Wavelet Adaptive Control for Robotic Manipulator With Input Deadzone

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Abstract. In this paper, the adaptive wavelet neural network (WNN) tracking control problem is investigated for robotic manipulator with input deadzone. The WNN is used to approximate the unknown nonlinear function and the derivative of virtual control in the system, which avoids the problem “explosion of complexity” in the traditional backstepping control methods, the requirement of input instruction signal is reduced. The robust term is designed to compensate the approximation errors of WNN and external disturbance. Since wavelet functions are used in WNN, its learning capability is precede to the traditional neural network for system identification. It is proved that the proposed controller can guarantee that all signals of the closed-loop system are uniformly ultimately bounded. Simulation results demonstrate the effectiveness of the proposed approach.

1. INTRODUCTION

In recent years, many research activities have emerged in the field of trajectory control of robotic manipulators. Some methods are proposed for the control problems of a single-link robotic manipulator in [1]-[5]. Such as, the disturbance observer method [1], the adaptive neural network control [2], the fuzzy logic control [3], the feedback linearization approach [4], and the slide model control method [5]. However, when designing the controller for the robotic systems, one of the most important issues to be addressed is external factors, such as input deadzone, saturation, and backlash. These non-linearities exist in the robotic actuators, and it is quite difficult to get their accurate models [6]. Ignoring these nonlinearities in order to simplify control design can lead to poor control effect and performance degradation [7].

Recently, many researchers have studied neural networks with new structures based on wavelet functions [8]. The learning ability of WNN is more effective than conventional neural networks in system identification because wavelet functions are spatial localized. The training algorithm of WNN has less convergence times than the conventional neural network [9]. Therefore, the control effect of the controller designed based on WNN is better than the traditional neural network.

There are indeed a lot of researchers dedicated to addressing or tackling nonlinear problems with input deadzone [10]. In [11], the WNN plays a significant role in approximating the unknown functions and unknown backlash in the transformed systems. However, few adaptive WNN control has been used for manipulator control problems with input deadzone.

Motivated by the above mentioned discussions, the main objective of this paper is to propose an adaptive WNN backstepping control approach for the robot manipulators with input deadzone. Compared with other existing methods, the WNN is used to approximate unknown nonlinear function in the system, which includes a subsystem of the derivative of the virtual control, avoiding the
“explosion of complexity” as mentioned in [12], at the same time, the requirement of input instruction signal is reduced, and the instruction signals are not required to be n-times differentiable. In addition, the robust term is designed to compensate for the deadzone and the negative influence of external disturbance on the robotic manipulator.

The surplus of this paper is divided into the following parts. Section 2 introduces the problem formulation and some preliminaries using the necessary lemmas, properties and assumptions. Section 3 is about the design of the adaptive WNN controller and stability analysis. Section 4 shows the simulation results using the proposed control method. Section 5 is the conclusion of this paper.

2. PROBLEM FORMULATION AND SOME PRELIMINARIES

2.1. Mathematical Model of the Robotic Manipulator

Considering a single link manipulator with motor characteristics [13]. The robotic manipulator model is shown in Fig. 1. From the Euler-Lagrange equation, the dynamic equation of such a system can be obtained as

\[
M_2 \ddot{\theta} + B \dot{\theta} - N f(x_3) = K_I I
\]

(1)

\[
L \dot{I} + R I + K_b = u - d(x,t)
\]

(2)

where \( \theta \) is the angular position of the link, \( M_r = J + \frac{1}{3} m l^2 + \frac{1}{10} M l^2 D \), \( N = m g l + M g l \), \( I \) is the armature current, \( f(x_i) (i=1,2,3) \) is a given nonlinear smooth function, and \( u \in \mathcal{U} \) is the output of the deadzone. \( B \) is the viscous friction coefficient of the bearing, \( J \) is the actuator torque, \( D \) is the load diameter, \( l \) is the link length, \( K_j \) is the torque constant, \( K_b \) is the back-emf constant, \( R \) is the armature resistance and \( L \) is inductance, \( m \) is the link mass, \( M \) is the load quality, \( d(x,t) \) is the unknown external disturbance, \( g \) is the acceleration of gravity.

![Fig. 1. The model of single link robotic manipulator](image)

Define the state variables \( x_1 = \theta \), \( x_2 = \dot{\theta} \), \( x_3 = I \), then (1) and (2) can be written as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= b_1 x_3 + f_1(x_1, x_2) \\
\dot{x}_3 &= b_2 u + f_2(x_1, x_2, x_3) + d(x,t) \\
y &= x_1
\end{align*}
\]

(3)
where \( f_1(x_1, x_2) = -\frac{B}{M_t} x_2 + \frac{N}{M_t} \sin(x_1) \), \( b_i = \frac{K_t}{M_t} \), \( f_2(x_1, x_2, x_3) = -\frac{R}{L} x_3 - \frac{K_t}{L} x_2 \), \( b_2 = \frac{1}{L} \).

On the basis of [14], \( u \) is the output of the deadzone, which is defined as follows

\[
\begin{cases} 
  m_i [v(t) - b_i] \quad & v(t) \geq b_i, \\
  0 \quad & -b_i < v(t) < b_i, \\
  m_i [v(t) + b_i] \quad & v(t) \leq -b_i,
\end{cases}
\]

where \( m_i \) represents the slope of the deadzone characteristic, \( v \in \mathcal{R} \) is the input to deadzone, \( b_i \) and \( b_i \) represent the breakpoints of the input deadzone. We can rewrite (4) as described in [15]

\[
\begin{cases} 
  -m_i b_i \quad & v(t) \geq b_i, \\
  -m_i v(t) \quad & -b_i \leq v(t) \leq b_i, \\
  m_i b_i \quad & v(t) \leq -b_i.
\end{cases}
\]

To facilitate the control system design, we need the following assumptions.

**Assumption 1** [10]: The deadzone coefficients \( m_i \), \( b_i \), \( b_i \) are unknown constants which satisfy the condition that \( b_i > 0 \), \( b_i > 0 \), \( m_i > 0 \).

**Assumption 2**: The external disturbance \( d(x, t) \) is bounded, and \(|d(x, t)| \leq \sigma \), \( \sigma \) is an unknown positive constant.

**Assumption 3**: The physical parameters of the robotic manipulator model, \( b_i \) and \( b_i \), are unknown but bounded by known positive constants \( b_{im} \), \( b_{im} \), which satisfies \( b_{im} \leq b_i \leq b_{im} \). \( f_1(x_1, x_2) \), \( f_2(x_1, x_2, x_3) \) are unknown smooth nonlinear functions.

### 2.2. WNN Identifier

The structure of WNN can be described as \( n \) inputs, one output, there are \( n \times l \) mother wavelet functions, \( l \) is the number of product layers. The first derivative of Gaussian function is selected as the mother wavelet function. Then the output of the product layer can be expressed as

\[
\Phi_i = \prod_{k=1}^{l} \phi_{ik} \left( x_k - m_{ik} \right) / d_{ik}, i = 1, \ldots, l.
\]

where \( x = [x_1, x_2, \ldots, x_n]^T \in \mathcal{R}^n \) is the input vector, \( \phi(\cdot) \) is the mother wavelet function. \( m_{ik} \) and \( d_{ik} \) are the dilation and translation parameters, respectively, \( \phi(x) = -x \exp((-1/2)x^2) \).

According to [16], the output of the WNN can perform the mapping as

\[
f(x) = W^T \Phi(x, d, m)
\]

where \( W \in \mathcal{R}^l \), \( \Phi(x, d, m) \in \mathcal{R}^l \), \( d \in \mathcal{R}^{neq} \), \( m \in \mathcal{R}^{neq} \). According to the universal approximation theorem proposed in [16], there is an ideal WNN identifier \( f^*(x) \), which is used to approximate \( f(x) \).

\[
f(x) = f^*(x) + \Delta = W^* \Phi(x, d^*, m^*) + \Delta
\]

where \( \Delta \) denotes the approximation error and is assumed to be bounded by \(|\Delta| \leq \Delta^* \), \( \Delta^* \) is positive constant, \( W^* \), \( d^* \), \( m^* \) are the optimal parameter vectors of \( W \), \( d \), \( m \), respectively. In fact, the optimal parameter vector used to estimate a given nonlinear function is difficult to determine. Hence, an estimate function is defined as

\[
\hat{f}(x, \hat{W}, \hat{d}, \hat{m}) = \hat{W}^T \Phi(x, \hat{d}, \hat{m})
\]
where \( \hat{W}, \hat{d}, \hat{m} \) are the estimation of \( W^*, d^*, m^* \), respectively. Define the estimated error as
\[
\hat{f} = f - \hat{f} = f^* - \hat{f} + \Delta = \hat{W}^T \hat{\Phi} + \hat{W}^T \hat{\Phi} + \hat{W}^T \hat{\Phi} + \Delta
\]
where \( \hat{W} = W^* - \hat{W}, \hat{d} = d^* - \hat{d}, \hat{m} = m^* - \hat{m} \).

According to Taylor linearization technique [17], \( \hat{\Phi} \) is linearized
\[
\hat{\Phi} = A^T \hat{d} + B^T \hat{m} + h
\]
where \( h \) is a vector of higher order terms.

\[
A = \begin{bmatrix} \frac{\partial \Phi}{\partial d} 
\quad \frac{\partial \Phi}{\partial d} 
\quad \vdots 
\quad \frac{\partial \Phi}{\partial m} \end{bmatrix}, 
B = \begin{bmatrix} \frac{\partial \Phi}{\partial m} 
\quad \frac{\partial \Phi}{\partial m} 
\quad \vdots 
\quad \frac{\partial \Phi}{\partial m} \end{bmatrix}.
\]

where \( \frac{\partial \Phi_i}{\partial d} \) and \( \frac{\partial \Phi_i}{\partial m} \) (\( i = 1, \cdots, l \)) are defined as follows
\[
\frac{\partial \Phi_i}{\partial d} = \left[ \frac{0}{(i-1)\times n} \frac{\partial \Phi_i}{\partial d_{i1}} \frac{0}{(i-1)\times n} \frac{\partial \Phi_i}{\partial d_{i2}} \frac{0}{(i-1)\times n} \frac{\partial \Phi_i}{\partial d_{in}} \right],
\frac{\partial \Phi_i}{\partial m} = \left[ \frac{0}{(i-1)\times n} \frac{\partial \Phi_i}{\partial m_{i1}} \frac{0}{(i-1)\times n} \frac{\partial \Phi_i}{\partial m_{i2}} \frac{0}{(i-1)\times n} \frac{\partial \Phi_i}{\partial m_{in}} \right].
\]

\[
\hat{f} = \hat{W}^T \left( \hat{\Phi} - A^T \hat{d} - B^T \hat{m} \right) + \hat{d}^T A \hat{W} + \hat{m}^T B \hat{W} + \epsilon
\]

3. ADAPTIVE WNN CONTROLLER DESIGN AND STABILITY ANALYSIS

Similar to other traditional backstepping methods [18], the specific process of controller design can be described in the following steps.

**Step 1:** The first tracking error of the subsystem is defined as
\[
e_1 = y - y_d
\]
Design the intermediate control function \( \alpha_1 \) as
\[
\alpha_1 = -k_1 e_1 + \hat{y}_d - u_{r1}
\]
where \( u_{r1} = \frac{e_1}{2\rho_1^2} \) is the robust item, \( \rho_1 > 0 \) is the design constant.

The second tracking error of the subsystem is defined as
\[
e_2 = x_2 - \alpha_1
\]
The time derivative of \( e_1 \) along with (3) and (13) is
\[
\dot{e}_1 = \dot{y} - \dot{y}_d = e_2 + \alpha_1 - \dot{y}_d
\]
Consider the following Lyapunov function candidate
\[
V_1 = \frac{1}{2} e_1^2
\]
The time derivative of \( V_1 \) along the solution of (14) and (16) is
\[
\dot{V}_1 = e_1 \dot{e}_1 = -k_1 e_1^2 + e_2 e_2 - \frac{e_1^2}{2\rho_1^2}
\]

**Step 2:** The third tracking error of the subsystem is defined as
\[
e_3 = x_3 - \alpha_2
\]
The time derivative of \( e_2 \) along with (3) and (15) is
\[
\dot{e}_2 = \dot{x}_2 - \dot{\alpha}_1 = b_1 e_3 + b_2 \alpha_2 + b_3 \overline{f}_1(x_1, x_2)
\]
where \( \overline{f}_1(x_1, x_2) \) is approximated by \( \hat{f}_1(x_1, x_2) = \hat{W}_1^T \hat{\Phi} \).
Define the robust term as 
\[ u_{r_2} = \frac{e_2}{2\rho_2}, \quad \rho_2 > 0 \]
the design constant. Then the intermediate control function \( \alpha_2 \) can be chosen as
\[ \alpha_2 = -k_2e_2 - e_1 - \hat{f}_1(x_1, x_2) - u_{r_2} \]  
(21)

Design the parameter adaptive laws as follows
\[ \dot{\hat{W}}_1 = \gamma_1e_2(\Phi_1 - A^T\hat{M}_1 - B^T\hat{m}_1), \quad \dot{\hat{d}}_1 = \gamma_1e_2A_1\hat{W}_1 + \hat{m}_1 = \gamma_1e_2B_1\hat{W}_1. \]  
(22)

Consider the following Lyapunov function candidate
\[ V_2 = V_1 + \frac{1}{2b_2}e_2^2 + \frac{1}{2\gamma_1}\hat{W}_1^T\hat{W}_1 + \frac{1}{2\gamma_1}\hat{m}_1\hat{m}_1 + \frac{1}{2\gamma_1}\hat{d}_1\hat{d}_1 \]  
(23)

where \( \gamma_1 \) is the learning rates with positive constants.

The time derivative of \( V_2 \) along the solutions of (20), (21) and (22) is
\[ \dot{V}_2 \leq -\sum_{j=1}^2 k_j e_j^2 + e_2e_3 + \frac{\delta e_1^2}{2} \]  
(24)

**Step 3:** The last step, the deadzone input \( v \) will be obtained. The time derivative of \( e_1 \) along with (3) and (19) is
\[ \dot{e}_1 = \dot{x}_1 - \alpha_2 = b_1(m_1v + \hat{m}_1 + \hat{h}_1 + \hat{h}_1) + b_2\hat{f}_1(x_1, x_2, x_3) + d(x, t) \]  
(25)

where \( \hat{m}_1 \) and \( \hat{h}_1 \) are the estimated values of the deadzone parameters, \( \hat{m}_1 = m_1 - \hat{m}_1 \) and \( \hat{h}_1 = h_1 - \hat{h}_1 \) are parameter errors, \( \hat{f}_1(x_1, x_2, x_3) = \frac{f_1(x_1, x_2, x_3) - \alpha_2}{b_2} \) is approximated by \( \hat{f}_1(x_1, x_2, x_3) = \hat{W}_2^T\Phi_1 \).

Define the robust term as \( u_{r_3} = \frac{e_3}{3\rho_3^3}, \quad \rho_3 > 0 \) is the design constant.

Deadzone input \( v \) and parameter adaptation functions can be designed as
\[ v = \frac{1}{\hat{m}_1}[-k_3e_3 - e_1 - \hat{f}_1(x_1, x_2, x_3) - \hat{h}_1 - u_{r_3}] \]  
(26)

Design parameter adaptation laws as follows
\[ \dot{\hat{W}}_2 = \gamma_2e_3(\Phi_2 - A_2^T\hat{M}_2 - B_2^T\hat{m}_2), \quad \dot{\hat{d}}_2 = \gamma_2e_3A_2\hat{W}_2 + \hat{m}_2 = \gamma_2e_3B_2\hat{W}_2 \]  
(27)

\[ \hat{m}_1 = \begin{cases} \delta e_1v & \text{if } |m_1| > M \\ -\delta e_1v & \text{if } |m_1| \leq M \\ \end{cases} \]  
(28)

where \( \gamma_2 > 0, \ M > 0 \) and \( \delta > 0 \) are design parameters.

**Theorem 1:** It is assumed that the system (3) can be controlled by the proposed control laws (26) and parameter adaptation laws (22), (27) and (28). If the proposed output feedback control system satisfies **Assumptions 1-3**, then all signals of the closed-loop system are uniformly ultimately bounded.

Consider the following Lyapunov function candidate as
\[ V = V_2 + \frac{1}{2b_2}e_2^2 + \frac{1}{2\gamma_2}\hat{W}_2^T\hat{W}_2 + \frac{1}{2\gamma_2}\hat{m}_2^T\hat{m}_2 + \frac{1}{2\gamma_2}\hat{d}_2^T\hat{d}_2 + \frac{1}{2\delta}\hat{m}_1\hat{m}_1 + \frac{1}{2\delta}\hat{h}_1^T\hat{h}_1 \]  
(29)

**Proof:** The time derivative of \( V \) along the solutions of (25), (26), (27) and (28) is
\[ \dot{V} \leq -\sum_{j=1}^{3} k_j e_j^2 + \sum_{j=2}^{3} \frac{\rho_j^2 e_{j-1}^2}{2} + |e_3|^2 \sigma \quad (30) \]

Notice the fact that for any positive constant \( \omega > 0 \)

\[ 2ab \leq \frac{1}{\omega} a^2 + \omega b^2 , \forall a, b > 0. \quad (31) \]

According to the inequality (30), one has

\[ \dot{V} \leq -\sum_{j=1}^{3} k_j e_j^2 + \sum_{j=2}^{3} \frac{\rho_j^2 e_{j-1}^2}{2} + \frac{1}{2\omega} e_3^2 + 2\omega \sigma^2. \quad (32) \]

Let \( a_0 = \min\{k_1, k_2, k_3 - \frac{1}{2\omega}\} \), \( D_0 = \sum_{j=2}^{3} \frac{\rho_j^2 e_{j-1}^2}{2} + 2\omega \sigma^2 \).

Then (32) can be rewritten as

\[ \dot{V} \leq -a_0 \sum_{j=1}^{3} e_j^2 + D_0 \quad (33) \]

When \( \sum_{j=1}^{3} e_j^2 > \frac{D_0}{a_0} \) is established, there is \( \dot{V} < 0 \).

The above analysis shows that \( V(t) \) is uniformly ultimately bounded. From (29), \( e_i, \tilde{W}_i, \tilde{d}_i, \tilde{m}_i \), \( i = 1, 2, 3 \), \( m_i \) and \( h_i \) are uniformly ultimately bounded. Considering that \( e_i = y - y_d, y_d \) and \( e_i = x_i - \alpha_{i-1} (i = 2, 3) \) are bounded, then \( x_i (i = 1, 2, 3) \) is bounded, hence all the signals in the closed-loop system are uniformly ultimately bounded, and the closed-loop system is stable.

4. SIMULATION

For the sake of argument of the proposed control method, simulation is designed to control the robotic system, where \( B = 1.5 \times 10^{-2} \) N m/s/\( \text{rad} \), \( L = 8 \times 10^{-4} \) H, \( D = 5 \times 10^{-2} \) m, \( R = 7.5 \times 10^{-2} \) \( \Omega \), \( m = 1 \times 10^{-7} \) kg, \( J = 5 \times 10^{-2} \) kg m\(^2\), \( l = 0.6 \) m, \( K_b = 8.5 \times 10^{-2} \) N m/A, \( M = 0.05 \) kg, \( K_t = 1 \), \( g = 9.8 \) N/kg, \( f(x_i) = \sin (x_i) \) is a given nonlinear smooth function, \( d(x, t) = 4 \sin(t) \) is the unknown external disturbance. The unknown deadzone parameters described in (4) are given as \( m_i = b_i = h_i = 1 \). The desired trajectory is chosen as \( y_d = \sin(t) \). The initial conditions are chosen as \( x_1(0) = x_2(0) = x_3(0) = 0, \tilde{W}_1(0) = \tilde{W}_2(0) = 0.25, \tilde{m}_1(0) = \tilde{m}_2(0) = 0.25, \tilde{d}_1(0) = \tilde{d}_2(0) = 0.25, \tilde{h}_1(0) = 0.25, \).

The design parameters in the control scheme are chosen as

\[ k_1 = 50, \quad k_2 = 5, \quad k_3 = 5, \quad \gamma_1 = 15, \quad \gamma_2 = 5, \]

\[ M = 0.4, \quad \delta = -0.001, \quad \rho_1 = 30, \quad \rho_2 = 10, \quad \rho_3 = 10. \]

The simulation results are given in Figs. 2-5. Fig. 2 shows the trajectories of the control \( u \), Fig. 3 shows the trajectories output \( y \) and reference signal \( y_d \), Fig. 4 shows the trajectories of the WNN used to estimate \( \tilde{f}_1(x_1, x_2) \) and \( \tilde{f}_2(x_1, x_2, x_3) \), Fig. 5 shows the trajectories of \( \hat{h} \) and \( \hat{m} \).
5. CONCLUSION
In this paper, an adaptive WNN control method is proposed for a robotic manipulator with input deadzone. By employing the backstepping method, the new WNN adaptive controller is designed to achieve the output trajectory. The WNNs are used to approximate the virtual control laws generated in the backstepping control procedure and the unknown nonlinear function of the system. All the signals in the closed-loop system can be proved to be uniformly ultimately bounded. Further research will focus on the WNN control methods for the output tracking error of the robotic manipulator with input deadzone converges to zero.

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