Multibeam x-ray optical system for high-speed tomography: supplementary material

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This document provides supplementary information to “Multibeam x-ray optical system for high-speed tomography,” https://doi.org/10.1364/OPTICA.384804. We give details on the design of the multibeam optical elements and the 3D reconstruction. Transmittance images and a tomogram of the tungsten wire sample are shown. Finally, the resolution of the present system is discussed.

1. Details of the multibeam optical elements

A hyperbola with the two foci $F_1$ and $F_2$ has the property that the angle between the line $F_1P$ from $F_1$ to an arbitrary point $P$ on the hyperbola and the normal of the hyperbola at $P$ is equal to the angle between the line $F_2P$ and the normal at $P$. This means that radiation emitted from $F_1$ towards $P$ that is Bragg reflected by planes normal to the surface of a crystal bent to a hyperbolic shape, pass through $F_2$. This explains the focusing behavior of the hyperbola in Laue geometry. Such a focusing device based on a Laue crystal (in a cylindrical approximation) has been used for dispersive X-ray absorption spectroscopy (XAFS) [1], dispersive X-ray reflectivity [2] and dispersive crystal truncation rod scattering measurements [3].

In the present case, the synchrotron radiation source is located at $F_1$ and the sample at $F_2$. The equation for a hyperbola with semimajor axis $a$ and semiminor axis $b$ is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

(S1)

where the semimajor axis $a$ was chosen to lie on the $x$-axis. To obtain $a$ and $b$ we need the distance $F_1F_2=2c$ and the width $2w$ of the incident synchrotron radiation beam at $F_2$. Using the relation $c^2 = a^2 + b^2$, we find that at $x=c, y=w = \frac{b^2}{2a}$. Then,

$$a = \frac{-w \pm \sqrt{w^2 + 4c^2}}{2},$$

(S2)

$$b = \frac{b^2}{2},$$

(S3)

where we choose the + sign in (S2).

The optical setup used in this work consists of three optical elements. The values of $a$, $b$ and the focal length of the optical element (here defined as the distance between the $x$-intercept of the hyperbola and $F_2$) are given in Table S1. The crystal used for optical element 3 was divided into two parts to avoid the small curvature radius near the $x$-axis. The crystals used for each optical element had 8 blades, 18 blades and 2×8 blades, respectively. The blades of optical element 1 had a size of 5×5 mm$^2$, the others 2×5 mm$^2$.

Table S1. Semimajor axis $a$, semiminor axis $b$, and focal length of the optical elements.

| Optical element | $a$ (mm) | $b$ (mm) | Focal length (mm) |
|-----------------|----------|----------|-------------------|
| 1               | 21925    | 1813.50  | 7487              |
| 2               | 21975    | 1048.21  | 2499              |
| 3               | 21987    | 74.141   | 1250              |

2. Transmittance images and 3D reconstruction

Figure S1 shows the transmittance images of the tungsten wire sample recorded with the multibeam optical system. The increase of the noise at large scattering angles is caused by the decrease of...
the beam intensity. An important reason for this decrease is the polarization factor, which decreases with the cosine of the scattering angle for the scattering geometry of the multibeam optics, assuming perfect Si crystals of sufficient thickness. Another effect is absorption of the low energy X-rays in the multiblade crystal, and the resulting importance of higher harmonics at large scattering angles (see below), which have a smaller Darwin width than the fundamental reflection.

Since the X-ray energy of each beam is determined by the Bragg condition at the crystal blade where they are diffracted, each transmittance image $T(\theta_i)(i = 1, 2, \ldots, 32)$ was measured with a different X-ray energy. Consequently, the transmittance of the...
sample is dependent on the projection angle $\theta_i$. For the reconstruction of the 3D structure of the tungsten wire, we assumed that the wire has a uniform density and composition, and a constant diameter. We aligned the top of the wire to the same position for each $T(\theta_i)$, computed the scaling factor $S(\theta_i)$ to perform the non-uniformity correction such that the transmittance at the top of the wire is the same for every $T(\theta_i)$, and multiplied each $T(\theta_i)$ by $S(\theta_i)$. No correction for possible image rotations was made.

For the Al tape sample, the transmittance was converted to a scale common to all beams using an effective absorption coefficient, which was obtained by measuring the absorption of aluminum films of known thickness. A homogeneous composition is assumed for the analysis, but the density or thickness of the sample does not need to be uniform. Using an effective absorption coefficient for the analysis, but the density or thickness of the sample does not need to be uniform. Using an effective absorption coefficient for projection images recorded with X-rays of more than a single energy (here, higher harmonics of the fundamental energy) is an approximation commonly used in tomography with white X-ray sources.

After the above calibration, a compressed-sensing CT reconstruction algorithm, the filtered back projection-preconditioned primal-dual iterative algorithm proposed by Kudo et al. [4, 5], was used to reconstruct the 3D structure. This algorithm minimizes a cost function consisting of the sum of the Total Variation (TV) norm and the least-squares data fitting term $f(x) = \beta \|x\|_{TV} + \|Ax - b\|^2$, where $x$ denotes the image and $b$ denotes the projection data. We adopted this algorithm, because it converges much faster compared to the other standard TV minimization methods.

Figure S2 shows the X-ray energy of each beam, depending on the scattering angle. The energy calculated from the Bragg angle of the 220 reflection of silicon (blue squares) decreases with absolute scattering angle according to Bragg’s law. To evaluate the effect of higher-harmonic energies, an effective energy (red circles) was estimated by comparing the effective absorption coefficients of aluminum films with literature values of the attenuation length [6, 7]. It is close to that calculated from the Bragg angle for scattering angles below 30°, approximates the energy of the second harmonic (blue triangles) around 40°-50°, but is nearly constant for larger angles. The reason for this behavior is that X-rays with low energies are mostly absorbed by the multibeam optics, while the detection efficiency for those with high energies are low.

3. Resolution

The resolution of the detector was estimated by convoluting the ideal projection profile of a cylindrical wire with a Gaussian point-spread function, and adjusting the width of the Gaussian to fit the observed profile in a projection image. This gave a FWHM of 65 μm.

The resolution of the experimental setup was evaluated using the tomogram of the tungsten wire shown in Fig. S3 (a). The line profiles through the wire in the $x$ and $y$ directions (Fig. S3 (b)) have the same width, showing that the resolution is nearly uniform. The solid line in the figure shows the convolution of the ideal profile of a cylindrical wire (a square function) with the point-spread function of the detector. The width is comparable to that of the line profiles. The resolution in the present setup is therefore dominated by the resolution of the detector.

In principle, different resolutions would be expected in the $x$ and $y$ directions, because of the non-uniform distribution of the projections and the lower signal-noise ratio at high scattering angles. These effects are expected to become important when a higher-resolution detector is used.

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