Acoustic streaming with heat exchange

A A Gubaidullin$^{1,2,3}$ and A V Pyatkova$^{1,2,4}$

1 Tyumen Branch of the Khristianovich Institute of Theoretical and Applied Mechanics, Siberian Branch of Russian Academy of Sciences, 74 Taymirskaya Street, Tyumen, Russia
2 Tyumen State University, 6 Volodarskogo Street, Tyumen, Russia
3 Tyumen industrial University, 38 Volodarskogo Street, Tyumen, Russia
4 Institute of Mechanics and Engineering, Kazan Science Center, Russian Academy of Sciences, 2/31 Lobachevskogo Street, Kazan, Russia

E-mail: annyakovenko@yandex.ru

Abstract. Acoustic streaming in a cylindrical cavity with heat exchange is numerically investigated. The cavity is filled with air. The boundaries of the cavity are maintained at constant temperature. The features of acoustic streaming manifesting with the decrease in the frequency of vibration in comparison with the resonant frequency are determined. The influence of the nonlinearity of process on acoustic streaming is shown. The nonlinearity is caused by the increase of the vibration amplitude.

1.Introduction

Acoustic streaming is a directed time average mass transfer by steady vortices formed in addition to the periodic motion of the medium in the sound field [1]. The first theoretical analysis of acoustic streaming in standing waves between parallel planes was performed by Rayleigh [2]. He analytically described stationary motion consisting of a series of vortices with a definite sense of rotation. Acoustic streaming inside the boundary layer is called the inner streaming or Schlichting streaming. Acoustic streaming outside the boundary layer is called outer streaming or Rayleigh streaming [3]. In [4, 5], for the example of vibration with the minimum resonance frequency, it was analytically demonstrated that in the case of a “narrow” tube, for which the ratio of the channel radius to the acoustic boundary layer thickness was below 5.7, only Schlichting streaming was present.

Due to the numerical methods and experimental investigations it became possible to describe the strongly nonlinear streaming, when the analytical solution is no longer valid. Nonlinear effects in acoustic streaming are widely investigated [6-11]. In [12-14] the nonlinear effects manifesting with the increase in vibration frequency at fixed vibration amplitude are described. In [15], the case of thermally insulated cavity walls is compared with the case of walls maintained at constant temperature for weakly nonlinear processes and vibration frequencies below resonance.

In the present study, acoustic streaming in a cylindrical cavity at below resonance vibration frequencies is investigated. The constant temperature walls are considered. For the five vibration frequencies the influence of heat exchange to the acoustic streaming is shown. Furthermore, for the closest to the resonant vibration frequency the influence of the nonlinearity on acoustic streaming is studied.

2.Problem formulation

A circular cylindrical cavity of length $L$ and radius $R_0$ with impermeable ends is considered (figure 1). The cavity is filled with a perfect viscous gas (air). Let gas in the cavity be initially at rest at constant
temperature \( T_0 \) and constant pressure \( p_0 \). The system is disturbed from equilibrium by vibration action \( A \cos(\omega t) \) with constant amplitude \( A \) and frequency \( \omega \), so that this action causes harmonic vibration of the entire cavity along its axis. We consider the case of walls maintained at constant temperature. The thermal conductivity, heat capacity and viscosity are assumed constant.

![Figure 1. Schematic of the problem.](image)

The system of equations describing gas motion with respect to the vibrating cavity in a cylindrical coordinate system in terms of dimensionless variables has the following form:

\[
\frac{\partial \tilde{\rho}}{\partial \tau} + \frac{\partial \tilde{\rho}U}{\partial X} + \frac{1}{R} \frac{\partial \tilde{\rho}V}{\partial R} = 0, \\
\frac{\partial \tilde{\rho}U}{\partial \tau} + \frac{\partial \tilde{\rho}UU}{\partial X} + \frac{1}{R} \frac{\partial \tilde{\rho}UV}{\partial R} = -\frac{\partial P}{\partial X} + N \left( \frac{\partial^2 \tilde{U}}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \tilde{U}}{\partial R} \right) \right) + \frac{1}{3} N \left( \frac{\partial^2 \tilde{U}}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial X} \left( \frac{\partial \tilde{V}R}{\partial R} \right) \right) + \tilde{\rho} \Omega^2 \cos(\Omega \tau), \\
\frac{\partial \tilde{\rho}V}{\partial \tau} + \frac{\partial \tilde{\rho}UV}{\partial X} + \frac{1}{R} \frac{\partial \tilde{\rho}VV}{\partial R} = -\frac{\partial P}{\partial R} + N \left( \frac{\partial^2 \tilde{V}}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \tilde{V}}{\partial R} \right) \right) + \frac{1}{3} N \left( \frac{\partial \tilde{V}}{\partial R} \left( R \frac{\partial \tilde{V}}{\partial R} \right) + \frac{\partial}{\partial X} \left( \frac{\partial \tilde{U}}{\partial X} \right) \right) - \frac{4}{3} N \frac{V}{R^2}, \\
\frac{\partial \tilde{\rho} \Theta}{\partial \tau} + \frac{\partial \tilde{\rho} U \Theta}{\partial X} + \frac{1}{R} \frac{\partial \tilde{\rho} V \Theta}{\partial R} = \gamma T \left( \frac{\partial^2 \tilde{\Theta}}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \tilde{\Theta}}{\partial R} \right) \right) - \gamma (y-1) P \left( \frac{\partial \tilde{U}}{\partial X} + \frac{1}{R} \frac{\partial \tilde{V}}{\partial R} \right) - \frac{2}{3} \gamma (y-1) N \left( \frac{\partial \tilde{U}}{\partial R} + \frac{\partial \tilde{V}}{\partial X} \right)^2 + \gamma (y-1) N \left( \frac{\partial \tilde{V}}{\partial R} \right)^2 + 2 \gamma (y-1) N \left( \frac{\partial \tilde{U}}{\partial X} \right)^2 + \frac{\tilde{V}^2}{R^2}, \\
P = \frac{\tilde{\rho} (\Theta + 1)}{\gamma}.
\]

The initial and boundary conditions are as follows:

\( \tau = 0 : \quad U = 0, \quad V = 0, \quad \Theta = 0, \quad P = 1/\gamma \approx 0.71, \quad \tilde{\rho} = 1, \)

\( X = 0 : \quad U = 0, \quad V = 0, \quad \Theta = 0, \)

\( X = 1 : \quad U = 0, \quad V = 0, \quad \Theta = 0, \)

\( R = R_0 : \quad U = 0, \quad V = 0, \quad \Theta = 0. \)

The following dimensionless variables and parameters are introduced:
Here, $t$ is time, $x$ and $r$ are the spatial coordinates, $u$ and $v$ are the axial and radial particle velocity components, respectively, $\rho$ is density, $p$ is pressure, $T$ is temperature, $c_0$ is the adiabatic speed of sound in the undisturbed medium, $R_g$ is the gas constant, $\nu$ is the kinematic viscosity and $\chi$ is the thermal diffusivity.

To calculate the velocity components of acoustic streaming (time-averaged rate of mass transfer), we use the formulas [6]:

$$
\bar{U}_{st} = \frac{\langle \bar{\rho} U \rangle}{\langle \bar{\rho} \rangle}, \quad \bar{V}_{st} = \frac{\langle \bar{\rho} V \rangle}{\langle \bar{\rho} \rangle},
$$

where $\langle \rstyle \rangle$ indicates averaging over the period $\bar{T} = \frac{2\pi}{\Omega}$, $U_{st}$ is the dimensionless axial velocity component of acoustic streaming and $V_{st}$ is the dimensionless radial velocity component of acoustic streaming.

### 3. Numerical method and parameters of calculations

The calculations were performed using an implicit numerical scheme correct to first order in time and space. Discretization of equations was performed by the control volume method with approximation of the convective diffusion flow by a power law. The numerical method is described briefly in [15] and in more detail on the example of the one-dimensional case in [16]. The following dimensionless parameters were used in the computations described below: $N = 8.6 \cdot 10^{-6}$, $\Gamma = 1.2 \cdot 10^{-5}$, $\gamma = 1.4$, $\tilde{R}_0 = 0.02$. The parameters of the gas corresponded to the thermophysical properties of air at a temperature of 300 K. The dimensional cavity length was 0.005 m. We considered the following vibration frequencies: $\Omega = 0.5, 1, 1.5, 2$ and 2.5; with a dimensionless resonance vibration frequency of about $\pi$. The amplitude of vibration $\tilde{A}$ was varied from 0.01 to 0.5. The computation grid for the vibration frequencies $\Omega = 0.5–2$ had 1002×22 nodes; for the frequency $\Omega = 2.5–1002\times42$ nodes. Thus, no less than 4 nodes were located within the acoustic boundary layer thickness, which is determined in the dimensionless form by the formula $\delta_r = \sqrt{\frac{2N}{\Omega}}$.

### 4. Results and discussion

Let us consider acoustic streaming at the case of weak nonlinearity at the vibration amplitude $\tilde{A} = 0.01$. At $\Omega = 2.5$, the value of the ratio between the cavity radius $\tilde{R}_0$ and acoustic boundary layer thickness $\tilde{\delta}_r$ is 7.6. Thus, acoustic streaming contains both Schlichting and Rayleigh streaming [4,5]. This is illustrated in figure 2(a). With decrease in vibration frequency, the ratio $\tilde{R}_0/\tilde{\delta}_r$ decreases and Rayleigh streaming vortices must be reduced, as compared to Rayleigh streaming vortices, and be disappeared with a further decrease of this ratio. This follows from the theory of acoustic streaming in the absence of heat exchange and was illustrated in [15] for the similar problem formulation at adiabatic boundary conditions. However, in the case of acoustic streaming with heat exchange, Schlichting streaming vortices decrease in size and disappear (figures 2(b-e)) with decrease in vibration frequency in comparison with the resonance frequency.
Figure 2. Streamlines of the streaming flow at $\tilde{A} = 0.01$; (a) – $\Omega = 2.5$, (b) – $\Omega = 2$, (c) – $\Omega = 1.5$, (d) – $\Omega = 1$, (e) – $\Omega = 0.5$.

At the lowest vibration amplitude from the considered range $\tilde{A} = 0.01$, the process intensity is low and the period average temperature, density and pressure coincide with their initial distributions. With the increase in vibration amplitude, the influence of the nonlinearity on the process increases and the distortion of acoustic streaming vortices is observed. Let us show this by the example of the frequency $\Omega = 2.5$. Figure 3 represents the period average distributions of temperature and streamlines of acoustic streaming with increase of the amplitude of vibration. It is seen that acoustic streaming is changed. The vortices, which were initially located near the lateral surface of the cylinder, are moved to the central part of the cavity. Also, additional small vortices are formed. It is seen that the period average temperature is negative (lower than the initial temperature) in the central part of the cavity. Note that the area of negative temperature decreases by increasing the frequency of vibration, but the minimum value of the period average temperature becomes closer to the initial temperature.

The period average density also becomes nonuniform with increasing of vibration amplitude (figure 4). In this case, there is also the area in the central part of the cavity in which the period average density is less than the initial value, but the size of this area changes not so quickly. The minimum value of the period average density decreases with the increase of the amplitude of vibration.
Figure 3. Period average distributions of temperature and streamlines at $\Omega = 2.5$;
(a) – $\tilde{A} = 0.05$, (b) – $\tilde{A} = 0.1$, (c) – $\tilde{A} = 0.2$, (d) – $\tilde{A} = 0.3$.

Figure 4. Period average distributions of density at $\Omega = 2.5$;
(a) – $\tilde{A} = 0.05$, (b) – $\tilde{A} = 0.1$, (c) – $\tilde{A} = 0.2$, (d) – $\tilde{A} = 0.3$.
5. Conclusions
Acoustic streaming in a cylindrical cavity at the vibration frequencies less than the resonant one and isothermal boundary conditions was investigated. When the frequency decreases from the resonance, it is possible to observe a decrease in size of the vortices of Schlichting streaming, as compared to the vortices of Rayleigh streaming, and their subsequent disappearance. For the case of near resonance frequency, it was shown that the enhancement of nonlinearity of the process by increasing the amplitude of vibration leads to the shift of the streaming vortices and the formation of additional vortices. Also, the period average temperature and density became nonuniform.

Acknowledgments
The study was performed by a grant from the Russian Science Foundation (project No. 15-11-10016).

References
[1] Nyborg W L 1965 Acoustic streaming Physical Acoustics vol 2 ed W P Mason (New York: Academic Press) chapter 11 pp 265–331
[2] Rayleigh L 1884 Philos. Trans. R. Soc. London 175 1–21
[3] Zarembo L K 1971 Acoustic streaming High-Intensity Ultrasonic Fields ed L D Rozenberg (New York: Plenum) part III pp 156–64
[4] Hamilton M F, Ilinskii Y A and Zabolotskaya E A 2003 J. Acoust. Soc. Am. 113 153–60
[5] Hamilton M F, Ilinskii Y A and Zabolotskaya E A 2003 J. Acoust. Soc. Am. 114 3092–101
[6] Aktas M K and Farouk B 2004 J. Acoust. Soc. Am. 116 2822–31
[7] Nabavi M, Siddiqui K and Dargahi J 2009 Wave Motion 46 312–22
[8] Daru V, Baltean-Carlès D, Weisman C, Debesse P and Gandikota G 2013 Wave Motion 50 955–63
[9] Reyt I, Daru V, Bailliet H, Moreau S, Valière J-C, Baltean-Carlès D and Weisman C 2013 J. Acoust. Soc. Am. 134 791–1801
[10] Reyt I, Bailliet H and Valière J-C 2014 J. Acoust. Soc. Am. 135 27–37
[11] Gubaidullin A A and Yakovenko A V 2015 J. Acoust. Soc. Am. 137 3281–7
[12] Gubaidullin A A and Yakovenko A V 2014 Thermophysics and Aeromechanics 21 589–99
[13] Gubaidullin A A and Yakovenko A V 2015 High Temp. 53 73–9
[14] Gubaidullin A A and Yakovenko A V 2014 High Temp. 52 264–70
[15] Gubaidullin A A and Pyatkova A V 2016 Acoust. Phys. 62 300–5
[16] Gubaidullin A A and Yakovenko A V 2013 Thermophysics and Aeromechanics 20 277–88