Study of the $s \to d\nu\bar{\nu}$ rare hyperon decays in the Standard Model and new physics\textsuperscript{*}

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Abstract: FCNC processes offer important tools to test the Standard Model (SM) and to search for possible new physics. In this work, we investigate the $s \to d\nu\bar{\nu}$ rare hyperon decays in SM and beyond. We find that in SM the branching ratios for these rare hyperon decays range from $10^{-14}$ to $10^{-11}$. When all the errors in the form factors are included, we find that the final branching ratios for most decay modes have an uncertainty of about 5% to 10%. After taking into account the contribution from new physics, the generalized SUSY extension of SM and the minimal 331 model, the decay widths for these channels can be enhanced by a factor of 2 ~ 7.

Keywords: branching ratios, rare hyperon decays, form factors, light-front approach, new physics

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1 Introduction

The flavor changing neutral current (FCNC) transitions provide a critical test of the Cabibbo-Kobayashi-Maskawa (CKM) mechanism in the Standard Model (SM), and allow to search for possible new physics. In SM, the FCNC transition $s \to d\nu\bar{\nu}$ proceeds through the Z-penguin and electroweak box diagrams, and thus the decay probabilities are strongly suppressed. In this case, a precise study allows to perform very stringent tests of SM and ensures large sensitivity to potential new degrees of freedom.

A large number of studies have been performed of the $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ processes, and reviews of these two decay modes can be found in [1–6]. On the theoretical side, using the most recent input parameters, the SM predictions for the two branching ratios are [7]

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{SM} = (8.4 \pm 1.0) \times 10^{-11},$$

(1)

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{SM} = (3.4 \pm 0.6) \times 10^{-11}.$$  

(2)

The dominant uncertainty comes from the CKM matrix elements and the charm contribution. On the experimental side, the NA62 experiment at the CERN SPS has reported the first search for $K^+ \to \pi^+\nu\bar{\nu}$ using the decay-in-flight technique, and the corresponding observed upper limit is [8]:

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\text{exp}} < 1.4 \times 10^{-10}, \text{ at 95\% CL.}$$

(3)

Similarly, the E391a collaboration reported the 90\% C.L. upper bound [9]

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\text{exp}} < 2.6 \times 10^{-8}.$$  

(4)

The KOTO experiment, an upgrade of the E391a experiment, aims at a first observation of the $K_L \to \pi^0\nu\bar{\nu}$ decay at J-PARC around 2020 [3, 10]. Given the goal of a 10\% precision by NA62, the authors of Ref. [11] intend to carry out lattice QCD calculations to determine the long-distance contributions to the $K^+ \to \pi^+\nu\bar{\nu}$ amplitude.

Analogous to $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, the rare hyperon decays $B_i \to B_f\nu\bar{\nu}$ also proceed via $s \to d\nu\bar{\nu}$ at the quark level, and thus offer important tools to test SM and to search for possible new physics. Compared to the widely considered $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, there are few studies devoted to rare hyperon decays $B_i \to B_f\nu\bar{\nu}$. This work aims to perform a preliminary theoretical research of the rare hyperon decays both in and beyond SM.

A study of the hyperon decays at the BESIII experiment is proposed using the hyperon parents of the $J/\psi$ de-
The electron–positron collider BEPCII provides a clean experimental environment. About $10^6$-$10^7$ hyperons, $\Lambda$, $\Sigma$, $\Xi$ and $\Omega$, are produced in the $J/\psi$ and $\psi(2S)$ decays with the proposed data samples at the BESIII experiment. Based on these samples, the sensitivity of the measurement of the branching ratios of hyperon decays is in the range of $10^{-5}$-$10^{-8}$. The author of Ref. [12] proposed that rare decays and decays with invisible final states may be probed.

The paper is organized as follows. In Sec. 2, our computing framework is presented. Sec. 3 is devoted to performing the numerical calculations. The branching ratios of several rare hyperon decays are calculated in SM. The new physics contribution, the Minimal Supersymmetric Standard Model (MSSM) and the minimal 331 model, are considered. We also discuss possible uncertainties from the form factors. The last section contains a short summary.

2 Theoretical framework

The next-to-leading order (NLO) effective Hamiltonian for $s \to d\bar{v}\bar{\nu}$ reads [13]:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \sum_{i=\pi,\rho,\omega} [V_{ts}^* V_{td} X_{NL}^i + V_{tb}^* V_{tb} X(x_t)],$$

where $X(x_t)$ and $X_{NL}^i$ are relevant for the top and the charm contribution, respectively. Their explicit expressions can be found in Ref. [13]. Here, $x_t = m_t^2/m_W^2$. To leading order in $a_s$, the function $X(x_t)$ relevant for the top contribution reads [14, 15]:

$$X(x_t) = x_0(x_t) + \frac{\alpha_s}{4\pi} X_1(x_t),$$

where

$$x_0(x_t) = \left( \frac{2 + x_t - 3x_t^2 + 1-x_t^2 \ln x_t}{1-x_t^2} \right),$$

$$X_1(x_t) = \left( \frac{2 + x_t - 3x_t^2 + 1-x_t^2 \ln x_t}{1-x_t^2} \right),$$

and

$$\frac{8 + 4x_t^2 + 3x_t^3 - 4x_t^3}{2(1-x_t^2)} \ln x_t - \frac{4x_t^3}{(1-x_t^2)} L_2(1-x_t) + 8\frac{\partial X_0(x_t)}{\partial x_t} \ln x_t,$$

where $x_t = \mu^2/M_W^2$ with $\mu = \mathcal{O}(m_t)$ and

$$L_2(1-x_t) = \int_1^\infty \frac{\ln t}{1-t} dt.$$

The function $X_{NL}^i$ corresponds to $X_1(x_t)$ in the charm sector. It results from the renormalization group (RG) calculation in next-to-leading-order logarithmic approximation (NLLA) and is given as follows:

$$X_{NL}^i = C_{NL} - 4B_{NL}^{(1/2)},$$

where $C_{NL}$ and $B_{NL}^{(1/2)}$ correspond to the $Z^0$-penguin and the box-type contribution, respectively, given as [16]:

$$C_{NL} = \frac{\mu}{m} \left[ \left( \frac{48}{7} K_+ + \frac{24}{11} K_+ - 196 \right) \left( \frac{4\pi}{\alpha_s(\mu)} \right) \right] + \frac{15212}{1875} \left[ (1 - K_+) + \left( 1 - \ln \frac{\mu^2}{m^2} \right) (16K_+ - 8K_-) \right] + \frac{1176244}{2302} K_+ - \frac{2302}{6875} K_- + \frac{3529184}{48125} K_{33},$$

$$B_{NL}^{(1/2)} = \frac{4\pi}{\alpha_s(\mu)} \left[ \frac{3}{2} (1 - K_2) \left( \frac{4\pi}{\alpha_s(\mu)} + \frac{15212}{1875} (1 - K_+^{-1}) \right) \right] - \ln \frac{\mu^2}{m^2} - r \ln r \left[ \frac{305}{12} + \frac{15212}{625} K_2 + \frac{15581}{7500} K_{33} \right],$$

where $r = m_t^2/m_\ell^2(\mu)$, $\mu = \mathcal{O}(m_t)$ and

$$K_+ = K_+^\pi, \quad K_- = K_-^\pi, \quad K_{33} = K_{33}^\pi.$$

In the following, we consider the transitions between the baryon octet ($\Xi, \Sigma, \Sigma$ and $N$) and the transitions from the baryon decuplet to the octet $\Omega^- \to \Xi^-$. The transition matrix elements of the vector and axial-vector currents between the baryon octets can be parametrized in terms of six form factors $f_1, f_2(q^2)$ and $g_1, g_2(q^2)$:

$$\langle B_k(P', S')|\bar{\psi}\gamma_{\mu}(1-\gamma_5)\psi|B_0(P, S)\rangle =$$

$$\bar{u}(P', S') \left[ \gamma_\mu f_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu M}{M} f_2(q^2) + \frac{q^\mu M}{M} f_3(q^2) \right] u(P, S),$$

$$\bar{u}(P', S') \left[ \gamma_\mu g_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu M}{M} g_2(q^2) + \frac{q^\mu M}{M} g_3(q^2) \right] \gamma_5 u(P, S),$$

where $q = P - P'$, and $M$ denotes the mass of the parent baryon octet $B_0$. The form factors for the $B_0 \to B_0$ transition, $f_i(q^2)$ and $g_i(q^2)$, can be expressed by the following formulas [17]:

$$f_m = a F_m(q^2) + b D_m(q^2),$$

$$g_m = a G_m(q^2) + b D_{m+1}(q^2), \quad (m = 1, 2, 3),$$

where $F_i(q^2)$ and $D_i(q^2)$, with $i = 1, 2, \cdots, 6$, are different functions of $q^2$ for each of the six form factors. Some remarks are necessary [17]:

- The constants $a$ and $b$ in Eq. (12) are the $SU(3)$ Clebsch–Gordan coefficients that appear when an octet operator is sandwiched between octet states.

- For $q^2 = 0$, the form factor $f_1(0)$ is equal to the electric charge of the baryon, therefore $F_1(0) = 1$ and $D_1(0) = 0$.

- The weak $f_2(0)$ form factor can be computed using the anomalous magnetic moments of proton and neutron ($\kappa_p$ and $\kappa_n$) in the exact $SU(3)$ symmetry. Here,
\[ F_2(0) = \kappa_p + \frac{1}{2}\kappa_n \quad \text{and} \quad D_2(0) = -\frac{3}{2}\kappa_n. \]

- \( g_1(0) \) is a linear combination of two parameters, \( F \) and \( D \).

- Since \( g_2^{\pi\pi} = F_3(q^2) + D_3(q^2) = 0 \) and \( g_2^{\pi\pi} = D_3(q^2) - F_3(q^2) = 0 \), we get \( F_3(q^2) = D_3(q^2) = 0 \). Therefore, all pseudo-tensor form factors \( g_2 \) vanish in all decays up to symmetry-breaking effects.

- In the \( s \to d\nu\bar{v} \) decay, the \( f_3 \) and \( g_3 \) terms are proportional to the neutrino mass and thus can be neglected for the decays considered in this work.

Since the invariant mass squared of lepton pairs in the hyperon decays is relatively small, it is expected that the \( q^2 \) distribution in the form factors has small impact on the decay widths. We list the expressions for \( f_1, f_2 \) and \( g_1 \) at \( q^2 = 0 \) in Table 1.

Hence, Eq. (11) can be rewritten as:

\[ \langle B'_\epsilon (P', S'_t)|d\gamma_\mu|B_\epsilon (P, S)\rangle = \bar{u}_\epsilon (P', S'_t) \left[ \gamma_\mu f_1(q^2) + i\sigma_\mu\nu q^2 q^2 f_2(q^2) - \gamma_\nu g_1(q^2)\gamma_5 \right] u_\epsilon (P, S). \]

(13)

The helicity amplitudes of the hadronic contribution are defined as

\[ H_{V,1,0}^V \equiv \langle B'_\epsilon (P', S'_t)|d\gamma_\mu|B_\epsilon (P, S)\rangle \epsilon^\mu_{\nu'}(\lambda_V), \]

(14)

\[ H_{A,1,0}^A \equiv \langle B'_\epsilon (P', S'_t)|d\gamma_\mu|B_\epsilon (P, S)\rangle \epsilon^\mu_{\nu'}(\lambda_A). \]

(15)

Here, \( \lambda^{\nu'} \) denotes the helicity of the parent (daughter) baryon in the initial (final) state, and \( \lambda_V \) is the helicity of the virtual intermediate vector particle. It can be shown that the helicity amplitudes \( H_{V,1,0} \) have the following simple forms [19]:

\[ H_{V,1,0}^V = -i \sqrt\frac{Q^2}{M^2 - q^2} \left[ (M + M') f_1 - \frac{q^2}{M} f_2 \right], \]

\[ H_{A,1,0}^A = -i \sqrt\frac{Q^2}{M^2 - q^2} (M - M') g_1, \]

\[ H_{V,1,1}^V = i \sqrt\frac{Q^2}{M^2} \left[ -f_1 + \frac{M + M'}{M} f_2 \right], \]

\[ H_{A,1,1}^A = -i \sqrt\frac{Q^2}{M^2} g_1. \]

(16)

In the above, \( Q = (M + M')^2 - q^2 \), and \( M (M') \) is the parent (daughter) baryon mass in the initial (final) state. The amplitudes for the negative helicity are obtained from the relations,

\[ H_{V,-1,0}^V = H_{V,1,0}^V, \quad H_{A,-1,0}^A = -H_{A,1,0}^A. \]

(17)

The complete helicity amplitudes are obtained by

\[ H_{V,1,0} = H_{V,1,0}^V - H_{A,1,0}^A. \]

(18)

Due to the lack of experimental data for the \( M_1 \) and \( E_2 \) transitions from the baryon decuplet to the octet, the vector transition matrix element for \( \Omega^- \to \Xi^- \) can not be determined. In this work we follow Ref. [18], and consider only the axial-vector current matrix element [18, 20, 21]:

\[ \langle \Xi^- (P', S'_t)|i\gamma_\mu\gamma_5|\Omega^- (P, S)\rangle = \bar{u}_\pi (P', S'_t) \left[ C_1^A(q^2) g_{\mu\nu} + C_3^A(q^2) q_{\mu\nu} + C_4^A(q^2) q_{\mu\nu} \right] \]

\[ \times \langle q_{\mu\nu} q_{\mu\nu} - q_{\mu\nu} q_{\mu\nu} \rangle u_\Omega^\mu (P, S). \]

(19)

Here, \( u_\Omega^\mu (P, S) \) represents the Rarita-Schwinger spinor that describes the baryon decuplet \( \Omega^- \) with spin \( \frac{3}{2} \).

In Ref. [22] it is shown that \( C_1^A(q^2) \) and \( C_2^A(q^2) \) are proportional to the mass difference of the initial and final baryons, and thus are suppressed. In the chiral limit, \( C_3^A(q^2) \) and \( C_4^A(q^2) \) are related by \( C_3^A(q^2) = M_0^2 C_4^A(q^2)/q^2 \) [20]. In our calculations, we use \( C_3^A(0) = 1.653 \pm 0.006 \) for \( \Omega^- \to \Xi^- \), which is the same as \( \Omega^- \to \Xi^- \) in the SU(3) limit [18]. The helicity amplitude can then be expressed as:

\[ H_{V,1,0}^A = \langle \Xi^- (P', S'_t)|i\gamma_\mu\gamma_5|\Omega^- (P, S)\rangle \epsilon^\mu_{\nu'}(\lambda_V), \]

(20)

\[ = \bar{u}_\pi (P', S'_t) \left[ C_1^A(q^2) g_{\mu\nu} + C_3^A(q^2) q_{\mu\nu} \right] u_\Omega^\mu (P, S) \epsilon^\mu_{\nu'}(\lambda_V). \]

(21)

Here, \( \lambda^{\nu'} \) and \( \lambda_V \) have the same definition as in Eqs. (14)-(15). It can be shown that the helicity amplitudes \( H_{V,1,0}^A \) have the following simple forms [19]:

\[ H_{V,1,0}^A = i \sqrt\frac{2Q^2}{M^2} C_3^A(q^2), \]

\[ H_{A,1,1}^A = i \sqrt\frac{2Q^2}{M^2} C_4^A(q^2), \]

\[ H_{A,1,1}^A = i \sqrt\frac{2Q^2}{M^2} C_4^A(q^2). \]

(22)

Table 1. The form factors for the \( B \to B' \) transition, \( f_1(0), f_2(0) \) and \( g_1(0) \) [17], where the experimental anomalous magnetic moments are \( \kappa_p = 1.793 \pm 0.087 \) and \( \kappa_n = -1.913 \pm 0.069 \) [18], with the two coupling constants \( F = 0.463 \pm 0.008 \) and \( D = 0.804 \pm 0.008 \) [18]. Here, \( g_1/f_1 \) is positive for the neutron decay, and all other signs are fixed using this sign convention.

| \( B \to B' \) | \( \Lambda \to n \) | \( \Sigma^+ \to p \) | \( \Xi^0 \to \Lambda \) | \( \Xi^0 \to \Sigma^0 \) | \( \Xi^- \to \Sigma^- \) |
|---|---|---|---|---|---|
| \( f_1(0) \) | \( \sqrt{2} \) | \( -3 \sqrt{2} \) | \( 1 \) | \( 1/\sqrt{2} \) | \( -1 \) |
| \( f_2(0) \) | \( -\sqrt{2} \) | \( \kappa_p + 2\kappa_n \) | \( -3 \sqrt{2} \) | \( \sqrt{2} (\kappa_p - \kappa_n) \) | \( 1/\sqrt{2} (\kappa_p - \kappa_n) \) |
| \( g_1(0) \) | \( -\sqrt{2} (F + D/3) \) | \( - (F - D) \) | \( F - D \) | \( F - D \) | \( F + D \) |
The differential decay width for $B \to B'\nu\bar{\nu}$ is given as:
\[
\frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}. \tag{23}
\]
Here, $d\Gamma_L/dq^2$ and $d\Gamma_T/dq^2$ are the longitudinal and transverse parts of the decay width, and their explicit expressions are given by
\[
\frac{d\Gamma_L}{dq^2} = N \frac{q^2 p'}{12(2\pi)^3 M^2} \left( |H_{1,0}^2| + |H_{-1,1}^2| \right), \tag{24}
\]
\[
\frac{d\Gamma_T}{dq^2} = N \frac{q^2 p'}{12(2\pi)^3 M^2} \left( |H_{1,1}^2| + |H_{-1,-1}^2| + |H_{1,-1}^2| + |H_{-1,1}^2| \right). \tag{25}
\]
In Eqs. (24) and (25), $p' = \sqrt{Q_s^2 - 2M}$ is the magnitude of the momentum of $B'$ in the rest frame of $B$, and $N = 2N_1(0) + N_1(m_t)$ with
\[
N_1(m_t) = \left[ \frac{G_F}{\sqrt{2}} \frac{\alpha}{\sin^2 \theta_W} \left( V_{cd}^* V_{cs} V_{\Lambda_1}^* e_N(0) + V_{td}^* V_{ts} X_{\Lambda_1} \right) \right]^2. \tag{26}
\]
Note that we have neglected the electron and muon masses.

One can then obtain the decay width
\[
\Gamma = \int_{0}^{M-M'} \frac{dq^2}{d\Omega} \frac{d\Gamma}{dq^2}. \tag{27}
\]

3 Numerical results and discussion

3.1 Calculations in SM

With the input parameters given in Table 2 and the formulae from the last section, the LO and NLO results for $\mu_c = 1$ GeV, $\mu_t = 100$ GeV, and $\mu_c = 3$ GeV, $\mu_t = 300$ GeV are listed in Table 3.

From the results in Table 3 one can see that:

- The branching ratios of the $s \to d\nu\bar{\nu}$ rare hyperon decays range from $10^{-14}$ to $10^{-11}$.

- For $\mu_c = 1$ GeV, $\mu_t = 100$ GeV, the NLO results are smaller than the LO ones by about 30%, while for $\mu_c = 3$ GeV, $\mu_t = 300$ GeV, the NLO results are larger than the LO ones by about 10%.

- The LO results vary by about 50% from $\mu_c = 1$ GeV, $\mu_t = 100$ GeV to $\mu_c = 3$ GeV, $\mu_t = 300$ GeV, while the NLO ones vary by about 30%. As expected, the NLO results depend less on the mass scales.

- The branching ratio of $\Omega^- \to \Xi^-\nu\bar{\nu}$ is the largest among the 6 channels. It is of the same order as for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$.

At present, there is a small number of experimental studies, and thus most experimental constraints are less severe. The prospects for rare and forbidden hyperon decays at BESIII were analyzed in a recent publication Ref. [12]. We quote the experimental sensitivity for all decay modes in Table 3. Unfortunately, one can see that the current BESIII experiment will not be able to probe these hyperon decays. We hope this may be improved at future experimental facilities like the Super Tau-Charm Factory.

3.2 Uncertainties of the form factors

Note that due to the Ademollo-Gatto theorem [25], the form factor $f_2(0)$ does not receive any $SU(3)$ symmetry breaking correction. However, $f_2(0)$ can be computed using the anomalous magnetic moments of proton and neutron ($\kappa_p$ and $\kappa_n$) in the exact $SU(3)$ symmetry. The experimental data for $\kappa_p$ and $\kappa_n$ already include the $SU(3)$ symmetry breaking effects [18]:
\[
\kappa_p \left[ O(m_0^3) \right] = 1.363 \pm 0.069, \quad \kappa_p \left[ O(m_s^3) \right] = -1.416 \pm 0.049, \quad \kappa_p \left[ O(m_t^3) \right] = 1.793 \pm 0.087, \quad \kappa_p \left[ O(m_s^3) + O(m_t^3) \right] = -1.913 \pm 0.069. \tag{28}
\]

The uncertainties from $\kappa_p$ and $\kappa_n$ in the effect of $SU(3)$ symmetry breaking is approximately 25%. We calculated the effect of $\kappa_p$ and $\kappa_n$ on the branching ratio of $\Sigma^+ \to p\nu\bar{\nu}$ in the case of NLO with the energy scale $\mu_c = 1$ GeV and $\mu_t = 100$ GeV such that:

| Physical constants and CKM parameters [23, 24] |
|-----------------------------------------------|
| $G_F = 1.16633787 \times 10^{-5}$ GeV$^{-2}$   |
| $\sin^2 \theta_W = 0.23122$                    |
| $\alpha(m_Z) = 0.1182$                        |
| $\alpha = \alpha(m_Z) = 1/128$               |
| $m_e = 9.17686$ MeV                           |
| $m_\mu = 1.7125$ GeV                         |
| $m_\tau = 173.0$ GeV                         |
| $m_W = 80.379$ GeV                           |
| $m_Z = 91.1876$ GeV                          |
| $\alpha = 0.122$                             |
| $\bar{\rho} = 0.122$                        |
| $\bar{\eta} = 0.355$                        |

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Table 2: The input parameters used in this work.

| The masses and lifetimes of baryons in the initial and final states [23] |
|---------------------------------------------------------------|
| $m_p = 938.2720813$ MeV                                       |
| $m_n = 939.5654133$ MeV                                       |
| $m_{\Lambda} = 1115.683$ MeV                                  |
| $m_{\Sigma^0} = 2.90 \times 10^{-10}$ s                      |
| $\tau_{\Sigma^0} = 2.632 \times 10^{-10}$ s                  |
| $m_{\Sigma^-} = 2.90 \times 10^{-10}$ s                      |
| $\tau_{\Sigma^-} = 2.632 \times 10^{-10}$ s                  |
| $m_{\Sigma^0} = 1189.207$ MeV                                  |
| $m_{\Sigma^-} = 1197.456$ MeV                                  |
| $m_{\Sigma^0} = 1192.642$ MeV                                  |
| $m_{\Sigma^-} = 1672.45$ MeV                                   |
| $m_{\Xi^-} = 0.821 \times 10^{-10}$ s                        |
| $\tau_{\Xi^-} = 0.821 \times 10^{-10}$ s                     |
| $m_{\Xi^-} = 173.0$ GeV                                       |
| $m_{\Xi^-} = 80.379$ GeV                                      |
| $m_{\Xi^-} = 91.1876$ GeV                                     |
| $A = 0.836$                                                   |
| $\lambda = 0.22453$                                           |
| $\bar{\rho} = 0.122$                                         |
| $\bar{\eta} = 0.355$                                         |
Table 3. The LO, NLO, NLO+SUSY and NLO+M331 results for the branching ratio of rare hyperon decays for $\mu_{c} = 1$ GeV, $\mu_{t} = 100$ GeV and $\mu_{t} = 3$ GeV, $\mu_{t} = 300$ GeV.

| Branching ratio | $\mathcal{B}(\Lambda \to n\bar{\nu})$ | $\mathcal{B}(\Sigma^{+} \to p\nu)$ | $\mathcal{B}(\Xi^{0} \to \Lambda\nu\bar{\nu})$ | $\mathcal{B}(\Xi^{0} \to \Sigma^{0}\nu\bar{\nu})$ | $\mathcal{B}(\Xi^{-} \to \Sigma^{-}\nu\bar{\nu})$ | $\mathcal{B}(\Omega^{-} \to \Xi^{-}\nu\bar{\nu})$ |
|-----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\mu_{c} = 1$ GeV | $2.85 \times 10^{-12}$ | $6.88 \times 10^{-13}$ | $1.06 \times 10^{-12}$ | $1.77 \times 10^{-13}$ | $2.17 \times 10^{-13}$ | $1.78 \times 10^{-11}$ |
| NLO             | $1.98 \times 10^{-12}$ | $5.01 \times 10^{-13}$ | $7.35 \times 10^{-13}$ | $1.24 \times 10^{-12}$ | $1.52 \times 10^{-12}$ | $1.93 \times 10^{-11}$ |
| NLO+SUSY (Set I) | $8.14 \times 10^{-12}$ | $2.06 \times 10^{-12}$ | $3.02 \times 10^{-12}$ | $5.08 \times 10^{-13}$ | $6.23 \times 10^{-13}$ | $7.94 \times 10^{-11}$ |
| $\mu_{t} = 100$ GeV | $3.78 \times 10^{-12}$ | $9.55 \times 10^{-13}$ | $1.40 \times 10^{-12}$ | $2.36 \times 10^{-13}$ | $2.89 \times 10^{-13}$ | $3.69 \times 10^{-11}$ |
| NLO+M331        | $1.24 \times 10^{-11}$ | $3.13 \times 10^{-12}$ | $4.59 \times 10^{-12}$ | $7.71 \times 10^{-13}$ | $9.45 \times 10^{-13}$ | $1.20 \times 10^{-10}$ |
| $\mu_{t} = 3$ GeV | $1.10 \times 10^{-12}$ | $2.65 \times 10^{-13}$ | $4.10 \times 10^{-13}$ | $6.83 \times 10^{-14}$ | $8.37 \times 10^{-14}$ | $1.07 \times 10^{-11}$ |
| NLO             | $1.20 \times 10^{-12}$ | $3.04 \times 10^{-13}$ | $4.46 \times 10^{-13}$ | $7.50 \times 10^{-14}$ | $9.19 \times 10^{-14}$ | $1.17 \times 10^{-11}$ |
| NLO+SUSY (Set I) | $5.85 \times 10^{-12}$ | $1.48 \times 10^{-12}$ | $2.17 \times 10^{-12}$ | $3.65 \times 10^{-13}$ | $4.47 \times 10^{-13}$ | $5.71 \times 10^{-11}$ |
| $\mu_{t} = 300$ GeV | $2.35 \times 10^{-12}$ | $5.94 \times 10^{-13}$ | $8.72 \times 10^{-13}$ | $1.47 \times 10^{-13}$ | $1.80 \times 10^{-13}$ | $2.29 \times 10^{-11}$ |
| NLO+M331        | $1.02 \times 10^{-11}$ | $2.58 \times 10^{-12}$ | $3.80 \times 10^{-12}$ | $6.37 \times 10^{-13}$ | $7.81 \times 10^{-13}$ | $9.95 \times 10^{-11}$ |

| BESIII sensitivity [12] | $3 \times 10^{-7}$ | $4 \times 10^{-7}$ | $8 \times 10^{-7}$ | $9 \times 10^{-7}$ | $- \times 2.6 \times 10^{-5}$ |

When all the above errors in the form factors are included, we find that the final branching ratios for most decay modes have an uncertainty of about 5% to 10%.

3.3 Contribution from MSSM

The effective Hamiltonian for $s \to d\bar{\nu}$ in the generalized supersymmetry (SUSY) extension of SM is given in Eq. (5), with $X(x_{t})$ replaced by [26]

$$X_{new} = X(x_t) + X_{H}(x_{H}) + C_{\chi} + C_{N}. \quad (35)$$

Here, $x_{H} = m_{2}/m_{H}^{}$, and $X_{H}(x_{H})$ corresponds to the charged Higgs contribution. $C_{\chi}$ and $C_{N}$ denote the chargino and neutralino contributions

$$C_{\chi} = X_{\chi}^{0} + X_{\chi}^{LH} R_{\chi}^{LH} \chi_{d} \chi_{d} + X_{\chi}^{LR} R_{\chi}^{LR} \chi_{d} \chi_{d},$$

$$C_{N} = X_{N} R_{\chi}^{LH} \chi_{d},$$

where $X_{\chi}$ and $X_{N}$ depend on the SUSY masses, and respectively on the chargino and neutralino mixing angles. The explicit expressions for $X_{\chi}(x_{t})$, $C_{\chi}$ and $C_{N}$ can be found in Ref. [26]. The $R$ parameters are defined in terms of mass insertions, and their upper limits are listed in Table 4 [26]. It should be mentioned that the phase $\phi$ of $R_{\chi}^{LH}$ and $R_{\chi}^{LR}$ is a free parameter which ranges from 0 to $2\pi$. We set $\phi = 0$ as a central result.

The parameters in Table 5 are adopted for detailed

Table 4. Upper limits for the $R$ parameters. Note that the phase of $R_{\chi}^{LH}$ and $R_{\chi}^{LR}$ is unconstrained.
calculations [27]. The assumption $M_1 \approx 0.5M_2$ was made [28]. With the above parameters, the branching ratios of hyperon decays are listed in Table 3, and are significantly enhanced compared with the SM results. Taking as examples the decays $\Lambda \to n\nu\bar{\nu}$ and $\Omega^- \to \Xi^- \nu\bar{\nu}$ with the energy scale $\mu_e = 1$ GeV and $\mu_1 = 100$ GeV, we obtain:

\begin{align}
NLO : & \quad B(\Lambda \to n\nu\bar{\nu}) = 1.98 \times 10^{-12}, \\
& \quad B(\Omega^- \to \Xi^- \nu\bar{\nu}) = 1.93 \times 10^{-11}, \quad (36) \\
SetI : & \quad B(\Lambda \to n\nu\bar{\nu}) = 8.14 \times 10^{-12}, \\
& \quad B(\Omega^- \to \Xi^- \nu\bar{\nu}) = 7.94 \times 10^{-11}, \quad (37) \\
SetII : & \quad B(\Lambda \to n\nu\bar{\nu}) = 3.78 \times 10^{-12}, \\
& \quad B(\Omega^- \to \Xi^- \nu\bar{\nu}) = 3.69 \times 10^{-11}. \quad (38)
\end{align}

Comparing the results of NLO+SUSY (Set I) and (Set II) with the ones of NLO, we see that all branching ratios are roughly enhanced by a factor of 4 and 2, respectively. However, none of these results can be probed at the ongoing experimental facilities, like the BESIII experiment [12].

3.4 Contribution from the minimal 331 model

The so-called minimal 331 model is an extension of SM at the TeV scale, where the weak gauge group of SM $SU(2)_L$ is extended to $SU(3)_L$. In this model, a new neutral $Z'$ gauge boson can give very important additional contributions, for it can transmit FCNC at the tree level. In Table 3, we denote this model as M331. More details of this model can be found in Ref. [29]. The minimal 331 model leads to a new term in the effective Hamiltonian [30]:

$$H_{\text{eff}} = \sum_{\ell=e,\mu,\tau} \frac{G_F}{\sqrt{2}} \frac{V_{32}V_{31}}{3} \left( \frac{M_Z}{M_Z'} \right)^2 \cos^2 \theta_W \cos^2 \theta_W \frac{1}{3} F_\nu^2 \frac{1}{4} \frac{\alpha}{3} \left( \frac{M_Z}{M_Z'} \right)^2 + \text{h.c.}, \quad (39)$$

with $M_{Z'} = 1$ TeV, $\text{Re}(V_{32}^*V_{31})^2 = 9.2 \times 10^{-6}$ and $\text{Im}(V_{32}^*V_{31})^2 = 4.8 \times 10^{-8}$ [30]. The other parameters are the same as the SM inputs [23, 24]. The function $X(x_t)$ in Eq. (5) can be redefined as $X(x_t) = X^{\text{SM}}(x_t) + \Delta X$ with

$$\Delta X = \frac{\sin^2 \theta_W \cos^2 \theta_W}{3} \frac{2}{\alpha} \frac{V_{32}^*V_{31}}{V_{12}^*V_{12}} \left( \frac{M_Z}{M_Z'} \right)^2. \quad (40)$$

With the modified function $X(x_t)$ and considering the NLO contribution, the branching ratios of rare hyperon decays in the minimal 331 model can be calculated, as shown in Table 3. The NLO+M331 predictions are much larger than the NLO results in SM, and are two and four times larger than the results of NLO+SUSY (Set I) and NLO+SUSY (Set II), respectively.

4 Conclusions

FCNC processes offer important tools to test SM and to search for possible new physics. The two decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ have been widely studied, while the corresponding baryon sector has not been explored. In this work, we studied the $s \to d\nu\bar{\nu}$ rare hyperon decays. We adopted the leading order approximations for the form factors for small $q^2$, and derived expressions for the decay width. We applied the decay width formula to both SM and new physics contributions. Different energy scales were considered. The branching ratios in SM range from $10^{-14}$ to $10^{-11}$, and the largest is of the same order as for the decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. When all the errors in the form factors are included, we found that the final branching ratios for most decay modes have an uncertainty of about 5% to 10%. After taking into account the contribution from MSSM, the branching ratios are enhanced by a factor of 2 – 4. The branching ratios of hyperon decays in the minimal 331 model are seven times larger than the SM results.

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