Study of Nuclear Radius and Back bending phenomenon of $^{20-30}$Mg Isotopes in sdpn Model Space

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Abstract: In present work, many parameters have been computed using NuShellX@ MSU code for ground state bands of $^{20-30}$Mg isotopes. These parameters were root mean square of nuclear radius, major and minor of ellipsoid axes (a and b), as well as to the different between them ($\Delta R$), deformation parameters ($\beta_2, \delta$), intrinsic quadrupole moment ($Q_0$) and back bending phenomenon. Model space(sdpn) for protons and neutrons orbits and cwhcnp interaction have been used to calculate above parameters. Effective charge of Bohr - Mottelson model has been used in this work. The results showed an brilliant agree with the obtainable investigational results all isotopes in this study. Also, the back bending phenomenon was well evident in our results for all isotopes used under study. In these calculations, new values have been theoretically designated for most parameters which were previously unknown in experimental information.

Key words: Model space, ground bands, NuShell X@ MSU Code, Deformation Parameter, back bending.

1. Introduction

In order to compute the nuclear states energies, many codes were used through higher numerical exactitude and to supply wave functions from which the calculation of other observables gets applicable. Mixing configuration of the nuclear shell-model orbits is the one of the common configurations utilized, for this object where the traditional shell-model codes provide approximately (1 keV) of numerical accuracy. In this mode the matrix is instituted upon all possible Slater determinants with diagonalization a comparatively small subset of valence nucleon orbits [1]. A set of single-particle energies and two-body interaction matrix elements or two-body matrix elements (TBME) have been represented one of essential demands of the configuration mixing calculations. These sets have recently been called effective interaction or model space Hamiltonian. Effective interaction may be distinguished in two ways: The first of them is the “realistic” which is manufactured for a given shell model space from known data on the free nucleon-nucleon force. While the second method is “empirical” that, it is established, for the parameters whose values have been specified by agreement between shell model eigenvalue and measured state energies [2,3]. Matrix components considered association to the free nucleon–nucleon interaction and only as parameters to be adjusted to accomplish convention with experimental spectra results [1,4]. The mass region $A = 17-39$ offers an ideal realm in which to explore the possibilities of accounting for experimentally observed features of nuclear arrangement with shell-model calculations[3]. Various nucleus in the main model space branch (sd) such as Mg isotopes are of interest for several reasons, like excitation spectra are sufficiently varied and moreover another many nuclear properties as electromagnetic properties, root mean four-sided of nuclear radius, main and minor of ellipsoid axes (a and b), as well as to the different between them ($\Delta R$), deformation parameters ($\beta_2, \delta$),
intrinsic quadrupole moment \(Q_0\) and also back bending phenomenon for nuclei with mass numbers \(A=20-30\) for Mg isotopes which contains of valance nucleons lies outside close core \(^{16}\)O nucleus and fill the orbits \(1d_{5/2}, 2s_{1/2}, 1d_{3/2}\) space. These isotopes, it have become a particularly interesting research in studies of nuclear structure [4] of several researchers such as: one of these studies is focused on the lowest 12 positive parity states in \(^{22}\)Ne and the lowest 15 in \(^{26}\)Mg [5], nuclear construction progress in Mg isotopes between proton and neutron trickle appearances are described by [6], also explanation of shell model calculations for negative parity states in sd - pf nuclei [7], finally study of ground-state properties that is fill of neutron Mg isotopes is studied by [8].

The main objective of this present work is to illustrate several nuclear parameters of \(^{20-30}\)Mg isotopes that have valance nucleons lie outside the closed core \(^{16}\)O nucleus, the calculations have been extracted by applying NuShell X @ MSU code.

2. Theory

Usually, the prevailing properties of the nuclei is the shape that these nucleons are disseminated across nuclei, where these nuclei can be recognized that either spherical nuclei or non-spherical nuclei called (deformation nuclei), where the distribution of charge in spherical nuclei is symmetric spherical, and these nuclei are situated near closed shells that are designated by numbers of protons or neutrons equal to (2, 8, 20, 28, 50, 82 and 126) of Mayer and Jensen and these numbers are called magic numbers and so that the nuclei with nucleons (the protons \(Z\) or neutrons \(N\)) the equals one of these numbers form closed shells and it has more stable than other nuclei also the nuclear binding energies in these nuclei are high and that these nuclei contain a large number of stable isotopes [9-11].

The distribution of electrical currents & charges produces multi-polar electrical and magnetic moments determined by \((2\lambda)\) when \((\lambda = 0)\) that is the orbital motion is equivalent to zero, then \((2^0 = 1)\) this means a monopole electrical field is generated known as the resulting Coulomb field. For the uniform spherical distribution of electric charge in the nuclei in the case of \((\lambda = 1)\), the orbital motion of the electric charge in closed orbits generates a dipole magnetic field. As well as in the case \((\lambda = 2)\), a quadrupole electric moment is produced [12-13]. Quadrupole electric moment is defined as the amount of deviation from the symmetric spherical distribution of the electrical charge inside the nucleus (protons) [14]. The quadrupole moments in spherical nuclei have been a small value that may reach zero in even-even isotopes with magic numbers, while in the case of deformed nuclei, that it is placed in the middle of the distance that is determined by closed shells, these deformed nuclei have electric moments of large values, the deformation in nuclei is increase as the quadrupole electric moment values increases, the intrinsic quadrupole electric moment is given according to the following relationship [15-16]:

\[
Q_0 = \frac{3}{\sqrt{5}} R_{av}^2 Z \beta_2 (1 + 0.36 \beta_2^2) \tag{1}
\]

The symbols in the above equation are defined as follows:

- \(R_{av}^2\): average radius of the nuclei, \(R_0 = 1.2\) fm. \(\beta_2\): deformation parameter parameter when in the first approximation is assumed as \(\beta_2 \ll 1\) and by ignoring the \(\beta_2^2\) time in eq. (1):

\[
Q_0 = \frac{3}{\sqrt{5}} R_{av}^2 Z \beta_2 \tag{2}
\]

There is a simplified relationship between \(\beta_2\) and \(\Delta R\) difference for (Semi-major axis \((b)\)) and (Semi-minor axis \((a)\)) is:

\[
\beta_2 = \frac{4}{3} \sqrt{\frac{\Delta R}{R_{av}}} \tag{3}
\]

\(\Delta R = (b - a)\) is represents the difference between (Semi-major axis \((b)\)) and (Semi-minor axis \((a)\)) in the deformed nucleus, in the case of spherical nuclei it is \((a = b)\) and the moment of the electrical quadrant \((Q_0)\) is equal to zero, when \((b > a)\) that the electrical quadrant moment \((Q_0) > 0\) of deformed nuclei and these nuclei take the prolate oval shape. While the one with \((Q_0)\) can be negative when \((Q_0) < 0\) in case \((b < a)\) and the nuclei in this case takes the oblate oval shape. Also, there are another relationship of the distortion factors \((\beta_2)\) with the decreased transmission possibility \(B\) (E2) by the formula following [14,17]:

\[
\beta_2 = \frac{4\pi}{32 R^2} \left(\frac{B(E2)}{T}\right)^{1/2} \tag{4}
\]

Where \(R^2 = 0.0144 A^{2/3}\) barn \((R:\) the nuclear charge radii \))
For a nuclear state, let $J$ be the total angular momentum and $k$ its projection on the body-fixed three-axis. The electric transition strength $B(E2)$ starting with a primary situation $|J_i, k\rangle$ to a last status $|J_f, k\rangle$ can be gets from the intrinsic moment $Q_0$ as following[18]:

$$B(E2 : J_i, k \rightarrow J_f, k) = \frac{5}{16\pi} Q_0 \langle J_i k; 20 | J_f k \rangle^2$$

(5)

Where $\langle J_i k; 20 | J_f k \rangle^2$ is represented the Clebsch-Gordan coefficient produce from the coupling of the angular momenta. The reduced electric quadrupole transition probability $B(E2)$ move likelihood, from the spin $0^+$ to first excited spin $2^+$ state is given by:

$$Q_0 = \frac{16\pi}{5} \times B(E2 ↑)$$

(6)

Manifest the coupling of the particle motion to the high-frequency quadrupole modes were supplied by the effective charge of reduced electrical quadrant transition possibility for low-energy transitions by Bohr and Mottelson formula for effective charges according as following[19,20]:

$$e_{\text{eff}}(r_0) = \{e(r_0) + e\delta e(r_0)\} = \{Z_A - 0.32 \frac{(N - Z)}{A} - 2tZ[0.32 - 0.3 \frac{(N - Z)}{A}]\}$$

(7)

nucleus shape gradation is different from sphere is represented quadru-pole distortion factor $\delta$ and may become as [21-22]

$$\delta = 0.75 \frac{Q_0}{(Z < r^2 >)}$$

(8)

The deformation parameter $\delta$ is related to the investigational quadrupole moment [21].

$$\delta \approx 0.946\beta_2$$

(9)

Value of $<r^2>$ evaluated using the following equation:

$$< r^2 > = 0.63R_0^2(1 + 10/3(\pi a_0/R_0)^2)/(1 + (\pi a_0/R_0)^2)$$

(10)

While (Semi-major axis(b)) and Semi-minor axis(a) can be calculated according following equation [20-21]:

$$a = \sqrt{\frac{< r^2 >}{3} \times (5 - \frac{28}{27})}$$

(11)

$$b = \sqrt{5(\pi a^2) - 2a^2}$$

(12)

There are method three for calculating the different between major and minor of ellipsoid axises (a,b) as following [22]:

$$\Delta R_1 = b - a$$

$$\Delta R_2 = \delta \times R_{av}$$

$$\Delta R_3 = \frac{\beta_2}{1.06} \times R_{av}$$

(13)

In 1971; back bending phenomenon has been explained by A.Johnson, et al. [23]. furthermore, they interpreted that inertia moment at specific angular momentum increases on the other hand, gradually followed a decrease of rotational energy value of some nuclear the reason for this is attributed that rotational energy is greater than the stopping power of nucleons couple of to couple[24]. This phenomena can be happened at high spins of even-even isotopes in the ground status rotational bands, which it has been studied extensively in many article by a lot of researcher such as Sharp backbending has recently been observed in the yrast structures of the transitional nuclei $^{190,192}$Pt nuclei by A.A. Raduta, et al.[25], also study the possibilities of E2 transmission in $^{198}$Pt [26]. Finally back-bending phenomena in even-even $^{110-118}$Te isotopes has been described by [27], A sudden decrease of the rotational frequency along with an anomalous increase in the moment of inertia has been found to occur in many deformed nuclei, rotational energy ($\hbar \omega$) and moment of inertia can be according as formula [24]:
\[ \hbar \omega = \frac{E(\lambda)}{\sqrt{\lambda(\lambda+1)}} \]  
\[ \frac{2\theta}{\hbar^2} = \frac{\lambda(\lambda+1)}{E(\lambda)} \]  

An equation (14) can be rewritten for ground-band and beta-band as follows [19]:

\[ \hbar \omega = \frac{E(\lambda \to \lambda-2)}{\sqrt{\lambda(\lambda+1) - \sqrt{\lambda-2}(\lambda-1)}} \]  

While the rotating energy for \( \gamma \)-band can be calculated according to the following formula:

\[ \hbar \omega = \frac{E(\lambda \to \lambda-1)}{\sqrt{\lambda(\lambda+1) - \sqrt{\lambda-1}(\lambda-2)}} \]  

For estimating the moment of inertia for ground-band and beta-band can be utilized, the following formula [24]:

\[ \frac{2\theta}{\hbar^2} = \frac{4\lambda-2}{E(\lambda \to \lambda-2)} \]  

While the moment of inertia for \( \gamma \)-band be according to the following relationship:

\[ \frac{2\theta}{\hbar^2} = \frac{2\lambda}{E(\lambda \to \lambda-1)} \]  

The symbols in above equations is refer to that \( E(\lambda \to \lambda-2) \) is determined the energy difference between any two states which have angular momentum \( \lambda \) and \( (\lambda-2) \) for ground and beta bands. While for \( \gamma \)-bands, \( E(\lambda \to \lambda-1) \) is defined that the energy difference between any two states having angular momentum \( \lambda \) and \( (\lambda-1) \)

3. Results and Discussion:
In our calculations, \( ^{20-30}\text{Mg} \) isotopes have been described by using (sdpn) model space and (cwhcdpn) interaction. NuShellX @ MSU code has been used, which showed a subset of programs have been programmed language Fortran for carrying out nuclear shell-model calculations. The researchers Brown and Rae have been written, this code can be utilized is as set of computer codes, these are used to obtain exact energies, eigenvectors and for low-lying states reduced electric quadrupole transition probability in shell model Hamiltonian matrix calculations with very large basis dimensions. It uses a J-coupled proton-neutron basis, and J-scheme matrix dimensions up to the order of 100 million [28]. Cwhcdpn is an effective interaction inculcated with NuShellX@ MSU code. Hamiltonians can be contained Coulomb force plus phenomenological isospin- nonconserving nucleon nucleon interactions. The hamiltonian parameters are isovector single-particle energies, overall Coulomb strength, isovector as well isotensor strengths of the nucleon-nucleon interaction. The matrix elements estimated with these wave functions are reduced in isospin space) that are Eigen-states of the isoscalar hamiltonian where cwhcdpn hamiltonian appropriated of the lie-states in region \( 18 \leq A \leq 34 \) range [29]. This interaction has supplied realistic sd-shell \( (0d_{5/2}, 0d_{3/2}, 1s_{1/2}) \) wave functions from 63 two-body matrix elements and three SPEs for the above subshells, Wildenthal's interaction or sd-universal called (USD) Hamiltonian is set up by fitting {380 energy data with investigational faults of 0.2 MeV or less from 66 nuclei} [30]. In this paper, It has described many properties of magnesium isotopes that whose numbers mass of range from 20 to 30, valance nucleons in \( ^{20-30}\text{Mg} \) isotopes occupying mode space \( (0d_{5/2}, 0d_{3/2}, 1s_{1/2}) \) outside \( ^{16}\text{O} \) closed core. Positive-parity states for energy levels of mixing configurations for ground band of many nuclear properties like energy levels, electric quadrupole transition probability (BE2), deformation parameters \( (\beta_2, \delta) \), rotational energy \( (\hbar \omega) \) and inertia moment \( (2\theta/\hbar^2) \) have been explained and comparing with experimental data. In addition the figures of the moment of inertia and the rotational energy of all nuclei have been drawn, these the drawn explained the back bending appearance of isotopes under this study.
3.1. Energy Levels

Table (1) displays the comparison between the calculated values and the experimental data [31-36] for energy levels at the ground states band of $^{20-30}$Mg isotopes. These calculations have been evaluated by using sdpn _model space and cwhcdpn interaction at NuShellX@MSU code. In this study, $^{20}$Mg isotope comprises two protons outside $^{16}$O closed core distributed on the (0d$^{3/2}$, 0d$^{5/2}$, and 1s$^{1/2}$) shell, while $^{22-30}$Mg isotopes have been included on two protons and neutrons equal (two, four, six, eight, and ten) respectively. In these isotopes, we have found somewhat agreement of the predicted results especially for level states (4$^+$ and 6$^+$) respectively for $^{22-28}$Mg isotopes, also confirmed spin(6$^+$) for experimental levels {8.201 and 8.439} MeV for $^{26-28}$Mg isotopes respectively. As well as, $^{20}$Mg isotopes have been designated a new levels with its spins such as {4$^+$ and 6$^+$; $^{20}$Mg}, {10$^+$; $^{22}$Mg}, {10$^+$ to 12$^+$; $^{24-26}$Mg}, and {8$^+$ to 12$^+$; $^{28-30}$Mg} in this present study. A state 8$^+$ is specified for experimental levels in unit MeV as {10.901, 11.904, and 12.958} respectively for $^{22-26}$Mg isotopes unassigned at spin and parity experimentally.

3.2. Deformation parameters ($\beta_2, \delta$) and Intrinsic quadrupole moment($Q_0$)

The expected values of intrinsic electric quadrupole moments are one of the most essential properties for measuring the nuclear deformation. From table 2., illustrated comparing the theoretic with the investigational deformation parameters results ($\beta_2, \delta$) and intrinsic quadrupole moment($Q_0$) $^{20-30}$Mg isotopes. In this work, Bohr and Mottelson formula for effective charges have been utilized for measuring all nuclear properties that above-mentioned. These comparison shows that the expected results agree well with the available experimental data [32-36, 37]. From table (2), we illustrate that the shape on $^{20-30}$Mg isotopes is prolate shape.

3.3 Root mean square of nuclear radius(Isotopes Shift):

The nuclear charge radius (rms) is represented as one of the most important nuclear properties of atomic nuclei especially nuclear ground-state of them, where it has been considered as the main key in studying the features of nuclei and theoretical models of them as well as in examining astrophysics and atomic physics [38]. Root mean square of nuclear radius was computed from equation (10) of $^{20-30}$Mg isotopes which comparison with experimental data [39] as showed in table (3). These comparison indicated that $^{24-26}$Mg isotopes for the expected theoretical results acceptable well with the available experimental data but of $^{20,22,28,30}$Mg isotopes have been appeared anew values of root mean square unassigned of experimentally. While the major and minor axes (a, b) were calculated using equations (11 and 12), respectively. The difference between them $\Delta R$ computed applying equation (13), the theoretical results are appeared in table (3). From these results showed that the $^{20-30}$Mg isotopes have a higher prolate shapes.

3.4. Rotational energy and lethargy moment

Information about the title above for each nucleolus takes part in to achieving the major significance of nuclear structure of many nuclei, which located in the periodic table. Moments of inertia and rotational energies values of $^{20-30}$Mg isotopes have been computed depended on (16) and (18) equations respectively. These predicted theoretical results were comparing with experimental values [31-36] of gamma energies, moments of inertia and rotational energies were tabulated in table (4). These comparisons have designated reasonable agreement with available experimental data[31-36]. Table (4) and the diagram is in figures (1-6) can be explained the nature of the back-bending properties of these nuclei in this study. For $^{20}$Mg isotope has been observed that there are rapidly increase of inertia moment $J=4^+$ with a decrease in the value of the rotational energy corresponding to it, and then a sudden decrease in the moment of inertia values at $J=6^+$ and an increase in the rotational energy very quickly. This is very clear that the back-bending occurred in the momentum values $J=4^+$ and $6^+$ respectively as showed fig.(1). In $^{22-30}$Mg isotopes have been observed that there is a gradual increase in the values of both the moment of inertia and the rotational energy of the above isotopes at the following momentum values {$(J=2^+, 4^+, 6^+$) for $^{22}$Mg isotope but {$(J=2^+, 4^+, 6^+$ and 8$^+$) for $^{24-30}$Mg isotopes respectively. While in $^{22-30}$Mg isotopes indicated that there are sudden decrease in the moment of inertia values at followed by an increase significantly in the rotational energy very quickly at momentum values {$(J=2^+ and 10^+$), $(J=10^+$), $(J=10^+ and 12^+$), $(J=10^+ and 12^+$) and $(J=10^+$)}
respectively of $^{22-30}$Mg isotopes. New values have been predicted each of moments of inertia rotational energies values in these present results relative to the unknown experimental values of $^{22-30}$Mg isotopes. As well as the phenomenon of back bending is explained in the following isotope in figures (2-6) Through the calculations, it is clear that this phenomenon (back bending) is more evident by increasing the number of neutrons per isotope of magnesium.

| Table (1). Illustrate the comparison between expected theoretical and experimental values of energy levels of $^{20-30}$Mg isotopes [31-36] |
|---|---|---|
| **nuclei** | **Theoretical values** | **Experimental values** |
| | ground band states | Energies(MeV) | ground band states | Energies(MeV) |
| $^{20}$Mg | $0^+_1$ | 0 | $0^+_1$ | 0 |
| & $2^+_1$ | 1.953 | $2^+_1$ | 1.598 |
| & $4^+_1$ | 3.414 | ----- | ----- |
| & $6^+_1$ | 11.011 | ----- | ----- |
| $^{22}$Mg | $0^+_1$ | 0 | $0^+_1$ | 0 |
| & $2^+_1$ | 1.450 | $2^+_1$ | 1.247 |
| & $4^+_1$ | 3.563 | $4^+_1$ | 3.308 |
| & $6^+_1$ | 6.613 | $6^+_1$ | 6.254 |
| & $8^+_1$ | 10.878 | ----- | 10.901 |
| & $10^+_1$ | 17.004 | ----- | ----- |
| $^{24}$Mg | $0^+_1$ | 0 | $0^+_1$ | 0 |
| & $2^+_1$ | 1.577 | $2^+_1$ | 1.368 |
| & $4^+_1$ | 4.635 | $4^+_1$ | 4.122 |
| & $6^+_1$ | 8.817 | $6^+_1$ | 8.114 |
| & $8^+_1$ | 11.914 | ----- | 11.904 |
| & $10^+_1$ | 19.470 | ----- | ----- |
| & $12^+_1$ | 26.862 | ----- | ----- |
| $^{26}$Mg | $0^+_1$ | 0 | $0^+_1$ | 0 |
| & $2^+_1$ | 2.041 | $2^+_1$ | 1.808 |
| & $4^+_1$ | 4.644 | $4^+_1$ | 4.318 |
| & $6^+_1$ | 8.552 | $(6^+_1)$ | 8.201 |
| & $8^+_1$ | 12.801 | ----- | 12.958 |
| & $10^+_1$ | 18.489 | ----- | ----- |
| & $12^+_1$ | 26.693 | ----- | ----- |
| $^{28}$Mg | $0^+_1$ | 0 | $0^+_1$ | 0 |
| & $2^+_1$ | 1.722 | $2^+_1$ | 1.473 |
| & $4^+_1$ | 4.445 | $4^+_1$ | 4.021 |
| & $6^+_1$ | 8.441 | $(6^+_1)$ | 8.439 |
| & $8^+_1$ | 12.516 | ----- | ----- |
| & $10^+_1$ | 19.065 | ----- | ----- |
| & $12^+_1$ | 27.233 | ----- | ----- |
| $^{30}$Mg | $0^+_1$ | 0 | $0^+_1$ | 0 |
| & $2^+_1$ | 1.815 | $2^+_1$ | 1.482 |
| & $4^+_1$ | 4.349 | $4^+_1$ | 3.381 |
| & $6^+_1$ | 8.014 | ----- | ----- |
| & $8^+_1$ | 11.867 | ----- | ----- |
| & $10^+_1$ | 20.775 | ----- | ----- |
Table 2 illustrate the comparison between expected theoretical and experimental values of deformation parameters ($\beta_2$, $\delta$) and intrinsic quadrupole moment ($Q_0$) of $^{20-30}$Mg isotopes

| Nuclei $^{12}$Mg$_N$ | Theoretical values | Experimental values |
|----------------------|---------------------|---------------------|
|                      | $b$ (fm)            | $Q_0$ theor. (b)    | $\delta$ theor. | $\beta_2$ exp. | $Q_0$ exp. | $\delta$ exp. |
| 8                    | 1.656               | 0.586               | 0.565           | 0.446         | 0.44±4     | ------         |
| 10                   | 1.682               | 0.809               | 0.831           | 0.641         | 0.67±13    | 0.60±11        |
| 12                   | 1.729               | 0.823               | 0.895           | 0.674         | 0.613±14   | 0.659±8        |
| 14                   | 1.707               | 0.729               | 0.837           | 0.618         | 0.484±11   | 0.554±12       |
| 16                   | 1.751               | 0.646               | 0.780           | 0.561         | 0.484±20   | 0.592±42       |
| 18                   | 1.771               | 0.552               | 0.697           | 0.489         | 0.41±3     | 0.544±24       |

Table 3 illustrate the comparison between expected theoretical and experimental values of root mean square and major and minor of ellipsoid axes (a and b), as well as to the different between them ($\Delta R$) of $^{20-30}$Mg isotopes [39]

| Nuclei $^{12}$Mg$_N$ | Theoretical values | Experimental values |
|----------------------|---------------------|---------------------|
|                      | $\langle r^2 \rangle^{1/2}$ (fm) | $a$ (fm) | $b$ (fm) | $\Delta R_1$ (fm) | $\Delta R_2$ (fm) | $\Delta R_3$ (fm) | $\langle r^2 \rangle^{1/2}$ (fm) |
| 8                    | 2.8112              | 1.3781             | 3.2028         | 1.8247         | 1.4528         | 1.801            | ------         |
| 10                   | 2.8460              | 0.8303             | 3.5849         | 2.7546         | 2.1553         | 2.5662           | ------         |
| 12                   | 2.8791              | 0.6973             | 3.6637         | 2.9664         | 2.3329         | 2.6875           | 3.0570        |
| 14                   | 2.9074              | 0.9233             | 3.5821         | 2.6586         | 2.1969         | 2.4448           | 3.0337        |
| 16                   | 2.9476              | 1.1127             | 3.5017         | 2.3890         | 2.0442         | 2.2551           | ------         |
| 18                   | 2.9844              | 1.3156             | 3.3853         | 2.0697         | 1.8233         | 1.9417           | ------         |
Table 4 illustrates the comparison between expected theoretical and experimental values of gamma energies, rotation frequency, and inertia moment of $^{20-30}\text{Mg}$ isotopes [31-36].

| nuclei | $\Gamma_i \rightarrow \Gamma_{f}$ | Theoretical values | Experimental values |
|--------|-------------------------------|---------------------|---------------------|
|        | $\text{Gamma Energies}$ (MeV) | $\hbar\omega$ (MeV) | $\Delta\hbar\omega$ (MeV)$^1$ | $\text{Gamma Energies}$ (MeV) | $\hbar\omega$ (MeV) | $\Delta\hbar\omega$ (MeV)$^1$ |
| $^{20}\text{Mg}$ | $2^+_1 \rightarrow 0^+_1$ | 1.953 | 0.797 | 3.072 | 1.598 | 0.652 | 3.754 |
|        | $4^+_1 \rightarrow 2^+_1$ | 1.461 | 0.722 | 9.582 | --- | --- | --- |
|        | $6^+_1 \rightarrow 4^+_1$ | 7.597 | 3.782 | 2.895 | --- | --- | --- |
| $^{22}\text{Mg}$ | $2^+_1 \rightarrow 0^+_1$ | 1.450 | 0.592 | 4.138 | 1.247 | 0.509 | 4.812 |
|        | $4^+_1 \rightarrow 2^+_1$ | 2.113 | 1.045 | 6.626 | 2.061 | 1.019 | 6.793 |
|        | $6^+_1 \rightarrow 4^+_1$ | 3.05 | 1.518 | 7.213 | 2.946 | 1.467 | 7.468 |
|        | $8^+_1 \rightarrow 6^+_1$ | 4.265 | 2.128 | 7.034 | 4.647 | 2.318 | 6.456 |
|        | $10^+_1 \rightarrow 8^+_1$ | 6.126 | 3.059 | 6.203 | --- | --- | --- |
| $^{24}\text{Mg}$ | $2^+_1 \rightarrow 0^+_1$ | 1.577 | 0.644 | 3.805 | 1.368 | 0.558 | 4.386 |
|        | $4^+_1 \rightarrow 2^+_1$ | 3.058 | 1.512 | 4.578 | 2.754 | 1.362 | 5.084 |
|        | $6^+_1 \rightarrow 4^+_1$ | 4.182 | 2.082 | 5.261 | 3.992 | 1.987 | 5.511 |
|        | $8^+_1 \rightarrow 6^+_1$ | 3.097 | 1.549 | 9.687 | 3.79 | 1.891 | 7.916 |
|        | $10^+_1 \rightarrow 8^+_1$ | 7.556 | 3.773 | 5.029 | --- | --- | --- |
|        | $12^+_1 \rightarrow 10^+_1$ | 7.392 | 3.692 | 6.223 | --- | --- | --- |
| $^{26}\text{Mg}$ | $2^+_1 \rightarrow 0^+_1$ | 2.041 | 0.833 | 2.939 | 1.808 | 0.738 | 3.319 |
|        | $4^+_1 \rightarrow 2^+_1$ | 2.603 | 1.287 | 3.578 | 2.51 | 1.241 | 5.578 |
|        | $6^+_1 \rightarrow 4^+_1$ | 3.908 | 1.946 | 5.629 | 3.883 | 1.933 | 5.666 |
|        | $8^+_1 \rightarrow 6^+_1$ | 4.249 | 2.119 | 7.060 | 4.757 | 2.373 | 6.306 |
|        | $10^+_1 \rightarrow 8^+_1$ | 5.688 | 2.840 | 6.681 | --- | --- | --- |
|        | $12^+_1 \rightarrow 10^+_1$ | 8.204 | 4.098 | 5.607 | --- | --- | --- |
| $^{28}\text{Mg}$ | $2^+_1 \rightarrow 0^+_1$ | 1.722 | 0.703 | 3.484 | 1.473 | 0.601 | 4.073 |
|        | $4^+_1 \rightarrow 2^+_1$ | 2.723 | 1.346 | 5.141 | 2.548 | 1.259 | 5.494 |
|        | $6^+_1 \rightarrow 4^+_1$ | 3.996 | 1.989 | 5.506 | 4.418 | 2.199 | 4.979 |
|        | $8^+_1 \rightarrow 6^+_1$ | 4.075 | 2.033 | 7.362 | --- | --- | --- |
|        | $10^+_1 \rightarrow 8^+_1$ | 6.549 | 3.269 | 5.802 | --- | --- | --- |
|        | $12^+_1 \rightarrow 10^+_1$ | 8.168 | 4.081 | 5.614 | --- | --- | --- |
| $^{30}\text{Mg}$ | $2^+_1 \rightarrow 0^+_1$ | 1.815 | 0.741 | 3.306 | 1.482 | 0.605 | 4.049 |
|        | $4^+_1 \rightarrow 2^+_1$ | 2.534 | 1.252 | 5.525 | 1.899 | 0.939 | 7.372 |
|        | $6^+_1 \rightarrow 4^+_1$ | 3.664 | 1.824 | 6.004 | --- | --- | --- |
|        | $8^+_1 \rightarrow 6^+_1$ | 3.853 | 1.922 | 7.786 | --- | --- | --- |
|        | $10^+_1 \rightarrow 8^+_1$ | 8.908 | 4.448 | 4.266 | --- | --- | --- |
Figure 1. Inertia moment as a function of rotational energy for \(^{20}\)Mg isotope.

Figure 2. Inertia moment as a function of rotational energy for \(^{22}\)Mg isotope.

Figure 3. Inertia moment as a function of rotational energy for \(^{24}\)Mg isotope.
Figure 4. Inertia moment as a function of rotational energy for $^{26}\text{Mg}$ isotope.

Figure 5. Inertia moment as a function of rotational energy for $^{28}\text{Mg}$ isotope.

Figure 6. Inertia moment as a function of rotational energy for $^{30}\text{Mg}$ isotope.
4. Conclusions:

Energy levels, deformation factors ($\beta_2$, $\delta$), central quadruple moments ($Q_0$), root mean square and major and minor of ellipsoid axes ($a$ and $b$), as well as to the different between them ($\Delta R$), rotational energies ($\hbar\omega$) and inertia moments ($2\hbar^2$) have been described of $^{20-30}$Mg isotopes mostly at ground state bands of them. Cwhcdpn interaction and (sdpn) space model used by applying NuShellX@MSU code. This effective interaction and space model (sdpn) have been very successful for to interpreting these properties. In our calculations, there has been well agreement of the expected theoretical results and experimental results of all isotopes. Total angular momenta and parities have been affirmed and determination of for some experimental energy levels. On other hand, new values of energy levels, deformation parameters ($\beta_2$, $\delta$), root mean square, rotational energies and inertia moments for $^{20-30}$Mg isotopes were specified through the theoretical results but these values were unspecified in the experimental data until now. These values can add more information for theoretical knowledge for all isotopes in this work. While, explained of the back bending phenomena in present study has indicated clearly $^{26}$Mg isotope at $J=6^+$, while ($^{20-30}$Mg ) isotopes have been displayed at ($J=2^+$ and $10^+$), ($J=10^+$ and $12^+$), ($J=10^+$ and $12^+$) and ($J=10^+$), respectively. All calculations of above nuclear properties for each nuclei in this study have been accomplished some information about the basic characteristics of nuclear structure of nuclei under study.

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