Plasmonic waves in two-dimensional electron gas

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Abstract. The electromagnetic waves applied on surfaces of materials with high
density of free charges induce the generation of plasmonic waves both in volume and
surface. Plasmonic waves in metals have been widely studied; also, recently plasmons
have been investigated in intrinsic and doped semiconductor materials. In this work,
plasmonic waves are simulated in a two-dimensional electron gas. Two-dimensional
electron gas may be created at the interfaces between $GaAs$ and $Al_xGa_{1-x}As$. The
plasmons propagation speed depends on Fermi velocity and Fermi vector of the two-
dimensional electron gas. The 2D plasmons dispersion relations deviate from light
dispersion at all frequencies and does not have an asymptotic character as in three-
dimensional electron gas; this allows operating plasmonic devices with speeds much
lower than the speed of light. At low frequencies the plasmon wave vector is much
smaller than the Fermi vector and the geometric capacitance dominates the quantum
capacitance. The propagation of plasmons in channels without gate is studied.

1. Introduction

In some heterostructures the electrons are confined in two dimensions. In figure 1 it
is seen that in the region that limits $Al_xGa_{1-x}As$ with $GaAs$ a quantum well appears,
where the energy of the electrons is quantized in the longitudinal direction. In the central
layer of $Al_xGa_{1-x}As/GaAs/Al_xGa_{1-x}As$ heterostructure a quantum well is formed when
the thickness of the central layer $GaAs$ is comparable to the Broglie wavelength $\lambda_B$ or
in the nanometric range as shown in the figure 2.

In materials where free charges are confined in two dimensions, there is less charge
concentration and the range of the Coulomb interaction is shorter than in three
dimensions. Therefore, charges have more kinetic energy and experience collective
oscillations easily. The dispersion relation of the plasmonic wave $\omega(k)$, deviates from
the light-line at all frequencies. This characteristic allows the development of structures
that operate at different frequencies from the optical regime.

The figure 3 shows the dispersion relation of plasmonic waves in the two-dimensional
electron gas. At gigahertz (GHz) frequencies, the propagation speed can be of the order
of 0.01 of the speed of light. As a consequence of the low speed, plasmons in 2D have wavelengths smaller than surface plasmons in 3D.

2. Transmission Line Model
The propagation medium of plasmonic waves in a two-dimensional electron gas 2DEG can be modeled as a transmission line, consisting of a distributed kinetic inductance per unit length of non-magnetic origin $L_k$ and a distributed capacitance per unit length $C$, where the capacitive effect is decomposed into two capacitances: classical capacitance and quantum capacitance $C^{-1} = C_c^{-1} + C_q^{-1}$. The collective oscillations of the electrons in 2D plasmonic wave give rise to the kinetic inductance. The electrical potential derived from the Coulomb restoration force in the plasmonic oscillations produces the electrical capacitance. Then, the kinetic inductance per unit length is derived from the inertial acceleration of the electrons and is equal to
Figure 4: Transmission line model for plasmons in 2D electron gas.

\[ L_k = \frac{m^*}{n_2 D e^2} W \]  \hspace{1cm} (1)

where \( m^* \) is the electron effective mass, \( n_{2D} \) the density of charges in two dimensions, \( W \) is the width of the channel and \( e \) is the electron charge. Taking into account that \( k_F = \sqrt{2\pi n_{2D}} \) for degenerate gas and \( m^* = \hbar k_F / v_F \), where \( k_F \) and \( v_F \) are the Fermi wavenumber and Fermi velocity, then

\[ L_k = \frac{2\pi \hbar}{e^2 v_F k_F} \frac{1}{W} \]  \hspace{1cm} (2)

The geometric classical capacitance per unit length is obtained by integrating the density of energy accumulated in the electric field in the 2D conductor assuming a sinusoidal charge density with the wavenumber \( k_p \). As a result of this procedure, the capacitance becomes

\[ C_c = 2e k_p W, \]  \hspace{1cm} (3)

where \( \epsilon \) is the dielectric constant of the material. The quantum capacitance distributed per unit length originates from the increase of the total kinetic energy of the electron gas by the addition of a charge to the system. The quantum capacitance of the 2D electronic gas is

\[ C_q = \frac{m^* e^2}{\pi \hbar^2} W \]  \hspace{1cm} (4)

Now, taking into account, that \( k_F = \sqrt{2\pi n_{2D}} \) and \( m^* = \hbar k_F / v_F \), then

\[ C_q = \frac{e^2 k_F}{\pi \hbar v_F} W \]  \hspace{1cm} (5)

3. Dispersion relation

From the transmission line model for the propagation of plasmons in a two-dimensional electron gas, the following dispersion relation is obtained
\[ \omega = \frac{k_p}{\sqrt{L k C}} = v_F k_F \sqrt{\frac{e^2}{4\pi\varepsilon_0 v_F k_F}} + \frac{1}{2} \left( \frac{k_p}{k_F} \right)^2 \]  

(6)

The first term under the root is associated with the classical capacitance and the second with the quantum capacitance. In the case of low frequencies, \( k_p \) is much less than \( k_F \) and the second term can be neglected, originating a simplified form of the equation (6).

\[ \omega = \sqrt{\frac{e^2 v_F k_F}{4\pi\varepsilon_0 h}} k_p \]  

(7)

4. Results and conclusions

Figure 5: Dispersion relation for plasmons in 2D electron gas with charge density \( n_{2D} = 10^{11} \text{ cm}^{-2} \) (low), \( n_{2D} = 4 \times 10^{11} \text{ cm}^{-2} \) (middle) and \( n_{2D} = 9 \times 10^{11} \text{ cm}^{-2} \) (upper).

The figure 5 shows the dispersion relation for the plasmonic wave in the two-dimensional electron gas with the following parameters: Charge density \( n_{2D} = 10^{11} \text{ cm}^{-2} \) (low), \( n_{2D} = 4 \times 10^{11} \text{ cm}^{-2} \) (middle) and \( n_{2D} = 9 \times 10^{11} \text{ cm}^{-2} \) (upper). The Fermi wavenumber \( k_F = \sqrt{2\pi n_{2D}} \text{ m}^{-1} \), the GaAs has a dielectric permittivity \( \varepsilon_r = 10.89 \) and electron effective mass \( m^* = 0.067m_0 \), where \( m_0 \) is the mass of the free electron. Then, for \( n_{2D} = 10^{11} \text{ cm}^{-2} \) and a frequency of 41 GHz, the plasmonic wavelength \( \lambda_p = 206 \mu\text{m} \) (see fig. 6).

The plasmonic waves were studied in a 2DEG with the transmission line model. These results qualitatively behave quite coincident with other models. The propagation speed of the plasmonic waves in this medium ranges between \( c/10 \) to \( c/40 \) for the charge concentration between \( 10^{11} \) and \( 10^{12} \text{ cm}^{-2} \).
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