Wave optical simulation of retinal images in laser safety evaluations

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Abstract
Lasers with wavelengths in the visible and near infrared region, pose a potential hazard to vision as the radiation can be focused on the retina. The laser safety standard IEC 60825–1:2014 provides limits and evaluation methods to perform a classification for such systems. An important parameter is the retinal spot size which is described by the angular subtense of the apparent source. In laser safety evaluations, the radiation is often described as a Gaussian beam and the image on the retina is calculated using the wave optical propagation through a thin lens. For coherent radiation, this method can be insufficient as the diffraction effects of the pupil aperture influence the retinal image. In this publication, we analyze these effects and propose a general analytical calculation method for the angular subtense. The proposed formula is validated for collimated and divergent Gaussian beams.

Keywords
accessible emission, IEC 60825–1, laser safety, retinal image, wave optics

1 | INTRODUCTION
Laser systems emitting radiation above the accessible emission limits of the laser safety standards [1, 2] or the exposure limits of the ICNIRP guidelines for laser radiation [3] represent a potential hazard for the human eye and skin.

For laser radiation in the visible and near-infrared wavelength region, defined from 400 nm to 1400 nm according to the laser safety standards, the radiation is imaged on the retina, the most sensitive part of the eye. The eye safety evaluation is based on the retinal image where the angular subtense of the apparent source is determined [4]. Thus, a laser classification on a theoretical basis depends on three main issues, the eye model, propagation method and analysis method. An eye model is necessary to simulate the retinal image that is produced by
the laser system as the damage takes place at the retina. For calculating the retinal image, an appropriate propagation method has to be applied where relevant aspects of laser propagation are taken into account. At last, the obtained retinal images have to be investigated with an analysis method which is in accordance with the laser safety standard. In Figure 1, the three main issues are summarized and different possible approaches are shown. The used models and calculation methods for this investigation are marked in color. The aim of this investigation is illustrated in Figure 2 where an analytical formula is proposed to perform an eye safety evaluation for Gaussian beams.

Generally, simulations use an air-equivalent eye model which consists of a circular aperture, an ideal lens with a varying focal length and a plane detector. This model emulates a human eye with a refractive index equal to one and a single lens [5, 6] is responsible for the refractive power. It is assumed that the human eye is able to accommodate between 10 cm and infinity. For the simulation of the retinal image, the choice of the appropriate propagation method is currently an open question. Software tools like Zemax Optic Studio [7] or TracePro [8] use the rules of geometrical optics. However, especially for laser radiation, diffraction effects can occur which impact the eye safety evaluation. Furthermore, the IEC/TR 60825-13 [9] suggests to consider wave optics rather than ray optics to determine the angular subtense of the apparent source. For this reason, we calculate the physically correct retinal images with the software VirtualLab Fusion [10] solving the diffraction equations numerically. At last, an analysis method has to be applied to the retinal images to obtain the angular subtense of the apparent source which is an important parameter in eye safety evaluations. Here, the laser safety standard provides different approaches where for example the \(d_{63}\) diameter of a Gaussian beam can be used or where an extensive image analysis is used involving either a rectangular or elliptical shaped measurement aperture [9].

Figure 2 illustrates the main aspect of this investigation with the aim of providing analytical equations to replace the wave optical simulation of retinal images in laser safety evaluations. As radiation we assume the fundamental mode of a Gaussian beam. In the simulation based approach, the retinal image is simulated with an air-equivalent eye model and by solving the diffraction equations. For these retinal images, an extensive analysis method has to be applied to obtain the angular subtense of the apparent source. As all calculation steps require a high computational effort, it would be beneficial to analytically describe the retinal image or to be more precise the angular subtense of the apparent source where the essential wave optical phenomena are included. In this investigation, a set of analytical equations is proposed and validated against the simulation based evaluation.

2 | GAUSSIAN BEAM PROPAGATION THROUGH THE AIR-EQUIVALENT EYE MODEL

The air-equivalent eye model is generally used in theoretical eye safety evaluations as well as experimental setups in laser safety test laboratories. Compared to realistic eye models where the complex point-spread function includes effects such as optical aberrations [11], the air-equivalent eye model provides idealized retinal images resulting in conservative eye safety evaluations. Another effect that is neglected in this model is thermal lensing which appears in the near-infrared wavelength region.
Due to a strong absorption of the vitreous body, local temperature changes lead to the development of a negative lens in case of Gaussian beams. As a result, the retinal spot sizes increase lowering the hazard. Thermal lensing is a relatively unexplored effect and is discussed in more detail with regard to experimental threshold data by Vincelette et al. [14].

In the following, the Gaussian beam propagation will be derived for the air-equivalent eye model. By using the paraxial Helmholtz equation, an analytical description of laser radiation can be derived [15]. The Helmholtz equation yields several modes of the beam whereas we focus on the fundamental mode, the TEM$_{00}$ mode. For this mode, the normalized spatial intensity distribution $I(r,z)$ is expressed by

$$I(r,z) = \frac{2}{\pi w(z)^2} \exp \left( -\frac{2r^2}{w(z)^2} \right).$$  \hspace{1cm} (1)

Here, it is assumed that the $z$-axis is the direction of propagation. The distance perpendicular to the $z$-axis is the radius $r$. The TEM$_{00}$ mode in Equation (1) shows a Gaussian distribution. The width of the distribution is defined by the beam radius $w(z)$ which describes a decrease by a factor of $e^2$. With this radius, the free space propagation of the Gaussian beam is defined by

$$w(z) = w_0 \sqrt{1 + \left(\frac{z - z_0}{z_R}\right)^2}.$$ \hspace{1cm} (2)

The radius $w_0$ is the beam waist located at $z_0$. The Rayleigh length $z_R$ can be calculated using

$$z_R = \frac{\pi}{\lambda} w_0^2.$$ \hspace{1cm} (3)

Only two parameters, namely the beam waist $w_0$ and the wavelength $\lambda$, are required to completely characterize a Gaussian beam. The relation between the waist and the divergence is given by the beam parameter product

$$w_0 \theta = \frac{2\lambda}{\pi}.$$ \hspace{1cm} (4)

Equation (2) describes the free space propagation of the Gaussian beam according to the Fresnel-Kirchhoff diffraction [16]. For the propagation through an optical system, the ray transfer matrices, also known as ABCD matrices can also be used for the Gaussian beam propagation [16, 17].

As shown in Figure 3, three essential matrices describe the propagation through the air-equivalent eye model. One matrix represent the refraction at an ideal lens and the other two are translation matrices.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/f & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix}.$$ 

**FIGURE 2** Comparison of the simulation based and analytical approach including a mapping of the essential equations. In the simulation, the propagation of the Gaussian beam is calculated using the diffraction equation and the retinal image is analyzed. The analytical procedure proposes several equations replacing the simulations.

**FIGURE 3** Description of the air-equivalent eye model in paraxial approximation using the ray transfer matrices.
It is assumed that the eye model is located at a distance $d_1$ from the waist of the Gaussian beam. According to the laser safety standard, the refractive power of the eye model shall be varied for accommodations between infinity and 10 cm. As the distance $d_2$ between the lens and the detector is set to 17 mm, the focal length $f$ of the lens varies between 14.52 and 17 mm. With the description in Figure 3, the retinal irradiance distribution is given by a Gaussian distribution with the beam radius $w_r$.

$$w_r = w_0 \sqrt{\left(1 - \frac{d_2}{f}\right)^2 + \left(\frac{1}{2R}\right)^2 \left(d_1 + d_2 \left(1 - \frac{d_1}{f}\right)\right)^2}. \quad (5)$$

The same result is described in a previous investigation by Galbiati in 2001 [18]. However, in this description the human pupil which is represented by a circular aperture is excluded. According to the laser safety standard, the circular aperture, also referred to as aperture stop, has a diameter of 7 mm. The applicability of Equation (5) is restricted to Gaussian beams much smaller than the pupil aperture. Generally, the propagation through the eye is affected by diffraction at the edges of the aperture. This results in a truncated Gaussian beam with a cut off of outer parts. In a previous publication [19], a more general formula was proposed to describe the truncated retinal image. The formula is obtained by multiplying a specific factor to Equation (5). This factor is calculated by the ratio between the radius $r_A$ of the aperture and the radius of the Gaussian beam at the aperture. The radius $w'_r$ of the truncated retinal image size is given by

$$w'_r = r_A \sqrt{\left(1 - \frac{d_0}{f}\right)^2 + \left(\frac{1}{2R}\right)^2 \left(d_1 + d_2 \left(1 - \frac{d_1}{f}\right)\right)^2} + \left(\frac{1}{2R}\right)^2 d_0^2. \quad (6)$$

This parameter $w'_r$ does not follow the previous diameter definition like in Equations (2) and (5) since it describes the location on the retina where the irradiance is cut off. Equations (5) and (6) describe two physically different propagations for a Gaussian beam entering the eye. Both are examined with regard to their application for eye safety calculations.

### 3 | WAVE OPTICAL PHENOMENA IN THE FIELD OF EYE SAFETY

Several aspects of the human vision like spherical aberrations or astigmatism can be explained using geometrical optics. However, especially for laser radiation the application of wave optics is more precise since two wave optical effects have an impact on the eye safety evaluation. One effect is the characteristic that the radiation cannot be focused on an infinitely small spot and the other effect is the truncation of the Gaussian beam at the pupil aperture. Both phenomena are shown in Figure 4.

In Figure 4 the size of the collimated Gaussian beam is larger than the aperture which leads to interference. In case the radiation is not focused on the retina, which is depicted in the left plot of Figure 4, the Gaussian beam is truncated and accompanied by oscillations. The calculation according to Equation (5) would result in a diameter $d_{\text{Airy}}$ of 2.3 mm whereas the hazard potential is underestimated compared to the real imaged radiation, which shows a smaller extent. With Equation (6) the diameter is approximated to 1.2 mm. Since this formula assumes a beam profile in which no oscillations occur, the question arises whether this approximation is sufficiently precise for eye safety calculations.

The other effect which is depicted in the right plot of Figure 4, represents the diffraction limit $d_{\text{Airy}}$ caused by the circular aperture of the pupil. The characteristic intensity pattern consists of a circular main lobe maximum, referred to as “Airy disc,” and several concentric sidelobes with decreased peak intensities. This intensity distribution is described by the far field approximation of

![Figure 4](image-url)
the diffraction equation, the Fraunhofer diffraction, whereas the far field image corresponds to the imaged radiation in the focal plane of the ideal lens. The calculation of the distribution in the right plot of Figure 4 is based on the Fourier transform of the circular aperture whereby the intensity peaks are given by the Bessel function of the first kind \([16]\). The width of the main lobe maximum can be estimated by

\[
d_{\text{Airy}} \approx 1.22 \frac{\lambda}{2r_A} d_2. \tag{7}
\]

According to this equation, the width of the “Airy disc” is calculated to approximately \(4.1 \, \mu m\) for the beam parameters in Figure 4. This limit cannot be obtained with the propagation formula for a Gaussian beam in Equation (5) because the diameter of the imaged Gaussian beam decreases continuously for increasing incident beam widths.

The laser safety standards \([1, 2]\) also include a minimum spot size. It is stated that laser sources which produce retinal spots smaller than the minimum spot size are referred to as point sources. The ICNIRP guidelines \([3]\) explain the imaging behavior of point sources due to the eye resolution \([20]\). The smallest spot is set to an angular subtense of 1.5 mrad which corresponds to an absolute size of 25.5 \(\mu m\) \([21]\). The deviation from the theoretical expectation according to Equation (7) of about one order of magnitude is due to the assumption of an idealized eye model. However, the question of the minimum spot size for the human eye is rather complex and currently there is no definite answer. Experimental measurements on rhesus monkeys show that there are marginal dependencies of the injury thresholds on spot sizes smaller than 5 mrad \([22, 23]\). In these measurements the spot size of 5 mrad could be interpreted as minimum spot size. The retinas of rhesus monkeys represent a suitable model to derive exposure limits as they are more sensitive compared to human retinas \([24–26]\). However, a possibly smaller spot size cannot be excluded for human retinas. For this reason, the thresholds of the above mentioned measurements are used for the exposure limits for point sources whereby a safety margin of 10 was applied \([27]\).

In addition, there are studies, which show that it is possible to produce smaller spot sizes on the retina. Experimental measurements with rabbit eyes done by Birngruber et al in 1979 \([28]\) demonstrate that an advantageous beam position leads to a minimum spot size of 7 \(\mu m\) representing the diffraction limit. However, due to scattering there is an intensity loss of about 50\%. It was found that also for the human eye a diffraction limited focus can be achieved \([29, 30]\). In another study which was done by Lund et al in 2008 \([31]\) measurements on non-human primates were performed by using wavefront-corrected beams to reduce refractive errors and aberrations. Though the results show lower thresholds than without using wavefront correction the retinal spot size could not be determined. More detailed examinations regarding the minimum spot size in the fields of laser safety can be found in further literature \([32–34]\).

In consequence of the uncertainties regarding the accurate determination of the minimum spot size, the description of point sources from the laser safety standards \([1, 2]\) is used in this investigation. The size of the analyzed intensity patterns is set to a minimum value of 25.5 \(\mu m\) in case the actual sizes have a lower value. The other effect, the truncation of the beam, is not mentioned in the current laser safety standards. The propagation of a Gaussian beam through a circular aperture is numerically studied in previous works \([35, 36]\). In this investigation we propose an analytical formula for the width of a truncated Gaussian beam which can directly be used for the determination of the angular subtense of the apparent source. We simulate the retinal images with the air-equivalent eye model using the software VirtualLab Fusion \([10]\) and compare them to the analytical descriptions according to Equations (5) and (6) in terms of eye safety. Furthermore, two wavelengths 400 and 1400 nm will be investigated as they define the borders of the wavelength range which is imaged on the retina according to the laser safety standards.

## 4 | LASER SAFETY EVALUATION OF RETINAL IMAGES

An eye safety evaluation for retinal images can be performed in two different ways, namely assuming a point source or an extended source. By assuming a point source, the whole radiation is considered to be contained within the minimum spot size. In case the retinal image shows an irradiance distribution that is much larger than the minimum spot size this approach is over restrictive. According to the laser safety standards, the accessible emission limits increase for larger spots of extended sources. The spot size is expressed by the angular subtense of the apparent source. However, an extensive calculation method has to be applied to the retinal image to obtain the angular subtense \([4, 37, 38]\). The size of an arbitrary retinal image could be characterized by different beam diameter definitions like the second moment diameter or the FWHM criterion used for different applications. In terms of the eye safety, such diameter definitions are not suitable to determine the angular subtense \([39]\). The tophat and the Gaussian profile are the only exceptions. For a tophat profile, the angular subtense is
equal to the extent of the profile. In case of a Gaussian profile, the laser safety standards state that the \( d_0 \) diameter can be used for a conservative evaluation. It should be emphasized that this method can only be used if the laser beam creates a Gaussian irradiance profile on the retina.

### 4.1 Image analysis for extended sources

For the classification of an extended source, the angular subtense \( \alpha \) is needed. This size is obtained by analyzing the retinal image. The general procedure is to determine the most restrictive size for the angle of acceptance \( \gamma \), which defines a field stop used to calculate the “accessible emission” (AE). This worst case is found by varying \( \gamma \) to maximize the ratio between AE and the “accessible emission limit” (AEL). This procedure is referred to as image analysis and is illustrated in Figure 5 for a Gaussian intensity distribution where the variation of the angle of acceptance \( \gamma \) and the angular subtense \( \alpha \) of the apparent source are shown. It is stated in the laser safety standards [1, 2], that the field stop can either be placed in front of a detector or close to the apparent source. In our simulation-based method the field stop is on the retinal image. There are two possible shapes of the field stop defined, namely an elliptical and a rectangular shape [9].

The AE and the AEL depend on the angular subtense \( \alpha \). By assuming a laser source with the total radiation power \( P_0 \), the AE can be given with

\[
AE(\alpha) = P_0\eta_{\text{pupil}}\eta_{\text{retina}}(\alpha). \tag{8}
\]

The factor \( \eta_{\text{pupil}} \) is the extraocular factor and gives the power percentage of \( P_0 \) that passes the pupil and enters the eye. The factor \( \eta_{\text{retina}}(\alpha) \) is the intraocular factor and is calculated by dividing the radiation energy contained within the field stop by the total energy of the retinal image. For an unknown angular subtense, the field stop is defined by the angle of acceptance \( \gamma_x \) and \( \gamma_y \) representing both orthogonal directions. By performing the image analysis, both \( \gamma_x \) and \( \gamma_y \) are varied within the limitations defined in the laser safety standards. In addition, the location of the field stop in the retinal image has to be varied for a complete analysis. It should also be noted that the field stops have to be rotated from 0° to 90°. However, rotating the field stops is not necessary in this investigation as the analyzed retinal images are rotationally symmetric.

The AEL increases with the angular subtense \( \alpha \) and the laser safety standard defines two different functional behaviors. Depending on the emission duration \( t \), the \( \alpha \) dependencies are given by the piecewise function

\[
\kappa(\alpha) = \begin{cases} 
\alpha, & \text{if } t \leq T_2(\alpha) \\
\alpha 10^{-5mrad^2}, & \text{if } t > T_2(\alpha)
\end{cases} \tag{9}
\]

with the time \( T_2(\alpha) \) defined in the laser safety standard IEC 60825–1:2014 [1].

\[
T_2(\alpha) = 10 \times 10^{[(\alpha-a_{\text{min}})/98.5mrad]}. \tag{10}
\]

In the following, the image analysis for \( t \leq T_2(\alpha) \) is referred to as “short-time method” and for the other case the term “long-time method” is used. As the time \( T_2 \) depends on the angular subtense, the value is determined during the image analysis. Here, for times greater than 10 seconds both the short and long-time have to be applied whereas the most restrictive result is used.

The angular subtense \( \alpha \) of the apparent source is not directly a characteristic of the source of the radiation. But it is derived from the retinal images the laser source produces. The image analysis is defined with

\[
\left\{ \gamma | \max_{\gamma} \eta_{\text{retina}}(\gamma) \right| _{\kappa(\gamma)} \right| _{\gamma = \alpha} \tag{11}
\]

in case of radially symmetric retinal images where \( \gamma_x \) equals \( \gamma_y \) and is illustrated in Figure 5. The calculation for the angular subtense can differ depending on the evaluation time according to Equation (9). Figure 6 illustrates the behavior for a Gaussian retinal image with varying widths.
The results of the image analysis for a symmetric Gaussian intensity distributions differ for the short- and long-time method ($\alpha_s$ and $\alpha_l$) as well as for the choice of the field stop (elliptical and rectangular). The x-axis shows different extents of the Gaussian distribution defined by the $d_{86}$-diameter and the y-axis shows the determined angular subtense expressed as $d_\circ$. The markers show all kinks in the curves.

![Figure 6](image)

**Figure 6** The results of the image analysis for a symmetric Gaussian intensity distributions differ for the short- and long-time method ($\alpha_s$ and $\alpha_l$) as well as for the choice of the field stop (elliptical and rectangular). The x-axis shows different extents of the Gaussian distribution defined by the $d_{86}$-diameter and the y-axis shows the determined angular subtense expressed as $d_\circ$. The markers show all kinks in the curves.

The kinks of all curves arise from the limitation of the angular subtense to 100 mrad. While both angular subtenses stay approximately constant for the short-time method an increasing curve can be seen for the long-time method. For the symmetric Gaussian intensity distribution, the choice of a rectangular field stop leads to a smaller angular subtense than for an elliptical field stop.

As the TEM$_{00}$ mode of the Gaussian beam is investigated, see Equation (1), the retinal images are radially symmetric. Thus, the retinal intensity distributions expressed in polar coordinates is $I_{RI}(r)$. The extraocular factor $\eta_{pupil}$ is given with

$$\eta_{pupil} = 1 - \exp \left( -\frac{r_A^2}{w_0^2} \left( 1 + \frac{d_2^2}{z_R^2} \right)^{-1} \right).$$

(12)

The intraocular factor $\eta_{retina}$ depends on the shape and size of the field stops. Due to radial symmetry, the center of the field stops coincide with the center of the intensity pattern and the shape is circular or square. Therefore, the factor is given by

$$\eta_{retina}^S(\alpha) = \frac{8}{P_{RI}} \int_0^{d_2 \tan(\alpha)} \int_0^{1 + \tan^2(\alpha) d_2^2} I_{RI}(r) r dr d\phi,$$

(13)

in case of a square field stop and

$$\eta_{retina}^C(\alpha) = \frac{2\pi}{P_{RI}} \int_0^{d_2 \tan(\alpha)} I_{RI}(r) r dr,$$

(14)

in case of a circular field stop. The radiation power $P_{RI}$ describes the total power within the retinal image and is equal to $P_0$ times the extraocular factor. The upper limit of the surface integral in Equations (13) and (14) depends on the angular subtense or in case of the image analysis on the angle of acceptance. This shows that each retinal image leads to a different behavior of AE.

All results are expressed by the maximum allowed power $P_0$. This power is calculated by equating AE and AEL. For emission durations greater than $T_2$ the emission limit from IEC 60825–1:2014 [1] is used to calculate a wavelength-independent power with

$$P_0 = \frac{7 \times 10^{-4} \frac{d_2}{w_0} \left( \frac{T_2(\alpha)}{l_s} \right)^{-\frac{1}{2}}}{\eta_{pupil} \eta_{retina}(\alpha)^{-1}} W.$$  

(15)

The wavelength of the radiation does not appear on the right side of Equation (15) and determines both correction factors $C_4$ and $C_7$. For wavelengths smaller than 700 nm, both factors are not defined according to the laser safety standard and they are set to one.

### 4.2 Proposed extended simplified method for Gaussian beams

For Gaussian beams, there is a procedure described in the laser safety standards [1, 2] to determine the angular subtense. The $d_{63}$ diameter can be used to calculate the angular subtense with the requirement that the total radiation power entering the eye is assumed to be contained in $\alpha$. With this requirement, the described method is more restrictive than the image analysis. However, as this method is only applicable to Gaussian intensity distribution on the retina, it is not suitable in case of a beam truncation. We propose a piecewise defined function to extend this method given by

$$\alpha = \begin{cases} 
2\arctan \frac{w_0}{\sqrt{d_2}} & \frac{w_0}{\sqrt{2}} \sqrt{1 + \frac{d_2^2}{z_R^2}} < r_A \\
2\arctan \frac{d_2 w_0}{\sqrt{2}} & \frac{w_0}{\sqrt{2}} \sqrt{1 + \frac{d_2^2}{z_R^2}} \geq r_A
\end{cases}$$

(16)

where both conditions on the right side ensure a continuous transition of the functions. The regions showing
which function from Equation (16) has to be used are illustrated in Figure 7 for different distances \(d_1\) and input beam radii \(w_0\). Analogous to Equation (15), the wavelength-independent maximum allowed power is determined with

\[
P_0 = \frac{7 \times 10^{-4} \cdot \alpha \cdot \left(\frac{T_1(\omega)}{18}\right)^{-\frac{4}{3}}}{\eta_{\text{pupil}}} W. \tag{17}
\]

The intraocular factor \(\eta_{\text{retina}}\) is set to one. This proposed extended method will be compared with the results from the image analysis to verify its applicability.

5 | RESULTS

In this section collimated and divergent Gaussian beams with the wavelengths 400 nm and 1400 nm are investigated with regard to eye safety. Here, the beam parameters as well as the parameters of the eye model are varied to simulate retinal images which are analyzed with the image analysis and compared to the proposed analytical formula.

5.1 | Collimated Gaussian beams

In the following, collimated Gaussian beam showing divergences smaller than 0.5 mrad are investigated. As the distance \(d_1\) is a negligible factor for the simulated retinal images it is set to zero. Therefore, the minimum beam waist \(w_0\), the wavelength \(\lambda\) and the focal length \(f\) of the eye model are the essential parameters to define all possible configurations. The free parameters and the eye model are shown in Figure 8.

As it was previously shown in Figure 4, there are two major diffraction effects. The diffraction limit occurs when focusing the radiation on the focal plane \((f = d_2 = 17 \text{ mm})\). The size of the simulated retinal image in dependence of the input beam waist as well as the theoretical expectations as defined in Equations (5) and (7) are shown for the two essential wavelengths in Figure 9.

For a sufficiently small beam radius \(w_0\), estimated to be smaller than 1.9 mm, the Gaussian beam propagation from Equation (5) where the circular aperture is excluded can be used for a correct calculation. However, for an increasing beam waist the diffraction at the aperture influences the image and the deviation increases.

FIGURE 7 Region plot using the conditions from Equation (16) to illustrate which case of the piecewise function applies for a specific distance \(d_1\) and beam radius \(w_0\). The plot is for a wavelength of 400 nm. Other wavelengths up to 1400 nm show a similar upper horizontal boundary line of \(w_r\). The lower diagonally rising boundary line of \(w_r\) shifts up to a factor of three for the \(w_0\) values.

FIGURE 8 The figure shows the free parameters to create all possible retinal images in case of collimated beams. The left image A, shows a configuration where the Gaussian beam is smaller than the eye pupil and in the right image B, the beam size exceeds the pupil which leads to non-negligible diffraction effects.

FIGURE 9 The plot shows the retinal image size expressed as the \(d_{63}\) diameter for different input beam waists. The solid lines show the wave optical simulation. The dashed lines correspond to Equation (5) and the dotted lines to the estimation of the diffraction limit using Equation (7).
larger input beam waists, the calculation for the diameter $d_{\alpha_1}$ is primarily based on the main lobe maximum and can be approximated by the diffraction limit from Equation (7). These retinal images show spots smaller than the minimum spot size and are classified as point sources according to the laser safety standards.

For focal lengths smaller than 17 mm the beam truncation occurs. This can be seen in Figure 10 where the retinal images show oscillations. The frequency and amplitudes of the oscillations differ significantly for both wavelengths. A wavelength of 1400 nm generally leads to lower frequencies and higher amplitudes compared to the wavelength of 400 nm. The angular subtense $\alpha$ is calculated with Equation (11) using a circular and square field stop. All results obtained by varying the focal length of the eye are shown in Figure 11 together with the analytical calculation from Equation (16).

The calculation using $w_r$ in Equation (5) yields much larger $\alpha$ values leading to an underestimation of the eye hazard. Here, the $w_r$ calculation from Equation (6) shows more accurate results. Furthermore, depending on the shape of the field stop, the image analysis gives slightly different angular subtenses. The circular field stop according to Equation (14) leads to larger $\alpha$ values than the square field stop from Equation (13). The figure depicts only the results for 400 nm as both wavelengths were found to be identical in case of the theoretical calculation which can be seen in Table 1. Regarding the image analysis, the differences are less than 1%.

**FIGURE 10** Retinal images for a Gaussian beam with an input beam waist of 7 mm and for 400 and 1400 nm where the normalized intensity distribution along a symmetry is plotted. The focal length of the eye model is set to 16.7 mm in plot A, and 14.5 mm in plot B. The simulation was performed with VirtualLab Fusion [10].

**TABLE 1** Calculation of the maximum allowed power for different approaches to perform eye safety evaluations

| Beam radius $w_0$ | $w_f$ | Theoretical calculation | Image analysis |
|-------------------|-------|-------------------------|---------------|
|                   | $w_r$ | $w_f$                   | Square      | Circular |
| 1 mm              | 3.71  | 18.36                   | 5.23        | 5.79     |
|                   | 3.71  | 18.67                   | 5.28        | 5.76     |
| 2 mm              | 7.58  | 18.40                   | 10.65       | 11.83    |
|                   | 7.59  | 18.42                   | 10.67       | 11.80    |
| 3 mm              | 12.31 | 19.83                   | 16.24       | 18.05    |
|                   | 12.31 | 19.84                   | 16.26       | 18.03    |
| 5 mm              | 29.31 | 29.18                   | 27.89       | 30.09    |
|                   | 29.30 | 29.18                   | 27.90       | 29.95    |
| 7 mm              | 66.97 | 47.09                   | 46.29       | 48.89    |
|                   | 66.96 | 47.09                   | 46.32       | 48.93    |
| 10 mm             | 304.90| 84.15                   | 83.58       | 87.61    |
|                   | 304.84| 84.15                   | 83.55       | 88.82    |
| 25 mm             | 8875  | 477.50                  | 477.60      | 498.20   |
|                   | 8875  | 477.50                  | 476.60      | 500.50   |
The results for a Gaussian beam with a varying beam waist at a fixed focal length of 14.53 mm can be seen in Figure 12. As a result of the truncation effect there is a maximum retinal image size. For beam waists above approximately 5 mm the angular subtense show a saturation value. For the circular field stop, a kink occurs and the saturation value is at about 72 mrad. The image analysis using the square field stop has a lower maximum value which is at 66 mrad. For beam waists smaller than 4 mm, Equation (5) which describes the radius \( w_r \) is more suitable to calculate the angular subtense. With the angular subtense \( \alpha \) and the intraocular factor, the wavelength-independent maximum allowed power can be calculated according to Equations (15) and (17). The obtained radiation power is shown in Figure 13.

For larger beam waists, the \( w_r \) calculation method gives an allowed power more than 10 times higher than with the image analysis. At about 7 mm, there is a discontinuity due to the limitation of the angular subtense to 100 mrad. The \( w_t \) calculation fits the results of the image analysis for large beam radii where the effect of truncation appears. Table 1 lists the maximum allowed power for specific beam waists regarding a wavelength of 400 and 1400 nm.

In general, it can be stated that the differences for both wavelengths 400 and 1400 nm are negligibly small. This means that the oscillations in the retinal images shown in Figure 10 have no decisive influence on the evaluation. Regarding both image analysis methods, the maximum allowed power is higher for a circular field stop than for a rectangular field stop. The difference between both methods decreases with increasing beam radius from 10% to about 4%.

In case of small beam waists where no truncation occurs, the \( w_r \) calculation from Equation (17) is more restrictive than both image analysis methods. In this beam radius region, the \( w_t \) calculation method leads to incorrect results. The piecewise function defined in Equation (16) contains both above mentioned calculation methods with corresponding conditions to define their scope. The application of the piecewise function is more restrictive than the image analysis using circular field stops. For a collimated Gaussian beam, the piecewise function can be extended. A further condition
checks if a classification according to an extended source has to be performed which is given by

$$f \leq \frac{d_2}{1 + \sqrt{\frac{25.5\mu m}{w_0}}}.$$  \hspace{1cm} (18)

In a second condition a boundary for the minimum beam radius is set to

$$w_0^B = \sqrt{r_A^2 + \sqrt{r_A^4 - \frac{\lambda^2}{\pi^2} d_1^2}}.$$  \hspace{1cm} (19)

In case the minimum beam waist $w_0$ is smaller than $w_0^B$ the upper case of Equation (16) applies and in the opposite case truncation occurs and the lower equation is used. The boundary $w_0^B$ is equal to the upper boundary line from Figure 7.

### 5.2 Divergent Gaussian beams

In comparison to collimated Gaussian beams, investigating divergent beams is more extensive as an additional degree of freedom, the distance $d_1$, has to be taken into account. Generally, a divergent Gaussian beam is obtained for a beam waist in the micrometer regime and is expressed by the full divergence angle which is calculated according to Equation (4). Figure 14 shows the free parameters varied in this investigation.

At first, the focal length $f$ is varied for a divergent beam illustrated in the right configuration of Figure 14. Here, the distance $d_1$ is 0.3 m and the Gaussian beam has a divergence angle of 50.9 mrad. This divergence is obtained for a beam waist of $w_0 = 5 \mu m$ in case of a wavelength of 400 nm and for $w_0 = 17.5 \mu m$ in case of 1400 nm. By accommodating to the beam waist, the minimal spot size is achieved. Accommodation to other distances result in larger retinal images with truncation. Figure 15 shows the truncation for two examples. Both curves show a sudden decrease in the intensity distribution. Varying the focal length leads to different retinal image sizes. By applying all calculation methods, the corresponding angular subtenses are shown in Figure 16. It was found that the curves for both wavelengths slightly differ from each other in case of the image analysis. In case of the theoretical calculations the results for both

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**FIGURE 14** The figure shows the free parameters to produce all possible retinal images. In the left image A, the beam has a low divergence leading to a small beam size at the pupil. In the right image B, the high divergence leads to diffraction effects at the pupil.

**FIGURE 15** Retinal images for accommodating to 10 cm A, and to infinity B. The intensity distributions for a wavelength of 400 nm and 1400 nm are shown. The results were obtained with VirtualLab Fusion [10].

**FIGURE 16** Determination of the angular subtense for a divergent Gaussian beam with a wavelength of 400 nm and a divergence angle of 50.9 mrad. The distance $d_1$ is 0.3 m.
wavelengths were identical. For this reason, only the curve for the wavelength 400 nm is plotted.

The angular subtense minimizes for a focal length of about 16.1 mm which corresponds to an accommodation to the beam waist. Using the \( w_r \) diameter from Equation (5) incorrectly results in a too large angular subtense. The \( w_r \) calculation from Equation (6) fits the image analysis results. With Equations (15) and (17) the maximum allowed power is calculated which is shown in Figure 17. For focal lengths around 16 mm, the maximum power is at 1.15 mW. This value is reached for all calculation methods due to the minimum spot size limitation from the laser safety standard. Table 2 lists the maximum allowed power for specific focal lengths and for both wavelengths of 400 and 1400 nm.

Though the differences for both wavelengths are generally larger than compared to the investigation done for collimated beams, the differences are still negligibly small. Therefore in case of divergent Gaussian beams, the effect of the oscillations can also be neglected. Furthermore, the analysis with circular field stops gives about 4% to 6% higher allowed powers than the analysis with square field stops. The \( w_r \) calculation turns out to be more restrictive in case of focal lengths from 16 mm to 17 mm than both image analysis methods. For other focal lengths, the results are slightly higher.

For the proposed extended simplified method from Equation (16), further conditions can be given. The first conditions are the inequalities

\[
f \leq \left[ \frac{d_1}{d_1^2 + z_R^2} + \frac{1}{d_2} - \frac{z_R}{d_2(d_1^2 + z_R^2)} \right] \sqrt{\frac{(25.5 \, \text{μm})^2}{2w_0^2}} \left( d_1^2 + z_R^2 - d_2^2 \right)^{-1}
\]

and

\[
f \geq \left[ \frac{d_1}{d_1^2 + z_R^2} + \frac{1}{d_2} + \frac{z_R}{d_2(d_1^2 + z_R^2)} \right] \sqrt{\frac{(25.5 \, \text{μm})^2}{2w_0^2}} \left( d_1^2 + z_R^2 - d_2^2 \right)^{-1}
\]

**FIGURE 17** Determination of the maximum allowed power for the investigation from Figure 15

**TABLE 2** Calculation of the maximum allowed power for specific focal lengths regarding the investigated beam from Figure 17 for both wavelengths 400 and 1400 nm
For focal lengths that meet the conditions, the retinal image is larger than the minimum spot size and a classification according to an extended source can be performed. Furthermore, a boundary beam waist can be defined with

\[
w_{0B} = \sqrt{r_A^4 - \frac{\lambda^2}{\pi^2} d_1^2}.
\]  

(22)

For beams waists smaller than \(w_{0B}\) the lower case of Equation (16) applies and for beam waists larger than \(w_{0B}\) the upper case is used. The boundary \(w_{0B}\) is equal to the lower boundary line from Figure 7.

The validity of these conditions is demonstrated in Figure 18 where the beam waist is varied in its size as well as in its position. Due to the variation of the distance \(d_1\), the boundary condition from Equation (22) for the correct calculation of the beam radius changes. For smaller distances the effect of the truncation occurs for a smaller beam radius. In addition, for 400 nm the truncation appears for smaller beam waists than for 1400 nm as the divergence angle appears to be generally higher for larger wavelengths. The theoretical calculation gives generally smaller angular subtenses than the image analysis. As the factor \(\eta_{\text{retina}}\) is set to one in the theoretical calculation, the results for the maximum allowed power are more restrictive than using the image analysis.

This procedure is applied to the Gaussian beam and compared with analytical equations from the laser safety standard where the \(d_{\text{63}}\) diameter is used to determine the angular subtense. For a coherent beam, it was found that the analytical calculation according to the standards can lead to significantly larger angular subtenses when truncation occurs due to the diffraction at the pupil. This results in an underestimation of the hazard as larger retinal spot sizes would be assumed.

We propose a new analytical formula to correctly calculate the angular subtense for possible configurations of collimated and divergent Gaussian beams. This proposed extended method was compared to the correct results from the simulative approach. It was found that our proposed analytical approach is sufficiently precise and can be applied to Gaussian beams in the visible and near-infrared wavelength region. The major advantage of this method is that the angular subtense \(\alpha\) can be directly calculated where there is no need to extensively simulate and analyze the retinal image.

In conclusion, it can be stated that the wave optical propagation is essential for a simulation-based classification of a laser system. The use of common Gaussian beam propagation equations can lead to wrong results and an underestimation of the hazard. For the fundamental mode of the Gaussian beam, an alternative calculation method was proposed and validated.

### 6 | SUMMARY AND CONCLUSION

A simulative approach to perform laser safety evaluations for radiation in the visible and near-infrared region is used to determine the angular subtense of the apparent source. The diffraction equations for radiation entering an air-equivalent eye model are solved to calculate the retinal image. An image analysis according to the laser safety standards is used to the retinal images to obtain the angular subtense.

### 7 | OUTLOOK

In a future study, the investigation can be expanded to elliptical or simple astigmatic coherent Gaussian beams where there are more free parameters. According to the laser safety standards, the angular subtense is calculated by averaging the size along both orthogonal directions. Here, the proposed formula can be applied to each direction to obtain the angular subtense. A comparison with the results of an image analysis could verify the applicability for these beams.
An important aspect of this study is that the investigated Gaussian beams are coherent. In reality, laser radiation deviates from ideal Gaussian beams which is described by the $M^2$ factor [40]. Partial coherent beams can be described with the Gaussian Schell-modell beams [41]. Here, the diffraction behaves different and the truncation effect is reduced [42]. In a further investigation, this model can be used to derive a more general formula to perform eye safety evaluations for Gaussian beams with an $M^2$ factor larger than one.

**CONFLICT OF INTEREST**

The authors declare that there are no conflicts of interest related to this article.

**DATA AVAILABILITY STATEMENT**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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**REFERENCES**

[1] IEC 60825–1 Ed. 3.0, Safety of Laser Products - Part 1: Equipment Classification and Requirements, International Electrotechnical Commission, Geneva, Switzerland, 2014.

[2] American National Standards Institute, ANSI, Z136.1–2014, American National Standard for the Safe Use of Lasers, Laser Institute of America, Orlando, FL, 2014.

[3] International Commission on Non-Ionizing Radiation Protection, Health Phys. 2013, 105(3), 271.

[4] S. Kotzur, S. Wahl, A. Frederiksen, Proc. SPIE 2020, 11238, 16. https://doi.org/10.1117/12.2545673.

[5] R. Henderson, K. Schulmeister, Laser Safety, CRC Press, Boca Raton, FL, 2003.

[6] C. P. Cain, G. D. Noojin, D. X. Hammer, R. J. Thomas, B. A. Rockwell, J. Biomed. Opt. 1997, 2(1), 88.

[7] Zemax Optic Studio.

[8] Trace Pro.

[9] IEC/TR 60825–13 Ed. 2.0, Safety of Laser Products - Part 13: Measurements for Classification of Laser Products, International Electrotechnical Commission, Geneva, Switzerland, 2011.

[10] VirtualLab Fusion: Optical Design Software from LightTrans.

[11] P. Artal, J. Santamaría, J. Bescós, J. Opt. Soc. Am. A 1988, 5(8), 1201.

[12] R. Vincelette, R. Thomas, B. Rockwell, A. Welch, Proc. SPIE 2007, 6435, 7.

[13] R. Vincelette, R. Thomas, B. Rockwell, C. Clark III, A. Welch, J. Opt. Soc. Am. A, Opt., Image Sci., Vision 2009, 26, 548.

[14] R. L. Vincelette, A. J. Welch, R. J. Thomas, B. A. Rockwell, D. J. Lund, J. Biomed. Opt. 2008, 13(5), 1.

[15] F. Pedrotti, L. Pedrotti, L. Pedrotti, Introduction to Optics, Cambridge University Press, Cambridge, 2017.

[16] W. Zinth, U. Zinth, Optik: Lichtstrahlen - Wellen - Photonen, De Gruyter, Berlin, Germany, 2013.

[17] F. Bodem, H.-D. Reidenbach, Opt. Quantum Electron. 1976, 8 (3), 207.

[18] E. Galbiati, J. Laser Appl. 2001, 13, 141.

[19] S. Kotzur, A. Frederiksen, StrahlenschutzPRAXIS 2018, 3.

[20] H. Gross, F. Blechinger, B. Achtner, Handbook of Optical Systems, Survey of Optical Instruments, Wiley, Weinheim, Germany, 2008.

[21] D. H. Sliney, M. L. Wolbarsht, Safety with Lasers and Other Optical Sources: A Comprehensive Handbook, Plenum Press, New York, 1980.

[22] D. J. Lund, P. R. Edsall, B. E. Stuck, K. Schulmeister, J. Biomed. Opt. 2007, 12(2), 024023.

[23] J. A. Zudlich, P. E. Edsall, D. J. Lund, B. E. Stuck, S. Till, R. C. Hollins, P. K. Kennedy, L. N. McLin, J. Laser Appl. 2008, 20(2), 83.

[24] J. Marshall, A. M. Hamilton, A. C. Bird, Br. J. Ophthalmol. 1975, 59, 610.

[25] B. Stuck, Letterman Army Inst. Res. 1984, 1, 67.

[26] K. Schulmeister, M. Jean, J. Laser Appl. 2017, 2017, 125. https://doi.org/10.2351/1.5056865.

[27] K. Schulmeister, B. Stuck, D. Lund, D. Sliney, Health Phys. 2011, 100(2), 210.

[28] R. Birngruber, E. Drechsel, F. Hillenkamp, V.-P. Gabel, Int. Ophthalmol. 1979, 3(3), 175.

[29] F. W. Campbell, R. W. Gubisch, J. Physiol. 1966, 186(3), 558.

[30] R. W. Gubisch, J. Opt. Soc. Am. 1967, 57, 407.

[31] B. J. Lund, D. J. Lund, P. R. Edsall, J. Laser Appl. 2008, 13(6), 064011.

[32] D. H. Sliney, Int. Laser Safety Conf., LIA, 2005, pp. 43–47.

[33] M. Jean, K. Schulmeister, J. Laser Appl. 2017, 29(3), 032004.

[34] M. Jean, K. Schulmeister, Modeling of Laser-Induced Thermal Damage to the Retina and the Cornea, CRC Press, Boca Raton, FL, 2014, p. 265. Ch. 15.

[35] P. Kuttner, Opt. Eng. 1986, 25(1), 180.

[36] C. Campbell, Opt. Eng. 1987, 26(3), 270.

[37] K. Schulmeister, J. Laser Appl. 2005, 2005, 91.

[38] K. Schulmeister, Int. Laser Safety Conf. 2015, 2015 (1), pp. 271–280.

[39] K. Schulmeister, R. Gilber, F. Edthofer, B. Seiser, G. Vees, Proc. SPIE 2006, 6101, 297. https://doi.org/10.1117/12.649233.

[40] ISO 11146:2005, Lasers and Laser-Related Equipment - Test Methods for Laser Beam Widths, Divergence Angles and Beam Propagation Ratios, International Organization for Standardization, Berlin Germany, 2005.

[41] R. J. Sudol, A. T. Friberg, in Coherence and Quantum Optics V (Eds: L. Mandel, E. Wolf), Springer US, Boston, MA 1984, p. 423.

[42] K. Schulmeister, Int. Laser Safety Conf. 2019 (1205), 293.

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