Study on frequency characteristics of wireless power transmission system based on magnetic coupling resonance

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Abstract. In order to study the frequency characteristics of the wireless energy transmission system based on the magnetic coupling resonance, a circuit model based on the magnetic coupling resonant wireless energy transmission system is established. The influence of the load on the frequency characteristics of the wireless power transmission system is analysed. The circuit coupling theory is used to derive the minimum load required to suppress frequency splitting. Simulation and experimental results verify that when the load size is lower than a certain value, the system will appear frequency splitting, increasing the load size can effectively suppress the frequency splitting phenomenon. The power regulation scheme of the wireless charging system based on magnetic coupling resonance is given. This study provides a theoretical basis for load selection and power regulation of wireless power transmission systems.

1. Introduction

Wireless power transmission technology (WPT), also known as non-contact power transmission technology is a power supply mode that transfers power from the power supply to the electrical equipment by means of a space medium. WPT avoids the connection between the electrical equipment and the grid and it achieves full electrical isolation. The transmission process is safe, reliable, and flexible, providing the possibility for man to get rid of the wire. WPT is a basic research and an applied research of multi-disciplinary cross-over of electromagnetic field, power electronics, high-frequency electron, electromagnetic induction and coupled-mode theory. It is a revolutionary progress in energy transmission [1]. WPT is important in military, aerospace, oilfield, mine, underwater operations, industrial robots, electric vehicles, wireless sensor networks, medical devices, household appliances and RFID identification. And WPT also has important scientific and research value and practical significance in the development of electromagnetic theory.

In 2007, Marin Soljacic of MIT proposed the principle of magnetic coupling resonant wireless power transmission technology. They used the theory to light up a 60 W bulb in the 2 m range successfully [2]. Since then, wireless power transmission technology has become a hot topic of domestic and foreign scholars.

At present, there are three main technologies for wireless power transmission: radio frequency or microwave wireless power transmission technology, electromagnetic induction wireless transmission technology and magnetic coupling resonant wireless power transmission technology (MCR-WPT).
Compared to the first two technologies, MCR-WPT has the following advantages: MCR-WPT has a larger transmission distance; Because of the use of strong coupling resonant technology, MCR-WPT can achieve higher reception power and efficiency; The system uses magnetic field coupling rather than electric field, and has no harm to the human body; MCR-WPT has good penetrability and is not affected by nonmetallic obstructions. Therefore, the magnetic coupling resonant wireless energy transmission system has become a new direction of development of wireless energy transmission technology, and more and more researchers carry out research work in this direction.

The key to the wireless energy transmission technology based on magnetic coupling resonant is that the system should be in a resonant state, to obtain a larger reception power and efficiency. Therefore, maintaining the stability of the resonant frequency is a key problem of the magnetic coupling resonant wireless power transmission technology. However, the resonant frequency of the magnetic coupling resonant wireless power transmission system is not only affected by the coil radius, coil shape and other factors, but also by the distance between the coil and the impact of the load and the resonant frequency of the system will split. [3-6] analyze the frequency splitting characteristics from different aspects. [7-9] propose the problem of efficiency reduction caused by frequency tracking to solve the problem of frequency splitting. [10] analyzes the maximum power transmission efficiency of a wireless power transmission system.

This paper considers the resonant frequency, load resistance and coil resistance and an equivalent model based on magnetic coupling resonant wireless power transmission is established. The effect of load resistance on the received power and energy transmission efficiency is analyzed, and the frequency characteristics of received power and efficiency are obtained. A magnetic coupling resonant wireless power transmission system is designed. The frequency characteristics of different load resistance values are simulated and verified.

2. System model and theoretical analysis
Magnetic coupling resonant wireless power transmission is based on the concept of near field coupling. The basic principle is that two objects with the same resonant frequency can achieve efficient energy exchange, while the energy exchange between no resonant objects is very weak. Figure 1 is a block diagram of a magnetic coupling resonant wireless power transmission system.

![Figure 1. Block diagram of magnetic coupling resonant wireless power transmission system.](image)

As can be seen from figure 1, $U_s$ is the three-phase grid voltage which is rectified and inverted equivalent to become the primary power source for wireless power transmission. The wireless power transfer between the transmitting coil and the receiving coil through the resonance coupling of the magnetic field of the space. This is the core of the entire wireless power transmission system and the level and characteristics of energy transfer are primarily determined by this.

The equivalent circuit based on the magnetic coupling resonant wireless power transmission system is shown in figure 2. $U_s$ is the equivalent voltage source of the primary side of the grid voltage
obtained by rectification and inverter. \( R_1, R_2 \) are the primary and secondary side coil equivalent resistance, respectively. \( R_L \) is the load resistance. \( C_1, C_2 \) are capacitors and \( L_1, L_2 \) are the coil equivalent inductances.

\[
\begin{align*}
R_1, R_2 & \text{ are the primary and secondary side coil equivalent resistance, respectively.} \\
R_L & \text{ is the load resistance.} \\
C_1, C_2 & \text{ are capacitors and } L_1, L_2 \text{ are the coil equivalent inductances.}
\end{align*}
\]

**Figure 2.** Equivalent circuit of wireless power transmission system.

At present, there are three main theories for quantitative analysis of wireless energy transmission, which are coupled-mode theory, circuit theory and band-pass filter theory. The paper uses the internal equivalent method of the circuit theory of the system for theoretical analysis.

When the coupling coefficient \( k \) between the two coils satisfies the formula (1), the system will appear frequency splitting phenomenon [12].

\[
k \geq \frac{1}{\sqrt{Q_1 Q_2}} \left[ \frac{1}{2} \left( \frac{Q_1}{Q_2} + \frac{Q_2}{Q_1} \right) \right]^{\frac{1}{2}} \tag{1}
\]

Where \( Q_1 \) and \( Q_2 \) are the quality factors of the transmitting coil and the receiving coil, respectively.

\[
\begin{align*}
Q_1 = \frac{\omega L_1}{R_1} \\
Q_2 = \frac{\omega L_2}{R_L}
\end{align*} \tag{2}
\]

Where \( \omega \) is the input frequency of the supply voltage.

Joint formulas (1) and (2) and formula (3) can be obtained by transformation.

\[
R_L \leq \sqrt{\frac{2 \omega^2 k^2 - \frac{R_1^2}{L_1^2} L_2^2}{L_1^2}} \tag{3}
\]

When \( R_L \) satisfies equation (3), the system will appear frequency splitting phenomenon. So the minimum or critical load value of the system to prevent frequency splitting is:

\[
R_{L_{\text{min}}} = \sqrt{\frac{2 \omega^2 k^2 - \frac{R_1^2}{L_1^2} L_2^2}{L_1^2}} \tag{4}
\]

It can be seen from the formula (4), for the different magnetic coupling coefficient \( k \), there is a minimum value \( R_{L_{\text{min}}} \) of the load. When the load is greater than \( R_{L_{\text{min}}} \), the system will not appear frequency splitting phenomenon.

Let the current flow through the transmitting coil and the receiving coil be \( I_1 \) and \( I_2 \) respectively. The mutual inductance between the two coils is expressed as \( M = k \sqrt{L_1 L_2} \). According to Kirchhoff’s theorem,

\[
\begin{align*}
U_0 &= Z_1 I_1 + j \omega M I_2 \\
0 &= j \omega M I_1 + Z_2 I_2
\end{align*} \tag{5}
\]
Where \( Z_1, Z_2 \) are the impedance of the transmitting coil and the receiving coil, respectively.

\[
\begin{align*}
Z_1 &= R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \\
Z_2 &= R_L + R_2 + j\omega L_2 + \frac{1}{j\omega C_2}
\end{align*}
\tag{6}
\]

When the system is in the resonant state, joint formulas (5) and (6) and we can get receipt power on the load as,

\[
P_L = I_2^2 R_L = \frac{(\omega M U_s)^2}{R_1[(R_L+R_2) + (\omega M)^2]} R_L
\tag{7}
\]

Since the equivalent resistance of the coil is small, its influence is negligible. Calculate partial differential of equation (7).

\[
\frac{\partial P_L}{\partial R_L} = 0
\tag{8}
\]

When the received power reaches its maximum value, the best match load is

\[
R_{L\text{max}} = \frac{k^2 L_2}{C_2 R_1}
\tag{9}
\]

It can be seen from equation (9) that the best matching load is not only related to the coil parameters, but also influenced by the magnetic coupling coefficient \( k \).

3. Simulation and analysis

To verify the correctness of the theoretical analysis, this paper uses MATLAB to simulate. In this paper, we set up a simulation circuit model based on the two-coil structure of the circuit in SIMULINK. The parameters of the simulation circuit are set as follows. \( L_1 = 233.6 \ \mu\text{H}, L_2 = 140 \ \mu\text{H}, C_1 = 15 \ \text{pF}, C_2 = 25 \ \text{pF}, R_1 = 0.5 \ \Omega, R_2 = 0.2 \ \Omega, U_s = 200 \ \text{V} \).

For a magnetic coupling coefficient \( k \), the wireless power transmission system will exhibit three states depending on the load: No frequency split phenomenon, which is called under-coupling; Just appear the phenomenon of frequency splitting, known as critical coupling; There is a frequency splitting phenomenon, known as over-coupling. Respectively, take \( k = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 \).

The frequency characteristics of the received power and efficiency at different loads are obtained by simulation as figure 3.
Figure 3. The frequency characteristic of received power.

From figure 3, when the load resistance is lower than the critical resistance, the frequency characteristics of the received power of the wireless power transmission exhibit two peaks. At this point, the system appears frequency splitting phenomenon. When the load resistance is greater and equal to the critical resistance, the received power of the system has only one peak with the frequency change. So, increasing the load resistance can achieve the purpose of suppressing frequency splitting.

Figure 4 shows the frequency characteristic curves of the system transmission efficiency corresponding to the three typical load resistance values when k is different. When the load resistance value changes from the undercounted state to the over-coupled state, the frequency characteristic of the efficiency gradually becomes smooth from the spikes. As the load resistance decreases, the transmission efficiency of the system increases. When the frequency of the system is split, the transmission efficiency of the system maintains a high value over a wide frequency range. However, when the load resistance is too low, the system transmission efficiency appears multiple peaks. This provides a reference for the power regulation of the wireless charging system.

Table 1 gives a comparison of the theoretical and simulated values of the minimum load resistance $R_{\text{Lmin}}$ for suppressing frequency splitting. From table 1, the difference between the theoretical calculated value and the simulated value of the critical resistance $R_{\text{Lmin}}$ is small and the error is kept within 2%. The simulation results verify the correctness of the theoretical analysis.
Figure 4. The relationship between transmission efficiency and system frequency.

Table 1. Comparison of theoretical value and simulation value of critical resistance $R_{LMin}$.

| $k$  | Theoretical Value /Ω | Simulation Value /Ω | Error/% |
|------|-----------------------|---------------------|---------|
| 0.05 | 5.27                  | 5.3                 | 0.57    |

Table 1. Comparison of theoretical value and simulation value of critical resistance $R_{LMin}$. 

| $k$  | Theoretical Value /Ω | Simulation Value /Ω | Error/% |
|------|-----------------------|---------------------|---------|
| 0.05 | 5.27                  | 5.3                 | 0.57    |
4. Experimental verification

To verify the correctness of the theoretical analysis of received power and efficiency frequency characteristics, this paper builds a wireless charging system based on magnetic coupling resonance. The resonant frequency of the system is 85 kHz. The equivalent inductance of the transmitting and receiving coils are $L_1 = 233.6 \mu H$, $L_2 = 140 \mu H$. Capacitance are $C_1 = 15 \mu F$, $C_2 = 25 \mu F$. Coil equivalent resistance are $R_1 = 0.5 \Omega$, $R_2 = 0.2 \Omega$. The equivalent supply voltage is $U_s = 200 V$. The experimental platform is shown in figure 5.

| $r$  | $P_r$ | $\eta$ | $\eta_1$ |
|------|-------|--------|----------|
| 0.1  | 10.57 | 10.8   | 2.10     |
| 0.15 | 15.85 | 16     | 0.95     |
| 0.2  | 21.14 | 21.5   | 1.70     |
| 0.25 | 26.42 | 26.5   | 0.30     |
| 0.3  | 31.71 | 31.5   | 0.66     |

Figure 5. Experimental platform of wireless energy transmission system based on magnetic coupling resonance.

Both the transmitting coil and the receiving coil adopt a circular coil structure. During the experiment, the diameter of the coils is kept constant, and the coil of the transmitting system is placed in parallel with the coil of the receiving system. The distance between the transmitting coil and the receiving coil is maintained at 9 cm. The magnetic coupling coefficient of the system is $k = 0.15$. Figure 6 shows the frequency characteristics of the system received power and transmission efficiency when the load is 5 \( \Omega \) and 10 \( \Omega \), respectively. The experimental results are in good agreement with the theoretical analysis, which verifies the correctness of the theoretical analysis.
5. Discussion on power regulation scheme based on wireless charging technology

From the previous analysis, we know that we can adjust the load to change the frequency characteristics of the system, so that the system avoids running in the frequency split state. However, for the wireless charging system, the load is a DC battery of which equivalent impedance is small and cannot be adjusted. Therefore, by adjusting the load resistance to improve the frequency characteristics of the method is no longer applicable. However, the analysis of frequency characteristics provides theoretical support for power regulation. There are two solutions to the wireless charging system to solve the power regulation.

- The frequency characteristic of the received power has a frequency splitting phenomenon, and the frequency characteristic does not appear frequency splitting. From the analysis above, the system at the resonant frequency can still achieve the maximum efficiency. And the efficiency can be maintained at a higher value in a wide frequency range near the resonant frequency. Therefore, we can adjust the system frequency near the resonant frequency, find the power to meet the system requirements. In this way, the system's demand for power and efficiency can be met.

- When the equivalent resistance of the battery is too small, the frequency characteristics of the system will also appear frequency splitting phenomenon. In this case, it is difficult to meet the requirements by adjusting the frequency of the system to adjust the power. So, we can set the frequency of the system at the resonant frequency, so that the system achieves maximum energy transfer efficiency. The output power is then adjusted by adjusting the input voltage of the system, thus enabling the system to meet the power and efficiency requirements.

6. Conclusion

In this paper, the theory and circuit model of wireless energy transmission system based on magnetic coupling resonant are established. The circuit model is analysed, and the frequency characteristics of the magnetic coupling resonant wireless transmission system are analysed. The equation of the minimum value of the load required to suppress the frequency splitting is deduced. The results show that for different coupling coefficients, increasing the load can suppress the occurrence of frequency splitting. The theory is simulated and experimentally verified. Simulation and experimental results verify the correctness of theoretical analysis. The power regulation based on resonant wireless charging system is discussed, and the power regulation scheme is proposed.
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