New Type of Vector Gauge Theory from Noncommutative Geometry

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Abstract

Using the formalism of noncommutative geometric gauge theory based on the superconnection concept, we construct a new type of vector gauge theory possessing a shift-like symmetry and the usual gauge symmetry. The new shift-like symmetry is due to the matrix derivative of the noncommutative geometric gauge theory, and this gives rise to a mass term for the vector field without introducing the Higgs field. This construction becomes possible by using a constant one form even matrix for the matrix derivative, for which only constant zero form odd matrices have been used so far. The fermionic action in this formalism is also constructed and discussed.

PACS Numbers: 11.30.Ly, 12.15.Cc, 12.50.Fk

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I. Introduction

In 1990, Connes and Lott [1] showed that the standard model can be obtained from noncommutative geometric gauge theory. In this noncommutative framework, the Dirac K-cycle plays an important role and the Higgs mechanism is implemented by a generalized Dirac operator acting on a discrete space. In Ref. [2], we showed that the same mechanism can be implemented via matrix derivative approach based on the superconnection concept [3, 4]. There already have been many works along this line [5, 6, 7], but in all these works only constant zero form odd matrices for the matrix derivative have been used in order to conform it to the Connes-Lott’s generalized Dirac operator which is in essence a matrix commutator with a constant odd matrix. In Ref. [2], we found the conditions that the matrix derivative should satisfy when the noncommutative geometric gauge theory is constructed via the superconnection formalism. There we found another type of the matrix derivative consisting of constant(closed) one form matrix which has nonvanishing elements in the even part and vanishing elements in the odd part. In this paper, we use this one form even matrix for the matrix derivative, and construct a new type of vector gauge theory possessing the usual gauge symmetry and a shift-like symmetry which in turn gives rise to a mass term for the vector gauge field without recourse to the Higgs field.

II. Noncommutative geometric gauge theory in the superconnection formalism

Superconnection was first introduced in mathematics by Quillen in 1985 [3]. However, in physics this concept was used earlier by Thierry-Mieg and Ne’eman without giving it a name in 1982 [8] under the notion of a generalized connection a la Cartan [9]. Then in 1990, Ne’eman and Sternberg [4] applied superconnections for the Higgs mechanism.

Let $V = V^+ \oplus V^-$ be a super (or $Z_2$-graded) complex vector space, then the algebra
of endomorphisms of $V$ is a superalgebra with the even or odd endomorphisms. Let $\mathcal{E} = \mathcal{E}^+ \oplus \mathcal{E}^-$ be a super (or $\mathbb{Z}_2$-graded) vector bundle over a manifold $M$, and $\Omega(M) = \oplus \Omega^k(M)$ be the algebra of smooth differential forms with complex coefficients. Then, the space of $\mathcal{E}$ valued differential forms on $M$, $\Omega(M, \mathcal{E})$, has a $\mathbb{Z} \times \mathbb{Z}_2$ grading, and here we are mainly concerned with its total $\mathbb{Z}_2$ grading.

In Ref. [2], we showed that a generalization of the superconnection concept can yield the matrix derivative of the noncommutative geometric gauge theory [3]. There, the generalized superconnection is given by

$$\nabla = d_t + \omega,$$  \hfill (1)

where $d_t = d + d_M$ is a generalization of the one form exterior derivative satisfying the derivation property, and $\omega$ is a generalized connection given by $\omega = \begin{pmatrix} \omega_0 & L_{01} \\ L_{10} & \omega_1 \end{pmatrix}$. Here, $\omega_0, \omega_1$ are matrices of odd degree differential forms and $L_{01}, L_{10}$ are matrices of even degree differential forms. The multiplication rule is given by

$$(u \otimes a) \cdot (v \otimes b) = (-1)^{|a||v|}(uv) \otimes (ab), \quad u, v \in \Omega(M), \quad a, b \in \mathcal{A},$$  \hfill (2)

where $\mathcal{A}$ is the endomorphisms of $V$.

In the matrix representation, $d = \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix}$ where $d$ inside the matrix denotes the usual 1-form exterior derivative times a unit matrix, and $d_M$ is given below. Since $d_M$ should behave as a part of the superconnection operator [10] in a sense, we write it as a (graded) commutator operator

$$d_M = [\eta, \cdot], \quad \eta \in \Omega(M, \mathcal{E}).$$  \hfill (3)

Now, $d_M$ should satisfy

$$d_M^2 = 0, \quad dd_M + d_Md = 0,$$

$$d_M(\alpha\beta) = (d_M\alpha)\beta + (-1)^{|\alpha|}\alpha(d_M\beta), \quad \alpha, \beta \in \Omega(M, \mathcal{E}).$$  \hfill (4)

In Ref. [2], two simple solutions satisfying the above conditions were given by

(1) $\quad \eta = \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix}$ where $u, v$ are odd degree closed forms with their coefficient matrices
satisfying $u^2 = v^2 \propto 1$ or $u^2 = v^2 = 0$,

(2) $\eta = \begin{pmatrix} 0 & m \\ n & 0 \end{pmatrix}$ where $m$, $n$ are even degree closed forms with their coefficient matrices satisfying $mn = nm \propto 1$ or $mn = nm = 0$.

If we take the second solution with 0-form $m$, $n$, then this choice yields the so-called matrix derivative $[1] \mathbf{d}_M = [\eta, \cdot]$ with $\eta = \begin{pmatrix} 0 & \zeta \\ \zeta & 0 \end{pmatrix}$ where $\zeta$, $\bar{\zeta}$ are 0-form constant matrices satisfying $\zeta \bar{\zeta} = \bar{\zeta} \zeta \propto 1$.

With the use of the generalized superconnection, the curvature is now given by

$$ F_t = (\mathbf{d}_t + \omega)^2 = \mathbf{d}_t \omega + \omega^2. \quad (5) $$

In this formulation, the Yang-Mills action is given by

$$ I_{YM} = \int_M \text{Tr}(F_t^* \cdot F_t) \quad (6) $$

where $\star$ denotes taking dual for each entries of $F_t$ as well as taking Hermitian conjugate.

The fermionic action is given by

$$ I_{sp} = \int_M \overline{\Psi} \gamma^\mu (\mathbf{d}_t + \omega)_\mu \Psi, \quad \Psi \in V \otimes S \quad (7) $$

where $S$ is a spinor bundle.

III. Massive vector gauge theory with unbroken symmetry

Now, we consider the first solution for $d_M$ given in the previous section with $\eta = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma' \end{pmatrix}$ where $\sigma$, $\sigma'$ are constant 1-form matrices whose squares are either proportional to a unit matrix or zero. For the generalized connection $\omega$, we set

$$ \omega = \begin{pmatrix} A & 0 \\ 0 & A' \end{pmatrix} \quad (8) $$

where $A$, $A'$ consist of one forms only.

For a definite understanding, we consider the case where $A$, $A'$ are SU(2) valued 1-form fields, and $\sigma$ and $\sigma'$ are proportional to a SU(2) Pauli matrix, say $\tau_3$:

$$ A = \frac{i}{2} A^a_\mu \tau_a dx^\mu \equiv A_\mu dx^\mu, \quad A' = \frac{i}{2} A'^a_\mu \tau_a dx^\mu \equiv A'_\mu dx^\mu, $$

$$ \sigma = \sigma' = \frac{i}{2} m \tau_3 n_\mu dx^\mu \equiv \sigma_\mu dx^\mu. \quad (9) $$


Here τ’s are Pauli matrices, \( n_\mu \) is a constant four vector, and \( m \) is a constant parameter. Throughout the paper, we use the metric \( g_{\mu\nu} = (-1, +1, +1, +1) \) and \( \epsilon_{0123} = +1 \), and the wedge product between forms is understood.

The curvature

\[
\mathcal{F}_t = d_\omega + \omega^2
\]

is now given by

\[
\mathcal{F}_t = \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} A & 0 \\ 0 & A' \end{pmatrix} + \left[ \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma' \end{pmatrix}, \begin{pmatrix} A & 0 \\ 0 & A' \end{pmatrix} \right] + \begin{pmatrix} A & 0 \\ 0 & A' \end{pmatrix} \cdot \begin{pmatrix} A & 0 \\ 0 & A' \end{pmatrix}.
\]

The first and third terms are the usual ones and the second term is a new piece due to the matrix derivative which we calculate below. Since all the odd parts are vanishing, the upper and lower diagonal parts do not mix. Hence, we mostly consider the upper part in our calculation.

Now,

\[
\sigma A + A \sigma = \frac{1}{2} \left\{ [\sigma_\mu, A_\nu] - [\sigma_\nu, A_\mu] \right\} dx^\mu dx^\nu
\]

\[
= -\frac{1}{4} m n_{\mu} \begin{pmatrix} 0 & A_1 - iA_2 \\ -A_1 - iA_2 & 0 \end{pmatrix} \nu \] \( dx^\mu dx^\nu \)

\[
= \frac{1}{2} A_{\mu\nu} dx^\mu dx^\nu.
\]

Thus the curvature is given by

\[
\mathcal{F}_t = \begin{pmatrix} F_t & 0 \\ 0 & F'_t \end{pmatrix}
\]

with

\[
F_t = \frac{1}{2} (F_{\mu\nu} + A_{\mu\nu}) dx^\mu dx^\nu,
\]

and \( F'_t \) is the same as \( F_t \) except that \( A \) is replaced by \( A' \), and \( F_{\mu\nu} \) is the usual one,

\[
F_{\mu\nu} = \partial_\mu A_\nu + [A_\mu, A_\nu].
\]
Following the same calculational step as in Ref. [7], we obtain the Yang-Mills type action of massive gauge fields from Eq. (6),

\[ I_{YM} = \int_{M} \text{Tr}(F_t^\ast \cdot F_t) \]

\[ = \frac{1}{2} \int_{M} d^4x \text{ Tr}\left[ (F_{\mu\nu} + A_{\mu\nu})(F_{\mu\nu} + A_{\mu\nu}) + \text{(terms with } A \rightarrow A') \right] \]

\[ = \frac{1}{2} \int_{M} d^4x \text{ Tr}\left[ F_{\mu\nu}F_{\mu\nu} + F_{\mu\nu}A_{\mu\nu} + A_{\mu\nu}F_{\mu\nu} + A_{\mu\nu}A_{\mu\nu} + \text{(terms with } A \rightarrow A') \right]. \]

The fourth term provides quadratic terms homogeneous in \( A_1 \) and \( A_2 \):

\[ \frac{1}{2} \text{Tr} A_{\mu\nu}A_{\mu\nu} = \frac{1}{2} m^2 \left[ -n_{\mu}n_{\nu}(A_{1\mu}A_{1\nu} + A_{2\mu}A_{2\nu}) + n_{\mu}n_{\nu}(A_{1}^\mu A_{1}^\nu + A_{2}^\mu A_{2}^\nu) \right]. \]

The second and third terms also give terms quadratic in \( A \) but mixed in \( A_1 \) and \( A_2 \):

\[ \frac{1}{2} \text{Tr} (F_{\mu\nu}A_{\mu\nu} + A_{\mu\nu}F_{\mu\nu}) = m\epsilon_{ab}(n_{\mu}A_{a\nu}\partial_\mu A_{b}^\nu - n_{\mu}A_{a\nu}\partial_\nu A_{b}^\mu) + O(A^3) \]

where \( a, b = 1, 2 \) and \( \epsilon^{12} = -\epsilon^{21} = 1, \epsilon^{11} = \epsilon^{22} = 0. \)

Before we perform diagonalization and obtain the propagators for these fields, we first identify the symmetry of the action. In Ref. [8], the so-called horizontality condition was used to analyze the BRST symmetry of the noncommutative geometric gauge theory. Since we use the same superconnection framework, the BRST analysis will be more convenient for finding the symmetry of the theory. In the Yang-Mills theory, the horizontality condition is given by [11, 12, 13, 14]

\[ \tilde{F} \equiv \tilde{d}\tilde{A} + \tilde{A}\tilde{A} = F, \]

where

\[ \tilde{A} = A_\mu dx^\mu + A_N dy^N + A_{\bar{N}} d\bar{y}^{\bar{N}} \equiv A + c + \bar{c}, \]

\[ \tilde{d} = d + s + \bar{s}, \quad d = dx^\mu \partial_\mu, \quad s = dy^N \partial_N, \quad \bar{s} = d\bar{y}^{\bar{N}} \partial_{\bar{N}}, \]

\[ F = dA + AA = \frac{1}{2} F_{\mu\nu}dx^\mu dx^\nu. \]

Here \( y \) and \( \bar{y} \) denote the coordinates in the direction of gauge orbit of the principal fiber whose structure-group is \( \mathcal{G} \otimes \mathcal{G} \), and \( c, \bar{c} \) are ghost and antighost fields. The above
horizontality condition now yields the BRST and anti-BRST transformation rules for the Yang-Mills theory.

\[
(dx)^1(dy)^1 : \quad sA_\mu = D_\mu c, \\
(dx)^1(d\bar{y})^1 : \quad \bar{s}A_\mu = D_\mu \bar{c}, \\
(dy)^2 : \quad sc = -cc, \\
(dy)^2 : \quad \bar{s}\bar{c} = -\bar{c}\bar{c}, \\
(dy)^1(d\bar{y})^1 : \quad s\bar{c} + \bar{s}c = -[c, \bar{c}].
\]  

(19)

In the superconnection framework, the horizontality condition is given as follows \[7\].

\[
\tilde{\mathcal{F}}_t \equiv \tilde{d}_t \tilde{\omega} + \tilde{\omega} \cdot \tilde{\omega} = \mathcal{F}_t
\]  

(20)

where

\[
\tilde{d}_t = d_t + s + \bar{s}, \\
\tilde{\omega} = \omega + \mathcal{C} + \bar{\mathcal{C}},
\]

(21)\hspace{1cm} (22)

and

\[
s = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix}, \quad \bar{s} = \begin{pmatrix} \bar{s} & 0 \\ 0 & \bar{s} \end{pmatrix}, \quad \mathcal{C} = \begin{pmatrix} c & 0 \\ 0 & c' \end{pmatrix}, \quad \bar{\mathcal{C}} = \begin{pmatrix} \bar{c} & 0 \\ 0 & \bar{c'} \end{pmatrix}.
\]

(23)

The above horizontality condition yields the following BRST and anti-BRST transformation rules:

\[
(dy)^1 : \quad s\omega = -d_t \mathcal{C} - \omega \cdot \mathcal{C} - \mathcal{C} \cdot \omega, \\
(d\bar{y})^1 : \quad \bar{s}\bar{\omega} = -d_t \bar{\mathcal{C}} - \bar{\omega} \cdot \bar{\mathcal{C}} - \bar{\mathcal{C}} \cdot \bar{\omega}, \\
(dy)^2 : \quad s\mathcal{C} = -\mathcal{C} \cdot \mathcal{C}, \\
(d\bar{y})^2 : \quad \bar{s}\bar{\mathcal{C}} = -\bar{\mathcal{C}} \cdot \bar{\mathcal{C}}, \\
(dy)^1(d\bar{y})^1 : \quad s\bar{\mathcal{C}} + \bar{s}\mathcal{C} + \mathcal{C} \cdot \bar{\mathcal{C}} + \bar{\mathcal{C}} \cdot \mathcal{C} = 0.
\]  

(24)\hspace{1cm} (25)\hspace{1cm} (26)\hspace{1cm} (27)\hspace{1cm} (28)

Since all the odd parts vanish as before, we again consider only the upper diagonal (even) parts in our calculation. Then, the BRST and anti-BRST transformation rules for the
fields appearing in the upper parts can be written as

\[
s A = -dc - [\sigma, c]_+ - [A, c]_+, \tag{29}
\]
\[
\bar{s} A = -d\bar{c} - [\bar{\sigma}, \bar{c}]_+ - [A, \bar{c}]_+, \tag{30}
\]
\[
s c = -cc, \tag{31}
\]
\[
\bar{s} \bar{c} = -\bar{c}\bar{c}, \tag{32}
\]
\[
s \bar{c} + sc + \bar{c} + \bar{c}c = 0, \tag{33}
\]

where

\[
c = \frac{i}{2} c_a \tau^a, \quad \bar{c} = \frac{i}{2} \bar{c}_a \tau^a, \quad a = 1, 2, 3.
\]

Now, one can check that the above BRST and anti-BRST transformations are nilpotent, \( s^2 = \bar{s}^2 = 0 \), and the total curvature \( F_t = dA + AA + \sigma A + A\sigma \) transforms as the usual curvature \( F = dA + AA \),

\[
sF_t = -[c, F_t]. \tag{34}
\]

Therefore, our Yang-Mills action,

\[
I_{Y M}^0 = \int_M \text{Tr} F_t^* F_t = \frac{1}{2} \int_M d^4x \ \text{Tr} [(F_{\mu\nu} + A_{\mu\nu})(F^{\mu\nu} + A^{\mu\nu})] \tag{35}
\]

where \( * \) denotes the Hodge dual, is invariant under the above given BRST(anti-BRST) transformation. Since the BRST and gauge transformations for classical fields are the same except for a switch between the classical gauge parameter and the ghost field, one can check that the action (35) is invariant under the following gauge transformation

\[
\delta A_\mu = \partial_\mu \varepsilon + [A_\mu, \varepsilon] + [\sigma_\mu, \varepsilon] \tag{36}
\]

where \( \varepsilon = \frac{i}{2} \varepsilon_a \tau^a \) (\( a = 1, 2, 3 \)) is a zero form gauge parameter. In order to obtain the propagators we use the following gauge fixing term for the action (35)

\[
\mathcal{L}_{g.f.} = \frac{1}{\xi} \text{Tr}(d\omega^*)^2, \tag{37}
\]

which is translated into the following condition for the action (35)

\[
\mathcal{L}_{g.f.}^0 = \frac{1}{\xi} \text{Tr}((\partial_\mu A^\mu + \sigma_\mu A^\mu - A_\mu \sigma^{\mu})^2
\]

8
where \( \sigma_\mu, A_\mu \) are given by Eq. (39). In terms of

\[
W_\pm = \frac{1}{\sqrt{2}} (A_1^\mu \mp i A_2^\mu),
\]

we obtain the propagators for \( W_\pm, A_3 \), after some calculation:

\[
W_\pm : \triangle_{\mu,\nu}^\pm = \frac{1}{(P^\pm)^2} \left( g_{\mu\nu} + (\xi - 1) \frac{P_\mu P_\nu}{(P^\pm)^2} \right),
\]

\[
A_3 : \triangle_{\mu,\nu}^3 = \frac{1}{p^2} \left( g_{\mu\nu} + (\xi - 1) \frac{p_\mu p_\nu}{p^2} \right),
\]

where \( P_\mu = p_\mu \pm mn_\mu \). If we set \( n \cdot p = n_\mu p^\mu = 0 \), then the denominator of the \( W \)-propagator becomes \( (P^\pm)^2 = p^2 + m^2 n^2 \). Thus, choosing \( n_\mu \) satisfying the above condition exhibits the \( W \) field having a mass; \( (\text{mass})^2 = m^2 n^2 \).

**IV. Fermionic action**

The fermionic action Eq. (7) shows that the spinor \( \Psi \) belongs to a \( Z_2 \)-graded vector space, thus we write \( \Psi = (\psi^+, \psi^-) \). Then the fermionic action,

\[
I_{sp} = \int_M d^4x \left[ \bar{\Psi} \gamma^\mu (d_\mu + \omega_\mu) \Psi + \bar{\Psi} \gamma^\mu \eta_\mu \Psi \right]
\]

where \( d_\mu = \begin{pmatrix} \partial_\mu & 0 \\ 0 & \partial_\mu \end{pmatrix} \) and \( \eta_\mu = \begin{pmatrix} \sigma_\mu & 0 \\ 0 & \sigma_\mu \end{pmatrix} \), can be written as

\[
I_{sp} = \int_M d^4x \left[ \bar{\psi}^+ \gamma^\mu (\partial_\mu + A_\mu) \psi^+ + \bar{\psi}^+ \gamma^\mu \sigma_\mu \psi^+ + (\text{terms with } + \rightarrow -) \right].
\]

Here, we assume \( A = A' \) for convenience, and \( \sigma_\mu = \frac{i}{2} m \gamma_3 n_\mu \) as before. The action given above is a massless fermionic action except for the term, \( \bar{\psi}^+ \gamma^\mu \sigma_\mu \psi^+ \), which looks similar to the usual mass term. In order to compare it with the usual mass term, \( m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) \), we now set \( \psi^+ = \psi_L = \frac{1 + \gamma_5}{2} \psi \) and \( \psi^- = \psi_R = \frac{1 - \gamma_5}{2} \psi \). Then

\[
\bar{\psi}^+ \gamma^\mu \sigma_\mu \psi^+ = \psi^+ \frac{1 + \gamma_5}{2} \gamma_0 \gamma^\mu \sigma_\mu \frac{1 + \gamma_5}{2} \psi = \psi^+ \gamma_0 \gamma^\mu \sigma_\mu \frac{1 + \gamma_5}{2} \frac{1 + \gamma_5}{2} \psi
\]

\[
= \bar{\psi} \gamma^\mu \sigma_\mu \frac{1 + \gamma_5}{2} \psi,
\]

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and the same way
\[ \bar{\psi}_- \gamma^\mu \sigma_\mu \psi_+ = \bar{\psi} \gamma^\mu \sigma_\mu \frac{1 - \gamma_5}{2} \psi. \] (44)

Thus the two terms containing \( \sigma_\mu \) become
\[ \bar{\psi}_+ \gamma^\mu \sigma_\mu \psi_+ + \bar{\psi}_- \gamma^\mu \sigma_\mu \psi_- = \bar{\psi} \gamma^\mu \sigma_\mu \psi. \] (45)

Notice that \( \sigma_\mu = \frac{i}{2} m n_\mu \tau_3 \) commutes with \( \gamma \)'s and only acts on the fundamental representation of \( \psi \), say a doublet \((u, d)\), and \( \gamma \)'s act on \( u \) or \( d \) itself. Thus this term has an extra factor of \( \gamma^\mu n_\mu \) compared with the usual mass term.

**V. Concluding remarks**

In this paper, we constructed a new type of vector gauge theory which possesses both the usual gauge symmetry and a shift-like symmetry which gives rise to a mass term for the vector gauge field. So far, in the noncommutative geometric gauge theory, only zero form constant odd matrices have been used for the matrix derivative, and this constant zero form odd matrix together with scalar fields appearing in the odd part of the gauge multiplet (or superconnection) give rise to the Higgs mechanism. In the Connes-Lott formalism, this constant odd matrix does the role of the generalized Dirac operator acting on a discrete space. However, constructing the noncommutative geometric gauge theory in the superconnection formalism, it is possible to use a constant one form even matrix for the matrix derivative [2]. This in turn makes it possible to construct a massive vector gauge model similar to the Proca’s while maintaining gauge symmetry. It is also possible to provide mass terms for all the gauge fields by properly choosing this constant one form even matrix. For instance, if we replace \( \tau_3 \) appearing in \( \sigma_\mu = \frac{i}{2} m n_\mu \tau_3 \) with \( \tau_1 + \tau_2 + \tau_3 \), then all \( A_1, A_2, A_3 \) fields become massive. However, if we use an identity matrix for \( \sigma_\mu \), then there will be no massive vector field.

Finally, we would like to mention two things which looked confusing in the previous sections. In Eq. (40), the propagators for \( W_\pm \) look apparently different by a term
involving the gauge fixing parameter $\xi$. This difference can be removed for the choice of $\xi = 1$, and for other $\xi$ values we expect that the contribution from this $\xi$ related term be cancelled by that of ghosts since the gauge fixing should not affect the physics.

In the case of the fermionic action, the presence of the extra factor $\gamma^\mu n_\mu$ in the “mass” term looks puzzling. However, the propagator for the $\psi$ field can be expressed as

$$\Delta = \frac{1}{-ip_\mu \gamma^\mu + \sigma_\mu \gamma^\mu} = \frac{ip_\mu \gamma^\mu + \sigma_\mu \gamma^\mu}{p^2 + \sigma^2},$$

and thus exhibits that $\psi$ has a mass; $(\text{mass})^2 = \sigma^2$.

Now, we leave the more rigorous work on these issues along with the investigation of the quantum effect for a future work.

Acknowledgments
I would like to thank D.S. Hwang for helpful discussions, B. Lee, W.T. Kim, P. Orland for helpful comments, and Y. Kiem for reading the manuscript and comments. This work was supported in part by the Ministry of Education, BSRI-97-2442, and KOSEF grant 971-0201-007-2.

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