Free-floating bike-sharing green relocation problem considering greenhouse gas emissions

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Abstract

This paper introduces the problem of green bike relocation considering greenhouse gas emissions in free-floating bike-sharing systems (FFBSSs) and establishes a mathematical model of the problem. This model minimizes the total imbalance degree of bikes in the FFBSS and the greenhouse gas emissions generated by relocation in the FFBSS. Before the relocation phase, the FFBSS is divided into multiple relocation areas using a two-layer clustering method to reduce the scale of the relocation problem. In the relocation phase, the relocation route problem is converted into a pickup and delivery vehicle-routing problem. Then, an adaptive variable neighbourhood tabu search algorithm with a three-dimensional tabu list is proposed, which can simultaneously solve the relocation problem and the routing problem. A computational study based on the actual FFBSS used in Shanghai shows that this method can effectively solve the green relocation problem of FFBSSs.

Keywords: free-floating bike-sharing system; greenhouse gas emissions; two-layer clustering method; adaptive variable neighbourhood tabu search algorithm

1. Introduction

The free-floating bike-sharing system (FFBSS) is a new emerging travel mode, which is convenient, fast, low-cost, and green, so it is welcomed by the public. An FFBSS treats public places as parking lots, which avoids the need for users to use vacant parking spaces at fixed stations to return bikes. The user can rent a bike near the starting point and return the bike to any public docking station near the destination after the ride is completed. The administrators use the satellite receiver on each bike to identify the location and usage status of the bike. FFBSSs can not only reduce the traffic pressure on the road network [1, 2] but also contribute to energy saving and emission reduction [3, 4]. In addition, FFBSSs can
complete the connection between various public transportation systems and solve the problem of ‘last-mile’ commuting [5, 6]. However, The FFBSS without a relocation system will become unbalanced. Unbalanced bike distribution may cause users in certain areas to be unable to use bikes.

Researchers are showing a growing interest in administrator-based relocation strategies in FFBSSs. Most of these studies have considered static relocation problems [7–9]. However, static relocation does not guarantee that the network will not fail during the day. To overcome this problem, dynamic relocation is used to relocate bikes according to the current bikes’ distribution. This approach is challenging because it involves relocating components and routing issues based on user activity during relocation operations [10–12]. In order to solve the relocation problem in large FFBSSs, some studies have followed the cluster-first and route-second procedure to solve the bike relocation problem for each bike relocation area [8, 13, 29]. This paper first divides the relocation system into multiple transportation analysis zones (TAZs), then uses a two-layer clustering method to aggregate the TAZs into multiple relocation areas and rebalances the bikes in each relocation area separately.

The advantages of shared bikes include a convenient journey, reduction of traffic congestion, CO₂ emission reduction in cities, flexible mobility and physical health benefits [14]. According to data released by the National Bureau of Statistics of China, greenhouse gases produced by urban transportation account for 13% of total global emissions, of which 30% to 40% is carbon dioxide. In the past 15 years, the average growth rate of the number of cars in China has been 17%. The emergence of shared transportation modes has greatly affected our commuting methods [15]. In 2016, the FFBSS in Shanghai saved 8358 tons of petrol and decreased CO₂ and NOₓ emissions by 25240 and 64 tons, respectively [4]. However, the current FFBSS contribution to emission reduction is still relatively limited, amounting to less than 0.1% [16]. Moreover, the relocation of shared bikes has a negative effect. The relocation of bikes is usually done by trucks/vans, which consume fossil fuels and generate emissions [9]. For example, due to extensive relocation efforts, London’s bike-sharing system results in an additional 766341 km of vehicle usage each year, which exceeds the travel-replacement revenue from bike-sharing system activation [17]. The greenhouse gas emissions from the relocation operation offset the environmental benefits of cycling. The impact of free-floating bike-sharing travel on urban traffic emission reduction needs further study.

In response to the environmental protection problem of FFBSS relocation, researchers have also proposed a number of variants of the FFBSS relocation problem. For example, Shui and Szeto [18] introduced a new dynamic green bike relocation problem, which determines the route and the number of bikes loaded and unloaded at each visited node over a multi-period operational horizon during which the cycling demand at each node varies. The problem simultaneously minimizes the total unmet demand of the bike-sharing system and the fuel and CO₂ emission costs of the relocation vehicle over an operational period. But Shui and Szeto [18] did not consider that increased bike rides would reduce the CO₂ emissions generated by users’ commuting. Luo et al. [29] proposed a framework for obtaining the optimal bike fleet size and rebalancing strategy to minimize the system’s life-cycle greenhouse gas emissions, integrating a simulation model for fleet-size estimation, an optimization model for bike rebalancing and a life-cycle assessment model to quantify the system’s greenhouse gas emission rate. However, their research is based on the station-based bike-sharing system, and no algorithm for the vehicle-routing problem is proposed. On these bases, this paper adds an analysis of the greenhouse gas emission reduction of shared bikes; at the same time, the maximum distance of vehicles during operation is fixed, thereby fixing the fuel consumption of vehicles.

The rest of the paper is organized as follows. Section 2 makes some considerations on the free-floating bike-sharing green relocation problem. The vehicle-routing model for the free-floating bike-sharing green relocation problem is established in Section 3. A new kind of adaptive variable neighbourhood tabu search (AVNTS) algorithm is introduced in Section 4. In Section 5, extensive computational studies are conducted to study the effectiveness of the proposed solution methodology. Finally, the conclusions from this study and directions for future research are presented in Section 6.

2. Basic considerations

2.1. Case study data

The data used in this paper is from a large-scale FFBSS in Shanghai, China, recorded over 14
continuous days from 26 August to 8 September 2018. The data was retrieved using global positioning system (GPS) devices installed on the bikes. Each bike record includes a unique bike ID, the time (to the second) of the record, the bike-lock status and the location (longitude and latitude) of the bike record, as shown in Table 1. For raw data cleaning and pre-processing, the following procedures were followed: (i) eliminate outlier trips whose OD locations are outside the city’s geographical boundaries (longitude 120.75–121.85 and latitude 30.2–31.3); (ii) for trips missing either origin or destination locations, assign the first or last point the status of ‘1’ as the O/D location, respectively; (iii) eliminate trips whose duration is less than 10 s; (iv) eliminate trips whose speed is greater than 40 km/h; and (v) eliminate trips whose distance is less than 10 m. After data cleaning, 10,088,094 trips were obtained.

2.2. Assumptions

A1) The entire spatial distribution is divided into a network of \( V \) TAZs, so each TAZ has an equal length and width. Let \( i \) or \( j \) represent the index of the TAZ, i.e. \( \{1, 2, 3, \ldots, i, j, \ldots \} \). Let \( x_i \) and \( y_i \) represent the horizontal index and vertical index of the TAZ \( i \) in the network.

A2) As the traffic conditions on workdays or weekends are quite different [19], the data set is divided into two categories: workdays and weekends. Assuming that the bike distribution is balanced initially, the difference between the bike distribution after a day of riding by the users and the bike distribution at the beginning of this day is the bike’s degree of imbalance. Equation (1) calculates the bike’s degree of imbalance in TAZ \( i \) at time \( T \), where \( T \) represents the end of the day, \( u_{it}^0 \) represents the number of bikes returned in TAZ \( i \) at time \( t \) and \( u_{it}^r \) represents the number of bikes rented in TAZ \( i \) at time \( t \):

\[
q_i = \sum_{t=1}^{t=T} (u_{it}^0 - u_{it}^r), \forall i \in V
\]  

(1)

In the FFBSS, there are a large number of TAZs that have a low degree of imbalance. In addition, except for the normal driving process, every stop and start of the vehicles will also produce greenhouse gas emissions. If vehicles are forced to rebalance the system more frequently to serve the low degree of imbalances, the system’s greenhouse gas emissions will increase [29]. Therefore, the relocation operation ignores those TAZs with low degrees of imbalance and still considers them to be balanced TAZs. The threshold \( \xi \) is used to sort out the TAZs with a high degree of imbalance. The situation of \( q_i \geq \xi \) shows that there are a lot of surplus bikes at the end of this day, and TAZ \( i \) can be called a surplus TAZ. The situation of \( q_i \leq -\xi \) shows that there are a lot of deficit bikes in the TAZ \( i \) at the end of this day, and the TAZ \( i \) can be called a deficit TAZ. A deficit or surplus TAZ can also be called an unbalanced TAZ. The operation of relocating the bikes in the unbalanced TAZ can be called rebalancing the TAZ. The situation of \( q_i = 0 \) shows that the number of bikes in TAZ \( i \) does not change after one day of riding, and the TAZ \( i \) can be called a balanced TAZ. The daily average of the degrees of imbalance of the TAZs in the FFBSS is calculated based on workday data from the two weeks, and the surplus TAZs and the deficit TAZs are the objects of rebalancing.

A3) Considering that the road structure in the city is grid-like, the distance between every two TAZs is calculated by the block distance, as shown in Equation (2), where \( x_i \) represents the position of TAZ \( i \), \( x_i \) and \( y_i \) represent the horizontal and vertical coordinates of TAZ \( i \).

\[
\| \bar{x}_i - \bar{x}_j \|_2 = |x_i - x_j| + |y_i - y_j|
\]  

(2)

A4) Each vehicle is of the same type, which means it has the same capacity, has the same quality, the same maximum distance and the same greenhouse gas emission rate. Each bike has the same quality.

A5) Each TAZ can be rebalanced multiple times by multiple vehicles to relocate bikes.

A6) There are no excess greenhouse gas emissions caused by uncertain factors such as road congestion.

A7) During the entire relocation operation at night, the bikes in any TAZ are not subject to user riding [20].

A8) There is one garage in each relocation area, which is located in the centre of the relocation area [29].

A8) The vehicles depart from their garages without any bikes.

2.3. Relocation area

To meet the requirements of dynamic relocation, the bike-sharing system is divided into multiple areas before relocation [9, 21, 22]. This paper uses the spatial clustering method to divide the entire
Table 1. Data sources

| Item          | Description                              | Example               |
|---------------|------------------------------------------|-----------------------|
| BIKE_ID       | ID code of the bike in the FFBSS         | 713ED790798C3233E0533C0BA8C91 |
| DATA_TIME     | Time                                     | 2018/8/27 00:04:53    |
| LOCK_STATUS   | Lock status (0: unlocked; 1: locked)     | 1                     |
| LONGITUDE     | Longitude of the bike in the FFBSS       | 121.4128915           |
| LATITUDE      | Latitude of the bike in the FFBSS        | 31.27068821           |

FFBSS into multiple relocation areas. Each vehicle belongs to only one relocation area, so vehicles cannot carry the bikes across different relocation areas, but there can be multiple vehicles within a relocation area. During the process of dividing the system into multiple relocation areas, two factors need to be kept in mind:

(i) The deficit TAZs in one relocation area must not be aggregated. If a deficit TAZ is scattered on the edge of the relocation area, it will cause the vehicles to rebalance the deficit TAZ on the edge of the area more frequently. Since vehicles cannot travel across different relocation areas, the irregular shape of a relocation area will cause the vehicle to run outside. After the vehicle unloads the bikes in deficit TAZs, it may not be possible to pick up bikes in surplus TAZs nearby. The vehicle can pick up excess bikes only by running back, which causes the empty driving rate to be very high.

(ii) The rebalancing task in each relocation area must not be unreasonable. If the relocation areas are divided without considering the bike’s degree of imbalance, it may cause a higher bike’s degree of imbalance in the area containing hot areas, but a lower bike’s degree of imbalance in the suburban areas. This may cause some vehicles to run overloaded and other vehicles to be in a long-term idle state. In addition, unreasonable rebalancing tasks may also result in the number of deficit bikes in the area being far greater than the number of surplus bikes, resulting in vehicles being unable to effectively rebalance the deficit TAZs in the relocation area.

In this section, the partitioning problem of the bike relocation areas is transformed into a two-layer clustering problem. We define a deficit TAZ $i$ with $q_i \leq -2\xi$ as a main deficit TAZ. The first clustering layer aims at minimum distances between the main deficit TAZs (Equation (4)), which aims to address the impact of the first situation; the second clustering layer aims at the minimum degree of aggregation in the relocation areas (Equation (3)), which aims to address the impact of the second situation. In Equation (3), $A_c$ represents the c-th relocation area, $N_c$ represents the number of unbalanced bikes in the c-th relocation areas, $\bar{\mu}_c$ represent the cluster centre of the c-th relocation area. Let $I$ represents the index of a main deficit TAZ.

$$\min \sum_{c=1}^{C} \sum_{i \in A_c} \| \bar{x}_i - \bar{x}_I \|_2 \times |N_c|^{\frac{\xi}{|\xi|+1}}$$

subject to

$$\min \sum_{c=1}^{C} \sum_{i \in A_c} \| \bar{x}_i - \bar{\mu}_c \|_2$$

It is necessary to comprehensively consider the distances between the deficit TAZs and the clustering centres and the number of unbalanced bikes in the relocation areas. So, the two-layer clustering method is proposed to balance the relationship between the two factors, which can make the number of TAZs in each relocation area more balanced. In order to allow the vehicle to access surplus TAZs nearby to load the bikes after dropping off in the deficit TAZs, the second clustering layer assigns the cluster label of the unbalanced TAZs to the main deficit TAZs with the smallest $\| \bar{x}_i - \bar{x}_I \|_2 \times |N_c|$ (Equation (4)). The second clustering layer does not affect the results of the first clustering layer and can ensure that the rebalancing tasks are appropriate.

3. Vehicle-routing model for free-floating bike-sharing green relocation problem

(Blank) The free-floating bike-sharing green relocation problem can be described as the FFBSS having been divided into multiple relocation areas, and bikes in each relocation area can be relocated by vehicles of the same type. Vehicles can rebalance any TAZ in one relocation area multiple times to relocate bikes. Each vehicle belongs to only one
garage, and there is only one garage in one relocation area. The vehicles start from their garages in the FFBSS and finally end at their garages within the maximum distance. The relocated bikes will increase the number of rides by the users the next day. The increased number of rides will reduce greenhouse gas emissions from using other alternative modes of commuting, while vehicle routing will produce greenhouse gas emissions. The problem aims to determine routes for the vehicles in the night and the loading and unloading quantities in each TAZ along the routes in order to minimize the degree of imbalance of bikes in the FFBSS and the greenhouse gas emissions due to relocation operations. The symbols needed to describe the vehicle-routing model for free-floating bike-sharing green relocation problem are shown in Table 2, and the established mathematical model is as follows:

\[
\text{minimize } F = \gamma_1 \cdot C_{\text{unbalance}} + \gamma_2 \cdot C_{\text{emission}} \quad (5)
\]

\[
C_{\text{unbalance}} = \sum_{i \in V} \left( q_i + \sum_{k \in K} y_{ik}^l - \sum_{k \in K} y_{ik}^u \right)^2 \quad (6)
\]

\[
C_{\text{emission}} = e_1 \cdot \sum_{i \in V} \sum_{j, k \in V} \left[ (m_k + m_b \cdot f_{ij}) \cdot d_{ij} \cdot x_{ijk} \right] - e_2 \cdot \sum_{k \in K} \sum_{i \in V} (y_{ik}^l \cdot r_i) \quad (7)
\]

subject to

\[
|q_i| \geq \sum_{k \in K} y_{ik}^l, \forall i \in S \quad (8)
\]

\[
|q_i| \geq \sum_{k \in K} y_{ik}^u, \forall i \in D \quad (9)
\]

\[
\sum_{j \in V} x_{ijk} = \sum_{h \in V} x_{ihk}, \forall i, j, h \in V, \forall k \in K \quad (10)
\]

\[
f_{ijk} = f_{ihk} + y_{ik}^l - y_{ik}^u - M \cdot (1 - x_{ijk}), \forall i, j, h \in V, \forall k \in K \quad (11)
\]

\[
0 \leq f_{ijk} \leq Q \cdot x_{ijk}, \forall i, j \in V, \forall k \in K \quad (12)
\]

\[
\sum_{i,j \in V} (d_{ij} \cdot x_{ijk}) \leq L, \forall k \in K \quad (13)
\]

\[
\sum_{i \in B} y_{ik}^l = \sum_{j \in C} y_{jk}^l, \forall k \in K \quad (14)
\]

\[
y_{ik}^l + y_{ik}^u \geq x_{ijk}, \forall i, j \in V, \forall k \in K \quad (15)
\]

\[
x_{ijk} \in \{0, 1\}, \forall i, j \in V, \forall k \in K \quad (16)
\]

\[
y_{ik}^l, y_{ik}^u > 0, \text{ integer}, \forall i \in V, \forall k \in K \quad (17)
\]

The first part of the objective function (5) represents the degree of imbalance in the FFBSS after relocated bikes, and the second part represents the greenhouse gas emissions. The sequential square calculation in equation (6) indicates that vehicles should be more inclined to balance the TAZs with high unbalanced agree. Equation (7) represents the greenhouse gas emissions including the greenhouse gas emissions from vehicles and the reduction of greenhouse gas emissions due to increased riding. Constraints (8) ensure that the number of bikes loaded in surplus TAZ cannot exceed its surplus quantities, Constraints (9) ensure that the number of bikes unloaded in deficit TAZ cannot exceed its deficit quantities. Equation (10) indicates that if vehicle k rebalances TAZ i, it must leave TAZ i later. Constraint (7) states the conservation of bikes in a vehicle. Constraint (11) ensures that each vehicle has the same maximum distance limitation. Constraint (14) ensures that all loaded bikes will eventually be unloaded. Constraint (15) indicates that if there is a vehicle visited a TAZ, a relocation operation must be performed. Constraints (16) and (17) are domains of decision variables.

4. AVNTS algorithm

As a variant of the pickup and delivery vehicle-routing problem, the proposed problem is also NP-hard, which raises a great computational challenge for large-size instances [23]. The tabu search algorithm has been successfully applied in solving the vehicle-routing problem [24] and simultaneous pickup and delivery problems. Previous studies on similar problems have shown that large-scale instances may not be solved by accurate algorithms, and meta-heuristic algorithms will provide a more effective solution. Our algorithm is based on the tabu search proposed by Glover in 1986. Since then, it has been used to solve many practical applications. The tabu search algorithm is a memory-based search strategy that can instruct the local search method to continue its search until it exceeds the local optimal value. One way to achieve this is to track the properties of
recent actions or solutions in the past in a tabu list. We proposed an AVNTS algorithm with a three-dimensional tabu list.

### 4.1. AVNTS algorithm framework

Since the free-floating bike-sharing green relocation problem in this paper is a multi-vehicle multi-trip routing problem, each TAZ can be rebalanced by multiple vehicles, so the effect of rebalancing is jointly determined by all the vehicles. Therefore, tabu search optimization cannot be performed on one of the vehicles alone, and there are multiple trips in a solution. In order to determine the absolute position of the neighbourhood operator (which node of which vehicle’s route), a three-dimensional tabu list is created by adding the serial number of the vehicle performing the neighbourhood operator, on the basis of the original two-dimensional tabu list by Glover in 1986.

For different scales of relocation areas, a single and fixed multiple neighbourhood operators may not achieve the best optimization efficiency. The AVNTS algorithm designs five neighbourhood operators and uses an adaptive method to adjust the probability of performing these neighbourhood operators. There are five adaptive factors corresponded to the five neighbourhood operators adopted by neighbourhood transformation, $\psi = \{\psi_s, s = 1, 2, \ldots, S\}$. In the initialization stage of the AVNTS algorithm, each adaptive factor is assigned the same initial value, $\psi_0$. The probability of selecting $s$-th neighbourhood operator as the neighbourhood transformation to generate the next candidate solution is proportional to $\psi_s$; with probability distribution $P = \{\psi_s, s = 1, 2, \ldots, S\}$. In other words, the greater the value of $\psi_s$, the greater the probability that the $s$-th neighbourhood operator will be selected. $\psi$ is updated by the attenuation coefficient $\alpha$ after each iteration, but in order for each adaptive factor to have a certain probability to be selected, it is necessary to limit $\psi_s \geq \psi_{\text{min}}$.

A pseudocode describing the AVNTS algorithm process used to solve the free-floating bike-sharing green relocation problem is presented in Algorithm 1. In Algorithm 1, $R_{\text{current}}$ represents the contemporary optimal solution, $R_{\text{global}}$ represents the global optimal solution. $P_{\text{stop}}$ is the maximum number of iterations.

### 4.2. Evaluation of solutions

There are two steps to evaluate each solution. First, transform $R$ into the effective rebalancing scheme: $\text{Transformation}(R, q, L) \rightarrow (Z, q, H_{\text{max}})$, $Z$ represents the set of rebalancing schemes, $H_{\text{max}}$ represents the set of the effective routing lengths of the vehicles and $q$ represents the set of the degrees of imbalance in the FFBSS after relocation. Next, the evaluation of $R$ is calculated according to

| Item | Description |
|------|-------------|
| $D$  | A set of deficit TAZs, $D = \{1, 2, \ldots, d\}$. |
| $S$  | A set of surplus TAZs, $S = \{1, 2, \ldots, s\}$. |
| $V$  | A set of TAZs, which include deficit TAZs, surplus TAZs and the garage, $V = D \cup S \cup \{0\}$. |
| $K$  | A set of vehicles, $K = \{1, 2, \ldots, k\}$. |

| Parameter | Description |
|-----------|-------------|
| $q_i$     | The average number of unbalanced bikes in TAZ $i$. |
| $r_i$     | The average riding distance of bikes rented in TAZ $i$. |
| $f_{ijk}$ | The number of bikes on vehicle $k$ when travelling from TAZ $i$ to TAZ $j$. |
| $Q$       | The capacity of the vehicles. |
| $m_0$     | The quality of a vehicle. |
| $m_b$     | The quality of a bike. |
| $d_{ij}$  | Travel distance from TAZ $i$ to TAZ $j$. |
| $L$       | The maximum distance of the vehicles. |
| $\gamma_1, \gamma_2$ | Two coefficients. |
| $e_1$     | Vehicle emission rate. |
| $e_2$     | Bike emission reduction rate. |
| $M$       | A huge positive number. |

| Variable | Description |
|----------|-------------|
| $\psi_{ik}$ | A binary variable that defines the route of vehicle $k$. If $\psi_{ik} = 1$, vehicle $k$ departs from place $i$ to place $j$; 0 otherwise. |
| $y^S_{ik}$ | The loading quantity onto vehicle $k$ in surplus TAZ $i$, $i \in S$. |
| $y^D_{ik}$ | The unloading quantity from vehicle $k$ in deficit TAZ $i$, $i \in D$. |
Algorithm 1. AVNTS algorithm

Input: the location of unbalance TAZ and degrees of imbalance in A_c, K_c vehicles in A_c
1. Construct the initial solution of K_c vehicles: R_c
2. Initialization: adaptive factor ψ, tabu list Ω and set R_{global} ← R_c; P = 1.
3. while P ≤ P_{stop}
   for n = 1 to N
      // Generate N candidate solutions: {R_n}n=1...N
      R_n is generated by n-th neighbourhood operator with the probability of ψ_n/sum(ψ)
   end
5. Find the non-tabu optimal solution R, optimal solution R_{best}
6. if f(R_{best}) < f(R_{global}) (Section 4.2)
   R_{global} ← R_{current} ← R_{best}
7. else
   R_{current} ← R_n
8. end
9. Update tabu list Ω; P ← P + 1.
10. Select the 10 candidate solutions with the best evaluation values and update the factors that were used to generate these 10 candidate solutions.
11. ψ ← max(ψ, ψ_{min}).
12. end
Output: R_{global}

Z and q. Its evaluation Equation (18) adds λ correction on the basis of Equation (5):

\[
f (R) = \frac{1}{2} \left[ \gamma_1 \sum_{i \in V} y_i + \gamma_2 \sum_{k \in K_c} \sum_{j \in V} \left( M + m \cdot f_j \right) \cdot x_{ijk} \right]
\] (18)

The set of rebalancing scheme Z is composed of the sub-rebalancing schemes of K vehicles: Z = \{Z_k\} \ , \ k \in K_c, the sub-rebalancing scheme of the vehicle k consists of a route L_k = {L_i, k} and a delivery scheme U_k = \{u_{ijk}\} : Z_k = \{L_k, U_k\}, \forall i, j \in V, \ k \in K. Since vehicles need to pick up bikes in surplus TAZs before they can go to drop off bikes in deficit TAZs, the route whose second place is a deficit TAZ is invalid. Therefore, this invalid route is punished by setting λ. If the second place in the route (the first place is garage) is a deficit TAZs then λ = 10; otherwise λ = 1.

4.2.1. Solution transformation. If the unbalanced bikes in a TAZ are higher than the capacity Q of the vehicle, then the unbalanced bikes q in this TAZ cannot be fully rebalanced by one relocation operation. One vehicle must visit TAZ i at least q_i/Q times to make it fully balanced. In order to allow the route to indicate that the vehicle can rebalance the highly unbalanced TAZ multiple times, this section divides the TAZ with a high rebalance degree into \{qi/Q\} virtual TAZs, which have the same geographic location. ⌈·⌉ means ‘integer up’. If a vehicle rebalances a virtual TAZ, the vehicle will perform loading and unloading operations in actual TAZ. Because there are too many unbalanced TAZs, this paper studies incomplete rebalancing, so not all TAZs will be visited by vehicles once. All TAZs in the relocation area will be in the R of the vehicles, but the route lengths of R exceeds the number of TAZs that the vehicles can reach within the maximum distance, and those TAZs outside the maximum distance will not be rebalanced.

Solution transformation is divided into four steps. In the c-th relocation area, R is composed of candidate routes of K_c vehicles: R = \{R_1, \ldots, R_{K_c}\}. The vehicles influence each other during the relocation operation, and the TAZ that has been balanced does not need to be rebalanced by other vehicles. Therefore, the rebalancing scheme for each vehicle is formulated in order, and their loading and unloading quantities are calculated in turn. In the first step, the effective rebalance route L_k and the number of TAZs in L_k are calculated based on the route R_k of vehicle k within the maximum distance L. In the second step, according to the route R_k of vehicle k and all unbalanced bikes q = \{q_i | i \in V\}, use algorithm 4 to derive the loading and unloading quantities of vehicle k, U_k = \{u_{ijk} | i, j \in V\}. The third step is to update the unbalanced bikes q after each vehicle's rebalancing scheme is derived. The fourth step, if k < K_c, return to the first step until k = K_c. Algorithm 2 shows the pseudocode of the process of transforming a route into a rebalancing scheme.

4.2.2. Maximum distance. In order to make the vehicle-routing model more realistic, a restriction on the maximum distance L of the vehicle is added to the model. In order to allow the vehicle to rebalance the TAZs within L, the route
Algorithm 2. Transformation

Input: $R, q, L, K_c$
1. for $k = 1$ to $K_c$
2. Access($R_k$, $L$) $\rightarrow (L_k, H_k, \text{max})$ (Section 4.2.2)
3. Derivation($L_k, H_k, \text{max}$) $\rightarrow U_k$ (Section 4.2.3) // Solve for loading and unloading quantities
4. update the unbalanced bikes: $(U_k, q) \rightarrow q$
5. end
Output: $(L_k, U_k, (H_k, \text{max}), q, k = 1, \ldots, K_c$

length in $R_k$ should be longer than the maximum distance. Although the vehicle does not need to visit the TAZs beyond the maximum distance, $R_k$ still retains complete routes for subsequent neighbourhood transformations in the AVNTS algorithm. When solving the loading and unloading quantities, it is necessary to calculate the actual reachable TAZs of the vehicle $k$ within the maximum distance to update $L_k$. The update process is shown in Algorithm 3.

4.2.3. Loading and unloading quantities. After determining the effective rebalance route, it is necessary to derive the corresponding loading and unloading quantities through Algorithm 4. The calculation principle of loading and unloading quantities is: when the vehicle reaches a deficit TAZ, all the bikes loaded on the vehicle drop off until the deficit TAZ is rebalanced. When the vehicle arrived at a surplus TAZ, the loading quantity was determined by Algorithm 5.

Algorithm 5 is used to calculate the number of bikes picked up in the surplus TAZs. The principle of Algorithm 5 is: the loading quantity must first meet the capacity of the vehicle. Second, the deficit TAZ where the loaded bike to be dropped off is calculated. Lastly, the route distance that the loaded bike will travel and the reduction of greenhouse gas emissions this bike will reduce can be obtained. Only when the reduced greenhouse gas emissions are larger than the greenhouse gas emissions caused by the vehicles to carry this bike, can this bike be picked up in the surplus TAZ.

4.3. Construction of the initial solution

The tabu search algorithm has a certain dependence on the initial solution. A good initial solution can help the tabu search algorithm find a good final solution in the solution space, while a poor solution can reduce the convergence rate of the tabu search algorithm. After generating an initial solution, the tabu search algorithm can be used to further improve the quality of the solution. A well-designed tabu search algorithm solution should not be strongly dependent on the quality of the initial solution, and if the tabu search algorithm uses multiple initial solutions, the diversity gained from different initial solutions may have some advantages [25]. In the initial solution constructed, the first rebalanced TAZ for each vehicle after leaving the garage must be a surplus TAZ, because of assumption A8, the bikes only can be picked up in surplus TAZs.

4.4. Neighbourhood transformation

Neighbourhood transformation operators choose one of five neighbourhood operators: random swaps of subsequences, random swaps of points, random insertions of subsequences, random insertions of points and reversing subsequences (Fig. 1). Before the neighbourhood transforma-

Algorithm 3. Access

Input: $X_{k, L}$
1. Initialization: $L_k \leftarrow \emptyset; d_k \leftarrow 0; H_k, \text{max} \leftarrow 0$
2. for $i = 2$ to $|R_k|$
3. 
4. if $d_k + d_{i-1,i} \geq L$
5. 
6. else
7. 
8. end
9. end
Output: $L_k, H_k, \text{max}$
Algorithm 4. Derivation

Input: $L_k, H_{k_{max}}$
Initialization: $f_{1,k} ← 0$

1. Initialization: $f_{1,k} ← 0$
2. for $i = 2$ to $H_{k_{max}}$
   3. if $L_{i,k} ∈ D$
   4. $y_{U_{ik}} ← \min(-q_i, f_{i-1,k}, y_{U_{ik}}) * 0$
   5. $f_{i-1,k} ← f_{i-1,k} - y_{U_{ik}}$
   6. $q_i ← q_i + y_{U_{ik}}$
   7. else $L_{i,k} ∈ S$
   8. $y_{L_{ik}} ← $ Emission reduction($L_k, Q, f_{i-1,k}, \{D_i, r_i | i ∈ V\}$; $y_{U_{ik}} ← 0$
   9. $f_{i-1,k} ← f_{i-1,k} + y_{L_{ik}}$
   10. $q_i ← q_i + y_{L_{ik}}$
11. $f_{i,k} ← f_{i,k} + y_{L_{ik}}$
12. $y_{U_{ik}} ← y_{L_{ik}}$
13. $U_k ← \{y_{L_{ik}}, y_{U_{ik}} | i = 1, ..., V, k ∈ K\}$

Output: $U_k$

Fig. 1. Five neighbourhood operators

Algorithm 5. Emission reduction

Input: $L_k, Q, f_{i-1,k}, \{q_i, R_i | i ∈ V\}$
Initialization: $m ← 0$

1. while $L_{i+mn,k} ∈ L_k$ and $y_{L_{i+mn,k}} > y_{L_{i+1,n,m}} + \sum_{n=1}^{m} (m \cdot d_{n,n+m} \cdot x_{n,n+m,k})$
   2. find $j ∈ \{L_{i,k}, ..., L_{i+mn,k}\}$
   3. if $\sum_{n=1}^{m} q_n > 0$ and $i < j$
      //loading bikes in the n-th TAZ can reduce greenhouse gas emissions
   4. $y_{U_{ik}} ← \min(\sum_{n=1}^{m} q_n, Q - f_{i-1,k})$
1. end
1. end

Output: $Y_k$
tion operators, five parameters $M_1$, $M_2$, $\Delta_1$, $\Delta_2$ and $k$ need to be randomly generated to determine the specific index position of the neighbourhood transformation, where $M_2 > M_1$, $0 \leq M_2 - M_1$ and $1 \leq k \leq K_c$. $M_1$ represents the starting position of the neighbourhood transformation; $M_2$ represents the end position of the neighbourhood transformation; $\Delta_1$ represents the length of the first sequence; $\Delta_2$ represents the length of the second sequence; $k$ represents the sequence index of the vehicle that performs the neighbourhood transformation operators.

Record $k$, $M_1$ and $M_2$ in the three-dimensional tabu list in sequence. Like the crossover operators, the neighbourhood transformation operators do not include the first and last indexes of the route sequence, and the positions of the first indexes of the crossover operators are within the practical route length. Fig. 1 shows the process of neighbourhood transformation operators. Only the positions of $M_1$, $M_2$, $\Delta_1$ and $\Delta_2$ in random swaps of subsequences are marked in Fig. 1a. The five parameters of the other neighbourhood transformation operators are the same as that in random swaps of subsequences: $M_1 = 3$, $M_2 = 9$, $\Delta_1 = 1$, $\Delta_2 = 2$. The specific process of each neighbourhood operator is detailed below.

(i) Random swaps of subsequences

The starting point of the first sequence is $i = M_1$, its length is $\Delta_1$; the terminal point of the second sequence is $j = M_2$, and its length is $\Delta_2$. Then the positions of the two sequences are reversed, i.e. $(2,3)$ is swapped with $(6,7,8)$ in Fig. 1a. The positions of the other points are unchanged.

(ii) Random swaps of points

In the condition of $\Delta_1 = \Delta_2 = 0$, the random swaps of subsequences become random swaps of points. Just swap the point of $i = M_1$ with the point of $j = M_2$, i.e. $(2)$ is swapped with $(8)$ in Fig. 1b. The positions of the other points are unchanged.

(iii) Random insertions of subsequences

The starting point of the subsequence is $j = M_2$, its length is $\Delta_2$ and it is inserted into the position of $i = M_1$, i.e. $(6,7,8)$ is inserted into the position of $L_k(3)$ in Fig. 1c. The positions of the other points are unchanged.

(iv) Random insertions of points

In the condition of $\Delta_1 = \Delta_2 = 0$, random insertions of a subsequence become random insertions of points. Just insert the point of $j = M_2$ to the position of $i = M_1$, i.e. $(8)$ in Fig. 1d insert to the position of $L_k(3)$. The positions of the other points are unchanged.

(v) Reversing subsequences

The terminal point of the subsequence is selected as $j = M_2$, and its length is $\Delta_2$. Then the sequence is reversed, i.e. the $(6,7,8)$ is reversed to $(8,7,6)$ in Fig. 1e. The positions of the other points are unchanged.

5. Computational study

As the traffic conditions on workdays or weekends are quite different [19], the data is divided into two categories: workdays and weekends. The data used contains only four weekends. Therefore, in order to make the statistical results more accurate, the computational study only discusses static rebalancing works during workdays. This section takes the average value of the data on the workdays.

5.1. Data analysis

Fig. 2 shows the average number of trips during each hour of the workday. The results show that there are two peaks of usage on workdays (09:00 and 20:00), which are consistent with the commuting hours of most jobs. In addition, there are still some people who ride bikes late at night (24:00 to 05:00), which may be produced by workers working overtime at night or entertainment activities late at night. Fig. 3 shows the average trip distance during each period of the workday. Interestingly, although the number of trips is small at night, the average travel distance per trip is long. This may be due to the shortage of public transportation at night so that people can only rely on the FFBSS for long-distance commuting.
Fig. 3. Trip distance

Fig. 4. Bike trip distance distribution

shows that the FFBSS provides a suitable choice for people travelling at night.

In the statistics of bike usage data, more than 95% of trips travel less than 4 km, and 50% of bikes travel less than 1 km. This shows that FFBSSs are mainly used for short-distance or medium-distance travel. Fig. 4 shows the distance distribution of the trips. In order to show the short-distance riding more clearly, Fig. 4 uses changing abscissas. For example, the first column indicates the number of trips with a riding distance of 0.01 km to 0.02 km, and the second column indicates the number of trips with a riding distance of 0.02 km to 0.04 km. It can be seen that most of the riding distances are between 0.77 km and 2.2 km, and only a few of the riding distances exceed 9 km.

Fig. 5 describes the average distance of trips from each TAZ, where the darker the blue represents the longer the average trip distance. It can be seen that the average trip distance of rides in the city centre is relatively short. There may be two reasons for this phenomenon. First, because the commercial density and residential density in the urban areas are relatively high, the starting and ending locations of trips are relatively close. Second, due to the convenient public transportation in urban areas, there are many other modes of travel besides the FFBSS. In the suburbs, the average trip distance of rides is longer, and the reason for this phenomenon is the opposite of that in the urban areas. With low building density in suburban areas, the commuting distances increase. In addition, the public transportation system in the suburbs is not as convenient as in the urban areas. Riding bikes can make up for the lack of public transportation.

Fig. 6 reveals the heat map of daily trips. More riding trips started or ended in reddish places. It can be seen that there are a large number of bikes in the urban areas of the city, while bike riding in the suburbs is less common. The hot spots of commuting lead to the clustering of bike peak locations, which provides a basis for the division of the relocation areas.

Fig. 7 shows the distribution of bike’s degree of imbalance in the FFBSS. Blue means there are deficit bikes in the TAZ, brown means there are
surplus bikes in the TAZ and white means the TAZ’s bike is self-balancing. It can be seen that there are self-balancing TAZs in the urban and suburban areas. In the urban areas, surplus TAZs and deficit TAZs alternated, and the deficit TAZs in the suburbs present a strip-shaped distribution extending from the urban areas to the suburbs. This may be due to users who took public transportation on the main roads and switched to the FFBSS in the suburbs to complete their last-mile commutes. There are a large number of deficit TAZs distributed in the suburbs, which makes it difficult to rebalance.

5.2. Parameter settings

In the following computational experiments, the key parameters in the model and algorithm are shown in Table 3. The vehicle emission rate is 2.13 carbon dioxide equivalent per ton per kilometre [26] and the bike emission reduction rate is 0.2026 carbon dioxide equivalent per ton per kilometre [27].

5.3. Algorithm performance analysis

The proposed solution methodology is coded in MATLAB2019 and run on a desktop computer with an Intel Core i5-3230M CPU 2.6 GHz with 4 GB of RAM. In order to prove that AVNTS can solve the vehicle-routing model efficiently, the performance of AVNTS is compared with that of TS and TS2. The information of each algorithm is shown in Table 4. In this section, the FFBSS is divided into 20 relocation areas using the two-layer clustering method. Computational experiments on these 20 relocation areas are conducted to assess the quality of the solutions.

Since at least one vehicle is required for rebalancing in each relocation area, the number of vehicles is greater than the number of relocation areas. Equation (19) is used to assign vehicles to each relocation area to work, according to the number of deficit bikes in each relocation area.

$$K_c = K \cdot \frac{\sum_{i \in A} q_i}{\sum_{i \in V} q_i}$$  \hspace{1cm} (19)

Each area is repeated 20 times and the computational results are presented in Table 5. The neighbourhood operators used in TS2 are the same as the neighbourhood operators used in the AVNTS (Section 4.4). The key parameters of every algorithm are $\bar{K} = 80$, $\alpha = 0.9$, $\psi_{min} = 2$. The numbers of TAZs and vehicles are shown in Table 5. Each area is repeated 20 times, the average computational times of CPU are shown in Table 5. Additionally, the ‘Mean’, ‘Best’ and ‘SD’ columns show the average value of the objective function, the best value of the objective function, and standard deviation, respectively. ‘Imp%’ referred to the improvement percentage against the TS-based method, based on the average values. As shown in Table 5, the AVNTS algorithm attains absolute advantages in 18 relocation areas out of 20 and the best means of 20 relocation areas. The average standard deviations of the AVNTS algorithm (278.3) are smaller than the other two algorithms (427.9 and 325.8). Among the 20 relocation areas, the AVNTS algorithm also has the smallest best value of average standard deviation.

In order to show the difference between the results calculated by the three algorithms, we selected the nine relocation areas with the largest SD after the TS calculation result for analysis (in Fig. 8). In each box, the light blue and dark blue boundary lines indicate the median value, the edges are the 25th and 75th percentiles, the upper edge of the light blue block and the lower edge of the dark blue block show the 85% confidence interval (CI) for the median, the whiskers extend to the most extreme non-outlier values and outliers are plotted individually (with black dots). Each box plots displays the results of the 20 simulations.

Observing Fig. 8, we can see that on the y-axis of each graph there is the objective function value. Nine groups can be observed on the x-axis, each one consisting of three box plots. Every group represents a different relocation area, the three box plots in each group represent the calculation
Table 3. Key parameters in our numerical studies

| Parameter | Description | Value | Unit |
|-----------|-------------|-------|------|
| $\xi$     | The threshold of unbalanced bikes | 10    | bikes |
| $Q$       | The capacity of the vehicles.     | 50    | bikes/vehicle |
| $m$       | The quality of the vehicles.      | 2000  | kg    |
| $m_b$     | The quality of the bikes.         | 20.5  | kg    |
| $L$       | The maximum distance of the vehicles. | 100  | km    |
| $e_1$     | Vehicle emission rate.           | 2.13  | CO$_2$-eq/(t km) |
| $e_2$     | Bike emission reduction rate.     | 0.2026| CO$_2$-eq/km |
| $\gamma_1$| The coefficient of degree of imbalance | 0.01  |       |
| $\gamma_2$| The coefficient of greenhouse gas emissions. | 1    |       |
| $P_{stop}$| The maximum number of iterations. | 100   |       |
| $\alpha$ | Attenuation coefficient.         | 0.9   |       |
| $\psi_{min}$| Minimum factor value.         | 2     |       |
| $N$       | Number of candidate solutions.    | $|R_k|$|       |
| TabuLength| The tabu length of the move for AVNTS. | $0.1\times P_{stop}$ |       |

Table 4. Information on the three algorithms

| Algorithm | Description |
|-----------|-------------|
| TS        | Tabu search algorithm proposed by Glover [28], with a single neighbourhood operator |
| TS2       | Tabu search algorithm proposed by Glover [28], with no adaptive neighbourhood operators |
| AVNTS     | The adaptive variable neighbourhood tabu search algorithm proposed in this paper |

Table 5. Computational results of the 20 relocation areas

| Area no. | TAZs | $K_c$ | TS | TS2 | AVNTS | Imp% | CPU(s) |
|----------|------|-------|-----|-----|-------|------|--------|
|          | Mean | Best  | SD  | Mean | Best  | SD   | Mean   | Best  | SD   |
| 14       | 41   | 428.0 | 314.7 | 102.4 | 392.8 | 325.0 | 44.0   | 371.9 | 297.9 | 42.8 |
| 19       | 51   | 1015.9| 600.3 | 317.6 | 593.4 | 456.0 | 78.4   | 526.3 | 416.6 | 52.5 |
| 3        | 63   | 1642.3| 1155.9| 290.0 | 1529.1| 1117.4| 204.2  | 1357.3| 1109.1| 125.9|
| 11       | 100  | 1750.0| 1502.2| 172.9 | 1698.7| 1422.4| 115.0  | 1602.1| 1354.1| 140.2|
| 1        | 90   | 2039.8| 1671.4| 275.2 | 1933.1| 1661.8| 162.6  | 1761.5| 1516.0| 179.8|
| 6        | 110  | 2370.8| 1978.5| 210.9 | 2276.3| 2049.7| 141.4  | 2185.4| 1921.2| 158.7|
| 10       | 71   | 1480.6| 1177.1| 160.9 | 1518.2| 1266.0| 128.5  | 1348.9| 1215.6| 97.2 |
| 12       | 91   | 1889.1| 1546.8| 143.6 | 1918.5| 1669.6| 172.4  | 1770.3| 1548.9| 160.0|
| 4        | 69   | 2538.0| 2037.1| 296.2 | 2468.1| 2114.2| 205.7  | 2352.6| 1963.0| 170.0|
| 18       | 51   | 2464.4| 1888.5| 289.0 | 2247.6| 1911.7| 247.7  | 2194.7| 1800.1| 293.7|
| 20       | 59   | 3287.3| 2719.5| 364.8 | 3416.1| 2728.7| 399.7  | 2740.2| 2259.5| 259.7|
| 7        | 75   | 3932.4| 3243.5| 560.8 | 3793.7| 2994.2| 475.7  | 3261.1| 2673.2| 473.5|
| 8        | 72   | 6186.7| 4601.4| 1023.1| 5791.2| 4618.5| 817.6  | 4956.2| 4476.0| 332.0|
| 9        | 32   | 1964.5| 1526.3| 311.3 | 1974.2| 1430.2| 338.0  | 1628.4| 1293.8| 186.7|
| 5        | 87   | 7188.4| 5920.6| 902.2 | 6796.9| 5931.3| 538.5  | 6093.3| 5101.5| 509.3|
| 13       | 79   | 5495.5| 4323.5| 635.2 | 5135.7| 4307.3| 409.2  | 4610.7| 3782.6| 378.7|
| 15       | 154  | 7227.8| 6308.1| 614.8 | 6955.5| 6212.3| 465.0  | 6485.4| 5593.3| 527.6|
| 16       | 57   | 3561.5| 3008.8| 320.9 | 3507.3| 2915.4| 393.6  | 3136.6| 2723.2| 263.5|
| 17       | 59   | 6071.2| 4245.8| 865.1 | 5180.6| 4386.1| 432.2  | 4694.3| 3105.2| 652.6|
| 2        | 74   | 6666.3| 5511.4| 700.9 | 6296.8| 5492.7| 746.0  | 5746.1| 4910.9| 561.9|
| AVG      |      |       | 427.9| 325.8| 278.3 |      |        |       |      |

The results of the three algorithms TS, TS2 and AVNTS. TS2 can generally reduce the length of the confidence interval, but there are exceptions (such as 2-th and 16-th relocation area); TS2 can also generally reduce the median value, except for 20-th relocation area. This also explains the limitations of TS2 for solving the vehicle-routing model in this article to some extent. The proposed AVNTS can make up for these demerits. The length of the AVNTS confidence interval is less than or equal to that of TS, and can get better median values and smaller best values at the same time. In addition,
when the results of TS is compared with those of AVNTS, it can be seen that the upper end and 75th percentiles of the confidence interval of AVNTS are both smaller than that of TS. And TS sometimes has poorly generated outliers, but AVNTS does not.

In order to illustrate the influence of adaptive variable factors on the selection of neighbourhood operators, Fig. 9 illustrates the changes in the proportion of the five factors of these neighbourhood operators within 100 iterations of the algorithm. In different iterations, the roles of these 5 neighbourhoods are different. For example, approximately at the 30-th iteration, the second neighbourhood operators create better candidate solutions. Approximately at the 40-th iteration, the fourth neighbourhood operators create better candidate solutions. The first, third and fifth neighbourhood operators have low proportions at the end of the iterations (Factor1: 10.9%, Factor2: 29.5%, Factor3: 10.9%, Factor4: 37.8%, Factor5: 10.9%). This may be because their operation methods are too complicated and not suitable for rebalancing in the selected 20 relocation areas.

5.4. Parameter analysis

In order to verify, the number of vehicles $K$ and the number of relocation areas $C$ can help administrators to formulate suitable rebalancing schemes for different FFBSSs. In this section, different values of $K$, $C$ and $\gamma_2$ are selected for comparison. Running 10 times for each relocation area to get the average value of $C_{\text{unbalance}}$ and the average value of $C_{\text{emission}}$. Then, the objective function value is calculated by $F = \gamma_1 \cdot C_{\text{unbalance}} + \gamma_2 \cdot C_{\text{emission}}$. The coefficient of $C_{\text{unbalance}}$ is fixed: $\gamma_1 = 0.01$, but the coefficient of $C_{\text{emission}}$ is changing: $\gamma_2 = \{1, 3, 5, 7, 9\}$. The performance indicators of interest in this section are the objective function, the average value of $C_{\text{unbalance}}$ of the FFBSS and the average value of $C_{\text{unbalance}}$ after the bikes are relocated.

First set $C = 20$, and choose six situations of the number of vehicles $K = \{30, 40, 50, 60, 70, 80\}$ and $\gamma_2 = \{1, 3, 5, 7, 9\}$. Fig. 10 shows the performance indicators of the system after the relocation operation. It can be seen from Fig. 10a that for different $\gamma_2$, when the number of vehicles exceeds 50, the marginal benefit (decline of the objective function value) of $C_{\text{unbalance}}$ for each additional vehicle decreases. In addition, with the higher $\gamma_2$, the decrease of $C_{\text{unbalance}}$ becomes smaller with the increase of the number of vehicles. That is, when $\gamma_2$ is larger, the sensitivity of $C_{\text{unbalance}}$ with the change of the number of vehicles decreases. It can be seen from Fig. 10b that if administrators increase the number of vehicles, they need to pay attention to greenhouse gas emissions, for its linearly increasing with the number of vehicles. As $\gamma_2$ continues to increase, $C_{\text{emission}}$ is less sensitive to the number of vehicles, which is similar with the features of $C_{\text{unbalance}}$. This rule can also be found in Fig. 10c, the interval between $\gamma_2 = 9$ and $\gamma_2 = 7$ is slightly smaller than the interval between $\gamma_2 = 1$ and $\gamma_2 = 3$. This is because with the linear increase of $\gamma_2$, the amount of change of $\gamma_1/\gamma_2$ actually decreases. Interestingly, at the situation of $\gamma_2 = 1$, the objective function decreases as the number of vehicles increases, although this decreasing trend gradually slows down. But at the situation $\gamma_2 = 9$, the objective function increases with the increase of the number of vehicles, especially the larger the number of vehicles, the greater the increase of the objective function. This is because the focus of the model has shifted to reducing greenhouse gas emissions, and the increase in the number of vehicles has a negative effect on greenhouse gas emissions that exceeds the rebalancing effect on the degree of imbalance. For administrators, this is a situation where the vehicles overly rebalanced the TAZs, which is not in the current interests and needs to be avoided. Specifically, if the administrator chooses $\gamma_1 = 0.01$, $\gamma_2 = 5$, then the number of vehicles cannot be more than 50.

Second, set $K = 80$, and choose seven situations of the number of relocation areas: $C = \{10, 15, 20, 25, 30, 35, 40\}$ and $\gamma_2 = \{1, 3, 5, 7, 9\}$. Similarly, Figs 11a and b draw the trend of $C_{\text{unbalance}}$ and the trend of $C_{\text{emission}}$ separately. It can be seen from Fig. 11a that when the number of vehicles exceeds 20, the decrease of $C_{\text{unbalance}}$ for each additional relocation area is very small; especially when the number of vehicles exceeds 30, the effect of increasing the number of relocation areas is negligible for $C_{\text{unbalance}}$. And for the situation of $\gamma_2 = 9$, $C_{\text{unbalance}}$ will fluctuate slightly in the range of $20 \leq C \leq 25$. This may be due to the fact that vehicles cannot rebalance the TAZs in other relocation areas, while the increased relocation areas limit the original rebalancing range of vehicles, resulting in overbalancing of certain vehicles within their relocation areas, thereby reducing the overall rebalancing efficiency. But when $\gamma_2$ decreases, this negative fluctuation slows down. The same situation can also be observed in Fig. 11b, but the impact of $\gamma_2$ is exactly the
opposite. When $\gamma_2 = 9$, $C_{\text{emission}}$ shows irregular fluctuations, which may be due to the unreasonable assigning method that lead to excessive greenhouse gas emissions in some relocation areas, and less vehicle greenhouse gas emissions in some relocated areas. However, when $\gamma_2$ decreases, that is, when the administrator shifts the rebalancing focus away from reducing greenhouse gas emissions, this negative fluctuation decreases, which is also accompanied by an increase in $C_{\text{emission}}$. It can be seen from Fig. 11c that the objective function value decreases as the number of relocation areas increases, when the number of relocation areas exceeds 30, the marginal benefit of each additional relocation area gradually decreases. In the situation of $C = 25$...
Fig. 10. Impacts of the number of vehicles: (a) impact on the degree of imbalance; (b) impact on greenhouse gas emissions; (c) impact on the objective function value

Fig. 11. Impacts of the number of relocation areas: (a) impact on the degree of imbalance; (b) impact on greenhouse gas emissions; (c) impact on the objective function value
there is a bottleneck period, then increasing relocation areas does not have a positive effect on the objective function, so administrators should avoid the negative situation. The advantage of increasing the number of relocation areas is that $C_{\text{emission}}$ and $C_{\text{unbalance}}$ fluctuations can be offset, and lead to a decline in the value of the objective function.

### 5.5. Coefficient analysis

Since $C_{\text{emission}}$ and $C_{\text{unbalance}}$ have different units, the two coefficients need to be used to balance their relationship in the rebalance model. This section chooses to fix the coefficient of degree of imbalance $\gamma_1 = 0.01$ in the case of 20 relocation areas and 50 vehicles, and change the value of the coefficient of greenhouse gas emissions $\gamma_2$. The comparison can be seen from Fig. 12, if $\gamma_1/\gamma_2$ decreases, the $C_{\text{unbalance}}$ shows an increasing trend, while the $C_{\text{emission}}$ shows a decreasing trend. This shows that the decrease in $C_{\text{emission}}$ is at the cost of an increase in $C_{\text{unbalance}}$. In order to consider the effects of both in the relocation operation, a suitable $\gamma_1/\gamma_2$ should be found. Further observation also shows that with the decrease of $\gamma_1/\gamma_2$, the increase of $C_{\text{unbalance}}$ first increases and then decreases; the decrease in $C_{\text{emission}}$ also first increases and then decreases. $\gamma_2 = 10$ or $\gamma_1/\gamma_2 = 0.001$ is the demarcation point of this change. We can also find that the 63.2% change in $C_{\text{unbalance}}$ and the 72.9% change in $C_{\text{emission}}$ can be realized in the interval of $\gamma_1/\gamma_2 \in [0.01, 0.0001]$. Therefore, administrators can adjust these two coefficients in different application situations in this interval according to different considerations of the environment and users.

### 5.6. The impact of garage location

Assumption A8 supposes that the garage is located in the centre of the relocation area. The vehicle starts from the garage and returns to the garage after rebalancing operation. A large number of garages means that there are more garages for parking vehicles, and the fixed cost in the construction stage is large. In order to reduce the construction cost of the garage and to divide more relocation areas, this section discusses the situation without assuming A8 that vehicles that may serve multiple relocation areas share the same garage. We define that if there are vehicles serving different relocation areas in a garage, then this garage is called the shared garage. The selection of shared garages location similar to two-layer clustering problem. However, in order to simplify, we assume that the location of shared garage is located in the centre of their relocation areas. That is to say, if the vehicles in the relocation area 1,2,3 start from shared garage A and return to shared garage A after rebalancing operation, then the shared garage A is located at the geometric centre of these relocation areas. The solution method is the same as Equations (3)–(4). As we saw in section 5.4, when the number of relocation areas is less than 20, the changing number of relocation areas has obvious influence on the objective function. So this section studies 20 cases of $C \in [1, 20]$ and sets $\gamma_1 = 0.01$, $\gamma_2 = 1$, $K = 50$.

Observing Fig. 13a, we can see that when $C \leq 5$, the changes of degree of imbalance and greenhouse gas emissions are relatively large; but when $C \geq 15$, the changes of degree of imbalance and greenhouse gas emissions are reduced a lot. If there are fewer garages, the vehicles need to travel a long distance to the relocation areas to which they are assigned, so the $C_{\text{unbalance}}$ will be larger. If there are more garages, the vehicle can reduce the empty driving distance and carry more bikes. The increase in the number of bikes loaded on vehicles will increase carbon emissions and reduce imbalances. Therefore, the changes of $C_{\text{unbalance}}$ and $C_{\text{emission}}$ show opposite trends. Fig. 13b illustrates that the objective function value has been decreasing with the increasing number of relocation areas, because the starting and ending points of the vehicle are closer to their relocation areas.

### 6. Conclusions and future research

This paper proposes a green relocation problem in FFBSSs relating to greenhouse gas emissions.
The problem is to reduce the overall degree of imbalance and greenhouse gas emissions of the FFBSS. Due to the large scale of the FFBSS, the FFBSS is first divided into multiple TAZs. The TAZs are then clustered into multiple relocation areas. Finally, vehicles are assigned according to the degree of imbalance of each relocation area, and the static bike relocation sub-problem is solved in each relocation area. The AVNTS algorithm is used to optimize the routing in each relocation area, and a heuristic algorithm for loading and unloading quantities is added to the algorithm to solve the loading and unloading sub-problems of a given route. We apply the proposed green relocation model to a large-scale FFBSS in Shanghai to illustrate the performance of the proposed algorithm and the nature of the problem. Compared with TS algorithm, AVNTS algorithm can increase 75% of the instances by more than 10%, and 35% of the instances can increase more than 20%. The performance analysis shows that the AVNTS algorithm is better than the tabu algorithm using a single neighbourhood operator, and it is better than the tabu algorithm using multiple neighbourhood operators without an adaptive mechanism.

A large number of relocation areas means that there are more garages for parking vehicles, and the fixed cost in the construction stage is large, while the number of vehicles indicates that the cost of vehicles in the running stage is large. The parameter analyses put forward a method to find the appropriate number of relocation areas and the appropriate number of vehicles for rebalancing different FFBSSs. For the FFBSS analysed in this study, when the number of relocation areas is less than 20, increasing the number of relocation areas is a good way to improve rebalancing efficiency, because it can offset the fluctuations of the greenhouse gas emissions and the degree of imbalance, and lead to a decline in the value of the objective function. But when the number of relocation areas is more than 20, any increase in the number of relocation areas needs to be carefully considered. For example, there is a bottleneck period in the situation of \( C = 25 \), where the increasing relocation areas does not have a positive effect on the objective function. In these situations, the administrators can increase the number of vehicles, if there is still a need to improve the effect of rebalancing. It is also not recommended to assign too many vehicles (over 80).

The coefficient analysis shows that the setting of coefficients is important for achieving a balance between two goals, which are minimizing the total degree of imbalance and minimizing the total greenhouse gas emissions. We can find that the 63.2% change in the degree of imbalance and the 72.9% change in the greenhouse gas emissions can be realized in the interval of \( \gamma_1 / \gamma_2 \in [0.01, 0.0001] \). Therefore, administrators can adjust these two coefficients in different application situations in this interval according to different considerations of the environment and users.

Although the established vehicle-routing model and the purposed algorithm have achieved success, there are still many constraints. Here, we simulate the use in an ideal build environment to support the development of rebalancing routes of the FFBSS in Shanghai. However, this may be expanded in at least four main ways, each of which will be the focus of future research. First, in this paper, TAZs are classified according to their geographical location and land-use characteristics. Other characteristics of each site, including road width and traffic flow, can also provide better potential to capture the use of bikes. Second, because the rebalancing operation is carried out at night, and the bicycle is a good alternative to public transport at night, future research can take into consideration the demand for bikes at night. This can meet the needs of those who come home from work at night and have to
choose long-distance bicycle riding because of unavailability of buses. Third, due to the fact that some urban roads will be closed at night, it will be more important to change the travel time between stations to combine with the real-time traffic conditions, which has an important impact on bike relocation. Fourth, the restriction that vehicles must return to the departure garage can be removed, so that the vehicles are allowed to rebalance in different relocation areas and be parked in other garages at the end. In the future, it may be worth considering the possibility of trying to use an enhanced branch and bound method to accurately solve the green rebalancing problem. In order to deal with the increase in the number of variables, the strategy based on column generation will be explored. Another interesting strategy might be to use the high-quality solution provided by AVNTS as the initial solution (upper bound) for MIP solvers once some recipe-enhancement mechanisms are implemented.

**Supplementary data**

Supplementary data is available at Transportation Safety and Environment online.

**Conflict of interest statement.** None declared.

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