Exchange terms in the two–nucleon induced non–mesonic weak decay of $\Lambda$–hypernuclei

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Abstract

The contribution of Pauli exchange terms to the two–nucleon induced non–mesonic weak decay of $^{12}_\Lambda$C hypernuclei, $\Lambda NN \rightarrow nNN \ (N = n \ or \ p)$, is studied within a nuclear matter formalism implemented in a local density approximation. We have adopted a weak transition potential including the exchange of the complete octets of pseudoscalar and vector mesons as well as a residual strong interaction modeled on the Bonn potential. The introduction of exchange terms turns out to reduce the two–nucleon induced non–mesonic rate by 18% and, jointly with an increase in the one–nucleon induced rate by the same magnitude, reveals to be significant for an accurate determination of the full set of hypernuclear non–mesonic decay widths in theoretical and experimental analyses.

Key words: $\Lambda$–Hypernuclei, Non–Mesonic Weak Decay, $\Gamma_n/\Gamma_p$ ratio, Two–Nucleon Induced Decay, Pauli Exchange Terms

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Hypernuclei, bound systems of neutrons, protons and hyperons, embody an important source of information able to match nuclear and particle physics. On the one hand, studies on the production mechanisms and the structure of hypernuclei [1] are of interest since they provide indications on the hyperon–nucleon and hyperon–hyperon strong interactions which cannot be determined with precision from scattering experiments (such experiments are very challenging due to the very short hyperon lifetimes). Hypernuclear weak decay is on the other hand the only available tool to acquire knowledge on the baryon–baryon strangeness–changing interactions [2–5]. The above subjects, which presuppose the solution of complex many–body problems, are in turn crucially related to the renormalization of hyperons and mesons properties in
the nuclear medium and are relevant in connection with neutron star studies [6].

A Λ–hypernucleus weakly decays via two distinct modes: the mesonic decay, which concerns the direct disappearance of the hyperon, $\Lambda \rightarrow \pi^- p$ and $\Lambda \rightarrow \pi^0 n$, and the non–mesonic decay, which occurs through the hyperon interactions with one or more nucleons of the medium: $\Lambda N \rightarrow nN$, $\Lambda NN \rightarrow nNN$, etc, where $N = n$ or $p$. The total non–mesonic decay rate is indicated by $\Gamma_{NM} = \Gamma_1 + \Gamma_2$ in the present study, $\Gamma_1 = \Gamma_n + \Gamma_p$ and $\Gamma_2 = \Gamma_{nn} + \Gamma_{np} + \Gamma_{pp}$ being the one– and two–nucleon induced widths, respectively. Moreover, we introduce the definitions: $\Gamma_n \equiv \Gamma(\Lambda n \rightarrow nn)$, $\Gamma_p \equiv \Gamma(\Lambda p \rightarrow np)$, $\Gamma_{nn} \equiv \Gamma(\Lambda nn \rightarrow nnn)$, $\Gamma_{np} \equiv \Gamma(\Lambda np \rightarrow nnp)$ and $\Gamma_{pp} \equiv \Gamma(\Lambda pp \rightarrow npp)$.

The non–mesonic weak decay of hypernuclei has been studied quite extensively to date both theoretically [2–4] and experimentally [5]. In the latest years a substantial amount of data has been collected at the KEK laboratory [7] and by the FINUDA experiment at Daphne [8]. Future measurements will be carried out at J–PARC [9, 10] and GSI [11]. The development of innovative experimental techniques and elaborated theoretical models has allowed to reach a reasonable agreement between data and predictions for the non–mesonic decay rate $\Gamma_{NM}$, the $\Gamma_n/\Gamma_p$ ratio and the intrinsic asymmetry parameter $a_\Lambda$ [12]. Concerning the most recent developments, we point out the finding that the inclusion of a two–pion–exchange mechanism (in both the uncorrelated and correlated fashions) in the weak transition potential seems to play a crucial role in solving the asymmetry puzzle [13]. A recent and different approach must be mentioned which proved that the exchange of the axial–vector $a_1$–meson is also relevant in asymmetry calculations [14].

Despite this progress, one should observe that no experimental identification has been obtained yet of two–nucleon stimulated decays, with the exception of a couple of rather indirect results [15, 16]. An indirect signature was found in KEK single and double–coincidence nucleon spectra from non–mesonic decay by Ref. [15], which however was based on a rather simplistic analysis regarding a supposed phase space argument allowing a uniform sharing of the decay $Q$–value among the two or three final nucleons in one– or two–nucleon induced decays. Such analysis provided an indication for a ratio $\Gamma_2/\Gamma_1 \simeq 0.7$ for $^{12}\Lambda C$, while values scattered in the interval 0.2–0.5 were obtained theoretically [17–22]. Another experiment [16], performed at BNL, obtained an upper limit for $^{4}\Lambda He$, $\Gamma_2/\Gamma_1 < 0.32$ (95% CL), which seems to be incompatible with the indication of Ref. [15]. It is thus clear that improved theoretical and experimental determinations of $\Gamma_2$ are necessary. An experiment with this purpose is indeed planned at J–PARC [9] and will adopt double– and triple–coincidence nucleon measurements. Possibly, such kind of study will enable an improved determination of the whole set of non–mesonic decay widths. This is an essential matter for a further development of the field, aimed at achieving a detailed
understanding of the reaction mechanisms underlying the various weak decay channels.

Various theoretical papers were dedicated to the calculation of the rates for the two–nucleon stimulated decay [17–23]. Some works also analyzed the effects of this mechanism on the observable non–mesonic decay nucleon spectra [19, 21, 24, 25]. The first paper which took into account this decay mode was Ref. [17]. There, within a nuclear matter scheme based on the polarization propagator method of Ref. [26], a phenomenological description of the two–particle two–hole (2p2h) polarization propagator was introduced by adapting previous results by the same authors on electron scattering off nuclei to nuclear pion absorption. This approach was improved in Ref. [18] and then in Ref. [19]. Again, the 2p2h configurations were not calculated exactly in Refs. [18, 19], but a phase space argument together with data on real pion absorption in nuclei was adopted. Subsequently, Ref. [23] evaluated microscopically the non–mesonic weak decay rates by means of a path integral method which allowed a classification of the 2p2h contributions according to the so–called boson loop expansion. For technical reasons, it was not possible to separate the total width $\Gamma_{NM} = \Gamma_1 + \Gamma_2$ into the partial contributions $\Gamma_1$ and $\Gamma_2$ in Ref. [23]. We also point out that only the decay channel $\Lambda np \rightarrow nnp$ was considered in the discussions of Refs. [17–19, 23]. On the contrary, all the three two–nucleon induced channels were explicitly taken into account in the microscopic approach of Ref. [20], which evaluated the corresponding direct 2p2h $\Lambda$ self–energy diagrams induced by ground state correlations. The adopted weak transition potential included the exchange of the full pseudoscalar ($\pi$, $\eta$, $K$) and vector meson ($\rho$, $\omega$, $K^*$) octets while the residual strong interaction was described by a well–tested Bonn potential model embodying the exchange of $\pi$, $\rho$, $\sigma$ and $\omega$ mesons. As a result of the distinct approaches followed in Refs. [18, 20], the kinematics of the nucleons emitted in two–nucleon induced decays turned out to be very different in these two studies, as discussed in Ref. [21], resulting in final spectra which could be well discriminated by a future triple–nucleon coincidence experiment.

We would also like to stress that a proper determination of the widths and the nucleon spectra for two–nucleon induced decays is essential, in turn, for an accurate determination of the $\Gamma_n/\Gamma_p$ ratio (undoubtedly, analyses of this ratio are also influenced by nucleon final state interaction effects [24]). On the one hand, this was demonstrated by Refs. [21, 25] in theoretical analyses of experimental double–coincidence nucleon spectra which allowed to derive values of $\Gamma_n/\Gamma_p$ in agreement with pure theoretical calculations. The extracted values of $\Gamma_n/\Gamma_p$ turned out to be rather sensitive to the input used for $\Gamma_2$. On the other hand, among the various non–mesonic decay rates, only $\Gamma_{NM} = \Gamma_1 + \Gamma_2$ is directly accessible to experiments, where it is derived from measurements of the total lifetime and the mesonic rate as $\Gamma_{NM} = \Gamma_T - \Gamma_M$. Therefore, again one sees that the decay rate $\Gamma_2$ plays an essential role in the determi-
nation of $\Gamma_n/\Gamma_p$: only after a genuine disentanglement between $\Lambda N \rightarrow nN$ and $\Lambda NN \rightarrow nNN$ experimental events one can proceed to an analysis of the nucleon spectra and deduce the experimental value of $\Gamma_n/\Gamma_p$.

In the present Letter the effect of Pauli exchange terms in the two–nucleon induced hypernuclear decay is investigated for the first time. We shall apply the calculation to the hypernucleus $^{12}\Lambda C$. Exchange corrections are genuine quantum mechanical effects due to the underlying Fermi–Dirac statistics which are expected to be important in the present case as for any fermionic many–body system. Exchange terms, required by fermion antisymmetrization, turned out to be relevant in one–nucleon induced decays, both in tree level [27, 28] and in final state interaction (RPA–like) diagrams [29]. We employ the non–relativistic nuclear matter formalism extended to finite nuclei by the local density approximation which was established in Refs. [20, 28]. The weak transition potential, whose formulation and weak coupling constants are taken from Refs. [30], contains the exchange of the mesons of the pseudoscalar and vector octets, $\pi$, $\eta$, $K$, $\rho$, $\omega$ and $K^*$. The strong coupling constants and cut–off parameters entering the weak transition potential are instead deduced from the Nijmegen soft–core interaction NSC97f of Ref. [31]. For the nucleon–nucleon strong interaction entering the 2p2h correlations we adopt, as in Ref. [20], a Bonn potential with the exchange of $\pi$, $\rho$, $\sigma$ and $\omega$ mesons [32, 33]. All the two–nucleon stimulated channels, $\Lambda nn \rightarrow nnn$, $\Lambda np \rightarrow nnp$ and $\Lambda pp \rightarrow npp$, are included in the calculation.

For the formal derivation of the one–nucleon induced decay widths, which includes both direct and exchange contributions, we refer to the original discussion of Ref. [28].

Consider then the two–nucleon induced decay width for a $\Lambda$–hyperon with four–momentum $k = (k_0, \mathbf{k})$ inside infinite nuclear matter with Fermi momentum $k_F$. Let us write it in a schematic way as follows:

$$
\Gamma_2(k, k_F) = \sum_f |\langle f | V^{\Lambda N \rightarrow NN} | 0 \rangle_{k_F} |^2 \delta (E_f - E_0),
$$

(1)

$|0\rangle_{k_F}$ and $|f\rangle$ denoting, respectively, the initial hypernuclear ground state (whose energy is $E_0$) and the possible final 3p2h states (with energy $E_f$) corresponding to three–nucleon emission. Moreover, $V^{\Lambda N \rightarrow NN}$ is the weak transition potential.

The two–nucleon induced decay rate for a finite hypernucleus is obtained by the local density approximation [26], i.e., after averaging the above partial width over the $\Lambda$ momentum distribution in the considered hypernucleus, $|\tilde{\psi}_\Lambda(k)|^2$, and over the local Fermi momentum, $k_F(r) = \{3\pi^2\rho(r)/2\}^{1/3}$, $\rho(r)$
being the density profile of the hypernuclear core. One thus has:

$$\Gamma_2 = \int dk |\tilde{\psi}_\Lambda(k)|^2 \int dr |\psi_\Lambda(r)|^2 \Gamma_2(k, k_F(r)) ,$$

(2)

where for \( \psi_\Lambda(r) \), the Fourier transform of \( \tilde{\psi}_\Lambda(k) \), we adopt the \( 1s_{1/2} \) harmonic oscillator wave–function with frequency \( \hbar \omega = 10.8 \) MeV adjusted to the experimental energy separation between the \( s \) and \( p \) \( \Lambda \)–levels in \( ^{12}C \). The \( \Lambda \) hyperon total energy in Eqs. (1) and (2) is given by \( k_0 = m_\Lambda + k^2/(2m_\Lambda) + V_\Lambda \), i.e., it also contains an experimental binding term \( V_\Lambda = -10.8 \) MeV.

The final states in Eq. (1) are restricted to three–particle emission. Since \( V^{NN\rightarrow NN} \) is a two–body operator, two–nucleon induced decays originates from ground state correlations due to the nucleon–nucleon interaction. The normalized hypernuclear ground state wave–function can be written as [22]:

$$|0\rangle_{k_F} = \mathcal{N}(k_F) \left| \right. - \sum_{p_4,p_3,h_2,h_3} \frac{\langle p_4p_3h_2h_3|V^{NN}| \rangle_{D+E}}{\varepsilon_{p4} + \varepsilon_{p3} - \varepsilon_{h2} - \varepsilon_{h3}} |p_4p_3h_2h_3\rangle \right) ,$$

(3)

where \( \left| \right. \) is the uncorrelated ground state wave–function, i.e., the Hartree–Fock vacuum, while the second term in the rhs represents \( 2p2h \) correlations and contains both direct \( (D) \) and exchange \( (E) \) matrix elements of the residual strong interaction \( V^{NN} \). Besides, the particle and hole energies are denoted by \( \varepsilon_i \) and \( \mathcal{N}(k_F) \) is the normalization factor

$$\mathcal{N}(k_F) = \left( 1 + \sum_{p_4,p_3,h_2,h_3} \frac{|\langle p_4p_3h_2h_3|V^{NN}| \rangle_{D+E}|^2}{\varepsilon_{p4} + \varepsilon_{p3} - \varepsilon_{h2} - \varepsilon_{h3}} \right)^{-1/2} .$$

(4)

The particular labeling of Eqs. (3) and (4) is easily understood from the direct \( 2p2h \) \( \Lambda \) self–energy diagram of Fig. 1.

![Fig. 1. Direct Goldstone diagram contributing to the two–nucleon induced decay width.](image)

Inserting Eq. (3) into Eq. (1) one obtains:
$$\Gamma_2(k, k_F) = \mathcal{N}^2(k_F) \sum_{p_1, p_2, p_3, p_4, h_2, h_3} \left| \langle p_1 p_2 p_3 h_2 h_3 | V^{AN-NN} | p_\Lambda p_4 p_3 h_2 h_3 \rangle_{D'+E'} \right|^2 \times \frac{\langle p_\Lambda p_4 p_3 h_2 h_3 | V^{NN} | p_\Lambda \rangle_{D+E}}{\varepsilon_{p_4} + \varepsilon_{p_3} - \varepsilon_{h_2} - \varepsilon_{h_3}} \delta(E_{|p_1 p_2 p_3 h_2 h_3|} - E_0),$$

where also the matrix elements of the weak transition potential $V^{AN-NN}$ appears in the antisymmetrized form. In the $V^{NN}$ matrix elements the $\Lambda$ is acting as a spectator and so the particle $p_3$ and the holes $h_2, h_3$ in the $V^{AN-NN}$ matrix elements. In order to lighten the notation, we omit the spectators and write:

$$\Gamma_2(k, k_F) = \mathcal{N}^2(k_F) \times \sum_{p_1, p_2, p_3, p_4, h_2, h_3} \left[ \langle p_1 p_2 | V^{AN-NN} | p_\Lambda p_4 \rangle_{D'} + \langle p_1 p_2 | V^{AN-NN} | p_\Lambda p_4 \rangle_{E'} \right] \times \left[ \frac{\langle p_4 p_3 | V^{NN} | h_2 h_3 \rangle_D}{\epsilon_{2p2h}} + \frac{\langle p_4 p_3 | V^{NN} | h_2 h_3 \rangle_E}{\epsilon'_{2p2h}} \right] \delta(E_{|p_1 p_2 p_3 h_2 h_3|} - E_0),$$

where $\epsilon_{2p2h}$ and $\epsilon'_{2p2h}$ are the energy denominators which correspond to the direct and exchange matrix elements. In order to make more evident the set of terms contributing to the two–nucleon induced rate, it is convenient to expand the above expression as follows:

$$\Gamma_2(k, k_F) = \mathcal{N}^2(k_F) \times \sum_{p_1, p_2, p_3, p_4, h_2, h_3} \left[ \langle h_2 h_3 | (V^{NN})^\dagger | p_4 p_3 \rangle_D + \langle h_2 h_3 | (V^{NN})^\dagger | p_4 p_3 \rangle_E \right] \times \left[ \langle p_\Lambda p_4 | (V^{AN-NN})^\dagger | p_1 p_2 \rangle_{D'} + \langle p_\Lambda p_4 | (V^{AN-NN})^\dagger | p_1 p_2 \rangle_{E'} \right] \times \left[ \langle p_1 p_2 | V^{AN-NN} | p_\Lambda p_4 \rangle_{D'} + \langle p_1 p_2 | V^{AN-NN} | p_\Lambda p_4 \rangle_{E'} \right] \times \left[ \frac{\langle p_4 p_3 | V^{NN} | h_2 h_3 \rangle_D}{\epsilon_{2p2h}} + \frac{\langle p_4 p_3 | V^{NN} | h_2 h_3 \rangle_E}{\epsilon'_{2p2h}} \right] \delta(E_{|p_1 p_2 p_3 h_2 h_3|} - E_0).$$

The product of the four direct plus exchange pieces makes a total of sixteen contributions to the decay rate of Eq. (5). We identify each one of them by the notation $\Gamma_2^{PQRS}$, where $P, Q, R, S = D$ (direct) or $E$ (exchange). For instance, $P$ refers to the direct or exchange character of the matrix element $\langle h_2 h_3 | (V^{NN})^\dagger | p_4 p_3 \rangle_P$ (the meaning of $Q, R$ and $S$ is thus self–evident). Several of these contributions turn out to be equal among each other (for instance, $\Gamma_2^{DD'E'D} = \Gamma_2^{EE'D'E} = \Gamma_2^{ED'D'E} = \Gamma_2^{E'E'D'E} = \Gamma_2^{E'E'E'E}$) and there are a total of five different corresponding self–energy diagrams. They are depicted in Fig. 2 and amount to the following partial rates:
\[
\Gamma_{2}^{{dd'}{d'}} \equiv \frac{1}{4} (\Gamma_{2}^{D'D'D'D'} + \Gamma_{2}^{D'E'D'\bar{D}'} + \Gamma_{2}^{E'D'E'D'} + \Gamma_{2}^{E'E'E'\bar{E}'} ) = \Gamma_{2}^{D'D'D'},
\]
\[
\Gamma_{2}^{{dd'}{e'}} \equiv \frac{1}{4} (\Gamma_{2}^{D'D'E'\bar{D}'} + \Gamma_{2}^{E'D'D'E'} + \Gamma_{2}^{E'D'E'\bar{E}'} + \Gamma_{2}^{E'E'E'E'} ) = \Gamma_{2}^{D'D'E'},
\]
\[
\Gamma_{2}^{{dd'}{e}} \equiv \frac{1}{4} (\Gamma_{2}^{D'D'E'\bar{D}'} + \Gamma_{2}^{D'E'D'E'} + \Gamma_{2}^{E'D'E'\bar{D}'} + \Gamma_{2}^{E'E'E'E'} ) = \Gamma_{2}^{D'E'E'},
\]
\[
\Gamma_{2}^{{de'}{e'}} \equiv \frac{1}{4} (\Gamma_{2}^{D'E'E'E'} + \Gamma_{2}^{E'E'E'E'} ) = \frac{1}{2} \Gamma_{2}^{D'E'E'}. 
\]

Fig. 2. Direct and exchange Goldstone diagrams induced by ground state correlations and contributing to the two–nucleon induced decay width of Eqs. (6) and (7).

Details on the calculation of the partial decay rates of Eq. (6) will be given elsewhere.

The two–nucleon stimulated decay rate of Eqs. (2) and (5) is therefore obtained
as:

\[ \Gamma_{2}^{pp} = \Gamma_{2}^{dd'd'd} + \Gamma_{2}^{dd'd'e} + \Gamma_{2}^{dd'e'd} + \Gamma_{2}^{dd'e'e} + \Gamma_{2}^{de'e'e}, \]  

(7)

where the use of the pp apex is due to the fact that in the previous derivation we have limited ourself to self-energy diagrams (given in Figure 2) with the two weak transition potentials connected to the same particle line. By denoting with \( \Gamma_{2}^{pp} \) the direct and exchange decay rates corresponding to the result of Eq. (7), it follows that

\[ \Gamma_{2}^{pp} = \Gamma_{2}^{pp} + \Gamma_{2}^{pp'} + \Gamma_{2}^{pp E} + \Gamma_{2}^{pp E'}, \]

and, in turn, each \( \Gamma_{2}^{ij} \) (\( \Gamma_{2}^{ij} \)) is obtained from four (twelve) direct (exchange) diagrams.

\[ \Gamma_{2} = \Gamma_{2}^{pp} + \Gamma_{2}^{pp'} + \Gamma_{2}^{pp} + \Gamma_{2}^{pp E} + \Gamma_{2}^{pp E'} + \Gamma_{2}^{pp E} + \Gamma_{2}^{pp E'} + \Gamma_{2}^{pp E} + \Gamma_{2}^{pp E'} + \Gamma_{2}^{pp E} + \Gamma_{2}^{pp E'} + \Gamma_{2}^{pp E}. \]

(8)

The three isospin channels, \( \Lambda nn \rightarrow nnn, \Lambda np \rightarrow nnp \) and \( \Lambda pp \rightarrow npp \), contribute to each term \( \Gamma_{2}^{ij} \):

\[ \Gamma_{2}^{ij} = \Gamma_{nn}^{ij} + \Gamma_{np}^{ij} + \Gamma_{pp}^{ij} \quad (P = D \text{ or } E), \]

(9)

In Table 1 we give our results for the partial rates \( \Gamma_{2}^{ij} \) of Eq. (6) and (7) for the case of \( \Lambda C \). We emphasize that these predictions have been obtained by an exact calculation from the Goldstone rules applied to the diagrams of Figure 2. In the first three lines we supply the separate contributions to the three isospin channels; the summed results are listed in the last line. As expected, the dominant contribution is provided by the direct term \( \Gamma_{2}^{dd'd'd} \) while among the exchange terms the more important ones turn out to be \( \Gamma_{2}^{dd'd'e} \) and \( \Gamma_{2}^{de'e'e} \). Their negative sign is due to the odd number of crossing between fermionic lines. The global effect of antisymmetry is to decrease by 32% the
We now proceed to discuss our calculation of the full set of two–nucleon induced contributions of Eqs. (8) and (9). All the direct terms have been obtained from the corresponding Goldstone diagrams of Figure 3. We remind the reader that the direct terms $\Gamma_{2}^{pp}$, $\Gamma_{2}^{ph}$, and $\Gamma_{2}^{hh}$ were calculated for the first time in Ref. [20] (Ref. [22]). Contrarily, we have not evaluated exactly the exchange terms $\Gamma_{2}^{pp'}$, $\Gamma_{2}^{ph'}$, $\Gamma_{2}^{pp''}$, $\Gamma_{2}^{ph''}$, and $\Gamma_{2}^{hh''}$. We note that these contributions are expected to be considerably smaller than the one we calculated exactly, $\Gamma_{2}^{pp'}$. Moreover, each $\Gamma_{2}^{ij}$ is smaller in absolute

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Channel & $\Gamma_{2}^{dd'd'd'}$ & $\Gamma_{2}^{dd'd'd'}$ & $\Gamma_{2}^{dd'd'd'}$ & $\Gamma_{2}^{dd'd'd'}$ & $\Gamma_{2}^{dd'd'd'}$ & $\Gamma_{2}^{dd'd'd'}$ \\
\hline
$\Lambda nn \rightarrow nnn$ & 0.012 & -0.002 & -0.002 & $\sim 0$ & 0.001 & 0.009 \\
$\Lambda np \rightarrow nnp$ & 0.184 & -0.047 & -0.039 & 0.009 & 0.016 & 0.123 \\
$\Lambda pp \rightarrow npp$ & 0.036 & -0.008 & -0.007 & 0.002 & 0.002 & 0.025 \\
\hline
sum & 0.232 & -0.057 & -0.048 & 0.011 & 0.019 & 0.157 \\
\hline
\end{tabular}
\end{table}

This relation remains almost unaltered when only the direct terms are taken into account.
value than $\Gamma_{ij}^{jjD}$. Therefore, we anticipate a limited effect of all the exchange contributions but $\Gamma_{ij}^{ppE}$.

Anyhow, we have evaluated all the exchange terms apart from $\Gamma_{ij}^{ppE}$ in an approximated way through the following strategy. We introduce a Landau–Migdal model in which the residual strong interaction $V_{NN}$ and the weak transition potential $V_{\Lambda N\pi N}$ are modified by the addition of spin–isospin $g'$ and $g'_\Lambda$ parameters: $V_{NN}(q) \rightarrow V_{NN}(q) + (f_{\pi}/m_{\pi})^2 g' F^2(q) \sigma \cdot \sigma' \cdot \tau \cdot \tau'$ and $V_{\Lambda N\pi N}(q) \rightarrow V_{\Lambda N\pi N}(q) + G_F m_{\pi} (f_{\pi}/m_{\pi}) (B_{\pi}/2M) g'_\Lambda F^2(q) \sigma \cdot \sigma' \cdot \tau \cdot \tau'$. In these expressions, a form factor $F(q) = (\Lambda^2 - m^2_{\pi})/((\Lambda^2 - q_0^2 + q^2)$ with cut–off $\Lambda = 1.75 \text{ GeV}$ is included, while $G_F$ is the Fermi constant, $B_{\pi}$ is the constant which defines the parity–conserving $\Lambda \pi N$ weak vertex and $M$ is the average between the nucleon and $\Lambda$ masses. The values of $g'$ and $g'_\Lambda$ are fixed by using the results we obtained microscopically for the $\Gamma_{ij}^{pdf\tau'\tau}$'s entering Eq. (7). The direct term $\Gamma_{ij}^{dd'd'd'}$ has to be calculated both with and without the modified $V_{NN}$ and $V_{\Lambda N\pi N}$ interactions. The condition that the calculation with the modified $V_{NN}$ ($V_{\Lambda N\pi N}$) provide the same result of the sum $\Gamma_{ij}^{dd'd'd'} + \Gamma_{ij}^{dd'd'e}$ ($\Gamma_{ij}^{dd'd'd'} + \Gamma_{ij}^{dd'e'd'}$) computed with the original interactions allows us to have a first determination of the Landau–Migdal parameter $g'$ ($g'_\Lambda$). By maintaining unaltered their ratio, the values of the two parameters are then fine–tuned in order to give the same result of the sum $\Gamma_{ij}^{dd'd'd'} + \Gamma_{ij}^{dd'd'e} + \Gamma_{ij}^{dd'e'd'} + \Gamma_{ij}^{dd'e'e} + \Gamma_{ij}^{dd'd'e}$ when implemented together in the direct term $\Gamma_{ij}^{dd'd'd'}$. The so determined $g'$ and $g'_\Lambda$ are then used to evaluate the other exchange terms via the relation:

$$\Gamma_{ij}^{jjE} = \Gamma_{ij}^{jjD}(g', g'_\Lambda) - \Gamma_{ij}^{jjD}(g' = g'_\Lambda = 0) \quad (ij \neq pp). \quad (10)$$

We stress that, formally, the above computational method makes use of the same Landau–Migdal phenomenology which have been widely adopted in many applications to take care of a number of effects such as the baryon–baryon short range correlations and the LLEE effect. Here we have applied this model only to obtain approximate results for those exchange diagrams that we have not evaluated microscopically. This fact is explained by the smallness of the parameters that we have obtained: $g' = 0.05$ and $g'_\Lambda = 0.09$. We also point out that the above mentioned fine–tuning has required a modification of $g'$ and $g'_\Lambda$ by only 10%, thus demonstrating a certain reliability of our approximate procedure.

In Table 2 we report our predictions for the two–nucleon induced terms $\Gamma_{ij}^{jiP}$ entering Eqs. (8) and (9). For the three isospin channels the dominant contribution is given by $\Gamma_{ij}^{jjD}$. The next contributions in order of importance are $\Gamma_{ij}^{jjD}$ and then the exchange terms that we have calculated exactly, i.e., $\Gamma_{ij}^{ppE}$. Among the exchange contributions that we have evaluated approximately, the most important ones are $\Gamma_{ij}^{ppE}$. Each exchange term is smaller in absolute value than the corresponding direct term and $|\Gamma_{ij}^{jjE}/\Gamma_{ij}^{jjD}| = 0.2$-
0.5. By neglecting the exchange terms that we have calculated with the approximated procedure, the total two–nucleon induced decay width would be \( \Gamma_2 = 0.287 \) instead of 0.252. This comparison shows that the exact calculation of all the exchange terms may be important for a precise determination of \( \Gamma_2 \). From the final results of the last column of Table 2 we see that \( \Gamma_{np} : \Gamma_{pp} : \Gamma_{nn} = 0.83 : 0.12 : 0.04 \), a relation which remains practically unchanged when limiting to the direct part only.

Table 2
Partial contributions to the two–nucleon induced decay width of Eqs. (8) and (9) for \(^{12}_\Lambda C\) in units of the free \( \Lambda \) rate. Values smaller than 0.0005 are represented by \( \sim 0 \).

| Channel         | \( \Gamma_{pp}^D \) | \( \Gamma_{pp}^E \) | \( \Gamma_{ph}^D \) | \( \Gamma_{ph}^E \) | \( \Gamma_{hh}^D \) | \( \Gamma_{hh}^E \) |
|-----------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| \( \Lambda n \rightarrow nnn \) | 0.012 | −0.003 | \( \sim 0 \) | \( \sim 0 \) | 0.002 | −0.001 |
| \( \Lambda n \rightarrow nnp \) | 0.184 | −0.061 | −0.003 | 0.001 | 0.027 | −0.006 |
| \( \Lambda p \rightarrow npp \) | 0.036 | −0.011 | \( \sim 0 \) | \( \sim 0 \) | 0.006 | −0.001 |
| sum             | 0.232 | −0.075 | −0.003 | 0.001 | 0.035 | −0.008 |

| Channel | \( \Gamma_{pp'}^D \) | \( \Gamma_{pp'}^E \) | \( \Gamma_{ph'}^D \) | \( \Gamma_{ph'}^E \) | \( \Gamma_{hh'}^D \) | \( \Gamma_{hh'}^E \) | \( \Gamma_2 \) |
|----------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| \( \Lambda n \rightarrow nnn \) | 0.002 | −0.001 | \( \sim 0 \) | \( \sim 0 \) | \( \sim 0 \) | \( \sim 0 \) | 0.011 |
| \( \Lambda n \rightarrow nnp \) | 0.081 | −0.021 | 0.010 | −0.004 | 0.004 | −0.002 | 0.210 |
| \( \Lambda p \rightarrow npp \) | 0.001 | \( \sim 0 \) | \( \sim 0 \) | \( \sim 0 \) | \( \sim 0 \) | 0.031 |
| sum       | 0.084 | −0.022 | 0.010 | −0.004 | 0.004 | −0.002 | 0.252 |

Our predictions for the total one– and two–nucleon induced rates are reported in Table 3 for both the full (direct plus exchange) calculation and for the evaluation limited to the direct part only. Note that in the Direct Only calculation the normalization of the ground state wave–function of Eq. (4) only contains direct matrix elements, while in the evaluation of the direct contributions of Table 2 (where also exchange contributions are given) the ground state incorporates both direct and exchange matrix elements. We see that the antisymmetrization property implies a reduction of 18% in the decay rate \( \Gamma_2 \). This is a rather important effect, especially when combined with the finding that the introduction of exchange terms produces an increase of 19% in the value of \( \Gamma_1 \). We thus find that antisymmetrization entails a substantial decrease, of 31%, in the value of \( \Gamma_2/\Gamma_1 \), while having less influence on the total non–mesonic decay width \( \Gamma_{NM} \) and the \( \Gamma_n/\Gamma_p \) ratio. The complete result for \( \Gamma_{NM} \) is in agreement with the most recent KEK datum only within 1.7 \( \sigma \) deviations, while it agrees well with the older KEK experiment. Further theoretical and experimental analyses are thus necessary to improve the agreement between predictions and data. A forthcoming experiment planned at J–PARC [9] will measure the ratio \( \Gamma_2/\Gamma_1 \) for the first time. In addition, new data on \( \Gamma_{NM} \) will also help the comparison between theory and experiment (the data in Table 3...
are indeed only barely compatible between each other). Before concluding we note that our result for the ratio between the neutron– and the proton–induced rates, $\Gamma_n/\Gamma_p = 0.327$, is in agreement with the value $(\Gamma_n/\Gamma_p)^{\text{Exp}} = 0.29 \pm 0.14$ deduced in Ref. [25] by fitting KEK–E508 double–coincidence nucleon spectra. We also point out that the rate $\Gamma_n$ is predicted to be smaller than $\Gamma_{np}$: $\Gamma_n = 0.154$, $\Gamma_{np} = 0.210$. As a final comment, we mention that from the notable weight obtained for the exchange contributions we anticipate important modifications in the measurable nucleon spectra, which indeed depend on the kinematics of the intermediate nucleons in the $\Lambda$ self–energy diagrams (we expect such kinematics to be strongly influenced by fermion antisymmetrization).

Table 3
One– and two–nucleon stimulated non–mesonic weak decay widths of $^{12}\Lambda C$ in units of the free $\Lambda$ rate. The most recent experimental results are given for the total non–mesonic rate.

|            | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_{NM}$ | $\Gamma_n/\Gamma_p$ | $\Gamma_2/\Gamma_1$ |
|------------|------------|------------|----------------|---------------------|---------------------|
| Direct Only| 0.524      | 0.308      | 0.832          | 0.310               | 0.588               |
| Direct + Exchange | 0.624 | 0.252 | 0.876 | 0.327 | 0.404 |
| KEK–E508 [34] |            |            | 0.929 \pm 0.027 \pm 0.016 |            |            |
| KEK–E307 [35] |            |            | 0.828 \pm 0.056 \pm 0.066 |            |            |

In this Letter we have studied the effects of Pauli exchange terms in the two–nucleon induced weak decay of $^{12}\Lambda C$ hypernuclei. All the possible stimulating channels, $\Lambda nn \rightarrow nnn$, $\Lambda np \rightarrow nnp$ and $\Lambda pp \rightarrow npp$, have been considered. A non–relativistic nuclear matter scheme has been adopted together with a local density approximation. The employed weak transition potential (residual strong interaction, modeled on a Bonn potential) contains the exchange of the full set of mesons of the pseudoscalar and vector octets ($\pi$, $\rho$, $\sigma$ and $\omega$ mesons). We predict that $\Gamma_p : \Gamma_{np} : \Gamma_n : \Gamma_{pp} : \Gamma_{nn} = 0.54 : 0.24 : 0.18 : 0.04 : 0.01$. The exchange contributions turn out to reduce by 18% the value of the two–nucleon induced rate obtained without fermion antisymmetrization. The global effect of antisymmetrization is of reducing $\Gamma_2/\Gamma_1$ by 31% and increasing $\Gamma_{NM} = \Gamma_1 + \Gamma_2$ and $\Gamma_n/\Gamma_p$ by 5%, while maintaining practically unchanged the $\Gamma_{np} : \Gamma_{pp} : \Gamma_{nn}$ relation. A careful evaluation of the exchange contributions is thus of special relevance for a correct separation of the total non–mesonic decay rate into the one– and two–nucleon induced parts.

In order to achieve a more exhaustive understanding of non–mesonic weak decay, for the future we plan to extend the microscopic calculation of the two–nucleon induced decay modes to the whole set of Pauli exchange diagrams. Another consequential prosecution of such a study will concern the calculation of those nucleon spectra which are essential for a meaningful comparison with
experiment as well as for extracting, from data, the whole set of partial non–mesonic decay widths. From the results presented here we can anticipate a pronounced modification, induced by antisymmetrization, in these measurable nucleon spectra. Such a study will also involve the use of a model accounting for the nucleon final state interactions occurring after the weak decay.

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