DESIGN OF ONE TYPE OF LINEAR NETWORK PREDICTION CONTROLLER FOR MULTI-AGENT SYSTEM

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Abstract. In this paper, to solve network delay and network tracking control problems in multi-agent system communication, a design method of network prediction controller was introduced based on state difference estimation and output tracking error. This design method not only effectively compensates for influences of network delay on the system but also ensures stability of the closed-loop system and realizes the same tracking performance among multi-agents. The effectiveness of the proposed method was proven by simulation experiment. The key innovation in the paper is that the influence of network delay on the system was actively compensated and a network prediction tracking control mechanism was proposed to guarantee the stability of the closed-loop system. The proposed method achieved the same tracking with the local tracking control system under certain conditions.

1. Introduction. With recent extensive applications of multi-agent system, abundant research on multi-agent system control has been reported, especially on network-induced delay of data exchange among shared network equipment (i.e., sensor, controller, and actuator). To guarantee the control performance of network multi-agent systems, Chinese and foreign scholars have introduced numerous methods, such as robust control, sampling scheduling, queuing approach, random optimal control, event-driven control, and hybrid system [2–4, 8–10, 13]. Investigations on network tracking control should pay more attention on broad and current issues, such as analysis of system stability, state estimation, and network scheduling. References [1, 11] considered the tracking control problem of the $H_\infty$ model reference in the network control system; however, the design process required online measurement and might have bounded tracking error. Ping Li et al. studied the distributed the $H_\infty$ tracking control problem of a “storehouse” network regardless of network characteristics and its applications in air traffic flow network model [5]. Hui Zhang et al. studied the step-change tracking control problem of the discrete Takagi–Sugeno fuzzy system under the Markov delay conditions, and proposed the design method for tracking controller related with time delay [12].

However, these methods are highly conservative and most of them use passive ways to reduce the influences of network delay on the system. Research involving passive stabilization of network tracking control is limited. Based on state difference

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estimation and output tracking error, a network prediction tracking control mechanism was proposed, which could effectively compensate for influences of network delay on the system. The entire design process did not depend on network delay and could guarantee the stability of the closed-loop system. The proposed method achieved the same tracking with the local tracking control system under certain conditions.

1.1. System description. The following discrete linear control system was considered:

\[
\begin{aligned}
\begin{cases}
x(t+1) = Ax(t) + Bu(t) + v(t) \\
y = Cx(t)
\end{cases}
\end{aligned}
\]

where \(x(t) \in \mathbb{R}^n\) is the state vector, \(y(t) \in \mathbb{R}^m\) is the output vector, \(u(t) \in \mathbb{R}^l\) is the control vector, \(A \in \mathbb{R}^{n \times n}\), \(B \in \mathbb{R}^{n \times l}\), and \(C \in \mathbb{R}^{m \times n}\) are known parameter matrixes of the object model, and \(v(t) \in \mathbb{R}^n\) is the step-change interference, that is

\[
v(t) = \begin{cases}
\text{const} & t \geq 0 \\
0 & t < 0
\end{cases}
\]

The differences of various parameters are defined as

\[
\begin{aligned}
\delta_x(t) &= x(t) - x(t-1) \\
\delta_u(t) &= u(t) - u(t-1) \\
\delta_y(t) &= y(t) - y(t-1) \\
\delta_v(t) &= v(t) - v(t-1)
\end{aligned}
\]

Evidently in Equation (3), \(\delta_v(t) = 0\) when \(t \geq 1\).

The differences are calculated on both sides of Equation (1), as shown as follows:

\[
\begin{aligned}
\begin{cases}
\delta_x(t+1) = A\delta_x(t) + B\delta_u(t) \\
\delta_y(t) = C\delta_x(t), \quad t \geq 1
\end{cases}
\end{aligned}
\]

If the reference input and output for tracking control are \(r(t)\) and \(y(t)\), then the tracking error is \(e(t) = r(t) - y(t)\). The differences on both sides are calculated and combined with Equation (4) as follows

\[
e(t+1) = e(t) - CA\delta_x(t) - CB\delta_u(t) + \delta_r(t+1)
\]

where \(\delta_r(t+1) = r(t+1) - r(t) \in \mathbb{R}^m\).

Let

\[
z(t) = \begin{bmatrix}
\delta_x(t) \\
e(t)
\end{bmatrix} \in \mathbb{R}^{(n+m) \times 1}
\]

Equations (4) and (5) are combined, and the state equation on \(z(t)\) can be obtained as follows

\[
\begin{aligned}
\begin{cases}
z(t+1) = \bar{A}z(t) + \bar{B}\delta_u(t) + M\delta_r(t+1) \\
\delta_y(t) = \bar{C}z(t)
\end{cases}
\end{aligned}
\]

where \(\bar{A} = \begin{bmatrix} A & 0_{n \times m} \\ -CA & I_m \end{bmatrix}\), \(\bar{B} = \begin{bmatrix} B \\ -CB \end{bmatrix}\), \(M = \begin{bmatrix} 0_{n \times m} \\ I_m \end{bmatrix}\), \(\bar{C} = \begin{bmatrix} C & 0_{m \times m} \end{bmatrix}\).

Equation (7) shows that the output tracking of the system (1) could be converted into the stable control problem of the system (7). In other words, if the stability of the closed-loop system can be ensured, then the same tracking of multiple agents can be achieved under certain conditions. The structure of the studied network tracking system is shown in Fig. 1.
2. Control method. For the convenience of analyzing and designing the network prediction tracking controller, the following conditions are hypothesized in this paper:

(a) The sensor, controller, and actuator are driven and synchronous at all times.
(b) The upper limit of the network delay between the controller and the actuator in the forward path is \( f \).
(c) The upper limit of the network delay between the sensor and the controller in the feedback path is \( b \).
(d) All data packets in the network transmission holds a time stamp.

To estimate the state differences of the controlled objects, the following observer is designed at the controller end:

\[
\begin{align*}
\dot{\delta}_x(t+1|t) &= A\delta_x(t|t-1) + B\delta_u(t) + L(\delta_y(t) - \hat{\delta}_y(t)) \\
\dot{\delta}_u(t) &= C\dot{\delta}_x(t|t-1) 
\end{align*}
\]  

(8)

where \( \hat{\delta}_x(t-i|t-j) \in \mathbb{R}^n \) is the predicted value of the state difference by \( t-j \) at \( t-i \); \( \delta_u(t) \) is the input vector of the observer; \( \hat{\delta}_y(t) \) is the output vector of the observer at \( t \); and \( L \in \mathbb{R}^{n \times m} \) is the gain matrix of the designing observer.

After the state difference of the controlled object is estimated, the predicted value of the control difference gained by the feedback control strategy based on the observer is

\[
\hat{\delta}_u(t-b|t-b) = K \hat{z}(t-b|t-b-1) 
\]  

(9)

where \( \hat{z}(t-b|t-b-1) = [\hat{\delta}_x^T(t-b|t-b-1) \ e^T(t-b)]^T \) and \( K \in \mathbb{R}^{l \times (n+m)} \) is the designing gain matrix.

Based on the iterative operation through Equations (7) and (9), the predicted values of the control differences until \( t+f \) are

\[
\begin{align*}
\dot{\hat{z}}(t-b+i|t-b-1) &= \tilde{A}\hat{z}(t-b+i-1|t-b-1) \\
+ B\dot{\delta}_u(t-b+i-1|t-b-1) + M\delta_r(t-b+i) \\
\hat{\delta}_u(t-b+i|t-b) &= K \hat{z}(t-b+i|t-b-1), i = 1, 2, \ldots, b+f 
\end{align*}
\]  

(10)

Based on Equation (10), the sum of the predicted values of the control differences is

\[
\hat{\delta}_u(t+f|t-b) = \sum_{j=0}^{b+f} \hat{\delta}_u(t-b+j|t-b) 
\]  

(11)
The control input at the actuator end is
\[ u(t) = u(t - b - f - 1) + \hat{\delta}_u(t|t - b - f) \] (12)

3. Stability and performance analysis. Let the reference input be \( r(t) = 0 \) to verify the stability of the closed-loop multi-agent system. Based on the aforementioned network prediction tracking control strategy, the control difference of system (7) at \( t \) is
\[ \delta_u(t) = \hat{\delta}_u(t|t - b - f - 1) = K\dot{z}(t|t - b - f - 1) \] (13)
Equation (8) of the observer can yield
\[ \dot{\delta}_x(t + 1|t) = A\dot{\delta}_x(t|t - 1) + B\delta_u(t) + L(\delta_y(t) - C\dot{\delta}_x(t|t - 1)) \] (14)
Calculating the difference between Equations (4) and (14) yields
\[ \varepsilon(t + 1) = (A - LC)\varepsilon(t) \] (15)
where \( \varepsilon(t) = \delta_x(t) - \hat{\delta}_x(t|t - 1) \).
Therefore, Equation (5) can be rewritten as follows
\[ e(t + 1) = e(t) - CA(\varepsilon(t) + \delta_x(t|t - 1)) - CB\delta_u(t) \] (16)
Combining Equations (14)–(16) yields the observer equation of system (7), as shown as follows
\[ \dot{\hat{z}}(t + 1|t) = \hat{A}\hat{z}(t|t - 1) + \hat{B}\delta_u(t) + E\varepsilon(t) \] (17)
where \( \hat{z}(t|t - 1) = \begin{bmatrix} \hat{\delta}_x(t|t - 1) \\ \varepsilon(t) \end{bmatrix} \in R^{(n+m)\times 1} \), and \( E = \begin{bmatrix} LC \\ -CA \end{bmatrix} \in R^{(n+m)\times n} \).
Based on Equation (13), Equation (10) can be rewritten as
\[ \dot{\hat{z}}(t + 1|t - b - f - 1) = \hat{A}\hat{z}(t|t - b - f - 1) + \hat{B}\delta_u(t) \] (18)
Let \( \hat{z}(t - i) = \hat{z}(t - i|t - i - 1) - \hat{z}(t - i|t - b - f - 1) \). Calculating the difference between Equations (17) and (18) yields
\[ \hat{z}(t + 1) = \hat{A}\hat{z}(t) + E\varepsilon(t) \] (19)
where \( i \in \{ I | I \leq b + f; I \in Z \} \).
According to Equation (19), that is
\[ \hat{z}(t) = \hat{z}(t|t - 1) - \hat{z}(t|t - b - f - 1) = \sum_{i=1}^{b+f} \hat{A}^{i-1}E\varepsilon(t - i) \] (20)
Combining Equations (7), (13), and (20) yields
\[ z(t + 1) = Az(t) + B\delta_u(t) \]
\[ = \hat{A}\hat{z}(t) + \hat{B}K\dot{z}(t|t - b - f - 1) \]
\[ = (\hat{A} + \hat{B}K) z(t) + \hat{B}K (z(t) - \hat{z}(t|t - 1) - \hat{z}(t)) \]
\[ = (\hat{A} + \hat{B}K) z(t) + \hat{B}K \left(-F\varepsilon(t) - \sum_{i=1}^{b+f} \hat{A}^{i-1}E\varepsilon(t - i) \right) \] (21)
where \( F = [I_n \ 0]^T \in R^{(n+m)\times n} \).
Let \( Z(t) = [z^T(t) \; \varepsilon^T(t) \; \varepsilon^T(t-1) \; \cdots \; \varepsilon^T(t-b-f)]^T \in R^{n(b+f+2+m)\times 1} \). When Equations (15) and (21) are combined, the simplified equation of the closed-loop network prediction tracking control system can be obtained, as shown as follows

\[
Z(t+1) = \begin{bmatrix}
\hat{A} + \hat{B}K & \Phi(b,f) \\
0 & 0
\end{bmatrix} Z(t)
\]

(22)

where \( \Phi(b,f) = -\begin{bmatrix}
\hat{B}KF & \hat{B}KE & \hat{B}K\hat{A}E & \cdots & \hat{B}K\hat{A}^{b+f}E
\end{bmatrix} \in R^{(n+m)\times n(b+f+1)} \), and

\[
\Xi = \text{diag}(A - LC \; \cdots \; A - LC) \in R^{n(b+f+1)\times n(b+f+1)}.
\]

**Theorem 3.1.** The necessary and sufficient condition for the asymptotic stabilization of the closed-loop system (22) is that the characteristic roots of matrix \( \hat{A} + \hat{B}K \) and \( A - LC \) are in the unit circle.

**Proof.** The closed-loop equation (22) describes an upper triangle system. According to [6], the necessary and sufficient condition for the asymptotic stabilization of the upper triangle system is that the submatrix blocks on the diagonal are stable. Apparently, the closed-loop system (22) achieves asymptotic stability if and only if \( \hat{A} + \hat{B}K \) and \( \Xi \) are asymptotically stable. Thus, the characteristic roots of matrix \( \hat{A} + \hat{B}K \) and \( A - LC \) are in the unit circle.

To sum up, Equation (22) describes the closed-loop dynamic characteristics based on the network prediction tracking control mechanism; thus, Theorem 1 shows that the stability of the closed-loop network prediction tracking control system is only related with the control gain matrix \( (K) \) and gain matrix of the difference observer \( (L) \), but is unrelated with network delay. Moreover, Theorem 1 reflects that when this closed-loop system approaches stability, synchronous tracking among multiple agents can be achieved.

The design steps of the network prediction tracking controller of the multi-agent system are as follows

**Step 1.** Design the state difference observer. The gain matrix of the state difference observer \( (L) \) can be designed according to the design steps of the local difference system (4).

**Step 2.** Design the feedback controller. The feedback gain matrix \( (K) \) can be designed with reference to the design method of the local control system (7).

4. **Simulation results.** The effectiveness of the network prediction tracking control algorithm was verified by the servo motor control system in [7]. This servo motor control system comprises a DC dynamo, a load board, and an angular transducer; it is mainly responsible for rotating the load board to the preset angle. A system identification method is used to determine the discrete transmission function model of the control voltage \( (V) \) and the angle position \( (\degree) \), as shown as follows

\[
G(z^{-1}) = \frac{-0.00886z^{-1} + 1.268227z^{-2}}{1 - 1.66168z^{-1} + 0.6631z^{-2}}
\]

The above equation can be rewritten as the state–space equation as follows

\[
\begin{cases}
x(t+1) = Ax(t) + Bu(t) \\
y = Cx(t)
\end{cases}
\]
where \( A = \begin{bmatrix} 1.6617 & -0.6631 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \) and \( C = \begin{bmatrix} -0.0089 & 1.2682 \end{bmatrix}. \)

The initial state of the system is \( x(0) = \begin{bmatrix} -31.762 \\ -31.762 \end{bmatrix}^T \) (corresponding to \( y(0) = -40 \)). The initial state of the state difference observer is \( \hat{\delta}x(0|\cdot -1) \) = \( \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \), and the reference signal \( r(t) \) is the square signal ranging between \(-40\) and 40.

The expected poles of the state difference observer (7) and the closed-loop system (8) are \( \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}^T \) and \( \begin{bmatrix} 0.76 + 0.28j & 0.76 - 0.28j & 0.32 \end{bmatrix}^T \), respectively. The pole configuration method is used to obtain \( L \) and \( K \), as shown as follows

\[
L = \begin{bmatrix} 0.8220 \\ 0.7640 \end{bmatrix} ; \quad K = \begin{bmatrix} -0.8223 & 0.4532 & 0.0734 \end{bmatrix}.
\]

A simulation test was conducted based on two aspects.

4.1. **Network control without prediction tracking mechanism.** Suppose the delay parameter of the forward channel is \( f = 2 \), and no time delay exists in the feedback channel, the network delay in the communication channel is not compensated. Therefore, the control law uses the state feedback based on the following delay information:

\[
\delta u(t) = K \hat{z}(t - 2 | t - 3)
\]

Simulation results are shown in Fig. 2. Network delay significantly reduces the tracking performance of the controller, thus resulting in system instability.

4.2. **Compensation for network delay by network prediction tracking control.** The delay parameters of the control loop are set as \( f = 2 \) and \( b = 2 \). The simulation results are presented in Fig. 3.

In Fig. 3, \( y(t) \) (LOTC) is the output curve of the local tracking control system (no delay of the control loop), and \( y(t) \) (NPOTC) is the output curve of the network prediction tracking control system. The comparison results of the two output curves reflect that the network closed-loop system is stable, and the network prediction tracking control system can achieve the same tracking with the local tracking control system.

**Figure 2.** Network tracking control without prediction tracking mechanism
5. **Conclusion.** Based on the idea of prediction control, a prediction tracking control mechanism is introduced for the network control system with network delay in both forward channel (from the controller to the actuator) and feedback channel (from the sensor to the controller). The design method of the network prediction tracking controller is proposed, which can actively compensate for network delay. The entire design process is unrelated with network delay, and effectively ensures stability and tracking performance of the network closed-loop multi-agent system.

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