Progress on six dimensional ionization muon cooling with relatively small rings of magnets is described. Lattices being explored include scaling sector cyclotrons with edge focusing and strong focusing, fixed field alternating gradient (FFAG) rings. Ionization cooling is provided by high pressure hydrogen gas which removes both transverse and longitudinal momentum. Lost longitudinal momentum is replaced using radio frequency (RF) cavities, giving a net transverse emittance reduction. The longer path length in the hydrogen of higher momentum muons decreases longitudinal emittance at the expense of transverse emittance. Thus emittance exchange allows these rings to cool in all six dimensions and not just transversely. Alternatively, if the RF is located after the ring, it may be possible to cool the muons by stopping them as they spiral adiabatically into a central swarm. As $p \to 0$, $\Delta p \to 0$. The resulting cooled muons can lead to an intense muon beam which could be a source for neutrino factories or muon colliders.

Keywords: beam cooling; cyclotron; muon; black hole.

1. Introduction

Muons at rest have a 2.2 $\mu$s lifetime; cooling an ensemble of muons must be completed faster than that. Ionization cooling can help. Random muon motion is removed by passage through a low Z material, such as hydrogen, and coherent motion is added with RF acceleration. Designs to cool muons in six dimensions using linear
helical channels\(^2\) at 100 MeV kinetic energies and using frictional cooling\(^3\) at 1 keV kinetic energies are under investigation. A number of muon cooling rings have been simulated at various levels.\(^4\) In a ring structure, the same magnets and RF cavities may be reused each time a muon orbits. Transverse cooling can naturally be exchanged for longitudinal cooling by allowing higher momentum muons to pass through more material. Thus rings can cool in all six dimensions. The rings reported here are the smallest to date, and are basically radial sector cyclotrons.

Small emittance bunches of cold muons are useful for a neutrino factory\(^5\) and are required for a muon collider.\(^6\) At a neutrino factory, accelerated muons are stored in a racetrack to produce neutrino beams (\(\mu^- \rightarrow e^- \nu_e \nu_\mu\) and \(\mu^+ \rightarrow e^+ \nu_e \nu_\mu\)). Neutrino oscillations have been observed\(^7\) and need more study. Further exploration at a neutrino factory could reveal CP violation in the lepton sector,\(^8\) and will be particularly useful if the \(\nu_e\) to \(\nu_\tau\) coupling, \(\theta_{13}\), is small.\(^9\) A muon collider can do s-channel scans to split the \(H^0\) and \(A^0\) Higgs doublet.\(^10\) Above the ILC’s 800 GeV there are a large array of supersymmetric particles that might be produced\(^11\) and, if large extra dimensions exist, so could mini black holes.\(^12\) Note that the energy resolution of a 4 TeV muon collider is not smeared by beamstrahlung.

2. Hydrogen Gas Filled Sector Cyclotrons with Internal RF

A low field ring is being designed using ICOOL\(^13\) for optimization and SYNCH\(^14\) for lattices. It could be built to demonstrate six dimensional muon cooling. Relatively constant \(\beta\) functions allow a continuous placement of the energy absorber. Hydrogen gas cools best and also prevents breakdown in the 201 MHz RF cavities.\(^2\) Skew quadrupole magnets mix vertical and horizontal betatron oscillations. The parameters for the ring appear in Table 1. Simulated 6D cooling is shown in Fig. 2.

The equation describing transverse cooling (with energies in GeV) is:

\[
\frac{d\epsilon_n}{ds} = -\frac{1}{\beta^2} \frac{dE_\mu}{ds} \frac{\epsilon_n}{E_\mu} + \frac{1}{\beta^3} \frac{\beta_\perp (0.014)^2}{2 E_\mu m_\mu L_R},
\]

\(1\)

Fig. 1. A way of fitting a pair of large 201 MHz RF cavities into a small four sector cyclotron. The cavities fill the center of the cyclotron.

Fig. 2. \(X, Y, \) and \(Z\) emittance versus orbits for the low field ring with four sectors that is described in Table 1.
Table 1. Scaling sector cyclotron parameters. The cooling merit factors include transmission loss; the X, Y, and Z cooling factors do not. The low field ring is designed to test 6D cooling.

|                  | Low Field Ring | High Field Ring | FFAG Ring |
|------------------|---------------|-----------------|----------|
| Dipole Field     | 1.8 T         | 5.2 T           | 2.6 T    |
| Magnetic Elements| hard edge     | hard edge       | hard edge          |
| Sectors          | 4             | 6               | 12       |
| Focusing         | edge          | edge            | alternating gradient |
| $\beta_x$ Range | 38 $\rightarrow$ 92 cm | 26 $\rightarrow$ 36 cm | 44 $\rightarrow$ 65 cm |
| $\beta_y$ Range | 54 $\rightarrow$ 66 cm | 30 $\rightarrow$ 32 cm | 26 $\rightarrow$ 42 cm |
| $p$(central orbit) | 172 MeV/c | 250 MeV/c       | 250 MeV/c |
| Hydrogen Pressure| 40 Atm. @ 3000 K | 100 Atm. @ 3000 K | 100 Atm. @ 3000 K |
| Peak RF Gradient | 14 MV/m       | 45 MV/m         | 8 MV/m   |
| Total RF Length  | 1.6 m         | 0.8 m           | 3.6 m    |
| Ring Circumference | 3.81 m    | 1.95 m          | 6.0 m    |
| X Aperture       | $\pm$20 cm    | $\pm$25 cm      | $\pm$25 cm |
| Y Aperture       | $\pm$15 cm    | $\pm$15 cm      | $\pm$15 cm |
| $p_x$ Acceptance | $\pm$10 MeV/c | $\pm$10 MeV/c  | $\pm$10 MeV/c |
| Orbits           | 100           | 250             | 40       |
| X, Y, Z Cooling Factors | 2.4, 2.3, 7.4 | 2, 13, 20        | 16, 15, 2 |
| Muon Decay Included | no           | yes             | yes      |
| Cooling Merit Factor | 20          | 400             | 120      |

where $\beta = v/c$, $\epsilon_n$ is the normalized emittance, $\beta_\perp$ is the betatron function (focal length) at the absorber, $dE_p/ds$ is the energy loss, and $L_R$ is the radiation length of the material. The first term in this equation is the cooling term, and the second is the heating term due to multiple scattering. This heating term is minimized if $\beta_\perp$ is small (strong-focusing) and $L_R$ is large (a low-Z absorber). The equilibrium emittance is achieved when the heating and cooling terms balance.

A higher field ring with smaller values of $\beta_\perp$ can give a higher cooling merit factor. Simulations show that high RF gradients are required as noted in Table 1.

A scaling fixed field, alternating gradient (FFAG) ring\textsuperscript{15} can allow the use of lower RF gradients and magnetic fields. For the FFAG ring in Table 1, a focusing parameter, $n = -(r/B)(dB/dr) = -0.6$, was used. Each sector has a focusing-defocusing-focusing (FDF) geometry with three magnets. There are no open slots between sectors, so the RF cavities would have to be placed within the magnets.

3. Sector Anti–Cyclotrons with External RF

An anti-cyclotron has been used to slow LEAR anti-protons at CERN\textsuperscript{16,17}. An annular quasipotential well, $U(r, z)$, is formed which ferries anti-protons towards the center of an azimuthally symmetric cyclotron. The radius of the annulus decreases with the decreasing angular momentum of the $p$.

$$U(r, z) = V(r, z) - (1/(2\eta r^2)) (L_g/M + \eta r A_\theta)^2,$$

(2)

where $\eta = e/M$ and $L_g = L_z - e r A_\theta$ is a generalized angular momentum. The radial well deepens with decreasing radius and the vertical well grows shallower
(see Fig. 2 of Ref. 16). Particles must adiabatically spiral to the center. If \( \frac{dE}{dx} \) is too large, particles will not stay in the magnetic wells. The final \( \bar{p} \) swarm has a radius of 1.5 cm, a height of 4 cm, and a kinetic energy of 2 keV. A long bunch train is coalesced into a single swarm, which is roughly the same diameter as the incoming beam. The spiral time is 20 \( \mu s \) with 0.3 mbar hydrogen and about 1 \( \mu s \) with 10 mbar hydrogen. Given the dependence of the cyclotron frequency on mass, \( f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m} \), the spiral time for a muon is nine time less than for a \( p \). The gas pressure in the center must be low, both to allow a particle to spiral all the way in before stopping, and to allow reasonable kicker voltages for extraction. An 80 ns electric kicker pulse rising to 500 V/cm in 20 ns is employed. The \( p \)'s move 32 cm in 500 ns. Given that \( F = ma \), muons will go farther.

The anti-cyclotron has now been moved from LEAR at CERN to PSI where it is being used to slow negative muons to kinetic energies of a few keV.\(^\text{18}\) Three centimeter diameter beams with 30000 \( \mu^-/s \) below 50 keV and 0.8 cm diameter beams with 1000 \( \mu^-/s \) in the 3 to 6 keV kinetic energy range are output for use. A static electric field continuously ejects the muons. The energy absorber and the electrode consist of a single 30 \( \mu g/cm^2 \) formvar foil with a 3 nm nickel film produced by 30 minutes of sputtering.

A scaling sector anti-cyclotron could allow greater acceptance than the azimuthally symmetric anti-cyclotron now running at PSI. For a given \( \int B \cdot d\ell \), the ra-
Table 2. Slow muon sticking solutions. Charged foils might repel muons and prevent them from stopping in the foils. In muon catalyzed fusion, a $\mu^-$ is typically freed 150 times before it sticks to an $^4\text{He}$ nucleus.

| Positively charged foils for $\mu^+$. Helium to slow $\mu^+e^-$ formation. Laser disassociate $\mu^+e^-$. |
|--------------------------------------------------------------------------------------------------|
| Negatively charged foils for $\mu^-$. DT or $D_2$ gas. Fusion frees $\mu^-$. |

Table 3. Emittance reduction goals for two anti-cyclotrons used in series. Between the anti-cyclotrons, Busch’s theorem gives a $\Delta p_x$ kick, muons are accelerated, and $\Delta p_z$ is traded for $\Delta x$ and $\Delta p_y$ for $\Delta y$. Emittance goes as $(\Delta p_x \Delta x)(\Delta p_y \Delta y)(\Delta p_z \Delta z)$. A muon collider needs a factor of $10^6$ in cooling.

| First Anti-Cyclotron | Second Anti-Cyclotron |
|----------------------|-----------------------|
| $\Delta p_x$         | $50 \rightarrow 1 \text{MeV/c}$ | $15 \rightarrow 1 \text{MeV/c}$ |
| $\Delta p_y$         | $50 \rightarrow 1 \text{MeV/c}$ | $15 \rightarrow 1 \text{MeV/c}$ |
| $\Delta p_z$         | $15 \rightarrow 10 \text{cm}$  | $5 \rightarrow 3 \text{cm}$    |
| $\Delta x$           | $15 \rightarrow 10 \text{cm}$  | $5 \rightarrow 3 \text{cm}$    |
| $\Delta y$           | $1000 \rightarrow 10 \text{cm}$| $10 \rightarrow 3 \text{cm}$  |

In summary, progress on large acceptance tabletop muon rings is underway, including energy loss injection (see Fig. 4), 6D cooling, and electric kicker extraction. Many thanks to Juan Gallardo and Franz Kottmann for useful suggestions.

References

1. A. Skrinsky and V. Parkhomchuk, Sov. J. Part. Nucl. 12, 223 (1981); D. Neuffer, Part. Accel. 14, 75 (1983); Nucl. Instrum. Meth. A532, 26 (2004);
6 D. J. Summers

G. Penn and J. S. Wurtele, *Phys. Rev. Lett.* **85**, 764 (2000);  
K. Kim and C. Wang *Phys. Rev. Lett.* **85**, 760 (2000); *ibid.* **88**, 184801 (2002);  
G. Franchetti, *Phys. Rev. ST Accel. Beams* **4**, 074001 (2001); G. Dugan, 104001.

2. Y. Derbenev, R. Johnson, *NIM* **A532**, 470 (2004); *AIP Conf. Proc.* **671**, 328 (2003).

3. R. Galea *et al.*, *Nucl. Instrum. Meth.* **A524**, 27 (2004); *J. Phys.* **G29**, 1653 (2003);  
A. Caldwell, *J. Phys.* **G29**, 1569 (2003); H. Abramowicz *et al.*, *physics/0410017*.

D. Taqqu, *AIP Conf. Proc.* **372**, 301 (1996); M. Muhlbauer *et al.*, *Nucl. Phys. Proc. Suppl.* **51A**, 135 (1996); *Hyperfine Interact.* **119**, 309 (1995).

4. V. I. Balbekov and A. Van Ginneken, *AIP Conf. Proc.* **441**, 310 (1998);  
J. S. Berg, R. C. Fernow, and R. B. Palmer, *J. Phys.* **G29**, 1657 (2003);  
R. B. Palmer, *J. Phys.* **G29**, 1577 (2003); *Nucl. Instrum. Meth.* **A532**, 255 (2004).

5. A. Blondel *et al.*, *Nucl. Instrum. Meth.* **A451** (2000) 102; R. Palmer *et al.*, *ibid.*, 265;  
D. Ayres *et al.*, *physics/9911009*; N. Holtkamp *et al.*, “A neutrino source based on a muon storage ring,” Fermilab-Pub-00-108-E; S. Ozaki *et al.*, “Study II,” BNL-52623.

6. G. Budker, *AIP Conf. Proc.* **352**, 4 (1996); **352**, 5 (1996); A. Skrinsky, **352**, 6 (1996).

7. R. Davis *et al.* (Homestake), *AIP Conf. Proc.* **441**, 310 (1998);  
Y. Fukuda *et al.* (SuperK), *Phys. Rev. Lett.* **81**, 1562 (1998); **93**, 101801 (2004);  
Q. R. Ahmad *et al.* (SNO), *Phys. Rev. Lett.* **89**, 011301 (2002); **92**, 181301 (2004);  
K. Eguchi (KamLAND), *Phys. Rev. Lett.* **90**, 021802 (2003); *hep-ex/0406035*.

M. Ahn *et al.* (K2K), *Phys. Rev. Lett.* **90**, 041801 (2003); E. Aliu, *hep-ex/0411038*.

8. S. Geer, *Phys. Rev.* **D57**, 6989 (1998); C. Albright *et al.*, *hep-ex/0008064*; V. Barger *et al.*, *Phys. Rev. Lett.* **45**, 2084 (1980); A. Cervera *et al.*, *Nucl. Phys.* **B579**, 17 (2000).

9. C. Albright *et al.*, *physics/0411123*; M. Maltoni *et al.*, *hep-ph/0405122*.

10. V. Barger *et al.*, *Phys. Rev. Lett.* **75**, 1462 (1995); *Phys. Rept.* **286**, 1 (1997);  
D. Atwood and A. Soni, *Phys. Rev.* **D52**, 6271 (1995); J. F. Gunion, *hep-ph/9802258*.

11. J. Ellis, *LCWS 04*, *hep-ph/0409140*; P. W. Higgs, *Phys. Rev.* **145**, 1156 (1966).

12. R. Godang *et al.*, *hep-ph/0411248*; M. Cavaglia and S. Das, *hep-th/0404050*.

13. R. C. Fernow, *AIP Conf. Proc.* **721**, 90 (2004); PAC99, eConf C990329, THP31.

14. A. A. Garren *et al.*, *AIP Conf. Proc.* **297**, 403 (1994).

15. M. Craddock, *CERN Cour.* **44N6**, 23 (2004); D. J. Summers, *physics/0411218*.

K. Symon, D. Kerst, L. Jones, L. Laslett, K. Terwilliger, *Phys. Rev.* **103**, 1837 (1956);  
J. S. Berg, *AIP Conf. Proc.* **642**, 213 (2003); D. Trbojevic *et al.*, *ibid.* **530**, 333 (2000);  
S. Koscienliak, C. Johnstone, *AIP Conf. Proc.* **721**, 467 (2004); *NIM* **A523**, 25 (2004);  
E. Keil, A. Sessler, *NIM* (2005); Y. Mori, *ICFA Beam Dyn. Newslett.* **29**, 20 (2002).

16. J. Eades and L. M. Simons, *Nucl. Instrum. Meth.* **A278**, 368 (1989).

17. J. Eades *et al.*, *Nucl.Phys.Proc.Suppl.* **8**, 457 (1989); E. Aschenauer *et al.*, *Sov. J. Nucl. Phys.* **55**, 856 (1992); L. M. Simons *et al.*, *Springer Proc. Phys.* **59**, 33 (1992);  
*Phys. Scripta* **T22**, 90 (1988); *Hyperfine Interact.* **81**, 253 (1993); *Phys. Bl.* **48**, 261 (1992); *Nucl. Instrum. Meth.* **B87**, 293 (1994); D. Horváth *et al.*, **B85**, 736 (1994).

18. P. DeCecco *et al.*, *Nucl. Instrum. Meth.* **A394**, 287 (1997); Franz.Kottmann@psi.ch.

19. A. Girard *et al.*, *Rev. Sci. Instrum.* **75**, 1381 (2004); A. Zelenski *et al.*, 1535.

20. H. Busch, *Annalen Phys.* **81**, 974 (1926); A. W. Chao and M. Tigner, *Handbook of Accelerator Physics and Engineering*, page 101 (1999).