Stability Analysis of Slotted Aloha with Opportunistic RF Energy Harvesting

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Abstract

Energy harvesting (EH) is a promising technology for realizing energy efficient wireless networks. In this paper, we utilize the ambient RF energy, particularly interference from neighboring transmissions, to replenish the batteries of the EH enabled nodes. However, RF energy harvesting imposes new challenges into the design of wireless networks. In this work, we investigate the performance of a slotted Aloha random access wireless network consisting of two types of nodes, namely type I which has unlimited energy supply and type II which is solely powered by an RF energy harvesting circuit. The transmissions of a type I node are recycled by a type II node to replenish its battery. Our contribution in this paper is multi-fold. First, we generalize the stochastic dominance technique for analyzing RF EH-networks. Second, we characterize an outer bound on the stable throughput region of RF EH-networks under the half-duplex and full-duplex energy harvesting paradigms. Third, we investigate the impact of finite capacity batteries on the stable throughput region. Finally, we derive the closure of the outer bound over all transmission probability vectors.

Index Terms—Wireless networks, slotted Aloha, opportunistic energy harvesting, interacting queues.

I. INTRODUCTION

One of the prominent challenges in the field of communication networks today is the design of energy efficient systems. In traditional networks, wireless nodes are powered by limited capacity...
batteries which should be regularly charged or replaced. Energy harvesting has been recognized as a promising solution to replenish batteries without using any physical connections for charging. Nodes may harvest energy through solar cells, piezoelectric devices, RF signals, etc. In this paper, we focus on RF energy harvesting. Recent studies present experimental measurements for the amount of RF energy that can be harvested from various RF energy sources. Two main factors affect the amount of RF energy that can be harvested, namely, the frequency of the RF signal and the distance between the “interferer” and the harvesting node, e.g., see Table I in [1].

Recently, an information-theoretic study of the capacity of an additive white Gaussian noise (AWGN) channel with stochastic energy harvesting at the transmitter has shown that it is equal to the capacity of an AWGN channel under an average power constraint [2]. This, in turn, motivated the investigation of optimal transmission policies [3] for single user [4]–[6] and multi-user [7]–[9] energy harvesting networks. The optimal policy that minimizes the transmission completion time was studied in [4]. In [5], the authors studied the problem of maximizing the short-term throughput and have shown that it is closely related to the transmission completion time problem [4]. The authors in [6] studied the optimal transmission policies for energy harvesting networks under fading channels. Moreover, [7]–[9] extends the analysis to broadcast, multiple access, and interference channels, respectively. The authors in [10] introduced the concept of energy cooperation where a user can transfer portion of its energy, through a separate channel, to assist other users.

Significant research has also been conducted on RF energy harvesting. In [11], the author discusses the fundamental trade-offs between transmitting energy and transmitting information over a single noisy link. The author derives the capacity-energy functions for several channels. The authors in [12], extend the point-to-point results of [11] to multiple access and multi-hop channels. Recently, several techniques were proposed for designing RF energy harvesting networks (RF-EHNs) [13]. The RF energy harvesting process can be classified as follows: i) Wireless energy transfer, where the transmitted signals by the RF source are dedicated to energy transfer, ii) Simultaneous wireless information and power transfer, where the transmitted RF signal is utilized for both information decoding and RF energy harvesting, and iii) Opportunistic energy harvesting, where the ambient RF signals, considered as interference for data transmission, are utilized for RF energy harvesting. The receiver architecture may also vary as follows [13], [14]: a) Co-located receiver architecture, where the radio receiver and the harvesting circuit
use the same antenna for both decoding the data and energy harvesting, and \( b \) \textit{Separated receiver architecture}, where the radio receiver and the harvesting circuit are separated, and each is equipped with its own antenna.

In \cite{14}, the authors discuss the practical limitations of implementing a simultaneous wireless information and power transfer (SWIPT) system. A major issue is that energy harvesting circuits are not able to simultaneously decode information and harvest energy. Hence, the authors in \cite{14} proposed and analyzed two modes of operation for the co-located receiver architecture, i.e., time switching and power splitting. Furthermore, the RF energy harvesting transceivers may also be classified as: \( a \) \textit{Half-duplex energy harvester}, where a co-located RF energy harvesting transceiver can either transmit data or harvest RF signals at a given instant of time, and \( b \) \textit{Full-duplex energy harvester}, where a node is equipped with two independent antennas and can transmit data and harvest RF signals, simultaneously. In this paper, we investigate opportunistic energy harvesting under the half-duplex and full-duplex modes of operation.

The cornerstone of medium access control (MAC) protocols is the Aloha protocol \cite{15}, which is widely studied in multiple access communication systems because of its simplicity. It is also considered as a benchmark for evaluating the performance of more sophisticated MAC protocols. Based on the Aloha protocol, nodes contend for the shared wireless medium and cause interference to each other. Hence, the service rate of a node depends on the backlog of other nodes, i.e., the nodes’ queues become \textit{interacting} as originally characterized in \cite{16}. Tsybakov and Mikhailov \cite{17} were the first to analyze the stability of a slotted Aloha system with finite number of users. Rao and Ephremides \cite{18} characterized the sufficient and necessary conditions for queue stability of the two user case, using the so called \textit{stochastic dominance} technique. In addition, they established conditions for the stability of the symmetric multi-user case. Other works followed and studied the stability of slotted Aloha with more than two users \cite{19}–\cite{22}. The authors in \cite{23} extended the stability analysis under the collision model to a symmetric multi-packet reception (MPR) model, which was later generalized to the asymmetric MPR model in \cite{24}.

Perhaps the closest to our work is \cite{25} which characterizes the stability region of a slotted Aloha system with energy harvesting capabilities, under the multi-packet reception model. The authors considered a system where the nodes harvest energy from the environment at a fixed rate and, thus, the energy harvesting process is modeled as a Bernoulli process.
In this work, we investigate the performance of an Aloha random access wireless network consisting of nodes with and without RF energy harvesting capability. Specifically, we consider a wireless network consisting of two nodes, namely a node of type I which has unlimited energy supply and a node of type II which is powered by an RF energy harvesting circuit. The RF transmissions of the type I node are harvested by the type II node to replenish its battery. Our contribution in this paper is multi-fold. First, we generalize the stochastic dominance technique for analyzing RF EH-networks. Second, we characterize an outer bound on the stable throughput region of RF EH-networks under the half-duplex and full-duplex energy harvesting paradigms. Third, we investigate the impact of finite capacity batteries on the stable throughput region. Finally, we derive the closure of the outer bound over all transmission probability vectors.

The rest of this paper is organized as follows. In Section II, we present the system model and the assumptions underlying our analysis. In Section III, we describe the energy harvesting model. The stability region of the opportunistic RF EH-Aloha is given in Section IV. The proofs of our main results are presented in Section V. In Section VI, we investigate the impact of finite capacity batteries and full-duplex energy harvesting on the stability region of our system. Finally, we draw our conclusions and point out directions for future research in Section VII.

II. SYSTEM MODEL

We consider a wireless network consisting of two source nodes and a common destination, as shown in Fig. 1. We consider a slotted Aloha multiple access channel, where time is slotted and the slot duration is equal to one packet transmission time. We assume two types of nodes in our system: type I node has a data queue, $Q_1$, and unlimited energy supply, while type II node has a data queue, $Q_2$, and a battery queue, $B$, as shown in Fig. 1. Moreover, packets arrive to the data queues, $Q_1$ and $Q_2$, according to independent Bernoulli processes with rates $\lambda_1$ and $\lambda_2$, respectively. The transmission probabilities of type I and II nodes are $q_1$ and $q_2$, respectively.

We assume perfect data channels, i.e., the destination successfully decodes a data packet, if only one node transmits. If two nodes transmit simultaneously, a collision occurs and both packets are lost and have to be retransmitted in future slots. At the end of each time slot, the destination sends an immediate acknowledgment (ACK) via an error-free feedback channel.

Data packets of type I and II nodes are stored in queues $Q_1$ and $Q_2$, respectively. The evolution
of the queue lengths is given by

\[ Q_i^{t+1} = \max\{Q_i^t - Y_i^t, 0\} + X_i^t, \quad i = 1, 2. \]  

(1)

where \( X_i^t \in \{0, 1\} \) is the arrival process for data packets and \( Y_i^t \in \{0, 1\} \) is the departure process independent of \( Q_i \) status, i.e., \( Y_i^t = 1 \) even if the data queue is empty. \( X_i^t \) is a Bernoulli process with rate \( \lambda_i \) and \( \mathbb{E}[Y_i^t] = \mu_i \).

We assume that type II nodes operate under a half-duplex energy harvesting mode, i.e., they either harvest or transmit but not both simultaneously. Hence, the harvesting opportunities are in those slots when a type II node is idle while a type I node is transmitting. The channel between two source nodes is a block fading channel, where the fading coefficient remains constant within a single time slot and changes independently from a slot to another. For Rayleigh fading, the instantaneous channel power gain \( h_t \) at time slot \( t \) is exponentially distributed, i.e., \( h_t \sim \text{Exp}(1) \).

Let \( P_j \) be the transmission power of node \( j \), and \( l \) be the distance between the two source nodes. We consider the non-singular\(^2\) pathloss model with \( (1 + l^\alpha)^{-1} \), where \( \alpha \) is the pathloss exponent.

In the next section, we develop a discrete-time stochastic process to model RF energy harvesting.

III. ENERGY HARVESTING MODEL

For energy harvesting from the nature, renewable energy resources, such as thermal, solar and wind energy, are exploited. Previously, the harvesting process was modeled as an independent

\(^1\)We extend the analysis to the case of full-duplex RF EH in Section VI.

\(^2\)In order to harvest significant amount of RF energy, \( l \) is typically small. Hence, we use a non-singular (bounded) pathloss model instead of a singular (unbounded) pathloss \( l^{-\alpha} \) model, because the singular pathloss model is not correct for small values of \( l \) due to singularity at 0. [20].
and identically distributed (i.i.d.) Bernoulli process with a certain rate \([25], [27]\), for analytical tractability. Typically the RF energy harvesting process is not i.i.d., since a unit of energy is accumulated over multiple slots. However, for mathematical tractability we model RF energy harvesting as an i.i.d Bernoulli process having the same average interarrival time as the exact process, i.e., it takes the same average number of slots to harvest one energy unit under the i.i.d Bernoulli and the exact harvesting processes. This assumption enables us to derive a closed form expression for the stability region. Let \(p_{h|M}\) be the rate of the i.i.d Bernoulli process, where the set of transmitting nodes \(M\) is either \(\{1\}\), \(\{2\}\), or \(\{1, 2\}\).

A. The RF energy Harvesting Model

The received power at type II node from the transmissions of type I node at time slot \(t\) is given by

\[
P_R(t) = \eta P_1 h_t (1 + I^\alpha)^{-1},
\]

where \(P_1\) is the transmission power of type I node and \(\eta\) is the RF harvesting efficiency \([13]\). The efficiency of an RF energy harvester depends on the efficiency of the harvesting antenna, impedance matching circuit and the voltage multiplier. The harvesting efficiency \(\eta\) typically ranges from 0.5 to 0.7 \([28]\).

Let \(\gamma\) joules be the amount of received energy needed for harvesting one energy unit. Typically,
we need to harvest RF energy over multiple slots in order to accumulate \( \gamma \) joules. We assume that an energy unit arrives to the battery queue at the end of the time slot in which the accumulated energy exceeds \( \gamma \), see Fig. 2. Also, we assume that the interarrival times are independent. Hence, once the accumulated energy exceeds \( \gamma \), the excess energy is discarded and does not contribute to the next energy arrival.

Given the above assumptions, we characterize next the exact RF energy harvesting arrivals process and discuss its complexity. Let \( Z \) be a random variable that represents the number of slots needed for harvesting one energy unit. In other words, \( Z \) is a random variable representing the inter-arrival times of energy units to the battery queue. If the accumulated received energy over \( k \) slots is greater than, or equal to \( \gamma \) while the accumulated received energy up to slot \( k - 1 \) is less than \( \gamma \), then we need \( k \) slots to harvest one energy unit. Let

\[
\theta = \frac{\gamma (1 + \eta \alpha)}{\eta P_1}.
\]

Thus, the probability mass function (pmf) of \( Z \) is given by

\[
P[Z = k] = P \left[ \sum_{t=1}^{k} P_R(t) \geq \gamma, \sum_{t=1}^{k-1} P_R(t) < \gamma \right]
\]

\[
\begin{align*}
&\overset{(a)}{=} P \left[ \sum_{t=1}^{k-1} h_t < \theta \right] - P \left[ \sum_{t=1}^{k} h_t < \theta \right] \\
&\overset{(b)}{=} e^{-\theta} \frac{\theta^{k-1}}{(k-1)!}, \quad k = 1, 2, \ldots,
\end{align*}
\]

where (a) follows from applying the law of total probability, i.e,

\[
P \left[ \sum_{t=1}^{k-1} h_t < \theta \right] = P \left[ \sum_{t=1}^{k-1} h_t < \theta, \sum_{t=1}^{k} h_t > \theta \right] + P \left[ \sum_{t=1}^{k-1} h_t < \theta, \sum_{t=1}^{k} h_t < \theta \right],
\]

and

\[
P \left[ \sum_{t=1}^{k-1} h_t < \theta, \sum_{t=1}^{k} h_t < \theta \right] = P \left[ \sum_{t=1}^{k} h_t < \theta \right].
\]

The distribution of the sum of independent exponential random variables is an Erlang Distribution. Hence, \( \sum_{t=1}^{k} h_t \sim \text{Erlang}(k, 1) \), where \( k \) is the shape parameter and \( h_t \sim \text{Exp}(1) \). From, the cumulative distribution function (CDF) of the Erlang distribution, we know that

\[
P \left[ \sum_{t=1}^{k} h_t < \theta \right] = 1 - \sum_{j=0}^{k-1} \frac{e^{-\theta} \theta^j}{j!},
\]

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Hence, (b) is obtained by substituting (4) in (2). From (3), we notice that the distribution of the inter-arrival times of the exact energy harvesting process is a shifted Poisson distribution, i.e., $Z = V + 1$, where $V$ is a Poisson random variable with mean $\theta$. The expected inter-arrival time of the exact harvesting process is given by $\mathbb{E}[Z] = 1 + \theta$. In order to better understand the harvesting process, we consider a battery queue with independent interarrival times distributed according to (3). The energy arrival at slot $k$ is a Bernoulli random variable $H^k$, since an energy packet arrives to the battery whenever the accumulated energy exceeds the energy threshold $\gamma$. Also, the probability of having an energy arrival at slot $k$, i.e., $\mathbb{P}[H^k = 1]$, depends on the time slot of the last energy arrival, since the distribution of interarrival times is not memoryless. In other words, the energy arrivals are dependent Bernoulli random variables. Assuming the departures of the battery are Bernoulli random variables with mean $\mu$, the battery queue can be modeled by a G/M/1 queue. G/M/1 queues are analyzed by embedded Markov chains [29]. The probability of having an empty battery $(1 - \sigma)$ is the unique root of $L_Z[\mu(1 - \sigma)] = \sigma$, where $L_Z[.]$ is the discrete Laplace transform of the pmf of interarrival times. The previous equation has no closed form solution for our case, where the interarrival times are distributed according to (3).

To circumvent this hurdle and in order to have a closed form expression for the steady state distribution of the battery queue, we propose an equivalent i.i.d. Bernoulli process that has the same expected inter-arrival time. Let $p_{h|\{1\}}$ be the mean of the i.i.d. Bernoulli process, which is interpreted as the probability of success in harvesting one energy unit given that type I node is transmitting. Since the inter-arrival time of an i.i.d. Bernoulli process is geometrically distributed, the mean inter-arrival time is $1/p_{h|\{1\}}$. Hence, the relationship between the exact harvesting process and the equivalent Bernoulli process is given by

$$p_{h|\{1\}} = \frac{1}{1 + \theta}. \tag{5}$$

In this paper, we assume that the harvesting process is modeled by an i.i.d. Bernoulli process of rate $p_{h|M}$, where $p_{h|M}$ is the probability of harvesting one energy unit given a set of nodes $M$ are transmitting. Under half-duplex energy harvesting, type II node can only harvest when it is not transmitting, i.e., the probability of harvesting one energy unit given type II node is transmitting $p_{h|\{2\}} = 0$, and the probability of harvesting one energy unit given both types of
nodes are transmitting is $p_{h|\{1,2\}} = 0$. For convenience, we denote $p_{h|\{1\}}$ by $p_h$.

B. Analyzing the Battery queue

We assume that a type II node opportunistically harvests RF energy, in units of fixed size, from the transmissions of type I nodes. Also, transmitting a single data packet costs one energy unit. Let $H^t$ denote the energy harvesting process modeled as a Bernoulli process. Assuming half-duplex harvesting\(^3\), the average harvesting rate is the difference between the fraction of time slots in which the type I node is transmitting and the fraction of time slots in which both nodes are transmitting, i.e.,

$$
\mathbb{E}[H^t] = q_1 p_h \mathbb{P}[Q_1 > 0] - q_1 q_2 p_h \mathbb{P}[Q_1 > 0, B > 0, Q_2 > 0].
$$

(6)

The battery queue evolves as\(^2\)

$$
B^{t+1} = B^t - \mu_B^t + H^t,
$$

(7)

where $\mu_B^t \in \{0, 1\}$ represents the energy consumed in the transmission of a data packet at time $t$. In order to characterize the probability that the battery queue is non-empty, we assume that the data queues $Q_1$ and $Q_2$ are backlogged. Hence, the average rate of harvesting becomes

$$
\mathbb{E}[H^t] = q_1 p_h (1 - q_2 \mathbb{P}[B > 0|Q_1 > 0, Q_2 > 0]).
$$

(8)

Now, the energy harvesting rate is only dependent on the battery queue status. Thus, if the battery queue is empty, the energy harvesting rate is $q_1 p_h$, otherwise it is reduced to $q_1 p_h (1 - q_2)$ because of the half-duplex operation. Hence, the battery queue forms a decoupled discrete-time Markov chain, as shown in Fig. 3. By analyzing the Markov chain we find the probability that the battery is non-empty, as given by the following lemma.

**Lemma 1.** For a half-duplex RF energy harvesting node with infinite capacity battery, the probability that the battery is non-empty, given that the data queues $Q_1$ and $Q_2$ are backlogged, is given by

$$
\mathbb{P}[B > 0|Q_1 > 0, Q_2 > 0] = \min \left\{ \frac{q_1 p_h}{q_2 (1 + q_1 p_h)}, 1 \right\}.
$$

(9)

\(^3\)In the sequel, we will extend the model to full-duplex as well.
Fig. 3: Markov chain model of the battery queue given that the data queues $Q_1$ and $Q_2$ are backlogged. Note that $\delta' = 1 - q_1 p_h - q_2$ and $\delta = 1 - q_1 p_h (1 - q_2) - q_2$.

Proof: Let $\pi = [\pi_0, \pi_1, \ldots]$ be the steady-state distribution of the Markov Chain shown in Fig. 3. By applying the detailed balance equations we obtain

$$\pi_k = \left(\frac{q_1 p_h}{q_2}\right)^k (1 - q_2)^{k-1} \pi_0, \ k = 1, 2, \ldots.$$ 

Therefore, by substituting in the normalization condition $\sum_i \pi_i = 1$, we get the utilization factor

$$\rho = 1 - \pi_0 = \frac{q_1 p_h}{q_2 (1 + q_1 p_h)}.$$ 

Hence, the probability that the battery is non-empty is given by

$$\mathbb{P}[B > 0 | Q_1 > 0, Q_2 > 0] = \min\{\rho, 1\}.$$ 

Remark: Note that $\mathbb{P}[B > 0 | Q_1 = 0, Q_2 > 0] = 0$, since the energy supply of the type II node is solely dependent on the transmissions of type I node.

IV. Main Results

In this section, we present our main results pertaining to the stable throughput region of opportunistic RF energy harvesting slotted Aloha networks. We adopt the notion of stability proposed in [22], where the stability of a queue is determined by the existence of a proper limiting distribution. A queue is said to be stable if

$$\lim_{t \to \infty} \mathbb{P}[Q^t < x] = F(x) \text{ and } \lim_{x \to \infty} F(x) = 1. \quad (10)$$

The stability of a queue is equivalent to the recurrence of the Markov chain modeling the queue length. Loynes’ Theorem [30] states that if the arrival and service processes of a queue are strictly jointly stationary and the average arrival rate is less than the average service rate,
then the queue is stable. Also, if the average arrival rate is greater than the average service rate, then the queue is unstable and the queue size $Q^t$ approaches infinity almost surely. The stable throughput region of a system is defined as the set of arrival rate vectors, $(\lambda_1, \lambda_2)$ for our system, for which all data queues in the system are stable.

Next, we establish the necessary conditions for the stability of opportunistic RF energy harvesting Aloha. Assuming half-duplex energy harvesting and unlimited battery capacity, the stability region is characterized by the following theorem.

**Theorem 1.** An outer bound on the stable throughput region of opportunistic RF energy harvesting Aloha is the triangle OBD, shown in Fig. 4. Assuming half-duplex energy harvesting and unlimited battery size, the region is characterized by

$$S_o = \left\{ (\lambda_1, \lambda_2) \mid \lambda_1 \leq q_1 \left(1 - \frac{\lambda_2}{1 - q_1}\right), \lambda_2 \leq \frac{(1 - q_1) \min \left\{ \frac{q_1 p_h}{1 + q_1 p_h}, q_2 \right\} \lambda_1}{q_1 \left(1 - \min \left\{ \frac{q_1 p_h}{1 + q_1 p_h}, q_2 \right\} \right)} \right\}. \quad (11)$$

**Proof:** The proof is established in the next section.

**Corollary 1.** The closure of $S_o$ over all transmission probability vectors $q = [q_1, q_2]$ is characterized by

$$C(S_o) = \bigcup_q S_o(q) = \left\{ (\lambda_1, \lambda_2) \mid \lambda_2 \leq \frac{p_h \lambda_1}{2} \left(1 - \lambda_1 + \lambda_2 + \sqrt{(1 + \lambda_1 - \lambda_2)^2 - 4\lambda_1} \right) \right\}. \quad (12)$$

**Proof:** The proof is established in Section V-E.

V. STABILITY ANALYSIS

For the majority of the prior work on the stability analysis of interacting queues, the service rate of a typical node decreases with respect to the transmissions of other nodes in the system. Perhaps, the most basic example is the conventional slotted Aloha system [18], where increasing the service rate of an arbitrary node comes at the expense of decreasing the service rate of other nodes. For our purposes, we refer to such systems without energy limitations as “interference-limited” systems.

On the other hand, in our RF energy harvesting system, transmissions from interfering nodes give rise to two opposing effects on type II (RF energy harvesting) nodes. Similar to classic interference limited systems, the interfering nodes create collisions and, thus, decrease the
Fig. 4: The energy limited and interference limited sub-regions of the stable throughput region are characterized by the triangles BCO and BCD, respectively.

service rate of RF energy harvesting nodes. Meanwhile, transmissions from interfering nodes are exploited by RF energy harvesting nodes to opportunistically replenish their batteries. Therefore, from the perspective of an RF energy harvesting node, a fundamental trade-off prevails between the increased number of energy harvesting opportunities and the increased collision rate, which are both caused by interference. As will be shown formally, this fundamental trade-off splits the stable throughput region for RF energy harvesting Aloha networks into two sub-regions, a sub-region where interference is advantageous for the RF energy harvesting node and another sub-region where it is harmful. These two sub-regions map directly to two modes of operation for our system and are characterized as follows:

1) Energy-limited mode: is the sub-region of the stable throughput region in which the transmissions of interfering nodes enhance the throughput of the RF energy harvesting node, i.e, the throughput enhancement due to the increased harvesting opportunities outweighs the degradation due to collisions created by the interfering nodes.

2) Interference-limited mode: is the sub-region of the stable throughput region in which the transmissions of interfering nodes degrade the throughput of the RF energy harvesting node, i.e, the throughput degradation due to collisions outweighs the throughput increase due to the increased energy harvesting opportunities.

The two parts of the stable throughput region of our system are shown in Fig. 4 where the energy-limited region is enclosed by the triangle BCO and the interference-limited region is
enclosed by the triangle BCD.

In order to derive the stability region, we go through the following three steps discussed next. First, we characterize the average service rates of the two interacting data queues. Second, we generalize the Stochastic dominance technique proposed in [18] to capture our system dynamics and multi-mode operation. Finally, we derive the necessary conditions for stability using the generalized stochastic dominance approach.

A. Service Rates of the Interacting Queues

In our system, we have three interacting queues, namely $Q_1$, $Q_2$ and $B$. The average service rates of the data queues, $Q_1$ and $Q_2$, are given by

$$
\mu_1 = q_1 (1 - q_2 P[B > 0, Q_2 > 0|Q_1 > 0]),
$$

$$
\mu_2 = q_2 P[B > 0, Q_1 = 0|Q_2 > 0] + q_2 (1 - q_1) P[B > 0, Q_1 > 0|Q_2 > 0].
$$

where the service rate of the type I node, $\mu_1$, is the fraction of time in which type I node decides to transmit, excluding the fraction of time in which type II node is also transmitting. A type II node transmits, if it is active, i.e., $B > 0$ and $Q_2 > 0$, and decides to transmit. Similarly, the service rate of the type II node, $\mu_2$, is the fraction of time in which type II node has a non-empty battery and it decides to transmit, excluding the fraction of time in which both nodes are transmitting.

In order to analyze the interaction between $Q_1$ and $Q_2$, we decouple the battery queue, $B$, by substituting the probability of battery queue is non-empty with the conditional probability given by Lemma 1. Hence, the average service rates of the data queues $Q_1$ and $Q_2$ are given as

$$
\mu_1 = q_1 (1 - q_2 P[B > 0|Q_1 > 0, Q_2 > 0] P[Q_2 > 0|Q_1 > 0])
= q_1 \left(1 - \min \left\{ \frac{q_1 P_h}{(1 + q_1 P_h)}, q_2 \right\} P[Q_2 > 0|Q_1 > 0] \right).
$$

$$
\mu_2 = q_2 P[B > 0, Q_1 = 0|Q_2 > 0] + q_2 (1 - q_1) P[B > 0|Q_1 > 0, Q_2 > 0] P[Q_1 > 0|Q_2 > 0]
= (1 - q_1) \min \left\{ \frac{q_1 P_h}{(1 + q_1 P_h)}, q_2 \right\} P[Q_1 > 0|Q_2 > 0].
$$
The probability that $Q_i$ is non-empty given that $Q_j$ is saturated (always backlogged) is given by

$$P[Q_i > 0 | Q_j > 0] = \frac{\lambda_i}{\mu^*_i}, \quad i = 1, 2, \quad i \neq j$$

where $\mu^*_i$ is the service rate of $Q_i$ given that both data queues are saturated. We derive $\mu^*_i$, $i = 1, 2$ in Section V-C.

From (13), we note that $\mu_1$ decreases with increasing $P[Q_2 > 0 | Q_1 > 0]$. Recall that for the interference-limited region, increasing the service rate of one node comes at the expense of decreasing the service rate of other nodes. The system is interference-limited from the perspective of type I node, since increasing $\lambda_2$ comes at the expense of decreasing $\mu_1$. Also, from (14), we observe that $\mu_2$ is increasing in $P[Q_1 > 0 | Q_2 > 0]$. Hence, increasing $\lambda_1$ increases $\mu_2$ until both data queues are saturated. Thus, the system is energy-limited from the perspective of the type II node until the data queues become saturated.

In Fig. 4, we depict the stable throughput region $S_o$ of our system. The boundary between the energy-limited part and the interference limited part, from the perspective of the type II node is $\lambda_1 = \tilde{\lambda}_1$, where $\tilde{\lambda}_1$ is the arrival rate for $Q_1$ at which both data queues $Q_1$ and $Q_2$ become saturated. Accordingly, the energy-limited sub-region is characterized by

$$S_{o|\lambda_1 \leq \tilde{\lambda}_1} = \left\{ (\lambda_1, \lambda_2) \in S_o | \lambda_1 \leq \tilde{\lambda}_1 \right\},$$

and the interference limited sub-region is characterized by

$$S_{o|\lambda_1 > \tilde{\lambda}_1} = \left\{ (\lambda_1, \lambda_2) \in S_o | \lambda_1 > \tilde{\lambda}_1 \right\}.$$
system.
Recall, from (14), that the service rate of $Q_2$ increases with $\lambda_1$. Thus, it is straightforward to show that, using classic stochastic dominance arguments, saturating $Q_1$ increases the service rate of $Q_2$. Hence, the queue length in this hypothetical system (particularly $Q_2$) no longer dominates (i.e. could be smaller) its counterpart in the original system and, thus the classic argument fails. For example, if we consider the case where $\lambda_1 = 0$ and $\lambda_2 > 0$, we observe that $\lambda_2 \leq q_2$ belongs to the stable throughput region, which contradicts (14), where for $\lambda_1 = 0$, we get $\lambda_2 = 0$.

Hence, we modify the hypothetical system to be constructed as follows:
- The arrivals at data queues $Q_1$ and $Q_2$ occur exactly at the same instants as in the original system.
- The transmission decisions, determined by independent coin tosses, are identical to those in the original system.
- In the energy-limited region, type I node does not transmit dummy packets. Thus, the energy arrivals to the battery queue of the harvesting node occurs exactly at the same instants as in the original system.
- In the interference-limited region, if the data queue is empty and the node decides to transmit, a dummy packet is transmitted, if the node has sufficient energy to transmit.

From the construction of the hypothetical system above, it can be noticed that it behaves like the original system in the energy-limited region and dominates the original system in the interference-limited region. The hypothetical system dominates the original system because the transmissions of dummy packets collide with the transmission of the other node. Also, the transmissions of dummy packets consume energy without contributing to the throughput of type II node.

C. Establishing the Necessary Conditions for Stability

In order to derive the necessary conditions for stability of the original system, we construct two dominant systems, where in the first dominant system type II node is backlogged and in the second dominant system type I node is backlogged only in the interference-limited region. As discussed in [25], in case of a system with batteries, the stability conditions, derived using the stochastic dominance techniques, are necessary conditions for the stability of the original system.
1) First dominant system: In this hypothetical system, we consider the case where the type II node continues transmitting dummy packets whenever its data queue, \( Q_2 \), is empty given that its battery is non-empty. Since the system is interference-limited from the perspective of type I node, our dominant system is identical to the one proposed in [18]. Hence, the saturated service rate of \( Q_1 \) is given by

\[
\mu_1^s = q_1 \left( 1 - \min \left\{ \frac{q_1 p_h}{(1 + q_1 p_h)} , q_2 \right\} \right). \tag{15}
\]

Also, by substituting \( \mathbb{P}[Q_1 > 0|Q_2 > 0] = \lambda_1/\mu_1^s \) in (14), the service rate of \( Q_2 \) becomes

\[
\mu_2 = \frac{(1 - q_1) \min \left\{ \frac{q_1 p_h}{(1 + q_1 p_h)} , q_2 \right\} \lambda_1}{q_1 \left( 1 - \min \left\{ \frac{q_1 p_h}{(1 + q_1 p_h)} , q_2 \right\} \right)}. \tag{16}
\]

Therefore, the stable throughput region of the first dominant system \( S_1 \) is given by

\[
S_1 = \left\{ (\lambda_1, \lambda_2) \mid \lambda_1 \leq q_1 \left( 1 - \min \left\{ \frac{q_1 p_h}{(1 + q_1 p_h)} , q_2 \right\} \right), \lambda_2 \leq \frac{(1 - q_1) \min \left\{ \frac{q_1 p_h}{(1 + q_1 p_h)} , q_2 \right\} \lambda_1}{q_1 \left( 1 - \min \left\{ \frac{q_1 p_h}{(1 + q_1 p_h)} , q_2 \right\} \right)} \right\}. \tag{17}
\]

Also, since the system becomes interference-limited from the perspective of node II, when the data queues, \( Q_1 \) and \( Q_2 \), are backlogged, \( \tilde{\lambda}_1 \) is given by

\[
\tilde{\lambda}_1 = \mu_1^s = q_1 \left( 1 - \min \left\{ \frac{q_1 p_h}{(1 + q_1 p_h)} , q_2 \right\} \right). \tag{18}
\]

2) Second dominant system: In this hypothetical system, \( Q_1 \) is backlogged only in the interference-limited region from the perspective of type II node. In the interference-limited part of \( S_2 \), the saturated service rate of \( Q_2 \) is given by

\[
\mu_2^s = (1 - q_1) \min \left\{ \frac{q_1 p_h}{(1 + q_1 p_h)} , q_2 \right\}. \tag{19}
\]

Similarly, by substituting \( \mathbb{P}[Q_2 > 0|Q_1 > 0] = \lambda_2/\mu_2^s \) in (13), we obtain

\[
\mu_1 = q_1 \left( 1 - \frac{\lambda_2}{1 - q_1} \right). \tag{20}
\]
Therefore, the stable throughput region of the interference-limited part of the second dominant system is given by

\[ S_{2|\lambda_1 \geq \bar{\lambda}_1} = \left\{ (\lambda_1, \lambda_2) \mid \bar{\lambda}_1 \leq \lambda_1 \leq q_1 \left(1 - \frac{\lambda_2}{1 - q_1}\right), \lambda_2 \leq (1 - q_1) \min \left\{ \frac{q_1 p_h}{1 + q_1 p_h}, q_2 \right\} \right\}. \tag{21} \]

The stable throughput regions of the dominant systems \( S_1 \) and \( S_{2|\lambda_1 \geq \bar{\lambda}_1} \) are shown in Fig. 4.

3) Stability region of the original system: In the following lemma we derive the relationship between the stable throughput region of the dominant systems \( S_1 \) and \( S_{2|\lambda_1 \geq \bar{\lambda}_1} \) and the original system \( S_o \).

**Lemma 2.** The stable throughput region of the original system \( S_o \) is given by the union of the stable throughput region of the first dominant system and the interference-limited part of the second dominant system, i.e.,

\[ S_o = S_1 \cup S_{2|\lambda_1 \geq \bar{\lambda}_1}. \tag{22} \]

**Proof:** The stable throughput region of the original system is the union of the two dominant systems, based on [18], i.e., \( S_o = S_1 \cup S_2 \). From the construction of the second dominant system, we know that the energy-limited region is identical to the original system, i.e., \( S_{2|\lambda_1 < \bar{\lambda}_1} = S_{o|\lambda_1 < \bar{\lambda}_1} \).

Hence, we have

\[ S_2 = S_{o|\lambda_1 < \bar{\lambda}_1} \cup S_{2|\lambda_1 \geq \bar{\lambda}_1}, \quad S_o = S_1 \cup S_{o|\lambda_1 < \bar{\lambda}_1} \cup S_{2|\lambda_1 \geq \bar{\lambda}_1}. \]

Now, assume that the rate pair \( (x_1, x_2) \in S_{o|\lambda_1 < \bar{\lambda}_1} \). Hence, \( x_1 < \bar{\lambda}_1 \) and \( x_2 \leq \mu_2 \). Since we achieve the maximum service rate \( \mu_2 \) by backlogging \( Q_2 \), from (17) we obtain

\[ x_2 \leq \frac{(1 - q_1) \min \left\{ \frac{q_1 p_h}{1 + q_1 p_h}, q_2 \right\} x_1}{q_1 \left(1 - \min \left\{ \frac{q_1 p_h}{1 + q_1 p_h}, q_2 \right\}\right)}. \]

Therefore, the rate pair \( (x_1, x_2) \in S_1 \) and \( S_{o|\lambda_1 < \bar{\lambda}_1} \subseteq S_1 \).


D. Discussion

According to [25], the stability conditions, derived using the stochastic dominance technique, are necessary but not sufficient, since the sample path dominance is not guaranteed due to the fact that the transmissions of dummy packets alter the dynamics of the battery. Thus, there exists instants at which type II node is no more able to transmit in the hypothetical system due to lack of energy, while it can transmit in the original system, which may imply higher success rate for type I node in the hypothetical system. Hence, the conditions are only necessary conditions for stability.

The authors in [25], characterized an inner bound by saturating both data queues, and derived the closure of the outer and inner bound over all transmission probabilities \((q_1, q_2)\). They found that the closure of the inner bound, and the closure of the outer bound are identical. Hence, the outer bound can be achieved and it is in fact the exact stability region. However, in our system, saturating both data queues is no longer an inner bound, since the dummy packets sent by type I node increase the energy supply at type II node. Hence, we need an alternative approach to prove the achievability of our outer bound, which is left as a future research direction.

E. The Closure over all Transmission Probabilities

In this section, we prove Corollary 1. The closure of \((S_o)\), is defined by the union of all stability regions for a given \([q_1, q_2]\), i.e.,

\[
C_{(S_o)} = \bigcup_{[q_1, q_2]} S_o([q_1, q_2]).
\]

In our system, the service rate of type II node is always lower than the arrival rate of type I node, i.e., \(\mu_2 < \lambda_1\). Also, from (11), we note that \(\mu_2\) is increasing in \(\min\{\frac{q_1 p_h}{1 + q_1 p_h}, q_2\}\), while \(\mu_1\) is not affected by \(q_2\). Hence, in order to find the closure \(C_{(S_o)}\), we need to find \(q_2\) that maximizes \(\mu_2\). From (11), we observe that for maximizing \(\mu_2\), the transmission probability of type II node \(q_2\) should be greater than or equal to \(q_2^* = \frac{q_1 p_h}{1 + q_1 p_h}\). Also, increasing \(q_2\) beyond \(q_2^*\) does not affect the service rate \(\mu_2\).

Interestingly, we can interpret \(q_2^*\) using Renewal reward theorem [31]. Assume that the data queues are backlogged and type II node transmits whenever it receives an energy packet, i.e., \(q_2 = 1\). Let the expected reward \(R\) that type II node obtains, be the transmission of one data
packet, i.e., \( \mathbb{E}[R] = 1 \). Also, the expected number of time slots, \( T \), needed for the transmission of one data packet is one slot for transmission, and \((q_1 p_h)^{-1}\) slots are needed for harvesting one energy packet. Hence, the expected time needed for a transmission \( \mathbb{E}[T] = 1 + (q_1 p_h)^{-1} \). Using the renewal reward theorem, we find that the effective transmission rate of type II node is given by

\[
q_{2, \text{eff}} = \frac{\mathbb{E}[R]}{\mathbb{E}[T]} = \frac{q_1 p_h}{1 + q_1 p_h}.
\]

(23)

Therefore, the previous expression represents the maximum possible transmission rate of type II node, which is the minimum transmission probability \( q_2 \) that maximizes \( \mu_2 \).

Now, the problem of finding the closure \( C_{(S_o)} \), reduces to finding the closure of \( S_o \left( [q_1, q_2^*] \right) \) over all \( q_1 \), i.e., \( C_{(S_o)} = \bigcup_{q_1 \in [0, 1]} S_o \left( [q_1, q_2^*] \right) \), where \( S_o \left( [q_1, q_2^*] \right) \) is given by

\[
\left\{(\lambda_1, \lambda_2) \mid \lambda_1 \leq q_1 \left(1 - \frac{\lambda_2}{1 - q_1}\right), \lambda_2 \leq (1 - q_1)p_h \lambda_1\right\},
\]

which represents the triangle OBD in Fig. 6, where \( D = (q_1, 0) \), and

\[
B = \left(\frac{q_1}{1 + q_1 p_h}, \frac{q_1 p_h (1 - q_1)}{1 + q_1 p_h}\right).
\]

Since, we know that the region \( S_o \) consist of two line segments, the closure \( C_{(S_o)} \) can be found by taking the union of the closures of the line segments \( \overline{OB}, \overline{BD} \). First, we find the closure of the line segment \( \overline{OB} \) by solving \( x = \frac{q_1}{1 + q_1 p_h} \), and \( y = \frac{q_1 p_h (1 - q_1)}{1 + q_1 p_h} \). The solution represents the trace of the point \( B \) for \( q_1 \in [0, 1] \), which is \( y = p_h x \left(1 - \frac{x}{1 - p_h x}\right) \). Hence, the closure of \( \overline{OB} \) is given by

\[
C_{\overline{OB}} = \left\{(\lambda_1, \lambda_2) \mid \lambda_2 \leq \lambda_1 p_h \left(1 - \frac{\lambda_1}{1 - \lambda_1 p_h}\right)\right\},
\]

which is represented in Fig. 5 by \( C_{\overline{OB}}^1 \) and \( C_{\overline{OB}}^2 \). Note that \( C_{\overline{OB}} \) is a convex region, since it is a hypograph of a concave function. Next, in order to find the closure of \( \overline{BD} \), we solve

\[
\max_{q_1 \in [0, 1]} q_1 \left(1 - \frac{\lambda_2}{1 - q_1}\right).
\]
The solution gives us the closure of the line segment extending \( \overline{OB} \) to the \( \lambda_2 \)-axis, represented by \( C_{OB}^1 \) and \( C_{BD}^2 \) in Fig. 5. However, since we want the closure of \( \overline{BD} \) and not the extension, \( C_{BD} \) is limited from the left by the trace of the point \( B \). Thus, the closure of \( \overline{BD} \) is the region bounded from the left by \( C_{OB}^2 \) and bounded by \( C_{BD}^1 \) from the right. It is characterized by

\[
C_{BD} = \left\{ (\lambda_1, \lambda_2) \in \Lambda \mid \sqrt{\lambda_1} + \sqrt{\lambda_2} \leq 1 \right\},
\]

\[
\Lambda = \left\{ \lambda_1 > \frac{1+2p_h-\sqrt{1+4p_h}}{2p_h^2}, \lambda_2 > \lambda_1 p_h \left(1 - \frac{\lambda_1}{1 - \lambda_1 p_h}\right) \right\}.
\]

Note that the first condition in \( \Lambda \) can be found by solving the two equations of \( C_{OB}^2 \) and \( C_{BD}^1 \).

Finally, the closure \( C_{(S_o)} \) is the union of \( C_{OB} \) and \( C_{BD} \), which is represented by \( C_{OB}^1 \) and \( C_{BD}^1 \) in Fig. 5. After some algebraic manipulations, we obtain (12).

**Remark:** (12) can also be obtained by maximizing the service rate of type II node \( \mu_2 \), subject to the stability condition of type I node \( \lambda_1 \leq \mu_1 \), i.e,

\[
\max_{q_1 \in [0,1]} (1 - q_1)p_h \lambda_1 \quad s.t \quad \lambda_1 \leq q_1 \left(1 - \frac{\lambda_2}{1 - q_1}\right).
\]

Hence, the closure of our system is equivalent to maximizing \( \mu_2 \), because \( \mu_2 \) is upper bounded by \( \mu_1 \). Thus, by maximizing \( \mu_2 \), we implicitly maximize \( \mu_1 \).
VI. MODEL EXTENSIONS

In this section, we discuss two extensions to the previous stability analysis. First, we investigate the impact of having a finite capacity battery on the stable throughput region. Second, we investigate the effect of having full-duplex energy harvesting node.

A. Finite Capacity Battery

In this subsection, we investigate the impact of having a finite capacity battery on the stable throughput region obtained in Theorem 1. Let $M$ be the capacity of type II node battery. Thus, the battery evolution equation becomes

$$B^{t+1} = \min \{ B^t - \mu_B^t + H^t, M \} \quad (24)$$

Similar to the unlimited capacity case, the battery queue forms a decoupled discrete time Markov chain given that the data queues are backlogged. By analyzing the Markov chain we find the probability that the battery is non-empty.

**Lemma 3.** For a half-duplex RF energy harvesting node with battery of size $M$, the probability that the battery is non-empty $\zeta$, given that the data queues $Q_1$ and $Q_2$ are backlogged, is given by

$$\zeta = \begin{cases} \rho \left( \frac{1 - \left( \frac{q_1 p_h (1 - q_2)}{q_2} \right)^M}{1 - \rho \left( \frac{q_1 p_h (1 - q_2)}{q_2} \right)^M} \right), & q_2 \neq \frac{q_1 p_h}{1 + q_1 p_h}, \\ 1, & q_2 = \frac{q_1 p_h}{1 + q_1 p_h} \end{cases} \quad (25)$$

where $\rho = \frac{q_1 p_h}{q_2 (1 + q_1 p_h)}$ is the probability that the battery is non-empty in the infinite capacity battery case.

**Proof:** Along the lines of Lemma 1.

Next, we apply the same procedure used in proving the stability region for the infinite capacity case.

**Theorem 2.** An outer bound on the stable throughput region of the opportunistic RF energy harvesting Aloha is the triangle OED, shown in Fig 6. Assuming half-duplex energy harvesting and a battery of size $M$, the region is characterized by the following equation

$$S_o = \left\{ (\lambda_1, \lambda_2) \mid \lambda_1 \leq q_1 \left( 1 - \frac{\lambda_2}{1 - q_1} \right), \lambda_2 \leq \frac{q_2 (1 - q_1) \zeta \lambda_1}{q_1 (1 - q_2 \zeta)} \right\} \quad (26)$$
Fig. 6: The stable throughput region of opportunistic energy harvesting Aloha with finite capacity battery is characterized by the triangle OED.

Proof: Along the lines of Theorem 1.

The reduction in the stability region due to finite capacity battery is the triangle OEB shown in Fig. 6.

B. Full-Duplex Energy Harvesting

Now, we investigate the effect of having a full-duplex energy harvesting type II node, i.e., the harvesting circuit is separated from the transmission circuit. Hence, a node can transmit and harvest simultaneously. Also, full-duplex energy transmission is advantageous because harvesting self-interference may introduce high energy gain. In the full-duplex energy harvesting paradigm, we have three types of harvesting opportunities. First, harvesting the transmissions of type I node while type II node is silent. Similar to the half-duplex case, we model this case by a Bernoulli process with mean $p_{h|1}$. Second, harvesting the self-interference of type II node while type I node is silent, which is modeled by a Bernoulli process with mean $p_{h|2}$. Third, harvesting both the transmissions of type I node and the self-interference of type II node, which is modeled by a Bernoulli process with mean $p_{h|1,2}$.

In order to characterize the probabilities $p_{h|2}$ and $p_{h|1,2}$, we use a similar approach to the one used in characterizing $p_{h|1}$ in Section III. We assume that the loopback interference coefficient $c \in [0,1]$ is known [32]. Also, we assume a Rayleigh fading channel between the transmit antenna and the harvesting antenna, i.e., $g_t \sim \exp(1)$. In case only self-interference is present, the received power at time slot $t$ is equal to $P_R(t) = \eta P_2 c g_t$. Hence, using the same
approach as in the half-duplex case, we obtain

\[ p_{h|\{2\}} = \frac{1}{1 + \frac{\gamma}{\eta P_{2e}}} \]  

(27)

In case both transmissions of type I node and the self-interference are present, the received power at time slot \( t \) is equal to \( P_R(t) = \eta(P_1 h(t)(1 + l^\alpha)^{-1} + P_2 c g_t) \). The probability \( p_{h|\{1,2\}} \) can be characterized in a similar fashion.

For a full-duplex harvesting node, the average energy harvesting process of the battery queue is given by

\[
\mathbb{E}[H^t] = q_1 p_{h|\{1\}} \mathbb{P}[Q_1 > 0] + q_2 p_{h|\{2\}} \mathbb{P}[Q_1 = 0, B > 0, Q_2 > 0]
\]

\[
+ (q_2 p_{h|\{2\}} + q_1 q_2 (p_{h|\{1,2\}} - p_{h|\{2\}} - p_{h|\{1\}})) \mathbb{P}[Q_1 > 0, B > 0, Q_2 > 0],
\]

where \( p_{h|M} \) is the harvesting probability given a set \( M \) of nodes are transmitting. The battery queue forms a decoupled Markov chain given that the data queues are backlogged. By analyzing the Markov chain we find the probability that the battery is non-empty, which is given by

**Lemma 4.** For a full-duplex RF energy harvesting node with infinite capacity battery, the probability that the battery is non-empty \( \Psi \), given that the data queues \( Q_1 \) and \( Q_2 \) are backlogged, is given by

\[
\Psi = \min \left\{ \frac{q_1 p_{h|\{1\}}}{q_2 (1 - q_1 (p_{h|\{1,2\}} - p_{h|\{2\}}) - (1 - q_1) p_{h|\{1\}})}, 1 \right\}.
\]  

(29)

**Proof:** Along the lines of Lemma 1.

We notice that the probability of non-empty battery for the full-duplex case is higher than that of the half-duplex case, i.e,

\[
\Psi \geq \frac{q_1 p_{h|\{1\}}}{1 + q_1 p_{h|\{1\}}}.
\]

Also, note that \( \mathbb{P}[B > 0|Q_1 = 0, Q_2 > 0] = 0 \). Since, once the battery is empty it will remain empty given \( Q_1 \) is empty. The stable throughput region of the system is given by the following theorem

**Theorem 3.** An outer bound on the stable throughput region of the opportunistic RF energy

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4In order to characterize \( p_{h|\{1,2\}} \), we need the distribution of the sum of independent gamma distributed random variables, all with integer shape parameters and different rate parameters, which is called the generalized integer gamma distribution (GIG) [33].
harvesting Aloha, under full-duplex energy harvesting mode and infinite capacity battery, is the region characterized by

\[ S_0 = \left\{ (\lambda_1, \lambda_2) \mid \lambda_1 \leq q_1 \left(1 - \frac{\lambda_2}{1 - q_1}\right), \lambda_2 \leq \frac{q_2(1 - q_1)\Psi\lambda_1}{q_1 (1 - q_2\Psi)} \right\} \]  

(30)

**Proof:** Along the lines of Theorem 1.

C. Numerical Analysis

In Fig. 7-9, we compare the stable throughput region of the RF EH-Aloha system and the conventional slotted Aloha with unlimited energy supply. The stability regions are shown for \(q_1 = 0.4, p_{h|1} = 0.2, p_{h|2} = 0.2,\) and \(p_{h|1,2} = 0.35.\) Also, we consider different values for \(q_2\) to compare between full-duplex and half-duplex energy harvesting. We observe that the stability region of slotted Aloha is significantly reduced when RF energy harvesting is implemented, due to the energy limited sub-region. Also, for small \(q_2,\) i.e., \(q_2 < \frac{q_1 p_{h|1}}{1 + q_1 p_{h|1}}\), we observe that the stability regions of half-duplex and full-duplex EH-Aloha are identical, because the service rate of type II node is limited by the transmission probability \(q_2.\) On the other hand, for large \(q_2,\) i.e., \(q_2 > \Psi,\) full-duplex energy harvesting expands the stability region, which agrees with intuition.

From Fig. 7, we notice that increasing \(q_2\) beyond \(\Psi\) enhances the throughput of node 2 in
Fig. 8: The stability regions of RF EH-Aloha under half/full-duplex modes vs. slotted Aloha with unlimited energy supply for \( \frac{q_1 p_h(1)}{1 + q_1 p_h(1)} < q_2 < \Psi \).

Fig. 9: The stability regions of RF EH-Aloha under half/full-duplex modes vs. slotted Aloha with unlimited energy supply for \( q_2 < \frac{q_1 p_h(1)}{1 + q_1 p_h(1)} \).
the slotted Aloha system. However, the throughput of the type II node in full-duplex EH-Aloha is limited by $\Psi$.

VII. CONCLUSION

In this paper, we studied the effects of opportunistic RF energy harvesting on the stability of a slotted Aloha system. We considered a system consisting of two types of nodes, namely type I which has unlimited energy supply and type II which is solely powered by an RF energy harvesting circuit. The transmissions of the type I node are recycled by the type II node to replenish its battery. In order to study the stability of RF EH-networks, we generalize the stochastic dominance technique. Then, we characterize an outer bound on the stable throughput region of RF EH-networks under the half-duplex and full-duplex energy harvesting paradigms. Moreover, we investigate the impact of finite capacity batteries on the stable throughput region. Finally, we derive the closure of the outer bound over all transmission probability vectors.

It is difficult to extend our analysis to the multi-user case, because of the complex interaction between the nodes. Alternative approaches should be investigated, which is left as a future direction. Another important future direction, is to prove the achievability of the outer bound, in order to characterize the exact stability region.

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