Breakup Reactions of Neutron Drip Line Nuclei Near $N=20$

Takashi Nakamura
2-12-1 O-Okayama, Meguro, Tokyo 152-8551, Japan
E-mail: nakamura@phys.titech.ac.jp

Abstract. Coulomb breakup at intermediate energies is a useful experimental tool for investigating the microscopic structure of neutron drip-line nuclei. Here, results from the inclusive Coulomb breakup experiment of $^{31}$Ne on a lead target at RIBF (RI Beam Factory) at RIKEN are presented. The experiment was performed as one of day-one campaign experiments at RIBF, using a $^{48}$Ca primary beam at 345 MeV/nucleon. A unique feature of a halo nucleus is the enhanced electric dipole strength of the order of 1 W.u. (Weisskopf unit) at very low excitation energies around 1 MeV (soft $E1$ excitation). Owing to high sensitivity of the Coulomb breakup to the soft $E1$ excitation, a measurement of inclusive Coulomb breakup cross section can be used to identify the halo structure of a certain drip-line nucleus. We have indeed observed a strong enhancement of the Coulomb breakup cross section of 540(70) mb for $^{31}$Ne on Pb at 230 MeV/nucleon, nearly as high as that for the known halo nucleus $^{19}$C, thereby giving evidence of the halo structure in $^{31}$Ne. The finding of a new halo structure for such a heavy system, compared to the known halo nuclei, is the first step for the understanding of halo phenomena along the neutron drip line towards heavier nuclei. We discuss also the change of shell structure in $^{31}$Ne, as a nucleus in the island of inversion.

1. Introduction

Coulomb breakup is a useful experimental tool for investigating neutron halo structure. This reaction can be understood as a photo-absorption process of a projectile when a projectile passes a high-$Z$ target at intermediate/high energies. Since a halo nucleus is weakly bound, the nucleus as a projectile is broken up into a core fragment and one or two neutrons, following the excitation by absorbing a virtual photon. In the conventional equivalent-photon approach [1, 2], the Coulomb breakup cross section for the $E1$ excitation can be written as

$$\frac{d\sigma(E_1)}{dE_x} = \frac{16\pi^3}{9hc} N_{E1}(E_x) \frac{dB(E1)}{dE_x},$$

(1)

This relation illustrates that the $B(E1)$ strength can be mapped as a function of $E_x$. This is realized by measuring all the momentum vectors of the outgoing breakup particles in coincidence (exclusive measurement), to reconstruct the invariant mass of the excited intermediate state. We show here that even inclusive Coulomb breakup measurements, where only the integrated cross section is extracted, can be a useful tool. In fact, as later shown, an inclusive cross section can possess significant information on the halo structure.

In Sec. 2, we review the exclusive Coulomb breakup measurement of a 1n halo nucleus $^{11}$Be and that of a 2n halo nucleus $^{11}$Li, in order to illustrate characteristic features of the soft $E1$...
excitation. Then, in Sec. 3, we show how the inclusive measurement works, and present the experimental results of the inclusive Coulomb breakup of $^{31}$Ne at RIBF. Finally, in Sec. 4, concluding remarks will be provided.

2. Characteristic features of Coulomb breakup and soft E1 excitation of halo nuclei
2.1. Exclusive Coulomb breakup of the 1n halo nucleus - Case of $^{11}$Be

The E1 response of halo nuclei is characterized by strong E1 transitions at low excitation energies, called soft E1 excitation. Figure 1(a) shows the E1 strength distribution for $^{11}$Be, extracted by the Coulomb breakup of $^{11}$Be at 70 MeV/nucleon at RIKEN [3]. The spectrum is dominated by a strong asymmetric peak with significantly large E1 strength of $1.05\pm0.06$ e$^2$fm$^2$ (3.29±0.19 W.u.) for $E_{\text{rel}} \leq 3.5$ MeV. Such a spectrum is now well understood by the direct breakup mechanism. The solid curve, representing the calculation of the direct breakup mechanism for $^{11}$Be, is in excellent agreement with the data.

In the direct breakup mechanism, E1 strength distribution can simply be described as a matrix element:

$$
\frac{dB(E1)}{dE_{\text{rel}}} = | \langle \Phi_f(\vec{r}, \vec{q}) | \frac{Ze}{A} \rho Y^{(1)}(\Omega) | \Phi_i(\vec{r}) \rangle |^2,
$$

where $\Phi_i(\vec{r})$ and $\Phi_f(\vec{r}, \vec{q})$ represent the wave function of the ground state, and that of the final state in the continuum, respectively. The initial and final states are functions of the relative coordinate, $\vec{r}$, of the valence neutron relative to the core. The final state, $\Phi_f(\vec{r}, \vec{q})$, is also a function of the relative momentum $q = \sqrt{2\mu E_{\text{rel}}}/\hbar$. We note that this equation corresponds to the Fourier transform of the ground state radial wave function multiplied by $r$, assuming the plain wave approximation for $\Phi_f(\vec{r}, \vec{q})$. The point is that the $B(E1)$ spectrum is sensitive only to the tail part of the radial wave function, which allows us to use the $B(E1)$ spectrum to extract the spectroscopic information. For $^{11}$Be, the ground state wave function $\Phi_i(\vec{r})$ ($^{11}$Be($1/2^+; \text{g.s.}$)) can be written as,

$$
|^{11}\text{Be}(1/2^+; \text{gs})\rangle = \alpha^{10}\text{Be}(0^+) \otimes \nu 2s_{1/2} + \beta^{10}\text{Be}(2^+) \otimes \nu 1d_{5/2} + ....
$$

Here, the first term represents the halo configuration. The large amplitude in the tail part of the radial wave function due to this term gives rise to the enhanced $B(E1)$ strength at small $E_{\text{rel}}$ (equivalently $E_x$). The amplitude of the $B(E1)$ distribution can then be used to extract the spectroscopic factor $\alpha^2$ of the halo configuration. In Ref. [3], $\alpha^2$ value of 0.72±0.04 was extracted, which is largely consistent with the results from other Coulomb breakup measurements [4, 5].

The spectral shape is also useful in the spectroscopy of halo nuclei, as demonstrated in the Coulomb breakup experiment of $^{19}$C [6]. There, the dominance of $|^{18}\text{C}(0^+) \otimes \nu 2s_{1/2}\rangle$ configuration, the spin-parity of $J^p=1/2^+$, and the separation energy $S_n$ of about 500 keV, for the $^{19}$C ground state were determined. Such a method is possible since the shape of the $B(E1)$ spectrum depends strongly on the angular momentum $\ell$ of the valence neutron in the ground state and the final state, and on $S_n$.

Characteristic features of the analytical forms of the $B(E1)$ distribution can be seen by examining an approximate but analytical form of the matrix element. We assume that the ground state wave function is an asymptotic form (spherical Hankel function), and that the final state is a plain wave (spherical Bessel function). Such theoretical studies [7, 8, 9] show that the $B(E1)$ spectrum at small $E_{\text{rel}}$ can be well approximated as

$$
\frac{dB(E1)}{dE_{\text{rel}}} \propto E_{\text{rel}}^{\ell_f+1/2},
$$

and $dB(E1)/dE_{\text{rel}}$ reaches maximum at $E_{\text{rel}} = kS_n$, where $k$ depends on $\ell_i$ and $\ell_f$. For instance, for the $s \rightarrow p$ transition, $B(E1)$ peaks at $E_{\text{rel}} = (3/5)S_n$. Such studies demonstrate that the
exclusive Coulomb breakup of a $1n$ halo nucleus is very useful in extracting the single particle nature of the halo neutron. Namely, $S_n$, $\ell$, and the single particle configuration and associated spectroscopic factors can be extracted.

![Figure 1](image-url)

**Figure 1.** $B(E1)$ spectrum obtained as a function of relative energy $E_{\text{rel}}$ for the one-neutron halo nucleus $^{11}\text{Be}$ (a), and that for the two-neutron halo nucleus $^{11}\text{Li}$. For $^{11}\text{Be}$ (a), the calculation based on the direct breakup mechanism is compared with the data. The amplitude is directly related to the spectroscopic factor of the $s$-wave neutron halo configuration, which was determined to be $0.72(4)$ [3]. For $^{11}\text{Li}$ (b), the solid curve represents the calculation based on the three-body model including the $nn$ correlation and the final state interaction [10].

2.2. **Exclusive Coulomb breakup of the $2n$ halo nucleus - Case of $^{11}\text{Li}$**

The soft $E1$ excitation of $2n$ halo nuclei bears more complicated but interesting aspects, compared to the case of the $1n$ halo nucleus. For the $2n$ halo nucleus, not only the core-$n$ motion, but also $nn$ correlation should be taken into consideration. From the experimental point of view, coincidence detection of two neutrons at very forward angles becomes an issue since a fast neutron easily cause cross-talk signals, and the $1n$ events may be identified mistakenly as $2n$ coincidence events. In the Coulomb breakup experiment of $^{11}\text{Li}$ at 70 MeV/nucleon at RIKEN [11], we introduced a novel method of distinguishing two neutrons by the kinematical condition, which enables us to measure the $E1$ strength distribution down to $E_{\text{rel}} \sim 0$ MeV, unambiguously.

Figure 1(b) shows the $B(E1)$ distribution obtained in this Coulomb breakup experiment [11]. The $B(E1)$ strength amounted to $B(E1)=1.42(18)$ e²fm², corresponding to 4.5(6) W.u. ($E_{\text{rel}} \leq 3$ MeV), which is about 40% larger than the case of $^{11}\text{Be}$. The data are compared with the three-body theory by H. Esbensen and G.F. Bertsch [10], where the calculation with
full three-body correlation (\(nn\) correlation and final state interaction) is in excellent agreement with the data. Recently, a revised three-body calculation was made by H. Esbensen et al., where the recoil effect is incorporated. The overall agreement with the data has been obtained, although a slight deviation appeared around \(E_{rel} \sim 0.5\) MeV [12]. The recent calculation by T. Myo et al. [13], which took into consideration the core polarization effect, reproduced better the peak region. These calculations also show the strong \(nn\) spatial correlation. As such, the significance of \(nn\) correlation is indicated through these comparisons.

The spatial two neutron correlation in the ground state of the two neutron halo nuclei can be quantitatively estimated by using the non-energy weighted \(E1\) cluster sum rule. The sum rule can be described as,

\[
B(E1) = \frac{3}{4\pi} \left( \frac{Ze}{A} \right)^2 \langle r_1^2 + r_2^2 + 2r_1 \cdot r_2 \rangle = \frac{3}{\pi} \left( \frac{Ze}{A} \right)^2 \langle r_{c,2n}^2 \rangle. \tag{5}
\]

Here, \(\vec{r}_i\) is the position vector of the valence neutron relative to the center of mass (c.m.) of the core, while \(\vec{r}_2\) is that for the other valence neutron. The distance \(r_{c,2n}\) is that between the core and the c.m. of the two halo neutrons, which is half the \(|\vec{r}_1 + \vec{r}_2|\). It should be noted that the term \(r_1 \cdot r_2\) involves the opening angle \(\theta_{12}\) between the two position vectors for the valence neutrons. The value of \(\langle r_{c,2n}\rangle\), and hence \(B(E1)\), becomes larger for the smaller spatial separation of the two neutrons, when \(\theta_{12}\) approaches 0°. The integrated \(B(E1)\) thus provides a good measure of the two-neutron spatial correlation. The extrapolated integral value of \(B(E1)\), according to the theoretical curve in Fig. 1(b), amounts to 1.78±0.22 \(e^2fm^2\) for \(E_x \geq S_{2n}\), which corresponds to \(\sqrt{\langle r_{c,2n}^2 \rangle} = 5.01 \pm 0.32\) fm. Adopting the sum rule value, 1.07 \(e^2fm^2\), for the two non-correlated neutrons [10], the value 1.78±0.22 \(e^2fm^2\) was found to correspond to \(\langle \theta_{12} \rangle = 48^{\pm 14}\) degrees. This angle is significantly smaller than the average opening angle of 90 degrees expected for the two non-correlated neutrons, which indicates an appreciable two-neutron spatial correlation for the two halo neutrons. The theoretical calculation in Ref. [12] estimated \(\sqrt{\langle r_{c,2n}^2 \rangle} = 5.22\) fm, while that in Ref. [13] estimated \(\sqrt{\langle r_{c,2n}^2 \rangle} = 5.69\) fm, which are close to the experimental estimation. The important point of the soft \(E1\) excitation of 2\(n\) halo nucleus is that the larger polarization of the charge due to the stronger dineutron-like correlation enhances the \(E1\) strength at low excitation energies.

3. Inclusive Coulomb breakup of \(^{31}\text{Ne}\)

3.1. Issues with \(^{31}\text{Ne}\)

The heaviest \(1n\) halo nucleus experimentally established has been \(^{19}\text{C}\) for about a decade. Since the first observation of \(^{31}\text{Ne}\) in 1996 [14], \(^{31}\text{Ne}\) has been the next candidate of the \(1n\) halo nucleus due to its small \(1n\) separation energy. The recent direct mass measurement at GANIL indeed showed the small separation energy (\(S_n=0.29\pm 0.04\) MeV [15]).

Here, we address the following questions: Can we observe many halo cases in heavier nuclei? How and in what form can halo states appear in heavier nuclei? In this context, the study of \(^{31}\text{Ne}\) is very important, which is significantly heavier than \(^{19}\text{C}\). The large \(\ell\) value in heavier nuclei may often hinder the formation of halo, blocked by the large centrifugal barrier. For \(^{31}\text{Ne}\), in the conventional shell order for the \(N=21\) nucleus, the ground state of \(^{31}\text{Ne}\) should not possess a halo structure due to the dominant \(1f_{7/2}\) orbital. Hence, the formation of a halo of this nucleus implies some changes of shell structure.

It it interesting to note that \(^{31}\text{Ne}\) (\(N=21\)) has been predicted to be inside “the island of inversion”, where shell model shows that \(N=20\) magicity is lost and significant \(2p-2h\) configurations are mixed [16, 17, 18]. The neighboring nuclei \(^{32}\text{Mg}\) [19], \(^{32}\text{Na}\) [20], \(^{30,32}\text{Ne}\) [21, 22] have been found within this island experimentally. The halo formation of \(^{31}\text{Ne}\) may thus be
possible when the significant $p_{3/2}$ component is involved in the ground state of $^{31}$Ne due to such a shell evolution.

3.2. Inclusive Coulomb breakup

The inclusive Coulomb breakup can be a signal of a halo structure. As was mentioned, the exclusive (full kinematical) measurement requires a measurement of four momentum vectors of all the outgoing particles. For that, we need a coincidence measurement of the neutron, which requires the beam intensity of the order of 10-100 counts/sec. On the other hand, an inclusive measurement is possible with the order of one counts/second, suitable for the earlier stage of the new facility RIBF. In the experiment of Ref. [23], typical $^{31}$Ne beam intensity was about 5 counts per second and the data was recorded only for about 10 hours.

The inclusive Coulomb breakup cross section can be written as,

$$
\sigma(E1) = \int_{S_n}^{\infty} \frac{16\pi^3}{9\hbar c} N_{E1}(E_x) \frac{dB(E1)}{dE_x} dE_x.
$$

(6)

Here, the product of $N_{E1}(E_x)$ and $dB(E1)/dE_x$ is involved in the cross section. As shown in Fig. 2(top), the photon spectrum falls exponentially with $E_x$. Thus the $\sigma(E1)$ value becomes significant only when the large $E1$ strength is concentrated at low excitation energies (soft $E1$ excitation), which uniquely appears for the case of halo nuclei. The bottom part of Fig.2 shows the comparison of $B(E1)$ strength distributions calculated for the assumed halo nucleus (soft $E1$ excitation) and for the assumed ordinary nucleus (GDR). For the GDR, we assume that the GDR is located at 21.6 MeV with a width of 5 MeV (Gaussian), which exhausts the full TRK (Thomas Reigh Kuhn) sum of 420 MeV-mb for the nucleus with $A=31$ and $Z=10$. In this case, the total Coulomb breakup cross section $\sigma(E1)$ for $^{31}$Ne on Pb target at 230 MeV is only 58 mb.

On the other hand, assuming that $^{31}$Ne is a halo nucleus and has a soft $E1$ strength caused by the valence neutron in $p_{3/2}$ with $S_n=0.5$ MeV, the total Coulomb breakup cross section up to $E_x=10$ MeV is 510 mb, which is about one order of magnitude larger than that of the case of the GDR. In the experiment, we measured $1n$ removal Coulomb breakup cross section, whose integration ranges up to $E_x \sim S_2n$ instead of infinity. In this range, the GDR cross section becomes zero, while the cross section assuming the $p$-wave halo state is calculated to be 470 mb. With such a distinctive difference, the inclusive cross section of Coulomb breakup can be used as a signal for a halo state.

3.3. Experiment and Results

The $1n$ removal cross sections of $^{31}$Ne on Pb and C targets were measured at RIBF operated by the RIKEN Nishina Center and the Center for Nuclear Study, University of Tokyo [23]. The Secondary beam of $^{31}$Ne was produced by the fragmentation of $^{48}$Ca primary beam on a Be production target. Isotopic separation was made by the first stage of BigRIPS (Big Riken Projectile-fragment Separator), and the projectile was identified event-by-event in the second stage by using the information of $\Delta E$ in an ionization chamber, TOF (Time of Flight), and the $B_\rho$ measurement at the dispersive focus [24, 25]. As shown in Fig. 1 of Ref. [23], $^{31}$Ne was distinguished unambiguously from other isotopes, which impinged on the Pb (3.37 g/cm$^2$) and the C (2.54 g/cm$^2$) targets. The mean energy of the secondary $^{31}$Ne beam was 234 MeV/nucleon for Pb and 230 MeV/nucleon for C. We also measured $1n$ removal cross section for $^{19,20}$C as a reference. In addition, we have obtained data on $2n$ removal cross sections for $^{20}$C and $^{22}$C. The latter result provided evidence for $2n$ halo structure of $^{22}$C, whose details will be presented elsewhere.

Figure 3 shows the $1n$ removal cross sections of $^{19,20}$C and $^{31}$Ne on Pb and C targets at about 230 MeV/nucleon. The cross sections of $^{19}$C and $^{31}$Ne were found both significantly larger than
that for $^{20}$C. Secondly, the ratio of the cross section for Pb to that for C is 9.0±1.1 for $^{31}$Ne and 7.4±0.4 for $^{19}$C, much larger than the ratio for nuclear breakup only, which is estimated to be about 1.7–2.6. This result shows that the cross section for Pb target is dominated by Coulomb breakup for $^{19}$C and $^{31}$Ne.

The Coulomb breakup component of the 1n removal cross section on Pb was deduced by subtracting the nuclear component estimated from $\sigma_{-1n}(C)$. For this purpose, it was assumed that $\sigma_{-1n}(C)$ arises entirely from the nuclear contribution, and that the nuclear component for the Pb target scales with the parameter $\Gamma$ as

$$\sigma_{-1n}(E1) = \sigma_{-1n}(Pb) - \Gamma \sigma_{-1n}(C),$$

(7)

where $\Gamma$ was estimated to be $\sim$1.7–2.6. The lower limit is from the ratio of target+projectile radii, as in Ref. [26], while the upper bound is the ratio of radii of two targets as in the Serber model [27]. The Coulomb breakup cross section for $^{31}$Ne was thus obtained to be $\sigma_{-1n}(E1)=540\pm70$ mb, which takes into consideration the ambiguity arising from the choice of these two models. The dominance of the Coulomb breakup for $^{31}$Ne on Pb and the deduced $\sigma_{-1n}(E1)$ of about 0.5 b, nearly as high as the established halo nucleus $^{19}$C, provides evidence of the existence of large E1 strength at low excitation energies (soft E1 excitation) for $^{31}$Ne. This result is thus indicative of the 1n halo structure of this nucleus.

According to the direct breakup mechanism, the single-particle structure of the ground state of $^{31}$Ne can be examined. Figure 4 compares the experimentally deduced $\sigma_{-1n}(E1)$ with calculations for possible valence-neutron configurations as a function of $S_n$. Due to a large experimental uncertainty in $S_n$, the calculated results are shown as a function of $S_n$. 

![Figure 2](image-url)
**Figure 3.** One neutron removal cross sections for $^{19}$C, $^{20}$C, and $^{31}$Ne on Pb (diamonds) and C (circles), and extracted Coulomb breakup cross sections (squares). The data for $^{19}$C and $^{31}$Ne are from Ref. [23]. The data for $^{20}$C are preliminary.

The figure deals with direct-breakup calculations for a pure single particle configuration $C^2S = 1$ for the valence neutron either in the $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$, or $2p_{3/2}$ orbital, being coupled to the ground state of $^{30}$Ne. More detailed analysis including possible configurations of the valence neutron coupled to $^{30}$Ne($2^+_1$) state is presented in Ref. [23]. In any cases, the essential point is included in this figure. Namely, the comparison in Fig. 4 shows that the data can be reproduced by the configuration of $[^{30}\text{Ne}(0^+_1) \otimes \nu 2p_{3/2})(J^\pi=3/2^-)$ or $[^{30}\text{Ne}(0^+_1) \otimes \nu 2s_{1/2})(J^\pi=1/2^+)$, but not by $[^{30}\text{Ne}(0^+_1) \otimes \nu 1d_{3/2})$ nor $[^{30}\text{Ne}(0^+_1) \otimes \nu 1f_{7/2})$. The significant contribution from the low-$\ell$ valence neutron is again consistent with the 1n halo structure of $^{31}$Ne. The other important

**Figure 4.** The Coulomb breakup cross section of $^{31}$Ne on Pb at 234 MeV/nucleon is compared with direct-breakup calculations for configurations of the valence neutron in $2s_{1/2}$, $2p_{3/2}$, $1d_{3/2}$, $1f_{7/2}$ coupled to $^{31}$Ne(g.s.) for $C^2S=1$ as a function of $S_n$. The case of the lower $C^2S$ value ($C^2S = 0.5$), shown by the dot-dashed curve for $2p_{3/2}$ configuration, is also shown.
implication of this result is that the conventional shell model configuration of \( ^{30}\text{Ne}(0^+_1) \otimes \nu 1f_{7/2} \) for the \( N = 21 \) nucleus is not a dominant configuration of the \( ^{31}\text{Ne} \) ground state. Large-scale Monte-Carlo Shell Model (MCSM) calculations using the SDPF-M effective interaction [18] support the assignment of \( J^\pi = 3/2^- \) having a \( ^{30}\text{Ne}(0^+_1) \otimes \nu 2p_{3/2} \) contribution.

The same calculations also show the large configuration mixing, where \( ^{30}\text{Ne}(0^+_1) \otimes \nu 2p_{3/2} \) can be mixed with \( ^{30}\text{Ne}(2^+_1) \otimes \nu 2p_{3/2} \) and \( ^{30}\text{Ne}(2^+_1) \otimes \nu 1f_{7/2} \). Such a large configuration mixing can be described by a deformed model. Recently, the current data were interpreted in terms of deformation [28]. There, the 21st neutron \((p_{3/2})\) can be in the Nilsson levels \([330]1/2^-\) or \([321]3/2^-\). The possibility of the \( s\)-wave valence neutron \((|200]|2^+)\) with large deformation \((\beta > 0.59)\) remains, though. It is interesting that the direct breakup mechanism and the enhancement of \( B(E1) \) at low excitation energies (soft \( E1 \) property) holds true for the halo wave function with a deformed potential.

4. Concluding Remarks
We have shown that the Coulomb breakup is a very useful tool to investigate the halo structure due to its sensitivity to the low-lying \( E1 \) transitions. By measuring all the momenta of outgoing particles in coincidence (exclusive measurement), one can determine the \( B(E1) \) distribution, which can be related to the microscopic structure of halo nuclei. For the \( 1n \) halo nuclei, the shape and the amplitude of the spectrum can be understood in the framework of the direct breakup mechanism. There, the spectrum is sensitive to \( S_n \), \( \ell \) of the valence neutron, and the spectroscopic factor for the halo configuration. The \( s \) or \( p \) configuration is essential for the formation of halo due to none or very low centrifugal barrier. As an exclusive experiment, the typical example of \( ^{11}\text{Be} \) was shown. For the \( 2n \) halo nuclei, \( nn \) spatial correlation is also essential to enhance the \( B(E1) \) strength as was shown for the non-energy weighted cluster sum rule. For the \( ^{11}\text{Li} \) case, we showed the possibility of dineutron correlation in this nucleus from the integrated \( E1 \) strength.

The recent inclusive Coulomb breakup experiment of \( ^{31}\text{Ne} \) at RIBF at RIKEN was then presented. The large Coulomb breakup cross section observed for \( ^{31}\text{Ne} \) gave evidence of \( 1n \) halo structure of \( ^{31}\text{Ne} \). On the other hand, the dominance of the \( f_{7/2} \) configuration, expected from the conventional shell order for the \( N=21 \) nucleus, was excluded. This is consistent with that the \( N=20 \) and \( 28 \) shell gaps are melted in \( ^{31}\text{Ne} \).

As demonstrated, the inclusive measurement is in particular important when the beam intensity is not sufficient. It provides us with the first approach to the exotic property of extremely neutron rich nuclei whose beam intensity is generally very weak. However, for the full understanding of the microscopic structure of \( ^{31}\text{Ne} \), the exclusive Coulomb breakup experiment would be desired, where \( C^2S \) and \( S_n \) can be extracted for this \( 1n \) halo nucleus. At RIBF at RIKEN, such experiments will be realized soon by the completion of SAMURAI/NEBULA project.

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