Privacy-Utility Management of Hypothesis Tests

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Abstract—The trade-off of hypothesis tests on the correlated privacy hypothesis and utility hypothesis is studied. The error exponent of the Bayesian composite hypothesis test on the privacy or utility hypothesis can be characterized by the corresponding minimal Chernoff information rate. An optimal management protects the privacy by minimizing the error exponent of the privacy hypothesis test and meanwhile guarantees the utility hypothesis testing performance by satisfying a lower bound on the corresponding minimal Chernoff information rate. The asymptotic minimum error exponent of the privacy hypothesis test is shown to be characterized by the infimum of corresponding minimal Chernoff information rates subject to the utility guarantees.

I. INTRODUCTION

Privacy-utility trade-offs have been studied in different contexts [1]–[9]. The privacy leakage can be modeled as a statistical inference and measured by the mutual information [1]–[3], [6], [7], divergence [4], differential privacy [9], or variance [8]. Depending on the application, the utility measure can be the expectation of cost [5], [6], [8], divergence [4], [7], or data rate [1], [3]. With the privacy and utility measures, the trade-off problem can be formulated as a worst case analysis [2], [4], [6]–[8] or zero-sum game [5].

The asymptotic error exponent of the simple binary hypothesis test with i.i.d. observations is characterized by a Kullback-Leibler divergence under the Neyman-Pearson criterion [10] or a Chernoff information under the Bayesian criterion [11]. It was shown in [12] that the asymptotic error exponent of a Bayesian m-ary hypothesis test with i.i.d. observations is characterized by the minimal Chernoff information among all conditional probability distribution pairs. In more general cases, the observations depend on correlated hypotheses. The asymptotically optimal composite hypothesis tests on one of the correlated hypotheses were studied under the Neyman-Pearson criterion, e.g., the Hoeffding test based on a Kullback-Leibler divergence statistic [13] and the mismatched test based on a more relaxed mismatched divergence statistic [14].

In this paper, we consider a novel privacy-utility trade-off scenario where the Bayesian composite hypothesis tests on the correlated privacy hypothesis and utility hypothesis are made based on the same sequence of random observations1. Firstly we prove and show that the error exponents of the Bayesian composite hypothesis tests on the privacy hypothesis and utility hypothesis can be characterized by their corresponding minimal Chernoff information rates. We then study the optimal management which degrades the privacy hypothesis test while guarantees a certain utility hypothesis testing performance, and further show that the asymptotic minimum error exponent of the privacy hypothesis test can be characterized by the infimum of corresponding minimal Chernoff information rates. In this initial study, we develop the results by considering a binary utility hypothesis and a binary privacy hypothesis. The extension to m-ary hypothesis tests, m > 2, is our future work. In the context of distributed hypothesis test, a similar problem has been studied in our previous work [15], where the Bayesian risks are used to measure the hypothesis testing performances and an optimal privacy-constrained distributed hypothesis testing network design is characterized. In the context of smart meter privacy, we characterized the optimal privacy-preserving energy management with an adversarial hypothesis test [16].

II. PROBLEM STATEMENT

The model in Fig. 1 shows a management of hypothesis tests on the correlated privacy hypothesis and utility hypothesis in the presence of an independent noise. Let U denote the binary utility hypothesis and P denote the binary privacy hypothesis that has to be protected. W.l.o.g., we assume the following hypothesis alphabets $U = P = \{0, 1\}$. Let $p_{U,P}$ denote the joint prior probability distribution of the random hypothesis

1A such scenario in practice is the smart meter privacy problem, where the smart meter readings consist of the utility information, e.g., the future energy consumption from the grid, as well as the consumers’ privacy, e.g., their life styles, and can be used by the authorized data recipient, e.g., the energy provider or grid operator, to make a utility hypothesis test and an illegitimate privacy hypothesis test.
pair. Given a hypothesis pair realization \((u, p) ∈ U × P\), \(X_i\) is i.i.d. generated following the pmf \(p_{X|u=p=p}\), which will be denoted by \(p_{X|u,p}\) in the following. We further assume that the four different pmfs \(\{p_{X|u,p}\} (u,p) ∈ U × P\) are defined on the same finite support set \(X\). An independent random noise \(Z_i\) defined on the finite alphabet \(Z\) is i.i.d. generated following the pmf \(p_z\). Over an \(n\)-slot time horizon, the management unit (MU) employs a randomized management policy \(φ^n : X^n × Z^n → X^n\), which maps the input sequence \(x^n\) and noise sequence \(z^n\) to a processed random observation sequence \(Y^n ∈ X^n\) subject to the following constraint:\(^2\)

\[0 ≤ \frac{1}{n} ∑_{i=1}^{n} y_i + z_i - x_i ≤ s.\]  

Bayesian hypothesis tests on the utility hypothesis \(U\) and the privacy hypothesis \(P\) are made with the objective to minimize their error probabilities based on the processed random sequence \(Y^n\).

There are two objectives in the management policy design: Enhance the utility hypothesis test; and degrade the privacy hypothesis test. The two objectives are generally conflicting to each other. Therefore, the trade-off of hypothesis tests needs to be studied in the management policy design.

## III. ERROR EXPONENT OF BAYESIAN COMPOSITE HYPOTHESIS TEST

In this section, we characterize fundamental bounds on the error exponent of Bayesian composite hypothesis test. The results derived in this section serve as the basis for the remaining discussions.

### A. Asymptotic Error Exponent with I.I.D. Observations

Let \(α_U (X^n)\) and \(α_P (X^n)\) denote the minimal error probabilities of the Bayesian composite hypothesis tests on \(U\) and \(P\) based on the i.i.d. sequence \(X^n\), e.g., a deterministic management policy \(φ^n (x^n, z^n) = x^n\) is employed when \(z^n\) is a deterministic sequence of zeros. The corresponding asymptotic error exponents are characterized in the following Theorem 1, which is an extension of Chernoff theorem [11] to the Bayesian composite hypothesis test. To this end, we first introduce a function \(T(Q_1||Q_2; Q_3)\), which is the minimum Kullback-Leibler divergence \(T(Q_1||Q_2; Q_3) = \min_{Q ∈ Q} D(Q||Q_2)\) with \(Q = \{Q : D(Q||Q_1) ≤ D(Q||Q_2), D(Q||Q_1) ≤ D(Q||Q_2)\}\).

**Definition 1.** Given three pmfs \(Q_1, Q_2,\) and \(Q_3\) with the same support set \(S\), we define

\[T(U_1, U_2, U_3) = - \log \sum_{a ∈ S} Q_{1,a}^{U_1}(a)Q_{2,a}^{U_2}(a)Q_{a}^{U_3}(a),\]

\[T(Q_1||Q_2; Q_3) = \max_{1 ≤ U_2, U_3 ≥ 0} T(U_1, U_2, U_3).\]

Conceptually, the function \(T\) is the extension for the Bayesian composite hypothesis test of the Chernoff information used in the standard Bayesian binary hypothesis test.

**Proposition 1.**

\[\lim_{n → ∞} \frac{1}{n} \log \frac{1}{α_U (X^n)} = \min_{u, p ∈ \{0, 1\}} \left\{ T(p_{X|u,p} || p_{X|1−u,p}; p_{X|1−u,1−p}) \right\}.\]  

The proof of Proposition 1 follows from Sanov’s theorem [10, Theorem 11.4.1] and is presented in the full version. Likewise, the result can be derived for the asymptotic error exponent of privacy hypothesis test with the i.i.d. sequence \(X^n\). Let \(C(Q_1||Q_2)\) denote the Chernoff information of pmfs \(Q_1\) and \(Q_2\) as defined in [11] as

\[C(Q_1||Q_2) = \max_{1 ≥ μ ≥ 0} − \log \sum_{α ∈ S} Q_1^α(α)Q_2^{1−μ}(a).\]

**Lemma 1.** Given pmfs \(Q_1, Q_2,\) and \(Q_3\) with the same support set, we have

\[\min \left\{ T(Q_1||Q_2; Q_3) \right\} = \min \left\{ C(Q_1||Q_2) \right\}.\]

The proof of Lemma 1 follows from the joint concavity of \(T_μ, ν\) over \((μ, ν) ∈ R^2\) and is presented in the full version. The following theorem is a direct consequence of Proposition 1 and Lemma 1, and shows that the asymptotic error exponents of Bayesian composite hypothesis tests on \(U\) and \(P\) with the i.i.d. sequence \(X^n\) are characterized by their corresponding minimal Chernoff information.

**Theorem 1.**

\[\lim_{n → ∞} \frac{1}{n} \log \frac{1}{α_U (X^n)} = \min_{p, μ \in \{0, 1\}} \left\{ C(p_{X|1,1} || p_{X|0,1}) \right\},\]

\[\lim_{n → ∞} \frac{1}{n} \log \frac{1}{α_P (X^n)} = \min_{a, α \in \{0, 1\}} \left\{ C(p_{X|0,1} || p_{X|α,1}) \right\}.\]

**Remark 1.** Theorem 1 cannot be implied from the asymptotic error exponent of the Bayesian multiple hypothesis test [12] since the minimal Chernoff information corresponding to a Bayesian composite hypothesis test in (4) is among four combinations of pmfs rather than all six combinations of pmfs. However, the proof ideas in [12] can be used to show the asymptotic error exponents of the Bayesian composite hypothesis tests in Theorem 1: Bound the minimal error probability of a composite hypothesis test by the minimal error probabilities of binary hypothesis tests; and then use the Chernoff theorem [11] to bound the asymptotic error exponent of the Bayesian composite hypothesis test by the Chernoff informations of the binary hypothesis tests.
B. Lower Bound on the Error Exponent

The problem shown in Fig. 1 considers hypothesis tests on the utility hypothesis and the privacy hypothesis based on the processed (not necessarily i.i.d.) sequence $Y^n$. When a randomized management policy $\phi^n_\alpha$ is used, let $\alpha_U(Y^n, \phi^n_\alpha)$ and $\alpha_P(Y^n, \phi^n_\alpha)$ denote the minimal error probabilities of the Bayesian composite hypothesis tests on $U$ and $P$ based on $Y^n = \phi^n_\alpha(X^n, Z^n)$. The following proposition gives lower bounds on the error exponents in terms of the corresponding Chernoff information rates when a management policy $\phi^n_\alpha$ is used.

Proposition 2. Given a management policy $\phi^n_\alpha$ and the resulting pmfs $p_{Y^n|0,0}$, $p_{Y^n|0,1}$, $p_{Y^n|1,0}$, $p_{Y^n|1,1}$, then we have

$$\frac{1}{n} \log \frac{1}{\alpha_U(Y^n, \phi^n_\alpha)} \geq \frac{1}{n} \min_{p, \overline{p} \in \{0,1\}} \left\{ C(p_{Y^n|0,1}||p_{Y^n|0,\overline{p}}) \right\} - \frac{\log sp_{\max}}{n},$$

$$\frac{1}{n} \log \frac{1}{\alpha_P(Y^n, \phi^n_\alpha)} \geq \frac{1}{n} \min_{a, \overline{a} \in \{0,1\}} \left\{ C(p_{Y^n|0,\overline{a}}||p_{Y^n|\overline{a},0}) \right\} - \frac{\log sp_{\max}}{n},$$

where $p_{\max} = \max_{a, \overline{a} \in \{0,1\}} \left\{ p_U(u, p) \right\}$.

Note that $p_{\max} \geq \frac{1}{4}$ and $\log sp_{\max} \geq \log 2 \geq 0$. The proof of Proposition 2 is given in the appendix.

IV. Optimal Management of Hypothesis Tests

We denote a management policy $\phi^n_\alpha$ by $\phi^n_{s,\lambda}$ if the resulting pmfs $p_{Y^n|0,0}$, $p_{Y^n|0,1}$, $p_{Y^n|1,0}$, $p_{Y^n|1,1}$ jointly satisfy the following hypothesis testing utility guarantee:

$$\frac{1}{n} \min_{p, \overline{p} \in \{0,1\}} \left\{ C(p_{Y^n|0,1}||p_{Y^n|0,\overline{p}}) \right\} \geq \lambda + \frac{\log sp_{\max}}{n}. \quad (6)$$

From Proposition 2, it follows that a policy $\phi^n_{s,\lambda}$ also satisfies a guarantee on the error exponent of the utility hypothesis test:

$$\frac{1}{n} \log \frac{1}{\alpha_U(Y^n, \phi^n_{s,\lambda})} \geq \lambda. \quad (7)$$

Remark 2. Instead of (7), the stronger utility hypothesis testing guarantee in (6) is imposed here to make the following asymptotic analysis tractable.

Let $\Phi^k_{s,\lambda}$ denote the set of all feasible $n$-slot policies satisfying the constraints in (1) and (6). In order to protect the privacy, an optimal management policy within $\Phi^k_{s,\lambda}$ is used to achieve the maximum minimal error probability of privacy hypothesis test as

$$\alpha^n_P(Y^n, s, \lambda) = \max_{\phi^n_{s,\lambda} \in \Phi^k_{s,\lambda}} \alpha_P(Y^n, \phi^n_{s,\lambda}), \quad (8)$$

or equivalently the minimum error exponent $\frac{1}{n} \log \frac{1}{\alpha^n_P(Y^n, s, \lambda)}$. The problem (8) tradeoffs the hypothesis testing performance by minimizing the error exponent of the privacy hypothesis test and meanwhile guaranteeing a lower bound on the utility hypothesis testing performance.

The formulated problem (8) corresponds to the practical scenario where the adversary is authorized and informed, e.g., a compromised grid operator in the smart meter privacy problem. In this case, the privacy leakage rate is usually more meaningful for the privacy measure when the adversary has a sequence of observations. In the following theorem, the asymptotic minimum error exponent of the privacy hypothesis test is characterized.

Theorem 2. Given feasible $s \geq 0$ and $\lambda \geq 0$, we have

$$\liminf_{n \to \infty} \frac{1}{n} \log \frac{1}{\alpha^n_P(Y^n, s, \lambda)} = \inf_{k \to \infty} \min \frac{1}{k} \left\{ C(p_{Y^n|0,1}||p_{Y^n|0,\overline{a}}) \right\} \geq \lambda + \frac{\log sp_{\max}}{k} \geq \lambda + \frac{\log sp_{\max}}{k}. \quad (9)$$

Proof: Given any $k \in Z_+$, any $\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}$, the resulting pmfs $p_{Y^n|0,0}$, $p_{Y^n|0,1}$, $p_{Y^n|1,0}$, and $p_{Y^n|1,1}$, let $\phi^k_{s,\lambda}$ denote a $k$-slot policy which repeatedly uses $\phi^k_{s,\lambda}$ for $l$ times. The policy $\phi^k_{s,\lambda}$ satisfies the constraint in (1) over the $k$-slot time horizon. Further, the resulting pmfs of the $k$-slot policy $\phi^k_{s,\lambda}$ jointly satisfy

$$\frac{1}{k} \min_{p, \overline{p} \in \{0,1\}} \left\{ C(p_{Y^n|0,1}||p_{Y^n|0,\overline{p}}) \right\} \geq \lambda + \frac{\log sp_{\max}}{k} \geq \lambda + \frac{\log sp_{\max}}{k}.$$ 

Therefore, we have $\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}$ since the policy $\phi^k_{s,\lambda}$ also satisfies the constraint in (6). It follows that

$$\liminf_{n \to \infty} \frac{1}{n} \log \frac{1}{\alpha^n_P(Y^n, s, \lambda)} \leq \liminf_{l \to \infty} \frac{1}{l} \log \frac{1}{\alpha^n_P(Y^{kl}, s, \lambda)} \leq \liminf_{l \to \infty} \frac{1}{l} \log \frac{1}{\alpha^n_P(Y^{kl}, (\phi^k_{s,\lambda}))} \leq \min_{a, \overline{a} \in \{0,1\}} \left\{ C(p_{Y^n|0,1}||p_{Y^n|\overline{a},0}) \right\} , \quad (10)$$

where the inequality (a) follows from the optimization in the definition (8); and the equality (b) follows from Theorem 1. The inequality (10) holds for all $k \in Z_+$ and all $\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}$. Therefore, we have

$$\liminf_{n \to \infty} \frac{1}{n} \log \frac{1}{\alpha^n_P(Y^n, s, \lambda)} \leq \inf_{k \to \infty} \min \frac{1}{k} \left\{ C(p_{Y^n|0,1}||p_{Y^n|\overline{a},0}) \right\} . \quad (11)$$

Suppose that the policy $\phi^n_{s,\lambda} \in \Phi^n_{s,\lambda}$ achieves $\alpha^n_P(Y^n, s, \lambda)$, i.e.,

$$\alpha^n_P(Y^n, s, \lambda) = \alpha_P(Y^n, \phi^n_{s,\lambda}).$$
Let $p_{Y^n|0,0^t}$, $p_{Y^n|0,1}$, and $p_{Y^n|1,0^t}$ denote the resulting pmfs. It follows from Proposition 2 that

$$
\frac{1}{n} \log \frac{1}{\alpha_P(Y^n, s, \lambda)} 
\geq \frac{1}{n} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\} - \frac{\log p_{\text{max}}}{n} 
\geq \min_{\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}} \frac{1}{n} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\} - \frac{\log p_{\text{max}}}{n}.
$$

In the asymptotic regime as $n \to \infty$, we have the following lower bound:

$$
\liminf_{n \to \infty} \frac{1}{n} \log \frac{1}{\alpha_P(Y^n, s, \lambda)} 
\geq \liminf_{n \to \infty} \frac{1}{n} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\} - \frac{\log p_{\text{max}}}{n}.
$$

(12)

From the definitions of infimum and limit infimum, we have

$$
\inf_{k \in \mathbb{Z}^+} \min_{\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}} \frac{1}{n} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\} 
\leq \liminf_{n \to \infty} \frac{1}{n} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\}.
$$

(13)

The inequalities (11), (12), and (13) jointly lead to the asymptotic minimum error exponent in (9).

\textbf{Remark 3.} The second equality in Theorem 2 can be alternatively justified by the following inequality: For all $k, l \in \mathbb{Z}^+$, we have

$$
\min_{\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}} \frac{1}{k} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\} 
\geq \min_{\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}} \frac{1}{k} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\}.
$$

(14)

where $n = kl \geq k$.

The proof ideas of the inequality (14) are: Construct a $kl$-slot policy $(\phi^{k,\lambda})$ where $(\phi^{k,\lambda})$ achieves the minimal Chernoff information rate over policies in $\Phi^k_{s,\lambda}$; show that the constructed policy $(\phi^{k,\lambda})$ attains the same minimal Chernoff information rate as $(\phi^{k,\lambda})$; and prove the inequality by the fact that $(\phi^{k,\lambda})$ does not necessarily achieve the minimal Chernoff information rate over policies in $\Phi^k_{s,\lambda}$.

Theorem 2 shows that the asymptotic minimum error exponent of the privacy hypothesis test is the infimum of corresponding minimal Chernoff information rates subject to the utility hypothesis testing guarantees. Theorem 2 also shows that the infimum of minimal Chernoff information rates in the general case is taken at the limit of the block length $n \to \infty$. Therefore, the numerical evaluation of the asymptotic minimum error exponent of the privacy hypothesis test and the design of an asymptotically optimal management policy are difficult tasks.

For all $k \in \mathbb{Z}^+$, the minimal Chernoff information rate $
\min_{\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}} \frac{1}{k} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\}$ is the asymptotic error exponent of the privacy hypothesis test when the block-wise i.i.d. management policy $(\phi^{k,\lambda})$ is used with

$$
\phi^{k,\lambda} = \arg \min_{\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}} \frac{1}{k} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\}.
$$

i.e.,

$$
\lim_{l \to \infty} \frac{1}{kl} \log \frac{1}{\alpha_P(Y^{kl}, (\phi^{k,\lambda})^l)} 
= \min_{\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}} \frac{1}{k} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\}.
$$

(15)

On the other hand, the minimal Chernoff information rate $
\min_{\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}} \frac{1}{k} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\}$ is an upper bound on the asymptotic minimum error exponent of the privacy hypothesis test and therefore can be seen as an asymptotic privacy guarantee. Then, the evaluation of an asymptotic privacy guarantee and the design of the corresponding block-wise i.i.d. policy are tractable.

\textbf{V. NUMERICAL EXAMPLE}

Fig. 2 illustrates the trade-off of the asymptotic privacy guarantee $\min_{\phi^k_{s,\lambda} \in \Phi^k_{s,\lambda}} \frac{1}{k} \min_{u, \bar{u} \in \{0,1\}} \left\{ C(p_{Y^n|u,1}^*||p_{Y^n|\bar{u},0}) \right\}$ and the utility hypothesis testing guarantee $\lambda$ in a simple model with $X = \mathbb{Z} = \{0,1\}$. The parameters are set as: $0 \leq \lambda \leq 0.16$, $s = 1.2$, $p_{\text{max}} = 0.2$, $p_{X|0,0}(0) = 0.1$, $p_{X|0,1}(0) = 0.25$, $p_{X|1,0}(0) = 0.8$, $p_{X|1,1}(0) = 0.9$, $p_{Z}(0) = 0.2$. As expected, the value of the asymptotic privacy guarantee increases as the value of the utility hypothesis testing guarantee increases; and a greater value of $s$ leads to a better asymptotic privacy guarantee. We can also learn from Fig. 2 that an asymptotic privacy guarantee is not necessarily convex or concave of the utility hypothesis testing guarantee $\lambda$. Since the asymptotic minimum error exponent of the privacy hypothesis test is the infimum of the asymptotic privacy guarantees, its convexity property is not clear.
VI. CONCLUSION

We showed that the error exponent of a Bayesian composite hypothesis test can be characterized by the corresponding minimal Chernoff information rate. With the optimal management, we further proved that the asymptotic minimum error exponent of the privacy hypothesis test can be characterized by the infimum of the corresponding minimal Chernoff information rates subject to utility hypothesis testing guarantees. The studied optimal management with privacy-utility trade-off can be applied in many practical scenarios, e.g., smart metering system.

APPENDIX

A. Proof of Proposition 2

Proof: For all $\mu_1, \mu_2, \mu_3, \mu_4 \in [0, 1]$, we have upper bounds on the minimal error probability of the utility hypothesis test as

$$\alpha_U(Y^n, \phi^n)$$

i.e., the error exponent of the hypothesis test on $U$ has the following lower bound:

$$\frac{1}{n} \log \frac{1}{\alpha_U(Y^n, \phi^n)} \geq \frac{1}{n} \min_{p, \bar{p} \in \{0,1\}} \left\{ C(pY^n|1,\bar{p}) \mid pY^n|0,\bar{p} \right\} - \frac{\log 8 p_{\max}}{n}.$$  

The proof of the lower bound on the error exponent of the privacy hypothesis test is similar and therefore is omitted. □

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