Sticky physics of joy: on the dissolution of spherical candies

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Abstract
Assuming a constant mass decrease per unit surface and unit time we provide a very simplistic model for the dissolution process of spherical candies. The aim is to investigate the quantitative behaviour of the dissolution process throughout the act of eating the candy. In our model we do not take any microscopic mechanism of the dissolution process into account, but rather provide an estimate which is based on easy-to-follow calculations. Having obtained a description based on this calculation, we confirm the assumed behaviour by providing experimental data for the dissolution process. Besides a deviation from our prediction caused by the production process of the candies below a diameter of 2 mm, we find good agreement with our model-based expectations. Delicate questions on the optimal strategy for enjoying a candy will be addressed, such as whether it is wise to split the candy by breaking it with the teeth or not.

Introduction
Let us consider a three-dimensional spherical sugar-made candy of given mass $m_0$, radius $r_0$ and density $\rho$, as depicted in figure 1. The mass-transfer rate $c$ is assumed to be constant, i.e. within an infinitesimal time-step, the total amount of mass that is dissolved depends on the surface area of the candy at this very instant only. If one enjoys such a candy, the dissolution process kicks in as soon as the candy is embedded in the saliva-reservoir of the oral cavity of the lucky connoisseur. However, the time of joy due to tastiness is quite finite. In order to provide a rough estimate for the time the candy survives in this hostile environment, let us try to quantify this problem. Clearly, the mass decreases due to the dissolution process. The constant mass-transfer rate decreases the surface area, which in turn reduces the overall reduction of mass. We thus assume that the candy keeps its shape throughout the whole dissolution process. As long as the shape of the candy stays the same, we can use the same formulae for its surface, volume, mass and radius throughout the dissolution process. As revealed by the actual measurement of spherical candy dissolution, the assumption of the candy keeping its shape is fulfilled quite well, leading to good predictions with this model.
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Figure 1. A spherical sugar-made homogeneous and isotropic candy. How long does it take until the candy is dissolved and how does it vanish?

A simplistic model
The desired quantity is the function that describes the decrease of the mass of the candy in time

$$\frac{dm}{dt} = cs(t),$$

(1)

where $s(t)$ is the surface area of the candy for a given time $t$ and $c$ ($c \in \mathbb{R}$, $c < 0$) is the mass-transfer rate, which is assumed to be constant. That is, the change of the mass in time is proportional to the surface area of the candy expressed in terms of its mass via the mass-transfer rate. In order to proceed, we have to express the surface area of the candy as a function of a given (time-dependent) mass, which can be done due to the assumption of constant density. By using the following equations

$$v(t) = \frac{4\pi}{3} r(t)^3,$$

(2)

$$s(t) = 4\pi r(t)^2,$$

(3)

$$\rho = \frac{m(t)}{v(t)},$$

(4)

$$r(t) = \left( \frac{3m(t)}{4\pi \rho} \right)^{\frac{1}{3}},$$

(5)

for the volume, surface, density and radius of the candy at an instant, we can express the surface area of the three-sphere as

$$s(t) = 4\pi \left( \frac{3m(t)}{4\pi \rho} \right)^{\frac{2}{3}}.$$

(6)

Plugging equation (6) into (1) we have

$$\frac{dm}{dt} = -|c| 4\pi \left( \frac{3m(t)}{4\pi \rho} \right)^{\frac{1}{3}}.$$  

(7)

Equation (7) is a homogeneous first-order nonlinear ordinary differential equation. By solving it we obtain the mass as a function of time. All relevant parameters of the candy (volume, surface, radius) follow then directly by resubstituting the mass at a given time into equations (2), (3) and (5) respectively. We employ $m(t = 0) = m(0) = m_0$ as the initial condition, which is the mass of the yet untouched candy. The (real) solution of (7) can be obtained easily by separation of variables, where we abbreviate the constant prefactor as $A = |c| 4\pi \left( \frac{3}{4\pi \rho} \right)^{\frac{1}{3}}$:

$$\frac{dm}{dt} = -Am(t)^{\frac{2}{3}},$$

(8)

$$m(t) = \left( \frac{m_0}{3} - \frac{A}{3} t \right)^{\frac{3}{2}},$$

(9a)

$$m(t) = m_0 - A^2/3 t + \frac{1}{3} A^2 m_0^{1/3} t^2 - \frac{1}{27} A^3 t^3.$$  

(9b)

This solution now gives access to all relevant parameters of the candy. Hereby, the parameters (radius, surface, volume) expressed in terms of the mass inherit their time dependence from the mass. Already by looking at equation (7), we can tell that the dissolution of the candy sphere’s mass does not occur exponentially, since equation (7) is not of the type $\dot{m} \propto m(t)$.

Furthermore, equation (7) possesses two additional solutions which are complex. They occur due to the root on the right-hand-side of the equation and are complex conjugate to each other. They do not seem to hold any relevant information, thus we concentrate on the real solution only, where we restrict the domain to positive or vanishing mass values.

Figure 2 shows the solution (9) for a given set of arbitrarily chosen parameters. The solution (9) is an equation of a line raised to the third power. Thus, since $r$ in equation (5) is proportional to the cubic root of $m$, it follows that the radius decreases linearly in time.

The radius (or to be more precise, the diameter) of the sphere is an experimentally excellent accessible quantity, thus we will use the
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Figure 2. The mass, radius, surface area and volume of a spherical candy with an initial mass $m_0 = 10$ mg, density $\rho = 0.8$ mg mm$^{-3}$ and a mass-transfer rate $c = -0.003$ mg s$^{-1}$ mm$^{-2}$.

prediction of the linear dissolution in terms of the diameter and seek experimental proof for this statement.

Methods

In ‘A simplistic model’ we proposed that within our idealized model the candy vanishes linearly in time when considering the radius or diameter to describe the state of the candy in an instant. To prove this behaviour, we constructed an experimental setup, allowing us to determine the diameter of the candy without interfering with the dissolution process. Thus the candy dissolves under conditions close to our theoretical assumptions. The experimental conditions should correspond to the situation where the candy in the mouth is kept in constant motion by moving it gently with the tongue, while applying as little pressure on the candy as possible. Saturation effects in the immediate vicinity of the candy should then be negligible, as they are in our experimental setup.

We used ordinary tap water in a bowl as the environment for the dissolution. The pH of water at room temperature is roughly 7, which corresponds to the pH of saliva [1]. Adequate samples in the form of homogeneous spherical sugar-made candies are provided by [2]. The experimental setup is depicted in figure 3. A power supply drives a small electric motor which stirs the water contained in the bowl slightly. Without stirring, the candies produce clouds of saturation in their immediate vicinity, such that additional diffusion effects become dominant. Directly above the water bowl a digital camera is mounted on a tripod [3]. The camera is equipped with the capability of taking a series of photos at fixed time intervals. By taking the time interval to be 1 min, and by using the date and time stamp feature of the camera, we get a series of pictures showing the candies in the water bowl in a top-down view in time–distances of 1 min. The bottom of the water bowl has been covered by black foam rubber to enhance the contrast for the candies. Once the candies are dissolved,
the frames taken throughout the measurement are processed on a computer. By taking a 50-Euro cent coin as a reference, a distance-per-pixel calibration value was determined. The setup even allows for unattended measurements.

The calibration shot, as well as the ongoing dissolution process of a certain measurement is shown in figure 4. From figure 4 it follows that under these experimental conditions the assumption that the candies maintain their spherical shape holds quite well. Of course, there were small deviations from the ideal sphere, but on average the shape was to a good approximation spherical. The diameter of the candies in terms of pixels was determined manually using the freely available image manipulation program GIMP [4]. Furthermore, an estimate of the error of the diameter is provided by the number of pixels situated between the last one definitely belonging to the candy and the first one definitely belonging to the black bottom of the bowl, accounting for the uncertainty as to where the actual rim of the candy is located.

**Results**

Figure 5 shows the results of the measurement depicted in figure 4. Above the 2 mm diameter mark, the overall behaviour of the candy diameter as a function of time shows a linear decrease to a good approximation. By applying a linear regression to these data points we get the following fit parameters for the diameter $d$: $d = 7.0732 - 0.20123t$ with $-0.986157$, $d = 6.6637 - 0.24202t$ with $-0.9867724$ and $d = 6.6049 - 0.20325t$ with $-0.980611$ for the fit function and correlation coefficient for candies 1, 2 and 3 respectively. With correlation coefficients of more than 98% we get a strong correlation for the linear decrease for all three candies. The slope of candy 2 differs by about 20% from the slopes of candies 1 and 3, but it still shows a linear behaviour. The deviation in the slope and thus in the transfer rate can have several causes. Each candy ‘feels’ its local flow provided by the stirring motor, making a candy feeling a flow of higher rate vanish faster. Thus, the position of the candy in the bowl has an impact on the dissolution process. The candies may also differ in terms of density or chemical properties, which might also affect the rate at which they are dissolving. The important fact is that their radius decreases linearly, as predicted in this simple model.

**Discussion**

In this study we raised, investigated and answered the question of how spherical candies dissolve in time. Providing a rather simple model for the dissolution process we claimed that such a candy should vanish linearly when characterized by its diameter. After the derivation and discussion of the model we constructed an experimental setup which allowed us to investigate the dissolution process of the candies quite close to the model’s assumptions. We found good agreement with our model above roughly 2 mm in diameter. Below the 2 mm diameter mark we considered the existence of a core with different physical or chemical properties as a possible explanation for the deviation from the linear decrease of the diameter, even though the candy looks like it is homogeneous when broken apart. As finally confirmed by the manufacturer of the candy samples, a core is indeed used in the production process of the candies, which validated our assumption. The outer material of the shell is sugar-coated onto this core. However, above the core radius, our model produces a valid prediction in an easy-to-follow approach.

Furthermore, we would like to stress that the underlying mechanism might also apply to other phenomena, as long as the mass-transfer rate is constant and the object dissolving or growing keeps its shape throughout the process.
Figure 4. A series of pictures taken throughout a certain measurement. (a) Calibration with a 50-Euro cent coin; the dissolution of the candies after (b) 0, (c) 1, (d) 7, (e) 10 and (f) 15 min. The stripes are in the rubber background. The candies change their position slightly in time due to the flow provided by the stirring motor.

Conclusion

Finally, we would like to address the question proposed at the very beginning of this study: what is the best strategy for eating such a candy? This delicate question may be phrased as a mathematical optimization problem, where the amount of mass that is transferred away in an instant acquires either a maximum or a minimum: if the candy is to ‘live’ as long as possible, the eater of the candy should try to maintain the spherical shape of the candy at all costs. Since the effect of mass transfer is driven by the surface and a sphere possesses the smallest surface for a given volume among all possible shapes, any deviation of the spherical shape increases the process of losing mass. This situation thus accounts for a
Figure 5. The result of the measurement depicted in figure 4. The three candies dissolve more or less linearly in time, as proposed by our simple model. Below 2 mm in diameter, the behaviour of the dissolution changes drastically. These points have been excluded from the fit area.

minimum of mass transfer, yielding the maximal time of joy. On the other hand, suppose you break the candy with your teeth into many pieces. The surface area increases and in an instant the mass that is transferred away from the fragments becomes huge as well, thus accounting for a maximum in the mass-transfer per unit time. This might amplify the effect of tastiness and joy, even though the life-time of the candy is considerably shorter with this approach. Even though we now know how candies dissolve in time, we stress that the best thing to do when eating a candy is to forget about these considerations, since they draw your attention away from what candies are made for—enjoyment.

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References
[1] Rhoades R A and Bell D R 2009 Medical Physiology—Principles for Clinical Medicine (Philadelphia, PA: Lippincott Williams & Wilkins)
[2] Liebesperlen Schokoladenwerk Berggold GmbH, Heinerle Spiel- und Süßwaren GmbH, Raniser Straße 11, 07381 Pößneck
[3] Nikon Corporation Nikon Coolpix P510 www.nikon.com
[4] GIMP–GNU image manipulation program www.gimp.org

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