Simple nonstationary generalization of Gaussian Shell-model beams

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ABSTRACT
Model of partially coherent pulses, based on the concept of “hidden coherence”, introduced recently by Picozzi and Haelterman in framework of parametric wave mixing, is presented. The nonuniform and nonstationary phase shift, while completely deterministic, results in the beam properties, which are typical for partially coherent light — low contrast of interference effects, increase of spectral width and so on — i.e. light becomes effectively non-coherent. The proposed model is studied in framework of coherent mode decomposition, its main properties and limitations of the model are discussed.

Keywords: Coherence, pulsed beam, phase shift, Shell-model

1. INTRODUCTION AND MOTIVATION
Nowadays there is a constant interest to use the partially coherent light in optical systems. Among the important problems related to partially coherent light are study of light propagation, image formation, pulse shaping, light interaction with nonlinear media, etc. All of these problems require adequate models of radiation, which should describe its spatial, spectral and statistical properties; at the same time analytically described models are advantageous. For stationary beams such model is well-known: it is the Shell model, especially Gaussian Shell-model (GSM) and its generalizations. Pulsed partially coherent beams is also described by various generalizations of the Shell-model, usually with the Gaussian profile of temporal dependence of correlation function.

In the present work, another model of partially coherent pulses is presented. It is based on the concept of “hidden coherence”, introduced recently by Picozzi and Haelterman in the framework of parametric wave mixing. Such radiation is completely coherent along some spatio-temporal trajectories in 4D space, but is neither completely spatially nor temporally coherent (from the point of view of usual interferometric experiments, such as Young double-slit interferometer). In the other words, wavefields with complex spatio-temporal structure are “operationally” equivalent to partially coherent beams.

As an example, nonuniform and nonstationary phase shift (completely deterministic, such as in non-linear self-phase modulation, or, for relatively large characteristic time intervals — by mechanical motion of lenses, mirrors, or phase screens) results in the beam properties, which are typical for partially coherent light — low contrast of interference effects, increase of spectral width and so on — i.e. light becomes effectively non-coherent.

In present paper, a quite a simple model of effectively partially coherent pulses is studied in framework of coherent mode decomposition, its main properties and limitations of the model are discussed.

2. MODAL APPROACH TO PARTIAL COHERENCE DESCRIPTION (KARHUNEN-LOÈVE DECOMPOSITION)
A complex field (analytic signal) of a light beam cross-section is considered as sum of factorized, mutually orthogonal (and normalized) components

$$E(r, t) = \sum_k \nu_k \, E_k(r) \, e_k(t),$$

$$\nu_k$$ is amplitude of $$k$$th component.

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• Envelopes and modal amplitudes are calculated as eigenfunctions and eigenvalues of dual integral equations where kernels are spatial and temporal correlation functions

\[ |\nu_k|^2 \mathcal{E}_k(r) = \int dr \Gamma_S(r, r') \mathcal{E}^*(r'), \]

\[ |\nu_k|^2 e_k(t) = \int dt \Gamma_T(t, t') e^*(t'). \]

• Decomposition gives optimal representation (most fast converging expansion).

• Effective number of modes

\[ N_{\text{eff}} = \sum_k |\nu_k|^4 / \left( \sum_k |\nu_k|^2 \right), \]

\[ N_{\text{eff}} = \frac{\left( \int_{-\infty}^{\infty} dx \Gamma_S(x, x) \right)^2}{\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' \Gamma_S(x, x')^2}, \]

\[ = \frac{\left( \int_{-\infty}^{\infty} dt \Gamma_T(t, t) \right)^2}{\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \Gamma_T(t, t')^2} \]

(or overall degree of coherence \( \mu = 1/N_{\text{eff}} \)) characterize contrast of interference effects (e.g., speckles)

\[ C = N_{\text{eff}}^{-1/2} \]

3. 1D MODEL

Light pulse with the spatially nonuniform and nonstationary phase shift:

\[ E(x, t) = \frac{A}{\sqrt{4\pi a^2 \tau^2}} \exp \left( -\frac{x^2}{4a^2} - \frac{t^2}{4\tau^2} + \frac{ix}{\eta} \right) \exp(-i\omega_0 t), \]

\( \eta \) — phase shift parameter.

Its spatial and temporal correlation functions

\[ \Gamma_S(x, x') = \int_{-\infty}^{\infty} dt E(x, t) E^*(x', t), \]

\[ \Gamma_T(t, t') = \int_{-\infty}^{\infty} dx E(x, t) E^*(x, t') \]

are of GSM form in both domains

\[ \Gamma_S(x, x') = \frac{A^2}{\sqrt{2\pi a^2}} \exp \left( -\frac{x^2 + x'^2}{4a^2} - \frac{(x - x')^2}{2\sigma_S^2} \right), \]

\[ \Gamma_T(t, t') = \frac{A^2}{\sqrt{\pi \tau^2}} \exp \left( -\frac{t^2 + t'^2}{4\tau^2} - \frac{(t - t')^2}{2\sigma_T^2} \right) \]

with \( \sigma_S = \eta / \tau, \sigma_T = \eta / a. \)

Note, that an observer, moving with a constant velocity \( \propto 1 / \eta \) across the beam, will treat it as fully coherent.
4. THE MODEL PROPERTIES

- Fully deterministic.
- Spatio-temporal symmetry.
- Any degree of coherence.
- Could be used in quasi-1D (or 1 + 1 + 1D) problems, e.g. with strip-source illumination.

Disadvantage: the proposed model requires large phase shifts at beam edges and on pulse rise and fall.

Generation: using specially designed moving DOE (diffractive optical element). Possibly together with lens and a (chirp-like) phase modulator.

Approximation: Nonlinear self-phase modulation
\[ E_{\text{out}}(x, t) = E_{\text{in}}(x, t) \exp \left[ i \alpha / \eta |E_{\text{in}}(x, t)|^2 \right] \]
(\(\alpha \approx 3\)) converts initially coherent pulsed beam \(E_{\text{in}}(x, t) = \mathcal{E}(x)e(t)\) into light with nonstationary and nonuniform phase shift and with approximately Gaussian correlation function (error is within 5%).

5. COHERENT MODE DECOMPOSITION

Spatial and temporal decomposition functions are already known — Hermite-Gaussian functions
\[
\mathcal{E}_k(x) = \left( \frac{2c}{\pi} \right)^{1/4} \frac{1}{(2k!)^{1/2}} H_k(x(2c)^{1/2}) \exp(-cx^2)
\]
\[
e_k(t) = \left( \frac{2d}{\pi} \right)^{1/4} \frac{1}{(2k!)^{1/2}} H_k(t(2d)^{1/2}) \exp(-dt^2)
\]
and modal weights
\[ |\nu|^2 = A^2 \frac{\eta^2 + b}{\eta^2 + 2a^2\tau^2 + b} \left( \frac{2a^2\tau^2}{\eta^2 + 2a^2\tau^2 + b} \right)^n \]
where \(H_k(x)\) are Hermitian polynomials, \(b = \sqrt{\eta^2 + 4a^2\tau^2}\) and
\[
c = \frac{\eta^2 + 4a^2\tau^2}{16a^4\eta^2}, \quad d = \frac{\eta^2 + 4a^2\tau^2}{16\tau^4\eta^2}.
\]

Number of coherent modes
\[ N_{\text{eff}} \propto a\tau/\eta, \quad \text{for } \eta \ll 1. \]

6. DISCUSSION OF 2D CASE

One more disadvantage of the model: difficulty of its generalization to more realistic case of 2D aperture. It is preferable, that field is Rotationally invariant, then use of term like
\[ E(x, y, t) \propto \exp \left( i \eta \sqrt{x^2 + y^2} t \right) \]
leaves to a phase singularity near \(x = y = 0\). An alternative variant with
\[ E(x, y, t) \propto \exp \left( i \eta (x^2 + y^2) t \right) \]
does not lead to Gaussian shape of spatial correlation function.

Moreover, in general, taking into account properties of modal decomposition of 1D (temporal \(\Gamma_T(t, t')\)) and 2D spatial \(\Gamma_S(x, x')\) kernels, it is possible to show, that deterministic double-Gaussian Shell-model pulsed beams are impossible.
Acknowledgement
The work has been supported by Belarusian Fund for Fundamental Research, projects No. F03MS-066 and F05K-056.

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