Simulation analysis of reflective performance of marking glass beads

Zhengwei Leng, Luwan Wang*, Huayang He and Rui Wang
Research Institute of Highway, Ministry of Transport, Beijing, China

*Corresponding author e-mail: hy.he@rioh.cn

Abstract. Road traffic markings are made of glass beads embedded in paint. To analyze the effects of glass-bead radius, embedment depth, correlated color temperature of light sources, and incident angle on the reflective performance of glass beads, a numerical model is established using the geometric optics method, effective light conditions are analyzed, calculation methods for optical power devised, and non-standard planes of incident light calculated. Calculations were also carried out. Results show that the retroreflective properties of glass beads are positively correlated with the correlated color temperature of the light source and radius of the beads. The glass-bead radii, embedment depths, and incident angles have little effect on their reflective performance. Moreover, the correlated color temperature of the light source is an important factor affecting the accuracy of measuring glass-bead reflective performance. When using retro reflectometers, standard retro reflectance-measuring devices for retro reflectance measurement, and other equipment to evaluate the reflective performance of road traffic markings, the correlated color temperature of the light source of the equipment should be as close as possible to 2856 K.

1. Introduction
Road traffic marking is an important measure that enhances driving safety [1]. Adding glass beads to marking paints is an important means of ensuring night-time visibility of road traffic markings [2]. To ensure the quality of glass beads for road traffic markings and that such beads meet the requirements for the use of retroreflective markings, China has issued GB/T24722, "Glass beads for road markings," and other national standards that stipulate the performance requirements of refractive index, radius, roundness ratio, and density of glass beads [3].

Qi et al. analyzed the roundness ratio, range of radius, matching problem of spreading time, and specific gravity of glass beads, as well as their dispersion and refractive index [4]. Kang et al. reviewed the recent trends in the manufacturing technology of glass beads suitable for autonomous driving age as well as road-marking regulations [5]. Based on Airy’s theory, Bo Jiangkang et al. measured the refractive index of glass beads and deduced the influence of their radius on the refractive index, and proposed that the change in the glass-bead radius has little effect on it [6]. Duan et al. used WebCamera to design and fabricate a visual inspection system for the roundness ratio of glass beads for road markings, and realized the detection of the roundness of such beads [7]. Pike et al. evaluated the retroreflectivity of 19 pavement-marking panels, and found that an increase in bead refractive index had a larger effect on retroreflectivity than did an increase in bead size [8]. Shin et al. assessed the refractive
index of glass beads for use in road-marking applications via retroreflectance measurements[9]. Researchers have mainly focused on the effects of refractive index, roundness ratio, and radius of glass beads on the reflective performance of road traffic markings, and less often discuss other influencing factors, e.g., embedment depth of glass beads in the marking paints, incident angle of detection light, and correlated color temperature.

To analyze the effects of glass-bead radius and embedment depth, correlated color temperature of light sources, and incident angle on the reflective performance of glass beads, a simulation analysis was performed on the reflective performance of glass beads.

2. Theoretical basis and calculation process of GLASS-BEAD simulation

2.1. Basic conditions for glass-bead simulation

Suppose that the light source is a Standard A light source and its correlated color temperature is in the range 2806–2906 K, its incident angle is in the range 75–90°, and the angular distance of incident on the glass-bead surface is 5.73°. The material of the glass beads is BK7 with a refractive index of 1.51. The transmittance for the incident light is 0.9 and the bead radii are in the range 300–600 μm. The embedment depths of the beads in marking paint are in the range 0–30%. The refractive index of the marking paint is approximately 2.5.

2.2. Theoretical basis for simulation

According to the light source, there is no need to consider the coherence. Because the radii of the glass beads are much larger than the wavelength of visible light, geometric optics can be used for simulation. First, given the input point \( P_{in} = (x_{in}, y_{in}, z_{in}) \) on the spherical surface of any glass bead and the direction of the incident light ray \( \theta_{ray} \), the center of the glass bead was set as the origin of the coordinates. Combining the refractive indices of the glass beads and marking paint, the exit point \( P_{out} = (x_{out}, y_{out}, z_{out}) \) of the reflected light ray from the surface of the glass bead, as well as the directions of the incident-light ray \( \theta_{ray} \); and optical path that the light ray travel in the glass bead, \( d_1 \) and \( d_2 \), respectively, can be calculated. The light-propagation path is shown in Figure 1. The light-absorption rate of glass beads can be obtained by combining the properties of the glass beads and marking paint. Finally, the intensity of the reflected light relative to the intensity of the incident light is calculated.

The above process is repeated until all the incident angles and spherical incident point positions are exhausted. For the same incident angle, the light intensities of different spherical incident points entering and exiting the glass beads can be superimposed to obtain the desired situation for light exiting.

![Figure 1. Light propagation path on standard plane.](image)
2.3. Calculation

The geometric relationship indicates that
\[
\vec{R}_\text{in} \cdot \vec{O} = (0, y_{\text{in}}, z_{\text{in}}),
\]
and the direction of the incident angle of the light is
\[
\vec{n}_m = (0, -r \times \cos \theta_{\text{ray}}, r \times \sin \theta_{\text{ray}}).
\]

The incident angle is
\[
\theta_{\text{in}} = \frac{\vec{n}_m \cdot \vec{P}_m \cdot \vec{O}}{\abs{\vec{n}_m} \abs{\vec{P}_m \cdot \vec{O}}},
\]

From Snell's Law, one can obtain
\[
n_0 \cdot \sin \theta_{\text{in}} = n_1 \cdot \sin \theta_{\text{refract}},
\]
\[
\theta_{\text{refract}} = \arcsin \frac{n_0 \cdot \sin \theta_{\text{in}}}{n_1},
\]
so that in the triangle \( P_	ext{in} O P_m \), the geometric relationship is
\[
P_	ext{in} P_m = d_1 = \frac{r}{\sin(\theta_{\text{refract}})} \sin(180 - 2\theta_{\text{refract}}).
\]

In addition, \( P \) is on the sphere surface, and thus
\[
\begin{aligned}
P_	ext{in} P_m &= d_1 = d_2 = \sqrt{(y_m - y_{\text{in}})^2 + (z_m - z_{\text{in}})^2} \\
\tan(\theta_{\text{ray}}) &= -\frac{z_m}{y_m},
\end{aligned}
\]
and the solution is \( P_m = (x_m, y_m, z_m) \).

Similarly, for points \( P_{\text{out}} = (x_{\text{out}}, y_{\text{out}}, z_{\text{out}}) \), by using
\[
\begin{aligned}
P_m P_{\text{out}} &= d_1 = d_2 = \sqrt{(y_m - y_{\text{out}})^2 + (z_m - z_{\text{out}})^2} \\
\tan(\theta_{\text{ray}}) &= -\frac{z_m + z_{\text{in}}}{y_m + y_{\text{in}}},
\end{aligned}
\]
the solution is \( P_{\text{out}} = (x_{\text{out}}, y_{\text{out}}, z_{\text{out}}) \).

3. Calculation condition and method

3.1. Effective light-beam condition

A light ray incident on the surface of a glass bead is not always able to enter the bead. Even if it enters the glass bead, the exiting light ray is not always able to exit in parallel to the incident one. Therefore, it is necessary to determine the effective entrance and exit conditions of light rays.

Suppose the depth of the glass beads in the marking paint is \( h \); that is, the percentage is \( h_{\text{percent}} = (r - h)/r \times 100 \).

As shown in Figure 2, the incident light has a certain width, and the outermost light rays are restricted by the following two conditions.

1. At the highest point: \( \vec{n}_m \cdot \vec{P}_\text{in} \cdot \vec{O} = 0 \) at the tangent.
2. For incident rays above the coating, i.e., \( z_{\text{in}} > (h - r) \).
Figure 2. Method to determine whether or not light is effective.

As the embedment depth is known, if $z_{out} < (h - r)$, the light is blocked by the coating material, so that light is not reflected in parallel and is not received by the detector. If $z_{out} > (h - r)$, the light can exit in parallel and be received by the detector. Thus, the effective incident and reflected light rays are obtained.

3.2. Method of calculation of optical power

The formula for calculating the intensity of the reflected light is

$$I_{out} = I_{in} T e^{-ad_1} \times Re^{-ad_1} T,$$

where $I_{in}$ and $I_{out}$ are the intensities of the incident and reflected light, respectively, and $T$ and $R$ are the light transmittances at the air–glass-bead interface and the reflectivity (reflectance) on the glass-bead–marking-paint interface, respectively. These are affected by the wavelength of the incident light and material comprising the medium. The relevant formula can be obtained by consulting the literature.

3.3. Calculation of incident light from non-standard plane

Figure 3 shows a frontal view of a cross-section of a glass bead. The plane with the largest section area of the glass beads is defined as the standard plane. As shown in the figure, the blue line represents the light incident ray on the standard plane and the orange line the light ray incident on the non-standard plane. The light rays incident on the non-standard plane are more complicated.

Figure 3. Frontal view of cross-section of glass bead.

$$r^2 = x^2 + y^2 + z^2.$$

Letting the normal vector of the plane be $n_t = (n_{tx}, n_{ty}, n_{tz})$, then $\overrightarrow{OP} \perp n_t$ and $\overrightarrow{Pm} \perp n_t$. One can determine the non-standard plane; that is, the refraction-reflection plane that satisfies the following equations,
The above equations can uniquely determine the normal vector $n_t = (n_{tx}, n_{ty}, n_{tz})$ of the non-standard plane, so the problem can be simplified to a standard-plane problem by rotating the non-standard plane. The direction of rotation is $n_t \rightarrow (1,0,0)$. After obtaining the corresponding result, the location of the real exit point can be obtained by rotating the light in the opposite direction.

According to the geometric relationship,

$$\overline{P_{in}O} = (-x_{in}, y_{in}, z_{in}),$$

the direction angle of the incident light is

$$\overline{n_{in}} = (0, -r \times \cos \theta_{ray}, r \times \sin \theta_{ray}),$$

and the incident angle is

$$\theta_{in} = \frac{\overline{n_{in}} \cdot \overline{P_{in}O}}{\|\overline{n_{in}}\| \|\overline{P_{in}O}\|}.$$

According to Snell’s Law,

$$n_0 \cdot \sin \theta_{in} = n_1 \cdot \sin \theta_{refract},$$

$$\theta_{refract} = \arcsin \frac{n_0 \sin \theta_{in}}{n_1},$$

the following geometric relationship exists in the triangle $P_{in} OP_{in}$,

$$P_{in} P_{m} = d_1 = \frac{r}{\sin (\theta_{refract})} \sin (180^\circ - 2\theta_{refract}).$$

Because the points $P_m$ and $P_{out}$ are on the sphere’s surface, $P_{in}$ is $P_{in}$ rotating about the vector $n_t$ (clockwise or counterclockwise) by $180^\circ - 2\theta_{refract}$, and the exit point $P_{out}$ is the point obtained by rotating $P_{in}$ about the vector $n_t$ (in the opposite direction) by $(4\theta_{refract})$.

Thus, $P_m(x_m, y_m, z_m)$ and $P_{out}(x_{out}, y_{out}, z_{out})$ can be solved.

4. Simulation results and analysis

4.1. Overall numerical calculation process

The overall numerical simulation calculation process proceeds as follows. (1) Define the light source. (2) Select an incident direction angle $\alpha$. (3) Select the radius $r$ of the glass bead. (4) Select the glass-bead embedment depth in percent, $h_{\text{percent}}$, and determine the height of the lowest entry and exit positions of the entry and exit points as $z_{\text{min}} = h_{\text{percent}} \times r$. (5) Select the point in the incident direction $P_{in} = (x_{in}, y_{in}, z_{in})$. (6) Determine whether or not the selected point is in the incident direction; if not, set the result to 0 and return to (5); if yes, continue. (7) Calculate the position of exit. (8) Determine whether or not the calculated position is an exit; if not, set the result to 0 and return to (4); if yes, continue. (9) Calculate the position of the reflection point and the optical path within the glass bead. (10) Determine the type of reflection cross-section (type 1 or type 2). (11) Calculate the intensity of the reflected light and save the result. (12) Determine whether or not all the light incident points are included; if not, return to (5), and if yes, return to (4). (13) Determine whether or not all the embedment depths are included; if not, return to (4), and if yes, return to (3). (14) Determine whether or not all the radii are
included; if not, return to (3), and if yes, return to (2). (15) Determine whether or not all the directions are included; if not, return to (2), and if yes, continue. (16) End the calculation.

The format of the saved data in this study is \([\lambda, \alpha, r, h_{\text{percent}}, I_{\text{out}}]\).

4.2. Numerical calculation results

Numerical simulation results are shown in Figures 4–7. It can be seen from Figure 4 that the relationship between the intensity of the reflected light and the degree of embedment is nonlinear. When the degree of embedment is less than 15%, the glass-bead reflective ability and degree of embedment are positively correlated. When the degree of embedment is greater than 15%, the reflective ability of glass beads and the degree of embedment are negatively correlated. When the degree of embedment is in the range 5–30%, the ratio of the maximum and minimum values of the reflected light intensity is close to 1. In this case, the degree of embedment is not the main factor affecting the reflective performance of glass beads.

![Figure 4](image1.png)

**Figure 4.** Ratios of reflected-light intensity to that of incident light of glass beads with different degrees of embedment when light is incident normally to marking paint.

It can be seen from Figure 5 that the intensity of the reflected light is positively linearly correlated with the correlated color temperature. For the design color temperature of 2856 K, when the absolute deviation of the correlated color temperature is 50 K, the absolute deviation of the intensity of the reflected light can reach 20%. Therefore, when using retroreflectance meters, standard retroreflective measurement devices, and other equipment to evaluate the reflective performance of road traffic markings, the correlated color temperatures of the equipment light sources should be as close as possible to 2856 K.

![Figure 5](image2.png)

**Figure 5.** Ratios of reflected-light intensity to that of incident light with different color temperatures of glass beads when light is incident normally to marking paint.
It can be seen from Figure 6 that the relationship between the intensity of the reflected light and the bead radius is linear, and the intensity of the reflected light relative to the incident light has a positive correlation with the bead radius. When the bead radius is in the range 300–600 μm, the ratio of the maximum and minimum values of the reflected light intensity is close to 1, and the bead radius is not the main factor affecting the reflective performance of the glass beads.

Figure 6. Ratios of reflected-light intensity from glass beads of different radii to that of incident light when light is incident normally to marking paint.

It can be seen from Figure 7 that in the process of increasing the incident angle, the intensity of the reflected light relative to the incident light first decreases rapidly and then slowly, and then increases slowly. Generally speaking, a small incident angle is conductive to light exiting. In the incident-angle range 75–90°, the ratio of maximum to minimum reflected light is approximately 1.007. Obviously, the incident angle is not the main factor that affects the reflective performance of glass beads.

Figure 7. Ratios of reflected-light intensity to that of incident light at different incident angles.

5. Conclusion
The relationship between the reflective performance of glass beads and the degree of embedment is not linear and the maxima exist in a certain range. Small incident angles are conducive to light exiting. The reflective performance of glass beads is positively correlated with the glass-bead radii and the correlated color temperature of the light source. Overall, the radius and embedment depth of the glass beads, as well as the incident angle, have little effect on the reflective performance of glass beads, while the
correlated color temperature of the light source is an important factor affecting the measurement accuracy of the reflective performance of glass beads.

References
[1] Guo YY, Liu P, Wu Y, Yu H. Evaluation of freeway traffic safety facility system based on attribute recognition [J]. Journal of Southeast University (Natural Science Edition), 2013,43(06):1305-1311. (in Chinese) doi:10.3969/j.issn.1001-0505.2013.06.032.
[2] Guo DH, Ma J, Ma XF. Status and trend of test techniques on glass beads for road markings[J]. Journal of Safety Science and Technology, 2011,7(11):174-180. (in Chinese) doi: 10.3969/j.issn.1673-193X.2011.11.034.
[3] Guo DH. Research on glass beads for road markings[J]. Communications Standardization, 2010(20):13-15. (in Chinese) doi:10.3869/j.issn.1002-4786.2010.20.004.
[4] Qi XJ, Guan LS, Zhang C. The analysis of the influencing factors for pavement marking reflective performance [J]. Shanxi Science & Technology of Communications, 2014(05):93-95+103. (in Chinese).
[5] Kang Byeongguk, Kang Seunggu. Trend in glass bead and regulation of road marking, and suggestions for preparing an autonomous vehicle age. 2019, 29(5):229-237.
[6] Bo JK, Li DH, Guo DH, Lv H, Liu X, Wang QH. Relationship between size and refractive index of the high refractive glass beads [J]. Optics & Optoelectronic Technology, 2011,9(06):45-49. (in Chinese) doi:10.3969/j.issn.1672-3392.2011.06.010.
[7] Duan ML, Xiao Q, Yang JQ, Liu C. Vision detecting system of roundness of glass beads based on WebCamera for pavement marking [J]. Transducer and Microsystem Technologies, 2011,30(10):93-95+98. (in Chinese) doi:10.3969/j.issn.1000-9787.2011.10.029.
[8] Adam M. Pike, Songjukta Datta. Effect of Glass Bead Refractive Index on Pavement Marking Retroreflectivity Considering Passenger Vehicle and Airplane Geometries. 2020, 2674(10):438-447.
[9] Sang Yeol Shin, Ji In Lee, Woon Jin Chung, et al. Assessing the Refractive Index of Glass Beads for Use in Road-marking Applications via Retroreflectance Measurement. 2019, 3(5):415-422.