On The Possibility of Observation of $a_0$-$f_0$ Mixing in the $pn \rightarrow da_0$ Reaction

A.E.Kudryavtsev, V.E.Tarasov

Institut of Theoretical and Experimental Physics
B.Cheremushkinskaya 25, 117259 Moscow, Russia
e-mail: kudryavtsev@vxitep.itep.ru, tarasov@vxitep.itep.ru

Abstract

It is shown that, if isospin is not conserved in $a_0$- and $f_0$-meson mixing, forward-backward asymmetry arises in the reaction $pn \rightarrow da_0$. This effect increases near the reaction threshold. The asymmetry is estimated within the framework of a model in which the $a_0$ -- $f_0$ mixing is due to the virtual $\pi^0 \leftrightarrow \eta$ transition and the difference in masses of the charged and neutral kaons in decay channels. The angular asymmetry near the threshold of the $pn \rightarrow da_0$ reaction was found to be large, of the order of 8 ÷ 15 %.

PACS: 13.60.Le, 13.75.-n, 14.40.Cs

The origin of the lightest, virtually mass-degenerate, scalar mesons $a_0$ (980) ($I^GJ^{PC} = 1^{-0^{++}}$) and $f_0$ (980) ($0^{++}$) is one of the most important problems of hadron physics. Different assumptions exist about the structure of these mesons, from the standard $q\bar{q}$ states [1] and their modifications (see, e.g., [2] and reference therein) to the 4-quark configurations [3] and the lightest scalar mesons as ”minions” in the Gribov confinement model [4]. The problem of the structure of $a_0$ and $f_0$ mesons is closely related to the problem of $a_0$- $f_0$ mixing. The dynamical mechanism of this mixing was suggested around 20 years ago in Ref. [5]. The first evidence of this phenomena came from the recent data of the WA102 collaboration at CERN [6]. These data on the central production of $a_0$ and $f_0$ in the reaction $pp \rightarrow p_s Mf$ were interpreted in Ref. [7] as the evidence of the $a_0$-$f_0$ mixing.

If the $a_0$ and $f_0$ mesons have close structures, then the mixing with violation of isospin conservation could be large. Along with the direct $a_0 \leftrightarrow f_0$ transition due to isospin violation in the quark sector, these mesons can mix due to isospin-violating interaction in the decay channels. Different mixing mechanisms are illustrated in Fig. 1. Note that the vertex of the direct $a_0 \rightarrow f_0$ interaction in Fig. 1a depends on the quark content of scalar mesons and should be extracted from the experiment. At the same time, the mixing due to the decay processes presented in Fig. 1b and 1c can be estimated rather reliably.

It is convenient to examine $a_0$ -- $f_0$ mixing in the reaction of production of a neutral $a_0$ meson:

$$ p n \rightarrow da_0. \quad (1) $$

1 This is the modified version of the paper published in JETP Lett. 72 (2000) 410.
Note that the forward-backward asymmetry in reaction (1) is absent if the isospin is conserved [8]. As will be shown below, observation of this asymmetry would testify to the presence of isospin-violating $a_0^0 - f_0$ mixing, and the asymmetry effect should be stronger near the threshold of the reaction (1).

If isospin is conserved, an isovector $a_0$ meson can be produced near the threshold of the reaction (1) only in the $p$ wave with respect to the deutron. At the same time, if an isoscalar $f_0$ meson is produced in the reaction

$$pn \rightarrow df_0,$$  \hspace{1cm} (2)

the final orbital angular momentum $L$ of the $df_0$ system may be zero. This conclusion follows from the isospin ($I$), parity ($P$), and angular momentum ($J$) conservation laws. The possible quantum numbers for reactions (1) and (2) yielding final systems with the smallest orbital angular momenta ($p$ and $s$ waves for the $da_0^0$ and $df_0$ systems, respectively) are listed in the table. The total spin of the system is denoted by $S$. The quantum numbers presented in the table are consistent with the requirement for antisymmetry of the system with respect to the initial fermions.

|       | $pn \rightarrow da_0^0$ | $pn \rightarrow df_0$ |
|-------|-------------------------|------------------------|
| $I$   | 1                       | 1                      |
| $S$   | 1                       | 1                      |
| $L$   | 1, 3                    | 1, 0, 2                |
| $P$   | -1                      | 1                      |

Thus, if isospin is conserved, reactions (1) and (2) should have different energy and angular dependences. In particular, for the near-threshold production of stable mesons, one has

$$\sigma(pn \rightarrow da_0^0) \sim Q^{3/2}, \quad \sigma(pn \rightarrow df_0) \sim Q^{1/2} \quad (Q = \sqrt{s - m_d - \bar{m}}),$$  \hspace{1cm} (3)

where $Q$ is the energy release in the respective reaction; $\sqrt{s}$ is the total CM energy; and $m_d$ and $\bar{m}$ are the deuteron and meson masses, respectively.

Let us assume that the $f_0 \rightarrow a_0^0$ transition can proceed without isospin conservation. Then, the $a_0$ meson in reaction (1) can be produced in the $s$ wave with respect to the final deuton. As is seen from the table, the initial spin state of nucleons in both reactions $pn \rightarrow da_0^0$ and $pn \rightarrow df_0 \rightarrow da_0^0$ is the same ($S = 1$). Therefore, the $p$-wave amplitude of the main process (1) interferes with the $s$-wave amplitude of the isospin-violating process $pn \rightarrow df_0 \rightarrow da_0^0$. Due to this interference, an asymmetry arises in the forward-backward escape of the $a_0$ meson in reaction (1). In this case, the process with isospin conservation is energetically suppressed in the range of low $Q$ values, as follows from Eqs. (3). For this reason, the angular asymmetry in the $a_0$-meson production may be large near the threshold.

2 Note that, for the $pn \rightarrow d\pi^0$ reaction discussed in [9], the angular asymmetry of the near-threshold cross section is suppressed. This is due to the fact that the process with isospin conservation yields the $d\pi^0$ system in the $s$ wave.
The asymmetry $A$ for reaction (1) is defined as

$$A = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}, \quad \sigma_{\pm} = \frac{d\sigma}{d\Omega}(z = \pm 1), \quad z = \cos \theta,$$

(4)

where $\theta$ is the polar CM angle of $a_0^0$-meson escape, with the polar axis coinciding with the initial beam.

For the numerical estimations of asymmetry $A$, we first consider the $\Lambda_{af}$ vertex determined by the diagram in Fig. 1b for the $a_0^0 \rightarrow f_0$ transition. The $\lambda_{\pi\eta}$ vertex corresponding to the $\pi^0 \leftrightarrow \eta$ transition in this diagram is known from the theoretical analysis of the $\eta \rightarrow 3\pi^0$ reaction (see also (3)). We take $\lambda_{\pi\eta} \approx -5000\, \text{MeV}^2$ as an estimate, which is the average of the theoretical values given in (4). Direct calculation of the diagram in Fig. 1b yields the following result for the contribution $\Lambda_{\pi\eta}$ of the process indicated in Fig. 1b to $\Lambda_{af}$:

$$\Lambda_{\pi\eta} = \frac{\lambda_{\pi\eta} g_{a\pi\eta} g_{f\pi^0\eta^0}}{16\pi^2 m^2} \left( \frac{m^2}{m^2_\pi} \ln \frac{m^2 - m^2_\eta}{m^2_\pi} - \ln \frac{m^2 - m^2_\eta}{m^2_\eta} + i\pi \right) \approx (118 - 186\, i)\, \text{MeV}^2$$

(5)

Here, $m_\eta$ is the $\eta$ mass, $m = 980\, \text{MeV}/c^2$ is the mass of the $a_0$ and $f_0$ mesons, and $g_{a\pi\eta}$ and $g_{f\pi^0\eta^0}$ are the vertices of the $a_0 \rightarrow \pi\eta$ and $f_0 \rightarrow 2\pi^0$ decays. Estimate (6) was obtained with zero pion mass $m_\pi = 0$ and under the assumption that the width of the $a_0$ and $f_0$ mesons with nominal mass $\bar{m}$ are $\Gamma(\bar{m}) \equiv \Gamma_0 = 50\, \text{MeV}/c^2$ and determined only by the decays through the $\pi\eta$ and $\pi\pi$ channels, respectively. Then, $g_{a\pi\eta}^2 = 8\pi m^2/\Gamma_0 g_{\pi\eta}$, $g_{f\pi^0\eta^0}^2 = 8\pi m^2/\sqrt{3}$, where $g_{\pi\eta}$ and $g_{\pi\pi}$ are the relative momenta in the $\pi\eta$ and $\pi\pi$ systems. For the $a_0^0 - f_0$ mixing angle, Eq. (3) gives the estimate $\sin \theta_{af} \approx |\Lambda_{\pi\eta}/\bar{m}\Gamma_0| \approx 0.0045$.

The mechanism of external mixing due to the $K\bar{K}$ decay channel, discussed in Ref. [13] and in recent paper [14], shows that the kinematic isospin violation due to the difference in masses of the $K^\pm$ and $K^0$ mesons is large and considerably stronger than that due to the $\pi-\eta$ mixing. At the same time, the strong isospin violation is concentrated in a narrow range of the $a_0$-meson masses near the thresholds of the decays through the $K\bar{K}$ channels.

The vertex $\Lambda_{KK}$ corresponding to the $a_0^0 \leftrightarrow f_0$ transition (Fig. 1c) has the form

$$\Lambda_{KK}(\bar{m}) = \left( \frac{g_{aKK} g_{fKK}^*}{32\pi} \right) \left( \sqrt{\frac{m - 2m_{K^+}}{m_{K^+}}} + i0 \right) - \left( \sqrt{\frac{m - 2m_{K^0}}{m_{K^0}}} + i0 \right),$$

(6)

where $m_{K^+} = 493.7\, \text{MeV}/c^2$ and $m_{K^0} = 497.7\, \text{MeV}/c^2$ are the masses of, respectively, charged and neutral kaons; $m_{K} = (m_{K^+} + m_{K^0})/2$; and $g_{aKK}$ and $g_{fKK}$ are the vertices of the $a_0 \rightarrow K\bar{K}$ and $f_0 \rightarrow K\bar{K}$ decays, respectively. In addition, $g_{aKK}^2 = 2g_{aK^+K^-}$, $g_{fKK}^2 = 2g_{fK^+K^-}$, $g_{aK^0K^0} = -g_{aK^+K^-}$, and $g_{fK^0K^0} = g_{fK^+K^-}$. To numerically estimate $\Lambda_{KK}$, we set $g_{aKK} = g_{a\pi\eta}$, in agreement with the experimental restriction $g_{aKK}^2/g_{a\pi\eta}^2 = 0.91\pm0.11$ [14]. We also assume that $g_{fKK}^2 = g_{aKK}^2$ and $g_{aK^+K^-} = -g_{fK^+K^-}$ (the relative signs of the $a_0$- and $f_0$-meson decay vertices are consistent with the predictions of SU(3) symmetry) [14]. The function $\Lambda_{KK}(\bar{m})$ is shown in Fig. 2. This function sharply changes near

---

3 It is assumed that the $a_0$ is a triplet and $f_0$ is a singlet and belong to the same scalar meson octet.
the thresholds of the $K^+K^-$ and $K^0\bar{K}^0$ systems within the mass intervals comparable to the $2m_{K^0} - 2m_{K^+} \approx 8\text{ MeV}/c^2$ difference, which is considerably smaller than the width of the $a_0$ and $f_0$ mesons. It is seen that $|\Lambda_{KK}| \gg |\Lambda_{\pi\eta}|$ near the $K\bar{K}$ thresholds.

Let us assume that the signal from the $a_0^0$-meson production is detected by identifying the $\pi^0\eta$ final state. In this case, the use of perturbation theory for the isospin-violating interaction gives three diagrams dominating the cross section for the process. They are shown in Fig. 3. The $M_1$ diagram corresponds to the process with isospin conservation. The $M_2$ and $M_3$ diagrams are first-order corrections in the isospin-violating interaction to the diagram $M_1$. The $M_2$ diagram is irrelevant the $a_0$-meson production but does contribute to the $s-p$ interference under discussion. This contribution corresponds to the interference of diagrams $M_2$ and $M_1$ through the $a_0^0-f_0$ mixing mechanism (Fig. 1b). The respective amplitudes $M_{1,2,3}$ are

\begin{equation}
M_1 = M_a G_a g_{a\pi\eta}, \quad M_2 = M_f G_f \frac{g_{f_a\pi^0}^0 \Lambda_{\pi\eta}}{m_0^2 - m_\pi^2}, \quad M_3 = M_f G_f \Lambda_{af} G_a g_{a\pi\eta},
\end{equation}

where $\Lambda_{af} = \Lambda_{dir} + \Lambda_{\pi\eta} + \Lambda_{KK}$ is the vertex corresponding to the $a_0^0 \leftrightarrow f_0$ transition. The vertex $\Lambda_{dir}$ of direct interaction (Fig. 1a) was not taken into account; i.e., $\Lambda_{af} = \Lambda_{\pi\eta} + \Lambda_{KK}$.

In Eqs. (7), $G_a$ and $G_f$ are the propagators of the $a_0$ and $f_0$ mesons, respectively:

\begin{equation}
G_a = G_f = \frac{1}{2m} \cdot \frac{1}{m - m + i\Gamma(m)/2}, \quad \Gamma(m) = \Gamma_0 + \frac{g_{aKK}^2}{8\pi m^2} \sqrt{m_K(m - 2m_K) + i\delta},
\end{equation}

where the width $\Gamma(m)$ takes into account that the resonances may decay through the $K\bar{K}$ channel \( \Box \) and $m$ is the mass of the $\pi\eta$ system. In Eqs. (7), $M_a$ and $M_f$ denote the amplitudes of production of the $a_0$ and $f_0$ mesons in reactions (1) and (2), respectively. These amplitudes can be estimated from the diagrams of the impulse approximation (Fig. 4). With allowance made only for the \( s \)-wave component of the deuteron wave function (WF), the expressions for $M_a$ and $M_f$ antisymmetrized over the initial nucleons have the form

\begin{equation}
M_a = g_{aNN} \sqrt{m_N} (u(q_1) - u(q_2)) X, \quad M_f = g_{fNN} \sqrt{m_N} (u(q_1) + u(q_2)) X.
\end{equation}

Here, $g_{aNN}$ and $g_{fNN}$ are the vertices for the $a_0$- and $f_0$-meson coupling to the nucleon; $m_N$ is the nucleon mass; $u(q)$ is the deuteron WF; $q_1(q_2)$ is the relative momentum in the deuteron vertex corresponding to the emission of an $a_0^0$ or $f_0$ meson by the initial proton (neutron); and $X = \varphi_p^T \sigma_2 \epsilon \cdot \sigma \varphi_n$ is the spin factor, where $\varphi_p$ and $\varphi_n$ are the proton and neutron spinors, respectively, and $\epsilon$ is the deuteron polarization vector. Let $p$ and $k$ be the CM 3-momenta of the initial proton and the final $\pi\eta$ system, respectively. For $m \simeq m$, one has $k \ll p$ and $q_{2,2}^2 \approx p^2 \pm (E_N/m_N)(p \cdot k)$ near the threshold $Q = \sqrt{s} - m_d - m \simeq 0$ of the reaction (1), where $E_N \approx m_N + m/2$ is the total nucleon CM energy. Then, Eqs. (9) take the form

\begin{equation}
M_a = 2 g_{aNN} \sqrt{m_N} \frac{du(p)}{dp^2} \frac{E_N}{m_N} (p \cdot k) \cdot X, \quad M_f = 2 g_{fNN} \sqrt{m_N} u(p) \cdot X.
\end{equation}

It follows from these expressions that at small $k$ values the amplitude $M_a$ of $a_0$-meson production is much smaller than the amplitude $M_f$ of $f_0$-meson production. Note that the
relative contributions of the $p$- and $s$-wave amplitudes $M_a$ and $M_f$ to the cross section for reaction (1) depend on the width of the $a_0$ and $f_0$ mesons and on the restrictions on the mass range of the final $\pi\eta$ system. In what follows, we set $g_{aNN} \equiv g_{fNN}$ in Eq. (10). For the deuteron WF with momenta $p \sim 1$ GeV/c, we take $u(p) \sim p^{-n}$. The Hulthen WF corresponds to $n = 4$; i.e., $du(p)/dp^2 = -2p^{-2}u(p)$. This approximation for the deuteron WF and the impulse approximation for the amplitudes $M_a$ and $M_f$ cannot give reliable estimates for the absolute values of the cross sections and are used only for estimating the asymmetry $A$. The differential cross section for the reaction $pn \rightarrow d\pi^0\eta$ can be written as

$$\frac{d\sigma}{d\Omega} = N \int_{m_{\text{min}}}^{m_{\text{max}}} |M_1 + M_2 + M_3|^2 k \, dm.$$  \hspace{1cm} (11)

The $m$ dependence of the amplitudes $M_{1,2,3}$ is taken into account for $\Lambda_{KK}$ (Eq.(3)); $G_a$, $G_f$ and $\Gamma(m)$ (Eqs.(8)); and for the relative momentum $k = \sqrt{2\mu(Q + \bar{m} - m)}$, where $\mu = m_d\bar{m}/(m_d + \bar{m})$. To a common factor, the amplitudes $M_a$ and $M_f$ for the unpolarized particles can be written as $M_a = -2(E_{N}/m_N)(k/p)z$ and $M_f = 1$. The normalization constant $N$ in Eq. (11) contains weakly varying factors, and its magnitude is of no interest to us. The quantities $p$ and $E_N$ are calculated at the threshold; i.e., $E_N = m_N + \bar{m}/2$ and $p = \sqrt{\bar{m}(m_N + \bar{m}/4)} \simeq 1$ GeV/c.

The width of the mass interval $(m_{\text{min}}, m_{\text{max}})$ of the $\pi^0\eta$ system (more precisely, its lower limit $m_{\text{min}}$), in which the $a_0$ meson is detected, is an important factor for estimating asymmetry $A$ (1). When the mass of the $\pi^0\eta$ system decreases (below the nominal mass $\bar{m}$), the momentum $k$ increases, resulting in both the enhancement in the $p$-wave amplitude $M_a$, as compared to the $s$-wave amplitude $M_f$, and the dependence of the asymmetry $A$ on $m_{\text{min}}$. In what follows, we specify integral (11) between the limits $m_{\text{max}} = Q + \bar{m}$ (kinematic boundary) and $m_{\text{min}} = \bar{m} - C(\Gamma_0/2)$, where $C$ is a variable parameter. The calculated (by Eqs. (3) and (11)) asymmetry $A$ of the $\pi^0\eta$-system production is shown in Fig. 5 as a function of energy release $Q$. The calculations were carried out for two lower limits (corresponding to $C = 1$ and $C = 2$) on the mass of the $\pi^0\eta$ system. A decrease in the effect with increasing parameter $C$ is due to the increase in the role of the main (isospin-concerning) process of $a_0$-meson production in the $p$ wave. Note that the contribution of the $p$-wave amplitude to the cross section dominates over the $s$-wave contribution in both variants if the $a_0$-meson width is taken into account. For this reason, the relative contribution of the $s-p$ interference and, hence, the asymmetry, decrease upon enhancing the $p$ wave. The dashed (dotted) curves in Fig. 5 correspond to the asymmetry calculations taking account of only one mixing mechanism given by the diagram in Fig. 1b (Fig. 1c), i.e., for $\Lambda_{df} = \Lambda_{\pi\eta}$ ($\Lambda_{df} = \Lambda_{KK}$). The diagram $M_2$ (Fig. 2) is automatically ignored in calculating the dotted curves.

As is seen from our calculations in Fig. 5, the asymmetry in the nearthreshold $a_0$-meson production is rather large (about $8 \div 15\%$), which enables one to believe that it can be experimentally observed. Note that nonresonance production of the $\pi^0\eta$ system in the reaction $pn \rightarrow d\pi^0\eta$ looks like a background to the reaction $pn \rightarrow da_0^0 \rightarrow d\pi^0\eta$. Fortunately this background gives no contribution to the discussed asymmetry $A$, see, e.g.
Ref. \[8\] for more details.

Note in conclusion that our estimates of the asymmetry may be improved. This is primarily true for the calculation of the amplitudes $M_a$ and $M_f$ of the $a_0$- and $f_0$-meson production. An approach with the inclusion of diagrams describing intermediate rescattering processes (see, e.g., \[13\]) seems to be more reliable than the inclusion of the pole diagrams (Fig. 4) that depend on the behavior of the deuteron WF at high momenta $\sim 1 \text{ GeV/c}$, where the WF is poorly known.

There is also a possibility to test the $a_0 - f_0$ mixing in the reaction $dd \rightarrow a_0^0 \, ^4\text{He}$, which is forbidden by isospin conservation. Note that the analogous forbidden reaction $dd \rightarrow \pi^0 \, ^4\text{He}$ was not observed. However, it could proceed through $\eta-$ and $\eta'$-production reactions due to $\pi-$-$\eta$- and $\pi-$-$\eta'$-mixing mechanisms \[16\]. We expect that the $a_0$-production process in the nearthreshold region has some privileges in comparison to the $\pi^0$ production. Note that the final $a_0^0$ meson may be produced in $s$ wave with respect to $^4\text{He}$ from the initial $s$-wave $dd$ state. That is why the formation of $^4\text{He}$ wave function should not be accompanied by the rearrangement of the orbital motion of nucleons. In the case of $\pi^0$ production the situation is opposite: $\pi^0$ meson is produced in $s$ wave, initial $dd$ system is in $p$ wave. That is why the formation of $^4\text{He}$ wave function from initial $dd$ state is accompanied by rearrangement of the orbital motion of nucleons. Thus, the process of $\pi^0$ production is dynamically suppressed versus $a_0$ case.

We are grateful to all participants of the Workshop on the ANKE-Spectrometer Program held at the ITEP (July 2000) and particularly to K.G.Boreskov and V.P.Chernyshev for helpful discussions and interest in this study. We also thank N.N. Achasov for useful discussion and information about his recent papers, related to this subject.

References

[1] N.A. Tornquist, Phys.Rev.Lett., 49, 624 (1982).
[2] V.V. Anisovich, hep-ph/9712504, 1997.
[3] N.N. Achasov and G.N. Shestakov, Usp.Fiz.Nauk 161, 53 (1991) [Sov.Phys.Uspr. 34, 471 (1991)]; Usp.Fiz.Nauk 142, 361 (1984) [Sov.Phys.Uspr. 27, 161 (1984)]; N.N. Achasov and G.N. Shestakov, Z.Phys.C 41, 309 (1988); N.N. Achasov, Nucl.Phys.A 675, 279 (2000); hep-ph/9910540.
[4] V.N. Gribov, LU-TP-91-7; ORSAY Lectures on Confinement 1, LPT HE-ORSAY-92-60; V.N. Gribov, Eur.Phys.J. C 10, 91 (1999); F.E. Close, Yu.L. Dokshitzer, V.N. Gribov, et al., Phys.Lett.B 319, 291 (1993).
[5] N.N. Achasov, S.A. Devyanin and G.N. Shestakov, Phys.Lett.B 88, 367 (1979); Yad.Fiz. 33, 1337 (1981) [Sov.J.Nucl.Phys. 33, 715 (1981)].
[6] D. Barberis, F.G. Binon, F.E. Close et al., hep-ex/0007013, 2000.
[7] F.E. Close and A. Kirk, Phys.Lett.B 489, 24 (2000).
[8] G.A. Miller, B.M.K. Nefkens and I. Slaus, Phys.Rep. 194, 1 (1990).

[9] J.A. Niskanen, nucl-th/9809009, Few Body Syst. 26, 241 (1999); Acta Phys. Polon. B 31, 2683 (2000).

[10] S.A. Coon and M.D. Scadron, Phys.Rev.C 51, 2923 (1995); S.A. Coon, B.H.J. McKellar and M.D. Scadron, Phys.Rev.D 34, 2784 (1986); Ll. Ametller, C. Ayala and A. Bramon, Phys.Rev.D 30, 674 (1984).

[11] B. Kerbikov and F. Tabakin, nucl-th/0006017, 2000; Phys.Rev.C 62, 064601 (2000).

[12] Review of Particle Physics. Eur.Phys.J. C 3, 1 (1998).

[13] S. Teige, B.B. Brabson, R.R. Crittenden, et al., Phys.Rev.D 59, 012001 (1999).

[14] S.M. Flatte, Phys.Lett., 63, 224 (1976).

[15] V.Yu. Grishina, L.A. Kondratyuk, E.L. Bratkovskaya, et al., nucl-th/0007074, 2000; submitted to EJP A.

[16] S.A. Coon and B.M. Preedom, Phys.Rev.C 33, 605 (1986).
Figure captions

**Fig. 1.** Different types of interactions resulting in the $a_0^0 - f_0$ mixing: (a) direct (or contact), (b) due to the virtual $\pi^0 \leftrightarrow \eta$ transition, and (c) due to the mass difference between $K^\pm$ and $K^0$ mesons.

**Fig. 2.** Vertex function $\Lambda_{K\bar{K}}$ vs. $a_0$-meson mass $m$. The solid, dashed and dotted curves correspond to its absolute value, real part, and imaginary part, respectively. The dashed (dotted) curve coincides with the solid curve for $m < 2m_{K^+} (m > 2m_{K^0})$.

**Fig. 3.** Zero- and first-order isospin-violating diagrams of the $pn \rightarrow d\pi^0\eta$ process.

**Fig. 4.** Diagrams for the $a_0(f_0)$-meson production in impulse approximation.

**Fig. 5.** Plots of the asymmetry $A$ of $a_0^0$-meson production in reaction (1) vs. the energy release $Q$ (3). Curves 1 and 2 correspond to two lower limits (specified by $C = 1$ and $C = 2$ (see text)) on the interval of $a_0^0$ masses. The dashed (dotted) curves are calculated by taking account of the $a_0^0 - f_0$ mixing through only the $\pi^0 \leftrightarrow \eta$ transition ($K\bar{K}$ decay channel). The solid curves are obtained with allowance made for both mixing mechanisms.
Fig. 1
Fig. 2
Fig. 3

Fig. 4