Sensorless Control of sPMSM Using Identification Theory

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Abstract. A novel sensorless control strategy is proposed for the surface type permanent magnetism synchronous motor (sPMSM) based on system identification. The grey method is used to detect the rotor position and rotor speed by the predict current in the two-phase rotor estimation frame. And then, the sensorless vector control system of sPMSM is set up based on this position estimation approach. The experimental operation and simulation have verified the validity of the proposed sensorless control scheme.

1. Introduction

Most permanent magnet synchronous motor control systems use sensors such as generator or photoelectric encoders to detect the speed and position feedback. This not only increases the cost of the drive, but also increases the connection lines and interface circuits between the motor and the control system. It makes the system susceptible to environmental disturbances and reduces reliability. The non-mechanical sensor-control technology developed in recent years provides a good way to solve these problems. It eliminates the magnetic pole position and speed sensors, simplifies the system structure, reduces the cost, improves the control and calculation accuracy, and is reliable, strong. It has become a research hot spot in recent years.

Motor less-sensor control technology refers to the use of or detecting electrical signals in motor winding without the installation of electromagnetic or microelectronics mechanical sensors in the motor rotors and bases, through direct calculation, parameter identification, state estimation, indirect measurement, etc. From the starter voltage, current, flux, and back electromotive force, the speed and position related quantities are extracted. Using these detected quantities and the mathematical model of the motor, the position and speed of the rotor of the motor are inferred to replace the mechanical sensor to realize the closed loop of the motor control.

High-precision motor control systems place high demands on speed and position detection, and correspondingly increased sensor requirements. At present, the existence of mechanical sensors in the motor control system hinders the development of the motor to high speed and miniaturization. Therefore, the research of sensor-less technology has important significance in the control of high-speed motors and micro-motors. Speed sensor-less technology was developed after the emergence of digital signal processor DSP. DSP high-speed information processing capabilities enable sophisticated algorithms without speed sensor control technology to be implemented.

The sensor-less technology currently used for PMSM is mainly divided into two categories: (1) the use of high-frequency voltage and high-frequency current [1-3]. (2) Utilizing the fundamental component of voltage and current [4-6]. The former method is also effective at stationary and low speeds because the high-frequency signal used for position estimation is independent of the rotational speed, and some methods can also be independent of electrical parameters. The latter method mainly uses the fundamental component of the signal to estimate the position, does not cause rotational speed
fluctuations and noise, is more effective at medium and high speeds, but depends on electrical parameters.

Based on the analysis of PMSM model, this paper proposes a new estimation algorithm based on gray theory. The gray system theory has been widely used in various fields of prediction. It is characterized by gray mathematics to deal with the uncertainty to quantify, and make full use of known information to seek the law of system motion. The main idea of the gray system is to process the original information data sequence through a certain mathematical method and convert it into a differential equation to describe the objective law of the original system. Using the gray system theory to predict the rotor position and rotor speed, this method can not only improve the robustness to the change of motor parameters, but also effectively solve the problem of insufficient accuracy of position estimation in the transient process. This method detects the starter voltage and current, without additional control equipment and devices, without additional signals, using software-only methods to achieve sensor-less control.

2. Permanent magnet synchronous motor mathematical model

\[ sPMSM \text{ at } \alpha - \beta \]

The coordinate system voltage equation is:

\[
\begin{align*}
\frac{di_a}{dt} &= -\frac{R_s}{L} i_a(t) + \frac{1}{L_i} v_a(t) + \frac{\omega(t)}{L} \psi_f \sin \theta(t) \\
\frac{di_\beta}{dt} &= -\frac{R_s}{L} i_\beta(t) + \frac{1}{L_i} v_\beta(t) - \frac{\omega(t)}{L} \psi_f \cos \theta(t)
\end{align*}
\]

(1)

Among them, \(i_a, i_\beta\) Winding current \(\alpha, \beta\) Axis components, \(v_a, v_\beta\) For winding voltage \(\alpha, \beta\) Axis components, \(R_s\) For stator coil equivalent internal resistance, \(\omega\) For the rotor electrical angular speed, \(\psi_f\) For the permanent magnet equivalent excitation flux. \(\theta\) For the rotor position electrical angle.

\[ sPMSM \text{ at } d - q \]

The coordinate system voltage equation is:

\[
\begin{align*}
\frac{di_d}{dt} &= \frac{-R_s}{L} \omega i_d(t) + \frac{1}{L_i} v_d(t) + \frac{0}{L} v_q(t) \\
\frac{di_q}{dt} &= \frac{-R_s}{L} \omega i_q(t) + \frac{1}{L_i} v_q(t) - \frac{0}{L} v_d(t)
\end{align*}
\]

(2)

In the formula, \(i_d, i_q\) is winding current \(d, q\) is axis components, \(v_d, v_q\) is winding voltage \(d, q\) Axis component. If \(\tilde{\theta}\) indicates the rotor position detection value, and note: \(\Delta \theta = \theta - \tilde{\theta}\), then rotate through the coordinates (the coordinate rotation angle is \(\tilde{\theta}\)) can be detected in the rotating coordinate system \(\gamma - \delta\). The voltage equation of the lower winding:

\[
\begin{align*}
\frac{di_\gamma}{dt} &= \frac{-R_s}{L} \frac{d\tilde{\theta}}{dt} i_\gamma(t) + \frac{1}{L_i} v_\gamma(t) \\
\frac{di_\delta}{dt} &= \frac{-R_s}{L} \frac{d\tilde{\theta}}{dt} i_\delta(t) + \frac{1}{L_i} v_\delta(t) - \frac{\omega(t)}{L} \frac{-\sin \Delta \theta}{\cos \Delta \theta}
\end{align*}
\]

(3)

The discretion form of motor equation (3) is as follows:

\[
\begin{bmatrix}
    i_\gamma(n+1) \\
    i_\delta(n+1)
\end{bmatrix} = \begin{bmatrix}
    A & B \\
    C & D
\end{bmatrix} \begin{bmatrix}
    i_\gamma(n) \\
    i_\delta(n)
\end{bmatrix} + \begin{bmatrix}
    v_\gamma(n) \\
    v_\delta(n)
\end{bmatrix} + \begin{bmatrix}
    C_1 \\
    C_2
\end{bmatrix}
\]

(4)
In the matrix
\[
\begin{bmatrix}
A & B \\
C & \Theta
\end{bmatrix}
\]
In them
\[
A = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} = \begin{bmatrix}
1 - \Delta T R_1 / L & \tilde{\theta}(n+1) - \tilde{\theta}(n) \\
-\tilde{\theta}(n+1) + \tilde{\theta}(n) & 1 - \Delta T R_2 / L
\end{bmatrix},
\]
\[
B = \begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix} = \begin{bmatrix}
\Delta T / L & 0 \\
0 & \Delta T / L
\end{bmatrix},
\]
\[
C = \begin{bmatrix}
c_1 \\
c_2
\end{bmatrix} = \begin{bmatrix}
\omega \Delta T \psi_f \sin \Delta \theta / L \\
-\omega \Delta T \psi_f \cos \Delta \theta / L
\end{bmatrix},
\]
\[
\Theta = \begin{bmatrix}
a_{11} & a_{12} & b_{11} & b_{12} & c_1 \\
a_{21} & a_{22} & b_{21} & b_{22} & c_2
\end{bmatrix}.
\]
\[
Y = \begin{bmatrix}
i_r(n+1) \\
i_\delta(n+1)
\end{bmatrix}^T, \quad Z = \begin{bmatrix}
i_r(n) \\
i_\delta(n) \\
v_r(n) \\
v_\delta(n) \\
1
\end{bmatrix}^T.
\]

Equation (4) can be rewritten as:
\[
Y = \Theta Z
\]

Identify the matrix in equation (5) \( \Theta \) need to know vector \( Y \), while vector \( Y \) value of current in the future will not be measured. We use the gray forecasting method to predict and will be described in Section 4.

3. Rotor position and speed estimation

Assumed known \( \tilde{\theta} \), Equation (5) in vector \( Z \) can pass measurement \( a - b - c \). The voltage and current of the coordinate system are obtained by coordinate conversion; vector \( y \) predicted using the gray method. Such a matrix to be identified \( \Theta \) can be known by vector \( Y \) with \( Z \) identification, the recursive least square method proposed in literature[7], can make the prediction error squared \( (Y - \hat{\Theta} Z)^2 \) the smallest. Equations (6) and (7) can recursively identify the parameter matrix \( \Theta \), \( \lambda \) is the defined weighting factor.

\[
\hat{\Theta}(k) = \hat{\Theta}(k-1) + (Y - \hat{\Theta}(k-1)Z)Z^T P(k)
\]
\[
P(k) = \frac{1}{\lambda} \left[ P(k-1) - P(k-1)Z \times (\lambda + Z^T P(k-1)Z)^{-1} Z^T P(k-1) \right]
\]

Study parameter matrix \( \Theta \), it can be found:
\[
c_1^2 + c_2^2 = (\omega \Delta T \psi_f / L)^2, \quad c_1 / c_2 = -\tan \Delta \theta.
\]

Set speed \( \omega(t) \) Direction is positive (counterclockwise), then
\[
\text{sign}(c_1) = \text{sign}(\sin \Delta \theta), \quad \text{sign}(c_2) = \text{sign}(-\cos \Delta \theta).
\]

For the identified parameter matrix \( \Theta \) the speed estimation formula and the rotor position estimation formula can be obtained:

\[
\hat{\omega} = (L / \Delta T \psi_f) \sqrt{c_1^2 + c_2^2}
\]
\[
\Delta \theta = \begin{cases} 
\arctan \left( -\frac{c_1}{c_2} \right) & c_1 > 0, c_2 < 0 \\
\arctan \left( \frac{c_1}{c_2} \right) + \pi & c_1 < 0, c_2 < 0 \\
\arctan \left( \frac{c_1}{c_2} \right) - \pi & c_1 < 0, c_2 > 0 \\
\arctan \left( -\frac{c_1}{c_2} \right) + \pi & c_1 > 0, c_2 > 0
\end{cases}
\]
\[
\hat{\theta} = \hat{\theta} + \Delta \theta
\]

The rotor position and speed estimation process is shown in Figure 1. In theory \( \tilde{\theta} \) the initial value can be given casually, but because the algorithm uses gray prediction, it must wait until the current value has at least 4 sampling values before it can be predicted, so the initial position of the rotor is still required, on the initial position of the rotor. There are many methods for calculation. This article adopts the method proposed in literature [8].
4. Using the gray prediction model GM(1, 1) to predict the current value

Gray system GM(1,1) solves non-negative sequences $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}$ , begging $x^{(0)}(n+1)$ problem. The original sequence $X^{(0)}$, Construct a sequence:

$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\}$, (among them, $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$ ) make

$$
W = \begin{bmatrix}
  x^{(0)}(2) \\
  x^{(0)}(3) \\
  \vdots \\
  x^{(0)}(n)
\end{bmatrix},
B = \begin{bmatrix}
  -\frac{1}{2}(x^{(0)}(1) + x^{(0)}(2)) & 1 \\
  -\frac{1}{2}(x^{(0)}(2) + x^{(0)}(3)) & 1 \\
  \vdots & \vdots \\
  -\frac{1}{2}(x^{(0)}(n-1) + x^{(0)}(n)) & 1
\end{bmatrix},
\Phi = \begin{bmatrix}
  \hat{a} \\
  \hat{u}
\end{bmatrix}^T,

$$

Construction equation $W = B\Phi$, to identify parameter vectors $\Phi$ Can be obtained by least squares method, ie

$$
\hat{\Phi} = [\hat{a} \quad \hat{u}]^T = (B^T B)^{-1} B^T W
$$

In order to get the formula for the prediction of the series:

$$
\dot{x}^{(0)}(n+p) = (1 - e^{\hat{a}}) [x^{(0)}(1) - \frac{\hat{u}}{\hat{a}}] e^{-\hat{a}(n+p-1)}, \ p = 1, 2, \ldots
$$

To predict the value of the next step, $p = 1$ Substituting (13), similarly predict the value of the next two steps. $p = 2$ Substitution and so on. Since the GM(1,1) model requires a series $X^{(0)}$ Non-negative, can be recorded in actual use $x^{(0)}_{\min} = \min X^{(0)}$. Such as $x^{(0)}_{\min} < 0$, then the series $X^{(0)}$ All elements plus $|x^{(0)}_{\min}|$. After the prediction is complete, the sequence $X^{(0)}$ All elements are subtracted $|x^{(0)}_{\min}|$.

Electric current $i_x(n+1), i_x(n+1)$ The forecasting method is as follows:
1. Take \( i_\gamma \) (or \( i_\delta \)) A total of 4 sampling values at the current moment and before.
2. The variable minX=0.
3. If there are less than zero in the four sampled values, assign minX to the smallest negative value. At the same time, the absolute value of minX is added to the 4 current sampling values.
4. According to (12) (13) formula for prediction.
5. The predicted value of step 4 should subtract the absolute value of minX to calculate \( i_\gamma (n+1) \) or \( i_\delta (n+1) \).
6. Return to step 1 to make the next forecast.

Here, the length of the original sequence is 4, which is the minimum length required by the gray method and can be changed as appropriate.

5. sPMSM sensor-less control based on identification theory

From the above analysis, the system can be designed as a rotor field oriented rotor-less position sensor-less control system. The system control principle is shown in Figure 2.

![Figure 2](image)

**Figure 2.** control block diagram of PMSM sensor-less control

6. Simulation and experimental studies

![Figure 3](image)

**Figure 3.** simulation results under load disturbance

In this paper, the experimental system is designed using TMS320LF2407A DSP as the control core. The motor parameters are: rated power 1800W, main magnetic pole flux \( \psi_f = 0.175\) Wb, moment of inertia \( J = 0.0012\) kg \( \cdot\) m\(^2\), pole pair number \( p=4\), winding resistance \( R_s = 1.8\) \( \Omega\), direct-axis synchronous conductor \( L_d = 8.5\) mH, quadrature axis synchronous conductor \( L_q = 8.5\) mH.
Digital simulation is implemented using Matlab/Simulation under the same conditions as the experiment. The sampling period $\Delta T = 300\mu s$. Figure 3 shows the speed set point 700 r/min, no-load starting, and sudden increase at 0.5 s $\times$ m. Figure 3(a) shows the actual speed curve, speed estimation curve and speed estimation error, and Figure 3(b) gives the actual rotor position curve, rotor position estimation curve and rotor. Location estimation is errant. It can be seen that the speed estimation error is about 8 rpm, which appears in the start-up phase, and the error is about 3 rpm after stable operation. The estimation error of the speed at start-up moment is about 8 rpm, and the estimation error of the rotor position is about 12 degrees. The error after stable operation is about 5 degrees.

Figure 4 shows the simulation of the variable speed process with a given speed of 100 r/min, no-load start, a given speed of 0.2 s at 700 r/min, and a speed of 0.6 s for a given speed of 200 r/min. The method is effective. Figure 4(a) gives the actual speed curve, speed estimation curve and speed estimation error, and Figure 4(b) gives the actual rotor position curve, rotor position estimation curve and rotor position estimation error.

Figure 5 shows that the speed set point is 700 r/min, the system speed waveform at no-load start (Fig. 5a) and the actual rotor position measured by the photoelectric encoder (Fig. 5b). It can be seen that the system is stable and relatively good dynamic and static performance.

![Simulation Results](image1)

![Experimentation Results](image2)

Figure 4. simulation results under variety speed

Figure 5. experimentation results

7. Summary

The sensor-less control method of permanent magnet synchronous motor given in this paper solves the problem of rotor speed and position estimation in the control process. The proposed method using the gray system theory to predict current is effective, and the rotor-less position and velocity sensor control system is simple and easy to construct. Simulation and experimental results verify the proposed method. The rotor position estimated by the strategy proposed in this paper has nothing to do with the electric parameters, but the estimated speed is related to the electric parameters, and further research is needed in the future.
8. Acknowledgement
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9. References
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