Signs of Dynamical Dark Energy in Current Observations

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Abstract. Investigations on dark energy (DE) are currently inconclusive about its time evolution. Hints of this possibility do however glow now and then in the horizon. Herein we assess the current status of dynamical dark energy (DDE) in the light of a large body of updated SNIa+$H(z)$+BAO+LSS+CMB observations, using the full Planck 2015 CMB likelihood. The performance of the ΛCDM model (with equation of state $w = -1$ for Λ) is confronted with that of the general XCDM and CPL parametrizations, as well as with the traditional φCDM model based on the scalar field potential $V \sim \phi^{-\alpha}$. In particular, we gauge the impact of the bispectrum in the LSS and BAO parts, and show that the subset of CMB+BAO+LSS observations may contain the bulk of the DDE signal. The departure from $w = -1$ is significant: roughly $2.6\sigma$ for XCDM and $2.9\sigma$ for φCDM. In both cases the full Bayesian evidence is found to be positive even for a prior range of the DDE parameters extending over several standard deviations from the mean when the bispectrum is taken into account. Positive signs follow as well from the preliminary results of Planck 2018 data using the compressed CMB likelihood. As a bonus we also find that the $\sigma_8$-tension becomes reduced with DDE.

Keywords: cosmological parameters, dark energy, large-scale structure of universe
1 Introduction

Even if the existence proof of the DE is not iron-clad, the great majority of cosmologists are agreed that the universe is in accelerated expansion \cite{1,2} and that some physical cause must be responsible for it. The canonical picture in GR is to assume that such cause is the presence of a cosmological constant (CC) term, \( \Lambda \), in Einstein’s equations. No reason, however, is given for its constancy throughout the entire cosmic history. In fact, \( \Lambda = \text{const.} \) is not required by the cosmological principle. Such oversimplification might be at the root of the Cosmological Constant Problem, namely the appalling discrepancy between the measured value of the vacuum energy density, \( \rho_\Lambda = \Lambda/(8\pi G) \sim 10^{-47} \text{ GeV}^4 \) (\( G \) being Newton’s constant), and the incommensurably larger value predicted in quantum field theory (QFT) \cite{3,5}. The cosmological constant problem stays unsolved right now, even after more than a full century after \( \Lambda \) was first introduced by Einstein. Most likely the mystery will remain for a long time still. In this work, we will not attempt to solve it. Our main aim is much more humble. Somehow we wish to follow the original phenomenological approach that made possible to unveil that \( \Lambda \) is nonvanishing, irrespective of its ultimate physical nature. The method to substantiate that \( \Lambda \) is non-null was largely empirical, namely \( \Lambda \) was assumed to be a parameter and then fitted directly to the data. Of course a minimal set of assumptions had to be made, such as the validity of the Cosmological Principle and hence of the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, with the ensuing set of Friedmann equations for the scale factor. In our case, we wish of course to keep these minimal assumptions and make a phenomenological case study of the next-to-leading possibility, which is that the \( \Lambda \), or in general the DE, might be not just a parameter but a slowly varying cosmic variable that stays close to a constant for cosmic spans of time and therefore capable of mimicking the \( \Lambda \text{CDM} \) like behavior.

This is all the most interesting if we take into account that, aside from theoretical problems, a number of persisting tensions with the data (particularly on the local value of the Hubble parameter \cite{6} and the large scale structure formation data \cite{7}) suggest that the concordance or \( \Lambda \text{CDM} \) model, with rigid \( \Lambda \)-term, might be performing insufficiently at the observational level. The possibility to alleviate some of these problems by assuming that \( \rho_\Lambda \), or in general the DE, might be (slowly) dynamical is a natural option to test, see e.g. \cite{8,11}. Here we are primarily spurred by these phenomenological considerations and the disparity of different results. For instance, hints of DDE were highlighted in \cite{12,14}. In addition, nonparametric studies of the DDE suggest positive evidence at about 3.5\( \sigma \) c.l., see \cite{15}. Such confidence level is actually comparable to that achieved through specific models of DDE, in which attempts were made to cure the mentioned tensions – see e.g. \cite{16,19} and references therein. Other recent works, however, find a variety of levels of evidence in different contexts or simply no sizeable deviation from the \( \Lambda \text{CDM} \), see \cite{20,26}, for example.

In this Letter, we reassess the current situation on DDE. The novelty is that we pay due attention to the sensitivity of the DDE signal to potentially relevant features sitting in the BAO and LSS data which have not been explored in previous studies. We try to clarify why some authors find evidence in contrast to others by noting that the specific ingredients of the data
The remaining two conventional parameters (ζ and CPL, as well as with a traditional scalar field model (φCDM considering the reaction of well-known generic parametrizations of the DDE, such as the XCDM with the power spectrum), may develop more sensitivity to DDE. We prove that this is the case by usually not taken into account (most noticeably the inclusion of the matter bispectrum together basis of a large set of updated SNIa+

\[ \varphi \text{CDM} = \Omega_{\text{cdm}} h^2 \text{ and } \varphi_0 = \Omega_b h^2 \text{ for cold dark matter and baryons. For convenience, instead of } A_\alpha \text{ we list the values of } \sigma_8(0). \]

The remaining two conventional parameters (\( n_+, \tau \)) are not quoted.

Table 1: The mean fit values for the considered models using dataset DS1, i.e. all SNIa+H(z)+BAO+LSS+CMB data, with full Planck 2015 CMB likelihood. In all cases a massive neutrino of 0.06 eV has been included. The first block involves BAO+LSS data using the matter (power) spectrum (SP) and is labelled DS1/SP. The second block includes both spectrum and bispectrum, and is denoted DS1/BSP (see text). We display the fitting results for a few parameters only, among them those that characterize the considered DDE models: the EoS parameter \( \varphi_0 \) for XCDM, \( \varphi_0 \) and \( \varphi_1 \) for the CPL, the power \( \alpha \) of the potential and \( \beta \equiv \kappa [\text{Mpc}/(\text{km/s/Mpc})]^2 \) for φCDM, as well as three conventional parameters: \( H_0 \) (Hubble parameter), \( \omega_{\text{cdm}} = \Omega_{\text{cdm}} h^2 \) and \( \omega_b = \Omega_b h^2 \) for cold dark matter and baryons. For convenience, instead of \( A_\alpha \) we list the values of \( \sigma_8(0) \).

2 DDE parametrizations and models

We consider two generic parametrizations of the DDE, together with a well-known φCDM model, and confront them to a large and updated set of SNIa+H(z)+BAO+LSS+CMB observations. Natural units are used hereafter, although we keep explicitly Newton’s \( G \), or equivalently the Planck mass: \( M_P = 1/\sqrt{G} = 1.2211 \times 10^{19} \text{ GeV} \). Flat FLRW metric is assumed throughout: \( ds^2 = -dt^2 + a(t)(dx^2 + dy^2 + dz^2) \), where \( a(t) \) is the scale factor as a function of the cosmic time.

2.1 XCDM and CPL

The first of the DDE parametrizations under study is the conventional XCDM [27]. In it both matter and DE are self-conserved (non-interacting) and the DE density is simply given by \( \rho_X(a) = \rho_0 a^{-3(1+w_0)} \), where \( \rho_0 = \rho_\Lambda \) is the current value and \( w_0 \) the (constant) equation of state (EoS) parameter of the DE fluid. For \( w_0 = -1 \) we recover the XCDM model with a rigid CC. For \( w_0 \gtrsim -1 \) the XCDM mimics quintessence, whereas for \( w_0 \lesssim -1 \) it mimics phantom DE. It is worth checking if a dynamical EoS for the DE can furnish a better description of the observational
data. So we consider the well-known CPL parametrization \[28, 29\], which is characterized by the following EoS:

\[ w(a) = w_0 + w_1 (1 - a) = w_0 + w_1 \frac{z}{1 + z}, \tag{1} \]

where \( z = a^{-1} - 1 \) is the cosmological redshift. The corresponding Hubble rate \( H = \dot{a}/a \) normalized with respect to the current value, \( H_0 = H(a = 1) \), takes the form:

\[ E^2(a) = \frac{H^2(a)}{H_0^2} = (\Omega_b + \Omega_{cdm}) a^{-3} + \Omega_\gamma a^{-4} + \frac{\rho_\nu(a)}{\rho_c} + \Omega_\Lambda a^{-3(1+w_0+w_1)} e^{-3w_1(1-a)}. \tag{2} \]

Here \( \Omega_i = \rho_i/\rho_c \) are the current energy densities of baryons, cold dark matter, photons and CC/DE normalized with respect to the present critical density \( \rho_c \). The neutrino contribution, \( \rho_\nu(a) \), is more complicated since it contains a massive component, \( \rho_{\nu,m}(a) \), apart from the massless ones. During the expansion of the universe, the massive neutrino transits from a relativistic to a nonrelativistic regime. This process is nontrivial and has to be solved numerically. In the above expression, for \( w_1 = 0 \) we recover the XCDM, and also setting \( w_0 = -1 \) we are back to the ΛCDM.

### 2.2 φCDM

Let us now briefly summarize the theoretical framework of the φCDM, which has a well-defined local Lagrangian description. The DE is described here in terms of a scalar field, \( \phi \), which we take dimensionless. Such field is minimally coupled to curvature (\( R \)) and the generic action is just the sum of the Einstein-Hilbert action \( S_{EH} \), the scalar field action \( S_{\phi} \), and the matter action \( S_m \):

\[ S = S_{EH} + S_{\phi} + S_m = \frac{M_P^2}{16\pi} \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} g^{\mu\nu} \partial_{\mu}\phi \partial_{\nu}\phi - V(\phi) \right] + S_m. \tag{3} \]

Dots indicate derivatives with respect to the cosmic time. The energy density and pressure of \( \phi \) follow from its energy-momentum tensor, \( T^{\phi}_{\mu\nu} = -(2/\sqrt{-g})\delta S_{\phi}/\delta g^{\mu\nu} \), and the fact that \( \phi \) is an homogeneous scalar field which depends only on the cosmic time. Thus,

\[ \rho_\phi = T_{00}^\phi = \frac{M_P^2}{16\pi} \left( \frac{\dot{\phi}^2}{2} + V(\phi) \right) \quad p_\phi = T_{ii}^\phi = \frac{M_P^2}{16\pi} \left( \frac{\dot{\phi}^2}{2} - V(\phi) \right). \tag{4} \]

Notice that within our conventions \( V(\phi) \) has dimension 2 in natural units.

The field equation for \( \phi \) is just the Klein-Gordon equation in curved spacetime, \( \Box \phi + \partial V/\partial \phi = 0 \), which for the FLRW metric leads to

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 0. \tag{5} \]

The corresponding Einstein’s field equations read:

\[ 3H^2 = 8\pi G (\rho_m + \rho_r + \rho_\phi) \tag{6} \]

\[ 3H^2 + 2\dot{H} = -8\pi G (p_m + p_r + p_\phi). \tag{7} \]

Here \( \rho_m = \rho_b + \rho_{cdm} + \rho_{\nu,m} \) involves the pressureless contributions from baryons and cold dark matter as well as the massive neutrino contribution. The latter evolves during the cosmic expansion
Figure 1: Likelihood contours for the XCDM parametrization (left) and the considered $\phi$CDM model (right) in the relevant planes ($\omega_{\text{cdm}}, w_0$) and ($\omega_{\text{cdm}}, \alpha$) respectively, after marginalizing over the remaining parameters. Dataset DS2/BSP is used in both cases. The various contours correspond to 1$\sigma$, 2$\sigma$ and 3$\sigma$ c.l. The central values in both cases are shifted $>2.5\sigma$ away from the $\Lambda$CDM, i.e. from $w_0 = -1$ and $\alpha = 0$ in each case. Marginalization also over $\omega_{\text{cdm}}$ increases the c.l. up to 2.6$\sigma$ and 2.9$\sigma$, respectively (cf. first block of Table 2).

from the relativistic regime (where $p_m = p_{\nu,m} \neq 0$) to the nonrelativistic one (where $p_{\nu,m} \simeq 0$). On the other hand, $p_r = \rho_r/3$ is the purely relativistic part from photons and the massless neutrinos.

As a representative potential for our analysis we borrow the traditional quintessence potential by Peebles and Ratra [30],

$$V(\phi) = \frac{1}{2} \kappa M_P^2 \phi^{-\alpha},$$  \hspace{1cm} (8)

where $\kappa$ is a dimensionless parameter. In this context, the value of the CC at present ($t = t_0$) is given by $\rho_\Lambda = \rho_\phi(t_0)$. We expect $\kappa$ to be positive since we know that $\rho_\Lambda > 0$ and the potential energy at present is dominant. In what follows, when referring to the $\phi$CDM model we will implicitly assume this form of the potential. The power $\alpha$ in it should be positive as well, but sufficiently small so that $V(\phi)$ mimics an approximate CC slowly evolving with time. In our case we take $\alpha$ as a free parameter while $\kappa$ is a derived one, as discussed below.

The Klein-Gordon equation with the potential (8) can be written in terms of the scale factor as follows (prime stands for $d/da$, and we use $d/dt = aHd/da$):

$$\phi'' + \left(\frac{H'}{H} + \frac{4}{a}\right) \phi' - \frac{\alpha \kappa M_P^2 \phi^{-(\alpha+1)}}{2(aH)^2} = 0.$$  \hspace{1cm} (9)

When the cosmic evolution is characterized by the dominant energy density $\rho(a) \propto a^{-n}$ ($n = 3$ for the matter-dominated epoch and $n = 4$ for radiation-dominated), i.e. when the DE density is negligible, one can search for power-law solutions of the Klein-Gordon equation, i.e. solutions of the form $\phi \propto a^\alpha$. However, around the current time the effect of the potential is significant and the solution has to be computed numerically. Equation (9) is coupled to the cosmological equations under appropriate initial conditions. We can set these conditions at the radiation epoch, where we can neglect the DE. We find

$$\phi(a) = \left[\frac{3\alpha(\alpha + 2)^2 \kappa M_P^4}{64\pi(\alpha + 6)} (\rho_r(a) + \rho_{\nu,m}(a))^{-1}\right]^{1/(\alpha+2)}.$$  \hspace{1cm} (10)
BAO+LSS+CMB data. These ingredients may be particularly sensitive to the DDE, as shown in these data, see [19]. In this study, however, we wish to isolate also the effect from the triad of that we take into account all the known correlations among data.

It is important to emphasize point from the Weak Lensing observable $S_f$ cosmic chronometers [36–43]; iii) 16 effective BAO points [44–48]; iv) 19 effective points from LSS, $\sigma_s(0)$ method, see [33] for a detailed exposition, what allowed us to consistently determine the value of $\phi$ replacing a boundary value problem with an initial value one through a large number of iterations of the initial conditions until finding the optimized solution. For the $\phi$CDM, the initial conditions are determined by (10) and its time derivative, which are both explicitly dependent on $\phi$. Thus, Eq.(10) together with $\rho, \rho_{\nu,m}$ is the total radiation energy density, which includes also the massive neutrino contribution. At the radiation epoch, however, $\rho_{r,\text{tot}}$ behaves very approximately as $\sim a^{-4}$. Then, $\rho_r + \rho_{\nu,m}$ is the total radiation energy density, which includes also the massive neutrino contribution. At the radiation epoch, however, $\rho_{r,\text{tot}}$ behaves very approximately as $\sim a^{-4}$. Thus, Eq.(10) together with $\phi(a)$ obtained from it can be used for the numerical solution of Eq. (9) and of the EoS of the scalar field at any subsequent epoch: $w_\phi(a) = p_\phi(a)/\rho_\phi(a)$.

Notice that $\rho_{r,\text{tot}} \equiv \rho_r + \rho_{\nu,m}$ is the total radiation energy density, which includes also the massive neutrino contribution. At the radiation epoch, however, $\rho_{r,\text{tot}}$ behaves very approximately as $\sim a^{-4}$. Thus, Eq.(10) together with $\phi(a)$ obtained from it can be used for the numerical solution of Eq. (9) and of the EoS of the scalar field at any subsequent epoch: $w_\phi(a) = p_\phi(a)/\rho_\phi(a)$.

To compare the theoretical predictions of the different models under study with the available observational data we have made use of the Boltzmann code CLASS [31] in combination with the powerful Monte Carlo Markov chain (MCMC) sampler MontePython [32]. In the particular instance of $\phi$CDM, we have conveniently modified CLASS such as to implement the shooting method, see [33] for a detailed exposition, what allowed us to consistently determine the value of $\kappa$ for each value of the free parameter $H_0$. As it is well-known, such numerical technique consists in replacing a boundary value problem with an initial value one through a large number of iterations of the initial conditions until finding the optimized solution. For the $\phi$CDM, the initial conditions are determined by (10) and its time derivative, which are both explicitly dependent on $\kappa$.

## 3 Data

To generate the fitting results displayed in Tables 1 and 2 we have run the MCMC code MontePython, together with CLASS, over an updated data set SNIa+$H(z)$+BAO+LSS+CMB, consisting of: i) 6 effective points on the normalized Hubble rate (including the covariance matrix) from the Pantheon+MCT sample [34, 35], which includes 1063 SNIa; ii) 31 data from $H(z)$ from cosmic chronometers [36, 43]; iii) 16 effective BAO points [44, 48]; iv) 19 effective points from LSS, specifically 18 points from the observable $f(z)\sigma_8(z)$ (mostly RSD) [45, 48, 58] and one effective point from the Weak Lensing observable $S_8 \equiv \sigma_8(0)/\Omega_m^{0.3}$ [59]; v) Finally we make use of the full CMB likelihood from Planck 2015 TT+lowP+lensing [2]. It is important to emphasize that we take into account all the known correlations among data.

The total data set just described will be referred to as DS1. For a more detailed discussion of these data, see [19]. In this study, however, we wish to isolate also the effect from the triad of BAO+LSS+CMB data. These ingredients may be particularly sensitive to the DDE, as shown in

| Model | $H_0$ (km/s/Mpc) | $\omega_{cdm}$ | $\omega_x$ | $w_\phi$ | $(\alpha, 10^{-3}r)$ | $\sigma_s(0)$ |
|-------|-----------------|----------------|-----------|---------|------------------|-------------|
| DS2 with Bispectrum (DS2/BSP) | | | | | | |
| $\Lambda$CDM | 68.20$^{+0.38}_{-0.41}$ | 0.117$^{+0.0008}_{-0.0009}$ | 0.02230 $\pm$ 0.00019 | -1 | - | 0.805 $\pm$ 0.007 |
| XCDM | 66.36$^{+0.76}_{-0.86}$ | 0.1155$^{+0.0014}_{-0.0012}$ | 0.02247 $\pm$ 0.000241 | -0.911$^{+0.035}_{-0.034}$ | - | 0.788 $\pm$ 0.010 |
| $\phi$CDM | 66.45 $\pm$ 0.74 | 0.1154 $\pm$ 0.0013 | 0.02248$^{+0.000020}_{-0.000621}$ | - | $\left(0.240^{+0.006}_{-0.102}, 32.0^{+0.8}_{-1.4}\right)$ | 0.789 $\pm$ 0.010 |

Table 2: As in Table 1, but using dataset DS2/BSP, which involves BAO+LSS+CMB data only. In the first block we use the full likelihood for Planck 2015, whereas in the second we use the compressed CMB likelihood for the more recent Planck 2018 data. See text for more details.
Figure 2: Contour lines for the XCDM in the planes $(\omega_{cdm}, w_0)$ and $(\Omega_m, w_0)$ obtained with different data set combinations and using the compressed Planck 2018 data \cite{63, 64} instead of the full Planck 2015 likelihood \cite{2}. More concretely we show the results obtained with the DS2/BSP data set (CMB+BAO+LSS, in grayish shades, see the second block of Table 2), the DS1/BSP (ALL DATA, dashed red contours), and the DS1/BSP but without LSS (CMB+BAO+SNIa+$H(z)$, solid green contours). The solid points indicate the location of the best-fit values in each case. When the LSS data points with bispectrum are considered in combination with the CMB and BAO constraints the DDE signal exceeds the 2\sigma c.l., regardless we use the Planck 2015 or 2018 information. See the text for further details.

the previous references. Such subset of DS1 will be called DS2 and contains the same data as DS1 except SNIa+$H(z)$. In the next section we define further specifications of these two datasets with special properties.

4 Spectrum versus bispectrum

The usual analyses of structure formation data in the literature are performed in terms of the matter power spectrum $P(k)$, referred to here simply as spectrum. As we know, the latter is defined in terms of the two-point correlator of the density field $D(k)$ in Fourier space, namely $\langle D(k) D(k') \rangle = \delta(k - k') P(k)$, in which $\delta$ is the Dirac delta of momenta. For a purely Gaussian distribution, any higher order correlator of even order decomposes into sums of products of two-point functions, in a manner very similar to Wick’s theorem in QFT. At the same time, all correlators of odd order vanish. This ceases to be true for non-Gaussian distributions, and the first nonvanishing correlator is then the bispectrum $B(k_1, k_2, k_3)$, which is formally connected to the three-point function

$$\langle D(k_1) D(k_2) D(k_3) \rangle = \delta(k_1 + k_2 + k_3) B(k_1, k_2, k_3). \quad (11)$$

The Dirac $\delta$ selects in this case all the triangular configurations. Let us note that even if the primeval spectrum would be purely Gaussian, gravity makes fluctuations evolve non-Gaussian. Such deviations with respect to a normal distribution may be due both to the evolution of gravitational instabilities that are amplified from the initial perturbations, or even from some intrinsic non-Gaussianity of the primordial spectrum. For example, certain implementations of inflation...
(typically multifield inflation models) unavoidably lead to a certain degree of non-Gaussianity \[60\]. Therefore, in practice the bispectrum is expected to be a nonzero parameter in real cosmology, even if departing from perfect Gaussianity, which is in no way an absolute condition to be preserved. On the other hand, such departure should, of course, be small. But the dynamics of the DE is also expected to be small, so there is a possible naturalness relationship between the two.

The bispectrum (BSP) has been described in many places in the literature, see e.g. \[60, 61\] and references therein. Here we wish to dwell on its impact as a potential tracer of the DDE. Observationally, the data on BAO+ LSS (more specifically, the \(f\sigma_8\) part of LSS) including both the spectrum (SP) and bispectrum (BSP) are taken from \[45\], together with the correlations among these data encoded in the provided covariance matrices. The same data including SP but no BSP has been considered in \[62\]. In this study, we analyze the full dataset DS1 with spectrum only (dubbed DS1/SP) and also the same data when we include both SP and BSP (denoted DS1/BSP for short). In addition, we test the DDE sensitivity of the special subset DS2, which involves both SP+BSP components (scenario DS2/BSP). Other combinations will be presented elsewhere in an extended study.

The numerical fitting results that we have obtained for the DDE under study and the \(\Lambda\)CDM are displayed in Tables 1 and 2. While we use Planck 2015 CMB data with full likelihood throughout most of our analysis, in the second block of Table 2 we report on the preliminary results obtained from the recent Planck 2018 CMB data under compressed likelihood \[63, 64\]. Let us note that the full likelihood for Planck 2018 CMB data is not public yet. We discuss the results that we have obtained and their possible implications in the next two sections.
Figure 4: The EoS for the considered DE models within the corresponding 1σ bands and under scenario DS2/BSP. For the XCDM the EoS is constant and points to quintessence at $\sim 2.6\sigma$ c.l (cf. Table 2). For the $\phi$CDM the EoS evolves with time and is computed through a Monte Carlo analysis. The current value is $w(z = 0) = -0.925 \pm 0.026$ and favors once more quintessence at $\sim 2.9\sigma$ c.l.

5 Confronting DDE to observations

As we can see from Tables 1 and 2, the comparison of the $\Lambda$CDM with the DDE models points to a different sensibility of the data sets used to the dynamics of the DE. If we focus on DS1/SP there is no evidence that the DDE models perform better than the $\Lambda$CDM. The XCDM yields a weak signal which is compatible with $w_0 = -1$ (i.e. a rigid CC). This is consistent e.g. with the analysis of [23]. The $\phi$CDM model remains also inconclusive under the same data. The upper bound of $\alpha < 0.092$ at 1σ (0.178 at 2σ) – see our Table 1 – is consistent, too, with recent studies [24]. However, both the XCDM parametrization and the $\phi$CDM model fare better than the $\Lambda$CDM if we consider the dataset DS1/BSP, i.e. upon including the bispectrum component of the BAO+LSS data. The DDE signature is about $2\sigma$ c.l. However, it is further enhanced within the restricted DS2/BSP dataset, where these models reach in between $2.5 - 3\sigma$ c.l. (cf. Table 2). As for the CPL parametrization, Eq. (1), we record it explicitly in Table 1 for the DS1/BSP case. One can check that the errors in the EoS parameters are still too big to capture any clear sign of DE dynamics, owing to the additional parameter. Specifically, we find $w_0 = -0.934^{+0.067}_{-0.075}$ and $w_1 = -0.045^{+0.273}_{-0.204}$, hence fully compatible with a rigid CC ($w_0 = -1, w_1 = 0$).

In Fig. 1 we display the contour plots for the XCDM and $\phi$CDM models at different confidence levels, which are obtained with the DS2/BSP data set. The EoS parameter $w_0$ of the XCDM is seen to be shifted upwards more than $2.5\sigma$ away from $-1$ and hence it lies in the quintessence region. In the $\phi$CDM case we consistently find $\alpha > 0$ at a similar (actually higher) c.l. In Figs. 2 and 3 we show the corresponding results obtained for these models under different data set combinations, but now making use of the Planck 2018 compressed CMB data. These plots make palpable the importance of the bispectrum influence on the study of the DE. Indeed, it is only
when the bispectrum is included in the analysis, together with the CMB and BAO data sets, that a non-negligible signal in favor of the dynamical nature of the DE clearly pops out. In Fig. 4 we plot the EoS of the various models in terms of the redshift near our time and within the 1σ error bands for the XCDM and φCDM, again for the DS2/BSP case. These bands have been computed from a Monte Carlo sampling of the \( w(z) \) distributions. The behavior of the curves shows that the quintessence-like behavior is sustained until the present epoch. For the \( \phi \)CDM we find \( w(z = 0) = -0.925 \pm 0.026 \), thus implying a DDE signal at \( \sim 2.9 \sigma \) c.l., which is consistent with the XCDM result. Finally, in Fig. 5 we compute the matter power spectrum and the temperature anisotropies for the DDE models using Planck 2015 data, and also display the percentage differences with respect to the ΛCDM. As can be seen, the differences of the CMB anisotropies between the DDE models and the ΛCDM remain safely small, of order \( \sim 1\% \) at most for the entire range.

### 6 Bayesian evidence of DDE

The main results of this work are synthesized in Tables 1 and 2, and in Figs. 1-5. Here we wish to quantify the significance of these results from the statistical point of view. This is done in detail in Fig. 6. In what follows we explain the meaning of this figure and review some basic facts on Bayesian analysis which can be helpful at this point. As remarked before, in the absence of BSP data the DDE signs are weak and we find consistency with previous studies. However, when we include BSP data and focus on the LSS+BAO+CMB observables (i.e. scenario DS2/BSP)
the situation changes significantly. Both models XCDM and $\phi$CDM then consistently point to a $2.5 - 3\sigma$ effect. Specifically, the evolving EoS of the $\phi$CDM takes a value at present which lies $\sim 3\sigma$ away from the $\Lambda$CDM ($w = -1$) into the quintessence region $w \gtrsim -1$ (cf. Fig. 4). The significance of these results can be further appraised by computing the Bayesian evidence, which is based on evaluating the posterior probability of a model $M$ given the data $x$ and the priors, see e.g. [61,65]. The relevant quantity we are looking for is the so-called Bayes factor, which does not depend on the arbitrarily assigned prior probability to the given model $M$. If $\theta$ is the set of free parameters of such model, Bayes theorem allows us to compute the probability of measuring a distribution of values of these parameters given the measured dataset $x$ and the model $M$. It is called the posterior probability, and it is given by

$$p(\theta|x,M) = \frac{p(x|\theta,M)p(\theta|M)}{p(x|M)}.$$  \hspace{1cm} (12)

In words, it says that the posterior probability of $\theta$ is equal to the probability of the data $x$ given the parameters of the model $M$ (i.e. the likelihood $p(x|\theta,M)$) times the prior probability of $\theta$, $p(\theta|M)$, divided by the probability of the data $x$ (usually in the form of a probability distribution function, PDF). Obviously the latter does not depend on the values of $\theta$. Therefore, if we normalize the posterior probability to one and integrate over all the parameters $\theta$ on both sides of Eq. (12), the corresponding integral in the numerator on the r.h.s. must be equal to $p(x|M)$:

$$p(x|M) = \int d\theta p(x|\theta,M) p(\theta|M).$$  \hspace{1cm} (13)

This likelihood integral over all the values that can take the parameters $\theta$ is called the marginal likelihood or evidence [61,65]. An analogous formula to (12) can also be applied to estimate the posterior probability that a model $M$ is true given a measured data set $x$. Thus, following the same scheme, the posterior probability of the model given the data, $p(M|x)$, must be equal to the probability of the data given the model (irrespective of the values of $\theta$) – i.e. the marginal likelihood [13] — times the prior probability of the model, divided by the PDF of the data. Writing this relation for two cosmological models $M_i$ and $M_j$ that are being compared, we find that the ratio of posterior probabilities of these models is equal to the ratio of model priors times a factor:

$$\frac{p(M_i|x)}{p(M_j|x)} = \frac{p(M_i)}{p(M_j)} B_{ij}.$$  \hspace{1cm} (14)

Such factor $B_{ij} = p(x|M_i)/p(x|M_j)$ is the so-called Bayes factor of the model $M_i$ with respect to model $M_j$. It gives the ratio of marginal likelihoods (i.e. of evidences) of the two models. Note that it coincides with the ratio of posterior probabilities of the models only if the prior probabilities of these models are the same. This is generally assumed (“Principle of Insufficient Reason”) and hence the comparison of the two models $M_i$ and $M_j$ is usually performed directly in terms of $B_{ij}$.

In the literature it has been customary to define the Bayes information criterion (BIC) through the parameter $\text{BIC} = \chi^2_{\text{min}} + n \ln N$, where $\chi^2_{\text{min}}$ is the minimum of $\chi^2$, $n$ is the number of independent fitting parameters and $N$ is the total number of data points [66,67]. One can show that the Bayes factor can be estimated through $B_{ij} = e^{\Delta \text{BIC}/2}$ where $\Delta \text{BIC} = \text{BIC}_j - \text{BIC}_i$ is the
\[ \Delta \text{BIC} = 2 \ln B_{ij} \]

Bayesian evidence of model \( M_i \) versus \( M_j \)

| \( \Delta \text{BIC} \)       | Observational Support                                      |
|------------------------------|------------------------------------------------------------|
| \( 0 < \Delta \text{BIC} < 2 \) | weak evidence (consistency between both models)           |
| \( 2 < \Delta \text{BIC} < 6 \) | positive evidence                                          |
| \( 6 < \Delta \text{BIC} < 10 \) | strong evidence                                            |
| \( \Delta \text{BIC} \geq 10 \) | very strong evidence                                       |
| \( \Delta \text{BIC} < 0 \)    | counter-evidence against model \( M_i \)                  |

Table 3: Conventional ranges of values of \( \Delta \text{BIC} \) used to judge the observational support for a given model \( M_i \) with respect to the reference one \( M_j \). See [66,67] for more details.

The difference between the values of BIC for models \( M_i \) and \( M_j \), i.e.

\[ \Delta \text{BIC} = \Delta \chi^2_{\text{min}} + \Delta n \ln N. \]  \hspace{1cm} (15)

Here \( \Delta n \) is the difference in the number of independent fitting parameters of \( M_i \) and \( M_j \), both describing the same \( N \) data points. The added term to \( \Delta \chi^2_{\text{min}} \), i.e. \( \Delta n \ln N \), represents the penalty assigned to the model having the largest number of parameters (\( \Delta n > 0 \)), i.e. we could say that it helps to implement quantitatively Occam’s razor. Even though the simple and very economic formula (15) is useful and has been applied in many places of the literature, see e.g. [16–19] and references therein, it is only an approximate formula. The exact value of \( \Delta \text{BIC} \) associated to the full Bayes factor requires to evaluate

\[ \Delta \text{BIC} = 2 \ln B_{ij} = 2 \ln \frac{\int d\theta_i p(x|\theta_i, M_i) p(\theta_i|M_i)}{\int d\theta_j p(x|\theta_j, M_j) p(\theta_j|M_j)}, \]  \hspace{1cm} (16)

where \( \theta_i \) and \( \theta_j \) are the two sets of free parameters integrated over for each model. We refer to this (exact) value of \( \Delta \text{BIC} \) as the full “Bayesian evidence” of model \( M_i \) versus the reference model \( M_j \). It represents the optimal implementation of Occam’s razor. Formula (15) provides an estimate (sometimes good, sometimes rough) of the cumbersome expression (16). The evaluation of the latter is carried out in practice using the MCMC’s for the statistical analysis of the model and it can be a formidable numerical task. Fortunately it can be speeded up with the help of the recent numerical code MCEvidence [68]. In Table 3 we collect the ranges of values of \( \Delta \text{BIC} \) that are conventionally used in the literature to quantify the observational support of a given model \( M_i \) with respect to some reference one \( M_j \) [66][67]. Positive values of \( \Delta \text{BIC} \) in the indicated intervals favor model \( M_i \) over model \( M_j \). Small positive values near one just denote that the two models are consistent (i.e. that there is no marked preference of \( M_i \) over \( M_j \)), but larger values increase the support of \( M_i \) versus \( M_j \). Negative values of \( \Delta \text{BIC} \), in contrast, would lead to the opposite conclusion, i.e. that the tested model \( M_i \) is penalized as compared to the reference model \( M_j \) and hence that the latter is preferred.

In Fig. 6 we compute the full Bayesian evidence curves as a function of the prior range. We assume flat priors for the parameters in each model (we have no reason to assume otherwise). Thus \( p(\theta|M) \) is constant and cancels out for all common \( \theta \) in the two models, rendering \( B_{ij} \) a pure ratio.

\[^1\text{We thank Y. Fantaye for helpful advice in the use of the numerical package MCEvidence.}\]
Figure 6: The full Bayesian evidence curves for the various models as compared to the \( \Lambda \)CDM as a function of the (flat) prior range (in \( \sigma \) units, see text). The curves are computed using the exact evidence formula, Eq. (16), with the data indicated in Table 1 and the first block of Table 2 (i.e. with full Planck 2015 CMB likelihood). The marked evidence ranges conform with the conventional definitions of Bayesian evidence shown in Table 3.

As indicated, we utilized the code MCEvidence [68] to compute the exact \( \Delta \text{BIC} \) values in Fig. 6. We have actually compared these results with those obtained for the evidences computed with Gaussian (Fisher) likelihoods for the parameters and we have found that they are qualitatively similar but with non-negligible numerical differences that were expected from the mild departures of Gaussianity from the exact distributions. The full evidence curves in Fig. 6 reconfirm the mentioned result that with the dataset comprising only the power spectrum (DS1/SP) the evidence is weak. But at the same time it is remarkable to find a long tail of sustained positive evidence both for the XCDM and \( \phi \)CDM within the scenario DS2/BSP, which corroborates the higher sensitivity of the BAO+LSS+CMB data to DDE. Incidentally, these evidence results resemble those reported sometime ago in Ref. [12], but for very different models and using older (Planck 2013+WMAP) data, and with no inclusion of BSP (not available at that time). The fact that with the Planck 2015 data and with traditional models and parametrizations of the DE we can still reach such significant level of evidence is encouraging. Even more reassuring is the fact that with the preliminary Planck 2018 data we can reach similar levels of evidence (cf. Table 2, second block, and also Figs. 2 and 3), which will have to be refined when the full Planck 2018 likelihood will be made public.

Let us finally note that, in the presence of BSP data, the values of \( \sigma_8(0) \) shown in the tables (specially in Table 2, corresponding to the DS2 set) are smaller for the DDE models than for the
ΛCDM, what contributes to alleviate the $\sigma_8$-tension \cite{7}. We have checked that a similar effect is possible within the ΛCDM (under DS1/BSP) for a sum of neutrinos masses of $\sum m_\nu = 0.195\pm0.076$ eV, leading to $\sigma_8(0) = 0.785 \pm 0.014$, which is however unfavored by the usual constraints on neutrino oscillations since it requires unnatural fine-tuning. The DDE option is thus more natural.

7 Conclusions

In this work we have tested the performance of cosmological physics beyond the standard or concordance ΛCDM model, which is built upon a rigid cosmological constant. We have shown that the global cosmological observations can be better described in terms of models equipped with a dynamical DE component that evolves with the cosmic expansion. Our task has focused on three dynamical dark energy (DDE) models: the general XCDM and CPL parametrizations as well as a traditional scalar field model ($\phi$CDM), namely the original Peebles & Ratra model. We have fitted them (in conjunction with the ACDM) to the same set of cosmological data based on the observables SNIa+BAO+$H(z)$+LSS+CMB. Apart from the global fit involving all the data, we have also tested the effect of separating the expansion history data (SNIa+$H(z)$) from the features of CMB and the large scale structure formation data (BAO+LSS, frequently interwoven), where LSS includes both the RSD and weak lensing measurements. We have found that the expansion history data are not particularly sensitive to the dynamical effects of the DE, whereas the BAO+LSS+CMB are more sensitive. Furthermore, we have evaluated for the first time the impact on the dynamics of the DE from the bispectrum component in the matter correlation function. We have done this by including BAO+LSS data that involve both the conventional power spectrum and the bispectrum. The outcome is that when the bispectrum component is not included our results are in perfect agreement with previous studies of other authors, meaning that in this case we find a weak signal of dynamical DE or no signal at all depending on the model/parametrization used. In contrast, when we activate the bispectrum component in BAO+LSS (along with the corresponding covariance matrices) we find that the dynamical DE signal is significantly enhanced. We conclude that the bispectrum can be a very useful tool to track possible dynamical features of the DE through its time-dependent influence on the formation of structures in the linear regime.

A surplus of our analysis is that we have also found noticeably lower values of $\sigma_8(0)$ in the presence of the bispectrum, see the last column of Table 2. For a long time it has been known that there is an unexplained ‘$\sigma_8$-tension’ in the framework of the LCDM, which is revealed through the fact that the LCDM tends to provide higher values of $\sigma_8(0)$ than those obtained from RSD measurements. Dynamical DE models, therefore, seem to provide a possible alleviation of such tension, specially when we consider the combined CMB+BAO+LSS measurements and with the inclusion of the bispectrum, together with the power spectrum.

Finally, let us remark that although we have used the full Planck 2015 CMB likelihood in our work, we have also advanced the preliminary results involving the recent Planck 2018 data under compressed CMB likelihood, still awaiting for the public appearance of the full Planck 2018 likelihood. The preliminary results that we have obtained consistently keep on favoring DDE versus a rigid cosmological constant.
To summarize, our study shows that it is possible to reach significant signs of DDE with the current data, provided we use the bispectrum in combination with the power spectrum. The former might be a good tracer of dynamical DE effects and ultimately of the “fine structure” of the DE. The details of this analysis will be presented elsewhere.

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