The Fluctuation-Dissipation Theorem of Colloidal Particle's energy on 2D Periodic Substrates: A Monte Carlo Study of thermal noise-like fluctuation and diffusion like Brownian motion

Amin Najafi

Department of physics, Islamic Azad University, (Hamedan Branch) Hamedan,734,Iran

Najafi.physic@gmail.com

Abstract. Using the Monte Carlo simulations, we have calculated mean-square fluctuations in statistical mechanics, such as those for colloids energy configuration are set on square 2D periodic substrates interacting via a long range screened Coulomb potential on any specific and fixed substrate. Random fluctuations with small deviations from the state of thermodynamic equilibrium arise from the granular structure of them and appear as thermal diffusion with Gaussian distribution structure as well. The variations are showing linear form of the Fluctuation-Dissipation Theorem on the energy of particles constitutive a canonical ensemble with continuous diffusion process of colloidal particle systems. The noise-like variation of the energy per particle and the order parameter versus the Brownian displacement of sum of large number of random steps of particles at low temperatures phase are presenting a markovian process on colloidal particles configuration, too.

1. Introduction

More recently, colloidal assemblies interacting with 2D periodic substrates have been investigated [1,2, 3, 5, 6,7]. Simulations of colloids on 2D periodic substrates have considered the case where the number of colloids is an integer multiple of the number of potential minima [2] There are also a two-stage melting transition for the trimer state at a filling of three colloids per minima for the square substrate [2]. Colloidal particle assemblies in two dimensions (2D) are an ideal system to study ordering and melting, since quantities such as diffusion, dislocation dynamics, and local ordering can be directly observed, in which is typically not the case for atomic and molecular systems [4]. More recently, colloidal assemblies interacting with 2D periodic substrates have been investigated as well [2, 3, 5, 6].

2. The correlation function and the fluctuation-Dissipation Theorem

The correlation function [7],[8] contains important information about a phase transition. There is an order-parameter density \( m(r) \), such that the order parameter can be written as:

\[
M = \left\langle \int d^3 r \, m(r) \right\rangle
\]  

(1)

Where \( <> \) denotes ensemble average. The correlation function is defined as:

\[
\Gamma(r) = \langle m(r)m(0) \rangle - \langle m(r) \rangle \langle m(0) \rangle
\]

(2)

We can rewrite the order parameter more explicitly in the form:

\[
\frac{M}{V} = \frac{1}{V} \int d^3 r \, \frac{Tr[m(r)e^{-H/\beta}]}{Tr e^{-H/\beta}} = \frac{Tr[m(0)e^{-H/\beta}]}{Tr e^{-H/\beta}}
\]

(3)
The last relation is obtained under the assumption that the system is translationally invariant. Assume that the magnetic field \( H \) is coupled linearly to the order parameter:

\[
H = H_0 - H \int d^3r m(r)
\]  

(4)

Where \( H_0 \) is the Hamiltonian for \( H=0 \), Differentiating (3) with respect to \( H \), we obtain:

\[
\chi = \frac{1}{kT} \int d^3r [\langle m(r)m(0) \rangle - \langle m(0) \rangle^2]
\]  

(5)

Comparison with the definition of the correlation function gives:

\[
\chi = \frac{1}{kT} \int d^3r \Gamma(r)
\]  

(6)

By Monte Carlo simulations, we have calculated mean-square fluctuations for colloids energy configuration are set on square 2D periodic substrates interacting via a long range screened Coulomb potential on any specific and fixed substrate. The correlation function for ensemble average of colloids energy is defined as:

\[
\Gamma(r) = \langle E(r)E(0) \rangle - \langle E(r) \rangle \langle E(0) \rangle
\]  

(7)

And then the heat capacity is defined as:

\[
C_v = \frac{1}{kT} \int d^3r [\langle E(r)E(0) \rangle - \langle E(0) \rangle^2]
\]  

(8)

and comparison with definition of the correlation function gives:

\[
C_v = \frac{1}{kT} \int d^3r \Gamma(r)
\]  

(9)

That is a special case of fluctuation- dissipation theorem with relation between the heat capacity and the energy fluctuations of colloids interacting via a long range screened Coulomb potential on square 2D periodic substrates. Here, we have calculated this theorem for any specific and fixed substrate of colloidal configurations separately.

3. Simulation of diffusion of colloidal particle and results

3.1. Model of simulation

Colloidal ordering and melting on 2D periodic square and triangular substrates using Langevin simulations have been studied by C. Reichhardt [1][11][12]. They also studied colloidal ordering and disordering on two-dimensional periodic substrates where the number of colloids per substrate minima is two or three by Langevin simulations [11,12]. Here we use the Monte Carlo simulation and metropolis algorithm for survey of particles’ motion and obtain the statistical quantities and colloidal particle configuration. 10^6 Loops of Monte Carlo are used in this experiment as the number of particles’s motion. We consider a 2D system of N=150 colloidal particles interacting with a repulsive Yukawa potential in the presence of a substrate. We perform a Monte Carlo simulation of a 2D system with periodic boundary conditions in the x and y directions. There are N colloids interacting with a 2D periodic substrate which has \( N_{\min} \) minima. We focus on the case \( \frac{N}{N_{\min}} = 1 \).
The overdamped equation of motion for a single colloid $i$ is:

$$\frac{dr_i}{dt} = f_i + f_s + f_T,$$

Where,

$$f_i = \sum_{j \neq i} N_i \nabla V(r_{ij})$$

Is the interaction force from the other colloids, which we take to be a Yukawa or screened Coulomb form:

$$V(r_{ij}) = \frac{Q_i Q_j}{\kappa |r_i - r_j|} \exp(-\kappa |r_i - r_j|)$$

The colloid-colloid interaction is a long range Yukawa or screened Coulomb potential. Here $Q$ is the charge of the particles, which we set to 1.0, $1/\kappa$ is the screening length, and $r_{ij}$ is the position of particle $(i, j)$. The system length is measured in units of the lattice constant $a_0$ and we take the screening length $\frac{1}{\kappa} = \frac{a_0}{10}$. We consider a square periodic substrate with a lattice constant $a_0$ and strength $A$.

In this work we keep $a_0$ fixed and vary $A$. The thermal force $f_T$ is a randomly fluctuating force from random kicks. The results of our simulation have been obtained on the (table1) and (table2) and 'figures1', 'figure2','figure3','figure4' and 'figure5' as well.

### 3.2. Simulation of diffusion of colloidal particle configuration

In the project we verify the simulation law in Brownian motion of colloidal particle interacting Yukawa potential on 2D periodic substrate as well. 'figure 1a','figure1b','figure 1c' and 'figure1d' show different stages of the stochastic diffusion process in low& (critical)&high phases for substrate strength($A=1$) with topological defects[13] such as dislocation and declination[14,15,16,17,18].In 'figures2',the Specific heat of the colloidal particle configurations vs T has been done in low temperatures phase. The variation versus temperature is performed for substrate strength ($A=3$) as well. The specific heat has two Gaussian-like peaks at low critical temperature phase. Two successive peaks on the variation of heat capacity of colloids strongly indicate to two consecutive stage of diffusion in low critical temperatures phase. Actually curves of 'figures2','figure3'represent data from 'figures1' with Gaussian-like distribution. The Brownian displacement of diffusion colloids in 'figures2' and 'figure3', in which is a sum of a large number of random steps, has a Gaussian distribution. Obtained results in (table1), confirm the diffusion law and the fluctuation-Dissipation theorem for colloidal particle's energy interacting with Yukawa potential on 2D periodic substrate numerically too. In this here, we have obtained results of diffusion of colloidal particle for different phase consist of low temperature phases, critical phase and high temperature phase for different potential strength of colloidal particles configuration. Two-step process of continuous diffusion occurs at critical temperatures phase: 'figures1b','figure1c'. In
very low temperatures phase particles are completely positionally and orientationally ordered: 'figure1a'. After a two-step process of continuous diffusion of colloids during intermediate critical temperature phase 'figure1a' 'figure1b', configurations of particles are completely disordered in high temperatures phase: 'figure1d'.

4. The numerically result of the Fluctuation-Dissipation Theorem of Colloidal Particle's energy

Thermodynamic quantities are supposed to be constant when the system is in thermal equilibrium. If we measure them with high precision, however, we will notice that they undergo small fluctuations. For example, the pressure a gas exerts on a wall fluctuates because of the randomness of atomic impacts. The internal energy of the gas fluctuates because it exchanges energy with the environment via atomic collisions. These fluctuations arise from the granular structure of matter and appear as thermal noise. We have calculated the mean-square fluctuations in statistical mechanics, such as those for the system's energy. We can usually ignore them because they are vanishingly small in the thermodynamic limit. We surveyed the relation of fluctuation-dissipation of energy to form:

\[ \beta(\langle E^2 \rangle - \langle E \rangle^2) = TC_v \]  

(14)

We'll investigate the actual range of the system's energy, in which is likely to be diffusion. In order to, we must write the average energy of canonical ensembles as follow:
\[ U \equiv \langle E \rangle = \frac{\sum_{r} E_r \exp(-\beta E_r)}{\sum_{r} \exp(-\beta E_r)} \quad (15) \]

By assume the energy \( E_r \) as the constant quantity; we will take the derivative \( U \) with respect to \( \beta \)

\[
\frac{\delta U}{\delta \beta} = \frac{\sum_{r} E_r^2 \exp(-\beta E_r)}{\sum_{r} \exp(-\beta E_r)} + \left[ \frac{\sum_{r} E_r \exp(-\beta E_r)}{\sum_{r} \exp(-\beta E_r)} \right] - \langle E^2 \rangle - \langle E \rangle^2 = \left( \frac{\delta U}{\delta \beta} \right) = kT^2 \left( \frac{\delta U}{\delta T} \right) = kT^2 C_v \quad (16)
\]

Thus we have:

\[
\langle (\Delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left( \frac{\delta U}{\delta \beta} \right) = kT^2 \left( \frac{\delta U}{\delta T} \right) = kT^2 C_v \quad (17)
\]

Using the relation above, for fluctuation of the relative root mean square of \( E \), we will have:

\[
\frac{\sqrt{\langle (\Delta E)^2 \rangle}}{\langle E \rangle} = \frac{\sqrt{kT^2 C_v}}{U} \quad (18)
\]

With \( O(N^{-1/2}) \), that \( N \) is the number of particles in the system. By expansion of the logarithm of the probability density \( P(E) \) around the most probable of energy value, \( E^* \approx U \), will have:

\[
\ln \left[ e^{-\beta E} g(E) \right] = \left( -\beta U \frac{S}{K} + \frac{1}{2} \frac{\partial^2}{\partial E^2} \ln \left[ e^{-\beta E} g(E) \right] \right)_{E=U} (E-U)^2 + ...
\]

\[
= -\beta (U-TS) - \frac{1}{2kT^2 C_v} (E-U)^2 + ...
\]

\[
(19)
\]

As result, will have:

\[
P(E) a e^{-\beta E} g(E) = e^{-\beta(U-TS)} \exp \left\{ \frac{(E-U)^2}{2kT^2 C_v} \right\} = e^{-\beta(\Delta E)} \exp \left\{ \frac{\langle (\Delta E)^2 \rangle}{2kT^2 C_v} \right\} \quad (20)
\]

This function is represented a Gaussian distribution at \( E \), with average value \( U \) and variance \( \sqrt{kT^2 C_v} \). For quantity\n
\[
\frac{\Delta E}{U} = \sqrt{\frac{k}{U} \frac{T^2 C_v}{U}}
\]

the distribution is also Gaussian that its average value is unit with variance \( \sqrt{kT^2 C_v} / U \) actually \( O(N^{-1/2}) \).

As for limit \( N \to \infty \), we will approach to the Dirac delta function.

In (table1), we have such result for our colloidal configuration. For example, for \( A=0 \) and \( \beta=1 \), will have: \( U = \langle E \rangle \)

\[
=212.54, \quad \sqrt{\langle (\Delta E)^2 \rangle} = \Delta E = 0.351511024, \quad \sqrt{\frac{(\Delta E)^2}{\langle E \rangle}} = \frac{\Delta E}{\langle E \rangle} = \frac{\sqrt{kT^2 C_v}}{U} = 0.001653858 \ll 1
\]

For \( A=1 \) and \( \beta=1 \), will have:

\[
U = \langle E \rangle = 211.05, \quad \sqrt{\langle (\Delta E)^2 \rangle} = \Delta E = 0.348883935, \quad \sqrt{\frac{(\Delta E)^2}{\langle E \rangle}} = \frac{\Delta E}{\langle E \rangle} = \frac{\sqrt{kT^2 C_v}}{U} = 0.001653087 \ll 1
\]
We easily see that, \( U = \langle E \rangle \) for \( A=0 \) \( >U = \langle E \rangle \) for \( A=1 \).

More results are obtained in Table1 completely. With more study of results obtained of (table1) numerically, we will obtain such \( O(N^{-1/2}) \) for topological Defect Colloidal Particles on square 2D Periodic Substrates interacting via a long range screened Coulomb potential. Finally, by survey of partition function of canonical ensemble of this system, will have:

\[
Q_N(V,T) = e^{-\beta(U-TS)} \int_0^\infty \frac{(E-U)^2}{2kTC} dE = e^{-\beta(U-TS)} \sqrt{2kT^2C} \int_0^\infty e^{-\beta x^2} dx = e^{-\beta(U-TS)} \sqrt{2kT^2C} (21)
\]

Therefore:

\[
-kT \ln Q_N(V,T) \approx F \approx (U-TS) - \frac{1}{2} kT \ln(2\pi kT^2C) \Rightarrow F \approx U - S \tag{22}
\]

The last sentence is order of \( \ln N \), whereas the others are order of \( N \). Therefore, we can ignore the last sentence with respect the others. As the result, we obtain a thermodynamic sentence according to the formulation of canonical ensembles. The following equation is used to calculate the order parameter's variation of particles:

\[
\overline{m} = \cos \left( \frac{2\pi}{A} \sum_{i=1}^{N} x_i + \frac{2\pi}{A} \sum_{i=1}^{N} y_i \right)
\]

\[
M = \overline{m} / N \tag{23}
\]

These formulas being applied to average magnetization of all particle and will be then calculated the order parameter of any particle.

**Table1.** By looking at the tables, we can easily such relation between the heat capacity and energy fluctuation of colloids (for different substrates: \( A=0,1 \), \( N=150 \); number of colloids) duration temperature interval: \( T \sim 0.9KT \) to \( T \sim 1 \), are actually the critical temperatures interval.

| T   | E   | \( \Gamma(r) \) \( A=0 \) | \( C_v \) \( A=0 \) | \( C_v \) \( A=1 \) | \( \sqrt{\langle \Delta E \rangle} \) \( A=0 \) | \( \Delta E \) / \( \langle E \rangle \) \( A=0 \) | M   | \( \sqrt{\langle \Delta E \rangle} \) \( A=1 \) | \( \Delta E \) / \( \langle E \rangle \) \( A=1 \) |
|-----|-----|-----------------|-----------------|-----------------|-------------------------------|-----------------|-----|-------------------------------|-----------------|
| 1.00 | 212.54 | 0.1235           | 0.01527         | 0.8698          | 0.351511024                   | 0.001653858     | 211.05 | 0.12172                       | 0.04187          | 0.8771 | 0.348883935                  | 0.001653087     |
| 0.99 | 212.51 | 0.1077           | 0.01178         | 0.8729          | 0.328603104                   | 0.001546295     | 211.03 | 0.10628                       | 0.011409         | 0.8796 | 0.326006135                  | 0.001544833     |
| 0.98 | 212.50 | 0.10967          | 0.01227         | 0.8885          | 0.331164612                   | 0.001558422     | 211.01 | 0.11201                       | 0.012801         | 0.8768 | 0.334678951                  | 0.001586081     |
| 0.97 | 212.45 | 0.17419          | 0.03127         | 0.8730          | 0.417360755                   | 0.001964513     | 210.97 | 0.16725                       | 0.028828         | 0.8840 | 0.408962101                  | 0.001938485     |
| 0.96 | 212.46 | 0.11091          | 0.01281         | 0.8777          | 0.33303153                    | 0.001567502     | 210.85 | 0.27592                       | 0.079256         | 0.8776 | 0.525280877                  | 0.002491254     |
| 0.95 | 212.44 | 0.11585          | 0.01411         | 0.8747          | 0.340367449                   | 0.001602182     | 210.96 | 0.1093                        | 0.02562           | 0.8802 | 0.330605505                  | 0.001567148     |
| 0.94 | 211.85 | 0.30929          | 0.10161         | 0.8749          | 0.556138472                   | 0.002625152     | 210.94 | 0.106                         | 0.01935          | 0.8736 | 0.325576412                  | 0.001543455     |
| 0.93 | 212.31 | 0.26160          | 0.07342         | 0.8700          | 0.511468474                   | 0.002409064     | 210.91 | 0.1149                        | 0.014165         | 0.8791 | 0.33896025                   | 0.001607174     |
| 0.92 | 211.97 | 0.37456          | 0.15204         | 0.8683          | 0.612013072                   | 0.002887263     | 210.4  | 0.36164                       | 0.14173           | 0.8787 | 0.601365114                  | 0.002858199     |
| 0.91 | 211.61 | 0.13240          | 0.01919         | 0.8754          | 0.363868108                   | 0.001719522     | 210.41 | 0.38153                       | 0.15935           | 0.8707 | 0.617681148                  | 0.002935607     |
| 0.908 | 211.60 | 0.14175          | 0.02222         | 0.8786          | 0.376497012                   | 0.001770286     | 210.1  | 0.13422                       | 0.019918         | 0.8783 | 0.366360478                  | 0.001743743     |
| 0.90 | 212.54 | 0.12634          | 0.01783         | 0.8761          | 0.355443385                   | 0.001680107     | 211.05 | 0.12172                       | 0.04187          | 0.8771 | 0.348883935                  | 0.001653087   |
Here, we have calculated the ensemble average of colloid's energy. The correlation function for ensemble average of colloid's energy colloids and the mean-square fluctuations of colloid energy during temperature interval between $T \sim 0.8KT$ to $T \sim 1KT$ (critical interval) is calculated for substrates strength ($A=0,1$). By examining of the numerically results obtained of (tables 1), the special case of "fluctuation- dissipation theorem" for energy of colloidal particle configurations will be satisfied as well. We can readily write relation between the heat capacity and the energy fluctuation of our colloidal particle configuration according to 'equations (14-20)'.

\begin{align*}
\text{Figure 2.} \quad & \text{The graph of the energy and the Specific heat fluctuation of the colloidal particle configurations vs $T$ in low temperatures phase for $A=3$, (N=150). Two successive peaks on the variation of them strongly indicate resembles and present that of KT-like of first order phase transition, the theory of Kosterlitz, Thouless, Halperin, Nelson and Young (KTHNY) defect-mediated topological phase transitions.}[14-18].
\end{align*}

\begin{align*}
\text{Figure 3.} \quad & \text{The graph of the Specific heat of the colloidal particle configurations between (T=0.8KT to T=2KT),(N=150). Here both low temperature and high temperature phase are considered. Two successive phase transition (Tc1~0.9224KT, Tc2~0.94148KT for A=0[17,18]) occurred over low critical temperatures (T \sim 0.8KT to T \sim 1KT) confirming of Gaussian distribution structure. But after T=1KT the fluctuations of Cv have mild and fairly constant trend over this range of temperature (T \sim 1KT to T \sim 2KT) in comparison with the previous low temperature phase(T \sim 0.8KT to T=1KT). In high temperature phase (T \sim 1KT-T \sim 2KT),}
\end{align*}
configurations have no kind of ordering and topological structures of defects (disclination and dislocation[14-18]) have been totally destroyed and the diffusion process has been done completely. 'figure3a' refer to ensemble average of colloid's energy vs T. 'figure3b' indicate to fluctuation of heat capacity vs temperature with Gaussian distribution structure indicating linear form of the Fluctuation-Dissipation Theorem. 'figure 3d', 'figure3e' refer to variation of energy and heat capacity of colloidal configuration at all temperature regions(low, critical and high temperature: $T \sim 0.3$ to $T \sim 5$) for $A=0$.

Random fluctuations with small deviations from the state of thermodynamic equilibrium arise from the granular structure of them and appear as thermal diffusion with Gaussian distribution structure, are showing linear form of the Fluctuation-Dissipation Theorem constitutive a canonical ensemble. The data obtained of (table1) is completely similar to 'equations (14-20)', with Gaussian distribution structure; all are confirming of linear form of the Fluctuation-Dissipation Theorem constitutive a canonical ensemble.

According to 'figure 4', there are relatively well comparisons between various substrate strength of colloid energy configuration versus temperature at low temperatures phase pointing to critical low temperature points.

![Figure 4](image)

**Figure 4.** There is relatively well comparison between different substrate strength of the colloids's energy versus temperature at low temperatures phase (N=150). As can be seen, as $A$ increases from zero to $A=3$, the energy decrease reversely for all the temperatures given in low temperature phase. Actually with increasing of substrate strength, the energy of colloidal particle configuration monotonically decreases. The calculated energy of colloidal particles on (table1) for $A=0$ and $A=1$, also confirm the results of 'figure4' as well.

5. Simulation of noise-like fluctuation of colloids

In this here, according to the number of motion of each particle in system of colloidal configuration the variation of order parameter is characterized with 'equation (23)' at the end of particles motion.

Actually, the above equation is showing the phase relationship between different particles on the critical temperatures phase ($T \sim 0.8KT - T \sim 1KT$) with different substrates strength. The variation of the order parameter versus the number of particle's motion is calculated at the definite critical temperatures [17, 18] with various substrates strength. We perform $10^6$Loops of Monte Carlo as the number of particles' motion in this experiment. In (table2), for example: $A=1$, $T \sim 1$, the energy per particle is calculated for 1000000 loops (the number of particle's motion) separately. The variations of order parameter versus the number of particle's motion (Markov Points) all are confirming of noise-like fluctuation and Brownian motion of colloidal particles as well. The fluctuations arise from the granular structure of matter and appear as thermal noise 'figures 5'. There is the large number of displacement in the coordinate of Brownian particles 'figures 5'. If we now want to measure of the energy and the order parameter, as obtained in 'figures5' for different temperatures, we will finally get a Gaussian distribution of energy performed in 'figures 2', 'figure3'and(table1) as well.
Figure 5. The potential energy per particle, and the order parameter versus the number of particles' motion in the critical temperatures interval (critical temperatures $T \sim 0.8 KT$ to $T \sim 1 KT$) confirming of noise-like fluctuation and diffusion like Brownian motion of colloidal particles. The variation of energy per particle versus the number of moving of particles: For $A=0,N=150$: (5a) at $T \sim 0.92274 KT$. (5b) The variation of the order parameter per particle versus the number of moving of particles for $A=3,N=150$: (5c) at $T \sim 0.9320 KT$. (5d) at $T \sim 0.9605 KT$.

Table 2. The energy per particle is calculated for 1000000 the number of particle's motion as loops of Monte Carlo Simulation separately ($A=1,T=1,N=150$) .

| The number of motion | Energy per particle | The number of motion | Energy per particle | The number of motion | Energy per particle | The number of motion | Energy per particle | The number of motion | Energy per particle |
|----------------------|---------------------|----------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 100000              | 211.0299            | 300000              | 211.0417            | 500000              | 211.0875            | 700000              | 211.037             | 900000              | 211.206             |
| 200000              | 211.1988            | 400000              | 210.9702            | 600000              | 211.1137            | 800000              | 211.003             | 1000000             | 211.209             |

6. Conclusion

Using the Monte Carlo simulations, we have calculated mean-square fluctuations in statistical mechanics, such as those for colloids energy configuration are set on square 2D periodic substrates interacting via a long range screened coulomb potential for different substrate. Random fluctuations with small deviations from the state of thermodynamic equilibrium arise from the granular (colloidal) structure of them and appear as thermal diffusion with a Gaussian distribution structure. The obtained results in the paper are showing linear form of the Fluctuation-Dissipation Theorem on the particle's energy constitutive a canonical ensemble with continuous diffusion process of colloidal particles system. The numbers on the (table1) write the numerically results of the Fluctuation-Dissipation Theorem of Colloidal Particle's energy interacting with Yukawa potential on 2D periodic
substrate. The observed results of both graph of colloidal particles configuration 'figure2' and the graph of the variation of statistical quantities of them 'figure3' at three phase of low, critical and high temperatures represent diffusion process of colloids completely [17,18]. We can easily that, the continuous process of phase transition of colloidal particles happened in the low temperatures phase 'figure3'. A good comparison is done between different substrates strength of the energy configuration of colloids versus temperature at low temperatures phase 'figure4'. As can be seen, as A increases from zero to A=3, the energy decrease all the temperatures given in low phase reversely. With the increasing substrate strength the colloidal particle's energy will monotonically decreases. The colloidal particles's energy for A=0, A=1 on the (table1) are also confirming the issue. The variations of energy per particle were calculated according to the random Brownian motion of colloidal particles (or loops of Monte Carlo Simulation (table2)) as well. The obtained results on the (table2) are actually consistent with results of 'figure5a', 'figure5b', too. The variations of the order parameter versus the number of particle's motion 'figures5c', 'figure5d' are also confirming of noise-like fluctuation of colloidal particles as well.

7. References
[1] Reichhardt C and Olson C J 2002 Phys. Rev. Lett. 88 248301.
[2] Brunner M and Bechinger C 2002 Phys. Rev. Lett. 88 248302.
[3] Murray C A and Winkle D H V 1987 Phys. Rev. Lett. 58 1200 Zahn K, Lenke R and Maret G 1999 Phys. Rev. Lett. 82 2721 Pertsinidis A and Ling X S 2001 Nature (London) 413 147.
[4] Mangold K, Leiderer P and Bechinger C 2003 Phys. Rev. Lett. 90 158302.
[5] Korda P T, Spalding G C and Grier D G 2002 Phys. Rev. B 66 024504 Korda P T, Taylor M B and Grier D G 2002 Phys. Rev. Lett. 89 128301.
[6] M. Brunner and C. Bechinger, Phys. Rev. Lett. 88, 248302 (2002); K. Mangold, P. Leiderer and C. Bechinger, ibid. 90, 158302 (2003).
[7] Prof. Matthias Troyer; Computational PhysicsII-ETH Zurich, SS 2004.
[8] W. Janke: Monte Carlo in classical statistical Methods physic, Lect. notes phys 739, 79-140 (2008), Springer-Verlag Berlin Heidelberg 2008.
[9] Reichhardt C, Olson C J, Scalettar R T and Zimányi G T 2001 Phys. Rev. B 64 144509.
[10] M Mikulis, C. J. Olson Reichhardt, C. Reichhardt, R T Scalettar and G T Zimányi, J. Phys: Condens. Matter 16, 7909 (2004).
[11] C. Reichhardt, C.J. Olson Reichhardt, Europhys. Lett. 68, 303 (2004).
[12] A. Libál, C. Reichhardt and C. J. Olson Reichhardt1, Phys. Rev. E 75, 011403 (2007).
[13] D.R. Nelson, Defects and Geometry in Condensed Matter Physics (Cambridge Univ. Press, Cambridge, 2002).
[14] Kosterlitz J M and Thouless D J 1973 J. Phys. C: Solid State Phys. 6 1181.
[15] Nelson D R and Halperin B I 1979 Phys. Rev. B 19 2457–84.
[16] Halperin B I and Nelson D R 1978 Phys. Rev. Lett. 41 121–4.
[17] Najafi Amin et al, Proceeding of 2nd IPM Conference on Soft Matter, Biological and Statistical Physics, School of Physics, IPM, Tehran (2013).
[18] Najafi Amin, Proceeding of XXV IUPAP International Conference on Statistical Physics. STATPHYS25, Seoul, Korea (2013).