Abstract

We present some predictions for the $\gamma^* N \rightarrow N^*$ transition amplitudes, where $N$ is the nucleon, and $N^*$ is a nucleon excitation from the third resonance region. First we estimate the transition amplitudes associated with the second radial excitation of the nucleon, interpreted as the $N(1710)$ state, using the covariant spectator quark model. After that, we combine some results from the covariant spectator quark model with the framework of the single quark transition model, to make predictions for the $\gamma^* N \rightarrow N^*$ transition amplitudes, where $N^*$ is a member of the $SU(6)$-multiplet [70,1−]. The results for the $\gamma^* N \rightarrow N(1520)$ and $\gamma^* N \rightarrow N(1535)$ transition amplitudes are used as input to the calculation of the amplitudes $A_{1/2}$, $A_{3/2}$, associated with the $\gamma^* N \rightarrow N(1650)$, $\gamma^* N \rightarrow N(1700)$, $\gamma^* N \rightarrow \Delta(1620)$, and $\gamma^* N \rightarrow \Delta(1700)$ transitions. Our estimates are compared with the available data. In order to facilitate the comparison with future experimental data at high $Q^2$, we derived also simple parametrizations for the amplitudes, compatible with the expected falloff at high $Q^2$.

Keywords

Nucleon resonances · Electromagnetic structure · Form factors · Valence quarks · Third resonance region

1 Introduction

One of the challenges in the modern physics is the description of the internal structure of the baryons and mesons. The electromagnetic structure of the nucleon $N$ and the nucleon resonances $N^*$ can be accessed through the $\gamma^* N \rightarrow N^*$ reactions, which depend of the (photon) momentum transfer squared $Q^2$. The data associated with those transitions are represented in terms of helicity amplitudes and have been collected in the recent years at Jefferson Lab, with increasing $Q^2$. The new data demands the development of theoretical models based on the underlying structure of quarks and quark-antiquark states (mesons). Those models may be used to guide future experiments as the ones planned for the Jlab–12 GeV upgrade, particularly for resonances in the second and third resonance region [energy $W = 1.4–1.8$ GeV] (see Fig. 1).

An example of a model appropriated for the study of the electromagnetic structure of resonances at large $Q^2$ is the covariant spectator quark model. In the covariant spectator quark model the baryons are described in terms of covariant wave functions based on quarks with internal structure (constituent quarks). Following previous studies for the nucleon and the first radial excitation of the nucleon, we use the covariant spectator quark model to calculate the transition amplitudes associated with the second radial excitation of the nucleon.
In a different work, we use the results of the covariant spectator quark model for the $\gamma^* N \rightarrow N(1520)$ and $\gamma^* N \rightarrow N(1535)$ transition amplitudes [11; 12] to estimate the transition amplitudes associated with four negative parity nucleon resonances from the $SU(6)$-multiplet [70, 1$^{-}$], in the third resonance region. This study is possible due to the combination with the single quark transition model, which allows the parametrization of the amplitudes $A_{1/2}$, $A_{3/2}$ for six resonances from the $SU(6)$-multiplet [70, 1$^{-}$], based on only three coefficients dependent of $Q^2$ [13].

### 2 Covariant spectator quark model

In the covariant spectator quark model, baryons are treated as three-quark systems. The baryon wave functions are derived from the quark states according with the $SU(6) \otimes O(3)$ symmetry group. A quark is off-mass-shell, and free to interact with the photon fields, and other two quarks are on-mass-shell [6; 7; 8]. Integrating over the quark-pair degrees of freedom we reduce the baryon to a quark-diquark system, where the diquark can be represented as an on-mass-shell spectator particle with an effective mass $m_D$ [6; 8; 11; 12].

The electromagnetic interaction with the baryons is described by the photon coupling with the constituent quarks in the relativistic impulse approximation. The quark electromagnetic structure is represented in terms of the quark form factors parametrized by a vector meson dominance mechanism [6; 8; 14]. The parametrization of the quark current was calibrated in the studies of the nucleon form factors data [6] and by the lattice QCD data for the decuplet baryon [8]. The quark electromagnetic form factors encodes effectively the gluon and quark-antiquark substructure of the constituent quarks. The quark current is decomposed as $j^\mu_q = j^\mu_1 + j^\mu_2$, where $j_i$ ($i = 1, 2$) are the Dirac and Pauli quark form factors, and $M$ is the nucleon mass. In the $SU(2)$-flavor sector the functions $j_i$ can also be decomposed into the isoscalar ($f_{i+}$) and the isovector ($f_{i-}$) components: $j_i = \frac{1}{2} f_{i+} + \frac{1}{2} f_{i-} \tau_3$, where $\tau_3$ acts on the isospin states of baryons (nucleon or resonance). The details can be found in Ref. [6; 7; 8].

When the nucleon wave function ($\Psi_N$) and the resonance wave function ($\Psi_R$) are both expressed in terms of the single quark and quark-pair states, the transition current is calculated in the relativistic impulse approximation, integrating over the diquark on-mass-shell momentum, and summing over the intermediate diquark polarizations [6; 8; 11]. In the study of inelastic transitions we use the Landau prescription to ensure the conservation of the transition current [11; 12; 13].

Using the relativistic impulse approximation, we can express the transition current in terms of the quark electromagnetic form factor $f_{i\pm}$ ($i = 1, 2$) and the radial wave functions $\psi_N$ and $\psi_R$ [6; 11; 12]. The radial wave functions are scalar functions that depend on the baryon ($P$) and diquark ($k$) momenta and parametrize the momentum distributions of the quark-diquark systems. From the transition current we can extract the form factors and the helicity transition amplitudes, defined in the rest frame of the resonance (final state), for the reaction under study [1; 2; 11; 12].
The covariant spectator quark model was used already in the study of several nucleon excitations including isospin 1/2 systems N(1410), N(1520), N(1535) \[6\,11\,12\] and the isospin 3/2 systems \[14\,15\,16\,17\]. In Fig. 1 the position of the nucleon excitations are represented and compared with the bumps of the cross sections. The model generalized to the SU(3)-flavor sector was also used to study the octet and decuplet baryons as well as transitions between baryons with strange quarks \[18\,19\]. Based on the parametrization of the quark current \(j^{\mu}_{q} \) in terms of the vector meson dominance mechanism, the model was extended to the lattice QCD regime (heavy pions and no meson cloud) \[14\,15\], to the nuclear medium \[16\] and to the timelike regime \[20\]. The model was also used to study the nucleon deep inelastic scattering \[8\,21\] and the axial structure of the octet baryon \[22\].

Most of the works refereed below, are based on the valence quarks degrees of freedom, as consequence of the relativistic impulse approximation. There are however some processes such as the meson exchanged between the different quarks inside the baryon, which cannot be reduced to simple diagrams with quark dressing. Those processes are regarded as arising from a meson cloud corrections to the hadronic reactions \[8\,12\,18\]. In some cases one can use the covariant spectator quark model to infer the effect of the meson cloud effects based on empirical information or lattice QCD simulations \[11\,14\,12\,18\,22\,23\].

3 Resonance \(N(1710)\)

We discuss now the \(\gamma^{*}N\rightarrow N^{*}\) transitions, where \(N\) is the nucleon state and \(N^{*}\) is a radial excitation of the nucleon based on a \(S\)-state wave function. We are excluding where the \(J^{P} = \frac{3}{2}^{-}\) \(N^{*}\) states based on spin-isospin wave functions with mixed symmetry. The \(N^{*}\) state share then with the nucleon, the structure of spin and isospin, and differ only in the radial structure (radial wave function \(\psi_{R}\)). Therefore, if we exclude the meson cloud effects, in principle relevant only at low \(Q^{2}\), we can estimate the transition form factors using the formalism already developed for the nucleon \[8\], replacing the radial wave function of the nucleon, labeled here as \(\psi_{N0}\), in the final state, by the radial wave function of the resonances. Since the spin and isospin structure is the same for \(N\) and \(N^{*}\) the orthogonality between the radial excitations is a consequence of the orthogonality of the radial wave functions. Labeling the radial wave functions of the first and second excitations, as \(\psi_{N1}\) and \(\psi_{N2}\) respectively, we can express the orthogonality condition as \[8\,10\]

\[
\int_{k} \psi_{N1}(k) \psi_{N0}(k) \bigg|_{Q^{2}=0} = 0, \quad \int_{k} \psi_{N2}(k) \psi_{N0}(k) \bigg|_{Q^{2}=0} = 0, \quad \int_{k} \psi_{N2}(k) \psi_{N1}(k) \bigg|_{Q^{2}=0} = 0, \quad (1)
\]

where the subindex \(Q^{2}=0\) indicates that the integral is calculated in the limit \(Q^{2}=0\).

The functions \(\psi_{N1}, \psi_{N2}\) can be defined in terms of the momentum range parameters \(\beta_{1}, \beta_{2}\) of the nucleon radial wave function, \(U(k)\), where \(\beta_{1}\) regulates the long-range structure and \(\beta_{2}\) regulates the short-range structure. If we choose a radial wave function that preserves the short-range structure of the nucleon wave function, we can determine all parameters of the functions \(\psi_{N1}, \psi_{N2}\) using Eqs. \[11\,10\]. In this case no parameters have to be adjusted, and the model provide true predictions for the transition form factors or the helicity amplitudes. This method was already used for the \(N(1440)\) state (the Roper), where we concluded, that, the model gives a very good description of the \(Q^{2} > 1.5\) GeV\(^{2}\) data \[11\,24\,25\], the region where the valence quark degrees of freedom dominate.

We tested then if the model could also be extended for the second radial excitation of the nucleon. Based on the quantum numbers \((P_{11})\), we assumed that the second radial excitation of the nucleon could be the \(N(1710)\) state. The model predictions for the \(N(1710)\) transition amplitudes are presented in Fig. 2 up to 12 GeV\(^{2}\) pointing for the upper limit of the Jlab-12 GeV upgrade. The results are compared with the available results for the \(N(1440)\) state. In order to obtain a comparison between elastic (nucleon) and inelastic transition amplitudes we also present the results of the nucleon equivalent amplitudes. The nucleon equivalent amplitudes are defined by \(A_{1/2} = \sqrt{2} R G_{M} \) and \(S_{1/2} = \sqrt{2} e \sqrt{M_{N}} G_{E} \), where \(G_{M}, G_{E}\) are the nucleon form factors, \(\tau = \frac{Q^{2}}{M_{N}}, \) and \(R\) is a function dependent of the \(N(1440)\) variables (mass \(M_{R}\)), \(R = \frac{K}{M_{R}} \frac{M_{R} - M_{N}^{2} + Q^{2}}{M_{R} M_{N}}, \) where \(K = \frac{M_{R}^{2} - M_{N}^{2}}{2M_{R}}\) \((e\) is the elementary electric charge\). In the amplitudes the factor \(\sqrt{2}\) was included for convenience.
In Fig. 2, one can see, that, the results for the amplitudes are similar for all the system for $Q^2 > 4$ GeV$^2$ (large $Q^2$). The exception is the result for amplitude $S_{1/2}$ for the nucleon, which vanishes for $Q^2 \approx 7$ GeV$^2$, because $G_E \simeq 0$ \cite{6}. One can interpret the approximated convergence of results for large $Q^2$ as a consequence of the correlations between the excited radial wave functions and the nucleon radial wave function \cite{10}.

Since the equivalent amplitude $A_{1/2}$ for the nucleon is known already for very large $Q^2$ (using the $G_M$ data), this result can be compared with the estimates for the $N(1440)$ and $N(1710)$ systems, for very large $Q^2$, as presented in Fig. 3. As shown in the figure, one predicts that the amplitude $A_{1/2}$ follows closely the equivalent amplitude of the nucleon, for large $Q^2$. Future experiments in the range $Q^2 = 4–10$ GeV$^2$ can confirm or deny this prediction.

The $\gamma^*N \to N(1710)$ transition amplitudes were determined for the first time in Ref. \cite{27}. The data is compared with our estimates in Fig. 4. From the figure, one can conclude that our estimate differs from the data in magnitude for $A_{1/2}$ and in sign for $S_{1/2}$, at least for $Q^2 \leq 4$ GeV$^2$. The new data suggests that, or, our estimate is valid only for larger values of $Q^2$, to be confirmed by new data for $Q^2 > 4$ GeV$^2$, or, our interpretation of $N(1710)$ as the second radial resonance of the nucleon, is not valid. It is worth to mention that our calculations are consistent with others estimates based on the assumption of the second nucleon radial excitation, where $A_{1/2} > 0$ and $S_{1/2} > 0$ \cite{28}. The signs, and magnitudes of the data, are however consistent with the estimates of the hypercentral quark model \cite{27,29}.

From the theoretical point of view there are other possible interpretations for the $N(1710)$ state. Although the interpretation of the states from the third resonance region based on the quark structure are partially tentative, the state $N(1710)$ can be interpreted as an excitation associated with a state with mixed symmetry \cite{30,31}. Several other suggestions have been made for the composition of the

![Fig. 2 Helicity amplitudes for the nucleon, $N(1440)$ and $N(1710)$. See definition of the nucleon equivalent amplitudes in the text.](image1)

![Fig. 3 Amplitude $A_{1/2}$ for the nucleon, $N(1440)$ and $N(1710)$, compared with the data extracted from the nucleon magnetic form factor \cite{26}.](image2)
N(1710) state, such as the mixture of states \( \pi N - \pi N \), the mixture \( \pi N - \sigma N \), a \( \sigma, N \) state, where \( \sigma \) represent a vibrational state of the \( \sigma \), and even a \( gN \) mixture, where \( g \) is a gluon \([10]\). Another possibility is that \( N(1710) \) is a dynamically generated resonance as suggested by Ref. \([32]\), although the estimated mass is larger \((1820\text{ MeV})\). Finally in the interacting quark diquark model the \( N(1710) \) is described as a quark-axial diquark system in a relative \( S \)-wave configuration. The prediction of the mass is respectively, \( 1640\text{ MeV} \) in the non relativistic version of the model \([33]\) and \( 1776\text{ MeV} \) in the relativistic version of the same model \([34]\).

Contrarily to the nucleon and the \( N(1440) \) that are part of the \( SU(6) \)-multiplet \([56, 0^+]\), the \( N(1710) \) state is more frequently associated with the \( J^P = 1^+ \) multiplet \([70, 0^+]\) \([23, 24, 30, 31, 33]\). In those conditions the second radial excitation of the nucleon should correspond to an higher mass resonance. The state \( N(1880) \) listed by the particle data group (PDG) \([30]\) is at the moment the best candidate. Within the quark models it is interesting to note that hypercentral quark model is the only model which admits two radial excitations of the nucleon belonging to \([56, 0^+]\) multiplets \([24, 32, 33]\).

We expect, that, future experiments provide accurate data for the transverse \((A_{1/2})\) and longitudinal \((S_{1/2})\) amplitudes, which may be used to test the expected falloff associated with the valence quark effects: \( A_{1/2} \propto 1/Q^2 \) and \( S_{1/2} \propto 1/Q^3 \), for large \( Q^2 \) \([37]\). If this is not case, and \( N(1710) \) is in fact a mixture of meson-baryon states, the \( 1/Q^3 \) falloff cannot be observed, and the interpretation of \( N(1710) \) as a state dominated by valence quark has to be questioned.

### 4 Resonances from the \([70, 1^-]\) \( SU(6) \)-multiplet

The combination of the wave functions of a baryon (three-quark system) given by \( SU(6) \otimes O(3) \) group and the description of electromagnetic interaction in impulse approximation leads to the so-called single quark transition model (SQTM) \([38, 39]\). In this context single means that only one quark couples with the photon. In those conditions the SQTM can be used to parametrize the transition current between two multiplets, in an operational form that includes only four independent terms, with coefficients exclusively dependent on \( Q^2 \).

In particular, the SQTM can be used to parametrize the \( \gamma N \rightarrow N^* \) transitions, where \( N^* \) is a \( N \) (isospin \( 1/2 \)) or a \( \Delta \) (isospin \( 3/2 \)) state from the \([70, 1^-]\) multiplet, in terms on three independent functions of \( Q^2 \): \( A, B \), and \( C \) \([38, 39]\). The relations between the functions \( A, B \), and \( C \) and the amplitudes are presented in the Table 1. In this analysis we do not take into account the constraints at the pseudo-threshold when \( Q^2 = -(M_R - M)^2 \) as in Ref. \([40]\). Using the results for the \( \gamma N \rightarrow N(1535) \) and \( \gamma N \rightarrow N(1520) \) amplitudes, respectively \( A_{S11}^{1/2}, A_{D13}^{1/2}, \) and \( A_{D13}^{3/2} \) in the spectroscopic notation, we can write, using \( \cos \theta_D = 0.995 \approx 1 \)

\[
A = 2 \frac{A_{S11}^{1/2}}{c} + \sqrt{2} A_{D13}^{1/2} + \sqrt{6} A_{D13}^{3/2}, \quad B = 2 \frac{A_{S11}^{1/2}}{c} - 2 \sqrt{2} A_{D13}^{1/2}, \quad C = -2 \frac{A_{S11}^{1/2}}{c} - \sqrt{2} A_{D13}^{1/2} + \sqrt{6} A_{D13}^{3/2}.
\]

### Fig. 4 \( \gamma^* N \rightarrow N(1710) \) transition amplitudes compared with the data from Park et al \([27]\). See discussion in the text.
We use then the amplitudes $A_{1/2}^{S11}$, $A_{3/2}^{D13}$ and $A_{3/2}^{D11}$, determined by the covariant spectator quark model for the $\gamma^*N \to N(1520)$ and $\gamma^*N \to N(1535)$ transitions $^{11,12,13}$, to calculate the coefficients $A$, $B$ and $C$. After that we can predict the amplitudes associated with the remaining transition for $[70, 1^-]$ states, namely for the the transitions $\gamma^*N \to N(1650)$, $\gamma^*N \to N(1700)$, $\gamma^*N \to \Delta(1620)$ and $\gamma^*N \to \Delta(1700)$. Since the covariant spectator quark model breaks the $SU(2)$-flavor symmetry, we restrict our study to reactions with proton targets (average on the SQTM coefficients). Based on the amplitudes used in the calibration we expect the estimates to be accurate for $Q^2 \gtrsim 2$ GeV$^2$ $^{13}$.

From the study of the $\gamma^*N \to N(1520)$ transition within the covariant spectator quark model it is possible to conclude that the contributions for the amplitude $A_{3/2}$ due to valence quarks are very small $^{12,13}$. This conclusion in consistent with others estimates from quark models, where the valence quark component is about 20–40% $^{12,41}$. An accurate description of the $\gamma^*N \to N(1520)$ transition requires, then, a significant meson contribution for the amplitude $A_{3/2}$, as suggested also by the EBAC/Argonne-Osaka model $^{42}$. Therefore, in Ref. $^{12}$, we developed an effective parametrization of the amplitude $A_{3/2}$, inspired on the meson cloud contributions for the $\gamma^*N \to \Delta$ transition $^{12,13,20}$.

From the relations $^{2}$, we can conclude in the limit where no meson cloud is considered ($A_{3/2}^{D13} = 0$), one has $C = -A$. The last condition defines the model 1, dependent only of the parameters $A$ and $B$. Since, as discussed, the description of the $\gamma^*N \to N(1520)$ is limited when we neglect the meson cloud contributions for $A_{3/2}$, we consider a second model (model 2) where in addition to the valence quark contributions for the amplitudes $A_{1/2}$ and $A_{3/2}^{D11}$, we include a parametrization for $A_{3/2}^{D13}$ derived in Ref. $^{12}$.

| State          | Amplitude      | $\frac{1}{\sqrt{2}}(A + B - C) \cos \theta_S$ |
|----------------|----------------|------------------------------------------------|
| $N(1535)$      | $A_{1/2}$      | $\frac{1}{\sqrt{2}}(A + B - C) \cos \theta_D$ |
| $N(1520)$      | $A_{1/2}$, $A_{3/2}$ | $\frac{1}{\sqrt{2}}(A + B - C) \sin \theta_S$ |
| $N(1650)$      | $A_{1/2}$      | $\frac{1}{\sqrt{2}}(A + B - C) \sin \theta_D$ |
| $\Delta(1620)$ | $A_{1/2}$      | $\frac{1}{\sqrt{2}}(3A + B + C)$ |
| $N(1700)$      | $A_{1/2}$      | $\frac{1}{\sqrt{2}}(A - B - C) \sin \theta_D$ |
| $\Delta(1700)$ | $A_{1/2}$      | $\frac{1}{\sqrt{2}}(3A + 2B + C)$ |

| State          | Amplitude      |
|----------------|----------------|
| $N(1520)$      | $A_{1/2}$      |
| $N(1535)$      | $A_{1/2}$      |
| $N(1650)$      | $A_{1/2}$      |
| $\Delta(1620)$| $A_{1/2}$      |
| $N(1700)$      | $A_{1/2}$      |
| $\Delta(1700)$| $A_{1/2}$      |

**Table 1** Amplitudes $A_{1/2}$ and $A_{3/2}$ estimated by SQTM for the proton targets ($N = p$) $^{13,15}$. The angle $\theta_S$ is the mixing angle associated with the $N_{\frac{1}{2}^-}$ states ($\theta_S = 31^\circ$). The angle $\theta_D$ is the mixing angle associated with the $N_{\frac{3}{2}^-}$ states ($\theta_D = 6^\circ$).

**Fig. 5** Results for $N(1650)$. The models 1 and 2 gave the same result (solid line).

**Fig. 6** Results for $\Delta(1620)$. Model 1 (dashed line) and model 2 (solid-line).
The models are compared with the available data for the amplitudes associated with the $[70,1^-]$ $SU(6)$-multiplet. The data available for those resonances, from CLAS [24], MAID [25], PDG [30] and others [35], is very scarce particularly for $Q^2 > 1.5$ GeV$^2$. Nevertheless, we conclude that for the cases $N(1650)$ and $\Delta(1620)$, the models 2 gives a good description of the data (see Figs. 5 and 6).

Based on the expected behavior for large $Q^2$ given by $A_{1/2} \propto 1/Q^2$ and $A_{1/2} \propto 1/Q^4$ in accordance with perturbative QCD arguments [35], we parametrize the amplitudes as

$$A_{1/2}(Q^2) = D \left( \frac{A^2}{A^2 + Q^2} \right)^{3/2}, \quad A_{3/2}(Q^2) = D \left( \frac{A^2}{A^2 + Q^2} \right)^{5/2}. \quad (3)$$

The coefficients $D$ and the cutoff $A$ are determined in order to be exact for $Q^2 = 5$ GeV$^2$. The results of the parametrizations are in Table 2. Those parametrizations may be useful to compare with future experiments at large $Q^2$, as the ones predicted for the Jlab-12 GeV upgrade [1].

For the amplitude $A_{1/2}$ associated with the $\Delta(1620)$ state, it is not possible to find a parametrization consistent the power 3/2 for the amplitude $A_{1/2}$. This is because, for that particular amplitude, there is a partial cancellation between the leading terms (on $1/Q^3$) of our $A, B$ and $C$ parametrization, due to the difference of sign between the amplitudes $A_{1/2}^{S11}$ and $A_{1/2}^{D13}$ used in the determination of the SQTM coefficients (see dashed line in Fig. 6). As consequence, the amplitude $A_{1/2}$ for the state $\Delta(1620)$ is dominated by next leading terms (on $1/Q^5$) or contributions due to meson cloud effects ($A_{3/2}^{D13}$). It is clear in Fig. 6 that, when we neglect the contributions from $A_{3/2}^{D13}$ the result is almost zero (model 1; dashed line). This result shows that in the $\gamma^* N \rightarrow \Delta(1620)$ transition, contrarily to what is usually expected, there is a strong suppression of the valence quark effects for $Q^2 = 1–2$ GeV$^2$. A better representation of the $\gamma^* N \rightarrow \Delta(1620)$ data is obtained using $A_{1/2} \propto \left( \frac{A^2}{A^2 + Q^2} \right)^{5/2}$, where $A^2 = 1$ GeV$^2$ (note the power 5/2, instead of the expected 3/2). For a more detailed discussion see Ref. [13].

### 5 Summary and Conclusions

Answering to the challenge raised by recent experimental results and by the experiments planed for a near future, for resonances in the region $W \approx 1.6–1.8$ GeV, with high photon virtualities, we present the more recent results from the covariant spectator quark model. The model is covariant, it is based on the valence quark degrees of freedom, and therefore potentially applicable in the region of the large energies and momenta. The estimates for the second radial excitation of the nucleon, interpreted as the $N(1710)$ state, are still under discussion, both theoretically and experimentally. At the moment it is more likely that the second radial excitation of the nucleon correspond to an higher mass resonance.

For the negative parity resonances from the $[70,1^-] SU(6)$-multiplet, we provide predictions for the transition amplitudes based on the combination with the SQTM. Our predictions compare well with the $Q^2 > 2$ GeV$^2$ data, for $N(1650)$ and $\Delta(1620)$. As for the remaining resonances, the predictions for the transverse amplitudes $A_{1/2}$ and $A_{3/2}$ have to wait for future high $Q^2$ data, such as the data expected from the Jlab-12 GeV upgrade.

| State        | Amplitude | $D(10^{-3}$ GeV$^{-1/2}$) | $A^2$(GeV$^2$) |
|--------------|-----------|---------------------------|---------------|
| $N(1650)$    | $A_{1/2}$ | 68.90                     | 3.35          |
| $\Delta(1620)$ | $A_{1/2}$ | ...                       | ...           |
| $N(1700)$    | $A_{1/2}$ | −8.51                     | 2.82          |
|              | $A_{3/2}$ | 4.36                      | 3.61          |
| $\Delta(1700)$ | $A_{1/2}$ | 39.22                     | 2.69          |
|              | $A_{3/2}$ | 42.15                     | 8.42          |

Table 2. Parameters from the high $Q^2$ parametrization given by Eqs. (3).
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