A Spin-Isospin Dependent 3N Scattering Formalism

in a 3D Faddeev Scheme

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Abstract

We have introduced a spin-isospin dependent three-dimensional approach for formulation of the three-nucleon scattering. Faddeev equation is expressed in terms of vector Jacobi momenta and spin-isospin quantum numbers of each nucleon. Our formalism is based on connecting the transition amplitude $T$ to momentum-helicity representations of the two-body $t$-matrix and the deuteron wave function. Finally the expressions for nucleon-deuteron elastic scattering and full breakup process amplitudes are presented.

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I. INTRODUCTION

During the past years, the three-dimensional (3D) approach has been developed for few-body bound and scattering problems [1-15]. The motivation for developing this approach is introducing a direct solution of the integral equations avoiding the very involved angular momentum algebra occurring for the permutations, transformations and especially for the three-body forces. Furthermore above the pion production threshold and the GeV region the three-nucleon (3N) scattering with the 3D formalism is simpler for more complex few body system by providing a strictly finite number of coupled 3D integral equation. Conceptually the 3D formalism considers all partial wave channels automatically.

In the case of the 3N scattering the Faddeev equation has been formulated for three identical bosons as a function of vector Jacobi momenta, with the specific stress upon the magnitudes of the momenta and the angle between them [1], [2]. Adding the spin-isospin to the 3D formalism is a major additional task, which will increase more degrees of freedom. This carried out by Fachruddin et al. by considering leading order of Faddeev equation for breakup process. Recently a 3D formalism based on operator form has been introduced for formulation of the 3N scattering [16]. In this paper we have attempted to formulate the full breakup and the elastic scattering of nucleon-deuteron (Nd) by including directly the spin-isospin degrees of freedom. To this end we have formulated the Faddeev equation for the 3N scattering with the advantage of using the helicity representation of NN forces such as the AV18, Bonn-B and the Chiral potentials [4], [17].

This manuscript is organized as follows. In sect. II we have derived Faddeev equation in a realistic 3D scheme as a function of Jacobi momenta vectors and the spin-isospin quantum numbers. In sect. III we have derived two expressions for amplitudes of elastic scattering and breakup process respectively. Finally in sect. IV a summary and an outlook have been presented.

II. FADDEEV EQUATION FOR THE 3N SCATTERING IN THE 3D APPROACH

Faddeev equation for the three identical particle is given by [18]:

\[ T|\phi\rangle = tP|\phi\rangle + tG_0 PT|\phi\rangle, \]  

(1)
where \( P = P_{12}P_{23} + P_{13}P_{23} \) is the sum of cyclic and anti-cyclic permutations of the three nucleons, \( t \) denotes the NN transition matrix determined by a two-body Lippman-Schwinger equation and \( G_0 \) is the free 3N propagator which is defined as:

\[
G_0 = \frac{1}{E - H_0 + i\varepsilon}, \quad E = E_d + \frac{3}{4m}q_0^2 = E_d + \frac{2}{3}E_{lab},
\]

where \( H_0 \) and \( E \) are the free 3N hamiltonian and the total energy in the center of mass frame respectively, \( E_d \) is the deuteron bounding energy and \( q_0 \) is the relative momentum of the projectile nucleon to the deuteron. In order to solve eq. (1) in the momentum space we introduce the 3N free basis states in a 3D formalism as [7], [13]:

\[
|pq\gamma\rangle \equiv |pq_{m_s_1,m_s_2,m_t_1,m_t_2,m_t_3}\rangle \equiv |q_{m_s_1,m_t_1}\rangle|p_{m_s_2,m_s_3,m_t_2,m_t_3}\rangle,
\]

(3)

The basis states involve two standard Jacobi momenta \( p \) and \( q \) which are the relative momentum in the subsystem and the momentum of the spectator with respect to the subsystem respectively [19]. \( |\gamma\rangle \equiv |m_{s_1}m_{s_2}m_{s_3}m_{t_1}m_{t_2}m_{t_3}\rangle \) is the spin-isospin parts of the basis states where the quantities \( m_{s_i} (m_{t_i}) \) are the projections of the spin (isospin) of each three nucleons along its quantization axis. The introduced basis states are completed and normalized as:

\[
\sum_\gamma \int dp \int dq |pq\gamma\rangle\langle pq\gamma| = 1, \quad \langle p'q'\gamma'|pq\gamma\rangle = \delta(p' - p)\delta(q' - q)\delta_{\gamma'\gamma}.
\]

(4)

Projecting the Faddeev equation on to Jacobi momenta leads to:

\[
\langle pq\gamma|T|q_0 m_s^0 m_t^0 \Psi_d^{M_d}\rangle = \langle pq\gamma|tP + tG_0PT|q_0 m_s^0 m_t^0 \Psi_d^{M_d}\rangle = \langle pq\gamma|tP|q_0 m_s^0 m_t^0 \Psi_d^{M_d}\rangle + \langle pq\gamma|tG_0PT|q_0 m_s^0 m_t^0 \Psi_d^{M_d}\rangle,
\]

(5)

where:

\[
|\phi\rangle \equiv |q_0 m_s^0 m_t^0 \Psi_d^{M_d}\rangle \equiv |q_0 m_s^0 m_t^0|\Psi_d^{M_d}\rangle,
\]

(6)

is the initial state where \( m_s^0 (m_t^0) \) is spin (isospin) projection of projectile nucleon along quantization axis and \( |\Psi_d^{M_d}\rangle \) is the antisymmetrized deuteron state with \( M_d \) being the projection of total angular momentum along quantization axis. We start by inserting the completeness
relation twice into the first term of eq. (5):

\[
\langle pq | tP | q_0 m_s^0 m_t^0 \psi_d^{M_d} \rangle = \sum_{\gamma'} \int dp' \int dq' \langle pq | tP | p' q' \gamma' \rangle \sum_{\gamma''} \int dp'' \int dq'' \langle p' q' \gamma' | P | p'' q'' \gamma'' \rangle \\
\times \langle p'' q'' \gamma'' | q_0 m_s^0 m_t^0 \psi_d^{M_d} \rangle.
\]

(7)

The matrix elements of the permutation operator \( P \) are evaluated as [7]:

\[
\langle p'' q'' \gamma'' | P | p' q' \gamma' \rangle = \delta(p'' - \frac{1}{2} q'' - q') \delta(p' + q'' + \frac{1}{2} q') \delta_{m''_1 m'_1} \delta_{m''_2 m'_2} \delta_{m''_3 m'_3} \delta_{m''_4 m'_4} \delta_{m''_5 m'_5} \delta_{m''_6 m'_6} \\
+ \delta(p' + \frac{1}{2} q'' + q') \delta(p'' - q'' - \frac{1}{2} q') \delta_{m''_1 m'_1} \delta_{m''_2 m'_2} \delta_{m''_3 m'_3} \delta_{m''_4 m'_4} \delta_{m''_5 m'_5} \delta_{m''_6 m'_6}.
\]

(8)

We have used these relations for the matrix elements of the two-body \( t \)-matrix and the deuteron wave function:

\[
\langle pq | t | p' q' \gamma' \rangle = \langle pm s m s m t m t | t(z) | pm' s m s m t' m t' \rangle \delta(q - q') \delta_{m s m s m t m t},
\]

(9)

\[
\langle p'' q'' \gamma'' | q_0 m_s^0 m_t^0 \psi_d^{M_d} \rangle = \langle p'' m'' s m'' s m'' t m'' t | \psi_d^{M_d} \rangle \delta(q'' - q_0) \delta_{m''_1 m'_1} \delta_{m''_2 m'_2} \delta_{m''_3 m'_3} \delta_{m''_4 m'_4} \delta_{m''_5 m'_5} \delta_{m''_6 m'_6},
\]

(10)

where the two-body subsystem energy in a 3N system is \( z = E - \frac{3}{4m} q^2 \). Substituting eqs. (8), (9) and (10) into eq. (7) yields:

\[
\langle pq | tP | q_0 m_s^0 m_t^0 \psi_d^{M_d} \rangle = \sum_{m'_2 m'_2} \langle pm s m s m t m t | t(z) | - \pi m'_2 m'_2 m'_2 m'_2 m'_2 \rangle \langle pm' s m' s m' t m' t \rangle \langle \psi_d^{M_d} \rangle \\
+ \sum_{m'_3 m'_3} \langle pm s m s m t m t | t(z) | - m'_3 m'_3 m'_3 m'_3 m'_3 \rangle \langle - \pi m'_3 m'_3 m'_3 m'_3 m'_3 \rangle \langle \psi_d^{M_d} \rangle \\
= \sum_{m'_t m'_t} \left\{ \langle pm s m s m t m t | t(z) | P_{23} | - \pi m'_s m'_s m'_t m'_t \rangle \langle \psi_d^{M_d} \rangle \\
+ \langle pm s m s m t m t | t(z) | P_{23}^{-1} \rangle \langle \pi m'_s m'_s m'_t m'_t | \psi_d^{M_d} \rangle \right\} \\
= - \sum_{m'_t m'_t} \langle pm s m s m t m t | t(z) | (1 - P_{23}) | - \pi m'_s m'_s m'_t m'_t \rangle \langle \psi_d^{M_d} \rangle.
\]

(11)

In the last equality we have used the antisymmetry of the deuteron state \( |\psi_d^{M_d}\rangle \) and also we have considered:

\[
\pi = -\frac{1}{2} q + q_0, \quad \pi' = q + \frac{1}{2} q_0.
\]

(12)
The antisymmetrized two-body $t$-matrix is given by [4]:

$$a\langle p' m' s_1 m' s_2 m' t_1 m' t_2 | t | p m s_1 m s_2 m t_1 m t_2 \rangle_a = \langle p' m' s_1 m' s_2 m' t_1 m' t_2 | t(1 - P_{12}) | p m s_1 m s_2 m t_1 m t_2 \rangle,$$

(13)

where $| p m s_1 m s_2 m t_1 m t_2 \rangle_a$ is the antisymmetrized two-body state which is defined as:

$$| p m s_1 m s_2 m t_1 m t_2 \rangle_a = \frac{1}{\sqrt{2}} (1 - P_{12}) | p m s_1 m s_2 m t_1 m t_2 \rangle.$$

(14)

Therefore eq. (11) can be written as:

$$\langle pq | t P | q_0 m_0^0 m_t^0 \Psi_d^M \rangle = - \sum_{m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2}} \langle p m s_1 m s_2 m t_1 m t_2 | t(z) | \pi m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2} \rangle_a \langle \pi' m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2} | \Psi_d^M \rangle.$$

(15)

Now by inserting the completeness relation twice into the second term of eq. (15) and using eqs. (8) and (9) we have obtained:

$$\langle pq | t G_{0P} T | q_0 m_0^0 m_t^0 \Psi_d^M \rangle$$

$$= \sum_{\gamma'} \int dp'' \int dq'' \langle pq | t G_0 | p'' q'' \gamma'' \rangle \sum_{\gamma} \int dp' \int dq' \langle p'' q'' \gamma'' | P | p' q' \gamma' \rangle$$

$$\times \langle p' q' \gamma' | T | q_0 m_0^0 m_t^0 \Psi_d^M \rangle$$

$$= \sum_{m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2}} \int dq' \frac{1}{E - \frac{\pi^2}{m} - \frac{3\pi^2}{4m} + i\epsilon} \langle p m s_1 m s_2 m t_1 m t_2 | t(z) | \pi m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2} \rangle_a$$

$$\times \langle -\pi' q' m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2} | T | q_0 m_0^0 m_t^0 \Psi_d^M \rangle$$

$$+ \sum_{m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2}} \int dq' \frac{1}{E - \frac{\pi^2}{m} - \frac{3\pi^2}{4m} + i\epsilon} \langle p m s_1 m s_2 m t_1 m t_2 | t(z) | -\pi m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2} \rangle_a$$

$$\times \langle \pi' q' m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2} | T | q_0 m_0^0 m_t^0 \Psi_d^M \rangle$$

$$= \sum_{m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2}} \int dq' \frac{1}{E - \frac{\pi^2}{m} - \frac{3\pi^2}{4m} + i\epsilon} \left\{ \langle p m s_1 m s_2 m t_1 m t_2 | t(z) | \pi m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2} \rangle_a \right.$$
where we have considered:

\[ \tilde{\pi} = \frac{1}{2} q + q', \quad \tilde{\pi}' = q + \frac{1}{2} q'. \]  \hspace{1cm} (17)

Final expression for the Faddeev equation is explicitly written as:

\[
\begin{align*}
\langle pqm_s m_s t m_t | T | \Psi_{Md} \rangle \\
&= - \sum_{m', m'} \left\{ \langle p m_s m_s t m_t | t(z) | \pi m'_s m'_s m'_t \rangle a \langle \pi' m'_s m'_s m'_t | \Psi_{Md} \rangle \\
&\quad + \sum_{m'_1, m'_1} \int dq' \langle p m_s m_s t m_t | t(z) | \pi m'_s m'_s m'_t \rangle a \\
&\quad \times \frac{\langle \tilde{\pi}' q | m'_s m'_s m'_t | T | \Psi_{Md} \rangle}{E - \frac{1}{m}(q^2 + q \cdot q' + q'^2) + i\varepsilon} \right\}. \hspace{1cm} (18)
\end{align*}
\]

Since the transition operator \( T \) is needed for all of \( q \) values and We know that the two-body interaction supports a bound state, which is characterized by a pole in the two-body \( t \)-matrix at the two-body binding energy \( E_d \), thus we need to consider this pole, which is located at \( z = E_d \). The residue at the pole can be explicitly extracted by defining:

\[ \hat{t}^a = (z - E_d) t^a, \]  \hspace{1cm} (19)

Since the pole of \( t^a \) will be present in \( T \), we define:

\[ \hat{T} = (z - E_d) T, \]  \hspace{1cm} (20)

Therefore eq. (18) can be written as:

\[
\begin{align*}
\langle pqm_s m_s t m_t | \hat{T} | \Psi_{Md} \rangle \\
&= - \sum_{m', m'} \left\{ \langle p m_s m_s t m_t | \hat{t}(z) | \pi m'_s m'_s m'_t \rangle a \langle \pi' m'_s m'_s m'_t | \Psi_{Md} \rangle \\
&\quad + \sum_{m'_1, m'_1} \int dq' \langle p m_s m_s t m_t | \hat{t}(z) | \pi m'_s m'_s m'_t \rangle a \\
&\quad \times \frac{\langle \tilde{\pi}' q | m'_s m'_s m'_t | \hat{T} | \Psi_{Md} \rangle}{E - \frac{3}{4m}q^2 - E_d + i\varepsilon} \right\}. \hspace{1cm} (21)
\end{align*}
\]
As a simplification we rewrite eq. (21) as:

\[
\hat{T}^{m_0^0 m_1^0, M_d}_{m_{s_1} m_{s_2} m_{s_3} m_{t_1} m_{t_2} m_{t_3}}(p, q; q_0) = -\sum_{m_{t_1}' m_{t_2}' m_{t_3}'} \left\{ \hat{i}_{a_{m_{s_1} m_{s_2} m_{s_3} m_{t_1} m_{t_2} m_{t_3}}(p, \pi; z) \psi^{M_d}_{m_{s_1} m_{t_1} m_{t_2} m_{t_3}'}(\pi') \right. \\
+ \sum_{m_{t_1}' m_{t_2}' m_{t_3}'} \int dq'' \frac{\hat{i}_{a_{m_{s_1} m_{s_2} m_{s_3} m_{t_1} m_{t_2} m_{t_3}}(p, \pi, z) \hat{T}^{m_0^0 m_1^0, M_d}_{m_{s_1} m_{t_1} m_{t_2} m_{t_3}'}(\pi', q'', q_0)}}{E - \frac{1}{m}(q^2 + q \cdot q'' + q''^2) + i\varepsilon} \left. \hat{T}^{m_0^0 m_1^0, M_d}_{m_{s_1} m_{t_1} m_{t_2} m_{t_3}'}(\pi', q'', q_0) \right\} 
\]

(22)

For solving this integral equation one needs the matrix elements of the deuteron wave function and the antisymmetrized two-body \(t\)-matrix. We have connected these to their helicity representations in appendices A and B respectively.

III. DERIVATION OF THE 3D EXPRESSIONS FOR THE 3N ELASTIC AND BREAKUP SCATTERING AMPLITUDES

A. Elastic scattering

In the Faddeev scheme the operator \(U\) for the Nd elastic scattering is defined by:

\[
U = PG_0^{-1} + PT, 
\]

(23)

Differential cross section for Nd elastic scattering in the center of mass frame is given by:

\[
\frac{d\sigma}{d\Omega} = (2\pi)^4 \frac{2m_1^2}{3} \frac{1}{6} \sum_{m_0^0 m_1^0 M_d} |U^{M_d', M_d}_{m_0^0 m_1^0 M_d'}(q, q_0)|^2, 
\]

(24)

where \(U^{M_d', M_d}_{m_0^0 m_1^0 m_0^0 m_1^0}(q, q_0)\) is the elastic scattering amplitude which is defined as:

\[
U^{M_d', M_d}_{m_0^0 m_1^0 M_d'}(q, q_0) = \langle q_0 m_s m_t \psi^{M_d'}_{d}|U|q m_s m_t \psi^{M_d}_d \rangle \\
= \langle q_0 m_s m_t \psi^{M_d'}_{d}|(PG_0^{-1} + PT)|q_0 m_s m_t \psi^{M_d}_d \rangle \\
= \langle q_0 m_s m_t \psi^{M_d'}_{d}|PG_0^{-1}|q_0 m_s m_t \psi^{M_d'}_{d} \rangle + \langle q_0 m_s m_t \psi^{M_d'}_{d}|PT|q_0 m_s m_t \psi^{M_d}_d \rangle, 
\]

(25)

where:

\[
|q_0 m_s m_t \psi^{M_d'}_{d} \rangle \equiv |q_0 m_s m_t \psi^{M_d'}_{d} \rangle, \quad q m_s m_t \psi^{M_d'}_{d} \rangle \equiv \langle q m_s m_t |\psi^{M_d'}_{d} \rangle, 
\]

(26)
are the initial and the final states respectively. For derivation of an expression for the elastic scattering amplitude we start by inserting the completeness relation twice into the first term of eq. (25) as:

\[
\langle q m_s m_t \Psi_d^{M_d'} | P \Psi_d^{M_d'} \rangle
= \sum_{\gamma''} \int dp'' \int dq'' \langle q m_s m_t \Psi_d^{M_d'} | p'' \Psi_d^\gamma' \rangle \sum_{\gamma'} \int dp' \int dq' (E - \frac{p'^2}{m} - \frac{3q'^2}{4m}) \times \langle p'' \Psi_d^\gamma' | P | p' \Psi_d^{\gamma'} \rangle \langle p' \Psi_d^{\gamma'} | q_0 m_s m_t \Psi_d^{M_d'} \rangle. \tag{27}
\]

with considering eq. (8) and following relations:

\[
\langle q m_s m_t \Psi_d^{M_d'} | p'' \Psi_d^\gamma'' \rangle = \langle \Psi_d^{M_d'} | p'' m_s m_t m_t' m_t'' | \Psi_d^{M_d'} \rangle \delta(q - q'') \delta_{m_s m_t} \delta_{m_t m_t''}, \tag{28}
\]

\[
\langle p' \Psi_d^{\gamma'} | q_0 m_s m_t \Psi_d^{M_d'} \rangle = \langle p' m_s m_t m_t' m_t'' | \Psi_d^{M_d'} \rangle \delta(q' - q_0) \delta_{m_t' m_t''} \delta_{m_t m_t'}, \tag{29}
\]

Equation (27) can be written as:

\[
\langle q m_s m_t \Psi_d^{M_d'} | P \Psi_0 | q_0 m_s m_t \Psi_d^{M_d'} \rangle
= (E - \frac{\pi^2}{m} - \frac{3q_0^2}{4m}) \sum_{m_s' m_t'} \langle \Psi_d^{M_d'} | \pi m_s m_t m_t' m_t'' \rangle \langle -\pi' m_s m_t m_t' m_t'' | \Psi_d^{M_d'} \rangle
+ \sum_{m_s' m_t'} \langle \Psi_d^{M_d'} | -\pi' m_s m_t m_t' m_t'' \rangle \langle \pi' m_s m_t m_t' m_t'' | \Psi_d^{M_d'} \rangle
= (E - \frac{\pi^2}{m} - \frac{3q_0^2}{4m}) \sum_{m_s' m_t'} \left\{ \langle \Psi_d^{M_d'} | \pi m_s m_t m_t' m_t'' \rangle \langle \pi' m_s m_t m_t' m_t'' | P_{23}^{-1} \Psi_d^{M_d'} \rangle \right. \\
+ \langle \Psi_d^{M_d'} | P_{23} \pi m_s m_t m_t' m_t'' \rangle \langle \pi' m_s m_t m_t' m_t'' | \Psi_d^{M_d'} \rangle \left\}
= -2(E - \frac{\pi^2}{m} - \frac{3q_0^2}{4m}) \sum_{m_s' m_t'} \langle \Psi_d^{M_d'} | \pi m_s m_t m_t' m_t'' \rangle \langle \pi' m_s m_t m_t' m_t'' | \Psi_d^{M_d'} \rangle
= -2(E - \frac{1}{m}(q^2 + q \cdot q_0 + q_0^2)) \sum_{m_s' m_t'} \langle \Psi_d^{M_d'} | \pi m_s m_t m_t' m_t'' \rangle \langle \pi' m_s m_t m_t' m_t'' | \Psi_d^{M_d'} \rangle. \tag{30}
\]
Now by inserting the completeness relation twice into the second term of eq. (25) and using eqs. (8) and (28) we have obtained:

\[
\langle q m_s m_t \Psi_d^M | P T | q_0 m_0 m_0 \Psi_d^M \rangle \\
= \sum_{\gamma'\gamma''} \int dp' \int dq' \langle q m_s m_t \Psi_d^M | p' q' \gamma' \rangle \sum_{\gamma} \int dp'' \int dq'' \langle p'' q'' \gamma'' | P | p' q' \gamma' \rangle \\
\times \langle p' q' \gamma' | T | q_0 m_0 m_0 \Psi_d^M \rangle \\
= \sum_{m'_1 m'_2} \int dq' \left\{ \sum_{m'_3} (\Psi_d^{M'_d} | \bar{\pi} m_{s_1} m_{s_2} m_{t_1} m_{t_2} | - \pi q m_{s_1} m_{s_2} m_{t_1} m_{t_2}) | T | q_0 m_0 m_0 \Psi_d^M \rangle \right\} \\
+ \sum_{m'_3 m'_2} \langle \Psi_d^{M'_d} | - \bar{\pi} m_{s_3} m_{t_3} m_{t_3}' | (\pi' q m_{s_1} m_{s_3} m_{t_1} m_{t_3}' | T | q_0 m_0 m_0 \Psi_d^M \rangle \right\} \\
= \sum_{m'_1 m'_2 m'_3} \int dq' \left\{ (\Psi_d^{M'_d} | \bar{\pi} m_{s_1} m_{s_2} m_{t_1} m_{t_2} \langle \pi' q m_{s_1} m_{s_3} m_{t_1} m_{t_3}' | T | q_0 m_0 m_0 \Psi_d^M \rangle \right\} \\
+ \langle \Psi_d^{M'_d} | P_{23} | \bar{\pi} m_{s_1} m_{s_2} m_{t_1} m_{t_2} | q_0 m_0 m_0 \Psi_d^M \rangle \right\} \\
= -2 \sum_{m'_1 m'_2 m'_3} \int dq' \langle \Psi_d^{M'_d} | \bar{\pi} m_{s_1} m_{s_2} m_{t_1} m_{t_2} \rangle (\pi' q m_{s_1} m_{s_3} m_{t_1} m_{t_3}' | T | q_0 m_0 m_0 \Psi_d^M \rangle \right\} \\
= -2 \sum_{m'_1 m'_2 m'_3} \int dq' \langle \Psi_d^{M'_d} | \bar{\pi} m_{s_1} m_{s_2} m_{t_1} m_{t_2} \rangle \frac{\langle \pi' q m_{s_1} m_{s_3} m_{t_1} m_{t_3}' | T | q_0 m_0 m_0 \Psi_d^M \rangle}{E - \frac{3}{4 m} q^2 - E_d + i \varepsilon}. \quad (31) 
\]

Final expression for the matrix elements of elastic scattering amplitude has been obtained as:

\[
\langle q m_s m_t \Psi_d^M | U | q_0 m_0 m_0 \Psi_d^M \rangle \\
= -2 \sum_{m'_1 m'_2} \left\{ (E - \frac{1}{m} (q^2 + q \cdot q_0 + q_0^2)) \langle \Psi_d^{M'_d} | \pi m_0 m_0 m_0 \Psi_d^M \rangle \right\} \\
+ \sum_{m'_1 m'_2} \int dq' \langle \Psi_d^{M'_d} | \bar{\pi} m_{s_1} m_{s_2} m_{t_1} m_{t_2} \rangle \frac{\langle \pi' q m_{s_1} m_{s_3} m_{t_1} m_{t_3}' | T | q_0 m_0 m_0 \Psi_d^M \rangle}{E - \frac{3}{4 m} q^2 - E_d + i \varepsilon}. \quad (32) 
\]

As a simplification we rewrite eq. (32) as:

\[
U_{m_{s_1} m_{s_2} m_{t_1} m_{t_2}}^{M'_d M_d}(q, q_0) \\
= -2 \sum_{m'_1 m'_2} \left\{ (E - \frac{1}{m} (q^2 + q \cdot q_0 + q_0^2)) \langle \Psi_d^{M'_d} | \pi m_0 m_0 m_0 \Psi_d^M \rangle \right\} \\
\times \langle \Psi_d^{M'_d} | \bar{\pi} m_{s_1} m_{s_2} m_{t_1} m_{t_2} \rangle \frac{\langle \pi' q m_{s_1} m_{s_3} m_{t_1} m_{t_3}' | T | q_0 m_0 m_0 \Psi_d^M \rangle}{E - \frac{3}{4 m} q^2 - E_d + i \varepsilon}. \quad (33) 
\]
B. Full breakup process

In this stage we have derived an expression for the matrix elements of the breakup process. In the Faddeev scheme the operator $U_0$ for the Nd breakup process is given by:

$$U_0 = (1 + P)T.$$  \hfill (34)

Differential cross section for the Nd breakup process is defined by [7]:

$$\frac{d\sigma}{d\Omega_d dq} = (2\pi)^4 \frac{m^2}{3q_0} p q^2 \frac{1}{6} \sum_{M_d m_{t_t}^0 \gamma} d\hat{p}|U_0^{M_d m_{t_t}^0 \gamma}(p, q; q_0)|^2,$$  \hfill (35)

where the matrix elements of the full breakup amplitude $U_0^{M_d m_{t_t}^0 \gamma}(p, q; q_0)$ is defined as:

$$U_0^{M_d m_{t_t}^0 \gamma}(p, q; q_0) = \langle pq\gamma|U_0|q_0 m_s^0 m_t^0 \Psi_d^M\rangle$$

$$\equiv \langle pq\gamma|(1 + P)T|q_0 m_s^0 m_t^0 \Psi_d^M\rangle + \langle pq\gamma|T|q_0 m_s^0 m_t^0 \Psi_d^M\rangle$$

$$+ \langle pq\gamma|(P_{12} P_{23})T|q_0 m_s^0 m_t^0 \Psi_d^M\rangle + \langle pq\gamma|(P_{13} P_{23})T|q_0 m_s^0 m_t^0 \Psi_d^M\rangle,$$  \hfill (36)

where $|q_0 m_s^0 m_t^0 \Psi_d^M\rangle$ and $\langle pq\gamma|$ are initial and final states respectively. By applying the permutation operator $P_{12} P_{23}$ and $P_{13} P_{23}$ to the final state, eq. \hfill (36) can be written as [7]:

$$U_0^{M_d m_{t_t}^0 \gamma}(p, q; q_0)$$

$$\equiv \langle pq s_1 s_2 m_{s_2} m_{t_1} m_{t_2} m_{t_3}|T|q_0 m_s^0 m_t^0 \Psi_d^M\rangle$$

$$+ \langle(-1)^2 p - \frac{3}{4} q)(p - 1/2 q)m_s m_{s_2} m_{s_1} m_{t_1} m_{t_2} m_{t_3}|T|q_0 m_s^0 m_t^0 \Psi_d^M\rangle$$

$$+ \langle(-1/2 p + \frac{3}{4} q)(-p - 1/2 q)m_s s_1 s_2 m_{t_2} m_{t_3} m_{t_4}|T|q_0 m_s^0 m_t^0 \Psi_d^M\rangle$$

$$= \langle pq s_1 s_2 m_{s_2} m_{t_1} m_{t_2} m_{t_3}|\hat{T}|q_0 m_s^0 m_t^0 \Psi_d^M\rangle$$

$$\frac{E - \frac{3}{4} m q^2 - E_d}{E - \frac{3}{4} m (p - \frac{3}{4} q)^2 - E_d}$$

$$+ \langle(-\frac{1}{2} p - \frac{3}{4} q)(p - \frac{1}{2} q)m_s m_{s_2} m_{s_3} m_{t_2} m_{t_3} m_{t_4}|\hat{T}|q_0 m_s^0 m_t^0 \Psi_d^M\rangle$$

$$\frac{E - \frac{3}{4} m (p - \frac{3}{4} q)^2 - E_d}{E - \frac{3}{4} m (p - \frac{3}{4} q)^2 - E_d}$$

$$+ \langle(-\frac{1}{2} p + \frac{3}{4} q)(-p - \frac{1}{2} q)m_s s_1 m_{s_2} m_{s_3} m_{t_2} m_{t_3}|\hat{T}|q_0 m_s^0 m_t^0 \Psi_d^M\rangle$$

$$\frac{E - \frac{3}{4} m (p - \frac{3}{4} q)^2 - E_d}{E - \frac{3}{4} m (p - \frac{3}{4} q)^2 - E_d}.$$  \hfill (37)
As a simplification eq. (37) has been rewritten as:

\[
U_0^{M_d m_s \gamma} (p, q; q_0) = \frac{\hat{T}^{m_0^0 m_1^0, M_d}_{m_2 m_3 m_4 m_5 m_6 m_7} (p, q; q_0) - \frac{1}{2} p + \frac{3}{4} q, -p - \frac{1}{2} q; q_0)}{E - \frac{3}{4m} q^2 - E_d}
\]

IV. SUMMARY AND OUTLOOK

We extended the recently developed formalism for a new treatment of the 3N bound state in three dimensions for the Nd scattering [13]. We propose a new representation of 3D Faddeev equation for the 3N scattering including the spin and isospin degrees of freedom in the momentum space. This formalism stays closely to the bosonic structure where the spin and isospin degrees of freedom are ignored. This is an important step forward since our 3D formalism avoids the very involved angular momentum algebra occurring for the permutations and transformations and it is more efficient and less cumbersome for considering the 3N forces. This formalism enables us to realistically handle more complexity in 3N scattering calculations. This work provides the necessary formalism for the calculation of 3N scattering observables which is under preparation [20].

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Appendix A: Connection of the deuteron wave function to its helicity representation

In our formulation, we need the matrix elements of the deuteron wave function \( \Psi^{M_d}_{m_1 m_2 m_3} (p) \). We have connected these matrix elements to those in the momentum-helicity basis. The momentum-helicity basis state which is parity eigenstate and antisym-
is worked out as: the momentum-helicity basis state. The normalization of the momentum-helicity basis state and the completeness relation of this state is defined by:

$$|\mathbf{p}; \hat{\mathbf{p}} S\lambda; t\rangle^\pi_a = \frac{1}{\sqrt{2}}(1 - P_{12})|\mathbf{p}; \hat{\mathbf{p}} S\lambda\rangle |t\rangle$$

$$= \frac{1}{\sqrt{2}}(1 - \eta_\pi(-)^{S+t})|\mathbf{p}; \hat{\mathbf{p}} S\lambda\rangle |t\rangle,$$

(A1)

Here $S$ is the total spin and $\lambda$ is the spin projection along relative momentum of two nucleons. $|t\rangle \equiv |tm_t\rangle$ is the total isospin state of the two nucleons, where $t$ is the total isospin and $m_t$ is the isospin projection along its quantization axis, which tells also the total electric charge of system. For simplicity $m_t$ is suppressed since electric charge is conserved. $P_{12}$ is the permutation operator which exchanges the two nucleons labels in all spaces i.e. momentum, spin and isospin spaces. $|\mathbf{p}; \hat{\mathbf{p}} S\lambda\rangle^\pi_a$ is parity eigenstate which is given by:

$$|\mathbf{p}; \hat{\mathbf{p}} S\lambda\rangle^\pi_a = \frac{1}{\sqrt{2}}(1 + \eta_\pi P_\pi)|\mathbf{p}; \hat{\mathbf{p}} S\lambda\rangle,$$

(A2)

where $P_\pi$ is parity operator, $\eta_\pi = \pm 1$ are the parity eigenvalues and $|\mathbf{p}; \hat{\mathbf{p}} S\lambda\rangle$ is the momentum-helicity basis state. The normalization of the momentum-helicity basis state is worked out as:

$$\sum_{S\lambda t} \int d\mathbf{p} |\mathbf{p}; \hat{\mathbf{p}} S\lambda; t\rangle^\pi_a \frac{1}{4} \pi_a |\mathbf{p}; \hat{\mathbf{p}} S\lambda; t\rangle = 1.$$  

(A4)

Inserting the completeness relation in the momentum helicity basis yields:

$$\Psi_{m_s m_s 2 m_t m_t 2}^{M_d}(\mathbf{p}) \equiv \langle \mathbf{p} m_s m_s 2 m_t m_t 2 |\Psi_{d}^{M_d}\rangle$$

$$= \frac{1}{4} \sum_{\lambda=-1}^{1} \int d\mathbf{p}' \langle \mathbf{p} m_s m_s 2 m_t m_t 2 |\mathbf{p}'; \hat{\mathbf{p}}'1\lambda; 0 \rangle^{1a} 1^{a} \langle \mathbf{p}'; \hat{\mathbf{p}}'1\lambda; 0 |\Psi_{d}^{M_d}\rangle.$$  

(A5)

where we have used the deuteron properties, i.e. $S = 1$, $t = 0$ and even parity. $\Phi_{\lambda}^{M_d}(\mathbf{p}') \equiv ^{1a} \langle \mathbf{p}'; \hat{\mathbf{p}}'1\lambda; 0 |\Psi_{d}^{M_d}\rangle$ is the deuteron wave function component in the momentum-helicity basis. The overlap of the momentum-helicity basis state with the state $|\mathbf{p} m_s m_s 2 m_t m_t 2\rangle$ is given by [4]:

$$\langle \mathbf{p} m_s m_s 2 m_t m_t 2 |\mathbf{p}'; \hat{\mathbf{p}}' S\lambda; t\rangle^\pi_a$$

$$= \frac{1}{2}(1 - \eta_\pi(-)^{S+t}) C(\frac{1}{2}; m_t m_t 2) C(\frac{1}{2}; m_s m_s 2 \lambda_0)$$

$$\times e^{-i\lambda_0 \phi} d^{S}_{\lambda_0 \lambda}(\theta) [\delta(\mathbf{p} - \mathbf{p}') + \eta_\pi \delta(\mathbf{p} + \mathbf{p}')]$$  

(A6)
where $C$ is the Clebsh-Gordan coefficient and $d^S_{\lambda_0\lambda}(\theta)$ are rotation matrices \cite{21}. Substituting eq. (A6) into eq. (A5) yields:

$$
\Psi_{M_d m_s m_t m'_s m'_t}(p) = \frac{1}{4} C(\frac{1}{2}; m_t m'_t) C(\frac{1}{2}; m_s m'_s m'_t 0) \sum_{\lambda = -1}^{1} \left\{ e^{-i\lambda_0\varphi} d^1_{\lambda_0\lambda}(\theta) \Phi^M_d(p) + e^{-i(\lambda_0 + \varphi)} d^1_{\lambda_0\lambda}(\pi - \theta) \Phi^M_d(-p) \right\}.
$$

(A7)

By considering these relations:

$$
d^S_{\lambda_0\lambda}(\pi - \theta) = (-)^{S+\lambda_0} d^S_{\lambda_0 - \lambda}(\theta), \quad \Phi^M_d(p) = -\Phi^M_d(-p),
$$

(A8)

Equation (A7) can be written as:

$$
\Psi^M_d_{m_s m_t m'_s m'_t}(p) = \frac{1}{2} C(\frac{1}{2}; m_t m'_t) C(\frac{1}{2}; m_s m'_s m'_t 0) e^{-i\lambda_0\varphi} \sum_{\lambda = -1}^{1} d^1_{\lambda_0\lambda}(\theta) \Phi^M_d(p).
$$

(A9)

The azimuthal dependency of $\Phi^M_d(p)$ is:

$$
\Phi^M_d(p) \equiv e^{iM_d\varphi} \Phi^M_d(p, \theta)
$$

(A10)

Finally the connection between the $\Psi^M_d_{m_s m_t m'_s m'_t}(p)$ and those in the momentum-helicity basis state is given by:

$$
\Psi^M_d_{m_s m_t m'_s m'_t}(p) = \frac{1}{2} C(\frac{1}{2}; m_t m'_t) C(\frac{1}{2}; m_s m'_s m'_t 0) e^{-i(\lambda_0 - M_d)\varphi} \sum_{\lambda = -1}^{1} d^1_{\lambda_0\lambda}(\theta) \Phi^M_d(p, \theta).
$$

(A11)

It should be mentioned that the deuteron wave function component $\Phi^M_d(p, \theta)$ obeys a set of coupled equations which are solved numerically in ref. \cite{22}.

**Appendix B: Anti-symmetrized NN t-matrix and connection to its helicity representation**

The connection of Anti-symmetrized two-body $t$-matrix to those in the momentum-helicity basis is given in ref. \cite{7}, here we prepare this connection according to the notation to be used in our work and then we have derived an expansion in momentum-helicity basis for
two-body $t$-matrix for $z \rightarrow E_d$. Based on momentum-helicity basis states the NN $t$-matrix element is defined as:

$$t_{\lambda\lambda'}^{\pi \pi}(p, p'; z) \equiv \pi \alpha(p, \hat{p} S \lambda; t|z|p'; \hat{p}' S \lambda'; t) \pi \alpha.$$  \hspace{1cm} (B1)

As shown in ref. [7], the selection of $p'$ parallel to the $z$-axis allows, together with the properties of the potential, that the angular dependencies of the NN $t$-matrix elements can be simplified as:

$$t_{\lambda\lambda'}^{\pi \pi}(p, p'; z) = e^{-i\Omega_{pp'}} t_{\lambda\lambda'}^{\pi \pi}(p \hat{n}_{pp'}, p' \hat{z}; z) = e^{-i\Omega_{pp'}} e^{i\phi_{pp'}} t_{\lambda\lambda'}^{\pi \pi}(p, p', \cos \theta_{pp'}; z) \equiv e^{i(\lambda' \phi_{pp'} - \lambda \Omega_{pp'})} t_{\lambda\lambda'}^{\pi \pi}(p, p', \cos \theta_{pp'}; z), \hspace{1cm} (B2)$$

The direction $\hat{n}_{pp'}$ can be determined by the spherical and the polar angles $\theta_{pp'}$ and $\varphi_{pp'}$, where

$$\cos \theta_{pp'} = \cos \theta_p \cos \theta_{p'} + \sin \theta_p \sin \theta_{p'} \cos(\phi_p - \phi_{p'}),$$

$$\sin \theta_{pp'} e^{i\phi_{pp'}} = -\cos \theta_p \sin \theta_{p'} + \sin \theta_p \cos \theta_{p'} \cos(\phi_p - \phi_{p'}) + i \sin \theta_p \sin(\phi_p - \phi_{p'}), \hspace{1cm} (B3)$$

and the exponential factor $e^{i(\lambda' \phi_{pp'} - \lambda \Omega)}$ is calculated as:

$$e^{i\Omega_{pp'}} = \frac{\sum_{N=-S}^{S} D_{N \lambda}^{S}(\phi_p \theta_p 0) D_{N \lambda'}^{\ast S}(\phi_{p'} \theta_{p'} 0)}{D_{N \lambda'}^{\ast S}(\phi_{pp'} \theta_{pp'} 0)},$$

$$e^{i(\lambda' \phi_{pp'} - \lambda \Omega_{pp'})} = \frac{\sum_{N=-S}^{S} e^{iN(\phi_p - \phi_{p'})} d_{N \lambda}^{S}(\theta_p) d_{N \lambda'}^{\ast S}(\theta_{p'})}{d_{N \lambda'}^{\ast S}(\theta_{pp'})}. \hspace{1cm} (B4)$$

In the above expressions, $D_{N \lambda}^{S}(\phi, \theta 0)$ are the Wigner D-functions. Finally the connection between the $t$-matrix elements $\langle p m_{s_1} m_{s_2} m_{t_1} m_{t_2} | \hat{t}(z) | p' m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2} \rangle_a$ and those in the momentum-helicity basis, namely $t_{\lambda\lambda'}^{\pi \pi}(p, p'; z)$, is given as:

$$\langle p m_{s} m_{s} m_{t_1} m_{t_2} | \hat{t}(z) | p' m'_{s_1} m'_{s_2} m'_{t_1} m'_{t_2} \rangle_a = \frac{1}{4} [\delta_{m_{t_1} + m_{t_2} m'_{t_1} + m'_{t_2}}] (z - E_d)$$

$$\times e^{-i(\lambda_0 \phi_p - \lambda' \phi_{p'})} \sum_{S \pi t} (1 - \eta_{\pi}(-)^{S+t})$$

$$\times C(\frac{1}{2}, \frac{1}{2}; S; m_{s_1} m_{s_2}) C(\frac{1}{2}, \frac{1}{2}; t; m'_{t_1} m'_{t_2})$$

$$\times C(\frac{1}{2}, \frac{1}{2}; t; m_{t_1} m_{t_2}) C(\frac{1}{2}, \frac{1}{2}; S; m'_{s_1} m'_{s_2})$$

$$\times \sum_{\lambda \lambda'} d_{\lambda_0 \lambda}^{S}(\theta_p) d_{\lambda_0 \lambda'}^{\ast S}(\theta_{p'}) t_{\lambda\lambda'}^{\pi \pi}(p, p'; z). \hspace{1cm} (B5)$$
It should be mentioned that the fully off-shell two-body $t$-matrix $t^{\pi_{St}}_{\lambda\lambda'}(p, p', \cos \theta_{pp'}; z)$ obeys a set of coupled Lippman-Schwinger equations which for $S = 0$ it is a single equation and for $S = 1$ it is a set of two coupled equations which are solved numerically in ref. [?].

In this stage we have derived an expression in the momentum-helicity basis for NN $t$-matrix for $z \rightarrow E_d$. It is clear that:

$$\lim_{z \rightarrow E_d} (z - E_d)t = V|\Psi_d^{M_d}\rangle \langle \Psi_d^{M_d}|V.$$ \hspace{1cm} (B6)

Projecting this equation on momentum-helicity basis states yields:

$$\lim_{z \rightarrow E_d} (z - E_d)t^{110}_{\lambda\lambda'}(p, p')$$

$$= \frac{1}{4} \sum_{\chi''} \int dp'' V^{110}_{\lambda\lambda'}(p, p') \Phi^{M_d}_{\chi''}(p'') \{ \frac{1}{4} \sum_{\chi''} \int dp'' V^{110}_{\lambda\chi''}(p''', p') \Phi^{M_d}_{\chi''}(p''') \}.$$ \hspace{1cm} (B7)

By considering vector $q'$ along $z$ axis we have obtained:

$$e^{i\lambda' \varphi} \lim_{z \rightarrow E_d} (z - E_d)t^{110}_{\lambda\lambda'}(p, p', x)$$

$$= \frac{1}{4} \sum_{\chi''} \int dp'' e^{-iM_d\varphi''} V^{110}_{\lambda\lambda''}(p, p') \Phi^{M_d}_{\chi''}(p'') \{ \frac{1}{4} \sum_{\chi''} \int dp'' e^{-iM_d\varphi''} e^{i\lambda' \varphi''} V^{110}_{\lambda'\chi''}(p''', p', x'') \Phi^{M_d}_{\chi''}(p''', x'') \},$$ \hspace{1cm} (B8)

where we have used the azimuthal behavior of the potential and the deuteron wave function as:

$$V^{\pi_{St}}_{\lambda\lambda'}(p'', p', \hat{z}) = e^{i\lambda' \varphi''} V^{\pi_{St}}_{\lambda'\lambda'}(p'', p', x''),$$

$$\Phi^{M_d}_{\chi''}(p'') = e^{iM_d\varphi''} \Phi^{M_d}_{\chi''}(p'', x'').$$ \hspace{1cm} (B9)
Equation (B8) can be rewritten as:

\[
\begin{align*}
\lim_{z \to E_d} (z - E_d)t^{110}_{\lambda\lambda'}(p, p', x) &= \left\{ \frac{1}{4} \sum_{\lambda''} \int dP'' e^{-iM_d(\varphi - \varphi'')} V^{110}_{\lambda\lambda'}(p, P'') \Phi^{M_d}_{\lambda''}(p'', x'') \right\} \\
&\times \left\{ \frac{1}{4} \sum_{\lambda''} \int dP'' e^{-iM_d(\varphi - \varphi)} e^{i\lambda'(\varphi'' - \varphi)} V^{110}_{\lambda''\lambda'}(p', p, x'') \Phi^{M_d}_{\lambda''}(p'', x'') \right\} \\
&= \left\{ \frac{1}{4} \sum_{\lambda''} \int_0^\infty dp'' p'^2 \int_{-1}^1 dx'' \upsilon^{110,M_d}_{\lambda''\lambda'}(p', p, p'', x'') \Phi^{M_d}_{\lambda''}(p'', x'') \right\} \\
&\times \left\{ \frac{1}{4} \sum_{\lambda''} \int_0^\infty dp'' p'^2 \int_{-1}^1 dx'' V^{110}_{\lambda''\lambda'}(p'', p', x'') \int_0^{2\pi} d\varphi'' e^{i(\lambda' - M_d)(\varphi'' - \varphi')} \right\},
\end{align*}
\]

where we have introduced:

\[
\upsilon^{110,M_d}_{\lambda''\lambda'}(p, x, p'', x'') = \int_0^{2\pi} d\varphi'' e^{-iM_d(\varphi - \varphi'')} V^{110}_{\lambda\lambda'}(p, P''),
\]

Finally we have obtained:

\[
\begin{align*}
\lim_{z \to E_d} (z - E_d)t^{110}_{\lambda\lambda'}(p, p', x) &= \left\{ \frac{1}{4} \sum_{\lambda''} \int_0^\infty dp'' p'^2 \int_{-1}^1 dx'' \upsilon^{110,M_d}_{\lambda''\lambda'}(p, x, p'', x'') \Phi^{M_d}_{\lambda''}(p'', x'') \right\} \\
&\times \left\{ \frac{\pi}{2} \delta_{\lambda',M_d} \sum_{\lambda''} \int_0^\infty dp'' p'^2 \int_{-1}^1 dx'' V^{110}_{\lambda''\lambda'}(p'', p', x'') \Phi^{M_d}_{\lambda''}(p'', x'') \right\},
\end{align*}
\]

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