Experimental Study of Steady Flows in Centrifuged Immiscible Liquids in Rotating Horizontal Cylinder

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Abstract. The steady time-averaged flows in a rotating system of immiscible fluids are studied experimentally. The fluids fill a horizontal cylindrical container and are brought into a centrifuged state, when a less dense fluid forms a cylindrical core, and a denser fluid is distributed in the form of a coaxial layer. Under the action of the gravity field, the core is steadily displaced along the radius in such a way that its axis is oriented parallel to the container axis and remains stationary in the laboratory frame of reference. In the frame rotating with the container, the core and the fluid surrounding it oscillate. This leads to the generation of a steady flow in the form of differential rotation of the fluid with respect to the container. To study the structure of the flow, light-scattering tracers of neutral buoyancy are added to the liquid. The flow rate is calculated from their displacement. Measurements show that at the interface, from the side of the external fluid, a Stokes boundary layer is formed, at the boundaries of which a tangential discontinuity of the averaged velocity is formed.

1. Introduction
Flows in rotating drum are related to a wide range of scientific research areas and technological processes [1]. There is no doubt this is due to the numerous, rather complex inertial effects [2]. At the combination of rotation and oscillations, sound mean vibrational effects are observed [3, 4].

A subset of problems on the rotating-drum flows may be distinguished where they consider centrifuged two-phase systems, which consist of a less dense core and a denser coaxial fluid layer. In such systems, the core–annulus interface plays the crucial role as it becomes the driver for many types of flows and, in many aspects, it determines the stability of the system. The rotating two-phase systems possess a large variety of oscillatory eigenmodes [5, 6], and this gives room for different ways to excite the oscillations within. One of the easiest ways is the action of a static external force field perpendicular to the rotation axis. As it was demonstrated in a pioneering work [7], the action of gravity on a centrifuged liquid layer with the free surface triggers two independent inertial effects: a steady displacement of the air core and a wavy disturbance of its surface. Both effects result in surface oscillations relative to the container and the generation of steady streaming in the centrifuged layer [8–10]. In the case when the two phases are immiscible liquids of different densities, these two effects persist but the dynamics becomes more complex because new parameters come into play, such as ratios of densities and of viscosities [11]. Analysis of experiments on the interface differential rotation that was done in [11] proves that the flow structure in a centrifuged two-liquid system is an important object for the investigation. In the present work, the steady streaming generated by fluid oscillations due to the action of gravity is studied experimentally.
1.1. Theoretical background

Under the action of gravity the light cylindrical core undergoes a radial displacement with the amplitude $b_1$, steady in the laboratory frame. The mathematical expression for $b_1$ for the case of a solid core, taking into account its density, may be found in [12]:

$$b_1 = \frac{1}{2} \Gamma R_1 \left( 1 - \frac{\rho_1}{\rho_2} \right) \left( 1 - \frac{R_1^2}{R_2^2} \right).$$

(1)

Here, $\rho_1$ is the core density, $\rho_2$ is the density of the heavier liquid, $R_1$ and $R_2$ are the radii of the interface and of the container, respectively.

In the reference frame that rotates with the container, this situation is equivalent to a solid-like motion of the core along a circular trajectory with the radius $b_1$. In a more general view, the radial displacement causes the oscillations of the interface with the frequency $-\Omega$. Consequently, due to the oscillations of the outer liquid, tangential to the interface, in the viscous boundary layers on the interface, an averaged mass force is generated that brings the fluid in the lagging differential rotation relative to the container with the angular speed $\Delta \Omega = (\Omega_{\text{fluid}} - \Omega) < 0$. This differential motion entrains the core itself, as well, and if the core is solid, it rotates with the rate determined by the following expression [12]:

$$\Delta \Omega_{\text{core}} = -\frac{1}{4} \Omega \frac{R_2}{\delta} \Gamma^2 \left( 1 - \frac{\rho_1}{\rho_2} \right)^2 \left( 1 - \frac{R_1^2}{R_2^2} \right).$$

(2)

Here, $\delta = \sqrt{2\nu_2 / \Omega}$ is the thickness of the viscous boundary layer formed on the interface, $\nu_2$ is the kinematic viscosity of the liquid in the annulus.

At this point, the analogy between the cores formed from different phases is limited, because the structure of the boundary layers on the interface is quite different for the following three cases: the core is gaseous, liquid or solid. For instance, a comparison of experimental results with the theoretical expression (2), made in [11], demonstrated that experimental values of $\Delta \Omega_{\text{core}} / \Omega_\tau$ scale as $\Gamma^2 \left( R_1 / \delta \right) \left( 1 - \rho_1 / \rho_2 \right)^2 \left( 1 - (R_1 / R_2)^2 \right)$ for some given viscosities of liquids, while experimental points split when the viscosities are varied. According to the theoretical works [13, 14], on the liquid–liquid interface, outside the thin boundary layers the steady streaming has a tangential discontinuity, expressed in a general case as follows:

$$\nabla v_2 - \nabla v_1 = \frac{3}{4} v_{\text{osc}}^2 A \frac{\partial A}{\partial x} \frac{\rho_2 \sqrt{\nu_2} - \rho_1 \sqrt{\nu_1}}{\rho_2 \sqrt{\nu_2} + \rho_1 \sqrt{\nu_1}}.$$

(3)

Here, $\nabla v_1$ and $\nabla v_2$ are time-averaged tangential velocities in different liquids, $v_{\text{osc}} = b \Omega_{\text{osc}}$, $\Omega_{\text{osc}}$ is the radian frequency of oscillations, $A = V_{\tau_2} - V_{\tau_1}$ is the discontinuity of the amplitudes of tangential-oscillation velocities in the inviscid domain, $x_\tau$ is the coordinate tangential to the interface.

Comparison of expressions (2) and (3) brings us to the idea that the difference between the dynamics of a solid and a liquid core should be related to the discontinuity of the tangential velocities of the steady streaming. In the present work we investigate experimentally the velocity field of the steady streaming that forms in a centrifuged two-liquid system in a rotating cylindrical container. The study is primarily focused on the situation when the core is radially displaced due to the gravity and retains the circular shape in the cross-section.

2. Experimental setup and technique

Two immiscible liquids fill a horizontal cylindrical container of radius $R_2$ that is rotated with the angular frequency $\Omega_\tau$ sufficiently fast, so that the interface between liquids is centrifuged and takes the form of a column with circular cross-section of radius $R_1$ (figure 1a). The liquid with lower density ($\rho_1$) forms a cylindrical core extended along the container axis, while the denser liquid ($\rho_2$) forms a coaxial layer.
limited by the cylindrical wall and the liquid–liquid interface. The gravity, which is directed perpendicularly to the rotation axis, disturbs the interface, whose dynamics is determined by the ratio between the gravity and the centrifugal force \( \Gamma = g / (\Omega^2 R_1) \). During the experiment, the container rotation rate decreases stepwise until a wave appears at the interface. At each step, the velocity of the interface and the velocity profile are studied.

In experiments, a transparent container 1 (figure 1b), made of acrylic glass, is used; its inner dimensions are \( R_2 = 3.0 \) cm and length \( L = 7.4 \) cm. The column radius \( R_1 \) is determined from the ratio of liquids’ volumes \( q = R_1^2 / R_2^2 \). The container is mounted in racks with bearings 2 and is fixed on an aluminum platform 3. The rotation with the radian frequency \( 3 < \Omega < 63 \) (s\(^{-1}\)) is provided by a stepper motor 4 of type FL86STH118-6004A through a coupling 5. An SMD-4.2 driver, commanded by a ZET-210 DAC module, assures the motor rotation rate with the precision of 0.01 rad/s and is powered by a NES-350-48 power supply. Liquid dynamics is registered through the front end of the container with the use of a high-speed video camera 6 of type Optronis CamRecord CL600x2.

![Figure 1. Problem formulation: a – configuration of the centrifuged two-liquid layer; b – diagram of the experimental setup (1 – container, 2 – supports, 3 – platform, 4 – motor, 5 – coupling, 6 – camera)](image)

The working liquids were industrial oil (its density and kinematic viscosity were, respectively, \( \rho_1 = (0.831 \pm 0.005) \) g/cm\(^3\), \( \nu_1 = (11.4 \pm 0.1) \) mm\(^2\)/s) and aqueous solutions of glycerol (solution #1 had \( \rho_2 = (1.243 \pm 0.003) \) g/cm\(^3\), \( \nu_2 = (15.2 \pm 0.1) \) mm\(^2\)/s, and solution #2: \( (1.185 \pm 0.004) \) g/cm\(^3\), \( (14.8 \pm 0.1) \) mm\(^2\)/s). The liquid density and viscosity were measured by means of a hydrometer with the precision 0.001 g/cm\(^3\) and of a viscometer with the precision 0.01 mm\(^2\)/s, respectively. The experiments were conducted with the liquids’ volume ratio \( q = 0.4 \).

To study the dynamics of the fluids, a high-speed camera and stroboscopic illumination were used. In order to observe the flows the liquids were seeded with fine tracer particles. Motion of the particles was studied using a 2D-PTV technique. Particle observation was carried out in the cross sections, which were transversal to the axis of rotation and illuminated by a laser light sheet. The video recording of experiments were split into separate frames that were processed digitally. To study the velocity profiles...
of the steady streaming, the frames were sampled with the frequency of container rotation. Due to this the rotation of the container was subtracted and the motion of fluid was considered immediately in the rotating frame of reference. Through the frame sequence, the motion of each particle was tracked individually. Their velocity was calculated from the variation of coordinates and the sampling frequency.

To study the rotation rate of the liquid–liquid interface, the visualizing particles were added in the light liquid. The size of these particles was 0.3–0.7 mm, their density was intermediate between the densities of liquids, thus they situated at the interface. The rotation rate of the particles was measured by means of synchronization with a stroboscopic illumination. The flicker frequency of the stroboscopic lamp was set using a waveform generator Rudnev-Shilayev GSPF-052 with the precision 0.001 Hz. The stroboscope flicker frequency was adjusted so as to be equal to the rotation frequency of particles, and so that the latter were seen immobile.

3. Experimental results

Initially the system is spin up to the rate about \( \Omega_0 \sim 60 \text{ s}^{-1} \), and the light liquid forms a circular column (figure 2a). The measurement from the photograph reveals that the core is displaced from the container axis to a distance of the order \( 10^{-1} \sim 10^0 \) mm (figure 3). With the decrease in \( \Omega_0 \), the displacement of the core \( b_3 \) increases. It scales with the dimensionless acceleration \( \Gamma \) in good agreement with expression (1), as one may notice when comparing the experimental points and the theoretical line (figure 3). At the approach to some critical value of the rotation rate, \( \Omega^* \), the interface is slightly disturbed (figure 2b), and after trespassing the threshold a centrifugal wave emerges, which in the present experiments had 3 crests on the circumference (figure 2c).

![Figure 2](image)

**Figure 2.** The photographs of the centrifuged liquids (\( \rho_2 = 1.24 \text{ g/cm}^3 \), \( v_2 = 15.2 \text{ mm}^2/\text{s} \)) at rotation with the rate \( \Omega_0 = 56.5 \text{ s}^{-1} \) (a), \( 29.8 \text{ s}^{-1} \) (b), \( 29.4 \text{ s}^{-1} \) (c), the corresponding values of the dimensionless acceleration are \( \Gamma = 0.16 \), \( 0.58 \), \( 0.60 \). a – steady interface (radially shifted), b – slightly deformed interface approximately at the threshold value of \( \Omega_0 \), c – unsteady interface, instantaneous picture of a centrifugal wave.

The amplitude of the steady core displacement increases linearly with \( \Gamma \) (figure 3), in good agreement with expression (1) that was found taking into account only the buoyancy of the core. In experiments, however, the viscosity of the liquids has its impact, in particular, it affects the azimuthal direction of the displacement. The air core is shifted vertically downwards in the ideal, inviscid case [7], but it is shifted at some angle when the viscosity is accounted for in both the cases of the air core [8] and the liquid core [11]. The fact the amplitude of the shift is not sensible to the viscosity proves that this effect is mainly driven by the inertial properties of the studied system.
3.1. Differential rotation of interface
The radian velocity of the interface, $\Delta \Omega_i$, measured by synchronization with stroboscope is presented in figure 4. The velocity is measured at the distance $z = (0.5 - 1)$ cm from the front flange of the container (points 1) and $z = (3.5 - 4)$ cm (points 2). The variation of the differential rate $|\Delta \Omega_i|$ with $\Omega_i$ is similar for both regions: near the flange and in the middle. In the entire studied range of the rotation rate, the interface moves faster relative to the container in the middle than near the flange. The difference between the rates is smaller than $|\Delta \Omega_i|$ almost by an order of magnitude and decreases with $\Omega_i$ (points 3). The variation in the rotation rate along the $z$ axis seems natural, because part of the energy of rotation is dissipated in the viscous boundary layers on the flanges, also known as Ekman layers [2]. This is related to the viscous exchange of momentum between the fluid and the container end-walls. The width of the Ekman layers may be estimated as the square root from the Ekman number $E = v/(\Omega_i L^2)$. In the present experiments, $E \sim 10^{-4}$, which means each of the Ekman layers is about 0.01L. As one may notice from this estimation, the end-wall effects should vanish rather quickly with the axial coordinate, and according to the stroboscopic measurements, they really do. This means that the interface velocity is quasi-two-dimensional, in other words, virtually constant along the axial coordinate $z$ and just slightly varying near the flanges. This result agrees well with the results on the dynamics of centrifugal waves that proved to be two-dimensional in a similar experimental configuration in the earlier study [11]. This equally means that the detailed study of the steady-streaming velocity in the middle of the container's length will be representative of the flow in the entire fluid volume, except the small domains in the immediate contact with the end-walls.

3.2. Velocity profile of steady flows
The measurements show that, in the container frame, the tracers move along circular trajectories at a constant angular velocity. For this reason, it was possible to take an average of the angular velocities along each circular trajectory. The radial profiles of the steady angular velocity, averaged along the azimuth, are given in figure 5a. Visually, the velocity profiles consist of two parts. The one on the left corresponds to the flow inside the core and is limited on the right by the coordinate of the interface, $r / R = 0.63$. In the central part of the core, at $r / R < 0.3$, the fluid rotates at some almost constant rate,
while upon approaching the interface the flow accelerates in the container frame. The fastest motion of the inner liquid is registered in the immediate contact with the outer liquid. The right part of the velocity profiles corresponds to the flow in the annulus. A gap is observed between the interface and the maximum measured velocity, then $|\Delta \Omega|$ decreases until virtually zero upon the approach to the container boundary at $r/R = 1$. The radial distance between the two peaks of $|\Delta \Omega|$ is comparable to the Stokes layer thickness $\delta$. The flow velocity within the boundary layer could not be measured, because no tracer particles were seen within it.

**Figure 4.** Differential rotation rate of the interface $\Delta \Omega_1$, measured near the front edge (points 1) and in the middle of the container’s length (points 2), as a dependence of the container rotation rate; points 3 represent the difference between the values at the front and the middle.

The profiles of the fluid rotation rate in figure 5a demonstrate that the flow is characterized by a “discontinuity” of the steady angular velocity, $|\Delta \Omega| = |\Delta \Omega_2 - \Delta \Omega_1|$. Here, $|\Delta \Omega_1|$ is the maximum rotation rate of the inner liquid, while $|\Delta \Omega_2|$ is for the outer liquid. With a decrease in the rotation rate of the container (equivalent to an increase in $\Gamma$) the discontinuity becomes larger (figure 5b). At some critical value of the rotation rate, $\Omega_*^\prime$, centrifugal waves are excited on the interface. The velocity profiles are not studied beyond the threshold of wave excitation.

**4. Discussion of results**

Several effects, observed in the present experiments, are worth a discussion. The tangential “discontinuity” of the steady streaming confirms the theoretical predictions [13, 14]. The term “discontinuity” is applied in this case to the flow in the inviscid domain, outside the viscous boundary layer on the interface. Figure 5a demonstrates that the peaks of the fluid velocity profile approximately coincide with the limits of the Stokes boundary layer. The flow that makes the two velocity peaks match inside the boundary layer has not been studied and is not discussed here.
Figure 5. Angular velocity of the steady streaming measured by processing images: a – radial profiles of the differential rate of fluid rotation, $\Delta \Omega$ at different values of $\Omega$; b – the dependence of the two maxima of $\Delta \Omega$ (indices 1 and 2 make reference to the inner and the outer liquids, respectively) on the rotation rate of the container.

Interestingly, the Stokes viscous layer is located entirely in the outer liquid and lacks any tracer particles. The absence of the boundary layer in the liquid core means that the fluid does not oscillate relative to the interface inside the core. This behavior can be explained by the fact that, at the steady-in-the-laboratory-frame displacement of the core, the latter oscillates relative to the rotating container as a whole, the same way a solid cylindrical core does [12]. In this situation, considering the motion in the reference frame of the core, one would expect a solid-body differential rotation of the inner liquid. Mathematically, that would mean $\Delta \Omega = \text{const}$ in the domain $0 < (r / R) < \sqrt{g}$, but this is not observed in the experiments (figure 5a). This contradiction will not arise if one takes into account that the flows, observed in the experiments, are not purely two-dimensional. Indeed, some variation of the flow rotation rate along the $z$ axis is observed (figure 4, points 3). The steady streaming has the negative direction in the container frame (figure 4). This means that, in the frame of the rotating fluid, the end-walls of the container lead the liquid. From the theoretical point of view, such situation should result in the Ekman pumping with the Coriolis force directed positively along the radius [2]. The radial expulsion of the fluid in the Ekman layers should be balanced by the axial inflow towards the end-walls in the central part of the core. Then, to satisfy the fluid continuity, one should suggest that at some distance $z$ such that $z / L \gg E^{-1/2}$, the Ekman pumping should result in a radial flow towards the core center. When an element EF of rotating fluid displaces radially inwards, its rotation rate $\Omega_{EF}$ must increase in order to compensate for the decrease in $r$, because of the conservation of momentum $m_{EF} r^2 \Omega_{EF}$. The decrease in $|\Delta \Omega|$ in the domain $r / R < 0.63$ in figure 5a means that the fluid in the core rotates faster than the interface, if considered from the laboratory frame. Thus, the radial profile of the steady streaming inside the core confirms the idea on the role of the Ekman pumping.

To sum up, the analysis of the profiles of the differential fluid velocity points to a hypothesis that the core oscillations are effectively two-dimensional and the differential rotation is generated in the Stokes boundary layer in the outer liquid. The average motion of the latter, in its turns, entrains the interface and the entire core into the rotation. However, the steady streaming is also affected by the Ekman pumping. Some additional experiments and a more detailed analysis are foreseen to verify this conclusion.
5. Conclusion
The steady flows generated in a coaxial two-liquid system due to the radial displacement of the core have been studied experimentally. In the considered problem formulation, the flows are quasi-two-dimensional, and the focus has been made on the velocity fields in the cross-section of the container. The PTV measurements have revealed that the lagging differential rotation of the liquids is established, during which the fluids move along circular trajectories. The radial profile of the differential rotation rate has a discontinuity that is localized in the outer liquid at the interface. Topologically, the discontinuity coincides with the Stokes boundary layer. The maximum rates of fluid retrograde rotation are not equal in the inner and outer liquids, the latter moving faster. Due to the radial displacement of the core the fluid in the outer liquid oscillates relative to the interface. This leads to the generation of the average mass force in the Stokes boundary layer, and the fluid is forced to the steady time-average differential rotation. The motion of the outer liquid entrains the liquid core. The exchange of momentum between the liquid and the end-walls of the container generates the Ekman pumping that is added to the differential rotation, and this renders the radial velocity profile more complicated.

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