Effects of The Initial Hadron in \( B \to J/\psi + X \)

J.P. Ma

*Institute of Theoretical Physics,*
*Academia Sinica,*
*P.O.Box 2735, Beijing 100080, China*
*e-mail: majp@itp.ac.cn*

**Abstract**

In the framework of HQET inclusive decays of b-flavored hadrons can be handled as decays of the free b-quark approximately. We analyse the correction to this approximation for the inclusive decays into a polarized \( J/\psi \), the correction is characterized by two matrix elements defined in HQET. For the \( J/\psi \) we use NRQCD to parameterize the formation of the \( c\bar{c} \) pair into \( J/\psi \). Numerically the correction is remarkably large, it can be at the level of 30% and it is mainly due to the Fermi-motion of the \( b \)-quark. With this correction we give a new determination for combinations of two NRQCD matrix elements.

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Inclusive decay of a b-flavored hadron into a charmonium is an interesting process in the aspect that two effective theories derived from QCD are used to factorize nonperturbative effects. For the charmonium, because the relative velocity \( v_c \) between the \( c^- \) and \( \bar{c}^- \)-quark inside the charmonium is small in the rest-frame, one can use NRQCD \([1]\) to handle the nonperturbative effect in the formation of the free \( c\bar{c} \) pair into the charmonium, while for the initial hadron one can employ an expansion in the inverse of \( m_b \), the b-quark mass, because \( m_b \) is very large in comparison with the nonperturbative energy-scale \( \Lambda \). The expansion can be done with the heavy quark effective theory (HQET) \([2]\), see for a review, see e.g. \([3]\). At the leading order the decay can be considered as the decay of the free b-quark, hence the decay width is the same for all initial hadrons. Corrections from higher orders can be systematically added, they will depend on the type of initial hadrons. It is the purpose of this work to study the corrections from the next-to-leading order, i.e., the effect of initial hadrons.

It should be kept in mind that the factorization mentioned before is not well established in comparison with the factorization for totally inclusive decays and for inclusive semilepton-decay, in these decays one can use the operator product expansion (OPE) to factorize nonperturbative effects. With OPE the effect of the initial hadron in inclusive decays are studied, for example, in \([4,5]\). For the process considered here OPE cannot be used. However we can use the diagram expansion proposed in \([6]\) to perform the factorization and assume that the factorization still holds, i.e., the perturbatively calculated coefficients are free from infrared divergences. We will use the expansion and work at the leading order of \( \alpha_s \).

We consider the process:

\[
H_b(P) \rightarrow J/\psi(k, \lambda) + X
\]

where we denote the b-flavored hadron as \( H_b \), which contains a b-quark. The momenta are given in the brackets. \( \lambda \) denotes the polarization of the \( J/\psi \), \( \lambda = L \) is for the longitudinal polarization and \( \lambda = T \) is for the transversal polarization. To analyze the decay three expansions are used, they are expansion in \( \alpha_s \), in \( v_c \) and in the inverse of \( m_b \). In \([7,8]\) the decay is studied at the leading order of all expansions. At the leading order of \( v_c \) and of the inverse of \( m_b \) the one-loop correction is calculated in \([3]\) for the unpolarized \( J/\psi \). The corrections from the next-to-leading order of \( v_c \) is analyzed in \([10]\) at the leading order of \( \alpha_s \) for the polarized \( J/\psi \), they can be very large. Because of lacking the detailed information about the formation of the \( c\bar{c} \) pair into a \( J/\psi \), i.e., the precise values of NRQCD matrix elements, a detailed prediction seems impossible without help of some models. In this work we analyze the corrections from the next-to-leading order of the inverse of \( m_b \) to complete the analysis of the correction from the next-to-leading order of all expansions. By completing this analysis the last unknown effect in the theory for the process can be estimated, in which nonperturbative effect is represented by matrix elements defined in HQET and in NRQCD. In \([11]\) models to account the effect are used to explain the \( J/\psi \)-spectrum.

The effective weak Hamiltonian for the decay is:

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=s,d} \{ V_{cb} V_{cq}^* \frac{1}{3} C_{[1]}(\mu) \bar{c} \gamma^\mu (1 - \gamma_5) c \bar{q} \gamma_\mu (1 - \gamma_5) b \\
+ C_{[8]}(\mu) \bar{c} T^a \gamma^\mu (1 - \gamma_5) c \bar{q} T^a \gamma_\mu (1 - \gamma_5) b \}.
\]
We neglected the contributions of QCD penguin operators in \( H_{\text{eff}} \). \( T^a(a = 1, \cdots 8) \) is SU(3) color-matrix. The coefficients \( C_{[1]} \) and \( C_{[8]} \) are related to the usual \( C_\pm \) by

\[
C_{[1]}(\mu) = 2C_+ (\mu) - C_- (\mu), \quad C_{[8]}(\mu) = C_+ (\mu) + C_- (\mu).
\]

(3)

With the effective Hamiltonia a \( c\bar{c} \) pair can not only be produced in the color-singlet state, but also in the color-octet state. Although a \( c\bar{c} \) pair in the color-octet state can be transmitted into a \( J/\psi \) at higher orders in \( \alpha_s \), than a \( c\bar{c} \) pair in the color-singlet state does, its contributions to the decay rate are important, because the \( c\bar{c} \) pair is more likely in the color-octet state than in the color-singlet state by the fact \( C_{[1]}(m_b) : C_{[8]}(m_b) \approx 0.18 : 1. \) We will take the color-octet states into account and write the decay width as

\[
\Gamma_\lambda(J/\psi) = \Gamma_\lambda^{(1)} + \Gamma_\lambda^{(8)},
\]

(4)

where the index 1 or 8 stands for color singlet contributions or for color-octet contributions, respectively. With the \( H_{\text{eff}} \) and in the approach of the diagram expansion at the leading order in \( \alpha_s \), the contributions to the decay width can be written:

\[
\Gamma_\lambda^{(1)} = \frac{G^2_F C^2_{[1]} |V_{cb}|^2}{18} \int d\Gamma \int dx^4 e^{iq \cdot x} \sum \chi
\cdot \langle 0 | \bar{c}(0)\gamma^\mu (1 - \gamma_5) c(0) | J/\psi + X \rangle \langle J/\psi + X | c(x)\gamma^\nu (1 - \gamma_5) c(x) | 0 \rangle \\
\cdot \langle H_b | b(0) \gamma_\mu (1 - \gamma_5) \gamma \cdot q \gamma_\nu (1 - \gamma_5) b(x) | H_b \rangle,
\]

(5)

where we used \( |V_{cd}|^2 + |V_{cs}|^2 \approx 1. \) The integral \( \int d\Gamma \) is

\[
\int d\Gamma = \int \frac{d^3 k}{(2\pi)^3} \frac{d^4 q}{(2\pi)^4} 2\pi \delta(q^2).
\]

(6)

In the above equations we neglect the mass of light quarks and take the nonrelativistic normalization for \( H_{\text{eff}} \) and \( J/\psi \)-state. Similarly the color-octet contribution is

\[
\Gamma_\lambda^{(8)} = \frac{G^2_F C^2_{[8]} |V_{cb}|^2}{2} \int d\Gamma \int dx^4 e^{iq \cdot x} \sum \chi
\cdot \langle 0 | \bar{c}(0)T^a \gamma^\mu (1 - \gamma_5) c(0) | J/\psi + X \rangle \langle J/\psi + X | c(x)T^b \gamma^\nu (1 - \gamma_5) c(x) | 0 \rangle \\
\cdot \langle H_b | b(0)T^a \gamma_\mu (1 - \gamma_5) \gamma \cdot q T^b \gamma_\nu (1 - \gamma_5) b(x) | H_b \rangle.
\]

(7)

In both contributions the dependence on the initial hadron appears in a matrix element, which is given by:

\[
T_{ij}(x) = \langle H_b | \bar{b}_j(0) b_i(x) | H_b \rangle,
\]

(8)

where the indices \( i, j \) stand for color- and Dirac indices and the average of the spin is implied if \( H_b \) has a non-zero spin. This matrix element represents the nonperturbative effects related to the initial hadron, it can be expanded in the inverse of \( m_b \) in the framework of HQET. The expansion is performed to expand the Dirac fields \( \bar{b}(0) \) and \( b(x) \) with the fields in HQET. We use the expansion proposed in [12], which is slightly different than the standard approach in
\( b(x) = e^{-im_b v \cdot x} \left\{ 1 + \frac{1}{2m_b} i \gamma \cdot D_T + \frac{1}{4m_b^2} v \cdot D \gamma \cdot D_T - \frac{1}{8m_b^2} (\gamma \cdot D_T)^2 \right\} h(x) + \mathcal{O}(\frac{1}{m_b^3}) \\
+ \text{(terms for anti-quark)}, \tag{9} \)

where \( v^\mu \) is the velocity of \( H_b \), \( D^\mu \) is the covariant derivative \( \partial^\mu + igG^\mu(x) \). \( D_T^\mu \) is defined as

\[ D_T^\mu = D^\mu - v^\mu v \cdot D. \tag{10} \]

\( h(x) \) is the field of HQET and its equation of motion reads:

\[ \left\{ iv \cdot D - \frac{1}{2m_b} (\gamma \cdot D_T)^2 \right\} h(x) = 0 + \mathcal{O}(\frac{1}{m_b^2}). \tag{11} \]

With the expansion in Eq. (9) the operator \( N \) for the number of b-quarks is

\[ N = \int d^3x \bar{h}(x) h(x). \tag{12} \]

Using the expansion in Eq. (9) the matrix element \( T_{ij}(x) \) can be written as

\[ T_{ij}(x) = e^{-im_b v \cdot x} \left\{ \langle H_b | \bar{h}_{ij}(0) h_i(x) | H_b \rangle + \frac{1}{2m_b} \langle H_b | \bar{h}_{ij}(0) (i \gamma \cdot D_T h(x))_i - i \gamma \cdot D_T \bar{h}_{ij}(0) h_i(x) | H_b \rangle \right. \\
+ \left. \frac{1}{4m_b^2} \langle H_b | \bar{h}_{ij}(0) (v \cdot D \gamma \cdot D_T h(x))_i + v \cdot D \gamma \cdot D_T \bar{h}_{ij}(0) h_i(x) + \gamma \cdot D_T \bar{h}_{ij}(0) (\gamma \cdot D_T h(x))_i \\n- \frac{1}{2} \bar{h}_{ij}(0) ((\gamma \cdot D_T)^2 h_i(x))_i - \frac{1}{2} (\gamma \cdot D_T)^2 \bar{h}_{ij}(0) h_i(x) | H_b \rangle \right\} + \mathcal{O}(\frac{1}{m_b^3}). \tag{13} \]

The dominant \( x \)-dependence of \( T_{ij}(x) \) is \( \exp(-im_b v \cdot x) \), while the \( x \)-dependence of matrix elements with \( h(x) \) is controlled by the energy scale \( \Lambda \), which is much smaller than \( m_b \), we can expand this \( x \)-dependence in \( x \). In this expansion we obtain a tower of local operators \( O_n \). The equation of motion Eq. (11) should be used to eliminate redundant operators. The contribution from a given operator \( O_n \) to the decay width will be suppressed by the power \( \Lambda^{d_n-3} \), where \( d_n \) is the dimension of the operator \( O_n \). For example, the first matrix element in Eq. (13) is expanded as

\[ \langle H_b | \bar{h}_{ij}(0) h_i(0) | H_b \rangle = \langle H_b | \bar{h}_{ij}(0) h_i(0) | H_b \rangle + x_\mu \langle H_b | \bar{h}_{ij}(0) \partial^\mu h_i(0) | H_b \rangle \\
+ \frac{1}{2} x_\mu x_\nu \langle H_b | \bar{h}_{ij}(0) \partial^\mu \partial^\nu h_i(0) | H_b \rangle + \cdots. \tag{14} \]

With the Lorentz covariance and properties of the field \( h(x) \), the matrix element can be written

\[ \langle H_b | \bar{h}_{ij}(0) h_i(x) | H_b \rangle = \frac{1}{12} (1 + \gamma \cdot v)_{ij} \langle H_b | \bar{h} h | H_b \rangle + \frac{1}{12} v \cdot x (1 + \gamma \cdot v)_{ij} \langle H_b | \bar{h} v \cdot \partial h | H_b \rangle \\
+ \frac{1}{72} (1 + \gamma \cdot v)_{ij} x_\mu x_\nu [g^{\mu\nu} \langle H_b | \bar{h} \partial_\mu^2 h | H_b \rangle \\
- v^\mu v^\nu \langle H_b | \bar{h} (\partial_\mu^2 - 3(v \cdot \partial)^2 h | H_b \rangle] + \cdots. \tag{15} \]
Because of Eq.(12) the first matrix element in the r.h.s. of the above equation is 1:

$$\langle H_b | \bar{h} h | H_b \rangle = 1. \quad (16)$$

This results in that the decay of $H_b$ at the leading order of $m_b^{-1}$ is the decay of the free $b$-quark. Similarly we can expand the other matrix elements in Eq.(13) and we obtain:

$$T_{ij}(x) = e^{-i m_{b} x} \left\{ \frac{1}{12} (1 + \gamma \cdot v)_{ij} + i \frac{x^\mu}{2m_b} (1 + \gamma \cdot v)_{ij} v_\mu \right. \right.$$

$$\left. + \frac{1}{18} (\gamma_\mu - v_\mu \gamma \cdot v) \langle H_b | \bar{h} (\gamma \cdot D_T)^2 h | H_b \rangle + \frac{1}{72} x^\mu x^\nu (g_{\mu \nu} - v_\mu v_\nu) \right.$$  

$$\cdot (1 + \gamma \cdot v)_{ij} \langle H_b | \bar{h} D_T^2 h | H_b \rangle - \frac{1}{24} \langle H_b | \bar{h} (\gamma \cdot D_T)^2 h | H_b \rangle \right\} + \cdots$$  

(17)

where we have replaced the derivative $\partial_\mu$ with the covariant one $D_\mu$ and the equation of motion was used to eliminate the redundant operators. The $\cdots$ stand for contributions from operators with the dimension higher than 5, which will lead to contributions at the order higher than $\Lambda^2$.

For the charmonium $J/\psi$ we will work at the leading order in $v_c$ and we have $M_{J/\psi} = 2m_c$. To expand the Dirac field $c(x)$ with the field of NRQCD, we make a Lorentz boost to transform the relevant matrix element into the $J/\psi$-rest frame and then make the expansion. We obtain for the color-singlet part:

$$\sum_X \langle 0 | \bar{c}(0) \gamma^\mu (1 - \gamma_5) c(0) | J/\psi + X \rangle \langle J/\psi + X | c(x) \gamma^\nu (1 - \gamma_5) c(x) | 0 \rangle$$

$$= e^{i k \cdot x} \frac{M_{J/\psi}}{k^0} \varepsilon^\mu (\varepsilon^\nu (\lambda)) \frac{1}{3} (\mathcal{O}^{J/\psi}_{1}(3S_1)) + \mathcal{O}(v_c^2), \quad (18)$$

where the factor $M_{J/\psi}/k^0$ is due to the boost. The definition of the matrix element $\mathcal{O}^{J/\psi}_{1}(3S_1)$ can be found in [1].

With Eq.(17) and Eq.(18) we can perform the $x$-integration in Eq.(5). After the integration we obtain a $\delta$-function for the momentum conservation, and also derivatives of the $\delta$-function like

$$\frac{\partial}{\partial q_\mu} (2\pi)^4 \delta^4 (m_b v - q - k). \quad (19)$$

Such derivatives can be eliminated by partial integrations when the integration of the phase-space is performed. For example, the contribution related the above derivative can be written:

$$\int d^4 q \delta(q^2) \frac{\partial}{\partial q_\mu} \delta^4 (m_b v - q - k) = -(2q^\mu \delta'(q^2) f(q) + \delta(q^2) \frac{\partial}{\partial q_\mu} f(q))|_{q=m_b v - k}, \quad (20)$$

where $\delta'(x) = \frac{d}{dx} \delta(x)$. Performing these integrations the final results for the color-singlet contribution can be obtained. To present our results we write:

$$\langle H_b | \bar{h} D_T^2 h | H_b \rangle = -\mu_\pi^2,$$

$$\langle H_b | \bar{h} (\gamma \cdot D_T)^2 h | H_b \rangle = \langle H_b | \bar{h} D_T^2 h | H_b \rangle + \langle H_b | \bar{h} \frac{1}{2} g G_{\mu \nu} \sigma^{\mu \nu} h | H_b \rangle + \cdots$$

$$= -\mu_\pi^2 + \mu_g^2 + \cdots, \quad (21)$$
where \( \cdots \) stand for higher-order contributions. The color-singlet contribution \( \Gamma^{(1)}_\lambda \) can be written as

\[
\Gamma^{(1)}_\lambda = \frac{m_b^3 G_F^2 C_W^2 |V_{cb}|^2}{432 \pi m_c} \langle O_1^{J/\psi} (3 S_1) \rangle \cdot \left\{ F_\lambda (y) + \frac{\mu_\pi^2}{m_b^2} H_\lambda (y) + \frac{\mu_\rho^2}{m_b^2} G_\lambda (y) \right\}
\]  

(22)

where \( y = 4m_c^2/m_b^2 \) and the functions are:

\[
F_L (y) = (1 - y)^2,
\]

\[
F_T (y) = y (1 - y)^2,
\]

\[
G_L (y) = \frac{1}{6} (7 - 4y + 7y^2 - 10y^3),
\]

\[
G_T (y) = \frac{1}{6} y (11 - 26y + 15y^2),
\]

\[
H_L (y) = \frac{1}{6} (-53 + 4y - y^2 + 26y^3),
\]

\[
H_T (y) = \frac{1}{6} y (-11 + 14y - 27y^2). \tag{23}
\]

From the above results the effect of the initial hadron is represented by two matrix elements defined in HQET. It should be noted that the effect is not suppressed by the power of \( m_b^{-1} \), but rather by the power of \( (m_b^2 - 4m_c^2)^{-1} \). This can be seen by comparing the coefficients given above.

For the color-octet contribution we expand the the color-octet matrix element related to \( J/\psi \) with NRQCD fields. The result at the leading order of \( \nu_c \) is:

\[
\sum_X \langle 0 | \bar{c}(0) T^a \gamma^\mu (1 - \gamma_5) c(0) | J/\psi + X \rangle \langle J/\psi + X | c(x) T^b \gamma^\nu (1 - \gamma_5) c(x) | 0 \rangle
\]

\[
= \epsilon^{k \cdot x} \frac{M_{J/\psi}}{k^0} \frac{1}{24} \delta_{ab} \left\{ \frac{k^\mu k^\nu}{M_{J/\psi}} \langle O_8^{J/\psi} (1 S_0) \rangle + \varepsilon^\nu (\lambda) (\varepsilon^{\nu} (\lambda))^* \langle O_8^{J/\psi} (3 S_1) \rangle \right\}
\]

\[
+ (-g^\mu + \frac{k^\mu k^\nu}{M_{J/\psi}} - \varepsilon^\nu (\lambda) (\varepsilon^{\nu} (\lambda))^*) \frac{1}{m_c^2} \langle O_8^{J/\psi} (3 P_1) \rangle \right\} + \mathcal{O}(\nu_c^4). \tag{24}
\]

At this order, the color-octet \( c \bar{c} \) pair with the quantum number \( 1 S_0, \ 3 S_1 \) and \( 3 P_1 \) can form a \( J/\psi \) through emission or absorption of soft gluons, the probability is at order of \( \nu_c^4 \). Perform similar calculations as for the color-singlet contribution we obtain the color-octet contribution:

\[
\Gamma^{(8)}_\lambda = \frac{m_b^3 G_F^2 C_W^2 |V_{cb}|^2}{288 \pi m_c} \left\{ F^{(8)}_\lambda (y) + \frac{\mu_\pi^2}{m_b^2} H^{(8)}_\lambda (y) + \frac{\mu_\rho^2}{m_b^2} G^{(8)}_\lambda (y) \right\}, \tag{25}
\]

where the functions are:

\[
F^{(8)}_L (y) = (1 - y)^2 \langle O_8^{J/\psi} (1 S_0) \rangle + \langle O_8^{J/\psi} (3 S_1) \rangle + 2y(1 - y)^2 \frac{1}{m_c^2} \langle O_8^{J/\psi} (3 P_1) \rangle,
\]

\[
F^{(8)}_T (y) = (1 - y)^2 \langle O_8^{J/\psi} (1 S_0) \rangle + y \langle O_8^{J/\psi} (3 S_1) \rangle + (1 - y)^2 (1 + y) \frac{1}{m_c^2} \langle O_8^{J/\psi} (3 P_1) \rangle,
\]
\[ G_L^{(8)}(y) = \frac{1}{6}((7 - 12y + 15y^2 - 10y^3)\langle O_8^{J/\psi}(S_0) \rangle + (7 - 4y + 7y^2 - 10y^3)\langle O_8^{J/\psi}(S_1) \rangle + 2(-1 - 14y + 15y^2)\frac{1}{m_c^2}\langle O_8^{J/\psi}(P_1) \rangle), \]

\[ G_T^{(8)}(y) = \frac{1}{6}((7 - 12y + 15y^2 - 10y^3)\langle O_8^{J/\psi}(S_0) \rangle + y(11 - 26y + 15y^2)\langle O_8^{J/\psi}(S_1) \rangle + (7 - 17y + 5y^2 + 5y^3)\frac{1}{m_c^2}\langle O_8^{J/\psi}(P_1) \rangle), \]

\[ H_L^{(8)}(y) = \frac{1}{6}((53 - 12y - 9y^2 + 26y^3)\langle O_8^{J/\psi}(S_0) \rangle + (53 + 4y - y^2 + 26y^3)\langle O_8^{J/\psi}(S_1) \rangle + 2y(1 + 2y - 27y^2)\frac{1}{m_c^2}\langle O_8^{J/\psi}(P_1) \rangle), \]

\[ H_T^{(8)}(y) = \frac{1}{6}((53 - 12y - 9y^2 + 26y^3)\langle O_8^{J/\psi}(S_0) \rangle + y(-11 + 14y - 27y^2)\langle O_8^{J/\psi}(S_1) \rangle + (-53 + 17y - 11y^2 - y^3)\frac{1}{m_c^2}\langle O_8^{J/\psi}(P_1) \rangle). \] (26)

With the results given above we complete our analysis for the effect of the initial hadron. As mentioned before, because the NRQCD matrix elements, especially the color-octet matrix elements, are not known precisely and also there are possibly large relativistic corrections for charmonium, we can not give a precise prediction for the decay width. However, the effect of the initial hadron can be estimated numerically. For this we take \( m_b = 4.8 \text{GeV} \) and \( m_c = 1.5 \text{GeV} \). We obtain for the color singlet contribution:

\[ \Gamma_T^{(1)} = \frac{m_b^3 G_F C_{[\pi]}^2 |V_{cb}|^2}{432\pi m_c} \langle O_1^{J/\psi}(S_1) \rangle \cdot \left\{ 0.15 - 0.63 \frac{\mu_{\pi}^2}{m_b^2} + 0.20 \frac{\mu_{\pi}^2}{m_b^2} \right\}, \]

\[ \Gamma_L^{(1)} = \frac{m_b^3 G_F C_{[\pi]}^2 |V_{cb}|^2}{432\pi m_c} \langle O_1^{J/\psi}(S_1) \rangle \cdot \left\{ 0.37 - 8.34 \frac{\mu_{\pi}^2}{m_b^2} + 0.98 \frac{\mu_{\pi}^2}{m_b^2} \right\}. \] (27)

It is remarkable that the coefficient in the front of \( \mu_{\pi}^2 \) is very large for \( \Gamma_L^{(1)} \) in comparison with others. This indicates that the effect of the initial hadron may be substantial. Taking \( H_b = B \) as an example, the parameter \( \mu_{\pi}^2(B) \) can be determined by the mass splitting between \( B \) and \( B^* \) \([13]\), it gives \( \mu_{\pi}^2(B) \approx 0.36 \text{GeV}^2 \), while \( \mu_{\pi}^2(B) \) can be determined from the QCD sum rules \([14]\) and from an analysis of spectroscopy of heavy hadrons \([15]\). These determinations give a numerical range from 0.3GeV\(^2\) to 0.54GeV\(^2\) for \( \mu_{\pi}^2(B) \). There is also a constraint for \( \mu_{\pi}^2(B) \), \( \mu_{\pi}^2(B) \leq \mu_{\pi}^2(B) \) \([16]\). We take the value \( \mu_{\pi}^2(B) \approx \mu_{\pi}^2(B) \) to estimate the effect of the initial \( B \). With these values, the effect in \( \Gamma_T^{(1)} \) is at 4% level, which is negligible, while the effect in \( \Gamma_L^{(1)} \) is to reduce the width at 30% level in comparison with \( \Gamma_L^{(1)} \) at the leading order, which is significant. Taking the same quark masses we obtain for the color-octet contribution:

\[ \Gamma_T^{(8)} = \frac{m_b^3 G_F C_{[\pi]}^2 |V_{cb}|^2}{288\pi m_c} \cdot \left\{ 0.37 - 8.02 \frac{\mu_\pi^2}{m_b^2} + 0.67 \frac{\mu_{\pi}^2}{m_b^2} \right\} \langle O_8^{J/\psi}(S_0) \rangle \]

\[ + (0.15 - 0.63 \frac{\mu_{\pi}^2}{m_b^2} + 0.20 \frac{\mu_{\pi}^2}{m_b^2}) \langle O_8^{J/\psi}(S_1) \rangle \]
In the color-octet contribution there are also large numbers in the front of $\lambda^2$ in different production channels. This results in that the corrections are significant. We also take $B$-meson as an example and we find:

$$\Gamma_L^{(8)}(B) \approx \frac{m_c^3 G_F^2 C_{[8]}^2 |V_{cb}|^2}{288 \pi m_c} \cdot \{(0.37 - 0.11) \langle O_8^{J/\psi}(1S_0) \rangle + (0.15 - 0.007) \langle O_8^{J/\psi}(3S_1) \rangle + (0.52 - 0.12) \frac{1}{m_c} \langle O_8^{J/\psi}(3P_1) \rangle \}$$

$$+ (0.37 - 0.11) \langle O_8^{J/\psi}(3S_1) \rangle + (0.29 - 0.004) \frac{1}{m_c} \langle O_8^{J/\psi}(3P_1) \rangle \}, \quad (29)$$

where the second number in $(\cdots)$ is for the corrections. From these numbers one can see that the corrections are large, especially in the $1S_0$ channel, which can be at 30% level. The large corrections arise in both color-singlet and color-octet contributions from terms with $\lambda^2$, that indicates that the Fermi-motion of the $b$-quark inside $B$ affects the decay width substantially.

If we collect all known results to compare the experimental result $^{17}$

$$Br(B \to J/\psi + X)_{\text{exp}} = 0.80 \pm 0.08\%,$$
$$Br(B \to \psi' + X)_{\text{exp}} = 0.34 \pm 0.05\%, \quad (30)$$

we have serious problems with the one-loop QCD correction and with the relativistic correction. With the one-loop correction the color-singlet contribution becomes negative $^{11}$. The relativistic correction is analyzed in $^{10}$ and it can have large effect, but the detailed size is unknown because there are four unknown matrix elements of NRQCD. If we neglect it we have:

$$Br(B \to \psi + X) = 0.073 \cdot 10^{-2} \langle O_1^{\psi}(3S_1) \rangle$$

$$+ 0.19 \langle O_8^{\psi}(3S_1) \rangle + 0.33 \langle O_8^{\psi}(1S_0) \rangle + \frac{0.34}{m_c} \langle O_1^{\psi}(3P_1) \rangle$$

$$- (0.068 \cdot 10^{-2} \langle O_8^{\psi}(3S_1) \rangle + 0.071 \langle O_8^{\psi}(1S_0) \rangle$$

$$+ 0.202 \langle O_8^{\psi}(3S_1) \rangle + \frac{0.053}{m_c} \langle O_1^{\psi}(3P_1) \rangle \}, \quad (31)$$
where we use $\psi$ to denote $J/\psi$ or $\psi'$. In the first two lines there are the results from the leading order in the $m_b^{-1}$-expansion. The numbers are from [4] and they contain one-loop QCD corrections. They are slightly modified due to the $m_b^{-2}$ corrections in the semileptonic decay width. It should be noted that the color-singlet contribution is improved by adding certain contributions from higher orders and with the improvement it becomes positive. The results of this work are given in the last two lines. With the correction the contributions from all channels become smaller than those without the corrections. If we take a larger value of $\mu^2$, the color-singlet contribution becomes negative. Among the four matrix elements in Eq.(31) the color-single one is calculated with potential models [18] and with lattice QCD [19], whose value is [18]

$$\langle O_J^{\psi}(3S_1) \rangle = 1.16 \text{GeV}^3,$$
$$\langle O_{\psi'}^{3S_1}(3S_1) \rangle = 0.76 \text{GeV}^3. $$

(32)

The matrix element $\langle O_8^{\psi}(1S_0) \rangle$ is determined by experiments at Tevatron and at Hera [20–23], but the uncertainty can be large. We take the value:

$$\langle O_8^{\psi}(3S_1) \rangle = 1.06 \cdot 10^{-2} \text{GeV}^3,$$
$$\langle O_8^{\psi'}(3S_1) \rangle = 0.44 \cdot 10^{-2} \text{GeV}^3. $$

(33)

The other two are not well determined, only certain combination of them is known with large uncertainty. With the experimental results in Eq.(30) we can determine a combination of them:

$$\langle O_8^{\psi}(1S_0) \rangle + \frac{1.13}{m_c^2} \langle O_1^{\psi}(3P_1) \rangle = 2.4 \cdot 10^{-2} \text{GeV}^3,$$
$$\langle O_8^{\psi'}(1S_0) \rangle + \frac{1.13}{m_c^2} \langle O_{\psi'}(3P_1) \rangle = 1.0 \cdot 10^{-2} \text{GeV}^3. $$

(34)

Comparing the combinations determined without the corrections [8] the change is significant, the combinations becomes 80% larger than those without the corrections and they are closer to those determined from other experiments [20,23]. However, the determination of the combinations should be regarded as a rough one because possibly large corrections from higher orders in $v_c$ are neglected.

To summarize: We have analyzed the effect of the initial hadron in the process $H_b \rightarrow \psi + X$. With our results the three expansions used for the process are all completed at the next-to-leading order. The effect of the initial hadron can be at 30% level for $B \rightarrow \psi + X$ and it reduces the decay width. The effect is mainly due to the Fermi-motion of the $b$-quark inside the initial hadron. Including this effect we have determined combinations of two NRQCD matrix elements, which are 80% larger than those without the effect and they are closer to those determined in other experiment. However, the results for the combinations should be taken with caution because neglected effects can be large.

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