Flavor and Little Higgs

Otto C.W. Kong
Department of Physics, National Central University, Chung-li, TAIWAN 32054
E-mail: otto@phy.ncu.edu.tw

Abstract

We discuss the proper starting point to look into flavor physics under the perspective of solving the hierarchy problem with the little Higgs mechanism — the construction of anomaly free fermionic spectra of the effective theory of extended electroweak gauge symmetry around the TeV scale. Anomaly cancellations among the three SM families, plus extra fermions, are typically needed, giving family non-universal flavor structures. Such a completed and consistent fermionic spectrum supplements a scalar sector little Higgs model, turning the latter into a realistic model the phenomenological viability of which can then be investigated. The implications for flavor physics are specially interesting. We give more elaborated discussion here on our basic perspective, trying to clarify on issues that are in our opinion under-appreciated. We also add a brief comment of little Higgs versus supersymmetry.

* Invited Talk presented at ICFP 2003 (Oct 6–11), KIAS, Korea
— submission for the proceedings.
Flavor and Little Higgs

Otto C. W. Kong
Department of Physics, National Central University,
Chung-li, Taiwan 32054

We discuss the proper starting point to look into flavor physics under the perspective of solving the hierarchy problem with the little Higgs mechanism — the construction of anomaly free fermionic spectra of the effective theory of extended electroweak gauge symmetry around the TeV scale. Anomaly cancellations among the three SM families, plus extra fermions, are typically needed, giving family non-universal flavor structures. Such a completed and consistent fermionic spectrum supplements a scalar sector little Higgs model, turning the latter into a realistic model the phenomenological viability of which can then be investigated. The implications for flavor physics are specially interesting. We give more elaborated discussion here on our basic perspective, trying to clarify on issues that are in our opinion under-appreciated. We also add a brief comment of little Higgs versus supersymmetry.

I. INTRODUCTION

This is second talk on little Higgs in the conference program. We will take advantage of the first talk from T. Han[1], in which the little Higgs idea has been reviewed. Instead of repeating what has been said there, we would rather focus on our main concern — the construction of an anomaly free fermionic spectrum as a necessary completion of a little Higgs model and the proper starting point to think about flavor physics under the model. To set a clear perspective, however, we supplement a brief comment of the apparent difference between the main physics of the first talk and that to be addressed here.

The simplest way to look at the question is to say that we are talking about different (kinds of) little Higgs models, namely, we focus on models from Kaplan and Schmaltz[2] with $SU(N)_L \times U(1)_X$ ($N = 3$ and 4) extended electroweak (gauge) symmetries. For instance, for the $N = 3$ case, we have actually a scalar (Higgs) sector of a $[SU(3)]^2/[SU(2)]^2$ nonlinear sigma model, as versus that of the $SU(5)/SO(5)$, model[3] on which the first talk is mainly based. In our opinion, the models discussed here actually have structures that are more carefully worked out, especially after our effort on which the current talk is mainly based[4, 5]. However, we certainly believe that the issues addressed here have relevance for generic little Higgs model building as well as little Higgs flavor physics. Such issues have not been well addressed by earlier authors on the subject, who might be focusing more on establishing and illustrating the little Higgs mechanism rather than furnishing complete particle physics models. For example, the full fermionic content of all the multiplets under the global as well as extended gauge symmetries of the $SU(5)/SO(5)$ model has not been presented. To elaborate more, only one extra fermion, the heavy top quark $T$ needed to cancel the quadratic divergence (of the contribution to Higgs mass) from the SM $t$ quark at 1-loop is discussed. It is taken to be vectorlike under the SM gauge symmetry. But there should be a lot more to the story. The $T$ quark, or rather its chiral components, has to join the SM chiral components to form multiplet(s) under the extended symmetries. There would then be other extra fermions from the full multiplet(s). It is also very unlikely there one can arrange the little Higgs mechanism with the $T$ quark, as well as all the other extra fermions, vectorlike under the full gauge symmetry of the model. Neither is that exactly a desirable feature. If the extra states are fully vectorlike, instead of forming vectorlike pairs only after the gauge symmetry is broken to that of the SM, there is no reason for such states, including the $T$, to have masses below the 10-50 TeV cutoff scale. For the kind of models we discussed here below, we will spell out the complete multiplets necessary to house all the fermions — SM ones plus $T$ as a minimal list. For each particular model, the list is chiral to begin with while yielding the SM chiral list plus vectorlike states including the $T$ as SM gauge symmetry becomes the only surviving gauge symmetry. The latter is exactly what is required to be a consistent extended symmetry model embedding the SM. Furthermore, we will see that such a list is essentially dictated by the particular realization of the little Higgs mechanism and the condition that chiral fermionic spectrum be gauge anomaly free. The gauge anomaly cancellation conditions constitute what we believe to be the best understanding we have on the question of why there is what there is. And, in our opinion, it is apparently much under-appreciated by many physicists.

Han and collaborators has performed some interesting phenomenological studies of the $SU(5)/SO(5)$ little Higgs model[1]. Most parts of those results are valid, as they are independent of the complete fermionic spectrum, i.e. when one assumes that such a consistent spectrum can be obtained, and any extra fermion to be included has a mass very close to the cutoff and/or has no significant coupling to the SM spectrum. However, looking at the little Higgs models
from a model-building point of view, one should wonder if we can do a more complete job. On the other hand, if our concern is flavor physics, the detailed representation assignment of the SM fermions themselves under the extended symmetries would have strong implications on the flavor structure of a model. Hence, it becomes an unavoidable subject. Not to say that the extra fermions may also play some role. What we are doing here is only a small starting step in the direction — a direction that certainly deserves more attention if we want to consider any little Higgs model as a realistic particle physics model though.

II. THE $SU(3)_L \times U(1)_X$ LITTLE HIGGS MODEL

Let us first give a summary of the particular realization of little Higgs mechanism our discussion explicitly based on, the model(s) from Kaplan and Schmaltz[2]. The latter paper, as well as Ref.[6], presents a simple group theoretical approach to little Higgs model building, which might look more transparent and easier to follow for beginners. The construction focuses only on the TeV scale effective field theory, independent of its strong couplings parent theory or so-called UV-completion. It is the same bottom-up model building perspective that we adopted here, only pushing it all the way to the end. We are looking into the structure of the little Higgs model as a complete particle physics model — effective field theory, living between the electroweak scale and the scale of the yet unknown UV-completion. We ask for a consistent and comprehensive description of its physics content. The phenomenological viability of the model may then be carefully studied and tested experimentally.

Kaplan and Schmaltz considered a scalar sector with two (anti)triplets $\Phi_1$ and $\Phi_2$ each taken as carrying its own $SU(3)$ symmetry. The two (anti)triplets are assigned VEVs aligned within a combination of the two $SU(3)$’s that is gauged together with an extra $U(1)_X$. The two sets of five Goldstone states from the two $SU(3) \to SU(2)$ symmetry-breakings then consists of an unphysical set to be eaten by the extra “electroweak” gauge bosons in the $SU(3)_L \times U(1)_X$ beyond their SM electroweak cousins. The orthogonal set contains pseudo-Nambu-Goldstone bosons (PNGBs) under the remaining part of the symmetries which are global and really (can be) only approximate in the full Lagrangian. The PNGBs include the SM Higgs doublet and an extra singlet scalar. Explicitly, there is the following nonlinear sigma model description:

$$
\Phi_1 = e^{i\theta/f} \begin{pmatrix} 0 & 0 & f \\ 0 & 0 & f \end{pmatrix}, \quad \Phi_2 = e^{-i\theta/f} \begin{pmatrix} 0 & 0 & f \\ 0 & 0 & f \end{pmatrix},
$$

$$
\theta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & h^\dagger \hfill \\
0 & 0 & h \hfill \\
h & h & 0 \hfill 
\end{pmatrix} + \frac{\eta}{4} \begin{pmatrix} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -2 \end{pmatrix}.
$$

To take care of the top sector contribution to Higgs mass correction at 1-loop, the extra $T$ quark is introduced with a chiral component living inside a triplet of the extended electroweak gauge symmetry. This is exactly the triplet that contains the $(t,b)$ SM doublet, hence linking $T$ to $t$. We have Yukawa terms involving the two top-like quarks given as

$$
L_{top} = y_t \bar{T}_a^i \Phi_1 Q^a + y_b \bar{T}_a^i \Phi_2 Q^a
$$

$$
= f (y_t \bar{t}^i + y_b \bar{\bar{T}}^i) T + \frac{i}{\sqrt{2}} (y_t \bar{t}^i - y_b \bar{\bar{T}}^i) h \begin{pmatrix} t \\
b \end{pmatrix} + \cdots,
$$

where $Q^a = (T^a, t^a, b^a)^T$, and $\bar{t}_a^i$ and $\bar{\bar{T}}_a^i$ denote colored singlet states. For details on the functioning of the little Higgs mechanism, we refer readers to the original paper[2]. From our discussion at the beginning, we are led to the question of how to complete the group theoretical description of the full fermion spectrum. All SM doublets would have to come from nontrivial representations of the $SU(3)_L$. The first point to note here is that simply repeating the structure here with quarks for the lighter two families does not work. The simple embedding produces $SU(3)_L$ gauge anomaly rendering the model inconsistent. Nor is such an embedding desirable, as we will show below.

III. GAUGE ANOMALY VS FLAVOR STRUCTURE — A DETOUR

To set a good reference background for the problem at hand, we take a detour here to re-examine the SM spectrum and the questions of flavor structure. The spectrum of SM fermion in one family is like perfection, essentially dictated by gauge anomaly cancellation conditions. And there is the million-dollar question, or fundamental question of flavor physics — Why three families? To illustrate the point of view, we recall our earlier argument[7]. Assuming that...
there exist a minimal multiplet carrying nontrivial quantum numbers of each of the component gauge groups, one can obtain the one-family SM spectrum as the unique solution by asking for the minimal consistent set of chiral states. A vectorlike set is trivial but not as interesting. Only chiral states are protected from heavy gauge invariant masses.

The above suggested derivation of the one-family SM spectrum goes as follow. We are essentially starting with a quark doublet, with arbitrary hypercharge normalization. The two $SU(3)_C$ triplets require two antitriplets to cancel the anomaly. Insisting on the chiral spectrum means taking two quark singlets here, with hypercharges still to be specified. Now, $SU(2)_L$ is real, but has a global anomaly. Cancellation requires an even number of doublet, so at least one more beyond the three colored components in the quark doublet. There are still four anomaly cancellation conditions to take care of. They are the $[SU(3)_C]^2U(1)_Y$, $[SU(2)_L]^2U(1)_Y$, $[grav]^2U(1)_Y$, and $[U(1)_Y]^3$. We are however left with three relative hypercharges to fit the four equations actually without a possible solution. A rescue comes from simply adding a $U(1)_Y$-charged singlet. But the four equation for four unknown setting is misleading. The $[U(1)_Y]^3$ anomaly cancellation equation is cubic in all the charges, with no rational solution guaranteed. The SM solution may actually be considered a beautiful surprise.

We would also like to take the opportunity here to briefly sketch the next step taken in Ref.[7], to further illustrate our perspective. The results there also may be considered a worthy comparison with our little Higgs motivated flavor/family spectrum presented below, from the point of view of the origin of the three families. The major goal of Ref.[7] is to use a similar structure with an extended symmetry to obtain the three families. For example, one can start with some $SU(4) \times SU(3) \times SU(2) \times U(1)$ gauge symmetry and try to obtain the minimal chiral spectrum contain a $(4, 3, 2)$ multiplet — the simplest one with nontrivial quantum number under all component groups. Having a consistent solution is not enough though. In order for the spectrum be of interest, we ask the spectrum to yield the chiral spectrum of three SM families plus a set of vectorlike states under a feasible spontaneous symmetry breaking scenario, i.e., when the gauge symmetry is broken to that of the SM. Ref.[7] has only partial success. A consistent group theoretical SM embedding could be obtained with a slight addition to the minimal chiral spectrum obtained from anomaly cancellation considerations alone. We give an example in Table I.

To conclude the section, we emphasize that anomaly cancellation conditions play a role of paramount importance in constraining the fermionic spectrum. Having a complete model with an extended gauge symmetry above the electroweak scale successfully, even though only group theoretically speaking, embedding the SM states is not at all trivial. Types of consistent models of the kind in the literature are quite limited. However, such exercises might, and have been, used as a way to probe the most fundamental question of flavor physics — why 3 families? We return to the little Higgs model(s) below, to get exactly such a job finished. We will then see how the requirement for an anomaly free chiral fermion spectrum dictates a specific flavor structure to a particular model. Readers are invited to consider if the model, or its fermion spectrum, obtained is as esthetically appealing as the (one-family) SM or the models obtained in Ref.[7]. We do believe that such esthetic concerns may have some relevance in fundamental physics. After all, it cannot be emphasized enough that the anomaly cancellation requirement is the best, and might be only, theoretical tool we have up to the moment to address the question of why there is what there is in terms of the fermionic sector.

Table 1. A $SU(4)_A \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_X$ anomaly free chiral fermion spectrum with three SM families.

| multiplets | $X$ | $U(1)$-grav. | $[SU(4)]^2U(1)$ | $[SU(3)]^2U(1)$ | $[SU(2)]^2U(1)$ | $[U(1)]^3$ |
|----------|----|-----------|----------------|----------------|----------------|----------|
| $(4, 3, 2)$ | 1 | 24 | 6 | 8 | 12 | 24 | 3 $(Q)$ -5 $(Q')$ |
| $(4, 3, 1)$ | 5 | 60 | 15 | 20 | 121500 | 3 -4 $(\bar{d})$ | 2 $(\bar{d})$ |
| $(4, 1, 2)$ | 3 | 24 | 6 | 12 | 216 | 3 -3 $(L)$ | 3 $(\bar{L})$ |
| $(4, 1, 1)$ | 9 | 36 | 9 | 2916 | 3 -6 $(\bar{E})$ | 0 $(N)$ |
| $(6, 1, 1)$ | -18 | -108 | -36 | -34992 | 3 $(\bar{6}(E))$ | 3 $(12(S))$ |
| $(1, 3, 2)$ | -10 | -60 | -20 | -30 | -6000 | 5 $(\bar{Q})$ |
| $(1, 3, 1)$ | -4 | -12 | -4 | -192 | 2 $(\bar{d})$ |
| $(1, 3, 1)$ | -4 | -12 | -4 | -192 | 2 $(\bar{d})$ |
| $(1, 1, 2)$ | 6 | 12 | 6 | 432 | -3 $(\bar{L})$ |
| $(1, 1, 1)$ | 24 | 72 | 41742 | 3 $(12(S))$ |
| $(1, 1, 1)$ | -12 | -36 | -5184 | 3 $(\bar{6}(E))$ |
| Total | 0 | 0 | 0 | 0 | 0 | 0 |
IV. ANOMALY FREE \(SU(N)_L \times U(1)_X\) SPECTRA

In our recent papers\cite{4,5}, we have illustrated that an anomaly free fermionic spectrum with consistent embedding of the three family SM fermions can be obtained. In fact, we have given a recipe for an infinite number of similar fermionic spectra of with an \(SU(N)_L \times U(1)_X\) extended electroweak symmetry for any \(N > 2\). The basic feature of such a spectrum is to have a SM quark doublet embedded into a fundamental representation of \(SU(N)_L\) while the lighter two quark doublets embedded into anti-fundamental, as inspired by Ref.\cite{8}. It exploits the fact that

\[ N_c = N_f; \]

namely, the number of SM families (of fermions) \(N_f\) happens to coincide with the number of colors. Indeed such a spectrum may be considered rather as the reason behind the number of SM family. We emphasize, again, that the need for gauge anomaly cancellation, for the SM or otherwise, is the best we have in addressing the question if why there is what there is — a perspective we try to illustrate in the last section. The \(1 + 2\) quark family embedding has net \(SU(N)_L\) anomaly to be canceled exactly by embedding the three leptonic doublets into fundamentals. The simplest way to complete the spectrum with consistent SM embedding is to put anything else in singlets, with just enough of them to produce vector-like pairs for all single fermion states at the QED and QCD level. Neutrinos may be sort of exceptions to the last statement. Carrying no conserved gauge quantum number, they can be considered self-conjugate. When checking the other gauge anomalies of such a spectrum, it looks like a miracle upon a first glance. There are prefect cancellations for all, as illustrated by the \(N = 3\) example given in Table II. Two other example spectra given in Table III below, in which we skip the explicit presentation of the gauge anomalies. Readers can easily checked that the cancellations do work. In fact, a careful checking of the algebra\cite{5} would show that the recipe as outlined here always works for any \(N\). Looking at the possible ways of obtaining the hypercharge \(U(1)\) out of the \(N - 1\) \(U(1)\)s inside the \(SU(N)_L\) and the still extra \(U(1)\) of \(U(1)_X\), one will see that there is exactly a \(N - 2\) parameter degree of freedom and choosing such a model spectrum. That can be taken as the freedom for choosing the electric charges of the \(N - 2\) extra quark states within the same \(SU(N)_L\) multiplet as the \(t\)-doublet.

Table II: The \(SU(3)_C \times SU(3)_L \times U(1)_X\) spectrum with little Higgs. Electroweak doublets are put in [.]’s.

| Fermion multiplets | \(U(1)\)-grav. | \([SU(3)]^3\) | \([SU(3)]^2U(1)\) | \([SU(3)c]^2U(1)\) | \([U(1)]^3\) | \(U(1)_Y\) states |
|-------------------|----------------|-------------|----------------|----------------|-------------|----------------|
| \((3c, 3c, \frac{1}{3})\) | \(\frac{1}{3} \times 9 \times 1\) | \(3 \times 1\) | \(\frac{1}{3} \times 3 \times 1\) | \(\frac{1}{3} \times 3 \times 1\) | \(\frac{1}{3} \times 9 \times 1\) | \(\frac{1}{3}[Q] \frac{2}{3}(T)\) |
| \((3c, 3c, 0)\) | \(0 - 3 \times 2\) | \(0\) | \(0\) | \(0\) | \(2\) | \(\frac{1}{2}[Q] \frac{1}{2}(D, S)\) |
| \((3c, 3c, \frac{2}{3})\) | \(\frac{1}{3} \times 3 \times 3\) | \(1 \times 3\) | \(-\frac{1}{3} \times 3\) | \(-\frac{1}{3} \times 3\) | \(3\) | \(\frac{1}{3}[L] 3\) 0(N) |
| \((3c, 1c, \frac{2}{3})\) | \(-\frac{2}{3} \times 3 \times 4\) | \(-2\) | \(-\frac{1}{3} \times 4\) | \(-\frac{1}{3} \times 4\) | \(4\) | \(-\frac{2}{3}(\bar{u}, \bar{c}, \bar{t}, T)\) |
| \((3c, 1c, \frac{1}{3})\) | \(\frac{1}{3} \times 3 \times 5\) | \(-\frac{1}{3} \times 5\) | \(-\frac{1}{3} \times 5\) | \(-\frac{1}{3} \times 5\) | \(5\) | \(\frac{1}{3}(d, s, b, \bar{D}, S)\) |
| \((3c, 1c, \frac{1}{3})\) | \(\frac{1}{3} \times 3\) | \(1 \times 3\) | \(-\frac{1}{3} \times 3\) | \(-\frac{1}{3} \times 3\) | \(3\) | \(1(e^+, \mu^+, \tau^+)\) |

We have summarized in the last paragraph the possible anomaly free fermionic spectra with \(SU(N)_L \times U(1)_X\) extended electroweak gauge symmetries. We would certainly love to arrive at a conclusion of only one or two of such spectra are possible, instead of having infinitely many candidates at hand. However, not all of such spectra would be relevant to little Higgs, which is the major motivation of our study here. The first obvious criterion for such a fermionic spectrum to fit into a little Higgs model is the existence of the \(T\) quark — a heavy top quark that couples to the SM Higgs in such a way as to allow the cancellation of quadratic divergence. Following what is illustrated in Ref.\cite{2} [cf. Eq.(3)], we simply ask for a \(T\) quark within the \(Q^a\) multiplet — the \(SU(N)_L\) multiplet containing the \(t-b\) doublet. (Note also that in the Tables, \(Q\) stands for a SM quark doublet instead.) For the case of \(N = 3\), that fix the spectrum as given in Table II as the only option.

Naively, one would prefer to stay with a smaller \(N\). The unique \(N = 3\) case fitted with the Higgs sector structure introduced in Ref.\cite{2}, however, has some problem with getting the Higgs quartic coupling, as noted by the authors. Interesting enough, another group published a different little Higgs model with again the \(SU(3)_L \times U(1)_X\) extended electroweak symmetry\cite{9}. In the latter case, the gauge symmetry is embedded into a global \(SU(9)\). It certainly looks like the fermionic spectrum could play a role in the \(SU(9)\) model too, though we are not really ready to comment of that here.
V. $SU(3)_C \times SU(4)_L \times U(1)_X$ MODELS

To surmount the quartic coupling barrier of the $N = 3$ case, Ref.[2] turns to the construction of a $N = 4$, with simply two $T$ quark instead of one. Our analysis of the fermionic sector suggests the first spectrum given in Table III works as a completion of the model. Another interesting question that arises is on the exact identity of the fourth quark in the multiplet of $t$-$b$ and the single $T$ always required. While we have not investigated all options in any detail, we believe the second spectrum given in Table III would be a feasible and interesting alternative, with a heavy bottom $B$ quark instead of a second $T$. At the electroweak level, the type of $N = 4$ little Higgs model has two SM Higgs doublet. The modified Higgs sector we introduced in Ref.[5] to accompany the second $N = 4$ fermionic spectrum looks more like a two Higgs doublet model with natural flavor conservation and a naturally large $\tan\beta$. The latter are phenomenologically desirable[10].

| First model spectrum | Second model spectrum |
|----------------------|-----------------------|
| $U(1)_Y$-states      | $U(1)_Y$-states       |
| $(3C, 4L, \frac{5}{2})$ | $\frac{1}{2}[Q]$ | $\frac{1}{2}[Q]$ |
| $2(3C, 4L, \frac{1}{2})$ | $\frac{1}{2}[2 Q]$ | $\frac{1}{2}[2 Q]$ |
| $3(1C, 4L, \frac{1}{2})$ | $\frac{1}{2}[3 L]$ | $\frac{1}{2}[3 L]$ |
| $5(3C, 1L, \frac{1}{2})$ | $\frac{1}{2}[3 L]$ | $\frac{1}{2}[3 L]$ |
| $7(3C, 1L, \frac{1}{2})$ | $\frac{1}{2}[3 L]$ | $\frac{1}{2}[3 L]$ |
| $3(1C, 1L, 1)$ | $\frac{1}{2}[3 L]$ | $\frac{1}{2}[3 L]$ |

VI. LITTLE HIGGS VS SUPERSYMMETRY

From the pure theoretical side, such models seem to hold great promise. Like supersymmetry (SUSY), we have consistent complete model(s) of TeV scale physics successful handling the notorious hierarchy problem. While the question of family tripling of SM fermions is not at all touch by supersymmetrizing the SM, the bosonic symmetry scheme of little Higgs adds fermions instead of sfermions, complicating the issues of gauge anomaly cancellation. We see here that solving the latter problem sheds some light on the number of family problem, at least for the class of little Higgs models we focus on here. A similar construction of anomaly free spectra does not work for the number of family being not three (as the number of color). While the supersymmetric SM is quite unique, there are already many little Higgs models on the market. Some models may not share much common structure with the ones discussed here, though the basic anomaly cancellation concern is always relevant. SUSY adds to the phenomenological flavor problems. Flavor changing neutral current (FCNC) constraints impose stringent conditions on the soft SUSY breaking parameters. FCNC constraints are certainly no less demanding on little Higgs models. Precision electroweak constraints are generally more difficult for the latter[1]. However, it is only with completed models as those exhibited here that one can launch a careful study of such constraints and hence the phenomenological viability of the models concerned. The minimal SUSY model keeps the SM neutrinos massless. Incorporating R-parity violation is arguably natural, at the expense of introducing many, many new couplings[11]. Our little Higgs fermionic spectra above all contain extra (singlet) neutrino states and hence the possibility of fixing the experimentally required neutrino properties at the TeV scale too.

VII. SOME IMPLICATIONS TO FLAVOR PHYSICS

First of all, the full quantum numbers characterizing a fermion multiplet will dictate what couplings it can have within the model. The gauge quantum numbers dictate the admissible gauge invariant couplings in the model Lagrangian. The latter is of course the very starting point to look into the phenomenology of the fermionic states involved, including those of the SM. The only other important issue here is the quantum numbers under the global symmetries. However, as the latter are only approximate symmetries and have no constraints like anomaly cancellations, there are more ambiguities and rooms to play around with. Let us illustrate some possible features with the models discussed.

Let us first restrict ourselves to the $SU(3)_L \times U(1)_X$ model originally introduced in Ref.[2]. We have the full list of fermion gauge quantum numbers as given in Table II. Next, we have scalar multiplets with gauge and global quantum numbers fixed by the requirement of the little Higgs mechanism itself [cf. discussion on Eq.(3)]. The minimal, and
certainly safe, strategy is to consider no other scalar multiplets. Gauge quantum numbers alone say that only the following direct Yukawa couplings are admissible:

\[ 1_L \Phi_i 3_L \quad \text{and} \quad 1_L \Phi_i^\dagger 3_L. \]

Only the \( t \) and \( T \) quarks come from representations of the right \( X \)-charges to have a coupling of the first from; while for the second, only the \( d \) and \( s \) quarks fit in. At the next level, we have

\[ 1_L \Phi_i^\dagger \Phi_j^\dagger 3_L \quad \text{and} \quad 1_L \Phi_i \Phi_j 3_L. \]

The first admits \( b \) and \( B \) Yukawas, as well as those for the charged leptons; the second admits those for \( u \) and \( c \). The global symmetry requirement for the ‘top’ Yukawa naturally admits the above ‘bottom’ Yukawa, the higher dimensionality of which might be a source of the smaller numerical coupling of the latter case. The \( 3_L \)'s containing the \( d \) and \( s \) quark would have to bear extra global quantum number constraints to push for high dimensional Yukawa couplings. Such coupling suppressions have effects also on the extra \( D \) and \( S \) quarks, which are likely to have strong implications to \( b \) physics then.

Finally, we comment on the \( SU(4)_L \times U(1)_X \) model with the second spectrum given in Table III. The model doubles the list of SM fermions, in much the same way as SUSY double the particle spectrum (with different spin though). The modified little Higgs part has two pairs of aligned-VEV \( \Phi \)'s with different \( X \)-charges to couple to the top and bottom sector separately. The quadratic divergence cancellation then works for the \( t-T \) pair as well as for the \( b-B \) pair in similar fashion. Large bottom Yukawa is naturally admitted. Without further global symmetry constraints, the ‘top-Higgs’ couples directly to \( d \)- and \( s \)-sectors while ‘bottom-Higgs’ to \( u \)-and \( c \)-sectors. Flavor physics would be very different from that of the \( SU(3)_L \) model above or that of the other \( SU(4)_L \) model also given in Table III. The latter is expected to be quite similar to the \( SU(3)_L \) case. More detailed analysis of such models have to start with feasible complete global quantum number assignment to all multiplets.

The bottom line here is that sensible discussion of flavor physics of a little Higgs model is not possible before the full fermion spectrum is spelt out. The latter is constrained by gauge anomaly cancellation. We exhibit some completed models here on which detailed flavor physics still have to be studied. Building models of the kind, and studying their phenomenology in details should be a worthy endeavor.

Our work is partially supported by the National Science Council of Taiwan, under grant number NSC 91-2112-M-008-044.

[1] T. Han, talk at ICFP 2003, this proceedings, and references therein.
[2] D.E. Kaplan and M. Schmaltz, hep-ph/0302049.
[3] N. Arkani-Hamed et al, JHEP 07, 034 (2002).
[4] O.C.W. Kong, NCU-HEP-k009, hep-ph/0307250.
[5] O.C.W. Kong, NCU-HEP-k010, hep-ph/0308148.
[6] M. Schmaltz, hep-ph/0210415.
[7] O.C.W. Kong, Mod. Phys. Lett. A11, 2547 (1996); Phys. Rev. D55, 383 (1997).
[8] P.H. Frampton, Phys. Rev. Lett. 69, 2889 (1992). See also M. Singer et al, Phys. Rev. D22, 738 (1980); F. Pisano and V. Plieitez, Phys. Rev. D46, 410 (1992); R. Foot et al, Phys. Rev. D50, R34 (1994).
[9] W. Skiba and J. Terning, hep-ph/0305302.
[10] See, for example, K. Cheung et al, Phys. Rev. D64, 111301(R), (2001) ; K. Cheung and O.C.W. Kong, Phys. Rev. D68, 053003 (2003).
[11] See, for example, O.C.W. Kong, hep-ph/0205205, Int. J. Mod. Phys. A (2003) to be published, and S.K. Kang and O.C.W. Kong, hep-ph/0206009, Phys. Rev. D to be published.