Influence of Phase Diffuser Dynamics on Scintillations of Laser Radiation in Earth Atmosphere: Long-Distance Propagation

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Abstract

The effect of a random phase diffuser on fluctuations of laser light (scintillations) is studied. Not only spatial but also temporal phase variations introduced by the phase diffuser are analyzed. The explicit dependence of the scintillation index on finite-time phase variations is obtained for long propagation paths. It is shown that for large amplitudes of phase fluctuations, a finite-time effect decreases the ability of phase diffuser to suppress the scintillations.

1 Introduction

Studies of laser beams propagating through turbulent atmospheres are important for many applications such as remote sensing, tracking, and long-
distance optical communications. However, fully coherent laser beams are very sensitive to fluctuations of the atmospheric refractive index. The initially coherent laser beam acquires some properties of Gaussian statistics in course of its propagation through the turbulence. As a result, the noise/signal ratio approaches unity for long-distance propagation. (See, for example, Refs.[1]-[6]). This unfavourable effect limits the performance of communication channels. To mitigate this negative effect the use of partially (spatially) coherent beams was proposed. The coherent laser beam can be transformed into a partially coherent beam by means of a phase diffuser placed near the exit aperture. This diffuser introduces an additional phase (randomly varying in space and time) to the wave front of the outgoing radiation. Statistical characteristics of the random phase determine the initial transverse coherence length of the beam. It is shown in Refs. [7],[8] that a considerable decrease in the noise/signal ratio can occur under following conditions: (i) the ratio of the initial transverse coherence length, $\lambda_c$, to the beam radius, $r_0$, should be essentially smaller than unity; and (ii) the characteristic time of phase variations, $\tau_d$, should be much smaller than the integration time, $T$, of the detector. However, only limiting cases ($\tau_d/T \to 0$ and ($\tau_d/T \to \infty$ have been considered in the literature. (See, for example, Refs. [7],[8] and Ref. [9], respectively). It is evident that the inequality $\tau_d << T$ can be easily satisfied by choosing a detector with very long integration time. At the same time, this kind of the detector cannot distinguish different signals within the interval $T$. This means that the resolution of the receiving system might become too low for the case of large $T$. On the other hand, there is a technical restriction on phase diffusers: up to now their characteristic times, $\tau_d$, are not smaller than $10^{-7}$s. Besides that, in some specific cases (see, for example, Ref. [10]), the spectral broadening of laser radiation due to the phase diffuser ($\Delta \omega \sim \tau_d^{-1}$) may become unacceptably high.

The factors mentioned above impose serious restrictions on the physical characteristics of phase diffusers which could be potentially useful for suppressing the intensity fluctuations. An adequate choice of diffusers may be facilitated if we know in detail the effect of finite-time phase variation, introduced by them, on the photon statistics. In this case, it is possible to control the performance of communication systems. In what follows, we will obtain theoretically the dependence of scintillation index on $\tau_d/T$ without any restrictions on the value of this ratio. This is the main purpose of our paper. Further analysis is based on the formalism developed in Ref. [8] and modified here to understand the case of finite-time dynamics of the phase
2 The method of photon distribution function in the problem of scintillations.

The detectors of the absorbed type do not sense the instantaneous intensity of electromagnetic waves \( I(t) \). They sense the intensity averaged over some finite interval \( T \) i.e.

\[
\bar{I}(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} I(t') dt'.
\]  

(1)

Usually, the averaging time \( T \) (the integration time of the detector) is much smaller than the characteristic time of the turbulence variation, \( \tau_A \), \( (T \ll \tau_A \sim 10^{-2} - 10^{-3} s) \). Therefore, the average value of the intensity can be obtained by further averaging of Eq. (1) over many measurements corresponding various realizations of the refractive-index configurations.

The scintillation index determining the mean-square fluctuations of the intensity is defined by

\[
\sigma^2 = \frac{\left[ \frac{1}{T^2} \int_{t-T/2}^{t+T/2} \int_{t-T/2}^{t+T/2} dt' dt'' \langle : I(t') I(t'') : \rangle - \langle \bar{I} \rangle^2 \right]}{\langle \bar{I} \rangle^2} = 1,
\]

(2)

where the symbol \{ : ... : \} indicates the normal ordering of the creation and annihilation operators which determine the intensity, \( I(t) \). (See more details in Refs. [8],[11]). The brackets \( < ... > \) indicate quantum-mechanical and atmospheric averagings.

The intensity \( I \) depends not only on \( t \), but also on the spatial variable \( r \). Therefore, the detected intensity is the intensity \( I(r,t) \) averaged not only over \( t \) as in Eq. (1) but also over the detector aperture. For simplicity, we will restrict ourselves to calculations of the intensity correlations for coinciding spatial points that correspond to "small" detector aperture. This simplification is quite reasonable for a long-distance propagation path of the beam.

In the case of quasimonochromatic light, we can choose \( I(r,t) \) in the form

\[
I(r,t) = \frac{1}{V} \sum_{q,k} e^{-ikr} b_{q+k/2}^+(t) b_{q-k/2}(t),
\]

(3)

where \( b_{q+k/2}^+(t) \) and \( b_{q}(t) \) are the creation and annihilation operators of photons.
with momentum $\hbar \mathbf{q}$. They are given in the Heisenberg representation. $V = L_x L_y L_z$ is the volume of the system.

It follows from Eqs. [23] that $\sigma^2$ can be obtained if one knows the average

$$
\langle : I(\mathbf{r}, t)I(\mathbf{r}, t') : \rangle = \frac{1}{V^2} \sum_{q, k, q', k'} e^{-i(k + k')^2} \langle b^+_{q + \frac{k}{2}}(t)b^+_{q - \frac{k}{2}}(t')b_{q' + \frac{k'}{2}}(t')b_{q' - \frac{k'}{2}}(t) \rangle.
$$

(4)

It is a complex problem to obtain this value for arbitrary turbulence strengths and propagation distances. Nevertheless, the following qualitative reasoning can help to do this in the case of strong turbulence. We have mentioned that the laser light acquires the properties of Gaussian statistics in the course of its propagation through the turbulent atmosphere. As a result, in the limit of infinitely long propagation path, $z$, only “diagonal” terms, i.e. terms with (i) $k = k' = 0$ or (ii) $q + k/2 = q' - k'/2$, $q - k/2 = q' + k'/2$ contribute to the right part of Eq. [4]. For large but still finite $z$, there exist small ranges of $k, k' \neq 0$ in case (i) and $q + k/2 \neq q' - k'/2$, $q - k/2 \neq q' + k'/2$ in case (ii) contributing into the sum in Eq. [4]. The presence of the mentioned regions is due to the two possible ways of correlating of four different waves (see Ref. [12]) which enter the right hand side of Eq. [4]. As explained in Ref. [13], the characteristic sizes of regions (i) and (ii) depend on the atmospheric broadening of beam radii as $(\Delta R_b)^{-1}$, thus decreasing with increasing $z$. In the case of long-distance propagation, $(\Delta R_b)^{-1}$ is much smaller than the component of photon wave-vectors perpendicular to the $z$ axis. The last quantity grows with $z$ as $z^{1/2}$. (See Ref. [8].) For this reason, the overlapping of regions (i) and (ii) can be neglected. In this case Eq. [4] can be rewritten in the convenient form:

$$
\langle : I(\mathbf{r}, t)I(\mathbf{r}, t') : \rangle = \frac{1}{V^2} \sum_{|k| < k_0} \sum_{q, q'} \langle : e^{-ik r b^+_{q + \frac{k}{2}}(t)b_{q - \frac{k}{2}}(t)} \rangle
$$

(5)

$$
e^{-ik r b^+_{q + \frac{k}{2}}(t')b_{q' - \frac{k'}{2}}(t')} + e^{-ik r b^+_{q + \frac{k}{2}}(t)b_{q - \frac{k}{2}}(t')} e^{-ik r b^+_{q + \frac{k'}{2}}(t')b_{q' - \frac{k'}{2}}(t')} : \rangle,
$$

where the value $k_0$, confining summation over $k, k'$, is chosen to be greater than $R_b^{-1}$ but much smaller than the characteristic transverse wave vector of the photons; this is consistent with the above explanations. The two terms in the right-hand side correspond to the two regions of four-wave correlations.

The quantity

$$
f(\mathbf{r}, \mathbf{q}, t) = \frac{1}{V} \sum_{|k| < k_0} e^{-ik r b^+_{q + \frac{k}{2}}(t)b_{q - \frac{k}{2}}(t)}
$$

(6)
entering the right side of Eq. 5 is the operator of photon density in phase space (the photon distribution function in \( r, q \) space). It was used in Refs. [8], [13] and [14] for the description of photon propagation in turbulent atmospheres. By analogy, we can define the two-time distribution function

\[
f(r, q, t, t') = \frac{1}{V} \sum_{|k| < k_0} e^{-ikr} b^+_{q + \frac{k}{2} + \frac{k}{2} - \frac{k}{2}}(t) b_{q - \frac{k}{2}}(t').
\]  

(7)

Then Eq. 5 can be rewritten in terms of the distribution functions as

\[
\langle : I(r, t) I(r, t') : \rangle = \sum_{q, q'} \langle : f(r, q, t) f(r, q', t') + f(r, q, t) f(r, q', t') f(r, q', t', t) : \rangle.
\]  

(8)

Let us represent \( f(r, q, t, t') \) in the form \( f(r, q, t, t + \tau) \). We assume that \( \tau \leq T \ll \tau_A \), as explained in the text after Eq. 11. In this case the Hamiltonian of photons in a turbulent atmosphere can be considered to be independent of time. As a result, both functions defined by Eqs. 6 and 7 satisfy the same kinetic equation, i.e.

\[
(\partial_t + c_q \partial_r + F \partial_q) f(r, q, t) = 0
\]

(9)

\[
(\partial_t + c_q \partial_r + F \partial_q) f(r, q, t, t + \tau) = 0,
\]

where \( c_q = \partial \omega_q / \partial q \) is the photon velocity, \( F \) is a random force, caused by the turbulence. This force is equal to \( \omega_0 \partial n(r) / \partial r \), where \( \omega_0 \) is the frequency of laser radiation. \( n(r) \) is the refractive index of the atmosphere. The general solution of the equation for \( f(r, q, t, t + \tau) \) can be written in the form

\[
f(r, q, t, t + \tau) = \frac{1}{V} \sum_{|k| < k_0} e^{-ikr(t_0)} b^+_{q(t_0) + \frac{k}{2} + \frac{k}{2} - \frac{k}{2}}(t_0) b_{q(t_0) - \frac{k}{2}}(t_0 + \tau),
\]  

(10)

where

\[
r(t_0) = r + \int_t^{t_0} \dot{r}(t') dt'
\]

(11)

\[
q(t_0) = q + \int_t^{t_0} \dot{q}(t') dt'.
\]

(12)

The functions \( r(t') \) and \( q(t') \) obey the equations of motion

\[
\dot{r}(t') = c_q(t'), \quad \dot{q}(t') = F(r(t'))
\]
with the boundary conditions \( r(t' = t) = r, q(t' = t) = q \). The instant \( t_0 \) is equal to \( t - z/c \), where \( c \) is the speed of light. \( t_0 \) is the time of the exit of photons from the source. This choice of \( t_0 \) makes it possible to neglect the influence of the turbulence on the initial values of operators \( b^+(t_0), b(t_0 + \tau) \) (their dependence on time is as in vacuum).

The term for \( f(r, q, t) \) can be obtained from Eq. 12 by putting \( \tau = 0 \). Substituting both distribution functions into Eq. 8, we obtain

\[
\langle : I(r, t) I(r, t') : \rangle = \frac{1}{V^2} \sum_{|k| \leq |k'| < k_0} \sum_{q \neq q'} \langle : e^{-ikr_0 q - ikr_0 q'} : \rangle \times
\]

\[
[b^+^{q(t_0)} \psi_{q(t_0)}^+(t_0) b^{q'(t_0)} \psi_{q'(t_0)}^-(t_0 + \tau) + b^+^{q(t_0)} \psi_{q(t_0)}^-(t_0 + \tau) b^{q'(t_0)} \psi_{q'(t_0)}^+(t_0 + \tau)] : ,
\]

where \( r_q(t_0) \) and \( r_{q'}(t_0) \) are solutions of Eqs. 12 with the initial conditions \( r(t' = t) = r, q(t' = t) = q \) and \( r(t' = t) = r, q(t' = t) = q' \), respectively.

### 3 A phase diffuser with finite correlation time

The operators on the right side of Eq. 13 are related through matching conditions with the amplitudes of the exiting laser radiation (see Ref. 8) by the relation

\[
b_{q \perp q_0} = b(L_x L_y)^{-1/2} \int dr_\perp e^{-ip_\perp r_\perp} \Phi(r_\perp),
\]

where \( b \) is the operator of the laser field which is assumed to be a single-mode field and the subscript \( (\perp) \) means perpendicular to the \( z \)-axis component. The function \( \Phi \) describes the profile of the laser mode, which is assumed to be Gaussian-type function \( \Phi = (2\pi)^{-1/2} r_0^{-1} e^{-r_0^2/r_0^2} \). \( r_0 \) describes the initial radius of the beam.

To account for the effect of the phase diffuser, a factor \( e^{i\varphi(r_\perp, t_0)} \) or \( e^{i\varphi(r_\perp, t_0 + \tau)} \) should be inserted into the integrand of Eq. 14. The quantity \( \varphi(r_\perp, t) \) is the random phase introduced by the phase diffuser. A similar consideration is applicable to each of four photon operators entering both terms in square brackets of Eq. 13. It can be easily seen that the factor

\[
\Upsilon = e^{i\varphi(r, t_0) - \varphi(r', t_0) + \varphi(r_1, t_0 + \tau) - \varphi(r'_1, t_0 + \tau)},
\]

\( 15 \)
Figure 1: Schematics of random phase variations along the $x$ direction. Two curves correspond to different instants $t$ and $t + \tau$.

describing the effect of phase screen on the beam, enters implicitly the integrand of Eq. 13 (the indices $\perp$ are omitted here for the sake of brevity). There are integrations over variables $r, r', r_1, r'_1$ as shown in Eq. 14. Furthermore, the brackets $< ... >$, which indicate averaging over different realizations of the atmospheric inhomogeneities, also indicate averaging over different states of the phase diffuser. As long as both types of averaging do not correlate, the factor (15) entering Eq. 13 must be averaged over different instants, $t_0$.

To begin with, let us consider the simplest case of two phase correlations $\langle e^{i[\varphi(r,t_0) - \varphi(r',t_0)]} \rangle$.  

It is evident that in the case $\langle |\varphi| \rangle >> 1$, as shown schematically in Fig. 1, the factor (16) is sizable if only points $r$ and $r'$ are close to one another. Therefore, the term given by Eq. 16 can be replaced by

$\langle e^{i\frac{\partial \varphi(r,t_0)}{\partial r}(r-r')} \rangle = e^{-\lambda_c^{-2}(r-r')^2}$,  

(17)

where $\frac{\partial \varphi(r,t_0)}{\partial r}$ is considered to be a Gaussian random variable with the mean-square values given by $\langle (\frac{\partial \varphi(r,t_0)}{\partial x})^2 \rangle = \langle (\frac{\partial \varphi(r,t_0)}{\partial y})^2 \rangle = 2\lambda_c^{-2}$, where $\lambda_c$ is the correlation length of phase fluctuations. (See Fig.1). As we see, in this case the effect of phase fluctuations can be described by the Schell model [7]-[9],[15]-[17].

A somewhat more complex situation is for the average value of $\Upsilon$ given by Eq. 13. There is an effective phase correlation not only in the case of
coincident times, but also for differing times. For $\lambda_c \ll r_0$, two different sets of coordinates contribute considerably to phase correlations. This can be described mathematically as

$$\langle \Upsilon \rangle = \langle e^{i[\varphi(r,t_0) - \varphi(r',t_0) + \varphi(r_1,t_0+\tau) - \varphi(r_1',t_0+\tau) - \varphi(r_1',t_0+\tau)]} \rangle \approx \langle e^{i[\varphi(r,t_0) - \varphi(r',t_0)]} \rangle \times$$

$$\langle e^{i[\varphi(r_1,t_0+\tau) - \varphi(r_1',t_0+\tau)]} \rangle + \langle e^{i[\varphi(r,t_0) - \varphi(r_1',t_0)]} \rangle \langle e^{i[\varphi(r_1,t_0+\tau) - \varphi(r',t_0)]} \rangle.$$

Repeating the arguments leading to Eq. 17, we represent the difference in the last term $\varphi(r,t_0) - \varphi(r_1',t_0 + \tau)$ as

$$\frac{\partial \varphi(r,t_0)}{\partial r}(r - r_1') - \frac{\partial \varphi(r,t_0)}{\partial t_0} \tau. \quad (19)$$

Then, considering the random functions $\frac{\partial \varphi(r,t)}{\partial r}$ and $\frac{\partial \varphi(r,t)}{\partial t}$ as independent Gaussian variables, we obtain a simple expression for $\langle \Upsilon \rangle$. It is given by

$$\langle \Upsilon \rangle = e^{-\lambda_c^2[(r-r')^2 + (r_1-r_1')^2]} + e^{-\lambda_c^2[(r-r')^2 + (r_1'-r_1)^2]} - 2\nu^2\tau^2, \quad (20)$$

where $\langle \left[ \frac{\partial \varphi(r,t)}{\partial t} \right]^2 \rangle = 2\nu^2$.

As we see, the effect of the phase screen can be described by two parameters, $\lambda_c$ and $\nu$, which characterize the spatial and temporal coherence of the laser beam. In the limiting case, $\nu \to \infty$, the second term in Eq. 20 vanishes and the problem is reduced to the case considered in Refs. [7], [8]. In the opposite case, $\nu \to 0$, both terms in Eq. 20 are important. This is shown in Ref. [9]. In what follows, we will see that these two limiting cases have physical interpretations where where $\nu T \gg 1$ (slow detector) and $\nu T \ll 1$ (fast detector), respectively.

There is a specific realization of the diffuser in which a random phase distribution moves across the beam. (This situation can be modeled by a rotating transparent disk with large diameter and varying thickness.) The phase depends here on the only variable $r - vt$, i.e.

$$\varphi(r,t) \equiv \varphi(r - vt), \quad (21)$$

where $v$ is the velocity of the drift. Then we have

$$\langle \Upsilon \rangle \approx e^{-\lambda_c^2[(r-r')^2 + (r_1-r_1')^2]} + e^{-\lambda_c^2[(r-r_1'+vt)^2 + (r'-r_1'+vt)^2]}$$

Comparing Eqs. 20 and 22, we see that the quantity, $v/\lambda_c$, stands for the characteristic parameter describing the efficiency of the phase diffuser. The
criterion of “slow” detector requires \((vT/\lambda_c) >> 1\). Qualitatively, the two scenarios of phase variations, given by Eqs. 20 and 22, affect in a similar way the intensity fluctuations. In what follows, we consider the first of them as the simplest one. (This is because the spatial and temporal variables in \(\langle \Upsilon \rangle\), given by Eq. 20, are separable.)

![Figure 2](image)

Figure 2: The scintillation index \(\sigma^2\) vs propagation distance \(z\) in the case of “slow” detector: \(vT \to \infty\). The parameter \(r_1^2/r_0^2\) indicates different initial coherence length. In the absence of phase diffuser \((r_1/r_0)^2 = 1\) (solid line). \(C_n^2\) is the conventional parameter describing a strength of the atmospheric turbulence.

Substituting the expressions for operators given by Eq. 14 with account for the phase factors \(e^{\pm i\varphi(r_{\perp}, t_0)}\) and averaging over time as shown in Eq. 1, we obtain

\[
\langle \hat{I}(t)^2 \rangle = \left( \frac{2\pi r_1^2}{L_x L_y V} \right)^2 \langle b^+ b^+ b b \rangle \Psi(vT) \sum_{|k|,|k'|<k_0} \sum_{q,q'} \left\langle e^{-i\mathbf{k}_q(t_0)-i\mathbf{k}'_{q'}(t_0)} \times \left[ e^{(k^2+k'^2)r_0^2/8-(q_0^2+q'^2)_r^2/2} + e^{(K_0^2+K'^2)\mathbf{r}_0^2/8-(Q_0^2+Q'^2)\mathbf{r}_0^2/2} \right] \right\rangle,
\]

(23)
where the notation \(< \ldots >\) after sums indicates averaging over different realizations of the atmospheric refractive index. The parameter \(r_1^2 = r_0^2(1 + 2r_0^2\lambda_c^{-2})^{-1}\) describes the initial coherence length modified by the phase diffuser. Other notations are defined by following relations

\[
\Psi(\nu T) \equiv 1 + \frac{\Gamma(1/2) - \Gamma(1/2, 2\nu^2T^2)}{\sqrt{2}\nu T} - \frac{1 - e^{-2\nu^2T^2}}{2\nu^2T^2}
\]

\[
K_{t_0} = q_{t_0} - q'_{t_0} + \frac{k + k'}{2}, K'_{t_0} = q'_{t_0} - q_{t_0} + \frac{k + k'}{2},
\]

\[
Q_{t_0} = \frac{q_{t_0} + q'_{t_0}}{2} + \frac{k - k'}{4}, Q'_{t_0} = \frac{q_{t_0} + q'_{t_0}}{2} - \frac{k - k'}{4},
\]

Further calculations follow the scheme described in Ref [8]. Fig. 2 illustrates the effect of the phase diffuser on scintillations in the limit of a "slow" detector \((\nu T \to \infty)\). We can see a considerable decrease in \(\sigma^2\) caused by the phase diffuser. At the same time, the effect of the phase screen on \(\sigma^2\) becomes weaker for finite values of \(\nu T\). Moreover, comparing the two upper curves in Fig. 3, we see the opposite effect: slow phase variations \((\nu T = 1)\) result in increased scintillations. There is a simple explanation for this phenomenon: the noise generated by the turbulence is complemented by the noise arising from the random phase screen. The integration time of the detector, \(T\), is not sufficiently large for averaging phase variations generated by the diffuser.

The function, \(\Psi(\nu T)\), has a very simple form in the two limits: (i) \(\Psi(\nu T) \approx 1 + \frac{\sqrt{\pi}}{\sqrt{2}\nu T}\), when \(\nu T >> 1\); and (ii) \(\Psi(\nu T) \approx 2\), when \(\nu T << 1\). Then, in case (i) and for small values of the initial coherence \([r_1^2/r_0^2] << 1\], the asymptotic term for the scintillation index \((z \to \infty)\) is given by

\[
\sigma^2 \approx \frac{r_1^2}{r_0^2} + \frac{\sqrt{\pi}}{\sqrt{2}\nu T}.
\]

The right-hand side of Eq. 24 differs from analogous one in Ref. [8] by the value \(\frac{\sqrt{\pi}}{\sqrt{2}\nu T}\) that is much less than unity but, nevertheless, can be comparable or even greater than \((r_1/r_0)^2\).

In case (ii), the asymptotic value of \(\sigma^2\) is close to unity, coinciding with the results of Refs. [6] and [9]. This agrees with well known behavior of the scintillation index to approach unity for any source distribution, provided the response time of the recording instrument is short compared with the source coherence time. (See, for example, survey [4]).
A similar tendency can be seen in both Figs. 3 and 4: the curves with the smallest $\nu T$, used for numerical calculations ($\nu T = 1$), are close to the curves “without diffuser” in spite of the small initial coherence length $[(r_1/r_0)^2 = 0.2; 0.05]$. It can also be seen that all curves approach their asymptotic values very slowly.

Figure 3: Dependence of scintillation index on the distance $z$ for different values of the parameter $\nu T$ describing diffuser dynamics. The solid curve is calculated for $(r_1/r_0)^2 = 1$ (without diffuser). Other curves are for $(r_1/r_0)^2 = 0.2$.

4 Discussion

It follows from our analysis that the scintillation index is very sensitive to the diffuser parameters, $\lambda_c$ and $\nu$, for long propagation paths. On the other hand, the characteristics of the irradiance such as beam radius, $R$, and angle-of-arrival spread, $\Delta \theta$, do not depend on the presence of the phase diffuser for large values of $z$. To see this, the following analysis is useful.
Figure 4: The same as in Fig. 3 but for smaller initial coherence length: \((r_1/r_0)^2 = 0.05\).

The beam radius expressed in terms of the distribution function is given by

\[
R^2(z) = \frac{\sum_q \int d\mathbf{r}_\perp r_\perp^2 \langle f(\mathbf{r}, \mathbf{q}, t) \rangle}{\sum_q \int d\mathbf{r}_\perp \langle f(\mathbf{r}_\perp, \mathbf{q}, t) \rangle}.
\]  

(25)

Straightforward calculations using Eq. (10) with \(\tau = 0\) (see Ref. [8]) result in the following explicit form:

\[
R^2(z) = \frac{r_0^2}{2} + \frac{2z^2}{q_0^2 r_1^2} + 4Tz^3,
\]

(26)

where \(T = 0.558C_n^2l_0^{1/3}\) and \(l_0\) is the inner radius of turbulent eddies, which in our previous calculations was assumed to be equal \(2\pi \times 10^{-3}\) m. As we see, the third term does not depend on the diffuser parameters and it dominates when \(z \to \infty\).

A similar situation holds for the angle-of-arrival spread, \(\Delta \theta\). (This physical quantity is of great importance for the performance of communication systems based on frequency encoded information [10].) It is defined by the
distribution function as

\[(\Delta \theta (r_\perp, z))^2 = \frac{\sum q q^2}{q_0^2 \sum q} \langle f(r_\perp, z, q, t) \rangle \]. \hspace{1cm} (27)

Simple calculations, which are very similar to those while obtaining \(R^2\), result in

\[[\Delta \theta (r_\perp = 0, z)]^2 = \frac{2}{r_1^2 q_0^2} + 12Tz - \frac{4z^2}{q_0^2 R^2} (r_1^2 + 3T q_0^2 z)^2]. \hspace{1cm} (28)

For long propagation paths, Eq. 28 reduces to \(3Tz\), which like \(R^2\) does not depend on the diffuser parameters.

As we see, for large distances \(z\), the quantities \(R^2\) and \(\Delta \theta\) do not depend on \(\lambda_c\) and \(\nu\). This contrasts with the case of the scintillation index. So pronounced differences can be explained by differences in the physical nature of these characteristics. It follows from Eq. 2 that the functional, \(\sigma^2\), is quadratic in the distribution function, \(f\). Hence, four-wave correlations determine the value of scintillation index. The main effect of a phase diffuser on \(\sigma^2\) is to destroy correlations between waves exited at different times. (See more explanations in Ref. [14]). This is achieved at sufficiently small parameters \(\lambda_c\) and \(\nu^{-1}\).

In contrast, \(R^2\) and \(\Delta \theta\) depend on two wave-correlations, both waves being given at the same instant. Therefore, the values of \(R^2\) and \(\Delta \theta\) do not depend on the rate of phase variations [\(\nu\) does not enter the factor (17) describing the effect of phase diffuser]. Moreover, these quantities become independent of \(\lambda_c\) at long propagation paths because light scattering on atmospheric inhomogeneities prevails in this case.

The plots in Figs. 3 and 4 show that the finite-time effect is quite sizable even for very “slow” detectors (\(\nu T = 20\)). Our paper makes it possible to estimate the actual utility of phase diffusers in several physical regimes.

5 Conclusion

We have analyzed the effects of a diffuser on scintillations for the case of large-amplitude phase fluctuations. This specific case is very convenient for theoretical analysis because only two parameters are required to describe the effects of the diffuser. Phase fluctuations may occur independently in space as well as in time. Also, our formalism can be applied for the physical situation in which a spatially random phase distribution drifts across the beam. (See
Eq. 22) Our results show the importance of both parameters, $\lambda_c$ and $\nu$, on the ability of a phase diffuser to suppress scintillations.

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