Higher-twist contributions to neutrino-production of pions

B. Z. Kopeliovich, Iván Schmidt and M. Siddikov
Departamento de Física, Universidad Técnica Federico Santa María,
y Centro Científico - Tecnológico de Valparaíso, Avda. España 1680, Valparaíso, Chile

In this paper we estimate the size of twist-3 corrections to the deeply virtual meson production in neutrino interactions due to the chiral odd transversity Generalized Parton Distribution (GPD). We conclude that in contrast to pion electroproduction, in neutrino-induced reactions these corrections are small. This happens due to large contribution of unpolarized GPDs $H, E$ to the leading-twist amplitude in neutrino production. We provide a computational code, which can be used for evaluation of the cross-sections accounting for these twist-3 corrections with various GPD models. Our results are particularly relevant for analyses of the pion and kaon production in the MINERvA experiment at FERMILAB.

PACS numbers: 13.15.+g,13.85.-t
Keywords: Single pion production, generalized parton distributions, neutrino-hadron interactions

I. INTRODUCTION

Nowadays one of the key objects used to parametrize nonperturbative structure of the target are the generalized parton distributions (GPDs). For the kinematics where the collinear factorization is applicable [1, 2], they allow evaluation of the cross-sections for a wide class of processes. Today all information on GPDs comes from the electron-proton and positron-proton measurements performed at JLAB and HERA, in particular from deeply virtual Compton scattering (DVCS) and deeply virtual meson production (DVMP) [3–16]. The forthcoming CLAS12 upgrade at JLAB will help to improve our understanding of the GPDs [16]. However, in practice, the procedure of extraction of GPDs from experimental data is subject to the uncertainties, like large BFKL-type logarithms in the next-to-leading order (NLO) corrections [17] in the HERA kinematics; contributions of the higher-twist components of GPDs and the pion distribution amplitudes (DAs) in the JLAB kinematics [16–21], or uncertainties in vector meson DAs in case of $\rho$- and $\phi$-meson production.

From this point of view, consistency checks of the GPD extraction procedure from experimental data, especially of their flavor structure, are important. Earlier we proposed to study the GPDs in deeply virtual neutrino production of the pseudo-Goldstone mesons ($\pi, K, \eta$) [22] with the high-intensity NUMI beam at Fermilab, which recently switched to the so-called middle-energy (ME) regime with an average neutrino energy of about 6 GeV, and potentially is able to reach energies up to 20 GeV, without essential loss of luminosity. The $\nu$DVMP measurements with neutrino and antineutrino beams are complementary to the electromagnetic DVMP. In the axial channel, due to the chiral symmetry breaking we have an octet of pseudo-Goldstone bosons, which act as a natural probe of the flavor content. Due to the $V-A$ structure of the charged current, in $\nu$DVMP one can access simultaneously the unpolarized GPDs, $H$, $E$, and the helicity flip GPDs, $\tilde{H}$ and $\tilde{E}$. Besides, using chiral symmetry and assuming closeness of parameters of pion and kaon, full flavour structure of the GPDs can be extracted.

It is worth reminding that the cross-sections were evaluated in [22] in the leading twist approximation, and for a correct extraction of the GPDs at the energies of MINERvA in ME regime, an estimate of the higher twist effects is required. The first twist-3 correction arises due to contribution of the transversely polarized intermediate virtual bosons and is controlled by convolution of poorly known transversity GPDs $H_T, E_T, H_T, E_T$ and twist-3 DAs of pion. While this correction vanishes at asymptotically large $Q^2$, at moderate $Q^2$ in electroproduction it gives a sizable contribution, which was confirmed in the CLAS experiment [16]. In case of neutrino-production situation is different since due to $V-A$ structure of the weak currents there is an additional and numerically dominant contribution of the unpolarized GPDs $H$, $E$ to the leading twist amplitude. In this paper we analyze the relative size of the twist-3 contributions to the neutrino-production of pions and demonstrate that they are indeed small. In this respect we differ from [23], where the contribution of chiral odd GPDs was assumed to be numerically dominant.

The paper is organized as follows. In Section II we evaluate the Goldstone meson production by neutrinos on nucleon targets accounting for higher twist effects. In Section III for the sake of completeness we highlight the properties of the GPD parametrization used for evaluations. In Section IV we present numerical results and make conclusions.
II. CROSS-SECTION OF THE $\nu$DVMP PROCESS

The cross-section of the Goldstone mesons production in neutrino-hadron collisions has the form

$$\frac{d\sigma}{dt \, dx_B dQ^2 d\phi} = \frac{d\sigma_T}{dt \, dx_B dQ^2 d\phi} + \frac{d\sigma_{LT}}{dt \, dx_B dQ^2 d\phi} + \frac{d\sigma_{TT}}{dt \, dx_B dQ^2 d\phi} + \frac{d\sigma_{L'TT}}{dt \, dx_B dQ^2 d\phi} + \frac{d\sigma_{TT'}}{dt \, dx_B dQ^2 d\phi},$$

(1)

where $t = (p_2 - p_1)^2$ is the momentum transfer to baryon, $Q^2 = -q^2$ is the virtuality of the charged boson, $x_B = Q^2/(2p \cdot q)$ is Bjorken $x$, $\phi$ is the angle between the lepton and meson production scattering planes, and we introduced shorthand notations

$$\epsilon = \frac{1 - y - z^2/4}{1 - y + z^2/4}, \quad \gamma = \frac{2m_Nx_B}{Q}, \quad y = \frac{Q^2}{sx_B}.$$

In the asymptotic Bjorken limit the cross-section is dominated by the first angular independent term $\epsilon \, d\sigma_L/\epsilon \, dt \, dx_B dQ^2 d\phi$ which was studied in our previous paper [22] and is a straightforward extension of the electroproduction of pions studied in [20, 21, 24–28]. As we will see below the twist-3 corrections are small, for this reason it is convenient to normalize all the cross-sections in (1) to this term,

$$\frac{d\sigma}{dt \, dx_B dQ^2 d\phi} = \frac{d\sigma_L}{dt \, dx_B dQ^2 d\phi} \sum_n (c_n \cos n\phi + s_n \sin n\phi)$$

(2)

and discuss higher-twist effects in terms of harmonics $c_n, s_n$. In what follows, it is convenient to introduce a photon helicity matrix $\sigma_{\alpha\beta}$ defined as

$$\sigma_{\alpha\beta} = \frac{1}{2} \sum_{\nu \nu'} A_{\nu \nu', \alpha} A_{\nu' \nu, \beta},$$

(3)

where $A_{\nu \nu', \alpha}$ is the amplitude of the corresponding process in helicity basis, and $\nu, \nu'$ are the polarizations of the initial and final baryon. In terms of $\sigma_{\alpha\beta}$ the cross-sections in (1) can be written as

$$\frac{d\sigma_L}{dt \, dx_B dQ^2 d\phi} = \Gamma \sigma_{00}$$

(4)

$$\frac{d\sigma_T}{dt \, dx_B dQ^2 d\phi} = \Gamma \left( \frac{\sigma_{++} + \sigma_{--}}{2} - \mu \sqrt{1 - \epsilon^2 \sigma_{++} - \sigma_{--}} \right)$$

(5)

$$\frac{d\sigma_{LT}}{dt \, dx_B dQ^2 d\phi} = \Gamma \left( \text{Re} (\sigma_{0+} - \sigma_{0-}) - \mu \sqrt{1 - \epsilon} \text{Re} (\sigma_{0+} + \sigma_{0-}) \right)$$

(6)

$$\frac{d\sigma_{TT}}{dt \, dx_B dQ^2 d\phi} = -\Gamma \text{Re} (\sigma_{+-})$$

(7)

$$\frac{d\sigma_{L'TT}}{dt \, dx_B dQ^2 d\phi} = -\Gamma \left( \text{Im} (\sigma_{0+} + \sigma_{0-}) + \mu \sqrt{1 - \epsilon} \text{Im} (\sigma_{0-} - \sigma_{0+}) \right)$$

(8)

$$\frac{d\sigma_{TT'}}{dt \, dx_B dQ^2 d\phi} = -\Gamma \text{Im} (\sigma_{+-})$$

(9)

where we introduced shorthand notations $\Gamma$ and $\mu$, which for the charged current (CC) and neutral current (NC) are defined as

$$\Gamma_{CC} = \frac{G_F^2 f_M^2 x_B^2 \left( 1 - y + \frac{y^2}{2} + \frac{\gamma^2 y^2}{4} \right)}{64\pi^4 Q^2 (1 + Q^2/M_W^2)^2 (1 + \gamma^2)^{3/2}},$$

(10)

$$\Gamma_{NC} = \frac{G_F^2 f_M^2 x_B^2 \left( 1 - y + \frac{y^2}{2} + \frac{\gamma^2 y^2}{4} \right)}{64\pi^4 \cos^4 \theta_W Q^2 (1 + Q^2/M_\pi^2)^2 (1 + \gamma^2)^{3/2}},$$

(11)

$$\mu_{\nu, \nu'} = \pm \frac{1}{2}.$$

(12)
\(f_\pi\) is the pion decay constant, \(G_F\) is the Fermi constant, \(\theta_W\) is the Weinberg angle, and \(M_W\), \(M_Z\) are the masses of the \(W\) and \(Z\) bosons.

From (49) we can see that the terms \(\sim \sin \phi\) and \(\sim \cos \phi\) appear due to interference of the leading twist and twist-3 contributions, whereas all the other contributions appear entirely due to the twist-3 effects. Since the twist-3 amplitudes are suppressed by \(1/Q\) compared to the leading twist result, we expect that in the large-\(Q\) limit

\[
c_1, s_1 \sim 1/Q, \quad c_0 - 1, c_2, s_2 \sim 1/Q^2.
\]

Due to the factorization theorem the amplitude \(A_{\nu,0,\nu}\) in (53) may be written as a convolution of the hard and soft parts,

\[
A_{\nu,0,\nu} = \int_{-1}^{+1} dx \sum_{q,q'} \sum_{L,\lambda'} H^{q,q'}_{\nu,\lambda',\nu} C_{\lambda,0,\lambda',\nu},
\]

where \(x\) is the average light-cone fraction of the parton, \(\lambda, q (\lambda', q')\) are the corresponding helicity and flavour of the initial (final) partons, the helicity amplitude, \(H^{q,q'}_{\nu,\lambda',\nu}\) is the process- and baryon-dependent soft part which will be specified later, and \(C_{\lambda,0,\lambda',\nu}\) is the hard coefficient function.

In the leading twist, four GPDs, \(H^{q,q}, E^{q,q}, \hat{H}^{q,q}\) and \(\tilde{E}^{q,q}\) contribute to \(H_{\nu,\lambda',\nu}\). They are defined as

\[
\frac{\hat{P}^+}{2\pi} \int dz e^{ix\hat{P}^+z} \left( B(p_2) \left| \bar{\psi}_{q'} \left( -\frac{z}{2} \right) ; \gamma_+ \psi_q \left( \frac{z}{2} \right) \right| A(p_1) \right) = \left( H^{q,q} (x, \xi, t), N(p_2) \gamma_+ N(p_1) \right) + \frac{\Delta_I}{2m} E^{q,q} (x, \xi, t) i\sigma_{+,k} N(p_1) \right)
\]

\[
\frac{\hat{P}^+}{2\pi} \int dz e^{ix\hat{P}^+z} \left( B(p_2) \left| \bar{\psi}_{q'} \left( -\frac{z}{2} \right) ; \gamma_+ \gamma_5 \psi_q \left( \frac{z}{2} \right) \right| A(p_1) \right) = \left( \hat{H}^{q,q} (x, \xi, t), N(p_2) \gamma_+ \gamma_5 N(p_1) \right) + \frac{\Delta_I}{2m} \tilde{E}^{q,q} (x, \xi, t) \tilde{N}(p_2) N(p_1) \right),
\]

where \(\hat{P} = p_1 + p_2, \Delta = p_2 - p_1\) and \(\xi = -\Delta^+ / 2 \hat{P}^+ \approx x_2/(2-x_2)\) (see e.g. [11] for the details of kinematics). In the case when the baryon remains intact, \(A = B\), the corresponding GPDs are diagonal in the flavour space, \(H^{q,q} \sim \delta_{qq} H^q, \text{etc.}\) In general, when \(A \neq B\), in the right-hand side (r.h.s.) of Eqs. (15), (16) there might be extra structures which otherwise are forbidden by \(T\)-parity in the case of \(A = B\) [11]. In what follows we assume that the target \(A\) is either a proton or a neutron, and the recoil \(B\) belongs to the same lowest \(SU(3)\) octet of baryons. In this case, all such terms are parametrically suppressed by the current quark mass \(m_q\) and vanish in the limit of exact \(SU(3)\), so we will disregard them. In this special case, we can rely on the \(SU(3)\) relations and express the nondiagonal transitional GPDs as linear combinations of the GPDs of the proton \(H^q, E^q, \tilde{H}^q, \tilde{E}^q\) [29], so (14) may be effectively rewritten as

\[
A_{\nu,0,\nu} = \int_{-1}^{+1} dx \sum_{q} \sum_{\lambda'} H^{q}_{\nu,\lambda',\nu} C_{\lambda,0,\lambda',\nu},
\]

The twist-3 correction is controlled by the chiral odd transversity GPDs defined as [30]

\[
\frac{\hat{P}^+}{2\pi} \int dz e^{ix\hat{P}^+z} \left( B(p_2) \left| \bar{\psi}_{q'} \left( -\frac{z}{2} \right) ; i\sigma^{+j} \psi_q \left( \frac{z}{2} \right) \right| A(p_1) \right) = \left( \hat{H}^{q,q}_{+} (x, \xi, t), N(p_2) i\sigma^{+j} N(p_1) \right) + \hat{H}^{q,q}_{-} \frac{\Delta^+ \Delta^- - \Delta^+ \hat{P}^+}{2m^2} + \hat{E}^{q,q}_{+} \frac{\Delta^+ \Delta^- - \Delta^+ \hat{P}^+}{2m^2} + \hat{E}^{q,q}_{-} \frac{\Delta^+ \Delta^- - \Delta^+ \hat{P}^+}{2m^2} + \hat{E}^{q,q}_{-} \frac{\Delta^+ \Delta^- - \Delta^+ \hat{P}^+}{2m^2},
\]

where \(j = 1, 2\) is the transverse index. Similar to the leading-twist case, the flavour structure may be simplified with the help of \(SU(3)\) relations and rewritten in terms of the transversity GPDs of the proton.

The matrix \(H^{q,q}_{\nu,\lambda',\nu}\) in (17) is a linear combination of the helicity-odd and even GPDs,

\[
H^{q,q}_{\nu,\lambda',\nu} = \frac{2\delta_{\lambda,\lambda'}}{\sqrt{1 - \xi^2}} \left( \left( 1 - \xi^2 \right) \tilde{H}^q - \xi^2 \tilde{E}^q \right) \left( \frac{\Delta^+ \Delta^- \xi E^q}{2m^2} \right) + \text{sgn}(\lambda') \left( \left( 1 - \xi^2 \right) \tilde{H}^q + \xi^2 \tilde{E}^q \right) \left( \frac{\Delta^+ \Delta^- \xi E^q}{2m^2} \right) + \left( n_{\nu,\nu}^q \delta_{\lambda,\lambda'} + n_{\nu,\nu}^q \delta_{\lambda,\lambda'} \right),
\]
where the coefficients \( m_{\pm}^q \) and \( n_{\pm}^q \) are given by

\[
m_{-}^q = \frac{-t'}{4m} \left[ 2\hat{H}_T^q + (1 + \xi) E_T^q - (1 + \xi) \bar{E}_T^q \right],
\]

\[
m_{+}^q = \frac{t'}{4m^2} \hat{H}_T^q,
\]

\[
m_{-}^q = \frac{-t'}{4m} \left[ H_T^q - \frac{\xi}{1 - \xi^2} E_T^q + \frac{\xi}{1 - \xi^2} \bar{E}_T^q - \frac{t'}{4m^2} \hat{H}_T^q \right],
\]

\[
m_{+}^q = \frac{t'}{4m} \left[ 2\hat{H}_T^q + (1 - \xi) E_T^q + (1 - \xi) \bar{E}_T^q \right],
\]

\[
n_{-}^q = -\frac{-t'}{4m} \left( 2\hat{H}_T^q + (1 - \xi) E_T^q + (1 - \xi) \bar{E}_T^q \right),
\]

\[
n_{+}^q = \frac{t'}{4m} \left( H_T^q - \frac{\xi}{1 - \xi^2} E_T^q + \frac{\xi}{1 - \xi^2} \bar{E}_T^q - \frac{t'}{4m^2} \hat{H}_T^q \right),
\]

\[
n_{-}^q = \frac{t'}{4m} \hat{H}_T^q,
\]

\[
n_{+}^q = -\frac{-t'}{4m} \left( 2\hat{H}_T^q + (1 + \xi) E_T^q -(1 + \xi) \bar{E}_T^q \right),
\]

and we introduced a shorthand notation \( t' = -\Delta^2 / (1 - \xi^2) \), where \( \Delta_{\perp} = p_{\perp,1} - p_{\perp,2} \) is the transverse part of the momentum transfer.

Evaluation of the hard coefficient function \( C_{\lambda,0,\lambda^I}^q \) is quite straightforward, and in the leading order over \( \alpha_s \) is given by the four diagrams shown schematically in Figure 1.

![Figure 1: Leading-order contributions to the DVMP hard coefficient functions.](image)

A key ingredient in evaluation of the coefficient functions are the distribution amplitudes (DAs) of the produced pion. Since we are interested in making evaluations up to twist-3 effects, we have to take into account both twist-2 and twist-3 pion DAs, defined respectively as [31]

\[
\phi_2 (z) = \frac{1}{f_\pi \sqrt{2}} \int \frac{du}{2\pi} e^{iz(2-0.5)u} \left< 0 \left| \tilde{\psi} \left( -\frac{u}{2} \right) \bar{n} \gamma_5 \psi \left( \frac{u}{2} \right) \right| \pi (q) \right>,
\]

\[
\phi_3 (z) = \frac{1}{f_\pi \sqrt{2}} \frac{m_u + m_d}{m_\pi^2} \int \frac{du}{2\pi} e^{iz(2-0.5)u} \left< 0 \left| \tilde{\psi} \left( -\frac{u}{2} \right) \gamma_5 \psi \left( \frac{u}{2} \right) \right| \pi (q) \right>,
\]

\[
\phi_3 (z) = \frac{3i}{\sqrt{2} f_\pi} \frac{m_u + m_d}{m_\pi^2} \int \frac{du}{2\pi} e^{iz(2-0.5)u} \left< 0 \left| \tilde{\psi} \left( -\frac{u}{2} \right) \sigma_+ \gamma_5 \psi \left( \frac{u}{2} \right) \right| \pi (q) \right>.
\]

As one can see from (28), the pseudovector and pseudoscalar DAs \( \phi_2 ; \pi \), \( \phi_3^{(p)} ; \pi \) are chiral even (symmetric w.r.t. \( z \to 1 - z \)), whereas the tensor DA \( \phi_3^{(c)} ; \pi \) is chiral odd. Straightforward evaluation of the diagrams shown in Figure 1 yields for the coefficient function

\[
C_{\lambda,0,\lambda^I}^q = \delta_{\mu \nu} \delta_{\lambda \lambda'} \sum_{k=\pm} \left[ \eta_{(2)} q \xi, k \right] (x, \xi) + \text{sgn}(\lambda) \eta_{(2)} q \xi, k \right] (x, \xi) + \delta_{\mu,0} \delta_{\lambda,-} (S_A^q - S_V^q) + \delta_{\mu,-} \delta_{\lambda,0} (S_A^q + S_V^q) + O \left( \frac{m^2}{Q^2} \right),
\]

(31)
where we introduced shorthand notations

$$S_A^q = \int dz \left( \left( \eta_A^{q - c_{3\, s}^{(3\, p)}}(x, \xi) - \eta_A^{q - c_{3\, p}^{(3\, p)}}(x, \xi) \right) + 2 \left( \eta_A^{q - c_{3\, s}^{(3\, p)}}(x, \xi) + \eta_A^{q - c_{3\, p}^{(3\, p)}}(x, \xi) \right) \right),$$  \hspace{1cm} (33)

$$S_V^q = \int dz \left( \left( \eta_V^{q + c_{3\, s}^{(3\, p)}}(x, \xi) + \eta_V^{q - c_{3\, p}^{(3\, p)}}(x, \xi) \right) + 2 \left( \eta_V^{q + c_{3\, s}^{(3\, p)}}(x, \xi) - \eta_V^{q - c_{3\, p}^{(3\, p)}}(x, \xi) \right) \right),$$  \hspace{1cm} (34)

$$c_{i \pm}^{(2\, 3\, s)}(x, \xi) = \left( \int dz \frac{\phi_2(z)}{z} \right) \frac{8\pi i \alpha_s f_{\pi}}{Q} \frac{1}{x \pm \xi \mp i0},$$  \hspace{1cm} (35)

$$c_{i \pm}^{(3\, 3\, s)}(x, \xi) = \frac{4\pi i \alpha_s f_{\pi} \xi}{9 Q^2} \int_0^1 dz \frac{\phi_3(z)}{z(x + \xi)^2},$$  \hspace{1cm} (36)

and the process-dependent flavor factors $\eta_{q \pm}^V$, $\eta_{q \pm}^V$ are presented in table \[3]. As was discussed above, for the processes, in which either initial or final baryon is different from proton, we used $SU(3)$ relations \[29], valid up to the corrections in current quark mass $\sim O(m_q)$. In the leading twist, due to the underlying $SU(3)$ relations there are identities relating the neutrino-hadron and antineutrino-hadron cross-sections. In the next-to-leading order these relations are broken due to the weak isospin-dependent factor $\mu$ in \[12], and corresponding $SU(3)$ identities are valid only for the cross-sections $d\sigma_{TT}$ and $d\sigma_{TT'}$ (harmonics $c_2$, $s_2$). For certain processes either $\eta_1^V$ or $\eta_1^V$ might vanish. In this case from the definition \[33] \[34] we may see that $S_A = \pm S_V$, and the matrix element $\sigma_{\pm}$ which controls the cross-sections $d\sigma_{TT}$, $d\sigma_{TT'}$ \[7] \[9] vanishes identically.

Using symmetry of $\phi_0$ and antisymmetry of $\phi_0$ with respect to charge conjugation, we can show that dependence on the pion DAs factorizes in the collinear approximation and contributes only as the minus first moment of the order of 10\% (see e.g. \[32, 33\] and reviews in \[33\] \[34\]). We see from \[17] \[36\] that, excluding the very special case when all the transversity GPDs vanish at $x = \pm \xi$, the transverse amplitude suffers from a collinear singularity at these two points. In order to regularize it, we follow \[19\] and introduce a small transverse momentum of the quarks inside the meson. Such regularization modifies \[36\] to

$$c_{i \pm}^{(3\, 3\, s)}(x, \xi) = \frac{4\pi i \alpha_s f_{\pi} \xi}{9 Q^2} \int_0^1 dz \frac{\phi_3(z)}{z(x + \xi)^2}, \hspace{1cm} (37)$$

$$c_{i \pm}^{(3\, 3\, s)}(x, \xi) = \frac{4\pi i \alpha_s f_{\pi} \xi}{9 Q^2} \int_0^1 dz \frac{\phi_3(z)}{z(1 - z)(x - \xi)^2}, \hspace{1cm} (38)$$

where $l_\perp$ is the transverse momentum of the quark, and we tacitly assume absence of any other transverse momenta in the coefficient function.

### III. GPD AND DA PARAMETRIZATIONS

The pion DAs are one of the main sources of uncertainty in the present analysis. For the leading twist DA $\phi_{2\pi}(x)$, the currently available data on meson photoproduction formfactor $F_{\pi \gamma\gamma}(Q^2)$ are compatible with an asymptotic form $\phi_{as}(z) = 6\sqrt{2} f_{\pi\gamma}(1 - z)$, with a typical uncertainty in the minus-first moment of the order of $\sim 10\%$ (see e.g. \[32, 33\] and reviews in \[32, 33\]).

For the twist-3 contribution, as was discussed in Section III, the DAs $\phi_{3\, p}(z, l_\perp)$ and $\phi_{3\, \sigma}(z, l_\perp)$ contribute in a linear combination

$$\phi_3(z, l_\perp) = \phi_{3\, p}(z, l_\perp) + 2\phi_{3\, \sigma}(z, l_\perp).$$  \hspace{1cm} (40)

For the sake of simplicity, we parametrize \[40\] in the form

$$\phi_3(z, l_\perp) = \frac{2\alpha_0^2}{\pi^3} l_\perp \phi_{as}(z) \exp \left(-a_{\phi}^2 l_\perp^2 \right),$$  \hspace{1cm} (41)
Table I: The flavour coefficients $\eta^q_{\pm}$ for various processes ($q = u, d, s, ...$). For the case of CC mediated processes, take $\eta^q_{\pm} = \eta^q_\pm$, $\eta^q_{\mp} = -\eta^q_\pm$. For the case of NC mediated processes, take $g_\nu$ corresponding to vector current coupling $g^V_\nu$ and axial-vector current coupling $g^A_\nu$ for the helicity odd and even GPDs respectively.

| Process | type | $\eta^u_+$ | $\eta^d_+$ | $\eta^u_-$ | $\eta^d_-$ | $\eta^s_+$ | $\eta^s_-$ |
|---------|------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\nu p \to \mu^+ \pi^- p$ | CC | $V_{ud}\delta_{uu}$ | $V_{ud}\delta_{ud}$ | $V_{ud}\delta_{ud}$ | $V_{ud}\delta_{dd}$ | $V_{ud}\delta_{dd}$ | $V_{ud}\delta_{dd}$ |
| $\bar{\nu} p \to \mu^- \pi^+ n$ | CC | $V_{ud}\delta_{ud}$ | $V_{ud}\delta_{ud}$ | $V_{ud}\delta_{ud}$ | $V_{ud}\delta_{dd}$ | $V_{ud}\delta_{dd}$ | $V_{ud}\delta_{dd}$ |
| $\nu p \to \mu^+ \pi^0 n$ | NC | $V_{ud}\delta_{uu}$ | $V_{ud}\delta_{ud}$ | $V_{ud}\delta_{ud}$ | $V_{ud}\delta_{dd}$ | $V_{ud}\delta_{dd}$ | $V_{ud}\delta_{dd}$ |
| $\nu p \to \nu \pi^0 n$ | NC | $g_\nu (\delta_{uu} - \delta_{ud})$ | $g_\nu (\delta_{uu} - \delta_{ud})$ | $g_\nu (\delta_{uu} - \delta_{ud})$ | $g_\nu (\delta_{uu} - \delta_{ud})$ | $g_\nu (\delta_{uu} - \delta_{ud})$ | $g_\nu (\delta_{uu} - \delta_{ud})$ |
| $\bar{\nu} p \to \mu^- \pi^0 n$ | NC | $V_{ud}\delta_{uu}$ | $V_{ud}\delta_{ud}$ | $V_{ud}\delta_{ud}$ | $V_{ud}\delta_{dd}$ | $V_{ud}\delta_{dd}$ | $V_{ud}\delta_{dd}$ |
| $\nu p \to \nu \pi^0 p$ | NC | $g_\nu (\delta_{uu} - \delta_{ud})$ | $g_\nu (\delta_{uu} - \delta_{ud})$ | $g_\nu (\delta_{uu} - \delta_{ud})$ | $g_\nu (\delta_{uu} - \delta_{ud})$ | $g_\nu (\delta_{uu} - \delta_{ud})$ | $g_\nu (\delta_{uu} - \delta_{ud})$ |

where the numerical constant $a_p$ is taken as $a_p \approx 2 \text{GeV}^{-1}$ in analogy with [19, 20]. More than a dozen of different parametrizations of GPDs have been proposed in the literature [7, 12, 28, 36–44]. While we neither endorse nor refute any of them, for the sake of concreteness we use the parametrization [26–28], which successfully described HERA [45] and JLAB [26–28] data on electroproduction of different mesons, so is expected to provide a reasonable description of $\nu$DVMP. The parametrization is based on the Radyushkin’s double distribution ansatz. It assumes additivity of the valence and sea parts of the GPDs,

$$H(x, \xi, t) = H_{\text{val}}(x, \xi, t) + H_{\text{sea}}(x, \xi, t),$$

which are defined as

$$H^q_{\text{val}} = \int_{|x|+|\beta|\leq 1} d\beta d\alpha \delta (\beta - x + \alpha \xi) \frac{3\theta(\beta) ((1 - |\beta|)^2 - \alpha^2)}{4(1 - |\beta|)^3} q_{\text{val}}(\beta) e^{(b_i - \alpha, \ln |\beta|) t},$$

$$H^q_{\text{sea}} = \int_{|x|+|\beta|\leq 1} d\beta d\alpha \delta (\beta - x + \alpha \xi) \frac{3 sgn(\beta) ((1 - |\beta|)^2 - \alpha^2)^2}{8(1 - |\beta|)^5} q_{\text{sea}}(\beta) e^{(b_i - \alpha, \ln |\beta|) t},$$

and $q_{\text{val}}$ and $q_{\text{sea}}$ are the ordinary valence and sea components of PDFs. The coefficients $b_i, \alpha_i$ as well as the parametrization of the input PDFs $q(x)$, $\Delta q(x)$ and pseudo-PDFs $e(x)$, $\bar{e}(x)$ (which correspond to the forward limit of the GPDs $E, \bar{E}$) are discussed in [26–28]. The unpolarized PDFs $q(x)$ are adjusted to reproduce the CTEQ PDFs in the limited range $4 \leq Q^2 \leq 40 \text{GeV}^2$. Notice that in this model the sea is flavor symmetric for asymptotically large
\[ Q^2, \]

\[ H^u_{\text{sea}} = H^d_{\text{sea}} = \kappa (Q^2) H^s_{\text{sea}}, \]  \hspace{1cm} (44)

where

\[ \kappa (Q^2) = 1 + \frac{0.68}{1 + 0.52 \ln (Q^2/Q^2_0)}, \quad Q^2_0 = 4 \text{GeV}^2. \]

The equality of the sea components of the light quarks in (44) should be considered only as a rough approximation, since in the forward limit the inequality \( \bar{d} \neq \bar{u} \) was firmly established by the E866/NuSea experiment \[46\]. For this reason, the predictions made with this parametrization of the GPDs for the \( p \to n \) transitions in the region \( x_{\text{ Bj}} \in (0.1...0.3) \) might slightly underestimate the data.

The transversity GPDs in the parametrization \[20\] are obtained using a familiar double-distribution based parametrization \[12,23\], and the forward limit of these GPDs is parametrized as

\[ H^0_T(x,0,0) = N^a_{M} \sqrt{x(1-x)} (q^a(x) + \Delta q^a(x)), \]

\[ \bar{E}_T(x,0,0) \equiv E_T(x,0,0) \equiv 2 \bar{H}_T(x,0,0) = N^a_{E_x} x^{-\alpha}(1-x)^{\beta}, \]

where the values of the parameters \( N^a_{M}, \alpha, \beta \) are fixed from the lattice data. Since in the parametrization \[20\], as well as in any other parametrization of chiral-odd GPDs available in the literature, \( s \)-quarks are not included, we do not make any predictions for strangeness production.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section we would like to present numerical results for the twist-3 corrections to pion production using the Kroll-Goloskokov parametrization of GPDs \[19,22,28\], briefly discussed in the section \[III\]. As was discussed in section \[II\] of the consequences of the higher twist corrections is the appearance of the azimuthal angular dependence of the DVMP cross-sections. Since the twist-3 corrections are small, in what follows we prefer to discuss the results in terms of the angular harmonics \( c_n, s_n \) defined in (2). The most important harmonics is \( c_0 \), because its deviation from unity affects the extraction of GPDs in the leading twist approximation, and extraction of the leading twist result requires the experimentally challenging Rosenbluth separation with varying energy neutrino beam. All the other harmonics generate nontrivial angular dependence and can be easily separated from the leading-twist contribution. For example, the angle-integrated cross-section \( d\sigma/d\ln x_B dt dQ^2 \) is not sensitive to those harmonics at all.

In Figure 2 we show the harmonics \( c_n, s_n \) for processes without change of baryon state. The processes shown in the lower row are isospin conjugate to processes in the upper row. While in the leading twist the cross-sections of the former and the latter coincide, with the account of the twist-3 corrections this is no longer valid due to the difference in weak isospin of \( \nu \) and \( \bar{\nu} \). In all cases at \( x_B \leq 0.5 \), where the cross-section is the largest, the harmonics are small and do not exceed few per cent. The largest twist-3 contribution is due to the \( c_1 \) harmonics, which may reach up to twenty per cent. This is different from the electroproduction experiments, where \( c_1 (\sim \sigma_{LT}) \) is very small. This result can be understood from (4): due to parity nonconservation in weak interactions we have for the interference term \( \sigma_{0+} \neq \sigma_{0-} \). A positive value of \( c_1 \) for most processes implies that pion production correlates with the direction of the produced muon (scattered neutrino) in the case of CC (NC) mediated processes. The interference term also yields a relatively large harmonics \( s_1 \) which appears due to the interference of the vector and axial vector contributions.

In the region of \( x \gtrsim 0.5 \) all the harmonics increase, but the cross-sections for both the leading twist and subleading twist results are suppressed there due to increase of \( |t_{\text{min}}| \) and are hardly accessible with ongoing and forthcoming experiments.

In the Figure 3 we present the harmonics \( c_n, s_n \) for processes with change of the baryon state. As was discussed in [22], in the leading twist these processes are sensitive to the valence quarks distributions. As we can see, similar to the previous case, all the harmonics are small and except the region of \( x_B \sim 1 \) do not exceed few per cent.

For strangeness production we obtained qualitatively similar results (corrections are small), however we refrain from making predictions because the corresponding amplitudes are sensitive to the strange component of the chiral odd GPDs which are unknown at this moment.

V. CONCLUSIONS

In this paper we estimated the contributions of the twist-3 corrections due to the chiral odd GPDs. One of the manifestations of the twist-3 corrections is the appearance of the dependence on the angle between the lepton
Figure 2: (color online) Pion production on nucleons without change of the baryon state. Processes in the lower row differ from the processes in the upper row due to isospin conservation breakdown by higher-twist corrections.

Figure 3: (color online) Pion production on nucleons with change of the baryon state. Processes in the lower row differ from the processes in the upper row due to isospin-breaking by higher-twist corrections.
scattering and pion production planes. We found that the largest harmonics is $c_1$, which can reach up to twenty per cent, however it does not affect the angular integrated cross-section $d\sigma/dx_B dt dQ^2$. All the other harmonics are small and do not exceed few per cent. This happens because in case of neutrino interactions, in contrast to electroproduction of pions, there are large contributions of unpolarized GPDs $H, E$ to the leading-twist amplitude. Notice that the similar angular harmonics may be generated by interference of the leading twist result with the electromagnetic corrections. At moderate virtualities of the order a few GeV$^2$ this mechanism also gives small harmonics (of the order few per cent), however those corrections grow rapidly as a function of $Q^2$, and already at $Q^2 \sim 100$ GeV$^2$ electromagnetic mechanism becomes dominant.

To summarize, we conclude that deeply virtual production of pions and kaons on protons and neutrons by neutrinos with typical values of $Q^2$ of the order few GeV$^2$ provide a clean probe for the GPDs, with various corrections of the order of few per cent. Our results are relevant for analysis of the pion and kaon production in the Minerva experiment at Fermilab as well as for the planned Muon Collider/Neutrino Factory. An ideal target for study of the GPDs could be a liquid hydrogen or deuterium. For other targets there is an additional uncertainty due to the nuclear effects which will be addressed elsewhere.

We provide a computational code, which can be used for evaluation of the cross-sections with inclusion of the twist-3 corrections employing various GPD models.

Acknowledgments

This work was supported in part by Fondecyt (Chile) grants No. 1130543, 1100287 and 1120920.

[1] X. D. Ji and J. Osborne, Phys. Rev. D 58 (1998) 094018 [arXiv:hep-ph/9801260].
[2] J. C. Collins and A. Freund, Phys. Rev. D 59, 074009 (1999).
[3] D. Mueller, D. Robaschik, B. Geyer, F. M. Dittes and J. Horjesi, Fortsch. Phys. 42, 101 (1994) [arXiv:hep-ph/9812448].
[4] X. D. Ji, Phys. Rev. D 55, 7114 (1997).
[5] X. D. Ji, J. Phys. G 24, 1181 (1998) [arXiv:hep-ph/9807358].
[6] A. V. Radyushkin, Phys. Lett. B 380, 417 (1996) [arXiv:hep-ph/9604317].
[7] A. V. Radyushkin, Phys. Rev. D 56, 5524 (1997).
[8] A. V. Radyushkin, arXiv:hep-ph/0101225.
[9] J. C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. D 56, 2982 (1997).
[10] S. J. Brodsky, L. Frankfurt, J. F. Gunion, A. H. Mueller and M. Strikman, Phys. Rev. D 50, 3134 (1994).
[11] K. Goeke, M. V. Polyakov and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47, 401 (2001) [arXiv:hep-ph/0106012].
[12] M. Diehl, T. Feldmann, R. Jakob and P. Kroll, Nucl. Phys. B 596, 33 (2001) [Erratum-ibid. B 605, 647 (2001)] [arXiv:hep-ph/0009255].
[13] A. V. Belitsky, D. Mueller and A. Kirchner, Nucl. Phys. B 629, 323 (2002) [arXiv:hep-ph/0112108].
[14] M. Diehl, Phys. Rept. 388, 41 (2003) [arXiv:hep-ph/0307382].
[15] A. V. Belitsky and A. V. Radyushkin, Phys. Rept. 418, 1 (2005) [arXiv:hep-ph/0504030].
[16] V. Kubarovsky [CLAS Collaboration], Nucl. Phys. Proc. Suppl. 219-220, 118 (2011).
[17] D. Y. Ivanov, arXiv:0712.3193 [hep-ph].
[18] S. Ahmad, G. R. Goldstein and S. Liuti, Phys. Rev. D 79 (2009) 054014 [arXiv:0805.3568 [hep-ph]].
[19] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C 65, 137 (2010) [arXiv:0906.0469 [hep-ph]].
[20] S. V. Goloskokov and P. Kroll, Eur. Phys. J. A 47, 112 (2011) [arXiv:1106.4897 [hep-ph]].
[21] G. R. Goldstein, J. O. G. Hernandez and S. Liuti, arXiv:1201.6088 [hep-ph].
[22] B. Z. Kopeliovich, I. Schmidt and M. Siddikov, Phys. Rev. D 86 (2012), 113018 [arXiv:1210.4825 [hep-ph]].
[23] G. R. Goldstein, O. G. Hernandez, S. Liuti and T. McAskill, AIP Conf. Proc. 1222, 248 (2010) [arXiv:0911.0455 [hep-ph]].
[24] M. Vanderhaeghen, P. A. M. Guichon and M. Guidal, Phys. Rev. Lett. 80, 5064 (1998).
[25] L. Mankiewicz, G. Piller and A. Radyushkin, 10, 307 (1999) [hep-ph/9812467].
[26] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C 50, 829 (2007) [hep-ph/0611290].
[27] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C 53, 367 (2008) [arXiv:0708.3569 [hep-ph]].
[28] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C 59 (2009) 809 [arXiv:0809.4126 [hep-ph]].
[29] L. L. Frankfurt, P. V. Pobylitsa, M. V. Polyakov and M. Strikman, Phys. Rev. D 60 (1999) 014010 [hep-ph/9901249].
[30] M. Diehl, Eur. Phys. J. C 19, 485 (2001) [hep-ph/0101335].
[31] B. Z. Kopeliovich, Iván Schmidt and M. Siddikov, Nucl. Phys. A 918, 41 (2013) [arXiv:1108.5654 [hep-ph]].
[32] A. V. Pimikov, A. P. Bakulev, S. V. Mikhailov and N. G. Stefanis, arXiv:1208.4754 [hep-ph].
[33] A. P. Bakulev, S. V. Mikhailov, A. V. Pimikov and N. G. Stefanis, Phys. Rev. D 86 (2012) 031501 [arXiv:1205.3770 [hep-ph]].
[34] S. J. Brodsky, F.-G. Cao and G. F. de Teramond, Phys. Rev. D 84, 075012 (2011) [arXiv:1105.3999 [hep-ph]].
[35] S. J. Brodsky, F.-G. Cao and G. F. de Teramond, Phys. Rev. D 84, 033001 (2011) [arXiv:1104.3364 [hep-ph]].
[36] K. Kumericki, D. Muller and A. Schafer, JHEP 1107, 073 (2011) [arXiv:1106.2808 [hep-ph]].
[37] K. Kumericki and D. Mueller, Nucl. Phys. B 841, 1 (2010) [arXiv:0904.0458 [hep-ph]].
[38] M. Guidal, Phys. Lett. B 693, 17 (2010) [arXiv:1005.4922 [hep-ph]].
[39] M. V. Polyakov and K. M. Semenov-Tian-Shansky, Eur. Phys. J. A 40, 181 (2009) [arXiv:0811.2901 [hep-ph]].
[40] M. V. Polyakov and A. G. Shuvaev, hep-ph/0207153.
[41] A. Freund, M. McDermott and M. Strikman, Phys. Rev. D 67, 036001 (2003) [hep-ph/0208160].
[42] G. R. Goldstein, J. O. G. Hernandez and S. Liuti, arXiv:1311.0483 [hep-ph].
[43] R. Manohar, A. Mukherjee and D. Chakrabarti, Phys. Rev. D 83, 014004 (2011) [arXiv:1012.2627 [hep-ph]].
[44] P. D. Aaron et al. [H1 Collaboration], JHEP 1005 (2010) 032 [arXiv:0910.5831 [hep-ex]]. Phys. Rev. D 82, 033001 (2010) [arXiv:1004.5484 [hep-ph]].
[45] E. A. Hawker et al. [FNAL E866/NuSea Collaboration], Phys. Rev. Lett. 80 (1998) 3715 [hep-ex/9803011].
[46] B. Z. Kopeliovich, I. Schmidt and M. Siddikov, Phys. Rev. D 87, 033008 (2013) [arXiv:1301.7014 [hep-ph]].
[47] J. C. Gallardo, R. B. Palmer, A. V. Tollestrup, A. M. Sessler, A. N. Skrinsky, C. Ankenbrandt, S. Geer and J. Griffin et al., eConf C 960625 (1996) R4.
[48] C. M. Ankenbrandt, M. Atac, B. Autin, V. I. Balbekov, V. D. Barger, O. Benary, J. S. Berg and M. S. Berger et al., Phys. Rev. ST Accel. Beams 2 (1999) 081001 [physics/9901022].
[49] M. M. Alsharooa et al. [Muon Collider/Neutrino Factory Collaboration], Phys. Rev. ST Accel. Beams 6 (2003) 081001 [hep-ex/0207031].