Horizontal-plane Trajectory-tracking Control of an Underactuated Unmanned Marine Vehicle in the Presence of Ocean Currents

Zaopeng Dong1*, Lei Wan1*, Tao Liu1 and Jiangfeng Zeng1

1 Science and Technology on Underwater Vehicle Laboratory, Harbin Engineering University, Harbin, Heilongjiang, China
*Corresponding author(s) E-mail: dongzaopeng/hrbeu.edu.cn; wanlei103@163.com

Received 17 January 2016; Accepted 12 April 2016
DOI: 10.5772/63634
© 2016 Author(s). Licensee InTech. This is an open access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

Based on an integral backstepping approach, a trajectory-tracking control algorithm is proposed for an underactuated unmanned marine vehicle (UMV) sailing in the presence of ocean-current disturbance. Taking into consideration the UMV model’s fore/aft asymmetry, a nonlinear three-degree-of-freedom (3DOF) underactuated dynamic model is established for the horizontal plane. First, trajectory-tracking differences between controllers designed based on symmetric and asymmetric models of the UMV are discussed. In order to explicitly study the effect of ocean-current interference on the trajectory-tracking controller, the ocean current is integrated into the kinematic and dynamic models of the UMV. Detailed descriptions of distinct trajectory-tracking control performances in the presence of different ocean-current velocities and direction angles are presented. The well-known persistent exciting (PE) condition is completely released in the designed trajectory-tracking controller. A mild integral item of trajectory tracking error is merged into the control law, and global stability analysis of the UMV system is carried out using Lyapunov theory and Barbalat’s Lemma. Simulation experiments in the semi-physical simulation platform are implemented to confirm the effectiveness and superiority of the excogitated control algorithm.

Keywords Unmanned Marine Vehicle (UMV), Trajectory-Tracking Control, Persistent Exciting (PE) Condition, Underactuated System, Ocean Currents

1. Introduction

For many years, the scientific, commercial, and naval sectors have shown considerable interest in the design and development of unmanned marine vehicles (UMVs), which can be used to perform a multitude of different tasks, such as mineral resources sampling, offshore oil and gas operations, ocean engineering maintenance, and military reconnaissance [1-4]. As described in [5], UMV is usually used as a generic term to describe unmanned/autonomous underwater vehicles (UUV/AUV) and unmanned/uninhabited surface vessels (USV). Although many advancements have been realized in this area, the demand for more
advanced navigation, guidance and control (NGC) systems for UMVs continues to grow, as more and more vehicle autonomy is required.

A UMV is generally underactuated, as the number of control inputs is less than the degrees of freedom, and there is a nonintegrable acceleration constraint in the UMV system. In addition, underactuated UMVs’ kinematic and dynamic models are highly nonlinear and coupled; therefore, classic linear methodologies cannot be applied. A typical and active research topic on UMV motion control is trajectory tracking, which is concerned with the design of control laws that force an UMV to reach and follow a time-parameterized reference trajectory. Note that when moving in the horizontal plane, AUVs present similar dynamic behaviour to USVs. Towards general research results, this paper addresses trajectory-tracking control of UMVs in the horizontal plane. In recent years, various nonlinear control approaches for trajectory-tracking control of UMVs have been proposed, such as sliding-mode control, backstepping techniques, neural network control, hybrid control, and the linear algebra methodology. These typical methods are not only used in marine vehicle control systems, but also widely in aerial vehicle control systems, ground vehicle control systems, and other complex nonlinear control systems [6-9].

Several nonlinear sliding-mode approaches are proposed in [10-12] for trajectory-tracking control of an underactuated USV. A robust sliding-mode controller and a second sliding-mode controller are separately proposed in [13] and [14] for trajectory-tracking control of an AUV. The controller in [14] comprises an equivalent controller and a switching controller, where the switching controller compensates for the uncertainties of the vehicle’s hydrodynamic and hydrostatic parameters. Backstepping techniques are utilized in [15-20] to design trajectory-tracking controllers for UMVs. An observer is constructed to provide an estimation of unknown disturbances in [18]. Some neural network controllers for trajectory-tracking control of an UMV are devised in [21-23]. In order to achieve optimal tracking performance, a reinforcement learning scheme is designed in [23] with two neural networks: one compensates for model uncertainties, and the other estimates the evaluation function.

In addition, a variety of hybrid controllers are excogitated in [24-33] for trajectory-tracking control of UMVs. An adaptive supervisory control algorithm that combines a switching method with an iterative Lyapunov technique is proposed in [24]; a stable adaptive neural network controller combined with a backstepping technique and Lyapunov theory is designed in [25]; a state feedback adaptive backstepping fuzzy logic controller is addressed in [27]; a hybrid sliding-mode control strategy based on a bio-inspired model is developed in [24]; a suboptimal robust control methodology is presented in [30]; and a hybrid control algorithm based on neural network and dynamic surface control is presented in both [28] and [33]. Moreover, some linear algebra and other methodologies are formulated in [34-42]. Based on searching for conditions under which a system of linear equations has an exact solution, linear algebra methodologies are proposed in both [34] and [35]. A global k-exponential convergence tracking controller is designed in [37]: an output feedback controller combined with a state feedback controller and a reduced-order observer is presented in [39]: a novel finite-time switching trajectory-tracking controller is developed in [41]; and a nonlinear predictive control technique is proposed in [42] for trajectory-tracking control of an USV with state and input constraints.

However, much of the early work in this area was related to developing trajectory-tracking controllers for an UMV with both port/starboard and fore/aft symmetry [10-14, 17, 19-21, 24, 26, 28, 31, 34, 36, 41], as most UMV models adopt the simplifying assumption of diagonal damping and inertia matrices. Ship hydrodynamic analysis shows that, under the assumption of a UMV with both port/starboard and fore/aft symmetry, all off-diagonal elements of the UMV’s damping and inertia matrices are zero. In this case, the trajectory-tracking controller design would be more convenient, and this is one of the main reasons for the assumption. In fact, most UMVs do have port/starboard symmetry, but they do not have symmetry fore/aft; models will therefore include off-diagonal matrix elements. Nonzero off-diagonal elements of the damping and inertia matrices could lead to difficulties in system analysis and trajectory-tracking controller design. A trajectory-tracking controller is not hard to devise for a fully actuated UMV, but this is not true for an underactuated case. Though many trajectory-tracking controllers have been excogitated in different papers for underactuated UMVs, few studies have taken into consideration the asymmetry fore/aft of the UMV model. Some exceptions are [15, 17, 22, 23, 25, 37, 42], but these studies do not present any details about the trajectory-tracking differences between the symmetric and asymmetric model of the UMV. Meanwhile, as we can see from [2], ocean current is one of the most important environmental disturbances for an UMV working in the infinitely vast ocean; however, few studies have explicitly addressed ocean current in controller design, and no detailed descriptions of distinct trajectory-tracking control performances in the presence of different ocean-current velocities and direction angles have been presented. The well-known persistent exciting (PE) condition is required in [13, 17-19, 22, 24, 28, 38, 41], though the methods proposed in [11, 37] only need a mild PE condition; complete elimination of the PE condition is still difficult in trajectory-tracking control of an underactuated UMV. In the papers [25, 39, 40], the trajectory-tracking problem is decomposed into several subproblems, separately considering course control and position control; this could lead to loss of the global stability of the overall system, meaning the system would only be stable under certain conditions.
Here, motivated by the above considerations, a trajectory-tracking controller based on a nonlinear backstepping technique is proposed for an UMV with asymmetry fore/aft sailing in the horizontal plane in the presence of ocean currents. In order to study the distinct trajectory-tracking control performances in the presence of different ocean-current speeds and directions, in contrast to [43, 44], a frequently used simplification that assumes the ocean current is irrotational and constant in an inertial coordinate system is adopted in this paper, as in [2, 43-47]. In the context of the existing research results on trajectory-tracking controllers designed for UMVs, the main contributions of this paper are as follows: (i) a UMV without symmetry fore/aft is considered, and the trajectory-tracking differences between the symmetric model and the asymmetric model of the UMV are discussed; (ii) detailed descriptions of distinct trajectory-tracking control performances in the presence of different ocean-current speeds and directions are provided; (iii) a novel and ingenious coordinate transformation is achieved in the paper, which simplifies the trajectory-tracking system and overcomes the difficulties brought about by model asymmetry; (iv) a mild but effective integral term of the tracking error is introduced into the trajectory-tracking controller, which enhances the convergence and convergence rate of the control system; (v) the well-known PE condition is completely released in the proposed trajectory-tracking control algorithm; (vi) a new backstepping technique-based control algorithm is proposed for trajectory-tracking control of an underactuated UMV sailing in the presence of ocean currents.

Simulation experiments are carried out to verify the efficacy of the contributions and to evaluate the control performance of the designed trajectory-tracking controller.

The remainder of the paper is organized as follows. An underactuated UMV model without symmetry fore/aft is established in section 2, and the trajectory-tracking control problem is formulated. In section 3, a novel tracking controller is designed based on an integral backstepping approach, and the asymptotic stability of the closed-loop system is proven using Lyapunov theory and Barbalat’s Lemma. Simulation experiments are carried out on an UMV in a semi-physical simulation platform in section 4, and conclusions are given in section 5.

2. Problem Formulation

In this section, the trajectory-tracking control problem of an underactuated UMV sailing in the horizontal plane in the presence of ocean currents is formulated. The kinematic and dynamic equations of the UMV without symmetry fore/aft are presented in section 2.1, while the trajectory-tracking control problem statement for the underactuated UMV is provided in section 2.2.

2.1 UMV Modelling

An underactuated UMV without symmetry fore/aft is considered here, assuming that the ocean current is irrotational and constant. The state of the UMV is given by the vector \( \eta^T = [x, y, \phi]^T \), where \( \eta^T = [x, y, \phi]^T \) describes the position and the orientation of the UMV with respect to the inertial frame \( \{I\} \) as shown in Fig. 1.

![Inertial reference frame and body-fixed reference system](image)

**Figure 1.** Inertial reference frame and body-fixed reference system.

The vector \( \nu = [u, v, r]^T \) contains the linear and angular velocities of the vehicle defined in the body-fixed frame \( \{b\} \), where \( u \) is the surge velocity, \( v \) is the sway velocity, and \( r \) is the yaw rate. All the state variables, \( x, y, \phi, u, v, r \) can be obtained by shipboard sensors, such as global positioning systems (GPS) or acoustic positioning systems (APS), inertial navigation systems (INS), magnetic compasses, and so on. The ocean-current velocity in \( \{I\} \) is \( V_c \) and \( \beta \) denotes its direction. In navigation and control problems involving ocean currents, it is useful to introduce the relative velocity \( \nu_r \) which is defined in \( \{b\} \):

\[
\nu_r = \nu - \nu_c = [u - u_c, v - v_c, r]^T = [u_c, v_c, r]^T
\]

(1)

where \( u_c \) and \( v_c \) denote the surge and sway velocity of the current in \( \{b\} \), \( u_c \) is the relative surge velocity, and \( v_c \) is the relative sway velocity. The current velocity in \( \{b\} \) can be described as follows:

\[
\nu_c = [u_c, v_c, 0]^T = [V_c \cos(\beta - \phi), V_c \sin(\beta - \phi), 0]^T
\]

(2)

According to [48], the kinematic and dynamic equations of the asymmetric underactuated UMV sailing in the horizontal plane can be expressed as:

\[
\eta = R(\phi)\nu
\]

(3)

with \( \nu \) and \( \eta \) being the control input and the state of the system, respectively.
desired surge and yaw velocity

\[ \eta = [x, y, \varphi]^T \]

coordinate transformation

\[ \tau = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_r \end{bmatrix} \]

actuator

control command transformation

UMV

control force

actuator

control command transformation

UMV

controller

control input

backstepping method

virtual variables

control force

desired surge, yaw velocity

intermediate state variables

trajectory tracking errors

UMV state variables

desired trajectory

ocean currents

\[ \mathbf{R}(\varphi) = \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

and

\[ \mathbf{M}_o + \mathbf{C}(\mathbf{u}) \mathbf{u}_o + \mathbf{D}_o = \tau \]

with

\[ \mathbf{M} = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix} \]

\[ \mathbf{C}(\mathbf{u}) = \begin{bmatrix} 0 & 0 & -m_{22} \varphi \ - m_{23} \tau \\ m_{22} \varphi + m_{23} \tau \ & m_{11} \mathbf{u} \ & 0 \end{bmatrix} \]

where \( \tau_x \) and \( \tau_y \) are the surge force and yaw moment, \( m_{11}, m_{22}, m_{23}, m_{32}, \) and \( m_{33} \) are the UMV’s inertia coefficients, including added mass effects and \( m_{23} = m_{32} \) and \( d_{11}, d_{22}, d_{23}, d_{32} \) are hydrodynamic damping coefficients. For an UMV with both port/starboard and fore/ aft symmetry, all off-diagonal elements of the damping and inertia matrices are zero, which means \( \mathbf{M} \) and \( \mathbf{D} \) are diagonal matrices and \( m_{23}, m_{32}, d_{23}, \) and \( d_{32} \) are zero. In this situation, the controller design would be more convenient, but the accuracy of the controller would be greatly reduced.

2.2 Problem formulation

The general trajectory-tracking control problem of an underactuated UMV considered in this paper can be formulated as follows:

Consider an arbitrary trajectory expressed in \{t\} with desired state \( \mathbf{u}_d = [x_d(t) \ y_d(t) \ \varphi_d(t)]^T \) and directly given surge and yaw reference velocities \( u_d(t) \) and \( \varphi_d(t) \), while the reference sway velocity \( v_d(t) \) is generated by a virtual vehicle as described in [19]:

\[
\begin{align*}
\dot{x}_d(t) &= u_d(t) \cos(\varphi_d(t)) - v_d(t) \sin(\varphi_d(t)) \\
\dot{y}_d(t) &= u_d(t) \sin(\varphi_d(t)) + v_d(t) \cos(\varphi_d(t)) \\
\dot{\varphi}_d(t) &= r_d(t) \\
\dot{v}_d(t) &= -m_{22} u_d(t) r_d(t) + d_{22} v_d(t) + d_{32} \tau_d + m_{33} r_d^2 / m_{32} 
\end{align*}
\]

Define the tracking errors as follows:

\[
\begin{align*}
\mathbf{e}_u &= \begin{bmatrix} x - x_d \\ y - y_d \\ \varphi - \varphi_d \end{bmatrix} \\
\mathbf{e}_v &= \begin{bmatrix} u - u_d \\ v - v_d \\ r - r_d \end{bmatrix} 
\end{align*}
\]

Thus, the control objective for trajectory tracking of the UMV is to design control laws \( \tau_x \) and \( \tau_y \) to ensure the tracking errors \( \mathbf{e}_u \) and \( \mathbf{e}_v \) converge to an arbitrarily small neighbourhood of zero as \( t \to \infty \).

3. Controller Design

A trajectory-tracking control law for an underactuated UMV is presented in this section. The main process of the controller design and its implementation is shown in Figure 2.

3.1 Coordinate transformation

In order to solve the difficulties brought about by model asymmetry, coordinate transformation needs to make the kinematic and dynamic equations of the UMV easier, and simplify the trajectory-tracking error system.

Step 1:
Expression (3) can be rewritten as:
\[ \dot{\eta} = R(\varphi)\dot{u} = R(\varphi)(\dot{u}_c + \dot{u}_s) \] (7)
Differentiating both sides of (7) results in:
\[ \ddot{\eta} = R(\varphi)\ddot{u} + R(\varphi)\dot{u}_c + R(\varphi)\dot{u}_s \]
\[ = R(\varphi)\ddot{u} + R(\varphi)\dot{u}_c + R(\varphi)\dot{u}_s \] (8)

Note that:
\[ R(\varphi)\dot{u}_c + R(\varphi)\dot{u}_s = 0 \] (9)

Expression (4) can be rewritten as:
\[ \ddot{u}_s = -M^{-1}\left[C(u)\dot{u}_c + Du_e\right] + M^{-1}\tau \] (10)
Substituting (10) into (8) yields:
\[ \ddot{\eta} = R(\varphi)\ddot{u} + R(\varphi)\left[-M^{-1}\left[C(u)\dot{u}_c + Du_e\right] + M^{-1}\tau\right] \] (11)
and (11) can be equivalent to:
\[ \text{MR}^{-1}(\varphi)\ddot{\eta} - \text{MR}^{-1}(\varphi)R(\varphi) - C(u)\dot{u}_c - D\dot{u}_e = \tau \] (12)

Expanding (12) makes for:
\[ \begin{bmatrix} m_1 \cos(\varphi)\dddot{x} + m_{11} \sin(\varphi)\dddot{y} + \alpha_1 = \tau_x \\
-m_{12} \sin(\varphi)\dddot{x} + m_{12} \cos(\varphi)\dddot{y} + m_{13}\dddot{\varphi} + \alpha_2 = 0 \\
-m_{13} \sin(\varphi)\dddot{x} + m_{13} \cos(\varphi)\dddot{y} + m_{13}\dddot{\varphi} + \alpha_3 = \tau_r \end{bmatrix} \] (13)

where:
\[ \alpha_1 = d_1\dddot{u} + (m_1 - m_{22})u \dddot{r} - m_{12} \dddot{r} + d_1 \dot{u}_r + d_1 \dot{u}_r - (m_1 - m_{22})u \dot{r} \]
\[ \alpha_2 = d_2\dddot{u} + (m_1 - m_{22})u \dddot{r} - m_{12} \dddot{r} + d_2 \dot{u}_r + d_2 \dot{u}_r - (m_1 - m_{22})u \dot{r} \]
\[ \alpha_3 = d_3\dddot{u} - (m_1 - m_{22})u \dddot{r} + d_3 \dot{u}_r + d_3 \dot{u}_r - (m_1 - m_{22})u \dot{r} \]
Step 2:
Consider the following new variables:
\[ \begin{bmatrix} x_1 & x_2 & x_3 \\
 x_4 & x_5 & x_6 \end{bmatrix} = \begin{bmatrix} x & y & \varphi \\
 \dot{x} & \dot{y} & \dot{\varphi} \end{bmatrix} \] (14)
Follow the control input transformation as follows:
\[ \begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \dot{x}_3 \\
 \dot{x}_4 & \dot{x}_5 & \dot{x}_6 \end{bmatrix} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\varphi} \end{bmatrix} \]

The kinematic and dynamic equations of the UMV, expressions (3) and (4), can be transformed into:
\[ \begin{bmatrix} \dot{x}_1 = x_4 \\
 \dot{x}_2 = x_5 \\
 \dot{x}_3 = x_6 \\
 \dot{x}_4 = u_1 \\
 \dot{x}_5 = -\sec(x_4)\dot{x}_2 z_4 + u_1 \tan(x_4) - m_{23} u_2 \sec(x_4) / m_{22} \\
 \dot{x}_6 = u_2 \] (16)
Step 3:
The desired trajectory in (5) can be transformed into the following form in the same way:
\[ \begin{bmatrix} \dot{x}_1 = x_4 \\
 \dot{x}_2 = x_5 \\
 \dot{x}_3 = x_6 \\
 \dot{x}_4 = x_{4d} \\
 \dot{x}_5 = x_{5d} \\
 \dot{x}_6 = x_{6d} \] (17)
From (16) and (17), new tracking errors can be designed as follows:
\[ e_i = x_i - x_{id}, i = 1, 2, ..., 6 \] (18)
Differentiating both sides of (18) leads to:

\[
\begin{aligned}
\dot{e}_1 &= e_4 \\
\dot{e}_2 &= e_3 \\
\dot{e}_3 &= e_6 \\
\dot{e}_4 &= \delta + \tau_1 \tan(e_3 + x_{3d}) - m_{12} \tau_2 \sec(e_3 + x_{3d}) / m_{22} \\
\dot{e}_5 &= \tau_2
\end{aligned}
\]

where:

\[
\tau_1 = u_1 - u_{1d} , \tau_2 = u_2 - u_{2d} ,
\]

\[
\delta = [ - \sec(e_3 + x_{3d}) \chi_{d2} / m_{22} + m_{12} \tan(e_3 + x_{3d}) - m_{22} \sec(e_3 + x_{3d}) x_{3d} / m_{12} ] \\
= [ d_{22}(e_3 + x_{3d}) + (m_{12} - m_{22})(e_3 + x_{3d}) ] / m_{22} - [ \sec(x_{3d}) \chi_{d2} / m_{12} + u_{1d} \tan(x_{3d}) - m_{22} x_{3d} / m_{12} ]
\]

Differentiating both sides of (18) leads to:

\[
\phi = e_5 + \delta - e_1(c_3 + x_{3d}) \sec^2(c_3 + x_{3d}) + m_{22} e_2(c_3 + x_{3d}) \sec(c_3 + x_{3d}) \tan(c_3 + x_{3d}) / m_{22}
\]

where \(\delta\) is a positive constant. Differentiating both sides of (20) results in:

\[
\begin{aligned}
\dot{e}_1 &= e_4 - k_1[e_2 + e_3 \tan(e_3 + x_{3d}) + m_{22} e_4 \sec(e_3 + x_{3d}) / m_{22}] \\
\dot{e}_2 &= e_3 - k_1[e_1 + e_3 \tan(e_3 + x_{3d}) + m_{22} e_4 \sec(e_3 + x_{3d}) / m_{22}] \\
\dot{e}_3 &= e_4 - k_1[e_1 + e_3 \tan(e_3 + x_{3d}) + m_{22} e_4 \sec(e_3 + x_{3d}) / m_{22}] \\
\dot{e}_4 &= e_5 - k_1[e_1 + e_3 \tan(e_3 + x_{3d}) + m_{22} e_4 \sec(e_3 + x_{3d}) / m_{22}]
\end{aligned}
\]

3.2 Controller design

Step 1:

Motivated by [15-20, 49-51], the following virtual error can be designed:

\[
e_3 = e_4 - k_1[e_2 + e_3 \tan(e_3 + x_{3d}) + m_{22} e_4 \sec(e_3 + x_{3d}) / m_{22}]
\]

Differentiating both sides of (19) leads to:

\[
\dot{e}_1 = e_2 + \omega_1 - k_\phi \\
\dot{e}_2 = e_3 = \omega_2 + k_\phi
\]

where \(\omega_1\) and \(\omega_2\) are positive constants, and expression (21) leads to:

\[
\dot{e}_1 = e_2 + \omega_1 - k_\phi
\]

Differentiating both sides of the third expression of (22) results in:

\[
\dot{e}_1 = e_2 + \omega_1 - k_\phi
\]
\[
\frac{d\theta}{d\xi} = \frac{d_2}{m_2} \tan(e + x_{sd}) \quad \frac{m_2 - (m_{11} - m_{22})(e + x_{ad})}{m_2} - \tan(e + x_{sd}) \sec^2(e + x_{sd})
\]

\[
\frac{d\phi}{d\xi} = \frac{1}{m_2} \tan(e + x_{sd}) \quad \frac{m_2 - (m_{11} - m_{22})(e + x_{ad})}{m_2} - \tan(e + x_{sd}) \sec(e + x_{sd})
\]

\[
\frac{d\theta}{d\xi} = \frac{1}{m_2} \tan(e + x_{sd}) = \frac{1}{m_2} \sec(e + x_{sd}) = \frac{1}{m_2} \sec(e + x_{sd}) 
\]

\[
(\partial X_{sd}/\partial c_{a}) = \frac{-d_{22} + (m_{11} - m_{22})x_{ad}x_{sd}}{x_{sd} \sin(\beta - x_{sd})} + d_{22} V \cos(\beta - x_{sd})
\]

\[
(\partial X_{sd}/\partial c_{b}) = d_{22} \cos(x_{sd}) + (m_{11} - m_{22})x_{ad} \sin(x_{sd})
\]

\[
\alpha_t = 1 - k_1(\partial\phi/\partial\xi) - k_1 \tan(e + x_{sd})(\partial\phi/\partial\xi)
\]

\[
\omega_\xi = k_1[(\partial\phi/\partial\xi)\dot{e}_1 + (\partial\phi/\partial\xi)\dot{e}_2 + (\partial\phi/\partial\xi)\dot{e}_3 + (\partial\phi/\partial\xi)\delta + \omega_\xi]
\]

Step 2:

As in step 1, the following variables can be designed:

\[
e_1 = e_1 - k_2 [e_1 + e_2 \tan(e + x_{sd}) + m_2 e_2 \sec(e + x_{sd})/m_22]
\]

\[
\dot{e}_1 = \int_0^1 e_1 dt
\]

\[
\alpha_t = -\rho_2 e_2 - \lambda_2 \gamma_2 + k_2 \phi
\]

\[
\alpha_t = e_4 - e_2
\]

where \(k_2, \rho_2\) and \(\lambda_2\) are all positive constants.

Differentiating both sides of (25) results in:

\[
\dot{e}_3 = e_3 - k_3 [e_1 + e_2 \tan(e + x_{sd}) - x_{sd} e_2 \sec^2(e + x_{sd}) + m_2 e_2 \sec(e + x_{sd})/m_22 + m_2 e_2 \sec(e + x_{sd})/m_22]
\]

\[
\dot{e}_4 = \int_0^1 e_4 dt
\]

\[
\alpha_t = -\rho_3 e_3 - \lambda_3 \gamma_3 + k_3 \phi
\]

\[
\alpha_t = e_5 - e_3 - k_3 \phi
\]

Differentiating both sides of the third expression of (26) leads to:

\[
\dot{e}_3 = e_3 - k_3 [e_1 + e_2 \tan(e + x_{sd}) - x_{sd} e_2 \sec^2(e + x_{sd}) + m_2 e_2 \sec(e + x_{sd})/m_22 + m_2 e_2 \sec(e + x_{sd})/m_22 + m_2 e_2 \sec(e + x_{sd})/m_22]
\]

\[
\dot{e}_4 = \int_0^1 e_4 dt
\]

\[
\alpha_t = -\rho_3 e_3 - \lambda_3 \gamma_3 + k_3 \phi
\]

\[
\alpha_t = e_5 - e_3 - k_3 \phi
\]
\[ \hat{e}_i = \dot{e}_i - \dot{e}_i \]

\[ \tau_1 + \rho_2 e_i + \lambda_2 j_2 - k_j \phi = \tau_2 + \rho_2 e_1 + \lambda_2 j_2 - k_j (\dot{\phi} / \dot{\varepsilon}) e_1 + \phi (\dot{\phi} / \dot{\varepsilon}) e_1 + (\dot{\phi} / \dot{\varepsilon}) e_1 + (\dot{\phi} / \dot{\varepsilon}) e_1 + (\dot{\phi} / \dot{\varepsilon}) e_1 \]

\[ = -k_1 (\dot{\phi} / \dot{e}) - k_2 \tan(e_1 + x_{m_2}) (\dot{\phi} / \dot{e}) \tau_1 + [1 + k_1 \times (\dot{\phi} / \dot{e}) m_2 \sec(e_1 + x_{m_2}) / m_2 - k_2 (\dot{\phi} / \dot{e})] \tau_2 + \rho_2 \times e_1 + \lambda_2 j_2 - k_1 e_1 (\dot{\phi} / \dot{e}) + e_2 (\dot{\phi} / \dot{e}) + e_3 (\dot{\phi} / \dot{e}) + \delta (\dot{\phi} / \dot{e}) + \omega_1 ] \]

\[ \Omega_2 = -k_1 [e_1 \dot{\phi} / \dot{e} + e_2 \dot{\phi} / \dot{e} + e_3 \dot{\phi} / \dot{e} + \delta (\dot{\phi} / \dot{e}) + \omega_1 ] \]

\[ \omega_1 = k_1 (\dot{\phi} / \dot{e}) (e_1 + x_{m_2}) \sec(e_1 + x_{m_2}) / m_2 - k_2 (\dot{\phi} / \dot{e}) \]

\[ \Omega_2 = -k_1 [e_1 \dot{\phi} / \dot{e} + e_2 \dot{\phi} / \dot{e} + e_3 \dot{\phi} / \dot{e} + \delta (\dot{\phi} / \dot{e}) + \omega_1 ] \]

\[ \text{Step 3:} \]

From expressions (24) and (28), the controls \( \tau_1 \) and \( \tau_2 \) can be designed as follows:

\[ \tau_1 = \left[ \omega_1 (\rho_2 e_1 + \lambda_2 j_2 + \Omega_2) + e_1 + \rho_2 e_1 - \omega_1 (\rho_2 e_1 + \lambda_2 j_2 + \Omega_2) \right] \]

\[ \tau_2 = \left[ \omega_1 (\rho_2 e_1 + \lambda_2 j_2 + \Omega_2) + e_1 + \rho_2 e_1 - \omega_1 (\rho_2 e_1 + \lambda_2 j_2 + \Omega_2) \right] \]

where \( \rho_2 \) and \( \rho_1 \) are both positive constants. Then, the actual control inputs \( \tau_u \) and \( \tau_r \) can be obtained as follows:

\[ \tau_u = m_1 \sec(x_1 (\tau_2 + u_2) - m_1 m_2 \tan(x_1) X_2) / m_2 \]

\[ + \chi_1 - m_1 \tan(x_1) X_2 / m_2 \]

\[ \tau_r = (m_2 m_3 - m_1 X_2) (\tau_2 + u_2) / m_2 - m_2 X_2 / m_2 + X_3 \]

\[ \text{3.3 Stability analysis} \]

\[ \text{Theorem 3.1} \]

The control inputs \( \tau_u \) and \( \tau_r \) given in (30) can achieve trajectory tracking of an arbitrary reference trajectory for the USV with the dynamics given in (3) and (4). In particular, for any initial conditions \( \eta(0) = [\eta(0) / (0) / (0) / (0)] \) and \( \psi(0) = [\psi(0) / (0) / (0) / (0)] \), the trajectory-tracking errors \( \eta = [\eta(t) / \eta(t) / \eta(t) / \eta(t)] \) and \( \psi = [\psi(t) / \eta(t) / \eta(t) / \eta(t)] \) will globally asymptotically converge to zero as \( t \to \infty \) under the operation of the control law given in (30).

\[ \text{Proof:} \]

Theorem 3.1 can be proven in three steps. The first step is to prove that the closed-loop system consisting of (23), (24), (27), and (28) shows asymptotic stabilization under the control inputs \( \tau_1 \) and \( \tau_2 \) in (29). In the second step, the tracking errors \( e_1 / \eta_i, i = 1, \ldots, 6 \) described in (18) can be proven to converge to zero as \( t \to \infty \). Finally, the trajectory-tracking errors \( \eta = [\eta(t) / \eta(t) / \eta(t) / \eta(t)] \) and \( \psi = [\psi(t) / \eta(t) / \eta(t) / \eta(t)] \) can be proven to globally asymptotically converge to zero in the third step.

\[ \text{Step 1:} \]

Substituting (29) into (24) and (28), combining (22), (23), (26), and (27) gives:

\[ \hat{e}_1 = e_1 \]

\[ \hat{e}_2 = e_2 \]

\[ \hat{e}_3 = e_3 \]

\[ \hat{e}_4 = -e_1 - \rho_2 e_1 - \lambda_2 j_2 \]

\[ \hat{e}_5 = e_4 - \rho_2 e_4 \]

\[ \hat{e}_6 = e_6 - \rho_2 e_6 \]

\[ \text{Consider the following Lyapunov function candidate:} \]

\[ V = \frac{1}{2} \lambda_1 e_1^2 + \frac{1}{2} \lambda_2 e_2^2 + \frac{1}{2} e_3^2 + \frac{1}{2} e_4^2 + \frac{1}{2} e_5^2 \leq 0 \]

\[ \text{Differentiating both sides of (32) based on the solutions of (31) results in:} \]

\[ \dot{V} = -2 \rho_2 e_1 e_2 - 2 \rho_2 e_1 e_4 - 2 \rho_2 e_4 e_2 - 2 \rho_2 e_4 e_4 \]

\[ \text{Step 2:} \]

This step aims to prove that the tracking errors \( e_1, \ldots, 6 \) described in (18) converge to zero. As \( e_1 \) converges to zero and \( k_1 \) is just an arbitrarily positive parameter, equation (20) implies that:

\[ \lim_{t \to \infty} e_1 = \lim_{t \to \infty} e_1 + e_1 - e_1 \tan(e_3 + x_{m_2}) = 0 \]

Similarly, equation (25) implies that:
\[
\lim_{t \to \infty} e_2 = \lim_{t \to \infty} [e_2 + e_3 - e_4 \tan (e_3 + x_{1m})] = 0
\]  
(36)

Equations (22) and (26) imply that:
\[
\lim_{t \to \infty} e_4 = \lim_{t \to \infty} e_6 = \lim_{t \to \infty} \phi = 0
\]  
(37)

Equations (19), (21), (35), (36) and (37) imply that:
\[
\lim_{t \to \infty} e_2 = \lim_{t \to \infty} e_5 = 0
\]  
(38)

This proves that the tracking errors \( e_i, i = 1, ..., 6 \) described in (18) converge to zero.

Step 3:
This step aims to prove that the control laws \( \tau_u \) and \( \tau_v \) in (30) can have tracking errors \( \eta = [x(t) y(t) \phi(t)]^T \) and \( \nu = [u(t) v(t) r(t)]^T \) converging to zero as \( t \to \infty \). As it is proven in step 2 that \( e_i, i = 1, 2, 3 \) converge to zero, so it is proven that the tracking errors \( \eta = [x(t) y(t) \phi(t)]^T \) converge to zero as \( t \to \infty \). Equation (3) can be rewritten as:
\[
\begin{align*}
\dot{x} &= u \cos(\phi) - v \sin(\phi) \\
\dot{y} &= u \sin(\phi) + v \cos(\phi) \\
\dot{\phi} &= r
\end{align*}
\]
and then
\[
\begin{align*}
u &= \dot{x} \cos(\phi) + \dot{y} \sin(\phi) \\
v &= \dot{y} \cos(\phi) - \dot{x} \sin(\phi) \\
r &= \dot{\phi}
\end{align*}
\]
So, \( e_i, i = 3, 4, 5, 6 \) converge to zero, implying that \( \psi, \dot{x}, \dot{y}, \) and \( \phi \) converge to \( \psi_d, \dot{x}_d, \dot{y}_d, \) and \( \phi_d \); then, \( u, v, \) and \( r \) converge to \( u_d, v_d, \) and \( r_d \). Thus, it is proven that \( \nu = [u(t) v(t) r(t)]^T \) converge to zero as \( t \to \infty \).

The proof of Theorem 3.1 is complete.

4. Simulation Experiment
In order to verify and illustrate the effectiveness of the trajectory-tracking control schemes proposed for the underactuated UMV, several computer simulation experiments are carried out on a UMV model with hydrodynamic parameters: \( m_1 = 47.5, m_2 = 94.1, m_3 = 13.6, m_3 = 5.2, m_3 = 5.2, d_{11} = 13.5, d_{22} = 50.2, d_{33} = 27.2, d_{22} = 41.4, \) and \( d_{33} = 17.3 \). The initial position and heading angle of the UMV is chosen as: \( x(0) = 10m, y(0) = -10m, \) and \( \phi(0) = 0rad \), with initial velocities \( u(0) = \dot{x}(0) = 0 \). The velocity of the constant ocean current is chosen as \( V_c = 1m/s \) and the direction angle is chosen as \( \beta_c = 30^\circ \). The reference velocities are chosen as \( u_g(t) = 3m/s \) and \( r_g(t) = 0.05rad/s \) with the initial desired state variables \( x_d(0) = y_d(0) = \phi_d(0) = 0 \).

Parameters of the trajectory-tracking controller designed above are chosen as: \( k_1 = 10, k_2 = 5, \lambda_1 = 0.5, \lambda_2 = 0.2, \rho_1 = 1.5, \rho_2 = 3, \rho_3 = 2, \) and \( \rho_4 = 1 \). The criteria used to select the parameters are based on the following procedure: (1) parameters \( p_1, p_2, p_3, \) and \( p_4 \) are selected to tune the convergence rate of variables \( \epsilon_1, \epsilon_2, \epsilon_3 \) and \( \epsilon_4 \) to zero, respectively; (2) parameters \( \lambda_1, \lambda_2 \) are chosen to adjust the performance of the integral action of \( \epsilon_1 \) and \( \epsilon_3 \) in order to enhance the convergence rate of \( \epsilon_1 \) and \( \epsilon_3 \); (3) parameters \( k_1, k_2 \) are used to regulate the convergence to zero of variables \( \epsilon_1, \epsilon_3 \), and then variables \( \epsilon_2, \epsilon_4, \epsilon_5, \) and \( \epsilon_6 \) converge to zero accordingly.

Case 1: In this case, the underactuated UMV model without symmetry fore/aft is taken into consideration, and the simulation results are shown below in Figs. 3-6. In Fig. 6, as the initial heading angle of the UMV \( \phi(0) \) and the initial desired heading angle \( \phi_d(0) \) are both 0, the desired trajectory of the manoeuvring of the UMV is a curve; the ocean-current disturbance in the direction of surge and sway velocity will change with the heading angle, and therefore the control input is oscillated. However, as the UMV’s actual state gradually converges to the desired state, the control input gradually converges to a stable value.

![Figure 3. Trajectory-tracking results of Case 1](image-url)

Case 2: For a more detailed analysis and discussion of the trajectory-tracking differences between the controller based on the symmetric model and that based on the asymmetric model, a simulation experiment with a controller designed based on the symmetric model is carried out in Case 2. The simulation results are shown below in Fig. 7. The symmetric-model-based controller is designed the same way as the asymmetric-model-based controller, as presented in this paper; only, the off-diagonal elements of the damping and inertia matrices \( m_{22}, m_{33}, d_{22}, d_{33} \) are all treated as zero during the design process of the former.
Case 3: In order to provide a detailed description of the distinct implications of different ocean-current velocities for trajectory-tracking control of the underactuated UMV, two other ocean-current velocities, \( V_c = 0.5 \text{ m/s} \) and \( V_c = 1.5 \text{ m/s} \), are chosen for simulation experiments with the direction angle \( \beta_c = 30 \); the initial state and desired state of the UMV are the same as in Case 1. As the velocities of ocean currents in most areas are less than 1.5 m/s [53], no velocities greater than this are chosen. The simulation results of different ocean-current velocities are shown in Fig. 8 below:

![Figure 8. Trajectory-tracking results of Case 3](image)

Case 4: With the purpose of providing a detailed description of the distinct implications of different ocean-current direction angles for trajectory-tracking control of the underactuated UMV, two other ocean current-direction angles, \( \beta_c = 45 \) and \( \beta_c = 60 \), are chosen for simulation experiments, with the velocities both chosen as \( V_c = 1 \text{ m/s} \); the initial state and desired state of the UMV are the same as in Case 1. As the tracking results of the angles \( \beta_c = -30 \), \( \beta_c = -45 \), and \( \beta_c = -60 \) are almost similar, and the results of \( \beta_c \in (-90, 90) \) are similar to those of \( \beta_c \in (90, 270) \), no more direction angles are chosen. The simulation results for different ocean-current direction angles are shown in Fig. 9 below:

![Figure 9. Trajectory-tracking results of Case 4](image)

Case 5: To verify that the well-known PE condition is completely released in the designed trajectory-tracking controller, a desired straight-line trajectory is selected for simulation. The initial position of the UMV is chosen as \( x(0)=3 \text{ m} \), \( y(0)=4 \text{ m} \), and \( v_x(t)=3 \text{ m/s} \) and \( r(t)=0 \) are chosen
in order to produce the desired straight-line trajectory. The initial state and desired state of the UMV are the same as in Case 1. The simulation results are shown below in Figs. 10-13.

A series of simulation experiments were carried out to demonstrate the presented theorem and verify the effectiveness and superiority of the control algorithm proposed in this paper. The simulation results shown in Figs. 3 and 10 indicate that the well-known PE condition is completely released in the proposed trajectory-tracking control algorithm, and both curved and straight-line trajectories can be tracked. In addition, the results shown in Figs. 4-6 and 11-13 show that all the state variables converge to the desired stable value, and the control input convergence to a stable nonzero value compensates for the effects of the ocean currents on the UMV.

The simulation results shown in Fig. 7 show that design of the trajectory-tracking controller without consideration of asymmetry fore/aft of the UMV model will lead to obvious tracking errors; asymmetry fore/aft of the UMV model therefore needs to be taken into consideration for precise trajectory-tracking control. Moreover, from the results shown in Figs. 8 and 9, it is obvious that the trajectory-tracking controller designed in this paper is robust to both the velocity and the direction angle of the ocean current; faint distinct effects of different velocities and direction angles are shown. Furthermore, the results in Figs. 8 and 9 suggest that the integral backstepping-based control...
approach may be suitable for trajectory-tracking control of
the UMV in the presence of unsteady ocean currents. This
can be taken as an indication of profitable future research
directions.

5. Conclusions

Based on an asymmetrical vehicle model, a nonlinear
integral backstepping algorithm has been proposed for
trajectory-tracking control of an underactuated UMV
sailing in the presence of irrotational and constant currents.
A mild but effective integral term of the tracking error is
introduced into the controller to improve the asymptotic
stability of the trajectory-tracking control system. The well-
known PE condition is completely released in the presented
control algorithm, and both curved and straight-line
trajectories can be tracked by the designed trajectory-
tracking controller. Asymmetry fore/aft of the UMV model
needs to be taken into consideration for precise trajectory-
tracking control, as considerable differences in tracking
control performance are revealed between the controller
based on a symmetric model and that based on an asym-
metric model. This has been discussed in an original way
in this paper. The designed trajectory-tracking controller is
robust to both velocity and direction angle of ocean current;
faint distinct effects of different current velocities and
direction angles on control performance are shown. Future
work will focus on expanding the integral backstepping
control approach for trajectory-tracking control of the UMV
in the presence of unsteady ocean currents, as suggested by
the analysis of the distinct control performance implica-
tions of different current velocities and direction angles.
With the aid of Lyapunov theory and Barbalat’s Lemma,
the asymptotic stability of the UMV trajectory-tracking
control system has been theoretically proven here in theory
as well as demonstrated in simulation experiments with an
underactuated UMV model without asymmetry fore/aft.

6. Acknowledgements

We would like to acknowledge the support of the National
High Technology Research and Development Programme
863 of PR China (Nos. 2012AA09A304 and 2014AA09A509)
and the National Natural Science Foundation of China
(Nos. 51409054, 51409059, 51409061, 51579022, 51509057).

7. References

[1] Sharma S. K, Sutton R, Motwani A et al. (2014) Non-
linear control algorithms for an unmanned surface
vehicle. Proceedings of the Institution of Mechani-
cal Engineers, part M. Journal of Engineering for the
Maritime Environment, vol. 228, no. 2, pp. 146-155.

[2] Caharija W, Pettersen K. Y, Sorensen A. J et al. (2014)
Relative velocity control and integral line of sight
for path following of autonomous surface vessels:
Merging intuition with theory. Proceedings of the
Institution of Mechanical Engineers, part M. Journal
of Engineering for the Maritime Environment, vol.
228, no. 2, pp. 180-191.

[3] Singh W, Ornolfsdottir E. B, Stefansson G. (2014) A
small scale comparison of Iceland scallop size
distributions obtained from a camera based auton-
omous underwater vehicle and dredge survey. Plos
One, vol. 9, no. 10, pp. 1-10.

[4] Marzinelli E. M, Williams S. B, Babcock R. C et al.
(2015) Large-scale geographic variation in distribu-
tion and abundance of Australian deep-water kelp
forests. Plos One, vol. 10, no. 2, pp. 1-12.

[5] Sutton R, Sharma S. K (2014) Special issue on
unmanned marine vehicles. Proceedings of the
Institution of Mechanical Engineers, part M. Journal
of Engineering for the Maritime Environment, vol.
228, no. 2, pp. 107.

[6] Farrell J. A, Polycarpou M, Sharma M, Dong W. J.
(2009) Command filtered backstepping. IEEE
Transactions on Automatic Control, vol. 54, no. 6,
pp. 1391-1395.

[7] Xu B, Shi Z. K, Yang C. G, Sun F. C. (2014) Composite
neural dynamic surface control of class of uncertain
nonlinear systems in strict-feedback form. IEEE
Transactions on Cybernetics, vol. 44, no. 12, pp.
2626-2634.

[8] Xu B, Yang C. G, Pan Y. P (2015) Global neural
dynamic surface tracking control of strict-feedback
systems with application to hypersonic flight
vehicle. IEEE Transactions on Neural Networks and
Learning Systems, vol. 26, no. 10, pp. 2563-2575.

[9] Xu B, Guo Y. Y, Yuan Y, Fan Y. H, Wang D. W (2015)
Fault-tolerant control using command-filtered
adaptive back-stepping technique: Application to
hypersonic longitudinal flight dynamics. Interna-
tional Journal of Adaptive Control and Signal
Processing. DOI: 10.1002/acs.2596

[10] Soltan R. A, Ashrafiuon H, Muske K. R (2009) State
dependent trajectory planning and tracking control
of unmanned surface vessels. In: Proceedings of
American Control Conference (ACC2009), pp.
3597-3602, IEEE, St. Louis, MO, USA, Jun 10-12.

[11] Yu R. T, Zhu Q. D, Xia G. H et al. (2012) Sliding mode
tracking control of an underactuated surface vessel.
IET Control Theory and Applications, vol. 6, no. 3,
pp. 461-466.

[12] Fahimi F, Van Kleeck C (2013) Alternative trajectory
tracking control approach for marine surface
vessels with experimental verification. Robotica,
vol. 31, no. 1, pp. 25-33.

[13] Yang Y. C, Yang K. S, Chen C. Y et al. (2013) Robust
trajectory control for an autonomous underwater
vehicle. In Oceans-IEEE, pp. 1-9, IEEE, Bergen,
Norway, Jun 10-14.

[14] Joe H, Kim M, Yu S. C (2014) Second-order sliding-
mode controller for autonomous underwater
vehicle in the presence of unknown disturbances. Nonlinear Dynamics, vol. 78, pp. 183-196.

[15] Do K. D, Pan J (2005) Global tracking control of underactuated ships with nonzero off-diagonal terms in their system matrices. Automatica, vol. 41, no. 1, pp. 87-95.

[16] Tee K. P, Ge S. Z. S (2006) Control of fully actuated ocean surface vessels using a class of feedforward approximators. IEEE Transactions on Control Systems Technology, vol. 14, no. 4, pp. 750-756.

[17] Repoulias F, Papadopoulos E (2007) Planar trajectory planning and tracking control design for underactuated AUVs. Ocean Engineering, vol. 34, no. 11-12, pp. 1650-1667.

[18] Yang Y, Du J. L, Liu H. B et al. (2014) A trajectory tracking robust controller of surface vessels with disturbance uncertainties. IEEE Transactions on Control Systems Technology, vol. 22, no. 4, pp. 1511-1518.

[19] Liao Y. L, Su Y. M, Cao J. (2014) Trajectory planning and tracking control for underactuated unmanned surface vessels. Journal of Central South University, vol. 21, no. 2, pp. 540-549.

[20] Xie W. J, Ma B. L (2015) Universal practical tracing control of a planar underactuated vehicle. Asian Journal of Control, vol. 17, no. 3, pp. 1016-1026.

[21] Zhang L. J, Qi X, Pang Y. J et al. (2013) Adaptive output feedback control for trajectory tracking of AUV in wave disturbance condition. International Journal of Wavelets Multiresolution and Information Processing, vol. 11, no. 3, pp. 1-15.

[22] Pan C. Z, Lai X. Z, Yang S. X et al. (2013) An efficient neural network approach to tracking control of an autonomous surface vehicle with unknown dynamics. Expert Systems with Applications, vol. 40, pp. 1629-1635.

[23] Cui R. X, Yang C. G, Li Y et al. (2014) Neural network based reinforcement learning control of autonomous underwater vehicles with control input saturation. In: 2014 UKACC International Conference on Control (CONTROL), pp. 50-55, IEEE, Loughborough, United Kingdom, Jul 09-11.

[24] Aguiar A. P, Hespanha J. P (2007) Trajectory-tracking and path-following of underactuated autonomous vehicles with parametric modeling uncertainty. IEEE Transactions on Automatic Control, vol. 52, no. 8, pp. 1362-1379.

[25] Dai S. L, Wang C, Luo F (2012) Identification and learning control of ocean surface ship using neural networks. IEEE Transactions on Industrial Informatics, vol. 8, no. 4, pp. 801-810.

[26] Sun B, Zhu D. Q, Ding F et al. (2013) A novel tracking control approach for unmanned underwater vehicles based on bio-inspired neurodynamics. Journal of Marine Science Technology, vol. 18, no. 1, pp. 63-74.

[27] Chen X. T, Tan W. W (2013) Tracking control of surface vessels via fault-tolerant adaptive backstepping interval type-2 fuzzy control. Ocean Engineering, vol. 70, pp. 97-109.

[28] Zhu D. Q, Sun B (2013) The bio-inspired model based hybrid sliding-mode tracking control for unmanned underwater vehicles. Engineering Applications of Artificial Intelligence, vol. 26, no. 10, pp. 2260-2269.

[29] Zhao Z, He W, Ge S. S (2014) Adaptive neural network control of a fully actuated marine surface vessel with multiple output constraints. IEEE Transactions on Control Systems Technology, vol. 22, no. 4, pp. 1536-1543.

[30] Sarkara M, Nandy S, Shome S. N (2015) Energy efficient trajectory tracking controller for underwater applications: A robust approach. Aquatic Procedia, vol. 4, pp. 571-578.

[31] Shojaei K, Arefi M. M (2015) On the neuro-adaptive feedback linearising control of underactuated autonomous underwater vehicles in three-dimensional space. IET Control Theory and Applications, vol. 9, no. 8, pp. 1264-1273.

[32] Miao B. B, Li T. S, Luo W. L (2013) A DSC and MLP based robust adaptive NN tracking control for underwater vehicle. Neurocomputing, vol. 111, pp. 184-489.

[33] Park B. S (2015) Neural network-based tracking control of underactuated autonomous underwater vehicles with model uncertainties. Journal of Dynamic Systems, Measurement, and Control, vol. 137, no. 2, pp. 1-7.

[34] Serrano M. E, Scaglia G. J. E, Mut V et al. (2013) Tracking trajectory of underactuated surface vessels: a numerical method approach. Control Engineering and Applied Informatics, vol. 15, no. 4, pp. 15-25.

[35] Serrano M. E, Scaglia G. J. E, Godoy S. A et al. (2014) Trajectory tracking of underactuated surface vessels: a linear algebra approach. IEEE Transactions on Control Systems Technology, vol. 22, no. 3, pp. 1103-1111.

[36] Sanyal A, Nordkvist N, Chyba M (2011) An almost global tracking control scheme for maneuverable autonomous vehicles and its discretization. IEEE Transactions on Automatic Control, vol. 56, no. 2, pp. 457-462.

[37] Ma B. L, Xie W. J (2013) Global asymptotic trajectory tracking and point stabilization of asymmetric underactuated ships with non-diagonal inertia/damping matrices. International Journal of Advanced Robotic Systems, vol. 10, pp. 1-9.

[38] Fischer N, Hughes D, Walters P et al. (2014) Non-linear RISE-based control of an autonomous
underwater vehicle. IEEE Transactions on Robotics, vol. 30, no. 4, pp. 845-852.

[39] Katayama H, Aoki H (2014) Straight-line trajectory tracking control for sampled-data underactuated ships. IEEE Transactions on Control Systems Technology, vol. 22, no. 4, pp. 1638-1645.

[40] Mukherjee K, Kat I. N, Bhatt R. K. P (2015) Region tracking based control of an autonomous underwater vehicle with input delay. Ocean Engineering, vol. 99, pp. 107-114.

[41] Wu Y. Q, Zhang Z. C, Xiao N (2014) Global tracking controller for underactuated ship via switching design. Journal of Dynamic Systems Measurement and Control-Transactions of the ASME, vol. 136, no. 9, pp. 1-7.

[42] Guerreiro B. J, Silvestre C, Cunha R et al. (2014) Trajectory tracking nonlinear model predictive control for autonomous surface craft. IEEE Transactions on Control Systems Technology, vol. 22, no. 6, pp. 2160-2175.

[43] Almeida J, Silvestre C, Pascoal A (2010) Cooperative control of multiple surface vessels in the presence of ocean currents and parametric model uncertainty. International Journal of Robust and Nonlinear Control, vol. 20, no. 4, pp. 1549-1565.

[44] Wang H, Wang D, Peng Z. H (2016) Adaptive neural control for cooperative path following of marine surface vehicles: state and output feedback. International Journal of Systems Science, vol. 47, no. 2, pp. 343-359.

[45] Dong Z. P, Wan L, Li Y. M et al. (2015) Point stabilization for an underactuated AUV in the presence of ocean currents. International Journal of Advanced Robotic Systems, vol. 12, pp. 1-13.

[46] Aguiar A. P, Pascoal A. M (2007) Dynamic positioning and way-point tracking of underactuated AUVs in the presence of ocean currents. International Journal of Control, vol. 80, no. 7, pp. 1092-1108.

[47] Borhaug E, Pavlov A, Pettersen K. Y (2008) Integral LOS control for path following of underactuated marine surface vessels in the presence of constant ocean currents. In: Proceedings of the 47th IEEE Conference on Decision and Control, pp. 4984-4991, IEEE, Cancun, Mexico, Dec 09-11.

[48] Fossen T. I (2011) Handbook of marine craft hydrodynamics and motion control. John Wiley & Sons.

[49] Morishita H. M, Souza C. E. S (2014) Modified observer backstepping controller for a dynamic positioning system. Control Engineering Practice, vol. 33, no. 12, pp. 105-114.

[50] Rudra S, Baral R. K, Maitra M (2014) Nonlinear state feedback controller design for underactuated mechanical system: a modified block backstepping approach. ISA Transactions, vol. 53, no. 2, pp. 317-326.

[51] Gayaka S, Lu L, Yao B (2012) Global stabilization of a chain of integrators with input saturation and disturbances: a new approach. Automatica, vol. 48, no. 7, pp. 1389-1396.

[52] Isidori A (1995) Nonlinear control systems (Vol. 1). Springer.

[53] Baidu. Baidu-baike: ocean current [Internet]. Available from: http://baike.baidu.com/link?url=S69w6CtoubYDyNrzYoTlniA8oJigYeMOrLE-MoTBCL1UmV018shZnsauk3YebifyO514VUZ-bEUUIb6HXCf8b4a Accessed on 10 Jul 2015.