Supplementary Information

Sensitivity to initial conditions

We determine how sensitive the final parameters are to variations in the initial parameters. For a quantity \( A \), we express the difference between its value for model \( i \) \( (A^i) \) and the best model \( (A^1) \) as \( \Delta \log A = \log(A^i/A^1) \), where \( A \) can be an initial property \( (N_0 \text{ or } \rho_{h0}) \) or an observable property. For the latter we use \( N_{\text{cluster}} \) and the number density within the half-light radius: \( \rho_{\text{eff}} = 3 N_{\text{cluster}}/(8 \pi R_{\text{eff}}^3) \). We can write the variation in the final properties in terms of the initial properties as

\[
\begin{pmatrix}
\Delta \log N_{\text{cluster}} \\
\Delta \log \rho_{\text{eff}}
\end{pmatrix}
= \Sigma
\begin{pmatrix}
\Delta \log N_0 \\
\Delta \log \rho_{h0}
\end{pmatrix}
\]

(4)

Here \( \Sigma \) is a matrix that contains the constants that relate variations in initial parameter to variations in the final parameters. We find the four elements of \( \Sigma \) from the two models that are nearest to the observations (i.e. wBH-2, wBH-3 and noBH-2, noBH-3)

\[
\Sigma_{\text{wBH}} = \begin{pmatrix}
-5.74 & -1.56 \\
-7.19 & -1.57
\end{pmatrix},
\Sigma_{\text{noBH}} = \begin{pmatrix}
1.47 & 38.4 \\
-0.71 & 122
\end{pmatrix}.
\]

(5)

Absolute values of 1 mean that a fractional change in an initial parameter leads to the same fractional change in the final parameter. The absolute values > 1 in the left column of \( \Sigma_{\text{wBH}} \) mean that the final properties are most sensitive to \( N_0 \), which is because of the collisional nature of the wBH models. The > 1 value in the right column of \( \Sigma_{\text{noBH}} \) show that the final parameters are most sensitive to the initial density, which is because of the collisionless nature of these models. Taking the inverse of the \( \Sigma \) matrices, and assuming small variations, i.e. \( \Delta \log A \simeq \log(1 + \epsilon_A) \propto \epsilon_A \), with \( \epsilon_A \ll 1 \), we can write

\[
\begin{pmatrix}
\epsilon_{N_0} \\
\epsilon_{\rho_{h0}}
\end{pmatrix}_{\text{wBH}} \simeq \begin{pmatrix}
0.711 & -0.707 \\
-3.26 & 2.60
\end{pmatrix}
\begin{pmatrix}
\epsilon_{N_{\text{cluster}}} \\
\epsilon_{\rho_{\text{eff}}}
\end{pmatrix}
\]

(6)

and

\[
\begin{pmatrix}
\epsilon_{N_0} \\
\epsilon_{\rho_{h0}}
\end{pmatrix}_{\text{noBH}} \simeq \begin{pmatrix}
0.590 & -0.186 \\
0.00345 & 0.00713
\end{pmatrix}
\begin{pmatrix}
\epsilon_{N_{\text{cluster}}} \\
\epsilon_{\rho_{\text{eff}}}
\end{pmatrix}.
\]

(7)

This shows that the level of fine-tuning to obtain the correct \( N_0 \) is similar in both models, albeit more sensitive to variations in \( \rho_{\text{eff}} \) for the noBH models. However, we find different behaviour for the initial density, \( \rho_{h0} \): for the wBH models, variations in \( N_{\text{cluster}} \) and \( \rho_{\text{eff}} \) allow for larger variations in \( \rho_{h0} \), meaning that a relatively large range of initial densities can contribute to the error bars on the present-day properties. The results of the noBH models are extremely sensitive to the initial density, because variations in the observed properties correspond to variations of less than a per cent in the initial density. We can also estimate what fraction of the parameter space of the initial conditions is covered by the uncertainties in \( N_0 \) and \( \rho_{h0} \). We assume that the initial cluster properties are sampled from power-law distributions with indices \(-2\) for \( N_0 \) and \(-1\) for \( \rho_{h0} \) in the ranges \( 10^5 \leq N_0 \leq 5 \times 10^6 \) and \( 1 \leq \rho_{h0}/(M_\odot \text{pc}^{-3}) \leq 10^4 \). Then we find that the initial conditions of wBH-1(noBH-1) that contribute to the error circle cover a fraction \( 1/200(1/3.6 \times 10^5) \) of the initial conditions. Given that the Milky Way has \( \sim 150 \) GCs, this exercise shows that finding Pal 5 is probable in the wBH scenario, while the probability in the noBH scenario is \( 10^{-3} \).
Supplementary Figure 1: **Comparison of stream properties.** Comparison between stream properties of wBH-1 and noBH-1 and the observed stream from Erkal et al.\textsuperscript{3} The stream width (top) of both $N$-body models is similar over the range included in the observations and shows some systematic deviations from the observed width. The density profile (bottom) of the $N$-body models is smoother than the observed profile, which shows signatures of over/under-densities. The decline in the density of the trailing arm (large RA) is faster in wBH-1 than in noBH-1, which agrees more with the observed decline. Shaded regions indicate the 67% confidence intervals.
| Name     | Distance [kpc] | Tail data          | Name     | Distance [kpc] | Tail data          |
|----------|----------------|--------------------|----------|----------------|--------------------|
| NGC 3201 | 4.9            | long tidal tails   | NGC 288  | 8.9            | long tidal tails   |
| NGC 6205 | 7.1            | tidal tails        | Pal 1    | 11.1           | tidal tail         |
| NGC 6341 | 8.3            | long tidal tails   | NGC 6101 | 15.4           | -                  |
| NGC 362  | 8.6            | tidal feature      | NGC 5466 | 16.0           | tidal tails        |
| NGC 6779 | 9.4            | -                  | NGC 5053 | 17.4           | tidal feature      |
| NGC 2808 | 9.6            | -                  | IC 4499  | 18.8           | -                  |
| NGC 5272 | 10.2           | -                  | BH 176   | 18.9           | -                  |
| NGC 4590 | 10.3           | long tidal tails   | Pal 12   | 19.0           | tidal tails        |
| NGC 7078 | 10.4           | -                  | NGC 6426 | 20.6           | -                  |
| NGC 2298 | 10.8           | tidal feature      | Rup 106  | 21.2           | -                  |
| NGC 7089 | 11.5           | -                  | ESO 280  | 21.4           | -                  |
| NGC 5286 | 11.7           | -                  | Ter 7    | 22.8           | -                  |
| NGC 1851 | 12.1           | long tidal tails   | Pal 5    | 23.2           | long tidal tails   |
| NGC 1904 | 12.9           | tidal tails        | IC 1257  | 25.0           | -                  |
| NGC 6934 | 15.6           | -                  | Pal 13   | 26.0           | long tidal tails   |
| NGC 1261 | 16.3           | long tidal tails   | Ter 8    | 26.3           | -                  |
| NGC 5024 | 17.9           | -                  | NGC 7492 | 26.3           | tidal tails        |
| NGC 6981 | 17.0           | -                  | Arp 2    | 28.6           | -                  |
| NGC 4147 | 19.4           | tidal feature      | AM 4     | 32.2           | -                  |
| NGC 6864 | 20.9           | -                  | Pyxis    | 39.4           | -                  |
| NGC 5634 | 25.2           | -                  | Pal 15   | 45.1           | tidal tails        |
| Pal 2    | 27.2           | -                  | Pal 14   | 76.5           | tidal tails        |
| NGC 6229 | 30.5           | -                  | Eridanus | 90.1           | tidal tails        |
| NGC 5824 | 32.1           | -                  | Pal 3    | 92.5           | tidal feature      |
| NGC 5694 | 35.0           | -                  | Pal 4    | 108.7          | tidal feature      |
| NGC 7006 | 41.2           | -                  | AM 1     | 123.3          | -                  |

Supplementary Table 1: Summary of results of stream searches for GCs at distances > 8 kpc from the Galactic center. The classification 'compact' (left) and 'fluffy' (right) is from Baumgardt et al.[22]. All clusters are sorted in distance from the Sun. Among the compact clusters, no tidal tails nor tidal features were found for clusters that are more than 20 kpc away, while they were found for about half of the fluffy clusters beyond 20 kpc.
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