Model of heterogeneous reinforced fiber foam concrete in bending

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Abstract. The paper considers a model of heterogeneous reinforced fiber foam concrete, given the bimodule in the elastic formulation. Based on the proposed model the analysis of stress-strain state of an I-beam of reinforced fiber foam concrete, arbitrarily supported and loaded arbitrarily. The dependence of maximum stress on the number of rods and their location in the compressed and stretched zones. The neutral line does not coincide with the main central axis in the bimodule materials, then the values of the maximum normal stresses also vary depending on the elastic moduli relations for tension and compression. This allows us to reasonably regulate the structural features of beams when the properties of fibrous concrete are taking into account. The value of the maximum normal stresses is compared with and without the bimodule material of the beam, from which it follows that the design load capacity of the beam increases with the bimodulity of the material taken into account. The magnitude of the maximum normal stresses in reinforced beams made of concrete of a cohesive structure with allowance for bimodulity was numerically investigated. It has been established that the normal tensile stress arising in a fiber-reinforced concrete beam is higher than in a beam of concrete of a fused structure by 42%. The value of the normal compressive stress is lower by an average of 38%.

1. Introduction

Reinforced concrete beams are widely used in the construction of frame structures of buildings with a height of 1 or more floors. In the structures, they provide the ability to withstand large values of bending moments. To date, these products are widely used in the construction of retail, entertainment, industrial and logistics facilities. I-beams are used for roofs and floors with long spans.

Many materials have different mechanical characteristics in tension and compression. The property of bimodularity is characterized by a significant discrepancy between the value of the elastic modulus and the Poisson's ratio under tension and compression. Bimodularity is established for alloys of cast iron, bronze, and steel. In steel, bimodularity manifests itself insignificantly, the difference in the Young's modulus under tension and compression is no more than 3-5%, in cast iron it can reach 30% [1-3]. Concrete as the widespread building material has strongly expressed property of bimodularity. For some types of fine-grained concrete, the tensile modulus is two to three times lower than under compression, for example, AFB-1 concrete: $E_C = 1.75 \times 10^4$ MPa , $E_P = 0.75 \times 10^4$ MPa [4].
Because of the establishment of bimodality of materials, it became necessary to study the influence of bimodality on the stress-strain state of structural elements and building structures. Models of bimodule materials are used to study the dynamics laminated composite cylindrical panels with various boundary conditions [5]. The mechanical characteristics of bimodule materials can be determined by nondestructive methods [6-8].

2. Materials and methods

The heterogeneous model of a reinforced beam with bimodule property is represented as a beam consisting of three parts: a stretched, compressed and reinforcing bar. Considering the beam as statically indeterminate, we get equilibrium equation for a heterogeneous beam

\[ M_y = M_{yp} + M_{yc} + M_{ya} \]  \hspace{1cm} (1)

and condition of consistency of deformations of a heterogeneous beam

\[ \frac{1}{\rho} = \frac{1}{\rho_p} = \frac{1}{\rho_c} = \frac{1}{\rho_a} \]  \hspace{1cm} (2)

where \( M_y, \frac{1}{\rho} \) - bending moment and curvature of the beam; \( M_{yp}, \frac{1}{\rho_p} \) - the bending moment and curvature of the beam of the stretched zone; \( M_{yc}, \frac{1}{\rho_c} \) - the bending moment and curvature of the beam of the compressed zone; \( M_{ya}, \frac{1}{\rho_a} \) - bending moment and curvature of reinforcing bars.

The equilibrium condition \( \sum M_y = 0 \) expressed in terms of normal stresses has the form

\[ M_y = \int_A \sigma z dA = \int_{A_p} \sigma_p z dA + \int_{A_c} \sigma_c z dA + \int_{A_a} \sigma_a z dA = M_{yp} + M_{yc} + M_{ya}, \]  \hspace{1cm} (3)

where \( \sigma_p, A_p, \sigma_c, A_c, \sigma_a, A_a \) - respectively, the normal stress and cross-sectional area of the beam of the stretched zone, the compressed zone and the reinforcement.

The proposed model of a heterogeneous bimodule material can be used for various cross-sectional shapes. In this article, the model is considered for the I-beam.

3. Results

Substituting the normal stresses \( \sigma_p = \frac{E_p z}{\rho}, \sigma_c = \frac{E_c z}{\rho}, \sigma_a = \frac{E_a z}{\rho} \) in (3), we obtain the curvature formula of the neutral line for the beam from the bimodule material

\[ M_y = \frac{1}{\rho} \left( E_p f_y + E_c f_y^c + E_a (n_p (f_y^p + A_p^p c_p^2) + n_c (f_y^c + A_c^c c_c^2)) \right) \]  \hspace{1cm} (4)

\( E_c, E_p, E_a \) - moduli of elasticity of material in the zone of compression, tension and reinforcement; \( f_y^p, f_y^c \) - the moment of inertia of that part of the cross-section that lies in the stretched and compressed zones, with respect to the neutral axis; \( f_y^{p,c}, f_y^{c} \) - the moment of inertia of the cross section of the reinforcement, which lies in the stretched and compressed zones, relative to its own central axis; \( n_p, n_c \) - number of reinforcing bars in the tension and compressed zones; \( A_{p}^{p}, A_{c}^{c} \) - cross-sectional area of the reinforcement in the stretched and compressed zones; \( c_p, c_c \) - distance from the reinforcement in the stretched and compressed zones to the neutral axis.

The general curvature formula for a beam

\[ \frac{1}{\rho} = \frac{M_y}{D} = \frac{M_{yp} + M_{yc} + M_{ya}}{D}, \]  \hspace{1cm} (5)

where \( D \) - the stiffness of the beam from a heterogeneous material.

Expression of reduced rigidity for reinforced beams of heterogeneous material
\[ D = E_p f_p^y + E_c f_c^y + E_a \left( n_p \left( j_p^y + A_p^c c_p^2 \right) + n_c \left( j_c^y + A_c^c c_c^2 \right) \right) \]  \hspace{1cm} (6)

To determine the position of the neutral line, consider one more equation of statics - the projection on the axis of the rod \( \sum F_x = 0 \)

\[ \int_A \sigma dA = \int_{A_p} \sigma_p dA + \int_{A_c} \sigma_c dA + \int_{A_a} \sigma_a dA = 0 \]  \hspace{1cm} (7)

Substituting \( \sigma_p, \sigma_c, \sigma_a \) in (7), we obtain

\[ (E_p S_p^y + E_c S_c^y + E_a (n_p A_p^c c_p + n_c A_c^c c_c)) = 0 \]  \hspace{1cm} (8)

where \( S_p^y, S_c^y \) - the static moment of that part of the cross-section that lies in the stretched and compressed zones, with respect to the neutral axis.

Formulas of normal stresses with allowance for (4), (5) have the form [5-8]

\[ \sigma_p = \frac{E_p M_y}{D} z, \quad \sigma_c = \frac{E_c M_y}{D} z, \quad \sigma_a = \frac{E_a M_y}{D} z \]  \hspace{1cm} (9)

4. Discussion

The paper considers an arbitrarily supported and arbitrary loaded reinforced concrete I-beam (Figure 1). The beam material possesses the bimodule property, i.e. the elastic moduli for tension and compression are different, but the material is isotropic. It is proved, that for such materials the hypotheses and formulas the mechanic of materials and the theory of elasticity are true.

Characteristics of fiber concrete \( E_c = 2.25 \times 10^3 \) MPa , \( E_p = 5.0 \times 10^3 \) MPa . Characteristics of reinforcement bars \( E_a = 2.06 \times 10^5 \) MPa, \( d_p = 12 \) mm, \( d_c = 8 \) mm. Cross-section characteristics \( h = 0.89 \) m; \( b_p = 0.28 \) m; \( t_p = 0.15 \) m; \( d = 0.12 \) m; \( b_c = 0.28 \) m; \( t_c = 0.08 \) m.

To determine the position of the neutral line in the I-beam, we obtain a quadratic equation with respect to \( h_c \) (the distance from the neutral line to the maximally remote point in the compressed zone)

\[ a h_c^2 + b h_c + c = 0, \]

\[ a = \frac{d}{2} (1 - k); \quad b = (k d t_c - d (h - t_p) - b_p t_p - k b_c t_c); \quad k = E_c / E_p \]

\[ c = \frac{d}{2} (h - t_p)^2 - k \frac{d}{2} t_c^2 + b_p t_p \left( h - \frac{t_p}{2} \right) + k b_c \frac{t_c^2}{2} + (n_c c_c A_c + n_p c_p A_p) \frac{E_a}{E_p}. \]
A numerical study of stress-strain state for a reinforced concrete I-beam of fiber foam concrete was carried out. The elastic moduli for tension and compression were obtained experimentally [9-11].

Table 1. Dependence of maximum normal stresses on the location and number of reinforcing rods without taking into account the bimodulity of fibro-concrete $E_c = E_p = 2.25 \cdot 10^3 \text{ MPa}$

| $n_c$ | $n_p$ | $\frac{|\sigma_{\text{max}}^p|}{M_{\text{maxy}}}$ | $\frac{|\sigma_{\text{max}}^c|}{M_{\text{maxy}}}$ | $EJ_y$ | $h_p$, m, $10^{-2}$ | $h_c$, m, $10^{-2}$ |
|-------|-------|---------------------------------|---------------------------------|--------|-----------------|-----------------|
| 0     | 0     | 33.83                           | 37.99                           | 27880  | 41.9            | 47.1            |
| 0     | 2     | 25.89                           | 35.39                           | 32680  | 37.6            | 51.4            |
| 0     | 4     | 19.35                           | 32.41                           | 38688  | 33.3            | 55.7            |
| 0     | 6     | 16.02                           | 31.11                           | 42481  | 30.2            | 58.8            |
| 2     | 2     | 24.48                           | 29.72                           | 36943  | 40.2            | 48.8            |
| 2     | 4     | 19.91                           | 28.46                           | 41394  | 36.6            | 52.4            |
| 2     | 6     | 17.06                           | 27.66                           | 44776  | 33.9            | 55.1            |
| 4     | 4     | 19.06                           | 24.57                           | 45901  | 38.9            | 50.1            |
| 4     | 6     | 16.28                           | 23.86                           | 49886  | 36.1            | 52.9            |
| 6     | 6     | 15.70                           | 21.27                           | 54162  | 37.8            | 51.2            |

Table 1 shows the dependence of the normal tensile and compressive stresses on the number of reinforcing rods in the stretched and compressed zones, without taking into account the bimodulity of fibro-foam concrete. A modulus of elasticity is accepted for the elastic modulus of elasticity under compression. As can be seen from Table 1, the compressive stress is always reduced for any arrangement of the reinforcing bars, depending on their number, and the tensile stresses depend on both the location and the number of reinforcing rods.
Table 2. Dependence of maximum normal stresses on the location and number of reinforcing rods taking into account the bimodality of fibro-concrete $E_c = 2.25 \cdot 10^5 \text{MPa}$, $E_p = 5.0 \cdot 10^5 \text{MPa}$

| $n_c$ | $n_p$ | $\sigma_{p \text{max}}^{\uparrow}$ | $\frac{1}{M_{\text{max} \gamma}}$ | $\sigma_{c \text{max}}^{\downarrow}$ | $\frac{1}{M_{\text{max} \gamma}}$ | $EJ_c$ | $h_p$, m, $10^2$ | $h_c$, m, $10^2$ |
|-------|-------|------------------|-----------------|------------------|-----------------|-------|----------------|----------------|
| 0     | 0     | 41.03            | 32.21           | 38752            | 31.8            | 57.2  |                |                |
| 0     | 2     | 36.04            | 32.36           | 41223            | 29.7            | 59.3  |                |                |
| 0     | 4     | 32.38            | 31.79           | 43196            | 28              | 61    |                |                |
| 0     | 6     | 29.08            | 31.16           | 45263            | 26.3            | 62.7  |                |                |
| 2     | 2     | 33.60            | 27.26           | 47254            | 31.8            | 57.2  |                |                |
| 2     | 4     | 30.13            | 26.74           | 49698            | 29.9            | 59.1  |                |                |
| 2     | 6     | 26.97            | 26.17           | 52280            | 28.2            | 60.8  |                |                |
| 4     | 4     | 28.44            | 23.12           | 55756            | 31.7            | 57.3  |                |                |
| 4     | 6     | 25.93            | 22.75           | 58178            | 30.2            | 58.8  |                |                |
| 6     | 6     | 24.65            | 20.07           | 64258            | 31.7            | 57.3  |                |                |

Table 2 shows that the qualitative image of the dependence of the normal stresses on number of the reinforcement bars with and without bimodule is preserved, but with allowance for bimodality, the value of the tensile normal stresses increases, and the magnitude of the compressive normal stresses decreases in comparison with the values of the corresponding quantities without taking into account the multiplicity of fibro-foam concrete.

Table 3. Dependence of maximum normal stresses on the location and number of reinforcing rods without taking into account the bimodality of fibro-concrete $E_c = E_p = 5.0 \cdot 10^5 \text{MPa}$

| $n_c$ | $n_p$ | $\sigma_{p \text{max}}^{\uparrow}$ | $\frac{1}{M_{\text{max} \gamma}}$ | $\sigma_{c \text{max}}^{\downarrow}$ | $\frac{1}{M_{\text{max} \gamma}}$ | $EJ_c$ | $h_p$, m, $10^2$ | $h_c$, m, $10^2$ |
|-------|-------|------------------|-----------------|------------------|-----------------|-------|----------------|----------------|
| 0     | 0     | 33.83            | 37.99           | 61958            | 41.9            | 47.1  |                |                |
| 0     | 2     | 29.72            | 36.66           | 67039            | 39.8            | 49.2  |                |                |
| 0     | 4     | 26.29            | 35.45           | 72077            | 37.9            | 51.1  |                |                |
| 0     | 6     | 23.87            | 34.68           | 76004            | 36.3            | 52.7  |                |                |
| 2     | 2     | 28.87            | 33.72           | 71096            | 41.1            | 47.9  |                |                |
| 2     | 4     | 25.57            | 32.63           | 76457            | 39.1            | 49.9  |                |                |
| 2     | 6     | 23.20            | 31.91           | 8074             | 37.5            | 51.5  |                |                |
| 4     | 4     | 24.93            | 30.212          | 80699            | 40.2            | 48.8  |                |                |
| 4     | 6     | 22.62            | 29.53           | 85335            | 38.6            | 50.4  |                |                |
| 6     | 6     | 22.18            | 27.68           | 89246            | 39.6            | 49.4  |                |                |

Table 3 shows the dependence of normal tensile and compressive stresses on the number of reinforcing rods in the stretched and compressed zones, without taking into account the bimodality of fibro-foam concrete. The elastic modulus of elasticity is taken as the elastic modulus of elasticity. As can be seen from Table 1 and Table 3, the compressive and tensile normal stresses are equal for an unreinforced beam. For a reinforced beam, the normal stresses are greater when the elastic modulus is equal to the tensile modulus of elasticity.
For aluminum-phosphate concrete AFB-1, in contrast to fibro foam concrete, the modulus of elasticity on compression is greater than the tensile modulus: $E_c = 1.75 \cdot 10^4 \text{MPa}, E_p = 0.75 \cdot 10^4 \text{MPa}$.

Table 4. Dependence of normal stresses on the location and number of reinforcing bars, taking into account the bimodality of AFB-1 concrete $E_c = 1.75 \cdot 10^4 \text{MPa}, E_p = 0.75 \cdot 10^4 \text{MPa}$

| $n_c$ | $n_p$ | $\frac{|\sigma_{max}^p|}{M_{max}^y}$ | $\frac{|\sigma_{max}^c|}{M_{max}^y}$ | $EJ_c$ | $h_{p, m}$ | $h_{c, m}$ |
|-------|-------|----------------------------------|----------------------------------|-------|-----------|-----------|
| 0     | 0     | 28.8                             | 45.7                             | 137981| 53.0      | 36.0      |
| 0     | 2     | 26.3                             | 44.3                             | 147258| 51.7      | 37.3      |
| 0     | 4     | 24.3                             | 43.1                             | 156183| 50.5      | 38.5      |
| 0     | 6     | 22.5                             | 42.0                             | 164710| 49.4      | 39.6      |
| 2     | 2     | 26.1                             | 43.1                             | 149794| 52.1      | 36.9      |
| 2     | 4     | 24.0                             | 41.9                             | 159116| 50.9      | 38.1      |
| 2     | 6     | 22.4                             | 41.1                             | 166771| 49.9      | 39.1      |
| 4     | 4     | 23.9                             | 40.9                             | 161251| 51.3      | 37.7      |
| 4     | 6     | 22.1                             | 39.9                             | 170157| 50.2      | 38.8      |
| 6     | 6     | 22.1                             | 39.1                             | 171929| 50.6      | 38.4      |

The normal tensile stress occurring in the fibre-foam concrete beam is higher than in the concrete beam of the AFB-1 brand by 42%, but the normal compressive stress is lower by an average of 38%. In this case, it is necessary to emphasize once again that in fibro-concrete, the modulus of elasticity on compression is an order of magnitude lower than the modulus of elasticity for compression in concrete AFB-1.

5 Conclusions
The paper considers a model of heterogeneous reinforced fiber foam concrete, given the bimodule in the elastic formulation. The use of the method of calculating reinforced beams from bimodule materials makes it possible to improve the properties of various concretes, changing the moduli of elasticity to tension and compression, based on the influence of different modularity on the strength of structural elements [12-14].

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