Spin-Allowed Chern-Simons Theory of Fractional Quantum Hall States for Odd and Even Denominator Filling Factors

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By allowing the spin degrees of freedom, we present a generalized spin allowed $U(1) \times U(1)$ Chern-Simons theory of fractional quantum Hall effects for odd and even denominator filling factors in single layers. This theory is shown to reproduce all possible odd denominator filling factors corresponding to spin-unpolarized, partially polarized, and fully polarized fractional quantum Hall states. Closely following our earlier work, we derive the formal expressions of electromagnetic polarization tensors and Hall conductivity for the spin-unpolarized and partially polarized fractional quantum Hall states. Finally we report the computed spectra of collective excitations for both the even and odd denominator filling factors for which Kohn’s theorem is satisfied.

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1. INTRODUCTION

The observed fractional quantum Hall effects (FQHE) encompass a large number of both even and odd denominator filling factors at low temperatures and strong magnetic fields $\nu$. The quantum Hall effect (QHE) at the even-denominator filling factor $\nu = 5/2$ appeared amidst numerous studies which pay attention to QHE at odd denominator filling factors. Tilted field experiments showed that the $5/2$ FQH state is destroyed by Zeeman coupling [4], indicating that the incompressible state of $\nu = 5/2$ is not fully polarized. Earlier Halperin [5] suggested that even denominator FQH states can exist if one allows extra degrees of freedom such as spin and layer index. Indeed the FQHE at $\nu = 1/2$ was observed in double layer systems [6], and is relatively well understood [7,8]. For single layer systems a great deal of attention [9–13] has been paid to the even-denominator filling factor of $\nu = 1/2$, of which pay attention to the odd denominator filling factors of $\nu = 1/2$ states [14] of $\nu = 1/(2m + 1)$ with $m$ integer are still partially polarized. Considering that the $\nu = 1/2$ state as a spin singlet state [13] may have some relevance to the experimentally observed $\nu = 5/2$ state, we present a Chern-Simons coupling matrix to reproduce the even denominator filling factors of $\nu = 1/2$ and $5/2$. Focusing mainly on the odd denominator filling factors we derive the formal expressions of spin-allowed electromagnetic response functions, Hall conductivity and compressibility. In the present work we do not allow the spin-flip and the spin-wave mode [20]. Thus we do not consider the promotion of an electron from one effective Landau level (LL) to another by permitting a spin-flip, nor the case of the spin reversed state at the same effective LL to allow a spin-wave mode. Finally for the sake of comparison, we discuss the computed results of collective excitations for the spin-unpolarized, partially polarized, and fully polarized states of single Chern-Simons coupling matrix and the other for the revelation of differences in collective excitations between the cases of the fully polarized states.
and the spin-unpolarized or partially polarized states for the odd denominator filling factors.

II. SPIN UNPOLARIZED, PARTIALLY POLARIZED AND FULLY POLARIZED FQHE

For the odd-denominator filling factors Jain proposed the CF wave function \[\Psi_c\]  
\[
\chi_{p/(2mp±1)} = \prod_{j<k=1}^{N_e} (z_j - z_k)^{2m} \chi_{±p} . \tag{1}
\]

Here \(m\) is a positive integer; \(z_j = x_j + iy_j\), the coordinate of the \(j\)th electron and \(N_e\), the total number of electrons. \(\chi_{±p}\) represents a state of integer quantum Hall effect (IQHE) with integer filling factor \(p\) and \(+(-)\) stands for vortex (anti-vortex) attachment to electrons. Considering the spin degrees of freedom but neglecting the Zeeman coupling, \(\chi_{±p}\) with \(p = p^z + p^\parallel\) contains a \(p^z\)

\[
S = \int d^3z \sum_\sigma \left\{ \psi_\sigma(z) [iD_0^\sigma + \mu] \psi_\sigma(z) - \frac{1}{2m_b} |D^\sigma \psi_\sigma(z)|^2 \right\}
+ \sum_{\sigma\sigma'} \frac{\alpha^{\sigma\sigma'}}{2} \int d^3z e^{\mu\lambda} a_\sigma^\mu a_{\sigma'}^\lambda - \int d^3z \sum_\sigma \psi_\sigma(z) \frac{q}{2} \mu_B B \sigma^z \psi_\sigma(z)
- \frac{1}{2} \int d^3z \int d^3z' \sum_{\sigma,\sigma'} (|\psi_\sigma(z)|^2 - \bar{\rho}_\sigma) \nabla \sigma \cdot (\bar{z} - \bar{z}') (|\psi_{\sigma'}(z')|^2 - \bar{\rho}_{\sigma'})
\]  

with

\[
D_\mu^\sigma = \partial_\mu + i \frac{e}{c} (A_\mu^\sigma - a_\mu^\sigma). \tag{4}
\]

Here \(\sigma\) and \(\sigma'\) are the spin indices and \(\sigma^z = +1\) for up-spin and \(\sigma^z = -1\) for down-spin. \(\psi_\sigma(z)\) is a second quantized Fermi field; \(\mu\), the chemical potential and \(\bar{\rho}_\sigma\), the average particle density with spin \(\sigma\). \(A_\mu^\sigma\) is the electromagnetic gauge potential and \(a_\mu^\sigma\), the statistical (Chern-Simons) gauge field; \(A_\mu^\sigma(z) = A_\mu^{-\sigma}(z)\) and \(a_\mu^\sigma(z) \neq a_\mu^{-\sigma}(z)\) by allowing the independent density fluctuations of spin-up electrons and spin-down electrons. The second term in \(S\) is the Chern-Simons term with the coupling matrix, \(\alpha^{\sigma\sigma'}\) to be introduced below. The Zeeman coupling is introduced in the third term with \(\mu_B\), the Bohr magneton. The Zeeman energy can be treated as a spin dependent chemical potential \[\mu_\sigma\], thus allowing the effective chemical potential, \(\mu_\sigma = \mu + g/2\mu_B B\sigma\) in \(S\) above.

The Chern-Simons action in \(S\) causes the statistical transmutation of each particle, by allowing an additional exchange phase as a result of attaching flux quanta to the particle. In the Chern-Simons term, we introduce the following coupling matrix, 

\[
\chi_{p/(2mp±1)} = \prod_{j<k=1}^{N_e} (z_j - z_k)^{2m} \chi_{p^z+p^\parallel} . \tag{2}
\]

For the case of \(p^z = p^\parallel = p/2\) with \(p\) even, the above state represents a spin-unpolarized (spin-singlet) state. For \(p\) odd it is a partially or fully polarized spin state. One notes from \(S\) that each electron of up-spin or down-spin sees an even number \(2m\) of flux quanta attached to other electrons.

Based on the above CF picture \[\Psi_c\] in \(S\), we present a generalized spin-allowed \(U(1) \times U(1)\) Chern-Simons theory of FQHE in order to predict all the known odd denominator filling factors. For a correlated electron system made of both up- and down-spin electrons in a magnetic field \(B\), we write the action for the systems of a single layer by allowing the spin degrees of freedom,

\[
\alpha^{\sigma\sigma'} = \frac{e}{\phi_0 \varepsilon} \begin{pmatrix} 2m + i \varepsilon & -2m \\ -2m & 2m - i \varepsilon \end{pmatrix} . \tag{5}
\]

Here \(\phi_0\) is the flux unit. \(m\) is an integer and \(\varepsilon\), an arbitrarily small real number which is introduced to avoid singularity.

The spin-allowed Maxwell’s equation, i.e., Euler-Lagrange equation of motion for the statistical gauge field \(a_0^\sigma\) is from \(S\), \(\rho^\sigma(z) - \frac{1}{2} \left\{ \alpha^{\sigma\sigma'} b_{\sigma'}(z) + \frac{1}{2} (\alpha^{d+} + \alpha^{d+}) b^{-\sigma}(z) \right\} = 0\), where \(b^\sigma(z)\) is the statistical ‘magnetic field’ given by \(b^\sigma(z) = \varepsilon_{ij} \partial_i a_j^\sigma(z)\). We now use the fact \(\alpha^{d+} = \alpha^{d+}\) to write it in a simplified form

\[
\rho^\sigma(z) = \frac{1}{e} \alpha^{\sigma\sigma'} b_{\sigma'}(z) . \tag{6}
\]

We rewrite \(S\) above

\[
b^\sigma(z) = e(\alpha^{\sigma\sigma'})^{-1} \rho^\sigma(z) = \phi_0 \begin{pmatrix} 2m - i \varepsilon & 2m \\ 2m & 2m + i \varepsilon \end{pmatrix} . \tag{7}
\]

Now by taking the limit of \(\varepsilon \to 0\) we explicitly result in

\[
\alpha^{\sigma\sigma'} = \frac{e}{\phi_0 \varepsilon} \begin{pmatrix} 2m + i \varepsilon & -2m \\ -2m & 2m - i \varepsilon \end{pmatrix} . \tag{5}
\]
Now we see that the expression (8) leads to the same interpretation of flux attachment to electrons as the CF picture theory described earlier; namely, an even number of flux quanta \( 2m \) is attached to both up- and down-spin electrons. Even allowing independent density fluctuations for spin-up and spin-down electrons, we find that the symmetry, \( b^\sigma(z) = b^{-\sigma}(z) \) holds as can be readily seen from the relation (8) above. We note from (8) that various possible spin polarized states can arise depending on the individual densities of spin-up and spin-down electrons. They are, namely, the spin-unpolarized, partially polarized, and fully polarized states.

After integrating out the fermion degrees of freedom in the spin-allowed partition function \( \mathcal{Z}[A_\mu] = \int D\psi D\bar{\psi} D\sigma_\mu \exp iS(\psi, \bar{\psi}, A_\mu) \) and using (3) for the last term of (8), we obtain the effective action (in the natural units of \( e = c = \hbar = 1 \)) (8),

\[
\mathcal{S}_{\text{eff}} = -i \sum_\sigma \text{tr} \ln \left\{ iD_0^\sigma + \mu_\sigma + \frac{1}{2m_\text{b}} (D^\sigma)^2 \right\} \\
+ \sum_{\sigma\sigma'} \frac{\alpha^\sigma \alpha'^\sigma}{2} \int d^3z \epsilon^{\mu\nu\lambda} a^\sigma_\mu \partial_\nu a^{\alpha'^\sigma}_\lambda \\
- \frac{1}{2} \int d^3z \int d^3z' \left( \alpha^\sigma b^\sigma (z) - \overline{\rho^\sigma} \right) \left( V_{\sigma\sigma'} (|z - z'|) \left( \alpha^{\sigma'} b^{\sigma'} (z') - \overline{\rho^{\sigma'}} \right) \right).
\]

(9)

Using the saddle point approximation for the stationary configurations of \( \mathcal{S}_{\text{eff}} \) with respect to the small fluctuations of the statistical gauge field \( a^\mu_\sigma, \frac{\delta \mathcal{S}_{\text{eff}}}{\delta a^\mu_\sigma} |_{a^\mu_\sigma} = 0 \) and \( \frac{\delta \mathcal{S}_{\text{eff}}}{\delta a^{\sigma}_\mu} |_{a^{\sigma}_\mu} = 0 \), we readily obtain the following mean field results,

\[
\langle \phi^\sigma (x) \rangle = \alpha^\sigma \langle b^\sigma (x) \rangle \\
\langle \phi^\sigma (x) \rangle = \alpha^\sigma \langle b^{\sigma}_\mu (x) \rangle \\
- \int d^3z' \alpha^\sigma V_{\sigma\sigma'} (|z - z'|) \times \\
\left[ \alpha^{\sigma'} b^{\sigma'} (z') - \overline{\rho^{\sigma'}} \right].
\]

(10)

where \( e(z) = -\partial_t a(z) \) is the ‘statistical electric field’. Here it should be noted that our approach differs from the CS theory of Lopez and Fradkin [7,9,23], that is, in deriving (10) above we did not take the gauge field shift and avoided the condition of the vanishing average of fluctuating electromagnetic field. Since the gauge shift alters the effective action (of course, it does not affect the partition function), our action without the gauge shift preserves its original characteristics (for further details we refer readers to our earlier work [8]). The effective (or residual) magnetic field \( B^\text{eff}_\sigma \) is given by the difference between the external magnetic field \( B \) and the statistical magnetic field \( b \), that is, \( B^\text{eff}_\sigma = B - \langle b^\sigma \rangle = B - (\alpha^\sigma \partial^\sigma) - \rho^\sigma = \hbar \omega_{\text{eff}} m_\text{b} / e \). Thus the total number of effective magnetic flux quanta, \( N^\sigma_{\text{eff}} \) seen by the composite fermions of spin \( \sigma \) is, from the use of (8),

\[
N^\sigma_{\text{eff}} = N_\phi - 2mN^\sigma_e - 2mN^{-\sigma}_e = N_\phi - 2mN_e
\]

(11)

with \( N_e = N^+_e + N^-_e \). Here \( N_\phi \) is the total number of magnetic flux quanta and \( N^\pm_e \), the total number of electrons with spin \( \sigma \),

The system becomes incompressible due to the presence of an energy gap as a result of the complete filling of an integer number \( p^\sigma \) of effective LL’s by the non-interacting composite fermions of spin \( \sigma \). Obviously \( N^p_{\phi_{\text{eff}}} = N^p_{\phi_{\text{eff}}} \) from (11). By definition we have \( \nu = N_\phi / N_e = (N^+_e + N^-_e) / N_e = \nu^+ + \nu^- \). Realizing that \( p^\sigma = N^p_{\phi_{\text{eff}}} \) for the effective filling factor of composite fermions with spin \( \sigma \) and \( \nu^\sigma = N^\sigma / N_e \) for the filling factor of bare electrons with spin \( \sigma \), we obtain from (11) the filling factors for the electrons of spin \( \sigma \),

\[
\nu^\sigma = \frac{1}{p^\sigma} + 2m + 2m \frac{N^\sigma_{\text{eff}}}{N_e}.
\]

(12)

Noting that \( N^+_e = N^+_e \), we get \( N^+_e = p^\sigma / p^\sigma \). Thus we obtain from (12),

\[
\nu = \frac{1 + N^+_e / N^-_e}{1 + 2m (p^+ + p^-)}.
\]

(13)

Now with some illustrations we check the validity of the expression (13) whether the above expression yields all possible spin states for odd denominator filling factors. For the unpolarized FQH states we have \( p^\sigma = p^- = n \) in (13), thus, obtain the even numerator filling factors,

\[
\nu = \frac{2n}{1 + 4mn}.
\]

(14)

For example, by choosing \( n = m = 1 \), the existence of the spin-unpolarized \( \nu = 2/5 \) state with even numerator is predicted. If we take \( p^- = p^+ + 1 = n \) for the highest effective LL filled with spin-down electrons in (13), we obtain the odd numerator filling factors,
\[
\nu = \frac{2n - 1}{1 + 4mn - 2m}.
\] (15)

By choosing \(n = 2\) and \(m = 1\), the partially polarized state of \(\nu = 3/7\) with odd numerator is predicted. Finally with the choice of \(n = 1\) and thus \(p^\dagger = 1\) and \(p^\dagger = 0\), we correctly obtain from (14) the filling factor of
\[
\nu = \frac{1}{2m + 1}
\] (16)
for the well-known fully polarized Laughlin states including \(\nu = 1/3\). Recently Mandal and Ravishankar suggested a doublet model for arbitrarily polarized FQH states \([22]\). However they used two different CS coupling matrices; one for the unpolarized states and the other for the partially polarized states. On the other hand, in our approach it is quite gratifying to note that one can extract from the single coupling matrix in (6) all possible odd-denominator filling factors for all possible spin-allowed states.

Soon after the experimental evidence \([2]\) of the incompressible state for \(\nu = 5/2\), there have been attempts for its explanation \([14,13]\). Noting that the spin unpolarized FQH state of \(\nu = 5/2 = 2 + 1/2\) at relatively low magnetic fields can be considered as a \(\nu = 1/2\) state at the 2nd LL with a sufficiently large LL spacing compared to correlation energy so that filled Landau levels are inert \([10]\). Belkhir and Jain \([13]\) showed that the \(\nu = 1/2\) ground state with a short range interaction can be incompressible for a wide range of repulsive interactions. Their trial wave function is

\[
\psi_{1/2} = \left[ \prod_{j<k=1}^{N} (z_j - z_k) \right] \left[ \prod_{j<k=1}^{N/2} (z_j - z_k) \right] \left[ \prod_{j<k=\frac{N}{4}+1}^{N} (z_j - z_k) \right] \chi_2.
\] (17)

Here \(z_j\)'s with \(j \leq N/2\) are the positions of spin-up electrons and \(z_j\)'s with \(j \geq N/2 + 1\), the positions of spin-down electrons. \(\chi_2\) represents the IQH state with two filled effective Landau levels, one for spin-up electrons and the other for spin-down electrons. \(\psi_{1/2}\) above is a spin-singlet state corresponding to the fully occupied lowest LL for the \(\nu = 1/2\) state.

In order to readily visualize the above trial state in terms of the CF picture, we choose an electron of, say, spin-up with coordinate \(z_1\). Then \(\psi_{1/2}\) above contains a factor \([13]\)
\[
(z_1 - z_2)(z_1 - z_3) \cdots (z_1 - z_{m^{\dagger}})^2 \times
\]
\[
(z_1 - z_{m^{\dagger}+1})(z_1 - z_{m^{\dagger}+2}) \cdots (z_1 - z_{N})
\]
We are now able to easily understand from this expression that two kinds of flux attachments are ‘seen’ by the electron of up-spin. Namely, this electron sees an even number (two) of flux quanta attached to each up-spin electron and an odd number (one) of flux quanta attached to each down-spin electron.

We introduce the CS coupling constant,
\[
\alpha_{\sigma,\sigma'} = \frac{e}{(4m - 1)\phi_0} \begin{pmatrix} 2m & -(2m - 1) \end{pmatrix} \begin{pmatrix} -2m - 1 \end{pmatrix}
\] (18)
where \(m\) is a positive integer. It is easy to see that the CS action with \(\alpha_{\sigma,\sigma'}\) above leads to the same interpretation of flux attachment as the composite fermion picture of the \(\psi_{1/2}\) wave function shown in the expression (17).

The above spin-allowed CS coupling matrix is similar, in form, to the \((3,1,1)\) mode treated for bilayer systems \([6,8]\) with the fully polarized spin configuration. Following a procedure similar to the case of the odd denominator filling factors, we obtain for the FQHE systems of even denominator filling factors,

\[
b^\sigma(z) = 2m\phi_0\rho^\sigma + (2m - 1)\phi_0\rho^{-\sigma}(z).
\] (19)

This result is in complete agreement with the CF picture of Belkhir and Jain shown in (17). That is, each electron with spin \(\sigma(\uparrow or \downarrow)\) sees the \(2m\) number of statistical flux quanta attached to other electrons of the same spin \(\sigma\) and the \((2m - 1)\) number of statistical flux quanta attached to the electrons of the opposite spin \(-\sigma\). Allowing independent density fluctuations for spin-up and spin-down electrons, we readily find from (13) that \(b^\sigma(z) \neq b^{-\sigma}(z)\) in general.

The total number of effective magnetic flux quanta seen by the composite fermions of spin \(\sigma\) is given by
\[
N_{\sigma,\nu} = N_{\sigma} - (2m)N_{e}^\sigma - (2m - 1)N_{e}^{-\sigma}
= N_{\sigma} - (2m)N_{e} + N_{e}^{-\sigma}.
\] (20)

The filling factor \(\nu\) is then, with \(\nu^\sigma = \rho^\sigma/\left(1 + 2mp^\sigma + (2m - 1)p^{-\sigma}\right)\)
\[
\nu = \nu^\uparrow + \nu^\downarrow
\] (21)
\[
= \frac{p^\uparrow}{1 + 2mp^\uparrow + (2m - 1)p^{-\uparrow}} + \frac{p^\downarrow}{1 + 2mp^\downarrow + (2m - 1)p^{-\downarrow}}.
\]

Indeed, only the unpolarized states can arise, that is, the relation (13) allows \(\langle \rho^\sigma(z) \rangle = \langle \rho^{-\sigma}(z) \rangle\) for the single layer constraint of \(\langle b^\sigma(z) \rangle = \langle b^{-\sigma}(z) \rangle\). Quite encouragingly, we find from (21) that the spin singlet (spin-unpolarized) state of \(p^\sigma = p^{-\sigma} = 1\) leads to the following even denominator filling factor,
\[
\nu = \nu^\uparrow + \nu^\downarrow = \frac{1}{4m} + \frac{1}{4m} = \frac{1}{2m}.
\] (22)
with $\nu^\dagger = \nu^\dagger = 1/(4m)$. With the choice of $m=1$ the predicted filling factor from (22) is $\nu = 1/2$. By setting $\nu^\sigma + 1$ for the electrons of spin $\sigma$ in the $\nu^\sigma = 1/4$ state at the second LL, we get the $\nu = 5/2$ filling factor. It is gratifying to note that the filling factors such as the unobserved 3/2 and 7/2 states can not be predicted from the use of the CS coupling matrix in (13). Unlike the case of the odd denominator filling factors for which all possible spin-allowed states are available with the symmetry, $b^\sigma(z) = b^{-\sigma}(z)$ in the limit of $\varepsilon \to 0$, the CS action with (13) for the even denominator filling factors is applicable only to the spin-uncoupled ground states associated sufficiently weak magnetic field to preserve the symmetry condition of $\langle b^\sigma(z) \rangle = \langle b^{-\sigma}(z) \rangle$ mentioned above.

$$\begin{align*}
S_{\text{eff}}(\vec{a}_\mu^\sigma, \vec{A}_\mu^\sigma) &= \frac{1}{2} \int d^3x \int d^3y \left( \vec{A}_\mu^\sigma(x) - \vec{a}_\mu^\sigma(x) \right) \Pi_{\mu \nu}^\sigma(x, y) \left( \vec{A}_\nu^\sigma(y) - \vec{a}_\nu^\sigma(y) \right) \\
&- \frac{1}{2} \int d^3x \int d^3y \, b_\mu(x) \alpha^{\sigma \tau} V_{\sigma \epsilon}(|x-y|) \alpha^{\epsilon \tau \nu} b_\nu(y) \\
&+ \frac{\alpha_{\sigma \epsilon}}{2} \int d^3x \, e^{\nu \lambda} \mu_{\sigma \epsilon} \delta_{\nu \lambda} \tilde{a}_{\epsilon}^\sigma,
\end{align*}$$

(23)

The polarization tensor $\Pi_{\mu \nu}^\sigma(x, y)$ here is simply the linear response kernel to the fluctuations of the effective gauge field $\vec{A}_\mu^\sigma \equiv \vec{A}_\mu^\sigma - \vec{a}_\mu^\sigma$ for the system made of composite fermions. Here $\vec{A}_\mu^\sigma (\vec{a}_\mu^\sigma)$ represents the fluctuations of electromagnetic (statistical) gauge field. By allowing the spin degrees of freedom and using the procedure of Lopez and Fradkin (18), the Fourier transform of the polarization tensor, $\Pi_{\mu \nu}^\sigma(q, p) = \int d^3x d^3y e^{i(q_\mu x_\mu - q \cdot x)} e^{i(p_\nu y_\nu - p \cdot y)} \Pi_{\mu \nu}^\sigma(x, y)$ leads to

$$\Pi_{\mu \nu}^\sigma(q, p) = (2\pi)^3 \delta^3(q + p) \Pi_{\mu \nu}^\sigma(q),$$

(24)

with

$$\begin{align*}
\Pi_{\mu \nu}^\sigma(q) &= q^2 \Pi_{0 \sigma}^\sigma(q), \\
\Pi_{\mu 0}^\sigma(q) &= \omega q_j \Pi_{0 \sigma}^\sigma(q) + i \epsilon_{\mu j k} q_k \Pi_{1 \sigma}^\sigma(q), \\
\Pi_{0 \mu}^\sigma(q) &= \omega q_j \Pi_{0 \sigma}^\sigma(q) - i \epsilon_{\mu j k} q_k \Pi_{1 \sigma}^\sigma(q), \\
\Pi_{ij}^\sigma(q) &= \omega^2 \delta_{ij} \Pi_{0 \sigma}^\sigma(q) - i \omega \epsilon_{ij k} q_k \Pi_{1 \sigma}^\sigma(q) + (q^2 \delta_{ij} - q_i q_j) \Pi_{2 \sigma}^\sigma(q),
\end{align*}$$

(25)

where $\Pi_{\mu \nu}^\sigma(q) \equiv \delta_{\sigma \epsilon} \Pi_{\mu \nu \rho \sigma}(\omega, q)$ with $l = 0, 1, 2$ and with $\rho^\sigma$ indicating the dependency of the polarization tensor on the effective filling factor $\rho^\sigma$ for spin $\sigma$.

For the Gaussian integration over $a_{\mu}^\sigma$, we add a gauge-fixing term $(1/2\beta) \partial_{\mu} a_\mu^\sigma)^2 \partial_{\mu} a_\mu^\sigma$ into (23) in order to avoid singularity in the inversion of the matrix involving the quadratic term in $a_{\mu}^\sigma$. Then we obtain the following $6 \times 6$ ‘hyper-matrix’ for the quadratic term in $a_{\mu}^\sigma$ in the momentum representation (18),

$$M = \begin{pmatrix}
q^2 \Pi_{0 \sigma}^\sigma + \omega q_j \Pi_{0 \sigma}^\sigma + \omega^2 \Pi_{2 \sigma}^\sigma & \omega q_k \Pi_{0 \sigma}^\sigma - \omega q_j \Pi_{0 \sigma}^\sigma & -\omega q_2 \Pi_{2 \sigma}^\sigma + \omega q_1 \Pi_{2 \sigma}^\sigma \\
\omega q_k \Pi_{0 \sigma}^\sigma - \omega q_j \Pi_{0 \sigma}^\sigma - \omega^2 \Pi_{2 \sigma}^\sigma + \omega q_2 \Pi_{2 \sigma}^\sigma & \omega q_k \Pi_{1 \sigma}^\sigma + \omega^2 \Pi_{2 \sigma}^\sigma & -\omega q_2 \Pi_{2 \sigma}^\sigma - \omega q_1 \Pi_{2 \sigma}^\sigma \\
\omega q_k \Pi_{0 \sigma}^\sigma + \omega q_j \Pi_{0 \sigma}^\sigma + \omega^2 \Pi_{2 \sigma}^\sigma & -\omega q_k \Pi_{1 \sigma}^\sigma - \omega q_j \Pi_{1 \sigma}^\sigma & \omega q_2 \Pi_{2 \sigma}^\sigma - \omega q_1 \Pi_{2 \sigma}^\sigma
\end{pmatrix},$$

(26)
With \( p^\parallel = p^\parallel \), \( \Pi_0^{\sigma\sigma'} \) \((l = 0, 1, 2)\) in (24) and (26) becomes simply the \( 2 \times 2 \) diagonal matrix of identical elements \( \Pi_1^{\uparrow\uparrow} = \Pi_1^{\downarrow\downarrow} \) due to the involvement of the \( 2 \times 2 \) unit matrix. Consequently all these \( 2 \times 2 \) submatrices in (24) commute with each other and this commutability aids matrix inversion. Thus for the cases of both unpolarized \( (p^\parallel = p^\parallel) \) and fully polarized FQH states, the Gaussian integral over \( a_\mu \) can be performed by using effectively reduced \( 3 \times 3 \) matrices or by directly taking the inverse of the \( 6 \times 6 \) matrix. However for the partially polarized states we deal directly with the above \( 6 \times 6 \) matrix.

After the Gaussian integration over the statistical degrees of freedom \( a_\mu^{\sigma} \), we obtain the effective action for the electromagnetic fluctuations \( \tilde{A}_\mu \),

\[
S_{\text{eff}}^{\text{EM}}(\tilde{A}_\mu) = \frac{1}{2} \int d^2 x \int d^2 y \sum_{\sigma\sigma'} \tilde{A}_\mu^{\sigma}(x) K_{\sigma\sigma'}^{\mu\nu}(x, y) \tilde{A}_\nu^{\sigma'}(y) .
\] (27)

Both spin-up and spin-down electrons are coupled to the same electromagnetic fluctuations; \( \tilde{A}_\mu^\uparrow = \tilde{A}_\mu^\downarrow = \tilde{A}_\mu \). Here \( K_{\sigma\sigma'}^{\mu\nu} \) is the electromagnetic polarization tensor which measures the electromagnetic response of the FQHE system to a weak electromagnetic perturbation. The components of the electromagnetic polarization tensor in the momentum space are obtained (1) as follows:

\[
K_{00}^{\sigma\sigma'} = q^2 K_0^{\sigma\sigma'}(\omega, q) ,
\]

\[
K_{0j}^{\sigma\sigma'} = \omega q_j K_0^{\sigma\sigma'}(\omega, q) + i \epsilon_{jk} q_k K_{1}^{\sigma\sigma'}(\omega, q) ,
\]

\[
K_{j0}^{\sigma\sigma'} = \omega q_j K_0^{\sigma\sigma'}(\omega, q) - i \epsilon_{jk} q_k K_{1}^{\sigma\sigma'}(\omega, q) ,
\]

\[
K_{ij}^{\sigma\sigma'} = \omega^2 \epsilon_{ij} K_0^{\sigma\sigma'}(\omega, q) - i \epsilon_{ij} \omega K_{1}^{\sigma\sigma'}(\omega, q) + (q^2 \delta_{ij} - q_i q_j) K_{2}^{\sigma\sigma'}(\omega, q) .
\] (28)

For the case of spin-unpolarized states, we obtain for \( K_{l}^{\sigma\sigma'}(\omega, q) \) \((l = 0, 1, 2)\)

\[
K_0^{\sigma\sigma'}(\omega, q) = - \Pi_0^{\sigma\sigma'}(\alpha^{l\prime} \tau\tau) \left( D^{\tau\tau'}(\omega, q) \right)^{-1} ,
\]

\[
K_1^{\sigma\sigma'}(\omega, q) = \alpha^{l\prime\tau} + \alpha^{\sigma\tau} \left( \Pi_1^{\tau\tau'} + \Pi_1^{\sigma\sigma'} + \Pi_2^{\sigma\sigma'} \right) ,
\]

\[
K_2^{\sigma\sigma'}(\omega, q) = - \alpha^{l\prime\tau} \left[ \Pi_2^{\tau\tau'} + V(q) \right] ,
\] (29)

Here \( D^{\tau\tau'}(\omega, q) \) is given by

\[
D^{\tau\tau'}(\omega, q) = \omega^2 \Pi_0^{\tau\tau'} \Pi_0^{\sigma\sigma'} - (\Pi_1^{\tau\tau'})^2 + \Pi_1^{\sigma\sigma'} \Pi_2^{\tau\tau'} - \alpha^{l\prime\tau} V(q) \alpha^{\sigma\tau} q^2 .
\] (30)

We obtain, in the limit of \( q^2 \rightarrow 0 \) and \( \omega \rightarrow 0 \),

\[
\Pi_0^{\sigma\sigma'}(0, 0) = \frac{p^0 m_b}{2 \pi B_{\text{eff}}} \delta^{\sigma\sigma'} = \frac{p^0 m_b}{2 \pi (B - b)} \delta^{\sigma\sigma'} ,
\]

\[
\Pi_1^{\sigma\sigma'}(0, 0) = \frac{p^0}{2 \pi} \delta^{\sigma\sigma'} ,
\]

\[
\Pi_2^{\sigma\sigma'}(0, 0) = - \frac{(p^0)^2}{2 \pi m_b} \delta^{\sigma\sigma'} ,
\] (31)

\[
\lim_{\omega \rightarrow 0} \lim_{q \rightarrow 0} K_{1}^{\sigma\sigma'}(\omega, q) = \alpha^{l\prime\tau} \left( \Pi_1^{\tau\tau'}(0, 0) \right)^{-1} \left[ \Pi_1^{\sigma\sigma'}(0, 0) \right] = \frac{1}{2 \pi} \frac{1}{1 + 2m(2n) + \varepsilon^2 n^2} \left( \frac{n + 2mn^2 + \varepsilon n^2}{-2mn^2} \right) ,
\] (32)

for the odd-denominator filling factors. We obtain in the limit of \( q^2 \rightarrow 0 \) and \( \omega \rightarrow 0 \),

\[
K_1(0, 0) = \sum_{\sigma\sigma'} K_1^{\sigma\sigma'}(0, 0) = \lim_{\varepsilon \rightarrow 0} \frac{1}{2 \pi} \frac{2n}{1 + 2m(2n) + \varepsilon^2 n^2} (33)
\]

where \( m_b \) is the band mass. Here it should be noted that the denominator in \( \Pi_0^{\sigma\sigma'}(0, 0) \) above is given by \( B_{\text{eff}} = B - b \), that is, the effective magnetic field seen by the composite fermions. On the contrary, it is given by the statistical magnetic field in the CS theory of Lopez and Fradkin (see their equation, B3 in Ref. 9).

Using (31), (30) and (31), we obtain from (29) with \( p^\parallel = p^\parallel = n \),

\[
\frac{1}{2 \pi} \frac{2n}{1 + 4mn} = \frac{\nu}{2 \pi} .
\]

For a weak external electric field \( \tilde{E}_j \), the induced current is \( J_i = K_1(0, 0) \epsilon_{ij} \tilde{E}_j \). Thus \( K_1(0, 0) \) is the same as the Hall conductance \( \sigma_{xy} \) of the FQHE system, which
correctly represents a fractional multiple of $\frac{1}{r^2}$. Using \( [24, 20] \) and \( [31] \) for \( [22] \) the compressibility for the unpolarized states is simply $\kappa = \lim_{q \to 0} K_{00}(\omega, q) = 0$, indicating the incompressible states.

\[
\lim_{\omega \to 0} K_1^{\sigma'\sigma}(\omega, q) = \frac{1}{2\pi} \frac{1}{1 + 2m(2n) + (4m - 1)n^2} \left( \frac{n + 2mn^2}{-(2m - 1)n^2} \right),
\]

and $K_1(0, 0) = 1/(2\pi 2m)$ in this case with $n = 1$. Although we do not write the explicit form of $K_0^{\sigma'\sigma}(\omega, q)$ ($l = 0, 1, 2$) here for the partially polarized states, one can find from a similar procedure the Hall conductance and incompressibility.

### IV. COMPUTED SPECTRA OF COLLECTIVE EXCITATIONS

There have been several theoretical attempts to obtain the collective excitations of fully polarized states. By using the single mode approximation (SMA) developed in analogy with the Feynman’s theory of superfluid helium, Girvin, MacDonald, and Platzman \([24]\) calculated collective excitations of the Laughlin ($\nu = 1/(2m + 1)$) states with $m$ positive integer and found the existence of relatively deep minima in the energy dispersion for $m = 3, 5$ and 7. Recently Kamilla, Wu, and Jain used the CF wave functions to investigate the collective excitations of various FQH states \([25]\) and also found the deep minima. Based on the CF picture, Chiu-Simons field theoretical approaches have been used to obtain the collective excitations of the FQH states: Lopez and Fradkin found a series of collective modes for fully polarized incompressible states \([26]\), and taking into account large mass renormalization, Simon and Halperin \([27]\) modified the earlier ran-

dom phase approximation (RPA) of Halperin, Lee, and Read \([11]\). They obtained a correct frequency scale for low-energy excitations and found a shallower magnetoroton minimum for the $\nu = 1/3$ Laughlin state compared to other studies \([24, 23]\).

#### A. Collective excitations for odd denominator FQHE

In the odd denominator FQH states for which the effective filling factor is an integer in the IQH state of CF, there exist three different cases of spin-allowed states: the spin unpolarized state, $p^+ = p^\dagger$; the partially polarized state, $p^+ \neq p^\dagger \neq 0$ and the fully polarized state, $p^\dagger = 0$.

Earlier the collective modes of the fully polarized FQHE were examined by Lopez and Fradkin \([23]\). Their computed energy dispersion was accurate only at low values of $q$ due to the use of the lowest order term in $q$. In the present study, in order to examine collective excitations for the wider range of $q$ we include higher order terms in $q$ and compute the poles of $K_{00}^{\sigma'\sigma}$ numerically. We adopt the components of one-particle polarization tensor derived by Lopez and Fradkin, which can be extended to the following generalized expressions to allow for the spin-unpolarized, partially polarized, and fully polarized FQH states \([27]\).

\[
\Pi_0^{\sigma'\sigma}(\omega, q) = -\delta_{\sigma\sigma'} \frac{B_{00}}{(2\pi)m_b} e^{-\frac{1}{2} \delta_{\sigma\sigma'}} \sum_{m=p}^{m'=p'} \sum_{m''=0}^{m'-1} \frac{\omega^2 - (\omega_m - \omega_{m''})^2}{m! m''!} q^{2(m-m'')} (L^{m'-m''} m'' (q^2))^2,
\]

\[
\Pi_1^{\sigma'\sigma}(\omega, q) = -\delta_{\sigma\sigma'} \frac{B_{10}}{(2\pi)m_b} e^{-\frac{1}{2} \delta_{\sigma\sigma'}} \sum_{m=p}^{m'=p'} \sum_{m''=0}^{m'-1} \frac{\omega^2 - (\omega_m - \omega_{m''})^2}{m! m''!} q^{2(m-m'')} L^{m'-m''} m'' (q^2),
\]

\[
\times \left\{ q^2 (L^{m-m'')(q^2) + 2L^{m-m'-1}(q^2)(1-\delta_{m'',0})] - (m-m')L^{m-m'')(q^2) \right\},
\]

\[
P_2^{\sigma'\sigma}(\omega, q) = -\delta_{\sigma\sigma'} \frac{B_{20}}{(2\pi)2m_b} e^{-\frac{1}{2} \delta_{\sigma\sigma'}} \sum_{m=p}^{m'=p'} \sum_{m''=0}^{m'-1} \frac{\omega^2 - (\omega_m - \omega_{m''})^2}{m! m''!} q^{2(m-m'')} (L^{m'-m''} m'' (q^2))^2,
\]

\[
\times \left[ L^{m-m'} m' (q^2) + 2L^{m-m'-1}(q^2)(1-\delta_{m',0}) \right]
\times \left\{ q^2 (L^{m-m'')(q^2) + 2L^{m-m'-1}(q^2)(1-\delta_{m'',0})] - 2(m-m')L^{m-m'')(q^2) \right\}.
\]
with \( q^2 \equiv q^2/(2B_{\text{eff}}) = q^2/2(B-b) \). Differences in the effective filling factors \( p^\sigma \) in the expression (25) here permit three different spin allowed FQH states mentioned above. Due to the neglect of spin wave mode and spin flip, the Zeeman coupling does not affect the density response kernel since the spin dependent chemical potential contribution disappears in the denominator of \( \Pi^{\sigma'}(\omega, q) \) (\( l = 0, 1, 2 \)). In the present study, by keeping virtual excitations up to the 9th effective LL in \( \Pi(\omega, q) \) in (28) above we compute collective excitations from the poles that appear in the density-density correlation functions \( K^{\sigma'}_{00}(\omega, q) \).

1. Fully polarized FQH state: \( p^\uparrow = 0, p^\downarrow \neq 0 \)

For the fully polarized Laughlin states, \( p^\uparrow = 0 \), the one-particle polarization tensor \( (33) \) has non-vanishing components only for spin-down electrons, \( \Pi^{\downarrow\downarrow}(\omega, q) \). The poles of the density-density correlation functions \( K^{\sigma'}_{00}(\omega, q) \) in (28) are obtained from the zeroes of the determinant \( (25) \) of \( ((\alpha^{\sigma'})^{-1}D^{\sigma'}(\omega, q)) \) as can be seen from (29) and (30). Obviously the expression of frequency dispersion at small \( q \) values for the fully polarized states is the same as that of Lopez and Fradkin (23). Thus we avoid repetition here.

In Fig. 1 we display numerical calculations of collective excitation spectrum for \( \nu = 1/3 \). As long as the short range interaction energy is in the order of the mean effective field gap \( \omega_{\text{eff}} \), we find that the dispersion \( \omega(q) \) does not change appreciably. We choose a constant value of \( v(q) = 4/5\hbar c \) with \( B \sim 10T \) which is in the same order of magnitude as the Coulomb repulsion energy \( e^2/\epsilon_0 \). Another choice of the interaction potential, \( v(q) \sim 1/q \) does not appreciably alter the energy dispersion. This is, indeed, consistent with the earlier studies based on the Laughlin wave function (13). The computed results are in excellent agreement with the unrenormalized RPA results of Simon and Halperin (26). In Fig. 1 the width of striped band represents the \( q^{-2} \) times the weight of poles of the density response function \( K^{00}(\omega, q) \). The width becomes negligibly small when \( q \) becomes larger than \( 5/\epsilon_0 \). Such damped collective excitation is already pointed out by Lopez and Fradkin (23), which is thought to occur when the excitation energy becomes equal to an energy to create the lowest available two-particle (particle-hole pair) state at sufficiently large \( q \).

2. Unpolarized FQH state: \( p^\uparrow = p^\downarrow \)

To the best of our knowledge, there exists no report on comparative studies between the spin unpolarized states and the spin fully polarized states for the choice of the same filling factors. We first consider the excitation mode for the unpolarized case with weak Zeeman coupling. As an example, we consider the filling factor of \( \nu = 2/5 \) with \( p^\uparrow = p^\downarrow = 1 \) and \( m = 1 \) in (5). The poles of the density-density correlation functions \( K^{\sigma'}_{00}(\omega, q) \) in (28) are obtained from the zeroes of the determinant \( (25) \) of \( ((\alpha^{\sigma'})^{-1}D^{\sigma'}(\omega, q)) \) as can be seen from (29) and (30) or from the zeroes of the determinant of \( 6 \times 6 \) hypermatrix (26) for the spin partially polarized states. We obtain the collective excitations for the unpolarized state by finding the zeroes of the determinant through either of the approaches above.

\[
\lim_{\varepsilon \to 0} U(\omega, q) = \left[ \omega^2 \Pi_0^2 - \left( \Pi_1 + \frac{1}{2\pi} \frac{1}{4m} \right)^2 + \Pi_0 \left( \Pi_2 - \frac{1}{4\pi^2} \frac{1}{4m} \right) (v_{\uparrow\uparrow} + v_{\downarrow\uparrow}) \right] q^2,
\]

where

\[
det M = -\frac{(2\pi)^2(4m)^2(\omega^2 + q^2)^4}{\epsilon^4 \beta^2} U(\omega, q) \quad (36)
\]
with $\Pi_l = \Pi^\dagger_{l\dagger} = \Pi^\dagger_{l\dagger}$. 

![Figure 2](image-url)

**FIG. 2.** Collective excitation spectrum for both the spin fully polarized and unpolarized FQH states at $\nu = 2/5$ with the short range potential $v(0) = 4/5 \hbar \omega_c$. The center of each stripe shows the pole and width of striped band represents $q^{-2}$ times the weights (residues) of the pole in $K_{00}$.

Here $v_{\uparrow\uparrow}$ and $v_{\uparrow\downarrow}$ are the zeroth order coefficients of the Fourier transform of the interaction potential between the composite fermions of the same spin or opposite spins and are identical, i.e. $v_{\uparrow\uparrow} = v_{\uparrow\downarrow} = v$. The energy dispersion relation obtained from the zeroes of (37) then becomes similar, in form, to the fully polarized case [23] of $\nu = 1/5$ except the difference in the potential energy dependent term ($2v$ for $\nu = 2/5$ and $1v$ for $\nu = 1/5$). There exists only one mode whose residue is proportional to $q^2$ at the cyclotron frequency, $\omega_c = 5\omega_{\text{eff}}$. By keeping only the lowest order term in $q$ and using (35) and (37), its frequency dispersion of $\omega(q)$ is given by

$$\omega(q) = \left[ \omega_c^2 + \frac{q^2}{2B_{\text{eff}}} \right] \omega_{\text{eff}}^2 \left[ \frac{16}{3} + \frac{4m_0 v}{2\pi} \right]^{1/2},$$

(38)

with the residue (weight of poles) for the density response function $K_{00}$

$$\text{Res}(K_{00}, \omega_{\text{res}}(q)) = -q^2 \omega_c \left( \frac{1}{2\pi} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right),$$

(39)

thus clearly satisfying the Kohn’s theorem [29], which demands that there should be only one mode converging to the cyclotron frequency with residue proportional to $q^2$ in the long-wavelength limit [30]. From the direct analogy to the fully polarized case, one can also show that the $\nu = 2/5$ state satisfies the $f$-sum rule [23].

We computed the case of fully polarized FQH states at $\nu = 2/5$ (see Fig. 2) and found a clear minimum for the lowest excitation mode. Unlike the case of the spin fully polarized state for $\nu = 2/5$ there is an almost flat minimum for the lowest mode for the choice of spin unpolarized state, as shown in Fig. 3. One of the two modes at the zero momentum excitation energy of $\omega_c = 5\omega_{\text{eff}}$ rapidly increases with $q$; this computed mode follows analytic behavior of $\omega(q) \sim q^2$ near $q = 0$. Besides we note that there exist differences in the number of subbands for higher lying collective excitation modes between the two different states. Both cases show Kohn’s modes at the long-wavelength limit. On the other hand the weight of the lowest excitation mode for the fully polarized case is somewhat larger than that of the unpolarized case.

3. Partially polarized FQH state: $p^\dagger = p^\dagger + 1$

It is not clear [31] whether the FQHE occurs with the fully or partially polarized states of odd denominator filling factors if the spin wave mode or spin flip excitations can happen. However it is thought that the incompressible states of IQHE at odd integer filling factors are possible if there exist large correlation effects between electrons; Sondhi et al. [32] showed that the fully polarized odd integer $\nu = 1$ state always has a gap due to the correlation effects, even if the Zeeman energy vanishes and spin flip is allowed. Similarly such FQHE state corresponding to the odd integer effective filling factors (e.g. $\nu = 1/3$ or $3/7$) in the CF picture may be also possible if there exist large correlation effects between composite fermions [31]. In the present calculations we did not consider the correlation effects between the composite fermions for the fully or partially polarized FQH states. Further we would like to point out that the computed energy dispersions represent collective excitations without the consideration of spin wave mode and spin-flip processes. For sufficiently large effective LL spacing, FQH states for the odd effective filling factors naturally occur.

With $p^{-\sigma} = p^\sigma + 1$ for the partially polarized states, the diagonal elements of $\Pi_\sigma^0$ are no longer identical, that is, $\Pi^\dagger_{l\dagger} \neq \Pi^\dagger_{l\dagger}$ in (35) due to the difference in population between the up-spin electrons and down-spin electrons. The poles of the density-density correlation functions $K_{00}^{\sigma\sigma}(\omega, q)$ are obtained from the zeroes of the determinant of the $6 \times 6$ hyper-matrix (35). We obtain the determinant through the symbolic calculation using Mathematica [33].

9
where

\[
\lim_{\epsilon \to 0} \frac{P^{\sigma \sigma'}(\omega, \mathbf{q})}{\epsilon^4} = \omega^2(\Pi_0^\dagger + \Pi_0^\dagger)^2 - \left( \Pi_0^\dagger + \Pi_0^\dagger + \frac{1}{2}\frac{1}{2m} \right)^2
+ \left[ \Pi_0^\dagger \Pi_0^\dagger + \Pi_0^\dagger \Pi_0^\dagger + \Pi_0^\dagger \Pi_0^\dagger - \frac{1}{4\pi^2} \frac{1}{4m^2} (\Pi_0^\dagger + \Pi_0^\dagger) v^{\uparrow \downarrow} \right] \mathbf{q}^2
- (v^{\uparrow \downarrow} - v^{\downarrow \uparrow}) \mathbf{q}^2 \times \left[ \frac{\omega^2}{\Pi_0^\dagger \Pi_0^\dagger (\Pi_0^\dagger + \Pi_0^\dagger)} - \Pi_0^\dagger \Pi_0^\dagger (\Pi_0^\dagger + \Pi_0^\dagger) - \frac{1}{4\pi^2} \frac{1}{4m^2} \Pi_0^\dagger \Pi_0^\dagger (v^{\uparrow \downarrow} + v^{\downarrow \uparrow}) \right].
\]

\[
\det M = -\frac{(2\pi)^2 (2m)^2 (\omega^2 + \mathbf{q}^2)^4}{\beta} P^{\sigma \sigma'}(\omega, \mathbf{q})
\]

FIG. 3. Collective excitation spectrum for both the spin fully polarized and partially polarized states at \(\nu = 3/7\) with the short range potential \(v(0) = 4/5 \omega_c\). The center of each stripe shows the pole and width of striped band represents \(q^{-2}\) times the weights (residues) of the pole in \(K_{00}\).

The dispersion of the collective excitations for the \(\nu = 3/7\) with \(m = 1\) and \(v^{\uparrow \downarrow} = v^{\downarrow \uparrow}\) is plotted in Fig. 3. The numerically obtained Kohn’s mode \(\omega = 7\omega_{\text{eff}} \equiv \omega_c\) at the zero momentum is quite satisfactory with a maximum width as \(q^2 \to 0\), as shown in Fig. 3. We note that besides the differences in the energy spectrum the lowest excitation mode for the spin partially polarized state of \(\nu = 3/7\) has a much shallower minimum compared to the case of the fully polarized state.

There exists discrepancy between our predicted energy gap and the experimentally observed energy gap; by choosing a typical value of magnetic field \(B \sim 10T\), our predicted energy gaps are about \(0.8\omega_{\text{eff}} \sim 50K\) with band mass \(m_b = 0.066m_e\) (\(m_e\), the electron bare mass) as shown in Figs. 3 through 7 which is about 2 to 4 times larger than the experimental values \([15,16]\) of \(\sim 0.1\epsilon^2/\varepsilon_{\text{lo}} \sim 16K\) with the choice of \(\epsilon = 13\) for dielectric constant.

For the fully polarized states this problem is remedied by the mass-renormalized RPA study of Simon and Halperin \([17]\). Their finding is that the magnetoroton minima are much less pronounced than those calculated from the unrenormalized RPA, and further comparison of the mass-renormalized RPA method with the results from the exact diagonalization method \([18]\) showed a good agreement particularly at low energies. In the present study, we noted marked differences particularly in the lowest collective excitation mode between the fully polarized states and the spin unpolarized or partially polarized states. Judging from the shallower magnetoroton minima with other than the spin fully polarized states, we believe that a study of finite-size system through an exact diagonalization procedure \([20]\) for the spin-unpolarized or partially polarized states of odd denominator filling factors may still lead to shallower magnetoroton minima than the fully polarized case.

B. Collective excitation for \(\nu = 1/2\) state

For the \(\nu = 1/2\) state, we consider the case of the effective filling factor, \(p^\dagger = p^\dagger = 1\) and \(m = 1\) in \([18]\) with sufficiently small Zeeman coupling. The effective cyclotron frequency is \(\omega_{\text{eff}}^\dagger = \omega_{\text{eff}}^\dagger = \omega_c/4\).

The poles of \(K_{00}\) are obtained from the zeroes of the determinant of \([20]\).

\[
\det M = -\frac{(2\pi)^2 (2m)^2 (\omega^2 + \mathbf{q}^2)^4}{\beta} I(\omega, \mathbf{q}) \times O(\omega, \mathbf{q}),
\]

where the determinant is factorized into two parts, in order to obtain,
\[
I(\omega, q) = \omega^2 \Pi_0^2 - (\Pi_1 + \frac{1}{2\pi \frac{1}{3}})^2 \\
+ \Pi_0 \left[ \Pi_2 - \frac{1}{4\pi^2 9} (v_{\uparrow\uparrow} + v_{\downarrow\downarrow}) \right] q^2 = 0,
\]
which leads to the ‘in-phase’ residues and
\[
O(\omega, q) = \omega^2 \Pi_0^2 - (\Pi_1 + \frac{1}{2\pi})^2 \\
+ \Pi_0 \left[ \Pi_2 - \frac{1}{4\pi^2} (v_{\uparrow\uparrow} - v_{\downarrow\downarrow}) \right] q^2 = 0.
\]
The latter part (44) yields the ‘out-of-phase’ residues; that is,
\[
Res(K_{00}, \omega(q)) \sim \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.
\]

Unlike the case of bilayer system [9], the residues for the out-of-phase mode from (44) yield null contributions to the density response function after the summation over the exchange energy may deserve some attention utilizing the present theory in the future.

In the present study, we introduced a spin-allowed \(U(1) \times U(1)\) Chern-Simons theory to extract all the known odd denominator filling factors from a single Chern-Simons coupling constant (matrix) and derived the formal expressions of spin-allowed electromagnetic polarization tensors corresponding to all the odd denominator filling factors. Further we computed the collective excitation spectra for various odd denominator filling factors in order to examine differences in the collective modes between the different cases of the fully polarized states and the unpolarized or partially polarized states.

One of the salient features from the present CS theory is that all possible odd denominator filling factors corresponding to the spin-unpolarized, partially polarized, and fully polarized states can be generated from a single CS coupling matrix. By comparing the collective excitation modes of \(\nu = 2/5\) and \(3/7\) for the cases of the spin-fully polarized and unpolarized or partially polarized states, we find that both the unpolarized and partially polarized FQH states have much shallower minima for the lowest collective mode compared to the fully polarized states. The Kohn’s theorem [23] was satisfied for all the predicted collective excitation modes. Judging from the predicted shallower magnetoroton minima with other than the spin fully polarized states, we believe that a study of finite-size system through the exact diagonalization procedure [23] for the spin-unpolarized or partially polarized states of odd denominator filling factors may also lead to shallower magnetoroton minima than the fully polarized states. In this paper, we did not allow the spin-flip and spin-wave mode. Taking into account the spin-flip and spin-wave mode, the contributions of exchange energy may deserve some attention utilizing the present theory in the future.

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![FIG. 4. Collective excitation spectrum for both the spin unpolarized state of \(\nu = 1/2\) with the short range potential \(v(0) = 4/5\omega_c\). The center of each stripe shows the pole and width of striped band represents \(q^{-2}\) times the weights (residues) of the pole in \(K_{00}\).](image)
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[28] In order to compute the poles of \( K_{\alpha\beta}^{\sigma\sigma'}(\omega, q) \), we use the fact that the product of \( D^{\sigma\sigma'}(\omega, q) \) cancels its direct dependency on \( q^{\sigma\sigma'} \) for the limiting case of \( \varepsilon \to 0 \).

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