Formation of a mixture with the specified quality characteristics from the available small batches of multicomponent mixtures, provided that the total batch of the largest volume is created

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Abstract. Statements, algorithms of the decision and the corresponding examples of tasks of forming of new mixes with the set characteristics or closest to the set characteristics from the available parties of multicomponent mixtures are given in work. The specified tasks have arisen in the course of the analysis of problem of drawing up by livestock complexes of the balanced nutritious mixes from the available small consignments of mixes. Solutions of the given tasks can be demanded also in other spheres of production. For example, in metallurgy at production of alloys, pharmacology at production of drugs.

1. Introduction
During production of multicomponent mixtures in different spheres of production there are various questions, connected, first with the analysis of quality of the received mixes [1-5]. Some of these questions can be resolved by means of mathematical modelling.

2. Problem definition
Let's consider the following situation.
Is available n mixes A₁,...,Aₙ, containing in addition to insignificant components k significant components B₁,...,Bₖ.

a₁,...,aₙ – volumes (masses) of mixes A₁,...,Aₙ respectively.

bᵢⱼ – relative maintenance (concentration) of component Bᵢ (i = 1,...,k) in mix Aⱼ, (j = 1,...,n), i.e. relation of the volume (weight) of component Bᵢ to the volume (weight) of mix Aⱼ.

\[
0 \leq bᵢⱼ \leq 1, \sum_{j=1}^{n} bᵢⱼ \leq 1
\]

From volumes x₁,...,xₙ, \((0 \leq x_j \leq a_j)\) mixes A₁,...,Aₙ new mix is formed A with relative contents b₁,...,bₖ components B₁,...,Bₖ, where sizes b₁,...,bₖ are defined from the system of the equations:
\[ \sum_{j=1}^{n} b'_i x_j = b_i \sum_{j=1}^{n} x_j, \]
\[ i = 1..k, \]

(1)

According to this system of the equations:
\[ \min_j \left( b'_i \right) \leq b_i \leq \max_j \left( b'_i \right), \quad i = 1..k, \quad j = 1..n \]

From the geometrical point of view set of sizes \((x_1, \ldots, x_k)\) represents coordinates of point of rectangular parallelepiped \( \Pi_x = [0 \leq x_1 \leq a_1] \times [0 \leq x_2 \leq a_2] \times \cdots \times [0 \leq x_n \leq a_n], \ n \) – measured space with the Cartesian system of coordinates \( Ox_1x_2 \ldots x_n \).

Similarly, \((b_1, \ldots, b_k)\) – point coordinates in \( k \) – measured cube:
\[ \Pi_x = [0 \leq x_1 \leq a_1] \times [0 \leq x_2 \leq a_2] \times \cdots \times [0 \leq x_n \leq a_n]. \]

Let's call this point of concentration. There are following questions.
1. Whether it is possible the choice of volumes \( x_1, \ldots, x_n \) to make mix with preset values \( b_1, \ldots, b_k \) concentration. If yes, that what at the same time mix volumes are possible \( A \)?
2. If the first issue is resolved negatively, then what it is possible to mix with the closest values \( b_1, \ldots, b_k \) to required sizes \( b_1^*, \ldots, b_k^*, \) and what at the same time are possible mix volumes \( A \)?
3. Whether it is possible to achieve the set relation \( b_1 : b_2 : \ldots : b_k = q_1 : q_2 : \ldots : q_n \) concentration of components in mix \( A \). If yes, that what maximum and minimum values of sizes \( b_1, \ldots, b_k \), and what volumes of mix answer them?
4. If the third issue is resolved negatively, then what most close relation \( b_1 : b_2 : \ldots : b_k \) to the set relation \( q_1 : q_2 : \ldots : q_n \) it is possible to achieve what at the same time concentration turn out \( b_1, \ldots, b_k \) and mix volumes \( A \)?

All questions formulated here are the statements of tasks 1-4 connected with search of extremum of the corresponding functions in the limited closed areas.

The private option of task 1 is completely solved in work [6] for values \( n = 3, \ k = 1 \).

3. Algorithm of the task solution

Solution of task 1

Task 1. We bring out of system (1): homogeneous linear system
\[ \sum_{j=1}^{n} \tilde{b}'_i x_j = 0, \quad \tilde{b}'_i = b'_i - b_i, \quad i = 1..k, \]

(2)

must have the nonzero decision with non-negative values of masses \( x_1, \ldots, x_n \). From here we receive necessary condition of the solution of task 1: matrix rank \( \tilde{B} \) systems (2) has to be less \( n \), i.e.
\[ \text{rang}(\tilde{B}) = \text{rang}(\tilde{b}'_i)_{j=1..n} = s < n \]

(3)

Let the requirement (3) was fulfilled. Then only \( s \) the equations of system (2) the others are essential, and \( k - s \) the equations are the investigations specified \( s \) equations.

Without restriction of community of the decision we consider independent the first \( s \) equations of system (2). If it not so, then we rearrange the equations of system (2) in the appropriate order. Having discarded the insignificant equations, we will receive system:
which allows to express $s$ unknown from set $x_1, \ldots, x_n$ through the others $n-s$ unknown. Let, for definiteness, $x_n, x_{n-1}, \ldots, x_{n-s+1}$ are expressed through $x_1, \ldots, x_{n-s}$.

Then the solution of system is representable in look:

$$
\begin{align*}
&x_1 = q_1^1 c_1 \\
&\ldots \\
&x_{n-s} = q_{n-s}^{n-s} c_{n-s} \\
&x_{n-s+1} = q_1^{n-s+1} c_1 + q_2^{n-s+1} c_2 + \ldots + q_{n-s+1}^{n-s+1} c_{n-s+1} \\
&\ldots \\
&x_n = q_1^n c_1 + q_2^n c_2 + \ldots + q_n^n c_{n-s} 
\end{align*}
$$

where $q_i^m$ - numerical coefficients ($l = n-s+1..n; m = 1..n-s$), $c_1, \ldots, c_{n-s}$ — any constants representing point coordinates from the limited closed area $\Omega$, defined owing to restrictions $0 \leq x_1 \leq a_1$, $0 \leq x_2 \leq a_2$, $\ldots$, $0 \leq x_n \leq a_n$, system of inequalities:

$$
\begin{align*}
&0 \leq c_1 \leq a_1 \\
&\ldots \\
&0 \leq c_{n-s} \leq a_{n-s} \\
&0 \leq q_1^l c_1 + q_2^l c_2 + \ldots + q_{n-s+1}^l c_{n-s+1} \leq a_{n-s+1} \\
&\ldots \\
&0 \leq q_1^n c_1 + q_2^n c_2 + \ldots + q_{n-s}^n c_{n-s} \leq a_n
\end{align*}
$$

Mix volume $A$ it is equal:

$$
V = x_1 + x_2 + \ldots + x_n = c_1 + c_2 + \ldots + q_1 c_1 + q_2 c_2 + \ldots + q_{n-s} c_{n-s} = f(c_1, c_2, \ldots, c_{n-s}).
$$

Thus, finding of the greatest value of size $V$ comes down to search of maximum of function $f(c_1, c_2, \ldots, c_{n-s})$ in the area $\Omega$. It is standard problem of the mathematical analysis, solving which we find values

$$
c_1 = c_1^*, \ c_2 = c_2^*, \ \ldots, \ c_{n-s} = c_{n-s}^*
$$

at which it is reached $\max V$.

Substituting the received sizes $c_1, \ldots, c_{n-s}$ in equalities (4) and (6), we will find required volumes $x_1 = x_1^*, \ldots, x_n = x_n^*$ mixes $A_1, \ldots, A_n$, at which the maximum volume of mix turns out $A$.

All possible volumes of mix $A$ with the set point of concentration $(b_1, \ldots, b_k)$ are defined by formula:

$$
V = t(x_1^* + x_2^* + \ldots + x_n^*), \ t \in (0; 1],
$$

also turn out at:
\[ x_i = t \bar{x}_i^*, \ldots, x_n = t \bar{x}_n^*, \ t \in (0; 1] \]

**Example 1.** There are mixes \( A_1, A_2, A_3, A_4 \), the containing two essential components \( B_1, B_2 \). Volumes of mixes and concentration of components are respectively equal in them:

\[ a_1 = 3.6, a_2 = 4.4, a_3 = 3.8, a_4 = 5.2; \]
\[ b_1^1 = 0.5, b_2^1 = 0.1; \ b_1^2 = 0.2, b_2^2 = 0.4; \ b_1^3 = 0.3, b_2^3 = 0.2; \ b_1^4 = 0.4, b_2^4 = 0.3. \]

It is required to make mix of the largest volume with concentration of components \( b_1 = 0.33, b_2 = 0.29 \). The decision we will pass according to the algorithm stated above decisions of **task 1**. The equations (1) in this case will register in look:

\[
\begin{aligned}
0.5x_1 + 0.2x_2 + 0.3x_3 + 0.4x_4 &= 0.33(x_1 + x_2 + x_3 + x_4) \\
0.1x_1 + 0.4x_2 + 0.2x_3 + 0.3x_4 &= 0.29(x_1 + x_2 + x_3 + x_4)
\end{aligned}
\]

The common decision of this system is representable in look:

\[
\begin{aligned}
x_1 &= 2c_1 \\
x_2 &= 2c_2 \\
x_3 &= 3c_2 - 5c_1 \\
x_4 &= 5c_2 - 7c_1,
\end{aligned}
\]

where any constants \( c_1, c_2 \) – are point coordinates from the closed limited area \( D \), determined by inequalities

\[
\begin{aligned}
0 &\leq c_1 \leq 1.8 \\
0 &\leq c_2 \leq 2.2 \\
0 &\leq 3c_2 - 5c_1 \leq 3.8 \\
0 &\leq 5c_2 - 7c_1 \leq 5.2.
\end{aligned}
\]

Mix volume \( V = x_1 + x_2 + x_3 + x_4 = 10(c_2 - c_1) = f(c_1, c_2) \), as show calculations, reaches the greatest value in point \( c_1 = \frac{29}{35}, c_2 = \frac{11}{5} \) in areas \( D \):

\[
\max V = \frac{96}{7} \approx 13.714\ldots
\]

This value of volume is reached when mixing volumes

\[
x_1 \approx \frac{58}{35}, \quad x_2 = \frac{22}{5} = 4.4, \quad x_3 \approx \frac{86}{35} \approx 2.457, \quad x_4 = \frac{26}{5} = 5.2.
\]

**Solution of task 2**

**Task 2.** Now we consider condition (3) outstanding. In this case the system (2) has only the zero decision \( x_1 = x_2 = \ldots = x_n = 0 \), i.e.

\[
\text{rang}(\bar{B}) = n,
\]

what is possible only at \( k \geq n \).
The received condition serves as the answer to question when it is impossible to achieve mix with the set concentration \( b_1, \ldots, b_k \).

Let's find out now what concentration in this case mix can have \( A \).

Let's notice that in all points \( (x_1, \ldots, x_n) \), beam

\[
\begin{aligned}
x_i &= x_i^+ t \\
\ldots \\
x_n &= x_n^+ t
\end{aligned}
\]

concentration \( b_1, \ldots, b_k \) have the same values. It follows from the fact that the system of the equations (1) does not change after replacement \( x_1, \ldots, x_n \) on \( tx_1, \ldots, tx_n \).

Let's consider set \( G_x \) points \( (x_1, \ldots, x_n) \) planes

\[
x_1 + x_2 + \ldots + x_n = 1,
\]

in the area \( x_1 \geq 0, \ldots, x_n \geq 0 \). Any beam (9) crosses set \( G_x \) in some point. Therefore, points \( (x_1, \ldots, x_n) \in G_x \) the specified site of the plane define all possible points \( (b_1, \ldots, b_k) \) concentration of mix.

For the specified points of the plane (10) we bring out of system (1):

\[
\begin{aligned}
b_i &= \sum_{j=1}^n b_j^i x_j, \quad (i = 1 \ldots k), \\
x_n &= 1 - x_1 - \ldots - x_{n-1}.
\end{aligned}
\]

Let's substitute the received expression \( x_n \) in the equations of system (11). As a result, we will receive:

\[
\begin{aligned}
b_i &= b_i^m + \sum_{m=1}^{n-1} b_i^m x_m, \quad (p_i^m = b_i^n - b_i^m, \quad i = 1 \ldots k, \quad m = 1 \ldots n-1).
\end{aligned}
\]

All points \((x_1, \ldots, x_{n-1})\) the limited closed area:

\[
D = \{(x_1, \ldots, x_{n-1}) | \ x_i \geq 0, \ (i = 1 \ldots n - 1), \ x_1 + \ldots + x_{n-1} \leq 1\}
\]

determine all possible points by formulas (12) \((b_1^*, \ldots, b_k^*)\) concentration of components of mix \( A \).

By means of Gauss's method of the smallest squares we will be engaged in search of point \((b_1^*, \ldots, b_k^*)=(b_1^*, \ldots, b_k^*)\), the closest to point \((b_1^*, \ldots, b_k^*)\). For this purpose, we will consider function:

\[
\left( p_1 x_1 + \ldots + p_1^{n-1} x_{n-1} + b_1^n - b_1^* \right)^2 + \ldots + \left( p_k x_1 + \ldots + p_k^{n-1} x_{n-1} + b_k^n - b_k^* \right)^2 = f(x_1, \ldots, x_{n-1}).
\]

The minimum of this function on the closed limited set (13) gives point \((x_1^*, \ldots, x_{n-1}^*)\), on which of formula (12) there is required point \((b_1^*, \ldots, b_k^*)\) concentration.

Size \( d = \sqrt{(b_1^* - b_1^*)^2 + \ldots + (b_k^* - b_k^*)^2} \) serves as measure of proximity of points \((b_1^*, \ldots, b_k^*), (b_1^*, \ldots, b_k^*)\).

All possible mass of mix with point of concentration \((b_1^*, \ldots, b_k^*)\) it is possible to find on the following algorithm.
1) By means of the beam equations
\[
\begin{align*}
x_1 &= x_1^* t \\
\cdots \\
x_{n-1} &= x_{n-1}^* t \\
x_n &= x_n^* t
\end{align*}
\]
we find its point \((x_1^*, \ldots, x_n^*)\) exit from parallelepiped \(\Pi_x\).

2) The formula (7) completes search of all possible volumes of mix \(A\) with concentration \((b_1^*, \ldots, b_k^*)\).

It is necessary to use function \(f(x_1, \ldots, x_{n-1})\) it is possible to use at the solution of task 1. If it appears \(d = 0\), point \((b_1^*, \ldots, b_k^*)\) is among points \((b_1, \ldots, b_k)\) possible concentration also serves as the answer.

Example 2. Mixes \(A_1, A_2, A_3\) with volumes \(a_1 = 3.4, a_2 = 3.6, a_3 = 4.8\) contain four components \(B_1, B_2, B_3, B_4\) with concentration:

\[
\begin{align*}
b_1^2 &= 0.2, b_2^2 = 0.15, b_3^2 = 0.24, b_4^2 = 0.16; \\
b_1^3 &= 0.15, b_2^3 = 0.21, b_3^3 = 0.19, b_4^3 = 0.20; \\
b_1^4 &= 0.18, b_2^4 = 0.20, b_3^4 = 0.13, b_4^4 = 0.22.
\end{align*}
\]

It is required to make new mix \(A\) with concentration \(b_1, b_2, b_3, b_4\), the closest to sizes \(b_1^* = 0.18, b_2^* = 0.18, b_3^* = 0.18, b_4^* = 0.20\), and to find all possible volumes of new mix with the found concentration.

In such problem definition of the equation (3) at values \(b_1 = b_1^*, b_2 = b_2^*, b_3 = b_3^*, b_4 = b_4^*\) will register in look:

\[
\begin{align*}
0.02x_1 - 0.03x_2 &= 0 \\
-0.03x_1 + 0.03x_2 + 0.02x_3 &= 0 \\
0.06x_1 + 0.01x_2 - 0.05x_3 &= 0 \\
-0.04x_1 + 0.02x_2 &= 0
\end{align*}
\]

This system has only the zero decision \(x_1 = x_2 = x_3 = 0\) and, therefore, mix \(A\) with concentration \(b_1 = b_1^*, b_2 = b_2^*, b_3 = b_3^*, b_4 = b_4^*\) it is impossible to make.

Let's be engaged in search of admissible point \((b_1, b_2, b_3, b_4)\), the closest to point \((b_1^*, b_2^*, b_3^*, b_4^*)\). For this purpose, on formula (14) we find function

\[
f(x_1, x_2) = \left[ (b_1 - b_1^*)^2 + \ldots + (b_4 - b_4^*)^2 \right] = 10^{-4} \left[ (2x_1 - 3x_2)^2 + (2 - 5x_1 + x_2)^2 + (-5 + 11x_1 + 6x_2)^2 + (2 - 6x_1 - 2x_2)^2 \right],
\]

and then point of minimum of this function in the limited closed area:

\[
D = \{(x_1, x_2) \mid x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 1\}.
\]

This point is:
Example 3. Mixes $A_1, A_2, A_3, A_4$ with volumes $a_1 = 28, a_2 = 36, a_3 = 27, a_4 = 36$ contain three components $B_1, B_2, B_3$ with concentration

$$b_1^1 = 0.10, b_1^2 = 0.16, b_1^3 = 0.20;$$
$$b_2^1 = 0.16, b_2^2 = 0.14, b_2^3 = 0.26;$$
$$b_3^1 = 0.16, b_3^2 = 0.18, b_3^3 = 0.24;$$
$$b_4^1 = 0.12, b_4^2 = 0.10, b_4^3 = 0.32.$$
It is required to make new mix \( A \) with concentration \( b_1, b_2, b_3 \), satisfying to the relation \( b_1 : b_2 : b_3 = 1:1:2 \). Let's find the greatest and smallest values of sizes \( b_1, b_2, b_3 \) with this relation and the mix volumes answering to them \( A \).

In the reviewed example of the equation (16) will correspond in the form of system
\[
\begin{align*}
0.03x_2 + 0.04x_3 - 0.04x_4 &= 0 \\
0.06x_1 + 0.01x_2 + 0.06x_3 - 0.06x_4 &= 0
\end{align*}
\]
with the common decision
\[
\begin{align*}
x_1 &= 7(c_2 - c_1) \\
x_2 &= 12(c_2 - c_1) \\
x_3 &= 9c_1 \\
x_4 &= 9c_2
\end{align*}
\]
where \( c_1, c_2 \) - are any constants.

Restrictions \( 0 \leq x_1 \leq 28, 0 \leq x_2 \leq 36, 0 \leq x_3 \leq 27, 0 \leq x_4 \leq 36 \) bring to the following closed limited area \( D \) planes of variables \( c_1, c_2 \).

\[
D: \begin{cases} 
0 \leq c_1 \leq 3 \\
0 \leq c_2 \leq 4 \\
0 \leq c_2 - c_1 \leq 3
\end{cases}
\]

Any point \((c_1;c_2) \neq (0;0)\) from area \( D \) on formulas (17) gives mix \( A \) with the required relation of concentration \( b_1 : b_2 : b_3 = 1:1:2 \).

Let's find the maximum and minimum values of concentration \( b_1, b_2, b_3 \). For this purpose, as it is stated above, it is enough to find the maximum and minimum values of size \( b_1 \).

In our case
\[
b_1 = \frac{0.10x_1 + 0.16x_2 + 0.16x_3 + 0.12x_4}{x_1 + x_2 + x_3 + x_4} = 0.02 \cdot \frac{18x_1 - 59c_2}{28c_2 - 10c_1} = f(c_1, c_2).
\]
Calculations have shown that:

1) maximum value of function \( f(c_1, c_2) \) in the area \( D \) equally 0.14 also it is reached on border \( 0 < c_1 = c_2 \leq 3 \). To the specified area points \( D \) volumes answer \( 0 \leq V = x_1 + ... + x_4 \leq 54 \) mixes \( A \) at values \( x_1 = x_2 = 0, x_3 = x_4 = 9\tau, \ 0 \leq \tau \leq 3 \);  

2) minimum value of function \( f(c_1, c_2) \) in the area \( D \) equally \( 37/280 \approx 0.1321 \). It is reached on border \( c_1 = 0, 0 < c_2 \leq 3 \). To the specified area points \( D \) volumes answer \( 0 \leq V = x_1 + ... + x_4 \leq 57 \) mixes \( A \) at values \( x_1 = 0, x_2 = 0, x_3 = x_4 = 9\tau, \ 0 \leq \tau \leq 3 \).

**Solution of task 4**

**Task 4.** Now we will consider case when matrix rank \( C \) systems (16) it is equal \( n \), and necessary relation \( q_1 : q_2 : ... : q_n \) concentration of components of mix cannot be received. Let's be engaged in search of admissible point of concentration \((b_1,...,b_k) = (\hat{b}_1,...,\hat{b}_k)\), which in the best way provides the required relation \( q_1 : q_2 : ... : q_n \). Let's make it by means of method of the smallest squares again.

Let's designate \( u_i = \frac{q_i}{q_k}, v_i = \frac{b_i}{b_k}, \ i = 1..k-1 \). We bring out of system (1)
\[
\begin{align*}
\left\{ \begin{array}{l}
\sum_{j=1}^{n} b^i_j x_j \\
\sum_{j=1}^{n} b^k_j x_j
\end{array} \right.,
\quad (i = 1..k-1).
\end{align*}
\]

Let's enter function
\[
f(x_1,\ldots,x_n) = \sum_{i=1}^{k-1} (v_i - u_i)^2 = \sum_{i=1}^{k-1} \left( \sum_{j=1}^{n} b^i_j x_j \right)^2 - u_i^2 = \sum_{i=1}^{k-1} \left( \sum_{j=1}^{n} e^i_j x_j \right)^2,
\]

where \( e^i_j = b^i_j - u_i b^j_k \).

Let's consider now that on beam (9) of concentration \( b_1,\ldots,b_k \) do not change and, therefore, sizes do not change \( v_i \). The set of all specified beams defines all possible points \( (b_1,\ldots,b_k) \) concentration. Let's consider set \( W \) points \( (x_1,\ldots,x_n) \) planes
\[
b^1_k x_1 + b^2_k x_2 + \ldots + b^n_k x_n = q,
\]
in the area \( x_1 \geq 0,\ldots,x_n \geq 0 \). Positive constant \( q \) is chosen so that to simplify the subsequent calculations.

Each beam (9) crosses set \( W \) in some point. Therefore, set points \( W \) define all possible points of concentration of mix. In these points function \( f(x_1,\ldots,x_n) \) has the constant denominator which is not influencing search of its extremum on set \( W \) and, therefore, it can be replaced with new simpler function:
\[
F(x_1,\ldots,x_n) = \frac{1}{q} \sum_{i=1}^{k-1} \left( \sum_{j=1}^{n} e^i_j x_j \right)^2.
\]

Let's reduce number of variables of this function. We consider \( b^n_k \neq 0 \) also we find expression of weight from the equation (19) \( x_n \),
\[
 x_n = \frac{1}{b^n_k} \left( q - b^n_k x_1 - \ldots - b^{n-1}_k x_{n-1} \right).
\]

After substitution of the found expression of variable \( x_n \) in function \( F(x_1,\ldots,x_n) \) let's receive new function:
\[
\varphi(x_1,\ldots,x_{n-1}) = \sum_{i=1}^{k-1} \left( \varepsilon^0_i + \sum_{j=1}^{n-1} \varepsilon^j_i x_j \right)^2,
\]

where \( \varepsilon^0_i = \frac{q e^i_n}{b^n_k} \), \( \varepsilon^j_i = e^j_i - \frac{e^n_i b^j_k}{b^n_k} \).

Point \( (x_1,\ldots,x_{n-1}) \) function minimum \( \varphi(x_1,\ldots,x_{n-1}) \) on the closed set:
\[ G = \left\{ (x_1, \ldots, x_n) \mid x_1 \geq 0, \ldots, x_{n-1} \geq 0; 0 \leq b_{k1} x_1 + b_{k2} x_2 + \ldots + b_{kn} x_n \leq q \right\}, \]

together with value \( x_n^* \), found from (17) in sizes \( x_1^*, \ldots, x_{n-1}^* \), allows to receive all required sizes on formula (15) \( v_1 = v_1^*, \ldots, v_{k-1} = v_{k-1}^* \), the closest to the set sizes \( u_1, \ldots, u_{k-1} \).

At the same time formula:

\[ d = \sqrt{\left(v_1 - u_1\right)^2 + \ldots + \left(v_{k-1} - u_{k-1}\right)^2}, \]

it is possible to consider characteristic of proximity of sizes \( v_1^*, \ldots, v_{k-1}^* \) to sizes \( u_1, \ldots, u_{k-1} \).

Problem of finding of point \((b_1, \ldots, b_k)\) with the maximum values \( b_1, \ldots, b_k \), answering to the found relations \( v_1^*, \ldots, v_{k-1}^* \) concentration arises in that case when the points answering to these relations \((x_1^*, \ldots, x_n^*)\) more than one. In this case for all points \((x_1^*, \ldots, x_n^*)\), corresponding to sizes \( v_1^*, \ldots, v_{k-1}^* \), on formulas (1) there are all points \((b_1, \ldots, b_k)\), from which it is easy to choose the necessary point.

Possible volumes of mix with the relations \( v_1^*, \ldots, v_{k-1}^* \) concentration of components in mix are determined by the following algorithm:

- for each of the points stated above \((x_1^*, \ldots, x_n^*)\) by means of the beam equations

\[
\begin{align*}
&x_1 = x_1^* t \\
&\ldots, t > 0 \\
&x_n = x_n^* t,
\end{align*}
\]

there is point \((x_1^*, \ldots, x_n^*)\) beam exit from parallelepiped \( \Pi_k \) and the maximum volume answering to it \( V = x_1^* + \ldots + x_n^* \). Further all possible volumes of mixes with the found concentration are determined by formula (7) \( b_1, \ldots, b_k \).

Example 4. Mixes \( A_1, A_2, A_3 \) with volumes \( a_1 = 1.2, a_2 = 2.4, a_3 = 1.9 \) contain four components \( B_1, B_2, B_3, B_4 \) with concentration

\[
\begin{align*}
b_1^1 &= 0.15, \quad b_2^1 = 0.10, \quad b_3^1 = 0.18; \\
b_1^2 &= 0.09, \quad b_2^2 = 0.20, \quad b_3^2 = 0.14; \\
b_1^3 &= 0.18, \quad b_2^3 = 0.36, \quad b_3^3 = 0.30; \\
b_1^4 &= 0.11, \quad b_2^4 = 0.16, \quad b_3^4 = 0.16.
\end{align*}
\]

It is required to make new mix \( A \) with concentration \( b_1, b_2, b_3, b_4 \), which relation \( b_1 : b_2 : b_3 : b_4 \) in the best way would satisfy to the relation \( b_1 : b_2 : b_3 : b_4 = 1:1:2:1 \). Let's find the greatest and smallest values of sizes \( b_1, b_2, b_3, b_4 \) with this relation and the mix volumes answering to them \( A \).

In this example of equality (18) will appear in look:
\[
\begin{align*}
\begin{cases}
  v_1 &= \frac{0.15x_1 + 0.10x_2 + 0.18x_3}{0.11x_1 + 0.16x_2 + 0.16x_3} \\
  v_2 &= \frac{0.09x_1 + 0.020x_2 + 0.14x_3}{0.11x_1 + 0.16x_2 + 0.16x_3} \\
  v_3 &= \frac{0.18x_1 + 0.036x_2 + 0.30x_3}{0.11x_1 + 0.16x_2 + 0.16x_3}
\end{cases}
\end{align*}
\]

It is easy to check that this system has no decisions at \( v_1 = v_2 = 1, \ v_3 = 3 \).
Let’s consider function:
\[
f(x_1, x_2, x_3) = (v_1 - 1)^2 + (v_2 - 1)^2 + (v_3 - 2)^2 = \\
= 4 \left( (2x_1 - 3x_2 + x_3)^2 + ((-x_1 + 2x_2 - x_3)^2 + (-2x_1 + 2x_2 - x_3)^2 \right)
\]

In points of the site of the plane \( 11x_1 + 16x_2 + 16x_3 = 16 \), lying in the area of values \( x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0 \), function \( f(x_1, x_2, x_3) \) after expression substitution:
\[
x_3 = 1 - \frac{11}{16} x_1 - x_2,
\]
will correspond in the form of function
\[
\varphi(x_1, x_2) = \left( \frac{2}{256} \right)^2 \left[ (2x_1 - 64x_2 + 16)^2 + ((-5x_1 + 48x_2 - 16)^2 + (-2x_1 + 48x_2 - 16)^2 \right]
\]
where
\[
(x_1, x_2) \in D, \quad D = \left\{ 0 \leq x_1 \leq 1.2; \ 0 \leq x_2 \leq 2.4; \ \frac{11}{16} x_1 + x_2 \leq 1 \right\}
\]
Calculations show that the minimum value in the area \( D \) function \( \varphi(x_1, x_2) \) accepts in point:
\[
\left( x_1 = x_1^\circ = \frac{176}{2297} \approx 0.0766, \ x_2 = x_2^\circ = \frac{728}{2297} \approx 0.3169 \right)
\]
as a result, we get the values:
\[
x_3 = x_3^\circ = 1 - \frac{11}{16} x_1^\circ - x_2^\circ = \frac{1448}{2297} \approx 0.6304
\]
In sizes \( x_1 = x_1^\circ, \ x_2 = x_2^\circ, \ x_3 = x_3^\circ = 1.9 \) there are concentration:
\[
b_1 = \frac{2249}{14700} \approx 0.153, \quad b_2 = \frac{2276}{14700} \approx 0.155, \quad b_3 = \frac{4551}{14700} \approx 0.3096, \quad b_4 = \frac{2297}{14700} \approx 0.156
\]
brought best closer to the set relation \( 1:1:2:1 \).
Then there is point:
\[
\left( x_1 = x_1^* = \frac{418}{1810} \approx 0.23, \ x_2 = x_2^* = \frac{1725}{1810} \approx 0.955, \ x_3^* = 1.9 \right)
\]
beam exit \{ x_1 = x_1^t, \quad x_2 = x_2^t, \quad x_3 = x_3^t, \quad t > 0 \} \text{ from parallelepiped:}

\[ \Pi_x = \{0 \leq x_1 \leq 1.2; \ 0 \leq x_2 \leq 2.4; \ 0 \leq x_3 \leq 1.9 \}. \]

All volumes \( V = x_1 + x_2 + x_3 \) decide on the received relation of concentration by sizes:

\[ x_1 = x_1^\ast \tau, \quad x_2 = x_2^\ast \tau, \quad x_3 = x_3^\ast \tau, \]

where \( 0 < \tau \leq 1 \).

The largest volume of mix \( A \) it is respectively equal \( V = x_1^\ast + x_2^\ast + x_3^\ast \approx 3.0834 \).

4. Conclusions

The stated above tasks and algorithms of their decision can be of interest and to be demanded is livestock production, and other spheres of production, For Example, at production of foodstuff, technical mixes, medicines, alloys, and other cases.

It is possible to give the following as examples of application of such tasks [1-6]:

– in the food industry – it is preparation of flour mix for bakery goods of a specific type,

– in pharmacochemistry – parameters of exact proportion in mixes of medicines and result of their complex application in treatment,

– in the tool industry – qualitative characteristics of preparation of metal furnace charge and parameters of use of the tool,

– in chemical technologies – set of different parameters of forming of uniformity of mixes of bulks and parameters of their end use, etc.

The offered approaches and algorithms of the solution of these tasks are provided for the first time and their relevance is caused by more and more high requirements imposed by consumers to composite products and goods with its application.

5. References

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