Chiral Transparency

L. FRANKFURT

School of Physics and Astronomy, Tel Aviv Univ., 69978, Israel
Institute for Nuclear Physics, St. Petersburg, Russia

T-S.H. LEE

Physics Div., Argonne National Lab, Argonne, IL 60439

G. A. MILLER

Physics Department, BOX-351560, University of Washington
Seattle, Washington 98195-1560, USA

M. STRIKMAN

Department of Physics, Pennsylvania State University, University Park, PA 16802
Institute for Nuclear Physics, St. Petersburg, Russia

Abstract

Color transparency is the vanishing of initial and final state interactions, predicted by QCD to occur in high momentum transfer quasielastic nuclear reactions. For specific reactions involving nucleons, the initial and final state interactions are expected to be dominated by exchanges of pions. We argue that these interactions are also suppressed in high momentum transfer nuclear quasielastic reactions; this is “chiral transparency”. We show that that studies of the $e^3{\text{He}} \to e' \Delta^{++}nn$ reaction could reveal the influence of chiral transparency.
1 Introduction and Outline

Effective Chiral Lagrangians have the same symmetries, unitarity and cluster decomposition properties as QCD, and therefore the two theories are expected to yield the same predictions [1]. Furthermore, a chiral perturbation theory treatment of the effective Lagrangians provides an organizing principle for handling the very strong interactions typical of low momentum transfer processes; see e.g. the reviews [2,3,4].

The utility of perturbation theory is due to a diminishing of the strong interaction which occurs as a straightforward consequence of approximate spontaneously broken chiral symmetry. Suppose, we lived in a chiral world in which the up and down quark masses were exactly zero. In this world the pion would be a Goldstone boson and there are theorems [5] that certain pion emission amplitudes at threshold must vanish or be constant. The pion mass is not actually zero—but it is small compared to typical hadronic scales. This suggests that the amplitude can be described in a systematic manner as an expansion in $(k/\Lambda)$, where $k$ is a typical (small) momentum or energy scale $k \sim m_\pi$ and $\Lambda$ is a typical (large-mass) hadronic scale $\sim 1$ GeV, such as $M_N$, $m_\rho$ or $4\pi f_\pi$. This systematic expansion is called chiral perturbation theory ($\chi$PT). This theory has been developed systematically for interactions of mesons and for interactions of mesons with a baryon; see the reviews [2]-[4]. Furthermore, chiral perturbation theory has been extended to systems of more than one baryon [6, 7] so that now chiral perturbation theory can be used to describe the nucleon-nucleon strong force in a qualitative fashion [7].

In principle, one ought to be able to apply the power counting in a completely systematic manner and work consistently to some given order. However, it is necessary to introduce counter terms to eliminate the divergent terms in loop diagrams and therefore represent the short distance physics. Once these counter terms are determined, one can predict other observables. This approach has been used with great success in describing the properties of the pseudoscalar mesons [8, 9] as well as more recently baryons [10]. Thus there seems to be a nice representation of low momentum transfer nuclear physics that is motivated by a fundamental theory. One uses pionic
exchanges for the long range physics and the short distance physics is represented by counter terms.

The present paper is concerned with a specific extension of $\chi$PT to the regime of high momentum transfer physics in which a large momentum transfer scale $Q^2$, much larger than the characteristic scale $\Lambda^2$ of $\chi$PT is introduced. Consider for example, elastic electron-proton scattering. At low momentum transfer one may try to describe the system in terms of the pion cloud of extent of the pion Compton radius $\frac{1}{m_\pi}$. But when $Q \gg m_\pi$ such effects do not enter. Instead one uses quarks and gluons to describe the interaction of the virtual photon with the proton. According to perturbative QCD, the high $Q^2$ process proceeds by components in which the quarks are close together. Such components have been called point like configurations[1].

The applicability of perturbative QCD to medium energy processes ($Q^2 \sim 1 - 5 GeV^2$) has been questioned[12]. As a result three of us developed a criteria[13] to determine whether or not a point like configuration is formed in non-perturbative models of the nucleon. We found that point like configurations are formed for all realistic quark models - those in which there are correlations between the quarks. Point like configurations are also formed in the Skryme model, in which baryonic degrees of freedom are represented by pionic solitons. We especially recall that the nucleon’s pion cloud provides negligible contributions to the form factor if $Q \gg m_\pi$[14].

The salient feature of point like configurations is that these do not undergo strong interactions for coherent low momentum transfer processes. This is because small color (neutral) singlets have small forward scattering amplitudes. As originally conceived within the two-gluon exchange model of the Pomeron[15], this results from the sum of the gluon emission amplitudes cancelling if the quarks and gluons of a color singlet are close together. See Ref. [16] for further references and a discussion of how this cancellation is treated within QCD.

The new feature we wish to explore here is that $\pi$ interactions with a color neutral point like configuration are suppressed. This is because the underlying interactions involved in producing or absorbing a pion are also gluonic in origin. One
example [17] is the ratio of decay widths for the Υ′ and Ψ′ to decay to their ground states via pion emission. The ratios are given by

\[
\frac{\Gamma(\Upsilon' \rightarrow \Upsilon\pi\pi)}{\Gamma(\Psi' \rightarrow \Psi\pi\pi)} \approx \frac{<r_{\Upsilon'}^2>}{<r_{\Psi'}^2>} \approx \frac{0.2^2}{0.8^2} \approx \frac{1}{16}.
\] (1)

This ratio can be explained naturally using the idea that pion production arises from gluon emissions from the \(b\) and \(\bar{b}\) which tend to cancel. Another very similar example occurs in the ratio

\[
\frac{\Gamma(\Psi' \rightarrow \Psi\pi\pi)}{\Gamma(\rho' \rightarrow \rho\pi\pi)} \approx \frac{135\text{KeV}}{200\text{MeV}}[18].
\] (2)

The small nature of this ratio was explained by Gottfried [19] and Goldberg [20] using the long wavelength approximation to the color multipole expansion and the relative sizes of the charmed and light quark systems. Another example, related to systems of only light quarks, comes from the work of deKam and Pirner [21] in which the suppression of pion emission from bags of small size is used as a mechanism to provide stability against the collapse of the bag. Without this suppression, the bag would collapse under the pressure of pions outside the bag. Weise et al [22] invoked a similar mechanism by assuming the pion quark coupling constant vanishes at the center of the nucleon bag. This was obtained by PCAC and the idea that the square of the pion mass is proportional to the mass \(M\) of the quark, which depends on the distance \(r\) between the quark and the center of the nucleon. Motivated by the asymptotic freedom idea that light quarks are free and nearly massless when they are close together, the function \(M(r)\) was taken to vary as a power of \(r\). However, this freedom occurs only for color singlet systems so that the motivation is very close to that for color transparency. See Ref. [11] for further discussion of the concept that pions are not absorbed or emitted by point like configurations.

Thus the notion that very small color neutral objects do not emit pions seems to be consistent with diverse phenomena. Consequently, point like configurations, produced in high momentum transfer quasielastic reactions, have no pionic cloud and are not expected to interact by pion exchange. We call this failure to interact “chiral transparency”. For an early discussion of chiral transparency see Ref. [13].
One possible example of chiral transparency occurs for the process

\[ e + {}^3\text{He} \rightarrow \Delta^{++} (\vec{q} - \vec{p}_t) + n(\vec{p}_t) + n + e'. \]  

(3)

where $\vec{q}$ is the spatial momentum of the incident virtual photon, $\vec{p}_t$ is the transverse ($\vec{p}_t \cdot \vec{q} = 0$) momentum of the detected neutron. Under chiral transparency the cross section vanishes at large values of momentum transfer. Another example, is that the production of a non-resonant uncorrelated proton and a $\pi^+$ is also expected to be suppressed.

To see how this suppression may come about, start by first considering the conventional approach. One expects that this reaction proceeds by various terms. The virtual photon could land on the proton ($p$) converting it to a high momentum $p$ or $\Delta^+$. The $p$ or $\Delta^+$ then undergoes a charge exchange reaction, i.e. $pp \rightarrow \Delta^{++}n$ or $\Delta^+p \rightarrow \Delta^{++}n$. Such reactions are dominated by pion exchanges\[23]. The photon could also be absorbed by a contact interaction of the form $\gamma^* p \rightarrow \Delta^{++}\pi^-$. In both of these processes the reaction proceeds by $\pi$ exchange in the final state. Another process would involve a $\Delta^{++}N$ component of the initial wave function. Such components are strongly suppressed by the $\Delta$ mass and systematic searches for such components have never succeeded. The relatively large mass of the $\Delta$, combined with the effects of short range repulsions, causes a vast reduction in the influence of $\Delta's$ in the initial state as compared with those produced by final state interactions.

Under chiral transparency the absorption of the virtual photon forms a point like configuration, PLC, which cannot emit a pion. In that case the cross section for quasielastic production of the $\Delta^{++}$ would vanish. This is chiral transparency. There is a complication because the point like configuration expands as it moves, so that it may indeed emit a pion some distance away from the point where it is produced. This physics is modeled by allowing the $\pi$-coupling constant to be a function of the propagation length, see below.

At $Q^2$ high enough to produce a PLC $Q^2 \sim 1 \text{ GeV}^2$, but not very large, the momentum of the produced PLC is small and the PLC expands to full baryonic size. The $\pi$-coupling is therefore of normal strength. However, as $Q^2$ increases, Lorentz
time dilation takes over and the $\pi$-coupling is reduced and the $\Delta^{++}$ production cross section is suppressed. Thus the basic idea is that under chiral transparency the cross section goes to zero, as $Q^2$ is increased, much faster than does a conventionally computed cross section.

The production of a $\Delta^{++}$ can also proceed via $\rho$ meson exchange. But the $\rho$ coupling to the point like configuration is also expected to be suppressed. Thus although the relative importance of $\pi$ and $\rho$ meson exchange is somewhat model dependent, transparency effects should occur for either either meson exchange. Our numerical work is based on the detailed baryon-baryon interaction model of Lee\cite{24}. In that model, the pion exchange effects are much more important than are those of rho meson exchange. Thus we shall ignore the effects of $\rho$ meson exchange in the remainder of the present paper.

The net result of this is that if the high momentum transfer virtual photon interacts with a proton bound in a nucleus a point like configuration is produced which does not interact with the surrounding nucleons. Thus the chiral physics undergoes a qualitative change; the interaction vertex is suppressed by a factor smaller than the usual $k/\Lambda$. The crucial issue is the value of $Q^2$ needed to turn on this transparency. If the value $Q^2$ were too high, chiral transparency would have no observable effect. On the other hand, if this number were too low, chiral perturbation theory would have diminished relevance because quark physics would enter at lower momenta than expected.

There are at least two major practical problems in observing chiral transparency. The first is that the PLC must not expand too fast. This disruptive effect can be reduced by using the smallest target nucleus possible-this is $^3$He. An additional advantage of using this target is that good wave functions for the ground state are available.

The second problem is that one needs to use the a reasonably accurate version of the conventional process of $\Delta^{++}$ production. Lee’s\cite{24} interaction accounts for the real and imaginary parts of nucleon-nucleon phase shifts, and mixing parameters for energies up to $s_{NN} = 8 \text{ GeV}^2$ This is high enough for the present purpose of
providing motivation for an experiment at the Jefferson Laboratory because $Q^2 = (s_{NN} - 4M_N^2)/(1 + 2\frac{M_N - M\Delta}{M\Delta})$ for quasileastic production of a $\Delta$.

The basic idea is that at $Q^2$ near 1 GeV$^2$ we expect that the conventional theory will work. This can be tested by comparing with relevant data when such becomes available. As $Q^2$ increases from 1 to $\sim$ 6 GeV$^2$ one expects that chiral transparency will become important. We shall compute the ratio of cross sections with and without the effects of chiral transparency.

It is necessary to consider the question of whether or not the existence of PLC is already ruled out. If so, chiral transparency could not exist. Thus, it is worthwhile to briefly review the current status of the color transparency experiments most closely related to the electron-nuclear interaction of the present paper. Color transparency (CT) and color coherent effects have been recently under intense experimental and theoretical investigation. The (p,pp) experiment of Carroll et al. [25] found evidence for color transparency [26] while the NE18 (e,e’p) experiment [27] did not. The appearance of color transparency depends on formation of a point-like configuration (PLC) by hard scattering. The $Q^2$ of the NE18 experiment (1 $\leq Q^2 \leq$ 7 GeV$^2$) seem to be large enough to form a small color singlet object. We believe that this failure to observe significant color transparency effects is caused by the rapid expansion of the point like configuration to nearly normal size (and nearly normal absorption ) at the relatively low momenta of the ejected protons [28],[29]. In particular, models of color transparency which reproduce the (p,2p) data and include expansion effects predicted small CT effects for the NE18 kinematics, consistent with their findings, see the discussion in Ref.[16].

Thus the ability to observe color transparency effects at intermediate values of $Q^2$ (between 1 and 7 GeV$^2$ ) rests on finding ways to avoid the effects of PLC expansion. One may use light nuclei and kinematics that require double scattering Ref. [30, 31]. The idea is that color transparency effects suppress the double scattering terms so that the cross section vanishes, in contrast with the predictions of the usual Glauber model.

The ability to observe chiral transparency rests also on requiring a final state
interaction mediated by pion exchange, in this case a charge exchange reaction. We immediately take the target nucleus to be small, so that the expansion of the PLC should not play a dominant role. Thus the observation of a significant cross section at large values of $Q^2$ would rule out the existence of chiral transparency.

We outline the remainder of the paper. Section 2 is concerned with a formal but schematic derivation of chiral transparency. Section 3 deals with the development of formulae necessary to compute cross sections both in the conventional approach and using the chiral transparency idea. The results are presented in Section 4, and summarized and discussed in Section 5.

2 Chiral Transparency

This section is intended to further specify the assumptions that underlie chiral transparency. We shall use a schematic notation to simplify the discussion as much as possible. Our starting point is the description of the nucleon wave function in the hadronic Fock state basis:

$$|N> = \sqrt{Z} [|N>_0 + C_{N,\pi}|N,\pi>_0 + C_{\Delta,\pi}|\Delta,\pi>_0 + \cdots]$$  \hspace{1cm} (4)

The states labelled with subscript 0 are eigenstates of the Hamiltonian $H_0$ given by

$$H_0 = H - H_{\pi,q} - \tilde{V},$$  \hspace{1cm} (5)

where $H$ is the complete Hamiltonian, $H_{\pi,q}$ represents the pion quark interaction, and $\tilde{V}$ is that part of the two (or three ) nucleon force that is not generated by various iterations of $H_{\pi,q}$. The interaction $H_{\pi,q}$ can be taken to be pseudovector pion-quark coupling, with the modification that there is no interaction when all quarks in the system are at the same location. For example,

$$H_{\pi,q} = \frac{1}{2f_\pi} \int d^3r \frac{r^2}{<r^2>} \bar{\psi}(\vec{r})\gamma_\mu\gamma_5\tau \cdot \partial^\mu \phi_\pi(\vec{r})\psi(\vec{r}).$$  \hspace{1cm} (6)

Here $\psi$ and $\phi_\pi(r)$ are the quark and pion field operators, the center of the nucleon is at $\vec{r} = 0$, and $<r^2>$ is the nucleonic expectation value of the $r^2$. The suppression factor
arises by starting with an operator \( \propto \sum_{i \neq j} (\vec{r}_i - \vec{r}_j)^2 \) and assuming spherical wave functions used in a mean field approximation so that the \( \vec{r}_i \cdot \vec{r}_j \) term averages to zero. The particular form \( r^2/ <r^2> \) is meant to be a schematic representation of any function of \( r^2 \) and \( <r^2> \) that vanishes at \( \vec{r} = 0 \) and has an expectation value of unity in the nucleon wave function. Equation (6) is meant only as an illustration; a complete derivation of the pion-quark interaction is beyond the scope of this paper.

The ability of the Fock state expansion of eq(4) to represent the nucleon wave function in a few terms (in particular, ignoring states with two or more pions) depends on underlying dynamical assumptions. In this paper we use Lee's model[24] in which the pion-nucleon form factor is a dipole with from the cloudy bag model because a dipole form factor with \( \Lambda = 650 \) MeV. This soft form factor corresponds to a large three-quark confinement (bag) radius of 1.33 fm. In this case, cloudy bag model studies [32] show that it very safe to ignore states with two or more pions and that \( Z \approx 1 \).

We now turn to how the nucleon state of eq.(4) responds to an external electromagnetic probe denoted by \( T_H(Q^2) \). Numerical work [14] indicates that the contribution of the pion cloud is negligible for values \( Q^2 > 0.5 \) (GeV/c)^2. Thus we may write

\[
T_H(Q^2)|N > \approx \sqrt{Z}|N >_0, \quad Q^2 > 0.5 \text{ (GeV/c)}^2.
\] (7)

We note that \( 0 < N|H_{\pi,q}|N >_0 \) gives the ordinary (lowest order) pion-nucleon emission vertex, which is not suppressed.

Chiral transparency requires higher values of \( Q^2 \). We rely on earlier work [13] which indicates that

\[
T_H(Q^2)|N > \approx |PLC >, \quad Q^2 > 1 \text{ (GeV/c)}^2,
\] (8)

where \( |PLC > \) represents a point like configuration, one in which the quarks are close enough together for significant suppression of the pion quark interaction to occur. In particular, the pion cloud absent according to Eq. (7) remains absent at higher values of \( Q^2 \). In particular, Eq. (8) implies that

\[
H_{\pi,q}|PLC > = 0.
\] (9)
This is the formal statement of chiral transparency. The state \( |PLC > \) is not an eigenstate of the Hamiltonian, so it will evolve to another state, one which is necessarily larger and which therefore interacts via \( H_{\pi,q} \). Thus we need to consider the time evolution. Suppose a PLC is produced at a position \( \vec{r} \) and moves to a position \( \vec{r}' \). This requires a time \( \tau \approx |\vec{r}' - \vec{r}| \), since we are interested in rapidly moving PLC’s. The effects of this time evolution can be incorporated in by using the Heisenberg representation so that the time-dependent pion-quark interaction \( H_{\pi,q}(|\vec{r}' - \vec{r}|) \) is given by

\[
H_{\pi,q}(|\vec{r}' - \vec{r}|) = e^{iH_0|\vec{r}' - \vec{r}|} H_{\pi,q} e^{-iH_0|\vec{r}' - \vec{r}|}. \tag{10}
\]

The relevant matrix elements for producing or absorbing pions is then

\[
0 < B|H_{\pi,q}(|\vec{r}' - \vec{r}|)|B' > 0,
\]

where \( B,B' \) represents the baryonic states \( N,\Delta \cdots \). This quantity is determined by a coupling constant which depends on \( |\vec{r}' - \vec{r}| \): \( g_{\pi B,B'}(|\vec{r}' - \vec{r}|) \). The explicit evaluation of Eq. (10) must necessarily involve many detailed model assumptions, and is not a subject of the present work. Instead we rely on the related experience of color transparency in which the the explicit evaluation of an equation similar to Eq. (10) using a sufficiently large hadronic basis \[33\] yielded results similar to that of a model based on quantum diffusion (qdm)\[28\]. The notion behind the qdm is that the interaction of the produced PLC is proportional to \( |\vec{r}' - \vec{r}| \) for \( \vec{r}' \approx \vec{r} \), but approaches normal strength for larger values of \( |\vec{r}' - \vec{r}| \). Furthermore, the size (and interaction strength) of the initially produced PLC depends on \( Q^2 \). With these features in mind we assume a form:

\[
g_{\pi B,B'}(|\vec{r}' - \vec{r}|) = g_{\pi B,B'}^0 \left( 1 - \kappa e^{-|\vec{r}' - \vec{r}|/l_c} \right), \tag{11}
\]

where \( l_c \) is the length (or time) scale required for the PLC to evolve to a configuration of nearly normal hadronic size. The idea that this length is given by the Lorentz time dilation of a relevant rest frame time leads to

\[
l_c = \frac{2 \mu}{p^2}, \tag{12}
\]
where $p$ is the longitudinal momentum of the PLC ($\approx |q|$). In the (e,e’p) reaction $\mu^2 \approx 0.7\text{ GeV}^2$, but in the present situation $\mu$ may be smaller than that. We estimate that $l_c \approx r_{\pi} \frac{p}{m_N}$ which arises from using the pion radius as the rest frame time for expansion and $\frac{p}{m_N}$ as the time dilation factor. This gives $\mu^2 = m_n/r_{\pi} \approx 0.3\text{ GeV}^2$.

The parameter $\kappa$ is related to the feature that high $Q^2$ is required to form a PLC that does not emit pions. We use the form

\[
\lambda_0 = 1 \\
\kappa = \begin{cases} 0, & Q^2 < Q_0^2 \\ 1 - \frac{Q_0^2}{Q^2}, & Q^2 \geq Q_0^2 \end{cases}
\]

$Q_0^2$ is a parameter that controls the momentum transfer at which the PLC is assumed to be formed. In particular, if $Q_2 < Q_0^2$ the pion-baryon coupling constants have their normal strength and chiral transparency does not occur. We shall use several values of $Q_0^2$ in this initial investigation.

We need to consider how to use Eq. (11) in computing the relevant scattering amplitudes. Consider first the amplitude $\mathcal{M}$ for the ordinary process in which the $\Delta^{++}$ is produced by the absorption of a photon followed by a final state charge exchange operator. Then schematically

\[\mathcal{M} \sim \sum_{B'} \int d^3r_1'd^3r_1d^3r_2\Psi_f^*(\vec{r}_1', \vec{r}_2)V_{\Delta^{++}n;B'^+,p}(|\vec{r}_1'-\vec{r}_2|)G_{B'}(|\vec{r}_1'-\vec{r}_1|) < B'|T_H(Q^2)|p> \Psi_i(\vec{r}_1, \vec{r}_2),\]

(13)

where $\Psi_i$ represents the initial wave function of the bound pp system and $\Psi_f$ represents the final state $\Delta^{++}n$ wave function. The final state charge exchange interaction, $V_{\Delta^{++}n;B'^+,p}$ contains the effects of the pion-baryon operator generated by the pion quark interaction Hamiltonian. The input wave functions and charge exchange interaction are specified in the next section.

Under chiral transparency one uses Eq. (10) and the related ansatz Eq. (11). In the quantum diffusion model the argument of $g^0_{\pi B, B'}$ is the same as that of the intermediate baryonic Green’s function, $G_B$, so that the effects of chiral transparency can be included simply by multiplying $G_B$ by $g^0_{\pi B, B'}(|\vec{r}'-\vec{r}|)/g^0_{\pi B, B'}(0)$. In operational
Thus when chiral transparency is invoked, the amplitude is given by
\[ M^\chi \sim \sum_{B'} \int d^3r_1' d^3r_1 d^3r_2 \Psi_f^\dagger(\vec{r}_1', \vec{r}_2) V_{\Delta^++n;B',p}(|\vec{r}_1'-\vec{r}_2|) G_B^\chi(|\vec{r}_1'-\vec{r}_1|) <B'|T_H(Q^2)|p> \Psi_i(\vec{r}_1, \vec{r}_2). \] (15)

The difference between Eqs. (13) and (15) is that in the latter equation the pion-baryon interaction is suppressed for small values of $|\vec{r}' - \vec{r}|$. This effect is carried by the different Green’s functions, in the present simple version of the theory.

## 3 Detailed Formalism

So far we have been concerned with the general description of chiral transparency. To make further progress, it is necessary to choose a specific reaction and display how the numerical results are obtained. The absorption of the photon by a $1^S_0$ pp-pair in $^3He$ is the example we choose because the target is the lightest stable one with two bound protons.

We proceed by first describing the formalism for the conventional treatment. The effects of chiral transparency are included simply by replacing the propagator of the intermediate baryon $G_{B'}$ by $G_B^\chi$ according to Eqs. (14) and (17).

The absorption of the photon by a $1^S_0$ pp-pair in $^3He$ is governed by a basic mechanism: $\gamma + (pp) \to Bp \to \Delta^{++}n$ with $B = p, \Delta^+$. The amplitude for this transition is
\[ T = \sum_{B=p,\Delta^+} \phi_{pp} J_{\gamma p,B} \cdot \epsilon_\Lambda |Bp \rangle \cdot G_{Bp}(E) <Bp|t(E)|\Delta^{++}n>, \] (16)
where $\epsilon_\Lambda$ is the photon polarization vector, $\phi_{pp}$ the wavefunction of the pp-pair, $J_{\gamma p,B}$ a one-body transition current operator, and $t$ the two-baryon transition t-matrix. (Our notation here is that $T = M^\chi$.)

The current $J_{\gamma p,p}$ is the standard empirical proton form, while $J_{\gamma p,\Delta^+}$ was determined in studies of $N(e,e'\pi)$ reaction Refs. [34] and [35]. The $B'N \to \Delta N$ transition
amplitude (a 3 by 3 matrix) $t(E)$ is calculated from a unitary model\textsuperscript{24} of the coupled $NN \oplus N\Delta \oplus \pi NN$ reactions.

The produced $\Delta^{++}$ decays into a detected $\pi^+p$ state. Thus the total production cross section must be integrated over all of the possible invariant masses of the resonant $\pi^+p$ system. The invariant mass $W$ of the final coupled $\Delta N \oplus \pi NN$ system is related to the photon four momentum $q = (\omega, \vec{q})$ in $^3He$ rest frame and $q_c = (\omega_c, \vec{q}_c)$ in the $\gamma^* - BB$ c.m. frame by

$$W = \omega_c + E_d(q_c) = (\omega + M_d)^2 - \vec{q}^2$$

(17)

The photon four-momentum satisfies the relation $q^2 = \omega^2 - \vec{q}^2 \equiv q_c^2 = \omega_c^2 - \vec{q}_c^2$. Thus the mass of the final state $\Delta^{++}(\rightarrow p, \pi^+)$ can vary between the ranges given by $W - m_n \geq m_\Delta \geq m_\pi + m_n$, so that the total production cross section is given by

$$\frac{d\sigma}{d\Omega_{c.m.}} = \int_{m_n + m_\pi}^{W - m_n} dm_\Delta \frac{d\sigma}{d\Omega dm_\Delta}(W)$$

(18)

The cross section for production of a $\Delta^{++}n$ final state from an initial $pp$ pair with a relative wavefunction $\phi^{T,MT}_{LSJ,MJ}$ can be derived in a straightforward way from Eq. (16). The result is

$$\frac{d\sigma}{d\Omega dm_\Delta}(W) = \frac{(2\pi)^4}{v_i} \frac{1}{(2J + 1) \cdot 2} \sum_m \rho_{N\Delta}(p_0) \rho_{\pi N}(p_0, k_0) \times |\langle \vec{q}_\lambda, JM, TM_T | T \bar{p}_0 m_s \Delta m_{\tau_\Delta} m_{s_2} m_{\tau_2} \rangle|^2$$

(19)

where $v_i = (q_c/\omega_c + q_c/E_d(q_c))$ is the relative velocity between the photon and the initial $NN$ pair. The $N\Delta$ relative momentum $\vec{p}_0$ and the $\pi N$ relative momentum $\vec{k}_0$ are defined by their corresponding invariant masses $W$ and $m_\Delta$

$$p_0 = \frac{1}{2W} \left[ (W^2 - m_\Delta^2 - m_N^2)^2 - 4 m_\Delta^2 m_N^2 \right]^{1/2}$$

$$k_0 = \frac{1}{2m_\Delta} \left[ (m_\Delta - m_N^2 - m_\pi^2)^2 - 4 m_N^2 m_\pi^2 \right]^{1/2}$$

(20)

In Eq. (19), we have introduced the phase-space factors for the final $N\Delta \oplus \pi NN$ subsystem. By using a dynamical description of the $N\Delta$ propagator and the $\Delta \rightarrow \pi N$ vertex determined in Ref.\textsuperscript{24}, we obtain

$$\rho_{N\Delta}(p_0) = \frac{\rho_{N}(p_0) \rho_{\Delta}(p_0)}{\rho_{N}(p_0) + \rho_{\Delta}(p_0)}$$

(21)
and
\[
\rho_{\pi N}(p_0, k_0) = \frac{E_\pi(k_0)E_N(k_0)k_0}{E_N(k_0) + E_\pi(k_0)} \left| \frac{h(k_0)}{W - E_N(p_0) - E_\Delta(p_0) - \Sigma_\Delta(p_0, W)} \right|^2 \tag{22}
\]

where \(\Sigma(p_0, W)\) is the complex \(\Delta\) self-energy evaluated in the presence of a spectator nucleon, \(h(k_0)\) is a \(\Delta \to \pi N\) vertex function. This is a dipole with \(\Lambda = 650\) MeV. The heart of the calculation is the \(\gamma^*pp \to Bp \to \Delta n\) transition amplitude. Explicitly, we have

\[
\langle \vec{q}\lambda, JM \; TM_T|T(W)|m_{s\Delta}m_{\tau\Delta}m_{s_2}m_{\tau_2}\vec{p} \rangle \\
= \sum_{B=p,\Delta^+} \sum_{m} \langle JM|L'S'M'_s|s_1', s_2', m_1', m_2'\rangle \langle TM_T|\tau_1'\tau_2', 1/2 \; m_{\tau_1}m_{\tau_2}\rangle \times \int d\vec{p}' \phi^{L'S'}_{J\lambda}(\vec{p}') Y_{L'M'_s}\left(\vec{p}' - \vec{q} \frac{\vec{p}'}{2}\right) \langle \vec{p}' - \vec{q}, m_1', m_2' | J_{\gamma N, B} | \vec{p}'m_{sB}m_{\tau B}\rangle \\
\times G_{Bp}(\vec{p}, W) \langle \vec{p}'m_{sB}m_{\tau B}m_{s_2}m_{\tau_2}|t(W)|p_0m_{s\Delta}m_{\tau\Delta}m_{s_2}m_{\tau_2}\rangle \tag{23}
\]

In the above equation, \(\phi^{L'S'}_{L'S}(p)\) is the radial part of the initial \(pp\) relative wavefunction.

We now discuss the propagators appearing in Eqs.(16) and (23). The \(N\Delta\) propagator is defined by

\[
G_{N\Delta}(\vec{p}', W) = \frac{1}{W - E_N(p') - E_\Delta(p') - \Sigma_\Delta(\vec{p}', W)}, \tag{24}
\]

and the intermediate NN propagator by

\[
G_{NN}(\vec{p}', W) = \frac{1}{W - E_N(p') - E_N(p') + i\epsilon}. \tag{25}
\]

This completes the specification of the conventional theory, so that we may now turn to the effects of chiral transparency. As discussed above, we may simply change the coupling constant so that its growth with the expansion of the wave packet is modeled as a function of the propagation length. The present formalism is in momentum space so that we need to re-express Eq.(14) in momentum space. To do this we first note that Eqs.(24), (25) can be re-expressed as

\[
G_{N,\alpha}(\vec{p}', W) = \frac{C(p_0^2(\alpha))}{p_0^2(\alpha) - p'^2 + i\epsilon}, \; \alpha = N, \Delta, \tag{26}
\]

where \(p_0(N, \Delta)\) is the position of the pole of \(p'\). The Fourier transform of these Green’s functions \(\sim e^{ip_0r}/r\). Under chiral transparency these are replaced by \(\sim\)
$e^{ip_0}/r(1-\kappa e^{-r/l_c})$. This means that the effects of chiral transparency can be included by replacing the above $G_{N,\alpha}$ by $G^\chi_{N,\alpha}$ given by

$$G^\chi_{N,\alpha}(\vec{p}\,'W) = \frac{C(p_0^2(\alpha))}{p_0^2 - p'^2 + i\epsilon} - \frac{\kappa C(p_0^2(\alpha))}{(p_0 + i/l_c)^2(\alpha) - p'^2 + i\epsilon}. \quad (27)$$

The transition matrix can be calculated according to Eq. (26) to obtain results for the conventional theory, or by using Eq. (27) to obtain the results for chiral transparency.

4 Results

We now discuss the results of explicit calculations which are focused on the quasielastic production of the $\Delta^{++} n$ system. That is both initial protons are taken to have momentum nearly equal to zero, i.e. at the peak of the initial state wave function, so that the cross section is maximized. This is achieved by setting the invariant mass of the produced $\Delta^{++}$ equal to the physical $\Delta$ mass of 1236 MeV, and by setting the momentum of the produced neutron $p$ according to the relativistic constraint

$$\alpha = \frac{\sqrt{m_N^2 + p^2 - p_z^2}}{m_N} = 1 \quad (28)$$

where $p_z$ is the neutron momentum projected in the direction of photon. Standard electron scattering kinematics are used. Setting the energy-transfer $\omega \sim 300$ MeV of the virtual photon to emphasize the excitation of a proton to a $\Delta$ state, choosing $Q^2$ and using Eq. (28) to determine the angle between the outgoing $\Delta$ and the incident virtual photon determines the necessary kinematics. We shall present our results in the form of a ratio of cross sections defined by

$$CT \equiv \frac{d\sigma^\chi}{d\sigma}. \quad (29)$$

The first set of results, shown in Fig. 1, are determined by the choice $Q_0^2 = 0.55$ GeV$^2$, and the variation with $\mu^2$ is displayed. Harmonic oscillator wave functions are used with $b=1$ fm. The ratio $CT$ differs significantly from zero at fairly low values of $Q^2$ for all of the values of $\mu^2$ displayed.
The dependence on $Q_0^2$ is displayed in Fig. 2. This parameter determines the $Q^2$ necessary for a PLC to be formed and for chiral transparency to set in. We have argued above and elsewhere that $Q_0^2 \approx 1 \text{ GeV}^2$, but the results shown in Fig. 2 indicate that experiments could determine whether or not the $1 \text{ GeV}^2$ estimate is anywhere near correct.

The sensitivity to the initial pp wave function can be studied by examining Fig. (3). This figure shows the sensitivity to the harmonic oscillator parameter $b$. Increasing this number increases the size of the system, and therefore also the influence of PLC expansion. One therefore expects that the value of $CT$ increases with increasing $b$. This is indeed the case. The figure also compares the effects of using harmonic oscillator wave functions with those of using the Paris $s$-state deuteron wave function. This is done to examine the influence of short range correlations. While there is sensitivity to using different wave functions, the significant feature is that $CT$ can be much less than unity for all of the wave functions studied.

The net result is that the predicted effects of chiral transparency seem to be very significant.

5 Summary

Color transparency involves the suppression of initial or final state interactions. This effect enhances $(e,e'p)$ and $(p,pp)$ reactions but suppresses reactions $(e,e'pp)$ and $(e,e'\Delta^{++} n)$ in which a second interaction is required for the reaction to occur. The production of a $\Delta^{++}$ requires a $\pi^-$ emission. The suppression of this process represents an example of chiral transparency. Measuring chiral transparency effects would provide evidence that high $Q^2$ electron-proton scattering proceeds via the formation of a PLC. Potentially large signals are available at Jefferson Lab.

This work is partially supported by the U.S. Department of Energy, Nuclear Physics Division (grants DE-FG06-88ER 40427, DE-FG02-93-ER 40771, and contract W-31-109-ENG-38) and by the US-Israeli Bi-National Science Foundation grant - 9200126.
References

[1] S. Weinberg, Physica **96A**, 327 (1979).

[2] V. Bernard, N. Kaiser and Ulf-G. Meissner, Int. J. Mod. Phys. E4, 193 (1995).

[3] A. Pich, Rept. Prog. Phys. **58**, 563, 1995; G. Ecker, Prog. Part. Nucl. Phys. **35**, 1 (1995).

[4] J. Bijnens, Int. J. Mod. Phys. 8, 3045 (1993)

[5] See e.g. the reprint collection “Current Algebras”, S. L. Adler and R.F. Dashen, 1968

[6] S Weinberg, Phys. Lett. **B251**, 288 (1990).

[7] C. Ordóñez, L. Ray and U. van Kolck, Phys. Rev. Lett. **72**, 1982 (1994); Phys. Rev. **C53**, 2086 (1996).

[8] J. Gasser and H. Leutwyler Ann. Phys. **158**, 142 (1984).

[9] J. Gasser and H. Leutwyler Nucl. Phys. **B250**, 465 (1985).

[10] E. Jenkins, A. V. Manohar Phys. Lett. **B255** 558, 1991.

[11] L. Frankfurt and M. Strikman, Nucl. Phys. **B250**(1985) 147

[12] N. Isgur and C.H. Llewellyn-Smith, Phys. Rev. Lett. 52, 1080 (1984); Phys. Lett. B217, 535 (1989); A.V. Radyushkin, Acta Phys. Polon. B15, 403 (1984); G.P. Korchemski, A.V. Radysushkin, Sov. J. Nucl. Phys. 45, 910 (1987) and refs. therein.

[13] L. Frankfurt, G.A. Miller and M. Strikman, Comm. Nucl. Part Phys. **21**(1992) 1; Nucl. Phys. A555(1993)752-764.

[14] E. Oset, R. Tegen and W. Weise, Nucl. Phys. A426, 456 (1984). Erratum - ibid. A453, 751 (1986).
[15] F. E. Low, Phys. Rev. D12, 163 (1975); J. F. Gunion and D. E. Soper, Phys. Rev.D15,2617 (1977); S. Nussinov Phys. Rev. Lett 34, 1286 (1975).

[16] L.L. Frankfurt, G.A. Miller and M. Strikman, Ann. Rev. Nucl. Part. Sci. 45(1994) 501

[17] M.S. Chanowitz, Proc. Summer School in particle physics, SLAC -240 (1981) ed. by A. Mosher

[18] Particle Data Group, Phys. Rev. D54, 1 (1996).

[19] K. Gottfried, Phys. Rev. Lett. 40, 598 (1978).

[20] H. Goldberg, Phys. Rev. Lett. 35, 605 (1975).

[21] J. de kam and H. Pirner Nucl. Phys. A389 , 640 (1982).

[22] See e.g. the review W. Weise, Int. Journ. Nucl. Phys. 1, 58 (1984).

[23] J.J. Aubert et al. Phys. Lett.69B, 372 (1977); V. Dmitriev, O. Shushkov and C. Gaarde, Nucl.Phys. A459, 503 (1986); the contribution of rho meson exchange dominates only at ISR energies or higher.

[24] T-S.H. Lee, Phys. Rev. C 29(1983) 195

[25] A.S. Carroll et al., Phys. Rev. Lett. 61(1988) 1698

[26] B.K. Jennings and G.A. Miller, Phys. Lett. B318(1993) 7

[27] N.C.R. Makins et al., Phys. Rev. Lett. 72(1994) 1986; T.G. O’Neill et al., Phys.Lett.B351, 87 (1995).

[28] G.R. Farrar et al., Phys. Rev. Lett. 61(1988) 686

[29] B.K. Jennings and G.A. Miller, Phys. Lett. B236(1990) 209; Phys. Rev. D44,(1990) 692;

[30] K. Egiyan et al., Nucl. Phys. A580(1994) 365
[31] L. Frankfurt, W.R. Greenberg, G.A. Miller, M.M. Sargsyan and M.I. Strikman, Z. Phys. A. 352, 97 (1995); Phys.Lett. B369, 201, (1996).

[32] S. Théberge, A. W. Thomas and G. A. Miller, Phys. Rev. D22 (1980) 2838; D23 (1981) 2106(E). A. W. Thomas, S. Théberge, and G. A. Miller, Phys. Rev. D24, 216 (1981); A. W. Thomas, Adv. Nucl. Phys. 13 (1984) 1; G. A. Miller, p 190 in Int. Rev. Nucl. Phys. 2 (1984).

[33] B.K. Jennings and G.A. Miller, Phys. Rev. Lett. 70(1992) 3619

[34] S. Nozawa and T.-S. H. Lee, Nucl. Phys. A513, 459(1990)

[35] T.-S. H. Lee, to appear Phys. Rev C. September, 1996.
Figure 1: Chiral transparency ratio $CT$ of Eq. (29). The transverse momentum of the neutron is 0.3 GeV/c. Harmonic oscillator wave functions are used with $b=1$ fm. The parameter $\mu^2$, which determines the value of $l_c$, is varied.

Figure 2: Chiral transparency ratio $CT$ of Eq. (29). The transverse momentum of the neutron is 0.3 GeV/c. Harmonic oscillator wave functions are used with $b=1$ fm. The parameter $Q_0^2$, which determines the value of $\kappa$, of eq. (11) is varied.

Figure 3: Chiral transparency ratio $CT$ of Eq. (29). The transverse momentum of the neutron is 0.3 GeV/c. Harmonic oscillator wave functions are used and the variation with $b$ is studied. Results obtained by using the Paris $s$-state deuteron wave function are shown.