VELOCITY AND DENSITY SPECTRA OF THE SMALL MAGELLANIC CLOUD

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ABSTRACT

This Letter reports results on the statistical analysis of H I turbulence in the Small Magellanic Cloud (SMC). We use 21 cm channel maps, obtained with the Australia Telescope Compact Array and the Parkes telescope, and analyze the spectrum of observed intensity fluctuations as a function of the velocity slice thickness. We confirm recent predictions by Lazarian & Pogosyan on the change of the power-law index and establish the spectra of three-dimensional density and velocity. The obtained spectral indices, $-3.3$ and $-3.4$, are slightly more shallow than the predictions for the Kolmogorov spectrum. This contrasts with the predictions for the shock-type spectra that are steeper than the Kolmogorov one. The nature of the energy injection in the SMC is unclear, as no distinct energy injection scales are observed up to the entire scale of the SMC.

Subject headings: galaxies: individual (Small Magellanic Cloud) — ISM: general — ISM: kinematics and dynamics — ISM: structure — turbulence

1. INTRODUCTION

Many observations in the past decade have challenged the traditional picture of the interstellar medium (ISM). Instead of a two-level hierarchical system, consisting of clouds uniformly dispersed in the intercloud medium (the so-called standard cloud model; Spitzer 1978, p. 1), the ISM shows an astonishing inhomogeneity, with many levels of hierarchy. In order to consider real density functions in physical processes, a better understanding of the inventory and topology of the ISM, as well as of the processes responsible for their creation, is essential. Having an extremely gas-rich ISM, dwarf irregular galaxies are particularly suitable for such studies. Here we discuss the inventory of the ISM in the Small Magellanic Cloud (SMC), using the spatial power spectrum, and point to several processes that may be involved in the sculpturing of its ISM.

The SMC is a nearby,1 extremely gas-rich, dwarf irregular galaxy. Taking part in an interacting system of galaxies (with other members being the Large Magellanic Cloud [LMC] and our Galaxy), the SMC’s morphology, dynamics, and evolution are very complex. Being one of our closest neighbors, the SMC is attractive for various astrophysical aspects. The inventory of the cool ISM in the SMC was recently studied using the high-resolution radio observations of neutral hydrogen (H I; Stanimirović et al. 1999). In addition, the dust properties of the SMC were investigated in detail, as well as the relationship between the cool gas and dust, using both H I and infrared (IR) observations (Stanimirović et al. 2000).

As a dwarf, irregular galaxy, the SMC is different from our own Galaxy in many respects: its interstellar radiation field is more than 4 times stronger (Lequeux 1989), its heavy-element abundance is almost 10 times lower (Sauvage & Vigroux 1991), its dust grains are on average smaller by $\approx 30\%$ (Rodrigues et al. 1997) and significantly hotter (Stanimirović et al. 2000), and the cool atomic phase of H I is only one-half as abundant (Dickey et al. 2000). In spite of all the above, the SMC appears to have similar interstellar turbulence properties, in that it has a similar power-law index to that of the Galaxy for the two-dimensional spatial power spectrum of its H I and dust column density distributions (Stanimirović et al. 2000).

What do the two-dimensional spatial power spectra of the intensity fluctuations mean? Relating the fluctuations of intensity in position-position-velocity data cubes and the underlying three-dimensional velocity and density statistics is a problem that has been recently addressed in Lazarian & Pogosyan (2000), where it was shown that, changing the thickness of the velocity slice, one can recover both spectra of turbulent velocity and density. The SMC provides an ideal testing ground for this theory, and we apply theoretical predictions to the data. Elsewhere, we plan to apply alternative tools for turbulence studies, e.g., principal component analysis (Heyer & Schloerb 1997; Brunt & Heyer 2001), the A-variance (Stutzki et al. 1998), and the spectral correlation function (Rosolowsky et al. 1999; see review by Lazarian 1999), to the SMC data.

The structure of this Letter is organized as follows. In § 2 we describe briefly the H I observations of the SMC. In § 3 we review previous results of the statistical investigation of H I in the SMC, using the spatial power spectrum, and investigate the influence of velocity fluctuations on the intensity statistics. Comparison with earlier work and discussion on the origin of the turbulence in the SMC are given in § 4.

2. H I DATA

The small-scale H I structure in the SMC was observed with the Australia Telescope Compact Array (ATCA), a radio interferometer, in a mosaicking mode (Staveley-Smith et al. 1997). Observations of the same area were obtained also with the 64 m Parkes telescope, in order to map the distribution of large-scale features. Both sets of observations were then combined (Stanimirović et al. 1999), resulting in the final H I data cube with angular resolution of 98", velocity resolution of 1.65 km s$^{-1}$, and 1 $\sigma$ brightness temperature sensitivity of 1.3 K, to the continuous range of spatial scales between 30 pc and 4 kpc. The velocity range covered with these observations is 90–215 km s$^{-1}$. For details about the ATCA and Parkes observations, data processing, and data combination (short-
spacings correction) see Staveley-Smith et al. (1997) and Stanimirović et al. (1999).

### 3. STATISTICAL INVESTIGATION

The two-dimensional spatial power spectrum, \( P(k) \), of the \( H\alpha \) emission fluctuations \((l)\) in the SMC was first derived in Stanimirović et al. (1999). This was the first such study for an entire galaxy. The power spectrum is defined as the Fourier transform of the autocorrelation function of the \( H\alpha \) emission fluctuations:

\[
P(k) = \int \int (\langle l(x)l(x') \rangle)e^{-ik\cdot L} dL, \quad L = x - x',
\]

with \( k \) being the spatial frequency, measured in wavelengths, and \( L \) being the distance between two points. To derive \( P(k) \), the channel maps (approximately 5 km s\(^{-1}\) wide) were Fourier-transformed. The average value of the square of the modulus \( |P(k)|^2 \) was then measured in 18 annuli\(^2\) of the transform, \( k \), was then measured in 18 annuli\(^2\) assuming radial equal width in \((u^2 + v^2)^{1/2} \) (\( u \) and \( v \) are the coordinates in the Fourier plane, measured in wavelengths) assuming radial symmetry. It was shown that the two-dimensional power spectra were remarkably well fitted by a power law, \( P(k) \propto k^n \), over the continuous range of spatial scales \( \sim 30 \) pc\(\sim 4 \) kpc and over the velocity range \( 110-200 \) km s\(^{-1}\) (several noisy channels, without significant line emission, were excluded from the analyses). The power-law index was remarkably constant over the given velocity range, and the average slope was estimated with \( \langle n \rangle = -3.04 \pm 0.02 \). No change of the power-law slope was seen on either the large- or small-scale end of the spectrum.

However, an examination of the power spectrum of the \( H\alpha \) column density distribution, obtained after integrating along the whole velocity range, showed a change of the power-law slope, with \( \gamma \) equal to \(-3.31 \pm 0.01 \) (Stanimirović et al. 2000).

The power-law behavior of the spatial power spectrum was seen before in the case of our Galaxy, for several ISM tracers (Crovisier & Dickey 1983; Green 1993; Gautier et al. 1992; Waller & Varosi 1997; Schlegel, Finkbeiner, & Davis 1998; Deshpande, Dwarkanath, & Goss 2000), and very recently in the case of the LMC (Elmegreen, Kim, & Staveley-Smith 2001). The integrated \( H\alpha \) column density distribution of the SMC has a steeper spatial power spectrum (\( \gamma = -3.3 \pm 0.01 \)), which is consistent with the spectrum of the dust column density (\( \gamma = -3.1 \pm 0.2 \); Stanimirović et al. 2000). In several cases, a steepening of the slope for higher spatial frequencies was found (Crovisier & Dickey 1983; Waller & Varosi 1997; Elmegreen et al. 2001). The difference between the power-law slopes derived from channel maps and the integrated \( H\alpha \) column density, noticed in Stanimirović et al. (2000), has not been found previously.

#### 3.1. Velocity Modification of the \( H\alpha \) Power Spectrum

The two-dimensional spatial power spectrum of the intensity (or emissivity) fluctuations provides important information about the underlying three-dimensional statistical properties of the ISM. However, as the spectral line data cubes have two spatial axes and one velocity axis, both three-dimensional density and velocity fluctuations can contribute to the two-dimensional statistics. Indeed, because of the velocity fluctuations, two clumps along the same line of sight at different distances may appear in the same velocity channel, hence doubling the measured intensity. In order to relate the two-dimensional statistics to the underlying three-dimensional statistics of the ISM, one must disentangle density from velocity influence on the power spectrum.

The velocity statistics is probably the most interesting and significant, partly because it directly represents the dynamics of the media and partly because other methods of turbulence study, i.e., scintillation technique (Spangler 1999), fail to deliver this piece of information. The problem of disentangling velocity and density fluctuations was addressed in Lazarian & Pogosyan (2000) (see also Lazarian 1999 for a review of the subject). They found that the relative influence of velocity and density fluctuations on the intensity fluctuations depends on the spectral index of density fluctuations and on the thickness of channel maps. The latter is easy to understand qualitatively, as it is clear that line integration should decrease the influence of velocity fluctuations. The former is less intuitive, but Lazarian & Pogosyan (2000) showed that, if the three-dimensional density spectrum is \( P \propto k^n \), two distinct regimes are present: (1) when \( n > -3 \) and (2) when \( n < -3 \).

Analytical results of the Lazarian & Pogosyan (2000) study are summarized in Table 1. It is easy to see that in both cases 1 and 2 the power-law index gradually steepens with the increase of velocity slice thickness. In the thickest velocity slices the velocity information is averaged out, and it is natural that we get the density spectral index \( n \). The velocity is the most important in thin slices, and, if the three-dimensional velocity power spectrum is \( k^{3-n} \), then the index \( m \) can be found from thin slices. Note that the notion of thin and thick slices depends on the turbulence scale under study and the slice can be thick for small-scale turbulent fluctuations and thin for large-scale ones. The formal criterion for the slice to be thin is that the dispersion of turbulent velocities on the scale studied be less than the velocity slice thickness. The slice is thick otherwise.

#### 3.2. Thin and Thick Slices

When velocity information is averaged out (for a more precise definition see Lazarian & Pogosyan 2000), the slice becomes...
very thick and the velocity fluctuations become independent of density.

Predictions of Lazarian & Pogosyan (2000) were successfully tested using numerical MHD data in Vazquez-Semadeni et al. (2001), and thus we can confidently apply the theory to observations.

To test the predictions by Lazarian & Pogosyan (2000) in the case of the SMC, we have determined the two-dimensional power spectrum slope, \( \langle \gamma \rangle \), while varying the velocity slice thickness from \( \sim 2 \) to \( \sim 100 \) km s\(^{-1} \). A significant variation of \( \langle \gamma \rangle \) was found, shown in Figure 1, consistent with the predictions: \( \langle \gamma \rangle \) gradually decreases with an increase of velocity slice thickness. The thickest velocity slice gives \( n \approx -3.3 \), which corresponds to the three-dimensional density power index (see Table 1). Using thin slices, however, we can find the slope of velocity fluctuations to be \( m \approx 0.4 \), which means that the three-dimensional velocity spectrum index is \( -3.4 \), which is very close to the three-dimensional density spectral index. The transition point between density- and velocity-dominated regimes is equal to the velocity dispersion on the scale of the whole SMC (\( \sim 4 \) kpc), which is \( \sim 22 \) km s\(^{-1} \) (S. Stanimirović et al. 2001, in preparation).

4. DISCUSSION

As the spatial power spectrum shows the importance of structure on various spatial scales, its power-law behavior suggests the hierarchical structure organization in the ISM, without preferred spatial scales. This phenomenon is usually ascribed to the interstellar turbulence (Scalo 1987; Elmegreen 1999). However, without velocity information one can always wonder whether we deal with a static structure or a real turbulence. Indeed, the distribution of sizes of sand grains on a beach also follows a power law, but no one would call this “turbulence.” The velocity information changes the picture dramatically, hence the extreme importance of the techniques that relate the observed two-dimensional power spectrum with the underlying three-dimensional statistics of both density and velocity. Here we have tested the theoretical predictions for such a technique (Lazarian & Pogosyan 2000) and as a result proved, for the first time, the presence of an active turbulence in the SMC.

In view of the theoretical results in Lazarian & Pogosyan (2000), it is now appropriate to reanalyze all the earlier data. These data were obtained without much concern about the thickness of velocity slices. Therefore, the observed variations of the power index can be due to transitions from “thin” to “thick” and to “very thick” slices. In the case of the data of Green (1993), additional complications are related to a divergent line-of-sight geometry. A more detailed discussion of the available data will be given elsewhere. We note that, although the description in terms of power spectra is common in hydrodynamics and the MHD theory, it has certain limitations, as discussed in Lazarian (1999). For example, the power spectrum analysis does not include information about the phase distribution, dealing only with the modulus of the Fourier transform, nor does it contain information about the structure connectivity (Scalo 1987). Other methods, hence, should be used as complementary statistical descriptors. The power spectrum as it is can provide us with an important insight into what kind of turbulence we deal with, e.g., distinguish the turbulence originating from shock waves from hydrodynamic turbulence.

Attempts to test the Lazarian & Pogosyan (2000) theory were made recently in Elmegreen et al. (2001) using H\( \text{I} \) observations of the LMC. In agreement with theoretical predictions, the steepening of spectrum was observed for high spatial frequencies. The puzzling thing discovered by Elmegreen et al. (2001) was the flattening of the spectra for velocity-integrated intensity, which was interpreted as an effect of the finite LMC disk thickness. This is an interesting explanation, which entails that the LMC spectrum at scales larger than 100 pc becomes essentially two-dimensional. Our study has not noticed a systematic change in the velocity-integrated power spectrum at large spatial scales. This may reflect the fact that the SMC, unlike the LMC, is essentially a three-dimensional entity.

Another approach in relating the two-dimensional with the three-dimensional statistics in the case of the SMC was presented in Goldman (2000). There it is assumed that the density fluctuations are a “passive scalar,” being driven by the velocity fluctuations, and hence having the same power spectrum. If we accept that the intensity fluctuations are due to the density fluctuations, then the corresponding spectral index \( q \) of intensity fluctuations in a two-dimensional slice can then be related to the three-dimensional density spectrum index \( n \) as

\[
q = n + 1,
\]

which for the SMC data produces \( n \approx -4 \). However, we note that the data used in Goldman (2000) are not in the real space \( (x, y, z) \) for which his treatment would be correct but are in the velocity space \( (x, y, v) \). In this situation the Lazarian & Pogosyan (2000) treatment is appropriate, and it provides a different result, namely, velocity index \( \approx -3.4 \) and density index \( \approx -3.3 \). Equation (2) also predicts that the difference in the power slope between thin and thick slices is equal to 1. This is inconsistent with Figure 1. We also note that for the Kolmogorov spectrum the predictions in Lazarian & Pogosyan (2000) (see Table 1) coincide with \( n \) calculated using equation (2), but this correspondence is accidental.

An interesting application of the power spectrum of H\( \text{I} \) opacity fluctuations was made by Deshpande (2000), in order to explain the long-standing puzzle of the tiny-scale structure

\[ \sim \]

2 This would correspond to the spectrum of shock waves. To avoid possible confusion we point out that we talk about power spectrum, which differs from the energy spectrum by \( k^2 \). The Kolmogorov turbulence corresponds to the power spectrum of \(-11/3\).
in H\textsc{i} (Heiles 1997). Assuming a single power spectrum of the opacity fluctuations, with a slope of 2.75 over the range of ∼0.02–∼4 pc, Deshpande (2000) obtained opacities consistent with the observations of small-scale H\textsc{i} structure in Deshpande et al. (2000). This is very encouraging and requires reinterpretation of previous observations of the small-scale structure in a similar way. A preliminary investigation of the small-scale structure found so far by Heiles (2000) suggested, though, a more complex structure function, with a significant change of slope for scales smaller than 0.01 pc.

What can drive the turbulence in the SMC? The natural assumption would be that it is due to the stirring of the ISM produced by a large number of expanding shells found in the SMC. The shell sizes range from ∼30 pc to ∼2 kpc. Hence, one scenario could be that the largest shells drive the turbulent cascade down to the smallest observed scales. However, processes such as shell fragmentation and/or shell propagation from the smaller scales (bottom-up scheme; see Scalo 1987) may play a significant role too. The main problem in pinning down the exact mechanisms is that, so far, we have not observed changes in the power spectrum, at any scale up to the entire size of the SMC, which would be indicative of energy injection. An alternative explanation was suggested in Goldman (2000), whereby the large-scale turbulence is induced by instabilities in the large-scale flows during the last SMC-LMC encounter. Future comparison with simulations of different types of turbulent cascades are essential to resolve this question.

5. CONCLUSIONS

We have successfully tested predictions of the Lazarian & Pogosyan (2000) study on the change of slope of the intensity fluctuation spectrum with velocity slice thickness. The SMC spectrum appears to be due to active turbulent motions rather than just a static hierarchical structure. We found the index for the three-dimensional velocity spectrum to be −3.4 and that for the density spectrum to be −3.3.

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