Measurement of the Microwave Refractive Index of Materials Based on Parallel Plate Waveguides

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Abstract. An electrical field scanning apparatus based on a parallel plate waveguide method is constructed, which collects the amplitude and phase matrices as a function of the relative position. On the basis of such data, a method for calculating the refractive index of the measured wedge samples is proposed in this paper. The measurement and calculation results of different PTFE samples reveal that the refractive index measured by the apparatus is substantially consistent with the refractive index inferred with the permittivity of the sample. The proposed refractive index calculation method proposed in this paper is a competitive method for the characterization of the refractive index of materials with positive refractive index. Since the apparatus and method can be used to measure and calculate arbitrary direction of the microwave propagation, it is believed that both of them can be applied to the negative refractive index materials, such as metamaterials or “left-handed” materials.

1. Introduction
Metamaterials are one of the most popular research fields in recent decades. In theory, metamaterials have many peculiar properties, such as backward wave properties, negative refraction properties, resonant wave absorption properties, etc., which is a breakthrough in traditional electromagnetic phenomena. New high-performance electronic components or devices can be designed according to such peculiar properties. For example, a perfect copy of an object exists on the opposite side of the planar slab made of metamaterials with negative refractive index under idealized conditions [1]. In 2006, D. R. Smith et al. proposed that the electromagnetic cloaking structure could realize the electromagnetic wave diffraction propagation and achieve the stealth effect on the metal target of the microwave band [2]. In many of the electromagnetic properties of metamaterials, the refractive index is one of the important quantifiable characteristic parameters.

In 2001, R.A. Shelby and D.R. Smith et al. [3] developed a device for measuring the negative refractive index for left-handed materials. The prismatic sample was placed between two circular aluminum plates (30 cm in diameter) with a plate spacing of 1.2 cm. The rotating detector was used for measuring the microwave beam refracted by the prismatic samples. In this way, the transmitted power spectrum of the measured sample was measured as a function of angle from the interface normal, and then the refractive index of the measured sample was obtained. However, the device was not widely recognized or used after then.
In 2006, D. R. Smith et al. [2, 3] proposed a parallel plate waveguide apparatus at X-band frequencies thus constraining the electromagnetic fields to two dimensions. This apparatus can be used for characterizing the microwave anomalous propagation characteristics of metamaterials.

In this paper, we modified a similar apparatus based on the parallel plate waveguide method, and proposed a method for calculating the refractive index of the measured samples on the basis of measured data.

2. Measurement technique and calculation method

2.1. Measurement theory

The apparatus that we build and utilize is similar in concept to previous literatures [2, 4-7]. In the parallel plate waveguide chamber, the microwave scattering is reduced to two dimensions by confining electromagnetic waves between two conducting planes, so long as only lowest transverse electromagnetic (TEM) mode is excited. For this mode, the electric and magnetic fields do not vary along the axis between the plates (z-axis). Therefore, a map of the fields at any plane within the parallel plate waveguide chamber should provide an equivalent characterization of the microwave scattering. This equivalence is convenient, because it allows us to detect the microwave scattering just above the measured samples rather than inside, where it would be difficult to insert a probe antenna that would not also alter the field pattern.

2.2. Measurement system

The core of the apparatus that we build is the parallel plate waveguide chamber (as shown in Figure 1), consisting of an upper and a lower aluminium plate made from 10 mm tooled aluminium sheet. The mapping area within the chamber is set by a ring of microwave absorber (Dalian Tianyu microwave products) cut into a saw-toothed pattern and fixed to the lower plate, serving to reduce reflections from the waveguide edges back into the mapping region.

Figure 1. Photograph of the parallel plate waveguide chamber in the open position.

A planar beam can be excited by introducing microwaves into the chamber by an X-band waveguide adapter fixed to the lower plate. The adapter was connected to port 1 of the Vector Network Analyzer (VNA) (Agilent, model number E8363B). Since the measured sample is also placed on the lower plate, the samples and the sources do not move relative to each other during the scan. The bottom plate was fixed to a customized 2D automatic displacement platform. The electromagnetic fields in the chamber are detected by a coaxial antenna mounted onto the upper plate. The coaxial antenna was connected to port 2 of the VNA. The VNA provides the source microwave signal and phase sensitive detection of the return signal. A customized LabVIEW program coordinates the
motion of the platform and scan with the data acquisition of the VNA. The transmitted signal (S21) data, along with the position and frequency information, are stored as complex values in matrices.

2.3. Calculation Method of the Refractive Index
To calculate the refractive index of the measured sample, the field of the test area as shown in Fig. 2 was scanned and then the matrices were stored as shown below.

\[
\begin{bmatrix}
  f & x_1 & x_2 & \cdots & x_{n-1} & x_n \\
  y_1 & a_{11} & a_{12} & \cdots & a_{1,n-1} & a_{1,n} \\
  y_2 & a_{21} & a_{22} & \cdots & a_{2,n-1} & a_{2,n} \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
  y_{n-1} & a_{n-1,1} & a_{n-1,2} & \cdots & a_{n-1,n-1} & a_{n-1,n} \\
  y_n & a_{n,1} & a_{n,2} & \cdots & a_{n,n-1} & a_{n,n}
\end{bmatrix}
\text{and}
\begin{bmatrix}
  f & x_1 & x_2 & \cdots & x_{n-1} & x_n \\
  y_1 & \phi_{11} & \phi_{12} & \cdots & \phi_{1,n-1} & \phi_{1,n} \\
  y_2 & \phi_{21} & \phi_{22} & \cdots & \phi_{2,n-1} & \phi_{2,n} \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
  y_{n-1} & \phi_{n-1,1} & \phi_{n-1,2} & \cdots & \phi_{n-1,n-1} & \phi_{n-1,n} \\
  y_n & \phi_{n,1} & \phi_{n,2} & \cdots & \phi_{n,n-1} & \phi_{n,n}
\end{bmatrix}
\]

where \( f \) is the test frequency; \( x_i \) and \( y_j \) is the \( x \) coordinates of the \( i \)th row and the \( y \) coordinates of the \( j \)th column of the test point, respectively. \( a_{ij} \) and \( \phi_{ij} \) \( (i = 1, 2, \ldots, n-1, n; j = 1, 2, \ldots, n-1, n) \) are the amplitude and phase of S21 data acquired by the VNA, corresponding to the test point in the \( i \)th row and the \( j \)th column.

\[\theta_1 = \arccot(k) + \theta_2\]  

The refractive index, \( n \), can then be calculated according to the Snell’s law.
\[ n = \frac{\sin \theta_1}{\sin \theta_2} \]  

(2)

3. Results and discussion

Two samples of PTFE samples with a wedge of 15° and 30° sharp angles have been measured using the apparatus that we build. Each sample has a thickness of about 10 mm. The permittivity at the frequency of about 12 GHz were measured by a split-cavity resonator method [8] and shown around \( \varepsilon_{\text{PTFE}} = 2.31 \pm 0.06 \). When the permittivity, \( \varepsilon \), and permeability, \( \mu \), of a material are simultaneously positive, the refractive index is given by \( n = \sqrt{\varepsilon / \mu} \) (\( \varepsilon_0 \) and \( \mu_0 \) are the free-space permittivity and permeability, respectively) [9]. Therefore, the refractive index of the present PTFE samples would be predicted for \( n_{\text{PTFE}} = 1.52 \pm 0.02 \).

The normalized amplitude and phase map at the frequency of 12 GHz of the 15° wedge sample are shown in Figure 3 and Figure 4, respectively. Similar results were also obtained for another 30° wedge sample, but due to the length of the article, they were not given herein.

![Figure 3](image)

**Figure 3.** The normalized amplitude map at the frequency of 12.0GHz of the 15° wedge sample.

![Figure 4](image)

**Figure 4.** The normalized phase map at the frequency of 12.0GHz of the 15° wedge sample.

The refractive index of the measured wedge sample was calculated according to the above-mentioned method. The slopes of the linear fittings, \( k \), were shown in Figure 5 and Figure 6, respectively. And the refractive indexes, along with the process parameters in the calculation were listed in Table 1. It is clear that the refractive index measured by the apparatus is substantially consistent with the refractive index inferred with the permittivity of the sample. Obviously, this apparatus and the proposed method can be applied to measure and calculate arbitrary direction of the microwave propagation in the parallel plate waveguide chamber. Therefore, it is believed that this apparatus and the corresponding calculation method of the refractive index can be applied to the negative refractive index materials, such as metamaterials or “left-handed” materials.
Figure 5. The least square linear fitting result of the 15° wedge sample.

Figure 6. The least square linear fitting result of the 30° wedge sample.

Table 1. The process parameters of the refractive index calculated on the basis of the amplitude and phase matrices at the frequency of 12.0GHz of the wedge samples measured using the apparatus that we build.

| Sample        | incident angle θ₂ | slope k    | refractive angle θ₁ | Refractive index n |
|---------------|-------------------|------------|---------------------|--------------------|
| 15° wedge     | 15°               | -6.765     | 8.409°              | 1.535              |
| 30° wedge     | 30°               | -2.940     | 18.785°             | 1.504              |

4. Conclusions
We constructed a parallel-plate-waveguide based measurement setup to scan the amplitude and phase of the parallel-plate field. Measurement results of PTFE positive refractive index materials show that the refractive index calculation method proposed in this paper is a competitive method for the characterization of metamaterials with negative refractive index.

References
[1] Smith D R, Pendry J B and Wiltshire M C K 2004 Science 305 788.
[2] Schurig D, Mock J J, Justice B J, Cummer S A, Pendry J B, Starr A F and Smith D R 2006 Science 314 977.
[3] Shelby R A, Smith D R and Schultz S 2001 Science 292 77.
[4] Justice B J, Mock J J, Guo L H, Degiron A, Schurig D and Smith D R 2006 Optics Express 14 8694.
[5] Smith D R, Schurig D, Mock J J, Kolinko P and Rye P 2004 Appl. Phys. Lett. 84 2244.
[6] Parimi P V, Lu W T, Vodo P, Sokoloff J, Derov J S and Sridhar S 2004 Phys. Rev. Lett. 92 127401.
[7] Vodo P, Parimi P V, Lu W T and Sridhar S 2005 Appl. Phys. Lett. 86 201108.
[8] Janezic M D and Baker-Jarvis J 1999 IEEE Trans. Microw. Theory Tech. 47 2014.
[9] Pendry J B 2000 Phys. Rev. Lett. 85 3966.