The effective $g_A$ in the $pf$-shell

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We have calculated the Gamow-Teller matrix elements of 64 decays of nuclei in the mass range $A = 41–50$. In all the cases the valence space of the full $pf$-shell is used. Agreement with the experimental results demands the introduction of an average quenching factor, $q = 0.744 \pm 0.015$, slightly smaller but statistically compatible with the $sd$-shell value, thus indicating that the present number is close to the limit for large $A$.

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The observed Gamow-Teller strength appears to be systematically smaller than what is theoretically expected on the basis of the model independent “$3(N-Z)$” sum rule. Much work has been devoted to the subject in the last fifteen years [1–4]. The heart of the problem can be summed up by defining the reduced transition probability as

$$B(\text{GT}) = \left( \frac{g_A}{g_V} \right)^2 \langle \sigma \tau \rangle^2, \quad \langle \sigma \tau \rangle = \frac{\langle f \mid \sum_k \sigma_k^+ t_k^+ \mid i \rangle}{\sqrt{2J_i + 1}},$$

(1)

and asking: Is the observed quenching due to a renormalization of the $g_A$ coupling constant —originating in non nucleonic effects—or is it the $\sigma \tau$ operator that should be renormalized because of nuclear correlations?

The analysis of some $pf$-shell nuclei for which very precise data are available and full $0\hbar\omega$ calculations are possible, strongly suggests that most of the theoretically expected strength has been observed [11]. The quenching factor necessary to bring into agreement the calculated and measured values is directly related to the amplitude of the $0\hbar\omega$ model space components in the exact wave functions. This normalization factor can also be obtained from $(d,p)$ or $(e,e')$ reactions and reflects the reduction in the discontinuity at the Fermi surface in a normal system. As such, it is a fundamental quantity, whose evolution with mass number is of interest.

In principle there are two ways of extracting it from Gamow Teller processes. One is to equate it to the fraction of strength seen in the resonance region in $(p,n)$ reactions. The alternative is to calculate lifetimes for individual $\beta$ decays and show that they correspond to the experimental values within a constant factor. The latter procedure is more precise, but demands high quality shell model calculations that until recently were available only up to $A = 40$ [12,13].

Our aim is to extend these analyses to the lower part of the $pf$ shell. Full $0\hbar\omega$ diagonalizations are done using the antoine code [14] with the effective interaction KB3, a minimally monopole modified version [12] of the original Kuo Brown matrix elements [13]. We refer to [14] for details of the shell model work.

Following ref. [14] we define quenching as follows: for beta decays populating well-defined isolated states in the daughter nucleus, the square root of the ratio of the experimental measured rate to the calculated rate in a full $0\hbar\omega$ calculation is called the quenching factor. An average quenching factor, $q$, implies an average over many transitions, and may be incorporated into an effective axial vector coupling constant:

$$q = \frac{g_{A,\text{eff}}}{g_A},$$

(2)

where $g_A$ is the free-nucleon value of $-1.2599(25)$ [14]. Following ref. [14] we define

$$M(\text{GT}) = [(2J_i + 1) B(\text{GT})]^{1/2},$$

(3)

so as to have quantities independent of the direction of the transition. Note here that our reduced matrix elements follow Racah’s convention [15]. In table 1 we list the $M(\text{GT})$ values and compare them with the experimental results. The table contain all the transitions known experimentally. We also include the quantum numbers of the final states, the $Q$-values, the branching ratios and the experimental log $ft$ values from which the $B(\text{GT})$ values were obtained using

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In order to obtain the effective values are systematically larger than the experimental ones. The theoretical ones for 
respectively \[16,7\].

A quick look to the table shows that the calculated values are systematically larger than the experimental ones. In order to obtain the effective \(g_A\), first we normalize the \(M(GT)\) to the “expected” total strength, \(W\) (listed in table \[1\]) and defined by

\[
W = \left\{ \begin{array}{ll}
|g_A/gV| \sqrt{(2J_i+1)|N_i - Z_i|} & \text{for } N_i \neq Z_i, \\
|g_A/gV| \sqrt{(2J_f+1)|N_f - Z_f|} & \text{for } N_i = Z_i.
\end{array} \right.
\]

In figure \[1\] are plotted the experimental values versus the theoretical ones for

\[R(GT) = M(GT)/W.\]

The points follow nicely a straight line whose slope gives the average quenching factor, \(q = 0.744 \pm 0.015\). Most \(R(GT)\) values are much smaller than 1, reflecting the fact the strength in the decay window is small and fragmented. As a consequence, each individual decay may be sensitive to small uncertainties in the calculations, which can be averaged out by summing the total strength for each nucleus. Therefore we introduce a new quantity

\[T(GT) = \left[ \sum_f R^2(GT, i \rightarrow f) \right]^{1/2}.\]

In the corresponding plot in figure \[2\] the points again follow closely the \(q = 0.744\) line.

Comparing with the results in other regions, is suggestive

- \(pf\) shell, \(q = 0.744 \pm 0.015\) this work,
- \(sd\) shell, \(q = 0.77 \pm 0.02\) \[3\],
- \(p\) shell, \(q = 0.82 \pm 0.015\) \[3\].

In the figures, both the lines for \(q = 0.744\), and \(q = 0.77\) are drawn and it is clear that there is not much to choose between them, and indeed, the average quenching factor of 0.77 has been extensively used in many \(pf\)-shell calculations, either in direct diagonalizations \[14\] or Shell Model Monte Carlo studies \[15\], leading to agreement with global Gamow-Teller strengths (as measured in \((n,p)\) and \((p,n)\) reactions) and lifetimes. Nevertheless, the results in the three regions point to a decrease of \(q\) with mass number, and the closeness of the \(sd\) and \(pf\) values suggests that we have reached the large-A regime. This observation is quite consistent with the numbers extracted by Osterfeld from \((p,n)\) data in heavier nuclei (see fig. 6 in \[3\]).

Whatever its origin, the \(q\) factor is a fundamental quantity telling us about correlations that are so well hidden, precisely behind overall renormalizations such as \(q\), that their existence may be doubted.

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FIG. 1. Comparison of the experimental matrix elements $R(GT)$ with the theoretical calculations based on the “free-nucleon” Gamow-Teller operator. Each transition is indicated by a point in the $x$-$y$ plane, with the theoretical value given by the $x$ coordinate of the point and the experimental value by the $y$ coordinate.

FIG. 2. Comparison of the experimental values of the sums $T(GT)$ with the corresponding theoretical value based on the “free-nucleon” Gamow-Teller operator. Each sum is indicated by a point in the $x$-$y$ plane, with the theoretical value given by the $x$ coordinate of the point and the experimental value by the $y$ coordinate.

TABLE I. Experimental and theoretical $M(GT)$ matrix elements. The experimental data have been taken from [19]. $I_\beta + I_\epsilon$ are the branching ratios. All other quantities explained in the text.

| Process | $2J_n^u, 2T_n^u$ | $Q$ (MeV) | $I_\beta + I_\epsilon$ (%) | $\log ft$ | $M(GT)$ | $W$ |
|---------|-----------------|-----------|---------------------------|-----------|---------|-----|
| $^{41}$Sc($\beta^+$)$^{41}$Ca | $^7, 1$ | 6.496 | 99.963(3) | 3.461(7) | 2.999 | 4.083 | 6.172 |
| $^{42}$Sc$^*(\beta^+)$$^{42}$Ca | $^{12^+, 2}$ | 3.851 | 100 | 4.17(2) | 2.497 | 3.389 | 11.127 |
| $^{42}$Ti($\beta^+$)$^{42}$Sc | $^{2^+, 0}$ | 6.392 | 55(14) | 3.17(12) | 2.038 | 2.736 | 3.086 |
| $^{43}$Sc($\beta^+$)$^{43}$Ca | $^{7^-, 3}$ | 2.221 | 77.5(7) | 5.03(2) | 0.677 | 0.764 | 6.172 |
| $^{44}$Sc($\beta^+$)$^{44}$Ca | $^{4^+, 4}$ | 0.998 | 1.04(4) | 5.15(3) | 0.466 | 0.205 | 6.901 |
| $^{45}$Ca($\beta^-$)$^{45}$Sc | $^{7^-, 3}$ | 0.258 | 99.9981 | 5.983(1) | 0.226 | 0.079 | 11.127 |
| $^{45}$Ti($\beta^+$)$^{45}$Sc | $^{7^-, 3}$ | 2.066 | 99.685(17) | 4.591(2) | 1.123 | 1.551 | 6.172 |
| $^{46}$Sc($\beta^-$)$^{46}$Ti | $^{5^-, 3}$ | 0.654 | 0.060(9) | 5.81(5) | 0.276 | 0.397 | 6.172 |
| $^{47}$Ca($\beta^-$)$^{47}$Sc | $^{7^-, 5}$ | 0.400 | 0.05(5) | 5.60(4) | 0.351 | 0.712 | 6.172 |
| $^{47}$Sc($\beta^-$)$^{47}$Ti | $^{7^-, 3}$ | 0.441 | 68.4(6) | 5.28(1) | 0.508 | 0.611 | 6.172 |
| Process | $2J_n^*-2T_n^*$ | $Q$ (MeV) | $I_\beta + I_e$ (%) | log $ft$ | $M(GT)$ | Exp. | Th. | W |
|---------|-----------------|-----------|---------------------|---------|--------|------|-----|---|
| $^{47}$V($\beta^+$)$^{47}$Ti | $5^+_1$, 3 | 2.928 | 99.552(15) | 4.901(5) | 0.555 | 0.896 | 4.365 |
| | $5^+_1$, 3 | 1.378 | 0.049(6) | 6.08(6) | 0.143 | 0.107 |
| | $5^+_1$, 3 | 1.1377 | 0.285(10) | 5.10(2) | 0.442 | 0.563 |
| | $5^+_1$, 3 | 0.765 | 0.071(3) | 5.36(2) | 0.327 | 0.514 |
| | $5^+_1$, 3 | 0.761 | 0.009(7) | 6.25(4) | 0.118 | 0.278 |
| | $5^+_1$, 3 | 0.402 | 0.0172(9) | 5.41(3) | 0.309 | 0.202 |
| | $5^+_1$, 3 | 0.379 | 0.0067(5) | 5.77(4) | 0.204 | 0.204 |
| | $5^+_1$, 3 | 0.134 | 0.0021(6) | 5.18(9) | 0.403 | 0.780 |
| $^{47}$Cr($\beta^+$)$^{47}$V | $3^-$, 1 | 7.451 | 96.1(13) | 3.70(2) | 0.942 | 1.186 | 4.365 |
| | $3^-$, 1 | 7.363 | 3.9(13) | 5.1(2) | 0.442 | 0.646 |
| $^{48}$Sc($\beta^-$)$^{48}$Ti | $12^-$, 4 | 0.661 | 90.0(3) | 5.532(13) | 0.484 | 0.780 | 22.256 |
| | $12^-$, 4 | 0.485 | 9.85(9) | 6.010(17) | 0.279 | 0.331 |
| $^{48}$V($\beta^+$)$^{48}$Ti | $8^+_1$, 4 | 1.719 | 89.0(9) | 6.175(7) | 0.192 | 0.345 | 9.259 |
| | $6^+$, 4 | 0.791 | 3.33(7) | 6.565(10) | 0.123 | 0.090 |
| | $8^+_1$, 4 | 0.775 | 7.76(9) | 6.180(6) | 0.191 | 0.181 |
| $^{48}$Cr(EC)$^{48}$V | $2^+$, 2 | 1.233 | 100 | 4.294(7) | 0.559 | 0.709 | 5.346 |
| $^{48}$Mn($\beta^+$)$^{48}$Cr | $8^+_1$, 0 | 11.670 | 6.5(25) | 5.4(2) | 0.469 | 0.527 | 9.259 |

**TABLE I. Continuation.**

| Process | $2J_n^*-2T_n^*$ | $Q$ (MeV) | $I_\beta + I_e$ (%) | log $ft$ | $M(GT)$ | Exp. | Th. | W |
|---------|-----------------|-----------|---------------------|---------|--------|------|-----|---|
| $^{49}$Ca($\beta^-$)$^{49}$Sc | $3^-$, 7 | 2.178 | 91.5(7) | 5.075(4) | 0.455 | 1.007 | 13.093 |
| | $5^+_1$, 7 | 1.190 | 7.0(7) | 5.12(5) | 0.392 | 0.209 |
| | $1^-$, 7 | 0.869 | 0.66(7) | 5.42(5) | 0.306 | 0.757 |
| | $5^+_1$, 7 | 0.524 | 0.21(6) | 5.3(2) | 0.351 | 0.591 |
| $^{49}$Sc($\beta^-$)$^{49}$Ti | $7^-$, 5 | 1.994 | 99.94(1) | 5.71(1) | 0.309 | 0.469 | 16.331 |
| | $9^-$, 5 | 0.371 | 0.010(3) | 6.9(2) | 0.079 | 0.072 |
| | $5^-$, 5 | 0.232 | 0.05(1) | 5.6(1) | 0.351 | 0.389 |
| $^{49}$V(EC)$^{49}$Ti | $7^-$, 5 | 0.602 | 100 | 6.2(1) | 0.176 | 0.130 | 10.691 |
| $^{49}$Cr($\beta^+$)$^{49}$V | $7^-$, 3 | 2.631 | 12(2) | 5.6(1) | 0.304 | 0.335 | 5.346 |
| | $5^+_1$, 3 | 2.540 | 37(2) | 5.02(2) | 0.593 | 0.817 |
| | $3^-$, 3 | 2.478 | 50(2) | 4.81(2) | 0.755 | 1.033 |
| | $5^+_1$, 3 | 1.116 | 0.081(9) | 5.80(4) | 0.242 | 0.312 |
| | $3^+_2$, 3 | 0.969 | 0.028(6) | 6.15(8) | 0.161 | 0.182 |
| | $5^+_1$, 3 | 0.396 | 0.0011(2) | 6.75(7) | 0.081 | 0.264 |
| | $3^+_2$, 3 | 0.322 | 1.9(7) 10^-4 | 7.3(2) | 0.043 | 0.195 |
| $^{49}$Mn($\beta^+$)$^{49}$Cr | $5^-$, 1 | 7.715 | 93.6(26) | 3.67(3) | 1.364 | 1.704 | 5.346 |
| | $7^-$, 1 | 7.443 | 6.4(26) | 4.8(2) | 0.764 | 0.623 |
| $^{50}$Ca($\beta^-$)$^{50}$Sc | $2^+$, 8 | 3.118 | 99.0(13) | 4.14(2) | 0.667 | 0.956 | 6.901 |
| | $8^+$, 6 | 4.213 | 8.4(18) | 6.7(1) | 0.116 | 0.208 | 20.471 |
| $^{50}$Sc($\beta^-$)$^{50}$Ti | $12^+$, 8 | 3.689 | 88.4(15) | 5.39(1) | 0.525 | 0.572 |
| | $8^+$, 8 | 2.741 | 0.58(4) | 7.01(4) | 0.081 | 0.125 |
| | $10^+$, 8 | 2.007 | 1.58(5) | 5.99(2) | 0.263 | 0.358 |