Measured User Correlation in Outdoor-to-Indoor Massive MIMO Scenarios

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ABSTRACT The performance of a multi-user multiple-input multiple-output (MIMO) system is mainly determined by the correlation of user channels. Applying channel models without spatial correlation or spatially inconsistent channel models for performance analysis leads to an under-estimation of spatial correlation and therefore to overly optimistic system performance. This is especially pronounced when the wireless channel is modeled as i.i.d. Rayleigh fading for users in a rich scattering environment, for example, for indoor users. To analyze the spatial channel correlation in massive MIMO systems, we performed wireless channel measurements in three different scenarios. In each scenario, we measure 148 receiver positions, spread over a length of almost 9 m. Since the wireless channel statistics change with receiver position, we perform a stationarity analysis. We provide a statistical analysis of the measured channel in terms of amplitude distribution and user-side spatial correlation within the region of stationarity. This analysis shows that, even for a deep-indoor user location, the spatial correlation is significantly higher compared to a Rician channel model with the same $K$ factor. We model the measured channel by means of a Rician channel model and a spatially consistent channel model, which is based on the scenario geometry, to provide an insight for the observed propagation phenomena. Results show that the user correlation is not negligible for massive MIMO in outdoor-to-indoor scenarios. The achievable spectral efficiency with linear precoding is 20% lower compared to the Rician channel model, even for large inter-user distances of 6 m.

INDEX TERMS MIMO, mobile communications, channel measurements, channel modeling.

I. INTRODUCTION

The challenge to meet all requirements of envisioned future mobile communications networks is manifold. Typical use cases for 5th generation (5G) mobile communications and beyond are: ultra-reliable low-latency communication (uRLLC) for mission critical applications such as vehicular-to-vehicular (V2V) communication, enhanced mobile broadband (eMBB) to meet the steadily increasing demand for data volume and data rate, and massive machine type communication (mMTC) for the implementation of the Internet of things (IoT). A key technology for tackling these diverse challenges is enhancing the well-established technology of MIMO communication to a large-scale, which is then referred to as massive MIMO [1], [2]. As the number of antennas at the base station (BS) side is increased and many users are served in a multi-user MIMO way, the sum rate and the energy efficiency are promised to increase simultaneously [3]–[5]. Since a large-scale antenna array at the BS leads to channel hardening in a wireless fading channel, the reliability and latency are also enhanced when scaling up a MIMO system with respect to the number of BS antennas [6].

Many of the theoretical works reporting the aforementioned desired improvements assume wireless channels following an i.i.d. Rayleigh fading behavior [1], [3], [6]. Especially for indoor mobile users, an uncorrelated i.i.d. channel model...
is justified with the argument of a dense scattering environment around the users. Since massive MIMO is a multi-user technology, the spatial channel correlation of simultaneously served users is an important factor for the performance of such systems [7]. The assumption of uncorrelated user channels is an optimistic one that leads to an over-estimation of system performance as spatial correlation hampers spatial multiplexing of users in close proximity. Measurements show that real-world wireless MIMO channels differ from i.i.d. Rayleigh fading channels in terms of spatial correlation [8]. Several works investigate the problem of spatial separation in MIMO communications [9]–[15], showing that users can be separated in line of sight (LOS) channel conditions.

The importance of spatial inter-user channel correlation has led to research in the area of spatially consistent wireless channel models [15]–[20]. Spatial consistency in this context refers to the fact that two users experience correlated channels when they are physically close to each other. These channel models provide a representation of spatially correlated channels that is more realistic than i.i.d. Rayleigh fading channel models and, therefore, facilitate more accurate performance analysis of massive MIMO systems.

To investigate propagation characteristics of real-world wireless channels in the context of massive MIMO, many measurements were conducted in the past years, see [8], [12], [21]–[28] to name just a few examples. Many of these measurement campaigns were performed entirely outdoors [8], [12], [21]–[24], or indoors [12], [25]–[27], while only few consider an outdoor-to-indoor scenario, for example [28].

**CONTRIBUTION**

We sounded the wireless channel of one outdoor-to-outdoor and two outdoor-to-indoor, large-scale MIMO scenarios at 2.5 GHz. Measuring the channel at 148 user positions per scenario, located on straight lines of 8.82 m length, allows us to investigate the spatial channel correlation depending on inter-user distance. We perform a stationarity analysis in the spatial domain and characterize the regions of wide-sense stationarity in terms of fading distribution and spatial correlation. We also provide a fit of a spatially consistent channel model, which is based on the measurement scenario geometry, to explain the observed propagation effects. We show that the achievable spectral efficiency of two users in an outdoor-to-indoor massive MIMO system is 20% lower compared to a Rician channel model with the same $K$ factor, even at large inter-user distances of 6 m.

**NOTATION**

We denote vectors by lowercase boldface letters, such as $\mathbf{x}$, and matrices by uppercase boldface letters, such as $\mathbf{X}$. The entry from the $n^{th}$ row and the $k^{th}$ column of matrix $\mathbf{X}$ is denoted by $X_{n,k}$. We denote the Frobenius norm by $\|\cdot\|_F$. The absolute value of a scalar as well as the cardinality of a set are denoted by $|\cdot|$. The transpose of a vector or matrix is denoted by $(\cdot)^T$ while the conjugate transpose of a vector or matrix is denoted by $(\cdot)^H$. The expectation of a random variable $X$ is denoted by $\mathbb{E}\{X\}$ and the trace of a matrix is denoted by $\text{tr}\,(\cdot)$. The operator $\lfloor \cdot \rfloor$ rounds down to the nearest integer and the operator $\lceil \cdot \rceil$ rounds up to the nearest integer.

**II. MEASUREMENT CAMPAIGN**

A wireless channel measurement campaign with three different measured scenarios was conducted in January 2020 at TU Wien in downtown Vienna, Austria. The scenarios measured will be described in detail later in this section. Channel soundings are performed with the Vienna MIMO testbed [30], [31] at a center frequency of 2.5 GHz (free space wavelength $\lambda \approx 120$ mm). The transmit side antenna array is located outdoors on a rooftop and is oriented parallel to the building’s facade, facing the receiver location, see Fig. 1. The distance between the transmit antenna array and the array at the receive side is $R \approx 155$ m.

**FIGURE 1.** Aerial photograph of the measurement scenario at TU Wien [29]. The red lines indicate the orientation of the antenna arrays. The coordinate system orientation is indicated in blue.

To obtain measurements of a massive MIMO channel, a uniform linear array with 40 elements at half-wavelength spacing is employed at the transmitter side. Since the Vienna MIMO Testbed is limited to four radio frequency (RF) chains, the transmit side array is implemented as a virtual antenna array. This means that a single antenna element is sequentially re-positioned in order to virtually assemble an antenna array, given that the wireless channel and the measurement hardware behave time-invariant [32]. The antenna element at the transmit array is a vertical dipole in front of the center of a 75 cm $\times$ 75 cm square aluminum reflector plate, see Fig. 2(a). We sound the wireless channel at 148 receiver positions per measurement scenario to investigate the spatial correlation as a function of inter-user distance. The receive antenna is mounted on a mechanical guide with a total length of 9 m, see Fig. 2(b). These receiver positions are arranged on a line with half a wavelength in between measurement points, resulting in a total user range of 8.82 m. As receive antenna element, an omni-directional monopole antenna with a circular groundplane with a diameter of approximately 2.5 $\lambda$ is employed.

An orthogonal frequency division multiplexing (OFDM) signal with a subcarrier spacing of $F = 15$ kHz is employed as channel sounding sequence. To keep the peak-to-average power ratio of the sounding sequence low, the transmit signal...
is designed using Newman phases, as in [33]. A transmit symbol is given by

$$x(m) = \text{Re} \left( \sum_{q=-Q/2}^{Q/2-1} e^{-i2\pi q M/2} e^{i\pi q^2/2} \right),$$  \hspace{1cm} (1)$$

where $i$ is the imaginary unit, $Q$ is the number of subcarriers, $m = 0, \ldots, M - 1$ denotes the time index and $q$ is the subcarrier index. Here, $M$ is the number of samples per OFDM symbol.

A sounding sequence consists of 101 repetitions of the described OFDM symbol. The first symbol is exploited as cyclic prefix (CP) and is discarded at the receiver. Given the scenario geometry and the CP duration, this leads to inter-symbol interference free transmission [34]. Assuming the channel to be time-invariant, we average over the remaining 100 received symbols in order to achieve an averaging gain of 20 dB in terms of signal to noise ratio (SNR).

To measure the phase drift between the transmitter clock and the receiver clock, an additional reference radio link is established. This reference link employs directional antennas at fixed positions on both ends to obtain a wireless channel that is flat in time and frequency. More details of the measurement system’s phase stability are provided in Appendix A.

We employ $Q = 10$ subcarriers for the wireless channel of interest and $Q = 20$ subcarriers for the reference link. Channel estimates for both wireless links are obtained via least squares estimation. The phases of the channel estimates obtained via the reference link are averaged over all 20 subcarriers and employed to compensate the phase drift of the channel estimates of the desired wireless channel. The phase compensated channel estimates of the desired radio link are averaged over all 10 subcarriers since the channel’s coherence bandwidth is much larger than 150 kHz. This improves the channel estimation error by 10 dB.

In total, we achieve 20 dB SNR at the receiver with the Vienna MIMO Testbed, additional 20 dB SNR through averaging in time and another 10 dB improvement in channel estimation error through averaging in frequency. This yields a total normalized mean squared channel estimation error of approximately $-50$ dB.

### A. LOS SCENARIO

In this scenario, the transmitter and the receiver are located on rooftops with no obstacles or blockages in between them. This scenario is therefore considered as a LOS propagation scenario. A photograph of the receive antenna on the building’s rooftop and the mechanical linear guide for antenna positioning is provided in Fig. 2(b). Buildings in between the transmitter and receiver location, see Fig. 1, are lower in height compared to the rooftops at transmit and receive sites. The first Fresnel zone, with a maximum diameter of $D = \sqrt{\frac{R\lambda}{2}} \approx 2$ m, is undisturbed. The receive antenna height above the rooftop is 1.7 m. At both, the transmitter and the receiver, the antennas are located sufficiently high above the handrails to obtain an unobstructed connection. Let us assume a coordinate system with its origin located at the first antenna position of the transmit side antenna array as shown in Fig. 1. In this coordinate system, the first user position is located at $(x, y, z) = (151 \text{ m}, 31 \text{ m}, 0 \text{ m})$ for this scenario.

### B. LABORATORY SCENARIO

This scenario is an outdoor-to-indoor scenario with the receiver located in an indoor laboratory environment. A photograph of the receive antenna in the laboratory is shown in Fig. 3(a). The receiver positions in terms of a floorplan...
of the 5th floor (two floors below the roof) of the receiver site building are shown in Fig. 3(b). In this scenario, the receiver positions are parallel to a wall, which is facing the transmitter array. The receive antenna height above the floor is 1 m, while the lower end of the windows is 90 cm above the floor. All window shades were in a fully opened position during the measurement. Therefore, there is a visual LOS path between the transmitter and the receiver for receive antenna positions directly in front of a window, obstructed only by two layers of uncoated glass. The first user position is at \((x, y, z) = (152 \text{ m}, 25 \text{ m}, -8 \text{ m})\).

C. HALLWAY SCENARIO

In this scenario, the receiver is located indoors in an empty hallway. The receiver positions are shown in Fig. 3(b) in terms of a floorplan. Looking at this floorplan, the hallway scenario is “deep indoor” compared to the laboratory scenario. A photograph of the receiver environment is shown in Fig. 3(c). All doors were closed during the measurement. The receive antenna height above the floor is 1.3 m. The first user position is at \((x, y, z) = (157 \text{ m}, 31 \text{ m}, -8 \text{ m})\).

III. STATISTICAL ANALYSIS

We perform a stationarity analysis and identify regions of spatial quasi-stationarity in the sense of wide-sense stationarity (WSS) in this section. To identify these regions, we employ the channel matrix collinearity metric [35]. We analyze the channel statistics in terms of amplitude distribution, spatial correlation and time correlation within the stationary regions. We illustrate the spatial and the temporal channel correlation within the stationary regions by means of the channel vector inner product.

A. STATIONARITY ANALYSIS

Previous massive MIMO channel measurements show that the wireless channel is not stationary in the spatial domain across the entire aperture of a large scale antenna array [21], [25], [26]. Due to the large physical size of such an array, not all antenna elements see the same scattering elements or even users. Such non-stationary effects are not captured by most current MIMO channel models, but are important for massive MIMO transceiver design. In the recent work [36], the authors consider different channel statistics across a huge antenna array for the design of hybrid beamforming schemes. In our measurements, the channel statistics change across the receiver positions since they are arranged on a long line of 8.82 m length. For example in the laboratory scenario, the channel statistics depend on whether a receiver position is located in front of a window or not.

The performance of a multi-user MIMO system is mainly determined by the correlation of user channels. Therefore, we analyze our channel measurements in terms of spatial correlation with respect to inter-user distance. We aim to find stationary regions at the receiver side and evaluate the wireless channel statistics within them. There exist two popular metrics for the stationarity analysis of measured wireless channels [37]: the mean channel power from [38] or the correlation matrix distance (CMD) from [39]. The former metric calculates the mean channel power through the average power delay profile (APDP). The latter metric is the similarity of channel correlation matrices. Since a measured
wireless channel is never perfectly stationary, these metrics need to be compared against a threshold after evaluating them in time, frequency or space. In our case, the domain of interest is the spatial domain. Authors of [26] apply both metrics for their massive MIMO channel stationarity analysis. They conclude that the channel’s spatial variation can be well characterized with both metrics. Since APDP is calculated by averaging in the delay domain, the CMD is better suited for our narrow-band channel measurements.

From the channel measurements, we obtain the MIMO channel matrix $\mathbf{H} \in \mathbb{C}^{N \times U}$, describing the wireless channel between all $N$ transmit antenna positions and all $U$ user positions. Please note that we obtain one MIMO channel matrix per measured scenario. We, however, omit a scenario index of the MIMO channel matrix for readability. We consider the channel coefficients, obtained via measurement, to be from a random process, sampled in space. We assume this process to be stationary in the spatial domain at the transmitter side. This is an intuitive assumption, as there are no obstacles in close proximity of the transmit antennas, which therefore all see the same scattering objects within the wireless channel. Under this assumption, the sample auto-correlation matrix is given by

$$\mathbf{R} = \frac{1}{N} \sum_{n=1}^{N} \tilde{\mathbf{h}}_{n} \tilde{\mathbf{h}}_{n}^H \in \mathbb{C}^{U \times U},$$

(2)

which serves as an estimate for the user-side auto-correlation matrix. In (2), $\tilde{\mathbf{h}}_{n}^U \in \mathbb{C}^{1 \times U}$ denotes the $n^{th}$ row vector of the channel matrix $\mathbf{H}$. Please note that the spatial correlation is of course analyzed for each measurement scenario individually, however, the scenario index of the correlation matrix is omitted in this section for readability.

By comparing a local auto-correlation at different positions, we identify regions of spatial quasi-stationarity in the WSS sense. Therefore, we introduce the CMD [39] between the correlation matrices $\mathbf{A}$ and $\mathbf{B}$ as

$$\text{CMD} (\mathbf{A}, \mathbf{B}) = \frac{\text{tr} (\mathbf{A}^H \mathbf{B})}{\|\mathbf{A}\| \|\mathbf{B}\|}.$$  

(3)

Due to the properties of the correlation matrices $\mathbf{A}$ and $\mathbf{B}$, $0 \leq \text{CMD} \leq 1$, as shown in Appendix B. Equality, that is, CMD = 1, is achieved if the matrices $\mathbf{A}$ and $\mathbf{B}$ are collinear, that is, $\mathbf{A} = \alpha \mathbf{B}$ with $\alpha \in \mathbb{R}$.

Let us consider a spatial averaging region of size $W$, with $W$ being a positive integer. For the ease of notation we define $L_L = \left\lfloor \frac{W-1}{2} \right\rfloor$ and $L_U = \left\lceil \frac{W-1}{2} \right\rceil$ such that $W = L_L + L_U + 1$. A local mean auto-correlation vector $\tilde{\mathbf{r}}_{p}^W \in \mathbb{C}^{W \times 1}$ at position index $p$ is then obtained by averaging along the diagonals of the sample auto-correlation matrix $\mathbf{R}$ in a region from $p-L_L$ to $p+L_U$. The $W^{th}$ element of $\tilde{\mathbf{r}}_{p}^W$ is given by the mean over the $W^{th}$ diagonal of $\mathbf{R}$, that is

$$\tilde{\mathbf{r}}_{p}^W [l] = \frac{1}{W-l+1} \sum_{u=-l+1}^{p+L_U-l+1} \mathbf{R} [u + l - 1, u]$$

(4)

for $l \in \{1, \ldots, W\}$ and $p \in \{L_L + 1, \ldots, U - L_U\}$.

The spatial averaging region size $W$ impacts the local mean auto-correlation vector $\tilde{\mathbf{r}}_{p}^W$. Averaging the auto-correlation matrix $\mathbf{R}$ over large spatial region sizes $W$ reduces fluctuations in the local mean correlation, thereby making it a smooth function. As we increase $W$, however, $\tilde{\mathbf{r}}_{p}^W$ becomes more similar to the global mean scenario correlation. Small spatial averaging regions make the local mean correlation representative for a local spatial region, but also lead to strong fluctuations of the local correlation with respect to location. We therefore choose the spatial averaging region size just large enough to identify few but large stationarity regions through a threshold on the CMD, see below. For the LOS and the laboratory scenario, a small value of $W = 5$ leads to a sufficiently smooth CMD curve. In the hallway scenario, we employ a larger value of $W = 15$ to obtain a smooth result in terms of CMD. Our chosen values for $W$ are summarized in Tab. 1 for all measured scenarios.

A locally averaged auto-correlation matrix $\overline{\mathbf{R}}_p^W$ is defined as a Hermitian Toeplitz matrix, with $\tilde{\mathbf{r}}_{p}^W$ as its first column vector and $\left(\tilde{\mathbf{r}}_{p}^W\right)^H$ as its first row vector. For a globally averaged auto-correlation function, an averaging region size of $U$ is employed, which leads to the mean correlation vector $\overline{\mathbf{r}}_{U}^{(W/2)}$. Defining a squared Hermitian Toeplitz matrix through the first $W$ elements of $\overline{\mathbf{r}}_{U}^{(W/2)}$ as explained before, yields the globally averaged auto-correlation matrix $\overline{\mathbf{R}} \in \mathbb{C}^{W \times W}$. To identify regions of spatial stationarity, we calculate the CMD between the globally averaged auto-correlation matrix $\overline{\mathbf{R}}$ and the local average $\overline{\mathbf{R}}_p^W$ as

$$c(p) = \text{CMD} \left( \overline{\mathbf{R}}_p^W, \overline{\mathbf{R}} \right)$$

(5)

for $p \in \{L_L + 1, \ldots, U - L_U\}$.

![Figure 4. CMD for all three scenarios with the corresponding threshold. We chose a threshold that results in few large quasi-stationary regions for further analysis.](image-url)
For a stationary channel, the CMD equals one for all possible positions. Since a measured channel, however, is not perfectly stationary and also contains measurement noise, the CMD is smaller than one. Therefore, we consider quasi-stationarity for the measured wireless channel. High values of the CMD (close to one) indicate a high level of quasi-stationarity, that is, an almost stationary channel. We employ a threshold on the CMD to determine the regions of quasi-stationarity that will be considered for the evaluation of the mean spatial correlation function. If this threshold is chosen low, the resulting correlation function is not representative for the quasi-stationary region, since a high level of fluctuation in the second-order statistic is still included in the same stationarity region. Choosing the threshold too high, only a small spatial region is considered as quasi-stationary, containing a small number of measured samples. Calculating the correlation function from a small number of samples leads to a mean correlation function that is again not representative for the quasi-stationarity region, since a high level of fluctuation in the second order statistic is still included. This is therefore a trade-off.

The CMD is plotted against the receiver position in Fig. 4 for the three measured scenarios. The position \( p \) on the abscissa consists of the position index \( p \) and the spacing of \( \frac{t}{2} \) between two receiver positions. We chose a threshold of \( t = 0.7 \) for the CMD to determine regions of quasi-stationarity. For this threshold, only few but large regions of quasi-stationarity are obtained, facilitating the calculation of a mean correlation function for the measured scenarios. The quasi-stationarity regions are indicated with arrows in Fig. 4. A stationary region \( S_i \) for scenario \( i \) is therefore defined as

\[
S_i = \{p \in \{L_i + 1, \ldots, U - L_i\} \mid c(p) \geq t\}.
\]

For the LOS scenario, the CMD is flat and the entire measured scenario is considered stationary. This is not surprising as there are no scattering objects in between the transmitter and the receiver or around the receiver.

In the laboratory scenario, the CMD shows three clearly distinguishable stationary regions, which coincide with the physical positions of the windows in the measured indoor office environment, see Fig. 3(a)-(b). Therefore, the three stationary regions show a high similarity to the scenario mean in terms of receiver side auto-correlation.

The hallway scenario is considered as a deep indoor scenario, which explains that the CMD does not show clearly distinguishable regions compared to the laboratory scenario. Since there is no position in this scenario that allows a direct LOS path to the transmitter, the hallway scenario is expected to be invariant with respect to the position within the hallway. There are no obstacles present in the hallway that would explain a local change of second order statistics. Still, there is a region at approximately 5 m which shows a lower CMD compared to the overall scenario mean.

Please note that low values of the CMD in Fig. 4 do not indicate non-stationary regions. Since we compare the scenario averaged correlation function to a locally averaged correlation, averaged within a region of size \( W \), high values of the plotted CMD indicate a high self-similarity of the local correlation. Therefore, a CMD \( c(p) \) value of one means that the local correlation is identical to the average scenario correlation. This is approximately the case for LOS scenarios. Although low values of \( c(p) \) do not mean that the channel behaves non-stationary at position index \( p \), we focus our remaining analysis to the regions where the CMD exceeds the chosen threshold. These regions dominate the second order statistics of the respective scenario.

### B. CHANNEL COEFFICIENT DISTRIBUTION

For the channel coefficient distribution analysis, we normalize the channel coefficients per stationary region as \( \hat{H} = cH \).

We chose the scaling coefficient \( c > 0 \) such that

\[
\frac{1}{N|S_i|} \sum_{n=1}^{N} \sum_{u \in S_i} |\hat{H}[n, u]|^2 = 1.
\]

The empirical cumulative distribution functions (ECDFs) of the channel coefficients’ magnitude \( |\hat{H}[n, u]| \) with \( n \in \{1, \ldots, N\} \) and \( u \in S_i \) for the respective scenario \( i \) are shown in Fig. 5. We perform maximum likelihood (ML) estimation with the measured channel’s amplitudes as data set to obtain a fit to a Rician distribution. We parametrize the Rician probability density function (PDF) with the Rician \( K \) factor and the power \( \Omega \), see [40, p. 76]. Therefore, the ML estimate yields the estimate \( \hat{K}_{ML} \) for the Rician \( K \) factor and the estimate \( \hat{\Omega}_{ML} \) for the power \( \Omega \). Due to the channel normalization (7), the estimated power is \( \hat{\Omega}_{ML} = 1 \). The fitted Rice distributions and the respective estimated Rician \( K \) factors are also shown in Fig. 5. The estimated Rician \( K \) factors for all measured scenarios are further summarized in Tab. 1.
TABLE 1. Channel parameters of the measured wireless channel, the Rician channel model and the scattering channel model.

|                          | LOS     | laboratory | hallway |
|--------------------------|---------|------------|---------|
| Rician K factor          | 10 dB   | 0 dB       | -28 dB  |
| averaging region size W  | 5       | 5          | 14      |
| number of scattering objects | 200    | 60         | 56      |
| scattering object gain γ | 2.5     | 12         | 50      |

The fit to a Rician distribution is perfect for the LOS scenario where the estimated K factor is high with $\hat{K}_{\text{ML}} \approx 10$ dB. In the indoor laboratory scenario, the amplitude distribution also shows a good fit to a Rician distribution. The estimated K factor here, $\hat{K}_{\text{ML}} \approx 0$ dB, is significantly lower compared to the LOS scenario although the considered region of quasi-stationarity is located next to the windows. For the hallway scenario, the Rician amplitude distribution provides a reasonable approximation for the observed amplitude distribution. The deviation from a Rician distribution comes from the fact that the channel statistics change more severely depending on the position within the quasi-stationary region as they do in the other two scenarios, see Fig. 4. As one would expect, the estimated Rician K factor of $\hat{K}_{\text{ML}} \approx -28$ dB is very low in this deep indoor scenario.

C. CHANNEL CORRELATION

1) TEMPORAL CORRELATION

To demonstrate the time stability of our measurement hardware, we performed the previously described measurement of channel coefficients twice for each user position. For the acquisition of all channel coefficients, the user side antenna is moved across the linear guide once, sequentially stopping at $U = 148$ positions to enable channel sounding. At each of these positions, the transmit antenna is sequentially moved to the $N = 40$ antenna positions. In the second measurement run, the receive antenna is moved sequentially across the linear guide again, in reverse direction. During this second run, the same wireless channel is observed once more, at the same physical antenna position $u$ but with a time difference $\Delta T_u$ compared to the first measurement run. Since the direction of movement is reversed, the time in between two channel measurements $\Delta T_u$, taken at the same user position index $u$, depends on the user position. This dependence is expressed via the subscript $u$ of $\Delta T_u$. For user positions at the end of the linear guide (large position indices), the time difference to the second measurement is smaller compared to the beginning (small position indices) of the linear guide. We denote the $u$th column vector of the channel matrix $\mathbf{H}$ by $\mathbf{h}_u \in \mathbb{C}^{N \times 1}$. Further, we denote the channel vector obtained in the second measurement at a time difference $\Delta T_u$ later compared to the first measurement run by $\mathbf{h}_u(\Delta T_u) \in \mathbb{C}^{N \times 1}$.

To investigate the change in correlation of channel coefficients at the same positions over time, we again employ the normalized inner product of channel vectors at identical positions, measured at different times

$$r_t(u) = \frac{|\mathbf{h}_u^H(\Delta T_u)\mathbf{h}_u|}{\|\mathbf{h}_u\| \|\mathbf{h}_u(\Delta T_u)\|}.$$  \hspace{1cm} (8)

The magnitude of (8) is plotted over time in between two channel vector measurements in Fig. 6 for all three scenarios. Please mind the axis scaling in this figure. The smallest measured value of channel correlation is 0.977, showing the high temporal stability of the measurement system.

2) SPATIAL CORRELATION

We employ the channel matrix collinearity as metric for spatial user correlation, similar to other works in the context of correlation of MIMO channels [27], [41]–[43]. This collinearity measure is defined via (3) with the matrices $\mathbf{A}$ and $\mathbf{B}$ being two MIMO channel matrices [44]. In our special case of massive MIMO with $N$ antennas at the base station and users with a single antenna, the absolute value of this metric simplifies to the normalized inner product of channel vectors for users $u$ and $u'$

$$r(u, u') = \frac{|\mathbf{h}_u^H\mathbf{h}_{u'}|}{\|\mathbf{h}_u\| \|\mathbf{h}_{u'}\|}.$$  \hspace{1cm} (9)

where $\mathbf{h}_u \in \mathbb{C}^{N \times 1}$ again denotes the $u$th column vector of channel matrix $\mathbf{H}$. Please note that the squared inner product corresponds to the interference power in a MIMO downlink system with two users and maximum ratio transmission, see Appendix C. Similar to (4), a mean channel vector inner product is then estimated as

$$\bar{r}(d) = \frac{1}{U-d+1} \sum_{u=1}^{U-d+1} r(u+d-1, u),$$  \hspace{1cm} (10)

for inter-user distances $d \in \{1, \ldots, U\}$. 

FIGURE 6. Magnitude of the temporal channel correlation for channel vectors measured at the same position but at different times. The lowest measured value of channel correlation is 0.977, showing the high temporal stability of the measurement system.
Plots for the mean channel vector inner product (10) within the respective stationary region are provided in Fig. 7. The inner product for a free space propagation model is also provided as a reference. This free space channel model is a small scale fading model, considering the phase between the transmit and the receive antennas. A channel coefficient from transmit antenna \( n \in \{1, \ldots, N\} \) to user \( u \in \{1, \ldots, U\} \) of the free space channel \( \mathbf{H}_{\text{free}} \in \mathbb{C}^{N \times U} \) is given by

\[
\mathbf{H}_{\text{free}}[n, u] = \exp\left(-i\frac{2\pi}{\lambda} \xi_{n,u}\right),
\]

with distance \( \xi_{n,u} \) between the \( n^{th} \) transmit antenna and the \( u^{th} \) user positions.

For the LOS scenario, the channel correlation is similar to a free space propagation model. The channel vector inner product is slightly lower compared to the pure LOS case, indicating that there are scattering objects present in the wireless channel.

For the laboratory scenario, the measured correlation function is very different from the correlation obtained from the free space channel model, since the estimated Rician \( K \) factor is only approximately 0 dB in this scenario. The decorrelation occurs slower with distance compared to the LOS scenario. The channel vector inner product behaves similar to the free space case in a region of 3 m to 6 m, but remains higher at larger distances.

This situation becomes even more pronounced for the hallway scenario. The spatial channel correlation starts out (for the first non-zero distance) at a lower level compared to the other measured scenarios, but shows almost no decorrelation with increasing distance. This is a remarkable result as one expects the spatial correlation to decrease quickly with distance in an indoor scenario.

IV. PERFORMANCE RESULTS AND CHANNEL MODELING

In this section, we present results for the achievable spectral efficiency within the stationary regions for all three scenarios. We also model the measured channel by means of a Rician channel model and a geometry based scattering channel model. We compare these channel models to the measured channel in terms of channel correlation and achievable spectral efficiency.

The Rician channel model is given by

\[
\mathbf{H}_{\text{Rice}} = \sqrt{\frac{K}{1+K}} \mathbf{H}_{\text{free}} + \frac{1}{\sqrt{1+K}} \mathbf{H}_{\text{sid}},
\]

where \( K \) denotes the Rician \( K \) factor. The free space channel \( \mathbf{H}_{\text{free}} \) is defined through (11). The i.i.d. Rayleigh channel matrix \( \mathbf{H}_{\text{sid}} \in \mathbb{C}^{N \times U} \) consists of independent complex Gaussian random variables with \( \mathbf{H}_{\text{sid}}[n, u] \sim \mathcal{CN}(0,1) \forall n \in \{1, \ldots, N\}, u \in \{1, \ldots, U\} \). Please note that the Rician channel model includes spatial correlation through the free space channel component. The amount of spatial correlation is dependent on the Rician \( K \) factor.

For a comparison with a spatially consistent channel model, we employ the channel model from [15]. Since this model is based on the multiple scattering channel model idea [45], we will refer to this model as scattering model within this work. For this channel model, a channel coefficient from transmit antenna \( n \in \{1, \ldots, N\} \) to user \( u \in \{1, \ldots, U\} \) is given by

\[
\mathbf{H}_{\text{scatter}}[n, u] = \alpha_{u,n} + \sum_{k \in \mathcal{C}} \beta_{u,k} \alpha_{k,n},
\]

where \( \mathcal{C} \) denotes the set of scattering objects. The coefficient \( \alpha_{u,n} \) describes the propagation from transmit antenna \( n \) to user \( u \), the coefficient \( \alpha_{k,n} \) describes the propagation from transmit antenna \( n \) to scattering object \( k \) and the coefficient \( \beta_{u,k} \) describes the propagation from scattering object \( k \) to user \( u \). These propagation coefficients are defined as

\[
\alpha_{k,j} = \frac{\lambda}{4\pi \| \mathbf{r}_k - \mathbf{r}_j \|} e^{i \frac{2\pi}{\lambda} |\mathbf{r}_k - \mathbf{r}_j|},
\]

\[
\beta_{j,k} = \frac{\delta_k}{\sqrt{4\pi \| \mathbf{r}_j - \mathbf{r}_k \|}} e^{i \frac{2\pi}{\lambda} |\mathbf{r}_j - \mathbf{r}_k|}.
\]

The vectors \( \mathbf{r}_j \in \mathbb{R}^2 \) contain the position of either transmit antenna \( j \), user \( j \) or scattering object \( j \). The scattering event is modeled via the scattering factor \( \delta_k = \gamma e^{i \phi_k} \), where the scattering gain \( \gamma \) describes the strength of the scattering events and \( \phi_k \) is a uniformly distributed angle, that is, \( \phi_k \sim \mathcal{U}(0, 2\pi) \). Please note that this channel model is defined via the actual position of the transmit antennas, scattering objects and user positions. This model is therefore inherently spatially consistent [15].

There are three major channel parameters for the scattering channel model: the scattering gain \( \gamma \), the number of scattering elements \( |\mathcal{C}| \) and the position of the scattering elements. We employ a uniformly distributed random placement of scattering elements within a limited spatial region for each.
FIGURE 8. Environments for the scattering channel model. (a) For the LOS scenario, scattering objects are uniformly distributed between transmitter and receiver. (b) In the laboratory scenario, the scattering object placement is restricted to positions that are visible through the window. (c) In the hallway scenario, scattering objects are uniformly distributed around the receiver positions.

scenario. These regions correspond to the positions of dominant scattering objects within the measured wireless channel. We provide further details on the scattering element placement in the scenario descriptions below.

To fit the scattering channel model to the channel measurements, we perform Monte Carlo simulations to find a match in terms of Rician $K$ factor and spatial channel correlation. The employed values for the number of scattering elements $|K|$ and the scattering gain $\gamma$ for each measurement scenario are summarized in Tab. 1. A large scattering gain and a high number of scattering elements yield dominant non-line-of-sight (NLOS) components and therefore a small Rician $K$ factor. When scattering elements are spread out widely in space, dominant NLOS components lead to a low spatial correlation. In case the scattering elements form a concentrated cluster, dominant NLOS paths result in high spatial correlation. This allows to fit wireless channels with low Rician $K$ factor that still show a large spatial correlation.

For the LOS scenario, the positions of the scattering objects are two dimensional uniformly distributed on a disk with radius 50 m between the transmitter and the receiver, see Fig. 8(a). These scattering objects account for the fact that the measured propagation environment is not a free space one. As the scattering elements are spread out widely in space, the NLOS components have a large angular spread and low correlation. Since, however, the Rician $K$ factor in this scenario is high, the LOS component is dominating in the channel model. The specific spatial distribution of the scattering elements has therefore little impact on the channel correlation in this scenario.

For the laboratory scenario, the scattering objects are clustered densely and closely to the direct LOS path, see Fig. 8(b). This models the fact that only scattering objects within a limited angular region are visible to the receiver through the windows. The positions of the scattering objects correspond to a building in between the transmitter and the user positions, see Fig. 1. The similar incident angles of scattered paths lead to a high level of spatial correlation over extended inter user distances at the receiver side, even at a low $K$ factor.

In the hallway scenario, a host of scattering objects is uniformly distributed within a disk of radius 30 m around the user positions, see Fig. 8(c). The radius of this disk has negligible impact on the channel model as long as the scattering elements are uniformly distributed around the user positions. This element placement models a dense scattering environment with a low Rician $K$ factor. A large scattering gain of $\gamma = 50$ and a rather small number of scattering elements of $|K| = 56$ allows a good fit of the large spatial correlation in this scenario.

A. SPATIAL CHANNEL CORRELATION

We provide results for the spatial channel correlation of the measured channel within the stationarity regions, the Rician channel model (12) and the scattering channel model (13) in Fig. 9. The Rician $K$ factors for both of these channel models are chosen to fit the ML estimated $K$ factor of the respective measured channel. The channel correlation of the free space channel model (11) is also provided as reference.

The spatial channel correlation for the LOS scenario is shown in Fig. 9(a). Although there are no obstacles in between the transmitter and the user in the measured LOS scenario, there are inevitably scattering objects present in the measured scenario. Therefore, the spatial channel correlation deviates from the free space model. Both, the Rician channel model as well as the scattering channel model, show a good fit to the measured channel in terms of the spatial channel correlation for the LOS scenario.

For the laboratory scenario, the resulting spatial correlation is shown in Fig. 9(b). The measured spatial correlation is not well explained by the fitted Rician channel model due to the low $K$ factor in this scenario. Scattering objects in the
corresponding scattering channel model scenario are located within a narrow angular range around the direct path. Thereby we achieve a higher spatial correlation for a low $K$ factor with the scattering channel model, providing a good match with the measured channel.

In case of the deep indoor hallway scenario, the spatial correlation function is shown in Fig. 9(c). Since the estimated Rician $K$ factor is very low with approximately $-28$ dB, the fitted Rician channel model is very close to an i.i.d. Rayleigh fading channel. Therefore, the Rician channel model shows almost zero spatial correlation, independent of the user separation distance. While the measured channel also shows a low level of spatial correlation, it is still significantly higher compared to the Rician channel model. For the scattering channel model, we model the indoor environment by scattering elements distributed uniformly all around the receiver positions. This scattering element placement leads to scattered paths arriving from all directions at the receiver. This yields scattered paths with low spatial correlation as the randomly fading channel has no self-similarity for different receiver positions. To fit the comparably large spatial correlation of the measured channel with the scattering channel model, we employ few scattering elements with large scattering gain in this model. Achieving a spatial correlation as low as in an i.i.d. Rayleigh channel requires a scattering environment with a huge number of scatterers. Our results show that such a large number of scattering elements is not necessarily found in a deep indoor scenario. The fact that we achieve a match with scattering elements uniformly distributed around the receiver additionally shows that the spatial correlation behavior does not necessarily come from waveguiding effects along the hallway.

**B. ACHIEVABLE SPECTRAL EFFICIENCY**

In order to illustrate the impact of channel correlation on a massive MIMO mobile communications system, we provide results for the achievable spectral efficiency (SE) within the regions of WSS in this section. A system model and the definition of the achievable SE [46, p. 9], [47], employed for the simulations in this section, are provided in Appendix C.

The mean achievable sum SE for the case of two users with minimum mean squared error (MMSE) precoding is shown in Fig. 10. This mean is plotted against the distance $d$ between the users for the measurement within the stationary regions, the scattering channel model, the Rician channel model and the free space channel model. The mean value of achievable sum SE is inferred by averaging over all possible positions pairs with distance $d$, similar to the mean channel inner product (10).

The SNR without beamforming gain in this simulation scenario is set to 0 dB. Please note that although the SNR varies between the scenarios, the SNR was set to the same value for all scenarios in this simulation for a better comparison.

In the LOS scenario, the achievable SE curve for the measured channel is very close to the free space channel model since this scenario is a LOS one, see Fig. 10(a). The presence of scattering objects leads to an SE that is higher than the SE of the free space channel model for small inter-user distances. The Rician channel model and the scattering channel model provide a good fit to the measured channel in the LOS scenario.

The achievable SE for the laboratory scenario is shown in Fig. 10(b). The measured channel decorrelates with inter-user distance, leading to an increased SE at large distances. As a consequence of the low Rician $K$ factor in this scenario, the Rician channel model includes little spatial correlation and therefore leads to a too high SE compared to the measurement. The scattering channel model includes higher spatial correlation (having the same, low Rician $K$ factor) and therefore describes the trend in sum SE better.

The SE for the hallway, or deep-indoor, measurement scenario is shown in Fig. 10(c). The measured wireless channel in this scenario has an amplitude distribution with a very low Rician $K$ factor and is therefore similar to an uncorrelated i.i.d. Rayleigh channel in terms of channel coefficient distribution. Still, the measured spatial correlation is much higher.
compared to the fitted Rician channel model with the same $K$ factor. Therefore, the resulting SE of the Rician channel model is too high. The scattering channel model provides more accurate spatial correlation. Therefore, the SE obtained by the scattering channel model matches the measurement.

Further, the achievable SE shows little dependence on inter-user distance in the hallway scenario. While one expects that a deep-indoor channel decorrelates quickly with distance, the effect of spatial correlation remains significant for several meters in this scenario. The achievable SE of the measured channel is 20% lower than the achievable SE of the Rician channel model with the same $K$ factor for an inter-user distance as large as 6 m, see Fig. 10(c).

Similar numbers were found in other MIMO measurements before. In [48], the authors measure an outdoor-to-outdoor massive MIMO scenario. Their results show that approximately 80% of the i.i.d. Rayleigh sum rate is achieved with MMSE precoding at 0 dB SNR with 112 antennas. Authors of [8] also report to achieve 80-90% of the i.i.d. Rayleigh channel dirty paper coding capacity in an outdoor-to-outdoor scenario with zero forcing (ZF) precoding and 128 antennas. In [49], the authors measure a $4 \times 4$ MIMO outdoor-to-indoor scenario. They report a median capacity of 80% compared to an i.i.d. Rayleigh channel at an SNR of 5 dB. We observe this remarkable performance gap also for a deep indoor user in an outdoor-to-indoor scenario where one would expect a low spatial correlation.

V. CONCLUSION

Spatial correlation heavily depends on the scenario and the user environment. The amount of spatial correlation and the decorrelation behavior with distance are mainly determined by the location of scattering objects visible to the receiver. When incident paths arrive from a narrow angular range at a certain user location, a high level of spatial correlation is experienced even when the amplitude statistic shows a very low Rician $K$ factor. For the measured outdoor-to-indoor hallway scenario, the observed amount of spatial correlation is significantly higher than one would obtain when modeling such a rich scattering environment with a Rician (or i.i.d. Rayleigh) channel model. Since spatial correlation between users occurs even in such dense scattering environments, user scheduling is an important tool for massive MIMO systems. Further, when a scattering environment yields spatially correlated channels for large user distances, massive MIMO with LOS is preferable to massive MIMO with NLOS. The achievable SE of a multi-user MIMO system is significantly over-estimated when using Rician channel models, such as (12), or spatially inconsistent channel models. The spatially consistent scattering channel model (13), based on the positions of the scattering elements, leads to a good match with the measurement. Thus, one is advised to use spatially consistent channel models to predict the performance of multi-user MIMO systems accurately.

APPENDIX A

MEASUREMENT HARDWARE STABILITY

The measurement principle of virtual antenna array measurements reduces hardware complexity in terms of number of required RF chains since only a single antenna is employed. However, the virtual array concept requires a measurement system that is stable over time, since channel measurements are performed sequentially. The obtained channel measurements are then joined into a channel matrix or channel vector as if they were all measured at the same time with a full antenna array. This requires both, a time-invariant wireless channel and measurement hardware that behaves completely stable over time. The former condition is achieved by performing wireless channel measurements out of office hours, during night, when there are no persons present at the faculty where the measurements were performed. The latter requirement on time stability of the measurement hardware includes the considerations below [50]:

- **Time Synchronization:** Since triggered measurements are performed at each transmit antenna position and each
receiver position, a repeatable triggering mechanism and time synchronization is required. The Vienna MIMO Testbed employs a hardware based timing synchronization unit [51], [52] that achieves a timing accuracy of ±10 ns. This timing offset is negligible compared to the employed OFDM symbol duration of approximately 66.67 µs.

- **Frequency Synchronization:** Global Positioning System (GPS) disciplined Rubidium frequency standards at the transmitter and the receiver ensure a frequency deviation of ±5·10⁻¹¹. At a center frequency of 2.5 GHz this leads to a worst case frequency difference between transmitter and receiver of Δf = 10⁻¹¹ × 2.5 GHz = 0.25 Hz. Compared to the employed subcarrier spacing of 15 kHz of the employed OFDM channel sounding signal, this frequency deviation is negligible.

- **Phase Synchronization:** To obtain a measured MIMO channel matrix with correct phase relation between all transmit antennas and all receive antennas from a virtual array measurement, phase stability is required. Although our testbed employs highly accurate Rubidium frequency standards, the worst case frequency difference between transmitter and receiver is of Δf = 10⁻¹¹ × 2.5 GHz = 0.25 Hz. Compared to the employed subcarrier spacing of 15 kHz of the employed OFDM channel sounding signal, this frequency deviation is negligible.

**APPENDIX B**

**PROPERTIES OF THE CMD**

Let’s assume matrices A, B ∈ ℂⁿˣⁿ are correlation matrices and therefore Hermitian and positive definite. Due to properties of the tr(·) operator and Hermitian matrices

\[
\text{tr} \left( A^H B \right) = \text{tr} \left( AB^H \right) = \text{tr} \left( B^H A \right) = \text{tr} \left( A^H B \right)^H, \quad (15)
\]

from which follows \( \text{tr} \left( A^H B \right) \in \mathbb{R} \). Let us consider the matrix factorizations \( B = U D U^H \) and \( A = Q V Q^H \), where D, Q ∈ ℂⁿˣⁿ are diagonal matrices and the matrices U and Q are unitary. The square root of B is given by \( B^{\frac{1}{2}} = U D^{\frac{1}{2}} U^H \) such that \( B = B^{\frac{1}{2}} B^{\frac{1}{2}} \). From this follows

\[
\text{tr} \left( A^H B \right) = \text{tr} \left( B^{\frac{1}{2}} A^H B^{\frac{1}{2}} \right) = \text{tr} \left( B^{\frac{1}{2}} V Q V^H B^{\frac{1}{2}} \right). \quad (16)
\]

Since the argument of the \( \text{tr}(·) \) is positive-definite, it further follows that \( \text{tr} \left( A^H B \right) > 0 \) and therefore \( \text{tr} \left( A^H B \right) = \left| \text{tr} \left( A^H B \right) \right| \). The CMD from (3) may be equivalently re-written as

\[
\text{CMD} \left( A, B \right) = \frac{\left| \text{tr} \left( A^H B \right) \right|}{\|A\| \|B\|}. \quad (17)
\]

With \( \|A\| = \sqrt{\text{tr} \left( A^H A \right)} \) and the Cauchy-Schwarz inequality, it follows \( 0 \leq \text{CMD} \left( A, B \right) \leq 1 \).

**APPENDIX C**

**SYSTEM MODEL AND ACHIEVABLE SPECTRAL EFFICIENCY**

We consider a multi-user massive MIMO downlink scenario with N transmit antennas at the BS. There are U users, each with a single omni-directional antenna. The received signal of user u is

\[
y_u = \sum_{j=1}^{U} h_{u,j} f_u \sqrt{\rho_u} x_j + w_u. \quad (18)
\]

where \( w_u \) is additive white Gaussian noise with \( w_u \sim \mathcal{CN} \left( 0, \sigma_w^2 \right) \). The real positive power scaling factors are denoted by \( \rho_u \) for users \( u \in \{1, \ldots, U\} \). The vector \( h_u \) contains all channel coefficients from N transmit antennas to user u. The independent random transmit symbol is denoted by \( x_u \) and is of unit power, that is, \( \mathbb{E} \{ x_u x_u^* \} = 1 \). The precoding vector of user u is denoted by \( f_u \). A power constraint \( \mathbb{E} \left\{ \sum_{u=1}^{U} f_u \sqrt{\rho_u} w_u \| f_u \|^2 \right\} \leq P_T \) is considered, which leads to \( \| f_u \|_2^2 = 1 \) and \( \sum_{u=1}^{U} \rho_u = P_T \) with the sum transmit power \( P_T \). The precoding matrix \( F = (f_1, f_2, \ldots, f_U) \in \mathbb{C}^{N \times U} \) is normalized per-user such that \( \| f_u \|_2 = 1 \), \( \forall u \in \{1, \ldots, U\} \). The signal to interference and noise ratio (SINR) of user u is given by

\[
\text{SINR}_u = \frac{\rho_u \left| h_{u}^H f_{u} \right|^2}{\sum_{j \neq u}^{U} \rho_j \left| h_{u}^H f_{j} \right|^2 + \sigma_w^2}. \quad (19)
\]

The achievable sum SE in bits/s/Hz is

\[
\text{SE} = \sum_{u=1}^{U} \log_2 \left( 1 + \text{SINR}_u \right). \quad (20)
\]

In this work, we consider MMSE precoding, with a precoding matrix calculated as

\[
\hat{F}^{\text{MMSE}} = H \left( H^H H + \frac{U \sigma_w^2}{P_T} I_U \right)^{-1}, \quad (21)
\]
with the channel matrix $\mathbf{H} = (\mathbf{h}_1, \ldots, \mathbf{h}_U)$ and the precoding matrix $\mathbf{F} = (\mathbf{f}_1, \ldots, \mathbf{f}_U)$. According to the sum transmit power constraint, the precoding matrix is normalized as

$$\mathbf{f}_u = \frac{\mathbf{f}_u}{\| \mathbf{f}_u \|_2} \quad \forall u \in \{1, \ldots, U\}. \quad (22)$$

Further, we apply equal power to all users, that is, we choose

$$\rho_u = \frac{\nu_{u}}{\nu_{1}} \quad \forall u \in \{1, \ldots, U\}.$$
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