Optimal Rate of Printing of 3D Printed Concrete Columns and Walls to Avoid Buckling

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Abstract. Extrusion based construction of concrete structures has been identified as one of the alternative construction technologies which reduces the construction costs, delays in construction and the material consumption. The most attractive feature of 3D printed constructions is the ease of developing intricate forms and shapes (which otherwise require skilled manpower). Since this is a relatively new technology, not much research has been done with regard to its structural performance and stability. One of the major concerns is the possibility of buckling (under its self-weight) of the column or wall during the extrusion process. This sets a limitation of the acceptable height, thickness and rate of extrusion process. In this study simple ways (second order analysis using matrix methods) have been explored to determine the critical height and the optimal rate of extrusion. Matrix methods have inherent limitations due to the simplification adopted by ignoring the higher-order terms in Taylor’s series. The study uses the evolution of modulus of elasticity of concrete as proposed in literature. The analysis method has been validated using pertinent buckling tests reported in literature. Parameters such as modulus of elasticity and length are considered to determine the critical height to thickness of the wall and the optimal rate of extrusion. The problem is solved using the MATLAB software. This method requires minimal input from the user such as material properties, geometry and printing speed. The analysis method used in this study provides a useful design tool to estimate the limiting parameters of the structural walls and columns manufactured through the process of 3D printing.

1. Introduction

Additive manufacturing is used in various disciplines such as medical devices, aerospace, automotive, bio-printing and construction. The research going in the field of 3D printing has increased drastically due to its benefits to the community [1]. The additive manufacturing of concrete structures is now gaining momentum and acceptance for the construction of full scale structures [2]. This method of construction can be especially useful for low cost construction by reducing the material consumption and manpower requirement. One of the major challenges faced during the extrusion process is the possibility of buckling of walls or columns. This happens because the concrete is still fresh and has not gained adequate strength. Tests have shown that when more layers are added to the member, at a relatively fast rate, the whole wall can buckle [3]. Hence it is of interest to know the critical height of the element, for a given thickness and printing speed to ensure buckling can be prevented.

Additive manufacturing of concrete is a process in which desired structure is built in layers of suitable concrete paste which is forced through a tube. This tube is guided by a machine that takes the design details from the user. Various studies have been done to get the structural properties of the 3D printed concrete using finite element analysis and differential equations, which are computationally expensive [3]. In this study, the use of stiffness matrix methods to estimate the buckling height of the 3D printed elements is explored using the procedure described in the literature [4].
In the extrusion process, as every new layer is laid, the hydration process initiates and the concrete starts gaining strength. The strength gain is generally dependent on the quantity of accelerator added in the concrete paste. With strength, the modulus of elasticity ($E$), also increases. The evolution of $E$ with time can be defined using various functions: linear, exponential or quadratic. In the present study, where the buckling problem is solved, the modulus of elasticity is considered as the input. It is reasonable to assume that there is no variation of the stiffness properties along the length of the wall ($z$-direction) (see figure 1). This implies that the major variation of stiffness properties occurs along the vertical direction ($y$-direction), i.e. in the direction of extrusion. Therefore, the early age modulus of elasticity of concrete varies for each layer. This needs to be accounted for in the buckling calculations.

The presence of axial compression reduces the flexural stiffness of the element, and conversely, the presence of axial tension enhances the flexural stiffness. A critical condition of buckling instability is likely to occur when the flexural stiffness of the column or wall reduces to zero under axial compression [4]. The axial compression load is referred to as the critical buckling load, $P_{cr}$. This instability, in literature, is referred to as elastic instability. To estimate the critical buckling load of the member, a primary stiffness matrix in conjunction with a geometric stiffness matrix is employed. The total stiffness matrix for each element is shown in equation (1). The terms in the primary stiffness matrix can be derived using the slope-deflection method, while the terms in the geometric stiffness matrix can be derived using the stability functions. This is represented in figure 2.

![Figure 1. Schematic representation of a 3D printed wall.](image)

To make the problem convenient to solve it is assumed that the elements are adequately braced against out of plane and torsional modes of buckling. The bottom of the wall is assumed to be fixed (rigid support). The results of the study have been compared with the experiments published in the literature [4].

![Figure 2. Simplified stiffness coefficients (for small $\mu L$; axial compression).](image)
where, $k(i)$ is the total stiffness matrix of the $i^{th}$ element, $E(t,i)$ is the modulus of elasticity of the $i^{th}$ element at time $t$, $I(i)$ is the area moment of inertia of the $i^{th}$ element, $h(i)$ is the height of $i^{th}$ element, $p$ is the critical buckling load of the entire member and $w$ is the self-weight of each layer.

2. Methodology
An algorithm used to determine the critical buckling height of the wall is shown in figure 3. First, the geometric parameters such as thickness of wall ($t$), length of wall ($L$) and height of each layer ($h$) are decided. The material parameters for the concrete used in the 3D printing process such as the variation of modulus of elasticity of concrete with time $E(t)$, Poisson's ratio ($\nu$) and density of fresh concrete ($\rho$) are chosen. Further, the rate of printing speed ($\nu$) is also decided based on concrete rheology and printer capability. With this as the input, the analysis is carried out for the $i^{th}$ layer and the critical buckling load ($P_{cr,i}$) is evaluated. This is checked against the self-weight of the element ($P_{sw}$) and verified whether $P_{cr,i} < P_{sw}$. If the condition fails, the program is executed for the next layer, by suitably modifying the parameters based on the elastic analysis. Once again, for the two layers, the $E$ is evaluated and used in the elemental stiffness matrix to determine the $P_{cr,i}$. This exercise is carried out till the critical buckling height is reached. Figure 4 shows a schematic representation of the layer creating in the numerical model.
The steps involved in assessing the critical height of the member are:

a. Assigning input values of the code such as length of the wall (L), thickness (t), height (h), density of the concrete (or any material) (\( \rho \)), initial modulus of elasticity (E), curing rate of concrete (\( \nu \)), velocity of the printer head (v).

b. Weight of each layer is calculated using the given dimensions of the section and density of the material.

c. Based on time taken to print a layer (from printer head velocity and length of the wall) variation of modulus of elasticity with time is determined with the help of specified curing function and curing rate constant.

d. Deformation of each layer due to the self-weight of concrete layers above the layer is determined by including the Poisson’s effect.

e. After computing all the deformed dimensions, the stiffness matrix of every layer is calculated and combined to get a global stiffness matrix.

f. The determinant of the global stiffness matrix is equated to zero and if the minimum value of the load carried by the wall is greater than zero we add/print next layer and run the loop again and if it is less than zero the wall buckles by its self-weight and previous layer is assumed to be the critical buckling height for the given initial conditions.

\[
\begin{align*}
\text{Figure 4. (a) Schematic representation of the critical height evaluation for each layer, (b) } P_{cr} - P_{sw} \text{ values for each layer.}
\end{align*}
\]

Figure 4(a) shows the creating of each layer in the numerical program to evaluate the buckling load of the element. In figure 4(b) it is assumed that the self-weight of each layer is acting at the bottom of every layer of the member, therefore the top layer also carries just compressive load and the bottom-most layer carries the self-weight of the member and the additional compressive load. When the critical load carried by the wall is less than or equal to the self-weight of the wall then it is assumed that the member has failed due to self-buckling.

The limitations of the numerical program are:

a. The viscous effects of the concrete paste are not considered
b. The modulus of elasticity of concrete is assumed to be a constant for the entire length of the layer which is clearly not the case for long walls
c. The slip between layers is ignored
d. Poisson deformation is assumed to be uniform on all the sides
e. The curing function of concrete needs to suitably simulated to get numerical values close to the experimental data.

3. Validation using test reported in the literature

Wolf and Suiker (2019) reported tests carried out on 3D printed walls which failed during the extrusion process with a buckling mode of failure. In this paper, the proposed numerical approach is adopted to evaluate the buckling height of the tested walls. Figure 5 shows the comparison of the numerical results with the experimental results of 1m, 5m and 10.4m long walls. It is seen that for all the three layers the numerical results under-estimate the buckling height. This discrepancy could be attributable to the approximation made in the modulus of elasticity and Poisson’s ratio of fresh concrete. Further, the viscous effects of fresh concrete are ignored. The deviation is further pronounced in the 10.4m length
wall and this could be attributable to the fact that the numerical program is capable of handling only the instability due to height and not due to length. The 10.4m length wall buckled due to the length instability. Further, the numerical program assumes a constant value of modulus of elasticity of concrete for the entire layer at a given time instant. However, for longer walls, this is a highly simplified approach.

4. Parametric study

In order to establish the critical buckling height of the wall for a given thickness and rate of printing, a parametric study is carried out by adopting three different rates of printing (10mm/s, 20mm/s and 100mm/s), for two different lengths: 1m and 5m. The height of each layer is assumed to be 9.5mm. The evolution of the modulus of elasticity is assumed using a function described in equation (2).

\[
E(t) = E_0(1 + \chi t)
\]

where \(E(t)\) is the modulus of elasticity at any time \(t\) in seconds, \(E_0\) is the initial modulus value and \(\chi\) is the rate function (here assumed to be \(7.2 \times 10^{-4}\)).

In the present study, a simple rectangular section is considered, and the moment of inertia is calculated for each layer. However, in practice, more complex/optimized cross-sections are possible. The proposed numerical method is capable of solving the buckling problem for any cross-section geometry.

Figure 6 and 7 shows the critical buckling height versus thickness plots for three different rates of printing for walls having lengths of 1m and 5m respectively. As expected, it is seen from both figures 6 and 7, that as the thickness of the wall increases, the critical self-buckling height also increases. The effect of the slow speed of printing versus the fast rate of printing is evident. It is seen that for a given thickness, slow rates of printing, result in higher critical self-buckling height.

A clear distinction between the three rates of printing is evident in the 5m long wall (as seen in figure 7). This is due to the fact that time taken to print each layer increases with the length of the wall. This means that there is a significant difference in the modulus of elasticity of the subsequent layers in the member.

![Figure 6. Self-buckling height of 1m long wall versus thickness for different rates of printing.](image-url)
4.1. Optimal rate of printing
A critical choice in the 3D printing process is the optimal rate of printing. Slower printing enables us to build taller members but causes time delays. Hence, based on the practical parameters governing the concrete mixture properties and the printer capability, the optimal rate of printing can be determined. Based on the parametric study carried out, for the given variation of modulus of elasticity with time, it is seen that the rate of printing has negligible influence for a 1m long wall. However, for 5m long wall, there is a significant difference.

4.2. Optimal thickness of the wall
As indicated above, the geometrical parameters (length and thickness of the wall) and the rate of printing dictate the self-buckling height of the wall during the printing process. If the rate of printing and the length of the wall are predetermined the only choice left to the designer is to alter the thickness of the wall. It is interesting to know what is the optimal thickness which needs to be used for the wall to avoid self-buckling. The present study shows the procedure using which this can be determined.

5. Conclusions
The following conclusions can be drawn from this study:
- A numerical procedure to evaluate the critical buckling height of a wall during its 3D printing process is developed using matrix methods of analysis.
- The proposed analytical procedure is validated using tests reported in the literature.
- It is seen that for the choice of the evolution of modulus of elasticity a function used in the present study, there is no significant difference in the critical buckling heights for different rates of printing for 1m long wall. However, there is a significant difference in the 5m length wall.
- Using the present study, an optimal rate of printing or an optimal thickness of wall can be determined to avoid buckling due to its self-weight during the extrusion process.
- This procedure can be extended to any cross-section of the wall or column element.

More studies are needed to ascertain the correct behaviour of the wall, by including the effects of viscosity of fresh concrete and realistic modulus of elasticity of fresh concrete.

6. References
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