Spherical Top-Hat Collapse of Viscous Modified Chaplygin Gas in Einstein’s Gravity and Loop Quantum Cosmology

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Abstract: In this work, we focus on the collapse of a spherically symmetric perturbation, with a classical top-hat profile, to study the nonlinear evolution of only viscous modified Chaplygin gas (VMCG) perturbations in Einstein’s gravity as well as in loop quantum Cosmology (LQC). In the perturbed region, we have investigated the nature of equation of state parameter, square speed of sound and another perturbed quantities. The results have been analyzed by numerical and graphical investigations.

I. INTRODUCTION

Recent years have witnessed the emergence of the idea of accelerating Universe and due to some observational results\textsuperscript{1,2} it is now established that the Universe is accelerating. This acceleration is caused by some unknown matter dubbed as dark energy. This dark energy has the positive energy density and strong negative pressure that satisfies equation of state parameter \( w = p/\rho < -1/3 \). The present acceleration is also confirmed by other observations like large scale structure\textsuperscript{3}, CMBR\textsuperscript{4,5} and WMAP\textsuperscript{5,6}. The observations predict that the present Universe occupies \( 4\% \) ordinary matter, \( 74\% \) dark energy and \( 22\% \) dark matter. The most simple candidate of dark energy is the cosmological constant \( \Lambda \) which satisfies the EoS parameter \( w = -1 \). Another candidates of dark energy are quintessence (where EoS parameter satisfies \( -1 < w < -1/3 \))\textsuperscript{8} and phantom (where EoS parameter satisfies \( w < -1 \))\textsuperscript{9}. A brief review of dark energy models is found in the ref.\textsuperscript{10}. The unified dark fluid (UDF) model\textsuperscript{11,12} was investigated extensively in the recent years. The main features of the UDF model are that it combines cold dark matter and dark energy and that it behaves like the cold dark matter and the dark energy at an early epoch and a late time respectively however the approach remains purely phenomenological. In recent years, an increasing number of cosmological observations suggest that our universe is filled with imperfect fluid which contains bulk viscosity which has negative pressure\textsuperscript{13,14}. The viscous generalized Chaplygin gas model\textsuperscript{15,16} is one of the suitable candidate of unified dark energy and cold dark matter as a unique imperfect dark fluid\textsuperscript{15,16}.

While dark energy may be modeled as a fluid with negative pressure acting against gravitational collapse, dark matter (in its cold version) is a dust-like fluid with no pressure, therefore enhancing the collapse of matter perturbations\textsuperscript{17,18}. It is very important to investigate the evolutions of density perturbations within realistic cosmological model (for recent reviews, see for instance\textsuperscript{19} and references therein). Fabris et al\textsuperscript{20} studied the evolution of density perturbations in a universe dominated by the Chaplygin gas. Their model shows the required density contrast observed in large scale structure of the universe, however their approach remained Newtonian. Similar investigations were performed by Carturan and Finelli for the generalized Chaplygin gas\textsuperscript{21} (see also\textsuperscript{22}). Fernandes et al\textsuperscript{23} have investigated the nonlinear collapse of generalized Chaplygin gas in the frame of spherical top-hat collapse (STHC). Recently Carames\textsuperscript{24} investigated the spherical collapse model using viscous generalized Chaplygin gas. Li et al\textsuperscript{15,16} have extended the above work by considering bulk viscosity in the general Chaplygin gas model. Besides the parameter \( \alpha \), they have also analyzed the effect of bulk viscosity on the structure formation of the variable generalized Chaplygin gas model which has a spherically symmetric perturbation.

In our work, we focus on the collapse of a spherically symmetric perturbation, with a classical top-hat profile, to study the nonlinear evolution of viscous modified Chaplygin gas perturbations in Einstein’s gravity as well as in loop quantum Cosmology in sections II and III respectively. The conclusion is present in the last section IV.

II. SPHERICAL TOP-HAT COLLAPSE MODEL OF VISCOS MODIFIED CHAPLYGIN GAS IN EINSTEIN’S GRAVITY

A. Basic Equations

We consider a flat Friedmann Robertson-Walker (FRW) universe described by the following metric

\[ ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]. \]  

(1)

Here \( a(t) \) is the scale factor of the universe. We assume that the spacetime is filled with only one component fluid having a bulk viscosity. The Einstein’s field equations are given by

\[ H^2 = \frac{8\pi G}{3} \rho, \]  

(2)

and

\[ \dot{H} = -4\pi G (\rho + p), \]  

(3)

where \( H = \frac{\dot{a}}{a} \) is the Hubble parameter, \( \rho \) is the energy density and the effective pressure \( p \) can be expressed as follows

\[ p = p_d + \Pi, \]  

(4)

where \( p_d \) is the dust pressure and \( \Pi \) is the pressure due to bulk viscosity.
which is the sum of the equilibrium pressure \( p_{de} \) and the bulk pressure \( \Pi = -\xi u^\gamma \), where \( u^\gamma \) is the four velocity of the fluid and \( \xi \) is the coefficient of bulk viscosity and is a function of energy density. The first attempts at creating a viscosity theory of relativistic fluids were executed by Eckart [26] and Landau and Lifshitz [27] who considered only a first-order deviation from equilibrium. The bulk viscous pressure \( \Pi \) is represented by the Eckart’s expression which is proportional to the Hubble parameter \( H \) with proportionality factor identified as the bulk viscosity coefficient \( \xi \) which is defined by \( \xi = \xi_0 \rho' \), \( \xi_0 \) and \( \nu \) are constants. For simplicity, choosing \( \nu = \frac{1}{2} \), \( \Pi \) can be written as

\[
\Pi = -3\xi_0 H \sqrt{\rho}. \tag{5}
\]

The Chaplygin gas is generally characterized by the equation of state (pressure is inversely proportional to the energy density): \( p = -A/\rho \). This equation of state has an interesting connection with the d-branes which are expressed via Nambu-Goto action [29]. It also enjoys connections with the Newtonian hydrodynamical equations. Further the Eddington-Born-Infeld model can be seen as an affine connection version for the Chaplygin gas approach [29]. The Chaplygin gas has been extensively studied within the unified dark energy-dark matter models as well [24]. Therefore we choose the equation of state for modified Chaplygin gas as [30]

\[
p_a = A\rho - \frac{B}{\rho^\alpha}, \tag{6}
\]

where \( A, B \) and \( \alpha \) are constants which are constrained by the astrophysical data (for recent constraints, see [31]). The modified Chaplygin gas satisfactorily accommodates an accelerating phase followed by a matter dominated phase of the universe. It is also consistent with the observational studies dealing with the large scale structure [31]. Hence combining the equations (4) to (6), we get

\[
p = A\rho - \frac{B}{\rho^\alpha} - 3\xi_0 H \sqrt{\rho}, \tag{7}
\]

which is called viscous modified Chaplygin gas (VMCG) [32]. Hence by choosing the above equation of state, we extend previous works dealing with viscous linear equation of state \( (B = 0) \) and viscous generalized Chaplygin gas \( (A = 0) \) [12, 16]. It may also be termed as a ‘viscous unified dark fluid’. The conservation equation is given by

\[
\dot{\rho} + 3H(\rho + p) = 0 \tag{8}
\]

Now using equations (7) and (8), we obtain the solution

\[
\rho = \left( \frac{B}{1 + A - \sqrt{3} \xi_0} + \frac{C}{a^{3(1+\alpha)(1-A-\sqrt{3} \xi_0)}} \right)^{\frac{1}{1+\alpha}}, \tag{9}
\]

where \( C \) is a constant of integration. Now using the redshift formula \( z = \frac{1}{a} - 1 \) (choosing \( a_0 = 1 \)), equation (9) can be re-written in the form:

\[
\rho(z) = \rho_0 \left[ A_s + (1 - A_s)(1 + z)^3(1+\alpha)(1-A-\sqrt{3} \xi_0) \right]^{\frac{1}{1+\alpha}}, \tag{10}
\]

where \( \rho_0 \) is the present value of the density and \( A_s = \frac{B}{(1+A-\sqrt{3} \xi_0)C + B} \) with \( 0 < A_s < 1 \) and \( 1 + A > \sqrt{3} \xi_0 \). Hence the Hubble parameter is obtained as

\[
H(z) = H_0 \left[ \Omega_0 \left\{ A_s + (1 - A_s) \times \left( 1 + z \right)^3(1+\alpha)(1-A-\sqrt{3} \xi_0) \right\} \right]^{\frac{1}{1+\alpha}}, \tag{11}
\]

where \( \Omega_0 = \frac{8\pi G \rho_0}{3H_0^2} \) is the present value of density parameter and \( H_0 \approx 72 \text{ km s}^{-1} \text{ Mpc}^{-1} \). The equation of state parameter is given by

\[
w = \frac{p}{\rho} = A - \frac{B}{\rho^{1+\alpha}} - \frac{3\xi_0 H}{\sqrt{\rho}}, \tag{12}
\]

while the adiabatic sound speed reads

\[
c_s^2 = \frac{\partial p}{\partial \rho} = A + \frac{\alpha B}{\rho^{1+\alpha}} - \frac{\xi_0 H}{2\sqrt{\rho}}. \tag{13}
\]

Note that the Chaplygin gas model is a dynamical dark energy model as well and dark energy perturbations play crucial role in dynamical dark energy models as well [34].

**B. Spherical top-hat collapse model**

The spherical collapse (SC) which provides a way to glimpse into the nonlinear regime of perturbation theory was introduced firstly by Gunn and Gott [33]. The SC describes the evolution of a spherically symmetric perturbation embedded in a homogeneous background, which can be static, expanding or collapsing. One assumes a spherical ‘top-hat’ profile for the perturbed region, i.e., a spherically symmetric perturbation in some region of space with constant density. Following the assumption of a top-hat profile, namely the density perturbation is uniform throughout the collapse, so the evolution of perturbation is only time-dependent.

The perturbed quantities \( \rho_c \) and \( p_c \) are related to their background counterparts by \( \rho_c = \rho + \delta \rho \) and \( p_c = p + \delta p \). Now the perturbed equation of state \( w_c \) is given by [12, 16, 23]

\[
w_c = \frac{p + \delta p}{\rho + \delta \rho} = \frac{w + c_s^2 \delta}{1 + \delta}, \tag{14}
\]

where \( \delta = \frac{\delta \rho}{\rho} \) and perturbed square of the sound speed is given by

\[
c_s^2 = \frac{\delta p}{\delta \rho} = \frac{\rho_c - \rho}{p_c - p} = \frac{A + \frac{B[(1 + \delta)^{1+\alpha} - 1]}{\delta(1 + \delta)^{1+\alpha}}} \frac{3\xi_0 H}{(\sqrt{1 + \delta} + 1)\sqrt{\rho}}. \tag{15}
\]

In the spherical top-hat collapse model, the background evolution equations are still in the forms [15, 16, 23]:

\[
\dot{\rho} = -3H(\rho + p), \tag{16}
\]

\[
\frac{\dot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p), \tag{17}
\]

For the perturbed region, the basic equations which depend on local quantities can be written as

\[
\dot{\rho}_c = -3b(\rho_c + p_c). \tag{18}
\]
\[ \frac{\ddot{r}}{r} = \frac{4\pi G}{3} (\rho_c + 3p_c) \]  

where \( h = \frac{\dot{r}}{r} \) is the local expansion rate and \( r \) is the local scale factor and furthermore, \( h \) relates to local expansion rate in the STHC framework [23, 35].

\[ h = H + \frac{\theta}{3a} \]  

where \( \theta \equiv \nabla \cdot \vec{v} \) is the divergence of the peculiar velocity \( \vec{v} \).

After some calculations [23 24], the dynamical equations of density contrast \( \delta \) and \( \theta \) can be obtained in the following forms:

\[ \delta' = -\frac{3}{a} (c_e^2 - w) \delta - [1 + w + (1 + c_e^2) \delta] \frac{\theta}{a^2 H}, \]  

\[ \theta' = -\frac{\theta}{a} - \frac{\theta^2}{3a^2 H} - \frac{3H}{2} \Omega (1 + 3c_e^2) \]  

where, \( \Omega = \frac{8\pi G \rho_0}{3H^2} \) and \( ' \) represents the derivative w.r.t. scale factor \( a \). The above two equations can be re-written in the forms:

\[ \frac{d\delta}{dz} = \frac{3(c_e^2 - w) \delta}{1 + z} + [1 + w + (1 + c_e^2) \delta] \frac{\theta}{H(z)}, \]  

\[ \frac{d\theta}{dz} = \frac{\theta}{1 + z} + \frac{\theta^2}{3H(z)} + \frac{3H(z)}{2(1 + z)^2} \Omega \delta (1 + 3c_e^2). \]  

We have drawn the time varying parameters \( \delta, \theta, w, c_e, c_e^2, h \) vs redshift \( z \) in figures 1-7 respectively for Einstein’s gravity. In all the figures we have taken the values of the parameters \( A = 0.9, B = 0.6, C = 0.2, \xi_0 = 0.2, \alpha = 0.5, H_0 = 72, \Omega_0 = 1 \). From fig.1 we observe that the parameter \( \delta \) increases from 0 to the value \( \sim 1.8 \) as the redshift \( z \) decreases from 5 to -1. Similar nature happens for the quantity \( \theta \) (increases from 0 to \( \sim 3.3 \)) which is shown in fig.2. The equation of state parameter \( w \) vs \( z \) is drawn in figure 3. We see that \( w \) decreases from 0.15 to -1. So the VMCG generates initially normal fluid and after certain stage, it generates quintessence dark energy but phantom barrier does not happen, but meanwhile the perturbed equation of state parameter \( w_c \) does not so change and its value \( \sim 0.9 \) all over the time. The square speed of sound \( c_e^2 \) for our VMCG fluid system first increases from 0.61056 for \( z \sim 2 \) and ultimately increases to the value 0.61059 at \( z \sim -0.5 \) (fig.5). But, perturbed square speed of sound \( c_e^2 \) actually increases from 0.74 (at \( z \sim 5 \)) to 0.9 after \( z \sim 4.8 \) (fig.6). Also the expansion rate \( h \) vs \( z \) has been drawn in figure 7. We observe that \( h \) first increases from 0 (from \( z \sim 5 \)) to \( \sim 60.1 \) around \( z \sim 0.4 \) and after that suddenly decreases to zero (nearly \( z = -1 \)).

III. SPHERICAL TOP-HAT COLLAPSE MODEL OF VISCOUS MODIFIED CHAPLYGIN GAS IN LOOP QUANTUM COSMOLOGY

Loop Quantum Gravity (LQG) is a canonical quantization of gravity based upon Ashtekar connection variables. LQG is an important frontier to explore the quantum
and
\[ H = -4\pi G (\rho + p) \left( 1 - \frac{2p}{\rho_1} \right), \quad (26) \]
where \(\rho_1 = \sqrt{\frac{\Lambda}{8\pi G}}\) is called the critical loop quantum density. \(\gamma\) is the dimensionless Barbero-Immirzi parameter. Now the fluid is considered as viscous modified Chaplygin gas and in this case, the density is given in equation (10) and in LQC, the Hubble parameter is obtained as
\[ H(z) = H_0 \left[ \Omega_0 \left\{ A_s + (1 - A_s)(1 + z)^3(1 + \alpha_0)(1 + A - \sqrt{\delta_0}) \right\} \right]^{\frac{1}{3}} \]
\[ \times \left[ 1 - \frac{3H_0^2\Omega_0}{\rho_1} \left\{ A_s + (1 - A_s)(1 + z)^3(1 + \alpha_0)(1 + A - \sqrt{\delta_0}) \right\} \right]^{\frac{1}{2}} \]
Similar to the above section, the equation (23) will be the same and equation (24) modifies to the form
\[ \frac{d\theta}{dz} = \frac{\theta}{1 + z} + \frac{\theta^2}{3H(z)} + \frac{3H(z)}{2(1 + z)^2} \Omega \delta \times \left[ 1 + 3c_s^2 \right] - \frac{6\Omega H^2(z)}{8\pi G \rho_1} \left\{ 3(1 + \delta)(1 + 3c_s^2) + (3w - \delta + 1) \right\} \]
(28)
In this case also, the equation of state \(w_c\) and square of the sound speed are given in (14) and (15). We have drawn the time varying parameters \(\delta, \theta, w, w_c, c_s^2, c_r^2, h\) vs redshift \(z\) in figures 8-14 respectively for LQC. In all the figures we have taken the values of the parameters \(A = 0.9, B = 0.6, C = 0.2, \delta_0 = 0.2, \alpha = 0.5, H_0 = 72, \Omega_0 = 1\). From fig.8 we observe that the parameter \(\delta\) decreases from a certain value (around \(z \sim 4\)). Also the quantity \(\theta\) (increases from 0 to \(\sim 4.4\)) during \(z = 5\) to \(z = 4.8\) and after that \(\theta\) decreases to zero (upto \(z \sim -1\)) which is shown in fig.9. The equation of state parameter \(w\) vs \(z\) is drawn in figure 10 which shows that \(w\) decreases from some negative value to \(-0.84\) as \(z\) decreases. So the VMCG in our considered LQC model generates like quintessence dark energy but phantom barrier does not happen. In the meanwhile the perturbed equation of state parameter \(w_c\) changes from some positive value (<1) to \(\sim -0.9\) (fig.11). The square speed of sound \(c_s^2\) for our VMCG fluid system first decreases to the value 0.61 at \(z \sim -0.5\) (fig.12). But, perturbed square speed of sound \(c_s^2\) actually increases from some fractional value (<0.5) to \(\sim 0.02\) (fig.13). Also the expansion rate \(h\) vs \(z\) has been drawn in figure 14. We observe that \(h\) first increases from some positive value (from \(z \sim 5\)) to \(\sim 4\) around \(z \sim 4.8\) and after that it decreases to zero (nearly \(z = -1\)).

**IV. CONCLUSIONS**

In this work, we mainly focused on the collapse of a spherically symmetric perturbation, with a classical top-hat profile, to study the nonlinear evolution of only viscous modified Chaplygin gas (VMCG) perturbations in Einstein’s gravity as well as in loop quantum Cosmology (LQC). The background model is considered as flat FRW metric. Since we know that modified Chaplygin

gravity effects in cosmology. Some of its implications includes the prediction of cosmic inflation in the early universe [38], late time cosmic acceleration [37] and primordial gravitational waves [38]. The field equations of LQC admit attractor solutions which are of enormous cosmological interest. The cosmological perturbation theory within LQC has also been explored in [39]. We here consider the flat homogeneous and isotropic universe described by FRW metric, so the modified Einstein’s field equations in LQC are given by [40 [42]
\[ H^2 = \frac{8\pi G \rho}{3} \left( 1 - \frac{\rho}{\rho_1} \right), \quad (25) \]
gas (MCG) is the unified model of dark matter and dark energy. So we have not assumed any dark matter separately. In this occasion, we have assumed viscous modified Chaplygin gas (VMCG) (which is also the unified model as established) by including the viscosity term in the equation of state in MCG. The density and the Hubble parameter have been calculated in terms of redshift $z$ in both gravity models. The equation of state parameter, square speed of sound have also been calculated for our considered VMCG model in Einstein’s gravity and LQC also. Next step, the density perturbation for our top-hat collapsing profile has been investigated. In the perturbed region, we have investigated the natures of equation of state parameter,
square speed of sound and another perturbed quantities like δ, θ, h etc. The dynamical equations of density contrast δ and θ have been found in both gravity models. Analytically, it is not possible to find out the nature of the perturbed quantities. So numerically and graphically we analyzed the nature of the perturbed quantities which are given as follows:

- **Einstein’s Gravity**: We have drawn the time varying parameters δ, θ, w, w, c^2_s, c^2_e, h vs redshift z in figures 1-7 respectively for Einstein’s gravity. In all the figures we have taken the values of the parameters A = 0.9, B = 0.6, C = 0.2, ξ_0 = 0.2, α = 0.5, H_0 = 72, Ω_0 = 1. From fig.1 we observe that the parameter δ increases from 0 to the value ~ 1.8 as the redshift z decreases from 5 to ~ 1. Similar nature happens for the quantity θ (increases from 0 to ~ 3.3) which is shown in fig.2. The equation of state parameter w vs z is drawn in figure 3. We see that w decreases from 0.15 to ~ 1. So the VMCG generates initially normal fluid and after certain stage, it generates quintessence dark energy but phantom barrier does not happen, but meanwhile the perturbed equation of state parameter w, does not so change and its value ~ 0.9 all over the time. The square speed of sound c^2_e for our VMCG fluid system first increases from 0.61056 for z ∼ 2 and ultimately increases to the value 0.61059 at z ∼ −0.5 (fig.5). But, perturbed square speed of sound c^2_e actually increases from 0.74 (at z ∼ 5) to 0.9 after z ∼ 4.8 (fig.6). Also the expansion rate h vs z has been drawn in figure 7. We observe that h first increases from 0 (from z ∼ 5) to ~ 60.1 around z ∼ 0.4 and after that suddenly decreases to zero (nearly z = −1).

- **LQC**: We have drawn the time varying parameters δ, θ, w_s, c^2_s, c^2_e, h vs redshift z in figures 8-14 respectively for LQC. In all the figures we have taken the values of the parameters A = 0.9, B = 0.6, C = 0.2, ξ_0 = 0.2, α = 0.5, H_0 = 72, Ω_0 = 1. From fig.8 we observe that the parameter δ decreases from a certain value (around z ∼ 4). Also the quantity θ (increases from 0 to ~ 4.4) during z = 5 to z = 4.8 and after that θ decreases to zero (upto z ∼ −1) which is shown in fig.9. The equation of state parameter w vs z is drawn in figure 10 which shows that w decreases from some negative value to −0.84 as z decreases. So the VMCG in our considered LQC model generates like quintessence dark energy but phantom barrier does not happen. In the meanwhile the perturbed equation of state parameter w_s, changes from some positive value (< 1) to −0.9 (fig.11). The square speed of sound c^2_e for our VMCG fluid system first decreases to the value 0.61 at z ∼ −0.5 (fig.12). But, perturbed square speed of sound c^2_e actually increases from some fractional value (< 0.5) to ~ 0.02 (fig.13). Also the expansion rate h vs z has been drawn in figure 14. We observe that h first increases from some positive value (from z ∼ 5) to ~ 4 around z ∼ 4.8 and after that it decreases to zero (nearly z = −1).

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