High-sensitivity monitoring of micromechanical vibration using optical whispering gallery mode resonators

A Schliesser\textsuperscript{1}, G Anetsberger\textsuperscript{1}, R Rivière, O Arcizet and T J Kippenberg\textsuperscript{2,3}

Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany
E-mail: tjk@mpq.mpg.de

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Abstract. The inherent coupling of optical and mechanical modes in high finesse optical microresonators provides a natural, highly sensitive transduction mechanism for micromechanical vibration. Using homodyne and polarization spectroscopy techniques, we achieve shot-noise limited displacement sensitivities of $10^{-19}$ m Hz$^{-1/2}$. In an unprecedented manner, this enables the detection and study of a variety of mechanical modes, which are identified as radial breathing, flexural and torsional modes using three-dimensional finite element modeling. Furthermore, a broadband equivalent displacement noise is measured and found to agree well with models for thermorefractive noise in silica dielectric cavities. Implications for ground-state cooling, displacement sensing and Kerr squeezing are discussed.

\textsuperscript{1} These authors contributed equally to this work.
\textsuperscript{2} Author to whom any correspondence should be addressed.
\textsuperscript{3} Present address: Swiss Federal Institute of Technology (EPFL), Lausanne, Switzerland.
1. Introduction

The transduction and measurement of small displacements of mechanical oscillators is important in a variety of studies ranging from macroscale gravitational wave detection [1] to micron-scale cantilever-based force measurements [2]. One embodiment that is particularly amenable to measurement of small displacements is the parametric coupling of a high-$Q$ electrical or optical resonance to a mechanical oscillator [1]. In the case of a Fabry–Perot interferometer with a harmonically oscillating end-mirror, this parametric coupling manifests itself as a position-dependent shift of the resonance frequency, thereby allowing the transduction of mechanical motion into a change of the phase of a resonant field probing the Fabry–Perot cavity.

However, even for a perfect measurement apparatus, the sensitivity is limited by the laws of quantum mechanics. Fundamentally, any linear measurement process entails a backaction onto the mechanical oscillator, as first discussed by Braginsky [3, 4]. Braginsky identified two types of backaction: quantum backaction and dynamical backaction. Dynamical backaction occurs when the optical interferometer (or cavity) is excited in a detuned manner. In this case, the radiation pressure force exerted by the light in the cavity can become viscous and leads to either amplification or cooling of the mechanical motion. This effect is entirely classical and was first observed in 2005 in the case of radiation-pressure amplification and oscillation [5, 6], and in 2006 also for radiation-pressure cooling [7–9]. For a recent review see [10, 11]. For the case of resonant excitation of the cavity or circuit, this effect can, however, in principle be entirely suppressed. In contrast, quantum backaction cannot be suppressed and arises from the quantum fluctuations of the light (or electric current) involved in the measurement process. Quantum fluctuations of the intracavity light field cause a random force that drives the mechanical oscillator and thereby leads to perturbation of its position. This quantum backaction provides a limit to continuous position measurements and leads to the so-called standard quantum limit (SQL) [3, 12]. At the SQL, the measurement imprecision at the mechanical resonance frequency is equal to the zero-point motion of the mechanical oscillator, to which both shot noise and quantum backaction contribute in equal amounts. Over the past decades, significant progress has been made in approaching this quantum limit of motion measurement in the context of both electromechanical and optomechanical experiments.

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Researchers have investigated a variety of mechanisms and devices as motion transducers of mesoscopic oscillators, for example, quantum point contacts [13, 14], superconducting single-electron transistors [15, 16], atomic point contacts [17], microwave interferometers [1, 18] and magnetomotive transducers [19, 20]. In terms of demonstrated sensitivity, optical transducers [21–23] have been unsurpassed, measuring mechanical displacement down to the $10^{-19}$ m Hz$^{-1/2}$ level for measurement bandwidths up to 1 MHz.

Important to the detection process is that the optical transducer is typically not only sensitive to a single mechanical mode of interest, but also sensitive to any differential change in the cavity’s optical path length. Therefore, all mechanical modes of all cavity boundaries are simultaneously measured, and the signal from the mode of interest may be spectroscopically extracted based on its Fourier frequency. The thermal excitation of other modes, and other effective cavity length fluctuations may constitute a measurement background in excess of the quantum noise of the light used to monitor the mode of interest. Assessing this background therefore proves particularly important if one mode is laser-cooled below the bath temperature as recently achieved [7–9], [23–29]. It is emphasized that any background of effective cavity length fluctuations not only limits the detection sensitivity, but also provides a limit for backaction cooling, as for detuned excitation it gives rise to backaction force fluctuations in excess of quantum fluctuations [29].

In this paper, we provide a broadband analysis of the noise mechanisms in the optomechanical transduction of radial displacements in toroidal silica microcavities, revealing in detail the influence of all other mechanical modes. Toroidal microcavities [30] are optical resonators that host both high-quality optical and mechanical modes in one and the same device, and have been used to demonstrate radiation-pressure dynamical backaction for the first time [5, 6]. Efficient cooling by dynamical backaction [31] has been demonstrated both in the ‘Doppler’ [9] and the resolved-sideband regime [29, 32, 33], rendering them particularly interesting for the goals of the emerging discipline of cavity quantum optomechanics [10, 11], which pertains to studying quantum phenomena of mesoscopic mechanical oscillators. We report broadband interferometric measurement of their radio-frequency mechanical modes based on parametric coupling to the optical whispering-gallery modes (WGMs). Using adaptations of both the Hänisch–Couillaud polarization spectroscopy [34] and optical homodyne measurement, displacement sensitivities at the level of $10^{-19}$ m Hz$^{-1/2}$ are achieved over a measurement bandwidth of up to 20 MHz.

We find sparse spectra of mechanical modes which enables a detailed understanding of the modes using three-dimensional finite-element simulations. More than 20 mechanical modes are observed between dc–100 MHz comprising radial breathing, flexural and torsional modes. We furthermore identify a broadband noise background which is attributed to thermorefractive noise, as previously observed in silica microspheres [35]. The detailed understanding of these noise processes (both due to mechanical modes and thermorefractive noise) is particularly important for studies, such as pondermotive squeezing [36], ground-state cooling [32, 33], as well as squeezing using the third order Kerr nonlinearity of glass [37].

2. Whispering gallery modes for optical motion transduction

Decades of research in the field of gravitational wave astronomy have brought major fundamental [3, 12] and technological advances in interferometric transduction of mechanical displacements. More recent efforts [21, 22, 38] have shown that these techniques are well suited...
Figure 1. Motion transduction with a WGM resonator. (a) Changes $x$ in the cavity radius change the resonance condition for the field $E_{in}$ evanescently coupled to the WGM. Information on the displacement $x$ is therefore imprinted on the field $E_{out}$ transmitted through the fiber taper. (b) Optical microscope image of tapered fiber coupled toroidal microresonator (top view, field of view ca 100 μm × 100 μm).

for application to much lighter oscillators operating at the microscale. Such oscillators are expected to display quantum effects at significantly higher temperatures. In the following, we will briefly review the limits for the interferometric detection of micromechanical oscillations using the ultra-high-quality WGMs in silica toroidal resonators. These devices (figure 1) possess ultra-high-$Q$ optical modes which are confined by total internal reflection [30]. In addition, microcavities also exhibit structural resonances giving rise to high frequency vibrational modes [5, 6, 39]. Mechanical modes which affect the circumference of the cavity shift the optical resonance frequency, and thereby couple to the optical degree of freedom. This was recognized in early experiments that demonstrated the parametric oscillation instability [5, 6] and provides, as shown here, a natural way for highly sensitive motion transduction.

A change of the major radius $R$ by a small displacement $x$ induces a relative shift of the resonance frequency $\omega_0$ of the WGMs located in the rim of the toroid by an amount $\Delta \omega_0/\omega_0 = x/R$. This induces a change in the properties of a field launched into this mode. In the usual coupling geometry using a tapered fiber, the field transmitted through the tapered fiber reads [40]

$$E_{out}(t) = \frac{\tau_{ex} - \tau_0 + 2i(\omega - \omega_0)\tau_{ex}\tau_0}{\tau_{ex} + \tau_0 + 2i(\omega - \omega_0)\tau_{ex}\tau_0} E_{in}(t),$$

where $\tau_{ex}$ and $\tau_0$ are the inverse cavity decay rates due to coupling to the taper and due to other losses. The condition with $\tau_{ex} = \tau_0$ is usually referred to as critically coupled or impedance matched. To first order, the transmitted amplitude is not affected by small mechanically induced resonance frequency shifts $\Delta \omega_0$ for a resonant laser $\omega = \omega_0$, the phase of the field, however, is. By comparison with a phase reference in an interferometric measurement, the displacement $x$ can be thereby be detected. For example, the output field may be brought to interference with a strong field $E_{lo}$ at the same frequency $\omega$ using a half-transmissive beam splitter. Choosing the appropriate phase of the reference field $E_{lo}$, the photon fluxes detected at the two output ports of the splitter are determined by $|E_{out}(t) \pm i E_{lo}(t)|^2/\sqrt{2}$. Subtraction of these two

4 The times $\tau_{ex}$ and $\tau_0$ can also be expressed in terms of the probability of transmission $T$ to the taper and loss $L$ to the environment within the time $\tau_{rt}$ of one round-trip via $\tau_{ex} = \tau_{rt}/T$ and $\tau_0 = \tau_{rt}/L$. 

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simultaneously measured fluxes yields a differential signal
\[
h(x) = \frac{8\omega_0\eta_c \sqrt{P_{in}} P_{lo}}{\kappa R \hbar \omega} x
\]  
(2)
to first order in \(x\), where \(\kappa = \tau_0^{-1} + \tau_{ex}^{-1}\) is the cavity’s total decay rate (i.e. linewidth) and \(P_{in}/\hbar \omega\) and \(P_{lo}/\hbar \omega\) are the photon fluxes corresponding to the fields \(E_{in}\) and \(E_{lo}\). The coupling efficiency
\[
\eta_c \equiv \frac{\tau_0}{\tau_0 + \tau_{ex}}
\]  
(3)
can take values 0 . . . 1, approaching 0 for undercoupling \((\tau_{ex} \to \infty)\) and 1 in the case of overcoupling \((\tau_{ex} \to 0)\), whereas at critical coupling it is equal to 1/2. Physically this quantity thus describes the probability of an intracavity photon coupling to the output fiber. The fundamental noise source in this detection scheme arises from the quantum phase noise of the light being detected. This leads to a spectral density of flux (and thus signal) fluctuations of \(P_{lo}/\hbar \omega\) and \(P_{lo}/\hbar \omega\) are the photon fluxes corresponding to the fields \(E_{in}\) and \(E_{lo}\). The coupling efficiency
\[
\delta x_{min} \frac{\sqrt{\Delta f}}{\lambda_{16} \pi \eta_c \mathcal{F}} = \frac{\tau_{ex} \kappa^2 R}{8 \omega_0 \sqrt{P_{in}} \hbar \omega} \frac{1}{16 \pi \eta_c \mathcal{F} \sqrt{P_{in}} / \hbar \omega},
\]  
(4)
where the finesse \(\mathcal{F} = c / n R \kappa\) of the WGM was introduced, \(\lambda = 2\pi c / n \omega\) is the optical wavelength in glass and \(\Delta f\) the measurement bandwidth. Note that the finesse \(\mathcal{F}\) is also affected by the coupling \(\eta_c\) via \(\mathcal{F} = \mathcal{F}_0 (1 - \eta_c)\), where \(\mathcal{F}_0\) is the ‘intrinsic’ finesse in the undercoupled limit \(\tau_{ex} \to \infty\), so that the optimum sensitivity is achieved at critical coupling \(\eta_c = 1/2\).

We note that the same result is obtained following a more formal approach, using the linearized quantum Langevin equations (QLE) of the coupled optomechanical system around a stable working point \([36, 41, 42]\). This allows in particular the calculation of the fluctuations in any arbitrary quadrature of the output field. Comparison of the resulting quantum fluctuations and mechanically induced fluctuations in the output phase quadrature yields a minimum displacement of
\[
\delta x_{min} (\Omega) = \frac{\lambda}{16 \pi \eta_c \mathcal{F} \sqrt{P_{in}} / \hbar \omega} \left( \frac{1}{\kappa/2} + \frac{\Omega}{\kappa/2} \right)^2.
\]  
(5)
Compared to (4), this calculation adds only a correction due to the finite response time of the cavity for Fourier frequencies \(\Omega\) exceeding the cavity cutoff \(\kappa/2\). We note that the full expression (5) can also be derived from a classical calculation of the signal, considering the amplitude of the motional sidebands of the field coupling out of the cavity and comparing it with detection shot noise \([29]\). The detection limit (5) corresponds to a spectral density of measurement imprecision
\[
S_x (\Omega) = \left( \frac{\lambda}{16 \pi \eta_c \mathcal{F}} \right)^2 \left( \frac{1 + (\Omega/(\kappa/2))^2}{P_{in}/\hbar \omega} \right).
\]  
(6)
As first pointed out by Braginsky \([3]\), this measurement inevitably exerts backaction on the mechanical device. In the case of an optical transducer, quantum backaction is enforced by the fluctuations of radiation pressure due to a fluctuating intracavity photon flux. From the
linearized Langevin equations, the intracavity force fluctuation spectrum can be calculated to have the form \( k = \omega_n/c \) is the wavenumber of the laser light in the cavity medium:

\[
S_F(\Omega) = 16\eta_c^2 \frac{P_{\text{in}}/\hbar \omega}{1 + (\Omega/(\kappa/2))^2} \hbar^2 k^2.
\]

(7)

It is noted that \( S_x(\Omega) \) and \( S_F(\Omega) \) fulfill the expected uncertainty relation

\[
S_x(\Omega)S_F(\Omega) = \frac{\hbar}{4\eta_c} \geq \frac{\hbar}{4},
\]

(8)

with an equality in the limit \( \eta_c \rightarrow 1 \) \( \leftrightarrow (\tau_0 > \tau_{\text{ex}}) \), that is, for a strongly overcoupled cavity. The fluctuating quantum backaction force drives the mechanical oscillator, and therefore induces position fluctuations. Assuming the fluctuations in the amplitude and phase quadratures of the input light field are uncorrelated, the position fluctuations induced by \( S_F \) are uncorrelated with the apparent position fluctuations \( S_x \). Thus, the total measurement uncertainty is given by

\[
S_{\text{tot}}(\Omega) = S_x(\Omega) + |\chi(\Omega)|^2 S_F(\Omega),
\]

(9)

where

\[
\chi(\Omega) = \frac{1}{m_{\text{eff}}(\Omega_m^2 - \Omega^2 - i\Omega \Gamma_m)}
\]

(10)

is the susceptibility of the mechanical oscillator, \( m_{\text{eff}} \) its effective mass [43] and \( \Gamma_m \) the mechanical damping rate. It is important to note that this susceptibility is modified when the optical resonance is excited in a detuned manner [44]. However, for resonant probing as considered here, it is not modified. The total measurement uncertainty is minimized for an input flux of

\[
P_{\text{opt}}^{\text{in}}/\hbar \omega = \eta_c^{-3/2} \frac{\lambda^2}{128\pi^2 F^2} \frac{1 + (\Omega/(\kappa/2))^2}{\hbar |\chi(\Omega)|},
\]

(11)

yielding

\[
S_{x_{\text{SQL}}} (\Omega) = \frac{\hbar |\chi(\Omega)|}{\sqrt{\eta_c}} = \frac{\hbar}{m_{\text{eff}} \sqrt{\eta_c} (\Omega_m^2 - \Omega^2)^2 + \Gamma_m^2 \Omega_m^2},
\]

(12)

called the SQL [3, 12] in the case \( \eta_c = 1 \). Its peak value calculated at \( \Omega_m \) yields

\[
S_{x_{\text{SQL}}} (\Omega_m) = \frac{\hbar}{\sqrt{\eta_c} m_{\text{eff}} \Gamma_m \Omega_m}.
\]

(13)

In this calculation, we have explicitly considered the effect of the coupling conditions to the cavity, which can—as a unique feature for an optical resonator—be varied continuously in the experiment by adjusting the gap between the coupling waveguide and the WGM resonator. The SQL is approached most closely in the overcoupled limit \( \tau_{\text{ex}} \ll \tau_0 \). It is noteworthy that the fiber-taper coupling technique to microtoroids can deeply enter this regime, and 100 \( \cdot \tau_{\text{ex}} < \tau_0 \) (\( \eta_c = 99\% \)) has been demonstrated [45]. On the other hand, such a strong coupling reduces the cavity finesse and thus comes at the expense of a higher optimum power \( P_{\text{opt}}^{\text{in}} \). Working with weaker coupling, such as critical coupling as typically pursued in this work, brings only a moderate penalty as (12) shows, for example, a factor of \( \sqrt{2} \) for \( \eta_c = 1/2 \).

In our experiment performed at room temperature, the noise induced by quantum backaction is masked by thermal noise due to a fluctuating Langevin force with \( S_{f}(\Omega) = \hbar m_{\text{eff}} \Gamma_m |\Omega| \coth (\hbar |\Omega|/2k_B T) \approx 2\Gamma_m m_{\text{eff}} k_B T \) as \( k_B T \gg \hbar \Omega_m \) with the Boltzmann constant \( k_B \).

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This adds a third term to the total apparent displacement noise \[36, 41\],

\[
S_{\omega}^\text{tot}(\Omega) = S_c(\Omega) + |\chi(\Omega)|^2 \left(S_F(\Omega) + S_F^h(\Omega)\right)
\]

\[
= \left(\frac{\lambda}{16\pi\eta_cF}\right)^2 \left(\frac{1 + (\Omega/(\kappa/2))^2}{P_{\text{in}}/\hbar\omega}\right)
\]

\[
+ |\chi(\Omega)|^2 \left(16\eta_cF^2 \frac{P_{\text{in}}/\hbar\omega}{1 + (\Omega/(\kappa/2))^2} \hbar^2 k^2 + 2\Gamma_m m_e k_B T\right).
\]

This expression describes the spectrum that is measured by analyzing the phase quadrature of the transmitted light past the microresonator \[41\].

3. Experimental implementation

Detection of the phase of a light field with quantum-limited sensitivity is a standard task in quantum optics, and several techniques have been developed to achieve this. In the following section, two techniques which were successfully implemented for motion transduction in WGM microresonators are described.

3.1. Homodyne spectroscopy

The most common method for quantum-limited phase measurement is a balanced homodyne receiver \[46\] as employed in previous optomechanical experiments \[21, 47, 48\]. We briefly discuss the experimental protocol used for homodyne spectroscopy. This method is adapted to the ring topology of our resonator by sending the laser beam that is transmitted through the taper to a beam splitter, where it is brought to interference with a strong local oscillator (LO), see figure 2. Signal and LO beams are derived from a monolithic Nd:YAG laser operating at \(\lambda = 1064\) nm. This source exhibits quantum-limited amplitude and phase noise at Fourier frequencies \(\Omega/2\pi \gtrsim 5\) MHz and power levels \(P_{\text{LO}} + P \lesssim 5\) mW of interest. Due to its limited tuning speed and range, we use a home-built external-cavity diode laser for pre-characterization of several samples until a suited toroid is found. The Nd:YAG laser beam is split using a polarizing beam splitter (PBS0), the signal beam is sent through the coupling taper in the near field of the excited cavity mode and its phase is shifted depending on the mutual laser-cavity detuning. The LO travels in the reference arm of an effective Mach–Zehnder interferometer and is recombined with the signal beam at a polarizing beam splitter (PBS1). Spatial matching of the incident modes is facilitated by using single-mode fiber as mode filter on the LO. After spatial recombination, interference is enforced using a retarder plate and another polarizing beam splitter (PBS2).

It was found advantageous to actively stabilize the phase of the LO at the combining beam splitter. Since the phase of the signal beam depends on the detuning of the laser from the cavity, which may itself be subject to drifts and fluctuations, it is not suited as a phase reference. However, by purposely introducing a small polarization mismatch (cf figure 2(b)) between the light in the taper region and the either predominantly TE- or TM-like WGM modes of the microcavity, it is possible to utilize the signal’s polarization component orthogonal to the cavity WGM mode as a phase reference, in order to lock the Mach–Zehnder interferometer to the desired phase angle between WGM and LO fields. WGM and locking polarization components
in the signal beam are separated by the first beam splitter (PBS1), after compensation of fiber-induced polarization rotation. An error signal is created by detecting the interference between the locking and the LO beam in a third PBS (PBS3), which provides a feedback signal to the piezoelectric transducer displacing a mirror.

In order to reach quantum-limited detection sensitivity of the signal beam’s phase, the power of the LO has to be chosen such that quantum shot noise exceeds the receiver noise, $P_{LO} \gg \text{NEP}(\Omega)^2/\eta h \omega$, where NEP($\Omega$) is the noise-equivalent power of the receiver and $\eta$ the detection efficiency. The employed commercial InGaAs receivers provide a nominal $\eta(1064\text{ nm}) \sim 87\%$, NEP $\sim 10 \text{ pW Hz}^{-1/2}$ between 0 and 80 MHz. Balancing the detectors avoids the saturation of the amplifiers by the large dc-field of the LO. Excess losses due to mode matching and tapered fiber imperfections further reduce the total detection efficiency to (an unoptimized) $\eta_{\text{tot}} \sim 50\%$.

An advantageous feature of the homodyne signal is that its dc-component directly provides a dispersive error signal

$$h(\Delta \omega) = \frac{2\eta_c \kappa \Delta \omega}{\Delta \omega^2 + (\kappa/2)^2} \sqrt{P_{\text{cav}} P_{\text{LO}}},$$

which allows a very stable lock at the low signal powers used here. An example of an experimentally obtained error signal is shown in figure 2(d). Simultaneously, the fluctuations in the differential photocurrent induced by both optical shot noise in the signal and the thermal noise in the cavity displacement can be frequency-analyzed using a high-performance electronic spectrum analyzer. For calibration purposes, we frequency-modulate the laser using a LiNbO$_3$ phase modulator external to the laser. The frequency modulation is given by $\delta \omega = \beta \Omega_{\text{mod}}$ for known modulation depth $\beta$ and frequency $\Omega_{\text{mod}}$, and generates the same signal as would be induced by a radius modulation of $\delta x = R \delta \omega / \omega$ [21, 29, 35, 38], independent of cavity linewidth and coupling conditions. If the cavity linewidth is known in addition, the spectra can be absolutely calibrated at all Fourier frequencies, taking into account the reduced sensitivity beyond the cavity cutoff at $\kappa/2$.

### 3.2. Polarization spectroscopy

A simplified setup may be obtained by co-propagating the LO in the same spatial, but orthogonal polarization mode as compared to the signal beam [29]. Since the WGM modes have predominantly TE or TM character and are not degenerate, this guarantees that the LO is not affected by the cavity. Due to common-mode rejection of many sources of noise in the relative phase between signal and LO (for example, frequency noise in the optical fiber), the passive stability is sufficiently enhanced to enable operation without active stabilization (figure 3).

Enforcing interference between LO and signal beams then corresponds to polarization analysis of the light (comprising both signal and LO) emerging from the fiber taper. While novel in the present context of a tapered fiber coupled microcavity, this is a well established technique to derive a dispersive error signal from the light reflected from a Fabry–Perot type reference cavity, named after its inventors Hänsch and Couillaud [34].

If fiber birefringence is adequately compensated, the error signal reads

$$h(\Delta \omega) = \frac{2\eta_c \kappa \Delta \omega}{\Delta \omega^2 + (\kappa/2)^2} \sqrt{P_{\text{cav}} P_{\text{LO}}},$$

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Figure 2. (a) Optical interferometric displacement transducer based on homodyne spectroscopy of light transmitted past the cavity (‘µ-toroid’). The phase of the LO is actively stabilized (‘LO phase control’). Details are given in the text. PBS0-PBS3, polarizing beam splitters. (b) Cross section through the fiber taper and the toroidal rim in the coupling region. The polarization in the taper is slightly mismatched with the polarization of the cavity mode. Thus only part of the total field $E_{\text{cav}}$ couples to the WGM, the other component $E_{\text{lock}}$ can be used for the stabilization of the LO phase. The components $E_{\text{cav}}$ and $E_{\text{lock}}$ are separated in PBS1. (c) Signal in the balanced receiver for a scanning LO phase (dotted, blue) at low power, and for the locked LO (red). The shown locked trace was recorded for about 5 s. (d) Typical experimental error signal in the balanced receiver when the laser is scanned over a cavity resonance with the LO locked to the appropriate phase.

identical to (16), and a typical trace is shown in figure 3(c). This is used to lock the laser at resonance $\Delta \omega \equiv 0$ with a bandwidth of about 10 kHz. Calibration of the spectra may be performed as described in the previous section.

While this approach obviously allows the reduction of the complexity of the experiment, this arrangement proved more sensitive to slow temperature drifts in the polarization mode dispersion of the fibers employed, due to the large ratio of signal and LO powers, the magnitudes of which are only defined by the polarization state of the light in the fiber taper region. Improved stability may be obtained by reducing fiber length to its minimum of about 0.5 m. For reasons of flexibility and convenience, the actual fiber length totaled several metres in our experiment. Nonetheless, sensitivities of $10^{-18}$ m Hz$^{-1/2}$ are achieved in toroids using this method [29]. The intrinsic polarization selectivity of WGM renders the introduction of an additional polarizer, mandatory in the original implementation [34], obsolete. In an earlier experiment with a

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Figure 3. Optical interferometric displacement transducer based on polarization spectroscopy of light transmitted in the taper past the cavity (‘µ-toroid’). (a) After phase modulation with an electro-optic modulator, the polarization is prepared with a first polarization control unit (PCU1). The cavity WGM defines signal and LO polarizations. A second polarization control unit (PCU2) compensates for fiber birefringence. Polarization analysis using a λ/4 plate and a polarizing beam splitter enforces interference of the signal and LO fields. (b) Due to the polarization nondegeneracy of the WGM in the cavity, only one polarization component of the light interacts with the mode. (c) Typical error signal obtained when the laser is scanned over a cavity resonance.

Fabry–Perot cavity [49], the losses associated with an intracavity polarization element limited the finesse, and therefore the attained sensitivity to ∼10⁻¹⁴ m Hz⁻¹/².

4. Observation and analysis of quantum and thermal noise

In this section, we present characteristic results obtained with silica microtoroids of typical major radii between 25 and 50 µm. Figure 4 shows an example of a broadband measurement using homodyne detection. If the taper is retracted from the proximity of the cavity, quantum shot noise dominates over electronic noise in the receiver. It was verified that the photocurrent noise √S₁ scales with the square root of the total power as expected for shot noise. While this noise is spectrally flat, the equivalent displacement noise exhibits a calculated \( \sqrt{1 + \Omega^2/(\kappa/2)^2} \) frequency dependence beyond the cavity cutoff at \( \kappa/2 \approx 2\pi \times 17 \text{ MHz} \).

When the laser is coupled and locked to a WGM resonance, a substantially different spectrum is observed (figure 4). Its equivalent displacement noise is calibrated in absolute terms using an a priori known phase modulation at 36 MHz, and taking the cavity cutoff into account. The equivalent displacement noise of the cavity exceeds the shot noise at all frequencies for a high enough power in the signal beam, leading to a background level equivalent
Figure 4. Broadband spectrum of the equivalent displacement noise in a silica toroidal cavity. Red, measured trace with laser coupled to a cavity resonance. The peak at 36 MHz is due to phase modulation for the purpose of absolute displacement calibration. Gray, measured shot noise with taper retracted from the cavity. The frequency dependence arises from the calculated reduced sensitivity at frequencies beyond cavity cutoff. Black, measured electronic detector noise. Orange and green lines are models for mechanical noise and thermorefractive noise, respectively, and the blue trace is a sum of the two models and the shot-noise background.

to a displacement noise of $\sqrt{S_x} \sim 10^{-18}$ m Hz$^{-1/2}$. The superimposed sparse spectrum of peaks fits the sum of several Lorentzians which arise from the thermal noise of several mechanical modes, $\sum_i |\chi_i(\Omega)|^2 S_{th,i}(\Omega)$. In the following, we discuss the features of the spectrum in more detail.

4.1. Thermorefractive noise

The broadband, low-frequency background noise is attributed to thermorefractive noise, the fluctuations in refractive index induced by the fluctuations $\langle \Delta T^2 \rangle = k_B T^2/\rho c_p V$ of temperature on a microscopic volume $V$ [50]. Here, $k_B$ denotes Boltzmann’s constant, $T$ temperature, $\rho$ density and $c_p$ specific heat capacity. This leads to fluctuations of the resonance frequency via both the dependence of the refractive index $n$ on temperature, and the thermal expansion of the material. At room temperature, however, the coefficient of thermal expansion $\alpha$ is more than 20 times smaller than $dn/dT$, so the analysis can be restricted to the resonance frequency fluctuations induced by thermorefractive fluctuations.

Introducing a fluctuating thermal source field in the heat diffusion equation similar to a Langevin approach [51] it is possible to derive the spectrum of refractive index fluctuations.
sampled by a WGM in a silica microsphere [35]. For high frequencies, an approximate analytic expression, neglecting also the boundary conditions for thermal waves, can be obtained. The result

$$S_{bn/n}(\Omega) \approx \frac{k_B T^2 D}{\pi^{5/2} n^2 \rho c_p R} \frac{2}{\sqrt{d^2 - b^2}} \left( \frac{dn}{dT} \right)^2 \int_0^{+\infty} q^2 e^{-(q^2 b^2)/2} \frac{dq}{D^2 q^4 + \Omega^2 2\pi}$$

was found in good agreement with the experimental data obtained on a silica microspheres [35] between 100 Hz and 100 kHz. Here, $R$ is the cavity radius, $d$ and $b$ are transverse mode dimensions and $D$ is the thermal diffusivity of silica. For comparison with the toroid measurement calibrated as effective radial displacement, $R \sqrt{S_{bn/n}(\Omega)}$ has to be evaluated. Inserting the material parameters of fused silica and the radius of the employed toroid into the model (18), the data between 100 kHz and 20 MHz can be quantitatively reproduced if no parameters except $b$ and the absolute magnitude are adjusted by factors of the order of 2 (figure 4). These corrections are justified considering the approximations made in the derivation, and potential differences in surface effects in spheres and toroids. It is interesting to calculate the resulting experimental root-mean-square fluctuations of the cavity’s refractive index $\sqrt{\int_{-\infty}^{+\infty} S_{bn/n}(\Omega) d\Omega}$ to be of order $10^{-10}$, as it constitutes a detection limit for resonance frequency shifts induced by molecules in the evanescent field [52].

For the purposes of cavity quantum optomechanics, thermorefractive fluctuations constitute a background noise, at room temperature rolling off to a level of $\sim 10^{-19}$ m Hz$^{-1/2}$ at $\Omega/2\pi > 50$ MHz, where the high-quality radial breathing modes (RBMs) typically reside. Practically, such experiments are performed in a cryogenic environment, leading to significant changes in the material properties. A level of $R \sqrt{S_{bn/n}(\Omega)} \sim 10^{-20}$ m Hz$^{-1/2}$ at $T \sim 1$ K may be estimated, assuming a reduction of $|dn/dT|$ to $\lesssim 0.8 \times 10^{-6}$ K$^{-1}$ as indicated by recent measurements [53]. While other sources of noise, such as thermoelastic and photothermal noise [54, 55] are not expected to exceed this value, thorough experimental characterization at cryogenic temperatures is necessary.

Such a study is also an important pre-study for experiments aiming at the demonstration of generating broadband intracavity Kerr squeezing. While above-threshold parametric oscillations have been observed in these cavities [56, 57], the room temperature experiments reported here indicate that the thermorefractive noise exceeds the quantum noise of the light in the cavity, as evidenced by the homodyne measurement of the cavity output. It therefore has to be suppressed to achieve squeezing of the cavity field.

4.2. Mechanical modes

The most prominent features in the noise spectrum are the mechanical modes of the microtoroids. To date, the variety of mechanical modes in silica microcavities (toroids and spheres) has been investigated only to a limited extent [10, 29], in most cases by driving modes with particularly strong optomechanical coupling into the parametric oscillatory instability [5, 6, 39, 58, 59]. Using a Fabry–Perot cavity, however, it was shown that the Brownian motion of a wealth of intrinsic mechanical modes of a cylindrical mirror can be studied [47]. The high sensitivity methods presented in the previous section enable monitoring the Brownian motion of around twenty different mechanical modes in silica microtoroids over a frequency range spanning from below 1 MHz to above 100 MHz. Figure 5 shows a noise spectrum obtained by Hänsch–Couillaud spectroscopy revealing a total of 16 mechanical
Figure 5. Mechanical modes in a silica toroidal cavity. (a) Displacement noise spectrum obtained by polarization spectroscopy. The calibration peak at 40 MHz corresponds to a displacement spectral density of $1.0 \times 10^{-32}$ m$^2$ Hz$^{-1}$. (b) Frequency zoom on 16 Lorentzian peaks identified in the spectrum (linear scale). The bar below each spectrum indicates a 100 kHz frequency span. The expected two-fold degeneracy of some of the modes is lifted (mode 1, 7, 9, 12 and 17) which is attributed to eccentricity. The panels are enumerated to facilitate comparison with (c) showing the spatial displacement patterns (highly exaggerated, also indicated in the color code) for the mechanical modes, as obtained from FEM. The identification of the modes is made via frequency comparison (see figure 6). The pattern of mode 4 is illustrated showing the displacement of initially parallel slices in order to depict its mainly torsional motion. Modes 6, 13 and 18 involve mainly motion of the silicon pillar and have not been observed experimentally.
Simulated against measured frequencies of mechanical modes (○) of monolithic silica microcavities ranging from 0–100 MHz. The measured frequencies are in excellent agreement with the values obtained by FEM simulation, showing a relative deviation of on average less than 2% (■). The corresponding mode patterns are given in figure 5.

In order to attribute the observed peaks with the appropriate mechanical mode patterns, a 3D finite element model (FEM) of the microtoroid is employed and implemented. Extracting the geometry parameters using an optical microscope (accuracy ±5%) and matching observed and simulated mechanical frequencies, the FEM simulation allows the identification of all observed peaks in the spectrum. Thus, all 16 observed modes were identified with the corresponding simulated mode patterns as depicted in figure 5. Figure 6 gives an overview of simulated frequencies and the frequencies deduced from the spectrum shown in figure 5 revealing excellent agreement. The relative frequency deviation between measurement and simulation is on average less than 2%. Moreover, almost all simulated modes are observed experimentally. Only three out of 19 modes (number 6, 13 and 18) cannot be observed, most likely due to low mechanical $Q$ factors (<10) and/or weak optomechanical coupling.

Due to its composite structure comprising several geometric objects microtoroids exhibit a diverse set of eigenmodes. In order to characterize the noise spectra, understanding the complex mode structure of microtoroids is in particular important as all modes contribute to a background noise floor. Indeed, various mode-families can be identified in which the motion of the silica disk, the silica torus and the silicon pillar partially decouples. The mode showing the strongest optomechanical coupling is the RBM (mode 14 in figure 5), and most previous work has focused on this mode [5, 9, 29]. An equivalent mode has been studied in a microdisk structure [60] where it was termed radial contour mode. In contrast, the optomechanical coupling of the torsional mode (mode number 4) where the silica disk shows an in-plane rotation vanishes to first order. Interestingly, this torsional mode can nevertheless be observed experimentally.

The commercial software Comsol Multiphysics was employed.
One particular mode family that can be identified are the radially symmetric flexural modes (modes 2 and 8) in which the motion of the free standing part of the silica disk resembles the modes of a cantilever. The fundamental frequencies of a cantilever of length $L$ can in general be expressed as $\Omega_i/2\pi = C \cdot \sqrt{k_i}/2\pi$, where $C$ is a material constant and the $k_i$ are given by the solutions of \[ \cos(k_i \cdot L) \cdot \cosh(k_i \cdot L) + 1 = 0. \] Figure 7 shows the measured frequencies of the two lowest order flexural modes (modes 2 and 8) and, in addition, the first five flexural modes of a different sample plotted as a function of the wavenumber $k_i$, where the free standing part of the silica disk is taken as equivalent cantilever length $L$ (13.2 and 39.6 $\mu$m, respectively). Both sets of data allow an accurate single quadratic fit of the fundamental radially symmetric modes. Thus, the latter can indeed be regarded as cantilever modes following a uniform quadratic dispersion even for microtoroids of different sizes. In particular, the quadratic dispersion rules out the presence of radial tensile stress within the silica disk as this would imply a linear dispersion relation.

Another obvious mode family which can be distinguished is characterized by sinusoidal oscillations of the torus itself (modes 1, 3, 5, 7, 11 and 16). The dispersion diagram of these modes, which we refer to as crown-modes, is depicted in figure 7 for two different samples. The respective wavelength $\lambda$ is given by twice the distance between two adjacent nodes of each mode. The frequencies $\Omega_i/2\pi$ of the crown modes observed in microtoroids of different circumference and torus diameter ($2\pi \times 23/5.3 \mu$m and $2\pi \times 45/5.7 \mu$m) allow a simultaneous quadratic fit with the frequencies $\Omega_i/2\pi$ following a quadratic dependence on the wavenumber $k_i = 2\pi/\lambda_i$. This uniform dependence shows that in this mode family the silica torus, despite its attachment to the silica disk, behaves effectively like an independent element. The observed quadratic dispersion relation rules out the presence of tensile stress within the torus in radial direction as this would lead to linear dispersion characteristic of a vibrating string. Since the
Figure 8. Spectra and simulated mode patterns of the second and third order crown modes. They exhibit very high mechanical quality factors of 58 000 and 51 000, which is attributed to low clamping losses [62].

Microtoroids undergo a reflow process [30] indeed all potentially present stresses should get relaxed during this fabrication step. As such, the observed quadratic dispersion characteristic for a rigid ring structure confirms this expectation.

The study of clamping losses in the RBM and the optimization of its mechanical $Q$ factors [62] also lead to the observation of very high $Q$ crown modes. For these measurements, the cavities are operated in low-pressure environment ($p < 1 \text{ mbar}$) in order to reduce viscous damping, which limited the $Q$’s of the previously discussed modes to values $\lesssim 3000$. For such a measurement, figure 8 shows the second and third order crown modes with $Q$ factors exceeding 50 000 at frequencies below 10 MHz. These high-$Q$ factors are attributed to low clamping losses which are studied in detail in [62].

5. Conclusions

In conclusion, we have shown that high-finesse WGMs are extraordinarily well suited as transducers for micromechanical motion. Sensitivities on the order of $10^{-19} \text{ mHz}^{-1/2}$ are achieved, on par with the best reported values [21, 22]. The small dimensions of the WGM resonators allow in addition the extension of the measurement bandwidth by more than an order of magnitude compared to earlier work [21, 22]. This enlarged range coincides with the resonance frequencies of the mechanical modes of interest present in the device. For example, the RBM, particularly amenable to optomechanical effects due to its strong optomechanical coupling ($m_{\text{eff}} \sim 10 \text{ ng}$), typically exhibits a resonance frequency around 50 MHz. At this frequency, average thermal phonon occupancies below unity are achieved at temperatures $\sim 2 \text{ mK}$. Approaching such temperatures appears feasible using a combination of conventional cryogenics and resolved-sideband cooling [29]. It is interesting that the expected spectrum of even the zero-point fluctuations of this mode peaks at a value of $\sqrt{S_x^{\text{ZPF}}(\Omega_m)} = \sqrt{\hbar Q / m_{\text{eff}} \Omega_m^2} \sim$
$10^{-19}$ m Hz$^{-1/2}$, assuming an effective $Q = 100$ after cryogenic and laser cooling. Such signal levels may be detected on top of the background of the thermal noise studied here, providing a route to experimental tests of the theory of quantum measurements on mesoscopic objects. Finally, we note that the advantageous properties of WGM resonators may also be exploited for motion transduction of a mechanical oscillator external to the cavity, for example, by bringing it into the near field of the WGM.

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References

[1] Braginsky V B and Manukin A B 1977 *Measurement of Weak Forces in Physics Experiments* (Chicago, IL: University of Chicago Press)
[2] Rugar D, Budakian R, Mamin H J and Chui B W 2004 Single spin detection by magnetic resonance force microscopy *Nature* 430 329–32
[3] Braginsky V B and Khalili F Y 1992 *Quantum Measurement* (Cambridge: Cambridge University Press)
[4] Giffard R P 1976 Ultimate sensitivity of a resonant gravitational wave antenna using a linear motion detector *Phys. Rev. D* 14 2478–85
[5] Kippenberg T J, Rokhsari H, Carmon T, Scherer A and Vahala K J 2005 Analysis of radiation-pressure induced mechanical oscillation of an optical microcavity *Phys. Rev. Lett.* 95 033901
[6] Rokhsari H, Kippenberg T J, Carmon T and Vahala K J 2005 Radiation-pressure-driven micro-mechanical oscillator *Opt. Express* 13 5293–301
[7] Gigan S, Böhm H R, Paternosto M, Blaser F, Langer G, Hertzberg J B, Schwab K C, Bäuerle D, Aspelmeyer M and Zeilinger A 2006 Self-cooling of a micromirror by radiation pressure *Nature* 444 67–70
[8] Arcizet O, Cohadon P-F, Briant T, Pinard M and Heidmann A 2006 Radiation-pressure cooling and optomechanical instability of a micromirror *Nature* 444 71–4
[9] Schliesser A, Del’Haye P, Nooshi N, Vahala K J and Kippenberg T 2006 Radiation pressure cooling of a micromechanical oscillator using dynamical backaction *Phys. Rev. Lett.* 97 243905
[10] Kippenberg T J and Vahala K 2007 *Cavity opto-mechanics* *Opt. Express* 15 17172–205
[11] Kippenberg T J and Vahala K J 2008 *Cavity optomechanics: back-action at the mesoscale* *Science* 321 1172–6
[12] Caves C M 1980 Quantum-mechanical radiation-pressure fluctuations in an interferometer *Phys. Rev. Lett.* 45 75–9
[13] Cleland A N, Aldridge J S, Driscoll D C and Gossard A C 2002 Nanomechanical displacement sensing using a quantum point contact *Appl. Phys. Lett.* 81 1699–701
[14] Poggio M, Jura M P, Degen C L, Topinka M A, Mamin H J, Goldhaber-Gordon D and Rugar D 2008 An off-board quantum point contact as a sensitive detector of cantilever motion *Nat. Phys.* 4 635–38
[15] Knobel R G and Cleland A N 2003 Nanometre-scale displacement sensing using a single-electron transistor *Nature* 424 291–93
[16] LaHaye M D, Buu O, Camarota B and Schwab K C 2004 Approaching the quantum limit of a nanomechanical resonator *Science* 304 74–7
[17] Flowers-Jacobs N E, Schmidt D R and Lehner K W 2007 Intrinsic noise properties of atomic point contact displacement detectors *Phys. Rev. Lett.* 98 096804
[18] Regal C A, Teufel J D and Lehnert K W 2008 Measuring nanomechanical motion with a microwave cavity interferometer Nat. Phys. 4 555–60

[19] Ekinci K L, Yang Y T, Huang X M H and Roukes M L 2002 Balanced electronic detection of displacement in nanoelectromechanical systems Appl. Phys. Lett. 81 2253–5

[20] Gaidarzhy A, Zolfagharkhani G, Badzey R L and Mohanty P 2005 Spectral response of a gigahertz-range nanomechanical oscillator Appl. Phys. Lett. 86 254103

[21] Hadjar Y, Cohadon P F, Aminoff C G, Pinard M and Heidmann A 1999 High-sensitivity optical measurement of mechanical Brownian motion Europhys. Lett. 47 545–51

[22] Arcizet O, Cohadon P-F, Briant T, Pinard M, Heidmann A, Mackowski J-M, Michel C, Pinard L, Francais O and Rousseau L 2006 High-sensitivity optical monitoring of a micromechanical resonator with a quantum-limited optomechanical sensor Phys. Rev. Lett. 97 133601

[23] Corbitt T, Wipf Ch, Bodiya T, Ottaway D, Sigg D, Smith N, Whitcomb S and Mavalvala N 2007 Optical dilution and feedback cooling of a gram-scale oscillator to 6.9 mK Phys. Rev. Lett. 99 160801

[24] Cohadon P-F, Heidmann A and Pinard M 1999 Cooling of a mirror by radiation pressure Phys. Rev. Lett. 83 3174–7

[25] Kleckner D and Bouwmeester D 2006 Sub-kelvin optical cooling of a micromechanical resonator Nature 444 75–8

[26] Corbitt T, Chen Y, Innerhofer E, Muller-Ebhardt H, Ottaway D, Rehbein H, Sigg D, Whitcomb S, Wipf Ch and Mavalvala N 2007 An all-optical trap for a gram-scale mirror Phys. Rev. Lett. 98 150802

[27] Poggio M, Degen L, Mamin J J and Rugar D 2007 Feedback cooling of a cantilever’s fundamental mode below 5 mK Phys. Rev. Lett. 99 017201

[28] Thomson J D, Zwickl B M, Jayich A M, Marquardt F, Girvin S M and Harris J G E 2008 Strong dispersive coupling of a high finesse cavity to a micromechanical membrane Nature 452 72–5

[29] Schliesser A, Rivière R, Anetsberger G, Arcizet O and Kippenberg T 2008 Resolved-sideband cooling of a micromechanical oscillator Nat. Phys. 4 415–9

[30] Armani D K, Kippenberg T J, Spillane S M and Vahala K J 2003 Ultra-high-Q toroid microcavity on a chip Nature 421 925–8

[31] Braginsky V B and Vyatchanin S P 2002 Low quantum noise tranquilizer for Fabry–Perot interferometer Phys. Lett. A 293 228–34

[32] Wilson-Rae I, Nooshi N, Zwerger W and Kippenberg T J 2007 Theory of ground state cooling of a mechanical oscillator using dynamical backaction Phys. Rev. Lett. 99 093901

[33] Marquardt F, Chen J P, Clerk A A and Girvin S M 2007 Quantum theory of cavity-assisted sideband cooling of mechanical motion Phys. Rev. Lett. 99 093902

[34] Hánsch T W and Couillaud B 1980 Laser frequency stabilization by polarization spectroscopy of a reflecting reference cavity Opt. Commun. 35 441–4

[35] Gorodetsky M L and Grudinin I S 2004 Fundamental thermal fluctuations in microspheres J. Opt. Soc. Am. B 21 697–705

[36] Fabre C, Pinard M, Bourzeix S, Heidmann A, Giacobino E and Reynaud S 1994 Quantum-noise reduction using a cavity with a movable mirror Phys. Rev. A 49 1337–43

[37] Slusher R E, Hollberg L W, Yurke B, Mertz J C and Valley J F 1985 Observation of squeezed states generated by four-wave mixing in an optical cavity Phys. Rev. Lett. 55 2409–12

[38] Tittonen I, Breitenbach G, Kalkbrenner T, Müller T, Conradt R, Schiller S, Steinsland E, Blanc N and de Rooij N F 1999 Interferometric measurements of the position of a macroscopic body: towards observations of quantum limits Phys. Rev. A 59 1038–44

[39] Carmon T and Vahala K 2007 Modal spectroscopy of optoexcited vibrations of a micron-scale on-chip resonator at greater than 1 GHz frequency Phys. Rev. Lett. 98 123901

[40] Haus H A 1984 Waves and Fields in Optoelectronics (Englewood Cliffs, NJ: Prentice-Hall)

[41] Giovannetti V and Vitali D 2001 Phase-noise measurement in a cavity with a movable mirror undergoing quantum Brownian motion Phys. Rev. A 63 023812

New Journal of Physics 10 (2008) 095015 (http://www.njp.org/)
[42] Walls D F and Milburn G J 2008 Quantum Optics (Berlin: Springer)
[43] Pinard M, Hadjar Y and Heimann A 1999 Effective mass in quantum effects of radiation pressure Eur. J. Phys. D 7 107–16
[44] Braginskii V B and Manukin A B 1967 Ponderomotive effects of electromagnetic radiation Sov. Phys.—JETP Lett. 25 653–5
[45] Spillane S M, Kippenberg T J, Painter O J and Vahala K J 2003 Ideality in a fiber-taper-coupled microresonator system for application to cavity quantum electrodynamics Phys. Rev. Lett. 91 043902
[46] Yuen H P and Chan V W S 1983 Noise in homodyne and heterodyne detection Opt. Lett. 8 177–9
[47] Briant T, Cohadon P -F, Heidmann A and Pinard M 2003 Optomechanical characterization of acoustic modes in a mirror Phys. Rev. A 68 033823
[48] Caniard T, Verlot P, Briant T, Cohadon P-F and Heidmann A 2007 Observation of back-action noise cancellation in interferometric and weak force measurements Phys. Rev. Lett. 99 110801
[49] Hahtela O, Nera K and Tittonen I 2004 Position measurement of a cavity mirror using polarization spectroscopy J. Opt. A: Pure Appl. Opt. 6 S115–20
[50] Landau L D and Lifshitz E M 1980 Statistical Physics (Course of Theoretical Physics vol 5) 3rd edn (Oxford: Pergamon)
[51] Braginsky V B, Gorodetsky M L and Vyatchanin S P 2000 Thermo-refractive noise in gravitational wave antennae Phys. Lett. A 271 303–7
[52] Schröter B, Reich C, Arcizet O, Rädler J O, Nickel B and Kippenberg T J 2008 Chip based, lipid bilayer functionalized microresonators for label-free, ultra sensitive and time-resolved molecular detection, submitted
[53] Park Y-S and Wang H 2007 Regenerative pulsation in silica microspheres Opt. Lett. 32 3104–6
[54] Braginsky V B, Gorodetsky M L and Vyatchanin S P 1999 Thermodynamical fluctuations and photo-thermal shot noise in gravitational wave antennae Phys. Lett. A 264 1–10
[55] Matsko A B, Savchenko A A, Yu N and Maleki L 2007 Whispering-gallery-mode resonators as frequency references. I. fundamental limitations J. Opt. Soc. Am. B 24 1324–35
[56] Kippenberg T J, Spillane S M and Vahala K J 2004 Kerr-nonlinearity optical parametric oscillation in an ultrahigh-Q toroid microcavity Phys. Rev. Lett. 93 083904
[57] Del’Haye P, Schliesser A, Arcizet O, Wilken T, Holzwarth R and Kippenberg T 2007 Optical frequency comb generation from a monolithic microresonator Nature 450 1214–7
[58] Ma R, Schliesser A, Del’Haye P, Dabirian A, Anetsberger G and Kippenberg T 2007 Radiation-pressure-driven vibrational modes in ultrahigh-Q silica microspheres Opt. Lett. 32 2200–2
[59] Park Y S and Wang H L 2007 Radiation pressure driven mechanical oscillation in deformed silica microspheres via free-space evanescent excitation Opt. Express 15 16471–7
[60] Clark J R, Hsu W T, Abdelmoneum M A and Nguyen C T-C 2005 High-Q UHF micromechanical radial-contour mode disk resonators J. Micromechanics. Syst. 14 1298
[61] Cleland A N 2003 Foundations of Nanomechanics (Berlin: Springer)
[62] Anetsberger G, Rivière R, Schliesser A, Arcizet O and Kippenberg T J 2008 Demonstration of ultra low dissipation optomechanical resonators on a chip Nat. Photonics to appear (arXiv:0802.4384)