Effect of Interdot Coulomb Repulsion on Charge Transport of Parallel Two Single-Electron Transistors

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(Received September 15, 2005; accepted December 23, 2005; published online April 7, 2006)

The charge transport behaviors of two single-electron transistors (SETs) sitting in parallel are investigated using the Anderson model with two impurity levels. Interdot Coulomb interactions as well as intradot Coulomb interactions are included in the model. The nonequilibrium Keldysh–Green’s function technique is used to calculate the current–voltage characteristics of this system. Considering the lowest order coupling between dots and electrodes, analytic spectral functions are calculated using the equation of motion method. We found that the interdot Coulomb interactions not only lead to interdot Coulomb blockade effects but also create negative differential conductances. In addition, the hysteretic effects arising from interface traps or defects are also investigated. SETs could be good detectors for determining interface acceptor-type or donor-type traps of quantum dots (QDs). [DOI: 10.1143/JJAP.45.2881]

KEYWORDS: single electron transistors, Coulomb blockade, quantum dots

1. Introduction

Modern advanced techniques, such as molecular beam epitaxy and metal organic chemical vapor deposition coupled with e-beam lithography, can provide good control of the size and shape of components in semiconductor circuits down to the nanometer scale, where the quantum effect becomes important. The interplay of quantum confinement and electron correlation leads to intriguing effects such as the Coulomb blockade and Kondo effects. Recently, many studies have been devoted to the understanding of transport properties in quantum dot nanostructures\(^1,5\) since quantum-dot (QD)-based systems have great potential applications in optoelectronic devices, such as infrared detectors,\(^6\) semiconductor lasers,\(^7\) and single-photon generators.\(^8\) These devices prefer III–V compound semiconductors. Apart from the applications of optoelectronics, nanometer-sized QDs can also be used to implement single-electron transistors (SETs) with extremely low power consumption. Unlike conventional transistors, in which severe and unavoidable short-channel effects occur as the devices are scaled down, SETs might play a crucial role in the next-generation integrated circuit of nanometer scale.\(^9\)

A SET is composed of three leads (source, drain, and gate) and one or more QDs, which can be a normal metal, superconductor, ferromagnetic material or semiconductor. The behavior of the tunneling current is sensitive to the properties of the QDs. For a metallic QD or QD formed by two-dimensional electron gas, the charging energy \(\Delta U\) is typically larger than the energy level spacing \(\Delta E\) caused by quantum confinement.\(^10,11\) Consequently, the tunneling current displays a periodic Coulomb oscillation with respect to the gate voltage, which is a result of homogeneous \(\Delta U\) and negligible \(\Delta E\) in a large QD. However, such a Coulomb oscillation was observed only at very low temperatures due to a small charging energy. Note that the charging energy must be greater than the thermal energy and tunneling rate multiplied by \(h\) in order for a SET to operate properly. However, for application at high temperatures, nanoscale QDs with a large charging energy are desired for SETs.

To date, silicon (Si) or germanium (Ge) QDs can be miniaturized to reach a nanometer-scale size using advanced fabrication technology, and the tunneling current of Si/Ge SETs displays nonperiodic Coulomb oscillations.\(^12–15\) This nonperiodic feature originates from the nonhomogeneous \(\Delta E\) and stochastic \(\Delta U\) in the nanometer-sized Si/Ge QDs of asymmetrical shapes. Although the tunneling current of Si/Ge SETs at high temperatures has been reported by several groups, it is difficult to gain full understanding of tunneling current characteristics due to the multivalley nature of Si/Ge and the unknown Si/Ge–SiO\(_2\) interface properties such as defects or traps. In addition, many theoretical studies have been devoted to the understanding of transport properties of individual SETs composed of two-dimensional electron gas, including the dc and ac tunneling current, using the Anderson model and Keldysh–Green’s function method.\(^4,5,16\) Nevertheless, only a handful of theoretical studies considered the coupling between SETs.\(^17\)

It is important to understand the interactions between coupled SETs in the viewpoint of the limits to semiconductor device integration in closely packed architectures. The mechanism of interactions between two SETs mainly arises from the interdot interactions, such as interdot Coulomb repulsion and interdot electron tunneling. The latter is negligible as a result of a high-potential barrier and a sufficient dot separation distance. The main purpose of this work is to theoretically study the charge transport of SETs sitting in parallel as depicted in Fig. 1, where SET-A (with dot A) is regarded as a detector SET and SET-B (with dot B) is regarded as a target SET (or trap). Each SET consists of a single dot and two electrodes (source and drain). The schematic diagram shown in Fig. 1 is very similar to that shown in ref. 17, where the shot noise and

![Fig. 1. Schematic diagram for parallel two single-electron transistors with tunneling rates \(\Gamma_{A,L}\) and \(\Gamma_{A,R}\) between QDs and electrodes; single-particle energy levels \(E_{A,L}\) and \(E_{A,R}\); and interdot Coulomb interaction \(U_{AB}\).](image-url)
tunneling current of coupled semiconductor dots are studied using a master equation with infinite intradot Coulomb interactions being considered. In this article, we consider not only finite intradot Coulomb interactions, but also interdot Coulomb interactions. Using the equation of motion method to calculate the retarded Green’s function, we obtain analytic spectral functions, which determine the tunneling current through individual dots. Since defects or traps essentially exist in the process of Si/Ge–SiO₂ fabrication, the effects of hysteresis arising from traps on the tunneling current of SET-A are also investigated.

2. Tunneling Current

In this study, we consider the Ge QDs embedded into the SiO₂ matrix contacted with metals or heavily doped semiconductors. The following Hamiltonian is employed to describe the system shown in Fig. 1,

\[ H = \sum_{\ell,\alpha} \varepsilon_{\ell} a^{\dagger}_{\ell,\alpha} a_{\ell,\alpha} + \sum_{\ell,\alpha} \sum_{\beta,\gamma} \bar{U}_{\ell,\alpha\beta\gamma} a^{\dagger}_{\ell,\alpha} a_{\ell,\beta} a_{\ell,\gamma}^\dagger + \sum_{\ell,\alpha} \sum_{\beta,\gamma} \bar{U}_{\ell,\alpha\beta\gamma} a_{\ell,\alpha} a^{\dagger}_{\ell,\beta} a^{\dagger}_{\ell,\gamma} \]

where \( a^{\dagger}_{\ell,\alpha} \) (\( a_{\ell,\alpha} \)) and \( b^{\dagger}_{\ell,\alpha} \) (\( b_{\ell,\alpha} \)) create (destroy) an electron of the momentum \( \ell \) and spin \( \sigma \) with the energy \( \varepsilon_{\ell} \) in the \( \ell \) electrode of SET-A and SET-B, respectively. The free electron model is considered in the electrodes. \( d^{\dagger}_{\ell,\alpha} (d_{\ell,\alpha}) \) creates (destroys) an electron inside SET’s QDs with the orbital energy \( E_{\ell} \). Only the ground state level for electrons \( E_{A} \) (\( E_{B} \)) is considered in SET-A (SET-B) because of a very strong confinement effect in the Ge/SiO₂ QDs. \( U_{ij} \) indicates the intradot and interdot Coulomb interactions. \( u_{\ell,\alpha} \) indicates the coupling between the band states and the Ge QDs. The Hamiltonian given by eq. (1) is based on the Anderson model with two energy levels.\(^{16}\) The time-independent tunneling current from the left leads (source electrodes) can be expressed as:1\(^{16}\)

\[ J = \frac{2e}{\hbar} \sum_{i=A,B} \int \frac{d\epsilon}{2\pi} \Gamma_{L,i}(\epsilon) \text{Im} \left[ \frac{1}{2} G_{iA}^{\text{re}}(\epsilon) + f_{L,i}(\epsilon) G_{iA}^{\text{im}}(\epsilon) \right] \]

Substituting eq. (3) into eq. (2), we obtain the tunneling current

\[ J = \frac{2e}{\hbar} \sum_{i=A,B} \int \frac{d\epsilon}{2\pi} \frac{\Gamma_{L,i}(\epsilon)}{\Gamma_{L,i}(\epsilon) + \Gamma_{R,i}(\epsilon)} \text{Im} G_{iA}^{\text{im}}(\epsilon) \]

\[ J = J_{A} + J_{B}. \]

(4)

For simplicity, these tunneling rates are assumed to be energy-and bias-independent. Therefore, the calculation of tunneling current is entirely determined by the spectral function \( A_{i} = \text{Im} G_{iA}^{\text{re}}(\epsilon) \), which is the imaginary part of the retarded Green’s function \( G_{iA}^{\text{re}}(\epsilon) \). The expression of the retarded Green’s function \( G_{iA}^{\text{re}}(\epsilon) \) can be obtained by the equation of motion method.\(^{18-21}\) The lowest order coupling between the electrodes and QDs are considered in the calculation of \( G_{iA}^{\text{re}}(\epsilon) \). The equation of motion for \( G_{iA}^{\text{re}}(\epsilon) \) leads to

\[ \left( \epsilon - E_{i} + i \frac{\Gamma_{i}}{2} \right) G_{iA}^{\text{re}}(\epsilon) = 1 + \sum_{j=A,B} U_{ij} G_{jA}^{\text{re}}(\epsilon) + U_{ij} G_{jA}^{\text{re}}(\epsilon) G_{jB}^{\text{re}}(\epsilon) \]

\[ = N_{i} G_{jA}^{\text{re}}(\epsilon) + N_{i} G_{jB}^{\text{re}}(\epsilon), \]

(5)

and

\[ \left( \epsilon - E_{i} + i \frac{\Gamma_{i}}{2} \right) G_{iB}^{\text{re}}(\epsilon) = N_{i} G_{jA}^{\text{re}}(\epsilon) + N_{i} G_{jB}^{\text{re}}(\epsilon), \]

(6)

The notations \( N_{i} \), \( N_{i} \) and \( N_{i} \) are the electron occupation numbers for the \( i \) and \( j \) dots. Note that \( i \) is not equal to \( j \) in eqs. (5)-(8). Now the two-particle Green’s functions are coupled with the three-particle Green’s functions \( G_{ij}^{\text{re}}(\epsilon) = \langle n_{i} n_{i} n_{j} d_{i\sigma} d_{i\sigma}^{\dagger} \rangle \), \( G_{ij}^{\text{re}}(\epsilon) = \langle n_{i} n_{i} n_{j} d_{i\sigma}^{\dagger} d_{i\sigma} \rangle \), and \( G_{ij}^{\text{re}}(\epsilon) = \langle n_{i} n_{i} n_{j} d_{i\sigma} d_{i\sigma}^{\dagger} \rangle \). The equation of motion of the three-particle Green’s functions leads to coupling with the four-particle Green’s functions, where the hierarchy terminates. Thus, these three particle Green’s functions can be expressed in the following closed form

\[ G_{ij}^{\text{re}}(\epsilon) = N_{i} G_{ij}^{\text{re}}(\epsilon) \]

\[ = \frac{1 - N_{j}}{\epsilon - (E_{i} + U_{ij} + U_{ij}) + i \frac{\Gamma_{i}}{2}}. \]

(7)
depend on electron occupation numbers, which determine the probability for each channel. The electron occupation numbers of eq. (12) can be solved in a self-consistent way using

$$N_{i\sigma} = \int \frac{d\epsilon}{2\pi} \text{Im} G_{i\sigma}^\epsilon(\epsilon)$$

(13)

$$N_{i\sigma}$$ is limited to the region of $0 \leq N_{i\sigma} \leq 1$. The electron occupation number $N_A$ ($N_B$) affects the tunneling current through dot B (A) due to interdot Coulomb interactions. According to eq. (12), the tunneling current through individual QDs is significantly influenced by the magnitudes of particle Coulomb interactions and energy levels. Therefore, we attempt to calculate the particle Coulomb interactions in the next section.

3. **Energy Levels and Particle Interactions**

The electronic structure of individual QDs with multi-valleys, such as Ge or Si QDs, has been studied by several groups considering spherical QDs.\(^{22,23}\) For example, the conduction band of Ge has four equivalent valleys aligned along $[1,1,1]$, $[1,-1,1]$, $[-1,1,-1]$, and $[-1,-1,1]$.\(^{24}\) The complex conduction band structures of Ge QDs may contain more interesting physics for transport properties than those of single-valley semiconductors. However, due to the lack of information on the precise shape of individual dots and the strain effect between the SiO$_2$ and Si/Ge QDs,\(^{12-15}\) we ignore the effect of multivalleys in this calculation. Our purpose is to estimate the ratio of the interdot Coulomb interactions to the intradot Coulomb interactions. To calculate the electron wavefunctions of individual QDs, we introduce a cylindrical Ge QD with the radius $R_0$ and height $h = 2R_0$. Within the effective mass model, the Hamiltonian is

$$\begin{align*}
  -\frac{\hbar^2}{2m^*}\nabla^2 + V_{QD}(\rho, z) \psi(\mathbf{r}) &= E_0\psi(\mathbf{r}), \\
  m^* &= m_0, \\
  m_0 &= \frac{1}{3}\frac{m_1 + 2}{m_1 - m_2}, \\
  m_1 &= 1.59m_e, \\
  m_2 &= 0.0823m_e.
\end{align*}$$

(14)

where $m^*\psi(\rho, z)$ denotes the position-dependent electron effective mass. Hence, $m_0 = m_2$ for SiO$_2$ and $m_0 = 1/3$ for Ge, where $m_1 = 1.59m_e$ and $m_2 = 0.0823m_e$ are the electron longitudinal and transverse effective masses of bulk material.\(^{25}\) $V_{QD}(\rho, z)$ is approximated using the constant potential $V_0 = 3.5$ eV in the QD region. Its value is determined by the conduction band offset between Ge and SiO$_2$. Owing to a very strong confinement effect, we use a perturbation to calculate the Coulomb interactions

$$U_{i,j} = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \frac{\epsilon^3 n_i(\mathbf{r}_1)n_j(\mathbf{r}_2)}{\epsilon_0(\mathbf{r}_1; \mathbf{r}_2)|\mathbf{r}_1 - \mathbf{r}_2|},$$

(15)

where $n_i(\mathbf{r}) = |\psi(\rho, z, \phi)|^2$ is the particle density of individual QDs, and $\epsilon_0(\mathbf{r}_1; \mathbf{r}_2)$ denotes the position-dependent static dielectric constant. For the purpose of constructing approximate wave functions, we place the system in a large cylindrical confining box with the length $L$ and radius $R$ ($L$ and $R$ must be markedly larger than those of cylindrical Ge QDs). Here $L = 60$ nm and $R = 40$ nm are adopted. We solve the eigenfunctions of the effective-mass Hamiltonian by the Ritz variational method. The wave functions are
Fig. 2. Lowest two energy levels of cylindrical Ge/SiO$_2$ QD for $\ell = 0$ as a function of dot radius $R_0$.

expanded in a set of basis functions, which are chosen to be products of Bessel functions and sine waves $\psi_{\ell,n}(\rho, z, \phi) = J_\ell(\beta_\ell \rho) e^{in\phi} \sin(k_m(z + L/2))$, where $k_m = m\pi/L, m = 1, 2, 3, \ldots$. $J_\ell$ is the Bessel function of the order of $\ell$ and $\beta_\ell R$ is the $n$th zero of $J_\ell$. The same set of basis functions has been used by Marzin and Bastard$^{25}$ to calculate the quantum confined states in a conical QD. The expression of the matrix elements of the Hamiltonian of eq. (14) can be readily obtained. Forty sine functions multiplied by fifteen Bessel functions for each angular function ($\ell = 0$ or 1) are used to diagonalize the Hamiltonian. Figure 2 shows the lowest two energy levels of a single dot as a function of the radius $R_0$ for $\ell = 0$. Note that the lowest energy level for $\ell = 1$, not shown in Fig. 2, is higher than the energy level denoted by the dashed line. If the energy level spacing between the ground state and the first excited state is much larger than the thermal energy of the temperature $T$, we can consider the lowest energy levels $E_A$ and $E_B$ for dots A and B ($E_A \neq E_B$), which could correspond to slightly different QD sizes, in the Hamiltonian of eq. (1). To calculate the tunneling current, we determine the dot sizes and adopt dot A of radius $R_0 = 4.6$ nm and the dot B of $R_0 = 4.4$ nm, which give, respectively, $E_A = 108$ meV and $E_B = 118$ meV. The QD sizes considered are the averaged sizes that we could experimentally demonstrated thus far.$^{15}$

Based on eq. (15), Fig. 3 shows the intradot Coulomb interactions and interdot interactions as functions of dot separation ($ds$), which is defined as the distance between the bottom of dot A and that of dot B. The solid and dashed lines denote the intradot Coulomb interactions of the dots B and A ($U_B = 37.18$ meV and $U_A = 36.43$ meV), the dotted line denotes the interdot Coulomb interactions ($U_{AB}$). The intradot Coulomb interactions are almost independent of dot separation indicating the very localized wavefunctions of QDs. In addition, it is also found that the interdot electron hopping energy $t_{AB}$ is smaller than 0.1 meV for $ds \geq 16$ nm, which is not readily be achieved by the current e-beam lithography technique. This negligible $t_{AB}$ is due to the high-potential barrier between Ge QDs and SiO$_2$. Despite, the energy levels and particle Coulomb interaction magnitudes shown in Figs. 2 and 3 are smaller than realistic values since we ignored, in eq. (14), the image charge effects resulting in the difference in dielectric constant between Ge QDs and SiO$_2$. Nevertheless, the results shown in Figs. 2 and 3 still provide us reliable $U_{AB}/E_{A(B)}$, $U_{A(B)}/E_{A(B)}$ and $t_{AB}/E_{A(B)}$ values, allowing us to justify the transport regime.$^{17}$

4. Results and Discussion

In this section, we perform the detailed numerical calculation of current–voltage ($I$–$V$) characteristics. For simplicity, we assume that the tunneling rates $\Gamma_{\alpha \lambda} = \Gamma_{A,R} = \Gamma_{B,L} = \Gamma_{B,R} = \Gamma$ are bias-independent. Meanwhile, the chemical potentials of four electrodes are assumed to be 5 meV below the ground-state energy level $E_A$ of dot A at zero bias. Considering the dot separation distance $ds = 16$ nm, the interdot Coulomb interaction is $U_{AB} = 11.06$ meV. Substituting eq. (12) into eq. (13), the electron occupation numbers $N_A$ and $N_B$ (for a fixed electron spin) could be solved in a self-consistent way. Figure 4 shows $N_A$ and $N_B$ as functions of the applied voltage $V_a$ at zero temperature. To exhibit the interdot Coulomb repulsion effect, the dashed lines for $U_{AB} = 0$ are also plotted in Fig. 4. In the absence of $U_{AB}$, the retarded Green’s function of eq. (12) has only two poles.

$$G_{\alpha \sigma}^R(\epsilon) = \frac{1 - N_{\alpha - \sigma}}{\epsilon - E_\sigma + i\Gamma_\alpha/2} + \frac{N_{\alpha - \sigma}}{\epsilon - E_\sigma - U_\alpha + i\Gamma_\alpha/2}$$

Therefore, the staircase behavior of $N_i$ is generated by the intradot Coulomb interactions $U_i$. Owing to $\Gamma_{\alpha \lambda} = \Gamma_{\alpha \rho}$ considered, $N_i$ reaches 0.33 when the applied voltage crosses the energy level $E_i$; however it is not sufficient for overcoming the charging energy $U_i$. This is the well-known

Fig. 3. Particle Coulomb interaction strengths as a function of dot separation $ds$.

Fig. 4. Electron occupation numbers as a function of applied voltage $V_a$. The solid lines denote $U_{AB} = 11.06$ meV and the dashed lines denote $U_{AB} = 0$. We adopt $\Gamma = 0.1$ meV and consider zero temperature.
Coulomb blockade effect. When the applied voltage overcomes the Coulomb blockade, \( N_i \) reaches 0.5. These fractional occupation numbers are the typical statistic feature of an open system.

As mentioned in Introduction, SET-A as a detector is for measuring the charge distribution of dot B via the electrostatic Coulomb interactions. For \( U_{AB} \neq 0 \), both \( N_A \) and \( N_B \) show a complex behavior. The results shown in Fig. 4 indicate that the charge detector (SET-A) and the target (SET-B) influence each other. The suppression of \( N_A \) around \( V_{ad} \) corresponds to dot B with single tunneling charges. It is unexpected that the “turn-on” voltage of SET-B (\( V_{ad} \approx 15 \text{ mV} \)) remains unchanged even in the presence of dot A electrons; however, we note that the original occupation number \( N_B = 0.33 \) is suppressed as \( N_B = 0.22 \) for \( U_{AB} \neq 0 \) at \( V_{ad} \). The plateau between \( V_{ad} \) and \( V_{as} \) for \( N_B \) indicates the interdot Coulomb blockade; the charge distribution of dot B increases when the applied voltage \( V_a \) reaches the resonant energy level \( U_B + U_{AB} \) and overcomes the interdot charging energy caused by \( U_{AB} \). The notations \( V_{as} \) and \( V_{ad} \) label, respectively, the applied voltages for overcoming the interdot charging energies of SET-A and SET-B. The first and second plateaus of the staircase at a high-voltage region can be regarded as the one-electron and two-electron detection process, respectively. The one-particle and two-particle processes coexist as a result of the open system. From the experimental viewpoint, the tunneling currents are more interesting than the electron occupation numbers. Therefore, we calculate the tunneling currents by substituting eq. (12) into eq. (4).

Figure 5 shows the tunneling current as a function of the applied voltage \( V_a \); the solid line denotes the tunneling current of SET-A (\( J_A \)), and the dashed line denotes the tunneling current of SET-B (\( J_B \)). The dotted line denotes the sum of \( J = J_A + J_B \) which corresponds to the case shown in ref. 17, where the sources A and B (the drains A and B) merge. We observe that the interdot Coulomb interactions not only create new resonant channels for SET-A and SET-B, but also yield negative differential conductances (defined as \( DG = dJ/(V_a)/dV_a \)). For a detailed comparison between the results shown in ref. 17 and ours, we show the tunneling current \( J \) and oscillatory differential conductances in Fig. 6.

The behavior of the tunneling current can be understood by analyzing the energy poles of the retarded Green’s function given in eq. (12). The first two peaks of the oscillatory differential conductances \( (P_1 \text{ and } P_2) \) correspond to the resonant energy levels \( E_A \) and \( E_B \); while the third peak and the fourth peak \( (P_3 \text{ and } P_4) \) correspond to \( E_A + U_{AB} \) and \( E_B + U_{AB} \), respectively. We cannot observe the resonant energy levels \( E_A + 2U_{AB} \) and \( E_B + 2U_{AB} \) at low applied voltages, since single electrons pass through each QD. Our results are in good agreement with those shown in ref. 17 where the range of applied voltage is limited so as inhibit the intradot charging energies. At high applied voltages, there are six resonant energy levels, namely, \( E_A + U_A \), \( E_A + U_A + U_{AB} \), \( E_A + U_A + 2U_{AB} \), \( E_B \), \( E_B + U_B \), \( E_B + U_B + U_{AB} \), and \( E_B + 2U_B + 2U_{AB} \). The peak \( P_6 \) is caused by a pair of poles at \( \epsilon = E_B + U_B \) and \( \epsilon = E_A + U_A + U_{AB} \), since the magnitude of \( E_B - E_A \) is very close to that of \( U_{AB} \) for the present case. The peak \( P_7 \) is caused by another pair of poles at \( \epsilon = E_A + U_A + 2U_{AB} \) and \( \epsilon = E_B + U_B + U_{AB} \). The last peak located near \( V_a = 75 \text{ mV} \) is due to the resonant tunneling through the energy level at the pole \( \epsilon = E_B + U_B + 2U_{AB} \).

In the above discussion, we considered \( \Gamma_{BL} = \Gamma_{BB} \). For dot B under \( \Gamma_{BL} \gg \Gamma_{BR} \), SET-B behaves as a closed system and dot B can be individually manipulated as the following three charge states; the empty state \( |B, 0 \rangle \), one-electron state with spin up or down \( |B, \uparrow \rangle \text{ and } |B, \downarrow \rangle \) and two-electron state \( |B, \uparrow, \downarrow \rangle \). Figure 7 shows the tunneling current and differential conductance as functions of applied voltage for...
various charge states of dot B; the solid line denotes \(|B, 0\rangle\), the dashed line denotes \(|B, 1\rangle, \langle B, 0\rangle\) and the dotted line denotes \(|B, 1, D\rangle, \langle B, 0\rangle\). When dot B situates in a closed system, there is only one plateau caused by intradot charging energy, whereas the “turn-on” voltage of SET-A depends on the charge numbers of dot B.

Thus far, we have not considered the effects of defects or traps on the tunneling current of SET-A. However, in the fabrication of Ge-SETS or (Si-SETSs), unavoidable impurities introduce random background charges or traps, which can significantly affect the transport properties of SETs and thus break the desired operation of single-electron devices and circuits. If we can clearly clarify the properties of traps, it is more possible for us to suppress random background charges on SETs. To understand the effects of trap on the tunneling current of SETs, in the Hamiltonian of eq. (1) we view \(E_g = E_{g,a}\) as a trap energy level, which generally is a degenerate acceptor-type state and below the bottom of the Ge QD conduction band. SET-A is expected to interact with a trap state via the effective electrostatic Coulomb repulsion \(U_{AT}\). Owing to many trap states around Ge QDs, we use the Hartree approximation to justify the effect of \(U_{AT}\) on the tunneling current in the calculation of \(G_{a,B}\), thus, the retarded Green’s function of eq. (12) is rewritten as

\[
G_{a,B}(\epsilon) = \frac{1 - N_{A,-\sigma}}{\epsilon - \epsilon_a - U_{AT}N_T + i\frac{\Gamma_A}{2}} + \frac{N_{A,-\sigma}}{\epsilon - \epsilon_a - U_{AT}N_T - U_A + i\frac{\Gamma_A}{2}},
\]

where \(N_T\) is the occupation number of a trap state and \(\epsilon_a = E_a - \beta V_g + \alpha V_i\); here, \(\beta\) and \(\alpha\) are the scaling factors of the applied field due to capacitance effects. In addition to the applied voltage \(V_i\), we also introduce the gate voltage \(V_g\) to manipulate the energy levels of QD. We use \(\beta = 0.5\) and \(\alpha = 0.5\). From the results obtained using eq. (16), we observe that electrons residing at dot A interact with traps through \(U_{AT}N_T\), which depends on the occupation number \(N_T\) and indicates a mean value of different interaction strengths between dot A and traps. Note that \(N_T\) is limited in the range of \(0 \leq N_T \leq 1\). It is apparent that this treatment is adequate for many trap states, but not for a single trap state. According to the observed hysteretic effects, \(N_T\) is the multivalue function of the applied gate voltage. We ignore the detailed calculation of \(N_T\) and phenomenologically assume that \(N_T = 1\) when the gate voltage increases to the threshold voltage \(V_{th}\) from zero; on the other hand, \(N_T = 0\) when the gate voltage decreases from \(V_{th}\). \(N_T\) is a single value when the gate voltage is larger than \(V_{th}\). The origin of hysteretic effects may be the interplay between the interface trap states of Ge/SiO\(_2\) QD and the interface trap states of Si/SiO\(_2\) electrodes. Using eq. (16), we calculate the tunneling current as a function of gate voltage at room temperature and \(V_g = 20\) mV. Figure 8 shows the hysteretic tunneling current through SET-A, in which we assumed the Fermi energy of electrodes below the ground-state energy level \(E_a\) 80 meV at zero bias \(V_i = 0\), \(U_{AT} = 25\) meV, \(U_A = 125\) meV, and \(\Gamma_L = \Gamma_R = 1\) meV. The hysteretic current loop displays a counterclockwise behavior, which has been observed by Li et al. However, if traps belong to donor-type states, the tunneling current of SET-A will display a clockwise hysteretic loop due to a negative \(U_{AT}\).

5. Conclusions

For parallel double SETs implemented by QDs embedded into SiO\(_2\), a interdot Coulomb repulsion is more important than interdot electron tunneling as a result of a high potential barrier. Interdot Coulomb interactions not only create interdot Coulomb blockade effects but also produce negative differential conductances. We also found that the current–voltage characteristics of the detector SET-A depend on the charge distribution of dot B situated in an open system or a closed system. Owing to highly sensitive charges, a SET is a useful detector for investigating the charge properties of trap states, such as donor-type traps or acceptor-type traps.

Acknowledgments

This work was supported by the National Science Council of Republic of China under Contract Nos. NSC-93-2215-E-008-014, NSC 93-2120-M-008-002, and NSC-94-2215-E-008-027.

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