ENHANCING GROUND RESOLUTION OF TM6 BASED ON MULTI-VARIATE REGRESSION MODEL AND SEMI-VARIOGRAM FUNCTION

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KEY WORDS multi-variate regression model; semi-vario gramm function; image fusion; template window; V C++ programming

ABSTRACT It is well known that Landsat TM images are the most widely used remote sensing data in various fields. Usually, it has 7 different electromagnetic spectrum bands, among which the sixth one has much lower ground resolution compared with the other six bands. Nevertheless, it is useful in the study of rock spectrum reflection, geo-thermal resources exploration, etc. To improve the ground resolution of TM6 to the level as that of the other six bands is a problem. This paper presents an algorithm based on the combination of multi-variate regression model with semi-vario gram function which can improve the ground resolution of TM6 by "fusing" the data of other six bands. It includes the following main steps: (1) testing the correlation between TM6 and one of TM1-5, 7. If the correlation coefficient between TM6 and another one is greater than a given threshold value, then select the band to the regression analysis as an argument. (2) calculating the size of the template window within which some parameters needed by the regression model will be calculated; (3) replacing the original pixel values of TM6 by those obtained by regression analysis; (4) using image entropy as a measurement to evaluate the quality of the fused image of TM6. The basic mechanism of the algorithm is discussed and the V C++ program for implementing this algorithm is also presented. A simple application example is given in the last part of this paper, showing the effectiveness of the algorithm.

1 Introduction

It is well known that Landsat TM images are probably the most important remote sensing images for geological purposes (Chen et al., 1994). Usually, there are 7 different electromagnetic spectrum bands in TM images, among which three bands cover the visible range of the electromagnetic spectrum, three bands cover the infrared or near infrared range and one in the thermal infrared range (Richard, 1986). Except for the sixth all the other bands have the ground resolution of 30 m × 30 m. That is, each pixel in the images of these bands represents a ground area with 30 m × 30 m, while TM6’s represents 120 m × 120 m, i.e., the area is 16 times bigger than that of the other six bands. Regardless its lower resolution, TM6 is commonly used for the study of rock spectrum and geo-thermal resources exploration. Nevertheless, TM6’s lower resolution limits its utility greatly. In most cases, image analysts simply neglect the sixth band, or regard it as a special one that is acquired by other different sensors, regardless the potential relationship that will exist with the other bands. Obviously, to treat the sixth band like this will lose information that will probably be useful.

As summarized by Luo Z et al. (1998) and Liu J G
et al. (1998), many approaches have been developed in order to improve the ground resolution of TM6, among which the most promising one is based on the principles of image fusion.

Image fusion has been a hot topic since the 1980's. Remotely sensed data with the properties of multi-platform, multi-layer, multi-temporal stage, multi-spectrum, multi-angle and multi-resolution have become more and more popular (Sun J B et al., 1998). All these image data from the same area form the so-called image pyramid (Richard J A, 1986; Sun J B et al., 1998) of the area. How to use these data with various properties utmostly in order to extract useful information is one of emergent and interesting research topic in the field of multi-spectral image processing. The main merit of image fusion compared with image addition that used to be adopted is information optimization: using this technique the objects being sensed can be recognized more clearly.

A very basic and important principle of data fusion is to select a suitable algorithm for a special application. Though it limits the applications of data fusion for lack of integral theoretical framework and methodology, the commonly used algorithms are those derived from non-deterministic mathematics such as probability theory, multi-variate statistic, etc. (Kang, 1997).

Many algorithms for data fusion have been developed, mainly for military purposes. Lots of new algorithms can be found in various literatures. A summarization of these methods were given by Kang Yaohong (1998).

Though many methods of data fusion can be listed for various applications, it is difficult to find one that is suitable for ground resolution enhancement of TM6 by image fusion. Luo Z et al. (1998) provided an algorithm based on regression analysis, but a few critical problems were not solved satisfactorily, such as correlation testing between TM6 and the other bands, determining the size of template window, evaluating the quality of the fused images, etc.

This paper will resolve all the problems mentioned above, as well as revise the mathematical model provided by Luo et al.. A V C ++ program is also developed for implementing the algorithm. The program is designed with the idea of object-oriented programming and all the algorithms are written by the form of class, which is convenient to port, maintain and expand.

2 Correlation test

It is intuitively understandable that all the bands of a scene of TM image are statistically correlated if the radiometric errors are completely removed (Winkler, 1995). If this is the case, we can have the following assumption: the resolution of TM6 is as high as that of the other six bands. In other words, its ground resolution is also 30 m × 30 m per pixel. But we assume it has been degraded by some unknown reasons and therefore the observed image of TM6 is hazed so that its ground resolution is decreased to 120 m × 120 m per pixel. In order to recover the "true pixel values" of TM6, one may spontaneously think of using regression model to estimate them by regarding pixel values of TM6 as dependent variable and those of TM1 ~5 and TM7 as arguments in the regression model, then replacing the pixel values of TM6 by these regressed data.

So the first step that must be done is to test the correlation between TM6 and each one of TM1 ~5 and TM7. This is quite easy for computation, though it may take a few minutes if large images are taken into consideration. The formulae of calculating correlation coefficient is as the following:

\[
\rho_{6j} = \frac{s_{6j}}{\sqrt{s_{66}} \sqrt{s_{jj}}}
\]

Where \( j = TM1 \sim 5 \) and TM7; \( n \) is the total number of pixels in each band; \( s_{6j} \) is actually the covariance between TM6 and each band of TM1 ~5, TM7. The correlation coefficients between TM6 and TM1 ~5 and TM7 then can be written as follows

\[
\rho_{6j} = \frac{s_{6j}}{\sqrt{s_{66}} \sqrt{s_{jj}}}
\]

where \( \rho \) represents correlation coefficient; \( s_{66} \) is the variance of pixel values of TM6; \( s_{jj} \) is the variance of band 1 ~ 5 and band 7.

3 The algorithm

As mentioned above, if the correlation of the pixel values between TM6 and TM1,2,3,4,5 and 7 is significant by comparing with the given threshold.
value, then the “true pixel values” of TM6 can be obtained by regression analysis, in which pixel values of TM1, 2, 3, 4, 5 and 7 are arguments and those of TM6 are the corresponding dependent variable. In symbols, it can be written as

\[
y'_a = b_0 + b_1 x_1(i,j) + b_2 x_2(i,j) + b_3 x_3(i,j) + b_4 x_4(i,j) + b_5 x_5(i,j) + b_7 x_7(i,j)
\]

where \( y'_a \) is the regressed pixel value of TM6; \( x_l(i,j) \) is the pixel value of \( l \)th row and \( j \)th column in the \( l \)th image of corresponding TM; \( b_i \) is the regression coefficient. It can be calculated from the following formula (Peng, 1983):

\[
B^T = (C^T C)^{-1} C^T Y
\]

where \( B^T \) is the transpose of vector \( B = (b_0, b_1, b_2, b_3, b_4, b_5, b_7) \); \( (C^T C)^{-1} \) is the inverse of \( C^T C \). Matrices \( C \) and \( Y \) can be written as follows.

\[
C = \begin{bmatrix}
  x_{11} - \bar{x}_1 & x_{21} - \bar{x}_2 & \cdots & x_{m1} - \bar{x}_m \\
  x_{12} - \bar{x}_1 & x_{22} - \bar{x}_2 & \cdots & x_{m2} - \bar{x}_m \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{1n} - \bar{x}_1 & x_{2n} - \bar{x}_2 & \cdots & x_{mn} - \bar{x}_m
\end{bmatrix}
\]

where \( \bar{x}_i (i = 1, 2, 3, 4, 5, 7) \) is the mean of pixel value of the \( i \)th band; \( n \) is the total number of pixels;

\[
Y = \begin{bmatrix}
  y_1 - \bar{y} \\
  y_2 - \bar{y} \\
  \vdots \\
  y_m - \bar{y}
\end{bmatrix}
\]

where \( \bar{y} \) is the mean of the pixel value of TM6.

Now, the rest is to calculate the mean values of TM1, 2, 3, 4, 5, 6, 7. If the mean values are calculated, then the vector \( B \) can be easily obtained from Eq. (2). However, a problem that is common in image processing may arise whether calculating these mean values by using all the pixel values within an image or firstly segmenting the whole image into several un-overlapped sub-image blocks, calculating mean values within these sub-image blocks and then calculating the total mean value according to those calculated from sub-image blocks. In this case it is obvious that the whole image should be segmented into different sub-image blocks at first. The reason for doing this is intuitive. For simple instance, suppose that an image is composed of \( 4 \times 4 = 16 \) pixels, which can be segmented into two continuous parts with pixel values of 0 to all the pixels in one part and 255 to all the pixels in another. If display this image, then one part is white while the other is black. If the mean value of the image is calculated by the whole image, then, obviously, the variance of these two parts (by comparing the individual pixel value of these two parts respectively with the mean value) will be beyond the tolerable level. In fact, the mean value of this image should be calculated according to the two continuous parts respectively. The problem arising here is how to determine the size of the template window within which the mean value and other parameters of the image are calculated.

Many literatures have pointed out that the determination of the size, shape and orientation of the template window is so important that results will greatly depend on it (Wang, 1995; Franklin et al., 1996; Dong et al., 1997). A promising strategy is to determine the template window’s size and shape on the basis of image itself, rather than to employ arbitrary window size that may be optimal for one specific application in a specific region, but suboptimal elsewhere. Franklin et al., (1996) summarized this problem thoroughly and pointed out that the concept of geographic window proposed by Merchant (1984) was suitable for determining the template window. Meanwhile, he provided a way to fix the size of the geographic window automatically according to the remotely sensed image itself. In doing so, the main idea that Franklin, et. al adopted is semivariogram function.

Semivariogram function is the basic concept from spatial statistics, which was created firstly by G. Matheron in 1960s. The theory found its wide utilizations in the field of spatial data processing and analysis. It was introduced to the field of remote sensing image processing first by Curren in 1988. From then on, lots of literatures have been published about the application of regionalized variable to remote sensing analysis (Curren, 1988; Franklin et al., 1996; Carr et al., 1996; Dong et al., 1996).

A typical semivariogram is shown in Fig. 1. It is the graph of semivariance versus sample spacing. Parameters of a fitted mathematical function (semi-
variogram model) may include a range, a nugget and a sill, as shown in Fig 1. The range of the semi-
variogram indicates a spatial scale of the pattern, the nugget is an indication of the level of uncorre-
lated noise in the data and the sill reveals the total variation (van der Meer F. et al., 2000). Semi-
variograms of remotely sensed measurements should be interpreted with care. Some major points for semi-
variogram interpretation were given by De Jong & Burrough (1995).

According to Curren's suggestion, in remote sens-
ing, semi-variance $\gamma(h)$ estimates the variability of the radiance of ground objects as a function of spa-
tial separation. If the spatial relationship exists, the value of $\gamma(h)$ increases as the separation distance, $h$, increases. The range can be used as a measure-
ment of spatial dependency or homogeneity. In more general sense, the range may be used to esti-
mate the geographic dimensions of optimal win-
dows for use in image processing and analysis.

Therefore the range of the semivariogram of a re-
motely sensed image can be employed as the tem-
plate window size, the range in horizontal orienta-
tion of the image is the height of the window while the range in vertical orientation of the image is the width of the window.

The total number of sub-image blocks into which the whole image should be segmented can be deter-
mined by the following steps:

(1) Calculating the horizontal semivariogram, and determine the range (in pixels) in this orienta-
tion, which can be denoted by $r_h$;

(2) Calculating the vertical semivariogram, and
determining the range (in pixels) in this orienta-
tion, which can be denoted by $r_v$;

(3) The whole image should be segmented into $M \times N$ sub-image blocks. The total number of sub-
image blocks along vertical orientation is $N$, while along horizontal orientation is $M$. The values of $M$ and $N$ are determined by the following formulae

$$M = \left[ \frac{\text{row}}{r_h} \right], N = \left[ \frac{\text{column}}{r_v} \right]$$

where row is the width of the image (in pixels); column is the length of the image (in pixels); $[a]$ is to round off $a$.

Now that the total number of sub-image blocks has been determined, the next step is to calculate the average values of pixels of TM1,2,3,4,5,6,7. There are two methods that are being used widely (Lou, 1999): local block method and local pixel method. The principles of these two methods are described as below.

1) Local block method
Denote the $M \times N$ sub-image blocks by $S, S = \{B_{k,l}, k = 1,2,\cdots,N; l = 1,2,3,\cdots,M\}$, then

$$\bar{x}_{r}^{k,l} = \frac{1}{16} \sum_{(i,j) \in B_{k,l}} x(i,j)$$

where $r = \text{TM1,2,3,4,5,7}$

$$\bar{y} = \frac{1}{N \times M} \sum_{B_{k,l}} y(i,j)$$

2) Local Pixel method
Denote the $M \times N$ sub-image blocks by $S, S = \{B_{k,l}, k = 1,2,\cdots,N; l = 1,2,3,\cdots,M\}$, select
two pixel values $x(r = \text{TM1,2,3,4,5,7})$ from $B_{k,l}$ randomly, then

$$\bar{x}_{r} = \frac{1}{N \times M} \sum_{B_{k,l}} \bar{x}_{r}^{k,l}$$

where $r = \text{TM1,2,3,4,5,7}$.

$$\bar{y} = \frac{1}{N \times M} \sum_{B_{k,l}} \bar{y}^{k,l}$$

Up to now, all the parameters in Eq. (1) have been obtained. The regressed values of TM6 can be calculated easily from the equation. Replacing the original pixel values of TM6 by the fused ones, the result image of TM6 with the ground resolution of
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30 m × 30 m per pixel then can be acquired.

4 Test of significance of the regression equation and quality evaluation of the fused image

The essential idea of the above described algorithm is to predict the "true pixel values" of TM6 by multi-variate regression model. An un-avoidable problem to use the regression equation is to test the significance of the regression analysis. F testing is adopted here, which is the most significant method of hypothesis testing in regression analysis (Peng, 1983). Given a significant level, through looking up the F distribution table, then the critical value (threshold value) can be found. A significant level of 0.001 is given here, the first free degree of F test is 6 while the second can be regarded as infinite, for the number of pixels in any real remotely sensed images is large enough to do so.

Quality evaluation of the fused image is another problem that must be answered to determine whether the fused images are better than the original ones. Usually, image analysts can appraise the fused images visually through analyzing the features in the images, which may help them extract information they are interested in. Nevertheless, this approach does not benefit for automatically image analysis. Several indices have been developed, and among them the image entropy seems to be the most useful one (Jin et al., 1992).

According to Shannon’s information theory, the information content of an image can be measured by image entropy. The greater the entropy is, the more information the image contains. In symbols, the image entropy can be written as follows.

\[ E = - \sum_{i=0}^{255} P_i \log P_i \]

where \( P_i \) is the occurrence probability of the pixel of \( i \)th gray level in an image. In most cases, remotely sensed images are digitized by 8 bits per pixel, so the range of gray level is from 0 to 255.

5 The V C++ program and application of the algorithm

The above mentioned algorithm as well as the auxiliary functions, such as semi-variogram calculating, image entropy calculating, correlation testing, etc., are implemented by V C++. The program is a Windows 95/NT based, object-oriented one, which supports the image format BMP at present. More image formats supporting is unnecessary, because most PC-based image formats are public, it is without difficulty to include these code in the program.

A case study is carried out to test the effectiveness of the algorithm as well as the program. The Landsat TM data are from Weixi Area, Yunnan province, with the size of 512 pixels × 512 pixels. The original images of TM1 and TM6 are shown in Fig. 2 and Fig. 3.

![Fig. 2 Original image of band 1 (512 pixels × 512 pixels)](image)

![Fig. 3 Original image of band 6 (512 pixels × 512 pixels)](image)

The values of correlation coefficient between TM6 and TM1 ~ 5 and TM7 are listed in Table 1.

Table 1 shows that TM6 is significantly correlated with all the bands of TM1 ~ 5 and TM7, so the values of the six bands are involved in regression analysis. If TM6 is correlated only with some of TM1 ~ 5 and TM7, then only those bands correlated with TM6 should be selected. If all the bands
of TM1 ~ 5 and TM7 are poorly correlated with TM6 then the method is invalid. The threshold value for judging the correlation degree can be determined by the user. In practice, if the minimum of the values of correlation coefficient is less than 0.5, then the situation of semi-correlation should be considered.

The vertical semi-variogram is shown as Fig. 4. From the figure, it can be determined that vertical range of the given template window is 12 pixels. After testing the correlation between TM6 and TM1 to TM5, TM7, and calculating the vertical and horizontal semi-variogram to determine the range values in these two directions, the regression analysis can be carried out. The significance of the regression equation should be tested before the "true pixel values" of band 6 are estimated by using it, which is very significant in this case, and then to estimate the pixel values of TM6 from the values of TM1 to TM5 and TM7. The resultant images by using Local Pixel Method and Local Block Method are shown as Fig. 5 and Fig. 6.

In order to evaluate the quality of the fused images, values of image entropy are calculated, and listed in Table 2.

Table 2 shows that the image entropy of both fused images by Local Block Method and Local Pixel Method is much greater than the originals.

### Table 2: Comparison of image entropy between different bands

| Images              | Image Entropy |
|---------------------|---------------|
| Image of TM1        | 2.757 842     |
| Image of TM2        | 2.303 347     |
| Image of TM3        | 3.084 615     |
| Image of TM4        | 4.019 727     |
| Fused image by LB   | 5.041 303     |
| Image of TM5        | 4.158 576     |
| Image of TM6        | 3.589 186     |
| Image of TM7        | 3.601 356     |
| Fused image by LP   | 5.038 577     |

### 6 Conclusions and suggestions

The algorithm presented here is suitable for estimating the "true pixel values" of TM band 6. The approach to determining the size of template window is improved greatly, which can be expanded to
any image processing procedures. The case study shows the effectiveness of this algorithm, the fused images are valuable for further processing. Furthermore, the concept of image fusion is embodied by the algorithm in some sense, and reflects an important thought that it is necessary to orient to practical problems when one wants to adopt the ideas from image fusion. However, some problems remain unsolved, for instance, finding out a suitable way to evaluate the error caused by replacing pixel values of band 6 by fused data. In addition, more case studies also should be demonstrated as well as further results obtained by processing the fused images such as classification should be analyzed. All these problems are listed in the agenda of our further studies.

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