Membranes on Calibrations

Chethan KRISHNAN\footnote{Chethan.Krishnan@ulb.ac.be} and Carlo MACCAFERRI\footnote{cmaccafe@ulb.ac.be}

Physique Théorique et Mathématique, International Solvay Institutes, Université Libre de Bruxelles, ULB Campus Plaine C.P. 231, B–1050 Bruxelles, Belgium

M2-branes can blow up into BPS funnels that end on calibrated intersections of M5-branes. In this quick note, we make the observation that the constraints required for the consistency of these solutions are automatic in Bagger-Lambert-Gustavsson (BLG) theory, thanks to the fundamental identity and the supersymmetry of the calibration. We use this to explain how the previous ad hoc fuzzy funnel constructions emerge in this picture, and make some comments about the role of the 3-algebra trace form in the derivation.

KEYWORDS: Extended Supersymmetry, M-Theory
1 Introduction

The difficulty with M-theory is that essentially all the things we know about it are indirect. We know that its low energy limit is 11 dimensional supergravity, we know some of its dual string theories under various compactifications, and we know that its solitonic extended objects are membranes and five-branes. But unlike in the case of string theory D-branes, the lack of a worldsheet description prevents us from explicitly constructing the low energy field theory living on these branes.

But indirect arguments can sometimes go a long way. We can construct (say) membrane solutions in the 11 D supergravity and try to deduce the properties of the M2-worldvolume theory by investigating the SUGRA solution. First of all, since the brane breaks only half of the 32 supersymmetries of 11 D supergravity, we expect that the theory has 16 supersymmetries. Next, we can take a near horizon limit which corresponds to looking at the low energy theory on the worldvolume. The result of the near-horizon limit is a $AdS_4 \times S^7$ background. The isometries of $AdS_4$ correspond precisely to the conformal group in 2+1 dimensions, and the isometries of $S^7$ should give rise to an $SO(8)$ R-symmetry. In short, we expect that the theory on the worldvolume of M2-branes is a maximally supersymmetric superconformal theory in 2+1 dimensions with an $SO(8)$ R-symmetry.

But for a while, nobody knew how to build such a theory and in fact it was shown that conventional approaches were doomed to fail [2]. The situation changed recently, when Bagger & Lambert [3] and Gustavsson [4] managed to construct such a theory using ideas inspired by non-associative algebras. This theory is the first ingredient in our work.

The second ingredient consists of BPS funnels constructed using M-branes. Fuzzy funnels are static configurations were one set of branes expands into another. In the case of string theory, such solutions have been written down explicitly for the case where D1-branes expand into a single D3-brane (BIon) [5] and into multiple intersecting D3-branes [6]. This latter case can be interpreted also as D1-branes ending on a calibrated cycle wrapped by the D3 [7]. In all these cases the configurations are solutions of the BPS equations (also called Nahm equations) for the low energy theory on the D1-branes which is two dimensional maximally supersymmetric Yang-Mills with $U(N)$ gauge group.

The generalization of the BIon to the case of the M-theory branes was done first by Basu and Harvey [8]. They wrote down an equation which was an ad hoc generalization of the BPS equation, without an underlying supersymmetric action to start with. But it captured the expected features of an M2-brane expanding into an M5. This construction was instrumental in the recent progress in M-brane theory. The Basu-Harvey equation was generalized to the case of multiple intersecting M5s by Berman and Copland in [9]. They found that for general calibrations the configuration has to satisfy some additional constraints (invisible in the Basu-Harvey case) if the BPS equation had to be compatible with the equation of motion.

When the Bagger-Lambert-Gustavsson theory came along, one of its interesting aspects was that it could reproduce the Basu-Harvey equation directly as a BPS equation. The question we address in this paper is how the more general BPS funnels of [9] fit into the theory. In fact, we will see that the analogues of the constraints found in [9] for general calibrations are automatic in the BLG theory because of the structure of the 3-algebra and supersymmetry. Therefore, the previous fuzzy funnel solutions can be constructed in the

\[^3\text{See [1] for an exception to this.}\]
BLG framework by relating the bracket structures. We also make some comments about
the conditions on the trace form of the 3-algebra if we want to derive the BPS equations
by following a Bogomol’nyi ansatz on the energy functional. In particular, we discuss the
Lorentzian trace forms discussed recently in the literature [18].

Recent work on M2-branes include investigations of their moduli spaces [10], the relation
of M2 branes to D2 branes [11], uniqueness theorems of the 3-algebra [12], and explanations
for the uniqueness by looking at the boundary of the open membrane version of the theory
[13]. Related references are [14]. Useful reviews on the pre-BLG state of the art on M-branes
and their interactions are [15].

2 M2-brane Field Theory

The Lagrangian for Bagger-Lambert-Gustavsson theory is written using the “structure con-
stants” $f_{abcd}$ of a 3-algebra. For the moment, they can be thought of as coeffic ients that dictate
the couplings between the internal components of various fields. With this understanding the
Lagrangian is written as

$$L = -\frac{1}{2} D^\mu X^a I D_\mu X^a I + \frac{i}{2} \bar{\Psi}^a \Gamma^\mu D_\mu \Psi_a + \frac{i}{4} \bar{\Psi}_b \Gamma_{I J} X^a I X^b I X^c I \Psi_a f^{abcd}$$

$$-V(X) + \frac{1}{2} \varepsilon^{\mu \nu \lambda} \left( f^{abcd} A_{\mu ab} \partial_\nu A_{\lambda cd} + \frac{2}{3} f^{cda} g f^{efg} A_{\mu ab} A_{\nu cd} A_{\lambda ef} \right).$$

(1)

Upper case Latin indices denote the transverse directions of the M2-brane ($I, J, K = 3, ..., 10$),
and lower case denotes internal indices in the 3-algebra. Greek indices denote the worldvol-
ume directions ($\mu, \nu, \lambda = 0, 1, 2$). It is useful to think of the Gamma matrices as those of
11 dimensions, constructed as tensor products of $SO(2,1)$ and $SO(8)$ Gamma matrices. The
scalar potential takes the explicit form $V(X) = \frac{1}{12} f_{abcd} f^{efg} X^a I X^b I X^c I X^d I X^e I X^f I X^g I$. The covariant derivative is $D_\mu X^I = \partial_\mu X^I - \hat{A}_\mu c d X^c I$, where $\hat{A}_\mu c d = A_{\mu ab} f^{abc d}$. The gauge field ensures
that the supersymmetry algebra can be closed, but it has no propagating degrees of freedom.
This is as it should be because supersymmetry forbids more bosons. On shell, the Lagrangian
above is closed under the following SUSY variations

$$\delta X^I_a = i \bar{\epsilon} \Gamma^I \Psi_a$$

$$\delta \Psi_a = D_\mu X^I_a \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X^I_a X^J_b X^K_c f^{abcd} f^{efg} A_{\mu ab} A_{\nu cd} A_{\lambda ef} + f^{efb} g f^{fcd} + f^{efc} g f^{fda}.$$  

(2)

if the structure constants satisfy the so-called fundamental identity: $f^{efg} f^{abcd} g = f^{efg} f^{abcd}$ +
$f^{efb} g f^{fcd} + f^{efc} g f^{fda}$. In the above, $\epsilon$ is the 32 component real spinor of 11 dimensions, which
satisfies the condition $\Gamma_{012} \epsilon = \epsilon$ because the other 16 supersymmetries are broken by the M2s.
This chirality condition picks out only the $\Gamma_{012} \Psi = - \Psi$ components in the superpartner
fermions. The equations of motion that follow from the closure of the SUSY variations (and
also from the action) are,

\[ \Gamma^\mu D_\mu \Psi_a + \frac{1}{2} \Gamma^I J X^I_c X^J_d \Psi_{fcd} \equiv 0 \]

\[ D^2 X^I_a - \frac{i}{2} \bar{\Psi}_c \Gamma^I J X^J_d \Psi_{fcd} - \frac{\partial V}{\partial X^I_a} = 0 \] (3)

\[ \tilde{F}_{\mu \nu}^{\, b} a + \varepsilon_{\mu \nu \lambda} (X^J_c D^\lambda X^J_d + \frac{i}{2} \bar{\Psi}_c \Gamma^\lambda \Psi_d) f^{cfd} a = 0. \]

with the gauge field strength \( \tilde{F}_{\mu \nu}^{\, b} a = \partial_\nu \tilde{A}_{\mu}^{\, b} a - \partial_\mu \tilde{A}_{\nu}^{\, b} a - \tilde{A}_{\mu}^{\, b} c \tilde{A}_{\nu}^{\, c} a + \tilde{A}_{\nu}^{\, b} c \tilde{A}_{\mu}^{\, c} a. \)

When investigating fuzzy funnel solutions we will not be interested in gauge fields and fermions. In this case it is often instructive to write the action in terms of the 3-algebra directly instead of using explicit internal components:

\[ L_B = -\frac{1}{2} \text{Tr} \left( \partial_\mu X^I, \partial^\mu X^I \right) - \frac{1}{12} \text{Tr} \left( [X^I, X^J, X^K], [X^I, X^J, X^K] \right) \] (4)

This is to be understood in terms of generators \( T^a \) \((a = 1, ..., N)\) such that \( X^I \equiv X^I_a T^a. \) These generators satisfy \([T^a, T^b, T^c] = f^{abc}_d T^d\), and have a trace form \( h^{ab} = \text{Tr}(T^a, T^b). \) The trace form can be used to raise indices in \( f^{abcd}. \) In fact to write down the above action (4), we have implicitly assumed the existence of such a trace form and assumed also that the \( f^{abcd} \) constructed that way is fully antisymmetric in all indices. In terms of the generators, this anti-symmetry condition means that \( \text{Tr}(T^a, [T^b, T^c, T^d]) = -\text{Tr}([T^a, T^b, T^c], T^d). \) It should be emphasized that if we forget about the action and merely want to close the supersymmetry variations, we only need the fundamental identity and neither the existence of the trace form nor the full antisymmetry. For future use, we also write down the fundamental identity in terms of the 3-algebra generators:

\[ [T^a, T^b, [T^c, T^d, T^e]] = [[T^a, T^b, T^c], T^d, T^e] + [T^c, [T^a, T^b, T^d], T^e] \]

\[ + [T^c, T^d, [T^a, T^b, T^e]], \] (5)

\[ [T^a, T^b, T^c, T^d, T^e]] \]

\[ \text{3 Nahm Equations and Calibrations} \]

We now proceed to construct the BPS equations for fuzzy funnels in Bagger-Lambert-Gustavsson theory following a Bogomol’nyi ansatz. Similar constructions are well known in the literature \( \text{[9]} \), the point here is only that we would like to work exclusively with the 3-algebra structure.

We are interested in static solutions, so we start with the Hamiltonian for (4):

\[ E = \frac{1}{2} \int d^2 \sigma \left( \text{Tr}(\partial_\sigma X^I, \partial_\sigma X^I) + \frac{1}{3!} \text{Tr}([X^I, X^J, X^K], [X^I, X^J, X^K]) \right) \] (6)

where \( \sigma_2 \) can be gauge-fixed to be \( X^2. \) The aim is to write this as

\[ E = \frac{1}{2} \int d^2 \sigma \left( \text{Tr} \left( \partial_\sigma X^I - \frac{g_{IJKL}}{3!} [X^J, X^K, X^L] \right)^2 + T \right) \] (7)
where $T$ is a total derivative, so that we can identify the perfect square piece with a BPS equation. By direct computation, it turns out that in order to do this we need the constraint

\[
\frac{1}{3!} g_{IJKL} g_{IPQR} \text{Tr}([X^J, X^K, X^L], [X^P, X^Q, X^R]) = \text{Tr}([X^I, X^J, X^K], [X^I, X^J, X^K])
\]

(8)
to be satisfied. If this holds, then we can conclude that the BPS configurations of the theory are given by

\[
\partial_\sigma X^I - \frac{g_{IJKL}}{3!}[X^I, X^K, X^L] = 0.
\]

(9)

These equations are the Bagger-Lambert-Gustavsson version of the (generalized) Nahm equations. The coefficients $g_{IJKL}$ are determined by the specific calibration on which the the M2-branes end. Notice that the conventional Nahm equations are written in terms of matrix commutators [6], but here the brackets stand for the 3-product of the 3-algebra.

The coefficients $g_{IJKL}$ characterize the specific BPS solution under consideration. In practice this corresponds to having M-brane configurations that leave some of the supersymmetry unbroken. For BLG theory, from [2], setting $\delta \Psi = \partial_\mu X^I \Gamma^I \epsilon - \frac{1}{6} [X^I, X^J, X^K] \Gamma^{IJK} \epsilon$ to zero results in

\[
\sum_{I<J<K} [X^I, X^J, X^K] \Gamma^{IJK} (1 - g_{IJKL} \Gamma^{IJKL}) \epsilon = 0,
\]

(10)

This can be solved as in [6, 9] by defining projectors. We will not repeat the details, the end result is that once we have a set of mutually commuting set of supersymmetries, we can rewrite the above equation in the form

\[
\sum_{g_{IJKL}=0} [X^I, X^J, X^K] \Gamma^{IJK} \epsilon = 0.
\]

(11)

The sum is over $I, J, K$ such that $g_{IJKL} = 0$. This can be re-expressed as a set of conditions to be satisfied by the 3-brackets.

Now we will show that the supersymmetry of the calibration, together with the fundamental identity is enough to show that the constraint equations are satisfied in Bagger-Lambert-Gustavsson theory. Together with the identification of the 3-algebra structure with the Nambu 4-bracket proposed in [8, 9], this will imply that the fuzzy funnels on these calibrations are naturally thought of as solutions of BLG. The $g_{IJKL}$ have a direct correspondence to the calibrating forms of the cycle on which the M5 wraps, and therefore are fully antisymmetric in their indices. When $g_{IJKL} = \epsilon_{IJKL}$, we end up with Basu-Harvey and the constraints are identities. For other calibrations $g_{IJKL}$ considered in [9] with more $X^I$ active, things are a bit more complicated. But it turns out that for each of them, after using the conditions arising from supersymmetry [11], we can write the constraints [8] in terms of the 3-algebra as

\[
[[X^L, X^I, X^J], X^K, X^L] = 0,
\]

(12)

\[
[[X^M, X^I, X^J], X^K, X^L] = 0.
\]

(13)

\footnote{Note that this assumes implicitly that the trace form in the 3-algebra is positive definite. This means that we are working with the 3-algebra $A_4$ with structure constants $\epsilon^{abcd}$ in this section [12].}
Here \( \{ \cdot, \cdot \} \) stands for antisymmetrization, and the activated \( I, J, K, L \) depend crucially on the chosen calibration. But the general structure is all we need to demonstrate our point. The first of these relations follows immediately upon setting \( a = e \) (or equivalently, symmetrizing in those indices) in (5). After a bit of massaging, the second relation can also be shown to hold due to the fundamental identity. Some useful relations are collected in the Appendix.

There is another constraint that we need to check for full consistency of the solution. This arises from the BLG equations of motion (3). With the gauge field and the fermion set to zero, the remaining constraint comes from the \( A_a^{\mu} \) equation of motion:

\[
X_a^I \partial_\sigma X_b^J f^{abc}_{\quad d} = 0.
\]

(14)

This can be reinterpreted as the statement that the 3-bracket \( [X^I, \partial_\sigma X^J, Z] \) vanish for arbitrary \( Z \). If we use the generalized Nahm equation (9) to rewrite \( \partial_\sigma X^I \), this becomes

\[
g_{IJKL}[X^I, [X^J, X^K, X^L], Z] = 0.
\]

(15)

But because of the antisymmetry of \( g_{IJKL} \), this is nothing but the (19) form of the fundamental identity.

### 3.1 Relation to Fuzzy 3-spheres

What we have shown is that the BLG theory can in principle have BPS solutions corresponding to various calibrations. Now we show how the 3-algebra structure incorporates the known fuzzy 3-sphere solutions. For a given calibration \( g_{IJKL} \), we get a solution to (9) if we can solve the 3-algebra equation

\[
\frac{1}{6} g_{IJKL} [A^I, A^K, A^L] = A^I,
\]

(16)

because then \( X^I(\sigma) = f(\sigma) A^I \) is a solution to (9) for \( f(\sigma) \) satisfying \( \partial_\sigma f(\sigma) = f^3(\sigma) \). If we imagine that the M5-branes are at \( X^2(= \sigma) = 0 \), then \( f(\sigma) = i \sqrt{2} \sigma \).

Now we show that the previous solutions \([8, 9]\) can all be seen as specific fuzzy 3-sphere realizations of the 3-algebra equation (16) for the case when the 3-algebra is \( A_4 \). We first notice from earlier work that the fuzzy 3-sphere coordinates defined in terms of matrices \( G^a(a = 1, 2, 3, 4) \) furnish a representation of the 3-algebra \( A_4 \) if one takes the 3-bracket to be defined by (upto some constant scalings which we ignore)

\[
[G^a, G^b, G^c] \equiv [G^*, G^a, G^b, G^c] \sim \epsilon^{abcd} G^d
\]

(17)

Here the 4-bracket operation is the one used by \([8, 9]\) and is defined through ordinary matrix multiplication as the Nambu 4-bracket: \( [[A_1, A_2, A_3, A_4]] = \sum_{\text{permutations}} \text{sgn}(\sigma) A_{\sigma_1} A_{\sigma_2} A_{\sigma_3} A_{\sigma_4} \). Once we have such an explicit representation of \( A_4 \), we can use it as a building block, and form various direct sums of these matrices to construct solutions of (16) and this is the standard recipe of \([8]\). Each block gives rise to a single copy of the fuzzy 3-sphere funnel. So the full configuration corresponds to many expanding fuzzy 3-spheres each giving rise to an M5-brane: the fact that the 3-spheres intersect results in the calibrated intersection of M5s.

---

5See \([16]\) for the details of the construction and the definitions of the matrices \( G^*, G^a \).
Before we end this section, we mention that the relevance of the dimensionality of the fuzzy 3-sphere representation here has to be better understood. Naively, if one would translate the BIon intuition for fuzzy 2-spheres to the M2-M5 system, one would expect that the sum of the dimensions of the fuzzy 3-sphere representations in each block should give rise to the total number of M2 branes. But since BLG theory for the 3-algebra $A_4$ has no free parameters (apart from the level of the Chern-Simons piece) analogous to the rank of the gauge group, this interpretation is far from obvious. In particular, the work of $[10, 11]$ seems to suggest that the theory corresponds to two M2 branes. The fuzzy funnel picture and this picture seem to be at odds. Maybe the mystery can be resolved by a better understanding of the algebraic structures on M2-branes and their representation theory. A reference which tries to address the issue of number of degrees of freedom of fuzzy 3-spheres in this context is $[17]$.

4 Signature of the Trace Form

The assumption that we made during the Bogomol’nyi construction, namely that the trace is positive definite in the 3-algebra is an incredibly strong one. In fact, it is shown in $[12]$ that the only positive definite finite-dimensional 3-algebra with fully anti-symmetric structure constants of this form is $A_4$ (and its disconnected copies). Fortunately, as we saw in the last section, this was enough to incorporate all the M2-brane funnel solutions in the literature.

It turns out that the constraint relations (8) can be obtained without relying on the Bogomol’nyi trick. The BPS equations arise from setting the SUSY variation $\delta \Psi = 0$. Therefore an equation linear in derivatives which preserves some combination of the supersymmetries has to take the form (9). One can convince oneself that this is the case by looking at specific configurations of branes (like the ones considered in $[9]$) that leave specific supersymmetries unbroken and imposing the resulting relations on $\delta \Psi = 0$. Once we accept that the BPS conditions are given by (9), one has to check that they are consistent with the equations of motion. From (3), we see that the relevant EOM takes the form $\partial_2 X^I_a - \frac{\partial V}{\partial X^I_a} = 0$. Taking a derivative of the BPS equation, getting rid of the derivatives, and finally taking a trace, one ends up precisely with the constraint relation (8). In this derivation we never needed to use the positivity of the norm.

But of course, the constraints are not the whole story. Lets restrict our attention to classical, static solutions. If we demand that a state is BPS, then it means that it satisfies $Q|\psi\rangle = 0$ for some supercharge. This in turn means that $\langle \psi|\{Q, Q\}|\psi\rangle \sim \langle \psi|H|\psi\rangle = 0$ in any positive definite Hilbert space. The energy of any state in a supersymmetric theory is non-negative, but it seems like the Hamiltonian written earlier (6) is unbounded below if the trace is not positive definite. So the theory is pathological, at least classically.

Recently, many more solutions to the fundamental identity were constructed with a norm that was Lorentzian $[18]$, by simply augmenting the structure constants of classical Lie algebras. Interestingly, these theories do not suffer from the problem we mentioned above. The

$^6$If we believe the picture that M2-branes have to end on M5-brane configurations, the 3-algebra seems forced to be $A_4$. One way to see this is to look at the target space of the boundary self-dual string of the BLG theory with boundary. One sees that it is 5+1 dimensional only if the 3-algebra structure constants are those of $A_4$ $[15]$. But perhaps the generic non-positive 3-algebras correspond to something more exotic.
scalar part of the Hamiltonian density for these theories can be written as

\[ \mathcal{H} = \frac{1}{2} \text{Tr} \left( \partial_\sigma X^I \partial_\sigma X^I \right) - \partial_\sigma X^I \partial_\sigma X^I + \frac{1}{12} \text{Tr} \left( X_+^I [X^J, X^K] + X_+^J [X^K, X^I] + X_+^K [X^I, X^J] \right)^2 \]  

where the troublesome negative norm direction has been split off in a “light-cone” notation. The interesting point is that after an integration by parts, \( X_+^I \) shows up only as a Lagrange multiplier enforcing the constraint \( \partial^2 \sigma X_+^I = 0 \). If we solve this with \( X_+^I \sim \sqrt{\lambda} \), where \( \lambda \) is a positive parameter, then the Hamiltonian is schematically that of a \( \lambda \phi^4 \) theory. Since \( \lambda \sim (X_+^I)^2 \), we have a positive definite Hamiltonian. It would be interesting to explore these theories along the lines considered here.

**Acknowledgments**

It is a pleasure to thank David Berman for a set of lectures at KU Leuven that inspired us to think about M-theory branes and also for encouragement and advice through emails during the initial stages of this project. We also thank Giuseppe Milanesi for discussions. This work is supported in part by IISN - Belgium (convention 4.4505.86), by the Belgian National Lottery, by the European Commission FP6 RTN programme MRTN-CT-2004-005104 in which the authors are associated with V. U. Brussel, and by the Belgian Federal Science Policy Office through the Interuniversity Attraction Pole P5/27.

**Appendix**

Some ways to rewrite the fundamental identity are collected here,

\[ f^{abc} f^{efg} d = 0, \]  \hspace{1cm} (19)

\[ f^{abc} f^{efg} d = 3 f^{efg} f^{abc} d. \]  \hspace{1cm} (20)

The equivalence of (19) to the fundamental identity is shown in Gran, Nilsson and Petersson [11]. Elementary, but useful symmetry relations are

\[ [mkl] = \frac{1}{3} (m[kl] + l[mk] + k[lm]) \]  \hspace{1cm} (21)

\[ [abcde] = \frac{1}{5} ([abcd]e + [eabc]d + [deab]c + [cdea]b + [bcde]a), \]  \hspace{1cm} (22)

where square brackets stand for anti-symmetrization.

**References**

[1] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, “M theory as a matrix model: A conjecture,” Phys. Rev. D 55, 5112 (1997) [arXiv:hep-th/9610043].

[2] J. H. Schwarz, “Superconformal Chern-Simons theories,” JHEP 0411, 078 (2004) [arXiv:hep-th/0411077].
REFERENCES

[3] J. Bagger and N. Lambert, “Modeling multiple M2’s,” Phys. Rev. D 75, 045020 (2007) [arXiv:hep-th/0611108]. J. Bagger and N. Lambert, “Gauge Symmetry and Supersymmetry of Multiple M2-Branes,” Phys. Rev. D 77, 065008 (2008) [arXiv:0711.0955 [hep-th]]. J. Bagger and N. Lambert, “Comments On Multiple M2-branes,” JHEP 0802, 105 (2008) [arXiv:0712.3738 [hep-th]].

[4] A. Gustavsson, “Algebraic structures on parallel M2-branes,” arXiv:0709.1260 [hep-th]. A. Gustavsson, “Selfdual strings and loop space Nahm equations,” arXiv:0802.3456 [hep-th].

[5] N. R. Constable, R. C. Myers and O. Tafjord, “Fuzzy funnels: Non-abelian brane intersections,” arXiv:hep-th/0105035. N. R. Constable, R. C. Myers and O. Tafjord, “The noncommutative bion core,” Phys. Rev. D 61, 106009 (2000) [arXiv:hep-th/9911136].

[6] N. R. Constable and N. D. Lambert, “Calibrations, monopoles and fuzzy funnels,” Phys. Rev. D 66, 065016 (2002) [arXiv:hep-th/0206243].

[7] G. W. Gibbons and G. Papadopoulos, “Calibrations and intersecting branes,” Commun. Math. Phys. 202, 593 (1999) [arXiv:hep-th/9803163]. J. P. Gauntlett, N. D. Lambert and P. C. West, “Branes and calibrated geometries,” Commun. Math. Phys. 202, 571 (1999) [arXiv:hep-th/9803216].

[8] A. Basu and J. A. Harvey, “The M2-M5 brane system and a generalized Nahm’s equation,” Nucl. Phys. B 713, 136 (2005) [arXiv:hep-th/0412310].

[9] D. S. Berman and N. B. Copland, “Five-brane calibrations and fuzzy funnels,” Nucl. Phys. B 723, 117 (2005) [arXiv:hep-th/0504044].

[10] M. Van Raamsdonk, “Comments on the Bagger-Lambert theory and multiple M2-branes,” arXiv:0803.3803 [hep-th]. N. Lambert and D. Tong, “Membranes on an Orbifold,” arXiv:0804.1114 [hep-th]. J. Distler, S. Mukhi, C. Papageorgakis and M. Van Raamsdonk, “M2-branes on M-folds,” arXiv:0804.1256 [hep-th].

[11] S. Mukhi and C. Papageorgakis, “M2 to D2,” arXiv:0803.3218 [hep-th]. U. Gran, B. E. W. Nilsson and C. Petersson, “On relating multiple M2 and D2-branes,” arXiv:0804.1784 [hep-th].

[12] M. A. Bandres, A. E. Lipstein and J. H. Schwarz, “N = 8 Superconformal Chern–Simons Theories,” JHEP 0805, 025 (2008) [arXiv:0803.3242 [hep-th]]. G. Papadopoulos, “M2-branes, 3-Lie Algebras and Plucker relations,” arXiv:0804.2662 [hep-th]. J. P. Gauntlett and J. B. Gutowski, “Constraining Maximally Supersymmetric Membrane Actions,” arXiv:0804.3078 [hep-th].

[13] D. S. Berman, L. C. Tadrowski and D. C. Thompson, “Aspects of Multiple Membranes,” arXiv:0803.3611 [hep-th].

[14] A. Morozov, “On the Problem of Multiple M2 Branes,” arXiv:0804.0913 [hep-th]. P. M. Ho, R. C. Hou and Y. Matsuo, “Lie 3-Algebra and Multiple M2-branes,” arXiv:0804.2110 [hep-th]. J. Gomis, A. J. Salim and F. Passerini, “Matrix Theory of Type IIB Plane Wave from Membranes,” arXiv:0804.2186 [hep-th]. E. A. Bergshoeff, M. de Roo and O. Hohm, “Multiple M2-branes and the Embedding Tensor,” arXiv:0804.2201 [hep-th]. K. Hosomichi, K. M. Lee and S. Lee, “Mass-Deformed Bagger-Lambert Theory and its
BPS Objects,” arXiv:0804.2519 [hep-th]. G. Papadopoulos, “On the structure of k-Lie algebras,” arXiv:0804.3567 [hep-th]. P. M. Ho and Y. Matsuo, “M5 from M2,” arXiv:0804.3629 [hep-th]. A. Morozov, “From Simplified BLG Action to the First-Quantized M-Theory,” arXiv:0805.1703 [hep-th]. Y. Honma, S. Iso, Y. Sumitomo and S. Zhang, “Janus field theories from multiple M2 branes,” arXiv:0805.1895 [hep-th]. H. Fuji, S. Terashima and M. Yamazaki, “A New N=4 Membrane Action via Orbifold,” arXiv:0805.1997 [hep-th]. P. M. Ho, Y. Imamura, Y. Matsuo and S. Shiba, “M5-brane in three-form flux and multiple M2-branes,” arXiv:0805.2898 [hep-th].

[15] D. S. Berman, “M-theory branes and their interactions,” Phys. Rept. 456, 89 (2008) arXiv:0710.1707 [hep-th]. N. B. Copland, “Aspects of M-Theory Brane Interactions and String Theory Symmetries,” arXiv:0707.1317 [hep-th].

[16] Z. Guralnik and S. Ramgoolam, “On the polarization of unstable D0-branes into non-commutative odd spheres,” JHEP 0102, 032 (2001) arXiv:hep-th/0101001]. S. Ramgoolam, “On spherical harmonics for fuzzy spheres in diverse dimensions,” Nucl. Phys. B 610, 461 (2001) arXiv:hep-th/0105006. S. Ramgoolam, “Higher dimensional geometries related to fuzzy odd-dimensional spheres,” JHEP 0210, 064 (2002) arXiv:hep-th/0207111.

[17] D. S. Berman and N. B. Copland, “A note on the M2-M5 brane system and fuzzy spheres,” Phys. Lett. B 639, 553 (2006) arXiv:hep-th/0605086.

[18] J. Gomis, G. Milanesi and J. G. Russo, “Bagger-Lambert Theory for General Lie Algebras,” arXiv:0805.1012 [hep-th]. S. Benvenuti, D. Rodriguez-Gomez, E. Tonni and H. Verlinde, “N=8 superconformal gauge theories and M2 branes,” arXiv:0805.1087 [hep-th]. P. M. Ho, Y. Imamura and Y. Matsuo, “M2 to D2 revisited,” arXiv:0805.1202 [hep-th].