Data Valuation for Vertical Federated Learning: An Information-Theoretic Approach

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Abstract

Federated learning (FL) is a promising machine learning paradigm that enables cross-party data collaboration for real-world AI applications in a privacy-preserving and law-regulated way. How to valuate parties’ data is a critical but challenging FL issue. In the literature, data valuation either relies on running specific models for a given task or is just task irrelevant; however, it is often requisite for party selection given a specific task when FL models have not been determined yet. This work thus fills the gap and proposes FedValue, to our best knowledge, the first privacy-preserving, task-specific but model-free data valuation method for vertical FL tasks. Specifically, FedValue incorporates a novel information-theoretic metric termed Shapley-CMI to assess data values of multiple parties from a game-theoretic perspective. Moreover, a novel server-aided federated computation mechanism is designed to compute Shapley-CMI and meanwhile protects each party from data leakage. We also propose several techniques to accelerate Shapley-CMI computation in practice. Extensive experiments on six open datasets validate the effectiveness and efficiency of FedValue for data valuation of vertical FL tasks. In particular, Shapley-CMI as a model-free metric performs comparably with the measures that depend on running an ensemble of well-performing models.

Keywords: federated learning, data valuation, private set intersection

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1. Introduction

Recent years have witnessed an increasing interest in collaborative machine learning with data fused from multiple data holders for Artificial Intelligence (AI) applications [1, 2]. In practice, however, it is yet very hard to integrate data from different parties, owing to industry competitions, security requirements or sophisticated administrative procedures [1, 3]. Federated learning (FL) is an emerging paradigm towards privacy-preserving collaborative machine learning [3, 4]. Basically, FL allows various parties to perform local computations on their private data and only communicate insensitive information, e.g., gradients of self-learned deep neural networks [5], with others. It benefits from all parties’ data to achieve high-performance machine learning without leaking any party’s data assets.

Horizontal FL (HFL) and vertical FL (VFL) are two typical types of FL mechanisms [3]. In HFL tasks, various parties share a same set of features and perform secure collaborative learning with different data samples [6, 7, 8]. Comparatively, parties in VFL tasks own different features for a same set of data samples, and collaboratively learn models by taking advantage of integrated features from other parties [9, 10]. While existing FL studies focus more on HFL to alleviate the training data scarcity problem, VFL is also requisite to many practical data-driven businesses [9]. For example, banks may willingly collaborate with mobile network operators or online commerce platforms, to collect more indicative information (e.g., phone bills and shopping logs) about their customers and improve credit assessment performance [11]. In brief, if any party (namely task party) needs to train a better business model via exploiting more feature information about its data samples, it can launch a VFL task by inviting some parties with valuable data (namely data parties).

This work concentrates on VFL tasks, which are widely needed in practice but still under-investigated [9]. In particular, it attempts to tackle a fundamental issue in VFL: data valuation, which aims at quantifying the potential
contributions of different data parties to a specific VFL task, given the task party’s self-owned data and before the launch of the VFL task. On one hand, data valuation can help task parties to determine which data parties are valuable to collaborate with for a specific task. On the other, it can help to incentivize high-quality data parties to join a VFL task if their contributions can be fairly priced, which is deemed crucial to the success of building legal data marts [3]. In this vein, data valuation can provide the imperative information for establishing contribution-based incentive mechanisms, so that both the task party and data parties in a VFL task can gain fair benefits.

The problem of data valuation in VFL, however, is non-trivial and presents two unique challenges. The first challenge comes from building a task-specific but model-free data valuation method. Some context-independent metrics have been proposed to valuate data by their intrinsic properties, including completeness, precision, uniqueness or timeliness [12], which however cannot estimate how much each of the data parties can improve a specific business task and therefore can hardly be used for precise data pricing. In this sense, a task-specific data valuation metric that can help differentiate the contribution of various parties is in great need for a VFL task. There also exist some context-aware feature importance methods, e.g., SHAP value [13], that can be applied to valuate data parties; but they often depend on running a bundle of specific models, which is typically infeasible in the early stage of an FL collaboration where basic trusts between different parties are still missing. Therefore, a task-specific but model-free data valuation measure is urgently required for VFL.

The second challenge comes from the privacy-preserving requirement for data valuation in VFL. In principle, data valuation for a VFL task should be mandatorily compliant with data protection regulations and should prevent the leakage of raw data from any parties. However, assessing the contributions of data parties in collaborative modeling often requires some knowledge about parties’ interactions, such as data correlations. How to acquire the interactive information among various parties without disclosing their raw data remains a great challenge. The secure multiparty computation (MPC) protocols provide
a promising solution by introducing some third-parties for privacy-preserving data valuation, but some key points remain unaddressed. For example, what information can or must be transmitted to achieve data valuation without privacy leakage, and what if the third-parties are not completely trustworthy, e.g., honest-but-curious or even malicious. Hence, to protect data valuation from being violated by third-party servers’ malicious behaviors in an MPC framework also needs a delicate design.

This paper overcomes the above challenges and makes several contributions:

• To the best of our knowledge, this is the first work that proposes a privacy-preserving, task-specific and model-free data valuation mechanism, named FedValue, for VFL tasks.

• We propose a novel data valuation metric Shapley-CMI for FedValue based on both information theory and game theory. Shapley-CMI can effectively quantify multiple data parties’ values in a specific learning task without relying on any particular model.

• We also design a novel dual-server-aided federated computation mechanism for FedValue, which can accomplish federated computation of Shapley-CMI without leaking any party’s raw data. Besides, we also demonstrate how to use some computational acceleration methods to enhance the efficiency and scalability of FedValue.

We conduct extensive experiments on six real-life datasets and verify the effectiveness of FedValue for data valuation in VFL. The results demonstrate that Shapley-CMI can effectively valuate data parties given a specific task, and the dual-server-aided federated computation mechanism can precisely compute Shapley-CMI in a privacy-preserving manner. Moreover, we evaluate the computational efficiency of FedValue by various parameters, such as the numbers of data samples, features in a party as well as data parties, and show the effects of acceleration techniques for practical FedValue computation. In particular, we show that FedValue can compute Shapley-CMI for a VFL task with five parties and 100,000 samples in 10 minutes by a laptop with ordinary configurations.
2. Problem Formulation

Generally speaking, a federated learning system makes use of distributed data from a few parties to collaboratively learn a model while ensuring that each party keeps its raw data unexposed to other parties \([3]\). In this work, we consider a vertical federated learning context where a task party (who seeks for task-specific models) and a set of data parties (who can provide data for collaborative modelling) are involved.

**Definition 1 (Task Party).** A task party \(t\) holds a group of data samples with a set of features \(X_t\) and a task label \(Y_t\). The set of sample IDs of \(t\) is denoted as \(I_t\).

In practice, a task party can hardly train a satisfactory model with its self-owned limited data features. However, it can get improved by inviting some data parties with incentives to launch a VFL task.

**Definition 2 (Data Party).** A data party \(d\) has no target labels but can provide data samples with features \(X_d\) to predict the task label \(Y_t\). The set of sample IDs of \(d\) is denoted as \(I_d\). The set of all available data parties is denoted as \(\mathcal{D}\).

Hereinafter, we assume the common data sample IDs between the task party and the data parties have already been identified with a data alignment method in any VFL protocol \([4]\), and only these common samples are kept for the VFL task. In other words, we assume all the available data parties share the sample IDs with the task party, and we will concentrate on the essential problem of vertical federated learning.

**Definition 3 (Multi-party VFL).** Given a task party \(t\) with prediction task of \(Y_t\), multiple data parties \(d \in \mathcal{D}\) with sample IDs \(I_d = I_t = I\), and a centralized model \(\hat{y}_t[i] = \mathcal{F}_{CEN}(x_{\{t\} \cup \mathcal{D}[i]}; \Theta_{CEN})\) trained by aggregating all parties’ data with sample ID \(i \in I\), a multi-party VFL task aims to train a model \(\hat{y}_t[i] = \mathcal{F}_{FED}(x_{[i]}; x_{d[i]}, \forall d \in \mathcal{D}; \Theta_{FED})\) by a decentralized collaborative modelling scheme so that 1) the performance of \(\mathcal{F}_{FED}\) is close to that of \(\mathcal{F}_{CEN}\), and 2) each party’s raw data will not be accessed by any other parties.
Data valuation is usually a key phase before launching a multi-party VFL task. From a task party perspective, it can use the valuation to determine whether it is worth inviting other parties and how to price their contributions properly. As to a data party, it regards the pricing to its data as incentives to decide whether to join a VFL task. In this sense, data valuation is considerably important to both task and data parties and their decisions on whether to launch or join a VFL task.

We note that a task-specific data valuation method is requisite, as the contribution of a party’s data highly relates to specific prediction tasks. Besides, we usually cannot foreknow the best model for a specific VFL task in its early phase of data valuation for party determination when the parties have not reached an agreement yet on the collaborative modeling; therefore, the VFL model \( F_{\text{FED}} \) would be arbitrary during the data valuation phase, which requires a data valuation method to be model-free. Finally, a VFL system typically assumes honest-but-curious (or equivalently, semi-honest) parties \([3]\); that is, all the parties will follow the predefined mechanism to conduct computations, but try their best to infer other parties’ private information with their obtained intermediate computation results. Therefore, we should seek for a secure data valuation method that can keep each data party’s raw data unexposed to other parties. With the above considerations, we now formulate our research problem as follows.

**Problem Definition (Data Valuation for Multi-party VFL).** Given a VFL task with a task party \( t \) and multiple data parties \( d \in D \), we aim to find a data valuation method for the data parties, which is task-specific, model-free and secure for decentralized data.

### 3. Method: FedValue

In this section, we propose FedValue, a privacy-preserving, task-specific but model-free framework, for fair data valuation in a VFL task. Two major components of FedValue, i.e., a data valuation metric and a federated computation mechanism for the metric, are carefully designed with this purpose. Concretely, given a specific learning task, we first design a model-free metric to assess the
value of data provided by various data parties without considering the data privacy issue. We then discuss how to compute the metric in a VFL system where the raw data of any party must be preserved locally for privacy concerns. Finally, some techniques are proposed to accelerate FedValue in practice.

3.1. Data Valuation Metric

As stated in Section 2, it is non-trivial to value a data party for a VFL task in a task-specific but model-free manner. The valuation task becomes even more challenging when we consider the involvement of multiple data parties. We therefore begin with the data valuation of only one data party, and then design a general metric that can take account of multiple data parties.

3.1.1. Metric with One Data Party

Suppose we have a task party \( t \) who owns features \( X_t \) and task label \( Y_t \), and a data party \( d \) who owns features \( X_d \). Intuitively, we can take the marginal improvement of \( X_d \) to the overall modelling performance as the data value of \( X_d \). This idea, however, requires to specify the learning model and therefore violates the model-free constraint.

An alternative method is to approximate the modelling performance with the coupling tightness between \( X_d \) and the task label \( Y_t \), denoted as \( C(X_d, Y_t) \), which not only avoids running specific models but also meets the task-specific requirement by using \( Y_t \). Nevertheless, \( C(X_d, Y_t) \) might still over-estimate the contribution of \( X_d \) for it might correlate positively with the task party’s self-owned features \( X_t \). Therefore, we should pursue a more accurate metric formulated like \( C(X_d, Y_t; X_t) \), which is the coupling tightness of \( X_d \) with respect to \( Y_t \) given the already possessed \( X_t \). Note that when there is only one data party \( d \), the marginal improvement of \( X_d \) to the modelling performance is right \( C(X_d, Y_t; X_t) \), and the remainder of our work is to define a proper \( C \) function.

Conditional Mutual Information (CMI), a fundamental metric used for feature selection [15], is an excellent candidate of the \( C \) function. Basically, CMI measures the shared information between two variables given a third variable.
In the VFL case with only one data party, CMI can be properly applied to assess the coupling tightness between the task label $Y_t$ and the data party’s features $X_d$ given the task party’s features $X_t$, and written as:

$$I(X_d; Y_t | X_t) = \sum_{x_d \in X_d} p(x_d) \sum_{x_t \in X_t} \sum_{y_t \in Y_t} p(x_d, y_t | x_t) \log \frac{p(x_d, y_t | x_t)}{p(x_d | x_t) p(y_t | x_t)},$$

where $X_d$, $X_t$, and $Y_t$ denote the sets of possible values of $X_d$, $X_t$, and $Y_t$, respectively.

According to Eq. (1), we need to estimate the distributions of $p(x_d, y_t | x_t)$, $p(x_d | x_t)$ and $p(y_t | x_t)$ for CMI computation. For discrete features, the maximum likelihood method, which divides the occurrence frequency of $x_t$ by the total number of data samples, can be adopted to estimate these distributions. As for continuous features, we can convert them to discrete ones and then apply the maximum likelihood method for estimation. In this vein, the CMI can be computed as:

$$\hat{I}(X_d; Y_t | X_t) = \sum_{x_d \in X_d} \sum_{x_t \in X_t} \sum_{y_t \in Y_t} \hat{p}(x_d, x_t, y_t) \log \frac{\hat{p}(x_d, y_t | x_t)}{\hat{p}(x_d | x_t) \hat{p}(y_t | x_t)},$$

$$= \frac{1}{n} \sum_{x_d \in X_d} \sum_{x_t \in X_t} \sum_{y_t \in Y_t} N(x_d) N(x_t, y_t) \log \frac{N(x_t) N(x_d, y_t)}{N(x_d) N(y_t)},$$

where $\hat{p}$ is the maximum likelihood probability estimation, $N(\cdot)$ is the number of data samples with the given variable values, and $n$ is the total number of data samples. Note that CMI can be estimated by Eq. (3) precisely when the number of data samples gets sufficiently large, referring to the Strong Law of Large Numbers.

Remark: The CMI in Eq. (2) is a task-specific but model-free metric. Compared with model-dependent metrics like SHAP value, the computation of CMI in Eq. (3) could be much more efficient without running models physically, and the loss of valuation precision is tolerable, as will be shown in our experimental part. It is also noteworthy that the computation of CMI is non-trivial in the VFL setting as $X_d$ and $X_t$ ($Y_t$) are typically stored in two parties privately. We will revisit this point in Section 3.2.
3.1.2. Metric for Multiple Data Parties

Although CMI can be used for data valuation when there is only one data party; it cannot be directly applied to the VFL scenarios with multiple data parties. One may suggest separately valuating each of the multiple data parties in a VFL task and taking the sum of a set of data parties’ values as their aggregated value to the task party. Nevertheless, this method neglects the feature correlations among data parties and likely leads to inaccurate data valuation for the parties’ collaborative modeling. To consider the correlations, one may assume data parties enter the VFL system orderly and then sequentially valuate a data party conditioning on the task party and early-joining data parties. Unfortunately, this method tends to depreciate the late-coming data party if it shares some similar features with the early-coming ones. Consider an extreme case where two data parties \( d \) and \( d' \) have exactly the same set of features, i.e., \( X_d = X_{d'} \). In this example, while \( d \) and \( d' \) should have the same data value to the task party, CMI will assign all the credits to the prior estimated data party and zero to the latter. In brief, an ideal data valuation method for VFL with multiple data parties should be able to incorporate the parties’ correlations but be insensitive to their estimation order.

To achieve an order-irrelevant estimation, we refer to the game theory of multiple parties [16] and average the marginal values of a game party by considering all possible game-joining orderings of the parties. Let \( \mathcal{D} \) denote the set of all parties in a game, and \( D \subseteq \mathcal{D} \) denote the set of parties that have joined the game and yielded the game value \( \text{val}(D) \). Then for a new party \( d \), its game value can be estimated by

\[
\varphi_d = \sum_{D \subseteq \mathcal{D} \setminus \{d\}} \frac{|D|!(|\mathcal{D}| - |D| - 1)!}{|\mathcal{D}|!} (\text{val}(D \cup \{d\}) - \text{val}(D)). \tag{4}
\]

Eq. (4) approximates the expected marginal improvement of party \( d \) to a game by taking the average on all possible combinations of \( D \). The combinatorial numbers in the equation are used to characterize the weight of each combination. Specifically, given the data parties in \( D \cup \{d\} \), the possible number of permutations with \( d \) as the last-in party is \( |D|! \), and the possible number of
permutations for $(|D| + 1)$ parties selected from $\mathcal{D}$ is $|\mathcal{D}|!/(|\mathcal{D}| - |D| - 1)!$. Hence, the occurrence probability of $D \cup \{d\}$ with $d$ as the last-in party is $|D|!(|\mathcal{D}| - |D| - 1)!/|\mathcal{D}|!$, which is right the weight of marginal improvement by $d$ in Eq. (4).

We then go back to the VFL scenario. From the perspective of data valuation, we can apply CMI to measure the marginal value of a data party $d$ to the task party given a party joining-order, and compute an average marginal value by enumerating all data party orders to valuate the party $d$. In other words, the CMI metric is adopted as the game value $\text{val}(\cdot)$ in Eq. (4), that is
\[
\text{val}(D) = I(X_D; Y_t | X_t),
\]
where $X_D$ denotes the set of features of all the data parties in $D$. Regarding the chain rule, we can easily have
\[
I(X_d; Y_t | X_D X_t) = I(X_d X_D; Y_t | X_t) - I(X_D; Y_t | X_t).
\]
As a result, by replacing the $\text{val}(D)$ in Eq. (4) by the $\text{val}(D)$ in Eq. (5) and using the chain rule in Eq. (6), we finally have the data value of $d$ in the case of multiple data parties as
\[
\varphi_d = \sum_{D \subseteq \mathcal{D} \setminus \{d\}} \frac{|D|!(|\mathcal{D}| - |D| - 1)!}{|\mathcal{D}|!} I(X_d; Y_t | X_D X_t).
\]
We call $\varphi_d$ in Eq. (7) as Shapley-CMI to recognize the influence of Shapley’s game theory \cite{16} to this metric. It is easy to note that when there is only one data party $d$, we have $D = \emptyset$ and $\mathcal{D} = \{d\}$, and $\varphi_d$ in Eq. (7) thus reduces to $I(X_d; Y_t | X_t)$, which is consistent with the CMI metric for one data party. As a result, Shapley-CMI is a general-purpose metric for data valuation of VFL with an arbitrary number of data parties.

Remark. To our best knowledge, Shapley-CMI is the first task-specific but model-free data valuation metric for multi-party VFL tasks. Inherited from CMI, Shapley-CMI is task-specific by incorporating task labels. It is also a model-free metric that can value data of multiple parities without running specific models. Finally, Shapley-CMI is a sound metric that takes the game behavior of different parties into consideration. These merits make Shapley-CMI
a promising tool for data pricing prior to data transactions, which is particularly important to fostering legitimate data markets, a considered trillion-level business in the digital economy era [17].

In what follows, we highlight some properties possessed by Shapley-CMI, which ensures its interpretability and validity.

**Property 1 (Additivity):** The sum of all the data parties’ Shapley-CMI equals the CMI between the task label and all the data parties’ features given the task party’s features, i.e.,

\[
\sum_{d \in \mathcal{D}} \varphi_d = I(\mathbf{X}_\mathcal{D}; Y_t | \mathbf{X}_t).
\] (8)

**Property 2 (Missingness):** If a party’s features are useless in predicting \( Y_t \), then its Shapley-CMI value equals zero, i.e.,

\[
\varphi_d = 0, \text{if } I(\mathbf{X}_d; Y_t | \mathbf{X}_D \mathbf{X}_t) = 0, \forall D \subseteq \mathcal{D} \setminus \{d\}.\] (9)

**Property 3 (Consistency):** Two parties with the features of the same contribution have the same Shapley-CMI values, i.e.,

\[
\varphi_d = \varphi_{d'}, \text{if } I(\mathbf{X}_d; Y_t | \mathbf{X}_D \mathbf{X}_t) = I(\mathbf{X}_{d'}; Y_t | \mathbf{X}_D \mathbf{X}_t), \forall D \subseteq \mathcal{D} \setminus \{d, d'\}.\] (10)

### 3.2. Dual-server-aided Federated Computation for Shapley-CMI

In general, Shapley-CMI can be directly computed if the data from various parties can be gathered. However, in VFL contexts where all the parties keep their data private, we propose a novel dual-server-aided private set intersection (PSI) mechanism to attain privacy-preserving Shapley-CMI computation. Subsequently, we first analyze how federated Shapley-CMI computation is related to PSI. We then elaborate on how our novel dual-server-aided PSI mechanism can boost the computation of Shapley-CMI in VFL.

#### 3.2.1. Analysis of Federated Shapley-CMI Computation

Here, we analyze the key of the federated Shapley-CMI computation problem. Without loss of generality, we assume the VFL task has multiple data
parties. Following the maximum likelihood method in Eq. (3), we can rewrite $I(X_d; Y_t | X_D X_t)$ in Eq. (7) as

$$ I(X_d; Y_t | X_D X_t) = \sum_{x_d \in X_d, x_D, x_t, y_t} \hat{p}(x_d x_D x_t y_t) \log \frac{\hat{p}(x_d y_t | x_t x_D)}{\hat{p}(x_d | x_t x_D) \hat{p}(y_t | x_t x_D)} \quad (11) $$

$\hat{p}$ denotes the set of parties that joined the VFL task ahead of the party $d$, and $x_D = \{ x_d' | d' \in D \}$ is a set of stacked features from the parties in $D$. In this vein, the problem of federated Shapley-CMI computation can be converted to calculating the cardinality of intersection among various parties without gathering or revealing any party’s raw data, i.e.,

$$ N(x_d x_D x_t y_t) = \left| \left\{ i | \langle x_t[i], y_t[i] \rangle = \langle x_t, y_t \rangle \right\} \cap \left\{ i | x_d'[i] = x_d \right\} \right|, \quad (13) $$

where $i$ denotes the $i$-th data sample whose feature values in the task party $t$, data party $d$, and data parties $D$ are $\langle x_t, y_t \rangle$, $x_d$, and $x_D$, respectively. When there is only one data party $d$, i.e., $D = \emptyset$, the components $\sum_{x' \in X_d, y_t} N(x'_d x_D x_t y_t)$ and $\sum_{x' \in X_d} N(x'_d x_D x_t y_t)$ in Eq. (12) respectively degenerate to $N(x_t)$ and $N(x_t y_t)$, which can be computed within the task party only.

### 3.2.2. Analysis of Potential Solution

Secure multi-party computation (MPC) protocols are one class of solutions that can support joint computations of data from multiple parties but reveal nothing other than the computational results to any of the parties. Private Set Intersection (PSI) is a specific MPC application, which allows each party to learn the intersection of item sets among various parties. The PSI result can make each party easily calculate the intersection cardinality for Shapley-CMI computation, whereas it would also violate the privacy requirement of VFL. Let suppose that a data party $d$ has a set of samples with features $x_d = \langle 1, 0 \rangle$, and
a task party $t$ owns a set of samples with label $y_t = 1$. PSI could be applied to let $d$ and $t$ acquire their intersection samples whose features and label are respectively $x_d = \langle 1, 0 \rangle$ and $y_t = 1$ for intersection cardinality computation; nevertheless, the intersection samples’ label information $y_t = 1$ is exposed from $t$ to $d$, and their feature information $x_d = \langle 1, 0 \rangle$ is leaked from $d$ to $t$, which should be prohibited in VFL tasks.

A few recent studies design MPC protocols to compute arbitrary functions (e.g., cardinality) over the intersection set but avoid disseminating its elements to attendees for information protection. Most of these methods, however, can only operate on MPC tasks with two parties [18], and thus a new MPC protocol is desired to compute the cardinality of the intersection set of multiple parties for data valuation in VFL tasks. It is worth noting that designing an MPC protocol for secured intersection cardinality computation is more challenging than that for the intersection element computation [18]. The essential difficulty lies in how each party discriminates the veracity of its received intersection cardinality when there exist malicious participants in MPC. We will articulate the difficulties in the sections to follow.

In summary, we require a secure, multi-party, PSI cardinality computation mechanism for federated Shapley-CMI calculation, so that each party can obtain an undisputed result about the cardinality of various parties’ intersection without sharing any raw data.

### 3.2.3. Multi-party PSI Cardinality Computation with Semi-honest Server

Server-aided and serverless solutions are two typical classes of MPC solutions. In general, server-aided MPC solutions are more efficient in large-scale computations, which are deemed as the typical scenario in real-life VFL practices [19, 18, 20]. For example, the commercial VFL platform FDN (federated data network) [1] is required to support millions of samples. We adopt the server-aided strategy in this work accordingly. Specifically, we first propose a simple

\[ \text{https://fdn.webank.com/} \]
and efficient solution based on a semi-honest aided server that would abide by
the designed protocol. We then enhance the solution in the subsequent section
to defend against untrustful servers that may perform maliciously.

Basically, if we have a semi-honest third-party server, we could refer to some
PSI protocols to let the server collect encrypted sample sets from all parties and
perform secure intersection computations [19]. While PSI likely leads to infor-
mation leakage by revealing the computed intersection to all parties, we only
return the cardinality of intersection to each party instead for data protection.
Detailed steps are as follows.

**Preparation.** All the data parties and the task party agree on the same
encryption scheme and encrypt their sample IDs, so that the original sam-
ple IDs cannot be re-identified by the server.

**Step 1.** All the parties send their own encrypted ID sets to a semi-honest
computation server for intersection computation.

**Step 2.** The server computes the intersection of all the received ID sets
and then returns its cardinality to all the parties.

This procedure is simple and effective, only if the computation server is
semi-honest and will not falsify the result.

### 3.2.4. Multi-party PSI Cardinality Computation with Untrustful Server

In practice, the computation server could be untrustful and may mislead
the task party’s data valuation maliciously. For instance, if the task party’s
set cardinality is \( n_t \), the task party barely discriminates the server’s malicious
report with any forged intersection cardinality \( \hat{n} \) \((\leq n_t)\). To deal with this, in
what follows, we propose a novel dual-server-aided PSI cardinality computation
mechanism, as shown in Fig. [1].

We first introduce a new validation server that incorporates an ID Dupli-
cation mechanism to detect the computation server’s malicious report with a
non-zero value. Basically, the ID duplication mechanism requires the parties to
generate \( q \) different encryption IDs for each sample ID, and let the computation server operate on all the encryption IDs. Note that the computation server is ignorable but the validation server is informed about how many and which encryption IDs correspond to the same sample IDs. The computation server respectively sends the encryption IDs of intersection set to the validation server and feeds back the intersection cardinality to the parties. Following the widely accepted assumption in server-aided MPC that the validation server will not collude with the computation server\(^2\), we can count on the validation server to detect the computation server’s malicious behavior by mapping its received encryption IDs back to the sample IDs. In principle, if the computation server is veracious, the validation server would obtain \( q \) duplicates for each sample ID; otherwise, the computation server may be regarded malicious. If the computation server intends to successfully deceive with a forged intersection cardinality \( \hat{n} \), it needs to correctly link each of the \( \hat{n} \) sample IDs to its \( q \) encryption IDs. Without knowing the exact mappings between the samples and their encryption IDs, it is nearly impossible for the computation server especially when \( q \) is large.

Unfortunately, the validation server is yet incompetent to discriminate the credibility of the computation server’s result with a zero intersection cardinal-

\(^2\)We can leverage the two servers in two competitive cloud services (e.g., Microsoft Azure and Amazon Web Services) and use contracts to regularize such collusion behaviors [20, 19].
ity. Besides, after running several rounds of intersection operations, untrustful servers could probably learn the feature distribution of data, which brings exposure risks in practice. To address these issues, we enhance the PSI cardinality computation with a randomized adversarial data augmentation method. Simply speaking, this method produces a random number \( n_r \) of adversarial data samples shared by all the parties and mixes them with real samples before computation. Then, the computed result of intersection cardinality should be a number no less than \( n_r \) but larger than zero. The difference between the computed intersection cardinality and \( n_r \) is the clear cardinality result. In other words, the parties could easily identify the computation server’s malicious behavior if it returns a number less than \( n_r \).

Besides, the adversarial data samples would protect the real feature distribution from being learned. More specifically, if we generate more adversarial data samples, the server-observed feature distribution will be closer to that of the adversarial samples but deviate more from the real one. Nevertheless, more adversarial data samples will incur a higher data transmission workload. As a result, we should carefully set the number of adversarial data samples to strike a balance between the risks from malicious attacks and the costs of data transmissions in practice.

Algorithm 1 illustrates the federated Shapley-CMI computation process. We describe the proposed protocol as follows.

### Preparation.
First, all the data parties and the task party make agreements on the same encryption scheme, the number of encryption IDs for each sample (i.e., \( q \)), and the random number of adversarial data samples (i.e., \( n_r \)). Then, each of the data parties (i.e., \( d \) or \( d' \in D \)) and the task party extract the subset of targeted samples with certain data values (i.e., \( x_d, x_{d'} \) or \( \langle x_t, y_t \rangle \)), and accordingly produce \( n_r \) adversarial sample IDs with the same values as the targeted samples. Furthermore, every party constitutes an encryption ID set by generating \( q \) different encryption IDs for each of the targeted and adversarial samples. Finally, the validation
Algorithm 1: Federated Computation of Shapley-CMI

**Input**: \( D \): data parties, each data party \( d \) with features \( X_d \);
\( T \): task party with features \( X_t \) and label \( Y_t \).

**Output**: \( \varphi = \{ \varphi_1, \varphi_2, \ldots \} \): Shapley-CMI values for all the data parties.

1. \( \pi \leftarrow \text{all_permutations}(D) \);
2. \( \varphi = [0] \ast |D| \);
3. for \( O \in \pi \) do
   4. \( D = [] \);
   5. for \( k \in [1, 2, \ldots, |D|] \) do
      6. \( d = O[k] \); \hspace{1em} // data party \( d \)
      7. \( \varphi[d] + = I(X_d; Y_t | X_D X_t) \);
      8. \( D.\text{append}(d) \);
   9. end
10. end
11. \( \varphi = \varphi / |\pi| \);

Step 1. The data parties and the task party send their encryption ID sets to the computation server for intersection computation.

Step 2. The computation server computes the intersection ID set \( I_{\text{inter}} \) and transmits it to the validation server (Step 2.1); it meanwhile calculates the intersection cardinality \( n_c = |I_{\text{inter}}| \) and feeds it back to the parties (Step 2.2).

Step 3. The validation server checks whether the encryption IDs in \( I_{\text{inter}} \) can be mapped to \( |I_{\text{inter}}| / q \) samples with \( q \) duplicates for each; If the answer is yes, it sends \( n_v = |I_{\text{inter}}| \) to the parties; otherwise, it sends \(-1\).

Step 4. The parties compare \( n_c \) and \( n_v \). If \( n_c = n_v \), the intersection cardinality is \( n_c / q - n_r \); otherwise, there must be certain malicious server behaviors.
Remark. To the best of our knowledge, this is the first work that designs a federated mechanism for CMI as well as Shapley-CMI computation. In particular, we first convert the federated CMI computation issue to the PSI cardinality calculation problem, and then propose a novel dual-server-aided mechanism to compute the multi-party PSI cardinality without leaking raw data from any parties. It should be emphasized that this mechanism can serve various tasks beyond CMI computation where a PSI cardinality calculation is required for multiple parties. In this light, it also contributes to the PSI research area.

3.3. Practical Considerations

The Shapley-CMI metric and its secure multi-party PSI cardinality computation mechanism together constitute our FedValue framework. While FedValue is theoretically feasible for data valuation in VFL tasks, it still encounters some computational challenges in practice. First of all, the federated Shapley-CMI computation in Eq. (7) needs to enumerate every possible party-joining order, and hence its computational complexity is \(O(2^{|\mathcal{D}|}-1)\), where \(\mathcal{D}\) is the set of all data parties. This indicates that the federated Shapley-CMI computation could be practically difficult for a VFL task with a large number of parties. Besides, if the parties’ data are of very high dimensions, the maximum likelihood estimation of Shapley-CMI in Eq. (7) may not be reliable and thus the data valuation is likely imprecise [15]. Accordingly, we employ two practical techniques, i.e., Shapley sampling and feature dimension reduction, to address these practical issues in FedValue computation.

3.3.1. Shapley Sampling

Inspired by the prior Shapley value computation work [21], we leverage the approximation technique by sampling and enumerating a subset of permutations of data parties. Specifically, a party \(d\)’s value \(\hat{\varphi}_d\) is estimated by the average of CMI values given a sampled set of data party permutations. That is,

\[
\hat{\varphi}_d = \frac{1}{|\pi|} \sum_{\sigma \in \pi} I(X_d; Y_t | X_{\sigma \setminus d} X_t),
\]                                  \hspace{1cm} (14)
Table 1: Dataset Statistics

|        | #features | #labels | #samples |
|--------|-----------|---------|----------|
| Wine   | 13        | 3       | 178      |
| Parkinsons | 22    | 2       | 195      |
| Spect  | 22        | 2       | 267      |
| Breast | 30        | 2       | 569      |
| Music  | 57        | 10      | 1000     |
| Credit | 23        | 2       | 30000    |

where $\pi$ is a sampled set of permutations of data parties, $\mathcal{O}$ is a specific permutation of data parties in $\pi$, and $\mathcal{O}_d$ is the set of data parties that precede the party $d$ in $\mathcal{O}$.

3.3.2. Feature Dimension Reduction

Feature dimension reduction techniques are widely used in machine learning tasks, which can reduce the computation overhead and may even boost the learning performance [22]. In our case, before calculating Shapley-CMI, all the parties can firstly run feature dimension reduction methods such as PCA [23] and reduce the feature space to a controllable small scale, so that the Shapley-CMI computation can be accelerated. The PCA can be performed on each party independently and thus will not lead to any privacy leakage. Then, the proposed FedValue can work with the principal components obtained by PCA for data valuation in VFL. In the experiment, we will empirically investigate how feature dimension reduction impacts the Shapley-CMI computation results.

4. Experiments

We conduct extensive experiments to verify FedValue with six real-world datasets, including Wine [24], Parkinsons [25], Spect [26], Breast [27], Music [3], and Credit [28]. The dataset statistics are summarized in Table 1. We discretize

https://www.kaggle.com/andradaolteanu/gtzan-dataset-music-genre-classification
the continuous features of the datasets into five equal-width-bin categorical features to facilitate CMI computation following the literature [15]. Next, we successively evaluate the effectiveness of FedValue, its computational efficiency, and the effects of practical computational techniques.

4.1. Effectiveness of FedValue

We evaluate the effectiveness of our FedValue framework from two aspects. We first verify the effectiveness of Shapley-CMI metric in data valuation for a VFL task. We then examine whether the dual-server-aided federated computation mechanism of FedValue can obtain precise Shapley-CMI for each party.

4.1.1. The Effectiveness of Shapley-CMI in Data Valuation

**Experimental setup.** Since there is no ground-truth about the data values in real-life practices, we need to carefully design a reasonable reference method to validate the effectiveness of Shapley-CMI. Intuitively, the results of a model-free data valuation method should be comparable to those of the model-dependent ones. However, the valuation results of model-dependent methods often vary with the adopted models [29, 30]. To deal with this, prior research suggests training a set of models and learning an ensemble feature importance based on the well-performing ones [30]. Inspired by this, we construct a model-dependent reference method for data party valuation, which takes all the features of a party as an aggregated feature and valuates the party by the ensemble model-dependent feature importance of its aggregated feature.

Specifically, we first train a set of prediction models with various machine learning methods, including support vector machine (SVM), gradient boosting decision tree (GBT), logistic regression (LR), random forest (RF), and neural networks (NN). Table 2 shows the prediction accuracy of different models on various datasets. Let $\text{acc}^*$ denote the best model’s accuracy and $\epsilon$ denote a threshold for model selection. For each dataset, we select the models with a prediction accuracy larger than $(\text{acc}^* - \epsilon)$ as the well-performing ones (denoted as $M_{\text{well}}$), where $\epsilon$ is set to 0.02 and 0.05 respectively by default. Then, given all
Table 2: Well-performing model selection by prediction accuracy. For each dataset, the accuracy of the best model is in bold; the models with a double-underlined accuracy would be selected when threshold $\epsilon = 0.02$; all the models with an (both single- and double-) underlined accuracy would be selected when threshold $\epsilon = 0.05$. For example, in the Parkinsons dataset, the SVM, GBT, and RF models are selected into $M_{\text{well}}$ when $\epsilon$ is set to 0.05.

|          | SVM   | GBT   | LR    | RF    | NN    |
|----------|-------|-------|-------|-------|-------|
| Wine     | 0.967 | 0.944 | 0.967 | 0.944 | 0.967 |
|          | ±0.027| ±0.030| ±0.011| ±0.025| ±0.041|
| Parkinsons| 0.897 | 0.908 | 0.821 | 0.928 | 0.856 |
|          | ±0.043| ±0.053| ±0.036| ±0.030| ±0.058|
| Spect    | 0.833 | 0.819 | 0.819 | 0.822 | 0.826 |
|          | ±0.012| ±0.032| ±0.041| ±0.022| ±0.045|
| Breast   | 0.951 | 0.937 | 0.961 | 0.937 | 0.949 |
|          | ±0.012| ±0.020| ±0.007| ±0.013| ±0.010|
| Music    | 0.605 | 0.643 | 0.596 | 0.653 | 0.613 |
|          | ±0.021| ±0.016| ±0.034| ±0.024| ±0.030|
| Credit   | 0.809 | 0.812 | 0.808 | 0.795 | 0.805 |
|          | ±0.004| ±0.005| ±0.006| ±0.006| ±0.005|

well-performing models in $M_{\text{well}}$, we use a model-dependent feature importance metric to valuate each party’s aggregated feature and average the valuation results by different models as reference.

In general, Permutation Importance (PI) \[31\] and SHAP \[13\] are two widely used model-dependent feature importance metrics. However, it is reported that PI cannot appropriately valuate the data parties that share some features\[4\]. In this vein, we ensemble SHAP values of a party’s aggregated feature given the well-performing models as a reference of the party’s value, denoted as $\text{SHAP-ensemble}$. We use the Pearson correlation to measure the similarity between Shapley-CMI and SHAP-ensemble, with the assumption that the valuation re-

\[4\]A detailed discussion can be found in Sec. 5.6 of *Interpretable Machine Learning* \[https://christophm.github.io/interpretable-ml-book/feature-importance.html\].
result of Shapley-CMI should be similar to that of SHAP-ensemble.

Based on the evaluation standard, we implement a task party and multiple data parties to carry out experiments on each of the six datasets in Table 1. For simplicity, a dataset’s features are randomly and evenly distributed among all the parties in our experiments. Specifically, the number of features in a party is set to 1, 2 and 3, respectively. Besides, we only select 80% samples of a dataset at random to run an experiment for each setting; by doing so, we can repeat the experiment many (i.e., 50) times with different data samples to obtain the mean and standard deviation of evaluation results for robustness check.

Main results. Fig. 2 displays the correlations between Shapley-CMI and SHAP-ensemble in data valuation when the well-performing model selection threshold $\epsilon$ is set to 0.05 and 0.02, respectively. We can observe that their data valuation correlations are almost larger than 0.8 on all datasets under different settings, indicating that Shapley-CMI indeed can effectively valuate each party’s contribution in VFL tasks. Recall that the computation of SHAP-ensemble requires data parties to collaboratively train some real models, which could be
(a) Each party with 1 feature
(b) Each party with 2 features
(c) Each party with 3 features

Figure 3: Correlations between Shapley-CMI and model-dependent SHAP values.

very costly or even infeasible before the launch of the VFL task. Shapley-CMI avoids specifying any concrete models and thus is more suitable for VFL in practice.

Fig. 3 shows the correlations between Shapley-CMI and SHAP values given various models (denoted as SHAP-\(m\) where \(m\) can be SVM, GBT, LR, RF, or NN). It is interesting that the correlations between Shapley-CMI and various model-dependent SHAP values are quite different. For example, the correlation of Shapley-CMI with SHAP-GBT is higher than 0.8 but lower than 0.6 with SHAP-NN on the Parkinsons dataset. This indicates that the data valuation by SHAP value varies with the adopted models obviously, and some of the model-
dependent SHAP values hardly reveal the essential value of a data party. Hence, a model-free data valuation method like Shapley-CMI would be more attractive. Besides, in the experiments we find that Shapley-CMI correlates more with the SHAP values of the high-performance models. For instance, while the model GBT (0.908) obtains a higher prediction accuracy than the model NN (0.856) does (shown in Table 2), Shapley-CMI has a larger correlation with SHAP-GBT than SHAP-NN on the Parkinsons dataset. This finding further validates the effectiveness of Shapley-CMI for data valuation in VFL.

4.1.2. The Effectiveness of Dual-server-aided Federated Computation

Ideally, FedValue’s Shapley-CMI computation with the designed dual-server-aided federated computation mechanism (denoted as Shapley-CMI\textsubscript{FED}) should obtain the same results as the centralized Shapley-CMI computation when all parties’ data can be gathered (denoted as Shapley-CMI\textsubscript{CEN}). We here empirically compare Shapley-CMI\textsubscript{FED} with Shapley-CMI\textsubscript{CEN} by Pearson correlation.

Fig. 4 shows the correlations of data valuation results between Shapley-CMI\textsubscript{FED} and Shapley-CMI\textsubscript{CEN} on all datasets. The observation verifies that Shapley-CMI\textsubscript{FED}, which allows all parties to preserve their own raw data, can achieve exactly the same data valuation results as Shapley-CMI\textsubscript{CEN}, which needs to aggregate all parties’ data. These results validate the effectiveness of the designed dual-server-aided federated computation paradigm in FedValue.
4.2. Computational Efficiency of FedValue

There are two classes of key parameters that may influence the computational efficiency of FedValue. The first class of parameters relates to the Shapley-CMI computation, including the numbers of data parties, features and data samples. The other class of parameters is set against the malicious servers, including the ID duplication times and the randomized adversarial data samples. We vary the parameters to examine the computation efficiency of FedValue. By default, there are 5 data parties; every data party holds a binary feature, while the task party owns a binary feature and a binary task label; the total number of samples is 100,000, including 10% (10,000) real data samples and 90% (90,000) adversarial data samples; the ID duplication times is set to 3. The experimental platform is an ordinary laptop with AMD Ryzen R7 4800HS (2.9GHz) and 16GB RAM.

Fig. 5 shows the computation time of FedValue with three varied parameters
for Shapley-CMI computation. In particular, Fig. 5a and Fig. 5b report the computation time over the number of data parties and the number of a party’s features, respectively. The results show that the computation time of FedValue increases almost exponentially with the two parameters. These observations suggest the importance of using practical techniques to reduce the computation time when there are a large number of data parties or the parties possess many features. Fig. 5c shows that the running time of FedValue increases linearly with the number of data samples. Specifically, for a VFL task with 100,000 data samples (10,000 real samples and 90,000 adversarial samples) and 5 parties, we can obtain the valuation results in 10 minutes; even if the number of data samples increases to 1,000,000 (100,000 real samples and 900,000 adversarial samples), the computation time is not beyond 2 hours.

Fig. 6a and Fig. 6b show how the computation time of FedValue varies with the ID duplication times and the number of adversarial data samples, respectively. These two parameters essentially control the overall amount of data used in the computation. It is common sense that adding more data would sacrifice the computation efficiency for the protection of data privacy. Nevertheless, we observe that the computation time is also linearly associated with both of the privacy-preserving parameters.

4.3. The Effectiveness of Practical Computation Techniques

Here, we evaluate the practical computation techniques for FedValue acceleration. In particular, we compare the data valuation results in terms of Shapley-CMI between the original FedValue and the FedValue that adopts any practical computation techniques (denoted as fast-FedValue). On one hand, we expect that the data valuation results of fast-FedValue could be close to those of the original FedValue. Accordingly, we adopt the MAPE (Mean Absolute Percentage Error) measure to characterize this closeness. On the other, we expect that the computation efficiency of FedValue could be improved with practical computation techniques. Since the datasets Breast and Music in Table I have the largest numbers of features, we use them as the representatives to examine
the techniques of Shapely sampling and feature dimension reduction.

**Shapley sampling.** Theoretically, the fast-FedValue with Shapley sampling cannot attain precise Shapley-CMI if the number of Shapley sampling is small. The increase of samplings will decrease the Shapley-CMI differences (MAPE) between the fast-FedValue and the original FedValue. Therefore, we have interests in how many samplings can help fast-FedValue achieve a small enough MAPE value. Fig. 7a displays how the MAPE of Shapley-CMI between FedValue and fast-FedValue changes with the number of samplings. The results show that the MAPE drops dramatically when the number of samplings increases from 100 to 1200, and it steadily converges towards a small value if the number of samplings continues growing. Fig. 7b reports how the number of samplings would affect the efficiency of Shapley-CMI computation. We notice
that the computation time is nearly linear to the number of samplings. By integrating both observations regarding effectiveness and efficiency evaluations, we leverage 5,000 samplings for Shapley-CMI computation in our experiment, as it can achieve a small MAPE within tolerable computation time.

**Feature dimension reduction.** In this experiment, we construct fast-FedValue using PCA to reduce the feature dimensions of data parties. Fig. 8a displays that the MAPE between FedValue and fast-FedValue decreases rapidly when the retained number of PCA components increases from 2 to 4. This observation indicates that the fast-FedValue can better approximate the true Shapley-CMI when it keeps more informative components. Fig. 8b compares fast-FedValue and FedValue by the computation time. We can observe that the computation time of FedValue can be decreased exponentially when each party holds less number of PCA components. In brief, there exists a trade-off between the effectiveness and efficiency of FedValue by leveraging feature dimension reduction techniques.

5. Related Work

Preserving private information is a fundamental requirement in nowadays’ AI applications [32, 33]. Federate learning (FL) is a privacy-preserving distributed learning paradigm originally proposed by Google [4]. After its initial proposal, FL has rapidly attracted a huge amount of attention from both academia and industry [3]. Generally speaking, FL conducts collaborative learning with data from multiple parties and meanwhile ensures that no party’s data are leaked to any others. According to the data dimension (i.e., feature or sample) that parties collaborate on, FL tasks are categorized into two main classes, i.e., HFL (horizontal FL) and VFL (vertical FL). Specifically, in HFL, parties have different data samples with the same features; while parties usually hold different features of the same data samples in VFL [3].

Prior FL research efforts are mostly devoted to designing various algorithms in an FL manner. The seminal work [5] presents two widely-used HFL learning
algorithms, FedSGD and FedAvg, for training neural networks in a federated manner. With FedSGD, HFL parties send every step of gradient descents over local data samples to a server for aggregating models. Comparatively, FedAvg lets each party transmit the average of gradient updates on local data samples for multiple steps to a server for model aggregation. Besides, HFL algorithms are proposed for machine learning models other than neural networks. For instance, a federated matrix factorization algorithm is proposed to achieve a similar performance as the centralized matrix factorization \cite{34}; an efficient HFL algorithm is also designed for building boosting decision trees with locality-sensitive hashing \cite{35}. While VFL is also of significant practical business value \cite{3}, it is much under-investigated compared to HFL \cite{9}. Cheng et al. \cite{14} propose the first decision tree algorithm for VFL. The security of the VFL tree is further enhanced by concealing sensitive information of final tree models (e.g., the split thresholds of internal tree nodes) \cite{9}. Hu et al. \cite{36} design an asynchronous stochastic gradient descent algorithm for learning VFL logistic regression and neural network models.

In general, FL algorithms require parties to exchange some insensitive intermediate results (e.g., gradients); however, researchers have found that attackers may still recover raw training data, e.g., images and texts, from gradients \cite{37}. To overcome this pitfall, advanced privacy-preserving computation techniques including homomorphic encryption \cite{38}, secret sharing \cite{39}, and differential privacy \cite{40} have been adopted in FL mechanisms. What’s more, due to the importance of FL systems in practice, various open-source privacy-preserving FL systems, such as PySyft\footnote{https://github.com/OpenMined/PySyft}, FedML \cite{41}, FATE\footnote{https://fate.fedai.org/} and FedEval \cite{42}, have been developed and deployed recently.

In addition to the FL algorithms and systems aforementioned, quantifying the contribution of different parties, i.e., data valuation, is another fundamental issue to build a healthy FL ecosystem. With effective data valuation methods,
proper incentive mechanisms can then be designed to encourage parties to join the FL community \[43, 44, 45\]. A few prior studies on FL data valuation mainly focus on the HFL scenario \[6, 7, 8, 46\], where the Shapley value \[16\] is usually adopted in evaluating each party’s contribution. Specifically, in a Shapley-value-based HFL valuation framework, each party’s value is estimated as the average marginal contribution (i.e., prediction accuracy on a separate test set) to every possible subset of other parties’ data samples \[47\]. To address the data valuation problem for the VFL scenario, a pioneering study \[10\] suggests using a model-dependent feature importance metric, SHAP \[13\], to valuate each party’s contribution, which requires a specific training model (e.g., neural networks or SVM). However, data valuation is often a prerequisite before model training for party selection. Our FedValue incorporates a novel model-free valuation metric, Shapley-CMI, and can thus support VFL data valuation before model training.

How to calculate a joint probability of multiple features from different parties in a privacy-preserving manner is one major challenge of FedValue. Some research approximates such probabilities based on probability distribution assumptions like Gaussian mixture models \[48\]. However, these methods will incur obvious approximation errors when the assumption does not hold. Without making any assumptions on the distribution, we leverage MPC (secure Multi-Party Computation) \[20\] to calculate the multi-feature joint probability via maximum likelihood estimation across parties. Furthermore, we analyze that the key step of the maximum likelihood joint probability estimation is computing the cardinality of the intersection of different parties’ data sample ID sets, which is related to the research topic of PSI (Private Set Intersection) in MPC. Specifically, PSI studies how to obtain intersection elements of multiple sets from different parties in a privacy-preserving manner \[49\], and the state-of-the-art PSI mechanism can support large-scale sets including billions of elements \[19\]. However, returning intersection elements will still incur VFL parties’ data leakage, and thus PSI mechanisms cannot be directly applied for data valuation. More recently, a few studies start exploring the problem of returning a certain function (e.g., cardinality) over PSI, instead of returning the intersection ele-
ments. The state-of-the-art work [18] can compute the intersection cardinality without leaking the intersection elements via secret sharing; however, this protocol [18] works for only two parties and cannot support VFL data valuation including more than two parties. Our designed dual-server-aided mechanism overcomes the limitation of party number and can efficiently compute PSI cardinality for an arbitrary number of parties.

6. Conclusion

In this paper, we proposed a novel data valuation method named FedValue for VFL. Specifically, we first designed Shapley-CMI, a task-specific but model-free data valuation metric based on information theory and game theory. We then proposed a new dual-server-aided mechanism to calculate Shapley-CMI in a federated manner and ensure that no party’s private data will be leaked during the computation process. Finally, we accelerated FedValue with some practical computation techniques. Extensive experiments on six real-life datasets verified the effectiveness of FedValue.

As a pilot study on model-free data valuation of VFL, our study may inspire a series of future work as follows: (i) While we propose some practical acceleration methods for FedValue, its performance on different datasets varies obviously, calling for further research on the efficiency issue; (ii) Our work provides a prototype design for VFL data valuation, while implementing it in reality may face other challenges. For example, some parties may encounter communication problems and fail to upload their information — how to deal with such connection losses needs careful design.

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References

[1] Y. Liu, T. Fan, T. Chen, Q. Xu, Q. Yang, Fate: An industrial grade platform for collaborative learning with data protection, Journal of Machine Learning Research 22 (226) (2021).

[2] D. Fisch, M. Jänicke, E. Kalkowski, B. Sick, Learning from others: Exchange of classification rules in intelligent distributed systems, Artificial Intelligence 187-188 (2012) 90–114.

[3] Q. Yang, Y. Liu, T. Chen, Y. Tong, Federated machine learning: Concept and applications, ACM Transactions on Intelligent Systems and Technology 10 (2) (2019) 12.

[4] J. Konečný, H. B. McMahan, F. X. Yu, P. Richtárik, A. T. Suresh, D. Bacon, Federated learning: Strategies for improving communication efficiency, arXiv preprint arXiv:1610.05492 (2016).

[5] B. McMahan, E. Moore, D. Ramage, S. Hampson, B. A. y Arcas, Communication-efficient learning of deep networks from decentralized data, in: Artificial intelligence and statistics, PMLR, 2017, pp. 1273–1282.

[6] S. Wei, Y. Tong, Z. Zhou, T. Song, Efficient and Fair Data Valuation for Horizontal Federated Learning, in: Federated Learning, Vol. 12500, Springer International Publishing, Cham, 2020, pp. 139–152.

[7] T. Song, Y. Tong, S. Wei, Profit Allocation for Federated Learning, in: IEEE International Conference on Big Data, IEEE, 2019, pp. 2577–2586.

[8] T. Wang, J. Rausch, C. Zhang, R. Jia, D. Song, A Principled Approach to Data Valuation for Federated Learning, in: Federated Learning, 2020, pp. 153–167. arXiv:2009.06192

[9] Y. Wu, S. Cai, X. Xiao, G. Chen, B. C. Ooi, Privacy preserving vertical federated learning for tree-based models, VLDB 13 (11) (2020) 2090–2103. arXiv:2008.06170
[10] G. Wang, C. X. Dang, Z. Zhou, Measure Contribution of Participants in Federated Learning, in: IEEE International Conference on Big Data, IEEE, 2019, pp. 2597–2604.

[11] X. Han, R. Ding, L. Wang, H. Huang, Creditprint: Credit investigation via geographic footprints by deep learning, arXiv preprint arXiv:1910.08734 (2019).

[12] C. Batini, C. Cappiello, C. Francalanci, A. Maurino, Methodologies for data quality assessment and improvement, ACM computing surveys (CSUR) 41 (3) (2009) 1–52.

[13] S. M. Lundberg, S.-i. Lee, A Unified Approach to Interpreting Model Predictions, in: NIPS, no. Section 2, 2017, pp. 1–10.

[14] K. Cheng, T. Fan, Y. Jin, Y. Liu, T. Chen, Q. Yang, Secureboost: A lossless federated learning framework, ArXiv abs/1901.08755 (2019).

[15] G. Brown, A. Pocock, M.-J. Zhao, M. Lujan, Conditional Likelihood Maximisation: A Unifying Framework for Information Theoretic Feature Selection, JMLR 13 (2012) 27–66.

[16] L. S. Shapley, A value for n-person games, Contributions to the Theory of Games 2 (28) (1953) 307–317.

[17] M. Chui, D. Farrell, K. Jackson, How government can promote open data and help unleash over $3 trillion in economic value, Innovation in Local Government: Open Data and Information Technology 2 (2014).

[18] P. H. Le, S. Ranellucci, S. Gordon, Two-party private set intersection with an untrusted third party, Proceedings of the 2019 ACM SIGSAC Conference on Computer and Communications Security (2019).

[19] S. Kamara, P. Mohassel, M. Raykova, S. S. Sadeghian, Scaling private set intersection to billion-element sets, in: Financial Cryptography, 2014.
[20] P. Bogetoft, D. L. Christensen, I. Damgård, M. Geisler, T. P. Jakobsen, M. Kroigaard, J. D. Nielsen, J. Nielsen, K. Nielsen, J. Pagter, M. Schwartzbach, T. Toft, Secure multiparty computation goes live, in: Financial Cryptography, 2009.

[21] E. Strumbelj, I. Kononenko, Explaining prediction models and individual predictions with feature contributions, Knowledge and Information Systems 41 (2013) 647–665.

[22] I. K. Fodor, A survey of dimension reduction techniques, Tech. rep., Lawrence Livermore National Lab., CA (US) (2002).

[23] A. M. Martinez, A. C. Kak, PCA versus LDA, IEEE Transactions on Pattern Analysis and Machine Intelligence 23 (2) (2001) 228–233.

[24] S. Aeberhard, D. Coomans, O. De Vel, Comparative analysis of statistical pattern recognition methods in high dimensional settings, Pattern Recognition 27 (8) (1994) 1065–1077.

[25] M. Little, P. McSharry, S. Roberts, D. Costello, I. Moroz, Exploiting nonlinear recurrence and fractal scaling properties for voice disorder detection, Nature Precedings (2007) 1–1.

[26] L. A. Kurgan, K. J. Cios, R. Tadeusiewicz, M. Ogiela, L. S. Goodenday, Knowledge discovery approach to automated cardiac spect diagnosis, Artificial intelligence in medicine 23 (2) (2001) 149–169.

[27] O. L. Mangasarian, W. N. Street, W. H. Wolberg, Breast cancer diagnosis and prognosis via linear programming, Operations Research 43 (4) (1995) 570–577.

[28] I.-C. Yeh, C.-h. Lien, The comparisons of data mining techniques for the predictive accuracy of probability of default of credit card clients, Expert Systems with Applications 36 (2) (2009) 2473–2480.
[29] L. Breiman, Statistical modeling: The two cultures (with comments and a rejoinder by the author), Statistical science 16 (3) (2001) 199–231.

[30] A. Fisher, C. Rudin, F. Dominici, All models are wrong, but many are useful: Learning a variable’s importance by studying an entire class of prediction models simultaneously., J. Mach. Learn. Res. 20 (177) (2019) 1–81.

[31] A. Altmann, L. Toloşi, O. Sander, T. Lengauer, Permutation importance: a corrected feature importance measure, Bioinformatics 26 (10) (2010) 1340–1347.

[32] T. Tassa, T. Grinshpoun, A. Yanai, Pc-syncbb: A privacy preserving collusion secure dcop algorithm, Artificial Intelligence 297 (2021) 103501.

[33] F. Fioretto, P. Van Hentenryck, K. Zhu, Differential privacy of hierarchical census data: An optimization approach, Artificial Intelligence 296 (2021) 103475.

[34] D. Chai, L. Wang, K. Chen, Q. Yang, Secure federated matrix factorization, IEEE Intelligent Systems (2020).

[35] Q. Li, Z. Wen, B. He, Practical federated gradient boosting decision trees, in: Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 34, 2020, pp. 4642–4649.

[36] Y. Hu, D. Niu, J. Yang, S. Zhou, Fdml: A collaborative machine learning framework for distributed features, in: Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, 2019, pp. 2232–2240.

[37] L. Zhu, S. Han, Deep leakage from gradients, in: Federated learning, Springer, 2020, pp. 17–31.

[38] Y. Aono, T. Hayashi, L. Wang, S. Moriai, et al., Privacy-preserving deep learning via additively homomorphic encryption, IEEE Transactions on Information Forensics and Security 13 (5) (2017) 1333–1345.
[39] K. Bonawitz, V. Ivanov, B. Kreuter, A. Marcedone, H. McMahan, S. Patel, D. Ramage, A. Segal, K. Seth, Practical secure aggregation for privacy-preserving machine learning, Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security (2017).

[40] R. C. Geyer, T. Klein, M. Nabi, Differentially private federated learning: A client level perspective, arXiv preprint arXiv:1712.07557 (2017).

[41] C. He, S. Li, J. So, M. Zhang, H. Wang, X. Wang, P. Vepakomma, A. Singh, H. Qiu, L. Shen, et al., Fedml: A research library and benchmark for federated machine learning, arXiv preprint arXiv:2007.13518 (2020).

[42] D. Chai, L. Wang, K. Chen, Q. Yang, Fedeval: A benchmark system with a comprehensive evaluation model for federated learning, arXiv preprint arXiv:2011.09655 (2020).

[43] H. Yu, Z. Liu, Y. Liu, T. Chen, M. Cong, X. Weng, D. Niyato, Q. Yang, A fairness-aware incentive scheme for federated learning, in: Proceedings of the AAAI/ACM Conference on AI, Ethics, and Society, 2020, pp. 393–399.

[44] L. U. Khan, S. R. Pandey, N. H. Tran, W. Saad, Z. Han, M. N. Nguyen, C. S. Hong, Federated learning for edge networks: Resource optimization and incentive mechanism, IEEE Communications Magazine 58 (10) (2020) 88–93.

[45] M. Cong, H. Yu, X. Weng, S. M. Yiu, A game-theoretic framework for incentive mechanism design in federated learning, in: Federated Learning, Springer, 2020, pp. 205–222.

[46] Y. Liu, Z. Ai, S. Sun, S. Zhang, Z. Liu, H. Yu, Fedcoin: A peer-to-peer payment system for federated learning, in: Federated Learning, Springer, 2020, pp. 125–138.

[47] A. Ghorbani, J. Zou, Data shapley: Equitable valuation of data for machine learning, in: International Conference on Machine Learning, PMLR, 2019, pp. 2242–2251.
[48] M. Jia, C. Shen, Privacy-preserving distributed joint probability modeling for spatial-correlated wind farms, ArXiv abs/1812.09247 (2018).

[49] M. J. Freedman, K. Nissim, B. Pinkas, Efficient private matching and set intersection, in: International conference on the theory and applications of cryptographic techniques, Springer, 2004, pp. 1–19.

Data and Code

We have released the code and data for reproducing our experiment results at https://github.com/wangleye/FedValue/