Codimension-2 brane cosmology

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Abstract. We study cosmological purely codimension-2 braneworlds in a 6d Lovelock theory, with topological matching conditions where the extrinsic curvature on the brane has no jump. For an arbitrary smooth regularisation of the defect, we show that the brane equations of motion do not close, as there is energy exchange between the brane and the 6d bulk spacetime. In the limit that the deficit angle of the defect is approximately constant, the system closes and there is no leakage of energy in the bulk. We analyse the modified Friedmann equation and we see that there are certain corrections coming from the non-zero extrinsic curvature on the brane. We establish the presence of geometric self-acceleration and a possible curvature domination wedged in between the period of matter and self-acceleration eras as signatures of codimension-2 cosmology.

1. Introduction
Codimension-2 distributional defects are particularly interesting because of the purely topological effect of their self-gravity. A straight distributional string carrying only tension is described by a codimension-2 conical singularity [1], while the ambient bulk spacetime remains flat. Each point of the string worldsheet generates a 2d cone with a non-trivial deficit angle related to the string tension. This topological property of codimension-2 defects generated an activity on this subject mostly in relation to the self-tuning paradigm [2] in brane worlds.

When considering however, non-trivial matter on the defect (as e.g., a cosmological fluid), one cannot find in the context of General Relativity (GR) a distributional solution [3]. This is an old paradox already noticed in 4d GR [4] (see [5] for a mathematical discussion on distributional sources in GR). One had to introduce finite thickness effects [6], which made the problem considerably more difficult, and more importantly, plagued the generality of the result and, hence, its physical relevance. Indeed, the key point when studying distributional sources is that they describe the important main features of brane dynamics for an arbitrary family of finite width regularisations, at the limit of infinitesimal thickness.

The way out of this problem can be found when one understands that it was not the defect construction which is problematic, but rather the gravity theory itself does not have the relevant differential complexity in order to describe complicated distributional solutions. Although 4d GR is the unique tensor theory to have at most second derivative (and not higher than second derivative) field equations, in 5d or 6d one has to add an extra term, the Gauss-Bonnet term,
to the action. This generic second order derivative tensor gravity theory is in fact well-known to be Lovelock’s theory [7].

The clear-cut hint that this completion of GR in higher dimensions may be relevant to codimension-2 braneworlds came with the work of [8, 9], where it was noticed that upon considering a Gauss-Bonnet bulk theory, one could have, at least in principle, a non trivial energy momentum tensor fuelling geometric junction conditions for a codimension-2 conical defect. This activity came to a halt when it was claimed that the full set of junction conditions plus bulk field equations at the location of the brane led to an inconsistent system [10] or introduced important constraints on the admissible matter on the brane [11], under certain restrictive initial conditions on the braneworld surface [8]. Relaxing these initial conditions in the recent publication [12], revealed that the above problems disappear. Furthermore, it was shown in [12] that the brane matching conditions of [8] inevitably lead to extra codimension-1 distributional matter, whereas the the brane matching conditions of [9] (referred to as topological) lead to a purely codimension-2 defect.

The aim of this talk is the study the cosmological evolution of a purely codimension-2 defect, studied in detail in [12]. For this purpose, we will use the topological matching conditions of [9]. After a brief introduction to the brane equations, i.e., the brane junction conditions and the leading bulk equations evaluated at the brane position, we will go on with investigating the cosmological equations on the brane. We will show that they have the form of modified Lemaître-Friedmann-Robertson-Walker (LFRW) equations, with several interesting new features: a geometric self-acceleration (in agreement with the maximally symmetric solutions of [13]), brane bending effects depending on the equation of state of matter, and energy exchange between the bulk and the brane triggered by a dynamical conical deficit angle.

2. Setup and brane equations of motion for the topological matching conditions

A general system of 6d Lovelock gravity coupled to localised brane sources is described by the total action

$$ S = \frac{1}{2\kappa_6^2} \int d^6x \sqrt{-|g|} \left( R + \frac{\alpha}{6} L_{GB} \right) + S_{\text{bulk}}[g] + S_{\text{brane}}[g], \quad (1) $$

where $S_{\text{bulk}}[g]$ is the action of regular bulk matter, $S_{\text{brane}}[g]$ is the action of distributional brane matter and $\alpha$ is the coupling in front of the Gauss-Bonnet Lagrangian density

$$ L_{GB} = R^2 - 4R_{MN}R^{MN} + R_{MNKL}R^{MNKL}. \quad (2) $$

In the above, the calligraphic quantities refer to the bulk metric tensor $g$, while the regular ones to the brane metric tensor $g$. The field equations arising from the action (1) are

$$ G^N_M - \frac{\alpha}{6} \left[ \frac{1}{2} L_{GB} \delta^N_M - 2R^N_M + 4R_{MK}^{\phantom{MK}N} \right] = \kappa_6^2 (T^N_M + T^N_M), \quad (3) $$

where $T^N_M$ is a regular bulk energy-momentum tensor and $T_{MN}$ is the distributional brane energy-momentum tensor. In the following, we will fix the notation to $3\kappa_6^2 = 4\pi$ to simplify the equations.

A crucial assumption about the dynamics of the system has to do with the matching conditions that one assumes at the brane position. This ambiguity is always present in defects of codimension greater than one. In particular, in the general case one may chose that the extrinsic curvature has a jump when passing from infinitesimally outside the defect to the core of the defect. Such a condition was imposed for example in [8]. The simplest case is the one
where the extrinsic curvature is continuous from the core of the defect to outside. These matching conditions were studied in [9] and were referred to as topological.

It has been shown in [12], that the general matching conditions combined with the other bulk equations evaluated the brane position require always some codimension-1 distributional matter and therefore do not describe a pure codimension-2 brane. For this reason in the following we will focus on the topological matching conditions.

Assuming that there is axial symmetry in the bulk, the bulk metric ansatz can be written in the brane Gaussian-Normal coordinates as

$$ds^2 = dr^2 + L^2(x, r)d\theta^2 + g_{\mu\nu}(x, r)dx^\mu dx^\nu ,$$

(4)

As usually done when dealing with distributional sources, the matching conditions are derived by integrating around the singular space. Here, we seek to integrate over the conical space, say $C$, with angular metric function

$$L(x, r) = \beta(x)r + O(r^2) ,$$

(5)

where $\delta = 2\pi(1 - \beta(x))$ is the conical deficit. Notice, the coefficients are allowed to depend on the “brane” coordinates $x^\mu$. The angle $\theta$ varies in the interval $[0, 2\pi)$.

Since we are interested in the pure codimension-2 case, the distributional brane energy-momentum tensor should have a Dirac singularity as

$$T_{MN} = \frac{\delta(r)}{2\pi L} \delta_M^\mu \delta_N^\nu .$$

(6)

Then, by integrating the equations of motion, the only non-zero contribution yielding the codimension-2 matching conditions reads [9],

$$\alpha \frac{1}{4\pi} \int d^2 y \sqrt{g_2} R_2 \left[ G_{\mu\nu} + W_{\mu\nu} - \frac{3}{2\alpha} g_{\mu\nu} \right] = T_{\mu\nu} ,$$

(7)

where $R_2 = -2L''/L$ is the curvature of the 2d internal space, and a $'$ denotes $\partial_r$. The Einstein equation is corrected with the following contribution from the extrinsic curvature $K_{\mu\nu} \equiv \frac{1}{2} \partial_r g_{\mu\nu}$ on the brane

$$W_{\mu\nu} = K_\nu^\alpha K_{\mu\lambda} - K K_{\mu\nu} + \frac{1}{2} g_{\mu\nu} (K^2 - K^2_{K\lambda}) ,$$

(8)

which is identical to the one in [8].

In order to evaluate the integral of (7) which is singular at the tip, we assume a family of smooth regularisations $C_\lambda$ of the cone, parametrised by $\lambda$, with smooth $C^2$ everywhere caps (for details see [14]), pasted smoothly to the rest of the cone at a boundary $\partial C$, which we assume does not depend on the parameter $\lambda$ (see left part of Fig.1). The distributional limit is obtained at the $\lambda \to \lambda_c$ limit, where the cap tends to the singular conical space. The width of the regularised defect is supposed to be small enough so that the $r$-dependent brackets can be approximated with the leading term in their $r$-expansion, which is just the value of the brackets at $r = 0$. Then the integral has to be performed over only the internal curvature. The Chern-Gauss-Bonnet theorem relates the geometric curvature for arbitrary $\lambda$, to the Euler characteristic $\chi$ of the regularised cap and the integral of the geodesic curvature $k_\gamma$ of the boundary $\partial C$. Then, for every cap labeled by $\lambda$, the following holds

$$\frac{1}{4\pi} \int_{C_\lambda} d^2 y \sqrt{g_2} R_\lambda = \chi - \frac{1}{2\pi} \int_{\partial C_\lambda} k_\gamma d\theta = 1 - \beta ,$$

(9)
Figure 1. The conical singularity is regularised by a family of regular interiors $C_\lambda$ smoothly connected to the exterior solution of the cone at the boundary $\partial C$. The above result only used the fact that the expansion of $L$ close to the conical defect has the expansion (5) and is independent of the smooth regularisation of the interior of the defect. Finally, with the use of the above theorem, the codimension-2 matching conditions for the topological case is

$$G_{\mu \nu} + W_{\mu \nu} - \frac{3}{2\alpha} g_{\mu \nu} = \frac{1}{\alpha (1 - \beta)} T_{\mu \nu} .$$

(10)

Let us now look at the leading $O(1/r)$ terms of the equations of motion. These come from the contributions of terms of the equations of motion which are multiplied by $L'/L = 1/r + O(1)$, or $\nabla_\mu L'/L = (\nabla_\mu \beta/\beta) \cdot 1/r + O(1)$, with $\nabla_\mu$ the covariant differentiation with respect to the metric $g_{\mu \nu}$.

With the natural assumption that the energy momentum tensor $T_{MN}$ does not blow up close to the distributional singularity (otherwise the singularity would not be distributional), we obtain that the $O(1/r)$ terms of the $(rr)$ and the $(r\mu)$ equations yield respectively

$$K^{\mu \nu} \left( G_{\mu \nu} + W_{\mu \nu} - \frac{3}{2\alpha} g_{\mu \nu} \right) = 0 ,$$

(11)

$$\frac{\nabla^\nu \beta}{\beta} \left( G_{\mu \nu} + W_{\mu \nu} - \frac{3}{2\alpha} g_{\mu \nu} \right) + \nabla^\nu W_{\mu \nu} = 0 .$$

(12)

These two equations act as constraint equations for the two quantities $K_{\mu \nu}, \beta$, which have to be determined and substituted back to the brane Einstein equation (10). In the general case, there are 11 independent unknown functions, but there are 5 independent constraint equations in (11), (12). Therefore, there will in general be functions which cannot be determined by the local equations around the brane, but need to be determined by the bulk solution. This, in fact, is clearly visible in the case of isotropic cosmology on the brane, as we will see in the next section.

Finally, if we differentiate the brane Einstein equation (10) and use the constraint (12), we can derive the energy conservation equation

$$\nabla^{\nu} T_{\mu \nu} = - \frac{\nabla^\nu \beta}{\beta (1 - \beta)} T_{\mu \nu} .$$

(13)

From the above, we see that the energy is not strictly conserved on the brane, but can radiate in the bulk if the deficit angle changes. We can therefore anticipate that cosmological evolutions,
close to the standard 4d one, will have $\beta$ almost constant. We will come to this point in the next section.

3. Codimension-2 Cosmological Evolution

Let us apply now the brane equations that we found in the previous section to a brane with cosmological symmetries (homogeneity and isotropy along the brane spatial directions). For this purpose we adopt the following metric ansatz for the 4d metric $g_{\mu\nu}$ in (4) describing LFRW cosmology

$$ds^2 = -n^2(t,r)dt^2 + a^2(t,r)dr^2 + \kappa + L^2(t,r)d\theta^2,$$

with usual maximally symmetric spatial line element for $\kappa = \pm 1, 0$ curvatures. The expansion of $n$ and $a$ around the brane is

$$n(t,r) = 1 + Nr + O(r^2), \quad a(t,r) = a[1 + Ar + O(r^2)],$$

where with $a = a(t)$ we denote the scale factor on the brane. We will assume that the energy-momentum tensor on the brane is the one of perfect fluid $T_{\mu\nu} = (-\rho, P, P, P)$. The energy density and pressure may contain contributions from the vacuum energy of the brane.

Then for the smoothly regularised brane with topological boundary conditions we obtain that the equations (10), (11), (12) give the following system

$$\frac{\rho}{3\alpha(1-\beta)} = H^2 + \frac{\kappa}{a^2} + \frac{1}{2\alpha} - A^2,$$  

$$\frac{\rho + 3P}{6\alpha(1-\beta)} = \frac{\ddot{a}}{a} + \frac{1}{2\alpha} - fA^2,$$  

$$f = \frac{N}{A} = \frac{3P}{\rho},$$  

$$2A\ddot{A} + 2H(1-f)A^2 = \frac{\dot{\beta}}{\beta 3\alpha(1-\beta)}.$$

In order to derive the equations (18) and (19), we have used the first one (16). Here, we recognize the first two equations as a modified version of the 4d Friedmann and acceleration equations. Note that $\beta$ is generically a function of time which can make the effective 4d Planck scale time varying. A second important point is the appearance of an effective cosmological constant in the face of $\alpha$, which is of geometric origin. A last difference from the standard 4d equations is the presence of the extrinsic curvature correction parametrised by $A$.

The above system of equations has evidently one free function, which we can take to be $\beta(t)$. This function is expected to be fixed by the asymptotic dynamics of the bulk and cannot be determined by the local equations of motion. As mentioned previously, the time dependence of $\beta(t)$ is translated to energy exchange between the brane and the bulk, since the brane conservation equation (13) reads

$$\dot{\rho} + 3H(\rho + P) + \frac{\dot{\beta}}{\beta(1-\beta)}\rho = 0.$$

This variation of $\beta$ induces a variation of the effective 4d gravitational constant read in (10) and is constrained during the early cosmology by the primordial abundances at the nucleosynthesis epoch [15]. Even more stringent constraints expected from solar system observations from the
fact that the theory with varying $\beta$ is similar to a scalar-tensor theory. Not knowing the full family of solutions in the bulk, we choose to consider the case where $\beta$ is approximately constant.

This, in fact, will be a consequence of the equations of motion, had we considered a ring regularisation of the topological matching condition case [12], instead of the smooth one that we used in this talk. This case is bound to give us, at least seemingly, acceptable 4d cosmology, and has the merit that the system of equations is closed and does not depend on undetermined (by the matching conditions) functions.

When $\beta$ is constant, we can solve for the extrinsic curvature $A$ for a given, constant equation of state for the brane matter. For that purpose, we split the energy density and pressure to a part which has the form of brane vacuum energy $\lambda$ and part which describes brane matter $\rho = \lambda + \rho_m$, $P = -\lambda + P_m$, \(^{(21)}\)

\[ A^2 = \frac{C^2}{(\lambda + \rho_m)^2} \rho_m^{\frac{8}{3(1+w)}} , \]

\(^{(22)}\)

with $C^2$ a positive integration constant. With this expression, the Friedmann and acceleration equations (16), (17) become

\[ H^2 + \frac{\kappa}{a^2} = \frac{\kappa^2}{3} \rho_m + \left( \frac{\kappa^2}{3} \lambda - \frac{1}{2\alpha} \right) + \frac{C^2}{(\lambda + \rho_m)^2} \rho_m^{\frac{8}{3(1+w)}} , \]

\[ \frac{\dot{a}}{a} = -\frac{\kappa^2}{6}(1+3w)\rho_m + \left( \frac{\kappa^2}{3} \lambda - \frac{1}{2\alpha} \right) + 3\frac{C^2}{(\lambda + \rho_m)^3} \left( w\rho_m - \lambda \right) \rho_m^{\frac{8}{3(1+w)}} , \]

\(^{(23)}\)

\(^{(24)}\)

where $\kappa^2 = 1/\alpha(1 - \beta)$, which show a non-trivial correction to the cosmological equations as a function of $w$. The magnitude of this correction depends on the integration constant $C^2$.

In the following, we will try to see whether it is possible for large enough $C^2$, consistent with observations, to have some interesting modification to cosmology. From the integration of the continuity equation (20) for $\beta=$const. we obtain

\[ \rho_m = \frac{\rho_0(1 - \beta)}{\beta \alpha^{3(1+w)}} , \]

\(^{(25)}\)

which holds also for the special $w = -1$ case.

For $C^2 \neq 0$, our 6d bulk geometry has a genuine curvature singularity at $r = 0$ (apart from the distributional one). In fact, this is to be expected from purely geometrical considerations (see [16]). Higher codimension defects, when considered in their zero width limit, will develop curvature singularities. These are expected to be smoohtable once we take finite width corrections into account. Therefore, we expect finite width effects to be important at the UV sector and our distributional approximation to break down even though this will not show up necessarily in the field equations themselves!

Let us now analyse some particular cases of interest.

### 3.1. Self-accelerating branes

The first interesting case is the one in which the tension of the brane is vanishing $\lambda = 0$. In this case, we see from (23) that there is an asymptotic accelerating phase for $\alpha < 0$. This acceleration is purely due to the geometric Gauss-Bonnet term and therefore is a case of self-acceleration.
This corresponds to generalisation of the self-accelerating solutions of [13] with the addition of matter. The correction to the Friedmann equation is given by

\[ A^2 = C^2 \frac{2(1-3w)}{\rho_m(1+w)} = \frac{C^2}{a^{2(1-3w)}}. \]  

Note that the second equality holds also for \( w = -1 \). In the table in Fig.2 we list the form of corrections that we obtain for different constant equations of state. It is worth noting three interesting limits of the cosmological evolution in this case.

First is the one where the brane is dominated by cosmological constant. This may go against the initial assumption that the tension of the brane vanishes, because of the ambiguity to separate the vacuum part of the energy density from the matter energy density. If a matter component behaving as inflaton or dark energy dominates at some period of the history of the Universe, it will have the behaviour that we note here. So, for this case the extrinsic curvature plays the role of a dark radiation squared term \( a^{-8} \) that will dominate the cosmology at early times.

A second interesting limit is when the matter equation of state is that of radiation \( w = 1/3 \). Then, the extrinsic curvature correction is of the form of constant vacuum energy. One would be tempted to use this vacuum energy to drive early Universe inflation. This seems to be possible, but around matter-radiation equality the Universe will be dominated by this new vacuum energy, making the phenomenology of the model problematic.

Finally, when the matter equation of state is that of dust \( w = 0 \), the extrinsic curvature correction is of the form of curvature \( (a^{-2}) \). In this case, we can see a possibility of having an observable signature of the codimension-2 cosmology, if the constant \( C^2 \) is chosen so that there is a brief period of curvature domination around the vacuum energy domination period. This scenario is depicted in Fig. 3. In order to obtain the standard epochs, i.e., matter domination and then cosmological constant domination, we need to have that \( \rho_m(a_{eq}) > 1/(2|\alpha|) \) and \( (\kappa_4^2/3)\rho_m(a_{eq}) > A^2(a_{eq}) \), where \( a_{eq} \) is the scale factor at radiation-matter equality and \( a_{acc} \) the one at the moment of cosmological constant domination. In order that the brief period of curvature domination is observable, we need \( A^2(a_{acc}) > 1/(2|\alpha|) \). Therefore combining these inequalities we see that we need

\[ \frac{1}{2|\alpha|} < A^2(a_{acc}) < \frac{\kappa_4^2}{3} \rho_m(a_{eq}) . \]  

Then, a straightforward manipulation with the help of (25), (26) provides us with the following constraint on \( C^2 \)

\[ \left( \frac{\rho_0(1-\beta)}{\beta} \right)^{-2/3} \frac{a_{acc}^2}{2|\alpha|} < C^2 < \kappa_4^2 \left( \frac{\rho_0(1-\beta)}{\beta} \right)^{1/3} \left( \frac{a_{acc}^2}{a_{eq}^3} \right) . \]  

**Figure 2.** The functional dependence on the scale factor of the energy density and the extrinsic curvature correction to the Friedmann equation for different equations of state of the brane matter in the self-accelerating case. From top to bottom we have the equations of state for cosmological constant, dust and radiation.
\[
\ln \rho_m = \ln \left( \frac{1}{(2|\alpha|)} \right) - \frac{a^{-2}}{2|\alpha|} - \frac{a^{-8}}{2|\alpha|}\ln a_{eq}
\]

Figure 3. The log-log evolution of the energy densities as a function of the scale factor. With solid lines, there are the contributions of the standard matter and the geometrically induced \(1/(2|\alpha|)\) cosmological constant. With dashed lines are the evolutions of the extra component \((\propto A^2)\) in the Friedmann equation due to the extrinsic curvature of the brane. The dependence of this extra component on the scale factor is noted. This dependence changes whenever a new matter component becomes dominant, i.e., at matter domination at \(a_{eq}\) and at the cosmological constant domination at \(a_{acc}\).

It is not yet clear if such a brief period of curvature domination is phenomenologically viable and it requires certainly further study to see the potential observational signatures of such a case.

3.2. Self-tuning branes

A second interesting case is when self-tuning can be realised. For that possibility, the topological quantity \(\beta\) is used to accurately cancel the vacuum energy contribution of the brane to the effective cosmological constant. In more details, as discussed in [13], by tuning \(\beta\) one can make the two vacuum energy contributions, the geometrical one \(1/2|\alpha|\) and the brane tension one \(\lambda\), cancel

\[
\frac{\kappa^2}{3} \lambda - \frac{1}{2|\alpha|} = 0.
\]

This limit is orthogonal to the self-accelerating case that we studied before. The effective Friedmann equation for the present case is given by the expression

\[
H^2 + \frac{\kappa}{a^2} = \frac{\kappa^2}{3}\rho_m + \frac{C^2}{(\frac{3}{2}\alpha^2 + \rho_m)^2} \rho_m^{\frac{8}{3(1+w)}},
\]

which includes a non-trivial correction beyond the standard linear to energy density term. Let us note here that the actual self-tuning is not visible from this Friedmann equation. This is because the present is valid for \(\beta = \text{const}\). Instead, one should consider the time-varying deficit angle case in order to see how (29) can be dynamically achieved at the early Universe evolution. Since before nucleosynthesis there are no constraints on the variation of Newton’s constant, one could have an acceptable cosmology with varying \(\beta\). However, then, the brane equations do not close and the mechanism of self-tuning should be dictated by bulk boundary conditions.
4. Conclusions
In this talk we have described the cosmological equations for a codimension-2 brane in 6d bulk Lovelock gravity. These were obtained by considering the junction conditions on the codimension-2 brane and the leading parts of the bulk equations evaluated at the brane position. For that we used the, so called, topological matching conditions [9], where the extrinsic curvature is continuous outside the defect and at the core of the defect, since these are the ones that do not require any codimension-1 extra sources [12].

Studying the cosmology of these conical branes, we have found several interesting features. Firstly, quite generically the higher order terms give ordinary LFRW equations, as though one included an induced gravity term on the brane [17]. This is quite different from codimension-1 cosmology [18], where the ordinary behaviour is recovered only at late times. Corrections to this standard evolution are threefold: there is a geometric acceleration scale related to $1/\alpha$, which restates the problem of a minute cosmological constant as a gravitational hierarchy between the bulk Gauss-Bonnet and Einstein-Hilbert term. Secondly, extrinsic curvature corrections are apparent and they are dependent of the equation of state of the perfect-fluid matter. In other words, as the brane evolves in the bulk, a dust equation of state can lead to an extra component behaving as curvature, or a radiation equation of state to a cosmological constant term, and so on. To put it in a nutshell, each matter fluid introduces two differing fluid components in the modified LFRW equations. Last, but not least, as the brane evolves it can generically radiate in the bulk. Again, this is unlike the vacuum bulk codimension-1 braneworlds.

The above characteristics can be used to constrain the model in question with respect to cosmological observations. Furthermore, on the theoretical side it would be interesting to have particular bulk solutions manifesting the cosmology evolution we have found, and in particular setting the boundary conditions in order to fix $\beta$. The most important information we have given here in this direction is that it is pure 6d Gauss-Bonnet gravity that gives the dynamical LFRW-like evolution on the brane. The 6d Einstein term gives the possibility for a phase of geometric acceleration. Furthermore, a varying $\beta$ could have interesting consequences for a cosmological self-tuning scenario following equation (29). However, note that the evolution in $\beta$ would then leave an imprint on the cosmological evolution equations. These are amongst the open, interesting questions that this work puts forward and which we hope will be answered in the near future.

Acknowledgments
I would like to thank Christos Charmousis and Georgios Kofinas for a very enjoyable collaboration. A.P. is supported by the Marie Curie Intra-European Fellowship EIF-039189.

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