Gravitational wave echoes from strange stars for various equations of state

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The tentative Gravitational Wave Echo (GWE) at a frequency of about 72 Hz has been recently claimed at 4.2σ significance level in the GW170817 event [1]. GWEs can be used as a tool to study the characteristics of ultra-compact stellar objects. Considering the final ultra-compact, post-merger object as a strange star, the GWE frequency can be calculated. However, GWEs are observed for only those compact stellar structures whose compactness lies between 0.33 and 0.44. Alternatively, GWE can be obtained for those compact stars which feature a photon sphere and compactness not crossing the Buchdahl’s limit radius \( R_B = \frac{9}{4}M \). A photon sphere is a surface located at \( R = 3M \), \( R \) being the radius and \( M \) is the total mass of the ultra-compact object. Recently using the simplest MIT Bag model Equation of State (EoS) it has been reported that strange stars can produce GWEs with frequencies of tens of kilohertz [2]. In view of this, for a comparative study, we have calculated the respective echo frequencies associated with strange stars by considering three models of strange star EoSs, viz., MIT bag model, linear and polytropic EoSs [3]. We found that, not being too stiff the polytropic EoS can not emit GWE, whereas the MIT Bag model and the linear EoSs can emit GWEs at a frequency range of about tens of kilohertz. Also, GWE frequency increases with the increase in values of bag constant \( B \) and decreases with the increasing values of linear constant \( b \). So a model-dependent nature of GWE frequencies is observed.

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I. INTRODUCTION

The possibilities that compact objects emitting Gravitational Wave Echoes (GWEs) are shown by various authors in the very recent past [2–5]. Among the other compact objects, strange stars (SSs) are able to draw notable attention in last few years. The very unique structural and also compositional behaviours of SSs are responsible for the increasing attraction towards such hypothetical stars. It is shown that, an SS can be an ultra-compact star having compactness large enough to emit GWE [2, 4]. The echo signal originating from ultra-compact star was first reported in [4]. Considering the final ultra-compact object formed in GW170817 event as an SS, the corresponding echo frequency is reported in [2] using the MIT Bag model EoS. In this GW170817 event the tentative GWE at a frequency of about 72 Hz has been recently claimed with 4.2σ significance level [1]. In this paper we have pointed out the possibilities of GWEs from SSs formed in GW170817 event depicted by various EoSs and re-examined the possibilities of using the MIT Bag model with large values of Bag constants.

After this introductory section, rest of the paper is organised as follows: In Sec. II the considered EoSs are described. In Sec. III we have discussed the echoes emitted by SSs, which is followed by Sec. IV for the result and discussion. In this work we have chosen the natural unit system, in which \( c = \hbar = 1 \). Also we have assumed \( G = 1 \) and the metric convention \((- , +, +, +)\) is adopted.

II. EQUATIONS OF STATE

There is no single EoS which could correctly explain strange quark matters till now. So, in this study we have chosen three EoSs which are found to be quite well in describing the states of such dense matters. The MIT Bag model EoS is the simplest EoS to describe strange matters. In our case we have chosen the stiffer form of this equation as [2],

\[ p = \rho - 4B, \]

(1)

We have taken three feasible values of bag constant \( B \) as \((190 \text{ MeV})^4\), \((217 \text{ MeV})^4\) and \((243 \text{ MeV})^4\). The parameter \( B \) does not affect the compactness of the stellar configuration [6], so any feasible \( B \) value gives star with compactness approximately 0.354 [3]. Another EoS that is used to describe strange matter is linear EoS of the form,

\[ p = b(\rho - \rho_s), \]

(2)

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where $b$ and $\rho_s$ are linear constant and the surface energy density respectively. Here we have chosen $b = 0.910, 0.918$ and 0.926. These values respects the conditions for emitting GWE frequency and the condition for causality. According to which 0.710 is the minimum and 1.000 is the maximum values of $b$ respectively [3]. Again we can consider SSs as polytropic spheres having polytropic EoS,

$$p = k \rho^\Gamma,$$

where $k$ is the polytropic constant, $\Gamma$ is the polytropic exponent with $\Gamma = 1 + 1/n$, $n$ being the polytropic index. In this work we have chosen $\Gamma = 1.5, 1.67$ and $2$ as guess values to describe the structure of SSs.

We have considered SSs as spherically symmetric, isotopic, stable-static configurations and neglected the stellar rotation and possible temperature effects on the EoSs. In such cases, the interior structure of the star can be obtained by solving the equations of Tolman, Oppenheimer and Volkoff (TOV), which are given as

$$\frac{d\chi}{dr} = -\frac{2}{\rho + p} \frac{dp}{dr},$$

$$\frac{dm}{dr} = 4\pi\rho r^2,$$

$$\frac{dp}{dr} = -(\rho + p) \left( \frac{m}{r^2} + 4\pi\rho r \right) \left( 1 - \frac{2m}{r} \right)^{-1},$$

where $\rho = \epsilon/c^2$ is the energy density.

### III. ECHOES FROM STRANGE STARS

In GW merging events, two massive objects lead to the formation of ultra-compact objects. As mentioned earlier, recently in the GW170817 event the tentative GWE at a frequency of about 72 Hz has been claimed at 4.2σ significance level [1]. The nature of final ultra-compact object of this event is not confirmed yet. It is possible to consider it as an SS. Considering it as an SS the corresponding emitted echoes can be calculated. Now to emit GWE, the ultra-compact star should feature a photon sphere and should have the compactness ($M/R$) larger than $1/3$ and smaller than $4/9$. The typical echo time can be given as the light crossing time from the centre of the astrophysical object to the photon sphere [2, 4],

$$\tau_{echo} = \int_0^{3M} \frac{1}{\sqrt{\epsilon \chi(r)(1 - 2m/r)/r}} \, dr.$$  

(7)

From this equation, the characteristic echo time, the echo frequency can be calculated using the relation $\omega_{echo} = \pi/\tau_{echo}$ and the corresponding repetition frequency of the echo signal can be calculated using the relation $\omega_{repetition} = 1/(2 \tau_{echo})$. 

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**FIG. 1:** MR curves of stars for all three EoSs (left plot), MIT Bag model with different $B$ values (middle plot) and Linear EoS with different $b$ values (right plot) along with the photon sphere limit, Buchdahl’s limit and black hole limit lines. Here EoS1, EoS2 and EoS3 represent the MIT Bag model, Linear EoS and Polytropic EoS respectively.
TABLE I: Mass, radius, compactness and estimated echo times, frequencies and repetition frequencies of the echo signals for SSs predicted by different EoSs.

| EoSs         | Model Parameter | Radius R (km) | Mass M (M⊙) | Compactness M/R | Echo time (ms) | Echo Frequency (kHz) | Repetition Frequency (kHz) |
|--------------|-----------------|---------------|-------------|-----------------|----------------|----------------------|---------------------------|
| MIT Bag model | B = (190 MeV)⁴  | 13.766        | 3.295       | 0.3540          | 0.078          | 39.91                | 6.35                      |
|              | B = (217 MeV)⁴  | 10.630        | 2.544       | 0.3540          | 0.060          | 51.70                | 8.23                      |
|              | B = (243 MeV)⁴  | 8.456         | 2.024       | 0.3540          | 0.048          | 64.98                | 10.34                     |
| Linear EoS   | b = 0.910       | 7.535         | 1.775       | 0.3484          | 0.043          | 72.90                | 11.60                     |
|              | b = 0.918       | 7.816         | 1.844       | 0.3489          | 0.044          | 70.21                | 11.18                     |
|              | b = 0.926       | 8.128         | 1.920       | 0.3494          | 0.046          | 67.42                | 10.73                     |
| Polytropic EoS | Γ = 1.50     | 11.200        | 0.814       | 0.1081          | -              | -                    | -                         |
|              | Γ = 1.67        | 7.980         | 0.964       | 0.1790          | -              | -                    | -                         |
|              | Γ = 2.00        | 7.500         | 1.350       | 0.2600          | -              | -                    | -                         |

IV. DISCUSSION

In this study we have examined the possibilities of GWE from SSs for different EoSs. From this study it can be concluded that, not all SSs can emit GWEs. SSs with MIT Bag model and linear EoS which fulfil the criterion for compactness and posses photon sphere can only emit GWEs. These frequencies are found to be in the range of about tens of kilohertz. Whereas SSs with polytropic EoS cannot give required compact configuration to echo GWs, hence no echo frequency is observed for polytropic SSs. The echo frequency and repetition frequency of GWs have shown a distinctive variation with different values of Bag constant B and linear constant b. It increases with the increase in values of B whereas decreases with the increase in values of b. So a model-dependent nature of GWE frequencies is observed. Again for smaller B value we have obtained larger structures. On the other hand larger b values are corresponding to larger SS structures. In Table I and Fig. 1 these results are summarised. In the first panel of Fig. 1, the comparison of mass-radius relations of stars among all three EoSs is shown. In the second and third plots mass-radius relationships are showing for MIT Bag model and linear EoS respectively.

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