Abstract

We study the monopole oscillation in the bose-fermi mixed condensed system by performing the time-dependent Gross-Pitaevsky (GP) and Vlasov equations. We find that the big damping exists for the fermion oscillation in the mixed system even at zero temperature.

It is one of the most excited theme in recent physics to study the time-dependent dynamical motions of trapped atoms under the existence of the Bose-Einstein condensates (BEC) [1, 2], such as the collective motions [3], quantum vortices [4] and atomic novae [5]. In the view of the theoretical research, these phenomena are very important also to construct and to examine the transport theory in finite many body system. In fact many atomic system must give good probes for such study because the fundamental interaction is clear and weak.

In this work we construct the transport model including the condensed bosons and fermions, and as a first step study the collective monopole motion.

Here we briefly explain our formalism. First we define the Hamiltonian for boson-fermion coexistent system as follows.

\[ H = H_B + H_F + H_{BF} \]  

with

\[ H_B = \int d^3x \{ -\frac{1}{2} \phi^\dagger(\mathbf{x}) \nabla^2 \phi(\mathbf{x}) + \frac{1}{2} x^2 \phi^\dagger(\mathbf{x}) \phi(\mathbf{x}) + \frac{g_B}{2} (\phi^\dagger \phi)^2 \}, \]  

\[ H_F = \int d^3x \{ -\frac{\hbar^2}{2m_f} \psi^\dagger \nabla^2 \psi + \frac{1}{2} m_f \omega_f^2 \psi^\dagger \psi \}, \]  

\[ H_{BF} = \hbar_{BF} \int d^3x (\phi^\dagger \psi^\dagger \psi), \]

where \( \phi \) and \( \psi \) are boson and fermion fields, respectively. Fermion mass \( m_f \) and the trapped frequency \( \omega_f \) are normalized with boson fields, respectively. Fermion mass \( m_f \) and the trapped frequency \( \omega_f \) are normalized with boson mass \( M_B \) and the boson trapped frequency \( \Omega_B \), respectively. The
coordinates are normalized by \( \xi_B = (\hbar/M_B \Omega_B)^{1/2} \). The coupling constants \( g_B \) and \( h_{BF} \) are given as

\[
g_B = 4 \pi a_{BB} \xi_B^{-1}, \\
h_{BF} = 2 \pi a_{BF} \xi_B^{-1}(1 + m_f^{-1}),
\]

where \( a_{BB} \) and \( a_{BF} \) are the scattering lengths between two bosons and between the boson and the fermion, respectively.

The total wave function \( |\Phi(\tau)\rangle \) has \( N_c \) condensed bosons, whose wave function \( \phi_c \) is defined as

\[
\phi_c(x, \tau) = \langle \Phi|\phi_B(x, \tau)|\Phi \rangle,
\]

where \( \tau \) is the time coordinate normalized with \( \Omega_B^{-1} \). In this work the wave function \( \phi_c \) is expanded with the harmonic oscillator wave function \( u_n(x) \) as

\[
\phi_c(x, \tau) = \sum_{n=0}^{N_{base}} A_n e^{i \theta_n} u_n(bx^2)e^{-\frac{1}{2} \nu x^2},
\]

where \( N_{base} + 1 \) is the number of the harmonic oscillator bases. We define the Lagrangian with the collective coordinates as

\[
L(A_n, \theta_n, b, \nu) = \langle \Phi(\tau) | \{ i \frac{\partial}{\partial \tau} - H \} | \Phi(\tau) \rangle.
\]

Instead of solving the time-dependent GP equation directly, we take \( A_n, \theta_n, b, \nu \) as time-dependent variables and solve the Euler-Lagrange equations with respect to these variables.

The many fermion system can be described with the Thomas-Fermi approximation, which is given in the limit \( \bar{\hbar} \to 0 \). Here we define the phase-space distribution function as

\[
f(x, p, \tau) = \int d^3z < \Phi|\psi(x + \frac{1}{2}z, \tau)\psi^\dagger(x - \frac{1}{2}z, \tau)|\Phi \rangle e^{-ipz}.
\]

In this classical limit this phase-space distribution function satisfies the following Vlasov equation:

\[
\frac{d}{d\tau} f(x, p, \tau) = \{ \frac{\partial}{\partial \tau} + \frac{p}{m_f} \nabla_x - [\nabla_x U_F(x)][\nabla_p] \} f(x, p; \tau) = 0.
\]

In the actual calculation we solve the above Vlasov equation with the test particle method with the test particle method.

Thus we can get the time-evolution of the condensed bosons and the fermions by solving the time-dependent Gross-Pitaevskii equation and the Vlasov equation. As a first step we apply this method to the monopole vibration.

We calculate the monopole vibration of the system \(^{39}\text{K}-^{40}\text{K}\); the number of the bosons \(^{39}\text{K}\) and the fermions \(^{40}\text{K}\) are taken to be 100,000 and 1,000, respectively. The trapped frequencies are taken as \( \Omega_B = 100 \) (Hz) and \( \omega_f = 1 \). Furthermore we use the interaction parameters as \( a_{BB} = 4.22(\text{nm}) \) and \( a_{BF} = 2.51(\text{nm}) \).

Using the root-mean-square radius (RMSR) \( R \), we here define

\[
\Delta r(\tau) = R(\tau)/R_0 - 1,
\]

where \( R_0 \) is the RMSR of the ground state, and \( \tau \) is the time variable normalized by \( \Omega_B^{-1} \). In Fig. 1 we show the time-dependence of this observable for bosons \( \Delta r_B \) and fermions \( \Delta r_F \). In this calculation the initial condition is taken to be \( \Delta r_B = 0 \) and \( \Delta r_F = -0.1 \).
Fig. 1: Time evolution of $\Delta r$ for boson (a) and fermion (b).

We see a fast damping in the fermion oscillation. The bosonic oscillation, too, is not harmonic. The latter should originate from the bose-fermi coupling, as otherwise the bosonic one is almost damping free at zero temperature. [9].

In order to study this phenomenon, further, we examine the spectrum which is obtained by the Fourier transformation

$$F(\omega) = A_C | \int d\tau R^2(\tau) e^{i\omega \tau} |^2,$$

(13)

where $A_C$ is a normalization factor. In Fig. 2 we give two typical results for the spectra obtained by integrating over the regions $0 < \tau < 60$ (Fig. 2a and 2b) and over $100 < \tau < 400$ (Fig. 2c and 2d).

In Fig. 2a we see that the boson vibration mainly has two large peaks at the frequency $\omega = 1.91$ and 2.21 in the early time stage. In this time region the fermion oscillation has one peak at $\omega = 1.91$ with broad width (Fig. 2b).

Then we can consider the phenomena in the early time stage as follows. At first stage only the fermions moves, and this fermion oscillation triggers the boson vibration. The intrinsic frequencies of the boson and fermion oscillations are $\omega \approx 2.2$ and 1.9, respectively. Thus the motion of the bosons corresponds to the forced vibration, and the beat appears in the bosonic vibration. Furthermore the boson motion scatters fermions, so that bosons mediate fermion-fermion interaction and causes a damping in the fermion vibration.
In later time stage the strength of the boson vibration distributes to higher frequency modes while the strength for the fermion almost concentrates to one mode with $\omega = 1.91$. In addition the beat of the boson vibration disappears in this later time stage (Fig. 1a).

![Graphs](image)

**Fig. 2:** Spectra of the boson oscillation in upper columns (a,c) and fermion oscillation in down columns (b,d). The spectra deduced from the evolution in $0 < \tau < 60$ are shown in the left columns and that in $100 < \tau < 400$ are in the right columns.

The behavior of this oscillation is never repetitive. In the actual calculation the fermion energy $< H_F + H_{BF} >$ does not change in the time evolution process. The oscillation energy of the fermions is turned to another kind of energy, which might be the thermal energy. We will investigate this possibility more in future.

**References**

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