Per-antenna power minimization in symbol-level precoding for the multibeam satellite downlink

Danilo Spano¹ | Symeon Chatzinotas¹ | Stefano Andrenacci¹ | Jens Krause² | Björn Ottersten¹

¹SnT-securityandtrust.lu, University of Luxembourg, 29 Avenue J.F. Kennedy, Luxembourg City L-1855, Luxembourg
²SES, Chateau de Betzdorf, Betzdorf, Luxembourg

Correspondence
Danilo Spano, SnT-securityandtrust.lu, University of Luxembourg, 29 Avenue J.F. Kennedy, L-1855 Luxembourg City, Luxembourg. Email: danilo.spano@uni.lu

Funding information
European H2020 project SANSA; Fond National de la Recherche Luxembourg (FNR)

Summary
This paper addresses the problem of multiuser interference in the forward downlink channel of a multibeam satellite system. A symbol-level precoding scheme is considered, to exploit the multiuser interference and transform it into useful power at the receiver side, through a joint utilization of the data information and the channel state information. In this context, a per-antenna power minimization scheme is proposed, under quality-of-service constraints, for multilevel modulation schemes. The consideration of the power limitations individually for each transmitting radio frequency chain is a central aspect of this work, and it allows to deal with systems using separate per-antenna amplifiers. Moreover, this feature is also particularly relevant for systems suffering nonlinear effects of the channel. This is the case of satellite systems, where the nonlinear amplifiers should be properly driven to reduce the detrimental saturation effect. In the proposed scheme, the transmitted signals are designed to reduce the power peaks, while guaranteeing some specific target signal-to-noise ratios at the receivers. Numerical results are presented to show the effectiveness of the proposed scheme, which is compared both with the state of the art in symbol-level precoding and with the conventional minimum mean square error precoding approach.

KEYWORDS
constructive interference, multibeam satellite downlink, nonlinear systems, per-antenna power minimization, symbol-level precoding

1 | INTRODUCTION

One of the biggest challenges in the current research on satellite communication (SatCom) systems is the need to break the existent throughput gridlock, to fulfill the ever-increasing demand for interactive services and multimedia content delivery. In particular, the throughput requirements for the next generation of SatCom systems are striving towards the terabit/s capacity.¹⁻⁵ To achieve this target, it is pivotal to find feasible technical solutions taking into account that the wireless spectrum is a scarce resource, which is getting more and more congested. The state of the art in high throughput SatCom relies on multibeam architectures, which exploit the spatial degrees of freedom offered by antenna arrays to aggressively reuse the available spectrum, thus realizing a space division multiple access scheme.⁶ As a matter of fact, aggressive frequency reuse schemes are possible only if advanced signal processing techniques are developed, with the objective of handling the multiuser interference (MUI) arising in multibeam systems and deteriorating their performance. Such signal processing techniques are commonly referred to as multuser multiple-input multiple-output and, in the satellite context, also as multibeam joint processing. In this context, linear precoding (or beamforming) techniques have been a prolific recent area in the recent years, showing to be an effective way to manage the MUI while guaranteeing some specific service requirements.⁷⁻¹² The benefits of using precoding techniques for managing the interference at the gateway in SatCom are also considered in the most recent extensions of broadband multibeam SatCom standards.¹³
The conventional precoding approach exploits the knowledge of the channel state information (CSI) to design a precoder to be applied to the multiple data streams, thus mitigating the MUI. Since the precoder depends only on the CSI, it remains constant for a whole block of symbols whose length is related to the coherence time of the channel. Therefore, we can refer to this scheme as block-level precoding. The most relevant closed-form solutions for block-level precoding are the well-known zero-forcing precoding\cite{16,17} and minimum mean square error (MMSE) precoding.\cite{16,17} Over-the-air demonstrations for such schemes have been recently proposed for SatCom systems.\cite{18} Beyond these closed-form schemes, a number of precoding strategies based on the numerical solution of optimization problems have been considered in the literature, which allow to optimize the system with respect to specific objectives. The optimal precoding algorithm for the minimization of the total transmit power, while guaranteeing some quality-of-service (QoS) targets at each user, was given in Bengtsson and Ottersten,\cite{10,12} while the max-min fair problem under sum power constraints was optimally solved in Schubert and Boche.\cite{11} The latter strategy aims to maximize the minimum signal-to-interference-plus-noise ratio (SINR) among the users, to preserve the fairness of the system. The research work on block-level precoding was extended in Yu and Lan\cite{19} accounting for per-antenna power constraints and in Dartmann et al\cite{20} and Zheng et al\cite{21} considering generalized power constraints. Furthermore, the problem of block-level precoding in a multigroup multicast framework has been tackled in Christopoulos et al.\cite{22}

A different precoding strategy, considered more recently in the literature, is known as symbol-level precoding.\cite{23-28} In this approach, the transmitted signals are designed on the basis of the knowledge of both the CSI and the data information (DI), constituted by the symbols to be delivered to the users. Since the design exploits also the DI, the objective of symbol-level precoding is not to eliminate the interference, but rather to control it so to have a constructive interference effect at each user. The classification of the interference as constructive or destructive was given in Masouros and Alsusa,\cite{24} where a selective channel inversion scheme was proposed to eliminate the destructive interference. A more advanced symbol-level precoding scheme was proposed in Masouros,\cite{24} based on the rotation of the constructive interference, with the aim to transform it into useful power. Different optimization approaches have been proposed in the literature for symbol-level precoding. In Alodeh et al,\cite{25} the sum power minimization and the max-min fair problem were solved for M-PSK modulations. Furthermore, symbol-level precoding has been considered for multicast-based systems and for multilevel modulations, including also flexible schemes accounting an imperfect knowledge of the CSI, as well as relaxed detection regions.\cite{27-29} The potential of symbol-level precoding has been further explored by the authors in previous studies,\cite{20,31} where the per-antenna power limitations of the transmitter are accounted on a symbol basis, for M-PSK modulation schemes.

Extending the work of Spano et al\cite{30,31} this paper addresses the problem of per-antenna power minimization in symbol-level precoding accounting for multilevel modulation schemes. The consideration of the power limitations individually for each transmitting radio frequency (RF) chain is a central aspect of this work and is motivated by the practical implementation of most systems that rely on precoding. In fact, a common practice in multiantenna systems is the use of individual per-antenna amplifiers, and this implies a lack of flexibility in sharing energy resources among the antennas of the transmitter. In spite of the possibility of using flexible amplifiers so to handle this issue, specific communication systems cannot afford this design. Typical per-antenna power limited systems can be found in multibeam SatCom,\cite{32} where flexible onboard payloads are difficult to implement. In addition to this, the proposed per-antenna power minimization approach is also particularly relevant for systems with nonlinear channels. This is the case of SatCom, where the nonlinear effects are usually introduced by the on-board per-antenna traveling-wave-tube amplifiers (TWTAs), which result in a distortion on the transmitted waveforms.\cite{13,33-35} A typical solution to this problem in single-user links relies on predistortion techniques,\cite{33,36,37} but their extension to multibeam systems relying on precoding is not straightforward, because of the mutual correlation between the data streams induced by the precoding schemes. About this, a joint predistortion algorithm for multibeam systems is given in Mengali et al.\cite{38} In this context, the proposed precoding strategy allows to design more robust signals to the nonlinear effects. In particular, the symbol-level design allows to have a control on the instantaneous per-antenna transmit power, thus reducing the power peaks among the different antennas. In this direction, different symbol-level strategies devised for nonlinear channels have been recently proposed in Spano et al.\cite{39,40} where however the objective is not the minimization of the per-antenna transmit power. Overall, the main contributions of this paper can be summarized as follows:

- A novel symbol-level precoding strategy is proposed, performing a per-antenna power minimization under QoS constraints, guaranteeing the required SINR to each user. Compared with the state-of-the-art symbol-level techniques, this design allows the reduction of the power peaks among the different antennas, for each symbol slot, and therefore improves the robustness of the signals to the nonlinear effects.
- First, the algorithm is formalized and solved for the case of single-level modulations (ie, M-PSK). Then, the scheme is generalized for multilevel constellations.
- The performance of the proposed approach is assessed through simulations in a realistic SatCom scenario.

The remainder of the paper is organized as follows. In Section 2, the system and signals communication model is delineated. In Section 3, the symbol-level peak power minimization problem is proposed and solved. In Section 4, the proposed approach is validated through simulation results. Finally, in Section 5, conclusions are drawn.

Notation: We use upper-case and lower-case boldfaced letters to denote matrices and vectors, respectively. \((\cdot)^T\) denotes the transpose of \((\cdot)\), \(|\cdot|\) and \(\xi(\cdot)\) denote the amplitude and the phase of \((\cdot)\), respectively, while \(\text{Re}(\cdot)\) and \(\text{Im}(\cdot)\) are the real and imaginary parts of \((\cdot)\), and \(i\) is used to denote the imaginary unit. \(|\cdot|\) and \(\|\cdot\|_\infty\) represent the Euclidean norm and the \(\infty\) norm of \((\cdot)\), respectively. Moreover, \(\Pr(\cdot)\) denotes the probability of an
event, $\text{diag}(\cdot)$ denotes a diagonal matrix whose diagonal entries are the elements of $\langle \cdot \rangle$, and $\cdot$ is used for denoting the element-wise Hadamard operations. Finally, $\geq$ is used as a generalized inequality for the optimization constraints, to be read as $\geq$ or as $=$ depending whether the constraint is referred to a boundary symbol or to an internal symbol of the constellation, respectively.

# 2 SYSTEM AND SIGNALS MODEL

We consider a multiuser multiple-input multiple-output satellite system, and we denote by $K$ the number of antennas of the on-board transmitter. These antennas allow the generation of $K$ independent beams through a parabolic reflector. Let us also assume that $K$ single-antenna users are served in a specific instant, namely, considering one user per beam.\(^1\) We consider a general multilevel modulation, and we assume a channel vector $h_j \in \mathbb{C}^{1 \times K}$ between the transmitting antennas and the $j$th user. The received signal at the $j$th user in the symbol slot $n$ can be written as

$$y_j[n] = h_j x[n] + z_j[n],$$

where $x[n] \in \mathbb{C}^{K \times 1}$ represents the transmitted signal vector from the $K$ transmit antennas and $z_j[n]$ is a random variable distributed as $\mathcal{CN}(0, \sigma_z^2)$, modeling the zero mean additive white Gaussian noise measured at the $j$th user’s receiving antenna.

By collecting the received signals by all the users in a vector $y[n] \in \mathbb{C}^{K \times 1}$, the above model can be rewritten in a compact form as

$$y[n] = H x[n] + z[n],$$

where $H = [h_1^T, \ldots, h_K^T]^T \in \mathbb{C}^{K \times K}$ represents the system channel matrix and $z[n] \in \mathbb{C}^{K \times 1}$ collects the additive white Gaussian noise components for all the users.

The transmitted signal vector $x[n]$ is obtained as output of a precoding module, which takes as input the CSI, which is an estimate of $H$, and the DI $d[n] \in \mathbb{C}^{K \times 1}$, namely, the data symbols to be conveyed to the users. The data symbols are assumed to be uncorrelated and taken from a constellation represented by the symbol set $\mathcal{D}$, having unit average power, i.e., $\mathbb{E}[|d|^2] = 1$. In the transmission scheme, we assume a framing structure including a preamble of pilot symbols. Such pilots are exploited by each user to estimate the related channel vector, and the resulting CSI is fed back to the gateway to be available for the precoding operation.

## 2.1 Multibeam channel model

The complex matrix $H$, modeling the multibeam satellite channel, can be written as follows:

$$H = L \cdot B \cdot \Phi,$$

where $B \in \mathbb{R}^{K \times K}$ models the gains related to the multibeam satellite radiation pattern, $\Phi \in \mathbb{C}^{K \times K}$ models the signal phase rotations induced by the on-board RF chains and by the propagation paths, while the matrix $L \in \mathbb{R}^{K \times K}$ includes the link budget coefficient for each antenna-user pair.

The elements of $B$ depend on the multibeam radiation pattern at hand and on the users position. Regarding the phase rotations, they are assumed independent for each antenna-user pair, so as to take into account the different propagation paths between the transmitter and the users, as well as the different on-board RF chains.\(^2\) Accordingly, the generic element of the matrix $\Phi$ in (3) is modeled as $[\Phi]_{jk} = e^{j \phi_{jk}}$, where $\phi_{jk}$ are independent random variables uniformly distributed in $[0, 2\pi)$ for $j,k=1,\ldots,K$.

Finally, considering the generic $(j,k)$ element of the matrix $L$, related to the link between the $k$th beam’s antenna and the $j$th user’s antenna, it will be given by

$$[L]_{jk} = \sqrt{G_{jk}} \left(4\pi d_{jk} \lambda \right)^{-1},$$

with $G_{jk}$ being the $j$th user’s receiving antenna gain, $d_{jk}$ being the distance between the $j$th user and the $k$th beam’s antenna, and $\lambda$ the wavelength.

## 2.2 Nonlinearities of the satellite channel

As anticipated, it should be taken into account how the system model introduced in (2) is actually degraded by the nonlinear effects introduced by the on-board per-antenna TWTA. To model such nonlinear effects, we write in polar coordinates the input signal to the TWTA on the generic $i$th RF chain of the transmitter as

\(^1\)This is reasonable if the systems resort to a time-division multiplexing scheme, to serve all the users in each beam.

\(^2\)In particular, we take into account phase shifts introduced on each RF chain due to on-board payload imperfections and/or different on-board propagation paths. This justifies the assumption of independent phases among the different RF chains. Nonetheless, the main conclusions of this work are still valid if a different model for $\Phi$ is considered.

\(^3\)To ease the notation, hereafter the time index $n$ is omitted in formulas.
where $r_i$ and $\theta_i$ are the amplitude and the phase of $x_i$, respectively. Then, the output signal of the TWTA can be written as

$$\hat{x}_i = f_A(r_i) \exp(f_P(r_i)) \exp(i\theta_i),$$

with $f_A(\cdot)$ and $f_P(\cdot)$ denoting the amplitude-to-amplitude and amplitude-to-phase conversions, respectively. The resulting system model is shown in Figure 1.

An analytical description of the amplitude and phase distortion induced by the TWTA is given in Saleh, with the well-known Saleh model. Further, different numerical models for the input-output characteristics of the amplifiers are provided in previous studies. We take as a reference the common nonlinearized TWTA model of previous study, whose normalized amplitude-to-amplitude and amplitude-to-phase characteristics are shown in Figure 2. The characteristics clearly show the saturation effect and the introduced phase shifts.

The on-board TWTA need to be operated as close as possible to saturation, to efficiently exploit the scarce available power. As a consequence, the need of controlling the power level of the transmitted waveforms is crucial to reduce the detrimental effect of the nonlinearities of the channel. Moreover, in systems using separate per-antenna TWTA, a different phase distortion is induced over the various data streams, because of the different instantaneous power carried out by the symbols of each stream. This additional issue, which we refer to as differential phase shift, is particularly relevant when precoding is applied, since the power imbalances between the transmitting antennas are usually not controlled. Therefore, a reduced power variation of the precoded waveforms is needed also in the space dimension, i.e., between the different antennas. In this direction, besides accounting for the per-beam power limitations, the proposed symbol-level precoding approach allows to reduce the power imbalances between the different transmitting antennas, thus improving the robustness of the waveforms to the channel nonlinearities.

FIGURE 1 Block scheme of the transmitter relying on symbol-level precoding, for a generic symbol slot. CSI, channel state information; TWTA, traveling-wave-tube amplifier

FIGURE 2 Normalized amplitude-to-amplitude (AM-AM) and amplitude-to-phase (AM-PM) characteristics of the on-board traveling-wave-tube amplifiers (nonlinearized model) [Colour figure can be viewed at wileyonlinelibrary.com]
3 | SYMBOL-LEVEL PRECODING FOR PEAK POWER MINIMIZATION

The aim is to design the transmitted vector $x$, based on the CSI and the DI, ensuring that the received signal lies in the detection region of the desired symbol, for each user. This way, the interfering signals are forced to constructively contribute to the useful received power, in line with the definition of constructive interference provided in Alodeh et al.\textsuperscript{25} In this context, the objective is to minimize the per-desired symbol, for each user. This way, the interfering signals are forced to constructively contribute to the useful received power, in line with per-beam QoS constraints, for each symbol slot. Such minimization can be seen as the minimization of the peak power between the antennas; therefore, we will refer to the proposed scheme as symbol-level precoding for peak power minimization (SL-PPM). We start by addressing the problem for M-PSK modulations, in the fashion of Spano et al.\textsuperscript{30} and Spano et al.\textsuperscript{31} Then, the problem will be extended to the general case of multilevel modulation schemes.

3.1 | SL-PPM for single-level modulations

Focusing on the case of single-level modulations, i.e., M-PSK, the SL-PPM optimization problem can be written as follows:

$$
\mathbf{x}(d, H, y) = \arg \min_x \max_{i = 1, \ldots, K} \{ |x|^2 \}
$$

subject to

$$
\begin{align*}
C1: & \quad |h_i x|^2 \geq \gamma \sigma_i^2, \quad j = 1, \ldots, K, \\
C2: & \quad \angle h x = \angle d_j, \quad j = 1, \ldots, K.
\end{align*}
$$

where $\gamma$ is the target SINR that should be granted for the $j$th user and $y = [y_1, \ldots, y_K]^T$ stacks the target SINR for all the users. The set of constraints $C1$ represents a QoS constraint for each user, while the set of constraints $C2$ represents the constructive interference condition, guaranteeing that each user receives the desired data symbol with the correct phase. To write the optimization problem in a more tractable form, we carry out the following steps, following the method of Kalantari et al.\textsuperscript{42}

First of all, by defining $a_j = \tan(\angle d_j)$ $\forall j = 1, \ldots, K$, the constraint $C2$ in (7) can be rewritten, by applying the tangent operator,\textsuperscript{4} as

$$
\frac{\text{Im}(h x)}{\text{Re}(h x)} = a_j, \quad j = 1, \ldots, K.
$$

However, since the tangent is not a one-to-one function, the following conditions should be added, to ensure that the received symbol and the intended one lie in the same quadrant:

$$
\begin{align*}
\text{Re}(d_j) \text{Re}(h x) & \geq 0, \quad j = 1, \ldots, K, \\
\text{Im}(d_j) \text{Im}(h x) & \geq 0, \quad j = 1, \ldots, K.
\end{align*}
$$

Second, the QoS constraint $C1$ in the problem (7) can be reformulated referring to the amplitude levels of the in-phase and quadrature components of the corresponding symbols, as follows:

$$
\begin{align*}
|\text{Re}(h x)| & \geq \sigma_2 \sqrt{\gamma} |\text{Re}(d_j)|, \quad j = 1, \ldots, K, \\
|\text{Im}(h x)| & \geq \sigma_2 \sqrt{\gamma} |\text{Im}(d_j)|, \quad j = 1, \ldots, K,
\end{align*}
$$

where the absolute value is necessary for accounting negative components. By multiplying both the members of the above equations by $\text{Re}(d_j)$ and $\text{Im}(d_j)$, respectively, and by taking into account the conditions in (9), the above constraints become

$$
\begin{align*}
\text{Re}(d_j) \text{Re}(h x) & \geq \sigma_2 \sqrt{\gamma} \text{Re}^2(d_j), \quad j = 1, \ldots, K, \\
\text{Im}(d_j) \text{Im}(h x) & \geq \sigma_2 \sqrt{\gamma} \text{Im}^2(d_j), \quad j = 1, \ldots, K.
\end{align*}
$$

With these new constraints, and resorting to the concept of $L_\infty$-norm, the problem (7) can be rewritten as:

$$
\mathbf{x}(d, H, y) = \arg \min_x \| x \|_\infty
$$

subject to

$$
\begin{align*}
C1: & \quad \text{Re}(d_j) \text{Re}(h x) \geq \sigma_2 \sqrt{\gamma} \text{Re}^2(d_j), \quad j = 1, \ldots, K, \\
C2: & \quad \text{Im}(d_j) \text{Im}(h x) \geq \sigma_2 \sqrt{\gamma} \text{Im}^2(d_j), \quad j = 1, \ldots, K, \\
C3: & \quad \frac{\text{Re}(h x)}{\text{Im}(h x)} = a_j, \quad j = 1, \ldots, K,
\end{align*}
$$

and, in a more compact form, as follows:

\footnote{This does not apply for data symbols laying on the imaginary axis, since the tangent is not defined in such case. Although this case can be easily handled, it is not considered herein, since we can always assume a phase offset preventing this situation.}
where $D=\text{diag}(d)$, $A=\text{diag}(a_1,\ldots,a_K)$, $\beta_j = \sigma_j \sqrt{\text{det}(d)}$ ², $\beta_i = \sigma_i \sqrt{\text{det}(d)}$.

The problem (13) can be written as a second-order cone programming (SOCP) in the stacked variable $\hat{x}=[\text{Re}(x)^T, \text{Im}(x)^T]^T \in \mathbb{R}^{2K}$. To this end, the objective function should be written as

$$\| \hat{x} \|_\infty = \max_{i=1,\ldots,K} |x_i| = \max_{i=1,\ldots,K} \| E_i \hat{x} \|.$$  

where $E_i \in \mathbb{R}^{2 \times 2K}$ is a matrix used for selecting $\text{Re}(x_i)$ and $\text{Im}(x_i)$ in the stacked vector $\hat{x}$ and, $\forall i=1,\ldots,K$, is in turn defined as

$$E_i = \begin{bmatrix} e_i & 0_K \; \; 0_K & e_i \end{bmatrix}.$$  

with $e_i$ being the $i$th row of an identity matrix with size $K$ and $0_K$ being the all zero entries vector in $\mathbb{R}^{K \times 1}$.

By defining the $H_1=[\text{Re}(H_{1}),\text{Im}(H_{1})]$ and $H_2=[\text{Im}(H_{1}),\text{Re}(H_{1})]$, the problem (13) becomes

$$\hat{x}(d, H, \gamma) = \arg \min_{x} \max_{i=1,\ldots,K} \| E_i \hat{x} \|$$

subject to

$$c1: \text{Re}(D)H_1 \hat{x} \geq \beta_i,$$

$$c2: \text{Im}(D)H_2 \hat{x} \geq \beta_i,$$

$$c3: (AH_1-H_2)\hat{x} = 0.$$  

Finally, by introducing a slack variable $r$, the SL-PPM problem can be formulated as an SOCP as follows:

$$\hat{x}(d, H, \gamma) = \arg \min_{x} \quad r$$

subject to

$$c1: \| E_i \hat{x} \| \leq r, \quad i=1,\ldots,K,$$

$$c2: \text{Re}(D)H_1 \hat{x} \geq \beta_i,$$

$$c3: \text{Im}(D)H_2 \hat{x} \geq \beta_i,$$

$$c4: (AH_1-H_2)\hat{x} = 0.$$  

This optimization problem can be efficiently solved using standard convex optimization tools. For this work, we rely on the CVX tool.

### 3.2 SL-PPM for multilevel modulations

We consider now a generalized version of the SL-PPM problem (7), assuming a multilevel modulation scheme for the DI. In this case, while imposing the constructive interference constraints, the symbol-level optimization strategy will have to take into account whether the DI symbols at hand are internal points of the constellation or border ones. Accordingly, the optimization problem can be generalized as follows:

$$x(d, H, \gamma) = \arg \min_{x} \quad \max_{i=1,\ldots,K} \{ |x_i|^2 \}$$

subject to

$$c1: |h_j x|^2 \geq \kappa^2 |d_j|^2, \quad j=1,\ldots,K,$$

$$c2: \text{Re}(H_{1}x) = \text{Re}(d), \quad j=1,\ldots,K,$$

where $\kappa_j = |d_j|/\sqrt{|E_2||d_j|^2}$ is a magnitude scaling factor for the symbol $d_j$ which allows to account the different amplitudes of the symbols in the multilevel constellation $d$. The assumption to have symbols with unit average power implies that $\kappa^2=|d|$. As anticipated, the notation $\geq$ is used as a generalized inequality that allows to unify different constraints for the internal points of the constellation, for which we impose an equality, and for the border ones, for which an inequality is considered (generalized inequalities related to the different detection regions can be also found in previous studies).

Analogously to the previous case, the constraints of (18) can be reformulated referring to the in-phase and quadrature components of the corresponding symbols. In particular, the constraint $c1$ can be written as
\begin{align}
|\text{Re}(h_j)| & \leq \sigma_j \sqrt{|\text{Re}(d_j)|}, \ j = 1, \ldots, K, \\
|\text{Im}(h_j)| & \leq \sigma_j \sqrt{|\text{Im}(d_j)|}, \ j = 1, \ldots, K.
\end{align}

Applying the same procedure as before, the problem can be reformulated as an SOCP, as follows:

\[ \hat{x}(d, H, \gamma) = \arg \min_r r \]

subject to

\begin{align}
C_1: \quad & ||E\hat{x}|| \leq r, \ i = 1, \ldots, K, \\
C_2: \quad & \text{Re}(D)H_1\hat{x} \succeq \beta_i, \\
C_3: \quad & \text{Im}(D)H_2\hat{x} \succeq \beta_i, \\
C_4: \quad & (AH_1-H_2)\hat{x} = 0,
\end{align}

and this final formulation can be efficiently solved resorting to standard convex optimization tools, such as CVX.\textsuperscript{43}

\section{Numerical Results}

In this section, some numerical results are presented, to evaluate the performance of the proposed approach. The performance is evaluated both in terms of transmit power (peak and average) and in terms of achieved SINR at the receivers.\textsuperscript{5} The numerical analysis is based on a 71-beam antenna satellite system ($K=71$) operating in the Ka band with carrier frequency of 19.5 GHz and user bandwidth of 500 MHz. A single polarization is assumed, along with a full frequency reuse scheme. The satellite is located on the Geostationary Earth Orbit at longitude of 30°E. The beam centers are located in the positions marked in Figure 3, and the users are assumed positioned in the centers of the respective beams, unless specified otherwise. The receiving antennas are assumed having a diameter of 0.6 m, while the noise temperature is set to 240 K. Moreover, a 16-APSK modulation scheme is assumed for the DI, while a perfect knowledge of the CSI is assumed at the transmitter. Further, for simplicity, the considered channel does not account for fading.\textsuperscript{6}

\subsection{Benchmark schemes}

First, we consider as benchmark the symbol-level precoding scheme of previous studies,\textsuperscript{25,26,29} which performs symbol-level sum power minimization (SL-SPM), comparing the achieved performance in terms of peak and average transmit power. This benchmark allows to assess the proposed scheme in terms of instantaneous power distribution among the antennas. It should be highlighted that the power distribution is

They work in this work, we have chosen the SINR as figure of merit, since a performance evaluation in terms of achieved rate is not straightforward under a finite alphabet assumption. An evaluation of the achievable information rate for symbol-level precoding is left for the future work.

\textsuperscript{6}However, the gains of the proposed approach hold if a flat fading effect is modeled (see Spano et al\textsuperscript{40}). Besides, recent extensions of symbol-level precoding (see Spano et al\textsuperscript{44}) allow to handle the intersymbol interference; thus, frequency-selective fading channels are foreseen to be tackled in the future work.
evaluated before the nonlinear TWTAs on the transmit RF chains, as a more uniform distribution would improve the robustness of the waveforms to the nonlinear effects and in particular to the problem of differential phase shift. For the sake of completeness, we provide in the following a formulation for the SL-SPM optimization problem:

\[
x(d, H, y) = \arg \min_x \|x\|^2
\]

subject to

\[
c1: \ |h_j x|^2 \leq k_j^2 \gamma, j = 1, \ldots, K.
\]

\[
c2: \ \angle h_j x = \angle d_j, j = 1, \ldots, K.
\]

Afterwards, the proposed approach is compared with the conventional MMSE precoding scheme in terms of attained SINR at the receivers, so as to observe the constructive interference effect of the symbol-level precoding strategy. A comparison in terms of per-beam transmit power is also presented. For the sake of completeness, we shall mention that in the MMSE case the transmit vector \( x \) is calculated by weighting the original symbols through a precoding matrix \( W \), as \( x=Wd \), and that the precoding matrix can be written as \( W=H^t(HH^t+\eta I)^{-1} \). In turn, \( \eta \) represents a regularization parameter inversely proportional to the signal-to-noise ratio. The power constraints of the system at hand can be accounted by accordingly rescaling the precoding matrix \( W \).

### 4.2 Comparison with the SL-SPM scheme

Herein, we present some numerical results comparing the proposed approach with the SL-SPM scheme formulated in (21), in terms of peak and average transmit power. The symbol-level average power transmitted by each antenna is defined as \( P_{av} = \frac{\|x\|^2}{K} \), while the symbol-level peak power among the antennas will be \( P_{peak} = \|x\|_\infty^2 \). By taking an average of such quantities over a large number of symbol slots, we obtain the frame-level average power and peak power, which are used as performance metric hereafter.

Figure 4 shows the introduced power metrics, in dBW, as a function of the target SINR, assumed the same for all the users for the sake of simplicity. The result shows how the required transmit power increases with the target SINR and how the proposed SL-PPM approach attains better performance in terms of peak power with respect to the SL-SPM scheme. On the other hand, the proposed approach requires a higher average transmit power. Overall, the SL-PPM approach allows to reduce the instantaneous power peaks among the different beams at the expense of a higher average transmit power, achieving a more uniform instantaneous power distribution among the antennas. As already mentioned, this is particularly important with respect to the problem of nonlinear TWTAs. For completeness, we present in Figure 5 an analogous result obtained by considering the users’ locations uniformly distributed within the respective beams and by averaging over several realizations for the users’ distribution. The result shows how, in this general case, the gains of the proposed approach in terms of reduced peak power are even higher with respect to the case of users located in the center of the beams. It can also be noticed how, given the target SINR, the required transmit power is considerably higher (both for the proposed scheme and for the benchmark) with respect to the scenario with central users, because of the higher interference experienced.

\[\text{FIGURE 4} \quad \text{Frame-level transmit power in dBW vs SINR target in dB. SINR, signal-to-interference-plus-noise ratio; SL-PPM, symbol-level precoding for peak power minimization; SL-SPM, symbol-level sum power minimization [Colour figure can be viewed at wileyonlinelibrary.com]}
\]
FIGURE 5  Frame-level transmit power in dBW vs SINR target in dB, for uniformly distributed users in the beams. SINR, signal-to-interference-plus-noise ratio; SL-PPM, symbol-level precoding for peak power minimization; SL-SPM, symbol-level sum power minimization [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 6  Instantaneous per-beam transmit power, in W, for a certain data information vector. SL-PPM, symbol-level precoding for peak power minimization; SL-SPM, symbol-level sum power minimization [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 7  Average SINR, in dB, vs available per-beam power. MMSE, minimum mean square error; SINR, signal-to-interference-plus-noise ratio; SL-PPM, symbol-level precoding for peak power minimization [Colour figure can be viewed at wileyonlinelibrary.com]
To better illustrate the performance of the proposed strategy in terms of instantaneous power distribution, we show in Figure 6 the instantaneous power utilization for each of the 71 beams, for a fixed symbol slot. The SINR target is set at 10 dB. Such representation clearly shows how, sacrificing some average power, the proposed approach leads to a more uniform distribution of the power between the antennas, resulting in a lower peak power.

4.3 | Comparison with the MMSE precoding

Hereafter we focus on the comparison between symbol-level precoding, in particular the proposed SL-PPM scheme, and MMSE precoding. The evaluation is done in terms of achieved average and per-beam SINR, as well as in terms of per-beam transmit power.

Figure 7 shows the achieved SINR, averaged between the different beams, as a function of the available per-beam power. Such power represents a per-beam constraint on the power consumption, intended as the transmit power averaged over the temporal dimension. It is interesting to notice how the SL-PPM scheme outperforms MMSE precoding and how the gap between the 2 approaches increases with the per-beam power. This result can be explained by considering that the symbol-level precoding exploits the constructive interference effect, which allows to have a considerable increase in the SINR at the receivers. Moreover, it shall be highlighted how a raise in the transmit power determines also a raise in the interference level among the beams: While this is a harmful factor in the MMSE case, it actually allows a better interference exploitation in the SL-PPM strategy, and this justifies the dependence of the achieved SINR gain on the power. We also show in Figure 8 the SINR comparison in the case of users uniformly distributed within the respective beams, averaging over several realizations for the users’ locations. In this general case, it can be seen how the higher interference level due to the different users’ location determines a further increase in the gap between the symbol-level scheme and the block-level one.

![Figure 8](wileyonlinelibrary.com)

**FIGURE 8** Average SINR, in dB, vs available per-beam power, for uniformly distributed users in the beams. MMSE, minimum mean square error; SINR, signal-to-interference-plus-noise ratio; SL-PPM, symbol-level precoding for peak power minimization [Colour figure can be viewed at wileyonlinelibrary.com]

![Figure 9](wileyonlinelibrary.com)

**FIGURE 9** Achieved per-beam SINR, in dB, for an available per-beam power of 16 dBW. MMSE, minimum mean square error; SINR, signal-to-interference-plus-noise ratio; SL-PPM, symbol-level precoding for peak power minimization [Colour figure can be viewed at wileyonlinelibrary.com]
In Figure 9, the distribution of the achieved SINR among the different beams is illustrated, when a per-beam available power of 16 dBW is considered. From this result, it is apparent how the SL-PPM approach outperforms MMSE for each beam and not only in average.

In the comparative analysis at hand, further insights can be given by observing how the power consumption varies among the different beams. Considering again a per-beam available power of 16 dBW, Figure 10 displays the transmit power (averaged over time) for each beam, in dBW, both for SL-PPM and MMSE. Interestingly, the symbol-level approach exploits more efficiently the available power than the MMSE scheme, where the power consumption results less balanced. This fact can be identified as an additional reason of the improved SINR performance of the proposed scheme.

For the sake of completeness, we also compare the distribution of the peak power among the different beams for the proposed approach and the MMSE scheme. In particular, considering again a per-beam available average power of 16 dBW, we show in Figure 11 the empirical evaluation of the cumulative distribution function (CDF) of the instantaneous peak power obtained with the 2 approaches. The CDF of the peak power $P_{\text{peak}}$ is a function of a variable $P_0$ defined as the probability of $P_{\text{peak}}$ being less than $P_0$, i.e., $\text{CDF}_{P_{\text{peak}}}(P_0) = \Pr(P_{\text{peak}} < P_0)$. The result highlights how the peak power of the proposed scheme is always below the one of the MMSE precoder, with a gap of approximately 1.5 dB in the median values. To better illustrate this effect, we consider in Figure 12 the instantaneous power distribution among the different beams for a fixed symbol slot. It is visible how, also compared with the MMSE precoding, the proposed approach determines a more uniform distribution of the instantaneous power, making the transmitted signal more robust to the nonlinear effects of the TWTA.

As mentioned above, all the presented results are obtained under the assumption of perfect CSI knowledge at the transmitter. However, it is well known from the literature that the available CSI usually has some level of uncertainty, due to errors in the channel estimation and to the non-ideal feedback link. Accordingly, it should be kept in mind that the performance of precoding in a practical scenario would be degraded with respect to the considered ideal case. In particular, the achieved SINR values presented in Figures 7 to 9 would be reduced both for the proposed
SL-PPM scheme and for the MMSE approach. While a quantitative analysis of this effect is out of the scope of this contribution, the results herein presented provide a performance benchmark and reveal the most suitable scenarios where precoding can be further developed accounting for nonideal conditions. However, we stress that the gains of the proposed scheme in terms of peak power reduction would hold even with an imperfect CSI, because of the applied optimization strategy. Further considerations on channel errors in the context of symbol-level precoding can be found in Masouros and Zheng, where a robust power minimization scheme is proposed.

4.4 Discussion on complexity

We conclude this section by discussing the complexity of the proposed approach. In this regard, a first fundamental remark to make is that symbol-level precoding schemes require the calculation of the transmit signal vector \( \mathbf{x} \) for each symbol slot, as outlined in Figure 1. This implies that the symbol-level precoding algorithm needs to be applied for every symbol slot; thus, its switching rate coincides with the baud rate. On the other hand, in block-level precoding schemes, the precoder remains constant for a whole block of symbols whose length is related to the coherence time of the channel. Accordingly, symbol-level precoding schemes are inherently more complex than the block-level ones, as discussed in Alodeh et al.

Herein, we focus on symbol-level schemes, and specifically we compare the proposed SL-PPM approach (20) with the SL-SPM one (21) in terms of complexity. Since the proposed optimization problems are tackled resorting to numerical convex optimization tools, analytical expressions for the complexity are hard to derive. Therefore, we evaluate the complexity on the basis of the average running time of the algorithms over the same machine. The complexity assessment, shown in Figure 13, has been carried out as a function of the system size \( K \), ie, the number of transmit antennas as well as of users, by averaging over several random channel matrices. As expected, the proposed SL-PPM scheme turns out to be more complex than the SL-SPM one. Interestingly, the complexity gap between the 2 approaches tends to increase with \( K \). It shall be stressed that the provided running time values are used solely as a comparative numerical assessment of the complexity, while they should not

![Figure 12](image1.png) Instantaneous per-beam transmit power, in W, for a certain symbol vector. MMSE, minimum mean square error; SL-PPM, symbol-level precoding for peak power minimization. [Colour figure can be viewed at wileyonlinelibrary.com]

![Figure 13](image2.png) Average running time of the symbol-level sum power minimization (SL-SPM) and symbol-level precoding for peak power minimization (SL-PPM) algorithms, in s, versus \( K \). [Colour figure can be viewed at wileyonlinelibrary.com]
be read as the actual processing time of the precoding schemes in a real-world implementation. In fact, the actual processing time would depend on the specific algorithms chosen to solve the problems (20) to (21) and on their hardware implementation. The processing time is a critical parameter towards the practical utilization of the schemes, and in real systems, it should be much lower than the round trip time, in order not to make the CSI outdated. A thorough study in this regard falls out of the scope of this paper; however, it is part of the ongoing work.45,46

5 CONCLUSIONS

In this work, a novel technique for symbol-level precoding has been proposed, taking into account the per-antenna power limitations that arise typically in satellite systems. In particular, the problem of minimization of the peak power among the transmitting antenna, under QoS constraints, is formulated and solved, to have a more uniform distribution of the transmitted power with respect to the state-of-the-art symbol-level techniques. The optimization scheme has been derived first for the case of single-level modulations and then extended to the general case of multilevel modulations. The considered peak power minimization design is suitable for systems corrupted by nonlinear effects, such as satellite ones, where the power peaks reduction, and more in general the control on the transmitted power, implies relevant benefits. The performance of the proposed scheme is assessed through numerical results, in comparison with the SL-SPM scheme and to the MMSE precoding. In the former case, it is shown how the proposed approach gains in terms of a reduced transmitted peak power across the antennas, at the expense of a higher required average power. In the latter case, the SINR gains of symbol-level precoding over the conventional MMSE are demonstrated, together with the improved behavior in terms of power peaks among the antennas.

ACKNOWLEDGMENT

This work is supported by European H2020 project SANSA (Shared Access Terrestrial-Satellite Backhaul Network enabled by Smart Antennas) and by Fond National de la Recherche Luxembourg (FNR), under the projects SeMiGd (Spectrum Management and Interference Mitigation in Cognitive Radio Satellite Networks), SATSENT (SATellite SEnsor Networks for spectrum monitoring), and BroadSat (AFR project).

ORCID

Danilo Spano  http://orcid.org/0000-0001-8677-555X

REFERENCES

1. Gaynard JD. Terabit satellite: myth or reality? In: Advances in Satellite and Space Communications, 2009. SPACOMM2009. First International Conference; 2009; Colmar, France:1-6. https://doi.org/10.1109/SPACOMM.2009.17.
2. Mignolo D, Re E, Ginesi A, Alamanac AB, Angeletti P, Harveryson M. Approaching terabit/s satellite capacity: A system analysis. In: Proc. Ka Broadband Conf.; 2011; Palermo, Italy.
3. Evans B, Thompson P. Key issues and technologies for a terabit/s satellite. In: 28th AIAA International Communications Satellite Systems Conference (ICSSC-2010); 2010; Anaheim, California.
4. Thompson P, Evans B, Castenet L, Bousquet M, Mathiopoulos T. Concepts and technologies for a terabit/s satellite. In: SPACom, 2011, The Third International Conference on Advances in Satellite and Space Communications; 2011; Budapest, Hungary:
5. Vidal O, Verelst G, Lacan J, Alberty E, Radzik J, Bousquet M. Next generation throughput satellite system. In: Satellite Telecommunications (ESTEL), 2012 IEEE First AESS European Conference; 2012:1-7. https://doi.org/10.1109/ESTEL.2012.6400146.
6. Roy R, Ottersten B. Spatial division multiple access wireless communication systems. US Patent 5,515,378, https://www.google.com/patents/US5515378; May 1996.
7. Liu YF, Dai YH, Luo ZQ. Coordinated beamforming for MISO interference channel: complexity analysis and efficient algorithms. IEEE Trans Signal Process. 2011;59(3):1142-1157.
8. Björnson E, Bengtsson M, Ottersten B. Optimal multiuser transmit beamforming: a difficult problem with a simple solution structure [lecture notes]. IEEE Signal Process Mag. 2014;31(4):142-148. https://doi.org/10.1109/MSP.2014.2312183.
9. Gershman AB, Sidiropoulos ND, Shahbazpanahi S, Bengtsson M, Ottersten B. Convex optimization-based beamforming. IEEE Signal Process Mag. 2010;27(3):62-75. https://doi.org/10.1109/MSP.2010.936015.
10. Bengtsson M, Ottersten B. Optimal and suboptimal transmit beamforming. In: Godara LC, ed. Handbook of Antennas in Wireless Communications: CRC Press; 2001:18-1-18-33.
11. Schubert M, Boche H. Solution of the multiuser downlink beamforming problem with individual SINR constraints. IEEE Trans Veh Technol. 2004;53(1):18-28. https://doi.org/10.1109/TVT.2003.819629.
12. Bengtsson M, Ottersten B. Optimal downlink beamforming using semidefinite optimization. In: Proc. of Annual Allert. Conf. on Commun. Control and Computing, Vol. 37; 1999; Citeseer:987-996.
13. ETSI EN 302 307-2. Digital video broadcasting (DVB); second generation framing structure, channel coding and modulation systems for broadcasting, interactive services, news gathering and other broadband satellite applications. part 2: DVB-S2 extensions (DVB-S2X).
14. Yoo T, Goldsmith A. Optimality of zero-forcing beamforming with multiuser diversity. IEEE Int Conf Commun (ICC 2005). 2005/1. https://doi.org/10.1109/ICC.2005.1494410.
Danilo Spano (S’14) was born in Lecce, Italy, in 1989. He received the BSc degree (cum laude) in Information Engineering and the MSc degree (cum laude) in Telecommunications Engineering from the University of Salento, Lecce, Italy, in 2012 and 2014, respectively. He is currently pursuing the PhD degree in electrical engineering at the Interdisciplinary Centre for Security, Reliability, and Trust, University of Luxembourg, Luxembourg. His research interests include signal processing for multiuser wireless communications, satellite communications, interference management, and wireless localization. Mr Spano was awarded in 2015 with the AFR scholarship from Luxembourg National Research Fund, for the study of interference management techniques in multi-antenna systems.

Symeon Chatzinotas (S’06–M’09–SM’13) is currently the Deputy Head of the SIGCOM Research Group, Interdisciplinary Centre for Security, Reliability, and Trust, University of Luxembourg, Luxembourg and Visiting Professor at the University of Parma, Italy. He received the MEng degree in telecommunications from the Aristotle University of Thessaloniki, Thessaloniki, Greece, in 2003, and the MSc and PhD degrees in electronic engineering from the University of Surrey, Surrey, UK, in 2006 and 2009, respectively. He was involved in numerous Research and Development projects for the Institute of Informatics Telecommunications, National Center for Scientific Research Demokritos, the Institute of Telematics and Informatics, Center of Research and Technology Hellas, and the Mobile Communications Research Group, Center of Communication Systems Research, University of Surrey. He has over 300 publications, 2500 citations, and an H-Index of 27 according to Google Scholar. His research interests include multiuser information theory, co-operative/cognitive communications, and wireless networks optimization. He was a co-recipient of the 2014 Distinguished Contributions to Satellite Communications Award, and the Satellite and Space Communications Technical Committee, the IEEE Communications Society, and the CROWNCOM 2015 Best Paper Award.

Stefano Andrenacci received his MS Degree in Telecommunication Engineering (cum laude) from the Polytechnic University of Marche, Ancona (Italy), in 2008 and his PhD on Telecommunication Engineering at the Department of Biomedical Engineering, Electronics and Telecommunications of the same University, in 2011. From July 2011 to January 2015, he was a Post-Doctoral Researcher at the Department of Electrical and Information Engineering (DEI) “Guglielmo Marconi” of the University of Bologna, where he worked on interference management techniques, channel estimation algorithms, synchronization procedures and on hardware implementation of satellite terminals. Since February 2015, he is a Post-Doctoral Researcher at the Interdisciplinary Centre for Security, Reliability and Trust of the University of Luxembourg. His research activities are mainly focused on interference management techniques, synchronization procedure design and synchronization techniques for digital receivers, beamforming and precoding for multi-beam satellite systems, DVB-S2/S2x, DVB-RCS2, Software Defined Radios (SDR), and spread spectrum systems.

Jens Krause was born in Werdohl, Germany, in 1963. He received the Dipl.-Ing. degree in 1987 and the PhD degree in 1993, both in electrical engineering from University of Karlsruhe, Germany. He has held a scientific employee position at University of Karlsruhe from 1988 to 1993. From 1994 to 1996, he has been an R&D engineer in the cable TV department of Richard Hirschmann GmbH in Germany. Since 1996, he works at the satellite operator SES S.A. in Luxembourg, with various positions in systems engineering. He represents SES in standardization organizations including ETSI and DVB. His research interests include satellite communications in general, modulation and coding, and signal processing for satellite communications.
Björn Ottersten (S’87–M’89–SM’99–F’04) was born in Stockholm, Sweden, in 1961. He received the MS degree in electrical engineering and applied physics from Linköping University, Linköping, Sweden, in 1986, and the PhD degree in electrical engineering from Stanford University, Stanford, CA, USA, in 1990. He has held research positions with the Department of Electrical Engineering, Linköping University, the Information Systems Laboratory, Stanford University, the Katholieke Universiteit Leuven, Leuven, Belgium, and the University of Luxembourg, Luxembourg. From 1996 to 1997, he was the Director of Research with ArrayComm, Inc, a startup in San Jose, CA, USA, based on his patented technology. In 1991, he was appointed a Professor of signal processing with the Royal Institute of Technology (KTH), Stockholm, Sweden. From 1992 to 2004, he was the Head of the Department for Signals, Sensors, and Systems, KTH, and from 2004 to 2008, he was the Dean of the School of Electrical Engineering, KTH. He is currently the Director for the Interdisciplinary Centre for Security, Reliability and Trust, University of Luxembourg. As Digital Champion of Luxembourg, he acts as an Adviser to the European Commission. He was a recipient of the IEEE Signal Processing Society Technical Achievement Award in 2011 and the European Research Council advanced research grant twice, in 2009 to 2013 and in 2017 to 2022. He has co-authored journal papers that received the IEEE Signal Processing Society Best Paper Award in 1993, 2001, 2006, and 2013, and 7 other IEEE conference papers best paper awards. He has served as an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING and the Editorial Board of the IEEE Signal Processing Magazine. He is currently the Editor-in-Chief of EURASIP Signal Processing Journal and a member of the editorial boards of EURASIP Journal of Applied Signal Processing and Foundations and Trends of Signal Processing. He is a fellow of EURASIP.

How to cite this article: Spano D, Chatzinotas S, Andrenacci S, Krause J, Ottersten B. Per-antenna power minimization in symbol-level precoding for the multibeam satellite downlink. Int J Satell Commun Network. 2019;37:15–30. https://doi.org/10.1002/sat.1244