Universal Bimagic Squares and the day 10\textsuperscript{th} October 2010 (10.10.10)

Inder Jeet Taneja
Departamento de Matemática
Universidade Federal de Santa Catarina
88.040-900 Florianópolis, SC, Brazil.
e-mail: ijtaneja@gmail.com
http://www.mtm.ufsc.br/~taneja

Abstract

In this short note we have produced for the first time in the history different kinds of universal bimagic squares. This we have made using only the digits 0,1 and 2. The universal bimagic squares of order $8 \times 8$ and $16 \times 16$ are with the digits 0 and 1. The universal bimagic square of order $9 \times 9$ is with the digits 0, 1 and 2. It is interesting to note that the day October 10 have only the digits 0 and 1 if we consider it as 10.10.10. If we consider the date as 10.10.2010, then this has the digits 0, 1 and 2.

1 Details

In this work we shall present universal bimagic squares of order $8 \times 8$ and $16 \times 16$ having only the digits 0 and 1. A universal bimagic square of order $9 \times 9$ is also presented having three digits 0, 1 and 2. These magic squares are based on the date October 10 (10.10.10 or 10.10.2010).

1.1 Universal and Bimagic Squares

Here below are some definitions.

- **Magic square**

  A magic square is a collection of numbers put as a square matrix, where the sum of elements of each row, sum of elements of each column or sum of elements of each of two principal diagonals are the same. For simplicity, let us write this sum as $S_1$.

- **Bimagic square**

  Bimagic square is a magic square where the sum of square of each element of rows, columns or two principal diagonals are the same. For simplicity, let us write this sum as $S_2$.

- **Universal magic square**

  Universal magic square is a magic square with the following properties:
(i) **Upside down**, i.e., if we rotate it to 180 degrees, it remains magic square again;

(ii) **Mirror looking**, i.e., if we put it in front of mirror or see from the other side of the glass, or see on the other side of the paper, it always remains the magic square.

### 1.2 The date 10.10.10

It is interesting to note that at 10 hours, 10 minutes and 10 seconds of the day 10, month 10 and the year 10 have only the digits 1 and 0, i.e., 10-10-10-10-10-10. Let us divide it in two parts, i.e., 101010 – 101010. Thus we have two equal blocks of six algarisms. If we go only for the day, these digits repeats on others days too, such as

01-10-10; 01-01-10; 10-01-10, 11-10-10, etc.

If we go on hours, minutes and seconds we have many combinations of six algarisms only with the digits 0 and 1.

- **8 × 8 – Universal bimagic square of binary digits 0 and 1**

We can make $2 \times 2 \times 2 \times 2 \times 2$ or $2^6 = 64$ different numbers of six algarisms with the digits 0 and 1. Also, we can write $64 = 8 \times 8$. Here below is a universal bimagic square of order $8 \times 8$ having 64 different numbers using only the digits 0 and 1.

\[
\begin{array}{cccccccc}
001111 & 101000 & 100011 & 000100 & 011010 & 111101 & 110110 & 010001 \\
011001 & 111110 & 110101 & 010010 & 001100 & 101011 & 100000 & 000111 \\
000000 & 100111 & 101100 & 001011 & 010101 & 110010 & 111001 & 011110 \\
010110 & 110001 & 111010 & 011101 & 000111 & 100100 & 101111 & 001000 \\
100101 & 000100 & 001001 & 101110 & 110000 & 010111 & 011100 & 111101 \\
110011 & 010100 & 011111 & 110000 & 100110 & 000011 & 001010 & 101110 \\
101010 & 001101 & 000110 & 100001 & 111111 & 011000 & 010011 & 110100 \\
111100 & 011011 & 010000 & 111011 & 101001 & 001110 & 000101 & 100010 \\
\end{array}
\]

S1:=44444
S2:=44893328844

Also we have sum of each block of $2 \times 4 = 44444$ and the square of sum of each term in of each block of $2 \times 4 = 44893328844$
• 16 × 16 – Universal bimagic square of binary digits 0 and 1

Instead, considering six algarisms using only the digits 0 and 1, if we consider eight algarisms using we can make $2^8 = 256$ different numbers only with the digits 0 and 1. Also we can write 256 as $16 \times 16$. Here below is a universal bimagic square of order $16 \times 16$ with these 256 different numbers made from the digits 0 and 1:

![Binary Magic Square](image.png)

S1:=88888888
S2:=897867554657688

Also we have sum of each block of $4 \times 4 = 88888888$ and square of sum of each term in of each block of $4 \times 4 = 897867554657688$.

1.3 The date 10.10.2010

Instead, considering the year as 10, if we consider it as 2010, then we have three algarisms 0, 1 and 2. These digits happens on other days too, such as

02.01.2010; 02.02.2010; 20.10.2010; 02.10.2010; 12.10.2010; 2.10.2010, etc.

Still there are many other dates having only the digits 0, 1 and 2.
9 × 9 – Universal bimagic square of digits 0, 1 and 2

We can make exactly 81 different numbers having four algarisms from the three digits 0, 1 and 2, i.e, $3 \times 3 \times 3 \times 3 = 81$. Also we can write, $81 = 9 \times 9$. Here below is a universal bimagic square of order $9 \times 9$ having only the digits 0, 1 and 2 with 81 different numbers.

\[
\begin{array}{cccccccc}
 0000 & 0122 & 0211 & 1021 & 1100 & 1202 & 2012 & 2101 & 2200 \\
 1012 & 1101 & 1220 & 2000 & 2122 & 2211 & 0021 & 0110 & 0202 \\
 2021 & 2110 & 2202 & 0012 & 0101 & 0220 & 1000 & 1122 & 1211 \\
 0222 & 0011 & 0100 & 1210 & 1002 & 1121 & 2201 & 2020 & 2112 \\
 1201 & 1020 & 1102 & 2222 & 2011 & 2100 & 0210 & 0002 & 0121 \\
 2210 & 2002 & 2121 & 0201 & 0020 & 0112 & 1222 & 1011 & 1100 \\
 0111 & 0200 & 0022 & 1102 & 1221 & 1010 & 2120 & 2212 & 2001 \\
 1120 & 1212 & 1001 & 2111 & 2200 & 2022 & 0102 & 0221 & 0010 \\
 2102 & 2221 & 2010 & 0120 & 0212 & 0001 & 1111 & 1200 & 1022 \\
\end{array}
\]

$S_1 := 9999$

$S_2 := 17169395$

Also we have sum of each block of $3 \times 3 = 9999$ and square of sum of each term in of each block of $3 \times 3 = 17169495$

We observe that from the above magic square that if we make a rotation of 180 degrees the digits 2 remains the 2 but if we see it in the mirror 2 becomes 5. Obviously, in this case the sum $S_1$ and $S_2$ are not the same as given above. But still it is a magic square. If we want to have the same sum, we have to use 2 and 5 together (in the digital form) with either 0, 1 or 8. This study is given in the another work Taneja [5].

For more studies on magic and bimagic squares, we suggest to the readers the two sites [1], [2] where one can find a good collection of work, papers, books, etc. The idea of universal bimagic square is presented for the first time here.

References

[1] http://www.multimagie.com/indexengl.htm.

[2] http://recmath.org/Magic Squares.
[3] I.J. TANEJA – DIGITAL ERA: Magic Squares and 8th May 2010 (08.05.2010),
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[4] I.J. TANEJA – ERA DIGITAL E 50 ANOS DA UFSC,
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[5] I.J. TANEJA – DIGITAL ERA: Universal bimagic squares – Under preparation.