Color entanglement for $\gamma$-jet production in polarized pA collisions

Andreas Schäfer$^1$ and Jian Zhou$^1$

$^1$Institut für Theoretische Physik, Universität Regensburg, Regensburg, Germany

A more reliable treatment of transverse momentum dependent physics and in particular transverse single spin asymmetries is urgently required, e.g. for polarized pA physics including novel effects like color entanglement. We argue that the measurement of azimuthal angular correlations of photons and jets produced in pA collisions provides a direct access to the novel gluon distribution $G_4$ that enters into many such processes.

PACS numbers:

Keywords:

I. INTRODUCTION

While collinear QCD is by now well understood, the new frontier is a rigorous treatment of transverse momentum dependent (TMD) physics. If the detected transverse momenta are not very large (e.g. several tens of GeV) many novel, highly non-trivial effects are relevant which could, e.g., influence the interpretation of heavy ion data (the $A$ dependence of these effects is basically unknown, making the comparison of $p + p$, $p + A$ and $A + A$ data for $k_{\perp}$ dependent quantities highly problematic). One of the tasks of the proposed EIC accelerator [1, 2] is the investigation of this physics and of the associated differences between $e + p$ and $e + A$ collisions.

There exist many open questions, starting from such fundamental issues as TMD factorization, see, e.g. [3–5]. TMD parton distributions are expressed as matrix elements of non-local combinations of quark or gluon fields which are connected by gauge links. To take their effects into account a generalized version of TMD factorization (GTMD) [6] was proposed containing process dependent gauge links and thus a modified concept of universality. However, it was found that in hadron-hadron reactions, color entanglement leads to additional contributions which cannot be factorized even within GTMD factorization [7]. If these effects are large and cannot be reliably and quantitatively described by theory the cor-

The Sivers effect giving rise to the SSA in this process is particularly sensitive to color entanglement as its existence relies on the gauge link. We take into account one extra gluon exchange on the proton side, while the gluon re-scattering is resummed to all orders on the nucleus side by means of Wilson lines. This approximation is justified by the fact that the gluon number density in a nucleus is much larger than in a proton. We will further argue below that higher order gluon exchange with the remnant part of the proton is suppressed by a factor $\Lambda_{QCD}/Q_s$ in the semi-hard region, where $Q_s$ is the saturation momentum.

The spin dependent observables in pA collisions (e.g. the Sivers asymmetry) are generally affected by saturation. Therefore, measuring these observables might be a promising approach to firmly experimentally establish saturation. Recent developments in this direction include the calculation of quark/gluon Boer-Mulders functions [12] and the quark Sivers function [13] in the quasi-classical McLerran Venugopalan (MV) model [14], the derivation of small $x$ evolution equation for the gluon Boer-Mulders function [15], the determination of the asymptotic behavior of SSAs at small $x$ [16, 17], and the investigation of spin asymmetries in pA collisions beyond the Eikonal approximation [18]. SSAs in various processes in polarized pA collisions also have been studied within different frameworks [19–22]. More recently, a hybrid approach was formulated to compute the SSA for inclusive direct photon production in polarized pA collisions [11].

In this letter, we study the SSA for photon and jet production in polarized pA collisions by closely following the method presented in Ref. [11, 23]. The result can be decomposed into two parts, one of which is related to the normal dipole type TMD gluon distribution and an additional one related to the new gluon distribution function $G_4(x, k_{\perp})$ generated by color entanglement. We will show that the transverse momentum dependence of the differential cross section in the semi-hard region reads,

\[
\frac{d\hat{\sigma}^{PA\rightarrow\gamma q+X}}{d\hat{P}S.} \approx \sum_q H_{\text{Born}} \left\{ x f_q(x) x' G_{DP}(x', q_{\perp}) \
- \epsilon_{\perp}^{ij} S_{\perp}^{ij} q'_{\perp} x T_{F,g}(x, x) \frac{N^2_c + 1}{N^2_c - 1} \frac{\partial}{\partial q'_{\perp}} x' G_{DP}(x', q_{\perp}) \
+ \epsilon_{\perp}^{ij} g^i_{\perp} g_j_{\perp} x T_{F,g}(x, x) \frac{2N^2_c}{N^2_c - 1} \frac{\partial}{\partial q'_{\perp}} x' G_{4}(x', q_{\perp}) \right\}
\] (1)

where $f_q(x)$ and $G_{DP}(x', q_{\perp})$ are the integrated quark PDF in the proton and the dipole type gluon TMD distribution in the nucleus. $\epsilon_{\perp}^{ij}$ is a short-hand notation for the transverse epsilon tensor $\epsilon_{\perp}^{i+ij}$ defined with the convention $\epsilon_{\perp}^{+12} = 1$, $T_{F,g}(x, x)$ is the well known ETQS function [27, 28], $d\hat{P}S. = d_y x d^2 l_{\perp} d^2 l_{\perp}^q$, where $y_1, y_2$ are the rapidities of the two outgoing particles, and $l_{\perp}, l_{\perp}^q$ are the transverse momenta of the produced photon and quark, respectively. $q_{\perp}$ is their sum $q_{\perp} = l_{\perp} + l_{\perp}^q$. The hard coefficient is given by,

\[
H_{\text{Born}} = \frac{\alpha_s \alpha_{em} \epsilon_q^2}{N_c s^2} \left( -\frac{\hat{s}}{\hat{u}} - \frac{\hat{u}}{\hat{s}} \right)
\] (2)
The first two terms in Eq.(1) are already obtained in GTMD factorization while the last one describes the novel color entanglement effect. Experimentally, nothing is known about the $q^2_\perp$ slope of $G_4$ and it is unclear if this contribution to the SSA is sizable. Thus, it is very important that this SSA is measured at RHIC to find out whether $G_4$ is relevant or not for this observable. If it is, color entanglement effects could very well be important in general for $q_\perp$ dependent hadronic reactions, which would greatly complicate the interpretation of existing and future experimental data.

In the following we will present a few details of the calculation.

II. $\gamma$-JET SSA IN GTMD FACTORIZATION

We consider the kinematic region where the transverse momenta of the produced photon and jet are much larger than their transverse momentum imbalance which is often referred to as the correlation limit. The dominant partonic channel contributing in the forward region of the proton is:

$$g_p(xP + p_\perp) + g_A(x_g^p P + k_\perp) \rightarrow \gamma(l_\gamma) + q(l_q)$$

where $P^\mu = P^- n^\mu$ and $P^\mu = P^+ n^\mu$ with the usual light cone vectors $n^\mu$ and $p^\mu$, normalized according to $p \cdot n = 1$. The Mandelstam variables are defined as: $\hat{s} = (l_q + l_\gamma)^2$, $t = (xP - l_\gamma)^2$ and $\hat{u} = (xP - l_q)^2$. In the correlation limit an effective TMD factorization should apply. Neglecting the transverse momentum carried by the incoming quark the corresponding unpolarized Born cross section reads [24],

$$\frac{d\sigma}{dP.S.} = \sum_q x_f q(x) x_g^P G_{DP}(x_g^p, q) H_{Born}.$$  

Note that a $\cos 2\phi$ modulation will show up for the virtual photon-jet production [12]. The above cross section can also be derived in the color glass condensate (CGC) framework. By applying a corresponding power counting in the correlation limit, a complete matching between TMD factorization and the CGC calculation has been found [24]. This power expansion can actually be performed either in coordinate [24] or momentum space [25].

For the SSA we proceed in the same way, but include the incoming quark transverse momentum dependence. To compute the polarized cross section, one has to apply GTMD factorization as already without color entanglement the color flow is non-trivial. This results in,

$$\frac{d\sigma}{dP.S.} = \sum_q x_f q(x, k_\perp) x_g^P G_{DP}(x_g^p, k_\perp) H_{Born}$$

section for photon-jet production in pp collisions takes the same form [26]. At small $x_g^p$, the typical gluon transverse momentum in the nucleus is of order of the saturation scale $Q_s$, which is much larger than the intrinsic parton transverse momentum in the proton. We thus can approximate the cross section by a power expansion in $k_\perp/Q_s$ in the semi-hard region where $q_\perp$ is of the order of $Q_s$. The leading non-trivial contribution reads,

$$\frac{d\sigma}{dP.S.} \approx \sum_q H_{Born} \left\{ x_f q(x, x_g^p) x_g^P G_{DP}(x_g^p, q_\perp) \right\}$$

To arrive at the above expression, we have made use of the identity [26, 29, 30],

$$\frac{N_c^2 + 1}{N_c - 1} T_{F,g}(x, x) \approx \frac{d^2 k_\perp}{N_c^2} f_{1T}^{\perp, g\gamma \rightarrow \gamma q}(x, k_\perp).$$

The non-trivial color factor appearing on the left side of Eq.(7) is determined by the color topology of the involved partonic scattering diagram. It would be interesting to see how the cross section with color entanglement effect being incorporated deviates from Eq. 6.

III. THE COLOR ENTANGLEMENT CONTRIBUTION

Color entanglement effect results from the nontrivial interplay between gluon attachments from the proton and nucleus sides. In principle it would be necessary to resum gluon attach- ments on both sides to all orders simultaneously. However, such a calculation is technically out of reach. We simplify the task by taking into account only one extra collinear gluon from the proton while resumming gluon re-scattering on the nucleus side to all orders. We expect this approximation to be valid in the kinematic limit $q_\perp \sim Q_s \sim k_{2\perp} \gg \Lambda_{QCD} \sim k_{2\perp}$ for the following reason. We indicate the transverse momentum dependence of the Wilson lines by the notation $\Gamma(q_\perp - k_{2\perp})$. In the semihard region, one can Taylor expand this expression $\Gamma(q_\perp - k_{2\perp}) = \Gamma(q_\perp - 2k_{2\perp}) + \frac{\partial^2}{\partial q_\perp^2} \Gamma(q_\perp) + O(\frac{k_{2\perp}^2}{Q_s^2})$ and conclude that the $k_{2\perp}$ moment of the Sivers function gives rise to the leading power contribution. To include the $k_{2\perp}$ term it is sufficient to only consider one gluon exchange for the proton side.

Typically, multiple scattering between the incoming quark (or transversely polarized gluon) and the classical color field of the nucleus can be resummed into a Wilson line. However, this procedure does not apply if the incoming parton is a collinear gluon. The formula valid for a longitudinally polarized gluon scattering off a nucleus has been worked out in Ref. [33]. The expression for the gauge field created through the fusion of the incoming gluon from the proton and small x gluons from the nucleus contains both singular terms (proportional to $\delta(z^+)\delta(z^-)$) and regular terms: $A' = A_{reg}^\mu + \delta \mu - A_{sing}^\mu$. The detailed expression for $A_{reg}^\mu$ and $A_{sing}^\mu$ can be found in Ref. [23].
We first consider the gluon attachment on the left side of the cut. All possible insertions of the fields $A_A$, $A^\mu$ and $A_s^\mu$ on the quark line must be taken into account as illustrated in Fig.1, where $A_A$ is the classical field created by the nucleus alone. As compared to [11], the calculation of the hard scattering amplitude is greatly simplified since we do not need to keep track of the transverse momentum carried by the gluon from the proton side. The contribution from the initial interactions to the amplitude is given by,

$$
\mathcal{M}_I = i e g A_p^a \int d^2x_1 e^{i k_{\perp} \cdot x_{\perp} \perp} \times \bar{u}(l_q) \frac{d S_F(x_P - l_q) \gamma \cdot \gamma + f S_F(l_q + l_\gamma) \gamma \cdot \gamma}{x gP + i \epsilon} U(x_{\perp} \perp) u(x P) 
$$

where $A_p^a$ is the gauge field created by the proton with color index $a$, and $\gamma^\mu$ is the polarization vector of the produced photon. $S_F(x_P - l_q) = \frac{x P - l_q}{|x P - l_q|^2 + \epsilon}$ and $S_F(l_q + l_\gamma) = \frac{l_q + l_\gamma}{(l_q + l_\gamma)^2 + \epsilon}$ are the quark propagators. $\bar{U}(x_{\perp} \perp)$ and $U(x_{\perp} \perp)$ are the path ordered Wilson lines in the adjoint and fundamental representation

$$
\bar{U}(x_{\perp} \perp) = \mathcal{P} \exp \left[ ig \int_{-\infty}^{+\infty} dz^+ \bar{A}_A(z^+, x_{\perp} \perp) \cdot T \right] 
$$

$$
U(x_{\perp} \perp) = \mathcal{P} \exp \left[ ig \int_{-\infty}^{+\infty} dz^+ A_A(z^+, x_{\perp} \perp) \cdot t \right] 
$$

with $T$ and $t$ being the generators in the adjoint and fundamental representation. In the case of the SSA for inclusive direct photon production, the contribution from the final state interactions to the spin asymmetry is absent due to cancelation between mirror diagrams. However, for the process under consideration, the final state interactions shown in diagram (b) and (d) of Fig.1 also generate a spin asymmetry. The cancelation between mirror diagrams does not occur because the out-going quark jet momentum is not integrated over. The amplitude for final state interactions reads,

$$
\mathcal{M}_F = i e g A_p^a \int d^2x_1 e^{i k_{\perp} \cdot x_{\perp} \perp} \frac{\bar{u}(l_q) \ell S_F(x_P - l_q) \gamma \cdot \gamma + f S_F(l_q + l_\gamma) \gamma \cdot \gamma}{x gP - i \epsilon} \left[ U(x_{\perp} \perp) - 1 \right] u(x P) 
$$

Combining both contributions, we obtain,

$$
\mathcal{M} = \mathcal{M}_I + \mathcal{M}_F = i e g A_p^a \int d^2x_1 e^{i k_{\perp} \cdot x_{\perp} \perp} \times \left\{ \frac{1}{x gP - i \epsilon} \bar{u}(l_q) \left[ \ell S_F(x_P - l_q) \gamma \cdot \gamma + f S_F(l_q + l_\gamma) \gamma \cdot \gamma \right] \times t^\mu U(x_{\perp} \perp) u(x P) \right\}_{ba} 
$$

where a term which does not contain any Wilson line has been neglected. The contribution from such a term to the inelastic scattering cross section is suppressed according to the arguments made in Ref. [31]. Using the identity $\frac{1}{x gP - i \epsilon} = P \frac{1}{x gP + i \epsilon} = i \pi \delta(x gP)$, the real and imaginary part of the gluonic pole have been expressed separately in the above equation. The imaginary part provides the phase necessary for generating a non-vanishing SSA, whereas the real part is irrelevant for the polarized cross section. Note that the structures of the Wilson lines associated with the real and imaginary parts are different. After carrying out the $x_{\perp}$ integration, the gluon field $A_p^a$ is incorporated into the gauge links of the unpolarized quark TMD and the quark Sivers function.

At the order we consider, the spin dependent hard part is calculated from an interference of the scattering amplitude with one extra gluon attachment from the proton given in the above equation and the Born scattering amplitude derived in Ref. [32]. It is straightforward to include the contributions from the right cut diagrams. To arrive at a compact expression for the polarized cross section, we simplify the Wilson line structure using the approach introduced in [11]. As a result, two different Wilson line structures emerge. These can be related to the dipole type gluon TMD distribution, and a new gluon distribution $G_q$, respectively. We then proceed by extrapolating the result to the correlation limit with a power expansion procedure performed in momentum space [25]. After neglecting all terms suppressed by the power of $k_{\perp}^2/l_{\perp}^2$, the hard part is no longer dependent of the incoming parton transverse momenta, while the soft part is expressed as the convolution of the quark Sivers function and the gluon distributions $G_{DP}$ and $G_q$. As argued above, the single gluon exchange from the proton remains a good approximation in the
semihard region. In this kinematic region, the soft part can be further simplified by carrying out power expansion leading from Eq.(5) to Eq.(6). Eventually, one obtains Eq.(1).

The definition of $G_A(x'_g, k_{\perp})$ has been given in [11],

$$x'_g G_A(x'_g, k_{\perp}) = \frac{k^2 g^2 N_c}{2\pi^2 a_s} \int \frac{d^2 x^\perp d^2 y^\perp}{(2\pi)^2} e^{ik_{\perp} \cdot (x^\perp - y^\perp)}$$

$$\times \left\{ \frac{1}{N_c} \left[ \text{Tr}_c \left[ U(x^\perp) \right] \right] \left[ \text{Tr}_c \left[ U^\dagger(y^\perp) \right] \right] \right\} x'_g$$

$$= \frac{k^2 R_0^2}{2\pi a_s N_c} \int \frac{d^2 r^\perp}{(2\pi)^2} e^{ik_{\perp} \cdot r_{\perp} - \frac{4}{\pi} r^\perp G_{DP} G_{\perp}$$

$$= \frac{k^2}{N_c} x'_g G_{DP}(x'_g, k_{\perp})$$

where $R_0$ is the radius of the nucleus. One notices that $G_A$ is suppressed in the large $N_c$ limit as compared to the normal dipole type gluon distribution $G_{DP}$.

We close this section with two further remarks:

(i) The unpolarized cross section is not affected by the color entanglement effect at the order that we consider. The observed color entanglement effect is the consequence of the non-trivial interplay among the T-odd effect, the coherent multiple gluon re-scattering, and the non-Abelian feature of QCD.

(ii) The spin asymmetry is found to be proportional to the slope of both gluon distributions. The same feature was also found in [19] for a different process.

IV. CONCLUSIONS

We have shown that the SSA in photon-jet production in polarized $pA$ collisions in the correlation limit offers a promising opportunity to pin down experimentally the size of the factorization breaking color entanglement effect parameterized by the small $x$ gluon distribution $G_A$. We hope very much that such a measurement will be performed at RHIC [34].

Acknowledgments: We are grateful to Bowen Xiao and Feng Yuan for reminding us of the tadpole type contribution to the gluon distribution $G_A$ in the MV model. This work has been supported by BMBF (OR 06RY9191).

[1] D. Boer, M. Diehl, R. Milner, R. Venugopalan, W. Vogelsang, D. Kaplan, H. Montgomery and S. Vigdor et al., arXiv: 1108.1713 [nucl-th].
[2] A. Accardi, J. L. Albacete, M. Anselmino, N. Armesto, E. C. Aschenauer, A. Bacchetta, D. Boer and W. Brooks et al., arXiv:1212.1701 [nucl-ex].
[3] J. C. Collins and D. E. Soper, Nucl. Phys. B 193, 381 (1981) [Erratum-ibid. B 213, 545 (1983)] [Nucl. Phys. B 213, 545 (1983)].
[4] X. -d. Ji, J. -p. Ma and F. Yuan, Phys. Rev. D 71, 034005 (2005).
[5] M. G. Echevarria, A. Idilbi and I. Scimemi, JHEP 1207, 002 (2012).
[6] C. J. Bomhof, P. J. Mulders and F. Pijlman, Phys. Lett. B 596, 277 (2004).
[7] T. C. Rogers and P. J. Mulders, Phys. Rev. D 81, 094006 (2010).
[8] M. G. A.Buffing and P. J. Mulders, JHEP 1107, 065 (2011).
[9] M. G. A. Buffing, A. Mukherjee and P. J. Mulders, Phys. Rev. D 86, 074030 (2012).
[10] M. G. A. Buffing and P. J. Mulders, Phys. Rev. Lett. 112, 092002 (2014).
[11] A. Schäfer and J. Zhou, arXiv:1404.5809 [hep-ph].
[12] A. Metz and J. Zhou, Phys. Rev. D 84, 051503 (2011). A. Schäfer and J. Zhou, Phys. Rev. D 88, 074012 (2013).
[13] Y. V. Kovchegov and M. D. Sievert, arXiv:1310.5028 [hep-ph].
[14] L. D. McLerran and R. Venugopalan, Phys. Rev. D 49, 2233 (1994); Phys. Rev. D 49, 3352 (1994).
[15] F. Dominguez, J. -W. Qu, B. -W. Xiao and F. Yuan, Phys. Rev. D 85, 045003 (2012).
[16] A. Schäfer and J. Zhou, arXiv:1308.4961 [hep-ph].
[17] J. Zhou, Phys. Rev. D 89, 074050 (2014).
[18] T. Altinoluk, N. Armesto, G. Beuf, M. Martinez and C. A. Salgado, arXiv:1404.2219 [hep-ph].
[19] D. Boer, A. Dumitru and A. Hayashigaki, Phys. Rev. D 74, 074018 (2006).
[20] Z. -B. Kang and F. Yuan, Phys. Rev. D 84, 034019 (2011).
[21] Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D 86, 034028 (2012) [Erratum-ibid. D 86, 079906 (2012)].
[22] Z. -B. Kang and B. -W. Xiao, Phys. Rev. D 87, 034038 (2013).
[23] J. P. Blaizot, F. Gelis and R. Venugopalan, Nucl. Phys. A 743, 57 (2004).
[24] F. Dominguez, B. -W. Xiao and F. Yuan, Phys. Rev. Lett. 106, 022301 (2011). F. Dominguez, C. Marquet, B. -W. Xiao and F. Yuan, Phys. Rev. D 83, 105005 (2011).
[25] E. Akcakaya, A. Schäfer and J. Zhou, Phys. Rev. D 87, no. 5, 054010 (2013).
[26] A. Bacchetta, C. Bomhof, U. D’Alesio, P. J. Mulders and F. Murgia, Phys. Rev. Lett. 99, 212002 (2007).
[27] A. V. Efremov and O. V. Teryaev, Sov. J. Nucl. Phys. 36, 140 (1982) [Yad. Fiz. 36, 242 (1982)]; Phys. Lett. B 150, 383 (1985).
[28] J.-w. Qu and G. F. Sterman, Phys. Rev. Lett. 67, 2264 (1991).
[29] D. Boer, P. J. Mulders and F. Pijlman, Nucl. Phys. B 667, 201 (2003).
[30] C. J. Bomhof and P. J. Mulders, JHEP 0702, 029 (2007).
[31] A. Dumitru and J. Jalilian-Marian, Phys. Rev. Lett. 89, 022301 (2002).
[32] F. Gelis and J. Jalilian-Marian, Phys. Rev. D 66, 014021 (2002).
[33] J. P. Blaizot, F. Gelis and R. Venugopalan, Nucl. Phys. A 743, 13 (2004).
[34] E. C. Aschenauer, A. Bazilevsky, K. Boyle, K. O. Eyser, R. Fatemi, C. Gagliardi, M. Grosse-Perdekamp and J. Lajoie
et al., arXiv:1304.0079 [nucl-ex].