Dynamics of Vortices in Nano-Structured Superconductors with Periodic Arrays of Various Antidots

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Abstract. Stable vortex configurations and its dynamics in superconductors with various types of antidotes are examined, using the molecular dynamics simulation. A pinning potential function, which parameterizes spatial shapes and pinning potential structure of the antidot, is introduced. Dependence of saturation number, which is a maximum number of vortices pinned in a single antidot, on these spatial shape and potential structure are investigated.

1. Introduction

Vortex dynamics and stable structures of vortices in superconductors with antidots are important for application of superconductivity, since introduction of antidots or pinning sites in the superconductors is a useful way for stabilization of the vortices. There have been many studies about these vortex dynamics and structures in superconductors with random [1-3] or periodic [4-6] pinning sites, using the molecular dynamics method.

In the molecular dynamics method, vortices are treated as point particles and phenomenological force on vortices introduced. Usually shapes of pinning sites are assumed circular and the pinning force is assumed to be an elastic force. Because of the anisotropy of the pinning force, square or triangular antidotes have been fabricated, e.g. for a ratchet motion of vortices [7, 8]. In such cases, the shapes of the antidots are not sharp polygons but their corners are rounded. Also, the vertical shape of the pinning potential is not parabolic and they may have a flat bottom. In this study, therefore, we introduce a general form of pinning potential. And we examine how such potential shape difference affect the stable vortex structures and vortex dynamics, using the molecular dynamics simulation.

2. Model

We treat vortices as point particles. They feel strong dissipation and inertia is negligible. Therefore we start following equation of motion of vortices [1-6].

\[ \eta \frac{dr_i}{dt} = f_{ni} + f_{imp} + F_{ad} + f_{vi} + f_{Ti}, \] (1)
where $\eta$ is the viscosity and $f_{di}$, $f_{pi}^{\text{imp}}$, $f_{pi}^{\text{ad}}$, $f_{vi}$ and $f_{Ti}$ are driving force due to the current, the pinning force from impurities and anti-dots, the vortex-vortex interaction from other vortices and the thermal fluctuation force from environment, respectively. The vortex-vortex interaction is given as,

$$f_{vi} = f_0 \sum_{j \neq i} K_i \left( \frac{r_i - r_j}{\lambda} \right) \hat{r}_{ij}$$  \hspace{1cm} (2)

where $K_i$ is the first order modified Bessel function and $\hat{r}_{ij}$ is a unit vector from $j$-th vortex to $i$-th vortex. $f_0 = \Phi_0^2 (8\pi^2 \lambda^3)^{-1}$ is the strength of vortex-vortex interaction, where $\Phi_0 = hc/2e$ is the flux quanta and $\lambda$ is the penetration depth. The pinning force from random impurities is given as,

$$f_{pi}^{\text{imp}} = \frac{f_{pi}^{\text{imp}}}{r_{pi}^{\text{imp}}} \sum_{l=1}^{N_{\text{imp}}} |r_i - r_l^{\text{imp}}| \Theta \left( r_p^{\text{imp}} - \frac{|r_i - r_l^{\text{imp}}|}{\lambda} \right) \hat{r}_{il}$$  \hspace{1cm} (3)

where $r_l^{\text{imp}}$ is the position of $l$-th impurities, $N_{\text{imp}}$ is number of impurities and $\Theta(x)$ is a step function. $f_{pi}^{\text{imp}}$ and $r_{pi}^{\text{imp}}$ are the strength of pinning potential and the radius of the impurity, respectively. We assume circular shape of impurities and an elastic pinning force. The thermal fluctuation force satisfies following conditions,

$$\langle f_{Ti}(t) \cdot f_{Ti}(t') \rangle = 2\eta k_B T \delta(t-t')$$  \hspace{1cm} (4)

**Figure 1.** Shapes of antidots for (a) $v_q = 2$, (b) $v_q = 4$, (c) $v_q = 8$ and (d) $v_q = 1000$.

**Figure 2.** Spatial distribution of pinning potential of antidots for (a) $p_v = 2$, (b) $p_v = 4$ and (c) $p_v = 6$.

In order to consider various types of antidots, we use following pinning potential. For an antidot of which shape is a regular polygon with $2n$ sides ($2n$-gon), the pinning potential is
\[ V_{p}^{ad}(r) = -\frac{1}{p_{v}} \frac{f_{p}}{r_{p}^{n+1}} \left\{ \frac{1}{2} \sum_{i=1}^{2n} \left[ (r - r_{p}) \cdot n_{i} \right]^{v_{q}} \right\}^{\frac{1}{v_{q}}} \Theta \left( r_{p} - \frac{1}{2} \sum_{i=1}^{2n} \left[ (r - r_{p}) \cdot n_{i} \right]^{v_{q}} \right) \] (5)

where \( \{n_{i}\}, r_{p} \) and \( r_{p} \) are normal vectors of sides, the center position and the radius of the antidot, respectively. \( f_{p} \) is the strength of pinning potential and \( p_{v} \) and \( v_{q} \) are parameters determining pinning potential structures. \( v_{q} \) determines the spatial shape of the antidot. When \( v_{q} = 2 \), the shape of the antidot is a circle and when \( v_{q} = \infty \), the shape becomes exactly a regular \( 2n \)-gon. For values of \( v_{q} \) between them, the shape becomes a rounded \( 2n \)-gon. Examples of square antidots are shown in figure 1. \( p_{v} \) determines the potential shape. When \( p_{v} = 2 \), the pinning potential is parabolic and when \( p_{v} \) becomes larger, the potential bottom becomes flat. Examples of potential shapes for square antidots are shown in figure 2.

In this study, we show only results for square shaped antidots.

3. Results

3.1. Stable configuration of vortices in the superconductors with various shaped antidots

First, we investigate the maximum number of pinned vortices at the antidotes, i.e. the saturation number, without external current at temperature \( T = 0.01 f_{0} \lambda / k_{B} \). We set \( r_{p} = 1.4 \lambda \) and \( f_{p} = 2.0 f_{0} \). Nominal saturation number is just \( S / (\pi \lambda^{2}) = 0.63 \). In figure 3 (a)-(b), we show change of maximum numbers of vortices and the stable vortex structures with increasing \( v_{q} \) when \( p_{v} = 2 \). The saturation number increases from 2 to 3 when antidot shape becomes sharper. This is because that effective area of the antidot becomes larger when its corners become shaper. This tendency becomes clearer for \( p_{v} = 4 \) (figure 3 (d)-(f)). The saturation number increases from 2 to 5. For the pinning potential with flat bottom, vortices can be trapped around the corners, and therefore the effective area becomes much larger for sharper shape of antidot.

![Figure 3](image-url)
3.2. Dynamics of vortices in the superconductors with various shaped antidots

Next we show the results for dynamics of vortices. We apply small external current \( f_d = 0.2 f_0 \), which is parallel to the edge of antidots, to the stable configurations in figure 3. In figure 4, the values of \( v_q \) and \( p_v \) are same as those in figure 3. For the antidots with parabolic potential, vortices easily move out. This can be explained from fact that the potential with sharp corners and the flat bottom has larger pinning force around corners.

![Figure 4](image)

*Figure 4.* Vortices start to flow for (b). The orientation of the vortices in the antidotes change and interstitial vortices do not flow for the others. Potential parameters are same as those in figure 3.

### 4. Summary

We have investigated vortex structures and their dynamics using the molecular dynamics simulation. Our results show that sharer shape of antidots, the saturation number of the antidot becomes larger. And the more flat the bottom of the pinning potential is, the saturation number of the antidot becomes larger. Also saturation number is usually greater than \( S / \left( \pi \lambda^2 \right) \), which is determined mainly by the vortex-vortex interaction.

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