Perhaps the largest gap in our understanding of nature at the smallest scales is a consistent quantum theory underlying the Standard Model and General Relativity. Substantial theoretical research has been performed in this context, but observational efforts are hampered by the expected Planck suppression of deviations from conventional physics. However, a variety of candidate models predict minute violations of both Lorentz and CPT invariance. Such effects open a promising avenue for experimental research in this field because these symmetries are amenable to Planck-precision tests.

The low-energy signatures of Lorentz and CPT breaking are described by an effective field theory called the Standard-Model Extension (SME). In addition to the body of established physics (i.e., the Standard Model and General Relativity), this framework incorporates all Lorentz- and CPT-violating corrections compatible with key principles of physics. To date, the SME has provided the basis for the analysis of numerous tests of Lorentz and CPT symmetry involving protons, neutrons, electrons, muons, and photons. Discovery potential exists in neutrino physics.

A particularly promising class of Planck-scale tests involve matter–antimatter comparisons at low temperatures. SME predictions for transition frequencies in such systems include both matter–antimatter differences and sidereal variations. For example, in hydrogen–antihydrogen spectroscopy, leading-order effects in a 1S–2S transition as well as in a 1S–Zeeman transition could exist that can be employed to obtain clean constraints. Similarly, tight bounds can be obtained from Penning-trap experiments involving antiprotons.
1. Motivations

The study of the hydrogen (H) atom is closely associated with two of the most important achievements in 20th-century physics: the explanation of its discrete spectrum provided a great triumph for quantum theory, and many details of the H spectrum serve as a powerful testimonial for Special Relativity in the microscopic world of atoms. The combination of these two groundbreaking theories lies at the foundation of the most successful physical theory to date—the Standard Model of particle physics.

For roughly half a century, theoretical research in fundamental physics has been dominated by various approaches to synthesize the Standard Model and General Relativity into a single unified theory that incorporates, for example, a quantum description of the gravitational interaction. Although a substantial amount of progress has been achieved on the theoretical front, the expected Planck suppression of quantum-gravity effects hampers experimental research in this field: low-energy measurements are likely to require sensitivities of at least one part in $10^{17}$. This talk argues that the recent creation and observation of hot antihydrogen ($\bar{H}$), the subsequent production of cold $\bar{H}$ by the ATHENA and ATRAP collaborations, and the synthesis of antiprotonic Helium by the ASACUSA collaboration pave the way for tests that could shed some light on this issue. The basic ideas behind this belief are summarized in the remainder of this section.

The presumed minuscule size of candidate quantum-gravity signatures requires a careful choice of experiments. A promising avenue that one can pursue is testing physical laws that satisfy three primary criteria. First, one should consider presently established fundamental laws that are believed to hold exactly. Any measured deviations would then definitely indicate qualitatively new physics. Second, the chances of finding an effect are increased by testing laws that can be violated in credible candidate fundamental theories. Third, from a practical viewpoint, this law must be amenable to ultrahigh-precision tests.

An example of a physics law that satisfies all of these criteria is CPT invariance. As a brief reminder, this law requires that the physics remains invariant under the combined operations of charge conjugation ($C$), parity inversion ($P$), and time reversal ($T$). Here, the $C$ transformation links particles and antiparticles, $P$ corresponds to a spatial reflection of physics quantities through the coordinate origin, and $T$ reverses a given physical process in time. The Standard Model is CPT-invariant by construction, so that the first criterion is satisfied. With regards to criterion two, we
mention that a variety of approaches to fundamental physics can lead to CPT breakdown. Examples include strings, spacetime foam, nontrivial spacetime topology, and cosmologically varying scalars. The third criterion is met as well. Consider, for instance, the conventional figure of merit for CPT conservation in the Kaon system: its value lies currently at $10^{-18}$, as quoted by the Particle Data Group.

The CPT transformation relates a particle to its antiparticle. One would therefore expect that CPT invariance implies a symmetry between matter and antimatter. Indeed, one can prove that the magnitude of the mass, charge, decay rate, gyromagnetic ratio, and other intrinsic properties of a particle are exactly equal to those of its antiparticle. This prove can be extended to systems of particles and their dynamics. For example, atoms and anti-atoms must exhibit identical spectra and a particle-reaction process and its CPT-conjugate process must possess the same reaction cross section. It follows that experimental matter–antimatter comparisons can serve as probes for the validity of CPT symmetry. In particular, the extraordinary sensitivities offered by atomic spectroscopy suggest comparative tests with $\text{H}$ and $\overline{\text{H}}$ as high-precision tools in this context.

CPT violation in Nature would also lead to other, less obvious effects. The celebrated CPT theorem of Bell, Lüders, and Pauli states that CPT invariance arises under a few mild assumptions through the combination of quantum theory and Lorentz symmetry. If CPT invariance is broken one or more of the assumptions necessary prove the CPT theorem must be false. This leads to the obvious question which one of the fundamental assumptions in the CPT theorem has to be dropped. Since both CPT and Lorentz symmetry involve spacetime transformations, it is natural to suspect that CPT violation implies Lorentz-invariance breakdown. This has recently been confirmed rigorously in Greenberg’s “anti-CPT theorem,” which roughly states that in any local, unitary, relativistic point-particle field theory CPT violation implies Lorentz violation. Note, however, that the converse of this statement—namely that Lorentz violation implies CPT breaking—is not true in general. In any case, it follows that CPT tests also probe Lorentz symmetry. This result offers the possibility for a another class of CPT-violation searches in addition to instantaneous matter–antimatter comparisons: probing for sidereal effects in matter–antimatter and other systems.

This talk gives an overview of Lorentz and CPT violation as a tool in the search for underlying physics—possibly arising at the Planck scale. Section 2 discusses some forms of Lorentz and CPT violation that could be
considered when constructing test models. The Standard-Model Extension (SME), which is currently the standard and most general framework for CPT and Lorentz tests, is reviewed in Sec. 3. Section 4 gives two examples of how a Lorentz- and CPT-invariant model can lead to the violation of these symmetries in the ground state generating SME coefficients. In Sec. 5, some experimental tests of Lorentz and CPT invariance that involve antimatter are discussed. Section 6 contains a brief summary.

2. Types of Lorentz violation

The first step in constructing a test model parametrizing the breakdown of Lorentz and CPT symmetry is to determine possible types of Lorentz violation. Additional considerations for CPT breakdown are unnecessary by the anti-CPT theorem because general Lorentz violation will include CPT breaking. It turns out that a clear understanding of the fundamental principle of coordinate independence will provide us with a useful, rough classification of types of Lorentz violation. For this reason, it appears appropriate begin with a brief review of this fundamental principle and its implementation.

Coordinate independence is one of the most basic principles in physics. Its need in the presence of Lorentz breaking is well established, and it has served as the basis for the construction of the SME.\textsuperscript{14,15} However, this principle is sometimes not fully appreciated. For example, in some investigations of Lorentz and CPT violation coordinate-dependent physics emerges, and occasionally Lorentz-symmetry breakdown is identified with the loss of coordinate independence.

A given labeling scheme for events in space and time is called a coordinate system. Such a labeling typically depends on the observer choosing the coordinates, and it is thus arbitrary to a large extent. In other words, coordinate systems are mathematical tools for the measurement, description, and prediction of physical phenomena. But since they are a pure product of human thought, coordinates lack physical reality. It follows that the physics should remain unaffected by the choice of a particular coordinate system. This principle of coordinate independence is one of the most fundamental in science. Since it assures that the physics remains independent of the observer, it is also called observer invariance. It should therefore be possible to formulate the fundamental laws of physics in a coordinate-free language. For example, this can be achieved mathematically, when spacetime is given a manifold structure and physical quantities are represented by geometric
objects, such as tensors or spinors.

The principle of coordinate invariance is more fundamental than Lorentz symmetry. Consider, for instance, Newton’s second law of motion in non-relativistic classical mechanics

$$\vec{F} = m\ddot{\vec{v}}$$

as well as its relativistic version

$$F^\mu = m \frac{d\nu^\mu}{d\tau}$$

where $\tau$ denotes the mass’ proper time, and $F^\mu$ is the usual relativistic generalization of the force $\vec{F}$. Both laws are coordinate independent. Equation (1) takes the same form in all inertial galilean frames; it is formulated in the coordinate-free language of 3-vectors. Similarly, Eq. (2) remains of the same form in all inertial Minkowski coordinate systems; it is expressed in terms of 4-vectors. However, only Eq. (2) is Lorentz invariant. We conclude that coordinate independence is more general than Lorentz symmetry because there might very well be laws—such as Eq. (1)—that maintain coordinate invariance but violate Lorentz symmetry.

The above situation becomes even more transparent with the following observation. Lorentz symmetry does not only require coordinate independence, but it also dictates the transformations that relate different inertial frames. Although Eq. (1) is coordinate independent, inertial frames are related by Galilei instead of Lorentz transformations. Mathematically speaking, both cases allow a coordinate-invariant spacetime-manifold description, but the manifold structure is galilean in the case of Eq. (1) and lorentzian in the case of Eq. (2). The question which spacetime manifold is realized in Nature must be answered experimentally.

**Lorentz violation through non-lorentzian manifolds.** The above considerations lead to one possible type of Lorentz violation maintaining coordinate independence: the spacetime manifold could be non-lorentzian. Then, the fundamental physics laws have the same form in all inertial frames, but the frames are no longer related by the usual Lorentz transformations. This point of view is taken in the early relativity test model of Robertson and its extension by Mansouri and Sexl, as well as in the so called “doubly special relativities.” In the present work, we do not treat this type of Lorentz-symmetry violation separately because it is known that (at least some of) these effects are equivalent to those of another type of Lorentz violation discussed next. Moreover, such frameworks are typically purely kinematical precluding the analysis of atomic level shifts, for example.
Lorentz violation through a nontrivial vacuum. Most modern approaches to fundamental physics involve lorentzian manifolds, where inertial frames are related by the usual Lorentz transformations. Such approaches take Lorentz symmetry as a key ingredient, and the issue arises as to whether Lorentz violation can occur in such situations. It turns out that this is indeed the case if the vacuum contains a tensorial background. The primary emphasis in this section is to gain some intuitive understanding of Lorentz breakdown in the presence of such a background. The question of how to generate tensorial backgrounds in a Lorentz-invariant theory is deferred to Sec. 4.

It is again useful to consider a familiar example from classical physics. Suppose the particle described by Eq. (2) has charge \( q \) and is subjected to an external electromagnetic field \( F_{\mu\nu} \). We remind the reader that the components of \( F_{\mu\nu} \) are determined by the usual electric and magnetic fields \( \vec{E} \) and \( \vec{B} \). The left-hand side of Eq. (2) is now given by the Lorentz force, which reads \( qF_{\mu\nu}v^\nu \) in covariant form. The equation of motion determining the trajectory of our particle becomes

\[
qF_{\mu\nu}v^\nu = m \frac{dv^\mu}{d\tau}.
\]

Note that Eq. (3) remains valid in all inertial coordinate systems because it is a tensor equation. Invariance under Lorentz transformations of the coordinate system is therefore maintained. However, the external \( F_{\mu\nu} \) background breaks, for example, symmetry under arbitrary rotations of the charge’s trajectory. Among the consequences of this rotation-symmetry violation is the non-conservation of the particle’s angular momentum. Notice the difference to coordinate changes, which leave unaffected the physics: in the present case, only the trajectory is rotated, so that its orientation with respect to \( F_{\mu\nu} \) can change. One then says that particle Lorentz symmetry is violated, despite the presence of Lorentz coordinate independence.\(^{14,15}\)

Figures 1 and 2 illustrate the difference between particle Lorentz transformations and Lorentz coordinate transformations.

Although the above example captures the main features of Lorentz violation through background vectors or tensors, there is an important difference to situations where these vectors or tensors can be considered as part of the effective vacuum. Our above background \( F_{\mu\nu} \) is a local electromagnetic field caused by other 4-currents that can in principle be accessed experimentally. Such backgrounds are therefore not a feature of the vacuum, so that these situations cannot be considered as exhibiting fundamental Lorentz violation. This is to be contrasted with situations involving candi-
date underlying physics, where tensorial backgrounds can extend over the entire Universe and are outside of experimental control. Such backgrounds must be viewed as a property of the effective vacuum, which can then be considered as violating Lorentz symmetry (see Sec 4).

Figure 1. Coordinate independence. Two experimenters observe identical physical systems represented by the black “particle with spin.” Although they may choose to employ different coordinate systems to describe their observations, the outcome of the experiment remains unaffected by this choice. It must therefore be possible to relate observations recorded with respect to different reference frames by appropriate transformations of coordinates. The principle of coordinate independence therefore assures that the physics is independent of the observer.

Figure 2. Particle transformations. Tests of rotational invariance, for example, would not be carried out as in Fig. 1: identical experiments with the observer rotated. Instead, one would perform a suitable measurement, repeat it with a rotated apparatus, and then compare the two measurements. Under these types of transformations, which involve localized particles and fields and leave unchanged the background, symmetry can be lost because of the different orientation with respect to the vacuum structure.

As a result of the lorentzian structure of the underlying manifold and the usual Lorentz-covariant dynamics at the fundamental level, this approach appears closest to established theories. The physical effects in such Lorentz-breaking vacua are perhaps comparable to those inside certain crys-
tals: the physics remains independent of the chosen coordinate system, but particle propagation, for example, can depend upon the direction. An immediate consequence is that one can locally still work with the metric \( \eta^{\mu\nu} = \text{diag}(1, -1, -1, -1) \), particle 4-momenta are still additive and still transform in the usual way under coordinate changes, and the conventional tensors and spinors still represent physical quantities.

**Models with coordinate-dependent physics.** We have argued above that coordinate independence is a principle more fundamental than Lorentz symmetry. If one is willing to give up coordinate independence, the loss of Lorentz invariance is unsurprising. Although it seems to be impossible to perform meaningful scientific investigations involving coordinate-dependent physics, such approaches to Lorentz breaking have been considered in the literature. More specifically, there have been the following two attempts in the context of neutrino phenomenology: the first one forces the masses of particle and antiparticle to be different,\(^\text{16}\) while the second one suggests to construct a model from positive-energy eigenspinors only.\(^\text{17}\) Both of these approaches are known to involve coordinate-dependent off-shell physics.\(^\text{12,18}\) In what follows, we do not consider these models further.

3. The Standard-Model Extension

The next step after determining general low-energy manifestations of Lorentz violation is the identification of specific experimental signatures for these effects and the theoretical analysis of Lorentz-violation searches. This task is most conveniently accomplished employing a suitable test model. Many Lorentz tests are motivated and analyzed in purely kinematical frameworks allowing for small violations of Lorentz symmetry. Examples are the aforementioned Robertson’s framework and its Mansouri–Sexl extension, as well as the \( c^2 \) model and phenomenologically constructed modified dispersion relations. However, the implementation of general dynamical features significantly increases the scope of Lorentz tests. For this reason, the SME mentioned in the Sec. 1 has been developed. But the use of dynamics in Lorentz-violation searches has recently been questioned on grounds of test-framework dependence. We disagree with this assertion and begin with a few arguments in favor of a dynamical test model.

The construction of a dynamical test framework is constrained by the requirement that known physics must be recovered in certain limits, despite some residual freedom in introducing dynamical features compatible with a given set of kinematical rules. Moreover, it appears difficult and
may even be impossible to develop an effective theory containing the Standard Model with dynamics significantly different from that of the SME. We also point out that kinematical analyses are limited to only a subset of potential Lorentz-violating signatures from fundamental physics. From this viewpoint, it seems to be desirable to explicitly implement dynamical features of sufficient generality into test models for Lorentz and CPT symmetry.

**The generality of the SME.** In order to understand the generality of the SME, we review the main cornerstones of its construction. Starting from the usual Standard-Model lagrangian $L_{\text{SM}}$, Lorentz-violating modifications $\delta L$ are added:

$$L_{\text{SME}} = L_{\text{SM}} + \delta L,$$

where the SME lagrangian is denoted by $L_{\text{SME}}$. The correction term $\delta L$ is obtained by contracting Standard-Model field operators of any dimensionality with Lorentz-violating tensorial coefficients that describe the nontrivial vacuum discussed in the previous section. To guarantee coordinate independence, this contraction must give observer Lorentz scalars. It becomes thus apparent that all possible contributions to $\delta L$ yield the most general effective dynamical description of Lorentz violation at the level of observer Lorentz-invariant quantum field theory. For simplicity, we have focused on nongravitational physics in the above construction. We mention that the complete SME also contains an extended gravity sector.

Possible Planck-scale features, such as non-pointlike elementary particles or a discrete spacetime, are unlikely to invalidate the above effective-field-theory approach at currently attainable energies. On the contrary, the phenomenologically successful Standard Model is widely believed to be an effective-field-theory approximation for underlying physics. If fundamental physics indeed leads to minute Lorentz-breaking effects, it would seem contrived to consider low-energy effective models outside the framework of effective quantum field theory. We finally note that the necessity for a low-energy description beyond effective field theory is also unlikely to arise in the context of candidate fundamental models with novel Lorentz-invariant aspects, such as additional particles, new symmetries, or large extra dimensions. Lorentz-symmetric modifications can therefore be implemented into the SME, if needed.

**Advantages of the SME.** The SME allows the identification and direct comparison of virtually all currently feasible experiments searching for Lorentz and CPT breaking. Moreover, certain limits of the SME
correspond to classical kinematics test models of relativity (such as the previously mentioned Robertson’s framework, its Mansouri-Sexl extension, or the \( c^2 \) model). Another advantage of the SME is the possibility of implementing further desirable conditions besides coordinate independence. For example, one can choose to impose spacetime-translation invariance, \( SU(3) \times SU(2) \times U(1) \) gauge symmetry, power-counting renormalizability, hermiticity, and pointlike interactions. These requirements further restrict the parameter space for Lorentz violation. One could also adopt simplifying choices, such as a residual rotation symmetry in certain coordinate systems. This latter assumption together with additional simplifications of the SME has been considered in Ref. 21.

**Analyses performed within the SME.** To date, the flat-spacetime limit of the minimal SME has provided the basis for numerous experimental and theoretical studies of Lorentz and CPT violation involving mesons, baryons, electrons, photons, muons, and the Higgs sector. We remark that neutrino-oscillation experiments offer the potential for discovery. A number of these studies involve some form of antimatter. CPT and Lorentz tests with antimatter will be discussed further in Sec. 5.

### 4. Sample mechanisms for Lorentz and CPT violation

In the previous two sections, we have studied various general *types* of manifestations of Lorentz and CPT breakdown, as well as the *description* of the corresponding effects in a microscopic model, such as the SME. However, the question of *how* exactly a Lorentz- and CPT-invariant candidate theory can lead to the violation of these symmetries has thus far been left unaddressed. The purpose of this section is to provide some intuition about such mechanisms for Lorentz and CPT breaking in underlying physics. Of the various possible mechanisms mentioned in Sec. 1, we will focus on spontaneous Lorentz violation and Lorentz breakdown through varying scalars.

**Spontaneous Lorentz and CPT violation.** The mechanism of spontaneous symmetry breaking is well established in various subfields of physics, such as the physics of elastic media, condensed-matter physics, and elementary particle theory. From a theoretical perspective, this mechanism is very attractive because the invariance is essentially violated through a non-trivial ground-state solution. The underlying dynamics of the system governed by the Hamiltonian remains completely invariant under the symmetry. To gain intuition about spontaneous Lorentz and CPT breakdown,
we will consider three sample systems, whose features will gradually lead us to a better understanding of the effect. An illustration supporting these three examples is given in Fig. 3.

First, let us consider classical electrodynamics. Any electromagnetic-field configuration is associated with an energy density \( V(\vec{E}, \vec{B}) \) given by

\[
V(\vec{E}, \vec{B}) = \frac{1}{2} \left( \vec{E}^2 + \vec{B}^2 \right),
\]

where we have employed natural units, and \( \vec{E} \) and \( \vec{B} \) denote the electric and magnetic field, respectively. With Eq. (5), we can determine the field energy of any given solution of the Maxwell equations. Note that if the electric field, or the magnetic field, or both are nonzero somewhere in spacetime, the energy stored in these fields will be strictly positive. The field energy can only be exactly zero when both \( \vec{E} \) and \( \vec{B} \) vanish everywhere. The vacuum (or ground state) is usually identified with the lowest-energy configuration of a system. We see that in conventional electrodynamics the configuration with the lowest energy is the field-free one, so that the Maxwell vacuum is empty (disregarding quantum fluctuations).

Second, let us look at the Higgs field, which is part of the phenomenologically very successful Standard Model of particle physics. Unlike the electromagnetic field, the Higgs field is a scalar. In what follows, we can adopt some simplifications without distorting the features important in the present context. The expression for the energy density of our Higgs scalar \( \phi \) in situations with spacetime independence is given by

\[
V(\phi) = (\phi^2 - \lambda^2)^2,
\]

where \( \lambda \) is a constant. As in the Maxwell case discussed above, the lowest possible field energy is zero. Note, however, that this configuration requires \( \phi \) to be nonzero: \( \phi = \pm \lambda \). It follows that the vacuum for a system containing a Higgs-type field is not empty; it is, in fact, filled with constant scalar field \( \phi_{\text{vac}} = \langle \phi \rangle = \pm \lambda \). In quantum theory, the quantity \( \langle \phi \rangle \) is called the vacuum expectation value (VEV) of \( \phi \). One of the physical effects caused by the VEV of the Higgs is to give masses to many elementary particles. Note, however, that \( \langle \phi \rangle \) is a scalar and does not select a preferred direction in spacetime.

Third, we consider a vector field \( \vec{C} \) (the relativistic generalization is straightforward) not contained in the Standard-Model. Of course, there is no observational evidence for such a field at the present time, but fields like \( \vec{C} \) frequently arise in approaches to more fundamental physics. In analogy
Figure 3. Spontaneous symmetry breaking. In conventional electrodynamics (1), the lowest-energy state is attained for zero $\vec{E}$ and $\vec{B}$ fields. The vacuum remains essentially empty. For the Higgs-type field (2), interactions lead to an energy density $V(\phi)$ that forces a nonzero value of $\phi$ in the ground state. The vacuum fills with a scalar condensate shown in gray. Lorentz invariance still holds (other, internal symmetries may be violated though). Vector fields occurring, e.g., in string theory (3) can have interactions similar to those of the Higgs requiring a nonzero field value in the lowest-energy state. The VEV of a vector field selects a preferred direction in the vacuum, which has now properties paralleling those of a crystal.
to Higgs case, we take its expression for energy density in situations with constant \( \vec{C} \) to be

\[
V(\vec{C}) = (\vec{C}^2 - \lambda^2)^2.
\]

As in the previous two examples, the lowest possible energy is exactly zero. As for the Higgs, this lowest energy configuration is attained for nonzero \( \vec{C} \). More specifically, we must have \( \vec{C}_{\text{vac}} \equiv \langle \vec{C} \rangle = \vec{\lambda} \), where \( \vec{\lambda} \) is any constant vector satisfying \( \vec{\lambda}^2 = \lambda^2 \). Again, the vacuum is not empty, but filled with the VEV of our vector field. Since we have only considered constant solutions \( \vec{C}, \langle \vec{C} \rangle \) is also spacetime independent (\( x \) dependence would lead to positive definite derivative terms in Eq. (7) raising the energy density).

The true vacuum in our model therefore contains an intrinsic direction determined by \( \langle \vec{C} \rangle \) violating rotation invariance and thus Lorentz symmetry. We mention that interactions leading to energy densities like (7) are absent in conventional renormalizable gauge theories, but can be found in the context of strings, for example.

**Cosmologically varying scalars.** A spacetime-dependent scalar, regardless of the mechanism driving the variation, typically implies the breaking of spacetime-translation invariance. Since translations and Lorentz transformations are closely linked in the Poincaré group, it is reasonable to expect that the translation-symmetry violation also affects Lorentz invariance.

Consider, for instance, the angular-momentum tensor \( J^{\mu \nu} \), which is the generator of Lorentz transformations:

\[
J^{\mu \nu} = \int d^3x \left( \theta^{\mu \nu} x^\lambda - \theta^{\lambda \nu} x^\mu \right).
\]

Note that this definition contains the energy–momentum tensor \( \theta^{\mu \nu} \), which is not conserved when translation invariance is broken. In general, \( J^{\mu \nu} \) will possess a nontrivial dependence on time, so that the usual time-independent Lorentz-transformation generators do not exist. As a result, Lorentz and CPT symmetry are no longer assured.

Intuitively, the violation of Lorentz invariance in the presence of a varying scalar can be understood as follows. The 4-gradient of the scalar must be nonzero in some regions of spacetime. Such a gradient then selects a preferred direction in this region (see Fig. 4). Consider, for example, a particle that interacts with the scalar. Its propagation features might be different in the directions parallel and perpendicular to the gradient. Physically inequivalent directions imply the violation of rotation symmetry. Since
rotations are contained in the Lorentz group, Lorentz invariance must be violated.

Lorentz violation induced by varying scalars can also be established at the Lagrangian level. Consider, for instance, a system with a varying coupling $\xi(x)$ and scalar fields $\phi$ and $\Phi$, such that the Lagrangian $L$ contains a term $\xi(x) \partial^\mu \phi \partial_\mu \Phi$. The action for this system can be integrated by parts (e.g., with respect to the first partial derivative in the above term) without affecting the equations of motion. An equivalent Lagrangian $L'$ would then obey

$$L' \supset -K^\mu \phi \partial_\mu \Phi,$$

where $K^\mu \equiv \partial^\mu \xi$ is an external nondynamical 4-vector, which clearly violates Lorentz symmetry. We remark that for variations of $\xi$ on cosmological scales, $K^\mu$ is constant to an excellent approximation locally—say on solar-system scales.

5. Lorentz and CPT tests involving antimatter

Numerous Lorentz and CPT tests among those listed in Sec. 3 involve some form of antimatter. As pointed out earlier, certain matter–antimatter comparisons are extremely sensitive to CPT violations. This is unsurprising because CPT symmetry connects particles and antiparticles. CPT tests with subatomic particles typically involve quantum numbers like mass, charge, spin, etc. Atoms and their anti-atoms possess additional, qualitatively different properties, such as spectra, that can be compared. The possibility of $\bar{H}$ production combined with the ultrahigh sensitivities attainable in atomic spectroscopy and the simplicity of the two-body problem (antiproton nu-
cleus and orbiting positron) make this anti-atom particularly well suited for such investigations. The determination of SME predictions for such physical systems follows the outline shown in Fig. 5.

Figure 5. SME analysis of atomic spectra. The resulting modified Pauli equation for fermions and that for antifermions—which typically differ—are employed to describe the (anti)proton and the orbiting (anti)electron in the H (¯H) system.

**The unmixed 1S–2S transition.** The experimental resolution of the transition involving the unmixed spin states is expected to be about one part in $10^{-18}$. This sensitivity appears promising in light of potential Planck-suppressed quantum-gravity effects. On the other hand, the leading-order SME calculation shows identical shifts for free $H$ or $\bar{H}$ in the initial and final states with respect to the conventional levels. It follows that this transition is less suitable for the measurement of unsuppressed Lorentz- and CPT-breaking effects. The largest non-trivial contribution to this transition within the SME test framework arises through relativistic corrections, and it involves two additional powers of the fine-structure parameter $\alpha = \frac{1}{137}$. The expected energy shift—already at zeroth order in $\alpha$ anticipated to be
minute—comes therefore with an additional suppression by a factor of more than ten thousand. This further exemplifies the need and importance of a viable test model for Lorentz- and CPT-violation searches.

![Figure 6. Sidereal variations. Experiments are typically associated with an intrinsic direction. For instance, particle traps usually involve a magnetic field. As the Earth rotates, this direction will change if the experiment is attached to the Earth. In the above figure, a trapping field $\vec{B}$ pointing vertically upward is shown at two times separated by approximately 12 hours (gray arrows). The angle between the Lorentz-violating background (black $\vec{b}$ arrows) and the orientation of the experiment is clearly different at these two times. An observable, such as an atomic transition, may for example acquire a correction $\sim \vec{B} \cdot \vec{b}$ that leads to the shown sidereal modulation.](image)

The spin-mixed 1S–2S transitions. When $H$ or $\overline{H}$ is confined with magnetic fields—such as in a Ioffe–Pritchard trap—the 1S and the 2S levels are each split by the Zeeman effect. In the framework of the SME, one can show that in this situation the 1S–2S transition between the spin-
mixed states is affected by Lorentz and CPT violation at leading order. A disadvantage from a practical perspective is the field dependence of this transition, so that the experimental resolution is limited by the size of the inhomogeneity in the trapping magnetic field. The development of novel experimental techniques would appear necessary to achieve resolutions close to the natural linewidth.

Hyperfine Zeeman transitions within the 1S state. An alternative Lorentz and CPT test could measure the transition frequency between the Zeeman-split states within the 1S level itself. Even in the zero-magnetic-field limit, the SME predicts first-order effects for two of these transitions. Other transitions of this type, such as the conventional Hydrogen-maser line, can be well resolved in experiments.

Tests in Penning traps. The SME predicts that not only atomic energy levels can be shifted in the presence of Lorentz violation, but also proton and antiproton levels in Penning traps. A calculation shows that only one SME coefficient ($b\mu$ in the standard notation) leads to transition-frequency differences between the proton and its antiparticle. More specifically, the anomaly frequencies are changed in opposite directions for protons and antiprotons. This effect permits a clean observational bound on $b\mu$ for the proton.

Searches for sidereal variations. Another general signature for Lorentz and CPT breakdown is the variation of measured quantities with the sidereal day. The anti-CPT theorem implies that CPT breakdown always comes with Lorentz violation, which in turn is typically accompanied by the loss of isotropy. Thus, experimental effects will generally depend on the direction. As the laboratory is attached to the rotating Earth, its orientation will change continually leading to sidereal modulations of signals. This situation is schematically depicted in Fig. 6. Note that sidereal variation tests are not confined exclusively to H–H spectroscopy, but they can also be performed in the context of other rotation-violation searches. Recent experiments with H-masers employing ingenious experimental techniques are based on such modulations.\(^{37}\)

6. Summary

Although Lorentz and CPT symmetry are deeply ingrained in the currently accepted laws of physics, there are a variety of candidate underlying theories that could generate the breakdown of these symmetries. The sensitivity attainable in matter–antimatter comparisons offers the possibility for CPT-
violation searches with Planck precision. Lorentz tests open an additional avenue for CPT measurements because CPT breakdown implies Lorentz violation.

A potential source of Lorentz and CPT violation is spontaneous symmetry breaking in string theory. Since this mechanism is theoretically very attractive and since strings show great potential as a candidate fundamental theory, this Lorentz-violation source is particularly promising. Lorentz and CPT violation can also originate from spacetime-dependent couplings: the gradient of such couplings selects a preferred direction in the effective vacuum. This mechanism for Lorentz breaking might be of interest in light of recent claims of a time-dependent fine-structure parameter and the presence of varying scalar fields in many cosmological models.

The leading-order Lorentz- and CPT-violating effects resulting from Lorentz-symmetry breakdown in approaches to fundamental physics are described by the SME. At the level of effective quantum field theory, the SME is the most general dynamical framework for Lorentz and CPT breaking that is compatible with the fundamental principle of coordinate independence. Experimental investigations are therefore best performed within the SME.

Cold antiprotons are excellent high-sensitivity tools in experimental searches for Planck-scale physics. Suppressed and unsuppressed effects exist for 1S–2S transitions in H and H. Leading-order shifts are also predicted in the 1S hyperfine Zeeman levels, which offers the possibility of alternative measurements. Further possibilities for Lorentz- and CPT-violation searches with antiprotons exist in Penning traps, where anomaly frequencies are affected. In general, tests with cold antiprotons probe parameter combinations inaccessible by other experiments.

Acknowledgments

The author would like to thank Yasunori Yamazaki for organizing this stimulating meeting, for the invitation to attend, and for partial financial support.

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