Measuring the Distances to Quasars at High Redshifts with Strong Lensing

Kai Liao

School of Science, Wuhan University of Technology, Wuhan 430070, People’s Republic of China; liaokai@whut.edu.cn

Received 2019 July 2; revised 2019 July 26; accepted 2019 August 7; published 2019 September 17

Abstract

Strongly lensed quasars with time-delay measurements are well known to provide the “time-delay distances” $D_{\Delta t} = (1 + z_L)D_L D_t / D_{LS}$, and the angular diameter distances to the lens galaxies $D_L$. These two kinds of distances give stringent constraints on cosmological parameters. In this work, we explore a different use of time-delay observables: under the assumption of a flat universe, strong lensing observations can accurately measure the angular diameter distances to the sources $D_S$. The corresponding redshifts of the quasars may be up to $z_S \sim 4$ according to the forecast. The high-redshift distances would sample the Hubble diagram between SNe Ia and the cosmic microwave background, model-independently providing direct information on the evolution of the nature of our universe, for example, the dark energy equation of state parameter $w(z)$. We apply our method to the existing lensing system SDSS 1206+4332 and get $D_S = 2388_{-97}^{+263} \text{ Mpc}$ at $z_S = 1.789$. We also make a forecast for the era of Large Synoptic Survey Telescope. The uncertainty of $D_S$ depends on the redshifts of the lens and the source, the uncertainties of $D_{\Delta t}$ and $D_L$, and the correlation between $D_{\Delta t}$ and $D_L$. Larger correlation would result in tighter $D_S$ determination.

Key words: distance scale – gravitational lensing: strong – methods: data analysis

1. Introduction

In the standard cosmological model, i.e., the ΛCDM, the universe is flat, dominated by cold dark matter and dark energy with equation of state (EOS) parameter $w \equiv -1$ (Frieman et al. 2008). This concordance scenario is able to explain most of the cosmological observations. However, more and more issues have emerged (Moore 1994; Moore et al. 1999; Frieman et al. 2008; Ding et al. 2015). Especially, the Hubble constant ($H_0$) measurement based on Cepheids and SNe Ia from the local universe (Freedman 2017; Riess et al. 2019) has 4.4σ discrepancy with measurement from cosmic microwave background (CMB) that assumes the ΛCDM model when inferring $H_0$ (Planck Collaboration 2018). A recent independent determination of the $H_0$ based on the tip of the red giant branch seems to reduce the discrepancy (Freedman et al. 2019). The inconsistency problem would either be related to unknown systematic errors or reveal new physics beyond the standard model. Alternative cosmological models were proposed to solve these issues while new problems may be brought in.

Putting aside the models, from the observational perspective, it is crucial to reconstruct the expansion history of the universe directly and model-independently from the data (Shafieloo et al. 2006; Shafieloo 2007; Clarkson & Zunckel 2010). Cosmology-free calculations can also be seen in Bernal et al. (2016), Li et al. (2019), Arendse et al. (2019), and Denissenya et al. (2018). If we have distances measured at different $z$ redshifts, we can reconstruct the distance-redshift relation $D(z)$, i.e., the Hubble diagram using cosmology-independent methods, for example, the Gaussian process (Seikel et al. 2012; Shafieloo et al. 2012; Yang et al. 2015). In addition, we can also reconstruct other cosmological quantities evolving with redshift, for example, the Hubble expansion rate $H(z)$, the deceleration parameter $q(z)$, and the dark energy EOS parameter $w(z)$. These would, in turn, help us to understand the issues related with theoretical models. However, the reconstruction is limited by the maximum redshift $z_{\text{max}}$ of the data (L’Huillier et al. 2019). Note that current reliable data used to study cosmology are either at low redshifts $z < 2$, for example the SNe Ia (Betoule et al. 2014) and the Baryon Acoustic Oscillations (Anselmi et al. 2019), or very high redshift $z \sim 1000$, i.e., the CMB (Planck Collaboration 2018). Other cosmological approaches such as cosmic chronometers (Chen et al. 2017) and galaxy clusters (Chen & Ratra 2012) are also at low redshifts, therefore, it is important to get high-redshift data (hereafter we take $z > 2$ as “high-redshift”) to fill up the data desert between the farthest SN Ia and the CMB. The gamma-ray bursts (GRBs) can be observed up to $z \sim 8$ and may provide the distance measurements (Schaef er 2003; Izzo et al. 2009; Wei 2010), however, they need calibration by SNe Ia at low redshifts which is very uncertain. Issues about the physical motivation of GRBs as standard candles also exist (Wang et al. 2015). The gravitational waves by compact binary stars can also provide luminosity distances as standard sirens (Schutz 1986), however, the measurement uncertainty would increase remarkably at redshift $z > 2$ (Cai & Yang 2017; Zhao & Wen 2018).

Recently, the quasars at redshifts up to $z \sim 5$ were proposed to measure the luminosity distances with a method based on the X-ray and ultraviolet emission (Risaliti & Lusso 2019). The robustness of this method needs to be further confirmed.

Strongly lensed quasars by galaxies are an excellent tool to study astrophysics and cosmology (Treu 2010). The distant active galactic nucleus (AGN) with its host galaxy is lensed by the foreground elliptical galaxy, forming multiple images and the arcs of the host galaxy. With the measurements of the time-delays between these images, the “time-delay distance” which is a combination of three angular diameter distances $D_{\Delta t} = (1 + z_L)D_t D_S / D_{LS}$ can be determined (Refsdal 1964; Treu & Marshall 2016). It is known to determine the Hubble constant (Refsdal 1964) and other cosmological parameters, for example, the EOS parameter of dark energy (Linder 2011). The H0LiCOW collaboration (Suyu et al. 2017) has constrained the
$H_0$ at 2.4% precision level under a flat ΛCDM model and a weaker constraint in wCDM model due to the degeneracy between $H_0$ and the $w$ (Taubenberger et al. 2019; Wong et al. 2019). In addition to the $D_{\Delta t}$, the angular diameter distance to the lens galaxy ($D_L$) was proposed to be determined by combining time-delay measurements with the lens stellar velocity dispersion measurements (Paraficz & Hjorth 2009; Jee et al. 2015, 2016; Yildirim et al. 2019). Four lensing systems have been given the robust $D_s$ measurements (Wong et al. 2019). Note that, unlike SNe Ia which determine the relative distances, the strong lensing measures the absolute angular diameter distances, with which one can directly establish the Hubble diagram. However, this approach is limited by the relatively low redshifts of the lenses $z < 1.2$ (Jee et al. 2016).

Motivated by acquiring high-redshift data for studying the universe, we propose a method to measure the distances to the quasars based on strong lensing. The source quasars are located at high redshifts up to $z_s \sim 4$. This paper is organized as follows: in Section 2, we introduce the current status of time-delay strong lensing cosmology. In Section 3, we give the idea of measuring the distances to the sources. Then we apply our method to a realistic system SDSS 1206+4332 in Section 4. We also make a forecast for the lensing observations in the Large Synoptic Survey Telescope (LSST) era in Section 5. Finally, we summarize and make discussions in Section 6.

2. Lensed Quasars with Time-delays

According to the theory of strong gravitational lensing (Refsdal 1964; Treu 2010; Treu & Marshall 2016; Liao 2019), the arriving time difference (time-delay) between two images of the source measured from AGN light curves is related with the geometry of the universe and gravity field of the lens galaxy through:

$$\Delta t = \frac{D_{\Delta t}}{c} \Delta \phi (\xi_{\text{lens}}),$$

where $c$ is the light speed. $\Delta \phi = [ (\theta_2 - \theta_1)^2 / 2 - \psi (\theta_2) - (\theta_2 - \theta_1)^2 / 2 + \psi (\theta_1) ]$ is the Fermat potential difference between image A and image B. $\theta_2$ and $\theta_1$ are angular positions of the images. $\theta_2$ denotes the source angular position. $\psi$ is the two-dimensional lensing potential determined by the Poisson equation $\nabla^2 \psi = 2\kappa$, where the surface mass density of the lens $\kappa$ is in units of critical density $\Sigma_{\text{crit}} = c^2 D_s / (4\pi G D_c D_L s)$. $\Delta \phi$ is determined by the lens model parameters $\xi_{\text{lens}}$ which can be inferred with the high resolution imaging data. $D_{\Delta t}$ is the “time-delay distance” consisting of three angular diameter distances:

$$D_{\Delta t} = (1 + z_L) \frac{D_L D_s}{D_{LS}},$$

where $L, S$ stands for lens and source. Note that the line of sight (LOS) mass structure could also affect the time-delay distance measurements (Falco et al. 1985; Rusu et al. 2017). At the same time, the angular diameter distance ratio can be measured in a general form (not limited to a singular isothermal sphere (SIS) model as one usually takes):

$$\frac{D_{LS}}{D_S} = \frac{c^2 J (\xi_{\text{lens}}, \xi_{\text{light}}, \beta_{\text{ani}})}{\sigma^P}$$

where $\sigma^P$ is the LOS projected stellar velocity dispersion of the lens galaxy which provides extra constraints to the cosmographic inference. The parameter $J$ captures all the model components computed from angles measured on the sky (the imaging) and the stellar orbital anisotropy distribution. It can be written as a function of lens model parameters $\xi_{\text{lens}}$, the light profile parameters $\xi_{\text{light}}$, and the anisotropy distribution of the stellar orbits $\beta_{\text{ani}}$.

Combining Equations (1) and (3), the angular diameter distance to the lens can be measured by:

$$D_L = \frac{1}{1 + z_L} \frac{c^2 J (\xi_{\text{lens}}, \xi_{\text{light}}, \beta_{\text{ani}})}{(\sigma^P)^2}.$$

Note that the lensing analysis is quite complicated and we only show the key equations. For dealing with the real data, one should use a full Bayesian analysis considering covariances between quantities to calculate the posteriors of each parameter. We refer to Jee et al. (2015), Shaibj et al. (2018a, 2018b), Birrer et al. (2019), and Yildirim et al. (2019) for more details of such process.

Therefore, the lensed quasars with time-delays could constrain parameters in cosmological models through the measured $D_{\Delta t}$ and $D_L$. The H0LiCOW project (Suyu et al. 2017) in collaboration with the COSMOSGRAIL programme (Courbin et al. 2018) has assembled a sample of lensed quasar systems, six of which (B1608+656, RXJ1131-1231, HE 0435-1223, SDSS 1206+4332, WFI2033-4723, PG 1115+080) have been well-analyzed in the milestone paper (Wong et al. 2019). Among them, four systems have both $D_{\Delta t}$ and $D_L$ measurements. Currently, the H0LiCOW team has only published the posteriors of both $D_{\Delta t}$ and $D_L$ measurements for SDSS 1206+4332. Assuming a flat ΛCDM and through a blind analysis, they reported $H_0 = 73.3^{\pm 3.3}_{-2.3}$ km s$^{-1}$ Mpc$^{-1}$, a 2.4% precision including systematics. Detailed results in different models can be found in Wong et al. (2019). The previous results from only 4 systems can be found in Taubenberger et al. (2019).

3. Distances to the Sources

We propose in this work that strong lensing can also provide the angular diameter distances to the quasars ($D_s$). As long as one assumes the universe is flat, the three relevant angular diameter distances can be respectively expressed as:

$$D_L = \frac{c}{(1 + z_L) H_0} \int_{z_L}^{z_S} \frac{1}{E(z)} dz,$$

$$D_S = \frac{c}{(1 + z_S) H_0} \int_{z_S}^{z} \frac{1}{E(z)} dz,$$

and

$$D_{LS} = \frac{c}{(1 + z_S) H_0} \int_{z_L}^{z} \frac{1}{E(z)} dz = D_S - \frac{1 + z_L D_L}{1 + z_S},$$

where $E(z) = H(z)/H_0$. Therefore, with Equation (2), the $D_S$ can be determined by:

$$D_S = (1 + z_L) D_L D_{\Delta t}/(1 + z_L D_{\Delta t} - (1 + z_L) D_L).$$

In other words, if the lensing observations give $D_{\Delta t}$ and $D_L$, one can always measure (infer) $D_S$ equivalently. We emphasize that although determining $D_S$ in this way would not bring any benefits (extra information) for constraining parameters in
specific cosmological models, the measured $D_S$ at high-redshifts can be further used in the model-independent reconstruction of the expansion history of the universe (whereas $D_L$ can be replaced by other low-redshift data, for example, the SNe Ia.). When applying Equation (8), one should consider the correlation between $D_{DL}$ and $D_L$. Actually, from strong lensing observations, only one distance among $D_L$, $D_S$, and $D_{DL}$ is totally independent. While the community uses either $D_L$ or $D_{DL}$ we focus on $D_S$ in this work. In principle, rather than inferring it from $D_{DL}$ and $D_L$, one can directly take $D_S$ as the lensing parameter in the first place during the lensing analysis. For example, for the simplest case where the lens is described by an SIS model (Paraficz & Hjorth 2009), the density distribution is given by:

$$\rho_{SIS}(r) = \frac{\sigma^2}{2\pi Gr^2},$$

where $\sigma^2$ is the three-dimensional isotropic velocity dispersion. Then

$$D_{DL} = \frac{2c\Delta t}{\theta^t_A - \theta^t_B},$$

and

$$D_L = \frac{c^2\Delta l}{4\pi\sigma^2(1 + z_L)\Delta\theta}.$$  \hspace{1cm} (11)

Thus we can directly relate $D_S$ with the observations by combining Equations (8), (10) and (11).

While we take the SIS model for illustration purposes, one should note that realistic lenses are much more complicated. Different components in the mass models were explored both for lensing and for kinematics (Jiang & Kochanek 2007; Jing & Kochanek 2010; Shajib et al. 2018a, 2018b). For the macro mass model, one needs to consider more properties for individual lenses, for example, the ellipticity and the density slope. A singular elliptical power-law model may not be sufficient and one usually tries a composite model consisting of a baryonic component linked to the stellar light distribution plus an elliptical Navarro, Frenk and White (NFW) dark matter halo. In some cases, the lens is during a merger process, for example, B1608+656 shows two interacting lens galaxies (Suyu et al. 2010). Besides, substructures, for example, the satellites and the dark matter sub-halos would make the observations anomalous if one ignores them (Liao et al. 2018). Furthermore, the nearby galaxies and the LOS structure can also make the lens modeling complicated.

4. Measurement of SDSS 1206+4332

We apply our method to the system SDSS 1206+4332 which was discovered by Oguri et al. (2005). It is one of high-quality lensing systems in the catalog of the H0LiCOW and has been modeled by Birrer et al. (2019) within the program. This system consists of a doubly lensed quasar with the host galaxy forming a nearly complete Einstein ring. The image separation is $3.0''3$ and the time-delay measured from the light curve pair is $111.3 \pm 3$ days. The redshifts of lens and source are $z_L = 0.745$ and $z_S = 1.789$, respectively. The H0LiCOW team took a blind time-delay strong lensing cosmographic analysis of the system. They combined the time-delay measurement between the two AGN images, Hubble Space Telescope imaging, spectroscopic data of the lens galaxy and the LOS field measurements to give measurements of both $D_L$ and $D_{DL}$ with systematic errors under control. The two distances are provided on the website of the program in the form of tables of cosmological models, the measured $D_s$ at high-redshifts can be further used in the model-independent reconstruction of the expansion history of the universe (whereas $D_L$ can be replaced by other low-redshift data, for example, the SNe Ia.). When applying Equation (8), one should consider the correlation between $D_{DL}$ and $D_L$. Actually, from strong lensing observations, only one distance among $D_L$, $D_S$, and $D_{DL}$ is totally independent. While the community uses either $D_L$ or $D_{DL}$ we focus on $D_S$ in this work. In principle, rather than inferring it from $D_{DL}$ and $D_L$, one can directly take $D_S$ as the lensing parameter in the first place during the lensing analysis. For example, for the simplest case where the lens is described by an SIS model (Paraficz & Hjorth 2009), the density distribution is given by:

$$\rho_{SIS}(r) = \frac{\sigma^2}{2\pi Gr^2},$$

where $\sigma^2$ is the three-dimensional isotropic velocity dispersion. Then

$$D_{DL} = \frac{2c\Delta t}{\theta^t_A - \theta^t_B},$$

and

$$D_L = \frac{c^2\Delta l}{4\pi\sigma^2(1 + z_L)\Delta\theta}.$$  \hspace{1cm} (11)

Thus we can directly relate $D_S$ with the observations by combining Equations (8), (10) and (11).

While we take the SIS model for illustration purposes, one should note that realistic lenses are much more complicated. Different components in the mass models were explored both for lensing and for kinematics (Jiang & Kochanek 2007; Jing & Kochanek 2010; Shajib et al. 2018a, 2018b). For the macro mass model, one needs to consider more properties for individual lenses, for example, the ellipticity and the density slope. A singular elliptical power-law model may not be sufficient and one usually tries a composite model consisting of a baryonic component linked to the stellar light distribution plus an elliptical Navarro, Frenk and White (NFW) dark matter halo. In some cases, the lens is during a merger process, for example, B1608+656 shows two interacting lens galaxies (Suyu et al. 2010). Besides, substructures, for example, the satellites and the dark matter sub-halos would make the observations anomalous if one ignores them (Liao et al. 2018). Furthermore, the nearby galaxies and the LOS structure can also make the lens modeling complicated.

4. Measurement of SDSS 1206+4332

We apply our method to the system SDSS 1206+4332 which was discovered by Oguri et al. (2005). It is one of high-quality lensing systems in the catalog of the H0LiCOW and has been modeled by Birrer et al. (2019) within the program. This system consists of a doubly lensed quasar with the host galaxy forming a nearly complete Einstein ring. The image separation is $3.0''3$ and the time-delay measured from the light curve pair is $111.3 \pm 3$ days. The redshifts of lens and source are $z_L = 0.745$ and $z_S = 1.789$, respectively. The H0LiCOW team took a blind time-delay strong lensing cosmographic analysis of the system. They combined the time-delay measurement between the two AGN images, Hubble Space Telescope imaging, spectroscopic data of the lens galaxy and the LOS field measurements to give measurements of both $D_L$ and $D_{DL}$ with systematic errors under control. The two distances are provided on the website of the program in the form of tables of cosmological models, the measured $D_s$ at high-redshifts can be further used in the model-independent reconstruction of the expansion history of the universe (whereas $D_L$ can be replaced by other low-redshift data, for example, the SNe Ia.). When applying Equation (8), one should consider the correlation between $D_{DL}$ and $D_L$. Actually, from strong lensing observations, only one distance among $D_L$, $D_S$, and $D_{DL}$ is totally independent. While the community uses either $D_L$ or $D_{DL}$ we focus on $D_S$ in this work. In principle, rather than inferring it from $D_{DL}$ and $D_L$, one can directly take $D_S$ as the lensing parameter in the first place during the lensing analysis. For example, for the simplest case where the lens is described by an SIS model (Paraficz & Hjorth 2009), the density distribution is given by:

$$\rho_{SIS}(r) = \frac{\sigma^2}{2\pi Gr^2},$$

where $\sigma^2$ is the three-dimensional isotropic velocity dispersion. Then

$$D_{DL} = \frac{2c\Delta t}{\theta^t_A - \theta^t_B},$$

and

$$D_L = \frac{c^2\Delta l}{4\pi\sigma^2(1 + z_L)\Delta\theta}.$$  \hspace{1cm} (11)

Thus we can directly relate $D_S$ with the observations by combining Equations (8), (10) and (11).

While we take the SIS model for illustration purposes, one should note that realistic lenses are much more complicated. Different components in the mass models were explored both for lensing and for kinematics (Jiang & Kochanek 2007; Jing & Kochanek 2010; Shajib et al. 2018a, 2018b). For the macro mass model, one needs to consider more properties for individual lenses, for example, the ellipticity and the density slope. A singular elliptical power-law model may not be sufficient and one usually tries a composite model consisting of a baryonic component linked to the stellar light distribution plus an elliptical Navarro, Frenk and White (NFW) dark matter halo. In some cases, the lens is during a merger process, for example, B1608+656 shows two interacting lens galaxies (Suyu et al. 2010). Besides, substructures, for example, the satellites and the dark matter sub-halos would make the observations anomalous if one ignores them (Liao et al. 2018). Furthermore, the nearby galaxies and the LOS structure can also make the lens modeling complicated.
and the Hyper Suprime-Cam Survey. Moreover, the upcoming LSST (Oguri & Marshall 2010) will bring us thousands of lensed quasars, some of which will have long-time high-quality light curves for each lensed image. The Time Delay Challenge (TDC) program (Liao et al. 2015) has proved that with good algorithms, there will be \sim 400 well-measured time-delays with average precision \sim 3\% and the average bias <1\%. According to Equations (1) and (3), to obtain the distance information, ancillary data are needed in terms of a few percent measurement of the spatially resolved velocity dispersion of the lens galaxy, the LOS mass fluctuation, and the highly resolved imaging from space telescopes. Therefore, as in Jee et al. (2016), we set the following criteria: (1) the AGN image separation should be >1" to distinguish them; (2) the third brightest image should be bright enough, its i-band magnitude \( m_i < 21 \); (3) the lens galaxy should be bright enough \( m_l < 22 \); (4) quadruply imaged lenses which carry more information to break the source position transformation, such that the uncertainty of the lens modeling process is comparable with the time-delay measurements, leading to percent level distance measurements from individual lenses.

With these assumptions, there will be \sim 55 high-quality lenses of the 400 lenses mentioned above selected from the mock LSST catalog (Oguri & Marshall 2010) that can give both \( D_{\Delta z} \) and \( D_L \) measurements. However, this number may vary due to the real distributions of the lens galaxies and quasars. Besides, the telescope observation strategy limits the estimate, for example, the spectroscopic follow-up may require brighter lenses, for a shallower limit \( m_l < 21 \) of the lens galaxies, the number would be only \sim 35. In the best case, both time-delay measurements and the lens modeling should achieve several percent precision. We take \sim 5\% precision for the distances as in Jee et al. (2016), Linder (2011). We also consider \sim 10\% precision which is allowed by current techniques, for comparison. These lenses will be “blind analyzed” that can effectively control the systematic errors which would bias the results. The two-dimensional distributions of \( z_L \) and \( z_S \) can be found in Jee et al. (2016). We marginalize \( z_L \) and get the distribution of \( z_S \) in Figure 2. As one can see, a large part of the corresponding quasars have high redshifts. \sim 35 systems are with \( z_S > 2 \).

To make a forecast, we take a fiducial flat \( \Lambda \)CDM model with \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}, \Omega_M = 0.3 \) for the simulation. For each lensing system, given \( z_L \) and \( z_S \), we first calculate the fiducial values of \( D_{\Delta z} \) and \( D_L \), then randomly generate 10000 realizations for each of them. The noise levels follow Gaussian distributions with uncertainties 5\% and 10\%, respectively. The uncertainties of \( D_{\Delta z} \) and \( D_L \) primarily come from the external convergence and the velocity dispersion of the lens, respectively. However, since measurements of \( D_{\Delta z} \) and \( D_L \) are based on the same lens model, one should consider the correlation between them unless one of the distances have much larger uncertainty. According to the simulation by Yildirim et al. (2019), the correlation is positive. We therefore try different correlation amplitudes for \( D_{\Delta z} \) and \( D_L \) with correlation coefficients \( \rho = 0.1, 0.4, 0.7 \), respectively.

Then for each simulated \( D_{\Delta z} \) and \( D_L \) pair, we calculate \( D_S \) based on Equation (8). At last, we get the distribution of \( D_S \) along with its median value plus 16th and 84th percentiles. Figures 3 and 4 correspond to a typical case where \( z_L = 0.7, z_S = 2.5 \) for uncertainties 5\% and 10\% of \( D_{\Delta z} \) and \( D_L \), respectively. We plot the simulated \( D_{\Delta z} \) and \( D_L \) distributions with different correlations at the upper panels and calculate the corresponding probability density distribution of \( D_S \) at the bottom panels. One can see the constraint becomes tighter when the correlation is larger. To show the dependence on the redshifts of the lens and source, the uncertainties of \( D_{\Delta z} \) and \( D_L \), and the correlation amplitude for the whole samples, we plot Figure 5 where \( \sigma_{D_S} = (84\text{th percentile} - 16\text{th percentile})/2 \) is taken as an estimate of the uncertainty.

6. Summary and Discussions

In this work, we find another powerful cosmological application of strong gravitational lensing. We propose to measure the angular diameter distances to the quasars at high redshifts with strong lensing and apply the method to SDSS 1206+4332. We also explore the power in the future LSST era and give the constraint dependence on the properties of the systems. Rather than constraining a specific cosmological model, distances measured to the sources would benefit reconstructing the nature of the universe model-independently and directly at high redshifts. A further work will emulate the reconstruction. Note that the measured high-redshift angular
diameter distances can be used to find their maximum value and the corresponding redshift since unlike luminosity distance, angular diameter distance would decrease if the redshift is larger than certain value $z \sim 1.6$ (Salzano et al. 2015).

Very recently, Yildirim et al. (2019) presented a joint strong lensing and stellar dynamical framework for future time-delay cosmography purposes. With the observations of high signal-to-noise integral field unit from the next generation of telescopes, they proved that $D_\Delta t$ can be constrained with 2.3% uncertainty and $D_L$ with 1.8% at best for a system like RXJ1131. In such cases, we can acquire much more precise $D_S$ as well, making this idea very promising. Note that RXJ1131 is the best case whereas an ordinary lens system would give larger uncertainties.

Our method relies on the inputs of $D_\Delta t$ and $D_L$ measurements by the H0LiCOW-like lensing teams. With more and more precise measurements, the intrinsic (unknown) systematic errors would be quite important. If the $D_\Delta t$ and $D_L$ are biased, the inferred $D_S$ would also be biased. The H0LiCOW team has adopted a blind analysis to control systematics. Data challenges, e.g., the TDC (Liao et al. 2015) and the Lens Modeling Challenge (Ding et al. 2018) would reveal the systematics by algorithms. The systematics from unknown physical processes would be further revealed by independent approaches. Considering the 5% and 10% uncertainties assumed in this work, a small systematic error, for example, 2% would not bias the results. There are great concerns about the lens modeling systematics being dominated by systematics (Schneider & Sluse 2013; Birrer et al. 2016; Tie & Kochanek 2017), Combining

Figure 3. A typical case with $z_L = 0.7$ and $z_S = 2.5$. The uncertainties of $D_\Delta t$ and $D_L$ are set by 5%. The upper panels show the simulated $D_\Delta t$ and $D_L$ distributions with different correlation amplitudes: $\rho = 0, 0.1, 0.4, 0.7$, respectively. The bottom panels are the corresponding $D_S$ inferences.

Figure 4. The same as Figure 3 but for 10% uncertainties of $D_\Delta t$ and $D_L$. 
many lenses may bias cosmological results. In this work, we only focus on determining individual $D_S$.

I thank the anonymous referee for his/her efforts to improve the quality of the paper, and Simon Birrer for introducing the data of SDSS 1206 + 4332 on the H0LiCOW website. This work was supported by the National Natural Science Foundation of China (NSFC) No. 11603015 and the Fundamental Research Funds for the Central Universities (WUT:2018IB012).

ORCID iDs
Kai Liao https://orcid.org/0000-0002-4359-5994

References
Anselmi, S., Corasaniti, P.-S., Sanchez, A. G., et al. 2019, PRD, 99, 123515
Arendse, N., Agnello, A., & Wojtak, R. 2019, arXiv:1905.12000
Bernal, J. L., Verde, L., & Riess, A. G. 2016, JCAP, 10, 019
Bertoule, M., Kessler, R., Guy, J., et al. 2014, A&A, 568, A22
Birrer, S., Amara, A., & Refregier, A. 2016, JCAP, 08, 020
Birrer, S., Treu, T., Rusu, C. E., et al. 2019, MNRAS, 484, 4726
Cai, R.-G., & Yang, T. 2017, PRD, 95, 040424
Chen, Y., Kumar, S., & Ratra, B. 2017, ApJ, 835, 86
Chen, Y., & Ratra, B. 2012, A&A, 543, A104
Clarkson, C., & Zumack, C. 2010, PRD, 104, 211301
Courbin, F., Bonvin, V., Buckley-Geer, E., et al. 2018, A&A, 609, A71
Denissenya, M., Linder, E. V., & Shafieloo, A. 2018, JCAP, 03, 041
Ding, X., Biesiada, M., Cao, S., Li, Z., & Zhu, Z.-H. 2015, ApJL, 803, L22
Ding, X., Treu, T., Shajib, A. J., et al. 2018, arXiv:1801.01506
Falco, E. E., Gorenstein, M. V., & Shapiro, I. I. 1985, ApJL, 289, L1
Freedman, W. L. 2017, NatAs, 1, 0121
Freedman, W. L., Madore, B. F., Hatt, D., et al. 2019, ApJ, 882, 34
Freedman, W. L., Hatt, D., & Madore, B. F. 2019, ApJ, 882, 34
Frieman, J. A., Turner, M. S., & Huterer, D. 2008, ARA&A, 46, 385
Izzo, L., Capozziello, S., Govone, G., & Capaccioli, M. 2009, A&A, 508, 43
Jee, I., Komatsu, E., & Suyu, S. H. 2015, JCAP, 11, 033
Jee, I., Komatsu, E., Suyu, S. H., & Huterer, D. 2016, JCAP, 04, 031
Jiang, G., & Kochanek, C. S. 2007, ApJ, 671, 1568
L'Huillier, B., Shafieloo, A., Linder, E. V., & Kim, A. G. 2019, MNRAS, 485, 2783
Li, E.-K., Du, M., & Xu, L. 2019, arXiv:1903.11433
Liao, K. 2019, ApJ, 871, 113
Liao, K., Ding, X., Biesiada, M., Fan, X.-L., & Zhu, Z.-H. 2018, ApJ, 867, 69
Liao, K., Treu, T., Marshall, P., et al. 2015, ApJ, 800, 11
Linder, E. V. 2011, PRD, 84, 123529
Moore, B. 1994, Natur, 370, 629
Moore, B., Ghigna, S., Governato, F., et al. 1999, ApJL, 524, L19
Oguri, M., Inada, N., Hennawi, J. F., et al. 2005, ApJ, 622, 106
Oguri, M., & Marshall, P. J. 2010, MNRAS, 405, 2579
Paraficz, D., & Hjorth, J. 2009, A&A, 507, L49
Planck Collaboration 2018, arXiv:1807.06209
Refsdal, S. 1964, MNRAS, 128, 307
Riess, A. G., Casertano, S., Yuan, W., Macri, L. M., & Scolnic, D. 2019, ApJ, 876, 85
Risaliti, G., & Lusso, E. 2019, NatAs, 3, 272
Rusu, C. E., Fassnacht, C. D., Sluse, D., et al. 2017, MNRAS, 467, 4220
Salzano, V., Dabrowski, M. P., & Lazkoz, R. 2015, PRD, 114, 103104
Schaefer, B. E. 2003, ApJL, 583, L67
Schneider, P., & Sluse, D. 2013, A&A, 559, A37
Schutz, B. F. 1986, Natur, 323, 310
Seikel, M., Clarkson, C., & Smith, M. 2012, JCAP, 06, 026
Shafieloo, A. 2007, MNRAS, 380, 1573
Shafieloo, A., Alam, U., Sahni, V., & Starobinsky, A. A. 2006, MNRAS, 366, 1081
Shafieloo, A., Kim, A. G., & Linder, E. V. 2012, PRD, 85, 123530
Shajib, A. J., Birrer, S., Treu, T., et al. 2018a, MNRAS, 483, 5649
Shajib, A. J., Treu, T., & Agnello, A. 2018b, MNRAS, 473, 210
Suyu, S. H., Bonvin, V., Courbin, F., et al. 2017, MNRAS, 468, 2590
Suyu, S. H., Marshall, P. J., Auger, M. W., et al. 2010, ApJ, 711, 201
Taubenberger, S., Suyu, S. H., & Komatsu, E. 2019, A&A, 628, L7
Tie, S. S., & Kochanek, C. S. 2017, MNRAS, 473, 80
Treu, T. 2010, ARA&A, 48, 87
Treu, T., & Marshall, P. J. 2016, ARA&A, 24, 11
Wang, F. Y., Dai, Z. G., & Liang, E. W. 2015, NewAR, 67, 1
Wei, H. 2010, JCAP, 1008, 020
Wong, K. C., Suyu, S. H., Chen, G. C.-F., et al. 2019, arXiv:1907.04869
Yang, T., Guo, Z.-K., & Cai, R.-G. 2015, PRD, 91, 123533
Yildirim, A., Suyu, S. H., & Halkola, A. 2019, arXiv:1904.07237
Zhao, W., & Wen, L. 2018, PRD, 97, 064031