Impedance measurement: transformation of the probability density function of the results uncertainty

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Abstract. Monte Carlo method modeling allows determining the transformation of the probability density function of the uncertainty of the impedance components measurement results. Direct measurements of impedance components are based on the indicative form. The probability distribution density function of the uncertainty of direct measurement results is a constant. The components of algebraic and trigonometric forms of impedance are measured indirectly. The type of transformed probability density function for these forms is determined by calculating estimates of mean, standard uncertainty, extended uncertainty, asymmetry coefficient. Modeling allows to obtain reliable estimates of extended uncertainty of indirect measurements results of impedance components in accordance with current metrological standards.

1. Introduction
Impedancemetry is an effective method of investigating physical objects in various fields of science and technology \([1,2,3]\). The mathematical impedance model has three forms: indicative, algebraic, and trigonometric:

\[ \tilde{Z} = z \cdot e^{j\varphi} = R + jX = z \cdot \cos\varphi + j \cdot z \cdot \sin\varphi, \]

where \(z\) – is an impedance module \(\tilde{Z}\), \(\varphi\) – is an impedance argument (phase shift between voltage at the measurement object and current through it), \(R\), \(X\) and \(z \cdot \cos\varphi\), \(z \cdot \sin\varphi\) – are active and reactive components of complex resistance in algebraic and trigonometric forms, respectively[4].

The algebraic form of impedance is used in determining the structure and parameters of elements of the electrical scheme of physical object replacement \([5,6]\) and in studying the dynamic properties of a physical object by constructing a Nyquist diagram \([4,7]\). Trigonometric form of impedance is used in research of composite materials \([8]\).

The representative form is preferred in the measurement of impedance components. In this case, the measurement can be carried out by serial devices: AC voltmeter and phasometer. Impedance components in algebraic and trigonometric forms are calculated from the results of direct measurements of the impedance modulus and the impedance argument in indicative form using coupling equations and are the results of indirect measurements \([9,10]\).
The components of the indicative and algebraic forms are related by relations:

\[ R = \frac{z}{\sqrt{1 + \tan \phi}} \quad \text{and} \quad X = \frac{z}{\sqrt{1 + \cot \phi}}. \]

The components of the trigonometric form of impedance are calculated as multiplications \( z \cdot \cos \phi \) and \( z \cdot \sin \phi \).

Measurement results are necessarily accompanied by estimates of their uncertainty in accordance with international metrological standards. These standards establish the following uncertainty estimates: mean, standard uncertainty, extended uncertainty.

The measurement results of the impedance components in indicative form are used together to calculate the results of indirect measurements impedance components in algebraic and trigonometric forms. If the measurement results are used in conjunction with other measurement results, standard uncertainty is taken as the point characteristic of the measurement results uncertainty. Point characteristic of uncertainty has to be accompanied by indication of accepted approximation of probability density function of the results uncertainty. These rules apply to the uncertainty of direct measurements. The probability density function of the uncertainty of direct measurement results is a type of symmetric, single-mode, truncated function.

Generation of uncertainty estimates of indirect measurements results is based on hypothesis on uncertainty probability distribution of direct measurements according to law of uniform density. The uniform density law is characterized by much standard uncertainty from the distribution laws of the specified type. The second hypothesis is that there is no correlation between the values measured by the direct method and, accordingly, the uncertainties of their measurement results.

2. Problem definition

Results of direct measurements of impedance components are generated with known limits of extended uncertainty \( u(z) \) and \( u(\phi) \). The density distributions of the probabilities of these measurements have a uniform density.

Uncertainties results \( u(R) \) and \( u(X) \) of indirect measurements of impedance components in algebraic form are determined by differences:

\[ u(R) = \frac{z + u(z)}{\sqrt{1 + \tan (\phi + u(\phi))} - \frac{z}{\sqrt{1 + \tan \phi}} \quad \text{and} \quad u(X) = \frac{z + u(z)}{\sqrt{1 + \cot (\phi + u(\phi))} - \frac{z}{\sqrt{1 + \cot \phi}}. \]

Uncertainties results \( u \cdot (z \cdot \cos \phi) \) and \( u \cdot (z \cdot \sin \phi) \) of indirect measurements of impedance components in trigonometric form are determined by differences:

\[ u \cdot (z \cdot \cos \phi) = (z + u(z)) \cdot \cos (\phi + u(\phi)) - z \cdot \cos \phi \]

\[ \text{and} \]

\[ u \cdot (z \cdot \sin \phi) = (z + u(z)) \cdot \sin (\phi + u(\phi)) - z \cdot \sin \phi \]

Determining the approximation of the function of uncertainty probability density distribution of measurement results by analytical method requires cumbersome calculations and geometric constructions. The Monte Carlo method is more efficient than analytical methods in assessing uncertainty characteristics of indirect, joint and aggregate measurements when direct measurements of two or more values are processed [11]. For example, the Monte Carlo method obtained an estimate of the random error of the measurement results multiplication obtained by channels of measuring systems with a nominally linear conversion function [12].

3. Method

The Monte Carlo method allows determining the probability density function and estimating the uncertainty characteristics of indirect and aggregate measurements based on known direct measurement error characteristics. The typical error of the Monte Carlo method is 5-10%. Such accuracy fully satisfies the reliability requirements of the uncertainty estimation of the measurement results. The number of
implementations is determined depending on the measurement accuracy requirements and must be at least $10^6$.

The Monte Carlo method makes it possible to simulate the transformation of the probability density distribution of impedance measurement results in the transition from the indicative form to the algebraic or trigonometric form and to obtain graphs of the probability density functions of indirect measurement results.

The simulation algorithm has the following characteristics. Input variables: number of implementations $M$; number of intervals of the series of measurement results (number of columns of the histogram) $K$; results of direct measurements of module $z$ and phase shift $\phi$; limits of extended uncertainty of measurements results of impedance module and impedance phase shift. Program output variables: mean, standard uncertainty, asymmetry coefficient, extended uncertainty. Graphs of the surrounding histograms of probability density distribution of uncertainty of measurement results are also the result of simulation.

The model had parameters: $M=10^6$, $K=100$, $z=1$, $\phi = 0.1\pi, 0.25\pi, 0.4\pi$. The relative limits of the extended uncertainty of the measurement results $z$ and $\phi$ are $\pm 1\%$.

4. Results and discussion

Probability density functions of the measurement results uncertainty of impedance components in algebraic and trigonometric forms are shown in figure 1 and figure 2, respectively.

![Figure 1](image1.png)

**Figure 1.** The probability density distribution of uncertainty of impedance components measurement results at phase shift $\phi =0.1\pi$.

![Figure 2](image2.png)

**Figure 2.** The probability density distribution of uncertainty of impedance components measurement results at phase shift $\phi =0.4\pi$. 

Estimates of mean, standard uncertainty, asymmetry coefficient, and extended uncertainty of the uncertainty probability density distributions of the three forms of impedance component measurements are presented in the table 1.

**Table 1. Characteristics of measurement uncertainty.**

| Impedance components | Mean     | Standard uncertainty | Asymmetry coefficient | Expanded uncertainty, ± |
|----------------------|----------|----------------------|-----------------------|-------------------------|
|                      | $\varphi=0.1\ \pi$ |                      |                       |                         |
| $Z$                  | $8.97\cdot10^{-6}$ | $5.77\cdot10^{-4}$  | $-1.60\cdot10^{-3}$  | $9.5\cdot10^{-3}$       |
| $\Phi$               | $6.49\cdot10^{-7}$ | $1.81\cdot10^{-3}$  | $-1.60\cdot10^{-3}$  | $3.10\cdot10^{-3}$      |
| $R$                  | $7.92\cdot10^{-6}$ | $5.06\cdot10^{-3}$  | $-1.10\cdot10^{-3}$  | $8.4\cdot10^{-3}$       |
| $X$                  | $2.45\cdot10^{-6}$ | $3.08\cdot10^{-3}$  | $2.69\cdot10^{-3}$   | $5.5\cdot10^{-3}$       |
| $zcos\varphi$        | $6.77\cdot10^{-6}$ | $5.52\cdot10^{-3}$  | $-1.32\cdot10^{-3}$  | $9.1\cdot10^{-3}$       |
| $zsin\varphi$        | $2.89\cdot10^{-6}$ | $2.48\cdot10^{-3}$  | $9.83\cdot10^{-3}$   | $4.7\cdot10^{-3}$       |
|                      | $\varphi=0.25\ \pi$ |                      |                       |                         |
| $Z$                  | $8.97\cdot10^{-6}$ | $5.77\cdot10^{-4}$  | $-1.60\cdot10^{-3}$  | $9.5\cdot10^{-3}$       |
| $\Phi$               | $4.89\cdot10^{-7}$ | $4.54\cdot10^{-3}$  | $-3.21\cdot10^{-4}$  | $7.5\cdot10^{-3}$       |
| $R$                  | $4.34\cdot10^{-6}$ | $4.38\cdot10^{-3}$  | $2.57\cdot10^{-3}$   | $7.9\cdot10^{-3}$       |
| $X$                  | $4.71\cdot10^{-6}$ | $4.39\cdot10^{-3}$  | $3.82\cdot10^{-3}$   | $7.9\cdot10^{-3}$       |
| $zcos\varphi$        | $-1.31\cdot10^{-6}$ | $5.19\cdot10^{-3}$  | $8.33\cdot10^{-3}$   | $9.8\cdot10^{-3}$       |
| $zsin\varphi$        | $-5.76\cdot10^{-7}$ | $5.19\cdot10^{-3}$  | $9.77\cdot10^{-3}$   | $9.8\cdot10^{-3}$       |
|                      | $\varphi=0.4\ \pi$ |                      |                       |                         |
| $Z$                  | $8.97\cdot10^{-6}$ | $5.77\cdot10^{-4}$  | $-1.60\cdot10^{-3}$  | $0.01$                 |
| $\Phi$               | $9.49\cdot10^{-7}$ | $7.25\cdot10^{-4}$  | $-3.10\cdot10^{-4}$  | $0.012$                |
| $R$                  | $-3.47\cdot10^{-5}$ | $5.43\cdot10^{-3}$  | $2.45\cdot10^{-3}$   | $0.01$                 |
| $X$                  | $1.39\cdot10^{-5}$ | $5.66\cdot10^{-3}$  | $5.67\cdot10^{-3}$   | $0.01$                 |
| $zcos\varphi$        | $-6.26\cdot10^{-6}$ | $7.13\cdot10^{-3}$  | $6.59\cdot10^{-3}$   | $0.012$                |
| $zsin\varphi$        | $-1.62\cdot10^{-5}$ | $5.93\cdot10^{-3}$  | $1.42\cdot10^{-3}$   | $0.011$                |

Analysis of simulation results allows to draw the following conclusions.

The asymmetry coefficient in all cases does not exceed $10^{-2}$, therefore all distributions are symmetrical and meet the requirements of the standard.

The mean displacement of all distributions relative to direct measurements is negligible compared to point estimates of measurement uncertainty.

The standard and extended uncertainty of the probability density distributions of the uncertainty of the indirect measurement results is less than the corresponding uncertainty characteristics of the direct impedance measurement results.

Graphs of envelope of probability density distributions of measurement results uncertainty of impedance components in algebraic form have the form of trapezium. At $\varphi=0.25\pi$ envelopes coincide. The standard uncertainty of the active impedance component becomes less than the standard uncertainty of the reactive component at $\varphi<0.4\pi$. The opposite is true if $\varphi>0.4\pi$.

The envelope of the probability density distribution of uncertainty of measurements results of the impedance components in trigonometric form is transformed from a trapezium into a triangle when the
argument $\varphi$ is reduced. The envelopes are also the same at $\varphi=0.4\pi$. The standard uncertainty of the active impedance component in trigonometric form is greater than the standard uncertainty of the reactive component when the argument changes in the range of $\varphi <0.4\pi$. The opposite is true if $\varphi>0.4\pi$.

The standard uncertainty of distributions differs by 1.5-2 times for the measurement results of the impedance components in algebraic and trigonometric forms with the impedance argument measurements results $\varphi =0.1\pi$ and $\varphi =0.4 \pi$. Such differences cannot be considered insignificant.

The simulation showed that equality of probability density functions of impedance component measurement results in indicative form does not provide equality of these functions for impedance components in other forms. The ratio of standard component uncertainties in some cases exceeds 2.

5. Conclusion
Monte Carlo modeling allows determining the shape of the envelope and the numerical characteristics of the transformed probability density distributions of the impedance measurement results uncertainty in the transition from the indicative form of impedance to the algebraic or trigonometric form [13,14]. The standard uncertainty of the measurement results of impedance components in algebraic and indicative forms varies significantly with different ratios of modulus values and impedance phase shift in indicative form. Also, the standard uncertainty of these components differs significantly (up to 2 times) with some values of module and phase shift. Therefore, the estimation of uncertainty by the Monte Carlo method should be made for each measurement result of each of the impedance components in algebraic and trigonometric forms.

Metrology increasingly uses multidimensional models of physical objects and processes based on hypercomplex numbers [15,16]. This trend also extends to impedance measurement. Quaternion is a mathematical model of nonlinear thermo-dependent hyperimpedance, and octonion allows to synthesize a model of hyperimpedance including all basic values of SI system [17]. The Monte Carlo method is a practically non-alternative method of studying transformations of probability density functions of direct measurements results uncertainty of hyperimpedance components into hypercomplex model parameters.

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