Theoretical modeling of the vertical stiffness of a rolling lobe air spring

Liufeng Xu
School of Traffic and Transportation Engineering, Central South University, Changsha, China

Abstract
In order to study the characteristics of a rolling lobe air spring, a vertical stiffness analytical model is constructed based on thermodynamics and hydrodynamics. The merit of this vertical stiffness analytical model is that an analytical solution of geometric parameters is obtained by an approximate analytic method. Meanwhile, experimental tests are carried out to verify the accuracy of the vertical stiffness analytical model. The vertical stiffness analytical model can be used to qualitatively analyze the influence of geometric parameters on the vertical stiffness characteristics of a rolling lobe air spring. Therefore, the relationship between geometric parameters and the vertical stiffness characteristics is analyzed based on the proposed model. The conclusions show that the vertical stiffness analytical model can well predict the mechanical characteristics of a rolling lobe air spring and provide guidance for parameter design and vehicle ride comfort improvement.

Keywords
Rolling lobe air spring, vertical stiffness, analytical model, geometric parameters, approximate analytical method

Introduction
Rolling lobe air springs have significant advantages, attenuating the harmful vibrations and improving the vehicle’s safety and ride comfort. The rolling lobe air spring as a critical component for the secondary suspension system has been widely used in mainline railway vehicles, high-speed trains, and commercial vehicles.1 The rolling lobe air spring is located between the car body and bogie frame, and its structure is simple, as shown in Figure 1. It consists of a top plate, a rubber diaphragm, an emergency spring, and a bottom plate. The vertical stiffness and
damping characteristics of a rolling lobe air spring exhibit strong nonlinearity. Therefore, establishing an accurate model is the key to study its stiffness and damping characteristics and a vehicle’s mechanical properties.

In recent decades, extensive research has been done by many scholars on the air spring stiffness modeling and vehicle dynamics analysis. Quaglia and Sorli\(^2\) and Quaglia and Guala\(^3\) proposed a dimensionless linear model for pneumatic suspension using hydrodynamics, and some design considerations were offered to analyze the effect of parameters on the stiffness characteristics. Liu and Lee\(^4\) derived a theoretical model to investigate the dynamic stiffness and overall equivalent damping of an air spring using energy conservation and gas state equations. Docquier et al.\(^5\) presented a multidisciplinary modeling approach for the bellows–pipe–tank subsystem of railway pneumatic suspension. Nakajima et al.\(^6\) developed a nonlinear model of the air spring system based on thermodynamics, and the coupling effect of the leveling valves and differential pressure valves was considered. Zhu et al.\(^7,8\) developed an air spring vertical dynamic model to predict the dynamic characteristics of air springs, in which the thermodynamics of the bellows–pipe–tank pneumatic subsystem, effective friction, and viscoelastic damping of bellows rubber were taken into account. Facchinetti et al.\(^9\) developed two different modeling approaches for air spring suspension, a quasi-static one based upon mechanics analysis and a dynamic one based on thermodynamics. Wang and Zhu\(^10\) derived a dynamic stiffness model of an air spring based on hydrodynamics and thermodynamics. Li and Li\(^11\) and Li et al.\(^12\) deduced an analytical solution to express the vertical stiffness of air springs, and the influence of geometric parameters on vertical stiffness was analyzed in terms of this analytical solution. Chen et al.\(^13\) refined a new bellows model to describe the vertical stiffness of a convoluted air spring, and the structural parameters are processed using a geometrical analysis approach. Wenku et al.\(^14\) built a finite element model (FEM) of an air spring to study its static elastic characteristics, and the impact of geometric parameters on the elastic characteristics was studied using a sensitivity analysis. Alonso et al.\(^15\) constructed a test bench to describe the characteristics of the air spring system, and the relationship between the geometric parameters and ride comfort was explored. Moheyeldein et al.\(^16\) proposed a new
dynamical model incorporating the damping characteristics of an air spring to study the performance of the air spring. Mazzola and Berg\textsuperscript{17} defined an accurate component model of an air spring based on a combination of laboratory tests and model identification techniques. Jingyue et al.\textsuperscript{18} obtained a nonlinear air spring polynomial model based on the polynomial and radial basis curve fitting method.

These studies showed that there are mature research results on the vertical stiffness modeling for air springs, but studies on the vertical stiffness modeling of rolling lobe air springs are less. Furthermore, the geometric parameters were available from manufactures and experimental tests, which is unacceptable for the qualitative analysis of air spring stiffness characteristics. Therefore, a novel vertical stiffness analytical model of a rolling lobe air spring is constructed based on thermodynamics and hydrodynamics in this article, and its geometric parameters such as the effective area, the equivalent volume, and their derivative terms are deduced using an approximate analytical method.

The layout of this article is as follows. First, the motion model of a rolling lobe air spring is analyzed, and a vertical stiffness analytical model is established based on the thermodynamic theory, hydrodynamic theory, and approximate analytical method. Second, experimental tests are carried out to verify the proposed vertical stiffness analytical model. Third, the impact of geometric parameters on vertical stiffness is analyzed using a sensitivity analysis approach, and also the influence of geometric parameters on vehicle ride comfort is evaluated using the Sperling comfort index. Finally, the conclusion is presented showing that the vertical stiffness analytical model can accurately predict the mechanical characteristics of a rolling lobe air spring, and a guideline for the design of these springs and an improvement in vehicle ride comfort is obtained.

\section*{Modeling development}

From the perspective of control strategy design, the rolling lobe air spring is simplified as a variable stiffness spring and a damper, and its single-degree-of-freedom vertical motion model is shown in Figure 2.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{rolling_lobe_air_spring.png}
\caption{Single-degree-of-freedom model of a rolling lobe air spring.}
\end{figure}
According to Figure 2, the motion equation of the rolling lobe air spring can be expressed as

\[ m\ddot{X} + mg + C(\dot{X} - \dot{X}_0) - F_z = 0 \]  

(1)

where \( m \) is the mass of a quarter car, \( C \) is the damping, \( X \) is the vibration displacement of the car body, \( X_0 \) is the excitation displacement, and \( F_z \) is the resilience of the spring, which represents the stiffness characteristics of the rolling lobe air spring.

In order to precisely reproduce the stiffness and damping characteristics of the rolling lobe air spring, a vertical stiffness analytical model is deduced based on thermodynamics and hydrodynamics, and its geometric parameters are obtained through an approximate analytical method.

**Vertical stiffness modeling**

The rolling lobe air spring is different from the typical bellow–pipe–tank pneumatic system, and its auxiliary chamber is linked to the rubber diaphragm via an orifice. Referring to the literature,\(^2,8\) the vertical stiffness model is established on the basis of hydrodynamics and thermodynamics.

The resilience of the air spring \( F_z \) can be calculated as follows

\[ F_z = A_{ef}(P_1 - P_a) \]  

(2)

where \( A_{ef} \) is the effective area of the rubber diaphragm, \( P_1 \) is the current pressure in the rubber diaphragm, and \( P_a \) is the atmospheric pressure.

The gas in the rubber diaphragm can be assumed as an ideal gas, and thus it satisfies the laws of thermodynamics, which can be expressed as follows

\[ P_1V_1 = \dot{m}_1GT_1 \]  

(3)

\[ P_1\left(\frac{V_1}{m_1}\right)^n = P_{10}\left(\frac{V_{10}}{m_{10}}\right)^n \]  

(4)

where \( V_1, m_1, \) and \( T_1 \) are the current volume, air flow mass, and temperature of the gas in the rubber diaphragm, respectively; \( G \) is the molar gas constant; \( n \) is the polytropic coefficient within the range of 1–1.4, which represents an isothermal process and an adiabatic process; and the subscript “0” indicates the initial value.

According to equation (4), the differential equation of the pressure in the rubber diaphragm \( P_1 \) can be expressed as

\[ \dot{P}_1 = \frac{nGT_{10}}{V_1} \dot{m}_1 - \frac{nP_{10}}{V_1} \dot{V}_1 \]  

(5)

The mass flow rate \( \dot{m}_1 \) in the diaphragm satisfies the hydrodynamics and can be obtained as
\[ \dot{m}_1 = q_{in} + q_{out} \]  

where \( q_{in} \) and \( q_{out} \) represent the gas flowing into and out of the diaphragm, respectively.

Differentiating equation (2) and then substituting equation (5) into it, the vertical stiffness of the rolling lobe air spring can be obtained as follows

\[
K_{dyn} = \frac{dF_z}{dx} = \left( P_1 - P_a \right) \frac{dA_{ef}}{dx} + A_{ef} \dot{m}_1 \frac{nGT_10}{V_1} - \frac{nA_{ef}P_1}{V_1} \frac{dV_1}{dx}
\]

where \( K_{dyn} \) is the vertical stiffness of the rolling lobe air spring, \( A_{ef} \) is the effective area of the rubber diaphragm, \( dV_1/dx \) is the rate of change of the rubber diaphragm volume, \( dA_{ef}/dx \) is the rate of change of effective area, and \( dx \) is the vertical vibration displacement.

From equation (7), the vertical stiffness \( K_{dyn} \) is closely related to the effective area \( A_{ef} \), the rubber diaphragm volume \( V_1 \), and their derivative terms \( dA_{ef}/dx \) and \( dV_1/dx \), respectively.

**Analytical modeling**

As mentioned above, the geometric parameters play a key role in the vertical stiffness of the rolling lobe air spring. In order to qualitatively analyze the relationship between the vertical stiffness characteristics and geometric parameters, an approximate analysis method is used to process the geometric parameters.

Under a small vertical deflection, the rubber diaphragm changes from the initial position (black solid line) to the current state (red solid line). The simplified diagram of the vertical deformation of a rolling lobe air spring is shown in Figure 3.12,13

The symbols in Figure 3 are defined as follows: \( dR, S, ds, dx' \), and \( dx'' \) are intermediate variables, \( \phi \) is the angle of the constraint of the top plate, which is defined as the angle between the symmetry axis and the constraint, and \( \theta \) and \( \theta + d\theta \) are

![Figure 3. Schematic diagram of the vertical deformation of a rolling lobe air spring.](image)
the angles of the diaphragm before and after deflection, respectively. Furthermore, in order to obtain an analytical solution of the geometric parameters, the diaphragm is assumed to be a circular arc, and the length of the arc is not changed during the process of vertical deformation.

According to Figure 3 and the assumptions, the geometric variable equations can be obtained by comparing the arc length before and after deflection

\[ S + dx'' + 2r\theta = S + ds + 2(r - dr)(\theta + d\theta) \]

\[ dx'' - ds = 2rd\theta - 2\theta dr \]

where \( ds \) can be derived by calculating the lengths of BC and BC’ in the triangles ABC and A’BC’, as follows

\[ ds = 2(r - dr)\sin[\pi - (\theta + d\theta)]\sin\left\{\frac{\pi}{2} - [\pi - (\theta + d\theta)]\right\} - 2r\sin(\pi - \theta)\sin\left\{\frac{\pi}{2} - (\pi - \theta)\right\} \]

\[ ds = \sin2\theta dr - 2r\cos2\theta d\theta \]

Combining equations (9) and (11), we obtain

\[ 2r(1 - \cos2\theta)d\theta + (\sin2\theta - 2\theta)dr = dx'' \]

Comparing the lengths of AC and AC’ in the triangles ABC and A’BC’, \( dx' \) can be derived as follows

\[ dx' = 2\sin^2\theta dr - 2r\sin2\theta d\theta \]

From the deformation in Figure 3, the following relationship can be obtained

\[ dx' = \sin\phi dx \]

\[ dx'' = \cos\phi dx \]

Substituting equations (13)–(15) into equation (12), the following equations can be derived

\[ dr = \frac{(1 - \cos2\theta)\sin\phi + \sin2\theta\cos\phi}{4(\sin^4\theta + \sin^2\theta\cos^2\theta - \theta\sin\theta\cos\theta)} dx \]

\[ d\theta = \frac{2\sin^2\theta\cos\phi - (\sin2\theta - 2\theta)\sin\phi}{8r(\sin^4\theta + \sin^2\theta\cos^2\theta - \theta\sin\theta\cos\theta)} dx \]

where \( dR \) can be calculated in the triangles OBD and O’BD’ as follows

\[ dR = 2r\sin(2\theta - \phi)d\theta + \cos(2\theta - \phi)dr \]
Substituting equations (16) and (17) into equation (18), \( dR \) can be expressed as follows

\[
dR = \frac{\sin(2\theta - \phi)[2\sin^2\theta \cos\phi - (\sin2\theta - 2\theta)\sin\phi] + \cos(2\theta - \phi)[1 - \cos2\theta] \sin\phi + \sin2\theta \cos\phi]}{4(\sin^4\theta + \sin^2\theta \cos^2\theta - \theta \sin\theta \cos\theta)} \, dx
\]

(19)

The effective area \( A_{\text{ef}} \) is defined as follows

\[
A_{\text{ef}} = \pi R^2
\]

(20)

The effective area after deflection can be expressed as follows

\[
dA_{\text{ef}} = \pi (R_0 + dR)^2 - \pi R_0^2 = 2 \frac{A_{\text{ef}}}{R_0} dR
\]

(21)

Combining equations (19) and (21), the rate of change of the effective area can be derived as

\[
\frac{dA_{\text{ef}}}{dx} = \alpha \frac{A_{\text{ef}}}{R_0}
\]

(22)

where

\[
\alpha = \frac{\sin(2\theta - \phi)[2\sin^2\theta \cos\phi - (\sin2\theta - 2\theta)\sin\phi] + \cos(2\theta - \phi)[1 - \cos2\theta] \sin\phi + \sin2\theta \cos\phi]}{2(\sin^4\theta + \sin^2\theta \cos^2\theta - \theta \sin\theta \cos\theta)}
\]

Under a small vertical displacement excitation, the change rate of the rubber diaphragm volume can be simply expressed as follows

\[
\frac{dV_1}{dx} = A_{\text{ef}}
\]

(23)

Substituting equations (22) and (23) into equation (7) yields the analytical formula for the vertical stiffness of the rolling lobe air spring

\[
K_{\text{dyn}} = (P_1 - P_a)\alpha \frac{A_{\text{ef}}}{R_0} + A_{\text{ef}} \frac{n G T_{10}}{V_1} - \frac{n P_1}{V_1} A_{\text{ef}}^2
\]

(24)

where \( \alpha \) is the shape coefficient of the rolling lobe air spring given by

\[
\alpha = \frac{\sin(2\theta - \phi)[2\sin^2\theta \cos\phi - (\sin2\theta - 2\theta)\sin\phi] + \cos(2\theta - \phi)[1 - \cos2\theta] \sin\phi + \sin2\theta \cos\phi]}{2(\sin^4\theta + \sin^2\theta \cos^2\theta - \theta \sin\theta \cos\theta)}
\]

Experiments and simulation

The rolling lobe air springs studied in this article were designed and manufactured by the CRRC Zhuzhou Locomotive Company, and performance experiments were also performed at this company. In this section, the simulation results were
compared with the experimental data to validate the accuracy of the proposed vertical stiffness analytical model.

**Experimental data**

The structure and working principle of the experimental setup have been described in detail by Li et al.\(^{12}\) and Li and Qi,\(^{1}\) as shown in Figure 4. A load sensor was mounted in the upper part of the setup to measure the vertical force. A vibration platform was positioned at the bottom of the setup to simulate the track excitation, and the auxiliary chamber was connected to the air spring through a pipe. The working range of the displacement sensor is 75 mm, the axial sensitivity is 500 ± 5\% mV/g, and the response frequency is 0.3–2500 Hz. The geometric parameters of a rolling lobe air spring used in the setup are listed in Table 1.

According to the TB/T 2841-2010 air spring for railway vehicles,\(^{19}\) the vertical preload is 50–125 kN, representing the empty load case AW0 and the overload case AW3. The vertical excitation displacement was simulated using a set of sinusoidal waves with different amplitudes and frequencies. Table 2 summarizes the main parameters of the vertical excitation.

The experimental tests were conducted under a harmonic displacement excitation with a frequency range of 0–20 Hz and an amplitude range of 2–10 mm. The experimental data are plotted in Figure 5. It can be seen that the vertical stiffness mainly depends on the excitation amplitude and decreases with the increase in the excitation amplitude. The vertical stiffness has a little relationship with the excitation frequency.

**Simulation and validation**

Figure 6 shows the simulation results of the vertical stiffness analytical model, where the vertical stiffness increases with the increase in pressure. Comparing the simulation results of Figure 6(a) and (b), it can be seen that the vertical stiffness displays a significant reduction of about 61\% with the increase in the auxiliary chamber volume.

Figure 7 shows the comparison of the experimental data and simulation results. It is clear that the simulation results of the proposed vertical stiffness analytical

### Table 1. Geometric parameters of a rolling lobe air spring.

| Symbol | Description                                      | Value       |
|--------|--------------------------------------------------|-------------|
| R      | Distance between the center of the circle and the axis of symmetry | 325 mm      |
| V\(_1\) | Diaphragm volume                                 | 32.5 L      |
| V\(_2\) | Auxiliary chamber volume                         | 50 L        |
| \(\phi\) | Constraint angle of the top plate                | 0.523       |
| \(\theta\) | Angle of the diaphragm                           | 2.38        |
model and the experimental data show the same trend. When the auxiliary chamber volume $V_2$ is 0 L, the simulation result of the vertical stiffness is in good agreement with the experimental data, as shown in Figure 7(a). There is a small deviation between the stiffness values, and the reason is that the vertical stiffness analytical model was derived based on the above two assumptions.

When the auxiliary chamber volume $V_2$ is 50 L, the deviation between the experimental data and the simulation results increases as the pressure increases, as shown in Figure 7(b). The reason is that the damping force caused by the air flow through the pipe was considered in the experimental data, but the maximum relative error is less than 3.5%. This indicates that the proposed vertical stiffness model can well characterize the stiffness characteristics.

**Analysis and discussion**

In this section, the impact of the geometric parameters on the vertical stiffness characteristics of the rolling lobe air spring is qualitatively analyzed by a sensitivity analysis method.

**Impact of the rubber diaphragm volume**

Figure 8 shows the influence of the rubber diaphragm volume on the vertical stiffness. In the case of the auxiliary chamber volume of 0 L, the vertical stiffness
decreases dramatically as the diaphragm volume increases. When the diaphragm volume exceeds 30 L, the vertical stiffness reaches a small value and changes slightly, as shown in Figure 8(a). When the auxiliary chamber volume is 50 L, as shown in Figure 8(b), the recession curve of vertical stiffness tends to be smooth as the chamber volume increases. The reason for this behavior is that the damping force generated by the air flow through the orifice effectively alleviates the attenuation of the vertical stiffness. The simulation results show that the optimal rubber diaphragm volume of a rolling lobe air spring is within a range of 20–40 L.

Figure 5. Experimental data: (a) the AW0 case and (b) the AW3 case.
Impact of the auxiliary chamber volume

One of the principal influences on the vertical stiffness of an air spring is the auxiliary chamber volume. It can be seen from Figure 9 that the vertical stiffness decreases with the increase in the auxiliary chamber volume. According to the discussion in Figure 8, better stiffness and damping performance can be obtained by designing an appropriate volume ratio of the auxiliary chamber and the rubber diaphragm, which can be derived from the following formula.
Impact of the shape coefficient

The shape coefficient $\alpha$ consists of the constraint angle of the top plate $\phi$ and the diaphragm angle $\theta$. Figure 10 shows the relationship between the shape coefficient $\alpha$, the constraint angle $\phi$, and the angle $\theta$. Under a synergistic effect of the constraint

$$V_2 > \frac{n P_1 R_0 A_{ef}}{\alpha (P_1 - P_a)} - V_1$$

(25)

Figure 7. Comparison results: (a) $V_2 = 0 \text{ L}$ and (b) $V_2 = 50 \text{ L}$. 
angle $\phi$ and the angle $\theta$, the shape coefficient $\alpha$ increases as the constraint angle $\phi$ increases and the angle $\theta$ decreases, as shown in Figure 10(a). It can be seen from Figure 10(b) that the shape coefficient $\alpha$ changes from positive to negative as the angle $\theta$ increases in the range of $0.5$–$3$ ($(\pi/4) - (2\pi/3)$). It can be seen from Figure 10(c) that the shape coefficient $\alpha$ first decreases and then increases as the constraint angle $\phi$ increases from $0$ to $1.6$ ($0 - \pi/2$), and there is a minimum value at the inflection point of $0.6411$ ($\pi/5$).
Figure 11 shows the relationship between the vertical stiffness and the shape coefficient $\alpha$, and it is clear that the vertical stiffness decreases from positive to negative with the increase in the shape coefficient, as shown in Figure 11(a). The reason is the negative shape coefficient $\alpha$ indicating that the air spring is compressed, so that the vertical stiffness is positive. On the contrary, it means that the air spring is stretched and the vertical stiffness is negative.

From Figure 11(b), it can be seen that the vertical stiffness increases as the angle $\theta$ increases. The vertical stiffness increases first and then decreases with the increase in the constraint angle $\phi$, and there is a maximum value at the extreme point 0.6411 ($\pi/5$), as shown in Figure 11(c). Therefore, according to the discussion, the reasonable ranges of the diaphragm angle $\theta$ and the constraint angle $\phi$ for a rolling lobe air spring are within $((\pi/4) - (2\pi/3))$ and $0 - (\pi/2)$, respectively.

### Ride comfort analysis

A set of optimal solutions of geometric parameters can be obtained through the vertical stiffness analytical model, and the influence of the geometric parameters on the vertical stiffness characteristics can be quantitatively evaluated by the vehicle comfort index. In this section, the Sperling comfort index is used to assess the influence of the geometric parameters before and after optimization on the vertical stiffness behaviors.

### Sperling comfort index

The Sperling comfort index is widely used as an international standard to evaluate the ride comfort of vehicles, which is expressed as
Figure 10. Synergistic relationship of constrain angle and diaphragm angle versus shape coefficient (a), shape coefficient versus diaphragm angle (b), shape coefficient versus constrain angle(c).
Figure 11. Vertical stiffness versus shape coefficient (a), vertical stiffness versus diaphragm angle (b), vertical stiffness versus constraining angle (c)
where \( a \) is the vibration acceleration of the car body, \( f \) is the vibration frequency, and \( F(f) \) is the correction coefficient related to the vibration frequency. The correction coefficient is shown in Table 3.

According to the Sperling standard, the ride comfort index is divided into four grades, as shown in Table 4.

### Vibration acceleration

According to the motion equation (1) and vertical stiffness formula (7), the vertical resilience of a rolling lobe air spring \( F_z \) can be expressed by the following formula

\[
W_z = 0.896 \sqrt{\frac{a^3}{f^3} F(f)}
\]  

(26)
Combining equations (24) and (27), the vibration acceleration of the car body $a$ can be derived as follows

$$ a = \ddot{x} = \frac{K_{dy} (x - x_0) - C (\dot{x} - \dot{x}_0)}{m - g} $$

Through the above analysis and discussion, the geometric parameters with optimization and no optimization are listed in Table 5.

**Figure 12.** Simulation results of car-body accelerations (a) and ride comfort index (b).
Substituting the data in Table 5 into equation (28), the magnitude of the car-
body acceleration is shown in Figure 12. The simulation results show that the accel-
eration amplitude obtained in the case of the optimized parameters is obviously
smaller than that in the case of nonoptimized ones, and also the ride comfort index
is smaller than that in the case of nonoptimized ones, as shown in Figure 12(b).

Conclusion

In this article, a vertical stiffness analytical model of a rolling lobe air spring is con-
structed based on hydrodynamics and thermodynamics. The main advance of the
vertical stiffness analytical model is that the geometric parameters are processed
with an approximate analytical method. The proposed vertical stiffness analytical
model is verified by comparing with the experimental data of the rolling lobe air
spring. Meanwhile, the influence of the rubber diaphragm volume, auxiliary cham-
ber volume, and shape coefficient on the vertical stiffness characteristics is analyzed
and discussed, and also the impact of the geometric parameters on the vertical stiff-
ness characteristics is evaluated by the Sperling comfort index; some valuable con-
clusions are drawn as follows:

1. The mechanical characteristics of a rolling lobe air spring can be well char-
acterized using the vertical stiffness analytical model.
2. The geometric parameters of a rolling lobe air spring can be deduced using
an approximate analysis method.
3. An appropriate volume ratio of the auxiliary chamber and the rubber dia-
phragm can be derived from equation (25).
4. A quick estimate of the shape coefficient of a rolling lobe air spring can be
obtained based on the vertical stiffness analytical model, and the reasonable
ranges for the angle of diaphragm \(\theta\) and the constraint angle \(\phi\) are within
\((\pi/4) - (2\pi/3))\) and \((0 - (\pi/2))\), respectively.
5. A set of optimized geometric parameters can be obtained using the sensitiv-
ity analysis approach.

In summary, the proposed vertical stiffness analytical model can accurately pre-
dict the mechanical characteristics of a rolling lobe air spring. Some conclusions
are obtained to provide guidance for parameter design and vehicle ride comfort
improvement.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, author-
ship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication
of this article.
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**Author biography**

Liufeng Xu, who received his Master’s degree from Guangxi University, and he is currently a Ph.D. Candidate in the School of Traffic and Transportation Engineering at Central South University. On the other hand, his research interests are vehicle dynamics analysis, mathematical modeling, vehicle equipment condition monitoring and fault diagnosis.