Resistivity of a 2d quantum critical metal

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Abstract

We calculate resistivity in the paramagnetic phase just above the curie temperature in a 2d ferromagnetic metal. The required dynamical susceptibility in the formalism of resistivity is calculated within the Random Phase Approximation(RPA). The mechanism of resistivity is magnetic scattering in which s-band electrons are scattered off the magnetic spin fluctuations of d-band electrons. We use the s-d Hamiltonian formalism. We find that near the quantum critical point the resistivity in 2d scales as $T^{4\over 3}$, whereas in 3d it scales as $T^{5\over 3}$. In contrast to it, resistivity due to phonon scattering is given by $T^5$ in low temperature limit as is well known. Our RPA result agrees with the Self-Consistence Renormalisation(SCR) theory result.

1 Introduction

Resistivity in metals is generally due to either impurity scattering or phonon scattering or both mechanism working together. In impurity scattering electrons are scattered off the immobile impurities leading to their momentum randomization thus resistivity. In phonon scattering electrons scatter by absorbing or emitting phonons or lattice vibrations. This can be studied using Bloch-Boltzmann kinetic equation and one finds that resistivity is proportional to temperature($T$) when $T \gg \Theta_D$, where $\Theta_D$ is the Debye temperature. In the opposite limit $T \ll \Theta_D$, $\rho \propto T^5$.

However, the above scenario is not applicable to magnetic material tuned near their critical points. An alternative mechanism in which electrons scatter off magnetic spin fluctuations becomes important. Currently, there is renewed interest in the topic of magnetic critical points and physical properties near a magnetic instability. It has been shown that electron-magnetic-spin-fluctuation scattering in a 3d ferromagnetic metal tuned near its critical point leads to a resistivity which scales as $T^{5\over 3}$. This stands in sharp contrast to phonon scattering. It has also been shown within the Self-Consistence Renormalisation(SCR) theory that in 2d ferromagnetic case resistivity scales as $T^{4\over 3}$ near the critical point. In this paper we represent our calculation of resistivity in a 2d quantum critical metal.
ferromagnetic metal near its critical point by using Random Phase approximation instead to SCR theory. We report that our RPA result agree with the SCR theory result, that is $\rho \propto T^4$. One importance of our result is that the finite temperature renormalization effects taken into account in SCR theory are not important near a quantum critical point when one study transport properties. And RPA is as good as SCR theory in this very low temperature regime near QCP\cite{12,13,14,15}.

2 Formalism

$s$-electrons are treated as conduction electrons which scatter via localised $d$-electrons, and their interaction is modelled with $s$-$d$ Hamiltonian\cite{2}:

$$H_{int} = \frac{J}{N} \sum_{k,k'} \left\{ a_{k\uparrow}^\dagger a_{k\downarrow} S^-(k' - k) + a_{k'\downarrow}^\dagger a_{k\uparrow} S^+(k' - k) + (a_{k\downarrow}^\dagger a_{k'\uparrow} - a_{k'\downarrow}^\dagger a_{k\uparrow}) S^z (k' - k) \right\} \tag{1}$$

where $J$ is $s$-$d$ electron coupling constant, $N$ is the number of atoms in the system, the $a^\dagger$ and $a$ are the creation and annihilation operators for $s$-electrons. $S^-(k' - k)$ and $S^+(k' - k)$ are lowering and raising spin density operators of $d$-band electrons, $S^z (k' - k)$ represents $z$-component of spin density of $d$-band electrons and $S(k)$ is the Fourier transform of spin density $S(r)$ of $d$-electrons and it is defined by

$$S(k) = \int e^{-ik\cdot r} S(r) \, dr. \tag{2}$$

The transition probability\cite{16} that an $s$-electron with wave vector $k$ will be scattered to the state $k + q$ can be written as

$$W_{k+q-k} = \frac{2\pi}{\hbar} | \langle d(F) | \langle k + q | H_{int} | k \rangle | d(I) \rangle |^2 \rho_f. \tag{3}$$

The eigenstate of the system can be approximated by the product of the form $|k\rangle |d(I)\rangle$, where the function $|k\rangle$ describes the state of the $s$-electrons, and $|d(I)\rangle$ the states of the $d$-electrons system. By employing the Fermi Golden Rule, one finds that $s$-electron of wave vector $s$ with spin up/down is scattered into the state $k + q$ with spin down/up at the rate\cite{16}

$$W_{k+q-k}^{para} = \frac{2\pi J^2}{\hbar N^2} f_s(\epsilon_k)(1 - f_s(k + q)) \sum_{I,F} \left\{ <d(I)S^+(-q)|d(F)> <dF|S^-(-q)|d(I)> + <d(I)|S^-(q)|d(F)> <d(F)|S^+(-q)|d(I)> \right\} \delta \left( \epsilon_d(F) - \epsilon_d(I) + \epsilon_s(k + q) - \epsilon_s(k) \right). \tag{4}$$

Here $f_s(\epsilon_k)$ is $s$-electron Fermi distribution function. The last term in Hamiltonian from the $z$-component of spin density of $d$-electrons get cancelled due to scattering from the same spin state of $s$-electrons. We employ the identity $\delta \epsilon = \int e^{i\epsilon t} \frac{d\epsilon}{2\pi}$ and $e^{-iH_d(I)|d(I)\rangle} \geq e^{-i\epsilon_d(I)|d(I)\rangle}$. Therefore we obtain:

$$W_{k+q-k}^{para} = \frac{3}{2} \frac{J^2}{\hbar N^2} f_s(\epsilon_k)(1 - f_s(\epsilon_{k+q})) \int_0^\infty dt e^{-i\omega t} \left[ \langle S^+(-q)S^-(q,t) \rangle + \langle S^-(-q)S^+(q,t) \rangle \right]. \tag{5}$$

2
where $\hbar \omega = \epsilon_k - \epsilon_{k+q}$ is an energy transfer provided by s-electrons, $\epsilon_k = \frac{h^2 k^2}{2m_s}$, and $< ... >$ denotes the thermal average. Time evolution of operators implies Heisenberg representation: $S^z(q,t) = e^{iHdt}S^z e^{-iHdt}$. The integral term describes the Fourier transform of the correlation function of spin densities of d-band electrons. Using the Fluctuation-dissipation theorem[17], we express the transition probability $W_{k+q-k}^{para}$ in terms of the dynamical susceptibility $\chi^+(q,\omega)$ as

$$W_{k+q-k}^{para} = \frac{3}{2} \frac{J^2}{N^2} \int_{-\infty}^{+\infty} d\omega f_s(\epsilon_k)(1 - f_s(\epsilon_{k+q}))(-n(-\omega))Im\chi_s^+(q,\omega)\delta(h\omega - \epsilon_k + \epsilon_{k+q}) \quad (6)$$

where $n(-\omega) = \frac{1}{e^{\beta \epsilon_k} - 1}$ and $\chi_s^+(q,\omega)$ is the symmetric part of the complex susceptibility corresponding to a magnetic scattering of spins of d-band electrons with wave vector $q$ and frequency $\omega$, $\chi^+(q,\omega)$ the susceptibility of d-electrons is defined by

$$\chi^+(q,\omega) = \lim_{\epsilon \to 0} \frac{i}{\epsilon} \int_0^\infty e^{-i\omega t - \epsilon t} \langle [S^+(q,t), S^-(q)] \rangle dt. \quad (7)$$

This dynamical susceptibility is the Fourier transform of response function or retarded Green function defined with respect to spin densities of d-electrons post scattering. The susceptibility tensor is isotropic for paramagnetic system, therefore omitting anisotropy for the present system

$$\chi^+(q,\omega) = \frac{2}{3}(\chi^{xx} + \chi^{yy} + \chi^{zz}). \quad (8)$$

The standard transport theory[18] can now be used to calculate the transport property in terms of scattering probability $W_{k+q-k}$

$$\rho_{para}^{para} = \frac{1}{2kBT} \int \int (\Phi_k - \Phi_{k+q}) W_{k+q-k}^{para} dk dq \quad (9)$$

where $e$ is the electronic charge, $v_k = \frac{h k}{m_s}$ is the Fermi velocity of s-electrons, and $-\Phi_k \frac{\partial \rho_s(\epsilon_k)}{\partial \epsilon_k}$ is the measure of deviation from the equilibrium in the electron distribution, the trial function $\Phi_k$ itself a measure of this deviation. If the usual assumption $\Phi_k = \text{const.} \times q.u$ is made, and the variational integral in the denominator is solved by the assumption of isotropy in the electron distribution, then the resistivity expression[9] reduces to

$$\rho_{2d}^{para} = \frac{3J^2 k^2}{N^2 k_B T (en_s)^2} \int_{-\infty}^{+\infty} d\omega \int_0^{K_F} d^2k \int d^2q(u.q)^2 f^0(\epsilon_k)(1 - f^0(\epsilon_{k+q}))(-n(-\omega)) Im\chi_s^+(q,\omega) \delta(h\omega - \epsilon_k + \epsilon_{k+q}), \quad (10)$$

where $u$ is a unit vector parallel to the electric field and $n$ is the number of s-electrons per unit volume. Using property $f(x)\delta(x - a) = f(a)\delta(x - a)$, and writing $\int d^2k = \int_0^\infty dk \int_0^{2\pi} d\phi$ we get

$$\rho_{2d}^{para} = \frac{3(\pi)J^2 k^2}{N^2 k_B T (en_s)^2} \int_{-\infty}^{+\infty} d\omega (-n(-\omega)) \int q^3 dq Im\chi_s^+(q,\omega) \int_0^\infty k dk \times$$

$$f^0(\epsilon_k)(1 - f^0(\epsilon_k - h\omega)) \int_0^{2\pi} d\phi \delta(h\omega - \epsilon_k + \epsilon_{k+q}) \quad (11)$$
Simplifying the integral with respect to $\phi$ (appendix A) and writing $(u.q)^2 = \frac{q^2}{2}$ for unit vector $u$, which is parallel to electric field and $q$. The expression gives

$$
\rho_{2d}^{para} = \frac{3(2\pi)J^2}{N^2\hbar^2k_BT(e_n)^2} \int_{-\infty}^{+\infty} d\omega \ (-n(-\omega)) \int q^2 \ dq \ Im \chi^{+}(q,\omega) \times \frac{\int_{q_0}^{\infty} k \ dk f^0(\epsilon_k)(1 - f^0(\epsilon_k - \hbar\omega))}{\sqrt{k^2 - q_0^2(q,\omega)}} \tag{12}
$$

here $q_0 = \frac{q}{2} + \frac{m\omega}{2\hbar} < k_F$, $k_F$ is Fermi wave vector of $s$-electron and on performing \[18\] $k$ integral (appendix B) expression (12) reduces to

$$
\rho_{2d}^{para} = \sqrt{\frac{m}{2\mu}} \frac{3(2\pi)J^2}{N^2\hbar^2k_BT(e_n)^2} \int_{-\infty}^{+\infty} d\omega \int_0^{2k_F} q^2 \ dq \ Im \chi^{+}(q,\omega)\left(\frac{-n(-\omega)\omega}{e^{\beta\hbar\omega} - 1}\right) \tag{13}
$$

To proceed further some assumption about $\chi^{+}(q,\omega)$ has to be made. We use Hamiltonian for $d$-electrons \[2\] \[17\]

$$
H_d = \sum_{k,\sigma} \epsilon(k)C^\dagger_{k,\sigma}C_{k,\sigma} + \frac{1}{N} \sum_{q,k,k',\sigma,\sigma'} C^\dagger_{k+q,\sigma}C_{k,\sigma}C^\dagger_{k',\sigma',\sigma'}C_{k'+q,\sigma'} \tag{14}
$$

Here $I$ is the exchange interaction parameter for $d$-band electrons, $N$ is the number of lattice points. And $C$ and $C^\dagger$ are the annihilation and creation operators for $d$-electrons, $\epsilon(k)$ is electron energy of Bloch state $k$ of $d$-electrons. The susceptibility has been calculated in a famous paper by Izuyama et al. (1963) \[17\]. Using random phase approximation the transverse susceptibility is given by \[IKK\] \[17\]

$$
\chi^{+}(q,\omega) = \frac{\Gamma^{+}(q,\omega)}{1 - \Pi^{-+}(q,\omega)} \tag{15}
$$

where

$$
\Gamma^{+}(q,\omega) = \sum_{k} \frac{f(\epsilon_k) - f(\epsilon_{k+q})}{\epsilon(k + q) - \epsilon(k) - \hbar\omega} \tag{16}
$$

$\Gamma^{+}(q,\omega)$ is the susceptibility of non-interacting electrons. Employing identity $\lim_{\eta \to 0} \frac{1}{a \pm i\eta} = p\left(\frac{1}{a}\right) \mp i\pi \delta(a)$ to equation (16) the real part can be written as

$$
\Re(q,\omega) = 2p \sum_{k} \frac{f(\epsilon_k) - f(\epsilon_{k+q})}{\epsilon(k + q) - \epsilon(k) - \hbar\omega} \tag{17}
$$

Thus $\Re(q,\omega)$ is the real part of non-interacting $d$-electrons susceptibility with wave number $q$ and frequency $\omega$. $\Im(q,\omega)$ is the corresponding imaginary susceptibility

$$
\Im(q,\omega) = \pi \sum_{k} f(\epsilon_k) - f(\epsilon_{k+q})\delta[\epsilon(k + q) - \epsilon(k) - \hbar\omega] \tag{18}
$$

Thus the electron spin susceptibility of $d$-electrons in terms of real and imaginary part is written as

$$
\Gamma^{+}(q,\omega) = \Re(q,\omega) + i\Im(q,\omega) \tag{19}
$$
We note that $R(q, \omega)$ is an even function of $q$. It can be expressed as

$$R(q, \omega) = \sum_k -q \frac{\partial f_k}{\partial k} - \frac{1}{2}(q, \nabla)^2 f_k k - \frac{1}{8}(q, \nabla)^3 f_k k + \frac{1}{3} (q, \nabla)^2 \varepsilon |k| - \hbar \omega$$  \hspace{1cm}(20)$$

The expression can be approximated to

$$R(q, \omega) \simeq a'_2 N(0) + \pi \bar{q}^2 E_d N(0) a_{2d}$$  \hspace{1cm}(21)$$

where $N(0)$ is the density of states at Fermi level, $E_d$ is energy of $d$-electrons. $\bar{q} = \frac{q}{k_F}$ is dimensionless wave-vector. Here $a_{2d}$ and $a'_2$ are constants, which can be computed. The above approximation hold good under the conditions\textsuperscript{[17]}:

$$\frac{\hbar^2 k_F q}{m} = v_F q \gg |\hbar \omega|,$$  \hspace{1cm}(22)$$

and

$$\frac{q}{k_F} \ll 1.$$  \hspace{1cm}(23)$$

As there are relevant near a critical point\textsuperscript{[17]}, $k_F$ being the magnitude of Fermi wave vector of $s$-electrons. The condition (23) is applicabale since we are concerned with in the radius of fermi surface of $s$-electron which is much bigger than the radius of $d$-electrons Fermi surface.

On applying these conditions coefficients $a'_2$ and $a_{2d}$ of $R(q, \omega)$ becomes independent of $q$ and $\omega$ and real part of susceptibility approximated to

$$R(q, \omega) \simeq a'_2 N(0) + \pi a_{2d} \bar{q}^2 E_d N(0)$$  \hspace{1cm}(24)$$

The imaginary part of susceptibility($\Im(q, \omega)$) can be solved as

$$\Im(q, \omega) = \pi \sum_k (-q \frac{\partial f_k}{\partial k} \cos \phi) \delta \left( q \frac{\partial \varepsilon_k}{\partial k} \cos \phi - \hbar \omega \right)$$

$$ = \pi q \sum_k \left( -\frac{\partial f_k}{\partial \varepsilon_k} \right) \frac{\partial \varepsilon_k}{\partial k} \cos \phi \delta \left( q \frac{\partial \varepsilon_k}{\partial k} \cos \phi - \hbar \omega \right)$$  \hspace{1cm}(25)$$

Converting summation into integral as $\sum_k = \frac{1}{2\pi} \int N(0) d\varepsilon \int_0^{2\pi} d\phi$ and $N(0)$ defines the density of states in two dimension. We replace $-\frac{\partial f_k}{\partial \varepsilon_k} = \delta(\varepsilon - \varepsilon_F)$ and first order drive of enegry with resprect to $k$ vector by $\hbar v_F$. The integral equation becomes as follows:

$$\Im \simeq q \frac{\hbar v_F}{2} \int_0^{\varepsilon_F} N(0) \delta(\varepsilon - \varepsilon_F) d\varepsilon \int_0^{2\pi} d\phi \cos \phi \delta \left( q \hbar v_F \cos \phi - \hbar \omega \right)$$  \hspace{1cm}(26)$$

Using property of delta function $\int_0^{\varepsilon_F} N(0) \delta(\varepsilon - \varepsilon_F) d\varepsilon = \frac{N(0)}{2}$ for energy integral and using $\delta(ax) = \frac{1}{|a|} \delta(x)$ for $\phi$ integral, we get

$$\Im \simeq \frac{N(0)}{2} \int_0^{\pi} d\phi \cos \phi \delta \left( \cos \phi - \frac{\omega}{q v_F} \right)$$  \hspace{1cm}(27)$$
Using property $\delta(f(\theta)) = \sum_{\theta_0} \delta(\theta - \theta_0)$, and setting $\theta = \cos^{-1}(\frac{\omega}{qv_F}) = \theta_0$, we obtain

$$\mathcal{I} = \frac{N(0)}{2} \frac{(\frac{\omega}{qv_F})}{\sqrt{1 - (\frac{\omega}{qv_F})^2}}$$

(28)

Apply the condition $\omega << qv_F$, then $\mathcal{I}(q, \omega)$ reduces to

$$\mathcal{I} = \frac{N(0)}{2} (\frac{\omega}{qv_F}) = N(0) (\frac{\omega k_D^{-1}}{2qv_F})$$

(29)

Here $\tilde{q} = \frac{q}{k_d}$, $k_d$ is the d-electron Fermi vector, $\frac{\hbar^2 q_d^2}{2m_d} = E_d$ its fermi energy.

$$\mathcal{I} = \frac{1}{4} N(0) \frac{\hbar \omega}{qE_D}$$

(30)

Collecting the above information imaginary part of susceptibility (15) reduce to

$$\text{Im}\chi^{-+}(q, \omega) = \frac{\mathcal{I}}{(1 - \mathcal{R})^2 + \mathcal{I}^2}$$

(31)

Substituting real and imaginary part of transverse susceptibility from equations (24) and (28) into $\text{Im}\chi^{-+}(q, \omega)$, we have

$$\text{Im}\chi^{-+}(q, \omega) = \frac{N(0) \frac{\hbar \omega}{qE_D}}{[1 - a_{2d}a_{2d}^I N(0) + \pi a_{2d}E_d F E_d N(0)]^2 + \frac{\hbar^2 \omega}{4 qE_D} [N(0) \frac{\hbar \omega}{qE_D}]^2}$$

(32)

Here $k_0^2 = 1 - a_{2d}^I N(0) = 1 - \mathcal{I}$ denotes the inverse of the RPA exchange enhancement factor for d-band. We are interested in the behaviour of system for $k_0^2 = 0$ i.e. $c = c_F$. At $c = c_F$, one shift the critical point to a desired low temperature regime. In other words nearness to a QCP is about by chemical doping [2]. Here one can focus on the low temperature regime [19] for physical properties near the critical point. Therefore susceptibility takes the form:

$$\text{Im}\chi^{-+}(q, \omega) = \frac{N(0) \frac{\hbar \omega}{qE_D}}{[\pi a_{2d}E_d F E_d N(0)]^2 + \left(\frac{\hbar \omega}{4 qE_D}\right)^2}$$

(33)

3 Result

Writing 2d paramagnetic resistivity replacing $\text{Im}\chi^{-+}(q, \omega)$ from equation (33) in expression (13) we have

$$\rho_{\text{para}} = \sqrt{\frac{m}{2\mu N^2 \hbar^2 (m\text{a}_{2d})^2 k_B T} \int_0^{4E_D} \frac{\omega d\omega}{(e^{\beta \hbar \omega} - 1)(1 - e^{-\beta \hbar \omega})} \int_0^{2k_d} \frac{dq^2}{q^2} dq \left(\frac{q}{k_d^2 (\frac{4\pi a_{2d}E_d}{\hbar \omega})^2 + 1}\right)}$$

(34)

put $t = \frac{2}{k_d} (\frac{4\pi a_{2d}E_d}{\hbar \omega})^{1/4}$, $q^3 = t^3 k_d^3 (\frac{4\pi a_{2d}E_d}{\hbar \omega})^{-1}$ and write
prefactor $p_0 = \sqrt{\frac{12(2\pi)^2N(0)E_d}{2\pi N^2 k^4_{F}(\xi_{12})^2}}$, then
\begin{equation}
\rho_{\text{para}} = p_0 k_d^2 \left( \frac{\hbar}{4\pi E_d} \right)^\alpha \int_0^{\frac{4\pi E_d}{k_B T}} \frac{\omega^4 d\omega}{(e^{\beta \omega} - 1)(1 - e^{-\beta \omega})} \int_a^b \frac{t^3}{t^6 + 1},
\end{equation}
(35)

where limits for $t$-integral change to \[ l_a = \frac{(4\pi a_2)^{\frac{1}{2}}}{2} \left( \frac{\hbar \omega}{E_d} \right)^{\frac{1}{2}} \] and \[ l_b = 2 \left( \frac{4\pi a_2 E_d}{\hbar \omega} \right)^{\frac{1}{2}}. \] We put $\beta \hbar \omega = u$ to make integrals temperature independent. Then resistivity simplifies to
\begin{equation}
\rho_{\text{para}}^{2d} = p_0 k_d^2 \left( \frac{k_B T}{\hbar 4\pi E_d} \right)^{\frac{1}{4}} \int_0^{\frac{4\pi E_d}{k_B T}} \frac{u^\frac{3}{2} du}{(e^u - 1)(1 - e^{-u})} \int_a^b \frac{t^3}{t^6 + 1} \propto T^{\frac{1}{4}}
\end{equation}
(36)

where \[ l_a' = \frac{(4\pi a_2)^{\frac{1}{2}}}{2} \left( \frac{u k_B T}{E_d} \right)^{\frac{1}{2}}, \] and \[ l_b' = 2 \left( \frac{4\pi a_2 E_d}{u k_B T} \right)^{\frac{1}{2}}. \] Thus our final result is $\rho_{\text{para}}^{2d} \propto T^{\frac{1}{4}}$.

4 Conclusion

We have performed a calculation for electrical resistivity in a 2d metal which is tuned near to its ferromagnetic instability. The required dynamical susceptibility in the expression of resistivity is calculated using Random Phase approximation. In 2d we find that the real and imaginary parts of dynamical susceptibility are proportional to $q^2$ and $q$ respectively. This is similar to the 3d case. However, we find that the resistivity calculated for 2d case scales as $\rho_{\text{para}}^{2d} \propto T^{\frac{1}{4}}$ whereas in 3d case resistivity scales at $T^{\frac{1}{5}}$. Our RPA result ($\rho_{\text{para}}^{2d} \propto T^{\frac{1}{4}}$) agrees with the SCR result.

A Appendix: mathematical details of $\phi$ integral

To solve the term $\int_0^{2\pi} d\phi \delta(\cos \phi + \frac{\omega m k}{h k q} + \frac{q}{2k})$. We use delta function property $\delta(F(x)) = \sum_i \frac{\delta(x-x_i)}{|F'(x_i)|}$, We can put $\cos \phi + \frac{\omega m k}{h k q} + \frac{q}{2k} = 0$, $\phi = \cos^{-1}(-\frac{\omega m k}{h k q} - \frac{q}{2k}) = \phi_0(q, k, \omega)$ and here $\frac{\omega m k}{h k q} + \frac{q}{2k} < k$.
\begin{align*}
\int_0^{2\pi} d\phi \delta(\cos \phi + f(k, q, \omega)) &= \int_0^{2\pi} d\phi \left| \delta(\phi - \phi_0(q, k, \omega)) \right| \sin \phi|_{\phi_0} \\
&= \frac{1}{\sin \phi_0} \int_0^{2\pi} d\phi \delta(\phi - \phi_0) = \frac{1}{\sin \phi_0} \\
&= \frac{1}{\sqrt{1 - (\frac{\omega m k}{h k q} + \frac{q}{2k})^2}} = \frac{1}{\sqrt{k^2 - (\frac{\omega m k}{h k q} + \frac{q}{2k})^2}}
\end{align*}
(37)
Appendix: mathematical details of $k$ integral

$$I_k = \int_{q_0}^{\infty} k \, dk \, \frac{f^0(\epsilon_k)(1 - f^0(\epsilon_k - \hbar \omega))}{\sqrt{k^2 - q_0^2}}$$

(38)

Converting $k$ integral into energy $k = \sqrt{\frac{2m}{\hbar}} \, \epsilon$, $dk = \sqrt{\frac{m \, d\epsilon}{\hbar \sqrt{2}}} \, \epsilon$ and writing lower limit for energy $\epsilon_0 = \frac{\hbar^2 q_0^2}{2m}$ and upper limit for energy integral becomes infinite.

$$I_\epsilon = \frac{m}{h^2} \int_{\epsilon_0}^{\infty} \, d\epsilon \, \frac{f^0(\epsilon_k)(1 - f^0(\epsilon_k - \hbar \omega))}{\sqrt{\frac{2m}{\hbar^2 \epsilon^2} - q_0^2}}$$

(39)

Replacing the value Fermi function $f^0(\epsilon_k) = \frac{1}{e^{\epsilon - \mu} + 1}$.

$$I_\epsilon = \frac{\sqrt{m}}{h \sqrt{2}} \int_{\epsilon_0}^{\infty} \, \frac{d\epsilon}{\sqrt{\epsilon - \frac{\hbar^2 q_0^2}{2m}}} \left( \frac{1}{e^{\beta(\epsilon - \mu)} + 1} \right) \left( 1 - \frac{1}{e^{\beta(\epsilon - \mu - \hbar \omega)} + 1} \right)$$

$$= \frac{1}{h} \sqrt{\frac{m}{2}} \int_{\epsilon_0}^{\infty} \, \frac{d\epsilon}{\epsilon - \epsilon_0} \left( \frac{1}{e^{\beta(\epsilon - \mu)} + 1} \right) \left( \frac{e^{\beta(\epsilon - \mu - \hbar \omega)}}{e^{\beta(\epsilon - \mu - \hbar \omega)} + 1} \right)$$

(40)

To simplify it further we take $\alpha = e^{-\beta \hbar \omega}$, $u = e^{\beta(\epsilon - \mu)}$, and its log gives $\epsilon = \mu + \frac{1}{\beta} \log u$. In low temperature case if we apply condition $\epsilon_0 \ll \mu$ the lower limit for $u$ becomes zero and higher limit goes to infinity. The above expression converts in new form as

$$I_\epsilon = \frac{\alpha}{\beta \hbar} \sqrt{\frac{m}{2}} \int_{0}^{\infty} \, \frac{du}{\sqrt{\mu + \frac{1}{\beta} \log u - \epsilon_0}} \left( \frac{1}{u + 1} \right) \left( \frac{1}{\alpha u + 1} \right)$$

(41)

Using the above defined condition of low temperature limit, we can reduce the square root term as $\sqrt{\mu + \frac{1}{\beta} \log u - \epsilon_0} \simeq \sqrt{\mu}$.

$$I_\epsilon = \frac{\alpha}{\beta \hbar} \sqrt{\frac{m}{2}} \mu \int_{0}^{\infty} \, \frac{du}{(u + 1)(\alpha u + 1)}$$

(42)

This elementary integral reduces to

$$I_\epsilon = \frac{\alpha}{\beta \hbar} \sqrt{\frac{m}{2}} \mu \left[ \int_{0}^{\infty} \, \frac{du}{(u + 1)(1 - \alpha)} + \frac{\alpha}{\alpha - 1} \int_{0}^{\infty} \, \frac{du}{\alpha u + 1} \right]$$

$$= \frac{\alpha}{\beta \hbar} \sqrt{\frac{m}{2}} \mu \left( \frac{1}{1 - \alpha} \right) \log \left| \frac{1}{\alpha} \right|$$

(43)

writing $\alpha = e^{-\beta \hbar \omega}$, we have

$$I_\epsilon = \frac{\alpha}{\beta \hbar} \sqrt{\frac{m}{2}} \mu \left( - \log(e^{-\beta \hbar \omega}) \right) = \sqrt{\frac{m}{2}} \mu \left( \frac{\omega}{e^{\beta \hbar \omega} - 1} \right)$$

(44)

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