CHARGINO MASS AND $R_b$ ANOMALY. *

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Abstract

We re-examine the possible magnitude of the supersymmetric contribution to $R_b$ with imposed all available phenomenological constraints and demanding good quality of the global fit to the precision electroweak data. For low $\tan \beta$ we find a new region of the parameter space, with $M_2 \approx |\mu|$ and $\mu < 0$ where $R_b$ remains large, $\sim 0.2180$ even for the lighter chargino as heavy as 90 – 100 GeV. It is an interesting mixture of the up-higgsino and gaugino. The rôle of various phenomenological constraints is discussed in analytic form and importance of small but non-negligible left-right mixing in the stop sector is emphasized in this context. The large $\tan \beta$ option for enhancement of $R_b$ is also reviewed. The available data do not rule out this scenario.

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1. INTRODUCTION.

A considerable excitement has recently been inspired by the $R_b$ and $R_c$ anomaly \cite{1,2,3,4,5,6,7,8,9}. The successful tests of the Standard Model (SM), to a per mille level \cite{10}, are challenged by the measurements of the partial widths of the $Z^0$ decays into $\bar{b}b$ and $\bar{c}c$ quarks which disagree with the SM predictions for $m_t = 180(170)$ GeV at the level of 3.7(3.5) and 2.5(2.5) standard deviations, respectively \cite{11}. If both results are confirmed, the SM and its simplest supersymmetric extension, the Minimal Supersymmetric Standard Model (MSSM), are ruled out. However, since the $R_b$ anomaly is statistically more significant, it is also of interest to discuss the possibility of explaining only the larger than in the SM value of $R_b$.

Even if $R_c$ is fixed to its SM prediction, $R_c = 0.172$ the then measured value of $R_b = 0.2206 \pm 0.0016$ is still 3 standard deviations away from its SM value. The issue has been addressed in particular in the framework of the MSSM \cite{2,3,4,5,6,7,8,9}. It is well known already for some time that in the MSSM there are new contributions to the $Z^0\bar{b}b$ vertex which can significantly enhance the value of $R_b$ (but not $R_c$) if some superpartners are sufficiently light \cite{12,2,4,5,6,8,9}. More specifically, for low (large) $\tan \beta$ the dominant contributions are chargino–stop ($CP$–odd Higgs boson and chargino–stop) loops.

Any improvement in $R_b$ must not destroy the perfect agreement of the SM with the other precision LEP measurements and must be consistent with several other experimental constraints (which will be listed later on). It is, therefore, important to discuss the changes in $R_b$ in the context of global fits to the electroweak data (and with all additional constraints included). Such fits in the effective low energy MSSM (unconstrained by any GUT assumptions about the pattern of soft supersymmetry breaking scalar masses) have shown that it is realistic to obtain the values of $R_b$ up to $R_b = 0.2180(0.2190)$ for small (large) $\tan \beta$ values \cite{5}. Although still away by 1.5$\sigma$ (1.0$\sigma$) even from the central experimental value obtained with $R_c$ fixed to its SM value, those results provide an interesting improvement over the SM prediction $R_b = 0.2158 (0.2160)$ for $m_t = 180 (170)$ GeV. At the same time the overall best $\chi^2$ is smaller than in the SM fits by $\Delta \chi^2 \approx 4$ (5 for the fit with $R_c$ fixed to the SM value) (for $m_t = 170$ GeV). Also, the fitted value of $\alpha_s(M_Z)$ is modified by $\Delta \alpha_s(M_Z) \approx -4 \delta R_b$, i.e. lower than in the SM fits which give $\alpha_s(M_Z) = 0.123 \pm 0.005$ \cite{13,3,14,10}. This may look desirable \cite{7} in view of the apparent hint for some discrepancy between the value of $\alpha_s(M_Z)$ obtained from the SM fits to the precision electroweak data and the values $\alpha_s(M_Z) = 0.112 \pm 0.005$ and $\alpha_s(M_Z) = 0.112 \pm 0.007$ obtained from the deep inelastic scattering \cite{15} and lattice calculations \cite{16}, respectively \cite{1}. Furthermore, increase of $R_b$ in the MSSM implies some light

\footnote{The overall average gives $\alpha_s(M_Z) = 0.117 \pm 0.005$ and includes also the results}
superpartners, with masses of the order or even below the electroweak scale $M_Z$ ! (It is may be worth noting small differences between various viewpoints: One is to explain the measured value of $R_b$. The other is to take it as a statistical hint for values of $R_b$ somewhat higher then predicted by the SM. It is reasonable to take the latter and to consider an increase in $R_b$ in the range 0.2170–0.2180(90) as interesting.)

In this paper, encouraged by the importance of the $R_b$ anomaly for experimental search for supersymmetry, we take up this issue once again. We clarify certain points of the earlier analysis and provide further insight into properties of light superpartners predicted by such global fits. Furthermore, we clarify the rôle of various experimental constraints, including the recent new limits on the chargino mass from LEP1.5. Our main new result is that for low $\tan \beta$ the $\chi^2$ of the global fit and the value of $R_b$ depend very weakly on the chargino mass (for fixed stop mass) in the range 50–100 GeV. The $R_b$ remains at the level of 0.2178 for $m_{C_1}$ up to 90 GeV in the region where $M_2 \sim |\mu|$ and $\mu < 0$. We also discuss large $\tan \beta$ case.

2. LOW $\tan \beta$ REGION.

Our discussion will be divided into small and large $\tan \beta$ cases. To start with, let us, however, recall the basic facts from the global fits in the MSSM. In order to maintain good agreement of the SM with the bulk of the precision data, such as $\Delta \rho$, $M_W$, $\sin^2 \theta_{\text{eff}}^\text{lept}$, ... we must avoid new sources of the custodial $SU_V(2)$ symmetry breaking in the left currents. In the MSSM, this is assured when the left squarks of the third generation (and all left sleptons) are sufficiently heavy \footnote{\textsuperscript{2}The actual lower limits on left squarks and sleptons depend on $\tan \beta$ value \footnote{\textsuperscript{3}Such a hierarchy is very natural in models where soft scalar mass terms have their origin at the GUT scale. Even with universal initial squark mass values $m_{0}^2$, the renormalization group evolution with large top quark Yukawa coupling gives the hierarchy \footnote{\textsuperscript{4}} provided $m_{0}^2 \gg M_2^2$.}}\footnote{\textsuperscript{1}} \textsuperscript{1}, say, $> \mathcal{O}(500 \text{ GeV})$. At the same time, an increase in $R_b$ requires a light right stop. So, one needs a hierarchy \footnote{\textsuperscript{2}}

\[ M_{t_L} \gg M_{t_R} \quad \text{or} \quad M_{\tilde{t}_1} \gg M_{\tilde{t}_2} \tag{1} \]

(in our notation $\tilde{t}_2$ denotes the lighter stop) with small left-right mixing.

The second important fact to recall is the pattern of the chargino sector (masses and mixings). The chargino mass matrix

\[ \mathcal{L}_{\text{mass}} = -\frac{1}{2}(\chi^+, \chi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \chi^+ \\ \chi^- \end{pmatrix} + h.c. \tag{2} \]

with

\[ X = \begin{pmatrix} M_2 \\ \sqrt{2} M_W \cos \beta \\ \sqrt{2} M_W \sin \beta \end{pmatrix} \tag{3} \]

obtained from $\tau$ decay and jet physics \footnote{\textsuperscript{4}}.
is diagonalized by two unitary matrices $Z_+$ and $Z_-$: $Z^T X Z_+ = \text{diag}(m_{C_1}, m_{C_2})$ with $0 < m_{C_1} < m_{C_2}$ (we follow the convention and notation of ref. [20]) which determine the projection of the physical two-component states $\lambda^\pm (i = 1, 2)$ on the gaugino and higgsino two-component weak eigenstates $(-i\psi^+, h_2^+, -i\psi^-, h_1^-) \equiv (\chi^+, \chi^-)$

$$h_2^+ = Z_{+i}^2 \lambda_1^+, \quad h_1^- = Z_{-i}^2 \lambda_1^-$$

(4)

$$\psi^\pm = iZ_{\pm i}^1 \lambda_1^\pm$$

(5)

with the Dirac charginos defined as

$$C_i^- = \left( \frac{\lambda_1^-}{\lambda_i^-} \right)$$

(6)

It is important to notice that a physical state $C_i^-$ may contain different admixtures of the "up" and "down" higgsinos ($h_2$ and $h_1$ respectively). For instance, it may be almost pure up-higgsino (in which case $|Z_{+i}^2| >> |Z_{\pm i}^1|$) in its lower two-component spinor $\lambda_1^+$ and almost pure gaugino ($|Z_{\pm i}^2| << |Z_{\pm i}^1|$) in its upper two-component spinor $\lambda_1^-$. For small values of $\tan \beta$ the pattern of the gaugino-higgsino mixing in the physical states is not just determined by the ratio $r \equiv M_2/|\mu|$ but also crucially depends on the sign of $\mu$. In addition, for fixed $M_2$ and $\mu$ also the pattern of the physical masses depends on the sign of $\mu$ in a crucial way as shown in Fig.1. For better qualitative understanding of those points it is instructive to consider the chargino masses and mixing for $r = 1$. For both signs of the $\mu$ parameter and any value of $\tan \beta$ the chargino mass matrix (3) can be easily transformed into symmetric form which is diagonalized by an unitary (orthogonal in our case of real $\mu$ and $M_2$) matrix i.e. $Z_+^T = Z_-^T$. With both eigenvalues positive and ordered, $m_{C_1} < m_{C_2}$, (we use the fact that $\tan \beta > 1$), the complete diagonalizing matrices read:

for $\mu > 0$

$$Z_- = \begin{pmatrix} -c_\theta & s_\theta \\ s_\theta & c_\theta \end{pmatrix} \quad Z_+ = \begin{pmatrix} -s_\theta & c_\theta \\ c_\theta & s_\theta \end{pmatrix}$$

(7)

for $\mu < 0$

$$Z_- = \begin{pmatrix} s_\theta & -c_\theta \\ c_\theta & s_\theta \end{pmatrix} \quad Z_+ = \begin{pmatrix} c_\theta & -s_\theta \\ -s_\theta & -c_\theta \end{pmatrix}$$

(8)

where $s_\theta \equiv \sin \theta$, $c_\theta \equiv \cos \theta$. For the angle $\theta$ we get

for $\mu > 0$

$$\tan 2\theta = -\frac{2\mu}{\sqrt{2}M_W (\sin \beta - \cos \beta)} < 0$$

(9)
for $\mu < 0$

$$\tan 2\theta = -\frac{2\mu}{\sqrt{2}M_W (\sin \beta + \cos \beta)} > 0 \quad (10)$$

We see that for low values of $\tan \beta$ the two cases are very different. For $\mu > 0$ we get $\theta \sim \pi/4$ and $c_\theta \sim s_\theta \sim 1/\sqrt{2}$. Thus all two-component states are mixed. Moreover, the mass eigenstates are

$$m_{C_{1,2}} = \mu \mp \frac{1}{\sqrt{2}}M_W (\sin \beta + \cos \beta) + \frac{1}{4}(1 - \sin 2\beta)\frac{M_W^2}{\mu} + ... \quad (11)$$

for $\mu > M_W$ (smaller $\mu$ values are excluded by experimental constraints) and charginos are split in mass by $\Delta m \sim 2M_W$. Pure up-higgsino can only be obtained for $r \gg 1$. One can then check that the lighter chargino is higgsino-like in both two-component spinors i.e. $|Z_{11}^+| >> |Z_{11}^0|$ and the second chargino is heavy.

For $\mu < 0$ and $|\mu| \ll M_W$ we get $\theta \sim 0$ and $c_\theta \sim 1$. The mass eigenvalues for low $\tan \beta$ are

$$m_{C_1} = \sqrt{2} \cos \beta M_W \left(1 + \frac{1}{2 \cos \beta (\sin \beta + \cos \beta) M_W} \frac{\mu^2}{M_W^2} + ...ight)$$

$$m_{C_2} = \sqrt{2} \sin \beta M_W \left(1 + \frac{1}{2 \sin \beta (\sin \beta + \cos \beta) M_W} \frac{\mu^2}{M_W^2} + ...ight) \quad (12)$$

First of all, low values of $\mu$ are indeed allowed for $m_{C_1} > 65$ GeV and the heavier chargino is still very light. Moreover, the heavier chargino is pure up-higgsino (gaugino) in its lower (upper) two-component spinor. For large values of $r$ and/or large values of $\tan \beta$ the difference between $\mu > 0$ and $\mu < 0$ disappears.

We can study now the supersymmetric contributions to the $Z^0\tilde{t}\bar{b}$ vertex. In the low $\tan \beta$ region there are two types of relevant diagrams: with stop coupled to $Z^0$ and with charginos coupled to $Z^0$. Their (renormalized) contributions to the vector and axial-vector formfactors of the $Z^0\tilde{t}\bar{b}$ vertex $F_V$ and $F_A$ ($L_{Z^0\tilde{t}\bar{b}}^{\mu\nu} = \overline{\psi}_b \gamma^\mu (F_V - \gamma^5 F_A) \psi_b Z_0^\nu$) are (in the limit $m_b = 0$) given by (the sum is over the two stop and two chargino mass eigenstates):

$$\delta F_V^{(i)} = \frac{e}{4s_W c_W} \sum_{n,m,l} V_L^{l,n}_{Z\tilde{t}} \left(L_{b\tilde{t}C}^{l,m} F_{n,m} + R_{b\tilde{t}C}^{l,m} F_{n,m}^{*}\right)$$

$$\times f_{ssf}(M_Z^2; M_{\tilde{t}_i}, m_{C_{m}}, M_{\tilde{t}_i}) \quad (13)$$

$$\delta F_A^{(i)} = \frac{e}{4s_W c_W} \sum_{n,m,l} V_L^{l,n}_{Z\tilde{t}} \left(L_{b\tilde{t}C}^{l,m} F_{n,m} - R_{b\tilde{t}C}^{l,m} F_{n,m}^{*}\right)$$

$$\times f_{ssf}(M_Z^2; M_{\tilde{t}_i}, m_{C_{m}}, M_{\tilde{t}_i}) \quad (14)$$
diagrams with stops coupled directly to $Z$

coupling to $Z$

in which charginos are coupled to $Z$

$R$ enhancement of $t$

$s$ where $R$

The top squark mixing matrix $T^{ij}$ is defined by:

$$
\begin{pmatrix}
\tilde{t}_L \\
\tilde{t}_R
\end{pmatrix} =
\begin{pmatrix}
T^{11} & T^{12} \\
T^{21} & T^{22}
\end{pmatrix}
\begin{pmatrix}
\tilde{t}_1 \\
\tilde{t}_2
\end{pmatrix} =
\begin{pmatrix}
c_t & -s_t \\
s_t & c_t
\end{pmatrix}
\begin{pmatrix}
\tilde{t}_1 \\
\tilde{t}_2
\end{pmatrix}
$$

where $s_t \equiv \sin \theta_t$, $c_t \equiv \cos \theta_t$. In the case of small $\tan \beta$ the couplings $R^{n,l}_{b\bar{t}C}$ are negligible and we have $\delta F_V = \delta F_A$. When the lighter stop, $\tilde{t}_2$, is dominantly right-handed, as required for a large $b\bar{t}_2C$ coupling, its coupling to $Z^0$ is suppressed (it is proportional to $g \sin^2 \theta_W$). Therefore, diagrams with stops coupled directly to $Z^0$ cannot give any significant enhancement of $R_b$. Significant contribution can only come from diagrams in which charginos are coupled to $Z^0$. Their actual magnitude depend on
the interplay of the couplings in the \( C_1^- \tilde{b}_2 \) vertex and the \( Z^0 C_i^- C_j^- \) vertex. The first one is large only for charginos with large up-higgsino component, the second - for charginos with large gaugino component in at least one of its two-component spinors. As we have seen, this combination never happens for \( \mu > 0 \). Large \( R_b \) can then only be achieved at the expense of extremely light \( C_j^- \) and \( \tilde{t}_2 \), either already ruled out by the existing mass limits or in conflict with global \( \chi^2 \) and/or other constraints such as \( BR(b \rightarrow s\gamma) \), \( BR(t \rightarrow new) \). In addition, for fixed \( m_{C_1} \) and \( M_{i_2} \), \( R_b \) is larger for \( r > 1 \) i.e. for higgsino-like chargino as the enhancement of the \( C_1^- \tilde{t}_2 b \) coupling is more important than of the \( Z^0 C_1^- C_1^- \) coupling.

For \( \mu < 0 \) the situation is much more favourable. In the range \( r \approx 1 \pm 0.5 \) the second chargino which for low \( \tan \beta \), is very close in mass to the lighter one (Fig.1), has large up-higgsino component and gaugino component. Large couplings in both types of vertices of the diagram with charginos coupled to \( Z^0 \) give significant increase in \( R_b \) even for the lighter chargino as heavy as \( 80 - 90 \) GeV (similar increase in \( R_b \) for \( \mu > 0 \) requires \( m_{C_1} \approx 50 \) GeV and \( M_{i_2} \approx 50 \) GeV).

We now turn our attention to a global fit to the precision data and to the rôle of the following constraints: 1) \( \Gamma(Z^0 \rightarrow \chi_i^0 \chi_j^0) < 4 \) MeV (in addition to the inclusion of the decay mode \( Z^0 \rightarrow \chi_i^0 \chi_j^0 \) into the total \( Z^0 \) width in the \( \chi^2 \) fit), 2) \( BR(Z^0 \rightarrow \chi_i^0 \chi_j^0) < 10^{-4} \), 3) \( M_h > 60 \) (50) GeV for small (large) \( \tan \beta \), 4) \( BR(b \rightarrow s\gamma) \) in the range \((1.2 - 3.4) \times 10^{-4} \), 5) \( BR(t \rightarrow new) < 45\% \) (following ref. \[21\]) \[4\]), 6) Recent exclusion curves in the \((m_{N_1}, M_{i_2})\) plane from \( D0 \) obtained under the assumption \( m_{C_1} > M_{i_2} \). Since the uncertainty in most of these constraints is much larger than in the precision data, we perform the \( \chi^2 \) fit to the latter and impose the former as rigid constraints. Importance of the constraints is illustrated in Figs. 2 and 3 and should be discussed in more detail.

The limits \( \Gamma(Z^0 \rightarrow \chi_i^0 \chi_i^0) < 4 \) MeV and \( BR(Z^0 \rightarrow \chi_i^0 \chi_2^0) < 10^{-4} \) put a constraint on the \((M_2, \mu )\) parameter space provided we make an additional assumption about the values of \( M_1 \) (bino mass). In this paper, for a sake of definiteness, we adopt the GUT relation \( M_1 = (5/3)\tan^2\theta_W M_2 \). Then for low \( \tan \beta \) the two constraints eliminate (approximately) a band in the \( M_2, \mu \) plane bounded by \( -50 < \mu < 100 \) GeV. This is basically due to the fact that for \( M_2 \) and \( \mu \) in this region neutralinos are too light and/or have too strong coupling to \( Z^0 \). The values of \( M_2 \) and \( \mu \) chosen for Figs. 2,3 and 7 are outside the forbidden region. One should remember, however, that the limits on \( Z^0 \rightarrow N_1^0N_1^0 \) and \( Z^0 \rightarrow N_1^0N_2^0 \) are less constraining for larger values of \( M_1, M_1 > 0.5 M_2 \) (i.e. heavier LSP). Similarly, the

\[4\] This is a constraint mainly on the decay \( t \rightarrow \tilde{t}_2 N_1^0 \). In the large \( \tan \beta \) case, an important decay can also be \( t \rightarrow bH^+ \), with \( M_{H^±} \) close to \( M_W \). It is difficult to distinguish this decay mode from the standard one, \( t \rightarrow bW^+ \) \[22\]) and to put an experimental upper bound on this branching ratio.
bounds on $M_{t\tilde{t}}$ from the D0 exclusion curves gradually disappear for $M_1 > 0.5M_2$. Finally, the larger the $M_1$ the better the degeneracy between the lighter chargino and the lightest neutralino masses. This is important since the new LEP1.5 limit $m_{C_1} > 65$ GeV has been obtained under the assumption $m_{C_1} - m_{N_1} > 10$ GeV [24]. We see that the significance of various constraints crucially depends on the ratio $M_1/M_2$. With the GUT assumption, our results remain on the conservative side.

The role of the lower limit on the Higgs boson mass (for a compact formula for radiatively corrected lighter Higgs boson mass in the limit $M_A >> M_Z$ see [23]) depends on the mass of the heavier stop and the left-right mixing angle. For $M_{\tilde{t}_1} > 500$ GeV (as required for good quality of the global fit) and small mixing angles (necessary for large $R_b$) $M_h$ is above the experimental limit in a large range of the parameter space. Very small and large left-right mixing angles are, however, ruled out by this constraint. This is clearly seen in Figs. 2 and 3 where we show the allowed region in the $(M_{t\tilde{t}}, \theta_t)$ plane for fixed $M_2$, $\mu$ and $\tan \beta$.

The $b \to s\gamma$ decay is a very important constraint on the parameters space. In addition to the experimental error in the $BR(b \to s\gamma)$ (which we take at the $2\sigma$ level) there is large uncertainty in the theoretical prediction mainly due to its renormalization scale dependence [24]. In Figs. 2 and 3 we show the significance of the $b \to s\gamma$ constraint in two cases: when the renormalization scale $Q$ is fixed, $Q = m_b = 4.7$ GeV and with the theoretical uncertainty included. In the latter case we consider the result for the $BR(b \to s\gamma)$ as acceptable if any of the theoretical values obtained with $Q$ varying from $m_b/2$ to $2m_b$ falls into the $2\sigma$ experimental range. Moreover, the results for $BR(b \to s\gamma)$ show weak but nonnegligible dependence on the value of $\alpha_s(M_Z)$. Since the fitted value $\alpha_s(M_Z)$ depends on the change in $R_b$ in Figs. 2 and 3 for self-consistency we use the value $\alpha_s(M_Z) = 0.123 + \delta \alpha_s$ where $\delta \alpha_s = -4\delta R_b$ and the value 0.123 is obtained from the SM fit [13, 14, 10]. For comparison, in Fig. 2, we also show the regions excluded by this constraint with $\alpha_s(M_Z)$ fixed to two different values 0.114 and 0.135. An important message from Figs. 2 and 3 is the dependence of the $b \to s\gamma$ constraint on the left-right mixing in the stop sector. One should stress that we keep the mass of the CP-odd Higgs boson large, $M_A = 1$ TeV (as needed for large $R_b$) and in consequence the charged Higgs boson is also heavy and its exchange gives negligible contribution to the $BR(b \to s\gamma)$. The acceptable values of this branching ratio are obtained from the sum $|A_W^{b\to s\gamma} + A_{SUSY}^{b\to s\gamma}|$ of the SM $W^\pm$ exchange and the supersymmetric contribution of the $\tilde{t} - C^-$ loops which has to be of the opposite sign. There are two possible solutions: either

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5Important rôle of the experimental lower bound on $M_h$ in ref. [8] in constraining the potential increase of $R_b$ is due to the chosen upper bound $M_{t\tilde{t}} < 250$ GeV which, anyway, looks too low from the point of view of global fit.
\[ |A_{SUSY}^{b\to s\gamma}| \ll |A_W^{b\to s\gamma}| \] or \[ |A_{SUSY}^{b\to s\gamma}| \gg |A_W^{b\to s\gamma}|. \] The strong dependence of the supersymmetric contribution to the \( b \to s\gamma \) rate on the mixing angle \( \theta_t \) observed in Fig. 2 and 3 can be understood from the general formulae given in [27]. For \( r = 1 \) and small angles \( \theta_t \) (interesting values of \( \Delta R_6 \) are obtained only for small mixing angles \( \theta_t \); large angles are anyway eliminated by the constraint from the lighter Higgs boson mass), setting \( \sin^2 \theta_t \approx 0 \), neglecting the contribution of the heavier stop \( \tilde{t}_1 \) and using the explicit form of the matrices \( Z_+ \) and \( Z_- \), eqs. (18), we get the expression:

\[
A_{\gamma/gl, SUSY}^{b\to s\gamma} = -g_t^2 \frac{M_W^2}{m_{C_1}^2} f_{\gamma/gl}^{(1)} \left( \frac{M_{t_2}^2}{m_{C_1}^2} \right) - g_t^2 s_\theta g_t \left[ f_{\gamma/gl}^{(1)} \left( \frac{M_{t_2}^2}{m_{C_1}^2} \right) + \sigma \frac{M_W^2}{m_{C_2}^2} f_{\gamma/gl}^{(1)} \left( \frac{M_{t_2}^2}{m_{C_2}^2} \right) \right] + \sin 2\theta_t s_\theta g_t \left[ f_{\gamma/gl}^{(1)} \left( \frac{M_{t_2}^2}{m_{C_1}^2} \right) \right] \] (21)

where \( \sigma \equiv \text{sign}(\mu) \), \( g_t \equiv m_t / \sqrt{2} M_W \sin \beta \) and the expressions for the functions \( f_{\gamma/gl}^{(i)}(x) \) can be found in [27]. Recalling that, for the same values of the arguments, functions \( f_{\gamma/gl}^{(i)}(x) \) are roughly 5 times larger in absolute values than the \( f_{\gamma/gl}^{(1)}(x) \)'s (and both are negative) it is easy to see that for \( \mu > 0 \) \( A_{SUSY}^{b\to s\gamma} \) is small for \( \theta_t > 0 \) due to the cancellation of terms of order \( g_t^2 \) with those of order \( g_t \). For small negative \( \theta_t \) both types of terms add up and cancel with the standard \( W^\pm \) contribution making the total amplitude unacceptably small. Eventually, for large negative angles, the supersymmetric contribution overcomes the standard one making again the total amplitude of right magnitude. However this region is excluded by the experimental limits on the Higgs boson mass. For negative \( \mu \) the cancellation between terms of order \( g_t^2 \) and those of order \( g_t \), which makes \( A_{SUSY}^{b\to s\gamma} \) small enough, occurs for negative angles \( \theta_t \). For positive angles the supersymmetric contribution is large and cancels the standard one making the total amplitude too small.

Thus, there are two mechanisms for obtaining an acceptable value for \( BR(b \to s\gamma) \) in the MSSM. One is a cancellation between the \( H^\pm \) and supersymmetric contributions [28]. This mechanism is, however, essentially in conflict with the simultaneous increase of \( R_6 \) due to large negative contribution of the \( H^\pm \) exchange to \( R_6 \). The other mechanism is based on the choice of the proper range of the left-right mixing angles. It is certainly interesting to notice that for small \( \tan \beta \) the region of acceptable \( BR(b \to s\gamma) \) partly overlaps with the region of large \( R_6 \). Small asymmetry (with respect to \( \theta_t = 0 \) of the constant \( \Delta R_6^{SUSY} \) contours seen in Figs. 2 and 3 can also be traced back to the \( \theta_t \) dependence of the couplings \( L_{ij}^{n,l} \) eqn. (14).

The exclusion curve from the condition \( BR(t \to \tilde{t}_2 N_1^0) < 45\% \) is also
shown in Fig. 2. The dependence of $BR(t \to \tilde{t}_2N_i^0)$ on the left-right mixing in the stop sector, i.e. on the angle $\theta_t$, can be understood from the formula for the decay width $\Gamma(t \to \tilde{t}_2N_i^0)$:

$$\Gamma(t \to \tilde{t}_2N_i^0) = \frac{e^2}{s_W^2 c_W^2 64\pi} \frac{m_t}{s_W c_W} \sqrt{1 - 2(x_k + y_i) + (x_k - y_i)^2} \times \left[ (|c_L^{ki}|^2 + |c_R^{ki}|^2)(1 + y_i - x_k) + 4Re(c_L^{ki} c_R^{ki}) \sqrt{|y_i|} \right]$$  \hspace{1cm} (22)

where $x_k \equiv M_{t_k}^2/m_t^2$, $y_i \equiv m_{N_i}^2/m_t^2$ and the couplings read \cite{20}:

$$c_L^{ki} = \left( \frac{1}{3} s_WZ_{1i}^i + c_WZ_{2i}^i \right) T^{1k} + \frac{m_t}{\sin \beta M_Z} Z_{4i}^i T^{2k}$$

$$c_R^{ki} = -\frac{4}{3} s_WZ_{2i}^i T^{2k} + \frac{m_t}{\sin \beta M_Z} Z_{4i}^i T^{1k}$$  \hspace{1cm} (23)

and $Z_{4i}^i$ diagonalizes the neutralino mass matrix \cite{20}. For example, for $\mu < 0$ and $r = 1$, $\Gamma(t \to new)$ is dominated by $\Gamma(t \to \tilde{t}_2N_2^0)$ because the second neutralino has larger up-higgsino component. In that case

$$c_L^{22} \approx \frac{m_t}{\sin \beta M_Z} Z_{N_2}^{42} \cos \theta_t$$

$$c_R^{22} \approx -\frac{m_t}{\sin \beta M_Z} Z_{N_2}^{42} \sin \theta_t$$  \hspace{1cm} (24)

and

$$\Gamma(t \to \tilde{t}_2N_2^0) = \frac{e^2}{64\pi s_W^2 c_W^2 M_Z^2 \sin^2 \beta} Z_{N_2}^{42} \sqrt{1 - 2(x_2 + y_2) + (x_2 - y_2)^2} \times \left[ (1 + y_2 - x_2) - 2 \sin 2\theta_t \sqrt{|y_2|} \right]$$  \hspace{1cm} (25)

explaining larger $BR(t \to new)$ for negative $\theta_t$ seen in Fig. 2.

The $D0$ exclusion contours \cite{23} in the $(M_{t_2}, m_N^0)$ plane eliminate in Fig. 2 a band 70 GeV $< M_{t_2} < m_{C_1}$ (not shown) and a similar band in Fig. 3. This follows from Fig. 10 in ref. \cite{23} (in Fig. 2 $M_2 = -\mu = 58$ GeV, $m_{C_1} = 85$ GeV and $m_{N_1} \approx 30$ GeV (with the GUT assumption)). Finally, it is worth recalling that

$$M_{t_2}^2 = M_{t_R}^2 - \theta_t^2 M_{t_L}^2, \hspace{0.5cm} |\theta_t| = \left| \frac{m_t A_t}{M_{t_L}^2} \right| \ll 1$$  \hspace{1cm} (26)

where

$$M_{t_R}^2 = m_T^2 + m_t^2 + \frac{2}{3} \cos 2\beta (M_Z^2 - M_W^2)$$  \hspace{1cm} (27)

($m_T^2$ is the soft supersymmetry breaking mass term). If we required $m_T^2 > 0$ then with $M_{t_L} \approx 1$ TeV we get $|\theta_t| > 9^\circ$ for $M_{t_2} \sim \mathcal{O}(100$ GeV) and with
\[ M_{Z_L} \approx 1.5 \text{ TeV} \quad |\theta_t| > 6^\circ. \] The corresponding values of \( A_t \) are \( \sim 1 \) TeV and \( \sim 600 \) GeV respectively. The theoretical status of the condition \( m_T^2 > 0 \) remains, however, unclear and we do not impose it on our global fit (with this condition imposed the fit would not have changed too significantly in particular given the freedom in the choice of \( M_{L_L} \)).

The results of a global fit are presented in Figs. 4-6 as projections of \( \chi^2 \) as a function of \( m_{C_1} \) for several values of \( r \) and two different lower bounds for the scan in \( M_{L_L}^2 \). We also show the best values of \( R_b \) obtained with the restriction \( \chi^2 < \chi^2_{\text{min}} + 1 \) where \( \chi^2_{\text{min}} \) is the minimum of the \( \chi^2 \) for fixed \( m_{C_1} \). The fitting procedure is the same as in ref. [5]. In particular the fitted parameters include \( m_t, \tan \beta, M_2, \mu, \alpha_s(M_z) \), \( M_{\tilde{t}_1} \), and \( A_t \). The values of the parameters \( m_{C_1}, M_{A_0}, M_{\tilde{l}_1} \) provided large enough, are not relevant for the quality of the fit and we have fixed them at 1 TeV. Similarly, the parameters like \( M_{L_R}, M_{R_R}, \) masses and mixings of the two first generations of squarks to which the fit is not sensitive have been fixed at 1 TeV (in the large \( \tan \beta \) case the best fit is obtained for \( M_{R_R} = 130 \) GeV due to additional contribution of \( N_{i}^o - \tilde{b}_R \) loops to \( R_b \)).

The value of \( m_t = 170 \) GeV chosen for the plots is close to the best value obtained from the fit. Larger values of \( m_t \) give worse \( \chi^2 \) and this is a reflection of the well known from the SM fits correlation between the Higgs boson and the top quark masses [10, 14]. Here the \( M_h \) is constrained by supersymmetry and cannot follow the increase of \( m_t \). In our fit \( \tan \beta \) is bounded so that top quark Yukawa coupling remains perturbative up to the GUT scale (we take \( \tan \beta \geq 1.4 \) for \( m_t = 170 \) GeV and \( \tan \beta \geq 1.6 \) for \( m_t = 180 \) GeV). We impose this theoretical constraint to remain on the conservative side, as lower values of \( \tan \beta \) (for the same \( m_t \)) give larger \( R_b \). For a given \( m_t \) the best fit is obtained for the lowest value of \( \tan \beta \) (for very large \( \tan \beta \) see next section). One may also wish to impose the constraint \( M_2 \geq 50 \) GeV which follows from the lower experimental bound on the gluino mass \( m_{\tilde{g}} > 150 \) GeV and the assumed GUT relation for the gaugino masses. This constraint does not change the best values of \( R_b \) and \( \chi^2 \) (but it eliminates e.g. large part of our curve with \( r = 0.5, \mu < 0 \)).

All discussed earlier constraints are included in those results. The fit reveals several interesting facts. For both signs of \( \mu \), the value of \( \chi^2 \) eventually increases towards low values of \( m_{C_1} \). The reason for this behavior of \( \chi^2 \) is the contribution of the neutralinos to \( \Gamma_Z \) which becomes too large and correspondingly \( \sigma_{\text{hadr}} \) too small (at this point we should again remember about our assumption \( M_1 = (5/3) \tan^2 \theta_W M_2 \)). This effect is much stronger than the decrease of \( \Gamma_Z \) due to the chargino contribution to the \( Z^0 \) boson wave function renormalization [30]. So, the increase of \( R_b \) is bounded not only by the lower experimental limits on \( m_{C_1}, M_{\tilde{t}_1} \), but also by the quality of the fit to the precision data. Still, for \( \mu > 0 \), the best fit is obtained for \( m_{C_1} \sim 50 \) GeV, well below the new experimental limit from
LEP1.5, $m_{C_1} > 65$ GeV \[24\]. This limit is valid for $m_{C_1} - m_{N_1} > 10$ GeV. In principle, the degeneracy of the chargino and neutralino masses can be better than 10 GeV. This occurs for $r > 10$ with $M_1 \approx 0.5M_2$ (note however, that supersymmetric contribution to $R_b$ decreases for very large values of $r$) or for any value of $r$ for sufficiently large $M_1$. This is an obvious possibility which we do not discuss further. On the other hand, for $m_{C_1} = 65$ GeV the maximal reachable value of $R_b$ is only $R_b = 0.2173$.

For $\mu < 0$ due to the mechanism explained earlier, $\chi^2(R_b)$ is small (large) for much larger values of $m_{C_1}$. In fact the new LEP1.5 limit is totally irrelevant in this case. A chargino with mass $70 - 90$ GeV and with the composition described by $M_2 \approx |\mu|$ remains an interesting potential possibility. The dependence of $R_b$ on the stop mass can be inferred from comparison of Fig. 4 and 5. We see that, even for $M_{\tilde{t}_2} = m_{C_1} = 90$ GeV, $R_b > 0.2175$. Moreover, a significant enhancement in $R_b$ is consistent with both configurations: $M_{\tilde{t}_2} > m_{C_1}$ and $M_{\tilde{t}_2} < m_{C_1}$. In Fig. 1 we show the cross sections for chargino production as $\sqrt{s} = 190$ GeV. The corresponding cross sections for the production of the pair $N_1^0N_2^0$ are also shown.

3. LARGE $\tan \beta$.

Significant enhancement of $R_b$ is also possible for large $\tan \beta$ values, $\tan \beta \approx m_t/m_b$. In this case, in addition to the stop - chargino contribution there can be even larger positive contribution from the $h^0$, $H^0$ and $A^0$ exchanges in the loops, provided those particles are sufficiently light (in this range of $\tan \beta$ $M_h \approx M_A$) and non-negligible sbottom-neutralino loop contributions. Recently it has been argued \[8\] that large $\tan \beta$ option of an enhancement in $R_b$ is already ruled out by the constraints from $b \rightarrow c\tau\nu\tau$ and the absence of 4$b$ events in the $Z^0$ decay which should have been present as a signature for the bremsstrahlung production of a light pseudoscalar Higgs boson $A^0$ at LEP1: $Z^0 \rightarrow b\bar{b}A^0 \rightarrow b\bar{b}b\bar{b}$. It has been claimed that the combination of the two constraints rules out the region in the $(M_A$, $\tan \beta)$ plane which can give substantial increase of $R_b$ (see Fig. 2 in ref. \[8\]). It seems, however, that this conclusion cannot be maintained after proper account of experimental efficiencies in the search for the process $Z^0 \rightarrow b\bar{b}b\bar{b}$. The decay $b \rightarrow c\tau\nu\tau$ with $BR = 2.69 \pm 0.44$ % indeed puts an upper bound $\tan \beta < 0.52M_{H^\pm}/(1\text{GeV})$ \[22\] which translates into a bound on $M_A$ as a function of $\tan \beta$ which is given in ref. \[8\]. For instance, for $M_A = 55$ (65) GeV, allowed $\tan \beta < 63$ (68). However, with proper account of the experimental efficiency in search for 4$b$ events and with $10^7$ events from LEP1, one cannot expect to get for, say, $\tan \beta = 60$, a lower limit on $M_A$ from this production mechanism better than about 40 GeV (since the four $b$ tagging procedure is not 100% efficient, the
main source of the QCD background are $\bar{b}bgg$ and $\bar{b}b\bar{q}q$ final states)\(^6\). This is well below the reported limit of 55 GeV based on the analysis of the process $Z^0 \to A^0 h^0$\(^{33}\). Thus, an enhancement of $R_b$ in the large $\tan \beta$ region still remains an open possibility and we briefly summarize the relevant results.

The discussion of large $\tan \beta$ case is relatively simpler as most of the results is symmetric under the simultaneous change of sign of $\mu$ and $\theta_t$. The contribution to the $Z^0\bar{b}b$ vertex from charginos-stop loops to the vector and axial-vector formfactors are (in the limit $m_b = 0$) given by eqs. \(^{13, 14}\). Compact expressions for the remaining contributions are collected in Appendix A.

In the large $\tan \beta$ case, for both signs of $\mu$ the chargino composition is the same in the up and down Weyl spinors and is a monotonic function of the ratio $r$. For enhancement in $R_b$ the $\tilde{b}_2 C$ coupling is more important than the $Z^0 CC$ coupling, so higgsino-like chargino ($r \gg 1$) is more favourable. The chargino masses are then given by

$$m_{C_1} \approx |\mu| \left( 1 - \frac{M_W^2}{M_Z^2} + ... \right)$$

$$m_{C_2} \approx M_2 \left( 1 + \frac{M_W^2}{M_Z^2} + ... \right)$$

The diagonalizing matrices have the form

$$Z_\pm = \begin{pmatrix} -\sin \theta_\pm & \cos \theta_\pm \\ \cos \theta_\pm & \sin \theta_\pm \end{pmatrix}$$

with

$$\tan 2\theta_- = \frac{2\sqrt{2} M_W \mu}{M_2^2 + 2M_W^2 - \mu^2}, \quad \tan 2\theta_+ = \frac{2\sqrt{2} M_W M_2}{M_2^2 - 2M_W^2 - \mu^2}$$

so that indeed for $M_2 \gg \mu, M_W$ both angles are close to zero. Notice however, that $\theta_+$ (whose smallness is important for the coupling $\tilde{b}_2 C$) approaches zero much slower than $\theta_-$ as $r$ is increased. The rôle of the experimental constraints (listed in the previous section) on the parameter space is illustrated in Fig. 7. The regions ruled out by the $BR(b \to s\gamma)$ are easy to understand. Neglecting the contribution of the heavier chargino and stop and restricting to small angles $\theta_t$ in the general formula of \(^{27}\) we get for the $b \to s\gamma$ amplitude the following expression

$$A_{\gamma/gl, SUSY}^{b \to s\gamma} \approx -g_t^2 \cos^2 \theta_+ \frac{M_W^2 f_{\gamma/gl}^{(1)}}{m_{C_1}^2} \left( \frac{M_2^2}{m_{C_1}^2} \right)$$

\(^6\)S. Simion and P. Janot, private communication. Such an experimental analysis has not yet been performed. We thank J. Kalinowski, M. Krawczyk, M. Carena and C. Wagner for several discussions of this point.
\[ + \sin 2\theta_t \left( \frac{\tan \beta}{2\sqrt{2}} \right) g_t \cos \theta_+ \cos \theta - \frac{M_W}{m_{C_1}} f^{(3)}_{\gamma/g} \left( \frac{M_{t_2}^2}{m_{C_1}^2} \right) \]

\[ + \frac{1}{2} \frac{m_t^2}{M_{H^\pm}^2} f^{(2)}_{\gamma/g} \left( \frac{m_t^2}{M_{H^\pm}^2} \right) \]

(31)

With very light \( A^0 \), the charged Higgs boson contribution is large and adds up (with the same sign) to the standard \( W^\pm \)-top exchange amplitude. Thus we need a cancellation from the supersymmetric part of eq. (32). Due to the presence of large \( \tan \beta \) factor, the second term is strongly dominant. Therefore, for \( \theta_t \leq 0 \) supersymmetric contribution to the \( b \to s\gamma \) amplitude is of the opposite sign to the standard contribution. Moreover, for angles \( \theta_t \) not very close to zero, the absolute value of the supersymmetric contribution exceeds the standard one. Thus, for a given chargino mass there are only two very narrow bands seen in Fig. 7 in the plane \( (\theta_t, M_{t_2}) \) where the total amplitude is acceptable: in the right one the total amplitude is of the same sign as the standard \( W^\pm \)-top and charged Higgs boson amplitudes and in the left band it is of the opposite sign. From the Figure 7 it is however clear that for large \( \tan \beta \), the \( b \to s\gamma \) rate does not constrain the value of \( R_b \) at all [5].

The lower experimental bound on the lightest Higgs boson mass in the large \( \tan \beta \) scenario is \( \sim 40 \text{ GeV} \). Since in the MSSM \( M_{A^0} \approx M_{h^0} \), our results are not constrained by this bound.

In the parameter space which gives enhancement in \( R_b \), also the decays \( t \to \text{new} \) are enhanced. In addition to \( t \to \tilde{t}_2N_1^0 \), important is also the decay \( t \to bH^+ \). For instance, for \( m_t = 170 \text{ GeV} \), \( \tan \beta = 50 \), \( m_{C_1} = 65 \text{ GeV} \) and \( M_A = 55 \) (65) GeV we get \( BR(t \to bH^+) \) ranging from 37(34)% up to 49(46)% depending on the stop sector parameters. One should remember, however, that the decay \( t \to bH^+ \) is not easily distinguishable from the standard one, \( t \to bW^+ \), for \( M_{H^+} \) close to \( M_W \) [22] and we do not impose this constraint in the global fit.

In Figs. 8 and 9 we present the results of a global fit in the large \( \tan \beta \) case, together with the corresponding values of \( \tilde{R}_b \). Two important features of the global \( \chi^2 \) is the strong decrease in the quality of the fit for \( m_t = 180 \text{ GeV} \) (compared to the best fit for \( m_t = 170 \text{ GeV} \)) and for light charginos (below 60 GeV). Those effects are stronger than similar effects for low \( \tan \beta \).

The values of \( R_b \) are almost insensitive to the value of \( M_{t_2} \) in the range 50–100 GeV and show the expected dependence on \( M_A \) and \( m_{C_1} \) with the maximal values for very light charginos. It is, therefore, worth recalling that the new limit \( m_{C_1} > 65 \text{ GeV} \) is based on the assumption \( m_{C_1} - m_{N_1} > 10 \text{ GeV} \). As for low \( \tan \beta \), better degeneracy of the two masses can be achieved for \( r > 10 \) and/or for \( M_1 > 0.5M_0 \). In this case also the quality of the fit with charginos below 65 GeV is improved as heavier neutralinos contribute less to \( \Gamma_Z \). It is also remarkable, that due to the combined effect of neutral
Higgs exchange and the chargino-stop together with the neutralino-sbottom contributions, the $R_b$ remains greater than 0.2175 for masses well above the present experimental limits. E.g., for $m_{C_1} \approx M_{\tilde{t}_2} \approx M_A \approx 70$ GeV, $R_b = 0.2178$.

4. CONCLUSIONS.

In this paper we re-examined the possible magnitude of the supersymmetric contributions to $R_b$, with imposed all available phenomenological constraints and demanding good quality of the global fit to the precision electroweak data. For low $\tan \beta$ we have found a new region of the parameter space, with $M_2 \approx |\mu|$ and $\mu < 0$ where $R_b$ remains large, $\sim 0.2180$, even for the lighter chargino as heavy as 90 GeV. It is an interesting mixture of the up-higgsino and gaugino. The role of various phenomenological constraints is discussed in detail in analytic form and importance of small but non-negligible left-right mixing in the stop sector is emphasized in this context.

The large $\tan \beta$ option for enhancement of $R_b$ is also summarized, with similar conclusions to those presented in the earlier papers. The available data do not rule out the possibility of large $R_b$ in the large $\tan \beta$ case.

We conclude that the new LEP1.5 limit, $m_{C_1} > 65$ GeV still leaves open the possibility of a supersymmetric explanation of $R_b$ up to 0.2180. We also conclude that $R_b > 0.2175$ even for $m_{C_1} = M_{\tilde{t}_2} = 90$ GeV both in small $\tan \beta$ and large $\tan \beta$ cases. Thus LEP2 may not resolve this question.

Finally, we stress that a good quality of the global fit requires the hierarchy $M_{\tilde{t}_2} \ll M_{\tilde{t}_1}$ (i.e. $M_{\tilde{t}_2} \ll M_{\tilde{t}_1}$). This hierarchy is natural if the low energy values of the soft squark masses have their origin in the renormalization group evolution from the GUT scale with the initial condition $m_0^2 \gg M_2^2$. 


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Here we collect formulae for the remaining contributions to the $Z^0 bb$ vertex.

The contribution from charged Higgs boson is given by:

$$
\delta F_V^{(t)} = \frac{e^3}{8 s_W c_W} \left[ \left( 1 - \frac{4}{3} s_W^2 \right) X_b - \frac{4}{3} s_W^2 X_t \right] f_{ffs}(M_Z^2; m_t, M_{H^\pm}, m_t) \\
- \frac{e^3}{8 s_W c_W} \left[ \left( 1 - \frac{4}{3} s_W^2 \right) X_t - \frac{4}{3} s_W^2 X_b \right] m_t^2 c_0(m_t, M_{H^\pm}, m_t)
$$

(32)

$$
\delta F_A^{(t)} = -\frac{e^3}{8 s_W c_W} \left[ \left( 1 - \frac{4}{3} s_W^2 \right) X_b + \frac{4}{3} s_W^2 X_t \right] f_{ffs}(M_Z^2; m_t, M_{H^\pm}, m_t) \\
- \frac{e^3}{8 s_W c_W} \left[ \left( 1 - \frac{4}{3} s_W^2 \right) X_t + \frac{4}{3} s_W^2 X_b \right] m_t^2 c_0(m_t, M_{H^\pm}, m_t)
$$

(33)

$$
\delta F_V^{(H^\pm)} = -\frac{e^3}{8 s_W c_W} \left( 1 - 2 s_W^2 \right) (X_b + X_t) f_{ssf}(M_Z^2; M_{H^\pm}, m_t, M_{H^\pm})
$$

(34)

$$
\delta F_A^{(H^\pm)} = \frac{e^3}{8 s_W c_W} \left( 1 - 2 s_W^2 \right) (X_b - X_t) f_{ssf}(M_Z^2; M_{H^\pm}, m_t, M_{H^\pm})
$$

(35)

where we used the abbreviations

$$
X_b \equiv \left( \frac{m_b}{M_Z} \tan \beta \right)^2, \quad X_t \equiv \left( \frac{m_t}{M_Z} \cot \beta \right)^2
$$

(36)

As is well known, contribution of the charged Higgs boson to $R_b$ is negative [12]. Positive contribution to $R_b$ is provided by the neutral Higgs bosons.

$$
\delta F_V^{(b,A^0)} = -\frac{e^3}{16 s_W^3 c_W^3} \left( 1 - \frac{4}{3} s_W^2 \right) X_b f_{ffs}(M_Z^2; m_b, M_{A^0}, m_b)
$$

(37)

$$
\delta F_A^{(b,A^0)} = \frac{e^3}{16 s_W^3 c_W^3} X_b f_{ffs}(M_Z^2; m_b, M_{A^0}, m_b)
$$

(38)

$$
\delta F_V^{(b,H^0,h^0)} = -\frac{e^3}{16 s_W^3 c_W^3} \left( 1 - \frac{4}{3} s_W^2 \right) X_b \left[ \cos^2 \alpha f_{ffs}(M_Z^2; m_b, M_{H^0}, m_b) + \sin^2 \alpha f_{ffs}(M_Z^2; m_b, M_{h^0}, m_b) \right]
$$

(39)
\[
\delta F^{(b, H^0, h^0)}_A = \frac{e^3}{16 s_W^2 c_W^3} \left[ \cos^2 \alpha f_{ffs}(M_Z^2; m_b, M_{H^0}, m_b) + \sin^2 \alpha f_{ffs}(M_Z^2; m_b, M_{H^0}, m_b) \right]
\]

(40)

\[
\delta F^{(b, A^0, h^0)}_V = 0
\]

(41)

\[
\delta F^{(b, A^0, H^0, h^0)}_A = -\frac{e^3}{4 s_W^2 c_W^4} X_b \left[ \cos^2 \alpha f_{ssf}(M_Z^2; M_{A^0}, m_b, M_{H^0}) + \sin^2 \alpha f_{ssf}(M_Z^2; M_{A^0}, m_b, M_{h^0}) \right]
\]

(42)

where \( \alpha \) is the neutral \( CP \)-even Higgs bosons mixing angle \[31\].

For completeness we display also formulae for the neutralino - sbottom contribution.

\[
\delta F^{(b)}_V = -\frac{e}{4 s_W c_W} \sum_{n,m,l} V_{Zb}^{l,m} \left( L_{bN}^{m,n,\ast} L_{bN}^{l,n} + R_{bN}^{m,n,\ast} R_{bN}^{l,n} \right)
\times f_{ssf}(M_Z^2; M_{b_m}, m_{N_n}, M_{b_i})
\]

(43)

\[
\delta F^{(N)}_V = -\frac{e}{2 s_W c_W} \sum_{n,m,l} \left( R_{ZNN}^{l,m} L_{bN}^{n,\ast} L_{bN}^{n,m} + L_{ZNN}^{l,m} R_{bN}^{n,\ast} R_{bN}^{n,m} \right)
\times f_{ffs}(M_Z^2; m_{N_n}, M_{b_m}, m_{N_l})
+ \frac{e}{2 s_W c_W} \sum_{n,m,l} \left( L_{ZNN}^{l,m} L_{bN}^{n,\ast} L_{bN}^{n,m} + R_{ZNN}^{l,m} R_{bN}^{n,\ast} R_{bN}^{n,m} \right)
\times m_{N_n} m_{N_l} c_0(M_Z^2; m_{N_n}, M_{b_m}, m_{N_l})
\]

(45)

\[
\delta F^{(N)}_A = -\frac{e}{2 s_W c_W} \sum_{n,m,l} \left( R_{ZNN}^{l,m} L_{bN}^{n,\ast} L_{bN}^{n,m} - L_{ZNN}^{l,m} R_{bN}^{n,\ast} R_{bN}^{n,m} \right)
\times f_{ffs}(M_Z^2; m_{N_n}, M_{b_m}, m_{N_l})
+ \frac{e}{2 s_W c_W} \sum_{n,m,l} \left( L_{ZNN}^{l,m} L_{bN}^{n,\ast} L_{bN}^{n,m} - R_{ZNN}^{l,m} R_{bN}^{n,\ast} R_{bN}^{n,m} \right)
\times m_{N_n} m_{N_l} c_0(M_Z^2; m_{N_n}, M_{b_m}, m_{N_l})
\]

(46)
where the couplings can be found in [20] and read:

$$L^{l,m}_{ZNN} = Z_N^{4l} Z_N^{4m} - Z_N^{3l} Z_N^{3m}$$
$$R^{l,m}_{ZNN} = Z_N^{3l} Z_N^{3m} - Z_N^{4l} Z_N^{4m*}$$

(47)

$$V^{l,m}_{Z\tilde{b}\tilde{b}} = B^{1l} B^{1m} - \frac{2}{3} s_W^2 \delta^{lm}$$

(48)

$$L^{n,l}_{\tilde{b}\tilde{b}N} = \frac{e}{\sqrt{2} s_W c_W} \left[ B^{1n} \left( \frac{1}{3} s_W Z_N^{1l} - c_W Z_N^{2l} \right) + \frac{m_b}{M_Z \cos \beta} B^{2n} Z_N^{3l} \right]$$

$$R^{n,l}_{\tilde{b}\tilde{b}N} = \frac{\sqrt{2} c_W}{3 c_W} B^{2n} Z_N^{3l*} + \frac{e}{\sqrt{2} s_W c_W} \frac{m_b}{M_Z \cos \beta} B^{1n} Z_N^{3l*}$$

(49)

where the matrix $Z_N$ diagonalizes the neutralino mass matrix and the matrix $B^{ij}$ for sbottom quarks is defined in the same way as the matrix $T^{ij}$ for stops in eq. [20].
APPENDIX B.

Here we give expressions for the functions $f_{ssf}$ and $f_{fss}$ needed to compute SUSY contributions to the $Z^0\bar{b}b$ vertex.

$$f_{ssf}(s, m_1, m_2, m_3) = -\frac{1}{2} + \frac{1}{2s} (a_0(m_1) + a_0(m_3) - 2a_0(m_2))$$
$$+ \frac{s - m_1^2 - m_3^2 + 2m_2^2}{2s} b_0(s, m_1, m_3)$$
$$+ \frac{m_2^2 + \frac{(m_1^2 - m_2^2)(m_3^2 - m_2^2)}{s}}{s} c_0(m_1, m_2, m_3)$$
$$- \frac{1}{4} b_0(0, m_2, m_1) - \frac{1}{4} (m_2^2 - m_1^2) b'_0(m_2, m_1)$$
$$- \frac{1}{4} b_0(0, m_2, m_3) - \frac{1}{4} (m_2^2 - m_3^2) b'_0(m_2, m_3)$$

$$f_{fss}(s; m_1, m_2, m_3) = \frac{1}{2} + \frac{s + m_1^2 + m_3^2 - 2m_2^2}{2s} b_0(s, m_1, m_3)$$
$$- \frac{1}{2s} (a_0(m_1) + a_0(m_3) - 2a_0(m_2))$$
$$- \frac{(m_1^2 - m_2^2)(m_3^2 - m_2^2)}{s} c_0(m_1, m_2, m_3)$$
$$- \frac{1}{4} b_0(0, m_1, m_2) - \frac{1}{4} (m_1^2 - m_2^2) b'_0(m_1, m_2)$$
$$- \frac{1}{4} b_0(0, m_3, m_2) - \frac{1}{4} (m_3^2 - m_2^2) b'_0(m_3, m_2)$$

Standard two-point functions used in eqs. (51,52) read:

$$a_0(m) = m^2 \left( \eta - 1 + \log \frac{m^2}{Q^2} \right)$$

$$b_0(s, m_1, m_2) = \eta + \int_0^1 dx \log \frac{x(x-1)s + x m_1^2 + (1-x)m_2^2}{Q^2}$$

$Q^2$ is the $\overline{MS}$ renormalization scale and $\eta \equiv 2/(d-4)$. Derivative at $s = 0$ of $b_0(s, m_1, m_2)$ reads:

$$b'(m_1, m_2) = -\frac{1}{2} \frac{m_1^2 + m_2^2}{(m_1^2 - m_2^2)^2} + \frac{m_1^2 m_2^2}{(m_1^2 - m_2^2)^3} \log \frac{m_1^2}{m_2^2}$$

Finally, the $c_0(m_1, m_2, m_3)$ function is defined as:

$$c_0(m_1, m_2, m_3) = \int \frac{d^4k}{\pi^2} \frac{i}{|k^2 - m_1^2||k + p|^2 - m_2^2||(k + p + q)^2 - m_3^2|}$$

In the case of the $Z^0\bar{b}b$ vertex $(p + q)^2 = M_Z^2$ and we $p^2 = q^2 = m_b^2 \approx 0$. 

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Figure 1: Masses of the two charginos and two lightest neutralinos as a function of $\mu$ for $r = 1$ (solid lines) and $r = 3$ (dotted lines) for $\tan \beta = 1.4$. In the lower panels the corresponding production cross sections of $C_1^+ C_1^-$ and $N_1^0 N_2^0$ at LEP2 ($s^{1/2} = 190$ GeV) are shown. Masses of the sparticles exchanged in the $t$ channel are taken to be $M_{\tilde{\nu}_e} = M_{\tilde{\nu}_R} = 500$ GeV. For neutralinos also the case $M_{\tilde{\nu}_R} = 50$ GeV (dashed and dash-dotted lines for $r = 1$ and $r = 3$ respectively) is shown.
Figure 2: Contours of constant \( \delta R_{b}^{\text{SUSY}} \) (solid lines) and various constraints in the plane \((\theta_t, M_{\tilde{t}^2})\) for \(m_t = 170\) GeV, \(\tan \beta = 1.4\), \(M_2 = -\mu = 58\) GeV \(m_{C_1} = 85\) GeV, \(M_A = M_{\tilde{t}_1} = 1\) TeV. Dashed and dash-dotted lines show the \(b \to s\gamma\) constraint with different treatments of \(\alpha_s\): \(\alpha_s(R_b)\) denotes the curves obtained with \(\alpha_s(M_Z) = 0.123 - 4 \delta R_{b}^{\text{SUSY}}\) and with the renormalization scale \(Q\) varied in the range \((m_b/2, 2m_b)\). The curve with \(Q\) fixed correspond to \(Q = m_b = 4.7\) GeV and \(\alpha_s(M_Z) = 0.123\) (for more details see the text). Dotted lines illustrate the Higgs boson mass constraint. The allowed region is bounded from below by the \(BR(t \to \text{new}) = 45\%\) curve and the parabolic \(b \to s\gamma\) curve \(\alpha_s(R_b)\) and from the left- and right-hand sides by the dotted curves \(M_h = 60\) GeV. The area below the central dotted curve is also excluded.
Figure 3: As in Fig. 2 but for $\mu > 0$ and $r = 1.5$. $\mu = 85.5$ GeV ($m_{c_{t}} = 55$ GeV), $M_A = M_{\tilde{t}_1} = 1$ TeV. The allowed region is for $\theta_t > 0$, between the two dashed curves denoted by $\alpha_s(R_b)$ (with exclusion of the central area $40 < M_h < 60$ GeV) and for $\theta_t < 0$, below the dashed curve $\alpha_s(R_b)$ (bounded from the left by the dotted curve $M_h = 60$ GeV).
Figure 4: $\chi^2$ as a function of $m_{C_1}$ for $r \equiv M_2/|\mu| = 0.5$ (solid lines), 1 (dashed), 1.5 (dotted), and 3 (dash-dotted) for both signs of $\mu$ for $m_t = 170$ GeV, $\tan \beta = 1.4$, $M_A = M_{\tilde{t}_1} = 1$ TeV. In lower panels the best values of $R_b$ with the restriction $\chi^2 < \chi^2_{\min} + 1$ (here $\chi^2_{\min}$ denotes the best $\chi^2$ for fixed value of $m_{C_1}$) are shown. In addition we required $M_{\tilde{t}_2} > 60$ GeV.
Figure 5: As in Fig. 4 but with condition $M_{t_2} > 90$ GeV.
Figure 6: As in Fig. 4 but for $m_t = 180$. 
Figure 7: Contours of constant $\delta R^\text{SUSY}_b$ (solid lines) and allowed by $BR(b \to s\gamma)$ regions in the $(\theta_t, M_{t_2})$ plane for $m_t = 170$ GeV, $\tan \beta = 50$, $M_2 = 1.5\mu$, $m_{C_1} = 65$ GeV, $M_{t_1} = 1$ TeV, $M_A = 55$ GeV and $M_{\tilde{b}_R} = 130$ GeV. Contours of constant $\sum_i BR(t \to \tilde{t}N^0_i)$ are also shown (dotted lines).
Figure 8: $\chi^2$ as a function of $m_{C_1}$ ($\mu > 0$) for $r \equiv M_2/|\mu| = 1$ (solid lines), 1.5 (dashed), 3 (dotted), and 5 (dash-dotted) for $m_t = 170$ GeV, $\tan \beta = 50$ and $180$ GeV, $\tan \beta = 55$. $M_{\tilde{t}_1} = 1$ TeV, $M_A = 55$ GeV, $M_{\tilde{b}_R} = 130$ GeV. In lower panels the best values of $R_b$ with the restriction $\chi^2 < \chi^2_{\text{min}} + 1$ (here $\chi^2_{\text{min}}$ denotes the best $\chi^2$ for fixed value of $m_{C_1}$) are shown. In addition we required $M_{\tilde{t}_2} > 60$ GeV.
Figure 9: As in Fig. 8 but for $M_A = 65$ GeV.