Calculation of the binding energy of the lithium nucleus with the Yukawa potential and the Hellmann potential using WKB approximation

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Abstract. It had been conducted the solving of the Schrödinger equation for 6-body particle system to determine the binding energy of the Lithium nucleus using two different potentials, The Yukawa Potential, and The Hellmann Potential. To ease the calculation process is used the WKB Approximation. It is shown that the binding energy of Lithium equals to 32.00 MeV with the error was 0.02% compared to the experimental reference values for each parameter where $a=0.8$ and $V_0=40$ MeV lithium nucleus

1. Introduction

The binding energy of a nucleus is the energy needed to break the nucleus into its constituent nucleons (neutrons and protons) \[1\]. Electrostatically the nucleons in the nucleus of the atom will try to repel each other. Another fact shows the nucleons in the nucleus of the atoms as if they were fused at a very close distance (about 1.4 fm). This nucleus system can be studied through the unity of the nucleus constituent system as a result of the interaction between the compilers of the nucleus. Several ways can be done to find out the binding energy in the nucleus, and the number of nucleons, one by determining the binding energy per nucleon, which is the average energy required to remove any nucleons.

In the nucleus of an atom there is a force that binds nucleons in a nucleus called a strong force, and in its interaction protons and neutrons undergo a process called meson exchange so that under bonded conditions will produce a new nucleus called deuteron.

![Figure 1. Illustration of interactions between nucleons within the nucleus of atoms][2]
The lithium atom is one that has 3 electrons on its skin. In the excitation state, one electron occupies the 1s orbital and the other two electrons occupy the 2s orbital. In some cases to know the binding energy of the lithium atomic nucleus is considered very complex, it takes a simple way in the process of solving it. The meson exchange that occurs due to the interaction between proton and neutron was first coined by Yukawa in 1935 known as the Meson Field Theory. This Yukawa concept states that there are particles with mass parameters between the mass of the electrons and the nucleons of the nucleons responsible for the existence of the nucleus force. With some parameters of the concept of Yukawa is expected to provide a simple concept in knowing the binding energy in the atomic nucleus. Another simple concept is to use the Hellmann potential, this potential gives more accurate, simpler and more efficient results [3].

In this study the binding energy of the lithium atomic nucleus was solved analytically using two potentials of the normal Yukawa potential, and the Hellmann potential then compared the two lithium nucleic energy binding potentials obtained with the experimental reference results.

2. Methodology

The energy of a system of an atomic nucleus can be obtained by first resolving the Schrödinger equation. Schrödinger’s equation is an important pillar in studying the behavior of matter and its interaction, in its solution that is giving potential value with certain value on Schrödinger equation. Historically, there have been many attempts made to solve Schrödinger equations, one of the ways to do this is to use the WKB Method [4]. The WKB method is also used to solve equations so as to produce the basic energy of lithium atomic nuclei. By using two different potentials are Normal Yukawa Potential [5], and Hellmann Potential.

The algorithm used in this research is as follows.
1. Define the value of parameters used in the general equation to find the value of Energy
2. This undefined parameter is then used as a reference value to get the value of Ω
3. The process of declaration of certain a and Vo values
4. Next system will calculate the value of β and γ
5. Finally obtained a certain value of E
6. If the result is not yet close to the reference value of E experimental results, then re-calculate the value of E by changing the value of Vo.
7. If the result is appropriate and close to the reference value E experimental results, then the calculation is complete.
8. Finally obtained output value of E for each potential used.

Schrödinger’s Equation for D-Dimension [6]:

\[ -\frac{\hbar^2}{2\mu} \left( \frac{d^2}{dx^2} + \frac{D-1}{x} \frac{d}{dx} - \frac{l(l + D - 2)}{r^2} \right) R_{n,l}(r) + v(r)R_{n,l}(r) = E_{n,l}R_{n,l}(r) \]

where

\[ D = 3N - 3 \]
\[ D = 3(6) - 3 \]
\[ D = 18 - 3 \]
\[ D = 15 \]

N is the particle number (for Li = 6)
l is angular momentum
To know the energy value of the lithium nucleus of the nucleus is used several parameters such as normal Yukawa potential and Hellmann potential.

**Table 1.** The potential carrier used

| Normal Yukawa potential | Hellmann potential |
|-------------------------|--------------------|
| \[ V = -\frac{V_0 e^{-ar}}{r} \] | \[ V = \frac{V_0 e^{-ar}}{r} \] |

By substituting the above equation with each potential used then can be the result of formulation of lithium nucleus binding energy by using two potential:

3. **Result**

The binding energy produced by using two different potentials is presented in Table 2.

**Table 2. Energy Comparison of Lithium Nuclear Binding**

| Normal Yukawa Potential | Hellmann Potential |
|-------------------------|--------------------|
| \[ E = \frac{\hbar^2}{2\mu} \left( \frac{-\beta}{1 \pm \sqrt{1 + 4\gamma}} \right)^2 + aV_0 \] | \[ E = \frac{\hbar^2}{2\mu} \left( \frac{-\beta}{1 \pm \sqrt{1 + 4\gamma}} \right)^2 + aV_0 \] |
| Where \[ \beta = \frac{4\mu a}{\hbar^2} V_0 \] | Where \[ \beta = \frac{4\mu a}{\hbar^2} (k - V_0) \] |
| \[ \gamma = \frac{2\mu}{\hbar^2} \Omega \] | \[ \gamma = \frac{2\mu}{\hbar^2} \Omega \] |

As a benchmark, this analytical solution compares the results of previously completed calculations. Through the calculation, obtained comparison of results through computed analytical calculations of the experimental results as the following table.

**Table 3.a. Calculations to Experiments of Lithium Nuclear Bond** by Ghazvini, 2015 [5].

| A  | V_0 (MeV) | BE (Calculation) (MeV) | BE (Experiment) (MeV) | Error (%) |
|----|-----------|------------------------|----------------------|-----------|
| 0.70 | 45        | 31.92                   | 31.995               | 1.80%     |
| 0.60 | 50        | 29.90                   | 31.995               | 6.55%     |
| 0.64 | 49.3      | 31.90                   | 31.995               | 0.30%     |
| 0.70 | 50        | 34.90                   | 31.995               | 9.08%     |
| 0.75 | 40        | 29.93                   | 31.995               | 6.45%     |
| 0.50 | 50        | 24.90                   | 31.995               | 22.18%    |
| 0.55 | 50        | 27.40                   | 31.995               | 14.36%    |
| 0.80 | 30        | 23.96                   | 31.995               | 25.11%    |
| 0.80 | 40        | 31.98                   | 31.995               | 0.05%     |
| 0.59 | 50        | 29.40                   | 31.995               | 8.11%     |

The authors also tried to calculate with the results obtained is as this following table.
Table 3.b. Calculations and Experiments of Lithium Nuclear Bond by author

| A   | V₀ (MeV) | BE (Calculation) (MeV) | BE (Experiment) (MeV) | Error (%) |
|-----|----------|------------------------|-----------------------|-----------|
| 0.70| 45       | 31.50                  | 31.995                | 1.55%     |
| 0.60| 50       | 30.00                  | 31.995                | 6.24%     |
| 0.64| 49.3     | 31.55                  | 31.995                | 1.38%     |
| 0.70| 50       | 35.00                  | 31.995                | 9.39%     |
| 0.75| 40       | 30.00                  | 31.995                | 6.24%     |
| 0.50| 50       | 25.00                  | 31.995                | 21.86%    |
| 0.55| 50       | 27.50                  | 31.995                | 14.05%    |
| 0.80| 30       | 24.00                  | 31.995                | 24.99%    |
| 0.80| 40       | 32.00                  | 31.995                | 0.02%     |
| 0.59| 50       | 29.50                  | 31.995                | 7.80%     |

Calculating of lithium nucleus energy is done using a data processing program which is Matlab. This program initiated the values of a and V₀ with values ranges of 0.4 to 1.4 and 20.00 to 25.00, respectively. Through this program, the value of lithium nucleus binding energy is 32.00 MeV for each parameter a = 0.8 and V₀ = 40 MeV. At this point the value is close to the actual value when compared to the results of the experimental experiment with a relative error value of 0.02%. The final result shows that with the same parameter values, what is done by the authors that gives results with a relative error rate that is smaller than what the previous researcher has done.

4. Conclusion

Based on the analytical calculations, the lithium atomic binding energy at two potentials is the Yukawa Normal potential, and the Hellmann potential is obtained with the same result of 32.00 MeV. Comparison of analytic calculations with an experimental reference generated a relative error of 0.02%, with values a = 0.8 and V₀ = 40 MeV

References

[1] Schaeffer B 2013 World J. Nuclear Sci. and Tech. 3 79-82
[2] Beiser A 1987 Concept of Modern Physics (New York: McGraw-Hill)
[3] Roy A K, Jalbout A F, Pyornov E I 2008 J. Math.Chem. 44 2690-269
[4] Fabre C M, Gu'ery-Odelin G 2011 American J. Phys. 79 755
[5] Ghazvini M, Salehi N, Hassanabadi H, Rajabi AA 2015 Chi. Phys.C 39
[6] H. H. A. A. R. and S. Z 2008 Mod. Phys. Lett. A 23 527-537