Observational constraints on non-minimally coupled Galileon model

Mubasher Jamil1,a, Davood Momeni2,b, Ratbay Myrzakulov2,c

1Center for Advanced Mathematics and Physics (CAMP), National University of Sciences and Technology (NUST), H-12 Islamabad, Pakistan
2Eurasian International Center for Theoretical Physics, L.N. Gumilyov Eurasian National University, Astana 010008, Kazakhstan

Abstract As an extension of Dvali–Gabadadze–Porrati (DGP) model, the Galileon theory has been proposed to explain the “self-accelerating problem” and “ghost instability problem”. In this paper, we extend the Galileon theory by considering a non-minimally coupled Galileon scalar with gravity. We find that crossing of the phantom divide line is possible for such model. Moreover we perform the statefinder analysis and Om(z) diagnostic and constrain the model parameters from the latest Union 2 type Ia Supernova (SNe Ia) set and the baryonic acoustic oscillation (BAO). Using these data sets, we obtain the constraints $\Omega_{m0} = 0.263^{+0.031}_{-0.031}$, $n = 1.53^{+0.21}_{-0.37}$ (at the 95% confidence level) with $\chi^2_{\text{min}} = 473.376$. Further we study the evolution of the equation of state parameter for the effective dark energy and observe that SNe Ia + BAO prefers a phantom-like dark energy.

1 Introduction

The observational constraints can be used to probe the equation of state (EoS) of dark energy $w_X$ using the Supernovae Ia (SNe Ia) data [1–3], Cosmic Microwave Background radiations (CMB) [4, 5] and Baryonic Acoustic Oscillations (BAO) [6, 7].

Over the past decade, different kinds of the dynamical dark energy model have been discussed (see Refs. [8–11] for review). Some popular models are quintessence [12–14], $f(R)$ gravity [15–32], scalar field models [33–39], the Dvali–Gabadadze–Porrati (DGP) braneworld [40, 41] scenario, modified gravities [42], Gauss–Bonnet gravity [43–49], $f(R, G)$ gravity [50–62], $f(R, T)$ gravity (here $T$ is the trace of the energy-momentum tensor) [63–78], and so on. Physically we need to find an effective gravitational action, which can recover Einstein gravity [79–81]. These modifications must be free from any extra degree of freedoms due to the ghost [82–87]. In modified $f(R)$ theory and its reduction to scalar field models, we need to be careful in picking the mathematical forms of $f(R)$ or the field potentials functions in order to have compatibility with astrophysical observations [88, 89].

The scalar mode of the DGP theory is due to a longitudinal mode of a free massless spin 2 graviton with self interaction, $\Box \phi (\partial^\mu \phi \partial_\mu \phi)$, which is the mixing with the transverse graviton [90, 91]. The physical mechanism which is hidden behind such decoupling is the so-called Vainshtein mechanism [92]. It means that it is possible to recover Einstein gravity in a region of spacetime of the size of solar scales. The graviton interaction term of the form $\Box \phi (\partial^\mu \phi \partial_\mu \phi)$ satisfies the non-Lorentzian invariant form of the classical boost symmetry, and it resembles the Galilean local boost transformation $\nabla_\mu \phi \rightarrow \nabla_\mu \phi + c_\mu$, in flat spacetime.

The non-relativistic model, based on the Galilean symmetry, is called the “Galileon” [93]. It has been shown that there are only five field Lagrangians $L_i$ ($i = 1, \ldots, 5$) which are invariant under the Galilean symmetry. Their discussion was based on the Minkowski background. The equation of the motion (EOM) derived from this action is second-order. Consequently, the model seems free from extra unphysical degenerated modes.

The plan of this paper is as follows. In Sect. 2, we introduce our proposed model of non-minimal Galileon cosmology. In Sect. 3, we perform the statefinder and Om diagnostics on the model. In Sect. 4, we discuss the observational constraints on our model. In Sect. 5, we provide the conclusion of our paper.
2 Non-minimal Galileon cosmology

The covariant Galileon action reads [93]

\[
S = \int \! d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} (1 - \epsilon k^2 \pi^2) R + \frac{1}{2} \sum_{i=1}^{5} c_i L_i \right] + \int \! d^4x \mathcal{L}_M,
\]

(1)

with \( g \) as \( \text{det}(g_{\mu\nu}) \) in units of \( k^2 = 8\pi G \), and the Galileon coupling constants \( c_i \) are constants. The covariant Lagrangians \( L_i \) \( (i = 1, \ldots, 5) \) are given by [93]

\[
L_1 = M^3 \pi, \quad L_2 = (\nabla \pi)^2, \quad L_3 = (\Box \pi)(\nabla \pi)^2/M^3, \\
L_4 = (\nabla \pi)^2 [2(\Box \pi)^2 - 2\pi_{ij\mu} \pi^{ij\mu} - R(\nabla \pi)^2/2]/M^6, \\
L_5 = (\nabla \pi)^2 [(\Box \pi)^3 - 3(\Box \pi) \pi_{ij\mu} \pi^{ij\mu} + 2\pi_{ij\mu} \pi^{ij\nu} \pi^{\nu \rho} - 6\pi_{ij\mu} \pi^{ij\nu} \pi^{\nu \rho} G_{\nu\rho}]/M^9,
\]

(2)

where \( M \) is the mass parameter of Galileon model. Using the following standard metric:

\[
d^2s^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \right],
\]

(3)

the equations of motion read

\[
3H^2 = k^2 (\rho + \rho_m + \rho_r + \rho_K + \rho_G), \\
3H^2 + 2\dot{H} = -k^2 (P + \rho_r/3 + \rho_K/3 + P_{\xi}), \\
\dot{\rho}_m + 3H \rho_m = 0, \\
\dot{\rho}_r + 4H \rho_r = 0,
\]

(4-7)

\[
\rho_{\xi} = 6\xi \epsilon \pi H \dot{H} + 3\xi \epsilon \pi^2 H^2, \\
P_{\xi} = \frac{\epsilon}{2} (1 - 4\xi) \dot{\pi}^2 + 2\epsilon \xi \dot{\pi} - 2\xi (1 - 6\xi) \dot{H} \pi^2 - 3\xi \epsilon (1 - 8\xi) H^2 \pi^2 - \frac{1}{2} c_1 \xi \pi
\]

(8)

\[
\rho_{DE} = -c_1 M^3 \pi /2 - c_2 \dot{\pi}^2 /2 + 3c_3 H \dot{\pi}^3 /M^3 - 45c_4 H^2 \dot{\pi}^4 / (2M^6) + 21c_5 H^3 \dot{\pi}^5 /M^9,
\]

(9)

\[
P_{DE} = c_1 M^3 \pi /2 - c_2 \dot{\pi}^2 /2 - c_3 \dot{\pi}^2 H /M^3 + 3c_4 \dot{\pi}^3 [8H \dot{\pi} + (3H^2 + 2H) \ddot{\pi}] / (2M^6) - 3c_5 H \dot{\pi}^4 [5H \ddot{\pi} + 2(H^2 + H) \dot{\pi}] /M^9.
\]

(10)

The solutions of (6) and (7) are, respectively, given by

\[
\rho_m = \rho_{m0} a^{-3}, \quad \rho_r = \rho_{r0} a^{-4}.
\]

(11)

(12)

We must write (5) in the following form:

\[
(H^2)^{1/2} = -k^2 \left( \rho + P_{DE} + P_{DE} + \frac{4}{3} \rho_m + \rho_r + P_{\xi} \right),
\]

(13)

in this new representation the derivatives are written with respect to the e-folding \( N = \ln a \). Figure 1 shows the time evolutionary scheme of the metric and the Galileon gauges, numerically. Also, the agreement of the Hubble parameter in our model and LCDM model is obviously manifested on the right panel.

![Fig. 1](image-url) (Left) Time evolution of scale factor \( a(t) \) and Galileon scalar \( \pi \). (Right) Time evolution of the Hubble parameter \( H(t) \)
3 Statefinder analysis and $Om$ diagnostic

In order to classify the different dark energy models, Sanhi et al. [94, 95] proposed a geometrical diagnostic method by considering higher derivatives of the scale factor. The statefinder parameters \{r, s\} are defined

$$r \equiv \frac{\dddot{a}}{aH^3},$$

$$s \equiv \frac{r - 1}{3q - 1/2},$$

where $q \equiv -\frac{1}{H^2} \frac{\dddot{a}}{a}$ is the deceleration parameter. Apparently, $\Lambda$CDM model corresponds to a point \{1, 0\} in \{r, s\} phase space. The statefinder diagnostic can discriminate different models. For example, it can distinguish the quintom from other dark energy models [97]. On the panel of Fig. 1, we observe that the behavior of Hubble parameter can be approximated as

$$H \propto \frac{1}{t^n}, \quad n \geq 1.$$  \hspace{1cm} (16)

With this ansatz form, the behavior of statefinder parameters is

$$r = \frac{n^2 - 3n + 2}{n^2}, \quad s = \frac{2(3n - 2)t^n}{3n(n^2 - 2)}.$$  \hspace{1cm} (17)

For very far future, $t \to \infty$,

$$r = 1 - \frac{3n - 2}{n^2}, \quad s = \frac{2(3n - 2)}{3n^2},$$

which can be combined as

$$r = 1 - \frac{3s}{2}. \hspace{1cm} (19)$$

From (16), the scale factor evolves like

$$a(t) = a_0 \exp\left(\frac{t^{1-n} - t_0^{1-n}}{1-n}\right). \hspace{1cm} (20)$$

From this expression we obtain the Hubble parameter as a function of redshift:

$$H(z) = H_0 \left[1 + t_0^{n-1}(n-1)\log(1+z)\right]^\frac{n}{n-1}. \hspace{1cm} (21)$$

We assume that $t_0 = 1$. So, the parameter for our model is $n$. Indeed, $n = n(\xi)$. We interpolate

$$n(\xi) = \sum_{m=1}^{\infty} c_m \xi^m, \quad \xi \in \mathbb{Z}. \hspace{1cm} (22)$$

It is very interesting to investigate the behavior of the (20) in the limit of $n \to 1$. In this case for (20) we have

$$a(t) = a_0 \left(\frac{t}{t_0}\right), \quad H(t) = \frac{1}{t}. \hspace{1cm} (23)$$

The $Om(z)$ is another diagnostic of dark energy proposed by Sahni et al. [98]. It is defined as

$$Om(z) \equiv \frac{E^2(z) - 1}{(1+z)^3}.$$  \hspace{1cm} (24)

We define $E^2 = H^2/H_0^2$. Obviously, this diagnostic parameter depends only to the first derivative of the luminosity distance $D_L(z)$. We are able from this diagnostic to discriminate different dark energy models by interpolating the geometrical slope of $Om(z)$, although we do not know the precise value of $\Omega_m$.

Figure 2 shows different graphs of the deceleration parameter and effective EoS by indicating the DE behavior in the phantom era. Also the statefinder analysis is presented in Fig. 3.

![Fig. 2](image)

Fig. 2 (Left) Time evolution of deceleration parameter $q$. (Right) Time evolution of equation of state parameter $w_\pi$. 

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Fig. 3 The evolutionary curves of statefinder pair \((r, s)\) (left), pair \((r, q)\) (middle) and \(\Omega_m(z)\) (right) for (21) with \(\Omega_m^0 = 0.278, -1 < z < 4\). Here we plot for \(1.2 < n < 1.5\) and \(0.67 < \xi < 0.82\).

Fig. 4 The 1\(\sigma\) and 2\(\sigma\) contours for \((\Omega_m^0, n)\) parameter space arising from the SNe Ia + BAO. The constraints on Model (21) from SNe Ia + BAO. The regions corresponds to 1\(\sigma\) (blue-left), 2\(\sigma\) (yellow-right) confidence regions. Here red is the background color (Color figure online)

4 Observational constraints

We will now discuss the constraints on our model parameter \(n\) which appeared in (21) with (13). Here we perform the data analysis using SNe Ia, BAO and SDSS. For the SNe Ia data, we use the Union 2 compilation released by the Supernova Cosmology Project collaboration recently [99]. First we must review these data sets (see Appendix A of [96] for a review).

In (2010), the Supernova Cosmology Project collaboration [99] reported the Union 2 compilation, which consists of 557 SNe Ia data points. In fact this is the largest reported and spectroscopically confirmed SNe Ia sample. We use it to constrain the theoretical models in this paper based on the model (5). As usual, the results can be obtained by minimizing the \(\hat{\chi}^2\):

\[
\hat{\chi}^2 = \sum_{i=1}^{557} \frac{(\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i))^2}{\sigma_{\mu,i}^2},
\]

where \(\sigma_{\mu,i}^2\) are the errors due to the flux uncertainties, intrinsic dispersion of SNe Ia absolute magnitude and peculiar velocity dispersion. The luminosity distance \(D_L\) can be calculated by [100, 101]

\[
D_L \equiv (1 + z) \int_0^z \frac{dz'}{E(z')}.
\]

Calculating the \(\chi^2_{\text{SNe}}\), we find that the best fit values occur at \(\Omega_m^0 = 0.271, n = n(\xi) = 1.52\) with \(\chi^2_{\min} = 481.272\). The results have been presented in Fig. 4 for different confidence limits.

Now, we use the BAO data. The parameter \(A\) is represented using the BAO peak [100]. The constraints from SNe Ia + BAO are given by minimizing \(\chi^2_{\text{SNe}} + \chi^2_{\text{BAO}}\). The results are \(\Omega_m^0 = 0.263^{+0.031}_{-0.031}, n = 1.53^{+0.21}_{-0.37}\) (at the 95 % confidence level) with \(\chi^2_{\min} = 473.376\).
In this paper, using the observational data of SNe Ia + BAO, we constrained a non-minimally coupled Galileon gravity with Lagrangian \( L = \frac{L}{16\pi^2} \left( 1 - \kappa^2 \xi^2 \pi^2 \right) R + \frac{1}{2} \sum_c \xi_i \mathcal{L}_i \). Compared with references, we can see the constraint has the effect that SNe Ia + BAO data give can be compatible with other data (SNe Ia, H(z), BAO, CMB and so on). The SNe Ia + BAO data propose a new way to probe the cosmology of the Galileon fields. As we expect, the SNe Ia + BAO data alone cannot give a stringent constraint. There are at least some aspects that contribute to the error. Combining with SNe Ia and BAO, it gives \( n = \sum c_i \xi_i \approx 1.5 \), which contains the CDM model. Until now, we cannot distinguish it from the standard cosmology. For future study, in order to improve the constraint, we hope large survey projects can find more data. At the same time, a better understanding of the non-minimally coupled Galileon model can give us more stringent results and more information about Galileon gravity. We like to mention here that we have followed largely the exposition given in the Wu and Yu paper [102].

5 Conclusions

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