Impedance matched thin metamaterials make metals absorbing

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Metals are generally considered good reflectors over the entire electromagnetic spectrum up to their plasma frequency. Here we demonstrate an approach to tailor their absorbing characteristics based on the effective metamaterial properties of thin, periodic metallo-dielectric multilayers by exploiting a broadband, inherently non-resonant, surface impedance matching mechanism. Based on this mechanism, we design, fabricate and test omnidirectional, thin (<1 micron), polarization independent, extremely efficient absorbers (in principle being capable to reach A > 99%) over a frequency range spanning from the UV to the IR. Our approach opens new venues to design cost effective materials for many applications such as thermo-photovoltaic energy conversion devices, light harvesting for solar cells, flat panel display, infrared detectors, stray light reduction, stealth and others.

Absorbing materials are crucial in many applications such as thermo-photovoltaic energy conversion devices1, light harvesting for solar cells2, flat panel displays3, stealth, and stray light reduction in imaging devices4. In nature, black pigments are found in the compound eyes of most insects to reduce crosstalk5. Another case is the black color on the Papilio Ulysses butterfly wings that is due to a combination of black pigment and structured surface that traps the light6. Many venues have been proposed during recent years to achieve broadband, polarization-insensitive, omni-directional absorbers. Currently available broadband low-reflectance materials include gold black7, black nickels, including porous nickel phosphorus (NiP)8 and black paint. Black metals and paints tend to have rough, protruding surface features to trap the light causing it to undergo several reflections. A material with 15% reflectance can absorb >99% of the energy after just three reflections. Black materials with rough surfaces such as anodized aluminum perform best at near normal incidence and degrade significantly for large angles4. Recently, successful demonstration of an extremely dark material has been made from low density nano-tube arrays9–11. However, fabrication of mechanically and environmentally durable nano-tube coatings over large areas is still a challenge and involves costly processes. Emerging new technologies have focused on absorption mechanisms based on the excitation of localized and/or surface plasmons on nano-patterned metal surfaces. For example, in Ref. 12 the authors experimentally demonstrate an ultrathin plasmonic absorber over the entire visible spectrum, in Ref. 13 the authors numerically study a broadband, 1-D metal grating with almost perfect absorption at λ ~ 600 nm, in Ref. 14 broadband absorption is predicted for all-metallic 1-D and 2-D gratings, in Ref. 15 an absorber based on light propagation in a metamaterial forming an effective “black-hole” is discussed, in Ref. 16 it is proposed a negative-index based, wide angle absorber for IR radiation, and in Ref. 17 broadband absorbers and selective emitters based on plasmonic Brewster metasurfaces are numerically investigated.

In this work we propose a viable, yet effective, alternative to achieve efficient absorbers spanning from the UV to the near IR by matching the impedance of a metamaterial to the impedance of air. The geometry of the multilayer stack under consideration is depicted in Fig. 1. It consists of an N-period, symmetric structure whose elementary cell is made of a metallic layer sandwiched between two identical dielectric layers. The metamaterial does not require surface features or nanopatterning and can be deposited over large areas with traditional deposition techniques.

In Ref. 18 we have shown that in general any stratification with a symmetric elementary cell, but not limited to a metallo-dielectric stratification, or a symmetric three-layer elementary cell described in Fig. 1, is equivalent to an effective bulk metamaterial, having the same physical thickness of the equivalent multilayer and univocally described by an effective characteristic impedance (Z_{eff}) and the Bloch wave vector (K_B) of the elementary cell. The expression of Z_{eff}, K_B and the free space characteristic impedance (Z_0) are respectively:
For a metallo-dielectric stratification with the symmetric three-layer elementary cell described in Fig. 1, the effective impedance at normal incidence from Eq.(1) and (2) can be explicitly written as:

\[
\frac{Z_{\text{eff}}}{Z_0} = \frac{1}{n_D} \sqrt{\frac{\sin(d_w k_o n_D - \delta) + 2n_D n_0 \tan(n_D k_0 d_w) \cos(d_M k_0 n_M) - \varepsilon_D \tan^2(n_D k_0 d_w) \sin(d_M k_0 n_M)}{\sin(d_w k_o n_D - \delta) + 2n_D n_0 \tan(n_D k_0 d_w) \cos(d_M k_0 n_M) - \varepsilon_D \tan^2(n_D k_0 d_w) \sin(d_M k_0 n_M)}}
\]

where \( n_M \) and \( n_D \) are respectively the refractive index of the metal and dielectric layer, while \( \varepsilon_M \) and \( \varepsilon_D \) their respective relative electric permittivities. Here for simplicity we analyze theoretically the normal incidence case, nevertheless all the resulting considerations can be straightforwardly extended to any arbitrary incident conditions and polarizations. For thin metallic layers, i.e. \( k_0 d_M < 1 \), a first order expansion of the trigonometric functions in (3) gives the following expression for the effective impedance:

\[
\frac{Z_{\text{eff}}}{Z_0} = \frac{1}{n_D} \sqrt{\frac{d_M k_0 \varepsilon_M \tan(n_D k_0 d_D) - 2n_D \tan(n_D k_0 d_D) - d_M k_0 \varepsilon_D}{d_M k_0 \varepsilon_D \tan^2(n_D k_0 d_D) - 2n_D \tan(n_D k_0 d_D) - d_M k_0 \varepsilon_M}}
\]

Eq.(4) can be recast in a more convenient form as:

\[
\frac{Z_{\text{eff}}}{Z_0} = \frac{1}{n_D} \sqrt{f_N(x)}
\]

where the numerator function \( f_N(x) \) and the denominator function \( f_D(x) \) are two parabolas respectively given by:

\[
f_N(x) = x^2 + \varepsilon_M \delta - 2n_D x - \varepsilon_D \delta,
\]

\[
f_D(x) = x^2 + \varepsilon_D \delta - 2n_D x - \varepsilon_M \delta,
\]

with \( x = \tan(n_D k_0 d_D) \) and \( \delta = k_0 d_M < 1 \). For the sake of simplicity, in the analysis that follows we neglect the imaginary part of the permittivity of the metal (as we will see later on, the results that stem from this analysis hold also when realistic metallic dispersions are used). Obviously, when the radicand of Eq.(5.a) is negative the effective impedance will be a pure imaginary number and therefore the metamaterial will show metallic-like behavior with 100% reflectivity and complete impedance mismatch, while in the case that the radicand of Eq.(5.a) is positive the effective impedance will be positive and the structure will show a dielectric-like behavior. In order to study the sign of the radicand, Eq.(5.a) can be rewritten, using Ruffini’s theorem, as:

\[
\frac{Z_{\text{eff}}}{Z_0} = \frac{1}{n_D} \sqrt{\frac{\varepsilon_M (x - x_N) (x - x_N)}{\varepsilon_D (x - x_D) (x - x_D)}}
\]

where the zeros of \( f_N(x) \) and \( f_D(x) \) are respectively given by:

\[
x_{\pm N} = \frac{n_D}{\varepsilon_M \delta} \left( 1 \pm \sqrt{1 + \varepsilon_M \delta^2} \right),
\]

\[
x_{\pm D} = \frac{n_D}{\varepsilon_D \delta} \left( 1 \pm \sqrt{1 + \varepsilon_D \delta^2} \right).
\]

For \( \delta > 1/\sqrt{-\varepsilon_M} \) (Case I) life is rather dull, in fact the radicand of Eq.(6.a) will always be negative and the structure will show a metallic-like behavior. After all, this is the regime one would expect given the presence of metal layers. Luckily, things become much more interesting for \( \delta < 1/\sqrt{-\varepsilon_D} \) (Case II). In this latter case, the radicand becomes positive when either \( x_N < x < x_N \) or \( x_D < x < x_D \) and in those intervals the structure will show a dielectric-like behavior. In Fig. 2 we show a graphical representation that summarizes the results.
In particular, Case II of Fig. 2 shows the bands where the structure either behaves as a metal or as a dielectric. The two dielectric regions are bounded respectively by the zeroes and the poles of the effective impedance. Namely, the dielectric region \( x_{-N} < x < x_{+N} \) is bounded by \( Z_{\text{eff}}/Z_0 = 0 \) (zeros of \( f_0(x) \)), while the dielectric region \( x_{-D} < x < x_{+D} \) is bounded by \( Z_{\text{eff}}/Z_0 = \infty \) (zeros of \( f_D(x) \)). A further analysis of the effective dispersion of the conditions \( Z_{\text{eff}}/Z_0 = \infty \) and \( Z_{\text{eff}}/Z_0 = 0 \) shows that they can be approximately expressed as:

\[
\begin{align*}
Z_{\text{eff}}/Z_0 & = \infty \quad \text{for} \quad m = 0, 1, 2, \ldots , \quad \delta \epsilon_M + \frac{\epsilon_M}{\epsilon_D} + \frac{\delta^2}{1 + \epsilon_M \delta^2} \\
Z_{\text{eff}}/Z_0 & = 0 \quad \text{for} \quad m = 1, 2, 3, \ldots , \quad \frac{\epsilon_M}{\epsilon_D} + \frac{\delta^2}{1 + \epsilon_M \delta^2} \\
Z_{\text{eff}}/Z_0 & = \frac{\delta^2}{1 + \epsilon_M \delta^2} \quad \text{for} \quad m = 1, 2, 3, \ldots , \quad \frac{\epsilon_M}{\epsilon_D} + \frac{\delta^2}{1 + \epsilon_M \delta^2}.
\end{align*}
\]

Let us now analyze in particular the dielectric region \( x_{-D} < x < x_{+D} \), i.e. the one bounded by \( Z_{\text{eff}}/Z_0 = \infty \). The impedance at the center of this band is:

\[
\left( \frac{Z_{\text{eff}}}{Z_0} \right)_{x = x_{0D}} = \frac{2 \epsilon_D - \epsilon_M \delta^2 + \delta^2}{1 + \epsilon_M \delta^2}.
\]

By solving Eq.(8) under the condition of perfect impedance matching with vacuum, we find:

\[
\left( \frac{Z_{\text{eff}}}{Z_0} \right)_{x = x_{0D}} = 1 \iff \epsilon_D = \frac{1 + \sqrt{1 - \epsilon_M (1 + \epsilon_M \delta^2 - \delta^2)}}{1 + \epsilon_M \delta^2 - \delta^2}.
\]

Eq.(9) dictates what is the best choice of the permittivity of the dielectric material for perfect impedance matching in the dielectric band. Therefore, not only our structure possesses bands of dielectric-like behavior, but, in principle, it is also possible to match its impedance to the vacuum by a proper choice of the dielectric material, no matter how many metal layers the structure is made of. The structure could contain hundreds of metal layers and still be impedance matched with vacuum!

In Fig. 3 we plot, as an example, \( \epsilon_D \) vs. \( \lambda/\lambda_p \) according to (9) for metal layers with different thickness whose dispersion we model by using a generic, free-electron gas, lossless Drude model \( \epsilon_M = 1 - (\lambda/\lambda_p)^2 \) (\( \lambda_p = 2\pi c/\epsilon_0 \) is the metal plasma wavelength).

In Fig. 3, it can be seen that the values of the permittivity of the dielectric materials necessary to achieve perfect impedance matching are within the range of the permittivity of many dielectric and/or semiconductor materials. For example, in Fig. 4 we show the reflectance at normal incidence \( R = \left| 1 - (Z_{\text{eff}}/Z_0) / 1 + (Z_{\text{eff}}/Z_0) \right|^2 \) numerically calculated in the plane \( (d_M/\lambda_p, \lambda/\lambda_p) \) for \( d_M/\lambda_p = 1/30 \) and \( \epsilon_D = 2.25 \). As expected from the previous study, the structure...
shows two bands of dielectric-like behavior. The one that is bounded by the poles of the effective impedance shows perfect impedance matching. The white dashed lines represent the dispersion of the poles and the zeroes of the effective impedance calculated according to (7).

Now that we have analyzed in detail the properties of the effective impedance of our metamaterial and the impedance matching condition, we come to the analysis of the propagation length ($L_p$). As already mentioned in the introduction, a good absorber should be characterized by a short propagation length. At normal incidence for a semi-infinite (s-i) structure the absorption is given by the well-known formula: $\text{absorption} = 1 - \frac{1}{\text{transmittance}} = \frac{1}{1 + \frac{\text{reflection}}{\text{transmittance}}} = \frac{1}{\text{reflection}}$. By expanding the Bloch vector at the first order for thin metal layers, it can be demonstrated by simple mathematical manipulations that, at the center of the pass band of the Bloch vector, i.e. for $\cos(K_B) = 0$, $L_p$ can be approximated as:

$$\frac{L_p}{\Lambda} = \frac{n_D}{\text{Im}(\varepsilon_M)}$$

(10)

Eq.(10) tells us that the propagation length is inversely proportional to the imaginary part of the metal permittivity (dissipation), as one may expect, and directly proportional to the refractive index of the dielectric material. In order to have an efficient absorber a metal with high dissipation properties, such as Cu for example, would be desirable in conjunction with a low refractive index dielectric material, ideally as close as possible to vacuum. So far we have concentrated for the sake of generality on metals with nonspecific dispersion properties. We want now provide two examples in which metals with realistic dispersion properties are used. In particular, we concentrate on copper (Cu) and silver (Ag). In Fig. 5, we plot the bulk dispersion relations of the two metals, as reported in the book of Palik\textsuperscript{19}, it is worthwhile to note that copper shows intrinsic better absorption properties, i.e. higher imaginary part of the permittivity, with respect to Ag, as it is seen from the bulk dispersion properties of both materials reported in Fig. 5. Fig. 6(c) and 7(c) compare $L_p/\Lambda$ calculated at the center of the pass band according to the exact formula $L_p = (2\text{Im}(\varepsilon_M))^{-1}$ and according to (10).

The results on the propagation length suggest that the choice of the particular kind of metal is essential to obtain efficient absorption in short distances. While in fact, in the examples made, both structures ensure a high degree of impedance matching, the Cu-based structure is much more efficient in absorbing the e.m. radiation than the Ag-based structure. This means that, based on the respective propagation lengths, a 6-period Cu-based structure would be highly absorbing whereas the same structure made of Ag would be highly transparent. Although here we are interested in increasing the absorption properties of the structure, it is nevertheless interesting to note the great versatility of the impedance matching mechanism that we have elucidated: it makes possible to pass from a highly absorbing structure to a highly transparent structure just by a proper choice of the particular kind of metal.

**Results**

Based on the theoretical study presented in the previous section, we have fabricated two samples: sample1 and sample2 (see Methods for the fabrication details). In both cases the elementary cell is a three layer stratification with 10 nm of Cu sandwiched between two layers of SiO$_2$ 30 nm thick. Sample1 is made of 6 repetitions of the elementary cell for a total length $L \sim 3L_m$, sample2 is made of 10 repetitions of the elementary cell for a total length $L \sim 5L_m$. Both samples have been grown on a quartz substrate. The transmittance (T) of the two
samples has been measured between $0^\circ$ and $70^\circ$ over a wide spectral range (300 nm up to 800 nm) while the reflectance (R) has been measured between $20^\circ$ and $70^\circ$ over the same spectral range (see Methods for measurements details). In Fig. 8 and Fig. 9 we report (left panel) the measured average transmittance (T) and reflectance (R) (the average has been taken over TE and TM polarization) as well as the derived absorption ($A = 1 - R - T$) for sample1 and sample2 respectively in the $(\lambda, \delta)$ plane, in comparison with the same quantities as predicted by the theory (right panel). A close look at Fig. 8 and Fig. 9 reveals that the reflectance from the two samples is almost identical, while the transmittance is slightly higher for sample 1 and scales approximately as $T \sim \exp(-L/L_p)$ in agreement with the theoretical predictions based on the propagation length analysis. As expected, both samples show high degree of polarization independent absorption ($>90\%$), over a wide range of incident angles and a spectral band extending from UV to near IR. Note also that the first
sample has a total length of only 420 nm and a total metal thickness of just 60 nm. The experimental absorption near 750 nm is larger than predicted by theory and we attribute this to the possibility that the 10 nm thick Cu film is below the percolation threshold which could induce losses due to the excitation of localized surface plasmons.

**Discussion**

We have reported a practical and simple way to achieve broadband, omnidirectional, polarization-insensitive absorption using periodic, thin, metallo-dielectric, multilayers in their metamaterial operation regime. Those structures can be easily fabricated over large areas at low cost using standard sputtering or thermal evaporation techniques. We remark that, although in this work we have focused on SiO₂/Cu multilayers for absorbing purposes, different combination of metal/dielectric materials can be exploited. As an example, in Fig. 10 we present the theoretical results for an $N = 10$-period structure whose elementary cell is made of a tungsten (W) layer 5 nm thick sandwiched between two Ta₂O₅ layers 75 nm thick. In this case the structure shows an ultra-wide band of absorption ranging from 300 nm to 2.5 μm.

The possible applications for these structures are innumerable. Absorbing materials structured at the nanoscale level are a fundamental part in the design of any efficient thermo-photovoltaic...
energy conversion device or solar cell. We also remark that the mechanism exploited in this work is based on a non-resonant impedance matching phenomenon which is therefore independent of the particular nature of the substrate. This makes possible, for example, to use our thin metamaterial to cover bulky metallic structures to eliminate electromagnetic reflection over an extremely broad frequency range from UV to almost mid-IR. As an example, we have grown sample 2 on a reflecting surface made of an optically thick silver layer. A picture of the sample and measured absorption is shown in Fig. 11.

Moreover, the non-resonant nature of the phenomenon also allows us to use these metamaterials in conjunction with other known absorbing configurations, such as the classical Salisbury-mirror-like configuration, i.e. antireflection coating/absorber/mirror, to further enhance their performance.

We also mention that the same approach that here we have employed to obtain efficient absorbers metamaterials for the visible and near-IR could be employed as well to obtain efficient absorbers in the mid-IR by using polar material such as SiC, for example.

Methods
Metamaterial description of multilayers. The fields at the input and output surface of a single layer material (a Fabry-Perot etalon) are linked to each other through a matrix relation that contains information on the angle of incidence by the longitudinal momentum $k_z$ of the impinging wave, the material properties $\varepsilon$ and $\mu$, and the layer thickness $d^{\text{th}}$ (see Fig. 12):

![Sample 1](image-url)
For TE polarization, $\alpha = \mu$, $\Psi = E$, $\varphi = (1/\mu)\partial E/\partial z$. For TM polarization, $\alpha = \varepsilon$, $\Psi = H$ and $\varphi = (1/\varepsilon)\partial H/\partial z$. In a cascade of layers, the fields at the input and output interface are still related each other through a matrix relation. In this case the transfer matrix of the whole structure is the product of all the single layer transfer matrix. If we consider an elementary cell of length $L = \sum d_i$, we have:

$$
\left( \begin{array}{c}
\Psi_{\text{out}} \\
\varphi_{\text{out}}
\end{array} \right) = \left( \begin{array}{c}
\cos(k_i d_i) \\
\sin(k_i d_i)
\end{array} \right) \left( \begin{array}{c}
\Psi_{\text{in}} \\
\varphi_{\text{in}}
\end{array} \right)
$$

(11)

where $d_i$ is the thickness of the $i$th layer, $k_i$ is the longitudinal component of the $k$ vector in the $i$th layer, $\alpha = \mu_i$ for TE polarization or $\alpha = \varepsilon_i$ for TM polarization, being $\mu_i$ and $\varepsilon_i$ the bulk magnetic permeability and the electric permittivity of the $i$th layer.

For a symmetric structure, it is possible to define $k_{\text{eff}}$ and $\alpha_{\text{eff}}$ so that:

$$
\left( \begin{array}{cc}
\cos(k_{\text{eff}} L) & \frac{\alpha_{\text{eff}}}{k_{\text{eff}}} \sin(k_{\text{eff}} L) \\
\frac{k_{\text{eff}}}{\alpha_{\text{eff}}} \sin(k_{\text{eff}} L) & \cos(k_{\text{eff}} L)
\end{array} \right) \left( \begin{array}{c}
\Psi_{\text{in}} \\
\varphi_{\text{in}}
\end{array} \right) = \left( \begin{array}{c}
m_{11} \Psi_{\text{in}} \\
m_{12} \varphi_{\text{in}}
\end{array} \right)
$$

(13)

Note that in general Eq. (13) provides 4 equations for 2 unknown quantities and in general there is no solution, unless the elementary cell is symmetric.

In Eq. (13), $\alpha_{\text{eff}} = \mu_{\text{eff}}$ (effective permeability) for TE polarization, or $\alpha_{\text{eff}} = \varepsilon_{\text{eff}}$ (effective permittivity) for TM polarization. $k_{\text{eff}}$ is the longitudinal component of the $k$ vector in the effective medium: $k_{\text{eff}} = K_n = n_{\text{eff}} \sqrt{(\alpha/\varepsilon) - (k_{\text{avg}}/n_{\text{eff}})^2}$ where, $k_{\text{avg}}$ is the transverse component of the $k$ vector, that is continuous through the interfaces. $k_{\text{eff}}$ is also equal to the Bloch vector of the structure.
the transfer matrix is given by:

$$\left( \begin{array}{cc} m_{11}^{N} & m_{12}^{N} \\ m_{21}^{N} & m_{22}^{N} \end{array} \right) = \left( \begin{array}{cc} \cos(k_{\text{eff}}^{N}A) & \frac{s_{\text{eff}}^{N} \cos(k_{\text{eff}}^{N}A)}{k_{\text{eff}}^{N}} \\ \frac{k_{\text{eff}}^{N} \sin(k_{\text{eff}}^{N}A)}{s_{\text{eff}}^{N}} & \cos(k_{\text{eff}}^{N}A) \end{array} \right).$$  \tag{16}

In other words, the N-period structure is equivalent to a bulk of the same effective medium of the elementary cell and having the same length $L = NA$ of the N-period structure itself; i.e. the dispersion properties of the single, symmetric elementary cell and the dispersion of the N-period structure made of the same elementary cell are identical.

**Dispersion properties of thin metallic layers.** Metal layers with dimensions on the order of the electron mean-free path (l) or less, exhibit higher resistivity than bulk conductors. The effect was first observed experimentally by Thomson in 1901 and later theoretically explained by Fuchs in 1938 and Sondheimer in 1952. According to the Fuchs-Sondheimer theory, the thin layer conductivity ($\sigma_{\text{thin}}$) is related to the bulk conductivity ($\sigma$), through a function of the layer thickness (d) over the electron mean-free path ($\xi = dl$):

$$\frac{\sigma}{\sigma_{\text{thin}}} = \frac{\Phi(\xi)}{\xi}$$  \tag{17}

with

$$\frac{1}{\Phi(\xi)} = \frac{1}{\xi} - \frac{3}{4} \left( 1 - \frac{1}{12} \xi^2 \right) Ei(-\xi) - \frac{3}{8\xi} (1 - e^{-\xi}) - \left( \frac{5}{16\xi} + \frac{1}{16} \xi \right) e^{-\xi}.$$  \tag{18}

where $Ei(-\xi)$ is a special function known as exponential integral:

$$Ei(-\xi) = -\int_{\xi}^{\infty} e^{-t} \frac{dt}{t}.$$  \tag{19}

For a Drude metal, with plasma frequency $\omega_p$ and damping $\gamma$, the electric permittivity $\varepsilon_{\text{bulk}}$ takes the following expression:

$$\varepsilon_{\text{Drude}} = 1 - \frac{\omega_p^2}{\omega^2(\omega + i\gamma)}.$$  \tag{20}

The damping term is inversely proportional to the conductivity $\gamma \propto \frac{1}{l}$, therefore we can derive a thin layer damping ($\gamma_{\text{thin}}$) in analogy with the thin layer conductivity:

$$\frac{\gamma_{\text{thin}}}{\gamma} = \frac{\sigma}{\sigma_{\text{thin}}} = \frac{\Phi(\xi)}{\xi}.$$  \tag{21}

and a thin layer Drude dispersion for the electric permittivity $\varepsilon_{\text{thin}}$:

$$\varepsilon_{\text{thin}} \varepsilon_{\text{Drude}} = 1 - \frac{\omega_p^2}{\omega^2(\omega + i\gamma)}.$$  \tag{22}

Unfortunately, close to the plasma frequency most metal, including Cu and W, do not strictly follow a Drude dispersion. In order to take into account the increasing absorption of thin layers, we subtract from the experimental bulk data taken from the book of Palik19 ($\varepsilon_{\text{bulk}}$) the free-electron contribution ($\varepsilon_{\text{Drude}}$) and we subsequently reintroduce the free-electron contribution of the thin layer ($\varepsilon_{\text{thin}}$) calculated according to Eq.(22):

Figure 10 | Absorption calculated for a 10 period Ta$_2$O$_5$(75 nm)/W(5 nm)/Ta$_2$O$_5$(75 nm) structure.

From Eq. (13) we can derive

$$\begin{align*}
\left( \frac{m_{12}}{m_{21}} \right)^2 &= -\frac{m_{21}^0}{m_{12}^0} \\
\cos(k_{\text{eff}}^{N}A) &= m_{11}^0 = m_{22}^0.
\end{align*}$$  \tag{14}

The characteristic impedance in a generic medium takes the following expressions:

$$Z = \sqrt{\frac{\mu}{\varepsilon k_z}} \quad \text{for TE polarization}$$

$$Z = \frac{\varepsilon}{\mu k_x} = \frac{\mu}{\varepsilon k_z} \quad \text{for TM polarization}.$$  \tag{15}

Eq.(1) follows from Eq.(14) and (15). Finally, for N repetition of the elementary cell the transfer matrix is given by:

Figure 11 | [Left] Measured absorption of a SiO$_2$/Cu multilayer stack (sample 2) on a reflecting surface made of silver and [Right] picture of the sample taken outdoors under normal daylight. The reflection of the blue sky and clouds is apparent in the picture on the reflecting side of the sample.
The SiO₂ film was deposited using a sputtering system. The uniformity of about 5% across the substrate.

Figure 12 | Transfer matrix representation for a single layer and a generic three-layer structure.

\[
\begin{align*}
\Psi_{\text{out}} &= \begin{pmatrix} \cos(k_d) & -\sin(k_d) \end{pmatrix} \Psi_{\text{in}}, \\
\varphi_{\text{out}} &= \begin{pmatrix} -\sin(k_d) & \cos(k_d) \end{pmatrix} \varphi_{\text{in}},
\end{align*}
\]

Note that in the limit of thick layers, i.e. \(d \gg l\), Eq.(23) gives the correct limit to the experimentally measured data of the bulk dispersion. The electron mean free path for Cu is 39 nm and for W 19 nm at room temperature. According to data reported in literature\textsuperscript{26,27}.

Fabrication. The metal-dielectric stacks were fabricated using a Denton magnetron sputtering system. The SiO₂ film was deposited using a 3° SiO₂ target and the copper layer was deposited using a 5° copper target. A 700 micron thick quartz wafer was used as a substrate. The system can be used to sputter a 6° diameter substrate with uniformity of about 5% across the substrate.

Optical measurements. Spectral transmission and reflection measurements were done using a Woollam VASE spectroscopic variable angle ellipsometer. The system allows measurements for s (TE) and p (TM) polarization from 15 degrees to grazing incidence in reflection and 0 to grazing in transmission. Transmission measurements were taken with respect to no sample in place. Angular measurements allow measurements for s (TE) and p (TM) polarization from 15 degrees to grazing incidence in reflection and 0 to grazing in transmission.

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Author contributions
N.M. and G.D. conceived the idea, developed the analytical model, contributed to the writing of the paper, measurements and the preparation of the samples.

Additional information
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