New Gauge Symmetry in Gravity and the Evanescent Role of Torsion

H. Kleinert

Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, D-14195 Berlin, Germany
ICRANeT, Piazzale della Republica 1, 10 -65122, Pescara, Italy

If the Einstein-Hilbert action $\mathcal{L}_{\text{EH}} \propto R$ is re-expressed in Riemann-Cartan spacetime using the gauge fields of translations, the vierbein field $h^\alpha\mu$, and the gauge field of local Lorentz transformations, the spin connection $A_{\mu}{}^\alpha$, there exists a new gauge symmetry which permits reshuffling the torsion, partially or totally, into the Cartan curvature term of the Einstein tensor, and back, via a new multivalued gauge transformation. Torsion can be chosen at will by an arbitrary gauge fixing functional. There exist many equivalent ways of specifying the theory, for instance Einstein’s traditional way where $\mathcal{L}_{\text{EH}}$ is expressed completely in terms of the metric $g_{\mu\nu} = h^\alpha\mu h_{\alpha\nu}$, and the torsion is zero, or Einstein’s teleparallel formulation, where $\mathcal{L}_{\text{EH}}$ is expressed in terms of the torsion tensor, or an infinity of intermediate ways. As far as the gravitational field in the far-zone of a celestial object is concerned, matter composed of spinning particles can be replaced by matter with only orbital angular momentum, without changing the long-distance forces, no matter which of the various new gauge representations is used.

PACS numbers: 98.80.Cq, 98.80. Hw, 04.20.Jb, 04.50+h

1. In theoretical physics it often happens that a mathematical structure has a simple extension for which a natural phenomenon is waiting to be discovered. The most prominent example is the existence of a negative square root of the relativistic mass shell relation $p_0 = \sqrt{p^2 + m^2}$ which led Dirac to postulate the existence of a positron, discovered in 1932 by Carl Anderson [1]. Sometimes, this rule does not seem to work initially, only to find out later that nature has chosen an unexpected way to make it work after all. Here the best example is the existence of a solution of the above energy-momentum relation for negative $m^2$, which was interpreted by some theoreticians as the signal for the existence of a particle faster than light. Such particles were never found. A simple physical realization appeared, however, with the discovery of the Ginzburg-Landau field theory of phase transitions and its quantum versions (now referred to as Higgs field theory). Since there are always interactions, a negative parameter $m^2$ destabilizes the field fluctuations. The fields move to a new ground state, around which they fluctuate with positive $m^2$. The situation is completely analogous to what happens in any building if $\omega^2$ of one of its eigenfrequencies turns negative. The building collapses until the debris settles in a ruin, and that has only positive $\omega^2$’s. The collapse of an interacting field system with negative $m^2$ is observed as a phase transition to a state with stable fluctuations and positive $m^2$.

2. For many years, theorists have been wondering why Einstein’s theory of gravity represents such a perfect geometrization of the gravitational forces [2]. Since the work of Cartan in 1922 it is known that the Riemannian spacetime, in which the celestial objects move, has a “natural” extension to Riemann-Cartan spacetime. This possesses a further geometric property called torsion. Why is there no trace of it in the movements of planets? Einstein himself has asked this question and discussed it in letters with Cartan [3]. He set up a theory of teleparallelism which explains gravity by a theory in Riemann-Cartan spacetime, in which the total curvature tensor vanishes identically. The Einstein-Hilbert action is then equal to a combination of scalars formed from torsion tensors [4], and torsion forces provide us merely with an alternative way of describing gravitational forces, as emphasized in Refs. [5, 6].

3. Yet another extension of Einstein’s theory to Riemann-Cartan spacetime was advanced since 1959 [7–9]. It has the appealing feature that it can be rewritten as a gauge theory invariant under local Poincaré transformations, i.e., both local translations and local Lorentz transformations, thus bringing it to a similar form as the gauge theories of weak, electromagnetic, and strong interactions. This gauge theory treats torsion as an independent field which couples only to the intrinsic spin of the elementary particles in a celestial body. Unfortunately, however, such an approach has several unsatisfactory features. First, the theory is meant to be classical, but the spin carries a power of $\hbar$ which vanishes in the classical limit. So there is really no classical source of torsion. Indeed, if torsion couples to spin with the gravitational coupling strength, the factor $\hbar$ implies that it cannot play any sizable role in the forces between celestial bodies. For example, even if the earth consisted only of polarized atoms, its intrinsic spin would be $10^{-15}$ times smaller than the rotational spin around the axis.

Moreover, there exist severe conceptual problems. One was emphasized in Ref. [10]. As long as we do not know precisely the truly elementary particles, and it is doubtful that we ever will, many particles are described by effective fields, and it is impossible to specify whether the spin of those fields is caused by orbital motion or by the intrinsic spins of more elementary constituents. As
an example, the spin-one field of a $\rho$-meson contains a wave function of two spinless pions in a $p$-wave, which do not couple to torsion. But it also contains two spin-$\frac{1}{2}$ quarks in an $s$-wave which would couple. Another problem is that if torsion couples to all spins, the photon becomes massive. In order to avoid this, the authors advocating this approach postulate that the photon is an exception, and is not coupled to torsion. However, this contradicts the fact that roughly one percent of a photon is a virtual $\rho$-meson, which is strongly coupled to baryons. These, in turn, are supposed to be coupled to torsion (see Fig. 1), so the photon would become massive after all.

Thus the existence of an independent torsion field is highly dubious, and we may ask ourselves, whether the description of gravity in Riemann-Cartan spacetime proposed in Refs. [7–9] has really a chance of being true, or whether nature doesn’t have a deeper reason for avoiding the above problems. It is the purpose of this note to answer this question affirmatively. Inspiration comes from a simple model of gravity, a “world crystal” with defects [2, 11, 12], whose lattice constant is of the order of a Planck length. Some consequences of such a world crystal were pointed out in a recent study of black holes in such a scenario [13].

4. We begin by showing that in the absence of matter, a world crystal is a model for Einstein’s theory with a new type of extra gauge symmetry in which zero torsion is merely a special gauge. A completely equivalent gauge is the absence of Cartan curvature, which is found in Einstein’s teleparallel universe. Before presenting the argument, recall that a crystal can have two different types of topological line-like defects [8, 9], which in a four-dimensional world crystal are world surfaces (which may be the objects of string theory).

First, there are translational defects called dislocations (Fig. 2). These are produced by a cutting process due to Volterra: a single-atom layer is removed from the crystal, allowing the remaining atoms to relax to equilibrium under the elastic forces. A second type of topological defects is of the rotation type, the so-called disclinations (Fig. 3). They arise by removing an entire wedge from the crystal and re-gluing the free surfaces.

The defects imply a failure of derivatives to commute in front of the displacement field $u_i(x)$. In three dimensions, the dislocation density is given by the tensor

$$\alpha_{ij}(x) = \epsilon_{ikl} \nabla_k \nabla_i u_j(x).$$

If $\omega_i \equiv \frac{1}{2} \epsilon_{ijk} [\nabla_j u_k(x) - \nabla_k u_j(x)]$ denotes the local rotation field, the disclination density is defined by

$$\theta_{ij}(x) = \epsilon_{ijk} \nabla_k \omega_j(x).$$

The defect densities satisfy the conservation laws

$$\nabla_i \theta_{ij} = 0, \quad \nabla_i \alpha_{ij} = -\epsilon_{jkl} \theta_{kl}.$$  

These are fulfilled as Bianchi identities if we express $\theta_{ij}(x), \alpha_{ij}(x)$ in terms of plastic gauge fields $\beta_{kl}^p, \phi_{ij}^p$, setting $\theta_{ij} = \epsilon_{ikl} \nabla_k \phi_{ij}^p$, $\alpha_{ij} = \epsilon_{ikl} \nabla_k \beta_{ij}^p + \delta_{ij} \phi_{kl}^p - \phi_{kl}^p$. The defect densities are invariant under the gauge transformations $\beta_{kl}^p \rightarrow \beta_{kl}^p + \nabla_k u_l^p - \epsilon_{kl} \omega^p$, $\phi_{ij}^p \rightarrow \phi_{ij}^p + \partial_i \omega^p$, where $\omega_i^p = \frac{1}{2} \epsilon_{ijk} \nabla_j u_k^p$. Thus $h_{ij} \equiv \beta_{ij}^p + \epsilon_{ijk} \omega^p$ and $A_{ijkl} \equiv \phi_{ij}^p \epsilon_{kl}$ are translational and rotational defect gauge fields in the crystal [14].

The Volterra processes can be represented mathematically by multivalued transformations from an Euclidean crystal with coordinates $\tilde{x}^a$ to a crystal with defects and coordinates $x^\mu$, as illustrated in Figs. 4 and 5 for two-dimensional crystals.

For an edge dislocation the mapping is $\tilde{x}^1 = x^1, \tilde{x}^2 = x^2 + (b/2\pi) \phi(x)$, where $\phi(x) \equiv (1/2\pi) \arctan(x^2/x^1)$. Initially, this function has a cut from the origin towards left.
infinity. In a second step, the cut is removed and the multivalued version of the arctan is taken. This makes \( \theta(x) \) the Green function of the commutator \( [\partial_i, \partial_j] \): 

\[
(\partial_i \partial_j - \partial_j \partial_i) \phi(x) = \delta^{(2)}(x).
\]

For a wedge disclination, the mapping is 

\[
d\hat{\xi}^i = \delta^i_\mu [\hat{x}^\mu + (\Omega/2\pi)\xi^i \hat{x}^\nu \phi(x)].
\]

A combination of the two 

\[
\eta_{ij}(x) \equiv \theta_{ij}(x) - \frac{1}{2} \nabla_m \left( \epsilon_{min} \alpha_{jmn}(x) \right) + \{ij\} + \epsilon_{ijm} \alpha_{mn} \tag{4}
\]

forms the defect tensor 

\[
\eta_{ij}(x) \equiv \epsilon_{ikl} \epsilon_{jmn} \nabla_k \nabla_m u_{ln}^p(x), \quad u_{ln}^p \equiv \frac{1}{2} (\beta_{ln}^p + \beta_{mn}^p). \tag{5}
\]

It is a symmetric tensor due to the conservation laws \( \delta \), and represents the Einstein tensor \( G_{ij} \equiv R_{ij} - \frac{1}{2} g_{ij} R \) of the geometry of the world crystal.

The expressions can easily be defined on a simple-cubic world lattice if we replace \( \nabla_i \) by lattice derivatives, as shown in \( \delta \). There it is also shown that, in three spacetime dimensions, the disclination density \( \theta_{ij}(x) \) represents the defect tensor \( G_{ij}^\Omega \) associated with the Curvature tensor \( R_{ijk}^\Omega \) of the Riemann-Cartan geometry of the world crystal. The relation is 

\[
G_{ij}^\Omega(x) = \epsilon_{ikl} \nabla_k \nabla_l \omega_j(x) = \theta_{ij}(x). \tag{6}
\]

The dislocation density \( \alpha_{ij}(x) \) represents the torsion \( S_{ikj} = \frac{1}{2} (\Gamma_{ikj} - \Gamma_{kij}) \) of the Riemann-Cartan geometry. Here the relation is 

\[
\alpha_{ij} = \epsilon_{ikl} S_{ikj}. \tag{7}
\]

5. The standard form of a defect with Burgers vector \( b_l \) and Frank vector \( \Omega_q \) has a displacement field 

\[
u(x) = -\delta(x, V)[b_l + \epsilon_{lqr} \Omega_q(\hat{x} - \hat{x}_r)], \tag{8}\]

where \( \epsilon_{lqr} \) is the antisymmetric unit tensor, \( \hat{x}_r \) the axis of rotation of the disclination part, and \( \delta(x, V) \) is the delta function on the volume \( V \), i.e., in three dimensions:

\[
\delta(x, V) = \int_V d^3x' \delta^3(x - x'). \tag{9}
\]

Its derivative is the delta function on the Volterra surface \( S \) of \( V \):

\[
- \nabla \delta(x, V) = \delta(x, S) = \int_S dS' \delta^3(x - x'). \tag{10}
\]

For the new gauge symmetry, the crucial observation is that as a simple consequence of \( \delta \), a dislocation line in the world crystal can either be obtained by a Volterra process of cutting out a thin slice of material of thickness \( b \), or alternatively by cutting out a wedge of Frank vector \( \Omega \), and reinserting it at distance \( b \) from the cut. Thus the dislocation line is indistinguishable from a pair of disclination lines with opposite Frank vector \( \Omega \) whose axes of rotation are separated by a distance \( b \) (Fig. 6).

Conversely, a disclination line is equivalent to a stack of dislocation lines with fixed Burgers vector \( b \) (Fig. 6).

Analytically, this is most easily seen in the two-dimensional version of the relation (11):

\[
\eta_{33} = \theta_{33} + \epsilon_{3mn} \nabla_m \alpha_{3n}. \tag{11}
\]

Each term is invariant under the plane gauge transformations \( \beta_{kl}^p \rightarrow \beta_{kl}^p + \nabla_k u_{ln}^p - \epsilon_{kli} \omega_{ln}^p, \; \phi_i^p \rightarrow \phi_i^p + \partial_i \omega_{ln}^p \). The general defect has a displacement field

\[
u_i = -\delta(V_2)[b_l - \Omega_{3lr}(x_r - \hat{x}_r)]. \tag{12}\]

The first term is a dislocation, the second term a disclination. According to Fig. 6, the latter can be read as a superposition of dislocations with the same Burgers vector \( b_l = -\int_0^l dx' \Omega_{3lr} \). The former may be viewed as a dipole of dislocations: \(-\nabla_i [-\frac{1}{2} b_m \epsilon_{3km} \Omega_{3lr}(x_r - \hat{x}_r)]\).

6. Let us now derive the emerging gravitational forces in the world crystal. Consider the partition function, at unit temperature, of the world crystal which we take to be three-dimensional, for simplicity:

\[
Z = \sum_{n_{ij}} \prod_x \left[ \frac{du_{ij}(x)}{a} \right] e^{-H}. \tag{13}
\]

In linear elasticity, the energy depends quadratically only on the difference between the elastic and the plastic strain tensors \( u_{ij} = \frac{1}{2} (\nabla_i u_j + \nabla_j u_i) \) and \( \nabla_{ij} \), and reads on the lattice

\[
H = \frac{\mu}{4} \sum_{i < j} \sum_x \left[ \nabla_i u_j(x) + \nabla_j u_i(x) - n_{ij}(x) \right]^2. \tag{14}
\]

Here \( \mu \) is the elastic constant [10], and the integer numbers \( n_{ij} \) of are the lattice versions of \( 2u_{ij}^p \) in Eq. 15.

This partition function explains for low temperature the correct classical specific heat. If the temperature is increased, it reaches a point where the configuration entropy of the defects wins over the Boltzmann factors of their energy, and the world crystal melts.

We have shown in [5] that in order to arrive at the proper Newton forces at long distances we have to insert one more derivative in the lattice action and start out with what is called the floppy world crystal where

\[
H = \mu \sum_{i < j} \sum_x \left[ \nabla_i (\nabla_i u_j(x) + \nabla_j u_i(x) - n_{ij}(x)) \right]^2. \tag{15}
\]

The partition function depends on the defect configuration only via the defect tensor formed from \( n_{ij} \), i.e., on \( \eta_{ij} \). It is a functional of this tensor which can be expanded into powers of \( \eta_i^r, \eta_{ij}^r, \eta_{ijk}^r, \eta_{ijkl}^r, \ldots \). The expansion coefficients are proportional to powers of the Planck length,
for each \( \eta_{ij} \) two powers. Since \( \eta_{ij} \) is the defect representation of the Einstein tensor \( G_{ij} \), the partition function defines a gravitational action which is a power series of the Einstein tensor. To leading (second) order in the Planck length \( G = G_{ij} = -R \).

Note that the gravitational action arises in this model from the *entropy* of the fluctuations in the same way as rubber elasticity in polymer physics [17].

The defect tensor \( \eta_{ij} \), i.e., in terms of the torsion tensor \( S_{ijk} \), can be decomposed into an Cartan curvature and torsion in some well-defined intermediate formulations of the theory with both Riemann and Cartan curvature and torsion in some well-defined mixture.

7. Generalizing the defect relations [11] and [12] to \( D \geq 4 \) spacetime dimensions and allowing for large deviations from Euclidean space, we find [13]

\[
G_{\mu\nu} = G^C_{\mu\nu} - \frac{1}{2} D^{\alpha\lambda} \left( S_{\mu\nu,\lambda} - S_{\nu,\lambda,\mu} + S_{\lambda,\mu\nu} \right) = 0 .
\]

(16)

where \( G_{\mu\nu} \) is the Einstein tensor and \( G^C_{\mu\nu} \) its Cartan version, while \( S_{\mu\nu,\tau} \) is the Palatini tensor related to the torsion field \( S_{\mu\nu} \), by

\[
\frac{1}{2} S_{\mu\nu,\tau} = S_{\mu\nu}^\tau + \frac{1}{2} \left( \partial_\mu S_{\nu,\lambda}^\tau - \partial_\nu S_{\mu,\lambda}^\tau \right).
\]

(17)

The symbol \( D_\mu \) denotes the covariant derivative defined by \( D_\mu \psi = \partial_\mu \psi - \Gamma^\lambda_{\mu\nu} \psi \lambda \), \( D_\mu \psi = \partial_\mu \psi + \Gamma^\lambda_{\mu\nu} \psi \lambda \), and \( D_\mu^\ast = D_\mu + 2S_{\mu}^\kappa \). The defect conservation laws [8] read

\[
D_\mu^\ast G^C_{\lambda\mu\nu} + 2S^{\kappa\lambda\mu\nu} G^C_{\kappa\lambda\nu} - \frac{1}{2} D^{\kappa\mu\nu} R^C_{\lambda\mu\nu\kappa} = 0 ,
\]

(18)

\[
D^{\kappa\mu\nu} S_{\lambda,\mu,\nu} = G^C_{\lambda\kappa} - G^C_{\kappa\lambda}.
\]

(19)

They are Bianchi identities ensuring the single-valuedness of observables, connection \( \Gamma^\mu_{\nu\lambda} \) and metric \( g_{\mu\nu} \), via the integrability conditions \( \left[ \partial_\mu, \partial_\nu \right] \Gamma^\rho_{\mu\nu} = 0 \) and \( \left[ \partial_\mu, \partial_\nu \right] g_{\mu\nu} = 0 \).

In a four-dimensional Riemann-Cartan spacetime, the geometry is described by the direct generalizations of translational and rotational defect gauge fields \( h_{ij} \) and \( A_{ijk} \), which are here the vierbein field \( h^\alpha_{\mu} \), and the spin connection \( A_{\mu\nu} \). The square of the former is the metric \( g_{\mu\nu} = h^\alpha_{\mu} h_{\alpha\nu} \). The latter is defined by the covariant derivative \( D_\mu h_{\beta\nu} = \partial_\mu h_{\beta\nu} + \Lambda^\lambda_{\mu\nu} h_{\beta\lambda} + \frac{1}{2} \Gamma^\lambda_{\mu\nu} h_{\beta\lambda} \equiv D^\lambda h_{\beta\nu} + \Gamma^\lambda_{\mu\nu} h_{\beta\lambda} \). The field strength of \( A_{\mu\nu} \) is \( A_{\mu\nu} \equiv \left( A_{\mu} \right)_{\alpha\beta} \).

The relations [10], [12], and [13] follow from this.

8. The theory is gauge invariant under local Lorentz transformations as a direct consequence of the fact that the metric can alternatively be written as

\[
g_{\mu\nu} = h^\gamma_{\mu} \Lambda^\alpha_{\gamma} \Lambda^\beta_{\nu} h_{2\nu} .
\]

(22)

where \( \Lambda_{\alpha}^{\beta} \) is an arbitrary local Lorentz transformation, and that the Einstein-Hilbert Lagrangian \( \mathcal{L}_{EH} = - (1/2k) R \) is independent of \( \Lambda^\alpha_{\nu} \). The extra \( \Lambda_{\alpha}^{\beta} \) transforms the gauge field \( A_{\mu}^{\alpha} \) as

\[
A_{\mu}^{\alpha} \rightarrow A_{\mu}^{\alpha} + \Delta A_{\mu}^{\alpha} , \quad \Delta A_{\mu}^{\alpha} = \Lambda_{\alpha}^{\beta} \partial_\mu A_{\beta}.
\]

(23)

At this point we are ready to introduce the new gauge invariance announced in the title: we allow \( \Lambda_{\alpha} \) in Eq. (22) to be a *multivalued Lorentz transformation*. This is not integrable, so that \( \Delta A_{\mu}^{\alpha} \) is a nontrivial gauge field. Indeed, the rotational field strength \( F_{\mu\nu}^{\alpha} \) can be expressed as \( F_{\mu\nu}^{\alpha} = \Lambda_{\gamma}^{\alpha} \partial_\mu A_{\gamma} \neq 0 \) and yields a nonzero Cartan curvature \( R_{\kappa\mu\nu}^{\alpha} = 0 \). The important observation is that a multivalued \( \Lambda^\alpha_{\nu} \) is able to *change the geometry* [13]. The right-hand side of [13] is independent of the vector field \( A_{\mu}^{\alpha} \).

This allows us to shuffle torsion into Cartan curvature and back, fully or partially, by complete analogy with the defect transformations in two-dimensional crystals in Fig. 8. We may choose for \( A_{\mu}^{\alpha} \) any function we like. For example we may choose it to make the torsion vanish, and \( A_{\mu}^{\alpha} \) reduces to the usual spin connection of Einstein’s gravity, the well-known combination of the objects of anholonomity

\[
\Omega_{\mu\nu}^{\alpha} = \frac{1}{2} [h_{\alpha,\mu} \partial_\nu h^\alpha_{\nu} - (\mu \leftrightarrow \nu)] .
\]

(24)

In the opposite extreme \( A_{\mu}^{\alpha} = 0 \), the Cartan curvature is zero, spacetime is teleparallel, and the Lagrangian is equal to the combination of torsion tensors:

\[
\mathcal{L}_{S} = - \frac{1}{2k} \left( -4D_{\mu} S^{\mu} + S_{\mu\lambda} S^{\mu\lambda} + 2S_{\mu\lambda} S^{\mu\lambda} - 4S^{\mu} S_{\mu} \right),
\]

(25)

where \( S_{\mu} \equiv S_{\mu\nu} \).

In any of the new gauges, the correct gravitational field equations are derived by extremizing the Einstein-Hilbert action

\[
\mathcal{A}_{EH} = \frac{1}{2k} \int dx \sqrt{-g} R^C + \int dx \sqrt{-g} \mathcal{L}_{S} + \mathcal{A}_{GF} ,
\]

(26)

where \( \mathcal{A}_{GF} \) is a functional of \( h^{\alpha}_{\mu} \) and \( A_{\mu}^{\alpha} \) fixing some convenient gauge. For \( \mathcal{A}_{GF} = \delta \left[ A_{\mu}^{\alpha} \right] \) this leads to the teleparallel theory, and for \( \mathcal{A}_{GF} = \delta \left[ S_{\alpha,\beta,\gamma} \left(h^{\alpha}_{\mu}, A_{\mu}^{\alpha} \right) \right] \) we re-obtain Einstein’s original theory.

9. Adding matter fields of masses \( m \) to the Einstein Lagrangian, and varying with respect to \( h^{\alpha}_{\mu} \), we find in the zero-torsion gauge the Einstein equation

\[
G_{\mu\nu} = \kappa T_{\mu\nu} ,
\]

(27)

where \( T_{\mu\nu} \) is the sum over the symmetric energy-momentum tensors of all matter fields. Each contains
the canonical energy-momentum tensor \( m\Theta_{\mu\nu} \) and the spin current densities \( m\Sigma_{\mu\nu}^\lambda \) in the combination due to Belinfante [21],

\[
m_{T_{\mu\nu}} = m\Theta_{\mu\nu} - \frac{i}{4} D^\mu \left( m\Sigma_{\nu,\mu} - m\Sigma_{\mu,\nu} + m\Sigma_{\mu,\nu} \right),
\]

which is the matter analog of the defect relation (10).

The new gauge invariance of (17) has the physical consequence that the external gravitational field in the far-zone of a celestial body does not care whether angular momentum comes from rotation of matter or from internal spins. The off-diagonal elements of the metric in the far-zone, and thus the Lense-Thirring effect measured in [15], depend only on the total angular momentum \( J^\lambda = \int d^3x (x^\lambda \Theta^0 - x^\mu \Theta^\lambda) \), which by the Belinfante relation (28) is the sum of orbital angular momentum \( S^\lambda = \int d^3x \Sigma^\nu_{\lambda,\beta} \). A star consisting of polarized matter has the same external gravitational field in the far-zone as a star rotating with the corresponding orbital angular momentum. This is the universality of orbital momentum and intrinsic angular momentum in gravitational physics observed in Ref. [10].

Since torsion is merely a new-gauge degree of freedom in describing a gravitational field, it cannot be detected experimentally, not even by spinning particles. A field with arbitrary spin may be coupled to gravity via the experimentally, not even by spinning particles. A field in describing a gravitational field, it cannot be detected covariant derivatives case \( \Sigma \).

If \( \Theta \) is permitted by covariance. In order to see which \( q \) is the analog of the metric \( g_{\mu\nu} \). We may now trivially introduce an extra gauge structure by re-expressing the Lagrangian in terms of a complex field \( \psi = e^{i\theta} \rho \) and a gauge field \( A_\mu \) as \( L = \int (\partial_\mu - iA_\mu)\psi| - |\psi|^{2} + |\psi|^{4} \). Now we form the partition function \( Z = \int D\psi D\psi^* D_{\mu} \Phi e^{-\frac{i}{2} \int dx \dot{L}} \), where \( \Phi \) is an arbitrary gauge-fixing functional multiplied by the associated Faddev-Popov determinant. The new \( Z \) is completely equivalent to the original \( L \). Obviously there is no way of observing \( A_\mu \). The partition function \( Z \) plays the role of Einstein’s theory, whereas \( Z \) gives its reformulation in terms of a gauge field, which does not change the physical content of the theory. The decomposition \( \rho = \psi^* \psi = (\rho e^{-i\theta}) (e^{i\theta} \rho) \) is the analog of the decomposition (22).

12. Higher gradient terms in elastic energy of the world crystal will generate an extra action \( A_{\mu}^\beta \) of the gauge field \( A_{\mu}^\beta \). This would, in general, violate the new symmetry discussed above and give torsion a life of its own. However, as long as the gravitational effects of spinning constituents in celestial bodies are suppressed with respect to that of the orbital angular momenta by many orders of magnitude, there is not much sense in conjecturing explicit forms of \( A_{\mu} \), unless we want to compete with string theory in setting up an ultimate theory of everything as a substitute of religion.

13. In summary, we have shown that if the Einstein-Hilbert Lagrangian is expressed in terms of the translational and rotational gauge fields \( h_{\mu}^\alpha \) and \( A_{\mu}^\beta \), the Cartan curvature can be converted to torsion and back, totally or partially, by a new type of multivalued gauge transformation in Riemann-Cartan spacetime, a hypergauge transformation. In this general formulation, Einstein’s original theory is obtained by going to the zero-torsion hypergauge, while his teleparallel theory is in the hypergauge in which the Cartan curvature tensor vanishes. But any intermediate choice of the field \( A_{\mu}^\beta \) is also allowed.
Acknowledgment: I thank F.W. Hehl, J.G. Pereira, and especially Jan Zaanen for a critical reading of the manuscript.

[1] C.D. Anderson, Phys. Rev. 43, 49 (1933).
[2] See the letter exchanges in Physics Today, 60, 10, 16 (2006) following S. Weinberg’s article in Vol. 58, 31 (2005).
[3] R. Debever (ed.), Elie Cartan–Albert Einstein, Letters on Absolute Parallelism 1929-1932, Princeton University Press, Princeton 1979.
[4] K. Hayashi and T. Shirafuji, Phys. Rev. D 19, 3524 (1979).
[5] H.I. Arcos, and J.G. Pereira, (gr-qc/0408096v2); Int. J. Mod. Phys. D 13, 2193 (2004) (gr-qc/0501017v1); V.C. de Andrade, H.I. Arcos, and J.G. Pereira, (gr-qc/0412034);
[6] V.C. de Andrade and J.G. Pereira, Int. J. Mod. Phys. D 8, 141 (1999) (gr-qc/9708051).
[7] R. Utiyama, Phys. Rev. 101, 1597 (1956); T.W.B. Kibble, J. Math. Phys. 2, 212 (1961); D.W. Sciama, Rev. Mod. Phys. 36, 463 (1964); F.W. Hehl, P. von der Heyde G.D. Kerlick, and J.M. Nester, Rev. Mod. Phys. 48, 393 (1976); F.W. Hehl, J.D. McCrea, E.W. Mielke, and Y. Neemann, Phys. Rep. 258, 1 (1995); R.T. Hammond, Rep. Prog. Phys. 65, 599 (2002); Wei-Tou Ni, Rep. Prog. Phys. 73, 056901 (2010) (arXiv:0912.5057).
[8] H. Kleinert, Gauge Fields in Condensed Matter, Vol. II Stresses and Defects, World Scientific, Singapore 1989, pp. 744-1443 (www.physik.fu-berlin.de/~kleinert/b2)
[9] H. Kleinert, Multivalued Fields in Condensed Matter, Electromagnetism, and Gravitation, World Scientific, Singapore 2008, pp. 1-497 (www.physik.fu-berlin.de/~kleinert/b1)
[10] H. Kleinert, Gen. Rel. Grav. 32, 1271 (2000) (physik.fu-berlin.de/~kleinert/271/271j.pdf).
[11] H. Kleinert, Ann. d. Physik. 44, 117 (1987) (http://physik.fu-berlin.de/~kleinert/172/172.pdf).
[12] The entropy origin of the stiffness of spacetime has recently been emphasized by E.P. Verlinde (arXiv:1001.0785). It has previously been used to generate the stiffness of strings: H. Kleinert, Dynamical Generation of String Tension and Stiffness, Phys. Lett. B 211, 151 (1988); Membrane Stiffness from v.d. Waals forces, Phys. Lett. A 136, 253 (1989). This is of course just another formulation of good-old Sakharov’s idea. See also J.H. Schmidt, Gen. Rel. Grav. 32, 361 (2000) (www.springerlink.com/content/t51570769p123410/fulltext.pdf).
[13] P. Jizba, H. Kleinert, F. Scardiglì, Uncertainty Relation on World Crystal and its Applications to Micro Black Holes, (arXiv:0912.2255).
[14] E. Kröner, in The Physics of Defects, eds. R. Balian et al., North-Holland, Amsterdam, 1981, p. 264.
[15] See einstein.stanford.edu.
[16] We ignore here the second elastic constant since it is irrelevant to the argument.
[17] See Chapter 15 in H. Kleinert, Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets, World Scientific, Singapore, 2009 (www.physik.fu-berlin.de/~kleinert/b5).
[18] B.A. Billby, R. Bullough, and E. Smith, Proc. Roy. Soc. London, A 231, 263 (1955); K. Kondo, in Proceedings of the II Japan National Congress on Applied Mechanics, Tokyo, 1952, publ. in RAAG Memoirs of the Unified Study of Basic Problems in Engineering and Science by Means of Geometry, Vol. 3, 148, ed. K. Kondo, Gakujutsu Bunken Fukyu-Kai, 1962.
[19] The new freedom brought about by multivalued gauge transformations in many areas of physics is explained in the textbook [9]. For instance, we can derive the physical laws with magnetism from those without it, in particular the minimal coupling law. Similarly, we can derive the physical laws in curved space from those in flat space.
[20] F.J. Belinfante, Physica 6, 887 (1939). For more details see Sect. 17.7 in the textbook [9].
[21] See p. 1453 in Ref. 8 (physik.fu-berlin.de/~kleinert/b1/gifs/v1-1453s.html).