Reliability of one Strength-four Stresses for Lomax Distribution

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Abstract. In this paper, we find the reliability $R$ of a component when it is exposed to four independent stresses and it having one strength for Lomax distribution. the reliability $R$ was estimated by using four different estimations (MLE, RgE, LSE, and WLS) methods. A comparison was made between the results of estimating the reliability function by MSE and MAPE criteria, that will get from a Monte Carlo simulation study. We found that the performance of ML is the best to.

1. Introduction
In the reliability literature, stress-strength is an important model. In the statistical approach to the stress-strength model, considerations are based on the supposition that component strengths are independently and identically distributed (iid) and are subjected to common stress. Here the system reliability, when $X$ and $Y$ are independent and identical. Let $X$ is a strength random variable subjected to a common stress $Y$ then the reliability of the system contained one component is: $R = P(X > Y)$ [1, 2]. The reliability of a component (or a system) can be represented in various forms depending on the structure of the system, where the component (or the system) would fail if the stress exceeds its strength [3, 4]. The Lomax (1954), or Pareto II, distribution has been quite widely applied in a variety of contexts [5]. The Lomax distribution has been used for reliability modeling and life testing and applied to income and wealth distribution data. It has also found application in the biological sciences. Some authors have suggested the use of this distribution as an alternative to the exponential distribution when the data are heavy-tailed [6].

The present study considering stress – strength Lomax reliability of model for one component having one strength it is exposed to four independent stresses when the stress and strength are followed Lomax two-parameter random variables. Estimation of the parameters and the reliability model is made by MLE, RgE, LSE, and WLSE estimation methods. Finally, the Monte Carlo simulation study for comparing between them by MSE and MAPE are explained.

2. The Mathematical Formula of one Strength-Four Stress Component Reliability
The reliability of the stress-strength model defines the life of a component that has strength $X$ and exposed to stress $Y$. So that if stress overtake the strength $(Y > X)$ the component fails [7]:

$R = P(Y < X) = \int_0^\infty f(x)F_Y(x)dx$
In this article, the stress-strength reliability of one component subjected to four stress is examined. Let the strength r.v. of the component represented by X as a Lomax r.v. \((\alpha, \mu)\), and the component subjected stress r.v. are represented by \(Y_i\); \(i = 1, 2, 3, 4\) following Lomax distribution \((\alpha_i, \mu)\); \(i=1,2,3,4\) with unknown shape parameters \(\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4\) and known common scale parameter \(\mu\). The pdf and CDF of the r.v. for \(X \sim L(\alpha, \mu)\) are given as:

\[
f(x) = \frac{\alpha}{\mu} \left(1 + \frac{x}{\mu}\right)^{-(\alpha+1)} \quad x \geq 0; \alpha, \mu > 0 \quad \tag{1}
\]

\[
F(x) = 1 - \left(1 + \frac{x}{\mu}\right)^{-\alpha} \quad y \geq 0; \alpha, \mu > 0 \quad \tag{2}
\]

Also the CDF of \(Y_i \sim L(\alpha_i, \mu)\):

\[
F_1(y_1) = 1 - \left(1 + \frac{y_1}{\mu}\right)^{-\alpha_1} \quad y_1 \geq 0, \alpha_1, \mu > 0 \quad \tag{3}
\]

\[
F_2(y_2) = 1 - \left(1 + \frac{y_2}{\mu}\right)^{-\alpha_2} \quad y_2 \geq 0, \alpha_2, \mu > 0 \quad \tag{4}
\]

\[
F_3(y_3) = 1 - \left(1 + \frac{y_3}{\mu}\right)^{-\alpha_3} \quad y_3 \geq 0, \alpha_3, \mu > 0 \quad \tag{5}
\]

\[
F_4(y_4) = 1 - \left(1 + \frac{y_4}{\mu}\right)^{-\alpha_4} \quad y_4 \geq 0, \alpha_4, \mu > 0 \quad \tag{6}
\]

Now, we can obtain to find the reliability model as follows:

\[
R = P[\text{Max}(Y_1, Y_2, Y_3, Y_4) < X]
= \int_0^\infty \int_0^{y_1} \int_0^{y_2} \int_0^{y_3} \int_0^{y_4} f(y_1, y_2, y_3, y_4, x) dy_4 dy_3 dy_2 dy_1 dx \quad \tag{7}
\]

since r.v are independent and identical distributed; then:

\[
R = \int_0^\infty \int_0^{y_2} \int_0^{y_3} \int_0^{y_4} f(y_1) f(y_2) f(y_3) f(y_4) f(x) dy_4 dy_3 dy_2 dy_1 dx
= \int_{x=0}^\infty F_{Y_1}(x) F_{Y_2}(x) F_{Y_3}(x) F_{Y_4}(x) f(x) dx \quad \tag{8}
\]

The reliability model of Lomax distribution will get by substitution equations (1),(2), (3),(4),(5) and (6) in equation (8), as:

\[
R = \int_{x=0}^\infty \left[1 - \left(1 + \frac{x}{\mu}\right)^{-\alpha_1}\right] \left[1 - \left(1 + \frac{x}{\mu}\right)^{-\alpha_2}\right] \left[1 - \left(1 + \frac{x}{\mu}\right)^{-\alpha_3}\right] \left[1 - \left(1 + \frac{x}{\mu}\right)^{-\alpha_4}\right]
\cdot \frac{\alpha}{\mu} \left(1 + \frac{x}{\mu}\right)^{-(\alpha+1)} dx
\]

\[
R = \int_0^\infty \left[1 - \left(1 + \frac{x}{\mu}\right)^{-\alpha_1}\right] - \left(1 + \frac{x}{\mu}\right)^{-\alpha_2} - \left(1 + \frac{x}{\mu}\right)^{-\alpha_3} - \left(1 + \frac{x}{\mu}\right)^{-\alpha_4} + \left(1 + \frac{x}{\mu}\right)^{-(\alpha_1+\alpha_2)}
\]

\[
+ \left(1 + \frac{x}{\mu}\right)^{-(\alpha_1+\alpha_2)} + \left(1 + \frac{x}{\mu}\right)^{-(\alpha_1+\alpha_3)} + \left(1 + \frac{x}{\mu}\right)^{-(\alpha_1+\alpha_4)} + \left(1 + \frac{x}{\mu}\right)^{-(\alpha_2+\alpha_3)}
\]

\[
+ \left(1 + \frac{x}{\mu}\right)^{-(\alpha_2+\alpha_4)} + \left(1 + \frac{x}{\mu}\right)^{-(\alpha_2+\alpha_3)} - \left(1 + \frac{x}{\mu}\right)^{-(\alpha_1+\alpha_2+\alpha_3)} - \left(1 + \frac{x}{\mu}\right)^{-(\alpha_1+\alpha_2+\alpha_4)}
\]

\[
- \left(1 + \frac{x}{\mu}\right)^{-(\alpha_1+\alpha_2+\alpha_3+\alpha_4)} + \left(1 + \frac{x}{\mu}\right)^{-(\alpha_1+\alpha_2+\alpha_3+\alpha_4)}\right]\frac{\alpha}{\mu} \left(1 + \frac{x}{\mu}\right)^{-(\alpha+1)} dx
\]

\[
R = \int_0^\infty \frac{\alpha}{\mu} \left(1 + \frac{x}{\mu}\right)^{-(\alpha+1)} dx - \int_0^\infty \frac{\alpha}{\mu} \left(1 + \frac{x}{\mu}\right)^{-(\alpha+\alpha_1+1)} dx
- \int_0^\infty \frac{\alpha}{\mu} \left(1 + \frac{x}{\mu}\right)^{-(\alpha+\alpha_2+1)} dx
\]
The reliability of a component has strength exposed to four stresses of Lomax distribution can be expressed as:
\[
R = 1 - \left[ \frac{\alpha}{(\alpha + \alpha_1)} \right] - \left[ \frac{\alpha}{(\alpha + \alpha_2)} \right] - \left[ \frac{\alpha}{(\alpha + \alpha_3)} \right] - \left[ \frac{\alpha}{(\alpha + \alpha_4)} \right] + \left[ \frac{\alpha}{(\alpha + \alpha_1 + \alpha_2)} \right] + \left[ \frac{\alpha}{(\alpha + \alpha_1 + \alpha_3)} \right] + \left[ \frac{\alpha}{(\alpha + \alpha_1 + \alpha_4)} \right] + \left[ \frac{\alpha}{(\alpha + \alpha_2 + \alpha_3)} \right] + \left[ \frac{\alpha}{(\alpha + \alpha_2 + \alpha_4)} \right] + \left[ \frac{\alpha}{(\alpha + \alpha_3 + \alpha_4)} \right] + \left[ \frac{\alpha}{(\alpha + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)} \right] \ldots \space(9)
\]

3. Maximum likelihood Estimator (MLE)
The maximum likelihood estimation method can use to estimate the parameter of Lomax distribution [8]. Let \( x_1, x_2, \ldots, x_n \) be random samples from LD \((\alpha, \mu)\) of \( n \) sample size then the Likelihood function will be:
\[
L(x, \alpha, \mu) = \frac{n}{\mu^n} \prod_{i=1}^{n} \left( 1 + \frac{x_i}{\mu} \right)^{-(\alpha + 1)} \ldots \space(10)
\]

By taking the Log-likelihood function and derivative this function with respect to \( \alpha \), then we get the ML estimator for \( \alpha \):
\[
\hat{\alpha}_{ML} = \frac{n}{\sum_{i=1}^{n} \ln \left( \frac{x_i}{\mu} + 1 \right)} \ldots \space(11)
\]

In the same way; let \( y_{11}, y_{12}, \ldots, y_{1n}; y_{21}, y_{22}, \ldots, y_{2n}; y_{31}, y_{32}, \ldots, y_{3n} \) and \( y_{41}, y_{42}, \ldots, y_{4n} \) be the random samples from LD of \( n_1, n_2, n_3 \) and \( n_4 \) sample sizes. Then the ML estimators of \( \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha_4 \) are given as:
\[
\hat{\alpha}_{1ML} = \frac{n_1}{\sum_{i=1}^{n_1} \ln \left( \frac{y_{1i}}{\mu} + 1 \right)} \ldots \space(12)
\]
\[
\hat{\alpha}_{2ML} = \frac{n_2}{\sum_{i=1}^{n_2} \ln \left( \frac{y_{2i}}{\mu} + 1 \right)} \ldots \space(13)
\]
\[
\hat{\alpha}_{3ML} = \frac{n_3}{\sum_{i=1}^{n_3} \ln \left( \frac{y_{3i}}{\mu} + 1 \right)} \ldots \space(14)
\]
\[
\hat{\alpha}_{4ML} = \frac{n_4}{\sum_{i=1}^{n_4} \ln \left( \frac{y_{4i}}{\mu} + 1 \right)} \ldots \space(15)
\]

The ML estimator of \( R \) is given by substitute (11), (12), (13), (14), and (15) in equation (9) we get:
\[
\hat{R}_{ML} = 1 - \left[ \frac{\hat{\alpha}_{1ML}}{(\hat{\alpha}_{ML} + \hat{\alpha}_{1ML})} \right] - \left[ \frac{\hat{\alpha}_{2ML}}{(\hat{\alpha}_{ML} + \hat{\alpha}_{2ML})} \right] - \left[ \frac{\hat{\alpha}_{3ML}}{(\hat{\alpha}_{ML} + \hat{\alpha}_{3ML})} \right] - \left[ \frac{\hat{\alpha}_{4ML}}{(\hat{\alpha}_{ML} + \hat{\alpha}_{4ML})} \right] + \left[ \frac{\hat{\alpha}_{ML}}{\hat{\alpha}_{ML} + \hat{\alpha}_{1ML} + \hat{\alpha}_{2ML}} \right]
\]
\[ x_i = a + b u_i + e_i \]  

By taking the Log to equation (2):

\[ \ln \left( 1 - F(x_i) \right) = -\alpha \ln \left( 1 + \frac{x_i}{\mu} \right) \]

Use plotting position \( (P_i) \) instead of \( F(x_i) \) where \( P_i = \frac{i}{n+1} \), then:

\[ \ln(1 - P_i) = -\alpha \ln \left( 1 + \frac{x_i}{\mu} \right) \]

Substitute an equation (18) into the equation (17):

\[ z_i = \ln(1 - P_i), \quad a = 0, \quad b = \alpha, \quad u_i = -\ln \left( 1 + \frac{x_i}{\mu} \right) \]

Then calculate the estimator \( \hat{\beta} \):

\[ \hat{\beta} = \frac{n \sum_{i=1}^{n} x_i u_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} u_i}{n \sum_{i=1}^{n} (u_i)^2 - [\sum_{i=1}^{n} u_i]^2} \]

Now, since \( b = \alpha \), the estimator \( \hat{\alpha}_Rg \) is:

\[ \hat{\alpha}_{Rg} = \frac{-n \sum_{i=1}^{n} \ln(1-P_i) \ln(1+\frac{x_i}{\mu}) + \sum_{i=1}^{n} \ln(1-P_i) \sum_{i=1}^{n} \ln(1+\frac{x_i}{\mu})}{n \sum_{i=1}^{n} [\ln(1+\frac{x_i}{\mu})]^2 - [\sum_{i=1}^{n} \ln(1+\frac{x_i}{\mu})]^2} \]

In the same steps above, we will estimate \( \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha_4 \) are:

\[ \hat{\alpha}_{Rg1} = \frac{-n \sum_{i=1}^{n} x_{ij1} \ln(1-P_{ij1}) \ln \left( 1 + \frac{y_{ij1}}{\mu} \right) + \sum_{i=1}^{n} x_{ij1} \ln(1-P_{ij1}) \sum_{i=1}^{n} x_{ij1} \ln \left( 1 + \frac{y_{ij1}}{\mu} \right)}{n \sum_{i=1}^{n} [\ln(1+\frac{y_{ij1}}{\mu})]^2 - [\sum_{i=1}^{n} \ln(1+\frac{y_{ij1}}{\mu})]^2} \]

\[ \hat{\alpha}_{Rg2} = \frac{-n \sum_{i=1}^{n} x_{ij2} \ln(1-P_{ij2}) \ln \left( 1 + \frac{y_{ij2}}{\mu} \right) + \sum_{i=1}^{n} x_{ij2} \ln(1-P_{ij2}) \sum_{i=1}^{n} x_{ij2} \ln \left( 1 + \frac{y_{ij2}}{\mu} \right)}{n \sum_{i=1}^{n} [\ln(1+\frac{y_{ij2}}{\mu})]^2 - [\sum_{i=1}^{n} \ln(1+\frac{y_{ij2}}{\mu})]^2} \]

\[ \hat{\alpha}_{Rg3} = \frac{-n \sum_{i=1}^{n} x_{ij3} \ln(1-P_{ij3}) \ln \left( 1 + \frac{y_{ij3}}{\mu} \right) + \sum_{i=1}^{n} x_{ij3} \ln(1-P_{ij3}) \sum_{i=1}^{n} x_{ij3} \ln \left( 1 + \frac{y_{ij3}}{\mu} \right)}{n \sum_{i=1}^{n} [\ln(1+\frac{y_{ij3}}{\mu})]^2 - [\sum_{i=1}^{n} \ln(1+\frac{y_{ij3}}{\mu})]^2} \]

\[ \hat{\alpha}_{Rg4} = \frac{-n \sum_{i=1}^{n} x_{ij4} \ln(1-P_{ij4}) \ln \left( 1 + \frac{y_{ij4}}{\mu} \right) + \sum_{i=1}^{n} x_{ij4} \ln(1-P_{ij4}) \sum_{i=1}^{n} x_{ij4} \ln \left( 1 + \frac{y_{ij4}}{\mu} \right)}{n \sum_{i=1}^{n} [\ln(1+\frac{y_{ij4}}{\mu})]^2 - [\sum_{i=1}^{n} \ln(1+\frac{y_{ij4}}{\mu})]^2} \]

The Rg estimator of \( R \) is given by substitute (21), (22), (23), (24), and (25) in equation (9) we get:
5. Least Square Estimator (LSE):

This estimation method is very popular for model fitting, especially in linear and non-linear regression [10]. Suppose that the random sample \( x_1, x_2, \ldots, x_n \) of size \( n \) from LD. The LS estimator can be obtained by minimizing:

\[
\sum_{i=1}^{n} [\ln(1 - P_i)]^{-1} - \alpha \ln \left( 1 + \frac{x_i(0)}{\mu} \right)^2.
\]

By driving with respect to \( \alpha \) we get LS estimator of \( \alpha \) is

\[
\hat{\alpha}_{LS} = \frac{\sum_{i=1}^{n} [\ln(1 - P_i)]^{-1} \ln \left( 1 + \frac{x_i(0)}{\mu} \right)}{\sum_{i=1}^{n} \left[ \ln \left( 1 + \frac{x_i(0)}{\mu} \right) \right]^2} \quad \ldots (27)
\]

Also the LS estimators of \( \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha_4 \) are:

\[
\hat{\alpha}_{1LS} = \frac{\sum_{i=1}^{n} [\ln(1 - P_i)]^{-1} \ln \left( 1 + \frac{x_i(0)}{\mu} \right)}{\sum_{i=1}^{n} \left[ \ln \left( 1 + \frac{x_i(0)}{\mu} \right) \right]^2} \quad \ldots (28)
\]

\[
\hat{\alpha}_{2LS} = \frac{\sum_{j=1}^{n} [\ln(1 - P_j)]^{-1} \ln \left( 1 + \frac{x_j(0)}{\mu} \right)}{\sum_{j=1}^{n} \left[ \ln \left( 1 + \frac{x_j(0)}{\mu} \right) \right]^2} \quad \ldots (29)
\]

\[
\hat{\alpha}_{3LS} = \frac{\sum_{j=1}^{n} [\ln(1 - P_j)]^{-1} \ln \left( 1 + \frac{x_i(0)}{\mu} \right)}{\sum_{j=1}^{n} \left[ \ln \left( 1 + \frac{x_i(0)}{\mu} \right) \right]^2} \quad \ldots (30)
\]

\[
\hat{\alpha}_{4LS} = \frac{\sum_{j=1}^{n} [\ln(1 - P_j)]^{-1} \ln \left( 1 + \frac{x_j(0)}{\mu} \right)}{\sum_{j=1}^{n} \left[ \ln \left( 1 + \frac{x_j(0)}{\mu} \right) \right]^2} \quad \ldots (31)
\]

The LS estimator of \( R \) is given by substitute (27), (28), (29), (30), and (31) in equation (9) we get:

\[
\hat{R}_{LS} = 1 - \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS}} \right] - \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS} + \hat{\alpha}_{2LS}} \right] - \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS} + \hat{\alpha}_{3LS}} \right] - \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS} + \hat{\alpha}_{4LS}} \right] + \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS} + \hat{\alpha}_{2LS} + \hat{\alpha}_{3LS} + \hat{\alpha}_{4LS}} \right]
\]

\[
+ \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS} + \hat{\alpha}_{2LS} + \hat{\alpha}_{3LS}} \right] + \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS} + \hat{\alpha}_{2LS} + \hat{\alpha}_{4LS}} \right] + \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS} + \hat{\alpha}_{2LS} + \hat{\alpha}_{3LS} + \hat{\alpha}_{4LS}} \right] + \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS} + \hat{\alpha}_{2LS} + \hat{\alpha}_{3LS}} \right] + \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS} + \hat{\alpha}_{2LS}} \right] + \left[ \frac{\hat{\alpha}_{LS}}{\hat{\alpha}_{1LS}} \right] \quad \ldots (32)
\]
6. Weighted Least Square Estimator (WLSE):

It can be obtained by the same sample assumptions of LS estimation using the weight \( w_i \), which given as:

\[
w_i = \frac{1}{\text{var}[F(x_{(i)})]} = \frac{(n+1)^2(n+2)}{i(n-i+1)}, i = 1,2,\ldots, n
\]

The minimizing with respect to the unknown parameter \( \alpha \):

\[
\sum_{i=0}^{n} w_i \left[ \ln(1 - P_i)^{-1} - \alpha \ln \left( 1 + \frac{x_{(i)}}{\mu} \right) \right] = 0
\]

... (33)

The WLS estimator for \( \alpha \), is finally given as:

\[
\hat{\alpha}_{WLS} = \frac{\sum_{i=1}^{n} w_i \ln(1-P_i)^{-1} \ln \left( 1 + \frac{x_{(i)}}{\mu} \right)}{\sum_{i=1}^{n} \left( \ln \left( 1 + \frac{x_{(i)}}{\mu} \right)^2 \right)}
\]

... (34)

The WLS estimators of \( \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha_4 \) are:

\[
\hat{\alpha}_{1WLS} = \frac{\sum_{j=1}^{n} w_{j1} \ln(1-P_{j1})^{-1} \ln \left( 1 + \frac{y_{(j1)}}{\mu} \right)}{\sum_{j=1}^{n} w_{j1} \left( \ln \left( 1 + \frac{y_{(j1)}}{\mu} \right)^2 \right)}
\]

... (35)

\[
\hat{\alpha}_{2WLS} = \frac{\sum_{j=2}^{n} w_{j2} \ln(1-P_{j2})^{-1} \ln \left( 1 + \frac{y_{(j2)}}{\mu} \right)}{\sum_{j=2}^{n} w_{j2} \left( \ln \left( 1 + \frac{y_{(j2)}}{\mu} \right)^2 \right)}
\]

... (36)

\[
\hat{\alpha}_{3WLS} = \frac{\sum_{j=3}^{n} w_{j3} \ln(1-P_{j3})^{-1} \ln \left( 1 + \frac{y_{(j3)}}{\mu} \right)}{\sum_{j=3}^{n} w_{j3} \left( \ln \left( 1 + \frac{y_{(j3)}}{\mu} \right)^2 \right)}
\]

... (37)

\[
\hat{\alpha}_{4WLS} = \frac{\sum_{j=4}^{n} w_{j4} \ln(1-P_{j4})^{-1} \ln \left( 1 + \frac{y_{(j4)}}{\mu} \right)}{\sum_{j=4}^{n} w_{j4} \left( \ln \left( 1 + \frac{y_{(j4)}}{\mu} \right)^2 \right)}
\]

... (38)

The WLS estimator of R is given by substitute (34), (35), (36), (37), and (38) in equation (9) we get:

\[
\hat{R}_{WLS} = 1 - \frac{\hat{\alpha}_{WLS}}{(\hat{\alpha}_{WLS} + \hat{\alpha}_{WLS} + \hat{\alpha}_{WLS})} - \frac{\hat{\alpha}_{WLS}}{(\hat{\alpha}_{WLS} + \hat{\alpha}_{WLS} + \hat{\alpha}_{WLS})} - \frac{\hat{\alpha}_{WLS}}{(\hat{\alpha}_{WLS} + \hat{\alpha}_{WLS} + \hat{\alpha}_{WLS})} + \frac{\hat{\alpha}_{WLS}}{(\hat{\alpha}_{WLS} + \hat{\alpha}_{WLS} + \hat{\alpha}_{WLS})} + \frac{\hat{\alpha}_{WLS}}{(\hat{\alpha}_{WLS} + \hat{\alpha}_{WLS} + \hat{\alpha}_{WLS})} + \frac{\hat{\alpha}_{WLS}}{(\hat{\alpha}_{WLS} + \hat{\alpha}_{WLS} + \hat{\alpha}_{WLS})} + \frac{\hat{\alpha}_{WLS}}{(\hat{\alpha}_{WLS} + \hat{\alpha}_{WLS} + \hat{\alpha}_{WLS})} - \frac{\hat{\alpha}_{WLS}}{(\hat{\alpha}_{WLS} + \hat{\alpha}_{WLS} + \hat{\alpha}_{WLS})} - \frac{\hat{\alpha}_{WLS}}{(\hat{\alpha}_{WLS} + \hat{\alpha}_{WLS} + \hat{\alpha}_{WLS})} + \frac{\hat{\alpha}_{WLS}}{(\hat{\alpha}_{WLS} + \hat{\alpha}_{WLS} + \hat{\alpha}_{WLS})} \]

... (39)

7. Simulation study

The Monte Carlo Simulation is conducted to compare the performances of the Maximum likelihood, Regression, Least Square, and Weighted Least Square estimators of R (repeat 10000 times).

It is performed by assuming eight states of R, say: \((\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \mu) = (1.9,0.5,0.4,0.5,1.5,1.3) (1.9,0.5,0.4,0.5,1.5,1.3) (0.7,1.2,1.5,1.8,1.6,1.3), (0.7,1.2,1.5,1.8,1.6,1.3), (2.6,1.3,2.4,1.6,1.4,1.3), (2.6,1.3,2.4,1.6,1.4,1.3), (2.6,1.3,2.4,1.6,1.4,1.3)\) and (2.2,1.3,1.8,1.6,1.5,1.3) and (2.2,1.3,1.8,1.6,1.5,1.3) for different sample sizes \((n, n_1, n_2, n_3, n_4) = (10,10,15,15,15), (25,25,25,25,25), (35,35,20,20,25), (75,75,75,75,75) & (75,75,50,50,50)\). The real values of R and the (MSE & MAPE) values for these cases are recorded in tables from (1) to (8). The ML estimator gives the best performance in all tables.
Table (1): MSE and MAPE values of \([\{\alpha, \alpha_2, \alpha_3, \alpha_4, \mu\} = (1.9,0.5,0.4,0.5,1.5,1.3)]\) \(R = 1.2557\)

| Simple size | Criterion | MLE | RgE | LSE | WLSE |
|-------------|-----------|-----|-----|-----|------|
| (10, 10, 15, 15, 15) | Mean | 1.2044 | 1.228 | 1.0666 | 1.0461 |
| | MSE | 0.0036 | 0.0606 | 0.0390 | 0.0603 |
| | MAPE | 0.0457 | 0.1888 | 0.1515 | 0.1889 |
| | Mean | 1.1619 | 1.2475 | 1.0267 | 1.0203 |
| (25, 25, 25, 25, 25) | MSE | 0.0114 | 0.0712 | 0.0565 | 0.0603 |
| | MAPE | 0.0774 | 0.2147 | 0.1875 | 0.1908 |
| | Mean | 1.2597 | 1.2433 | 1.2573 | 1.2582 |
| (35, 35, 20, 20, 25) | MSE | 0.0107 | 0.0188 | 0.0132 | 0.0187 |
| | MAPE | 0.0657 | 0.0865 | 0.0728 | 0.0867 |
| | Mean | 1.2590 | 1.2976 | 1.2567 | 1.2613 |
| (75, 75, 75, 75, 75) | MSE | 0.0066 | 0.0132 | 0.069 | 0.0133 |
| | MAPE | 0.0506 | 0.708 | 0.0673 | 0.0708 |
| | Mean | 1.2396 | 1.2747 | 1.2631 | 1.2524 |
| (75, 75, 50, 50, 50) | MSE | 0.0040 | 0.0081 | 0.0051 | 0.0118 |
| | MAPE | 0.0414 | 0.0586 | 0.0488 | 0.0688 |

Table (2): MSE and MAPE values of \([\{\alpha, \alpha_2, \alpha_3, \alpha_4, \mu\} = (1.9,0.5,0.4,0.5,1.5,1.8)]\) \(R = 1.2557\)

| Simple size | Criterion | MLE | RgE | LSE | WLSE |
|-------------|-----------|-----|-----|-----|------|
| (10, 10, 15, 15, 15) | Mean | 1.2567 | 1.2644 | 1.2565 | 1.2543 |
| | MSE | 0.0356 | 0.0545 | 0.0421 | 0.0423 |
| | MAPE | 0.1222 | 0.1465 | 0.1294 | 0.1365 |
| | Mean | 1.2604 | 1.2587 | 1.2589 | 1.2590 |
| (25, 25, 25, 25, 25) | MSE | 0.0179 | 0.0290 | 0.0221 | 0.0269 |
| | MAPE | 0.0849 | 0.1088 | 0.0923 | 0.1046 |
| | Mean | 1.2589 | 1.2514 | 1.2569 | 1.2508 |
| (35, 35, 20, 20, 25) | MSE | 0.0111 | 0.0188 | 0.0189 | 0.0187 |
| | MAPE | 0.0659 | 0.0878 | 0.0890 | 0.0868 |
| | Mean | 1.2577 | 1.2958 | 1.2620 | 1.2956 |
| (75, 75, 75, 75, 75) | MSE | 0.0060 | 0.0122 | 0.0138 | 0.0129 |
| | MAPE | 0.0503 | 0.0723 | 0.0768 | 0.0725 |
| | Mean | 1.2590 | 1.2860 | 1.2629 | 1.2779 |
| (75, 75, 50, 50, 50) | MSE | 0.0039 | 0.0089 | 0.0120 | 0.0090 |
| | MAPE | 0.0444 | 0.0589 | 0.0669 | 0.0590 |

Table (3): MSE and MAPE values of \([\{\alpha, \alpha_2, \alpha_3, \alpha_4, \mu\} = (0.7,1.2,1.5,1.8,1.6,1.3)]\) \(R = 1.1678\)

| Simple size | Criterion | MLE | RgE | LSE | WLSE |
|-------------|-----------|-----|-----|-----|------|
| (10, 10, 15, 15, 15) | Mean | 1.1999 | 1.1640 | 1.1659 | 1.1670 |
| | MSE | 0.0131 | 0.0191 | 0.0169 | 0.0144 |
| | MAPE | 0.0780 | 0.0943 | 0.0890 | 0.0838 |
| | Mean | 1.1665 | 1.1655 | 1.1657 | 1.1660 |
| (25, 25, 25, 25, 25) | MSE | 0.0060 | 0.0103 | 0.0094 | 0.0070 |
| | MAPE | 0.0540 | 0.0688 | 0.0665 | 0.0586 |
| | Mean | 1.1676 | 1.1388 | 1.1620 | 1.1615 |
| (35, 35, 20, 20, 25) | MSE | 0.0043 | 0.0080 | 0.0076 | 0.0054 |
| | MAPE | 0.0445 | 0.0632 | 0.0583 | 0.0483 |
Table (4): MSE and MAPE values of $\{(\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \mu) = (0.7, 1.2, 1.5, 1.8, 1.6, 1.8)\}$ $R = 1.1678$

| Simple size | Criterion | ML | Rg | LS | WLS |
|-------------|-----------|----|----|----|-----|
| (75, 75, 75, 75) | Mean | 1.1676 | 1.1789 | 1.1676 | 1.1685 |
| | MSE | 0.0022 | 0.0048 | 0.0057 | 0.0032 |
| | MAPE | 0.0335 | 0.0469 | 0.0497 | 0.0356 |
| | Mean | 1.1670 | 1.1776 | 1.1676 | 1.1665 |
| (75, 75, 50, 50, 50) | MSE | 0.0014 | 0.0032 | 0.0042 | 0.0020 |
| | MAPE | 0.0268 | 0.0395 | 0.0440 | 0.0310 |

Table (5): MSE and MAPE values of $\{(\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \mu) = (2.6, 1.3, 2.4, 1.6, 1.4, 1.3)\}$ $R = 0.2037$

| Simple size | Criterion | MLE | RgE | LSE | WLS |
|-------------|-----------|-----|-----|-----|-----|
| (10, 10, 15, 15, 15) | Mean | 0.2079 | 0.2043 | 0.2087 | 0.2090 |
| | MSE | 0.0065 | 0.0106 | 0.0089 | 0.0076 |
| | MAPE | 0.3240 | 0.4097 | 0.3645 | 0.3456 |
| | Mean | 0.2060 | 0.1914 | 0.2040 | 0.2022 |
| (25, 25, 25, 25, 25) | MSE | 0.0035 | 0.0056 | 0.0051 | 0.0046 |
| | MAPE | 0.2387 | 0.3053 | 0.2879 | 0.2589 |
| | Mean | 0.2065 | 0.1924 | 0.2035 | 0.2040 |
| (35, 35, 20, 20, 25) | MSE | 0.0023 | 0.0034 | 0.0037 | 0.0029 |
| | MAPE | 0.1819 | 0.2314 | 0.2403 | 0.1977 |
| | Mean | 0.2083 | 0.2043 | 0.2092 | 0.2149 |
| (75, 75, 75, 75, 75) | MSE | 0.0005 | 0.0015 | 0.0027 | 0.0010 |
| | MAPE | 0.0989 | 0.1390 | 0.1892 | 0.1287 |
| | Mean | 0.2030 | 0.1922 | 0.2086 | 0.2013 |
| (75, 75, 50, 50, 50) | MSE | 0.0007 | 0.0017 | 0.0024 | 0.0011 |
| | MAPE | 0.1089 | 0.1590 | 0.1793 | 0.1245 |

Table (6): MSE and MAPE values of $\{(\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \mu) = (2.6, 1.3, 2.4, 1.6, 1.4, 1.8)\}$ $R = 0.2037$

| Simple size | Criterion | MLE | RgE | LSE | WLS |
|-------------|-----------|-----|-----|-----|-----|
| (10, 10, 15, 15, 15) | Mean | 0.2706 | 0.2690 | 0.2806 | 0.2713 |
| | MSE | 0.0070 | 0.0079 | 0.0118 | 0.0090 |
| | MAPE | 0.3882 | 0.4004 | 0.4475 | 0.3899 |
| | Mean | 0.2270 | 0.2189 | 0.2283 | 0.2391 |
Table (7): MSE and MAPE values of $[(\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \mu) = (2.2, 1.3, 1.8, 1.6, 1.5, 1.3)]$ $R = 0.2305$

| Simple size | Criterion | MLE | RgE | LSE | WLSE |
|-------------|-----------|-----|-----|-----|-----|
| (25, 25, 25, 25) | MSE | 0.0010 | 0.0015 | 0.0019 | 0.0021 |
| | MAPE | 0.1267 | 0.1321 | 0.1666 | 0.1897 |
| | Mean | 0.1517 | 0.1790 | 0.1945 | 0.2036 |
| (35, 35, 20, 20, 25) | MSE | 0.0035 | 0.0044 | 0.0042 | 0.0040 |
| | MAPE | 0.2200 | 0.2357 | 0.2251 | 0.2223 |
| | Mean | 0.2050 | 0.1897 | 0.2023 | 0.2023 |
| (75, 75, 75, 75) | MSE | 0.0010 | 0.0022 | 0.0025 | 0.0014 |
| | MAPE | 0.1268 | 0.1784 | 0.1935 | 0.1424 |
| | Mean | 0.2039 | 0.1896 | 0.2020 | 0.2015 |
| (75, 75, 50, 50) | MSE | 0.0007 | 0.0016 | 0.0022 | 0.0009 |
| | MAPE | 0.1068 | 0.1542 | 0.1758 | 0.1206 |

Table (8): MSE and MAPE values of $[(\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \mu) = (2.2, 1.3, 1.8, 1.6, 1.5, 1.8)]$ $R = 0.2305$

| Simple size | Criterion | MLE | RgE | LSE | WLSE |
|-------------|-----------|-----|-----|-----|-----|
| (10, 10, 15, 15, 15) | MSE | 0.0077 | 0.0132 | 0.0103 | 0.0095 |
| | MAPE | 0.3088 | 0.3862 | 0.3484 | 0.3302 |
| | Mean | 0.2332 | 0.2126 | 0.2315 | 0.2287 |
| (25, 25, 25, 25, 25) | MSE | 0.0042 | 0.0073 | 0.0065 | 0.0054 |
| | MAPE | 0.2359 | 0.2972 | 0.2834 | 0.2556 |
| | Mean | 0.1893 | 0.2321 | 0.2555 | 0.2633 |
| (35, 35, 20, 20, 25) | MSE | 0.0034 | 0.0045 | 0.0057 | 0.0043 |
| | MAPE | 0.2162 | 0.2264 | 0.2537 | 0.2257 |
| | Mean | 0.2313 | 0.2108 | 0.2287 | 0.2275 |
| (75, 75, 75, 75) | MSE | 0.0023 | 0.0027 | 0.0029 | 0.0026 |
| | MAPE | 0.1225 | 0.1665 | 0.1887 | 0.1380 |
| | Mean | 0.2313 | 0.2145 | 0.2277 | 0.2264 |
| (75, 75, 50, 50, 50) | MSE | 0.0008 | 0.0019 | 0.0027 | 0.0012 |
| | MAPE | 0.1063 | 0.1547 | 0.1730 | 0.1216 |

8. Conclusions
The reliability $R$ of a component being having $X$ Lomax distribution strength and exposed to $Y_1, Y_2, Y_3$ and $Y_4$ Lomax stresses the shape parameter and a common scale parameter is obtained. We conducted a comparison between the four different estimation methods using MSE and MAPE and found that MLE is best for estimating $R$. The performance of LSE and WLSE convergent in all tables.

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