Abstract. The blast wave model is applied to the recent data on HBT radii in pp collisions, measured by the ALICE collaboration. A reasonable description of data is obtained for a rather low temperature of the system, $T \leq 120$ MeV and the transverse profile corresponding to the emission from a shell of a fairly small width $2\delta \sim 1.5$ fm. The size and the life-time of the produced system are determined for various multiplicities of the produced particles.

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1. Introduction

Recently, the ALICE collaboration has presented an impressive collection of data on the Hanbury-Brown–Twiss (HBT) radii measured in pp collisions at the 7 TeV cms energy [1]. In the present paper we discuss to what extent this data is consistent with the blast-wave model [2, 3, 4] which has been used in analyses of the HBT correlations in relativistic heavy-ion collisions [5, 6, 7]. We note that the blast-wave model, originally introduced in [2], was inspired by the results of the hydrodynamic description of the hadron production processes. It was later adapted to ultra-relativistic energies in [3], for a mini review see also [4].

The agreement of the data with the blast-wave model predictions suggests that the produced matter exhibits thermal features such as local equilibration and hydrodynamic flow. As a matter of fact, with increasing beam energies, such as those presently available at the LHC, the final state hadron multiplicities grow significantly, and a thermodynamic/hydrodynamic description of hadron production may possibly be valid even in more elementary hadron+hadron and hadron+nucleus collisions, e.g., see Ref. [8, 9, 10]. Recently, the blast wave model has been used in this context to analyze high-multiplicity pp collisions at the LHC [11]. The authors of [11] found indications for the strong transverse radial flow in such events.

In the present paper we consider the blast wave model featuring a boost-invariant, azimuthally symmetric fluid expanding in the transverse direction according to the Hubble law [12]. We also accept that the momentum distribution of the particles emitted from the fluid element with the four-velocity $u$ at freeze-out is given by the Boltzmann formula

$$e^{-\beta E^*} = e^{-\beta p^\mu u_\mu},$$  \hspace{1cm} (1)

where $E^*$ is the energy of the emitted particle in the fluid element rest frame, $p^\mu$ is the particle four-momentum, and $T = 1/\beta$ is the temperature of the system.

The main conclusion of this work is that the blast-wave model can indeed account for the vast collection of the ALICE data [1]. However, a suitably chosen transverse profile for the distribution of matter in the transverse plane should be used in order to describe the data well. This profile corresponds to a shell of radius $R$ and width $2\delta$ (with $\delta < R$). Interestingly, such a shape helps to reproduce correctly the ratio $R_{\text{out}}/R_{\text{side}}$ of the two HBT radii measured at large values of the transverse momentum of the pion pair. The permanent problems with a correct reproduction of this ratio are known as the HBT puzzle. In heavy-ion physics these problems may be eliminated in practice if several improvements/modifications are done in the standard hydrodynamic codes [13, 14]. In this context, our present finding offers yet another hint on a possible solution of the HBT puzzle.

The paper is organized as follows: In the next Section we define the model by introducing the source function based on the Cooper-Frye formula and Hubble-like expansion of the fluid. The momentum distribution of particles, the HBT correlation
functions and the HBT radii are discussed in Sections 3 and 4. The results of the data analysis are described in Sections 5 and 6. The results are summarized in the last Section. The Appendix contains the tables and figures where the model results are compared with the data.

2. The Source function

Our starting point is the formula for the source/emission function

$$S(x, p) = \int d\Sigma_\mu(x) \, p^\mu f(x) e^{-\beta p^\mu u_\mu(x)}.$$  \(2\)

Here \(x\) and \(p\) are the spacetime position and four-momentum of the emitted particle (which we anticipate to be a pion) and \(d\Sigma_\mu(x)\) is an element of the freeze-out hypersurface which we take in the form

$$d\Sigma_\mu(x) = S_0 \sigma_\mu(x) \delta(\tau_f - \tau) d^3x = S_0 \sigma_\mu(x) \delta(\tau_f - \tau) d\tau d\eta d^2r,$$  \(3\)

where the variables \(\tau\) and \(\eta\) are the longitudinal proper time and the space-time rapidity

$$t = \tau \cosh \eta, \quad z = \tau \sinh \eta.$$  \(4\)

In a similar way, we define the particle radial distance from the beam axis and the azimuthal angle in the transverse plane

$$x = r \cos \phi, \quad y = r \sin \phi.$$  \(5\)

The four-vector \(\sigma^\mu = \sigma^\mu(x)\) defines the space-time orientation of an element of the freeze-out hypersurface

$$\sigma^\mu = (\cosh \eta, 0, 0, \sinh \eta).$$  \(6\)

The function \(f(x)\) in (2) describes the distribution of particles in space. Following Ref. \[15\] we assume that \(f(x)\) depends only on the transverse radius \(r\). Below we argue that the appropriate choice of the distribution \(f(r)\) is crucial for reproducing the experimental results.

Since the system is boost-invariant and cylindrically symmetric, the four-velocity \(u = u(x)\) has the form \[7\]

$$u = (\cosh \eta \cosh \theta, \sinh \theta \cos \phi, \sinh \theta \sin \phi, \sinh \eta \cosh \theta).$$  \(7\)

In addition, we assume that the transverse rapidity of the fluid element at freeze-out \(\theta(r)\) and its position \(r\) are related by the condition of the radial Hubble-like flow \[12\]. This leads to the expressions

$$\sinh \theta = \omega r, \quad \cosh \theta = \sqrt{1 + \omega^2 r^2},$$  \(8\)

where \(\omega\) is the parameter controlling the strength of the transverse flow.

The particle four-momentum is parameterized in the standard way in terms of rapidity, \(y\), transverse momentum, \(p_\perp\), transverse mass, \(m_\perp\), and the azimuthal angle in the transverse plane, \(\phi_p\).

$$p = (m_\perp \cosh y, p_\perp \cos \phi_p, p_\perp \sin \phi_p, m_\perp \sinh y).$$  \(9\)
The scalar product of \( p \) and \( u \) is
\[
p \cdot u = m_\perp \cosh(y - \eta) \cosh \theta - p_\perp \cos(\phi_p - \phi) \sinh \theta.
\] (10)

This form is used in the thermal Boltzmann distribution. In a similar way we obtain the factor \( p \cdot \sigma \) needed to define the element of the freeze-out hypersurface
\[
p \cdot \sigma = m_\perp \cosh(y - \eta).
\] (11)

The form of (11) follows directly from (9) and (10). Other forms are also possible here if one assumes different freeze-out conditions. Using (3) and (6) we follow the most popular version of the blast-wave model.

3. Momentum distribution and the HBT correlation functions

The integral of the source function \( S(x,p) \) over the space-time coordinates gives the momentum distribution
\[
\frac{dN}{dy d^2 p_\perp} = W(p) = \int d^4 x S(p, x).
\] (12)

The explicit calculation starting from Eq. (2) leads to the expression [3, 4]
\[
W(p_\perp) = m_\perp \int rdr f(r) K_1(U) I_0(V),
\] (13)
where \( K_1 \) and \( I_0 \) are the modified Bessel functions and
\[
U = \beta m_\perp \cosh \theta, \quad V = \beta p_\perp \sinh \theta.
\] (14)

Assuming that one can neglect correlations between the produced particles, the distribution of two identical bosons can be expressed in terms of the Fourier transform of the source function [16]
\[
W(p_1, p_2) = W(p_1)W(p_2) + |H(P,Q)|^2
\] (15)
with
\[
H(P,Q) = \int d^4 x e^{iQ \cdot x} S(x,P).
\] (16)

Here \( Q = p_1 - p_2 \) and \( \vec{P} = (\vec{p}_1 + \vec{p}_2)/2 \). The time-component of the four-vector \( P \) is not uniquely defined. We take \( P_0 = \sqrt{m^2 + |\vec{P}|^2} \) [14]. We shall work in the so-called LCMS system in which \( P_z = 0 \), i.e., \( p_{1z} = -p_{2z} \). In this reference frame the substitution \( p \to P \) in the source function \( S(x,p) \) is simply realized by the change \( m_\perp \to \sqrt{P_0^2 - P_z^2} = P_0 \).

Starting directly from (16) we find
\[
H(P,Q) = P_0 \int r dr f(r) \int d\phi \int d\eta \cosh \eta e^{-U \cosh \eta + V \cos \phi - r \Phi}
\] (17)
where now \( U = \beta P_0 \cosh \theta \) and \( V = \beta P_\perp \sinh \theta \), and the phase \( \Phi \) is given by the formula
\[
\Phi = -Q_0 t + Q_z z + Q_x x + Q_y y.
\] (18)

‡ From now on we shall omit all constant factors in the source function, since its normalization is irrelevant for the problems we discuss in this paper.
The phase $\Phi$ depends on the relative direction of $\vec{P} = (P_\perp, 0, 0)$ and $\vec{Q}$. Following the standard approach [17], we consider three regimes: *long*, *side* and *out*. It was shown in [15] that $H_d(P_\perp, q)$ can be explicitly expressed as integrals involving Bessel functions. We have

$$H_{\text{long}} = P_0 \int r dr f(r) I_0(V) U_l K_1(U_l) / U_l, \quad U_l = \sqrt{U^2 + Q_z^2 \tau_f^2},$$  

(19)

$$H_{\text{side}} = P_0 \int r dr f(r) I_0(V_s) K_1(U), \quad V_s = \sqrt{V^2 - Q_y^2 r^2}.$$  

(20)

If $V^2 < Q_y^2 r^2$, $V_s$ is imaginary and the function $I_0(V_s)$ should be replaced by $J_0(|V_s|)$. In the *out* direction we have

$$H_{\text{out}} = P_0 \int r dr f(r) I_0(V + iQ_x r) K_1(U - iQ_0 \tau_f).$$  

(21)

### 4. The HBT radii

Experiments usually measure the correlation function defined as

$$C(p_1, p_2) \equiv \frac{W(p_1, p_2)}{W(p_1)W(p_2)} - 1 = \frac{|H(P, Q)|^2}{W(p_1)W(p_2)}.$$  

(22)

Each HBT radius is obtained from a gaussian fit to the correlation function for the direction *long*, or *side*, or *out*:

$$C(p_1, p_2) = e^{-R_{\text{HBT}}^2 q^2},$$  

(23)

with $q$ being the component of the vector $\vec{Q}$ in the analyzed direction. This means that the radii can be evaluated analytically as the logarithmic derivatives of the correlation functions at $q = 0$

$$R_{\text{HBT}}^2 = - \frac{d \log[C(p_1, p_2)]}{dq^2} |_{q=0}.$$  

(24)

The formula gives mean square radii of the distribution of the particle emission points.

Using this definition and the formulae from the previous Section one obtains the expressions for the $R_{\text{HBT}}^2$ in all three directions, in the form of integrals involving the modified Bessel functions. They were given explicitly in [15] and, as they are rather lengthy, we shall not repeat them here.

### 5. Comparison with data

The HBT radii were measured by the ALICE collaboration for 6 intervals of pair transverse momentum and 8 intervals of multiplicity. This means that, at each multiplicity interval, there are 18 numbers to be explained. Our aim is thus to check if these 18 experimental numbers can be accounted for by the model, and — where possible — to determine the relevant physical parameters.

Two parameters, the temperature $T = 1 / \beta$ and $\omega$, (responsible for the transverse flow, c.f. Eq. (8) ) reflect the dynamics of the produced system, whereas the transverse profile $f(r)$ describes its geometry.
In our analysis we have assumed that the temperature is constant. It turned out that an acceptable description of data can be obtained only if $T$ does not exceed 120 MeV. In the final analysis we have taken $T = 100$ MeV.

5.1. The transverse profile

For the transverse profile we took a two-parameter function

$$f(r) \sim e^{-(r-R)^2/\delta^2}$$

(normalization is irrelevant for our purposes), i.e., we consider emission from a shell of radius $R$ and width $2\delta$. Note that this form includes, as a special case ($R = 0$), the gaussian profile, sometimes used in description of the heavy-ion data. We found, however, that to obtain a good description of data it is necessary to keep $R > \delta$.

5.2. The transverse momentum

Since the model must be consistent with the general features of data, it is necessary to demand the agreement with the measured (average) transverse momentum. This condition implies an additional relation between the parameters of the model.

To implement this condition we observe that, as seen from (13), the distribution of transverse momentum depends on three parameters: $T$, $\omega R$ (controlling the transverse flow) and $\Delta \equiv \delta/R$ (describing the shape of the transverse profile). Demanding that the average transverse momentum resulting from (13) agrees with data, one finds a relation between $\omega R$ and $\Delta$ (at a fixed value of $T$). Thus, effectively, we are left with 3 parameters: $\tau_f$, $R$ and $\delta = \Delta R$ for the description of the HBT radii. Qualitatively, one easily sees that, for a fixed (measured) $\langle P_{\perp} \rangle$, increasing temperature must be compensated by decreasing $\omega$, the flow parameter. Thus, if one wants to keep the flow at a decent level, the temperature cannot be too high.

The measurements by the CMS collaboration [18] give, approximately,

$$\langle P_{\perp} \rangle \approx [400 + 2.5(N_c - 10)] \text{ MeV},$$

(26)

where $10 \leq N_c \leq 50$ is the number of charged particles. Using (26) as input, one can find numerically the relation between $\omega R$ and $\Delta$. For $T = 100$ MeV it can be approximated by the formula

$$\omega R = a_0(N_c) + a_1(N_c)\Delta + a_2(N_c)\Delta^2 + a_3(N_c)\Delta^3$$

(27)

with the following coefficients:

$$a_0 = 0.695 + 0.00785 N_c - 0.0000075 N_c^2,$$
$$a_1 = -0.385 - 0.00395 N_c - 0.0000075 N_c^2,$$
$$a_2 = 0.0868 + 0.00062 N_c + 0.0000085 N_c^2,$$
$$a_3 = -0.00312 + 0.0000865 N_c - 0.00000272 N_c^2.$$
6. Description of the HBT radii

For each interval of $N_c$ the ALICE data consist of 18 measured numbers: $R_{\text{long}}$, $R_{\text{side}}$ and $R_{\text{out}}$ for 6 bins in $P_{\perp}$ [1]. For each of these intervals we have at our disposal three parameters ($\tau_f$, $R$ and $\delta = \Delta R$). To reduce further the uncertainties of the model we considered the simple idea of selecting $\delta$, the half-width of the "shell" from which particles are emitted, to be constant, independent of multiplicity (and thus also of the size of the system). As shown below, the present data are consistent with this assumption. It should be emphasized, however, that data do not restrict substantially $\delta$, particularly at low multiplicities. It is thus not excluded that the condition $\delta = \text{const}$, although appealing by its simplicity, may be challenged by more precise future measurements.

To determine $R$ and $\tau_f$ we looked for a minimum of $\chi^2$, using 5 intervals of $P_{\perp}$ (the lowest one was omitted for reasons explained below). It turned out that one could obtain in this way a very good description of data. With the value of $\delta$ fixed at 0.75 fm, the results for $R$ and $\tau_f$ are:

$$\langle N_c \rangle = 6.3, 13.9, 19.3, 25.2, 31.2, 37.6, 45.6, 59.9,$$
$$R = 1.15, 1.52, 1.77, 1.97, 2.14, 2.32, 2.49, 2.91 \text{ fm},$$
$$\tau_f = 1.90, 2.18, 2.37, 2.50, 2.63, 2.74, 2.80, 3.09 \text{ fm},$$
$$\chi^2 = 0.96, 1.90, 2.89, 4.06, 5.88, 5.45, 11.63, 8.48. \quad (29)$$

The multiplicity dependence of the parameters $R$ and $\tau_f$ is shown in Fig. 1.

One sees that the deviations from the experimental values are indeed very small. With increasing $\delta$ the description becomes worse, but it is still acceptable up to $\delta = 0.85 \text{ fm}$. When the smallest $P_{\perp}$ bin is included, the description is worse. The corresponding values of $\chi^2$ are

$$\chi^2_{\text{tot}} = 18.77, 11.08, 6.34, 4.86, 6.65, 5.79, 11.72, 9.7. \quad (30)$$

One can easily trace the discrepancy to the very bad description of $R_{\text{out}}$ at small $P_{\perp}$. Indeed, the data show an anomalous behavior: at small multiplicities $R_{\text{out}}$ increases with $P_{\perp}$, whereas the model predicts a steady decrease. A possible explanation of this "anomalous" effect is presented below.

The detailed comparison with data is given in the Appendix, where the full table of the radii evaluated from the model and of those measured by the ALICE collaborations is presented.

Using these results and formula (27) one can evaluate the Hubble parameter $\omega$, characterising strength of the radial flow. In Fig. 2 we show $\omega$ and $\omega R$ plotted vs. multiplicity $N_c$. One sees that the effect of the flow (as measured by $\omega R$) is non-negligible even at smallest multiplicities and increases substantially with increasing $N_c$ although the $\omega$ itself decreases.
6.1. Correlation functions for \( q \neq 0 \)

We evaluate the HBT radii using (24) and consequently they are sensitive to the \( q^2 \) dependence of the correlation functions only at very small \( q \). It is therefore essential to verify if the model provides a reasonable description of the correlation functions at larger values of \( q \). To this end we also evaluated, using (29), the correlation functions themselves in the region of \( q \) up to 800 MeV. It turns out that for \( q \geq 300 \) MeV both \( C_{\text{out}} \) and \( C_{\text{side}} \) are rather sensitive to the value of \( \Delta \), the relative width of the ”shell” from which the particles are emitted. At small \( \Delta \) they fall only very slowly with increasing \( q \). In addition, \( C_{\text{side}} \) shows large oscillations. This restricts \( \Delta \) from below and thus does not allow to take too small \( \delta \) (although the fit to the radii becomes even better for small \( \delta \)).

Using the parameters as explained above, we evaluated the correlation functions for various multiplicities and transverse momenta. They look reasonable, except at smallest \( P_\perp \), where the \( C_{\text{out}} \)'s exhibit heavy tails and thus differ substantially from Gaussians. This, naturally, may influence the experimentally fitted HBT radii. This is illustrated in Fig. 3 where \( \log[C_{\text{out}}(q)] \) for the second multiplicity bin \( (N_c=12–18) \) is plotted vs. \( q^2 \). One sees a rather dramatic difference between \( C_{\text{out}}(q) \) at \( \langle P_\perp \rangle = 163 \) MeV and
at $\langle P_\perp \rangle = 251$ MeV. It is clear from this figure that at $\langle P_\perp \rangle = 163$ MeV the fit to a Gaussian cannot provide a reliable value of the $R_{out}$. For the first rapidity bin the effect is even stronger, for the third it is significantly weaker. We feel that this may be a possible explanation of the discrepancy of our model with data at this smallest $P_\perp$. It will be interesting to verify this conjecture directly with the ALICE measurements of the correlation functions.

At larger values of $\langle P_\perp \rangle$ the deviations from Gaussians are important mostly in the region where the correlation functions are already rather small, and thus the effect seems to be contained within the (rather large) systematic errors quoted in [1].

6.2. The volume

The data were taken for $|\eta| \leq 1.2 \equiv \Delta \eta/2$. This allows us to evaluate the effective volume from which particles are emitted from the formula

$$V = \frac{2\pi \tau_f \Delta \eta \int r dr f(r) \sqrt{1 + \omega^2 r^2}}{f(r_s) \sqrt{1 + \omega^2 r_s^2}},$$

where $r_s$ is the point at which the function $f(r) \sqrt{1 + \omega^2 r^2}$ takes the maximal value. The numerical evaluation gives:

$$V = 46.2, 68.9, 86.8, 101.7, 116.6, 130.7, 142.9, 183.2 \text{ fm}^3,$$

$$r_s = 1.21, 1.06, 1.02, 0.99, 0.96, 0.94, 0.91, 0.90 \text{ fm},$$

for the eight multiplicity ranges considered, where the second line gives the radius of the sphere of volume $V/N_c$. One sees that at larger multiplicities the pions are somewhat more tightly packed.
The graphical representation of the dependence of the volume $V$ on the mean multiplicity is shown in Fig. 4. We note that $V$ is much larger than the product of the three HBT radii (at a given multiplicity). This is an expected result, the HBT radii measure the homogeneity lengths of the system rather than its physical dimensions [19]. The former are typically smaller than the latter.

![Figure 4.](image)

**Figure 4.** (Color online) Effective volume of the system as function of the multiplicity $N_c$.

### 7. Summary and conclusions

The main conclusion from our work is that the boost-invariant and azimuthally symmetric blast-wave model — with a suitably selected transverse profile — can account for the complicated structure of the HBT radii measured by the ALICE collaboration. In particular, it has been possible to explain (i) the general decrease of the the HBT radii with increasing transverse momentum of the pair and (ii) the so-called HBT puzzle, i.e., small values of the ratio $R_{\text{out}}/R_{\text{side}}$.

The blast-wave model, realizing explicitly (although only approximately) a well-defined physical picture, allows to determine the relevant physical parameters describing the state of the system at the kinematic freeze-out (as measured by the Bose-Einstein correlations). Our analysis shows that the relevant temperature must be rather low (below 120 MeV). To obtain reasonable agreement with data, it was also necessary to introduce a certain amount of transverse flow, which we assumed to be of the Hubble type (c.f. Eq. (8)).

The results concerning the geometry of the system at the (kinetic) freeze-out seem also interesting. The first observation is that the transverse profile is far from a Gaussian: it rather resembles a shell which can be chosen to be of an approximately constant width of about 1.5 - 1.7 fm. This feature (when combined with the transverse flow) makes it possible to explain the small ratio $R_{\text{out}}/R_{\text{side}}$. The radius of the shell increases steadily
BWM description of HBT radii

with increasing multiplicity (from $\sim 1$ fm to $\sim 3$ fm). Similarly, the proper time $\tau_f$ at which the freeze-out takes place increases with the multiplicity from $\sim 2$ fm to $\sim 3$ fm. The increase is not linear but tends to saturate at high multiplicities.

It should be realized that the formulae we use to determine the HBT radii describe the slope of the correlation function at $q^2 = 0$. In experiment, however, the radii are measured by fitting the observed correlation function to a Gaussian. These two procedures give identical results only if the correlation functions are indeed Gaussians. In our calculations the non-gaussian shape of the transverse profile implies, of course, deviations of the measured correlation functions from the simple gaussian shape. We have found that this effect is not significant for $R_{\text{long}}$. For $R_{\text{out}}$, however, it is essential at small $P_\perp$. This may perhaps explain the anomalous behavior of the measured $R_{\text{out}}$ in this region. Another interesting consequence of these deviations from the gaussian profile are substantial oscillations in the correlation function $C_{\text{side}}$ at large $q$.

In conclusion, we have shown the the blast-wave model provides a useful parametrization of the complicated pattern of data on HBT radii measured by the ALICE collaboration. This is achieved in terms of, basically, two parameters: the radius $R$ of the system and its (proper) life-time $\tau_f$, both showing a regular dependence on the multiplicity.

Let us add that the model provides also a host of predictions for the correlation functions at large $q$. Detailed comparison with the existing (although not yet published) ALICE data and with the future CMS data [20] in this $q$ region will provide a very strong test of the idea that the blast wave model can be applied even to such small systems as those produced in $pp$ collisions.

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8. Appendix

In the Tables 1–8 we list the values of the HBT radii (in fm) following from our fit, compared to those measured by the ALICE collaboration [1]. The first column: the mean transverse momentum of the pion pair, $P_\perp$; the second and third columns: $R_{\text{long}}$ from the model calculation and the experiment, respectively; the fourth and fifth columns: $R_{\text{side}}$; the sixth and seventh columns: $R_{\text{out}}$. The listed errors represent systematic and statistical errors added in quadrature. The quality of the fits is shown in Fig. 5 where the lines represent our model predictions, the central points of the bands correspond to the experimental results, and the width of the bands describes the experimental error.
### BWM description of HBT radii

#### Table 1. Model results for the HBT radii compared with the experimental results for the multiplicity class $N=1$–$11$.

| $P_\perp$ [GeV] | $R_{\text{long}}$ [fm] | $R_{\text{long}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{out}}$ [fm] | $R_{\text{out}}$ [fm] |
|-----------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
|                 | model exp.           | model exp.           | model exp.           | model exp.           | model exp.           | model exp.           |
| 0.163           | 1.54                 | 1.58±0.37            | 0.75                 | 0.75±0.10            | 0.95                 | 0.49±0.10            |
| 0.251           | 1.26                 | 1.23±0.16            | 0.74                 | 0.76±0.10            | 0.87                 | 0.79±0.13            |
| 0.348           | 1.05                 | 1.01±0.10            | 0.72                 | 0.73±0.10            | 0.77                 | 0.78±0.11            |
| 0.447           | 0.91                 | 0.91±0.10            | 0.69                 | 0.69±0.10            | 0.69                 | 0.69±0.14            |
| 0.547           | 0.80                 | 0.85±0.10            | 0.66                 | 0.66±0.10            | 0.63                 | 0.64±0.15            |
| 0.647           | 0.73                 | 0.80±0.19            | 0.63                 | 0.62±0.11            | 0.59                 | 0.57±0.23            |

#### Table 2. Same as Table 1 but for the multiplicity class $N=12$–$16$.

| $P_\perp$ [GeV] | $R_{\text{long}}$ [fm] | $R_{\text{long}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{out}}$ [fm] | $R_{\text{out}}$ [fm] |
|-----------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
|                 | model exp.           | model exp.           | model exp.           | model exp.           | model exp.           | model exp.           |
| 0.163           | 1.74                 | 1.80±0.29            | 0.98                 | 1.06±0.11            | 1.13                 | 0.78±0.12            |
| 0.251           | 1.41                 | 1.38±0.13            | 0.94                 | 0.97±0.10            | 1.00                 | 1.01±0.11            |
| 0.348           | 1.18                 | 1.15±0.10            | 0.89                 | 0.89±0.10            | 0.85                 | 0.84±0.10            |
| 0.448           | 1.02                 | 1.02±0.10            | 0.84                 | 0.83±0.10            | 0.74                 | 0.73±0.10            |
| 0.547           | 0.91                 | 0.95±0.10            | 0.79                 | 0.78±0.10            | 0.67                 | 0.61±0.10            |
| 0.647           | 0.83                 | 0.96±0.15            | 0.75                 | 0.77±0.10            | 0.62                 | 0.54±0.14            |

#### Table 3. Same as Table 1 but for the multiplicity class $N=17$–$22$.

| $P_\perp$ [GeV] | $R_{\text{long}}$ [fm] | $R_{\text{long}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{out}}$ [fm] | $R_{\text{out}}$ [fm] |
|-----------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
|                 | model exp.           | model exp.           | model exp.           | model exp.           | model exp.           | model exp.           |
| 0.163           | 1.86                 | 1.88±0.30            | 1.14                 | 1.18±0.11            | 1.24                 | 0.99±0.14            |
| 0.251           | 1.52                 | 1.49±0.14            | 1.08                 | 1.11±0.10            | 1.07                 | 1.11±0.12            |
| 0.349           | 1.27                 | 1.22±0.11            | 1.00                 | 0.98±0.10            | 0.90                 | 0.90±0.10            |
| 0.448           | 1.10                 | 1.12±0.11            | 0.94                 | 0.90±0.10            | 0.77                 | 0.72±0.10            |
| 0.548           | 0.98                 | 1.03±0.10            | 0.88                 | 0.87±0.10            | 0.69                 | 0.62±0.10            |
| 0.647           | 0.89                 | 1.00±0.16            | 0.83                 | 0.89±0.10            | 0.64                 | 0.54±0.14            |
### BWM description of HBT radii

Mult. class 23–28

| $P_\perp$ [GeV] | $R_{\text{long}}$ [fm] | $R_{\text{long}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{out}}$ [fm] | $R_{\text{out}}$ [fm] |
|-----------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
|                 | model                | exp.                 | model                | exp.                 | model                | exp.                 |
| 0.163           | 1.94                 | 1.99±0.31            | 1.26                 | 1.30±0.12            | 1.32                 | 1.18±0.17            |
| 0.251           | 1.58                 | 1.56±0.15            | 1.18                 | 1.21±0.10            | 1.12                 | 1.15±0.13            |
| 0.349           | 1.33                 | 1.29±0.11            | 1.09                 | 1.06±0.10            | 0.92                 | 0.93±0.10            |
| 0.448           | 1.15                 | 1.15±0.11            | 1.01                 | 0.99±0.10            | 0.79                 | 0.73±0.10            |
| 0.548           | 1.03                 | 1.05±0.11            | 0.94                 | 0.97±0.10            | 0.70                 | 0.63±0.10            |
| 0.648           | 0.93                 | 1.13±0.19            | 0.89                 | 0.91±0.12            | 0.65                 | 0.48±0.13            |

Table 4. Same as Table 1 but for the multiplicity class $N=23–28$.

Mult. class 29–34

| $P_\perp$ [GeV] | $R_{\text{long}}$ [fm] | $R_{\text{long}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{out}}$ [fm] | $R_{\text{out}}$ [fm] |
|-----------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
|                 | model                | exp.                 | model                | exp.                 | model                | exp.                 |
| 0.163           | 2.01                 | 1.98±0.31            | 1.36                 | 1.35±0.12            | 1.38                 | 1.23±0.17            |
| 0.251           | 1.64                 | 1.60±0.15            | 1.26                 | 1.30±0.10            | 1.15                 | 1.20±0.14            |
| 0.349           | 1.37                 | 1.32±0.11            | 1.16                 | 1.09±0.11            | 0.94                 | 0.90±0.10            |
| 0.448           | 1.20                 | 1.16±0.11            | 1.07                 | 1.06±0.11            | 0.80                 | 0.75±0.10            |
| 0.548           | 1.06                 | 1.18±0.11            | 1.00                 | 1.01±0.11            | 0.71                 | 0.61±0.10            |
| 0.648           | 0.97                 | 1.13±0.19            | 0.93                 | 1.05±0.13            | 0.65                 | 0.52±0.15            |

Table 5. Same as Table 1 but for the multiplicity class $N=29–34$.

Mult. class 35–41

| $P_\perp$ [GeV] | $R_{\text{long}}$ [fm] | $R_{\text{long}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{side}}$ [fm] | $R_{\text{out}}$ [fm] | $R_{\text{out}}$ [fm] |
|-----------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
|                 | model                | exp.                 | model                | exp.                 | model                | exp.                 |
| 0.163           | 2.06                 | 1.99±0.31            | 1.46                 | 1.43±0.12            | 1.44                 | 1.34±0.19            |
| 0.251           | 1.69                 | 1.63±0.15            | 1.35                 | 1.35±0.13            | 1.17                 | 1.22±0.14            |
| 0.349           | 1.41                 | 1.37±0.11            | 1.23                 | 1.17±0.11            | 0.95                 | 0.92±0.11            |
| 0.448           | 1.23                 | 1.22±0.11            | 1.13                 | 1.12±0.11            | 0.80                 | 0.75±0.11            |
| 0.548           | 1.10                 | 1.19±0.11            | 1.04                 | 1.07±0.11            | 0.71                 | 0.60±0.10            |
| 0.648           | 1.00                 | 1.15±0.20            | 0.98                 | 1.14±0.14            | 0.66                 | 0.54±0.15            |

Table 6. Same as Table 1 but for the multiplicity class $N=35–41$. 
### Table 7. Same as Table 1 but for the multiplicity class $N=42–51$. 

| $P_\perp$ [GeV] | $R_{\text{long}}$ [fm] model | $R_{\text{long}}$ [fm] exp. | $R_{\text{side}}$ [fm] model | $R_{\text{side}}$ [fm] exp. | $R_{\text{out}}$ [fm] model | $R_{\text{out}}$ [fm] exp. |
|-----------------|-------------------------------|-----------------------------|-------------------------------|-------------------------------|-----------------------------|-----------------------------|
| 0.163           | 2.07                          | 2.14±0.34                   | 1.55                          | 1.53±0.13                    | 1.48                        | 1.43±0.21                   |
| 0.251           | 1.70                          | 1.66±0.16                   | 1.41                          | 1.41±0.13                    | 1.18                        | 1.24±0.14                   |
| 0.349           | 1.42                          | 1.33±0.11                   | 1.28                          | 1.21±0.11                    | 0.94                        | 0.94±0.11                   |
| 0.448           | 1.24                          | 1.23±0.11                   | 1.17                          | 1.12±0.11                    | 0.80                        | 0.69±0.11                   |
| 0.548           | 1.10                          | 1.23±0.11                   | 1.08                          | 1.18±0.11                    | 0.71                        | 0.61±0.11                   |
| 0.648           | 1.00                          | 1.27±0.22                   | 1.01                          | 1.25±0.17                    | 0.66                        | 0.42±0.15                   |

### Table 8. Same as Table 1 but for the multiplicity class $N=52–151$. 

| $P_\perp$ [GeV] | $R_{\text{long}}$ [fm] model | $R_{\text{long}}$ [fm] exp. | $R_{\text{side}}$ [fm] model | $R_{\text{side}}$ [fm] exp. | $R_{\text{out}}$ [fm] model | $R_{\text{out}}$ [fm] exp. |
|-----------------|-------------------------------|-----------------------------|-------------------------------|-------------------------------|-----------------------------|-----------------------------|
| 0.163           | 2.21                          | 2.14±0.50                   | 1.78                          | 1.63±0.19                    | 1.59                        | 1.60±0.32                   |
| 0.251           | 1.81                          | 1.77±0.24                   | 1.60                          | 1.54±0.21                    | 1.23                        | 1.28±0.22                   |
| 0.349           | 1.52                          | 1.49±0.14                   | 1.43                          | 1.39±0.14                    | 0.97                        | 0.97±0.14                   |
| 0.449           | 1.33                          | 1.25±0.14                   | 1.30                          | 1.27±0.14                    | 0.81                        | 0.77±0.16                   |
| 0.548           | 1.18                          | 1.39±0.17                   | 1.20                          | 1.33±0.15                    | 0.73                        | 0.62±0.16                   |
| 0.648           | 1.08                          | 1.32±0.33                   | 1.12                          | 1.29±0.24                    | 0.67                        | 0.35±0.16                   |
Figure 5. (Color online) The model results (solid curves) compared to the experiment results (central points of the bands). The width of the bands represents the experimental error.
BWM description of HBT radii

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