Experimental Validation of the Mean Pitch Theory

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Abstract. There are several types of exterior ballistic models used to calculate projectile’s flight trajectories. The most complex 6 degree of freedom rigid body model has many disadvantages to using it to create firing tables or rapid calculations in fire control systems. Some of ballistic phenomena can be simplified by empirical equations without significant loss of accuracy. This approach allowed to create standard NATO ballistic model for spin stabilized projectiles named Modified Point of Mass Model (PM Model). For fin (aerodynamically) stabilized projectiles like mortar projectiles simple Point of Mass Model is commonly used. The PM Model excludes many flight phenomena in calculations. In this paper authors show the mean pitch theory as an approximation of the natural fin stabilised projectile pitch during flight. The theory allows for simple improvement of accuracy of the trajectories calculation. In order to validate the theory data obtained from shooting of supersonic mortar projectiles were used. The comparison of accuracy between simple PM Model and PM Model including mean pitch theory were shown. Results were also compared with the angle of response theory.

1. Introduction
One of the main parts of the exterior ballistic is modelling flight of the projectiles. Across the ages, during evolution of the exterior ballistic, new types of the flight models were created [1]. Many well-known physicists and mathematicians (Newton, Bernoulli) works refer to exterior ballistic problems. At the second part of the nineteenth century Siacci and Mayewski developed first modern drag shapes of projectiles used at these times [2]. That allowed to calculate the projectile flight using material point model with accuracy sufficient for these days. At the first part of the twentieth century researchers in many countries tried to develop the most complex model (including forces and moments) of the projectile flight – the 6 degree of freedom (6-dof) flight model. The first incomplete 6-dof model was published in 1920 by Fowler, Gallop, Lock and Richmond [3]. It was completed by Nielsen and Synge in 1943 [4] and it is used in that form, with minor modifications, to this day. 6-dof models are much more accurate that point mass models [2]. Main disadvantages of the 6-dof model are much higher computational cost (comparing with point mass model) and necessity of determining more aerodynamic coefficients. Increasing range of new types of guns and ammunition demands higher accuracy of computations and increasing their time. That is especially necessary for Fire Control Systems where time of the calculation is strictly limited. That time of calculations is a main disadvantage for using 6-dof model in FCS. The range increasing also concerns mortar ammunition. Nowadays self-propelled mortars have range above 10 km (RAK-Mortar) which make point mass model much less sufficient. Unlike of spin stabilized projectiles, the aerodynamically (fin) stabilized...
projectiles Did not have specialized compromised trajectory model like Modified Point Mass Trajectory Model [5]. The Point Mass model neglects many of the phenomena which influence the projectiles trajectories. One of the most important is neglecting of pitch. In the paper authors have shown mean pitch theory and its influence to trajectories calculation accuracy.

2. The pitch during flight
During CRAWFISH-AMMUNITON (RAK-AMUNICJA) program Military Institute of Armament Technology developed supersonic mortar projectiles family. The projectiles are designed for a modern CRAWFISH mortar which can also shoot a flat fire and its maximum muzzle velocity should be above 500 m/s for HE projectile. During creation of firing tables for the projectile low accuracy of the Point Mass model were detected in some cases. The analysis has shown significant influence of the pitch to the trajectories. The influence is visible especially during long range shooting or flat fire.

Lieskie and Reiter create Modified Point Mass Model [5] for spin stabilized projectiles. McCoy in his work [2] show equation derived from Lieskie works for angle of response for fin stabilized projectiles. The equation can be written as:

$$\alpha_R = \left( \frac{C_{Mq}}{C_{Ma}V^4} \right) \left[ \vec{v} \times (\vec{v} \times \vec{g}) \right]$$

(1)

The angle of response includes pitch and yaw of the projectiles during flight. To check accuracy of the equation comparison with 5-dof model (excluded spin of the projectile) data will be made. Pitch calculations were made for elevation angles from 10 to 80 degrees at velocities 150 m/s and 500 m/s. Atmosphere parameters measured during one of shooting tests will be used (real perturbations). To avoid influence of different external ballistic models (5-dof, point mass) $\alpha_R$ will be calculated from 5-dof model parameters. At figures 2 and 3 comparisons for 150 m/s velocity were shown and at figures 4 and 5 for 500 m/s. Values are named by elevation angles and “R” mark (response angle) means values calculated by equation (1).
Picture 2 - Comparison for $V_0 = 150$ m/s and elevation angles from 10 to 50 degrees

Picture 3 - Comparison for $V_0 = 150$ m/s and elevation angles from 60 to 80 degrees
Comparison for $V_0 = 500 \text{ m/s}$ and elevation angles from 10 to 50 degrees

Picture 3 - Comparison for $V_0 = 500 \text{ m/s}$ and elevation angles from 10 to 50 degrees

Comparison for $V_0 = 500 \text{ m/s}$ and elevation angles from 60 to 80 degrees

Picture 4 - Comparison for $V_0 = 500 \text{ m/s}$ and elevation angles from 60 to 80 degrees
The comparison has shown insufficient accuracy of equation (1) especially at high velocities and low elevation angles. Mean pitch theory is much more accurate.

3. Mean pitch theory

Most of mortar projectiles were aerodynamically stabilized. The aerodynamically stabilization generates a pitch (stabilization) moment if total angle of attack is non zero. The moment tries to decrease that angle to zero. Some perturbations (e.g. during leaving muzzle or wind) can give the projectile some angular velocity or increase the angle. As a result of the perturbations the projectiles begin damping total angle of attack fluctuations. Some perturbations can generate asymmetry of lift force generated by projectiles at beginning (e.g. aerodynamic jump) but after that stage total angle of attack should be damped and mean lift force should be approximate to zero. To simplify theoretical analysis the projectile flight will be established as a planar. At figure 5 factors generating pitch were shown.

Picture 5 - Main factors influencing pitch

After initial perturbations where total angle of attack increases, yaw and pitch as a their components will be damped (if no other perturbations will increase them during flight). Yaw equilibrium state will be 0° value but for pitch equilibrium state will be reached when the projectile rotation ω will be equal to the trajectory curvature (θ velocity). That condition can be written as:

$$\omega = \dot{\theta} \quad (2)$$

Also in that case pitching moment will be equal to damping moment:

$$M_\alpha = M_q \quad (3)$$

So:

$$\frac{\rho V^2 SC_{Ma}}{2I_y} = \frac{\rho V^2 SC_{Mq}}{2I_y} \quad (4)$$

After simplification:
Using coefficient linearization:

\[ C_{Ma} = C_{Mq} \]  \hspace{1cm} (5)

\[ C_{Ma} = C_{Ma} \dot{\alpha} \]  \hspace{1cm} (6)

\[ C_{Mq} = C_{Mq} \dot{\omega} \]  \hspace{1cm} (7)

Because of (2)

\[ C_{Mq} = C_{Mq} \dot{\theta} \]  \hspace{1cm} (8)

\( \dot{\theta} = \frac{a}{V} \cos(\theta) \)  \hspace{1cm} (9)

So

\[ C_{Ma} \dot{\alpha} = C_{Mq} \frac{a}{V} \cos(\theta) \]  \hspace{1cm} (10)

\[ \alpha = \frac{C_{Mq} \frac{a}{V} \cos(\theta)}{C_{Ma} V} \]  \hspace{1cm} (11)

Equation (11) should estimate mean value of pitch because any fluctuations should be around equilibrium state. To distinguish values from equations (1) and (11) pitch calculated from equation (11) will be named “mean pitch” \( \alpha_M \) instead of \( \alpha_R \) from (1). To check equation (11) accuracy a comparison with the 5-dof model and equation (1) results should be done. Values marked by “M” description means values calculated by equation (11). Results were shown at figures 6 to 9.

Picture 6 - Results for \( V_0 = 150 \text{ m/s} \) and elevation angles from 10 to 50 degrees
Results for $V_0 = 150 \text{ m/s}$ and elevation angles from 60 to 80 degrees

Picture 7 - Results for $V_0 = 150 \text{ m/s}$ and elevation angles from 60 to 80 degrees

Results for $V_0 = 500 \text{ m/s}$ and elevation angles from 10 to 50 degrees

Picture 8 - Results for $V_0 = 500 \text{ m/s}$ and elevation angles from 10 to 50 degrees
Results for \( V_0 = 500 \) m/s and elevation angles from 60 to 80 degrees

The comparison has shown that the mean pitch theory estimate pitch of mortar projectiles much better during flight. Big differences for high elevation angles are the result of low stability of the projectile at trajectory apogee. The reason of this is low projectile velocity (which affect stabilization moment value) and high curvature of the trajectory close to apogee. The main differences between angle of response and mean pitch theory is that the angle of response theory concentrates on trajectory apogee.

4. Experimental validation

To validate the mean pitch theory experimental shooting data were used. During creation of firing tables for CRAWFISH-AMMUNITION (RAK-AMUNICJA) HE projectile several shooting were made. For each combination of propellant charge and elevation angle several projectiles were fired and muzzle velocity, range and atmosphere parameters were measured Mean of measured parameters were used to take into account dispersion of the projectile. To check usefulness of the mean pitch theory in creation of firing tables differences of calculated and experimental values were compared. Ranges were calculated in standard PM model and PM model including mean pitch theory. Drag shape was fitted to give accurate ranges for elevation angle 45\(^\circ\) and highest tested charge (~340 m/s) and other elevation angles and charges were also tested. The PM model with mean pitch theory will include generation of lift force and drag force increase as a function of angle of pitch. The experimental muzzle velocities and the ranges were presented in the table 1.
Table 1. Experimental muzzle velocities and ranges

| Muzzle velocity [m/s] | Elevation angle [°] |
|-----------------------|---------------------|
|                       | 15  | 17  | 29  | 30  | 45  | 60  | 66  | 80  |
| Charge                |     |     |     |     |     |     |     |     |
| 0                     | 149,4 | 146,2 | 147,7 | 145,1 | 148 |
| 1                     | 207  | 208,9 | 207,5 | 206,6 |
| 2                     | 282,1 | 281,8 | 282  |
| 3                     | 341,7 | 340  | 340,7 | 340,6 |
| Range [m]             | Elevation angle [°] |
| 15  | 17  | 29  | 30  | 45  | 60  | 66  | 80  |
| Charge                |     |     |     |     |     |     |     |     |
| 0                     | 1088 | 1765 | 2034 | 1457 | 726 |
| 1                     | 1999 | 3349 | 2752 | 1304 |
| 2                     | 5687 | 6338 | 5445 | 2245 |
| 3                     | 5205 | 7333 | 8242 | 6135 |

In the table 2 ranges calculated by standard PM model and PM model including mean pitch were presented.

Table 2. Calculated ranges

| Mean pitch | Elevation angle [°] |
|------------|---------------------|
|            | 15  | 17  | 29  | 30  | 45  | 60  | 66  | 80  |
| PM         |     |     |     |     |     |     |     |     |
| 0          | 1097 | 1784 | 2073 | 1481 | 724 |
| 1          | 2045 | 3456 | 2836 | 1330 |
| 2          | 5793 | 6418 | 5520 | 2235 |
| 3          | 5138 | 7272 | 8242 | 6132 |

In most cases PM model including mean pitch was more accurate. The biggest differences were at high elevation angles and higher charges.

5. Conclusion

The mean pitch theory is useful for fast prediction of the fin stabilized projectile’s pitch during flight. The most significant disadvantage of the theory is the assumption of the equilibrium state. It was responsible for lower accuracy for short range shooting (low muzzle velocity and elevation angle). For long time of flight shootings it improved accuracy of the calculation significantly. Simple formula of calculating the mean pitch allows it to being used in the Fire Control Systems.

6. References

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