Twist-3 effect from the longitudinally polarized proton for $A_{LT}$ in hadron production from $pp$ collisions

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Abstract

We compute the contribution from the longitudinally polarized proton to the twist-3 double-spin asymmetry $A_{LT}$ in inclusive (light) hadron production from proton-proton collisions, i.e., $p^1\bar{p} \to hX$. We show that using the relevant QCD equation-of-motion relation and Lorentz invariance relation allows one to eliminate the twist-3 quark-gluon correlator (associated with the longitudinally polarized proton) in favor of one-variable twist-3 quark distributions and the (twist-2) transversity parton density. Including this result with the twist-3 pieces associated with the transversely polarized proton and unpolarized final-state hadron (which have already been calculated in the literature), we now have the complete leading-order cross section for this process.
1 Introduction

Twist-3 observables in high-energy semi-inclusive reactions provide us with an important opportunity to test theoretical frameworks for QCD hard processes and to understand the quark-gluon substructure of hadrons beyond the conventional parton model. Well-known examples are the experimental observation of hyperons with large transverse polarization produced in unpolarized proton-proton collision, \( pp \rightarrow \Lambda^+ X \) \cite{1-5}, and the transverse single-spin (or left-right) asymmetry (SSA) \( A_N \) of a produced hadron in the collision between a transversely polarized proton and an unpolarized proton, \( p^+p \rightarrow h X (h = \pi, K, \eta, \text{etc.}) \) \cite{6-16}. The magnitude of the asymmetries were as large as a few tens of percent in the forward direction. In collinear factorization, these SSAs appear as twist-3 observables. They are driven by multi-parton (quark-gluon or purely gluonic) correlations \cite{17, 18} either in the initial-state hadrons or in the final-state fragmentation process. The formalism for deriving the twist-3 cross section for SSAs has been well developed, and the formulae involve the relevant multi-parton correlation functions instead of the usual (twist-2) parton densities or fragmentation functions \cite{19-35}. The \( A_N \) data for \( \pi, K, \eta \), and jet production obtained at the Relativistic Heavy Ion Collider (RHIC) have been analyzed using this formalism \cite{20, 36-38}.

Besides these large SSAs, the double-spin asymmetry (DSA) \( A_{LT} \) for particle production (direct photon, Drell-Yan lepton pair, hadron, jet, etc.) in collisions between longitudinally and transversely polarized protons, \( p^+\bar{p} \rightarrow C X \), is also a twist-3 observable \cite{40-45}. Unlike SSAs, which are naively “T-odd” effects, DSAs like \( A_{LT} \) are naively “T-even,” which leads inherently to different forms for the corresponding twist-3 cross section (see the discussion below Eq. (2)). Therefore, \( A_{LT} \) and \( A_N \) probe different yet complimentary aspects of hadronic structure, and both are critical to test the underlying mechanism for these asymmetries. Surprisingly, RHIC has never run an experiment for \( A_{LT} \) despite being the only facility in the world with polarized proton beams and having measured every other combination of proton spins (\( A_N, A_L, A_{TT}, A_{LL} \)).

In this paper we compute the polarized cross section for \( A_{LT} \) in the production of an unpolarized (light) hadron \( h \) from proton-proton collisions,

\[
p(P, S_\perp) + p(P', \Lambda) \rightarrow h(P_h) + X, \tag{1}
\]

where \( S_\perp \) is the transverse spin vector for the nucleon \( A \), \( \Lambda \) is the helicity of the longitudinally polarized nucleon \( B \), and the momenta of the particles are shown. In the framework of collinear factorization, the first nonvanishing contribution to the cross section appears at twist-3, and it receives three contributions,

\[
d\sigma(P_h, S_\perp, \Lambda) = H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{h/c(2)} \\
+ H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{h/c(2)} \\
+ H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{h/c(3)}, \tag{2}
\]

where \( f_{a/A(3)} \) represents the twist-3 distribution function for parton species \( a \ (a = q, \bar{q}, g) \) in nucleon \( A \) with the subscript (3) indicating the twist (and similar for \( f_{b/B(3)} \)). Likewise, \( D_{h/c(3)} \) represents the twist-3 fragmentation function for the parton species \( c \) into the final-state hadron.

\[\text{1 Data from RHIC is on tape for } A_N \text{ in prompt photon production and several predictions exist for this asymmetry within collinear factorization} \cite{21, 37, 39}.\]

\[\text{2 } A_{LT} \text{ in } ep \text{ collisions is also an interesting twist-3 asymmetry and has been studied in Refs.} \cite{40, 48}.\]
The factors \( H, H', \) and \( H'' \) are the partonic hard cross sections for each contribution, and \( \otimes \) represents a convolution in the appropriate momentum fractions.

So far, the leading-order (LO) cross section was derived for the first term [43] and the third term [45] in Eq. (2). The first line of (2) involves twist-3 distributions in the transversely polarized nucleon coupled to the twist-2 helicity distribution. Unlike the SSA for \( p^1 p \rightarrow h X \), the partonic hard part for this term is given as a non-pole contribution [42, 43]. In the third line of (2), the real part of the unpolarized chiral-odd twist-3 quark-gluon fragmentation function couples to the transversity parton density [45]. This is in contrast to SSAs, where the imaginary part of the same quark-gluon twist-3 fragmentation function contributes [31, 32]. A recent analysis suggests that this imaginary part can be the main cause of the large \( A_N \) observed for pion production in \( pp \) collisions at RHIC [38]. This new insight is what motivated the calculation of the third line in Eq. (2) for the \( A_{LT} \) case [45]. Again we emphasize that \( A_{LT} \) in \( p^1 \vec{p} \rightarrow h X \) is a unique quantity that should be measured at RHIC.

To complete the LO cross section for the process (1), we will compute the second term in Eq. (2), where, as we will see in Sec. 3, chiral-odd twist-3 distributions for the longitudinally polarized nucleon enter along with the transversity parton density (the latter shows up when one employs QCD equation-of-motion and Lorentz invariance relations). Both of these couple to the transversity function for the transversely polarized nucleon. We note that two twist-3 terms analogous to the first two lines in Eq. (2) (with the fragmentation functions omitted) contribute to \( A_{LT} \) in Drell-Yan when one integrates over the transverse momenta of the lepton pair, and both pieces are of a similar magnitude [41]. Therefore, it is possible that the second term of (2) for hadron production is just as important as the first and brings a non-negligible contribution. In addition, as alluded to above, the third term might also be significant (as in \( A_N \)). Thus, a detailed numerical study of all three parts of \( A_{LT} \) will be needed and is the subject of future work.

The rest of this paper is organized as follows: in Sec. 2 we summarize the twist-3 distribution functions in the nucleon relevant for this computation and the relations among them. In Sec. 3, we derive the LO cross section for the second term of Eq. (2). We will see that, owing to a simple form of the partonic hard cross sections, the effect of the twist-3 quark-gluon correlation function in the longitudinally polarized nucleon can be expressed in terms of one-variable twist-3 quark distributions and the transversity parton density. Sec. 4 is devoted to a brief summary.

## 2 Twist-3 distribution functions for a longitudinally polarized proton

In this section we summarize the distribution functions in the nucleon relevant to our study. We first have a quark correlator in the nucleon that gives two chiral-odd polarized functions needed in our calculation [40],

\[
M_{ij}^{\sigma q}(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}_j(0)\psi_i(\lambda n)|PS\rangle
\]

\[
= \frac{1}{2} (\gamma_5 S_\perp \partial_\lambda)_{ij} h_1^q(x) + \frac{M_N}{2} \Lambda(i\gamma_5 \sigma^{\mu\nu})_{ij} h_T^q(x) + \cdots, \tag{3}
\]

where \( \psi_i \) is a quark field with spinor index \( i \), \( M_N \) is the nucleon mass, \( S \) is the nucleon spin vector normalized as \( S^2 = -1 \), and \( \Lambda = M_N (S \cdot n) \) is its helicity. We also introduced two lightlike vectors \( p^\mu \) and \( n^\mu \), where \( P = p + (M_N^2/2)n \) and \( p \cdot n = 1 \) with the only nonzero components \( p^+ = P^+ \)
M \text{ expressed in terms of the meaning of the derivative becomes clear. Using the QCD equation-of-motion,}

\[
M'^{\alpha \beta}_{ij}(x_1, x_2) = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle PS|\bar{\psi}(0)gF^{\alpha \beta}(\mu n)\psi(\lambda n)|PS\rangle
\]

\[
= i\frac{M}{2} g^{\alpha \beta} \Lambda(\gamma^5 \gamma^\beta \gamma^\alpha)_{ij} H_{FL}^q(x_1, x_2) + \cdots,
\]

where \( F^{\alpha \beta} \) is the gluon field strength tensor and \( g^{\alpha \beta} \equiv g^{\alpha \beta} - p^{\alpha} n^\beta - p^\beta n^\alpha . \) From Hermiticity and \( PT\)-invariance, \( H_{FL}(x_1, x_2) \) is shown to be real and satisfies the symmetry property

\[
H_{FL}^q(x_1, x_2) = -H_{FL}^q(x_2, x_1).
\]

The \( D\)-type twist-3 distribution \( H_{DL}(x_1, x_2) \) is defined by the replacement \( gF^{\alpha \mu}(\mu n) \rightarrow D^\alpha(\mu n) = \partial^\alpha - igA^\alpha(\mu n) \) in (4), and is related to \( H_{FL}^q(x_1, x_2) \) as

\[
H_{DL}^q(x_1, x_2) = \mathcal{P} \frac{1}{x_1 - x_2} H_{FL}^q(x_1, x_2) + \delta(x_1 - x_2) \tilde{h}^q(x_2),
\]

where \( \mathcal{P} \) indicates the principal value. The function \( \tilde{h}_L(x) \) is another real twist-3 distribution function, which is defined as

\[
M_{ij}^{\alpha \beta}(z) = \lim_{z \rightarrow 0} \int \frac{d\lambda}{2\pi} \frac{\partial}{\partial \lambda} \langle PS|\bar{\psi}(0)[0, \infty n][\infty n, \infty n + z] |\infty n + z, \lambda n + z\rangle \psi(\lambda n + z) |PS\rangle
\]

\[
= \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)D^\alpha(\lambda n)\psi(\lambda n) |PS\rangle + \int \frac{d\lambda}{2\pi} e^{i\lambda x} \int_{\lambda}^{\infty} d\mu \langle PS|\bar{\psi}(0)igF^{\alpha \beta}(\mu n)\psi(\lambda n) |PS\rangle
\]

\[
= i\frac{M}{2} g^{\alpha \beta} \Lambda(\gamma^5 \gamma^\beta \gamma^\alpha)_{ij} \tilde{h}^q(x) + \cdots,
\]

where in the first line we explicitly wrote the gauge links \([\infty n + z, \lambda n + z]\), etc., so that the meaning of the derivative becomes clear. Using the QCD equation-of-motion, \( h_L(x) \) can be expressed in terms of \( H_{FL}(x_1, x) \) and \( \tilde{h}_L(x) \) as

\[
h_L^q(x) = -\frac{1}{x} \int_{-1}^{1} dx_1 \left( H_{DL}^q(x_1, x) + H_{DL}^q(x, x_1) \right)
\]

\[
= -\frac{2}{x} \int_{-1}^{1} dx_1 \mathcal{P} \frac{1}{x_1 - x} H_{FL}^q(x_1, x) - \frac{2}{x} \tilde{h}^q_\perp(x).
\]

In addition, the operator product expansion gives another relation among \( h_L(x) \), \( h_1(x) \), and \( H_{FL}(x_1, x_2) \) as

\[
-x^2 \frac{d}{dx} \left( \frac{1}{x} h_1^q(x) \right) = 2h_1^q(x) + 2 \int_{-1}^{1} dx_1 \mathcal{P} \frac{1}{x - x_1} \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x_1} \right) H_{FL}^q(x, x_1).
\]

The combination of (8) and (9) leads to

\[
\frac{d\tilde{h}^q_\perp(x)}{dx} - h_1^q(x) + h_L^q(x) = 2 \int_{-1}^{1} dx_1 \mathcal{P} \frac{1}{(x - x_1)^2} H_{FL}^q(x, x_1),
\]

which is known as a Lorentz invariance relation in the literature (48). In Sec. 3, we will see the relations (8) and (10) lead to a simple form for the cross section for the second term of (2).
Figure 1: Generic diagrams for the contribution to the process (11) from the second term in Eq. (2). The correlators for the longitudinally polarized nucleon (upper blob) couple to the transversity distribution (lower blob). Diagram (a) gives rise to the first and second terms in (11), and (b) and (c) are for the third term in (11). Mirror diagrams of (b) and (c) also contribute, which are included in Eq. (11).

3 Calculation of the polarized cross section for $A_{LT}$

We now derive the cross section for the second term of Eq. (2). As mentioned before, the twist-3 cross section for the naively T-even $A_{LT}$ arises from non-pole contributions. The method of the calculation has been formulated both in Feynman gauge \[32,44\] and lightcone gauge \[31,42,43\], and it has been confirmed that they give identical results for the twist-3 cross section in terms of the gauge-invariant distribution and fragmentation functions defined in the previous section \[45,47,50\]. Here we follow the Feynman gauge formulation (but have checked that the same result is achieved in lightcone gauge), which has an advantage that the gauge invariant correlation functions appear manifestly. Since we are interested in the twist-3 effect from the longitudinally polarized nucleon, we factorize the transversity distribution $h_1(x)$ and the unpolarized fragmentation function for the hadron $D(z)$ from the rest of the cross section and perform a collinear expansion of the hard part. The generic diagrams for this contribution is shown in Fig. 1. According to the general formalism developed in \[32\], the twist-3 cross section is obtained as

\[
E_h \frac{d\sigma(S_\perp, \Lambda)}{d^3P_h} = \frac{1}{16\pi^2S} \int \frac{dx}{x} \int \frac{dz}{z^2} D(z) \left\{ \int dx' Tr \left[ M(x') S(x'p') \right] + i\omega_{\alpha} \int dx' Tr \left[ M^\beta_0(x') \frac{\partial S(k)}{\partial k^\alpha} \bigg|_{k=x'p'} \right] \right. \\
+ 2i\omega_{\beta} \int dx' \int dx'_1 P_\frac{1}{x'_1 - x'} Tr \left[ M^\beta_0(x'_1, x') S_{L\alpha}(x'_1p', x'p') \right] \right\},
\]

where $S = (P + P'^2)$ is the center-of-mass energy squared, $M(x')$, $M^\beta_0(x')$, and $M^\beta_F(x'_1, x')$ are, respectively, defined in Eqs. (3), (7), and (4) with $p$ and $n$ replaced by $p'$ and $n'$ (similarly defined...
for the momentum $P'$ by $P' = p' + (M_F^2/2) n'$ and $p' \cdot n' = 1$, and $\omega'^3 = \theta'^3 - p'^{\alpha} n'^{\beta}$. The partonic hard parts $S(k)$ and $S_{La}(x'_1 p', x'_p')$ are shown by the middle blobs of Fig. 1(a) and Fig. 1(b),(c), respectively. (It is understood that $S$ and $S_{La}$ also depend on $x p$ and $P_h/z$.) Here $S_{La}(x'_1 p', x'_p')$ represents the hard part for the diagram in which the coherent gluon line from $M_F^2(x'_1, x'_p)$ is located in the left of the cut, and the effect of the mirror diagrams is taken into account by the principal value prescription and the factor of 2 in the third term of Eq. (11).

By direct computation of all channels, we find that $\hat{h}_L$ through the factor 1/3 corresponds to the LO diagrams for the hard parts are shown in Figs. 2–4: they correspond to the $qq \rightarrow qq$ channel (Fig. 2), $qq \rightarrow q'q'$, $qq \rightarrow q'q$, $qq \rightarrow qq$, $qq \rightarrow qq$ channels (Fig. 3), and $qq \rightarrow gg$ channel (Fig. 4). Inspecting these diagrams, it is not difficult to find that $S_{La}(x'_1 p', x'_p')$ depends on $x'_1$ only through the factor $1/(x'_1 - x')$ and $1/x'_1$. Therefore the cross section can be decomposed as

$$E_h \frac{d\sigma(S, \Lambda)}{d^3 P_h} = \frac{2\alpha_s^2 M_N \Lambda}{S} (S \cdot P_h) \sum_i \sum_{a,b,c} \int_0^1 \frac{dx}{x} h^a_i(x) \int_0^1 \frac{dz}{z^3} D^c(z) \int_0^1 dx' \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \left[ h^b_L(x') \hat{\sigma}^i_L + \frac{h^b_L(x')}{x'} \hat{\sigma}^i_{ND} + \frac{\bar{h}^b_L(x')}{x'} \hat{\sigma}^i_D \right]$$

$$+ \frac{1}{x'} \int_{-1}^1 \frac{dx'}{x} \frac{1}{x'_1 - x'} \frac{H^b_{FL}(x'_1, x')}{x'_1 - x'} \hat{\sigma}^i_{F1} + 2 \frac{1}{x'_1 - x'} \frac{H^b_{FL}(x'_1, x')}{(x'_1 - x')^2} \hat{\sigma}^i_{F2}$$

$$+ \frac{1}{x'} \int_{-1}^1 \frac{dx'}{x} \frac{1}{x'_1 - x'} \frac{H^b_{FL}(x'_1, x')}{x'_1 - x'} \hat{\sigma}^i_{SFP} \right],$$

where $\sum_i \sum_{a,b,c}$ indicates a sum over channels $i$ and parton flavors in each channel (where $\{a, b\} \in \{q, \bar{q}\}$, $c \in \{q, \bar{q}, g\}$). The partonic hard cross sections $\hat{\sigma}_L$, $\hat{\sigma}_{ND}$, $\hat{\sigma}_D$, $\hat{\sigma}_{F1}$, $\hat{\sigma}_{F2}$, $\hat{\sigma}_{SFP}$ are independent of $x'_1$ and are functions of the Mandelstam variables

$$\hat{s} = (xp + x'p')^2, \quad \hat{t} = \left( x p - \frac{P_h}{z} \right)^2, \quad \hat{u} = \left( x' p' - \frac{P_h}{z} \right)^2.$$  

By extracting the $1/x'_1$ component of $S_{La}(x'_1 p', x'_p')$ we can see that $\hat{\sigma}_{SFP}$ has a structure identical to a SSA soft-fermion-pole (SFP) cross section (besides the projection tensor) with $x'_1 = 0$ [26, 34, 35]. By direct computation of all channels, we find that $\hat{\sigma}_{SFP} = 0$, $\hat{\sigma}_{ND} = \hat{\sigma}_{F1}$, and the contribution from Fig. 1(c) is identically zero. This vanishing $\hat{\sigma}_{SFP}$ is reminiscent of the fact that the SFP hard parts of the chiral-odd contribution to $p p \rightarrow \Lambda^+ X$ and $p^1 p \rightarrow \gamma X$ (i.e., the piece involving twist-3 distributions for the unpolarized proton) vanish [34, 35]. Accordingly, using Eqs. (8) and (10) in Eq. (12), one can eliminate $H_{FL}(x'_1, x')$ in favor of $h_1(x'), h_L(x')$, and $h_L(x')$ and obtain the twist-3 cross section as

$$E_h \frac{d\sigma(S, \Lambda)}{d^3 P_h} = \frac{2\alpha_s^2 M_N \Lambda}{S} (S \cdot P_h) \sum_i \sum_{a,b,c} \int_0^1 \frac{dx}{x} h^a_i(x) \int_0^1 \frac{dz}{z^3} D^c(z) \int_0^1 dx' \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \left[ h^b_1(x') \hat{\sigma}^i_1 + h^b_L(x') \hat{\sigma}^i_2 + \frac{d\bar{h}^b_L(x')}{dx'} \hat{\sigma}^i_3 \right],$$

with

$$\hat{\sigma}_1 \equiv \hat{\sigma}_{F2}, \quad \hat{\sigma}_2 \equiv \hat{\sigma}_L - \hat{\sigma}_{F2} - \frac{1}{2} \hat{\sigma}_{F1}, \quad \hat{\sigma}_3 \equiv \hat{\sigma}_D - \hat{\sigma}_{F2}.$$  

Here $ab \rightarrow cd$ implies that parton $a$ is from $p'$, $b$ is from $\bar{p}$, and $c$ fragments into the hadron $h$. 

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The partonic cross section for each channel reads:

(i) $qq \rightarrow qq$ channel:
\[
\hat{\sigma}_1 = -\frac{1}{N^3} \frac{\hat{t} - \hat{u}}{t \hat{u}}, \quad \hat{\sigma}_2 = \left( \frac{1}{N} + \frac{1}{N^3} \right) \frac{\hat{t} - \hat{u}}{2t \hat{u}}, \quad \hat{\sigma}_3 = -\frac{1}{N} \frac{1}{t} + \frac{1}{N^3} \frac{1}{\hat{u}}.
\]

(ii) $\bar{q}q \rightarrow q'q'$ channel:
\[
\hat{\sigma}_1 = \frac{\hat{t}}{s^2} + \frac{1}{N^2} \frac{2}{s}, \quad \hat{\sigma}_2 = -\frac{\hat{u}}{s^2} + \frac{1}{N^2} \frac{2\hat{u} - \hat{s}}{s^2}, \quad \hat{\sigma}_3 = \frac{1}{s} + \frac{1}{N^2} \frac{\hat{u} - 2\hat{s}}{s^2}.
\]

(iii) $\bar{q}q \rightarrow \bar{q}'q'$ channel:
\[
\hat{\sigma}_1 = -\frac{\hat{u}}{s^2} - \frac{1}{N^2} \frac{2}{s}, \quad \hat{\sigma}_2 = \frac{\hat{t}}{s^2} + \frac{1}{N^2} \frac{\hat{s} - 2\hat{t}}{s^2}, \quad \hat{\sigma}_3 = \frac{1}{N^2} \frac{2\hat{s} + \hat{u}}{s^2}.
\]

(iv) $\bar{q}q \rightarrow \bar{q}q$ channel:
\[
\hat{\sigma}_1 = \frac{\hat{t}}{s^2} + \frac{1}{N^2} \frac{2}{s} + \frac{1}{N} \frac{1}{s} - \frac{1}{N^3} \frac{1}{\hat{u}}.
\]

$N = 3$ is the number of colors and $C_F = (N^2 - 1)/2N = 4/3$. 

Figure 2: Feynman diagrams in the $qq \rightarrow qq$ channel for the partonic hard parts $S(k)$ and $S_{L\alpha}(x_1', x'p')$ in (11). Only the top two diagrams contribute to $S(k)$, while all the diagrams contribute to $S_{L\alpha}(x_1', x'p')$. The circled cross indicates the fragmentation insertion. For $S_{L\alpha}(x_1', x'p')$, it is understood for each diagram that the coherent gluon line coming out of the longitudinally polarized nucleon matrix element (upper side) attaches to one of the dots. Mirror diagrams also contribute, which is taken into account in (11).
Figure 3: The same as Fig. 2, but for the $\bar{q}q \rightarrow q'q'$, $qq \rightarrow q'q'$, $\bar{q}q \rightarrow \bar{q}q$, $\bar{q}q \rightarrow \bar{q}q$ channels. Only the first diagram contributes in the $\bar{q}q \rightarrow q'q'$ and $\bar{q}q \rightarrow \bar{q}q$ channels.

\[
\hat{\sigma}_2 = -\frac{\hat{u}}{s^2} + \frac{1}{N^2} \frac{2\hat{u} - \hat{s}}{s^2} - \frac{1}{N} \frac{\hat{t}}{2s\hat{u}} - \frac{1}{N^3} \frac{\hat{t} + 4\hat{u}}{2s\hat{u}}.
\]

\[
\hat{\sigma}_3 = \frac{1}{s} + \frac{1}{N^2} \frac{\hat{u} - 2\hat{s}}{s^2} - \frac{1}{N^3} \frac{\hat{u} - \hat{s}}{s\hat{u}}.
\]

(v) $\bar{q}q \rightarrow \bar{q}q$ channel:

\[
\hat{\sigma}_1 = -\frac{\hat{u}}{s^2} - \frac{1}{N^2} \frac{2\hat{u}}{s^2} - \frac{1}{N} \frac{\hat{t}}{s} + \frac{1}{N^3} \frac{3\hat{t}}{s},
\]

\[
\hat{\sigma}_2 = \frac{\hat{t}}{s^2} + \frac{1}{N^2} \frac{\hat{s} - 2\hat{t}}{s^2} + \frac{1}{N} \frac{\hat{u}}{2s\hat{t}} + \frac{1}{N^3} \frac{4\hat{t} + \hat{u}}{2s\hat{t}},
\]

\[
\hat{\sigma}_3 = \frac{1}{N^2} \frac{2\hat{s} + \hat{u}}{s^2} - \frac{1}{N} \frac{\hat{t}}{\hat{s}} + \frac{1}{N^3} \frac{3\hat{t}}{s}.
\]

(vi) $\bar{q}q \rightarrow gg$ channel:

\[
\hat{\sigma}_1 = C_F \frac{2(\hat{t}^3 - \hat{u}^3)}{s^2 \hat{t}u} - \frac{1}{N} \frac{\hat{t} - \hat{u}}{s^2},
\]

\[
\hat{\sigma}_2 = -C_F \frac{2(\hat{t} - \hat{u})(s^2 + \hat{u})}{s^2 \hat{t}u} + \frac{C_F^2 2(\hat{t} - \hat{u})}{N \hat{t}u} + \frac{1}{N} \frac{\hat{t} - \hat{u}}{s^2},
\]

\[
\hat{\sigma}_3 = C_F \frac{2(\hat{t}^2 - \hat{t}u - \hat{u}^2)}{s^2 \hat{t}u} - \frac{C_F^2 4}{N} \frac{\hat{t} - \hat{u}}{s^2} + \frac{1}{N} \frac{\hat{t} - \hat{u}}{s^2}.
\]

For the charge conjugated channels (where an antiquark comes from the longitudinally polarized proton) we find $\hat{\sigma}_{\bar{a}b\rightarrow\bar{c}d} = \hat{\sigma}_{ab\rightarrow cd}$, where $\hat{\sigma}_{ab\rightarrow cd}$ are given in Eqs. (16)–(21). As shown in Sec. 2, there are various twist-3 distributions which are not independent of each other. In particular, $h_L(x')$, $\tilde{h}_L(x')$, and $H_{DL}(x_1', x')$ can be expressed in terms of $H_{FL}(x_1', x')$ and the transversity distribution $h_1(x')$, and thus are “auxiliary” twist-3 distributions. However, the simple structure of the partonic cross section for $H_{FL}(x_1', x')$ allows us to rewrite the cross section in terms of $h_1(x')$, $h_L(x')$, and $\tilde{h}_L(x')$, as shown in Eq. (14), for the LO twist-3 cross section. We recall a similar simplification also occurred for the third term in Eq. (2) [45].

\footnote{We refer the reader to Ref. [45] for an extensive work on relations between twist-3 functions (including fragmentation ones) and their importance in showing the Lorentz invariance of twist-3 cross sections.}
4 Summary

In this paper we have derived the twist-3 contribution from the longitudinally polarized nucleon to $A_{LT}$ in $p^\uparrow \bar{p} \rightarrow h X$. Along with the other two twist-3 pieces derived in the literature\,[43, 45], we now have the complete LO cross section for this process at twist-3. Like in the case of the twist-3 fragmentation contribution for $A_{LT}$\,[45], we found that the twist-3 part for the longitudinally polarized proton can be also expressed in a simple form using one-variable quark distributions. This will be useful for phenomenological analyses. Given that $A_{LT}$ probes different yet equally important aspects of hadronic structure as $A_N$, and the fact that RHIC has never run an experiment for this asymmetry despite being the only accelerator in the world with polarized proton beams and having measured every other proton spin configuration, we plan to conduct such a numerical study in future work.

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Figure 4: The same as Fig. 2, but for the $\bar{q}q \rightarrow gg$ channel. Only the top nine diagrams contribute to $S(k)$, while all the diagrams contribute to $S_{Lo}(x_1p', x'p')$. 