The Non-linear Matter Power Spectrum in Warm Dark Matter Cosmologies

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ABSTRACT

We investigate the non-linear evolution of the matter power spectrum by using a large set of high-resolution N-body/hydrodynamic simulations. The linear matter power in the initial conditions is consistently modified to mimic the presence of warm dark matter particles which induce a small scale cut-off in the power as compared to standard cold dark matter scenarios. The impact of such thermal relics is examined at small scales $k > 1 \, h \, Mpc^{-1}$, at redshifts of $z < 5$, which are particularly important for the next generation of Lyman-$\alpha$ forest, weak lensing and galaxy clustering surveys. We measure the mass and redshift dependence of the warm dark matter non-linear matter power and provide a fitting formula which is accurate at the $\sim 2\%$ level below $z = 3$ and for particle masses of $m_{WDM} \geq 0.5$ keV. The role of baryonic physics on the warm dark matter induced suppression is also quantified. In particular, we examine the effects of cooling, star formation and feedback from strong galactic winds. Finally, we find that a modified version of the halo model describes the shape of the warm dark matter suppressed power spectra better than HALOFIT. In the case of weak lensing however, the latter works better than the former, since it is more accurate on the relevant, mid-range scales, albeit very inaccurate on the smallest scales ($k > 10 \, h \, Mpc^{-1}$) of the matter power spectrum.

Key words: Cosmology: theory – large-scale structure of the Universe – dark matter, methods: numerical – gravitational lensing: weak

1 INTRODUCTION

The increasing amount of observational data available and the numerical tools developed for their interpretation have given rise the so-called era of precision cosmology. At the present time, the cosmological concordance model based on a mixture of cold dark matter and a cosmological constant must thereby be tested in new regimes (both in space and in time) and using as many observations and techniques as possible in order to either confirm or disprove it.

Among the many different observables the non-linear matter power spectrum is a crucial ingredient since it allows us to describe the clustering properties of matter at small scales and low-redshift, where linear theory is not reliable. However, accurate modelling of non-linear physical processes is needed, in order to use this observable to gain quantitative results on the nature of dark matter.

Warm Dark Matter (WDM) is an intriguing possibility for a dark matter candidate. Its velocity dispersions are intermediate between those of cold dark matter and hot dark matter (e.g. light neutrinos). In this scenario, at scales smaller than WDM free-streaming scales, cosmological perturbations are erased and gravitational clustering is significantly suppressed. If such particles are initially in thermal equilibrium, they have a smaller temperature and affect smaller scales than those affected by neutrinos. In addition, WDM produces a distinctive suppression feature at such scales as compared to that induced by neutrinos. For example, thermal relics of masses at around 1 keV which constitute all of the dark matter have a free-streaming scale that is comparable to that of galaxies, well into the non-linear regime. Among the different WDM candidates a special role is played by the sterile neutrino with mass at the keV scale (Boyarsky et al. (2009a)). WDM was originally proposed to solve some putative problems that are present in cold dark matter scenarios at small scales (see Colin et al. (2000); Bode et al. (2001)), however it is at present controversial whether these tensions with cold dark matter predictions can be solved by modifying the nature of dark...
matter particles or by some other baryonic process (e.g. Trujillo-Gomez et al. (2010)).

In the present paper we wish to quantify the impact of a WDM relic on the non-linear power spectrum by using a set of N-body/hydrodynamic simulations of cosmological volumes at high resolution. Investigating WDM scenarios in a cosmological setting has been done by means of N-body codes in order to carefully quantify the impact of such a candidate in terms of halo mass function, structure formation, halo density properties (Bode et al. 2001; Colín et al. 2008; Colombi et al. 2009) and particular care needs to be placed on correctly addressing numerical/convergence issues (Wang & White 2007). In general, while the WDM induced suppression transfer function can be reliably estimated in the linear regime (e.g. Viel et al. (2005); Bovarsky et al. 2008; Lesgourgues & Tram (2011)), the non-linear suppression has not been investigated. However, a recent attempt to obtain the non-linear matter power at small scales by modifying the halo model is described in Smith & Markovid (2011).

The analysis of matter power spectra at small scales has been performed in recent year by different groups by focussing mostly on baryon physics such as feedback and cooling (e.g. Rudd et al. 2008; Guillet et al. 2010; Casarini et al. 2011; van Daalen et al. 2011). Recently, van Daalen et al. (2011) presented an extensive investigation of the effects from several different implementations of galactic feedback on the total matter power. Including feedback from Active Galactic Nuclei (AGN) was claimed to be most realistic since it matches optical and X-ray observations of groups of galaxies and solves the overcooling problem (see also related works by Puchwein et al. 2008; Fabjan et al. (2010); McCarthy et al. (2010); Teyssier et al. 2011). This scenario has a relatively large impact in terms of total matter power on the scales affected by the presence of a WDM candidate. In a subsequent paper, Semboloni et al. (2011) carefully analysed the impact of this feedback on weak lensing observables and concluded that it will not be possible to constrain WDM masses with future weak lensing surveys (as claimed in for example Markovic et al. (2011)). These findings are very interesting and show that a future measurement of the matter power at such small scales should be considered with an accurate model of baryonic physics and not only of the dark matter component. In the present work, although we will be exploring some feedback mechanisms that do not include the AGN feedback, the main focus is on the signature of the WDM suppression in terms of total matter power, presented as a ratio with a corresponding overall suppression in the halo model.

Different constraints can be obtained by using several astrophysical probes. For example by using Lyman-α observables such as the transmitted Lyman-α flux power, very competitive measurements in the form of lower limits (\(m_{\text{WDM}} > 4 \text{ keV} \), \(2\sigma \) C.L.) have been derived by using the SDSS flux power and other higher redshift and higher resolution data (Viel et al. 2005; 2006; Seljak et al. 2006): these constraints become much weaker if the WDM is assumed to account to only a fraction of the dark matter (Bovarsky et al. 2009) or if the initial linear suppression for a sterile neutrino is considered (Bovarsky et al. 2009b) (in this latter case basically any \(m_{\text{WDM}} > 1 \text{ keV} \) is allowed). Alternatively, constraints on WDM models can be placed using: the evolution and size of small scale structure in the local volume high resolution simulations (Tikhonov et al. 2009); simulated Milky Way haloes to probe properties of satellite galaxies (Polisensky & Ricotti 2011; Lovell et al. 2011); large scale structure data (Abazajian 2006); the formation of the first stars and galaxies in high resolution simulations (Gao & Theuns 2007); weak lensing power spectra and cross-spectra (Markovic et al. 2011; Semboloni et al. 2011); the dynamics of the satellites (Knebe et al. 2008); the abundance of sub-structures (Colín et al. 2008); the inner properties of dwarf galaxies (Strigari et al. 2006); mass function in the local group as determined from radio observations in HI (Zavala et al. 2009); the clustering properties of galaxies at small scales (Coil et al. 2008); the properties of satellites as inferred from semi-analytical models of galaxy formation (Macciò & Fontanot 2010); phase-space density constraints from dwarf galaxies (de Vega & Sanchez 2010).

We believe that most of the astrophysical probes used so far in order to constrain the small scale properties of dark matter could benefit from a comprehensive numerical modelling of the non-linear matter power. The present work aims at providing such a quantity by using N-body/hydrodynamic simulations. The findings could also be useful for future surveys such as PanSTARRS, DES, LSST, ADEPT, EUCLID, JDEM or eROSITA, WFXT and SPT.

The layout of the paper is as follows. In Section 2 we present our set of simulations and the code we use in order to investigate the non-linear suppression on the total matter power. Section 3 contains the main results of the present work and the description of the checks made in order to present a reliable estimate of the WDM non-linear suppression: we focus on numerical convergence, box-size, baryonic physics, particle velocities and the effect induced by cosmological parameters on the WDM power. As an application of the findings in Section 3 we present the weak lensing power and cross-spectra for a realistic future weak lensing survey in Section 4 and compare these results with those that could be obtained by using either linear-theory or halo models in Section 5. We conclude with a summary in Section 6.

2 THE SIMULATIONS

Our set of simulations has been run with the parallel hydrodynamic (TreeSPH: Tree-Smoothed Particle Hydrodynamics) code GADGET-2 based on the conservative ‘entropy-formation’ of SPH (Springel 2005). Most of the runs use the TreePM (Tree-Particle Mesh) N-body set-up and consist only of dark matter particles, however for a few runs, in order to test the impact of baryonic physics, we switched hydrodynamic processes on.

The cosmological reference model corresponds to a ‘fiducial’ CDM Universe with the following parameters, at \(z = 0 \): \(\Omega_m = 0.2711 \), \(\Omega_L = 0.7289 \), \(\Omega_b = 0.0451 \), \(n_s = 0.966 \), \(H_0 = 70.3 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \(\sigma_8 = 0.809 \). This model is in agreement with the recent constraints obtained by WMAP-7 year data (Komatsu et al. 2011) and by other large scale structure probes. The initial (linear) power spectrum is generated at \(z = 99 \) with the publicly available software CAMB.
The suppressed scales are highly non-linear and therefore require high-resolution as well as N-body techniques. However, to be conservative we present results for the following simulations (a) have been run with hydrodynamic processes (a simplified star formation recipe and radiative processes for the gas) and with full hydrodynamics with the standard multiphase modelling of the interstellar medium and strong kinetic feedback in the form of galactic winds. Simulations (a) have also been run at lower resolution $N_{\text{DM}} = 384^3$ and for different values of $\sigma_8$, $\Omega_m$ and $H_0$. Simulation (b) has been run by switching the initial velocities of warm dark matter particles off and by increasing the linear size of the PM grid by a factor 3. Simulation (c) has been run with $N_{\text{DM}} = 640^3$ dark matter particles with a softening of 1 kpc/h to $z = 0.5$.

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### Table 1. Summary of the simulations performed.

| linear size (Mpc/$h$) | $m_{\text{WDM}}$(keV) | soft. (kpc/$h$) |
|-----------------------|------------------------|-----------------|
| 12.5                  | 0.62                   |                 |
| 12.5                  | 1                      | 0.62            |
| 25                    | 1.25                   |                 |
| 25                    | 1                      | 1.25            |
| 50                    | 2.5                    |                 |
| 50                    | 1                      | 2.5             |
| 100                   | 5                      |                 |
| 100                   | 1                      | 5               |
| 25                    | 0.25                   | 1.25            |
| 25                    | 0.5                    | 1.25            |
| 25                    | 1                      | 1.25            |
| 25                    | 2                      | 1.25            |
| 6.25                  | 1                      | 0.625           |
| 6.25                  | 1                      | 0.33            |

and then modified to simulate warm dark matter (see below).

We consider different box sizes in order to address both the large scale power and (more importantly) the effect of resolution. The gravitational softening is set to be $1/40$-th of the mean linear inter-particle separation and is kept fixed in comoving units. The dimension of the PM grid, which is used for the long-range force computation, is chosen to be equal to $N_{\text{DM}}^3$ with $N_{\text{DM}} = 512^3$. Simulations (a) have been run with hydrodynamic processes (a simplified star formation recipe and radiative processes for the gas) and with full hydrodynamics with the standard multiphase modelling of the interstellar medium and strong kinetic feedback in the form of galactic winds. Simulations (a) have also been run at lower resolution $N_{\text{DM}} = 384^3$ and for different values of $\sigma_8$, $\Omega_m$ and $H_0$. Simulation (b) has been run by switching the initial velocities of warm dark matter particles off and by increasing the linear size of the PM grid by a factor 3. Simulation (c) has been run with $N_{\text{DM}} = 640^3$ dark matter particles with a softening of 1 kpc/h to $z = 0.5$.

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The initial conditions for warm dark matter particles are generated using the procedure described in [Viel et al. (2005)](http://camb.info/), which we briefly summarize here. The linear ACNM power is multiplied by the following function:

$$T_{\text{lin}}^2(k) \equiv P_{\text{WDM}}(k)/P_{\Lambda CDM}(k) = (1 + (\alpha k)^{\nu})^{-5/\nu},$$

where $\nu = 1.12$ and $\alpha$ has units of $h^{-1}$ Mpc (e.g. [Hansen et al. (2004)](http://camb.info/)). We stress that the above equation is an approximation which is strictly valid only at $k < 5 - 10 h$ Mpc$^{-1}$. Below this scale the warm dark matter power spectrum could be described by a more complicated function and acoustic oscillations are present (see for example the recent work in [Lesgourgues & Tram (2011)](http://camb.info/)).

Initial velocities for warm dark matter particles are drawn from a Fermi-Dirac distribution and added to the proper velocity assigned by linear theory: the r.m.s. velocity dispersion associated to their thermal motion is 27.9, 11.5, 4.4, 1.7, 0.7 km/s for $m_{\text{WDM}}=0.25,0.5,1,2,4$ keV, respectively. The typical r.m.s. velocity dispersion for the dark matter particles of the ACNM runs is $\sim 27$ km/s, so at least for masses above 1 keV the thermal WDM motion is a small fraction of the physical velocity dispersion assigned by the Zel’dovich approximation.

When baryonic physics is included, we consider the following processes: i) radiative cooling and heating, ii) star formation processes, iii) feedback by galactic winds. We note that metal cooling is not included in the simulations and only cooling from H and He is considered, while galactic winds are powered by massive stars and not AGN.

The rationale is to see at which level these processes impact the non-linear matter power at small scales in order to compare to the suppression effects of WDM. Thus, we are not aiming at exploring in a comprehensive way the impact of these processes on the non-linear power at small scales. (e.g. [van Daalen et al. 2011], [Casarini et al. 2011]): the baryonic simulations are used only to quantify the impact of such processes on the suppression induced by WDM w.r.t. cold dark matter scenarios.

Radiative cooling (H and He) as well as heating processes are assumed for a primordial mix of hydrogen and helium corresponding to a mean Ultraviolet Background similar to that produced by quasars and galaxies and implemented in [Katz et al. (1996)](http://camb.info/). This background naturally gives a hydrogen ionization rate $\Gamma_{\alpha} \sim 1$ at high redshift and an evolution of the physical state of the intergalactic medium (IGM) which is in agreement with observations (e.g. [Bolton et al. 2005]). The star formation criterion for the default runs is a very simple one that converts all the gas particles whose temperature falls below $10^5$ K and whose density contrast is larger than 500 into collisionless stars (more details can be found in [Viel et al. 2004]). This prescription is usually called “QLYA” (quick Lyman-\(\alpha\) ) since it is very efficient in quantitatively describing the Lyman-\(\alpha\) forest and the low density IGM. We also run a simulation with the full multi-phase description of the interstel-
Figure 1. “Visual” inspection of the redshift evolution of cosmic structures in the $\Lambda$CDM and WDM ($m_{\text{WDM}}=1$ keV) scenarios (left and right columns, respectively) for the defaults (25,512) runs. From the top to the bottom rows we show a 2.5 $h^{-1}$ Mpc thick slice of the projected dark matter density at $z=0,2,5$ respectively. At $z=0$ the clustering properties of the dark matter at scales $k<10$ $h$ Mpc$^{-1}$ are indistinguishable in the two scenarios, while at $z=2,5$ the WDM model has a suppression in power of about 5% and 25% at $k=10$ $h$ Mpc$^{-1}$. 
lar medium (ISM) and with kinetic feedback in the form of strong galactic winds as in Sprinch & Hernquist (2003). The chosen speed of the wind is 483 km/s and both the ISM modellling and this feedback mechanism is expected to impact on the distribution of baryons and thus on the total matter power spectrum. The wind particles temporarily decouple from the hydrodynamics: the maximum allowed time of the decoupling is t_{dec} = l/v_w with l=20 kpc/h and \( v_w = 483 \) km/s (see for example Dalla Vecchia & Schaye 2008 and Pierleoni et al. 2008) to understand the effects of this parameter in terms of feedback efficiency and on the properties of local HI-galaxies).

We note that simulations that include baryons are significantly slower than the default dark matter only runs and therefore our constraints will mainly be derived from the former simulations.

In the following, the different simulations will be denoted by two numbers, \((N_1, N_2)\): \(N_1\) is the size of the box in comoving Mpc/h and \(N_2\) is the cubic root of the total number of gas particles in the simulation. The mass per dark matter particle is \(8.7 \times 10^8 M_\odot/h\) for the default (25,512) simulations. This mass resolution allows to adequately sample the free-streaming mass for the models considered here.

In Figure 1 we show the projected dark matter density as extracted from the default (25,512) runs in the ΛCDM case (left) and WDM case (right) for \(m_{\text{WDM}}=1\) keV. This WDM particle mass is already ruled out at a significant level by Lyman-\(\alpha\) forest observations (e.g. Seljak 2003; Viel et al. 2003). The different rows refer to \(z = 0, 0.2, 0.5\) from top to bottom, respectively. In this Figure it is essential to see how the clustering proceeds differently in the two scenarios and while there are large differences below the Mpc scale at \(z = 5\) between the two cosmic webs, these differences are largely erased by non-linear evolution at \(z = 0, 2\).

The main features of the simulations are summarized in Table 1.

3 RESULTS

In this section we describe the main results obtained from our sample of simulations. The power is computed from the distributions of the different sets of particles (dark matter, gas and stars) separately and for the total matter component by performing a CIC (Cloud-In-Cell) assignment to a grid of the size of the PM grid. The CIC kernel is also deconvolved when getting the density at the grid points (e.g. Viel et al. 2010). We also show a small scale estimate \((k > 10 \ h Mpc^{-1})\) of the power obtained with the folding method described in Jenkins et al. 1998, Colombi et al. 2002, although this power will not be used quantitatively.

We will plot the suppression in power as a percentage difference between WDM and ΛCDM total matter power spectra, normalized by the default ΛCDM total matter power. The initial conditions for CDM and WDM have the same phases and cosmological/astrophysical parameters in order to highlight the effect of the warm dark matter free streaming.

3.1 Resolution and box-size

In Figure 2 we show the percentage difference between the total non-linear powers of WDM \((m_{\text{WDM}}=1\) keV) and ΛCDM runs. We subtract the shot-noise power from all the power spectrum estimates made. For our largest box-sizes the shot-noise power is comparable to the actual measured power at \(z = 0\) at \(k \sim 150 \ h Mpc^{-1}\), while for the default simulations (25,512) of \(m_{\text{WDM}}=1\) (0.25) keV the matter power is always above the shot-noise level for \(z < 10\) and for \(k < 20(7) \ h Mpc^{-1}\).

This figure focuses on the resolution and box-size effects and presents the percentage difference at four different redshifts \(z = 0, 1, 3, 5\) (bottom right, bottom left, top right and top left panels respectively) and for three different box-sizes \((100, 50, 25 \ h^{-1} \ Mpc\) shown as blue, black and green curves respectively). The dotted line represents the redshift independent linear cut-off of Eq.1 while the lower resolution \((25,384)\) run is also plotted in orange. Here, there are two estimates for the power: one at large scales (continuous curves), the second at smaller scales (dashed curves).

We are primarily interested in the power at scales \(k < 10 \ h Mpc^{-1}\) and therefore only the large scale estimate will be used. However, we also show the power at smaller scales since physical and numerical effects play a larger role in this range. We note that the linear theory suppression is a good approximation only at \(k < 1 \ h Mpc^{-1}\). From the figure one can see that there is convergence up to \(k = 50 \ h Mpc^{-1}\) (between (25,512) and (25,384) runs in the redshift range considered. The resolution used is thus sufficient for \(m_{\text{WDM}}=1\) keV particles. Note that van Daalen et al. (2011) recently found that (100,512) ΛCDM simulations have sufficiently converged at scales \(k < 10 \ h Mpc^{-1}\). At \(k = 3(10) \ h^{-1} Mpc\) and \(z = 5\) there is already a 5 (50)% difference between the linear and non-linear power. At \(z = 0, 1, 3\) the differences between WDM and ΛCDM power is below 1%, 2% and 5% respectively at \(k = 10 \ h Mpc^{-1}\). The maximum suppression dip is strongly influenced by resolution and moves to larger wavenumbers when the resolution increases. At \(k > 100 \ h Mpc^{-1}\) we note a steep (resolution dependent) turn-over in the suppression which is likely to be due to effects that impact on the halo structure and which has also been found in CDM numerical simulations that include a fraction of the matter content in the form of active neutrinos (Brandbyge et al. 2008, Viel et al. 2010).

We have checked that increasing the particle-mesh grid by a factor three (i.e. PM=1536) has negligible impact on the total matter power at scales \(k < 100 \ h Mpc^{-1}\). In order to test the robustness of our results in terms of shot-noise level we have also run a WDM simulation with \(m_{\text{WDM}}=1\) keV and \(N_{\text{DM}} = 640^3\) particles and compared the power spectra with the (25,512) and (25,384) runs: we confirm very good agreement between these simulations at \(k < 0.2 \ h Mpc^{-1}\) in the redshift range considered in the present work. More precisely, the (25,512) and (25,640) WDM runs agree below the one percent level at \(k < 100 \ h Mpc^{-1}\).

3.2 The effect of the mass of a warm dark matter particle

Here we address the effect of varying \(m_{\text{WDM}}\) on the non-linear matter power. The results are shown in Figure 3.
Figure 2. Percentage difference between warm dark matter non-linear power and cold dark matter for the different runs. The mass of the warm dark matter particle is kept fixed to $m_{\text{WDM}} = 1$ keV. Blue, black, green curves refer to $100, 50, 25 \, h^{-1} \, \text{Mpc}$ respectively and with a fixed number of particles $N_{\text{DM}} = 512^3$. The orange curves refer to $25 \, h^{-1} \, \text{Mpc}$ and has a fixed number of particles $N_{\text{DM}} = 384^3$. The continuous lines represent the large scale estimate of the power, while the dashed ones describe the small scale power obtained with the folding method (see text). The four panels represent different redshifts at $z = 0, 1, 2, 5$ (bottom right, bottom left, top right and top left, respectively). The dotted line plotted at $z = 0$ and $z = 5$ is the redshift independent linear suppression between the two models.

where we report five different masses for the $(25,512)$ default runs. The curves correspond to $m_{\text{WDM}} = 0.25, 0.5, 1, 2$ and $4 \, \text{keV}$ (orange, green, black, blue and red curves, respectively) at $z = 0, 1, 2$ and $5$ (bottom right, bottom left, top right and top left, respectively). The linear suppressions are also shown with dotted lines of the corresponding colors. At $z = 5$ we can see large differences between the models that become smaller with the redshift evolution. The 20% suppression at $k = 10 \, h \, \text{Mpc}^{-1}$ at $z = 5$ for the $m_{\text{WDM}} = 1 \, \text{keV}$ model becomes 2% at $z = 1$ and it is below 1% at $z = 0$: the clustering properties of the dark matter are the same at scales above $k \sim 10 \, h \, \text{Mpc}^{-1}$ at least for $m_{\text{WDM}} > 1 \, \text{keV}$. The $m_{\text{WDM}} = 0.5 \, \text{keV}$ model still presents a 7% suppression by $z = 0$, while the suppression is four times larger at $z = 2$. The linear suppression is a very poor approximation in the range of wavenumbers considered here even at high redshift. At $z = 1$, which is particularly interesting for weak lensing data, a 2% measurement of the non-linear power is likely to be able to exclude models below the 1 keV value (bottom left panel). The dip of the maximum suppression and the turn-over both move to larger scales as the mass decreases.

We have also investigated the importance of WDM velocities in the initial conditions by running a simulation without assigning a Fermi-Dirac drawn thermal velocity to the dark matter particles. We tested this for a $m_{\text{WDM}} = 1 \, \text{keV}$ model and found differences always below 1% in terms of total matter power at the scales of interest here.

3.3 Baryonic effects

In this section we explore the effects of baryonic physics on the warm dark matter suppression. Baryons amount to about 17% of the total matter content and we expect that astrophysical processes affecting their properties can impact...
the total matter power at small scales at some level. We identify three processes that are able to modify the clustering properties of baryons: radiative processes, star formation and galactic feedback. These processes are usually modelled by hydrodynamic simulations of galaxy formation. Here, the main goal is not to explore fully the many parameters governing these important physical aspects, but rather to address their impact in WDM models by adopting prescriptions that are widely used in the literature. There could well be other astrophysical processes (radiative transfer effects, feedback from active galactic nuclei, etc.) that can also affect the distribution of baryons and their clustering properties (see for example van Daalen et al. 2011).

In Figure 3 we plot the WDM suppression for the default simulation of $m_{\text{WDM}}=1$ keV for three different cases: pure dark matter (green curves); a hydrodynamic simulation that includes cooling (H and He only) and heating by an ultraviolet background as well as the simple star formation criterion able to simulate the Lyman-$\alpha$ forest (“BARYONS+QLYA” run in blue); a hydrodynamic simulation that includes the full star formation model based on the multi-phase description of the ISM (sophisticated compared to QLYA) and strong galactic feedback in the form of winds. (“BARYONS+SF+WINDS” in black). Unfortunately, due to the fact that hydrodynamic simulations are slower than dark matter only runs it was not possible to carry this last simulation down to $z=0$ and it was stopped at $z=1.2$. We also report the ratio between mass in stars $\Omega_{\text{WDM}}/\Omega_{\Lambda\text{CDM}}(z = 0, 1, 2, 3) = 0.85, 0.78, 0.7, 0.6$ for the “BARYONS+QLYA” runs, while we have $\Omega_{\text{WDM}}/\Omega_{\Lambda\text{CDM}}(z = 1.2, 3) = 0.87, 0.63$ for the “BARYONS+SF+WINDS” runs with galactic winds feedback. For both the WDM and $\Lambda$CDM runs the mass fraction

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**Figure 3.** Percentage difference between warm dark matter non-linear power and cold dark matter for the different runs. The resolution is kept fixed in this plot and only $25 h^{-1}$ Mpc boxes are considered. Orange, green, black, blue and red curves refer to $m_{\text{WDM}} = 0.25, 0.5, 1, 2, 4$ keV, respectively. The continuous lines represent the large scale estimate of the power, while the dashed ones describe the small scale power obtained with the folding method (see text). The four panels represent different redshifts at $z = 0, 1, 2, 5$ (bottom right, bottom left, top right and top left, respectively). The dotted coloured curves plotted at $z = 0$ and $z = 5$ are the redshift independent linear suppression between the different models.
in stars is reduced by a factor five when winds are included compared to the “BARYONS+QLYA” case.

All of these processes can significantly change the clustering of baryons especially at intermediate scales where baryon pressure is important ($k \sim 1 \, h \, \text{Mpc}^{-1}$), where they are not expected to trace the dark matter and at smaller scales due to the complex interplay between feedback and star formation processes. Cooling as well as heating modify the thermal properties of the gas and are important especially for the low density IGM; the star formation criterion determines how much gas is turned into stars within the potential wells of dark matter haloes; galactic winds displace gas out of the galaxies into the low density IGM, usually in a hot phase that prevents subsequent cooling. Since the cosmic structure is generally different in CDM and WDM models we do not expect the WDM suppression to be exactly the same between two simulations that share the same astrophysical prescriptions. From Figure 3 one can see that dark matter only simulations are in good agreement (at the percent level up to $k = 10 \, h \, \text{Mpc}^{-1}$) with simulations that include radiative cooling (metal cooling is not included) and QLYA star formation, while at smaller scales there are significant differences. It is clear that the presence of baryons and star formation greatly affects the maximum suppression and the turn-over. Note that differences much larger than 10% between simulations implementing different radiative pro-

Figure 4. Percentage difference between total WDM non-linear matter power and total ΛCDM non-linear matter power for runs that incorporate baryonic physical processes. The simulations refer to a $25 \, h^{-1} \, \text{Mpc}$ box and $m_{\text{WDM}}=1 \, \text{keV}$. The green curves refer to the pure dark matter simulations; blue curves refer to simulations that include baryons, cooling from H and He and a simplified recipe for star formation that turns into collisionless stars all the gas particle below $T=10^5$ K and denser than 1000 times the mean density (QLYA); black curves are instead obtained by using the default criterion of multi-phase star formation of Springel (2005) and feedback in the form of strong kinetic driven winds (this simulation was stopped at $z = 1.2$). The continuous lines represent the large scale estimate of the power, while the dashed ones describe the small scale power obtained with the folding method (see text). The four panels represent different redshifts at $z = 0, 1.2, 3, 5$ (bottom right, bottom left, top right and top left, respectively). In the $z = 0$ panel (note the different scale for the $y-$axis) we also show as the red and cyan curves the percentage difference between total matter power spectra that include and do not include cooling for ΛCDM (red) and WDM (cyan) models: namely the quantity $100 \times (P_{\text{baryons+QLYA}}^{\text{mat}} - P_{\text{DMONLY}}^{\text{mat}})/P_{\text{DMONLY}}^{\text{mat}}$. 

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cesses (e.g. metal cooling) or feedback recipes are expected at \( k > 20 \, h \, \text{Mpc}^{-1} \) in \( \Lambda \text{CDM} \) models (see e.g. Rudd et al. 2003; Guillet et al. 2010; van Daalen et al. 2011). Furthermore, in the case of AGN feedback at the level required to match observed gas fractions of groups a 10% difference is found already at \( k = 1 \, h \, \text{Mpc}^{-1} \) and a 1% reduction already at \( k = 0.3 \, h \, \text{Mpc}^{-1} \) (van Daalen et al. 2011).

In the \( z = 0 \) panel we also show the difference in the power spectra of \( \Lambda \text{CDM} \) and WDM models by normalizing to the corresponding dark matter only model, in order to highlight the effect of cooling produced by baryons as opposed to the WDM signature. The two percentage differences are shown as cyan (WDM) and red (\( \Lambda \text{CDM} \)) curves: the WDM universe when filled with baryons that can cool has more power than a corresponding \( \Lambda \text{CDM} \) universe filled with the same baryon fraction. The quantity \( P_{\text{nl, WDM, cooling}}/P_{\text{nl, WDM, dmonly}} \) is only about 5% larger than \( P_{\text{nl, \Lambda \text{CDM, cooling}}}/P_{\text{nl, \Lambda \text{CDM, dmonly}} \text{\, at } k = 10 \, h \, \text{Mpc}^{-1}} \) and \( z = 5 \), at \( z = 1 \) it becomes only 2% larger and by \( z = 0 \), there are no differences between the two quantities at \( k = 10 \, h \, \text{Mpc}^{-1} \). The cooling of baryons inside the potential wells of dark matter haloes produces further collapse of structures and in general increases the (total) matter power spectrum. It is thus likely than in the WDM model the baryons cool slightly more efficiently than in the corresponding \( \Lambda \text{CDM} \) since at high redshifts, the collapse of haloes around the WDM cutoff is rapid and small scale modes affected by cooling (H and He) grow more rapidly than in CDM: this is also the trend found by Gao & Theuns (2007) from the analysis of cooling at very high resolution and high redshift in hydrodynamic simulations.

The WDM suppression is thereby highly influenced by astrophysical effects at \( k > 100 \, h \, \text{Mpc}^{-1} \). In general we expect an additional suppression due to baryons of about 2-3% at \( k = 10 \, h \, \text{Mpc}^{-1} \) at \( z > 1.5 \) for \( m_{\text{WDM}} = 1 \, \text{keV} \), while this discrepancy becomes smaller at lower redshifts. The numbers above do not apply once AGN feedback is included and are greatly underestimated, if AGN feedback impacts the matter power at the level found by Gao & Theuns (2007) at \( z = 5 \) are largely canceled by the non-linear growth and are at the \( \pm 2\% \) level at \( z = 1 - 2 \) and at the 3% level at \( k = 10 \, h \, \text{Mpc}^{-1} \). Motivated by the present findings we regard our non-linear cutoff and its redshift dependence as robust at least for the range of cosmological parameters investigated at \( z < 3 \), for \( m_{\text{WDM}} \geq 0.5 \, \text{keV} \) and at \( k = 1 - 10 \, h \, \text{Mpc}^{-1} \); in fact the differences are at the \( \pm 2\% \) level and in the next section we will provide a fitting formula with a comparable level of accuracy. Larger masses for \( m_{\text{WDM}} \) will only result in smaller differences in terms of WDM suppression. We also notice that degenerate features with the non-linear WDM suppression might arise in the context of non-standard models of dark energy, as e.g. interacting dark energy scenarios (see e.g. Baldi 2010). The investigation of such possible degeneracies goes beyond the scope of the present paper.

### 3.4 Other cosmological parameters

To test the robustness of our results we extended the set of simulations by exploring also other cosmological parameters, namely: \( \Omega_m \), \( H_0 \) and \( \sigma_8 \). In order to do that we modify the input linear \( \Lambda \text{CDM} \) parameter calculated by CAMB and vary one parameter at a time. It is clear that some parameters like \( \sigma_8 \) (or \( A_s \)) do not have any impact at the linear level, while they could impact the non-linear power in a way that should be quantified with simulations. We choose the following parameters for the WDM and corresponding \( \Lambda \text{CDM} \) runs: \( \Omega_m = 0.22, 0.32 \), \( H_0 = 62, 78 \, \text{km/s/Mpc} \) and \( \sigma_8 = 0.75, 0.87 \). When calculating the suppression we always normalize both simulations to the same \( \sigma_8 \) value (\( \sigma_8 = 0.75, 0.809 \) and 0.87). Since the WDM suppression of the power spectrum has a relatively distinct shape and the cut-off scale is at much higher \( k \) than what is probed by \( \sigma_8 \) normalisation, we expect WDM to be nearly independent of any other parameter probed. The range explored by the \( H_0 \) values produces a maximum \( \pm 2\% \) difference in terms of the WDM suppression compared to the reference \( H_0 = 70.3 \, \text{km/s/Mpc} \) case at \( k = 1 - 10 \, h \, \text{Mpc}^{-1} \) and at \( z < 3 \), while at \( z = 5 \) there is a 5% difference at \( k = 10 \, h \, \text{Mpc}^{-1} \). The \( \Omega_m \) parameter produces a maximum difference of 1% at \( z < 3 \) in the same range of wavenumbers and about 5% at \( z = 5 \) and \( k = 10 \, h \, \text{Mpc}^{-1} \). A different choice of \( \sigma_8 \) has a slightly larger impact. This is seen in Figure 6, where the WDM induced suppression with such a choice of \( \sigma_8 \) is divided by the reference case of \( \sigma_8 = 0.809 \). It is clear from the figure that the large (10%) differences in place at \( z = 5 \) are largely canceled by the non-linear growth and are at the \( \pm 2\% \) level at \( z = 1 - 2 \) and at the 3% level at \( k = 10 \, h \, \text{Mpc}^{-1} \) today.

### 3.5 An analytical fitting formula

Inspired by the corresponding formula for the linear suppression, we have found the following fitting formula to be a good approximation of the late time evolution of the non-linear suppression with an accuracy at the 2% level at \( z < 3 \) and for masses larger than \( m_{\text{WDM}} = 0.5 \, \text{keV} \):

\[
T_{\text{nl}}^2(k) \equiv P_{\text{WDM}}(k)/P_{\text{\Lambda \text{CDM}}}(k) = (1 + (\alpha k)^{\nu})^{-s/\nu},
\]

\[
\alpha(m_{\text{WDM}}, z) = 0.0476 \left( \frac{1 \, \text{keV}}{m_{\text{WDM}}} \right)^{1.85} \left( \frac{1 + z}{2} \right)^{1.3}, \quad (2)
\]

with \( \nu = 3, l = 0.6 \) and \( s = 0.4 \).

We have chosen as a pivot redshift \( z = 1 \) since this is the redshift where accurate weak lensing data will be available. This formula has been derived from the dark matter only runs.

### 4 WEAK LENSING SHEAR POWER SPECTRA

Following Markovic et al. (2011) and Smith & Markovic (2011), we examine the effect of the fitting function in Equation 4 on the weak lensing power spectrum. Weak gravitational lensing is the distortion found in images of distant galaxies due to the deflection of light from these galaxies by the gravitational potential wells of intervening matter. For a review, see for example Bartelmann & Schneider (2001). The advantage of gravitational lensing is that unlike other large scale structure data, it does not require a knowledge of galaxy bias for the derivation of the properties of the underlying dark matter density field and is, at least on large scales, independent of baryonic physics.
In other words, the weak lensing power spectrum directly probes the matter power spectrum. However, weak lensing measures the matter power spectrum at low redshifts. For this reason it is necessary to have available robust models of non-linear structure. For a survey able to probe angular multipoles from $l \sim 20$ up to $l \sim 2 \times 10^4$, in the redshift range of $z = 0.5 - 2.0$, the corresponding range of wavenumbers must be $k \sim 0.005 - 15 \, h \, \text{Mpc}^{-1}$. Note that the matter power at $k > 10 \, h \, \text{Mpc}^{-1}$ only has a significant contribution to the weak lensing power spectrum at lower redshifts, where however the lensing power is lower.

Future weak lensing surveys accompanied by extensive photometric redshift surveys will be able to disentangle the contribution to weak lensing by dark matter at different redshifts, by binning source galaxies into tomographic bins (Hu 1999). By cross and auto correlating the lensing power in these bins, the three dimensional dark matter distribution can be reconstructed. An existing example of such a reconstruction is the COSMOS field (Massey et al. 2007). Such tomography probes the non-linear matter power spectrum at different redshifts.

We use HALOFIT (Smith et al. 2003) to calculate non-linear corrections to the approximate linear matter power spectrum (Ma 1996). We then apply Equation 2 to approximate the WDM effects and find the weak lensing power spectrum (e.g. Takada & Jain 2004):

$$C_{ij}(l) = \int_0^{\chi_l} d\chi_l W_i(\chi_l) W_j(\chi_l) \chi_l^{-2} P_{nl}(k = \frac{l}{\chi_l} \chi_l),$$

(3)

where $\chi_l(z_l)$ is the comoving distance to the lens at redshift $z_l$ and $W_i$ is the lensing weight in the tomographic bin $i$. The factor $\chi_l^{-2}$ corrects for the distance element.

$$W_i(z_l) = \frac{4 \pi G}{a_i(z_l)^2 c^2 \rho_{m,0}} \int_{z_{l}}^{z_{\text{max}}} n_i(z_s) \frac{\chi_{ls}(z_s, z_l)}{\chi_s(z_s)} dz_s,$$

(4)

where we assume a flat universe and $a_i(z_l)$ is the scale factor at the redshift of the lens, $\rho_{m,0}$ is the matter energy density today and $n_i(z_s)$ is the normalised redshift distribution of
The percentage WDM effect in auto- and cross-correlation power spectra of redshift bins at approximately $z = 1$ and $z = 1.6$, respectively. All the lines are calculated from non-linear matter power spectra modified for WDM by the fitting function in Equation (2011) and Smith & Markovic (2011) with 8 redshift bins in the range $z = 0.5 - 2.0$. We plot the resulting percentage differences between WDM and CDM weak lensing power spectra in Figure 6. It is important to note that the error bars in the figure do not fully characterise the sensitivity of the power spectra, since there are additional correlations between the error bars of different bin combinations. Additionally, there are correlations in the error bars on large $l$ (small scales) due to non-linearities. Further statistical tests using the entire covariance matrix must be used in order to fully account for the above correlations. For this plot we choose only the $5^{th}$ and $8^{th}$ redshift bins, whose source galaxy distributions have the mean at $z \sim 1.0$ and 1.6 respectively. These bins are chosen because they represent a range with the maximal WDM effect as well as lensing signal. Note that the upturn around $l \sim 10^3$ in the auto-correlation power spectra of bins 5 and 8 is due to the dominance of shot noise on those scales. This upturn is not present in the cross power spectrum, because through cross correlation this noise due to intrinsic galaxy ellipticities is eliminated.

In order to assess detectability of WDM by future weak lensing surveys, we calculate predicted error bars on the weak lensing power spectrum using the covariance matrix formalism (Takada & Jain 2004) and assuming errors for a future realistic weak lensing survey as in Markovic et al. (2011) and Smith & Markovic (2011) with 8 redshift bins in the range $z = 0.5 - 2.0$. We plot the resulting percentage differences between WDM and CDM weak lensing power spectra in Figure 6. It is important to note that the error bars in the figure do not fully characterise the sensitivity of the power spectra, since there are additional correlations between the error bars of different bin combinations. Additionally, there are correlations in the error bars on large $l$ (small scales) due to non-linearities. Further statistical tests using the entire covariance matrix must be used in order to fully account for the above correlations. For this plot we choose only the $5^{th}$ and $8^{th}$ redshift bins, whose source galaxy distributions have the mean at $z \sim 1.0$ and 1.6 respectively. These bins are chosen because they represent a range with the maximal WDM effect as well as lensing signal. Note that the upturn around $l \sim 10^3$ in the auto-correlation power spectra of bins 5 and 8 is due to the dominance of shot noise on those scales. This noise is due to intrinsic galaxy ellipticities and can be eliminated by cross-correlating different redshift bins, as can also be seen in Figure 6 (see also Takada & Jain 2004).

In the right panel of Figure 6 we plot the effects of a 1 keV WDM particle: in this case it is more difficult to distinguish from CDM (black dashed line), but the strongly affected cross power spectra are still significantly different from their expected values in $\Lambda CDM$.

In a recent paper, Semboloni et al. (2011) have explored the effect that AGN feedback has in terms of matter power and weak lensing power spectra finding that there is a suppression of about 30% at $k = 10 \ hMpc^{-1}$ when this feedback mechanism is included. Although they did not investigate WDM models, this result is important, since it shows that the effect could be much larger than the corresponding WDM induced suppression and comparable at $z = 0$ to the $m_{WDM}=0.25 \ \text{keV}$ case. It is clear that future weak lensing surveys aiming at measuring the matter power at these scales should carefully consider AGN effects since they could be degenerate with cosmological parameters such as the mass of the WDM particle.

5 COMPARISON WITH HALO MODEL

As described in Section 4 it is necessary to have a robust model of non-linear structure in order to take full advantage of future weak lensing data. For this reason we compare the non-linear matter power spectra extracted from our simulations with previously derived non-linear models. The halo model of non-linear structure is based on the
assumption that large scale structure is made up of individual objects occupying peaks in the matter overdensity field (Press & Schechter 1974; Seljak 2000; Cooray & Sheth 2002). The most important elements of this model, the mass function, the halo bias (Press & Schechter 1974) and the halo density profile (Navarro et al. 1995) are based on the assumptions that all dark matter in the universe is found in haloes and that there is no observable suppression of small scale overdensities from early-times free-streaming of dark matter particles or late-times thermal velocities.

These are characteristic properties of CDM, but do not apply to WDM. For this reason Smith & Markovic (2011) modified the halo model by applying a specific prescription to the non-linear contribution, in addition to suppressing the initial density field, modelled by applying a transfer function from Viel et al. (2005) to the linear matter power spectrum. Such prescription consists of: i) treating the dark matter density field as made up of two components: a smooth, linear component and a non-linear component, both with power at all scales; ii) introducing a cut-off mass scale, below which no haloes are found; iii) suppressing the mass function also above the cut-off scale and iv) suppressing the centers of halo density profiles by convolving them with a Gaussian function, whose width depended on the WDM relic thermal velocity.

Here, we do not attempt to explore each of these elements with simulations individually, but rather compare the final matter power spectra found from simulations with those from the WDM halo model of Smith & Markovic (2011).

Secondly, Smith et al. (2003) compared the standard CDM halo model to CDM simulations of large scale structure formation and developed an analytical fit to the non-linear corrections of the matter power spectrum, known as HALOFIT. We apply these corrections to a linear matter power suppressed by the Viel et al. (2005) WDM transfer function (see Equation 1).

We show the results of these comparisons in Figure 7. As before, we plot the percent differences between the WDM and CDM matter power spectra obtained from our simulations of WDM only. We show this for particle masses of $m_{\text{WDM}}=1 \text{ keV}$ (left panels) and $m_{\text{WDM}}=0.5 \text{ keV}$ (right panels) at redshifts $z = 1$ (top row) and $z = 0.5$ (bottom row). We find that the WDM halo model is closest to simulations at redshift 1 for 1 keV WDM, but that it over-estimates the suppression effect at redshift 0.5 for 0.5 keV.
It is evident that excluding such corrections causes a significant overestimation of the WDM effect. All the lines in this plot are calculated from cross power spectra of the 5th and 8th tomographic bins (corresponding to \( z = (1.0, 1.6) \) and \( m_{\text{WDM}} = 1 \) keV (left panel) and \( m_{\text{WDM}} = 0.5 \) keV (right panel)).

Figure 8. The comparison of the impact of using different models of non-linear power spectra from figure 7 on the weak lensing power spectrum. As above, the blue line is the fractional difference in percent between weak lensing power spectra calculated using the fitting function found in this work \([4]\). The green solid line is the weak lensing power spectrum calculated using the halo model modified for WDM. The dashed green line is the same using standard HALOFIT. The dotted line is calculated by omitting all non-linear corrections.

We additionally consider these models of non-linear WDM structure to calculate the weak lensing power spectra in order to explore the significance of using the correct model. We again plot percentage differences between WDM and CDM weak lensing power spectra in Figure 8. We show only curves representing the cross correlation power spectrum of redshift bins at \( z = 1 \) and \( z = 1.6 \) for consistency with Figure 6. We again examine WDM models with particle masses of \( m_{\text{WDM}} = 1 \) keV (left panel) and \( m_{\text{WDM}} = 0.5 \) keV (right panel). We also calculate the weak lensing power spectra without non-linear corrections to the matter power spectrum and note that this severely over-estimates the effect of WDM suppression. In the lensing calculation, the HALOFIT non-linear corrections applied to the WDM suppressed linear matter power spectrum seem to perform better in describing the results of our WDM simulations than the WDM halo model. This due to the fact that the range of wavenumbers that are better described by the HALOFIT corrections, namely \( k < 1 h \text{Mpc}^{-1} \) are significantly more relevant to the weak lensing power spectrum than the smaller scales where HALOFIT strongly deviates from the simulation results.

6 CONCLUSIONS

By using a large set of N-body and hydrodynamic simulations we have explored the non-linear evolution of the total matter power. The focus of the present work is on small scales and relatively low redshifts where non-linear effects are important and need to be properly modelled with simulations. We checked for numerical convergence and boxsizes/resolution effects in the range \( k = 1 - 10 h \text{Mpc}^{-1} \). We explored how different masses of a warm dark matter candidate affect the non-linear suppression as compared to a corresponding ΛCDM model that shares the same parameters and astrophysical inputs. Our findings can be summarized as follows:

- Cosmological volumes of linear size \( 25h^{-1} \) comoving Mpc and with \( 512^3 \) DM particles are sufficient to sample the WDM suppression for \( m_{\text{WDM}} \geq 1 \) keV at the percent level at \( k < 10 h \text{Mpc}^{-1} \).
- The non-linear suppression induced by WDM is strongly redshift dependent. However, by \( z = 0 \), up to \( k = 10 h \text{Mpc}^{-1} \), there are virtually no differences (below 1%) between ΛCDM and WDM models with \( m_{\text{WDM}} \geq 1 \) keV.
- At higher redshifts differences are larger, being closer to the linear suppression. At \( z \sim 1 \) there are differences of the order of a few percent between the non-linear WDM and ΛCDM power spectra.
- Baryonic physics and in particular radiative processes in the gas component, the star formation criterion and galactic feedback in the form of winds are likely to affect the matter power at the 2-3 % level in the range \( k = 1 - 10 h \text{Mpc}^{-1} \). However, a much stronger effect can be expected if AGN feedback is considered as in (van Daalen et al. (2011)).
- We investigate how a change in $\Omega_m$, $H_0$, and $\sigma_8$ impacts the non-linear power and WDM suppression in particular, when values different from our reference choice are used. Small differences are found (at the $\pm 2\%$ level) at the scales considered here. Thus the WDM cutoff has a distinctive feature which is not degenerate with other cosmological parameters also at a non-linear level.

- We provide a useful fit to the non-linear WDM induced suppression in terms of a redshift-dependent transfer function; this fitting formula should agree to the actual measured power at the $2\%$ level at $z < 3$ and for masses above 0.5 keV.

- Reaching a higher accuracy (percent level) in terms of WDM non-linear power would require a very extensive analysis of astrophysical aspects related to the baryonic component such as considering different feedback effects. Among these the most promising seems to be AGN feedback which happen to solve the overcooling problem and has a strong impact on the total matter power (see van Daalen et al. (2011), Semboloni et al. (2011)).

- We find that future weak lensing surveys will most likely be powerful enough to rule out WDM masses smaller than 1 keV, which is consistent with previous results of Markovic et al. (2011) and Smith & Markovic (2011). Ruling out models for masses larger than 1 keV would still be possible by using the cross-correlation signal between different redshift bins. However, measurement of the weak lensing power at these scales should also consider the effect of baryonic physics carefully and parameters could be biased as recently found in Semboloni et al. (2011), where it has been shown that AGN feedback produces a suppression which is larger than the one induced by WDM at the scales considered here.

- Non-linear corrections to the matter power spectrum in the WDM scenario obtained from HALOFIT correspond better to the results of the WDM only simulation at scales $k < 10 h\, \text{Mpc}^{-1}$, if compared to the non-linear corrections of the halo model from Smith & Markovic (2011). Because these scales are most relevant for weak lensing power spectra, using HALOFIT yields a better correspondence to the weak lensing power spectra calculated using our fitting function. However, on scales $k > 10 h\, \text{Mpc}^{-1}$, the halo model performs slightly better in that it better describes the shape of the suppression in the power spectrum, even if it does overestimate the effect. For this reason we believe that a further modification to the halo model may be needed, especially for weak lensing power spectra calculations.

As recently shown by van Daalen et al. (2011) and Semboloni et al. (2011) including AGN feedback has strong consequences in terms of matter power and weak lensing, a comprehensive analysis that aims at measuring the mass of a warm dark matter candidate should thus hope to lift the degeneracies present (i.e. suppression in terms of matter power) by exploiting the different redshift and scale dependencies and by fully exploring the astrophysical parameter space and marginalize over the nuisance parameters.

We believe that future efforts aiming at measuring the coldness of cold dark matter should investigate the non-linear matter power in the range $z = 0 - 5$ either using weak lensing observables or the small scale clustering of galaxies. These constraints can be particularly useful since they are complementary to those that can be obtained from high redshift Lyman-α forest data (e.g. BOSS/SDSS-III survey) or galactic and sub-galactic observables in the local universe.

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