Perturbative Analysis of Nonabelian Aharonov-Bohm Scattering

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ABSTRACT

We perform a perturbative analysis of the nonabelian Aharonov-Bohm problem to one loop in a field theoretic framework, and show the necessity of contact interactions for renormalizability of perturbation theory. Moreover at critical values of the contact interaction strength the theory is finite and preserves classical conformal invariance.
I  Introduction

The nonabelian generalization of the Aharonov-Bohm (AB) effect [1, 2] is essentially the scattering of particles carrying nonabelian charge by a tube carrying a nonabelian magnetic flux. The two body case has recently been solved exactly [3, 4, 5], by choosing a convenient basis in which the problem reduces to the abelian AB effect.

Recent interest in the abelian AB effect is due to the fact that anyons (particles which acquire fractional statistics through the AB effect) are useful for understanding the Fractional Quantum Hall Effect [6], and may play a role in High $T_c$ Superconductors [7]. The exact solution to the AB scattering problem has been known for over thirty years, yet it had until recently [8] resisted a perturbative treatment. Earlier attempts at a perturbative solution failed by missing the s-wave contribution in first order, and producing a divergence in second order [9]. The failure was explained in Ref. [10] by showing that the series expansion of the exact solution is ill defined for a zero diameter flux tube. In Ref. [8] a field theoretic model for the AB effect was presented. It is based on Hagen’s model [11], but also includes contact interactions. It was shown in that paper that, for a critical value of the contact interaction strength, perturbation theory is well defined and gives the correct conformally invariant scattering amplitude to one loop. It was also shown that the model possesses a conformal anomaly away from the critical point. Subsequently, Freedman et. al. showed that conformal invariance is preserved at this critical point to three loops [12].

The nonabelian generalization of this field theoretic model was first studied at the classical level in Ref. [13]. Quantization and the derivation of the two body Schrödinger equation for the nonabelian AB problem was carried out in [14]. So far, a perturbative treatment has not been attempted, but it is obvious that it will suffer the same difficulties as in the abelian problem.

The aim of our paper is to perform a perturbative analysis of the nonabelian AB problem in a field theoretic framework. The field theory we use is a slight generalization of the one studied in Ref. [13]. We shall show that the contact interaction is necessary for renormalizability of the theory, and for a correct treatment of the AB problem. In section II we introduce the field theory and review the resulting two body Schrödinger equation and its solution. In section III we compute the two particle scattering amplitude to one loop and show that in general renormalization is necessary resulting in a conformal anomaly. We shall also show that this theory possesses critical points at which the anomaly vanishes and conformal invariance is regained. At a particular critical point, namely the completely repulsive critical contact interaction, the perturbative scattering amplitude agrees with the exact one. Section IV is devoted to concluding remarks.

II  Field Theoretical Formulation

Nonrelativistic bosonic particles carrying nonabelian charges are described by the Lagrange density,

\[ \mathcal{L} = \frac{1}{2} (\partial \psi)^2 - \sum_i A_i \bar{\psi} \gamma_i \psi - \sum_{ij} \frac{g_{ij}}{2} (\bar{\psi} \gamma_i \gamma_j \psi)^2 \]
\[ L = -\kappa \epsilon^{\alpha \beta \gamma} \text{tr}(A_\alpha \partial_\beta A_\gamma + \frac{2g}{3} A_\alpha A_\beta A_\gamma) + i \phi^\dagger D_t \phi - \frac{1}{2m} (D \phi)^\dagger \cdot D \phi \]

\[ -\frac{1}{4} \phi^\dagger_{\mu} \phi^\dagger_{\tau} C_{\mu \tau \nu \sigma} \phi_{\nu} \phi_{\sigma}, \]

where \( \phi \) is a complex bosonic field transforming in an irreducible representation of the gauge group \( G \), generated by the matrices \( T_a \) with \( a = 1 \ldots \text{dim} G \), and \( A_\mu \equiv A_\mu^a T_a \). The matrices satisfy the Lie algebra

\[ [T_a, T_b] = f_{ab}^c T_c, \]

and are normalized by

\[ \text{tr} (T_a T_b) = -\frac{1}{2} h_{ab}, \]

where \( h_{ab} \) is a nonsingular group metric. This metric can be used to raise and lower group indices. The covariant derivatives are given by

\[ D_t = \partial_t + g A_0 \]

\[ D = \nabla - g A. \]

The contact interaction term describes a delta function interaction between the particles. Since the particles are bosons we can assume

\[ C_{n' m' n m} = C_{m' n' m n}, \]

and from the reality of the Lagrange density, the matrix \( C \) should be Hermitian,

\[ C_{n' m' n m}^* = C_{n m n' m'}. \]

To make the notation concise we shall drop the matter indices, and regard four indexed objects as components of matrices in the basis \( |n, m\rangle \), for example,

\[ C_{n' m' n m} \equiv \langle n', m' | C | n, m \rangle \]

\[ T^a_{n' n} T^b_{m' m} \equiv \langle n', m' | T^a \otimes T^b | n, m \rangle \]

\[ C^2_{n' m' n m} \equiv \sum_{l, l'} \langle n', m' | C | l, l' \rangle \langle l, l' | C | n, m \rangle. \]

The last definition in (II.8) agrees with the usual matrix multiplication. An additional restriction on the form of \( C \) comes from gauge invariance of the action,

\[ [T_a \otimes 1 + 1 \otimes T_a, C] = 0. \]

By Schur’s lemma, \( T^2 \equiv T_a T^a \propto 1 \), so

\[ [T^2 \otimes 1, C] = 0, \quad [1 \otimes T^2, C] = 0. \]

Using the identity

\[ T^a \otimes T_a = \frac{1}{2} (T \otimes 1 + 1 \otimes T)^2 - \frac{1}{2} T^2 \otimes 1 - \frac{1}{2} 1 \otimes T^2, \]

\[ (\text{II.11}) \]
To get the most general gauge invariant form of $C$, let us use a basis that simultaneously diagonalizes $T^a \otimes T_a$, all the other Casimir operators $B$ constructed from $T_a \otimes 1 + 1 \otimes T_a$, and a maximal set of mutually commuting operators $W$ chosen from the set of $T_a \otimes 1 + 1 \otimes T_a$:

$$T^a \otimes T_a |\alpha, \beta, w\rangle = \alpha |\alpha, \beta, w\rangle .$$

(II.13)

Here $\beta$ and $w$ represent the eigenvalues of the operators $B$ and $W$ respectively. Note that the matrix $C$ is also a Casimir constructed from $T_a \otimes 1 + 1 \otimes T_a$ [cf. (II.9), which holds for all $a$]. Since we already use all the Casimir operators in the construction of the basis, the Casimir $C$ is diagonalized in this basis and its eigenvalues do not depend on $w$:

$$C|\alpha, \beta, w\rangle = c(\alpha, \beta)|\alpha, \beta, w\rangle .$$

(II.14)

Hence the most general gauge invariant form of $C$ is given by

$$C = \sum_{\alpha \beta w} |\alpha, \beta, w\rangle c(\alpha, \beta)\langle \alpha, \beta, w| .$$

(II.15)

Quantization of this theory in the two particle sector in Coulomb gauge yields the following Schrödinger equation [5, 14]

$$i\partial_t \psi(r_1, r_2; t) = \left\{ -\frac{1}{2m} \left[ \nabla + \frac{ig^2}{\kappa} G(r_1 - r_2)T^a \otimes T_a \right]^2 + \left[ 1 \leftrightarrow 2 \right] + \frac{C}{2} \delta(r_1 - r_2) \right\} \psi(r_1, r_2; t) ,$$

(II.16)

where $G(r) = \frac{1}{2\pi} \nabla \times \ln r$, and in the $|n, m\rangle$ basis, $T^a \otimes T_a \psi$ and $C\psi$ are respectively

$$(T^a \otimes T_a \psi)_{nm} = (T^a)_{nn'}(T_a)_{mm'}\psi_{n' m'}$$

$$(C\psi)_{nm} = C_{nmn' m'}\psi_{n' m'} .$$

(II.17)

The components of the wavefunction in the diagonal basis are given by

$$\psi_{\alpha \beta w} = \sum_{n, m} \psi_{nm}\langle n, m|\alpha, \beta, w\rangle .$$

(II.18)

In this basis the nonabelian problem is reduced to the abelian one. The time independent Schrödinger equation in the center of mass frame is

$$\left[ -\frac{1}{m} \left( \nabla + 2\pi i\nu G(r) \right)^2 + \frac{c}{2} \delta(r) - E \right] \psi_{\alpha \beta w}(r) = 0 ,$$

(II.19)

where $r \equiv r_1 - r_2$, and $\nu \equiv -\frac{g^2 e}{2\pi \kappa}$. With the usual boundary condition $\psi_{\alpha \beta w}(0) = 0$ the contact term drops out, and the solution is [4, 2, 15]

$$\psi_{\alpha \beta w}(r, \theta) = e^{i(\varphi \cos \theta - \nu(\varphi - \pi))} - \sin \nu \pi e^{-i(|\nu|+1)\theta} \int_{-\infty}^{\infty} \frac{dt}{\pi} e^{ipt \cosh t} \frac{e^{-(\nu)t}}{e^{-i\theta} - e^{-t}} ,$$

(II.20)
where \([\nu]\) is the greatest integer part of \(\nu\), \(\{\nu\} = \nu - [\nu]\), and \(\tilde{\theta}(r) = \theta(r) - 2\pi n\) when \(2\pi n \leq \theta < 2\pi(n + 1)\). [The overall phase is fixed by the condition that in the partial wave expansion, each ingoing partial wave has the same phase as the plane wave. The vanishing boundary condition at the origin makes the delta function in (II.19) irrelevant.] The expression is manifestly single valued. The function \(\tilde{\theta}(r)\) is discontinuous along the positive \(x\)-axis, but the wavefunction is continuous.

The first term in (II.20) is not appropriate as an incoming wave since the discontinuity gives a singular contribution to the particle flux along the positive \(x\)-axis.\(^\dagger\)

If we assume a plane wave form for the incident wave, and use the identity\(^\ddagger\)

\[
e^{ipr\cos\theta} = \sum_{-\infty}^{\infty} (-i)^n e^{in(\theta - \pi)} J_n(pr)
\]

\[
\sim \left(\frac{2\pi}{ipr}\right)^{1/2} e^{ipr\delta(\theta)} \quad \text{as } r \to \infty , \tag{II.21}
\]

we can cast the solution in the large \(r\) limit as

\[
\psi_{\alpha\beta w}(r, \theta) \sim e^{ipr\cos\theta} + \frac{1}{r} e^{ipr+\pi/4} f_\alpha(\theta) , \tag{II.22}
\]

where

\[
f_\alpha(\theta) = -\frac{i}{\sqrt{2\pi p}} \left[ \sin \pi\nu \cot \frac{\theta}{2} - i \sin |\pi\nu| - 4\pi \sin^2 \frac{\pi\nu}{2} \delta(\theta) \right] . \tag{II.23}
\]

The delta function in the forward direction is crucial for unitarity of the scattering matrix\(^\vardagger\).

In the original basis, the c.o.m. scattering amplitude is given by

\[
f_{n_1n_2 \to n_3n_4}(\theta) = \langle n_3, n_4 | \mathcal{F}(\theta) | n_1, n_2 \rangle
\]

\[
\mathcal{F}(\theta) = -\frac{i}{\sqrt{2\pi p}} \left[ \sin(\pi\Omega) \cot \frac{\theta}{2} - i \sin |\pi\Omega| - 4\pi \sin^2 \frac{\pi\Omega}{2} \delta(\theta) \right] , \tag{II.24}
\]

where

\[
\Omega \equiv -\frac{g^2}{2\pi\kappa} T^a \otimes T_a = -\frac{g^2}{2\pi\kappa} \sum_{\alpha\beta w} |\alpha, \beta, w\rangle \langle \alpha, \beta, w| \]

\[
|\Omega| \equiv \frac{g^2}{2\pi|\kappa|} \sum_{\alpha\beta w} |\alpha, \beta, w\rangle |\alpha| \langle \alpha, \beta, w| \]

\[
\Omega^2 = \frac{g^4}{4\pi^2\kappa^2} T^a T^b \otimes T_a T_b = \frac{g^4}{4\pi^2\kappa^2} \sum_{\alpha\beta w} |\alpha, \beta, w\rangle \alpha^2 \langle \alpha, \beta, w| . \tag{II.25}
\]

The abelian result is regained if one sets \(T = \frac{i}{\sqrt{\kappa}}\) and \(g = \sqrt{2}e\).

\(^\dagger\) This discontinuity of \(\tilde{\theta}(r)\) was not noticed in previous treatments of the AB problem, which led to the incorrect conclusion that the phase-modulated plane wave was appropriate as an incoming wave in the scattering solution\(^\vardagger\) [3].

\(^\ddagger\) Contrary to the statement made by Hagen\(^\vardagger\), the asymptotic relation (II.21) holds by virtue of the fact that the scattering matrix for a free particle is given by \(S = 1\).
Taking into account the exchange symmetry, the total scattering amplitude is
\[ f_{n_1n_2 \rightarrow n_3n_4}^{\text{tot}} = \langle n_3, n_4 | F(\theta) | n_1, n_2 \rangle + \langle n_4, n_3 | F(\theta + \pi) | n_1, n_2 \rangle . \] (II.26)

In contrast to the claim in Ref. [4], the amplitude is single-valued. Let us compare with Ref. [5] where the scattering amplitude is obtained for \( SU(2) \). First of all, our formula has a contribution of the delta function while theirs does not. At \( \theta \neq 0 \), their \( \bar{F}(\theta) \) is related to \( F(\theta) \) in (II.24) by
\[ F(\theta) = e^{i\pi \Omega} \bar{F}(\theta) \] (II.27)

Note that this matrix multiplication factor cannot be ignored when one considers the scattering cross section, \( \frac{d\sigma}{d\theta} |_{n_1n_2 \rightarrow n_3n_4} \), because of the effect of phase interference between the diagonalized channels. Also, in Ref. [5], they did not exchange the particle labels \( n_3 \) and \( n_4 \) in their exchange amplitude.

The amplitude (II.24) depends on the momentum \( p \) only through the kinematical factor, which reflects the conformal invariance of the system [15]. In fact, the action gotten by integrating eq. (II.1) possesses an \( SO(2,1) \) conformal symmetry, generated by time dilation
\[ t' = at \]
\[ r' = \sqrt{a} r \]
\[ \psi'(r', t') = \frac{1}{\sqrt{a}} \psi(r, t) \]
\[ A'_\mu(x') = \frac{\partial x'^\nu}{\partial x'^\mu} A_\nu(x) \] (II.28)

conformal time transformation
\[ \frac{1}{t'} = \frac{1}{t} + a \]
\[ r' = \frac{r}{1 + at} \]
\[ \psi'(r', t') = (1 + at)e^{-\frac{im^2}{2(1 + at)}} \psi(r, t) \]
\[ A'_\mu(x') = \frac{\partial x'^\nu}{\partial x'^\mu} A_\nu(x) \] (II.29)

and the usual time translation. This symmetry is broken however in perturbation theory by quantum corrections, producing an anomaly.

### III Perturbation Theory

We analyze the nonabelian AB scattering problem perturbatively in a field theoretic approach. We add to the Lagrange density (II.1) a gauge fixing term
\[ \mathcal{L}_{gf} = -\frac{1}{\xi} \text{tr} (\nabla \cdot A)^2 , \] (III.1)
and a corresponding ghost term

$$\mathcal{L}_{gh} = \eta^* a \left( \nabla^2 h_{ab} + g f_{abc} A^c \cdot \nabla \right) \eta^b .$$  \quad (III.2)

The Feynman rules are derived from the total Lagrange density. Fig. (1) depicts the propagators of this theory, given in the limit $\xi \to 0$ by

$$D(p) = \frac{i}{p_0 - \frac{1}{2m} p^2 + i\epsilon} \quad (III.3)$$

$$G(p) = -\frac{i}{p^2} \quad (III.4)$$

$$G_{0i}(p) = -G_{i0}(p) = \frac{\epsilon_{ij} p^j}{\kappa p^2} \quad (III.5)$$

$$G_{00}(p) = G_{ij}(p) = 0 . \quad (III.6)$$

Fig. (2) depicts the interaction vertices, given by

$$\Gamma^{a,0} \quad \Gamma^{a,i} (p, q) \quad \Gamma^{abc,i} (q) \quad \Gamma^{ab,ij}$$

Figure 1. Propagators

Figure 2. Interaction Vertices
Before computing the scattering amplitude we need to check that there are no corrections, at least to one loop, to the gluon propagator. These would contribute unwanted divergences to the scattering amplitude. We already know from the abelian theory that there are no corrections to the boson propagator, and we don’t really care about the ghost propagator since it won’t contribute to the one loop boson 4-point function. Fig. (3) depicts the two contributions to the gluon self energy, which only has space-space components.

\[
\Pi_{ab,ij}^{(1)} = \frac{g^2}{2m} f^{acd} f_{cd} \int \frac{d^3k}{(2\pi)^3} \frac{k^i(k-p)^j - (i \leftrightarrow j)}{k^2(k-p)^2} 
\]

\[
\Pi_{ab,ij}^{(2)} = -\frac{g^2}{2m} f^{acd} f_{cd} \int \frac{d^3k}{(2\pi)^3} \frac{k^i(k-p)^j}{k^2(k-p)^2} .
\]

The total self energy is then

\[
\Pi_{ab,ij} = \Pi_{ab,ij}^{(1)} + \Pi_{ab,ij}^{(2)} = -\frac{g^2}{2m} f^{acd} f_{cd} \int \frac{d^3k}{(2\pi)^3} \frac{k^i(k-p)^j - k^j(k-p)^i}{k^2(k-p)^2} = 0 .
\]

We compute the scattering amplitude by applying the Feynman rules to calculate the 4-point function in the c.o.m. frame and multiplying the result by \(-i\). Fig. (4) depicts the
tree level contributions, resulting in the amplitude:

\[ A^{(0)} = \frac{C}{2} - \frac{i2\pi}{m} \Omega \cot \frac{\theta}{2}, \]  

(III.16)

where \( \theta \) is the scattering angle.

\[ \begin{align*}
A^{(1)}_{\text{box}}(p, p') &= \frac{16\pi^2}{m} \Omega^2 \int \frac{d^2k}{(2\pi)^2} \frac{(k \times p)(k \times p')}{(k + p)^2(k + p')^2(k^2 - p^2 - i\epsilon)} \\
A^{(1)}_{\text{triangle}}(p, p') &= \frac{8\pi^2}{m} \Omega^2 \int \frac{d^2k}{(2\pi)^2} \frac{(p \times k)(p \times k')}{(p + k)^2(p + k')^2(p^2 - k^2 - i\epsilon)} \\
A^{(1)}_{\text{tri-gluon}}(p, p') &= \frac{4\pi^2}{m} \Omega^2 \int \frac{d^2k}{(2\pi)^2} \frac{(k \times p)(k \times p')}{(k + p)^2(k + p')^2(k^2 - p^2 - i\epsilon)} \\
A^{(1)}_{\text{bubble}}(p, p') &= \frac{2\pi^2}{m} \Omega^2 \int \frac{d^2k}{(2\pi)^2} \frac{(p \times k)(p \times k')}{(p + k)^2(p + k')^2(p^2 - k^2 - i\epsilon)}
\end{align*} \]

(III.17)

where \( p \) is the incident momentum in the c.o.m. frame, and \( p' \) is the scattered momentum. Using the well known decomposition,

\[ \frac{1}{k^2 - p^2 - i\epsilon} = \frac{P}{k^2 - p^2} + i\pi \delta(k^2 - p^2), \]  

(III.18)

we can split the amplitude into a real part and an imaginary part. The real part is given by

\[ \text{Re} \left( A^{(1)}_{\text{box}}(p, \theta) \right) = -\frac{2\pi}{m} \Omega^2 \ln \left| 2 \sin \frac{\theta}{2} \right|. \]  

(III.19)
The computation of the imaginary part is somewhat subtle, but the result is crucial. We expect a divergence in the forward direction on the grounds of unitarity. Integrating over the angle and then taking the limit \( k^2 \rightarrow p^2 \) gives

\[
Im \left( A_{\text{box}}^{(1)}(p, \theta) \right) = -\frac{2\pi^2}{m} \Omega^2 \left[ 1 - 2\pi \delta(\theta) \right],
\]

(III.20)

reproducing the \( \delta \)-function of (II.24). In the field theoretic approach one implicitly assumes that the asymptotic states (incoming and outgoing) are free particles, i.e. plane waves, so this result is consistent.

The triangle and tri-gluon contributions are given by

\[
A_{\text{triangle}}^{(1)} = -\frac{g^4}{4mk^2} (T^a T^b + T^b T^a) \otimes (T_a T_b) \int \frac{d^2 k}{(2\pi)^2} \frac{k \cdot (k - q)}{k^2 (k - q)^2},
\]

(III.21)

\[
A_{\text{tri-gluon}}^{(1)} = -\frac{g^4}{2mk^2} f_{abc} T^a \otimes (T^b T^c) \int \frac{d^2 k}{(2\pi)^2} \frac{k^2 q^2 - (k \cdot q)^2}{q^2 k^2 (k - q)^2},
\]

(III.22)

where \( q \equiv p - p' \). Using

\[
T^a T^b + T^b T^a = 2T^a T^b - [T^a, T^b] = 2T^a T^b - f^{abc} T^c
\]

we split \( A_{\text{triangle}}^{(1)} \) in two parts, with different tensor structures,

\[
A_{\text{triangle},1}^{(1)} \propto f_{abc} T^a \otimes (T_b T_c)
\]

and

\[
A_{\text{triangle},2}^{(1)} \propto \left( T^a T^b \right) \otimes (T_a T_b).
\]

By using Feynman reparameterization and Euclidean space dimensional regularization we get

\[
A_{\text{triangle},1}^{(1)} + A_{\text{tri-gluon}}^{(1)} = 0 \quad \text{(III.23)}
\]

\[
A_{\text{triangle},2}^{(1)} = -\frac{\pi}{m} \Omega^2 \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{p^2} - 2 \ln \left| \frac{2 \sin \frac{\theta}{2}}{\epsilon} \right| - \gamma + \mathcal{O}(\epsilon) \right],
\]

(III.24)

where the dimension of space is taken to be \( 2 - 2\epsilon \), \( \mu \) is an arbitrary scale, and \( \gamma \) is the Euler constant. (At this point we note that without the contact term in the action the theory would not be renormalizable, since there is no parameter to absorb the \( 1/\epsilon \) divergence.)

The contribution of the bubble diagram is

\[
A_{\text{bubble}}^{(1)} = \frac{1}{4} mc^2 \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2 - p^2 - i\epsilon}
\]

\[
= \frac{1}{16\pi} mc^2 \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{p^2} + i\pi - \gamma + \mathcal{O}(\epsilon) \right],
\]

(III.25)

and the total one loop scattering amplitude is given by

\[
A^{(1)} = \frac{m}{16\pi} \left[ C^2 - \frac{16\pi^2}{m^2} \Omega^2 \right] \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{p^2} + i\pi - \gamma \right] + i \frac{2\pi^3}{m} \Omega^2 \delta(\theta).
\]

(III.26)
This amplitude is renormalized by redefining the contact interaction matrix $C$:

$$c(\alpha, \beta) = c_{\text{ren}}(\alpha, \beta) + \delta c(\alpha, \beta)$$

$$\delta c(\alpha, \beta) = -\frac{m}{8\pi} \left( \frac{1}{\epsilon + \ln 4\pi - \gamma} \right) \left[ c_{\text{ren}}^2(\alpha, \beta) - \frac{4g^4}{m^2\kappa^2\alpha^2} \right]$$

$$C_{\text{ren}} = \sum_{\alpha\beta w} |\alpha, \beta, w\rangle c_{\text{ren}}(\alpha, \beta) \langle \alpha, \beta, w| ,$$  \hspace{1cm} (III.27)

and the total renormalized amplitude is given by

$$A_{\text{ren}}(p, \theta, \mu) = -\frac{2\pi i}{m} \left[ \Omega \cot\left(\frac{\theta}{2}\right) + \frac{mC_{\text{ren}}}{4\pi} - \Omega^2\pi^2\delta(\theta) \right] + \frac{m}{16\pi} \left( C_{\text{ren}}^2 - \frac{16\pi^2}{m^2}\Omega^2 \right) \left( \ln\frac{\mu^2}{p^2} + i\pi \right).$$  \hspace{1cm} (III.28)

A conformal anomaly appears through dependence on an arbitrary scale. There exist however critical points at which the amplitude (III.28) is conformally invariant, given by

$$C_{\text{ren}}^2 - \frac{16\pi^2}{m^2}\Omega^2 = 0.$$  \hspace{1cm} (III.29)

Inserting (II.15) into (III.29), the solution in the diagonal basis is given by

$$C_{\text{ren}} = -\frac{2g^2}{m|\kappa|} \sum_{\alpha\beta w} \epsilon(\alpha, \beta) |\alpha, \beta, w\rangle |\alpha\rangle \langle \alpha, \beta, w| ,$$  \hspace{1cm} (III.30)

where $\epsilon(\alpha, \beta)$ is either $+1$ or $-1$ and does not depend on $w$. We still have the freedom to choose the sign in each irreducible block of $C_{\text{ren}}$. One solution corresponds to choosing $\epsilon(\alpha, \beta) = \alpha/|\alpha|$ which gives $C_{\text{ren}} = -\frac{2g^2}{m|\kappa|} T_a \otimes T^a = \frac{4\pi\kappa}{m|\kappa|}\Omega$. Dunne et. al. have found self-dual solitons for this solution [13]. Another solution is gotten by choosing $\epsilon(\alpha, \beta) = +1$, resulting in a purely repulsive contact interaction in the diagonalized two body Schrödinger equation. For the latter choice, the total scattering amplitude is simply

$$A(\theta) = -\frac{2\pi i}{m} \left[ \Omega \cot\left(\frac{\theta}{2}\right) - i|\Omega| - \Omega^2\pi^2\delta(\theta) \right] + O(\Omega^3),$$  \hspace{1cm} (III.31)

which agrees, up to a kinematical factor, with the exact result in (II.24) to $O(\Omega^3)$. Putting the matter indices back in gives

$$A(n_1n_2 \rightarrow n_3n_4, \theta) = \langle n_3n_4|A(\theta)|n_1n_2\rangle + \langle n_4n_3|A(\theta + \pi)|n_1n_2\rangle.$$  \hspace{1cm} (III.32)

for the total scattering amplitude.

**IV Conclusion**

The nonabelian AB scattering result is successfully obtained to one loop in field theoretic perturbation theory. We demonstrated that contact interactions are necessary for a renormalizable perturbation theory, even though they do not contribute in the exact treatment.
The Schrödinger equation (II.16) requires physical input in the form of a boundary condition to obtain an exact solution. Such a boundary condition cannot however be imposed in a perturbative treatment, but its physical content can be included in the form of a contact interaction.

At critical values of the contact interaction, the theory is finite and conformally invariant. For a purely repulsive critical contact interaction, the perturbative one loop result agrees to second order with the exact solution with vanishing boundary condition at the origin.

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