EXTENDED GAUGE SECTORS AT LINEAR COLLIDERS

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ABSTRACT

Signatures of extended gauge models at high energy $e^+e^-$ linear colliders are summarized.

The phenomenology of models with extended gauge symmetries is particularly rich. The existence of new gauge bosons is one of the hallmarks of extended electroweak theories and their discovery would be a definitive signal for physics beyond the Standard Model (SM). However, extra gauge bosons are not the sole signature of an extended gauge group. These models also contain new exotic fermions, which are required for anomaly cancellation, as well as an enlarged Higgs sector, to facilitate the extended symmetry breaking. In addition, Supersymmetry may also be present, particularly in Grand Unified Theories (GUTS), to solve the hierarchy problem and to ensure the consistency of coupling constant unification with present data. This talk will summarize the potential of high energy $e^+e^-$ linear colliders to investigate the physics of new gauge bosons, at mass scales both above and equal to the machine center of mass energy, and to discover signals for exotic fermions. The subjects of extended Higgs sectors and Supersymmetry will be left to other speakers.

We begin with a brief review of several extended gauge models which presently appear in the literature. The most appealing set of enlarged electroweak models are those which are based on GUTS, examples being the unifying groups $SO(10)$ and $E_6$. In $E_6$ string-inspired models additional $Z$-bosons arise from the breaking chain

$$E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi$$

$$\rightarrow SM \times U(1)_\theta,$$

where $U(1)_\theta$, which is a linear combination of $U(1)_\chi$ and $U(1)_\psi$, remains unbroken at low energies ($\lesssim 10$ TeV). The fermion couplings of the extra $Z$-boson, $Z'$, in this model take the form

$$\frac{g}{2\cos}\sqrt{5\sin^2\theta/\sqrt{6} - Q\cos\theta/\sqrt{10}},$$

where $\theta$ lies in the

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range \(-90^\circ \leq \theta \leq 90^\circ\), \(Q_{\psi, \chi}\) are determined by group theory, \(x_w = \sin^2 \theta_w\), and \(c_w = \cos \theta_w\). Special cases in this category are \(\theta = 0^\circ\) (model \(\psi\)), \(\theta = -90^\circ\) (model \(\chi\)), and \(\theta = \arcsin(\sqrt{3/8})\) (model \(\eta\)). The latter case represents the rank-5 model derived directly from the flux breaking of superstring theories.

\[ SO(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi \]

The first chain leads to the additional \(Z\)-boson, \(Z_\chi\), discussed above, while the second chain yields right-handed charged as well as neutral currents and is denoted as the left-right symmetric model (LRM). In the LRM, the \(Z'\) couples to \(\frac{\kappa}{2c_w}(\kappa - 1 + \kappa)x_w^{-1/2}[x_wT_{3L} + \kappa(1 - x_w)T_{3R} - x_wQ]\), with \(0.55 \leq \kappa \equiv g_R/g_L \leq 1 - 2\). \(T_{3L(R)}\) is the fermion left-(right-)handed isospin, and \(Q\) is the fermion electric charge. Note that for strict left-right symmetry, \(\kappa = 1\). Another extended model based on the second intermediate group above is the alternative left-right symmetric model (ALRM), which is embedded in \(E_6\) GUTS and switches the quantum number assignments between some of the ordinary fermions and exotic fermions contained in the \(27\) of \(E_6\). In this case the right-handed \(W\)-boson carries lepton number and has odd \(R\)-parity, thus avoiding the usual constraints on the mass of right-handed \(W\)'s.

There are, of course, many other models with extended electroweak sectors that are not based on GUTS. In order to get a feel for the variety of such models, we list a few of them here. For example, the sequential standard model (SSM) contains a \(Z'\) which is just a heavier version of the SM \(Z\). While this model is lacking in theoretical motivation, it does provide a useful benchmark in comparing experimental constraints and capabilities. A recently revived model, based on the gauge symmetry \(SU(3)_C \times SU(3) \times U(1)\), provides interesting production mechanisms at \(e^+e^-\) colliders which will be discussed further in this report. The un-unified model extends the electroweak gauge group to \(SU(2)_c \times SU(2)_q \times U(1)_Y\), where the quark and lepton generations transform under their own \(SU(2)\). Other models extend the color gauge group, such as that of Foot and Hernandez, which is based on \(SU(5)_C \times SU(2)_L \times U(1)_Y\). There are several theories based on horizontal interactions, such as the \(SP(6)_L \times U(1)_Y\) model of Bagneid et al. In the generational model, each generation transforms under its own \(SU(2)\). And in the Leptophilic model, differences in lepton number are gauged.

**New Gauge Bosons**

In all of the above models, the \(Z - Z'\) mass matrix takes the form

\[
\mathcal{M}^2 = \begin{pmatrix}
M_Z^2 & \gamma M_Z^2 \\
\gamma M_Z^2 & M_{Z'}^2
\end{pmatrix},
\]

where \(\gamma\) is determined within each model once the Higgs sector is specified. For example, \(\gamma_{LRM} = -\sqrt{1 - 2x_w}\) (for \(\kappa = 1\)), and \(\gamma_{E_6} = -2\sqrt{5x_w/3}[(\cos \theta/\sqrt{6} - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\l
\[
\sin \theta / \sqrt{10} \tan^2 \beta - \left( \cos \theta / \sqrt{6} + \sin \theta / \sqrt{10} \right) (1 + \tan^2 \beta)^{-1}, \]
where \( \tan \beta \) is the ratio of vacuum expectation values (VEVs) of the two Higgs doublets present in \( E_6 \) superstring models. The physical eigenstates are then
\[
Z_1 = Z' \sin \phi + Z \cos \phi, \\
Z_2 = Z' \cos \phi - Z \sin \phi, \tag{4}
\]
where \( Z_1 \) is currently being probed at LEP, and \( \tan 2 \phi = 2 \gamma M_Z^2 / (M_Z^2 - M_{Z'}^2) \) with \( |\phi| \lesssim 0.01 \) from LEP (Ref. [13]).

Precision measurements constrain\[13\] extended gauge sectors, by limiting the indirect \( Z_2 \) contributions to processes such as \( \mu \)-decay, low-energy neutral currents, atomic parity violation, the \( W \) mass, and the properties of the \( Z_1 \) boson. In Fig. 1, from Langacker and Luo\[13\], the the bounds on \( M_2 \) are presented as a function of the \( Z - Z' \) mixing angle \( \phi \) in \( E_6 \) models \( \chi \) and \( \eta \) for the case where \( m_{t_{top}} \) is unconstrained. The allowed regions are enclosed by the curves and the dotted curve represents the additional constraints imposed from the Higgs sector as described in the above discussion of the \( Z - Z' \) mass matrix. It should be noted that these limits are slightly sensitive to the effects of other possible indirect contributions, such as, the top-quark, Higgs bosons, supersymmetry, and exotic fermions. Future improvements in these indirect bounds will be obtained as the data becomes ever more precise and the top-quark is discovered.

Direct searches for new gauge bosons at hadron colliders are performed via the production mechanisms, \( p\bar{p} \to Z' \to \ell^+\ell^- \) and \( p\bar{p} \to W' \to \ell^\pm \not{p}_T \). From the 1988-89 Tevatron run with 4.7 \( \text{pb}^{-1} \), CDF has set a 95% C.L. bound\[14\] on the \( Z_2 \) mass of 412 (320, 340, 310) GeV in the SSM (Models \( \psi \), \( \chi \), \( \eta \), and the LRM, respectively), using both \( e \) and \( \mu \) data. A limit has also been set on a possible new \( W' \) with SM strength couplings of 520 GeV. The recent 1992-93 run with approximately 21 – 22 \( \text{pb}^{-1} \) of integrated luminosity yield\[15\] the very preliminary 95% C.L. constraint of \( M_2 > 495 \text{ GeV} \) in the SSM from the CDF electron data alone. Existing and future bounds obtainable at the Tevatron on new gauge bosons in \( E_6 \) models are shown in Fig. 2a as a function of \( \theta \) for various values of integrated luminosity. These calculations assume \( e + \mu \) data samples with CDF detector efficiencies, and include a 2-loop K-factor as well as a finite top-quark mass. Assuming 400 \( \text{pb}^{-1} \), we see that the Tevatron will be probing \( Z_2 \) masses of order 700-800 GeV in these models. At the Tevatron, one may also search for new gauge bosons in the \( Z', W' \to 2 \) jets channel, providing the large QCD background can be overcome. The dijet invariant mass spectrum at the Tevatron can give\[10\] additional bounds on the mass of new \( W \)-bosons, depending on their coupling strength, but the lepton data is found to give the best constraints on most new neutral gauge bosons. Hadron supercolliders, \( i.e. \), the SSC and LHC, can search for \( Z_2 \) bosons with masses up to 3-7 TeV, depending on the model. For example, Fig. 2b displays the discovery limit as a function of integrated luminosity for a \( Z_2 \) from the LRM at the SSC (solid curve) and LHC (dashed curve), assuming a 5\( \sigma \) signal in the \( e \) channel only and \( \kappa = 1 \). At design luminosity, the SSC(LHC) could detect a LRM \( Z_2 \) with \( M_2 \lesssim 6.1(4.9) \text{ TeV} \). We note
that in the search regions presented here, it is assumed that the $Z_2$ decays only into 3 generations of SM fermions.

If a $Z_2$ is discovered at a hadron collider, a much more difficult puzzle develops – the identification of the extended electroweak model from which the $Z_2$ originates. Numerous studies of this issue have been performed during the last few years and have been summarized at this meeting by Cvetič [17]. Several ideas have been proposed, including $Z_2 + \gamma, Z, W$ associated production, leptonic forward-backward and final state tau polarization asymmetries, 3-body $Z_2$ decays, $Z_2$ production with polarized beams, and extracting a signature from the $Z_2 \rightarrow 2$ jet channel. All of the techniques proposed thus far (with the exception of the leptonic forward-backward asymmetry) suffer from at least one of the following problems, (i) the event rate dies off rapidly unless the $Z_2$ mass is in the range 1-2 TeV, and (ii) it is difficult to extract the signal from the overwhelming background.

LEP II and future $e^+e^-$ colliders allow for indirect searches in the case $M_2 > \sqrt{s}$ by looking for possible deviations from SM expectations for cross sections and asymmetries associated with the reaction $e^+e^- \rightarrow f\bar{f}$. This is similar in concept to exploring modifications of QED predictions due to the SM $Z$ at PEP/PETRA energies. The limits obtained by this method are very model-dependent and are quite sensitive to the integrated luminosity, the value of $\sqrt{s}$, as well as the flavor of the final state fermion. The measurable quantities associated with $e^+e^- \rightarrow f\bar{f}$ that are sensitive to $Z_2$ exchange are (i) total cross section, $\sigma_f$, (ii) the ratio of hadronic to leptonic cross section, $\sigma_{had}/\sigma_\ell$, (iii) the forward-backward asymmetry, $A_{FB}^f$, (iv) final state polarization asymmetry of taus, $A_{\tau pol}$, and if longitudinal polarized beams are available, (v) the left-right asymmetry, $A_{LR}^f$, and (vi) the polarized forward-backward asymmetry, $A_{FB}^f(pol)$. As an example of the power of this search technique, Fig. 3a presents the possible bounds on $M_2$ in $E_6$ models as a function of $\theta$ from a 1$\sigma$ deviation in $\delta\sigma/\sigma$ for $\mu, c, b$ final states with 200 pb$^{-1}$ of integrated luminosity at LEP II. Note that the discovery region extends to values of the $Z_2$ mass that are $2-3 \times \sqrt{s}$ and is comparable to the future search reach of the Tevatron. A $\chi^2$ analysis has been performed in Ref. [19] for a $\sqrt{s} = 500$ GeV linear collider, where the above processes (i) and (iii)–(vi) have been examined, including 3-loop QCD corrections, oblique electroweak corrections, identification efficiencies of final state particles, and a beam polarization measurement error of $\delta P/P = 1\%$. Figure 3b displays the 95\% C.L. search limits from this study for a $Z_2$ arising from $E_6$ models as a function of the model parameter $\theta$ assuming $L = 10$ or 25 fb$^{-1}$ with or without beam polarization ($P = 90\%, 0$). In general, the discovery region reaches $3 - 6 \times \sqrt{s}$ for $L = 25$ fb$^{-1}$. The results of a separate investigation [20] are shown in Fig. 4 for a $Z_2$ originating from (a) $E_6$ models as a function of $\cos \beta$ where $\beta = \theta + 90^\circ$ and (b) LRM as a function of $\alpha_{LR}$ (where $\alpha_{LR}$ is related to $\kappa$) for various values of the center of mass energy. Here, QCD, electroweak corrections, initial state radiation, as well as final state identification efficiencies are included for the processes (i)–(iii) and (v) above. While high-energy $e^+e^-$ colliders do have a large discovery reach for extra gauge bosons, it is clear from these results that a $\sqrt{s} \sim 1$ TeV machine is needed in order
to be competitive with the hadron supercolliders.

How well does an $e^+e^-$ collider perform in determining the model of origin of a new neutral gauge boson? In Fig. 5a-c from [19] a $\chi^2$ distribution as a function of the $E_6$ model parameter $\theta$ is presented assuming a $Z_2$ from Model $\psi (\theta = 0^\circ)$ with $M_2 = 1 - 2 \text{ TeV}$ is present in the data at $\sqrt{s} = 500 \text{ GeV}$. This analysis examines the processes (i) and (iii)–(vi) above with an integrated luminosity of 5 or $25 \text{ fb}^{-1}$ and both polarized ($P = 90\%$) and unpolarized beams. The ‘measured’ range of $\theta$ at 95\% C.L. corresponds to the points where the solid horizontal line intersects the parabolic curves. A complimentary analysis[20] is shown in Fig. 5 (d-f), where the ability to discriminate between a $Z_2$ from $E_6$ Models and one from the LRM is examined. Here, the unshaded areas represent the regions in the $\alpha_{LR} - \beta$ plane where the two models are distinguishable and the single-hatched area corresponds to the region of confusion, using the quantities (i)–(iii) above. If one includes polarized beams and process (v), then the area of confusion is reduced to the double-hatched region. It is clear from these studies that $e^+e^-$ machines are capable of determining the couplings of a relatively light $Z_2 (\sim 1 \text{ TeV})$!

The existence of new gauge bosons may also modify the reaction $e^+e^- \rightarrow W^+W^-$, which is notably sensitive to the specific form of the gauge couplings. The $Z_2$ participates in this process only through $Z-Z'$ mixing and hence $e^+e^- \rightarrow W^+W^-$ can be used as a probe of the amount of mixing present, i.e., the value of $\phi$. Here, we consider two observables, the total cross section and the polarized left-right asymmetry. The 95\% C.L. search reach at a 1 TeV collider with 100 fb$^{-1}$ of integrated luminosity in $E_6$ models with $M_2 > \sqrt{s}$ is shown in Fig. 6a as a function of the parameter $\theta$ for various values of $\tan \beta$. (Recall that the $Z-Z'$ mixing angle $\phi$ is determined in $E_6$ models once the $Z_1$ and $Z_2$ boson masses, $\theta$ and $\tan \beta$ are known.) The relatively small search limits reflect the fact that $\phi$ is tightly constrained by LEP data, and the regions where is the search limit disappears, for example at $\theta = 0^\circ$ with $\tan \beta = 1$, is due to the fact that $\phi = 0$ close to these values of the parameters. The $Z_2$ discovery window from measurements of the left-right asymmetry is, of course, quite sensitive to the amount of beam polarization and its associated error. The anticipated error in $A_{LR}$ due to finite polarization is

$$\delta A_{LR} = \left[ \frac{1 - (PA_{LR})^2}{N_W P^2} \oplus \left( \frac{A_{LR} \delta P}{P} \right) \right]^{1/2},$$

where $N_W = \sigma \mathcal{L} \epsilon_W$ with $\epsilon_W$ being the $W$ identification efficiency, and ‘⊕’ represents that the errors are added in quadrature. The 95\% C.L. search reach from $A_{LR}$ and $\sigma$ combined is presented in Fig. 6b for the LRM as a function of $\kappa$, with $P = 90\%$ or 100\% and $\delta P/P = 0$ or 1\%. We see that as soon as the uncertainty in the amount of polarization is included, most of the sensitivity to $Z_2$ exchange is lost. $e^+e^- \rightarrow W^+W^-$ may be more interesting in the case where the $Z_2$ is on resonance, which will be discussed below.

In principle, new neutral gauge bosons will contribute to Bhabha and Moller scattering, see e.g., Ref. [3]. However, in practice these processes are dominated by
the $\gamma$-pole and are not very sensitive to indirect $Z_2$ contributions below production threshold.

Now, we turn our attention to on-resonance $Z_2$ production at a TeV $e^+e^-$ collider. We note that most of the discussion in the literature on the production and model identification of $Z_2$ bosons at hadron supercolliders has focused on the 1–2 TeV mass range, and clearly, the discovery of a $\sim 1$ TeV $Z_2$ at the SSC/LHC (which will presumably, but not necessarily, be built before a high-energy $e^+e^-$ linear accelerator!), would provide powerful motivation for building a TeV $e^+e^-$ machine.

The event rates are, of course, quite large on-resonance, yielding enhancements of 2–3 orders of magnitude over the usual falling SM cross sections. Such a machine would become a $Z_2$ factory in a manner similar to the $Z_1$ production at LEP I. In this case, unraveling the various couplings of the $Z_2$ and determining its model of origin would become a much easier puzzle to solve. The $Z_2$ properties which can be measured accurately (besides its mass) include (i) the $Z_2$ partial widths into all identifiable final states, $\Gamma_\ell, \Gamma_b, \Gamma_t$, and $\Gamma_{had}$, as well as perhaps $\Gamma(Z_2 \to \text{exotics, superpartners})$ if these channels are available. (ii) The total width, $\Gamma_{tot}$, which could also yield information on possible invisible decay channels. (iii) Universality should be verified, as it is violated in some extended models. (iv) Several asymmetries can be determined for every identifiable final state. For $f = \ell, b, \text{ or } t$, we have

\begin{align}
A^\ell_{FB} &= 3A_\ell A_f/4,
A^\ell_{FB}(pol) &= 3A_f/4, \\
A^\ell_{LR} &= A_\ell = A^\ell_{pol},
\end{align}

with the definition $A_i = 2v_i a_i/(v_i^2 + a_i^2)$. The forward-backward asymmetry for the process $e^+e^- \to \mu^+\mu^-$ is presented in Fig. 7a as a function of the $E_6$ parameter $\theta$, including the statistical errors calculated for $L = 25 \text{ fb}^{-1}$ at $\sqrt{s} = 1$ TeV. The values of $A^\mu_{FB}$ in the LRM (with $\kappa = 1$), the SSM, and the ALRM are also shown for comparison. We see from the figure that although this asymmetry can be well measured, it alone can not uniquely differentiate between the various models.

Some extended electroweak models possess specific $Z_2$ properties that are unique to a new neutral gauge boson originating within that particular scenario. For example, in $E_6$ theories, the $Z_2$-fermion vector and axial vector coupling constants obey the relations $v_\ell = -v_b$ and $a_\ell = a_b$, which yields the predictions

\begin{align}
\frac{\Gamma(Z_2 \to b\bar{b})}{\Gamma(Z_2 \to \ell^+\ell^-)} &= 3 \oplus \text{QCD corrections}, \\
A^\ell_{FB} &= -A^b_{FB}, \\
A^b_{FB}(pol) &= -\frac{3}{4}A^\ell_{LR},
\end{align}

while the $E_6$ property $v_{u,c,t} = 0$ implies

\begin{align}
A^\ell_{FB} &= 0 = A^\ell_{FB}(pol), \\
&= A^\ell_{LR}.
\end{align}
These relations are shown explicitly in Fig. 7b where the forward-backward asymmetry is presented for $\mu, b, t$ final states as a function of $\theta$. In the case of the LRM, the forward-backward asymmetry for $\mu, b, t$ final states as well as $A_{LR}^\mu$ vanish at two distinct values of $\kappa$, $\kappa = \kappa_{\text{min}} = \sqrt{x_w/(1-x_w)}$ and $\kappa = \sqrt{3x_w/(1-x_w)}$. The forward-backward asymmetry and left-right asymmetry for $\mu, b, t$ final states and the ratio $\Gamma_{b,t}/\Gamma_\ell$ are shown in Fig. 8a–c for the LRM as a function of $\kappa$. These figures clearly display that the asymmetries do indeed vanish for these two values of $\kappa$. The results presented here are essentially unmodified by small values of $Z-Z'$ mixing and finite values of $m_t/M_2$. These distinctive properties make excellent on-resonance tests of $E_6$ and left-right symmetric models!

As a further example of the power of these on-resonance tests, we show the areas of the $\Gamma_t/\Gamma_\mu - \Gamma_b/\Gamma_\mu$ and $\Gamma_b/\Gamma_\mu - A_{FB}^\mu$ planes that are populated by three different extended models in Fig. 9a–b, respectively. In each case, the solid curve represents $E_6$ models as the parameter $\theta$ is varied, the dotted curve corresponds to the LRM as $\kappa$ is varied, and the dashed curve is the un-unified model as its model parameter ($\sin \phi$, which represents the relative amount of mixing between the $SU(2)_e$ and $SU(2)_q$ factors). Note that these three models cover quite distinct regions in these planes.

As discussed above, the decay $Z_2 \rightarrow W^+W^-$ can be a useful probe of $Z-Z'$ mixing as well as the Higgs sector. The total cross section for $e^+e^- \rightarrow W^+W^-$ in the $E_6$ model $\chi$ with $M_\chi = 1$ TeV (dashed curve) contrasted to that of the SM (solid curve) as a function of $\sqrt{s}$ is shown in Fig. 10, from Ref. [22], for $\Delta M \equiv M_Z - M_1 = 50$ MeV. Clearly, the existence of a $Z_2$ greatly enhances the event rate; for more details see [3, 22].

New Fermions

There is a wealth of phenomenology associated with exotic fermions that are present in extended gauge models, particularly in the case of $E_6$ theories, and in the case where mirror fermions are present. So much research has been performed in this area [3, 23, 24] that it could well be the subject of a separate review. Here, we will restrict ourselves to work which was presented at this meeting and related material.

In $E_6$ models, each chiral generation of fermions is assigned to the 27 dimensional representation which contains the usual 16 of $SO(10)$ (the SM fermions plus the right-handed $\nu$) as well as 11 new fields. Since the superstring-inspired $E_6$ theories are supersymmetric, the corresponding scalar superpartners to these fermions are also present. For each family one then has the additional fermions,

$$\left( \begin{array}{c} N \\ E \end{array} \right)_L, \left( \begin{array}{c} N \\ E \end{array} \right)_c^L, h^c_L, h^c_L, S^c_L,$$

where $(N,E)$ are color singlet, vector-like leptons with $Q_N = 0$ and $Q_E = -1$, $h$ is a color triplet, iso-singlet fermion with $Q_h = -1/3$, and $S$ is a neutral color and iso-singlet. The most general superpotential allows for three possible baryon and
lepton number assignments for $h$, however, all terms in the superpotential can not be simultaneously present or low-energy baryon and lepton number violation will occur. Thus, $h$ is forced to be either a quark, a diquark, or a leptoquark.

These exotic fermions may be pair produced at $e^+ e^-$ colliders via s-channel $\gamma, Z$ exchange. Clean signatures are expected up to the kinematical limit, $m_F \simeq \sqrt{s}/2$. A thorough background study\[25\] of $e^+ e^- \rightarrow N \bar{N}$ has verified that this holds true for heavy neutrino production. If the pair production of new fermions is observed, then measurement of their forward-backward and left-right asymmetries can be used as a probe\[24\] of their electroweak properties. Possible mixing between the exotic and ordinary SM fermions can induce single exotic production\[3, 27\] via $e^+ e^- \rightarrow E \bar{e} + \bar{E} e, N \bar{\nu}_e + \bar{N} \nu_e, h d + \bar{h} d$. These processes are mediated through s-channel $Z_1$ and $Z_2$ exchange, as well as t-channel $Z_{1,2}(W)$ exchange in the case of $E \bar{e}(N \bar{\nu}_e)$ production, and are directly proportional to the degree of ordinary-exotic mixing. Present data restricts\[13\] this mixing to values $\theta_{mix} < \sim 0.1$. This yields cross sections\[27\] in the $10 - 1000 \text{ fb}^{-1}$ range for $m_F < 450 - 475$ GeV for $E \bar{e}$ or $N \bar{\nu}_e$ production with maximal mixing at a 500 GeV machine (with the neutral cross section being larger than that for the charged leptons). The cross sections without the t-channel contribution (i.e., single $h$ production) are somewhat smaller. Azuelos and Djouadi\[27\] have performed a Monte-Carlo study and have considered several background sources for both neutral and charged single lepton production. These authors found that the ratio of signal events to the square root of the background is greater than unity ($S/\sqrt{B} > 1$) for mixing angles $\theta_{mix} > 0.005(0.03)$ with $m_F = 350$ GeV for neutral(charged) lepton single production at $\sqrt{s} = 500$ GeV and $\mathcal{L} = 50 \text{ fb}^{-1}$.

Leptoquarks appear naturally in theories which place quarks and leptons on equal footing, including $E_6$ models as mentioned above. They couple to a leptoquark pair, with an $a$ priori unknown strength governed by the Yukawa coupling, $\lambda$. For calculational purposes, these Yukawa couplings are usually parameterized by $\lambda^2/4\pi = F_{\alpha_{em}}$. These particles can be light $\sim 100$ GeV and still avoid conflicts\[28\] with rapid proton decay and dangerously large flavor changing neutral currents. This is particularly true in models where each generation of fermions has its own leptoquark(s) which couples only within that generation. Present experimental bounds\[28\] on scalar leptoquarks are (i) $M_{LQ} > 116$ GeV at 95% C.L. if $B(LQ \rightarrow eu) = 100\%$ from $gg, q\bar{q} \rightarrow LQ \bar{LQ}$ production at CDF, and (ii) $M_{LQ} \gtrsim 175$ GeV at 95% C.L., where $\lambda$ takes on values equivalent to the electroweak coupling strength, from the production $eq \rightarrow LQ$ at HERA.

Leptoquarks may also be pair produced and observed up to the kinematic limit in $e^+ e^-$ collisions. For this case, QCD as well as QED (initial state radiation and beamstrahlung) corrections have been recently computed and reported at this meeting\[30\]. The angular distributions in leptoquark pair production are affected by the presence of the t-channel quark exchange diagram, and are hence sensitive to the Yukawa coupling. This can be seen explicitly in Fig. 11 from Blümlein and Rückl\[31\], where it is clear that the angular distributions can be used to distinguish between scalar and vector leptoquark production as well as between the existence of right-
left-handed couplings. Single production\cite{31, 32} occurs via the reaction $e\gamma \rightarrow LQ +$ jet and is proportional to the value of the Yukawa coupling. Cross sections at $\sqrt{s} = 500$ GeV for scalar leptoquarks using three different photon sources, (i) Weizsäcker-Williams distribution, (ii) beamstrahlung, and (iii) backscattered laser beam, are displayed in Fig. 12a from Ref. \cite{31} (including a $p_T$ cut on the associated jet of 10 GeV). Figure 12b shows the discovery region\cite{31} from this mechanism, demanding a 5$\sigma$ signal for both 10 and 25 fb$^{-1}$ of integrated luminosity. We see that with large values of the coupling strength discovery up to the kinematic limit is possible, rendering this process competitive with the pair production process.

If leptoquarks are too heavy to be produced directly, perhaps they can be detected through indirect effects. In principle, leptoquarks can participate in the reaction $e^+e^- \rightarrow q\bar{q}$ via u- and t-channel exchanges, due to the presence of the Yukawa couplings, and can produce deviations from the SM predictions for cross sections and asymmetries. The 95\% C.L. bounds which can be obtained at a 500 GeV $e^+e^-$ collider from such reactions is presented in Fig. 13 in the leptoquark coupling strength - mass plane for various values of integrated luminosity. Here, the search region corresponds to the area above the curves, and has been computed for scalar leptoquarks.

$e^-e^-$ Collisions

An interesting reaction in $e^-e^-$ collisions, a possible option at the next linear $e^+e^-$ collider, is inverse neutrinoless double beta decay, first proposed by Rizzo\cite{33}. Extended electroweak models with Majorana neutrinos and heavy iso-singlet neutral leptons can mediate $\Delta L = 2$ interactions; at low-energies such models can be probed indirectly by searching for rare processes such as neutrinoless double $\beta$-decay. High energy $e^-e^-$ collisions may provide a new window into the $\Delta L = 2$ sector of these models via the reaction $e^-e^- \rightarrow W^-_i W^-_i$, where the $W_i$ may represent either the SM $W_L$ boson or an additional charged gauge boson, such as the right-handed $W_R$ of the LRM. In the case where two SM $W^-_L$ bosons are produced, there is a danger of unitarity violation, which can only be cured if either extra neutral fields plus mixing are included, or a doubly charged Higgs boson ($\Delta$) is exchanged in the s-channel. The cross section in the former case is presented in Fig. 14a as a function of the heavy neutrino mass with $\sqrt{s} = 500$ or 1000 GeV from Rizzo\cite{33}. Note that these results should be scaled by four powers of the light-heavy neutrino mixing angle, which is expected to be $\lesssim \mathcal{O}(10^{-2})$.

This $\Delta L = 2$ reaction is quite natural in the LRM, as this model contains all the necessary ingredients for the preservation of unitarity: heavy right-handed neutrinos, a doubly-charged Higgs scalar, as well as right-handed $W_R$ bosons. In the LRM case, the sum of the asymptotic $\nu$ and $\nu^c$ contributions no longer cancel and a $\Delta$ exchange is required in order to restore unitarity. Here, $d\sigma|_{s\rightarrow\infty} \sim m_N$, where $N$ represents the heavy right-handed neutrino, and unitarity is maintained as long as $N$ (and $\Delta$) have a mass less than $\sim 2$ (10) TeV. Generally the cross section for $e^-e^- \rightarrow W^-_R W^-_R$ can be quite large, as is depicted in Fig. 14b as a function of $\sqrt{s}$.
from Maalampi et al. [33]. In this figure the mass of the right-handed $W_R$ is taken to be 500 GeV, $M_\Delta = 500$ GeV and $m_N = 0.5, 1, 1.5$ TeV. We see that the event rate can be sizable for integrated luminosities in the 100 fb$^{-1}$ range. Mixing in the $\nu - \nu^c$ mass matrix does not significantly contribute to $W_R^- W_R^+$ production, but can induce the mixed final state $W_L^- W_R^+$. In the absence of $W_L^- - W_R$ mixing, this process will proceed via $t$- and $u$-channel exchanges, but still obeys unitarity due to the opposite helicity structures at the two vertices. The cross section (from Rizzo [33]) for $e^- e^- \rightarrow W_L^- W_R^+$, is displayed in Fig. 14c as a function of the mass of the heavy neutrino, taking $m_{W_R} = 480$ GeV and $\kappa = 0.9$ and $\sqrt{s} = 1$ TeV. Here the result must be multiplied by two powers of the scaled mixing angle, $\theta_{mix}/0.01$. We see that this cross section is reasonably small, but might be observable depending on the value of the mixing.

A second interesting possibility in $e^- e^-$ collisions is dilepton production. As has been emphasized by Frampton [7, 34], dileptons naturally appear in various extended electroweak models, including $SU(15)$ GUTS and the $SU(3) \times U(1)$ model discussed in the introduction. Here, dileptons contribute in $s$-channel exchange, yielding a distinctive resonance peak as shown in Fig. 15 from Ref. [34] for a dilepton mass of 500 GeV.

Conclusions

In summary, we see that extended gauge sectors yield an abundance of exciting phenomenology at high energy $e^+ e^-$ colliders! The present bounds on additional gauge bosons are becoming stronger everyday, with mass limits being in the several
hundred GeV range. Hadron supercolliders can discover $Z_2$ bosons with masses up to several TeV, but will have more difficulty determining the extended model from which the $Z_2$ originates. $e^+e^-$ colliders can indirectly probe the existence of new neutral gauge bosons up to $M_2 \sim (3 - 6) \times \sqrt{s}$ for integrated luminosities $\mathcal{L} \simeq 25 \text{ fb}^{-1} \left[\sqrt{s}/500 \text{ GeV}\right]^2$. The possible discovery of a 'light' $Z_2$ ($\sim 1 \text{ TeV}$) at a hadron supercollider would provide an impetus for the construction of a TeV $e^+e^-$ linear collider, as $Z_2$ on-resonance physics has an overwhelming potential. In the case of exotic fermions, there are many exciting production mechanisms and signatures which are unique to $e^+e^-$ collisions. And, last, but not least, one should keep the $e^-e^-$ collider option open as a possible technique for finding striking and unique signatures for new physics.

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Fig. 1. The 90% C.L. allowed region in the $M_2 - \phi$ plane in (a) Model $\chi$ and (b) Model $\eta$ from precision data as described in the text. $\rho_0$ is the tree-level $\rho$ parameter, and the dotted curves represent the constraints from the minimal Higgs sector for various values of the ratios of VEVs.

Fig. 2. (a) Existing and expected future search limits at the Tevatron for new $Z$ bosons in $E_6$ models as a function of $\theta$ for various values of integrated luminosity as shown. (b) Discovery limit as a function of integrated luminosity at the SSC (solid curve) and LHC (dashed curve) in the LRM.

Fig. 3. Indirect $Z_2$ search limits as a function of the $E_6$ parameter $\theta$ at (a) LEP II with 200 pb$^{-1}$ of integrated luminosity using $\delta \sigma/\sigma$ alone with $\mu, b$ and $c$ final states, (b) at $\sqrt{s} = 500$ GeV, assuming $L = 10$ fb$^{-1}$ with $P = 0$ (dots) and $P = 90\%$ (dashes) and $L = 25$ fb$^{-1}$ with $P = 0$ (dashed-dots) and $P = 90\%$ (solid).

Fig. 4. $Z_2$ discovery limits in (a) $E_6$ models and (b) LRM as a function of the model parameters $\cos \beta$ and $\alpha_{LR}$, respectively for $\sqrt{s} = 190$ GeV with $L = 500$ pb$^{-1}$ (dashed-dotted), 500 GeV (solid), 1 TeV (dashed), and 2 TeV (dotted) with $L = 20$ fb$^{-1}$. In each case, the upper thin (lower thick) curves are with (without) beam polarization.

Fig. 5. $\theta$ determination for $E_6$ model $\psi$ assuming (a) $M_2 = 1$ TeV, (b) $M_2 = 1.5$ TeV, and (c) $M_2 = 2$ TeV. The inner (outer) pair of dashed ($P = 90\%$) and dotted ($P = 0$) curves correspond to $L = 25(5)$ fb$^{-1}$. The 95% C.L. is determined when the horizontal solid line intersects the parabolic curves. 95% C.L. distinction between $E_6$ and LRM $Z_2$ bosons for (d) $M_2 = 1.5$ TeV, (e) $M_2 = 2$ TeV, and (f) $M_2 = 2.5$ TeV with $\sqrt{s} = 500$ GeV with $L = 20$ fb$^{-1}$. The single (double) hatched regions denotes the area of confusion without (with) the availability of polarized beams.

Fig. 6. 95% C.L. $Z_2$ search limits from the reaction $e^+e^- \rightarrow W^+W^-$ with $\sqrt{s} = 1$ TeV and $L = 100$ fb$^{-1}$. (a) In $E_6$ models as a function of $\theta$ with $\tan \beta = 1(2,10)$ corresponding to dotted (dashed, solid) curve, using $\sigma$ alone, (b) In the LRM as a function of $\kappa$, using $\sigma$ and $A_{LR}$, with $P = 100\% \delta P/P = 0$ (solid), $P = 90\% \delta P/P = 0$ (dashed), and $P = 90\%$ or 100% $\delta P/P = 0.01$ (dashed-dotted).

Fig. 7. On-resonance forward-backward asymmetry in $E_6$ models as a function of $\theta$, assuming $\sqrt{s} = M_2 = 1$ TeV. (a) In $e^+e^- \rightarrow \mu^+\mu^-$, where the statistical errors are displayed for $L = 25$ fb$^{-1}$. $A_{FB}^\mu$ is also shown in the LRM, SSM, and ALRM, for comparison. (b) For $\mu$ (solid), $t$ (dotted), and $b$ (dashed) final states.

Fig. 8. On-resonance tests of the LRM as a function of $\kappa$ for final states $\mu$ (solid), $t$ (dotted), $b$ (dashed) with $\sqrt{s} = 1$ TeV. (a) The forward-backward asymmetry, (b) left-right asymmetry, and (c) the ratio of partial widths $\Gamma_q/\Gamma_t$.

Fig. 9. On-resonance model identification, where the regions populated by $E_6$ models as $\theta$ is varied (solid), LRM as $\kappa$ is varied (dotted), and the Un-unified model as $\sin \phi$ is varied (dashed) are displayed in the (a) $\Gamma_t/\Gamma_\mu - \Gamma_b/\Gamma_\mu$ and (b) $\Gamma_b/\Gamma_\mu - A_{FB}^\mu$ planes.

Fig. 10. The total $\sigma(e^+e^- \rightarrow W^+W^-)$ as a function of $\sqrt{s}$ for the $E_6$ model $\chi$ with $M_2 = 1$ TeV. The solid line represents the SM behavior.

Fig. 11. Angular distributions of a scalar ($S_1$) and vector ($U_1$) leptoquark with mass $M_{LQ} = 400$ GeV and $\sqrt{s} = 1$ TeV and the couplings $\lambda_L = \lambda_R = 0$ (solid), $\lambda_L/e = 0.3$ and $\lambda_R = 0$ (dashed), $\lambda_L = 0$ and $\lambda_R/e = 1$ (dotted).
Fig. 12. (a) Cross section for single leptoquark production via $e\gamma \to LQ + \text{jet}$ as a function of the leptoquark mass at $\sqrt{s} = 500$ GeV. The various photon sources are Weizsäcker-Williams (solid), beamstrahlung (dashed), and backscattered laser beam (dashed-dotted), with the upper (lower) curves corresponding to the values of the coupling parameter $F = 1$ ($F = 0.1$) in each case. (b) Discovery region (which lie above the curves) for single leptoquark production as a function of the leptoquark mass for the values of integrated luminosity as indicated.

Fig. 13. 95% C.L. indirect search limits on the leptoquark mass and coupling $\kappa = 2F$ from $e^+e^- \to q\bar{q}$, assuming $\sqrt{s} = 500$ GeV and $L = 5$ fb$^{-1}$. The discovery region lies above the curves.

Fig. 14. (a) $\sigma(e^-e^- \to W_L^-W_L^-)$ as a function of the heavy neutrino mass for $\sqrt{s} = 0.5 (1)$ TeV corresponding to the solid (dashed-dotted) curve. The cross section should be rescaled by 4 powers of the neutrino mixing angle. (b) Total cross section for $e^-e^- \to W_R^-W_R^-$ as a function of the CM-energy for various values of the mass of the heavy right-handed neutrino $\nu_2$. (c) $W_L^-W_R^-$ production as a function of the heavy neutrino mass $m_N$ with $\sqrt{s} = 1$ TeV. This result must be rescaled by two powers of the mixing angle ratio $\theta/0.01$.

Fig. 15. Total cross section for $e^-e^- \to e^-e^-$ in the presence of a 500 GeV dilepton as a function of the CM-energy.