Modeling of energy transfer between two crossing smoothed laser beams in a plasma with flow profile

A. Colaitis\textsuperscript{1}, S. H"uller\textsuperscript{2}, V. T. Tikhonchuk\textsuperscript{1}, D. Pesme\textsuperscript{2}, G. Duchateau\textsuperscript{1} and A. Porzio\textsuperscript{2,3}
\textsuperscript{1} Centre Lasers Intenses et Applications, Université de Bordeaux CNRS CEA, Talence, France
\textsuperscript{2} Centre de Physique Thorique, CNRS Ecole Polytechnique, Palaiseau, France
\textsuperscript{3} LAGA, Institut Galilée, Université Paris 13 CNRS, Villetaneuse, France
E-mail: hueller@cpt.polytechnique.fr

Abstract. We study the crossed beam energy transfer (CBET) between laser fields generated by optical smoothing methods. The energy transfer, as well as the angular distribution of the outgoing light fields are investigated for two incident smoothed laser beams in a plasma with a flow gradient, allowing for resonant transfer close to the sonic point. Simulations with the code \textsc{Harmony} based on time-dependent paraxial light propagation are compared to simulations using a new approach based on paraxial complex geometrical optics (PCGO). Both approaches show good agreement for the average energy transfer past a short transient period, which is a promising result for the use of the PCGO method as a module within a hydrodynamics code to efficiently compute CBET in mm-scale plasma configurations. Statistical aspects related to role of laser speckles in CBET are considered via an ensemble of different phase plate realizations.

1. Introduction
For both Direct Drive and Indirect Drive concepts of laser fusion, a reliable modeling of laser propagation is of crucial importance. Such a modeling has to take into account the potential energy exchange between laser beams, and between groups of laser beams.

For this purpose we show here results of a comparison between two approaches to model crossed beam energy transfer (CBET), namely (i) a standard modeling of laser plasma interaction using a paraxial wave solver coupled to non linear fluid code for the plasma motion and (ii) a new approach that is based on paraxial complex geometrical optics (PCGO) and which can be implemented in radiation hydrodynamics codes. In this approach the path of multiple beamlets is integrated by a set of model equations [1, 2] for each time step of the hydro code.

In particular we investigate the crossing of laser beams, whose fields have been generated by optical smoothing methods as random (or kinoform) phase plates (RPP, KPP). The energy transfer considered in this comparison corresponds hence to a realistic situation that may appear in laser fusion configurations where wide beams with speckle structure overlap in a plasma profile [3]-[8]. We concentrate to one of the most likely configurations where resonant exchange between beams of equal laser frequency occurs, namely in a plasma with a flow gradient in the vicinity of sonic point. We compare the amplification of the beam that receives transfer from its counterpart between both models and analyze the outgoing laser fields behind the crossing zone with respect to their angular distribution and direction, and we investigate the statistical role of speckles.
2. Comparison: paraxial complex geometrical optics vs. paraxial propagation

We have carried out simulations in a two-dimensional (2D) box with 4000 wavelengths in length and 1800 wavelengths in width. Two laser beams are incident from the left-hand-side boundary, as shown in Fig. 1, with an angle of ±10°, hence the angle θ = 20° between both beams. Beam 1 with the wave vector \( \mathbf{k}_1 \) and frequency \( \omega_1 \) is denoted as the pump beam, beam 2 with \( \mathbf{k}_2 \) and \( \omega_2 \) as the probe beam. As probe to pump ratio we denote the ratio between the average intensity (taken over the beam width) of both beams, \( 1:1 \), \( 1:8 \), and \( 1:64 \) are discussed here. The overlap of both beams forms a rhombus in the central part of the simulation box in which the plasma density is homogeneous along the common beam axis \( x | (\mathbf{k}_1 + \mathbf{k}_2) \). In the direction of the difference wave vector \( \mathbf{k}_1 - \mathbf{k}_2 \) the electron density follows a parabolic shape \( n_e(y) = n_0 \max\{0, 1 - [(y - 900\lambda)/1580\lambda]^2\} \) and has a flow with gradient as \( V_p(y) = c_s(y - 657\lambda)/200\lambda \), with \( n_0 = 0.1n_e \) standing for the maximum electron density, \( c_s \) the ion sound velocity, and \( \lambda \) for the vacuum wave length. The critical density \( n_c \) and the wavelength \( \lambda \) correspond to beam 1. In our simulations beam 2 has the same frequency as beam 1, \( \omega_1 = \omega_2 \). The resonance conditions for crossed beam energy transfer correspond to stimulated Brillouin scattering; the matching conditions for frequency and wave vector between the ponderomotively driven ion acoustic density perturbations and the two overlapping laser waves, \( \omega_s - \mathbf{k}_s \cdot \mathbf{V}_p = \omega_1 - \omega_2 \) and \( \mathbf{k}_s = \mathbf{k}_1 - \mathbf{k}_2 \), are fulfilled for a flow profile with velocities close to the sound speed \( c_s \) inside the rhombus of the crossing beam, as chosen in our simulations.

We have computed the energy transfer with two approaches, namely a time-dependent paraxial code, coupled to fluid equations for the plasma fluid, with the code HARMONY [9, 10] and with the paraxial geometrical optics approach, based on a stationary CBET model [2, 11, 12]. The amplification in power, \( T = P_{\text{probe, out}}/P_{\text{probe, in}} \) of the probe beam at the right-hand-side boundary (“out”) with respect to its incident power (“in”) is shown in Fig. 2 for three different probe:pump ratios and as a function of the overlapped flux of the incident beams in their focal planes, \( I_1 \lambda^2 \left[10^{14} \text{W cm}^{-2} \mu \text{m}^{-2}\right] \). The values from HARMONY have been taken after a transient period, see Fig. 2, lasting a few oscillation periods of \( 2\pi/\omega_s = \lambda/(2c_s \sin \theta/2) \). In the range of \( I_1 \lambda^2 \) shown here the agreement between both models is very good. The values obtained are also in relatively good agreement with the model developed by McKinstrie et al [12]. Assuming that the SBS gain is determined by inhomogeneous plasma flow with \( L_v = |V_p/\nabla V_p| \), given by \( G \approx 3.5(L_v/200\lambda)(n_e/0.1n_c)I_{14} \lambda^2/T_e(\text{keV}) \), at \( \theta = 20° \), the amplification in the probe beam

![Figure 1](image1.png)

**Figure 1.** Scheme of two crossing laser beams in a plasma profile with flow. Both the pump and the probe are smoothed laser beam. Ponderomotively driven density perturbations arise where both beams overlap.

![Figure 2](image2.png)

**Figure 2.** Probe amplification computed from simulations with the PCGO model (green curve) and with HARMONY (black) as a function of the overlapped beam flux \( I_{14} \lambda^2 \).
power is delimited by $\mathcal{T} = (\mathcal{R} \mathcal{G})^{-1} \log [1 + e^{\mathcal{G} (\mathcal{R} \mathcal{G} - 1)}]$ with $\mathcal{R}$ as the probe:pump ratio (see [12]). The latter eventually confirms the CBET algorithm chosen in the PCGO approach (see [2, 11]).

Both approaches use smoothed laser beams. Beam speckle patterns are generated by random numbers and therefore realization dependent. For the HARMONY simulations the beam shapes correspond to RPP speckle pattern in 2D while the beams in the PCGO mimic a speckle pattern shape by superposition of numerous (here: 100) beamlets, having usually a greater radius than laser speckles defined by the focusing $f$-number (here $f = 7$). Error bars indicate the standard deviation around the ensemble average due to the spread from different (here: 16) realizations.

Also the angular distributions of the outgoing fields have been compared, where we also find good agreement between both approaches. Figures 3 and 4 show that due to pump depletion, which transfers gradually its flux to the probe beam, the angular distribution changes with increasing probe amplification $\mathcal{T}$. Figure 3 shows both the distribution of 5 different RPP realizations and for a ‘regular’ beams without speckles, being closer to the PCGO results in Fig. 4 (also shown here for ‘regular’ beams). While the overall tendency is the same, the differences illustrate the role of speckles. Ensemble average ‘center of mass’ values in angle $\langle \vartheta \rangle$ of the distributions are computed from 16 realizations, listed together with the standard deviation in Table 1. The angular dispersion $\sigma_\vartheta$ from the ensemble average due to the speckles is still dominated by the angular width of the smoothed beams, which is more pronounced for RPP than for PCGO beams where the width depends on both the number and the size of beamlets chosen. It is remarkable that the center of mass of the pump beam is systematically shifted due to depletion, which gives rise to additional angular dispersion. The statistical standard deviation $\sigma_\mathcal{T}$ in $\mathcal{T}$ observed in the HARMONY simulations, Figs. 2 and 5, is proportional to the probe amplification itself, so that the relative deviation $\sigma_\mathcal{T}/\mathcal{T}$ is stable for a fixed $f$-number. Figure 5 illustrates the importance of the $f$-number: for small speckles with $f = 5$, the relative dispersion is reduced to $<10\%$ with respect to $13\%$ for larger speckles with $f = 7$. Note that $\sigma_\mathcal{T}/\mathcal{T}$ is overestimated in 2D simulations compared to 3D geometry. However, the greater angular spread in speckle beams compared to regular beams is an important feature both in 2D and 3D. For successive beam crossings, the importance of speckles to both the energy transfer and the beam direction may increase, depending on the number of encounters $n$ and the strength

![Figure 3. From HARMONY: angular distribution of transmitted pump and probe beams, with ‘regular’ beams without speckles, for $I_{14} \lambda^2 = 0.45$ (green), 0.9 (red), and 1.8 (blue). Blue thin lines: smoothed distributions corresponding to 5 RPP realizations at $I_{14} \lambda^2 = 1.8$.](image1)

![Figure 4. From PCGO: angular distribution of pump and probe beams, with ‘regular’ beams without pseudo-speckles [1], for $I_{14} \lambda^2 = 0.45$ (green), 0.9 (red), and 1.8 (blue), corresponding as in Fig. 3 to probe amplification values $\mathcal{T} = 2.8$, 4, and 5, respectively.](image2)
Table 1. Expectation values $\langle \vartheta \rangle$ and standard deviation $\sigma_\vartheta$ in angle of the transmitted pump and probe beams, for indicated values of overlapped laser flux $I_{14}\lambda^2$, and the corresponding probe amplification $T$, computed from 16 different realizations. The angles of the incident light are $\vartheta_{\text{pump}} = -\vartheta_{\text{probe}} = \theta/2 = 10^\circ$, each with an angular width $\pm (1/2) \tan^{-1}(1/2f) \approx \pm 2^\circ$ for $f=7$.

| $I_{14}\lambda^2$ | $T$ | $\langle \vartheta \rangle$ | $\sigma_\vartheta$ | $\langle \vartheta \rangle$ | $\sigma_\vartheta$ |
|------------------|-----|-----------------|-----------------|-----------------|-----------------|
| 0.45             | 2.8 | -9.7$^\circ$    | $\pm 1.7^\circ$ | 9.0$^\circ$    | $\pm 1.9^\circ$ |
| 0.9              | 4.0 | -10.0$^\circ$   | $\pm 2.2^\circ$ | 8.7$^\circ$    | $\pm 2.3^\circ$ |
| 1.8              | 5.0 | -10.8$^\circ$   | $\pm 2.5^\circ$ | 8.6$^\circ$    | $\pm 3.0^\circ$ |

Figure 5: Time history of probe amplification $(\pm \sigma_T)$ of 16 RPP realizations with HARMONY, for $f$-numbers $f = 5$ (red) and $f = 7$ (green); probe:pump = 1:8, at $I_{14}\lambda^2 = 1.8$.

of the transfer. Assuming independence of the successive beam crossing, the variance $\sigma_T^2$ will add up over all encounters, $\sum_{i=1}^{n} \sigma_{T,i}^2$. By disregarding decrease of $\sigma_{T,i}$ (e.g. pump depletion), we can estimate that the deviation due to speckle contributions increases with $\sqrt{n}$.

3. Conclusions
The comparison between simulations with HARMONY and the paraxial complex geometrical optics approach shows good agreement concerning the energy transfer, and the angular distribution of the transmitted fields. The PCGO approach is computationally less expensive and can be implemented in radiation hydro codes in a similar manner as standard ray tracing methods, so that PCGO can be efficiently used for mm-scale simulations for ICF design. We find that CBET from smooth laser beams is mainly governed by the parameters characterizing the average beam. Depletion of the pump beam that transfers energy to the probe beam leads to a modification of the angular distribution, and in particular to an angular deviation of the pump beam. The contribution to CBET due to speckles decreases with the number of speckles involved in the crossing volume, which depends on the focusing $f$-number.

Acknowledgments
This work was carried out within the framework of the EUROfusion Consortium. It has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053, and from the Agence Nationale de Recherche, project no. ANR-12-BS04-0006. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References
[1] Colaitis A et al 2015 Phys. Rev. E 91 013102
[2] Colaitis A 2015 Ph D thesis Université de Bordeaux, http://www.theses.fr/en/2015BORD0253
[3] Kirkwood R K et al 1996 Phys. Rev. Lett. 76 2065
[4] Krue W L et al 1996 Phys. Plasmas 3 382
[5] Michel P et al 2009 Phys. Rev. Lett. 102 025004 and Phys. Plasmas 16 042702
[6] Igumenshchev I V et al 2010 Phys. Plasmas 17 122708
[7] Igumenshchev I V et al 2012 Phys. Plasmas 19 056314
[8] Myatt J F et al 2014 Phys. Plasmas 21 055501
[9] Huller S et al 2006 Phys. Plasmas 13 22703
[10] Masson-Laborde P E et al 2014 Phys. Plasmas 21 032703
[11] Colaitis A et al 2016 Phys. Plasmas 23 032118, http://dx.doi.org/10.1063/1.4944496
[12] McKinstrie C J et al 1996 Phys. Plasmas 3 2686