The electromagnetic characteristics of the fractional quantum Hall states are studied by formulating an effective vector-field theory that takes into account projection to the exact Landau levels from the beginning. The effective theory is constructed, via bosonization, from the electromagnetic response of an incompressible and uniform state. It does not refer to either the composite-boson or composite-fermion picture, but properly reproduces the results of the standard bosonic and fermionic Chern-Simons approaches, thus revealing the universality of the long-wavelength characteristics of the quantum Hall states and the associated quasiparticles. In particular, the dual-field Lagrangian of Lee and Zhang is obtained without invoking the composite-boson picture. An argument is also given to verify, within a vector-field version of the fermionic Chern-Simons theory, the identification by Goldhaber and Jain of a composite fermion as a dressed electron.

I. INTRODUCTION

The fractional quantum Hall effect (FQHE) results from formation of incompressible quantum fluids that support quasiparticles carrying fractional charges and statistics, as all embodied in Laughlin’s wave functions and his reasoning resting on the gauge principle. The early approaches based on variational wave functions successfully clarified some fundamental aspects of the FQHE, which evolved into the descriptions of the FQHE in terms of electron–flux composites, composite bosons or composite fermions.

The bosonic and fermionic Chern-Simons (CS) theories are the field-theoretical frameworks that realize the composite-boson and composite-fermion pictures of the FQHE and have been successful in describing the long-wavelength characteristics of the fractional quantum Hall (FQH) states. In the wave-function approach it is crucial to use wave functions projected to the lowest Landau level, while no explicit account of such projection is taken in the CS approaches. One might naturally wonder if and how the latter are compatible with the Landau quantization such as Landau levels and quenching of the electronic kinetic energy.

The purpose of this paper is to present a field-theoretical approach to the FQH system, that takes account of the Landau quantization from the very beginning. The basic quantity we rely on is the electromagnetic response of an incompressible and uniform state, which we calculate by handling Landau-level mixing in a systematic way. We then use a bosonization technique and construct from this response an equivalent vector-field theory that describes the electromagnetic characteristics of the FQH states and the associated quasiparticles in the presence of the Coulomb interaction. This bosonic effective theory is derived without recourse to either the composite-boson or composite-fermion picture, but properly reproduces the random-phase-approximation results of the bosonic and fermionic CS theories, thus revealing the universality of the long-wavelength characteristics of the FQH states. We examine some consequences of it, especially for the composite fermions.

II. ELECTROMAGNETIC RESPONSE OF HALL ELECTRONS

Consider electrons confined to a plane with a perpendicular magnetic field, described by the action:

\[ S = \int dt d^2x \psi^\dagger(x, t) (i \partial_t - H) \psi(x, t) + S_{\text{Coul}}[\rho], \]

\[ H = \frac{1}{2M} (p + eA^B(x) + eA(x, t))^2 + eA_0(x, t). \]

The vector potential \( A^B(x) \) is taken to supply a uniform magnetic field \( B_\perp = B > 0 \) normal to the plane, and we make the Landau-gauge choice \( A^B = \frac{1}{2} B (y, 0) \) below. Our task in this section is to study the response of Hall electrons to weak external potentials \( A_\mu(x, t) = (A_0(x, t), A(x, t)) \). [We suppose that \( \mu \) runs over \((0, x, y)\) or \((0, 1, 2)\), and denote \( A = (A_x, A_y) = (A_1, A_2) \), etc. Remember that our \( A_0 \) equals minus the conventional scalar potential.] For notational simplicity, we shall write \( x = (t, x), d^2x = dt d^2x, A_\mu(x) = A_\mu(x, t), \) etc., when no confusion arises; the electric charge \( e > 0 \) will also be suppressed by rescaling \( eA_\mu \rightarrow A_\mu \) in what follows.

The Coulomb interaction is a functional of the density \( \rho(x) = \psi^\dagger(x) \psi(x), \)

\[ S_{\text{Coul}}[\rho] = -\frac{1}{2} \int d^3x d^3x' \delta \rho(x) V(x - x') \delta \rho(x'), \]

where \( \delta \rho(x) = \rho(x) - \bar{\rho} \) stands for the deviation from the average electron density \( \bar{\rho} \). It is convenient to write it...
in an equivalent form linear in \(\rho(x)\) using the Hubbard-Stratonovich field \(\chi(x)\),

\[
S_{\text{Coul}}[\rho] = \int d^3x \left[ \frac{1}{2} \chi \Gamma[-i \nabla] \chi - \chi \delta \rho \right],
\]

where \(\Gamma[p] = 1/V[p]\) in terms of the Fourier transform of \(V(x)\). Note that as for the electron sector the effect of this linearization is to replace \(A_0(x)\) by \(A_0(x) + \chi(x)\) in the Hamiltonian \(H\) of Eq. (2.3). Accordingly, in the rest of this section we shall denote \(A_0 + \chi\) as \(A_0\) and focus on the one-body part of \(S\). Explicit account of the Coulomb interaction will be taken in Secs. III and IV again.

The eigenstates of \(H\) with \(A_{\mu} = 0\) are Landau levels \(| N \rangle = | n, y_0 \rangle\) of energy \(\omega(n + \frac{1}{2})\), labeled by integers \(n = 0, 1, 2, \cdots\), and \(y_0 = \ell^2 p_x\), where \(\omega = eB/M\) is the cyclotron energy and \(\ell \equiv 1/\sqrt{eB}\) is the magnetic length. The external potentials \(A_{\mu}(x)\) modify this level structure, and the desired response of the electron is obtained by diagonalizing the Hamiltonian \(H\) with respect to the true Landau levels \(| n \rangle\) (or, projecting \(H\) into the true levels).

For actual calculations it is convenient to pass to the \(| N \rangle\) representation via a unitary transformation \(\langle x | N \rangle\), with the expansion \(\psi(x, t) = \sum_N \psi_N(t) \langle x | N \rangle\). The translation is simple. The relative coordinates \((y - y_0)/\ell\) and \(\ell \partial y_0\) turn into the matrices \(Y\) and \(P\) of a harmonic oscillator with \([Y, P] = i\). The arguments \(x = (x, y)\) of \(A_{\mu}(x, t)\) are replaced by the operators \(\hat{x} = (\hat{x}, \hat{y})\).

\[
\hat{x} \equiv x_0 + \ell P, \quad \hat{y} \equiv y_0 + \ell Y,
\]

with \([\hat{x}, \hat{y}] = 0\). Here \(y_0\) and \(x_0 \equiv i\ell^2 \partial / \partial y_0\) stand for the center coordinates of an orbiting electron with uncertainty \([x_0, y_0] = i\ell^2\). The action is thereby rewritten as

\[
S = \int dt \sum_{m,n=0}^{\infty} \psi_+^\dagger(y_0, t) \left( i \partial_{mn} \partial_t - \hat{H}_{mn} \right) \psi_0(y_0, t),
\]

\[
\hat{H} = \omega \left\{ [\hat{Z} - i\ell \hat{A}(\hat{x}, t)] [\hat{Z} + i\ell \hat{A}(\hat{x}, t)] + \frac{1}{2} \right\}
\]

\[
+ \frac{1}{2M} h_{\perp}(\hat{x}, t) + A_0(\hat{x}, t),
\]

with

\[
A(x) = \{ A_y(x) + i A_x(x) \}/\sqrt{2}, \quad \hat{A}(x) = A(x)\dagger;
\]

\[
h_{\perp}(x) = \partial_x A_y(x) - \partial_y A_x(x).
\]

In the above we have set \(\psi_N(t) = \psi_0(y_0, t)\) and defined \(Z = (Y + iP)/\sqrt{2}\) and \(\hat{Z} = Z^\dagger\) so that \(Z_{mn} = \sqrt{n} \delta_{m,n-1}\) and \([Z, \hat{Z}] = 1\). In this section we use the Landau gauge but all the manipulations are carried over to the symmetric gauge as well.

The Hamiltonian \(H\) is an infinite-dimensional matrix in Landau-level indices and an operator in \(y_0\) and \(x_0 = i\ell^2 \partial_{y_0}\). It has a gauge symmetry far larger than the electromagnetic gauge invariance: The change of bases in \(N\) space, \(|n, y_0\rangle \rightarrow |n', y'_0\rangle\), induces a unitary transformation \(G = \{ G_{mn}(x_0, y_0, t) \}\) of the field operator

\[
\psi_0^G(y_0, t) = \sum_{n=0}^{\infty} G_{mn}(x_0, y_0, t) \psi_0(y_0, t).
\]

The potentials \(A_\mu\) thereby undergo the transformation

\[
A_0^G = GA_0(\hat{x}, t)G^{-1} - (i/\ell) [G, Z]G^{-1},
\]

\[
A_0^G = GA_0(\hat{x}, t)G^{-1} - i G \partial_t G^{-1},
\]

which leaves the action \(S\) invariant. Let us write

\[
G = \exp[i\ell \eta],
\]

where \(\eta\) takes on values in the \(U(\infty)\) or \(W_\infty\) algebra

\[
\eta = \sum_{r,s=0}^{\infty} \eta_{rs}(x_0, y_0) \bar{Z}^r Z^s
\]

with operator-valued coefficients \(\eta_{rs}(x_0, y_0)\). This \(W_\infty\) transformation \(G\) in general mixes Landau levels. The original electromagnetic gauge invariance is realized when \(\eta\) is written in terms of \((\hat{x}, \hat{y})\), i.e., of the special form \(\eta(\hat{x}, \hat{y}, t)\).

The reason for displaying the symmetry structure underlying the Hall system is that it provides a powerful tool to systematically project the Hamiltonian into the exact Landau levels for weak fields \(|A_\mu| \ll \omega\). One may simply adjust \(\eta\) so that \(H^G = G(\hat{H} - i\partial_t G^{-1})G^{-1}\) is diagonal in level indices. Some calculations along this line were given earlier. Here we demonstrate another advantage of the method that, with a suitable transformation, the projection is done in a manifestly gauge-covariant manner.

The calculation itself, however, is rather independent of the main line of our discussion and is left for Appendix A. Here we quote only the result: The projected Hamiltonian, to \(O(A^2)\) and \(O(\partial^2)\), reads

\[
\hat{H}^G = (\bar{Z}Z + \frac{1}{2}) \left\{ \omega + \frac{1}{M}(h_z + \ell^2 h_2^2) + \frac{1}{2} \ell^2 \nabla \cdot E \right\}
\]

\[
+ A_0 - \frac{\ell^2}{2\omega} E^2 + \frac{1}{2} \ell^2 \epsilon^{\mu \nu \lambda} A_\mu \partial_\nu A_\lambda + \cdots,
\]

where all the fields depend on \((x_0, y_0, t)\) and total derivatives have been suppressed; \(E = -\partial_0 A + \nabla A_0\), and \(\epsilon^{\mu \nu \lambda}\) is a totally-antisymmetric tensor with \(\epsilon^{012} = 1\).

Since the Hamiltonian \(H^G\) is diagonal in \(n\), let us denote \(\langle H^G \rangle_m = (n + \frac{1}{2}) \omega + V_n(\hat{x}, \hat{y}, t)\) and focus on a single level \(n\). The cyclotron energy is now quenched and \(V_n\) describes the response of the Hall electron. Let us suppose that in the absence of external perturbations \((A_\mu = 0)\) a nondegenerate many-body state of uniform density is realized. Here we have in mind the integer and fractional quantum Hall states and understand that the Coulomb interaction is responsible for their formation. For such incompressible and uniform states it is not difficult to translate \(V_n(\hat{x}, \hat{y}, t)\) into this effective action or the partition function in \(x\) space. To this end, replace first \((x_0, y_0) \rightarrow (\hat{x}, \hat{y})\) in \(V_n\) to form \(\hat{V}_n \equiv V_n(\hat{x}, \hat{y}, t)\), and note that \(\hat{V}_n\) differs from \(V_n\) by total divergences, which
will soon turn out irrelevant in the end. Then go back to the $x$ space with $V_n$. This yields an interaction term of the form $\psi^\dagger V_n(x,y,t)\psi$, which, for an electron state of uniform density $\rho_n$ in the $n$th level, gives rise to the effective Lagrangian $L_{\text{eff}}^n = -\rho_n V_n(x,y,t)$, or

$$L_{\text{eff}}^n = \rho_n \left\{ -eA_0 + \frac{e^2}{2m} E^2 - (n + \frac{1}{4}) \frac{e^2}{M} h_x^2 - \frac{1}{2} \frac{e^2}{2m} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + \cdots \right\} \tag{2.13}$$

with the electric charge $e$ restored. Here all the fields are functions of $(x,y,t)$ and total divergences have been dropped. The $L_{\text{eff}}^n$ summarizes the $O(V_n)$ electromagnetic response of the incompressible and uniform state. It correctly reproduces earlier results for integer filling $\rho_n \to 1/(2\pi\ell^2)$. It is clear from our derivation that such a response is determined uniquely for an incompressible state of general $\rho_n$, independent of the detail of how it is formed; this is the key observation that enables one to use this response for the discussion of the FQHE.

### III. Bosonization and an Effective Theory

The action $S = S[\psi, \psi^\dagger, A_\mu]$ in Eq. (2.1) describes the electron field $\psi(x)$ minimally coupled to the external potentials $A_\mu(x) = (A_0(x), A_k(x))$, where $k$ runs over $(1,2) = (x,y)$. In this section we study this electron system from somewhat different angles and construct a bosonic effective theory of the FQHE. The electromagnetic response of the system is summarized in the partition function written as a functional integral

$$W[A] = \int [d\psi][d\psi^\dagger] e^{iS[\psi,\psi^\dagger,A_\mu]}. \tag{3.1}$$

[From now on $A$ simply refers to $A_\mu$, and not to $(A_y + iA_x)/\sqrt{2}$ any more.] The $W[A]$ encodes in its $A_\mu$ dependence the quantum effect of the electron field $\psi$. Once such $W[A]$ is known it is possible to reconstruct it through the quantum fluctuations of a boson field. Let us first briefly review this procedure, known as functional bosonization.

The gauge invariance of the action $S[\psi, \psi^\dagger, A_\mu] = S[e^{-i\epsilon_0 \psi^\dagger \psi}, A_\mu + \partial_\mu \xi]$ dictates that $W[A]$ is gauge-invariant, i.e., $W[A'] = W[A_\mu + \partial_\mu \xi]$, where $\partial_\mu = (\partial_0, \partial_1, \partial_2) = (\partial_t, \partial_x, \partial_y)$. Thus integrating $W[A_\mu]$ over $\xi$ amounts to an essential change in the overall normalization, yielding $W[A_{\text{eff}}] = \int [d\xi] W[A_\mu + \partial_\mu \xi]$. Now let us rewrite the integral $\int [d\xi]$ as an integral over a 3-vector field $v_\mu(x) = (v_0, v_1, v_2)$

$$W[A] = \int [dv_\mu] \delta(e^{i\mu_\nu \partial_\nu v_\mu}) W[A + v], \tag{3.2}$$

where the delta functional enforces the pure-gauge nature $v_\mu \sim \partial_\mu \xi$. One can disentangle the pure-gauge constraint by making use of a functional Fourier transform with another 3-vector field $b_\mu = (b_0, b_1, b_2)$. A subsequent shift $v_\mu \to v_\mu - A_\mu$ in the functional integral then yields the representation

$$W[A] = \int [db_\mu] e^{-i \int d^3x (A_\mu e^{i\mu_\nu \partial_\nu b_\nu}) + iZ[b_\mu]}, \tag{3.3}$$

$$e^{iZ[b_\mu]} \equiv \int [dv_\mu] e^{i \int d^3x (b_\mu e^{i\mu_\nu \partial_\nu v_\mu}) W[v_\mu]. \tag{3.4}$$

Here $Z[b]$ is obtained by Fourier transforming (given) $W[v]$. Note that the electron current $j_\mu = \delta S/\delta A_\mu$ is written as a rotation $e^{i\mu_\nu \partial_\nu b_\mu}$ in the $b$-boson theory.

An advantage of the bosonic theory is that the bosonization of the current holds exactly for the Coulomb interaction (2.3). To see this write the Coulomb interaction in linearized form $S_{\text{Coul}}[\rho] = 0$ and follow the bosonization procedure (with a shift $v_\mu \to v_\mu - A_\mu - \delta_\mu \chi$). Eliminating (or integrating over) the $\chi$ field then entails the replacement $\rho \to e^{i\chi} \partial_\mu b_\mu = \partial_t b_1 - \partial_x b_2 - \partial_y b_3 = b_{12}$ in $S_{\text{Coul}}[\rho]$, and the $b$-boson theory is described by the action

$$S_{\text{eff}}[b_\mu] = \int d^3x (-A_\mu \partial_\mu b_\mu) + Z[b] + S_{\text{Coul}}[b_{12}], \tag{3.5}$$

where $Z[b]$ is now calculated from the one-body part of the action $S$ in Eq. (2.1), i.e., from the $v$-field theory with the action

$$S_v = \int d^3x v_\mu b_\mu - iTr \log(i\partial_\mu - H[v]). \tag{3.6}$$

Here we have introduced compact notation $A_\mu e^{i\mu_\nu \partial_\nu b_\mu}$, etc., which will be used from now on.

Actually the second term of Eq. (3.5) has been calculated in the preceding section. Suppose that a nondegenerate many-body state of uniform density $\tilde{\rho} = \nu/(2\pi\ell^2)$ is formed via the Coulomb interaction (for $A_\mu = 0$); take, for generality, $n < \nu < n + 1$, so that the lower $n + 1$ Landau levels are filled up. Its response to weak electromagnetic potentials $A_\mu(x)$ is obtained by collecting Eq. (2.13) for the filled levels, yielding log $W[A] = i \int d^3x L[A;\nu]$ with

$$L[A;\nu] = \tilde{\rho} \left[ -A_0 - s_\mu \ell_\mu^2 \frac{1}{2} A_\mu \partial_\mu A_\mu + \frac{\ell_\mu^2}{2w}(A_{k0})^2 - \frac{\sigma(\nu) \ell_\mu^2}{2M} (A_{12})^2 \right] + O(\ell^3), \tag{3.7}$$

where $A_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ (so that $A_{k0} = E_k$ and $A_{12} = h_z$). Here $\sigma(\nu) = 1 + 2n - n(n + 1)/\nu$ at each integer interval $n < \nu \leq n + 1$; the modification needed for realistic spin-resolved levels is obvious. For later use we have generalized $L[A;\nu]$ to accommodate the sign $s_\mu$ $\pm$ sign$(B) = \pm 1$ of $B_z = \pm B$; remember that, when the magnetic field is reversed in direction, the CS term $\propto A_\mu \partial_\mu A_\mu$ (or the Hall conductance) changes in sign.

Let us now substitute this $L[A;\nu]$ into Eq. (3.4) and construct an effective theory of the $b_\mu$ field that recovers it. The exponent $\propto b_\mu \partial_\mu v + L[v;\nu]$ is essentially quadratic

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in $v_\mu$, and functional integration over $v_\mu$ is carried out exactly. Here it is necessary to fix a gauge. Fortunately there is a way to avoid such gauge-fixing complications, that works in the presence of CS couplings, as we explain below: First make a shift $v_\mu = v_\mu' + f_\mu[b]$ and choose $f_\mu[b]$ so that no direct coupling between $v_\mu'$ and $b_\mu$ remains in the exponent, which thereby is split into two terms $L[v';v] + L^{(0)}[b]$. The integration over $v_\mu'$ requires fixing a gauge but is done trivially, leaving no dependence on $b_\mu$. All the dependence on $b_\mu$ is now isolated in the background piece $f_\mu = (1/\ell^2) s_B b_\mu + O(\partial b)$ and the desired bosonic action is given by $Z[b] = \int d^4x L^{(0)}[b]$ with

$$L^{(0)}[b] = - \frac{1}{\ell^2} b_0 + \frac{s_B}{\nu} \pi \epsilon^{\mu\nu\lambda} \partial_\nu b_\lambda + \frac{\pi}{\nu \omega} (b_{0\nu})^2 - \frac{\sigma(\nu)}{\nu M} (b_{12})^2 + O(\partial^3),$$

(3.8)

where the filling factor $\nu = 2\pi \ell^2 \tilde{\rho}$. See Appendix B for details. An effective action analogous to the above was discussed for integer filling earlier in connection with the $v_\mu$ integration we remark that any simple choice of gauge for $v_\mu'$, e.g., $\partial_\mu v'_\mu = 0$, takes a rather unconventional form $\partial_\mu v_\mu = \delta_\mu [v_\mu]$ in terms of $v_\mu$. Still this is a perfectly legitimate choice, and it has the advantage of achieving separation of the background ($b_\mu$, here) dependent piece at the classical level.

The bosonic theory allows one to handle the Coulomb interaction exactly, as already remarked. It further admits inclusion of new degrees of freedom, vortices, which describe quasiparticle excitations over the FQH states.

Vortices arise only around special filling fractions, at which nondegenerate many-body ground states are realized, as revealed by Laughlin’s reasoning of arriving at an excited state by adiabatically piercing an exact many-body state with a thin solenoid of a unit flux quantum $\phi_D = 2\pi n/h/e$: recall that the key element there is the nondegeneracy of the ground state. The vortices reside in a topologically nontrivial component of the phase of the electron field. Let us isolate the nontrivial portion by writing $\psi(x) = e^{i\Theta(x)} \psi'(x)$ with $\epsilon^{\nu\delta} \partial_\nu \partial_\delta \Theta(x) \neq 0$, where $\psi'(x)$ describes the electrons away from the vortices so that they locally belong to the ground state with uniform density $\tilde{\rho}$. For vortices of vorticity $\{q_i\}$ and position $\{x^{(i)}(t)\}$ the vortex 3-current $\tilde{j}_\mu = (\tilde{\rho}, j_k) = (1/2\pi) \epsilon^{\mu\nu\rho} \partial_\nu \partial_\rho \Theta(\{x^{(i)}(t)\})$ is written as

$$\tilde{j}_\mu(x) = \sum_i (1, \partial_i x^{(i)}(t)) q_i \delta(x - x^{(i)}(t)).$$

(3.9)

The partition function (3.11), rewritten in terms of $\psi'(x)$, is given by $W[A + \partial \tilde{\rho}]$, which upon bosonization brings about the change $Ac\tilde{\rho} \rightarrow A c\tilde{\rho} + 2\pi \tilde{\rho} b_\mu$ in $S_{\text{eff}}[b]$. Thus the bosonic theory that takes into account both the Coulomb interaction and vortices is described by the Lagrangian

$$L_{\text{eff}}[b] = - A_\mu \epsilon^{\mu\nu\rho} \partial_\nu b_\rho - 2\pi \tilde{\rho} b_\mu + L^{(0)}[b] - \frac{1}{2} \delta b_{12} V \delta b_{12},$$

(3.10)

where $\delta b_{12} V \delta b_{12} = \int d^2 y \delta b_{12}(x) V(x - y) \delta b_{12}(y)$ for short and $\delta b_{12}(x) = b_{12}(x) - \tilde{\rho}$. Upon quantization the CS term $bc\tilde{\rho}b$ in $L^{(0)}[b]$ combines with the kinetic term $(b_{0\nu})^2$ to yield a “mass” gap $\omega$ while the $(b_{12})^2$ term yields only a tiny $O(\nabla^2)$ correction to it, as we shall see soon.

Note that the $A c\tilde{\rho}$ term combines with the $(1/\ell^2) b_0$ term in $L^{(0)}[b]$ to promote $A_\mu$ to the full vector potential $A_\mu + \ell b_\mu$. As a result, $L_{\text{eff}}[b]$ almost precisely agrees with the dual-field Lagrangian of Lee and Zhang (LZ), describing the low-energy features of the FQHE. The only difference to $O(\partial^2)$ lies in the $(b_{12})^2$ term, which, however, is unimportant at long wavelengths and actually negligible compared with the $(b_{0\nu})^2$ term, as indicated by the ratio $\omega/M \sim 10^{-7}$ typically. We have thus practically reproduced the dual Lagrangian of LZ.

The LZ approach relies on the Chern-Simons-Landau-Ginzburg (CSLG) theory realizing the composite-boson description of the FQHE: there the quantum fluctuations of the composite-boson field around the mean field $\rho \sim \tilde{\rho}$ are corrected, via the dual transformation of Lee and Fishman, into those of a 3-vector field $b_\mu^L$. The $b_\mu^L$ and the $b_\mu$ bosonize the electron current in formally the same way. In the CSLG theory no account of the Landau-level structure is taken, and the proper electromagnetic responses of the FQH states are obtained through the random-phase approximation (RPA) beyond mean field.

In contrast, our approach takes explicit account of Landau levels and relies on the electromagnetic response of a uniform-density state, which, via bosonization, is transformed into the dynamics of the field to study the long-wavelength characteristics of the FQH states.

Thus, the fact that we have arrived at the LZ dual Lagrangian without invoking the composite-boson picture would reveal the following: (1) The long-wavelength electromagnetic responses of the FQH states, as governed by the LZ dual Lagrangian, are determined universally by the filling factor and some single-electron characteristics, independent of the details of the FQH states. (2) In the CSLG theory the RPA properly recovers the effect of the Landau-level structure, crucial for determining the electromagnetic response.

Having established the connection to the composite-boson approach, let us now derive the electromagnetic response starting from the bosonic Lagrangian $L_{\text{eff}}[b]$. As before the $b$ integration is done by a suitable shift $b_\mu = b_\mu' + f_\mu$, with the result (in obvious compact notation)

$$L_{\text{eff}}[A;\nu] = \tilde{\rho} \ell^2 \left[ - \frac{1}{\ell^2} A_{0\nu} - s_B \frac{1}{2} A_{D\nu} + \frac{1}{2} \epsilon A_{\nu} - \frac{1}{\nu M} \frac{1}{4} A_{12} \Sigma A_{12} \right].$$

(3.11)

where

$$\Sigma = V + \frac{2\pi \sigma(\nu)}{\nu M}, \quad D = 1 + \frac{1}{\omega^2} \partial_0^2 - \frac{\nu}{2\pi \omega} \Sigma \nabla^2;$$

(3.12)

see Appendix B for details. This $L_{\text{eff}}[A;\nu]$ improves the original response $L[A;\nu]$ in Eq. (3.7) and agrees with
the RPA result in the composite-boson theory. One can read off, e.g., the density-density correlation function from the \( A_0A_0 \) portion of the \( A_{00}D^{-1}A_{00} \) term. One also learns from D that the Coulomb interaction modifies the dispersion of the cyclotron mode so that

\[
\omega(p) = \omega + \frac{1}{2} \partial^2 p^2 \left( V[p] + \frac{2\pi\sigma(\nu)}{\nu M} \right) \\
\approx \omega + \frac{1}{2} \partial^2 p^2 V[p]
\]

(3.13)
in accordance with Kohn’s theorem. Here we see explicitly that the \((b_{12})^2\) terms in \( L_{\text{eff}}[b] \) yields only a tiny nonleading correction.

Observe here that, in spite of the quenched electronic kinetic energy, the \( b_\mu \) field correctly acquires a dispersion of the Landau gap \( \omega \) (through level mixing caused by electromagnetic couplings). The cyclotron mode is readily identified by first isolating a mean-field piece from the quantum component \( b_\nu \) so that \( b_\nu = \delta b_\nu + \langle b_\nu \rangle \) with \( \langle b_\nu \rangle \) around \( \tilde{\rho} \). In the Coulomb gauge \( \delta b_\nu \) is zero one can write \( \delta b_\nu = e^{\nu(\partial_j/\sqrt{\nabla^2})} \zeta \) and show that \( \zeta \) is the canonical scalar field with the dispersion \( \omega(p) \).

In Eq. (3.11) we have set \( j_0 = 0 \) for simplicity, but the vortices are easily recovered by the substitution \( A_\mu \rightarrow A_\mu + \partial_\mu \Theta \) and \( A_{\mu\nu} \rightarrow A_{\mu\nu} + 2\pi e^{-\nu/2\pi} \tilde{\lambda} \). The dynamics of vortex is immediately read from \( L_{\text{effective}}[A_\mu, \partial_\mu \Theta; \nu] \). In particular, the \( A\Theta A \) term contains a vortex coupling like \( -\nu j_\mu A_\mu \), which shows that a vortex of vorticity \( q \) has charge \( -\nu q e \). (We have set \( s_B = 1 \) here.)

It is possible to read off the vortex charge directly from \( L_{\text{eff}}[b] \) in Eq. (3.11). There the vortex enters only through the \(-2\pi j_\mu b_\mu \) coupling and no direct coupling to \( A_\mu \) exists. As remarked in deriving Eq. (3.8), however, the \( b_\mu \) field acquires, in the presence of the electromagnetic coupling \(-A_\mu e^{i\nu/2\pi} \partial_\nu b_\mu \), a background component, i.e., \( b_\mu = b'_\mu + f_\mu \) with \( f_\mu = s_B (\nu/2\pi) A_\mu + O(\partial A) \), where the coefficient \( \nu/2\pi \) derives from the \( be\partial b \) term. Thus the vortex is coupled to \( A_\mu \), through the background piece

\[
-2\pi j_\mu b_\mu = -\nu j_\mu A_\mu + \cdots,
\]

(3.14)

which reveals the vortex charge of \(-\nu q e \).

**IV. COMPOSITE FERMIONS**

The composite fermions are electrons carrying an even number of flux quanta. In the CS approach the electron field \( \psi(x) \) is converted to the composite-fermion field \( \phi(x) \) by a singular gauge transformation that attaches an even number \( \alpha = 2\pi \) of flux quanta \( \phi_\alpha = 2\pi \hbar/e \) of the CS field \( C_\mu(x) \), with the Lagrangian

\[
L = \phi^\dagger \left[ i\partial_t - \frac{1}{2M} \Pi[A^B + A + C]^2 - eA_0 - eC_0 \right] \phi + \frac{1}{2} \delta \rho V \delta \rho - e \delta \rho \sqrt{\frac{2\pi \alpha}{e}} \Lambda_{\mu\nu\rho} \partial_\mu \partial_\nu \partial_\rho C_\lambda;
\]

(4.1)

\( \Pi[A] \equiv p + eA \). Here \( \delta \rho(x) = \rho(x) - \tilde{\rho} \) is written in terms of \( \rho(x) = \phi^\dagger(x)\phi(x) \), but one can effectively replace it by

\[
\delta \rho(x) = -\left(1/\alpha \delta \rho \right) \epsilon^{\alpha\beta\gamma} \partial_\beta C_\gamma(x) - \tilde{\rho}.
\]

(4.2)

To see this, express the Coulomb interaction in linearized form \( \Pi[A] \), again, and rewrite \( L \) in favor of \( \phi_0 \) and \( \chi \). The \( C\epsilon\partial C \) term thereby acquires a \( \chi \)-dependent piece, which, upon eliminating \( \chi \), gives the Coulomb interaction with \( \delta \rho(x) \) of Eq. (4.2). With this in mind we now start with Eqs. (3.11) and (3.14).

The mean-field treatment corresponds to an expansion in \( C_\mu(x) \) of \( L \) around the mean field \( \delta \rho \sim 0 \), or \( \langle C_\mu(x) \rangle \) with \( e^{\nu} \partial_\mu \langle C_\mu(x) \rangle = -\alpha \partial_\mu \tilde{\rho} \) and \( \langle C_0(x) \rangle = 0 \). Let us set \( C_\mu(x) = \langle C_\mu(x) \rangle + c_\mu(x) \) and rewrite the Lagrangian as

\[
L = \phi^\dagger \left[ i\partial_t - \frac{1}{2M} \Pi[A^\text{eff} + a]^2 - e\epsilon_0 \right] \phi + L_{\text{CS}}[c],
\]

(4.3)

\[
L_{\text{CS}}[c] = -\frac{e}{2\alpha \delta \rho} \partial_\mu e^{\mu\nu\lambda} \partial_\nu c_\lambda + \tilde{\rho} \epsilon_0 - \frac{1}{2} \delta \rho V \delta \rho,
\]

(4.4)

where \( a_\mu = A_\mu + c_\mu \) and \( \delta \rho(x) = -(1/\alpha \delta \rho) c_\mu(x) \).

The composite fermions, coupled to \( A^\text{eff}(x) = A^\text{eff}(x) + \chi(x) \), experience a reduced mean magnetic field \( B_{\text{eff}} = B - \alpha \delta \rho \tilde{\rho} \), and form Landau levels of smaller gap \( \omega_{\text{eff}} = e|B_{\text{eff}}|/M \) and state density \( 1/(2\pi^2 \ell^2) \), \( \ell = 1/\sqrt{|B_{\text{eff}}|} \). The fractional quantum Hall states of interacting electrons in a magnetic field \( B \) at the principal filling fractions

\[
\nu = 2\pi \ell^2 \tilde{\rho} = \nu_{\text{eff}}/(2\nu_{\text{eff}} \pm 1),
\]

(4.5)

where \( \pm \) refers to the sign of \( B_{\text{eff}}/B \), are thereby mapped into an integer quantum Hall state of composite fermions in the reduced field \( B_{\text{eff}} \) at integer filling \( \nu_{\text{eff}} = 2\pi \ell^2 \tilde{\rho} = 1,2,\cdots \). This Jain’s pictures of (supposedly weakly-interacting) composite fermions has a number of consequences that have been supported experimentally.

Consider now a many-body ground state of composite fermions at integer filling \( \nu_{\text{eff}} \). Its response to \( A_\mu \) is described by \( L[A_\mu + c_\mu; \nu_{\text{eff}}] \), i.e., Eq. (4.7) with \( A_\mu \rightarrow A_\mu + c_\mu + \delta \rho \). Note further that, since the ground state at integer filling is incompressible and non-degenerate, it supports vortices as elementary excitations over it, which as before are introduced via the replacement \( A_\mu \rightarrow A_\mu + (1/e)\partial_\mu \Theta \). The composite-fermion (CF) theory (4.3), generalized to accommodate vortices, is thus described by the effective Lagrangian

\[
L_{\text{CF}} = \left[ L[A_\mu + (1/e)\partial_\mu \Theta + c_\mu; \nu_{\text{eff}}] + L_{\text{CS}}[c] \right].
\]

(4.6)

This is immediately transcribed into an equivalent bosonic version of the CF theory, with the Lagrangian

\[
L_{\text{CF}}[b, c] = -e(A_\mu + c_\mu)\epsilon^{\mu\nu\rho} \partial_\nu b_\rho - 2\pi j_\mu b_\mu \\
+ L^{(0)}[b; B_{\text{eff}}] + L_{\text{CS}}[c],
\]

(4.7)

where \( b_\mu \) bosonizes the 3-current of the CF field \( \phi \); and \( L^{(0)}[b; B_{\text{eff}}] \) stands for \( L^{(0)}[b] \) of Eq. (3.8) with \( \ell \rightarrow
\( \ell_{\text{eff}}, \omega \to \omega_{\text{eff}}, \nu \to \nu_{\text{eff}} \) and \( s_B \to s_{B*} \), where \( s_{B*} = \pm 1 \) refers to the sign of \( B_{\text{eff}}/B \).

Let us here try to eliminate \( c_\mu \) from \( L_{\text{CF}}[b, c] \) and obtain an equivalent theory of the \( b_\mu \) field alone. With the shift \( c_\mu = c_\mu' - \alpha \phi_D b_\mu \), the \( c \bar{c} b \) term and \( L_{\text{CS}}[c] \) combine to yield

\[
\Delta L = 2\pi \alpha \left( \frac{1}{2} b \bar{c} b \delta - \bar{\rho} b_0 \right) - \frac{1}{2} \delta b_{12} V \delta b_{12}, \quad (4.8)
\]

which is combined with the rest of terms in \( L_{\text{CF}}[b, c] \) to give the desired \( b \)-field Lagrangian \( L_{\text{eff}}^{(\text{CF})}[b] \). Actually this \( L_{\text{eff}}^{(\text{CF})}[b] \) almost coincides with \( L_{\text{eff}}[b] \) in Eq. (3.10) and their apparent difference lies in

\[
\Delta L + L^{(0)}[b; B_{\text{eff}}] \leftrightarrow L^{(0)}[b]. \quad (4.9)
\]

As a matter of fact, both sides agree precisely for the first three leading terms of \( L^{(0)}[b]; \) i.e., \( \alpha \omega/\ell^2 + s_{B*}/\ell_{\text{eff}}^2 = 1/\ell^2 \) for the \( b_0 \) term, \( \alpha + s_{B*}/\nu_{\text{eff}} = 1/\nu \) for the \( b \bar{c} b \) term, and \( \nu_{\text{drift}} \omega_{\text{eff}} = \nu \omega \) for the \( (b_0)^2 \) term, with the sign \( s_{B*} = \pm 1 \) properly taken into account. Note, however, that \( \sigma(\nu)/\nu \neq \sigma(\nu_{\text{eff}})/\nu_{\text{eff}} \) in general. Thus there is a discrepancy in the \( (b_{12})^2 \) term, which fortunately is unimportant at long wavelengths, as noted for \( L_{\text{eff}}[b] \).

This leads to two important observations: First, the fact that both sides of Eq. (4.9) are practically the same shows the mutual consistency between the fermionic CS theory and our approach, and makes it clear again that the effective Lagrangian \( L_{\text{eff}}[b] \) of Eq. (3.10) rests on general grounds. The RPA response of the fermionic CS theory is obtained here from the bosonic \( L_{\text{CF}}[b, c] \) theory by integrations over \( b_\mu \) and \( c_\mu \), with the result given by \( L_{b}(A; \nu) \) of Eq. (3.11).

Secondly, a tiny discrepancy in the \( (b_{12})^2 \) term indicates that both sides of Eq. (4.9) actually differ beyond the long-wavelength regime. This discrepancy is ascribed to a mismatch between the Landau levels of the original electron and those of the composite fermion, which is inevitable because the procedures of CS-flux attachment and Landau-level projection do not commute. The shorter-wavelength regime, of course, is beyond the scope of both the fermionic CS theory and our approach, and it is already nontrivial that both descriptions are perfectly consistent in the long-wavelength regime.

In the CF description of the FQHE the composite fermion itself constitutes an elementary excitation in the FQH ground states. In particular, Goldhaber and Jain [2] identified a composite fermion as a dressed electron with bare charge \(-e\) and argued by exploiting the incompressible nature of Laughlin’s wave functions that the bare charge is screened in the CF medium to yield local charge equal to \(-\nu e\), consistent with Laughlin’s quasiparticles.

It is possible to substantiate such characteristics of composite fermions within the CS approach. As revealed by Laughlin’s reasoning, adiabatically piercing the CF ground state of integer filling \( \nu_{\text{eff}} \) with a thin flux of vorticity \( q = -s_{B*} = \mp 1 \) (depending on the sign of \( B_{\text{eff}}/B \)) introduces a hole per filled level and thus \( \nu_{\text{eff}} \) holes in total. It is quite obvious at this level of composite fermions that the quasiholes in Laughlin’s picture are nothing but the vacancies of the associated composite fermions, and that the quasiholes are the composite fermions themselves. The charges of these excitations are readily read off from the bosonic CF Lagrangian \( L_{\text{CF}}[b, c] \) in Eq. (4.7): As discussed in the previous section, the vortex is coupled to \( A_\mu \) through a background piece of \( b_\mu \),

\[
-2\pi j_\mu b_\mu = -\nu_{\text{eff}} \Delta \mu (A_\mu + c_\mu + \cdots), \quad (4.10)
\]

which indicates that each composite fermion has “bare” charge \(-e\) in response to \( A_\mu + c_\mu \).

Note next that, upon integration over \( b \), \( L_{\text{CF}}[b, c] \) gives rise to a CS term of the form \(-\frac{1}{2} \beta_\phi (A + c) \partial (A + c) + \beta_\phi = s_{B*} \nu_{\text{eff}} e^2/(2\pi) \), as seen also from \( L_{\text{eff}}^{(\text{CF})} \) in Eq. (4.4). This term combines with another CS term in \( L_{\text{CS}}[c] \), \(-\frac{1}{2} \beta_\epsilon c \bar{c} b \epsilon \) with \( \beta_\epsilon = e^2/(2\pi) \), to fix the background piece of \( c_\mu \) as \( c_\mu^* = -\beta_\epsilon/(-\beta_\phi + \beta_\epsilon) A_\mu + O(DA) \). As a result, the dressing (or renormalization) of the vortex coupling (4.10) by the CS field is substantial:

\[
A_\mu + c_\mu = s_{B*} \nu_{\text{eff}} A_\mu + \cdots, \quad (4.11)
\]

which shows that a composite fermion has fractional charge

\[
Q_{\text{CF}} = -\nu_{\text{eff}} e = -\frac{1}{2\nu \nu_{\text{eff}}} \pm 1 \quad (4.12)
\]

in response to \( A_\mu \), where \( \pm \) refers to \( s_{B*} \). Hence the bare charge \(-e\) coupled to \( A_\mu + c_\mu \) is the same as the renormalized charge \( Q_{\text{CF}} \) probed with \( A_\mu \). This special renormalization feature of the linear \( A_\mu + c_\mu \) coupling is a consequence of the dynamics in the fermionic CS theory. With this in mind one can directly learn from the original Lagrangian (4.4) that the CF field \( \phi \), when probed with \( A_\mu \), has fractional charge \( Q_{\text{CF}} \).

It will be worth remarking here that the coefficient \( \beta_\phi = s_{B*} \nu_{\text{eff}} e^2/(2\pi h) \) of the CS term mentioned above stands for the “bare” Hall conductance in response to \( A_\mu \), where \( \pm \) refers to \( s_{B*} \). Hence the bare charge \(-e\) feels an effective electric field \( E_{\text{eff}}^{A+} = E_y + c_{20} = s_{B*} (\nu/\nu_{\text{eff}}) E_y + \cdots \) in the effective magnetic field \( B_{\text{eff}} = s_{B*} (\nu/\nu_{\text{eff}}) B \) so that it drifts with the same velocity as the electron, as it should,

\[
\nu^\text{drift}_x = E_y A^+/B_{\text{eff}} = E_y / B. \quad (4.13)
\]

V. SUMMARY AND DISCUSSION

In this paper we have studied the electromagnetic characteristics of the FQH states by formulating an effective
vector-field theory that properly takes into account the Landau-level structure and quenching of the cyclotron energy. The effective theory has been constructed, via bosonization, from the electromagnetic response of Hall electrons, in which, as we have seen, the long-wavelength characteristics of the FQH states and the associated quasiparticles are correctly encoded. We have thereby reproduced the dual-field Lagrangian of Lee and Zhang without invoking the composite-boson picture.

Our approach does not refer to either the composite-boson or composite-fermion picture, and simply supposes a FQH ground state of uniform density (in the absence of an external probe). Our approach by itself does not tell at which filling fraction ν such a state is realized. Instead, it tells us that once such an incompressible state is formed its long-wavelength characteristics are fixed universally, independent of the composite-boson and composite-fermion pictures. In this sense, our approach is complementary to the CS approaches, both bosonic and fermionic, where the characteristic filling fractions are determined from the picture-specific condition for the emergence of composite-boson condensates or filled composite-fermion Landau levels.

All these three approaches are consistent at long wavelengths, as we have verified. This gives further evidence for the universality of the long-distance physics of the FQHE, as advocated in Ref. 25. Remember, however, that they start to deviate at shorter wavelengths; thus care is needed in studying the detailed features of the FQH states.

It could happen that a bosonic effective theory better reveals some basic features of the original fermionic theory. Indeed, we have derived a vector-field version of the fermionic CS theory and found a special renormalization pattern of the electron charge, which substantiates the identification by Goldhaber and Jain of the composite fermion as a dressed electron. Looking into the FQHE from various angles would promote our understanding of it.

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APPENDIX A: CALCULATION

In this appendix we outline the construction of the projected Hamiltonian (2.12). Let us begin with sorting the $U(\infty)$ basis in Eq. (2.11)
\[
\gamma_r^k = \frac{Z^r Z^{r+k}}{r!(r+k)!}, \quad \bar{\gamma}_r^k \equiv (\gamma_r^k)^*, \quad (A1)
\]
into diagonal components $\gamma_r^0$ and off-diagonal components $\gamma_r^k$ with $k \geq 1$. One can expand, e.g., $A(\mathbf{x}, t)$ in this basis as $A(\mathbf{x}, t) = e^{i\mathbf{p} \cdot \mathbf{x}} \to e^{i\mathbf{p} \cdot \mathbf{x}}$, which yields the expression
\[
A_{00}(x_0, y_0, t) = e^{\frac{i}{2} \bar{\gamma} \cdot \mathbf{A}} A_0(x_0, y_0, t). \quad (A3)
\]
Actually such reference to a particular ordering convention chosen disappears when one goes back to the $x$ space.

The gauge transformation law (2.9) or its first-order form $\langle A_G^{\eta} \rangle^\dagger = A_0 + [\eta, Z]$ tells us that, with a suitable choice of $G$, $A_G$ is brought to have only $\gamma_r^0$ components. This is the key to systematicizing the calculation. Indeed, to $O(A)$, $A_G$ is written in terms of the field strength $h_z$,
\[
i(A_G^{\eta})^\dagger = \sum_{r=0}^{\infty} \gamma_r^0 \frac{1}{2} (h_z_{00}^{(r)}) + \sum_{r=0}^{\infty} \sum_{k=2}^{\infty} \gamma_r k \bar{k}^{k-1} (h_z_{00}^{(r)}) \quad (A4)
\]
onlyx{upon choosing $\eta$ as
\[
\eta_{[1]} = [0]_0 + \sum_{r=1}^{\infty} \gamma_r^0 \frac{1}{2} (\partial A_{00}^{(r-1)} + \bar{\partial} \bar{A}_{00}^{(r-1)}) + \sum_{r=0}^{\infty} \sum_{k=1}^{\infty} \left( \gamma_r^k \partial A_{00}^{(r)} + \bar{\gamma}_r k \bar{k} A_{00}^{(r)} \right), \quad (A5)
\]
with $(h_z_{00}^{(r)}) = \langle \bar{\partial} \bar{\partial} (h_z)_{00} \rangle_{x_0, y_0, t}$, etc. At the same time the scalar potential $(A_0^G)^{(1)}_{00} \equiv A_0 - \partial_0 \eta$ reads
\[
(A_0^G)^{(1)} = (A_0')_{00} + \sum_{r=1}^{\infty} \gamma_r^0 \frac{1}{2} (E_{00}^{(r-1)} + \bar{E}_{00}^{(r-1)}) + \sum_{r=0}^{\infty} \sum_{k=1}^{\infty} \left( \gamma_r^k \partial E_{00}^{(r)} + \bar{\gamma}_r k \bar{k} E_{00}^{(r)} \right), \quad (A6)
\]
with $E_{00} = \langle E_{00} \rangle_{x_0, y_0, t}$, etc. The $(A_0')_{00} \equiv (A_0'_{00} - \partial_0 \eta_0)$ takes a gauge-invariant form $(A_0'_{00})_{00} = A_0 + g(\frac{1}{2} \bar{\partial} \bar{\partial} + \frac{1}{2} \nabla \cdot \mathbf{E})$, upon choosing $\eta_0 = g(\frac{1}{2} \bar{\partial} \bar{\partial} A + \bar{\partial} \bar{A})$, where $g(x) = (e^x - 1)/x$.

One can go to higher orders in $A_\mu$ by fixing $\eta$ so that $A_G$ has only $\gamma_r^0$ components. Here we construct $H_G$ to $O(A^2)$ and up to two derivatives. A direct calculation yields
$i(A^G)^2 = Z \left\{ \frac{3}{8} b_{\mu}^2 - \frac{1}{2} \partial \bar{\partial} (\bar{A}A) + \frac{1}{2} \text{Re}(\partial^2 A^2) \right\} + O_3,$

$(A_0^G)^2 = \frac{1}{2} [\epsilon^{\mu \nu \lambda} A_\mu \partial_\nu A_\lambda - \epsilon^{0jk} \partial_j (A_k A_0)] + Z O_2 + \bar{Z} O_2 + O_3,$

(A7)

where $O_n$ stands for terms with $n$ derivatives. Finally substitute these $A^G, A_0^G$ and $G h_z G^{-1} = h_z + (h_z)_{00} +$ total deriv. $+ \cdots$ into the transformed Hamiltonian $\bar{H}^G$, and make a further transformation to remove off-diagonal pieces $\propto Z, \bar{Z}$ from it. This leads to $\bar{H}^G$ in Eq. (2.12).

**APPENDIX B: INTEGRATION OVER CHERN-SIMONS FIELDS**

In the text we frequently calculate a functional integral over a 3-vector field $b_\mu$, with a Lagrangian of the form

$$L[b] = -A_\mu \epsilon^{\mu \nu \rho} \partial_\nu b_\rho - \frac{1}{2} \beta \frac{1}{2} b_\mu \epsilon^{\mu \nu \rho} \partial_\nu b_\rho + \frac{1}{2} b_\mu \Gamma b_\mu - \frac{1}{2} b_{12} \Sigma b_{12} - \kappa b_0,$$

(B1)

where $\beta$ and $\kappa$ are real constants; $\Gamma$ and $\Sigma$ may contain derivatives. Our task is to derive the response to an external potential $A_\mu$. The relevant integration is best carried out in the following way: First shift the field $b_\mu = b' \mu + f_\mu$ and choose $f_\mu$ so that no direct coupling between $A_\mu$ and $b' \mu$ remains, i.e.,

$$f_0 + (1/\beta) \Sigma f_{12} = -(1/\beta) A_0,$$

$$f_j - (1/\beta) \epsilon^{jk} \Gamma f_{k0} = -(1/\beta) A_j,$$

(B2)

This is readily solved for $f_{\mu \nu}$,

$$f_{k0} = \frac{1}{\beta D} \left\{ A_{k0} - \frac{1}{\beta} \epsilon^{kj} \Gamma \partial_j A_{00} - \frac{1}{\beta} \Sigma \partial_k A_{12} \right\},$$

$$f_{12} = -\frac{1}{\beta D} \left[ A_{12} - \frac{1}{\beta} \Gamma \partial_k A_{00} \right],$$

$$D = 1 + (1/\beta^2) (\Gamma^2 \partial^2_0 - \Gamma \Sigma \nabla^2),$$

(B3)

and for $f_\mu$ as well.

Integration over $b' \mu$ (which requires fixing a gauge) thereby becomes trivial, yielding no dependence on $A_\mu$. All the dependence on $A_\mu$ is now isolated in $f_\mu$, in terms of which the stationary action or the effective Lagrangian is given by $L_{\text{eff}}[A] = -\frac{1}{2} A \partial \bar{\partial} f - \kappa f_0$, or explicitly

$$L_{\text{eff}}[A] = \frac{1}{\beta} \frac{A}{D} \bar{\partial} A + \frac{1}{2\beta^2} \frac{F_{k0}}{D} \Gamma F_{k0} - \frac{1}{2\beta^2} \frac{F_{12}}{D} \Sigma F_{12} + \frac{\kappa}{\beta} A_0,$$

(B4)

apart from total derivatives. Remember that the leading part of the background piece $f_\mu = -(1/\beta) A_\mu + O(\partial A)$ is fixed from the first two terms in Eq. (B3); extensive use of this fact is made in the text. Note finally that Eq. (B4) loses sense for $\beta \to 0$. This shows that the presence of the CS term $\propto \beta b_0$ is crucial for the present method to work. For $\beta = 0$ one has to first fix a gauge for $b_\mu$ and calculate the response to $A_\mu$.

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1. D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. 48, 1559 (1982).
2. For a review, see The Quantum Hall Effect, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, Berlin, 1987).
3. R. B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
4. F. D. M. Haldane, Phys. Rev. Lett. 51, 605 (1983).
5. B. I. Halperin, Phys. Rev. Lett. 52, 1583 (1984).
6. S. M. Girvin and A. H. MacDonald, Phys. Rev. Lett. 58, 1252 (1987).
7. N. Read, Phys. Rev. Lett. 62, 86 (1989).
8. J. K. Jain, Phys. Rev. Lett. 63, 199 (1989); Phys. Rev. B 41, 7653 (1990).
9. S. C. Zhang, T. H. Hansson, and S. Kivelson, Phys. Rev. Lett. 62, 82 (1989).
10. D.-H. Lee and S.-C. Zhang, Phys. Rev. Lett. 66, 1220 (1991).
11. S. C. Zhang, Int. J. Mod. Phys. B 6, 25 (1992).
12. B. Blok and X. G. Wen, Phys. Rev. B 42, 8133 (1990).
13. A. Lopez and E. Fradkin, Phys. Rev. B 44, 5246 (1991); *ibid.* 47, 7080 (1993).
14. B. I. Halperin, P. A. Lee and N. Read, Phys. Rev. B 47, 7312 (1993).
15. K. Shizuya, Phys. Rev. B 45, 11 143 (1992); *ibid.* 52, 2747 (1995).
16. P. K. Panigrahi, R. Ray, and B. Sakita, Phys. Rev. B 42, 4036 (1990).
17. E. Fradkin and F. A. Schaposnik, Phys. Lett. B 338, 253 (1995).
18. F. A. Schaposnik, Phys. Lett. B 356, 39 (1995).
19. D. G. Barci, C. A. Linhares, A. F. de Queiroz, and J. F. Medeiros Neto, Int. J. Mod. Phys. A 15, 4655 (2000).
20. D. G. Barci and L. E. Oxman, Nucl. Phys. B 580, 721 (2000).
21. D.-H. Lee and M. P. A. Fisher, Phys. Rev. Lett. 63, 903 (1989).
22. W. Kohn, Phys. Rev. 123, 1242 (1961).
23. R. L. Willett, R. R. Ruel, K. W. West, and L. N. Pfeiffer, Phys. Rev. Lett. 71, 3846 (1993); W. Kang, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. 71, 3850 (1993); V. J. Goldman, B. Su, and J. K. Jain, Phys. Rev. Lett. 72, 2065 (1994).
24. A. S. Goldhaber and J. K. Jain, Phys. Lett. A 199, 267 (1995).
25. A. Lopez and E. Fradkin, Phys. Rev. Lett. 69, 2126 (1992).