Evidence for an alpha cluster condensed state in $^{16}$O$(\alpha, \alpha')$ at 400 MeV

T. Wakasa\textsuperscript{a} E. Ihara\textsuperscript{a} K. Fujita\textsuperscript{b} Y. Funaki\textsuperscript{c} K. Hatanaka\textsuperscript{b} H. Horiuchi\textsuperscript{b} M. Itoh\textsuperscript{d} J. Kamiya\textsuperscript{e} G. Röpke\textsuperscript{f} H. Sakaguchi\textsuperscript{g} N. Sakamoto\textsuperscript{b} Y. Sakemi\textsuperscript{b} P. Schuck\textsuperscript{h} Y. Shimizu\textsuperscript{b} M. Takashina\textsuperscript{i,c} S. Terashima\textsuperscript{c} A. Tohsaki\textsuperscript{b} M. Uchida\textsuperscript{j} H. P. Yoshida\textsuperscript{k} M. Yosoi\textsuperscript{b}

\textsuperscript{a}Department of Physics, Kyushu University, Fukuoka 812-8581, Japan
\textsuperscript{b}Research Center for Nuclear Physics, Osaka University, Osaka 567-0047, Japan
\textsuperscript{c}The Institute of Physical and Chemical Research, Wako, Saitama 351-0198, Japan
\textsuperscript{d}Cyclotron and Radioisotope Center, Tohoku University, Sendai, Miyagi 980-8578, Japan
\textsuperscript{e}Accelerator Group, Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195, Japan
\textsuperscript{f}Institut für Physik, Universität Rostock, D-18051 Rostock, Germany
\textsuperscript{g}Department of Engineering, Miyazaki University, Miyazaki 889-2192, Japan
\textsuperscript{h}Institut de Physique, Nucléaire, F-91406 Orsay Cedex, France
\textsuperscript{i}Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
\textsuperscript{j}Department of Physics, Tokyo Institute of Technology, Tokyo 152-8550, Japan
\textsuperscript{k}Center for Research and Advancement in Higher Education, Kyushu University, Fukuoka 810-8560, Japan

Abstract

Inelastic $\alpha$ scattering on $^{16}$O is studied at 400 MeV by using an ice target. Near the $4\alpha$ breakup threshold of 14.4 MeV, a broad peak is observed at an excitation energy of 13.6±0.2 MeV with a width of 0.6±0.2 MeV. The spin-parity is estimated to be $0^+$ from the momentum-transfer dependence. The observed width is significantly larger than those of the neighboring $0^+$ states indicating a state with a well-developed $\alpha$ cluster structure. The magnitude of the cross section is sensitive to the density distribution of the constituent $\alpha$ clusters. The observed cross section is consistent with the theoretical prediction for the $\alpha$ cluster condensed state characterized by its dilute density distribution with a large root-mean-square radius of about 4.3 fm.
In the last decade, Green’s function Monte Carlo (GFMC) calculations have successfully reproduced ground and low-lying excited state energies of light nuclei up to \( A = 10 \) [1,2,3] with an accuracy of 1–2\% using realistic two-nucleon [4] and three-nucleon potentials [5,6]. One of the most impressive results of these so-called first-principles calculations for \( A = 8 \) is that the intrinsic density distribution for the ground state of \( ^8\text{Be} \) has two peaks representing a 2\( \alpha \) cluster structure [1]. These two clusters are separated by about 4 fm, and thus they form a dilute system. It is natural to expect that there are also loosely bound dilute states with 3\( \alpha \) and 4\( \alpha \) clusters as excited states in \( ^{12}\text{C} \) and \( ^{16}\text{O} \), respectively. For \( ^{12}\text{C} \), several 3\( \alpha \) cluster calculations [7,8,9] almost a quarter century ago have shown that the second 0\(^+\) state has a larger root-mean-square (rms) radius than the ground state by about 1 fm. These results imply that this 0\(^+\) state has a gas-like structure of 3\( \alpha \) clusters predominantly in relative \( S \) waves [10,11].

Recently, using a new \( \alpha \) cluster wave function, a 4\( \alpha \) cluster state of dilute density has been theoretically predicted in \( ^{16}\text{O} \) near the 4\( \alpha \) breakup threshold [12]. The same wave function has also been used to predict a 3\( \alpha \)-cluster state with dilute density in \( ^{12}\text{C} \) near the 3\( \alpha \) breakup threshold [12,13], and there are strong indications [14] that the second 0\(^+\) state is this state. This new \( \alpha \) cluster wave function can represent a condensation of \( \alpha \) clusters in a spherically symmetric state. Because the \( \alpha \) cluster is a boson, an excited state with dilute density composed of weakly interacting \( \alpha \) clusters in relative \( S \) waves can be considered as a \( \alpha \) cluster condensed (ACC) state. This idea is based on theoretical investigations on the possibility of \( \alpha \) particle condensation in low-density nuclear matter [15]. It should be noted that, in the case of finite self-conjugate 4\( n \) nuclei, coherent phenomena such as super-fluidity observed in a Bose–Einstein condensed state are not expected to occur since \( n \) is very small, e.g., \( n = 3 \) in \( ^{12}\text{C} \). Nevertheless the idea of \( \alpha \) cluster condensation in nuclei is very important since the ACC state does seem to exist in self-conjugate \( n\alpha \) nuclei [12,16]. Therefore, it is very important to search for and investigate the theoretically predicted 4\( \alpha \) cluster condensed state in \( ^{16}\text{O} \). In this Letter we report experimental evidence for a broad 0\(^+\) state at an excitation energy of \( E_x = 13.6 \pm 0.1 \text{ MeV} \) with a width of \( \Gamma = 0.6 \pm 0.2 \text{ MeV} \) which can be interpreted as being the predicted exotic ACC state.

Measurements were carried out using the West–South Beam Line (WS-BL) [17] and the Grand Raiden (GR) spectrometer [18] at the Research Center for Nuclear Physics, Osaka University. The WS-BL provides a double-achromatic

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**Email address:** wakasa@phys.kyushu-u.ac.jp (T. Wakasa).
beam with zero lateral and angular dispersions. The beam energy was $T_\alpha = 400$ MeV (100 MeV/A) and its energy spread was 108 keV. The beam bombarded a windowless and self-supporting ice (H$_2$O) target [19] with a thickness of 9 mg/cm$^2$. The thickness was determined by comparing the elastic yield from the SiO$_2$ target with a thickness of 2.0 mg/cm$^2$. Scattered $\alpha$ particles from the target were momentum analyzed by the high-resolution GR spectrometer with a typical resolution of about 160 keV FWHM. The yields of the scattered $\alpha$ particles were extracted using the peak-shape fitting program ALLFIT [20].

The elastic differential cross sections of $^{16}$O are shown in Fig. 1. The data decrease almost monotonically as a function of momentum transfer over a momentum-transfer region up to 4.7 fm$^{-1}$. The data were analyzed by a phenomenological optical model potential (OMP). We adopt the single-folding model based on the nucleon–$\alpha$ interaction, in which an empirical nucleon–$\alpha$ potential [21] is folded with the nucleon density of $^{16}$O. Note that the resulting OMP is a complex potential because the nucleon–$\alpha$ potential is complex. The solid curve in Fig. 1 shows the result using the OMP generated by the folding. The single-folding potential reproduces the data well up to $q_{c.m.} \approx 3$ fm$^{-1}$ and slightly underestimates the data for $q_{c.m.} \gtrsim 3$ fm$^{-1}$. This underestimation is believed to be due to the energy dependence of the nucleon–$\alpha$ interaction and could be resolved by adjusting the parameters of the single-folding potential. However, in the following, we use the OMP for the analysis of the ACC without modification because the ACC data have been measured only up to $q_{c.m.} \approx 2.3$ fm$^{-1}$ where the elastic data are well reproduced by this OMP.

Figure 2(a) shows the excitation energy spectrum of $^{16}$O($\alpha, \alpha'$) scattering including $\theta_{c.m.} = 0^\circ$. The effects of the finite solid angle of GR have been taken into account in all the momentum transfer $q_{c.m.}$ values reported here. For Fig. 2(a), $q_{c.m.} \approx 0.2$ fm$^{-1}$ is obtained. At this small $q_{c.m.}$, the $J^\pi = 0^+$ state at $E_x = 12.0$ MeV is the most prominent peak and the $J^\pi = 2^+$ state at $E_x = 11.5$ MeV also forms a prominent peak. Both broad peaks at $E_x \approx 13.0$ and 14.0 MeV consist of several peaks. An ACC state is expected to be in this region. We performed peak fitting using ALLFIT to find evidence of the ACC state. In the peak fitting, the narrow peaks of $^{16}$O were described by a standard hyper-Gaussian line shape and the peaks with intrinsic widths were described as Lorentzian shapes convoluted with a resolution function based on the narrow peaks. The positions and widths were taken from Ref. [22]. It was found that the region below $E_x \approx 13$ MeV is well reproduced in the peak fitting, whereas the region around $E_x \approx 13.6$ MeV is significantly under-predicted because there is no known state in this region [22]. The under-prediction is common at all momentum transfers and indicates the existence of a new state in this region.

A wavelet analysis technique was employed to investigate the structure of the new state. This technique has been developed in signal theory [23] and has
also been used in nuclear physics for the extraction of characteristic scales in the fine structure of giant resonances [24,25]. The excitation energy spectrum $\sigma(E)$ was folded with a chosen wavelet function $\Psi$ and the coefficients

$$C(E_x, \Delta E) = \frac{1}{\sqrt{\Delta E}} \int \sigma(E) \Psi \left( \frac{E - E_x}{\Delta E} \right) dE$$  \hspace{1cm} (1)

were obtained in the continuum wavelet transform (CWT), where $E_x$ is the excitation energy and $\Delta E$ is the scale parameter of $\Psi$. We used the Morlet type function consisting of a cosine function with a Gaussian envelope,

$$\Psi(x) = \frac{1}{\sqrt{\pi}} \cos(kx) \exp \left( -\frac{x^2}{2} \right), \hspace{0.5cm} k = \frac{5}{2\pi}$$  \hspace{1cm} (2)

which is often used for the wavelet function [24,25].

Figures 2(b)–(d) show the results of CWT for the data at $q_{\text{c.m.}} = 0.2$ fm$^{-1}$. The correlation of the wavelet coefficients $C$ in Eq. (1) is displayed in Fig. 2(b) as functions of $E_x$ and $\Delta E$. For the bumps at $E_x \approx 13.0$ and 14.0 MeV consisting of several peaks, two maxima at scales of about 0.1 and 0.7 MeV are observed. The larger value corresponds to the total width of the bumps and the smaller value reflects the width of the constituent peaks as determined by the experimental energy resolution. These results indicate that the CWT technique can be applied to the investigation of the fine structure of the bump excited by $(\alpha, \alpha')$ scattering. Figure 2(c) shows $C$ at a scale of 0.12 MeV as a function of excitation energy. The structures of the Morlet type function are clearly found at $E_x \approx 9.8, 11.5,$ and 12.0 MeV, which correspond to the known states with $J^\pi = 2^+, 2^+$, and $0^+$, respectively. An uneven structure of $C$ is identified at $E_x \gtrsim 13$ MeV, displayed magnified in Fig. 2(d). The structure at $E_x \approx 13.6$ MeV indicates the existence of the new state because there is no known state in this region. The observed structure is well reproduced by assuming the excitation energy of the new state to be $E_x = 13.6$ MeV, as shown by the dashed curve in Fig. 2(d). The reproducibility is sensitive to the excitation energy of the new state and thus the uncertainty is estimated to be $\pm 0.1$ MeV. It is rather difficult to determine the width of the new state from the results of CWT because of interference from the neighboring bumps at $E_x \approx 13.0$ and 14.0 MeV.

In order to deduce the width of the new state, we performed peak fitting in this region by adding the new state at $E_x = 13.6$ MeV, and determined its width so as to reproduce the experimental data. Figure 3 shows the results of the fitting at $q_{\text{c.m.}} = 0.2$ fm$^{-1}$ for $\Gamma = 0.3, 0.6,$ and 0.9 MeV. The dashed curves represent the fits to the individual peaks while the straight line and solid curve represent the background and the sum of the peak fitting, respectively. In the case of $\Gamma = 0.3$ MeV, the region around the new state could not be well reproduced, and in the case of $\Gamma = 0.9$ MeV, the region at $E_x \approx 14.4$ MeV is overestimated.
The best reproduction is achieved for $\Gamma = 0.6$ MeV and its uncertainty is estimated to be about 0.2 MeV, calculated considering this result and the uncertainty of the excitation energy. We also performed peak fitting for other momentum-transfer data; the results are displayed in Fig. 4 for $q_{\text{c.m.}} \approx 0.2$, 0.5, and 1.2 fm$^{-1}$. At all momentum transfers, reasonable reproduction of the data is achieved by including the new state at $E_x = 13.6 \pm 0.1$ MeV with $\Gamma = 0.6 \pm 0.2$ MeV.

It should be noted that the width $\Gamma = 0.6 \pm 0.2$ MeV of the new state is significantly larger than that of the neighboring $0^+$ states at $E_x = 12.0$ and 14.0 MeV, with $\Gamma = 1.5$ and 185 keV [22], respectively. A recent theoretical investigation with the new $\alpha$ cluster wave function predicts the ACC state of $^{16}$O to be $E_x \approx 14.1$ MeV and $\Gamma \approx 1.5$–1.9 MeV [26]. Quantitative comparisons between experimental and theoretical results are rather difficult due to the parameter dependence of the theoretical predictions. However, the calculations show qualitatively that the ACC state should be around the $4\alpha$ break-up threshold of 14.4 MeV with a significantly large $\Gamma$ compared with those of the $0^+$ states described in the shell model. The ACC state is characterized by the diluteness of the constituent $\alpha$ clusters, and this diluteness results in the large $\Gamma$ value of the ACC. The observed new state is consistent with this theoretical prediction for the ACC. Note that the transition form factor to the ACC is almost independent of the parameters [26]. Thus, in the following, a qualitative comparison is made of the cross section in order to confirm that the new state is the ACC.

Figure 5 shows the cross section of the new state at $E_x = 13.6$ MeV as a function of momentum transfer. The error bars include both statistical and systematic uncertainties of the data, with the systematic uncertainties being dominant. The systematic uncertainties are estimated by taking into account the uncertainties of both the excitation energy and the width of the new state. The data cover a momentum-transfer region up to about 2.3 fm$^{-1}$, and have a maximum at $q_{\text{c.m.}} = 0.2$ fm$^{-1}$ ($\theta_{\text{c.m.}} \approx 0^\circ$). This forward-peaking of the cross section indicates that the transferred angular momentum $L$ is 0 and the spin parity of the new state is $0^+$ by considering the spin-scalar and isospin-scalar nature of $(\alpha, \alpha')$. We compare our cross section with a microscopic coupled-channel (MCC) calculation for the ACC state. The OMP is the same as that used for the analysis of the elastic scattering data, and the transition form factor is obtained using the ACC wave function described in Ref. [26]. The solid curve in Fig. 5 is the result of the calculation and reproduces the experimental data well without normalization. It should be noted that the magnitude of the form factor is sensitive to the rms radius of the ACC state, whereas its radial dependence is insensitive to the rms radius [26]. Therefore, for the corresponding cross section in $(\alpha, \alpha')$, the rms radius should be investigated through its magnitude rather than its momentum-transfer dependence [27]. Since we have used an OMP that reproduces the elastic scattering data well,
we can compare the absolute values of both the experimental and theoretical results. The good theoretical reproduction without any normalization shows that the rms radius of the new state is consistent with that of the ACC state used in the calculation. Therefore, the rms radius could be estimated to be same as the theoretical value of about 4.3 fm [26], which is significantly larger than that of the ground state of $^{16}$O of 2.7 fm. This large rms radius is due to the diluteness of the $\alpha$ clusters, and thus the new state can be interpreted as being the ACC state characterized by its dilute density distribution.

In conclusion, we have searched for evidence of the ACC state in $^{16}$O in $^{16}$O($\alpha, \alpha'$) scattering at 400 MeV using an ice target. A broad peak was found at $E_x = 13.6 \pm 0.1$ MeV with $\Gamma = 0.6 \pm 0.2$ and its spin-parity $J^\pi$ was estimated to be $0^+$ from the momentum-transfer dependence. The excitation energy is near the $4\alpha$ breakup threshold of 14.4 MeV, whereas the width is significantly larger than those of neighboring $0^+$ states. This large $\Gamma$ indicates that the $\alpha$ cluster structure is well-developed in the new state. The magnitude of the cross section is sensitive to the diluteness of the $\alpha$ clusters. The observed cross section, the excitation energy, and the width all strongly indicate the existence of an ACC state in $^{16}$O, characterized by a large rms radius of about 4.3 fm compared with that of the ground state of 2.7 fm.

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Fig. 1. Measurement of the cross section (solid circles) for $^{16}\text{O}(\alpha,\alpha)^{16}\text{O}(g.s.)$ at $T_{\alpha} = 400$ MeV. The solid curve is the theoretical prediction using the single folding potential for $^{16}\text{O}$, as explained in the text.
Fig. 2. (a) Spectrum of $^{16}$O($\alpha,\alpha'$) scattering at $T_{\alpha} = 400$ MeV and $q_{c.m.} = 0.2$ fm$^{-1}$. (b) Result of CWT analysis as a function of both excitation energy and scale parameter. (c) Wavelet coefficients at a scale parameter of 0.12 MeV. (d) Magnification of the region in (c) around the new state at 13.6 MeV.
Fig. 3. Results of peak fitting of $^{16}\text{O}(\alpha, \alpha')$ at $T_\alpha = 400$ MeV and $q_{c.m.} = 0.2$ fm$^{-1}$ with hyper-Gaussian and Lorentzian peaks and a continuum, with the width $\Gamma$ of the new state at 13.6 MeV taken to be (a) 0.3 MeV, (b) 0.6 MeV, and (c) 0.9 MeV.
Fig. 4. Reproduction of the excitation energy spectra for $^{16}\text{O} (\alpha, \alpha')$ at $T_\alpha = 400$ MeV and a momentum transfer $q_{c.m.}$ of (a) 0.2 fm$^{-1}$, (b) 0.5 fm$^{-1}$, and (c) 1.2 fm$^{-1}$. The width of the new state at 13.6 MeV is taken to be as 0.6 MeV.
Fig. 5. Measurement of the cross section of $^{16}\text{O}(\alpha,\alpha')^{16}\text{O}(13.6\text{MeV})$ at $T_\alpha = 400\text{MeV}$. The error bars include both statistical and systematic uncertainties of the data. The solid curve is the theoretical prediction for the ACC state in $^{16}\text{O}$, as described in the text.
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