Approximately $\mu - \tau$ Symmetric Minimal Seesaw Mechanism and Leptonic CP Violation

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We estimate the effect of leptonic CP violating phases in the minimal seesaw model with two heavy neutrinos $N$ by imposing the approximately $\mu - \tau$ symmetry on the model. For $N$ subject to the $\mu - \tau$ symmetry, we find that neutrinos show the normal mass hierarchy and obtain the general phase structure that contains only two phases which arise from $\mu - \tau$ symmetry breaking terms. To perform similar consideration for $N$ blind to the $\mu - \tau$ symmetry necessarily giving the inverted mass hierarchy, we assume the same phase structure. Our phases consist of six phases that need to take care of generic phases of flavor neutrino masses, from which one can obtain one Dirac phase and one Majorana phase as observable phases (because one neutrino is massless). The Dirac CP violating phase $\delta_{CP}(=\delta + \rho)$ depends on $\nu_e - \nu_x = (\nu_e - \nu_\mu)$ mixing phase $\delta$ (violating phase is suppressed because it depend on $\rho$, which is found to be associated with almost real quantity.

1. Introduction

The $\mu - \tau$ symmetry is the symmetry that provides the consistent sizes of mixing angles with those indicated by experimental data \cite{1}. In the minimal seesaw model \cite{2} with two heavy right-handed neutrinos \cite{3}, we obtain that 1) the normal mass hierarchy is realized if $N$ is subject to the $\mu - \tau$ symmetry while the inverted mass hierarchy is realized if $N$ is blind to the $\mu - \tau$ symmetry; 2) we understand how CP violation depends on phases of flavor neutrino masses; 3) Dirac CP violating phase depends on $\nu_e - \nu_x$ mixing phase $\delta$ and Majorana CP violation depends on $\nu_e - \nu_\mu$ phase $\rho$, which turns out to be suppressed.

The mass term for neutrinos in the minimal seesaw model is defined as follows:

$$L_{\text{mass}} = - \tau_R Y_\tau L H_d - \overline{N} Y_\nu L H_u - \frac{1}{2} N^T M_R N, \quad (1)$$

where $Y_\tau$ and $Y_\nu$ are Yukawa couplings, and $M_R$ is a mass matrix of $N$. Three flavor leptons are denoted by $L$ as SU(2)$_L$-doublets and by $e_R$ as SU(2)$_L$-singlets, $H_{u,d}$ denote two Higgs and two heavy flavor neutrinos are denoted by $N$ as SU(2)$_L$ singlets. If there are three flavorfor $N$, $N$ can be any combination of $N e, \mu, \tau$. We use $e, \mu, \tau$ as the suffix of $N$. We have chosen the base in which $Y_\nu$ is diagonalized. The coupling $Y_\nu$ and the mass matrix $M_R$ are parametrized by

$$Y_\nu = \begin{pmatrix} h_{f_1 e} & h_{f_1 \mu} & h_{f_1 \tau} \\ h_{f_2 e} & h_{f_2 \mu} & h_{f_2 \tau} \end{pmatrix}, \quad M_R = \begin{pmatrix} M_{R1f_1f_1} & M_{R1f_1f_2} & M_{R1f_2f_2} \\ M_{R1f_1f_1} & M_{R1f_1f_2} & M_{R1f_2f_2} \end{pmatrix}, \quad (2)$$

where $f_1 \neq f_2 = e, \mu, \tau$ or other possible combinations surviving as two “light” heavy neutrinos.

The seesaw mechanism yields a neutrino mass matrix:

$$M_\nu = - \langle H_u \rangle^2 Y_\nu^T M_R^{-1} Y_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix}, \quad (3)$$

which satisfies $\det (M_\nu) = 0$ in the minimal seesaw model. Hence, at least, one of neutrinos does not have a mass. $M_\nu$ is diagonalized by Pontecorvo-Maki-Nakagawa-Sakata Matrix $U_{PMNS} = U_\nu K$ \cite{4}. We parametrize $U_\nu$ and $K$ to be \cite{5}:

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & e^{i \gamma} \\ 0 & e^{-i \gamma} & 0 \end{pmatrix}, \quad K = \text{diag} \left( e^{i \beta_1}, e^{i \beta_2}, e^{i \beta_3} \right), \quad (4)$$

where $U_{PMNS}$ has three neutrino mixing angles $\theta_{12}, \theta_{13}$ and $\theta_{23}$ and six leptonic CP violating phases that are defined as Dirac CP phases: $\delta$, $\rho$ and $\gamma$ and Majorana CP phases: $\beta_1, \beta_2$ and $\beta_3$ \cite{6}, $c_{ij}$ and $s_{ij}$ represent $\cos \theta_{ij}$ and $\sin \theta_{ij}$. The PMNS unitary matrix converts the left-handed flavor neutrinos into massive neutrinos as $\nu_f = \sum_j (U_{PMNS})_{fj} \nu_j$, ($f = e, \mu, \tau$ i = 1,2,3). Note that $\gamma$ is the $\mu - \tau$ symmetery breaking quantity which is, therefore, generally small.

The phases $\rho$, $\gamma$ and one of Majorana phases are redundant parameters and therefore the PDG (particle data group) version \cite{7} of the PMNS matrix $U_{PDG}^{PMNS} = U_\nu^{PDG} K'$ can be defined by removing these redundant phases from $U_{PMNS}$ to be:

$$U_\nu^{PDG} = \begin{pmatrix} c_{13} c_{12} & c_{13} s_{12} & s_{13} e^{-i\delta_{CP}} \\ -s_{12} c_{23} & c_{12} c_{23} & s_{12} s_{23} e^{i\delta + i \rho} \\ -s_{13} c_{12} s_{23} & -s_{13} s_{12} c_{23} & c_{13} s_{23} \end{pmatrix}, \quad (5)$$

$$K' = \text{diag} \left( e^{i (\beta_1 - \rho)}, e^{i \beta_2}, e^{i \beta_3} \right), \quad (5)$$

with $\delta_{CP} = \delta + \rho$ as a Dirac CP violating phase.

We report main results obtained in Ref.\cite{8}. In the next section, we introduce the approximately $\mu - \tau$
symmetry, which is related to the invariance under the interchange of \( \nu_\mu \leftrightarrow -\sigma \nu_\tau \) \((\sigma = \pm 1)\). In particular, we discuss general phase structure for \( N \) subject to the \( \mu - \tau \) symmetry. In the third section, we argue how we describe the mass hierarchy in our model. In the fourth section, we estimate the CP violating phases in each mass hierarchy. The final section is devoted to summary and discussions.

2. \( \mu - \tau \) symmetry and it’s breaking

The mass matrix is devided into the \( \mu - \tau \) symmetric part \( M_{\mu}^{(+)} \) and the \( \mu - \tau \) symmetry breaking part \( M_{\mu}^{(-)} \):

\[
M_\mu = M_{\mu}^{(+)} + M_{\mu}^{(-)},
\]

\[
M_{\mu}^{(+)} = \begin{pmatrix} M_{\mu R} & M_{\mu e} \\ M_{\mu e}^* & M_{\mu e} \end{pmatrix},
\]

\[
M_{\mu}^{(-)} = \begin{pmatrix} \sigma M_{\mu e} \\ -\sigma M_{\mu e} \end{pmatrix},
\]

which is just an identity. The interchange of \( \nu_\mu \leftrightarrow -\sigma \nu_\tau \) leads to \( M_{\mu}^{(\pm)} \rightarrow \pm M_{\mu}^{(\pm)} \). We also use the notation: \( a_0 = M_{ee}, b_0 = M_{e\mu}, d_0 = M_{\mu\mu}^{(+)} \) and \( c_0 = M_{\mu\tau} \). The \( \mu - \tau \) symmetric part \( M_{\mu}^{(+)} \) gives the mixing angles as:

\[
s_{13} = 0, \quad s_{23} = \frac{\sigma}{\sqrt{2}}, \quad \tan 2\theta_{12} = \frac{2\sqrt{2}b_0}{a_0 - d_0 + \sigma c_0}. \tag{7}
\]

However, the Dirac CP phase vanishes because it appears as the coefficient of \( s_{13} \).

We introduce

\[
\nu_{\pm} = \nu_{U} \pm (-\sigma \nu_{\tau}) \quad N_{\pm} = N_{F_1} \pm (-\sigma N_{F_2}), \tag{8}
\]

to describe the effect of the \( \mu - \tau \) symmetry and it’s breaking, where \( f_1 = \mu \) and \( f_2 = \tau \) for \( N \) subject to the \( \mu - \tau \) symmetry. \( N_{\pm}, M_{R} \) and \( Y_{\nu} \) are parametrized as

\[
Y_{\nu} = Y_{\nu}^{(+)} + Y_{\nu}^{(-)},
\]

\[
Y_{\nu}^{(+)} = \begin{pmatrix} h_{\nu e}^{(+)} \\ h_{\nu e}^{(-)} \end{pmatrix},
\]

\[
Y_{\nu}^{(-)} = \begin{pmatrix} h_{\nu e}^{(-)} \\ h_{\nu e}^{(+)} \end{pmatrix},
\]

\[
M_R = M_R^{(+)} + M_R^{(-)},
\]

\[
M_R^{(+)} \equiv \text{diag}(M_{R++}, M_{R--}),
\]

\[
M_R^{(-)} = \begin{pmatrix} 0 & e^{i\omega_+} \\ e^{-i\omega_+} & 0 \end{pmatrix}, \tag{9}
\]

where the \( \mu - \tau \) symmetric parts are \( Y_{\nu}^{(+)} \) and \( M_R^{(+)} \) and the \( \mu - \tau \) symmetry breaking parts are \( Y_{\nu}^{(-)} \) and \( M_R^{(-)} \). The subscripts \( \pm \) represent terms for \( N_{\pm} \).

For \( N \) subject to the \( \mu - \tau \) symmetry, we require that \( M_{R++} \approx 0, \quad |h_{ij}^{(-)}| \ll 1 \) and that the mixing between \( N_+ \) and \( N_- \) be small. We obtain that, for \( \omega \) associated with the \( N_+ N_- \) mixing,

\[
Y_{\nu}^{(+)} = \begin{pmatrix} h_{\nu e}^{(+)} \\ h_{\nu e}^{(-)} \end{pmatrix},
\]

\[
Y_{\nu}^{(-)} = \begin{pmatrix} 0 \\ h_{\nu e}^{(-)} \end{pmatrix},
\]

\[
M_{ee} \approx -\omega^2 M_{ee}^{(-)},
\]

\[
M_{\mu e}^{(+)}, \quad M_{\mu e}^{(-)}, \quad M_{\mu \tau} \text{ are the mixing matrix}
\]

up to the first order of the breaking terms, where \( v = (0 \ H_u \ 0) \) and \( \omega \approx 0 \). In this case, we can absorb all phases of \( Y_{\nu}^{(+) \ 0} \) into phases of \( \nu_\mu, \nu_\tau, \) and therefore, the CP phases only arise from \( M_{\mu e}^{(-)} \). We use \( M_{\nu} \) by \( \alpha = \arg \left( M_{\mu e}^{(+) \ 0} \right) \) and \( \beta = \arg \left( M_{\mu e}^{(-)} \right) \). If we consider the second order of the breaking terms, which have been safely neglected, the \( \mu - \tau \) symmetric part \( M_{\mu e}^{(+) \ 0} \) includes CP phases. For \( N \) blind to the \( \mu - \tau \) symmetry, we obtain that

\[
Y_{\nu}^{(+)} = \begin{pmatrix} h_{\nu e}^{(+)} \\ h_{\nu e}^{(-)} \end{pmatrix},
\]

\[
Y_{\nu}^{(-)} = \begin{pmatrix} 0 \\ h_{\nu e}^{(-)} \end{pmatrix},
\]

\[
M_{ee} = -\omega^2 \left( M_{ee}^{(-)} \right)^2, \quad M_{\mu e}^{(+) \ 0}, \quad M_{\mu e}^{(-)}, \quad M_{\mu \tau} \text{ are the mixing matrix}
\]

(11)

where \( \theta \) is the \( N_+ N_- \) mixing angle \((c = \cos \theta, s = \sin \theta)\), up to the first order of the breaking terms of \( h_{\nu e}^{(-)}, h_{\nu e}^{(+)} \) and \( h_{\nu e}^{(-)} \). The \( \mu - \tau \) symmetric part of \( M_{\mu e}^{(+) \ 0} \) is not generally real. To performed similary consideration, we assume the same phase structure as the one for \( N \) subject to the \( \mu - \tau \) symmetry, namely \( M_{\mu e}^{(+) \ 0} \) is real.
We obtain neutrino masses from Eq.(A5) as
\[
    m_{1e}e^{-2i\beta_1} \approx \frac{1}{2} \left[ \left( e^{2i\eta_0} + d_0 - s_0 \right) + 2i \left( \Delta \gamma d_0 + (\nu + \Delta) d_0 e^{i\beta} \right) \right], \\
    -\frac{\sqrt{2}}{\sin 2\theta_{12}} \left[ \left( b_0 + i(\Delta + \gamma) \right) + b_0 e^{i\alpha} \right], \\
    m_{2e}^{-2i\beta_2} \approx \frac{1}{2} \left[ \left( e^{2i\eta_0} + d_0 - s_0 \right) + 2i \left( \Delta \gamma d_0 + (\nu + \Delta) d_0 e^{i\beta} \right) \right], \\
    +\frac{\sqrt{2}}{\sin 2\theta_{23}} \left[ \left( b_0 + i(\Delta + \gamma) \right) + b_0 e^{i\alpha} \right], \\
    m_{3e}^{-2i\beta_3} \approx \frac{1}{2} \left[ \left( d_0 + i\rho \right) - 2i(\nu + \Delta) d_0 e^{i\beta} \right], \\
\]
where we have parameterized \( M_\nu^{(-)} \) to be:
\[
    M_\nu^{(-)} = \begin{pmatrix} 0 & b_0 e^{i\alpha} & \sigma b_0 e^{i\alpha} \\ 0 & d_0 e^{i\beta} & 0 \\ \sigma b_0 e^{i\alpha} & 0 & -d_0 e^{i\beta} \end{pmatrix}. \\
\]

The parameter \( \Delta \) is defined by \( s_{23} = \sigma(1 - \Delta)/\sqrt{2}, \ c_{23} = (1 + \Delta)/\sqrt{2} \). For the normal mass hierarchy, \( m_2 \) can be further converted into
\[
    m_{2e}^{-2i\beta_2} \approx \frac{2\sqrt{2}}{\sin 2\theta_{12}} \left[ \left( b_0 + i(\Delta + \gamma) \right) + b_0 e^{i\alpha} \right], \\
\]
because of \( m_1 = 0 \).

3. Describing the mass hierarchy

In this section, we describe how the mass hierarchy is described by using Eq.(10) and Eq. (11). To perform numerical calculation, we have assumed \( \omega = 0 \) and the tri-bi maximal mixing for \( M_{\nu_{\mu}} \) which gives \( \sin^2 \theta_{12} = 1/3, \sin^2 \theta_{23} = 1/2, \sin^2 \theta_{13} = 0 \). There are three textures. These textures can reproduce respectively the normal mass hierarchy with \( m_1 \approx m_2 \) and the inverted mass hierarchy with \( m_1 \approx -m_2 \).

3.1. Normal mass hierarchy (\( m_2 \ll m_3 \))

As we can see from Eq.(10), we have the relation for \( N \) subject to \( \mu - \tau \) symmetry:
\[
    M_{ee}M_{\mu\mu}^{(+)} = \left( M_{e\mu}^{(+)} \right)^2, \\
\]
up to the first order of \( \mu - \tau \) symmetry breaking. This relation forbids us to use the inverted mass hierarchy because we can not reproduce the \( \mu - \tau \) symmetric mass matrix for the inverted mass hierarchy. We see that \( M_{ee}^{(+)} \approx 0 \) and \( M_{ee}, M_{\mu\mu}^{(+)} \neq 0 \) for the inverted mass hierarchy with \( m_1 \approx m_2 \) and that \( M_{ee}M_{\mu\mu}^{(+)} < 0 \) for the inverted mass hierarchy with \( m_1 \approx -m_2 \). We are only allowed to use the normal mass hierarchy.

The normal mass hierarchy is realized \([10]\) by
\[
    M_{ee}^{(+)} = m_0 \begin{pmatrix} 2 - pt \eta & \eta & -\eta \\ \eta & 1 & -\eta \\ -\eta & -\eta & 1 \end{pmatrix}, \\
\]
for \( |\eta| \ll 1 \). The condition \( det(M_{ee}) = 0 \) requires the relation \( p = 2/s \). We require that \( s = 2 \) for the tri-bi maximal mixing. The neutrino masses are computed from Eq.(12) and Eq.(14) giving
\[
    m_{2e}^{-2i\beta_2} \approx \frac{2\sqrt{2}M_{ee}^{(+)}m_0}{\sin 2\theta_{12}}, \\
    m_{3e}^{-2i\beta_3} \approx \left( m_0 (2 - s\eta) \right) \left( 2i\nu + \Delta m_0 + d_0 (\Delta - i\gamma) e^{i\beta} \right), \\
\]
(17)

3.2. Inverted mass hierarchy

The Eq.(11) gives the similar relation to Eq.(15) for \( N \) blind to the \( \mu - \tau \) symmetry:
\[
    M_{\mu\mu}^{(+)} = -\sigma M_{e\tau}, \\
\]
leading to \( m_3 = 0 \). Therefore Eq.(15) only allows the inverted mass hierarchy with either \( m_1 \approx \pm m_2 \).

**Type-I** (\( m_1 \approx m_2 \))

The type-I is realized \([10]\) by
\[
    M_{ee}^{(+)} = m_0 \begin{pmatrix} -2 + \eta & \eta & -\eta \\ \eta & 1 & -\eta \\ -\eta & -\eta & 1 \end{pmatrix}, \\
\]
for \( p = 1 \) to describe the tri-bi maximal mixing and \( |\eta| \ll 1 \). The neutrino masses are computed from Eq.(12) to be:
\[
    m_{1e}^{-2i\beta_1} \approx m_0 \left( 1 + e^{2i\rho} - \frac{1}{2} p \nu e^{2i\rho} - \frac{\sqrt{2}e^{i\rho}}{\sin 2\theta_{12}} \right), \\
    m_{2e}^{-2i\beta_2} \approx m_0 \left( 1 + e^{2i\rho} - \frac{1}{2} p \nu e^{2i\rho} + \frac{\sqrt{2}e^{i\rho}}{\sin 2\theta_{12}} \right), \\
\]
(20)

**Type-II** (\( m_1 \approx -m_2 \))

The type-II is realized \([10]\) by
\[
    M_{ee}^{(+)} = m_0 \begin{pmatrix} -2 + \nu & \nu & -\nu \\ \nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{pmatrix}, \\
\]
for \( q = 4 \) to describe the tri-bi maximal mixing and \( |\nu| \ll 1 \). The neutrino masses are also computed from Eq.(12) to be:
\[
    m_{1e}^{-2i\beta_1} \approx m_0 \left( \frac{2(1 - e^{2i\rho}) + \nu e^{2i\rho}}{\sin 2\theta_{12}} \right), \\
    m_{2e}^{-2i\beta_2} \approx m_0 \left( \frac{2(1 - e^{2i\rho}) + \nu e^{2i\rho}}{\sin 2\theta_{12}} \right), \\
\]
(22)

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4. Estimation of CP violating phases

In this section, we estimate sizes of CP phases. Our seesaw model has four phases: three phases from Yukawa couplings and one phase from Majorana phase for heavy neutrinos. These phases correspond to one Dirac phase and three Majorana phases.

The CP violating phases of $\delta$ and $\rho$ depend on $X$ and $Y$ \(\text{[A4]}\), which can be calculated to be:

\[
\begin{align*}
\delta_{13} & \approx \sqrt{2} \left( \frac{b_0 (a_0 - \sigma c_0 + d_0) + b_0 d_0 e^{i(\beta - \alpha)}}{\sqrt{2}} \right), \\
Y & \approx \sqrt{2} \left( \frac{(\sigma c_0 + a_0 d_0) + a_0 b_0 e^{-i\alpha}}{\sqrt{2}} \right),
\end{align*}
\]

where we neglect the second order of $\gamma$ and $\Delta$ because of $|\gamma|, |\Delta| \ll 1$. From Eq.\(\text{[A4]}\), we expect that $\rho$ will be suppressed if the term of $b_0 (a_0 - \sigma c_0 + d_0)$ in $X$ is not suppressed and this is the case of the present discussions.

We will show that the Dirac CP violating phase $\delta_{CP}$ depends on $\delta$ and that $\rho$ is suppressed because $X$ is almost real. Please see the detailed numerical results in \[5\].

4.1. Normal mass hierarchy

We obtain from \[16\]:

\[
\begin{align*}
\delta_{13} X & \approx m^2 \sqrt{2} \left( \frac{\eta^2 (p + s) + b_0 d_0 e^{i(\beta - \alpha)}}{\sqrt{2}} \right), \\
Y & \approx \sigma \sqrt{2} \left( \frac{m_0 (\Delta + i\gamma)}{\sqrt{2}} \right),
\end{align*}
\]

leading to

\[
\begin{align*}
\delta & \approx \alpha, \quad \rho \approx 0, \quad \varphi \approx 0.
\end{align*}
\]

4.2. Inverted mass hierarchy

In this case, we denote the majorana phase by $\varphi = \frac{1}{4} (\beta_1 - \beta_2)$.

Type-I

We approximately obtain $X$ and $Y$ from \[19\] as

\[
\begin{align*}
\delta_{13} X & \approx m^2 \sqrt{2} \eta, \\
Y & \approx \sigma m^2 \sqrt{2} b_0 e^{i\alpha},
\end{align*}
\]

which result in

\[
\begin{align*}
\varphi & \approx 0.
\end{align*}
\]

Type-II

We approximately obtain $X$ and $Y$ from \[21\] as

\[
\begin{align*}
\delta_{13} X & \approx \sqrt{2} m^2 \eta, \\
Y & \approx \sigma \sqrt{2} m^2 \left( q d_0 e^{i\beta} - 2 b_0 e^{i\alpha} \right),
\end{align*}
\]

leading to

\[
\begin{align*}
\delta & \approx \alpha, \quad \rho \approx 0.
\end{align*}
\]

In all three cases, the Majorana CP violating phase is found to be suppressed.

We have discussed how the size of leptonic CP violation is estimated from phases of flavor neutrino masses in the approximately $\mu - \tau$ symmetric minimal seesaw model. We have also discussed that, if $N$ is subject to the $\mu - \tau$ symmetry, neutrinos show the normal mass hierarchy while, if $N$ is blind to the $\mu - \tau$ symmetry, neutrinos show the inverted mass hierarchy. The clear dependance of leptonic CP violation on phases of flavor neutrino masses is presented in Eq.\[23\] for CP violation described by $\delta_{CP} = \rho + \delta$ with $\delta = - \arg (Y)$ and $\rho = - \arg (X)$ and in Eqs.\[12\] and \[14\] for Majorana CP violation described by $\varphi$. Since $X$ is found to be almost real (while $Y$ becomes generally complex), we observe that $\varphi$ is suppressed. We have obtain that $\varphi \approx - \rho/4$ for the normal mass hierarchy, $\varphi \approx 0$ up to $\rho^2$ for the inverted mass hierarchy with $r_1 \approx m_2$ and $\varphi \approx - \rho/6$ for the inverted mass hierarchy with $r_1 \approx m_2$ as long as the $\mu - \tau$ symmetric flavor neutrino masses are taken to be real. It should be noted that the real $\mu - \tau$ symmetric flavor neutrino masses arise as a general property of neutrinos exhibiting the normal mass hierarchy.

Since Majorana phase turns out to be suppressed because of $\rho \approx 0$, to expect larger Majorana CP violation, we may discuss another realization of the $\mu - \tau$ symmetry for neutrinos. For the same $M_{\nu}$ Eq.\[9\], $\sin \theta_{23} \approx - \sigma/\sqrt{2}$ is taken instead of the present value of $\sin \theta_{23} \approx \sigma/\sqrt{2}$ \[10\]. In this case, the larger Majorana CP violation is expected because of the role of $X$ and $Y$ is almost interchanged and we can find that $X$ becomes complex, yielding larger $\rho$ thereby, giving larger $\varphi$, and $Y$ becomes almost real. The detailed discussions based on this expectation will be presented elsewhere \[11\].
Acknowledgments

The author would like to thank Professor M. Yasuè for his patient reading of the manuscript and useful advices.

Appendix A: Useful formula

There are useful formula [5] for

\[
M \equiv M^\nu_\mu = \begin{pmatrix}
A & B & C \\
B^* & D & E \\
C^* & E^* & F \\
\end{pmatrix},
\]

which is expressed to be:

\[
M = M^{(+)} + M^{(-)},
\]

\[
M^{(+)} = \begin{pmatrix}
A & -B & -D & -E \\
B^* & D & E & 0 \\
0 & A & B^* & -D \\
\end{pmatrix},
\]

\[
M^{(-)} = \begin{pmatrix}
0 & A & B^* & -C \\
-\sigma B & -D & E & 0 \\
0 & -\sigma B & -D & E \\
\end{pmatrix},
\]

\[
A = [M_{\mu\mu}]_2^2 + 2 \left[ |M_{\mu\mu}^{(+)}|^2 + |M_{\mu\mu}^{(-)}|^2 \right],
\]

\[
B_\mu = M_{\mu\mu}^{(+)} M_{\mu\mu}^{(-)} \left( M_{\mu\mu}^{(-)} - \sigma M_{\mu\mu}^{(+)} \right) + M_{\mu\mu}^{(-)} M_{\mu\mu}^{(-)},
\]

\[
D_\mu = |M_{\mu\mu}^{(+)}|^2 + |M_{\mu\mu}^{(-)}|^2 + \left[ |M_{\mu\mu}^{(+)M_{\mu\mu}^{(-)}}|^2 \right] + |M_{\mu\mu}^{(-)}|^2,
\]

\[
E_\mu = \Re(\ell_j) = \Re \left( M_{\mu\mu}^{(-)} \right)^2 + 2 \left[ M_{\mu\mu}^{(-)} \right]^2 + \left[ |M_{\mu\mu}^{(+)M_{\mu\mu}^{(-)}}|^2 \right] + \left[ |M_{\mu\mu}^{(-)}|^2 \right] + |M_{\mu\mu}^{(-)}|^2,
\]

\[
E_\mu = \Im(\ell_j) = \Im \left( M_{\mu\mu}^{(-)} \right)^2 + 2 \left[ M_{\mu\mu}^{(-)} \right]^2 + \left[ |M_{\mu\mu}^{(+)M_{\mu\mu}^{(-)}}|^2 \right] + \left[ |M_{\mu\mu}^{(-)}|^2 \right] + |M_{\mu\mu}^{(-)}|^2,
\]

\[
(A2)
\]

\[
(A3)
\]

\[
(A4)
\]

\[
(A5)
\]

\[
(A6)
\]

References

[1] T. Fukuyama and H. Nishiura, in Proceedings of International Workshop on Masses and Mixings of Quarks and Leptons, Shizuoka, 1997 edited by Y. Koide (World Scientific, Singapore, 1997), p.272; “Mass Matrix of Majorana Neutrinos”, arXiv:hep-ph/9702253; R.N. Mohapatra and S. Nussinov, Phys. Rev. D 60 (1999) 013002; Z.Z. Xing, Phys. Rev. D 61 (2000) 057301; Phys. Rev. D 64 (2001) 093013; Phys. Rev. D 64 (2001) 093013; E. Ma and M. Raidal, Phys. Rev. Lett. 87 (2001) 011802; [Erratum-ibid 87 (2001) 159901]; C.S. Lam, Phys. Lett. B 507 (2001) 214; W. Grimus and L. Lavoura, JHEP 0107 (2001) 045; T. Kitabayashi and M. Yasuè, Phys. Lett. B 524 (2002) 308; P.F. Harrison and W.G. Scott, Phys. Lett. B 547 (2002) 219; T. Ohlsson and G.Seidl Nucl. Phys. B 643 (2002) 247.

[2] P.Minkowski, Phys. Lett. B 67 (1977) 421; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and Baryon Number in the Universe, ed. O. Sawada and A. Sugamoto (KEK report 79-18, 1979)p. 95;Prog. Theor. Phys. 64 (1980) 1870; M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity edited by P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam 1979),p.315; R.N. Mohapatra and G. SenjanovićPhys. Rev. Lett. 44 (1980) 912.See also,P. Minkowski, in proceedings of the XI International Workshop on Neutrino Telescopes in Venice, Venice, 2005, edited by M. Baldo Ceolin (Papergraf S.p.A.,Italy,2005),p.7.

[3] L.Lavoura and W. Grimus, JHEP 09 (007) 2007; T. Endoh, S.Kaneko, S.K. Kang, T. Morozumi, and T.Tanimoto, Phys. Rev. Lett. 89 (2002) 231601; P.H. Frampton, S.L. Glashow, and T. Yanagida, Phys. Lett. B 548 (2002) 119; M. Raidal and A. Strumia, Phys. Lett. B 553 (2003) 72; V. Barger, D.A. Dicus, H-J. He, and T. Li Phys. Lett. B 583 (2004) 173; R.G. Felipe, F.R. Joaquin, and B.M. Nobre, Phys. Rev. D 70 (2004) 085009. For a review, see, for example, W.L. Guo, Z.Z.Xing, and S.Zhou, Int. J. Mod. Phys. E 16 (2007) 1.

[4] B. Pontecorvo, Sov. Phys. JETP 7 (1958) 172 [Zh. Eksp. Teor. Fiz. 34 (1958) 247]; Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

[5] T. Baba and M. Yasiè, Phys. Rev. D 75 (2007) 055001.

[6] S.M. Bilenky, J. Hosek and S.T. Petcov, Phys. Lett. B 94 (1980) 495; J. Schechter and J.W.F. Valle, Phys. Rev. D 22 (1980) 2227; M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, Phys. Lett. B 102 (1981) 323.

[7] J. Eidelman et al. (Particle Data Group), Phys. Lett. B 592 (2004) 149. See also, L.-L. Chan and
W.-Y. Keung, Phys. Rev. Lett. 53 (1984) 1802.
[8] T. Baba and M. Yasuè, Phys. Rev. D 77 (2008) 075008.
[9] P.F. Harrison, D.H. Perkins, and W.G. Scott, Phys. Lett. B 530 (2002) 167; Z.Z. Xing, Phys. Lett. B 533 (2002) 85; P.F. Harrison and W.G. Scott, Phys. Lett. B 535 (2002) 163.
[10] K. Fuki and M. Yasuè, Phys. Rev. D 73 (2006) 055014; Nucl. Phys. B 783 (2007) 31.
[11] T. Baba and M. Yasuè, in preparation.