Muon Oscillations

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Abstract

Muons produced via the $\pi \rightarrow \mu \nu_\mu$ decay are in an entangled superposition of energy states because the $\nu_\mu$ is not a mass eigenstate. This presents an opportunity to access neutrino mixing parameters via muon decay. The oscillation period is long compared to the muon lifetime which presents some experimental challenges.

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Neutrino masses and mixings have become an area of intense study. Access to the neutrino parameters via experiments in which neutrinos do not need to be observed at large distances would permit verification and more precise measurements of the mixing parameters.

In the two body decay $\pi \rightarrow \mu \nu_\mu$ the recoiling muon has an energy given by:

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} - \frac{m_\nu^2}{2m_\pi}$$

Where $m_\nu$ is the recoiling neutrino mass. Since the $\nu_\mu$ is not a mass eigenstate the recoiling muon will be in an entangled superposition of closely spaced energy states.

If:

$$|\nu_\mu > = \cos(\theta)|\nu_1 > + \sin(\theta)|\nu_2 >$$

one expects:

$$|\mu(t)\nu > = e^{-(\Gamma/2+iE_1)t} \cos(\theta)|\mu(0) > |\nu_1 > + e^{-(\Gamma/2+iE_2)t} \sin(\theta)|\mu(0) > |\nu_2 >$$

A two component model has been used for illustration. Neutrino mixing seems to be dominated by two component mixing. Extension to three components is straightforward.

The decay time distribution should be:

$$e^{-\Gamma t} + 2 \cos(\theta) e^{-\Gamma t} \cos((E_2 - E_1)t) < \nu_2|\nu_1 >$$

$$e^{-\Gamma t} (1 + \sin(2\theta) \cos(\frac{m_\nu^2 - m_\nu^2 t}{2m_\pi}) < \nu_2|\nu_1 >$$

$$e^{-\Gamma t} (1 + \sin(2\theta) \cos(\Delta m_\nu^2 \frac{t}{2m_\pi}) < \nu_2|\nu_1 >$$

Temporal modulation of the standard exponential decay by $1 + \sin(2\theta) \cos(\Delta m_\nu^2 \frac{t}{2m_\pi})$ is avoided due to the orthogonality of the two neutrino mass states, $< \nu_2|\nu_1 > = 0$ so a pure exponential is observed.

The other orthogonal lepton flavor state is given by:

$$|\nu_x > = -\sin(\theta)|\nu_1 > + \cos(\theta)|\nu_2 >$$

One can rewrite the $\pi$ decay final state in the flavor basis by rewriting $|\nu_1 >$ and $|\nu_2 >$ in terms of $|\nu_\mu >$ and $|\nu_x >$.

$$|\nu_1 > = \cos(\theta)|\nu_\mu > - \sin(\theta)|\nu_x >$$
\[ |\nu_2> = \sin(\theta)|\nu_\mu> + \cos(\theta)|\nu_x> \]

This gives the two body \( \pi \to \mu \nu \) decay final state as:

\[
|\mu(t)\nu> = (e^{-(\Gamma/2+iE_1)t} \cos(\theta)^2 + e^{-(\Gamma/2+iE_2)t} \sin(\theta)^2)|\mu(0)> |\nu_\mu>
+ \cos(\theta) \sin(\theta)(e^{-(\Gamma/2+iE_2)t} - e^{-(\Gamma/2+iE_1)t})|\mu(0)> |\nu_x>
\]

A measurement of a \( |\nu_\mu> \) via a charged current interaction projects out the coherent superposition of energy states:

\[
|\mu(t)> = N(e^{-(\Gamma/2+iE_1)t} \cos(\theta)^2|\mu(0)> + e^{-(\Gamma/2+iE_2)t} \sin(\theta)^2|\mu(0)>)
\]

Where \( N \) is the normalization, \( N = 1 \).

The time dependence of this \( \nu_\mu \) tagged muon state is now seen to be:

\[
e^{-\Gamma t}(1 - \frac{\sin(2\theta)^2}{2}(1 - \cos(\frac{\Delta m^2}{2m_\pi} t)))
\]

The decay time distribution of the tagged muon is no longer exponential. The time distribution is modulated in amplitude and time by factors that depend on the neutrino mixing parameters, \( \theta \) and \( \Delta m^2_\nu \).

The mixing angle is expected to be large with \( |\sin(2\theta)| \approx 1 \). With \( \Delta m^2_\nu \approx 2.6 \times 10^{-3} \) eV\(^2\)\[1, 2\] and \( m_\pi = 139.57 \) MeV\[3\] the oscillation frequency is about \( \frac{\Delta m^2_\nu}{2m_\pi} \approx 1.42 \times 10^4 \) radians/s, which yields a period of about 442 \( \mu \)s. One should expect a node in the decay time distribution at \( \frac{1}{2} \) of the period. One can define \( x_\mu = \frac{\Delta m^2_\nu}{2m_\pi \Gamma} \approx .00312 \) which is a property of muons from pion decay and not an intrinsic property of muons. Muons from other two body decays, such as, \( K \to \mu \nu \) would have a different time structure.

If one were able to reliably tag the alternative, \( |\nu_x> \) neutrino type, muons with the \( \nu_x \) tag would have a time dependence of the form:

\[
e^{-\Gamma t}(\frac{\sin(2\theta)^2}{2}(1 - \cos(\frac{\Delta m^2}{2m_\pi} t)))
\]

Notice that:

\[ ||\mu(t)>_\mu|^2 + ||\mu(t)>_x|^2 = e^{-\Gamma t} \]

We conclude that temporal modulation of the muon decay time distribution can yield, in principle, information about neutrino oscillation parameters.
The oscillation period is many times longer than the muon lifetime so intense sources would be needed to observe the modulation of the exponential decay. Comparing the $\nu_\mu$ tagged decay time distribution with the decay time distribution for all decays may eliminate some forms of systematic error.

Due to the finite width of the $\pi$ and $\mu$ the energy, $E_\mu$ given above will not be exact. But any energy is split by (almost) the same amount due to the recoiling neutrino mass differences. Splitting energies are shifted by no more than $\delta m_\pi / m_\pi \approx 10^{-16}$. So all muons should have the same oscillation period.

An earlier draft of this manuscript had assumed that the $\pi$ decay final state was a simple direct product state of the $|\mu>$ and the $|\nu_\mu>$. The entangled state reflects the true nature of the muon energy splitting and the neutrino mass.

These ideas suggest a potential alternative to long baseline neutrino experiments for the extraction of neutrino mixing parameters. A tagged neutrino beam with the ability to observe the tagging muon combined with a modest near neutrino detector to provide the neutrino identification projection would be adequate. Observing an intense muon source for periods long compared to the muon lifetime[4] provides different experimental challenges to those needed for massive long baseline neutrino detectors.

The long period relative to the decay lifetime is a result of the choice of parent and daughter decay particles. It may be possible to find two body decays with a neutrino in the final state with much more favorable conditions. Beta beams utilize electron capture of an unstable parent nucleus to yield a neutrino beam, $P \to D\nu_e$. With $P$ and $D$ being the parent and daughter nucleus respectively. Ideally one would like $D$ formed in an excited state with a decay lifetime close to $\Delta m^2 / 2E_{\nu_\mu}$. Complications can arise in the case of nuclear decays if there are multiple initial or final states possible for the decay.

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