SUSY and Symmetry Nonrestoration at High Temperature

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ABSTRACT

The status of internal symmetry breaking at high temperature in supersymmetric models is reviewed. This phenomenon could solve some well known cosmological problems, such as the domain wall, monopole and false vacuum problems.

Introduction

The fascinating idea of symmetry nonrestoration at high temperature has been proposed long ago [1, 2], but is still a subject of ongoing research (see for example [3, 4] for recent reviews). As we will see, apart from being interesting by itself the phenomenon of symmetry nonrestoration could also naturally solve some cosmological problems. At first sight such a possibility seems forbidden in supersymmetric models due to the existence of a no-go theorem [5, 6]. I will show how this difficulty can be circumvented.

Let me first explain in detail the choice of the title, motivating in such a way the work in this field.

Why symmetry nonrestoration at high temperature? We know that the low-energy symmetry of the Standard Model is described by the group $H = SU(3)_c \times SU(2)_L \times U(1)_Y$. It is natural to expect that at some high scale
\( M_X \approx 10^{15} - 10^{16} \text{ GeV} \) the three gauge couplings unify into one coupling, i.e. that \( H \) is only a subgroup of a larger grandunified simple group \( G \) (for example \( SU(5), SU(6), SO(10), \text{etc.} \)). One would naively expect that at \( T \gg M_X \) the ground state of the universe was symmetric under this group \( G \), while it is clearly asymmetric at today temperatures \( T \ll M_X \). Technically this can be studied by an order parameter, which is for example in the case of \( G = SU(5) \) the vacuum expectation value (vev) of the 24-dimensional adjoint \( \Sigma \). Thus one usually thinks that \( < \Sigma > = 0 \) at high \( T \) (symmetry \( G \) restored), while \( < \Sigma > \neq 0 \) signals the symmetry breaking \( G \to H \) at small \( T \). This means that in between these two extremes a phase transition took place. Being \( G \) a simple group and having the low-energy group \( H \) a \( U(1) \) (hypercharge) factor, GUT monopoles were created during this phase transition via the well-known Kibble mechanism. However this would lead to a cosmological disaster: even if the monopoles were created during the phase transition at a rate of one per horizon, the fact that they could not decay sufficiently fast (so their energy density scaling only as \( T^3 \) instead as \( T^4 \) as for light particles) would be enough to have approximately 16 orders of magnitude more energy today in the form of monopoles than in the form of baryons (obviously such a universe would have closed itself long ago). A similar derivation for models with domain walls gives very similar results and an analogue problem.

Such cosmological problems can be trivially solved if one finds that there were no phase transitions, i.e. that the grandunified group \( G \) (for example \( SU(5) \)) was spontaneously broken already at high temperature \( T > M_X \) to its low-energy subgroup \( H \) (the SM group \( SU(3)_c \times SU(2)_L \times U(1)_Y \)). In spite of a claim to the opposite, in the case of global symmetries (domain wall problem) the majority of nonperturbative results confirm the possibility of symmetry nonrestoration. The question is still open in the case of gauge symmetries (monopole problem), where next to leading order corrections are big and tend to spoil the possibility of a perturbative expansion. This gives a reason more to look for new noncanonical ways of gauge symmetry nonrestoration at high temperature. As I will show later, such solutions can be found and, contrary to naive expectations, are even more natural in supersymmetric models.

Another possible solution to the monopole problem is that the low-energy subgroup \( H \) does not contain a \( U(1) \) factor below \( M_X \). In such a scenario electromagnetism would be spontaneously broken during a period of the his-
tory of the universe and would eventually get restored later. Anyway, both in the case that \( G \) is not restored and in the case that \( H \) does not contain a \( U(1) \) factor, we have symmetry breaking at high \( T \), a phenomenon which is counterintuitive but extremely interesting.

Of course, there are also other possible solutions of the monopole and domain wall problems. The most fancy is probably inflation. Obviously we still want to have inflation (for example we need the Higgs field to be homogeneous), but some constraints on possible inflationary models needed to solve the above cosmological problems can be relaxed. Two other interesting scenario were proposed recently: in [18] it was shown that unstable domain walls could sweep away monopoles, while in [19] moduli fields could have diluted the original monopole density.

It is important here to stress once more that we still want inflation to take place. After all it has still to solve the usual horizon problem, homogeneity, etc. The only suggestion here is that inflation does not solve the monopole problem. Only in this way there is a hope that some monopole could one day be detected and thus give a clear experimental signature of a GUT. In fact if inflation solves the monopole problem, it pushes the monopole number essentially to zero, while thermal production even in the case of symmetry nonrestoration (and thus no phase transition) can give a sensible but nondangerous monopole density today [20, 10]. More important, inflation needs more than just GUT, while symmetry nonrestoration could be obtained at least in principle from minimal models. We will see later that in SUSY such a solution does not only exist, but it is also very natural.

Why SUSY? The historical reason to consider seriously supersymmetric models is the solution to the hierarchy problem; on top of that the minimal \( SU(5) \) SUSY GUT predicts correct gauge coupling unification, which cannot be achieved in the nonsupersymmetric version of the Standard Model [21]. There are two special reasons which make the study of gauge symmetry nonrestoration in SUSY models at high temperature especially interesting.

The first one is the appearence of a new cosmological problem [22], usually ignored, but very important. It comes from the fact that SUSY models have (at \( T = 0 \)) often more than one vacuum, which are degenerate and disconnected. For example the minimal \( SU(5) \) with one adjoint has three degenerate vacua: the one with \( SU(5) \) symmetry (call it vacuum 1), one with \( SU(5) \) broken to \( SU(3) \times SU(2) \times U(1) \) (vacuum 2) and a third one with the symmetry \( SU(4) \times U(1) \) (vacuum 3). Obviously, at low energy (but
above the scale $M_W$) our universe was in vacuum 2. If one assumes as usual that at high $T$ the GUT symmetry was restored, when going from $T \gg M_X$ to $T \ll M_X$ the ground state would remain trapped in the wrong vacuum with $SU(5)$ unbroken. The point is that the barrier between two vacua is huge (of order $M_X^4$), so that tunneling is essentially impossible. Again, the problem could be solved if also at $T \gg M_X$ the GUT symmetry was spontaneously broken to the SM gauge group.

The second special reason to make SUSY models interesting is that apparently the program of symmetry nonrestoration does not work. There exists in fact a no-go theorem [5, 6]: any internal symmetry in a renormalizable SUSY model gets restored at high enough temperatures. The generalization to nonrenormalizable (effective) SUSY models has not been proven in general, but there is good evidence that it applies also in this case [23, 24], despite some previous claims on the opposite [25]. But, as is usually the case with no-go theorems, it is possible to evade them finding some more general examples, which were not considered by the authors of [5, 6]. In the following I will describe two such possibilities.

**Large external charge density**

The above no-go theorem considers only cases with vanishing charge density, so it cannot be applied here. The essential idea was developed for nonsupersymmetric models long ago, but it was realized only in [26] that it can be applied in SUSY models as well. Since here supersymmetry is not essential and does not change any conclusion, I will sketch only the nonsupersymmetric case. Consider a toy model with a $U(1)$ global symmetry with the potential

$$V_0 = \lambda |\phi|^4 .$$

At high temperature, the leading contribution is given by the term [27, 1, 28]

$$\Delta V_T = \frac{T^2}{24} \sum_i \frac{\partial^2 V_0}{\partial \phi_i^2} = \frac{\lambda T^2}{3} |\phi|^2 .$$

At high charge density $n$ one has to introduce also a chemical potential $\mu$, which modifies the effective potential by further new terms [29, 30].
\[ \Delta V_\mu = -\mu^2|\phi|^2 - \frac{\mu^2 T^2}{6} + \mu n. \]

The effective potential thus starts with
\[ V = \left( \frac{\lambda T^2}{3} - \mu^2 \right) |\phi|^2 + \ldots, \]
so that the vev of \( \phi \) becomes nonzero as long as the chemical potential is large enough (\( \mu_{\text{crit}} = T \sqrt{\lambda/3} \)). Of course, the right way to proceed is to impose the condition \( \partial V/\partial \mu = 0 \) and thus rewrite the potential in terms of the charge density \( n \) instead of the chemical potential \( \mu \). In such a way one can find the critical charge density
\[ n_{\text{crit}} = \sqrt{\frac{\lambda}{27}} T^3. \]

For \( n > n_{\text{crit}} \) the vev of \( \phi \) is nonzero and the global symmetry \( U(1) \) is spontaneously broken, while for \( n \leq n_{\text{crit}} \) the same vev is vanishing, thus restoring the global symmetry in question. Since both \( n \) and \( n_{\text{crit}} \) scale as \( T^3 \) during the history of the universe, the condition for an internal symmetry to be broken remains invariant as long as the charge is really conserved.

The most attractive way of implementing the above scenario is to have a large lepton number in the universe \([31, 32, 33]\). Notice that a large lepton number in the universe is experimentally allowed. From primordial nucleosynthesis constraints and galaxy formation one gets an approximate limit \( n_L \leq 70 T^3 \) for \( T \gg M_W \) \([34]\), which is an order of magnitude more than the critical density found in \([33]\). Also, such a large lepton number is consistent with a small baryon number (\( \approx 10^{-9} T^3 \)) since sphalerons are not operative when \( SU(2)_L \) is spontaneously broken (similarly to the usual \( T = 0 \) tunneling, it is exponentially suppressed). So the picture is consistent, but unfortunately one Higgs only in the SM is not enough to break completely the whole gauge group, leaving the system with electromagnetic gauge invariance at any temperature. Thus the SM itself does not possess the solution to the monopole problem. This difficulty can be however easily circumvented in the MSSM, where several boson fields can get nonzero vevs breaking partially or completely the gauge group and thus solve the monopole problem \([35]\).
In some type of GUT supersymmetric models the role of the lepton symmetry can be played by an R-symmetry \[36\]. A large R-charge would be eventually washed out at a later stage of the evolution of the universe due to supersymmetry breaking, so that contrary to the case with the lepton charge no signal of any R-charge would remain today. Nevertheless such a scenario would postpone the creation of monopoles to a nondangerous era.

**Flat directions at high temperature**

Recently a new idea for symmetry nonrestoration in SUSY models was proposed \[37, 38\]. At \(T = 0\) it is very common in supersymmetric theories to have flat directions. The point is that by definition they can be very softly coupled to the rest of the world (for example with higher dimensional nonrenormalizable terms), so that it is not difficult to find them out of thermal equilibrium at high temperature. This is exactly the case which was not considered by the no-go theorem: in \[4, 5\] all the fields were assumed to be in thermal equilibrium, i.e. to develop a positive mass term \(+T^2|\phi|^2\) in the potential, see (2). To get the idea, let me consider the toy model from \[38\] with the superpotential and Kähler potential given by

\[
W = \frac{\lambda}{3} q^3 + \frac{\phi^{n+3}}{(n+3)M^n}, \quad n \geq 1, \quad (6)
\]

\[
K = q^\dagger q + \phi^\dagger \phi + a \frac{(q^\dagger q)(\phi^\dagger \phi)}{M^2}. \quad (7)
\]

Obviously, for \(M \to \infty\) and \(\lambda \approx O(1)\), the field \(q\) is in thermal equilibrium (with itself through its self coupling), while \(\phi\) is a flat direction and not in thermal equilibrium with \(q\) (actually it is a free field). For a finite \(M\) (let me take \(M = M_{Pl}\) for simplicity) \(\phi\) is no more an exact flat direction, but let us assume that it is still out of thermal equilibrium, due to the smallness of its interaction. As we said before, \(q\) gets a positive mass term at high \(T\), which drives its vev to zero, \(<q> = 0\). Therefore in the evaluation of the effective potential the field \(q\) is never present as an external leg, but it runs in the loops, so we will denote it as \(\hat{q}\). The opposite happens to the out of equilibrium field \(\phi\): it does not run in thermal loops, but can have a nonzero
vev, so it can have external legs in a Feynman diagram. We will denote this background field as before with $\phi$.

The bosonic part of the Lagrangian is

$$\mathcal{L}_B = \left(1 + a\frac{|\phi|^2}{M^2}\right)|\hat{q}|^2 - \frac{\lambda^2 |\hat{q}|^4}{1 + a|\phi|^2/M^2} - \frac{|\phi|^{2(n+2)}/M^{2n}}{1 + a|\hat{q}|^2/M^2}.$$  \hspace{1cm} (8)

The kinetic term in (8) is not in the canonic form, so we have to rescale $\hat{q} \rightarrow \hat{q}/(1 + a|\phi|^2/M^2)^{1/2}$. After expanding the Lagrangian in inverse powers of $M$, one gets the most important interaction part

$$\mathcal{L}_{\text{int}} = 3a\lambda^2 \frac{|\hat{q}|^4|\phi|^2}{M^2} + ...,$$ \hspace{1cm} (9)

which generates nothing else than the “butterfly” diagram of [23]. However, its sign depends on the parameter $a$. After including also the fermion loops one gets

$$V_{\text{eff}} = -\frac{3a\lambda^2}{32} \frac{T^4}{M^2} |\phi|^2 + \frac{|\phi|^{2(n+2)}}{M^{2n}}.$$ \hspace{1cm} (10)

Clearly, for $a > 0$ the effective potential (10) has a minimum at

$$<\phi>^{n+1} = \left(\frac{3a\lambda^2}{32(n + 2)}\right)^{1/2} T^2 M^{n-1},$$ \hspace{1cm} (11)

which gives $<\phi> \approx T$ for $n = 1$ and even $<\phi> \gg T$ for $n > 1$.

It is important to stress again that such $a <\phi> \neq 0$ at high $T$ means symmetry breaking at high $T$, which could be a possible solution to the above mentioned cosmological problems of monopoles, domain walls and wrong vacuum. The essential point is that the temperature provides the necessary breaking of supersymmetry needed to give large vevs to the would have been flat directions.

Let me give few comments. First, the constraint $a > 0$ can be generalized in different models to a constraint for the Kähler potential. It is enough that $\partial^2 K/\partial q\partial\bar{q}$ grows with $\phi$. Such a constraint is very natural and not at all difficult to achieve. Second, the last term in the superpotential (7) is needed, since without it the vev of $\phi$ would tend to infinity. Finally, a more realistic case with $\phi$ a charged field under some gauge group was considered in [37] with similar conclusions.
Summary

I showed that generic cosmological problems are present in SUSY GUTs: the monopole problem and the wrong vacuum problem, while sometimes also the domain wall problem can be present. An appealing solution to all these problems is given by the symmetry nonrestoration at high temperature. I described two such possibilities: the presence of a large lepton (or some other charge) density or the use of out of thermal equilibrium flat directions.

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References

[1] Weinberg S., Phys. Rev. D9, 3357 (1974).

[2] Mohapatra R.N., and Senjanović G., Phys. Rev. Lett. 42, 1651 (1979); Phys. Rev. D20, 3390 (1979).

[3] Senjanović G., COSMO 97, ed. L. Roszkowski, Singapore: World Scientific, 1998, pp. 437-445, hep-ph/9805361.

[4] Bajc B., COSMO 97, ed. L. Roszkowski, Singapore: World Scientific, 1998, pp. 467-469, hep-ph/9805352.

[5] Haber H., Phys. Rev. D 46, 1317 (1982).

[6] Mangano M., Phys. Lett. 147B, 307 (1984).

[7] Kibble T., J. Phys. A9, 1387 (1976).

[8] Preskill J., Phys. Rev. Lett. 43, 1365 (1979).

[9] Zeldovich Ya.B., Kobzarev I.Yu., Okun L.B., JETP 40, 1 (1974).
[10] Dvali G., Melfo A., Senjanović G., *Phys. Rev. Lett.* **75**, 4559 (1995), hep-ph/9507230; Salomonson P., Skagertan B.S., Stern A., *Phys. Lett.* **B151**, 243 (1985).

[11] Dvali G., Senjanović G., *Phys. Rev. Lett.* **74**, 5178 (1995), hep-ph/9501387.

[12] Bimonte G., Iñiguez D., Tarancón A., Ullod C.L., *Phys. Rev. Lett.* **81**, 750-753 (1998), hep-lat/9802022.

[13] Roos T.G., *Phys. Rev.* **D54**, 2944 (1996); Amelino-Camelia G., *Phys. Lett.* **B388**, 776 (1996); Pietroni M., Rius N., Tetradis N., *Phys. Lett.* **B397**, 119 (1997); Orloff J., *Phys. Lett.* **B403**, 309 (1997), hep-ph/9611398.

[14] Jansen K., Laine M., *Phys. Lett.* **B435**, 166 (1998), hep-lat/9805024.

[15] Bimonte G., Lozano L., *Nucl. Phys.* **B460**, 155 (1996), hep-th/9509060.

[16] Gavela M.B., Pène O., Rius N., Vargas-Castrillon S., *Phys. Rev.* **D59**, 025008 (1999), hep-ph/9801244.

[17] Langancker P., Pi S.-Y., *Phys. Rev. Lett.* **45**, 1 (1980).

[18] Dvali G., Liu H., Vachaspati T., *Phys. Rev. Lett.* **80**, 2281 (1998); hep-ph/9710301.

[19] Brustein R., Maor I., preprint BGU-PH-98/14, hep-ph/9901290.

[20] Turner M., *Phys. Lett.* **115B**, 95 (1982).

[21] Dimopoulos S., Raby S., Wilczek F., *Phys. Rev.* **D24**, 1681 (1981); Ibáñez L.E., Ross G.G., *Phys. Lett.* **105B**, 439 (1981); Einhorn M.B., Jones D.R.T., *Nucl. Phys.* **B196**, 475 (1982); Marciano W.J., Senjanović G., *Phys. Rev.* **D25**, 3092 (1982).

[22] Weinberg S., *Phys. Rev. Lett.* **48**, 1776 (1982).

[23] Bajc B., Melfo A., Senjanović G., *Phys. Lett.* **B387**, 796 (1996), hep-ph/9607214.
[24] Bajc B., Senjanović G., *Nucl. Phys. Proc. Suppl.* **52A**, 246-250 (1997), hep-ph/9610352.

[25] Dvali D., Tamvakis K., *Phys. Lett.* **B378**, 141 (1996), hep-ph/9602336.

[26] Riotto A., Senjanović G., *Phys. Rev. Lett.* **79**, 349 (1997), hep-ph/9702319.

[27] Kirzhnitz D.A., *JETP Lett.* **15**, 529 (1972); Kirzhnitz D.A., Linde A.D., *Phys. Lett.* **B42**, 74 (1972).

[28] Dolan L., Jackiw R., *Phys. Rev.* **D9**, 3320 (1974).

[29] Haber H.E., Weldon H.A., *Phys. Rev.* **D25**, 502 (1982).

[30] Benson K.M., Bernstein J., Dodelson S., *Phys. Rev.* **D44**, 2480 (1991).

[31] Linde A.D., *Phys. Rev.* **D14**, 3345 (1976).

[32] Liu J., Segre G., *Phys. Lett.* **B338**, 259 (1994).

[33] Bajc B., Riotto A., Senjanović G., *Phys. Rev. Lett.* **81**, 1355-1358 (1998), hep-ph/9710413.

[34] Kang H.-S., Steigman G., *Nucl. Phys.* **B372**, 494 (1992), and references therein.

[35] Bajc B., Senjanović G., in preparation.

[36] Bajc B., Riotto A., Senjanović G., *Mod. Phys. Lett.* **A13**, 2955-2964 (1998); hep-ph/9803438.

[37] Dvali G., Krauss L., preprint CWRU-P31-98, hep-ph/9811298.

[38] Bajc B., Senjanović G., preprint IJS-TP-98-24, hep-ph/9811321.