Model of the Stochastic Vacuum and QCD Parameters

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Abstract

Accounting for the two independent correlation functions of the QCD vacuum, we improve the simple and consistent description given by the model of the stochastic vacuum to the high-energy pp and ¯pp data, with a new determination of parameters of non-perturbative QCD. The increase of the hadronic radii with the energy accounts for the energy dependence of the observables.

PACS Numbers : 12.38 Lg , 13.85 -t, 13.85 Dz , 13.85 Lg .
1. Profile Function for Hadron-Hadron Scattering

In the effort to explain the properties of soft high-energy scattering in a non-perturbative QCD framework, we reconsider the application of the model of the stochastic vacuum of non-perturbative QCD to hadron-hadron scattering. This approach combines the parameters describing properties of the QCD field (gluon condensate, correlation length) with those describing the colourless hadrons. The model presents the characteristic features of the pomeron exchange mechanism of Regge phenomenology, where vacuum quantum numbers are exchanged between hadronic structures. For all hadronic systems the total cross-sections increase with the energy somewhat between \( s^{0.0808} \) and \( \log^2(s/s_0) \) within the present experimental range, and this is explained in terms of the energy dependence of the effective hadronic radii.

Several models relate the total high-energy cross-sections and slope parameters to the hadronic radii. This is a characteristic feature also of the model of the stochastic vacuum which gives specific predictions for the size dependence of the observables for different hadronic systems. The knowledge of the hadronic structures required for the description of the soft high-energy data does not go beyond the information on their sizes, the simplest transverse wave-function giving all information required for the determination of the observables.

The model of the stochastic vacuum deals successfully with non-perturbative effects both in low-energy hadron physics and in soft high-energy scattering. The treatment of scattering is based on the concept of loop-loop interaction, which allows a gauge-independent formulation for the amplitudes. The loops, formed by the quark and antiquark light-like paths in a moving hadron, have their contributions added incoherently, with their sizes weighed by transverse hadronic wave-functions.

The principles and methods used for the evaluation of the observables of high-energy scattering in the model of the stochastic vacuum have been fully described before. In the present work we present the results of a more complete calculation in which both Abelian and non-Abelian contributions to the field correlator are taken into account. The dynamics
is generated by correlations in the QCD vacuum field, according to the pioneering work of Nachtmann. With the assumption that the correlator is independent of the reference point of the gauge field, its most general form contains two tensor structures, each with a correlation function, with a weight parameter $\kappa$ between them.

To write the expression for the profile function for hadron-hadron scattering, we introduce the notation $\vec{R}(I, J)$, where the first index $(I=1,2)$ specifies the loop, and the second specifies the particular quark or antiquark $(J=1$ or 2) in that loop. Fig.1 shows a projection of the collision of two mesons on the transverse scattering plane. The vectors $\vec{Q}(K, L)$ in the transverse plane connect the reference point $C$ to the positions of the quarks and antiquarks of the loops 1 and 2. The quantity $\psi(K, L)$ is the angle between $\vec{Q}(1, K)$ and $\vec{Q}(2, L)$.

The eikonal function of the loop-loop amplitude is written

$$\tilde{\chi}(\vec{b}, \vec{R}(1, 1), \vec{R}(2, 1)) = \kappa \left[ -\cos\psi(1, 1) I[Q(1, 1), Q(2, 1), \psi(1, 1)] 
- \cos\psi(2, 2) I[Q(1, 2), Q(2, 2), \psi(2, 2)] 
+ \cos\psi(1, 2) I[Q(1, 1), Q(2, 2), \psi(1, 2)] 
+ \cos\psi(2, 1) I[Q(1, 2), Q(2, 1), \psi(2, 1)] \right] 
+ (1 - \kappa) \left[ - W[Q(1, 1), Q(2, 1), \psi(1, 1)] - W[Q(1, 2), Q(2, 2), \psi(2, 2)] 
+ W[Q(1, 1), Q(2, 2), \psi(1, 2)] + W[Q(1, 2), Q(2, 1), \psi(2, 1)] \right].$$

(1)

The quantities $I$ which represent the non-Abelian contributions are given by integrations along the dashed lines of the figure

$$I[Q(1, K), Q(2, L), \psi(K, L)] = \frac{32}{9\pi} \left( \frac{3\pi}{8} \right)^2 \times \left\{ Q(1, K) \int_0^{Q(2, L)} \left[ Q(1, K)^2 + x^2 - 2xQ(1, K)\cos\psi(K, L) \right] dx 
+ Q(2, L) \int_0^{Q(1, K)} \left[ Q(2, L)^2 + x^2 - 2xQ(2, L)\cos\psi(K, L) \right] dx \right\},$$

(2)
with $Q(K, L) = |\tilde{Q}(K, L)|$. The quantities $W$, which come from the non-confining part of the correlator, are given by

$$W[Q(1, K), Q(2, L), \psi(K, L)] = \frac{32}{9\pi} \frac{2\times 3\pi}{8} \times [Q(1, K)^2 + Q(2, L)^2 - 2Q(1, K)Q(1, L)\cos\psi(K, L)]^{3/2}$$

$$K_3\left[\frac{3\pi}{8}\sqrt{Q(1, K)^2 + Q(2, L)^2 - 2Q(1, K)Q(2, L)\cos\psi(K, L)}\right].$$

(3)

From the eikonal function $\tilde{\chi}$ we form the loop-loop amplitude $\tilde{J}_{L_1 L_2}(\tilde{b}, \tilde{R}_1, \tilde{R}_2)$ through

$$\tilde{J}_{LL'}(\tilde{b}, \tilde{R}_1, \tilde{R}_2) \equiv \frac{1}{[g^2 FF]^2} J_{LL'}(\tilde{b}, \tilde{R}_1, \tilde{R}_2) = -\frac{[\tilde{\chi}(\tilde{b}, \tilde{R}_1, \tilde{R}_2)]^2}{144 \cdot 576},$$

(4)

where $\tilde{R}_1$ and $\tilde{R}_2$ are shorthand notations for $\tilde{R}(1, 1)$ and $\tilde{R}(2, 1)$ respectively.

The hadron-hadron amplitude is constructed from the loop-loop amplitude using a simple quark model for the hadrons, by smearing over the values of $\tilde{R}_1$ and $\tilde{R}_2$ with transverse wave-functions $\psi(\tilde{R})$. Taking into account the results of the previous analysis of different hadronic systems, in the present calculation we only consider for the proton a diquark structure, where the proton is described as a meson, in which the diquark replaces the antiquark. Thus these expressions apply equally well to meson-meson, meson-baryon and baryon-baryon scattering.

For the hadron transverse wave-function we make the simple ansatz

$$\psi_H(R) = \sqrt{2/\pi} \frac{1}{S_H} \exp (-R^2/S_H^2),$$

(5)

where $S_H$ is a parameter for the hadron size. We then write the reduced profile function of the eikonal amplitude

$$\tilde{J}_{H_1 H_2}(\tilde{b}, S_1, S_2) = \int d^2\tilde{R}_1 \int d^2\tilde{R}_2 \tilde{J}_{L_1 L_2}(\tilde{b}, \tilde{R}_1, \tilde{R}_2) |\psi_1(\tilde{R}_1)|^2 |\psi_2(\tilde{R}_2)|^2,$$

(6)

which is a dimensionless quantity. The dimensionless scattering amplitude is given by

$$T_{H_1 H_2} = is[(g^2 FF)a^4]a^2 \int d^2\tilde{b} \exp (i\tilde{q} \cdot \tilde{b}) \tilde{J}_{H_1 H_2}(\tilde{b}, S_1, S_2),$$

(7)

where the impact parameter vector $\tilde{b}$ and the hadron sizes $S_1$, $S_2$ are in units of the correlation length $a$, and $\tilde{q}$ is the momentum transfer projected on the transverse plane, in
units of $1/a$, so that the momentum transfer squared is $t = -|\vec{q}|^2/a^2$. For convenience we have explicitly factorized the dimensionless combination $\langle g^2 FF \rangle a^4$. The normalization for $T_{H_1H_2}$ is such that the total and differential cross-sections are given by

$$
\sigma^T = \frac{1}{s} \Im T_{H_1H_2}, \quad \frac{d\sigma^{\ell\ell}}{dt} = \frac{1}{16\pi s^2} |T_{H_1H_2}|^2. \tag{8}
$$

For short, from now on we write $J(b)$ or $J(b/a)$ to represent $\hat{J}(\vec{b},S_1,S_2)$. To write convenient expressions for the observables, we define the dimensionless moments of the profile function (as before, with $b$ in units of the correlation length $a$)

$$
I_k = \int d^2\vec{b} \ b^k \ J(b), \quad k = 0, 1, 2, ...
$$

which depend only on $S_1/a$, $S_2/a$, and the Fourier-Bessel transform

$$
I(t) = \int d^2\vec{b} \ J_0(ba \sqrt{|t|}) \ J(b), \tag{10}
$$

where $J_0(ba \sqrt{|t|})$ is the zeroth-order Bessel function. Then

$$
T_{H_1H_2} = is[\langle g^2 FF \rangle a^4]^2 a^2 I(t).
$$

Since $J(b)$ is real, the total cross section $\sigma^T$ and the slope parameter $B$ (slope at $t = 0$) are given by

$$
\sigma^T = I_0 \ [\langle g^2 FF \rangle a^4]^2 a^2, \quad B = \frac{d}{dt} \left( \ln \frac{d\sigma^{\ell\ell}}{dt} \right) \bigg|_{t=0} = \frac{1}{2} \frac{I_2}{I_0} a^2 \equiv Ka^2. \tag{11}
$$

The model conveniently factorizes the QCD strength and length scale in the expressions for the observables, and the correlation length appears as the natural unit of length for the geometric aspects of the interaction. These aspects are contained in the quantities $I_0(S_1/a,S_2/a)$ and $I_2(S_1/a,S_2/a)$, which depend on the hadronic structures and on the shapes and relative weights (parameter $\kappa$) of the two correlation functions. It has been shown before that for the case $\kappa = 1$ (pure confining correlator) the two moments have simple form as functions of $S/a$, and in the present work we have obtained similarly simple expressions for any value of $\kappa$. We concentrate on the range about $\kappa = 3/4$, which is indicated by lattice results.
It is important that the high-energy observables $\sigma^T$ and $B$ require only the two low moments $I_0$, $I_2$ of the profile functions. We observe that in the lowest order of the correlator expansion the slope parameter $B$ does not depend on the value of the gluon condensate $\langle g^2 FF \rangle$ and, once the proton radius $S$ is known, it may give a direct determination of the correlation length.

2. pp and $\bar{p}p$ systems and QCD Parameters

In order to have a wide range of data to extract reliable information on QCD parameters, we concentrate here on the pp and $\bar{p}p$ systems, for which $S_1 = S_2 = S$. The curves for $I_0 = \sigma^T / [\langle g^2 FF \rangle^2 a^{10}]$ and $K = B/a^2$ can be parametrized as functions of $S/a$ with simple powers, with good accuracy. The convenient expressions are

$$I_0 = \alpha \left( \frac{S}{a} \right)^\beta, \quad K = \eta + \gamma \left( \frac{S}{a} \right)^\delta.$$  \hspace{1cm} (12)

The numerical values for the parameters are given in table 1.

The parametrizations for $\sigma^T$ and $B$ are very convenient for comparison of the results of the model of the stochastic vacuum with experiment. In the present analysis we take into account all available data on $\sigma^T$ and $B$ in pp and $\bar{p}p$ scattering, which consist mainly of ISR (CERN) measurements at energies ranging from $\sqrt{s} = 23$ GeV to $\sqrt{s} = 63$ GeV, of the $\sqrt{s} = 541 - 546$ GeV measurements in CERN SPS and in Fermilab, and of the $\sqrt{s} = 1800$ GeV information from the E-710 Fermilab experiment. Besides these, there is a measurement of $\sigma^T = 65.3 \pm 2.3$ mb at $\sqrt{s} = 900$ GeV and there are the measurements of $\sigma^T = 80.6 \pm 2.3$ and $B = 17.0 \pm 0.25$ GeV$^{-2}$ in Fermilab CDF at $\sqrt{s} = 1800$ GeV which seem discrepant with the E-710 experiment at the same energy. A measurement by Burq et al. at $\sqrt{s} = 19$ GeV, which was taken as input in a previous calculation, seems to disagree with the ISR data, presenting a too high slope $B = 12.47 \pm 0.10$ GeV$^{-2}$.

In fig.2 we plot $\sigma^T$ and $B$ against each other. At the ISR energies we use $\sigma_{\text{pom}} = (21.70 \text{ mb}) s^{0.0808}$ and the pp slope parameter as representative of the non-perturbative contributions which are the concern of the model of the stochastic vacuum. At the highest ener-
gies (541-1800 GeV) the process is essentially non-perturbative and dominated by pomeron exchange. The relation between the two observables is fitted with the form

$$ B = B_\Delta + C_\Delta (\sigma^T)^\Delta. \quad (13) $$

This form, which is represented by the solid line in the figure, is suggested by the results of the calculations with the model of the stochastic vacuum \[1\], where an interpretation for the meaning of the parameters is given in terms of QCD and hadronic quantities.

The parameters in the model that are fundamentally related to QCD are the weight $\kappa$, the gluon condensate $\langle g^2 FF \rangle$ and the correlation length $a$. The hadronic extension parameter $S_H$ accounts for the energy dependence of the observables. We here show how these quantities can be evaluated using only high-energy scattering data.

The proton radius can be eliminated from eqs(11) and (12), and we obtain a relation between the observables $\sigma^T$ and $B$ at a given energy

$$ (B - \eta a^2) = \frac{a^2}{(\langle g^2 FF \rangle a^4)^{2\delta/\beta}} \frac{\gamma}{a^{\delta/\beta}} \left( \frac{\sigma^T_{\text{pom}}}{a^2} \right)^{\delta/\beta}. \quad (14) $$

This form is the same as given by eq.(13), with an obvious correspondence of parameters. We first remark that the exponent $\Delta = \delta/\beta$ does not depend on QCD quantities and is almost constant (equal to about 0.75) in the range of values of $\kappa$ that are obtained in lattice calculations ($\kappa \approx 3/4$). Then we fix $\Delta = \delta/\beta = 0.75$ and are left with only two free quantities in both energy independent relations (13) and (14), and they can be determined using as input the two clean experimental points for $\sigma^T$ and $B$ at 541 and 1800 GeV. The results for the parameters are nearly the same as obtained in a free fitting of all points. With $\sigma^T$ in mb we have

$$ B_\Delta = \eta a^2 = 5.38 \text{ GeV}^{-2} = 0.210 \text{ fm}^2, \quad C_\Delta = 0.458 \text{ GeV}^{-2}. \quad (15) $$

Of course these results are subject to uncertainties. We have adopted an ansatz for the correlation function, which is arbitrary (although numerically it could not have very different shape). There is some uncertainty also in the determination of the parameters.
\(\alpha, \beta\ldots\) representing the final results of the numerical calculation. On the other hand, the model gives a rather unique prediction for \(\Delta = \delta/\beta = 0.75\) and this value is well sustained by the data as shown in fig.2.

We have made calculations with several values of \(\kappa\), but to be specific, we borrow from lattice calculation the value \(\kappa = 3/4\), and then use as parameter values the numbers shown in table 1. Taking into account the experimental error bars in the input data at 541 and 1800 GeV, we obtain

\[
\kappa = \frac{3}{4}, \quad a = 0.32 \pm 0.01 \text{ fm}, \quad <g^2FF> a^4 = 18.7 \pm 0.4, \quad <g^2FF> = 2.7 \pm 0.1 \text{ GeV}^4.
\]  
(16)

With the value \(\kappa = 33/40\) obtained in more recent lattice results, the central values change only slightly to

\[
\kappa = \frac{33}{40}, \quad a = 0.33 \text{ fm}, \quad <g^2FF> a^4 = 19.2, \quad <g^2FF> = 2.6 \text{ GeV}^4.
\]  
(17)

The results of the pure \(SU(3)\) lattice gauge calculation for the correlator \(\langle F_{\mu \nu}(x, 0) F_{\rho \sigma}(0, 0) \rangle_A\) have been fitted with the same correlation function used in the present work. The correlation between the values of \(<g^2FF>\) and \(a\) that was then obtained can be well represented by the empirical expressions

\[
\Lambda_L = 1.1122 \ a^{1.310}, \quad <g^2FF> = \frac{0.01813}{a^{4.656}}, \quad <g^2FF> a^4 = 0.0172 \sqrt{\Lambda_L},
\]  
(18)

with the lattice parameter \(\Lambda_L\) in MeV, \(a\) in fm, and \(<g^2FF>\) in \(\text{GeV}^4\). This correlation is displayed in fig.3, for values of \(\Lambda_L\) in the usual range. The point representing our results in eq.(16) is marked in the same figure. The dashed line represents the relation with the string tension obtained in the application of the model of the stochastic vacuum to hadron spectroscopy; for our form of correlator, this relation is

\[
\kappa \langle g^2FF \rangle = \frac{81\pi}{8a^2\rho}.
\]  
(19)

As can we may see in the figure, the constraints from these three independent sources of information are simultaneously satisfied, building a consistent picture of soft high-energy \(pp\)
and \( \bar{p}p \) scattering. The (pure gauge) gluon condensate is well compatible with the expected value. The lattice parameter \( \Lambda_L \) and the string tension \( \rho \) are also in their acceptable ranges. As we describe below, the resulting proton size parameter \( S_p \) takes values quite close to the electromagnetic radius. The lattice parameter \( \Lambda_L \) and the string tension \( \rho \) are also in their acceptable ranges.

In this model the increase of the observables with the energy is due to a slow energy dependence of the hadronic radii. An explicit relation is obtained if we bring into eqs. (11) and (12) a parametrization for the energy dependence of the total cross-sections, such as the Donnachie-Landshoff form. In this case we obtain for the proton radius

\[
S_p(s) = a \frac{1}{\alpha^{1/\beta}} \frac{1}{\left( g^2 FF a^4 \right)^{1/2/\beta}} \left( \frac{21.7 \text{ mb}}{a^2} \right)^{1/\beta} s^{0.0808/\beta} .
\]

The energy dependence, given by a power \( 0.0808/\beta \) of \( s \) is very slow, and the values obtained for \( S_p \) are in the region of the proton electromagnetic radius, which is \( R_p = 0.862 \pm 0.012 \) fm. However the Donnachie-Landshoff parametrization for the total cross-sections is very convenient to identify the pomeron contributions at the ISR energies, but does not give the best representation of the cross-sections at higher energies. Using eqs. (11) and (12) and directly the data at 541 and 1800 GeV, we obtain the values for the proton radius that are shown by the small squares in fig.4, where a log scale is used for \( \sqrt{s} \). It is remarkable that we have an almost linear dependence, which can be represented by

\[
S_p(s) = 0.671 + 0.057 \log \sqrt{s} \text{ (fm)} ,
\]

with \( \sqrt{s} \) in GeV. With this form for the radius, which is shown in dashed line in fig.4, the cross-sections evaluated at very high energies rise with a term \( \log^{\beta} \sqrt{s} \), and are smaller than predicted by the power dependence of Donnachie-Landshoff. However, since \( \beta \approx 2.8 \), they still violate the bound \( \log^2 \sqrt{s} \). This may be corrected using a power \( 2/\beta \) instead of 1 in the parametrization for \( S_p(s) \), and we then obtain

\[
S_p(s) = 0.572 + 0.123 \left[ \log \sqrt{s} \right]^{0.72} \text{ (fm)} .
\]

This form is shown in solid line in fig.(4). Clearly it gives a good representation for the existing data. At 14 TeV, which is the expected energy in the future LHC experiments,
we obtain \( S_p(14 \text{ TeV}) = 1.19 \text{ fm} = 1.38 \) \( R_p = 3.7a \) and the model predictions for the observables are \( \sigma^T = 92 \text{ mb} \) and \( B=19.6 \text{ GeV}^{-2} \). The dashed line representing eq.(21) leads at the same LHC energy to \( \sigma^T = 97 \text{ mb} \) and \( B=20.1 \text{ GeV}^{-2} \), while the Donnachie-Landshoff formula leads to a higher value \( \sigma^T = 101.5 \text{ mb} \). A recent fit of all data gives a power \( 2.25 \pm 0.35 \) in the logarithm, and predicts \( \sigma^T = 112 \pm 13 \text{ mb} \) at 14 GeV.

The non-perturbative QCD contributions to soft high-energy scattering are expected to be dominant in the forward direction, thus determining the total cross-section (through the optical theorem) and the forward slope parameter. The model, as it is presented in this paper, leads to a negative curvature for the slope \( B(t) \), which decreases as \( |t| \) increases. The data however shows an almost zero curvature of the peak, so that above some value of \( |t| \) the model leads to too high values of the differential cross-section. Rather small changes in the form of the profile function \( J(b) \) that enters in the expression for the scattering amplitude may modify this behaviour. These changes can be made phenomenologically, introducing a form factor. However, in the present work we wish to keep the fundamental characteristics of the model, which is that of a pure QCD based calculation, with a unique set of quantities governing all systems at all energies.

3. Conclusions

Extending the previous calculation of high-energy observables, now including more data in the analysis, and taking into account the two independent correlators of the QCD gluon field, we give a unified and consistent description of all data on total cross-section and slope parameter for the pp and \( \bar{p}p \) systems, from \( \sqrt{s} \approx 20 \) to 1800 GeV. The non-perturbative QCD parameters determining the observables, the gluon condensate and the correlation length of the vacuum field fluctuations, are determined. The third quantity entering the calculations is the transverse hadron size, which has a magnitude close to the electromagnetic radius, and whose slow variation accounts for the energy dependence of the observables.

The model allows a very convenient factorization between the QCD and hadronic sectors. The relation between \( \sigma^T \) and \( B \) obtained by elimination of the hadron size parameter agrees
very well with experiment. Starting from two experimental energies as input (541 and 1800 GeV), this expression allows a determination of the correlation length and gluon condensate from high-energy data alone and gives good prediction of the remaining data. The results obtained are in good agreement with the correlations between the two QCD parameters obtained independently in lattice calculations and in the application of the stochastic vacuum model to hadronic spectroscopy.

The present calculation is restricted to the lowest order non-vanishing contribution in the expansion of the exponential with functional integrations, and we may conclude from our results that this is justified for the evaluations of total cross-section and slope parameter. The resulting amplitude is purely imaginary, and the \( \rho \)-parameter (the ratio of the real to the imaginary parts of the elastic scattering amplitude) can only be described if we go one further order in the contributions to the correlator.

It is interesting to compare our results with Regge phenomenology, where the relation between the observables is given by \( \sigma_{\text{Regge}}^T = \sigma_0^T e^{0.1616(B-B_0)} \), obtained from a Regge amplitude using the slope of the pomeron trajectory \( \alpha'(0)_{\text{pom}} = 0.25 \text{ GeV}^{-2} \). This relation requires an input pair \( \sigma_0^T \), \( B_0 \) at a chosen energy. Using as input the \( \sqrt{s} = 541 \text{ GeV} \) data \( \sigma_0^T = 62.20 \text{ mb} \), and \( B_0 = 15.52 \text{ GeV}^{-2} \) the line passes close to the CDF experimental point, instead of the E-710 point. If instead one uses as input \( \sigma_0^T \) and \( B_0 \) from the E-710 experiment at \( \sqrt{s} = 1800 \text{ GeV} \), the line shows a marked deviation at 541 GeV. This is rather intriguing, as it implies that the Regge formula favors the CDF experimental results at 1800 GeV.

**Acknowledgements**

Part of this work has been done while one of the authors (EF) was visiting CERN, and he wishes to thank the Theory Division for the hospitality. Both authors thank CNPq (Brazil) and FAPERJ (Rio de Janeiro, Brasil) for financial support. The authors are grateful to H. G. Dosch for discussions and criticism.
REFERENCES

1 H.G.Dosch, E.Ferreira and A.Krämer, *Phys. Lett.* **B289** (1992) 153; *Phys. Lett.* **B318** (1993) 197; *Phys. Rev. D** **50** (1994) 192.

2 A.Donnachie and P.V.Landshoff, *Phys. Lett.* **B296** (1992) 227.

3 A.Bueno and J.Velasco, *Phys. Lett.* **B380** (1996) 184.

4 B.Povh and J.Hüfner, *Phys. Rev. Lett.* **58** (1987) 1612, *Phys. Lett.* **B215** (1988) 772 and **B245** (1990) 653, *Phys. Rev. D** **46** (1992) 990.

5 C.Bourrely, J.Soffer and T.T.Wu, *Nucl. Phys.* **B247** (1984) 15; *Phys. Rev. Lett.* **54** (1985) 757; *Phys. Lett.* **B196** (1987) 237; J.Dias de Deus and P.Kroll, *Nuovo Cimento* **A37** (1977) 67, *Acta Phys. Pol.* **B9** (1978) 157; *J. Phys.* **G9** (1983) L81; P.Kroll, *Z. Phys.* **C15** (1982) 67; T.T.Chou and C.N.Yang, *Phys. Rev.* **170** (1968) 1591 and **D19** (1979) 3268; *Phys. Lett.* **B128** (1983) 457 and **B244** (1990) 113; E.Levin and M.G.Ryskin, *Sov. J. Nucl. Phys.* **34** (1981) 619.

6 H.G.Dosch, *Phys. Lett.* **B190** (1987) 177; H.G.Dosch and Yu.A. Simonov, *Phys. Lett.* **B205** (1988) 339.

7 O.Nachtmann, *Ann. Phys.* **209** (1991) 436.

8 Main references to the data.
   (a) N. Amos et al, *Nucl. Phys.* **B262** (1985) 689;
   (b) R. Castaldi and G. Sanguinetti, *Ann. Rev. Nucl. Part. Sci.* **35** (1985) 351;
   (c) C. Augier et al, *Phys. Lett.* **B316** (1993) 448;
   (d) M. Bozzo et al, *Phys. Lett.* **B147** (1984) 392; M. Bozzo et al, *Phys. Lett.* **B147** (1984) 385;
   (e) N. Amos et al, *Phys. Lett.* **B247** (1990) 127; *Phys. Rev. Lett.* **68** (1992) 2433; Fermilab-
9 Data Compilations

Particle Data Group, Review of Particle Properties, *Phys. Rev. D50* (1994) number 3, part I;

Review Articles

L.L. Jenkovszky, *Fort. Phys.* **34** (1986) 791;

M.M. Block and R.N. Cahn, *Rev. Mod. Phys.* **57** (1985) 563.

10 G. Alner, *Zeit. Phys.* **C32** (1986) 153.

11 F. Abe et al. *Phys. Rev. D50* (1994) 5550; *Phys. Rev. D50* (1994) 5518.

12 J.B. Burq et al, *Nucl. Phys. B217* (1983) 285.

13 A. Di Giacomo and H. Panagopoulos, *Phys. Lett. B285* (1992) 133.

14 A. Di Giacomo, E. Maggiolaro and H. Panagopoulos, [hep-lat/9603017](http://arxiv.org/abs/hep-lat/9603017), March 1996.

15 Proton radius: G.G. Simon et al. *Z. Naturforschung 35A* (1980) 1.

16 U. Grandel and W. Weise, *Phys. Lett. B356* (1995) 567.
Fig.1 - Geometrical variables of the transverse plane, which enter in the calculation of the eikonal function for meson–meson scattering. The points $C_1$ and $C_2$ are the meson centres. In the integration, $P_2$ runs along the vector $\vec{Q}(2,1)$, changing the length $z$, which is the argument of the characteristic correlator function. In analogous terms, points $P_1$, $\bar{P}_1$ and $\bar{P}_2$ run along $\vec{Q}(1,1)$, $\vec{Q}(1,2)$ and $\vec{Q}(2,2)$. This explains the four terms that appear inside the brackets multiplying $\kappa$ in the expression for the loop-loop amplitude. The length $z'$ of the dot-dashed line is the argument of the Bessel function arising from the non-confining correlator $D_1$; there are four such terms, appearing inside the brackets multiplying $(1 - \kappa)$.

Fig.2 - Relation between the two experimental quantities of the pp and $\bar{p}p$ systems. The values of $\sigma_T$ at energies up to 62.3 GeV shown in this figure are the $\sigma_{pom}^T$ values as given by the parametrization $\sigma_{pom}^T = (21.70 \text{ mb}) s^{0.0808}$ of Donnachie-Landshoff. The values taken for $B$ at the ISR energies are those for the pp system. The solid line represents $B = B_\Delta + C_\Delta (\sigma_T)^\Delta$, with values for $\Delta$, $B_\Delta$ and $C_\Delta$ obtained by fitting the data. The point at 1800 GeV used in the determination of the line is taken from the Fermilab E-710 experiment, but the CDF point is also shown.

Fig.3 - Constraints on the values of $\langle g^2 FF \rangle$ and of the correlation length $a$. The solid line is the fit of our correlator to the lattice calculation as given in eq. (13). The dashed line plots eq. (19), with $\rho = 0.16 \text{ GeV}^2$. The cross centered at $a = 0.32 \text{ fm}$, $\langle g^2 FF \rangle = 2.7 \text{ GeV}^4$ shows our results.

Fig.4 - Energy dependence of the proton radius. The marked points are obtained from the total cross-section data (at the ISR energies the total cross-sections are represented by the pomeron exchange contributions). The two representations for the radius dependence are indistinguishable with the present data, but give quite different predictions for the cross-section values at the LHC energies.
Table 1. Values of the parameters for eq.(12) for $\kappa = 3/4$ and $\kappa = 1$.

| $\kappa$ | $\alpha \times 10^2$ | $\beta$ | $\eta$ | $\gamma$ | $\delta$ | $\delta/\beta$ |
|----------|----------------------|---------|--------|---------|---------|---------------|
| $3/4$    | 0.6532               | 2.791   | 2.030  | 0.3293  | 2.126   | 0.762         |
| 1.0      | 0.6717               | 3.029   | 1.859  | 0.3118  | 2.183   | 0.721         |
$B = 5.38 + 0.46 \left( \sigma_{pom}^T \right)^{0.75}$
$S_p$ (fm)

$\sqrt{s}$ (GeV)

Proton e.m. radius = 0.862 fm

Solid line: $S_p = 0.572 + 0.123 (\log \sqrt{s})^{0.72}$ (fm)

Dashed line: $S_p = 0.671 + 0.057 \log \sqrt{s}$ (fm)