How much of the inflaton potential do we see?

Wessel Valkenburg

LAPTH, Université de Savoie & CNRS
9 chemin de Bellevue, BP110
F-74941 Annecy-le-Vieux Cedex
France
E-mail: wessel.valkenburg@lapp.in2p3.fr

We discuss the latest constraints on a Taylor-expanded scalar inflaton potential, obtained focusing on its observable part only. This is in contrast with other works in which an extrapolation of the potential is applied using the slow-roll hierarchy. We find significant differences. The results discussed here apply to a broader range of models, since no assumption about the invisible e-folds of inflation has to be made, thereby remaining conservative.

PACS: 98.80.Cq
1. Introduction

The cosmology of the early universe is hindered by the question about initial conditions. By inflating a region, so small that it does not sense a non flat universe, up to proportions at least larger than the presently observable universe, the paradigm of cosmic inflation wipes out any other initial conditions than an approximately spatially flat, homogeneous and isotropic universe [1]. At the same time it provides a mechanism for the generation of correlations in perturbations on all scales, observed today in the cosmic microwave background (CMB) and the large scale structure (LSS) [2].

In order to create the coherence on scales as large as the Hubble horizon today, inflation must last about 30 - 60 e-folds after the presently observed spectrum has left the Hubble horizon. The shape of the spectrum of these observed perturbations presently gives us the only hint on the physics that can have driven inflation. The observations alone though, give no information on inflation after the creation of the observed spectrum. Many previous works have used the formalism of flow equations [3] to extrapolate beyond the observed primordial power spectrum, in order to select only those spectra that correspond to a scalar-field potential that drives inflation long enough [4 - 6]. This extrapolation puts strong bounds on the shape of the potentials [6]. However, no work has focused on the shape of the potential within only the observable window, using present-day data.

In a recent work we presented new bounds on the shape of the inflaton potential within the observable window only, thereby remaining conservative about what (humble or exotic) mechanism drives inflation in the subsequent e-folds [7]. The results were obtained by matching a Taylor-expanded smooth scalar-field potential directly to the data numerically (the data being WMAP1 3-year data and the SDSS-LRG2 sample). These results apply to any theory, that within the observable window effectively has one scalar degree of freedom (one inflaton) with a smooth potential.

2. Conservative approach

In our calculations, we evolved the universe for a given \( V(\phi) \). During a stage of scalar field domination, the Friedmann equation and the equation of motion of the scalar field can be written with the usual quantities as

\[
\dot{\phi} = \frac{m_p^2}{4\pi} H^2(\phi) \; ;
\]

\[
H^2(\phi) \frac{12\pi m_p^2}{m_p^2} H^2(\phi) = \frac{32\pi^2}{m_p^2} V(\phi) \; ;
\]

where \( \dot{\cdot} \) denotes a time derivative and \( \phi \) denotes a derivative with respect to \( \phi \). Initial conditions for the evolution can be chosen in terms of \( V(\phi) \) and \( \phi \). By demanding that the attractor solution for \( \dot{\phi} \) is reached just before the observable scales leave the horizon, we chose to decrease the number of free parameters. The attractor solution is the solution in which the accelerating force on the inflaton and the Hubble friction precisely cancel. In practice this implies that inflation starts at least a few e-folds before observable modes leave the horizon. To do a Monte-Carlo simulation we used the

---

1 Wilkinson Microwave Anisotropy Probe [8, 9]
2 Sloan Digital Sky Survey - Luminous Red Galaxy, data release 4 [10]
Table 1: Bayesian 68% confidence limits for ΛCDM inflationary models with a Taylor expansion of the inflaton potential at order $v = 2, 3, 4$ (with the primordial spectra computed numerically). The last line shows the maximum likelihood value.

| Parameter | $v = 2$ | $v = 3$ | $v = 4$ |
|-----------|---------|---------|---------|
| $\Omega_b h^2$ | 0.022 0.001 | 0.022 0.001 | 0.022 0.001 |
| $\Omega_{cdm} h^2$ | 0.019 0.004 | 0.019 0.004 | 0.019 0.004 |
| $\theta$ | 0.0041 0.0041 | 0.0041 0.0041 | 0.0041 0.0041 |
| $\tau$ | 0.07 0.07 | 0.07 0.07 | 0.07 0.07 |
| $\ln \frac{128 \pi^{10} V^3}{3 V^2 m_p^2}$ | 3.06 0.06 | 3.07 0.06 | 3.11 0.08 |
| $\frac{V^2}{m_p^2}$ | $< 0.4$ | $< 0.4$ | $< 0.8$ |
| $\frac{V^0}{m_p^2}$ | 0.5 0.5 | 0.6 0.6 | 0.9 0.9 |
| $\frac{V^{20} V^0}{m_p^4}$ | 0 0 | 0 0 | 13 11 |
| $\frac{V^{20} V^0 2}{m_p^6}$ | 0 0 | 0 0 | 200 150 |
| $\ln L_{\text{max}}$ | 2688.3 2687.2 2687.2 |

3. Current bounds

The new bounds on the derivatives of the inflaton potential, are displayed in Table 1. The number of degrees of freedom in the potential is denoted by the parameter $v$, where $v = 2$ denotes an expansion up to the second derivative, $v = 3$ up to $V^{(3)}$, etc. Central values and error bounds are obtained by marginalization over all other parameters and given at 68% confidence level. All solutions are symmetric under a sign change in $V^0$ (the inflaton always rolls down) and $V^{(3)}$.

The best likelihoods of the simulations indicate that the improvement by adding the fourth derivative is negligible. As could be expected, the bounds on lower derivatives loosen when one allows higher derivatives. In Figure 1 all 2D-correlations between the potential derivatives are given, marginalized over all other parameters. In this figure results are compared with those obtained by inverting the slow-roll approximation, when probing a scalar and tensor primordial spectrum with a scalar tilt ($p = 2$) or an additional running of the tilt ($p = 3$). The covered area in parameter space is equally large in both approaches, however the overlap is not 100%, which shows that indeed information is lost when mapping from one basis to the other. This indicates that up to second order, the
How much of the inflaton potential do we see?

Wessel Valkenburg

4. Discussion

Previous works relied on an extrapolation of data over a range which is an order of magnitude broader than the range of the data itself, which is fine from one point of view. Focusing on the data only, these are the first results to constrain to potential up to such a high order.
In equations (2.1,2.2), one sees that \( V(\phi) \) does not unambiguously define \( H(\phi) \), however any \( H(\phi) \) would uniquely define \( V(\phi) \). Hence the defining quantity should be \( H(\phi) \), which will be subject to a future work \([12]\).

Acknowledgements

The author would like to thank the organisers of the Cargèse Summerschool on Cosmology and Particles Beyond The Standard Models for an excellent stay. This work was supported by the EU 6th Framework Marie Curie Research and Training network “UniverseNet” (MRTN-CT-2006-035863).

References

[1] A. A. Starobinsky. *Phys. Lett.*, B91:99–102, 1980; A. H. Guth. *Phys. Rev.*, D23:347–356, 1981; K. Sato. *Mon. Not. Roy. Astron. Soc.*, 195:467–479, 1981; S. W. Hawking and I. G. Moss. *Phys. Lett.*, B110:35, 1982; A. D. Linde. *Phys. Lett.*, B108:389–393, 1982; A. D. Linde. *Phys. Lett.*, B129:177–181, 1983.

[2] A. A. Starobinsky. *JETP Lett.*, 30:682–685, 1979; S. W. Hawking. *Phys. Lett.*, B115:295, 1982; A. A. Starobinsky. *Phys. Lett.*, B117:175–178, 1982; A. H. Guth and S. Y. Pi. *Phys. Rev. Lett.*, 49:1110–1113, 1982; A. D. Linde. *Phys. Lett.*, B116:335, 1982; J. M. Bardeen, P. J. Steinhardt, and M. S. Turner. *Phys. Rev.*, D28:679, 1983; L. F. Abbott and M. B. Wise. *Nucl. Phys.*, B244:541–548, 1984; D. S. Salopek, J. R. Bond, and J. M. Bardeen. *Phys. Rev.*, D40:1753, 1989.

[3] P. J. Steinhardt and M. S. Turner. *Phys. Rev.*, D29:2162–2171, 1984; D. S. Salopek and J. R. Bond. *Phys. Rev.*, D42:3936–3962, 1990; A. R. Liddle, P. Parsons, and J. D. Barrow. *Phys. Rev.*, D50:7222–7232, [astro-ph/9408015], 1994. S. M. Leach, A. R. Liddle, J. Martin and D. J. Schwarz. *Phys. Rev.*, D66:023515, [astro-ph/0202094], 2002.

[4] D. N. Spergel et al. *Astrophys. J. Suppl.*, 170:377, [astro-ph/0603449], 2007.

[5] H. Peiris and R. Easther. *JCAP*, 0607:002, [astro-ph/0603587], 2006; H. J. de Vega and N. G. Sanchez. astro-ph/0604136, 2006; W. H. Kinney, E. W. Kolb, A. Melchiorri, and A. Riotto. *Phys. Rev.*, D74:023502, [astro-ph/0605338], 2006; J. Martin and C. Ringleval. *JCAP*, 0608:009, [astro-ph/0605367], 2006; L. Covi, J. Hamann, A. Melchiorri, A. Slosar, and I. Sorbera. *Phys. Rev.*, D74:083509, [astro-ph/0606452], 2006; F. Finelli, M. Rianna, and N. Mandolesi. *JCAP*, 0612:006, [astro-ph/0608277], 2006; H. Peiris and R. Easther. *JCAP*, 0610:017, [astro-ph/0609003], 2006; C. Destri, H. J. de Vega, and N. G. Sanchez. [astro-ph/0703417], 2007; C. Ringleval. [astro-ph/0703486], 2007; A. Cardoso. *Phys. Rev.*, D75:027302, [astro-ph/0610074], 2007;

[6] R. Easther and H. Peiris. *JCAP*, 0609:010, [astro-ph/0604214], 2006;

[7] J. Lesgourgues and W. Valkenburg. *Phys. Rev.*, D75:123519, [astro-ph/0703625], 2007.

[8] B. A. Powell and W. H. Kinney, *JCAP* 0708 (2007) 006 [arXiv:0706.1982 [astro-ph]].

[9] L. Page et al. *Astrophys. J. Suppl.*, 170:335, [astro-ph/0603450], 2007; G. Hinshaw et al. *Astrophys. J. Suppl.*, 170:288, [astro-ph/0603451], 2007; N. Jarosik et al. *Astrophys. J. Suppl.*, 170:263, [astro-ph/0603452], 2007.

[10] M. Tegmark et al. *Phys. Rev.*, D74:123507, [astro-ph/0608632], 2006.

[11] A. Lewis and S. Bridle. *Phys. Rev.*, D66:103511, [astro-ph/0205436], 2002.

[12] A. A. Starobinsky, J. Lesgourgues and W. Valkenburg. *work in progress*. 

5