Generation and characterization of complex vector modes with digital micromirror devices: a tutorial

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Abstract
Complex vector light modes with a spatial variant polarization distribution have become topical of late, enabling the development of novel applications in numerous research fields. Key to this is the remarkable similarities they hold with quantum entangled states, which arises from the non-separability between the spatial and polarisation degrees of freedom (DoF). As such, the demand for diversification of generation methods and characterization techniques have increased dramatically. Here we put forward a comprehensive tutorial about the use of digital micromirrors devices (DMDs) in the generation and characterization of vector modes, providing details on the implementation of techniques that fully exploits the unsurpassed advantage of DMDs, such as their high refresh rates and polarisation independence. We start by briefly describing the operating principles of DMD and follow with a thorough explanation of some of the methods to shape arbitrary vector modes. Finally, we describe some techniques aiming at the real-time characterization of vector beams. This tutorial highlights the value of DMDs as an alternative tool for the generation and characterization of complex vector light fields, of great relevance in a wide variety of applications.

Supplementary material for this article is available online

Keywords: complex vector beams, digital micromirror devices

(Some figures may appear in colour only in the online journal)

1. Introduction
Complex vector light fields, also known as vector or classically-entangled modes, feature a non-homogeneous polarisation distribution, resulting from the non-separable coupling between the spatial and polarisation degrees of freedom [1–7]. Vector modes have drawn significant attention in the last two decades or so, in part due to their broad variety of applications, which extends to areas such as, classical and quantum communications, laser material processing, optical manipulations, optical metrology, amongst many others [8–22]. As such, different techniques have been proposed for their generation, which can be classified into geometric
or dynamic phase modulation. The former is based on the direct transformation from spin to orbital angular momentum (OAM), using an inhomogeneous and anisotropic material as the intermediary, a process known as spin-to-orbital conversion [21]. One of the first optical elements, a liquid crystal birefringent material widely known as q-plate, has the capability of directly transforming a linearly polarized beam into a complex vector beam [24, 25]. Geometric-phase elements are also fabricated using sub-wavelength gratings, with periods smaller than the wavelength of the input light, so that each groove acts as a polariser with a variant transmission axis across the transverse profile of the beam [26]. This spatially-varying polarisation transformation induces an optical phase delay linked to the geometry of the sub-wavelength structure, which is responsible for the conversion of arbitrary homogeneously polarised states of light into vector beams [27]. Noteworthy, even though the geometric-phase approach allows the generation of vector beams in an easy and direct way, they are limited in that they can only generate one particular optical field of specific intensity profile, polarisation distribution, wavelength and size. In other words, once fabricated, they are not re-configurable at all, which represents a serious drawback in many applications, such as in optical communications or optical metrology [19, 28]. In regards to the generation via the dynamic phase, most techniques rely on the interferometric superposition of two coherent orthogonal spatial modes, such as the Laguerre–Gaussian (LG) beam, of orthogonal polarisation. A way to achieve this is by splitting an input beam into two beams which propagate along different paths so that their polarisation, amplitude or phase, can be manipulated independently prior to their coaxial recombination. Manipulation of the state of polarisation is achievable through phase retarders, polarisers and wave plates, whereas manipulation of the spatial degree of freedom, phase and amplitude is achieved in a broad variety of ways. Nonetheless, computer-controlled devices, such as liquid crystal spatial light modulators (SLMs) and digital micromirror devices (DMDs), stand out as some of the most flexible since they allow almost unlimited freedom in the variety of spatial modes that can be generated [29–33]. SLMs, however, are polarization-dependent, allowing only the modulation of linear polarization (typically horizontal). Thus, for generating arbitrary vector modes with SLMs, the transverse profiles of both polarization components have to be manipulated independently, either in interferometric arrays containing one or two SLMs [34–40] or via a temporal sequence using a double pass over a single SLM [32, 41]. Besides, their high price, low refresh rate (60 Hz), polarization-sensitivity, wavelength dependence, and low efficiency, also represent serious disadvantages in many applications. On the contrary, recently proposed techniques based on the DMD technology, initially developed for projection systems, have shown unsurpassed advantages over SLMs, such as, high refresh rates, polarisation independence, low cost, wide operation range, amongst others. For the applications requiring fast reconfiguration of the generated vector modes, the refresh rates of DMDs, which can reach up to 30 kHz, provides the means to produce arbitrary vector beams fields at high speeds [42–49]. Nonetheless, in almost all the techniques proposed so far, the polarisation-insensitive attribute of DMDs has gone almost unnoticed. An exception to this is a recently proposed technique, which we will describe in detail in this tutorial [50]. It is also worth mentioning that even though the resolution of SLMs and DMDs is relatively low compared to diffractive optical elements, this does not represent an issue to generate structured light beams with high quality. In fact, light modes endowed with OAM can be generated using low-resolution optical elements [31]. Importantly, their resolution allows the generation of OAM modes with topological charges as high as 600. In addition, they also allow the simultaneous generation of multiple scalar and vector modes using a multiplexing approach [21, 34, 51].

Another important aspect of complex light fields is their characterization, which provides useful information about their purity or their spatial polarisation distribution. Nonetheless, due to the non-separable coupling between the spatial and polarisation DoFs, their characterisation is challenging. A well-known technique, aiming at the reconstruction of their entire transverse polarisation, is Stokes polarimetry (SP), which comprises a series of intensity measurements [52, 53]. A more modern approach, which exploits their similarity with quantum entangled states, utilizes well-established tools from quantum mechanics, namely, Concurrence (C) [54]. Such measure, which for vector modes is called vector quality factor (VQF), measures the degree of coupling between the spatial and polarisation DoF assigning a number in the range [0, 1], 0 to scalar modes with a null degree of coupling and 1 to vector modes with a maximum degree of coupling [28, 55–59]. This technique was first implemented with SLMs requiring the spatial separation of both polarisation components, followed by the projection of each beam onto a series of spatial filters (SFs) encoded on the SLM. Noteworthy, the polarisation-insensitivity property of DMDs enables the real-time and all-digital SP to reconstruct the transverse polarisation distribution as well as a simplified implementation of the VQF measurement [60–62]. As such, DMDs represent a powerful tool for the generation and characterisation of complex vector modes, of great relevance in industrial applications as well as in undergraduate optics laboratories.

In what follows, we provide a comprehensive description, in the form of a tutorial, of the use of DMDs to generate and characterise complex vector modes. First, in section 2 we briefly describe the working principle of DMDs, followed by a tutorial-style explanation of their use in controlling the amplitude and phase of light fields (section 2.1). In section 2.2 we describe a novel binary encoding scheme based on random spatial multiplexing to maximally exploit the high refresh rates of DMDs. Afterwards, in section 3, we describe in detail the experimental generation of complex vector modes, providing a variety of examples, such as, Laguerre-, Ince-, Helical Mathieu- and Travelling Parabolic-Gaussian vector beams [63–66]. Section 4 mainly focus on the description of characterization techniques where we first describe a real-time SP technique followed by the description of an alternative technique to determine the VQF. Both techniques will pave way for real-time characterisation of vector modes, of great relevance in applications where the non-separability can be
used as an optical sensor. Finally, in section 5, we expand a discussion in details about the comparison between SLMs and DMDs in shaping light field.

2. Light beam shaping with digital micromirror devices

2.1. Phase and amplitude binary holograms

To start with, it is worth emphasising the DMDs were initially designed as digital projection devices but nowadays are routinely used as SLMs. Figure 1(a) shows an example of the specific DMD (DLP Light Crafter 6500 from Texas Instrument), which was employed in all the DMD techniques introduced across the whole tutorial. A DMD consist of an array of millions of micro-sized mirrors ($\approx 8 \mu m$ in size), each of which can be turned to an ‘Off’ or ‘On’ state by tilting it $-12^\circ$ or $+12^\circ$, respectively. In this way, when the DMD is properly aligned, each mirror in the ‘On’ state reflects light in the desired direction [49, 67], as schematically illustrated in figure 1(b). Since each mirror can only be in two states, it is important to bear in mind that one of the principal requirements is to address the DMD with binary holograms, even though some devices can also display gray-scale holograms by allowing the micromirrors to oscillate between the ‘On’ or ‘Off’ states, at the expense of a dramatically reduced refresh rate.

The methods to shape both the amplitude and phase of a complex light field $u(x, y) = A(x, y) \exp[\phi(x, y)]$, with amplitude $A(x, y)$ and phase $\phi(x, y)$, via binary amplitude holograms where developed in the 1960s [68]. In particular, one of the most known methods, which was proposed by Lee, consist in creating a periodic binary amplitude grating, where the phase and amplitude information is encoded in the first diffraction order [69, 70]. Such binary grating is given by,

$$T(x, y) = \frac{1}{2} + \frac{1}{2} \text{sgn}\{\cos[p(x, y)] + \cos[\pi q(x, y)]\},$$  \hspace{1cm}(1)

where $\text{sgn}\{\cdot\}$ is the sign function that reflects the binary-amplitude modulation, which forces all arguments to fall to either 0 or 1. Hence, to generate the complex field $u(x, y) = A(x, y) \phi(x, y)$, the functions $p(x, y)$ and $q(x, y)$ are defined in terms of the amplitude and phase information as,

$$q(x, y) = \arcsin(A(x, y)/A_{max})/\pi,$$

$$p(x, y) = \phi(x, y) + 2\pi(x\nu + y\eta),$$  \hspace{1cm}(2)

respectively. Here the term $A_{max}$ represents the maximum amplitude value that normalises $A(x, y)$. Notice how the amplitude $A(x, y)$ and phase $\phi(x, y)$ of the complex light field are modulated locally as function of position. Figure 2(a) illustrates how such binary periodic grating diffracts an input beam into multiple orders. A typical example of a binary hologram for generating the specific LG mode ($LG_{\nu,\eta}$) with radial index $p = 1$ and azimuthal index $\ell = -1$ is shown in figure 2(b). Notice that the colors in all the binary holograms shown through the manuscript were inverted and the holograms were also scaled for display purposes. Hence, the black color represents the mirrors in the ‘On’ state that reflect the light in the desired direction, and the white color represents mirrors in the ‘Off’ state, which direct light away. In the actual experimental implementation, the negative of such transmission grating is displayed on the DMD. When this fork-like hologram is illuminated with an expanded light beam, several diffraction orders appear in the far field, five of which are shown in figure 2(c). The target beam appears in the first diffraction order, which for the sake of clarity is shown in figure 2(d). The intensity profiles shown in this figure correspond to numerical simulations performed through the Rayleigh–Sommerfeld diffraction theory [71]. Furthermore, the angle of the first diffraction order, and therefore its position $(U, V)$ in the Fourier plane can be adjusted via the spatial frequency of the grating $(\nu, \eta)$ under the relation, $U = \nu \lambda / f$ and $V = \eta \lambda / f$, where $\lambda$ and $f$ represent the wavelength of the laser and the focal length of the Fourier lens, respectively [29, 51]. So far, we have explained how to generate arbitrary scalar modes characterised by a homogeneous transverse polarisation distributions, in the following section we will explain how to generate complex vector light fields bearing non-homogeneous transverse polarisation distributions.
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Figure 3. The convolution of a random binary mask $R(x,y;a)$ (left) and a transmittance function $T(x,y)$ (middle) give rise to the
synthetic mask shown on the right. Here, we show the specific cases for $a \in [0.33, 1, 0.06]$ in (a), (b) and (c), respectively. The pixels staying in the ‘Off’ state represented with the colour of white will
direct the light away while the ones with black colour staying in the
‘On’ state will redirect the light to the desire direction. Notice how
the pixels with black colour in the random binary mask $R(x,y;a)$
switch to after convoluted with the transmittance function $T(x,y)$.

2.2. Spatial random multiplexing encoding

On the basis of the device introduced earlier, an encoding scheme of spatial random multiplexing, specially designed for the DMD working mechanism is introduced in this section. Crucially, such scheme allows to maximally exploit the high refresh rates of DMDs during the generation of complex vector modes. In this encoding scheme, we firstly define a random binary mask $R(x,y;a)$ with the exact resolution of the DMD \((1920 \times 1080\) pixels). Examples of such mask containing \(9 \times 9\) pixels are schematically shown in the left panels of figure 3. The pixels in the ‘On’ state, which are represented with the black colour, are spatially selected at random, the number of which is selected through the parameter $a \in [0,1]$. For example, the case $a = 0.5$, with half of the total pixels in the ‘On’ state, is shown in the left panel of figure 3(a), whereas the case $a = 1$, with all the pixels in the ‘On’ state, is shown in the left panel of figure 3(b). Finally the case $a = 0$, with all the pixels in the ‘Off’ state, is illustrated in the left panel of figure 3(c).

Thus, when such a random binary mask is displayed on the DMD, the effect of tilting the micromirrors to the ‘On’ or the ‘Off’ state is determined by the parameter $a$. Given that only the pixels in the ‘On’ state diffraight light in the desired direction, the parameter $a$ controls the amount of power in the generated mode. Notice that the power efficiency of any binary hologram is naturally limited to a maximum of \(\approx 10\%\), which in our technique is achieved when $a = 1$. Now, to generate a spatial mode given by a transmittance function $T(x,y)$ (equation (1)), the random binary mask $R(x,y;a)$ is convoluted with this transmittance function (see middle panels of figure 3). As result of such convolution, only the pixels which are in the ‘On’ state on both, $T(x,y)$ and $R(x,y;a)$ will remain in the ‘On’ state, the rest will be switched ‘Off’, as schematically illustrated in the right panels of figure 3.

We are now ready to explain how the principle of random spatial multiplexing can be applied to the simultaneous generation of two (or more) optical fields with independent amplitudes and phases \([48,72,73]\). At this stage, it is worth clarifying that each optical field must be associated to an independent transmittance function, namely, $T_1(x,y)$ and $T_2(x,y)$. To generate the final transmittance function $T_f(x,y)$, both transmittance function are carefully recombined ensuring the removal of any spatial overlap between the pixels associated to $T_1(x,y)$ and those associated to $T_2(x,y)$, here is where the random binary approach becomes relevant. To this end, we first define the complementary function $\bar{R}(x,y;a)$ as,

$$\bar{R}(x,y;a) = \mathbb{1} - R(x,y;1-a),\quad (3)$$

where $\mathbb{1}$ represents a binary mask with all its entries in 1, i.e. all the micromirrors in the ‘On’ State. Importantly, the direct product, entry-by-entry, of the matrix $R(x,y;a)$ and its complementary matrix yields a mask with all its entries in zero, that is, $R(x,y;a)\bar{R}(x,y;1-a) = \mathbb{1}$. To further clarify this, for $a = 1$ all the entries of $R(x,y;1)$ are in the ‘On’ state, whereas in $R(x,y;0)$ all are in the ‘Off’ state. On the contrary, for $a = 0$, all the pixels of $R(x,y;0)$ are in the ‘Off’ state, while in $R(x,y;1)$, all are in the ‘On’ state. Using this two random binary masks, the multiplexed binary mask is then defined as,

$$T_f(x,y) = R(x,y;a) * T_1(x,y) + \bar{R}(x,y;1-a) * T_2(x,y),\quad (4)$$

This synthetic hologram allows the simultaneous generation of two modes, with a power determined by the parameter $a$. For example, for $a = 0.5$, half of the pixels, spatially selected at random, are used to generate the mode encoded in $T_1(x,y)$, while the rest to generate the mode encoded in $T_2(x,y)$. As mentioned earlier, none of the pixels associated to $T_1(x,y)$ overlap spatially with those associated to $T_2(x,y)$. Figure 4 schematically illustrate this description, where we encode the specific mode $LG_j$ in the function $T_1(x,y)$ and $LG_{j-1}$ in $T_2(x,y)$, as shown in the left panel of figure 4(a) and the middle panel of figure 4(b), respectively. We further show an example of a random binary mask $R(x,y;a)$ and its complementary random mask $\bar{R}(x,y;1-a)$ in the middle panel of figure 4(a) and right panel of figure 4(b), an enlarged portion of both masks is shown as an inset to emphasize their complementary relation. As a result, the holograms of the convolution of $R(x,y;a) * T_1(x,y)$ and $\bar{R}(x,y;1-a) * T_2(x,y)$ are shown in the right and left panel of figures 4(a) and (b). Finally, the target binary hologram resulting from the superposition of both holograms is shown in figure 4(c). Importantly, such a multiplexing hologram enables to generate arbitrary vector mode at the speed only limited by the specific of DMDs.

As stated earlier, the intensity distribution of each generated beam varies as a function of parameter $a$. To further
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It is worth mentioning that the spatial degree of freedom encoded in the spatial modes $u_1(\vec{r})$ and $u_2(\vec{r})$ can be given by any of the solutions to the wave equation in the different coordinates systems. An example of such are the LG vector modes, which are natural solutions in cylindrical coordinates. From equation (5) the LG vector modes can be expressed as,

$$\vec{u}(\vec{r}) = \cos(\theta) LG_{p_1}^{m_1}(\vec{r}) \hat{r} + \sin(\theta) LG_{p_1}^{m_2}(\vec{r}) e^{im},$$

where $\vec{r} = (\rho, \phi)$ is the position vector of the cylindrical coordinates $(\rho, \phi)$ and $LG_{p}^{m}$ are the LG modes, carrying a well-defined amount of OAM, as defined in [74].

### 3.2. Experimental setup

Complex vector modes of various spatial distributions can be generated from a DMD in different ways. Here, we will describe an approach that takes full advantage of the polarisation-insensitive property of DMDs, which is fully detailed in [50]. The experimental setup for its implementation is schematically shown in figure 6(a). To begin with, a horizontally polarized laser beam $(\lambda = 532 \text{ nm})$ is collimated and expanded by lenses $L_1$ ($f_1 = 20 \text{ mm}$) and $L_2$ ($f_2 = 200 \text{ mm}$). Afterwards, the polarisation state of the expanded beam is rotated to the diagonal polarisation state with the help of a half-wave plate (HWP) at $22.5^\circ$. A Wollaston prism (WP) subsequently separates the beam into its horizontal and vertical polarization components, both of which are transformed to the circular polarization basis $(\hat{l}, \hat{r})$ after passing a quarter-wave plate (QWP). Alternatively, it is also possible to use a polarizing beam splitter to separate both polarisation components and a D-shaped mirror to redirect the vertical polarisation component to the DMD. In this option an extra optical path difference between both arms will be introduced, which can be easily corrected digitally by inserting a compensating phase term to the DMD. A 4f imaging system composed of lenses $L_3$ and $L_4$ ($f_3 = f_4 = 200 \text{ mm}$) redirects these two beams towards the centre of a DMD where they impinge at slightly different angles ($\approx 1.5^\circ$) but exactly at the same spatial location, the centre of the hologram. A multiplexed binary amplitude hologram based on the spatial random multiplexing method, which will be explained in the following section, is encoded on the DMD. Such multiplex hologram consist of the superposition of two individual holograms, one for each of the constituting scalar fields in equation (5), overlapped with a controllable linear diffraction grating. The period of the diffraction grating is carefully chosen to ensure the overlap of the first diffraction order of each beam along a common propagation path, where the desired complex vector field $\vec{u}(\vec{r})$ is generated (see supplementary material for additional details).

We can explain this process in a more detailed way using the schematic representation shown in figure 6(b). Here and for the sake of clarity the DMD is represented as a transmission device but as shown in figure 6(a) it is a reflection device. The two input beams with orthogonal circular polarization $\hat{l}, \hat{r}$, which emerge after the QWP, impinge in the centre of hologram displayed on DMD. Afterwards, each beam’s 0th diffraction order propagate in a divergent way from each other, as if they were simply reflected from a mirror, leaving both 1st orders propagating along the same axis where the desired beam is acquired. During the whole experimental alignment, the period of the linear diffraction grating is carefully adjusted to guarantee the overlap of both beams, as illustrated in figure 7. Once the two beams (Beam 1 and Beam 2) with orthogonal polarisation impinge on the centre of DMD encoded with a multiplexing hologram, four modes appear in the first diffraction order, where, modes $m_1$ and $m_2$ correspond to Beam 1 and modes $m_3$ and $m_4$ to Beam 2. Notice that $m_1$ and $m_4$ are two orthogonal spatial modes with orthogonal polarisation, hence, their overlap will generate the desired vector mode. To guarantee this overlap, the period of the diffraction gratings in the vertical direction is set to the same value for both beams, while the period of the horizontal grating is set to the same value but one as the negative of the other. Hence, in the observation plane, all beams will appear at the same height but on opposite sides of the 0th diffraction order, as illustrated in figure 7(a). Hence, the overlap of $m_2$ and $m_3$ is acquired by adjusting the diffraction grating along the horizontal direction, the spatial distance of modes $m_2$ and $m_3$ can go down to zero (shown in figure 7(b)), where the desire vector mode is generated when they overlap completely with each other. Noteworthy, this adjustment is performed in an all-digital by tuning the frequency of each linear diffraction grating. Finally, to remove all the undesired diffraction orders, a SF is placed in the far-field plane of a telescope composed of the lenses $L_5$ and $L_6$ ($f_5 = f_6 = 100 \text{ mm}$). For the sake of clarity, in figures 6(a) and (b) only the first and zero diffracting orders are shown. Noteworthy, once the device is properly aligned, it enables the

![Figure 6](https://example.com/figure6.png)
3.3. Experimentally generated vector modes

The generation of arbitrary vector modes will be exemplified using the set of LG vector modes and their representation on the higher-order Poincaré Sphere (HOPS). The HOPS provides a geometric way to visualize any vector mode on its surface by associating their inter-modal phase ($\alpha$) and weighting coefficient ($\theta$) to a point with coordinates ($2\alpha$, $2\theta$) [75].

A representative set of five $LG_\ell^p$ vector modes labelled from 1 to 5 and with specific coordinates ($0$, $0$), ($\pi/2$, $0$), ($\pi$, $0$), ($\pi/2$, $3\pi/4$) and ($3\pi/2$, $\pi/4$) on the HOPS, as represented on figure 8(a), are shown of figure 8(b). Here, the left column illustrates in a conceptual way and to scale the holograms required to generate these modes, additionally, the middle and right columns shows numerical simulation and experimental results, respectively, of the transverse intensity pattern overlapped with polarization distribution. For this example we used the spatial modes $LG_3^0$ and $LG_{-3}^0$. Additional examples of vector beams in different coordinate systems are shown in figure 9, from left to right they correspond to the Laguerre-, Ince-, Helical Mathieu- and Travelling Parabolic-Gaussian vector modes [63–65]. The first is a solution to the wave equation in cylindrical coordinates, the following two are solutions in elliptical coordinates and the last is a solution in the parabolic coordinates. More details of these vector modes are further detailed in supplementary material. In figure 9(a) we show, to scale, the required holograms to generate the vector modes shown in figures 9(b) and (c), theoretical and experimental, respectively, where the polarization distribution is overlapped with the transverse intensity profile. It is worth mentioning that the deviations of the experimentally generated beams from the numerical simulations, originate mainly from aberrations produced by the DMD screen. Nonetheless, they can be removed using advanced techniques, using advanced techniques, as those described in [49].

It is worth emphasising the vector modes shown above are just a representative example of the wide variety of modes that can be generated using DMDs. They can be used, for example, to generate non-paraxial tightly-focused
beams, for applications in optical tweezers [76, 77] or to generate nontrivial 3D topologies, such as, Möbius [78–80], ribbon strips [81] or knots [82]. They also enables connections between rays and waves, quantum and classical [83]. In addition, in recent time, the optical-analogous of skyrmions, topologically protected quasiparticles in high-energy physics and condensed matter [84, 85], featuring sophisticated vectorial structures, have received an increasing amount of interest by the photonics community [86–89]. Remarkably, such optical structures can be realized directly from complex vector modes. For example, the polarisation singularities known as ‘lemon’ and ‘star’ correspond to the skyrmion and anti-skyrmion, respectively [90]. Crucially, while the polarisation distribution of complex vector are 2D plots, the optical skyrmions are 3D plots [91, 92], nonetheless, both are obtained from of the Stokes parameters. Therefore, the existing techniques capable to generate complex vector modes from DMDs, applies identically to generate optical skyrmions.

4. Characterization of vector modes using a digital micromirror device

Even though recent years have witnessed a growing interest in the generation of vector light fields in a variety of ways, only a few techniques have been proposed to characterise them. Along this line, DMDs have also started to play a fundamental role, as such in this section we will present two techniques capable to monitor in real time their transverse polarisation distribution and their vector quality, respectively.

4.1. Real-time polarisation reconstruction through Stokes polarimetry

Stokes polarimetry is a powerful technique that allows to reconstruct the transverse polarisation distribution of any light field through a minimum of four intensity measurements. The information contained in these intensities is captured by a set of four parameters known as Stokes parameters, from which the polarisation distribution can be determined [52]. The relation between the intensities and the Stokes parameters is given as:

\[
\begin{align*}
S_0 &= I_0, \\
S_1 &= 2I_H - S_0, \\
S_2 &= 2I_D - S_0, \\
S_3 &= 2I_R - S_0,
\end{align*}
\]  

(7)

where \(I_0\) is the total intensity of the given optical field. \(I_H\) and \(I_D\) are the measured intensities of the field after a linear polarized orientated at 0 and \(\pi/4\), respectively. \(I_R\) is the intensity acquired after the combination of a QWP at \(\pi/4\) and a linear polariser at \(\pi/2\).

Traditionally, these intensities are obtained individually, one-by-one, at different times, as schematically shown in figure 10(a) limiting its performance to light beams with static states of polarisation. As such, in recent time we proposed a technique relying on a DMD that allows the real-time dynamic reconstruction of the SoP of any light field, which is described next. The key idea behind this novel technique relies on performing all the intensity measurements simultaneously. For this we exploit the polarisation-insensitive attribute of DMDs. To be more specific, a DMD is addressed with a multiplexed digital hologram, enabling to split the input beam into four identical copies propagating along different paths. In this way, all the four required intensities can be recorded simultaneously in a single shot with the help of the required optical filters and a CCD, as illustrated in figure 10(b). A dedicated software specifically designed to analyse the acquired images allows to reconstruct in real time the state of polarization of any light beam at speeds limited only by the specific CCD camera in use [60]. A schematic representation of the experimental setup implemented to demonstrate this idea is shown in figure 11, which is divided into two sections: Generation and Measurement. The required CV beam is generated from a linearly polarized Gaussian beam (\(\lambda = 532\) nm) via a q-plate (\(q = 1/2\)) in combination with a HWP1. A multiplex digital hologram with unique diffraction gratings is displayed on the DMD, to split the input beam into four identical copies, as illustrated in figure 12(a), from which, the polarisation distribution can be reconstructed. To remove the undesired diffraction orders, these beams are spatially filtered by four individual apertures and collimated to propagate parallel to each other using the set of lenses \(L_1\) and \(L_2\) (f1,2 = 200 mm). Afterwards, the required intensities \(I_0, I_H, I_D\) and \(I_R\) are measured as explained next (see figure 12(b)). The intensity \(I_{ID}\) is measured by inserting a linear polariser at \(\theta = 0^\circ\) (P1) from path ①. The intensity \(I_{ID}\) is obtained by using another linear polariser at \(\theta = 45^\circ\) (P2) in path ②. From path ③, \(I_R\) is measured by combining a QWP (QWP2) at 45° and a linear polariser at 90°. Finally, the intensity \(I_0\) is directly obtained from

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**Figure 10.** (a) The required intensities to reconstruct the polarisation distribution of any light field in standard polarimetry are commonly recorded one by one at different time \(t_1, t_2, t_3\) and \(t_4\). (b) Our technique enables a simultaneous measurement of the intensities by taking advantage of the polarisation-insensitive DMDs, which allows the splitting of the input beams into four identical copies.
Figure 11. Schematic representation of the setup implemented to reconstruct in real time the SoP of a light field. A CW Gaussian beam ($\lambda = 532$ nm) is transformed into a vector beam by the use of a q-plate ($q = 1/2$) and a half wave-plate (HWP1). The resulting beam is then split into four identical copies propagating along parallel paths using a DMD with the help of lenses $L_1$ and $L_2$. $I_H$ and $I_D$ are measured by inserting each path with the linear polarizers $P_1$ and $P_2$, respectively, while $P_3$ in combination with a quarter wave-plate (QWP2) filters $I_R$. Lens $L_3 = 200$ mm finally focuses these four beams into a CCD to capture all the intensities in one single shot. One example of this is shown in the inset box. The system E composed by a rotating half wave-plate inserted before the DMD, enables real-time evolution of the input beam’s SoP.

Figure 12. (a) Single-shot of the calibration image to find the centres of the beams. (b) An example of the intensity images of $I_0$, $I_H$, $I_D$ and $I_K$. (c) A set of four Stokes parameters computed through four intensity measurements from (b) simultaneously. (d) Reconstructed polarisation distribution from the Stokes parameters in (c).

Figure 13. Extracted frames of the real-time reconstruction of polarisation after passing the generated beam through a rotating HWP at $\theta = 0^\circ$, $15^\circ$, $30^\circ$ and $45^\circ$ for (a) experiment and (b) simulation.

4.2. Determination of the non-separability through the vector quality factor

In this section we will describe a technique which allows to quantify the purity of vector modes [55, 56]. This quantity is based on the mathematical similarities between classical and quantum entanglement. In quantum mechanics this quantity, known as Concurrence ($C$), measures the degree of entanglement of two photons, in classical optics it is known as VQF and also measures the degree of entanglement between the spatial and polarisation degrees of freedom, assigning 1 to vector modes, with a maximal degree of entanglement and 0 to scalar mode, with a null degree of entanglement. In general, the VQF is determined by first projecting the vector mode onto one DoF, either the spatial or polarisation, and tracing over the other by means of spatial or polarisation filters. The first experimental demonstration of this technique was performed with SLMs, for this, a given vector mode is first projected onto the ‘On’ state, as well as the period of the linear grating, as illustrated in figure 12(a). Here, the beams are labelled with the numbers ①, ②, ③ and ④ to identify their position in the experimental setup (figure 11). Figure 12(b) shows an example of the simultaneously measured intensities and the required Stokes parameters reconstructed from these intensities are shown in figure 12(d). This technique allows to monitor in real-time the polarisation evolution of dynamically-changing vector modes, as shown in figure 13. Here, a HWP, represented as E in figure 11, was inserted in the path of the system to modify in real-time the SoP of the input beam. As the rotating half wave-plate (RHWP) rotates from $0^\circ$ to $90^\circ$, an angle-dependent phase delay between both orthogonal polarisation components is introduced, resulting in a continuous evolution of the vector modes, from radial (at $0^\circ$ and $90^\circ$) to azimuthal (at $45^\circ$) polarisation. Experimental and numerical simulations are shown in figures 13(a) and (b), respectively, for the specific angle of $0^\circ$, $15^\circ$, $30^\circ$ and $45^\circ$. The position and intensity of each beam is previously calibrated with a vortex beam, by adjusting the micromirrors in path ④. To record all the required intensities in a single image, a third lens ($L_3 = 200$ mm) is added to focus the beams into a CCD camera (BC106N-VIS from Thorlabs).
two orthogonal polarisation components, afterwards, the resulting mode is passed through a series of six SFs encoded as holograms on the SLM. The VQF is computed from twelve measurements, six for each polarisation components of the on-axis intensity measured in the far field, all of which can be performed simultaneously using a multiplexing approach [56–58]. One of the main drawback in using SLMs is their polarisation dependence, which only allows to modulate linearly polarised states (typically the horizontal), which is the main reason why it is required to first project over the polarisation DoF, by splitting the input vector mode onto the two polarisation components. As such, in this section we introduce another affordable method to measure the VQF of any arbitrary vector mode by taking full advantage of the polarisation insensitive attribute of DMDs. Noteworthy, this technique enables a reduction in the number of required measurements, from 12 to 8. Additionally, it is low-cost, it can operate over a wide range of wavelengths, and it can measure the VQF in real-time.

To start with, let us remind that even though classical and quantum mechanics are considered as two completely different fields, they still share some similarities [93–96]. This is the case of entanglement, commonly associated to the quantum world, which captures the degree of non-separability of an entangled state. However, non-separability is not a distinct feature of quantum systems, classical systems can also feature non-separability [19, 97–99]. It is possible to distinguish between different types of entanglement, non-local and local. While the former happens between systems that are spatially separated from each other, the later happens between the internal degrees of freedom of a system. Crucially, local entanglement also applies to classically-entangled systems, e.g. vector beams non-separable in their spatial and polarisation degrees of freedom. Therefore, it is possible to quantify the non-separability of vector modes using well-established tools from quantum mechanics. It has also become common to use the bra-ket notation from quantum mechanics to describe classical non-separable states. In this way, equation (6) can be written as,

$$|u⟩ = \cos(θ)|u_R⟩ \otimes |R⟩ + \sin(θ)|u_L⟩ \otimes |L⟩,$$

(8)

where the kets $|u_R⟩$ and $|u_L⟩$ represents the spatial modes $LG_{0}^{\ell}$ and $LG_{0}^{\ell}e^{i\alpha}$. The kets $|R⟩$ and $|L⟩$ represents, respectively, the right and left circular polarisation vectors. The symbol $\otimes$ denotes the tensor product between the vectors representing the spatial and polarisation degrees of freedom. Now, to quantify the entanglement of a classical system we rely on the entanglement entropy, which is commonly used to quantify the quantum entanglement of a pure bipartite system [54]. The entanglement entropy can be computed through the von Neumann entropy as,

$$E(|u⟩) = - \left( \frac{1+s}{2} \right) \log \left( \frac{1+s}{2} \right) - \left( \frac{1-s}{2} \right) \log \left( \frac{1-s}{2} \right),$$

(9)



| Basis states | $|R⟩$ | $|L⟩$ | $|H⟩$ | $|D⟩$ | $|V⟩$ | $|A⟩$ |
|-------------|------|------|------|------|------|------|
| $|+\ell⟩$   | $I_{R\ell}^+$ | $I_{L\ell}^+$ | $I_{H\ell}^+$ | $I_{D\ell}^+$ | $I_{V\ell}^+$ | $I_{A\ell}^+$ |
| $|-\ell⟩$   | $I_{R\ell}^-$ | $I_{L\ell}^-$ | $I_{H\ell}^-$ | $I_{D\ell}^-$ | $I_{V\ell}^-$ | $I_{A\ell}^-$ |

Table 1. Standard 12 normalised intensity measurements $I_{mn}$ to determine the expectation values $⟨\sigma_i⟩$.

where $s$ is the length of the Bloch vector defined as,

$$s(\rho) = (\text{Tr}[\rho^2])^{1/2} = \left( \sum_{i=1}^{3} (\sigma_i)^2 \right)^{1/2}.$$

(10)

Here, $\rho$ is the reduced density matrix of one of the subsystems, which is obtained by tracing over the other degree of freedom. The parameters $⟨\sigma_1⟩$, $⟨\sigma_2⟩$ and $⟨\sigma_3⟩$ are the expectation values of the Pauli operators, which for the case of classically entangled beams can be obtained from a set of normalised intensity measurements.

Finally, to measure the degree of classical entanglement or non-separability in vector modes, we can use the concurrence $C$ and define the VQF as the real part of $C$ [56],

$$\text{VQF} = \text{Re}(C) = \text{Re} \left( \sqrt{1-s^2} \right) = |\sin(2θ)|.$$

(11)

As defined, the VQF is a positive number with values in the range $[0, 1]$. When $θ = 45°$, the VQF reaches its maximum value and represent a pure vector mode, whereas in the case $θ = 0°$ and $θ = 90°$, it takes its minimum value and represents scalar modes. The first experimental measurement of $⟨\sigma_1⟩$, $⟨\sigma_2⟩$ and $⟨\sigma_3⟩$ was performed by projecting the vector mode over the polarisation degree of freedom and tracing over the spatial [56]. Crucially, this process can also be performed in reverse, to explain this better and without the loss of generality, we will use the spatial degree of freedom encoded in the OAM basis. Hence, the vector beam generated in the OAM basis, is first projected onto a series of OAM filters encoded on a DMD, namely, $|+\ell⟩$ and $|-\ell⟩$ and then traced over the polarisation DoF. More precisely, the right, left-circular, horizontal, vertical, diagonal and anti-diagonal polarisation components, $|R⟩$, $|L⟩$, $|H⟩$, $|V⟩$, $|D⟩$ and $|A⟩$, respectively. Here it is worth mentioning that this procedure can not be performed with an SLM due to its polarisation dependence. The set of twelve intensities are shown in table 1 for the sake of clarity. Here for example, $I_{R\ell}^−$ represents the intensity after projecting the vector mode on the $|-\ell⟩$ OAM phase filter and passing it through a $|R⟩$ polarisation filter. The explicit form of $⟨\sigma_1⟩$, $⟨\sigma_2⟩$ and $⟨\sigma_3⟩$ in terms of this intensity measurements will be,

$$⟨\sigma_1⟩ = (I_{H\ell}^+ + I_{H\ell}^-) - (I_{D\ell}^+ + I_{V\ell}^-),$$

$$⟨\sigma_2⟩ = (I_{DE}^+ + I_{DE}^-) - (I_{DE}^+ + I_{DE}^-),$$

$$⟨\sigma_3⟩ = (I_{RE}^+ + I_{RE}^-) - (I_{DE}^+ + I_{DE}^-).$$

(12)

Importantly, the use of a DMD allows to significantly reduce the number of required intensities from 12 to only 8, as has been previously demonstrated [63]. To this end, we first
rewrite the second term of \( \langle \sigma_1 \rangle \) by adding the terms \( I_{HE^-} \)
and \( I_{HE^+} = 0 \) as,
\[
I_{V_-} + I_{V_+} = I_{V_+} + I_{HE^+} + I_{HE^-} + I_{HE^-} = -I_{HE^-} = -I_{HE^-} - I_{HE^-} - I_{HE^-} = -(I_{HE^-} + I_{HE^-}) + I_{HE^-} + I_{HE^-} + I_{HE^-} + I_{HE^-} = -(I_{HE^-} + I_{HE^-}) + I_{HE^-} + I_{HE^-},
\]
where \( I_{V_+} = I_{HE^+} + I_{HE^-} \) and \( I_{V_-} = I_{HE^-} + I_{HE^-} \). A similar procedure can be done to the second term of \( \langle \sigma_2 \rangle \) and \( \langle \sigma_3 \rangle \) to finally obtain the relations,
\[
\langle \sigma_1 \rangle = 2(I_{HE^+} + I_{HE^-}) - (I_{V_+} + I_{V_-}),
\langle \sigma_2 \rangle = 2(I_{DE^+} + I_{DE^-}) - (I_{V_+} + I_{V_-}),
\langle \sigma_3 \rangle = 2(I_{RE^+} + I_{RE^-}) - (I_{V_+} + I_{V_-}).
\]

To arrive to these equations, we have also used that \( I_{HE} = I_{RE^+} + I_{LE^+} = I_{HE^+} + I_{LE^+} = I_{DE^+} + I_{LE^-} \) with a similar relation for \( I_{HE} \). Hence, only the 8 intensities \( I_{HE^+}, I_{HE^-}, I_{DE^+}, I_{DE^-}, I_{RE^+}, I_{RE^-}, I_{LE^+} \) and \( I_{LE^-} \) are required to compute the VQF, which for the sake of clarity are shown in Table 2. It is worth noting that this is the minimum number of required measurements to compute the VQF, the reason being, the optimal basis for projections depend on the dimension of the space involved and can be computed as what is termed positive operator valued measurement. This is the reason, for example, why Stokes parameters can be determined with only four intensity measurements rather than six, where the measurements have to be carefully selected to avoid any information loss. For vector modes resulting from the tensor product of two DoFs, we need eight measurements, four for each circular polarisation [5].

As explained earlier, the VQF can be measured in a straight forward way through a series of normalised intensity measurements, all of which can be measured simultaneously using a multiplexing approach. Figure 14(a) schematically represents such a concept for the specific case of cylindrical vector modes, whereby, eight individual holograms are multiplexed with unique spatial frequencies to diffract each beam along different angles. As illustrated in this figure, the top four holograms perform the \( |+\ell\rangle \) projection, while the four on the bottom perform the \(-\ell\rangle \) projection. Each beam is then passed through a series of polarisation filters to trace the polarisation DoF, namely, the polarisation components \( |R\rangle, |L\rangle, |H\rangle \) and \( |D\rangle \). At last, a lens performs the far-field of all beams simultaneously and the on-axis intensity of each of them is measured to obtain the expectation values \( \langle \sigma_1 \rangle, \langle \sigma_2 \rangle \) and \( \langle \sigma_3 \rangle \) (see [62] for more details).

Figure 14(b) shows a schematic representation of the implemented experimental setup. Here, the input vector mode is first sent to the centre of DMD where the eight holograms are displayed to perform the projection onto the spatial DoF. As can be seen, after the DMD eight beams emerge along different angles, which are passed afterwards through a 4f system consisting of lenses \( L_1 \) and \( L_2 \) \((f_{1,2} = 200 \text{ mm})\). An aperture \((A)\) placed in the focusing plane of \( L_1 \) allows to filter only the first diffraction order of each beam. After performing the projection on spatial basis \(|+\ell\rangle\) (beams \( 1, 3, 5, 7 \)) and \(-\ell\rangle\) (beams \( 2, 4, 6, 8 \)), the next step is to trace over the polarisation DoF by inserting to each path specific different polarisation filters. To be more specific, \( I_{HE^+} \) and \( I_{HE^-} \) are obtained by passing beams \( 1 \) and \( 2 \) through a linear polariser \((P)\) at 0°, \( I_{DE^+} \) and \( I_{DE^-} \) are acquired by passing beams \( 3 \) and \( 4 \) through a linear polariser oriented at 45°. To measure the \( I_{RE^+} \) and \( I_{RE^-} \), a QWP at 45° in combination with a linear polariser at 90° are inserting into the paths \( 5 \) and \( 6 \). Finally, \( I_{LE^+} \) and \( I_{LE^-} \) are acquired by transmitting beams \( 7 \) and \( 8 \) through a QWP at −45° and a linear polariser at 90°. To measure the far field intensities in a ‘single shot’, all beams are focused onto a CCD camera (BC106N-VIS Thorlabs) with the help of a lens \((L_3, f_3 = 200 \text{ mm})\). The measurement of on-axis intensity is further detailed in supplementary material.

Figure 15 shows experimental results of the VQF as a function of \( \theta \), where the theory (equation (11)) is represented by the black continuous line and the experimental data by red points. Here, we show the VQF increasing continuously from 0 to 1 and back to 0, corresponding to the scalar \((\theta = 0)\), pure vector \((\theta = \pi/4)\) and scalar \((\theta = \pi/2)\) modes, respectively. The corresponding polarisation distribution for these three typical cases are shown in the insets of figure 15, where the orange and green colours correspond to the right and left polarisation
reconfiguration of the displayed hologram, their refresh rates is based on the liquid crystal technology which allows a rapid exotic shape and polarization distributions. While SLMs are rapid generation of complex light beams with almost any most versatile due to their high flexibility, which enable the devices, such as SLMs and DMDs, are perhaps among the most attractive. Their properties of DMDs and SLMs are summarised in table 3. As a final comment, a possible route we might explore is the combination of techniques which involve the use of both DMDs and SLMs, which might bring revolutionary advances to transferred proof-of-concept experiments from the laboratory to real world applications.

![Diagram](image)

**Figure 15.** VQF as function of the parameter $\theta$. The black solid line corresponds to the theory and the red data points to the experimental results. Here, we show the CV modes with continuous varying inter-modal phase $\theta \in [0, \pi/2]$. Notice the entanglement reaches its maximum for pure vector mode $\theta = \pi/4$ while it gains null for scalar mode $\theta = 0, \pi/2$. The insets show the polarization distribution of such three typical cases, where the colours of orange and green represent the right and left circular polarisation while the white stands for the linear.

states, respectively, and the white colour to the linear polarisation. Notice the high agreement of the experimental data with the theoretical prediction.

Crucially in recent time it was also proposed that the degree of coupling between the spatial and polarisation degrees of freedom can be measured directly from the Stokes parameters, by integrating these over the transverse plane [59]. In this way, no prior knowledge of the involved spatial modes is needed for the reason that these polarisation measurements for obtaining the required Stoke parameters are nevertheless affected by the spatial structure of the vector beam. More specifically, the degree of coupling can be mathematically expressed as $\sqrt{1 - (S_1^2 + S_2^2 + S_3^2)/S_0^2}$, where $S_i$ are the values of the Stokes parameters $S_i$ integrated over the entire transverse profiles. This approach offers a basis-independent way to infer the degree of coupling between both DoFs by using only the conventional Stokes parameters, representing a notable advance in characterizing these complex vector light fields.

5. Discussion

In the field of light beam shaping, computer-controlled devices, such as SLMs and DMDs, are perhaps some of the most versatile due to their high flexibility, which enable the rapid generation of complex light beams with almost any exotic shape and polarization distributions. While SLMs are based on the liquid crystal technology which allows a rapid reconfiguration of the displayed hologram, their refresh rates is limited to a maximum of 60 Hz. Here, a digital hologram with grey levels in the interval $[0, 255]$ is properly designed and displayed on the SLM’s screen to convert an input beam into a complex light field with the desired structure. Noteworthy, while SLMs are phase-only devices, the amplitude can also be modulated through well-known techniques, such as complex amplitude modulations [29, 100]. An advantage of SLMs are their ability to generate multiple scalar or vector beams simultaneously, via a multiplexing approach, where each can be manipulated independently [32, 34–41]. Among their disadvantages we can mention the wavelength-dependence of their modulation efficiency, even though some techniques have been proposed to overcome this drawback [102]. Another disadvantage is their polarization-dependence, which restricts them to the modulation of only linear polarization (typically horizontal). Even though this does not represent an issue to generate scalar beams, it complicates the techniques to generate complex light beams with non-homogeneous polarization distribution. Nonetheless, this issue can be solved in different ways, such as, using interferometric arrays with one or two SLMs or by passing the beam two times over a single SLM [32, 34–41]. Finally, their high prices also represent a serious disadvantage, which in many cases prohibits the use of such devices to, for example, undergraduate students. Noteworthy, a slighter newer device is the DMD, which provides many advantages in comparison to SLMs. A DMD is composed of around two millions of micromirrors arranged in a rectangular array, each of which with the capability to be controlled independently from an ‘On’ to an ‘Off’ state. Therefore DMDs can only be addressed with binary holograms to get access to their highest refresh rate, which allow the generation of structured light fields with refresh rates of up to 30 kHz. This property provides strong technical support for applications requiring fast reconfiguration of the generated beam, such as real-time detection in optical sensing. Additional advantages of DMDs include their polarization and wavelength independence, which are perhaps among the most attractive. Their polarization-independence property is crucial to simplify the optical setup required to generate vector modes, while their wavelength-independence enables the use of DMDs to application to new research fields, such as, space-time modulation of broadband structured pulses [103, 104]. In addition, the low cost of DMD, at least an order of magnitude cheaper than SLM, makes them very attractive for their use, for example in undergraduate laboratories. It is worth mentioning that the modulation efficiency of DMDs is very low ($\approx 4\%$), which might represent a serious drawback in some applications. Nonetheless, DMDs have many advantages, for example their low price, which makes them the ideal candidate for their integration into real-world applications. A comparison of some of the properties of DMDs and SLMs are summarised in table 3. As a final comment, a possible route we might explore is the combination of techniques which involve the use of both DMDs and SLMs, which might bring revolutionary advances to transfer proof-of-concept experiments from the laboratory to real world applications.
6. Conclusions

The techniques presented in this tutorial provides a framework for the generation and characterisation of vector modes employing a digital micromirror device. Even though SLMs have already shown their capabilities in shaping complex light fields, the unnoticed attribute of DMDs offers more flexibility and exhibits competitive advantage. Here, we firstly introduce a novel technique allowing the generation of arbitrary vector mode by fully taking advantage of the polarisation-insensitive attribute of DMDs. Together with the employment of spatial random multiplexing encoding scheme, this technique enables to thoroughly exploit the rather high refresh rate of DMDs during the generation process. We further demonstrated this technique not only by experimentally generating the well-known CV mode, but also by generating another three types of vector modes whose spatial basis are encoded with the Ince-, Helical Mathieu-, and Travelling Parabolic-Gauss beams. The high agreement between the experimental results and theoretical prediction exhibit the reliable performance of this technique. In terms of the characterisation of vector modes, we explained another two novel DMD-techniques capable to monitor in real time complex vector light field, one through the SP allowing to reconstruct the polarisation distribution of the generated beams; the other one through the VQF which provide a reliable way to determine in real-time the non-separability of the undetected beams. All the work present here provides new light on the field of structured light by employing DMDs with high refresh rates allowing advanced applications in fields such as optical communications, optical metrology and optical tweezers, to mention a few.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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Conflict of interest

The authors declare that there are no conflicts of interest related to this article.
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