Orbital evolution of the inner solar system towards the red giant phase of the Sun: Simultaneous production of axions and neutrinos with a non-zero magnetic dipole moment

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Abstract. We describe how the simultaneous production of axions and neutrinos with a nonzero magnetic dipole moment enlarge the solar radius and luminosity during the red and asymptotic giant phases and affect the physical state of the planets within the solar system. Numerical simulations were created by coupling the Eggleton stellar evolution code with a fourth-order Runge-Kutta algorithm, to calculate the orbital distance of each planet to the Sun and its physical properties. We compare the predictions of canonical stellar evolution against solar models that include an enhanced energy loss within their core induced by the production of axions and neutrinos, considering the current most restrictive limits for the coupling constant between axions and electrons and the magnetic dipole moment of neutrinos ($\alpha_a = 0.5 \times 10^{-26}$, $\mu_\nu = 2.2 \times 10^{-12} \mu_B$). The enhanced energy loss accelerates the expansion rate of the solar giant and ensuring that all the planets up to Mars become engulfed, at an earlier age than what is predicted by standard physics. Along with the increment of the solar radius, the solar bolometric luminosity could be up to 30% stronger, affecting the physical conditions of the remaining planets.

1. Introduction

The survival of Earth to the red giant phase of the Sun is a classical topic in stellar astrophysics. Schröder and Smith [1] revisited the problem with a much more accurate description of the mass lost during the red giant phase. They found that the loss of angular momentum, even in a small amount, would make Earth, Venus and Mercury spiral inwards just a few million years before the end of the red giant phase. Only astronomical bodies located at a minimum safe distance around 1.15 astronomical units (AU) would survive to the solar giant.

Most works relate Earth’s fate to the physics within the layers of matter surrounding the solar core, also known as the envelope (e.g. the current uncertainties in the solar opacity, the magnitude of the mixing length parameter, the amount of mass that will be expelled to space during the red giant phase or the fraction of angular momentum that will be lost due to the
resulting expansion of the solar giant). However, the dissipation of energy from the stellar core has a major role in defining the stellar bolometric luminosity and radius prior to the helium flash. Any energy lost will delay the helium flash and, as it is shown below, help to produce a larger, more luminous, red giant expanding at a faster phase.

Neutrinos, the lightest known massive particles, have an important role on the cooling of the core of stars as they low interaction rate with other particles matter is very low in most astrophysical scenarios dissipating energy from physical systems. Neutrino production either by nuclear [2] or thermal reactions [3, 4] implies that hydrostatic equilibrium has to accelerate the nuclear fusion to compensate for the energy lost.

The discovery of the neutrino mass points towards the possibility that neutrinos could have still undiscovered properties: if neutrinos have a non-zero magnetic dipole moment, there should be an increment on the decay rate of plasmons (a physical state in which photons can be encountered when they are produced by degenerate matter) as the degeneracy of the helium core grows towards the helium flash. Also, the possible existence of axions [5] would imply that more energy escapes from the stellar core. Even though the occurrence of these two cases was interesting mostly for particle physics, it was soon made clear [6–9] that stellar astrophysics could be used to get upper limits on the magnetic dipole moment of neutrinos and the coupling constant between axions and electrons. Currently, the most restrictive limits on the magnetic dipole moment and the axion electron coupling constant are: $\mu_{\nu} \leq 2.2 \times 10^{-12} \mu_B$ and $\alpha_a \sim 0.5 \times 10^{-26}$, based on the determination the maximum bolometric luminosity for red giants within globular clusters and supported by evidence gathered from the tip-RGB of $\omega$-Centauri and other well-populated globular clusters [10–12].

Bellow, the consequences on the simultaneous production of axions and neutrinos with a non-zero magnetic dipole moment, in the context of the future evolution of the solar system are displayed.

2. Metodology

This study uses the stellar evolution code created by Eggleton [13] as its computational basis. The present version, used by [11] and related works to constrain the neutrino magnetic dipole moment and the axion-electron coupling constant (see the references therein for more details).

The inclusion of the magnetic dipole moment on the emission rate of neutrinos due to plasmon decay, non-standard neutrino emission, was included accordingly to Raffelt et al. [8]:

$$\epsilon_{\text{pl}} = F_{\text{SM}} Q_{\text{pl}} \left(1 + \frac{F_{\mu_{\nu}}}{F_{\text{SM}}} \right);$$  \hspace{1cm} (1)

(in units of erg $\cdot$ g$^{-1} \cdot$ s$^{-1}$). $F_{\text{SM}}$ denotes the standard model coefficient. The coefficient $F_{\mu_{\nu}}$ depends on the parameterized magnetic dipole moment: $\mu_{12} = \mu_{\nu} / (10^{-12} \mu_B)$ and is given by:

$$F_{\mu_{\nu}} = 0.0713 \left[ \left( \frac{\mu_e}{2 \rho_6} \right)^2 + 0.641 \left( \frac{\mu_e}{2 \rho_6} \right)^{4/3} \right]^{1/2} \mu_{12};$$  \hspace{1cm} (2)

where $\rho_6$ represents matter density in units of $10^6$ g $\cdot$ cm$^{-3}$, and $\mu_e$ is the molecular weight.

For axion production processes we considered the reactions described by Raffelt et al. [9]. The axion emissivity of the Compton process is:

$$\epsilon_C = \alpha_{26} \times 33 \mu_e T_6^4 \left[ \frac{F}{\sqrt{1 + F^2}} \right];$$  \hspace{1cm} (3)
where F accounts for relativistic corrections, degeneracy and photon dispersion by the medium and is taken as F = 1, except at the start of the ascend towards the tip-RGB (in that case we use F = 3E_F T/p_F^2) and α_{26} is the axion-electron coupling constant, in units of 10^{-26}. For the Bremsstrahlung process, the emissivity is:

$$\epsilon_B = (\epsilon_{\text{non-deg}}^{-1} + \epsilon_{\text{deg}}^{-1})^{-1};$$  \quad (4)

where

$$\epsilon_{\text{non-deg}} = \alpha_{26} \times 297 T_s^2 \rho_6 \left[ (1 + X) + \frac{(1 + X)^2}{2\sqrt{2}} \right];$$  \quad (5)

represents axion emission in non-degenerate conditions (ignoring screening effects and considering a chemical composition of hydrogen-helium) and

$$\epsilon_{\text{deg}} = \alpha_{26} \times 10.8 \frac{Z^2}{A} T_s^4 F_B;$$  \quad (6)

is the energy loss rate when degeneracy can’t be ignored (A and Z represent atomic mass and number, respectively). For the Bremsstrahlung process:

$$F_B = \frac{2}{3} \log \left( \frac{2 + k^2}{k^2} \right) + \left[ \frac{2 + 5k^2}{15} \log \left( \frac{2 + k^2}{k^2} \right) - \frac{2}{3} \right] \beta_F^2 + O(\beta_F^4);$$  \quad (7)

with \(\beta_F = p_F/E_F\) and \(k^2 = 0.147 \rho_6^{1/3}/T_s\). Using these formulae, Rafelt et al. [9] constrained the axion-electron coupling constant to \(\alpha_{26} \sim 0.5\) (in units of 10^{-26}).

The mass-loss rate occurring during the red giant phase follows the empirical formula by Schröder et al. [14]. Mass-loss depends on the chromospheric flux of mechanical energy and is related to the surface gravity, bolometric luminosity and effective temperature. As non-standard energy losses affect these parameters, a recalibration of the mass-loss parameter becomes necessary, to ensure that the tip-RGB mass is the same for non-standard as for canonical models [10].

The doomsday scenario depicted below goes in the same path as [1] using the formulation for planetary orbits by [15]:

$$R_p = \frac{A_p^2}{M_p \Omega M_\odot};$$  \quad (8)

where \(R_p\), \(M_p\) and \(A_p\) represent the orbital distance between an specific planet and the Sun, its mass and angular momentum, while \(M_\odot\) and G are the mass of the Sun and the universal gravitational constant. Due to the gravitational drag caused by the expansion of the convective envelope of the Sun, once it becomes a red giant, the orbital angular momentum of the planets within the solar system decays according to the Equation:

$$\frac{dA_p}{dt} = \Gamma;$$  \quad (9)

where the decay rate

$$\Gamma = -6 \frac{\lambda_2}{t_f} q^2 M_\odot R_\odot^2 \left( \frac{R_\odot}{R_p} \right)^6 (\Omega - \omega).$$  \quad (10)
depends on the convective friction time, the planetary angular velocity and the planet to star mass ratio while it is assumed that the rotational angular momentum of the Sun, $\Omega$ is too small to be important.

The temporal evolution of planetary orbits is calculated in two stages: first the Eggleton code is used to evolve a ZAMS model through its main-sequence and up to the end of the asymptotic giant phase (there are two sets of models: the canonical scenario and the one in which axion production and enhanced neutrino emission happen simultaneously, following Equations 1-7). Afterwards, the change in the orbital distance of each planet is calculated, step by step, by the Runge-Kutta algorithm (using Equations 8-10).

3. Results
The solar models used in this work were set to have the reduced metallicity $Z = 0.0122$ suggested by Asplund et al. [16], instead of the traditional value $Z = 0.02$ [1], and the initial helium abundance $Y_i = 0.278$ by Serenelli and Basu 2010 [17], in accordance to the latest estimations on the initial chemical composition of the Sun.

Table 1. Canonical and non-standard solar tracks. Both evolve from the same ZAMS model and remain identical until the start of the red giant branch, when the enhanced energy losses can not be compensated by hydrostatic equilibrium.

| Phase         | Age [Gyrs] | $T_{\text{eff}}$ [$^\circ$ K] | $L_{\text{bol}}$ [$L_\odot$] | $R$ [$R_\odot$] | $M$ [$M_\odot$] |
|---------------|------------|--------------------------------|-----------------------------|----------------|----------------|
| ZAMS          | 0          | 5582                           | 0.6909                      | 0.8910          | 1.0000         |
| Present       | 4.6        | 5745                           | 1.0000                      | 1.0000          | 1.0000         |
| MS_{hottest}  | 8.2364     | 5785                           | 1.2676                      | 1.1240          | 1.0000         |

Canonical

| Phase | Age [Gyrs] | $T_{\text{eff}}$ [$^\circ$ K] | $L_{\text{bol}}$ [$L_\odot$] | $R$ [$R_\odot$] | $M$ [$M_\odot$] |
|-------|------------|--------------------------------|-----------------------------|----------------|----------------|
| Tip-RGB | 13.019     | 2672                           | 2545                        | 236            | 0.7218         |
| Tip-AGB | 13.152     | 3277                           | 1937                        | 142            | 0.5701         |

Non-standard

| Phase | Age [Gyrs] | $T_{\text{eff}}$ [$^\circ$ K] | $L_{\text{bol}}$ [$L_\odot$] | $R$ [$R_\odot$] | $M$ [$M_\odot$] |
|-------|------------|--------------------------------|-----------------------------|----------------|----------------|
| Tip-RGB | 12.199     | 2509                           | 3443                        | 312            | 0.7218         |
| Tip-AGB | 13.112     | 2988                           | 2846                        | 200            | 0.5922         |

The main features of the canonical and non-standard stellar tracks are shown in Table I. The enhanced cooling caused by the simultaneous production of axions and neutrinos with a non-zero magnetic dipole moment accelerates the evolution of stellar models by speeding up the reactions of the pp-chain happening on the hydrogen-burning shell (or in the double hydrogen/helium burning shells for the asymptotic giant phase). This reduces the time before the end each giant phase (0.82Gyrs for the first and 0.04Gyrs for the second). Both axions and neutrinos cool down the core and, as a consequence, it contracts, increasing its gravitational pull over the hydrogen-burning shell. The slight increment on density and temperature intensifies the helium production rate by the CNO-cycle. This, in turn, induces observable consequences on the surface parameters of the star: a brighter bolometric luminosity (around 900$L_\odot$ in both cases) accompanied by a larger stellar radius ($76R_\odot$ at the end of the first red giant phase and $58R_\odot$ at the end of the second). The resulting expansion leads also to slightly cooler effective temperature: the non-standard model has a cooler photosphere by 173$^\circ$K and 289$^\circ$K at the end of each giant phase.
Figure 1 shows the emission history of neutrinos and axions for the canonical (solid lines) and non-standard tracks (dashed lines). In the non-standard track, axion emission is active since the start of the main-sequence and steadily increases towards the tip-RGB, with the production of axions by the Bremsstrahlung process always being a factor of two more intense than that by the Compton reaction. The emissivity of neutrinos by the standard thermal reactions is much weaker and it becomes considerable only until the start of degeneracy within the helium core (with plasmon decay becoming the most important source of neutrinos). Non-standard plasmon decay develops quite differently to its canonical analog: it is allowed to start much earlier (just after the end of the main-sequence, with a neutrino emissivity three orders of magnitude larger) this early start allows it to build up a larger helium core.

Figure 2 shows the orbital evolution for the inner solar system from the RGB-phase to the end of the AGB (the upper panel correspond to canonical evolution while the one below shows the predictions under the enhanced cooling of the solar core, see also Table 2). The solar photosphere reaches Mercury and Venus 4.36 and 1.77 million years before the end of the red giant phase. During the second expansion phase, in which the solar giant losses more mass, the orbit of Mars expands again by around 0.6AU while the giant’s radius expands to a lesser degree than during the first red giant phase. Earth’s orbit expands initially as a consequence of the mass-loss suffered by the Sun. However, before the star loses one third of its mass, the photosphere expands beyond Earth’s orbit causing it to spiral inwards to 0.38AU (the current orbital distance of Mercury) just 0.15 million years before the end of the red giant phase. During the core Helium-burning phase, that follows the first time that the star becomes a red giant, Earth’s could be outside the photosphere. However, the planet would be engulfed again time during the second expansion of the Sun. The orbit of Mars remains outside the solar photosphere.
during both expansions. In the non-standard scenario, Earth and the innermost planets fall more quickly: Mercury and Venus spiral inwards 5.06 and 2.69 million years before the end of the red giant phase, while the solar photosphere reaches Earth about one million years after the fall of Venus. The final radius of the solar giant reaches almost to Mars’s orbit. However, due to the decreasing gravitational pull caused by the stellar mass-loss, Mars has moved 0.5AU. In this scenario, there is no chance for Earth to remain outside the solar photosphere after the end of the first red-giant phase. As in the canonical scenario, the orbit of Mars grows beyond the solar photosphere. The distance required to escape the expansion during the red giant phase of the Sun in the non-standard scenario corresponds to 1.3AU.

**Figure 2.** Orbital evolution of the inner solar system from the RGB-phase to the end of the AGB-phase. The solar photosphere is represented by the red marks. The orbits of Mercury (light bluish marks), Venus (magenta marks), Earth (blue marks) and Mars (green marks) correspond to the other symbols.
Table 2. Solar models and orbital distances for the planets within the inner solar system.

| Body | $R_p$(Now) [AU] | $R_p$(Tip-RGB) [AU] | $R_p$(Tip-AGB) [AU] | Fall Time |
|------|----------------|---------------------|---------------------|-----------|
| Sun  | 0.0046         | 1.0731              | 0.6458              | 13.0187   |
| Mercury | 0.3886      | 0.0006              | 0.0000              | 13.0129   |
| Venus | 0.7219         | 0.0048              | 0.0000              | 13.0167   |
| Earth | 1.0000         | 0.3877              | 0.0000              | 13.0187   |
| Mars  | 1.5241         | 2.0722              | 2.6203              | -         |

4. Conclusions
The simultaneous emission of axions and neutrinos, whose production rate is enhanced by the non-standard decay of plasmons due to a non-zero magnetic dipole moment, augments the radius and luminosity of the solar giant during the RGB and AGB phases. Bolometric luminosity increases by almost $900L_\odot$, or about 30%, of its total value, in both giant phases. Relevant changes are also produced on the effective temperature and stellar radius: the non-standard model for the Sun has a cooler (173$^\circ$K at the tip-RGB and 289$^\circ$K at the tip-AGB) and larger (76$R_\odot$, at the tip-RGB and 58$R_\odot$ at the tip-AGB) photosphere that expands more quickly. These changes make impossible for Earth, Venus and Mercury to escape and become engulfed in a shorter time than in the canonical scenario (although the expansion of the orbit of Mars allows it to move outwards and escape). But even if Mars does not fall within the Sun, the dramatic increment in the Solar bolometric luminosity would severely affect the amount of radiation reaching the planet’s surface. In general, our simulations show that the minimum safe distance, for a body with the mass and angular momentum of Earth to survive the expansion of the Sun in the non-standard scenario, corresponds to around 1.3 AU.

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