Selection of Suitable Lubricant for Sliding Contact Bearing and the Effect of Different Lubricants on Bearing Performance: A Review and Recommendations

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ABSTRACT

The word bearing implies to an element of the machine which allows one moving or vibrating part to support other rubbing surfaces by which the load can transmit. Depending upon in nature and application of load, an extensive variety of different bearing is available. They can generally be categorized into two kinds of bearing such as rolling element bearing and a sliding bearing. Lubrication is required in the middle of rubbing surfaces, which enacts as a medium between the moving surfaces to diminish friction and to take away heat for survival during the transmission of the load. In this paper, a literature review has been made on the influence of different lubricants on the performance of various bearing. The selection of an appropriate lubricant is a very critical decision to take for the efficient performance of a bearing because each lubricant has its own merits and demerits on various kinds of bearing. In the later portion of this paper, a selection process of right lubricant in sliding contact bearing is analyzed mathematically. This critical decision is being made by using PROMETHEE method (broadly Preference Ranking Organization Method for Enrichment Evaluations) hybridized by Entropy Weight Measurement method and Fuzzy scale. As per the ranking of Entropy-PROMETHEE method, the best suited lubricant for sliding contact bearing is SAE20W40.

Keywords: Lubrication systems; Lubricant Additives; Lubricants Selection; Entropy-PROMETHEE.

INTRODUCTION

For the purpose of running the machines or engines, wide varieties of bearings are required. General types of bearing contain two or more rotating metal or composite surfaces which revolves around each other. Here lubrication enacts a very vital role in bearing to work efficiently. Moreover, lubricant helps the bearing to withstand high speeds, loads, and incessantly increasing pressures and mechanical shearing forces conditions as implied by operating conditions [1]. The continuous sliding between the surfaces and the presence of a layer of foreign particles at the interface developed high friction and wear as a result of the high surface energy. To counter these effects, lubricants are applied to lessen the friction and wear between the interfaces [2].

The lubrication is categorized into two different types such as solid lubrication and fluid (liquid or gaseous) film lubrication. A solid lubricant protects damage during a relative movement by reducing the wear and coefficient of friction, which is used as a powder or a thin film of solid. The powder or solid lubricant shows hydrodynamic behaviour which is similar to the liquid lubricant after a certain point of the run. This condition of the solid lubricant is also recognized as 'quasi-hydrodynamic lubricant' because of attaining of semi-liquid conditions after a certain point of run [3]. To improve the performance of solid lubricant, various additives are used by which it can fulfill the requirements of engineering applications [4].

Lubricant fluid can be liquid or gaseous. Even a thick film of air transposed between two mating surfaces is a method of good lubrication [2]. A thin liquid film on the order of surface roughness of moving surfaces results in relatively low friction and wear, as compared to solid-solid contact. In some cases, a thin layer of lubricant can also mitigate some asperities of the surfaces of solid, when they come to close to each other for a moment. A solid-solid contact can also be separated by using a thick fluid film between two surfaces, which are in relative motion and can provide very low friction (the value of the coefficient of friction is between 0.001 and 0.003) and a very negligible wear [5].

From the above literature review, it has been perceived that the selection of a best-suited lubricant for a particular bearing is very important for efficient working of any part of the machine. This paper will present a review of the utility of different kinds of bearing lubricants and their effects on the various bearing performance and also recommended...
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the best lubricant for a bearing with the support of Entropy-PROMETHEE hybridized optimization technique.

**TYPES OF LUBRICATION**

A numerous forms of lubrication work on a different kind of lubrication regime. In light loading and high-speed condition of any bearing, a thick film of lube oil is used and the range of lube film thickness is greater than 0.25 μm and the range of coefficient of friction is 0.01-0.005 [1]. The thin film lubrication system is used, when the bearing operates on heavy loading, high speed and high temperature. The range of film thickness and range of coefficient of friction for thin film lubrication are 0.025 to 5 μm and 0.005 to 0.05 respectively. But the boundary lubrication is used when the bearing works on heavy loading condition and the range of film thickness and range of coefficient of friction are 1 to 3 nm and 0.05 to 0.15 respectively [2]. A summary of these lubrication system shown in figure 1.

**Hydrodynamic Lubrication**

The bearings, under hydrodynamic lubrication, sustain the load on thick film generated due to hydrodynamic mechanism between moving parts. A comparatively thick film of lubricants is generated due to hydrodynamic effect to detach the bearing surfaces of load carrying and also to preclude the contact between two metallic surfaces (as shown in figure 2 & 3) [2]. The hydrodynamic bearings are also recognized as self-acting bearings. The hydrodynamic lubrication is suitable in such conditions where the bearing is subjected to a sudden heavy load in various industrial machines because a thick film of lubrication is necessitated in this condition to endure the heavy loading [4].

**Hydrostatic Lubrication**

Hydrostatic lubrication is described as a form of lubrication mechanism in which the rotating surfaces of bearing are being detached by a thick film of oil by the peripheral pressure spawned by an oil pump used externally. Due to this, the bearings operated with hydrostatic lubricant are called "externally pressurized" bearing. Hydrostatic bearings are designed to operate with both compressible and incompressible fluids [2]. In many applications of industries, hydrostatic journal bearing have used very efficiently because of their improved characteristics as compared to the hydrodynamic bearing, such as high accuracy, low friction, very good vibration characteristics and even at low speed, it can rotate very smoothly. In an exceptional case, when the journal of the hydrostatic journal bearing doesn’t rotate, then hydrostatic lubricants also provides large values of fluid film stiffness and damping coefficient to withstand heavy loads [5].

**Boundary Lubrication**

The domain of the boundary lubrication surfaces is that situation when the shaft remains at rest and then it starts

![Figure 1: Different lubrication system.](image1)

![Figure 2: Mechanism of Hydrodynamic lubrication.](image2)

![Figure 3: Striebeck curve for different lubrication mechanisms.](image3)
moving with very low speed, very high load and with a lubricant having much lower viscosity. The boundary lubrication will be induced between the rotating surfaces when the solid surfaces come so close such that the interaction between the solid asperities and surface of mono or multi-molecular layers of lubricants (liquids or gases) will occur very smoothly [2]. It is shown in figure 3.

**Elastohydrodynamic Lubrication**

Elastohydrodynamic lubrication is mostly used in between the heavily loaded bearing surfaces, where loads spread over a small contact surface (which is equal to one-thousandth of the apparent area of a journal bearing approximately), such as the point contacts of ball bearings and of gear teeth and the line contacts of roller bearings [7]. The elastohydrodynamic lubrication will occur in the region between the rolling contact surfaces, (such as rolling bearings, gears) when a suitable lubricant is introduced into it. In case of elastohydrodynamic lubrication, a thinner film thickness will occur (ranges from 0.5μm to 5μm) as compared with conventional hydrodynamic lubrication [2], as shown in Figure 1.

**Solid-film Lubrication**

The bearing is operated at extreme environment i.e. extreme temperature and pressure, in which a film of solid lubricant (like molybdenum disulphide, graphite and PTFE etc.) may be used because, in this environment, the mineral oils are not suitable [8]. Other than these three solid lubricants, PFA (perfluoroalkoxy copolymer), tungsten disulfide, lead oxide, boron nitride, antimony oxide, FEP (fluorinated ethylene propylene), silver, lead, tin etc. can also be used as solid lubricant [2].

**Mixed Lubrication**

The transition zone between the boundary and elastohydrodynamic or hydrodynamic regimes is a grey area (as shown in Figure 3) and is called a mixed lubrication in which two unlike lubrication mechanisms having dissimilar properties may dominate [2]. The film thickness (nominal) of the lubricant in the regime of mixed lubrication is reduced, the coefficient of friction is increased and the speed of the journal is increased, then the mixed lubrication is converted to hydrodynamic lubrication [9]. Under the regime of mixed lubrication, both the hydrodynamic pressure and contact pressures of asperity are necessary to support the load applied to bearing [10].

**Wear Rate Comparison**

The estimation of bearing service life may be necessary in designing the bearings. The discussion about their service life in general, but it is quite difficult to estimate it reliably from an aspect of wear across various wear types and terms of service. Normally the approximate amount of wear is determined using a specific amount of wear currently accumulated in an experiment. Using this particular amount of wear, the service life is easily measured but care needs to be taken to take its numerical value [2-4]. The rate of wear of the bearings is varied with the various load at

**Figure 4:** Wear rate vs. Load/Temperature with respect to different lubricating mechanisms [14].

**BEARING LUBRICANTS**

In this section, discussion about numerous varieties of lubricants used in bearing with their physical properties and functions is carried out. Generally, the lubricants are the composition of 95% oil (which is called as base stock) and 5% additives. The main sources of base stock are mineral oil, biological oil and synthetic oil. The basic structure and physical properties of lubricant mainly are dominated by its major constituent i.e. base stock and the performance of the lubricants get improved by the use of additives [2]. The effect of various types of lubricants on different bearings are briefly discussed below.

**Mineral Oils**

The lubricant which is mostly used in the bearing is mineral oil, due to its easy availability and cheap cost. In bearing, the mineral oils are applied to keep it cool during working and it also flushes away debris or contaminant particles with it from the bearing [1]. That’s why the journal bearings operated with high-speed are always being mineral oil lubricated rather than lubricated by grease [11-19]. The grade of oil used as a lubricant and the change in load into the assembly of the bearing make a significant effect on the temperature and pressure of lubricating oil film. Besides these advantages, the mineral lubricating oils also have some shortcomings such as it contains various surface active polar compounds in a medium of non-polar hydrocarbon [20-21].

**Grease**

Grease is employed to journal bearings for the lubricating purpose when it runs at comparatively low speeds and cooling of the rubbing surfaces of the bearing are not a major factor. The key advantage of using grease is that it withstands the shock loading ensues in bearing assembly and grease is also beneficiary when the bearing repeatedly
stops and starts or reverses direction. The best-suited lubricant for pins and bushings is grease because it can resist shock loading, it protects the bearing against vibration, it can support static loads and it provides a thick semi-solid layer of lubricant as compared to oil [1]. Grease is also worked very efficiently to rolling contact bearings because it reduces the frictional coefficient between the moving surfaces, improves the corrosion resistance and ensures effective lubricant withholding [22]. The density of grease is much greater than that of oil. Because of that it always makes a thick layer between the moving surfaces. Grease has very good sealing property and it can’t be leaked out of the bearing and this property makes it very easily applicable [23]. A centralized lubricating system can be used to apply grease lubrication to supply grease continuously or even after a certain interval between the moving surfaces of bearing.

In recent time the grease oriented research of lubrication has reported that the key topographies of the classical theory of elastohydrodynamic oil lubrication far differ from the grease lubrication and the starvation property of grease lubrication is more noticeable. The rolling bearings lubricated with grease are applicable in many important electronic and mechanical devices like washing machines, conveyors and electric motors, adjusters of timing belt and bearings of the wheel of automotive, centrifugal pumps, bearings of train wheelset [24]. Though grease has many benefits, it also has some drawbacks such as its limited service life which is reliant on its operating temperatures and grease will lose its chemical structure in a high-temperature regime which causes damages on rolling bearing and failure of lubrication [22].

Synthetic Oil

Synthetic lubricating oil is an artificially made lubricating oil which consists chemical compounds. Synthetic lubricants are synthesized from petroleum by-products, which are chemically modified. Synthetic oils can also be extracted from various unrefined and organic substances. The synthetic oil is applied in bearing as a surrogate for other lube oils when bearings are operated at extreme temperature. The most used synthetic lubricants are polyphenyl ethers, diesters, phosphate ester, silicate esters, polyglycerol esters, polyglycerols, esters of fatty acid, neopentyl polyol esters, cyclo-aliphatic, fluoro esters, silicones, Sila hydrocarbons, perfluoropolyether, chlorofluorocarbon, polytetrafluoroethylenes, chlorotrifluoroethylene, perfluoropolyalkylethers etc. [1]. Recently, the demand for synthetic lube oils goes to a high extent due to its superior properties which it can endure hazardous environments like high humidity and vacuum, extreme temperature, high-pressure and fewer of a fire hazard. The synthetic oils can efficiently work up to a temperature of 370°C and for a little period of time it works up to 430°C [2].

Water-based Lubricants

Water as lubricant is a substitute of hydrocarbon oil. The bearing operated with a water-based lubricant makes the lubricating system free from the hazardous environment. The advantages of using water-lubricated bearing are a long lifespan, low runout, high stiffness and improved damping as compared to a rolling contact bearing, oil film or hydrodynamic bearing or gas bearing [25]. But, in case of rotor-bearing, water as a lubricant has a major problem with the stability of water film, particularly under the condition of non-laminar lubrication. The stability performance of bearing is being affected by various major factors, like Reynolds number, supply pressure, geometrical parameters and bearing configuration [12]. A research also reported that water as a lubricant is not so effective as mineral oil because the stiffness coefficient of oil film and the capacity to carry load are very less as compared with that of mineral oil under the same operating conditions [26].

Gaseous Lubricants

Recently, the bearings operated with gaseous lubricant are commonly used in different types of machinery and engines because the gaseous lubricants, as compared with liquid (oils and greases) lubricants, enhance the mechanical efficiency, lessen the frictional losses, decrease the wear rate and also mitigates the external environmental impact [27]. Moreover, the nominal film thickness of gaseous lubricant decreases with the rapidly increasing capabilities of damping and stiffness of the gaseous film [28]. With the increasing application of the bearings lubricated with gas in the industry, the evaluation of the performance and characteristics of the gas lubricated bearings are attracting more researchers as a field of study. The efficient working of the hydrostatic bearings lubricated with gas depends mainly on the uninterrupted supply of high pressurized gas to the gap provided for the lubricant the said bearing. The example of some self-acting gas lubricated bearings are as follows: step bearings, fixed tapered land bearings, tilting pad bearings, etc. Though these bearings do have their relative merits of self-acting, they can’t be a replacement for a hydrodynamic gas-lubricated bearing due to complex design and comparatively high maintenance and manufacturing cost [29-34].

Air as a Lubricant

The bearings with air, as a gaseous lubricant, have low heat generation, low frictional loss and very high precision as compared with traditional oil film bearings and roller or ball bearings. That’s why the air bearings are extensively used in the machines with high precision, machine tools with high accuracy, different measurement equipment and laboratory instruments [35].

Air bearings have some drawbacks. This type of bearings has the small load carrying capacity because air has an extremely low coefficient of viscosity. So, this bearing has a very limited field of application. But the above-written drawbacks can be overcome by properly designing the aerostatic bearings, which can increase the load carrying capacity of such bearings [36].
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Table 1: Lubricants with its properties [29, 30, 35, 47-49]

| Sl. No | Lubricant            | Boiling Point (°C) | Freezing Point (°C) | Viscosity Index | Thermal stability | Hydraulic stability | Corrosive Resistance | Resistance to Oxidation |
|--------|----------------------|--------------------|--------------------|-----------------|-------------------|---------------------|----------------------|-------------------------|
| 1      | SAE20W40             | 200                | -21                | Low             | Very High         | High                | Extremely High        | Extremely High           |
| 2      | Rapeseed oil         | 344                | -10                | Below Average   | High              | Very High           | Very High            | Very High               |
| 3      | Lithium-based grease | 220                | -45                | High            | Extremely High    | Fair                | High                 | High                    |
| 4      | Mobil SHC Pegasus    | 255                | -42                | High            | Very High         | High                | High                 | High                    |
| 5      | Molybdenum disulfide | 4640               | 1185               | Very High       | Extremely High    | Low                 | Low                  | Average                 |
| 6      | Liquid Air           | -194.35            | -215               | Average         | Very Low          | Extremely High      | Average              | Average                 |
| 7      | Mineral oil          | 310                | -15                | Average         | Low               | High                | High                 | High                    |

Vegetable Oil

The different vegetable oils used in the bearing are soybean oil, rapeseed oil, rice bran oil, canola oil, sunflower oil etc. [37]. It is noticed that the variation of pressure inside bearing using rapeseed oil and SAE 20W40 is similar with a variation of 10 to 20% and the plot of pressure distribution is very much similar with a small variation in the range of the pressure for rapeseed oil. The plot of pressure distribution, at the same operating conditions, is obtained that for soya bean oil is not similar to rapeseed oil and the maximum pressure range is varied from 50 to 75% [37]. But the heat generation is much higher in the case of soya bean oil as compared with rapeseed oil because soya bean oil has very low viscosity. The soya bean oil forms a very thin oil film and because of that, the metal-to-metal contact between the journal and shaft is more as compared with rapeseed oil. Therefore it can be concluded that soya bean oil doesn’t work efficiently in journal bearing [37].

Solid Lubricant

The solid lubricant always employed as a coating between the mating surfaces of bearing, to trim down the rate of friction and wear. These type of coatings contains PTFE (Polytetrafluoroethylene), graphite, gold, silver and MoS2 (Molybdenum disulphide). The solid lubricants are being employed in an extreme environment, where greases and other bearing oils are not suitable. These solid films of the lubrication are engineered for a certain use by means of its application [30]. In tribology, dry particulate (solid) lubricants are divided into two types i.e. powder lubricant and granular lubricant. Granular lubricants contain very hard, cohesionless and dry granular particles. Recently an innovative lubrication approach has been considered as powder lubrication because powder lubricants contain comparatively soft cohesive and dry particles which behave like hydrodynamic lubricant [33]. The extensive experiments have been revealed that a powder lubricant containing TiO2 (titanium dioxide) or MoS2 (Molybdenum disulphide) applied into the clearance of bearing can be responsible for more useful lubrication as compared with coating [30]. According to the hypothesis of the quasi-hydrodynamic system of lubrication, powders can also flow like viscous liquids. A film of powder is being formed between the two sliding or rubbing surfaces due to which hydrodynamic pressure can be generated and the load carrying capacity of bearing under powder lubricant is as like as liquid lubricant. For an example, the tungsten disulphide powder offered a very good resistance to wear of the journal and bearing [31]. The layer or film of powder lubricant generates lift to detach the mating surfaces of bearing and due to that side leakage of the bearing will occur. The advantage of the side leakage is that it takes away the major part of heat which is generated by shear. They generate lift and encourage side leakage, which lessens the friction and generation of heat in the bearing, thereby greatly mitigating the rate of wear [32].
Micropolar Lubricant

A special kind of liquid lubricant introduced which is called as micropolar fluid. It contains small solid particles in suspension which is in the form of a colloid. During any machining operation, the lubricant used in the machine is contaminated with dirt and small metal particles. Then the lubricant behaves like a non-Newtonian fluid. Then the micropolar fluid theory comes into action to overcome this problem of lubricant [6]. The micropolar lubricant is defined as the fluid containing a colloid or suspended particles which move with their individual motion [6]. These solid particles make an influence on the lubrication system of journal bearing, ball bearing or rolling bearings and thrust bearing, which will be described by the micropolar fluid theory. It has been stated that the rigid and solid particles present in the lubricant increase its effective viscosity and it will also improve the capacity of load carrying of the bearing [34, 51-53].

Some of the experiments conducted with micropolar lubricant suggested that the direct fluid film stiffness of lubricant and coefficient of damping of fluid film in rolling bearing and slot-entry hybrid journal bearings increased as compared with the same characteristics rolling bearing and slot-entry hybrid journal bearing lubricated using Newtonian lubricant [5-6].

Nano-Lubricant Additives

The tribologists around the globe have recommended the nanoparticles as a most apposite lubricant additive because of its ability to increase the friction and wear resistance behaviour between contact surfaces of bearings. A study reported that the load carrying capacity of the journal bearings has drastically increased (up to 40%) with the addition of TiO2 (Titanium Oxide) nanoparticle with engine oil, as compared with the plain (without any nanoparticle) engine oil [38]. However, the bearing surfaces under pure rolling friction would have been tarnished and corroded after continued operation due to the major change in the composition of the grease, which will conquer by the use of lithium as an additive with grease [39]. On the other hand, ultrafine additives contain ultrafine particles of fullerene, fluoropolymer and fullerene black, which are used as additives to decrease the coefficient of friction between journal and bearing, vibration level of bearing and increase the capacity of the journal bearing to carry the load. These properties of the ultrafine additives directly depend on the pseudoplastic behaviour of the used sample which is a mixer of bearing oil and ultrafine additives [40, 41]. In another case, the presence of talc particles as an additive with oil in the range of 3% to 5%, which improves the properties of oil in terms of reduced coefficient of friction, frictional heating and wear loss of bearings. But the talc particles gives a reverse effect when it mixed with oil in higher concentrations [54].

RESULTS AND DISCUSSION

Based on above literature review of different lubricants, we recommend a lubricant which acts perfectly on a sliding contact bearing in all aspects and in any circumstances [42-43]. Now, to acquire the best one, we use a hybrid opti-
attributes are being assigned a particular quantitative scale (Table 2). This data sheet consists of some qualitative measures which will be converted to quantitative measures by using a Fuzzy scale (Table 2).

Recommendation of Best Lubricant

The steps involved in the process adopted for recommendation are described as follows: All these performance parameters with the criteria and alternatives produce the decision matrix. The following decision matrix (Table-1) is developed based on real data which gives an idea that which alternative bearing oil performs in what fashion in a sliding contact bearing. The different criteria are already set in the decision matrix. At first, the relative weight factor among all the criteria is to be set and for which an Entropy method is used in this paper. After multiplying this relative weight to the fuzzified decision matrix, the weighted fuzzified decision matrix is developed. In this weighted fuzzified decision matrix, PROMETHEE method is used to rank the different alternative bearing oil lubricants and to select the proper one. From decision matrix (Table 1) fuzzified decision matrix (Table 3) conversion is done by the fuzzy scale sown in Table 2.

This data sheet consists of some qualitative measures which will be converted to quantitative measures by using a Fuzzy scale (Table 2).

Now, Table 1 is converted to the following table by using the Fuzzy scale in which the qualitative measures of attributes are being assigned a particular quantitative measured value.

### Entropy Weight Measurement Method

Determination of the weight of each attribute is a very important step in any optimization process. The two main weight measurement methods are Subjective Weight Measurement and Objective Weight measurement. This paper uses an objective Weight Measurement technique i.e. Entropy Weight Measurement method [43]. So, this method is used to determine the weightages of each attribute such as Boiling Point (°C), Freezing Point (°C), Viscosity Index, Thermal stability, Hydraulic stability, Corrosive Resistance and Resistance to Oxidation. The following Table-4 shows that the resultant weightages of the seven attributes and the calculations are shown in Appendix I (Table 7 and Table 8).

### PROMETHEE

The PROMETHEE methodology has been first proposed by Professor Jean-Pierre Brans in 1982 [45] and this is a kind of method which will make an outranking relation among the alternatives present in this work. In this method, the decision makers should give their preference to all alternatives as a function and the value of this function ranges from 0 to 1 [46]. The findings of the present work using Entropy-PROMETHEE method is given below.

The calculation of the Entering flow, Leaving flow and Net flow related to different alternatives has been tabulated according to the PROMETHEE method and the other steps involved in the proposed method is given in Appendix I (from Table 9 to Table 16). Now compare the net flow values given in Table 5 and then it is very clear that the lubricant SAE20W40 designated as the best choice among the different lubricants considered for the sliding contact bearing under the given circumstances. According to the Entropy-PROMETHEE method, the ranking of lubricants is as follows 1-3-2-5-7-4-6. The present results are compared and validated with the previously published results of Ref. [50].

### Table 5: Final Dominance Matrix with Leaving, Entering and Net Flow values for all alternatives

| Sl. No | Lubricants       | Leaving Flow (ɸ⁺) | Entering Flow (ɸ⁻) | Net Flow (ɸ⁺ - ɸ⁻) | Rank |
|--------|------------------|-------------------|--------------------|---------------------|------|
| 1      | SAE20W40         | 4.28572           | 0.99997            | 3.28575             | 1    |
| 2      | Rapeseed oil     | 3.14299           | 2.57122            | 0.57177             | 3    |
| 3      | Lithium-based grease | 3.14273       | 2.14301            | 0.99972             | 2    |
| 4      | Mobil SHC Pegasus| 2.42847           | 2.57147            | -0.14300            | 5    |
| 5      | Molybdenum disulfide | 1.14259        | 4.71448            | -3.57189            | 7    |
| 6      | Liquid Air       | 3.00015           | 3.14268            | -0.14253            | 4    |
| 7      | Mineral oil      | 2.00002           | 2.99984            | -0.99982            | 6    |
CONCLUSIONS

The increasing scientific application and importance of lubricants (mainly synthetic) including surface contacts where the absence of a rigorous proactive classification of the lubrication system in the field of industrial application is felt. This shortcomings led this review for the oils in terms of the theoretical assumptions and description of different lubricants with sufficient details and also to recommend a best-suited lubricant for the specific bearing.

In view of the literature survey, the majority of studies related to the impact on the performance of different types of bearings with a number of bearing lubricants. Every single lubricant has its own merits and demerits when they are pressed to perform towards different bearings. So improving the specific and significant properties of the lubricants, various types of additives are used with mineral and synthetic oils, which was discussed broadly in the present study. However, the key findings of the present review are as follows:

i. The solid lubricants such as MoS\textsubscript{2} and graphite are efficiently worked up to a temperature range of 350°C to 400°C. But, in high temperature ranging from 900°C to 1000°C, some solid oxides, nitrides and carbides are suitable for sliding contact bearing. On the other hand, ion-plated lead and gold are very effective solid lubricants for rolling contact bearing.

ii. Under micropolar lubricant, the performance characteristics of a journal bearing, rolling contact bearing and thrust bearing are being improved as compared to the customary Newtonian lubricant.

iii. Grease is comparatively and surprisingly best-suited lubricant for rolling contact bearings. The rolling bearings lubricated with grease are always in a condition of starved lubrication. It will occur inside the bearing when the film thickness decreases with the increase in speed of the bearing for a long period of time.

iv. Depending upon the past literature survey and present studies with Entropy-PROMETHEE method, the most influential attributes in lubricants are as follows:

- The viscosity index of the lubricants which is low for SAE20W40 compare to other lubricants.
- Thermal and hydraulic stability of the lubricant which is comparatively very high in case of SAE20W40.
- Last of all, the capability of corrosive and oxidation resistance of the lubricant which is extremely high for SAE20W40 as compare the same with other lubricants.

The area covered under the lubrication and its studies is a vast one and therefore, a judicious comment is made that SAE20W40 synthetic lubricant is the best choice for sliding contact bearing according to the Entropy-PROMETHEE method.

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Conflict of Interest

The authors declare that they have no conflict of interest.

Availability of data and material

The data used in this research work has been placed in its appropriate place.

Code availability

No code is used to pursue this work.

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Appendix I

Table 6 (a): Symbols used for attributes

| A (+) | B (-) | C (-) | D (+) | E (+) | F (+) | G (+) |
| (+) denotes Beneficial criteria | (-) denotes Non-beneficial criteria |

Boiling Point (°C) | Freezing Point (°C) | Viscosity Index | Thermal stability | Hydraulic stability | Corrosive Resistance | Resistance to Oxidation |

Table 6 (b): Symbols used for lubricant

| L1 | L2 | L3 | L4 | L5 | L6 | L7 |
|----|----|----|----|----|----|----|
| SAE20W40 | Rapeseed oil | Lithium-based grease | Mobil SHC Pegasus | Molybdenum disulfide | Liquid Air | Mineral oil |

Entropy method for finding the weight of attributes

Table 7: Normalized matrix

| Attributes | Boiling Point (°C) | Freezing Point (°C) | Viscosity Index | Thermal stability | Hydraulic stability | Corrosive Resistance | Resistance to Oxidation |
|------------|-------------------|---------------------|----------------|------------------|---------------------|---------------------|------------------------|
| Alternatives | A (+) | B (-) | C (-) | D (+) | E (+) | F (+) | G (+) |
| SAE20W40 (L1) | 0.08157 | 0.86142 | 1 | 0.85714 | 0.66667 | 1 | 1 |
| Rapeseed oil (L2) | 0.11135 | 0.85357 | 0.8 | 0.71428 | 0.83333 | 0.85714 | 0.85714 |
| Lithium-based grease (L3) | 0.08571 | 0.87857 | 0.2 | 0.85714 | 0.33333 | 0.85714 | 0.85714 |
| Mobil SHC Pegasus (L4) | 0.09294 | 0.87642 | 0.2 | 0.85714 | 0.66667 | 0.71428 | 0.71428 |
| Molybdenum disulfide (L5) | 1 | 0 | 0 | 1 | 0 | 0.14285 | 0.14285 |
| Liquid Air (L6) | 0 | 1 | 0.6 | 0 | 1 | 0 | 0 |
| Mineral oil (L7) | 0.10432 | 0.85714 | 0.6 | 0.14285 | 0.66667 | 0.71428 | 0.71428 |
Table 8: Weightages of each attributes

| Alternatives | Boiling Point (°C) | Freezing Point (°C) | Viscosity Index | Thermal stability | Hydraulic stability | Corrosive Resistance | Resistant to Oxidation | Total |
|--------------|-------------------|---------------------|-----------------|-------------------|---------------------|---------------------|------------------------|-------|
| L1 SAE20W40  | 0.05140           | 0.13927             | 0.33300         | 0.16667           | 0.13333             | 0.19445             | 0.19445                |       |
| L2 Rapeseed oil | 0.07017         | 0.13801             | 0.22222         | 0.20820           | 0.16667             | 0.16667             | 0.16667                |       |
| L3 Lithium-based grease | 0.05401     | 0.14161             | 0.05556         | 0.16667           | 0.23322             | 0.16667             | 0.16667                |       |
| L4 Mobil SHC Pegasus | 0.05856       | 0.14170             | 0.05556         | 0.16667           | 0.13333             | 0.13889             | 0.13889                |       |
| L5 Molybdenum disulfide | 0.63018 | 0.016168               | 0.16667         | 0.26376           | 0.19434             | 0.19438             |                        |       |
| L6 Liquid Air | 0                | 0.16168             | 0.16667         | 0.2               | 0                   | 0                   |                        |       |
| L7 Mineral oil | 0.06574         | 0.13858             | 0.16667         | 0.02778           | 0.13333             | 0.13889             | 0.13889                |       |

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Table 9: Preference Matrix 1 (Dominant to boiling point) (+) (Wt. = 0.14287)

| L1 L2 L3 L4 L5 L6 L7 |
|---------------------|
| L1 1 1 1 1 1 1 0    |
| L2 0 0 0 0 1 0       |
| L3 0 1 - 1 1 0 1     |
| L4 0 1 0 - 1 0 1     |
| L5 0 0 0 0 - 0 0     |
| L6 1 1 1 1 1 - 1     |
| L7 0 1 0 0 1 0 -     |

Table 10: Preference Matrix 2 (Dominant to freezing point) (-) (Wt. = 0.14283)

| L1 L2 L3 L4 L5 L6 L7 |
|---------------------|
| L1 - 1 0 0 1 0 0     |
| L2 0 - 0 0 1 0 0     |
| L3 1 1 - 1 1 0 1     |
| L4 1 1 0 - 1 0 1     |
| L5 0 0 0 0 - 0 0     |
| L6 1 1 1 1 1 - 1     |
| L7 0 1 0 0 1 0 -     |

Table 11: Preference Matrix 3 (Dominant to Viscosity Index) (-) (Wt. = 0.14285)

| L1 L2 L3 L4 L5 L6 L7 |
|---------------------|
| L1 - 1 1 1 1 1 1     |
| L2 0 - 1 1 1 1 1     |
| L3 0 0 - 0 1 0 0     |
| L4 0 0 0 - 1 0 0     |
| L5 0 0 0 0 - 0 0     |
| L6 0 0 1 1 1 - 0     |
| L7 0 0 1 1 1 0 -     |

Table 12: Preference Matrix 4 (Dominant to Thermal Stability) (+) (Wt. = 0.14281)

| L1 L2 L3 L4 L5 L6 L7 |
|---------------------|
| L1 - 1 0 0 0 1 1     |
| L2 0 - 0 0 0 1 1     |
| L3 0 1 - 0 0 1 1     |
| L4 0 1 0 - 0 1 1     |
| L5 1 1 1 1 - 1 1     |
| L6 0 0 0 0 - 0 0     |
| L7 0 0 0 0 0 - 0     |
Table 13: Preference Matrix 5  
(Dominant to Hydraulic Stability) (+)  
(Wt. = 0.14290)

|     | L1 | L2 | L3 | L4 | L5 | L6 | L7 |
|-----|----|----|----|----|----|----|----|
| L1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  |
| L2  | 1  | 0  | 0  | 1  | 1  | 0  | 0  |
| L3  | 0  | 0  | 0  | 0  | 1  | 0  | 0  |
| L4  | 0  | 0  | 1  | 1  | 1  | 1  | 1  |
| L5  | 0  | 0  | 0  | 0  | 0  | 1  | 0  |
| L6  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| L7  | 0  | 0  | 0  | 0  | 0  | 1  | 0  |

Table 14: Preference Matrix 6  
(Dominant to Corrosive Resistance) (+)  
(Wt. = 0.14285)

|     | L1 | L2 | L3 | L4 | L5 | L6 | L7 |
|-----|----|----|----|----|----|----|----|
| L1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  |
| L2  | 1  | 0  | 0  | 1  | 1  | 1  | 1  |
| L3  | 0  | 0  | 0  | 0  | 1  | 1  | 1  |
| L4  | 0  | 0  | 0  | 0  | 0  | 1  | 1  |
| L5  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| L6  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| L7  | 0  | 0  | 0  | 0  | 1  | 1  | 1  |

Table 15: Preference Matrix 7  
(Dominant to Resistance to Oxidation) (+)  
(Wt. = 0.14288)

|     | L1 | L2 | L3 | L4 | L5 | L6 | L7 |
|-----|----|----|----|----|----|----|----|
| L1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  |
| L2  | 1  | 0  | 0  | 1  | 1  | 1  | 1  |
| L3  | 0  | 0  | 0  | 1  | 1  | 1  | 1  |
| L4  | 0  | 0  | 0  | 0  | 1  | 1  | 1  |
| L5  | 0  | 0  | 0  | 0  | 0  | 1  | 1  |
| L6  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| L7  | 0  | 0  | 0  | 0  | 1  | 1  | 1  |

Table 16: Resulting preference indices with Leaving, Entering, Net Flow values and Ranks for all alternatives

|     | L1      | L2      | L3      | L4      | L5      | L6      | L7      | Leaving Flow (Φ+) | Net Flow (Φ±-Φ) | Rank |
|-----|---------|---------|---------|---------|---------|---------|---------|-------------------|-----------------|------|
| L1  |         | 0.85709 | 0.71435 | 0.57146 | 0.87570 | 0.71426 | 0.71429 | 4.28572           | 3.28575         | 1    |
| L2  | 0.14280 |         | 0.28575 | 0.57146 | 0.71431 | 0.71426 | 0.71429 | 3.14299           | 0.57177         | 3    |
| L3  | 0.14283 | 0.42851 |         | 0.57143 | 0.87570 | 0.42854 | 0.71426 | 3.14273           | 0.99972         | 2    |
| L4  | 0.14283 | 0.42851 | 0.14290 |         | 0.28575 | 0.57146 | 0.42854 | 2.42847           | -0.14300        | 5    |
| L5  | 0.14281 | 0.14281 | 0.14281 | 0.28575 |         | 0.57146 | 0.14281 | 1.14259           | -3.57189        | 7    |
| L6  | 0.42860 | 0.14280 | 0.28575 | 0.57145 | 0.42860 |         | 0.14281 | 3.00015           | -0.14253        | 4    |
| L7  |         |         |         |         |         |         | 2.00002 | -0.99982          | -              | 6    |

Entering Flow (Φ-): 0.85999  2.57122  2.57147  4.71448  3.14268  2.00002  -99982