Reliability of stellar inclination estimated from asteroseismology: analytical criteria, mock simulations and Kepler data analysis

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ABSTRACT
Advances in asteroseismology of solar-like stars, now provide a unique method to estimate the stellar inclination $i_{\star}$. This enables to evaluate the spin-orbit angle of transiting planetary systems, in a complementary fashion to the Rossiter-McLaughlin effect, a well-established method to estimate the projected spin-orbit angle $\lambda$. Although the asteroseismic method has been broadly applied to the Kepler data, its reliability has yet to be assessed intensively. In this work, we evaluate the accuracy of $i_{\star}$ from asteroseismology of solar-like stars using 3000 simulated power spectra. We find that the low signal-to-noise ratio of the power spectra induces a systematic under-estimate (over-estimate) bias for stars with high (low) inclinations. We derive analytical criteria for the reliable asteroseismic estimate, which indicates that reliable measurements are possible in the range of $20^\circ \lesssim i_{\star} \lesssim 80^\circ$ only for stars with high signal-to-noise ratio. We also analyze and measure the stellar inclination of 94 Kepler main-sequence solar-like stars, among which 33 are planetary hosts. According to our reliability criteria, a third of them (9 with planets, 22 without) have accurate stellar inclination. Comparison of our asteroseismic estimate of $v\sin i_{\star}$ against spectroscopic measurements indicates that the latter suffers from a large uncertainty possibly due to the modeling of macro-turbulence, especially for stars with projected rotation speed $v\sin i_{\star} \lesssim 5$ km/s. This reinforces earlier claims, and the stellar inclination estimated from the combination of measurements from spectroscopy and photometric variation for slowly rotating stars needs to be interpreted with caution.

Key words: asteroseismology – stars: rotation – stars: planetary systems – methods: data analysis – techniques: photometric

1 INTRODUCTION
Asteroseismology, the science that studies pulsation of stars, is a powerful tool to explore the stellar internal structure. Since it requires to observe bright stars over long period of time, however, its applicability has been rather limited, and the Sun has been the major target for decades.

The situation has drastically changed recently, thanks to the space missions, CoRoT (Baglin et al. 2006a,b) and Kepler (Borucki et al. 2010), which enabled dedicated photometric long-term monitoring of hundreds of thousands of stars. Asteroseismology is known to significantly improve the precision of fundamental parameters of stars, down to a few percents in radius and $\simeq 5$ percents in mass, respectively. Also asteroseismology helps improve the age estimate of stars down to $\simeq 15$ percents when combined with stellar modeling (Silva Aguirre et al. 2017).

This is why asteroseismology is becoming an important method to study stellar populations. For example, asteroseismology allows to provide a new insight on the formation and evolution history of the milky way (Casagrande et al. 2016; Miglio et al. 2016, 2017). Asteroseismology can also play an essential role in current and future surveys of exoplanets (Davies et al. 2016; Ragazzoni et al. 2016; Van Eylen et al. 2017). Indeed, the most basic parameters such as the mass and radius of exoplanets, are usually estimated on the basis of those from their host stars. Thus the precise determinations of the fundamental parameters such as mass and radius of host stars are essential.

Another important quantity that asteroseismology can
also deliver uniquely is the inclination angle of stellar spin axis with respect to our line-of-sight, $i_\star$. Indeed the correlation between the stellar spin vector $\vec{s}$ and the angular momentum vector of planetary orbit $\vec{l}$ is widely recognized as an important clue to the initial condition and migration history of exoplanetary systems (e.g., Winn & Fabrycky 2015, and references therein).

We refer to the angle $\psi$ between the stellar spin and planetary orbital momenta as the spin-orbit angle. Observationally $\cos\psi$ can be decomposed into the components perpendicular and parallel to the observer’s line-of-sight and expressed in terms of three angles as

$$\cos\psi = \sin i_\star \sin i_{\text{orb}} \cos \lambda + \cos i_\star \cos i_{\text{orb}}$$  \hspace{0.5cm} (1)

where $i_{\text{orb}}$ is the inclination angle of the planetary orbit, and $\lambda$ denotes the projected spin-orbit angle introduced by Ohta et al. (2005).

High-resolution spectroscopic observations of transiting planetary systems enable to measure $\lambda$ (Holt 1893; Rossiter 1924; McLaughlin 1924; Queloz et al. 2000). Since $i_{\text{orb}} \approx \pi/2$ for transiting planets, equation (1) reduces to

$$\cos\psi \approx \sin i_\star \cos \lambda.$$  \hspace{0.5cm} (2)

Clearly the measurement of $i_\star$ is essential to recover the true spin-orbit angle $\psi$, in addition to the projected angle $\lambda$. This is why asteroseismology is important (Toutain & Gouttebroze 1993; Gizon & Solanki 2003). Indeed $i_\star$ for planetary host stars has been measured by several authors, including Huber et al. (2013a), Chaplin et al. (2013), Benomar et al. (2014), and Campante et al. (2016). For example, Benomar et al. (2014) found that $i_\star \approx 30^\circ$ and $\psi \approx 120^\circ$ for HAT-P-7, and $i_\star \approx 65^\circ$ and $\psi \approx 30^\circ$ for Kepler-25.

The number of transiting exoplanetary systems with measured $\psi$ using asteroseismology is hampered by the required signal-to-noise ratio (SNR) and the relatively high stellar rotation rate. In reality, a majority of transiting exoplanets have been searched around F, G, and K type stars in their main-sequence phase. For such main-sequence stars, $i_\star$ is difficult to measure. Their oscillation modes exhibit low amplitudes and suffer from severe blending among modes (see e.g., Appourchaux et al. 2008). Therefore the systematic verification of the reliability of $i_\star$ derived from asteroseismology is of fundamental importance. As shown by Gizon & Solanki (2003) and Ballot et al. (2006, 2008) using a limited number of simulations, the inferred value of $i_\star$ is not so accurate if modes are of insufficient SNR or not properly identified. This is why we attempt here to perform systematic mock simulations to examine the reliability of $i_\star$ determination from asteroseismology. In the present paper, we focus on main-sequence stars, but our method could be applied to evolved stars as well. This is because the fundamental physical reason allowing us to infer the inclination are the same (Beck et al. 2012; Benomar et al. 2013; Mosser et al. 2017). In the future, it is worthwhile to extend our methodology for those evolved stars, as discussed in more details in section 6.2.

Section 2 briefly reviews the basic method of $i_\star$ determination proposed by Gizon & Solanki (2003), and then presents our approximations in generating mock power spectra. We consider several conditions for possible degeneracies in line profile fitting in section 3, and then compare those analytic conditions against the simulation results in section 4. In section 5, we perform asteroseismic analysis for Kepler stars with and without known planetary companions, and examine their reliability discussed in the previous section. Finally, section 6 discusses the implications of our findings and section 7 is devoted to summary and implications of the present paper.

2 DETERMINATION OF STELLAR INCLINATIONS FROM ASTEROSEISMOLOGY

In this section, we briefly describe the basic methodology of estimating $i_\star$ from asteroseismology following Gizon & Solanki (2003). In addition, we summarize several approximations that are commonly adopted in the real data analysis, and therefore in our mock simulations below as well.

2.1 Basic model

Kepler performed photometric monitoring of $\sim 1.5\times10^5$ stars over 4 years. For application of asteroseismology, the Kepler short cadence data with its one minute exposure is essential because it allows us to explore the frequency regime of pulsations for the solar-like stars in the main-sequence phase. The time-dependent light curve of each star is transformed into the corresponding power spectra $P(\nu)$, in which stochastically excited pulsation modes can be easily identified. In the frequency domain, each pulsation mode is characterized by a set of three indices $(n,l,m)$, corresponding to the spherical harmonics $Y_{n,m}(\theta,\phi)$ defined at each radial eigen-mode $n$. The indices $(n,l,m)$ are referred to as radial order, angular degree, and azimuthal order, respectively. Each oscillation mode is approximated by a Lorentzian line profile, and is characterized by its height $H(n,l,m,i_\star)$, width $\Gamma(n,l,m)$, and central frequency $\nu(n,l,m)$. Note that the dependence on $i_\star$ is entirely imprinted in its height $H(n,l,m,i_\star)$ as we describe later.

Thus the entire power spectra $P(\nu)$ are well approximated by a summation of different oscillation modes:

$$P(\nu) = \sum_{n=0}^{n_{\text{max}}} \sum_{l=0}^{l_{\text{max}}} \sum_{m=-l}^{l} \frac{H(n,l,m,i_\star)}{1 + 4[\nu - \nu(n,l,m)]^2/\Gamma^2(n,l,m)} + N(\nu),$$  \hspace{0.5cm} (3)

where $N(\nu)$ is a background model. The background is essentially due to the convection motion at the stellar surface. As discussed by e.g., Harvey (1985); Appourchaux et al. (2009); Karoff et al. (2013), the background model can be described as a sum of several semi-Lorentzian and of a white noise background. Each semi-Lorentzian relates to the flow motion at different spatial and temporal scales. The white noise corresponds to the noise limit, mostly due to the photon shot noise. Because the pulsation in main-sequence stars occurs at high frequency (e.g., in the Sun, the so-called 5 min oscillations), it suffices to consider only two semi-Lorentzians for the background model:

$$N(\nu) = \frac{A_1}{1 + (\tau_1\nu)^p} + \frac{A_2}{1 + (\tau_2\nu)^p} + N_0.$$  \hspace{0.5cm} (4)

Here $N_0$ is the white noise and $A$, $\tau$, and $p$ correspond
to the height, characteristic time scale, and slope of semi-
Lorentzian, respectively.

In the rotating frame of each star, the eigen-mode fre-
quency is independent of $m$, and given by $v(n,l)$. Thus the central frequency in the observer’s frame becomes

$$v(n,l,m) = v(n,l) + m\delta v_\star = \left[n + \frac{l}{2} + \varepsilon_{nl}\right] \Delta v + m\delta v_\star,$$  \hspace{1cm} (5)

where $\Delta v$ is referred to as a large separation (a fre-
quency spacing between consecutive radial modes), $\varepsilon_{nl}$ is a small correction of order unity (e.g., Tassoul 1980, 1990; Mosser et al. 2013), and $\delta v_\star$ is approximately the inverse of the stellar rotational period and called the stellar rotational splitting (see e.g., Appourchaux et al. 2008). Thus the degeneracy among $m$ can be broken due to the stellar rotation.

The height $H(n,l,m,i_\star)$ of the mode for the observer is known to be given by

$$H(n,l,m,i_\star) = \mathcal{E}(l,m,i_\star) \delta v_\star,$$  \hspace{1cm} (6)

where

$$\mathcal{E}(l,m,i_\star) = \frac{(l + |m|)!!}{(l + |m|)!} \left| P_{l}^{m} \right|^2 \cos^2 i_\star,$$  \hspace{1cm} (7)

and $P_{l}^{m}$ is the associated Legendre polynomials with degree $l$ and order $m$ (see Toutain & Gouttebroze 1993; Gizon & Solanki 2003). For instance,

$$\mathcal{E}(1,0,i_\star) = \cos^2 i_\star,$$

$$\mathcal{E}(1,\pm1,i_\star) = \frac{1}{2} \sin^2 i_\star,$$

$$\mathcal{E}(2,0,i_\star) = \frac{1}{4} \left(3 \cos^2 i_\star - 1\right)^2,$$

$$\mathcal{E}(2,\pm1,i_\star) = \frac{3}{8} \cos^2 i_\star \sin^2 i_\star,$$

$$\mathcal{E}(2,\pm2,i_\star) = \frac{3}{8} \sin^4 i_\star.$$

Therefore if each $m$-mode associated to the same degree $l$ is properly identified in $P(v)$, the ratio of their heights can be used to determine $i_\star$.

### 2.2 Conventional approximations

An asteroseismic analysis requires to identify the indices $(n,l,m)$ for each mode from the noisy spectra, and then preferably fit many lines simultaneously to determine the global parameters $i_\star$ and $\delta v_\star$. Therefore one has to adopt several approximations in order to reduce the number of free parameters as much as possible. We summarize conventional assumptions often adopted in asteroseismology.

Since it is known for the Sun that height ratio of non-radial modes ($l \neq 0$) and radial ($l = 0$) mode is uniform over the range of pulsation frequency (Salabert et al. 2011, and references therein), the intrinsic height of the oscillation for $l \neq 0$ is assumed to be

$$H(n,l) = V_\perp^2 H(n,l = 0),$$  \hspace{1cm} (9)

where $V_\perp^2$ is referred as the mode visibility and independent of the radial orders $n$. We adopt a slightly different sets of values, $(V_\perp^2, 1)=$(1.449, 0.6589) in simulations and (1.447, 0.5485) in the real data analysis of section 5, as described later.

We neglect the $m$-dependence of $\Gamma(n,l,m)$, and the remaining $l$-dependence is empirically modeled from the set of the fitted values of $\Gamma(n,l = 0)$ as follows. First we identify the modes $(n,0)$ for $n_{\min} \leq n \leq n_{\max}$ from the spectra, and obtain the corresponding eigen-frequency $v(n,0)$ and mode width $\Gamma(n,0)$. Then we construct a continuous function $F$ that linearly interpolates those discrete sets of parameters, i.e., $\Gamma(n,0) = F(v(n,0))$. Since earlier analysis of the Sun shows that the point $(v(n,l), \Gamma(n,l))$ stays approximately at the same trajectory of $(v(n,0), \Gamma(n,0))$ (Toutain & Froehlich 1992; Garcia et al. 2004), one simply replaces $\Gamma(n,l \neq 0)$ by $F(v(n,l \neq 0))$ evaluated at the fitted value of the eigen-frequency $v(n,l \neq 0)$.

In summary, under the above assumptions, the free parameters characterizing the entire power spectra include $H(n,l = 0)$, $\Gamma(n,l = 0)$, $v(n,l)$, the global parameters responsible for the shape of peaks ($i_\star$, $\delta v_\star$), and background parameters ($A_{1,2}$, $\tau_{1,2}$, $p_{1,2}$, and $N_0$). Accordingly, the total number of fitting parameters is $(n_{\max} - n_{\min} + 1)(l_{\max} + 1) + 9$.

### 3 ANALYTIC CRITERIA TO DISTINGUISH AMONG DIFFERENT AZIMUTHAL ORDERS

As suggested in the previous section, accurate measurement of $i_\star$ crucially depends on the ability of identifying the frequencies and heights of different $m$-modes associated with the same degree $l$. Ideally, the higher amplitude and the wider separation between different $m$-modes are required. More specifically, the former is represented by the ratio of the height $H(n,l,m,i_\star)$ and the noise level, and the latter, by the ratio of the stellar rotation splitting and the width, $\delta v_\star/\Gamma(n,l)$. This consideration may be translated into analytic criteria that are necessary to distinguish among different $m$-modes.

Because $l = 0$ modes are insensitive to either rotation ($\delta v_\star$) or inclination ($i_\star$), $i_\star$ can be determined by non-radial modes ($l \neq 0$). For a majority of main-sequence stars whose pulsations are detected, their visible modes are limited up to $l = 2$. Moreover, the amplitudes of $l = 1$ modes are roughly three times larger than those of $l = 2$ modes. Thus $l = 1$ modes dominate the ability to determine $i_\star$ in practice, and we consider analytic criteria to separate $m = 0$ and $m = \pm 1$ modes for $l = 1$.

A difficulty to distinguish among different $m$-modes may be understood from Figure 1, in which model profiles of power spectra around the central frequency $v_0$ for different values of $i_\star$ and $\delta v_\star/\Gamma$ are plotted; $i_\star = 30^\circ$, $60^\circ$, and $80^\circ$ from left to right panels, and $(\delta v_\star/\Gamma)/(\delta v_\star/\Gamma)_0 = 2$, 1, and 0.5 from top to bottom panels, with $(\delta v_\star/\Gamma)_0 \simeq 0.42\mu Hz/0.95\mu Hz = 0.44$ being the solar value near the maximum of mode amplitude. The horizontal axis corresponds to $(v - v_0)/\Gamma$ in units of $(\delta v_\star/\Gamma)_0$.

Hereafter, we note the contribution of $m = 0$ to the power spectra, $P_{l=1,m=0}(v)$. The contribution of $m = \pm 1$ modes is noted $P_{l=1,m=\pm 1}(v)$. From equations (3), (5) ~ (8), $P_{l=1,m=0}(v)$ and $P_{l=1,m=\pm 1}(v)$ are explicitly written as

$$P_{l=1,m=0}(v) = H(n,l = 1) \frac{\cos^2 i_\star}{1 + 4(v - v_0)^2/\Gamma^2},$$  \hspace{1cm} (10)

$$P_{l=1,m=\pm 1}(v) = H(n,l = 1) \frac{\sin^2 i_\star}{2[1 + 4(v - v_0 + \delta v_\star)^2/\Gamma^2)].}$$  \hspace{1cm} (11)

Figure 1 is normalized so that $H(n,l = 1) = 1$.
The reliability of the estimate of \(i_\ast\) and \(\delta v_\ast\) is crucially determined by how well one can separate the contributions from three different \(m\)-modes embedded in the total profile (black solid curve in Figure 1). More specifically, their separate contributions to the total power are computed as

\[
\int_0^\infty dv P_{l=1,m=0}(v) \approx \frac{H(n,l=1)\Gamma(n,l=1)}{2}\cos^2 i_\ast \delta v_\ast \quad (12)
\]

\[
\int_0^\infty dv P_{l=1,m=\pm1}(v) \approx \frac{H(n,l=1)\Gamma(n,l=1)}{4}\sin^2 i_\ast \quad (13)
\]

These need to be much larger than the resolvable element of the power, which is roughly given by the product of the rms noise level \(\sigma_n\) in the observed power spectra and the frequency resolution \(\delta f = 1/\mathcal{T}_{\text{obs}}\) with \(\mathcal{T}_{\text{obs}}\) being the total observation duration.

The above consideration leads to the following qualitative but analytic criteria.

(I) The identification of \(m = 0\) mode requires

\[
\frac{H(n,l=1)\Gamma(n,l=1)}{2}\cos^2 i_\ast > \alpha \sigma_n \delta f, \quad (14)
\]

where we introduce a fudge constant \(\alpha\) that will be empirically determined later through the comparison against mock simulation results. The condition (14) becomes

\[
\cos^2 i_\ast > \frac{2}{\text{SNR}} \frac{\delta f \Gamma}{\sigma_n}. \quad (15)
\]

where we define the signal-to-noise ratio SNR:

\[
\text{SNR} = \frac{H(n,l=1)}{\sigma_n}. \quad (16)
\]

Then the inequality (15) leads to an upper limit on the detectable \(i_\ast\):

\[
i_\ast < \cos^{-1} \left( \frac{2}{\text{SNR}} \frac{\delta f \Gamma}{\sigma_n} \right). \quad (17)
\]

In other words, one cannot reliably estimate the true value of \(i_\ast\) if it is larger than the threshold value in the right-hand-side of the above inequality. For instance, a reliable estimate of \(i_\ast = 90^\circ\) is very demanding and requires an ideal observation with either SNR = \infty or \(\delta f = 0\).

(II) Similarly the identification of \(m = \pm 1\) mode requires

\[
\frac{H(n,l=1)\Gamma(n,l=1)}{4}\sin^2 i_\ast > \beta \sigma_n \delta f, \quad (18)
\]
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covering a wide range of their values. In practice, we first choose KIC 12069424 (16 Cyg A) as our reference star, which is one of the brightest stars monitored by *Kepler*. It has one of the highest HBR among the observed main-sequence stars, and therefore is one of the most studied star in the *Kepler* field (Metcalf et al. 2012, 2014; Davies et al. 2015). The observed power spectrum for KIC 12069424 is shown in the left panel of Figure 2.

We extract the mode parameters (frequency, height, width, noise background parameters, rotational splitting and inclination) of the reference star by fitting a sum of Lorentzians and a noise profile as equation (3) in section 2.1. We use an MCMC sampling method based on an updated version to C++ of the algorithm from Benomar et al. (2009). We fit a total of 17 radial orders with associated degrees $l = 0, 1, 2$. While $l = 3$ degree is identifiable for this reference star, it is not the case for a majority of stars. Therefore we do not incorporate the modes of $l \geq 3$ for the reference star and for the simulated spectra. We verified that frequencies, rotational splitting and stellar inclination are all consistent with the result derived by Davies et al. (2015) within $2\sigma$. The right panel of Figure 2 shows the measured profile of HBR (HBR$_{\text{ref}}$) and the splitting-to-width ratio ($\delta\nu_{*\text{ref}}/\Gamma_{\text{ref}}$) of KIC 12069424 as a function of the radial mode frequency. These profiles are fairly representative of other solar-like stars (see e.g., Appourchaux et al. 2012).

As shown in the right panel of Figure 2, the mode height-to-background ratio HBR($n, l = 0$) as a function of the radial mode frequency has a peak at $v_{\text{max,ref}}$, and the corresponding peak value of HBR is defined as HBR$_{\text{max,ref}}$. We scale those reference parameters to generate mock spectra covering a range of HBR and $\delta\nu_{*}/\Gamma$ as described below. Hereafter, the subscript $\star$ indicates the variables of simulated stars scaled from a reference star.

The mode heights $H_{\star}(n, l = 0)$ of a simulated star are specified by its maximum value of HBR at $v_{\text{max,ref}}$, and are scaled as

$$H_{\star}(n, l = 0) = \frac{\text{HBR}_{\text{max,ref}}}{\text{HBR}_{\text{max,ref}}(n, l = 0)} N_0.$$

In reality, the noise background of the spectrum of actual main-sequence solar-like stars weakly depends on frequency (see Section 2.1). It typically decreases by a factor of a few between modes with the lowest measurable frequency and those with the highest frequency. The left panel of Figure 2 illustrates this for KIC 12069424 (red solid line). Here, for simplicity we neglect the frequency dependence in equation (22). We consider only the white noise $N_0$, and adopt the value of the reference star $N_0_{\text{ref}} = 0.0857\text{ppm}^2/\mu\text{Hz}$. The height $H_{\star}(n, l) = \Gamma(n, l) N_0$. The height $H_{\star}(n, l) = 1.449$ and $V_{lmax}^2 = 0.659$, from equations (9) and (22).

The other primary parameter that controls the reliability of the estimate of $i_{\star}$ is the splitting-to-width ratio $\delta\nu_{*}/\Gamma$. It measures the influence of the overlap between split components (see section 3) on the inclination. In the current simulation, we fix the width of the mode $\Gamma(n, l)$ to the reference value $\Gamma_{\text{ref}}(n, l)$. On the other hand, we modify the rotational splitting $\delta\nu_{*}$. Then,

$$\delta\nu_{*} = \gamma_{\star} \Gamma_{\text{max,ref}} .$$

4 MOCK SIMULATION TO EXTRACT STELLAR INCLINATIONS FROM OSCILLATION POWER SPECTRA

Measurement of the stellar inclination from asteroseismology is based on several complicated procedures and their validity can be examined quantitatively only through the analysis of simulated power spectra. We carry out intensive analyses of systematic mock spectra. We first describe how to generate simulated power spectra, and then present the results against our analytic criteria discussed in the previous section.

4.1 Generating mock power spectra scaled from a reference star KIC 12069424

As we demonstrated in the previous section, the precision and accuracy of the estimate of the stellar inclination depend sensitively on the splitting-to-width ratio $\delta\nu_{*}/\Gamma$ and on the signal-to-noise ratio SNR; in the present simulation, we have not implemented realistic noises except for the background noise level of $N_0$ in equation (4). Thus the signal-to-noise ratio $\text{SNR} = H(n, l = 1)/\sigma_V$, defined in section 3 is not easy to properly assign. Instead, we use height-to-background ratio $\text{HBR} = H(n, l)/N_0$ as a proxy for SNR throughout the following analysis. In practice, the ratio between HBR and SNR is expected to be incorporated by renormalizing the values of $\alpha$ and $\beta$ in inequalities (17) and (19).

We take the HBR and $\delta\nu_{*}/\Gamma$ as our primary variables in simulated power spectra, and generate realistic mock spectra covering a wide range of their values.
summarizes the observations of three stars

\[
\frac{\delta \nu}{\nu} \approx 15 \mu Hz
\]

Superimposed is the background level (red) and the fitted oscillation frequency (blue). The inset shows the modes of degree \( \ell=0,1,2 \) of highest amplitude. Right. Measured height-to-background (HBR; blue) and splitting-to-width ratio \( (\delta \nu_\ell / \Gamma; \text{red}) \) of the radial modes, as a function of the oscillation frequency \( \nu(n, \ell = 0) \). These reference profiles are scaled and used in simulated power spectra discussed in Section 4.1.

Table 1. Range of parameter values in simulated spectra with \( T_{\text{obs}} = 1 \) year and \( T_{\text{obs}} = 4 \) years.

| parameter       | range     | grid interval |
|-----------------|-----------|---------------|
| \( \Gamma_{\text{max, ref}} \) | \([0.30]\) | 1             |
| \( \gamma_\ast \delta \nu_\ast / \Gamma_{\text{max, ref}} \) | \([0.1, 1.0]\) | 0.1           |
| \( i_\ast (\text{deg}) \) | \([0.0, 90]\) | 10            |

where \( \Gamma_{\text{max, ref}} = 1.08 \mu Hz \) is the width of the mode that corresponds to \( \Gamma_{\text{max, ref}} \). Here, \( \gamma_\ast \) is the splitting-to-width ratio at \( \Gamma_{\text{max, ref}} \).

Obviously the observation duration \( T_{\text{obs}} \) is another important factor that defines the number of independent data points sampling a mode profile. The longer \( T_{\text{obs}} \) improves the frequency resolution \( \delta f \propto 1 / T_{\text{obs}} \). It also improves the description of the mode profile, which in turns enhances the accuracy on the stellar inclination. To assess this, we consider \( T_{\text{obs}} = 1 \) and 4 years, corresponding to the minimal and maximal observation duration of the LEGACY \(^1\) Kepler sample (Lund et al. 2017).

A grid with a total of 3000 artificial spectra is generated each for \( T_{\text{obs}} = 1 \) and 4 years. Table 1 summarizes the ranges of the three control parameters for the simulated spectra; \( \Gamma_{\text{max, ref}}, \gamma_\ast \) and the inclination angle \( i_\ast \).

4.2 Results of mock spectra analysis

Figure 3 plots the result of mock spectra analysis on \( i_\ast - \delta \nu_\ast / \Gamma \) plane. Specifically, it shows the difference between the true input value and the median of the inferred posterior probability distribution (PPD) for \( i_\ast \) and \( \delta \nu_\ast / \Gamma \). As we will show later, the median value does not necessarily represent the best-fit, but we use it here just for simplicity. The base of the black arrows indicates the input value, the tip is the measured median value. Left panels are for \( T_{\text{obs}} = 4 \) years, while right panels are for \( T_{\text{obs}} = 1 \) year. Top, middle, and bottom panels correspond to \( \Gamma_{\text{max, ref}} = 30, 5, \) and 3, respectively. Note that \( \Gamma_{\text{max, ref}} = 3 \sim 5 \) are representative of the maximum HBR of the modes for Kepler stars with pulsations. In practice, below \( \Gamma_{\text{max}} = 3 \) the noise makes difficult to observe the individual pulsation modes, so that the seismic analysis is often limited to the measurement of the central frequency at maximum power \( \nu_{\text{max}} \) and of the large separation \( \Delta \nu \). The case with \( \Gamma_{\text{max, ref}} = 30 \) corresponds to the best cases, such as KIC 12069424 (the reference star).

Clearly there exists a coherent pattern of arrow distribution over the plane, indicating the presence of the systematic bias in the parameter estimation. The length of each arrow reflects the amplitude of the bias. Labels of “a” to “i” in the top-left panel of Figure 3 indicate the locations of \( (i_\ast, \delta \nu_\ast / \Gamma) \) in the corresponding panels of Figure 1. The comparison of Figure 3 with Figure 1 helps intuitive understanding of the result.

To proceed further, we overlay the analytic criteria (I) ~ (III) on each panel of Figure 3. The three vertical lines in the right part indicate the criterion (I) with \( \alpha = 15 \) (left dashed), 10 (middle solid) and 5 (right dashed). Similarly the three vertical lines in the left part indicate the criterion (II) with \( \beta = 10 \) (left dashed), 15 (middle solid) and 20 (right dashed). Finally the horizontal dashed line corresponds to the criterion (III), \( \delta \nu_\ast / \Gamma = 0.5 \). In doing so, we set SNR=HBR\(_{\text{max}}\) just for simplicity. As we remarked, those criteria are not expected to be strict, and the adopted values of \( \alpha \) and \( \beta \) are merely empirical. Nevertheless the regions bounded by the criteria agree with those in which the input parameters are reproduced fairly accurately from the mock simulation. We also note that the length of the arrows for \( T_{\text{obs}} = 4 \) years becomes approximately half with respect to that for 1 year on average. This indicates that not only the uncertainty but also the accuracy of the estimate scales as \( 1 / \sqrt{T_{\text{obs}}} \).

On the basis of the above empirical comparison, we divide the observed Kepler stars into two different categories adopting \( \alpha = 10 \) and \( \beta = 15 \) in the next section.

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\(^1\) The LEGACY sample corresponds to the ensemble of Kepler main-sequence stars that show Sun-like pulsations and that were observed continuously for at least a year.
Figure 3. Comparison of asteroseismically derived values and true inputs of $i_\star$ and $\delta \nu_\star/\Gamma$ for simulated spectra. Arrows in each panel start from the true values and end at the estimated values. Left and right panels show the results for $T_{\text{obs}} = 4$ years and 1 year, respectively. Top, middle, and bottom panels correspond to HBR$_{\text{max,} \star} = 30$, 5, and 3. Nine alphabets in the top-left panel indicate the set of $(i_\star, \delta \nu_\star/\Gamma)$ assumed in the the corresponding panel of Figure 1. Blue lines, red lines and horizontal black dashed lines correspond to analytic criteria (I), (II), and (III) discussed in Section 3, respectively.

Before finishing this section, it is worth emphasizing the limitation of using the median value of the derived PPD in estimating the inclination angle. For that purpose, we compute the derived PPD of the inclination angle for a simulated star assigned $i_\star = 90^\circ$ as a true input. The resulting PPDs are plotted in Figure 4 for different values of $\delta \nu_\star/\Gamma$. Red and blue histograms correspond to 1 year and 4 years simulations, respectively, for HBR$_{\text{max,} \star} = 3$. While the true value ($90^\circ$) can be measured better for higher $\delta \nu_\star/\Gamma$ and longer observation, the measured value always becomes less than $90^\circ$. For example, the gray regions in the bottom-right panel brackets between the measured and true values. This is because we employ the sampling method for inclination over the range of $[0^\circ, 90^\circ]$. Similarly, the derived inclination for true inclination of $0^\circ$ is always greater than $0^\circ$. Therefore the bias indicated in Figure 3 may be partly, even though not entirely, due to the use of the median value of the entire PPD.

5 APPLICATION TO KEPLER DATA

5.1 Target star selection

We analyse stars monitored by Kepler during its initial 4 years mission. In total, we consider 33 stars with transiting planets, and 61 stars without known transiting planets.
Figure 4. Posterior probability distribution (PPD) of measured $i_*$ for simulated spectra with input $i_*=90^\circ$ with different input $\delta v_*/\Gamma$. HBR$_{\text{max},*}=3$ is assumed. These are shown as histograms, with the number of bins defined according to Scott’s normal reference rule (Scott 1992). Red and blue histograms show the results for $T_{\text{obs}}=1$ year and 4 years, respectively. Gray area in the lower-right panel indicates the region from the median of the 4 year PPD ($i_*=83.8^\circ$) to the input value ($i_*=90^\circ$).

The stars without known planets in this work are taken from LEGACY sample (Lund et al. 2017; Silva Aguirre et al. 2017). This sample consists of 66 main-sequence stars observed in short cadence for at least 1 year by Kepler. Out of the 66 stars, we select 61 stars that do not have known planets (the remaining 5 stars with planets are also analysed below).

We re-analyze 25 stars with planets from Campante et al. (2016), which include 4 stars (KIC 3632418, 9414417, 9955598, and 10963065) with planets from the LEGACY sample. In addition, we analyse 8 stars with planets whose asteroseismic analysis has not yet been published elsewhere, including one star (KIC 7296438) from LEGACY sample.

The power spectra prepared using the method of Handberg & Lund (2014), are downloaded from Kepler Asteroseismic Science Operations Center (KASOC) database (http://kasoc.phys.au.dk). There are two different spectra available, with and without weighting of the photometric flux with the flux uncertainty. We use unweighted spectra for 42 targets since their weighted spectra are not available. Otherwise we use the weighted spectra for 51 stars. For KIC 11401755, the latest weighted spectra are not available, and we decided to use its unweighted spectra.

We adopt $V_1^2=1.447$ and $V_2^2=0.5485$, the mean of visibility of the Sun in green and red VIRGO/SPM channels from Salabert et al. (2011). This averaging might give visibilities representative of the Kepler visibilities (Ballot et al. 2011).

### 5.2 Asteroseismic inference of Kepler stars

We perform the asteroseismic analysis using the MCMC method, and summarize the main results in Tables 2, 3, and 5 for stars with and without planets, respectively. We classify those stars as category A if their measured values of $(i_*,\delta v_*/\Gamma)$ satisfy the three analytic criteria (I), (II), and (III) with $\alpha=10$ and $\beta=15$. Otherwise the stars are classified as category B. There are 9 stars with planets of category A, and 22 without planets. The classification is admittedly not strict, because it is based on the measured median values neglecting the quoted errors, in addition to the qualitative nature of the criteria themselves. Nevertheless such a classification is useful as a rough measure of the reliability of the inference.

Upper and lower panels in Figure 5 plot the distribution of measured $i_*$ and $\delta v_*/\Gamma$ for stars with and without planets, respectively. Stars belonging to categories A and B are
Stellar inclination from asteroseismology

Figure 5. Measured values of $i_\star$ and $\delta\nu_\star/\Gamma$ for Kepler stars with (top) and without (bottom) known planetary companions. The black solid horizontal line represents $\delta\nu_\star/\Gamma = 0.5$ (equation 21). Filled circles indicate category A stars, while crosses correspond to category B stars. The numbers labelling the filled circles denote the KOI IDs for stars with planets.

Figures 6 and 7 show examples of power spectra and the resulting two dimensional PPD of $i_\star$ and $\delta\nu_\star$ for categories A and B stars. They present the difference of the constraining power on $i_\star$ and $\delta\nu_\star$ between our categories A and B. Note, however, that these two may be extreme examples, and in some cases the difference between A and B is milder.

5.3 Consistency of asteroseismically-derived parameters with other observations

Unlike for simulated stars, the true values of the stellar parameters for actual Kepler stars are obviously not known. Thus it is important to compare our asteroseismic estimates plotted in filled circles with error-bars and in crosses, respectively. Also we indicate the KOI number for category A stars with planets in the upper panel. Since the target selection is somewhat heterogeneous, we cannot put any strong conclusion at this point. Nevertheless it is interesting to note that the category A stars with planets are preferentially located around the large $i_\star$ region relative to those without planets, suggesting a spin-orbit alignment of transiting planets in general. Further implications of the results for the Kepler will be described in our next paper (Kamiaka, Benomar, and Suto, in preparation).
of stellar parameters against other independent observations, which is attempted in this subsection.

We first consider $v \sin i_\star$ that can be measured also from line widths of spectroscopically observed stars. For 33 stars with planets, we use the spectroscopic $v \sin i_\star$ from California-Kepler Survey (CKS; https://california-planetsearch.github.io/cks-website/), except one from Huber et al. (2013b).

For 61 stars without planets, we consider two different spectroscopic datasets from Bruntt et al. (2012) and Molenda-Zakowicz et al. (2013). First we adopt the data for 43 stars from Bruntt et al. (2012). Out of the remaining 18 stars not listed in their catalog, we adopt the data from Lund et al. (2017) for 11 stars. Next we repeat the same procedure starting with the dataset of Molenda-Zakowicz et al. (2013). In this case, we combine 46 stars from Molenda-Zakowicz et al. (2013), and 11 stars from Lund et al. (2017).

Note that $v \sin i_\star$ in Huber et al. (2013b) and Lund et al. (2017) is calculated on the basis of the Stellar Parameter Classification pipeline (SPC; Buchhave et al. 2012), while CKS, Bruntt et al. (2012), and Molenda-Zakowicz et al. (2013) develop their own pipeline in computing $v \sin i_\star$.

While $v \sin i_\star$ can be directly estimated from spectroscopic data, it is not the case for asteroseismology. We estimate the stellar radius $R_\star$ from the scaling relation calibrated with the Sun:

$$\frac{R_\star}{R_\odot} = \left( \frac{v_{\text{max}}}{v_{\text{max, } \odot}} \right) \left( \frac{\Delta v_{\nu}}{\Delta v_{\nu, \odot}} \right)^{-2} \left( \frac{T_{\text{eff}}}{T_{\text{eff, } \odot}} \right)^{1/2},$$

(24)

where $v_{\text{max}}$ is the frequency corresponds to the peak of the mode heights (see Figure 2), $\Delta v_{\nu}$ is the large separation, and $T_{\text{eff}}$ is the effective temperature of the star. Thus $R_\star$ can be estimated from the two asteroseismic observables, $v_{\text{max}}$ and $\Delta v_{\nu}$, along with $T_{\text{eff}}$, which leads to asteroseismic estimate.
of \( v \sin i_\star \): 

\[
v \sin i_\star \text{ (asteroseismology)} = 2 \pi R_\star \delta \nu_\star \sin i_\star.
\]  

We adopt \( v_{\text{max,}} = 3100 \mu\text{Hz}, \Delta \nu_\star = 134.9 \mu\text{Hz, and } T_{\text{eff,}} = 5777 \text{ K} \) (Broomhall et al. 2009; Gaulme et al. 2016).

Figure 8 compares \( v \sin i_\star \) from asteroseismic and spectroscopic data. The left panel shows planet-hosting stars in blue (32 from CKS and 1 from Huber et al. 2013b), and also stars without planets, 11 from Lund et al. 2017 in black, 43 from Bruntt et al. 2012 in red, and 46 from Molenda-Żakowicz et al. 2013 in green. A significant fraction of stars without planets overlaps in the two sources, and thus we distinguish them using different colors. Filled circles and crosses correspond to categories A and B stars, respectively. We do not quote error-bars for Bruntt et al. (2012) since they are not available from the published table.

The left panel of Figure 8 suggests that asteroseismic and spectroscopic \( v \sin i_\star \) are in reasonable agreement. However, a closer look at \( v \sin i_\star < 6 \text{km/s} \) data in the right panel reveals an interesting feature; the estimates by Bruntt et al. (2012) (red) are systematically larger than our asteroseismic values, while those by Molenda-Żakowicz et al. (2013)
Table 5. Results on *Kepler* planet-less stars in category B

| KIC    | HBR$_{\text{max}}$ | $\delta\nu_\star/T$ | $i_\star$ (16%, 50%, 84%) (deg) | $\delta\nu_\star$ (16%, 50%, 84%) (µHz) |
|--------|-------------------|----------------------|----------------------------------|----------------------------------------|
| 2837475| 1.74              | 0.46                 | [70.9, 76.6, 83.6]              | [3.01, 3.10, 3.20]                     |
| 3427720| 3.18              | 0.31                 | [12.4, 28.5, 61.0]              | [0.19, 0.43, 0.88]                     |
| 3456181| 3.52              | 0.28                 | [34.5, 48.6, 72.0]              | [0.75, 0.99, 1.36]                     |
| 3656476| 11.57             | 0.42                 | [34.6, 48.8, 73.1]              | [0.24, 0.31, 0.42]                     |
| 3735871| 1.42              | 0.38                 | [42.9, 66.2, 83.1]              | [0.62, 0.72, 0.98]                     |
| 5184732| 2.16              | 1.04                 | [1.7, 8.2, 27.6]                | [0.05, 1.15, 2.35]                     |
| 5950854| 2.16              | 1.04                 | [1.7, 8.2, 27.6]                | [0.05, 1.15, 2.35]                     |
| 6106415| 19.61             | 0.44                 | [67.0, 75.9, 85.1]              | [0.68, 0.71, 0.75]                     |
| 6116048| 12.58             | 0.42                 | [62.8, 73.0, 83.7]              | [0.61, 0.64, 0.70]                     |
| 6508366| 2.56              | 0.49                 | [80.9, 85.6, 88.7]              | [2.12, 2.19, 2.26]                     |
| 6603624| 17.93             | 0.44                 | [2.0, 4.2, 38.7]                | [2.00, 0.30, 1.44]                     |
| 6933899| 10.51             | 0.31                 | [48.9, 64.2, 81.0]              | [0.33, 0.37, 0.45]                     |
| 7106245| 1.65              | 0.43                 | [13.5, 28.6, 62.6]              | [0.27, 0.57, 1.36]                     |
| 7771282| 1.17              | 0.39                 | [48.6, 67.0, 82.0]              | [1.05, 1.19, 1.39]                     |
| 7940546| 9.25              | 0.35                 | [52.5, 63.0, 76.6]              | [0.97, 1.08, 1.23]                     |
| 8228742| 6.52              | 0.44                 | [29.9, 38.5, 58.6]              | [0.56, 0.83, 1.11]                     |
| 8694723| 7.59              | 0.46                 | [32.4, 37.4, 43.0]              | [1.10, 1.25, 1.46]                     |
| 8760414| 7.19              | 0.43                 | [2.1, 8.1, 40.4]                | [0.04, 0.48, 1.74]                     |
| 8938364| 8.91              | 0.31                 | [7.8, 25.0, 61.7]               | [0.10, 0.23, 0.65]                     |
| 9098294| 3.86              | 0.36                 | [30.8, 49.9, 75.1]              | [0.33, 0.43, 0.66]                     |
| 9206432| 1.62              | 0.30                 | [21.1, 36.2, 59.6]              | [1.06, 1.73, 2.77]                     |
| 9353712| 1.78              | 0.87                 | [21.0, 28.9, 53.8]              | [0.15, 0.49, 1.37]                     |
| 9410862| 1.66              | 0.78                 | [13.7, 21.2, 45.3]              | [0.47, 1.16, 2.01]                     |
| 9812850| 1.82              | 0.38                 | [50.7, 64.7, 81.3]              | [1.40, 1.57, 1.87]                     |
| 1009226| 11.91             | 0.40                 | [33.4, 41.9, 58.4]              | [0.58, 0.77, 0.96]                     |
| 10162436| 2.23             | 2.50                 | [49.8, 71.6, 84.0]              | [0.64, 0.75, 0.93]                     |
| 10454113| 3.23            | 0.42                 | [28.7, 44.6, 62.2]              | [0.48, 0.65, 1.02]                     |
| 10516096| 6.79             | 0.33                 | [53.2, 70.2, 83.8]              | [0.45, 0.49, 0.58]                     |
| 10644253| 1.86            | 0.21                 | [2.5, 13.9, 54.4]               | [0.06, 0.34, 1.36]                     |
| 11081729| 1.32           | 2.30                 | [80.7, 85.4, 88.6]              | [3.22, 3.40, 3.51]                     |
| 11772920| 1.93          | 0.49                 | [51.1, 67.0, 81.8]              | [0.28, 0.33, 0.40]                     |
| 12069127| 1.66           | 0.19                 | [16.9, 40.4, 70.3]              | [0.35, 0.65, 1.16]                     |
| 12069449| 29.29          | 0.37                 | [33.1, 47.0, 70.7]              | [0.27, 0.35, 0.49]                     |
| 12258514| 13.77          | 0.30                 | [19.9, 34.0, 64.6]              | [0.28, 0.46, 0.81]                     |
| 12317678| 2.74           | 0.20                 | [46.1, 62.4, 80.5]              | [0.92, 1.06, 1.34]                     |

Figure 7. Same as Figure 6, but for KIC 6196457.
Figure 8. Comparison of $v\sin i_*$ estimated from spectroscopy and those from asteroseismology. Filled circles and crosses correspond to categories A and B stars, respectively. Figure a (left) shows stars with planets (blue), with spectroscopic values from CKS (32 stars) and Bruntt et al. 2012 (1 star). Shown stars without planets use values from Lund et al. 2017 in black (11 stars), from Bruntt et al. 2012 in red (45 stars), and from Molenda-Zakowicz et al. 2013 in green (46 stars). Figure b (right) is an enlarged view for stars without planets whose $v\sin i_*$ is less than 6km/s.

(green) are systematically smaller. Our result are somewhere in-between, except for $v\sin i_* \lesssim 2$km/s. Since these authors have a large fraction of stars in common, the feature should not be due to differences in the stellar properties. We suspect that the difference between the two spectroscopic results comes from the subtle modeling of micro/macro-turbulence effects in spectroscopic data. We would like to point that “the roundest A-type star” KIC 11145123 (Kurtz et al. 2014; Gizon et al. 2016) presents an interesting example in this context. (Takada-Hidai et al. 2017) found that the spectroscopically measured value of $v\sin i_*=5$km/s suffers from systematic overestimate, and asteroseismically derived equatorial rotation velocity of $v\sin i_* \approx 1$km/s proved to be more reliable. This suggests that the spectroscopic measurement of $v\sin i_*$ for slowly rotating stars needs to be interpreted with caution, which is in good agreement with our conclusions from Figure 8. The importance of the careful calibration of turbulence has been well recognized in earlier publications, for instance by Bruntt et al. 2012. The lower panels of Figure 8 provide observational evidences of this problem. Incidentally, the overall consistency between asteroseismic and spectroscopic $v\sin i_*$ ($> 1$km/s) may also reinforce the nearly-uniform rotation of stars as stated by Benomar et al. 2015. This is because asteroseismology measures the stellar rotation averaged over its interior, while spectroscopy measures its surface rotation.

Figure 9 compares asteroseismic $\delta v_*$ and the inverse of stellar rotation period measured from photometric variability for 46 stars (García et al. 2014). While they agree reasonably on average, individual agreement is not good except for $\delta v_* \gtrsim 2\mu$Hz. Again both the photometric variation and rotational splitting are not reliably identified for slowly rotating stars.

Combining the spectroscopic $v\sin i_*$, asteroseismic $R_*$, and photometric $P_{\text{rot}}$, we can estimate $i_*$ as

$$i_\ast \text{ (combined)} = \sin^{-1} \left( \frac{P_{\text{rot}}}{2\pi R_\ast} v\sin i_\ast \right).$$

Figure 10 is similar to Figure 8, but instead, compares $i_\ast$ estimated from equation (26) with the asteroseismic $i_\ast$. Panel a (left) shows the stars whose $i_\ast$ is derived from the combined analysis, while panel b (right) shows planet-less stars with $v\sin i_\ast < 6$km/s alone, similarly to Figure 8. The large scatter, that is mainly due to the photometric variation uncertainty, makes it difficult to draw any definite conclusion at this point. Indeed, the lightcurve modulation attributed to spots could be affected by the fact that the number, lifetime and latitude of the spots are unknown. It is therefore difficult to identify the reason of the scatter at this stage. However, this is the current status of the mutual compari-
6 DISCUSSION

One of our main findings is that the seismology provides reliable stellar inclination only for stars with $20^{\circ} \lesssim i \lesssim 80^{\circ}$, $\delta v_{\ast}/\Gamma \gtrsim 0.5$, with high signal-to-noise ratio, and with longer observations. A significant bias arise when this is not the case, so that the stellar inclination could be overestimated for low inclinations and underestimated otherwise. Below we discuss more broadly its implication on previous results.

6.1 Inclinations on CoRoT stars

Although the statistics is low, it is interesting to note that the analysis of solar-like stars observed by CoRoT (Baglin et al. 2006a,b) often led to low and medium stellar inclinations. An isotropic distribution of spins in the sky should give instead a larger proportion of stars with high inclination. We have $i_{\ast} = 45^{\circ} \pm 4^{\circ}$ for HD 181420 (Barban et al. 2009), $i_{\ast} = 24^{\circ} \pm 3^{\circ}$ for HD 181906 (García et al. 2009), and they are based on the low SNR. On the other hand, we have $i_{\ast} = 17^{\circ} \pm 9^{\circ}$ or $i_{\ast} = 26^{\circ} \pm 7^{\circ}$ for HD 49933 (Benomar et al. 2009, 2015) and $i_{\ast} = 71^{\circ} \pm 6^{\circ}$ for HD 49385 (Deheuvels et al. 2010) with high SNR.

Those CoRoT stars were observed only for 90 to 180 days, with a signal-to-noise that does not exceed $\sim 5$. From Figure 3, we expect a substantial bias toward lower inclinations for most of the CoRoT stars, in agreement with the apparent excess of low to medium stellar inclinations. This is also largely consistent with our Kepler data analysis plotted in Figure 5, especially for stars without planet in which the correlation with the transiting planetary orbital plane should not exist.

6.2 Implication for evolved stars

While the current work is specifically dedicated to low-mass main-sequence stars, our results can be of importance also for evolved stars. Subgiants and red-giant stars show mixed modes, arising from the coupling between pressure modes and gravity modes. Mixed modes can be mostly sensitive either to the envelope (pressure-like modes) or to the interior (gravity-like modes). The large number of $l = 1$ mixed modes observed in evolved stars has enabled detailed studies of the interior and evolution of those stars (e.g., Deheuvels et al. 2012; Mosser et al. 2014).

Because pressure-like modes are short-lived and probe the slowly rotating envelope (e.g., rotations of the order $\sim 100$ days in red-giants), it is expected that split-components of $l = 1$ modes suffer from a severe blending ($\delta v_{\ast} \ll \Gamma$). On the contrary, gravity-like modes have lifetimes of the order of years and probe regions that mostly rotate faster than the envelope (Beck et al. 2012; Deheuvels et al. 2012; Benomar et al. 2013; Deheuvels et al. 2014, 2015; Mosser et al. 2017), so that split-components are well separated ($\delta v_{\ast} \gg \Gamma$).

In these conditions and as suggested by Figure 3, it is likely that gravity-like modes allow an accurate determination of the stellar inclination, provided that they have a significant signal-to-background ratio. However, one needs to be cautious when determining the stellar inclination from pressure-like modes. We also stress that when modes of evolved stars are fitted individually (e.g., using a local fit, the lifetime of the modes is inversely proportional to the mode width.

Figure 9. Comparison of $\delta v_{\ast}$ derived from asteroseismology and the inverse of the stellar rotation period derived from photometric variations, whenever available. Stars with and without planets are plotted in blue and red, respectively. Filled circles and crosses correspond to categories A and B stars.
rather than a global fit as performed in this study), inclinations of blends modes or low signal-to-background modes are expected to be significantly biased towards lower values.

This suggest that the seismic determination of the stellar inclination for the red-giant Kepler-56 (Huber et al. 2013a), reported to have a large $i_\star$ and to host multiple transiting planets, remains certainly accurate because the analysed split component of the $l = 1$ modes are clearly well resolved and of high signal-to-noise ratio (see their Figure 1). However, results from Corsaro et al. (2017) on spin alignment of star clusters may require a careful interpretation because they fit different modes independently and determine a posteriori the stellar inclination. In addition, the clusters consist of faint stars with modes of relatively low amplitudes. As suggested by Figure 3, this may bias stellar inclinations towards ~ 30 degrees. This indicates the importance of studying a potential bias on stellar inclination for subgiants and red-giants as we have performed for main-sequence stars.

7 SUMMARY

The measurement of the stellar inclination angle $i_\star$ is particularly important to probe the spin-orbit alignment of transiting exoplanetary systems in an independent and complementary manner to the projected angle $\lambda$ from Rossiter-McLaughlin measurement (Ohta et al. 2005). The statistical distribution of $i_\star$ and $\lambda$ provides a quantitative test for theories of origin and evolution of planetary systems.

While the majority of transiting exoplanets are found around F, G, and K type stars in their main-sequence phase, those are harder to measure $i_\star$ compared to evolved solar-like stars (red-giants and subgiants). This is mostly due to the relatively lower oscillation amplitude and the severe mode blending of main-sequence solar-like stars. Therefore, it is of fundamental importance to perform a systematic verification of the reliability of $i_\star$ derived asteroseismology for those stars.

We generated 3000 simulated oscillation power spectra scaled from a reference star KIC 12069424 (16 Cyg A) that span a wide range of the height-to-background ratio, rotational splitting $\delta \nu_\star$, and inclination angle $i_\star$, each for 1 year and 4 years observation duration. Then we performed systematic mock simulations of asteroseismic analysis, and examined the reliability of $i_\star$ derived from asteroseismic analysis with a Bayesian-MCMC sampling method.

We find that the low signal-to-noise ratio of the power spectra induces a systematic under-estimate (over-estimate) bias for stars with high (low) inclinations. The combination of analytical consideration and mock simulation results revealed three empirical criteria on $(i_\star, \delta \nu_\star / \Gamma)$ plane as a function of the power, height-to-background ratio HBR, and the observation duration $T_{\text{obs}}$, which are required for a reliable estimate of $i_\star$. The criteria indicate that reliable measure-
ments are possible in the range of $20° \lesssim \lambda \lesssim 80°$ for stars with high HBR, high $\delta v_i/\Gamma$, and/or longer $T_{obs}$.

We also performed asteroseismic analysis of 94 main-sequence stars in Kepler short cadence data using the same Bayesian-MCMC sampling method; 33 and 61 are stars with and without known planetary companions, respectively. We find that 9 stars with planet and 22 stars without planet satisfy the above criteria.

The stellar inclination and rotation, $i_*$ and $\delta v_*$, that we derived asteroseismically for those Kepler stars are compared with those derived photometrically and spectroscopically. We find that our asteroseismic $v \sin i_*$ is in agreement with the average of two independent spectroscopic analysis by Bruntt et al. (2012) and Molenda-Zakowicz et al. (2013). This suggests that a careful modelling of macroturbulence is crucial in estimating $v \sin i_*$ from spectroscopic data, especially for slowly rotating stars.

The rotation period $P_{rot}$ derived from the photometric variability of the stellar light curve shows reasonable, even if not good, agreement with $\delta v_*$. The combined estimate of $i_*$, however, is very limited both observationally and statistically, and does not show strong agreement with its asteroseismic estimate at this point, indicating that further quantitative study is necessary. The statistical discussion and implications of our asteroseismic result for the Kepler stars will be presented in a future study.

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