Mass reconstruction in disc like galaxies using strong lensing and rotation curves: The Gallenspy package

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Abstract

Two methods for mass profiles reconstruction in disc-like galaxies are presented in this work, the first is done with the fit of the rotation curve based on the data of circular velocity which are obtained observationally in a stars system, while the other method is focused in the Gravitational Lensed Effect (GLE). For these mass reconstructions, two routines developed in the language of programming python were used: one of them is \textit{Galrotpy} Granados et al. \cite{1}, which was built by members of the Galaxies, Gravitation and Cosmology group from the Observatorio Astronómico Nacional of the Universidad Nacional de Colombia and whose functionality is applied in the rotation curves, the other routine is \textit{Gallenspy} which was created in the development of this work \cite{2} and it is focused in the GLE. It should be noted that both routines perform a parametric estimation from the Bayesian statistics, which allows obtaining the uncertainties of the estimated values. Finally is shown the great power of combining galactic dynamics and GLE, for this purpose the mass profiles of the galaxies SDSSJ2141-001 and SDSSJ1331+3628 were reconstructed with \textit{Galrotpy} and \textit{Gallenspy} where these results obtained are compared with those reported by other authors regarding these systems.

Keywords: Mass reconstructions, GLE, rotational curves, mass profiles, \textit{Gallenspy}, \textit{Galrotpy}.

1. Introduction

The study of mass distribution in galaxies allows obtaining valuable information about the universe structure on a large scale and the process of stellar evolution. For this reason, the rotation curves and the GLE present in galaxies are very important, because they let us the analysis of the distribution of barionic and dark matter in these systems and in this way we can obtain significant restrictions of values such as cosmological densities, the Hubble constant and the cosmological constant among others.

The analysis of the rotation curves is done with base in the Newtonian gravitation theory, in this case it is important to point out that the flatness in the contours of these curves is the cause why the dark matter is included by other authors in different mass reconstructions \cite{1,3,4,5}. From this perspective, the dark matter reconcile the keplerian decrease with the observations done along the astronomy history, which forces to include the superposition of different mass components in the fitting of the rotation curves with observational data.

In addition to the rotation present in the galaxies, the GLE has been evidenced in many of them, which has to do with the deflection that the light coming from a background source presents due to gravitational potential of these stellar systems. Thanks to the achievements of the General Relativity Theory, is possible to estimate the mass distribution of these galaxies based on the deflector angle of the light beams \cite{6,7} and for this reason this effect can be complemented with the dynamical analysis in the reconstructions of mass profiles in galaxies.

\*https://github.com/ialopez7/Gallenspy

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Between the kinds of GLE observations present in galaxies and clusters, it is very common the formation of Einstein rings and giant arcs which are evidenced in this work for the mass reconstructions of the galaxies SDSSJ2141-001 and SDSSJ1331+3628, regarding to the rotational velocity data of this galaxies it is important to clarify that these were obtained of Dutton et.al. [3, 5] hence the mass profiles reconstruction was done with both methods.

Finally it is important to point out that in this work, the advantages of combining GLE and galactic dynamics are presented, where the complement between these methods is a powerful tool for the mass reconstructions.

2. Galactic Dynamics

In the last time, has been evidenced that the disc-like galaxies have different mass components, these can be classified into four kinds: dark matter halo, stellar halo, disc and bulge which interact between them in concordance with the Newtonian dynamics, where each mass distribution is essential in the understanding of the functional form of the gravitational potential for these stellar systems.

![Scheme of the main mass components of disc-like galaxies.](Image)

Since the gravitational force ($\vec{F}$) present in galaxies is conservative [8], the relation between this and the gravitational potential ($\Phi$) is:

$$\vec{F} = -\nabla \Phi,$$

(1)

this leads to the mass volumetric density ($\rho$) and $\Phi$ are related by means of the Poisson equation [1]:

$$\nabla^2 \Phi(\vec{r},t) = 4\pi G \rho(\vec{r},t),$$

(2)

where $G$ is the universal gravitation constant.

Due to the linearity of the Poisson equation [9] [11], in the case of a galaxy with $N$ components and respective volumetric mass densities $\rho_1$, $\rho_2$, ...$\rho_N$, the total density of this system is:

$$\rho = \sum_{i=1}^{N} \rho_i,$$

(3)

and this means that the total gravitational potential is expressed in this way $\Phi = \sum_{i=1}^{N} \Phi_i$.

As the circular velocity (in the equatorial plane) associated to a gravitational potential $\Phi(R, z = 0)$ in disc-like galaxies is described by:

$$V_c^2(R) = R \frac{\partial \Phi}{\partial r}_{r=R},$$

(4)
the total circular velocity of this kind of galaxies, is expressed as a superposition of the velocities belonging to each gravitational potential, which is evidenced in the equation 5

$$V_c^2 = \sum_{i=1}^{N} V_{c(i)}^2,$$  \hspace{1cm} (5)

and therefore is possible to make reconstructions of mass profiles in disc-like galaxies through the fitting of rotation curves with the observationally values of circular velocity, as such is evidenced in the figure 2.

![Figure 2: Fitting of rotational velocities with the rotation curve belonging to the galaxy NGC6361 (the observational data are the black dots and the fitting curve is the continuous line), in this case the Miyamoto-Nagai profile was used for the bulge (gray dotted line) and the stellar disc (red dotted line), while the Navarro-Frenk-White (green dotted line) belongs to the dark matter halo.](image)

3. Gravitational Lensing Effect

In the GLE, the relation between the coordinates of the images $\tilde{\theta}$ and the source $\tilde{\beta}$ is given by the equation:

$$\tilde{\beta} = \tilde{\theta} - \nabla_{\tilde{\theta}} \psi(\tilde{\theta}),$$ \hspace{1cm} (6)

where $\psi$ is the deflector potential, which has the information of the lens and the cosmological distances. For the case of mass profiles with spherical symmetry, it is possible assumed that $\psi(\tilde{\theta}) = 2 \int_{0}^{|\tilde{\theta}|} \theta' \kappa(\theta') |\ln(\frac{|\tilde{\theta}|}{\theta'})| d\theta'$, \hspace{1cm} (7)

for this case $\kappa$ is the convergence, which is defined as

$$\kappa = \frac{\Sigma}{\Sigma_{\text{crit}}}$$ \hspace{1cm} (8)

where $\Sigma$ is the superficial mass density and $\Sigma_{\text{crit}}$ is given by the relation

$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_s}{D_d D_{ds}},$$ \hspace{1cm} (9)

in this case, the term $c$ is the light velocity, $D_d$, $D_s$, and $D_{ds}$ are the angular diameter distances between observer-lens, observer-source and lens-source respectively.
Additionally in the GLE, the mapping between the planes of the lens and the source is done through the jacobian transformation matrix

$$A_{i,j} = \frac{\partial \beta_i}{\partial \theta_j},$$

(10)

for $i,j = 1,2$.

With this transformation matrix, is defined the magnification of the images by means of

$$\mu = \frac{1}{|\text{det}A|},$$

(11)

and therefore the critical curve is the set of points in the lens plane where $|\text{det}A| = 0$.

Note: The set of points in the source plane, which through the equation 6 belongs to the critical points is denominated caustic curve.

4. Combining GLE and Galactic Dynamics

Due to the superposition between the different mass distributions in galaxies, there are mass profiles reconstructions done with galactic dynamics and other with lensing where the obtention of parameters does not occur within acceptable reliability regions [3, 4].

A solution strategy for this problem has been proposed by other authors [11, 3], this has to do with the combination of these mass reconstructions methods, which means taking advantage of the geometry used for each one of them.

For this purpose it must be taken into account, that while the mass projection in the galactic dynamics is done in the equatorial plane of the galaxy, in the case of the GLE the operator $\Sigma$ is projected in the plane $(\theta_1, \theta_2)$ where the deflected images are formed.

![Figure 3: Illustration of the geometries used in galactic dynamics and GLE for the mass projection in 2-D based on what was stated by Dutton et al [4].](image)

In the figure 3 are illustrated the geometries belonging to the methods of mass reconstruction used in this work, wherein the GLE is done the projection of mass in a cylinder through the line of sight $z$ and within a radius $R_{eins}$ restricted by the position of the deflected images, which from the formalism of strong lensing is related to the formation of the Einstein ring [3] and it is described by the equation.

$$R_{eins} \approx \left( \frac{M_{eins}}{\pi \Sigma_{crit}} \right)^{1/2},$$

(12)

with $M_{eins}$ the mass projected in the cylinder of figure III.

In the case of mass projection from galactic dynamics, it is corresponding to enclosed spheres in different radius due to the circular velocities estimated in the galaxy. This geometrical approach is optimal, as long as the disc has an inclination, so that the observer can see it from its edges.
Based on the exposed, the combination of GLE and galactic dynamics for this work is proposed around the restrictions in the parameter space that both methods can provide, in such a way that it is possible to distinguish more clearly the gravitational contribution of each mass components in disc-like galaxies.

5. Gallenspy

Gallenspy is an open source code created in python, designed for the mass profiles reconstruction in disc-like galaxies using the GLE. It is important to note, that this algorithm allow to invert numerically the lens equation (equation 6) for gravitational potentials with spherical symmetry, in addition to the estimation in the position of the source \((\beta_1, \beta_2)\), given the positions \((\theta_1, \theta_2)\) of the images produced by the lens.

The main libraries used in this routine are: numpy [12] for the data handling, matplotlib [13] regarding the generation of graphic interfaces, galpy [14] to obtain mass superficial densities, as to the parametric adjust with Markov-Montecarlo chains (MCMC) is taken into account emcee [15] and for the graphics of reliability regions corner [16] is used.

The deflector potential is obtained numerically in Gallenspy by mean of the equation 7, which is very helpful because some mass distribution models do not have analytical solutions for the equations 3, 6 and 7.

Also it is important to note others tasks of Gallenspy as compute of critical and caustic curves and obtention of the Einstein ring. For a description more detailed, it is recommended to see the repository page which the source code is available together with its requirements and instructions for its use.

5.1. Tested case with Isotherm Singular Sphere (SIS)

A way by which Gallenspy was tested, is the comparison with the analytical solutions given by Hurtado [10] of the isotherm singular sphere (SIS) under certain specific conditions. In the figure 4 some of these comparisons are shown for the obtention of deflection angle, the images formation, and deflector potential in the case of a circular source of radius 1kpc to a distance of 2kpc respect to the observer, where this mass profile was modeled with a dispersion velocity of 100km/s.

As it is possible to observe in each comparative graphic, the results obtained numerically with Gallenspy present high reliability where the percentage error is of 0.1 due to the grid used in this case.

5.2. Gallenspy input

To start Gallenspy, it is important to give the values of cosmological distances in Kpc and critical density in \(M_\odot/Kpc^2\), which are introduced by means of a file named Cosmological_distances.txt. On the other hand, it is the file coordinates.txt where the user must introduced the coordinates of the observational images (in radians). (Note: for the case of a circular source it is present the file alpha.txt, where the user must introduced angles value belonging to each point of the observational images.)

5.3. Visual fitting with Gallenspy

Gallenspy present a interactive fitting of parameters through a routine developed in Jupyter Notebook with the name of Interactive_data, in this case the user has the possibility of made a free choose of the parametric range for each value, however it is suggested a parametric space illustrated in table 1 which is based on the values used by other authors and with the ones they modeled galaxies dwarfs and Milky way-like [1].

In figure 5 is illustrated the interactive panel, for the fitting of a Exponential Disk lens model and a circular source, this observational data belong to the galaxy J2141 which is analyzed later.

\[1\]https://github.com/ialopez/GalLenspy
5.4. Bayesian statistics with Gallenspy

In the mass reconstructions, Gallenspy allows assigning a mass profile to each component of the lens galaxy, where each free parameter has a range of values possible for the obtention of a set of initial values in the parametric exploration.

Also, it is important to point out that for the mass reconstruction of each component of the galaxy, it is possible to choose between different profiles for the parametric fitting with Gallenspy: for example, in the case of galactic disc it is possible choose between the options of Miyamoto-Nagai and Exponential Disc [1][8], for the dark matter halo between Navarro-Frenk-White (NFW) and Burket [1][8], while in the bulge is used the Miyamoto-Nagai even though in this profile are given two possible ranges of data.

When the positions $(\theta_1, \theta_2)$ of the GLE are known, the work with Gallenspy is to find the model and parameters set which can reproduce these provided data, for this reason in this routine the bayesian statistics is not only based on the exploration of all possible positions of the source.

For this parametric exploration, Gallenspy implements the Metropolis-Hasting algorithm through the MCMC [17], where is obtained a posterior probability distribution $P(p|D, M)$ for each parameter set of the lens model selected of the table [1] which is given by the relation:

$$P(p|D, M) = \frac{P(D|p, M)P(p|M)}{P(D|M)},$$

with:

- $P(p|D, M)$ the probability that this parameter set $p$ is appropriate for the model $M$ and the data $D$. 
### Range of values with Gallenspy

| Component         | Range of parameters | Units       |
|-------------------|---------------------|-------------|
| Bulge I           | \(a = 0\)           | kpc         |
|                   | \(0.0 < b < 0.5\)   | kpc         |
|                   | \(0.1 < M < 1.0\)   | \(10^{10} M_\odot\) |
| Bulge II          | \(0.01 < a < 0.05\) | kpc         |
|                   | \(0.5 < b < 1.5\)   | kpc         |
|                   | \(1 < M < 5\)       | \(10^{10} M_\odot\) |
| Disc thin         | \(1 < a < 10\)      | kpc         |
|                   | \(0.1 < b < 1.0\)   | kpc         |
|                   | \(0.5 < M < 1.5\)   | \(10^{11} M_\odot\) |
| Disc thick        | \(1 < a < 10\)      | kpc         |
|                   | \(0.1 < b < 15.0\)  | kpc         |
|                   | \(0.5 < M < 1.5\)   | \(10^{11} M_\odot\) |
| Exponential Disc  | \(2 < h_r < 6\)     | kpc         |
| Halo NFW          | \(1 < \Sigma_0 < 15\) | \(10^2 M_\odot/pc^2\) |
| Halo Burket       | \(0.1 < a < 30\)    | kpc         |
|                   | \(0.1 < M_0 < 10\)  | \(10^{11} M_\odot\) |
|                   | \(2 < a < 38\)      | kpc         |
|                   | \(0.1 < \rho_0 < 10\) | \(10^9 M_\odot/kpc^3\) |

| Table 1: Parametric space used in Gallenspy |

- \(P(D|p,M)\) the probability that the data \(D\) are obtained with the model \(M\) and the parameter set \(p\), is known as likelihood \([17]\) and it is denoted with \(L\).
- \(P(p|M)\) the prior and this is the reliability that the parameter set is correct for the model.
- \(P(D|M)\) is denoted as \(Z\), this is the normalization factor and is the probability of obtaining the data \(D\) with the model \(M\).

It is important to note that in Gallenspy the normalization factor is not considered, hence the fundamental work is the compute of the likelihood (this is because the prior for this parameter set have the same value). From this perspective the method for obtaining the initial values of \(P(D|p,M)\), is through a visual fitting in the Interactive data routine from which it is possible to make a first approximation between the data set \(D\) and the model values.

Later of this process, the user must to execute the code created in the respective file.py (for the estimation of the source source_lens.py and in the case of mass reconstruction parameters_estimation.py) where Gallenspy request to introduce the initial values obtained in the visual fitting. Next Gallenspy let to the user choose the number of steps and walkers, which is enough for the MCMC of this computational routine.

In Gallenspy the minimization function \(\chi^2\), for a source with a number \(n_i\) of images in the GLE is given by:

\[
\chi^2_i = \sum_{j=1}^{n_i} \frac{|\theta_{\text{obs}}^j - \theta^j(p)|^2}{\sigma_{ij}^2}, \tag{14}
\]

where in this equation \(\theta_{\text{obs}}^j\) is the position of observed image \(j\) in the data set \(D\), \(\sigma_{ij}\) the error in position \(\theta_{\text{obs}}^j\) due to the noise in the image and \(\theta^j(p)\) the image \(j\) predicted by the mass model used with the parameter set.

The index \(i\) appears in the function \(\chi^2\), this because in the GLE it is possible the formation of images for various sources, for this reason the likelihood for each explored parameter set is expressed through the
Gaussian distribution

\[ L = P(D|p, M) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_{ij}} \exp \left( -\frac{x_i^2}{2} \right), \]  

(15)

where N is the number of sources.

In figure 6, is shown the algorithm of Gallenspy where in the next section is illustrated an example of this with the SIS profile.

Finally it is important to note that Gallenspy generate a file.txt with the final parameters obtained: in the case of source estimation parameters_lens_source.txt and parameters_MCMC.txt, where these files are request by Gallenspy for other tasks as compute of Einstein ring and mass estimations.

5.5. Illustrative example with the SIS profile

Although the SIS is not included in the profiles of Gallenspy, it is possible to show an illustration of the mode which this routine performs the parameters exploration with this mass distribution.

Because of the analytical solutions, shown by Hurtado [10] to the lens equation in the SIS profile, the formation of images in the GLE are given by the relations:

\[ |\vec{\theta}_p| = |\vec{\beta}| + \frac{4\pi\sigma^2 D_{ds}}{c^2 D_s}, \]  

(16)

\[ |\vec{\theta}_n| = -|\vec{\beta}| + \frac{4\pi\sigma^2 D_{ds}}{c^2 D_s}, \]  

(17)

with \(\sigma\) the dispersion velocity, \(\theta_p\) and \(\theta_n\) are the positive and negative solutions respectively where the images are formed.
In the figure 7 is evidenced the images formation, when \( \sigma = 1 \times 10^5 \text{km/s} \) for a circular source of radius \( r = 1\text{arcs} \) whose center has coordinates \((h = 0.8\text{arcs}, k = 0.8\text{arc})\) and where the cosmological distances are \( D_{ds} = 1\text{Kpc} \) and \( D_s = 2\text{Kpc} \).

If for this specific case is applied the parametric exploration with Gallenspy, then it is necessary to suppose that the values of \( \sigma, r, h \) and \( k \) are not known and that the objective is the obtention of the parameters family which through of GLE can reproduce the images illustrated in figure VI, where in this example only values of \( |\vec{\theta}_p| \) are supposed to be known. This time, the ranges established for the parametric exploration were \( 10^4 \text{km/s} < \sigma < 2 \times 10^5 \text{km/s}, 0.1\text{arcs} < r < 2\text{arcs}, -8\text{arcs} < h < 8\text{arcs} \) and \( 8\text{arcs} < k < 8\text{arcs} \).

With the application of the function \( \chi^2 \) based on the equation 14 and with an error of 0.1 in the position of the images, was obtained the initial parameters set of the likelihood for the MCMC. In the figure 8a is evidenced the comparison between the images of \( |\vec{\theta}_p| \) with those produced by the parameters obtained of \( \chi^2 \).

From these values obtained with the multiparametric minimization, the MCMC was executed with Gallenspy where the best result was obtained for a number of 100 walkers and 1000 steps. In the figure VIII is shown the chain convergence in the obtention of parameter \( \sigma \), where it is possible to observe that from the obtained data in step 400 can be done the estimation of the parameters.
(a) Red Image formed in the GLE for a SIS with dispersion velocity $0.38 \times 10^3 \text{km/s}$ and a circular velocity of radius $2 \text{arc}$, the center of this source is in $(2, 2) \text{arcseg}$. Black Image to reproduce by means of Gallenspy.

(b) MCMC in the exploration of the $\sigma$ parameter.

Figure 8: Illustration of the arc belonging to the initial guess and evolution of the MCMC for the $\sigma$ parameter, in this case the values converge from the 300 step approximately.

(a) Results obtained with Gallenspy, in the exploration of parameters for the SIS profile.

(b) Graph comparative between the produced images by the SIS model and the images of $|\vec{\theta}_p|$.

Figure 9: Final results obtained with Gallenspy, for the fitting of the arc generated with a SIS model.
The graphs illustrated in figure 9a show the reliability regions, under which the parameters family was obtained for the reproduction of the images belonging to |$\vec{\theta}_p$|.

| Parameter | 95% |
|-----------|-----|
| SIS $\sigma$ ($10^5$km/s) | $0.999^{+4.456\times10^{-6}}_{-0.260}$ |

Table 2: Parameters obtained with Gallenspy for the SIS model.

In table 2 the parameters obtained are shown with its uncertainties for the quantile of 95%, these values are consistent with those established for the obtention of the images of |$\vec{\theta}_p$|, for this reason with this illustrative example was possible to observe the form which Gallenspy proceeds and its efficiency in the estimation of the parameters from the GLE.

Finally in figure 9b it is the superposition of images, where it is clear the reliability of results obtained with Gallenspy.

5.6. Curves criticals and caustics with Gallenspy

Another process that Gallenspy performs, is the obtention of criticals and caustics curves for distinct lens model. To understand this method, it is important to remember that $detA$ depends on the convergence $\kappa$ and the shear $\gamma$, which have relation with the deflector potential computed by Gallenspy in the fitting of images ($\theta_1, \theta_2$) [7, 10].

In this way, the first step given by Gallenspy is the obtention of $\gamma_1$ and $\gamma_2$ with the equation 18 and 19

$$\gamma_1(\vec{\theta}) = \frac{1}{2} \left( \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_1^2} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_2^2} \right),$$  

and from this results, Gallenspy computes the points of lens plane for those who $detA = 0$ based on the given relation by Hurtado [10]

$$A = \begin{pmatrix}
1 - \kappa - \gamma_1 & -\gamma_2 \\
-\gamma_2 & 1 - \kappa + \gamma_1
\end{pmatrix}$$

As the obtention of this critical points is not a trivial process, Gallenspy makes this process with the Bartelmann method [15], in which the critical curve in the lens plane is a border from which changes the sign of the determinant ($detA$) as illustrated the figure 10.

As is shown in this image, there are points of the plane ($\theta_1, \theta_2$) that although these do not belong to the critical curve, can be considered close to this: from this focus, if $S = Sign(detA)$ then a point $S_{00}$ is considered adjacent to the curve in a grid when $S$ changes between $S_{00}$ and any of these neighbors points.

In figure 10 is presented the estimation of each point for a grid with dimension $NXN$ based on the sign $S$, this means that for positive values $S_{i,j} = 1$ and in the case of negative values $S_{i,j} = -1$, where $i$ and $j$
are in a range of 0 to $N - 1$. From this perspective, the established restriction in Gallenspy to know if a point $S_{i,j}$ is adjacent to the critical curve in the grid, is based on the condition:

$$S_{i,j}(S_{i-1,j} + S_{i+1,j} + S_{i,j-1} + S_{i,j+1}) < 4. \quad (21)$$

This method is very effective in the compute of the critical curve as the grid is refined since it allows to reduce the range in which each critical point can be.

In figure 11 is the process flow diagram for the estimation of critical curves with Gallenspy, it is important to highlight that the error range is low due to the grid has a refinement of 100X100 points. From this perspective, the set of critical points with a percentage error of 0.1 is estimated as those points $S_{i,j}$ in which the condition of equation (21) is met.

When the critical points are computed by Gallenspy, the obtention of the caustic points is an easy process because with the application of the equation (6) is enough.

Going back to the example of SIS profile of the previous section, the critical curve was obtained with Gallenspy where the result is shown in figure 12.
This graph evidences that the critical curve belongs to a circle with 6.278 arcs of the radius. In this case, it is important to consider the analytical solution to the SIS profile for \( \det A = 0 \), where the critical curve is a circumference of radius \( r \) given by the relation

\[
r = \frac{4\pi\sigma^2 D_{LS}}{c^2 D_{OS}},
\]

where \( \sigma \) is the mass density of the SIS profile.

Base on the data provided above regarding to the SIS, the radius obtained of the equation (22) is of 6.283 arcs and this is very close to the obtained result with \textit{Gallenspy}, where it is possible to check a percentage error of 0.1 in this numerical process.

In this way, in the analytical solution of the SIS concludes that the caustic curve in this mass profile belongs to a point in the origin of the plane \((\beta_1, \beta_2)\), this is evidenced in the figure where the source is near to this caustic point and for this reason the magnification of the images is significant.

6. GALROTPY

\textit{Galrotpy} is an interactive tool whose work is focused on the visualization and exploration of parameters through MCMC, in such a way that it is possible to make mass reconstructions from the fitting of rotational curves in disc-like galaxies.

The main python packages used in this routine are: \texttt{matplotlib} for the generation of graphic environment, \texttt{numpy} in the data management, \texttt{astropy} which is useful for the units assignation, \texttt{Galpy} for the construction of rotational curves with each mass profile, \texttt{emcee} used in the exploration and fitting of data with MCMC, and \texttt{corner} for the reliability regions of the parametric fitting.
The parametric exploration space in this routine is the same as Gallenspy due to the reasons given in the previous section. On the other hand, the rotational velocity data from which are done the parametric fitting should be consigned in a file denominated \texttt{rot\_curve.txt}, in which the units belonging to the radial coordinates must be expressed in Kpc and the velocities in Km/s. In the figure 14a is shown the panel for the selection of gravitational potentials in Galrotpy, and below these are sliders with the ones, the parameters variation is done. Thus the user can do a visual fitting between the rotational curve and the observational data of rotational velocity (this is evidenced in figure 14b).

6.1. Bayesian statistics in Galrotpy

In the case of Galrotpy the MCMC has similar characteristics with Gallenspy even in the consideration of the prior and in their approach to evaluation of the likelihood.

However, the process of parametric exploration with Galrotpy presents greater facilities due to the visual fitting that is possible to do with this routine and for this reason, the parametric minimization done with Gallenspy is not necessary.

In this way, with the visual fitting of parameters Galrotpy proceed to run the MCMC where as in Gallenspy the user can choose the number of walkers and steps. It is important to point out that in this routine, the likelihood es given by the relation

\[
L = \exp \left( -\frac{1}{2} \sum_{i=1}^{N} \left[ \frac{v_{\text{obs}}^i - v_{\text{modelo}}^i}{v_{\text{error}}^i} \right]^2 \right),
\]

with:

- \( N \) the number of observationally obtained data.
- \( v_{\text{obs}} \) is the velocity observed.
- \( v_{\text{modelo}} \) the rotational velocity of the mass model chosen for the fitting.
• $v_{\text{error}}$ the error in the observational velocity data.

Once Galrotpy do the parametric exploration, the behavior of the MCMC is illustrated and the values with these uncertainties of 68% and 95% are shown. Finally Galrotpy generates two types of graphics, one in which is the fitting rotation curve and other where are the reliability regions.

6.2. Mass reconstruction of galaxy M33 with Galrotpy

M33 is a spiral galaxy without bar structure [1], and its data of rotational velocity were obtained from Corbelli et al. [20]. For the mass reconstruction with Galrotpy, it is important to point out that the parametric exploration was done with a number of 100 walkers and 3000 steps.

![Figure 15: Behavior of the MCMC in the exploration of parameter $h_r$.](image)

In figure 15, is presented the way of the MCMC in the exploration of parameter $h_r$, where the convergence is given in a number of steps less than in Gallenspy, this thanks to the visual fitting of Galrotpy.

![Figure 16: Rotation curves and reliability regions of the Galaxy M33 with Galrotpy, where the fitting was done with NFW profile for the dark matter halo and Exponential Disc in the case of baryonic matter.](image)

The figure 16 showed the fitting done with Galrotpy, where the NFW profile was used for the dark matter halo while the contribution of baryonic matter was analyzed with the Exponential Disc profile. In the right side of this figure are the reliability regions, and these values are consigned in the table 3.

These values obtained and its uncertainties are in concordance with the results reported by López Fune et.al. [21], where $M_*(X10^9 M_\odot) = 4.9 \pm 1.5$ for the Exponential Disc and $M_h (X10^{11} M_\odot) = 5.4 \pm 0.6$ in the case of dark matter halo. With this example, it was possible to show a set of results with high reliability in the use of this routine, and for this reason in the next section Galrotpy and Gallenspy are combined for the mass reconstructions of two disc-like galaxies.
Table 3: Estimated parameters with GalRotsy for the galaxy M33.

| Parameter                          | 68%          | 95%          |
|-----------------------------------|--------------|--------------|
|                                   |              |              |
| **Exponential Disc**              |              |              |
| $b_r$ (Kpc)                       | $1.52^{+0.10}_{-0.11}$ | $1.52^{+0.20}_{-0.23}$ |
| $\Sigma_0$ $(X10^2 \, M_\odot \, pc^{-2})$ | $2.50^{+0.37}_{-0.43}$ | $2.50^{+0.66}_{-0.99}$ |
| $M_*$ $(X10^9 \, M_\odot)$       | $3.61^{+0.96}_{-0.91}$ | $3.61^{+1.89}_{-1.74}$ |
| **NFW**                           |              |              |
| $a$ $(X10kpc)$                    | $1.46^{+0.42}_{-0.57}$ | $1.46^{+1.02}_{-2.19}$ |
| $M_0$ $(X10^{11} \, M_\odot)$    | $2.37^{+0.55}_{-2.56}$ | $2.37^{+0.94}_{-0.91}$ |
| $\rho_0$ $(X10^6 \, M_\odot Kpc^{-3})$ | $6.05^{+1.11}_{-1.17}$ | $6.05^{+1.98}_{-2.35}$ |
| $M_h$ $(X10^{11} \, M_\odot)$    | $4.16^{+0.72}_{-0.72}$ | $4.16^{+1.21}_{-1.24}$ |

7. Mass Reconstruction of galaxies J2141 and J1331

The galaxies SDSSJ2141-001(J2141) and SDSSJ1331+3628(J1331) are systems that present the strong lensing effect, and its rotation velocities data were given by Dutton et.al. [3, 5]; for these mass reconstructions were taken into account the profiles of Miyamoto-Nagai, Exponential Disc, and NFW [8, 1]. Regarding to the GLE, in the case of J2141 it was modeled an extended circular source due to the deflected image is an arc, while for J1331 it was considered a punctual source in which four images are produced in the lens plane.

As to the mass distribution of these galaxies, other authors have reported a high contribution of baryonic matter [5, 3] and this coincides with the obtained results in the use of Galrotpy and Gallenspy.

7.1. Galaxy J2141

J2141 is a spiral galaxy of type S0, where its dominant gravitational contribution is from the disc [5]. This object was initially observed in 2006 by mean of the Hubble Spatial Telescope(HST) [5], with a camera ACS in a filter F814 and an exposure time of 420 seconds. In 2009 the Keck telescope had again images of these object with a camera NIR and K filter.

From these images the GLE in this galaxy was evidenced, with the formation of an arc belonging to a source with a redshift different to J2141 ($z_L = 0.3180, z_s = 0.7127$) [5], and for this reason these data were considered important to study the mass distribution of this galaxy.

Figure 17: Images obtained of J2141 by mean of HST and KeckII telescopes with distinct filters [5].

The rotational velocity data of this galaxy were derived of spectral lines Mg b 5177, Fe II5270, Na D 5896, O II 3727 and $H_\alpha$ 6563, obtained of the Keck telescope. In figure [18a] the inclination of these emission lines are shown for the case of $H_\alpha$ 6563.

In table 4 are consigned the rotational velocity data for different values of galactocentric radius.

Figure 18: Images obtained of J2141 by mean of HST and KeckII telescopes with distinct filters [5].

The rotational velocity data of this galaxy were derived of spectral lines Mg b 5177, Fe II5270, Na D 5896, O II 3727 and $H_\alpha$ 6563, obtained of the Keck telescope. In figure [18b] the inclination of these emission lines are shown for the case of $H_\alpha$ 6563.

In table 4 are consigned the rotational velocity data for different values of galactocentric radius.
7.1.1. Data concerning the GLE

For the coordinates of the arc in the lens plane ($\theta_1, \theta_2$), the restriction of its contours was made in the way of lower the computational cost in Gallenspy.

This image treatment was made in python, through a pixel to pixel discrimination based on its position and luminosity, which can be possible to get the contour showing in figure 19.

As it is possible to observe, this obtained contour is incomplete given the noise of image 18a, even so, the number of gotten coordinates was enough for an appropriate adjustment with a circular source.

In this case the equivalence between arc seconds and pixels was made based on the observed scale of figure 18a, and in this way it was possible to get each coordinate of the image in radians as observed in figure 19.

For the determination of cosmological distances, the ΛCDM model was taken into account due to the fact that this one has been used by other authors in this galaxy [5]; in this way the current matter density is $\Lambda_m = 0.3$ and the Hubble parameter $H_0 = 70 \text{ km/s Mpc}^{-1}$.

| Radio(arcsec) | Radio(kpc) | Velocidad(km/s) | Error(km/s) |
|---------------|------------|-----------------|-------------|
| 0.000         | 0.00       | 3.5             | 5.3         |
| 0.593         | 1.45       | 114.1           | 5.8         |
| 1.185         | 2.89       | 153.8           | 2.9         |
| 1.778         | 4.33       | 212.7           | 2.6         |
| 2.370         | 5.78       | 243.8           | 2.6         |
| 2.963         | 7.22       | 259.8           | 2.3         |
| 4.148         | 10.11      | 254.9           | 7.5         |
| 4.740         | 11.56      | 263.4           | 2.3         |
| 5.333         | 13.00      | 265.9           | 3.5         |

Table 4: Rotational velocity values obtained of Dutton et.al. [5] for J2141.
Figure 19: Contours of arc generated by GLE in J2141. The scale of this plane is in $1 \times 10^6$ radians.

With this considerations, the cosmological distances given by Dutton et al. [5] are $D_{OL} = 497.6\, Mpc$, $D_{OS} = 1510.2\, Mpc$ and $D_{LS} = 1179.6\, Mpc$; what leads to $\Sigma_{\text{crit}} = 4285.3\, M_{\odot}pc^{-2}$.

In relation to the source, its ranges of possible values for the position and the radius are consigned in table 5.

| Parameter   | Range   | Units  |
|-------------|---------|--------|
| Radius      | $0.05<r<1.5$ | arcs  |
| x-center    | $-2.5<h<2.5$ | arcs  |
| y-center    | $-2.5<k<2.5$ | arcs  |

Table 5: Range of values to explore for the source.

7.1.2. Mass reconstruction of J2141.

In this mass reconstruction, the profiles selected of table 1 were Bulge I, Exponential Disc, and NFW. The first parametric exploration was done with Galrotpy, where the more reliable result for the MCMC was obtained for a number of 100 walkers and 1500 steps so that the parametric exploration roads are shown in figures 20, 21 and 22.

Figure 20: Exploration roads for the parameters of the Miyamoto-Nagai profile.
In this graphics, the initial values of the MCMC are in red lines, where for the dark matter halo and bulge is not evidenced a convergence of the values; this is pointed out by Dutton as the problem of degeneracy among the disc and different mass components of the galaxy.\[5\]

These authors, \[5\] affirm that the mean reason of this degeneracy is the gravitational dominance of disc in this system, and for this cause, it is possible to adjust the rotation curve only with the profile of this mass component and therefore it is very difficult to know with clarity the circular velocity contribution of other components.

Because of this situation, this mass reconstruction was done through the integration of galactic dynamics and GLE, where the restrictions of each method occur in different geometries and therefore this combination is a powerful tool for the breaking of this degeneracy.

In this way and with the arc coordinates, the parameters of the source were estimated as shown in figure 23a under a lens model of Exponential Disk, where these values are presented in table 6 with a radius of the source of 0.03arcs.

| Parameter | 68%       | 95%       |
|-----------|-----------|-----------|
| $h$ ($10^2$arcs) | $3.56^{+1.102}_{-1.108}$ | $3.56^{+2.336}_{-2.416}$ |
| $k$ ($10^3$arcs) | $5.938^{+2.372}_{-2.470}$ | $5.938^{+4.384}_{-4.840}$ |

Table 6: Source values estimated by Gallensepy for the case of J2141.
(a) Estimation of circular source with Gallenspy for the galaxy J2141.  
(b) Deflected image and Einstein ring (scale in arcsec).

Figure 23: Compute of Gallenspy for source parameters (left side) and Einstein radius (right side).

With the position and size of the source, the mass reconstruction of J2141 based on the GLE was done with the restrictions obtained of the rotation curves in relation with the parametric space. For this specific case, the parametric exploration established was three times of the table 1.

The combination of lensing and Galactic Dynamics was a process very efficient in the breaking of the degeneracy between mass components of J2141, and this allowed a mass density minor value for the disc as to is shown in figure 24. Based on the observed, it is important to point out, how the combination of Galropy and Gallenspy is a great alternative for galaxies where the gravitational contribution of each mass component is not easy to distinguish.

In table 7 the parameters value and its uncertainties are consigned, where the parameters with major dispersion belong to NFW profile, which is related to the degeneracy of this system.

| Parameter                  | 68%          | 95%          |
|----------------------------|--------------|--------------|
| $a$ (kpc)                  | $47.653^{+0.020}_{-0.016}$ | $47.653^{+0.036}_{-0.026}$ |
| $\mu_0 (X10^{11} M_\odot)$ | $2.171^{+0.120}_{-0.107}$ | $2.171^{+0.140}_{-0.129}$ |

Exponential Disc

| $h_e$ (Kpc)         | $29.312^{+0.020}_{-0.018}$ | $29.312^{+0.036}_{-0.026}$ |
| $\Sigma_0 (X10^9 M_\odot pc^{-2})$ | $4.368^{+0.012}_{-0.012}$ | $4.368^{+0.014}_{-0.014}$ |

Miyamoto-Nagai

| $b$ (Kpc) | $1.778^{+0.010}_{-0.006}$ | $1.778^{+0.016}_{-0.012}$ |
| $M (X10^9 M_\odot)$ | $1.492^{+0.376}_{-0.414}$ | $1.492^{+0.376}_{-0.360}$ |

Table 7: Values of parameters obtained in the mass reconstruction for J2141.
Figure 24: Contours obtained through the combination of restrictions between Galropy and Gallempy for the galaxy J2141.
Based on the values of these parameters, we computed the Einstein ring and Einstein radius ($\theta_{\text{Eins}}$). In figure 23b, this curve obtained by Gallenspy is evidenced, where the Einstein radius $\theta_{\text{Eins}}$ presented a value of $0.943^{+0.128}_{-0.144}$ respectively.

The compute of enclosed mass for J2141, was done taking into account the relation:

$$M = \frac{\Sigma_{cr}}{2} \int_S \nabla^2 \psi(\theta_1, \theta_2) d^2 \theta,$$

such that in table 8 are these estimated values.

| Parameter   | $\log_{10} \left( \frac{M}{M_\odot} \right)$ |
|-------------|-----------------------------------------------|
| $M_{\text{Eins}}$ | 10.906$^{+0.030}_{-0.100}$                  |
| $M_{\text{bar,Eins}}$ | 10.905$^{+0.023}_{-0.027}$                 |

Table 8: Enclosed mass with Einstein radius, M is the total mass and Mbar the baryonic matter.

These results are in concordance with the values range given by Dutton et al. [5], where they reported for J2141 a value of $\log_{10} \left( \frac{M_{\text{bar}}}{M_\odot} \right) = 10.99^{+0.11}_{-0.25}$ within the Einstein radius.

The results gotten in this work shown separately mass estimation of the bulge and disc, to difference with results of Dutton et al., where they obtain the stellar mass without discriminating each contribution of these baryonic matter components. These mass values are shown in Table 9 and the fitting made to the rotation curve and arc generated in the GLE with these results, it is illustrated in images 25a and 25b.

| Component of the galaxy | $\log_{10} \left( \frac{M}{M_\odot} \right)$ |
|-------------------------|-----------------------------------------------|
| Bulge                   | 8.004$^{+0.014}_{-0.009}$                     |
| Disc                    | 10.905$^{+0.034}_{-0.028}$                   |
| Dark Matter Halo        | 7.740$^{+0.009}_{-0.007}$                    |

Table 9: Mass values for each component of J2141.

In the fitting of the rotational curve, it is possible to evidence how the dark matter halo is dominant gravitationaly in a radius minor to 1.5Kpc, this observation could be done due to the combination of GLE and galactic dynamics, since the generated arc is in a near radius to this galaxy zone and therefore the combination of restrictions between Gallenspy and Galrotpy is a great option.

### 7.2. Galaxy J1331

SDSSJ1331+3628 (J1331) is a spiral galaxy with a counter-rotating massive core [3], where just like J2141, the inclination of this galaxy allows to get its rotational velocity values in function of the galactocentric radius.

J1331 is localet in RA = 202.9188° and DEC = 36.469990°, and in the observation of this system Treut et.al (2011) [22] observed 2 distinct redshifts within a radius of 1 arcs ($z_L = 0.113, z_s = 0.254$), given the GLE in this galaxy.

The images of this galaxy, were obtained by mean of the SWELLS(Sloan WFC Edge-on late-type lens survey)/[WFC-Wide Field Camera] [23]: In figure 26 these images are illustrated with the HST telescope in the F450W and F814W filters, where it is evidenced the high size of its core, and for this reason in the right side it is evidenced a slit done by Trick et. al [3], which they reconstructed the brightness surfaces.
(a) Comparison of the observational data and lens model data for a circular source in galaxy J2141 with \textit{Gallenspy}, in this case the lens model choose let us obtained a set of images which overlap the observational images.

(b) Fitting of rotation curve with the model data.

Figure 25: Fitting obtained for lensing and rotation curves through the combination of restrictions between \textit{Gallenspy} and \textit{Galrotpy}.

The produced images for the GLE are shown in figure 27\textit{a} which are pointed with letters A, B, C y D, also it is important to clarify that the other 3 unlabeled images do not belong to this group, since according to what indicates Trick et.al\textit{[3]} these are part of a stellar formation region.

Regarding to the dynamics aspect of J1331, Dutton et al.(2013) got its rotational velocity values with the use of Keck I telescope by mean of a spectrograph LRIS (Low Resolution Imaging Spectrograph)\textit{[?]}, where these data were obtained of the absorption lines $M\_gb$ (5177 Å), $Fe\_I\_f$(5270,5335 Å) and $Fe\_I\_f$ (5406 Å), while the gas velocity was estimated with emission lines $H\_a$ (6563 Å) and $N\_II$ (6583 Å).

An important aspect of J1331 is its supermassive core, since due to the exposed by other authors \textit{[3]} about half the brightness is enclosed in the effective radius illustrated in figure 27\textit{b}. For this reason, this galaxy has been interest object in different works \textit{[22, 24]}, where it is speculated a possible merger event in the past of this system which changes its structure and kinematics.

\textbf{7.2.1. Mass reconstruction based on the GLE}

Unlike J2141, the mass profiles for J1331 within its core can not be used by \textit{Galrotpy}, due to the rotational negative velocity values present in this part of the galaxy.

Because of this, the galaxy region enclosed in the effective radius was analyzed totally with lensing, such that the only restrictions applied in \textit{Gallenspy} are in the parametric ranges used with \textit{Galrotpy} for the fitting of the rotation curves in close radii to the galaxy periphery.

In this way, other authors allow watching how the breaking of the degeneracy is not an easy task \textit{[3, 4]}, and even if different advances had been obtained this objective has not been achieved yet.

The observational data of the images (A-D) are consigned in table 10 where it was necessary to express these coordinates in arcs for a scale 1 pixel = 0.05 arcs.

For the determination of the cosmological distances, the redshifts were taken into account in the numerical solution to the Dyer-Roeder equation \textit{[10]}, which \textit{Gallenspy} made based on the Jimenez code \textit{[25]}. For this
Figure 26: Images of J1331 obtained with the HST telescope in F450W and F814W filters. (Image take of Trick et.al [3])

(a) Quadruplet of images formed through the ELG for a point source, in this case G is the galactic center. (Image take of Trick et.al [3]).

(b) Rotational velocity values of J1331, where the effective radius is distinguished by Trick et.al with 2.6 arcs, within which is a supermassive and counter-rotating core [3].

Figure 27: Observational data of lensing and rotational velocities for the galaxy J1331.

In the case, it was used the cosmological model ΛCDM where the obtained results were $D_{LS} = 442.7 \times 10^3$Kpc, $D_{OL} = 422 \times 10^3$Kpc and $D_{OS} = 817.9 \times 10^3$Kpc.

The next step in Gallenspy was the estimation of the source position, and this obtained values are in table 11. In the parameters exploration for the mass reconstruction, the established ranges of the bulges I and II belonging to the table 1 were not enough for the fitting of the observational images, and this is shown in figure 28. For this reason a very massive bulge was considered, where the selected most appropriate profile is of the Miyamoto-Nagai with parametric exploration ranges of thick disc evidenced in table 1.
This process was done in Gallenspy with 100 walkers and 100 steps in the MCMC, in such a way that in figure 29 these contours are shown.

Under this estimation, the parameters set more appropriate for the mass distribution of J1331 obtained with Gallenspy is in table 12 and these values allow getting the fitting of figure 30b.

| Parameter                  | 95%         | 68%         |
|----------------------------|-------------|-------------|
| $a$ (Kpc)                  | 8.131±0.874 | 8.131±0.874 |
| $m_0$ (X10^{11} M_{\odot})| 5.734±0.874 | 5.734±0.874 |
| $h_t$ (Kpc)                | 9.902±0.700 | 9.902±0.700 |
| $\Sigma_0$ (10^{9} M_{\odot}Kpc^{-2}) | 7.487±0.471 | 7.487±0.471 |
| $b$ (kpc)                  | 4.647±0.471 | 4.647±0.471 |
| $a$ (kpc)                  | 2.826±0.471 | 2.826±0.471 |
| $M$ (10^{10} M_{\odot})   | 7.542±0.471 | 7.542±0.471 |

Table 11: Source position obtained with Gallenspy for the case of the GLE in J1331.

Table 12: Parameters set obtained with Gallenspy for J1331.

With this mass distribution, the critical and caustic curves and the Einstein ring are presented in figure 31a, 31b, and 30a where the Einstein radius and critical radii have values of 0.916arcsec, 0.280arcsec, and 0.410arcsec respectively. It is important to point out, that with this lens model was possible to estimate the mass within the effective radius under which Trick et al. got restrictions for the mass estimation through the luminosity of J1331.

In table 13 the mass values restricted by the Einstein radius are consigned; when it is doing a review of the results reported by Trick et al. under the lens model that they assumed, it is found what the Einstein radius estimated is 0.91±0.02arcsec with an enclosed mass of 7.8±0.3X10^{10}M_{\odot}, and this being in concordance with the results obtained in this work with Gallenspy.

Regarding the restriction within the critical radius, the values obtained of baryonic and dark matter are 2.190±0.205X10^{11}M_{\odot} and 2.179±0.288X10^{11}M_{\odot} respectively, these results are also consistent with the reported by Trick et al., where for the effective radius these values are 2.352±0.2X10^{11}M_{\odot} for the total mass and 1.970±0.39X10^{11}M_{\odot} of baryonic matter, also it should be clarified that these authors obtain their results with alternative methods to the GLE of J1331.
The results obtained with **Gallenspy**, show that this is a very effective tool for the mass reconstructions within the critical curve and Einstein radius. However for J1331, the estimation is not enough with radii greater than 2.6 arcs, and therefore in this case was used **Galrotpy**.

In other’s works [3, 4] is evidenced how the mass reconstructions for J1331 from a dynamics analysis have many complications due to the complexity of its rotation curve. It is the reason which Trick et al. [3] restricted this mass reconstruction to the effective radius, while Dutton et al. in 2013 [4] dedicated efforts in studying the periphery of the galaxy. Based on the exposed, each routine in this work was applied separately in different galaxy regions.

### 7.2.2. Mass reconstruction of J1331 with Galrotpy

The best result in the fitting of the rotation curve with **Galrotpy** was for a number of 20 walkers and 100 steps, wherein the image 32 the contours of each parameter obtained are presented and the figure 33 the curve obtained for these observational data.

In table 14 are presented these values and its uncertainties for each parameter. The estimated mass distribution with these indicated parameters was restricted to 7.56 arcs (in this radius are all data of rotational velocity), and therefore the enclosed mass in this amplitude was estimated in $\log_{10}\left(\frac{M}{M_\odot}\right) = 11.448^{+0.224}_{-0.131}$ where the baryonic matter has a value of $\log_{10}\left(\frac{M}{M_\odot}\right) = 10.898^{+0.303}_{-0.164}$.

The results given by Dutton et al. in 2013 [4] indicate that the baryonic matter in this radius is of $\log_{10}\left(\frac{M}{M_\odot}\right) = 11.03 \pm 0.07$ which is in concordance with the result obtained through of **Galrotpy**. Also, it is important to note, the great relation in the estimation of the bulge mass, where they report a value of...
Figure 29: Reliability regions of obtained parameters in the case of J1331.

(a) Einstein ring obtained with the mass distribution of J1331, the lens plane is in an arcs scale.

(b) Adjustment obtained with Gallenspy for J1331, where it is important to highlight how the model images are very closed with the observational images for the lens model choose.

Figure 30: Comparison of observational images with Einstein ring and lens model images computed in Gallenspy.
(a) Critical curves obtained with the mass distribution of J1331, the lens plane is in an arcsec scale.

(b) Caustic curves obtained with the mass distribution of J1331, the source plane is in a scale of 1X10^{6} radians.

Figure 31: Critical and caustic curves obtained with Galle spy for the lens model selected.

Figure 32: Credibility regions of obtained parameters with Galroty for J1331.
Table 14: Estimated parameters with Galrotpy for J1331 galaxy.

\[
\log_{10} \left( \frac{M}{M_\odot} \right) = 10.89 \pm 0.10 \text{ and in this work the obtained value is } \log_{10} \left( \frac{M}{M_\odot} \right) = 10.885^{+0.249}_{-0.520}.
\]

7.2.3. Analysis of the mass reconstruction for J1331

In the mass reconstructions performed for this galaxy with Galrotpy and Gallenspy, it follows that about 87% of the mass of J1331 is enclosed in the effective radius, this confirmed the presence of a supermassive core, which to the presenting a negative direction in its rotation opens the possibility to think that this galaxy is the result of a merger process between two stellar systems, with angular momentum oriented in distinct orientations.

Also it is important to mention the high effectiveness of Galrotpy in this process, where the obtained results for radii close to the galaxy periphery were very successful in comparison with the results of Dutton et al. in 2013 [4], all this taking into account that these estimations were done with the fitting from just 3 rotational velocity values.
Besides, it is important to remember that the degeneracy between the disc and halo is still a research topic [3, 4], and for this reason, the possibility of adjusting \textit{Galrotpy} for negative values of the rotational velocity is open, since this would allow combining lensing and galactic dynamics for similar galaxies to J1331.

8. HE 0435-1223 test case

An additional case of tested for \textit{Gallenspy} was the Quasar HE 0435-1223, in which is presented the GLE through a quadruply imaged belonging to a background source [26]. HE 0435-1223 was discovered by Wisotzki et al. (2000) and from there has been research object in distinct works [26, 27].

![Figure 34: Quadruply imaged formed through the GLE on the case of quasar 0435-1223. (Image take of Courbin et al. (2011)[26].)](image)

In figure 34 are shown these formed images by mean of the GLE, where the redshifts of the lens and source are $z_s = 1.689$ and $z_L = 1.4546$ respectively, with a value of 6.666 Kpc for the Einstein radius.

Based on these redshifts value, the cosmological distances estimated by \textit{Gallenspy} are $D_{ds} = 1070.3 Mpc$, $D_d = 1163.3 Mpc$ and $D_s = 1700.4 Mpc$. Also it is important to point out, that the positions of the images formed in the GLE were obtained of Courbin et al. (2011) [26], and of this manner it was possible to perform the mass reconstruction for this quasar with the Exponential Disc and NFW profiles, where the estimated parameters are in Table 15.

| Parameter | 95% | 68% |
|-----------|-----|-----|
| $a$ (Kpc) | 52.792$^{+0.133}_{-0.071}$ | 52.792$^{+0.122}_{-0.054}$ |
| $m_0$ ($10^{-11} M_\odot$) | 19.961$^{+0.037}_{-0.135}$ | 19.961$^{+0.030}_{-0.054}$ |
| $h_r$ (Kpc) | 11.999$^{+0.0003}_{-0.001}$ | 11.999$^{+0.0003}_{-0.0007}$ |
| $\Sigma_0$ ($10^2 M_\odot pc^{-2}$) | 2.999$^{+0.0004}_{-0.0025}$ | 2.999$^{+0.0003}_{-0.0007}$ |

**Table 15: Values of the obtained parameters with Gallenspy**

With these parameters, the obtained images are illustrated in figure 35, where the values of baryonic and dark matter are consigned in table 15. The results given by Courbin et al. (2011) for this system, reveal that the total mass of this quasar is of $3.16 \pm 0.31 X 10^{11} M_\odot$ which is very close to the obtained value in this work, other important aspect is the matter baryonic fraction which in this work is of $0.764 \pm 0.15$ while Courbin
et al. (2011) reported $0.65^{+0.13}_{-0.10}$ with the Sapelter IMF; in this way it is possible to confirm as Gallenspy is an efficient tool.

| Mass         | Value $\left(1\times10^{11} M_\odot\right)$ |
|--------------|-----------------------------------------------|
| Baryonic     | $2.395^{+0.0031}_{-0.0030}$                   |
| Dark         | $0.618^{+0.001}_{-0.0031}$                    |
| **Einstein Mass** | $3.014^{+0.066}_{-0.069}$                  |

Table 16: Mass Values within Einstein radius.

9. Conclusions

In this work was presented the Gallenspy code, which is very useful in mass reconstructions based on GLE; in this way it is important to highlight, the manner which this routine allows to obtain the mass distribution of bulge and disc separately unlike to the methods of reconstruction applied by other authors [5, 3, 4].

Also, the advantages of combining Lensing and Galactic Dynamics were illustrated with the use of Galrotpy and Gallenspy, in this case, with the restrictions given by each routine, it was possible to have significant progress in the breaking of the degeneracy in J1331 and J2141. Additionally, for J1331 was used Gallenspy in the mass reconstruction within the critic radius, while with Galrotpy the peripheral region was analyzed and although this degeneracy not could be break completely, the estimated parameters have concordance with the obtained results of other authors [3, 4], and this gives reliability to these routines constructed.

On the other hand, it is important to highlight the use of mass models with spherical symmetry, the ones are used by distinct authors [27, 26], and which allows getting very good results as in mass reconstructions as in estimations of Hubble parameter.

Regarding future improvements for Gallenspy, is considered the increase in the number of mass profiles used in this routine, besides there is the possibility of extending this code for reconstructions of superficial
brightness functions in lens galaxies, like the estimation of temporary cosmological delays for the study of the universe expansion.

Finally it is important to mention the advantages of performing visuals fitting with Galropy in the rotation curves, since through this process it is possible get the initial values set for the MCMC in both routines. For this reason, a step to follow with Galenpspy is related with its optimization, in such a way that in this code would be possible to make interactives fitting, the ones allows that in similar galaxies to J1331 can be given restrictions from the GLE for its dynamical analysis.

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