Optimal Discrete Beamforming of Reconfigurable Intelligent Surface

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Abstract—Utilizing reconfigurable intelligent surface (RIS) for communication service usually leads to non-convex optimization problems. Existing methods either suffer from significant scalability issues or lead to local optima. This paper focuses on optimal beamforming in RIS-aided single input single output (SISO) communications. We formulate the discrete beamforming into a discrete product maximization problem, a fundamental yet unexplored problem. A highly efficient divide-and-sort (DaS) search framework is developed. The proposed approach is guaranteed to find global optima with linear search complexity, both in the number of discrete levels and the length of vectors. This approach is seen as particularly effective for large scale problems. Numerical and experimental studies about the effectiveness and speed of DaS are also presented. Extensive trials show that, for moderate resolution quantization, e.g., 4-bits and above, there seems to be no noticeable difference between continuous and discrete phase configuration.

Index Terms—Reconfigurable intelligent surface, discrete, optimal phase configuration, inner product maximization.

I. INTRODUCTION

The reconfigurable intelligent surface (RIS) technique has recently demonstrated its great potential for reconfiguring wireless propagation environments [1]–[8]. The advantage, as compared to other competitive technologies, lies in the fact that RISs provide opportunities for the so-called passive relays. RISs consist of a large number of carefully designed electromagnetic cells and result in electromagnetic fields with controllable behaviors such as amplitude, phase, and polarization. It has recently been shown that RIS is a crucial enabler and presents a paradigm shift for future wireless networks.

RIS might be the solution to coverage enhancement through signal-to-noise ratio (SNR) improvement and interference suppression. To do that, we must develop efficient algorithms to find suitable phase configurations for all unit cells. A key issue for algorithm design is the highly non-convex unit modulus constraint induced by the phase shifter [4]–[7]. Various methods have been developed to tackle the unit modulus constraints, including, e.g., semi-definite relaxation (SDR) [8]–[12], manifold optimization (Manopt) based algorithms [10], [13]–[15], etc. These approaches produce continuous phase configurations that take any value within [0, 2π).

From an implementation viewpoint, one of the main obstacles is the phase quantization effects. The phase shifter takes only several discrete values. Extremely low resolution quantization schemes are employed in most existing prototypes [16]–[22]. The 1-bit phase quantization scheme is widely adopted due to its simplicity. It remains a challenging task to design highly efficient algorithms for optimal discrete beamforming in RIS-aided systems. There are two possible methodologies to solve it.

The first class, taking a continuous perspective, relaxes the discrete problem to the continuous and then performs discretization on the continuous beamforming solution. The main disadvantage of this heuristic strategy is the inherent quantization error, especially for extremely low resolution quantization schemes. It has recently been pointed out that this strategy can lead to an arbitrarily lousy performance in the worst-case scenario [23].

On the other hand, taking a discrete perspective, the second class develops efficient algorithms that run over the search space. A natural but intractable approach is to perform an exhaustive search with exponential complexity. Several acceleration methods that lead to locally optimal solutions were studied in [24], [25]. An approximation (APX) algorithm achieving near optimality for high resolution quantization schemes was proposed in [23].

To the best of our knowledge, no studies have been performed to design highly efficient search algorithms resulting in discrete phase configurations with global optimality. This paper focuses on optimal discrete beamforming in RIS-aided single input single output (SISO) communication systems. To handle it, we reformulate the discrete beamforming problem into a discrete inner product maximization problem, a fundamental yet unexplored problem. Then, a concise search framework is proposed to solve it.

A. Contributions

The main contributions of the paper can be summarized as follows:

- We focus on the beamforming in RIS-aided SISO system and formulate the discrete beamforming problem as a discrete inner product maximization problem, under the modeling of the classic cascaded channel.
- We propose an efficient algorithm to obtain the global optimal solution to the maximization problem, as the quantization scheme of RIS is 1-bit. The proposed algorithm has linear complexity \(O(N \log N)\).
- For more general, we provide a mathematically concise search framework, named divide-and-sort (DaS), for the
optimal solution of a class of discrete inner product maximization problems in low-bits RIS. The proposed approach (i) is guaranteed to find the global optimal discrete solution for the inner product maximization problem, and (ii) has only linear search complexity, both in the number of discrete levels and the length of vectors.

- We introduce other different channel models and formulate optimization problems to maximize the received signal power respectively. We prove that the proposed DaS method can be applied to such typical optimization problems by always transforming them into homogeneous quadratic programming problems.

- The SNR performance of the proposed approach is evaluated through numerical simulations and experiments. In simulations, we compare the performance with other optimization-based methods. The proposed DaS is superior to other methods and is seen as particularly effective for large scale problems. Extensive numerical simulations show that, for moderate resolution quantizations, e.g., 4-bits, there seems to be no noticeable difference between continuous and discrete phase configurations. Experimental results further prove the effectiveness of the proposed algorithm.

B. Outline

The remainder of the paper is organized as follows. We start with the models for signal enhancement in Section II. The beamforming of RIS could be formulated as an inner product maximization problem. In Section III, we focus on the 1-bit RIS and propose a divide-and-sort strategy to solve the binary discrete inner product maximization. A mathematically concise search framework is generalized and proposed to solve discrete inner product maximization problems in Section IV named ‘DaS’. We then extend the proposed DaS method to other discrete beamforming problems under various channel models in Section V. Section VI is devoted to numerical and experimental testing to evaluate the performance of the proposed algorithm. Finally, we conclude the paper in Section VII.

C. Reproducible Research

The simulation results can be reproduced using code available at: https://github.com/RuijingXiong/RIS_Optimization.git

D. Notations

The imaginary unit is denoted by $j$. Let $|\cdot|$ and $\mathbb{R}\{\cdot\}$ denote the modulus and real part of a complex number. Unless otherwise stated, lower and upper case bold letters stand for vectors and matrices. We use $A^H$ and $A^T$ to denote the conjugate transpose and transpose of $A$, respectively. Let $\|\cdot\|$ represent the Frobenius norm of a vector, and $\odot$ represent the Hadamard product.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider RIS-aided SISO communication scenarios in Fig. 1. A RIS consisting of $N$ passive cells is employed to improve the received signal power at the user. The received signal at the user can be written as

$$y = (h_s^H \mathbf{W} h_s + h_d)x + s,$$

(1)

where $x \in \mathbb{C}$ and $y \in \mathbb{C}$ are the transmitted and received signal, $\mathbf{W} = \text{diag}(e^{j\omega_1}, e^{j\omega_2}, \ldots, e^{j\omega_N}) \in \mathbb{C}^{N \times N}$ is the phase configuration matrix related to the RIS, and $s$ is the additive Gaussian white noise. The equivalent channel between BS-RIS, RIS-USER, and BS-USER links are denoted by $h_s \in \mathbb{C}^N$, $h_d^H \in \mathbb{C}^N$, and $h_d$, respectively.

B. Problem Formulation

For RIS-aided coverage enhancement, the goal is to maximize the received signal power, which can be formulated as

$$\max_{x, \mathbf{W}} \left| (h_s^H \mathbf{W} h_s + h_d)x \right|^2,$$

subject to $0 \leq \omega_i < 2\pi, \quad \forall i = 1, \ldots, N.$

(2)

For given phase configurations, it can be verified that the maximum-ratio transmission (MRT) is the optimal transmission solution [26]. Problem (2) can thus be simplified as

$$\max_{\mathbf{W}} \left\| h_s^H \mathbf{W} h_s + h_d \right\|^2,$$

subject to $0 \leq \omega_i < 2\pi, \quad \forall i = 1, \ldots, N.$

(3)

Let $\phi = \text{diag}(h_s^H h_s)$. We write (3) as

$$\max_{\mathbf{w}} \mathbf{w}^H \phi \mathbf{w} + h_d \phi^H \mathbf{w} + \mathbf{w}^H \phi h_d^H + |h_d|^2,$$

subject to $0 \leq \omega_i < 2\pi, \quad \forall i = 1, \ldots, N.$

(4)

Here $\mathbf{w} = [e^{j\omega_1}, e^{j\omega_2}, \ldots, e^{j\omega_N}]^H$. Due to the non-convex unit modulus constraints induced by the phase shifts, (4) is a non-convex quadratic programming problem. If we let $\tilde{\mathbf{w}} = [e^{j\omega_1}, e^{j\omega_2}, \ldots, e^{j\omega_N}]^H$ and $\tilde{\phi} = [\phi^T, h_d]^T$, (4) could be turned into a standard homogeneous quadratic programming problem

$$\max_{\mathbf{w}} \tilde{\mathbf{w}}^H \tilde{\mathbf{R}} \tilde{\mathbf{w}},$$

subject to $0 \leq \omega_i < 2\pi, \quad \forall i = 1, \ldots, N.$

(5)

Here $\tilde{\mathbf{R}} = \tilde{\phi} \tilde{\phi}^H$. We emphasize that $\tilde{\mathbf{R}}$ is a semi-positive definite matrix with rank one.

As mentioned earlier, the phase configuration is selected from a finite number of discrete values in practical implementations. For convenience, we denote by $u$ the discrete
configuration set \(\{0, \Omega, \ldots, (2^B - 1)\Omega\}\), where \(B\) is the phase quantization level and \(\Omega = \frac{2\pi}{2^B}\). Problem (5) may be rewritten as

\[
\max_{\omega_1, \ldots, \omega_N \in u} \tilde{w}^H \tilde{R} \tilde{w}.
\]

(6)

It is worth pointing out that if \(R\) is given a solution \(\tilde{w}_{\text{opt}}\) to (5), the optimal discrete phase configuration to the original coverage enhancement problem is given by \(\tilde{w}_{\text{opt}} = \tilde{w}_{\text{opt}}(1 : N)/\tilde{w}_{\text{opt}}(N + 1)\).

In some scenarios, the line of sight between BS and users may be blocked by obstacles, as shown in Fig. 1(b). We could set \(h_d = 0\). The problem of achieving the maximum received signal power is reduced as

\[
\max_{\omega_1, \ldots, \omega_N \in u} w^H R w,
\]

(7)

where \(R = \phi \phi^H\) is semi-positive definite and of rank one, again. A comparison of (6) and (7) shows that there is no significant difference except for the length of optimization variables. Without loss of generality, the next section focuses on efficient algorithms to solve the problem (7).

C. Discrete Inner Product Maximization

The problem (7) could be reduced further. It is well-known that, for any semi-positive definite matrix \(R\) of rank one, it has

\[
R = \lambda z z^H,
\]

where \(\lambda > 0\) is the only non-zero eigenvalue and \(z \in \mathbb{C}^N\) is the associated eigenvector. Thus, problem (7) becomes

\[
\max_{\omega_1, \ldots, \omega_N \in u} w^H \lambda z z^H w,
\]

which is equivalent to

\[
\max_{\omega_1, \ldots, \omega_N \in u} |w^H z|,
\]

(8)

The combinatorial optimization problem (8) is actually a discrete inner product maximization problem. In principle, any search algorithm can be used to solve it. An immediate but less efficient approach is to perform an exhaustive search of order \(O(2^B N)\) among all elements of \(w^N\). It is worth noting that the optimal solution to (8) is not unique. We see at once that, for any solution \(w_{\text{opt}}\), \(e^{jB} w_{\text{opt}}\) is also the optimal solution. Therefore, it is possible to develop efficient algorithms that rule out large parts of the search space.

III. Binary Phase Beamforming with 1-bit Quantization Scheme

We first consider the 1-bit RIS, in where the discrete configuration set is \(u = \{0, \pi\}\), each element in unknown vector \(w\) take the possible value \(w_i \in \{\pm 1\}\), \(i = 1, \ldots, N\). Perform a greedy search among all elements of \(\{\pm 1\}^N\) with time complexity \(O(2^N)\) is a natural but inefficient work. In this section, we propose a prompt and efficient binary beamforming algorithm to obtain the optimal solution, with a linear complexity \(O(N \log N)\).

For RIS with discrete configurations \(\{0, \pi\}^N\), we can first rewrite the optimization problem in (3) as

\[
\max_{\omega_1, \ldots, \omega_N \in [0, \pi]} |w^T z|.
\]

(9)

We observe that the optimal objective function value \(w^T z\) is a scalar. So we can introduce an auxiliary variable \(\psi \in [0, 2\pi)\) and rewrite the quantity to be maximized in (9) as

\[
|w^T z| = \max_{\psi \in [0, 2\pi)} \Re\{w^T z e^{-j\psi}\},
\]

(10)

let \(z = [z_1, z_2, \ldots, z_N]^T \in \mathbb{C}^N\) with polar decomposition:

\[
z_i = |z_i|e^{j\tau_i}, 0 \leq \tau_i < 2\pi, \quad i = 1, \ldots, N.
\]

Then there is

\[
\max_{\omega_1, \ldots, \omega_N \in \{0, \pi\}} |w^T z| = \max_{\omega_1, \ldots, \omega_N \in \{0, \pi\}} \left| \sum_{i=1}^{N} z_i |w_i| \cos(\omega - \tau_i) \right|.
\]

(11a)

\[
= \max_{\omega_1, \ldots, \omega_N \in \{0, \pi\}} \left\{ \sum_{i=1}^{N} |z_i| |w_i| \cos(\omega - \tau_i) \right\}.
\]

(11b)

In (11a), the fact is that for any \(\psi \in [0, 2\pi)\), the optimal \(w_{i, \text{opt}}\) is simply \(w_{i, \text{opt}}(\psi) = \text{sgn}(\cos(\psi - \tau_i))\), \(i = 1, 2, \ldots, N\). The final quantity \(\sum_{i=1}^{N} |z_i| |w_i| \cos(\omega - \tau_i)\) in (11b) is maximized for a particular value \(\psi_{\text{opt}} \in [0, 2\pi)\) and \(w_{\text{opt}} = [w_{1, \text{opt}}(\psi_1), \ldots, w_{N, \text{opt}}(\psi_N)]^T\) is the optimal binary vector we are searching for, in (9).

We will now show that we can always construct a set of \(N\) binary spreading candidates \(U = \{w_{1, \text{opt}}(\psi_1), \ldots, w_{N, \text{opt}}(\psi_N)\}\), and guarantee that \(w_{\text{opt}} \in U\). Therefore, the maximization in (9) can be performed on the set \(U\) of \(N\) candidates without loss of optimality.

We notice that for each \(1 \leq i \leq N\), \(|z_i|\) and \(\tau_i\) are discrete and known a priori. To construct the set \(U\), we can divide the complete circle into discrete parts according to \(\tau_i\). Then we run \(\psi\) over all parts.

Firstly, we partition the index set \(Z_N = \{1, \ldots, N\}\) into

\[
I_1 \triangleq \{i : \tau_i \in [0, \pi)\}, \quad I_2 \triangleq \{i : \tau_i \in [\pi, 2\pi)\} = Z_N \setminus I_1,
\]

and denote the angles into \([0, \pi)\)

\[
\hat{\tau}_i \triangleq \begin{cases} 
\tau_i, & i \in I_1 \\
\tau_i - \pi, & i \in I_2
\end{cases}, \quad i = 1, \ldots, N.
\]

(13)

Secondly, introduce a mapping function \(\mathcal{M}\) from \(\{1, \ldots, N\}\) to \(\{1, \ldots, N\}\), sorting the angles \(\hat{\tau}_1, \hat{\tau}_2, \ldots, \hat{\tau}_N\) in a non-decreasing order \(0 \leq \hat{\mathcal{M}}(1) \leq \hat{\mathcal{M}}(2) \leq \cdots \leq \hat{\mathcal{M}}(N) < \pi\). For each \(i\), the complete angle is divided into \(N\) parts by \(\hat{\mathcal{M}}(i) - \frac{\pi}{2}\). If we let \(\psi\) run over all the parts in \([0, 2\pi)\), different possible combinations of \(w_{1, \text{opt}}(\psi)\) can be obtained as

\[
\{w_{1, \text{opt}}(\psi_1), \ldots, w_{N, \text{opt}}(\psi_N)\} = \left\{ +1, \ldots, +1, -1, \ldots, -1 \right\}_{i}^{|N|} \hat{\mathcal{M}}(i) - \frac{\pi}{2} < \psi < \hat{\mathcal{M}}(i+1) - \frac{\pi}{2},
\]

(14)

where \(i = 1, \ldots, N\). Note that \(\hat{\mathcal{M}}(N+1)\) returns to \(\hat{\mathcal{M}}(1) + 2\pi\).
Finally, we construct a set $U$ consisting of $N$ candidates, i.e.,

$$U = \{ w_{1,\text{opt}}(\psi_i), \ldots, w_{N,\text{opt}}(\psi_i) \}, i \in I_1$$

Thus the original problem is reduced to

$$\max_{w_1, \ldots, w_N \in \{ \pm 1 \}} |w^T z| = \max_{w \in U} |w^T z|.$$  

We can easily obtain the optimization solution to (9)

$$w_{\text{opt}} = [w_{1,\text{opt}}(\psi), \ldots, w_{N,\text{opt}}(\psi)]^T.$$  

The complexity of the construction of $U$ is dominated by the complexity of the mapping function $M$, which is of order $O(N \log N)$.

We summarize the proposed algorithm in Algorithm 1

**Algorithm 1**: Optimal binary discrete inner product maximization in 1-bit RIS

**Input**: A complex vector $z$

**Output**: Optimal discrete phase configurations vector $w_{\text{opt}}$

1. For each $0 \leq i \leq N$, calculate $|z_i|$ and $\tau_i = \text{angle}(z_i)$
2. Classify all the $\tau_i$ and mark down the index, devote all of the angles $\tau_i$ into the interval $[0, \pi)$ as $\tilde{\tau}_i$ in (12) and (13)
3. Find the mapping function $M$ sorting $\tilde{\tau}_i$ and obtained possible combinations of $\psi_{\text{opt}}(\tilde{\tau}_i)$ as (14)
4. Recover and construct set $U$ consisting of $N$ candidates as (15)
5. Perform all the candidates and find $w_{1,\text{opt}}(\psi), \ldots, w_{N,\text{opt}}(\psi) = \arg \max_{w \in U} |w^T z|$.
6. return $w_{\text{opt}} = [w_{1,\text{opt}}(\psi), \ldots, w_{N,\text{opt}}(\psi)]^T$

IV. A GENERAL DIVIDE-AND-SORT SEARCH FRAMEWORK FOR DISCRETE INNER PRODUCT MAXIMIZATION

In this section, we generalize the divide-and-sort strategy and proposed the DaS algorithm for multi-bits RIS-aided wireless systems. With this algorithm, we can construct a search set of cardinality $2^B N$ which contains the optimal solution. An exhaustive search is then performed for the optimal solution to (8) among this set. It has linear search complexity, both in the number of discrete levels and the length of vectors in multi-bits RIS.

Recall a remarkable feature of the combinatorial optimization problem (8) is that $w^H z$ is a scalar. Thus

$$|w^H z| = \max_{\psi \in [0, 2\pi)} \Re \left\{ e^{-j\psi} w^H z \right\}.$$  

![Fig. 2. The approach to find a suitable configuration $\psi_{\text{opt}}$. (a) For any $\psi$ being given, the only step to determine the best phase configuration is to observe which part $\psi$ is lying. (b) The optimal solution $\psi_{\text{opt}}(\psi)$ is a piecewise constant function of $\psi$.](image)

![Fig. 3. For $N > 1$, there exist a collection of partitions. The complete angle is divided into $2^B N$ joint parts.](image)

We write $z_i = |z_i| e^{j\tau_i}$, then

$$\max_{\omega_1, \ldots, \omega_N \in \U} |w^H z|$$

$$= \max_{\omega_1, \ldots, \omega_N \in \U} \max_{\psi \in [0, 2\pi)} \Re \left\{ \sum_{i=1}^{N} |z_i| e^{j(\psi - \tau_i + \omega_i)} \right\}$$

$$= \max_{\psi \in [0, 2\pi)} \max_{\omega_1, \ldots, \omega_N \in \U} \left\{ \sum_{i=1}^{N} |z_i| \cos \left( \psi - (\tau_i + \omega_i) \right) \right\}$$

$$= \max_{\psi \in [0, 2\pi)} \left( \sum_{i=1}^{N} \max_{\omega_i \in \U} |z_i| \cos \left( \psi - (\tau_i + \omega_i) \right) \right).$$  

We next develop a practical approach to solve the subproblem

$$\omega_{i,\text{opt}}(\psi) = \arg \max_{\omega_i \in \U} |z_i| \cos \left( \psi - (\tau_i + \omega) \right).$$  

![Fig. 2(a)](image)

Recall that, for each $1 \leq i \leq N$, $|z_i|$ and $\tau_i$ are known a priori. It is essential to observe that, for any $\psi \in [0, 2\pi)$ being given, the value $\cos \left( \psi - (\tau_i + \omega) \right)$ increases as the difference angle $|\psi - (\tau_i + \omega)|$ decreases. The optimal solution to (18) can be identified by selecting $\omega_i$ such that the difference angle $|\psi - (\tau_i + \omega)|$ reaches the smallest value.

Fig. 2(a) demonstrates the basic idea of finding a suitable phase configuration $\omega_{i,\text{opt}}$. The complete angle of $2\pi$ is divided uniformly into $2^B$ parts, denoted by $P_n^0, P_n^1, \ldots, P_n^{2^B - 1}$, respectively. The $n$-th part $P_n^{k}$ is centered around $\tau_i + (n - \ldots$
For any $\psi$ being given, the only step to determine the best phase configuration $\omega_i$ is to observe which part $\psi$ is lying.

It is important to pay attention that, due to the discrete nature of the problem, the optimal solution $\omega_{i,\text{opt}}(\psi)$ is a piecewise constant function of $\psi$. The function $\max_{\omega_i \in U} |z_i| \cos (\psi - (\tau_i + \omega_i))$ has at most $2^B$ different values if we let $\psi$ run over $[0, 2\pi)$. We plot in Fig. 2(b) an example of the optimal phase configuration $\omega_{i,\text{opt}}(\psi)$ as a function of $\psi$.

When $N > 1$, there exist a collection of such partitions, as illustrated in Fig. 3. The complete angle is then divided into $2^B N$ joint parts $P_1, P_2, \ldots, P_{2^B N}$. We now consider the problem

$$g(\psi) = \sum_{i=1}^{N} \left( \max_{\omega_j \in u} |z_i| \cos (\psi - (\tau_i + \omega_i)) \right). \quad (19)$$

Similarly, for any $\psi$ being given, a simple computation gives rise to the best phase configurations $\omega_{1,\text{opt}}(\psi), \ldots, \omega_{N,\text{opt}}(\psi)$. Actually, with (17) and (19), we could have

$$\max_{\omega_{1}, \ldots, \omega_{N} \in U} |w^H z| = \max_{\psi \in [0, 2\pi)} g(\psi).$$

Recall that, for each $i$, $\omega_{i,\text{opt}}(\psi)$ is a piecewise constant function and has at most $2^B$ different values when $\psi$ runs over $[0, 2\pi)$. We then know, for $\psi \in P_i$, the vector $[\omega_{1,\text{opt}}(\psi), \omega_{2,\text{opt}}(\psi), \ldots, \omega_{N,\text{opt}}(\psi)]^T$ is constant. Consequently, $[\omega_{1,\text{opt}}(\psi), \omega_{2,\text{opt}}(\psi), \ldots, \omega_{N,\text{opt}}(\psi)]^T$ is a piecewise constant function and has at most $2^B N$ different values when $\psi$ runs over $[0, 2\pi)$. Finally, an exhaustive search on these $2^B N$ values can be performed to find the optimal value. We thus rule out large parts of the search space.

### A. Encoding partitions

We now turn to how to encode all partitions $P_1, P_2, \ldots, P_{2^B N}$ efficiently. Recall that the angles $\tau_1, \tau_2, \ldots, \tau_N$ are known a priori. For each $i$, the complete angle is divided uniformly into $2^B$ parts and the $n$-th part is centered around $\tau_i + (n - 1)\Omega, \quad n = 1, \ldots, 2^B$. For clarity, we illustrate all of the centers as

$$C^1 = \{\tau_1, \tau_1 + \Omega, \ldots, \tau_1 + (2^B - 1)\Omega\} \pmod{2\pi},$$
$$C^2 = \{\tau_2, \tau_2 + \Omega, \ldots, \tau_2 + (2^B - 1)\Omega\} \pmod{2\pi},$$
$$\vdots$$
$$C^N = \{\tau_N, \tau_N + \Omega, \ldots, \tau_N + (2^B - 1)\Omega\} \pmod{2\pi}.$$ \quad (20)

Without loss of generality, we assume the smallest element in the set $C^i$ is $\tau_i$. It is easily to see that $\tau_i$ belongs to the interval $[0, \Omega)$.

We then construct a one-to-one mapping $M$ from $\{1, \ldots, N\}$ to $\{1, \ldots, N\}$ such that

$$\tau_{M(1)} \leq \tau_{M(2)} \leq \cdots \leq \tau_{M(N)}. \quad (21)$$

Each partition could be encoded as

$$P^1 := [\tau_{M(1)} + \Omega, \tau_{M(2)} + \Omega, \cdots, \tau_{M(n)} + \Omega]^T,$$
$$\vdots$$
$$P^{N+1} := [\tau_{M(1)} + 2\Omega, \tau_{M(2)} + 2\Omega, \cdots, \tau_{M(n)} + 2\Omega]^T,$$
$$\vdots$$
$$P^{2B N} := [\tau_{M(1)} + 2^B \Omega, \tau_{M(2)} + 2^B \Omega, \cdots, \tau_{M(n)} + 2^B \Omega]^T.$$ \quad (22)

The advantage of using such coders is that we can easily find the optimal solutions to a sequence of local optimization problems

$$\max_{\omega_{1}, \ldots, \omega_{N} \in U} \max_{\psi \in P_i} \left\{ \sum_{i=1}^{N} |z_i| e^{j(\psi - \tau + \omega)} \right\} = \max_{\psi \in P_i} g(\psi).$$ \quad (23)

For convenience, we denote by $\omega_{1,\text{opt}}^i, \ldots, \omega_{N,\text{opt}}^i$ the optimal solution to the problem (23). Then, for $1 \leq i \leq N$,

$$\omega_{M(i),\text{opt}}^i = P^i(i) - \tau_{M(i)}.$$ \quad (24)

The important point to note here is that $\omega_{1,\text{opt}}^i, \ldots, \omega_{N,\text{opt}}^i$ are actually local optimal solutions to the problem (23). We finally construct a set consisting of $2^B N$ candidates, i.e.,

$$U = \{\{\omega_{1,\text{opt}}^1, \ldots, \omega_{N,\text{opt}}^1\}, \ldots, \{\omega_{1,\text{opt}}^{2^B N}, \ldots, \omega_{N,\text{opt}}^{2^B N}\}\}.$$ \quad (25)

Then the original problem is reduced to

$$\max_{\omega_{1}, \ldots, \omega_{N} \in U} |w^H z| = \max_{\{\omega_{1}, \ldots, \omega_{N}\} \in U} |w^H z|. \quad (26)$$

We highlight the main advantage of such manipulation is that we rule out large parts of the search space. We reduce the number of searches from $2^{2BN}$ to $2^B N$. The proposed DaS finds the global optimal solution with linear search complexity, both in the number of discrete levels and the length of vectors.

We generalize the proposed algorithm in Algorithm 2.

**Algorithm 2** Divide-and-Sort (DaS) for discrete inner product maximization

**Input:** A complex vector $z$ and the number of discrete levels $2^B$.

**Output:** Optimal discrete phase configurations $\omega_{1,\text{opt}}, \ldots, \omega_{N,\text{opt}}$

1: For each $0 \leq i \leq N$, calculate $|z_i|$ and $\tau_i = \text{angle}(z_i)$.
2: Divide the complete angle into $2^B$ uniformly parts based on each $\tau_i$ and construct the center set $C^i$ of (20).
3: Sort $\{\tau_1, \ldots, \tau_N\}$ and find the mapping $M$ in (21).
4: Based on the coder associated with each part, build the set $U$ consisting of $2^B N$ candidates.
5: Find $\omega_{1,\text{opt}}^i, \ldots, \omega_{N,\text{opt}}^i = \arg \max_{\{\omega_{1}, \ldots, \omega_{N}\} \in U} |w^H z|.$
6: return $w_{\text{opt}} = [e^{j \omega_{1,\text{opt}}^i}, \ldots, e^{j \omega_{N,\text{opt}}^i}]^H$. 

V. ATTENTION

The proposed framework is also efficient for other models in maximizing SNR, e.g., [23], [27]. Exactly, optimization problems can be transformed into Quadratic form maximization problems are all well resoluble. We now introduce another two maximizing problems under different signal models that the DaS can solve.

A. Channel Model 1

Model 1 is a statistical channel model introduced in [23]. Considering the scenario in Fig.1 let \( h_i \in \mathbb{C}, i = 1, \ldots, N \), be the cascaded channel from the transmitter to the receiver that is induced by the \( i \)-th reflective cell, \( h_0 \in \mathbb{C} \) be the superposition of all the channels that are not related to the RIS, called background channel. Each channel can be rewritten in an exponential form as

\[
 h_i = \beta_i e^{j\alpha_i}, \quad i = 0, \ldots, N,
\]

with the magnitude \( \beta_i \in (0, 1) \) and the phase \( \alpha_i \in [0, 2\pi) \). Denote the RIS phase configuration matrix as \( \text{diag}(e^{j\omega_1}, e^{j\omega_2}, \ldots, e^{j\omega_N}) \in \mathbb{C}^{N \times N} \), where each \( \omega_i \in [0, 2\pi) \) refers to the phase shift of the \( i \)-th reflective cell. The choice of each \( \omega_i \) is restricted to the discrete set

\[
 u = \{0, \Omega, \ldots, (2^B - 1)\Omega\}
\]

with the distance parameter

\[
 \Omega = \frac{2\pi}{2^B}.
\]

Where \( B \) is the phase quantization level. Let \( x \in \mathbb{C} \) be the transmit signal with the mean power \( P \), i.e., \( \mathbb{E}[|x|^2] = P \). The received signal \( y \in \mathbb{C} \) is given by

\[
y = \left( h_0 + \sum_{i=1}^{N} h_i e^{j\omega_i} \right) x + s, \quad i = 1, \ldots, N,
\]

where an i.i.d. random variable \( s \sim \mathcal{CN}(0, \sigma^2) \) models the additive thermal noise. The received SNR can be computed as

\[
\text{SNR} = \frac{\mathbb{E}[|y|^2]}{\mathbb{E}[|s|^2]} = \frac{P \left| h_0 + \sum_{i=1}^{N} h_i e^{j\omega_i} \right|^2}{\sigma^2}.
\]

Our goal is to maximize the SNR. With the MRT theory, the optimization problem can be stated as

\[
\max_{\omega_1, \ldots, \omega_N \in u} \left| h_0 + \sum_{i=1}^{N} h_i e^{j\omega_i} \right|^2.
\]

Denote \( h = [h_1, \ldots, h_N]^T, w = [e^{j\omega_1}, e^{j\omega_2}, \ldots, e^{j\omega_N}]^H \), the maximization problem can be rewritten as

\[
\max_{\omega_1, \ldots, \omega_N \in u} \left| h_0 + w^H h \right|^2.
\]

Introduce \( \tilde{w} = [e^{j\omega_1}, e^{j\omega_2}, \ldots, e^{j\omega_N}]^H \), (24) could be turned into a standard homogeneous quadratic programming problem as

\[
\max_{\omega_1, \ldots, \omega_N \in u} \tilde{w}^H \tilde{R} \tilde{w},
\]

where \( \tilde{R} = \tilde{\phi} \tilde{\phi}^H \), is rank one and semi-positive definite and \( \tilde{\phi} = [h^T, h_0]^T \). Thus can be solved by the proposed DaS algorithm.

B. Channel Model 2

Model 2 is an angle-based physical model considering the far-field regime. For a RIS-employed Communication SISO system, previous work constructs the equations of the scattered electric field of RIS reflecting [27]. Suppose the effect size of cells on the RIS board is much smaller than the income wavelength. As Fig.4 shows, if there is an incoming signal \( x(\theta^{Arr}) \) from \( \theta^{Arr} \), the received signal reflecting by the \( i \)-th cell of the linear RIS at position \( p_i \) can be written as

\[
x(\theta^{Arr}) \cdot w_i e^{2\pi j \frac{\lambda_i}{\lambda} \sin(\theta^{Arr})} = e^{j\omega_i} \text{ denotes the } i \text{-th reflection coefficient.}
\]

Likewise, considering RIS a uniform plane array, the multi-incident narrowband signals from different directions, we can write the received signals at \( \varphi^{Dep}(\theta^{Dep}, \phi^{Dep}) \) as (26) according to equations (III.2), (IV.2) and (IV.6) in [27].

where \( \eta \) denotes the attenuation of the path, \( \theta, \phi \) denote the elevation angle and azimuth angle, respectively. \( p_i = [x_i, y_i, z_i]^T, i = 1, \ldots, N \) is the location of \( i \)-th cell of RIS. The directional vector \( u \) gives

\[
u(\theta, \phi) = \begin{bmatrix} \sin(\theta) \cos(\phi) \\ \sin(\theta) \sin(\phi) \\ \cos(\theta) \end{bmatrix},
\]

where \( (\theta_1, \phi_1), \ldots, (\theta_M, \phi_M) \) denote the spatial sampling angle in radians. Signal component from angle \( (\theta_i^{Arr}, \phi_i^{Arr}) \) are represented by \( x(\theta_i^{Arr}, \phi_i, \eta) \), \( i = 1, \ldots, N \). Here we do not consider the noise. Maximize the received signal power corresponding to the optimization problem

\[
\max_{\omega_1, \ldots, \omega_N \in u} \left| y(\varphi^{Dep}(\theta^{Dep}, \phi^{Dep})) \right|^2.
\]

Let

\[
b = \left[ e^{j2\pi p_1^T u(\varphi^{Dep}(\theta^{Dep}, \phi^{Dep})) / \lambda}, \ldots, e^{j2\pi p_N^T u(\varphi^{Dep}(\theta^{Dep}, \phi^{Dep})) / \lambda} \right]^T,
\]

\[
B = \begin{bmatrix} e^{j2\pi p_1^T u(\varphi^{Dep}(\theta^{Dep}, \phi^{Dep})) / \lambda} \\
\vdots \\
\end{bmatrix},
\]

and \( x = [x(\theta_1^{Arr}, \phi_1^{Arr}), x(\theta_2^{Arr}, \phi_2^{Arr}), \ldots, x(\theta_M^{Arr}, \phi_M^{Arr})]^T \), the maximized optimization problem can be rewritten as

\[
\max_{\omega_1, \ldots, \omega_N \in u} \left| y(\varphi^{Dep}(\theta^{Dep}, \phi^{Dep})) \right|^2 = \max_{\omega_1, \ldots, \omega_N \in u} \left| B W B x \right|^2
\]

\[
= \max_{\omega_1, \ldots, \omega_N \in u} \tilde{w}^H (Q \circ P^T) \tilde{w},
\]

(28)

where \( \tilde{w} = [e^{j\omega_1}, e^{j\omega_2}, \ldots, e^{j\omega_N}]^H \), \( P = B x x^H B^H Q = b^H b \), and \( \tilde{R} = Q \circ P^T \) is of rank one and semi-positive definite.

The proposed DaS algorithms can also be used solve the optimization problem in (28).
\[ y(\rho^{\text{Dep}}, \phi^{\text{Dep}}) = \eta \left[ e^{j2\pi p_1 u(\theta^{\text{Arr}}, \phi^{\text{Arr}})/\lambda} \ldots e^{j2\pi p_N u(\theta^{\text{Arr}}, \phi^{\text{Arr}})/\lambda} \right] \]

\[ = e^{j\omega_1} \ldots e^{j\omega_N} \]

(26)

\[ \begin{bmatrix} e^{j2\pi p_1 u(\theta^{\text{Arr}}, \phi^{\text{Arr}})/\lambda} \\ \vdots \\ e^{j2\pi p_N u(\theta^{\text{Arr}}, \phi^{\text{Arr}})/\lambda} \end{bmatrix} \]

\[ \begin{bmatrix} e^{j\omega_1} \\ \vdots \\ e^{j\omega_N} \end{bmatrix} \]

VI. NUMERICAL AND EXPERIMENTAL RESULTS

In this section, we support the claim that the proposed DaS is highly effective (particularly at large scales) in practice through extensive numerical experiments. We compare our approach with several SOTA methods, including SDR-SDP, Manopt, and APX [23]. Both SDR-SDP and Manopt take a continuous perspective. They perform discretization on the continuous beamforming solutions. For clarity, these algorithms are referred to as discrete SDR-SDP and discrete Manopt, respectively. A standard CVX solver in Matlab [28] is used to solve the relaxed convex problems arising in SDR-SDP.

On the other hand, exhaustive search can find the optimal discrete phase configurations. Due to the exponential search complexity, it is only used to solve problems of small size. Recently, an approximation (APX) algorithm, which can achieve near optimality for high resolution quantizations, was proposed in [23]. We include these methods as benchmarks also. We must comment that the comparisons are far from complete. There exist algorithms that we do not involve in the comparison.

A. Signal power gains

We first test the performance in terms of SNR as a function of the number of cells. In the following numerical experiments, all equivalent channels are i.i.d. Gaussian of mean zero and variance \( c \). We record the power gain for each of the 400 trials with a fixed value of \( N \).

The average performance of various methods is demonstrated in Fig. 5. It can be seen that the proposed DaS outperforms the other algorithms. Specifically, the lower dashed block (Part I) shows that the proposed DaS always achieves the optimal discrete phase configurations because there is no difference from the results given by exhaustive search. On the other hand, when the number of cells is moderate, as demonstrated in Fig. 5(b), the SNR boosts of the proposed DaS are 0.5dB, 1dB, and 2dB on average over discrete SDR-SDP, discrete Manopt, and APX for 1-bit quantization scheme. We take ten Gaussian randomizations when using SDR-SDP here.

For understanding the SNR boost more clearly, we tested for 1000 trials with a fixed \( N = 200 \), and plot the cumulative distribution function of SNR, as shown in Fig. 6. It can be seen that the proposed DaS performs better, the SNR by DaS gains 1dB over that by Discrete Manopt.

The second test compares the performance in terms of SNR for different values of the quantization bits. The results
Fig. 7. SNR performance for different methods as a function of $N$. There seems to be no significantly noticeable difference between continuous and discrete phase configurations for moderate resolution quantizations, e.g., 4-bits and above. (a) 1-bit. (b) 2-bits. (c) 3-bits. (d) 4-bits.

Fig. 8. Cumulative distribution of SNR when the number of cells $N = 200$

Fig. 9. (a) The SNR boost of 2-bits over 1-bit as a function of $N$. (b) Relative SNR boost (in percentage).

Fig. 10. Overall execution-time of 100 trials for each $N$.

for Gaussian channels are demonstrated in Fig. 7. To better understand the performance degradation due to quantization, we plot in red the results of continuous phase configurations (given by Manopt). Figs. 7(a) and 7(b) show the fact that the proposed DaS outperforms the APK approach, especially for extremely low resolution quantization schemes. Figs. 7(c) and 7(d) indicate that, for moderate resolution quantizations, e.g., 4-bits and above, there seems to be no significantly noticeable difference between continuous and discrete phase configurations. It is only a 0.05dB loss with continuous phase configurations as the quantization is 4-bits. The same viewpoints can be concluded from the results in Fig.8. We can observe that 1-bit RIS loss of about 3dB than the continuous phase configurations (Manopt), and higher resolution quantizations, less received signal power loss.

Another interesting finding is that the SNR performance benefits less from higher resolution phase configurations for $N$ being large. For clarity, we plot the SNR boost of 2-bits over 1-bit as a function of $N$ in Fig. 9(a). The relative SNR boost (in percentage) is demonstrated in Fig. 9(b). There exists a tradeoff in the choice of the number of cells and discrete levels.

B. Overall execution-time comparison

We now test the performance in terms of execution-time as a function of the number of cells. For execution-time comparison, we tested for 100 trials for each $N$. The execution-time is recorded for every trial and the average time is then calculated, as demonstrated in Table 1 and Fig. 10.

The execution-time of discrete SDR-SDP grows dramatically as the number of cells increases. For the case of $N = 1000$, the CVX fails to provide a solution after running for several hours. For large scale problems, Fig. 10 indicates that the proposed DaS is much more efficient than all competitive algorithms. The APX stands at the second position. The Manopt spends much more time.

We finally emphasize that the proposed DaS is much more efficient than all competitive algorithms. When $N = 1000$, it costs only 0.1881s to find the optimal discrete phase configurations.
TABLE I
OVERALL EXECUTION-TIME COMPARISON

| Methods         | $N = 10$ | $N = 50$ | $N = 100$ | $N = 200$ | $N = 500$ | $N = 1000$ |
|-----------------|----------|----------|-----------|-----------|-----------|-----------|
| Exhaustive Search | 0.90s    |          |           |           |           |           |
| Discrete SDR-SDP | 169.26s  | 191.67s  | 312.45s   | 903.77s   | 10785.55s |           |
| Discrete Manopt  | 3.26s    | 26.11s   | 48.50s    | 80.95s    | 260.90s   | 950.24s   |
| APK             | 0.13s    | 0.24s    | 0.47s     | 1.28s     | 8.51s     | 66.81s    |
| Proposed DaS    | 0.03s    | 0.09s    | 0.17s     | 0.48s     | 3.59s     | 18.81s    |

C. Simulation results based on Model 1

As the one of the SOTA algorithms in discrete optimization, APK has illustrated its excellent SNR performance under Model 1 in [23]. We now look at the SNR performance when undertaking the proposed DaS method to solve the problem [25]. All the simulation parameters maintain conformity with that in [23]. The locations of the transmitter, RIS, and receiver are denoted by the 3-dimensional coordinate vectors $(50, -200, 20), (-2, -1, 0)$, and $(0, 0, 0)$ in meters, respectively. The number of reflective elements $N$ equals 200.

We detail the background channel as

$$h_0 = 10^{-PL_0/20} \cdot \zeta_0,$$

where $PL_0 = 32.6 + 36.7 \log_{10}(d_0)$ denote the path loss between the transmitter and the receiver, $d_0$ relates to the distance between them in meters, the Rayleigh fading component $\zeta_0$ is drawn from the Gaussian distribution $CN(0, 1)$.

The reflected channel related to RIS is given by

$$h_i = 10^{-PL_1 + PL_2/20} \cdot \zeta_i, \quad i = 1, \ldots, N,$$

where $PL_1$ and $PL_2$ are both based on the pathloss model $PL_k = 30 + 22 \log_{10}(d_k)$, with $d_k$ in meters respectively denoting the transmitter-to-RIS distance and the RIS-to-receiver distance, while the Rayleigh fading component $\zeta_i$ is drawn from the Gaussian distribution $CN(0, 1)$ independently across $i = 1, \ldots, N$. The transmit power level $P$ equals 30dBm. The background noise power level $\sigma^2$ equals $-90$dBm. We record the power gain for 1000 trials with a fixed value of $N = 200$. To better understand the SNR performance difference, we plot the results of continuous phase configurations (given by Manopt) with a dashed red curve. We take only one Gaussian randomization when using SDR-SDP as a benchmark here.

Fig. 11 shows the cumulative distribution of SNR when the quantization level $B = 1$ and number of cells $N = 200$, Model 1

![Fig. 11. Cumulative distribution of SNR with different quantization level](image1)

Fig. 12 shows. The RIS-aided wireless system operates in the band with a center frequency of 4.85 GHz, the wavelength $\lambda$ is 0.062m. The RIS board consists of 160(10 × 16) reflecting cells, Each of them with spacing 0.027m. The USRP produces a frequency-modulated signal and transmits it through the Tx antenna. After RIS reflecting, the signal is received by the Rx antenna.

Suppose a signal from $(\theta_{Arr}, \phi_{Arr}) = (0^\circ, 0^\circ)$, a single user located at $(\theta_{Dep}, \phi_{Dep}) = (30^\circ, 0^\circ), (45^\circ, 0^\circ)$ and $(60^\circ, 0^\circ)$, respectively. We first calculate and obtain the optimal discrete phase configurations by solving the optimization problem with DaS. It is worth mentioning that we need no channel state information but only the elevation and azimuth angle of the transmitter and the receiver to RIS to do that. These positions

D. Experimental results based on Model 2

For further verification of the proposed method, we applied the DaS to our 1-bit RIS-aided prototype wireless system in the real world. Our system mainly includes a reflective RIS, a USRP 2954R, and a pair of horn antennas (Tx and Rx), as Fig. 13 shows. The RIS-aided wireless system operates in the band with a center frequency of 4.85 GHz, the wavelength $\lambda$ is 0.062m. The RIS board consists of 160(10 × 16) reflecting cells, Each of them with spacing 0.027m. The USRP produces a frequency-modulated signal and transmits it through the Tx antenna. After RIS reflecting, the signal is received by the Rx antenna.

![Fig. 12. Cumulative distribution of SNR with different quantization level when $N = 200$, Model 1](image2)
Fig. 13. The prototype of the RIS-aided wireless communication system. Current experimental setting is, Tx antenna location at (0°, 0°) with distance of 2.8m to the center of RIS, Rx antenna location at (45°, 0°) with distance 3.96m to the center of RIS.

Fig. 14. RIS controlling matrices (codebook) related to the optimization phase configurations. (a) The codebook as the receiver located at elevation and azimuth angle (30°, 0°); (b) The receiver at (45°, 0°). (c) The receiver at (60°, 0°) are easy to obtain from the actual environment. Then, we transform the obtained configuration matrices into controlling matrices (codebook) and test the received signal power at the preset locations. The three codebooks are indicated in Fig. 14. Value ‘0’ and ‘1’ are related to phase configuration π and 0, corresponding to the conduction and block of diodes on the RIS board, respectively.

The experimental results are shown in Fig. 15. To provide a standard for comparison with the optimal codebook, we take the All-zeros (all the RIS cells taking the phase shift π on the incoming electromagnetic wave) and Random codebooks as the benchmark. We emphasize that the RIS can be considered as a copper plate and specular reflection occurs as All-zeros is taken. It is obvious that different codebook brings various signal power received. The optimization RIS achieves an average 10dB power gain over All-zeros RIS while the receiver is located at different positions.

VII. Conclusion

This investigation focuses on optimal beamforming through SNR improvement in RIS-aided SISO communications. We reformulate the discrete beamforming problem into a discrete inner product maximization problem from an implementation viewpoint. A highly efficient DaS search framework with linear time complexity, both in the number of discrete levels and the length of vectors, is developed. The proposed DaS is guaranteed to find the global optimal discrete solution for the inner product maximization problem. Numerical results demonstrate the superior performance of the proposed DaS over the existing SOTA algorithms in enhancing the received SNR. Experimental results indicate the proposed DaS method is efficient in real RIS-aided wireless systems.

APPENDIX

We now give proof of the equation in (28).

\[ |y(\theta_{\text{Dep}}, \phi_{\text{Dep}})|^2 = \text{tr}[(\theta_{\text{Dep}}, \phi_{\text{Dep}})^* y(\theta_{\text{Dep}}, \phi_{\text{Dep}})^H] \]
\[ = \text{tr}(bWb^{\text{H}}B^{\text{H}}W^{\text{H}}b^H) \]
\[ = \text{tr}(Bxx^HB^HW^{\text{H}}b^HbW) \]
\[ = \text{tr}(PW^HQW) \]
\[ = \text{tr}(W^HQWP) \]
\[ = \hat{w}^H(Q \odot P^T)\hat{w} \]
\[ = \hat{w}^H\hat{R}\hat{w}, \]

where \( \hat{w} = \text{diag}(W) \), \( P = Bxx^HB^HW^{\text{H}}Q = b^Hb \).

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