Analysis of finite fluctuations as an approach to mathematical remodeling

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Abstract. The paper introduces the interpretation of Analysis of Finite Fluctuations as an approach to Mathematical Remodeling. Fitting based on a given multivariable function a new function, connecting the response fluctuation with the fluctuations of its arguments, refers to Mathematical Remodeling problems and is a standard problem in Mathematical Analysis, which is usually solved approximately under the assumption of smallness of fluctuations. Some important applications have identified an importance of mathematical problems to represent finite (generally speaking, not small) fluctuations of a function (response) via finite fluctuations of its arguments. The way to solve the problem in this current formulation comes from classical Mathematical Analysis in the form of Lagrange and Bonnet theorems, operating with finite fluctuations.

1. Introduction
Before to present the concept of Mathematical Remodeling, it is worthwhile to recall the main stages of the mathematical modeling [19]: Stage I is to construct a mathematical model of the problem under consideration, to fit the dependency of an object or a system; Stage II is to study the constructed model by means of a suitable mathematical approaches and computer simulations; Stage III is to interpret results of the study and to evaluate model adequacy; Stage IV is to adjust the model in case of inadequacy or in connection with new data on-line inflow.

The purpose of the paper is to show that Mathematical Remodeling occurs already in the Stage II of the mathematical modeling and to present some aspects of Analysis of Finite Fluctuations as an approach to Mathematical Remodeling, which can replace the model connecting factors and response with a model based on their finite fluctuations.

The Section 2 presents a review on origins and development of the concept of Mathematical Remodeling with an emphasis on its traditional approaches, and the Section 6 introduces a review on applied aspects of Mathematical Remodeling. The Section 3 presents a basis of Analysis of Finite Fluctuations, the Section 4 gives ideas of Analysis of Finite Fluctuations, based on Lagrange mean value theorem, and the Section 5 provides Analysis of Finite Fluctuations based on Bonnet theorem. Conclusion summarizes the considerations of this research. It should be noted that performed at the Stage IV adjustment of the model also admits the interpretation
as an approach to Mathematical Remodeling, but this aspect is expected to devote a separate research.

2. Mathematical Remodeling Concept

The idea of Mathematical Remodeling, which consists in the transformation of some mathematical models in others, in accordance with defined reasons and criteria were stated in connection with the applied problems of the metallurgical production (cf. [15]) and is actively developing, including the same applications (e.g., [17, 11, 3]).

There are approaches which are close to remodeling scheme (remodeling, cf. [16]). They are surrogate modeling (cf. [13]), metamodeling (cf. [20]), co-simulation (cf. [12]) and some others similar approaches (re-run, repeat, reproduce, reuse, replicate, cf. [2]).

Many classical and modern approaches and methods of fundamental and applied mathematics allow to be interpreted as Mathematical Remodeling.

The Stage II of the mathematical modeling is usually divided into two sub-stages: before getting results to interpret in the Stage III, results obtained with the model constructed in the Stage I, could be remodeled by the structure from the remodeling class suitable for computer simulation and applying various mathematical approaches.

Traditionally remodeling is used to approximate static models using classical numerical methods of function approximation; in this case remodeling classes are Taylor polynomials, Fourier polynomials, etc. In particular, it is an approximate linearization; variables fluctuations in this case are assumed to be small. In case of not small but finite fluctuations, the exact linearization may be based on Lagrange mean value theorem (intermediate point theorem) and the second mean value Bonnet theorem; this is considered further.

Traditional remodeling is also used to approximate dynamic models using classical numerical methods of solving ordinary or partial differential equations, that is, the transformation (remodeling) of the continuous lumped or distributed models to discrete structures.

Traditional remodeling (approximation) could be used to solve nonlinear optimization problems, e.g., by using linearization or linear programming problems; as well as many other well known classical approaches.

The other traditional approach is an equivalent remodeling, in which, in contrast to the approximation, the model is not approximated, but substituted by the equivalent one.

An equivalent remodeling of various orders canonical system of ordinary differential equations, solved with respect to higher-order derivatives, which presents the transformation of the model of a continuous dynamic lumped system into an equivalent normal Cauchy system, in which all the equations have the first order, and the system has the order which is the sum of the orders of the original differential equations system. The similar approach could be mentioned for discrete lumped dynamic systems described by systems of ordinary finite-difference equations. This approach may also include the reduction of discrete distributed dynamical systems to the minimum basic models which are close to lumped models.

The equivalent remodeling is also associated with the transformation of dynamic systems into equivalent “input-state-output” models, i.e. the solution of the direct problem for these systems.

Traditional remodeling is also an equivalent transformation of linear, with constant coefficients, differential or finite-difference equations and systems of equations (linear stationary lumped dynamic systems) into an algebraic system using $L$ or $Z$–transformations (operation method). The similar way could be applied for distributed systems by using a multidimensional transformation.

Traditional approach of transforming distributed systems into lumped (evolutionary) models which is called “lumping” is also an approach to the remodeling. The example which should be mentioned here is the reduction of basic models of homogeneous linear discrete distributed
dynamic multidimensional MD-system to the associated with it “lumped” (in an associated time)
variable structure 1D-model (with increasing in an associated time dimensions of the composite
vectors and block matrices).

Many of mentioned above approaches to remodeling have an understandable and good
investigated background. One can note Analysis of Finite Fluctuations, which finds an
application in Economics within the transition from a planned to a market economy; the
transformation of MD-models to 1D-models, which allow to extend well-developed methods
of classical control theory to distributed systems; and some others.

Modern approaches to mathematical remodeling reported in [17, 11, 3, 16, 13, 20, 12, 2] are
extending classes of models which are used as remodeling structures (artificial neural networks
and neuro-fuzzy systems, inclusion functions, etc.) and extending a data domain as well
(idempotent mathematics).

3. Analysis of Finite Fluctuations

Analysis of Finite Fluctuations is one of the approaches to Mathematical Remodeling which has
a great practical importance. It could be understood as the transition from the initial values of
the model to the fluctuations of these values.

In a practical applications normally the argument \( v \) could be measured, and its measurement
\( \mu(v) \) could have different forms. The most used forms for transition from initial value \( v(0) = a \)
to the value \( v(1) = b \) are

- absolute increment \( \mu(v) = \Delta v = b - a \);
- index \( \mu(v) = i(v) = \frac{b}{a} \);
- relative increment \( \mu(v) = \delta(v) = \frac{b - a}{a} = \frac{\Delta(v)}{a} = i(v) - 1 \).

The central problem of Analysis of Finite Fluctuations is formulated as follows. Let us have
at the Stage I of the mathematical modeling the fitted model

\[
y = f(x) = f(x_1, ..., x_n), \quad x \in \mathbb{R}^n,
\]

which describes the connection between the response \( y \) and its arguments \( x_i, i = 1, ..., n \). It is
necessary to transform this model (1) to the model

\[
\mu(y) = \phi(\mu(x_1), ..., \mu(x_n)),
\]

which shows the connection between the fluctuation of the response \( \mu(y) \) and the fluctuations
\( \mu(x_i) \) of its arguments \( x_i, i = 1, ..., n \).

In many applied problems the fluctuations mentioned above could be considered as small.

In case of small increments Mathematical Analysis [9] gives an example of the approximative
remodeling. If the function \( y = f(x_1, ..., x_n) \) describing the model under consideration, is defined
and continuous in a closed domain and has continuous partial derivatives within this domain,
the approximate connection between the response small fluctuation and small fluctuations of its
arguments is

\[
\Delta(y) = f(x^{(0)} + \Delta x) - f(x^{(0)}) =
\]

\[
= f(x_1^{(0)} + \Delta x_1, ..., x_n^{(0)} + \Delta x_n) - f(x_1^{(0)}, ..., x_n^{(0)}) \approx
\]

\[
\approx \sum_{i=1}^{n} \frac{\partial f(x^{(0)})}{\partial x_i} \cdot \Delta(x_i).
\]

On the other hand, in some applied problems the fluctuations could not be considered as
small values, but could be interpreted as finite values. This is for example the problems of
Economic Factor Analysis in application to the transition from planned to market economy.
4. Lagrange Analysis of Finite Fluctuations

Mathematical Analysis gives the basis for Mathematical Remodeling and, in case of finite increments in \([10]\) in the paragraph “Mean value theorem” there is the model, which allows to remodel the model (1) to an exact connection between the finite fluctuation of the response and the finite fluctuations of its arguments. This is Lagrange mean value theorem (the formula of finite increments, intermediate value theorem of Differential Calculus) for multivariable functions, defined and continuous in a closed domain and having continuous partial derivatives inside this domain. It is formulated in the following manner:

\[
\Delta y = \sum_{i=1}^{n} \frac{\partial f(x^{(m)})}{\partial x_i} \cdot \Delta(x_i),
\]

\[
x^{(m)} = (x_1^{(m)}, \ldots, x_n^{(m)}), \quad x_i^{(m)} = x_i^{(0)} + \alpha \cdot \Delta(x_i),
\]

\[
i = 1, \ldots, n, \quad 0 < \alpha < 1.
\]

Here the mean (or intermediate) values of arguments \(x_i^{(m)} = x_i^{(0)} + \alpha \cdot \Delta(x_i), 0 < \alpha < 1,\) are defined by the value \(\alpha\).

In case of a single-factor model

\[
\Delta y = f(b) - f(a) = f'(c) \cdot (b - a) = f'(a + \alpha \cdot \Delta(x)) \cdot \Delta x = L \cdot \Delta(x),
\]

\[
c = a + \alpha \cdot \Delta(x), \quad 0 < \alpha < 1
\]

(here and further \(L\) comes from Lagrange).

Rename for the future reference

\[
x^{(0)} = a, \quad x^{(0)} + \Delta(x) = b,
\]

\[
\Delta(x) = b - a, \quad \Delta(y) = f(b) - f(a),
\]

\[
x^{(m)} = c = x^{(0)} + \alpha \cdot \Delta(x), \quad a < c < b.
\]

In mentioned further variants of Lagrange mean value theorem, the parameter values \(0 < \alpha < 1\) could differ, hence different notations will be used.

In [10] the paragraph 304 “Mean value theorem” gives Lagrange mean value theorem in its integral representation. If the function \(y = f(x)\) is continuous inside the interval \([a, b]\), there is a point \(c\ (a < c < b)\), such that

\[
\int_{a}^{b} f(x)dx = f(c) \cdot (b - a).
\]

Applying the fundamental theorem of Calculus to this theorem leads to the formula of the antiderivative \(F(x)\) for the function \(f(x)\):

\[
F(b) - F(a) = F'(c) \cdot (b - a).
\]

Renaming here and further \(F\) into \(f\), \(F'\) into \(f'\), we get the mean value theorem in its main (differential) form with the parameter \(\beta\) instead \(\alpha\).

Analogues of the model (3) for other measures of finite factors and response variations (indices and relative increments) can be obtained by the remodeling this model by the logarithmic substitution of variables [7]:

\[
i(y) = \prod_{i=1}^{n} (i(x_i))^{f^{(c)}(x^{(m)})}.
\]
The formula of finite increments allows us to represent each of them in the form of a Lagrange increment:

\[ \Delta f_j = ((\Delta f_j)_{\mathcal{L}})_{\alpha_j} = \sum_{i=1}^{m} \frac{\partial f_j}{\partial x_i} (x + \alpha_j \Delta x) \cdot \Delta x_i, \quad j = 1, \ldots, n, \tag{5} \]

\( \mathcal{L} \) from Lagrange with its own value for each Lagrange parameter \( \alpha_j \in (0, 1) \). Lagrange mean value theorem in a general case does not indicate a way to find these parameters; it is not constructive, it refers to pure existence theorems; however, as shown below, for many specific functions, the parameter values can be calculated by solving their equations

\[ \Delta f_j = ((\Delta f_j)_{\mathcal{L}})_{\alpha_j}, \quad j = 1, \ldots, n. \]

Now let \( p \) be a quantity, aggregating all the outputs of \( f_j, \ j = 1, \ldots, n \) in the form of a scalar function of the vector argument \( p = p(f) = p(\ldots, f_j, \ldots) \). For its direct increment \( \Delta p = p(f + \Delta f) - p(f) \) is fairly similar to the above representation in the form of the Lagrange increment

\[ \Delta p = ((\Delta p)_{\mathcal{L}})_{\beta} = \sum_{j=1}^{n} \frac{\partial p}{\partial f_j} (f + \beta \Delta f) \Delta f_j, \]

which for specific functions of \( p \) can be calculated the values of the Lagrange parameter \( \beta \). It should be noted that in applications sometimes this representation of the Lagrange increment of the aggregating quantity is used, applying direct responses increments.
Often it is more important — and more logical — to represent the Lagrange increment of the aggregating quantity, using Lagrange increments of responses. For the reason that each response has its own Lagrange increment, the last formula should be rewritten in this way:

$$\Delta p = ((\Delta p)_L)_\beta = \sum_{j=1}^{n} \frac{\partial p}{\partial f_j}(..., f_j + \beta \Delta f_j, ...) \Delta f_j.$$

Introducing the special symbol for the result of substitution into the last formula of Lagrange increments of individual outputs instead of direct outputs, we can write

$$\Delta p = (((((\Delta p)_L)_\beta)_L)_{\alpha_j}, ...) =$$

$$\sum_{j=1}^{n} \frac{\partial p}{\partial f_j}(..., f_j + \beta ((\Delta f_j)_L)_{\alpha_j}, ...)((\Delta f_j)_L)_{\alpha_j}.$$

Using expressions (5) for Lagrange increments of responses in the multipliers of this formula, it can be rewritten like that:

$$\Delta p = (((((\Delta p)_L)_\beta)_L)_{\alpha_j}, ...) =$$

$$\sum_{j=1}^{n} \frac{\partial p}{\partial f_j}(..., f_j + \beta ((\Delta f_j)_L)_{\alpha_j}, ...) \sum_{i=1}^{m} \frac{\partial f_j}{\partial x_i}(x + \alpha_j \Delta x) \cdot \Delta x_i$$

or changing the order of summation:

$$\Delta p = (((((\Delta p)_L)_\beta)_L)_{\alpha_j}, ...) =$$

$$\sum_{i=1}^{m} \left[ \sum_{j=1}^{n} \frac{\partial p}{\partial f_j}(..., f_j + \beta ((\Delta f_j)_L)_{\alpha_j}, ...) \frac{\partial f_j}{\partial x_i}(x + \alpha_j \Delta x) \right] \cdot \Delta x_i.$$ (6)

In this formula, the superposition character of the aggregating quantity $p$ is considered in a special way, which is a complex function (superposition or composition) of factors depending on them through the outputs.

5. **Bonnet Analysis of Finite Fluctuations**

In [10] in the paragraph 304 “Extended mean value theorem” and in the paragraph 306 “The second mean value theorem” are given forms of the mean value theorem in the integral representation, which are related to the integral of the product of two functions $f(x) \cdot g(x)$ (including Bonnet formulas). Requirements to the function $g(x)$ allow us here and further to assume $g(x) \equiv 1$. With that in mind, the extended mean value theorem from [10] leads again to the model (4) with the parameter $\gamma$ against $\alpha$. Further are mentioned variants of the mean value theorem in Differential Calculus to which (with $g(x) \equiv 1$) lead the forms of the mean value theorem in the Integral Calculus in [10] (including Bonnet formulas).

If the derivative $f'(x)$ of the function $f(x)$ is continuous, monotone decreasing and non-negative inside the interval $[a, b]$, than

$$\Delta y = B_1 \cdot \Delta x = f'(a) \cdot \rho \Delta x$$

(7)

(here and further $B$ comes from Bonnet).
If the derivative $f'(x)$ of the function $f(x)$ is continuous, monotone increasing and non-negative inside the interval $[a, b]$, than
\[
\Delta y = B_2 \cdot \Delta x = f'(b) \cdot \rho (1 - \sigma) \Delta x.
\] (8)

If the derivative $f'(x)$ of the function $f(x)$ is continuous and monotone inside the interval $[a, b]$, than
\[
\Delta y = B \cdot \Delta x = (f'(a) \cdot \tau + f'(b) \cdot (1 - \tau)) \Delta x.
\] (9)

In models (7)–(9) parameters $\rho, \sigma, \tau$ are, in general, different (it was mentioned above) values of the parameter $0 < \alpha < 1$ from the formula (2).

The basis for the model (7) is the formula (3) from [10]:
\[
\int_a^b f(x) \cdot g(x) = f(a) \cdot \int_a^\xi g(x) dx, \quad a < \xi < b,
\]
which (within $g(x) \equiv 1$, $\xi = a + \rho \cdot \Delta x$ and using the fundamental theorem of Calculus) could be written as follows (the model (7))
\[
\Delta y = f(b) - f(a) = f'(a) \cdot ((a + \rho \cdot \Delta x) - a) = f'(a) \cdot \rho \cdot \Delta x.
\] (10)

Similarly, the formula (8) could be obtained from the corresponding formula from [10] (the paragraph 306), and the formula (9) — from formula (4) ([10], paragraph 306). The formula (9) could be obtained from the formula (7), with replacing $f'(x)$ by $f'(x) - f'(b) \geq 0$, as well as from the formula (8), with replacing $f'(x)$ by $f'(x) - f'(a) \geq 0$ (in these two variants of the formula (9) values of the parameter $\tau$, generally speaking, may vary).

Lagrange mean value theorem in the general case is non-constructive. It is a pure existence theorem and gives no way to find the parameter $\alpha$ values in the equation (4) for an arbitrary function; this is, for example, connected with the difficulty of finding the parameter $\alpha$ value in the formula (4) for an arbitrary function solving the equation
\[
f'(a + \alpha \cdot \Delta x) = \frac{\Delta y}{\Delta x},
\] (10)
which includes $\alpha$ under the sign of the derivative of the considered function and desirable (although, generally speaking, insufficient) specification of the function in the equation (10).

In contrast, parameters $\rho, \sigma, \tau$ exist in formulas (7–9) directly and linear. And this fact (in case of applying mentioned formulas) allows to obtain
\[
\rho = \frac{\Delta y/\Delta x}{f'(a)}, \quad \sigma = 1 - \frac{\Delta y/\Delta x}{f'(b)}, \quad \tau = \frac{(\Delta y/\Delta x) - f'(b)}{f'(a) - f'(b)}.
\] (11)

Returning to the multivariable model (1), it should be noted that, if the partial derivatives of the function describing the model is continuous and monotonic on the relevant variables in hyperparallelepiped $\prod_{i=1}^n [a_i, b_i]$, taking into account the model (9), the model (3) could be remodeled into
\[
\Delta y = \sum_{i=1}^n \left( \frac{\partial f(a_1, ..., a_n)}{\partial x_i} \cdot \tau + \frac{\partial f(b_1, ..., b_n)}{\partial x_i} \cdot (1 - \tau) \right) \cdot \Delta x_i.
\] (12)
To find the parameter $\tau$ values an analogue of the last formula (11) could be applied. Thus, in the simplest case of the function $z = x \cdot y$, partial derivatives of which $z'_x = y$, $z'_y = x$ are continuous and monotone inside $[a, b] \times [c, d]$, this formula gives

$$
\tau = \frac{(bd - ac) - d(b - a) - b(d - c)}{(c - d)(b - a) + (a - b)(d - c)} = \frac{-(b - a)(d - c)}{-2(b - a)(d - c)} = \frac{1}{2}.
$$

This result is well known and interpreted in Economic Factor Analysis [7].

### 6. Applied Aspects of Mathematical Remodeling

Applied aspects of Analysis of Finite Fluctuations and Mathematical Remodeling are presented in [7, 5, 6, 18, 8, 4]. In the book [7] it deals with metallurgical production management problems and energy consumption questions. So, for the values assessment there were applied approaches of Chain Dynamic Economic Factor Analysis. In this connection, advantages of Lagrange Analysis of Finite Fluctuations comparing with the traditional methods of Economic Factor Analysis were shown (in particular, the method of chain substitutions). The book [5] presents problems associated with the financial condition of the organization assessment, with employees stimulation, and with the structural optimization of a traffic control system.

Regarding the latter mentioned problem, it is noted that in connection with the growth of population motorization, and consequently, its transport mobility, problems associated with traffic flow management are becoming more relevant. One of the visible reasons of the increasing number of personal vehicles is a traffic congestion at an urban roads intersection. To solve the problem of traffic congestion at signalized intersections the average traffic delay is used as an indicator of the operation quality. The transport delay is a one-variable or multivariable function. An important problem is to research the behavior of this function within its arguments fluctuations [7, 18]. The answer to this question could be obtained from the point of view of Lagrange Analysis of Finite Fluctuations for the aggregated transport delay function.

Consider a class of rational functions of a special form:

$$
f(x_1, \ldots, x_n) = \frac{a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} b_i x_i^2}{\sum_{i=1}^{n} x_i}, \quad x_i \geq 0, \quad a_0, a_i, b_i \geq 0. \tag{13}
$$

The initial values of arguments $x_i^0$, $i = 1, \ldots, n$ correspond the initial value of the response $f(x_1^0, \ldots, x_n^0)$. The arguments get a change of $\Delta x_i$ which generate new values of the arguments $x_i^0 + \Delta x_i$ and function response $f(x_1^0 + \Delta x_1, \ldots, x_n^0 + \Delta x_n)$.

Partial derivatives of the function (13) are

$$
\frac{\partial f(x_1, \ldots, x_n)}{\partial x_i} = a_i + 2b_i x_i + \frac{a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} b_i x_i^2}{\sum_{i=1}^{n} x_i}. \tag{14}
$$

Directly the increment of the function (13) could be found as

$$
f(x_1^0 + \Delta x_1, \ldots, x_n^0 + \Delta x_n) - f(x_1, \ldots, x_n), \tag{15}
$$

And from the other hand, Lagrange increment is

$$
\sum_{i=1}^{n} \frac{\partial f}{\partial x_i} (x_1^0 + \alpha \cdot \Delta x_1, \ldots, x_n^0 + \alpha \cdot \Delta x_n) \cdot \Delta x_i. \tag{16}
$$
Equating expressions (14) and (15) and solving the resulting equation for $\alpha$, we obtain:

$$\alpha_1 = \frac{-\sum_{i=1}^{n} x_i + \sqrt{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i^0 + \Delta x_i \sum_{i=1}^{n} x_i}}{\sum_{i=1}^{n} x_i}, \quad \alpha_2 = \frac{-\sum_{i=1}^{n} x_i - \sqrt{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i^0 + \Delta x_i \sum_{i=1}^{n} x_i}}{\sum_{i=1}^{n} x_i}. \quad (16)$$

Because of the assumption, that parameter values and its increments are positive, the value of $\alpha_1$ satisfying the condition of Lagrange mean value theorem of setting the intermediate point inside the interval $(0, 1)$ is used.

In the absence of the model connecting the response and its arguments it is possible to use retrospective statistical data on the process. In this case, it is appropriate based directly on fluctuations of these data to fit models. In the paper [4] it is indicated that a well known problem of the reduction of electric power systems as the transformation of the original mathematical model into the other (more simple, but preserves basic, essential for the system properties) model can be classified as the problem of equivalent Mathematical Remodeling. However, the complex system is divided into subsystems, for each its mathematical model is build separately. And then with these partial models are combined in the overall system model. It is efficient and focused on the computer modeling approach. Later this approach was extended to complex systems of different nature, production, technical, technological, economic, environmental, computational and other systems.

7. Conclusion and Outlook
Analysis of Finite Fluctuation is the next step to develop Economic Factor Analysis. In contrast to classical Mathematical Analysis, our approach operates, generally speaking, not with small but with finite fluctuations of the response and its arguments. In contrast to Statistical Factor Analysis, our approach is not aimed at the fitting the model of some dependency, but at the fitting a new model connecting the finite fluctuation of the response with finite fluctuations of its arguments. This idea corresponds to the concept of Mathematical Remodeling. The feature of Lagrange Analysis of Finite Fluctuations is the usage of Lagrange mean value theorem, operating with finite increments. Further development of this approach, as shown above, is based on the usage of Bonnet theorems.

It should be noted that in the book [5], when analyzing finite fluctuations there were used two approaches: on one hand, Quantum Calculus in the sense of [14], and on the other hand, four basic Cauchy equations presented in [1]. It is necessary to emphasize that these relations operate only finite fluctuations. In Quantum Calculus the derivative function is substituted by additive or multiplicative analogues. Cauchy functional equations define additive, multiplicative, additive-multiplicative and multiplicative-additive homomorphisms for real numbers set which are used as the basis for the transition from models connecting the response and its arguments to models for fluctuations.

There are relatively small number of publications devoted to mathematical basis of an applied Analysis of Finite Fluctuations and Lagrange Analysis of Finite Fluctuations. The classification and case study of particular applied problems as well as the development of methods and algorithms to solve them, were poorly researched. This paper brings some new result, considering Analysis of Finite Fluctuations with the position of a new approach in the mathematical modeling — the concept of Mathematical Remodeling.

Acknowledgments
The reported study was funded by RFBR and Lipetsk region according to the research project 17-47-480305-r_a.
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