A mathematical modeling of two-pot biomass cookstove
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Abstract
Biofuels are extensively used as primary fuel in cookstoves by the rural masses in India. About 70% of the population resides in the rural area, out of which 70% are entirely dependent on biofuels for their energy requirements. The traditional cookstoves are poorly designed causing inefficient burning. Hence, the efforts are needed to improve the quality of these cookstoves concerning of design and efficiency. The present work overcomes the difficulty of designing the two-pot biomass cookstove by presenting a simple mathematical approach to evaluate the thermal performance using the heat and mass transfer equations. The model predicts the heat interaction between the stove body and pots, combustion temperatures in different zones, the thermal efficiency and excess air ratio for different geometrical parameters of the stove. The model values are validated with the experimental results and found to be in close agreement. The findings of the model are likely to change the future cookstove modeling and design approach.

1. Introduction
Biofuels or biomass is the prime energy resource on which the majority of the population depends especially in developing countries, like India. Biomass contributes to one-third of the primary energy requirement in India. Biomass fuels are predominantly used in rural households as well as traditional and artisan industries for cooking and heating. According to the report of the National Council for Applied Economic Research (NCAER), biomass fuels contributed 90% energy in the rural areas and over 40% in the cities. Among the biomass energy sources, wood fuels are more prominent. It contributes 56 percent of the total biomass energy [1]. In rural areas, most of the biomass energy is used for cooking and water heating purposes. The traditional biomass cookstoves used in rural areas are poorly designed leading to inefficient burning and higher energy losses. Moreover, the exhausts from these cookstoves also cause deaths due to indoor air pollution (IAP). Around 4 million people die every year, and many are affected by birth due to IAP. So it is necessary to improve the efficiency of the existing cookstove for low use of fuel and low emissions. In India, the development for cookstove was started in the late 40’s with an objective to provide a better stove than the three stone fire and traditional stoves adopted in the villages. Many organizations came together for introducing a new design of cookstove and creating an awareness program about using biomass cookstove. Many attempts are made by the researcher to alter the design and performance parameter of single pot cookstove to work it efficiently [2–4]. Only a few experimental works are reported on the multipot biomass cookstoves [5–9]. Also, the reported experimental works are not supported with any energy system analysis based on heat and mass transfer equations.

The present work deals with the mathematical modeling of the two-pot biomass cookstove and its validation with the experimental results performed in the laboratory. The mathematical model for the two-pot biomass cookstove will help the user in deciding the dimensions of the cookstove be adopted and hence saving the resources.
Nomenclature

| Q_1      | Char Radiation loss          | Q_{17} | Radiative heat taken through the pot gap |
| Q_2      | Flame radiation loss         | Q_{18} | Heat supplied by the flue gas at pot side |
| Q_3      | Wall losses in zone 1        | Q_{19} | Convective heat taken through the pot gap |
| Q_4      | Heat loss due to hydrogen in fuel | Q_{20} | Radiative heat taken through the pot gap |
| Q_5      | Moisture loss                | Q_{21} | Heat lost by the flue to surrounding |
| Q_6      | Heat entering zone 2         | Q_i    | Heat gain by flue in zone 1 |
| Q_7      | Wall losses in zone 2        | Q_s    | Heat supplied by the fuel |
| Q_8      | Heat leaving zone 2          | Q_d    | Feeding door loss |
| Q_9      | Heat gain by the flue in zone 3 | Q_{t4} | Heat loss from flame to inner wall of stove |
| Q_{10}   | Wall losses in zone 3        | Q_{t6} | Heat loss from inner wall to outer wall |
| Q_{11}   | Heat leaving the zone 3      | Q_{t6} | Heat loss from outer wall to surrounding |
| Q_{12}   | Heat supplied by the flue gas at pot bottom | Q_{pot1} | Heat taken by pot 1 |
| Q_{13}   | Convective heat taken by pot 2 bottom | Q_{pot2} | Heat taken by pot 2 |
| Q_{14}   | Radiative heat taken by pot 2 bottom | P     | Firepower, kW |
| Q_{15}   | Heat supplied by the flue gas at pot gap |     | |
| Q_{16}   | Convective heat taken through the pot gap |     | |

2. Mathematical modelling

The two pot cookstove is divided into different heat zones. Three main zones are shown in Figure 1, where the basic heat and mass transfer equations are applied. The assumptions consider for the modeling of two-pot biomass cookstove are:

1. The cookstove operates at steady state since the average parameters over a length of operational time are more important than the instantaneous values.
2. The emissivity’s of the pot bottom and stoves inner surface is 1.
3. The flue gas properties were taken in the range of 273-1473K [10]
4. Mass of flue is constant throughout the different zones.

Figure 1. Heat balance for two-pot cookstove

Zone 1
Combustion Chamber

The fuel combustion takes place in this zone. It is the feeding zone, through which the fuel input is metered. Thus, the amount of air entering is controlled by changing the inlet area ratio (IAR) [4]. The losses like char radiation, flame radiation, heat losses through the walls and feed door (inlet opening) are taken into consideration. In addition, sensible heat loss of hydrogen and moisture present in fuel are substantial and are included in the modeling.

\[
IAR = \frac{\text{Area unoccupied by the feed door}}{\text{cross-sectional area of the chimney}}
\]  

(1)

Zone 1 (Combustion chamber):
Applying heat balance in the combustion zone we have,

\[
Q_i = Q_{s} - (Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6)
\]

(2)

Also, by using the heat analogy for heat transfer from flame to the surrounding we have,
Heat loss from flame to inner wall and innerwall to outer wall and outer wall to surrounding as same,

\[
Q_3 = Q_{f-i} = Q_{o-surrounding}
\]

(3)

Solving above we get, \(T_{fg1}, T_{wi1}, T_{wo1}\)

Zone 2 (Intermediate zone):
Subtracting the heat taken by pot1 from the heat gain by flue in zone 1 we have,

\[
Q_6 - Q_{pot1} = Q_{6}
\]

(4)

Solving above we get, \(T_{fg2}, T_{wi2}, T_{wo2}\)

Similarly, heat gain by the flue gas in zone 2 minus the wall losses in zone 2 gives the heat leaving the zone 2,

\[
Q_6 - Q_7 = Q_8
\]

(5)

Solving above we get, \(T_{fg3}, T_{wi3}, T_{wo3}, T_e\)

Zone 3 (Secondary zone)
Applying heat balance in zone 3 we have,

\[
Q_9 - Q_{10} - Q_{pot2} = Q_{11}
\]

(6)

Solving above we get, \(T_{e3}\)

Zone 4 (Pot bottom)
The heat lost by the flue leaving zone 3 is transferred to pot bottom.

\[
Q_{12} = Q_{13} + Q_{14}
\]

(7)

Solving above we get, \(T_{c3}\)

Zone 5 (Pot gap)
In this zone the heat loss from the flame to the pot gap is considered.

\[
Q_{15} = Q_{16} + Q_{17}
\]

(8)

Solving above we get, \(T_{o3}\)
Zone 6 (Pot side)
In this zone the heat loss from the flame to the sides of the pot is considered.

\[ Q_{18} = Q_{19} + Q_{20} + Q_{21} \quad (9) \]

Formulating above equations in matlab and solving simultaneously gives different flame temperatures, different heat encountered in the stove, thermal efficiency, and mass flow rate. Thus, the overall efficiency can be calculated by an indirect method as,

\[ \eta_o = \frac{Q_{pot 1} + Q_{pot 2}}{p} \times 100 \quad (10) \]

3. Experimental Method
The experimental setup is shown in Figure 2. The metallic (cast iron) stove was insulated with ceramic wool to prevent the heat losses through the walls of the stove. The 1.3 x 1.3 cm² cross-sectional wood was used for experimentation. The height of the stove used was 200 mm. The mass of fuel required to boil water from pot 1 was recorded. Simultaneously, the water temperature in the second pot was noted down. A calibrated weighing balance with a maximum uncertainty of ±0.06 kg was used. Two R-type thermocouples were used to measure the flame temperature. The velocity of the flue was measured with calibrated flue gas anemometer (Kanomax 6162). The time taken to boil water from pot 1 was noted down by a stopwatch. The weight of the charcoal left was recorded to measure the exact dry fuel consumed. The average firepower (FP) values obtained through the 3 sets of experimentation, were used as the input for the proposed model. The temperatures, velocity and thermal efficiency obtained through the experimentation were compared with the model results.

![Experimental setup](image)

**Figure 2.** Experimental setup

Model-predicted results
The equations were solved iteratively using a MATLAB script. For the given geometrical inputs and other operational constants, the iterative solution yields 13 temperatures values. The other output parameters were then determined from these temperatures values and flue gas properties. The calculation for FP = 2.202 kW for selected geometry is shown in Table 1. The relative percentage error was calculated [11] which is the ratio of the difference between the experimental and model predicted to the experimental results.
Table 1. Temperatures at different zones

| Sr.no. | Parameter | Description | Value (K) |
|--------|-----------|-------------|-----------|
| Zone 1 | \( T_{fg1} \) | Flame temperature in zone 1 | 743.60 |
|        | \( T_{wi1} \) | Inner wall surface temperature for zone 1 | 724.98 |
|        | \( T_{wo1} \) | Outer wall surface temperature in zone 1 | 497.63 |
| Zone 2 | \( T_{central1} \) | Temperature of flue entering zone 2 | 677.20 |
|        | \( T_{wi2} \) | Inner wall surface temperature for zone 2 | 512.99 |
|        | \( T_{wo2} \) | Outer wall surface temperature of zone 2 | 416.07 |
|        | \( T_{central2} \) | Temperature of flue leaving zone 2 | 666.49 |
|        | \( T_{fg2} \) | Average flame temperature in zone 2 | |
| Zone 3 | \( T_{fg3} \) | Flame temperature in zone 3 | 633.68 |
|        | \( T_{wi3} \) | Inner wall surface temperature in zone 3 | 482.23 |
|        | \( T_{wo3} \) | Outer wall surface temperature in zone 3 | 377.74 |
|        | \( T_{e3} \) | Temperature of flue leaving zone 3 | 600.87 |
|        | \( T_{c3} \) | Temperature of flue at the corner of pot 2 | 583.68 |
|        | \( T_{o3} \) | Temperature of flue leaving the side of pot 2 | 418.15 |

5. Results and Discussion

Three sets of experiment were conducted on the model stove and the average value of firepower was given as an input to the mathematical model. Table 2 shows the value of three sets with standard deviation value.

Table 2. Performance parameters with ±standard deviation

| Sr. No. | Parameter | Values | Average | Standard deviation |
|---------|-----------|--------|---------|--------------------|
| 01.     | Firepower (kW) | 1. 2.98  
2. 2.20  
3. 2.99 | 2.72 | ± 0.45 |
| 02.     | Time taken to boil (min) | 1. 23  
2. 38  
3. 24 | 28.33 | ± 8.39 |
| 03.     | Thermal Efficiency (%) | 1. 34.44  
2. 37.87  
3. 34.99 | 35.77 | ±1.84 |
| 04.     | CO (g/MJ\(_d\)) | 1. 8.71  
2. 10.87  
3. 10.22 | 9.93 | ±1.11 |

5.1 Flame temperatures

The model predicted temperature values are averaged over a zone, while the experimental values are taken at two points. Considering the complex combustion taking place in the stove, the deviation in the experimental values with the model is obvious. Figure 3 shows the experimental and mathematical model values for all three zones. It can be seen that the temperature in zone 1 (\( T_{fg1} \)) is much higher for the model but that is not the case for the zone 3 (\( T_{fg3} \)). As already stated the temperatures measured during the experimentation process are just point temperatures, so in case of zone 1, which is combustion zone, the values of temperature at different points are higher, the average
temperature variations with the mathematical modelling for zone 1, zone 2 and zone 3 is 10%, 3% and 8% respectively.

5.2 Thermal Efficiency

The value of thermal efficiency is the ratio of heating and evaporating water to the energy consumed by burning fuel. The thermal efficiency is calculated by the direct and indirect methods for the experimental and theoretical model, respectively. The deviation in the value of model and experimental results are in good agreement. The average deviation from the experimental values is 13%. It proves that the model predicted values can help to design a two pot cookstove which will save the resources.

5.3 Exhaust

Though the exhaust values cannot be predicted by the mathematical modelling, the exhaust was measured with the help of flue gas analyser (FGA-Indus make). The value of CO was calculated by WBT standards in g/MJ d. The average value of CO in g/MJ d for high power test was 9.93. Since the values of exhaust cannot be validated with the modelling results, they are not discussed in the paper.

6. Model limitation and Future scope

The proposed mathematical model helps to find the flame temperature, excess air ratio, thermal efficiency, flue velocity and percent oxygen in flue gas but it fails to predict the exact constituents of the exhaust gas. The model holds good only for two-pot cookstove natural draft biomass cookstove where the gap between pot 1 and combustion chamber is zero. The heat and mass transfer equations can be applied to various combinations of geometrical parameters and large sets of data can be generated and used to optimize the design which is the task to be taken in future.

7. Conclusion

The mathematical model is sufficiently accurate in determining the performance of cookstove without actually performing an experiment. The dimensions of the experimented stove were taken randomly to validate the mathematical model, but the MATLAB code helps to generate a possible
combination of design parameters and its performance for different geometrical and operational inputs. Thus, the present model can be considered as a beginning step for designing a two-pot cookstove.

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