INCLUDING CAHN AND SIVERS EFFECTS INTO EVENT GENERATORS

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It is demonstrated that event generators LEPTO and PYTHIA can be modified to describe some azimuthal modulations. The comparisons of results obtained with modified LEPTO with existing data in the current fragmentation region of SIDIS are presented for Cahn and Sivers effects as well as the predictions for the target fragmentation region. The predictions for Sivers effect in the Drell–Yan process obtained with modified PYTHIA are also presented. The concept of hadronization function is discussed.

1. Introduction

The spin (in)dependent azimuthal asymmetries arising in high energy reactions allow us to study the dynamical effects related to spin and transverse momentum of partons in target nucleon and in hadronization.

The Drell–Yan lepton pair production is the simplest process which does not include the hadronization dynamics and within QCD can be described using as a nonperturbative input only the parton distribution functions:\n
\begin{equation}
\frac{d\sigma}{d \Omega} h_1 h_2 \rightarrow \ell^+ \ell^- + X \sim \sum_q (f_{\bar{q}/h_1} f_{q/h_2} + 1 \leftrightarrow 2) \otimes d\sigma_{q+\bar{q} \rightarrow \ell^+ \ell^-}. \tag{1}
\end{equation}

The predictions for the Sivers\(^2\) effect for this process recently have been presented in\(^3\). Here it will be demonstrated that similar results are obtained using modified PYTHIA event generator.

More nonperturbative inputs are needed to describe the semi-inclusive DIS (SIDIS) processes within the QCD formalism. Namely, for the particles

\(^a\)In the following the notations of\(^1\) are used.
produced in the current fragmentation region (CFR) of SIDIS one needs to introduce fragmentation function, $D_h^q(z)$:

$$d\sigma_{\ell p \rightarrow \ell hX} \sim \sum_q f_q \otimes d\hat{\sigma}_{\ell q \rightarrow \ell q} \otimes D_h^q.$$ (2)

For the particles produced in the target fragmentation region (TFR) another nonperturbative input — the fracture functions, $M_{q/N}^h(x, z)$, are needed:

$$d\sigma_{\ell p \rightarrow \ell hX} \sim \sum_q d\hat{\sigma}_{\ell q \rightarrow \ell q} \otimes M_{q/N}^h.$$ (3)

In practice it is not easy to separate the two regions at moderate beam energies and final hadronic state invariant masses. An alternative approach which is able to describe the particles production in the whole phase space is based on the LUND string fragmentation model and adopted in the Monte Carlo event generators. Here the SIDIS cross section can be represented as

$$d\sigma_{\ell p \rightarrow \ell hX} \sim \sum_q f_q \otimes d\hat{\sigma}_{\ell q \rightarrow \ell q} \otimes H_{q/N}^h,$$ (4)

where the hadronization function, $H_{q/N}^h$, describes the particle production from the system formed by the struck quark and target remnant.

In the role of parton intrinsic motion in SIDIS processes in CFR within QCD parton model has been considered at leading order; intrinsic $k_{\perp}$ is fully taken into account in quark distribution functions and in the elementary processes as well as the hadron transverse momentum, $p_{\perp}$, with respect to fragmenting quark momentum.

The average values of $k_{\perp}$ for quarks inside protons and $p_{\perp}$ for final hadrons inside the fragmenting quark jet where fixed by a comparison with data on Cahn effect — the dependence of the unpolarized cross section on the azimuthal angle between the leptonic and the hadronic planes. The single spin asymmetry (SSA) $A_{UT}^{\sin(\phi - \phi_S)}$ recently observed by HERMES and COMPASS Collaborations was successfully described by Sivers mechanism.

Here it will be demonstrated that both Cahn and Sivers effects can be implemented into Monte Carlo event generators.

2. Including Cahn effect in LEPTO

In the simplest case, corresponding to LO approximation of parton model, event generation in LEPTO proceeds in several steps:
(1) the hard scattering kinematics is generated,
(2) the active quark inside the nucleon is chosen according to the quark density function \( f_q(x, Q^2) \),
(3) the transverse momentum of the final quark is simulated with Gaussian \( k_\perp \) and flat \( \phi \) distributions. Note that the transverse momentum of the final final quark is equal to that of initial quark for leading order hard subprocess.
(4) the string fragmentation machinery of JETSET program\(^7\) is applied to form the final hadrons.

The Cahn effect\(^11\) is a kinematical effect arising due to the presence of nonzero intrinsic transverse momentum of quarks in the nucleon. In the general case of non collinear kinematics Mandelstam variables depend on the quark transverse momentum and its azimuthal angle and at order \( O(k_\perp/Q) \) one has

\[
d\hat{\sigma}_{\ell q\to\ell q} \propto 1 - \frac{(2 + y)\sqrt{1 - y} k_\perp}{1 + (1 - y)^2} \frac{Q}{Q} \cos \phi. \tag{5}
\]

Eq. (5) shows that the azimuthal angle of the final quark (and of the string’s end associated with the struck quark) is now modulated with amplitude depending on \( y, Q \) and \( k_\perp \).

This effect can be introduced in the LEPTO event generator at the step (3) of the event generation, when the transverse momentum and azimuthal angle of the scattered quark are generated. To do this the generation of the quark transverse momentum, \( k_\perp \), is left unchanged and then the azimuthal angle is generated according to Eq. (5). This leads to azimuthal modulation of the string axis. The momentum conservation means that the transverse momentum of the quark is balanced by that of the target remnant, which in turn means that the azimuthal angle of the target remnant \( \varphi_{qq} = \varphi - \pi \). Hence, one expects that the azimuthal angle of the hadrons in the target fragmentation region (TFR), \( x_F < 0 \), will be modulated with a phase shifted by \( \pi \) with respect to that in CFR.

Data on azimuthal dependencies of SIDIS covering a large \( x_F \) range have been obtained by the EMC Collaboration\(^14\) for a beam energy of 280 GeV. The \( x_F \) dependence of \( \langle \cos \phi_h \rangle/w_1(y) \), where \( w_1(y) = (2 - y)\sqrt{1 - y}/(1 + (1 - y)^2) \), obtained by using modified LEPTO for EMC kinematics are presented in Fig. 1 together with data points from\(^14\) (left panel). The simulations has been done with LO setting of LEPTO (LST(8)=0) and with values of the parameters describing intrinsic \( k_T \) (PARL(3)=0.5) and
fragmentation $p_T$ ($\text{PARL}(21)=0.45$) as adopted in $^1$.

The predictions of modified LEPTO for $\langle \cos \phi_h \rangle$ of different hadron ($\pi^+$, $\pi^-$, $\pi^0$ and $p$) produced in SIDIS on a proton target at future CEBAF 12 GeV facility at JLab$^{15}$ are presented in Fig. 1 (right panel). One can see from Fig. 1 and that the predicted mean value of $\cos \phi_h$ in the CFR is negative $\langle \cos \phi_h \rangle_{CFR} < 0$, while in the TFR is positive $\langle \cos \phi_h \rangle_{TFR} > 0$, as suggested by arguments based on transverse momentum conservation.

3. Including Sivers effect in LEPTO

The azimuthal modulation of the string transverse momentum in the previous section was due to Cahn effect – the dependence of the non planar lepton-quark scattering cross section on the quark azimuth. The quark distribution, $f_q(x, k_\perp)$ itself is independent of quark azimuthal angle.

The situation is different when one considers SIDIS on a transversely polarized nucleon. Now a correlation between transverse momentum of quark, $k_\perp$ and target transverse polarization $S_T$ of type $S_T \cdot (\hat{P} \times \hat{k}_\perp)$ is possible – the so called Sivers effect$^2$.

The unpolarized quark (and gluon) distributions inside a transversely polarized proton can be written as:

$$f_{q/p}(x, k_\perp) + f_{\bar{q}/p}(x, k_\perp) = 2f_{q/p}(x, k_\perp),$$
$$f_{q/p}(x, k_\perp) - f_{\bar{q}/p}(x, k_\perp) = \Delta N f_{q/p}(x, k_\perp) S_T \cdot (\hat{P} \times \hat{k}_\perp).$$

Eq. (6) implies

$$f_{q/p}(x, k_\perp) + f_{\bar{q}/p}(x, k_\perp) = 2f_{q/p}(x, k_\perp),$$
$$f_{q/p}(x, k_\perp) - f_{\bar{q}/p}(x, k_\perp) = \Delta N f_{q/p}(x, k_\perp) S_T \cdot (\hat{P} \times \hat{k}_\perp),$$

Figure 1. Left: The $x_F$ dependence of $\langle \cos \phi_h \rangle/w_1(y)$ for charged hadrons compared with EMC data. Right: Predictions of modified LEPTO for $x_F$ dependence of $\langle \cos \phi_h \rangle$ for different hadrons produced in 12 GeV unpolarized SIDIS process.

The unpolarized quark (and gluon) distributions inside a transversely polarized proton can be written as:

$$f_{q/p}(x, k_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta N f_{q/p}(x, k_\perp) S_T \cdot (\hat{P} \times \hat{k}_\perp).$$

The unpolarized quark (and gluon) distributions inside a transversely polarized proton can be written as:

$$f_{q/p}(x, k_\perp) + f_{\bar{q}/p}(x, k_\perp) = 2f_{q/p}(x, k_\perp),$$
$$f_{q/p}(x, k_\perp) - f_{\bar{q}/p}(x, k_\perp) = \Delta N f_{q/p}(x, k_\perp) S_T \cdot (\hat{P} \times \hat{k}_\perp).$$

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where \( f_{q/p}(x, k_{\perp}) \) is the unpolarized parton density and \( \Delta^N f_{q/p'}(x, k_{\perp}) \) is referred to as the Sivers function. Notice that, as requested by parity invariance, the scalar quantity \( S_T \cdot (\hat{P} \times \hat{k}_{\perp}) \) singles out the polarization component perpendicular to the \( \mathbf{P} - \mathbf{k}_{\perp} \) plane. For a proton moving along \(-z\) and a generic transverse polarization vector \( S_T = |S_T| (\cos \phi_S, \sin \phi_S, 0) \) one has:

\[
S_T \cdot (\hat{P} \times \hat{k}_{\perp}) = |S_T| \sin (\phi - \phi_S) \equiv |S_T| \sin \phi_{Siv} ,
\]

(8)

where \( (\phi - \phi_S) = \phi_{Siv} \) is the Sivers angle.

The Sivers function for each light quark flavor \( q = u, d \) are parameterized in the following factorized form\(^1\):

\[
\Delta^N f_{q/p'}(x, k_{\perp}) = 2 N_q(x) h(k_{\perp}) f_{q/p}(x, k_{\perp}) ,
\]

(9)

where

\[
N_q(x) = N_q x^{a_q} (1 - x)^{b_q} \left( a_q + a_{q'} + b_q \right) \frac{a_q b_{q'}}{a_q' b_q} ,
\]

(10)

\[
h(k_{\perp}) = \sqrt{2e} \frac{k_{\perp}}{M} e^{-k_{\perp}^2/M^2} ,
\]

(11)

where \( N_q, a_q, b_q \) and \( M \) (GeV/c) are parameters. Then Eq. (6) can be rewritten as

\[
f_{q/p'}(x, k_{\perp}) = f_{q/p}(x, k_{\perp}) [1 + |S_T| N_q(x) h(k_{\perp}) \sin \phi_{Siv}] .
\]

(12)

Again, the Sivers effect is incorporated into LEPTO at the stage 3) of the event generation in the same way as for the Cahn effect but now the azimuthal angle is generated according to Eq. (12). For simulations the following set of parameters compatible with those obtained in\(^1,3\) have been used: \( N_u = N_{\bar{u}} = 0.5, N_d = N_{\bar{d}} = -0.2, a_q = 0.3, b_q = 2 \) and \( M^2 = 0.36 \) (GeV/c)\(^2\).

In Fig. 2 the results of simulation for HERMES experimental conditions are compared with observed Sivers asymmetries\(^12\) (left panel). Future facilities as Electron Ion Colliders or upgraded JLab will have larger kinematic coverage and will offer the possibility of studying the Sivers effect also with hadrons produced in the TFR. As an example, the simulations have been done for 12 GeV electron SIDIS of a proton target. The DIS cut \( Q^2 > 1 \) (GeV/c)\(^2\) and \( W^2 > 4 \) GeV\(^2\) and a cut on the produced hadron transverse momentum \( P_T > 0.05 \) GeV/c was imposed. The predictions for \( x_F \) dependence for JLab kinematics is presented in Fig. 2 (right panel). The \( x \) and \( P_T \) dependencies in the TFR are presented Fig. 3.
energies: planned GSI \( \bar{p}p \) obtained with modified transversely polarized proton target. The curves are the results of simulations obtained with modified LEPTO.

Right: Predicted dependence of \( A^{\sin(\phi_T - \phi_S)}_{UT} \) on \( x_F \) for different hadrons produced in SIDIS of 12 GeV electrons off a transversely polarized proton target.

Figure 3. Predicted dependence of \( A^{\sin(\phi_T - \phi_S)}_{UT} \) on \( x \), left panel, \( (p_T, \text{right panel}) \) for different hadrons produced in the TFR \( (x_F < -0.1) \) of SIDIS of 12 GeV electrons off a transversely polarized proton target.

In Fig. 4 the results for Sivers asymmetry in the Drell –YAn process obtained with modified PYTHIA generator are presented for two different energies: planned GSI \( \bar{p}p \) collider with \( \sqrt{s} = 14.4 GeV \) (left panel) and RHIC \( \sqrt{s} = 200 GeV \) (right panel). The results\(^b\) are similar to that obtained in\(^4\).

\(^b\)Note, that here the sign convention of\(^4\) is adopted.
4. Discussion and Conclusions

The advantage of this MC based approach compared to standard QCD factorized approach is the full coverage of produced hadron phase space. Figs. 1 and 2 demonstrate that the modified LEPTO event generator well describes the data in the CFR both for Cahn and Sivers asymmetries. One can notice in Fig. 1 that the integrated experimental value of $\langle \cos \phi_h \rangle$ for charged hadrons in the CFR is not compensated by that in TFR. It seems improbable that this imbalance can be compensated by larger values of $\langle \cos \phi_h \rangle$ of neutral hadrons at $x_F \simeq -1$.

Note, that in present approach a possible modifications of hadronization in the case of polarized target have been ignored. In\textsuperscript{8} it was shown that the hadronization functions in principle depends on polarization states of struck quark and target nucleon even for production of (pseudo)scalar or unpolarized particles. This dependence cannot be neglected at moderate energies and new nonperturbative input — the polarized hadronization functions, $\Delta H^h_{q/N}$, are needed to describe the polarized SIDIS. The expression for the SIDIS helicity asymmetry is then looks as:

$$A_h^h(x, z, Q^2) = \sum_q \epsilon_q^2 q(x, Q^2) H^h_{q/N}(x, z, Q^2) \left[ \frac{\Delta q(x, Q^2)}{\bar{q}(x, Q^2)} + \frac{\Delta H^h_{q/N}(x, z, Q^2)}{H^h_{q/N}(x, z, Q^2)} \right]$$

$$\sum_q \epsilon_q^2 q(x, Q^2) H^h_{q/N}(x, z, Q^2)[1 + \frac{\Delta q(x, Q^2)}{q(x, Q^2)} \frac{\Delta H^h_{q/N}(x, z, Q^2)}{H^h_{q/N}(x, z, Q^2)}].$$

(13)

Since the hadronization functions depend on target nucleon, active quark and produced hadron variables the new correlations as (a): $S_L \cdot [p_\perp \times k_\perp]$, (b): $s_L \cdot [p_\perp \times k_\perp]$, (c): $[S_T \times p_\perp] \cdot [p_T^h \times s_T]$, etc are possible. This correlations cannot be present separately in the distribution
and fragmentation functions. They will induce the azimuthal asymmetries in the SIDIS of (a): unpolarized lepton off longitudinally polarized target, (b): longitudinally polarized lepton off unpolarized target, (c): unpolarized lepton off transversely polarized target etc.

The new high statistic measurements in both CFR and TFR of SIDIS will allow to check the predictions of the approach presented here and better understand the effects of the quark intrinsic transverse momentum and hadronization mechanism in SIDIS. The study of single spin asymmetries in Drell-Yan process will provide an additional test of our understanding of spin dependent phenomena.

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References

1. M. Anselmino et al., Phys. Rev. D72 (2005) 054028, arXiv:hep-ph/0501196.
2. D. Sivers, Phys. Rev. D41 (1990) 83; D43 (1991) 261.
3. M. Anselmino et al., arXiv:hep-ph/0507181.
4. W. Vogelsang and F. Yuan, Phys. Rev. D71 (2005) 074006, arXiv:hep-ph/0507266.
5. E. Berger, Proceedings of the NPAS Workshop on Electronuclear Physics with Internal Targets (SIAC, 1987), SLAC report 316, eds R G Arnold and R C Mmehart, p 82; Preprint ANL-HEP-CP-87-45, April 30, 1987.
6. G. Ingelman, A. Edin and J. Rathsman, Comp. Phys. Commun. 101 (1997) 108.
7. T. Sjöstrand, Comp. Phys. Commun. 39 (1986) 347, 43 (1987) 367;
   T. Sjöstrand, PYTHIA 5.7 and JETSET 7.4: Physics and Manual, arXiv:hep-ph/9508391;
   T. Sjöstrand et al., Comp. Phys. Commun. 135 (2001) 238.
8. A. Kotzinian, Eur. Phys. J. C44 (1995) 211, arXiv:hep-ph/0410093.
9. A. Kotzinian, Nucl. Phys. B441 (1995) 234, arXiv:hep-ph/9412283.
10. A. Kotzinian, arXiv:hep-ph/0504081.
11. R. N. Cahn, Phys. Lett. B78 (1978) 269; Phys. Rev. D40 (1989) 3107.
12. HERMES Collaboration, A. Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002, arXiv:hep-ex/0408013; M. Diefenthaler, arXiv:hep-ex/0507013.
13. COMPASS Collaboration, V. Y. Alexakhin et al., Phys. Rev. Lett. 94 (2005) 202002, arXiv:hep-ex/0503002.
14. EMC Collaboration, M. Arneodo et al., Z. Phys. C34 (1987) 277.
15. Pre-Conceptual Design Report,
   http://www.jlab.org/12GeV/collaboration.html.