Pseudo-global Fitting of Gapped Helioseismic Data

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Abstract. Mode fitting or “peak-bagging” is an important procedure in helioseismology allowing one to determine the various mode parameters of solar oscillations. We have recently developed a new “pseudo-global” fitting algorithm as a way of reducing the systematic bias in the fits of certain mode parameters that are seen when using “local” fitting techniques to analyse “sun-as-a-star” p-mode data. This new algorithm has been designed specifically to gain the advantages of fitting the entire power spectrum, while retaining the efficiency of local fitting techniques.

Using simulated data with a full fill we have previously shown that the pseudo-global routine reduces the bias in estimates of the frequencies and asymmetries and in the estimates of the solar background when compared with a traditional fitting technique. Here we present results that show that the pseudo-global routine is also effective in reducing bias in the parameter estimates when the time-series has significant gaps. As such we are now able to employ the routine in order to fit ground based helioseismic data such as that collected by the Birmingham Solar Oscillations Network (BiSON).

1. Introduction

Being able to determine accurate estimates of the various solar mode parameters is an important goal in helioseismology. Using inversion techniques frequencies of the oscillation modes can be used to constrain estimates of the sound speed and density in the solar interior, while the rotational splitting of the modes can determine the internal rotation rate.

Over the years the quality and quantity of helioseismic data has improved significantly enabling the parameter values to be constrained with increasingly greater precision. However, with greater precision comes the need for greater accuracy and this can only be archived if the models used to fit the data are also accurate.

For low-degree (low-\(\ell\)) Sun-as-a-star observations, the most common method of fitting is to split the power spectrum into a series of fitting regions centered on \(\ell = 0/2\) and \(1/3\) pairs. The modes are then fitted, pair by pair, to determine the parameter values without the need to fit the entire spectrum simultaneously.

The main advantage of this “pair-by-pair” fitting method (hereafter abbreviated PPM) is its computational efficiency, since the number of parameters being varied is small. However, the cost of this efficiency is that the model used
to fit the data encompasses only those modes within the fitting region. Hence, any power from modes whose central frequencies lie outside this region will not be accounted for. This imperfect match between the fitting model and the underlying profile of the data can lead to significant biases in some of the fitted parameters.

In Fletcher et al. (2008a,b) (hereafter paper’s I and II) we introduced a modified fitting routine that takes into account the effect of modes that lie outside the fitting regions. This was done by employing a model which is valid for the entire spectrum. However, in order to retain computational efficiency only the parameters associated with those modes within the fitting region were allowed to vary. Hence we refer to this technique as a pseudo-global fitting method (hereafter abbreviated PGM).

In this paper we modify the PGM method further in order to fit spectra made from time series that have significant gaps. An overview of the changes made are given in section 2. Using simulated data we show that the PGM remains a robust fitting strategy. Being able to fit gap-affected data allows us to test the PGM on ground based data such as that collected by the Birmingham Solar Oscillations Network (BiSON). Results of the fits to both the simulated and BiSON data, along with comparisons with fits from the traditional PPM are given in Section 4.

2. Fitting Techniques

The PPM and PGM routines were described at length in Paper’s I and II so only the details of modifications made to treat gap-affected data are given here. The best way of accounting for gaps is to convolve the power spectrum fitting model with the spectral window and this technique was incorporated in both the PPM and PGM routines.

For the PPM, this is a fairly straight-forward modification. However, in the case of the PGM the convolution with the spectral window is a little more complicated. The background is accounted for by fitting a model to the low and high frequency ends of the power spectrum, well away from the regimes of the p-modes. Two background sources need to be considered for the model. The first of these is a granulation-like component that mimics the solar velocity continuum. The power spectral density of which may be described by a power-law model (see Harvey 1985). Secondly, a frequency independent term, $\sigma_s$, is added to account for the uncertainty in the velocity measurements caused by photon shot noise. Finally, when gapped data is being considered, the full model used to fit the background is convolved with the spectral window, $W$. Hence, the full power spectral density of the background, $n(\nu)$, is given by:

$$n(\nu) = \left[ \frac{2\sigma_g^2 \tau_g}{1 + (2\pi \nu \tau_g)^2} + \sigma_s \right] \otimes W$$

(1)

where the values of $\tau_g$ and $\sigma_g$ give the lifetime and amplitude of the granulation profile. Depending on how low in frequency one fits, further terms may be required to account for meso- and super-granulation.

Once the background fit is performed the main mode-fitting process can begin. A starting model is generated for the entire spectrum and is derived
from the results of an initial run of the pair-by-pair method and the results of the background fit. For gapped data this model is then convolved with the spectral window and fitted to the data, allowing only the parameters associated with the modes within the fitting window to vary.

Fig. 1 gives an updated flow diagram of the main steps involved in performing the PGM when fitting gap-affected data.

3. Data

We have performed a comparative analysis of the PPM and PGM described above using both real and simulated data. The real data consists of ground-based observations taken by the Birmingham Solar Oscillations Network (BiSON). The simulated data were created using the second-generation solarFLAG simulator described in Chaplin et al. (2008).

When using simulated data a common approach is to perform a full Monte-Carlo analysis whereby many independent realizations of the same underlying data are fitted. This enables one to take the average of the fitted mode parameters and compare with the input values in order to obtain an estimate of any bias. However, in this analysis we chose instead to perform fits to a single limit spectrum (i.e., the spectrum one would obtain in the limit of summing an infinite number of independently generated spectra). Toutain et al. (2005) showed that fitting the limit spectrum and comparing the results with the known input values gives a direct measure of any bias without the need to fit many independent spectra (see Paper II for more justification of this approach).

The simulated data were created in the same manner as explained in Paper II using the second-generation solarFLAG simulator described in Chaplin et al. (2008). As the main focus of this paper is to test the benefits of the PGM when applied to data with significant gaps, a BiSON-like window function was imposed on the simulated data. Since the simulated limit spectrum is actually
created in the frequency domain, this had to be done by convolving with the spectral window function (as opposed to simply multiplying the time series by the window function).

When fitting the real data a 3456-day BiSON time series was used. The data were collected between December 1998 and July 2008, and the fill is about 80 percent.

4. Results

In this section we first give the results of fits to the simulated data before giving the results for the real data. In both cases we compare the fitted parameters returned from both the PPM and PGM.

Fig. 2 shows the estimated asymmetries, frequencies and background when fitting simulated data with an imposed BiSON-like window function. The asymmetries returned by the PPM clearly give significantly biased estimates compared with the known input values as given by the solid line. This result can be explained by the fact that asymmetric modes lying outside a particular fitting region will introduce a different amount of excess power at the low frequency end of the fitting region compared with the high frequency end. Since this is not accounted for by the model in the PPM, the fitted asymmetry will be biased. In contrast the fits from the PGM, given by the black symbols, are considerably more accurate since the excess power is accounted for.

For the PPM the fact that the asymmetries are biased also leads to bias in the fitted frequencies. This is shown in the middle panel of Fig. 2 where, in order to give a direct measurement of the significance of the bias we have plotted the difference between the fitted values and the known input values (in the sense fitted - input) and divided by the estimated uncertainties, $\sigma_{\nu_{nl}}$.

The right-hand panel of Fig. 2 shows the fitted backgrounds and it is clear that the PPM return values that overestimate the true background, as given by the dashed line, by a considerable amount. In contrast the background fits from the PGM match much more closely, although some overestimation still remains. However, there is a very good agreement with the predicted background one obtains by fitting the power spectrum at low and high frequencies. The reason why there is a difference between the input and predicted background is explained in Paper II.

As would be expected, all these results are similar to what was obtained when fitting the full-fill data given in Paper II. The main difference is that the biases seen when applying the PPM are more significant. In the case of the background this is fairly straightforward to explain. The model in the PPM does not account directly for the power from modes lying outside the fitting regions. Therefore, the background is increased to account for the excess power. In the case of gap-affected spectra there is also excess power that is taken out of the mode peaks. Within the fitting regions this is accounted for by convolving the model with the spectral window. Outside the fitting regions, however, this extra power is unaccounted for and this will have a small but cumulative impact inside the fitting window. Therefore, the fitted background is increased even further to compensate. The PGM removes this problem by convolving the full spectrum model with the spectral window.
Figure 2. Results of fitting simulated limit spectra convolved with a BiSON-like spectral window. The left panel gives the fitted asymmetries with the dotted line giving the input asymmetries. The middle panel gives the differences between the fitted and input frequencies as described in the text while the solid and dashed lines give the means over four points (one of each degree). The right panel gives the fitted backgrounds with the dashed line giving the input background and the dotted line the predicted background (see text). In all cases the open symbols give the results from the PPM and the black symbols give the results from the PGM. Diamonds signify $\ell = 0$ modes, triangles $\ell = 1$, squares $\ell = 2$ and circles $\ell = 3$.

In Fig. 2 we show the fitted asymmetries, frequencies and backgrounds for the 3456-day BiSON data set. For the asymmetry and background results we have plotted the same quantities as in Fig. 2 whereas for the frequencies we have plotted the differences between the results returned from the two techniques (in the sense PGM minus PPM).

The plots show similar trends to those seen for the simulated data, with the estimates from the PGM being significantly different compared with those from the PPM. However, in the case of the frequencies the differences do not appear to be as significant. This is most likely due to the fact that the modes in the real data do not perfectly match the fitting model.

The background estimates from the PGM, while still far better than those from the PPM, do not match the predicted background (i.e., the background estimated from a fit to the spectrum at low and high frequencies) as well as in the simulated data. The dotted line in the background plot represents the predicted background when forcing the fitting model to take physical (i.e., non-negative) values. The dashed line is the predicted background when this restriction is not enforced. The fact there is a large difference between the two would seem to suggest that there may be some non-solar background source that is dependent on frequency that is not currently being included in the fitting model, (hence why a less physical model can currently more accurately represent the background). However, whichever background prediction is used the fits still lie above it, which again suggests we do not have a perfect model for the modes.

5. **Summary**

We have modified our pseudo-global fitting method (PGM) outlined in Paper II to enable the fitting of spectra created from time series with significant gaps. By using simulated data it was shown that the PGM still enabled less biased esti-
Results of fitting real 3456-day BiSON data. The left panel gives the fitted asymmetries. The middle panel gives the differences between the fitted frequencies returned by the two routines (in the sense PGM minus PPM) divided by the combined formal uncertainties with the solid line giving the mean over four points (one of each different degree). The right panel gives the fitted backgrounds with the dotted line giving the predicted background using physical fitting values and the dotted line giving the predicted background using non-physical fitting values. The symbols have the same meanings as in Fig 2.

Fits to real BiSON data showed similar difference in the returned parameter values between the PGM results and pair-by-pair fitting method (PPM) results as was seen in the simulated data. Given what was seen when fitting the simulated data we would expect the results returned from the PGM to be more accurate. Hence we intend to use the new PGM to fit a long stretch of BiSON data and publish a new set of updated BiSON frequencies.

Differences between the fitted frequencies from the PPM and PGM were not as significant as was seen for the simulated data. Also, in the real data the background fits from the PGM did not match the predicted background (i.e., the background estimated from a fit to the spectrum at low and high frequencies) as well as in the simulated data. These observation suggests that the fitting model for the modes is still not perfect.

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