Super-Virasoro Anomaly, Super-Weyl Anomaly and the Super-Liouville Action for 2D Supergravity

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Abstract

The relation between super-Virasoro anomaly and super-Weyl anomaly in $N = 1$ NSR superstring coupled with 2D supergravity is investigated from canonical theoretical viewpoint. The WZW action canceling the super-Virasoro anomaly is explicitly constructed. It is super-Weyl invariant but nonlocal functional of 2D supergravity. The nonlocality can be remedied by the super-Liouville action, which in turn recovers the super-Weyl anomaly. The final gravitational effective action turns out to be local but noncovariant super-Liouville action, describing the dynamical behavior of the super-Liouville fields. The BRST invariance of this approach is examined in the superconformal gauge and in the light-cone gauge.
1 Introduction

The characteristic feature of string theories at subcritical dimensions is that the conformal degree of the world-sheet metric variables being decoupled from the theory due to the local Weyl invariance at the classical level comes into dynamical play through the Weyl anomaly. As was shown by Polyakov in his classic paper [1], this leads to Liouville action in the conformal gauge. Motivated by his work, the Liouville quantum theory has been investigated extensively in refs. [2, 3, 4]. In the original functional approach of Polyakov there arose a difficulty in handling the path integral for the conformal mode due to the translation noninvariance of the functional measure. This led the authors of refs. [5, 6] to the analysis in the light-cone gauge. The conformal gauge was investigated by the authors of refs. [7, 8]. They noted that the gravitational scaling dimensions can be reproduced also in this gauge by imposing the functional measure ansatz that the Jacobian of the transformation between the translation noninvariant measure to translation invariant one is an exponential of a local action of Liouville type for the conformal mode. Their ansatz was examined by the heat kernel method in ref. [9].

The conventional approaches to subcritical string theories and/or 2D quantum gravity more or less rely on path integral formalism. It is certainly desired to have a better understanding of the path integral results from a consistent canonical viewpoint. In ref. [10], it was argued by noting the connection between the Virasoro anomaly and the Weyl anomaly that the Liouville action must be introduced as the Wess-Zumino-Witten (WZW) term to cancel the Virasoro anomaly to recover the world-sheet reparametrization invariance also in the canonical treatment. This approach turned out to reproduce the primary results both for the light-cone and the conformal gauges. The purpose of the present paper is to extend the work done for bosonic string to fermionic theory [11, 12]. 2D quantum supergravity coupled to superconformal matter has been investigated in the light-cone gauge in refs. [13, 14, 15] and in the superconformal gauge in refs. [8, 16]. Analyses based on the BRST formalism have been carried out in refs. [17, 18]. The connection between the (super-)Virasoro anomaly and the (super-)Weyl anomaly has been
extensively studied in refs. [19, 20, 21]. Formulation of 2D (super)gravity as anomalous
gauge theory has been argued in refs. [22, 23], where the (super-)Weyl anomaly is canceled
by introducing additional degrees of freedom identified with the (super-)Liouville mode.
The present work will give another exposition to the subject taken up in ref. [23].

This paper is organized as follows. In Section 2, we will describe the super-Virasoro
anomaly in canonical formalism and examine the local invariances of the string action.
The super-Virasoro anomaly will be related to the anomaly in the covariant conservations
of the stress tensor and the supercurrent, leading to the anomaly equations. In Section
3, we will examine the integrability of the super-Virasoro anomaly and solve the anomaly
equations by noting their covariance under reparametrizations and local supersymmetry.
The anomaly canceling WZW action is also constructed explicitly. Superfield formulation
of the super-Virasoro anomaly is argued in Section 4 to make clear the geometrical mean-
ing of the super-Virasoro anomaly and the WZW action. The counterterm derived there
is nonlocal in the 2D supergravity variables. We will describe in Section 5 the cancellation
of the nonlocality by the nonlocal covariant super-Liouville action. Quantization of 2D
supergravity in superconformal gauge and light-cone gauge will be argued in Section 6 to
see how the primary results found in the literature are reproduced in our approach. Our
main concern there is to examine the BRST invariance. Section 7 is devoted to summary
and discussion. We also provide some appendices. Appendix A deals with the summary
of covariant BRST transformations. The super-Virasoro algebra with the cosmological
term included is treated in Appendix B. The BRST gauge-fixing procedure is exposed in
Appendix C in some detail in the case of light-cone gauge.

2 Super-Virasoro constraints in 2D superstring

The fermionic string of Neveu-Schwarz-Ramond can be formulated as 2D supergravity
described by the action [12]

\[ S_X = - \int d^2 x \, e \left[ \frac{1}{2} (g^{\alpha \beta} \partial_\alpha X \partial_\beta X - \bar{\psi} \rho^\alpha \nabla_\alpha \psi) + \bar{\chi}_\alpha \rho^\beta \rho^\alpha \psi \partial_\beta X + \frac{1}{4} \bar{\psi} \bar{\chi}_\alpha \rho^\beta \rho^\alpha \chi_\beta \right], \]  (2.1)
where \( X^\mu \) and \( \psi^\mu \) \((\mu = 0, \cdots, D - 1)\) are, respectively, the bosonic and fermionic coordinates of string variables. The zweibein and the gravitino on the world-sheet are denoted by \( e_\alpha^a \) and \( \chi_\alpha \), respectively.

The classical action (2.1) is invariant under the reparametrizations and local supersymmetry on the world-sheet. It also possesses the invariances under local Lorentz transformations, Weyl rescalings and fermionic symmetry. The latter symmetries can be used to eliminate some degrees of freedom of \( e_\alpha^a \) and \( \chi_\alpha \) from (2.1). So it is convenient to define variables for zweibein by

\[
\lambda^\pm = \pm \frac{e_0^\pm}{e_1^\pm}, \quad \xi = \ln(-e_1^+e_1^-), \quad l = \frac{1}{2} \ln\left(-\frac{e_1^+}{e_1^-}\right),
\]

where \( e_\alpha^\pm = e_\alpha^0 \pm e_\alpha^1 \). As for the fermionic variables, we introduce \( \psi^\pm \) by

\[
\psi = \begin{pmatrix} (-e_1^-)^{-\frac{1}{2}} \psi_- \\ (e_1^+)^{-\frac{1}{2}} \psi_+ \end{pmatrix},
\]

and define the gravitino fields by

\[
\nu^\pm = \frac{\chi_0^\pm \pm \lambda^\pm \chi_1^\pm}{\sqrt{\pm e_1^\pm}}, \quad \Lambda^\pm = \frac{4\chi_1^\mp}{\sqrt{\pm e_1^\pm}},
\]

where \( \chi_{\alpha \mp} \) stands for the upper and lower components of \( \chi_\alpha \).

The superconformal gauge \( e_\alpha^a = \sqrt{\delta_\alpha^a}, \chi_\alpha = -\frac{1}{2} \rho_\alpha \rho^\beta \chi_\beta \) corresponds to the conditions

\[
\lambda^\pm = 1, \quad \nu^\mp = 0, \quad l = 0.
\]

Under the local Lorentz transformations \( \delta e_\alpha^\pm = \pm \lambda e_\alpha^\pm \), the Weyl rescaling \( \delta e_\alpha^\pm = \phi e_\alpha^\pm \) and the fermionic symmetry \( \delta \chi_\alpha = i \rho_\alpha \eta, X^\mu, \psi^\pm, \lambda^\pm \) and \( \nu^\pm \) are invariant, while \( \xi \), \( l \) and \( \Lambda_\pm \) are transformed as

\[
\delta \xi = 2 \phi, \quad \delta l = \lambda, \quad \delta \Lambda_\pm = -4 \eta_\pm,
\]

\*We choose \( \eta^{ab} = \text{diag}(-1,1) \) and \( \eta^{\mu\nu} = \text{diag}(-1,1,\cdots,1) \) for flat metrics. The world-sheet coordinates are denoted by \( x^\alpha = (\tau, \sigma) \) for \( \alpha = 0,1 \) and are assumed to take \( -\infty < \sigma < +\infty \). It is straightforward to make the analysis on a finite interval of \( \sigma \) so as to impose the Neveu-Schwarz or Ramond boundary conditions. We will use the notation \( \partial_\tau A \) and \( \partial_\sigma A \) for derivatives. Dirac matrices \( \rho^a \) \((a = 0,1)\) are chosen to be \( \rho^0 = \sigma_2, \rho^1 = i \sigma_1 \), and \( \rho^5 \equiv \rho^0 \rho^1 = \sigma_3 \), where \( \sigma_k \) \((k = 1,2,3)\) are Pauli matrices.
where $\eta$ is an arbitrary Majorana spinor and $\eta_{\pm}$ are the rescaled components of $\eta$ as defined by (2.3). In terms of these variables (2.1) can be written as

$$S_X = \int d^2x \left[ \frac{(\dot{X} - \lambda^+ X')(\dot{X} + \lambda^- X')}{\lambda^+ + \lambda^-} + \frac{i}{2} \psi_+ (\dot{\psi}_+ - \lambda^+ \psi'_+) + \frac{i}{2} \psi_- (\dot{\psi}_- + \lambda^- \psi'_-) 
+ \frac{2}{\lambda^+ + \lambda^-} \left\{ i(\dot{X} - \lambda^+ X') \psi_- \nu_+ - i(\dot{X} + \lambda^- X') \psi_+ \nu_- + \psi_+ \psi_- \nu_+ \nu_- \right\} \right]. \quad (2.7)$$

Since (2.1) is invariant under (2.6), (2.7) does not contain $\xi, l$ and $\Lambda_{\pm}$.

Let us denote by $P_\mu$ the canonical momentum conjugate to $X_\mu$ and assume the following set of fundamental super-Poisson brackets among $X, P$ and $\psi_{\pm}$

$$\{X^\mu(\sigma), P_\nu(\sigma')\} = \delta_\nu^\mu \delta(\sigma - \sigma'), \quad \{\psi_{\pm}^\mu(\sigma), \psi_{\pm}^\nu(\sigma')\} = -i\eta^{\mu\nu} \delta(\sigma - \sigma'). \quad (2.8)$$

Then the canonical theory of (2.1) is characterized by the quantities defined by

$$\varphi_{\pm} = \frac{1}{4}(P \pm X')^2 \pm \frac{i}{2} \psi_{\pm} \psi'_{\pm}, \quad \mathcal{J}_{\pm} = \psi_{\pm}(P \pm X'). \quad (2.9)$$

They satisfy the classical super-Virasoro algebra

$$\{\varphi_{\pm}(\sigma), \varphi_{\pm}(\sigma')\} = \pm(\varphi_{\pm}(\sigma) + \varphi_{\pm}(\sigma')) \partial_\sigma \delta(\sigma - \sigma'),$$

$$\{\mathcal{J}_{\pm}(\sigma), \varphi_{\pm}(\sigma')\} = \pm \frac{3}{2} \mathcal{J}_{\pm} \partial_\sigma \delta(\sigma - \sigma') \pm \mathcal{J}_{\pm}'(\sigma) \delta(\sigma - \sigma'), \quad (2.10)$$

$$\{\mathcal{J}_{\pm}(\sigma), \mathcal{J}_{\pm}(\sigma')\} = -4i \varphi_{\pm}(\sigma) \delta(\sigma - \sigma'),$$

and generate fixed $\tau$ reparametrizations and supertransformations on the canonical variables $X, P$ and $\psi_{\pm}$. If we consider $e_\alpha^a$ and $\chi_\alpha$ as dynamical variables, (2.9) appears as super-Virasoro constraints among canonical variables. This can be seen directly from the fact that these quantities can also be obtained from (2.7) by taking the variations with respect to $\lambda^\pm$ and $\nu_{\pm}$, i.e.,

$$\frac{\delta S_X}{\delta \lambda^\pm} = -\varphi_{\pm}, \quad \frac{\delta S_X}{\delta \nu_{\pm}} = \pm i \mathcal{J}_{\pm}. \quad (2.11)$$

For most part of this paper, however, we will regard $e_\alpha^a$ and $\chi_\alpha$ as classical background fields and do not consider (2.9) as constraints.\[\text{We refer to (2.9) as super-Virasoro constraints for simplicity.}\]
The canonical Hamiltonian can be expressed as a linear combination of the constraints

\[ H_0 = \int d\sigma \left[ \lambda^+ \varphi_+ + \lambda^- \varphi_- - i \nu_- J_+ + i \nu_+ J_- \right]. \]  

Then the super-Virasoro constraints (2.9) are subject to the equations of motion

\[ \dot{\varphi}_\pm \mp \lambda_\pm^\prime \varphi_\pm \mp 2 \lambda_\pm^\prime \varphi_\pm \mp \frac{3}{2} i \nu_\pm J_\pm \mp \frac{i}{2} \nu_\pm J_\pm^\prime = 0, \]  

\[ \dot{J}_\pm \mp \lambda_\pm J_\pm \mp \frac{3}{2} \lambda_\pm^\prime J_\pm \mp 4 \nu_\pm \varphi_\pm = 0. \]  

The symmetric stress tensor \( T_{\alpha\beta} \) and the supercurrent \( J_\alpha \) are defined by

\[ T_{\alpha\beta} = -\frac{2}{e} \frac{\delta S_X}{\delta g^{\alpha\beta}}, \quad J_\alpha = -\frac{1}{2e} \frac{\delta S_X}{\delta \chi_\alpha^\pm}. \]  

Each component of \( T_{\alpha\beta} \) and \( J_\alpha \) can be expressed as linear combinations of the constraints (2.9) as

\[ T_{00} = (\lambda^+)^2 (\varphi_+ + \frac{i}{4} \Lambda_+ J_+) + (\lambda^-)^2 (\varphi_- + \frac{i}{4} \Lambda_- J_-), \]

\[ T_{01} = T_{10} = \lambda^+ (\varphi_+ + \frac{i}{4} \Lambda_+ J_+) - \lambda^- (\varphi_- + \frac{i}{4} \Lambda_- J_-), \]

\[ T_{11} = \varphi_+ + \frac{i}{4} \Lambda_+ J_+ + \varphi_- + \frac{i}{4} \Lambda_- J_-, \]

\[ J_0^\pm = \frac{1}{2e} J_\pm, \quad J_1^\pm = \pm \frac{\lambda^\pm}{2e} J_\pm. \]  

We again see that the stress tensor and supercurrent constraints \( T_{\alpha\beta} = J_\alpha^\beta = 0 \) are fulfilled by imposing the super-Virasoro constraints \( \varphi_\pm = J_\pm = 0 \) for \( e_\alpha^\alpha \) and \( \chi_\alpha \) being dynamical variables.

From the expressions (2.16) it is easy to see that the traces of the stress tensor and the supercurrent vanish, i.e.,

\[ T_\alpha^\alpha = 0, \quad \rho_\alpha J_\alpha = 0, \]  

where \( J_\alpha \) is the two component spinor with \( J_\alpha^\alpha \) as upper and lower components, and use has been made of the relations \( g^{00}(\lambda^\pm)^2 \pm g^{01} \lambda^\pm + g^{11} = 0, \rho^\alpha \rho^\beta \rho_\alpha = 0 \). Eq. (2.17) is the direct consequence of the super-Weyl invariance of (2.1). The invariances of (2.1) under
reparametrizations and local supersymmetry can be restated by a kind of generalization of covariant conservations of stress tensor and supercurrent.

To see this let us denote by \( \delta_u \) the Lie derivative for the reparametrization \( x^\alpha \rightarrow x^\alpha + u^\alpha(x) \), then \( \lambda^\pm \) and \( \nu_\mp \) are transformed by

\[
\begin{align*}
\delta_u \lambda^\pm &= -u^\alpha \partial_\alpha \lambda^\pm - \lambda^\pm (\dot{u}^0 \mp \lambda^\pm u^0) \mp (\dot{u}^1 \mp \lambda^\pm u^1), \\
\delta_u \nu_\mp &= -u^\alpha \partial_\alpha \nu_\mp - \{ \dot{u}^0 \mp \lambda^\pm u^0 - \frac{1}{2} (u^1 \pm \lambda^\pm u^1) \} \nu_\mp. 
\end{align*}
\]

The infinitesimal supertransformations are given by

\[
\begin{align*}
\delta_\epsilon \lambda^\pm &= -4 i \epsilon_{\mp} \nu_\mp, \\
\delta_\epsilon \nu_\mp &= \dot{\epsilon}_{\mp} \mp \lambda^\pm \epsilon_{\mp} \pm \frac{1}{2} \lambda^\pm \epsilon_{\mp}, 
\end{align*}
\]

where the parameters \( \epsilon_{\mp} \) of local supersymmetry are the rescaled components of a two component Majorana spinor \( \epsilon \) as is defined in (2.3) for \( \psi \). The variations of (2.7) under these transformations are computed to

\[
\begin{align*}
\delta_u S_X &= \int d^2 x \left( \delta_u \lambda^+ \frac{\delta}{\delta \lambda^+} + \delta_u \lambda^- \frac{\delta}{\delta \lambda^-} + \delta_u \nu_+ \frac{\delta}{\delta \nu_+} + \delta_u \nu_- \frac{\delta}{\delta \nu_-} \right) S_X \\
&= \int d^2 x \left( -(u^1 + \lambda^+ u^0)(\dot{\varphi}_+ - \lambda^+ \varphi_+ - 2 \lambda^+ \varphi_+) + \frac{3}{2} i \nu_+ \mathcal{J}_+ + \frac{i}{2} \nu_- \mathcal{J}_- \\
&\quad + (u^1 - \lambda^- u^0)(\dot{\varphi}_- - \lambda^- \varphi_- + 2 \lambda^- \varphi_-) + \frac{3}{2} i \nu_+ \mathcal{J}_- + \frac{i}{2} \nu_- \mathcal{J}_+ \\
&\quad + i u^0 \nu_+ (\dot{\mathcal{J}}_+ - \lambda^+ \mathcal{J}_+ - \frac{3}{2} \lambda^+ \mathcal{J}_+) - 4 \nu_- \varphi_+) \\
&\quad - i u^0 \nu_-(\dot{\mathcal{J}}_+ - \lambda^+ \mathcal{J}_+ + \frac{3}{2} \lambda^+ \mathcal{J}_+) + 4 \nu_+ \varphi_- \right), \\
\delta_\epsilon S_X &= \int d^2 x \left( \delta_\epsilon \lambda^+ \frac{\delta}{\delta \lambda^+} + \delta_\epsilon \lambda^- \frac{\delta}{\delta \lambda^-} + \delta_\epsilon \nu_+ \frac{\delta}{\delta \nu_+} + \delta_\epsilon \nu_- \frac{\delta}{\delta \nu_-} \right) S_X \\
&= \int d^2 x \left( -i \epsilon_- (\dot{\mathcal{J}}_+ - \lambda^+ \mathcal{J}_+ - \frac{3}{2} \lambda^+ \mathcal{J}_+) - 4 \nu_- \varphi_+) \\
&\quad + \epsilon_+(\dot{\mathcal{J}}_+ - \lambda^+ \mathcal{J}_+ + \frac{3}{2} \lambda^+ \mathcal{J}_+) + \frac{3}{2} \lambda^+ \mathcal{J}_+) + 4 \nu_+ \varphi_- \right),
\end{align*}
\]

where use has been made of the equations of motion for \( X \) and \( \psi_\pm \). The local invariances of (2.1) thus lead to the equations of motion (2.13) and (2.14).

We now consider canonical quantization of string variables \( X, P \) and \( \psi_\pm \) by the quantization rule \( \{ , \} \rightarrow -i[ , ] \) in (2.8), where \([ , ] \) stands for supercommutator. The
super-Virasoro constraints (2.9) become quantum mechanical operators. They are, however, ill-defined unless an operator ordering is specified. One way to implement this is to Fourier expand field variables and then to define harmonic oscillators. By putting raising operators to the left of lowering operators, we obtain the normal ordered form of an arbitrary product of operators of equal \( \tau \). The operators \( P \pm X' \) and \( \psi_\pm \) can be divided into two parts, one containing only lowering operators, \((P \pm X')^-(\pm)\) and \(\psi^-_\pm\), and the other containing only raising operators \((P \pm X')^+(\pm)\) and \(\psi^+_\pm\). They are defined by

\[
(P + X')^{(\pm)}(\tau, \sigma) = \int d\sigma' \delta^{(\pm)}(\sigma - \sigma')(P + X')(\tau, \sigma') ,
\]

\[
(P - X')^{(\pm)}(\tau, \sigma) = \int d\sigma' \delta^{(\pm)}(\sigma - \sigma')(P - X')(\tau, \sigma') ,
\]

\[
\psi^{(\pm)}(\tau, \sigma) = \int d\sigma' \delta^{(\pm)}(\sigma - \sigma') \psi^+(\tau, \sigma') ,
\]

\[
\psi^{(\pm)}(\tau, \sigma) = \int d\sigma' \delta^{(\pm)}(\sigma - \sigma') \psi^-_+(\tau, \sigma') ,
\]

(2.22)

with \( \delta^{(\pm)}(\sigma) = \frac{1}{2\pi} \frac{\pm i}{\sigma \pm i\epsilon} \). Then by putting \((P \pm X')^-(\pm)\) and \(\psi^-_\pm\) to the left of \((P \pm X')^{(\pm)}\) and \(\psi^+_\pm\), we can define an operator ordering in the present case.

The super-Virasoro operator thus defined will develop super-Virasoro anomaly in their supercommutation relations, i.e.,

\[
[\varphi_\pm(\sigma), \varphi_\pm(\sigma')] = \pm i(\varphi_\pm(\sigma) + \varphi_\pm(\sigma')) \partial_\sigma \delta(\sigma - \sigma') \pm i\kappa_0 \partial^3_\sigma \delta(\sigma - \sigma') ,
\]

\[
[J_\pm(\sigma), \varphi_\pm(\sigma')] = \pm i\frac{3}{2} J_\pm \partial_\sigma \delta(\sigma - \sigma') \pm i J'_\pm(\sigma) \delta(\sigma - \sigma') ,
\]

\[
[J_\pm(\sigma), J_\pm(\sigma')] = 4\varphi_\pm(\sigma) \delta(\sigma - \sigma') + 8\kappa_0 \partial^2_\sigma \delta(\sigma - \sigma')
\]

(2.23)

where \( \kappa_0 \) is given by

\[
\kappa_0 = -\frac{D}{16\pi} .
\]

(2.24)

Due to the appearance of the super-Virasoro anomaly, the classical equations of motion (2.13) and (2.14) are modified to

\[
\dot{\varphi}_\pm + \lambda^\pm \varphi'_\pm + 2\lambda^\pm \varphi_\pm + \frac{3}{2} i \nu'_\pm J_\pm + \frac{i}{2} \nu'_\pm J'_\pm = \pm \kappa_0 \lambda^\pm ,
\]

(2.25)

\[
\dot{J}_\pm + \lambda^\pm J'_\pm + \frac{3}{2} \lambda^\pm J_\pm + 4\nu_\pm \varphi_\pm = \pm 8\kappa_0 \nu'_\pm .
\]

(2.26)
Putting these into (2.20) and (2.21), we obtain

\[
\delta_u S_X = -\kappa_0 \int d^2x \left( (u^1 + \lambda^+ u^0)\lambda^{++} + (u^1 - \lambda^- u^0)\lambda^{--} - 8i\nu^0 (\nu_- \nu_- + \nu_+ \nu_+) \right),
\]

(2.27)

\[
\delta_\epsilon S_X = -8i\kappa_0 \int d^2x \left( \epsilon_- \nu_- + \epsilon_+ \nu_+ \right).
\]

(2.28)

We see that invariances of (2.1) under the reparametrizations and local supertransformations are violated by the super-Virasoro anomaly unless the background 2D metric variables satisfy

\[
\partial^3 \sigma \lambda^\pm = 0, \quad \partial^2 \sigma \nu_\mp = 0.
\]

(2.29)

The superconformal gauge (2.5) is a special case satisfying these condition.

The reason for the noninvariance of \( S_X \) is rather obvious. The ordering prescription we are employing to define operator products involves equal \( \tau \) operators of different spatial coordinates and, hence, treats space- and time-coordinates asymmetrically. This violates the supercovariance on the world-sheet. Since we do not refer to any particular zweibein \( e^a_\alpha \) and gravitino \( \chi_\alpha \) in defining ordering prescription, the super-Weyl invariance remains intact upon quantization. At first sight, this conclusion seemed to be inconsistent with the well-known fact that the reparametrization invariance and local supersymmetry can be maintained upon quantization, while the super-Weyl invariance is violated by the super-Weyl anomaly. In Section 5 we will resolve this puzzling feature.

Before closing this section it is worth mentioning the survival symmetries of (2.1) at the quantum level. From (2.27) and (2.28), \( S_X \) is invariant for

\[
\partial_\sigma u^0 = \partial_\sigma^2 u^1 = 0, \quad \partial_\sigma^2 \epsilon_\mp = 0.
\]

(2.30)

As we will see in Section 4, (2.30) can be enlarged corresponding to the ten anomaly free components of the super-Virasoro generators.

### 3 Super-Virasoro anomaly

In the previous section we have argued that the super-Virasoro anomaly originates from the artificial definition of operator ordering. It is known, however, that (2.1) possesses
no anomalies under the reparametrizations and local supersymmetry. This implies that we can find a suitable redefinition of (2.1) to maintain these invariances at the quantum level. In this section we shall see that the super-Virasoro anomaly can be canceled by adding to (2.1) a counterterm, which is a functional of $\lambda^\pm$ and $\nu_\mp$.

The super-Virasoro anomaly in the equations of motion for the constraints (2.25) and (2.26) can be canceled if we modify the constraints (2.9) by

$$\bar{\phi}_\pm = \phi_\pm + \zeta_\pm, \quad \bar{J}_\pm = J_\pm + j_\pm,$$

(3.1)

where $\zeta_\pm$ and $j_\pm$ are the solutions to the equations

$$\dot{\zeta}_\pm \mp \lambda^\pm \zeta_\pm \mp 2\lambda^\pm \zeta_\pm = \mp \frac{3}{2} i \nu_\pm j_\pm + \frac{i}{2} \nu_\pm j_\pm = \mp \kappa_0 \lambda^\pm \zeta_\pm,$$

(3.2)

$$\dot{j}_\pm \mp \lambda^\pm j_\pm \mp \frac{3}{2} \lambda^\pm j_\pm + 4\nu_\pm \zeta_\pm = \mp 8\kappa_0 \nu_\pm.$$

(3.3)

They depend functionally on $\lambda^\pm, \nu_\mp$ and are assumed to vanish if there is no super-Virasoro anomaly in (2.25) and (2.26), i.e.,

$$\zeta_\pm = j_\pm = 0 \quad \text{for} \quad \lambda^\pm \zeta_\pm = \nu_\pm = 0.$$

(3.4)

The modification in (3.1) can be readily related to the counterterm canceling the super-Virasoro anomaly (2.27) and (2.28). Let us denote the counterterm by $S_V$, then $\zeta_\pm$ and $j_\pm$ can be obtained by

$$\frac{\delta S_V}{\delta \lambda^\pm} = -\zeta_\pm, \quad \frac{\delta S_V}{\delta \nu_\pm} = \pm ij_\pm,$$

(3.5)

as is inferred form (2.11). The $S_V$ is assumed to be a functional of $\lambda^\pm$ and $\nu_\mp$. Eq. (3.3) can be regarded as functional differential equations. In order for them to be integrable the super-Virasoro anomaly must satisfy the Wess-Zumino consistency conditions [24]. (See also [25, 19].) To see this let us define the generators of time-preserving reparametrization and local supersymmetry $L_u^\pm$ and $Q_\epsilon^\pm$ by

$$L_u^\pm = \int d^2 x \left( \delta_u \lambda^\pm \frac{\delta}{\delta \lambda^\pm} + \delta_u \nu_\mp \frac{\delta}{\delta \nu_\mp} \right),$$

(3.6)

$$Q_\epsilon^\pm = \int d^2 x \left( \delta_\epsilon \lambda^\pm \frac{\delta}{\delta \lambda^\pm} + \delta_\epsilon \nu_\mp \frac{\delta}{\delta \nu_\mp} \right),$$

(3.7)
where $\delta_r$ is given by (2.19) and $\delta_u$ is obtained from (2.18) for $u^0(x) = 0$ and $u^1(x) = u(x)$, i.e.,

$$
\delta_u \lambda^\pm = \mp \dot{u} + \lambda^\pm u' - u \lambda^\pm, \quad \delta_u \nu_\mp = -u' \nu_\mp + \frac{1}{2} u' \nu_\mp.
$$

(3.8)

These generators satisfy the classical super-Virasoro algebra

$$
[L^\pm_u, L^\pm_v] = L^\pm_{[u,v]}, \quad [L^\pm_u, Q^\pm_\epsilon] = Q^\pm_{u_\epsilon - \frac{1}{2} u_\epsilon'}, \quad [Q^\pm_\epsilon_1, Q^\pm_\epsilon_2] = 4L^\pm_{\mp i \epsilon_1 \epsilon_2},
$$

(3.9)

where we have defined $[u, v] = uv' - vu'$. Then (3.2) and (3.3) can be written as

$$
L^\pm_u S_{SV} = A^\pm_u, \quad Q^\pm_\epsilon S_{SV} = S^\pm_\epsilon,
$$

(3.10)

where $A^\pm_u$ and $S^\pm_\epsilon$ are the super-Virasoro anomaly given by

$$
A^\pm_u = \kappa_0 \int d^2x \ u \lambda^\pm''', \quad S^\pm_\epsilon = 8i\kappa_0 \int d^2x \ \epsilon \nu_\mp''.
$$

(3.11)

From (3.9) and (3.10) we obtain the Wess-Zumino conditions

$$
L^\pm_u A^\pm_v - L^\pm_v A^\pm_u = A^\pm_{[u,v]},
$$

$$
L^\pm_u S^\pm_\epsilon - Q^\pm_\epsilon A^\pm_u = S^\pm_{u_\epsilon - \frac{1}{2} u_\epsilon'},
$$

$$
Q^\pm_\epsilon_1 S^\pm_\epsilon_2 - Q^\pm_\epsilon_2 S^\pm_\epsilon_1 = 4A^\pm_{\mp i \epsilon_1 \epsilon_2}.
$$

(3.12)

Since (3.11) satisfies these conditions, we see that (3.5) is indeed integrable.

The solution to (3.2) and (3.3) satisfying (3.4) can be obtained by utilizing the transformation properties of $\zeta_\pm$ and $j_\pm$ under time-preserving reparametrizations and local supertransformations. Let us first consider a time- and orientation-preserving transformation given by $x^0 \rightarrow \tilde{x}^0 = x^0$, $x^1 \rightarrow \tilde{x}^1 = f(x)$ with $f'(x) > 0$. Under the coordinate change $\lambda^\pm$ and $\nu_\mp$ are transformed into

$$
\tilde{\lambda}^\pm(\tilde{x}) = \mp \dot{f}(x) + f'(x) \lambda^\pm(x), \quad \tilde{\nu}_\mp(\tilde{x}) = \sqrt{f'(x)} \nu_\mp(x).
$$

(3.13)

In the new coordinates $\tilde{x}$ the same type of equations as (3.2) and (3.3) must be satisfied by $\tilde{\zeta}_\pm(\tilde{x})$ and $\tilde{j}_\pm(\tilde{x})$. This unambiguously determines transformation properties

$$
\tilde{\zeta}_\pm(\tilde{x}) = (f'(x))^{-2}(\zeta_\pm(x) + \kappa_0 \mathcal{D} f(x)), \quad \tilde{j}_\pm(\tilde{x}) = (f'(x))^{-\frac{3}{2}} j_\pm(x),
$$

(3.14)
where $Df$ is the schwarzian derivative defined by
\[
Df = \frac{f'''}{f'} - \frac{3}{2} \left( \frac{f''}{f'} \right)^2 .
\] (3.15)

We see that $\zeta_\pm$ and $j_\pm$, respectively, are tensors of weight 2 and 3/2. The inhomogeneous term denotes the finite form of Virasoro anomaly analogous to the one well-known in conformal field theories.

We next consider the supertransformation (2.19). By requiring covariance of (3.2) and (3.3), we find that $\zeta_\pm$ and $j_\pm$ must be transformed by
\[
\delta_\epsilon \zeta_\pm = -\frac{i}{2} \epsilon_\mp j_\pm + 3i \epsilon_\mp j_\pm, \quad \delta_\epsilon j_\pm = \pm 4 \epsilon_\pm \zeta_\pm \mp 8 \kappa_0 \epsilon''_\pm .
\] (3.16)

The term proportional to $\kappa_0$ represents anomalous behavior of $j_\pm$. The finite form of these infinitesimal transformations can be obtained by straightforward integration. Let us denote by $\eta_\pm$ the components of a finite Majorana spinor $\eta$, then the supertransformation of $\lambda_\pm$ and $\nu_\pm$ by $\eta_\pm$ are given by
\[
\hat{\lambda}_\pm = \lambda_\pm - 4i \eta_\pm \nu_\mp - 2i \eta_\pm (\dot{\eta}_\mp \mp \lambda_\pm^\prime \eta_\mp) , \\
\hat{\nu}_\pm = \nu_\mp + \dot{\eta}_\mp \mp \lambda_\pm^\prime \eta_\mp^\prime \mp 3i \eta_\mp \eta_\mp^\prime \nu_\mp \mp i \eta_\mp \eta_\mp^\prime \eta_\pm .
\] (3.17)

For example, $\hat{\lambda}^+$ can be obtained by Taylor expanding $\lambda^+(\alpha) \equiv e^{\alpha Q^+_0} \lambda^+ e^{-\alpha Q^+_0}$ with respect to $\alpha$ and then putting $\alpha = 1$. Similarly, we find
\[
\hat{\zeta}_\pm = \zeta_\pm - \frac{3}{2} \eta_\mp j_\pm - \frac{i}{2} \eta_\mp j_\mp^\prime \mp 2i \eta_\pm \eta_\mp^\prime \zeta_\pm \mp 2i \kappa_0 (\eta_\mp \eta_\mp^\prime + 3\eta_\pm \eta_\mp^\prime) , \\
\hat{j}_\pm = j_\pm \mp 4 \eta_\mp \zeta_\pm \mp 3i \eta_\mp \eta_\mp^\prime j_\pm \mp 8 \kappa_0 (1 \mp i \eta_\mp \eta_\mp^\prime) \eta_\pm^\prime .
\] (3.18)

The $\kappa_0$-terms emerge from the anomalous transformation properties of $j_\pm$ and denote finite anomaly under local supertransformation.

All these properties are enough to solve (3.2) and (3.3). We first choose $\eta_\pm$ to satisfy $\hat{\nu}_\pm = 0$. We next consider a coordinate transformation $\tilde{x}^0 = x^0$, $\tilde{x}^1 = f_\pm(x)$ where $\tilde{\lambda}_\pm = f_\pm' \lambda_\pm + f_\pm \tilde{\lambda}_\pm = 0$ is satisfied. Since $\hat{\nu}_\pm$ also vanishes in this coordinates, (3.2) and (3.3) take the simplest forms $\partial_\tau \hat{\zeta}_\pm = \partial_\tau \hat{j}_\pm = 0$ with obvious solution
\[
\zeta_\pm = \hat{j}_\pm = 0 .
\] (3.19)
Since we know the transformation properties of $\zeta_\pm$ and $j_\pm$ as (3.14) and (3.18), we can completely determine $\zeta_\pm$ and $j_\pm$ from (3.19) as

$$\zeta_\pm = -\kappa_0 (1 \pm 2i\eta_\mp'\eta_\pm')Df_\pm \pm 2i\kappa_0 (\eta_\mp\eta_\pm'' + 3\eta_\mp'\eta_\pm'') ,$$

$$j_\pm = \pm 4\kappa_0 \eta_\mp Df_\pm \pm 8\kappa_0 (1 \mp i\eta_\mp'\eta_\pm')\eta_\pm'' ,$$  \hspace{1cm} (3.20)

where $f_\pm$ and $\eta_\mp$ are related to $\lambda_\pm$ and $\nu_\mp$ by the conditions $\hat{\nu}_\mp = \tilde{\lambda}_\pm = 0$. Solving these conditions with respect to $\lambda_\pm$ and $\nu_\mp$, we obtain

$$\lambda_\pm = (\pm 1 + 2i\eta_\mp'\eta_\pm')\frac{\dot{f}_\pm}{f_\pm} - 2i\eta_\mp \dot{\eta}_\mp ,$$

$$\nu_\mp = -(1 \mp i\eta_\mp'\eta_\pm')\dot{\eta}_\pm + \frac{\dot{f}_\pm}{f_\pm} \eta_\pm' - \frac{1}{2} \left( \frac{\dot{f}_\pm}{f_\pm} \right)' \eta_\mp .$$ \hspace{1cm} (3.21)

Eqs. (3.20) and (3.21) are the extension of the results obtained for Polyakov string [10] to superstring as one can easily see by putting $\eta_\mp = 0$. Since $f_\pm$ and $\eta_\mp$ depend only on $\lambda_\pm$ and $\nu_\mp$, they are invariant under the super-Weyl transformations.

We are now in a position to compute the counterterm $S_V$ satisfying (3.3). Since $\zeta_+$ and $j_+$ ($\zeta_-$ and $j_-$) depend only on $\lambda^+$ and $\nu_-$ ($\lambda^-$ and $\nu_+$), we can find a solution in the form

$$S_V = S_V^+ + S_V^- ,$$  \hspace{1cm} (3.22)

where $S_V^+$ ($S_V^-$) is a functional of $\lambda^+$, $\nu_-$ ($\lambda^-$, $\nu_+$) and must satisfy the variational equation

$$\delta S_V^\pm = \int d^2x \left( - \delta \lambda^\pm \zeta_\pm \pm 4i \delta \nu_\mp j_\pm \right) .$$  \hspace{1cm} (3.23)

Putting (3.20) into the rhs’ of these expressions and using the relations (3.21), we obtain the counterterms

$$S_V^\pm = \pm \kappa_0 \int d^2x \left[ \frac{1}{2} \left\{ \left( f''_\pm \right)^2 \frac{\dot{f}_\pm}{f_\pm^3} - \frac{f''_\pm \dot{f}_\pm}{(f'_\pm)^2} \right\} \mp 4i\dot{\eta}_\mp \eta_\pm'' + \pm 2i (\eta_\mp\eta_\pm'' + 3\eta_\mp'\eta_\pm'') \dot{f}_\pm f'_\pm - 2\dot{\eta}_\mp\eta_\pm'\eta_\pm'' \right] .$$ \hspace{1cm} (3.24)

This is the extension of the results for Polyakov string found in ref. [10] to superstring.

Since $S_V$ is manifestly invariant under super-Weyl rescalings, and cancels the super-Virasoro anomaly (2.27) and (2.28) in $S_X$ by construction, we see that the effective action
$S_X + S_V$ possesses all the local symmetries of the classical theory and there is no anomaly in super-Weyl symmetry. This seemed again to be inconsistent with the generally accepted fact that one can not maintain reparametrization invariance and local supersymmetry at the quantum level without sacrificing super-Weyl invariance. As was noted in ref. [10] for bosonic theory, this is not a contradiction. Since $f_\pm$ and $\eta_\mp$ satisfying (3.21) depend nonlocally on $\lambda^\pm$ and $\nu_\mp$, the counterterm $S_V$ is in general a nonlocal functional of these variables. It is, however, possible to choose the counterterm recovering reparametrization invariance and supersymmetry to be a local functional of $e_\alpha^a$ and $\chi_\alpha$ as is known by perturbative analysis. We will in fact show in Section 5 that we can reproduce super-Weyl anomaly by requiring the locality of the counteraction.

4 Superspace formulation of the super-Virasoro anomaly

In the previous section we have obtained the counterterm $S_V$ that cancels the super-Virasoro anomaly to recover reparametrization invariance and local supersymmetry at the quantum level. In this section we will make a small detour to superspace and argue the superspace formulation of the super-Virasoro anomaly. We wish to note the existence of somewhat unusual fermionic coordinates and supertranslations under which $\lambda^\pm$ and $\nu_\mp$ form supermultiplets and to clarify the geometrical meaning of what we have done in the previous section. We also see that the arguments of ref. [10] on the Virasoro anomaly in bosonic string can naturally extend to the supersymmetric case and the invariance of $S^\pm_V$ under the OSp(1,2) Kac-Moody like transformations reveals itself in the reformulation by superfields.

Let us consider superspaces with supercoordinates $z^\pm = (\tau, \sigma, \theta_\mp)$ for $\lambda^\pm$ and $\nu_\mp$, where $\theta_\mp$ are grassmannian variables. The infinitesimal supertranslations for $z^\pm$ are defined by

$$
\delta_\epsilon \tau = 0 , \quad \delta_\epsilon \sigma = \pm 2i \epsilon_+ \theta_\mp , \quad \delta_\epsilon \theta_\mp = \epsilon_\mp .
$$

The supercovariant derivatives are given by

$$
D_\pm = \pm i \frac{\partial}{\partial \theta_\mp} + 2 \theta_\mp \frac{\partial}{\partial \sigma} .
$$
They anticommute with the supercharges and satisfy $D_\pm^2 = \pm 2i\partial_\sigma$.

We now define superfields by

$$
\phi^\pm(z^\pm) = \lambda^\pm(x) + 4i\theta_\mp\nu_\mp(x), \quad G_\pm(z^\pm) = j_\pm(x) \mp 4\theta_\mp\zeta_\pm(x). \tag{4.3}
$$

Under the time-preserving supercoordinate transformations given by

$$
z^\pm = (\tau, \sigma, \theta_\mp) \rightarrow z^\pm = (\tau, g^\pm(z^\pm), \vartheta_\mp(z^\pm)) \quad \text{with} \quad D_\pm g^\pm \pm 2i\vartheta_\mp D_\pm \vartheta_\mp = 0, \tag{4.4}
$$

(4.3) are transformed into

$$
\phi^\pm(z^\pm) = (\mp iD_\pm \vartheta_\mp(z^\pm))^2(\phi^\pm(z^\pm) \mp \dot{g}^\pm(z^\pm) + 2i\vartheta_\mp(z^\pm)\dot{\vartheta}_\mp(z^\pm)), \\
G^\pm(z^\pm) = (\mp iD_\pm \vartheta_\mp(z^\pm))^{-3}(G_\pm(z^\pm) + 2i\kappa_0 D_\pm \vartheta_\mp(z^\pm)), \tag{4.5}
$$

where $D_\pm$ stand for superschwarzian derivatives defined by

$$
D_\pm \vartheta_\mp = \frac{D^4_\pm \vartheta_\mp}{D_\pm \vartheta_\mp} - 2\frac{D^2_\pm \vartheta_\mp D^3_\pm \vartheta_\mp}{(D_\pm \vartheta_\mp)^2}. \tag{4.6}
$$

In terms of (4.3), the anomaly equations (3.2) and (3.3) can be written as

$$
\dot{G}^\pm + \frac{i}{2}D_\pm \phi_\mp^2 G_\pm + \frac{i}{4}D_\pm \phi_\mp D_\pm G_\pm + \frac{3}{4}D^2_\pm \phi_\mp G_\pm = \frac{\kappa_0}{2}D^5_\pm \phi^\pm. \tag{4.7}
$$

The transformation properties for $G_\pm$ can be derived by noting the covariance of (4.7) under (4.4).

We can solve (4.7) by making use of the supercoordinate transformations which transform $\phi^\pm$ into 0. Let us denote such transformations by $z^\pm = (\tau, \sigma, \theta_\mp) \rightarrow \tilde{z}^\pm = (\tau, F^\pm(z^\pm), \Theta_\mp(z^\pm))$, then we find

$$
\Phi^\pm = (\mp iD_\pm \Theta_\mp)^{-2}(\mp \dot{F}^\pm - 2i\Theta_\mp \dot{\Theta}_\mp), \\
G^\pm = -2i\kappa_0 D_\pm \Theta_\mp. \tag{4.8}
$$

It is easy to see that $F^\pm$ and $\Theta_\mp$ are transformed as scalar superfields under the supercoordinate changes (4.4). Eqs.(4.8) correspond to the solutions (3.21) and (3.20) for the choices

$$
F^\pm = f_\pm \mp 2i\dot{f}_\pm^\prime \eta_\mp, \quad \Theta_\mp = \sqrt{f_\mp^\prime}\{\eta_\mp + \theta_\mp(1 \mp i\eta_\mp \eta_\mp^\prime)\}. \tag{4.9}
$$
where \( f_{\pm} \) and \( \eta_{\mp} \) have been introduced in the previous section.

We have furnished all the staffs needed to write down the counterterms (3.24) as functionals on the superspace. It is straightforward to retrace the procedure to obtain (3.24) from (3.23) in terms of the superfields. We find

\[
S_{\pm} = \pm i\kappa_0 \int d^3z \pm \left( \mp \frac{\hat{F}_{\pm} + 2i\Theta_{\pm} \dot{\Theta}_{\mp} D_{\pm} \Theta_{\mp} + 2iD_{\pm} \dot{\Theta}_{\mp} D_{\pm} \Theta_{\mp}}{(D_{\pm} \Theta_{\mp})^2} \right),
\]

where the superspace volume elements are defined by \( d^3z_{\pm} = d^2x \mp d\theta_{\mp} \). Under the coordinate changes (4.4), \( S_{\pm} \) are transformed into \( \mathcal{S}_{\pm} \) given by

\[
\mathcal{S}_{\pm} = S_{\pm} + 2i\kappa_0 \int d^3z \pm \dot{\phi}_{\mp} D_{\pm} \vartheta_{\mp} - 4\kappa_0 \int d^3z \mp \frac{D_{\pm} \dot{\vartheta}_{\mp} D_{\pm} \vartheta_{\mp}}{(D_{\pm} \vartheta_{\mp})^2}.
\]

We see that \( S_{\pm} \) is invariant, up to surface terms, under the transformations satisfying

\[
D_{\pm} \vartheta_{\mp} = 0.
\]

They correspond to time-dependent super-M"obius transformations on the coordinates \( \sigma \) and \( \theta_{\mp} \). These together with the obvious \( \tau \)-reparametrization \( \tau \to \tau(\tau) \) constitute the full symmetries of the integrated super-Virasoro anomaly.

As argued in Section 2, the string action \( S_X \) at the quantum level is invariant under the transformations satisfying (2.30), which are the special case of (4.12) for infinitesimal transformations given by \( \tau = \tau + u^0(x), \sigma = \sigma + u^1(x) \mp 2i\theta_{\mp} \epsilon_{\mp} \) and \( \vartheta_{\mp} = \left( 1 + \frac{1}{2}u^{1'}(x) \right)\theta_{\mp} + \epsilon_{\mp}(x) \). By construction the symmetries of \( S_X \) can be enlarged to those of (3.22) as mentioned before.

## 5 Super-Weyl anomaly and super-Liouville action

In Section 3 we have shown that the super-Virasoro anomaly of the original string action (2.1) can be cancelled by the counteraction (3.24). The total effective action turns out to possess all the local symmetries of the classical theory. It is, however, a nonlocal functional in 2D supergravity fields. As was discussed in ref. [10] for Polyakov string, we will introduce another counterterm which cancels the nonlocal terms of (3.24), and recover
the locality of the effective action. We must choose the new counterterm to be invariant under reparametrizations and local supersymmetry to maintain these symmetries.

To get some insight into the counterterm we will first investigate the nonlocality of (3.24) by using weak field expansion. Let us expand the 2D metric $g_{\alpha\beta} = e_\alpha^a e_\beta^a$ by linearizing fields as

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} .$$

(5.1)

We also assume $\chi_\alpha$ to be the same order as $h_{\alpha\beta}$. Then to lowest order in the weak fields one can easily obtain $\zeta_\pm$ and $j_\pm$ from (3.2) and (3.3)

$$\zeta_\pm = \pm 2\kappa_0 \frac{\partial^3}{\partial x^\pm} h_{\pm\pm} , \quad j_\pm = \pm 16\kappa_0 \frac{\partial^2}{\partial x^\pm} \chi_{\pm\pm} .$$

(5.2)

Putting these into (3.23) and then integrating the arbitrary variation of the counteraction, we find the lowest order form of $S_V$ as

$$S_V = \kappa_0 \int d^2x \left( \frac{1}{4} h_{++} \frac{\partial^3}{\partial x^+} h_{++} + \frac{1}{4} h_{--} \frac{\partial^3}{\partial x^-} h_{--} - 4i\chi_{++} \frac{\partial^2}{\partial x^+} \chi_{++} - 4i\chi_{--} \frac{\partial^2}{\partial x^-} \chi_{--} \right) + \cdots ,$$

(5.3)

where we have suppressed terms local in $h_{\alpha\beta}$ and $\chi_\alpha$. This is an extension of the bosonic string case observed in ref. [10] to fermionic string.

As is easily inferred from the case of bosonic string, the nonlocal terms of (5.3) can be canceled by the supersymmetric extension of the nonlocal Liouville action found by Polyakov [1, 23, 24, 13]. It can be defined by the super-Liouville action given by

$$S_L = - \int d^2x \left[ e \left\{ \frac{1}{2} (g^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi - i\bar{\Psi} \rho^\alpha \nabla_\alpha \Psi) + \bar{\chi}_\alpha \rho^\beta \rho^\alpha \bar{\chi}_\beta \Phi + \frac{1}{4} \bar{\Psi} \bar{\chi}_\alpha \rho^\beta \rho^\alpha \chi_\beta \right\} + eR \Phi - 4ie^{\alpha\beta} \bar{\chi}_\alpha \rho_5 \rho^\gamma \chi_\beta \partial_\gamma \Phi - 4e^{\alpha\beta} \bar{\chi}_\alpha \rho_5 \nabla_\beta \Psi \right] ,$$

(5.4)

where $R$ is the scalar curvature computed from the 2D metric $g_{\alpha\beta}$. The super-Liouville fields $\Phi$ and $\Psi$ are defined as the solutions to the classical field equations obtained from

\footnote{The light-cone coordinates are defined by $x^\pm = x^0 \pm x^1$. The flat metric $\eta_{ab}$ in this coordinates is given by $\eta_{++} = \eta_{--} = 0$, $\eta_{+-} = -\frac{1}{2}$. We will use the convention $\partial_\pm = \partial_0 \pm \partial_1$, hence $\partial_\pm x^\pm = 2$.}
this action and are assumed to satisfy $\Phi = \Psi = 0$ for $e_\alpha^a = \delta_\alpha^a$ and $\chi_\alpha = 0$. They depend functionally on the 2D supergravity fields and are, in general, nonlocal quantities.

The lowest order approximation of (5.4) can be computed in a straightforward manner. It is given by

$$S_L = -\frac{1}{2} \int d^2 x \left\{ R \frac{1}{\Box} R + 16 \chi \rho \cdot \Box \chi \right\},$$

where $R = \ddot{h}_{11} + h''_{00} - 2\dot{h}'_{01}$ is the linearized scalar curvature and we have defined $\Box \equiv -\partial_+ \partial_-$, $\rho \cdot \partial \equiv \delta_\alpha^a \rho^a \partial_\alpha$ and $\chi \equiv \epsilon^{\alpha\beta} \partial_\alpha \chi_\beta$. Then the nonlocality of (5.3) can be removed within the linearizing approximation by introducing the new counteraction

$$S_T = -\frac{\kappa_0}{2} S_L.$$  

(5.6)

Weak field expansion can be systematically carried out order by order and one can examine the cancellation of the nonlocal terms between $S_V$ and $S_T$ to any order, establishing the relation (5.6). We shall show in a moment that (5.6) is exact without relying on such approximation scheme.

Before going to the derivation of (5.6), we remark here the symmetry properties of $S_L$. It is obviously invariant under the reparametrization and local supersymmetry. This property is necessary to maintain these symmetries recovered by adding $S_V$ to $S_X$. It is, however, not invariant under the super-Weyl transformations. Under the infinitesimal Weyl rescaling $\delta e_\alpha^a = \frac{\phi}{2} e_\alpha^a$ and $\delta \chi_\alpha = \frac{\phi}{4} \chi_\alpha$, $S_T$ produces the Weyl anomaly, i.e.,

$$\delta S_L = \int d^2 x \left[ eR + 4i\epsilon^{\alpha\beta} \partial_\alpha (\chi_\beta \rho_5 \rho^7 \chi_7) \right] \phi.$$  

(5.7)

Similarly for the fermionic transformation $\delta e_\alpha^a = 0$ and $\delta \chi_\alpha = i\rho_\alpha \epsilon$ with $\epsilon$ being an infinitesimal Majorana field, we find the anomaly

$$\delta S_L = 16 \int d^2 x \epsilon^{\alpha\beta} \rho_5 \nabla_\alpha \chi_\beta.$$  

(5.8)

We thus see that the super-Weyl anomaly is correctly reproduced by requiring the locality of the counteraction.

To establish the locality of $S_V + S_T$ we will basically follow the idea used in ref. [10]. We first rewrite (5.4) in terms of the variables (2.2) – (2.4) as

$$S_L = \int d^2 x \left[ \frac{\dot{\Phi} - \lambda^+ \Phi'}{\lambda^+ + \lambda^-} + \frac{i}{2} \Psi^+ (\dot{\Psi}^+ - \lambda^+ \Psi') + \frac{i}{2} \Psi^- (\dot{\Psi}^- + \lambda^- \Psi') \right]$$

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and then perform the supertranslation by $\eta_\mp$ satisfying (3.21), hence $\dot{\nu}_\mp = 0$. In this special gauge the bosonic and fermionic sectors are decoupled in (5.9) and one can easily observe that

$$\hat{\Lambda} \equiv \begin{pmatrix} (-\dot{e}_1^-)^{-1/2} \dot{\Lambda}_- \\ (\dot{e}_1^+)^{-1/2} \dot{\Lambda}_+ \end{pmatrix}$$

transforms as a scalar under reparametrizations and as a spinor under local Lorentz transformations. The action can be simplified further if we work in the new coordinates $(\tilde{x})$ defined by

$$\tilde{x}^\pm = f_\mp(x),$$

where $f_\pm(x)$ are introduced in Section 3. As in the bosonic string case, the metric tensor becomes conformally minkowskian and is given by

$$\bar{g}_{\alpha\beta}(\tilde{x}) = \eta_{\alpha\beta} \frac{\hat{g}_{11}(x)}{f'_+(x)f'_-(x)}.$$  

The super-Liouville action (5.9) reduces to

$$S_L = \int d^2\tilde{x}\left\{ -\frac{1}{2}\eta_{\alpha\beta}\partial_\alpha\tilde{\Phi}\partial_\beta\tilde{\Phi} + i\bar{\Psi}_+(\dot{\tilde{\Psi}}_+ - \tilde{\Psi}'_+) + i\bar{\Psi}_-(\dot{\tilde{\Psi}}_- + \tilde{\Psi}'_-) \\
-\eta_{\alpha\beta}\partial_\alpha\tilde{\Phi}\partial_\beta\tilde{\Phi} - i\partial_+\tilde{\Phi} - i\dot{\tilde{\Lambda}}_+(\tilde{\Psi}_+ - \tilde{\Psi}'_+) - i\dot{\tilde{\Lambda}}_- (\tilde{\Psi}_- + \tilde{\Psi}'_-) \right\}.$$  

This immediately gives the classical solution for the super-Liouville fields as

$$\tilde{\Phi}(\tilde{x}) = \tilde{\xi}(\tilde{x}), \quad \tilde{\Psi}_\pm(\tilde{x}) = \tilde{\Lambda}_\pm(\tilde{x}).$$

(5.14)
Since the super-Liouville fields $\Phi$, $\Psi$ and $\hat{\Lambda}$ defined by (5.10) transform as scalars under world-sheet reparametrizations, we easily obtain from (5.14) the super-Liouville fields in the original coordinates $(x)$ as

$$
\hat{\Phi}(x) = \hat{\xi}(x) - \ln f_+'(x)f_-'(x), \quad \hat{\Psi}_\pm(x) = \hat{\Lambda}_\pm(x) .
$$

(5.15)

In deriving these we have used the relations (5.12). The super-Liouville action $S_L$ can be obtained from (5.13) by substituting the classical solutions (5.14) or (5.15) for the super-Liouville fields. In the $(x)$-coordinates all the nonlocalities of $S_L$ arise through the dependences of various variables on $f_\pm$ and $\eta_\pm$. We would like to separate the dependences on these variables.

To do this we first note that the scalar curvature density in the $(\tilde{x})$-coordinates is given by $\tilde{e}\tilde{R} = -\eta_{\alpha\beta}\partial_\alpha\partial_\beta\tilde{\xi}$. Putting this into the rhs of (5.13) and coming back to the $(x)$-coordinate system, we obtain

$$
S_L = S_a + S_b ,
$$

(5.16)

where $S_{a,b}$ are given by

$$
S_a = \int d^2x \left\{ \hat{\epsilon} \left( -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \hat{\xi} \partial_\beta \hat{\xi} + \hat{R}\hat{\xi} \right) - \left( \frac{\hat{\lambda}'^+ - \hat{\lambda}'^-}{\lambda^+ + \lambda^-} \right)^2 - \frac{i}{2} \hat{\alpha}_+(\hat{\lambda}_+ - \hat{\lambda}'_+) - \frac{i}{2} \hat{\lambda}_-(\hat{\lambda}_- + \hat{\lambda}'_-) \right\} ,
$$

(5.17)

$$
S_b = \int d^2x \left( \frac{f_+'' f'_+}{(f_+')^3} - \frac{f_+ f_+'''}{(f_+')^2} - \frac{(f_-')^2 f_-'}{(f_-')^3} + \frac{f_- f''_+}{(f_-')^2} \right) .
$$

(5.18)

We see that except for the second term in the integrand of (5.17) $S_a$ can be obtained from the super-Liouville action (5.9) by the following replacements

$$
\lambda^\pm, \xi, \nu_\mp, \Lambda_\pm \to \hat{\lambda}^\pm, \hat{\xi}, \hat{\nu}_\mp(= 0), \hat{\Lambda}_\pm ,
$$

$$
\Phi, \Psi \to \hat{\xi}, \hat{\Lambda}_\pm .
$$

(5.19)

Hence, let us define a family of variables parametrized by a real parameter $\alpha$ by the set of differential equations

$$
\frac{d\lambda^\pm(\alpha)}{d\alpha} = -4i\eta_\pm\nu_+(\alpha) ,
$$

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\[ \frac{d\nu_{\mp}(\alpha)}{d\alpha} = \dot{\eta}_{\mp} \mp \lambda^{\pm}(\alpha)\eta'_{\mp} \pm \frac{1}{2} \lambda^{\pm'}(\alpha)\eta_{\mp}, \]
\[ \frac{d\xi(\alpha)}{d\alpha} = i(\eta_+ \Lambda_-(\alpha) - \eta_- \Lambda_+(\alpha)), \]
\[ \frac{\lambda^+(\alpha) + \lambda^-(\alpha)}{4} \frac{d\Lambda_{\pm}(\alpha)}{d\alpha} = (\lambda^+(\alpha) + \lambda^-(\alpha))\eta'_{\mp} \mp \frac{1}{2} (\lambda^+(\alpha) - \lambda^-(\alpha))'\eta_{\mp} \pm \frac{1}{2} \eta_{\mp}(\dot{\eta}_{\pm}(\alpha) \pm \lambda^{\pm'}(\alpha)\eta'_{\mp} \pm \frac{1}{2} \lambda^{\pm'}(\alpha)\eta_{\mp}, \]
\[ \frac{d\Phi(\alpha)}{d\alpha} = i(\eta_+ \Psi_-(\alpha) - \eta_- \Psi_+(\alpha)), \]
\[ \frac{\lambda^+(\alpha) + \lambda^-(\alpha)}{4} \frac{d\Psi_{\pm}(\alpha)}{d\alpha} = (\lambda^+(\alpha) + \lambda^-(\alpha))\eta'_{\mp} \mp \frac{1}{2} (\lambda^+(\alpha) - \lambda^-(\alpha))'\eta_{\mp} \pm \frac{1}{2} \eta_{\mp}(\dot{\Phi}(\alpha) \pm \lambda^{\pm}\Phi') \pm \frac{i}{2} (\Psi_-(\alpha)\nu_+(\alpha) - \Psi_+(\alpha)\nu_-(\alpha))\eta_{\mp}, \]
\[ (5.20) \]

with the initial conditions
\[ \lambda^{\pm}(0) = \lambda^{\pm}, \quad \nu_{\mp}(0) = \nu_{\mp}, \quad \xi(0) = \Phi(0) = \xi, \quad \Lambda_{\pm}(0) = \Psi_{\pm}(0) = \Lambda_{\pm}. \quad (5.21) \]

Except for the equations for \( \Phi(\alpha) \) and \( \Psi_{\pm}(\alpha) \), (5.20) define a one-parameter family of 2D metrics and gravitinos connected by a local supersymmetry. At \( \alpha = 1 \), we obtain \( \lambda^{\pm}(1) = \dot{\lambda}^{\pm}, \xi(1) = \dot{\xi} \) and \( \nu_{\mp}(1) = \dot{\nu}_{\mp} = 0 \). On the other hand the last two equations together with the initial conditions (5.21) give
\[ \Phi(\alpha) = \xi(\alpha), \quad \Psi_{\pm}(\alpha) = \Lambda_{\pm}(\alpha). \quad (5.22) \]

Explicit solutions to (5.20) can be found in a straightforward manner and turn out to be polynomials (at most cubic) in \( \alpha \). For instance, \( \nu_{\mp}(\alpha) \) are given by
\[ \nu_{\mp}(\alpha) = \nu_{\mp} + \alpha(\dot{\eta}_{\mp} \mp \lambda^{\pm}\eta'_{\mp} \pm \frac{1}{2} \lambda^{\pm'}\eta_{\mp}) \mp 3i\alpha^2 \eta_{\mp}\eta'_{\mp}\nu_{\mp} \mp i\alpha^3 \eta_{\mp}\eta'_{\mp}\eta_{\mp}. \quad (5.23) \]

Let us now define a one-parameter family of actions by
\[ S(\alpha) = S_L(\alpha) - \int d^2x \frac{(\lambda^{+(\alpha)} - \lambda^{-'}(\alpha))^2}{\lambda^{+(\alpha)} + \lambda^{-}(\alpha)}, \quad (5.24) \]

where \( S_L(\alpha) \) is obtained form (5.9) by replacing all the variables by the corresponding \( \alpha \)-dependent ones introduced above. With this definition, we find
\[ S_\alpha = S(1). \quad (5.25) \]
To find the explicit form of (5.24) we first derive the differential equation satisfied by $S(\alpha)$. This can be found by the observation that $S_L(\alpha)$ would be independent of $\alpha$ if $\Psi_\pm(\alpha)$ obeyed the equation
\[
\frac{\lambda^+(\alpha) + \lambda^-(\alpha)}{4} \frac{d\Psi_\pm(\alpha)}{d\alpha} = \pm \frac{1}{2} \eta_\mp (\Phi(\alpha) \pm \lambda^\pm \Phi') \pm \frac{i}{2} (\Psi_-(\alpha) \nu_+(\alpha) - \Psi_+(\alpha) \nu_-) \eta_\mp . \tag{5.26}
\]
This implies that $S(\alpha)$ is subject to the differential equation
\[
\frac{dS(\alpha)}{d\alpha} = \int d^2x \left[ \Delta \Psi_+(\alpha) \frac{\delta S(\alpha)}{\delta \Psi_+(\alpha)} + \Delta \Psi_-(\alpha) \frac{\delta S(\alpha)}{\delta \Psi_-(\alpha)} - \frac{d}{d\alpha} \frac{(\lambda^+(\alpha) - \lambda^-(\alpha))^2}{\lambda^+(\alpha) + \lambda^-(\alpha)} \right] , \tag{5.27}
\]
where $\Delta \Psi_\pm(\alpha)$ stand for the deviations of the last equations in (5.20) from (5.26), i.e.,
\[
\Delta \Psi_\pm(\alpha) \equiv 4 \eta_\mp \mp \frac{2(\lambda^+(\alpha) - \lambda^-(\alpha))}{\lambda^+(\alpha) + \lambda^-(\alpha)} \eta_\mp . \tag{5.28}
\]
Surprisingly enough, many terms are canceled out in the rhs of (5.27) due to the supersymmetry mentioned above and we find
\[
\frac{dS(\alpha)}{d\alpha} = 16i \int d^2x (\nu_- (\alpha) \eta'''_+ + \nu_+ (\alpha) \eta'''_-) . \tag{5.29}
\]
This just corresponds to the anomaly equation (2.28) for the local supersymmetry.

Using (5.23), we can integrate (5.20) to obtain
\[
S_\alpha = S_L^\xi - \int d^2x \frac{(\lambda^+ - \lambda^-)^2}{\lambda^+ + \lambda^-} - 4 \int d^2x \left[ 2i \eta_+ \eta'''_+ - i \hat{f}^+ (3 \eta'_+ \eta''_+ + \eta_+ \eta'''_+) - \eta_- \eta''_+ \hat{\eta}_- - 2i \eta_+ \eta''_+ - i \hat{f}^- (3 \eta'_- \eta''_+ + \eta_+ \eta'''_+) + \eta_+ \eta'_- \hat{\eta}_+ \right] . \tag{5.30}
\]
In deriving (5.30), we have used the relations (3.21). The $S_L^\xi$ is defined from (5.9) with the replacements $\Phi \to \xi$ and $\Psi_\pm \to \Lambda_\pm$, hence is a local functional of zweibein and gravitino. The nonlocalities of $S_\alpha$ are completely isolated as terms containing $f_\pm$ and $\eta_\mp$. Combining (3.22), (3.24), (5.6), (5.16), (5.30) and (5.18), we finally obtain
\[
S_g = S_V + S_T = \kappa_0 \frac{g}{2} \int d^2x \left[ e \left\{ \frac{1}{2} (g^{\alpha \beta} \partial_\alpha \xi \partial_\beta \xi - i \chi_\alpha \rho^\alpha \nabla_\alpha \Lambda) + \bar{\chi}_\alpha \rho^\beta \rho^\alpha \Lambda \partial_\beta \xi + \frac{1}{4} \bar{\Lambda} \chi_\alpha \rho^\beta \rho^\alpha \chi_\beta \right\} \right.
- e R \xi - 4ie^{\alpha \beta} \bar{\chi}_\alpha \rho^\beta \rho^\gamma \chi_\gamma \partial_\beta \xi - 4e^{\alpha \beta} \bar{\chi}_\alpha \rho_5 \nabla_\beta \Lambda \right. + \left. \frac{2g_{11}}{\sqrt{-g}} \left\{ \left( g_{01} \right)^2 \right\} , \tag{5.31}
\]
where $\Lambda$ is defined by
\[
\Lambda \equiv \begin{pmatrix}
(-e_1^-)^{-1/2}\Lambda_-
\end{pmatrix}.
\]
(5.32)

This is the local super-Liouville action with $\xi$, $\Lambda$ identified with the super-Liouville fields and has been obtained in \[21, 23\]. As discussed in \[21, 23\], $S_g$ is not invariant under world-sheet reparametrizations and local supersymmetry since the super-Liouville fields $\xi, \Lambda$ do not possess simple transformation properties such as scalar and spinor. Instead it exactly cancels the super-Virasoro anomaly given in (2.27) and (2.28). Furthermore, we see from (5.7) and (5.8) that the super-Weyl anomaly is correctly reproduced by (5.31). In this sense $S_g$ can be regarded as a WZW term converting the super-Virasoro anomaly into the super-Weyl anomaly. It can also be considered as a seagull term for the action (2.1) to maintain the covariance. The requirement of locality, which is dictated by the perturbative analysis, is so restrictive that we can not maintain all the local symmetries of the classical action. We stress that the action (5.31) is local in contrast to the nonlocal action (5.33). This is only possible by sacrificing the covariance under reparametrizations and supersymmetry.

We will close this section by mentioning the inclusion of the auxiliary fields for the supermultiplets and the cosmological term. The latter arises through quantum effects, giving additional contributions to the super-Weyl anomaly. The auxiliary fields should be introduced to retain off-shell closure of the supersymmetry algebra, which ensures the off-shell nilpotency of the BRST transformations. As was stressed in ref. \[23\], this is crucial in order to achieve quantization of the model for general gauge-fixings. The action $S_X + S_g$, however, is not invariant under the supersymmetry transformations modified to satisfy off-shell closure condition because of the super-Weyl anomaly. This can be remedied by introducing additional term to the action which is quadratic in the auxiliary field for the zweibein multiplet and turns out to be related with the cosmological term \[1, 27, 23\]. Thus the additional action including the cosmological term is given by

\[
S_{\text{cosm}} = \frac{\kappa_0}{2} \int d^2x \left[ \frac{1}{2} A^2 + 2\mu (A - 2e^{-1} e^{\alpha \bar{\alpha}} \chi_{a\beta} \rho_{a\beta} \chi_{a\beta}) \right]
= \frac{\kappa_0}{2} \int d^2x \left[ \frac{1}{2} a^2 + \mu e^\xi \sqrt{2(\lambda^+ + \lambda^-)} a - 2i\mu e^\xi (\nu_+ \Lambda_+ + \nu_- \Lambda_-) \right]
\]

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where $A$ is the auxiliary field for the zweibein supermultiplet. We have introduced the rescaled variable $a$ given by

$$a = \sqrt{e}A$$  \hfill (5.34)$$

for later convenience. The auxiliary field can be eliminated by using the equation of motion $A = -2\mu$, leading to the standard form for the cosmological term

$$S_{\text{cosm}} = -\kappa_0 \int d^2x (\mu e + 2\mu e^{\alpha\beta}\chi_{\alpha\rho5}\chi_{\beta}) .$$  \hfill (5.35)$$

We should also include auxiliary fields for the string sector. Since they play no role in the subsequent section, we omit them here.

The action (5.31) together with (5.33) describe the dynamics of the super-Liouville mode of 2D supergravity. We may then regard $S_X + S_g + S_{\text{cosm}}$ as starting action to canonically quantize this system, which we will argue in the next section.

## 6 BRST Quantization

So far we have considered the zweibeins and the gravitinos as background classical fields and only the string variables are treated as quantized operators. In the previous sections we have shown that the quantum fluctuations of the string variables induce the super-Liouville action (5.31), which can be considered to describe the dynamics of the super-Liouville mode of the 2D supergravity. In this section we will promote them to dynamical variables and investigate their quantization. Since graviton and gravitino themselves including the superghosts associated with gauge-fixing also contribute to the super-Virasoro anomaly, we will replace the coupling constant $\kappa_0$ by an arbitrary parameter $\kappa$ in the starting action

$$S_0 = S_X + S_g + S_{\text{cosm}} .$$  \hfill (6.1)$$
Then, despite the apparent lack of invariances under reparametrizations and local supersymmetry, they will recover at the quantum level for an appropriate choice of $\kappa$. We will argue the BRST invariance of the theory described by the action in the superconformal gauge and in the supersymmetric light-cone gauge. Since these gauges have already been taken up in ref. [23] for essentially the same action, we will be brief and contain them here to make the paper self-contained.

6.1 Superconformal gauge-fixing

The Superconformal gauge is defined by the relations

$$e_\alpha^a = \sqrt{c} \delta_\alpha^a, \quad \chi_\alpha = -\frac{1}{2} \rho_\alpha^\beta \chi_\beta .$$

(6.2)

These correspond to the gauge conditions (2.5). We summarize the BRST transformations in covariant notation in Appendix A. The BRST gauge-fixed action in this gauge is then given by

$$S_{\text{eff}} = \int d^2x \left[ \frac{1}{2} \partial_+ X \partial_- X + \frac{i}{2} \psi_+ \partial_- \psi_+ + \frac{i}{2} \psi_- \partial_+ \psi_- \right] \\
+ \frac{\kappa}{2} \int d^2x \left[ \frac{1}{2} \partial_+ \xi \partial_- \xi + \frac{i}{2} \Lambda_+ \partial_- \Lambda_+ + \frac{i}{2} \Lambda_- \partial_+ \Lambda_- + \mu e^2 (2a + i \Lambda_- \Lambda_+) + \frac{1}{2} a^2 \right] \\
+ \int d^2x [-b_+ \partial_- c^+ - b_- \partial_+ c^- + \beta_+ \partial_+ \gamma_+ + \beta_- \partial_- \gamma_-] ,$$

(6.3)

where $c^\pm \equiv C^\pm$ and $\gamma_\mp \equiv \omega_\mp$ are superconformal ghosts, and $b_{\pm\mp}$ and $\beta_{\pm\mp}$ are their antighosts. Since the local Lorentz mode $l$ and the associated ghost fields are not propagating degrees, they have been integrated out via the equations of motion from the action.

The action (6.3) is invariant under the BRST transformations

$$\delta X = \frac{1}{2} c^+ \partial_+ X + \frac{1}{2} c^- \partial_- X - i (\gamma_- \psi_+ - \gamma_+ \psi_-) ,$$

$$\delta \psi_{\pm} = \frac{1}{2} c^\pm \partial_{\pm} \psi_{\mp} + \frac{1}{4} \partial_{\pm} c^\pm \psi_{\pm} \pm \gamma_{\mp} \partial_{\pm} X ,$$

$$\delta \xi = \frac{1}{2} \partial_+ c^+ + \frac{1}{2} \partial_- c^- + \frac{1}{2} c^+ \partial_+ \xi + \frac{1}{2} c^- \partial_- \xi - i (\gamma_- \Lambda_+ - \gamma_+ \Lambda_-) ,$$

$$\delta \Lambda_{\pm} = \frac{1}{2} c^\pm \partial_{\pm} \Lambda_{\mp} + \frac{1}{4} \partial_{\pm} c^\pm \Lambda_{\pm} \pm \gamma_{\mp} \partial_{\pm} \xi \pm 2 \partial_\mp \gamma_{\mp} \pm \frac{\mu}{2} \epsilon^2 \xi \Lambda_{\mp} - 2 \mu \epsilon^2 \gamma_{\pm} .$$
\[ \delta a = \frac{1}{2} c^+ \partial_+ a + \frac{1}{2} c^- \partial_- a + \frac{1}{4} \partial_+ c^+ a + \frac{1}{4} \partial_- c^- a - i(\gamma_+ \partial_- \Lambda_+ + \gamma_- \partial_+ \Lambda_-), \]
\[ \delta c^\pm = \frac{1}{2} c^\mp \partial_\pm c^\pm + 2i\gamma_\mp^2, \]
\[ \delta \gamma_\pm = \frac{1}{2} c^\mp \partial_\pm \gamma_\pm - \frac{1}{4} \partial_\pm c^\mp \gamma_\pm, \]
\[ \delta b_{\pm \pm} = T^X_{\pm \pm} + T^L_{\pm \pm} + T^{gh(2)}_{\pm \pm} + T^{gh(3/2)}_{\pm \pm}, \]
\[ \delta \beta_{\pm \pm} = \pm i(J^X_{\pm \pm} + J^L_{\pm \pm} + J^{gh}_{\pm \pm}), \] (6.4)

where the stress tensors \( T^X_{\pm \pm}, T^L_{\pm \pm}, T^{gh(2)}_{\pm \pm}, T^{gh(3/2)}_{\pm \pm} \) and the supercurrents \( J^X_{\pm \pm}, J^L_{\pm \pm}, J^{gh}_{\pm \pm} \) are given by

\[ T^X_{\pm \pm} \equiv \frac{1}{4}(\partial_\pm X)^2 + \frac{i}{4}\psi_\pm \partial_\pm \psi_\pm, \]
\[ T^L_{\pm \pm} \equiv \kappa \left[ \frac{1}{4}(\partial_\pm \xi)^2 - \frac{1}{2} \partial_\pm^2 \xi + \frac{i}{4} \Lambda_\pm \partial_\pm \Lambda_\pm \right], \]
\[ T^{gh(2)}_{\pm \pm} \equiv -b_{\pm \pm} \partial_\pm c^\pm + \frac{1}{2} \partial_\pm b_{\pm \pm} c^\pm, \]
\[ T^{gh(3/2)}_{\pm \pm} \equiv \frac{3}{4} \beta_{\pm \pm} \partial_\pm \gamma_\mp + \frac{1}{4} \partial_\pm \beta_{\pm \pm} \gamma_\mp, \]
\[ J^X_{\pm \pm} \equiv \psi_\pm \partial_\pm X, \]
\[ J^L_{\pm \pm} \equiv \frac{\kappa}{2}(\Lambda_\pm \partial_\pm \xi - 2\partial_\pm \Lambda_\pm), \]
\[ J^{gh}_{\pm \pm} \equiv \mp i\left( \frac{3}{4} \beta_{\pm \pm} \partial_\mp c^\mp + \frac{1}{2} \partial_\pm \beta_{\pm \pm} c^\pm - 4ib_{\pm \pm} \gamma_\mp \right). \] (6.5)

Careful reader might be sceptical over the BRST invariance of (6.3) since the action \( S_g \) is neither generally covariant nor locally supersymmetric at the classical level. This can be understood by noticing that the super-Virasoro anomaly appearing in (3.2) and (3.3) vanishes in this gauge as pointed out in Section 2.

The BRST charge generating the transformations (6.4) is given by

\[ Q_B = \int d\sigma \left[ c^+(T^X_{++} + T^L_{++} + \frac{1}{2} T^{gh(2)}_{++} + T^{gh(3/2)}_{++}) + c^-(T^X_{--} + T^L_{--} + \frac{1}{2} T^{gh(2)}_{--} + T^{gh(3/2)}_{--}) \right. \]
\[ \left. -i\gamma_-(j^X_{++} + j^L_{++}) + i\gamma_+(j^X_{--} + j^L_{--}) + 2ib_{++} \gamma_+ + 2ib_{--} \gamma_- \right], \] (6.6)

which is conserved correspondingly to the invariance of the action but not nilpotent under super-Poisson bracket, i.e., \( \{Q_B, Q_B\} \neq 0 \), even at the classical level due to the nonvanishing central term in the super-Virasoro algebra for the super-Liouville sector.

Quantum theory can be achieved by replacing fundamental super-Poisson brackets \( \{ , \} \) with supercommutator \( \frac{1}{i}[ , ] \). To define operator products, free field normal ordering
can be used for string coordinates and ghost fields. As for the super-Liouville sector, we should be careful in defining $T_{\pm \pm}$ and $J_{\pm \pm}$. Since we wish to retain superconformal symmetry, they must be defined to satisfy super-Virasoro algebra. One way to implement this is to reduce the super-Liouville fields to free ones by choosing the cosmological term to zero. In this case the stress tensor and the supercurrent become those of free field with coupling to background charge as in refs. [8, 16].

Another way to define these operators is to apply ordering prescription based on the decomposition (2.22) to the super-Liouville fields. We describe in some detail the derivation of super-Virasoro algebra in Appendix B. We show there that the Liouville sector contribute to the additional Virasoro central charge by $3/2$ due to the quantum fluctuation and gravitational dressing effect [7, 8], which was originally observed in ref. [2] for Liouville theory, is argued. Then the BRST charge satisfies the nilpotency for the vanishing total central charge. This leads to the condition for the coupling $\kappa$

$$\kappa = \frac{9 - D}{16\pi}.$$  \hspace{1cm} (6.7)

As is shown in Appendix B, the cosmological term in the super-Liouville action suffers from gravitational dressing effect to retain BRST invariance. We then arrive at the super-Liouville action

$$S_g + S_{\cosm} = \frac{\kappa}{2} \int d^2 x \left[ \frac{1}{2} \partial_+ \xi \partial_- \xi + \frac{i}{2} \Lambda_+ \partial_- \Lambda_+ + \frac{i}{2} \Lambda_- \partial_+ \Lambda_- ight. \\
\left. + 2\mu e^{\alpha} (a + i\alpha \Lambda_- \Lambda_+) + \frac{1}{2} a^2 \right] \\
= \frac{\kappa}{2} \int d^2 x d^2 \theta \left( iD_+ \Xi D_- \Xi + \frac{4\mu}{\alpha} e^{\alpha} \Xi \right),$$  \hspace{1cm} (6.8)

where we have defined a superfield $\Xi(x, \theta) = \xi + i(\theta_+ \Lambda_- - \theta_- \Lambda_+) + i\theta_+ \theta_- a$. The parameter $\alpha$, which is classically $\frac{1}{2}$, is determined by the condition that the operator $e^{\alpha} \Xi$ transforms as superconformal field of weight $(\frac{1}{2}, \frac{1}{2})$ so that the integral over the superspace becomes

\footnote{The $\partial_\mp$ and $D_\mp = \mp i \frac{\partial}{\partial \theta_\mp} \mp \theta_\pm \partial_\mp$, respectively, stand for the fermionic coordinates and covariant derivatives of the flat superspace.}
invariant under superconformal transformations. This combined with (6.7) leads to
\[ \alpha = \frac{9 - D \pm \sqrt{(1 - D)(9 - D)}}{4} . \]  
(6.9)

By the rescaling \( \sqrt{\frac{9 - D}{8}} \Xi \rightarrow \Xi \), we get the standard normalization for the action (6.8). We thus arrive at the minkowskian version of the principal results of refs. [8, 16]. The well-known \( c = 1 \) barrier is also observed through the definition of the cosmological term. In other respects our arguments seem to be insensitive in going beyond the barrier.

### 6.2 Supersymmetric light-cone gauge

Light-cone gauge is defined by the conditions
\[ e^+ = e^- = 1 , \quad e^+ = 0 , \quad \chi^- = \chi^- = 0 . \]  
(6.10)

In this gauge our parametrizations (2.2), (2.4) and (5.34) are reduced to
\[ \lambda^+ = 1 , \quad \nu^- = 0 , \quad \Lambda_+ = 4 \chi_+ , \]
\[ \lambda^- = \frac{1 - g_{++}}{1 + g_{++}} , \quad \nu_+ = \frac{2 \chi_{++}}{(1 + g_{++})^2} , \quad \Lambda_- = \frac{4 \chi_{++}}{\sqrt{1 + g_{++}}} , \]
\[ \xi = \ln(1 + g_{++}) , \quad l = -\frac{1}{2} \ln(1 + g_{++}) , \quad a = A , \]  
(6.11)

where \( e^+ = -g_{++} \) and \( \chi_{++} \) are the unfixed gravitational dynamical degrees.

To write down the gauge-fixed action is again a standard routine. In ref. [23] a slightly different gauge conditions, which are not manifestly invariant under the rigid Lorentz rotations on the two dimensional parameter space, are employed. Here we will adopt the Lorentz invariant gauge conditions (6.10) to make it manifest the rigid Lorentz covariance of the ghost and gauge-fixing parts of the action. We describe the handling of the ghost sector in Appendix C in some detail. We also show that all the ghost variables can be made free fields by suitable field redefinitions. As a sequel, the gauge-fixed action is found to be

\[ S_{\text{eff}} = \int d^2x \left[ \frac{1}{2} \partial_+ X (\partial_+ X + g_{++} \partial_- X) + \frac{i}{2} \psi_+ \partial_- \psi_+ \right] . \]
\[ + \frac{i}{2} \psi_+ (\partial_+ \psi_- + g_{++} \partial_- \psi_-) - 2i \chi_{++} \psi_- \partial_- X \]
\[ + \frac{\kappa}{2} \int d^2 x \left[ \frac{1}{2(1 + g_{++})} \left( (\partial_- g_{++})^2 - 2 \partial_- g_{++} (\ln(1 + g_{++}))' + 4 (\ln(1 + g_{++}))'' \right) \right. \]
\[ + \frac{8i}{1 + g_{++}} \chi_{++} \left( \partial_- \chi_{++} - \frac{2 \chi_{++}'}{1 + g_{++}} \right) + \frac{i}{2} \chi_{+} \partial_- \chi_{+} \]
\[ + \int d^2 x (-b_+ \partial_- c_+ - b \partial_- c_+ + \beta_+ \partial_- \gamma_+ + \beta_+ \partial_- \gamma_+) \]

where the \((-e_1^-)^{1/2} \psi_-\) given in (2.3) has been redefined as \(\psi_-\) and all the nonpropagating fields have been integrated out via the equations of motion. We have also set the cosmological mass \(\mu\) to zero for simplicity.

The gravitational part of the effective action can be endowed with an interpretation only in terms of \(f_-(x)\) and \(\eta_+(x)\) introduced in Section 3. As can be seen from (3.21) and (6.11), the \(f_+\) and \(\eta_-\) come to be decoupled from the gravitational variables in this gauge, and hence can be ignored from the beginning. The super-Liouville equations for \(\Phi\) and \(\Psi_\pm\) described by (5.4) can be solved in terms of \(f_-\) and \(\eta_+\) as

\[ \Phi = - \ln \partial_- f_- + i \xi_+ \partial_- \xi_+ , \quad \Psi_- = - \frac{2}{\sqrt{\partial_- f_-}} \partial_- (\sqrt{\partial_- f_-} \xi_+) \] \(\Psi_+ = 4 \chi_{+-}\),

with \(\xi_+ \equiv \sqrt{-e_1^-} \eta_+\). Inserting these solutions into (5.4), we obtain supersymmetric extension of the gravitational WZW action for the bosonic string case given in ref. [5].

It is interesting to note that the resulting super-Liouville action coincides, up to over all constant and \(\chi_{+-}\) kinetic term, with the one obtained from \(S_+\) by the replacements \(\partial_\tau, \partial_, f_+, \eta_- \rightarrow \frac{1}{2} \partial_\tau, \frac{1}{2} \partial_, f_-, \xi_+\). To see this explicitly, let us introduce light-cone superspace with supercoordinates \(z = (x^+, x^-, \theta_+\) and supercovariant derivative \(D_- = -i \frac{\partial}{\partial \theta_+} \theta_+ \partial_-\). The graviton superfield is defined by \(G_{++}(z^+) = g_{++}(x) - 4i \theta_+ \chi_{++}(x)\). In terms of the superfields \(\mathcal{F}_-(z)\) and \(\Xi_+(z)\) given by

\[ \mathcal{F}_-(z) = f_-(x) - i \theta_+ \xi_+(x) \partial_- f_-(x) \] \[ \Xi_+(z) = \sqrt{\partial_- f_-(x)} \{ \xi_+(x) + \theta_+(1 - \frac{i}{2} \xi_+(x) \partial_- \xi_+(x)) \} \]

**Though the cosmological term reduces to a constant in this gauge, it contributes to the equations of motion, hence to the BRST charge.**
the $G_{++}$ can be written as
\[ G_{++} = \frac{\partial_+ F_- - i\Xi_- \partial_+ \Xi_-}{(D_- \Xi_-)^2}. \] (6.15)

The super-Liouville action (5.4) is then given by
\[ -\kappa S_L = i\kappa \int d^2x d\theta_+ \left( \frac{\partial_+ F_- - i\Xi_- \partial_+ \Xi_-}{(D_- \Xi_-)^2} D_- \Xi_- - i\partial_+ D_- \Xi_- D^2 \Xi_- \right) + \cdots, \] (6.16)

where we have suppressed the $\chi_{+-}$ kinetic term. This result has already been noted in ref. [13, 29]. The general argument presented in the previous section shows that the nonlocalities of (6.16) cancel those of $S_V$ and the local terms left over by the cancellation constitute the gravitational action in (6.12). The light-cone gauge is retained under the pseudo-superconformal transformations $z = (x^+, x^-, \theta_+)$ $\rightarrow$ $z' = (x^+, g_+(z), \vartheta_+(z))$ satisfying $D_- g_+ = -2i\vartheta_+ D_- \vartheta_+$ if the $G_{++}$ is subject to the relation
\[ G'_{++}(z') = (iD_- \vartheta_+(z))^2 G_{++}(z) + \frac{1}{2} \{ \partial_+ g_+(z) - 2i\vartheta_+(z) \partial_+ \vartheta_+(z) \}. \] (6.17)

The super-Liouville action (6.16), however, is not symmetric under the general transformations due to the super-Weyl anomaly but it is invariant for the ones satisfying $D_- \vartheta_+ = 0$ as explained in Section 4 for $S_V^+$. Thus (6.16) is invariant up to surface term under the super-Möbius transformations with the coefficients being arbitrary function of $x^+$. This leads to the OSp(1,2) Kac-Moody (KM) symmetry, which can not be seen clearly in the local effective action (6.12). We can, however, extract the symmetry with the guideline of BRST invariance, to which we are to turn now.

In Appendix C we give the BRST transformations (C.8) using the free ghost varaibles. Their consistency with the BRST invariance which are supposed to recover at the quantum level leads to
\[ \kappa \partial_- g_{++} = 0, \quad \kappa \partial_- \chi_{++} = 0, \quad \kappa \partial_- \chi_+ = 0, \] (6.18)
from which the conservations of the gravitational stress tensor $T^a_{++}$ also follows. If we define conserved currents $J^a(x^+)$ and $\Psi^r(x^+)$ for $a = 0, \pm$, $r = -1/2, 1/2$ by
\[ g_{++} = -\frac{1}{2\kappa} [J^+(x^+) - 2x^- J^0(x^+) + (x^-)^2 J^-(x^+)], \]
\[ \chi_{++} = -\frac{1}{2\kappa}[\Psi^{-1/2}(x^+) + x^-\Psi^{1/2}(x^+)], \]  
(6.19)

then they can be shown from their BRST transformation to satisfy OSp(1,2) KM algebra under the super-Poisson brackets. In our canonical approach the OSp(1,2) KM symmetry arises in this way. At first sight, our derivation might appear rather indirect and irrelevant to the invariance of the action. The reason for this is due to the fact that this symmetry is not present in (6.12) at the classical level in contrast with (6.16). The BRST charge, however, can be regarded as the generator of the OSp(1,2) KM symmetry as noted in Appendix C, and the dynamics described by the action has been reflected to it through the equations of motion for such as the multiplier fields.

Quantization can be achieved by replacing the super-Poisson brackets with the supercommutators times \(-i\) for the fundamental brackets among the ghost variables and for the OSp(1,2) KM algebra. Free field operator ordering can be applied to the ghost variables to define operator products. As for the OSp(1,2) KM currents, we decompose them into lowering and raising operators by the positive and negative frequency parts, by which we define operator ordering. The quantum mechanical stress tensor \(T^g\) for gravity sector given in (C.9) can be written in Sugawara form

\[ T^g_{++} = -\frac{1}{\kappa'} : (\eta^a_{\bar{b}} J^a_{\bar{b}} + i\eta_{rs} \Psi^r \Psi^s) : -\frac{1}{2} \partial_+ J^0 + \frac{i\kappa'}{8} : \chi_+ \partial_+ \chi_+ : , \]  
(6.20)

where \(\eta^a_{\bar{b}}\) and \(\eta_{rs}\) are the inverse of the OSp(1,2) Killing metric. The parameter \(\kappa'\) is given by

\[ \kappa' = \kappa - \frac{3}{8\pi} . \]  
(6.21)

According to this, we have made a rescaling \(\sqrt{\kappa'}\chi_+ \rightarrow \sqrt{\kappa}\chi_+\) in (6.20). The stress tensor thus defined satisfies the Virasoro algebra with central charge

\[ c_g \equiv \frac{2k}{2k-3} + 6k + \frac{1}{2} , \]  
(6.22)

where \(k \equiv 4\pi\kappa\) is the central charge of the OSp(1,2) KM current algebra. The second term in the rhs of (6.22) is present already at the classical level. The first term denotes
the quantum Virasoro anomaly due to OSp(1,2) KM currents and the last one is due to $\chi_+$. The quantum theoretical BRST charge can be inferred from the classical expression (C.10), and is given by

$$Q_B = \int d\sigma \left[ c^+ \left( T_{++}^X + T_{++}^g + \frac{1}{2} T_{++}^{gh(2)} + T_{++}^{gh(0)} + T_{++}^{gh(3/2)} + T_{++}^{gh(1/2)} \right) ight. \\
+ c_+ \left( -J^- - ib\gamma_- \chi_+ + \frac{1}{8\kappa'} (\beta_+ \gamma_-)^2 \right) \\
+ \gamma_- \left( -i J_{++}^X - i\chi_+ J^0 + i \left( \kappa' + \frac{1}{8\pi} \right) \partial_+ \chi_+ + 2ib_+ \gamma_- - \frac{i}{\kappa'} \gamma_- (bJ^+ - \beta_+ \Psi^{-1/2}) \right) \\
+ \gamma_+ \left( -4i\Psi^{1/2} + 2ib\gamma_+ - \frac{i}{2} \beta_+ \gamma_- \chi_+ \right) \right]. \quad (6.23)$$

This satisfies the nilpotency condition if and only if the central charge of the total stress tensors vanishes. We thus arrive at the KPZ condition [6, 14, 17] for $N=1$ NSR superstring

$$c_{\text{tot}} \equiv \frac{2k}{2k-3} + 6k + \frac{1}{2} + \frac{3}{2}D - 18 = 0,$$

where the last two terms are the central charge of the string and the ghost sectors, respectively.

### 7 Summary and discussion

In this paper we have investigated the $N=1$ RNS superstring at subcritical dimensions as 2D supergravity coupled to superconformal matter with special emphasis on the role played by the super-Virasoro and the super-Weyl anomaly. Naive action defined by free field ordering is not invariant under reparametrizations and local supersymmetry due to super-Virasoro anomaly but is under local Weyl rescalings and fermionic transformations. We have shown that the super-Virasoro anomaly can be canceled by introducing an additional term to the string action, which was also invariant under the local Weyl and fermionic transformations by construction. It was, however, nonlocal in the graviton and gravitino variables. The covariant nonlocal super-Liouville action with appropriate coefficients just has been shown to cancel the nonlocality and to recover the super-Weyl
anomaly as expected from the perturbative analysis. This has lead to local but noncovariant super-Liouville action for the 2D supergravity, which can be interpreted as WZW term converting the super-Virasoro anomaly to the super-Weyl anomaly.

We have also studied the quantization of 2D supergravity based on the super-Liouville action within the framework of BRST formalism and examined the BRST invariance in the superconformal gauge and in the light-cone gauge. In refs. [7,8,16] the key observation leading to the (super-)Liouville action is that the resulting effective theory should be (super)conformally invariant. Since the (super)conformal invariance is equivalent to the BRST invariance in (super)conformal gauge, our approach naturally reproduces the basic results of refs. [7,8,16] as argued in refs. [22,23]. This can also be regarded as a canonical verification of the the functional measure ansatz of refs. [7,8,16]. We have also noted the subtlety concerning the cosmological term, which is not seen for the bosonic case [2]. It merely implies an artifact coming from the operator definition and could be resolved in principle in (super)conformally invariant formulation.

In the case of light-cone gauge the residual OSp(1,2) KM symmetry can be extracted in a rather different way compared with the approaches of refs. [13,14,15], where the effective action obtained by integrating out the string variables possesses manifestly the symmetry. It is, however, hidden in our effective action and is extracted as a result of BRST invariance. One advantageous point of our approach is that we can investigate local properties of the theory instead of referring to the nonlocal effective action.

We must reformulate our analysis for strings of finite length before the detailed comparison with the involved BRST analysis [17,18] and the computations of physical quantities such as mass spectrum and string amplitudes, which are beyond the scope of the present paper.

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A BRST transformations

In this appendix we summarize the BRST transformations in covariant notation. The complete BRST transformations are also given in the appendix of ref.\cite{23} with slightly different notation, where the scalar super-Liouville and super-Weyl ghost multiplets are included. In the present approach, these additional fields can be simply ignored since we do not impose the super-Weyl invariance at the quantum level.

Let us introduce the reparametrization ghosts $C^\alpha (\alpha = 0, 1)$, the local Lorentz ghost $C_L$ and the spinor ghost $\omega$ for the local supersymmetry. Then the BRST transformations are given by

\begin{align*}
\delta X &= C^\alpha \partial_\alpha X + \overline{\omega} \psi , \\
\delta \psi &= -\frac{1}{4} C_W \psi + C^\alpha \partial_\alpha \psi + \frac{1}{2} C_L \rho_5 \psi - i \rho^\alpha \omega (\partial_\alpha X - \overline{\chi}_\alpha \psi) + \omega F_X , \\
\delta F_X &= -\frac{1}{2} C_W F_X + C^\alpha \partial_\alpha F_X - i \overline{\omega} \rho^\alpha [\nabla_\alpha \psi + i \rho^\beta (\partial_\beta X - \overline{\chi}_\beta \psi) \chi_\alpha - \chi_\alpha F_X] , \\
\delta e_\alpha^a &= C^\beta \partial_\beta e_\alpha^a + \partial_\alpha C^\beta e_\beta^a + \epsilon^a_b C_L e_\alpha^b - 2 \overline{\omega} \rho^a \chi_\alpha , \\
\delta \chi_\alpha &= C^\beta \partial_\beta \chi_\alpha + \partial_\alpha C^\beta \chi_\beta + \frac{1}{2} C_L \rho_5 \chi_\alpha + \nabla_\alpha \omega - \frac{i}{4} \rho_5 \omega A , \\
\delta A &= C^\alpha \partial_\alpha A - 4 \epsilon_\alpha^\beta \overline{\omega} \rho_5 \nabla_\alpha \chi_\beta + i \overline{\omega} \rho^\alpha \chi_\alpha A , \\
\delta C^\alpha &= C^\beta \partial_\beta C^\alpha + i \overline{\omega} \rho^\alpha \omega , \\
\delta \omega &= C^\alpha \partial_\alpha \omega + \frac{1}{2} C_L \rho_5 \omega - i \overline{\omega} \rho^\alpha \omega \chi_\alpha , \\
\delta C_L &= C^\alpha \partial_\alpha C_L + \frac{1}{2} A \overline{\omega} \rho_5 \omega - i \epsilon_{ab} e^{\beta^a} \overline{\omega} \rho^\alpha (\partial_\alpha e^{b}_\beta - \partial_\beta e^{a}_\alpha - i \chi_\alpha \rho^b \chi_\beta) , \tag{A.1}
\end{align*}

where $F_X^\mu (\mu = 0, 1, \cdots, D - 1)$ and $A$ are, respectively, the auxiliary fields for the string and zweibein supermultiplets introduced to ensure the off-shell nilpotency of the BRST transformations. We should supplement the transformations for the anti-ghosts and multiplier fields, which have been omitted here.
B Derivation of super-Virasoro algebra

In this appendix we describe operator ordering of the super-Virasoro generator for the super-Liouville field and their commutation relations in the presence of the cosmological term.

We first note that the gravitational super-Virasoro generators can be defined in the manner similar to (2.11) as

\[ \Phi_\pm = -\frac{\delta S_g}{\delta \lambda^\pm} , \quad \mathcal{I}_\pm = \mp i \frac{\delta S_g}{\delta \nu_\mp} . \]  

(B.1)

When expressed in terms of the canonical variables, \( \xi \) and its conjugate momentum, for instance, these can be seen to satisfy super-Virasoro algebra with central charge \( 24\pi\kappa \) under the super-Poisson brackets. This situation is not changed when the contribution from the cosmological term are included.

For notational simplicity, we use here the rescaled variables \( \varphi = \gamma \xi \), \( \chi^\pm = \gamma \Lambda \pm \) and \( f = \gamma a \) with \( \gamma \equiv \sqrt{\frac{\kappa}{2}} \) for the action (6.8). We make also an appropriate translation of \( \varphi \) by a constant to keep fixed the cosmological constant \( \mu \). Then the super-Virasoro generators for the gravitational sector including the contributions from the cosmological term can be written as

\[ \Phi_\pm = \frac{1}{4} \Theta^2_\pm - \gamma \Theta'_\pm \pm \frac{i}{2} \chi_\pm \chi'_\pm + \mu^2 e^{2\beta\varphi} + i\mu\beta e^{\beta\varphi} \chi_+ \chi_- , \]

\[ \mathcal{I}_\pm = \pm \chi_\pm \Theta_\pm \mp 4\gamma \chi'_\pm \mp 2\mu e^{\beta\varphi} \chi^\pm , \]  

(B.2)

where \( \beta \equiv \frac{\alpha}{\gamma} \) and \( \Theta_\pm \equiv \varphi' \pm \pi_\varphi \) with \( \pi_\varphi = \dot{\varphi} \) being the canonical momentum conjugate to \( \varphi \). We have inserted a parameter \( \alpha \) to take quantum corrections into account. For the classical value \( \beta \gamma = \frac{1}{2} \), they coincide with \( T^L_{\pm\pm} \) and \( J^L_{\pm\pm} \) given in (3.8).

In quantum theory, we assume equal-time supercommutation relations

\[ [\varphi(\sigma), \pi_\varphi(\sigma')] = i\delta(\sigma - \sigma'), \quad [\chi_\pm(\sigma), \chi_\pm(\sigma')] = \delta(\sigma - \sigma') . \]  

(B.3)

To define operator ordering, we introduce harmonic oscillators \( a_k \) and \( b_k \) by

\[ \varphi(\sigma) = \frac{i}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{dk}{k} (a_k e^{-ik\sigma} + b_k e^{ik\sigma}) , \quad \pi_\varphi(\sigma) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{dk}{k} (a_k e^{-ik\sigma} + b_k e^{ik\sigma}) . \]  

(B.4)
We do not care for the infrared singularity due to the infinite region for $\sigma$ since most of
our arguments are irrelevant to this problem and it can be avoided by considering finite
interval for $\sigma$. These oscillators satisfy

$$[a_k, a_{k'}] = [b_k, b_{k'}] = k \delta(k - k') , \quad [a_k, b_{k'}] = 0 \ . \quad (B.5)$$

We then define $a_k$ and $b_k$ as lowering operators and $a_k^\dagger = a_{-k}$ and $b_k^\dagger = b_{-k}$ as rasing
operators for $k > 0$. We use similar decomposition for $\chi_{\pm}$. The lowering and rasing parts
of $\Theta_{\pm}$ and $\chi_{\pm}$ can be written in the manner similar to (2.22) as

$$\Theta_{+}^{(\pm)}(\sigma) = \int d\sigma' \delta^{(\mp)}(\sigma - \sigma') \Theta_{+}(\sigma') , \quad \Theta_{-}^{(\pm)}(\sigma) = \int d\sigma' \delta^{(\mp)}(\sigma - \sigma') \Theta_{-}(\sigma') ,$$

$$\chi_{+}^{(\pm)}(\sigma) = \int d\sigma' \delta^{(\mp)}(\sigma - \sigma') \chi_{+}(\sigma') , \quad \chi_{-}^{(\pm)}(\sigma) = \int d\sigma' \delta^{(\mp)}(\sigma - \sigma') \chi_{-}(\sigma') \quad (B.6)$$

This define an operator ordering.

Let us denote by $\Phi_{\pm}^{(0)}$ and $I_{\pm}^{(0)}$ the super-Virasoro operators for $\mu = 0$ defined by the
normal ordering, then they satisfy super-Virasoro algebra with central charge $\frac{3}{2} + 24\pi\kappa$.
The deviation from the classical value is due to quantum fluctuations of the super-Liouville
fields themselves.

We next define vertex-like operators by

$$V_{\beta} =: \exp \beta \phi : , \quad V_{\beta\pm} =: \exp \beta \phi \chi_{\pm} : , \quad V_{\beta+-} =: \exp \beta \phi \chi_{+}\chi_{-} : , \quad (B.7)$$

and examine the commutation relations with $\Phi_{\pm}^{(0)}$ and $I_{\pm}^{(0)}$. We find

$$[\Phi_{\pm}^{(0)}(\sigma), V_{\beta}(\sigma')] = \mp \frac{i}{2} \beta : \Theta_{\pm} V_{\beta} : \delta(\sigma - \sigma') \pm i\left(\beta \gamma - \frac{\beta^2}{8\pi}\right) \partial_{\sigma}(V_{\beta} \delta(\sigma - \sigma')) ,$$

$$[\Phi_{\pm}^{(0)}(\sigma), V_{\beta\mp}(\sigma')] = \mp \frac{i}{2} \beta : \Theta_{\mp} V_{\beta\pm} : \delta(\sigma - \sigma') \pm i\left(\beta \gamma - \frac{\beta^2}{8\pi}\right) \partial_{\sigma}(V_{\beta\mp} \delta(\sigma - \sigma')) ,$$

$$[\Phi_{\pm}^{(0)}(\sigma), V_{\beta\pm}(\sigma')] = \mp i \left(\frac{\beta}{2} \Theta_{\pm} V_{\beta\pm} + V_{\beta} \partial_{\sigma} \chi_{\pm} \right) : \delta(\sigma - \sigma') \pm i\left(\beta \gamma - \frac{\beta^2}{8\pi} + \frac{1}{2}\right) \partial_{\sigma}(V_{\beta\pm} \delta(\sigma - \sigma')) ,$$

$$[\Phi_{\pm}^{(0)}(\sigma), V_{\beta+-}(\sigma')] = \mp i \left(\frac{\beta}{2} \Theta_{\pm} V_{\beta+-} + V_{\beta} \partial_{\sigma} \chi_{+}\chi_{-} \right) : \delta(\sigma - \sigma') \pm i\left(\beta \gamma - \frac{\beta^2}{8\pi} + \frac{1}{2}\right) \partial_{\sigma}(V_{\beta+-} \delta(\sigma - \sigma')) ,$$
\[ [I_{\pm}^{(0)}(\sigma), V_\beta(\sigma')] = -i\beta V_{\beta \pm} \delta(\sigma - \sigma') , \]
\[ [I_{\pm}^{(0)}(\sigma), V_{\beta \mp}(\sigma')] = \mp i\beta V_{\beta \pm} \delta(\sigma - \sigma') , \]
\[ [I_{\pm}^{(0)}(\sigma), V_{\beta \pm}(\sigma')] = \pm : \Theta_{\pm} V_\beta : \delta(\sigma - \sigma') \mp \left(4\gamma - \frac{\beta}{2\pi} \right) \partial_\sigma (V_{\beta \mp} \delta(\sigma - \sigma')) , \]
\[ [I_{\pm}^{(0)}(\sigma), V_{\beta \mp}(\sigma')] = : \Theta_{\pm} V_{\beta \mp} : \delta(\sigma - \sigma') - \left(4\gamma - \frac{\beta}{2\pi} \right) \partial_\sigma (V_{\beta \mp} \delta(\sigma - \sigma')) . \] (B.8)

The operators (B.7) themselves satisfy
\[ [V_\beta(\sigma), V_\beta(\sigma')] = [V_\beta(\sigma), V_{\beta \pm}(\sigma')] = [V_\beta(\sigma), V_{\beta \mp}(\sigma')] \]
\[ = [V_{\beta \pm}(\sigma), V_{\beta \mp}(\sigma')] = [V_{\beta \mp}(\sigma), V_{\beta \pm}(\sigma')] = 0 , \] (B.9)

where we have used the relation
\[ V_\beta(\sigma)V_\beta(\sigma') =: V_\beta(\sigma)V_\beta(\sigma') = \exp \left[ -i \sigma \sum_{n=1}^{\infty} \frac{1}{n} \cos 2\pi n \sigma' \right] , \]
\[ [: \chi_+ \chi_-(\sigma) : = : \chi_+ \chi_-(\sigma') : ] = 0 , \] (B.10)

with \( L \) being the length of the interval for \( \sigma \) to avoid infrared divergence. The commutators among \( V_{\beta \pm} \) and \( V_{\beta \mp} \) are problematic since they contain a divergent quantity \( V_\beta^2 \), which can not be removed by the simple ordering prescription. This situation also occurs even in free theories. We do not pursue here the handling of the divergence of this type. Instead, we simply assume the existence of suitable way of defining operators like \( V_\beta^2 \) consistently with the superconformal transformations and content ourselves to write the commutators as
\[ [V_{\beta \pm}(\sigma), V_{\beta \pm}(\sigma')] = V_\beta^2 \delta(\sigma - \sigma') \quad [V_{\beta \mp}(\sigma), V_{\beta \mp}(\sigma')] = \pm V_\beta^2 \chi_\pm \delta(\sigma - \sigma') . \] (B.11)

We now define quantum super-Virasoro generators by
\[ \Phi_\pm = \Phi_0^{(0)} + \mu V_\beta^2 + i\mu \beta V_{\beta \mp} , \quad I_\pm = I_0^{(0)} \mp 2\mu V_{\beta \mp} \] (B.12)

Using (B.8) and (B.9), one can show the following commutation relations
\[ [\Phi_\pm(\sigma), \Phi_\pm(\sigma')] = \pm i(\Phi_\pm(\sigma) + \Phi_\pm(\sigma')) \partial_\sigma \delta(\sigma - \sigma') \mp i \left(2\gamma^2 + \frac{1}{16\pi} \right) \partial_\sigma^2 \delta(\sigma - \sigma') \]
\[ \pm 2i \left( \beta \gamma - \frac{\beta^2}{8\pi} - \frac{1}{2} \right) (\mu^2 V^2_\beta(\sigma) + i\mu \beta V_{\beta_-}(\sigma) + (\sigma \to \sigma')) \partial_\sigma \delta(\sigma - \sigma') , \]

\[
[\Phi_\pm(\sigma), I_\pm(\sigma')] = \pm \frac{3}{2} i I_\pm(\sigma) \partial_\sigma \delta(\sigma - \sigma') \pm \frac{i}{2} \partial_\sigma I_\pm(\sigma - \sigma') \\
-4i\mu \left( \beta \gamma - \frac{\beta^2}{8\pi} - \frac{1}{2} \right) \left( \frac{3}{2} V_{\beta_+}(\sigma) \partial_\sigma \delta(\sigma - \sigma') + \frac{1}{2} \partial_\sigma V_{\beta_+}(\sigma - \sigma') \right) ,
\]

\[
[I_\pm(\sigma), I_\pm(\sigma')] = 4\Phi_\pm(\sigma - \sigma') - 8 \left( 2\gamma^2 + \frac{1}{16\pi} \right) \partial^2_\sigma \delta(\sigma - \sigma') ,
\]

\[
[\Phi_\pm(\sigma), \Phi_-(\sigma')] = 2i \left( \beta \gamma - \frac{\beta^2}{8\pi} - \frac{1}{2} \right) \partial_\sigma (\mu^2 V^2_\beta + i\mu \beta V_{\beta_-} \delta(\sigma - \sigma') ,
\]

\[
[\Phi_\pm(\sigma), I_\pm(\sigma')] = -2i\mu \left( \beta \gamma - \frac{\beta^2}{8\pi} - \frac{1}{2} \right) \left( V_{\beta_+}(\sigma) \partial_\sigma \delta(\sigma - \sigma') - \partial_\sigma V_{\beta_+}(\sigma - \sigma') \right) ,
\]

\[
[I_+(\sigma), I_-(\sigma')] = -\frac{8i\mu}{\beta} \left( \beta \gamma - \frac{\beta^2}{8\pi} - \frac{1}{2} \right) \partial_\sigma V_\beta \delta(\sigma - \sigma') .
\]

We see that the contribution from the cosmological term does not shift the central charge and (B.12) fulfills super-Virasoro algebra for \( \beta \) satisfying

\[ \beta \gamma - \frac{\beta^2}{8\pi} = \frac{1}{2} . \]

This combined with (6.7) gives (6.9).

\[ \text{C Light-cone gauge-fixing} \]

We shall argue the light-cone gauge-fixing in some detail. The ghost and gauge-fixing part of the action is given by

\[
S_{gh} = \int d^2x \left( -C_++ \delta e_+ - \overline{C}_+ \delta e_+ - \overline{C}_- \delta e_- + \overline{\beta}_+ \delta \chi_- + \overline{\beta}_- \delta \chi_+ \right) ,
\]

\[
S_{GF} = \int d^2x \left[ -B_- (e_+ - 1) - B_+ e_- + B_-(e_- - 1) \right.
\]

\[ -\zeta_- \chi_- - \zeta_+ \chi_+ \right] ,
\]

(C.1)

where \( C_{aa}, \overline{\beta}_{++} \) are the anti-ghosts and \( B_{aa}, \zeta_{+} \) are the multiplier fields for the gauge conditions (6.10). They satisfy the BRST transformations

\[
\delta C_{aa} = -B_{aa} , \quad \delta \overline{\beta}_{++} = -\zeta_+ , \quad \delta B_{aa} = 0 , \quad \delta \zeta_+ = 0 .
\]

(C.2)
The BRST transformations of the zweibeins and gravitinos are given in (A.1). To find the BRST transformations of the anti-ghost variables, we need the multiplier fields, which can be found by taking variations of the total action with respect to the variables fixed by the gauge conditions and then imposing the light-cone gauge. For instance, the $\zeta_{+-}$ and $B_{+-}$ can be obtained as follows;

$$\zeta_{+-} = \frac{2}{\sqrt{-e_1^-}} \frac{\delta S_{GF}}{\delta \nu_-} \bigg|_{\text{l.c.g.}} - \frac{2}{\sqrt{-e_1^-}} \frac{\delta}{\delta \nu_-} (S_X + S_g + S_{\text{cosm}} + S_{\text{gh}}) \bigg|_{\text{l.c.g.}} = \frac{2i}{\sqrt{-e_1^-}} \left( J_- + I_- + i \frac{\delta S_{\text{gh}}}{\delta \nu_-} \right) \bigg|_{\text{l.c.g.}} = 8i \kappa \partial_- \chi_{++} + 4i \bar{C}^- \omega_- + \frac{1}{2} \partial_+ \bar{\beta}_+ C^+ - \frac{1}{4} \bar{\beta}_- \partial_+ C^+ + 2i \bar{\beta}_- \omega_- \chi_- ;$$

$$B_{+-} = \frac{2}{e_1^-} \left( \frac{\delta S_{GF}}{\delta \lambda_-} - \frac{1}{8} \sqrt{-e_1^-} \zeta_{+-} \Lambda_- \right) \bigg|_{\text{l.c.g.}} = 2e_1^- \left( -\delta \frac{\delta}{\delta \lambda_-} (S_X + S_g + S_{\text{cosm}} + S_{\text{gh}}) + \frac{i}{4} \Lambda_- \left( J_- + I_- + i \frac{\delta S_{\text{gh}}}{\delta \nu_-} \right) \right) \bigg|_{\text{l.c.g.}} = \frac{2}{e_1^-} \left( \varphi_- + \Phi_- + i \Lambda_- (J_- + I_-) - \frac{\kappa}{8} g_{11} A^2 - \frac{\delta S_{\text{gh}}}{\delta \lambda_-} + \frac{i}{4} \Lambda_- \frac{\delta S_{\text{gh}}}{\delta \nu_-} \right) \bigg|_{\text{l.c.g.}} = -\frac{\kappa}{2} \partial_- g_{++} + \frac{1}{2} \partial_+ \bar{C}^- C^+ + 2i \bar{C}^- \omega_- \chi_+ - \frac{1}{4} \bar{\beta}_- \omega_- A + \frac{\kappa}{4} A^2 ,$$

where $|_{\text{l.c.g.}}$ denotes restriction to the light-cone gauge and $\Phi_-, I_-$ are defined in (B.1).

In deriving these results, use has been made of the equaitons of motion for $g_{++}$ and $\chi_{++}$, i.e.,

$$\frac{\kappa}{4} \partial_- g_{++} = \frac{1}{g_{11}} \left( \varphi_- + \Phi_- \bigg|_{\text{l.c.g.}} + 4 \kappa \chi_{++} \partial_- \chi_{++} \right) ;$$

$$4 \kappa \partial_- \chi_{++} = \frac{1}{\sqrt{g_{11}}} \left( J_- + I_- \bigg|_{\text{l.c.g.}} \right).$$

The rest of the multipliers can be found in the same way.

We now turn to the ghost action in (C.1). In the light-cone gauge this reduces to

$$S_{\text{gh}} = \int d^2 x \left[ -\frac{1}{2} (\bar{C}^- g_{++} \bar{C}^+ + \bar{\beta}_+ \chi_{++} + \bar{\beta}_- \chi_- ) \partial_- C^+ \right]$$
\[-\frac{1}{2}C_+^-(\partial_+ C^+ + \partial_- C^- - 8i\omega_- \chi_+) + \frac{1}{2}C_-^-(\partial_\omega + \frac{1}{2}\omega_+ A) + \frac{1}{2}C_-^+(\partial_\omega - \frac{1}{2}\omega_- A), \quad (C.5)\]

where $C_+^-$ and $C_L$ have been eliminated by virtue of their equations of motion. From (C.5) we see that the ghost variables satisfy
\[\partial_- C^+ = 0, \quad \partial^2 C^- = 0, \quad \partial_\omega_+ = 0, \quad \partial^2 \omega_+ = 0 \quad (C.6)\]
in accord with the residual OSp(1,2) KM symmetry of the effective action. In deriving these use has been made of the equations of motion for $\chi_+^-$ and $A$. Then the BRST charge can be regarded as the infinitesimal generator of this symmetry.

Eqs. (C.6) suggest that all the ghost variables can be converted to free fields. This is implemented by the field redefinitions
\[C^+ = c^+, \quad C^- = c_+ - \frac{x^-}{2} \partial_+ c^+ + ix^- \gamma_- \chi_+ + \frac{i}{\kappa} (x^-)^2 b \gamma_+^2, \quad \omega_+ = \gamma_+ + \frac{x^-}{4\kappa} \beta_+ \gamma_+^2, \quad \omega_- = \gamma_-, \quad \overline{C}_{++} = 2 \left\{ b_{++} - \frac{x^-}{2} \partial_+ b + g_{++} b - \beta_+ \chi_{++} - \frac{1}{4} \beta_+ \chi_+ + \frac{x^-}{2\kappa} \beta_+ \gamma_+ + \frac{x^-}{16\kappa} \beta_+ \gamma_- \chi_+ + \frac{(x^-)^2}{8\kappa^2} b \beta_+ \gamma_- \right\}, \quad \overline{C}_{+-} = 2b, \quad \zeta_{++} = 2 \left\{ \beta_{++} + i x^- \beta_+ \chi_+ - \frac{x^-}{4\kappa} \beta_+ \gamma_- \right\}, \quad \zeta_{+-} = 2 \beta_+, \quad \chi_{+-} = \frac{1}{4} \chi_+ + \frac{x^-}{2\kappa} b \gamma_- \quad (C.7)\]

We thus arrive at the ghost part of the effective action (6.12).

It is straightforward to rewrite the BRST transformations in terms of these free ghost variables. They are give by
\[\delta g_{++} = \frac{1}{2} c^+ \partial_+ g_{++} + g_{++} \partial_+ c^+ + \frac{1}{2} \left( c_+ - \frac{x^-}{2} \partial_+ c^+ \right) \partial_- g_{++} - \frac{1}{2} \partial_+ \left( c_+ - \frac{x^-}{2} \partial_+ c^+ \right) \]
where the stress tensors and supercurrents are given by

\[
\begin{align}
T_{++}^X & \equiv \varphi_+ + \frac{1}{4}[(\partial_+ X + g_{++}\partial_- X)^2 + i\psi_+\partial_+\psi_+] , \\
J_{++}^X & \equiv J_+ (\partial_+ X + g_{++}\partial_- X) ,
\end{align}
\]
\[
T^g_{++} \equiv \frac{\kappa}{2} \left[ \frac{1}{4}(\partial_- g_{++})^2 - \frac{1}{2}g_{++} \partial_-^2 g_{++} - \frac{1}{2}(\partial_+ - \frac{x^-}{2} \partial_-) \partial_- g_{++} \right] \\
+ 4i\kappa \chi_+ \partial_- \chi_+ + \frac{i\kappa}{8} \chi_+ \partial_+ \chi_+ ,
\]
\[
T^{gh(2)}_{++} \equiv -\frac{1}{2} \partial_+ b_+ c^+ - b_+ \partial_+ c^+ ,
\]
\[
T^{gh(0)}_{++} \equiv \frac{1}{2} \partial_+ b c_+ ,
\]
\[
T^{gh(3/2)}_{++} \equiv \frac{3}{4} \beta_+ \partial_+ \gamma_+ + \frac{1}{4} \partial_+ \beta_+ \gamma_+ ,
\]
\[
T^{gh(1/2)}_{++} \equiv \frac{1}{4} \beta_+ \partial_+ \gamma_+ - \frac{1}{4} \partial_+ \beta_+ \gamma_+ .
\]

(C.9)

In (C.8) the transformations for string variables have been suppressed. Since the ghost variables are free fields, we can unambiguously find the classical BRST charge from the BRST transformations of the ghost variables as

\[
Q_B = \int d\sigma \left[ c^+ \left( T^X_{++} + T^g_{++} + \frac{1}{2} T^{gh(2)}_{++} + T^{gh(0)}_{++} + T^{gh(3/2)}_{++} + T^{gh(1/2)}_{++} \right) \\
+ c_+ \left( \frac{\kappa}{4} \partial_-^2 g_{++} - i b \gamma_- \chi_+ + \frac{1}{8\kappa} (\beta_+ \gamma_-)^2 \right) \\
+ \gamma_- \left( - i J^X_{++} - \frac{i \kappa}{2} \chi_+ \left( 1 - \frac{x^-}{2} \partial_- \right) \partial_- g_{++} + i \kappa \partial_+ \chi_+ + 2i b_+ \gamma_- \right) \\
+ 2i b \gamma_- \left( 1 - \frac{x^-}{2} \partial_- + \frac{(x^-)^2}{8} \partial_-^2 \right) g_{++} - 2i \beta_+ \gamma_- \left( 1 - \frac{x^-}{2} \partial_- \right) \chi_+ \right) \\
+ \gamma_+ \left( 4i \kappa \partial_- \chi_+ + 2i b \gamma_+ - \frac{i}{2} \beta_+ \gamma_- \chi_+ \right) .
\]

(C.10)
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