About the Poisson Structure for D4 Spinning String

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Abstract

The model of D4 open string with non-Grassmann spinning variables is considered. The non-linear gauge, which is invariant both Poincaré and scale transformations of the space-time, is used for subsequent studies. It is shown that the reduction of the canonical Poisson structure from the original phase space to the surface of constraints and gauge conditions gives the degenerated Poisson brackets. Moreover it is shown that such reduction is non-unique. The conception of the adjunct phase space is introduced. The consequences for subsequent quantization are discussed. Deduced dependence of spin $J$ from the square of mass $\mu^2$ of the string generalizes the "Regge spectrum" for conventional theory.

1. Introduction

The investigation of the constrained dynamical systems was started by Dirac \cite{1} and is continued in connection with the gauge theories development. There are many directions of the studies exist here; one of them is the modification of the conventional phase space conception (see, for example, \cite{2}). In this article, firstly, we suggest new viewpoint on the phase space for some kind of the gauge systems and, secondly, we apply the suggested conseqts to investigation of D4 string dynamics.

We start our studies with the following simple example. Let the space $\mathcal{H}_N$ be the phase space of any dynamical system with $N$ degrees of the freedom; any point $M \in \mathcal{H}_N$ has the coordinates $p_1, q_1, \ldots, p_N, q_N$ which diagonalize the standard non-degenerated Poisson brackets: $\{p_i, q_j\} = \delta_{ij}$, $\{p_i, p_j\} = \{q_i, q_j\} = 0$. Let us consider the subset $V \subset \mathcal{H}_N$: $V = \{M \in \mathcal{H}_N \mid p_1 = 0, q_1 > 0\}$. What is the Poisson structure of the set $V$? It is

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clear that such structure must be degenerated because codim \( V = 1 \). The simplest foliation of the set \( V \) will be following:

\[
V = \bigcup_{c>0} V^0_c,
\]

where set \( V^0_c = \{ M \in \mathcal{H}_N \mid p_1 = 0, q_1 = c, \ (c = \text{const}) \} \). It is well-known fact that the "correct" brackets for any set \( V^0_c \) will be the Dirac brackets \( \{\cdot, \cdot\}_1 \) for the pair (second type) constraints \( p_1 = 0, q_1 - c = 0 \). This bracket structure can be naturally extended on set \( V \); the function \( f_0 \equiv q_1 \) will be annihilator. The interesting fact is that the constructed brackets are non-unique. Indeed, we can introduce the other foliation of the set \( V : V = \bigcup_{c>0} V^f_c \), where the subsets \( V^f_c = \{ M \in \mathcal{H}_N \mid p_1 = 0, f(q_1; q_2, p_2, \ldots) = c, \ (c = \text{const}) \} \) were defined with help of some appropriate function \( f \) such that condition \( 0 < \frac{\partial f}{\partial q_1} < \infty \) holds. It is clear that corresponding Dirac brackets \( \{\cdot, \cdot\}_f \) differ from the brackets \( \{\cdot, \cdot\}_1 \); the annihilator for new brackets is the function \( f \). Thus, the reduction of same Poisson structure from the original phase space \( \mathcal{H}_N \) on some subset \( V \subset \mathcal{H}_N \) can be ambiguous if the reduced brackets degenerated.

In general, the situation is same if we consider the system of the first type constraints \( f_1, \ldots, f_k \), where \( k < N \), instead the single one \( p_1 = 0 \). Of course, this example is the special case of the general theory of degenerated Poisson brackets [3]. It was discussed in detail because the goal of our subsequent studies will be investigate this effect in a string theory.

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The satisfactory version of D4 quantum (super)string was a purpose of the theoretical studies for many authors (see, for example, [4, 5, 6, 7, 8]). Moreover, many authors construct the theories in arbitrary (non-critical) space-time dimensions [9, 10, 11]. Of course, this list is uncomplete: the detail review is impossible here. Suggested approach differs, in our knowledges, from other because it founded on the new conception of adjunct phase space.

So, we consider the following model here. Let the fields \( X_\mu(\xi^0, \xi^1) \) and \( \Psi^A_\pm(\xi^0, \xi^1) \) interact with two-dimensional gravity \( h_{ij}(\xi^0, \xi^1) \), where
\[ \xi^1 \in [0, \pi] \text{ and } \xi^0 \in (-\infty, \infty), \text{ such that dynamics is defined by the action constructed in accordance with the well-investigated manner } [12]: \]

\[ S = -\frac{1}{4\pi\alpha'} \int d\xi^0 d\xi^1 \sqrt{-h} \left\{ h^{ij} \partial_i X^\mu \partial_j X_\mu - i \Theta e^j_I (\Gamma^0)_{AB} \Psi^A \gamma_j \nabla^I \Psi^B \right\}. \quad (1) \]

The notations are following: \( h = \det(h^{ij}) \), the vectors \( e^j_I (\xi^0, \xi^1) \) are the vectors of two-dimensional basis such that the equalities \( h^{ij} = e^i_I e^j_I \) take place and the matrices \( \Gamma^\mu \) and \( \gamma_i \) are the Dirac matrices in the four- and two-dimensional space-time respectively. The field \( X = X_\mu t^\mu \) is the vector field in ("isotopical") Minkowski space-time \( E_{1,3} \); the fields \( \Psi^A \) with components \( \Psi^A_{\pm} \) are the spinor fields in two-dimensional space; index \( A \) is the spinor index in the space \( E_{1,3} \) such that the fields \( \Psi_{\pm} \) are the Majorana spinor fields in four-dimensional space-time. The numbers \( \Psi^A_{\pm} \) are the complex numbers, so there are no classical Grassmann variables in our action. The consideration of the spinning string without the Grassmann variables does not new (see, for example, [13]). In our opinion such approach is justified here because the new fundamental variables will be the complicated functions from the original fields \( X \) and \( \Psi \).

To fix the gauge arbitrariness we demand, as usually, \( e^j_I = \delta^j_I \) so that \( h^{ij} = \text{diag}(1, -1) \) and the equations of motion can be written in simplest form. For fields \( X \) and \( \Psi \) we have \( \partial_- \partial_+ X_\mu = 0, \quad \partial_\mp \Psi_{\pm} = 0 \); the equations of motion \( \delta S / \delta h^{ij} = 0 \) for gravity \( h \) lead as well-known to the equalities

\[ F_{1\pm}(\xi) \equiv (\partial_\pm X)^2 \pm \frac{i\Theta}{2} \Psi_{\pm} \partial_\pm \Psi_{\pm} = 0, \quad (2) \]

where \( \partial_\pm \) are derivatives with respect to cone parameters \( \xi_{\pm} = \xi^1 \pm \xi^0 \). The remained gauge freedom [12]

\[ \xi_{\pm} \rightarrow \tilde{\xi}_{\pm} = \pm A(\pm \xi_{\pm}), \quad (3) \]

must be fixed by means of additional conditions. For our subsequent consideration it is important that the function \( A(\xi) \) must satisfy the property

\[ A(\xi + 2\pi) = A(\xi) + 2\pi, \quad A' \neq 0 \]
in accordance with the standard boundary conditions for original variables \( X \) and \( \Psi \): \( X'_\mu(\xi^0,0) = X'_\mu(\xi^0,\pi) = 0 \), \( \Psi_+(\xi^0,0) = \Psi_-(\xi^0,0) \) and \( \Psi_+(\xi^0,\pi) = e\Psi_-(\xi^0,\pi) \), where \( e = \pm \).

The original phase space \( \mathcal{H} \) has the coordinates \( \dot{X}_\mu \equiv \partial_0 X_\mu, X_\mu, \Psi^+_\pm \) and \( \Psi^A_\pm \). As usually, canonical Poisson bracket structure is following:

\[
\{\dot{X}_\mu(\xi), X_\nu(\eta)\} = -4\pi\alpha' g_{\mu\nu} \delta(\xi - \eta),
\]

\[
\{\Psi^A_+(\xi), \Psi^B_+(\eta)\} = \frac{8\pi i\alpha'}{\Theta} (\Gamma^0)^{AB} \delta(\xi - \eta).
\]

2. The additional gauge conditions and the adjunct phase space.

The spinor variables give the additional possibilities to construct the natural Poincaré-invariant structures on the \((\xi^0,\xi^1)\)-plane\(^2\). For example, we can construct the following two-tensor:

\[
\Omega_{ij}(\xi^0,\xi^1) = \frac{1}{2} (h_{im}h_{jn} + h_{in}h_{jm} - h_{ij}h_{mn}) (\Gamma^0\Gamma^\mu)_{AB} \Psi^A_\gamma^m \Psi^B_\partial^n X_\mu.
\]

Another objects can be constructed too. The detail investigations these structures and the geometrical properties of the "extended" world-sheet \((\xi^0,\xi^1) \rightarrow (X_\mu(\xi^0,\xi^1), \Psi^A_\pm(\xi^\pm))\) in some complex space, probably, will be interesting, but lie outside the frameworks of this article. We include in our subsequent studies the string configurations \((X, \Psi)\) which give the positive-defined quadratic form \(\Omega_{ij}d\xi^i d\xi^j\) only. This demand means that two inequalities

\[
\pm \bar{\Psi}_\pm \Gamma^\mu_\xi \Psi_\pm \partial_\pm X_\mu > 0 \tag{4}
\]

hold for any point \((\xi^0,\xi^1)\). To destroy the gauge freedom (3) we select the string configurations \((X, \Psi)\) such that the conditions

\[
F_{2\pm}(\xi) \equiv \bar{\Psi}_\pm \Gamma^\mu_\xi \Psi_\pm \partial_\pm X_\mu = \pm \kappa^2 \tag{5}
\]

^2 not only first and second quadratic form of the world-sheet, as in the case of bosonic string
hold for any non-zero constant $\kappa = \kappa[X, \Psi]$. Note that the equalities (5) are invariant both under Poincaré and under scale transformations of the space-time $E_{1,3}$, so we assume that the resulting theory will be attractive. Such invariance is first point of the motivation for the conditions (5). Second point is to the gauge (5) generalizes naturally the well-known light-cone gauge in a string theory. We discuss this fact detailed in the end of section 4.

It should be stressed that the restriction

$$\kappa[X, \Psi] = q,$$

where $q$ is some fixed input parameter is not suitable for complete theory. Indeed, the different values of the constant $\kappa$ correspond to different orbit of the gauge transformations (3), so that $\kappa$ is Teichmüller-like parameter. Consequently, the ”strong“ restriction (6) will be not grounded because the gauge transformations (3) were forbidden by the ”weak“ conditions (5) (the discussion of this situation for general gauge systems can be found in the work [14]).

Note that the gauge (5) does not forbid the transformations (3) such that $A(\xi) \equiv \xi + c$, where $c = const$. Obviously, they give the shifts $\xi^0 \rightarrow \xi^0 + c$, which correspond to dynamics.

We are going to study the Poisson structure of the set $V$ of string configurations $(X(\xi^0, \xi^1), \Psi(\xi_{\pm}))$ which are selected by the constraints (2) and ”weak“ gauge conditions (5). It is non-trivial problem, because the variation $\delta(\kappa[X, \Psi])$ does not defined by the variations of the coordinates of original phase space. Let us introduce the auxiliary minimal subspace $H_1$ such that, firstly, the inclusion $V \subset H_1 \subset H$ holds and, secondly, the Poisson structure on $H_1$ is well-reduced from the original phase space $H$. Such subspace is given by the equalities

$$F_i^{(n)} = 0, \quad n \neq 0, \quad i = 1, 2.$$  \hspace{1cm} (7)

The constants $F_i^{(n)}$ are Fourier modes of $2\pi$-periodical functions

$$F_i(\xi) = \begin{cases} F_i^+(\xi), & \xi \in [0, \pi), \\ F_i^-(\xi), & \xi \in [-\pi, 0), \end{cases}$$
which are well-defined in accordance with the boundary conditions for the variables $X$ and $\Psi$. The canonical Poisson structure on original phase space $\mathcal{H}$ gives the following brackets:

$$\{F_{1}^{(n)}, F_{2}^{(n)}\} = 8\pi i\alpha' n F_{2}^{(0)}.$$ 

Because $F_{2}^{(0)} = \kappa^2/2 > 0$ in our theory, the system (7) will be second type system of constraints so that natural brackets on space $\mathcal{H}_{1}$ will be corresponding Dirac brackets $\{\cdot, \cdot\}_{1}$. The condition $F_{1}^{(0)} = 0$ gives the reduction on set $\mathcal{V}$; obviously, $\text{codim}\mathcal{V} = 1$. Analogously with the example, considered in the introduction, we can select the various foliations

$$\mathcal{V} = \bigcup_{q^2 > 0} \mathcal{V}_{q}^{f},$$

where the sets $\mathcal{V}_{q}^{f} \subset \mathcal{V}$ can be defined both by the restriction (6) and any more complicated conditions. As for the finite-dimensional case, any foliation (8) gives the Poisson structure on the set $\mathcal{V}$, which will be degenerated. Thus, the natural canonical structure of the original phase space $\mathcal{H}$ does not have the unique reduction to the set $\mathcal{V}$.

At first it seems that such indeterminacy can be ignored at the subsequent quantization. Indeed, we can quantize the brackets $\{\cdot, \cdot\}_{1}$ and construct the correspondent Fock space $\mathcal{H}_{1}$. After that we must select the physical vectors $|\psi\rangle \in \mathcal{H}_{1}$ which will be the solutions of the "Shrödinger equation" $F_{1}^{(0)} |\psi\rangle = 0 \ [\Pi]$. In our opinion, the ambiguity in determination of the Poisson structure of the manifold $\mathcal{V}$, which consists all physical information, leads to additional possibilities for quantization. Indeed, let any space $\mathcal{H}^{ad}$ be any Poisson manifold with the Poisson brackets $\{\cdot, \cdot\}^{0}$. Suppose that the finite number of some constraints $\Phi_{i}(\ldots) = 0$, $i = 1, \ldots, l$ give the first type system of constraints:

$$\{\Phi_{i}, \Phi_{j}\}^{0} = C_{ijk} \Phi_{k}.$$ 

Suppose, that for the surface of these constraints $\mathcal{W} \subset \mathcal{H}^{ad}$: $\mathcal{W} = \{M \in \mathcal{H}^{ad} \mid \Phi_{i} = 0, i = 1, \ldots, l\}$ the diffeomorfism $\mathcal{V} \approx \mathcal{W}$ takes place $\ [\exists].$

\[\text{this diffeomorfism must be conserved in the dynamics}\]
Thus, we have the following diagram:

\[
\mathcal{H}_1 \supset V \approx W \subset \mathcal{H}^{ad}.
\]  

We call any such space \( \mathcal{H}^{ad} \) the \textit{adjunct phase space}. From the classical viewpoint, the manifold \( W \) is tantamount to the manifold \( V \), because it has same information about the physical degrees of the freedom. As discussed above, the single-defined Poisson structure is absent both on the manifold \( V \) and the manifold \( W \), that is why there are no reasons prefer one to other. Consequently, we can fulfill the subsequent quantization of the theory in terms of the space \( \mathcal{H}^{ad} \). It should be stressed that \( \mathcal{H}_1 \not \approx \mathcal{H}^{ad} \) as the Poisson manifolds in general, even for case \( l = 1 \). This means that the results here can differ from the conventional ones. The simplest example of adjunct phase space will be the manifold \( D^* \mathcal{H}_1 \), where symbol \( D^* \) means any (not only Poisson) diffeomorphism. It is clear from the physical point of view. Indeed, if we have both physical and non-physical coordinates in some phase space, there are no reasons to consider only Poisson-conserved transformations of such space. The suggested definition of the space \( \mathcal{H}^{ad} \) is more general, of course.

The main goal of this article is construct the physically appropriate adjunct phase space for the dynamical system (1) and discuss the quantization. In our opinion, there are several reasons to refuse the description of string dynamics in term of the space \( \mathcal{H}_1 \) (or original phase space \( \mathcal{H} \) – it is equivalent): Firstly, the standard approach leads to the additional dimensions for space-time while the existence of such dimensions is not proven experimentally. Secondly, the conventional approaches (see, for example, [12]) lead as is well known to the linear Regge trajectories for free strings such that the slope \( \alpha' \) is input parameter in a theory. But the trajectories \( s = \alpha' \mu^2 + c \), where the value \( \alpha' \approx 0.9 \text{ Gev}^{-2} \) is the universal constant, describe the spectrum of real particles well but only approximately. Indeed, the linearity means that the width of any resonance is equal to zero; the universality of the slope \( \alpha' \) is connected with the absence of exotic particles [15]. In the meantime we have the stable experimental data on hadronic exotics now [16]. Some of them give direct
information about Regge trajectories with slopes $\alpha_g \neq 0.9 Gev^{-2}$ \cite{17}. As regards the form of the trajectory, the linear dependence gives a good approximation for light-flavoured mesons and baryons only (see, for example \cite{18}). Moreover, the width of any real resonance does not equal zero, of course.

As it seems, the construction of D4 free string model, while taking into account the small non-linearity of the trajectories and the existence of the different slopes, can be very interesting.

3. The world-sheet geometry.

We will define next the adjunct phase space $\mathcal{H}^{ad}$, the subset $\mathcal{W}$ and construct the corresponding diffeomorphism $\mathcal{V} \approx \mathcal{W}$ in accordance with diagram (9). The coordinates in the space $\mathcal{H}^{ad}$, which will be introduced in the following section, naturally fall into two groups: the finite number of ”external“ variables which are transformed as the tensors under the Lorentz transformations of the space-time $E_{1,3}$ and some ”internal“ scalar variables. In order to define these quantities, we consider in this section the geometrical construction which is quite natural for the studied model. Some parts of the section will be analogous to corresponding parts in the works \cite{19, 20}, so that we give some formulae without detail proof.

Let the constant $\kappa$ is the constant existing for given fields $X_{\mu}$ and $\Psi_{\pm}$ in accordance with the conditions (5). We introduce the tensors

$$ L_{\pm}^{\mu} = \frac{1}{\kappa} \bar{\Psi}_{\pm} \Gamma^{\mu} \Psi_{\pm}, \quad G_{\pm}^{\mu\nu} = \frac{i}{2\kappa} \bar{\Psi}_{\pm} (\Gamma^{\mu} \Gamma^{\nu} - \Gamma^{\nu} \Gamma^{\mu}) \Gamma^{\nu} \Psi_{\pm}, $$

which carry the full information about Majorana spinors $\Psi_{\pm}$ and satisfy the properties $L_{\mu} G^{\mu\nu} = 0$, $L^{\mu} L^{\nu} = G^{\mu\nu} G^{\rho\sigma}$. After that we define the pair of vectors $N_{\pm}^{\mu}$:

$$ N_{\pm}^{\mu} = \frac{1}{\kappa} \partial_{\pm} X^{\mu} + \frac{i\Theta}{2\kappa^2} \bar{\Psi}_{\pm} \partial_{\pm} \Psi_{\pm} L_{\pm}^{\mu}. $$

According to the equalities (2) and (5) these vectors are light-like and
satisfy the conditions
\[ L^\mu_\pm N^\mu_\pm = \pm \frac{1}{2}. \]  
(10)

Let us define eight vectors \( e^\mu_{\pm}, \) (\( \mu = 0, \ldots, 3 \)):
\[
\begin{align*}
(e^\mu_{0\pm})^\mu &= L^\mu_\pm \pm N^\mu_\pm; \\
(e^\mu_{3\pm})^\mu &= \pm L^\mu_\pm - N^\mu_\pm; \\
(e^\mu_{1\pm})^\mu &= \mp 2G^{\mu\nu}N_\pm^\nu; \\
(e^\mu_{2\pm})^\mu &= \varepsilon^{\mu\nu\lambda\rho} (e^\nu_3)_{\nu} (e^\lambda_0)_{\lambda} (e^\rho_1)_{\rho}.
\end{align*}
\]

The direct verification allows to state that these vectors give the pair of the orthonormal bases in space \( E_{1,3} \) for every point \((\xi^0, \xi^1)\) of the world-sheet. Further it is more convenient to deal with the vector-matrices
\[
E^\pm = e^\mu_\pm \sigma^\mu_\sigma, \quad (\sigma_0 = 1)
\]
instead the bases \( e^\mu_{\pm} \). So, we define \( SL(2, C) \)-valued chiral field \( K = K(\xi^0, \xi^1) \) by means of the formula
\[
E^- = KE^+K^+. \quad (11)
\]

The free equations of motion for original fields \( X^\mu \) and \( \Psi^\pm \) lead to the ”conservation laws“ \( \partial^\pm E^\mp = 0 \). As the consequence, we have the equation for chiral field \( K = K(\xi^0, \xi^1) \):
\[
\partial^- (K^{-1} \partial^+ K) = 0. \quad (12)
\]

It is special case for well-known Wess-Zumino-Novikov-Witten equation [21, 22]. The left and right currents
\[
Q^- = -(\partial^- K)K^{-1}, \quad Q^+ = K^{-1}(\partial^+ K)
\]
for the defined chiral field \( K \) satisfy the equations \( \partial^\pm Q^\pm = 0 \). As can be proven with help of the boundary conditions for original string variables \( X \) and \( \Psi \), the \( sl(2, C) \)-valued function
\[
Q(\xi) = \begin{cases} 
Q^+(\xi), & \xi \in [0, \pi], \\
-\sigma_1 Q^-(\xi)\sigma_1, & \xi \in [-\pi, 0], 
\end{cases}
\]
is continous and can be extended $2\pi$-periodically and continiously throughout the real axis.

Let us consider the following auxiliary linear system with $2\pi$-periodical coefficients:
\[
T'((\xi)) + Q(\xi)T((\xi)) = 0. \tag{13}
\]
This system plays central role for our subsequent considerations. This role is conditioned by the possibility of reconstruction of original string variables $\partial_{\pm}X_{\mu}$ and $\Psi_{\pm}$ through the matrix-solution $T((\xi))$ of the system (13).

Indeed, the chiral field $K((\xi^0,\xi^1))$ can be written in the form [22]:
\[
K((\xi^0,\xi^1)) = T_-(\xi_-)T_+^{-1}(\xi_+), \tag{14}
\]
for some matrices $T_\pm \in \text{SL}(2,C)$. Using the definition of the field $K$, we have the formulae
\[
E_\pm((\xi_\pm)) = T_\pm((\xi_\pm))E_0T_\pm^\dagger((\xi_\pm)), \tag{15}
\]
where $E_0 = t^\mu\sigma_\mu$ is the matrix representation of the stationary basis $t^\mu$. In accordance with the definition of matrix $Q$, we have the equalities
\[
T_+(\xi_+) = T(\xi_+); \quad T_-(\xi_-) = i\sigma_1T(-\xi_-),
\]
where $T((\xi))$ is matrix-solution of the system (13) such that condition $\det T = 1$ holds. So, we can reconstruct the vector-matrices $E_\pm((\xi_\pm))$ through the matrix $T((\xi))$. If the constant $\kappa$ is given, we can reconstruct the original string variables too. Thus the following one-to-one correspondence takes place:
\[
V/E_{1,3} \approx (T((\xi)),\kappa), \tag{16}
\]
where as $E_{1,3}$ we denote the group of the translations $X \to X + A$. The evident formulae for the reconstruction of original string variables can be
deduced analogously just as in the works \cite{20}. For example, we have for the matrices \( \partial_{\pm} \hat{X}(\xi_{\pm}) \equiv \partial_{\pm} X^\mu \sigma_\mu; \)

\[
\partial_{\pm} \hat{X}(\xi_{\pm}) = \pm T^+(\pm \xi_{\pm}) R(\pm \xi_{\pm}) T(\pm \xi_{\pm}),
\]

(17)

where the matrix \( R(\xi) = \text{diag}(\kappa, -2\Theta \text{Re} Q_{21}(\xi)) \). If we select the Weyl representation for \( \Gamma \)-matrices, the explicit expressions for reconstructed spinors are quite simple:

\[
\Psi_{\pm} = \sqrt{\kappa} \begin{pmatrix} \varphi_{\pm} \\ -\sigma_2 \varphi_{\pm}^* \end{pmatrix},
\]

(18)

where \( \varphi_{\pm}(\xi_{\pm}) = \begin{pmatrix} t_{21}(\pm \xi_{\pm}) \\ t_{22}(\pm \xi_{\pm}) \end{pmatrix} \) are the Weyl spinors which were expressed in terms of the elements \( t_{ij} \) of the matrix-solution \( T(\xi) \).

This section is finished by the following important statement \cite{19}. The boundary conditions for original variables \( X \) and \( \Psi \) were fulfilled, if the equality

\[
\mathcal{M}_1(Q) = \epsilon 1
\]

(19)

holds for monodromy matrix \( \mathcal{M}_1 \) of the system (13) defined in accordance with the equality \( T(\xi + 2\pi) = T(\xi) \mathcal{M}_1 \).

Thus we have as the result of this section the fact that the variables \( \partial_{\pm} X_\mu \) and \( \Psi_{\pm} \), constrained by the conditions (2) and (5), can be reconstructed through the matrix \( T \) – the matrix-solution of the linear \( 2\pi \)-periodical system (13); moreover it is need that the coefficients \( Q_{ij} \) of this system were constrained by equality (19).

4. The topological charge and the definition of space \( \mathcal{H}_1 \).

In this section we determine the adjunct phase space for the dynamical system (1). The starting point of our subsequent consideration is the

\footnote{It is important that resulting dependence from the "variable" \( \kappa \) differs here from the dependence in cited work}
correspondence (16). Note that the matrix-solution $T(\xi)$ is defined up to within the transformations

$$T(\xi) \rightarrow \tilde{T}(\xi) = T(\xi)B,$$

where constant matrix $B \in SL(2, C)$. It is clear from the formulae (17) and (18) that these transformations are the Lorentz transformations of space-time $E_{1,3}$. Thus we can write for every solution of the system (13):

$$T(\xi) = T_0(\xi)B_1(q_1, \ldots, q_6), \quad (20)$$

where the values $q_i$ parametrize the group $SL(2, C)$ somehow or other and the matrix $T_0$ is defined from the functions $Q_{ij}(\xi)$ by some unique manner. In order to give the correspondent definition of the matrix $T_0$, let us fulfill the Iwasawa expansion for the matrix-solution $T(\xi)$:

$$T = \mathcal{N}\mathcal{E}U,$$

where $U \in SU(2)$ and the matrices $\mathcal{E}$ and $\mathcal{N}$ are following

$$\mathcal{E} = \text{diag} \left( e^d, e^{-d} \right), \quad \mathcal{N} = \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix}.$$

After that we define the functions $j_a = j_a(\xi), \quad a = 1, 2, 3$:

$$j_a = -i \text{Tr} \sigma_a \left[ \mathcal{G}^{-1} Q \mathcal{G} + \mathcal{G}^{-1} \mathcal{G}' \right],$$

where $\mathcal{G} = \mathcal{N}\mathcal{E}$. Then the matrix $U$ satisfies the following linear system:

$$U' + \frac{i}{2} \left( \sum_{a=1}^{3} \sigma_a j_a \right) U = 0. \quad (21)$$

Because $U \in SU(2)$, the functions $j_a(\xi)$ are the real functions. It is more convenient to replace the function $d(\xi)$, which defines the matrix $\mathcal{E}$, with the function $j_0(\xi) \equiv d'(\xi)$ and the constant $d_0 = d(0)$.

We postulate the following six conditions to fix the matrix $T_0$:

$$\int_{0}^{2\pi} f(\xi) d\xi = 0, \quad d_0 = 0, \quad \mathcal{U}(0) = 1.$$
Thus we define four real \((j_a)\) and one complex \((f)\) function such that the correspondence \(Q \leftrightarrow (j_a; f)\) is one-to-one. Let us rewrite the condition (19) in terms of the introduced functions. So, the matrix \(T_0(\xi)\) will be \(2\pi\)-periodical if the functions \(f(\xi), j_0(\xi)\) are periodical and the equalities

\[
\int_0^{2\pi} j_0(\xi) d\xi = 0, \quad U(\xi + 2\pi) = \epsilon U(\xi)
\]

hold. The last equality means that the monodromy matrix \(\mathcal{M}\) for linear system (21) satisfies the condition

\[
\mathcal{M} = \epsilon 1.
\]

This constraint on the variables \(j_a\) leads to the appearance of the topological charge \(n\) in our model. Indeed, let us consider the spectral task

\[
U' + \frac{i\lambda}{2} \left( \sum_{a=1}^{3} \sigma_a j_a \right) U = 0.
\]

The condition (21) holds if and only if this task has a point \(\lambda_n = \lambda_n[j_a]\) of the periodical or antiperiodical spectrum such that \(\lambda_n = 1\) for certain number \(n\). The equivalent form of this condition is following:

\[
\Phi_m^m \equiv \arccos \left( \frac{1}{2} \text{Tr} \mathcal{M} \right) - \pi m = 0. \tag{22}
\]

Thus we state the one-to-one correspondence

\[
\mathbf{V}/E_{1,3} \approx (f(\xi), j_0(\xi), \ldots, j_3(\xi); q_1, \ldots, q_6; \kappa). \tag{23}
\]

The whole number \(n\) is the topological charge in our theory; the continuous deformation of the string configuration \((f(\xi), \ldots)\) for some \(n\) into the configuration \((f(\xi), \ldots)\) with other number \(m\) breaks either boundary conditions or gauge (5).

Our following step is to define six parameters \(q_i\) according to the representation (20). Moreover, we must to add four constants \(Z_\mu\) for the
reconstruction of the variables $X_\mu$ from the derivatives $\partial_{\pm}X_\mu$. Let us consider the usual Noether expressions for the energy-momentum $P_\mu$ and the moment $M_{\mu\nu}$:

$$P_\mu = \frac{1}{4\pi\alpha'} \int_{0}^{\pi} \dot{X}_\mu d\xi^1,$$

$$M_{\mu\nu} = \frac{1}{4\pi\alpha'} \int_{0}^{\pi} (X_\mu \dot{X}_\nu - X_\nu \dot{X}_\mu) d\xi^1 - \frac{i\Theta}{8\pi\alpha'} \sum_{\epsilon=\pm} \int_{0}^{\pi} \overline{\Psi}_\epsilon \left( \Gamma_{\mu} \Gamma_{\nu} - \Gamma_{\nu} \Gamma_{\mu} \right) \Psi d\xi^1.$$  

Let $w_\mu = -(1/2)\varepsilon_{\mu\nu\lambda\sigma} M^{\nu\lambda} P^\sigma$. In accordance with the formulae (17) and (18) we have the equalities:

$$(P)^2 = \left( \frac{\Theta}{4\pi\alpha'} \right)^2 \sum_{l=0}^{2} \left( \frac{\kappa}{\Theta} \right)^l D_l,$$  

$$(w)^2 = \frac{\Theta^6}{(4\pi\alpha')^4} \sum_{l=0}^{6} \left( \frac{\kappa}{\Theta} \right)^l F_l.$$  

It is important that the coefficients $D_l$ and $F_l$ in the polynomials (24) and (25) depend on the functions $f(\xi)$ and $j_\alpha(\xi)$ only. This fact means that these formulae give the $\kappa$-parametric form of "constraint"

$$\Phi_2(P^2, w^2; f, j_0, \ldots, j_3) = 0.$$  

The main idea is to use the components $P_\mu$ and $M_{\mu\nu}$ as an additional variables instead the constants $Z_\mu$, $q_i$ and $\kappa$. The exact statement is following.

**Corollary 1** Let two-parametric group $G_2$ was composed from the transformations:

1. rotations $X_\mu \rightarrow \Lambda_\mu^\nu(\phi)X_\nu$ in the space-like plane which is orthogonal with the vector $P_\mu$ and pseudo-vector $w_\mu$;

2. translations $X_\mu \rightarrow X_\mu + cP_\mu$. 

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Then, if the quantities \( f(\xi), j_a(\xi) \ (a = 0, \ldots, 3) \), \( P_\mu \) and \( M_{\mu\nu} \) are constrained by the equalities (22) and (26), the diffeomorphism

\[ V/G_2 \approx (f(\xi), j_0(\xi), \ldots, j_3(\xi); P_\mu, M_{\mu\nu}) \]

takes place.

The sketch of the proof is following. Let the auxiliary vector field \( X_{(0)} \) and the spinor fields \( \Psi_{(0)}^\pm \) were defined from the variables \( f(\xi) \) and \( j_a(\xi) \) with help of the formulae (17) and (18), where the replacement \( T(\xi) \to T_0(\xi) \) has been fulfilled. We define next the vector \( P_{(0)\mu} \) and pseudo-vector \( w_{(0)\mu} \) by means of the replacement \( X \to X_{(0)} \) and \( \Psi^\pm \to \Psi_{(0)}^\pm \) in the correspondent Noether expressions. Let \( P_\mu \) arbitrary time-like vector and \( M_{\mu\nu} \) arbitrary antisymmetrical tensor. Then, if the constraint (26) takes place, the matrix \( B \in SL(2, C) \) exists such that the equalities

\[ \hat{P} = B^+ \hat{P}_{(0)} B, \quad \hat{w} = B^+ \hat{w}_{(0)} B \]

hold. That is why we can reconstruct the matrix \( T = T_0 B \). Moreover, we can restore the integration constants \( Z_\mu \) because the full moment \( M_{\mu\nu} \) consists the information about center of mass of the string. Consequently, the original string variables \( X \) and \( \Psi^\pm \) can be restored from the variables \( f, j_a, P_\mu \) and \( M_{\mu\nu} \). More detail investigations show that this reconstruction will be smooth and two-parametric arbitrariness exists, so that the corresponding cosets appear.

To describe the degrees of the freedom connected with the group \( G_2 \), we introduce the additional coordinates \( q \) and \( \theta \), such that \( -\infty < q < \infty \) and \( \theta \in [0, 2\pi] \). Now we give the straightforward definition of the adjunct phase space \( \mathcal{H}^{ad} \) for the considered string model. This is manifold such that any point \( M \in \mathcal{H}^{ad} \) has the following coordinates: 1) \( 2\pi \)-periodical complex function \( f(\xi) \) without zero mode; 2) \( 2\pi \)-periodical real functions \( j_a(\xi) \ (a=0,1,2,3) \) such that the function \( j_0 \) has not zero mode; 3) 4-vector \( P_\mu \) such that the inequality \( P^2 > 0 \) holds; 4) antisymmetrical tensor \( M_{\mu\nu} \); 5) four additional coordinates \( q, \theta, p \) and \( \chi \). Let us define the Poisson
brackets
\[ \{ f(\xi), \mathcal{F}(\eta) \}^0 = \frac{\alpha'}{\Theta^2}\delta'(\xi - \eta), \quad \{ j_0(\xi), j_0(\eta) \}^0 = -2\frac{\alpha'}{\Theta^2}\delta'(\xi - \eta), \]
\[ \{ j_a(\xi), j_b(\eta) \}^0 = 2\frac{\alpha'}{\Theta^2} \left( -\delta_{ab}\delta'(\xi - \eta) + \varepsilon_{abc}j_c(\xi)\delta(\xi - \eta) \right) \]
(where \( a, b, c = 1, 2, 3 \) and also \( \delta(\xi) = \sum_n e^{in\xi} \)),
\[ \{ M_{\alpha\beta}, M_{\gamma\delta} \}^0 = g_{\alpha\delta}M_{\beta\gamma} + g_{\beta\gamma}M_{\alpha\delta} - g_{\alpha\gamma}M_{\beta\delta} - g_{\beta\delta}M_{\alpha\gamma}, \]
\[ \{ M_{\alpha\beta}, P_{\gamma} \}^0 = g_{\beta\gamma}P_{\alpha} - g_{\alpha\gamma}P_{\beta}, \]
\[ \{ p, q \}^0 = 1, \quad \{ \chi, \theta \}^0 = 1 \]
(The other possible brackets are equal to zero). With respect to defined brackets the space \( \mathcal{H}^{ad} \) is the Poisson manifold.

The manifold \( \mathbf{W} \) is defined as follows. As first we require, that the equalities
\[ \Phi_3 \equiv p = 0, \quad \Phi_4 \equiv \chi = 0 \]
hold. As \( \Phi^i_n \) we denote the constraint (22) for some topological number \( n \). Let the set \( \mathbf{W}_n \subset \mathcal{H}^{ad} \) be the surface of the constraints \( \Phi_i, \ i = 1, \ldots, 4, \) where \( \Phi_2 = \Phi^2_2 \). Then,
\[ \mathbf{W} = \bigcup_{n \in \mathbb{Z}} \mathbf{W}_n. \]

**Corollary 2** The constraints \( \Phi_i = 0, \ i = 1, \ldots, 4 \) will be first type constraints with respect to the brackets \( \{\cdot, \cdot\}^0 \).

Indeed, \( \{ \Phi_i, \Phi_j \} = 0 \) for \( i = 1, 2 \) and \( j = 3, 4 \), or \( i = 3 \) and \( j = 4 \). Let us prove that \( \{ \Phi_1, \Phi_2 \} \propto \Phi_1 \). We first note that the matrix \( \mathcal{M} \) depends on the variables \( j_a, \ a = 1, 2, 3 \) only. Let us calculate the brackets of the matrix elements of the matrices \( \mathcal{M} \) and \( Q_g = (i/2)\sum_a j_a\sigma_a \). The identity
\[ \left\{ (\mathcal{U}'(\xi) + Q_g(\xi)\mathcal{U}(\xi)) \otimes \mathcal{M} \right\}^0 \equiv 0 \]
holds on space $\mathcal{H}^{ad}$, therefore we can apply for such calculations the Leibniz rule \(\{AB, C\}^{0} = A\{B, C\}^{0} + \{A, C\}^{0}B\) and the definition of the matrix $\mathcal{M}$. As result we have the equality

\[
\left\{Q_{g}(\xi) \otimes \mathcal{M}\right\}^{0} = [1 \otimes \mathcal{M}, C(\xi)],
\]

where the square brackets denote the commutator $4 \times 4$ matrices. The explicit form of the matrix $C(\xi)$ does not important here because it is clear if $\mathcal{M} \propto 1$, than \(\{Q_{g}(\xi) \otimes \mathcal{M}\}^{0} \equiv 0\). Consequently, we have

\[
\{\Phi_{1}, A\}^{0} \propto \Phi_{1}
\]

for arbitrary function $A = A(f, j_{a}; P_{\mu}, M_{\mu\nu}; q, \theta)$, so that the Corollary is proven.

The dynamical equations

\[
\{H_{0}, X_{\mu}\}^{0} = \frac{\partial X_{\mu}}{\partial \xi^{0}}, \quad \{H_{0}, \Psi_{\pm}\}^{0} = \frac{\partial \Psi_{\pm}}{\partial \xi^{0}}
\]

hold for the hamiltonian

\[
H_{0} = \frac{\Theta^{2}}{2\pi \alpha'} \left( \int_{0}^{2\pi} |f(\xi)|^{2}d\xi + \frac{1}{4} \sum_{a=0}^{3} \int_{0}^{2\pi} j_{a}^{2}(\xi)d\xi \right).
\]

These formulae can be proven with help of the representation (17) and (18) for original string variables $X_{\mu}, \, \Psi_{\pm}$ and with help of the obvious equalities

\[
\{H_{0}, j_{a}\}^{0} = j_{a}', \quad \{H_{0}, f\}^{0} = f', \quad \{H_{0}, P_{\mu}\}^{0} = \{H_{0}, M_{\mu\nu}\}^{0} = 0.
\]

It can be verified directly that all constraints in our theory are co-ordinated with dynamics.

**Remark.** It is clear that the brackets of the variables $P_{\mu}$ and $M_{\mu\nu}$ are motivated by Poincaré algebra. Consequently, we have two annihilators here: $P_{\mu}P^{\mu}$ and $w_{\nu}w^{\nu}$. But every Poisson structure $\{\cdot, \cdot\}$ must be co-ordinated with the tensor property of all considered functions. So, for
instance, the equality \( \{ P_\mu, A_\nu \} = g_{\mu \nu} \) must holds for any 4-vector \( A_\mu \) in order to the dynamical variables \( P_\mu \) generate Poincaré translations. In our theory the integration of the formula (17) gives the expression for radius-vector \( X_\mu(\xi^0, \xi^1) : \)

\[
\hat{X}(\xi^0, \xi^1) = \hat{Z} + \frac{\xi^0 + q_0}{\pi} \hat{P} - \frac{i}{\pi} \sum_{n \neq 0} \hat{C}_n e^{in\xi^0} \cos n\xi^1,
\]

where \( \hat{C}_n = \int_0^{2\pi} T^\dagger(x) R(x) T(x) e^{-inx} dx \)

and \( Z_\mu = M_{\mu \nu} P^\nu / P^2 \). Therefore, we have the brackets

\[
\{ P_\mu, X_\nu \}^0 = g_{\mu \nu} - \frac{P_\mu P_\nu}{P^2},
\]

which are co-ordinated with the fact that function \( P^2 \) will be annulator. This means that for every constant 4-vector \( b_\mu \) the following formula takes place:

\[
e^{b_\mu \{ P_\mu, \ldots \}} X_\nu = X_\nu + b_\nu - \left( \frac{b_\rho P^\rho}{P^2} \right) P_\nu.
\]

Thus, with respect to the defined brackets, the variables \( P_\mu \) will generate the Poincaré translations on the correspondent cosets only. Same situation holds for the rotations, mentioned in the definition of the group \( G_2 \). The additional constraints \( \Phi_3 \) and \( \Phi_4 \) allow to reconstruct the correct co-ordination of the introduced Poisson brackets with translations and rotations. Indeed, let us consider Lie operator

\[
L^\mu(P) = \{ P^\mu, \ldots \} + \frac{P^\mu}{P^2} \{ \Phi_3, \ldots \}
\]

instead of the conventional operator of translation \( \{ P^\mu, \ldots \} \). In accordance with the definitions of the variable \( q \) and the constraint \( \Phi_3 \), the equality

\[
e^{\alpha_\mu L^\mu(P)} X_\nu = X_\nu + a_\nu
\]

holds. Analogously, Lie operator \( \{ M^{\mu \nu}, \ldots \} \) must be improved by means of adding the term with the operator \( \{ \Phi_4, \ldots \} \).
Let us discuss the quantization of the suggested model. We surmise that the structure of the fundamental Poisson brackets algebra $A_{cl}$ gives some information about the constructed space of the quantum states. In our model this algebra has the form

$$A_{cl} = A_{int} \oplus P,$$

where $A_{int}$ the Poisson brackets algebra of the "internal" variables $f(\xi), j_a(\xi)$ and $P$ is the Poincaré algebra. It should be emphasized that the energy-momentum and moment of the string (1) are independent fundamental variables, so there are no problems with the quantum ordering when we construct the quantum generators of Poincaré transformations.

The defined new variables are complicated functions from the original fields $X$ and $\Psi$, that is why the correct introduction of quantum fermionic fields is not so obvious here. The following proposition clarifies this question [20]

**Corollary 3** The equalities $\Psi_A^\pm(\xi) \equiv \text{const}$ hold if and only if the equalities $j_a(\xi) \equiv 0$ for $a = 0, \ldots, 3$ take place.

This statement means that, in spite of the complicated dependence of the variables $f$ and $j_a$ from the original variables $X_\mu$ and $\Psi$, the bosonic and fermionic degrees of the freedom are still non-mixed. It is natural to fulfill the quantization of the variables $j_a$ in terms of the fermionic fields with help of the bosonization procedure [22]. Thus the natural Hilbert space of the of the quantum states of the string will be following:

$$H = \bigoplus_{l,i,s} (H_b \otimes H_f \otimes H_{\mu^2,s}),$$

where the spaces $H_{\mu^2,s}$ are the spaces of irreducible representations of Poincaré algebra $P$, labeled by the eigenvalues of the Cazimir operators $P^\mu P_\mu$ and $w^\mu w_\nu; H_b$ - the Fock space of two-dimensional bosonic field in
the ”box” and $\mathbf{H}_f$ – the Fock space of two-dimensional fermionic field in the ”box”. The corresponding physical vectors of states must be selected with help of the ”Shrödinger equations”

$$\Phi_i \mid \psi_{phys} \rangle = 0,$$

where $\Phi_i$ are the quantum expressions for considered constraints.

The other consequence of this Corollary is that the suggested theory can be considered as the new spinning generalization of the standard bosonic string model with the light-cone gauge. Indeed, the standard light-cone gauge for bosonic string can be written in the form

$$n^\mu \partial_\pm X^\mu = \pm p_+/2,$$  \hspace{1cm} (27)

where light-like vector $n^\mu$ are selected usually as $(1, 0, 0, 1)$. In our case both spinors $\Psi_\pm$ are Majorana spinors in $D4$ space-time, so the vectors $n^\mu_\pm = \Psi_\pm \Gamma^\mu \Psi_\pm$ will be light-like always. The reduction $j_a \equiv 0$ means that these vectors are constant, moreover $n^\mu_+ = n^\mu_-$ in accordance with the usual boundary conditions for the spinor variables. Therefore, we have the theory with the gauge (27) where the light-like vector $n^\mu$ constant, but arbitrary. If we require $\Theta = 0$, the action (1) takes the standard bosonic form. The real and imaginary parts of the functions $f(\pm \xi_\pm)$ will be the (well-known) transversal components for vectors $\partial_\pm X$. With respect to the formulae (5)

$$\Omega_{ij} d\xi^i d\xi^j \propto \kappa^2 [(d\xi_+)^2 + (d\xi_-)^2],$$

so this form has a good limit when $\Theta \to 0$. In spite of this fact we assume that the two-metrics $\Omega_{ij}$ does not natural object for bosonic case $\Theta = 0$, because the spinor variables are absent here. This case was studied recently in the author’s work [23], where both classical and quantum version of the model investigated in detail. As result, we have Regge trajectories $\hbar \sqrt{s(s+1)} = \alpha_n \mu^2$, where the slopes $\alpha_n, \quad n = 1, 2, \ldots$ are the eigenvalues for some spectral task in the space of quantum states. The case $j_a \equiv 0$, but $\Theta \neq 0$ is quite similar technically, but it is more interesting, because leads to the more complicated trajectories.
Note that we can fulfill some unusual reduction \( f \equiv 0 \) in our model which corresponds to the string, where all bosonic degrees of the freedom are "frozen". Previous investigation of this case was made in the work [19], where the quantization was discussed too. This case more complicated because (nontrivial) topological condition (22). Author hopes to study the general quantum case in the future.

5. Concluding remarks.

In this paper we suggest new concept of adjunct phase space to investigate the open spinning string. It should be stressed that suggested approach leads to D4 covariant theory both in the classical and in the quantum cases. Main result is new non-trivial Regge spectrum which can be applied in our opinion to the description of the exotic particles. The dependence \( J = J(P^2) \), where the spin \( J = \sqrt{w^2/P^2} \), can be analysed already on classical level with help of the formulae (24) and (25). It will be essentially non-linear for small masses although for large \( P^2 \) we have the asymptotics \( J \propto P^2 + \mathcal{O}(\sqrt{P^2}) \).

Let us note that we have two fundamental constants in the theory: \( \alpha' \) and \( \Theta \). Because the spinor part of the action (1) vanishes on the equations of the motion, the constant \( \Theta \) can be introduced in the model not as the fixed constant but as the additional variable. The previous investigation of the theory with the original configuration space \( (X, \Psi; \Theta) \) instead of the space \( (X, \Psi) \) was fulfilled in the works [19, 20]. It should be stressed that such extension leads to the scale-invariant theory if we define the scale transformations as \( (X, \Psi; \Theta) \rightarrow (aX, \sqrt{a} \Psi; a\Theta) \). As a natural result, here the linear dependence \( J \equiv \sqrt{s(s+1)} = \alpha_n \mu^2 \) was deduced: we have the set of Regge trajectories with zero intercepts but with various slopes \( \alpha_n \). We consider the value \( \Theta \) as the constant but not as the variable in this work. Because the scale invariance is broken in this case the resulting Regge spectrum is more complicated than the spectrum in the articles [19, 20]. Note that the models of bosonic strings with non-standard spectrum were suggested last time in the works [24, 23].
It is known that the slope $\alpha'$ of the Regge trajectory can be connected with the tension $\tau$ of the string: $\alpha' \propto 1/\tau$. If the tension is constant and there are no other internal forces, the slope will be constant too. Thus the complicated form of the Regge trajectory bears a relation, probably, to some additional internal forces within the string. Note that the models, where spinning degrees of the freedom were connected with distributed charges and currents, were investigated early (see, for example, [26]). It appears that such interpretation is possible in our case too. The interesting moment here is that the model has topological charge $n$ which vanishes if the fermionic variables disappear.

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