Calculation of rocks permeability based on periodic models of porous media

D E Igoshin\textsuperscript{1,2} and D Yu Legostaev\textsuperscript{1,2}

\textsuperscript{1}Tyumen Branch of Khristianovich Institute of Theoretical and Applied Mechanics SB RAS, Taymyrskaya str., 74, Tyumen, 625026, Russia
\textsuperscript{2}Tyumen State University, 6 Volodarskogo st., 625003 Tyumen, Russia

igoshinde@gmail.com

Abstract. The paper proposes a method for calculating rock permeability based on periodic cubic models of porous media, taking into account the particle size distribution of the samples. Conditional permeability is calculated for each fraction. The permeability of the sample depends on the relative position of the fractions and can be calculated as the weighted average (arithmetic, geometric or harmonic). Next, an optimization problem is solved, which allows one to choose the “weight” of a particular method of averaging permeability over fractions inside the sample for all samples of the selected formation. Results were obtained for several formations from different fields.

1. Introduction
The main filtration-capacitive characteristics of porous media are porosity and permeability (absolute and phase). In the study of rocks, an important measured quantity, in addition to porosity and permeability, is the particle size distribution. The permeability of the rock is a complex function, determined by the granulometric composition of the grains composing the rock, their shape and relative position. As a rule, core data are many points on the “porosity – permeability” diagram, with a pronounced positive correlation [1].

2. Particle size distribution of core
The particle size distribution of the core sample is a histogram of the percentage of grains for \(N\) fractions. In Figure 1 shows the particle size distribution averaged over approximately two hundred samples from the UZ formation of one of the deposits in Western Siberia \((N = 14)\). Additionally, the function of the density distribution of grain sizes is given, taking into account the average size of the fractions and the width of their ranges. It can be seen that the highest distribution density is realized for grains less than 1 \(\mu m\) in size; for grains larger than 100 \(\mu m\), the distribution density drops sharply. A practically important task is the calculation of the permeability of core samples taking into account their particle size distribution.

3. Porosity and permeability of cubic structures
In this paper, we use the approach for determining the permeability of rocks using model periodic porous media described in [1, 2] with a skeleton of spherical segments based on cubic lattice systems: primitive cubic (PC), body-centered cubic (BCC) face-centered (FCC). The model parameters in these structures are the side of the cube \(L\) and the degree of intersection of the spheres \(\alpha\) (dimensionless). The table 1 shows the analytical expressions for the porosity of cubic structures and its limiting values.
At [4; 5] for the structures under consideration, hydrodynamic modeling of the stationary flow of a viscous incompressible fluid was carried out. Based on the numerical solution of the Navier – Stokes system of equations and the Darcy law for media formed by these structures, the permeability is determined depending on the dimensionless parameter $\alpha$: $k = k(\alpha)$ and porosity $m$: $k = k(m)$ at $L = 10^{-5}$ m.

**Figure 1.** The average particle size distribution in the reservoir (left) and the function of the grain density distribution over the average size of the fractions (right).

**Table 1.** Porosity of cubic structures.

| Structure | Porosity, $m = m(\alpha)$ | $m(0)$ | $\alpha^*$ | $m(\alpha^*)$ | $\beta^*$ |
|-----------|--------------------------|--------|-----------|---------------|----------|
| PC        | $1 - (\pi/12) \cdot \left[ 2 - 3\alpha^2(3 - \alpha) \right] / (1 - \alpha)^2$ | 0.4764 | 0.2929 | 0.0349 | 1 |
| BCC       | $1 - \left( \pi \sqrt{3}/8 \right) \cdot \left[ 1 - 2\alpha^2(3 - \alpha) - 1.5\alpha^2(3 - \alpha_2) \right] / (1 - \alpha)^2$ | 0.3198 | 0.1835 | 0.0055 | 3/4 |
| FCC       | $1 - \left( \pi \sqrt{3}/2 \right) \cdot \left[ 1 - 3\alpha^2(3 - \alpha) \right] / (1 - \alpha)^2$ | 0.2595 | 0.134 | 0.0359 | 1/2 |

According to the $\pi$-theorem, the permeability of the medium can be represented in the form of products of a dimensionless function and a scalable parameter.

$$k = d^2 \bar{k}.$$  \hfill (1)

The reduced permeability $\bar{k}$ depends only on the type of structure and does not depend on the characteristic grain size. For cubic periodic structures

$$d = \beta L / (1 - \alpha),$$  \hfill (2)

where $\beta$ value is given for each type of cubic structure in the table.

Based on system (1) – (2), we formally write down the permeability values of the $i$-th fraction and the values determined numerically in [3, 4]

$$k_i = d_i^2 \bar{k}, \quad k_{num} = \left[ \beta L / (1 - \alpha) \right]^2 \bar{k},$$

where $d_i$ is the average diameter of the $i$-th fraction, whence the conditional permeability of the $i$-th fraction is

$$k_i = \left[ (1 - \alpha) d_i / \beta L \right] k_{num}.$$  \hfill (3)

**4. Permeability at different locations of fractions**

If the fractions in the rock are located in parallel, then its permeability reaches a maximum and is determined by the arithmetic mean weighted by fractions. With a random mutual arrangement of fractions, the permeability is determined by the weighted geometric mean. With a sequential (layered)
arrangement of fractions, the permeability is determined by the weighted average harmonic and takes a minimum value.

Figure 2 shows the weighted arithmetic mean (WAM), weighted geometric mean (WGM) and weighted harmonic mean (WHM) permeability estimates, taking into account the particle size distribution based on the CP structure. It can be seen that the WHM permeability estimate is significantly closer to the core data than the other two estimates, which are 1-3 orders of magnitude higher. This is due to the fact that in the rock, large grains are usually surrounded by smaller grains that gravitate towards the necks of the pores and decisively affect the permeability of the medium. Moreover, the proportion of large pores is negligible. Flow through a porous media occurs mainly through a network of channels through small fractions.

\[
\sigma = \frac{k^c - k^f}{k^f} 
\]

\[
k^c = \bar{k}^\sigma \bar{k}^{1-\sigma}
\]

(4)

\[
\sigma = Am + B
\]

(5)

\[
J = \sum_{j=1}^{n} \frac{k_{f,j} - k_{c,j}}{k_{f,j}},
\]

(6)

indices \(f\) and \(c\) refer to actual and calculated values respectively, \(n\) is the number of core samples.
When approximating filtration-capacitive properties, the exponential dependence of permeability on porosity is often used:

\[ k = k_0 \exp \left( \frac{m}{m_0} \right) \]  

(7)

Figure 3 shows the results of the calculation of permeability for reservoir E1 and core data. It is seen that the proposed method, taking into account the particle size distribution, gives a better result than the exponential permeability estimate (7) based on the least squares method.

![Porosity-permeability diagrams for various averaging methods and for laboratory data.](image)

**Figure 3.** Porosity-permeability diagrams for various averaging methods and for laboratory data.

Table 2 shows the results of the optimization problem (4) - (6) and the parameters of the exponential dependence of permeability on porosity (7). In 6 cases out of 10, the proposed method gives the best results.

**Table 2.** Calculation results.

| Formation | E1 | E2 | E3 | P1 | P2 | N   | V1  | MV1 | MV2 | MZ  |
|-----------|----|----|----|----|----|-----|-----|-----|-----|-----|
| Cubic     |    |    |    |    |    |     |     |     |     |     |
| Object    | 1.558 | 1.586 | 1.329 | 0.556 | 0.277 | 0.551 | 1.906 | 0.452 | 1.776 | 0.472 |
| A         | 0.267 | 0.129 | 0.027 | 0.032 | 0.006 | 0.402 | -0.005 | 1.717 | -0.008 | 1389 |
| B         | -0.012 | 0.000 | -0.001 | -0.001 | 0.002 | -0.018 | 0.008 | -0.079 | 0.003 | -0.064 |
| Exponent  | 2.161 | 2.933 | 1.393 | 0.542 | 0.293 | 0.470 | 0.829 | 0.675 | 0.831 | 0.555 |
| k₀, md    | 3.8E-04 | 7.2E-04 | 1.7E-05 | 4.9E-04 | 1.8E-03 | 3.6E-04 | 2.1E-03 | 5.9E-03 | 6.6E-03 | 1.4E-02 |
| m₀        | 0.016 | 0.019 | 0.014 | 0.020 | 0.025 | 0.017 | 0.022 | 0.029 | 0.044 | 0.031 |

It can be assumed that a three-parameter model of a periodic medium [5] allows one to obtain a better approximation of permeability.
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