Calculation of the Unit Normal Vector Using the Cross-Curve Moving Mask for Measurement Data Obtained from a Coordinate Measuring Machine

Fang-Jung Shiou and Ruey-Tsung Lee
Department of Mechanical Engineering, National Taiwan University of Science and Technology, No. 43, Section 4, Keelung Rd. 106 Taipei, Taiwan, China
E-mail: shiou@mail.ntust.edu.tw

Abstract. This study presents a cross-curve moving mask method to calculate the unit normal vector based on 5 or 9 data points of a freeform surface measurement. The middle point and 4 or 8 neighboring points can be constructed as two crossed curves - longitude and latitude. The unit normal vector at the middle point can be determined by calculating the cross product of two tangent vectors along these 2 crossed curves. Different curve fitting methods passing 5 or 9 data points, such as Lagrange, parametric polynomial, Bezier and B-spline methodologies, are investigated. Two kinds of surfaces, namely, a sphere surface and a shoe-shaped geometric surface are selected for evaluating the accuracy of the calculated unit normal vectors.

1. Introduction
In product development, the 3-D geometric data of a sample part, prototype or final product can be digitized by a contact type probe on a coordinate measuring machine (CMM) or by a non-contact type 3-D optical measurement system. Processing software for 3-D data points is usually required for further data processing, such as editing of the data points, construction of curves and surfaces, or quality inspection of a freeform surface based on these 3-D data points [1, 2]. The accuracy of the 3-D data points measured by a CMM, is, in general, more reliable and accurate than if done by a 3-D optical measurement system. For the measurement with a CMM, the probe radius compensation must be calculated in order to get the data points on the workpiece surface with the information of the normal vector in each data point [3]. Traditionally, it is assumed that a composite surface passing through all these points can be constructed by the Coons’ method [4, 5] and the normal vector can then be determined at each data point. To manipulate a huge number of data points, a large memory space and long computing time are required for the Coons’ method. The other methods used to determine the normal vector were by averaging the normal values of the triangular meshes or edge-pairs [6, 7].

The moving mask method is usually used in digital image processing, in which a 3X3 pixel mask is taken for spatial convolution operation to process the gray-level of the middle pixel [8]. This method takes the middle pixel and the surrounding four or eight pixels to extract average characteristics and smooth the gray-level of the middle pixel. The 3X3 moving mask method for calculation of the normal vector at each data point was proposed using the similar idea in digital image processing, as shown in Figure 1 [9, 10]. Passing through these 3X3 grid points, a surface patch was constructed and the unit normal vector of the middle point was then calculated. The unit normal vector at each measured data point could then be calculated by moving the 3X3 grid point mask over the entire area of measured points.
2. Methodologies for the normal vector calculation by the cross-curve moving mask method

The 5-point cross-curve moving mask method is similar to the 3X3 grid point moving mask method and proposed in this study to calculate the unit normal vector. The proposed 5-point cross-curve moving mask method, only the middle point and the other four neighboring points are taken into account to construct the two crossed curves. Comparing to the 3X3 method, the four corner points in the 3X3 grid point moving mask method are disregarded. The unit normal vector \( \mathbf{N}_c \) at the middle point can be determined by calculating the cross product of 2 tangent vectors \( \mathbf{T}_u \times \mathbf{T}_w \) of the longitudinal and latitudinal crossed curves (Figure 2). Extending this method, the 9-point cross-curve mask method, which passes through five longitudinal points and five latitudinal points, can be constructed two crossed quartic curves.

Different mathematic interpolation methods, including Lagrange, polynomial, Bezier, and B-Spline method, can be applied to construct the longitudinal and latitudinal quadratic curves (2nd degree, 3rd order) passing through the 3 points or quartic curves (4th degree, 5th order) passing through the 5 points. The tangent vector \( \mathbf{T}_u' \) at the middle point along a quadratic parametric polynomial interpolated curve passing through 3 points \( P_0, P_1, \) and \( P_2 \) discussed in detail below [5, 11, 12].

**Polynomial:**

\[
P'(u) = \frac{(u_1 +1)}{u_1} P_0 + \frac{-1}{u_1(u_1-1)} P_1 + \frac{u_1}{(u_1-1)} P_2 + 2 \frac{1}{u_1} P_0' + \frac{1}{u_1(u_1-1)} P_1' + \frac{-1}{(u_1-1)^2} P_2'
\]

**Lagrange:**

\[
P'_1(u) = \frac{(u_1 -1)}{u_1} P_0 + \frac{2u_1 -1}{u_1(u_1-1)} P_1 + \frac{u_1}{(1-u_1)} P_2
\]

**Bezier:**

\[
P'_1(u) = -2(1-u_1) P_0 + 2(1-2u_1) (1-u_1)^2 P_0 + P_1 - u_1^2 P_2 + 2u_1 P_2
\]

The calculated tangent vectors at the middle point of interpolated curve passing through 3 or 5 points by the Polynomial, Lagrange and Bezier methods are exactly the same.

In general, a uniform nonperiodic (clamped) B-spline curve is exactly the same as a Bezier curve. Thus, the B-spline curve of order three or five passing through three or five points is the same as the Bezier curve. When passing through five points, a B-spline curve can be also constructed of order three or four. The B-spline curve of order three or four is different from Bezier curve passing through the same five points. The B-Spline curve of 3rd order (2nd degree) and 4th order (3rd degree) passing through five points, \( P_0, P_1, P_2, P_3, \) and \( P_4 \), can be written in matrix form as follows:
Where \( B_0, B_1, B_2, B_3 \) and \( B_4 \) are control points and \( N_{i,j}(u) \) is blending function. The tangent vector at the middle point for the B-Spline curve of order three and four passing through five points can be obtained based on the first derivative of above equations:

\[
P'(u) = 2(u-1)N_{3,1}B_0 + \left[(2-3u)N_{2,1} + (u-2)N_{3,1}\right]B_1 + \left[uN_{2,1} + (3-2u)N_{3,1} + (u-3)N_{4,1}\right]B_2 + \left[(u-1)N_{3,1} + (7-3u)N_{4,1}\right]B_3 + 2(u-2)N_{4,1}B_4
\]

and

\[
P''(u) = -3(1-u)^2 N_{3,1}B_0 + 3(u-1)^2 N_{4,1}B_4 +
\left[\left((-u)^2 - 2u(1-u) + (1-u)(2-u) - u(2-u) - u(1-u)\right)N_{3,1} - 3\left(2-u\right)^2 N_{4,1}\right]B_1 +
\left[\left(2u(1-u) + u^2\right)N_{3,1} + \left(2-u\right)^2 - 2u(2-u) - 2(1-u)(2-u)\right]N_{4,1}\right]B_2 +
\left[\left(3u^2\right)N_{3,1} + \left(\frac{2u(2-u) - u^2}{2} + \frac{(u-1)(2-u) + u(2-u) - u(u-1)}{2} - (u-1)^2 + 2(2-u)(u-1)\right)N_{4,1}\right]B_3
\]

3. The accuracy of the cross-curve moving mask method

For evaluating the accuracy of the calculated normal vector using the cross-curve moving mask method, two surface categories were selected, namely the sphere surface and the shoe-shaped model. The sphere surface is an analytic surface with a constant curvature – a convenient shape for checking the normal vector deviation. Three radii, namely R10, R100 and R1000 were applied. The specific check positions are defined by two angular values, \( \theta \) and \( \psi \), and the different pitches (d) of 5 or 9 data points are 1, 2 and 3mm shown on Figure 3. The angular deviations in degree, using 5- and 9-point cross-curve moving mask of the Bezier and the B-spline methods, are listed in Table 1. The angular deviations, \( \theta_\perp \), between the theoretical unit normal vectors \( \left(N_t\right) \) and the calculated unit normal vectors \( \left(N_c\right) \) can be expressed as \( \theta_\perp = \cos^{-1}\left(\frac{N_t \cdot N_c}{|N_t| \cdot |N_c|}\right) \).
### Table 1. Angular deviation between the theoretical normal vectors and the calculated normal vectors on a sphere surface using the cross-curve moving mask method.

| Sphere | Theoretical normal vector | 5+Pts-Bezier | 9+Pts-Bezier | 9+Pts-B-Spline (Order 3) | 9+Pts-B-Spline (Order 4) |
|--------|---------------------------|-------------|-------------|--------------------------|--------------------------|
| 0      | 0                        | 0           | 0           | 0                        | 0                        |
| 0      | 10                       | 0           | 0           | 0                        | 0                        |
|       | 100                      | 0           | 0           | 0                        | 0                        |
|       | 1000                     | 0           | 0           | 0                        | 0                        |
| 30     | 0                        | 0.5         | 0.5         | 0.5                      | 0.5                      |
|       | 10                       | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 100                      | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 1000                     | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
| 45     | 0                        | 0.5         | 0.5         | 0.5                      | 0.5                      |
|       | 10                       | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 100                      | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 1000                     | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
| 60     | 0                        | 0.5         | 0.5         | 0.5                      | 0.5                      |
|       | 10                       | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 100                      | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 1000                     | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
| 30     | 22.5                     | 0.5         | 0.5         | 0.5                      | 0.5                      |
|       | 10                       | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 100                      | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 1000                     | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
| 45     | 22.5                     | 0.5         | 0.5         | 0.5                      | 0.5                      |
|       | 10                       | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 100                      | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 1000                     | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
| 60     | 22.5                     | 0.5         | 0.5         | 0.5                      | 0.5                      |
|       | 10                       | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 100                      | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 1000                     | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
| 30     | 45                       | 0.5         | 0.5         | 0.5                      | 0.5                      |
|       | 10                       | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 100                      | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 1000                     | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
| 45     | 45                       | 0.5         | 0.5         | 0.5                      | 0.5                      |
|       | 10                       | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 100                      | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 1000                     | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
| 60     | 45                       | 0.5         | 0.5         | 0.5                      | 0.5                      |
|       | 10                       | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 100                      | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
|       | 1000                     | 0.0021      | 0.000998    | 0.00126                  | 0.00126                  |
The shoe-shaped geometric surface, which is an industrial geometric model with a freeform surface, is selected for investigating the error of the unit normal vector calculation. For a quality assessment of the freeform surface, the deviation of the distances between the machined freeform surface and its originally designed freeform surface is considered. First, the offset surfaces of the freeform surface on the shoe-shaped model are created on a CAD system with an offset distance of 10mm space. The data points \( (P_d) \) on these offset surfaces are generated with different pitches, \( d = 1, 3 \) and \( 5 \)mm (Fig. 4). These data points can be assumed to be the measured positions on the surface of a shoe last by a contact probe with a radius \( (r = 10\text{mm}) \), on a coordinate measuring machine. Based on the calculated unit normal vectors by cross-curve moving mask method, the compensated points \( (P_c) \) back to the original surface of the shoe last can be obtained after the calculation of probe radius compensation. The positions of these compensated data points can be calculated by \( P_c = P_d - r \cdot N_t \). These compensated data points are compared with the original surface of the shoe last. The distance deviation between these compensated data points and the original surface are evaluated and illustrated on Fig 5 and Table 2. The advantage of this evaluation process is that it can investigate the computation error and accuracy of the cross-curve moving mask method without considering the error generated by manufacturing and measuring, especially, the error caused by manufacturing is usually far greater than other sources of error. According to the result of Table 1 and 2, the 9-point cross-curve moving mask method using Bezier and B-Spline (4\textsuperscript{th} order) interpolation almost have the same accuracy. Considering the computation of the tangent vector, Bezier cross-curve method is easier and more efficient than B-Spline cross-curve method.

**Table 2.** Effects of grid distance on the accuracy (max. distant deviation) of the calculated normal vectors.

| Method          | Pitch | 5-point | 9-point | 9-pt B-Spline (Order = 3) | 9-pt B-Spline (Order = 4) |
|-----------------|-------|---------|---------|---------------------------|---------------------------|
| Bezier cross-curve | 1 mm  | 3.7e-4  | 3.47e-4 | 3.58e-4                   | 3.47e-4                   |
| Bezier cross-curve | 3 mm  | 0.00359 | 0.00156 | 0.002133                  | 0.00157                   |
| Bezier cross-curve | 5 mm  | 0.0155  | 0.0033  | 0.0046                    | 0.00329                   |
4. Conclusion
The 5- and 9-point cross-curve moving mask methods, using polynomial, Lagrange, Bezier, and B-Spline algorithms have been reviewed for the computation of the unit normal vector. The Bezier algorithm used with the cross-curve moving mask method has been shown to be the best methodology, considering its performance in the ease and speed of computation and the accuracy of results. These methods avoid matrix operations, and reduce the required memory space and calculating time for computations in comparison to the conventional composite Coons’ Bi-cubic surface method and the 3X3 or the 9X9 moving mask methods. This method can also be applied to reverse engineering measurements for offsetting the data points, which have been measured by a 3-D coordinate measuring machine with a ball probe tip. The results show that the error of the computed normal vector is increased when the pitch distance of the data points is increased, whereas the error decreased when the curvature radius of the freeform surface increased. In addition, the error of the normal vectors calculated with the cross-curve moving mask method, for data points with an appropriated pitch, is much smaller than the accuracy requirement of the freeform surface measurement.

References
[1] Chen LC and Lin GC 1997 An integrated reverse engineering approach to reconstructing free-form surface Computer Integrated Manufacturing System 10 49–60
[2] Choi BK, Shin HY, Yoo WW 1993 Visually smooth composite surfaces for an unevenly spaced 3D data array Computer-Aided Geometric Design 10 151–171
[3] Shiou F 1998 Strategien und Messverfahren zur Erfassung Dimensioneller Bauteilegroessen fuer die Laserbearbeitung (RWTH Aachen, Shaker Verlag)
[4] Faux ID and Pratt MJ 1979 Computational Geometry for Design and Manufacture (Ellis Horwood)
[5] Yamaguchim F 1988 Curves and surfaces in computer aided geometric design (Spinger-Verlag) 64–71
[6] Liang SR and Lin AC 2002 Probe-radius compensation for 3D data points in reverse engineering Computers in Industry 48 241–251
[7] Woo H, Kang E, Wang S, Lee KH 2002 A new segmentation for point cloud data International Journal of Machine Tools and Manufacturing 42 167–178
[8] Gonzalez RC and Woods RE 2002 Digital Image Processing (Prentice Hall)
[9] Shiou FJ and Lin YF 2002 Calculation of the Normal Vector using 3 X 3 Moving Mask Method for Freeform Surface Measurement and its Application International Journal of Advance Manufacturing Technology 19 516–524
[10] Lin YF 2000 The study of the calculation of the normal vector using the 9-point mask moving method for the free form surface measurement (National Taiwan University of Science and Technology: Taiwan)
[11] Lee K 1998 Principles of CAD/CAM/CAE system (Addison Wesley) 157–185
[12] McMahon C and Browne J 1998 CAD/CAM principles practice and manufacturing management (Addison Wesley) 55–71

Figure 5. The distance deviation between the original surface and the compensated points (pitch=1mm) calculated 5-point cross-curve moving mask Bezier method