Multiuser MIMO with Large Intelligent Surfaces: Communication Model and Transmit Design

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Abstract—This paper proposes a communication model for multiuser multiple-input multiple-output (MIMO) systems based on large intelligent surfaces (LIS), where the LIS is modeled as a collection of tightly packed antenna elements. The LIS system is first represented in a circuitual way, obtaining expressions for the radiated and received powers, as well as for the coupling between the distinct elements. Then, this circuitual model is used to characterize the channel in a line-of-sight propagation scenario, rendering the basis for the analysis and design of MIMO systems. Due to the particular properties of LIS, the model accounts for superdirectivity and mutual coupling effects along with near field propagation, necessary in those situations where the array dimension becomes very large. Finally, with the proposed model, the matched filter transmitter and the weighted minimum mean square error precoding are derived under both realistic constraints: limited radiated power and maximum ohmic losses.

Index Terms—Beamforming, holographic MIMO, large intelligent surfaces, super-directivity.

I. INTRODUCTION
Since the seminal paper by Marzetta [1], massive multiple-input multiple-output (MIMO) systems have moved from being an unrealistic idea to becoming a key enabling technology in 5G and future generations of wireless networks [2, 3]. The promising gain of these systems have given raise to a widespread interest in considering even a larger number of antennas than in conventional massive MIMO. Hence, new concepts such as holographic MIMO, large intelligent surfaces (LIS) or intelligent reflecting surfaces (IRS) have emerged as a natural evolution of classical MIMO.

The use of LIS (i.e., large arrays) for wireless networks may render considerable gains in terms of capacity, interference reduction and user multiplexing; but it also supposes a new paradigm from a system design point of view. Introducing a massive number of antennas in a limited surface leads to a small inter-element distance (ideally almost-continuous radiating surfaces [4]). Hence, phenomena that have been classically neglected in the analysis of MIMO systems, such as mutual coupling [5, 7] and superdirectivity effect [5, 8–11], become now much relevant.

Mutual coupling is inherent to arrays with closely-spaced antennas, affecting both the radiation pattern and the impedance of the antenna element, which implies ultimately a change on the received power [12]. However, this effect is not considered when designing the linear transmit and receive processing [13, 14], giving rise to solutions that might not be optimal in realistic conditions. Also, related to mutual coupling is the superdirectivity effect, which theoretically allows for the design of highly directive (ideally unbounded) arrays of closely-spaced antennas [9]. However, in practice, achieving such superdirectivity comes at the price of extremely large excitation currents, which considerably increases the losses and reduces the efficiency [8], and makes the array sensitive to small random variations in the excitation [15].

On a related note, as the array dimensions become large and the number of the antennas increases, some of the classical results for MIMO systems are no longer valid. For instance, in [16], it is proved that the widely accepted scaling law, i.e., the signal-to-noise ratio scales with the number of antennas, is only valid under the far-field assumption. Therefore, in order to properly analyze and design LIS-based MIMO systems, it is also necessary to consider near-field effects, specially in indoor scenarios or those situations where far-field conditions cannot be guaranteed due to the large LIS dimensions.

Although some works have considered the effects of superdirectivity arrays [5, 17] or mutual coupling [18], to the best of our knowledge, no model accounting for superdirectivity, coupling and near-field propagation has been presented in the literature. Aiming to fill this gap, we here propose a communication model for LIS-based MIMO, which considers the three aforementioned phenomena. To that end, we merge electromagnetic theory with classical MIMO system models, creating a link that allows to include all these effects in the channel matrix and paving the way to more detailed works. As a result, we obtain a characterization based on infinitesimal dipoles, which is independent of any physical antenna realization and can be used to model real deployments, e.g., metasurfaces [19]. Finally, we use the derived model to explore the design and performance of two transmission schemes: matched filtering (MF) and weighted minimum mean square error (WMMSE) [13].

Notation: $i$ is the imaginary unit, $\| \cdot \|$ is the euclidean norm, $| \cdot |$ is the absolute value and $^T$ and $^H$ are the transpose and Hermitian transpose respectively. Vectors are denoted by bold lowercase symbols, and matrices are denoted by bold uppercase symbols. Finally, $\Re\{ \cdot \}$, $\Tr\{ \cdot \}$ and $\mathbb{E}\{ \cdot \}$ are the real part, the trace and the expectation operator, respectively.

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where the voltage is simply the difference in electrical potential we are abstracting from any physical structure for the antennas, whereas the UEs are arbitrary placed in front of it.

To model ohmic losses within the LIS, of capital importance the currents and voltages is therefore given by

\[ z = j \text{ipt} \eta \text{tr} \text{m} \times M \times M \]

by individual ports carrying different currents and voltages.

every antenna element in the LIS and every UE is represented as a collection of \( M \) closely spaced antennas, emulating a near-continuous radiating surface, and centered at the origin of a cartesian coordinate system aligned with the \( yz \)-plane, whereas the UEs are arbitrary placed in front of it.

The antennas composing the LIS are modelled as identical and infinitesimal dipoles carrying a uniform current along a short line segment, where, by definition, the current distribution is independent of the surroundings. Note that, with this model, we are abstracting from any physical structure for the antennas, and just considering the antennas as uniform current sources where the voltage is simply the difference in electrical potential along the length of the dipoles. This keeps the mathematical complexity of the model under control and allows to capture near-field propagation effects without resorting to complicated electromagnetic simulations. Also, we only consider linear near-field cross-polarization terms is left for future work. As with the relation between \( z_{\text{tr}} \) and \( z_{\text{re}} \) in [2], the relation between the transmitted and received currents is given by

\[ j_r = -(Z_{\text{tr}} + I_M z_0)^{-1} Z_{\text{re}} j_t. \]

With the relation between \( j_t \) and \( j_r \), the time-averaged power received at the \( m \)-th UE is directly expressed as

\[ P_{\text{tm}} = \frac{2}{2} \left( j_{\text{trm}} \right)^2 \text{Re}\{z_0\}, \]

where \( j_{\text{trm}} \) and \( v_{\text{trm}} \) for \( m = 1, \ldots, M \) are the elements of \( j_r \) and \( v_r \), respectively.

On the transmitter side, the primary interest is the time-averaged power delivered to the network, which is given by

\[ P_{\text{f}} = \frac{2}{2} \left( j_{\text{tr}} Z_{\text{re}} j_t \right)^2 + \frac{2}{2} \left( j_{\text{tr}} Z_{\text{re}} j_t \right)^2 + \frac{2}{2} \left( j_{\text{tr}} Z_{\text{re}} j_t \right)^2, \]

In [6], the first term (labeled as internal) is the actual power that the transmitter delivers to the network, which is only

\[ \text{Re}\left\{ Z_{\text{tr}} - Z_{\text{tr}} Z_{\text{re}} Z_{\text{tr}}^{-1} Z_{\text{re}} \right\}. \]
impacted by the coupling matrix between the antenna elements in the LIS. The second term encompasses the coupling between the inducted currents at the UEs and the LIS, which may be relevant in near-field scenarios where the distance between the users and the transmitter is small. If all the UEs are placed far enough, then this second term can be neglected.

Finally, to model thermal losses at the transmitter, a loss resistance $r_1$ is attached to all ports corresponding to the LIS antennas. These losses, although usually ignored in MIMO works, play a pivotal role in beamforming analysis and design. As the antennas in the LIS are very closely spaced, optimal precoders result in very high currents, which leads to significant thermal losses even for very high efficiency antennas [6, 8, 15].

In our proposal, all the UE and LIS antennas are modelled as small dipoles and, hence, the entries of the impedance matrix $Z$ in (1) are given by (16). Similarly, introducing (16) in (5), we obtain the received power by an arbitrary user.

A consequence of the infinitesimal antenna model is that the imaginary part of the antenna’s self-impedance diverges to $\pm \infty$ depending on the direction of approach. This means that it is practically impossible to perform an impedance matching for an infinitesimal dipole. However, the model can be seen as a discretization of a physical system which could potentially be realised. As such, the model serves to represent an arbitrary design that is theoretically possible while being independent of practical implementation limitations. For instance, the same technique is observed in [21], where common antenna designs are split into sets of infinitesimal dipoles while successfully capturing the narrow-band radiation characteristics of the original design.

As shown in (5) and (6), the system performance is dominated by the real part of the impedance, whose maximum value is obtained as $r \rightarrow 0$ (corresponding to the value of $z_0$), i.e.,

$$\text{Re}\{z_0\} = \lim_{r \rightarrow 0} \left( \text{Re}\{z(r)\} \right) = \frac{kl^2 \eta}{3\lambda}.$$  

Without any loss of generality, the length of the short dipoles is chosen as to normalize the radiation resistance, and thus $l = 1.036 \lambda$, approximated as $\approx \frac{i l^2 \eta}{2\lambda r} \left( 1 - \frac{z^2}{r^2} - \frac{i}{kr} - \frac{1}{k^2 r^2} + \frac{i 3z^2}{kr^3} + \frac{3z^2}{k^2 r^4} \right).$  

At a receiver, the voltage across the receiving antenna is given by integration along the line segment of the receiver as

$$v(r) = -\int_{\frac{l}{2}}^{\frac{l}{2}} [0 \ 0 \ 1] E(r) \, dz,$$  

which, assuming again a short dipole, can be approximated as

$$v(r) \approx -[0 \ 0 \ i] G(r) [0 \ 0 \ jl]^T.$$  

Finally, dividing (15) by the source current yields the mutual impedance between two antennas separated by the distance vector $r$ as

$$z(r) = i \frac{l^2 \eta e^{-ikr}}{2\lambda r} \left( 1 - \frac{z^2}{r^2} - \frac{i}{kr} - \frac{1}{k^2 r^2} + \frac{i 3z^2}{kr^3} + \frac{3z^2}{k^2 r^4} \right).$$  

In our proposal, the lengths of the antennas are chosen as to normalize the radiation resistance, and thus $l = \sqrt{\frac{3\lambda}{k\eta}} \approx 0.036\lambda$.

IV. MIMO COMMUNICATION MODEL

With the analysis of the circuital model in Fig. 1 accomplished, the next step is linking it to a MIMO communication model that can be easily used for precoding analysis and design. To that end, we consider that the base station serves $M$ users simultaneously, and therefore the decoded signal vector $\hat{x} \in \mathbb{C}^{M \times 1}$ is expressed as

$$\hat{x} = A(HBx + n),$$  

where each element is explained in the following. Vector $x = [x_1 \ x_2 \ \ldots \ x_M]^T$ denotes the set of symbols intended to each user, represented as complex root mean square (RMS)
values with zero mean and covariance matrix $\mathbb{E}[x^H x] = I_M$, where $x_m \in \mathbb{C}$ denotes the symbol destined for $m$-th user. This set of symbols is passed through a transmit filter (precoding) represented by matrix $B = [B_1 \ldots B_M]$, with $B_m \in \mathbb{C}^{N \times 1}$ the beam targeted at $m$-th user. Coming back to the circuitual model in (1), the RMS value of the currents at the transmitter would be given then by $j_b/\sqrt{2} = Bx$. Note that these currents are time varying, but for simplicity we remove the temporal dependency. Given $j_b$, the currents induced at the receivers are given by (4), and, therefore, the channel matrix is expressed as

$$H = -(Z_m + I_M z_m^*)^{-1} Z_m = [h_1 \ h_2 \ldots \ h_M]^H,$$

where $h_m \in \mathbb{C}^{N \times 1}$ is the channel vector from the LIS to $m$-th user. At the receiver side, a diagonal filter matrix $A \in \mathbb{C}^{M \times M}$ is applied to the received symbols. Finally, $n \sim \mathcal{CN}(0_{M \times 1}, \sigma_n^2 I_M)$ is the noise term, which is independent of the transmitted symbols, i.e., $\mathbb{E}[n x^H] = 0_{M \times M}$.

Note that we have presented a formulation in terms of voltages can be obtained by applying the relations in superdirective systems [7], and it seems more realistic and actual radiated power [5]. On the other hand, considering two constraints that are of interest when designing LIS-based communications: radiated power (20) and ohmic losses (21). Considering the radiated power constraint instead of the traditional $\text{Tr}\{B^H B\}$ is important in highly coupled systems, since the latter may lead to a considerably larger actual radiated power [5]. On the other hand, considering ohmic losses is necessary since these losses are usually high in superdirective systems [7], and it seems more realistic and feasible than restraining the superdirectivity $Q$ factor [7].

### A. MF transmitter

For the matched filter, we follow the objective of [14] in maximizing the correlation between the received and transmitted symbol. The diagonal receive filter $A$ is set to the identity matrix $I_m$, as it does not affect the correlation. The optimization problem is then formulated as

$$\text{maximize} \quad \mathbb{E}[x^H x] = \text{Tr}\{HB\}$$

subject to \n
$$\text{Tr}\{B^H R_p B\} \leq P_R,$$

$$\left(\frac{1}{\sigma_r^2} - 1\right) \text{Tr}\{B^H B\} \leq P_L,$$

where $P_R$ and $P_L$ are the maximum allowed radiated power and ohmic losses, respectively. The Karush–Kuhn–Tucker (KKT) conditions are given as

$$B_{MF|\mu_1=0} = H^H \sqrt{\frac{P_L}{\text{Tr}\{HH^H r_r\}}},$$

$$B_{MF|\mu_2=0} = R_p^{-1} H^H \sqrt{\frac{P_R}{\text{Tr}\{HR_p^{-1}H^H\}}}.$$

To the best of authors’ knowledge, if both $\mu_1 \neq 0$ and $\mu_2 \neq 0$, a closed form solution cannot be obtained. Instead, we propose an algorithm which rapidly converges to a solution within a specified precision. To this end, a variable $\alpha = \frac{\mu_2}{\mu_1}$ is defined and inserted into (25), leading to

$$B_{MF|\mu_1, \mu_2 \neq 0} = (R_p + \alpha \gamma I_N)^{-1} H^H \frac{1}{2 \mu_1}.$$

Using (30), (26) and (27) are rewritten as

$$P_{RL} \mu_1^2 = \text{Tr}\{H (R_p + \alpha \gamma I_N)^{-1} R_p (R_p + \alpha \gamma I_N)^{-1} H^H\},$$

$$P_{RL} \mu_2^2 = r_l \text{Tr}\{H (R_p + \alpha \gamma I_N)^{-2} H^H\}.$$

The optimal value of $\alpha$ is obtained by dividing (31) and (32),

$$\frac{P_{RL}}{P_{RL} \mu_1^2} = \frac{r_l \text{Tr}\{H (R_p + \alpha \gamma I_N)^{-1} (R_p + \alpha \gamma I_N)^{-1} H^H\}}{\text{Tr}\{H (R_p + \alpha \gamma I_N)^{-1} (R_p + \alpha \gamma I_N)^{-1} H^H\}}.$$
We here follow a procedure similar to that in [13] to derive the
and $A$ with $W$ where $[13$, eqs. (7) and (38). Finally, conditioned on this result, we propose an heuristic solution given by $\sigma \beta$ satisfied, i.e., $\beta$ and the value of $\alpha$ is chosen so that both constraints are satis
deed, $P_L$ and $P_R$, respectively. The UEs are positioned at a distance $d = 9 \lambda$ or $d = 0.1 \lambda$, respectively. The UEs are positioned at a distance $d = 20 \lambda$ along the $x$-axis on a line along the $y$-axis of length $10 \lambda$. Hence, the larger the number of UEs, the smaller the separation between them. In this scenario, the WMMSE sum capacity in (23) is calculated in terms of the number of users with difference antenna efficiency and spacing between the LIS elements, as depicted in Fig. 2. We observe that the inter-UE coupling plays an important role in the performance. If we neglect it, i.e. $Z_n = 1_M$, then the sum capacity rises to a maximum and remains constant as more users are added. However, if we take into account this coupling, then the capacity raises up to a turning point, from which it starts decreasing as the number of users increases. Note, however, that the maximum value for the capacity is higher when coupling is present. Notably, with a relatively low antenna efficiency of $\epsilon_r = 0.8$, decreasing the spacing from $d = 0.5 \lambda$ to $d = 0.1 \lambda$ yields no gains in terms of sum capacity since the system is limited by ohmic losses.

On the other hand, Fig. 3 shows the received power from a four-by-four wavelength two-dimensional transmit array, populated by an increasing number of transmitters. A UE is aligned within the center of the LIS at a distance of $2 \lambda$ along the $x$-axis. The received power at the UE is determined along the $x$-axis of length $20 \lambda$, respectively. The UEs are positioned at a distance $d = 9 \lambda$ or $d = 0.1 \lambda$, respectively. The UEs are positioned at a distance $d = 20 \lambda$ along the $x$-axis on a line along the $y$-axis of length $10 \lambda$. Hence, the larger the number of UEs, the smaller the separation between them. In this scenario, the WMMSE sum capacity in (23) is calculated in terms of the number of users with difference antenna efficiency and spacing between the LIS elements, as depicted in Fig. 2. We observe that the inter-UE coupling plays an important role in the performance. If we neglect it, i.e. $Z_n = 1_M$, then the sum capacity rises to a maximum and remains constant as more users are added. However, if we take into account this coupling, then the capacity raises up to a turning point, from which it starts decreasing as the number of users increases. Note, however, that the maximum value for the capacity is higher when coupling is present. Notably, with a relatively low antenna efficiency of $\epsilon_r = 0.8$, decreasing the spacing from $d = 0.5 \lambda$ to $d = 0.1 \lambda$ yields no gains in terms of sum capacity since the system is limited by ohmic losses.

VI. NUMERICAL RESULTS

Finally, we present some simulated results for both proposed beamformers, namely MF and WMMSE, in order to show the impact of coupling, antenna efficiency and number of radiating elements in the number of supported simultaneous UEs. Throughout the whole section, perfect knowledge of the channel in (19) is assumed.

We consider first a one-dimensional transmit array with length of $4 \lambda$ along the $y$-axis, populated by either $N = 9$ or $N = 41$ transmitters, corresponding to a spacing of $d = 0.5 \lambda$ and $d = 0.1 \lambda$, respectively. The UEs are positioned at a distance of $20 \lambda$ along the $x$-axis on a line along the $y$-axis of length $10 \lambda$. Hence, the larger the number of UEs, the smaller the separation between them. In this scenario, the WMMSE sum capacity in (23) is calculated in terms of the number of users with difference antenna efficiency and spacing between the LIS elements, as depicted in Fig. 2. We observe that the inter-UE coupling plays an important role in the performance. If we neglect it, i.e. $Z_n = 1_M$, then the sum capacity rises to a maximum and remains constant as more users are added. However, if we take into account this coupling, then the capacity raises up to a turning point, from which it starts decreasing as the number of users increases. Note, however, that the maximum value for the capacity is higher when coupling is present. Notably, with a relatively low antenna efficiency of $\epsilon_r = 0.8$, decreasing the spacing from $d = 0.5 \lambda$ to $d = 0.1 \lambda$ yields no gains in terms of sum capacity since the system is limited by ohmic losses.

On the other hand, Fig. 3 shows the received power from a four-by-four wavelength two-dimensional transmit array, populated by an increasing number of transmitters. A UE is aligned within the center of the LIS at a distance of $2 \lambda$ along the $x$-axis. The received power at the UE is determined using the MF transmitter for three different efficiencies, whilst the transmit and loss power constraints are fixed to $P_R = 1$ and $P_L = 1$. As a reference, we represent also the results obtained from the models in [22] and [16]. In this scenario,
the LIS covers one sixth of all space seen from the perspective of the UE. As such, the model of [22], which models the physical aperture of the LIS as an electrical aperture, yields a received power of \( P_r = 1 \). Regarding Fig. 3 we see that, when \( e_r < 1 \), the performance approaches the result predicted by [22] as the number of antenna-elements increases (equivalently, \( d \) decreases). In turn, for \( e_r = 1 \), the super-directivity effect is not restricted, allowing electrical aperture to extend beyond the physical aperture and reach a point where the scattering from the UE, marked as external in (6), limits the radiated wave. Note that, if the scattering is not included in the model, the received power can extend beyond the transmitted power and energy is not conserved.

VII. CONCLUSIONS

We have introduced a new communication model for multi-user MIMO based on LIS, which takes into account several phenomena that has been classically neglected in MIMO analysis and design such as mutual coupling, superdirectivity and near-field effects. The proposed model, which merge communication and electromagnetic theory, may be a first step in the design of LIS systems in realistic conditions. Based on our proposal, we also provide, for first time in the literature, a transmit MF and WMMSE beamforming schemes that take into account two constraints: effective radiated power and ohmic losses. As shown in the provided numerical results, the impact of near-field propagation (namely, coupling between the users and the transmitter) as well as that of the antenna efficiency and ohmic losses are non-negligible in the system performance, so its characterization seems to be of key importance in the future design of LIS-based solutions.

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