A criterion of simultaneously symmetrization and spectral finiteness for a finite set of real $2 \times 2$ matrices

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Abstract

For $K \geq 1$, let there be given an arbitrary finite set $A$ consisting of real 2-by-2 matrices

$$A_0 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, A_1 = \begin{bmatrix} a_1 & r_1b \\ r_1c & d_1 \end{bmatrix}, \ldots, A_K = \begin{bmatrix} a_K & r_Kb \\ r_Kc & d_K \end{bmatrix},$$

and by $\rho(M)$ it stands for the spectral radius of a square matrix $M$. In this paper, we first show that if $bc > 0$ then $A$ may be simultaneously symmetrized. This then implies that if $bc \geq 0,$

$$\max\{\rho(A_0), \rho(A_1), \ldots, \rho(A_K)\} = \sup \{n \geq 1, \max_{A \in A_n} \sqrt[n]{\rho(M)}\} = 0;$$

that is, $A$ has the spectral finiteness property and then the stability of the switched system defined by $A$ is decidable.

Keywords: Symmetrization of matrices, generalized spectral radius, spectral finiteness.

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1. Introduction

In this paper, we study the simultaneously symmetrization and then the finite-step realizability of the generalized/joint spectral radius for a finite set of real $2 \times 2$ matrices.

1.1. Criterion of simultaneously symmetrization

For a real $d \times d$ matrix $A = [a_{ij}]_{i,j=1}^{d} \subseteq \mathbb{R}$, it is said to be symmetric if $a_{ij} = a_{ji}$ for all $1 \leq i, j \leq d$. A symmetric matrix has many good property like diagonalizion. So, symmetrization of matrices is very important for problems involving numerical computation of matrices. In this short paper, we first show a simultaneously symmetrization for a family of real $2 \times 2$ matrices, which may be stated as follows:

Theorem 1. Let there be arbitrarily given $K + 1$ real $2 \times 2$ matrices

$$A_0 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, A_1 = \begin{bmatrix} a_1 & r_1b \\ r_1c & d_1 \end{bmatrix}, \ldots, A_K = \begin{bmatrix} a_K & r_Kb \\ r_Kc & d_K \end{bmatrix}$$

where $K \geq 1$. If $bc > 0$, then one can find a nonsingular matrix $Q \in \mathbb{R}^{d \times d}$ such that $Q^{-1}A_0Q = 0 \leq k \leq K$, all are symmetric.

This provides a criterion of simultaneously symmetrizing a finite set of real $2 \times 2$ matrices.

Remark 2. In fact, our condition “$bc > 0$” is already very close to “necessary”, as shown by the following counterexample. Let

$$A_0 = \begin{bmatrix} -3 & 3.5 \\ -4 & 4.5 \end{bmatrix}, A_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix},$$

where $bc = -14 < 0$.

Although $A_0$ may be diagonalized and $A_1$ is already diagonal, yet it will be proved in Section 2 that $(A_0,A_1)$ cannot be simultaneously symmetrized.

As an application, we will see that Theorem 1 is important for the numerical computation of the generalized spectral radius of a family of real $2 \times 2$ matrices.

1.2. Spectral finiteness for a finite set of real $2 \times 2$ matrices

Throughout this paper, $\rho(M)$ will stand for the usual spectral radius of a square matrix $M$. For an arbitrary family of real matrices

$$A = [A_0, \ldots, A_K] \subseteq \mathbb{R}^{d \times d}$$

where $2 \leq d < +\infty$, its generalized spectral radius, first introduced by Daubechies and Lagarias in [15], is defined by

$$\rho = \sup_{n \geq 1} \max_{M \in A_n} \sqrt[n]{\rho(M)} = \lim_{n \to +\infty} \sup_{M \in A^n} \sqrt[n]{\rho(M)};$$

where

$$A^n = [M_1 \cdots M_n]: M_i \in A \text{ for } 1 \leq i \leq n \quad \forall n \geq 1.$$

According to the Berger-Wang spectral formula [2], this quantity is very important for many pure and applied mathematics
branches like numerical computation of matrices, differential equations, coding theory, wavelets, stability analysis of random matrix, control theory, combinatorics, and so on. See, for example, [115, 117].

Therefore, the following finite-step realization question for the accurate computation of the spectral radius \( \rho \) becomes very interesting and important.

**Problem 1.** Does there exist a finite-length word which realize \( \rho \) for \( A \); i.e.,

\[
\rho = \max_{n \geq 1} \max_{M \in A^n} \sqrt[n]{\rho(M)}?
\]

In other words, does there exist any \( M \in A^n \) for some \( n \geq 1 \) such that

\[
\rho = \sqrt[n]{\rho(M)}?
\]

If one can find some word, say \( M \in A^n \), for some \( n \geq 1 \), such that \( \rho = \sqrt[n]{\rho(M)} \), then \( A \) is said to possess the spectral finiteness property.

This problem is equivalent to the following stability question:

**Problem 2.** If the periodic stability (i.e. \( \rho(M) < 1 \) for any finite-length words \( M \in \bigcup_{n \geq 1} A^n \)) is satisfied then, does it hold the absolute stability:

\[
\max_{M \in A^n} \|M\| \to 0 \quad \text{as} \quad n \to +\infty?
\]

This spectral finiteness property, or equivalently, “periodic stability \( \Rightarrow \) absolute stability”, of \( A \) was conjectured, respectively, by Pyatnitskii (see e.g. [25, 27]), Daubechies and Lagarias in [15], Gurvits in [17], and by Lagarias and Wang in [23]. It has been disproved first by Bousch and Mairesse in [7], and then by Blondel et al. in [3], by Kozyakin in [21, 22], all offered the existence of counterexamples in the case where \( d = 2 \); moreover, an explicit expression for such a counterexample has been found in the recent work of Hare et al. [18].

However, an affirmative solution to Problem 1 (or equivalently, to Problem 2) is very important; this is because it implies an effective computation of \( \rho \) and decidability of stability of \( A \) by only finitely many steps of computations. There have been some sufficient (and necessary) conditions for the spectral finiteness property for some systems \( A \), based on and involving Barabanov norms, polytope norms, ergodic theory or some limit properties of \( A \), for example, in Gurvits [17], Lagarias and Wang [23], Guglielmi, Wirth and Zennaro [16], Kozyakin [22], Dai, Huang and Xiao [12], and Dai and Kozyakin [14]. But these theoretic criteria seems to be difficult to be directly employed to judge whether or not an explicit family \( A \) or even a pair \( [A, B] \subset \mathbb{R}^{2 \times 2} \) have the spectral finiteness property.

From literature, as far we know, there are only few results on such an explicit family of matrices \( A \).

**Theorem A** (Theys [28], also see [19, Proposition 4]). If \( A_0, \ldots, A_K \in \mathbb{R}^{d \times d} \) all are symmetric matrices, then the spectral finiteness property holds for \( A \). In fact, there holds

\[
\rho = \max_{0 \leq k \leq K} \rho(A_k).
\]

For any matrix \( A \), by \( A^T \) it denotes the transpose of \( A \). An generalization of Theorem A is the following

**Theorem B** (Plischke and Wirth [24, Proposition 18]). If the system \( A = [A_0, \ldots, A_K] \in \mathbb{R}^{d \times d} \) is symmetric, i.e. \( A_k^T \in A \) for all \( 0 \leq k \leq K \), then the spectral finiteness property holds for \( A \).

For a pair of matrices, there are the following results.

**Theorem C** (Jungers and Blondel [19]). If \( A_0, A_1 \) are \( 2 \times 2 \) sign-matrices, that is, \( A_0, A_1 \) belong to \([0, \pm 1]^{2 \times 2} \), then the spectral finiteness property holds for \([A_0, A_1]\).

A more general result than the statement of Theorem C is the following

**Theorem D** (Cicone et al. [8]). If \( A_0, A_1 \) are \( 2 \times 2 \) sign-matrices, that is, \( A_0, A_1 \) belong to \([0, \pm 1]^{2 \times 2} \), then the spectral finiteness property holds for \([A_0, A_1]\).

The followings are other type of results.

**Theorem E** (Dai et al. [10]). If one of \( A, B \in \mathbb{R}^{d \times d} \) is of rank one, then there holds the spectral finiteness property for \([A, B]\).

**Theorem F** (Dai, Huang and Xiao [13]). If, for \( A, B \in \mathbb{R}^{d \times d} \), there is a symmetric positive-definite matrix \( P \) such that

\[
P - A^T P A \geq 0 \quad \text{and} \quad P - B^T P B \geq 0,
\]

then the spectral finiteness property holds for \([A, B]\) in the case \( 2 \leq d \leq 3 \).

Using our symmetrization Theorem 1, we can prove the following finiteness result:

**Theorem 3.** Let there be arbitrarily given \( K + 1 \) real \( 2 \times 2 \) matrices

\[
A_0 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, A_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}, \ldots, A_K = \begin{bmatrix} a_K & b_K \\ c_K & d_K \end{bmatrix},
\]

where \( K \geq 1 \). If \( bc \geq 0 \) then \( A = [A_0, \ldots, A_K] \) has the spectral finiteness property and moreover

\[
\rho = \max_{0 \leq k \leq K} \rho(A_k).
\]

**Proof.** If \( bc = 0 \) then the statement holds trivially. Now let \( bc > 0 \). From Theorem 1, one can find some nonsingular matrix \( Q \) such that \( QA_kQ^{-1}, 0 \leq k \leq K \), all are symmetric. Then, the statement of Theorem 3 follows immediately from Theorem A, also from Theorem B.

As a result of Theorem 3, we can obtain the following

**Corollary 4.** Let \( A, B \in \mathbb{R}^{2 \times 2} \) be a pair of matrices such that

\[
A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}, \quad B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.
\]

If \( bc \geq 0 \) then there holds the spectral finiteness property for \([A, B]\). More precisely, if \( bc \geq 0 \) then \( \rho = \max(\rho(A), \rho(B)) \).

Without the constraint condition \( bc \geq 0 \) in Corollary 4, a special case might be simply observed.
Theorem 5. Let \( A, B \in \mathbb{R}_{2\times 2}^{2\times 2} \) be a pair of matrices such that 
\( A = \text{diag}(A_1, A_2) \) and \( B = \begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix} \). Then \( \{A, B\} \) has the spectral finiteness property with \( \rho = \max(\rho(A), \rho(B)) \).

Proof. Let \( \rho(A) = \max\{|\lambda_1|, |\lambda_2|\} < 1 \) and \( \rho(B) = \sqrt{bc} < 1 \). Let \((m_n, n_n)\) be an arbitrary sequence of positive integer pairs. We claim that
\[
\|A^{m_n} B A^{m_n} B^2 \cdots A^{m_n} B^k\|_2 \to 0 \quad \text{as} \quad k \to +\infty,
\]
where \( \| \cdot \|_2 \) denotes the matrix norm induced by the standard Euclidean vector norm on \( \mathbb{R}^2 \). In fact, the claim follows from
\[
A^n = \begin{bmatrix} a^n & 0 \\ c^n & d^n \end{bmatrix} \quad \text{and} \quad B^n = \begin{cases} (bc)^{n/2} I_2 & \text{if} \quad n = 2^r', \\ (bc)^{n/2} B & \text{if} \quad n = 2^n + 1. \end{cases}
\]
Then, this claim implies that \( \rho = \max(\rho(A), \rho(B)) \). \( \square \)

1.3. Outline

This paper is simply organized as follows. We will prove Theorem 1 and Remark 2 in Section 2. Finally, we will end this paper with some examples in Section 3.

2. Simultaneously symmetrization

This section is mainly devoted to proving our criterion of simultaneously symmetrizing, i.e., Theorem 1.

Proof of Theorem 1. Let there be arbitrarily given \( K + 1 \) real \( 2 \times 2 \) matrices
\[
A_0 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad A_1 = \begin{bmatrix} a_1 & r_1 b \\ r_1 c & d_1 \end{bmatrix}, \ldots, A_K = \begin{bmatrix} a_K & r_K b \\ r_K c & d_K \end{bmatrix},
\]
where \( K \geq 1 \), such that \( bc > 0 \). Let
\[
Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}
\]
such that \( q_1 q_2 \neq 0 \) and \( q_1 = q_2 = \sqrt{bc} \).

Then,
\[
QA_0 Q^{-1} = \begin{bmatrix} a & \sqrt{bc} \\ \sqrt{bc} & d \end{bmatrix},
\]
\[
QA_1 Q^{-1} = \begin{bmatrix} a_1 & r_1 \sqrt{bc} \\ r_1 \sqrt{bc} & d_1 \end{bmatrix},
\]
\[
\vdots 
\]
\[
QA_K Q^{-1} = \begin{bmatrix} a_K & r_K \sqrt{bc} \\ r_K \sqrt{bc} & d_K \end{bmatrix},
\]
they are symmetric. This proves Theorem 1.

We now turn to the proof of Remark 2. Let
\[
A_0 = \begin{bmatrix} -3 & 3.5 \\ -4 & 4.5 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{where} \quad bc = -14 < 0,
\]
as in Remark 2. Put
\[
Q = \begin{bmatrix} -0.5 & 1 \\ 0 & 1 \end{bmatrix}.
\]
then we have
\[
Q^{-1} = \begin{bmatrix} -2 & 2 \\ 0 & 1 \end{bmatrix}.
\]
So,
\[
B_0 := Q^{-1} A_0 Q = \begin{bmatrix} 1 & 0 \\ 2 & 0.5 \end{bmatrix}
\]
and
\[
B_1 := Q^{-1} A_1 Q = \begin{bmatrix} 0.5 & 1 \\ 0 & 1 \end{bmatrix}.
\]

According to Kozyakin [22, Theorem 10, Lemma 12 and Theorem 6], there follows that: There always exists a pair of real numbers \( a > 0, \beta > 0 \) such that \( \{aB_0, \beta B_1\} \) does not have the spectral finiteness property.

Thus, if \( \{A_0, A_1\} \) might be simultaneously symmetrized, then \( \{aA_0, \beta A_1\} \) and hence \( \{aB_0, \beta B_1\} \) have the spectral finiteness property from Theorem 3, for all \( a > 0, \beta > 0 \). This is a contradiction. Therefore, \( \{A_0, A_1\} \) cannot be simultaneously symmetrized.

This proves the statement of Remark 2. Meanwhile this argument shows that the constraint condition “\( bc \geq 0 \)” in Theorem 3 and even in Corollary 4 is crucial for the spectral finiteness property in our situation.

Given an arbitrary set \( A = \{A_0, \ldots, A_K\} \subset \mathbb{R}^{d \times d} \), although its periodic stability implies that it is stable almost surely in terms of arbitrary Markovian measures as shown in Dai, Huang and Xiao [11] for the discrete-time case and in Dai [9] for the continuous-time case, yet its absolute stability is generally undecidable; see, e.g., Blondel and Tsitsiklis [4, 5, 6].

However, Theorem 3 proved in Section 1.2 is equivalent to the statement — “periodic stability \( \Rightarrow \) absolute stability”, i.e., Problem 2, under suitable additional conditions.

Theorem 6. Let \( A = \{A_0, \ldots, A_K\} \subset \mathbb{R}_{2\times 2}^{2\times 2} \) be such that
\[
A_0 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, A_1 = \begin{bmatrix} a_1 & r_1 b \\ r_1 c & d_1 \end{bmatrix}, \ldots, A_K = \begin{bmatrix} a_K & r_K b \\ r_K c & d_K \end{bmatrix},
\]
where \( K \geq 1 \) and \( bc \geq 0 \). Then \( A \) is absolutely stable if and only if \( \rho(A_k) < 1 \) for all \( 0 \leq k \leq K \).

Proof. This statement comes immediately from Theorem 3. In fact, Theorem 3 implies \( \rho < 1 \) if \( \rho(A_k) < 1 \) for all \( 0 \leq k \leq K \) and hence \( A \) is absolutely stable if \( \rho(A_k) < 1 \) for all \( 0 \leq k \leq K \); see, e.g., [1, 17, 26]. \( \square \)

This shows that the absolute stability of the switched system induced by \( A \) is decidable in the situation of Theorem 6.

3. Examples of stability

In this section, we consider some explicit examples using Theorem 3 and Corollary 4.
3.1. Applications of Corollary 4

For any two real 2×2 matrices $A, B$, to utilize our Corollary 4, the first step is to diagonalize one of $A, B$. So, we need the Diagonalization Theorem: An $n \times n$ matrix $A$ is diagonalizable if and only if $A$ has $n$ linearly independent eigenvectors.

Example 7. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -\frac{5}{2} & 2\sqrt{2} - 1 \\ 1 & 4 \end{bmatrix}$. We assert that $\{A, B\}$ has the spectral finiteness property.

In fact, since

$$A_1 := \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$A_0 := \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{5}{2} & 2\sqrt{2} - 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & \sqrt{2} \\ 1 & 3 \end{bmatrix},$$

it follows, from Corollary 4, that $\{A, B\}$ has the spectral finiteness property with

$$\rho = \rho(B) = \frac{1}{2} \left( 3 + \sqrt{27 + 4\sqrt{3}} \right).$$

Example 8. Let $A = \begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix}$, where $b, c \in \mathbb{R}$ such that

$$225b^2 - 34bc + c^2 \leq 0.$$ 

We now consider the spectral finiteness property of $\{A, B\}$.

The eigenvalues of $A$ are 1 and 0.92, their corresponding eigenvectors are respectively $(3, 5)^T$ and $(1, -1)^T$. We put

$$P = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}.$$

Then

$$P^{-1} = \begin{bmatrix} 1/8 & 1/8 \\ 5/8 & -3/8 \end{bmatrix}$$

and

$$P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & 0.92 \end{bmatrix}.$$

Since

$$P^{-1}BP = \frac{1}{8} \begin{bmatrix} * & a \\ -c & 25b \end{bmatrix} \begin{bmatrix} c - 9b \\ * \end{bmatrix},$$

there follows $(c - 9b)(-c + 25b) \geq 0$. So, $\{A, B\}$ has the spectral finiteness property from Corollary 4 such that

$$\rho = \max \left\{ 1, \sqrt{bc} \right\}.$$ 

Example 9. Let $A_0 = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ and $A_1 = \begin{bmatrix} 1 & 0 \\ c & d \end{bmatrix}$, where the constants $a, b, c, d \in \mathbb{R}$ with $a \neq 1$.

If $ad = 0$ then either $\text{rank}(A_0) = 1$ or $\text{rank}(A_1) = 1$ and so $\{A_0, A_1\}$ has the spectral finiteness property from Theorem E.

If $bc = 0$ then either $b = 0$ or $c = 0$. So $\{A_0, A_1\}$ has the spectral finiteness property from Corollary 4.

Next, we let $bc \neq 0$ and define

$$Q = \begin{bmatrix} \frac{b}{a-1} & 1 \\ 0 & 1 \end{bmatrix}.$$

Then,

$$Q^{-1} = \begin{bmatrix} \frac{b}{a-1} & -\frac{b}{a-1} \\ 0 & 1 \end{bmatrix}$$

and

$$QA_0A^{-1} = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}, \quad QA_1A^{-1} = \begin{bmatrix} 1 + \frac{bc}{a-1} & \frac{(d-1)(a-1)-bc}{a-1} \\ \frac{bc}{a-1} & \frac{d-1}{a-1} \end{bmatrix}.$$

Note that

$$\frac{(d-1)(a-1)-bc}{a-1} \times \frac{bc}{a-1} \geq 0$$

if and only if

$$[(1-a)(1-d)-bc] \times bc \geq 0.$$ 

Hence, if either $a \neq 1$ or $d \neq 1$ and $[(1-a)(1-d)-bc] \times bc \geq 0$, then $\{A_0, A_1\}$ has the spectral finiteness property.

If $a = d = 1$ and $bc \geq 1$, then $\{A_0, A_1\}$ has the spectral finiteness property from Kozyakin [22, Theorem 10, Lemma 12 and Theorem 6].

3.2. Applications of Theorem 3

Applying Theorem 3, we consider the following

Example 10. Let

$$A_0 = \begin{bmatrix} \sqrt{2} & 1 \\ 2 & 1.3 \end{bmatrix}, \quad A_1 = \begin{bmatrix} \sqrt{2} & 10 \\ 20 & \sqrt{7} \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0.1 \\ 0.2 & \sqrt{5} \end{bmatrix}.$$ 

Then from Theorem 3, if follows that $\{A_0, A_1, A_2\}$ has the spectral finiteness property.

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