Acceleration time scale in an ultrarelativistic shock

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ABSTRACT
The acceleration mechanism at ultrarelativistic shocks is investigated using the Monte Carlo simulations. We apply a method of discrete small amplitude particle momentum scattering to reproduce highly anisotropic conditions at the shock and carefully describe the acceleration mechanism. The obtained acceleration times equal $1.0 \tau_g/c$ if the spectral index reach the value of 2.2, independent of physical conditions in the shock. Some other parameters of the acceleration process are also provided.

Key words: acceleration of particles – shock waves – cosmic rays – gamma-rays: bursts.

1 INTRODUCTION
Observations carried out by the Burst and Transient Source Experiment show that GRBs originate from cosmological sources (Meegan et al. 1992 and Dermer 1992). Identification of the host galaxy for the GRB 971214 (Kulkarni et al. 1998) and several other bursts causes there is little doubt now that some, and most likely all GRBs are cosmological. These phenomena are surely related to ultrarelativistic shocks with the Lorentz factors $\gamma > 10^2$.

Several papers suggested that ultrarelativistic shocks in GRBs could be sources of high energy cosmic rays (cf. Waxman 1995, Vietri 1995), and simulations done by Bednarz & Ostrowski (1998) showed that such shocks are able to accelerate charged particles and values of their energy spectral indices converge to $\sigma = 2.2$ when $\gamma \rightarrow \infty$ and/or magnetic turbulence amplitudes grow. Because the acceleration mechanism is quite different from that in the non-relativistic and mildly relativistic regime we distinguish a class of ultrarelativistic shocks if their Lorentz factors $\gamma > 1$.

Observations seem to confirm this mechanism. Waxman (1997) used a fireball model of GRBs and showed from the functional dependence of the flux on time and frequency that $\sigma = 2.3 \pm 0.1$ in the afterglow of GRB 970228. Galama et al. (1998) made two independent measurements of the electron spectrum index in the afterglow of GRB 970508 which was very close to 2.2.

2 ACCELERATION MECHANISM
A particle crossing the shock to upstream medium has a momentum vector nearly parallel to the shock normal. Then the particle momentum changes its inclination in two ways by: 1) scattering in an inhomogeneous magnetic field and 2) smooth variation in a homogeneous field component. Hereafter, the mean deflection angle in these two cases will be denoted by $\Delta \Omega_S$ and $\Delta \Omega_H$, respectively. The first process is a diffusive one and the second depends on time linearly. That means that with increasing shock velocity, keeping other parameters constant, $\Delta \Omega_S$ decreases slower as a square root of time in comparison with $\Delta \Omega_H$. The Lorentz transformation shows that with $\gamma \gg 1$ even a tiny angular deviation in the upstream plasma rest frame can lead to a large angular deviation in the downstream plasma rest frame. Let us denote a particle phase by $\phi$ and the angle between momentum and a magnetic field vector by $\theta$ both measured in the downstream plasma rest frame. Values of these parameters at the moment when a particle crosses the shock downstream determine if it is able to reach the shock again in the case of neglected magnetic field fluctuations downstream of the shock. In fact a motion in the homogeneous magnetic field carries a particle in such a way that it cannot reach the shock again. The magnetic field fluctuations upstream of the shock perturbing the momentum direction lead to broadening the $(\phi, \theta)$ range that allows particles to reach the shock again. Thus, as we show below for efficient scattering, when $\Delta \Omega_H$ becomes unimportant in comparison to $\Delta \Omega_S$, the spectral index and the acceleration time reach their asymptotic values.

The discussed relation between $\Delta \Omega_H$ and $\Delta \Omega_S$ is reproduced in our simulations and presented in Fig. 1. There are shown 11 points from $\gamma = 100$ to 320 and three additional for $\gamma = 640, 1280, 2560$. The expected linear dependence of these quantities can be noticed.

3 NUMERICAL SIMULATIONS
In simulations we follow the procedure used by Bednarz & Ostrowski (1996) with a hybrid approach used in Bednarz & Ostrowski (1998). Monoenergetic seed particles are injected at the shock and then their trajectories are derived...
in the perturbed magnetic field. The inhomogeneities are simulated by small amplitude particle momentum scattering within a cone with angular opening $\Delta \theta$ less than the particle anisotropy $\sim 1/\gamma$ (cf. Ostrowski 1991).

A particle is excluded from simulations if it escapes through the free-escape boundary placed far off the shock or reaches the energy larger than the assumed upper limit. These particles are replaced with the ones arising from splitting the remaining high-weight particles, preserving their physical parameters. Particles that exist longer than the time upper limit for simulations are excluded from simulations without replacing.

All computations are performed in the respective upstream or downstream plasma rest frame. When particles cross the shock their parameters are transformed to the current plasma rest frame and the weighted contribution divided by the particle velocity component normal to the shock ($\Omega$ particle density) is added to the time and momentum bin depending on particle parameters, as measured in the shock normal rest frame. For the considered continuous injection after initial time, the energy cut-off of the formed spectrum shifts toward higher energies with time. The resulting spectra allows one to fit spectral indices and derive acceleration time in the shock normal rest frame in units of downstream $r_g/c$ ( $r_g$ - particle gyroradius in the homogeneous magnetic field component, $c$ - speed of light; for details see Bednarz & Ostrowski 1996). We transform the acceleration time $t_{acc}$ to the downstream plasma rest frame. Hereafter, subscripts U or D mean that a parameter is measured in the upstream or downstream plasma rest frame respectively. We will use downstream $r_g$ as a distance and $r_g/c$ as a time units. The magnetic field inclination to the shock normal upstream of the shock, $\psi$, is measured in the upstream plasma rest frame.

Let us denote the ratio of the cross-field diffusion coefficient $\kappa_\perp$ to the parallel diffusion coefficient $\kappa_\parallel$ (the value is measured in the plasma rest frame). Simulations prove that fluctuations upstream of the shock (measured by $\tau^U$) and downstream of the shock (measured by $\tau^D$) influence the acceleration process independently. The minimum fluctuations upstream of the shock needed to run the acceleration process efficiently tend to zero when $\gamma \to \infty$. We checked by simulations with different $\tau^D$ that its value does not influence the spectral index considerably for a given $\tau^U$.

Our scattering model is very simple but also universal. In the model we are not able to discuss gyroresonant scattering. Upstream of the shock particles have not enough time to interact resonantly with low-frequency waves and the rough relation $\tau \sim (\delta B/B)^4$ (cf. Blandford & Eichler 1987) cannot be deduced from the interaction there. However, for growing $\tau^U$ and fixed $\Delta \theta$ the time between scattering acts decreases what is equivalent to increasing the magnetic field fluctuations.

4 RESULTS

In the following simulations we consider shocks with $\gamma = 20, 40, 80, 160, 320, 640, 1280, 2560$ and yield $\sigma = 2.5, 2.3$ and $2.2$ respectively.

Figure 1. The relation between the mean deflection angle upstream of the shock caused by the scattering in an inhomogeneous magnetic field ($\Delta \Omega_\parallel$) and by smooth variation in a homogeneous magnetic field ($\Delta \Omega_H$). Last three points for $\Delta \Omega_H$ below $1 \cdot 10^{-3}$ represent $\gamma = 640, 1280, 2560$ and yield $\sigma = 2.5, 2.3$ and $2.2$ respectively.

Figure 2. Simulated spectral indices as a function of magnetic field fluctuations upstream of the shock. Fluctuations downstream of the shock are neglected. The chosen $\tau^U$ value for $\gamma = 80$ and $\psi = 45^\circ$ is pointed by a dashed line. A second-degree polynomial fit is also marked by a dashed line.

Thus, as a first case we consider downstream conditions without magnetic field fluctuations. By simple data inspection (cf. Fig. 2) we look for minimum $\tau^U$ where the spectral index reaches its limit of 2.2 and we apply this value in further simulations. The relation between $\tau^U$, $\gamma$ and $\psi$ can be roughly fitted with the equation $\tau^U = 0.25 \gamma^{-1.2} \psi$ in the considered range of shock parameters. We repeated simulations for a number of cases with different $\gamma$ and $\psi$ and $\tau^D \neq 0$. The obtained results are in good agreement with the ones derived from the above equation up to $\tau^D = 0.11$.

Values of the acceleration time $t_{acc}$ for three amplitudes of magnetic field fluctuations downstream of the shock are presented in Fig. 3. In the figure one can see the lack of change of $t_{acc}$ with $\psi$, but it slowly decreases to the asymptotic value with $\gamma$. In the simulations we have observed tendency of $t_{acc}$ to grow when $\sigma$ increases up to 2.3-2.4 and no further change if magnetic field fluctuations upstream of the shock grow. For $\tau^D \leq 0.11$ the asymptotic value of the acceleration time is close to $r_g/c$. It occurs that $r_g/c$ is a good
unit provided that the homogeneous magnetic field dominates the randomly component. Unfortunately, when this condition fails the meaning of $t_{acc}$ becomes unclear in the simulations then. For this reason we will not discuss further the case of $\tau^D = 0.69$ any more.

Approximate calculations of Gallant & Achterberg (1999) showed that $t_U^D / t_D^H \simeq 1$, where $t_U^D$ is the particle mean residence time upstream of the shock (upper index) as measured in the upstream plasma rest frame (lower index), and D in $t_D^H$ stands for the downstream residence time. However, they were not able to consider the anisotropic particle momentum distribution and our results in Fig. 4 transformed to the upstream plasma rest frame with $t_U^D / t_D^H$ within the range $0.01 \sim 0.1$ are more adequate for real situations. Additionally, the above authors applied an extremely irregular magnetic field upstream of the shock represented by randomly oriented magnetic fields with field amplitude $B$ and they measured time in the upstream unit of $r_B(B)/c$. As a result they obtained that $t_U^D / t_D^H$ could be much larger than 1 in the case.

Just before the spectral index reaches its minimal value (cf. Fig. 2) $\Delta \Omega_s$ stabilizes near the limit which value does not further depend on the magnetic field inclination as is seen in Fig. 5. Momentum vectors of particles crossing downstream of the shock have similar distributions as measured in the downstream plasma rest frame if $\Delta \Omega_s$ approaches the maximum value. Then, it follows that parameters we consider below depend only on $\tau^D$.

For growing $\tau^D$ ($\tau^D = 0, 1.0 \cdot 10^{-3}, 1.1 \cdot 10^{-2}, 0.11$) the acceleration time is constant and accompanied by a slow increase of the mean energy gain in one cycle downstream-upstream-downstream $\langle \Delta E/E \rangle_D = 0.89, 0.94, 1.0, 1.1$, and a slight decrease of the fraction of particles that reach the shock again after crossing it downstream $\langle \Delta n/n \rangle = 0.51, 0.50, 0.48, 0.44$. Simultaneously the mean time a particle spends downstream of the shock grows as $t_D^H = 0.96, 1.0, 1.2, 1.35$. Time that a particle spends upstream of the shock can be neglected in this rest frame as is visible in Fig. 4. It implies, approximately, $t_{acc} = t_D^H / (\Delta E/E) D$ if one neglects correlations between these quantities (cf. Bednarz & Ostrowski 1996). Similarly we can roughly estimate the value of the energy spectral index of accelerated particles as

$$\alpha \simeq 1 - \ln(\langle \Delta n/n \rangle) / \ln(\langle \Delta E/E \rangle_D + 1).$$

The simulated maximum distance in downstream medium the particle is able to depart from the shock and reach it again is, respectively, $d_M^H = 0.84, 1.5, 2.5, 4.0$ (the values were derived from $\sim 10^5$ events). We calculated the average values of $\sin^2 \theta$ for returning particles wandering downstream of the shock and found, respectively, $\langle \sin^2 \theta \rangle = 0.676, 0.658, 0.650, 0.651$.

5 DISCUSSION

Because of newly found acceleration mechanism in ultrarelativistic shock waves we propose that some part of GRBs radiation could arise due to synchrotron radiation of electrons or electron pairs accelerated across the mechanism.

* Below, we provide the respective series of simulated parameter for this sequence of $\tau^D$

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We follow the internal shocks model of GRBs (cf. Kobayashi et al. 1997 for example). In the model two different shells have different Lorentz factors. The inner shell overtakes a slower outer shell and form a shock. The Lorentz factor of the shock as measured in the frame at rest with respect to the outer shell is assumed to be $\sim 2$. With only a part of kinetic energy converted into the internal energy the particle energy distribution downstream of the shock will be non-thermal with, possibly, a substantial fraction of relativistic particles. As a result an amount of relativistic particles will be present in the shock. The particles can be accelerated across the mechanism presented in the paper and could be observed in the afterglow (cf. Waxman 1997, Galama et al. 1998).

We derived some parameters of the process that could be used in GRBs models. The acceleration time $t_{acc} = 1.0 r_g/c$, measured in the downstream plasma rest frame in the unit of particle gyroradius in the homogeneous magnetic field component divided by the speed of light is the second important parameter besides the spectral index $\sigma = 2.2$. The values of $d_0^{3D}$ and $t_0^{3D}$ define the dimensions of the shock that allow the process to be effective, and the values of $\left\langle \sin^2 \theta \right\rangle$ shows how the synchrotron radiation can influence the process.

In the ejecta of the relativistic matter in the GRB model the outer shells can be faster than the following ones. In this case separated shocks with Lorentz factors reaching $\gamma \sim 10^3$ will be generated. The leading shock could produce seed protons with energies of $10^{14} - 10^{16}$ eV. These protons downstream of the first shock can interact with the following one to be reflected with energy gains $\sim \gamma^2$ (cf. Gallant & Achterberg 1999, Bednarz & Ostrowski 1999). For a constant reflection probability the spectrum of these reflected highest energy particles, above $10^{20}$ eV, will be only the shifted in energy spectrum of seed particles with the universal spectral index $\sim 2.2$. 

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