Nilpotent chiral superfield in $\mathcal{N} = 2$ supergravity
and partial rigid supersymmetry breaking

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Abstract

In the framework of $\mathcal{N} = 2$ conformal supergravity in four dimensions, we introduce a nilpotent chiral superfield suitable for the description of partial supersymmetry breaking in maximally supersymmetric spacetimes. As an application, we construct Maxwell-Goldstone multiplet actions for partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking on $\mathbb{R} \times S^3$, $\text{AdS}_3 \times S^1$ (or its covering $\text{AdS}_3 \times \mathbb{R}$), and a pp-wave spacetime. In each of these cases, the action coincides with a unique curved-superspace extension of the $\mathcal{N} = 1$ supersymmetric Born-Infeld action, which is singled out by the requirement of $U(1)$ duality invariance.
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1 Introduction

Inspired by the work of Antoniadis, Partouche and Taylor [1], Bagger and Galperin [2] constructed the Goldstone-Maxwell multiplet model for partially broken $\mathcal{N} = 2$ Poincaré supersymmetry in four spacetime dimensions (4D). Their model proved to coincide with the $\mathcal{N} = 1$ supersymmetric Born-Infeld action [3, 4]. Two years later, Roček and Tseytlin [5] re-derived the model of [2] using $\mathcal{N} = 2$ superfields, building on the earlier formulation
due to Roček [6] for the Volkov-Akulov Goldstino model [7] in terms of a nilpotent \( \mathcal{N} = 1 \) chiral superfield.

The \( \mathcal{N} = 2 \) Minkowski superspace is one of many maximally supersymmetric backgrounds in 4D \( \mathcal{N} = 2 \) off-shell supergravity. Such superspaces were classified in [10] building on the earlier analysis [11] of maximally supersymmetric backgrounds in 5D \( \mathcal{N} = 1 \) off-shell supergravity. The construction in [5] is down-to-earth in the sense that it is specifically designed to describe the partial breaking of \( \mathcal{N} = 2 \) Poincaré supersymmetry. Here we present a theoretical scheme which is suitable for the description of partial supersymmetry breaking in curved maximally supersymmetric backgrounds in 4D \( \mathcal{N} = 2 \) off-shell supergravity. As an application of this scheme, we construct Maxwell-Goldstone multiplet actions for partial \( \mathcal{N} = 2 \rightarrow \mathcal{N} = 1 \) supersymmetry breaking on \( \mathbb{R} \times S^3 \), AdS$_3 \times S^1$ (or its covering AdS$_3 \times \mathbb{R}$), and a pp-wave.

This paper is organised as follows. In section 2 we introduce a nilpotent chiral superfield coupled to \( \mathcal{N} = 2 \) conformal supergravity. In section 3 we explain how such a superfield can be used to construct a model for partially broken supersymmetry for certain maximally supersymmetric backgrounds of \( \mathcal{N} = 2 \) supergravity. The formalism developed is applied in section 4 to re-derive the Roček-Tseytlin construction. In section 5 we construct Maxwell-Goldstone multiplet actions for partial \( \mathcal{N} = 2 \rightarrow \mathcal{N} = 1 \) supersymmetry breaking on \( \mathbb{R} \times S^3 \), AdS$_3 \times S^1$ (or its covering AdS$_3 \times \mathbb{R}$), and a pp-wave. Concluding comments are given in section 6. The main body of the paper is accompanied by three technical appendices. In Appendices A and B, we present group-theoretic formulations for four-dimensional \( \mathcal{N} = 1 \) and \( \mathcal{N} = 2 \) superspaces over \( U(2) = (S^1 \times S^3)/\mathbb{Z}_2 \). The maximally \( \mathcal{N} = 2 \) supersymmetric background over \( \mathbb{R} \times S^3 \), which is used in section 5, is the universal covering space of the \( \mathcal{N} = 2 \) superspace over \( (S^1 \times S^3)/\mathbb{Z}_2 \). Appendix A also contains the group-theoretic description of \( \mathcal{N} = 1 \) superspace over \( U(1,1) = (AdS_3 \times S^1)/\mathbb{Z}_2 \). Appendix C is devoted to the discussion of a unique feature of the anti-de Sitter supersymmetry that distinguishes AdS$_4$ from the other maximally supersymmetric four-dimensional backgrounds.

\footnote{The same nilpotent chiral superfield was independently introduced, a few months later, by Ivanov and Kapustnikov [8] as a simple application of the general relationship between linear and nonlinear realisations of supersymmetry established in their earlier work [9].}
2 Nilpotent chiral superfield in $\mathcal{N} = 2$ supergravity

In the framework of four-dimensional $\mathcal{N} = 2$ conformal supergravity\(^2\) we introduce a nilpotent chiral superfield constrained by

\begin{align}
\bar{D}_i \dot{\alpha} Z &= 0 , \\
(D^{ij} + 4S^{ij}) Z - (\bar{D}^{ij} + 4\bar{S}^{ij}) \bar{Z} &= 4i G^{ij} , \\
Z^2 &= 0 ,
\end{align}

where $G^{ij}$ is a linear multiplet constrained by $G^{ij} G_{ij} \neq 0$. One may interpret $G^{ij}$ as the field strength of a tensor multiplet. The constraints (2.1a)–(2.1c) are invariant under the $\mathcal{N} = 2$ super-Weyl transformations \([12, 13]\) if $Z$ is considered to be a primary superfield of dimension 1.

A chiral superfield constrained by (2.1b) was considered in \([14]\) in the context of the dilaton effective action in $\mathcal{N} = 2$ supergravity. In the super-Poincaré case, chiral superfields obeying the constraint (2.1b) with a constant $G^{ij}$ naturally originate in the framework of partial $\mathcal{N} = 2 \to \mathcal{N} = 1$ supersymmetry breaking \([1, 15, 16]\).

We recall that the $\mathcal{N} = 2$ tensor multiplet is described in curved superspace by its gauge invariant field strength $G^{ij}$ which is a linear multiplet. The latter is defined to be a real $\text{SU}(2)$ triplet (that is, $G^{ij} = G^{ji}$ and $\bar{G}_{ij} := \bar{G}^{ij} = G_{ij}$) subject to the covariant constraints \([17, 18]\)

\begin{align}
D_{\alpha}^{(i} G^{jk)} &= \bar{D}_{\dot{\alpha}}^{(i} G^{jk)} = 0 .
\end{align}

These constraints are solved in terms of a chiral prepotential $\Psi$ \([19, 20, 21, 22]\) via

\begin{align}
G^{ij} &= \frac{1}{4} \left( D^{ij} + 4S^{ij} \right) \Psi + \frac{1}{4} \left( \bar{D}^{ij} + 4\bar{S}^{ij} \right) \bar{\Psi} , \\
\bar{D}_i \dot{\alpha} \Psi &= 0 ,
\end{align}

which is invariant under Abelian gauge transformations

\begin{align}
\delta_\Lambda \Psi &= i \Lambda ,
\end{align}

with $\Lambda$ a reduced chiral superfield,

\begin{align}
\bar{D}_i \dot{\alpha} \Lambda &= 0 ,
\end{align}

\(^2\)In this paper, we use Howe’s superspace formulation \([12]\) for $\mathcal{N} = 2$ conformal supergravity and follow the supergravity notation and conventions of \([13]\). In particular, the superspace covariant derivatives are denoted $\bar{D}_A = (D_a, \bar{D}_a, \bar{D}_i \dot{\alpha})$. We make use of the second-order differential operators $D^{ij} := D_{\alpha}^{(i} D^\alpha_{j)}$, $\bar{D}^{ij} := \bar{D}_{\dot{\alpha}}^{(i} \bar{D}^\dot{\alpha}_{j)}$. The $\text{SU}(2)$ triplet $S^{ij} = S^{ji}$ and its conjugate $\bar{S}_{ij} = \bar{S}^{ij}$ stand for certain components of the superspace torsion tensor.
\[(D_{ij} + 4S_{ij}) \Lambda - (\bar{D}_{ij} + 4\bar{S}_{ij}) \bar{\Lambda} = 0 \, . \tag{2.5b}\]

We recall that the field strength of an Abelian vector multiplet is a reduced chiral superfield \[23\].

The constraints on \(\Lambda\) can be solved in terms of the Mezincescu prepotential \[24\] (see also \[19\]), \(U_{ij} = U_{ji}\), which is an unconstrained real \(SU(2)\) triplet. The curved-superspace solution is \[25\]

\[\Lambda = \frac{1}{4} \bar{\Delta} \left( D_{ij} + 4S_{ij} \right) U_{ij} \, . \tag{2.6}\]

Here \(\bar{\Delta}\) denotes the chiral projection operator \[26,27\]

\[\bar{\Delta} = \frac{1}{96} \left( (D_{ij} + 16S_{ij}) \mathcal{D}_{ij} - (\mathcal{D}^{\dot{\alpha}\dot{\beta}} - 16Y^{\dot{\alpha}\dot{\beta}}) \mathcal{D}_{\dot{\alpha}\dot{\beta}} \right) \]

\[= \frac{1}{96} \left( \mathcal{D}_{ij} (D_{ij} + 16S_{ij}) - \mathcal{D}_{\dot{\alpha}\dot{\beta}} (\mathcal{D}^{\dot{\alpha}\dot{\beta}} - 16Y^{\dot{\alpha}\dot{\beta}}) \right) \, , \tag{2.7}\]

with \(\mathcal{D}^{\dot{\alpha}\dot{\beta}} := \mathcal{D}^{(\dot{\alpha} \mathcal{D}^{\dot{\beta})k}}. Its main properties can be formulated using a super-Weyl inert scalar \(V\). It holds that

\[\bar{D}^{\dot{\alpha}} \bar{\Delta} V = 0 \, , \tag{2.8a}\]

\[\delta_{\sigma} V = 0 \Longrightarrow \delta_{\sigma} \bar{\Delta} V = 2\sigma \bar{\Delta} V \, , \tag{2.8b}\]

\[\int d^4x \, d^4\theta \, d^4\bar{\theta} \, E \bar{V} = \int d^4x \, d^4\theta \, \mathcal{E} \bar{\Delta} V \, , \tag{2.8c}\]

where the real unconstrained parameter \(\sigma\) corresponds to the super-Weyl transformations \[13\]. Here \(E\) and \(\mathcal{E}\) denote the full superspace and chiral densities, respectively.

The constraints \((2.1a)\) and \((2.1b)\) define a deformed reduced chiral superfield. These constraints may be re-cast in the language of superforms as \(dF = H\), where \(F\) is a two-form and \(H\) is the three-form field strength, \(dH = 0\), describing the tensor multiplet \[27\], see also \[28\]. Switching \(H\) off, \(H = 0\), turns \(F\) into the two-form field strength of the vector multiplet.

The constraint \((2.1b)\) naturally originates as follows. Consider the model for a massive improved tensor multiplet coupled to \(N = 2\) conformal supergravity \[29,30\]. The action of this model in the form given in \[25\] is

\[S_{\text{tensor}} = - \int d^4x \, d^4\theta \, \mathcal{E} \left\{ \bar{\Psi} \mathcal{W} + \frac{1}{4} \mu (\mu + i\epsilon) \Psi^2 \right\} + \text{c.c.} \, , \tag{2.9}\]

\[3\]The parameter \(\sigma\) was denoted \(2U\) in \[13\].

\[4\]We are grateful to Joseph Novak for this observation.
where $\mu$ and $e$ are real parameters, with $\mu \neq 0$ (the tensor multiplet mass can be shown to be $m = \sqrt{\mu^2 + e^2}$). The kinetic term involves the composite \[ \mathcal{W} := -\frac{G}{8} (\bar{D}_{ij} + 4 \bar{S}_{ij}) \left( \frac{G^{ij}}{G^2} \right), \] \[ (2.10) \]
which proves to be a reduced chiral superfield.\footnote{The superfield \eqref{eq:composite} is one of the simplest applications of the powerful approach to generate composite reduced chiral multiplets which was presented in \cite{25}.} For $m = 0$ the above action describes the improved tensor multiplet \cite{31}. We introduce a St"uckelberg-type extension of the model \[ \tilde{S}_{\text{tensor}} = -\int d^4x d^4\theta \mathcal{E} \left\{ \bar{\Psi} \mathcal{W} + \frac{1}{4} \mu (\mu + ie) (\bar{\Psi} - iW)^2 \right\} + \text{c.c.}, \] \[ (2.11) \]
where $W$ is the field strength of a vector multiplet. The action is invariant under the gauge transformation \eqref{eq:gauge_transformation} accompanied by \[ \delta_{\Lambda}W = \Lambda. \] \[ (2.12) \]
The original action \eqref{eq:original_action} is obtained from \eqref{eq:stuckelberg_action} by choosing a gauge $W = 0$. Now one can see that the superfield $Z := W + i\Psi$ obeys the constraint \eqref{eq:constraint}. It is well known that the functional \[ i \int d^4x d^4\theta \mathcal{E} W^2 + \text{c.c.} \] \[ (2.13) \]
is a total derivative. Since the mass term in \eqref{eq:stuckelberg_action} is invariant under the gauge transformation \eqref{eq:gauge_transformation} and \eqref{eq:gauge_transformation_2}, it follows that, given a chiral superfield $Z$ constrained by \eqref{eq:constraint}, the functional \[ I = \int d^4x d^4\theta \mathcal{E} \left\{ \bar{Z} \Psi - \frac{i}{2} \Psi^2 \right\} + \text{c.c.} \] \[ (2.14) \]
is invariant under the gauge transformation \eqref{eq:gauge_transformation}, $\delta_{\Lambda}I = 0$. The constraints \eqref{eq:constraint} imply that, for certain supergravity backgrounds, the degrees of freedom described by the $\mathcal{N} = 2$ chiral superfield $Z$ are in a one-to-one correspondence with those of an Abelian $\mathcal{N} = 1$ vector multiplet. The specific feature of such $\mathcal{N} = 2$ supergravity backgrounds is that they possess an $\mathcal{N} = 1$ subspace of the full $\mathcal{N} = 2$ superspace. This property is not universal. In particular, there exist maximally $\mathcal{N} = 2$ supersymmetric backgrounds with no admissible truncation to $\mathcal{N} = 1$ \cite{10}.
3 Maximally $\mathcal{N} = 2$ supersymmetric backgrounds and partial supersymmetry breaking

So far we have discussed an arbitrary supergravity background. Now we restrict our consideration to a maximally supersymmetric background $M^{4|8}$ with the property that the chiral prepotential $\Psi$ for $G^{ij}$ may be chosen such that the following two conditions hold. Firstly, the complex linear multiplet

$$G^{ij}_+ := \frac{1}{4}(D^{ij} + 4S^{ij})\Psi$$

is covariantly constant and null,

$$D_A G^{ij}_+ = 0 ,$$
$$G^{ij}_+ G^{+ij} = 0 .$$

Secondly, the prepotential $\Psi$ may be chosen to be nilpotent,

$$\Psi^2 = 0 .$$

The null condition for $G^{ij}_+$ means that $G^{ij}_+ = q^i q^j$, for some isospinor $q^i$. It follows that $G^{ij} = G^{ij}_+ + G^{ij}_-$ is covariantly constant,

$$D_A G^{ij} = 0 ,$$

where we have denoted $G^{ij}_- := \frac{1}{4}(\bar{D}^{ij} + 4\bar{S}^{ij})\bar{\Psi}$.

We are going to show that the following functional

$$I = \int d^4x \, d^4\theta \, \mathcal{E} \, \Psi \, \mathcal{Z}$$

is supersymmetric. Here $\mathcal{Z}$ is the nilpotent chiral superfield (2.1), which is assumed to be a composite of the dynamical fields. The complex linear multiplet (3.1) and its chiral prepotential $\Psi$ are background fields associated with the background superspace $M^{4|8}$. Since the covariant derivatives $D_A$ are invariant under the isometry transformations of $M^{4|8}$, the fields $G^{ij}_\pm$ and $\Psi$ do not change under such transformations. Let $\xi$ be a Killing supervector field for $M^{4|8}$ (see section 6.4 of [32] and [33] for general discussions). Then

$$\delta_\xi I = \int d^4x \, d^4\theta \, \mathcal{E} \, \Psi \delta_\xi Z = - \int d^4x \, d^4\theta \, \mathcal{E} \, Z \delta_\xi \Psi .$$
We introduce a reduced chiral superfield $W$ by

$$Z = W + i\Psi, \quad W = \frac{1}{4} \bar{\Delta} \left( D^{ij} + 4S^{ij} \right) U_{ij}, \quad (3.8)$$

where $U_{ij}$ is the Mezincescu prepotential for the reduced chiral superfield $W$. Since $\Psi \delta_\xi \Psi = 0$, we have

$$\delta_\xi I = - \int d^4 x d^4 \theta \, \mathcal{E} \delta_\xi \Psi = - \int d^4 x d^4 \theta \, \mathcal{E} W \delta_\xi \Psi$$

$$= - \frac{1}{4} \int d^4 x d^4 \theta \, d^4 \bar{\theta} \, E U_{ij} \left( \bar{D}^{ij} + 4S^{ij} \right) \delta_\xi \Psi$$

$$= - \frac{1}{4} \int d^4 x d^4 \theta \, d^4 \bar{\theta} \, E U_{ij} \delta_\xi \left( \bar{D}^{ij} + 4S^{ij} \right) \Psi$$

$$= - \int d^4 x d^4 \theta \, d^4 \bar{\theta} \, E U_{ij} \delta_\xi G^{ij} = 0. \quad (3.9)$$

In the next two sections, it will be shown that the action

$$S = - \frac{i}{4} \int d^4 x d^4 \theta \, \mathcal{E} \Psi Z + \text{c.c.} \quad (3.10)$$

describes the Maxwell-Goldstone multiplet for partial $\mathcal{N} = 2 \to \mathcal{N} = 1$ supersymmetry breaking on the maximally supersymmetric backgrounds specified.

The above derivation does not use the null condition (3.3). The latter is introduced for the $\mathcal{N} = 2$ superspace $\mathbb{M}^{4|8}$ to possess an $\mathcal{N} = 1$ subspace.

### 4 Example: The super-Poincaré case

The simplest maximally supersymmetric background is $\mathcal{N} = 2$ Minkowski superspace. In this superspace, every constant real $\text{SU}(2)$ triplet $G^{ij}$ is covariantly constant,

$$D_A G^{ij} = 0, \quad (4.1)$$

where $D_A = (\partial_a, D^i_a, \bar{D}^{\dot{a}}_i)$ are the flat superspace covariant derivatives. Let $\Psi$ be a chiral prepotential for $G^{ij}$, $\bar{D}^{\dot{a}}_i \Psi = 0$. We represent

$$G^{ij} = G^{ij}_+ + G^{ij}_-, \quad G^{ij}_+ = \frac{1}{4} D^{ij} \Psi, \quad G^{ij}_- = \frac{1}{4} \bar{D}^{ij} \bar{\Psi}. \quad (4.2)$$

It is always possible to choose the prepotential $\Psi$ such that the following properties hold:

$$\Psi^2 = 0, \quad D_A G^{ij}_+ = 0, \quad G^{ij}_+ G^{ij}_+ = 0. \quad (4.3)$$
In $\mathcal{N} = 2$ Minkowski superspace, the constraints (2.1a)–(2.1c) turn into
\begin{align}
\bar{D}^{\dot{\alpha}} \mathcal{Z} &= 0 , \quad \text{(4.4a)}
D^{ij} \mathcal{Z} - \bar{D}^{ij} \bar{\mathcal{Z}} &= 4i G^{ij} , \quad \text{(4.4b)}
\mathcal{Z}^2 &= 0 . \quad \text{(4.4c)}
\end{align}
The action (3.10) becomes
\begin{equation}
S = -\frac{i}{4} \int d^4x \, d^4\theta \mathcal{Z} \Psi + \text{c.c.} \quad \text{(4.5)}
\end{equation}
Since $G^{ij}$ is constant, it is invariant under the $\mathcal{N} = 2$ supersymmetry transformations. In accordance with the analysis given in the previous section, the action is $\mathcal{N} = 2$ supersymmetric.

For the Grassmann coordinates $\theta^\alpha_i$ and $\bar{\theta}^{\dot{\alpha}}_i$ of $\mathcal{N} = 2$ Minkowski superspace, as well as for the spinor covariant derivatives $D^{i\alpha}$ and $\bar{D}^{i\dot{\alpha}}$, it is useful to label the values of their $R$-symmetry indices as $i, j = 1, 2$. Without loss of generality we can choose
\begin{equation}
G^{ij} = -i \delta^i_2 \delta^j_2 , \quad \Psi = i \theta^\alpha_2 \theta^{\dot{\alpha}}_2 . \quad \text{(4.6)}
\end{equation}
We can now reproduce the results of [2] from the $\mathcal{N} = 2$ setup described. In order to solve the constraints (4.4), it is useful to carry out a reduction to $\mathcal{N} = 1$ Minkowski superspace.

Given a superfield $U(x, \theta_i, \bar{\theta}^i)$ on $\mathcal{N} = 2$ Minkowski superspace, we introduce its bar-projection
\begin{equation}
U := U(x, \theta_i, \bar{\theta}^i)|_{\theta_2 = \bar{\theta}_2 = 0} , \quad \text{(4.7)}
\end{equation}
which is a superfield on $\mathcal{N} = 1$ Minkowski superspace with the Grassmann coordinates $\theta^\alpha = \theta^\alpha_1$ and $\bar{\theta}^{\dot{\alpha}} = \bar{\theta}^{\dot{\alpha}}_1$ and the spinor covariant derivatives $D_{\alpha} = D_{\alpha}^1$ and $\bar{D}^{\dot{\alpha}} = \bar{D}^{\dot{\alpha}}_1$. The background superfield $\Psi$ is characterised by the properties
\begin{equation}
\Psi| = 0 , \quad D^{2}_{\alpha} \Psi| = 0 . \quad \text{(4.8)}
\end{equation}
Since $\mathcal{Z}^2 = 0$, the constraints (4.4) imply
\begin{equation}
(D^{\alpha\dot{\alpha}} \mathcal{Z}) D^{\dot{\alpha}}_{\alpha} \mathcal{Z} + \mathcal{Z} \bar{D}^{\dot{\alpha}}_{\alpha} \bar{\mathcal{Z}} + 4 \mathcal{Z} = 0 . \quad \text{(4.9)}
\end{equation}
Taking the bar-projection of this constraint gives
\begin{equation}
X + \frac{1}{4} X D^2 \bar{X} = W^2 , \quad W^2 := W^\alpha W_\alpha . \quad \text{(4.10)}
\end{equation}
where we have introduced the $\mathcal{N} = 1$ components of $Z$:

$$X := Z|, \quad W_\alpha := -\frac{i}{2} D_{\dot{\alpha}}^2 Z|. \tag{4.11a}$$

These superfields satisfy the constraints

$$\bar{D}_{\dot{\alpha}} X = 0, \quad \bar{D}_{\dot{\alpha}} W_\alpha = 0, \quad D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}} W^\dot{\alpha}. \tag{4.11b}$$

The constraints on $W_\alpha$ tell us that it can be interpreted as the field strength of an Abelian $\mathcal{N} = 1$ vector multiplet. The constraint (4.10) is equivalent to the Bagger-Galperin constraint [2]. Its general solution is

$$X = W^2 - \frac{1}{2} D^2 \frac{W^2 \bar{W}^2}{(1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{4} B^2})}, \tag{4.12a}$$

$$A = \frac{1}{2} (D^2 W^2 + \bar{D}^2 \bar{W}^2), \quad B = \frac{1}{2} (D^2 W^2 - \bar{D}^2 \bar{W}^2). \tag{4.12b}$$

Upon reduction to $\mathcal{N} = 1$ superspace, the action (4.5) becomes

$$I = \frac{1}{4} \int d^4 x d^2 \theta X + \frac{1}{4} \int d^4 x d^2 \bar{\theta} \bar{X}. \tag{4.13}$$

This is the $\mathcal{N} = 1$ supersymmetric Born-Infeld action. Being manifestly $\mathcal{N} = 1$ supersymmetric, the action is also invariant under the second nonlinearly realised supersymmetry transformation [2]

$$\delta \epsilon W_\alpha = \epsilon_\alpha + \frac{1}{4} \epsilon_\alpha \bar{D}^2 \bar{X} + i \epsilon^{\dot{\beta}} \partial_{\dot{\alpha} \dot{\beta}} X \quad \Rightarrow \quad \delta \epsilon X = 2 \epsilon^\alpha W_\alpha. \tag{4.14}$$

For completeness, we re-derive this result.

Let $U$ be a scalar superfield on $\mathcal{N} = 2$ Minkowski superspace. Its isometry transformation is

$$\delta \xi U = -\xi U, \tag{4.15}$$

where

$$\xi = \xi^A D_A = \xi^\alpha \partial_\alpha + \xi_i D_i^\alpha + \xi_{\dot{\alpha}} \bar{D}_{\dot{\alpha}} \tag{4.16}$$

is a Killing supervector field of Minkowski superspace [6]

$$\xi^\alpha = -\frac{i}{8} \bar{D}_{\dot{\beta}} \xi^{\dot{\beta} \alpha}, \quad D^i_{(\alpha} \xi_{\beta)} = \bar{D}^{\dot{i}(\alpha} \xi_{\beta)} = 0, \quad D^i \xi_{\alpha} = 0. \tag{4.17}$$

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6It follows from (4.17) that $\xi^\alpha$ is chiral, $\bar{D}_{\dot{\beta}} \xi_{\alpha} = 0$. 
The Killing supervector field generating the supersymmetry transformation is characterised by the components

$$\xi^a = 2i(\theta_i \sigma^a \tilde{\epsilon}^i - \epsilon_i \sigma^a \tilde{\theta}^i), \quad \xi^\alpha = \epsilon^\alpha = \text{const.} \quad (4.18)$$

Applying this transformation to $Z$ gives $\delta_{\xi} Z = \xi^a \partial_a Z + \xi^\alpha D^\alpha Z$. We now consider only the second supersymmetry transformation by choosing $\epsilon^1 = 0$ and $\epsilon^2 = \epsilon^\alpha$. It acts on the $\mathcal{N} = 1$ superfields (4.11a) as follows

$$\delta_{\epsilon} X = -\xi^\alpha (D^\alpha Z)| = -2i\epsilon^\alpha W_\alpha, \quad (4.19a)$$

$$\delta_{\epsilon} W_\alpha = -\frac{i}{2}(D^\alpha \xi Z)| = -i\epsilon_\alpha - \frac{i}{4} \epsilon_\alpha \bar{D}^2 \tilde{X} - \epsilon^{\beta}_{\alpha \beta} X, \quad (4.19b)$$

where we have made use of the constraints obeyed by $Z$ and $X$. The supersymmetry transformation (4.14) follows from (4.19) upon a rescaling of $\epsilon^\alpha$.

5 Maxwell-Goldstone multiplet for partially broken rigid supersymmetry in curved space

We turn to applying the theoretical framework of section 3 to maximally supersymmetric curved backgrounds in $\mathcal{N} = 2$ supergravity.

5.1 Curved $\mathcal{N} = 2$ superspace backgrounds

We consider a maximally supersymmetric background $\mathbb{M}^{4|8}$ described by the following algebra of covariant derivatives $^7$

$$\{D^i_\alpha, D^j_\beta\} = \{\bar{D}^\dot{i}_{\dot{\alpha}}, \bar{D}^{\dot{j}}_{\dot{\beta}}\} = 0, \quad (5.1a)$$

$$\{D^i_\alpha, \bar{D}^{\dot{j}}_{\dot{\beta}}\} = -2i\delta^i_\alpha D^j_\beta + 4i G^{\gamma j} M_{\alpha \gamma} + 4i G_{\alpha \dot{\gamma}}^j \bar{M}^{\dot{\gamma} \beta}$$

$$\quad -4i\delta^i_\alpha G_{\alpha \beta kl} J_{kl} - 2i G_{\alpha \beta j} Y, \quad (5.1b)$$

$$\{D_\alpha, \bar{D}^{\dot{\beta}}_{\dot{j}}\} = (\bar{\sigma}_a)_{\gamma}^\dot{\beta} G_{\beta a}^j k D^k_{\gamma}, \quad [D_\alpha, \bar{D}^{\dot{\beta}}_{\dot{j}}] = -(\bar{\sigma}_a)^\gamma_{\alpha \beta} G_{\alpha \beta j} k \bar{D}^{\dot{\gamma} k}, \quad (5.1c)$$

where the torsion tensor $G_{\alpha}^{ij}$ is annihilated by the spinor covariant derivatives,

$$D^i_\alpha G^{jk}_b = 0, \quad \bar{D}^{\dot{i}}_\dot{\alpha} G^{jk}_b = 0. \quad (5.1d)$$

$^7$Here $M_{ab}$, $J^{kl}$ and $Y$ are the Lorentz, SU(2) and U(1) generators, respectively, defined as in [13].
This algebra is obtained from that corresponding to $\mathcal{N} = 2$ conformal supergravity, and given by eq. (2.8) in [13], by (i) switching off the components $S^{ij}, Y_{\alpha \beta}, W_{\alpha \beta}$ and $G_{\alpha \dot{\alpha}}$ of the torsion tensor; and (ii) imposing (5.1d). The constraints (5.1d) are required by the theorem [11] that all fermionic components of the superspace torsion tensor must vanish in maximally supersymmetric backgrounds.

In complete analogy with the 5D case [11], the constraints (5.1d) imply the following integrability condition

$$G_a^{\ k(i} G_{b^i)}_k = 0 .$$ (5.2)

As shown in [11], the general solution of the conditions (5.1d) and (5.2) is

$$G_b^{\ kl} = -\frac{1}{4} g_b s^{kl}, \quad \mathcal{D}_a g_b = 0 , \quad \bar{\mathcal{D}}_\dot{\alpha} g_b = 0 , \quad \mathcal{D}_A s^{kl} = 0 ,$$ (5.3)

for some real vector $g_b$ and real $\text{SU}(2)$ triplet $s^{kl}$. The latter may be normalised as

$$s^{ij} s_{ij} = 2 .$$ (5.4)

Since $g^2 = g^a g_a$ is constant, $\mathcal{D}_A g^2 = 0$, there are in fact three different superspaces described by the above algebra: (i) if $g_a$ is time-like, $g^2 < 0$, the bosonic body of $\mathbb{M}^{4|8}$ is $\mathbb{R} \times S^3$; (ii) if $g_a$ is space-like, $g^2 > 0$, the bosonic body of $\mathbb{M}^{4|8}$ is $\text{AdS}_3 \times \mathbb{R}$; (iii) in the null case, $g^2 = 0$, the spacetime geometry is a pp-wave. We will denote these superspaces as $\mathbb{M}_T^{4|8}$, $\mathbb{M}_S^{4|8}$ and $\mathbb{M}_N^{4|8}$, respectively. These backgrounds were constructed in [10], and they have 5D cousins [11].

In order to get some more insight into the structure of the superspace geometry (5.1), a specific value of $g^2$ has to be fixed. It suffices to consider the superspace $\mathbb{M}_T^{4|8}$, since the other two cases may be treated similarly. As a supermanifold, $\mathbb{M}_T^{4|8}$ is the universal covering of the 4D $\mathcal{N} = 2$ superspace introduced in Appendix B.

In the case $g^2 < 0$, it is possible to choose a Lorentz and $\text{SU}(2)_R$ gauge such that

$$g_a = (g, 0, 0, 0) , \quad s_i^j = -i(\sigma^3)_i^j = i(-1)^i \delta_i^j .$$ (5.5)

As shown in [10], the algebra of covariant derivatives is equivalent to

$$\{\mathcal{D}_a^i, \mathcal{D}_j^j\} = \{\bar{\mathcal{D}}_\dot{\alpha}^i, \bar{\mathcal{D}}_\dot{\beta}^\dot{\beta}\} = 0 , \quad \{\mathcal{D}_a^i, \bar{\mathcal{D}}_\dot{\alpha}^\dot{\beta}\} = -2i\delta^i_j (\sigma^a)_\alpha^{\beta} \mathcal{D}_a^j ,$$ (5.6a)

$$[\mathcal{D}_a^i, \mathcal{D}_b^j] = \frac{i}{2} \delta^{ij} (-1)^j (\sigma_a)_{\beta \dot{\beta}} g^{\beta \dot{\gamma}} \mathcal{D}_a^\gamma ,$$ (5.6b)

$$[\mathcal{D}_a^i, \mathcal{D}_b^j] = (-1)^{i+1} \delta^{ij} \varepsilon_{abc} d^c g^d \mathcal{D}_d^j ,$$ (5.6c)
where we have introduced the “improved” vector covariant derivatives

$$D_a^{(i)} := D_a + \frac{1}{2} g_{a \bar{s} k l} J_{k l} + (-1)^i \left( \frac{1}{4} \varepsilon_{a b c d} g^{b c d} M^{c d} + i g_a Y \right). \quad (5.7)$$

These (anti-)commutation relations correspond to the superalgebra $\mathfrak{su}(2|1) \times \mathfrak{su}(2|1)$.

The superspace geometry of $\mathbb{M}^{4|8}_T$ can be described, e.g., in terms of the covariant derivatives $\tilde{D}_A = (D^{(1)}_a, D^i_{\alpha}, \bar{D}^\dot{\alpha}_i)$. In accordance with (5.6), the operators $(D^{(1)}_a, \bar{D}^\dot{\alpha}_i)$ form a closed algebra isomorphic to that of the superalgebra $\mathfrak{su}(2|1)$. This property means that the $\mathcal{N} = 2$ superspace $\mathbb{M}^{4|8}_T$ possesses an $\mathcal{N} = 1$ subspace which will be denoted $\mathbb{M}^{4|4}_T$. In particular, this superspace allows the existence of covariantly constant complex $\text{SU}(2)$ triplets $G_{ij}$. Since the graded commutation relations for $\tilde{D}_A$ involve neither Lorentz nor $\text{SU}(2)$ curvature tensors, the Lorentz and $\text{SU}(2)$ connections may be gauged away. In such a gauge, every constant complex $\text{SU}(2)$ triplets $G_{ij}$ is covariantly constant.

Since the superspaces $\mathbb{M}^{4|8}_T$, $\mathbb{M}^{4|8}_S$ and $\mathbb{M}^{4|8}_N$ meet the requirements (3.2)–(3.4), the formalism of section 3 may be used to construct a Maxwell-Goldstone multiplet action for partial supersymmetry breaking. Instead of implementing the scheme directly, we will take a shortcut to constructing such actions on the $\mathcal{N} = 1$ subspaces of the superspaces $\mathbb{M}^{4|8}_T$, $\mathbb{M}^{4|8}_S$ and $\mathbb{M}^{4|8}_N$.

### 5.2 Goldstone multiplet for partially broken supersymmetry

We consider a maximally supersymmetric background $\mathbb{M}^{4|4}_T$ described by the following algebra of $\mathcal{N} = 1$ covariant derivatives:

\[
\begin{align*}
\{ D_\alpha, D_\beta \} &= 0, & \{ \bar{D}_\dot{\alpha}, \bar{D}_\dot{\beta} \} &= 0, & \{ D_\alpha, \bar{D}_\dot{\beta} \} &= -2i D_{\alpha \beta}, \\
[ D_\alpha, D_{\beta \dot{\beta}} ] &= i \varepsilon_{\alpha \beta} G^{\gamma \dot{\gamma}} D_{\gamma \dot{\gamma}}, & [ \bar{D}_{\dot{\alpha}}, D_{\beta \dot{\beta}} ] &= -i \varepsilon_{\dot{\alpha} \dot{\beta}} G_{\dot{\beta} \dot{\gamma}} \bar{D}_{\dot{\gamma} \dot{\gamma}}, \\
[ D_{\alpha \dot{\alpha}}, D_{\beta \dot{\beta}} ] &= -i \varepsilon_{\dot{\alpha} \dot{\beta}} G_{\dot{\gamma}} \gamma \gamma D_{\alpha \gamma} + i \varepsilon_{\alpha \beta} G_{\beta \dot{\gamma}} \bar{D}_{\gamma \dot{\gamma} \dot{\alpha}}.
\end{align*}
\]  

where the torsion tensor $G_a$ is covariantly constant,

$$D_A G_b = 0. \quad (5.8d)$$

This is a special case of the superspace geometry for $\mathcal{N} = 1$ old minimal supergravity [34] reviewed in [32]. The above algebra is obtained from the supergravity (anti-)commutation relations (5.5.6) and (5.5.7) in [32] by (i) switching off the chiral torsion superfields $R$ and $W_{\alpha \beta \gamma}$ and their conjugates; and (ii) imposing the condition (5.8d).
Since $G^2 = G^a G_a$ is constant, the geometry (5.8) describes three different superspaces, $\mathcal{M}^{4|4}_T$, $\mathcal{M}^{4|4}_S$, and $\mathcal{M}^{4|4}_N$, which correspond to the choices $G^2 < 0$, $G^2 > 0$ and $G^2 = 0$, respectively. These $\mathcal{N} = 1$ superspaces originate as the $\mathcal{N} = 1$ subspaces of the $\mathcal{N} = 2$ superspaces of $\mathcal{M}^{4|8}_T$, $\mathcal{M}^{4|8}_S$, and $\mathcal{M}^{4|8}_N$, respectively, considered in the previous subsection.

We recall that the Lorentzian manifolds supported by these super spaces are $\mathbb{R} \times S^3$, AdS$_3 \times S^1$ or its covering AdS$_3 \times \mathbb{R}$, and a pp-wave spacetime, respectively. As a supermanifold, $\mathcal{M}^{4|4}_T$ is the universal covering of the $\mathcal{N} = 1$ superspace $\mathcal{M}^{4|4}$ introduced in section A.1. The isometry group of $\mathcal{M}^{4|4}$ is $SU(2|1) \times U(2)$. As a supermanifold, $\mathcal{M}^{4|4}_S$ is the universal covering of the $\mathcal{N} = 1$ superspace $\tilde{\mathcal{M}}^{4|4}$ introduced in section A.2. The isometry group of $\tilde{\mathcal{M}}^{4|4}$ is $SU(1,1|1) \times U(2)$.

The superspace $\mathcal{M}^{4|4}$ allows the existence of covariantly constant spinors,

$$\mathcal{D}_A \epsilon_\alpha = 0 \ .$$

(5.9)

Such a spinor is constant in a gauge in which the Lorentz connection vanishes.

By analogy with the flat-superspace case, we consider the following $\mathcal{N} = 1$ supersymmetric theory with action

$$S = \frac{1}{4} \int d^4 x \ d^2 \theta \ E X + \text{c.c.} \ ,$$

(5.10)

where the covariantly chiral superfield $X$ is a unique solution of the constraint

$$X + \frac{1}{4} X \mathcal{D}^2 X = W^2 \ .$$

(5.11)

The superfield $W_\alpha$ is the chiral field strength of an Abelian vector multiplet and, together with its complex conjugate $\bar{W}_{\dot{\alpha}}$, it obeys the Bianchi identity

$$\mathcal{D}^\alpha W_\alpha = \mathcal{D}_a \bar{W}_{\dot{a}} \ .$$

(5.12)

The explicit solution of the constraint (5.11) is a covariantisation of that described in the previous section. It is given, e.g., in (38).

The action (5.10) is invariant under a second supersymmetry given by

$$\delta X = 2 \epsilon^\alpha W_\alpha \ ,$$

(5.13)
with the parameter $\epsilon_\alpha$ being constrained as in (5.9). Of course, this transformation should be induced by that of $W_\alpha$. The correct supersymmetry transformation of $W_\alpha$ proves to be

$$\delta_\epsilon W_\alpha = \epsilon_\alpha + \frac{1}{4} \epsilon_\alpha \mathcal{D}^2 \bar{X} + i \epsilon^\beta \mathcal{D}_{\alpha\beta} X - \bar{\epsilon}^\beta G_{\alpha\beta} X .$$

(5.14)

It has the correct flat superspace limit [2], compare with (4.14), and respects the Bianchi identity (5.12),

$$\mathcal{D}^\alpha \delta_\epsilon W_\alpha = \mathcal{D}_\alpha \delta_\epsilon \bar{W}_\alpha .$$

(5.15)

The dynamical system defined by eqs. (5.10) and (5.11) describes the Maxwell-Goldstone multiplet action for partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking in those curved spacetimes which are supported by the superspace geometry (5.8), including $\mathbb{R} \times S^3$, AdS$_3 \times S^1$ and its covering AdS$_3 \times \mathbb{R}$.

6 Concluding comments

There are five types of maximally supersymmetric backgrounds in four-dimensional $\mathcal{N} = 1$ off-shell supergravity, two of which are well known: Minkowski superspace $\mathbb{R}^{4|4}$ [39, 40] and anti-de Sitter superspace AdS$_{4|4}$ [41, 42, 43]. The remaining three superspaces, $\mathcal{M}_{T}^{4|4}$, $\mathcal{M}_{S}^{4|4}$ and $\mathcal{M}_{N}^{4|4}$, are described by the geometry (5.8) with different choices of $G_a$. All five $\mathcal{N} = 1$ superspaces possess $\mathcal{N} = 2$ extensions. The Maxwell-Goldstone multiplet on $\mathbb{R}^{4|4}$ for partially broken $\mathcal{N} = 2$ Poincaré supersymmetry was found long ago [2, 5]. In this paper, we have constructed the Maxwell-Goldstone multiplets which are defined on $\mathcal{M}_{T}^{4|4}$, $\mathcal{M}_{S}^{4|4}$ and $\mathcal{M}_{N}^{4|4}$ and describe partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking.

In Appendix C we demonstrate that no Maxwell-Goldstone multiplet action for partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking exists in the case of the anti-de Sitter (AdS) supersymmetry. The reason for this obstruction is the fact that every covariantly constant SU(2) triplet $G^{ij}_+$ must be proportional to the torsion tensor $S^{ij}$, which is real and covariantly constant in AdS$_{4|8}$ [44]. As a consequence, the conditions (3.2) and (3.3) are not compatible in AdS$_{4|8}$. Since the $\mathcal{N} = 1$ AdS superspace AdS$_{4|4}$ is naturally embedded in AdS$_{4|8}$ as a subspace [73], applying the formalism of section 2 to the case of AdS$_{4|8}$ allows us to derive a Maxwell-Goldstone multiplet for partially broken $\mathcal{N} = 2$ AdS supersymmetry. The corresponding technical details are spelled out in Appendix C. However, since the conditions (3.2) and (3.3) are not compatible in AdS$_{4|8}$, we cannot use this Maxwell-Goldstone multiplet to construct a supersymmetric invariant action.
There exists a one-parameter family of $\mathcal{N} = 1$ supersymmetric extensions of the Born-Infeld actions [4]. A unique extension is fixed by the requirement that the action should describe the Maxwell-Goldstone multiplet on $\mathbb{R}^{4|4}$ for partially broken $\mathcal{N} = 2$ Poincaré supersymmetry [2, 5]. The same extension is uniquely fixed by the requirement of $U(1)$ duality invariance [45, 46], which implies the self-duality under superfield Legendre transform discovered by Bagger and Galperin [2]. A curved-superspace extension of the $\mathcal{N} = 1$ supersymmetric Born-Infeld action is not unique. However, a unique extension is fixed by the requirement of $U(1)$ duality invariance [38]. It is given by the action (5.10) in which $X$ is a unique solution to the constraint

$$X + \frac{1}{4}X(\bar{D}^2 - 4R)\bar{X} = W^2,$$

(6.1)

with $R$ the chiral scalar torsion superfield. This action was first proposed in [47]. In the case of anti-de Sitter superspace AdS$^{4|4}$, the only non-zero components of the superspace torsion are $R$ and $\bar{R}$, which are constant. The corresponding $\mathcal{N} = 1$ supersymmetric Born-Infeld action possesses $U(1)$ duality invariance, however it is not invariant under a second nonlinearly realised supersymmetry, as demonstrated in Appendix C. Therefore, this action is not suitable to describe a partial breaking of the $\mathcal{N} = 2$ AdS supersymmetry.

In addition to the Maxwell-Goldstone multiplet of [2, 5], there exist other multiplets for partially broken $\mathcal{N} = 2$ Poincaré supersymmetry [48, 3, 49]. We believe these models can be generalised to the superspaces $M_{T}^{4|4}$, $M_{S}^{4|4}$ and $M_{N}^{4|4}$ to describe partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking. It would also be interesting to investigate whether some of these models can be extended to describe partially broken $\mathcal{N} = 2$ AdS supersymmetry.

Recently, there has been much interest in models for spontaneously broken local $\mathcal{N} = 1$ supersymmetry [50, 51, 52, 53, 54, 55, 56, 57, 58], which are based on the use of the nilpotent chiral Goldstino superfield proposed in [59, 60]. Other nilpotent Goldstino superfields can be used to describe spontaneously broken $\mathcal{N} = 1$ supergravity [61, 62, 63] (for an alternative approach to de Sitter supergravity, see [64]). At the moment it is not clear whether the nilpotent $\mathcal{N} = 2$ chiral superfield advocated in the present paper is suitable for the description of partial supersymmetry breaking in $\mathcal{N} = 2$ supergravity. It is certainly of interest to develop a superspace description for the models for spontaneous $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ local supersymmetry breaking pioneered in [65, 66] and further developed, e.g., in [67, 68].
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A $\mathcal{N} = 1$ superspaces over $U(2) = (S^1 \times S^3)/\mathbb{Z}_2$ and $(\text{AdS}_3 \times S^1)/\mathbb{Z}_2$

In this appendix we give supermatrix realisations for two maximally supersymmetric backgrounds in 4D $\mathcal{N} = 1$ supergravity.

A.1 $\mathcal{N} = 1$ superspace over $U(2) = (S^1 \times S^3)/\mathbb{Z}_2$

Here and in the next appendix, the supergroup $SU(2|1)$ is defined to consist of complex $(2|1) \times (2|1)$ supermatrices (with $A, D$ bosonic blocks and $B, C$ fermionic ones)

$$g = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

(A.1)

constrained by

$$g^\dagger \eta g = \eta , \quad \text{Ber } g = 1 , \quad \eta = \begin{pmatrix} 1_2 & 0 \\ 0 & -1 \end{pmatrix} .$$

(A.2)

We introduce a superspace $\mathcal{M}^{4|4}$ consisting of complex $(2|1) \times (2|0)$ supermatrices (with $h$ bosonic and $\Theta$ fermionic blocks)

$$p = \begin{pmatrix} h \\ \Theta \end{pmatrix}$$

(A.3)

constrained by

$$p^\dagger \eta p = 1_2 \iff h^\dagger h = 1_2 + \Theta^\dagger \Theta .$$

(A.4)
The supermanifold defined by this equation coincides with the 4D $\mathcal{N} = 1$ compactified Minkowski superspace (described in detail in section 3 of [69]) on which the superconformal group $\text{SU}(2, 2|1)$ acts by well-defined transformations. The bosonic body of the superspace is $\text{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$.

It is useful to switch from the variables $h$ and $\Theta$ to new ones, $\varphi \in \mathbb{R}$, $u$ and $\theta$, defined as follows:

$$\mathcal{P} = \begin{pmatrix} e^{i\varphi}u \\ e^{i\varphi}\theta \end{pmatrix}, \quad u^\dagger u = 1_2 + \theta^\dagger \theta, \quad \det u = \det u^\dagger = \left(1 + \theta\theta^\dagger\right)^{-\frac{1}{2}}. \quad (A.5)$$

We can represent

$$u = \hat{u}\sqrt{1_2 + \theta^\dagger \theta}, \quad \hat{u} \in \text{SU}(2). \quad (A.6)$$

The supermatrix (A.5) is invariant under the $\mathbb{Z}_2$ transformation $\varphi \to \varphi + \pi$, $\hat{u} \to -\hat{u}$ and $\theta \to -\theta$. This is the origin of $\mathbb{Z}_2$ in $\text{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$.

It turns out that the superspace $\mathcal{M}^{4|4}$ introduced above can be identified with the group manifold $\text{SU}(2|1)$. Indeed, it may be checked that every element $g \in \text{SU}(2|1)$ has the form (compare with a similar result in [70])

$$g = \frac{\left(e^{i\varphi}u \right)}{e^{i\varphi} \theta} \frac{e^{2i\varphi} (1 + \theta\theta^\dagger)^{-\frac{1}{2}} u^\dagger \theta^\dagger}{e^{2i\varphi} (1 + \theta\theta^\dagger)^{\frac{1}{2}}}, \quad (A.7)$$

where $u$ is constrained as in (A.5).

The isometry group of $\mathcal{M}^{4|4}$ is $\text{SU}(2|1) \times \text{U}(2)$. It acts on $\mathcal{M}^{4|4}$ as follows:

$$\mathcal{P} \to g_L \mathcal{P} g_R^{-1}, \quad g_L \in \text{SU}(2|1), \quad g_R \in \text{U}(2). \quad (A.8)$$

These transformations are holomorphic in terms of the variables $h$ and $\Theta$ (hence the isometry transformations act on a chiral subspace of the full superspace). The isometry group has two $\text{U}(1)$ subgroups that describe $R$-symmetry transformations and time translations. One subgroup corresponds to all diagonal supermatrices (A.7) with $u = 1_2$ and $\theta = 0$. The other subgroup is spanned by all diagonal matrices $e^{i\psi}1_2$ in $\text{U}(2)$.

On the group manifold $\text{SU}(2|1)$, we can define an action of $\text{SU}(2|1) \times \text{SU}(2|1)$ by the standard rule

$$g \to g_L g g_R^{-1}, \quad g_L, g_R \in \text{SU}(2|1). \quad (A.9)$$

These transformations leave invariant the supermetric

$$ds^2 = -\frac{1}{2} \text{Str} \mathcal{E}^2, \quad \mathcal{E} = g^{-1}dg. \quad (A.10)$$

However, such transformations map the chiral subspace (A.3) to itself only if $g_R \in \text{U}(2)$.
A.2 $\mathcal{N} = 1$ superspace over $U(1, 1) = (\text{AdS}_3 \times S^1)/\mathbb{Z}_2$

We define the supergroup $SU(1, 1|1)$ to consist of complex $(2|1) \times (2|1)$ supermatrices (with $A, D$ bosonic blocks and $B, C$ fermionic ones)

$$g = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

(A.11)

constrained by

$$g^\dagger \eta g = \eta , \quad \text{Ber} \ g = 1 , \quad \eta = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -1 \end{pmatrix} .$$

(A.12)

Every element $g \in SU(1, 1|1)$ can be written in the form

$$g = \begin{pmatrix} e^{i\varphi} u & e^{2i\varphi} (1 + \theta \sigma_3 \theta^\dagger)^{-\frac{1}{2}} u \theta^\dagger \\ e^{i\varphi} \theta & e^{2i\varphi} (1 + \theta \sigma_3 \theta^\dagger)^{\frac{1}{2}} \end{pmatrix} ,$$

(A.13)

where $u$ is constrained by

$$u^\dagger \sigma_3 u = \sigma_3 + \theta^\dagger \theta , \quad \det u = \det u^\dagger = (1 + \theta \sigma_3 \theta^\dagger)^{-\frac{1}{2}} .$$

(A.14)

We can represent

$$u = \hat{u} \sqrt{1_{12} + \sigma_3 \theta^\dagger \theta} , \quad \hat{u} \in SU(1, 1) .$$

(A.15)

The supermatrix defined by eqs. (A.13) and (A.15) is invariant under the discrete transformation $\varphi \to \varphi + \pi$, $\hat{u} \to -\hat{u}$ and $\theta \to -\theta$.

We introduce a four-dimensional superspace $\tilde{\mathcal{M}}^{4|4}$ consisting of complex $(2|1) \times (2|0)$ supermatrices (with $h$ and $\Theta$ being bosonic and fermionic blocks, respectively)

$$\mathcal{P} = \begin{pmatrix} h \\ \Theta \end{pmatrix} \equiv \begin{pmatrix} e^{i\varphi} u \\ e^{i\varphi} \theta \end{pmatrix} ,$$

(A.16)

where $\varphi$, $u$ and $\theta$ are defined as in (A.13). This superspace can be identified with the group manifold $SU(1, 1|1)$. Its bosonic body is $U(1, 1) = (\text{AdS}_3 \times S^1)/\mathbb{Z}_2$.

The isometry group of $\tilde{\mathcal{M}}^{4|4}$ is $SU(1, 1|1) \times U(2)$. It acts on $\tilde{\mathcal{M}}^{4|4}$ as follows:

$$\mathcal{P} \to g_L \mathcal{P} g_R^{-1} , \quad g_L \in SU(1, 1|1) , \quad g_R \in U(2) .$$

(A.17)
These transformations are holomorphic in terms of the variables $h$ and $\Theta$ (hence the isometry transformations acts on a chiral subspace of the full superspace), and leave invariant the supermetric
\[ ds^2 = \frac{1}{2} \text{Str} \, \mathcal{E}^2, \quad \mathcal{E} = g^{-1}dg. \] (A.18)
Unlike the superspace considered in the previous subsection, the dimension parametrised by $\varphi$ is now space-like.

Let us consider the coset space
\[ \text{AdS}_{(3|2,0)} := \text{SU}(1,1|1)/U(1), \] (A.19)
where the subgroup $U(1)$ of $\text{SU}(1,1|1)$ consists of all diagonal supermatrices (A.13) with $u = \begin{pmatrix} 1_2 \\ 1_2 \end{pmatrix}$ and $\theta = 0$. This coset space may be seen to coincide with the 3D $(2,0)$ anti-de Sitter superspace [71]. We recall that in three dimensions, $\mathcal{N}$-extended anti-de Sitter (AdS) superspace exists in several incarnations known as $(p,q)$ AdS superspaces, where the non-negative integers $p \geq q$ are such that $\mathcal{N} = p + q$. The conformally flat $(p,q)$ AdS superspace is
\[ \text{AdS}_{(3|p,q)} = \frac{\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})}{\text{SL}(2, \mathbb{R}) \times \text{SO}(p) \times \text{SO}(q)}. \] (A.20)
In the case $p = \mathcal{N} \geq 4$ and $q = 0$, non-conformally flat AdS superspaces also exist [72].

### B $\mathcal{N} = 2$ superspace over $U(2) = (S^1 \times S^3)/\mathbb{Z}_2$

$\mathcal{N} = 2$ superspace $\mathcal{M}^{4|8}$ over $U(2) = (S^1 \times S^3)/\mathbb{Z}_2$ can be realised as the quotient space
\[ \mathcal{M}^{4|8} := \mathcal{M}_{L}^{4|4} \times \mathcal{M}_{R}^{4|4} / \sim, \] (B.1)
where $\mathcal{M}_{L}^{4|4}$ and $\mathcal{M}_{R}^{4|4}$ denote two copies of $\mathcal{M}^{4|4}$. The equivalence relation is defined by the rule: two pairs $\mathcal{P} = (\mathcal{P}_L, \mathcal{P}_R)$ and $\mathcal{P}' = (\mathcal{P}'_L, \mathcal{P}'_R)$ are equivalent, $\mathcal{P} \sim \mathcal{P}'$, if
\[ \mathcal{P}'_L = \mathcal{P}_L h, \quad \mathcal{P}'_R = \mathcal{P}_R h, \] (B.2)
for some group element $h \in U(2)$.

The isometry group of $\mathcal{M}^{4|8}$ is
\[ G := G_L \times G_R \times U(1) = \text{SU}(2|1) \times \text{SU}(2|1) \times U(1). \] (B.3)
Given a group element \( g = g_L \times g_R \times e^{i\psi} \in G \), with \( \psi \in \mathbb{R} \), it acts on the pair \( \mathcal{P} = (\mathcal{P}_L, \mathcal{P}_R) \) by the rule:

\[
(\mathcal{P}_L, \mathcal{P}_R) \rightarrow (g\mathcal{P}_L, g\mathcal{P}_R), \quad g\mathcal{P}_L = g_L P_L e^{i\psi}, \quad g\mathcal{P}_R = g_R P_R e^{-i\psi}.
\]  

The equivalence relation allows us to choose \( \mathcal{P}_R \) in the form:

\[
\mathcal{P}_R = \left( \sqrt{1 + \psi^\dagger \psi} \right). \tag{B.5}
\]

The above construction can readily be modified in order to describe the \( \mathcal{N} = 2 \) superspace over \( U(1, 1) = (AdS_3 \times S^1)/\mathbb{Z}_2 \).

\section{Example: The anti-de Sitter supersymmetry}

In this appendix we show that the formalism of sections 2 and 3 can be used to define a Goldstone-Maxwell multiplet for partially broken 4D \( \mathcal{N} = 2 \) anti-de Sitter (AdS) supersymmetry with the following properties: (i) it is the standard Maxwell multiplet with respect to the \( \mathcal{N} = 1 \) AdS supersymmetry; (ii) it transforms nonlinearly under the second AdS supersymmetry. However, making use of this multiplet does not allow one to construct an invariant action describing the partial \( \mathcal{N} = 2 \to \mathcal{N} = 1 \) AdS supersymmetry breaking.

To start with, we recall a few definitions concerning the 4D \( \mathcal{N} = 2 \) AdS superspace

\[
AdS^{4|8} := \frac{OSp(2|4)}{SO(3, 1) \times SO(2)},
\]

which is a maximally symmetric geometry that originates within the off-shell formulation for \( \mathcal{N} = 2 \) conformal supergravity developed in [44]. For comprehensive studies of \( \mathcal{N} = 2 \) supersymmetric field theories in AdS$_4$, the reader is referred to [73, 74].

We assume that AdS$_{4|8}$ is parametrised by local bosonic \((x)\) and fermionic \((\theta, \bar{\theta})\) coordinates \(z^M = (x^m, \theta_\mu, \bar{\theta}_{\dot{\mu}})\) (where \(m = 0, 1, 2, 3, \mu = 1, 2, \dot{\mu} = 1, 2\) and \(i = 1, 2\)). The corresponding covariant derivatives

\[
\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}^i_a, \mathcal{D}_i^\beta) = E_A^M \partial_M + \frac{1}{2} \Omega_A^{bc} M_{bc} + \Phi_A^{ij} J_{ij}, \quad i, j = 1, 2
\]

obey the algebra [73]

\[
\{\mathcal{D}_a^i, \mathcal{D}^j_\beta\} = 4S^{ij} M_{a\beta} + 2\varepsilon_{a\beta} \varepsilon^{ij} S^{kl} J_{kl}, \quad \{\mathcal{D}_a^i, \bar{\mathcal{D}}^j_\beta\} = -2i\delta_j^i (\sigma^c)_\beta \mathcal{D}^c, \tag{C.2a}
\]
\[ [\mathcal{D}_a, \mathcal{D}_b^j] = \frac{i}{2}(\sigma_a)_{\beta i} S^{jk} \bar{D}^\gamma_k, \quad [\mathcal{D}_a, \mathcal{D}_b^\gamma] = \frac{i}{2}(\bar{\sigma}_a)^{\delta j} S_j k \mathcal{D}^k, \quad (C.2b) \]

\[ [\mathcal{D}_a, \mathcal{D}_b] = -S^2 M_{ab}. \quad (C.2c) \]

The SU(2) triplet \( S^{ij} \) is the only non-vanishing component of the superspace torsion in AdS\(^{4|8} \); it is **covariantly constant** and real

\[ \mathcal{D} A S^{ij} = 0, \quad \bar{S}^{ij} = S^{ij}. \quad (C.3) \]

The parameter \( S^2 := \frac{1}{2} S^{ij} S_{ij} = \text{const} \) is positive, and therefore (C.2) gives the algebra of covariant derivatives in AdS\(_4\).

The isometry transformations of AdS\(^{4|8} \) form the supergroup OSp(2|4). In the infinitesimal case, an isometry transformation is described by a Killing supervector field \( \xi^A \mathcal{E}_A \), with \( \mathcal{E}_A = \mathcal{E}_A^M \partial_M \), defined to obey the equation

\[ \left[ \xi + \frac{1}{2} l^{bc} M_{bc} + \rho S^{jk} J_{jk}, \mathcal{D}_A \right] = 0, \quad \xi := \xi^b \mathcal{D}_b = \xi^b \mathcal{D}_b + \xi^\alpha \mathcal{D}_\alpha + \bar{\xi}^i \bar{\mathcal{D}}^i, \quad (C.4) \]

for some real antisymmetric tensor \( l^{bc}(z) \) and scalar \( \rho(z) \) parameters. It turns out that the Killing equation (C.4) uniquely determines the parameters \( \xi^a, l^{cd} \) and \( \rho \) in terms of \( \xi^A \). A similar property exists for superspace isometry transformations in any number of dimensions [33]. The specific feature of the 4D \( N = 2 \) AdS superspace is that the parameters \( \xi^A \) and \( l^{ab} \) are uniquely expressed in terms of \( \rho \) [73].

Due to (C.2), the SU(2) gauge freedom can be used to choose the SU(2) connection \( \Phi_{A}^{ij} \) in (C.1) to look like \( \Phi_{A}^{ij} = \Phi_{A} S^{ij} \), for some one-form \( \Phi_{A} \) describing the residual U(1) connection associated with the generator \( S^{ij} J_{ij} \). Then \( S^{ij} \) becomes a constant iso-triplet, \( S^{ij} = \text{const} \). The remaining global SU(2) rotations can take \( S^{ij} \) to any position on the two-sphere of radius \( S \). We make the choice

\[ S^{\underline{12}} = 0, \quad \mu := -S^{\underline{22}}, \quad \bar{\mu} = -S^{\underline{11}}, \quad (C.5) \]

with \( |\mu| = S \). This choice must be used in order to embed an \( N = 1 \) AdS superspace, AdS\(^{4|4} \), into the full \( N = 2 \) AdS superspace [73].

As already mentioned, the choice \( S^{\underline{12}} = 0 \) is required for embedding AdS\(^{4|4} \) into AdS\(^{4|8} \). By applying certain general coordinate and local U(1) transformations in AdS\(^{4|8} \), it is possible to identify AdS\(^{4|4} \) with the surface \( \theta^\mu_2 = 0 \) and \( \bar{\theta}^\mu_2 = 0 \). The covariant derivatives for AdS\(^{4|4} \),

\[ \mathcal{D}_A = \left( \mathcal{D}_a, \mathcal{D}_\alpha, \mathcal{D}^\dot{\alpha} \right) = E_A^M \partial_M + \frac{1}{2} \Omega_A^{bc} M_{bc}, \quad (C.6) \]
are related to (C.1) as follows
\[ D_\alpha := \mathcal{D}_\alpha^\dagger, \quad \bar{D}^\dagger := \bar{\mathcal{D}}^\dagger, \] (C.7)
and similarly for the vector covariant derivative. Here the bar-projection is defined by
\[ U| := U(x, \theta, \bar{\theta}')|_{\theta_2 = \bar{\theta}_2 = 0}, \] (C.8)
for any $N = 2$ tensor superfield $U(x, \theta, \bar{\theta})$. It follows from (C.2) that the $N = 1$ covariant derivatives obey the algebra
\[ \{ D_\alpha, D_\beta \} = -4 \bar{\mu} M_{\alpha \beta}, \quad \{ \bar{D}_\dot{\alpha}, \bar{D}_\dot{\beta} \} = 4 \mu \bar{M}_{\dot{\alpha} \dot{\beta}}, \quad \{ D_\alpha, \bar{D}_\dot{\beta} \} = -2i \bar{\mu} \bar{M}_{\alpha \dot{\beta}}, \] (C.9a)
\[ [D_a, D_\beta] = -|\mu|^2 M_{ab}, \] (C.9b)
\[ [D_a, \bar{D}_\dot{\beta}] = i |\mu| \bar{M}_{\alpha \dot{\beta}} D_\alpha, \] (C.9c)
which indeed corresponds to the $N = 1$ AdS superspace (see [32] for more details). As a result, every $N = 2$ supersymmetric field theory in AdS$^{4|8}$ can be reformulated as some theory in AdS$^{4|4}$.

Given an $N = 2$ tensor superfield $U(x, \theta, \bar{\theta})$, its infinitesimal $\text{OSp}(2|4)$ transformation law is
\[ \delta_\xi U = -\left( \xi + \frac{1}{2} l^{bc} M_{bc} + \rho S^{jk} J_{jk} \right) U. \] (C.10)
Upon reduction to AdS$^{4|4}$, this transformation law turns into a superposition of several independent $N = 1$ transformations. Evaluating the bar-projection of $\xi$ gives
\[ \xi| = \lambda + \varepsilon^\alpha D_\alpha^\dagger| + \bar{\varepsilon} \bar{D}^\dagger, \quad \lambda = \lambda^\alpha D_\alpha = \lambda^\dot{\alpha} \bar{D}_\dot{\alpha}, \] (C.11a)
where we have introduced
\[ \lambda^\alpha := \xi^\alpha|, \quad \lambda^\dot{\alpha} := \xi^\dot{\alpha}|, \quad \bar{\varepsilon} := \varepsilon^\alpha|, \quad \bar{\varepsilon}_\dot{\alpha} := \xi^\dot{\alpha}|. \] (C.11b)
We denote the bar-projection of the parameters $l_{ab}$ and $\rho$ as
\[ \omega_{ab} := l_{ab}|, \quad \varepsilon := \rho|. \] (C.12)
It holds that
\[ \omega_{\alpha \beta} = \mathcal{D}_\alpha \lambda_\beta = \mathcal{D}_\beta \lambda_\alpha. \] (C.13)
Now, the bar-projection of (C.10) takes the form
\[ \delta_\xi U| = -\left( \lambda + \frac{1}{2} \omega_{ab} M_{ab} \right) U| - \left( \varepsilon^\alpha D_\alpha^\dagger U| + \bar{\varepsilon}_\dot{\alpha} \bar{D}_\dot{\alpha}^\dagger U| \right) + \varepsilon (\bar{\mu} J_{11} + \mu J_{22}) U|. \] (C.14)
The first term on the right is an infinitesimal $\text{OSp}(1|4)$ transformation generated by $\lambda$. The parameters $\lambda$ and $\omega^{bc}$ obey the equation

$$[\lambda + \frac{1}{2} \omega^{bc} M_{bc}, \mathcal{D}_A] = 0, \quad (C.15)$$

which defines the Killing supervector field of AdS$_4^{4|4}$ [32]. The second and third terms on the right of (C.14) prove to describe the second supersymmetry and U(1) transformations. The corresponding parameters $\varepsilon_\alpha$, $\bar{\varepsilon}_\dot{\alpha}$ and $\varepsilon$ have the properties

$$\varepsilon_\alpha = \frac{1}{2} \mathcal{D}_\alpha \varepsilon, \quad \mathcal{D}_\alpha \bar{\mathcal{D}}_{\dot{\alpha}} \varepsilon = 0, \quad (\mathcal{D}^2 - 4\bar{\mu}) \varepsilon = 0. \quad (C.16)$$

The parameter $\varepsilon$ was originally introduced in [75].

We are now prepared to analyse the nilpotent $\mathcal{N} = 2$ chiral superfield $Z$ constrained by (2.1) in the case that the background superspace is AdS$_4^{4|8}$. We recall that a necessary ingredient of the construction described in section 3 is that $G^{ij}$ is covariantly constant, $\mathcal{D}_A G^{ij} = 0$. We require this condition to hold in AdS$_4^{4|8}$, which implies that $G^{ij}$ is proportional to $S^{ij}$

$$G^{ij} = \kappa S^{ij}, \quad (C.17)$$

where $\kappa$ is a real constant. In accordance with (C.5), we have $G^{12} = 0$. The parameter $\kappa$ can be chosen to have any given non-zero value by means of rescaling the chiral superfield $Z$. We choose $\kappa = |\mu|$, and hence $G^{11} = -|\mu| \bar{\mu}$ and $G^{22} = -|\mu| \mu$.

The degrees of freedom described by $Z$ are those of an Abelian $\mathcal{N} = 1$ vector multiplet in AdS$_4^{4|4}$. Indeed, upon reduction to the $\mathcal{N} = 1$ AdS superspace, the $\mathcal{N} = 2$ chiral scalar $Z$ leads to two chiral superfields, $X$ and $W_\alpha$, defined as

$$X := Z|, \quad \bar{\mathcal{D}}_{\dot{\alpha}} X = 0, \quad (C.18a)$$

$$W_\alpha := -\frac{i}{2} \mathcal{D}_\alpha^2 Z|, \quad \bar{\mathcal{D}}_{\dot{\alpha}} W_\alpha = 0. \quad (C.18b)$$

One may check that the $\mathcal{N} = 2$ constraints (2.1) imply the Bianchi identity

$$\mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}, \quad (C.19a)$$

as well as the nonlinear constraint

$$-i \mu |\mu| X + \frac{1}{4} X (\mathcal{D}^2 - 4\bar{\mu}) \bar{X} = W^2, \quad W^2 := W^{\alpha} W_\alpha. \quad (C.19b)$$

Eq. (C.19a) tells us that $W_\alpha$ is the chiral field strength of a Maxwell multiplet in AdS$_4^{4|4}$. Eq. (C.19b) is of the same type as the constraint (6.1), which generates the $\mathcal{N} = 1$ locally
supersymmetric Born–Infeld action with \( \text{U}(1) \) duality invariance. The constraint (C.19b) is uniquely solved by expressing \( X \) in terms of \( W^2 \) and \( \bar{W}^2 \) and their covariant derivatives, in complete analogy with the general supergravity analysis of [38].

In accordance with (C.10), the infinitesimal \( \text{OSp}(2|4) \) transformation of \( Z \) is \( \delta Z = -\xi \mathcal{L} \). Using this result, it is straightforward to derive the transformation laws of \( X \) and \( W^\alpha \) under the second supersymmetry and \( \text{U}(1) \) transformations described by the superfield parameter \( \varepsilon \). Making use of the constraints obeyed by \( Z \) and \( X \), we obtain

\[
\delta_{\varepsilon} X = -2i\varepsilon^\alpha W^\alpha, \quad (C.20a)
\]

\[
\delta_{\varepsilon} W^\alpha = i\varepsilon_\alpha \left[ i\mu|\mu| - \frac{1}{4}(\mathcal{D}^2 - 4\mu)\bar{X} - \mu X \right] - \bar{\varepsilon}^\beta \mathcal{D}_{\alpha\beta} X + \frac{i}{2}\mu\varepsilon \mathcal{D}_\alpha X. \quad (C.20b)
\]

One can check that \( \delta_{\varepsilon} X \) and \( \delta_{\varepsilon} W^\alpha \) preserve the constraints (C.19). Due to (C.16), the variation \( \delta_{\varepsilon} W^\alpha \) can be rewritten in the form

\[
\delta_{\varepsilon} W^\alpha = -\frac{i}{8}(\mathcal{D}^2 - 4\mu) \left[ 2\left(\bar{X} - X + i|\mu|\right)\varepsilon_\alpha - \varepsilon\mathcal{D}_\alpha X \right], \quad (C.20c)
\]

which makes manifest the chirality of \( \delta_{\varepsilon} W^\alpha \). It follows from (C.20) that the second supersymmetry and \( \text{U}(1) \) transformations are nonlinearly realised.

Let us consider the supersymmetric and \( \text{U}(1) \) duality invariant Born–Infeld action in the \( \mathcal{N} = 1 \) AdS superspace\(^{10}\)

\[
S = -\frac{i}{4}|\mu|\mu \int d^4x d^2\theta \mathcal{E} X + \text{c.c.}, \quad (C.21)
\]

with \( X \) constrained by (C.19b). The action is manifestly invariant under the isometry transformations of \( \text{AdS}^{4|4} \), with the infinitesimal transformation law of \( W^\alpha \) being

\[
\delta W^\alpha = -\lambda W^\alpha - \omega^\beta_\alpha W^\beta. \quad (C.22)
\]

However, the action is not invariant under the transformation (C.20),

\[
\delta_{\varepsilon} S = 2|\mu|^3 \int d^4x d^2\theta d^2\bar{\theta} \mathcal{E} \varepsilon V, \quad (C.23)
\]

where the real scalar \( V \) denotes the unconstrained prepotential of the vector multiplet,

\[
W^\alpha = -\frac{1}{4}(\mathcal{D}^2 - 4\mu)\mathcal{D}_\alpha V. \quad (C.24)
\]

Eq. (C.23) is a unique feature that distinguishes \( \text{AdS}_4 \) from the other maximally supersymmetric backgrounds we have studied in this paper.

\(^{10}\)In accordance with (C.19b), the overall coefficient in (C.21) is chosen such that the kinetic term for the vector multiplet is canonically normalised, \( S = \frac{1}{4} \int d^4x d^2\theta d^2\bar{\theta} \mathcal{E} W^2 + \text{c.c.} + \text{interaction terms} \). It should be remarked that the functional \( \text{Re}(\mu \int d^4x d^2\theta d^2\bar{\theta} \mathcal{E} X) \) is a total derivative, in accordance with (C.19b).
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