A fast-estimation approach to multiple frequency excitation for online electromagnetic rail detection

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Abstract. This paper proposes a fast-estimation approach to demodulating multiple frequency signal for online electromagnetic rail detection. In order to speed up the demodulation, the approach uses the sine function’s expression to build Kalman filtering model, and by only one measured point, it can demodulate each component’s amplitude in multi-frequency signal induced in sensors. Instead of using all measured points in a period, the demodulating speed is faster than other demodulation methods, like FFT and fast correlation. Finally, the experimental result shows that this approach can follow the amplitude variation caused by the defect for online electromagnetic rail detection.

1. Introduction
With the development of mass railway transportation, the security of railway has been accounted for much attention and for that, the online detection of rail defect also has become more important than before [1]. Due to the non-contact and the high-speed detection, the electromagnetic method is suitable for online rail detection [1-3]. Furthermore, compared with the single-frequency excitation, the multi-frequency excitation performs better to detect the rail defect [4]. But if the signal process costs much time, the demodulation method will limit detecting rail defect online. So another problem is how to be efficient in signal processing, i.e. reducing demodulation time. To achieve that, the fast-estimation approach of this paper uses the known function, for example the sine function, to build state-prediction model in Kalman filtering for speeding up the demodulation. And the result shows that this approach can demodulate the induced signal’s amplitude in each frequency component by only one measured point. Therefore, this approach can apply to online rail detection.

2. Kalman-based approach
Assume that the multi-frequency signal in quadrature form is sampled with the angular frequency \( \omega_s \) as

\[
s[k] = d_0 + \sum_{m=1}^{M} \left( \alpha_m \sin \frac{2\pi k}{N_m} + \beta_m \cos \frac{2\pi k}{N_m} \right)
\] (1)
$N_m$ is the total points in one period of frequency component corresponding to $\omega_m$ when sampled at $\omega_s$, $N_m = \omega_s/\omega_m$; $\omega_m (\omega_m = 2\pi f_m)$ is the single angular frequency; $M$ is the number of multi-frequency excitation; $d_0$, $\alpha_m$ and $\beta_m$ are the function’s coefficients; $k$ means the time step or the sampling point.

According to the characteristic of sinusoidal function indicated by [5], we assume $x_k$ as the state variable at $k$ time instant. Then, $x_k$ is written by

$$x_k = \begin{bmatrix} \alpha_1 \cos k\theta_1 + \beta_1 \sin k\theta_1 \\ -\alpha_1 \sin k\theta_1 + \beta_1 \cos k\theta_1 \\ \alpha_2 \cos k\theta_2 + \beta_2 \sin k\theta_2 \\ -\alpha_2 \sin k\theta_2 + \beta_2 \cos k\theta_2 \\ \vdots \\ d_0 \end{bmatrix}$$

(2)

where $\theta_m = 2\pi/N_m$ and $x_k$ is a $(2M+1)$-dimension vector. Moreover, the state-variable equation at the $(k+1)$th time instant can be determined by

$$x_{k+1} = F\hat{x}_k + v_k$$

(3)

$\hat{x}_k$ is the optimal estimation at $k$ time instant, $v_k$ is the noise of predicting state, and $F$ is the state-transform matrix with $(2M+1)$ dimension,

$$F = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ 1 \end{bmatrix}$$

(4)

and

$$F_m = \begin{bmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{bmatrix}$$

(5)

Therefore, the covariance equation of state prediction at the $(k+1)$th time instant is

$$P_{k+1} = FP_{k}F^T + Q$$

(6)

$\hat{P}_k$ is the optimal covariance matrix of the state variable at $k$ time instant. $Q$ is the covariance matrix of the prediction noise and $Q = E[v_kv_k^T]$. In general, $v_k$ obeys a stable Gaussian distribution and the covariance matrix $Q$ doesn't change with time.

According to the induced signal sampled by ADC, it is measured as (1) that is the observable value $s_k$. It is found that each component of the induced signal is related to the odd number of elements in the state variable. Therefore, the observer equation can be described by

$$s_k = \begin{bmatrix} 1 & 0 & \cdots & 1 & 0 & 1 \end{bmatrix} x_k + u_k$$

(7)

$u_k$ is the observer noise, which is generally white noise produced by electronic components, and thus $u_k$ obeys Gaussian distribution as well. Then, to obtain the optimal estimation of each component of the multi-frequency signal, the predicted state variable can be updated by one measured point. And the updating equations are described by

$$\hat{x}_{k+1} = F\hat{x}_k + Kg(s_k - H^T x_{k+1})$$

$$Kg = FP_{k+1}H(HP_{k+1}H^T + R)^{-1}$$

(8)

(9)
\( \hat{P}_{k+1} = (I - Kg)P_{k+1} \) \hspace{1cm} (10)

Kg is Kalman gain which can be regarded as weight factor between the prediction and the observer, and \( R \) is the covariance matrix of observer noise and its off-diagonal element is zero due to the white noise \( u_k \).

Finally, through the process of Kalman filtering, the amplitude and phase of each component of the multi-frequency signal can be obtained. As is known from (1) and (2), both \( 2m \)th and \( (2m-1) \)th elements in state variable \( x_k \) contain the amplitude and phase terms corresponding to the \( m \)th component. Therefore, according to the expression of \( x_k \), the amplitude \( A_{m,k+1} \) of \( m \)th component at \( (k+1) \)th time instant is

\[
A_{m,k+1} = \sqrt{\hat{x}_{k+1}^2[2m-1] + \hat{x}_{k+1}^2[2m]} \hspace{1cm} (11)
\]

Kalman filtering is a recursive one and in every time-step, only one sample point will update the state variable, which can improve demodulation speed and thus detect the rail defect online.

3. Experimental Results

The electromagnetic rail detection is based on the principle of electromagnetic induction, and thus the acquisition signal also has multi-frequency signal when using multi-frequency excitation. There are the different sensitivity to defect in different frequency. When a defect is detected, each component’s amplitude will alter and its change scope is related to the defect’s shape, size, type and etc. Therefore, the rail defect can be detected by the change of amplitude when the electromagnetic technology is applied.

To evaluate the availability of this approach, the experiment is performed to online detect the defect on the surface of rail using multiple frequency excitation, as is shown in Figure 1. The raw data from the differential coil is shown in Figure 2(a). According to the acquisition data, the feature of the defect shows that the demodulated amplitude has two peaks that respectively occur when sensors move towards or away from defects. So, the amplitude variation of each frequency component should be as the same as the acquisition data due to the independence of the frequency components.

In Figure 2, the fast-estimation approach can separate all known-frequency components and obtain or track its amplitude in time. All of the known-frequency components show that their demodulated amplitude has two peaks, which are consistent the variation of the amplitude when the defect occurs. Observing the demodulation result of each frequency component, it’s found
that the anti-noise ability of the highest frequency component is better than other components. Furthermore, the high frequency excitation is generally a measure to speed up the detection of defect. Thus, if the train speed is higher, increasing excitation frequency can perform well in online electromagnetic rail detection.

4. Conclusion
By the known expression of sine function, this paper presents the mathematical model in Kalman filtering to demodulate the multi-frequency signal. The experimental result indicates that this approach can separate multiple known-frequency components and track their own amplitude by only one measured point. Moreover, the high-frequency excitation can further improve the performance of detection. So, this approach can apply to online detection of rail defect.

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