Ensemble averaging and stacking of ARIMA and GSTAR model for rainfall forecasting

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Abstract. Unpredictable rainfall changes can affect human activities, such as in agriculture, aviation, shipping which depend on weather forecasts. Therefore, we need forecasting tools with high accuracy in predicting the rainfall in the future. This research focus on local forecasting of rainfall at Jember in 2005 until 2016, from 77 rainfall stations. The rainfall here was not only related to the occurrence of the previous of its stations, but also related to others, it’s called the spatial effect. The aim of this research is to apply the GSTAR model, to determine whether there are some correlations of spatial effect between one to another stations. The GSTAR model is an expansion of the space-time model that combines the time-related effects, the locations (stations) in a time series effects, and also the location it self. The GSTAR model will also be compared to the ARIMA model that completely ignores the independent variables. The forecasted value of the ARIMA and of the GSTAR models then being combined using the ensemble forecasting technique. The averaging and stacking method of ensemble forecasting method here provide us the best model with higher accuracy model that has the smaller RMSE (Root Mean Square Error) value. Finally, with the best model we can offer a better local rainfall forecasting in Jember for the future.

1. Introduction

Extreme climate change is one of the impacts of global warming, the extreme impact of climate change, unconditional temperature and season. In addition, the climate change leads to the changes in air temperature, rainfall, air pressure and wind speed. Of these factors, the rainfall is the most affect on human life, directly. Accurate weather forecasting of future rainfall data is one of the main foundations of a plan, effective and efficient in weather forecasting. The one of popular rainfall forecasting method is Autoregressive Integrate Moving Average (ARIMA) or often called the Box-Jenkins time series method. ARIMA matches the observations of the time series (series time) in conjunction with each other (depending on) or it may be said that the ARIMA model works with the relation of past storm events to the forecast in the future. The ARIMA model does not contain any element of location in the forecasting model so that another model containing time element, and location to predict rainfall data in 4 rainfall station groups. The time-space model is one of the models that connects time-related elements, and location to a time series and location data. The time-space model developed by Pfeifer and Deutsch (1980a, 1980b) [1,2] has weaknesses on, which describes the locations, and the different times in a time series and location data. This weakness is chosen by Baravkova et al. (2002) [3] through a model known as the GSTAR model (Generalized Space Time
Autoregressive), which is a model that has an interrelationship of time and location where the location under study has various characteristics (heterogeneous). Along with the development of the times, rainfall forecasting model is growing. One is the concept of combining several forecasting models. The Ensemble model is a forecasting method created, either with different models or different multiple initial conditions which are then combined into a single unit for the assessment of uncertainty in the formulation of the initial model and conditions. The data applied to the Ensemble model in this case study is the rainfall data of Jember, which has certain characteristics for each cluster of locations. Given Jember is one of the best producers of sugar cane plants. The productivity of sugar cane plants depends on climate change. Extreme climate changes affect the rainfall pattern that is uncertain and above normal, can cause long drought that will also impact on the productivity of sugar cane plants. To minimize the impacts arising from the uncertain changes in rainfall by providing information on weather forecasts, especially opportunities through rainfall in an area within a certain period of time.

2. Timeseries Model
2.1. ARIMA
ARIMA is often called the Box-Jenkins time-series method. The Autoregressive Integrated Moving Average (ARIMA) model is a model that completely ignores independent variables in making forecasting. ARIMA uses past and present values of the dependent variable to produce accurate short-term forecasting. The basic assumption used in the ARIMA time series process discussion is a stationary process. To stationize the non-stationary data generation process is to do the first level difference, and so on. For example, the first or first difference is called \( W_t = W_t - W_{t-1} \), the result of each difference is called an integrated process. While the order of the process that gets stationary time series is determined by the number of differences (differencing) is done.

The general model for the pure mixture of AR \( (p) \) and MA \( (q) \) pure processes, as ARIMA \((1,0,1)\), also known as ARMA\((p,q)\) is expressed as follows, where \( X_t \) regressed on itself lagged by some periods,

\[
X_t = \mu + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \theta_q \epsilon_{t-q}
\]

(1)

with \( \mu \) is mean, \( \phi_i \) \((i = 1,2,...,p)\) are the autoregressive parameters, \( \theta_j \) \((j = 1,2,...,q)\) are the moving average parameters, and \( \epsilon_t \) is lagged errors.

Furthermore, there are four steps in ARIMA modeling; identification, estimation and information criteria, diagnostic checking and model’s use. Identification involves determining the order of the model required \((p, d, \text{ and } q)\) in order to capture the salient dynamic features of the data. This mainly leads to use graphical procedures (plotting the series, the ACF and PACF \([4]\), etc). Then the second step, estimation and information criteria, involves estimation of the parameters of the different models (using step 1) and proceeds to a first selection of models (using information criteria). The next, diagnostic checking involves determining whether the model(s) specified and estimated is adequate. Notably, one uses residual diagnostics. Lastly, the best model will be used as a forecast model. To determine the best model can be used the following standard error estimate:

\[
s = \left[ \frac{\text{sse}}{n-n_p} \right]^{1/2} = \left[ \frac{\sum_{t=1}^{n}(Y_t-\hat{Y}_t)^2}{n-n_p} \right]^{1/2}
\]

(2)

2.2. GSTAR
Generalized Space Time is the development of the STAR model that fits the data heterogeneity location. Mathematically, the notation of GSTAR \((p; \lambda_1, \ldots, \lambda_p)\) is the same as the model STAR \((p_1)_3[3]\). In the STAR model parameter values \( \phi_kl \) assumed to be similar for all locations, while at GSTAR \( \phi_kl \) parameter values in the same spatial lag allowed between different locations. GSTAR \((p; \lambda_1, \ldots, \lambda_p)\) can be written as follows:
\[ Z(t) = \sum_{k=1}^{p} [\phi_{k0} + \phi_{k1} \mathbf{W}] \mathbf{Z}(t-k) + \mathbf{e}(t) \]  

(3)

Where \( \phi_{k0} \): diag \((\phi_{k0}^{(1)}, ..., \phi_{k0}^{(n)})\) and \( \Phi_{k1} \): diag \((\phi_{k1}^{(1)}, ..., \phi_{k1}^{(n)})\). The weights are chosen so that \( W_{ii} = 0 \) and \( \sum_{i \neq j} W_{ij} = 1 \)

[5] said that the selection of spatial order of GSTAR model is generally limited to order 1, because higher order would be difficult to interpretation. While at the time order (autoregressive) can be determined with AIC (Akaike Information Criterion). The choice of best model order on GSTAR can be determined with the smallest AIC value. AIC calculations as according to Akaike (1973, 1974) [6], as:

\[ AIC(p) = \ln|\sum_{i} (p)| + \frac{2p}{n} K^2 \]  

(4)

On the other hand, the selection of weight location GSTAR model is based on these three weights location:

2.2.1 Weight location of uniform
Uniform weight in \( W_{ij} = \frac{1}{n_i} \) with \( n_i \) = the number border location to location \( i \). The weights this location give equal weight to each location. Therefore, the weight of this location often used on data that have a distance between the same location (homogeneous). [7]

2.2.2 Weight location of inverse distance
Weighting by the inverse distance refers to the distance between locations. Suppose the distance between the four locations are defined, \( r_1 = \) distance between location 1 to location 2; \( r_2 = \) distance between location 1 to location 3; \( r_3 = \) distance between location 1 to location 4; \( r_4 = \) distance between location 2 to location 3; \( r_5 = \) distance between location 2 to location 4; \( r_6 = \) distance between location 2 to location 3. Then, the matrix below are standardized in the form \( W_{ij} \) to get \( \sum_{i \neq j} W_{ij} = 1 \) [3]

\[
W_{ij} = \begin{bmatrix}
0 & \frac{r_2 + r_3}{r_1 + r_2 + r_3} & \frac{r_2 + r_3}{r_1 + r_2 + r_3} & \frac{r_2 + r_3}{r_1 + r_2 + r_3} \\
\frac{r_2 + r_3}{r_1 + r_2 + r_3} & 0 & \frac{r_2 + r_3}{r_1 + r_2 + r_3} & \frac{r_2 + r_3}{r_1 + r_2 + r_3} \\
\frac{r_2 + r_3}{r_1 + r_2 + r_3} & \frac{r_2 + r_3}{r_1 + r_2 + r_3} & 0 & \frac{r_2 + r_3}{r_1 + r_2 + r_3} \\
\frac{r_2 + r_3}{r_1 + r_2 + r_3} & \frac{r_2 + r_3}{r_1 + r_2 + r_3} & \frac{r_2 + r_3}{r_1 + r_2 + r_3} & 0 \\
\end{bmatrix}
= \begin{bmatrix}
0 & W_{12} & W_{13} & W_{14} \\
W_{21} & 0 & W_{23} & W_{24} \\
W_{31} & W_{32} & 0 & W_{34} \\
W_{41} & W_{42} & W_{43} & 0 \\
\end{bmatrix}
\]

2.2.3 Weight location of cross correlation
Estimates of the cross-correlation of this data sample is:

\[ r_{ij}(k) = \frac{\sum_{t=k+1}^{n} [Z_i(t) - \bar{Z}_i] [Z_j(t-k) - \bar{Z}_j]}{\sqrt{(\sum_{t=1}^{n} [Z_i(t) - \bar{Z}_i]^2)(\sum_{t=1}^{n} [Z_j(t) - \bar{Z}_j]^2)}} \]

Furthermore, determination of the weight location can be done with the normalization from amount of the cross-correlation between locations at the corresponding time. This process generally produces weight following locations:

\[ W_{ij} = \frac{r_{ij}(1)}{\sum_{k=1}^{n} r_{ik}(1)} \]
This weight fulfilling $\sum_{i \neq j} W_{ij} = 1$ weights locations using normalization of the magnitude of the cross-correlation between locations at the corresponding time allows all forms of possible relationships between locations. This weights also gives flexibility on the magnitude and sign of the relationship between different locations: positive and negative. The weights of this location is the weight location includes uniform weights and binary locations.

Furthermore, the parameter estimation of GSTAR model by all location weights are using Least Square Method. Based on the GSTAR model with the order $p = 1$ and the spatial order 1 where we can write $\phi_{kl} = \phi_{k}^{(l)}$ for $k = 0, 1$ be derived as:

$$Z_i(t) = \phi_{10}^{(10)} Z_i(t - 1) + \phi_{11}^{(1)} \sum_{j} W_{ij} Z_j(t - 1) + e_i(t)$$

where $Z_i(t)$ observation on $t = 0, 1, \ldots, T$, for location $i = 1, 2, \ldots, N$, so:

$$V_i(t) = \sum_{j=1}^{N} W_{ij} Z_j(t)$$

This applies for linear form, that is $Y_i = X_i \beta_i + e_i$:

$$Y_i = \begin{bmatrix} Z_i(1) \\ Z_i(2) \\ \vdots \\ Z_i(T) \end{bmatrix}, X = \begin{bmatrix} Z_i(0) \\ \vdots \\ Z_i(t-1) \end{bmatrix}, e_i = \begin{bmatrix} e_i(1) \\ \vdots \\ e_i(t) \end{bmatrix},$$

$$\beta = \left( \phi_{10}^{(1)}, \phi_{10}^{(2)}, \ldots, \phi_{10}^{(N)}, \phi_{11}^{(1)}, \phi_{11}^{(2)}, \ldots, \phi_{11}^{(N)} \right).$$

The modeling for all linear models can be write as $\phi_{10}^{(10)} = X \beta + e$, where

$$Y = (Y_1', \ldots, Y_N'), X = (X_1', \ldots, X_N'), \beta = (\beta_1', \ldots, \beta_N'), e = (e_1', \ldots, e_N')'.$$

So the form of least squares estimates of $\hat{\beta}_T$ is $\hat{\beta}_T = (X'X)^{-1}X'y$

The following step is to determine whether residuals meet white noise. The GSTAR model is considered feasible if it meets the constant variant (white noise). Assumption testing is necessary needed. The time-series of $r_t$ is said to be white noise if $r_t$ is an identically normal distributed sequence with a finite mean and variance. $r_t$ is normally distributed with mean 0 and variance $\sigma^2$, it can be said that if the AIC value in lag AR (0) and MA (0), the residual meets the residual of white noise assumption [8]. Fulfillment of white noise assumption can be done by using portmanteau test with significance level of 5%.

At last, the purpose of a forecast is to produce an optimum prediction with the smallest possible error. This leads to the value of the MSE (Mean Square Error) forecast to use. In general, MSE can be formulated in the following form

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_t - \hat{y}_t)^2}$$

(5)

Where $y_t$ the observation of $i$ time, $\hat{y}_t$ the prediction of $i$ time, $n$ sum of observation (testing data)
2.3. Ensemble
Forecasting ensemble is forecasting techniques combine the output of several methods of forecasting as a prediction value. In its development, in the last 15 years can be demonstrated that using metod ensemble forecasting has become one of the techniques that are widely used in weather forecasting [9]. There are two methods most commonly used in combining a different output of ensemble members [10], the average method (averaging) and merging (stacking).

2.3.1 Ensemble Averaging
Ensemble averaging using the average output of the ensemble obtained by calculating the average of the model output member ensemble models. Suppose \( N \) is the number of individual members models in an ensemble, a combination of function \( f \) that is

\[
\hat{y}_i = (\hat{y}_i^k), i = 1, 2, ..., m \tag{6}
\]

with \( \hat{y}_i \) is a predicted value of observation \( i \) obtained from the \( k \) models and forms of the function \( f \) is

\[
f = (\hat{y}_i^k) = \frac{1}{N} \sum_{k=1}^{N} \hat{y}_i^k \tag{7}
\]

2.3.2 Ensemble Stacking
Stacking is a common method that uses a combination of a model of high-level and lower level models to achieve a higher accuracy prediction. [10] suggest the minimization of a function \( G \) to improve the generalizability of the model, is

\[
G = \sum_{i=1}^{m} \left[ y_i - \sum_{k=1}^{N} c_k \hat{y}_i^k \right], \quad c_k > 0
\]

The coefficients \( \hat{c}_1, \hat{c}_2, \hat{c}_3, ..., \hat{c}_N \) in the above equation are estimated to form or construct a predictive value of the end of an ensemble, is

\[
\hat{y}_i = \sum_{k=1}^{N} c_k \hat{y}_i^k, \quad i = 1, 2, ..., m \tag{8}
\]

3. Research Methods
The data used in this research is rainfall dataset from 77 rainfall stations in Jember Regency from January 2005 to December 2016 in four clusters [11], which was kindly given by Central Bureau of Statistics (BPS) Jember. The dataset is divided into two kinds, namely in-sample data and out-sample data. In-sample data is used to form forecasting model. While the out-sample data is used to check the model's forecasting. Models used in this study are ARIMA, GSTAR and both combination models called Ensemble. Moreover, those forecasting models are applied to forecast another time periods (out-sample data). From this forecasting results, we will get the errors and compare each model’s error. The smaller the error, the better the model. Generally, this research method is shown on Figure 1.
4. Result and Discussion

Time-series plotting as shown in Figure 2 says that the data plot are seasonal spread and relative sufficiently homogeneous for among four clusters. This paper concerns on non seasonal ARIMA model and seasonal GSTAR model due to expect to see combination of seasonal and non seasonal two different models.

![Figure 1. Research Flow Chart](image)

**Figure 2.** Timeseries Plotting of Rainfall Data from 77 Rainfall Stations in Jember (2005-2016)

### 4.1. ARIMA Models

ARIMA modeling starts with stationary checking. From Figure 3, we know that every single cluster is stationar, both in mean and variance. It also shows us the possible order of ARIMA models from its lag. However, since ACF and PACF cut off after in certain time lags for each model, those order can be considered as space-time order candidates. ARIMA models also known as ARMA models as long as the timeseries data are already stationer in mean. In other words, we no need to do differencing in this case, so the d order is 0 (zero).
Table 1. ARMA Models In Every Single Cluster

| Cluster  | Model   | Parameter | Parameter Value |
|----------|---------|-----------|-----------------|
| 1st Cluster | ARMA (4,1) | AR1     | 1.1899          |
|          |         | AR2     | -0.332          |
|          |         | AR3     | -0.0710         |
|          |         | AR4     | -0.01878        |
|          |         | MA1     | -0.7702         |
|          |         | AR1     | 1.5655          |
| 2nd Cluster | ARMA (2,1) | AR1     | 1.6773          |
|          |         | AR2     | -0.9239         |
|          |         | MA1     | -1.1804         |
|          |         | MA2     | 0.3863          |
|          |         | AR1     | 1.0668          |
|          |         | AR2     | -0.3781         |
|          |         | AR3     | -0.0164         |
|          |         | AR4     | -0.0414         |
|          |         | AR5     | -0.2032         |
|          |         | MA1     | -0.6123         |

Figure 3. ACF and PACF Plot of ARIMA Models for four Clusters
Since there is no differencing in this case, then we could call the result models as ARMA (p,q). In addition, table 1 shows us the best ARMA model from all of space-time candidates in every single cluster based on Figure 3, with its parameter value. These models also have passed the diagnostic checking, so that it guarantees that they are white noise assumed.

4.2. GSTAR Models
In the other side, seasonal-GSTAR (1;1) is the simplest model of GSTAR (p;λ1,...,λp) model class defined in Eq. (3) because it is characterized by autoregressive terms lagged in time and spatial of order one. From Eq. (3), we can write seasonal-GSTAR (1;1) of four models for each cluster as follows:

\[ Z_k(t) = 0.234Z_k(t - 1) + 0.057Z_k(t - 1) + 0.039Z_k(t - 1) + 0.032Z_k(t - 1) + 0.129Z_k(t - 12) + 0.129Z_k(t - 12) + 0.088Z_k(t - 12) + 0.071Z_k(t - 12) + e_k(t) \]

\[ Z_k(t) = -0.053Z_k(t - 1) + 0.579Z_k(t - 1) - 0.038Z_k(t - 1) - 0.035Z_k(t - 1) + 0.106Z_k(t - 12) + 0.275Z_k(t - 12) + 0.076Z_k(t - 12) + 0.071Z_k(t - 12) + e_k(t) \]

\[ Z_k(t) = 0.072Z_k(t - 1) + 0.071Z_k(t - 1) + 0.311Z_k(t - 1) + 0.071Z_k(t - 1) + 0.098Z_k(t - 12) + 0.203Z_k(t - 12) + 0.098Z_k(t - 12) + e_k(t) \]

\[ Z_k(t) = 0.145Z_k(t - 1) + 0.176Z_k(t - 1) + 0.199Z_k(t - 1) + 0.119Z_k(t - 1) + 0.152Z_k(t - 12) + 0.185Z_k(t - 12) + 0.208Z_k(t - 12) + 0.057Z_k(t - 12) + e_k(t) \]

4.3. Ensemble Models
4.3.1. Ensemble Averaging
In this section, we combine some forecasting models to cover possible factors than can influence rainfall data. First method is ensemble averaging. This method computes the average of two forecasting models used before, which are ARMA and GSTAR, based on Eq. (7) in every single cluster. Further, this forecasting result will help us to find the best performance of forecasting models by looking for each RMSE of the forecasting models before. In this case, each model brings its own error that possibly has large difference within both. Hence, it makes error of ensemble averaging would not be minimum, even larger than individual model’s error. This could make ensemble averaging can not optimize forecasting result. Otherwise, ensemble averaging method will give better performance than individual forecasting model if one forecasting result negates the others so that the errors will be minimum. It will clearly stated in the next section.

4.3.2. Ensemble Stacking
In this section, we will forecast the rainfall data by Eq. (8), with estimasting the coefficient c_k by least square method. here means the models involved in this ensemble, in this case, we used two models before, ARMA and GSTAR. In addition, to estimate coefficients in ensemble stacking, we involved the rainfall dataset in the year of 2016. From Eq. (8), we gives these four forecasting models of ensemble stacking:

\[ Y_1 = 0.61 \text{ARMA} + 0.22 \text{GSTAR} \]
\[ Y_2 = 0.58 \text{ARMA} + 0.49 \text{GSTAR} \]
\[ Y_3 = 0.50 \text{ARMA} + 0.03 \text{GSTAR} \]
\[ Y_4 = 0.66 \text{ARMA} + 0.30 \text{GSTAR} \]

4.4. The Best Performance of Forecasting Models
For the purpose of forecasting model, it would be useful if we also consider their forecast performance. Therefore, in this section we will examine one-step-ahead forecasting performance for each model candidate using out-sample data of rainfall dataset. RMSE value based on Eq. (5) determines the best performance of forecasting models. The smaller the value of RMSE model, it can
be said the better the model. It can be seen on Table 2 that the model with the smallest RMSE is ensemble stacking of ARMA and GSTAR rather than individual models and ensemble averaging. Between individual models, ARMA and GSTAR, RMSE values show that ARMA gives the best performance (in average) based on the smaller values. It means that seasonal models on ARIMA, also known as SARIMA, do not give much effects in rainfall forecasting due to extreme climate changes lately.

The RMSE value in Table 2 shows that weights given by each coefficient on each models can minimize the RMSE value in all clusters. In addition, combined forecasting models in this study may also reduce the RMSE model values, except for ensemble averaging. Further, ensemble averaging provides larger RMSE than GSTAR. It means that GSTAR shows better performance than ensemble averaging, unless the difference error of GSTAR and ARMA are smaller. So it can be concluded that the best model in this study is ensemble stacking of ARMA and GSTAR.

Table 2. RMSE Comparisons: Real data, ARIMA, GSTAR, and Ensemble

| Cluster     | ARMA | GSTAR | ENSEMBLE Averaging | Ensemble Stacking |
|-------------|------|-------|--------------------|-------------------|
| 1st Cluster | 96,29834 | 100,05 | 90,88354 | 87,49765 |
| 2nd Cluster | 145,4629 | 151,692 | 132,3958 | 131,0503 |
| 3rd Cluster | 141,5264 | 129,6659 | 128,9629 | 87,7086 |
| 4th Cluster | 101,9235 | 111,4439 | 100,1336 | 98,98162 |

Figure 4 also shows us that there is a large different performance between ARMA (in green) and GSTAR (in red) model like previous description. In the other side, we can see that ensemble stacking (in light blue) have the same pattern with real rainfall dataset (in dark blue) rather than three other forecasting models. It supports earlier statement that RMSE values of ensemble stacking will be smaller than the others. Ensemble averaging also gives better performance than individual models (ARMA and GSTAR), but not good enough rather than ensemble stacking.

Figure 4. Plot of Forecasting Rainfall Dataset of 2015-2016 in Jember Regency
5. Conclusion
The conclusion from the above research is that each cluster has different RMSE values due to different topography of each region. The difference RMSE for each cluster is around 1,000 until 20,000. Ensemble stacking models of ARMA and GSTAR gives the best performance of forecasting models in all clusters rather than individual models and ensemble averaging of ARMA and GSTAR. While non seasonal ARMA model gives better performance in almost each cluster than GSTAR does. It tells us that in this case, seasonal models do not influence forecasting results since extreme climate changes occur lately.

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References
[1] Pfeifer P E and Deutsch S J A 1980a Three Stage Iterative Procedure for Space-Time Modeling, Technometrics 22 (1) 35-47
[2] Pfeifer P E and Deutsch S J 1980b Identification and Interpretation of First Order Space-Time ARMA Models Technometrics 22 (1) 397-408
[3] Borovkova S A Lopuhaa H P and Nurani B 2002 Generalized STAR Model With Experimental WEIGHTS In M Stasinopoulos & G Tou-louni (Eds.) Proc. of the 17th International Workshop on Statistical Modeling (Chania) 139-147
[4] Wei W W S 2006 Time Series Analysis Univariate And Multivariate Methods (Canada: Addison Wesley Publishing Company)
[5] Wutsqa D W Suhartono and Sujito B 2010 Generalized Space Time Autoregresive Proc. Of The 6th IMT-GT Conf. On Mathematics, Statistics And Its Application 120-131
[6] Nurcahyani F 2016 Pengelompokan Stasiun Curah Hujan Untuk Model Generalized Space Time Autoregresive (GSTAR) Pada Peramalan Curah Hujan Kabupaten Jember Dengan Tiga Pembobotan Digital Repository University of Jember
[7] Anggraeni D Prahutama A and Andari S 2013 Aplikasi Generalized Space Time Autoregressive (GSTAR) Pada Pemodelan Volume Kendaraan Masuk Tol Semarang. Media Statistika 6 (2) 71-80
[8] Tsay R S 2005 Analysis of Financial Time Series (New Jersey: John Wiley & Sons)
[9] Leutbecher M and Palmer T N 2008 Ensemble Forecasting Journal of Computational Physics 3515-39
[10] Zaier I, Shu C, Ouarda T, Seidou O and Chebana F 2010 Estimastion of Ice Thickness on Lakes Using Artificial Neural Network Ensembles Journal of Hydrology 383( ¾) 330-340
[11] Hadi A F Yudistira I Anggraeni D and Hasan M Geographical Clustering of The Rainfall Stations on Seasonal GSTAR Modeling for Rainfall Forecasting in Jember preprint Advanced Science Letters