Relativistic electromagnetic mass models in spherically symmetric spacetime

S.K. Maurya\textsuperscript{a}, Y.K. Gupta\textsuperscript{b}, Saibal Ray\textsuperscript{c} Vikram Chatterjee\textsuperscript{d}

\textsuperscript{a}Department of Mathematical & Physical Sciences, College of Arts & Science, University of Nizwa, Nizwa-Sultanate of Oman
\textsuperscript{b}Department of Mathematics, Jaypee Institute of Information Technology University, Sector-128 Noida (U.P.), India
\textsuperscript{c}Department of Physics, Government College of Engineering & Ceramic Technology, Kolkata 700010, West Bengal, India
\textsuperscript{d}Department of Physics, Central Footwear Training Centre, Kalipur, Budge Budge, South 24 Parganas 700138, West Bengal, India

Abstract

Under the static spherically symmetric Einstein-Maxwell spacetime of embedding class one we explore possibility of electromagnetic mass model where mass and other physical parameters have purely electromagnetic origin \cite{1,2,3}. This work is in continuation of our earlier investigation \cite{4} where we developed an algorithm and found out three new solutions of electromagnetic mass models. In the present letter we consider different metric potentials $\nu$ and $\lambda$ and analyzed them in a systematic way. It is observed that some of the previous solutions related to electromagnetic mass models are nothing but special cases of the presently obtained generalized solution set. We further verify the solution set and show that these are extremely applicable in the case of compact stars as well as for understanding structure of the electron.

Key words: General Relativity; electromagnetic mass; compact star; electron

Email addresses: sunil@unizwa.edu.om (S.K. Maurya), kumar001947@gmail.com (Y.K. Gupta), saibal@iucaa.ernet.in (Saibal Ray), vikphy1979@gmail.com (Vikram Chatterjee).
1 Introduction

The general theory of relativity (GTR), an outstanding extension of the special theory of relativity with non-uniform reference frame, was put forward by Einstein \[5\] in the year 1915 - exactly 100 years ago! Till date this is considered as the most profound and effective theory of gravitation. The field theoretical effect of this geometric theory has been described by Wheeler \[6\] in a poetic exposition as follows: "Matter tells space-time how to bent and space-time returns the complement by telling matter how to move".

In the present context we employ GTR as our background canvas to formulate solutions from class 1 metric and thereafter to investigate electromagnetic mass models. As the present work is a sequel of our earlier work \[4\] so we shall refer this article of Maurya et al. \[4\] for detailed discussions on class 1 metric as well as the electromagnetic mass models. However, in the present work we particularly give emphasis on the concept of electromagnetic mass models for which a brief historical and philosophical review can be obtained in the Ref. \[7\]. On the other hand, for the inclusion of charge in the spherical bodies one can look at the Ref. \[8\].

However, a special discussion on the electromagnetic mass models seems required as provided by Gautreau \[2\]. Along the line of thinking of Tiwari et al. \[1\] he shown that the Einstein-Maxwell field equations of GTR can be used to construct a Lorentzian model of an electron as an extended body consisting of pure charge and no matter \[9,10,11,12\]. However, in contrast with Lorentz’s approach using inertial mass, Gautreau \[2\] associated the mass of the electron with its Schwarzschild gravitational mass and thus the field equations for a Lorentz-type pure-charge extended electron could be obtained by setting the matter terms equal to zero in the field equations for a spherically symmetric charged perfect fluid. He examined several explicit solutions to the pure-charge field equations which we have shall use as a standard benchmark to compare our solution set.

In connection to the above work on the electromagnetic mass models it have been specially argued by Maurya et al. \[4\] that most of the investigators \[13,14,15,16,17,18,19,20,21,22,23\] consider an ad hoc equation of state \(\rho + p = 0\) (where \(\rho\) is the density and \(p\) is the pressure), which suffers from a negative pressure, and in the literature known as a false vacuum or degenerate vacuum or \(\rho\)-vacuum \[24,25,26,27\]. In the present investigation, however, for the construction of electromagnetic mass models following Maurya et al. \[4\] we also employ a different technique by adopting an algorithm. We shall see later on that this algorithm will act as general platform to generate physically valid solutions compatible with the spherically symmetric class one metric.
The plan of the present work can be outlined as follows: we have provided the static spherically symmetric spacetime and Einstein-Maxwell field equations in the Sec. 2. In the Sec. 3 an algorithm for class one metric has been developed from which we construct a set of new general solutions. As a particular case this solution set reduces to three sub-set as follows: (i) \( a = 0 \) which corresponds to the charge analogue of the Schwarzschild [28] interior solution, (ii) \( a = 2b \) with \( A = 0 \) which corresponds to the charge analogue of the Kohlar-Chao [29] interior solution, and (iii) \( a = b \) which corresponds to the concept of electromagnetic mass model as proposed by Lorentz [9] where \( a, b \) and \( A \) are some constants. In the next Sec. 5 boundary conditions are discussed to find out constants of integration. The Sec. 6 deals with the solutions where critical analysis has been performed to check several physical properties of the model whereas in the Sec. 7 we have particularly discussed some special features of the models, firstly regarding validity with the stellar structure, and secondly with the structure of the electron. We have made some remarks in the concluding Sec. 8.

2 The static spherically symmetric spacetime and Einstein-Maxwell field equations

The Einstein-Maxwell field equations can be provided as usual

\[
G^i_j = R^i_j - \frac{1}{2} R g^i_j = \kappa(T^i_j + E^i_j),
\]

where \( k = 8\pi \) is the Einstein constant (\( G = c = 1 \), in the relativistic units).

The matter distribution inside the star is assumed to be locally perfect fluid and consequently \( T^i_j \) and \( E^i_j \), the energy-momentum tensors for fluid distribution and electromagnetic field respectively, are defined by

\[
T^i_j = \left( \rho + p \right) v^i v^j - p \delta^i_j,
\]

\[
E^i_j = \frac{1}{4\pi} \left( -F^m_i F_{jm} + \frac{1}{4} \delta^i_j F^{mn} F_{mn} \right),
\]

where \( v^i \) is the four-velocity as \( e^{-\nu(r)/2} v^i = \delta^i_4 \), \( \rho \) is the energy density and \( p \) is the fluid pressure of the matter distribution.

Now the anti-symmetric electromagnetic field tensor, \( F_{ij} \), satisfies the Maxwell equations

\[
F_{ik,j} + F_{kj,i} + F_{ji,k} = 0,
\]
\[
\frac{\partial}{\partial x^k}(\sqrt{-g}F^{ik}) = -4\pi\sqrt{-g}j^i,
\]
(5)

where \( g \) is the determinant of quantities \( g_{ij} \) in Eq. (8) and is defined by \( g = -e^{(\nu + \lambda)}r^4\sin^2\theta \).

The only non-vanishing components of electromagnetic field tensor are \( F^{41} \) and \( F_{14} \) which describe the radial component of the electric field and are related as \( F^{41} = -F^{14} \). From Eq. (5), we can obtain the following expression for the electric field

\[
F^{41} = e^{-\frac{(\lambda + \nu)}{2}} q(r) \frac{1}{r^2}
\]
(6)

where \( q(r) \) represents the total charge contained within the sphere of radius \( r \) and is defined by

\[
q(r) = r^2 \sqrt{-F_{14}F^{14}} = r^2 F^{41} e^{(\lambda + \nu)/2} = r^2 E = 4\pi \int_0^r \sigma r^2 e^{\lambda/2} dr,
\]
(7)

where \( \sigma \) is the charge density.

Now, following the work of Maurya et al. [4] here we consider the static spherically symmetric metric in the form

\[
ds^2 = -e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) + e^\nu dt^2.
\]
(8)

The above metric represents spacetime of embedding class one if it satisfies the Karmarkar condition [30]

\[
R_{1414} = \frac{R_{1212}R_{3434} + R_{1224}R_{1334}}{R_{2323}}
\]
(9)

along with the constraint \( R_{2323} \neq 0 \) [31].

Therefore, by the application of above condition in Eq. (8) we obtain the following second order differential equation

\[
\frac{\lambda' \nu'}{(1 - e^\lambda)} = -2(\nu'' + \nu'^2) + \nu'^2 + \lambda' \nu' ; \quad e^\lambda \neq 1,
\]
(10)

where \( \nu(r) \) and \( \lambda(r) \) are metric potentials and depends only on the radial coordinate \( r \).
After manipulation, the solution of the second order differential equation (10), can be obtained as

\[ e^\lambda = \left(1 + K \frac{\nu'^2 e^\nu}{4}\right). \]  

(11)

Here \( K \) is a non-zero arbitrary constant, \( \nu'(r) \neq 0, e^{\lambda(0)} = 1 \) and \( \nu'(0) = 0 \).

For the above spherically symmetric metric (8), the Einstein-Maxwell field equations (1) can be expressed as the following set of ordinary differential equations [4,8]

\[ \frac{\nu'}{r} e^{-\lambda} - \frac{(1 - e^{-\lambda})}{r^2} = -\kappa T_1^1 = \kappa p - \frac{q^2}{r^4}, \]  

(12)

\[ -\kappa T_3^3 = \left[ \frac{\nu''}{2} - \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right] e^{-\lambda} = -\kappa T_2^2 = \kappa p + \frac{q^2}{r^4}, \]  

(13)

\[ \frac{\nu'}{r} e^{-\lambda} + \frac{(1 - e^{-\lambda})}{r^2} = \kappa T_4^4 = \kappa \rho + \frac{q^2}{r^4}, \]  

(14)

where the prime denotes differentiation with respect to \( r \).

Therefore, by incorporating Eq. (11) in the set of Eqs. (12) - (14), we get

\[ \frac{\nu'}{r^2(4 + K \nu'^2 e^\nu)}(4r - K \nu') = \kappa p - \frac{q^2}{r^4}, \]  

(15)

\[ \frac{4}{4 + K \nu'^2 e^\nu} \left( \frac{\nu'}{2r} - \frac{(K \nu' e^\nu - 2r)(2\nu'' + \nu'^2)}{2r(4 + K \nu'^2 e^\nu)} \right) = \kappa p + \frac{q^2}{r^4}, \]  

(16)

\[ \frac{K e^\nu \nu'}{4 + K \nu'^2 e^\nu} \left( \frac{4(2\nu'' + \nu'^2)}{(4 + K \nu'^2 e^\nu)} + \frac{\nu'}{r} \right) = \kappa \rho + \frac{q^2}{r^4}. \]  

(17)

Along with these Eqs. (12) - (14), we also include the pressure isotropy condition and pressure gradient as follows

\[ \left( \frac{K \nu' e^\nu}{2r} - 1 \right) \left( \frac{2\nu'}{r(4 + K \nu'^2 e^\nu)} - \frac{4(2\nu'' + \nu'^2)}{(4 + K \nu'^2 e^\nu)^2} \right) = \frac{2q^2}{r^4}, \]  

(18)

\[ \frac{dp}{dr} = -\frac{M_G(r)(\rho + p)}{r^2} e^{(\lambda - \nu)/2} + \frac{q}{4\pi r^4} \frac{dq}{dr}, \]  

(19)

where \( M_G \) is the gravitational mass within the radius \( r \) and is given by

\[ M_G(r) = \frac{1}{2} r^2 \nu' e^{(\nu - \lambda)/2}. \]  

(20)
The above Eq. (19) represents the charged generalization of the Tolman-Oppenheimer-Volkoff (TOV) \[32\] \[33\] equation of hydrostatic equilibrium or equation of continuity.

It have been argued by \[4\] that if charge vanishes in a charged fluid of embedding class one then survived neutral counterpart will only be either the Schwarzschild \[28\] interior solution (or its special cases de-sitter universe or Einstein’s universe) or Kohler-Chao \[29\] solution otherwise either charge cannot be zero or the survived space-time metric is flat.

3 The algorithm of electromagnetic mass models for class one metric

It can be shown that in the presence of electrical charge the fluid sphere under consideration can be defined by the following metric functions:

\[ e^{-\lambda} = 1 - \frac{2m(r)}{r} + \frac{q^2}{r^2}, \quad (21) \]

\[ \nu' = \left( \frac{\kappa r p + \frac{2m}{r^2} - \frac{2q^2}{r^3}}{1 - \frac{2m}{r} + \frac{q^2}{r^2}} \right), \quad (22) \]

where \( m(r) \) is the mass function which in the explicit form can be written as \[34\]

\[ m(r) = \frac{\kappa}{2} \int \rho r^2 dr + \frac{q^2}{2r} + \frac{1}{2} \int \frac{q^2}{r^2} dr. \quad (23) \]

Therefore, Eqs. (12) - (14) in terms of the above mass function \( m(r) \) can be provided as follows \[14\] \[18\]:

\[ -\frac{2m}{r^2} \left[ \frac{(1 + r \nu')}{r} \right] + \frac{\nu'}{r} + \frac{q^2(1 + r \nu')}{r^4} + \frac{q^2}{r^4} = \kappa p, \quad (24) \]

\[ \frac{-m'(r \nu' + 2)}{2r^2} - \frac{m}{2r^2} \left( \frac{2r^2 \nu'' + r^2 \nu'^2 + \nu' r - 2}{r} \right) \]

\[ + \left[ \frac{2rqq' \nu' - 2q^2 \nu' + 4qq' + (r^2 + q^2)(2r \nu'' + r \nu'^2 + 2 \nu')}{4r^3} \right] - \frac{2q^2}{r^4} = \kappa p \quad (25) \]

\[ \frac{2m'}{r^2} - \frac{2qq'}{r^3} = \kappa p \quad (26) \]
From Eqs. (24) and (25), the first order linear differential equation for \( m(r) \) in terms of \( \nu(r) \) and electric charge function \( q(r) \) can be provided as:

\[
m' + \frac{(2r^2 \nu'' + r^2 \nu'^2 - 3\nu' r - 6)}{r(\nu' + 2)} m = \frac{r(2r\nu'' + r \nu'^2 - 2\nu')}{2(\nu' + 2)} + f(r), \tag{27}
\]

where

\[
f(r) = \frac{g^2[2r^2 \nu'' + r \nu'(\nu' - 4) - 16]}{2r^2(\nu' + 2)} + \frac{qq'}{r}, \tag{28}
\]

which gives the mass \( m(r) \) as:

\[
m(r) = e^{-\int g(r) dr} \left[ \int [h(r) + f(r)] \left( e^{\int g(r) dr} \right) dr + A \right], \tag{29}
\]

where \( g(r) = \frac{(2r^2 \nu'' + r^2 \nu'^2 - 3\nu' r - 6)}{r(\nu' + 2)} \) and \( h(r) = \frac{r(2r\nu'' + r \nu'^2 - 2\nu')}{2(\nu' + 2)} \).

### 4 A set of new class of solutions

To find out a set of new class of solutions for electromagnetic mass models let us now consider the following forms of the metric potentials

\[
\nu = 2 \log[A + B \sqrt{1 + br^2}], \tag{30}
\]

\[
\lambda = \log \left( \frac{1 + ar^2}{1 + br^2} \right), \tag{31}
\]

where \( A \) and \( B \) are two positive constants with

\[
B = \frac{1}{b} \sqrt{\frac{(a - b)}{K}}, \tag{32}
\]

\( a \) and \( b \) being two real numbers.

Though the above forms of the metric potentials are chosen on the \textit{ad hoc} basis but later on one can see that these will lead us to very interesting and physically valid solutions.
Thus the expressions for electromagnetic mass and electric charge respectively can be provided as

\[
m(r) = \frac{1}{2} r^3 \left[ \frac{(a - b)}{1 + ar^2} + \frac{ar^2[C(r) - D(r)]}{F(r)} \right],
\]

\[
q(r) = Er^2 = \sqrt{a} r^3 \sqrt{\frac{[C(r) - D(r)]}{F(r)}},
\]

where \(C(r) = a(B + bBr^2 + A\sqrt{1 + br^2})\), \(D(r) = b(A\sqrt{1 + br^2} + 2B + 2Bbr^2)\) and \(F(r) = 2(1 + ar^2)^2\sqrt{1 + br^2}[A + B\sqrt{1 + br^2}]\).

The expressions for fluid pressure and energy density are respectively given by

\[
8\pi p = \left[ -a^2r^2 H(r) + 2b[H(r) + 2B(1 + br^2)] + aI_1 \right],
\]

\[
8\pi \rho = \left[ -6bH(r) + a^2r^2 H(r) + aI_2 \right],
\]

where \(H(r) = B(1 + br^2) + A\sqrt{1 + br^2}\), \(I_1 = A(-2 + br^2)\sqrt{1 + br^2} + 2B(-1 + br^2 + 2b^2r^4)\) and \(I_2 = A(6 - br^2)\sqrt{1 + br^2} + 6B(1 + br^2)\).

Therefore, the pressure and density gradients are

\[
\frac{dp}{dr} = 2r \left[ \frac{p_1 + p_2 + p_3}{4(1 + ar^2)^3\sqrt{1 + br^2}[A + B\sqrt{1 + br^2}]^2} \right],
\]

\[
\frac{d\rho}{dr} = -\frac{2A^2r}{8\pi} \left[ \frac{\rho_1 + \rho_2 + \rho_3}{4(1 + ar^2)^3\sqrt{1 + br^2}[A + B\sqrt{1 + br^2}]^2} \right],
\]

where

\[
p_1 = 4Ab^2B - ab[6A^2\sqrt{1 + br^2} + 16B^2(1 + br^2)^{3/2} + AB(22 + 15br^2)],
\]

\[
p_2 = 2a^3r^2[A^2\sqrt{1 + br^2} + B^2(1 + br^2)^{3/2} + 2AB(1 + br^2)],
\]

\[
p_3 = a^2[-2A^2(-3+br^2)\sqrt{1 + br^2} + AB(12+2br^2-7b^2r^4)-2B^2(-3+br^2+4b^2r^4)\sqrt{1 + br^2}],
\]
\[ \rho_1 = -b[22A^2 \sqrt{1 + br^2} + 24B^2(1 + br^2)^{3/2} + AB(46 + 47br^2)], \]

\[ \rho_2 = 2a^2r^2[A^2 \sqrt{1 + br^2} + B^2(1 + br^2)^{3/2} + 2AB(1 + br^2)], \]

\[ \rho_3 = a[-2A^2(-11 + br^2) \sqrt{1 + br^2} + 22B^2(1 + br^2)^{3/2} + AB(44 + 42br^2 - 3b^2r^4)]. \]

4.1 Specific results at a Glance

The metric (8), with the metric potentials (30) and (31), describes the following special cases:

4.1.1 \( a = 0 \)

If \( a = 0 \) then corresponding solution becomes the charge analogue of the Schwarzschild [28] interior solution. In this case of charged fluid sphere the metric potentials turn out to be \( e^\nu = (A + B\sqrt{1 + br^2})^2 \) and \( e^\lambda = (1 + br^2)^{-1} \).

4.1.2 \( a = 2b \) with \( A = 0 \)

If one put \( a = 2b \) and \( A = 0 \) in Eqs. (30) and (31) then corresponding solution becomes the charge analogue of the Kohlar-Chao [29] interior solution with the metric potentials \( e^\nu = B^2(1 + br^2) \) and \( e^\lambda = (1 + 2br^2)/(1 + br^2) \).

4.1.3 \( a = b \)

The case \( a \neq b \) gives charged perfect fluid sphere while the case \( a = b \) implies flat spacetime with \( B = 0 \), \( e^\nu = A^2 \) and \( e^\lambda = 1 \). As a consequence all the physical parameters, viz. mass, electric charge, corresponding pressure as well as density become zero. This result is consistent with the concept of electromagnetic mass model as proposed by Lorentz [9].

In the above analysis it would be curious, on the mathematical point of view, to look at the possibility of replacement \( a \) and \( b \) by \( ka \) and \( kb \) respectively where \( k \) is a constant. On making \( k = 0 \) the metric turns out to be zero, whatever may be the value of \( a \) and \( b \).
However, out of the above three cases we are interested for the third sub-case 4.1.3 which corresponds to the electromagnetic mass model having a long historical background with the structure of electron.

5 Boundary conditions

The arbitrary constants $A$, $B$ and $K$ can be obtained by using the boundary conditions. For the above system of equations the boundary conditions that applicable are as follows: the pressure $p = 0$ at $r = R$, where $r = R$ is the outer boundary of the fluid sphere. Actually, the interior metric should join smoothly at the surface of spheres ($r = R$) to the exterior Reissner-Nordström metric whose mass is $m(r = R) = M$, a constant, given by

$$ds^2 = - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2. \quad (39)$$

This requires the continuity of $e^\lambda$, $e^\nu$ and $Q$ across the boundary $r = R$, so that

$$e^{-\lambda(R)} = 1 - \frac{2M}{R} + \frac{Q^2}{R^2}, \quad (40)$$

$$e^{\nu(R)} = 1 - \frac{2M}{R} + \frac{Q^2}{R^2}, \quad (41)$$

$$q(R) = Q, \quad (42)$$

$$p(r = R) = 0. \quad (43)$$

The pressure is zero on the boundary $r = R$ and hence we obtain

$$\frac{B}{A} = \frac{(a - b)(2 + aR^2)}{[6b - a^2R^2 - a(2 - 4bR^2)]\sqrt{1 + bR^2}}. \quad (44)$$

Again, at the boundary $e^{-\lambda(R)} = e^{\nu(R)}$, which gives

$$A = \frac{(6b - a^2R^2 - 2a + 4abR^2)\sqrt{1 + bR^2}}{(4b + 3abR^2)\sqrt{1 + aR^2}}. \quad (45)$$

Also from Eqs. (44) and (45) one gets

$$B = \frac{(a - b)(2 + aR^2)}{(4b + 3abR^2)\sqrt{1 + aR^2}}. \quad (46)$$
For the third constant $K$, we use the Eqs. (32) and (46), which provides the required expression as

$$K = \frac{(4b + 3abR^2)^2(1 + aR^2)}{b^2(a - b)(2 + aR^2)^2}.$$  \hspace{1cm} (47)

### 6 Physical acceptability conditions for the isotropic stellar models

In this Sec. 6 we have critically verified our models by performing mathematical analysis and plotting several figures for some of the compact star candidates. All these indicate that the results are fantastically overcome all the barrier of the physical tests.

#### 6.1 Regularity and Reality Conditions

**6.1.1 Case 1**

It is expected that the solution should be free from physical and geometrical singularities i.e. the pressure and energy density at the centre should be finite and metric potentials $e^\lambda(r)$ and $e^\nu(r)$ should have non-zero positive values in the range $0 \leq r \leq R$. We observe that at the centre Eqs. (30) and (31) gives $e^\lambda(0) = 1$ and $e^\nu(0) = (A+B)^2$. These results suggest that the metric potentials are positive and finite at the centre. These features can be found explicitly from Fig. 1.

![Fig. 1. The behavior of metric potentials $\nu$ and $\lambda$ with respect to radial coordinate $r/R$](image)

**Fig. 1.** The behavior of metric potentials $\nu$ and $\lambda$ with respect to radial coordinate $r/R$
6.1.2 Case 2

For any physical valid solutions the density $\rho$ and pressure $p$ should be positive inside the star. Also the pressure must vanish on the boundary of the fluid sphere $r = R$. The other physical conditions to be maintained are as follows:

(1) $(dp/dr)_{r=0} = 0$ and $(d^2p/dr^2)_{r=0} < 0$ so that pressure gradient $dp/dr$ is negative for $0 \leq r \leq R$.

(2) $(d\rho/dr)_{r=0} = 0$ and $(d^2\rho/dr^2)_{r=0} < 0$ so that density gradient $d\rho/dr$ is negative for $0 \leq r \leq R$.

The above two conditions (1) and (2) imply that the pressure and density should be maximum at the centre and they should monotonically decrease towards the surface. All these are evident from Fig. 2.

![Fig. 2. The behavior of fluid pressure $p$ and energy density $\rho$ with respect to radial coordinate $r/R$, where $p_i = \kappa p$, $\rho_i = \kappa \rho$.](image)

6.2 Causality and Well Behaved Conditions

Inside the fluid sphere the speed of sound should be less than the speed of light i.e. $0 \leq \left(\frac{dp}{d\rho}\right) < 1$, which can be observed in Fig. 3. We observe from this figure that the velocity of sound monotonically is decreasing away from the centre.

6.3 Energy Conditions

It is, in general, argued that a physically reasonable energy-momentum tensor which represents an isotropic charged fluid sphere composed of matter must satisfy the following energy conditions:
The behavior of sound speed $v$ with respect to radial coordinate $r/R$.

(1) null energy condition (NEC): $\rho + \frac{E^2}{4\pi} \geq 0$
(2) weak energy condition (WEC): $\rho - p + \frac{E^2}{4\pi} \geq 0$
(3) strong energy condition (SEC): $\rho - 3p + \frac{E^2}{4\pi} \geq 0$

The behavior of these energy conditions are shown in Fig. 4. This figure clearly indicates that all the energy conditions in our model are satisfied throughout the interior region of the spherical distribution.

6.4 Stability Conditions

6.4.1 Tolman-Oppenheimer-Volkoff equation

The generalized Tolman-Oppenheimer-Volkoff (TOV) equation can be provided as

$$- \frac{M_G(\rho + p_r)}{r^2} e^{\lambda/2} - \frac{dp}{dr} + \sigma \frac{q}{r^2} e^{\lambda/2} = 0,$$ (48)
where $M_G$ is the effective gravitational mass given by

$$M_G(r) = \frac{1}{2} r^2 e^{\frac{\nu - \lambda}{2}} \nu'.$$  \hspace{1cm} (49)

This Eq. (48) describes the equilibrium condition for a charged perfect fluid subject to the gravitational ($F_g$), hydrostatic ($F_h$) and electric ($F_e$). In summary, we can write it as

$$F_g + F_h + F_e = 0,$$  \hspace{1cm} (50)

where

$$F_g = -\frac{1}{2} (\rho + p) \nu' = -\frac{rbB}{8\pi} \left[ \frac{2[A(a - b)\sqrt{1 + br^2} + Ba(1 + br^2)]}{(1 + ar^2)^2(1 + br^2)[A + B\sqrt{1 + br^2}]^2} \right],$$  \hspace{1cm} (51)

$$F_h = -\frac{dp}{dr} = -\frac{2r}{8\pi} \left[ \frac{p1 + p2 + p3}{4(1 + ar^2)^3\sqrt{1 + br^2}[A + B\sqrt{1 + br^2}]^2} \right],$$  \hspace{1cm} (52)

$$F_e = \frac{\sigma q}{r^2} e^{\lambda/2} = \frac{ar}{4\pi} \left[ \frac{F_{e1} + F_{e2} + F_{e3}}{4(1 + ar^2)^3\sqrt{1 + br^2}[A + B\sqrt{1 + br^2}]^2} \right],$$  \hspace{1cm} (53)

$$p1 = 4Ab^2 B - ab[6A^2\sqrt{1 + br^2} + 16B^2(1 + br^2)^{3/2} + AB(22 + 15br^2)],$$

$$p2 = 2a^3r^2[A^2\sqrt{1 + br^2} + B^2(1 + br^2)^{3/2} + 2AB(1 + br^2)],$$

$$p3 = a^2 [-2A^2(-3+br^2)\sqrt{1 + br^2} + AB(12 + 2br^2 - 7b^2r^4) - 2B^2(-3 + br^2 + 4b^2r^4)\sqrt{1 + br^2}],$$

$$F_{e1} = -b[6A^2\sqrt{1 + br^2} + 12B^2(1 + br^2)^{3/2} + AB(18 + 19br^2)],$$

$$F_{e2} = 2a^2r^2[A^2\sqrt{1 + br^2} + B^2(1 + br^2)^{3/2} + 2AB(1 + br^2)],$$

$$F_{e3} = a[-2A^2(-3+br^2)\sqrt{1 + br^2} + AB(12 + 6br^2 - 7b^2r^4) + 2B^2(3 + br^2 - 2b^2r^4)\sqrt{1 + br^2}].$$
We have shown the plots for TOV equation in Fig. 5 for different compact strange stars. From the figures it is observed that the system is in static equilibrium under four different forces, e.g. gravitational, hydrostatic, electric and anisotropic to attain overall equilibrium.

Fig. 5. The behavior of forces for the compact stars (i) Her X – 1 (Top Left), (ii) RX J 1856 – 37 (Top Middle) and (iii) 4U 1820 – 30 (Top Right) (iv) SAX J1808.4 – 3658(SS1) (Bottom Left), (v) SAX J1808.4 – 3658(SS2) (Bottom Middle) and (vi) PSR 1937 + 21 (Bottom Right) with respect to radial coordinate $r/R$.

6.4.2 Electric charge contain

In the present work the expression for electric charge can be given by Eq. (34) and following the work of Maurya et al. [4] we can figure out that the charge on the boundary is $1.15295 \times 10^{20}$ C and at the center it is zero. The charge profile has been shown in the Fig. 6 for different compact stars which starts from a minimum value at the centre and acquires the maximum value at the boundary. This feature is also evident from the Table 1 and compatible with the result of Ray et al. [37] where they studied the effect of electric charge in compact stars and found the upper bound as $\sim 10^{20}$ Coulomb.

Table 1
The profile of electric charge for different compact stars

| $r/a$ | Her. X-1 | RXJ 1856-37 | SAX-2 | SAX-1 | 4U 1820-30 | PSR 1937+21 |
|-------|-----------|-------------|-------|-------|------------|-------------|
| 0.0   | 0         | 0           | 0     | 0     | 0          | 0           |
| 1.0   | 0.9686    | 0.8706      | 1.4664| 1.4056| 2.3974     | 1.8996      |
Fig. 6. Behavior of electric charge $q$ with respect to radial coordinate $r/R$

6.4.3 Effective mass-radius relation

Buchdahl [38] has proposed an absolute constraint on the maximally allowable mass-to-radius ratio ($M/R$) for static spherically symmetric isotropic fluid spheres which amounts $2M/R \leq 8/9$. On the other hand, Böhm and Harko [39] proved that for a compact charged fluid sphere there is a lower bound for the mass-radius ratio

$$\frac{3Q^2}{2R^2} \left(1 + \frac{Q^2}{18R^2}\right) \leq \frac{2M}{R},$$

for the constraint $Q < M$.

This upper bound of the mass for charged fluid sphere was generalized by Andreasson [40] who proved that

$$\sqrt{M} \leq \frac{\sqrt{R}}{3} + \sqrt{\frac{R}{9} + \frac{Q^2}{3R}}.$$ (55)

In the present model, we find the effective gravitational mass as

$$M_{\text{eff}} = 4\pi \int_0^R \left(\rho + \frac{E^2}{8\pi}\right) r^2 dr = \frac{1}{2} R(1 - e^{-\lambda(R)}) = \frac{1}{2} R \left\lbrack \frac{(a - b)R^2}{(1 + aR^2)} \right\rbrack.$$ (56)

In terms of the compactness factor $u = M_{\text{eff}}/R$ we now define the surface red-shift $Z_s$ as

$$Z_s = (1 - 2u)^{-\frac{1}{2}} - 1 = e^{\frac{1}{2}\lambda(R)} - 1 = \sqrt{\frac{(1 + aR^2)}{(1 + bR^2)}} - 1.$$ (57)
We have demonstrated the behavior of surface redshift $Z_s$ with respect to radial coordinate $r/R$ in Fig. 7 which shows the desirable features [8].

![Graph showing the behavior of surface redshift $Z_s$ with respect to radial coordinate $r/R$.]

Fig. 7. The behavior of surface redshift $Z_s$ with respect to radial coordinate $r/R$

7 Some special features of the models

7.1 Stellar structure

In the previous Sec. 6, we have discussed several properties of our solutions in terms of various physical parameters based on some of the compact stars. In this Sub-section we provide two Tables 2 and 3 where we have figured out some other physical parameters as well as some constants of our models. Actually, the values of Table 2 have been used in Table 3 to find out the energy densities and pressure for different strange star candidates. It is worthwhile to mention that the densities $\sim 10^{15} \text{gm/cm}^{-3}$ and pressure $\sim 10^{35} \text{dyne/cm}^{-2}$ are in very good agreement with the observational data of the compact stars, specially Her X 1 [41,42].

7.2 Electronic structure

Let us now come down from macro-scale of the stellar structure to the micro-scale of the structure of the electron. Here we have performed a comparative study of the values of the physical parameters of electron between the data of the present work and that from the work of Gautreau [2] as shown in Table 4. Here also one can observe that the data of both works are exactly correspond to each other at least as far as order of magnitude are concerned. In is to note
Table 2
Values of the model parameters for different strange stars

| Strange star candidates | $M$ ($M_{\odot}$) | $R$ (Km) | $M/R$ | $a$ | $b$ | $A$ | $K$ |
|-------------------------|-------------------|----------|-------|-----|-----|-----|-----|
| Her X − 1              | 0.9800            | 6.70     | 0.2160| -0.00600 | 1.4566 | 485.1733 |
| RX J 1856 − 37         | 0.9000            | 6.00     | 0.2220| -0.00800 | 1.4137 | 377.0041 |
| SAX J1808.4 − 3658(SS1)| 1.4351            | 7.07     | 0.2990| -0.00800 | 1.2506 | 414.5739 |
| SAX J1808.4 − 3658(SS2)| 1.3235            | 6.35     | 0.3071| -0.00811 | 1.4506 | 351.2430 |
| PSR 1937 + 21          | 2.0830            | 11.40    | 0.2692| -0.00295 | 1.2546 | 1138.6000 |
| 4U 1820 − 30           | 2.2457            | 9.95     | 0.3325| -0.00400 | 1.2946 | 797.8075 |

Table 3
Energy densities and pressure for different strange star candidates

| Strange star candidates | Central energy density ($gm/cm^3$) | Surface energy density ($gm/cm^3$) | Central pressure ($dyne/cm^2$) |
|-------------------------|------------------------------------|-----------------------------------|-------------------------------|
| Her X − 1               | $1.8285 \times 10^{15}$            | $1.2590 \times 10^{15}$           | $1.7018 \times 10^{35}$       |
| RXJ 1856 − 37           | $2.3360 \times 10^{15}$            | $1.6223 \times 10^{15}$           | $2.3810 \times 10^{35}$       |
| SAX J1808.4 − 3658(SS1) | $2.4471 \times 10^{15}$            | $1.4404 \times 10^{15}$           | $4.5051 \times 10^{35}$       |
| SAX J1808.4 − 3658(SS2) | $3.4330 \times 10^{15}$            | $1.6506 \times 10^{15}$           | $5.0610 \times 10^{35}$       |
| PSR 1937 + 21           | $1.5176 \times 10^{15}$            | $7.2620 \times 10^{14}$           | $3.0741 \times 10^{35}$       |
| 4U 1820 − 30            | $7.9745 \times 10^{15}$            | $5.3441 \times 10^{14}$           | $1.2871 \times 10^{35}$       |

that the density of electron from our model turns out to be $8.541 \times 10^{10} gm/cm^3$
which seems to closer to the actual value of the density of electron.

Table 4
A comparative study of the values of the physical parameters of electron

| Physical parameter | In the present paper | Data from Gautreau [2] |
|--------------------|----------------------|------------------------|
| Mass               | $6.6772 \times 10^{-56}$ cm | $6.67 \times 10^{-56}$ cm |
| Radius             | $2.82 \times 10^{-13}$ cm | $2.82 \times 10^{-13}$ cm |
| Charge             | $1.95 \times 10^{-34}$ cm | $1.38 \times 10^{-34}$ cm |

For specific numerical values of the constant $\kappa = 8\pi G/c^4$ and other physical parameters we have used the data $G = 6.67 \times 10^{-8} cm^3/gs^{-2}$ and $c = 2.997 \times 10^{10} cm/s$ in the calculations of Tables 3 and 4.
8 Conclusion

Our sole aim in the present letter was to investigate nature of class 1 metric. For this purpose we have considered matter-energy distribution under the framework of Einstein-Maxwell spacetime. At first we developed an algorithm which has a general nature and thus can be reduced to three special cases, viz. (i) charge analogue of the Kohler-Chao [29] solution, (ii) charge analogue of the Schwartzschild [28] solution (i.e. the Reissner-Nordstro¨ om solution), and (iii) the Lorentz [9] solution of electromagnetic mass model.

By considering the third case of the Lorentz solution of electromagnetic mass model we have studied its properties through the following two basic physical testing, such as (i) regularity and reality conditions, and (ii) causality and well behaved conditions. Moreover, some other essential testing also have been performed, viz. (i) energy conditions, and (ii) stability conditions. In the case of energy conditions we have seen that the isotropic charged fluid sphere composed of matter satisfy the (i) null energy condition \( (\rho + \frac{E^2}{4\pi}) \geq 0 \), (ii) weak energy condition \( (\rho - p + \frac{E^2}{4\pi}) \geq 0 \), and (iii) strong energy condition \( (\rho - 3p + \frac{E^2}{4\pi}) \geq 0 \) (Fig. 4). On the other hand, in connection to stability conditions we critically have discussed the Tolman-Oppenheimer-Volkoff equation, electric charge contain, effective mass-radius relation of the charged spherical distribution. Here also we find that the results are in favour of the physical requirements (Figs. 5 - 8).

As some special features of the models we have presented here two-level applications in the following fields: (i) stellar structure, and (ii) electronic structure. The behavior of the compact stars (i) Her X – 1, (ii) RX J 1856 – 37, (iii) 4U 1820 – 30, (iv) SAX J1808.4 – 3658(SS1), (v) SAX J1808.4 – 3658(SS2), and (vi) PSR 1937 + 21 have been demonstrated through two Tables 2 and 3 which are quite satisfactory. Another application of the models have been done in the case of the electron. This is shown in the Table 4 where one can notice that the model data resembles with the observational data of the electron.

Acknowledgments

SKM acknowledges support from the authority of University of Nizwa, Nizwa, Sultanate of Oman. Also SR is thankful to the authority of Inter-University Center for Astronomy and Astrophysics, Pune, India for providing Associate-ship under which a part of this work was carried out.
References

[1] R.N. Tiwari, J.R. Rao, R.R. Kanakamedala, Phys. Rev. D 30 (1984) 489.

[2] R. Gautreau, Phys. Rev. D 31 (1985) 1860.

[3] Ø. Grøn, Phys. Rev. D 31 (1985) 2129.

[4] S.K. Maurya, Y.K. Gupta, S. Ray, S. Ray Chowdhury, arXiv:1506.02498 (2015).

[5] O'Connor, J.J., E.F. Robertson, E.F., General relativity, Mathematical Physics index, School of Mathematics and Statistics, University of St. Andrews, Scotland (1996).

[6] Wheeler, J.A., A Journey Into Gravity and Spacetime, Scientific American Library, San Francisco: W. H. Freeman (1990).

[7] Ray, S., Apeiron 14 (2007) 153.

[8] S.K. Maurya, Y.K. Gupta, S. Ray, arXiv:1502.01915 [gr-qc] (2015).

[9] H.A. Lorentz Proc. Acad. Sci., Amsterdam 6 (1904) (Reprinted in The Principle of Relativity, Dover, INC., p24, 1952)

[10] J.A. Wheeler, Geometrodynamics (Academic, New York, p. 25, 1962).

[11] R.P. Feynman, R.R. Leighton, M. Sands, The Feynman Lectures on Physics (Addison-Wesley, Palo Alto, Vol.II, Chap. 28, 1964).

[12] F. Wilczek, Phys. Today 52 (1999) 11.

[13] F.I. Cooperstock, V. de la Cruz, Gen. Relat. Grav. 9 (1978) 835.

[14] R.N. Tiwari, J.R. Rao, R.R. Kanakamedala, Phys. Rev. D 34 (1986) 1205.

[15] J. Ponce de Leon, J. Math. Phys. 28 (1987) 410.

[16] R.N. Tiwari, J.R. Rao, S. Ray, Astrophys. Space Sci. 178 (1991) 119.

[17] R.N. Tiwari, S. Ray, Astrophys. Space Sci. 180 (1991) 143.

[18] R.N. Tiwari, S. Ray, Astrophys. Space Sci. 182 (1991) 105.

[19] R.N. Tiwari, S. Ray, Gen. Relativ. Gravit. 29 (1997) 683.

[20] S. Ray, B. Das, Astrophys. Space Sci. 282 (2002) 635.

[21] S. Ray, B. Das, Mon. Not. R. Astron. Soc. 349 (2004) 1331.

[22] S. Ray, Int. J. Mod. Phys. D 15 (2006) 917.

[23] S. Ray, B. Das, F. Rahaman, S. Ray, Int. J. Mod. Phys. D 16 (2007) 1745.

[24] C W Davies Phys. Rev. D 30 (1984) 737.

[25] J.J. Blome, W. Priester, Naturwissenschaften 71 (1984) 528.
[26] C. Hogan, Nature 310 (1984) 365.
[27] N. Kaiser, A. Stebbins, Nature 310 (1984) 391.
[28] K. Schwarzschild, Phys.-Math. Klasse, 189 (1916).
[29] M. Kohler, K.L. Chao, Z. Naturforsch. Ser. A 20 (1965) 1537.
[30] K.R. Karmarkar, Proc. Ind. Acad. Sci. A 27 (1948) 56.
[31] S.N. Pandey, S.P. Sharma, Gene. Relativ. Gravit. 14 (1982) 113.
[32] R.C. Tolman, Phys. Rev. 55 (1939) 364.
[33] J.R. Oppenheimer, G.M. Volkoff, Phys. Rev. 55 (1939) 374.
[34] P.S. Florides, J. Phys. A: Math. Gen. 16 (1983) 1419.
[35] C.W. Misner, D.H. Sharp, Phys. Rev. B 136 (1964) 571.
[36] V. Canuto, In: Solvay Conf. on Astrophysics and Gravitation, Brussels (1973)
[37] S. Ray, A.L. Espindola, M. Malheiro, J.P.S. Lemos, V.T. Zanchin, Phys. Rev.
  D 68 (2003) 084004.
[38] H.A. Buchdahl, Phys. Rev. 116 (1959) 1027.
[39] C.G. Böhm and T. Harko, Gen. Relativ. Gravit. 39, (2007) 757.
[40] H. Andréasson, Commun. Math. Phys. 288 (2009) 715.
[41] R. Ruderman, Rev. Astron. Astrophys. 10 (1972) 427.
[42] S.K. Maurya, Y.K. Gupta, S. Ray, B. Dayanandan, Eur. Phys. J. C 75 (2015)
  225.