Simulation method of milling vibration based on cutting force segmentation model and Simulink

Hua Li¹, Weijie Du¹ and Jianxin Han²

¹Tianjin Jinhang Institute of Technical Physical, Tianjin 300300, China
²Tianjin University of Technology and Education, Tianjin 300222, China

Abstract. Modeling and Simulation of vibration of cutting process is the main way to control milling chatter. In this paper, an integral model of cutting force for milling process with helical cutter considering the sectional characteristics of cutting force is established. Based on the model, the dynamic analysis module of thin-walled milling process including the judgment module, signal module and definite integration module is built by using Simulink of MATLAB, which. The module can realize fast simulation in different cutting parameters and tool geometry conditions. This research can provide important reference for simulation analysis of cutting process.

1. Introduction
High material removal rates often cannot be achieved in practice due to the inherent instability of a cutting process known as chatter. The most powerful source of chatter is regeneration. The approach to gain stability lobe diagram (SLD) is the most established method for predicting and avoiding regenerative chatter. Some methods focused on disrupting the regenerative effect to enlarge the stable zone of the SLD. Refs. [1-4] adapted an analytical milling stability model for variable pitch cutters. Recently, in order to consider the cutter runout or variable pitch angle, a unified SD method [5] and an improved FDM [6] have been proposed successively for predicting the stability lobes of milling process with multiple delays. Jin et al. [7] used improved SD algorithm examine the effect of the tool geometries on the stability trends for different cutters.

The focus of the current manuscript is to propose a program which can consider the sectional characteristics of helical milling cutter to research the stability trend based on Simulink. The work of this paper is organized as follows. Firstly, an integral model of cutting force is proposed. Then, a program based on Simulink of MATLAB is conducted. The feasibility of the calculation model is analyzed. The final section describes the conclusions from this work.

2. Cutting force modeling
For milling of thin-walled parts, the stiffness in the direction of workpiece thickness is relatively small, if this direction is y, the dynamic equation of the workpiece is

\[ M \ddot{y}(t) + C \dot{y}(t) + K y(t) = F(t) \]  

(1)

where, \( M \) is modal mass, \( C \) is modal damping, \( K \) is modal stiffness. These parameters can be obtained by hammering experiment. \( F(t) \) is cutting force.
Figure 1. Micro-element cutting force of helical milling cutter.

The sectional characteristics of helical milling cutters have a transitional effect in the process of cutting in or out of workpiece. When one cutting occurs, cutting depth is increasing from 0 to the maximum value. However, cutting depth is decreasing from the maximum value to zero in the process of out of cutting, this can avoid the sudden change of cutting force and thus leads to relatively stable processing. As shown in figure 1, the rotation angles corresponding to the different cutting heights for the same cutter teeth are different due to the influence of the helical angle. The rotation angles of the j-th cutter teeth at cutting heights z are as follows

$$\phi_j(t, z) = \theta t + \frac{2\pi}{N} (j-1) - z \frac{\tan \beta}{R}$$

where $\beta$ is the helix angle of the tool. If a micro element $dz$ is chosen at any height $z$ on the j-th cutter tooth, the cutting force $dF$ on the micro element can be expressed as

$$dF_j(t, z) = dF_y \sin \phi_j(t, z) - dF_x \cos \phi_j(t, z)$$

where tangential and radial cutting forces can be derived by

$$dF_y(t, z) = k_f \sin \phi_j(t, z) + (y(t) - y(t-\tau)) \cos \phi_j(t, z) \, dz + k_c \, dz$$

$$dF_x(t, z) = k_f \sin \phi_j(t, z) + (y(t) - y(t-\tau)) \cos \phi_j(t, z) \, dz + k_c \, dz$$

Because the value of $\phi$ in Z direction for a helical milling cutter is changing on linearly, its integrating upper and lower limits is different. Thus, it is converted into angle integral here. Derivating of equation (2) with respect to $z$, $dz = -R / \tan \beta \, d\phi$ can be obtained. Bring $dz$ and equation (3) into equation (4), one can gain

$$F(t) = \sum_{j=1}^{N_x} G(\phi_j(t, z)) \frac{R}{\tan \beta} \int_{\phi_j(t, 0)}^{\phi_j(t, \tau)} (K_1(t)(y(t) - y(t-\tau)) + K_2(t)) \, d\phi$$

where $K_1(t)$ and $K_2(t)$ respectively represent dynamic and static cutting coefficients, which are

$$K_1(t) = k_c s - k_c c^2$$

$$K_2(t) = k_f s^2 + k_c s - k_f s c - k_c c$$
Compared with straight-tooth milling cutter, helical milling cutter reduces the discontinuity of milling process, but it can not change the cutting nature of discontinuity. Figure 2 shows the cutting process for a helix cutter, in which the green rectangular thin line is the projection of the contact surface between the workpiece and the cutter, and the black thick solid line represents the milling cutter. Obviously, the helical milling cutter has a longer cutting time than the straight milling cutter in the cutting state $\phi_{\text{lag}}$ ($\phi_{\text{lag}} = a_p \tan \beta / R$), the cutting judgment function is as follows.

$$G(t) = \begin{cases} 1, & \text{if } \phi_s(t) < \phi_j(t) < \phi_e + \phi_s \\ 0, & \text{otherwise} \end{cases}$$  (7)

If the integral interval is defined as $C$, named as the lower angle $\phi_{\text{j}}(t)$ and the upper angle $\phi_{\text{e}}(t)$, one can be obtained

$$C = \begin{cases} \phi_{\text{j}}(t), & \phi_{\text{j}}(t) = \max(\phi_j(t,a_p),\phi_s) \\ \phi_{\text{e}}(t), & \phi_{\text{e}}(t) = \min(\phi_j(t,0),\phi_s) \end{cases}$$  (8)

### 3. Simulation of milling vibration based on Simulink and cutting force subsection model

#### 3.1 Simulation master system

Figure 3 shows the main system diagram of milling Simulink simulation. For the convenience of introduction, all the modules are numbered. The module in the black box can realize the control of the number of cutter teeth. If the number of teeth needs to be increased, an execution module or a judgment module must be added based on the judgment condition 10. Numbers 2, 3, 6, 9 in figure 4 are complete judgment modules, in which number 2 is the judgment module, numbers 3, 6, 9 are the execution modules, numbers 4, 5, 7, 8 are the control data of modules 3 and 6 respectively. Numbers 22-25 are modules from the first to the fourth cutter, which contains two subsystems to simulate their milling displacement and cutting force signals respectively. Numbers 34 and 35 are used to simulate the milling vibration and cutting force signals of the main system. The other modules are all operation ones.
3.2 Integral judgment module

Before the calculation of definite integral, the process judgment is conducting firstly. Then, the integral boundary and the corresponding milling state whether it is in the process of cutting need to determine. As shown in figure 4, the subsystem of judgment module, number 16 can calculate the rotation angle and the milling area. Number 1 is the input module, where the periodic signals of rotation angle between \([0, 2\pi]\) need to input, and number 2 can input the values of time. Number 8 is the judgment condition, if the rotation angle satisfies the judgment condition, number 11 starts to executive, number 9 outputs 1, this means cutting state; if not, number 12 starts to executive, number 10 outputs 0, this means non-cutting state. Then, the milling region of the whole section is obtained by combining the values of above two sections. Finally, above all messages are displayed by module 14 and output by module 15.

Figure 4. Judgment module subsystem diagram

Figures 5 shows the results of the lower and upper limit angles used the theoretical derivation of part 1 in the cutting process under the condition as: tool radius 9.525mm, the number of teeth 3, cutter helix angle 30°, radial depth ratio 5.25 %, axial depth of cutting 4mm and feed per tooth 1.78*10^{-4} m/s.
Figure 6 shows the corresponding results by Simulink. Obviously, their results have the same changing trend about the lower and upper limit angles. Obviously, the Simulink model has good ability to capture the character of cutting force segmentation caused by the helix angle.

Figure 7 shows the simulation results of the cutting force time history and milling vibration displacement based on Simulink in the same condition above. According to the law of signal evolution, the cutting process is stable.

**4. Conclusion**

In this paper, an integral model of cutting force for milling process with helical cutter considering the sectional characteristics of cutting force is established. Based on the model, the dynamic analysis of thin-walled milling process considering the sectional characteristics of helical milling cutter is analyzed by one program which is built base on Simulink of MATLAB. This program includes the judgment module, signal module and definite integration module. Theoretical calculation and simulation result show that the Simulink model has good ability to capture the character of cutting force segmentation caused by the helix angle, and can realize fast simulation in different cutting parameters and tool geometry conditions. This research can provide important reference for simulation analysis of cutting process.
Figure 7. Cutting force signal (left) and milling vibration displacement signal (right)

Acknowledgments
This work is supported by Innovation Team Training Plan of Tianjin Universities and colleges (No. TD13-5096), Tianjin Natural Science Foundation (No. 18JCQNJC05123), Key Projects in the Tianjin Science and Technology Pillar Program (No. 17ZXZNGX00080).

Reference
[1] Slavicek J 1965 In: Proceedings of the 6th MTDR Conference, London, Pergamon Press.
[2] Altintas Y, Engin S and Budak E 1999 *J. Manuf. Sci. Eng* **121** 173-178.
[3] Budak E 2003 *J. Manuf. Sci. Eng* **125** 29-34.
[4] Budak E 2003 *J. Manuf. Sci. Eng* **125** 35-38.
[5] Wan M, Zhang WH, Dang JW, et al 2010 *Int. J. Mach. Tools Manuf* **50** 29-41.
[6] Ding Y, Zhu L, Zhang X and Ding H 2010 *Int. J. Mach. Tools. Manuf* **50** 502–509.
[7] Jin G, Qi HJ, Li ZJ and Han JX 2018 *Commun. Nonlinear. Sci* **63** 38-56.