PARTICLE-IN-CELL SIMULATIONS OF PARTICLE ENERGIZATION VIA SHOCK DRIFT ACCELERATION FROM LOW MACH NUMBER QUASI-PERPENDICULAR SHOCKS IN SOLAR FLARES

JAEHONG PARK1,2, CHUANG REN1,2,3, JARED C. WORKMAN1,4, AND ERIC G. BLACKMAN1,2
1 Department of Physics & Astronomy, University of Rochester, Rochester, NY 14627, USA
2 Laboratory for Laser Energetics, University of Rochester, Rochester, NY 14623, USA
3 Department of Mechanical Engineering, University of Rochester, Rochester, NY 14627, USA
4 Department of Physical & Environmental Sciences, Colorado Mesa University, Grand Junction, CO 81501, USA

Received 2012 October 19; accepted 2013 January 21; published 2013 February 27

ABSTRACT

Low Mach number, high beta fast mode shocks can occur in the magnetic reconnection outflows of solar flares. These shocks, which occur above flare loop tops, may provide the electron energization responsible for some of the observed hard X-rays and contemporaneous radio emission. Here we present new two-dimensional particle-in-cell simulations of low Mach number/high beta quasi-perpendicular shocks. The simulations show that electrons above a certain energy threshold experience shock-drift-acceleration. The transition energy between the thermal and non-thermal spectrum and the spectral index from the simulations are consistent with some of the X-ray spectra from RHESSI in the energy regime of \( E \lesssim 40 \sim 100 \text{ keV} \). Plasma instabilities associated with the shock structure such as the modified-two-stream and the electron whistler instabilities are identified using numerical solutions of the kinetic dispersion relations. We also show that the results from PIC simulations with reduced ion/electron mass ratio can be scaled to those with the realistic mass ratio.

Key words: acceleration of particles – instabilities – methods: numerical – plasmas – shock waves – Sun: flares

Online-only material: color figures

1. INTRODUCTION

Low Mach number (\( M \)), high plasma beta (\( \beta_p \)) fast mode shocks can occur in the magnetic reconnection outflows of solar flares. Hard X-ray data from Yohkoh and RHESSI have revealed that electrons are energized above flare loop tops and footpoints (e.g., Lin et al. 2003). Solar flares are diverse and the associated reconnection events are likely “acceleration environments” with potentially different mechanisms of particle acceleration operating on different scales. One potential source of particle acceleration, seen from analytic predictions (Blackman & Field 1994) and numerical simulations of reconnection configurations in which an obstacle was present (e.g., Forbes 1998; Workman et al. 2011), involves the presence of low Mach number fast shocks in reconnection outflows. These “termination” shocks may contribute to the high energy acceleration observed over some frequency range, studying the potential ways in which these shocks can accelerate particles is well motivated.

Mann et al. (2006, 2009) and Warmuth et al. (2009) suggested an electron energization via shock drift acceleration (hereafter SDA) in termination shocks. Guo & Giacalone (2010, 2012) performed hybrid simulations for termination shocks where test electrons were effectively energized via the interaction with pre-existing large-scale magnetic fluctuations. In our recent work (Park et al. 2012), we performed a full particle-in-cell (PIC) simulation for exactly perpendicular (i.e., magnetic field perpendicular to the shock normal), low \( M/high \beta_p \) shocks. We found that both electrons and ions participated in SDA.

Termination shocks may, however, deviate from exactly perpendicularity in solar flares, the extent to which is an open question. In the meantime, it is instructive to relax the constraint that the shocks are exactly perpendicular and consider the case of quasi-perpendicularity. The difference is significant because particles can cross back upstream for quasi-perpendicular shocks. There were analytical (Wu 1984; Krauss-Varban et al. 1989b) and hybrid simulation studies (Krauss-Varban et al. 1989a) of SDA in a nearly perpendicular bow shock for energetic electrons. Recently, Matsukiyo et al. (2011) performed one-dimensional (1D) PIC simulation for the electron SDA in low Mach number quasi-perpendicular shocks in galaxy clusters.

In this paper, we present the results of two-dimensional (2D) PIC simulations for quasi-perpendicular low \( M/high \beta_p \) shocks in solar flares. The upstream magnetic field makes angles of \( \theta_B = 80^\circ, 82^\circ, \) and \( 83.5^\circ \) respectively to the shock normal and the shocks satisfy the subluminal condition, \( V_{\text{sh}}/\cos \theta_B < c \), where \( V_{\text{sh}} \) is the shock speed in the upstream rest frame and \( c \) is the speed of light. In such a subluminal shock, electrons can then be reflected at the shock front due to the magnetic mirror effect and gain energy (e.g., Ball & Melrose 2001; Mann et al. 2006, 2009; Warmuth et al. 2009).

One difficulty in performing PIC simulations for SDA in solar flares is that they require sufficiently high energy electrons above the threshold energy for SDA in the simulation to obtain statistically reliable results. The commonly used Maxwellian distribution has too few above the threshold electrons to be used in a simulation. Herein we use a kappa distribution with \( \kappa = 10 \) for both injected ions and electrons. This increases the number of high energy electrons but does not significantly alter the original shock structure, which is determined by the bulk of the thermal particles. Although the kappa distribution is used for computational convenience in this paper, the kappa distribution may be physically a relevant distribution in the outflow driven electron–ion jet (e.g., Yoon et al. 2006; Kašparová & Karlický 2009; Mann et al. 2006, 2009; Warmuth et al. 2009).

The goals of this paper are two-fold: (1) to study the formation and structure of such shocks, including the turbulent dissipation mechanism for collisionless shock sustenance and entropy creation, and (2) to study the particle acceleration mechanism relevant for the soft/hard X-ray flux observations in solar flares. We observe the same modified-two-stream instability (Krall & Liewer 1971) in the shock transition region as in...
the perpendicular shocks (Park et al. 2012) that can provide the turbulent dissipation in the downstream. Furthermore, a temperature anisotropy after the shock transition region is found to drive the electron whistler instability (e.g., Gary 1993).

As shown below, more SDA-accelerated electrons are found in the present simulations of the quasi-perpendicular case compared to the previously simulations of the perpendicular case (Park et al. 2012). In this context, we also extend the theoretical analysis in Mann et al. (2006, 2009) and Warmuth et al. (2009) to include the electric potential jump at the shock front (e.g., Ball & Melrose 2001) and generalize the electron energy spectrum. Both our theoretical analysis and our simulations show a transition energy, $E_{\text{trans,p}}$, between the thermal and non-thermal photon spectrum that is determined by the minimum angle of $\theta_B$. We find that the transition energy $E_{\text{trans,p}}$ and the spectral index $\delta$ from theory and our simulation are consistent with some of the X-ray spectra of solar flares from RHESSI (e.g., Altyntsev et al. 2012) in the energy range of $E \lesssim 40 \sim 100$ keV. Beyond this range, to maintain the power-law distribution up to $E \sim$ MeV, additional mechanisms beyond SDA are required.

The rest of the paper is organized as follows. The simulation setup is described in Section 2. The shock structure and particle acceleration are described in Section 3. We summarize in Section 4.

2. SIMULATION SETUP

We use the fully-relativistic full PIC code OSIRIS (Fonseca et al. 2002) to study the formation of and particle energization in low-$M$, high-$\beta_p$ shocks, where $M$ is the Mach number and $\beta_p$ is the ratio of thermal to magnetic pressure. To launch a shock, we use the moving wall boundary condition (Langdon et al.1988; Park et al. 2012) at the right boundary of the 2D simulation box. The moving wall method generates a slowly propagating shock compared to the more standard fixed reflection boundary method, and allows for smaller box sizes and more efficient use of simulation time.

We adopt parameters typical of those found in solar flare reconnection outflows (Tsuneta 1996; Workman et al. 2011) as the upstream conditions for our shock. In particular, we use a plasma density $n = 5 \times 10^{10}$ cm$^{-3}$, electron and ion temperatures $T_e = T_i = 0.8$ keV (= 9.27 $\times 10^6$ K), and the magnetic field strength $B = 6$ G with $\beta_p \equiv 8\pi n(T_e + T_i)/B^2 = 8.93$. The magnetic field is in the $x$-$y$ plane ($\mathbf{B} = B_x \hat{x} + B_y \hat{y}$) and has an angle of $\theta_B = 80^\circ$, $82^\circ$, and $83.5^\circ$ from the shock normal ($x$-axis) in each simulation. We also performed another simulation where the upstream magnetic field is directed out of the simulation plane ($\mathbf{B} = B_x \hat{x} + B_z \hat{z}$) and has an angle of $\theta_B = 80^\circ$ to compare the electron energy spectrum with that in the “in-plane” $B$-field simulation. A reduced ion/electron mass ratio of $m_i/m_e = 30$ is used to reduce computational demands. The Alfvén Mach number is chosen to be $M_A \equiv V_1\sqrt{4\pi m_e n}/B = 6.62$, which equates to an upstream plasma flow velocity in the shock rest frame, $V_1 = 0.032c$, for $m_i/m_e = 30$. If the real mass ratio, $m_i/m_e = 1836$, is used for the same $M_A$, then the upstream flow velocity would be 1226 km s$^{-1}$ (= 0.0041c). Effects of a realistic mass ratio are further discussed in Section 3.3. The fast-magnetosonic Mach number $M$ satisfies $M \equiv M_A/\sqrt{1+(5/6)\beta_p} = 2.28$. The ratio of the electron cyclotron frequency to the electron plasma frequency is $\Omega_{ce}/\omega_{pe} = 0.0265$. With these upstream values of $M_A$ and $\beta_p$, the Rankine–Hugoniot relation (Tidman & Krall 1971) for the shocks with the angles used gives a compression ratio in the range of (2.06, 2.50), where the lower and the upper limits are calculated for 2D and 3D, respectively.

The simulation box is initialized with a kappa-distributed ion–electron plasma drifting with $V_d = 0.0213c$ and $T_e = T_i = 0.8$ keV, where $V_d$ is set to a smaller value than the upstream speed $V_1(=0.032c)$ in the shock rest frame anticipating that the shock will be traveling to the left. In the upstream rest frame, the energy distribution is a kappa distribution such as

$$f(E) = \frac{2^{5/2}}{\sqrt{\pi} } \frac{1}{(2\kappa - 3)^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)} \frac{E^{1/2}}{T^{3/2}} \times \left(1 + \frac{2}{2\kappa - 3} \frac{E}{T}\right)^{-\kappa - 1}, \quad (1)$$

where $\Gamma$ is the gamma function and $f(E)$ is normalized to 1, $\int_0^{\infty} dE f(E) = 1$. The kappa distribution approaches the Maxwellian distribution as $\kappa$ goes to $\infty$. Figure 1 shows the initial energy distribution in the upstream rest frame from the simulation. The dashed line is a kappa distribution in Equation (1) with $\kappa = 10$ and $T = 0.8$ keV and the dotted line is a Maxwellian distribution with the same temperature. The bulk parts ($E < 5$ keV) of the two distributions are very similar. The implementation of the kappa distribution in OSIRIS is described in Appendix A.

A uniform external $E_z$ field is set up along the $z$-axis with $E_z = -V_d B_y/c$. A new plasma of the same kappa distribution is constantly injected from the left boundary ($x = 0$) throughout the simulation. The box sizes are $L_x \times L_y = 250c/\omega_{pe} \times 50c/\omega_{pe}$. The grid size used is $dx = dy = 0.08c/\omega_{pe}$ and the time step used is $dt = 0.056/\omega_{pe}$. For each particle species, 196 particles per cell are used. The particle number/cell is fairly large to reduce numerical collisions and maintain the kappa distribution for a sufficiently long time.

A linear current deposition scheme is used for all simulations in this paper. A periodic boundary condition is used in the $y$-direction for both particles and fields. For fields, an open boundary condition is used in the $x$-direction. Particles that reach $x = 0$ are re-injected into the box with the initial kappa distribution. At $x = L_x$, a moving wall boundary condition is adopted (Park et al. 2012) and the moving wall speed is set to $V_{wall} = 0.004c$.

3. RESULTS AND ANALYSIS

3.1. Shock Structure

In this subsection, we present results for the shock structure for $\theta_B = 80^\circ$ in-plane case only but other simulations for $\theta_B = 82^\circ$ and $83.5^\circ$ also show similar shock structures. The

![Figure 1. Initial energy distribution in the upstream rest frame from the simulation (solid line). The dashed line is a kappa distribution with $\kappa = 10$ and $T = 0.8$ keV. The dotted line is a Maxwellian distribution with $T = 0.8$ keV. (A color version of this figure is available in the online journal.)](image-url)
electron energization via SDA shows different energy spectra for the different angles of $\theta_B$ as will be seen in the next subsection.

In Figures 2(a) and (b), we plot for both species the upstream to downstream ratios of density $n(x)/n_1$, flow speeds $V_i/V(x)$, and $y$-magnetic field, $B_y(x)/B_1$, in Figures 2(c)–(l), we plot the phase-space distributions of $p_x$, $p_y$, and $p_z$, the $y$-averaged flow velocity profiles of $V_x$, $V_y$, and $V_z$, and the temperature profiles of $T_x$, $T_y$, and $T_z$, where $\parallel$ and $\perp$ are perpendicular and parallel to the magnetic field $B(x) \equiv B_x(x)\hat{x} + B_y(x)\hat{y}$, respectively, for the electrons (the left column) and the ions (the right column) at $t = 11200/\omega_{pe}$ for the shock-rest frame and the other plots are obtained in the simulation frame. (m)–(o) show the $B_x$, $B_y$, and $B_z$ fields in the $x$–$y$ space.

Figures 2(m)–(o) show $B$ in the $x$–$y$ space. The $B_x$ and $B_z$ fields show oscillatory patterns along the $y$-axis near the shock front ($x \sim 150c/\omega_{pe}$) and the $B_y$ field shows a rippled surface along the $y$-axis. These variations are due to electron temperature anisotropy-driven instabilities, $T_{x\perp} > T_{y\parallel}$.

Figure 3 shows the $y$-averaged $E$, $B$, and the potential energy $|e|\Phi(x) = e \int_{x_1}^{x} E_y dx$ at $t = 11200/\omega_{pe}$, measured in the shock rest frame. The $E_y$, $E_z$, and $B_y$ fields are approximately constant across the shock while $E_x$ and $B_z$ oscillate with a wave number $k = 0.2\omega_{pe}/c$ downstream. The potential energy jump $|e|\Delta\Phi$ at the shock front is $\sim 3.5$ keV (Figure 3(d)). Some ions in the low energy tail reflect at the shock front and can drive the modified two-stream instability.

In Figures 4(a)–(d), we plot the Fourier spectra of the $E_x$, $B_x$, $B_y$, and $B_z$ fields, where $\perp$ and $\parallel$ are perpendicular and parallel to $B_0$, respectively. The $B_0(= B_{0\parallel}\hat{x} + B_{0\perp}\hat{y})$ is an averaged field over $140 < x < 170 (c/\omega_{pe})$ in Figure 2 and has an angle 86° from the x-axis. Two distinct dominant modes are excited by temperature anisotropy-driven instabilities ($T_{x\perp} > T_{y\parallel}$), one is along the $B_0$ axis with $k = 0.5\omega_{pe}/c$ and another is sightly deviated from the $B_0$ axis with $k = 0.8\omega_{pe}/c$. In Figure 4(d), the strong signals at $k_\perp = \pm 0.2\omega_{pe}/c$ are due to the oscillatory pattern of the $B_z$ field along the $x$-axis as seen in Figure 3(f).

To analyze the spectra from the simulation, we numerically solve the dispersion relation (e.g., Gary 1993) in Figures 4(e) and (f). Here we assume bi-Maxwellian electron and ion distributions and a uniform background magnetic field $B_0$.  

Figure 2. Ratios, $n/n_1$, $V_i/V(x)$, and $B_y/B_1$, momentum distribution of $p_x$, $p_y$, and $p_z$, $y$-averaged flow velocity of $V_x$, $V_y$, and $V_z$, and temperature $T_x$, $T_y$, $T_z$ for the electrons (left column) and the ions (right column) at $t = 11200/\omega_{pe}$. (a) and (b) are calculated in the shock-rest frame and the other plots are obtained in the simulation frame. (m)–(o) show the $B_x$, $B_y$, and $B_z$ fields in the $x$–$y$ space. (A color version of this figure is available in the online journal.)
Figure 3. $y$-averaged $E$ and $B$ fields, and the potential energy $|e|\Phi(x)$ at $t = 11200/\omega_{pe}$ measured in the shock rest frame.

Figure 4. (a)–(d) Fourier spectra of the $E$ and $B$ fields in the $k_x - k_y$ space from the simulation. (e)–(f) Numerically solved growth rates of temperature anisotropy-driven instabilities in the region, $140 < x < 170(c/\omega_{pe})$ of Figure 2.

We use the parameters extracted from the simulation with $B_0 = 13.5G$, $T_{e\parallel} = 1.25$ keV and $T_{e\perp} = 1.66$ keV for the electrons, and $T_{i\parallel} = 0.86$ keV and $T_{i\perp} = 2.55$ keV for the ions, where $\parallel$ and $\perp$ are parallel and perpendicular directions to $B_0$, respectively.

In Figure 4(e), the electron whistler modes are centered on the $B_0$ axis with $k = 0.45\omega_{pe}/c$, the ion cyclotron modes centered on the $B_0$ axis with $k = 0.1\omega_{pe}/c$, and the ion mirror modes are obliquely off the $B_0$ axis with $k = 0.12\omega_{pe}/c$. The maximum growth rates are $\omega_i = 5.6 \times 10^{-4}\omega_{pe}$, $2.5 \times 10^{-4}\omega_{pe}$, and $2.0 \times 10^{-4}\omega_{pe}$ for the electron whistler, the ion cyclotron, and the ion mirror modes, respectively. In Figure 4(f), the real frequencies for the electron whistler and the ion cyclotron modes are around $\omega_r = 0.012\omega_{pe}$ and $0.0012\omega_{pe}$, respectively, and the ion mirror modes have zero real frequency. These are consistent with the analytical results (e.g., Gary 1993), $\Omega_{ci} < \omega_r < |\Omega_{ce}|$ for the electron whistler modes and $0 < \omega_r < \Omega_{ei}$ for the ion cyclotron modes.

The linear theory based on bi-Maxwellian distributions with the parameters found in the PIC simulations show that electron mirror modes are marginally stable (e.g., Gary & Karimabadi 2006). The oblique modes found in Figures 4(a)–(d) could be due to some unknown modes. However, given the fact that the actual distribution is not exactly a bi-Maxwellian (e.g., with flows) and the uncertainty in the temperature anisotropy measurements in the simulation, the oblique modes with $k = 0.8\omega_{pe}/c$ seen in the simulation (Figures 4(a)–(d)) may possibly still be a electron mirror-type of modes for a non-bi-Maxwellian distribution. In addition, the ion cyclotron/mirror modes are not observed in this simulation because of the small box size $L_y = 50c/\omega_{pe}$ and/or their relatively weaker growth.

Figures 5(a) and (b) show the electron and ion distributions in the shock transition region in $135 < x < 155(c/\omega_{pe})$ at $t = 11200/\omega_{pe}$ from the simulation. We observe that 27% of the incoming ions are reflected at the shock front (Figure 5(a)) and the modified two-stream instability (MTSI; e.g., Matsukiyo & Scholer 2003, 2006; Umeda et al. 2010, 2012) can be excited. Here we solve the MTSI in the electrostatic limit with its wave vector along the $x$-axis (Appendix B). The electrons have a temperature of $T_{e\parallel} = 1.65$ keV and drift with $V_{e\parallel} = 0.0064c$ in the simulation frame (Figure 5(b)). We fit the distributions using drifting Maxwellians. The electrons are
magnetized with $B = 10$ G while the ions are assumed to be non-magnetized. In the electron rest frame, the drift velocities for the incoming and reflecting ions are $V_{\text{in}} = 0.0116c$ and $V_{\text{re}} = -0.0134c$, respectively. Both incoming and reflecting ions have a temperature of $T_i = 0.98$ keV. In Figures 5(c) and (d), we solve the kinetic dispersion relation for the MTSI. The maximum growth rate is $k_i = 3.2 	imes 10^{-4} \omega_{\text{pe}}$ at $k = 0.2c/\omega_{\text{pe}}$ and the real frequency is $\omega_r = -0.0015\omega_{\text{pe}}$.

Turbulent dissipation is needed to randomize the upstream flow to form collisionless shocks (e.g., Wu 1982). The macroscopic jump conditions across a shock are essentially independent of the source of microphysical turbulence, as long as there is such a source. Plasma instabilities in the shock transition region are natural sources for this needed turbulent dissipation and we consider three possible instabilities: (1) lower hybrid instability (the excited modes are in the $B \times \mathbf{v}$ direction; e.g., Zhou et al. 1983), (2) whistler instability, and (3) MTSI (e.g., Papadopoulos et al. 1971; Wagner et al. 1971). The lower hybrid instability is precluded in these 2D in-plane simulations where the $B$-field is in the $x$-$y$ simulation plane. In addition, we notice that the shock structure (i.e., the compression ratio) in the simulation with an in-plane $B$ field is the same as that with the out-of-plane $B$ field which then precludes the whistler instability. We are led to conclude that the MTSI is the most likely candidate to provide the needed turbulence (e.g., Park et al. 2012).

3.2. Particle Heating via Shock Drift Acceleration

In Figure 6, we plot the electron and ion energy distributions, $f(E) = dN/dE$ upstream ($50 < x < 155c/\omega_{\text{pe}}$ (b) and (e)) and downstream ($155 < x < 250c/\omega_{\text{pe}}$ (c) and (f)) at $t = 8400/\omega_{\text{pe}}$ for $\theta_B = 80^\circ$. We fit the thermal (bulk) part of the distributions using a kappa distribution with $\kappa = 10$ (dashed lines), yielding $T_e = T_i = 0.8$ keV upstream and $T_e = 1.58$ keV and $T_i = 1.8$ keV downstream. In Figures 6(a) and (d), we plot the electron and ion phase–space distributions in $E - x$, overlaid with $B_\|$, field to indicate the location of the shock front. Abundant non-thermal electrons are seen ahead of the shock at $x = 155c/\omega_{\text{pe}}$, traveling as far back as $x = 10c/\omega_{\text{pe}}$ (Figure 6(a)).

In Figure 6(b), the electron energy spectrum shows a deviation from a thermal distribution at $E \sim 12$ keV and the spectrum becomes steeper at $E \sim 50$ keV. The dotted line in Figure 6(b) is the theoretical energy distribution via SDA when the electric potential energy jump $|e|\Delta \Phi \approx 3.5(keV)$ at the shock front is considered (see Section 3.2.1). In Figure 6(e), the ion energy spectrum shows a deviation from a thermal distribution at $E \sim 15$ keV and the spectrum becomes steeper at $E \sim 30$ keV.

In Figures 7(a)–(d), we plot a typical track of an electron experiencing SDA, overlaid with the shock front and the subsequent compression peaks (Figure 7(a)). The electron is reflected at the shock front (Figure 7(a)) and gains energy from 8 keV to 50 keV (Figure 7(c)). After the reflection, the electron drifts along the upstream magnetic field lines (Figure 7(b)). In Figure 7(d), we plot the electron in the $v_\perp - v_\parallel$ phase–space. The electrons in the region I are transferred to the region II after the reflection.

In Figures 7(e) and (f), we plot a typical track for an ion experiencing SDA. Whether or not an ion gains energy via SDA depends on its incident speed and angle of incidence at the shock front (Kirk 1994). When the ion meets the shock front, it turns back toward the upstream with a larger gyro-radius (Figure 7(a)) and is accelerated by the $E_z$ field. The kinetic energy of the ion increases from 5 keV up to 20 keV (Figure 7(b)). A detailed analysis for ions experiencing SDA in perpendicular shocks was described in our previous work (Park et al. 2012).

3.2.1. Electron Spectrum via SDA

In this subsection, we generalize the electron energy spectrum via SDA by Mann et al. (2006, 2009) and Warmuth et al. (2009) to include the electric potential energy, $e \Phi$, at the shock front (e.g., Ball & Melrose 2001) and compare with the PIC simulation results.

First, we consider the de Hoffmann–Teller (dHT) frame (denoted by $'$) where the motional $E_z = -V_{\text{sh}}/cB_\|$, field vanishes. The dHT frame is obtained by boosting with $V_{\text{sh}} = V_{\text{in}}/\cos \theta_B$ along the magnetic field line in the upstream rest frame. (Here we consider the negative shock speed, $V_{\text{sh}} < 0$, for a shock traveling to the $-\hat{x}$ direction.) The maximum $\theta_B$ for the existence of the dHT frame is given by $\theta_{B_{\text{max}}} = \cos^{-1}(V_{\text{sh}}/c) < 90^\circ$.

In the dHT frame, the condition for an electron to reflect at the shock front is (e.g., Ball & Melrose 2001; Mann et al. 2006, 2009) $\beta_\perp > \beta_\parallel \tan \alpha_0$ and $\beta_\parallel > 0$, where $\perp$ and $\parallel$ are perpendicular and parallel to the upstream $B_\parallel$, respectively, $\beta_\parallel = v_\parallel/c$, and $\alpha_0 = \sin^{-1}\sqrt{B_\perp/B_\parallel} = \sin^{-1}\sqrt{|e|\Phi/m_e c^2}$. If we include the potential energy $e \Phi < 0$ at the shock front, the reflection condition, Equation (2), can be written in the non-relativistic limit by (e.g., Ball & Melrose 2001)

$$\beta_\perp > \sqrt{\beta_\parallel^2 - 2e \Phi/m_e c^2} \tan \alpha_0,$$

where $\perp$ and $\parallel$ are perpendicular and parallel to the upstream $B_\parallel$, respectively, $\beta_\parallel = v_\parallel/c$, and $\alpha_0 = \sin^{-1}\sqrt{B_\perp/B_\parallel} = \sin^{-1}\sqrt{|e|\Phi/m_e c^2}$. If we include the potential energy $e \Phi < 0$ at the shock front, the reflection condition, Equation (2), can be written in the non-relativistic limit by (e.g., Ball & Melrose 2001)

$$\beta_\perp > \sqrt{\beta_\parallel^2 - 2e \Phi/m_e c^2} \tan \alpha_0,$$

where $\perp$ and $\parallel$ are perpendicular and parallel to the upstream $B_\parallel$, respectively, $\beta_\parallel = v_\parallel/c$, and $\alpha_0 = \sin^{-1}\sqrt{B_\perp/B_\parallel} = \sin^{-1}\sqrt{|e|\Phi/m_e c^2}$. If we include the potential energy $e \Phi < 0$ at the shock front, the reflection condition, Equation (2), can be written in the non-relativistic limit by (e.g., Ball & Melrose 2001)

$$\beta_\perp > \sqrt{\beta_\parallel^2 - 2e \Phi/m_e c^2} \tan \alpha_0,$$

where $\perp$ and $\parallel$ are perpendicular and parallel to the upstream $B_\parallel$, respectively, $\beta_\parallel = v_\parallel/c$, and $\alpha_0 = \sin^{-1}\sqrt{B_\perp/B_\parallel} = \sin^{-1}\sqrt{|e|\Phi/m_e c^2}$. If we include the potential energy $e \Phi < 0$ at the shock front, the reflection condition, Equation (2), can be written in the non-relativistic limit by (e.g., Ball & Melrose 2001)
Figure 6. (a) and (d) Energy distribution vs. $x$-ranges where the $B_y$ field is over-plotted with an arbitrary scale. The electron ((b) and (c)) and ion ((e) and (f)) energy distributions in upstream ($50 < x < 155c/\omega_{pe}$) and downstream ($155 < x < 250c/\omega_{pe}$) at $t = 8400/\omega_{pe}$. In (b), the dotted line is a theoretical energy distribution when the potential energy at the shock front is $e\Phi = -3.5$(keV). We fit the thermal distributions with kappa distributions with $\kappa = 10$ (dashed lines).

(A color version of this figure is available in the online journal.)

Figure 7. (a)–(d) A typical electron tracking experiencing SDA. (e) and (f) A typical ion tracking experiencing SDA.
where $\beta_s = v_s/c = V_{sh}/c \cos \theta_B$ and $\gamma_s = 1/\sqrt{1 - \beta_s^2}$.

Equation (3) is transformed in the upstream rest frame into

$$\beta_1 > \gamma_s \tan \alpha_0 \left[ \left( \beta_{1\|} - \beta_s \right)^2 - \frac{2e\Phi}{m_e c^2} \left( 1 - \beta_s \beta_r \right)^2 \right]^{1/2},$$

$$\beta_{1\perp} > \beta_s.$$  \hspace{0.5cm} (5)

The velocity of the electron after reflection in the dHT frame is given by $\beta_{1\|} = -\beta_1^\prime \|$ and $\beta_{1\perp} = \beta_1^\prime \perp$, where the indices $i$ and $r$ represent the incoming and reflected electron, respectively. In the upstream rest frame, one gets (Mann et al. 2006, 2009)

$$\beta_{1\|} = \frac{2\beta_i - \beta_1^\prime \| (1 + \beta_r^2)}{1 - 2\beta_1^\prime \| \beta_i + \beta_r^2}, \beta_{1\perp} = \gamma_s \left[ 1 - 2\beta_1^\prime \beta_i + \beta_r^2 \right].$$  \hspace{0.5cm} (6)

In Figure 8, we plot the SDA condition in the $\beta_{1\perp} - \beta_{1\|}$ space for $\beta_1 = -0.293$ as an example (e.g., Mann et al. 2006, 2009). Equation (5) defines the region I (light shaded region) and the hyperbolic curve is given by the potential energy $e\Phi = -3.5$ keV at the shock front. When $e\Phi$ goes to zero, the hyperbolic curve approaches the straight dotted lines as seen in Mann et al. (2006, 2009). Equation (6) implies that the incoming electrons in the region I are transferred to the region II (dark shaded region) after reflection. As $\theta_B$ increases to $\Theta_{th} = \cos^{-1}(V_{sh}/c)$, $\beta_r$ goes to $-1$ in Figure 8 and the number of electrons satisfying the reflection condition in Equation (5) decrease to zero. Therefore, no electron reflects at the shock front in the superluminal shocks (where $|\beta_1| \geq 1$) or the perpendicular shocks.

The threshold energy $E_{thres}$ for SDA is given by the shortest distance from the origin to the hyperbolic curve, $\bar{\beta}_{thres}(= \bar{\beta}_{thres})$ in Figure 8, namely

$$E_{thres} = \left( \frac{1}{\sqrt{1 - \beta_{thres}^2}} - 1 \right) m_e c^2,$$

$$\bar{\beta}_{thres} = \sqrt{\frac{D^2 \left[ (1 - \gamma_s^2) D^2 P^2 \left[ P - (1 + \beta_r^2) \right] + \beta_r^2 - P \right]}{1 + D^2 \left[ 1 - P \beta_r^2 \right]}}.$$

where $D = \gamma_s \tan \alpha_0$ and $P = 2e\Phi/m_e c^2$.

The transition energy $E_{trans,e}$ between thermal and non-thermal electron populations is given by the distance from the origin to the point $\bar{\beta}_{trans}(= \bar{\beta}_{trans})$, in Figure 8 such as

$$E_{trans,e} = \left( \frac{1}{\sqrt{1 - \beta_{trans}^2}} - 1 \right) m_e c^2,$$

$$\beta_{trans} = \beta_{i\|}^\prime \left[ 1 - \beta_i^\prime \right] (2e\Phi/m_e c^2) \tan^2 \alpha_0.$$  \hspace{0.5cm} (8)

Beyond $E = E_{trans,e}$, the spectral index $\delta$, defined as in $f(E) \propto E^{-\delta}$, increases. Figure 9 shows how the transition energy $E_{trans,e}$ for $e\Phi = -3.5$ keV (solid) and 0 keV (dashed) from Equation (8). The energy point $E_{2,e}$ for $e\Phi$ (dot-dashed) varies with $\theta_B$ when $V_{sh} = 0.0041c$, $r = 2.5$ and $m_i/m_e = 1836 (M_A = 6.62$ and $\beta_p = 8.93)$.

The reflected electron distribution in the upstream rest frame is written as

$$f_r(\beta_r) = f_i(\beta_i(\beta_r)) \left( \frac{d^3 \beta_i}{d^3 \beta_r} \right) \Theta(\beta_s - \beta_r) \times \Theta(\beta_{1\perp} - \gamma_s \tan \alpha_0 \left[ \beta_s \beta_r \right]^2 - \frac{2e\Phi}{m_e c^2} (1 - \beta_s \beta_r) \right]^{1/2},$$

where $\Theta$ is the step function and the term $d^3 \beta_i/d^3 \beta_r$ is given by the Jacobian determinant,

$$\frac{d^3 \beta_i}{d^3 \beta_r} = \frac{\beta_{1\perp}}{\beta_{1\perp}} \left| \frac{\partial(\beta_{1\perp}, \beta_{1\|})}{\partial(\beta_{1\perp}, \beta_{1\|})} \right| = \frac{(1 - \beta_r)^4}{(1 - 2\beta_{1\perp} \beta_i + \beta_r^2)^2}. $$

Here we let the incoming distribution in Equation (10) be a semi-relativistic kappa distribution (Mann et al. 2006, 2009),

$$f_i(\beta_i) = \delta \left( 1 + \frac{2(\gamma_i - 1) m_e c^2}{2\delta - 3} T \right)^{-\delta - 1},$$

where $\gamma_i = 1/\sqrt{1 - \beta_i^2}$ and $\delta = 1/ \int d^3 \beta f_i(\beta_i)$. We calculate the upstream electron energy distributions, $f(E) = f(\beta) d\beta/dE$, in the upstream rest frame using
Equations (6)–(12) to obtain

\[
 f(\beta) = \begin{cases} 
 2\pi\beta^2 \int_{-1}^{1} dt f_i(\beta) & \text{for } \beta < \beta_{\text{thres}} \\
 2\pi\beta^2 \left( \int_{-1}^{0} dt + \int_{0}^{1} dt \right) f_i(\beta) & \text{for } \beta_{\text{thres}} \leq \beta < \beta_{\text{trans}}, \\
 2\pi\beta^2 \int_{-1}^{1} dt \frac{\partial f_i}{\partial \beta} f_i(\beta, \beta) & \text{for } \beta_{\text{trans}} \leq \beta, \\
 +2\pi\beta^2 \left( \int_{-1}^{0} dt + \int_{0}^{1} dt \right) f_i(\beta) & \text{for } \beta \geq \beta_{\text{trans}}, 
\end{cases}
\]

(13)

where \( t = \cos \theta \) and \( \theta \) is the electron’s pitch angle, i.e., the angle between \( \beta \) and \( B_1 \). The boundaries \( t_1(= \cos \theta_1), t_2(= \cos \theta_2), \) and \( t_3(= \cos \theta_3) \) as shown in Figure 8 are given as \( t_2 = \beta_s / \beta \), and roots of the equation,

\[
 t^2_{1,3} \left[ 1 + D^2 \left( 1 - P\beta^2 \right) \right] - 2\eta_1 D^2 \frac{B_i}{\beta} \left( 1 - P \right) + \frac{D^2}{\beta^2} \left( \beta^2_s - P \right) - 1 = 0.
\]

(14)

In Figures 10(a)–(c), we plot the upstream electron distribution in the \( \beta_1 - \beta_3 \) space of the upstream rest frame from the simulations for \( \theta_B = 80^\circ, 82^\circ, \) and \( 83.5^\circ \). Electrons in region I are transferred to region II. As \( \theta_B \) increases, the number of electrons participating in SDA decreases since the energy threshold for SDA in Equation (7) increases.

In Figures 10(d)–(f), we plot the normalized upstream electron energy distributions, \( f(E) = (1/N)(dN/dE) \), in the upstream rest frame for \( \theta_B = 80^\circ, 82^\circ, \) and \( 83.5^\circ \). We compare the results of Equation (13) for \( e\Phi = 0 \) (dashed) and \(-3.5\) keV (dot-dashed) with the simulation result (solid). The dotted line is the incoming kappa distribution with \( T = 0.8 \) keV and \( \kappa = 10 \). Using Equation (8), the transition energy points are given by

\[
 E_{\text{trans},e} = 11.3 \text{ keV}, 16.5 \text{ keV}, \text{ and } 24.2 \text{ keV for the angles, } \theta_B = 80^\circ, 82^\circ, \text{ and } 83.5^\circ, \text{ respectively, when } e\Phi = -3.5 \text{ keV.}
\]

Using Equation (9), the energy points \( E_{\text{2},e}'s \), beyond which the spectral index increases, are given by \( E_{\text{2},e} = 34 \text{ keV}, 56 \text{ keV}, \text{ and } 93 \text{ keV for the angles, } \theta_B = 80^\circ, 82^\circ, \text{ and } 83.5^\circ, \text{ respectively. Here } V_{\text{sh}} = -0.032c, r = 2.5 \text{ and } m_i/m_e = 30 (M_A = 6.62 \text{ and } \beta_p = 8.93). \)

In Figure 11, we plot the upstream electron energy distribution in the upstream rest frame from the out-of-plane upstream \( B \)-field simulation (solid) (\( B_1 = B_{1z}, z+B_{1x}, x \)). The dashed and dot-dashed lines are the theoretical results for \( e\Phi = 0 \) and \( e\Phi = -3.5 \text{ keV at the shock front, respectively.} \)

(A color version of this figure is available in the online journal.)
the shock front can contribute to further electron acceleration, possibly via the interaction between electrons and perturbed magnetic fields. There is a still discrepancy between the theory with $\Phi = -3.5$ keV and the simulation around the transition energy, $E_{\text{trans}} = 11.3$ keV. This is probably due to the effects of small-scale waves generated in the transition region as indicated by Matsukiyo et al. (2011).

### 3.2.2. Bremsstrahlung Radiation

Electrons accelerated via SDA move along the magnetic field lines as can be seen from the electron tracking in Figure 7(b) and will collide with ions to emit bremsstrahlung radiation. In the low-frequency limit of $\omega b / \gamma v \ll 1$, where $\omega$ is a photon angular frequency, $b$ is the impact parameter, $v$ is the electron speed, and $\gamma = 1/\sqrt{1 - (v/c)^2}$, the number of photons per unit frequency per unit volume per unit time produced by the bremsstrahlung process is (e.g., Rybicki & Lightman 1979)

$$\frac{dN}{d\omega dV dt} = \frac{16Z^2 e^4 n_i n_e}{3c^3 \hbar^2 \omega} \int_{v_{\text{min}}}^{c} \frac{\gamma}{v} \ln \left( \frac{\gamma m_e v^2}{\hbar \omega} \right) f(v) dv, \quad (15)$$

where $f(v)$ is a normalized electron distribution and $v_{\text{min}}$ is determined by the equation $\hbar \omega = (\gamma (v_{\text{min}}) - 1) mc^2$.

Given the electron distributions in Figures 10(d)–(f), we calculate the number of photons per unit time (s) per unit energy (keV) using Equation (15) for the ion density of $n_i = n_e$ and the volume of the region emitting the X-rays, $V = 10^{27} \text{ cm}^3$. In Figures 12(a) and (b), we show the photon distribution for $\theta_B = 80$ (solid), 82 (dashed), and 83.5 (dotted) when the electron distribution is given by the theoretical distribution in Equation (13) with $\Phi = -3.5$ keV (Figure 12(a)) and by the simulation (Figure 12(b)). The dotted line is the photon distribution when the electron distribution is given by a kappa distribution with $T = 0.8$ keV and $\kappa = 10$. Here $V_{sh} = -0.032c, r = 2.5$ and $m_i/m_e = 30 (M_A = 6.62$ and $\beta_p = 8.93)$.

In Figure 12(c), we plot the averaged photon distribution of $80^\circ \leq \theta_B \leq \theta_{\text{max}} = 88.17^\circ$ from the theoretical distribution in Equation (13) with $\Phi = 0$ (dashed) and $\Phi = -3.5$ keV (solid). In Figure 12(d), we plot the averaged photon distribution of $\theta_B = 80, 82,$ and 83.5 from the theoretical distribution in Equation (13) with $\Phi = 0$ (dashed) and $\Phi = -3.5$ keV (dotted), and from the simulation (solid). In Figure 12(d), we notice that the theoretical result with $\Phi = -3.5$ keV (dotted) is in good agreement with the simulation result (solid).

In Figure 12(d), the simulation result shows that the transition energy between the thermal and non-thermal photon spectrum is $E_{\text{trans,p}} \approx 10$ keV and the energy point beyond which the photon spectrum becomes steeper is $E_{2,p} \approx 40$ keV. The spectral index is $\delta = 3$ (simulation) and $\delta = 2.2$ (theory with $\Phi = -3.5$ keV) in $10$ keV $< E < 40$ keV and $\delta = 7$ (simulation) in $E > 40$ keV. For emission from multiple shocks with different $\theta_B$’s, the transition energy $E_{\text{trans,p}}$ and $E_{2,p}$ would be dominated by the shock with the minimum $\theta_B$. Note that the transition energy $E_{\text{trans,p}}$ for the photon distribution is a bit smaller than $E_{\text{trans,n,e}}$ for the electron distribution because bremsstrahlung photons are produced by electrons with higher energies than the photon energy (e.g., Holman 2003). The energy point $E_{2,p}$ is approximately given by Equation (9).

*RHESSI* data for several solar flares (Table I in Altyntsev et al. 2012) show that the spectral index $\delta$ is in the range $2.5 < \delta < 3$ and the transition energy is in the range $12.1$ keV $< E_{\text{trans,p}} < 29.2$ keV. Therefore, the electron energization via SDA well explains some of the *RHESSI* X-ray spectra for the energy regime, $E < E_{2,p}$ and how the transition energy is related with the shock geometry, i.e., the minimum $\theta_B$.

The observed *RHESSI* spectra do not show a steepening beyond $E = E_{2,p}$, and thus the theory herein by itself cannot account for the electron acceleration to produce those photons. This indicates that additional mechanisms, such as diffusive shock acceleration, are required for further electron energization to maintain the power-law spectrum up to $E \sim \text{MeV}$. It is
not unreasonable to expect that solar flares involve multiple acceleration mechanisms operating on a range of scales.

3.3. Effects of a Realistic Proton/Electron Mass Ratio on the Spectra

With the actual ion/electron mass ratio of $m_i/m_e = 1836$, the shock speed in the upstream rest frame is reduced by $\sqrt{30/1836}$ compared to our simulations when the Mach number $M$ and the plasma $\beta_p$ are fixed. The compression ratio $r$ and the electric potential energy $e\Phi$ are unchanged for a fixed $M$ and $\beta_p$ (e.g., Hoshino 2001). Therefore, the shock structure is not expected to change for a realistic mass ratio simulation.

The electron energy spectrum in Equation (6) depends only on $\beta_e$, $\alpha_0$, and $e\Phi$. For a fixed $M$ and $\beta_p$, $\alpha_0(\equiv \sin^{-1}\sqrt{B||/B_0})$ and $e\Phi$ are unchanged. From the definition of $\beta_e(\equiv V_{\text{sh}}/c\cos \theta_B)$, we only need to change the angle from $\theta_B$ into $\theta_B'$ to obtain the electron energy spectrum for the mass ratio $m_i/m_e = 1836$ from that with $m_i/m_e = 30$,

$$\theta_B' = \cos^{-1}\left[\frac{\cos \theta_B}{\sqrt{1836/30}}\right].$$

For example, the averaged spectrum with $80^\circ \leq \theta_B < \theta_{B\text{max}} (= 88.73^\circ)$ for $m_i/m_e = 30$ corresponds to that with $88.73^\circ \leq \theta_B \leq 89.77^\circ$ for $m_i/m_e = 1836$.

In Figure 13, we plot the averaged photon distribution of $\theta_{B\text{min}} \leq \theta_B \leq \theta_{B\text{max}} (= 89.77^\circ)$ for a real ion/electron mass ratio, $m_i/m_e = 1836$, from the electron distribution given by Equation (13) with $e\Phi = -3.5$ keV. Here $V_{\text{sh}} = 0.0041c$ and $r = 2.5$ ($M_A = 6.62$ and $\beta_p = 8.93$). The different values of $\theta_{B\text{min}}$, from $\theta_{B\text{min}} = 88.73^\circ$ to $89.43^\circ$, give the different transition energy points, from $E_{\text{trans},p} = 10$ to 35 keV. The power indices of the photon spectrum are nearly the same as $\delta \sim 2.2$ in $E_{\text{trans},p} < E < E_{2,p}$, where $E_{2,p}$ runs from 40 to 150 keV.

4. CONCLUSION

In summary, we studied quasi-perpendicular, low $M$/high $\beta_p$ shocks with full PIC 2D simulations using a reduced ion/electron mass ratio $m_i/m_e = 30$. The shock compression ratio we found was in agreement with the Rankine–Hugoniot relation. Whistler instabilities driven by downstream temperature anisotropy were observed. A modified two-stream instability due to the incoming and reflecting ions in the shock transition region was also observed.

Abundant non-thermal electrons accelerated via SDA were observed upstream. We compared the electron energy distribution from the simulations with the distributions derived by extending a theoretical model (Mann et al. 2006, 2009; Warmuth et al. 2009) and found that they reasonably agree with each other.

In the perpendicular shocks, however, SDA can be achieved only by particles transmitting into the downstream, and their energy gains are smaller than those of the reflected ones in quasi-perpendicular shocks here (e.g., Ball & Melrose 2001). Therefore, such abundant non-thermal electrons observed in this paper were not seen in the perpendicular shocks (e.g., Park et al. 2012).

We calculated the photon flux via bremsstrahlung radiation from the electron distributions from both the theory and the simulations. We showed that a transition energy, $E_{\text{trans},p}$, marking the transition from a thermal to a non-thermal part of the photon spectrum, is determined by the minimum $\theta_B$ from multiple shocks with different $\theta_B$’s in $\theta_{B\text{min}} \leq \theta_B \leq 90^\circ$. Different solar flares have different $\theta_{B\text{min}}$’s in their termination shocks and therefore can show different transition energy points.

From the simulations, the averaged photon spectrum of $\theta_B = 80^\circ$, $82^\circ$, and $83.5^\circ$ gives a spectral index $\delta \sim 3$ in $10 \text{ keV} < E < 40 \text{ keV}$ and the spectral index increases beyond $E = 40 \text{ keV}$. The spectral index $\delta \sim 3$ as well as the transition energy, $E_{\text{trans},p} = 10$ keV, well explains some of the RHESSI X-ray spectra in the energy regime, $E < 40 \text{ keV}$. To account for the spectral index of the RHESSI X-ray spectrum up to $E \sim \text{MeV}$, however, additional mechanisms other than SDA are required.

Note that although our simulations were performed using $m_i/m_e = 30$, we analytically scaled the results to the realistic ion/electron mass ratio, and found that the predicted photon spectra are indeed insensitive to this mass ratio.

This work was supported by NSF under Grant PHY-0903797, by DOE under Grant No. DE-FG02-06ER54879 and Cooperative Agreement No. DE-FG52-08NA28302, and by NSFC under Grant No. 11129503. We also thank the OSIRIS consortium for the use of OSIRIS. The research used resources of NERSC.

APPENDIX A

GENERATION OF A KAPPA DISTRIBUTION

To generate a kappa distribution, we use the random number distribution by Leitner et al. (2011). We denote \{Ni(0, 1)|i = 1, 2, ..., N\} as a set of normally distributed random numbers with the mean of 0 and the deviation of 1, and \{Ui(0, 1)|i = 1, 2, ..., N\} as a set of uniformly distributed random numbers between [0, 1]. Then a sequence of the random number \xi given by

$$xi = b_1 Ni(0, 1) + b_2 U_i(0, 1)xi_{i-1}$$

(A1)

generates a 1D kappa distribution with the mean of 0 and the deviation of 1, and the coefficients, $b_1$ and $b_2$ determine the index $\kappa$ (Figure 8 in Leitner et al. 2011).

To implement the kappa generator in OSIRIS, we determine the particle’s initial momentum $\mathbf{p}$ as

$$\mathbf{p}_i = \mathbf{p}_{0i} + \mathbf{p}_d,$$

(A2)
where $x_i$ is given by Equation (A1), $p_{th}$ and $p_d$ are the thermal and the drift momentum, respectively. For $k = 10$, we choose $b_1 = 0.58$ and $b_2 = 1.15$. Then a sequence of $p_i$ in Equation (A2) generates a 3D kappa distribution in Figure 1.

APPENDIX B

DISPERSION RELATION FOR THE MODIFIED TWO-STREAM INSTABILITY

The kinetic dispersion relation for electrostatic instabilities is

$$1 + \sum_{s=\infty,1} K_s(k, \omega) = 0,$$  \hspace{1cm} (B1)

where $K_s(k, \omega)$ is the susceptibility. For magnetized electrons with isotropic Maxwellian distributions, $K_s(k, \omega)$ is given by (Gary 1993)

$$K_s(k, \omega) = \frac{\omega_p^2}{v_{th}^2 k^2} \left[ 1 + \xi_0 e^{-\lambda_e} \sum_{m=-\infty}^{\infty} I_m(\lambda_e) Z(\xi_e^m) \right],$$  \hspace{1cm} (B2)

where $v_{th}$ is the thermal velocity, $I_m$ is the modified Bessel function of the first kind, $\lambda_e = k_{\parallel} v_{th}^2 / \Omega_e$, $\Omega_e = eB / (m_e c) (\epsilon < 0)$, and $Z(\xi_e^m) = (\omega - m \Omega_e) / (\sqrt{2} k_{\parallel} v_{th})$. For unmagnetized ions with drifting Maxwellian distributions,

$$K_i(k, \omega) = -\frac{\omega_p^2}{2 v_{th}^2 k^2} Z'(\xi_i),$$  \hspace{1cm} (B3)

where $\xi_i = (\omega - k \cdot V_{id}) / (\sqrt{2} k v_{th})$ and $'$ is the derivative with respect to $\xi_i$.

Here we consider that $B \approx B_{\parallel} \hat{y}$, $k = k_{\parallel} \hat{x} V_{id} = V_{sd} \hat{x}$. Then the term $\xi_0^0 Z(\xi^0)$ in Equation (B2) becomes $-\omega / (\omega - m \Omega_e)$ as $k_{\parallel}$ goes to 0. The dispersion relation for the MTSI in the electrostatic limit becomes

$$1 + \frac{\omega_p^2}{v_{th}^2 k^2} \left( 1 - e^{-\lambda_e} \sum_{m=-\infty}^{\infty} I_m(\lambda_e) \frac{\omega}{\omega - m \Omega_e} \right)$$

$$- \sum_{s=\infty,1} \frac{\omega_p^2}{2 v_{th}^2 k^2} Z'(\xi_s) = 0.$$  \hspace{1cm} (B4)

REFERENCES

Altyntsev, A. A., Fleishman, G. D., Lesovoi, S. V., & Meshalkina, N. S. 2012, ApJ, 758, 138
Ball, L., & Melrose, D. B. 2001, PASA, 18, 361
Blackman, E. G., & Field, G. B. 1994, PRL, 73, 3097
Fonseca, R. A., Silva, L. O., Tsang, F. S., et al. 2002, in OSIRIS: A Three-dimensional, Fully Relativistic Particle in Cell Code for Modeling Plasma-based Accelerators, Lecture Notes in Computer Science (Berlin, Heidelberg: Springer), 342
Forbes, T. G. 1998, SoPh, 117, 97
Gary, S. P. 1993, Theory of Space Plasma Microinstabilities (Cambridge: Cambridge Univ. Press)
Gary, S. P., & Karimabadi, H. 2006, JGRA, 111, A11224
Guo, F., & Giacalone, J. 2010, ApJ, 715, 406
Guo, F., & Giacalone, J. 2012, ApJ, 753, 28
Holman, G. D., Sui, L., Schwartz, R. A., & Emslie, G. 2003, ApJL, 595, L97
Hoshino, M. 2001, PThPS, 143, 149
Kašparová, J., & Karlický, M. 2009, A&A, 497, L13
Kirk, J. G. 1994, in Plasma Astrophysics, ed. J. G. Kirk, D. B. Melrose, & E. R. Priest (Berlin: Springer), 225
Krall, N. A., & Liewer, P. C. 1971, PhRvA, 4, 2094
Krauss-Varban, D., Burgess, D., & Wu, C. S. 1989a, JGR, 94, 15089
Krauss-Varban, D., & Wu, C. S. J. 1989b, JGR, 94, 15367
Langdon, B., Arons, J., & Max, C. 1988, PRL, 61, 7
Leitner, M., Leubner, M. P., & Vörös, Z. 2011, PhyA, 390, 1248
Lin, R. P., Krucker, S., Hurford, G. J., et al. 2003, ApJL, 595, L69
Mann, G., Aurass, H., & Warmuth, A. 2006, A&A, 454, 969
Mann, G., Warmuth, A., & Aurass, H. 2009, A&A, 494, 669
Matsukiyo, S., Ohira, Y., Yamazaki, R., & Umeda, T. 2011, ApJ, 742, 47
Matsukiyo, S., & Scholer, M. 2003, JGRA, 108, 1459
Matsukiyo, S., & Scholer, M. 2006, JGRA, 111, A06104
Papadopoulos, K., Wagner, C. E., & Haber, I. 1971, PRL, 27, 982
Park, J., Workman, J. C., Blackman, E. G., Ren, C., & Siller, R. 2012, PPhI, 19, 062904
Rybicki, G. B., & Lightman, A. P. 1979, Radiative Processes in Astrophysics (New York: Wiley)
Tidman, D. A., & Krall, N. A. 1971, Shock Waves in Collisionless Plasmas (New York: Wiley)
Tsuneta, S. 1996, ApJ, 456, 840
Umeda, T., Kidani, Y., Matsukiyo, S., & Yamazaki, R. 2012, JGRA, 117, A03206
Umeda, T., Kidani, Y., Yamao, M., Matsukiyo, S., & Yamazaki, R. 2010, JGRA, 115, A10250
Wagner, E., Papadopoulos, K., & Haber, I. 1971, PhLA, 35, 440
Wang, A., Mann, G., & Aurass, H. 2009, A&A, 494, 677
Workman, J. C., Blackman, E. G., & Ren, C. 2011, PhPh, 18, 092902
Wu, C. S. 1982, SSRv, 32, 83
Wu, C. S. 1984, JGR, 89, 8857
Yoon, P. H., Rhee, T., & Ryu, J. 2006, JGRA, 111, A09106
Zhou, Y. M., Wong, H. K., Wu, C. S., & Winske, D. 1983, JGR, 88, 3026