Star Formation in Semi-Analytic Galaxy Formation Models with Multiphase Gas

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ABSTRACT
We implement physically motivated recipes for partitioning cold gas into different phases (atomic, molecular, and ionized) in galaxies within semi-analytic models of galaxy formation based on cosmological merger trees. We then model the conversion of molecular gas into stars using empirical recipes motivated by recent observations. We explore the impact of these new recipes on the evolution of fundamental galaxy properties such as stellar mass, star formation rate (SFR), and gas and stellar phase metallicity. We present predictions for stellar mass functions, stellar mass vs. SFR relations, and cold gas phase and stellar mass-metallicity relations for our fiducial models, from redshift $z \sim 6$ to the present day. In addition we present predictions for the global SFR, mass assembly history, and cosmic enrichment history. We find that the predicted stellar properties of galaxies (stellar mass, SFR, metallicity) are remarkably insensitive to the details of the recipes used for partitioning gas into H\textsubscript{i} and H\textsubscript{2}. We see significant sensitivity to the recipes for H\textsubscript{2} formation only in very low mass halos, which host galaxies that are not detectable with current observational facilities except very nearby. The properties of low-mass galaxies are also quite insensitive to the details of the recipe used for converting H\textsubscript{2} into stars, while the formation epoch of massive galaxies does depend on this significantly. We argue that this behavior can be interpreted within the framework of a simple equilibrium model for galaxy evolution, in which the conversion of cold gas into stars is balanced on average by inflows and outflows. Star formation in low mass galaxies is strongly self-regulated by powerful stellar driven outflows, so the overall galaxy-scale star formation efficiency is nearly independent of the H\textsubscript{2} depletion time. Massive galaxies at high redshift have not yet had time to come into equilibrium, so the star formation efficiency is strongly affected by the H\textsubscript{2} depletion time.

Key words: galaxies: formation; galaxies: evolution; galaxies: high redshift

1 INTRODUCTION
While the ΛCDM (Cold Dark Matter plus cosmological constant Λ) model (Blumenthal et al. 1984) now provides us with a well-motivated framework for predicting the abundances and properties of dark matter halos and the large scale structures in which they are embedded, all galactic or larger scale simulations must rely on “sub-grid” recipes in order to treat processes such as star formation and stellar feedback. Cosmological simulations are unable to directly resolve individual stars or, usually, even Giant Molecular Clouds (GMC). In order to model the conversion of cold gas into stars, up until recently, both numerical and semi-analytic cosmological simulations typically utilized a very simple empirical sub-grid recipe based on observations most famously by Schmidt (1959, 1963) and Kennicutt (1989, 1998) (often referred to as the “Kennicutt-Schmidt” (KS) relation). These observations showed that the surface density of star formation $\Sigma_{\text{SFR}}$ was proportional to the surface density of cold gas to a power $N_{KS}$. Observations also showed that the efficiency of star formation dropped rapidly below a critical gas surface density (Martin & Kennicutt 2001). There has been debate about whether this critical surface den-
sity is best described in terms of a Toomre stability criterion \citep{Toomre1964} or a constant critical density, and indeed about the physical origin of this critical density \citep{Schaye2004, Leroy2008}.

From the pioneering work of \citet{Katz1992} up until recently, cosmological simulations of galaxy formation, both numerical and semi-analytic, have implemented a star formation recipe in which “cold” gas (typically with $T < 10^4$ K) with volume density $\rho_{\text{gas}}$ is assumed to form stars at a rate per unit volume:

$$\dot{\rho}_* = \epsilon_* \rho_{\text{gas}}$$

with $N \approx 1.5$ and $\epsilon_*$ usually treated as a free parameter, tuned to match the observed Kennicutt relation. A common variant assumes $\dot{\rho}_* \propto \rho_{\text{gas}}/t_{\text{ff}}$, which is approximately equivalent because the local free-fall time $t_{\text{ff}} \propto \rho^{-0.5}$. Motivated by the observational evidence described above, many models incorporated either a critical surface density or volume density into their star formation recipe, which proved to be important in order to reproduce the observed high gas fractions in low-mass galaxies.

Beginning about a decade ago, our understanding of how star formation on $\sim 100$ pc–kpc scales depends on local conditions began to undergo a revolution. \citet{WongBlitz2002} showed that the correlation between $\Sigma_{\text{SFR}}$ and the surface density of molecular gas $\Sigma_{\text{HI}}$ was stronger than that between $\Sigma_{\text{SFR}}$ and the total gas density $\Sigma_{\text{gas}}$ in molecule rich galaxies. In the past five years, this field has advanced rapidly with the availability of galaxy-wide, high resolution maps of the star formation and multi-phase (HI and H$_2$) gas in reasonably large samples of nearby galaxies, e.g. from the THINGS (The HI nearby galaxy survey; \citealt{Walter2008}) combined with BIMA SONG (BIMA survey of Nearby Galaxies; \citealt{Helfer2003}) and HERACLES (HERA CO-Line Extragalactic Survey; \citealt{Leroy2009}). Based on these observations, it has been shown \citep{Bigiel2008, Leroy2008, Bigiel2011, Schruba2011} that, when averaged over scales of $\sim 700$ pc, the star formation density is tightly correlated with the molecular gas density to a nearly linear power, and that there is almost no correlation between $\Sigma_{\text{SFR}}$ and the density of atomic gas, so that the correlation between $\Sigma_{\text{SFR}}$ and $\Sigma_{\text{gas}}$ (the traditional KS relation) breaks down badly in the HI-dominated parts of galaxies (typically in galaxy outskirts). These results highlight the importance of modeling the partition of gas into different phases, i.e. atomic vs. molecular, which has not be attempted in most cosmological simulations of galaxy formation to date.

At the same time, there has been significant progress in understanding and modeling the formation of molecular hydrogen and star formation on galactic scales. \citet{BlitzRosolowsky2004, BlitzRosolowsky2006} showed that the fraction of atomic to molecular gas in a sample of nearby disk galaxies was tightly correlated with the midplane pressure (determined by the density of both stars and gas), and this result has been confirmed in larger samples such as THINGS \citep{Leroy2008}. \citet{RobertsonKravtsov2008} implemented low-temperature ($T < 10^4$ K) cooling, photo-dissociation of H$_2$, and an H$_2$-based SF recipe in hydrodynamic simulations of isolated disk galaxies of various masses. \citet{Krumholz2009} presented analytic models for the formation of H$_2$ as a function of total gas density and metallicity, supported by numerical simulations with simplified geometries \citep{Krumholz2008, Krumholz2009a}, emphasizing the importance of metallicity as a controlling parameter in H$_2$ formation. \citet{GnedinKravtsov2010, GnedinKravtsov2011} included detailed chemistry and low temperature cooling as well as a simplified treatment of radiative transfer and an H$_2$-based SF recipe in cosmological “zoom-in” Adaptive Mesh Refinement (AMR) simulations of small regions, and presented analytic fitting functions to their results as a function of total gas density, metallicity, and the strength of the local UV background. \citet{Christensen2012} used a similar approach to implement chemistry and simplified radiative transfer in Smoothed Particle Hydrodynamics (SPH) zoom-in simulations of galaxy-sized regions, which include a blast-wave treatment of supernova feedback.

A somewhat different view has been presented by \citet{Ostriker2010}, who propose that heating of the Interstellar Medium (ISM) by the stellar UV background plays a key role in regulating star formation. In their model, the thermal pressure in the diffuse ISM, which is proportional to the UV heating rate, adjusts until it balances the midplane pressure set by the vertical gravitational potential. This could provide an explanation for the strong empirical correlation between H$_2$ fraction and disk midplane pressure found by \citet{BlitzRosolowsky2006}.

Although detailed simulations are crucial in order to understand the complex physical processes involved, extremely high resolution is required in order to obtain reliable results (see e.g. \citealt{Christensen2012}), implying that it will be feasible to simulate only small numbers of galaxies with these techniques for the next few years. Meanwhile, large surveys of cold gas in nearby and distant galaxies with new and upcoming facilities such as the Atacama Large Millimeter/submillimeter Array (ALMA) and the SKA (Square Kilometer Array) and its pathfinders are already being planned and pilot projects are underway. As a result, it is important to develop computationally efficient techniques that can incorporate physically motivated treatments of gas partitioning into its atomic, molecular, and possibly ionized phases and H$_2$-based star formation recipes into simulations of cosmological volumes.

Semi-analytic models (SAMs) provide an alternative approach to this problem. In semi-analytic merger tree models, a merger tree represents the formation and growth of a dark matter halo that is identified at some redshift of interest; these merger trees may be extracted from dissipationless N-body simulations or created using analytic techniques (e.g. \citealt{SomervilleKolatt1999}). Simplified but physically motivated recipes are used to track the rate of gas cooling into galaxies, and these recipes have been tested against fully numerical hydrodynamic simulations (e.g. \citealt{Hirschmann2012}). These models use angular momentum based arguments to track the radial sizes of forming disks \citep{Mo1998, Somerville2008}.
and can then implement recipes for how cold gas is converted into stars, and how energy and momentum from massive stars and supernovae is returned to the Interstellar Medium (ISM). This “feedback” from stars and SNe is assumed to drive large-scale winds that can remove gas from the galaxy. The production, ejection, and recycling of metals is also tracked. Thus our existing semi-analytic modeling framework provides the main quantities (total gas density in disks, gas metallicity) needed to implement physically motivated recipes for partitioning gas into an atomic and molecular component and then implementing an H$_2$-based star formation recipe.

Several efforts along these lines have already been presented in the literature. Ohbuchi & Rawlings (2009) implemented a prescription to estimate the H$_2$ fraction based on the empirical pressure-based recipe of Blitz & Rosolowsky (2006) applied in post-processing to the Millennium simulations of De Lucia & Blaizot (2007). However, in this approach, the star formation in the simulations was still based on a traditional KS recipe using the total gas density, not self-consistently on the estimated H$_2$ gas density. Fu et al. (2010, 2012) modeled the partitioning of gas into H and H$_2$ in radial bins in each galaxy, using both the metallicity-dependent recipes of Krumholz et al. (2009a) and the pressure-based recipe of Blitz & Rosolowsky (2006), and self-consistently implemented an H$_2$-based star formation recipe, within the semi-analytic modeling framework of Guo et al. (2011). Lagos et al. (2011b) also estimated gas partitioning into an atomic and molecular component, and implemented an H$_2$-based star formation recipe, within the GALFORM semi-analytic models of Baugh et al. (2005), Bower et al. (2006). Similar modeling efforts utilizing a somewhat more simplified framework (i.e., only the mass accretion history of the largest progenitor is tracked, rather than the full merger tree) have been presented by Dutton et al. (2014) using the pressure-based Blitz & Rosolowsky (2006) approach, and Krumholz & Dekel (2012) using the Krumholz et al. (2009b) metallicity-based approach.

It is already clear that the results of this kind of exercise may depend on the other ingredients of the modeling, in particular on the treatment of stellar feedback, chemical evolution, and potentially on feed from Active Galactic Nuclei (AGN). In this work, we present new models that incorporate a metallicity or pressure oriented treatment of atomic-molecular gas partitioning and an H$_2$-based star formation recipe within the semi-analytic modeling framework developed by the Santa Cruz group (Somerville & Primack 1999; Somerville et al. 2000, 2008a, 2012).

The current generation of semi-analytic models (incorporating some form of “quenching” in massive halos, e.g. from AGN feedback) has been fairly successful at reproducing a variety of galaxy observations, but suffer from generic problems. Both the successes and problems seem to be common to the semi-analytic models developed by many different groups as well as to large-volume cosmological hydrodynamic simulations (see Somerville & Davé 2014, for a discussion). Significant successes include the ability to match the observed stellar mass function or luminosity functions from the UV to the NIR at z = 0, while simultaneously matching the gas fraction as a function of stellar mass for nearby disk galaxies (e.g. Somerville et al. 2008a, 2012; Lu et al. 2014). Observations show that massive galaxies form their stars early, and that the star formation in many of these massive objects is quenched early, so that their stars evolve largely passively. There is some tension in the ability of models to produce enough massive galaxies at early times (z > 2), and a dearth of very rapidly star forming objects observed in the sub-mm and FIR (Somerville et al. 2012; Niemi et al. 2012). However, the evolution of the number of massive “quenched” galaxies in models with AGN feedback seems to match observations reasonably well (Kimm et al. 2009; Brennan et al. 2013).

Low mass galaxies seem to present a more thorny set of problems, which we refer to collectively as the “dwarf galaxy conundrum”. Models that reproduce the low-mass end of the observed stellar mass function locally, generically overproduce low-mass ($m_* \sim 10^{10} M_\odot$) galaxies at redshifts 0.5 ≤ z ≤ 2. Moreover, low-mass galaxies apparently have (specific) star formation rates that are too low over the same redshift range. The stellar ages predicted by our models for these galaxies are too old compared to those derived for nearby galaxies based on “archaeological” evidence. A summary of these problems, demonstrated for several independently developed semi-analytic models, was presented in Fontanot et al. (2009). Weinmann et al. (2012) presented a similar study that showed that the same problem also occurs in numerical hydrodynamic simulations, and recently Somerville & Davé (2014) showed that the problem persists to varying degrees in most state-of-the-art SAMs and cosmological hydrodynamical simulations. It has been suggested that these problems might be due to inaccurate recipes for star formation, and that they might be cured by implementing metallicity dependent recipes for H$_2$ formation and H$_2$-based star formation (Krumholz & Dekel 2012; Kuhlen et al. 2012). This was one of the original motivations for the work we present here.

The purpose of this paper is to present the details of how we incorporate partitioning of gas into an atomic, molecular and (optionally) ionized component in our existing semi-analytic models, how we self-consistently implement an H$_2$-based star formation recipe, and how sensitive our results are to details of the implementation. We explore three different recipes for the partitioning of gas into different phases: the pressure-based recipe of Blitz & Rosolowsky (2006, BR) and two metallicity-based recipes, that of Krumholz et al. (KMT) and that of Guedin & Kravtsoy (2011, GK). We compare the predictions of these three new models with those using the “classic” Kennicutt-Schmidt (KS) star formation recipe with no gas partitioning. In addition, we explore several different empirically motivated H$_2$-based star formation recipes.

This paper is part of a series of related works. In Popping et al. (2014, PST14), we presented predictions for the atomic and molecular gas content of galaxies, and its evolution with redshift from z = 6–0, using the same models presented here. Popping et al. (2014b) ex-
tended these models by carrying out radiative transfer calculations to predict sub-mm line emission luminosities from several atomic and molecular species, including CO, HCN, C\textsuperscript{+}, and [OI]. In Berry et al. (2014), we presented predictions for the properties of objects that would be selected as Damped Lyman-\alpha systems (DLAS) in absorption against background quasars, again using the same model framework described here. In this paper, we focus on quantities pertaining to the stellar content, SFR, and metal content of galaxies and their evolution since $z \sim 6$. In addition, we explore a wider variety of model variants than presented in the earlier works.

The outline of the paper is as follows. In §2.1 we outline the basic framework of the semi-analytic models and the treatment of structure formation, gas cooling and infall, chemical evolution, and starbursts and morphological transformation via galaxy mergers. In §2.2 we describe our approaches for partitioning cold gas into an atomic, molecular, and (optionally) ionized component, in §2.3 we describe the new H\textsubscript{2}-based star formation recipes, and in §2.4 we describe our implementation of metal enhanced winds. In §2.5 we describe how we choose the values of the free parameters in our models, and summarize their values. In §3.1 we show how the star formation histories and build-up of stars, gas, and metals as a function of halo mass are impacted by the different recipes for gas partitioning and star formation, and other details of our model implementation. In §3.2 we show predictions for the relationship between total gas density and SFR density in our models. In §3.3 we present predictions for the stellar mass functions and stellar fractions, specific star formation rates, gas depletion timescales, and gas and stellar phase metallicities over cosmic time from $z \sim 6$ to the present. We discuss our results in §4 and summarize and conclude in §5.

2 MODEL DESCRIPTION

The semi-analytic models used here have been described in detail in Somerville & Primack (1999), Somerville et al. (2001) and most recently in Somerville et al. (2008a, hereafter S08) and Somerville et al. (2012, S12). The Santa Cruz modeling framework has also recently been described in Porter et al. (2014). We refer the reader to those papers for details.

2.1 The Semi-Analytic Model Framework

This section describes the aspects of the semi-analytic models that have been documented in previous papers. Therefore we give a relatively brief description of these ingredients here.

In this work, the merging histories (or merger trees) of dark matter haloes are constructed based on the Extended Press-Schechter (EPS) formalism using the method described in Somerville & Kolatt (1999), with improvements described in S08. These merger trees record the growth of dark matter haloes via merging and accretion, with each “branch” representing a merger of two or more haloes. We follow each branch back in time to a minimum progenitor mass $M_{\text{res}}$, which we refer to as the mass resolution of our simulation. Our SAMs give nearly identical results when run on the EPS merger trees or on merger trees extracted from dissipationless N-body simulations (Lu et al. 2014; Porter et al. 2014). We use EPS merger trees here because they allow us to attain extremely high resolution, which is important for this study. We resolve haloes down to $M_{\text{res}} = 10^9 M_\odot$ for all root haloes, and below root halo masses of $M_{\text{res}} = 10^8 M_\odot$, we set $M_{\text{res}} = 0.01 M_{\text{root}}$, where $M_{\text{root}}$ is the mass of the root halo. Our root haloes cover a range from $M_h = 5 \times 10^8 M_\odot$ to $5 \times 10^{14} M_\odot$.

When dark matter haloes merge, the central galaxy of the largest progenitor becomes the new central galaxy, and all others become ‘satellites’. Satellite galaxies lose angular momentum due to dynamical friction as they orbit and may eventually merge with the central galaxy. To estimate this merger timescale we use a variant of the Chandrasekhar formula from Bovian-Kolchin et al. (2008). Tidal stripping and destruction of satellites are also modeled as described in S08.

Before the Universe is reionised, each halo contains a mass of hot gas equal to the universal baryon fraction times the virial mass of the halo. After reionisation, the photo-ionising background suppresses the collapse of gas into low-mass haloes. We use the fitting functions provided by Gnedin (2000) and Kravtsov et al. (2004), based on their hydrodynamic simulations, to model the fraction of baryons that can collapse into haloes of a given mass after reionisation, and assume that the universe was fully reionized by $z = 11$.

When a dark matter halo collapses, or experiences a merger that at least doubles the mass of the largest progenitor, the hot gas is assumed to be shock-heated to the virial temperature of the new halo. This radiating gas then gradually cools and collapses. The cooling rate is estimated using a simple spherically symmetric model similar to the one originally suggested by White & Frenk (1991). Details are provided in S08.

We assume here that the cold gas is accreted only by the central galaxy of the halo, although in reality satellite galaxies probably also continue to accrete some cold gas after they cross the virial radius of their host. In addition, we assume that all newly cooling gas initially collapses to form a rotationally supported disc. The scale radius of the disc is computed based on the initial angular momentum of the gas and the halo profile, assuming that angular momentum is conserved and that the self-gravity of the collapsing baryons causes contraction of the matter in the inner part of the halo (Blumenthal et al. 1986; Flores et al. 1993; Mo et al. 1998). This approach has been shown to reproduce the observed size versus stellar mass relation for disc-dominated galaxies from $z \sim 0$–2 (Somerville et al. 2008). In PST14 we also showed that our models reproduce the sizes of HI disks in nearby galaxies, and sizes of CO disks out to $z \sim 2$.

Star formation occurs in two modes, a normal “disc” mode in isolated discs, and a merger-driven “starburst” mode. Star formation in the disc mode is modelled as described in Section 2.3 below. The efficiency and timescale of the merger driven burst mode is modeled as described in S08 and is a function of the merger mass ratio and the
gas fractions of the progenitors. The treatment of merger-driven bursts is based on the results of hydrodynamic simulations of binary galaxy mergers (Robertson et al. 2006; Hopkins et al. 2009a).

Some of the energy from supernovae and massive stars is assumed to be deposited in the ISM, resulting in the driving of a large-scale outflow of cold gas from the galaxy. The mass outflow rate is

\[
\dot{m}_{\text{out}} = \epsilon_{\text{SN}} \left( \frac{V_0}{V_c} \right)^{\alpha_{\text{SN}}} \dot{m}_* \tag{2}
\]

where \(V_c\) is the maximum circular velocity of the galaxy (here approximated by \(V_{\max}\) of the dark matter halo), \(\dot{m}_*\) is the star formation rate, \(\epsilon_{\text{SN}}\) and \(\alpha_{\text{SN}}\) are free parameters, and \(V_0 = 200\) km/s is an arbitrary normalization constant. Some fraction of this ejected gas escapes from the potential of the dark matter halo, while some is deposited in the hot gas reservoir within the halo, where it becomes eligible to cool again. The fraction of gas that is ejected from the disc but retained in the halo, versus ejected from the disc and halo, is a function of the halo circular velocity (see S08 for details), such that low-mass haloes lose a larger fraction of their gas.

The gas that is ejected from the halo is kept in a larger reservoir, along with the gas that has been prevented from falling in due to the photoionizing background. This gas is assumed to accrete onto the halo on a timescale that is proportional to the halo dynamical time (see S08 for details).

Each generation of stars produces heavy elements, and chemical enrichment is modelled in a simplified manner using the instantaneous recycling approximation. For each parcel of new stars \(\text{d}m_*\), we also create a mass of metals \(\text{d}M_g = y \text{d}m_*\), which we assume to be instantaneously mixed with the cold gas in the disc. The yield \(y\) is assumed to be constant, and is treated as a free parameter. When gas is removed from the disc by supernova driven winds as described above, a corresponding proportion of metals is also removed and deposited either in the hot gas or outside the halo, following the same proportions as the ejected gas. Ejected metals also “re-accrete” into the halo along with the ejected gas, as described above.

Mergers are assumed to remove angular momentum from the disc stars and to build up a spheroid. The efficiency of disc destruction and spheroid growth is a function of progenitor gas fraction and merger mass ratio, and is parameterized based on hydrodynamic simulations of disc-disc mergers (Hopkins et al. 2009a). These simulations indicate that more “major” (closer to equal mass ratio) and more gas-poor mergers are more efficient at removing angular momentum, destroying discs, and building spheroids. Note that the treatment of spheroid formation in mergers used here has been updated relative to S08 as described in (Hopkins et al. 2009b and Porter et al. 2014). We do not include a disk instability driven mode for spheroid growth in the models presented here.

In addition, mergers drive gas into galactic nuclei, fueling black hole growth. Every galaxy is born with a small “seed” black hole (BH; here we adopt \(M_{\text{seed}} \sim 1.0 \times 10^5 M_\odot\)). Following a merger, any pre-existing black holes are assumed to merge immediately, and the resulting hole grows at its Eddington rate until the energy being deposited into the ISM in the central region of the galaxy is sufficient to significantly offset and eventually halt accretion via a pressure-driven outflow. This results in self-regulated accretion that leaves behind black holes that naturally obey the observed correlation between BH mass and spheroid mass or velocity dispersion. Our models produce good agreement with the observed luminosity function of X-ray/optical/IR detected quasars and AGN (Hirschmann et al. 2012).

A second mode of black hole growth, termed “radio mode”, is associated with powerful jets observed at radio frequencies. In contrast to the merger-triggered, radiatively efficient mode of BH growth described above (sometimes called “bright mode” or “quasar mode”), in which the BH accretion is fueled by cold gas in the nucleus, here, hot halo gas is assumed to be accreted according to the Bondi-Hoyle model (Bondi 1952). This leads to accretion rates that are typically \(\lesssim 10^{-3}\) times the Eddington rate, so that most of the BH’s mass is acquired during episodes of “bright mode” accretion. However, the radio jets are assumed to couple very efficiently with the hot gas, and to provide a heating term that can partially or completely offset cooling during the “hot flow” mode (we assume that the jets cannot couple efficiently to the cold, dense gas in the infall-limited or cold flow regime).

### 2.2 Multi-phase Gas Partitioning

In this section, we describe in detail the updates to the model ingredients that are explored in this paper. These include partitioning of the cold gas in galactic disks into an ionized (H\(_i\)), atomic (H\(_\text{I}\)), and molecular (H\(_2\)) component, an option to include metal enhanced winds, and a set of new H\(_2\)-based star formation recipes.

At each timestep, we compute the scale radius of the cold gas disc using the angular momentum based approach described above, and assume that the total (H\(_i\) + H\(_2\) + H\(_\text{I}\)) radial cold gas distribution is described by an exponential with scale radius \(r_{\text{gas}}\). We do not attempt to track the scale radius of the stellar disk separately, but make the simple assumption that \(r_s = \chi_{\text{gas}} r_{\text{gas}},\) with \(\chi_{\text{gas}}\) fixed to match observed stellar scale lengths at \(z = 0\).

Bigiel & Blitz (2012) showed that this is a fairly good representation, on average, of the disks of nearby spirals (see also Kravtsov 2013). We then divide the gas disk into radial annuli and compute the fraction of molecular gas, 

\[
j_{\text{HI}}(r) \equiv \Sigma_{\text{HI}}(r)/[\Sigma_{\text{HI}}(r) + \Sigma_{\text{H}_2}(r)],\]

in each annulus, as described below. We use a fifth order Runge-Kutta integration scheme to compute the integrated mass of H\(_i\) and H\(_2\) in the disk, and the integrated SFR, at each timestep.

#### 2.2.1 Ionized gas associated with galaxies

Most previous semi-analytic models have neglected the ionized gas associated with galaxies, which may be ionized either by an external background or by the radiation field from stars within the galaxy. Here we investigate a simple analytic model motivated by the work of Gnedin.
We assume that some fraction of the total cold gas in the galaxy, $f_{\text{ion, int}}$, is ionized by the galaxy’s own stars. In addition, a slab of gas on each side of the disk is ionized by the external background radiation field. Assuming that all gas with a surface density below some critical value $\Sigma_{\text{HI}}$ is ionized, we have

$$f_{\text{ion}} = \frac{\Sigma_{\text{HI}}}{\Sigma_0} \left[ 1 + \ln \left( \frac{\Sigma_0}{\Sigma_{\text{HI}}} \right) + 0.5 \left( \ln \left( \frac{\Sigma_0}{\Sigma_{\text{HI}}} \right) \right)^2 \right]$$

where $\Sigma_0 \equiv m_{\text{cold}}/(2\pi r_{\text{gas}})^2$ is the central surface density of the cold gas (where $m_{\text{cold}}$ is the mass of all cold gas in the disk and $r_{\text{gas}}$ is the scale radius of the gas disk). We typically assume $f_{\text{ion, int}} = 0.2$ (as in the Milky Way) and $\Sigma_{\text{HI}} = 0.4 \, M_\odot \, pc^{-2}$, as in (Gnedin 2012). Applying this model within our SAM gives remarkably good agreement with the ionized gas fractions in galaxies. The first is based on local sources and the cosmic background. An alternate approach based on similar physics processes was presented in a series of papers by Krumholz and collaborators (Krumholz et al. 2009a,b). Krumholz et al. (2009b) developed an analytic model for the molecular fraction in galaxies, based on the ansatz that the interplay between the interstellar radiation field and molecular self-shielding determines the molecular fraction. They presented a fitting function:

$$f_{\text{H}_2} = \left[ 1 + \frac{\tilde{\Sigma}}{\Sigma_{\text{HI}} + \Sigma_{\text{H}_2}} \right]^{-2}$$

where

$$\tilde{\Sigma} = 20 \, M_\odot \, pc^{-2} \left( \frac{D_{\text{MW}}}{\sqrt{1 + U_{\text{MW}} D_{\text{MW}}^2}} \right)^{4/7}$$

$$\Lambda = \ln(1 + g D_{\text{MW}}^{4/7}(U_{\text{MW}/15})^{4/7})$$

$$g = 1 + os + s^2$$

$$s = \frac{D_* + D_{\text{MW}}}{U_{\text{MW}/2}^3}$$

$$D_* = 1.5 \times 10^{-3} \ln(1 + (3U_{\text{MW}})^{1.7})$$

We take the dust-to-gas ratio to be proportional to the cold gas phase metallicity in solar units $D_{\text{MW}} = Z/Z_\odot$. The local UV background relative to the MW is assumed to scale in proportion with the global SFR of the galaxy in the previous time step relative to the MW SFR, $U_{\text{MW}} = SFR_{\text{MW}}$, where we choose $SFR_{\text{MW}} = 1.0 \, M_\odot \, yr^{-1}$ (Murray & Rahman 2010; Robitaille & Whitmore 2010). We refer to this as our ‘fiducial’ GK model. We also investigate the results of keeping $U_{\text{MW}}$ fixed to the Milky Way value, which we refer to as the GKFUV model (fixed UV field).

An alternate approach based on similar physical processes was presented in a series of papers by Krumholz and collaborators (Krumholz et al. 2009a,b). Krumholz et al. (2009a,b) developed an analytic model for the molecular fraction in galaxies, based on the ansatz that the interplay between the interstellar radiation field and molecular self-shielding determines the molecular fraction. They presented a fitting function:

$$f_{\text{H}_2} = 1 - \left[ 1 + \frac{3}{4} s \left( \frac{1}{1 + s} \right)^{-5/3} \right]^{-1/3}$$

where

$$s = \ln(1 + 0.6) / (0.04 \Sigma_{\text{comp,0}} Z''), \chi = 0.77(1 + 3.1(0.36)) \, \delta = 0.07(0.18 + 0.675)^{-2.8}, \Sigma_{\text{comp,0}} = \Sigma_{\text{comp}} / (1 M_\odot \, pc^{-2})$$

The quantity denoted $\Sigma_{\text{comp}}$ is the surface density within a $\sim 100$ pc sized atomic-molecular cloud complex. Krumholz et al. (2009a) suggest using a “clumping factor” to apply the model to simulations with spatial resolution coarser than 100 pc; i.e. $\Sigma_{\text{comp}} \rightarrow c \Sigma_{\text{HI}} + \Sigma_{\text{H}_2}$, where $c \rightarrow 1$, and $\Sigma_{\text{HI}} + \Sigma_{\text{H}_2}$ is the neutral gas surface density on some larger scale. The appropriate value of $c$ depends on this spatial scale, where $c \rightarrow 1$ as the scale approaches 100 pc. We refer to this as the KMT gas partitioning recipe. Both Gnedin & Kravtsov (2011) and Krumholz et al. (2009a) note that the fitting functions, as well as perhaps (in the case of KMT) the
underlying assumptions of the model, begin to break down at metallicities lower than about 1/50th of the Solar value.

The KMT and GK fitting functions above characterize the formation of H$_2$ on dust grains, which is the dominant mechanism once the gas is enriched to more than a few hundredths of Solar metallicity. Other channels for the formation of H$_2$ in primordial gas must be responsible for producing the molecular hydrogen out of which the first stars were formed. Hydrodynamic simulations containing detailed chemical networks and analytic calculations have shown that H$_2$ can form in metal-free gas in dark matter halos above a critical mass $M_{\text{crit}} \sim 10^6 M_\odot$ (e.g., Nakamura & Umemura 2001; Glover 2013). This gas can then form “Pop III” stars which can enrich the surrounding ISM to $Z_{\text{pre-enrich}} \sim 10^{-3} Z_\odot$ (Schneider et al. 2002, Greif et al. 2010, Wise et al. 2012). These processes take place in halos much smaller than our resolution limit. We represent them by setting a “floor” to the molecular hydrogen fraction in our halos, $f_{H_2,\text{floor}}$. In addition, we “pre-enrich” the initial hot gas in halos, and the gas accreted onto halos due to cosmological infall, to a metallicity of $Z_{\text{pre-enrich}}$. We adopt typical values of $f_{H_2,\text{floor}} = 10^{-4}$ and $Z_{\text{pre-enrich}} = 10^{-3} Z_\odot$ (Haiman et al. 1996, Bromm & Larson 2004). We explore the sensitivity of our results to these parameters in Section 5.4. Note that observations of resolved stars in the halo of our Galaxy and local dwarfs have revealed stars with metallicities below $Z \sim 10^{-2} Z_\odot$ (Tolstoy et al. 2009, Schöck et al. 2004, Kirby et al. 2011), precluding much higher values for $Z_{\text{pre-enrich}}$.

2.3 Star Formation Recipes

2.3.1 The Kennicutt-Schmidt (KS) Recipe

The KS recipe (Kennicutt 1998) assumes that the surface density of star formation in a galaxy is a function of the total surface density of the cold neutral gas (atomic and molecular), above some threshold surface density $\Sigma_{\text{crit}}$.

The star formation rate density (per unit area) for $\Sigma_{\text{gas}} > \Sigma_{\text{crit}}$ is given by:

$$\Sigma_{\text{SFR}} = A_{\text{SF}} \Sigma_{\text{gas}}^{N_{\text{SF}}},$$  \hspace{1cm} (5)

where $\Sigma_{\text{SFR}} = 0$ for $\Sigma_{\text{gas}} < \Sigma_{\text{crit}}$. This recipe is the same one used in most of our previously published SAMs (S08, S12), and is similar to recipes commonly adopted in many other SAMs. The values of the free parameters are given in Table 1.

2.3.2 Molecular Hydrogen-based Recipes

In the same spirit as the KS recipe, we use empirical relationships from observations to motivate our H$_2$-based recipes. Bigiel et al. (2008) found, based on observations of spiral galaxies from the THINGS survey, that the star-formation rate surface density can be directly related to the surface density of molecular gas, i.e.

$$\Sigma_{\text{SFR}} = \left( \frac{A_{\text{SF}}}{10 \ M_\odot \text{pc}^{-2}} \right) \Sigma_{H_2}^{N_{\text{SF}}}.$$  \hspace{1cm} (6)

with $N_{\text{SF}} \approx 1$ (see also Bigiel et al. 2011, Leroy et al. 2013). Observations of higher density environments suggest that above some critical H$_2$ surface density, the slope of the relation described in Eqn. 6 steepens (Narayanan et al. 2012). We therefore also consider a two-part scaling law given by:

$$\Sigma_{\text{SFR}} = A_{\text{SF}} \left( \frac{\Sigma_{H_2}}{10 \ M_\odot \text{pc}^{-2}} \right) \left( 1 + \frac{\Sigma_{H_2}}{\Sigma_{H_2,\text{crit}}} \right)^{N_{\text{SF}}}.$$  \hspace{1cm} (7)

The values of the parameters $A_{\text{SF}}, N_{\text{SF}},$ and $\Sigma_{H_2,\text{crit}}$ are given in Table 1.

A star formation relation that changes slope above a critical density is also expected based on theoretical grounds. Krumholz et al. (2009b) adopt the star formation relation:

$$\Sigma_{\text{SFR}} = A_{\text{SF}} \Sigma_{H_2}^{\frac{\Sigma_{\text{gas}}}{\Sigma_{\text{crit}}}}^{N_{\text{SF}}}.$$  \hspace{1cm} (8)

where $N_{\text{SF}} = -0.33$ for $\Sigma_{\text{gas}}/\Sigma_{\text{crit}} < 1$ and $N_{\text{SF}} = 0.33$ for $\Sigma_{\text{gas}}/\Sigma_{\text{crit}} > 1$. KMT adopt $A_{\text{SF}} = 1/2.6$ Gyr$^{-1}$ and $\Sigma_{\text{crit}} = 85 M_\odot \text{pc}^{-2}$. We adopt the same parameter values in our “KMT” model.

A comparison of various star formation relations in the literature, and used in this work, is shown in Fig. 1.

2.4 Metal Enhanced Winds

Most of our previous models have assumed that metals are ejected from galaxies with the same efficiency as the
gas, i.e. with the same mass loading factor \( \eta \equiv \dot{m}_{\text{out}} / \dot{m}_* \). However, since metals are produced by the same massive stars and supernovae that are believed to drive galactic outflows, it is possible that metals are preferentially ejected (i.e., have a higher effective mass loading factor than the gas averaged over the whole disk). Because two of our recipes for gas partitioning depend on the gas metallicity, the dispersal of metals in our models has a potentially important impact on our results. We therefore include an optional treatment of metal-enhanced winds in our models.

We base our parameterization of metal-enhanced winds on the approach used in Krumholz & Dekel (2012), in part because we want to be able to compare our results with theirs. The fraction of metals that is ejected is parameterized by:

\[
\zeta = \zeta_0 \exp(-M_b/M_{\text{vir}})
\]

where both \( \zeta_0 \) and \( M_{\text{vir}} \) are free parameters, and \( M_b \) is the virial mass of the halo. The modified equation for the evolution of the mass in metals in the cold gas phase is then:

\[
\dot{M}_Z = y(1-R)(1-\zeta)\dot{m}_* + Z_{\text{hot}}\dot{m}_{\text{inf}} - Z_{\text{cold}}\dot{m}_{\text{out}}
\]

where \( R \) is the recycled fraction, \( y \) is the chemical yield, \( Z_{\text{hot}} \) is the metallicity of the hot gas, \( Z_{\text{cold}} \) is the metallicity of the cold gas, and \( \dot{m}_* \), \( \dot{m}_{\text{inf}} \), and \( \dot{m}_{\text{out}} \) are the star formation rate, inflow rate of gas from the hot halo into the disk, and the outflow rate of gas from the disk, respectively.

### 2.5 Calibrating the Free Parameters

As in any sub-grid physics in our models. In keeping with common practice, we choose the values of the free parameters by tuning to a subset of observations in the local universe. In this subsection we summarize the values of the free parameters used here (see Table 1) and the observations we used to constrain them. The parameters are the same as those used in the models presented in PST14.

We assume values for the cosmological parameters consistent with the five year WMAP results (WMAP5): \( \Omega_m = 0.28 \), \( \Omega_\Lambda = 0.72 \), \( H_0 = 70.0 \), \( \sigma_8 = 0.81 \), and \( n_s = 0.96 \) (Komatsu et al. 2009). We note that these values are generally consistent with those obtained from the analysis of the seven-year WMAP data release (Komatsu et al. 2010). The adopted baryon fraction is 0.1658. We assume a recycled fraction of \( R = 0.43 \), as appropriate for a Chabrier stellar Initial Mass Function (Chabrier 2003).

As discussed in S08 (see also White et al. 2014), in our models the supernova feedback parameters mainly control the low-mass end of the stellar mass function (\( m_* \lesssim M_{\text{char}} \), where \( M_{\text{char}} \) is the characteristic mass of the “knee” in the Schechter function describing the stellar mass function), or equivalently, the fraction of baryons that is turned into stars in halos with \( M_* \gtrsim 10^{12} M_\odot \). On the other side, the efficiency of the “radio mode” AGN feedback (one can think of this schematically as the efficiency with which radio jets couple to and heat the hot intragroup and intracluster medium) controls the number density of massive galaxies \( m_* \gtrsim M_{\text{char}} \), or the fraction of baryons that are able to turn into stars in massive halos (\( M_h \gtrsim 10^{12} M_\odot \)). We tune the parameters controlling supernova feedback and AGN feedback to reproduce the observed stellar mass function at \( z = 0 \) in our traditional KS model (see S08 for details). These parameters are then kept fixed as we explore the effects of varying the modeling of gas partitioning and star formation.

The parameters of the star formation recipe mainly control the fraction of cold gas in galaxies, and do not strongly affect the \( z = 0 \) stellar mass function (White et al. 2014). We require the parameters characterizing our star formation recipes to lie within the observational uncertainties from recent empirical constraints, and tune them within these limits to match the total gas fractions as a function of galaxy stellar mass at \( z = 0 \) (see PST14).

The chemical yield \( y \) could in principle be obtained from stellar evolution models, but these model yields are uncertain by a factor of \( \sim 2 \), and the single-element instantaneous recycling approach to chemical evolution that we are using here is somewhat crude, so we instead treat the chemical yield as a free parameter (though we restrict it to be in the expected range). We tune our yield to match the normalization of the observed stellar metallicity vs. mass relation of Gallazzi et al. (2005).

We take the parameter values for the BR gas partitioning recipe from the observational results of Leroy et al. (2008). We implement the GK recipe as it is given in Gaedt & Kravtsov (2011), with no tunable parameters. The KMT recipe has one free parameter, the clumping factor of the gas. We adopt \( c = 5 \), following Krumholz & Dekel (2012).

Our new models predict the fraction of cold gas in different phases: ionized, atomic, and molecular. We showed the predictions of our two fiducial models (GK and BR) for the fraction of H\textsc{i} and H\textsc{ii} as a function of galaxy internal stellar density and stellar mass in PST14. It is encouraging that our new models reproduce these observed scalings for nearby galaxies quite well without any additional tuning. We emphasize that the only new free parameters in the gas partitioning recipe have been taken directly from observations (in the case of the BR recipe) or from numerical simulations (in the case of KMT and GK).

### 3 RESULTS

#### 3.1 Effects of Varying Model Ingredients, Parameter Values, and Resolution

In this subsection we explore the sensitivity of our model results to new model ingredients and parameter values related to gas partitioning and star formation, as well as to our numerical resolution. To illustrate these effects, we show the properties of the largest progenitor galaxy as a function of time (or redshift) in a set of halos with masses at \( z = 0 \) ranging from \( \log M_h/M_\odot = 10.0\)–\(11.5 \) (except in one case, where we show a more massive halo with \( \log M_h/M_\odot = 12.5 \)). The variations that we
Table 1. Summary of Model Parameters

| parameter      | description                        | section defined | value        |
|----------------|------------------------------------|-----------------|--------------|
| $\epsilon_{\text{SN}}$ | supernova feedback efficiency      | §2.1            | 1.5          |
| $\alpha_{\text{SN}}$ | supernova feedback slope            | §2.1            | -2.2         |
| $y$             | chemical yield                     | §2.1            | 1.6 $Z_\odot$|
| $\kappa_{\text{AGN}}$ | radio mode AGN feedback           | S08 §2.11, Eqn. 20 | $3.8 \times 10^{-3}$ |
| $\chi_{\text{gas}}$ | ratio of stellar to gas scale length | §2.2            | 0.59         |
| $\Sigma_{\text{HII}}$ | critical density for ionized gas    | §2.2            | 0.4 $M_\odot pc^{-2}$ |
| $f_{\text{ion, int}}$ | internal fraction of ionized gas   | §2.2            | 0.2          |
| $P_0$           | pressure scaling in BR recipe      | §2.2            | 4.23 $k_B cm^3 K$ |
| $a_{\text{BR}}$ | slope in BR recipe                 | §2.2            | 0.8          |
| $c$             | clumping factor in KMT recipe      | §2.2            | 5            |
| $f_{\text{H2, floor}}$ | primordial H$_2$ fraction         | §2.2            | $10^{-4}$    |
| $Z_{\text{pre-enrich}}$ | metallicity due to Pop III stars | §2.2            | $10^{-3}$    |
| $\zeta_{\text{tot}}$ | metal enhanced winds normalization | §2.2            | 0.1          |
| $M_{\text{tot}}$ | metal enhanced winds mass scale    | §2.2            | 0.9          |
| $A_{\text{SF}}$ (KS) | star formation relation normalization | §2.3, Eqn. 5 | $1.1 \times 10^{-4}$ |
| $N_{\text{SF}}$ (KS) | star formation relation slope      | §2.3, Eqn. 5    | 1.4          |
| $\Sigma_{\text{crit}}$ (KS) | critical density for SF           | §2.3, Eqn. 5    | 6 $M_\odot pc^{-2}$ |
| $A_{\text{SF}}$ (Big1, Big2) | star formation relation normalization | §2.3, Eqn. 6 | $4.0 \times 10^{-3}$ |
| $N_{\text{SF}}$ (Big1, Big2) | star formation relation slope      | §2.3, Eqn. 6    | 1.0          |
| $\Sigma_{\text{H2, crit}}$ | critical H$_2$ density            | §2.3, Eqn. 7    | 70 $M_\odot pc^{-2}$ |

Table 2. Summary of Model Variants

| model       | H$_1$/H$_2$ partitioning | SF law | metal-enhanced winds |
|-------------|---------------------------|--------|----------------------|
| KS “fiducial” | none                      | KS     | N                    |
| BR “fiducial” | BR                        | Big2   | N                    |
| GK “fiducial” | GK, $U_{MW} \propto SFR$ | Big2   | N                    |
| GK + Big1   | GK                        | Big1   | N                    |
| GKFUV       | GK, $U_{MW} = 1$          | Big2   | N                    |
| BR + Big1   | BR                        | Big1   | N                    |
| KMT + Big1  | KMT                       | Big1   | N                    |
| KMT         | KMT                       | KMT    | N                    |
| KMT + MEW   | KMT                       | KMT    | Y                    |

explore here mainly affect lower mass halos, and produce no significant differences for galaxies in halos more massive than those shown. We use the same merger trees for each model, and fix all model properties that are chosen from random distributions to their average values. Each panel shows the average over 60 different realizations of halos with the specified final mass. For each experiment we show the stellar mass, total neutral cold gas mass (H$_1$ + H$_2$), SFR, H$_2$ fraction, and gas phase metallicity. To facilitate comparison, the stellar mass, gas mass, and SFR are normalized by dividing by the root halo mass at $z = 0$.

As a first basic check, we test the impact of varying the mass resolution of our merger trees (Fig. 4). Note that what we mean by the ‘mass resolution’ here is the mass of the smallest halos that are tracked in the merger tree. This is not equivalent to the particle mass in an N-body simulation, but rather to the smallest halo mass that can be robustly identified. We see from this test that in order to robustly reconstruct the whole halo mass accretion history back to $z \sim 10$, we require a minimum halo mass of $\sim 1/100$ of the root mass at the output redshift. Accordingly, we impose this condition on all halos in the runs used in this work. It is reassuring to see that, once the halo mass accretion history is well resolved, our SAM predictions converge extremely well (note that we do not retune the free parameters when we change the mass resolution).

In Fig. 5 we test for possible sensitivity to the values of two parameters that we introduce to simulate the formation of stars in primordial gas, the metallicity of the “pre-enriched” gas $Z_{\text{pre-enrich}}$ (most relevant for the metallicity-based recipes), and the primordial molecular hydrogen fraction $f_{\text{H2, floor}}$ (most relevant for the BR recipe). Leaving all other settings of the fiducial GK model fixed, we vary $Z_{\text{pre-enrich}}$ from its fiducial value of $10^{-3}$ by one order of magnitude downwards, to $10^{-4}$, and upwards to 0.01. Overall, the impact of even such
Figure 2. From top to bottom, colored lines show the stellar mass, cold neutral gas mass (H\textsubscript{i} + H\textsubscript{2}), and SFR normalized by the mass of the root halo at $z = 0$ for the largest progenitor galaxy as a function of cosmic time (redshift). The H\textsubscript{2} fraction ($f_{H2} \equiv m_{H2}/(m_{H2} + m_{HI})$) and gas phase metallicity (in solar units) are also shown. Gray lines show the maximum baryon fraction in the halo, $f_b M_h(t)$, where $M_h(t)$ is the mass of the largest progenitor halo at time $t$ and $f_b$ is the universal baryon fraction (different gray lines correspond to different resolutions; lower resolution runs cannot resolve the halo mass accretion history as far back in time). In this experiment, we test the dependence of our results on the mass resolution of our merger trees, varying the mass resolution by two orders of magnitude. Results are shown for a mass resolution of $10^{10}$, $M_\odot$ (solid purple), $10^9$, $M_\odot$ (dashed orange), and $10^8$, $M_\odot$ (dotted green). The mass accretion histories are well-resolved when the mass resolution is at least 1/100 the mass of the root halo.

In a related experiment, we run our (otherwise) fiducial GK model with metal-enriched winds, described in Section 2.4. Here, the metallicity of stellar driven outflows can be metal-enhanced relative to the ISM by a factor that depends on the halo mass. As seen in Fig. 4, we find that metal enhanced winds can significantly delay the formation of H\textsubscript{2} and stars in very low-mass halos ($\log M_h/M_\odot < \sim 10^9$), and cause the build-up of slightly more cold gas in low-mass halos ($\log M_h/M_\odot \lesssim 10.5$); note however that these halos host galaxies that are well below the detection limits of most...
Figure 3. Same as Fig. 2, except here we compare the results of our fiducial GK model with different values for the “pre-enriched” gas metallicity ($Z_{\text{pre-enrich}}$), as shown in the key. Our results are quite insensitive to the value of this parameter within a reasonable range.

surveys except in the very nearby Universe ($m_\star \approx 10^7 - 10^8 M_\odot$). Metal-enriched winds can also delay the build-up of metal-enriched gas even in more massive halos ($\log M_h/M_\odot \lesssim 11.5$).

A new ingredient we have introduced into our models is the tracking of gas that is photoionized either by an external radiation field or by internal sources. This gas is not eligible to form $H_2$ or stars. In Fig. 5 we show the galaxy properties in the fiducial GK model with and without tracking of HII. Although our model predicts that galaxies contain a substantial amount of HII (see Fig. 2 in Popping et al. 2014c), partitioning this gas into a separate reservoir has a very weak effect on our results. The only noticeable effect is slightly lower $H_2$ fractions at high redshift, particularly in the lowest mass halos.

In the next experiment, shown in Fig. 6 we investigate several different recipes for converting cold molecular gas into stars within the fiducial GK model. We consider two variants of the empirical recipe based on the observations by Bigiel et al. (2008). In addition, we consider the recipe proposed by KMT based on theoretical arguments (see Section 2.3 for details). Big1 refers to Eqn. 6 and Big2 to Eqn. 7. In contrast to most of our other experiments, these variations have almost no discernable effect on the low mass halos. However, the Big2 recipe leads to significantly earlier build-up of stellar mass, more efficient star formation at high redshift, and earlier metal enrichment in massive halos ($\log M_h/M_\odot \gtrsim 12.0$). This is owing to the non-linear dependence of the star formation efficiency on $H_2$ density at high densities in this model, and the positive correlation between halo mass and galaxy surface density in our models.

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Figure 4. Same as Fig. 2, except here we compare the results of our fiducial GK model with and without metal enhanced winds. Metal enhanced winds can delay enrichment and, to a lesser extent, H$_2$ and star formation in low mass halos.

Next we experiment with changing the recipe for partitioning gas into H$_2$ (Fig. 7). All other ingredients are the same as our fiducial GK models. We show the pressure-based BR model as well as two alternate metallicity based models. Recall that in the fiducial GK model, \( f_{H_2} \) depends on the total gas density and metallicity as well as the local UV radiation field (which we scale with the global galaxy SFR). In the GKUV model, we remove the UV radiation field dependence by using the Milky Way value in all galaxies. In the KMT model, \( f_{H_2} \) depends only on total gas density and metallicity, and has different dependencies on these quantities than the GK model. The results of this experiment are quite interesting. The predictions of the BR and fiducial GK models are quite similar, although the GK model tends to predict higher gas masses, lower H$_2$ fractions, and lower metallicities at early times in the two lowest mass halo bins. The KMT and GKUV models also produce similar results, as expected based on the findings of Krumholz & Gnedin (2011), who also showed the two models to be very similar. However, the KMT and GKUV models predict significantly suppressed H$_2$ formation, leading to lower star formation rates and stellar masses, reduced chemical enrichment, and higher gas fractions in the two lowest halo mass bins. This is because galaxies in low-mass halos tend to have lower metallicities but also lower SFR. Therefore in our fiducial GK models, the lower metallicity, which makes H$_2$ formation less efficient, is mitigated by the lower SFR, which leads to weaker photo-dissociation and relatively higher H$_2$ fractions.

In our penultimate experiment (Fig. 8), we compare our two new fiducial multiphase gas recipes with the “classic” Kennicutt-Schmidt recipe (see Section 2.3), in which all cold gas above a fixed surface density is eligible
Figure 5. Same as Fig. 2 except here we compare the GK model with and without the partitioning of ionized gas (H\textsc{ii}) into a separate reservoir. Our results are nearly unchanged whether or not we include the ionized gas component.
Figure 6. Same as Fig. 2 except here we compare different recipes for converting molecular gas into stars within our fiducial GK models: Big2, Big1, and KMT. Note the different range of halo masses from the other plots. Here, the strongest effect seen is the more efficient production of stars and earlier enrichment in massive halos in the Big2 model. This is owing to the non-linear dependence of the star formation efficiency on $H_2$ density at high densities in this model.

3.2 Star Formation Relations

Fig. 10 shows the relationship between the total neutral cold gas surface density $\Sigma_{HI+H_2}$ and star formation rate density $\Sigma_{SFR}$ in our two new fiducial models with multiphase gas partitioning. In our previous generation of models, galaxies had a deterministic relation between $\Sigma_{HI+H_2}$ and $\Sigma_{SFR}$ given by the assumed KS relation (as plotted in Fig. 1), and $\Sigma_{SFR}$ was set to zero below the critical gas surface density $\Sigma_{ crit}$ (also shown in Fig. 1). In our new models, neutral gas is ‘partitioned’ into HI and $H_2$, and only $H_2$ is allowed to participate in star forma-

those used by other SAMs in the literature, so these may not correspond to the actual predictions of those models. This exercise is intended to shed some light on the effect of choosing different recipes for gas partitioning and conversion of $H_2$ into stars in a controlled environment where all other aspects of the models are held fixed. Most other SAMs to date that have attempted to track multi-phase gas with a metallicity-based approach have done so using the KMT recipe. Our experiment shows that this may result in more delayed star formation in low-mass galaxies, particularly at early times.
Figure 7. Same as Fig. 2 except here we compare our fiducial GK and BR models with the KMT recipe for gas partitioning and the GK recipe for gas partitioning with a fixed UV background field. The KMT recipe and the GKFUV recipes, which both neglect the dependence of H$_2$ formation on the local UV radiation field, both predict much lower H$_2$ fractions in low mass halos, especially at high redshift, leading to later stellar mass assembly, slightly higher overall gas fractions, lower SFE, and later metal enrichment.

3.3 Evolution of Galaxy Populations
3.3.1 Stellar Mass Functions and Stellar Fractions

In this sub-section we examine the evolution of galaxy populations, which are directly comparable with observations. We consider three main model variants: the fiducial versions of the GK, BR and KS models (see Table 2). The KS model is the star formation recipe used in previously published Santa Cruz SAMs (e.g. Somerville et al. 2008a, 2012; Porter et al. 2014). The GK and BR models...
Figure 8. Same as Fig. 2 except here we compare the three “fiducial” models, GK, BR, and KS. We find remarkably similar results among all three models. The largest differences are in the predicted overall gas fraction and the H\textsubscript{2} fraction at high redshift in the lowest mass halos.

are the same as the models used in Popping et al. (2014c) and Berry et al. (2014). Selected results for other model variants are shown in the Appendix.

In Fig. 8 we present predictions for the stellar mass function of galaxies from \( z = 0 \) to \( z = 6 \). We compare these predictions with a compilation of observations as described in the figure caption. The similarity of the three model predictions is striking, particularly on the low mass end where we might have expected the differences to be largest. The most notable differences are instead at high masses, particularly at redshifts \( z > \sim 2 \). Here, the KS model produces significantly lower number densities of massive galaxies at \( z > \sim 1 \), with the deviation growing with increasing redshift. It is important to note that we have not accounted for the expected errors in the observational estimates of the stellar masses when comparing to the models — this would tend to lead to an apparent increase in the number of massive galaxies due to Eddington bias (see Lu et al. 2014, e.g.). However, even if this effect were included, the earlier formation of massive galaxies predicted by the GK and BR models is clearly in better accord with recent observations. All three models suffer from the familiar excess of low-mass galaxies at \( z \sim 0.5-2 \) which, as we have already discussed, is a widespread problem in both semi-analytic models and numerical hydrodynamic simulations (Fontanot et al. 2009; Weinmann et al. 2012; White et al. 2014; Somerville & Davé 2014). One of the important conclusions of this paper is that varying the star formation efficiency according to physically motivated recipes does not appear to be able to cure this problem within the current model framework. A possi-
bly related problem is that over this redshift range, the predicted gas fractions in low mass galaxies in these same models may be too low (White et al. 2014; Somerville & Davé 2014; Popping et al. 2014c, Popping et al. in prep). Fig. 12 shows a related quantity, the stellar fraction (stellar mass divided by halo mass; \( f_{\text{star}} \equiv m_\ast/M_h \)) over the same redshift range. Our model predictions are now compared with constraints from (sub)-halo abundance matching from Behroozi et al. (2013). The conclusions are similar to the ones above, unsurprisingly since the \( f_{\text{star}} \) constraints are derived from observational estimates of stellar mass functions (though not exactly the same ones plotted in our Fig. 11). The median stellar fractions are nearly identical in the three models in low mass halos (\( \log M_h/M_\odot < 11 \)) and are very similar in the GK and BR model over the whole range of halo masses. The median value of \( f_{\text{star}} \) in the KS model is much lower in massive halos than in the other two models, and the difference increases with redshift to about 0.4–0.5 dex at \( z = 6 \).

Although the median values of \( f_{\text{star}} \) are similar in the three models, the distributions differ significantly for low mass host halos. Fig. 13 and 14 show the distribution of \( f_{\text{star}} \) in halo mass and redshift bins, for central

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1 An important caveat, however, is that the gas mass estimates that lead to this conclusion are based on indirect methods. It remains to be seen whether this is confirmed by direct observations of cold gas in these low-mass galaxies.
and satellite galaxies respectively. For all models and at all epochs, the distribution of $f_{\text{star}}$ becomes broader and more skewed with decreasing halo mass. For massive halos, the width of the distribution becomes slightly narrower with increasing time, while for low mass halos, a more noticeable tail towards lower values of $f_{\text{star}}$ develops with time. In the two intermediate halo mass bins, this tail is more prominent in the GK models than in the other two models. The predicted broadening in $f_{\text{star}}$ has potentially important implications for empirical halo-based models, which generally assume a narrow and fixed scatter in $f_{\text{star}}(M_h)$. We show the results for these two types of galaxies separately because a) central and satellite galaxies are treated differently in SAMs. For example, in our models, satellite galaxies are not allowed to accrete new gas from the IGM; b) our predictions for differences between stellar fractions for satellites and centrals can provide useful input to empirical models such as Halo Occupation Distribution (HOD) and abundance matching models. Many such models do not distinguish between satellite and central galaxies.

### 3.3.2 Star Formation rates and gas depletion times

Fig. 15 shows the total gas depletion time $t_{\text{dep}} \equiv (m_{\text{HI}} + m_{\text{H}_2})/\dot{m}_i$ in the fiducial GK and BR models, and in the GK+Big1 model. Here we compute the depletion time using only the SFR due to the ‘disc’ mode of SF, i.e. not including star formation due to merger-triggered bursts, but the plot looks very similar when the burst mode is included. Observations of nearby galaxies show that $t_{\text{dep}}$ increases with decreasing stellar mass, i.e. the conversion of cold gas into stars is less efficient in low

Figure 10. Relation between total cold neutral gas density ($\text{H}_1 + \text{H}_2$) and star formation rate density (SFRD). The solid black line shows the input $\text{H}_2$-based star formation recipe (Big2). Slanted dotted lines show a star formation efficiency of 1%, 10%, and 100% per $10^8$ yr. Gray contours show observational estimates from the 13 Spiral galaxies from the THINGS+Heracles sample presented in Leroy et al. (2008). Colored dots show a selection of 25 galaxies in our fiducial GK (left) and BR (right) models (at $z = 0$), with a stellar mass range chosen to match the THINGS sample. Each point shows the value in an annulus with radius 500 pc. The points are color-coded with the average gas phase metallicity of the galaxy. Note that more noticeable tail towards lower values of $Z_{\text{gas}}$ with time. In the two intermediate halo mass bins, the predicted broadening in $f_{\text{star}}$ is common to most cosmological models of galaxy formation (Somerville & Davé 2014). Our results show that this relation is extremely robust to changing the metallicity gas have lower SFRD for a given total gas density, because a smaller fraction of the gas is predicted to be in the form of $\text{H}_2$. In the GK model, a direct dependence of $Z_{\text{gas}}$ on metallicity is assumed. In the BR model, a dependence on disk midplane pressure is assumed, but this quantity turns out to be highly correlated with metallicity in our models.

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mass galaxies (e.g. Leroy et al. 2008). A similar trend is indicated by the empirical estimates of total gas depletion time from Popping et al. (2014a), also shown in Fig. 17 for comparison. The empirical estimates are based on a SFR-halo mass relation inferred from abundance matching, and an indirect estimate of the H\textsubscript{i} and H\textsubscript{2} masses from inverting the SFR density. These estimates rely on a number of assumptions (e.g., that disk cold gas radial profiles are well-represented by exponentials), and on the observed relationship between size and stellar mass in disk-dominated galaxies. In Popping et al. (2014a), the empirical predictions are shown only up to \(z \sim 3\), because it is not known whether these assumptions and empirical relations hold at higher redshift. Here we show the results of extrapolating the same method to \(z \sim 6\), but these should be considered highly uncertain.

Our three fiducial models reproduce the same qualitative trends indicated by the observations and by the empirical predictions. First, depletion times are longer in lower-mass galaxies. In detail, the physics that is responsible for this trend is different in the three fiducial models. Low mass galaxies have lower gas and stellar surface density on average. In the KS model, gas below the critical surface density is not allowed to make stars, and low mass galaxies tend to have a larger fraction of their gas below this threshold. Low mass galaxies also tend to have lower gas-phase metallicities, and in the GK model, this results in less efficient formation of H\textsubscript{2} and thus of stars. In the BR model, the lower SFE in low-mass galaxies is due to their lower stellar surface density, which leads to a lower disc mid-plane pressure, and again a lower H\textsubscript{2} fraction. Second, in all models, \(t_{\text{dep}}\) at a given stellar mass was lower in the past and increases with cosmic time. The GK and BR models show more pronounced evolution and shorter depletion times (higher SFE) at high redshift, particularly in massive galaxies. This is due to the steeper dependence of the SFR density on gas density adopted in our GK and BR models (slope \(N_{\text{SF}} = 2\) instead of 1.4). High-redshift galaxies contain higher surface density gas overall, and so the results are more sensitive to the slope of the SF relation at high gas densities. This result explains the less efficient formation of stars in massive galaxies at high redshift in the KS model relative to the GK and BR models, seen in Fig. 11 and 12. Note that already by \(z \sim 3\), and increasingly so at higher redshift, the predictions using the Big1 recipe for SF are inconsistent with the empirical constraints. This suggests that the assumption of a constant H\textsubscript{2} depletion time in galactic disks (which is inherent in the Big1 recipe) may not be universally applicable.
Figure 12. The stellar mass of central galaxies divided by the total mass of their dark matter halo, in redshift bins from \(z = 0\)–\(6\). Dark gray lines and shaded areas show constraints from halo abundance matching from Behroozi et al. (2013). The purple solid line shows the results of the GK model, the orange dashed line shows the BR model, and the green dotted line shows the KS model.

3.3.3 Mass-metallicity relations

Fig. 19 and Fig. 20 show the mass-metallicity relation (MZR) for stellar and cold gas phase metallicities, respectively. Recall that the chemical yield parameter in our models has been adjusted to approximately reproduce the normalization of the stellar MZR measured by Gallazzi et al. (2005). Our models naturally predict a slope for the stellar MZR that is in fairly good agreement with observations (Woo et al. 2008; Kirby et al. 2013) down to very low stellar masses \((m_{\text{star}} \sim 10^7 M_\odot)\). The three fiducial models make very similar predictions for the stellar phase MZR, except that massive galaxies become enriched much earlier in our two new models (GK and BR) than in the KS model. This is owing to the steeper slope of our Big2 star formation recipe at high gas densities (see discussion in §3.1, especially Fig. 6, and above). It is also interesting to note that our models predict a much smaller dispersion in the stellar MZR than the observational dispersion estimated by Gallazzi et al. (2005).

In Fig. 20, the model results shown are for central, star forming galaxies selected using the same criteria described above. This is because the observational estimates of gas-phase metallicity are based on emission line diagnostics from H\(\text{II}\) regions, which are only detectable in star forming galaxies. A compilation of observational estimates for the gas phase MZR is shown. We have converted the observed values of \(12 + \log(O/H)\) to \(Z/Z_\odot\) assuming \(12 + \log(O/H) = 8.76\) for the Sun (Caffau et al. 2011), and \(Z_\odot/Z_\odot = (O/H)/(O/H)_\odot\). Note that the observed MZR is uncertain by a factor of 2–3 as different calibration methods produce different zero-points and, to some extent, different slopes (Kewley & Ellison 2008). The predictions of our three models are again quite similar, except that the KS model again produces later enrichment of massive galaxies, so the MZR is shallower at \(z \sim 3–6\). All three models produce a cold gas phase MZR that is, taken at face value, considerably steeper than the observed gas phase MZR. Moreover, the predicted evolution of gas phase metallicity in our models is quite different from that implied by current observations. The models predict that the gas phase metallicity for galaxies of fixed stellar mass declines slightly with decreasing redshift, while observations indicate an increase of almost a factor of two between \(z \sim 2.2\) and \(z \sim 0\).
discrepancy was shown previously by White et al. (2014) for our KS models; we see here that the results are qualitatively similar for our new fiducial GK and BR models. We discuss possible reasons that our models reproduce the stellar MZR fairly well but seem to fail to reproduce the observed gas phase MZR in §4.2.

3.3.4 Evolution of Global Quantities

In Fig. [21] we show our model predictions for the evolution of the global SFR density, global stellar mass density, and average metallicity of cold gas and stars over cosmic time. The figure shows that the three models predict almost identical global SFR densities at low redshift, while at \( z \gtrsim 2 \), the BR model produces the highest SFR density, and the KS model the lowest, with the GK model in between. We also see that our model predictions are in reasonably good agreement with observations at \( z \lesssim 2 \) and \( z \gtrsim 6 \), though this is in part a fortuitous cancellation — the models overproduce galaxies with low SFR but underproduce ones with high SFR. Moreover, the observational estimates of the SFR density from Madau & Dickinson (2014) are integrated only down to 0.03 \( L_\star \), while our theoretical predictions are integrated over all galaxies. This same behavior is echoed in the build-up of the global stellar mass density. The largest difference in the three models appears in the evolution of the stellar and cold gas phase metallicity. The models differ in the normalization and evolution of the mean metallicity for gas and stars. In addition, in the KS model, the mean metallicity of cold gas and stars is very similar, while there is a much larger difference between the stellar and gas phase metallicities of gas and stars in the BR model and GK models. As one can see in Fig. [19] and [20] at stellar masses above \( m_{\text{star}} \sim 10^7 M_\odot \), the models make similar predictions for

\[
\log (f_{\text{star}}) = \log (f_{\text{star}}|M_h) \quad \text{for central galaxies}
\]

[275x764]Star Formation in Models with Multiphase Gas

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Figure 13. Distribution functions for the stellar fraction \( f_{\text{star}} = m_{\text{star}}/M_h \) of central galaxies in bins of halo mass \( \log M_h = 9.25 - 9.75, 9.75 - 10.25, 10.25 - 10.75, 10.75 - 11.25 \) and redshift as indicated on the panels. The purple solid line shows the results of the GK model, the orange dashed line shows the BR model, and the green dotted line shows the KS model.
the relative metallicities of gas and stars. The differences seen in Fig. 21 are entirely due to very low mass halos.

4 DISCUSSION

4.1 Interpreting our results: the equilibrium model

One of the main conclusions of our work is that modifying the recipes for how cold gas is converted into stars has very little effect on the properties of low-mass galaxies \( m_* \lesssim M_{\text{char}} \), where \( M_{\text{char}} \) is the “knee” in the stellar mass function. Instead, modifying the star formation recipe mainly changes the ratio of cold gas to stars in galaxies. Similar conclusions were reached in the study by White et al. (2014), in which more extreme (though in some cases less physically motivated) modifications to the SF recipe in similar models were made. This is because in our models, star formation in low-mass galaxies is strongly self-regulated: if star formation is made less efficient, less gas is ejected by stellar winds, leading to more efficient star formation, and vice versa.

A number of recent works have pointed out this property of self-regulation, which is a rather generic feature of modern galaxy formation models, arising from the broadly adopted hypothesis of efficient stellar-driven feedback (Schaye et al. 2010; Haas et al. 2013; White et al. 2014; Somerville & Davé 2014). A useful analytic framework for understanding the behavior of the fairly complex intertwined suite of physical processes at play in state-of-the-art galaxy formation simulations is the “equilibrium model”, sometimes called the “bath-tub model” (e.g. Davé et al. 2012; Dekel et al. 2013; Dekel & Mandelker 2014). The basic assumption in this model is that due to self-regulation, on some timescale \( t_{\text{eq}} \), galaxies establish an equilibrium state in which the rate of change of their cold gas reservoir is small, i.e. \( \dot{m}_{\text{cold}} \approx 0 \). Once equilibrium is established, the star for-
Star Formation in Models with Multiphase Gas

**Figure 15.** Mean specific star formation rate \( \text{sSFR} \equiv \dot{m}_* / m_* \), as a function of stellar mass for our three fiducial models (purple solid: GK; orange dashed: BR; green dotted: KS), in redshift bins from \( z = 0 \)–6. The blue contours show the conditional sSFR in the GK model. The horizontal gray line shows the sSFR corresponding to \( 1/(3t_H) \), where \( t_H \) is the Hubble time at that redshift. Only galaxies with sSFR \( > 1/(3t_H) \) are included in the mean. Symbols show a compilation of observations for star forming galaxies as follows: \( z = 0 \) – Salim et al. (2007, open circles); \( z = 1 \) – Whitaker et al. (2014, pentagons, interpolated in redshift from the published results); \( z = 4 \) – Steinhardt et al. (2014, crosses); \( z = 4 \) and \( z = 6 \) – Salmon et al. (2014, circles); all panels – fit to data compilation from Speagle et al. (2014, squares).

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stellar mass and gas phase metallicity (assumed to be universal) and predicted the stellar MZR, finding fairly good agreement with observations. However, Muñoz & Peeples (2014) did a more realistic calculation using a similar approach, but accounting for stochasticity in the SF histories and quenching, and found more significant tension between the gas and stellar phase MZR.

We can see by comparing Fig. 19 and 20 that our models predict that the stellar metallicity in galaxies is about a factor of 1.5–1.7 lower than the gas phase metallicity, with weak trends on stellar mass and redshift. We can also see from the figures in §3.1 that the gas phase metallicity tends to increase rapidly and monotonically with time in our models. The stellar metallicity is effectively a mass-weighted average over the chemical enrichment history of the galaxy, so it makes sense that the stellar metallicities are slightly, but not enormously, lower than the gas phase metallicities at any given time. Taken at face value, the observational results — which imply that $Z_{\text{gas}}/Z_{\text{star}}$ is as high as a factor of $\sim 10$ or more, and is a fairly strong function of stellar mass — may be difficult to reproduce in cosmological models without invoking accretion of highly metal pre-enriched gas.

An alternative explanation is that the normalization, and possibly the slope, of the gas and stellar phase MZRs are not accurately calibrated to the same system. Indeed, some gas phase metallicity indicators do yield a gas MZR normalization and slope that is more consistent with the stellar MZR (Kewley & Ellison 2008; Muñoz & Peeples 2014). Another potential issue is that we have plotted stellar mass weighted metallicities, while the observed stellar metallicities are luminosity weighted. However, other studies have found that the luminosity weighted stellar MZR does not differ significantly in slope from the stellar mass weighted MZR (Trager & Somerville 2009; Peeples & Somerville 2013).

Another issue is that different observational probes are sensitive to different chemical elements. The stellar
Figure 17. Total gas depletion time, defined as the total cold neutral gas mass \((\text{H}_1 + \text{H}_2)\) divided by the star formation rate. The purple solid lines show the predictions of the fiducial GK model, the orange dashed lines show the BR model, and the green dotted lines show the KS model. The lavender dot-dashed lines show the GK+Big1 model. The 16th, 50th, and 84th percentiles are shown for central star forming galaxies in the models. The horizontal black dashed line shows the age of the Universe at that redshift. The open diamonds in the \(z = 0\) panel show observational estimates for galaxies in the THINGS+Heracles sample from Leroy et al. (2008). The horizontal gray line shows the average molecular gas depletion time estimated by Leroy et al. (2013) for nearby galaxies; the shaded gray area indicates the uncertainty in the measurement due to the uncertain conversion factor between CO and \(\text{H}_2\). The gray circles show the estimates obtained via the empirical method of Popping et al. (2014a). These are plotted with open symbols at \(z > 3\) to indicate that the estimates are quite speculative in this redshift regime (see text for more details). All three models reproduce the observed trend of decreasing depletion time with increasing stellar mass, but different underlying physics are responsible for the trends in the different models.

Metallicities derived by Gallazzi et al. (2005) are sensitive to a combination of Fe and Mg, and the stellar MZR derived by Kirby et al. (2013) measures \([\text{Fe/H}]\). Gallazzi et al. (2005) quote their results in terms of \(Z_{\text{star}}/Z_\odot\) and claim that their results are independent of \(\alpha/\text{Fe}\) (A. Gallazzi, priv. comm.). The observational gas phase MZR are primarily sensitive to \(\alpha\) elements and are quoted in terms of oxygen abundance \((12 + \log(\text{O}/\text{H}))\). We have assumed that \(Z_{\text{gas}}/Z_\odot = (\text{O}/\text{H})/(\text{O}/\text{H})_\odot\) and \(Z_{\text{star}}/Z_\odot = (\text{Fe}/\text{H})/(\text{Fe}/\text{H})_\odot\) for the Kirby et al. (2013) observations. This is equivalent to assuming that all the stars in our model galaxies have \((\alpha/\text{Fe}) = (\alpha/\text{Fe})_\odot\). However, \((\alpha/\text{Fe})\) is known to differ significantly from the Solar value in stars in our own Galaxy (e.g. Stoll et al. 2013), in nearby dwarf galaxies (Toft et al. 2004, and references therein), and in giant ellipticals (Thomas et al. 2005; Trager et al. 2000).

In the models presented here, we track the total metallicity assuming a constant yield, and we also adopt the instantaneous recycling approximation. Metals are produced in direct proportion to the formation of stars, so enrichment in our models probably most closely traces \(\alpha\) elements, but we normalized our yield parameter to observations that are also sensitive to Fe (see above). This simple version of single-element chemical enrichment with the instantaneous recycling approximation is the standard approach adopted in semi-analytic models. However, physical processes in the models are actually dependent on different elements in potentially significant ways. For example, the (also widely adopted in SAMs) cooling tables of Sutherland & Dopita (1993) used to model the cooling rate of hot halo gas are parameterized via \([\text{Fe/H}]\) and adopt assumed \([\text{Fe/H}]-dependent abundance ratios. However, \(\text{H}_2\) formation is more closely tied to \(\alpha\) elements such as oxygen and carbon, which are primary coolants in the interstellar medium (e.g. Glover & Clark 2014). One conclusion of the work presented here is that if we wish to include more realistic physics in our models, it is important to track multiple chemical elements and their production via different channels (stars of various masses, Type II SNe and prompt and delayed Type Ia SNe) and on dif-
Figure 18. $H_2$ depletion time, defined as the $H_2$ mass divided by the star formation rate. Models shown (colored lines) are as in Fig. 17. The horizontal black dashed line shows the age of the Universe at that redshift. The horizontal gray line shows the average molecular gas depletion time estimated by Leroy et al. (2013) for nearby galaxies; the shaded gray area indicates the uncertainty in the measurement due to the uncertain conversion factor between CO and $H_2$. Our models predict that at high redshift, molecular gas depletion times were significantly shorter, and there is a much stronger trend between galaxy stellar mass and $H_2$ depletion time.

Different timescales. Several groups have developed SAMs that include more detailed chemical evolution models (Arrigoni et al. 2010; Yates et al. 2013). We have integrated the more sophisticated chemical evolution models presented in Arrigoni et al. (2010) within our new SAMs, including the new metallicity-dependent $H_2$ formation and $H_2$-based star formation recipes as described here, and plan to investigate the predicted metallicities of cold gas and stars and their evolution in these models in a future work (Peeples et al. in prep).

In the meantime, we can perform an empirical correction for the variation of $\alpha$/Fe to see if this is a plausible explanation for the discrepancy. If we assume that the metallicity tracked in our models is actually Fe, and apply the empirical relation presented by Stoll et al. (2013) to our model galaxies to “convert” to [O/H], we find much better agreement between our predicted gas phase MZR and at least some calibrations of the observed MZR (see Fig. 20). Note that, conceptually following Muñoz & Peeples (2014), we are effectively assuming that a relationship between [Fe/H] and [O/H] derived for individual stars in the Milky Way holds for the average stellar and gas metallicities in galaxies with a variety of star formation histories — an assumption that may well not be valid. However, it suggests that properly accounting for non-Solar $\alpha$/Fe and its possible trends with other galaxy properties (such as stellar mass and metallicity) may at least partially relax the tension between the stellar and gas phase MZR seen in our models and others.

4.3 Caveats and limitations of our models

Understanding how the “small scale” processes of star formation and stellar feedback interact with cosmological scale processes such as galactic scale inflows and outflows to shape the observable properties of galaxies is currently one of the major unsolved problems in the study of galaxy formation and evolution. The models presented here neglect a large number of processes that are thought to be important in influencing how efficiently gas can form molecules, and in turn how stars form within molecular gas. For example, we do not consider the possible impact of the local shear field, and non-axisymmetric perturbations such as spiral arms and bars. Nor do we attempt to model the “local” effects of stellar feedback (through stellar winds, supernovae, and $H_2$ regions) on star formation. We instead assume that the efficiency of converting molecular gas into stars is roughly constant, as suggested by observations of nearby spiral galaxies.

$^3$ [Fe/H] = $-0.34 + 1.25$[O/H]
As pointed out by Krumholz et al. (2012) and many others, the efficiency of forming stars within GMCs is surprisingly low, only about 1% per free fall time. Our picture is that local feedback processes are responsible for setting that efficiency, and that when we smooth over several 100 pc regions of the ISM, it averages out to a nearly universal value.

However, there have been some recent studies that suggest that the galaxy-averaged value of this molecular star formation efficiency (often expressed as a depletion time, $t_{\text{dep}, \text{H}_2} \equiv m_{\text{H}_2}/\dot{m}_*\) may vary significantly from galaxy to galaxy, and may have a strong dependence on global galaxy properties. Saintonge et al. (2011) find that in the COLD GASS sample, $t_{\text{dep}, \text{H}_2}$ is weakly correlated with the galaxy stellar mass and stellar surface density, and rather strongly (anti-) correlated with sSFR. However, Saintonge et al. (2011) adopted a constant (Galactic) value for the conversion factor between CO and H$_2$ ($\alpha_{\text{CO}}$). Leroy et al. (2013) found similar correlations in their sample of 30 disk galaxies from the HERACLES survey, but found that most of the correlation disappeared when they applied a theoretically motivated dependence of $\alpha_{\text{CO}}$ on the dust-to-gas ratio. They found smaller residual variations in $t_{\text{dep}, \text{H}_2}$, mainly associated with nuclear gas concentrations. This suggests that there may be different values of $t_{\text{dep}, \text{H}_2}$ in undisturbed disk galaxies and in galaxies experiencing mergers and interactions (see also Daddi et al. 2010; Genzel et al. 2010), possibly due to a super-linear dependence of SFRD on H$_2$ density, as assumed in our fiducial models. We do include an enhancement of SFE in mergers in our models, but the treatment is based on hydrodynamic simulations of binary mergers with a rather outdated treatment of sub-grid physics, so this is clearly an area that should be explored with state-of-the-art high-resolution hydrodynamic simulations.

Another limitation of our approach is that although we compute the H$_2$ fraction and $\Sigma_{\text{SFR}}$ in radial annuli in each disk, and integrate over the disk to obtain the global properties, we do not store the information on the stellar, gas, and metal content of each annulus over different timesteps. Therefore we assume that both the gaseous and stellar disks have radial exponential profiles, and adopt a simple fixed factor relating the size of the gaseous and stellar disk. While this may be a reasonable approximation on average, it may miss important trends. We also use the global values of the gas phase metallicity and SFR (which we use as a proxy for the UV radiation field) in the GK recipe, instead of the local values of these quantities in annuli. In future work, we plan to construct more detailed models of disks in which all of these quantities are tracked as a function of radius, along the lines of work by Fu et al. (2010) and Dutton et al. (2010).

### 4.4 Comparison with previous work

Several other groups have carried out studies similar to this one. Lagos et al. (2011b) considered two models without gas partitioning. The first unpartitioned model
Figure 20. Stellar mass vs. cold gas phase metallicity. Black symbols show observational estimates of the gas-phase metallicity: $z = 0.1$ – Peeples et al. (2014, pluses); Andrews & Martini (2013, filled circles). In all panels, the filled squares show the compilation of observational estimates from Zahid et al. (2013). For comparison, we also show the observational estimates of stellar metallicity, as in Fig. 19, with gray star symbols. Colored lines show model predictions, as in Fig. 19. The dot-dashed purple line shows the GK model with an approximate correction for varying $\alpha$/Fe (see text). Taken at face value, our predicted gas-phase MZR appears to be much steeper than the observational estimates. However, properly accounting for varying abundance ratios of $\alpha$ versus Fe elements may at least partially remove this tension (see text).

Figure 21. Global history of star formation, stellar mass assembly, and metallicity as a function of redshift. The solid purple line shows the results of our fiducial GK model, the dashed orange line shows the BR model, and the dotted green line shows the KS model. Left: global star formation rate density; gray lines show the fit to the compilation of observational estimates from Madau & Dickinson (2014). Middle panel: global stellar mass density; symbols show selected observational estimates taken from Table 2 of Madau & Dickinson (2014). Right: mean mass-weighted metallicity; thick lines show the metallicity of the cold gas component and thin lines show the stellar component. The diamond symbol at $z \sim 0$ is the mean stellar metallicity derived by Gallazzi et al. (2005).

adopts the original SF relations implemented in the Baugh et al. (2005) and Bower et al. (2006) GALFORM models, in which the SFR was assumed to be proportional to the total cold gas mass divided by a timescale $\tau_s$. In the Baugh et al. (2005) models, $\tau_s$ was scaled with the galaxy circular velocity to a power, and in the Bower et al. (2006) models, also with the galaxy dynamical time. The second unpartitioned model used a Kennicutt-Schmidt SF recipe similar to the one we have adopted here. They also considered two models with gas partitioning: one
with the pressure-based BR recipe, and one with the metallicity-based KMT recipe. In their BR model, they adopted SF relations similar to our Big1 and Big2 recipes. In the KMT model, they used the SF relation given by KMT. They did not attempt to separate the effects of the different gas partitioning recipes versus H$_2$-based SF recipes. We focus on the results of the Bower et al. (2006) variant of the GALFORM models, which are more similar to our models than the Baugh et al. (2005) variant. Lagos et al. (2011b) do not show stellar mass function predictions, but find that the rest frame K-band luminosity function at $z = 0, 1,$ and 2 shows little change between the six different star formation recipes they explored, consistent with our results. Lagos et al. (2011b) emphasize differences in the SFR distributions as a function of stellar mass, in particular the prominence of a passive population in models with different star formation recipes. We find very small differences in the SFR distributions between our different models, and note that the prominence and location of a passive population will certainly also be very sensitive to the treatment of AGN feedback. It is also interesting to note that Lagos et al. (2011b) end up favoring their BR models and strongly disfavor the KMT recipe because they find that it does not reproduce the observed H$\alpha$ and H$_2$ gas scaling relations. However, we showed in PST14 that both our BR and GK models do about equally well at reproducing available observations of cold neutral gas in galaxies. Lagos et al. (2011b) do not show metallicity predictions.

Fu et al. (2012) conducted a similar study, investigating four SF recipes and separately studying two gas partitioning recipes, BR and KMT, within the framework of the models developed by Fu et al. (2010) based on the MPA Millennium SAMs (Croton et al. 2006;Guo et al. 2011). Their “Bigiel” SF recipe is similar to our Big1 recipe, but depends on the H$_2$ fraction, steepening below a critical value. Their “Kennicutt” recipe is similar to our KS recipe. Their “Genzel” recipe contains a linear scaling of $\Sigma_{\text{SFR}}$ with $\Sigma_{\text{H}_2}$, but also scales as the inverse galaxy dynamical time (which is redshift dependent). They also consider the KMT SF recipe. Again, their results are quite consistent with ours. The stellar and H$_2$ mass functions are almost identical for all models, while the largest difference between models is in the H$\alpha$ content. This is the same conclusion that we reach based on PST14 and this study. All of their models underestimate massive galaxies at high redshift ($z \gtrsim 2$), as we found with similar SF prescriptions to the ones they adopted. Fu et al. (2012) do not show their $\dot{m}_{\text{out}}$-SFR relation, but they show that the different star formation recipes can lead to substantial deviation between model predictions for the cosmic SFR density at high redshift ($z \gtrsim 3-4$), as we also find. Interestingly, Fu et al. (2012) find that their gas phase MZR is too shallow compared with observations — the opposite problem to the one we encounter in our models. This is probably due to their adopted scaling for stellar driven winds. They assume that the mass outflow rate $\dot{m}_{\text{out}} \propto \dot{m}_*$, i.e., a fixed mass loading factor, while we assume that the mass loading factor $\dot{m}_{\text{out}}/\dot{m}_*$ scales approximately with inverse circular velocity squared. This dependence of MZR slope on wind scaling parameters is well known (Peeples & Shankar 2011). They also find that the redshift evolution of the MZR is quite sensitive to the SF recipe adopted; in particular, recipes in which SFE scales with galaxy dynamical time predict very weak or no evolution in the MZR, while their models without dynamical time scalings predict stronger evolution. None of our models contain an explicit scaling with dynamical time, but the non-linear slope of our Big2 and KS recipes has a similar effect, consistent with the weak evolution in the MZR seen in our models.

Krumholz & Dekel (2012) implemented an updated version of the KMT recipe within simplified semi-analytic models that only follow the mass accretion history of the main branch, and do not track the full merger trees. In their fiducial model they additionally assume a constant mass loading factor for stellar-driven winds (no dependence on galaxy mass or circular velocity) and strongly metal-enhanced winds. When we implement similar ingredients in our models (except that we retain our “energy driven” stellar wind scalings), we obtain qualitatively similar results. Namely, a metallicity-dependent formation efficiency for H$_2$ and self-consistent H$_2$-based star formation recipe can significantly suppress and delay star formation and metal enrichment in very low-mass halos. We find that this effect is considerably stronger in our KMT+MEW models than in our fiducial GK models. This is because a) in our fiducial GK model, the effect of a varying UV background partially compensates for the metallicity dependence of H$_2$-formation; and b) the metal enhanced winds delay enrichment of the cold gas, further suppressing star formation (see Fig. 8.1). However, we stress that a noticable effect is seen only in halos with virial mass $M_h \gtrsim 10^{9.5} M_\odot$ and at early times ($z \gtrsim 1$). Therefore, although Krumholz & Dekel (2012) do not show their predicted stellar mass functions or stellar fractions, it is likely that their model also still suffers from the overdensation of low-mass galaxies ($m_{\text{star}} \sim 10^{9-10} M_\odot$) at intermediate redshifts (0.5 $z \lesssim 4$) that we have discussed extensively above (see also Fig. 22 and 23). Certainly they see the same qualitative problems with predicted specific star formation rates being too low at intermediate redshifts that we have described here.

Galaxies hosted by halos in the strongly affected mass range have typical stellar masses of $m_{\text{star}} \sim 10^{7-8} M_\odot$, and it is probably not feasible to obtain complete samples of these objects at high redshift with existing facilities. However, this strong suppression of star formation in low mass halos could have implications for reionization, future observations with the James Webb Space Telescope, and stellar archaeology in local dwarf galaxies. Moreover, Berry et al. (2014) showed that the strong suppression of H$_2$ formation and star formation in low-mass halos predicted by the KMT-like models would lead to a large population of “barren” halos that never experience significant star formation, and so are filled with H$\alpha$ (see also Kuhlen et al. 2012). Berry et al. (2014) found that the existence of this population is in apparent tension with observations of H$\alpha$ absorption systems at high redshift.

We found that the inclusion of metal-enhanced winds (MEW) produces a steeper MZR, leading to an even greater tension with observations. Krumholz & Dekel (2012) also found that their models produce a steeper
gas phase MZR than observations, in spite of their adopted constant mass loading factor which generally leads to shallower predicted MZR. Interestingly, Krumholz & Dekel (2012) find significant evolution in the gas phase MZR from $z \sim 2-0$ in their models, in better apparent agreement with observations.

5 SUMMARY AND CONCLUSIONS

We have presented new models of galaxy formation, set in the framework of cosmological merger trees, which include physically motivated recipes for the partitioning of cold gas into an atomic, molecular, and ionized phase. These models have several advantages: first, we can make explicit predictions for the atomic and molecular gas properties of galaxies over cosmic time, which can be directly confronted with observations from current and upcoming facilities. The first of these predictions, for HI and H$_2$ gas observed in emission, were presented in Popping et al. (2014c), and predictions for HI gas observed in absorption were presented in Berry et al. (2014). Popping et al. (2014b) extended these models to predict sub-mm line emission from several atomic and molecular species. These predictions may be directly compared with observations from current and upcoming facilities such as ALMA (Popping et al. in prep), and also used to plan future observations with these facilities. Second, we can implement more physically motivated H$_2$-based recipes for star formation within our models. In this paper we focussed on the predictions for the stellar mass content, star formation rate, and metallicity of galaxies in our new models, and compared these with available observational estimates from $z \sim 0-6$.

We summarize our main conclusions below:

- We performed a number of tests of the robustness of our models to resolution and various parameter values and ingredients. We find that our code is very well converged with respect to variation of the mass resolution of our merger trees. In addition, our results do not depend sensitively on the assumed values of the metallicity of pre-enriched gas or the molecular hydrogen floor (within a reasonable range of values).
- Accounting for a reservoir of ionized gas (due to an internal and external photo-ionizing radiation field) in our models, which is not allowed to form molecular hydrogen or stars, does not significantly change our predictions.
- We explored the effect of adopting different H$_2$-based star formation recipes. All the recipes we considered gave similar results for low-mass galaxies. Models that adopted a “two-slope” recipe (Big2), which has a linear dependence of star formation rate density on molecular gas surface density below a critical value, steepening to $\Sigma_{\text{SFR}} \propto \Sigma_{\text{H}_2}^2$ above a critical value of $\Sigma_{\text{H}_2}$, produced more efficient star formation and metal enrichment in massive galaxies at high redshift. Models that implement the Big2 recipe appear to be in better agreement with current observational estimates of the number density of massive galaxies at high redshift.
- We explored the effect of different recipes for partitioning gas into an atomic and molecular component. The metallicity-based GK recipe and the pressure-based BR recipe gave surprisingly similar results, perhaps because of the strong correlation between disk mid-plane pressure ($\propto \Sigma_*$, to first order) and metallicity in our models. The KMT and GKFUV recipes, which do not include a dependence on the FUV radiation background as our fiducial GK recipe does, predicted less efficient formation of H$_2$, less star formation and metal enrichment at early times, and later stellar mass assembly. These differences are only noticeable, however, in very low mass halos ($\log(M_h/M_\odot) \lesssim 10.5$).
- Both of our new fiducial models (GK and BR) reproduce the curvature in the relationship between total cold gas surface density and $\Sigma_{\text{SFR}}$ seen in observations of nearby spiral galaxies. Our results illustrate the difficulty of disentangling the physical processes that are responsible for the scatter in this relationship, however, because of the strong correlations in galaxy properties.
- The stellar mass function and mean stellar fractions ($f_{\text{star}} \equiv m_*/M_h$) of galaxies in our three fiducial models (the “classic” KS model, the metallicity-based GK model and the pressure-based BR model) are almost identical for low-mass galaxies at all redshift $z \sim 0-6$. Both of the new models (GK and BR) predict earlier formation of massive galaxies, in better agreement with observational estimates than models that adopt the KS recipe.
- Although the median values of $f_{\text{star}}$ are very similar in low-mass halos in all three models, the models can have significantly differently shaped distribution functions $P(f_{\text{star}}|M_h)$. In particular, the GK model tends to have a much more pronounced tail to low values of $f_{\text{star}}$ in low-mass halos ($\log(M_h/M_\odot) \lesssim 10.5$). The predicted broadening in $f_{\text{star}}$ has potentially important implications for halo occupation models, which generally assume a narrow and fixed scatter in $f_{\text{star}}(M_h)$.
- All three fiducial models produce nearly identical predictions for the relationship between stellar mass and SFR at all redshifts. Even the distributions of sSFR at a given stellar mass are very similar. The KS model predicts a slightly narrower distribution of sSFR in high-mass galaxies at $z \gtrsim 1$ than the other two models.
- All three models predict a weak dependence of the gas depletion time ($t_{\text{dep}} \equiv (m_{\text{HI}} + m_{\text{H}_2})/m_*$) on stellar mass, in agreement with observations of nearby normal disk galaxies and empirical estimates. The predicted $t_{\text{dep}}$ decreases by about 1.3 dex from $z \sim 0$ to $z \sim 6$. The KS model predicts milder evolution in the depletion time for massive galaxies to high redshift, resulting in longer depletion times in massive high redshift galaxies compared to the other two models.
- All three models predict quite good agreement with the observed $z = 0$ stellar mass versus metallicity relation (MZR) for stellar metallicities, but predict a gas phase MZR that is much steeper than observational estimates taken at face value. However, this tension may perhaps be relieved by properly accounting for the possible dependence of $[\alpha/\text{Fe}]$ on galaxy properties. The KS model predicts later metal enrichment of massive galaxies, leading to a shallower MZR at high redshift. In tension with observational results, all of our models predict a nearly
constant or slightly declining metallicity for galaxies selected at fixed stellar mass from $z \sim 4$–0.

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APPENDIX: RESULTS FOR ADDITIONAL MODEL VARIANTS

In this Appendix we show results for the stellar mass functions and stellar fractions in several additional model variants, in order to aid the interpretation of the results presented in the main text. In Fig. 22, we show the stellar mass function at $z = 0, 1, 2$, and 6 for our fiducial GK model (the same one shown in Fig. 11), compared with the GK model with a fixed value of the UV radiation background $U_{MW} = 1$ (GKFUV), the GK model for gas partitioning with the Big1 SF relation (GK+Big1), and a model with the KMT recipes for gas partitioning and a KMT SF relation (see Table 2). Fig. 23 shows the median stellar fraction as a function of halo mass for central galaxies, in the same suite of models, with the addition of the GK model with photo-ionization “squelching” switched off shown in the $z = 0$ panel only.

These plots illustrate several points, which we already discussed in §4.5. First, the metallicity and UV radiation field dependence in the GK recipe partially counteract each other (lower mass galaxies have lower metallicity, resulting in less efficient H$_2$ formation, but also a lower SFR, resulting in less efficient H$_2$ destruction). Recipes that do not account for the effect of a varying UV radiation field (GKFUV and KMT) predict that H$_2$ formation becomes so inefficient in low mass galaxies that the stellar mass function actually turns over at $\log (\text{mass}/\text{M}_\odot) \approx 8$. Similarly, $f_{\text{star}}(\text{mass})$ declines sharply at $\log (\text{mass}/\text{M}_\odot) \approx 10$. Note that although in the models shown, the abundance of very low-mass galaxies is actually probably too small compared with observations, we could probably adjust our recipes for stellar feedback or photo-ionization squelching to fix this. However, it does appear that even in these models, the excess of low-mass galaxies ($m_{\text{stellar}} \sim 10^{8-9} \text{M}_\odot$) at intermediate redshift ($0.5 \lesssim z \lesssim 4$) persists. This indicates that the halo mass scale where star formation can become inefficient enough to break the self-regulation equilibrium is smaller than the one where the discrepancy with current observations appears.

The second point is that the differences between models seen in more massive halos are almost solely due to the assumed scaling of the star formation relation at large gas surface densities. The model with the steepness dependence of $\Sigma_{\text{SFR}}$ on $\Sigma_{\text{H}_2}$ (Big2, with $N_{\text{SF}} \rightarrow 2$ in dense gas) has the highest number density of massive...
Figure 22. Stellar mass function evolution with redshift. Symbols show observational estimates as detailed in Fig. 11. The purple solid line shows the results of the fiducial GK model, the cyan dotted line shows the GKFUV model, the dashed lavender line shows the GK Big1 model and the long-dashed blue line shows the KMT model (see text and Table 2).

galaxies and the highest values of $f_{\text{star}}$ in massive halos. The GKFUV model is almost identical to the fiducial GK model at high masses. The KMT model is the next highest ($N_{\text{SF}} \rightarrow 1.4$ in dense gas), then the GK Big1 model ($N_{\text{SF}} = 1$). This is because regardless of the gas partitioning recipe, gas in these galaxies is dense enough that it is nearly all molecular. Stellar driven winds cannot efficiently escape these deep potential wells. Therefore there is a strong dependence on the gas depletion time (star formation efficiency).

It is also clear from Fig. 23 that modeling of photoionization squelching will have an extremely degenerate effect with that of gas partitioning and stellar feedback on stellar properties. However, observations of gas content should help break these degeneracies.
Figure 23. The stellar mass of central galaxies divided by the total mass of their dark matter halo, in redshift bins from $z = 0$–6. Dark gray solid lines and shaded areas show constraints from abundance matching from Behroozi et al. (2013). Models shown are as in Fig. 22. In addition, the dot-dashed purple line (shown in the $z = 0$ panel only) shows the fiducial GK model with photo-ionization squelching switched off.