3D magnetization currents, magnetization loop, and saturation field in superconducting rectangular prisms

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Abstract

Bulk superconductors are used in many applications and material characterization experiments, with the bulk shape of the rectangular prism being the most frequent. However, the magnetization currents are still mostly unknown for this kind of 3D shape, especially below the saturation magnetic field. Knowledge of the magnetization currents in this kind of sample is needed to interpret the measurements and the development of bulk materials for applications. This article presents a systematic analysis of the magnetization currents in square-based prisms of several thicknesses. We make this study by numerical modeling using a variational principle that enables a high number of degrees of freedom. We also compute the magnetization loops and saturation magnetic field, using a definition that is more relevant for thin prisms than previous ones. The article presents a practical analytical fit for any aspect ratio. For applied fields below the saturation field, the current paths are not rectangular, presenting 3D bending. The thickness-average results are consistent with previous modeling and measurements for thin films. The 3D bending of the current lines indicates that there could be flux-cutting effects in rectangular prisms. The component of the critical current density in the applied field direction may play a role, with the magnetization currents being different in a bulk and a stack of tapes.

Keywords: superconducting bulk, electromagnetic modeling, variational principle, magnetization loop, saturation magnetic field

(Some figures may appear in colour only in the online journal)

1. Introduction

Bulk superconductors are used in many applications, such as compact magnets for magnetic separation, superconducting motors and generators, and levitation systems [1–4]. In addition, bulk samples are commonly used to study the material properties of superconductors [5, 6]. Rectangular-shaped prisms are frequently chosen for both applications and material characterization.

REBCO\(^1\) superconducting bulks have been shown to trap large magnetic fields [7, 8], reaching 17.6 T [8]. Magnesium diboride (MgB\(_2\)) bulks are also promising, although the maximum trapped field is only 5.4 T [9]. The advantages of MgB\(_2\) are homogeneity, isotropy and, more important, higher power-law exponent in the relation between the electric field, \(E\), and the current density, \(J\). This is crucial for nuclear magnetic resonance magnets, where the generated magnetic field should be highly stable.

Superconducting bulks can be magnetized in several ways. The most common is field-cool magnetization, consisting of placing a bulk in a DC magnet where the sample is above the critical temperature, \(T_c\), to decrease the temperature below \(T_c\), and reduce the applied magnetic field from a certain initial value to zero. The maximum trapped field in the superconductor is reached when the initial applied field is above the saturation field [10]. Alternatively, the sample could be cooled down at zero applied field, increase the
applied field, and come back to zero. In this case, achieving the maximum trapped field requires a peak applied field that is twice the bulk-saturation field. These DC magnetization methods are often used for material characterization or testing the maximum possible trapped field. The present article provides insight into this kind of magnetization. In certain applications, such as motors, it could be more convenient to use pulse magnetization [10]. This method usually implies a combination of electromagnetic and thermal processes, which is not the goal of this paper.

Although there are several works on 3D modeling of superconducting bulks [10, 11], the magnetization currents in rectangular prisms of finite thickness remains mostly unknown. Infinite rectangular prisms in the critical-state model (CSM) were analytically solved in [12]. Thin rectangular films have been studied in [13, 14] and [15, 16] for an isotropic power-law $E(J)$ relation and the CSM, respectively. Computations for a rectangular prism with a hole have been published in [17] for a power-law $E(J)$ relation. Reference [18] presented approximated solutions for a cube in the CSM, assuming square current paths. The trapped field of an array of rectangular prisms is computed in [19]. 3D modeling has been applied to finite cylinders with holes [20]; under transverse applied field, both as a stand-alone superconductor [21] and interacting with ferromagnetic disks and rings [22]; and under axial applied field but non-homogeneous critical current density [23, 24]. Another 3D studied case is that of two rectangular superconducting filaments coupled by a normal metal in between [25], although a single prism was not studied in that work.

Therefore, a systematic study of the magnetization process in finite rectangular prisms is needed. This article presents a detailed analysis of the 3D current flow, as well as the magnetization loops and saturation magnetic field. We focus on square-based rectangular prisms of several thicknesses. The analysis is done by numerical modeling using the minimum electromagnetic entropy production method in 3D (MEM-EP3D), since it has been shown to be able to manage the high number of degrees of freedom necessary for this study [26].

Section 2 outlines the main features of the numerical model and the assumed material properties. We discuss the results in section 3, emphasizing the qualitative shape of the current paths below saturation (section 3.1). Section 3.2 presents the magnetization loops and the saturation magnetic field calculated from them. There, we introduce a practical analytical fit of the saturation field that is useful for any aspect ratio, including thin prisms. We present our conclusions in section 4.

2. Model

2.1. Material properties and physical situation

In this work, we consider an isotropic power law as

$$E(J) = E_c \left( \frac{|J|}{J_c} \right)^n \frac{J}{|J|},$$

where $E_c$ is an arbitrary constant, usually $10^{-4}$ V m$^{-1}$, $J_c$ is the critical current density, and $n$ is the power-law exponent. The limit of $n \to \infty$ corresponds to the isotropic CSM, which assumes a multi-valued $E(J)$ relation.

In this article, we assume uniform applied fields $\mathbf{B}_a$; although the presented variational principle is also valid for non-uniform applied fields. We consider that the applied field follows the $z$ direction and is generated by a long racetrack coil in the $y$ direction and high in the $z$ direction. The resulting

$$\mathbf{B}_a = B_y \mathbf{e}_y$$

is the void permeability. In the text, we use ‘magnetic field’ to refer to both the magnetic field and magnetic flux density.

Figure 1. $J_y$ at the midplane ($y = 0$) for square-based rectangular prisms and thickness-over-width ratio $c = d/w = 1, 0.5, 0.2, 0.1$ in (a), (b), (c), (d), respectively. For all cases, the applied field is $B_y = 0.484 B_c$, being $B_c$ the saturation field in table 1.

We do not take magnetic materials into account, and hence the magnetic field and magnetic flux density are proportional $\mathbf{H} = \mathbf{B}/\mu_0$, being $\mu_0$ the void permeability.
applied vector potential $A_a$ in Coulomb’s gauge, defined as appendix B in [11], is

$$A_a(r) \approx B_a x e_y,$$

where $B_a$ is such that $B_a = B_a e_x$ and $e_y, e_z$ are the unit vectors in the y and z directions, respectively.

### 2.2. Numerical method

We use the MEMEP3D variational method, detailed in [26]. This method is valid for any combination of applied field and transport current and any vector $(E, J)$ relation, including the case of magnetic-field-dependent parameters in the $(E, J)$ relation like $J_c$ and $n$.

Outlining, for no transport current, the current density is taken as a function of the effective magnetization $T$ as

$$J = \nabla \times T.$$

When increasing the time from a certain $t_0$ to $t_0 + \Delta t$, the applied vector potential changes by $\Delta A_a$, which causes a change in $T$ by $\Delta T$. Then, $J = J_0 + \nabla \times \Delta T$, where $J_0$ is $J$ at time $t_0$. This $\Delta T$ is obtained by numerically minimizing the functional

$$L[\Delta T] = \int_V \int_{V'} \frac{\mu_0}{8\pi} \frac{(\nabla \times \Delta T) \cdot (\nabla' \times \Delta T')}{|r - r'|} dV + \int_V dV \left( \frac{\Delta A_a}{\Delta t} \cdot \nabla \times T + U(J_0 + \nabla \times \Delta T) \right).$$

where the dissipation factor $U(J)$ is defined as

$$U(J) = \int_0^J dJ' \cdot E(J').$$

For the power-law $E(J)$ relation of (1), the dissipation factor becomes

$$U(J) = \frac{E_c J_c}{n + 1} \left( \frac{J_c}{J} \right)^{n+1}.$$

As demonstrated in [26], this functional always presents a minimum and that minimum is unique. Minimizing this functional is equivalent to solving Faraday’s law

$$\nabla \times E = -\frac{\partial B}{\partial t}.$$  

We minimize the functional of (4) by the iterative parallel algorithm of [26] in a self-programmed implementation. In a previous work, we tested the numerical method with the analytical limits of thin strips and disks, showing very good agreement [26]. Mutual benchmarking with other numerical methods will be the focus of future work (benchmark 5 in [27]).

### 2.3. Magnetization

The average magnetization $M$ is defined as the total magnetization $m$ per unit sample volume $V$ as $M = m/V$. The magnetic moment is

$$m = \frac{1}{2} \int dV \ r \times J.$$
Figure 3. Current flux lines in a thin prism \((d/w = 0.1)\) do not follow square loops [plots for \(z/d = 0, -0.09, -0.18, -0.27, -0.36, -0.45\) in (a), (b), (c), (d), (e), (f), respectively]. The graphs show the projection of the 3D current lines in the \(xy\) plane and the magnitude of the current density (colormap), since the 3D flux lines show the direction of the current density, but not its magnitude. Current lines start at \(y = 0\) and the \(z\) coordinate of the plotted map, but bend in the \(z\) direction, belonging to other \(z\) layers close to the diagonals (see 3D lines in figure 5). Computed case for power-law exponent 100 and \(B_y = 0.484B_0\).

Figure 4. In a thin prism \((d/w = 0.1)\), there appear regions with \(J_z\) at all heights except at the midplane \(z = 0\) [plots for \(z/d = 0, -0.09, -0.18, -0.27, -0.36, -0.45\) in (a), (b), (c), (d), (e), (f), respectively], causing current lines with 3D bending (see figure 5). The computed situation is the same as in figure 3.
penetration front close to the central $z$ decreases with the prism height.

The current density in figure 1 is qualitatively similar to cylinders [31, 32]. Two main differences are the presence of a non-zero $J_x$ component (see figure 2) and current paths that do not always follow the shape of the sample boundary, and hence they are non-square (figures 3, 5 and 7(c)).

The $J_z$ component reaches values as high as 30% of $J_c$ (figure 2). The highest magnitude of $J_z$ is close to the diagonal of the prisms. This $J_z$ bends the current flux lines, as can be seen in the 3D current loops in figure 5. The cause of this $J_z$ component is the self-field. In cylinders, the radial component of the self-field, perpendicular to the current loops, is balanced by higher current penetration close to the ends [32]. That is possible thanks to the cylindrical symmetry, which causes the radial field to be uniform in any circular loop. This no longer applies to rectangular prisms. The magnetic field created by rectangular loops at their diagonal is higher than closer to their straight parts at the same distance from the lateral faces [13]. Thus, higher current penetration close to the ends following rectangular loops cannot fully cancel the self-field. Close to the diagonals, the additional perpendicular self-field pointing inwards is canceled by a $J_z$ component that changes its sign at the diagonal. For applied fields well above the saturation field, the self-field is not relevant, and hence the current paths follow rectangular loops in the whole sample.

Let us analyze in detail the thinnest sample, with $d/w = 0.1$. Figures 3 and 4 show the maps of $|J|$ and $J_z$ for all computed heights in the lower half of the prism. Figure 3 also shows the projections in the $xy$ plane of the 3D current flux lines. An interesting issue is that the current loops do not close within the same plane (figure 5). The roughly straight segments close to one side (point 1 at figures 3(c) and 4(c)) bend downwards when approaching the diagonal (point 2 in figures 3(c) and 4(c)), where $J_z < 0$, reaches a lower layer, where $|J|$ is slightly below $J_c$, and bends in the $xy$ plane.

### 3. Results and discussion

This section presents the current density, saturation field, and magnetization loops for rectangular prisms with several height-to-width aspect ratios $c \equiv d/w$ and square base, being $w$ and $d$ the sample width and thickness. For all cases, we consider a power-law exponent of 100, unless stated otherwise.

#### 3.1. Current density

We compare the current density in prisms of several aspect ratios and the same applied field relative to the saturation field $B_s/B_c = 0.484$. We define the $z$ axis as that of the applied field (see sketch in figure 7). The saturation field, $B_s$, is presented in section 3.2.

The $J_z$ component of the current density at the central plane of constant $y$ for several aspect ratios is as in figure 1. The highest penetration depth is at the top and bottom, being the smallest at the middle. The roughly flat part of the

**Figure 6.** Thickness–average current density for the sample with aspect ratio 0.1 (same situation as figure 3) is consistent with both previous calculations for thin films [14, 15] and current density extracted from magnetic-field imaging [28–30]. The lines are current lines for the thickness-averaged $J$.
Figure 7. Current density magnitude (a), (b), (c) and $J_z$ (d) at several cross-sections of a cube, $d/w = 1$. The lines are 3D current flux lines projected on the plotted plane that start at $y = 0$ in (a), (b), (c) and $x = 0.5w$ in (d), representing the direction of the current density, but not its magnitude. Computed case for power-law exponent 100 and applied field $B_a = 0.484B_s$, being $B_s$ the saturation field given in table 1. The sketch shows the taken geometry and axis.

Non-square current paths in thin films with regions with $|J| < J_c$ were found in [14, 15] from numerical modeling and in [28–30] from the inversion of measured magnetic-field maps. Although thin-film models assume uniform $\mathbf{J}$ along the film thickness, their predictions are valid for the thickness-averaged $\mathbf{J}$ of samples with small but finite thickness. Our calculations for the thinnest sample ($d/w = 0.1$) qualitatively agree with the thin-film calculations and measurements (figure 6). The thickness-averaged current paths from thin films cannot be obtained by superposing square paths at several heights, because they always result in square current paths. Therefore, non-square current paths need to exist in prisms, at least below their saturation field. In addition, the arguments above justifying the non-zero $J_c$ are valid for any aspect ratio. This contrasts with earlier predictions in [18] for a cube, where in-plane square loops were assumed. This assumption was supported by taking into account that $|J|$ follows $|J| = J_c$ or 0 only, while the CSM allows any $|J| \leq J_c$. Current densities with magnitude slightly below $J_c$ are enough to bend $\mathbf{J}$ vertically and obtain the necessary $J_z$ to shield the self-field. The assumption of rectangular current loops should still produce fairly accurate results of the magnetic moment, although the obtained magnetic field at the surface will present significant errors.

For a cube, the current flux lines are practically square at the center (figure 7(a)). Close to the ends, they change progressively from square next to the side surfaces to circular at the center (figure 7(c)). At the intermediate height $z/d = -0.39$, there is a slight bending of the current lines (figure 7(b)). The non-zero $J_c$ bends the current lines vertically, as can be seen in figure 7(d) for a plane close to the side surface. After increasing the applied field to the effective saturation field, defined in section 3.2, the magnitude of $J_c$ reduces and the current flux lines become closer to squares (figure 8). Well above the effective saturation field, the current flux lines are perfectly square (figure 10).

For a cube with lower power-law exponent, $n = 30$, the current density is approximately the same as for $n = 100$. Then, the finite power-law exponent is not the cause of the $J_c$ component, since samples with substantially different power-law exponent present $J_c$ of similar magnitude (see figures 2(a) and 11(e)). In contrast, the modulus of $\mathbf{J}$ for $n = 30$ is larger than that for $n = 100$, overcoming substantially $J_c$ [figures 7(a), (b), (c), 11(a), (b), (c) and figures 1(a), 12]. The cause is that, for the same electric field caused by the applied field variation, lower $n$ exponents result in higher current...
densities. For the CSM, the maximum $|J|$ will be $J_c$ and $|J_z|$ will be slightly lower, especially away from the diagonals.

### 3.2. Magnetization loops and saturation field

We calculated the magnetization loops for several aspect ratios $c \equiv d/w$, assuming constant $J_c$. Figure 13 shows the hysteresis loops for $c = 1, 0.5, 0.2, 0.1$ and 1 T of peak applied field. From these loops, we obtained the saturation field, $B_s$, in table 1; which we define as the applied magnetic field that causes $M = 0.99M_s$, $M_s$ being the saturation magnetization. The value of $|M_s|$ is taken as the maximum $|M|$ at the initial curve. This definition of $B_s$ is more relevant than previous ones, being the applied field that fully saturates the sample with critical current [$33$] or the self-field at the sample center at full current penetration [$34$]. The cause is that $M$ for thin samples practically saturates at applied fields much lower than $B_s$ from the previous definitions [$35$]. Indeed, a perfectly thin film requires an infinite applied field to be fully saturated with current [$36$–$38$]. Alternatively, [$39$] proposes the magnetic-field amplitude at the maximum of the imaginary AC susceptibility, $H_{m,pk}$, as the penetration field for cylinders (see note at the end 16 in [$39$]). The reason is that infinite cylinders saturate at this applied field and thin cylinders present magnetization $M = 0.9903M_s \sim M_c$. However, the situation for strips and slabs is a bit different. Infinite slabs already saturate at 3/4 of $H_{m,pk}$, while strips at applied field $H_{m,pk}$ present $M = 0.9856M_s \sim M_c$. Taking into account that the behavior of rectangular prisms will present similarities to both cylinders, strips and slabs, and in order to use a simple criterion for all shapes, we choose $M = 0.99M_s$ as the penetration field.

Figure 8. The same as figure 7 for (a), (b), (c), (d), and figure 2(a) for (e) but for an applied field $B_a = B_s$; defined as $M(B_a) = 0.99M_s$, $M_s$ being the saturation magnetization.

Figure 9. The same as figure 1(a) but for for $B_a = B_s$.
Figure 10. The same as figure 7 for (a), (b), (c), (d), and figure 2(a) for (e) but for an applied field $B_a = 2.0B_c$. At high applied fields, the current lines are square and $J_z$ vanishes, as in CSM predictions for long bars.

Figure 11. The same situation as figure 7 for (a), (b), (c), (d,) and figure 2(a) for (e) but with power-law exponent 30.
moment. The finite power-law exponent of 100 slightly enhances this effect, because $|J|$ is slightly above $J_c$ at certain regions.

The saturation field increases with the aspect ratio (figure 14) because, for the same width, thinner prisms generate lower maximum self-fields. Figure 14 also shows the infinite-bar limit, calculated from the analytical magnetization curve of the CSM [12], and the thin square situation, computed by our model. The saturation field of the latter is proportional to the film thickness. The results for finite thickness approach the infinite bar and thin limits for high and low $c = d/w$ aspect ratios, respectively. The following analytical fit agrees exactly with both infinite and thin limits and differs from the intermediate cases by less than 3%.

\[ B_s(c) = \mu_0 I_w a_1 \left[ 1 + a_2 \exp \left( \frac{\ln^2(a_3 c)}{2a_4^2} \right) \right] \tanh(a_5 c) \]  

with $a_1 = 0.492$, $a_2 = -0.26$, $a_3 = 2.56$, $a_4 = 0.75$, and $a_5 = 1.92$.

The calculated saturation field with a finite power-law exponent is expected to slightly differ from that from the ideal CSM. Tests for 2D cross-sectional calculations for cylinders with the method in [40] show that the results with power-law exponent $n = 100$ are over-estimated by only a few percent compared to the CSM.

4. Conclusions

This article has presented a systematic study of the magnetization currents in rectangular prisms of several thickness-to-width aspect ratios. This study has been done by 3D electromagnetic modeling by means of the variational method.
MEMEP3D [26], which enables modeling the high number of degrees of freedom required for the computations. For applied magnetic fields below the saturation field, we have found that the current lines do not follow rectangular paths. These current paths are possible thanks to bending in the direction of the applied magnetic field. Although the assumption of rectangular current paths may be fair to predict the magnetization loops, it will introduce significant errors in the prediction of generated magnetic field at the surface. Calculations for a relatively thin sample show that the average of the current density over the thickness is compatible with previous solutions for thin films and those extracted from magnetic-field imaging experiments.

This article has also presented the magnetization loops and saturation magnetic field, defined by the applied field that causes a magnetization of 99% of the saturation magnetization. This definition is more practical than previous ones, especially for thin prisms, as previous definitions substantially over-estimated the saturation field. The numerical computations of this article have provided insight into current penetration. The 3D bending of the current lines indicates that there could be force-free and flux-cutting effects [41, 42] in superconducting rectangular prisms. The component of the critical current density in the applied field direction may play a role, with the magnetization currents being different in a bulk and a stack of tapes. The cube represents a simply truly 3D shape to benchmark numerical methods between each other (benchmark 5 in [27]).

The possibility to solve 3D situations with a relatively high number of degrees of freedom enables us to systematically analyze other situations, such as cross-field demagnetization or coupling effects in multi-ribbons, flywheels and transportation Supercond. Sci. Technol. 25 014007

[5] Hecher J, Baumgartner T, Weiss J D, Tarantini C, Yamamoto A, Jiang J, Hellstrom E E, Larbalestier D C and Eisterer M 2015 Small grains: a key to high-field applications of granular Ba-122 superconductors? Supercond. Sci. Technol. 29 025004

[6] Mishev V, Zehetmayer M, Fischer D X, Nakajima M, Eisaki H and Eisterer M 2015 Interaction of vortices in anisotropic superconductors with isotropic defects Supercond. Sci. Technol. 28 102001

[7] Tomita M and Murakami M 2003 High-temperature superconductor bulk magnets that can trap magnetic fields of over 17 tesla at 29 K Nature 421 517–20

[8] Durrell J H et al 2014 A trapped field of 17.6 T in melt-processed, bulk Gd-Ba-Cu-O reinforced with shrink-fit steel Supercond. Sci. Technol. 27 082001

[9] Fuchs G, Häßler W, Wenk K, Scheiter J, Perner O, Handstein A, Kanai T, Schultz L and Holzapfel B 2013 High trapped fields in bulk MgB2 prepared by hot-pressing of ball-milled precursor powder Supercond. Sci. Technol. 26 122002

[10] Ainslie M D and Fujishiro H 2015 Modelling of bulk superconductor magnetization Supercond. Sci. Technol. 28 053002

[11] Grilli F, Pardo E, Stenvall A, Nguyen D N, Yuan W and Gömöry F 2014 Computation of losses in HTS under the action of varying magnetic fields and currents IEEE Trans. Appl. Supercond. 24 8204033

[12] Chen D-X and Goldfarb R B 1989 Kim model for magnetization of type-II superconductors J. Appl. Phys. 66 2489–500

[13] Brandt E H 1995 Square and rectangular thin superconductors in a transverse magnetic field Phys. Rev. Lett. 74 3025–8

[14] Brandt E H 1995 Electric field in superconductors with rectangular cross section Phys. Rev. B 52 15442

[15] Prigozhin L 1998 Solution of thin film magnetization problems in type-II superconductivity J. Comput. Phys. 144 180–93

[16] Navas C, Sanchez A, Del-Valle N and Chen D X 2008 Alternating current susceptibility calculations for thin-film superconductors with regions of different critical-current densities J. Appl. Phys. 103 113907

[17] Pecher R, McCulloch M D, Chapman S J, Prigozhin L and Elliott C M 2003 3D-modelling of bulk type-II superconductors using unconstrained H-formulation European Conf. on Applied Superconductivity (EUCAS) (Sorrento, Italy, September 2003) 1418

[18] Badía-Majós A and López C 2005 Critical state model in superconducting parallelepips Appl. Phys. Lett. 86 202510

[19] Zhang M and Coombs T A 2012 3D modelling of high-Tc superconductors by finite element software Supercond. Sci. Technol. 25 015009

[20] Lousberg G P, Ausloos M, Geuzaine C, Dular P, Vanderemboden P and Vanderheyden B 2009 Numerical simulation of the magnetization of high-temperature superconductors: a 3D finite element method using a single time-step iteration Supercond. Sci. Technol. 22 055005

[21] Campbell A M 2014 Solving the critical state using flux line properties Supercond. Sci. Technol. 27 124006

[22] Fagnard J-F, Morita M, Nariki M, Teshima H, Caps H, Vanderheyden B and Vanderemboden P 2016 Magnetic moment and local magnetic induction of superconducting/ferromagnetic structures subjected to crossed fields: experiments on GdBCO and modelling Supercond. Sci. Technol. 29 125004

[23] Komí Y, Sekino M and Ohsaki H 2009 Three-dimensional numerical analysis of magnetic and thermal fields during pulsed field magnetization of bulk superconductors with inhomogeneous superconducting properties Physica C 469 1262–5

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References

[1] Hull J R and Murakami M 2004 Applications of bulk high-temperature superconductors Proc. IEEE 92 1705–18

[2] Murakami M 2007 Processing and applications of bulk RE-Ba-Cu-O superconductors Int. J. Appl. Ceram. Technol. 4 225–41

[3] Zhou D, Izumi M, Miki M, Felder B, Ida T and Kitano M 2012 An overview of rotating machine systems with high-temperature bulk superconductors Supercond. Sci. Technol. 25 103001

[4] Werfel F N, Floegel-Delor U, Rothfeld R, Riedel T, Goebel B, Wippich D and Schirmeister P 2012 Superconductor
Ainslie M D, Fujishiro H, Ujiie T, Zou J, Dennis A R, Shi Y H and Cardwell D A 2014 Modelling and comparison of trapped fields in (RE)BCO bulk superconductors for activation using pulsed field magnetization Supercond. Sci. Technol. 27 065008

Grilli F, Stavrev S, Le Floch Y, Costa-Bouzo M, Vinot E, Klutsch I, Meunier G, Tixador P and Dutoit B 2005 Finite-element method modeling of superconductors: from 2-D to 3-D IEEE Trans. Appl. Supercond. 15 17–25

Pardo E and Kapolka M 3D computation of non-linear eddy currents: Variational method and superconducting cubic bulk arXiv:1611.04752 [cond-mat.supr-con]

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Romero-Salazar C, Jooss C and Hernandez-Flores O A 2010 Reconstruction of the electric field in type-II superconducting thin films in perpendicular geometry Phys. Rev. B 81 144506

Wells F S, Pan A V, Golovchanskiy I A, Fedoseev S A and Rozenfeld A 2017 Observation of transient overcritical currents in YBCO thin films using high-speed magneto-optical imaging and dynamic current mapping Sci. Rep. 7 40235

Brandt E H 1998 Superconductor disks and cylinders in an axial magnetic field. I. Flux penetration and magnetization curves Phys. Rev. B 58 6506

Sanchez A and Navau C 2001 Magnetic properties of finite superconducting cylinders. I. uniform applied field Phys. Rev. B 64 214506

Bean C P 1962 Magnetization of hard superconductors Phys. Rev. Lett. 8 250–3

Forkl A and Kronmüller H 1994 A contribution to the analysis of the current-density distribution in elongated hard type-II superconductors with rectangular cross-section Physica C 228 1–14

Pardo E, Chen D-X, Sanchez A and Navau C 2004 The transverse critical-state susceptibility of rectangular bars Supercond. Sci. Technol. 17 537

Halse M R 1970 AC face field losses in a type II superconductor J. Phys. D: Appl. Phys. 3 717–20

Brandt E H and Indenbom M 1993 Type-II-superconductor strip with current in a perpendicular magnetic field Phys. Rev. B 48 12893–906

Zeldov E, Clem J R, McElfresh M and Darwin M 1994 Magnetization and transport currents in thin superconducting films Phys. Rev. B 49 9802–22

Palau A et al 2004 Simultaneous inductive determination of grain and intergrain critical current densities of YBa2Cu3O7−x coated conductors Appl. Phys. Lett. 84 230–2

Pardo E, Šouc J and Frolek L 2015 Electromagnetic modelling of superconductors with a smooth current–voltage relation: variational principle and coils from a few turns to large magnets Supercond. Sci. Technol. 28 044003

Clem J R, Weigand M, Durrell J H and Campbell A M 2011 Theory and experiment testing flux-line cutting physics Supercond. Sci. Technol. 24 062002

Badía-Majós A and López C 2015 Modelling current–voltage characteristics of practical superconductors Supercond. Sci. Technol. 28 024003