Mathematical model of motor vehicle air suspension with a combined damping system

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Abstract. The article presents a design scheme and a mathematical model of a spatial four-support pneumatic suspension of a car with a combined damping system, which includes: a hydraulic shock absorber of small capacity, air damping, an inertial-friction shock absorber and a dynamic damper.

1. Introduction
Currently, various types of elastic elements and telescopic hydraulic shock absorbers (HSA) are used in motor vehicle suspensions (MV). At the same time to improve smoothness of movement metal springs are replaced with adjustable pneumatic springs (PS) with rubber-cord shells (RCS) and hydraulic absorbers are tuned to the mode of damping oscillations of a loaded car.

However, the uncontrollability of the characteristics of the HSA does not allow optimal damping of body and wheel vibrations under various conditions of suspension operation, which results in decreasing smoothness of the HSA and significant energy losses in them. One of the possible ways to solve this problem is the use of combined damping systems of various types (hydraulic, pneumatic, friction, inertial and dynamic), which are automatically activated depending on the oscillation modes and provide effective vibration protection. Therefore, development and study of such a pneumatic suspension with a combined damping system (CDS), enhancing vibration-proof properties of an MV suspension, remains an urgent problem [1 ... 22].

2. Description of the mathematical model
A spatial 4-way model of the air suspension of the vehicle with a combined air, hydraulic, inertial and dynamic damping has been developed to study the effect of the combined damping on the vibration-protective properties of the suspension system (Figure 1). In this suspension, inertial and dynamic damping are realized due to flywheel dampers (FD) 12 and 15, they are made in the form of inertial-friction shock absorbers, elastically installed on unsprung masses 6 and 7 in the suspension of front and rear wheels.

The following assumptions were made in developing this model:
- a car of full mass is considered, the body is symmetric about the longitudinal axis x, the torsional and bending frame deformations can be neglected;
- characteristics of stiffness of the front and rear suspensions, characteristics of shock absorbers and radial load characteristics of tires are linearized;
- dry friction in springs and suspension elements is reduced to viscous;
- the movement of the car is stationary, the center of gravity of the body is located in the longitudinal plane, the center of gravity of a uniformly distributed load corresponds to its geometric center of gravity;
- only vertical impact is taken into account; the transverse and longitudinal reactions of the road are neglected;
- axles (bridges) move in planes perpendicular to the plane frame, and their moments of inertia about the axis of rotation of the wheels are equal to zero;
- contact of the tire with the road point, takes into account only one force, characterized by the ordinate road surface under the center of the wheel;
- wheels have two-way communication with the road, i.e. there is no wheel breakaway from the road.

Figure 1. Scheme of 4-axle car suspension with combined damping system: 1 – pressed mass; 2, 3 and 4, 5 – front and rear elastic elements and shock absorbers; 6 and 7 – front and rear unsprung mass; 8, 10 and 9, 11 – front and rear tires with elastic and damping properties; 12, 15 – front and rear FD; 13, 14 and 16, 17 – elastic and damping element of the anterior and posterior FD; \( a \) – distance between the axis of rotation of the FD lever; \( M \) – spring-loaded mass (body); \( J_x \) and \( J_y \) are moments of inertia of the body relative to the axes \( x \) and \( y \); \( l \) – length of lever; \( l_1, l_2 \) – distance from the center of the car body to the front and rear axes; \( B \) – distance from the middle of the car body to the wheels along the \( y \) axis; \( i_k \) – gear ratio of FD; \( m_{FD} \) – mass of FD; \( c_{FD} \) – stiffness of the FD spring; \( k_{FD} \) – coefficient of resistance of HSA FD; \( T \) – Dry friction of RCS; \( 2c_1 \) and \( 2c_2 \) – rigidity of front and rear wheels; \( 2a_1 \) and \( 2a_2 \) – coefficient of resistance of the above the front and rear wheels; \( 2k_{t1}, 2k_{t2} \) – damping in the front and rear tires; \( 2c_{t1} \) and \( 2c_{t2} \) – tire strength of the front and rear wheels; \( 2m_1, 2m_2 \) – mass of front and rear wheels.

Dynamic system shown in Figure 1 simulates three body movements: translational along the \( y \) axis and rotating with respect to the \( x \) and \( z \) axes. Thus three degrees of freedom can be designated in the pressed part of the car: in the coordinate \( y \), that is, the vertical movement of the body on the springs together with the center of mass; by a generalized coordinate \( \theta \), which characterizes the longitudinal-angular oscillations, that is, the turns of the body relative to the \( z \)-axis; by a generalized coordinate \( \varphi \), which characterizes the transverse angular vibrations of the body relative to the \( x \) axis. Taking into account the vertical displacements of unsprung masses along the coordinates and the flywheel shifter by the coordinates \( \varsigma \) and \( \theta \), the dynamic model of the car is a system with 15 degrees of freedom, which is described by the system of equations(1):
\[ 
\dot{z} + k_1 \cdot L_1 + k_2 \cdot L_2 + Mg - Mg \cdot O1 \cdot E1 - p_{atm} \cdot S \cdot D1 - D1 - Mg \cdot O1 \cdot E2 - p_{atm} \cdot S \cdot D2 - MG \cdot O2 \cdot E3 - p_{atm} \cdot S \cdot D3 - Mg \cdot O2 \cdot E4 - p_{atm} \cdot S \cdot D4 + \frac{M_{fr}}{l} \cdot F1 + \frac{M_{fr}}{l} \cdot F2 + \frac{M_{fr}}{l} \cdot F3 + \frac{M_{fr}}{l} \cdot F4 = 0
\]

\[ 
J_y \ddot{\psi} + k_1 \cdot L_1 \cdot l_1 - k_2 \cdot L_2 \cdot l_2 - O3 \cdot E1 - p_{atm} \cdot S \cdot l_1 \cdot D1 \cdot O4 \cdot E2 - p_{atm} \cdot S \cdot l_1 \cdot D2 \cdot O5 \cdot E3 - p_{atm} \cdot S \cdot l_2 \cdot D3 + O6 \cdot E4 + p_{atm} \cdot S \cdot l_2 \cdot D4 + \frac{M_{fr}}{l} \cdot F1 + \frac{M_{fr}}{l} \cdot F2 + \frac{M_{fr}}{l} \cdot F3 + \frac{M_{fr}}{l} \cdot F4 = 0
\]

\[ 
F_4 \cdot \frac{B}{2} = 0,
\]

\[ 
m_{1/2} \ddot{\xi}_{11} + k_{11} \left( \ddot{\xi}_{11} - \ddot{\theta}_{11} \right) + c_{11} \left( \dot{\xi}_{11} - \dot{\theta}_{11} \right) - k_1 \left( \ddot{z} + \dot{\phi}_l - \frac{\psi}{z} - \ddot{\xi}_{11} \right) + Mg \frac{l_2}{l_1 + l_2} - Mg \frac{l_2}{l_1 + l_2} \cdot E1 - p_{atm} \cdot S \cdot D1 - \frac{M_{fr}}{l} \cdot \lambda \cdot F1 = 0.
\]

\[ 
m_{1/2} \ddot{\xi}_{1r} + k_{1r} \left( \ddot{\xi}_{1r} - \ddot{\theta}_{1r} \right) + c_{1r} \left( \dot{\xi}_{1r} - \dot{\theta}_{1r} \right) - k_1 \left( \ddot{z} + \dot{\phi}_l - \frac{\psi}{z} - \ddot{\xi}_{1r} \right) + Mg \frac{l_2}{l_1 + l_2} - Mg \frac{l_2}{l_1 + l_2} \cdot E2 - p_{atm} \cdot S \cdot D2 - \frac{M_{fr}}{l} \cdot \lambda \cdot F2 = 0.
\]

\[ 
m_{1/2} \ddot{\xi}_{21} + k_{21} \left( \ddot{\xi}_{21} - \ddot{\theta}_{21} \right) + c_{21} \left( \dot{\xi}_{21} - \dot{\theta}_{21} \right) - k_2 \left( \ddot{z} + \dot{\phi}_l - \frac{\psi}{z} - \ddot{\xi}_{21} \right) + Mg \frac{l_1}{l_1 + l_2} - Mg \frac{l_1}{l_1 + l_2} \cdot E3 - p_{atm} \cdot S \cdot D3 - \frac{M_{fr}}{l} \cdot \lambda \cdot F3 = 0.
\]

\[ 
m_{1/2} \ddot{\xi}_{2r} + k_{2r} \left( \ddot{\xi}_{2r} - \ddot{\theta}_{2r} \right) + c_{2r} \left( \dot{\xi}_{2r} - \dot{\theta}_{2r} \right) - k_2 \left( \ddot{z} + \dot{\phi}_l - \frac{\psi}{z} - \ddot{\xi}_{2r} \right) + Mg \frac{l_2}{l_1 + l_2} - Mg \frac{l_2}{l_1 + l_2} \cdot E4 - p_{atm} \cdot S \cdot D4 - \frac{M_{fr}}{l} \cdot \lambda \cdot F4 = 0.
\]

\[ 
\begin{align*}
\text{Where } E1 & = \left( \frac{l_0}{l_0 + z + \phi_l + \psi_2 - \zeta_{21}} \right)^n; \\
E2 & = \left( \frac{l_0}{l_0 + z + \phi_l_2 + \psi_2 - \zeta_{21}} \right)^n; \\
E3 & = \left( \frac{l_0}{l_0 + z + \phi_l + \psi_2 - \zeta_{21}} \right)^n;
\end{align*}
\]

\[ 
\begin{align*}
E4 & = \left( \frac{l_0}{l_0 + z + \phi_l + \psi_2 - \zeta_{21}} \right)^n; \\
D1 & = \left( \frac{l_0 - (l_0 + z + \phi_l + \psi_2 - \zeta_{21})}{l_0 + z + \phi_l + \psi_2 - \zeta_{21}} \right)^n; \\
D2 & = \left( \frac{l_0 - (l_0 + z + \phi_l + \psi_2 - \zeta_{21})}{l_0 + z + \phi_l + \psi_2 - \zeta_{21}} \right)^n; \\
D3 & = \left( \frac{l_0 - (l_0 + z + \phi_l + \psi_2 - \zeta_{21})}{l_0 + z + \phi_l + \psi_2 - \zeta_{21}} \right)^n; \\
F1 & = \text{sgn} \left( \ddot{z} + \dot{\phi}_l + \frac{B}{z} - (1 - \lambda) \zeta_{11} - \lambda \dot{\zeta}_{11} - \dot{\theta}_{11} \frac{l}{l_1} \right);
\end{align*}
\]

\[ 
\begin{align*}
O2 & = Mg \frac{l_1}{l_1 + l_2}; \\
F2 & = \text{sgn} \left( \ddot{z} + \dot{\phi}_l + \frac{B}{z} - (1 - \lambda) \zeta_{1r} - \lambda \dot{\zeta}_{1r} - \dot{\theta}_{1r} \frac{l}{l_1} \right);
\end{align*}
\]

\[ 
\begin{align*}
F3 & = \text{sgn} \left( \ddot{z} + \dot{\phi}_l + \frac{B}{z} - (1 - \lambda) \zeta_{21} - \lambda \dot{\zeta}_{21} - \dot{\theta}_{21} \frac{l}{l_1} \right); \\
O8 & = Mg \frac{l_1}{l_1 + l_2}; \\
F4 & = \text{sgn} \left( \ddot{z} + \dot{\phi}_l + \frac{B}{z} - (1 - \lambda) \zeta_{2r} - \lambda \dot{\zeta}_{2r} - \dot{\theta}_{2r} \frac{l}{l_1} \right); \\
L1 & = \left( 2 \ddot{z} + 2 \dot{\phi}_l - \zeta_{11} - \dot{\zeta}_{11} \right); \\
L2 & = \left( \psi_2 - \zeta_{11} + \dot{\zeta}_{11}; \lambda = \frac{l}{a} \right)
\end{align*}
\]
\[ L4 = (\ddot{\psi}B - \zeta_{2l} + \zeta_{2r}); \quad O4 = \frac{Mgl_1}{l_1 + l_2}; \quad O5 = \frac{Mgl_2}{l_1 + l_2}; \quad O6 = \frac{Mgl_1}{l_1 + l_2}; \quad O7 = \frac{Mgl_2}{l_1 + l_2}; \]

\[ \dot{z}, \ddot{z} - \text{vertical movements, speeds and accelerations of the pressurized mass}; \quad \psi, \dot{\psi}, \ddot{\psi} - \text{displacement, speed and acceleration of the sprung mass regarding to the longitudinal axis x and the transverse axis y}; \quad \zeta_{1l}, \zeta_{1r}, \zeta_{2l}, \zeta_{2r} - \text{movement of FD over the front and rear right and left wheels}; \quad \theta_{1l}, \dot{\theta}_{1l}, \ddot{\theta}_{1l}, \theta_{1r}, \dot{\theta}_{1r}, \ddot{\theta}_{1r}, \theta_{2l}, \dot{\theta}_{2l}, \ddot{\theta}_{2l}, \theta_{2r}, \dot{\theta}_{2r}, \ddot{\theta}_{2r} - \text{angle, speed and acceleration of the FD flywheel above each wheel}; \quad \zeta_{1l}, \zeta_{1r}, \zeta_{2l}, \zeta_{2r} - \text{moving the speed and acceleration of the wheels.} \]

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