theory there is no $U$-duality in the full theory. Assuming that $U$-duality is true for $l \geq 2$, our approach provides an algebraic tool for identifying the minimal non-perturbative states at every level once the perturbative states are listed as in Table 1.

There seems to be a non-perturbative 11D structure lurking behind the theory. In view of the existence of a classical membrane theory with some promise of its consistency at the quantum level, or a related M-theory, searching for hidden 11 dimensional structure is an interesting challenge. There is mounting evidence that 11D is present in the 10D theory, including the work we presented here and in [3][13]. We have seen that $U$-duality is distinct from this 11D structure, although in some cases they appeared to imply each other. We have found cases where there is a clash between the two if one or the other is restricted to a minimal set of non-perturbative states. We have shown, at least in one example, that the conflicts may be resolved by adding more non-perturbative states (non-minimal ones [3]). But nevertheless this example clearly shows that 11D and $U$-duality are quite distinct from each other. If they are both true, their combined effect is quite restrictive on the non-perturbative structure of the theory. Whether the conflict can be resolved generally is a major question raised by our work.

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