PLASMA INDUCED NEUTRINO RADIATIVE DECAY
INSTEAD OF NEUTRINO SPIN LIGHT

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Abstract

The conversion $\nu_L \rightarrow \nu_R\gamma^*$ of a neutrino with a magnetic moment is considered, caused by the additional Wolfenstein energy acquired by a left-handed neutrino in medium, with an accurate taking account of the photon $\gamma^*$ dispersion in medium. It is shown that the threshold arises in the process, caused by the photon (plasmon) effective mass. This threshold leaves no room for the so-called “neutrino spin light” in the most of astrophysical situations.

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The most important event in neutrino physics of the last decades was the solving of the Solar neutrino problem, made in the unique experiment on the heavy-water detector at the Sudbury Neutrino Observatory [1]. This experiment, together with the atmospheric and the reactor neutrino experiments [2,3], has confirmed the key idea by B. Pontecorvo on neutrino oscillations [4]. The existence of non-zero neutrino mass and lepton mixing is thereby established. The Sun appeared in this case as a natural laboratory for investigations of neutrino properties.

There exists a number of natural laboratories, the supernova explosions, where gigantic neutrino fluxes define in fact the process energetics. It means that microscopic neutrino characteristics, such as the neutrino magnetic moment, the neutrino dispersion in an active medium, etc., would have a critical impact on macroscopic properties of these astrophysical events.

This is the reason for a growing interest to neutrino physics in an external active medium. In an astrophysical environment, the main medium influence on neutrino properties is defined by the additional Wolfenstein energy $W$ acquired by a left-handed neutrino [5]. The general expression for this additional energy of a left-handed neutrino with the flavor $i = e, \mu, \tau$ is [6–8]

$$W_i = \sqrt{2} G_F \left[ \left( \delta_{ie} - \frac{1}{2} + 2 \sin^2 \theta_W \right) (N_e - \bar{N}_e) ight.$$

$$+ \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) (N_p - \bar{N}_p) - \frac{1}{2} (N_n - \bar{N}_n) + \frac{1}{2} \sum_{\ell=e,\mu,\tau} (N_{\nu_\ell} - \bar{N}_{\nu_\ell}) \right],$$

(1)

where the functions $N_e, N_p, N_n, N_{\nu_\ell}$ are the number densities of background electrons, protons, neutrons, and neutrinos, and $\bar{N}_e, \bar{N}_p, \bar{N}_n, \bar{N}_{\nu_\ell}$ are the densities of the corresponding antiparticles. To find the additional energy for antineutrinos, one should change the total sign in the right-hand side of Eq. (1).

For a typical astrophysical medium, except for the early Universe and a supernova core, one has $\bar{N}_e \simeq \bar{N}_p \simeq \bar{N}_n \simeq N_{\nu_\ell} \simeq 0$, and $N_p \simeq N_e = Y_e N_B$, $N_n \simeq (1 - Y_e) N_B$, where $N_B$ is the barion density. One obtains

$$W_e = \frac{G_F N_B}{\sqrt{2}} (3 Y_e - 1),$$

(2)

$$W_{\mu,\tau} = - \frac{G_F N_B}{\sqrt{2}} (1 - Y_e).$$

(3)

As $Y_e < 1$, the additional energy acquired by muon and tau left-handed neutrinos is always negative. At the same time, the additional energy of electron left-handed neutrinos becomes positive at $Y_e > 1/3$. And vice versa, the additional energy for electron antineutrinos is positive at $Y_e < 1/3$, while it is always positive for the muon and tauon antineutrinos. On the other hand, right-handed neutrinos and their antiparticles, left-handed antineutrinos, being sterile with respect to weak interactions, do not acquire an additional energy.

The additional energy $W$ from Eq. (2) gives an effective mass squared $m_L^2$ to the left-handed neutrino,

$$m_L^2 = \mathcal{P}^2 = (E + W)^2 - \mathbf{p}^2,$$

(4)

where $\mathcal{P}$ is the neutrino four-momentum in medium, while $(E, \mathbf{p})$ would form the neutrino four-momentum in vacuum, $E = \sqrt{\mathbf{p}^2 + m_\nu^2}$.

Given a $\nu \nu \gamma$ interaction, the additional energy of left-handed neutrinos in medium opens new kinematical possibilities for the radiative neutrino transition:

$$\nu \rightarrow \nu + \gamma.$$  

(5)

It should be self-evident, that the influence of the substance on the photon dispersion must be taken into account, $\omega = |\mathbf{k}|/n$, where $n \neq 1$ is the refractive index.
First, a possibility exists that the medium provides the condition $n > 1$ (the effective photon mass squared is negative, $m_\gamma^2 \equiv q^2 < 0$) which corresponds to the well-known effect [14–16] of “neutrino Cherenkov radiation”. In this situation, the neutrino dispersion change under the medium influence is being usually neglected, because the neutrino dispersion is defined by the weak interaction while the photon dispersion is defined by the electromagnetic interaction.

Pure theoretically, one more possibility could be considered when the photon dispersion was absent, and the process of the radiative neutrino transition $\nu \to \nu \gamma$ would be caused by the neutrino dispersion only. As the left-handed neutrino dispersion is only changed, transitions become possible caused by the $\nu \nu \gamma$ interaction with the neutrino chirality change, e.g. due to the neutrino magnetic dipole moment.

Just this situation called the “spin light of neutrino” ($SL\nu$), was first proposed and investigated in detail in an extended series of papers [9–13]. However, in the analysis of this effect the authors overlooked such an important phenomenon as plasma influence on the photon dispersion. As will be shown below, this phenomenon closes the $SL\nu$ effect for all real astrophysical situations.

In this Letter, we reanalyse the process $\nu_L \to \nu_R \gamma$ taking into account both the neutrino dispersion and the photon dispersion in medium. Having in mind possible astrophysical applications, it is worthwhile to consider the astrophysical plasma as a medium, which transforms the photon into the plasmon, see e.g. Ref. [17] and the papers cited therein.

To perform a kinematical analysis, it is necessary to evaluate the scales of the values of the left-handed neutrino additional energy $W$ and of the photon (plasmon) effective mass squared $m_\gamma^2$.

One readily obtains from Eq. (2):

$$W \simeq 6 \text{ eV} \left( \frac{N_B}{10^{38} \text{ cm}^{-3}} \right) (3 Y_e - 1),$$

where the scale of the barion number density is taken, which is typical e.g. for the interior of a neutron star.

On the other hand, a plasmon acquires in medium an effective mass $m_\gamma$, which is approximately constant at high energies. For the transversal plasmon, the value $m_\gamma^2$ is always positive, and is defined by the so-called plasmon frequency. In the non-relativistic classical plasma (i.e. for the solar interior) one has:

$$m_\gamma \equiv \omega_{pl} = \sqrt{\frac{4\pi \alpha N_e}{m_e}} \simeq 4 \times 10^2 \text{ eV} \left( \frac{N_e}{10^{26} \text{ cm}^{-3}} \right)^{1/2}.$$ (7)

For the ultra-relativistic dense matter one has:

$$m_\gamma = \sqrt{\frac{3}{2}} \omega_{pl} = \left( \frac{2\alpha}{\pi} \right)^{1/2} \left( 3 \pi^2 N_e \right)^{1/3} \simeq 10^7 \text{ eV} \left( \frac{N_e}{10^{37} \text{ cm}^{-3}} \right)^{1/3}. $$ (8)

In the case of hot plasma, when its temperature is the largest physical parameter, the plasmon mass is:

$$m_\gamma = \sqrt{\frac{2\pi \alpha}{3}} T \simeq 1.2 \times 10^7 \text{ eV} \left( \frac{T}{100 \text{ MeV}} \right). $$ (9)

One more physical parameter, a great attention was payed to in the $SL\nu$ analysis [9–13], was the neutrino vacuum mass $m_\nu$. As the scale of neutrino vacuum mass could not exceed essentially a few electron-volts, which is much less than typical plasmon mass scales for real astrophysical situations, see Eqs. (7)–(9), it is reasonable to neglect $m_\nu$ in our analysis.

Thus, in accordance with (4), a simple condition for the kinematical opening of the process $\nu_L \to \nu_R \gamma$ is:

$$m_L^2 \simeq 2 EW > m_\gamma^2.$$ (10)
This means that the process becomes kinematically opened when the neutrino energy exceeds the threshold value,
\[ E > E_0 = \frac{m_e^2}{2W}. \] (11)
Let us evaluate these threshold neutrino energies for different astrophysical situations.

For the solar interior \( N_B \simeq 0.9 \times 10^{26} \text{ cm}^{-3}, \) \( Y_e \simeq 0.6, \) and the threshold neutrino energy is
\[ E_0 \simeq 10^{10} \text{ MeV}, \] (12)
to be compared with the upper bound \( \sim 20 \text{ MeV} \) for the solar neutrino energies.

For the interior of a neutron star, where \( Y_e \ll 1, \) the Wolfenstein energy for neutrinos \( (2), (3) \) is negative, and the process \( \nu_L \rightarrow \nu_R \gamma \) is closed. On the other hand, there exists a possibility for opening the antineutrino decay. Taking for the estimation \( Y_e \simeq 0.1, \) one obtains from \( (6) \) and \( (8) \) the threshold value
\[ E_0 \simeq 10^7 \text{ MeV}, \] (13)
to be compared with the typical energy \( \sim \text{ MeV} \) of neutrinos emitted via the URCA processes.

For the conditions of a supernova core, the additional energy of left-handed electron neutrinos can be obtained from Eq. \( (1) \) as follows:
\[ W_e = \frac{G_F N_B}{\sqrt{2}} (3 Y_e + Y_{\nu e} - 1), \] (14)
where \( Y_{\nu e} \) describes the fraction of trapped electron neutrinos in the core, \( N_{\nu e} = Y_{\nu e} N_B. \) Taking typical parameters of a supernova core, we obtain
\[ E_0 \simeq 10^7 \text{ MeV}, \] (15)
to be compared with the averaged energy \( \sim 10^2 \text{ MeV} \) of trapped neutrinos.

In the early Universe, when plasma was almost charge symmetric, the Wolfenstein formula \( (1) \) giving zero should be changed to a more accurate expression for the additional energy which is identical for both neutrinos and antineutrinos \([6, 18]\)
\[ W_i = -7 \sqrt{2} \pi^2 G_F T^4 \left( \frac{1}{m_Z^2} + \frac{2 \delta_{ie}}{m_W^2} \right) E. \] (16)
The minus sign unambiguously shows that in the early Universe, in contrast to the neutron star interior, the decay process is forbidden both for neutrinos and antineutrinos.

Thus, the above analysis shows that the nice effect of the “neutrino spin light”, unfortunately, has no place in real astrophysical situations because of the photon dispersion. The sole possibility for the discussed process \( \nu_L \rightarrow \nu_R \gamma \) to have any significance could be connected only with the situation when an ultra-high energy neutrino threads a star. Obviously it could have only a methodical meaning. Let us calculate the process width for these purposes correctly.

A neutrino having a magnetic moment \( \mu_\nu \) interacts with photons, and the Lagrangian of this interaction is
\[ \mathcal{L} = -i \frac{\mu_\nu}{2} (\bar{\nu} \sigma_{\alpha \beta} \nu) F^{\alpha \beta}, \] (17)
where \( \sigma_{\alpha \beta} = (1/2) (\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha), \) and \( F^{\alpha \beta} \) is the tensor of the photon electromagnetic field.

With the Lagrangian \( (17) \), the invariant amplitude squared for the process \( \nu_L \rightarrow \nu_R \gamma, \) summarized over the transversal plasmon polarizations, can be obtained by the standard way:
\[ |\mathcal{M}|^2 = 4 \mu_\nu^2 E^2 \left[ 2 W^2 \left( 1 - \frac{\omega}{E} \right) - m_\gamma^2 \sin^2 \theta \right], \] (18)
where \( \omega \) is the plasmon energy, \( \theta \) is the angle between the initial neutrino momentum \( \mathbf{p} \) and the plasmon momentum \( \mathbf{k}. \) It should be stressed that discussing ultra-high energy neutrinos, and
consequently the high plasmon energies, one can consider with a good accuracy the plasmon mass $m_\gamma$ as a constant depending on the plasma properties only, see Eqs. (7)-(9). This is in contrast to the left-handed neutrino effective mass squared $m_\nu^2$, which is the dynamical parameter, see Eq. (4).

The differential width of the process $\nu_L \rightarrow \nu_R \gamma$ is defined as:

$$d\Gamma = \frac{|\mathcal{M}|^2}{8E(2\pi)^2} \delta(E + W - E' - \omega) \delta^{(3)}(p - p' - k) \frac{d^3p'd^3k}{E'\omega},$$

where the plasmon energy $\omega$ cannot be taken the vacuum one ($\omega = |k|$), as it was done in the $SL\nu$ analysis [9–13], but it is defined by the dispersion in plasma, $\omega = \sqrt{k^2 + m_\gamma^2}$.

Performing a partial integration in Eq. (19), one obtains for the photon (i.e. transversal plasmon) spectrum

$$d\Gamma = \frac{\alpha}{4} \frac{(\mu_\nu}{\mu_B})^2 \frac{m_\nu^2 W}{m_e^2} f(x, \varepsilon) dx \left( \frac{1}{\varepsilon} \leq x \leq 1 \right),$$

$$f(x, \varepsilon) = \varepsilon (1 - x) + 2 \left( 1 - \frac{\varepsilon + 1}{\varepsilon x} + \frac{1}{\varepsilon x^2} \right),$$

where $\mu_B = e/(2 m_e)$ is the Bohr magneton, and the notations are used $x = \omega/E$, and $\varepsilon = E/E_0$. Recall that $E_0 = m_\gamma^2/(2W)$ is the threshold neutrino energy for the process to be opened. In Fig. 1 the function $f(x, \varepsilon)$ is shown for some values of the ratio $\varepsilon$.

![Diagram](image)

Figure 1: The function $f(x, \varepsilon)$ defining the spectrum of plasmons from the left-handed neutrino decay for different values of the ratio $\varepsilon = E/E_0$ of the neutrino energy to the threshold neutrino energy, $\varepsilon = 10$ (solid line), $\varepsilon = 5$ (dotted line), and $\varepsilon = 2$ (dashed line).

Instead of the photon energy spectrum (20), one can obtain also the spatial distribution of final photons. As the analysis shows, all the photons are created inside the narrow cone with the opening angle $\theta_0$,

$$\theta < \theta_0 \approx \frac{\varepsilon - 1}{\varepsilon} \frac{W}{m_\gamma}. $$

(21)
Performing the final integration in Eq. (20), one obtains the total width of the process

$$\Gamma = \alpha \frac{2}{8} \left( \frac{\mu_\nu}{\mu_B} \right)^2 \frac{m_\gamma^2 W}{m_e^2} F(\varepsilon) \quad (\varepsilon \geq 1),$$

$$F(\varepsilon) = \frac{1}{\varepsilon} \left[ (\varepsilon - 1)(\varepsilon + 7) - 4(\varepsilon + 1) \ln \varepsilon \right]. \quad (22)$$

It should be noted that in the situation when $W < 0$, and the transition $\nu_L \to \nu_R \gamma$ is forbidden, the crossed channel $\nu_R \to \nu_L \gamma$ becomes kinematically opened. As the analysis shows, the plasmon spectrum and the total decay width are described in this case by the same Eqs. (20) and (22), with the only substitution $W \to |W|$. To illustrate the extreme weakness of the effect considered, let us evaluate numerically the mean free path of an ultra-high energy neutrino with respect to the radiative decay, when the neutrino threads a neutron star. For the typical neutron star parameters, $N_B \simeq 10^{38} \text{cm}^{-3}$, $Y_e \simeq 0.05$, we obtain

$$L \simeq 10^{19} \text{cm} \times \left( \frac{10^{-12} \mu_B}{\mu_\nu} \right)^2 \left[ F \left( \frac{E}{10 \text{TeV}} \right) \right]^{-1}, \quad (23)$$

where the neutrino energy $E > E_0$, $E_0 \simeq 10 \text{TeV}$ is the threshold energy for such conditions. The mean free path (23) should be compared with the neutron star radius $\sim 10^6 \text{cm}$.

In conclusion, we have shown that the effect of the “neutrino spin light” has no place in real astrophysical situations because of the photon dispersion in plasma. The photon (plasmon) effective mass causes the threshold which leaves no room for the process. For a pure theoretical situation when an ultra-high energy neutrino threads a star, the total width of the process $\nu_L \to \nu_R \gamma$ is calculated with a correct taking account of the photon dispersion in plasma. The extreme weakness of the effect considered is illustrated.

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References

[1] Q. R. Ahmad et al. (SNO Collab.), Phys. Rev. Lett. 87, 071301 (2001); ibid. 89, 011301; 011302 (2002).

[2] Y. Fukuda et al. (Super-Kamiokande Collab.), Phys. Lett. B433, 9 (1998); ibid. B436, 33 (1998); Phys. Rev. Lett. 82, 2644 (1999).

[3] K. Eguchi et al. (KamLAND Collab.), Phys. Rev. Lett. 90, 021802 (2003).

[4] B. Pontecorvo, Zh. Eksp. Teor. Fiz. 33, 549 (1957) [Sov. Phys. JETP 6, 429 (1957)]; Zh. Eksp. Teor. Fiz. 34, 247 (1958) [Sov. Phys. JETP 7, 172 (1958)].

[5] L. Wolfenstein, Phys. Rev. D17, 2369 (1978).

[6] D. Nötzold and G. Raffelt, Nucl. Phys. B307, 924 (1988).

[7] P. B. Pal and T. N. Pham, Phys. Rev. D40, 259 (1989).
[8] J. F. Nieves, *Phys. Rev.* **D40**, 866 (1989).

[9] A. Lobanov and A. Studenikin, *Phys. Lett.* **B564**, 27 (2003).

[10] A. Lobanov and A. Studenikin, *Phys. Lett.* **B601**, 171 (2004).

[11] A. Studenikin and A. Ternov, *Phys. Lett.* **B608**, 107 (2005).

[12] A. E. Lobanov, *Phys. Lett.* **B619**, 136 (2005).

[13] A. Grigoriev, A. Studenikin and A. Ternov, *Phys. Lett.* **B622**, 199 (2005).

[14] W. Grimus and H. Neufeld, *Phys. Lett.* **B315**, 129 (1993).

[15] J.C. D’Olivo, J.F. Nieves and P.B. Pal, *Phys. Lett.* **B365**, 178 (1996).

[16] A.N. Ioannisian and G.G. Raffelt, *Phys. Rev.* **D55**, 7038 (1997).

[17] E. Braaten and D. Segel, *Phys. Rev.* **D48**, 1478 (1993).

[18] P. Elmfors, D. Grasso and G. Raffelt, *Nucl. Phys.* **B479**, 3 (1996).