Analytical calculation of adiabatic processes in real gases

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Abstract. The impact of gases nonideality in the compression and expansion processes on the specific heat ratio and the heat capacity is analyzed. The specific heat ratio variation leads to temperature variation during compression in the compressor and expansion in the turbine and, consequently, the gas turbine cycle efficiency factor variation. It is also essential to consider the gases nonideality in the compression and expansion processes in the compression processes in compressor. Generally it is assumed during calculations that the heat capacities depend only on temperature, in this case the reference data presented by various authors differs markedly. In the real processes the heat capacity and the specific heat ratio depend on temperature and within the particular temperatures and pressures range depend on pressure. Consequently, the operating fluid nonideality in the gas turbine cycle should be considered.

1. Introduction
The processes of gases compression in the noncooled compressor and expansion in the turbine can be assumed as adiabatic to a good approximation. Usually during the thermodynamic analysis of the processes mentioned above it is assumed that the operating fluid is an ideal gas with the thermophysical properties depending only on temperature.

In real processes the heat capacity and, consequently, the specific heat ratio depend heavily on temperature and within the particular temperatures and pressures range depend on pressure, it means that the operating fluid nonideality in the gas turbine cycle should be considered.

The thermodynamic efficiency improvement in the modern gas turbine units is achieved by means of the increase in the compression pressure ratio in the compressor and the combustion products’ temperature rise before the turbine, which can result in the necessity of the gas nonideality consideration. For instance, the change of 2% in the specific heat ratio as compared to the $k$ value being equal to $1.4$ at the compression pressure ratio $\beta = 25$, at the temperature before the turbine of $1600$ K, at the relative efficiency of the compressor and the turbine $\eta_c = \eta_t = 0.85$ results in the change of $35$ K and $30$ K in temperature at the end of the compression process in the compressor and at the end of the expansion process in the turbine respectively, in this case the gas turbine cycle efficiency factor changes in 2%. Therefore the accurate determination of the specific heat ratio values is of high importance.

2. The pressure dependence of the specific heat ratio for the nonideal gas
Usually during the calculations the heat capacity is taken to be not depending on pressure and the temperature dependences at the pressure of $0.1$ MPa, presented in the guidelines [1-3] are used. It should be mentioned that sometimes the reference data presented by various authors differ markedly, as it is seen in Fig.1, where the dependences mentioned above are given for air.
The pressure dependence of the air heat capacity was approximated in [4] by the polynomials, obtained by the table data processing, but the heat capacity at constant volume, which is included in the specific heat ratio expression was calculated by the Meyer relation $c_p - c_v = R$ being true only for the ideal gas.

The present expression is not appropriate for the real gas and the relation between the heat capacities is written as

$$c_p = c_v + T \left( \frac{\partial p}{\partial T} \right)_v \left( \frac{\partial v}{\partial T} \right)_p = c_v - T \left( \frac{\partial p}{\partial v} \right)_T \left( \frac{\partial v}{\partial T} \right)_p$$

The derivatives in the specific heat expression can be calculated only if the real gas state equation is known.

The pressure dependence at different temperatures was considered with a correction $\varphi(p,T)$ to the temperature dependence of the heat capacity at pressure of 0.1 MP, $c_p^0(T)$,

$$c_p(p,T) = \varphi(p,T) \times c_p^0(T). \quad (1)$$

The correction to the pressure dependence was approximated by data [3] as follows

$$\varphi(p,T) = A(p) \cdot \exp \left[ \left( 0,1 - p \right) \Theta \cdot \sum_{i=0}^{4} a_i(p) \Theta^i \right], \quad (2)$$

where $\Theta = \frac{T - 250}{100}$, $A(p) = b_0 + b_1 p$, $a_i(p) = \sum_{j=0}^{2} \alpha_{ij} p^j$. \quad (3)
The analytical temperature dependence at pressure of 0.1 MPa was accepted in accordance with [3]:

\[ c_p^0(T) = c_0 + c_1T + c_2T^2 + \frac{c_3}{T^2} \]

In this case the Redlich-Kwong and Diterichi equations [2] are the most adequate equations [2]. The deviation from ideality can be also taken into consideration by the compressibility factor

\[ z = \frac{p
\nu}{RT} \]

The analytical expression of which can be obtained by the reference data processing [3].

The processing results for air are presented in Fig. 2.

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**Figure 2.** Temperature dependence of the air compressibility at different pressures

In Diterichi state equation

\[ p(\nu - b) \cdot \exp \left( \frac{a}{RT\nu} \right) = RT \]

the corrections for the interactions between molecules and their volume, defined from the condition that the first and the second order derivatives are equal to zero

\[ \left( \frac{\partial p}{\partial \nu} \right)_T = \left( \frac{\partial^2 p}{\partial \nu^2} \right)_T = 0 \]

in the critical point, are as follows

\[ a = 2RT\nu, \quad b = \frac{\nu}{2} \alpha \]

The results of the specific heat ratio calculation by the Diterichi equation are presented in Fig. 3.

The nonideality consideration at the pressure of 4MPa results in the specific heat ratio change from \( k=1.4 \) to \( k=1.5 \) value, which results in the temperature increase from 832 to 932 K at the end of compression at the compression pressure ratio \( \beta = 25 \) and at the relative efficiency of the compressor \( \eta_\nu^c = \eta_\nu^t = 0,85 \).
Figure 3. Temperature dependence of the air specific heat ratio at different pressures

The pressure dependence of the heat capacity is sufficient for temperatures less than \( \sim 700 \) K, which are typical for the end of compression processes in the compressor.

3. Conclusion

Thus, the accurate determination of the specific heat ratio values with consideration of both temperature and pressure dependences of the heat capacity is of high importance. It is particularly true for the compression processes in the compressor as far as at the temperatures typical for the expansion processes in the turbine the pressure dependence is not so essential and the gas can be assumed to be ideal, and in this case only the temperature dependence of the heat capacities can be considered.

This work was carried out at UrFU and financially supported by the Russian Science Foundation (project number 14-19-00524).

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