Lambda-proton correlations in relativistic heavy ion collisions

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The prospect of using \(\Lambda p\) correlations to extract source sizes in relativistic heavy ion collisions is investigated. It is found that the strong interaction induces a large peak in the correlation function that provides more sensitive source size measurements than \(pp\) correlations under some circumstances. The prospect of using \(\Lambda p\) correlations to measure the time lag between lambda and proton emissions is also studied.

Two-particle correlations have proven to be a powerful tool for determining source sizes and lifetimes in heavy ion collisions. At low energies, correlations of protons, neutrons and intermediate mass fragments have provided information on the space-time extent of the collision systems \(^1\). At relativistic energies, pion, kaon and proton correlations have greatly enhanced our understanding of the dynamics of heavy-ion collisions \(^2\): these correlations provide different but complementary information. For instance, heavier particles are more affected by collective flow, thus making the mass-dependence of source sizes a test of our picture of explosive flow in heavy ion collisions; freeze-out conditions may be different for pions, kaons and protons, thus comparing parameters inferred from their correlations allows one to test the conjecture of sequential freeze-out.

In this letter, we explore lambda-proton (\(\Lambda p\)) correlations as a candidate of interferometric study. We find that an enhancement to the correlation function at low relative momentum allows one to infer the size of the emitting source. The inferred lambda source parameters may provide valuable information because lambda are strangeness carrying baryons. Unlike two-proton (\(pp\)) system, the \(\Lambda p\) system has no repulsive Coulomb interaction. Thus the enhancement from the strong interaction better survives when source sizes become large. We illustrate the sensitivity of \(\Lambda p\) correlations and show that for large sources, they might be more sensitive than \(pp\) correlations, but not as sensitive as coalescence measurements. We also study the possibility to determine whether lambdas and protons are emitted simultaneously by comparing the correlations for positive and negative values of the projected outward relative momentum.

The correlation of two particles from a chaotic source may be estimated by assuming that they interact only with each other after they are emitted from space time points \(x_a\) and \(x_b\) \(^3\).

\[
C(p_a, p_b) = \frac{P(p_a, p_b)}{P(p_a)P(p_b)} 
\approx \frac{\int d^4x_a d^4x_b S_a(p_a, x_a)S_b(p_b, x_b)\phi_{\text{rel}}(p_b - p_a)^2}{\int d^4x_a d^4x_b S_a(p_a, x_a)S_b(p_b, x_b)}
\]

In principle, the correlation depends on the size and shape of the source described by function \(S(p, x)\), which provides the differential probability of emitting particles of momentum \(p\) at a space-time point \(x\). However, for the purposes of our study we will ignore the momentum dependence of the source functions and assume a Gaussian form for \(S\).

\[
S(x_a) = \delta(t) \exp \left(\frac{-x^2 + y^2 + z^2}{2R_g^2}\right),
\]

where \(R_g\) is the Gaussian size of the source.

The correlation function’s sensitivity to the source size depends on the form of the relative wave function, \(\phi_{\text{rel}}\). The relative wave function is determined by the relative strong interaction, the relative Coulomb interaction, and in the case of identical particles, symmeterization constraints. Since the \(\Lambda p\) system involves non-identical particles, and interacts mostly through the s-wave channel at low relative momentum, it is largely insensitive to details about the shape of the emitting source. However, since the strong interaction is short range, the enhancement of the correlation function at low relative momentum is highly sensitive to the size. Furthermore, the lack of a Coulomb repulsion, which dominates \(pp\) correlations at low relative momentum, allows the \(\Lambda p\) correlation function to remain sensitive to the volume for fairly large sources.

An Urbana-type potential which was motivated from low energy \(\Lambda p\) scattering and hypernuclei bind energy data \(^4\) is used to generate the relative wave functions.

\[
V_{\Lambda p} = V_C - \left(\vec{V} - \frac{1}{4} V_\sigma \sigma_\Lambda \cdot \sigma_p\right) T_\pi^2,
\]

where \(V_C\) is a Woods-Saxon repulsive core.

\[
V_C = W_C \left[1 + \exp \left(\frac{r - R}{d}\right)\right]^{-1},
\]

with \(W_C=2137\) MeV, \(R=0.5\) fm, \(d=0.2\) fm. The modified one-pion exchange tensor potential

\[
T_\pi = \left(1 + \frac{3}{x} + \frac{3}{x^2} \right) \frac{e^{-x}}{x} \left(1 - e^{-cr^2}\right)^2,
\]
where \( x=0.7r \) and \( c=2 \text{ fm}^{-2} \). The spin-independent part of the attractive potential is characterized by \( V = 6.2 \pm 0.05 \text{ MeV} \), while the spin-dependent part is small, \( V_\sigma = 0.25 \pm 0.25 \text{ MeV} \), and not well determined.

For illustration, we have used the same source size for both lambda and proton, and assumed thermal momentum distributions. To eliminate the Lorentz factor effect and also for computational reasons, we have used thermal temperature \( T=3 \text{ MeV} \). However, the Lorentz factor effect is small due to the large lambda and proton masses. For instance, the difference in the correlation functions between \( T=3 \) and \( 300 \text{ MeV} \) is less than 5%.

An experimental correlation function often resorts to a mixed-event technique for uncorrelated pairs. Since both the height and the width of a \( \Lambda p \) correlation function are sensitive to the source size, it is important to normalize the correlation function properly. Since lambda and proton exhibit no correlation at large \( k \), an experimental \( \Lambda p \) correlation function may be normalized to unity at large \( k \).

The \( \Lambda p \) correlation function is one fourth spin singlet \((S=0)\) and three fourths spin triplet \((S=1)\). To illustrate the spin dependence, correlation functions are presented for a \( R_\sigma=4 \text{ fm} \) source in the upper panel of Fig. 2, separately for \( S=0 \) and \( S=1 \) pairs. If \( V_\sigma \) in Eq. (3) were zero, the two contributions would be identical. As \( V_\sigma \) is not well understood, \( 0 < V_\sigma < 0.5 \text{ MeV} \), measuring the spin dependence of the correlation function could in principle determine \( V_\sigma \).

The correlation function is largely determined by the scattering length and effective range of the potential \( \bar{V} \). The scattering length and effective range corresponding to the potential \( \bar{V} = 6.2 \text{ MeV}, V_\sigma = 0.25 \text{ MeV} \) are \(-2.88 \text{ fm} \) and \( 2.92 \text{ fm} \) for the spin singlet, and \(-1.66 \text{ fm} \) and \( 3.78 \text{ fm} \) for the spin triplet, respectively. They are in reasonable agreement with those from Refs. [4,7]. Using these values and an analytical approximation similar for neutron-proton correlations \( pp \), we obtain \( \Lambda p \) correlation functions that are consistent with the results in the upper panel of Fig. 2.

The sensitivity of the spin-averaged correlation function to the parameters \( \bar{V} \) and \( V_\sigma \) is illustrated in the lower panel of Fig. 2. Using the stated uncertainties \( \bar{V} = 6.2 \pm 0.05 \text{ MeV} \) and \( V_\sigma = 0.25 \pm 0.25 \text{ MeV} \), the correlation function for a \( R_\sigma=4 \text{ fm} \) source is shown for a range of parameters. The results suggest that the uncertainties in the potential parameters translate into an approximately \( \pm 0.5 \text{ fm} \) uncertainty in the source size extracted from the correlation function at \( k<25 \text{ MeV}/c \),
while the large \( k \) tail has better constrain on the source size.

The displacement direction, \( \hat{z} \), is then defined by the direction of \( v \), or that of the pair momentum relative to the source. Thus, if \( \Lambda p \) correlations can be measured with an accuracy of a few percent at \( k \sim 30 \text{ MeV/c} \), the conjecture that strange and non-strange baryons are emitted simultaneously can be addressed quantitatively.

One should remember that the residual interaction of the proton with the Coulomb field of the nuclear sources might distort the result. Unlike \( pp \) correlations, where both particles feel the identical force, only the proton experiences the Coulomb field. This issue has been previously considered in the context of \( pn \) correlations where the distortion was shown to be small for fast-moving pairs. This effect should be smaller at RHIC where the excess charge at midrapidity is expected to be smaller than observed at SPS and AGS energies.

Finally, we make a brief note regarding \( \Lambda \Lambda \) and \( \bar{\Lambda}p \) correlations. In the view of our \( \Lambda p \) results, we expect that the \( \Lambda \Lambda \) strong interaction would also give sizeable correlations. These correlations are of great interest because of the predicted existence of a \( \Lambda \Lambda \)-like particle. In conjunction with \( \Lambda p \), \( \bar{\Lambda}p \) correlations may reveal valuable information on low energy \( \Lambda p \) annihilation cross sections which are presently unknown but indispensable in modeling certain aspects of heavy-ion collisions.

In summary, \( \Lambda p \) correlations may provide a useful characterization of the space-time structure of relativistic heavy ion collisions should one be able to gather sufficient statistics. The lack of a relative Coulomb interaction allows the strong interaction to produce a large peak in the correlation function even for large sources, to which \( pp \) correlation loses its sensitivity. Furthermore, by binning according to the sign of the projected relative momentum in the direction defined by that of the displaced pair. An example is illustrated in Fig. 3, where both sources are assumed to be characterized by a size of 4 fm, but are separated by \( \Delta z = 1.5 \text{ fm} \) or 3.0 fm. In Fig. 3 the differences of the correlation functions, \( C_+(k) - C_-(k) \), is plotted against \( k \) where the transverse component of \( k \) is required to be less than 10 MeV/c. Here, \( C_+ \) refers to the correlation function constructed with the requirement that \( k_z > 0 \), while \( C_- \) is constructed with the opposite constraint.

A displacement \( \Delta z \) can result from a displacement in time, \( \Delta \tau = \Delta z/v \), where \( v \) is the velocity of the \( \Lambda p \) pair.

FIG. 2. Upper panel: the spin decomposition of the \( \Lambda p \) correlation function for a \( R_p = 4 \text{ fm} \) source, with the spin-averaged result represented by the thick solid line. Lower panel: the sensitivity of the correlation function to the uncertainties in the potential parameters. Default values (\( V = 6.2 \text{ MeV}, V_z = 0.25 \text{ MeV} \)) are represented by the thick solid line. Changing \( V \) yields the results represented by the squares; changing \( V_z \) yields the results represented by the circles. The uncertainties in the parameters translate to a \( \pm 0.5 \text{ fm} \) uncertainty in the extracted source size.

Recently, Lednicky et al. have shown that correlations of non-identical particles can provide information revealing whether the particles are emitted simultaneously. If the lambdas are emitted before the protons in such a way that the probability cloud describing the protons lags that for the lambdas of the same velocity, the correlation function then depends on the sign of the relative momentum in the direction defined by that of the displacement of the lambda and proton clouds. An example is illustrated in Fig. 3, where both sources are assumed to be characterized by a size of 4 fm, but are separated by \( \Delta z = 1.5 \text{ fm} \) or 3.0 fm. In Fig. 3 the difference of the correlation functions, \( C_+(k) - C_-(k) \), is plotted against \( k \) where the transverse component of \( k \) is required to be less than 10 MeV/c. Here, \( C_+ \) refers to the correlation function constructed with the requirement that \( k_z > 0 \), while \( C_- \) is constructed with the opposite constraint.

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momentum, one might address the question of whether lambda and protons are emitted simultaneously. However, the interpretation of the correlation function could benefit from a more precise parameterization of the Λp interaction.

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