Generalized mathematical model of red muds’ thickener of alumina production

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Abstract. The article describes the principle of a generalized mathematical model of the red mud’s thickener construction. The model of the red muds’ thickener of alumina production consists of sub-models of flocculation zones containing solid fraction feed slurry, free-fall and cramped sedimentation zones or effective sedimentation zones, bleaching zones. The generalized mathematical model of thickener allows predicting the content of solid fraction in the condensed product and in the upper discharge. The sub-model of solid phase aggregation allows one to count up average size of floccules, which is created during the flocculation process in feedwell. The sub-model of the free-fall and cramped sedimentation zone allows one to count up the concentration profile taking into account the variable cross-sectional area of the thickener. The sub-model of the bleaching zone is constructed on the basis of the theory of the precipitation of Kinc, supplemented by correction factors.

1. Introduction
Repartition of the condense process and leaching of red muds is one of the most important repartitions of alumina production [1]. Uncontrolled repines and big inertness in the process of information obtainement significantly complicate the administration task.

The main technological parameter which defines the work of the red mud’s thickener is turbidity of the upper discharge of the thickener. The functions of the existing slurry thickening control systems consist in stabilizing the parameters of the primary process loops, such as volumetric pulp flow, volumetric flow of thickened slurry, volumetric flocculant consumption, solid content in the slurry, etc. But the process of condensation is characterized by a large number of output indicators, many of which are highly correlated with each other and controlled by single-loop algorithms and manually inefficient.

The actual direction of the scientific research is the creation and improvement of models and control systems for the thickening process, but existing models do not fully take into account the presence of disturbing effects, in particular particle size distribution in nutrition, the size of flocculated particles, the shape of the floccula formed, overgrowth and thickening of the thickener. The generalized mathematical model presented in this paper describes the behavior of the suspension in the main zones of the thickener.

2. Flocculation zone model
The generalized model of the thickener includes a sub-model of the process of aggregation of particles, a sub-model of the zone of free deposition and compaction of the sediment, a sub-model of
the bleaching zone.

The model of the flocculation zone allows calculating the average size of floccules $D_{\text{floc}}$ depending on the volume flow rate of flocculant $Q_{\text{floc}}$ and its dilution. Dependence was determined by the results of numerical experiments in the program "The model of the population balance of Excel" ("Project AMIRA p266: Improving the technology of thickening") and has the following view:

$$D_{\text{floc}} = k \cdot (840.2 + 0.701 \cdot Q_{\text{floc}} - 250.7 \cdot \varphi_0 - 0.00019 \cdot Q_{\text{floc}}^2 - 0.081 \cdot Q_{\text{floc}} \cdot \varphi_0 + 22.8 \cdot \varphi_0^2)$$  \hspace{1cm} (1)

where $k$ is the coefficient of floccule sphericity; $\varphi_0$ is the content of the solid fraction in the feeding pulp. The program is based on the equation of balance and population dynamics - the equation of the population balance (PBE - the equation of the balance of the population).

At the heart of the program there are the equations of balance and dynamics of populations. The average diameter of the floccule is 158 $\mu$m; the distribution range of the floccule in the feeding glass after flocculation for the test process is 10 - 600 $\mu$m (Figure 1).

$$\varphi = - \frac{\Delta \rho \varphi_{\text{uf}}}{S(z) \sigma_f} \varphi_f$$ \hspace{1cm} (2)

where $\varphi$ – content of solid fraction, $Q$ – volumetric flow of pulp, $\Delta \rho = \rho_S - \rho_L$, $\rho_S$ – solid phase density, $\rho_L$ – liquid phase density, $\varphi_{\text{uf}}$ – content of solid fraction in condensed product, $S(z)$ - cross-sectional area of the thickener, $\sigma_f$ – limit of liquidity under compression, $f_{\text{bk}}$ – Kinch density function.

The basic equation, which describes speed of free-fall sedimentation of spherical particles in liquid environment is Stokes equation [3]:

$$U_{\infty} = \frac{D_{\text{floc}}^2 \cdot g \cdot \Delta \rho}{18 \cdot \mu}$$ \hspace{1cm} (3)

where $\mu$ – pulp’s astringency at operating temperature $- 0.0021$ Pa$\cdot$s, $D_{\text{floc}}$ – diameter of floccul, created in thickener’s feedwell.

When the concentration of solid particles in liquid becomes high enough (occurs transition in cramped sedimentation regime), Stokes and Batchelor’s equation ceases to work [2, 3, 4, 5].

In the presence of suitable conditions, particles in suspensions can interact, forming floccules, consisted of solid particles and liquid. Further this congregates can transform in a larger and stable state of matter. To calculate the final density of such aggregate formations, it is necessary to take into account it both within them and between them. The first time Michael and Bolger in 1962 suggested such emissions in the water. Predicting or calculating the thickness of this layer is quite difficult. The
The amount of liquid depends on the degree of concentration of the floccule, as well as the rate of their deposition. The more complex is the form of aggregate formation (the form of floccule), the greater is the value of the index, which, in turn, presupposes the presence of more fluid. This outer layer of liquid should not be confused with the liquid in each bunch. The number of such liquids is also an important variable, and its calculation is an important parameter for optimizing the deposition process, as Nuke wrote in 1986. There is a lot of controversy about "enveloping" the liquid of particles or "stagnating" around them, flowing liquid "through" particles or "over" it, but the question on this topic remains relevant [2, 3, 4, 5].

The velocity in the zone of the constrained deposition of $U_S$ is described using the Richardson and Zaki empirical equation of 1954 [2, 3]:

$$U_S = U_{\infty}(1 - \varphi)^n \quad (4)$$

where $n$ - index of cramped sedimentation (Richardson-Zaki index) $= 89.39$.

The function of Kinch density is equal to [2, 3]:

$$f_{pk}(\varphi) = U_{\infty}\varphi(1 - \varphi)^n \quad (5)$$

Effective compression of solid substance can be expressed exceptionally like a concentration function. The only way to transfer the moment directly from the particle to the particle during deposition is through the collision of particles. Therefore, for solids concentrations below the critical level, the particles in the liquid are in a suspended state, effective compression is a constant. With a concentration above the critical concentration, the formed particle structure transfers forces directly between the particles in the sediment. This phenomenon can be characterized by the following constraints for effective compression of a solid or yield stress with compression $\sigma_e(\varphi)$ [2, 3]:

$$\sigma_e(\varphi) = \begin{cases} 
\text{Const}, & \varphi < \varphi_c \\
\sigma_0 \left( \frac{\varphi}{\varphi_c} \right)^c - 1, & \varphi \geq \varphi_c 
\end{cases} \quad (6)$$

where $\sigma_0$, $c$ – the characteristic coefficients of the yield stress for the compression of the sediment are 2 Pa and 6.5 respectively, $\varphi_c$ – the critical concentration or the gel point – 0.028 v/v for the control object under study.

The stationary state in the unloading zone, $z = 2.35$ m, describes the content of the solid phase in condensed product $\varphi_{UF}$. The calculation of the concentration profile occurs in the direction of decreasing depth, provided that $\varphi_{UF} > \varphi_c$; otherwise the process is modeled as a constrained precipitation with a constant concentration in all zones.

The stationary state in the unloading zone, $z = 2.35$ m, describes the content of the solid phase in condensed product $\varphi_{UF}$. The model is solved using an algorithm developed in the software product MatLab and includes:

- model of the flocculation process;
- correction factor for the calculation of the Stokes velocity of the floccules. The limiting velocity of the particle motion is used to calculate the Reynolds number of the particle, from which the correction coefficient is derived from the correlation relations derived for mass transfer data to fixed spherical particles;
- coefficient of sphericity of the floccule, affecting the concentration profile in the thickener (Figure 2a);
- calculation of the concentration profile in the thickener with a variable cross-sectional area $S(z)$, i.e. the model takes into account the cylindrical and conical parts of the apparatus.

The concentration profile obtained with the same initial data for the cylindrical and cylindrical cone thickener is shown in Figure 2b. In the developed model, the height of the thickener bed is calculated from the concentration profile [2-5].
4. Model of bleaching zone

The concentration of solid fraction $C_{OF}$ in bleached feedwell of the thickener is expressed in following way [3]:

$$C_{OF} \approx K \cdot b_{PSD} \cdot \frac{Q_{F} \cdot \varphi_{F}}{S_{0} \left( S_{0} + \sum S_{x} \right)}$$  \hspace{1cm} (7)

where $Q_{F}$ – volumetric flow of feed pulp, $\varphi_{F}$ – content of solid in feeding, $Q_{D}$ – volumetric expense of condensed product, $S_{0}$ – the cross-sectional area of the cylindrical part of the thickener, $K$, $b_{PSD}$ – dimensionless adjusting coefficient, physical meaning of which is described below in this paper.

Coefficient $b_{PSD}$ allows one to consider the influence of the distribution range of floccules, emerging in the fluctuation process in feedwell, on massive expense of solid fraction with upper discharge of the thickener. It was obtained from the results of a series of experiments in the software complex ANSYS Fluent when comparing the precipitation of suspensions, containing monodisperse and polydisperse particles and equal to 1.17 for the object under study.

Coefficient $K$ reflects the proportional dependence of the yield of the solid fraction in the clarified discharge on the concentration of solid fraction at the feed point for the sample thickener, which is characterized by the concentration of solid fraction in the upper sludge and the solid content in the condensed product within the specified ranges and maximum productivity. It depends on the process under study and can be determined by comparing calculated and archived data. For the process under study, $K = 0.14$.

5. Mathematical model of a thickener

The thickener is an inertial object. The stationary nonlinear model solves the problem of one-dimensional modeling of the process of precipitation of flocculated suspension in a thickener, described by a strictly degenerate diffusion equation with flow discontinuities.

For the subsequent synthesis of a control system based on a regulator with a predictive model, it is necessary to use a dynamic model, since the main principle of the predictive regulator's work - the forecast is a few steps ahead.

The structure of the dynamic generalized thickener model is represented by a simple Wiener-Hammerstein cascade model and consists of series-connected dynamic linear elements (DLEs) represented by two first-order aperiodic links and a static nonlinear element (SNE) represented by a stationary nonlinear model expressed by equations (1) - (7). The DLE model reproduces the dynamic properties of the control object under study; the SNE makes it possible to calculate the gain by various channels, depending on the values of the state variables of the object.
Figures 3 and 4 are based on calculations of the generalized model and archive data on the parameters "concentration of solid fraction in the upper sink" and "solid content in the condensed product".

The coefficient of linear correlation $R$ of experimental and calculated data on the parameter "concentration of solid fraction in the upper discharge" is 0.8368; in the parameter "solid fraction content in the condensed product", it is 0.8447.

6. Conclusion

The developed generalized dynamic model includes a solid phase flocculation model, a model of a restricted deposition zone and effective sediment compression, a clarification zone model and predicts a solid fraction content in the lower product in the range of 0.01-0.10 v/v, in the upper plume in the range of 0-1000 mg / l.

The sub-model of the solid phase aggregation zone allows calculating the average size of the flocules formed during flocculation in the feeding glass. The sub-model of the zone of restricted deposition and compaction of the sediment makes it possible to calculate the concentration profile taking into account the variable cross-sectional area of the thickener. The modeling of the process of thickening the conical part of the thickener for the control object under study allows a 30% reduction in the error in calculating the height of the bed for the thickener under study.

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