EXTENDING RECOVERY OF THE PRIMORDIAL MATTER POWER SPECTRUM

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ABSTRACT

The shape of the primordial matter power spectrum encodes critical information on cosmological parameters. At large scales, in the linear regime, the observable galaxy power spectrum \( P_{\text{obs}}(k) \) is expected to follow the shape of the linear matter power spectrum \( P_{\text{lin}}(k) \), but on smaller scales the effects of nonlinearity and galaxy bias make the ratio \( P_{\text{obs}}(k)/P_{\text{lin}}(k) \) scale-dependent. We develop a method that can extend the dynamic range of the primordial matter power spectrum recovery, taking full advantage of precision measurements on quasi-linear scales, by incorporating additional constraints on the galaxy halo occupation distribution (HOD) from the projected galaxy correlation function \( w_p(r_p) \). We devise an analytic model to calculate observable galaxy power spectrum \( P_{\text{obs}}(k) \) in real-space and redshift-space, given \( P_{\text{lin}}(k) \) and HOD parameters, and we demonstrate its accuracy at the few percent level with tests against a suite of populated N-body simulations. Once HOD parameters are determined by fitting \( w_p(r_p) \) measurements for a given cosmological model, galaxy bias is completely specified, and our analytic model predicts both the shape and normalization of \( P_{\text{obs}}(k) \). Applying our method to the main galaxy redshift samples from the Sloan Digital Sky Survey (SDSS), we find that the real-space galaxy power spectrum follows the shape of the nonlinear matter power spectrum at the 1–2% level up to \( k = 0.2\ h\,\text{Mpc}^{-1} \) and that current observational uncertainties in HOD parameters leave only few percent uncertainties in our scale-dependent bias predictions up to \( k = 0.5\ h\,\text{Mpc}^{-1} \). These uncertainties can be marginalized over in deriving cosmological parameter constraints, and they can be reduced by higher precision \( w_p(r_p) \) measurements. When we apply our method to the SDSS luminous red galaxy (LRG) samples, we find that the linear bias approximation is accurate to 5% at \( k \leq 0.08\ h\,\text{Mpc}^{-1} \), but the strong scale-dependence of LRG bias prevents the use of linear theory at \( k \geq 0.08\ h\,\text{Mpc}^{-1} \). Our HOD model prediction is in good agreement with the recent SDSS LRG power spectrum measurements at all measured scales \( (k < 0.2\ h\,\text{Mpc}^{-1}) \), naturally explaining the observed shape of \( P_{\text{obs}}(k) \) in the quasi-linear regime. The phenomenological “Q-model” prescription is a poor description of galaxy bias for the LRG samples, and it can lead to biased cosmological parameter estimates when measurements at \( k \geq 0.1\ h\,\text{Mpc}^{-1} \) are included in the analysis. We quantify the potential bias and constraints on cosmological parameters that arise from applying linear theory and Q-model fitting, and we demonstrate the utility of HOD modeling of high precision measurements of \( P_{\text{obs}}(k) \) on quasi-linear scales, which will be obtainable from the final SDSS data set.

Subject headings: cosmology: theory — dark matter — galaxies: halos — large-scale structure of universe

1. INTRODUCTION

In the linear regime, the power spectrum of matter fluctuations encodes information about the physics of early universe (e.g., the potential of the field that drives inflation) and about the matter and energy contents of the cosmos. The power spectrum of galaxies can be biased relative to the power spectrum of matter (Kaiser 1984; Bardeen et al. 1986), but fairly general theoretical arguments imply that the shape of galaxy power spectrum should approach the shape of the linear matter power spectrum \( P_{\text{lin}}(k) \) at sufficiently large scales, i.e.,

\[
P_{\text{g}}(k) = b_0^2 P_{\text{lin}}(k) + N_0,
\]

where \( b_0 \) is a constant galaxy bias factor and \( P_{\text{g}}(k) \) denotes the real-space galaxy power spectrum (Coles 1993; Fry & Gaztanaga 1994; Weinberg 1995; Mann, Peacock & Heavens 1998; Scherrer & Weinberg 1998; Narayan et al. 2000; Berlind & Weinberg 2002; Schulz & White 2006). The additive “shot noise” term \( N_0 \) reflects both galaxy discreteness and small scale clustering (Scherrer & Weinberg 1998; McDonald 2006; Smith, Scoccimarro & Sheth 2007); in general, it can differ from a simple Poisson sampling correction. In the linear regime, distortions of redshift-space structure by peculiar velocities also alter the amplitude but not the shape of galaxy power spectrum (Kaiser 1987). There have therefore been great efforts to measure the galaxy power spectrum on large scales from angular catalogs and redshift surveys and to use the results to test cosmological models (e.g., Yu & Peebles 1968; Baugh & Fry 1991; Feldman et al. 1994; Park et al. 1994; Lin et al. 1996; Sutherland et al. 1999). The enormous size of the Two Degree Field Galaxy Redshift Survey (2dFGRS; Colless et al. 2001) and the Sloan Digital Sky Survey (SDSS; York et al. 2000) relative to earlier samples allows much higher precision measurements. The current state-of-the-art power spectrum measurements are Cole et al.’s (2005) analysis of the power spectrum in the 2dFGRS, Padmanabhan et al.’s 2007, and Blake et al.’s...
analyses of luminous red galaxies (LRGs) with photometric redshifts in the SDSS, and Percival et al. (2007) and Tegmark et al.’s (2006) measurements from the SDSS redshift survey of main sample galaxies and LRGs.

This paper investigates the problem of going from the galaxy power spectrum to the linear matter power spectrum, and hence to cosmological conclusions. The latest observational analyses yield impressive statistical precision on scales near the transition from the linear to the non-linear regime, e.g., typical 1-σ errors of 5–10% in $P(k)$ at $k \approx 0.15 h \text{Mpc}^{-1}$. The critical uncertainty in cosmological interpretation is therefore the accuracy of equation (1) on these scales. The effects of nonlinearity and redshift-space distortions on the matter power spectrum can be computed using numerical simulations or tuned analytic models (Smith et al. 2003, and references therein), but details of galaxy formation physics can influence the relation between galaxy and matter power spectra in this regime. Percival et al. (2007) find that linear theory fits imply different cosmological parameters if applied to measurements with $k \leq 0.06 h \text{Mpc}^{-1}$ or with $k \leq 0.15 h \text{Mpc}^{-1}$, indicating that nonlinear effects have become significant in this regime. Furthermore, Cole, Sánchez & Wilkins (2005) analyze the SDSS and 2dFGRS galaxy samples and find that the measured shapes of galaxy power spectra differ at a level that cannot be explained by the expected cosmic variance. They show that the likely source of the discrepancy is different scale-dependence of galaxy bias, originating from the different color distributions of galaxies in the SDSS and 2dFGRS samples. Cole et al. (2005), Tegmark et al. (2006), and Padmanabhan et al. (2007) approach this problem by fitting a parametrized model of scale-dependent bias,

$$P_{\text{gal}}(k) = b_0^2P_{\text{lin}}(k)^{1+Qk^2} + \frac{1}{1+Ak},$$

where we use $P_{\text{gal}}(k)$ to represent the galaxy power spectrum, which can be either in real-space or redshift-space. The functional form is devised for convenience to approximate the scale-dependent bias of galaxy samples obtained by populating the Hubble volume simulation (Evrard et al. 2002) using a semi-analytic model of galaxy formation (Benson et al. 2000). Here $A = 1.4 h^{-1} \text{Mpc}$ or $1.7 h^{-1} \text{Mpc}$ for real-space power spectrum $P_{\text{r}}(k)$ or angle-averaged redshift-space power spectrum $P_{\text{p}}(k)$ measurements, respectively, and $Q$ is treated as a free parameter that is marginalized over in deriving cosmological parameter constraints. This approach is adequate if equation (2) is a sufficiently accurate description of scale-dependent bias for some value of $Q$, but it could yield biased parameter estimates or incorrect error bars if the actual scale-dependence is different. It also gives up on extracting cosmological information from scales where bias might be mildly scale-dependent. For example, Tegmark et al. (2006, hereafter, T06) find that cosmological parameters remain unaffected by changes in power spectrum measurements at $k \geq 0.1 h \text{Mpc}^{-1}$ once they marginalize over the value of $Q$. This implies that the statistical constraining power on cosmological parameters is lost at $k \geq 0.1 h \text{Mpc}^{-1}$ by the marginalization process.

In this paper, we present an alternative approach to recovering the shape of the linear matter power spectrum, both more aggressive and more robust than “marginalizing over $Q$.” Our approach is based on the halo occupation distribution (HOD) framework, which describes the nonlinear relation between galaxies and matter by specifying the probability $P(N|M)$ that a halo of mass $M$ hosts $N$ number of galaxies of a given type, together with specification of the relative spatial and velocity distributions of galaxies within halos. The HOD formalism has emerged as a powerful method of modeling galaxy bias (Jing et al. 1998, Seljak 2000, Ma & Fry 2004, Peacock & Smith 2004, Scoccimarro et al. 2001, Berlind & Weinberg 2002) because the dynamics of dark matter halos can be accurately calculated using analytic approximations or $N$-body simulations, and the effects of galaxy formation physics can be parametrized in terms of an HOD and inferred by fitting observational data.

Our strategy for extending recovery of the primordial matter power spectrum is to use complementary information from the measurements of the projected correlation function $w_p(r_p)$ as a constraint to obtain HOD parameters given a cosmological model. We then predict the galaxy power spectrum $P_{\text{gal}}(k)$ and study the scale-dependent bias

$$b^2(k) \equiv \frac{P_{\text{gal}}(k)}{P_{\text{lin}}(k)}.$$  

For each cosmological model, fitting $w_p(r_p)$ measurements determines HOD parameters and we can then compute a unique prediction of $P_{\text{gal}}(k)$, both shape and normalization (which is essentially pinned to the amplitude of $w_p(r_p)$). Uncertainties in HOD parameters introduce uncertainty in $P_{\text{gal}}(k)$ and $b^2(k)$, but these uncertainties can be accurately computed and marginalized over. Therefore, we can extend the wavenumber range over which $P_{\text{gal}}(k)$ measurements can be used for cosmological parameter constraints, taking full advantage of precision measurements on quasi-linear scales. In practice, we are just using the measured $P_{\text{gal}}(k)$ and $w_p(r_p)$ to simultaneously constrain HOD parameters and the cosmological parameters and marginalizing over the former. Relative to the $Q$-model approach, our method adopts a more physically motivated computation of $P_{\text{gal}}(k)$ and $b^2(k)$, requiring only the validity of the adopted HOD parametrization, and it brings in the additional information present in $w_p(r_p)$ rather than using only the $P(k)$ shape itself to constrain the scale-dependence of bias.

In principle, the power spectrum $P(k)$ and correlation function $\xi(r)$ contain the same information. However, they are in practice measured via different estimators and on different scales, where their signal-to-noise ratios are highest and systematic errors are relatively well understood. The information in $P(k)$ and $\xi(r)$ measurements on these non-overlapping scales is therefore not identical, but complementary. Furthermore, the projected correlation function $w_p(r_p)$ is measured to ease the difficulty in interpreting nonlinear redshift-space distortion of correlation function measurements on small scales. Therefore, the addition of $w_p(r_p)$ measurements at $r_p \leq 30h^{-1} \text{Mpc}$ brings new information that is not present in $P(k)$ measurements at $k \leq 1 h \text{Mpc}^{-1}$.

A different approach to this problem is to develop an analytic model for predicting the scale-dependence of galaxy bias by using higher-order perturbation theory (e.g., McDonald 2006, Smith, Scoccimarro & Sheth 2007). This approach is

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8 Throughout this paper, the term ‘halo’ refers to a dark matter structure of overdensity $\rho/\rho_m \simeq 200$, in approximate dynamical equilibrium.
elegant and transparent in nature, since it is based on linear theory and its extension to higher-order, while our approach is less \textit{ab initio} in the sense of incorporating elements calibrated by numerical $N$-body simulations in our analytic model. However, the critical uncertainty for this approach based on higher-order perturbation theory is its applicability on quasi-linear scales ($\gtrsim 0.1 h$/Mpc$^{-1}$), where first-order linear theory is known to be inaccurate, but the measurement precision is highest in practice. In contrast, our approach is fully nonlinear, and phenomenological in nature, so it can be applied down to small scales, limited only by the point at which uncertainties in the HOD parameters introduce systematic uncertainty in the $P(k)$ recovery.

Analyses of galaxy redshift surveys typically estimate the angle-averaged power spectrum $P_\theta(k)$, i.e., the monopole of the redshift-space power spectrum (e.g., Cole et al. 2005; Percival et al. 2007). Redshift-space distortions do not alter the shape in linear theory, but they do change the shape in the trans-linear regime (e.g., Cole et al. 1994), and finger-of-god (FoG) effects have impact out to large scales (e.g., Scoccimarro et al. 2001; Padmanabhan et al. 2007) and Blake et al. (2007) deproject the angular clustering measurements of the SDSS LRG sample using photo-$z$ catalogs to estimate the real-space power spectrum, independent of redshift-space distortions. Tegmark et al. (2004, 2006) use a linear combination of the redshift-space monopole, quadrupole, and hexadecapole that recovers the real-space power spectrum in the linear regime. We will denote this “pseudo real-space” power spectrum $P_{2 \rightarrow \text{sym}}(k)$. The redshift-space power spectrum estimators can be applied directly to galaxy redshift data or applied after compressing FoG effects. We will investigate $P_{\text{gal}}(k)$ and $b^2(k)$ for all of these cases.

To this end, we develop an analytic model in § 2 for calculating real-space and redshift-space galaxy power spectra given $P_{\text{lin}}(k)$ and a galaxy HOD, drawing on the Tinker et al. (2007) model for redshift-space distortion, which improves on previous work (e.g., Seljak 2001; White 2001; Kang et al. 2002; Cooray 2004). Tinker (2007) tests the model for computing the redshift-space correlation function against a series of populated $N$-body simulations. Here we extend the model and present additional tests of its applicability to modeling redshift-space power spectra in § 3.

In this paper, we use HOD parameters for volume-limited galaxy samples that have well defined classes of galaxies, focusing on SDSS main galaxy samples with absolute-magnitude limits $M_r < -20$ and $M_r < -21$ (Zehavi et al. 2005b) in § 4 and SDSS LRG samples with absolute-magnitude limits $-23.2 < M_r < -21.2$ and $-23.2 < M_r < -21.8$ (Eisenstein et al. 2001; Zehavi et al. 2005a; Zheng et al. 2008) in § 5 for application of our method. More complete modeling of the conditional luminosity function (Yang et al. 2003) might allow use of flux-limited galaxy catalogs, though it requires more free parameters to provide complete descriptions of the galaxy samples. Here we only consider volume-limited galaxy samples, whose results can be combined to improve statistical precision. We summarize our main results in § 6.

2. CALCULATIONAL METHODS

2.1. Numerical Model

We use the $N$-body simulations of Tinker et al. (2006) to test our analytic model calculations of the correlation function and the power spectrum in real-space and redshift-space. These are five simulations of a flat $\Lambda$CDM universe using the publicly available tree-code \textsc{gadget} (Springel et al. 2001), and all the simulations are performed with identical cosmological parameters except for the random seed numbers used to generate initial conditions. The initial scale-invariant ($n_s = 1$) power spectrum is modified by the transfer function of Eftakharzadeh et al. (1992) with shape parameter $\Gamma = 0.2$. The simulation was evolved from an expansion factor $a = 0.01$ to $a = 1.0$ with $\Omega_m = 0.1$, $\Omega_{\Lambda} = 0.9$ and $\sigma_8 = 0.95$ at $z = 0$. To cover a range of parameter space spanned by $\Omega_m$ and $\sigma_8$, we use earlier outputs to represent different cosmological models from the simulations. Our choices for the earlier expansion factors are $a_{\text{out}} = 0.84$, 0.64, 0.49, and 0.40. These outputs correspond respectively to simulations with different parameter combinations ($\Omega_m$, $\sigma_8$) = (0.16, 0.90), (0.30, 0.80), (0.48, 0.69), and (0.63, 0.60) with the identical power spectrum shape ($\Gamma = 0.2$) but evolved beginning at expansion factor $a = 0.01/a_{\text{out}}$. This procedure correctly provides the density field that would be obtained from an independent simulation evolved to $z = 0$ with the corresponding parameter combination of $\Omega_m$ and $\sigma_8$ (Zheng et al. 2002). We evolve $360^2$ particles in a volume of comoving side length $253 h^{-1}$ Mpc to take into consideration that the lowest mass halos that host galaxies with $M_r < -20$ contain at least 32 particles. Dark matter halos are identified by using the friends-of-friends algorithm (FoF; Davis et al. 1985) with a linking length of 0.2 times the mean interparticle separation, i.e., $140 h^{-1}$ kpc.

To populate dark matter halos with galaxies in $N$-body simulations, we use HOD parameters listed in Table 1 that are chosen to match the mean number density $\bar{n}_g$ and projected correlation functions $w_\rho(r_p)$ of the SDSS galaxy samples with absolute-magnitude limits $M_r < -20$ and $M_r < -21$ (Zehavi et al. 2005b). In our standard HOD parametrization, the number of central galaxies is a step function changing from zero to one at a minimum halo mass $M_{\text{min}}$. Therefore, halos of mass $M < M_{\text{min}}$ lack galaxies. We assume $\langle N_{\text{gal}} \rangle \propto M$ at high masses with a smooth cutoff at low mass. Therefore, the number of satellite galaxies is

$$\langle N_{\text{sat}} \rangle_M = \frac{M}{M_1} \exp\left(-\frac{M_{\text{cat}}}{M-M_{\text{min}}}\right).$$

for a halo of mass $M \geq M_{\text{min}}$, and the distribution of satellite galaxy number $P(N_{\text{sat}}|\langle N_{\text{sat}} \rangle_M)$ is assumed to be Poisson (Kravtsov et al. 2004, Zheng et al. 2005). This parametrization is well suited to our purposes, but we also investigate the effect of adopting a more flexible HOD parametrization in § 4.

We replace halos identified by the FoF algorithm by spherical NFW halos (Navarro, Frenk & White 1997) with identical mass, truncated at virial radius $R_{\text{vir}}$, within which the mean density is 200 times the mean matter density. The concentration parameters $c_{\text{min}}$ of dark matter halos are computed using the relation of Bullock et al. (2001) and are scaled to account for the different definition of halo overdensity adopted here. This NFW-replacement method reduces numerical artifacts caused by finite force resolution in our simulations. We place a central galaxy at the center of mass of halos. Assuming that satellite galaxies trace the dark matter distribution within halos, we place satellite galaxies following the NFW profile of halos.
In redshift-space, galaxies are displaced because of peculiar velocity. Central galaxies are assumed to be at rest relative to the halo center; no velocity bias is assumed for central galaxies. For satellite galaxies, we add line-of-sight velocities drawn from a Gaussian distribution with zero mean and dispersion

\[ \sigma_v(M) = \left( \frac{GM}{2R_{\text{vir}}} \right)^{1/2} \]  

to the velocity of the halo center of mass. This procedure is exact for isotropic singular isothermal halos and is reasonably accurate for NFW profiles (see Tinker et al. 2006 for detailed tests).

Finally, we compute the density contrast field by cloud-in-cell weighting the particle distribution onto 360° grids and use the publicly available fast Fourier transform (FFT) code, FFTW, to obtain the Fourier components in units of the fundamental mode of our simulation box, \( \Delta k = 0.02 h \text{Mpc}^{-1} \). We deconvolve the cloud-in-cell weighting function \( W_{\text{CIC}}(k) \), and subtract shot-noise contributions \( 1/N_{\text{gal}} \) to compute the power spectrum at each \( k \):

\[ P_{\text{gal}}(k) = P_{\text{FFT}}(k)/W_{\text{CIC}}^2(k) - \frac{1}{N_{\text{gal}}}, \]  

where the weighting function is

\[ W_{\text{CIC}}(k) = \left[ \sum_{i=1}^{3} \sin\left(\pi k_i/2k_N\right) \right]^2 \]  

and \( N_{\text{gal}} \) is the number of galaxies, \( k_i \) is the \( i \)-th component of wavenumber \( k \), and \( k_N = 4.56 \text{Mpc}^{-1} \) is the Nyquist wavenumber of our simulations. For computations in redshift-space, we simply displace particles using the z-component of the peculiar velocity scaled by the Hubble constant as they would appear to a distant observer at \( z = -\infty \). This procedure satisfies the distant observer approximation we adopt here, and it ensures periodic radial velocity fields in the simulation volume, appropriate for FFT. Redshift-space multipoles are extracted by least-squares fitting to the Legendre polynomial coefficients (eq 13). We repeat the procedure for the \( x \)- and \( y \)-axes, treating each axis as the line-of-sight, and we average the resulting power spectra over the three line-of-sight directions.

### 2.2. Analytic Model

Our analytic calculation of the real-space galaxy autocorrelation function \( \xi_0(r) \) follows Tinker et al. (2005), which improves the methods of Zheng (2004) with more accurate treatments of scale-dependent halo bias and halo exclusion. For a given galaxy sample with its projected correlation function measurements \( w_p(r_p) \), we obtain HOD parameters by fitting the mean space density \( \bar{n}_g \) and \( w_p(r_p) \), computed by

\[ \bar{n}_g = \int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn}{dM} \langle N \rangle_M, \]  

where \( dn/dM \) is the halo mass function of Jenkins et al. (2001) and we use \( z_{\text{max}} = 40 \text{h}^{-1} \text{Mpc} \) as adopted in SDSS clustering measurements (e.g., Zheng et al. 2004, 2005). The real-space galaxy power spectrum \( P_g(k) \) is computed by taking the Fourier transform of the correlation \( \xi_0(r) \).

We compute the redshift-space correlation function \( \xi(r ||, r _\perp) \) using the probability distribution of galaxy pairwise velocities \( f(v_z, r) \),

\[ 1 + \xi(r ||, r _\perp) = \int_{-\infty}^{\infty} dv_z [1 + \xi(r)] f(v_z, r), \]  

where \( r _\perp \) is the projected separation, \( r || \) is the line-of-sight separation in redshift-space, and \( v_z = 100 \text{km s}^{-1} (r || - z)/h^{-1} \text{Mpc} \) is the pairwise velocity of galaxies separated by \( r = (r _\perp^2 + r _\perp^2)^{1/2} \). Equation (10) is called the streaming model and has been used to measure the mean matter density (Peacock et al. 2001; Cole et al. 2005) and to model redshift-space correlations (White 2001; Seljak 2001). It is valid in the linear and nonlinear regime (Fisher 1995; Scoccimarro 2004) provided that the correct \( f(v_z, r) \) is used. We adopt the probability distribution function of Tinker (2007) for the galaxy pairwise velocities, which is an analytic model with some elements calibrated on N-body simulations. We refer the reader to the work by Tinker (2007) for extensive discussion and tests.

### 2.3. Redshift-Space Multipoles

Using the analytic model, we compute the redshift-space correlation function \( \xi(r ||, r _\perp) \) given \( P_g(k) \) and a set of HOD parameters, and we expand \( \xi(r ||, r _\perp) \) with Legendre polynomials,

\[ \xi(r, \mu) = \sum_{l=0}^{\infty} L_l(\mu) \xi_l(r), \]  

where \( r = (r _\perp^2 + r _\perp^2)^{1/2} \) and \( \mu = r _|| / r \) is the direction cosine of the separation and line-of-sight vectors. The redshift-space multipole component is then

\[ \xi_l(r) = \frac{2l+1}{2} \int_{-1}^{1} d\mu L_l(\mu) \xi(r, \mu). \]  

We use \( L_l(\mu) \) to denote Legendre polynomials to avoid confusion with redshift-space multipole power spectra \( P_f(k) \) defined below. The redshift-space power spectrum \( P_f(k) \) can be similarly expanded using Legendre polynomials.

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10 In general the correlation \( \xi^{(2)}_l(r) \) from galaxy pairs in two distinct halos is computed already in Fourier space, and one only needs to add the Fourier transform of the correlation \( \xi^{(2)}_l(r) \) from galaxy pairs in the same halo to compute \( P_f(k) \). However, \( \xi^{(2)}_l(r) \) is modified to account for the finite extent of halos and the scale-dependence of halo bias, and hence the contributions to \( P_f(k) \) from pairs in two distinct halos cannot be expressed in a simple form in Fourier space (for details, see Zheng 2006, Tinker et al. 2008, Yoo et al. 2006).
where $k = (k_x^2 + k_y^2)^{1/2}$ and $\mu = k_x/k$, in analogy to quantities in configuration space. The reflection symmetry of the correlation function and power spectrum ensures that multipoles with odd $l$ vanish on average. Making use of the fact that $\xi(l, r)$ and $P(k, l)$ are Fourier counterparts, each redshift-space multipole component is computed by

$$P_l = 4\pi r^2 \int_0^{\infty} r^2 \xi_l(r, j_l(kr)) dr,$$  \hspace{1cm} (14)

where $j_l(x)$ are spherical Bessel functions \cite{Cole1994, Hamilton1998}. Note that the quadrupole ($l = 2$) components of $P(k)$ and $\xi(r)$ have opposite sign.

Equation (14) requires knowledge of $\xi_l(r)$ on large scales, while our analytic model is only tested at $r \leq 40h^{-1}\text{Mpc}$. Therefore, we compute $\xi_l(r)$ at $r \geq 40h^{-1}\text{Mpc}$ with the linear approximation for redshift-space distortion. In the linear regime, the multipole expansion of $\xi_l(r)$ has only three nonzero multipoles: monopole $\xi_0$, quadrupole $\xi_2$, and hexadecapole $\xi_4$ \cite{Kaiser1987}, which are in turn related to $C_l$.

$$\xi_0(r) = C_0 \xi_0(r),$$

$$\xi_2(r) = C_2 (\xi_2(r) - \xi_0(r)),$$

$$\xi_4(r) = C_4 \left( \xi_4(r) + 2.5 \xi_2(r) - 3.5 \xi_0(r) \right),$$  \hspace{1cm} (15)

where $C_0 = 1 + \frac{1}{2} \beta + \frac{1}{2} \beta^2$, $C_2 = \frac{1}{2} \beta + \frac{1}{2} \beta^2$, $C_4 = \frac{8}{35} \beta^2$, $\beta = \Omega_m^{0.6}/b_0$, $b_0$ is the asymptotic galaxy bias factor, and the barred correlations are

$$\bar{\xi}_0(r) = \frac{3}{r^3} \int_0^r s^2 \xi_0(s) ds,$$  \hspace{1cm} (16)

$$\bar{\xi}_2(r) = \frac{5}{r^5} \int_0^r s^4 \xi_2(s) ds.$$  \hspace{1cm} (17)

\cite{Hamilton1992}. We compute the three multipoles at $r \geq 40h^{-1}\text{Mpc}$, rather than $\xi_0(r)$ itself. We first compute the values of $C_l$ at $r = 40h^{-1}\text{Mpc}$, where the deviations from the linear theory predictions are less than 5%, then we smoothly transition $C_l$ values to $r_{\text{lin}}$ beyond which the adopted $C_l$ values exactly become the linear theory predictions. The redshift-space multipoles $\xi_l(r)$ at $r \geq 40h^{-1}\text{Mpc}$ are then obtained by using equations (15) with $C_l$, in place of constant $C_l$, the linear theory prediction. We simply set $r_{\text{lin}} = 500h^{-1}\text{Mpc}$, and $P_l(k)$ values are insensitive to the choice of $r_{\text{lin}}$ as long as $r_{\text{lin}} > 100h^{-1}\text{Mpc}$. We adopt the \cite{Smith2003} prescription for the matter power spectrum $P_{\text{lin}}(k)$ and its Fourier transform $\xi_{\text{lin}}(k)$ to compute $\xi_l(k)$ on large scales, and the asymptotic galaxy bias factor is computed by

$$b_0 = \frac{1}{n_g} \int_0^{\infty} dM \frac{dn}{dM} \langle N \rangle_M b_h(M),$$  \hspace{1cm} (18)

where $b_h(M)$ is the bias factor of halos of mass $M$. We use the \cite{ShethMoTormen2001} formulation with coefficients obtained by \cite{Tinker2005}, which yield a better fit to the simulations.

3. RECOVERING THE REAL-SPACE GALAXY POWER SPECTRUM

Before turning to the bias between the galaxy power spectrum and the linear matter power spectrum, we investigate how well the method used by \cite{Tegmark2004, Tegmark2006} recovers the true real-space galaxy power spectrum. Along the way, we test the accuracy of our analytic model prediction for the redshift-space power spectrum against the results obtained from the $N$-body galaxy catalogs described in \S2.1. Our basic approach to predicting the galaxy power spectra is that we first determine HOD parameters for an observed galaxy sample given a cosmological model, then calculate the galaxy power spectra using this inferred relation between galaxies and dark matter halos. We obtain HOD parameters by fitting the mean number densities $n_g$ and projected correlation function measurements $w_p(r_p)$ of the SDSS $M_B \leq -20$ and $M_B \leq -21$ galaxy samples, taking into account the full covariance matrix, estimated through jackknife resampling of the observational sample \cite{Zehavi2005b}.

Figure 1 shows the projected correlation functions of the two SDSS galaxy samples, where the solid lines are the analytic model predictions for the five different cosmological models (listed in Table 1), obtained by fitting the $\xi_{\text{lin}}(p)$ measurements shown as symbols. The error bars show only the diagonal elements of the covariance matrix of the $w_p(r_p)$ measurements, and the analytic model fits to the measurements are acceptable over a wide range of parameter combinations $(\Omega_m, \sigma_8)$ when the full covariance matrix is considered, since the errors between data points are strongly correlated. However, there is strong degeneracy between the shape $\Gamma$ and spectral index $n_s$ of the power spectrum, and the best-fit HOD parameters are insensitive to $\Gamma$ and $n_s$, as discussed in \S4.2 below. Changing the adopted value of $z_{\text{max}}$.
TABLE 1

| Model | $\Omega_m$ | $\sigma_8$ | $M_{20}$ (h^{-1}M_{\odot}) | $M_{21}$ (h^{-1}M_{\odot}) | $M_{22}$ (h^{-1}M_{\odot}) |
|-------|------------|------------|-----------------|-----------------|-----------------|
| 1     | 0.10       | 0.95       | 2.95 × 10^{11}  | 5.37 × 10^{12}  | 1.40 × 10^{13}  |
| 2     | 0.16       | 0.90       | 4.80 × 10^{11}  | 8.03 × 10^{12}  | 1.37 × 10^{13}  |
| 3     | 0.30       | 0.80       | 9.33 × 10^{11}  | 1.32 × 10^{13}  | 1.38 × 10^{13}  |
| 4     | 0.47       | 0.69       | 1.44 × 10^{12}  | 1.62 × 10^{13}  | 1.78 × 10^{13}  |
| 5     | 0.63       | 0.60       | 1.89 × 10^{12}  | 1.60 × 10^{13}  | 2.46 × 10^{13}  |

Note: The HOD parameters of the five N-body models are determined to reproduce the same clustering $w_{p}(r_p)$ of the SDSS galaxy samples with $M_r \leq -20$ and $M_r \leq -21$, and to match the number densities $n_i = 5.74 \times 10^{-3}$ (h^{-1}Mpc)^{-3}$ for the $M_r \leq -20$ sample and $n_i = 1.17 \times 10^{-3}$ (h^{-1}Mpc)^{-3}$ for the $M_r \leq -21$ sample, respectively.

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Figure 2. Dimensionless real-space and redshift-space multipole power spectra $\Delta^2(k)$ for the HOD parameters appropriate to $M_r \leq -20$ galaxies. Various curves represent the analytic predictions of the central model for the corresponding galaxy power spectra indicated in the legend, and shaded regions show the statistical uncertainty on the real-space galaxy power spectrum computed from the five N-body simulations. The light, short dashed curve, labeled $P_{l = 0}$ (see, eq. [19]), is a linear combination of redshift-space multipoles that reduces to $P_0(k)$ in linear regime; it is largely obscured by the solid curve. Thick dashed curves represent the linear (long) and non-linear (short) matter power spectra. Note that the quadrupole $P_2(k)$ crosses zero at $k = 0.3h\text{Mpc}^{-1}$, and $-P_2(k)$ is plotted at larger $k$.

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has little effect on the HOD parameters inferred by fitting $w_{p}(r_p)$, though it affects the $\chi^2$ values of these fits slightly. We use $z_{\text{max}} = 40h^{-1}\text{Mpc}$ for further analyses. The shaded regions represent the statistical uncertainty in the mean value of $w_{p}(r_p)$, computed from the dispersion among the five independent populated N-body simulations, for the central model ($\Omega_m = 0.3$, $\sigma_8 = 0.8$; see Table 1). Our analytic model predictions for $w_{p}(r_p)$ agree with the N-body results within 5% fractional differences.

Using the analytic model and assuming the central cosmological model, we illustrate the dimensionless power spectra $\Delta^2(k) = k^3 P(k)/2\pi^2$ of the $M_r \leq -20$ galaxy sample in Figure 2 where the thick solid line represents the real-space galaxy power spectrum $P_0(k)$ and the shaded region is the statistical uncertainty in the mean value of $P_0(k)$. The finite box size of the simulations puts a limit on Fourier modes $k \geq k_{\text{box}} \equiv 2\pi/L_{\text{box}} = 0.025h\text{Mpc}^{-1}$ that we can measure from the populated halo catalogs. We test our analytic model predictions for galaxy power spectra below. The two thick dashed lines are the matter power spectra of the assumed cosmological model; the long-dashed line is the linear matter power spectrum $P_{\text{lin}}(k)$ and the short-dashed line is the nonlinear matter power spectrum $P_{\text{nlin}}(k)$, computed by using the Smith et al. (2003) prescription. On large scales, $P_0(k)$ has the same shape as $P_{\text{lin}}(k)$ with normalization differing by $b_0^2$, the square of the large-scale galaxy bias factor, as predicted by the linear bias approximation (eq. [1]). However, on scales $k \gtrsim 0.2h\text{Mpc}^{-1}$, where $P_{\text{nlin}}(k)$ departs from $P_{\text{lin}}(k)$ or $\Delta^2_{\text{lin}}(k) \simeq 1$, $P_0(k)$ follows more closely the $P_{\text{nlin}}(k)$ shape than the $P_{\text{lin}}(k)$ shape.

We also present the analytic model predictions for the redshift-space multipole power spectra $\Delta^2_{l}(k) = (l^2 P_{l}(k))/2\pi^2$. The dotted, long-dashed, and dot-dashed lines in Figure 2 represent the redshift-space monopole $P_0(k)$, quadrupole $P_2(k)$, and hexadecapole $P_4(k)$, respectively. On large scales, coherent peculiar velocities produce redshift-space distortion, where the overdense regions shrink and the underdense regions inflate in redshift-space. Therefore, when averaged over angle, $P_0(k)$ is larger than $P_0(k)$. However, on small scales, nonlinear collapse and random motions in virialized objects stretch systems along the line-of-sight, giving rise to the Finger-of-God (FoG) effect, which inflates overdense regions and depresses their density contrast. Therefore, $P_0(k)$ is smaller than $P_0(k)$ on small scales.

In the linear regime, the redshift-space power spectrum can be written as $P(k, \mu) = (1 + \beta \mu^2) P_0(k)$, and the real-space power spectrum can be reconstructed by using a linear combination of the three redshift-space multipoles to remove the unknown variable $\beta$,

$$P_{2-\nu}(k) = P_0(k) - \frac{1}{2} P_2(k) + \frac{3}{8} P_4(k), \quad (19)$$

which exactly reduces to $P_0(k)$ if the linear theory approximation holds (Kaiser 1987). The upper panel of Figure 3 tests the ability of equation (19) to recover the true $P_0(k)$. $P_{2-\nu}(k)$ recovers $P_0(k)$ to 5% at $k \leq 0.4h\text{Mpc}^{-1}$, while it substantially underestimates $P_0(k)$ at $k > 0.4h\text{Mpc}^{-1}$. This pseudo real-space power spectrum $P_{2-\nu}(k)$ can be used to correct for the effect of redshift-space distortions and to estimate $P_0(k)$. A k-by-k application of equation (19) is, essentially, the procedure referred to as the disentanglement approach in Pegram et al. (2004, 2006), and our $P_{2-\nu}(k)$ corresponds to $P_{\text{lin}}(k)$ in their notation. Their alternative, modeling approach, is to construct two more power spec-
results for the corresponding galaxy power spectra. Shaded regions represent linear theory to match distortions. Note that caused by the finite simulation volume. The other power spectra have larger too closely on large scales to be separated from the thin solid line.

parameters that fit the SDSS galaxy sample with certainties in the other two power spectra at each statistical errors because it effectively marginalizes over un-
terapproximation on nonlinear scales, but it yields larger cancellation of deviations in the multipoles from the linear approximation (Tegmark et al. 2004, 2006). The former method gives a more robust approximation to $P_R(k)$ after scaling the constant factor $C_0$ in $\delta(k)$. The scaled $P_0(k)$ is depressed by 5% at $k = 0.1 h$Mpc$^{-1}$ compared to $P_R(k)$. FoG compression helps at $k \approx 0.1 h$Mpc$^{-1}$, but the difference between $P_0^{\text{FoG}}(k)$ and $P_R(k)$ reaches 5% at $k \approx 0.2 h$Mpc$^{-1}$ and 10% at $k \approx 0.3 h$Mpc$^{-1}$.

Having shown the agreement between the analytic model predictions and the $N$-body results in $w_p(r_p)$, we now test the accuracy of the analytic model predictions for real-space and redshift-space power spectra against the $N$-body simulations. The bottom panel shows the fractional difference in galaxy power spectra between the analytic model and the $N$-body results, where the shaded region shows only the statistical uncertainty in the mean value of $P_R(k)$ from the simulations. Note that the uncertainties on the other power spectra are larger and are not shown. The analytic model calculation of $P_R(k)$ is accurate to better than a few percent at $k > 0.08 h$Mpc$^{-1}$, while it is difficult to assess the statistical significance at $k < 0.08 h$Mpc$^{-1}$, where the simulations only have few Fourier modes due to the finite box size. Since linear theory should become accurate on large scales, it would be surprising if the analytic model became less accurate on this regime. Our analytic model also provides accurate predictions for $P_0(k)$ and $P_{Z-R}(k)$, both with and without FoG compression, at $k > 0.1 h$Mpc$^{-1}$.

4. RECOVERING THE LINEAR MATTER POWER SPECTRUM

We now turn to our principal results, the scale-dependent bias relation $b^2(k) = P_{\text{obs}}(k)/P_{\text{lin}}(k)$ between observable galaxy power spectra $P_{\text{obs}}(k)$ and the linear matter power spectrum $P_{\text{lin}}(k)$. As potentially observable power spectra, we consider $P_R(k)$ (inferred from the angular clustering power spectrum), $P_{Z-R}(k)$, and $P_0(k)$, with varying levels of FoG comp-
pression for the latter two. Here we use HOD constraints for the \( M_r \leq -20 \) and \( M_r \leq -21 \) samples of Zehavi et al. (2005b) based on their \( w_p(r_p) \) measurements from SDSS Data Release 2. These could be further improved with \( w_p(r_p) \) measurements from subsequent SDSS data and with constraints from the group multiplicity function (Berlind et al. 2006). We investigate the uncertainties in the power spectrum recovery associated with our HOD modeling and with variations of the assumed cosmological model. We use a CMBFAST transfer function (Seljak & Zaldarriaga 1996) to compute \( P_{\text{lin}}(k) \) for a given cosmology, in place of the Efstathiou et al. (1992) parametrization used in § 3 to test our analytic model against the N-body simulations (which used these initial conditions). Unless explicitly stated otherwise, we use a cosmological model with \( \Omega_m = 0.3, \Omega_{\Lambda} = 0.7, n_s = 1.0, h = 0.7, \Omega_h^2 = 0.02 \), and \( \sigma_8 = 0.9 \) in this section.

4.1. Scale-Dependent Bias of the \( M_r \leq -20 \) and \( M_r \leq -21 \) Galaxy Samples

Figure 4 plots the scale-dependent bias functions \( b^2(k)/b_0^2 \) of the galaxy sample with \( M_r \leq -20 \), in which \( b_0 \) is the asymptotic galaxy bias factor computed via equation (13). Figure 4 shows the real-space \( P_R(k) \) (solid line) and the nonlinear matter power spectrum \( P_{\text{nl}}(k) \) (circles), which follow each other remarkably closely. A suppression of \( P_{\text{nl}}(k) \) at \( k \approx 0.1 \text{hMpc}^{-1} \) relative to \( P_{\text{lin}}(k) \) results from the nonlinear damping of linear perturbations. The physical origin of the \( P_R(k) \) shape is discussed by Smith, Scoccimarro & Sheth (2007). Note that we compute the nonlinear matter power spectrum \( P_{\text{nl}}(k) \) using the Smith et al. (2003) prescription, modified to utilize a \textsc{Cmbfast} transfer function. \( P_{\text{lin}}(k) \) retains the baryonic acoustic oscillations (BAO) imprinted by sound waves in the baryon-photon plasma before recombination, but nonlinear evolution washes out the oscillations at higher \( k \) (see, Eisenstein, Seo, White 2007). The bias functions in Figure 4 exhibit small oscillations at \( k > 0.1 \text{hMpc}^{-1} \) because the smoother nonlinear power spectra are divided by a reference \( P_{\text{lin}}(k) \) that retains the BAO features at their original strength.

When normalized to the large-scale amplitude, the real-space power spectrum of \( M_r \leq -20 \) galaxies falls below \( P_{\text{nl}}(k) \) by \( \approx 5\% \) at \( k \approx 0.1 \text{hMpc}^{-1} \), climbs above by \( 5\% \) at \( k \approx 0.2 \text{hMpc}^{-1} \), and rises rapidly thereafter. However, for this galaxy sample, the assumption that \( P_R(k) \) traces the nonlinear matter power spectrum remains quite accurate, to \( 1\% \) at \( k \leq 0.2 \text{hMpc}^{-1} \) and \( 7\% \) at \( k = 0.3 \text{hMpc}^{-1} \). The dotted curve shows the \( Q \)-model prediction of equation (2) with \( Q = 10.6 \left( h^{-1} \text{Mpc} \right)^2 \), which gives a least-squares fit to
our predicted bias curve over the range $0.01h\text{Mpc}^{-1} < k < 0.3h\text{Mpc}^{-1}$. The largest difference is at $k \approx 0.04h\text{Mpc}^{-1}$, where the $Q$-model predicts a 5% deviation from $P_{\text{lin}}(k)$ while we find 3%. The $Q$-model accurately traces the predicted bias shape beyond $k = 0.1h\text{Mpc}^{-1}$. However, if one normalized the curves to match at $k \approx 0.04h\text{Mpc}^{-1}$, which might well happen in practice because of the large statistical uncertainties at low $k$, then deviations in the bias shape would be smaller at $k < 0.1h\text{Mpc}^{-1}$ and larger at $k > 0.1h\text{Mpc}^{-1}$.

Figure 4 plots $P_{Z-R}(k)$ and $P_0(k)$ with no FoG compression. The bias shape for $P_{Z-R}(k)$ is qualitatively similar to the true real-space $P_R(k)$, but it follows the $P_{\text{lin}}(k)$ shape more closely than the $P_{\text{nl}}(k)$ shape at $k \lesssim 0.05h\text{Mpc}^{-1}$. $P_{Z-R}(k)$ is poorly described by the $Q$-model prescription, with differences of 6% at $k \approx 0.05h\text{Mpc}^{-1}$. The bias shape for $P_0(k)$ is completely different from those for $P_{R}(k)$ and $P_{Z-R}(k)$. By $k = 0.1h\text{Mpc}^{-1}$, $P_0(k)/b_0^2$ is below $P_{\text{lin}}(k)$ by 10% and $P_{\text{nl}}(k)$ by 5%, and it remains below $P_{\text{lin}}(k)$ by 10-15% to $k = 0.5h\text{Mpc}^{-1}$. The best-fit $Q$-model has $Q = 2.2 \left( h^3\text{Mpc}^3 \right)$, and it has several percent differences from our $P_0(k)$ prediction at $k < 0.1h\text{Mpc}^{-1}$.

Figures 4 and 5 illustrate the effects of FoG compression on $P_{Z-R}(k)$ and $P_0(k)$. Changes to $P_{Z-R}(k)$ are minor, though the agreement with the true $P_R(k)$ is significantly improved at $k \gtrsim 0.25h\text{Mpc}^{-1}$. Changes to the monopole $P_0(k)$ are much more substantial, especially at $k \gtrsim 0.2h\text{Mpc}^{-1}$: suppression of $P_0(k)$ by the virial motions of satellites is much stronger than for $P_{Z-R}(k)$. However, the shape of $P_0(k)$ remains quite far from that of the true $P_R(k)$, even for the aggressive FoG compression with $\sigma_h \gtrsim 400\text{km s}^{-1}$.

Figure 5 plots the scale-dependent bias functions of the galaxy sample with $M_r \lesssim -21$, in the same format as Figure 4. Compared to the $M_r \lesssim -21$ galaxy sample, this galaxy sam-ple has a large-scale bias factor higher by 20%, and hence a power spectrum amplitude higher by nearly 50%. However, the shape of $b^2(k)/b_0^2$ is nearly identical for the two galaxy samples over a large dynamic range $k \lesssim 0.3h\text{Mpc}^{-1}$, as one can see by using the nonlinear matter power spectrum (circles) as a reference. In each case, the best-fit $Q$-model reproduces our predictions for $P_R(k)$, $P_{Z-R}(k)$, and $P_0(k)$ at roughly the 5% level for $k \lesssim 0.3h\text{Mpc}^{-1}$.

4.2. Impact of HOD Uncertainties

Our method for calculating $P_{\text{obs}}(k)$ and then $b^2(k)$ is that we first determine HOD parameters by fitting $w_p(r_p)$ measurements given a cosmological model. With perfect knowledge of HOD and cosmological parameters, it should be possible in principle to calculate $P_{\text{obs}}(k)$ exactly and fit the observed
Fig. 6.— Impact of HOD parameter uncertainties on bias shape. We compare our best-fit HOD models (solid) to ten HOD models (gray) for each galaxy sample that have $\Delta \chi^2 \leq 1$ relative to the best-fit models, randomly chosen from the Monte Carlo Markov Chain. Left-hand panels show mean occupation functions and right-hand panels show $b^2(k)$.

To investigate these uncertainties, we adopt the 5-parameter HOD model of Zheng et al. (2005) instead of the 3-parameter model described in § 2.1, so that we do not underestimate HOD uncertainties because of an overly restrictive form. We determine best-fit parameters using this parametrization and the CMBFAST power spectrum. Then we generate a Markov Chain Monte Carlo based on the Metropolis-Hastings algorithm by fitting the $w_p(r_p)$ measurements with covariance matrix. In this parametrization, the mean occupation function for central galaxies is

$$\langle N_{cen}\rangle_M = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\log M - \log M_{\text{min}}}{\sigma_{\log M}} \right) \right],$$

where erf(x) is the error function, which corresponds to a Gaussian scatter of width $\sigma_{\log M}$ in $\log L$ at fixed halo mass (Zheng, Coil & Zehavi 2007). The sharp cut-off of § 2.1 is the limiting case of $\sigma_{\log M} \rightarrow 0$. The distribution of $N_{cen}$ about the mean is a nearest-integer or Bernoulli distribution. The mean occupation function for satellite galaxies is

$$\langle N_{sat}\rangle_M = \left( \frac{M - M_0}{M'} \right)^\alpha,$$

for $M > M_0$, and halos of $M \leq M_0$ are devoid of satellite galaxies, so the slope of the satellite occupation is now free instead of fixed to one as in § 2.1. The number of satellites is Poisson-distributed about $\langle N_{sat}\rangle_M$.

The induced errors for the $M_r \leq -20$ sample are even smaller, by about a factor of two, because of the smaller measurement errors in $w_p(r_p)$. Hence its impact on cosmological parameter constraints is small at $k \leq 0.2\hMpc^{-1}$, and we discuss the
impact as we include measurements at higher $k$ in § 5. Note that even these small uncertainties in $P_{\text{obs}}(k)$ associated with HOD parameter uncertainties can be reduced using $w_p(r_p)$ measurements from larger SDSS samples, and by bringing in additional constraints such as the group multiplicity function. These uncertainties are unlikely to make a major contribution to overall error budget in $P_{\text{lin}}(k)$ recovery.

Our model fitting assumes that the galaxy HOD is independent of large-scale environment. However, recent studies show that the clustering of galaxy mass halos has a substantial dependence on halo formation time as well as halo mass (e.g., Gao et al. 2005; Harker et al. 2005), opening the door to environment-dependent HODs. Croton et al. (2005) use the Millennium simulation of Springel et al. (2005) to investigate the effect of environment-dependent HODs on galaxy clustering in semi-analytic galaxy models and find that environmental effects change galaxy bias factors for luminosity thresholded samples by a few percent, but with virtually no scale-dependence (see also Zhu et al. 2006). Our HOD parameters are insensitive to the large-scale amplitude of $w_p(r_p)$, being driven mainly by shape of $w_p(r_p)$ in a regime where the contributions of galaxy pairs from single halos and two distinct halos are comparable. Therefore, we suspect that environment dependence at the level predicted by Croton et al. (2005) would have few percent impact on the predicted asymptotic galaxy bias factor $b^2_2$ but probably much smaller effect on the scale-dependence of $b^2(0)/b^2_2$. Furthermore, studies of void probabilities (Tinker et al. 2007) and color-separated correlation functions in “scrambled” group catalogs (Blanton et al. 2006) suggest that the Croton et al. (2005) models overpredict the environmental dependence of galaxy HODs. Nevertheless, the environment-dependent issue merits further investigation in future work, as the statistical precision of the measurements themselves sets better.

4.3. Sensitivity of Galaxy Power Spectrum to $P_{\text{lin}}(k)$

In approach to real observations, we will need to simultaneously fit cosmological parameters and HOD parameters using $P_{\text{obs}}(k)$ and $b^2(k)$ as constraints and to marginalize over HOD parameters. The ability to constrain cosmological parameters depends on the measurement errors in $P_{\text{obs}}(k)$ and on the sensitivity of $P_{\text{obs}}(k)$ to $P_{\text{lin}}(k)$. Here we investigate the sensitivity of $P_{\text{obs}}(k)$ to $P_{\text{lin}}(k)$ given the constraints of $w_p(r_p)$. We discuss the impact on cosmological parameter estimation in § 5.

In Figures 7 and 8 we illustrate the changes in $P_{\text{obs}}(k)$ and $b^2(k)$ for cosmological model variations using the $M_r \leq -21$ sample. We adjust HOD parameters to fit $w_p(r_p)$ with each cosmological model shown in the legend. Figure 7 plots $b^2(0)$ for models with $\sigma_8 = 0.8$ and $\sigma_8 = 1.0$. Note that for specified cosmology, fitting $w_p(r_p)$ completely determines both the shape and amplitude of $b^2(k)$. The thick dashed curve shows $b^2(k)$ for the $\sigma_8 = 1.0$ model scaled to have the same large-scale bias factor as the $\sigma_8 = 0.8$ model. The few percent difference in scaled $b^2(k)$ at $k \approx 0.1-0.2 h\text{Mpc}^{-1}$ results from the shape variation in the Smith et al. (2003) $P_{\text{nl}}(k)$ prescription for different $\sigma_8$ values.

Figures 7 and 8 plot two more model sequences with variations in $n_s$ and $h$, which change the shape of $P_{\text{lin}}(k)$ in similar but not identical ways. Roughly speaking, fitting $w_p(r_p)$ fixes the $P_{\text{obs}}(k)$ amplitude at $k \approx 0.5 h\text{Mpc}^{-1}$ for the $M_r \leq -21$ sample. The large-scale bias factors are therefore slightly different for distinct $n_s$ or $h$ values. At $k \leq 0.1 h\text{Mpc}^{-1}$, the shapes of $b^2(k)$ are nearly identical and nearly scale-independent, among the $n_s$ and $h$ model sequences. On smaller scales, the $b^2(k)$ curves separate as HOD parameters adjust to try to produce similar $w_p(r_p)$ from models with different matter power spectra.

Figure 8 plots $P_{\text{lin}}(k)$ and $P_R(k)$ in the upper panels and their...
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els. The T06 main galaxy 0.8 can be almost exactly compensated by a change in the
mental variation, merits further investigation in future work.

ratios relative to the fiducial model power spectra in the bottom panels, for the same model sequences considered in Figure 7. Thus, this figure shows the degree to which a change in the linear power spectrum produces a detectable change in the observable power spectrum once we allow the galaxy HOD to vary in a way that reproduces \( w_p(r_p) \). A reduction in \( \sigma_8 \) to 0.8 can be almost exactly compensated by a change in the HOD — in linear theory, of course, such a change could be exactly compensated by the linear bias factor \( b_\text{lin} \). Linear bias would not change the shape of the observable power spectrum, but we see in Figure 10 that the \( P_R(k) \) curves for different \( n_s \) and \( h \) converge towards a common shape at \( k \geq 0.1 h^{-1}\text{Mpc}^{-1} \), though they resemble \( P_{\text{lin}}(k) \) at larger scales. Thus, nonlinear dynamical evolution and the freedom to introduce scale-dependent bias via HOD variation reduces the discriminatory power of \( P_{\text{obs}}(k) \). However, with the HOD constrained by \( w_p(r_p) \), the amplitude of \( P_R(k) \) is offset by 5% or more for \( 0.1 h^{-1}\text{Mpc} \leq k \leq 0.5 h^{-1}\text{Mpc} \) in the \( n_s = 0.9 \) and \( h = 0.6 \) models. The T06 main galaxy \( P_{\text{obs}}(k) \) measurements have typical uncertainty of about 10% per data point, so this level of offset could provide significant additional sensitivity to cosmological parameters if the measurements are extended to \( k = 0.5 h^{-1}\text{Mpc}^{-1} \). The caveat is that environmental dependence of the HOD (not allowed in our models here) might be able to change the large-scale amplitude of \( P_{\text{obs}}(k) \) relative to \( w_p(r_p) \), erasing the predicted offset. The systematic uncertainty due to this effect could be removed by adding and marginalizing over a multiplicative normalization factor (essentially a “bias offset”) so that only the shape of \( P_{\text{obs}}(k) \) provides constraints. This technique, and the impact of plausible levels of environmental variation, merits further investigation in future work.

5. BIAS AND POWER SPECTRUM OF LUMINOUS RED GALAXIES

While we have focused on the SDSS main galaxy samples so far, the most powerful measurements in galaxy power spectrum come from the luminous red galaxy (LRG) samples (Eisenstein et al. 2001) because of the large effective volume that the LRG samples probe. We consider the bias shape and power spectrum of the LRG samples separately from those of the main galaxy samples for several reasons. First, LRGs are physically distinct from SDSS main galaxies: they are mainly central galaxies, occupying massive halos of \( M \gtrsim 10^{13.5} h^{-1} M_\odot \), and they appear to have much stronger scale-dependent bias (e.g., T06 find higher \( Q \) values). Second, constraints on HOD parameters and systematic uncertainties in their values have not been as extensively investigated; here we draw on parameter constraints from the recent study of Zheng et al. (2008). Third, few simulations with the necessary large volume and dynamic range are available for testing our analytic model predictions for the LRG samples. The tests of our analytic model in this regime that we present here should be regarded as a first step. However, our modeling allows us to understand how the scale-dependence of LRG bias may differ from that of the SDSS main galaxies. More thorough investigation of HOD parameters and their uncertainties is necessary before applying our method to LRG \( P_{\text{obs}}(k) \) measurements to infer cosmological parameters. Here we simply adopt HOD parameters from Zheng et al. (2008) and ignore the impact of uncertainties in the parameters for the moment. Zheng et al. (2008) obtain the HOD parameters of the LRG samples with absolute-magnitude limit \(-23.2 \leq M_g \leq -21.2 \) and \(-23.2 \leq M_r \leq -21.8 \) by matching the projected correlation function \( w_p(r_p) \) and mean space density \( \bar{n}_g = 9.7 \times 10^{-5} (h^{-1}\text{Mpc})^{-3} \) and \( 2.4 \times 10^{-5} (h^{-1}\text{Mpc})^{-3} \), and by accounting for the error covariance matrix taken from Zehavi et al. (2005). (see Appendix B of Zheng et al. 2008 for the HOD parameters). The mean redshift of the LRG samples is \( z = 0.3 \), and the two LRG samples are essentially luminosity-thresholded samples. The \( M_g \leq -21.2 \) sample is dominated by fainter galaxies, so it probes a smaller volume than the
Marginal discrepancy at the 10% level, but it is difficult to assess the statistical significance of the discrepancy with only one simulation. Figure 10 shows the analytic predictions for $P(k)$ and $P_0(k)$, where the statistical uncertainties are computed by using the eight octants as independent measurements. The analytic model provides good approximations to $P(k)$ and $P_0(k)$ at $k < 0.3h$Mpc$^{-1}$. The shot-noise power spectrum dominates the measurements of $P(k)$ and $P_0(k)$ at $k \gtrsim 0.3h$Mpc$^{-1}$, and the discrepancy at $k \gtrsim 0.3h$Mpc$^{-1}$ may indicate that our shot-noise subtraction scheme is imperfect. Nevertheless, more simulations are necessary to better quantify the statistical significance of the deviation at $k \gtrsim 0.3h$Mpc$^{-1}$. The N-body test at $z = 0$ demonstrates that our analytic model can be used to compute the correlation functions and power spectra in the LRG regime with reasonable accuracy, and we suspect that these predictions would remain accurate at $z = 0.3$.

Figures 11 and 12 plot the bias shapes of the LRG samples, computed by using our analytic model and accounting for the mean redshift $z = 0.3$ of the LRG samples. We use a new large N-body simulation (Warren et al. 2006) to test our analytic model predictions for the LRG samples. The simulation is performed with the Hashed Oct-Tree code (Warren & Salmon 1993), evolving $1024^3$ particles in a volume of comoving $1086h^{-1}$Mpc on a side from $z = 34$ to the present, using a $\Lambda$CDM cosmology ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_b = 0.046$, $h = 0.7$, $n_s = 1$, and $\sigma_8 = 0.9$). Dark matter halos are identified using the friends-of-friends algorithm with $b_{fof} = 0.2$. Since we only have the simulation output at $z = 0$, we cannot directly compute quantities of interest at the mean redshift of LRGs ($z = 0.3$), but we can test whether our analytic model is accurate.

Figure 9 plots the real-space and redshift-space monopole correlation functions measured from the N-body simulation at $z = 0$, populated by using the same HOD parameters of the LRG sample obtained by fitting $w_p(r_p)$ measurements at $z = 0.3$. The evolution of cosmic structure from $z = 0.3$ to $z = 0$ increases the abundance of high mass halos, so with these parameters we set higher $n_g$ than observed. Note, however, that we use the simulation just for the purpose of testing our analytic model. The error bars are computed by jackknife resampling of the eight octants of the cube. Our analytic predictions are in good agreement with the populated N-body simulation. The marginally significant discrepancy at $r = 4h^{-1}$Mpc could indicate that our halo exclusion treatment is inaccurate at the 10% level, but it is difficult to assess the statistical significance of the discrepancy with only one simulation. Figure 11 shows the analytic predictions for $P_{\text{z,IG}}(k)$ and $P_{\text{z,GR}}(k)$, where the statistical uncertainties are computed by using the eight octants as independent measurements. The analytic model provides good approximations to $P_{\text{z,IG}}(k)$ and $P_{\text{z,GR}}(k)$ at $k < 0.3h$Mpc$^{-1}$. The shot-noise power spectrum dominates the measurements of $P_{\text{z,IG}}(k)$ and $P_{\text{z,GR}}(k)$ at $k \gtrsim 0.3h$Mpc$^{-1}$, and the discrepancy at $k \gtrsim 0.3h$Mpc$^{-1}$ may indicate that our shot-noise subtraction scheme is imperfect. Nevertheless, more simulations are necessary to better quantify the statistical significance of the deviation at $k \gtrsim 0.3h$Mpc$^{-1}$. The N-body test at $z = 0$ demonstrates that our analytic model can be used to compute the correlation functions and power spectra in the LRG regime with reasonable accuracy, and we suspect that these predictions would remain accurate at $z = 0.3$.
Fig. 12.—Bias shapes $P_R(k)$, $P_{Z-R}(k)$, and $P_0(k)$ of the luminous red galaxy (LRG) sample with $-23.2 \leq M_g \leq -21.8$, in the same format as Fig. 11. The dotted curves are computed with $Q = 15$. The solid curves plot the $Q$-model prescriptions with $Q = 20$ and $15 \,(h^{-1}{\text{Mpc}})^2$ for the faint and bright samples, which would closely follow the nonlinear matter power spectrum (circles). This is mainly because the fraction of satellite galaxies is small in the LRG samples, approximately 5%, and because their host halos are massive ($>10^{13}h^{-1}M_\odot$) and hence highly clustered. The bright LRG sample is more biased by 15% compared to the faint LRG sample, but it shows less scale-dependence at $k \lesssim 0.1h{\text{Mpc}}^{-1}$. The low fraction of satellite galaxies also suppresses the redshift-space multipoles arising from virial motions of satellite galaxies in halos, and the $P_{Z-R}(k)$ and $P_0(k)$ shapes have little difference compared to the $P_R(k)$ shape. In the bottom panels of Figures 11 and 12, relatively small changes in the bias shapes arise between two thresholds of FoG compression, because both cases basically suppress all the halos that host two or more LRGs.

The dotted curves plot the $Q$-model prescriptions with $Q = 20$ and 15 $(h^{-1}{\text{Mpc}})^2$ for the faint and bright samples, which approximately follow our calculations. These values are smaller than the best-fit value $Q = 30.3 \,(h^{-1}{\text{Mpc}})^2$ found by T06. Because of the relative statistical weights of the $P(k)$ multipoles, the quantity $P_{\text{gg}}(k)$ that they measure is intermediate between our $P_{Z-R}(k)$ and $P_0(k)$, but closer to the latter (see the discussion in § 3 above). The scale-dependence of LRG bias that we predict from HOD modeling is therefore weaker than that inferred by T06 by fitting the power spectrum. The discrepancy may reflect the difference between the “defogging” procedure used by T06 and the perfect FoG compression assumed here, though since we find that FoG compression has little impact this explanation would imply that the T06 method overcorrects FoGs. (More direct evidence for such overcorrection is presented by B. Reid et al. [in preparation]). It should also be emphasized that no values of $Q$ can fit our calculations with 5% accuracy up to $k = 0.2h{\text{Mpc}}^{-1}$. However, we do confirm a basic result of T06 model fitting: scale-dependence of LRG bias becomes substantial at $k > 0.1h{\text{Mpc}}^{-1}$, and it is much stronger than that of less luminous, main sample galaxies.

Proper application of our method to observations will require several key steps beyond the scope of our current investigation. First and foremost is careful matching of galaxy samples analyzed for $P_{\text{obs}}(k)$ measurements and HOD modeling. T06 use “flux-limited” LRG and main galaxy catalogs from SDSS DR4 to obtain $P_{\text{obs}}(k)$ measurements and constraints on the scale-dependent bias, while we use Zheng et al.’s (2008) “volume-limited” galaxy samples to constrain HOD parameters and predict $P_{\text{obs}}(k)$. Second, as mentioned earlier, we need more large-volume simulations at $z > 0$ for
testing the accuracy of our analytic model in a regime adequate for LRG clustering. Third, we need to more fully investigate HOD uncertainties associated with analytic model fitting of $w_p(r_p)$ and with possible environmental dependence of the LRG HOD.

With these caveats in mind, we present preliminary results on comparison of our analytic model predictions to the T06 measurements of the LRG power spectrum in Figure 13. Since it is hard to correctly account for the difference in our FoG compression and their defogging processes, we instead use the $P_{\text{obs}}(k)$ measurements without defogging (see Fig. 22 in T06 for the no-defog data). Points with error bars show the SDSS measurements.

In Figure 13 the shaded gray bands show $P_{\text{gg}}(k) \approx 0.8P_0(k) - 0.07P_2(k) + 0.006P_3(k)$ for the two LRG samples, constructed by using our analytic model predictions of $P_0(k)$, $P_2(k)$, and $P_3(k)$, with the lower bound computed from the faint sample and the upper bound from the bright sample (see the Appendix of T06 for the empirical formula for $P_{\text{gg}}(k)$). Considering the difference in the analyzed galaxy samples, the shaded region is likely to encompass the true $P_{\text{obs}}(k)$, and it is indeed in good agreement with the measurements. Note that our analytic model prediction allows no freedom in the asymptotic bias $b_0$, and hence in the normalization of $P_{\text{gg}}(k)$, once HOD parameters are pinned down by fitting $w_p(r_p)$.

This is in contrast to the linear or $Q$-model fit, in which $b_0$ can be arbitrarily adjusted to fit the data. The solid and dashed lines show $b_0^2P_{\text{lin}}(k)$ of the fiducial cosmological model and the $Q$-model prescription with $Q = 30.3 \left( h^{-1}\text{Mpc} \right)^2$ quoted in T06, which best fits their “defogged” measurements. At $k < 0.09 h^{-1}\text{Mpc}^{-1}$, all predictions are consistent with the linear model (solid) at roughly the 10% level, justifying the applicability of the linear model with the same accuracy. However, significant scale-dependence of LRG bias prevents the use of the linear model at $k \geq 0.1 h^{-1}\text{Mpc}^{-1}$. The vertical dotted lines show the nominal nonlinear scale quoted by T06; they fit $Q$-models up to $k = 0.2 h^{-1}\text{Mpc}^{-1}$ but note that data beyond $k = 0.1 h^{-1}\text{Mpc}^{-1}$ provided little leverage on cosmological parameters and mainly constrained $Q$.

We further investigate this point by considering variations of the fiducial cosmological parameters in Figure 14. For each cosmological model variation, we first re-fit $w_p(r_p)$ to obtain best-fit HOD parameters and then compute $P_{\text{gg}}(k)$. The solid lines are computed by averaging $P_{\text{gg}}(k)$ of the two LRG samples with weight by their number density. Figure 14 shows a sequence of $P_{\text{gg}}(k)$ with varying $\sigma_8$ values. The predictions are virtually identical, showing that the combination of $P_{\text{gg}}(k)$ and $w_p(r_p)$ has no direct constraining power on $\sigma_8$ given the flexibility of our 5-parameter HOD model. Since the predictions are obtained by applying our analytic model of $P_{\text{gg}}(k)$ to Zheng et al.’s (2008) HOD fits to $w_p(r_p)$, both of which depend on $\sigma_8$ in a complex way, this constancy of $P_{\text{gg}}(k)$ is a reassuring consistency check.

In Figures 13 and 14, we consider sequences with varying $n_s$ and $h$, respectively. Once we have fit $w_p(r_p)$ for a specific cosmology, there is no freedom to adjust the shape or amplitude of the model $P_{\text{gg}}(k)$, except within the observational uncertainties on the HOD parameters. We see from Figures 14 and 14 that normalizing to $w_p(r_p)$ effectively forces different models to agree at $k \approx 0.08 h^{-1}\text{Mpc}^{-1}$. The largest separation among the models comes at large scales, $k \lesssim 0.02 h^{-1}\text{Mpc}^{-1}$. However, the models also differentiate noticeably on smaller scales, especially when $n_s$ is varied. In linear theory fits, these scales would be discarded because the model predictions are unreliable. In Q-model fits, the information they contain would largely be lost by marginalizing over $Q$. The varying $n_s$ models also predict different $w_p(r_p)$ for $r_p > 20 h^{-1}\text{Mpc}$, but these scales carry little weight in the HOD fitting.

These results suggest that fitting $P_{\text{obs}}(k)$ to small scales with HOD parameter constraints from $w_p(r_p)$ could yield significantly improved constraints on cosmological parameters. To investigate this point, and to compare linear theory, $Q$-model, and full HOD-model fitting, we generate synthetic $P_{\text{obs}}(k)$ measurements equal to the predictions of our analytic model for the fiducial cosmological parameters and the corresponding Zheng et al.’s (2008) HOD parameters. We assign observational errors that are a factor of two smaller than those found by T06 at the same value of $k$, roughly approximating the improvement that might come from the future full SDSS data set. At $k > 0.2 h^{-1}\text{Mpc}^{-1}$ (where T06 did not present measurements), they fit $Q$-models up to $k = 0.2 h^{-1}\text{Mpc}^{-1}$ but note that data beyond $k = 0.1 h^{-1}\text{Mpc}^{-1}$ provided little leverage on cosmological parameters and mainly constrained $Q$.
measurements), we assign fractional uncertainties equal to those at \( k = 0.2 \) hMpc\(^{-1}\).

Figure 15 shows the synthetic measurement and its errors with our analytic model prediction (dotted) in the left panels and the cosmological parameter constraints inferred by fitting these data using the three different models in the right panels. For simplicity, we fixed the combinations \( \Omega_m h^2 = 0.1272 \) and \( \Omega_b h^2 = 0.0222 \), which are best constrained from cosmic microwave background measurements, and we varied \( n_s \) and \( h \) as two free parameters, assuming a flat \( \Lambda \) CDM universe. In Figure 15, the solid and dashed lines show the best-fit linear model and \( Q \)-model using the measurements only at \( k \leq k_{\text{max}} = 0.09h\) Mpc\(^{-1}\). We marginalize over \( b_0 \) for linear theory and \((b_0, Q)\) for \( Q \)-model fitting. Both models can describe the data reasonably well (the dashed line is largely obscured by the solid line). The contours and shaded ellipses in Figure 15 show the 1-\( \sigma \) confidence levels (\( \Delta \chi^2 = 2.3 \)) for all three models, and the symbols represent the best-fit parameters. To compute the parameter constraints of the HOD model, we use the Fisher matrix formalism and marginalize over a cosmological parameter \( \sigma_8 \) and two sets of five HOD parameters for each LRG sample. Note that analytic model fitting of \( w_p(r_p) \) assumes a \( \sigma_8 \) value, which is largely degenerate in \( P_{\text{obs}}(k) \) measurements.

The HOD model (shaded region) provides tighter constraints than linear theory (dashed contour), because we have additional information from \( w_p(r_p) \) measurements. The best-fit parameters (circle) of the linear theory fit are only marginally consistent at 1-\( \sigma \) level with the true values (asterisk), because the linear bias approximation is no longer an accurate description of LRG bias even at \( k \leq 0.09h\) Mpc\(^{-1}\), given the small uncertainties in \( P_{\text{obs}}(k) \) assumed here. The \( Q \)-model prescription (solid contour) yields relatively less biased best-fit parameters (triangle). However, marginalizing over \( Q \) compromises statistical constraining power, and this fit has the largest uncertainties in parameter estimates among the three models.

The bottom panels compare the three models with \( k_{\text{max}} = 0.2 \) and \( 0.4h\) Mpc\(^{-1}\), demonstrating the utility of higher \( k \) measurements. The best-fit \( P_{\text{obs}}(k) \) (dashed line) of linear theory in Figure 15 shows large deviations, and the best-fit parameters (circles) in Figure 15 are significantly different from the true values (out of range \( n_s > 1.1 \) for \( k_{\text{max}} = 0.4h\) Mpc\(^{-1}\)). The linear theory fit prefers higher \( n_s \) to compensate the deficit in \( P_{\text{obs}}(k) \) on small scales, while the overall \( P_{\text{obs}}(k) \) shape and baryon wiggles keep the \( h \) value relatively unchanged. Clearly, linear theory is an invalid description of \( P_{\text{obs}}(k) \) for LRGs when \( k_{\text{max}} > 0.09h\) Mpc\(^{-1}\).

A similar but less extreme trend of bias in best-fit parameters is found when the \( Q \)-model prescription is applied to \( P_{\text{obs}}(k) \) with \( k_{\text{max}} \geq 0.2h\) Mpc\(^{-1}\). The best-fit parameters of the \( Q \)-model fit are \((1.0, 0.74)\), deviating from the true value and from the best-fit values with \( k_{\text{max}} = 0.09h\) Mpc\(^{-1}\) at the 1-\( \sigma \) level. Higher \( n_s \) is again favored, but a higher \( Q \) can partially balance the trend of higher \( n_s \). With \( k_{\text{max}} = 0.4h\) Mpc\(^{-1}\), the best-fit parameters are \((1.09, 0.74)\) and biased at the 3-\( \sigma \) level. The \( Q \)-model prescription also develops substantial bias in best-fit parameters as \( k_{\text{max}} \) increases. In contrast, while our HOD prediction is accurate by construction in this experiment, it makes full use of the variations of \( P_{\text{obs}}(k) \) on small scales, providing tighter constraints on cosmological parameters as small scale measurements are included.

Since we have generated the synthetic data from the HOD model, we have adopted the most optimistic scenario in which the HOD model with correct parameters can give a perfect description of the data. Nevertheless, Figure 15 conveys two key points. First, \( Q \)-model fitting to the LRG \( P_{\text{obs}}(k) \) may yield unreliable parameter estimates for \( k_{\text{max}} = 0.2h\) Mpc\(^{-1}\) (and linear theory may be problematic even at \( k_{\text{max}} = 0.09h\) Mpc\(^{-1}\)). Second, increasing \( k_{\text{max}} \) from \( 0.09h\) Mpc\(^{-1}\) to \( 0.2h\) Mpc\(^{-1}\) with \( HOD \) model fitting can significantly reduce statistical uncertainties in cosmological parameters, and increasing to \( 0.4h\) Mpc\(^{-1}\) produces a modest further gain.

6. CONCLUSIONS

We have developed an analytic model to predict observable galaxy power spectra \( P_{\text{obs}}(k) \) for specified cosmological and galaxy HOD parameters, and we have verified its accuracy using \( N \)-body simulations. As potentially observable power spectra \( P_{\text{obs}}(k) \), we have considered the real-space \( P_{\text{g}}(k) \), the redshift-space monopole \( P_{\text{g}}(k) \), and the pseudo real-space \( P_{\text{g}}(k) \), with varying levels of Finger-of-God (FoG) compression for the latter two. Once HOD parameters are determined by fitting the number density \( n_s \) and projected correlation function \( w_p(r_p) \) of the observed SDSS galaxy samples, given a specified cosmological model, our analytic model can
be used to predict $P_{\text{obs}}(k)$ measurements.

The large-scale normalization of our predictions is also fixed in the process of fitting $w_p(r_p)$, providing a unique prediction for each combination of cosmological and HOD parameters. In practice, one can simultaneously fit cosmological and HOD parameters using $P_{\text{obs}}(k)$ and $w_p(r_p)$ as constraints, then marginalize over the HOD in deriving cosmological parameters. By implementing a complete physical model of nonlinear galaxy bias and drawing on the additional information in $w_p(r_p)$, our method allows one to take full advantage of precision measurements of $P_{\text{obs}}(k)$ on quasi-linear scales ($k = 0.1 - 0.4h$ Mpc$^{-1}$), where linear theory or the phenomenological $Q$-model may be insufficiently accurate. Our main findings are as follows:

1. Our analytic model for calculating $w_p(r_p)$ follows the method described in Tinker et al. (2006), with the improved treatment of the scale-dependent halo bias and ellipsoidal halo exclusion corrections. Drawing on the Tinker (2007) model for redshift-space distortion, the analytic model is extended to incorporate calculating real-space and redshift-space power spectra. We have tested its predictions for $w_p(r_p)$ and $P_{\text{obs}}(k)$ against populated $N$-body simulations spanning cosmological parameter range $\Omega_m = 0.1 - 0.63$ and $\sigma_8 = 0.6 - 0.95$, with HOD parameters matched to represent two SDSS galaxy samples with absolute-flux limits $M_r \leq -20$ and $M_r \leq -21$ (Zehavi et al. 2005b). The analytic model reproduces the numerical results of $w_p(r_p)$ to 5% or better, and the predictions of $P_{\text{obs}}(k)$ are consistent with the numerical results to 2% at $k = 0.1 - 1h$ Mpc$^{-1}$ and to 10% at $k = 0.025 - 0.1h$ Mpc$^{-1}$, though the finite box size of the simulations makes it difficult to assess the statistical significance of differences on large scales.

2. For the $M_r \leq -20$ galaxy sample, the pseudo real-space power spectrum $P_{Z-R}(k)$ recovers the true $P_k(k)$ to 2% at $k \leq 0.2h$ Mpc$^{-1}$, while the deviation between $P_k(k)$ and the scaled monopole $P_0(k)$ is already 10% at $k = 0.1h$ Mpc$^{-1}$. However, the deviation of $P_{Z-R}(k)$ from $P_k(k)$ becomes substantial at $k \geq 0.3h$ Mpc$^{-1}$. This deviation can be partly remedied by FoG compression, which suppresses nonlinear behavior of the redshift-space multipoles caused by the random motions of satellite galaxies within halos. With FoG compression threshold $\sigma_h = 750 \text{ km s}^{-1}$, $P_{Z-R}(k)$ can recover $P_k(k)$ to 5% at $k \leq 0.45h$ Mpc$^{-1}$, and at higher $k$ for $P_{Z-R}(k)$. FoG compression also reduces nonlinearity of the monopole power spectrum, but $P_{\text{1d}}^0(k)$ can only achieve 10% accuracy at $k \leq 0.3h$ Mpc$^{-1}$. We conclude that the pseudo real-space method of Tegmark et al. (1998) is an effective tool for recovering the nonlinear real-space galaxy power spectrum from redshift-space measurements, especially if it is combined with
accurate FoG compression.

3. The nonlinear matter power spectrum describes the nonlinear real-space galaxy power spectra to 1% at \( k \lesssim 0.2h \text{Mpc}^{-1} \) for the \( M_\ast \leq -20 \) and \( M_\ast \leq -21 \) galaxy samples, up to an overall bias factor \( b_0^2 \). The shape of the scale-dependent bias function \( b^2(k)/b_0^2 \) for \( P_{\text{z} \rightarrow \text{r}}(k) \) is qualitatively similar to \( P_\rho(k) \) at \( k \lesssim 0.3h \text{Mpc}^{-1} \), but the shape for \( P_\rho(k) \) is completely different over the entire range we consider here. FoG compression makes little difference to \( b^2(k)/b_0^2 \) for \( P_{\text{z} \rightarrow \text{r}}(k) \), but a large difference for \( P_\rho(k) \). For these SDSS main galaxy samples, the \( Q \)-model prescription traces our calculation of \( P_\rho(k) \) relatively well at \( k \gtrsim 0.1h \text{Mpc}^{-1} \), but its shape on large scales differs, so it might induce some overall bias in cosmological parameters when fitted to \( P_{\text{obs}}(k) \) measurements that have large uncertainties at \( k \lesssim 0.05h \text{Mpc}^{-1} \). Similar trends but with larger discrepancy are found in comparison to our \( P_{\text{z} \rightarrow \text{r}}(k) \) and \( P_\rho(k) \) calculations.

4. Uncertainties in computing \( P_{\text{obs}}(k) \) in our method arise from observational uncertainties in the HOD parameters and from uncertainty in the adopted parametrization itself. We have examined these uncertainties by adopting a flexible HOD parametrization with freedom to explore a wider range of plausible halo occupation functions. For the \( M_\ast \leq -20 \) sample with the \( \Xi_{\text{zhavi}}(2005b) \) uncertainties in \( w_p(r_p) \), the uncertainty in the predicted \( P_{\text{obs}}(k) \) is 2% at \( k = 0.2h \text{Mpc}^{-1} \), becomes progressively smaller at lower \( k \), and climbs up to 4% at \( k = 0.5h \text{Mpc}^{-1} \). The uncertainty is a factor of two smaller for the \( M_\ast \leq -21 \) sample, roughly the ratio of the fractional \( w_p(r_p) \) measurements of the two samples. We have not investigated the uncertainties associated with possible environmental variations of the HOD (Croton et al. 2006). Based on work to date, we expect that such variations might lead to few percent uncertainties in the overall normalization predicted for \( P_{\text{obs}}(k) \) after fitting \( w_p(r_p) \), but that the impact on scale-dependence of \( b^2(k)/b_0^2 \) would be smaller.

5. Moving to the LRG regime, we have tested our analytic model predictions against the \( z = 0 \) output of a large volume, \( 1024^3 \)-particle N-body simulation (Warren et al. 2006), populated based on Zheng et al.’s (2008) HOD fits to \( w_p(r_p) \) for two volume limited SDSS LRG samples (Zehavi et al. 2005b). The analytic model predicts \( \xi_\theta(r) \) and \( \xi_p(r) \) to 5% or better over the range 0.1-4h\text{Mpc} \leq r \lesssim 30h^{-1}\text{Mpc} \, \text{and the predictions for } P_\rho(k) \text{ and } P_\rho(k) \text{ have similar accuracy over the range } 0.01h\text{Mpc}^{-1} \leq k \lesssim 0.3h\text{Mpc}^{-1} \.

6. For the LRG samples, the linear (scale-independent) bias approximation remains accurate at the 5% level to \( k \leq 0.08h \text{Mpc}^{-1} \) for the \( -23.2 \leq M_g \leq -21.2 \) sample and to \( k \leq 0.1h \text{Mpc}^{-1} \) for the \( -23.2 \leq M_g \leq -21.8 \) sample. There is little variation among \( P_{\text{z} \rightarrow \text{r}}(k) \), \( P_\rho(k) \), and \( P_\rho(k) \), because LRGs are mainly central galaxies in massive halos, so random motions of satellite galaxies have little impact. Similarly, FoG compression has only a small impact on \( b^2(k)/b_0^2 \) for these samples. Both samples show strong scale-dependence of bias at \( k \gtrsim 0.1h \text{Mpc}^{-1} \), much more than for main sample galaxies.\(^{15} \) If we fit \( b^2(k)/b_0^2 \) from our HOD models with the best \( Q \)-model over the range \( k \approx 0.01 - 0.2h \text{Mpc}^{-1} \), the largest deviation is 7%.

7. We have presented a preliminary comparison of our analytic model predictions to the T06 measurements of the LRG \( P_{\text{obs}}(k) \), with no FoG compression. The difference between our volume-limited samples and the T06 flux-limited sample precludes a full quantitative assessment, but the qualitative agreement is remarkably good over the full range of the measurements, \( k = 0.01 - 0.2h \text{Mpc}^{-1} \) (Fig. 13). Fits with different cosmological parameters differ on large scales and, to a smaller degree, at \( k \gtrsim 0.2h \text{Mpc}^{-1} \), indicating that measurements to smaller scales would provide additional discriminatory power.

8. Looking to the future, we have generated synthetic \( P_{\text{obs}}(k) \) data from our analytic model with error bars half those of T06, then fit them to successively higher \( k_{\text{max}} \) with linear theory, the \( Q \)-model, and the HOD model. Cosmological parameters from linear theory fits are badly biased for \( k_{\text{max}} \gtrsim 0.1h \text{Mpc}^{-1} \), while for \( k_{\text{max}} = 0.09h \text{Mpc}^{-1} \) they are biased at less than 1σ. Parameters from the \( Q \)-model are minimally biased for \( k_{\text{max}} = 0.09h \text{Mpc}^{-1} \), biased by 1.2σ for \( k_{\text{max}} = 0.2h \text{Mpc}^{-1} \), and biased by many-σ for \( k_{\text{max}} = 0.4h \text{Mpc}^{-1} \). Since the synthetic data are generated from the HOD model, the HOD parameter estimates are unbiased, and the error bars in cosmological parameters shrink steadily as \( k_{\text{max}} \) is increased from 0.1 to 0.2 to 0.4h\text{Mpc}^{-1}.

Results 7 and 8 are especially encouraging. Using only the HOD model and the information in \( w_p(r_p) \), our method predicts exactly the scale-dependent bias for LRGs that is required to transform the linear power spectrum from WMAP3 into the SDSS galaxy power spectrum measured by T06. This is in contrast to a\( Q \)-model fitting, where a phenomenological parameter (motivated by simulation results but with no clear physical interpretation) is introduced specifically to account for the difference between the linear theory \( P(k) \) and the observed power spectrum.

Despite the clear evidence that scale-dependent bias affects the LRG power spectrum beyond \( k = 0.1h \text{Mpc}^{-1} \), result 8 shows that one can gain substantial additional leverage on cosmological parameters with HOD modeling of power spectrum measurements up to \( k = 0.2 - 0.4h \text{Mpc}^{-1} \), and possibly beyond. Realizing this opportunity will require several investigations beyond those presented here. First, we will need more large volume simulations to test and, if necessary, refine our analytic model to the level of accuracy demanded by the final SDSS data set. Second, we must explore more thoroughly the uncertainties associated with the HOD fitting, including alternative parametrizations, the impact of velocity bias on redshift-space predictions, and the possible impact of environmental variations of the HOD. Given the growth of current and future galaxy surveys in depth and redshift, these investigations will be needed to go beyond linear theory. Precise measurement of the primordial matter power spectrum will play a crucial role in constraining cosmological parameters and testing dark energy theories.

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15 This result, obtained by fitting HODs and computing \( P_{\text{obs}}(k) \) with our analytic model, confirms the result of T06 inferred by fitting \( P_{\text{obs}}(k) \) with \( Q \)-models. However, the actual scale-dependence we find for LRGs is somewhat weaker than that inferred by T06.
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