Pulsar radiation belts and transient radio emission

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ABSTRACT

It is proposed that radiation belts similar to the ones in the planetary magnetosphere can exist for a pulsar with a relatively long period and a strong magnetic field. In the belts located in the closed field line region near the light cylinder relativistic pairs are trapped and maintained at a density substantially higher than the local Goldreich–Julian corotation density. The trapped plasma can be supplied and replenished by either direct injection of relativistic pairs from acceleration of externally supplied particles in a dormant outer gap or in situ ionization of the accreted neutral material in the trapping region. The radiation belts can be disrupted by waves that are excited in the region as the result of plasma instabilities or emitted from the surface due to starquakes or stellar oscillations. The disruption can cause an intermittent particle precipitation towards the star producing radio bursts. It is suggested that such bursts may be seen as rotating radio transients.

Key words: acceleration of particles – radiation mechanisms: non-thermal – pulsars: general.

1 INTRODUCTION

The recent discovery of a new class of radio transient sources, known as rotating radio transients (RRATs) (McLaughlin et al. 2006), suggests that radio transient phenomena may be quite common for typical pulsars. About 11 such sources have been detected so far; they are characterized by short outbursts of a typical from 2 to 30 ms duration with an average interval between two consecutive bursts from a few minutes to a few hours. 10 RRATs have their periods and period derivatives determined and three of them have their period derivatives identified. Observations appear to suggest that these objects are part of the normal pulsar population, with periods of a few seconds, within the period range of typical radio pulsars, and magnetic fields up to \(10^{10}\) T, close to a lower limit of the magnetic field of the known magnetars. No transient X-rays or optical counterpart has been observed, though thermal X-ray emission was detected from one RRAT, which appears to be similar to X-ray dim isolated neutron stars (Popov, Turolla & Possenti 2006; Reynolds et al. 2006).

Three RRATs with both periods and period derivatives identified overlap with normal radio pulsars in the \(P - \dot{P}\) (pulsar period versus period derivative) distribution, which strongly suggests that RRATs may have similar properties to radio pulsars and should have normal radio emission as well. However, regular radio pulses have not been detected from any of these sources, suggesting that either (1) these radio bursts are distinct from the normal radio emission in aspects of emission geometry or processes or (2) normal radio pulses are too weak to be detectable (Weltevrede et al. 2006a). For case (1), one could postulate, for example, that radio emission jumps between two alternative beaming directions, one of which intersects the Earth. For case (2), RRATs may be considered as extreme pulses and the ‘missing’ normal radio pulses can be detected with a sufficiently long observing time (Weltevrede et al. 2006a). Weltevrede et al. (2006a,b) specifically considered PSR B0656+14, which is a nearby pulsar that has pulsed X-rays and intense bursty radio pulses similar to RRATs. They suggested that if this pulsar were at a distance similar to the RRATs, one would see the strong radio bursts as RRATs and its normal radio emission becomes non-detectable.

If RRATs are indeed pulsars, one has a major difficulty in interpretation of the observed intense radio bursts on the basis of conventional polar cap models because most of the RRATs have long periods and pair production is not effective. Cordes & Shannon (2006) proposed that the bursty emission by RRATs is due to circumstellar asteroids randomly straying into the magnetosphere; the neutral material is evaporated and ionized, leading to a sudden downward flow of charged particles in the polar region, which in turn ignites a transient pair cascade towards the PC. The low-mass disc hypothesis has some observational support from the recent discovery of the fall-back disc of a magnetar, (Wang, Chakrabarty & Kaplan 2006). One main ingredient of the Cordes & Shannon (2006) model is that a cascade is assumed to be due to acceleration in the outer gap. However, RRATs are in the regime in the \(P - \dot{P}\) distribution where the outer gap is inactive for pair production (through photon–photon collision) except for the case of a nearly orthogonal inclination angle where the outer gap may form close to the PC and pair production through a single photon decay in the magnetic field is important. In this latter case, the cascade has to be close to the PC and backward emission from the cascade may not be able to propagate freely through the intervening plasma near the star in the closed
field line region (CFLR) where induced three-wave interactions are expected to be strong (Luo & Melrose 2006).

In this paper, we propose that typical pulsars may have radiation belts in the CFLR where relativistic plasmas are trapped. The trapping regions bear many similarities to the Earth’s radiation belts. For example, they are similarly subject to various low-frequency disturbances that disrupt the trapped plasma causing intense precipitation. While in the case of the Earth’s van Allen belts, which are subject to disruption by geomagnetic storms due to the solar wind, the proposed main disturbances to the pulsar’s radiation belts are low-frequency Alfvén waves generated from the star’s surface as a result of stellar oscillations (McDermott, van Horn & Hansen 1988) or shear waves in the neutron star’s crust due to starquakes (Blaes et al. 1989). It is suggested here that transient radio emission (TRE) similar to RRATs can be generated as a result of catastrophic disruption of the trapping region and that the coherent emission is produced by particles precipitating towards the star.

The density of the trapped plasma can be maintained at a much higher value than the local Goldreich-Julian (GJ) density. In the trapping region the magnetic field is relatively weak and the synchrotron decay time is long so that the plasma can be replenished, leading to a build-up in the density. The major sources of the trapped plasma can be particle acceleration in a dormant outer gap or ablation and ionization of the accreted neutral matter. We assume that low-level accretion of neutral grains from a dust disc or from the interstellar medium (ISM) occurs and that the accreted matter is ionized inside the light cylinder (LC) feeding charged particles directly to both the open field line region (OFLR) (Cheng 1985; Ruderman & Cheng 1988; Cordes & Shannon 2006) and CFLR (Cordes & Shannon 2006). Note that there were also previous discussions on accretion of neutral material from the ISM and possible effects on radio emission by ionization of the neutral matter in the pulsar magnetosphere (Tsyyan 1977; Wright 1979). Charged particles created from destruction and ionization of neutral material in the CFLR can accumulate and be trapped in the region. Charged particles created in the OFLR can be accelerated in the outer gap and relativistic pairs can be injected into the trapping region. High-energy gamma-rays emitted by the accelerated particles produce pairs in the trapping region on the thermal radiation emitted from the surface. The pair-production rate is rather slow, but provided that it exceeds the loss rate, it can accumulate plasma near the LC in the CFLR. Note that re-ignition of a latent outer gap was discussed by Ruderman & Cheng (1988) in the context of gamma-ray bursts. However, they considered nearly aligned rotators with a period much shorter than that considered here. In their model, injection of externally supplied particles in the outer gap is assumed to lead to efficient pair creation, a case that does not apply to RRATs or magnetars.

In Section 2, it is argued that plasma trapping regions similar to the Earth’s radiation belts can exist in the CFLR of typical pulsars with a long period. Injection of charged particles to the trapping region due to latent particle acceleration in an outer gap or direct ionization of accreted neutral dust grains is discussed in Sections 3 and 4. Section 5 discusses the stability of the pulsar radiation belts and possible mechanisms for particle precipitation. Application to RRATs is discussed in Section 6.

2 RADIATION BELTS

It is argued here that radiation belts similar to that in planetary magnetospheres may exist in pulsar magnetospheres with relatively weak magnetic fields at the LC.

2.1 Magnetic mirror

Particles injected with non-zero perpendicular momenta can be trapped in the CFLR due to the magnetic mirror effect. For a particle with a perpendicular momentum $p_\perp$, the first adiabatic invariant can be expressed as $p_\perp^2 / B = \text{constant}$. In this section, we ignore the loss of perpendicular energy due to cyclotron decay, which can be important for pulsars and is considered in Section 2.2. From a quantum mechanical view point, this corresponds to a particle remaining in the same Landau level. The magnetic field in the CFLR forms a ‘magnetic bottle’ and particles injected at a radial height $\eta_1 < (r/L_{\text{LC}})$ can be trapped at $\eta < 1$ provided that their perpendicular momenta satisfy the condition

$$ p_\perp > \left( \frac{\eta}{\eta_1} \right)^{3/2}, $$  

where $p = (p_\perp^2 + p_i^2)^{1/2}$ with $p_i$ the parallel momentum. For example, particles injected with $p_\perp / p_i > 0.35$ at the LC ($\eta_1 = 1$) can be trapped; they bounce back and forth between two opposite hemispheres in the CLFR above $\eta = 0.5$. The right-hand side can be written into the form $\sin \alpha_i = (\eta / \eta_1)^{3/2}$, where the angle $\alpha_i$ defines a conal surface in the momentum space, referred to as the loss cone at $\eta$. Particles with a pitch angle smaller than $\alpha_i$ (inside the cone) pass through the point $\eta$ and those with a pitch angle larger than $\alpha_i$ (outside the cone) are reflected above $\eta$. For particles moving along the last closed field lines, the typical bounce time is (cf. Appendix A)

$$ \tau_b \approx \frac{2 \pi}{P}, $$

which is not sensitive to the particle’s initial pitch angle $\alpha_i$ at the injection but required to satisfy the condition

$$ \alpha_i \geq \left( \frac{R_0}{R_{\text{LC}}} \right)^{3} \approx 10^{-12} \left( \frac{P}{2s} \right)^{-3}, $$

that is, the reflection radius must be above the stellar radius. Equation (3) provides a much smaller value than the minimum loss-cone angle of the van Allen belts in the Earth’s magnetosphere, which is a few degrees. However, it is shown in Section 2.2 that the minimum loss-cone angle determined by cyclotron decay can be much larger than that given by equation (3).

Since the dipole magnetic field is inhomogeneous, the guiding centres of trapped particles undergo both a gradient drift and curvature drift across the field lines (Northrop & Teller 1960). These drifts cause particles to circulate around the star forming a ring current, similar to the ring current in the Earth’s radiation belts. The current gives rise to perturbations to the dipole field at $\eta = 1$:

$$ \delta B \approx \frac{N_i \mu_0 c^2 \gamma}{2U_{\text{BL}}} \approx 10^{-6}, $$

where $U_{\text{BL}} = B_0^2 / \mu_0$ is the magnetic energy density at the LC, $N_i = N_i 0^\theta_0 \delta_0 = (R_0/R_{\text{LC}})^{3/2} \approx 0.012 (P/1.5 \text{s})^{-1/2}$ is the half-opening angle of the PC and $N_{\text{GJ}} \approx 2.3 \times 10^{17} (B_0/10^9 \text{T})(P/1.5 \text{s})^{-1} \text{m}^{-3}$ is the GJ density at the PC where the magnetic field is assumed to be $B_0$. In practical situations where the plasma density can be much higher than the GJ density (Section 3), the perturbations can exceed the estimate given by (4). Since the ring current can vary temporally as a result of instabilities in the trapped plasma (cf. Section 5), $\delta B$ should be time dependent. The effect of such perturbations on pulsar electrodynamics and their possible contribution to pulsar timing noise will be discussed elsewhere.
& Melrose 2000), where \( p_{\perp,0} \) and \( p_{\parallel,0} \) are the perpendicular and parallel components of the particle’s dimensionless initial momentum. This gives \( \tau_s \approx \tau_0 / \gamma_c \), which has the following numerical form:

\[
\tau_s \approx 1.4 \times 10^4 \gamma_c^{-1} \left( \frac{B_s}{10^8 \text{T}} \right)^{-2} \left( \frac{P}{2 \text{s}} \right)^6 \left( \frac{\eta}{0.5} \right)^6 \text{ s.} \quad (6)
\]

One has \( \tau_s \sim \tau_P \) if a particle is injected into the trapping region with a large initial pitch angle \( \tan \alpha_0 = p_{\perp,0} / p_{\parallel,0} \approx 1 \). However, one has a much shorter cooling time if the initial pitch angle is small \( \alpha_0 \ll 1 \).

In the cyclotron regime, the particle radiates at a much slower rate, relaxing to the ground Landau state on a time \( \tau_s \gamma_c^2 \sim \tau_0 \gamma_c^2 \).

Equating the cyclotron decay time to the bounce time, one finds the minimum pitch angle for a particle to be reflected at the mirror point. This angle is estimated to be

\[
\alpha_i \geq 0.04 \left( \frac{B_s}{10^8 \text{T}} \right)^{1/2} \left( \frac{P}{2 \text{s}} \right)^{-1/2}. \quad (7)
\]

For particles to remain trapped for time much longer than \( \tau_b \), the pitch angle at injection needs to be much larger than (7).

Pitch-angle scattering by waves can lead to particle diffusion to small pitch angles and limit the lifetime of the trapped particles. Since the decay time decreases rapidly with decreasing altitude according to \( \tau_s \propto \eta^4 \), particles scattered to small pitch angles can reach low altitudes rapidly radiating away their perpendicular energy and eventually reach the star. For example, particles with \( \gamma_c \sim 1 \), which are scattered into the loss cone of \( \alpha_0 = 0.01 \), can travel down to an altitude \( \eta = 0.05 \eta_i \) where the decay time is only \( \tau_s \sim (100 \text{ ms}) \eta_i \).

The sources of the waves can be internal, that is, they are generated in situ through plasma instabilities, or external, that is, they are generated outside the region. Since particles with small pitch angles escape through both ends of the ‘magnetic bottle’, there is an excess of particles with a large pitch angle; this gives rise to an ideal condition for instabilities to develop (driven by an inversion in the particle pitch-angle distribution). Waves generated from instabilities can cause pitch-angle diffusion, further enhancing particle precipitation. In principle, this process can impose an upper limit to the density of the trapped plasma (Kennel & Petschek 1966). However, since the efficiency of pitch-angle scattering depends explicitly on the wave intensity that in this case is limited to much smaller than \( \eta_i \), one needs a relatively high plasma density for such upper limit to be significant (cf. Section 5).

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### 2.2 The loss-cone angle

The lifetime of trapped particles is limited by cyclotron decay and pitch-angle scattering, both of which violate the first invariant. The electron’s synchrotron cooling time, that is, the time in which it loses half of its energy \((m_e c^2) / \gamma_c \) is \( \tau_s = \tau_0 / \gamma_c \) with

\[
\tau_0 = \frac{3c}{4r_e \Omega_c}, \quad (5)
\]

where \( r_e \approx 2.8 \times 10^{-13} \) m is the classical radius of the electron and \( \Omega_c \) is the electron cyclotron frequency. Fig. 1 shows a pulsar distribution in \( P-P/B_s \) with the decay time \( \tau_0 \) indicated. Six examples where \( \tau_0 \) is calculated are listed in Table 1. For fast-rotating young pulsars or millisecond pulsars, (5) is very short \( \sim 10^{-6} \) s, and for typical pulsars it can be quite long \( \sim 10^5 - 10^7 \) s. Since in the synchrotron regime, the pitch angle remains constant, the relevant time that constrains particle trapping is the cooling time in the limit \( p_{\perp} \to 1 \). \( \gamma \to \gamma_c \equiv \sqrt{1 + p_{\perp,0}^2 / (1 + p_{\parallel,0}^2)} \) \( 1/2 \) (Luo, Shibata

### 2.3 Sources for the trapped plasma

Particle loading into the trapping region can be achieved through a process that is not constrained by the first adiabatic invariant.

**Table 1.** The magnetic field \( B_s \) (at the LC), \( N_L = N_G l / B_s \), synchrotron decay time \( \tau_b \), and charge loading time \( \tau_L \). The magnetic field on the polar cap surface is obtained from \( B_s = 6.4 \times 10^{15} (P/P_1)^{1/2} \) (Uslov & Melrose 1995), twice the value quoted by McLaughlin et al. (2006). The time \( \tau_0 \) is calculated at 0.5 \( R/\Omega_1 \).

| RRATs/pulsars | \( P \) (s) | \( B_s \) (10^8 T) | \( B_L \) (10^-8 T) | \( N_L \) (m^-3) | \( \tau_0 \) (s) | \( \tau_L \) (s) |
|---------------|-------------|---------------------|---------------------|-----------------|----------------|----------------|
| J1317−5759 | 2.64 | 1.2 | 5.8 | 7.8 x 10^4 | 2.7 x 10^4 | 1.5 x 10^9 |
| J1819−1458 | 4.26 | 10 | 11.8 | 9.8 x 10^4 | 6.5 x 10^4 | 93.7 |
| J1913+1333 | 0.92 | 0.54 | 64 | 2.4 x 10^6 | 2.3 x 10^4 | 10.8 |
| Typical magnetars | 7 | 10^2 | 18 | 2.5 x 10^5 | 1.3 x 10^4 | 34 |
| The Crab | 0.033 | 0.6 | 1.3 x 10^6 | 9.6 x 10^1 | 4 x 10^-6 | – |
| PSR1957+21 | 0.0015 | 8 x 10^-5 | 2.2 x 10^6 | 4.5 x 10^13 | 2 x 10^-6 | – |
We consider two possible sources for the trapped plasma, both of which involve extrinsic charged particles: (1) pair production in the trapping region by high-energy gamma-rays emitted by particles accelerated in an outer gap and (2) direct ionization of neutral material in the thermal radiation field in the CFLR. Both cases may involve ionization of neutral matter migrating into the magnetosphere due to accretion of a disc (Cordes & Shannon 2006; Li 2006) or neutral dust grains in the ISM (Cheng 1985). For (1), the outer gap, which is usually dormant for typical pulsars, is re-ignited with the supply of charged particles. Although the gap is rather inefficient for pair production, it can lead to a build-up of particles in the CFLR over a time much longer than the pulsar period (cf. Section 4). For (2), charged particles from the ionization can be accelerated through either inward (towards the star) cross-field diffusion or resonant wave–particle interactions, which is similar to the particle acceleration in the van Allen belts in the Earth’s magnetosphere (Horne et al. 2005).

In principle, high-energy cosmic rays can also inject pairs into the trapping region. (Note that cosmic rays contribute to trapped plasmas in the van Allen belts through decay of upward deflected neutrons produced by high-energy cosmic rays.) For example, cosmic protons with a gyroradius larger than \( R_{LC} \) can drift into the pulsar magnetosphere. For a typical pulsar, the proton needs to have a Lorentz factor \( \gamma > \Omega_p/\Omega \sim 5 \times 10^4 \), where \( \Omega_p \) is the proton cyclotron frequency at the LC and \( \Omega = 2\pi/P \). Ultrarelativistic protons produce pairs on the thermal radiation through \( \gamma^2 \to p + e^\pm \). However, it can be shown that such a mechanism is not effective. Assuming a typical proton flux density similar to that received on the Earth, \( F_p \sim 5 \times 10^{-7} \text{ m}^{-2} \text{ s}^{-1} \) at 10 GeV, one finds a pair injection rate about \( 10^8 \text{ s}^{-1} \). It would take much longer than the Hubble time to fill the magnetosphere to the GJ density.

### 3 Dormant Outer Gap

We show that a dormant outer gap can supply pairs to the trapping regions. It is generally thought an outer gap exists between the neutron star and the pulsar magnetosphere. For a typical pulsar, the proton needs to have a Lorentz factor \( \gamma > \Omega_p/\Omega \sim 5 \times 10^4 \), where \( \Omega_p \) is the proton cyclotron frequency at the LC and \( \Omega = 2\pi/P \). Ultrarelativistic protons produce pairs on the thermal radiation through \( \gamma^2 \to p + e^\pm \). However, this can be shown to be inefficient. Assuming a typical proton flux density similar to that received on the Earth, \( F_p \sim 5 \times 10^{-7} \text{ m}^{-2} \text{ s}^{-1} \) at 10 GeV, one finds a pair injection rate about \( 10^8 \text{ s}^{-1} \). It would take much longer than the Hubble time to fill the magnetosphere to the GJ density.

#### 3.1 Acceleration in the outer gap

The accelerating electric field along the field lines in an outer gap can be written as a fraction of the maximum potential drop \( \Phi_m \) across the PC:

\[
E_{\parallel} = \frac{\Phi_m w_p}{R_{LC}},
\]

where \( w_p \leq 1 \) is a parameter characterizing the geometry of the gap and \( \Phi_m = 0.5\eta_p^2 B_0 R_0 \approx 1.7 \times 10^{13} \text{ V} (B_0/10^9 \text{ T})(P/2s)^{-2} \). Since pair production can change the gap geometry, the parameter \( w_p \) is usually determined self-consistently by taking into account the effect of the accelerating potential by pair creation (Takata et al. 2006). As we consider here specifically slowly rotating pulsars, in contrast to rapidly rotating young pulsars such as the Crab pulsar, pair production has little effect on the gap electrodynamics and thus, one can treat \( w_p \) as a constant.

Assume that the outer gap is located along the last open field line starting from the null surface at a radius \( r_{NS} \) and consider acceleration of an electron towards the star. Possible extrinsic sources of charged particles are discussed in Section 3.3. When curvature radiation is the dominant energy-loss process, an assumption used in the usual outer gap models (Cheng et al. 1986; Romani 1996), the radiation-reaction-limited Lorentz factor is

\[
\gamma_c = 2^{1/2}(w_p \gamma_m r_{NS})^{1/4} \left( \frac{R_{LC}}{r_c} \right)^{1/4}
\]

\[
\approx 4.1 \times 10^7 w_p^{1/4} r_{NS,0.5} \frac{P}{1.5 \text{ s}}^{-1/4} \left( \frac{B_p}{10^9 \text{ T}} \right)^{1/4},
\]

where \( \gamma_m = e\Phi_m/m_e c^2 \approx 3.2 \times 10^7 (B_0/10^9 \text{ T})(P/2s)^{-2} \), \( r_{NS} = 0.5 r_{NS,0.5} \), the curvature radius at \( r_{NS} \) is assumed to be \( R_{LC} = (4/3) r_{NS}^2 R_{LC} (1 - 3\eta_p/4)^{3/2} (1 - \eta_p r_{NS,2}/2) \), and \( r_c \approx 2.8 \times 10^{-15} \text{ m} \) is the classical radius of the electron. The characteristic energy \( (m_e c^2)^2 \) of curvature photons emitted by these electrons is

\[
e_c = \frac{3}{2} \frac{\lambda_C}{R_{LC}} \gamma^8 \approx 349 \eta_{NS,0.5}^{1/4} \frac{P}{1.5 \text{ s}}^{-7/4} \left( \frac{B_p}{10^9 \text{ T}} \right)^{3/4},
\]

where \( \gamma = \sqrt{1 + \eta_c} \), \( \eta_c = \hbar/m_e c \approx 3.9 \times 10^{13} \text{ m} \) is the Compton wavelength divided by \( 2\pi \). One may estimate the pulsar period range where the acceleration is limited by curvature radiation:

\[
P < 1.8 \frac{w_{p}^{7/4} \eta_{NS,0.5}^{-1/7}}{B_{p}^{1/9}} \text{ s}.
\]

The probability of a photon being converted to a pair through collision with thermal photons is tiny, and hence the bulk of curvature photons escape as an MeV gamma-ray flux with a luminosity \( L_{\gamma} \approx \varepsilon_{\nu} n_{LC} c R_{LC} T_{\gamma} \approx m_e^2 c^2 \approx 10^{39} \text{ J s}^{-1} \). For slowly rotating pulsars this flux may be too low to be detectable.

For pulsars with rotation periods that do not satisfy (11), primary particles may lose energy principally through inverse Compton scattering (ICS) on thermal radiation from the star’s surface. For a surface temperature \( T_s \), the number density of thermal photons with a characteristic energy \( k_B T_s \) is

\[
r_{\text{ph}} \approx \frac{2}{\pi^2} \left( \frac{\Theta_T}{\lambda_C} \right)^3 \left( \frac{R_0}{R_{LC}} \right)^2
\]

\[
\approx 1.3 \times 10^{17} \eta^{-2} \left( \frac{T_s}{10^6 \text{ K}} \right)^3 \left( \frac{P}{2 \text{ s}} \right)^{-2} \text{ m}^{-3},
\]

where \( \Theta_T = k_B T_s/m_e c^2 \approx 1.7 \times 10^{-4} (T_s/10^6 \text{ K}) \). Since the thermal photons propagate radially, backward moving electrons undergo head-on scattering on these photons. In the electron rest frame, the thermal photon energy is \( \gamma \Theta_T \gg 1 \), implying that the scattering is in the Klein–Nishina (KN) regime. The energy-loss rate is

\[
\frac{dy}{dt} \sim \frac{c r_T^2}{16 \lambda_C^2 T_s^2 \eta^2} \ln (2\gamma \Theta_T),
\]

with \( \sigma_T \) the Thomson cross-section.

#### 3.2 Pair creation

The only viable channel for pair creation far from the star’s surface is through photon–photon collision. The primary photons can be
produced from curvature radiation or IC. First, consider pair production due to curvature radiation. The number of curvature photons with energy $\epsilon > \epsilon_c$ emitted by a primary electron over a distance $\Delta \rho R_{LC}$ is written as

$$n_c \approx \frac{3^{3/2}a_t}{8\pi} \frac{\Delta \rho y}{\eta_{NS}} I(\epsilon/\epsilon_c),$$  \hspace{1cm} (14)

where $\Delta \rho < 1$ is the thickness (in units of $R_{LC}$) of the shell region, $a_t \approx 1/137$ is the fine constant and $K_{\nu}(\cdot)$ is the modified Bessel function. The number of curvature photons emitted decreases rapidly with increasing $\epsilon/\epsilon_c$. One has $n_c \approx 6.4 \times 10^3$ for $\epsilon \geq \epsilon_c$ and $n_c \approx 1.7 \times 10^3$ for $\epsilon \geq 2\epsilon_c$ and $n_c \approx 2 \times 10^2$ for $\epsilon \geq 4\epsilon_c$, with $\Delta \rho = 0.2$, $\eta_{NS} = 0.5$ and $\gamma = 3.2 \times 10^7$. The number of pairs per electron produced near the LC, that is, the multiplicity denoted by $M_{\gamma}$, is estimated as $M_{\gamma} \approx n_n \sigma_{\gamma \gamma} R_{LC} \Delta \rho$ where $\sigma_{\gamma \gamma} \approx 2 \times 10^{-2} \text{m}^2$ is the cross-section of pair production via photon–photon collision. Since pairs are produced on the Wien tail of the thermal spectrum, in steady pulsar period. As examples, from equation (17) one obtains $M_L \approx 4.3 \times 10^{-3} w_{\gamma}^{1/4} \frac{\epsilon}{10^7 \text{K}}^3 \left(\frac{P}{1.5 \text{s}}\right)^{-5/4} \left(\frac{B_{\gamma}}{10^9 \text{T}}\right)^{1/4} \left(\frac{T_c}{10^5 \text{K}}\right)^{2/3}$ \hspace{1cm} (17)

with $\Delta \rho_{0.2} = 0.2 \Delta \rho$. The multiplicity increases rapidly with decreasing pulsar periods. As examples, from equation (17) one obtains $M_L \approx 0.27$ for J1913+1333 with $T_c = 10^7 \text{K}$ and $M_L \approx 0.015$ for J1819–1458 with $T_c = 1.4 \times 10^6 \text{K}$, where $\eta_{NS} = 0.5$ and $\Delta \rho = 0.2$ is assumed.

Pair production due to IC becomes significant if the curvature photon energy is too low to satisfy the threshold condition (16). Since the scattered photon energy is $\epsilon_{\nu} \sim \gamma$ in the KN regime, the production rate of the scattered photons with energy $\epsilon_{\nu}$ is $dN_{\nu}/d\nu \approx -\gamma^{-1} d\gamma/d\nu$. The number of photons emitted through ICs over a distance $\eta_{NS} R_{LC}$ is

$$n_{ICS} \approx \sigma_{\gamma \gamma} R_{LC} \frac{\epsilon^2 \Delta \rho}{16\pi^3} \frac{\ln(2\gamma/\epsilon_c)}{\eta_{NS} y_{\gamma}},$$  \hspace{1cm} (18)

Similarly, one has $M_{\rho} \approx n_{ICS} \eta_{\rho} \sigma_{\gamma \gamma} R_{LC} \Delta \rho$, which takes the following numerical form:

$$M_L \approx 2.1 \times 10^{-4} \left(\frac{T_c}{10^7 \text{K}}\right)^5 \left(\frac{B_{\gamma}}{10^9 \text{T}}\right)^{-1} \frac{\Delta \rho_{0.2}}{\eta_{NS} 0.5}$$  \hspace{1cm} (19)

Pair production is strongly dependent on the surface temperature but insensitive to the pulsar period.

In the outer gap models, the primary particles are assumed to be created in a pair cascade in the gap (Cheng et al. 1986; Romani 1996) and the primary flux is limited to a value near the GJ flux. To allow a substantial energy transfer from primaries to pairs, as this is the case for high-energy pulsars, one must have the pair multiplicity

$$n_{\rho} \approx 3^{3/2} a_t \frac{\Delta \rho y}{\eta_{NS}} I(\epsilon/\epsilon_c),$$  \hspace{1cm} (14)

where $\eta_{NS}$ is the efficiency in converting the gravitational potential drop.

3.3 Source of primary particles

Low-level accretion of neutral grains from a dust disc or the ISM can provide extrinsic charged particles. The accreted neutral material can be ionized inside the pulsar magnetosphere by the thermal radiation from the star’s surface and particles produced from ionization are then channelled to the outer gap or trapped in the CFLR. The maximum flux that can be tolerated in the gap is about the GJ flux $cN_L/\eta_{NS}$ with the gap location assumed to be along the last open field lines starting from the null surface at the radius $\eta_{NS}$. It is unlikely that the supply of charged particles matches exactly the GJ flux and the acceleration may become oscillatory so that the net flux is maintained at about the GJ level. For the case of accretion of the ISM dust grains, the accretion rate is $M \approx (GM)^2 \rho_{ISM}/\epsilon^3 \sim 10^4 \text{kg s}^{-1}$, where $G$ is the gravitational constant, $M$ is the neutron star mass, $\epsilon$ is the pulsar velocity and $\rho_{ISM}$ is the ISM density. Dust grains of small size ($<0.1 \mu\text{m}$) may not survive sputtering by protons when crossing the bow shock and those with size $>0.1 \mu\text{m}$ have a better chance to reach the LC (Cheng 1985). Assuming that flux rate $F_{\rho}$ of electrons into the gap is 10 per cent of this flux rate, one estimates $F_{\rho} = cN_L \sim 0.1 c/2\pi R_{LC}^2 (M/\rho_{ISM}) \approx 5 \times 10^{13} \text{m}^{-2} \text{s}^{-1}$. Here we assume $v = 100 \text{km s}^{-1}$ and $\rho_{ISM} = 10^{-23} \text{kg m}^{-3}$. One finds this flux is considerably larger than the GJ flux $cN_L \sim 2.4 \times 10^{13} \text{m}^{-2} \text{s}^{-1}$. The accelerating electric field changes sign if the flux exceeds the GJ flux (Hirotani & Shibata 2002). Although a divergent solution for $E_{\rho}$ is possible (Hirotani & Shibata 2002), here we assume that the maximum accelerating potential is limited by (8) determined by the vacuum potential drop.

For pulsars with $T_c > 10^7 \text{K}$, the ISM grains may not survive disintegration (Cheng 1985) and in this case the disc is the main source of the accreted neutral matter. The disc model was discussed in detail by Cordes & Shannon (2006). The recent discovery of a fall-back disc around a magnetar lends support for the existence of such discs around long-period pulsars.

3.4 Particle loading in the CFLR

Despite the slow rate of pair production, a significant number of particles can be accumulated in the CFLR over a time $\gg P$. A schematic of pair injection by a latent outer gap is shown in Fig. 2.

One may estimate the time required for the plasma density to increase to the GJ density in the trapping region. Assume a particle flux in the gap is $cN_L/\eta_{NS}$. Acceleration of these particles can inject pairs into the CFLR at a rate $\pi N_{NS} c \delta \epsilon_{\gamma} R_{LC}^2/\eta_{NS}$ with the gap’s cross-sectional area approximated by $\pi \delta_1 \eta_{NS} R_{LC}^2$ (where $\delta_1 < 1$). A continuous injection of pairs can lead to build-up of the plasma density in the trapping region. The time for the density to reach the GJ density $N_{\rho}$ is

$$\tau_L = \frac{\Delta \rho P}{2\pi M \delta_1},$$  \hspace{1cm} (20)

We refer to (20) as the pair loading time. Numerical examples of $\tau_L$ are shown in Table 1, where we assume $\Delta \rho = 0.2$, $\eta_{NS} = 0.5$ and $\theta = \pi /3$. The surface temperature is assumed to be $T_c = 10^7 \text{K}$ for J1913+1333, $T_c = 1.4 \times 10^8 \text{K}$ for J1819–1458 (Reynolds et al. 2006) and $T_c = 1.5 \times 10^6 \text{K}$ for a magnetar. One has $\epsilon_c \gg \epsilon_{\gamma}$ for
these three pulsars. For J1317−5759, one finds $\tau_L \sim 1.5 \times 10^7$ s for $\theta = \pi/2$, $T_e = 1.5 \times 10^6$ K and $\eta_{NS} = 0.2$.

Alternatively, the plasma can be injected near the star through photon decay in the magnetic field (Wang et al. 1998). Such case was also considered by Rafikov & Goldreich (2005) for the double pulsar system. In their model, they considered pitch angle increase due to absorption of the coherent radio waves from the other pulsar in the system.

4 INJECTION THROUGH IONIZATION

Radiation belts can form directly through evaporation and ionization of neutral grains accreted from a disc or the ISM. One can show that the density of trapped plasmas can easily exceed the local GJ density with a slow accretion rate. For example, to accumulate a plasma with the GJ density over a time $\tau_L$, a rate as low as $cN_L/\tau_L$ would be sufficient. The trapped particles can be accelerated to relativistic energy through either cross-field diffusion towards the star or interactions with waves near the cyclotron resonance. Both processes are believed to be responsible for particle acceleration in the Earth’s van Allen belts where electrons are accelerated up to 10 MeV energy (Horne et al. 2005). In the cross-field diffusion, extremely low-frequency disturbances (ELFWs) in resonance with the particle drift frequency (around the star) can force particles to diffusion across the field lines inward (towards the star) where the magnetic field increases. Conservation of both the first adiabatic invariant $p^2/\mathcal{B}$ and the second adiabatic invariant $\oint dp_z$ (bouncing back and forth between two mirror points) implies an increase in both $p_1 \propto 1/r^3$ and $p_2 \propto 1/r^3$; thus, $p$ increases as $\tan \alpha = p_z/p_\parallel \propto 1/r^{1/2}$ increases with decreasing $r$ (inward), that is, the particles are accelerated. For the Earth’s radiation belts, ELFWs are generated due to the large-scale spatial disturbances by the solar wind. In the pulsar case, the most plausible candidate for such low-frequency waves is Alfvén waves (cf. Section 6).

5 STABILITY OF THE TRAPPED PLASMA

The pulsar radiation belts have many similarities to planetary radiation belts. For example, in both cases the stability of the trapped plasma can be strongly affected by change of the global magnetic field structure, that is, distortion of the ‘magnetic bottle’, or strong pitch-angle scattering of particles on waves. For the Earth’s van Allen belts, such activities are due to the disturbances from the solar wind, while activities in the pulsar radiation belts are powered by the rotational energy or magnetic energy. In magnetars, strong starquakes can lead to significant distortion of magnetic field lines, but such major events may not be common for normal pulsars. Here we only discuss low-level starquakes or neutron star oscillations (cf. Fig. 3), which can lead to pitch-angle scattering by Alfvén waves generated at the surface.

5.1 Pitch-angle scattering

A formal theory for diffusion of particles in momentum space is the quasi-linear diffusion formalism in which the processes arising from a feedback from growth of linear waves are determined by diffusion coefficients. In momentum space, such diffusion can be separated into diffusion in pitch angle and in $p$. The general formalism is outlined in Appendix B. Here we consider specifically the pitch-angle diffusion coefficient $\mathcal{D}_{\alpha\alpha}$ (cf. Appendix B). Quasi-linear diffusion occurs effectively through resonant wave–particle interactions. The most widely discussed process in the context of cosmic ray confinement is the scattering due to cyclotron resonance $\omega - k_z v_z - s\Omega_{e}/\gamma = 0$ with $s = \pm 1$ (Melrose 1980). In this case, the wave fluctuations can be regarded as predominantly magnetic and as the zeroth-order approximation in the expansion on $\omega/kc \ll 1$, the process can be viewed as elastic scattering of particles by magnetic fluctuations, causing change of their motion direction. Since the frequency of waves generated from starquakes (Blaes et al. 1989) or stellar oscillations (McDermott et al. 1988) is low, about kHz as compared to the cyclotron frequency $> 10$ MHz, the cyclotron resonance
condition is generally not satisfied except for the cases where these waves can cascade into high-frequency waves, for example, through three-wave interactions. Here we only consider pitch-angle scattering due to the Cerenkov resonance.

Unlike wave–particle interactions in the cyclotron resonance, the Cerenkov resonance only changes $p_f$ not $p_i$. In general, it leads to an increase in $p_f$, corresponding to a decreasing pitch angle $\alpha$. There are two relevant low-frequency wave modes: the X mode and the LO, corresponding, respectively, to the fast mode and the Alfvén mode in the magnetohydrodynamics. Pitch-angle diffusion due to the fast mode waves in Cerenkov resonance was discussed by Schlickeiser & Miller (1998). Since there is a non-zero $\delta B$ along the mean magnetic field, fast particles in the Cerenkov resonance bounce back and forth between two successive magnetic compressions (acting as two magnetic mirrors) in the wave frame. On average, particles gain energy causing an increase in $p_f$. This process, also called transit-time damping, works for a low Alfvén speed $v_A/v \ll 1$, a condition not satisfied for pulsars. So, instead we consider a low-frequency LO mode, which we refer to as the Alfvén wave. In kinetic theory in the limit $v_A \rightarrow c$, which is appropriate for pulsar magnetospheric plasmas, Alfvén waves have a non-zero $\delta E$ along the magnetic field (Arons & Barnard 1986; Melrose & Gedalin 1999) and particles can be accelerated through the Cerenkov resonance (Volokitin, Krasnosel’skikh & Machabeli 1985). The scattering time $t_s = 1/D_{\text{sc}}$ may be written in terms of the ratio of the magnetic energy density $U_B = B^2/2\mu_0$ of the wave to the plasma kinetic energy density $U_p = m_e c^2/\gamma$ (Appendix B)

$$t_s \approx \frac{1}{2\pi\omega} \left( \frac{U_B}{U_p} \right) \frac{\omega_p}{\omega} \frac{1}{\sin^3 \theta \sin^2 \alpha \cos \alpha}.$$

(21)

Considering a plasma consisting of electrons and positrons, for $U_B \sim U_p$, $\omega = 0.1\omega_p$, $\theta = 0.3$ and $\alpha = \pi/4$, one estimates $t_s \approx 1$ s. One concludes that diffusion due to particles in Cerenkov resonance with a low-frequency Alfvén wave can effectively transfer particles to small pitch angles.

It is appropriate to comment here that growth or damping of Alfvén waves through the Cerenkov resonance is generally not effective because the parallel component of the wave polarization is rather small, $e_p \sim (\sigma_0/\dot{\sigma_0})^2 \ll 1$. Thus, quasi-linear diffusion as a result of such linear wave growth is not significant, limited by $U_B/U_p \ll 1$ (cf. equation 21). This has been the main reason that such two-stage diffusion mechanism is not favoured for the interpretation of cosmic ray propagation, which, as inferred from observations, is subject to strong pitch-angle scattering in the ISM (Melrose 1980). However, such limit is not applicable in our case as Alfvén waves are assumed to be generated externally, not through the Cerenkov resonance, with magnetic energy density comparable with $U_p$, thus, efficient diffusion occurs even for $(\sigma_0/\dot{\sigma_0})^2 \ll 1$.

5.2 Precipitation

Disturbances to the radiation belt can be catastrophic if the wave that propagates to the region is in the form of short bursts producing $U_B \approx U_p$ in the trapping region. As shown from (21), the scattering time can be as short as seconds. The requirement of $U_B \approx U_p$ is rather modest. For example, for $(\gamma) = 5$, one has $U_p \approx 5 \times 10^{-7}$ J m$^{-3}$, giving a luminosity $\Delta \sigma R_p^2 U_p/\Delta t \approx 10^{16} \Delta \sigma (1/s/\Delta t) W$ where $\Delta t$ is the pulse duration. As this luminosity is minuscule compared with transient emission in magnetars (typically $10^{25}$ W), these waves are not significant in producing high-energy emission. Strong pitch-angle diffusion channels most particles into the much smaller loss cone through which particles can reach a region close to the star where their synchrotron decay time is short. This can cause a sudden intense precipitation of particles on a time-scale shorter than the pulsar period. One may estimate the required wave amplitude in terms of magnetic fluctuations as $\delta B/\sigma \approx (U_p/U_{\text{lin}})^{1/2} \approx 2 \times 10^{-3}$, implying low-intensity quakes or oscillations would be sufficient for triggering disruption of the trapped plasma.

If the triggering mechanism is starquakes, the re-occurrence time of such particle storms is essentially the re-occurrence time of the quakes. The physics of neuron starquakes is not well understood. A possibility is that a strong toroidal field exists in the crust underneath the surface and its stress can be released as quakes transferring the magnetic energy to the magnetosphere in Alfvén waves (Blaes et al. 1989; Thompson & Duncan 1995). One expects that such starquakes are relatively more frequent for a high-field, young pulsar. If generation of low-intensity Alfvén waves is so frequent that the re-occurrence time is much shorter than the pulsar period, trapped plasmas do not have enough time to accumulate and radiation belts may not form. This may be the case for magnetars with active transient emission.

6 APPLICATION TO RRATs

An important consequence of the formation of pulsar radiation belts is transient emission–radio bursts that can be produced as particles precipitate and such transient emission may be relevant to RRATs.

6.1 Coherent emission

RRATs are bright radio bursts with a flux density up to a few Jy at 1.4 GHz. As for normal radio pulses, the emission mechanism must be coherent. Since there is no widely accepted mechanism for the pulsar radio emission, it is expected that many uncertainties would remain in identifying the specific emission mechanism for the transient case. None the less, in the following we consider two possibilities in particular: two-stream instability and cyclotron/synchrotron maser.

6.1.1 Two-stream instability

The two-stream instability has been discussed extensively as a possible mechanism for pulsar radio emission, but the growth rate is not sufficient for the instability to develop in the polar region, mainly because of the very high Lorentz factor of the primary particle beam (Usov 1987). It is shown here that the plasma conditions created during the particle precipitation is favourable for such an instability to occur.

Two-stream instability can develop when particles traverse a corotating background plasma which is assumed to be stationary. Assuming the precipitating plasma density is $2M_{\odot}$ times the local GJ density $N_\odot \eta^{-3}$ and the spread in the particle momentum distribution is ignored (this spread can reduce the growth rate, cf. discussion below). Consider wave growth due to the Cerenkov resonance $\omega \approx k_1 v_1$. The characteristic frequency is determined by the denser plasma. For $2M_{\odot} \ll 1$, the characteristic frequency is the frequency $\omega_p = (e^2 N_\odot/\epsilon_0 m_e)^{1/2} \eta^{-3/2}$ of the stationary background, corotating plasma, with the backflowing plasma regarded as a weak beam satisfying the resonance condition. The growth rate is then estimated to be

$$\Gamma \approx \frac{\sqrt{3}}{2} \left( \frac{M_{\odot}^{1/3}}{2\eta} \right) \omega.$$  

(22)
The growth rate reaches maximum at $\gamma \rightarrow \gamma_c$ in the limit $p_\perp \rightarrow 1$. For $2M_L \gg 1$, the growth rate can be estimated in the rest frame of the backflowing plasma assuming that the corotating background plasma is a beam in Cerenkov resonance. One finds

$$\Gamma \approx \frac{\sqrt{3}}{2} \frac{\omega}{(4M_L)\gamma^{1/3}}. \quad (23)$$

where $\omega = \omega_0(2M_L \gamma)^{1/2}$ is the characteristic frequency seen in the pulsar frame.

Note that for two-stream instability in the polar cap model, the energetic particle beam is usually identified as those primary particles from the PC, with a Lorentz factor as high as $\sim 10^9$ and instability is strongly suppressed (Cheng & Ruderman 1977; Usov 1987). In the model discussed here $\gamma_c$ is about a few and the growth rate (22) or (23) can be quite fast. For $\gamma \rightarrow \gamma_c = 2M_L = 0.1$, (22) gives $\Gamma/\omega \approx 0.1$. Although the growth rate can be reduced due to a spread in the particle momentum distribution, it seems that the instability can develop rapidly. It should be pointed out here that radio emission from two-stream instability is a two-stage process. The instability can generate Langmuir waves which can in turn be converted to electromagnetic waves and at the same time facilitate the particle precipitation.

### 6.1.2 Cyclotron/synchrotron maser

Since particles with small pitch angles escape through the loss cone, particle precipitation creates an inversion in the pitch-angle distribution of the upward flowing particles, driving maser emission (Hewitt, Melrose & Rönnmark 1982; Melrose & Dulk 1982) or synchrotron maser (Zheleznyakov & Suvorov 1972; Yoon 1990; Hoshino & Arons 1991). This mechanism was discussed widely in the interpretation of planetary radio bursts (Melrose 1993; Ergun et al. 2000) and solar bursts (Melrose & Dulk 1982). Synchrotron maser is applicable in the relativistic regime $p_\perp > 1$, while cyclotron maser may be the more relevant when such inversion occurs close to the star during the precipitation when their gyration is in the non-relativistic regime. The relevant radio wave can grow when the frequency satisfies the cyclotron resonance condition. For cyclotron resonance at the fundamental harmonic $\omega = \Omega_c/\gamma - k_{\parallel}v_{\parallel} = 0$, the growth rate is given by (Melrose & Dulk 1982)

$$\Gamma \approx \frac{N_b}{N_L} \left( \frac{\omega_0}{\Omega_c} \right) \frac{\gamma^2}{\nu_0}. \quad (24)$$

where $N_b$ is the number density of particles in cyclotron resonance, and $\nu_0$ is the velocity of particles in cyclotron resonance. For a typical pulsar with $P = 1.5$ s and $B_0 = 10^6$ T, this condition can be satisfied at a radius $0.1R_{LC}$. It is worth noting that the related but different process–cyclotron instability due to the anomalous Doppler effect, which has been considered in the literature for pulsar radio emission (Kazbegi, Machabeli & Millikidze 1991), is not applicable here; it requires an energetic beam ($\gamma \sim 10^9$–$10^{10}$) of particles that satisfy the anomalous Doppler condition.

### 6.2 Beam properties

Bursts produced from particle precipitation are characterized by sharp spikes. In the case of two-stream instability, since emission arising from the instability is confined to a thin shell region in the CFLR near the last open field lines, one expects the beaming direction of the radio emission to be similar to that of reverse emission in the conventional polar cap models (Dyk & Dyks 2007). Consider that a trapping region has a radial range from $R_{LC}(1 - \Delta \psi)$ to $R_{LC}$. The inward tangent angles at $\eta$ are, respectively, given by $\psi_2$ and $\psi_1$. One has a conal structure with an angular thickness as $\Delta \psi = \psi_2 - \psi_1 \approx (3/4)\Delta \phi \gamma^{1/2}$ for $\Delta \phi \ll 1$. For the emission to be in radio, the emission radius has to be $\eta \sim 10^{-2}$, giving $\Delta \psi \approx 0.05$ for $\Delta \phi = 0.1$. Thus, the model predicts a rather narrow burst profile. Since particle precipitation and subsequent burst emission can be highly localized and occurs over a large range of altitudes, such bursts can appear on a wide range of pulsar phase.

For cyclotron maser, the beamwidth is $\Delta \theta \sim \nu_0/c \ll 1$ (Hewitt et al. 1982); thus, the maser emission can produce narrow spiky profiles. Since cyclotron maser emission occurs near particle reflection, the emission is beamed at a large angle to the field lines. This beaming feature is very different from the usual relativistic beaming ($1/\gamma \ll 1$) along the field lines. Such unusual beaming feature can be tested observationally if both bursts and weak emission (i.e. the usual pulsed emission) can be detected and relative phase between the two components can be determined.

### 6.3 Propagation

Propagation of radio emission in the CFLR is constrained by induced scattering by electrostatic waves excited by the radio emission itself (Luo & Melrose 2006). Assuming a brightness temperature $T_b$ and a beaming solid angle $\Delta \Omega$, one may define a radius of an opaque sphere (Luo & Melrose 2006),

$$\eta_{\text{cr}} \approx 3 \times 10^{13} \left( \frac{\Delta \Omega T_b}{10^{24} \text{K}} \right)^{2/7} \left( \frac{\nu}{1.4 \text{GHz}} \right)^{-2/7} \left( \frac{P}{2 \text{s}} \right)^{-10/7} \times \left( \frac{B}{10^9 \text{T}} \right)^{3/7}, \quad (25)$$

such that the wave with a frequency $\nu$ can only propagate outside the sphere $\eta > \eta_{\text{cr}}$. Here we assume that the plasma density in the propagation path is the local GJ density.

Although $\Delta \Omega T_b$ is not well constrained, the condition (25) is quite robust. For an observed flux density $S_{\text{obs}}$ and a pulsar distance $D_L$, the brightness temperature times the beaming solid angle can be estimated from

$$\Delta \Omega T_b \approx 2.3 \times 10^{23} \left( \frac{\Delta \Omega}{10^4 \text{sr}} \right)^{-2} \left( \frac{1.4 \text{GHz}}{\nu} \right)^2 \times \left( \frac{D_L}{3 \text{kpc}} \right)^2 \left( \frac{S_{\text{obs}}}{10^4 \text{Jy}} \right), \quad (26)$$

where $\Delta \Omega$ is the distance between the scattering region and the emission region. In the case of inward emission due to two-stream instability, (25) gives a lower limit to the emission radius; for the emission to be potentially visible, the emission radius must be $\eta > 4\eta_{\text{cr}}$. (Luo & Melrose 2006).

### 7 CONCLUSIONS AND DISCUSSION

We consider possible existence of transient radiation belts in the magnetospheres of pulsars with low LC magnetic fields. It is suggested that particle acceleration in latent outer gaps can lead to creation of pairs with large pitch angles in the CFLRs near the LC where they are trapped due to the magnetic mirror effect. In the trapping regions where the magnetic fields are weak ($\sim 10^{-4} T$), particles radiate away their perpendicular energy over a time much longer than a typical pulsar period. Thus, the plasma density can build up over...
a synchrotron cooling time, which is much longer than the pulsar period. Similar to the planetary radiation belts, pulsar radiation belts can be disrupted due to disturbances of waves propagating into the regions. Catastrophic disruption of these regions may lead to intense particle precipitation, which in turn generates radio bursts that can be seen as radio transients. The main triggering mechanism is scattering of trapped particles into a loss cone as a result of interactions with waves generated in plasma instabilities in the trapping regions or Alfvén waves emitted at the surface due to low-intensity starquakes or oscillations. The plasma that rushes through the background corotating plasma can produce bursts of coherent radio emission through streaming instability or cyclotron/synchrotron maser. Existence of such radiation belts, if observationally confirmed, would imply that some of the bursty phenomena of pulsar radio emission may be due to particle precipitation. In particular, we suggest that such TRE may be seen as RRATs.

A notable feature of our model, as applied to RRATs, is that in comparison with conventional polar cap models, the transient emission mechanism considered here is less constrained by the pair-production efficiency; it predicts that pulsars below the ‘deathline’ can still produce radio bursts. One of the major problems with polar cap models is the low efficiency of pair production. Most RRATs have long periods and for pulsars with such long periods, there is insufficient supply of pairs needed for coherent emission. Although an outer gap can supply additional pairs through acceleration of charged particles supplied externally (Cheng 1985; Ruderman & Cheng 1988; Cordes & Shannon 2006), the pair-production efficiency is too low to produce a substantial downflow of pairs; the only possible location for reverse emission in the polar regions is near or in the polar gap where a downward pair cascade can occur; for example, externally supplied particles flush the polar gap resulting in a downward transient pair cascade followed by quenching of the gap (Cordes & Shannon 2006). However, such a scenario would predict an emission region very close to the star and any reverse emission may be eclipsed by either the star or a dense plasma near the surface. In our model, since particle precipitation occurs in the CFLR, it does not quench the polar gap, allowing the possibility of observing both burst and normal radio emission.

In application to magnetars with active high-energy transient emission, one expects low-intensity quakes to occur much more frequently than in normal pulsars (Thompson & Duncan 1995). Such frequent disruption may prevent a radiation belt forming in a magnetar magnetosphere, even though the synchrotron decay time is relatively long near the LC. This scenario seems to be supported by lack of any detection of RRAT-like radio bursts from the known magnetars. Although TRE was recently detected from the magnetar XTE J1810−197, polarization study suggests that the emission geometry resembles that of young pulsars (Camilo et al. 2007).

In principle, our model may also apply to old, long-period pulsars. The loading time is significantly longer than the synchrotron decay time. The plasma trapping can be due to in situ ionization of the accreted neutral grains in the CFLR. Since the synchrotron decay time is long, for example, τs ≈ 16 yr for PSR J2144−3933 with P = 8.5 s and B = 4 × 10^9 T, a nearby long-period pulsar, accumulation of plasma can occur over a long time (provided that the re-occurrence time of the triggering mechanism is also long). Since old pulsars are not energetic, the predicted radio luminosity is low. As it takes a long time to replenish the trapped plasma, such transient events can be rather infrequent but may still be detectable provided these pulsars are located relatively nearby.

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APPENDIX A: BOUNCING TIME

From the first adiabatic invariant \( p^2 / B = \text{constant} \) and the energy conservation \( p^2 = \text{constant} \), one has

\[
p^2 = \frac{B(\eta)}{B(\eta_0)} p^2_\perp(\eta) + p^2_\parallel(\eta), \tag{A1}
\]

where the relevant quantities are functions of \( \eta \). Assuming \( \sin \alpha_0 = p_\perp(\eta_0) / p \), the time to travel from the injection radius \( \eta_i \) to the reflection radius \( \eta_f \) and back to \( \eta_i \) can be written in terms of integration along the particle’s path \( d s \):

\[
\Delta t = \frac{2}{v} \int \frac{d s}{1 - B(\eta)/B(\eta_0)} \approx \frac{R_{1C}}{v} \int_{\eta_i}^{\eta_f} \left( \frac{4 - 3\eta}{1 - \eta} \right)^{1/2} \frac{d \eta}{1 - (\eta_0 / \eta)^{3/2}}, \tag{A2}
\]

where we assume a particle follows the last closed field line of a dipole field and \( \eta_i < \eta_f \) satisfies the condition \( 1 - [B(\eta)] / [B(\eta_0)] \) \( \sin^2 \alpha_i = 0 \). The integration yields approximately \( \Delta t \approx \eta_i P / \eta_i \). One may assume without loss of generality \( \eta_i = 1 \) to show that the bounce time (back and forth between two mirror points) is \( \tau_0 = 2\Delta t \approx 2P / \eta_i \). The bounce time is not sensitive to the pitch angle at the injection provided that the condition \( \alpha_0 \geq \theta_i^0 \) is satisfied; this condition requires the reflection radius to be larger than the stellar radius, \( \eta_i \geq R_0 / R_{1C} \).

APPENDIX B: PITCH-ANGLE SCATTERING

We outline quasi-linear theory for pitch-angle scattering. Let \( f(p, \alpha) \) be a gyrophase-averaged distribution in the particle momentum space. In the quasi-linear diffusion formalism one has

\[
\frac{df}{dt} = \frac{1}{\sin \alpha} \frac{\partial}{\partial \alpha} \left[ \sin \alpha \left( D_{\alpha \alpha} \frac{\partial}{\partial \alpha} + \frac{\partial}{\partial \alpha} D_{\alpha p} \frac{\partial}{\partial \alpha} \right) f \right] + \frac{1}{p^2} \frac{\partial}{\partial \alpha} \left[ p^2 \left( D_{\alpha p} \frac{\partial}{\partial \alpha} + D_{pp} \frac{\partial}{\partial \alpha} \right) f \right], \tag{B1}
\]

where the diffusion coefficients are written in terms of the scattering probability \( w(s, p, k) \) and the wave occupation number \( N(k) \) (Melrose 1980):

\[
\begin{bmatrix}
D_{\alpha \alpha} \\
D_{\alpha p} \\
D_{pp}
\end{bmatrix} = \sum_k \int \frac{dk}{(2\pi)^3} w(s, k, p) N(k) \begin{bmatrix}
(\Delta \alpha)^2 \\
\Delta \alpha \Delta p \\
(\Delta p)^2
\end{bmatrix} \tag{B2}
\]

with \( D_{\alpha p} = \rho_{\alpha p} \Delta \alpha = h(\omega \cos \alpha - k_\parallel v) / (p \sin \alpha) \), and \( \Delta p = h \nu / v \). The scattering probability is

\[
w(s, p, k) = \frac{2\tau e^2}{e \nu_\alpha} \left| \mathbf{e} \cdot \mathbf{V}(s, p, k) \right|^2 \times \delta(\omega - k_\perp v - s \Omega_\perp / \gamma), \tag{B3}
\]

\[
V(s, p, k) = \left( \frac{1}{2} v_\perp \left[ e^{i \varphi} J_{s-1}(z) + e^{-i \varphi} J_{s+1}(z) \right], \right.
\]

\[
- \frac{1}{2} i \delta_s v_\parallel \left[ e^{i \varphi} J_{s-1}(z) - e^{-i \varphi} J_{s+1}(z) \right], \left. v_s J_s(z) \right), \tag{B4}
\]

with \( \delta_s = 1 \) for positrons and \( \delta_s = -1 \) for electrons, \( R \) is the ratio of the electric to total wave energy density, \( z = k_\parallel v \gamma / \Omega_\parallel \) and \( k = (k_\perp, \cos \phi, k_\parallel, \sin \phi, k_\parallel) \). In the following we consider the small \( z \ll 1 \) limit. The Bessel function has an approximation \( J_i \approx (z / 2)^i / (i + 1) \) for \( s \geq 1 \). This gives \( J_0 = -J_1 = J_1 \).

The polarization of a low-frequency Alfvén wave can be written as

\[
e_\perp \approx (\cos \phi, \sin \phi, e_{\lambda \parallel}), \tag{B5}
\]

where \( e_{\lambda \parallel} \approx (\omega / \nu_\parallel)^2 (k_\perp / k) m \) (Arons & Barnard 1986; Melrose & Gedalin 1999), and \( n \) is the refractive index. It can easily be verified that \( e_\perp \cdot V = e_{\lambda \parallel} v_\parallel \) for \( s = 0 \). Assuming \( R = 1 / 2 \) and \( \sin \theta = k_\perp / k \), one obtains

\[
w(0, p, k) = \frac{7\tau e^2 v_\parallel^2}{2e \nu_\alpha} \left( \frac{\omega}{\nu_\alpha} \right)^4 n^2 \sin^2 \theta \delta(\omega - k_\perp v). \tag{B6}
\]

The diffusion coefficient is

\[
D_{\alpha \alpha} \approx 2\tau \omega \left( \frac{U_\parallel}{U_\perp} \right) \left( \frac{k_\parallel}{\Delta k_\parallel} \right) \left( \frac{\nu_\alpha}{\omega} \right)^2 \sin^2 \theta \sin^2 \alpha \cos \alpha, \tag{B7}
\]

where \( \omega \approx k_\perp C, U_\parallel = \Delta B_\perp^2 / 2 \rho_0 \) is the magnetic energy density of the wave concerned, and \( U_\parallel = m_e c^2 (\gamma) N_\parallel M_\parallel \) is the density of the plasma kinetic energy. It should be emphasized here that the diffusion in pitch angle is caused by change in \( p_\parallel \) not \( p_\perp \).

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