On Super RS Algebra and Drinfeld Realization of Quantum Affine Superalgebras

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Abstract

We describe the realization of the super Reshetikhin-Semenov-Tian-Shansky (RS) algebra in quantum affine superalgebras, thus generalizing the approach of Frenkel-Reshetikhin to the supersymmetric (and twisted) case. The algebraic homomorphism between the super RS algebra and the Drinfeld current realization of quantum affine superalgebras is established by using the Gauss decomposition technique of Ding-Frenkel. As an application, we obtain Drinfeld realization of quantum affine superalgebra \( U_q[osp(1|2)^{(1)}] \) and its degeneration – central extended super Yangian double \( DY_h[osp(1|2)^{(1)}] \).

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1 Introduction

Recently we introduced [1], without elaboration, a super version of the Reshetikhin-Semenov-Tian-Shansky (RS) algebra [2]. Using this super RS algebra and a super analogue of the Gauss decomposition formula of Ding-Frenkel [3], we obtain Drinfeld realization [4] of quantum affine superalgebra $U_q[gl(m|n)]$ (see also [5] for the special case of $m = n = 1$).

In this paper, we adopt a more conceptual way of proceeding. We shall show that the super RS algebra can be realized in quantum affine superalgebras, which enables extension of the work by Frenkel-Reshetikhin [6] to the supersymmetric (and twisted) case. The algebra homomorphism between the super RS algebra and the Drinfeld realization of the quantum affine superalgebras is achieved by using the Gauss decomposition technique of Ding-Frenkel. As an application, we obtain the Drinfeld realization of $U_q[osp(1|2)]$ and its degenerated algebra – central extended super-Yangian double $DY_{h}[osp(1|2)]$.

2 Super RS Algebra and its Realization in Quantum Affine Superalgebras

Let us start with the definition of the super RS algebra. Let $R(z) \in \text{End}(V \otimes V)$, where $V$ is a $\mathbb{Z}_2$ graded vector space, be a matrix obeying the weight conservation condition $R(z)_{\alpha\beta,\alpha'\beta'} \neq 0$ only when $[\alpha'] + [\beta'] + [\alpha] + [\beta] = 0 \mod 2$, and the graded Yang-Baxter equation (YBE)

$$R_{12}(z)R_{13}(zw)R_{23}(w) = R_{23}(w)R_{13}(zw)R_{12}(z). \quad (2.1)$$

The multiplication rule for the tensor product is defined for homogeneous elements $a, b, a', b'$ by

$$(a \otimes b)(a' \otimes b') = (-1)^{[b][a']} (aa' \otimes bb'), \quad (2.2)$$

where $[a] \in \mathbb{Z}_2$ denotes the grading of the element $a$. We introduce [1]

Definition 1 : Let $R(\frac{z}{w})$ be a $R$-matrix satisfying the graded YBE (2.1). The super RS algebra $U(R)$ is generated by invertible $L^\pm(z)$, satisfying

$$R(\frac{z}{w})L^+_1(z)L^+_2(w) = L^+_2(w)L^+_1(z)R(\frac{z}{w}),$$

$$R(\frac{z}{w_-})L^+_1(z)L^+_2(w) = L^-_2(w)L^+_1(z)R(\frac{z}{w_-}). \quad (2.3)$$

where $L^+_1(z) = L^\pm(z) \otimes 1$, $L^+_2(z) = 1 \otimes L^\pm(z)$ and $z_\pm = zq^{\pm\frac{\lambda}{2}}$. For the first formula of (2.3), the expansion direction of $R(\frac{z}{w})$ can be chosen in $\frac{z}{w}$ or $\frac{w}{z}$, but for the second formula, the expansion direction must only be in $\frac{z}{w}$. 


The algebra $U(R)$ is a graded Hopf algebra: its coproduct is defined by
\[
\Delta(L^\pm(z)) = L^\pm(zq^{\pm 1} \otimes q^{\pm 1}) \otimes L^\pm(zq^{\pm 1} \otimes 1),
\] (2.4)
and its antipode is
\[
S(L^\pm(z)) = L^\pm(z)^{-1}.
\] (2.5)

We now consider a realization of $U(R)$ in quantum affine superalgebra $U_q[G^{(k)}]$, generalizing the Frenkel-Reshetikhin description to the supersymmetric case. Let us first of all recall some facts about the affine superalgebra $G^{(k)}$. For simplicity, we restrict ourselves to the case of $k \leq 2$. Let $G_0$ be the fixed point subalgebra under the diagram automorphism $\hat{\tau}$ of $G$ of order $k$. In the case of $k = 1$, we have $G_0 \equiv G$. For $k = 2$ we may decompose $G$ as $G_0$ plus a $G_0$-representation $G_1$ of $G$. Let
\[
\psi = \begin{cases} 
\text{highest root of } G_0 \equiv G, & \text{for } k = 1, \\
\text{highest weight of the } G_0 - \text{representation } G_1, & \text{for } k = 2.
\end{cases}
\] (2.6)

Following the usual convention, we denote the weight of $G^{(k)}$ by $\Lambda \equiv (\lambda, \kappa, \tau)$, where $\lambda$ is a weight of $G_0$. With this convention the nondegenerate form $(,)$ induced on the weights can be expressed as
\[
(\Lambda, \Lambda') = (\lambda, \lambda') + \kappa \tau' + \kappa' \tau.
\] (2.7)

Let $h_\rho$ denote the unique element of the Cartan subalgebra of $G^{(k)}$ satisfying $h_\rho(\alpha_i) = \frac{1}{2}(\alpha_i, \alpha_i)$, where $\alpha_i$ ($0 \leq i \leq r$) are simple roots of $G^{(k)}$. Then $h_\rho$ is given by
\[
h_\rho = h_\rho + gd,
\] (2.8)
where $g = \frac{k}{2}(\psi, \psi + 2\rho)$, $\rho$ is the graded half-sum of positive roots of $G_0$ and $d$ is the usual level operator.

We shall not give the relations obeyed by the simple generators \{\(h_i, E_i, F_i, d, 0 \leq i \leq r\)\} of $U_q[G^{(k)}]$, but mention that $U_q[G^{(k)}]$ is endowed with a graded Hopf algebra structure with coproduct and antipode given by
\[
\Delta(h_i) = h_i \otimes 1 + 1 \otimes h_i, \quad \Delta(d) = d \otimes 1 + 1 \otimes d,
\]
\[
\Delta(E_i) = E_i \otimes q^{-h_\rho} + q^{h_\rho} \otimes E_i, \quad \Delta(F_i) = F_i \otimes q^{-h_\rho} + q^{h_\rho} \otimes F_i,
\]
\[
S(a) = -q^{-h_\rho} a q^{h_\rho}, \quad \forall a = d, h_i, E_i, F_i.
\] (2.9)

We denote by $\hat{R}$ the universal R-matrix of $U_q[G^{(k)}]$, which satisfies the graded YBE:
\[
\hat{R}_{12} \hat{R}_{13} \hat{R}_{23} = \hat{R}_{23} \hat{R}_{13} \hat{R}_{12}
\] (2.10)
and the coproduct properties:

\[
\Delta \otimes 1 \hat{R} = \hat{R}_{13} \hat{R}_{23}, \quad 1 \otimes \Delta \hat{R} = \hat{R}_{13} \hat{R}_{12},
\]
\[
\Delta^T \otimes 1 \hat{R} = \hat{R}_{23} \hat{R}_{13}, \quad 1 \otimes \Delta^T \hat{R} = \hat{R}_{12} \hat{R}_{13},
\]

(2.11)

where \(\Delta^T \equiv T \Delta\), \(T\) is the graded twist map so that for \(a = \sum_i a_i, \ b = \sum_j b_j\),

\[
T(a \otimes b) = \sum_{i,j} (-1)^{|a_i||b_j|} (b_j \otimes a_i).
\]

(2.12)

Let \(\hat{R}_{21} \equiv \hat{R}^T \equiv T \hat{R}\). Note that \((\hat{R}^T)^{-1} = (\hat{R}^{-1})^T\) also satisfies the coproduct properties (2.19). Both \(\hat{R}\) and \((\hat{R}^T)^{-1}\) satisfy the intertwining property

\[
\hat{R} \Delta(a) = \Delta^T(a) \hat{R}, \quad (\hat{R}^T)^{-1} \Delta(a) = \Delta^T(a) (\hat{R}^T)^{-1}, \quad \forall a \in U_q[\mathcal{G}^{(k)}].
\]

(2.13)

The Hopf superalgebra \(U_q[\mathcal{G}^{(k)}]\) contains two important Hopf subalgebras \(U_q^+\) and \(U_q^-\) which are generated by \\{\(E_i, h_i, d|i = 0, 1, \cdots, r\)\} and \\{\(F_i, h_i, d|i = 0, 1, \cdots, r\)\}, respectively. By Drinfeld’s quantum double construction, the universal R-matrix \(\hat{R}\) can be written in the form

\[
\hat{R} = \left( I \otimes I + \sum_t a^t \otimes a_t \right) \cdot q^{-\sum_i H^i \otimes H_i - c \otimes d - d \otimes c},
\]

(2.14)

where \(\{a^t\} \in U_q^+\), \(\{a_t\} \in U_q^-\) do not depend on \(d\), and are generated by \{\(E_i, h_i\)\} and \{\(F_i, h_i\)\}, respectively; \(c\) is given by \(\frac{\Delta c}{h} = h_0 + h_\psi\), and \{\(H^i\), \(H_i\) \((i = 1, 2, \cdots, r)\) satisfy

\[
\sum_{i=1}^r \Lambda(H^i) \Lambda'(H_i) = (\lambda, \lambda').
\]

(2.15)

Let us remark that another form of the universal R-matrix corresponding to the same coproduct reads

\[
\hat{R}' = \left( I \otimes I + \sum_t a'_t \otimes a''_t \right) \cdot q^{\sum_i H_i \otimes H^i + c \otimes d + d \otimes c},
\]

(2.16)

where \(\{a'_t\} \in U_q^-\) and \(\{a''_t\} \in U_q^+\) do not depend on \(d\). \(\hat{R}'\) can be identified with \((\hat{R}^T)^{-1}\).

Following Frenkel and Reshetikhin \(\Box\), we define the ‘normalized’ universal R-matrix by the formula

\[
\hat{R} = \hat{R} \cdot q^{c \otimes d + d \otimes c}.
\]

(2.17)

Then it can be shown that \(\hat{R}\) satisfies the following ‘normalized YBE’

\[
\hat{R}_{12} q^{-d \otimes c \otimes 1} \hat{R}_{13} q^{d \otimes c \otimes 1} \hat{R}_{23} = \hat{R}_{23} q^{-1 \otimes c \otimes d} \hat{R}_{13} q^{1 \otimes c \otimes d} \hat{R}_{12},
\]

(2.18)
and the coproduct properties:

$$(\Delta \otimes 1)R = R_{13}q^{-c \otimes d}R_{23}q^{c \otimes d}, \quad (1 \otimes \Delta)R = R_{13}q^{-d \otimes c}R_{12}q^{d \otimes c},$$

$$(\Delta^T \otimes 1)R = R_{23}q^{-1 \otimes d}R_{13}q^{1 \otimes d}, \quad (1 \otimes \Delta^T)R = R_{12}q^{-d \otimes c}R_{13}q^{d \otimes c}.$$ (2.19)

Let $z$ be a formal variable. Define an automorphism $D_z$ of $U_q[G^{(k)}]$ by

$$D_z(e_i) = z^{d_i} e_i, \quad D_z(f_i) = z^{-d_i} f_i, \quad D_z(h_i) = h_i, \quad D_z(d) = d.$$ (2.20)

It is worth noting that the automorphism $D_z$ is related to the generator $d$ by the formula:

$$D_z(a) = z^d a z^{-d}, \quad \forall a \in U_q[G^{(k)}]$$ (2.21)

which is easily checked by means of the commutation relations,

$$[d, e_i] = \frac{\delta_{i0}}{k} e_i, \quad [d, f_i] = -\frac{\delta_{i0}}{k} f_i, \quad [d, h_i] = 0.$$ (2.22)

We define a universal $R$-matrix $R(z)$ depending on the formal parameter $z$ by the formula

$$R(z) = (D_z \otimes 1)R = (1 \otimes D_{z^{-1}})R.$$ (2.23)

Then (2.18) implies the following relation for $R(z)$:

$$R_{12}(z)R_{13}(z)q^{-c_2}R_{23}(w) = R_{23}(w)R_{13}(z)q^{-c_2}R_{12}(z),$$ (2.24)

where $c_2 = 1 \otimes c \otimes 1$. One can also show that $R(z)$ enjoys

$$(S \otimes 1)(R(z)) = R(zq^{-c_1})^{-1}, \quad (1 \otimes S^{-1})(R(z)) = R(zq^{1 \otimes c})^{-1},$$ (2.25)

and $(S \otimes S)(R(z)) = R(zq^{1 \otimes c-1})$.

For a finite dimensional representation $\pi_V$ supplied by the graded vector space $V$, we define $R(z) \in End(V \otimes V)$ by

$$R(z) = (\pi_V \otimes \pi_V)R(z).$$ (2.26)

Since for any finite dimensional representation $V$, $\pi_V(c) = 0$. It follows from (2.23) that $R(z)$ obeys the graded YBE (2.1). Following Frenkel-Reshetikhin [6], we define the ‘right’ dual module $V^*$ and ‘left’ dual module $^*V$ of $V$ by

$$\pi_{V^*}(a) = \pi_V(S(a))^s t, \quad \pi_{^*V}(a) = \pi_V(S^{-1}(a))^s t,$$ (2.27)

respectively. Here $st$ is the supertransposition operation defined by

$$(A_{ab})^{st} = (-1)^{[a][b]+[d]} A_{ba}.$$ (2.28)
Note that in general \(((A_{ab})^{st})^{st} = (-1)^{|a|+|b|}A_{ab} \neq A_{ab}\). Let \(ist\) be the inverse operation of \(st\) such that \(((A_{ab})^{st})^{ist} = ((A_{ab})^{ist})^{st} = A_{ab}\). Then
\[
(A_{ab})^{ist} = (-1)^{|b|(|a|+|b|)}A_{ba} = (-1)^{|a|+|b|}(A_{ab})^{st},
\]
(2.29)
or \(A^{ist} = \eta A^{st} \eta\), where \(\eta\) is a diagonal matrix with elements \(\eta_{ab} = (-1)^{|a|} \delta_{ab}\).

By means of (2.27) and (2.28), one can show that
\[
R^{V^{\ast}\ast V}(z) = (R(z)^{-1})^{st_1}, \quad R^{V^{\ast\ast}V}(z) = (R(z)^{-1})^{st_2}.
\]
(2.30)
From the representations for \(R^{V^{\ast\ast}V}(z)\) and \(R^{V^{\ast\ast}V}(z)\), and the formulae for the action of the square of antipode:
\[
S^2(a) = q^{-2h_\rho} D_{q^{-2\rho}}(a) q^{2h_\rho}, \quad S^{-2}(a) = q^{2h_\rho} D_{q^{2\rho}}(a) q^{-2h_\rho}, \quad \forall a \in U_q[\mathcal{G}(k)],
\]
(2.31)
which can be checked on the generators [remembering that the simple roots associated with \(e_0, f_0\) are \(\alpha_0 = \pm (\frac{1}{2} \delta - \psi)\), respectively, where \(\delta = (0, 0, 1)\)], it follows that for any finite dimensional representation \(V\), the R-matrix satisfies the following crossing-unitarity relations:
\[
\begin{align*}
((R(z)^{-1})^{st_1})^{-1}^{st_1} &= (\pi_V(q^{-2h_\rho}) \otimes 1)((R(z q^{-2\rho}))^{st_1} \pi_V(q^{2h_\rho}) \otimes 1), \\
((R(z)^{-1})^{st_2})^{-1}^{st_2} &= (1 \otimes \pi_V(q^{2h_\rho}))((R(z q^{2\rho}))^{st_2} (1 \otimes \pi_V(q^{-2h_\rho})).
\end{align*}
\]
(2.32)
Note also that
\[
(\pi_V(q^{-2h_\rho}) \otimes \pi_V(q^{2h_\rho})) R(z) = R(z)(\pi_V(q^{2h_\rho}) \otimes \pi_V(q^{-2h_\rho})).
\]
(2.33)
We introduce the graded permutation operator \(P\) on the tensor product module \(V \otimes V\):
\[
P(v_\alpha \otimes v_\beta) = (-1)^{[\alpha][\beta]}(v_\beta \otimes v_\alpha), \quad \forall v_\alpha, v_\beta \in V.
\]
Using the irreducibility of \(V(z) \otimes V\), where \(\pi_V(z)(a) = \pi_V(D_z(a)) \forall a \in U_q[\mathcal{G}(k)]\), and the crossing-unitarity relations (2.32), one can show that the R-matrix \(R(z)\) is unitary, that is,
\[
R_{12}(\frac{z}{w})\overline{R}_{21}(\frac{w}{z}) = 1,
\]
(2.34)
where \(R_{21}(z) = P_{12}R_{12}(z)P_{12}\).

Now we are going to realize the super RS algebra in \(U_q[\mathcal{G}(k)]\). Let
\[
L^+(z) = (\pi_V \otimes 1)\mathcal{R}(z q^{\frac{\psi}{2}}),
\]
\[
L^-(z) = (\pi_V \otimes 1)\mathcal{R}_{21}(z^{-1} q^{-\frac{\psi}{2}})^{-1},
\]
(2.35)
where $c_2 = 1 \otimes c$. It is worth noting that our definition (2.39) of $L^\pm(z)$ is different from that of Frenkel and Reshetikhin \[8]. Then, from (2.24) and (2.34) we obtain:

$$R(\frac{z}{w})L^+_1(z)L^+_2(w) = L^+_2(w)L^+_1(z)R(\frac{z}{w}),$$
$$R(\frac{z+}{w_-})L^+_1(z)L^+_2(w) = L^+_2(w)L^+_1(z)R(\frac{z+}{w_+}),$$ (2.36)

which are nothing but the defining relations (2.3) of the super RS algebra $U(\mathcal{R})$. Moreover, from the formulae for the action of the coproduct and antipode on $\mathcal{R}(z)$ we derive:

$$\Delta^\tau(L^\pm(z) = L^\pm(z q^{\pm 1 \otimes \frac{1}{2}}) \otimes L^\pm(z q^{\mp 1 \otimes \frac{1}{2}}),$$
$$S^{-1}(L^\pm(z)) = L^\pm(z)^{-1}. \quad (2.37)$$

We thus arrive at

**Proposition 1**: Equations (2.37) give a realization of $U(\mathcal{R})$ in $U_q[\mathcal{G}^{(k)}]$, with the opposite Hopf algebra structure on $U_q[\mathcal{G}^{(k)}]$.

Now let $\mathcal{R}(z)$ be the R-matrix associated with the minimal $M$-dimensional defining representation $V$ of $U_q(\mathcal{G})$. We have

**Theorem 1**: $L^\pm(z)$ has the following unique Gauss decomposition

$$L^\pm(z) = \begin{pmatrix}
1 & \cdots & 0 \\
\varepsilon^+_2,1(z) & \ddots \\
\varepsilon^+_3,1(z) & \ddots & \vdots \\
\vdots & \ddots & \ddots \\
\varepsilon^+_M,1(z) & \cdots & \varepsilon^+_M,M-1(z) & 1
\end{pmatrix}
\times
\begin{pmatrix}
1 & f^+_1(z) & \cdots & f^+_1,M(z) \\
\vdots & \ddots & \cdots & \vdots \\
0 & \cdots & f^+_M-1,M(z) \\
& & & 1
\end{pmatrix}, \quad (2.38)$$

where $\varepsilon^\pm_{i,j}(z), f^\pm_{j,i}(z)$ and $k^\pm_i(z)$ ($i > j$) are elements in the super RS algebra and $k^\pm_i(z)$ are invertible. Let \[9\]

$$X^+_i(z) = f^+_i,i+1(z+) - f^-_{i,i+1}(z_-),$$
$$X^-_i(z) = e^+_i,i+1(z+) - e^-_{i,i+1}(z_-), \quad (2.39)$$

\footnote{Note that our notations here for $X^+_i(z)$ are different from those used in previous papers \[1, 7\] where $X^+_i(z)$ are defined as $X^+_i(z) = e^+_i,i+1(z_-) - e^+_i+1,i(z_+)$ and $X^-_i(z) = f^+_i,i+1(z+) - f^-_{i,i+1}(z_-)$, respectively.}
where \( z_\pm = z^q \pm \bar{z}^q \), then the defining relations of \( U_q[G(k)] \) can be derived, through difference combinations, from relations satisfied by \( q^{\pm \bar{z}} \), \( X_i^\pm(z) \), \( k_j^\pm(z) \), \( i = 1, 2, \ldots, M - 1 \), \( j = 1, 2, \ldots, M \).

In particular, for the R-matrix \( R(z) \) associated with the \((m+n)\)-dimensional representation of \( U_q[gl(m|n)] \), we have

**Theorem 2** [4]: Then the relations satisfied by \( \{ q^{\pm \bar{z}}, X_i^\pm(z), k_j^\pm(z), i = 1, 2, \cdots, m + n - 1, j = 1, 2, \cdots, m + n \} \) are nothing but the defining relations of \( U_q[gl(m|n)(1)] \).

**Remark**: The theorems are supersymmetric generalizations of that of Ding-Frenkel [3] for the bosonic case. The Gauss decomposition implies that the elements \( e_{i,j}^\pm(z) \), \( f_{j,i}^\pm(z) \) \((i > j)\) and \( k_i^\pm(z) \) are uniquely determined by \( L^\pm(z) \). In the following we will denote \( f_{i,i+1}^\pm(z) \), \( e_{i+1,i}^\pm(z) \) as \( f_i^\pm(z) \), \( e_i^\pm(z) \), respectively.

### 3 Ungrading Multiplication Rule of Tensor Products

We define the matrix elements of \( R(z) \) and \( L^\pm(z) \) by

\[
R(z)(v_{\alpha'} \otimes v_{\beta'}) = R(z)_{\alpha\beta,\alpha'\beta'}(v_{\alpha} \otimes v_{\beta}), \quad \quad \quad L^\pm(z)v_{\alpha'} = L^\pm(z)_{\alpha\alpha'} v_{\alpha}. \tag{3.1}
\]

In matrix form, (2.3) carries extra signs due to the graded multiplication rule of tensor products:

\[
R(\frac{z}{w})_{\alpha\beta,\alpha'\beta'} L^\pm(z)_{\alpha'\alpha} L^\pm(w)_{\beta\gamma,\beta'} (-1)^{|\alpha'|(|\beta|)+|\beta'|}) = L^\pm(w)_{\beta\gamma,\beta'} L^\pm(z)_{\alpha\alpha'} R(\frac{z}{w})_{\alpha'\beta',\alpha'\beta'} (-1)^{|\alpha|(|\beta|)+|\beta'|}),
\]

\[
R(\frac{z}{w})_{\alpha\beta,\alpha'\beta'} L^\pm(z)_{\alpha'\alpha} L^\pm(w)_{\beta\gamma,\beta'} (-1)^{|\alpha'|(|\beta|)+|\beta'|}) = L^\pm(w)_{\beta\gamma,\beta'} L^\pm(z)_{\alpha\alpha'} R(\frac{z}{w})_{\alpha'\beta',\alpha'\beta'} (-1)^{|\alpha|(|\beta|)+|\beta'|}). \tag{3.2}
\]

We introduce matrix \( \theta \):

\[
\theta_{\alpha\beta,\alpha'\beta'} = (-1)^{|\alpha|(|\beta|)} \delta_{\alpha\alpha'} \delta_{\beta\beta'}. \tag{3.3}
\]

With the help of this matrix \( \theta \), one can cast (3.2) into the usual matrix equations,

\[
R(\frac{z}{w}) L^\pm_1(z) \theta L^\pm_2(w) \theta = \theta L^\pm_2(w) \theta L^\pm_1(z) R(\frac{z}{w}),
\]

\[
R(\frac{z}{w}) L^\pm_1(z) \theta L^\pm_2(w) \theta = \theta L^\pm_2(w) \theta L^\pm_1(z) R(\frac{z}{w}). \tag{3.4}
\]
Now the multiplications in (3.4) are simply the usual matrix multiplications, that is the tensor products in (3.4) carry no grading.

The following matrix equations can be deduced from (3.4):

\[
\begin{align*}
R_{21}\left(\frac{z}{w}\right)\theta L_2^+(z)\theta L_1^+(w) &= L_1^+(w)\theta L_2^+(z)\theta R_{21}\left(\frac{z}{w}\right), \\
R_{21}\left(\frac{z}{w}\right)\theta L_2^+(z)\theta L_1^+(w) &= L_1^-(w)\theta L_2^-(z)\theta R_{21}\left(\frac{z}{w}\right), \\
R_{21}\left(\frac{z}{w}\right)\theta L_2^-(z)\theta L_1^+(w) &= L_1^+(w)\theta L_2^-(z)\theta R_{21}\left(\frac{z}{w}\right),
\end{align*}
\]

(3.5) \quad (3.6) \quad (3.7)

\[
\begin{align*}
\theta L_2^-(z)^{-1}\theta L_1^+(w)^{-1}R_{21}\left(\frac{z}{w}\right) &= R_{21}\left(\frac{z}{w}\right)L_1^+(w)^{-1}\theta L_2^-(z)^{-1}\theta, \\
\theta L_2^+(z)^{-1}\theta L_1^+(w)^{-1}R_{21}\left(\frac{z}{w}\right) &= R_{21}\left(\frac{z}{w}\right)L_1^-(w)^{-1}\theta L_2^+(z)^{-1}\theta, \\
\theta L_2^-(z)^{-1}\theta L_1^+(w)^{-1}R_{21}\left(\frac{z}{w}\right) &= R_{21}\left(\frac{z}{w}\right)L_1^+(w)^{-1}\theta L_2^-(z)^{-1}\theta,
\end{align*}
\]

(3.8) \quad (3.9) \quad (3.10)

\[
\begin{align*}
L_1^+(w)^{-1}R_{21}\left(\frac{z}{w}\right)\theta L_2^+(z)\theta &= \theta L_2^+(z)\theta R_{21}\left(\frac{z}{w}\right)L_1^+(w)^{-1}, \\
L_1^-(w)^{-1}R_{21}\left(\frac{z}{w}\right)\theta L_2^+(z)\theta &= \theta L_2^+(z)\theta R_{21}\left(\frac{z}{w}\right)L_1^-(w)^{-1}, \\
L_1^+(w)^{-1}R_{21}\left(\frac{z}{w}\right)\theta L_2^-(z)\theta &= \theta L_2^+(z)\theta R_{21}\left(\frac{z}{w}\right)L_1^+(w)^{-1}.
\end{align*}
\]

(3.11) \quad (3.12) \quad (3.13)

As in (3.4), the multiplications in (3.5 – 3.13) are usual matrix multiplications.

4 Drinfeld Current Realization of $U_q[osp(1|2)^{(1)}]$ 

We take $R(\frac{z}{w}) \in \text{End}(V \otimes V)$ to be the R-matrix with $V$ being the 3-dimensional vector representation of $U_q(osp(1|2))$. Let basis vectors $v_1, v_3$ be even and $v_2$ odd. The R-matrix has the following form:

\[
R(\frac{z}{w}) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a & 0 & b & 0 & 0 & 0 & 0 \\
0 & 0 & d & 0 & c & 0 & r & 0 \\
0 & f & 0 & a & 0 & 0 & 0 & 0 \\
0 & 0 & g & 0 & e & 0 & c & 0 \\
0 & 0 & 0 & 0 & a & 0 & b & 0 \\
0 & 0 & s & 0 & g & 0 & d & 0 \\
0 & 0 & 0 & 0 & f & 0 & a & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix},
\]

(4.1)

where

\[
a = \frac{q(z - w)}{zq^2 - w}, \quad b = \frac{w(q^2 - 1)}{zq^2 - w}, \quad c = \frac{q^{1/2}w(q^2 - 1)(z - w)}{(zq^2 - w)(zq^3 - w)}.
\]
\begin{align*}
d &= \frac{q^2(z-w)(qz-w)}{(qz^2-w)(qz^3-w)}, \\
e &= a - \frac{zw(q^2-1)(q^3-1)}{(qz^2-w)(qz^3-w)}, \\
f &= \frac{z(q^2-1)}{qz^2-w}, \\
g &= -\frac{q^{5/2}z(q^2-1)(z-w)}{(qz^2-w)(qz^3-w)}, \\
r &= \frac{w(q^2-1)[q^2z + q(z-w)-w]}{(qz^2-w)(qz^3-w)}, \\
s &= \frac{z(q^2-1)[q^2z + q^2(z-w)-w]}{(qz^2-w)(qz^3-w)}. \\
\end{align*}

\(R_{21}(\frac{z}{w}) = R(\frac{w}{z})^{-1}\) takes the form

\[
R_{21}(\frac{z}{w}) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a & 0 & f & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & d & 0 & -g & 0 & s & 0 & 0 \\
0 & b & 0 & a & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -c & 0 & e & 0 & -g & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a & 0 & f & 0 \\
0 & 0 & r & 0 & -c & 0 & d & 0 & 0 \\
0 & 0 & 0 & 0 & b & 0 & a & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}.
\tag{4.3}
\]

We will construct Drinfeld current realization of \(U_q[osp(1|2)^{(1)}]\). We first note that in the present case, the theorem \[\text{[\ref{4}]}\] implies the following decomposition for \(L^\pm(z)\):

\[
L^\pm(z) = \begin{pmatrix}
1 & 0 & 0 \\
e_1^\pm(z) & 1 & 0 \\
e_{3,1}^\pm(z) & e_2^\pm(z) & 1
\end{pmatrix}
\begin{pmatrix}
k_1^\pm(z) & 0 & 0 \\
0 & k_2^\pm(z) & 0 \\
0 & 0 & k_3^\pm(z)
\end{pmatrix}
\begin{pmatrix}
1 & f_1^\pm(z) & f_{1,3}^\pm(z) \\
0 & 1 & f_2^\pm(z) \\
0 & 0 & 1
\end{pmatrix}
= \begin{pmatrix}
k_1^\pm(z) & k_1^\pm(z)f_1^\pm(z) & k_1^\pm(z)f_{1,3}^\pm(z) \\
e_1^\pm(z)k_1^\pm(z) & k_2^\pm(z) + e_1^\pm(z)k_1^\pm(z) & f_1^\pm(z) \\
e_{3,1}^\pm(z)k_1^\pm(z) & v^\pm & x^\pm
\end{pmatrix},
\tag{4.4}
\]

where,

\[
\begin{align*}
u^\pm &= k_2^\pm(z)f_2^\pm(z) + e_1^\pm(z)k_1^\pm(z)f_{1,3}^\pm(z), \\
v^\pm &= e_2^\pm(z)k_2^\pm(z) + e_{3,1}^\pm(z)k_1^\pm(z)f_1^\pm(z), \\
x^\pm &= k_3^\pm(z) + e_2^\pm(z)k_2^\pm(z)f_2^\pm(z) + e_{3,1}^\pm(z)k_1^\pm(z)f_{1,3}^\pm(z).
\end{align*}
\tag{4.5}
\]

The inversions \(L^\pm(z)^{-1}\) of (4.4) are easily seen to be

\[
L^\pm(z)^{-1} = \begin{pmatrix}
y^\pm & \tilde{x}^\pm & \tilde{u}^\pm \\
\tilde{y}^\pm & k_2^\pm(z)^{-1} + f_2^\pm(z)k_3^\pm(z)^{-1}e_2^\pm(z) & -f_2^\pm(z)k_3^\pm(z)^{-1} \\
\tilde{v}^\pm & -k_3^\pm(z)^{-1}e_2^\pm(z) & k_3^\pm(z)^{-1}
\end{pmatrix},
\tag{4.6}
\]
where

\[
\begin{align*}
\tilde{u}^\pm &= [f_2^\pm(z)f_2^\pm(z) - f_3^\pm(z)]k_3^\pm(z)^{-1}, \\
\tilde{v}^\pm &= k_3^\pm(z)^{-1}[e_2^\pm(z)e_1^\pm(z) - e_3^\pm(z)], \\
y^\pm &= k_1^\pm(z)^{-1} + f_1^\pm(z)k_2^\pm(z)^{-1}e_1^\pm(z) + [f_1^\pm(z)f_2^\pm(z) - f_1^\pm(z)] \\
&\quad \times k_3^\pm(z)^{-1}[e_2^\pm(z)e_1^\pm(z) - e_3^\pm(z)], \\
\tilde{x}^\pm &= -f_1^\pm(z)k_2^\pm(z)^{-1} + [f_1^\pm(z) - f_1^\pm(z)f_2^\pm(z)]k_3^\pm(z)^{-1}e_2^\pm(z), \\
y^\pm &= -k_2^\pm(z)^{-1}e_1^\pm(z) + f_2^\pm(z)k_3^\pm(z)^{-1}e_1^\pm(z) - e_2^\pm(z)e_1^\pm(z). \quad (4.7)
\end{align*}
\]

By means of (3.4, 3.3 - 3.13) and after tedious calculations, we derive

\[
\begin{align*}
k_1^\pm(z)k_1^\pm(w) &= k_1^\pm(w)k_1^\pm(z), \\
k_1^\pm(z)k_1^\mp(w) &= k_1^\mp(w)k_1^\pm(z), \\
k_2^\pm(z)k_2^\pm(w) &= k_2^\pm(w)k_2^\pm(z), \\
k_3^\pm(z)k_3^\pm(w) &= k_3^\pm(w)k_3^\pm(z), \\
k_3^\pm(z)k_3^\mp(w) &= k_3^\mp(w)k_3^\pm(z), \\
k_2^\pm(z)k_2^\mp(w) &= k_2^\mp(w)k_2^\pm(z), \\
\frac{z_\pm - w_\pm}{z_\mp q^2 - w_\pm}k_1^\pm(z)k_2^\pm(w) &= \frac{z_\mp - w_\pm}{z_\mp q^2 - w_\pm}k_2^\pm(w)k_1^\pm(z), \\
\frac{z_\pm - w_\pm}{z_\mp q^2 - w_\pm}k_1^\pm(z)k_3^\pm(w)^{-1} &= \frac{z_\mp - w_\pm}{z_\mp q^2 - w_\pm}k_3^\pm(w)^{-1}k_1^\pm(z), \\
\frac{z_\pm - w_\pm q}{z_\mp q - w_\mp}k_3^\pm(z)k_2^\pm(w) &= \frac{z_\mp - w_\pm q}{z_\mp q - w_\mp}k_2^\pm(w)k_3^\pm(z), \\
\frac{z_\pm - w_\pm}{z_\mp q^2 - w_\mp}k_2^\pm(z)k_3^\pm(w)^{-1} &= \frac{z_\mp - w_\pm}{z_\mp q^2 - w_\mp}k_3^\pm(w)^{-1}k_2^\pm(z)^{-1}, \\
\frac{z_\pm - w_\pm q}{z_\mp q^2 - w_\mp}k_3^\pm(z)k_3^\pm(w) &= \frac{z_\mp - w_\pm q}{z_\mp q^2 - w_\mp}k_3^\pm(w)k_3^\pm(z), \\
\frac{z_\pm - w_\pm}{z_\mp q^2 - w_\mp}k_3^\pm(z)k_3^\pm(w)^{-1} &= \frac{z_\mp - w_\pm}{z_\mp q^2 - w_\mp}k_3^\pm(w)^{-1}k_3^\pm(z)^{-1}. \quad (4.8)
\end{align*}
\]
\[ k_1^+(z)X_2^-(w)k_1^+(z)^{-1} = \frac{z \pm q^3 - w}{q(z \pm q - w)} X_2^-(w), \]
\[ k_1^+(z)^{-1}X_2^+(w)k_1^+(z) = \frac{z \pm q^3 - w}{q(z \pm q - w)} X_2^+(w), \]
\[ k_2^+(z)X_2^-(w)k_2^+(z)^{-1} = \frac{(z \pm wq)(z \pm q^2 - w)}{q(z \pm q - w)(z \pm - w)} X_2^-(w), \]
\[ k_2^+(z)^{-1}X_2^+(w)k_2^+(z) = \frac{(z \pm wq)(z \pm q^2 - w)}{q(z \pm q - w)(z \pm - w)} X_2^+(w), \]
\[ k_3^+(z)X_2^-(w)k_3^+(z)^{-1} = \frac{z \pm wq^2}{q(z \pm w - w)} X_2^-(w), \]
\[ k_3^+(z)^{-1}X_2^+(w)k_3^+(z) = \frac{z \pm wq^2}{q(z \pm w - w)} X_2^+(w), \]

(4.9)

\[ \frac{z - w}{zq^2 - w} X_1^-(z)X_2^-(w) + \frac{z - wq}{zq^3 - w} X_2^-(w)X_1^-(z) = 0, \]
\[ \frac{z - wq}{zq - w} X_1^-(z)X_1^-(w) + \frac{z - wq^2}{zq^2 - w} X_1^-(w)X_1^-(z) = 0, \]
\[ \frac{z - wq}{zq - w} X_2^-(z)X_2^-(w) + \frac{z - wq^2}{zq^2 - w} X_2^-(w)X_2^-(z) = 0, \]
\[ \frac{z - wq}{zq^3 - w} X_1^+(z)X_2^+(w) + \frac{z - w}{zq^2 - w} X_2^+(w)X_1^+(z) = 0, \]
\[ \frac{z - wq^2}{zq^2 - w} X_1^+(z)X_1^+(w) + \frac{z - wq}{zq - w} X_1^+(w)X_1^+(z) = 0, \]
\[ \frac{z - wq^2}{zq^2 - w} X_2^+(z)X_2^+(w) + \frac{z - wq}{zq - w} X_2^+(w)X_2^+(z) = 0, \]

(4.10)

\[ \{X_1^-(w), X_1^+(z)\} = (q - q^{-1}) \left[ -\delta\left(\frac{z}{w}q^c\right) k_2^+(z_+) k_1^+(z_+)^{-1} \right. \]
\[ \left. + \delta\left(\frac{z}{w}q^{-c}\right) k_2^-(w_+) k_1^-(w_+)^{-1} \right], \]

\[ \{X_2^-(w), X_2^+(z)\} = (q - q^{-1}) \left[ \delta\left(\frac{z}{w}q^c\right) k_3^+(z_+) k_2^+(z_+)^{-1} \right. \]
\[ \left. - \delta\left(\frac{z}{w}q^{-c}\right) k_3^-(w_+) k_2^-(w_+)^{-1} \right], \]

\[ \{X_2^-(w), X_1^+(z)\} = (q - q^{-1}) q^{1/2} \left[ -\delta\left(\frac{z}{w}q^{c+1}\right) k_2^+(z_+) k_1^+(z_+)^{-1} \right. \]
\[ \left. + \delta\left(\frac{z}{w}q^{-c+1}\right) k_2^-(w_+) k_1^-(w_+)^{-1} \right], \]

\[ \{X_1^-(z), X_2^+(w)\} = (q - q^{-1}) q^{1/2} \left[ \delta\left(\frac{w}{z}q^{-c-1}\right) k_2^+(z_-) k_1^+(z_-)^{-1} \right. \]
\[ \left. - \delta\left(\frac{w}{z}q^{c-1}\right) k_3^-(w_-) k_2^-(w_-)^{-1} \right], \]

(4.11)

where \(\{X, Y\} \equiv XY + YX\) denotes an anti-commutator, and

\[ \delta(z) = \sum_{l \in \mathbb{Z}} z^l \]  

(4.12)
is a formal series. \( \delta(z) \) enjoys the following properties:

\[
\delta\left(\frac{z}{w}\right) = \delta\left(\frac{w}{z}\right), \quad \delta\left(\frac{z}{w}\right)f(z) = \delta\left(\frac{z}{w}\right)f(w).
\]  

The last relation in (4.11) can be recast into the following form

\[
\{X_1^-(w), X_2^+(z)\} = (q - q^{-1})q^{\frac{1}{2}} \left[ \delta\left(\frac{z}{w}q^{-1}\right)k_2^+(z+q^{-1})k_1^+(z+q^{-1})^{-1} - \delta\left(\frac{z}{w}q^{-1}\right)k_3^-(w+q)k_2^-(w+q)^{-1} \right].
\]  

We define the following difference combinations:

\[
X^\pm(z) = (q - q^{-1}) \left[ X_1^\pm(z) + X_2^\pm(zq) \right].
\]  

Then (4.19 – 4.14) can be rewritten as

\[
k_1^+(z)X_1^-(w)k_1^+(z)^{-1} = \frac{zq^2-w}{q(z_q-w)}X^-(w),
k_1^+(z)X_2^+(w)k_1^+(z) = \frac{zq^2-w}{q(z_q-w)}X^+(w),
k_2^+(z)X_1^-(w)k_2^+(z)^{-1} = \frac{(z_q-w)q^2(z_q-w)q-w}{q(z_q-w)(z_q-w)q}X^-(w),
k_2^+(z)X_2^+(w)k_2^+(z) = \frac{(z_q-w)q^2(z_q-w)q-w}{q(z_q-w)q(z_q-w)q}X^+(w),
k_3^+(z)X_1^-(w)k_3^+(z)^{-1} = \frac{zq-wq^3}{q(z_q-w)q}X^-(w),
k_3^+(z)X_2^+(w)k_3^+(z) = \frac{zq-wq^3}{q(z_q-w)q}X^+(w),
\]  

\[
\frac{z-wq}{zq-w}X^-(z)X_1^-(w) + \frac{z-wq}{zq^2-w}X^-(w)X^-(z) = 0,
\frac{z-wq^2}{zq^2-w}X^+(z)X_2^+(w) + \frac{z-wq}{zq-w}X^+(w)X^+(z) = 0,
\]

\[
\{X^-(w), X^+(z)\} = \frac{-1}{q-q^{-1}} \left[ \delta\left(\frac{z}{w}q^{-e}\right) \left( (1 + q^{-\frac{1}{2}} - q^{\frac{1}{2}})k_2^+(z_q)k_1^+(z_q)^{-1} - k_3^+(z_q+k)k_2^+(z_q+k)^{-1} \right) - \delta\left(\frac{z}{w}q^{-e}\right) \left( k_2^-(w_q)k_1^-\left(w_q\right)^{-1} - (1 + q^{-\frac{1}{2}} - q^{\frac{1}{2}})k_3^-\left(w_q\right)k_2^-\left(w_q\right)^{-1} \right) \right].
\]  

Further defining

\[
\phi_i(z) = k_i^{+}(z)k_i^{-}(z)^{-1},
\psi_i(z) = k_{i+1}^{+}(z)k_i^{-}(z)^{-1}, \quad i = 1, 2,
\phi(z) = (1 + q^{-\frac{1}{2}} - q^{\frac{1}{2}})\phi_1(z) - \phi_2(zq),
\psi(z) = \psi_1(z) - (1 + q^{-\frac{1}{2}} - q^{\frac{1}{2}})\psi_2(zq),
\]

then we have
**Theorem 3**: \( q^{\pm \frac{\zeta}{\zeta}}, X^\pm(z), \phi(z), \psi(z) \) give the defining relations of \( U_q[osp(1\vert 2)^{(1)}] \). More precisely, \( U_q[osp(1\vert 2)^{(1)}] \) is an associative algebra with unit 1 and the Drinfeld generators: \( X^\pm(z), \phi(z) \) and \( \psi(z) \), a central element \( c \) and a nonzero complex parameter \( q \). \( \phi(z) \) and \( \psi(z) \) are invertible. The gradings of the generators are: \([X^\pm(z)] = 1\) and \([\phi(z)] = [\psi(z)] = [c] = 0\). The relations are given by

\[
\begin{align*}
\phi(z)\phi(w) &= \phi(w)\phi(z), \\
\psi(z)\psi(w) &= \psi(w)\psi(z), \\
\phi(z)\psi(w)\phi(z)^{-1}\psi(w)^{-1} &= \frac{(z_q - w_q)(z_q + w_q)(z_q - w)(z_q^2 - w)}{(z_q - w_q)(z_q - w)(z_q^2 - w)}, \\
\phi(z)X^-(w)\phi(z)^{-1} &= \frac{(z_q - w_q)(z_q + w_q)}{(z_q^2 - w)(z_q - w)} X^-(w), \\
\phi(z)^{-1}X^+(w)\phi(z) &= \frac{(z_q - w_q)(z_q - w)}{(z_q^2 - w)(z_q - w)} X^+(w), \\
\psi(z)X^-(w)\psi(z)^{-1} &= \frac{(z_q - w_q)(z_q - w)}{(z_q - w_q)(z_q^2 - w)} X^-(w), \\
\psi(z)^{-1}X^+(w)\psi(z) &= \frac{(z_q - w_q)(z_q + w_q)}{(z_q^2 - w)(z_q^2 - w)} X^+(w), \\
\frac{z - w q}{z q - w} X^-(z) X^-(w) + \frac{z - w q^2}{z q^2 - w} X^-(w) X^-(z) &= 0, \\
\frac{z - w q^2}{z q^2 - w} X^+(z) X^+(w) + \frac{z - w q}{z q - w} X^+(w) X^+(z) &= 0, \\
\{X^+(z), X^-(w)\} &= \frac{1}{q - q^{-1}} \left[ \delta(w_q)\psi(w_q) - \delta(w_q)\phi(z_q) \right]. \quad (4.20)
\end{align*}
\]

Super-Yangian doubles with center \( DY_h[G^{(k)}] \) are the degenerated cases of the corresponding quantum affine superalgebras \( U_q[G^{(k)}] \). The Drinfeld realization of \( DY_h[G^{(k)}] \) can also be obtained by means of the super RS algebra with a rational R-matrix \( R(z) \) and the Gauss decomposition theorem. In [4], we gave the defining relations of \( DY_h[gl(m\vert n)^{(1)}] \) in terms of Drinfeld current generators (see also [8] for the simplest case of \( m = n = 1 \)). We now introduce \( DY_h[osp(1\vert 2)^{(1)}] \).

**Theorem 4**: \( DY_h[osp(1\vert 2)^{(1)}] \) is an associative algebra over the ring of formal power series in the variable \( h \) and the Drinfeld generators: \( X^\pm(u), \phi(u) \) and \( \psi(u) \), and a central element \( c \). \( \phi(u) \) and \( \psi(u) \) are invertible. The gradings of the generators are: \([X^\pm(u)] = 1\) and \([\phi(u)] = [\psi(u)] = [c] = 0\). The defining relations are given by

\[
\begin{align*}
\phi(u)\phi(v) &= \phi(v)\phi(u), \\
\psi(u)\psi(v) &= \psi(v)\psi(u), \\
\phi(u)\psi(v)\phi(u)^{-1}\psi(v)^{-1} &= \frac{(u_+ - u_- + h)(u_+ - u_- - h)(u_+ - u_- - 2h)(u_+ - u_- + 2h)}{(u_+ - u_- - h)(u_+ - u_- + h)(u_+ - u_- + 2h)(u_+ - u_- - 2h)},
\end{align*}
\]
\[
\phi(u)X^-(v)\phi(u)^{-1} = \frac{(u_+ - v - 2h)(u_+ - v + \hbar)}{(u_+ - v + 2h)(u_+ - v - \hbar)} X^-(w), \\
\phi(u)^{-1}X^+(v)\phi(u) = \frac{(u_- - v - 2h)(u_- - v + \hbar)}{(u_- - v + 2h)(u_- - v - \hbar)} X^+(w), \\
\psi(u)X^-(v)\psi(u)^{-1} = \frac{(u_- - v - 2h)(u_- - v + \hbar)}{(u_- - v + 2h)(u_- - v - \hbar)} X^-(w), \\
\psi(u)^{-1}X^+(v)\psi(u) = \frac{(u_+ - v - 2h)(u_+ - v + \hbar)}{(u_+ - v + 2h)(u_+ - v - \hbar)} X^+(w),
\]

\[
\begin{align*}
\frac{u - v - \hbar}{u - v + \hbar} X^-(u)X^-(v) + \frac{u - v - 2\hbar}{u - v + 2\hbar} X^-(v)X^-(u) &= 0, \\
\frac{u - v - 2\hbar}{u - v + 2\hbar} X^+(u)X^+(v) + \frac{u - v + \hbar}{u - v - \hbar} X^+(v)X^+(u) &= 0,
\end{align*}
\]

\[
\{X^+(u), X^-(v)\} = \frac{1}{2\hbar} [\delta(u_- - v_+)\psi(v_+) - \delta(u_+ - v_-)\phi(u_+)], \tag{4.21}
\]

where \(u_\pm = u \pm \frac{1}{2}\hbar c\) and

\[
\delta(u - v) = \sum_{l \in \mathbb{Z}} u^l v^{l-1} \tag{4.22}
\]

is a formal series.

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