Topological plasmon-polariton on a Dirac magnet helical state

I V Iorsh¹, G R Rakhmanova¹ and M Titov²,1
¹ITMO University, Saint Petersburg 197101, Russia
²Radboud University, Institute for Molecules and Materials, NL-6525 AJ Nijmegen, The Netherlands

Abstract. A one-dimensional plasmon-polariton which is characterized by non-linear dispersion is supported by electromagnetic field interacting with topologically protected helical state. The electronic helical state arises at the surface of topological insulator in the close proximity of the ferromagnet. In a two-dimensional Dirac magnet these electro-optical excitations are limited by domain walls. Topological electronic imaging of domain walls can provide generally a novel tool to couple magnetic dynamics with both transport and optical properties of helical electronic states. An exact dispersion relation for the topological plasmon-polariton were obtained.

1. Introduction
In 1987 edge magnetoplasmons excitations were investigated of an inhomogeneous 2D system in the quantum Hall effect regime [1]. We considered a system, where similar quasiparticles arise. It is a surface of 3D topological insulator or topological semimetal in a close proximity to a ferromagnet thin film. A one-dimensional domain wall in the ferromagnet is imaged in the Dirac electron system as a zero mass line that supports a helical electronic state. The helical state at the domain wall results from the anomalous Hall effect in a Dirac magnet and the properties of such a state are similar to those of a quantum Hall edge state. Along the helical state one-dimensional plasmon-polariton excitations can be created by light polarized along the domain wall. We studied the properties of this hybrid light-matter quasiparticles. Moreover, these states can be used for the all-optical imaging of the domain wall structure of the ferromagnets using scattering SNOM technique (see Fig. 1). The physics can be realized in ZrSiS thin crystal that is weakly coupled to a ferromagnet film.

Figure 1. Optical imaging of the ferromagnetic domain wall via the excitation of the plasmon-polariton in the topological semimetal.
2. Dispersion of the topological plasmon polariton

The Hamiltonian of this system

\[ H = m\sigma_z + v_F[\sigma \times p]_z \]  

(1)

where the direction \( z \) is chosen perpendicular to the 2D surface, \( p \) is the momentum operator, \( \sigma \) is the three-dimensional vector of Pauli matrices, \( m \) is a magnetization of a ferromagnet, in the semimetal it is responsible for the energy gap, \( v_F \approx 5 \times 10^5 \text{ m/s} \) is the effective velocity of Dirac quasiparticles.

Due to the interaction of the magnetization in the ferromagnet with the spins in the semimetal an energy gap is responsible for the excitation of a 2D surface plasmon. Plasmon has linear dispersion (\( \sigma \))

\[ \omega_q = \frac{e v_F q}{c} \]

(2)

where \( v_F \) is the effective velocity of Dirac quasiparticles.

\[ \sigma_{xy}(z, z', x, x', \omega, q_y) = \frac{i e^2}{h} v_F \delta(z) \delta(z') f^2(x) f^2(x') \int_{m/hv_F}^{p_F} d\epsilon_y \frac{n_f(\epsilon_{ky}) - n_f(\epsilon_{ky+q_y})}{q_y(w-v_F q_y)} \]

(3)

where \( n_f \) is the Fermi-Dirac distribution function, \( q_y \) is the wave-vector in the y direction. The integral is almost a constant and equal to 1. The expression for the surface conductivity then simplifies. It can be seen that conductivity tensor has the only non-zero component, corresponding to both current and electric field aligned along the domain wall. This conductivity has a pole, where it goes to infinity, this pole corresponds to the excitation of a directed plasmon. Plasmon has linear dispersion (\( \omega = v_F q_y \)).

THz radiation applied to the system is capable of creating one-dimensional plasmon-polariton excitation along helical state. To find its dispersion it is necessary to solve the Maxwell equation:

\[ \text{rotrot } \vec{E}(\vec{r}, \omega) = k_0^2 \vec{E} - \frac{4\pi}{c^2} \partial_t j(\vec{r}, \omega) \]

(4)

where the linear response of the current density \( j \) to the electric field \( E \) is defined in the frequency domain by \( j = \int d^3\vec{r}' \sigma(\vec{r}, \vec{r}', \omega) \vec{E}(\vec{r}', \omega) \)

This equation has formal solution:

\[ \vec{E}(\vec{r}, \omega) = i \frac{4\pi k_0}{c} \int d^3(\vec{r}'') G(\vec{r}, \vec{r}'', \omega) \int d^3\vec{r}'' \sigma_{\alpha\beta}(\vec{r}', \vec{r}'', \omega) E_\beta(\vec{r}'', \omega) \]

(5)
where $G$ - is the dyadic Green’s function in vacuum. That allows us to get the linear algebraic equation system for \( \Lambda(\omega, q_y) = \int dx dz f^2(x) \delta(z) E_y : \)

\[
[1 - i \frac{e \Lambda B}{c} \delta(\omega, q_y)] \int dx' f^2(x') G_{yy}(x, x', 0, 0, q_y, \omega) \Lambda(\omega, q_y) = 0
\]  

(10)

The determinant of this matrix gives the dispersion equation for the topological plasmon-polariton. After some algebra, the dispersion equation can be written as

\[
1 - \bar{\nu} \bar{q} = \alpha \bar{\nu}(\bar{q}^2 - 1) S(\omega, q_y, a_0, \bar{a})
\]  

(11)

Where \( \bar{\nu} = \frac{v_p}{c}, \bar{q} = \frac{q_v}{c}, \alpha \approx 1/137 \) - is the fine structure constant. The quantity \( S \) is given by

\[
S = \int_0^\infty dx \left[ \frac{r(\bar{a}(1+i\bar{q}/2))}{r(\bar{a})} \right]^4,
\]

(12)

where \( \kappa = \frac{h \nu \omega}{m} \sqrt{\bar{q}^2 - 1} \)

As \( \bar{a} \rightarrow 0 \) the dispersion equation is simplified to

\[
1 - \bar{\nu} \bar{q} = \frac{\alpha \bar{\nu}(\bar{q}^2-1)}{2} \left[ \frac{\tanh^{-1} \kappa'}{\kappa'^2} - \frac{1}{\kappa'^2} + \frac{\tanh^{-1} \kappa'}{\kappa'^3} \right]
\]  

(13)

For the case of and some generic ferromagnetic with saturation magnetization of 0.5 T and domain wall width \( a_0 \approx 10 \text{ nm} \), we find that \( m \approx 1.5 \text{ meV}, v_p \approx 4.3 \times 10^5 \text{ m/s} \), \( \alpha \approx 0.06 \) and thus approximate dispersion equation Eq. (13) is valid. In Fig. 2 we plot the dispersion of the plasmon polariton for different values of \( a_0 \) and thus \( \bar{a} \). It can be seen that the dispersion for the case of \( \bar{a} = 0.06 \) almost fully coincides with the analytical expression for \( \bar{a} = 0 \) given by Eq. (13). The discrepancies in the dispersion become visible at large \( \bar{a} \approx 1 \). The dispersion has asymmetry. It depends on the ratio of the Fermi velocity and the speed of light. The faster the electrons move along the structure, the more asymmetric the picture.

![Figure 2. Dispersion of the topological plasmon polariton at different values of \( \bar{a} \). Black line corresponds to \( \bar{a} = 0 \), green to \( \bar{a} = 0.06 \), dotted red - to \( \bar{a} = 1 \) and dashed blue to \( \bar{a} = 2 \).](image)

3. Experiment simulation

We considered excitation of a helical plasmon-polariton by a point source of terahertz radiation and looked as the scattered field of such structure looks (see Fig. 3). The dipole oriented along axis and placed at the position \( r_0 = (x_0, 0, z_0) \) in the vicinity of the domain wall.

The \( y \) component of the electric field is then given by

\[
E_y(q_y, r) = G_{yy}(x - x_0, z - z_0, q_y) + \alpha \bar{\nu} \int dx' G_{yy}(x - x', z - 0) f^2(x') \Lambda(q_y, \omega, x_0, z_0)
\]

(14)
and \( \Lambda \) is given by

\[
\Lambda(q_y, \omega, x_0, z_0) = \int [dx''dz''g_{yy}(x''-x_0, z''-z_0)e^{i(x'')\delta(z'')} \frac{1}{1-\psi q} \frac{1}{2\pi} \frac{\tanh^{-1} q}{q} \frac{1}{\tanh^{-1} q'} \frac{\tanh^{-1} q'}{q'}]
\] (15)

In Fig. 3 the dipole field that quickly fades out is shown and it excites a plasmon-polariton runs along a domain wall. As can be seen in the inset the interference between the incident field and the excited plasmon-polariton leads to occurring fast oscillations. The field from a source has its own wave vector, the phase changes with own speed, but the plasmon-polariton field has wave vector is much larger, the phase changes quickly. The spatial period of the oscillations can be estimated as \( \frac{\pi}{k_0-q_p} \), where \( q_p \) is the plasmon-polariton wavevector. so it can be find out what the plasmon wave vector is equal to.

To sum up we considered a helical electron state which supports edge plasmon-polariton, it can be formed along at a surface of topological insulator or topological semimetal that is in a close proximity to a ferromagnet thin film. Such a state is analogous to the quantum Hall edge state but arise due to the anomalous Hall effect. The exact dispersion equation for topological plasmon-polariton is derived. It is possible to get imaging of domain wall structure of the ferromagnetic via the expatiation of the edge plasmon polariton in the semimetal.

References
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