Torsional vibrations of shafts of mechanical systems

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Abstract. The aim of the research is to compare the calculated dependencies for determining the equivalent rigidity of a mechanical system and to come to an agreement on the methods of compiling dynamic models for systems with elastic reducer couplings in applied and classical oscillation theories. As a result of the analysis, it was revealed that most of the damage in the mechanisms and their details is due to the appearance of oscillations due to the dynamic impact of various factors: shock and alternating loads, unbalanced parts of machines, etc. Therefore, the designer at the design stage, and the engineer in the process of operation should provide the possibility of regulating the oscillatory processes both in details and machines by means of creating rational designs, as well as the use of special devices such as vibration dampers, various vibrators with optimal characteristics. A method is proposed for deriving a formula for determining the equivalent stiffness of a double-mass oscillating system of a multistage reducer with elastic reducer links without taking into account the internal losses and inertia of its elements, which gives a result completely coinciding with the result obtained by the classical theory of small mechanical oscillations and allows eliminating formulas for reducing the moments of inertia of the flywheel masses and the stiffness of the shafts.

1. Introduction
Calculation of oscillations arising as a result of the dynamic impact of various factors (shock and alternating loads, unbalanced parts of machines, etc.), rotating shafts is a very important part of the calculation of transmissions of tractors, cars, agricultural machines, gear drives and any other mechanisms and machines, having elastic reducer connections. First of all, the calculation is necessary for determining the frequencies, shapes and amplitudes of the free torsional vibrations of the shaft in order to exclude the phenomenon of resonance, in the event of occurrence of considerable deformations and strains in unfavourable circumstances, as well as rapid wear of the structural elements and even their destruction.

The creation of rational designs (the rational choice of the ratios of the elastic inertial characteristics of individual components and gear units), as well as special devices, such as vibration dampers, the selection of the optimum parameters of various vibrators widely used in modern technology, satisfactory for reasons of serviceability and durability, is based on the provisions established by the theory of oscillations [10].

The content of publications [2, 8] forms an applied theory of oscillations, the basis of which is the formula for reducing the stiffness of the shafts and the moments of inertia of the flywheel masses. The purpose of reduction is the simplification of computational processes due to the formal exclusion from the multi-mass system of elements of elastic reducer bonds.

In this case, the development of an equivalent system is based on the energy method, when the value of the kinetic energy of the oscillatory system is used only to calculate the moments of inertia of the flywheel mass, and the potential energy is used only to calculate the reduced shaft rigidities. The problems associated
with this constitute the essence of the problem of constructing adequate dynamic models that reflect the
dynamic phenomena in the reducer with the necessary completeness.

It turned out that because of the variety of different dynamic processes, the solution of such problem in
the general case is ambiguous. However, it is obvious that for its solution, it is expedient to have an
ensemble of correct dynamic models, since the effectiveness of the research largely depends on the validity
of the computational models and the effectiveness of the methods used.

Analysis of the force effects on the elements of multi-mass systems adopted in the applied theory of small
mechanical vibrations [2, 8] shows the inconsistency of some of its provisions with the mechanical notions
of gear reducers, in particular, the ratio of torque at the input and output shafts and the calculated
dependences for determining the equivalent rigidity [3, 4].

2. Results and Discussion
The main goal of the proposed work is to confirm the methods of matching the applied and classical theories
of small mechanical torsional oscillations of the shafts presented in publications [3, 4], using the example of
a two-mass oscillatory system (Fig.).

On the basis of the applied theory of oscillations, the torsional rigidity of the reduction gear shafts are
equal [8]:

\[ C_1 = C_1'; \quad C_2 = \frac{C''_1}{u_{12}}, \]

where \( C_1', C_1'' \) are the torsional rigidity of the reduction gear shafts; \( u_{12} = \frac{z_2}{z_1} \) is the gear ratio [1];
\( z_1, z_2 \) are the number of teeth of the gear wheels. Rigidity of an equivalent system with single-stage gear connection or equivalent torsional stiffness [8] is represented as:

\[ C_{12} = \frac{1}{\frac{1}{C_1} + \frac{u_{12}}{C_2}}. \]  \hspace{1cm} (1)

The classical theory of small mechanical oscillations is based on solutions of the fundamental Lagrange
equation of the second kind for a system with holonomic constraints [7]:

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} = -\frac{\partial \Pi}{\partial \dot{q}_i} \quad (i = 1, 2, 3 \ldots n), \]  \hspace{1cm} (2)

where \( T \) and \( \Pi \) are the kinetic and potential energies of the vibrational system; \( q_i \) is the generalized
cordinate; \( \dot{q}_i \) is the generalized speed; \( n \) is the number of degrees of freedom of the system.

The use of equation (2) for the analysis of the dynamic parameters of a single-mass and two-mass
systems with elastic gear couplings [4] produced the following expression for equivalent torsional rigidity:

\[ C_{12} = \frac{C'_1 C''_1}{u_{12}^2 C'_1 + C''_1} = \frac{1}{\frac{u_{12}^2 C'_1}{C''_1} + \frac{1}{C''_1}}. \]  \hspace{1cm} (3)

It can be seen that the equality of the equivalent torsional rigidity according to (1) and (3) is not observed.
The equivalent rigidity of the system is obtained in another way, comparing the compliance of the real
one (Fig. a) and conditional (equivalent) schemes (Fig.b).
Let us apply a single torque to the flywheel mass with the moment of inertia $J_1$ [5], assuming the flywheel mass with the moment of inertia $J_2$ being stationary.

The shaft with rigidity $C''_{12}$ without taking into account internal losses will be influenced by the torque [6]:

$$T_2 = u_{12}T_1,$$

where $T_1$ is the torque on the shaft with a rigidity of $C'_{12}$.

This will cause its twisting by an angle of $u_{12}/C'_{12}$. In this case, the angle of rotation of the shaft with rigidity $C'_{12}$ due to the action of a single torque is equal to $1/C'_{12}$.

Then taking into account the whole reducer, the angle of rotation of flywheel mass $J_1$ relative to the fixed flywheel mass $J_2$ is equal to:

$$\frac{l}{C'_1} + \frac{z_2}{z_1} = \frac{l}{C'_1} + \frac{u_{12}^2}{C''_{12}}$$

and should be equal to the angle of its rotation on an equivalent shaft equal to:

$$\frac{l}{C_{12}}.$$  

Comparing (4) and (5), one will obtain:

$$C_{12} = \frac{1}{\frac{1}{C'_{12}} + \frac{u_{12}^2}{C''_{12}}}.$$  

Thus, to calculate the equivalent rigidity of the reducer, it is necessary to use equation (3) since it is obtained by two independent methods, which confirms its reliability.

All this indicates a defect in the applied theory of small mechanical torsional vibrations of the systems with elastic reducer links and the need for further development of this theory.

For a two-mass oscillatory system (Fig.), the differential equations of free oscillations on the basis of the applied theory of torsional oscillations will be written as follows:

$$\begin{align*}
J_1\ddot{\phi}_1 + C_{12}(\phi_1 - \phi_2) &= 0, \\
J_2\ddot{\phi}_2 - u_{12}^2C_{12}(\phi_1 - \phi_2) &= 0,
\end{align*}$$

where $J_1, J_2$ are the real moments of inertia; $\phi_1, \phi_2$ are the rotation angles of the flywheel masses; $\dot{\phi}_1, \dot{\phi}_2$ are the angular velocity of the flywheel masses; $\ddot{\phi}_1, \ddot{\phi}_2$ are the angular acceleration of flywheel masses.

On the basis of practical considerations [6], equations (6) will have the form:

$$\begin{align*}
J_1\ddot{\phi}_1 + T_1 &= 0, \\
J_2\ddot{\phi}_2 - T_2 &= 0.
\end{align*}$$

To calculate free frequency $\omega_c$, let us use the particular solutions of equations (6) in the form:

$$\begin{align*}
\phi_1 &= A_1\sin(\omega_c t + \delta), \\
\phi_2 &= A_2\sin(\omega_c t + \delta)
\end{align*}$$

where $A_1, A_2$ are the angular amplitudes of free oscillations of the flywheel masses; $\delta$ is the phase angle of the oscillation phases.
Substituting equations (8) into (6) and solving them, let us obtain the dependence for calculating the free vibration frequency:

$$\omega_c = \sqrt{\frac{C_{12} \left(u_{12}^2 J_1 + J_2 \right)}{J_1 J_2}}.$$

The corresponding value of the amplitude distribution coefficient for the main oscillation is:

$$\eta = \frac{A_2}{A_1} = \frac{C_{12} - J_1 \omega_c^2}{C_{12}} = \frac{u_{12}^2 C_{12}}{u_{12}^2 C_{12} - J_2 \omega_c^2} = -u_{12}^2 \frac{J_1}{J_2}.$$

The ratio of the moments, according to (7), will be equal to $$T = u_{12}^2 T_j$$, which confirms the authors’ criticism [4] that the analysis of the force effects of flywheel masses, conducted from the position of the applied theory of small mechanical vibrations, shows the inconsistency of the torques on the input and output shafts of the gear reducer.

For a double-mass oscillatory system (Fig.) with an elastic reducer connection, the kinetic energy is:

$$T = \frac{1}{2} \left( J_1 \phi_1^2 + J_2 \phi_2^2 \right)$$

and potential energy is:

$$\Pi = \frac{1}{2} C'_{12} (\phi_1 - \phi_1')^2 + \frac{1}{2} C''_{12} (\phi_2 - \phi_2')^2$$

where $$\phi_1', \phi_2'$$ are the angles of rotation of the gears.

From the relation between the moments:

$$C'_{12} (\phi_1 - \phi_1') u_{12} = C''_{12} (\phi_2 - \phi_2'),$$

where kinematic relation $$\phi_1' = u_{12} \phi_2'$$ is taken into account, let us find $$\phi_1'$$ and $$\phi_2'$$:

$$\phi_1' = \frac{u_{12} (C'_{12} \phi_1 + C''_{12} \phi_2)}{u_{12}^2 C'_{12} + C''_{12}}, \quad \phi_2' = \frac{u_{12} (C'_{12} \phi_1 + C''_{12} \phi_2)}{u_{12}^2 C'_{12} + C''_{12}}$$

and substituting them into the expression for the potential energy after the transformations, one obtains:

$$\Pi = \frac{1}{2} \frac{C'_{12} C''_{12}}{u_{12}^2 C'_{12} + C''_{12}} (\phi_1 - u_{12} \phi_2')^2.$$

It is obvious that:

$$\frac{C'_{12} C''_{12}}{u_{12}^2 C'_{12} + C''_{12}} = \frac{1}{\frac{1}{C'_{12}} + \frac{u_{12}^2}{C''_{12}}} = C_{12}.$$

After the transformations, the value of the free oscillation frequency is obtained:

$$\omega_c = \sqrt{\frac{C_{12} \left(u_{12}^2 J_1 + J_2 \right)}{J_1 J_2}}.$$
and the corresponding value of the amplitude distribution coefficient for the main oscillation is:

$$\eta = \frac{A_2}{A_1} = \frac{C_{12} - J_1 \omega_1^2}{u_{12} C_{12}} = \frac{u_{12} C_{12}}{u_{12}^2 C_{12} - J_2 \omega_2^2} = -u_{12} \frac{J_1}{J_2}.$$  

3. Conclusion

It is proved that both approaches to the solution of the problem produce the same expression for the free frequency of oscillations, but the correct value of the amplitude distribution coefficient corresponding to the relation between moments $T_2 = u_{12}T_1$ is obtained only when properly formulated equations are used, which corresponds to the results and conclusions of \cite{3, 4, 9}.

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