Manifestation of one- and two-body currents in longitudinal and transverse response functions of the $^{12}$C nucleus at $q = 300$ MeV/c

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Abstract The experimental values of longitudinal and transverse response functions of the $^{12}$C nucleus have been obtained at the 3-momentum transfer $q = 300$ MeV/c. The data are compared with the calculations made with due regard to the dynamics of all the nucleons constituting the $^{12}$C nucleus, and also, to the contributions of both the one-body currents only, and their combination with two-body currents.

1 Introduction

The calculations of the longitudinal $R_L(q, \omega)$ and transverse $R_T(q, \omega)$ response functions, which are in good agreement with experiment, were performed for several nuclei (see Refs. [1–4]). In the present work, the calculations of $R_L(q, \omega)$ and $R_T(q, \omega)$ of the $^{12}$C nucleus obtained in Ref. [5] are tested by experimental data.

In paper [5], the calculations for the response functions were performed on the basis of the AV18+IL7 combination of two and three-nucleon potentials and accompanying set of two-body electromagnetic currents. The Green’s function Monte Carlo methods and maximum-entropy techniques were used in the calculation. In case of the longitudinal response function, the consideration of contributions from one-body currents only, or from a combination of one- and two-body currents, causes an insignificant change in $R_L(q, \omega)$ only in the vicinity of the threshold. However, since the two-body currents generate a large excess of strength in $R_T(q, \omega)$ over the whole $\omega$-spectrum, the comparison with the experimental data could be a good test of the calculations.

The calculations of response functions in Ref. [5] were compared with the experimental response functions of $^{12}$C, determined from the world data analysis of Jourdan [6,7] and, for $q = 300$ MeV/c, from the Saclay data [8,9]. The data of the mentioned works differ widely. In view of this, it should be noted that the experimental data on the functions $R_L(q, \omega)$ and $R_T(q, \omega)$ of the $^{12}$C nucleus were obtained in Saclay [8,9] at constant momentum transfers $q$ ranging from 200 to 550 MeV/c. In his papers [6,7], Jourdan has reanalyzed the primary data from Refs. [8,9] and the measured data obtained at SLAC [10–12], which were then used for determining the “world” response function values of the $^{12}$C nucleus. However, not all researchers were content with the results of the reanalysis [6,7]. For example, Morgenstern and Meziani have carried out their own reanalysis of the experimental data for a variety of nuclei, and demonstrated [13] that the results changed only insignificantly with the combination of the SLAC and Saclay data.

It follows from the above that for testing the calculations of Ref. [5], there is a need to use other experimental data on the response functions of the $^{12}$C nucleus, which would be independent of the ones in Refs. [6–9]. These data are derived in the present work and are used for comparison with the calculations [5].

2 Experimental procedure

The experimental response functions of the present work were obtained from processing the spectra of electrons scattered by the $^{12}$C nucleus, which were measured at the NSC KIPT linac LUE-300. We have used here eight spectra, the characteristics of which are given below in Table 1. We measured the spectra over the entire range of energy transfer by taking measurements at successive spectrometer central momenta. We combined the individual measurements following the procedure of Ref. [14] (see p. 1272). By way of illustration, two of the spectra are shown in Fig. 1.

In experiments, the measurement conditions and the characteristics of the facility were as follows.

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Table 1 Kinematic conditions for measuring the spectra of scattered electrons. $\omega_{el}$—elastic scattering peak position, $\omega_{\text{max}}$—maximum spectrum length

| $E_0$, MeV | $\theta$, $^\circ$ | $\omega_{el}$, MeV | $\omega_{\text{max}}$, MeV |
|------------|------------------|------------------|------------------|
| 149.30     | 140              | 3.45             | 94               |
| 166.65     | 140              | 4.30             | 112              |
| 196.60     | 140              | 5.90             | 125              |
| 207.65     | 140              | 6.60             | 125              |
| 259.70     | 67.30            | 3.70             | 127              |
| 259.30     | 74               | 4.30             | 127              |
| 259.60     | 82               | 5.10             | 125              |
| 260.00     | 90               | 5.90             | 126              |

The current pulse frequency of the electron linac LUE-300 amounted to 50 Hz, the pulse length being 2 $\mu$s. The current value of the electron beam incident on the target was measured to be about 0.2 $\mu$A, and the beam diameter on the target was 0.2 cm. The charge that passed through the target per exposure was determined with a secondary-emission monitor, which was calibrated against the Faraday cup. The charge measurement accuracy was no worse than 1%. In the experiment under discussion, there was no need for high energy resolution. Therefore, for purposes of increasing the current and reducing the radiation background in the experimental hall, the monochromator was in the open state during the measuring run, and the beam monochromaticity was determined to be 0.5% FWHM in this case. Two graphite discs, of thicknesses 0.1460 g/cm$^2$ and 0.2872 g/cm$^2$, were used as nuclear targets. The targets were arranged so that the normal to the discs’ plane bisected the scattering angle. For measurements at small angles ($\theta_1 \leq 90^\circ$) the thin disc was used, while the thick disc was used for large-angle measurements ($\theta_2 = 140^\circ$). A part of electrons scattered by the $^{12}$C nucleus comes to the spectrometer’s collimator having the acceptance angle $d\Omega = 2.89 \times 10^{-3}$ sr. The spectrometer’s magnet produces a uniform magnetic field having the second-order focusing in the horizontal and vertical planes. The spectrometer resolution was determined to be 0.11%, the dispersion in the focal plane being 1.37 cm/%, and the momentum acceptance $\pm$ 5% of the central electron trajectory in the spectrometer (for more detail, see Ref. [15]).

The electrons are registered by eight detectors located in the focal plane of the spectrometer, spaced at intervals of 0.46% and having the energy acceptance of 0.31%. After passing through the scintillators, the electrons hit the organic-glass Cherenkov detector. The pulses from the photomultipliers of the scintillation/Cherenkov detectors are registered as the channel occupancy and are passed to the coincidence circuit with the time resolution of 9 ns.

In order that the measured spectra could be used in nuclear physics research, it is essential that the contributions from different background processes accompanying the measurements should be subtracted from the spectra.

The physical background (PB) represents detector registration of the electrons and gamma-quanta that originate from scattering of a part of the beam by structural elements of the facility. The PB value is determined in the measurements, where the target is removed from the beam trajectory. In each spectrum of the present experiment, the PB value was measured at a few exposures. In the region of the quasielastic scattering (QES) peak maximum, in the spectra taken at $\theta_2$ the PB contribution amounted to $\sim$1% of the effect, whereas in the spectra measured at small $\theta_1$, the PB contribution attained 10%.

The random coincidence background (RCB) stems from manifestation of the static probability of instrumental regis-
tration of two uncoupled pulses over the resolution time of the coincidence circuit. These pulses may come from both the particles of the spectrum measured, and the background particles, or the photo-electron multiplier noise. This results in the occurrence of spurious counts in the spectrum, which were taken into account with the use of the equations from Ref. [16]. The RCB value in the region of the elastic scattering peak and at the QES peak maximum amounted to 1% of the effect for the case of small scattering angles, while it reached 1.2% and 1%, respectively, for the large-angle scattering spectra.

Electron scattering by the target is accompanied by photoproduction of $e^+e^-$-pairs in the target substance. The electrons of the pairs give rise to an additional background. The latter is measured through reversing the spectrometer magnet polarity and registering the positron spectrum, which is identical to the spectrum of electrons from the $e^+e^-$-pairs. This measurement has been performed in the present experiment, yet the positrons could not be revealed. Perhaps, this might be due to their low yield under the measurement conditions of the experiment. To check the conclusion, we have estimated the positron yields by the calculation method of Ref. [17]. The estimates have demonstrated that the manifestation of the electron-positron background in our measurements is much lower than the experimental errors.

Subsequent to consideration of different background contributions, radiation correction (rad. correction) of the spectra was carried out, which took into account the electron energy losses for ionization of target atoms, and also, for the gamma emission in the interaction with the target substance or the scattering nucleus. The correction was made with the methods of Refs. [18,19], which offered two calculation variants: exact and approximate. The exact rad. correction calls for measuring a series of spectra at the same angle $\theta$ as that for the corrected spectrum. This is not necessary for the approximate variant.

We have applied the exact rad. correction to the available set of spectra measured at $\theta_2 = 140^\circ$, and the approximate variant—to the spectra measured at small angles. To evaluate the error introduced by applying the approximate rad. correction, this correction was applied to the $\theta_2 = 140^\circ$ spectra. The comparison between the data on applying two rad. correction variants has shown that the difference between them was about 2% on the average.

The analysis of rad. correction equations has shown that with a decreasing scattering angle, the differences in the calculations by the both rad. correction variants get reduced, too. In the case of small angles considered here, the difference under discussion may be estimated as 1.3% and less. Taking account of this error cannot change appreciably the sum of all other errors of the present results.

The measured data were normalized with the normalizing factor $k = F_2^2(q)/F_1^2(q)$, where $F_1(q)$ denotes our measured ground-state form factor values of the nucleus under study, and $F_2(q)$ stands for the data from Ref. [20]. In this case, consideration was given to the 3% correction for the data of Ref. [20], which was found in Ref. [21].

We extracted the response functions in the same manner as in Refs. [22,23]. According to Ref. [24], the longitudinal $R_L(q, \omega)$ and transverse $R_T(q, \omega)$ response functions of the nucleus are related to the double differential electron scattering cross-section $d^2\sigma/d\Omega d\omega$ by the expression

$$R_0(q, \omega) = \frac{d^2\sigma}{d\Omega d\omega}(\theta, E_0, \omega)/\sigma_M(\theta, E_0)$$

$$= \frac{q^4}{q^4} R_L(q, \omega) + \left[ \frac{1}{2} \frac{q^2}{q^2} + \tan^2 \frac{\theta}{2} \right] R_T(q, \omega).$$

(1)

Here $R_0(q, \omega)$ is the angular response function, $E_0$ is the initial energy of the electron scattered by the angle $\theta$, with the transfer to the nucleus of: energy $\omega$, the effective 3-momentum

$$q = \{4E_{\text{eff}}[E_{\text{eff}} - \omega] \sin^2(\theta/2) + \omega^2 \}^{1/2}$$

and the effective 4-momentum $q_\mu = (q^2 - \omega^2)^{1/2}$, $\sigma_M(E_0, \theta) = e^4 \cos^2(\theta/2)/[4E_0^2 \sin^4(\theta/2)]$ is the Mott cross-section, $e$ is the electron charge. The $E_{\text{eff}}$ value in the definition of the effective 3-momentum transfer denotes the effective energy, which is the sum of the initial energy $E_0$ and the correction $E_C$ that takes into account the influence of the nuclear Coulomb field on the incident electron. According to Ref. [25], this correction is written as $E_C = 1.33Ze^2 \langle r^2 \rangle^{-1/2}$, where $Z$ and $\langle r^2 \rangle^{1/2}$ are the nuclear charge and the r.m.s. radius, respectively.

Each $R_0$-function has the corresponding dependence of the momentum transfer on the energy transfer. The dependences corresponding to the $R_0$-functions used in the present work are shown in Fig. 2. As may be inferred from the figure, in the range of energy transfers under study the two angular response functions $R_0(q, \omega)$, which were measured at different scattering angles $\theta_1$ and $\theta_2$, have one pair of common arguments $q'$ and $\omega'$ (the intersection point of the dependences). If a system of two equations of form Eq. (1) is set up for the values of the functions $R_{\theta_1}(q', \omega')$ and $R_{\theta_2}(q', \omega')$, then the solution of this system will determine the values of the longitudinal $R_L(q', \omega')$ and transverse $R_T(q', \omega')$ response functions at the arguments $q'$ and $\omega'$. To obtain the functions $R_L(q, \omega)$ and $R_T(q, \omega)$ as a sequence of $\omega$-values, there is a need to carry out some interpolations.

The applicability of interpolations in the experiments aimed to determine the $R_{T/L}$-functions has been discussed in Refs. [14,22]. According to the authors, in the region of $\omega$-values, where the peaks of nuclear excited states are dominant, the lines of interpolation in the plane of the $q\omega$-
arguments must be of the form of \( \omega = \varepsilon_j + (q^2 - \omega^2)/(2M_n) \),
where \( \varepsilon_j \) is the nuclear excitation energy, \( M_n \) is the nuclear mass. At higher energy transfers, we have the following expression for the line of optimum interpolation \( \omega = C + K_j (q^2 - \omega^2) \), where \( C = \text{const} \), and \( K_j \) corresponds to different interpolation lines (see Fig. 2).

Interpolation between the angular functions \( R_\theta(q, \omega) \) requires the invariance of the angle \( \theta \) in Eq. (1). For this angle we took here \( \theta_2 = 140^\circ \). As a result of interpolations between the angular functions measured at \( \theta_2 = 140^\circ \), we find the \( R_\theta(q_i, \omega) \) values at those \( q_i \) and \( \omega \), which are the arguments of a certain small-angle response function \( R_{\theta_1}(q_i, \omega) \). Then, from the obtained values and with the use of the set of equations of form (1) we obtain the response functions \( R'_L(q_i, \omega) \) and \( R'_T(q_i, \omega) \). If we carry out a similar procedure for the arguments \( q \) and \( \omega \) of each of the small-angle \( \theta_1 \) functions shown in Fig. 2, then we shall obtain \( R'_L(q_i, \omega) \) and \( R'_T(q_i, \omega) \) along 4 curves of the arguments \( q_i = q(\omega, \theta_1) \).

With the range of the determined \( R_L(q_i, \omega) \) values we carry out the interpolation of the data to the line of the constant value, \( q = q_c \) (in a similar way for the set of data for \( R_T(q_i, \omega) \)). As a result, we determine the sought for \( R_L(q_c, \omega) \) and \( R_T(q_c, \omega) \) at a given constant value of the 3-momentum \( q_c = 300 \text{ MeV/c} \).

According to Ref. [5], the calculation of the longitudinal response function of the \(^{12}\text{C} \) nucleus weakly depends on which of the two nuclear-current models is used. However, for the transverse response functions the calculation variants are very much different. So, for testing the results of work [5] it is necessary, first of all, to have the experimental values of the \( R_T(q_c, \omega) \) function. According to Eq. (1), the \( R_T(q_c, \omega) \) function is expressed in terms of the measurement data as
\[
R_T(q_c, \omega) = [R_{\theta_2}(q_c,\omega) - R_{\theta_1}(q_c,\omega)]/[\tan^2\theta_2/2 - \tan^2\theta_1/2].
\] (3)

In the foregoing we mentioned a high PB in the measurements at small angles \( \theta_1 \). This raises the question of how the error in the background values (\( \Delta P_{\theta_1} \)) affects the accuracy of the experimental values of the \( R_T(q_c, \omega) \) function. With the use of Eq. (3) it is easy to present the relative error of the \( R_T(q_c, \omega) \) function as
\[
\delta R_T = \sqrt{[(\Delta R_{\theta_1})^2 + (\Delta R_{\theta_2})^2]/[R_{\theta_2} - R_{\theta_1}]}.
\] (4)

where in the case under consideration, the \( \Delta R_{\theta_1} \) and \( \Delta R_{\theta_2} \) must be replaced by the corresponding \( \Delta P_{\theta_1} \) and \( \Delta P_{\theta_2} \).

We now estimate the numerical \( \delta R_T \) values at the intersection point of the \( \omega \)-dependences of \( q \) for the \( R_\theta(q, \omega) \) functions, one of which was measured at \( \theta_1 = 82^\circ \) and \( E_0 = 259.6 \text{ MeV} \), and the other at \( \theta_2 = 140^\circ \) and \( E_0 = 196.6 \text{ MeV} \). The coordinates of the cross point are \( q = 296 \text{ MeV/c}, \omega = 89 \text{ MeV} \). For these arguments we have \( R_{\theta_1} = 0.041 \text{ MeV}^{-1} \) and \( R_{\theta_2} = 0.151 \text{ MeV}^{-1} \).

The maximum background values in the small-angle \( (\theta_1) \) and large-angle \( (\theta_2) \) response functions were determined to be 10% and 1%, respectively. We assume the error of each of the background value to be 50%. In other words, the error values of the angular response functions associated with the error values of the backgrounds are estimated to be \( \Delta P_{\theta_1} = 0.0021/\text{MeV}^{-1}, \Delta P_{\theta_2} = 0.00075/\text{MeV}^{-1} \). Then, according to Eq. (4), in the experimental value of the transverse response function the relative error due to the mentioned background errors is expected to be \( \delta R_T \approx 2\% \).

As regards the systematic error in the transverse response function \( \delta R_T \) value, it is mainly specified by the systematic errors of the \( \delta R_\theta \) angular response functions. In the case under consideration, the error \( \delta R_\theta \) is about 3% for both angular response functions. According to Eq. (4), the error for \( R_T(q, \omega) \) will make about 4%.

At determining the experimental \( R_T/L \) function values, the data go sequentially through two interpolations. The effect of this procedure on the resulting data has been discussed in Refs. [22,23,26]. The interpolations were investigated there with the use of the models of scattered electron spectra. Variations were made in the \( q \) distances between the adjacent spectra, in the measurement statistics, and the formulas for the functions used in the interpolations. Measurements in the ranges of \( q = 170 - 250 \text{ MeV/c} \) [26] and \( q = 150 - 320 \text{ MeV/c} \) [23] were simulated for \(^4\text{He} \) and \(^6\text{Li} \), respectively. The contribution from the interpolation procedures to the systematic error for the response functions was estimated

Fig. 2 Plane of response function arguments. Dashed and solid lines represent the coordinates of \( R_\theta \)-functions measured at small angles \( \theta_1 \leq 90^\circ \) and at big angle \( \theta_2 = 140^\circ \), respectively; the solid thick horizontal line is for the functions \( R_{T/L} \) at a constant 3-momentum transfer; the dotted line shows the peaks location of elastic scattering of electrons by the nucleus. Line coordinates of the interpolation optimum for the spectral region, where the peaks of nuclear excited states prevail (dash-and-dot lines), and for the region of the QES peak (dash-double-dotted lines).
to be about 0.7% in Ref. [26] and 1% in Ref. [23]. In LUE-300 experiments for finding the $R_{T/L}$ function values, the measurement conditions and the data processing techniques were similar for all the nuclei studied. Therefore, the conclusion concerning the contribution of the interpolation procedure to the systematic error of the experimental response functions, which was made in Refs. [23,26], can be referred to the present data, too.

3 Results and discussion

The above-described processing of the measured data has resulted in obtaining the experimental values of the functions $R_L(q, \omega)$ and $R_T(q, \omega)$ of the $^{12}$C nucleus at a constant momentum transfer $q = 300 \text{ MeV/c}$. They are shown in Fig. 3a and 3b as being divided by the square of the proton charge form factor (see Ref. [27]) and with statistical errors included. The experimental data from Saclay and world data are also shown in this figure. It can be seen that there is actually no difference between our data and the Saclay data on the $R_T$-function.

Figure 4 shows the comparison between the data of the present work and the calculations of Ref. [5] for the longitudinal and transverse response functions of the $^{12}$C nucleus. The dash-and-dot line represents the plane-wave impulse-approximation (PWIA) calculation using the single-nucleon momentum distribution [28]. The other calculations are based on the realistic dynamic pattern of the description of nucleus for the cases with consideration of only one-body (O1b) currents in the electromagnetic operator, and also, with the combination of one- and two-body currents (O1b-2b). In the last calculations the AV18+IL7 combination of two- and three-nucleon potentials is used.

The $R_L(q, \omega)$ calculation data of Ref. [5] are close to our present experimental data. The comparison of the $R_L(q, \omega)$ calculations with the experiment can be represented in terms of the $V_{te} = S_{th}/S_{exp}$ ratio, where $S_{th}$ is the area under the calculated function $R_L(q, \omega)$ on the 26–120 MeV interval, and $S_{exp}$ is the area defined by the experimental points on this interval. For the case of the calculation [5] in the O1b-2b variant, we denote the ratio $V_{te}$ as $V_{2e}$, and find $V_{2e} = 0.95 \pm 0.05$. For comparison with the PWIA calculation, we write down the ratio $V_{te}$ as $V_{Pe}$. Then we have $V_{Pe} = 1.28 \pm 0.07$. This approach clearly shows how much closer to the experiment is the calculation from Ref. [5] than the PWIA calculation from Ref. [28]. The both calculations make use of the free proton form factor. However, the agreement of the calculation from Ref. [5] with the experiment renders the hypothesis about the proton modification in the nuclear matter unnecessary. This hypothesis has been widely discussed over the years (e.g., see Refs. [5,29]). We also note that the exact description of QES is of importance for the analysis of the neutrino experiment data [30].

Unlike the $R_L(q, \omega)$ case, in the $R_T(q, \omega)$ case (see Fig. 4b), the calculations with the contribution of only one-body currents or with the contribution from combination of one- and two-body currents show quite a difference, thereby
Thus, in the present study, the experimental functions \( R_L(q, \omega) \) and \( R_T(q, \omega) \) of the \(^{12}\)C nucleus at \( q = 300 \text{ MeV/c} \) have been determined. The obtained results are independent of the data of Refs. [6–9], which were earlier used for testing the calculations of Ref. [5]. Our experimental values of the transverse response function correspond to the calculation variant of Ref. [5], in which the combination of one- and two-body currents was taken into account.

**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The experimental results from this work are freely available from the authors on request.]

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