Solar energetic particles show a rich variety of spectra and relative abundances of many ionic species and their isotopes. A long-standing puzzle has been the extreme enrichments of $^{3}\text{He}$ ions. The most extreme enrichments are observed in low-fluence, the so-called impulsive, events which are believed to be produced at the flare site in the solar corona with little scattering and acceleration during transport to the Earth. In such events, $^{3}\text{He}$ ions show a characteristic concave curved spectra in a log–log plot. In two earlier papers of Liu et al., we showed how such extreme enrichments and spectra can result in the model developed by Petrosian and Liu, where ions are accelerated stochastically by plasma waves or turbulence. In this paper, we address the relative distributions of the fluences of $^{3}\text{He}$ and $^{4}\text{He}$ ions presented by Ho et al., which show that while the distribution of $^{4}\text{He}$ fluence (which we believe is a good measure of the flare strength) like many other extensive characteristics of solar flare is fairly broad, the $^{3}\text{He}$ fluence is limited to a narrow range. These characteristics introduce a strong anticorrelation between the ratio of the fluences and the $^{4}\text{He}$ fluence. One of the predictions of our model presented in the 2006 paper was the presence of steep variation of the fluence ratio with the level of turbulence or the rate of acceleration. We show here that this feature of the model can reproduce the observed distribution of the fluences with very few free parameters. The primary reason for the success of the model in both fronts is because fully ionized $^{3}\text{He}$ ion, with its unique charge-to-mass ratio, can resonantly interact with plasma modes not accessible to $^{4}\text{He}$ and be accelerated more readily than $^{4}\text{He}$. Essentially in most flares, all background $^{3}\text{He}$ ions are accelerated to few MeV/nucleon range, while this happens for $^{4}\text{He}$ ions only in very strong events. A much smaller fraction of $^{4}\text{He}$ ions reach such energies in weaker events.

**Key words:** acceleration of particles – plasmas – Sun: flares – Sun: particle emission – turbulence – waves

**Online-only material:** color figures

1. INTRODUCTION

Solar flares are excellent particle accelerators. Some of these particles on open field lines are observed as solar energetic particles (SEPs) at 1 AU or produce type III and other radio radiation. Those on closed field lines can be observed by the radiation they produce as they interact with solar plasma and fields. Electrons produce nonthermal bremsstrahlung and synchrotron photons in the hard X-ray and microwave range, while protons (and other ions) excite nuclear lines in the 1–7 MeV range or may produce higher energy gamma rays via $\pi^{0}$ production and its decay. It appears that stochastic acceleration (SA) of particles by plasma waves or turbulence plays an important role in the production of high-energy particles and consequent plasma heating in solar flares (e.g., Ramaty 1979; Möbius et al. 1980, 1982; Hamilton & Petrosian 1992; Miller 1997; Petrosian & Liu 2004, hereafter PL04). This theory was first applied to the acceleration of nonthermal electrons in solar flares (Miller & Ramaty 1987; Hamilton & Petrosian 1992). It has been shown that it can produce many of the observed radiative signatures such as broadband spectral features (Park et al. 1997; PL04) and the commonly observed hard X-ray emission from the tops of flaring loops (Masuda et al. 1994; Petrosian & Donaghy 1999). It is also commonly believed that the observed relative abundances of ions in SEPs favor a SA model (e.g., Mason et al. 1986; Mazur et al. 1995), and more recent observations have confirmed this picture (see Mason et al. 2002a, 2002b; Reames et al. 1994, 1997; Miller 2003). One of the most vexing problem of SEPs has been the enhancement of $^{3}\text{He}$ ions in the so-called impulsive or $^{3}\text{He}$-rich events, which sometimes can be 3–4 orders of magnitude above the photospheric value. There have been many attempts to explain this enhancement. Most of the proposed models, except the Ramaty & Kozlovsky (1974) model based on spallation (which has many problems), rely on resonant wave–particle interactions and the unique charge-to-mass ratio of $^{3}\text{He}$ (see, e.g., Ibragimov & Kocharov 1977; Fisk 1978; Temerin & Roth 1992; Miller & Viñas 1993; Zhang 1995; Paesold et al. 2003). Most of these models assume the presence of some particular kind of waves which preferentially heats $^{3}\text{He}$ ions to a higher temperature than $^{4}\text{He}$ ions, which then become seeds for subsequent acceleration by some (usually) unspecified mechanism (for more detailed discussion, see Petrosian 2008 and Wiedenbeck et al. 2009). None of these earlier works did a comparison of model spectra with observations.

In two more recent papers, Liu et al. (2004, 2006; LPM04 and LPM06, respectively) have demonstrated that a SA model by parallel propagating waves can explain both the extreme enhancement of $^{3}\text{He}$ and can reproduce the observed $^{3}\text{He}$ and $^{4}\text{He}$ spectra. In LPM06, it was shown that the relative fluences of these ions, and to a lesser extent their spectral indexes, essentially in most flares, all background $^{3}\text{He}$ ions are accelerated to few MeV/nucleon range, while this happens for $^{4}\text{He}$ ions only in very strong events. A much smaller fraction of $^{4}\text{He}$ ions reach such energies in weaker events.

6 In addition, there is charge-to-mass ratio dependent enhancement relative to the photospheric values of heavy ions in SEPs, and in a few flares gamma-ray line emission also points to anomalous abundance pattern of the accelerated ions (Share & Murphy 1998; Hua et al. 1989). We will not be dealing with these anomalies in this paper.

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depend on several model parameters so that in a large sample of events one would expect some dispersion in the distributions of fluences and spectra. Ho et al. (2005; Ho05) analyzed a large sample of events and provide distributions of $^3$He and $^4$He fluences and the correlations between them. Our aim here is to explore the possibility of explaining these observations by the above-mentioned dependence of the fluences on the model parameters. In particular, we would like to explain the observations reproduced in Figure 1. The two striking features of these data are (1) that there is no or very weak correlation between the two fluences (left panel) and (2) that the $^3$He fluence distribution appears to be relatively narrow (and follows a lognormal distribution) while $^4$He distribution is much broader and may have a power-law distribution in the middle of the range, where the observational selection effects are unimportant (middle panel). These two basic aspects of the observations introduce a strong anticorrelation between the fluence ratio of $^3$He to $^4$He fluence (right panel). Often the SEP events are divided into two classes; impulsive-high enrichment and gradual-normal abundance classes. However, as evident from the left panel of the above figure there is a continuum of enrichment extending over many orders of magnitude.

In the next section, we describe some of the model characteristics that can explain these observations. These features were introduced in LPM06. Here, we describe the importance of these features in describing the observations summarized in Figure 1. For convenience of the reader, we reproduce some of the relevant figures from LPM06. In Section 3, we compare the model predictions with the observations, specifically the distributions of the fluences. A brief summary and conclusion are given in Section 4.

2. MODEL CHARACTERISTICS

The model used in LPM04 and LPM06, which successfully described the enrichment and spectra in several flares, has several free parameters. As usual we have the plasma parameters density $n$, temperature $T$, and magnetic field $B_0$. It turns out that the final results are insensitive to the temperature as long as it is higher than $2 \times 10^6$ K (see Figure 4 below), which is the case for flaring coronal loops. It also turns out that only a combination of density and magnetic field ($\sqrt{n}/B_0$) comes into play in the acceleration model. We express this as the ratio of plasma to gyrofrequency of electrons, $\alpha = \omega_{pe}/\Omega_e$, which is related to the Alfvén velocity in unit of speed of light, $v_A = \delta^{1/2}/\alpha$, where $\delta = m_e/m_p$ is the ratio of the electron to proton masses. So, in reality, we have only one effective free plasma parameter $\alpha$ or $\beta_A$. On the other hand, several parameters are required to describe the spectrum of the turbulence. Following the above papers, we assume broken power laws for the two relevant plasma modes, the proton cyclotron (PC) and the He cyclotron (HeC), with an inertial range $k_{\text{max}} < k < k_{\text{min}}$, and similar power-law indexes $q$ and $q_h$ in and beyond the inertial range, respectively. The only difference between the two branches is that the wave numbers $k_{\text{max}}$ and $k_{\text{min}}$ for the PC mode are two times higher than those for the HeC mode. Finally, there is the most important parameter related to the total energy density of turbulence, $\mathcal{E}_0$, which determines both the rate of acceleration and, when integrated over the volume of the source region, determines the intensity or the strength of the event. This parameter is the characteristic timescale $\tau_p$, or its inverse the rate defined as (see, e.g., Pryadko & Petrosian 1997) \footnote{The $^4$He distribution shows a weak sign of bimodality but this is not statistically significant. In this paper, we will ignore this feature.}

$$\tau_p^{-1} = \frac{\pi}{2} \Omega_e \left[ \frac{4\mathcal{E}_0}{B_0^2/8\pi} \right] \quad \text{with} \quad \mathcal{E}_0 = \frac{(q-1)\mathcal{E}_{\text{tot}}}{(k_{\text{min}}/\Omega_e)^{1-q}},$$

for each mode. The factor of 4 arises from having two branches (PC and HeC) and two propagation directions of the waves (see LPM06 for details).

As shown in LPM04 and LPM06 papers, the main difference between the acceleration process of $^3$He and $^4$He is in the difference between their acceleration rates or timescales ($\tau_a$). The other relevant timescales, namely the loss ($\tau_{\text{loss}}$) and escape $^8$.

\footnote{In LPM06, we also have an index $q_l$ describing the power law below the inertial range which is of minor consequence. For all practical purposes, we can assume a sharp cutoff below $k_{\text{min}}$, which means $q_l \to \infty$.}

\footnote{Note that in Pryadko & Petrosian (1997), the wave vector is dimensionless expressed in units of $\Omega_e/c$.}
(Tesc) times are essentially identical for the two ions (e.g., see the left panel of Figure 7 of LPM06). The acceleration timescales are mainly different at low energies (typically below 1 MeV nucleon$^{-1}$), where the acceleration time of $^4$He is longer (by 1–2 orders of magnitude). As a result, at these low energies $^4$He is accelerated more comparably or longer than the loss time, which makes it difficult to accelerate $^3$He ions. Most of $^4$He ions are piled up below some energy (roughly where $\tau_a = \tau_{\text{loss}}$), and only a few of them are accelerated into the observable range (e.g., see the right panel of Figure 7 of LPM06). However, because the acceleration times scale as $\tau_p$ while the loss time does not, for higher level of turbulence (larger $\mathcal{E}_\alpha$), the acceleration time may fall below the loss time so that $^3$He ions can be then accelerated more readily (see Figure 3 below). On the other hand, essentially independent of values of any of the above parameters, the $^3$He acceleration time at all energies, in particular at low energies, is always far below its loss time so that in all cases (except for very high densities or very low values of $\tau_p^{-1}$) $^3$He ions are accelerated easily to high energies. The relative values of the escape and acceleration times (for both ions) determine their high-energy spectral cutoffs.

Figure 2 shows variation with energy of acceleration times of $^3$He (thick lines) and $^4$He (thin lines) and their dependence on parameters $k_{\text{min}}, \alpha$, and $q$. The remaining parameters $q_b$ and $k_{\text{max}}$ only affect the slope of the low-energy end of $^3$He, which does not affect the spectra noticeably. It is evident that the general behavior of the acceleration timescales described above (consisting of a low- and a high-energy monotonically increasing branches with a declining transition in between) is present in all models. These features change only quantitatively and often by small amounts. As expected lowering $k_{\text{min}}$ decreases the acceleration times at the high-energy branch (left panel). This is because the lower $k_{\text{min}}$ waves interact resonantly with higher energy ions. On the other hand, a lower value of $\alpha$ (or larger Alfvén velocity or magnetization) decreases the times at the low-energy branch (middle panel). Steeper spectra in the inertial range enhance the dependence on the energy of turbulence (see the dependence of $\tau_p^{-1}$ on $q$ in Equation (1)) and decrease the overall acceleration timescales (right panel).

Note that in this and subsequent figures, $k_{\text{max}}$ is in units of $\Omega_p/c$ so that $k_{\text{max}} = 2\alpha \delta^{-1/2}$ in the labels means an actual $k_{\text{max}} = 2\Omega_p/v_A = \sqrt{2p}/r_{g,p}$, where $\beta_p = 2(v_{th,p}/v_A)^2$ is the plasma beta, and $v_{th,p} = k_B T/m_p$ and $r_{g,p} = v_{th,p}/\Omega_p$ are the proton thermal velocity and gyroradius. The scale of $k_{\text{max}}$ is clearly beyond the MHD regime (in the MHD regime, the wave frequency $\omega = v_A k \ll \Omega_p$), but is below the proton gyroradius for the chosen parameters.

Using these acceleration rates one can calculate spectra of the two ions (as in LPM04 and LPM06) for a range of parameters. Figure 3 shows three sets of spectra of escaping ions (two from LPM06 and one new) where we vary $k_{\text{min}}, \alpha$, and $\tau_p^{-1}$. In each panel, the solid lines are for the fiducial model ($\alpha = 0.5, k_{\text{max}} = 2\alpha \delta^{-1/2} = 10k_{\text{min}}, q = 2$, and $q_b = 4$) chosen to fit the spectra observed by ACE/ULEIS for 1999 September 30 event. The spectral variations here reflect the above-described variations of the acceleration timescales. Lower $k_{\text{min}}$ (or larger inertial range) yields a larger tail for both ions (left panel). Variation of $\alpha$ has a similar and smaller effect on $^3$He spectra, but it dramatically affects the $^4$He spectra; for $\alpha \sim 1$ essentially there is no $^4$He acceleration, but the model with $\alpha \sim 1/4$ accelerates a large number of $^4$He ions beyond 0.1 MeV nucleon$^{-1}$ and into the observable range (middle panel). This effect is even more pronounced for increasing values of $\tau_p^{-1}$, where a factor of few increase in the general rate of acceleration (or the level of turbulence) causes a large increase of the fluence of $^4$He (right panel), because, as stated above, its acceleration time becomes shorter than its loss time even at low energies. All these spectra show the following general characteristic. While most $^3$He ions are accelerated to high energies for essentially all model parameters appropriate for solar coronal conditions and reasonable level of turbulence, $^4$He ions show a characteristic low-energy bump with a nonthermal hard tail. The lower energy bump is below the observation range except for low $\alpha$ and high values of $\tau_p^{-1}$. Since a high level of turbulence is expected for stronger events (with high-SEP fluences), this means that we get smaller $^3$He/$^4$He flux or fluence ratios for stronger events. Note that the spectra in such cases may not agree with observations but this is not troublesome, because as is well established, the stronger events (the so-called gradual events) are associated with larger and faster coronal mass ejections (CMEs) and perhaps stronger shocks which most likely will modify the depicted spectra of ions escaping the corona. The spectra shown here should be considered as seeds for such further acceleration during the transport from the lower corona to the Earth, which
Figure 3. Dependence of spectra of accelerated $^4$He and $^3$He escaping the solar flare site on $k_{\text{min}}$ (left), $\alpha$ (middle), and $\tau_p^{-1}$ (right; $\tau_p^{-1}$ in units of $\tau_p^{-1} = 0.0055$ s$^{-1}$). The lines are labeled with the corresponding numbers of each parameter. In each case, the solid lines are for the fiducial model with $\alpha = 0.5$, $k_{\text{max}} = 2\alpha \delta^{-1/2} = 10 k_{\text{min}}$, $q = 2$, and $q_{\delta} = 4$ that is chosen to fit the data point shown for the 1999 September 30 event observed by ACE. Note that for a better indication of what energy particles dominate the spectra in a log–log plot, we plot $E \times F$, the particle (energy/nucleon) times fluence, vs. $E$ (energy/nucleon). Left and middle panels from LPM06.

(A color version of this figure is available in the online journal.)

Figure 4. Variation of the accelerated $^3$He to $^4$He fluence ratio (at $E = 1$ MeV nucleon$^{-1}$) with background plasma temperature $T$ (left) and $\tau_p^{-1}$ (middle and right) for several values of other specified model parameters. The lines are labeled with the corresponding numbers of each parameter. In each case the open circle stands for the model that fits spectra of the 1999 September 30 event, and the solid lines are for the fiducial model with $\alpha = 0.5$, $k_{\text{max}} = 2\alpha \delta^{-1/2} = 10 k_{\text{min}}$, $q = 2$, and $q_{\delta} = 4$. Note the weak dependence on the temperature for $T > 2 \times 10^6$ K and a strong dependence on $\tau_p^{-1}$ for all model parameters which saturates at chromospheric values of the ratio. The horizontal dot–dash line shows the highest ratio observed so far (see Figure 1, left). From LPM06.

(A color version of this figure is available in the online journal.)

becomes more likely, and is expected to change the above spectra more significantly, for more energetic events. This is consistent with the association of narrow CMEs and/or jets in some of the larger $^3$He-rich events (Wang et al. 2006; Pick et al. 2006, Kahler et al. 2001). The subsequent interaction in the associated CME shocks could modify the spectral form of the population escaping the turbulent coronal site without affecting the relative abundances. This combination of processes could cause a blurring of the two classes of SEP events (impulsive and highly enriched and gradual with normal abundances), leading to the continuum of fluences and abundance enrichments shown in Figure 1 (see also, e.g., Cliver 2008). The above scenario implies that the main process determining the abundance ratios is the acceleration by turbulence in the flaring loops. For weak (low fluence and perhaps short) events the resultant SEP spectra are unaffected during the transport to Earth, while for the stronger (high fluence, longer lasting) events the flare-generated spectra are modified by subsequent interactions in CME shocks while the relative abundances are that of the seed particles injected at the flare site.

From the spectra, we can calculate the ratio of $^3$He to $^4$He fluences for different models which could be then compared with the observed ratios shown in Figure 1. Inspection of observed spectra indicates that a representative ion energy would be 1 MeV nucleon$^{-1}$. In Figure 4, we show the variation of this ratio with temperature (left panel) and $\tau_p^{-1}$ (middle and right panels) for several values of other important parameters. As evident, this ratio is most sensitive to the value of $\tau_p^{-1}$, which represents the general rate of acceleration or the level of turbulence. The ratio can change from the highest observed value ($\sim 30$) to near photospheric value ($\sim 2 \times 10^{-4}$) for only a factor of 30 change in $\tau_p^{-1}$. It is natural to expect higher level of turbulence generation (i.e., a larger value of $\tau_p^{-1}$) in stronger events. Therefore, this predicted anticorrelation is in agreement with the general trend of observation shown in Figure 1 (right panel), if the strength of an event is measured by the observed fluence of $^4$He ions and
most other ions such as carbon, nitrogen, and oxygen.\textsuperscript{10} This seems reasonable and calls for more quantitative comparison with observations and model prediction. In the next section, we present one such comparison.

3. DISTRIBUTIONS OF FLUENCES

We have seen that the general observed behavior of the ratio of the fluences is similar to the model predictions. In this section, we try to put this result on a firmer quantitative footing by considering the observed distributions of the fluences of both ions as shown in Figure 1 (middle panel). As shown in the left panel of Figure 1, except for the minor truncation at high values of \(^3\)He fluence (denoted here by \(F_3\)), the observed distribution of \(^3\)He fluences (denoted here by \(F_3\)) seems to be almost bias free and not significantly affected by the observational selection effects. For example, there are well defined and steep decline both at the high and low fluences away from the peak fluence value of \(F_0 \approx 10^{3.7}\) particles cm\(^{-2}\) MeV nucleon\(^{-1}\). This is not what one would expect if the data suffered truncation due to a low-observation threshold. In such a case, one would observe a distribution increasing up to the threshold followed by a rapid cutoff below it. Our model results described above also seem to predict this observed behavior. As stressed in previous section, the \(^3\)He spectra and fluxes appear to be fairly independent of model parameters because essentially under all conditions most \(^3\)He ions are accelerated. Thus, we believe that it is safe to assume that the observed \(^3\)He distribution is a true representation of the intrinsic distribution (as produced on the Sun). This distribution can be fitted very nicely with a lognormal expression. If we define the logs of the fluences and their ratio as

\[
\text{LF}_3 \equiv \ln(F_3/F_0), \quad \text{LF}_4 \equiv \ln(F_4/F_0), \quad \text{LR} \equiv \ln(F_3) - \ln(F_4),
\]

then from fitting the observed distribution of \(^3\)He by a lognormal form we obtain\textsuperscript{11}

\[
\psi_3(\text{LF}_3) = \phi_0 \exp \left( \frac{\text{LF}_3}{\sigma_3} \right)^2 \quad \text{with} \quad \sigma_3 \approx 0.22,
\]

which is shown in the right panel of Figure 5.

Using this distribution we now derive the distribution of \(^4\)He fluences, \(\psi_4(\text{LF}_4)\). For this we use the model predicted relationship between the two fluences as shown in Figure 4 above. We will use the two panels of this figure showing the dependence of the log of the fluence ratio LR on \(\tau_p^{-1}\). It turns out that most of these curves can be fitted by a simple three parameter function:

\[
\text{LR} = \ln R_0 + \frac{A}{\ln \left( \frac{\tau_p^{-1}}{\tau_{p0}^{-1}} \right)}.
\]

The left panel of Figure 5 shows fits to the curves in the right panel of Figure 4 with the indicated values of the fitting parameters \(A, R_0\), and \(\tau_{p0}^{-1}\) (which is not the same as the \(\tau_{p,0}^{-1} = 0.0055 \text{ s}^{-1}\) in Figure 3). We shall use this relation to transfer the \(^3\)He fluences and distribution to those of \(^4\)He.

For a given value of \(\tau_{p}^{-1}\), the number of events with \(^3\)He log-fluences between \(\text{LF}_3\) and \(\text{LF}_3 + d(\text{LF}_3)\) (i.e., \(\psi_3(\text{LF}_3)d(\text{LF}_3)\)) is equal to \(\psi_3(\text{LF}_3)d(\text{LF}_3)\), the number of events with \(^3\)He log-fluence \(\text{LF}_3 = \text{LF}_3 + \text{LR}(\tau_{p}^{-1})\) and \(\text{LF}_3 + d(\text{LF}_3)\), where \(d(\text{LF}_3) = d(\text{LF}_4)\) and \(\text{LR}(\tau_{p}^{-1}) = \ln R_0 + A/\ln \left( \frac{\tau_{p,0}^{-1}}{\tau_{p}^{-1}} \right)\). Thus, we have

\[
\psi_4(\text{LF}_4) = \psi_3(\text{LF}_4 + \text{LR}(\tau_{p}^{-1})) = \phi_0 \times \exp \left\{ \left( \frac{\text{LF}_4 + \text{LR}(\tau_{p}^{-1})}{\sigma_3} \right)^2 \right\}.
\]

However, we expect not a single value for \(\tau_{p}^{-1}\), which as stated above is a proxy for the strength of the event, but a broad distribution of events with different strengths, say \(f(\tau_{p}^{-1})\). Since, as argued above, the \(^3\)He fluence distribution \(\psi_3(\text{LF}_3)\) is independent of \(\tau_{p}^{-1}\), then for a population of events we have

\[
\psi_4(\text{LF}_4) = \int_{0}^{\infty} \phi_0 \exp \left\{ - \left( \frac{\text{LF}_4 + \text{LR}(\tau_{p}^{-1})}{\sigma_3} \right)^2 \right\} f(\tau_{p}^{-1})d\tau_{p}^{-1}.
\]

Every term in the above equations is determined by observations and our models except the function \(f(\tau_{p}^{-1})\), which describes the distribution of the level of the turbulence. The level of turbulence multiplied by the volume of the turbulent acceleration region (which does not affect the \(^3\)He/\(^4\)He ratio) is related to the overall strength of the event. Observations of solar flares show that most extensive characteristics which are a good measure of the flare strength, such as X-ray, optical, or radio fluxes, appear to obey a steep power-law distribution, usually expressed as a cumulative distribution \(\Phi(\geq F_i) \propto F_i^{-n}\) (or differential distribution \(\phi(F_i) \propto F_i^{-n-1}\)) with typically \(n \approx 1.5\) (see, e.g., Dennis 1985 and reference therein). Such a distribution seems to roughly agree with the prediction of the so-called avalanche model proposed by Lu & Hamilton (1991). Now assuming that \(\tau_{p}^{-1}\) also obeys such a power-law distribution, i.e., \(f(\tau_{p}^{-1}) \propto (\tau_{p}^{-1})^{-1(n+1)}\), we can write the distribution of \(^4\)He as

\[
\psi_4(\text{LF}_4) = \int_{0}^{\infty} \phi_0 \exp \left\{ - \left( \frac{\text{LF}_4 + \text{LR}_0 + A/x}{\sigma_3} \right)^2 \right\} e^{-nx}dx,
\]

with \(x \equiv \ln \left( \frac{\tau_{p,0}^{-1}}{\tau_{p}^{-1}} \right)\).

Using the above relations, we have calculated the \(^4\)He fluence distribution. The results for three models are compared with the observations in the right panel of Figure 5. Given the other model parameters \((k_{\text{min}}, \alpha, \text{ etc.})\) we have only one free parameter, namely the index \(n\) for this fit. The solid line obtained for the top curve of the left panel \((k_{\text{min}} = 0.2k_{\text{max}}, \alpha = 0.5)\), and for \(n = 2\) provides a good fit to the observed distribution of \(^4\)He fluences. In order to demonstrate the sensitivity of the results to the parameters, we also show two other model predictions based on slightly different parameter values. These results provide additional quantitative evidence (besides those given in LPM04 and LPM06) on the validity of the SA of SEPs by turbulence, and indicate that with this kind of analysis one can begin to constrain model parameters.

\textsuperscript{10} It should be noted that while the observations are for fluences integrated from 0.2 to 2.0 MeV nucleon\(^{-1}\), our theoretical ratios are calculated at 1 MeV nucleon\(^{-1}\) which is near the geometric or algebraic mean of the range.

\textsuperscript{11} The truncation shown by the shaded area in the middle panel of Figure 1 introduces a slight bias against detection of low fluences. We estimate that, because there are fewer events at the high \(^4\)He fluence end, this means a 10%–20% underestimation of the distribution of the three lowest bins of the \(^3\)He histogram (right panel, Figure 1). We will ignore this small correction, whose main effect is to increase the value of \(\sigma_3\) by a small amount.
4. SUMMARY AND CONCLUSIONS

In this paper, we have carried out further comparison with observations of the predictions of the model based on SA of SEP ions by turbulence. In our earlier works (LPM04 and LPM06), we demonstrated that the extreme enrichments of $^3\text{He}$ and spectra of $^4\text{He}$ and $^3\text{He}$ observed in several events can be naturally described in such a model. Using the results based on this model, here we consider the relative distributions of $^4\text{He}$ and $^3\text{He}$ fluences derived from a large sample of event by Ho05. We show that with some simple and reasonable assumptions we can explain the general features of these observations as well.

These are clearly preliminary results and are intended to demonstrate that in addition to modeling only a few bright events it is also important to look at population as a whole and ascertain that a model which can explain the detail characteristics of individual events can also agree with the distributions of observables for a large sample of events. Here, we have shown how the dispersion in one parameter, namely the acceleration rate or the strength of the flare, can account for the observed distributions of fluences. The key assumption here is that the amount of produced turbulence (represented by $\tau_p^{-1}$) has a wide dispersion and obeys a power-law distribution similar to that observed for other extensive parameters that give a measure of the strength of a flare. The dispersion in other model parameters can also influence the final outcome. However, the dispersion of most of the other important parameters, such as intensive parameters temperature, density, and magnetic field, are expected to be much smaller than that of an extensive parameter such as the overall strength of the event, the amount of turbulence produced, the flare volume, etc. In addition, as shown in the previous section, most of the intensive parameters play a lesser role than the $\tau_p^{-1}$ (related to the total turbulence) in determining the relative characteristics of $^3\text{He}$ and $^4\text{He}$. Given the dispersion of any other parameter one may carry out similar integration over its range. However, for the reasons given above we expect smaller changes in the shapes of predicted distributions due to dispersion of most of the intensive parameters. Given a more extensive set of data such improvements may be needed and can be carried out.

The existing data may be used to test some of our assumptions, in particular, the assumption of constancy of the $^3\text{He}$ distribution. We intend to address these in future works. We can also make the above results more robust by using model fluences integrated over the same spectral range as the observations instead of fluences at 1 MeV nucleon$^{-1}$. One can also expand this approach and address the distributions of other characteristics besides the fluence, such as the spectral indices or break energies (if any). The available data contain some of this information but require more analysis.

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