CP Asymmetries of $B \to \phi K$ and $B \to \eta^{(t)} K$ Decays

Using a Global Fit in QCD Factorization

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Abstract

We analyze the CP asymmetries of $B \to \phi K$ and $B \to \eta^{(t)} K$ modes in the QCD improved factorization framework. For our calculation we use the phenomenological parameters predetermined from the global fit for the available $B \to PP$ and $VP$ modes (without the quark-level subprocess $b \to s\bar{s}s$). We show that the large negative $\sin(2\phi_1)_{\phi K}$ and the large branching ratio for $B^\pm \to \eta' K^\pm$ can be simultaneously explained in the context of supersymmetry (SUSY). The R-parity conserving SUGRA models are used and their parameter space is constrained with the observed dark matter relic density along with other experimental constraints. The R-parity violating SUSY models are also used to show that they can provide solutions. We calculate the CP asymmetries for different $B^{\pm(0)} \to \phi K^{\pm(0)}$ and $B^{\pm(0)} \to \eta^{(t)} K^{\pm(0)}$ modes and show that the SUSY model predictions are consistent with the available experimental data.
Recent results from Belle and BaBar on $B$ decays provide an opportunity to test the source of CP violation. The standard model (SM) source for CP violation arises from the CKM matrix which has only one phase. The SM predictions for CP asymmetry for different $B$ decay modes are being tested at the $B$ factories and discrepancy seems to be emerging. For example, the modes $B \rightarrow \phi K_s$ and $B \rightarrow J/\Psi K_s$ are uniquely clean in their theoretical interpretations. Among these decay modes, $B \rightarrow \phi K_s$ occurs only at one loop level in the SM and hence is a very promising mode to see the effects of new physics. In the SM, it is predicted that the CP asymmetries of $B \rightarrow \phi K_s$ and $B \rightarrow J/\Psi K_s$ should measure the same $\sin(2\phi_1)$ with negligible $\mathcal{O}(\lambda^2)$ difference \cite{1}. The Belle and BaBar experiments measure: \cite{2}

$$\sin(2\phi_1)_{J/\Psi K_s} = 0.734 \pm 0.055,$$

(1)

and \cite{3,4}

$$\sin(2\phi_1)_{\phi K_s} = -0.96 \pm 0.5^{+0.09}_{-0.11} \text{ (Belle)}, \quad 0.45 \pm 0.43 \pm 0.07 \text{ (BaBar)}.$$  

(2)

The world average shows a $2.7\sigma$ disagreement between $\sin(2\phi_1)_{\phi K_s}$ and $\sin(2\phi_1)_{J/\Psi K_s}$. The $\sin(2\phi_1)_{J/\Psi K_s}$ being a tree level process is in excellent agreement with the SM theoretical prediction, $\sin(2\phi_1)_{\text{SM}} = 0.715^{+0.055}_{-0.045}$ \cite{5}.

Experimental data is also available for the decay modes $B^{\pm(0)} \rightarrow \eta^{(0)} K^{\pm(0)}$ which involve the quark-level subprocess $b \rightarrow s \bar{s}s$ as in $B \rightarrow \phi K_s$. The world average value of the measured branching ratio (BR) is $\mathcal{B}(B^{\pm} \rightarrow \eta^{(0)} K^{\pm}) = (77.6 \pm 4.6) \times 10^{-6}$ \cite{6,7,8}, which is larger than the predicted SM value. The results also exist for $\sin(2\phi_1)_{\eta^{(0)} K_s} = 0.33 \pm 0.34 \pm 0.11$ \cite{9,10} and the CP rate asymmetries $A_{\text{CP}}$ for different $B^{\pm} \rightarrow \phi K^{\pm}$ and $B^{\pm} \rightarrow \eta^{(0)} K^{\pm}$ modes \cite{11}.

In this letter we try to find a consistent explanation for all the observed data in charmless hadronic $B \rightarrow PP$ and $B \rightarrow VP$ decays [$P(V)$ denotes a pseudoscalar (vector) meson] in the framework of QCD factorization (QCDF). We calculate the BRs and the CP asymmetries for the decay processes $B^{\pm(0)} \rightarrow \phi K^{\pm(0)}$ and $B^{\pm(0)} \rightarrow \eta^{(0)} K^{\pm(0)}$ in the SM and its SUSY extensions with R-parity conservation (SUGRA models), and with R-parity violation. The required QCDF input parameters for the calculation are determined by using a global fit of all possible $B$ decay modes without the subprocess $b \rightarrow s \bar{s}s$, since this process may involve new physics.

Previously, new interactions were invoked to explain the large BRs of $B^{\pm(0)} \rightarrow \eta^{(0)} K^{\pm(0)}$ and the large negative $\sin(2\phi_1)_{\phi K_s}$ \cite{12,13,14,15,16,17}. However, attempts of simultaneous
explanation for the observed data of these decay modes were made only by using the naive factorization technique \cite{13, 15}. We adopt the newly developed QCD improved factorization \cite{18} for the calculation in this work. This approach allows us to include the possible non-factorizable contributions. In the heavy quark limit \( m_b >> \Lambda_{QCD} \), the hadronic matrix element for \( B \to M_1 M_2 \) due to a particular operator \( O_i \) can be written in the QCDF as follows:

\[
\langle M_1 M_2 | O_i | B \rangle = \langle M_1 M_2 | O_i | B \rangle_{NF} \cdot \left[ 1 + \sum_n r_n (\alpha_s)^n + O \left( \frac{\Lambda_{QCD}}{m_b} \right) \right],
\]

where \( NF \) denotes the naive factorization. The second and third term in the square bracket represent the radiative corrections in \( \alpha_s \) and the power corrections in \( \Lambda_{QCD}/m_b \). The decay amplitudes for \( B \to M_1 M_2 \) can be expressed as

\[
\mathcal{A}(B \to M_1 M_2) = \mathcal{A}^f(B \to M_1 M_2) + \mathcal{A}^a(B \to M_1 M_2),
\]

where

\[
\mathcal{A}^f(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_{i=1}^{10} V_{pb} V_{pq}^* a_i^p \langle M_1 M_2 | O_i | B \rangle_{NF},
\]

\[
\mathcal{A}^a(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} f_B f_{M_1} f_{M_2} \sum_{p=u,c} \sum_{i=1}^{10} V_{pb} V_{pq}^* b_i.
\]

Here \( \mathcal{A}^f(B \to M_1 M_2) \) includes vertex corrections, penguin corrections, and hard spectator scattering contributions which are absorbed into the QCD coefficients \( a_i \), and \( \mathcal{A}^a(B \to M_1 M_2) \) includes weak annihilation contributions which are absorbed into the parameter \( b_i \). The explicit expressions of \( a_i \) and \( b_i \) can be found in Refs. \cite{17, 18, 19}. The relevant end-point divergent integrals are parametrized as \cite{18}

\[
X_{H,A} \equiv \int_0^1 \frac{dx}{x} \equiv \left( 1 + \rho_{H,A} e^{i \phi_{H,A}} \right) \ln \frac{m_B}{\Lambda_h},
\]

where \( X_H \) and \( X_A \) denote the hard spectator scattering contribution and the annihilation contribution, respectively. Here the phases \( \phi_{H,A} \) are arbitrary, \( 0^0 \leq \phi_{H,A} \leq 360^0 \), and the parameter \( \rho_{H,A} \leq 1 \), and the scale \( \Lambda_h = 0.5 \text{ GeV} \) assumed phenomenologically \cite{18}. In principle, the parameters \( \rho_{H,A} \) and \( \phi_{H,A} \) for \( B \to PP \) decays can be different from those for \( B \to VP \) decays.

We use the global analysis to determine the QCD parameters \( X_H \) and \( X_A \) \cite{17}. In the global analysis we exclude the decay modes whose (dominant) internal quark-level process
is $b \to s\bar{s}s$: for example, $B \to \phi K$ and $B \to \eta^{(0)} M$, where $M$ denotes a light meson, such as $\pi$, $K$, $\rho$, $K^*$. The reason for such a global analysis is due to the fact that the $b \to s\bar{s}s$ mode may require the existence of new physics. We use twelve $B \to PP$ and $VP$ decay modes, including $B \to \pi \pi$, $\pi K$, $\rho \pi$, $\rho K$, $\omega \pi$, $\omega K$, to determine the QCD parameters, $\rho^{PP, PV}_{A,H}$ and $\phi^{PP,PV}_{A,H}$. The parameters are distinguished by their superscripts $P$ and $V$ which denote final state mesons. Now, if $\rho_{A,H}$ and $\phi_{A,H}$ are large, the effects of $X_{A,H}$ can be large and give rise to a large annihilation (and/or hard spectator scattering) contribution, which indicates the theory becomes less reliable and suspicious due to that large non-perturbative contribution. We find that it is possible to obtain a good global fit ($\chi^2_{min}=7.5$) with large $X_{A,H}$ effects. In this scenario, the BR for $B^+ \to \eta' K^+$ is $7.47 \times 10^{-6}$, which saturates the experimental limit with the very large $X_{A,H}$ effects [17] and the BR for $B^+ \to \phi K^+$ is small: $4.0 \times 10^{-6}$. It is possible to obtain successful fits to all $B \to \phi K$ and $B \to \eta' K$ data in our SUGRA models. But, since the effects of $X_{A,H}$ are large, we will just comment on the fits in this scenario in our result sections.

In Ref. [17], we generated a fit (with $\chi^2_{min}=18.3$) with the (relatively) small $X_A$ and $X_H$ effects. The corresponding theoretical inputs for this fit are as follows (we will use these values in this work):

$$
\lambda = 0.2198, \quad A = 0.868, \quad \phi_3 = 86.8^0, \quad |V_{ub}| = 3.35 \times 10^{-3},
$$

$$
\mu = 2.1 \text{ GeV}, \quad m_s(2 \text{ GeV}) = 85 \text{ MeV}, \quad f_B = 220 \text{ MeV},
$$

$$
F_{B\pi} = 0.249, \quad R_{\pi K} = 1, \quad A_{B\rho} = 0.31,
$$

$$
\rho^{PP}_{A} = 0, \quad \rho^{VP}_{A} = 0.5, \quad \rho^{PP}_{H} = 1, \quad \rho^{VP}_{H} = 0.746,
$$

$$
\phi^{VP}_{A} = -6^0, \quad \phi^{VP}_{H} = \phi^{PP}_{H} = 180^0. \quad (7)
$$

Note that in this case the effect of the weak annihilation parameter $X_A$ is relatively small (i.e., $\rho^{PP}_{A} = 0$ and $\rho^{VP}_{A} = 0.5$), and the effect of the hard spectator scattering parameter $X_H$ is also very small, because $\rho^{PP}_{H} = 1, \rho^{VP}_{H} = 0.746$, and $\phi^{PP}_{H} = \phi^{VP}_{H} = 180^0$ so that the terms 1 and $\rho_{H}e^{i\phi_{H}}$ in $X_{H}$ cancel each other in Eq. (3). Based on the above inputs, the BRs and CP asymmetries for $B \to \phi K$ and $B \to \eta' K$ are predicted and shown in Table I. We see that the predicted $\mathcal{B}(B^{+(0)} \to \eta' K^{+(0)})$ are smaller than the measured values and the predicted $\sin(2\phi_1)_{\phi K_s}$ is also very different from the world average. The predicted $\mathcal{B}(B^+ \to \eta K^+)$ is smaller than the experimental data as well.

In the following two sections, we will discuss the CP asymmetries and BRs of $B \to \phi K$
TABLE I: The branching ratios (in unit of $10^{-6}$) and CP asymmetries of $B \to \phi K$ and $B \to \eta^{(')}K$ decays are shown. The values in the parenthesis are experimental numbers [3, 4, 6, 7, 8, 9, 10, 11, 20]. Here the inputs for the fit with the small effects of $X_A$ and $X_H$ are used. The first row of the last column shows the $\sin(2\phi_1)$ for $B \to \phi K_s$ and the last row in the same column shows the value for $B \to \eta K_s$.

| Decay mode | BR  | $A_{CP}$ | BR  | $A_{CP}$ | $\sin(2\phi_1)$ |
|------------|-----|----------|-----|----------|-----------------|
| $B^+ \to \phi K^+$ | 7.3 | 0        | $B^0 \to \phi K^0$ | 6.7 | 0.01 | $\phi K_s$: 0.68 |
|            | $(9.2 \pm 1.0)$ | $(0.03 \pm 0.07)$ | | $(7.7 \pm 1.1)$ | $(0.19 \pm 0.68)$ | (Eq. 2) |
| $B^+ \to \eta^{(')}K^+$ | 51  | 0.01     | $B^0 \to \eta^{(')}K^0$ | 46.8 | 0.016 | $\eta^{(')}K_s$: 0.57 |
|            | $(77.6 \pm 4.6)$ | $(0.02 \pm 0.04)$ | | $(60.6 \pm 7)$ | $(0.8 \pm 0.18)$ | $(0.33 \pm 0.34)$ |
| $B^+ \to \eta K^+$ | 1.9 | $-0.16$  | $B^0 \to \eta K^0$ | 1.7  | $-0.16$ | <4.6 |
|            | $(3.1 \pm 0.7)$ | $(-0.32 \pm 0.20)$ | | | | |

and $B \to \eta K$ modes in the context of SUSY models.

[1] R-parity violating SUSY case

The R-parity violating (RPV) part of the superpotential of the minimal supersymmetric standard model can have the following terms

$$W_{RPV} = \kappa_i L_i H_2 + l_{ijk} L_i L_j E_k^c + l'_{ijk} L_i Q_j D_k^c + l''_{ijk} U_i^c D_j^c D_k^c$$

where $E_i$, $U_i$ and $D_i$ are respectively the $i$-th type of lepton, up-quark and down-quark singlet superfields, $L_i$ and $Q_i$ are the SU(2)$_L$ doublet lepton and quark superfields, and $H_2$ is the Higgs doublet with the appropriate hypercharge.

For our purpose, we will assume only $l'$-type couplings to be present. Then, the effective Hamiltonian for charmless hadronic $B$ decay can be written as

$$H_{eff}^N(b \to d_j d_k d_n) = d_{jkn}^R \left[ \bar{d}_{\alpha} \gamma_{\mu} L d_{j} \beta \bar{d}_{k} \gamma_{\mu R} b_{\alpha} + d_{jkn}^L \left[ \bar{d}_{\alpha} \gamma_{\mu L} b_{\beta} \bar{d}_{k} \gamma_{\mu R} d_{j\alpha} \right] \right]$$

$$H_{eff}^N(b \to u_j u_k d_n) = u_{jkn}^R \left[ \bar{u}_{\alpha} \gamma_{\mu L} u_{j} \beta \bar{d}_{k} \gamma_{\mu R} b_{\alpha} \right]$$

Here the coefficients $d_{jkn}^{LR}$ and $u_{jkn}^R$ are defined as

$$d_{jkn}^R = \sum_{i=1}^{3} \frac{l'_{ijk} l'_{in}}{8m_{\tilde{\nu}_L}^2}, \quad d_{jkn}^L = \sum_{i=1}^{3} \frac{l''_{ijk} l''_{in}}{8m_{\tilde{\nu}_L}^2}, \quad (j, k, n = 1, 2)$$
\[ u_{jkn}^R = \frac{3}{8\pi^2} \sum_{i=1}^{f} \frac{g_{i\mu\nu}^j}{\tilde{m}_{i\mu\nu}^j}, \quad (j, k = 1, n = 2) \]  

(10)

where \(\alpha\) and \(\beta\) are color indices and \(\gamma_{R,L}^\mu \equiv \gamma^\mu (1 \pm \gamma_5)\). The leading order QCD correction to this operator is given by a scaling factor \(f \approx 2\) for \(m_\phi = 200\) GeV. We refer to Refs. [21, 22] for the relevant notations.

The RPV SUSY part of the decay amplitude of \(B^- \to \phi K^-\) is given by

\[ A_{\phi K}^{RPV} = (d_{222}^L + d_{222}^R) \tilde{a} A_\phi , \]

where the coefficient \(\tilde{a}\) is expressed as

\[ \tilde{a} = \frac{1}{N_c} \left[ 1 - \frac{4\pi}{4\pi} \left( V_\phi + 12 - \left[ \frac{4}{3} \ln \frac{m_\phi}{\mu} - G_\phi(0) \right] + \frac{4\pi^2}{N_c} H(BK, \phi) \right) \right] . \]  

(12)

It has been noticed [22] that the RPV part of the decay amplitude for \(B \to \eta'K\), \(A_{\eta'K}^{RPV}\), is proportional to \((d_{222}^L - d_{222}^R)\), while the RPV part of the decay amplitude for \(B \to \phi K\), \(A_{\phi K}^{RPV}\), is proportional to \((d_{222}^L + d_{222}^R)\). It has been also pointed out [22] that the opposite relative sign between \(d_{222}^L\) and \(d_{222}^R\) in the modes \(B \to \eta'K\) and \(B \to \phi K\) appears due to the different parity in the final state mesons \(\eta'\) and \(\phi\), and this different combination of \((d_{222}^L - d_{222}^R)\) and \((d_{222}^L + d_{222}^R)\) in these modes plays an important role to explain both the large BRs for \(B \to \eta'K\) and the large negative value of \(\sin(2\phi_1)_{\phi K}\) at the same time.

We define the new coupling terms \(d_{222}^L\) and \(d_{222}^R\) as follows:

\[ d_{222}^L \propto |\lambda_{332}'\lambda_{322}'| e^{i\theta_L} , \quad d_{222}^R \propto |\lambda_{322}'\lambda_{323}'| e^{i\theta_R} , \]

(13)

where \(\theta_L\) and \(\theta_R\) denote new weak phases of the product of new couplings \(\lambda_{332}'\lambda_{322}'\) and \(\lambda_{322}'\lambda_{323}'\), respectively, as defined by \(\lambda_{332}'\lambda_{322}' \equiv |\lambda_{332}'\lambda_{322}'| e^{i\theta_L}\) and \(\lambda_{322}'\lambda_{323}' \equiv |\lambda_{322}'\lambda_{323}'| e^{i\theta_R}\).

We consider two different cases as follows.

**Case (a):** \(d_{222}^L \neq 0\) and \(d_{222}^R = 0\), *i.e.*, \(|\lambda_{323}'| = 0.077 , \ |\lambda_{332}'| = 0.077 , \ |\lambda_{322}'| = 0\), \(\theta_L = 1.5 \), \(m_{\text{SUSY}} = 200\) GeV.

(14)

Our results are summarized in Table II. We use the negative \(d_{222}^L\) and the scaling factor [22]. We find that \(\sin(2\phi_1)_{\phi K}\) can be brought down to 0 at most. The BRs, \(B(B^+ \to \eta K^+)\) and \(B(B^0 \to \eta K^0)\), are larger compared to the experimental values, but the experiments (Belle, BaBar and CLEO) are not quite in agreement and the experimental errors are also large.
TABLE II: Case (a): the branching ratios (in unit of 10^{-6}) and CP asymmetries of $B \to \phi K$ and $B \to \eta^{(')} K$ decays are calculated in the framework of R-parity violating SUSY.

| Decay mode $\rightarrow$ | BR $\times 10^{-6}$ | $A_{CP}$ | Decay mode $\rightarrow$ | BR $\times 10^{-6}$ | $A_{CP}$ | $\sin(2\phi_1)$ |
|------------------------|------------------|----------|------------------------|------------------|----------|------------------|
| $B^+ \rightarrow \phi K^+$ | 8.9 | -0.17 | $B^0 \rightarrow \phi K^0$ | 8.2 | -0.18 | $\phi K_s$: -0.03 |
| $B^+ \rightarrow \eta^{(')} K^+$ | 72.0 | 0.16 | $B^0 \rightarrow \eta^{(')} K^0$ | 66.0 | 0.16 | $\eta^{(')} K_s$: -0.2 |
| $B^+ \rightarrow \eta K^+$ | 10.5 | 0.25 | $B^0 \rightarrow \eta K^0$ | 9.7 | 0.25 | $\eta K_s$: -0.32 |

TABLE III: Case (b): the branching ratios (in unit of 10^{-6}) and CP asymmetries of $B \to \phi K$ and $B \to \eta^{(')} K$ decays are calculated in the framework of R-parity violating SUSY.

| Decay mode $\rightarrow$ | BR $\times 10^{-6}$ | $A_{CP}$ | Decay mode $\rightarrow$ | BR $\times 10^{-6}$ | $A_{CP}$ | $\sin(2\phi_1)$ |
|------------------------|------------------|----------|------------------------|------------------|----------|------------------|
| $B^+ \rightarrow \phi K^+$ | 10.2 | -0.05 | $B^0 \rightarrow \phi K^0$ | 9.5 | -0.04 | $\phi K_s$: -0.61 |
| $B^+ \rightarrow \eta^{(')} K^+$ | 74.0 | 0.11 | $B^0 \rightarrow \eta^{(')} K^0$ | 67.7 | 0.11 | $\eta^{(')} K_s$: 0.48 |
| $B^+ \rightarrow \eta K^+$ | 6.7 | 0.06 | $B^0 \rightarrow \eta K^0$ | 6.1 | 0.06 | $\eta K_s$: 0.48 |

for these modes. In Table II, we used $\delta' = 0$, but if we use $\delta' = 30^0$, the $\sin(2\phi_1)_{\phi K_s}$ can be larger negative: $-0.2$, after satisfying all the constraints especially the $B(B^+ \to \eta K^+)$ and $B^0 \to \eta K^0$.

Case (b): $d_{L222}^{L} \neq d_{R222}^{R} \neq 0$, i.e.,

$$
\begin{align*}
|\lambda_{322}^{'}| &= 0.076 , \\
|\lambda_{332}^{'}| &= 0.076 , \\
|\lambda_{323}^{'}| &= 0.064 , \\
\theta_L &= 1.32 , \\
\theta_R &= -1.29 , \\
m_{SUSY} &= 200 \text{ GeV}.
\end{align*}
$$

(15)

Our results are summarized in Table III. We find that $\sin(2\phi_1)_{\phi K_s}$ can be large negative. In addition to the parameters given in Eq. (15), we also used the additional strong phase $\delta' = 30^0$, which can arise from the power contributions of $\Lambda_{QCD}/m_b$ neglected in the QCDF scheme, and whose size can be in principle comparable to the strong phase arising from the radiative corrections of $O(\alpha_s)$. If $\delta' = 0$ is used, we obtain $\sin(2\phi_1)_{\phi K_s} = -0.2$.

In the case of the large $X_{A,H}$ effects with $\chi^2_{min} = 7.5$ where the BR of $B^+ \to \eta K^+$ is large, we can use the R-parity violating SUSY couplings to raise the BR of $B^+ \to \phi K^+$ (which is small, $4.0 \times 10^{-6}$ to begin with). It is possible to raise $B(B^+ \to \phi K^+)$ to $(8 - 9) \times 10^{-6}$, but its CP rate asymmetry $A_{CP}(B^+ \to \phi K^+)$ is large $\sim -0.4$ and $\sin(2\phi_1)_{\phi K_s}$ can be brought down to at most $-0.16$. 

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The RPV terms can arise in the context of SO(10) models which explain the small neutrino mass and has an intermediate breaking scale where $B - L$ symmetry gets broken by $(16 + \bar{16})$ Higgs. These additional Higgs form operators like $16_H 16_m 16_m / M_{pl}$ ($16_m$ contains matter fields) and generate the RPV terms [23].

[2] R-parity conserving SUSY case

As an example of the R-parity conserving (RPC) SUSY case, we will consider the supergravity (SUGRA) model with the simplest possible non-universal soft terms which is the simplest extension of the minimal SUGRA (mSUGRA) model. In this model the lightest SUSY particle is stable and this particle can explain the dark matter content of the universe. The recent WMAP result provides [24]:

$$\Omega_{CDM} h^2 = 0.1126^{+0.008}_{-0.009},$$

and we implement $2\sigma$ bound in our calculation.

In the SUGRA model, the superpotential and soft SUSY breaking terms at the grand unified theory (GUT) scale are given by

$$W = Y^U Q H_2 U + Y^D Q H_1 D + Y^L L H_1 E + \mu H_1 H_2,$$

$$\mathcal{L}_{\text{soft}} = -\sum_i m_i^2 |\phi_i|^2 - \left[ \frac{1}{2} \sum_{\alpha} m_{\alpha} \bar{\lambda}_\alpha \lambda_\alpha + B \mu H_1 H_2 \right.$$

$$+ (A^U Q H_2 U + A^D Q H_1 D + A^L L H_1 E) + \text{H.c.} \big],$$

where $E$, $U$ and $D$ are respectively the lepton, up-quark and down-quark singlet superfields, $L$ and $Q$ are the SU(2)$_L$ doublet lepton and quark superfields, and $H_{1,2}$ are the Higgs doublets. $\phi_i$ and $\lambda_\alpha$ denote all the scalar fields and gaugino fields, respectively. The parameters in the mSUGRA model, a universal scalar mass $m_0$, a universal gaugino mass $m_{1/2}$, and the universal trilinear coupling $A$ terms are introduced at the GUT scale:

$$m_i^2 = m_0^2, \quad m_\alpha = m_{1/2}, \quad A^{U,D,L} = A_0 Y^{U,D,L},$$

where $Y^{U,D,L}$ are the diagonalized $3 \times 3$ Yukawa matrices. In this model, there are four free parameters, $m_0$, $m_{1/2}$, $A_0$, and $\tan \beta \equiv \langle H_2 \rangle / \langle H_1 \rangle$, in addition to the sign of $\mu$. The parameters $m_{1/2}$, $\mu$ and $A$ can be complex, and four phases appear: $\theta_A$ (from $A_0$), $\theta_1$ (from the gaugino mass $m_1$), $\theta_3$ (from the gaugino mass $m_3$), and $\theta_\mu$ (from the $\mu$ term).

The mSUGRA model can not explain the large negative value of $\sin(2\phi_1)\phi_K$, because in this model, the only source of flavor violation is in the CKM matrix, which can not provide a
From the experimental constraints, we find that \( \theta \), LEP bounds on masses of SUSY particles and the lightest Higgs, relic density measurements, \( K \), unless \( \sin(2\theta) \) where \( \Delta A \), are 3 \( \times \) 3 complex matrices and \( \Delta A \), unless \( (i, j) = (2, 3) \) or \( (3, 2) \). It is obvious that the mSUGRA model is recovered if \( \Delta A = 0 \).

It has been noticed [16] that the SUSY contribution mainly affect the Wilson coefficients \( C_{8g(7\gamma)} \) and \( \tilde{C}_{8g(7\gamma)} \) and these coefficients do not change the weak annihilation effects arising from the SM calculation. In our analysis, we consider all the known experimental constraints on the parameter space of the model, as in Ref. [16]. Those constraints come from the radiative \( b \to s \) transition in the processes \( B \to \phi K \) [16], neutron and electron electric dipole moments (\( d_n < 6.3 \times 10^{-26} e cm, d_e < 0.21 \times 10^{-26} e cm \) [27]), relic density measurements, \( K^0 - \bar{K}^0 \) mixing (\( \Delta M_K = (3.490 \pm 0.006) \times 10^{-12} \) MeV [27]), LEP bounds on masses of SUSY particles and the lightest Higgs (\( m_h \geq 114 \) GeV).

We consider only non-zero (2,3) elements in \( A \) terms as a simplest extension of the mSUGRA model. This new piece enhances the left-right mixing of the second and third generation. The \( A \) terms with only non-zero (2,3) elements can be expressed as

\[
A^{U,D} = A_0 Y^{U,D} + \Delta A^{U,D},
\]

where \( \Delta A^{U,D} \) are 3 \( \times \) 3 complex matrices and \( \Delta A^{U,D}_{ij} = |\Delta A^{U,D}_{ij}| e^{i\phi^{U,D}_{ij}} \) with \( |\Delta A^{U,D}_{ij}| = 0 \) unless \( (i, j) = (2, 3) \) or \( (3, 2) \). It is obvious that the mSUGRA model is recovered if \( \Delta A^{U,D} = 0 \).

| \( |A_0| \) | 800 | 600 | 400 | 0 | \( \Delta A^{D}_{23(32)} \) |
|---|---|---|---|---|---|
| \( m_{1/2} = 300 \) | -0.55 | 9.9 | -0.57 | 9.2 | -0.55 | 9.1 | -0.54 | 8.1 | 66 - 74 |
| \( m_{1/2} = 400 \) | -0.56 | 9.9 | -0.53 | 9.6 | -0.56 | 9.2 | -0.58 | 8.5 | 150 - 168 |
| \( m_{1/2} = 500 \) | -0.37 | 9.9 | -0.39 | 9.9 | -0.42 | 10.0 | -0.43 | 8.1 | 244 - 256 |
| \( m_{1/2} = 600 \) | -0.32 | 7.6 | -0.30 | 7.5 | -0.30 | 7.5 | -0.05 | 7.1 | 270 - 304 |
though the value of $m$ are set to be zero. We also implement to find solutions, we do not need to use the additional strong phase: i.e., $m$ variations from 300 GeV to 600 GeV, and the value of $m$ for $φ$ of sin(2 $δ$) can be large and negative. If we use the additional strong phase $δ’ = 0$. The value of $m_1/2$ varies from 300 GeV to 600 GeV, and the value of $|A_0|$ varies from 0 to 800 GeV. Even though the value of $m_0$ is not explicitly shown, it is chosen for different $m_1/2$ and $A_0$ such that the relic density constraint is satisfied. (We satisfy the relic density constraint using the stau-neutrino co-annihilation channel [28].) The value of $m_0$ increases as $m_1/2$ increases. The value of $|A_0|$ increases as $m_1/2$ does. The phases $φ_{23}^D$ and $φ_{32}^D$ are approximately $−40^0$ to $−15^0$ and $165^0$ to $180^0$, respectively.

Table IV shows the BRs for $B^± → φK^±$ (right column) and $sin(2φ_1)φK_+$ (left column), calculated for various values of the parameters $m_1/2$ and $|A_0|$. The value of $sin(2φ_1)φK_+$ can be large and negative. If we use the additional strong phase $δ’ ≠ 0$, the magnitude of $sin(2φ_1)φK_+$ can be even larger. For example, if $δ’ = −30^0$, then $sin(2φ_1)φK_+ = −0.65$ for $m_1/2 = 400$ GeV and $A_0 = −800$ GeV. In Table V, we show the CP rate asymmetries

### Table V: $A_{b→s+γ}$ (left column) and $A_{φK^±}$ (right column) at $tan β = 10$.

| $|A_0|$ | 800  | 600  | 400  | 0    |
|-------|------|------|------|------|
| $m_1/2 = 300$ | 0.011| 0.21 | 0.017| 0.22 | 0.02 | 0.22 |
|       | 0.017| 0.21 | 0.028| 0.21 | 0.027| 0.22 |
|       | 0.033| 0.18 | 0.034| 0.18 | 0.033| 0.19 |
|       | 0.023| 0.2  | 0.023| 0.2  | 0.023| 0.2  |

### Table VI: $B(B^± → η’K^±) × 10^6$ (left column) and $B(B^± → ηK^±) × 10^6$ (right column) at $tan β = 10$.

| $|A_0|$ | 800  | 600  | 400  | 0    |
|-------|------|------|------|------|
| $m_1/2 = 300$ | 79.6 | 3.67 | 81.0 | 3.76 | 79.6 | 3.66 | 79.0 | 3.62 |
|       | 78.2 | 4.4  | 83.0 | 3.85 | 79.0 | 3.69 | 81.0 | 3.69 |
|       | 84.8 | 3.90 | 83.7 | 3.85 | 81.0 | 3.71 | 77.0 | 3.50 |
|       | 73.0 | 3.26 | 71.0 | 3.18 | 70.0 | 3.10 | 70.0 | 3.00 |
with other available data on CP asymmetry for different $B$ that the R-parity violating SUSY models can provide solutions. Our results are consistent observed dark matter relic density along with other experimental constraints. We also show We use R-parity conserving SUGRA models and constrain the parameter space with the

| $|A_0|$ | 800 | 600 | 400 | 0 |
|-----|-----|-----|-----|-----|
| $m_{1/2} = 300$ | 0.0 | −0.24 | 0.0 | −0.23 | 0.0 | −0.23 | −0.001 | −0.22 |
| $m_{1/2} = 400$ | −0.026 | −0.31 | −0.003 | −0.22 | −0.004 | −0.24 | −0.003 | −0.23 |
| $m_{1/2} = 500$ | −0.005 | −0.21 | −0.005 | −0.21 | −0.008 | −0.22 | −0.002 | −0.22 |
| $m_{1/2} = 600$ | −0.0004 | −0.22 | −0.0006 | −0.22 | −0.001 | −0.22 | −0.009 | −0.195 |

$A_{b \to s^+\gamma}$ (left) and $A_{\phi K^\pm}$ (right). Since $A_{23}$ contributes to $b \to s \gamma$, the CP asymmetry gets generated, but the asymmetry is small. So far we have assumed that $\Delta A^U_{23,32} = 0$. But if we use $\Delta A^U_{23,32} \neq 0$ and $\Delta A^D_{23,32} = 0$, the value of $\sin(2\phi)_\eta K_s$ is mostly positive. The BRs for $B^\pm \to \eta' K^\pm$ and $B^\pm \to \eta K^\pm$ are shown in Table V and the CP asymmetries are shown in Table VII. They are consistent with the experimental data. The value of $\sin(2\phi)_\eta K_s$ is $0.6-0.7$ for all these scenarios. The BRs are: $B(B^0 \to \eta' K^0) \sim (63-73) \times 10^{-6}$ and $B(B^0 \to \eta K^0) \sim (2.25-3.37) \times 10^{-6}$. The CP asymmetries are: $A_{\eta' K^0} \sim A_{\eta K^+}$ and $A_{\eta K^0} \sim A_{\eta K^+}$. We see that the CP asymmetries are also in good agreement with the experimental values shown in Table I. As a final comment, we note that in the case of the large $X_{A,H}$ effects with $\chi^2_{\text{min}} = 7.5$, it is possible to raise the BR for $B^+ \to \phi K^+$ to $(8-9) \times 10^{-6}$. However, in that case, $A_{\text{CP}}(B^+ \to \phi K^+) \sim -0.2$ and $\sin(2\phi_1)_\phi K_s \geq -0.34$.

In conclusion, we have analyzed the CP asymmetries of $B \to \phi K$ and $B \to \eta(0) K$ modes in the QCDF framework. The phenomenological parameters $X_H$ and $X_A$ arising from endpoint divergences in the hard spectator scattering and weak annihilation contributions are determined by the global analysis for twelve $B \to PP$ and $VP$ decay modes, such as $B \to \pi\pi$, $\pi K$, $\rho \pi$, $\rho K$, etc, but excluding the modes whose (dominant) internal quark-level process is $b \to s\bar{s}s$. We found that it is possible to explain large negative $\sin(2\phi_1)_\phi K_s$ simultaneously with the large BRs for $B^{\pm(0)} \to \eta' K^{\pm(0)}$ in the context of supersymmetry. We use R-parity conserving SUGRA models and constrain the parameter space with the observed dark matter relic density along with other experimental constraints. We also show that the R-parity violating SUSY models can provide solutions. Our results are consistent with other available data on CP asymmetry for different $B \to \phi K$, $B \to \eta' K$, and $B \to \eta K$ modes.
The work of B.D was supported by Natural Sciences and Engineering Research Council of Canada. The work of C.S.K. was supported in part by Grant No. R02-2003-000-10050-0 from BRP of the KOSEF and in part by CHEP-SRC Program. The work of S.O. and G.Z. was supported by the Japan Society for the Promotion of Science (JSPS).

[1] Y. Grossman and M. P. Worah, Phys. Lett. B 395, 241 (1997).
[2] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 89, 201802 (2002); K. Abe et al. [Belle Collaboration], Phys. Rev. D 66, 071102 (2002).
[3] T. E. Browder, arXiv:hep-ex/0312024
[4] K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0308035.
[5] A. J. Buras, arXiv:hep-ph/0210291
[6] A. Gordon et al. [Belle Collaboration], Phys. Lett. B 542, 183 (2002).
[7] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 91, 161801 (2003).
[8] R. A. Briere et al. [CLEO Collaboration], Phys. Rev. Lett. 86, 3718 (2001).
[9] G. Hamel De Monchenault [BABAR Collaboration], arXiv:hep-ex/0305055.
[10] K. Abe et al. [BELLE Collaboration], Phys. Rev. D 67, 031102 (2003).
[11] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0303029.
[12] D. Atwood and A. Soni, Phys. Lett. B 405, 150 (1997); A. L. Kagan and A. A. Petrov, arXiv:hep-ph/9707354; M. R. Almady, E. Kou and A. Sugamoto, Phys. Rev. D 58, 014015 (1998); D.-s. Du, C. S. Kim and Y.-d. Yang, Phys. Lett. B 426, 133 (1998); I. Halperin and A. Zhitnitsky, Phys. Rev. Lett. 80, 438 (1998); D. Choudhury, B. Dutta and A. Kundu, Phys. Lett. B 456, 185 (1999); A. Kundu and T. Mitra, arXiv:hep-ph/0302123.
[13] B. Dutta, C. S. Kim and S. Oh, Phys. Rev. Lett. 90, 011801 (2003).
[14] G. Hiller, Phys. Rev. D 66, 071502 (2002); A. Datta, Phys. Rev. D 66, 071702 (2002); M. Ciuchini and L. Silvestrini, Phys. Rev. Lett. 89, 231802 (2002); S. Baek, Phys. Rev. D 67, 096004 (2003); C. W. Chiang and J. L. Rosner, arXiv:hep-ph/0302094; K. Agashe and C. D. Carone, arXiv:hep-ph/0304229; G. L. Kane, P. Ko, H. b. Wang, C. Kolda, J. h. Park and L. T. Wang, Phys. Rev. Lett. 90, 141803 (2003); D. Chakraverty, E. Gabrielli, K. Huitu and S. Khalil, arXiv:hep-ph/0306076; J. F. Cheng, C. S. Huang and X. h. Wu, arXiv:hep-ph/0306086; D. Atwood and G. Hiller, arXiv:hep-ph/0307251; A. K. Giri and R.
[15] S. Khalil and E. Kou, Phys. Rev. D 67, 055009 (2003); arXiv:hep-ph/0307024
[16] R. Arnowitt, B. Dutta and B. Hu, Phys. Rev. D 68, 075008 (2003).
[17] B. Dutta, C. S. Kim, S. Oh and G. Zhu, arXiv:hep-ph/0312388
[18] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999);
    Nucl. Phys. B 591, 313 (2000); Nucl. Phys. B 606, 245 (2001).
[19] D. Du, H. Gong, J. Sun, D. Yang and G. Zhu, Phys. Rev. D 65, 074001 (2002); ibid. 65,
    094025 (2002) [Erratum-ibid. D 66, 079904 (2002)].
[20] M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003).
[21] D. Choudhury, B. Dutta and A. Kundu, Phys. Lett. B 456, 185 (1999).
[22] B. Dutta, C. S. Kim and S. Oh, Phys. Lett. B 535, 249 (2002).
[23] R. N. Mohapatra, arXiv:hep-ph/9604414
[24] C. L. Bennett et al., arXiv:astro-ph/0302207
[25] M. S. Alam et al. [CLEO Collaboration], Phys. Rev. Lett. 74, 2885 (1995).
[26] A. J. Buras, A. Czarnecki, M. Misiak and J. Urban, Nucl. Phys. B 631, 219 (2002).
[27] K. Hagiwara et al., Phys. Rev. D 66, 010001 (2002).
[28] J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Srednicki, Phys. Lett. B 510, 236 (2001);
    R. Arnowitt, B. Dutta and Y. Santosso, Nucl. Phys. B 606, 59 (2001); M. E. Gomez and
    J. D. Vergados, Phys. Lett. B 512, 252 (2001); H. Baer, C. Balazs, A. Belyaev, J. K. Mizukoshi,
    X. Tata and Y. Wang, arXiv:hep-ph/0210441; A. B. Lahanas and D. V. Nanopoulos,
arXiv:hep-ph/0303130