1. INTRODUCTION

Properties of hot and/or dense QCD matter has been extensively studied within chiral approaches \([1]\). Our knowledge on the phase structure is however still limited and the description of the matter around the phase transitions does not reach a consensus, where a typical size of the critical temperature and chemical potential is considered to be of order \(T_{\text{QCD}}\). The phases of QCD are characterized by symmetries and their breaking pattern: QCD at asymptotically high density leads to the color-flavor-locked phase as the true ground state under the symmetry breaking pattern, \(SU(3)_c \times SU(3)_L \times SU(3)_R\) down to the diagonal subgroup \(SU(3)_{c+L+R}\) \([2]\). The residual discrete symmetries characterize the spectra of excitations.

At zero temperature and density, an alternative pattern of spontaneous chiral symmetry breaking was suggested in the context of QCD \([3, 4, 5]\). This pattern keeps the center of chiral group unbroken, i.e. \(SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \times (Z_{N_f})_A\), where a discrete symmetry \((Z_{N_f})_A\) is the maximal axial subgroup of \(SU(N_f)_L \times SU(N_f)_R\). The \(Z_{N_f}\) symmetry protects a theory from condensate of quark bilinears \(\langle \bar{q}q \rangle\). Spontaneous symmetry breaking is driven by quartic condensates which are invariant under both \(SU(N_f)_V\) and \(Z_{N_f}\) transformation. Although meson phenomenology with this breaking pattern seems to explain the reality reasonably \([3]\), this possibility is strictly ruled out in QCD both at zero and finite temperatures but at zero density since a different way of coupling of Nambu-Goldstone bosons to pseudo-scalar density violates QCD inequalities for density-density correlators \([6]\). However, this does not exclude the unorthodox pattern in the presence of dense baryonic matter. There are several attempts which dynamically generate a similar breaking pattern in an \(O(2)\) scalar model \([7]\) and in \(\mathcal{N} = 1\) Super Yang-Mills theory \([8]\).

Within the Skyrme model on crystal, a new intermediate phase where a skyrmion turns into two half skyrmions was numerically found \([9]\). This phase is characterized by a vanishing quark condensate \(\langle \bar{q}q \rangle\) and a non-vanishing pion decay constant. Recently, another novel view of dense matter, Quarkyonic Phase, has been proposed based on the argument using the large \(N_c\) counting where \(N_c\) denotes the number of colors \([10]\). In the large \(N_c\) limit there are three phases which are rigorously distinguished using the Polyakov loop expectation value \(\langle \Phi \rangle\) and the baryon number density \(\langle N_B \rangle\). The quarkyonic phase is characterized by \(\langle \Phi \rangle = 0\) indicating the system confined and non-vanishing \(\langle N_B \rangle\) above \(\mu_B = M_B\) with a baryon mass \(M_B\). The separation of the quarkyonic from hadronic phase is not clear any more in a system with finite \(N_c\). Nevertheless, an abrupt change in the baryon number density would be interpreted as the quarkyonic transition which separates meson dominant from baryon dominant regions. This might appear near the boundary for chemical equilibrium at which one would expect a rapid change in the number of degrees of freedom \([10]\) \([11]\).

A steep increase in the baryon number density and the corresponding maximum in its susceptibility \(\chi_B\) are driven by a phase transition from chirally broken to restored phase in most model-approaches. Interplay between (de)confinement and chiral symmetry breaking has been studied within a Nambu–Jona-Lasinio model with Polyakov loops \([12]\) which describes how the deconfinement and chiral phase boundaries are changed from \(N_c = \infty\) down to \(N_c = 3\) \([13]\). The model study shows that the chiral phase transition at \(T = 0\) appears just above mass threshold \(\mu_B = M_B\) and thus a large \(\chi_B\) is associated with the chiral phase transition. However, a constituent-quark picture does not directly describe the thermodynamics of hadronic matter and there are no a priori reasons that the quarkyonic transition should
be accompanied by chiral phase transition. Besides, it seems unlikely that the chiral symmetry is (even partially) restored slightly above the freeze-out curve where the baryon density is not high enough to drive a phase transition. From this perspective, further investigations of dense baryonic matter and a possible appearance of the quarkyonic phase in QCD with \( N_c = 3 \) require a modeling in terms of dynamical hadronic-degrees of freedom in a systematic way.

In this paper we will address this issue under the alternative pattern of chiral symmetry breaking in dense hadronic matter. We will show a possible intermediate phase between chiral symmetry broken and its restored phases with analyses using a general Ginzburg-Landau free energy. This leads to multiple critical points and one of them is associated with restoration of the center symmetry in \([14]\) where their 4-quark states are considered for a system with 2- and 4-quark states \( \Sigma \) in the adjoint representation as

\[
M_{ij} = \frac{1}{\sqrt{2}} (\sigma \delta_{ij} + i \phi^a \tau^a_{ij}),
\]

\[
\Sigma_{ab} = \frac{1}{\sqrt{3}} \chi \delta_{ab} + \frac{1}{\sqrt{2}} \epsilon_{abc} \psi_c,
\]

where \( \sigma \) and \( \chi \) represent scalar fields and \( \phi \) and \( \psi \) pseudoscalar fields, and \( \epsilon_{ijk} \) is the total anti-symmetric tensor with \( \epsilon_{123} = 1 \). In general the field \( \Sigma \) contains an isospin 2 state. One can take appropriate parameters in a Lagrangian in such a way that this exotic particle is very heavy. Thus, we will consider only isospin 0 (\( \chi \)) and 1 (\( \psi \)) states in this paper. The fields transform under \( SU(2)_L \times SU(2)_R \) as chiral non-singlet,

\[
M \rightarrow g^{(2)}_L M g^{(2)\dagger}_R, \quad \Sigma \rightarrow g^{(3)}_L \Sigma g^{(3)\dagger}_R.
\]

This transformation property implies that the field \( M \) changes its sign under the center \( Z_2 \) of \( SU(2)_L \) (or \( SU(2)_R \)), while \( \Sigma \) is invariant:

\[
M \rightarrow -M, \quad \Sigma \rightarrow \Sigma.
\]

Up to the fourth order in fields one obtains a potential,

\[
V(M, \Sigma) = -\frac{m^2}{2} \text{Tr} [MM^\dagger] + \frac{\lambda^2}{4} (\text{Tr} [MM^\dagger])^2 - \frac{m^2}{2} \Sigma_{ab} \Sigma^T_{ba} + \frac{\bar{\lambda}^2}{4} \Sigma_{ab} \Sigma^T_{ba} \Sigma_{cd} \Sigma^T_{dc} + \frac{\bar{\lambda}^2}{4} (\Sigma_{ab} \Sigma^T_{ba})^2 + 2g_1 \Sigma_{ab} \text{Tr} [T_a M T_b M^\dagger] + g_2 \Sigma_{ab} \Sigma^T_{ba} \text{Tr} [MM^\dagger]
\]

\[
+ g_3 \text{Det} \Sigma + g_4 (\text{Det} M + h.c.).
\]

The last term violates the \( U(1)_A \) symmetry. The coefficients of the quartic terms are positive for this potential to be bounded. Other parameters \( g_i \) can be both positive and negative and will determine the topology of the phase structure. An explicit chiral symmetry breaking can be introduced through, e.g.,

\[
V_{SB}(M, \Sigma) = -h\sigma - \alpha h^2 \chi,
\]

with constants \( h \) and \( \alpha \). Note that a similar Lagrangian was considered for a system with 2- and 4-quark states under the symmetry breaking pattern without unbroken center symmetry in \([14]\) where their 4-quark states are chiral singlet and the potential does not include quartic terms in fields.

### 2.2. Ginzburg-Landau effective potential

We first study possible phases derived from the effective potential (2.6) taking

\[
M_{ij} = \frac{1}{\sqrt{2}} \sigma \delta_{ij}, \quad \Sigma_{ab} = \frac{1}{\sqrt{3}} \chi \delta_{ab}.
\]

2. A MODEL FOR 2-QUARK AND 4-QUARK STATES

We construct a chiral Lagrangian for 2- and 4-quark states under the following pattern of symmetry breaking,

\[
SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V \times (Z_{N_f})_A \rightarrow SU(N_f)_V.
\]

In this paper we will restrict ourselves to a two-flavor case.

#### 2.1. Lagrangian

We introduce a 2-quark state \( M \) in the fundamental and a 4-quark state \( \Sigma \) in the adjoint representation as \#1

\[
M_{ij} \sim \bar{q}_{R,j} q_{L,i},
\]

\[
\Sigma_{ab} \sim \bar{q}_{L} \gamma^\mu q_{L} \bar{q}_{R} \gamma^\mu q_{R}, \quad (2.2)
\]

where the flavor indices run \((i,j) = 1,2\) and \((a,b,c) = 1,2,3\) and Pauli matrices \( \tau^a = 2T^a \) with \( \text{tr}[T^a T^b] = \delta^{ab}/2 \). The \( M \) and \( \Sigma \) are expressed as

\[
M_{ij} = \frac{1}{\sqrt{2}} (\sigma \delta_{ij} + i \phi^a \tau^a_{ij}),
\]

\[
\Sigma_{ab} = \frac{1}{\sqrt{3}} \chi \delta_{ab} + \frac{1}{\sqrt{2}} \epsilon_{abc} \psi_c,
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\]

\#1 We consider \( \Sigma \) as any linear combination of \( \bar{q}q \)-\( \bar{q}q \) and \( \bar{q}q \)-\( \bar{q}q \) type fields allowed by symmetries.
FIG. 1: Phase diagram with \( D = F = 0 \) and \( h = 0 \). The solid and dashed lines indicate first and second order phase boundaries, respectively. One tricritical point, TCP\(_1\), is located at \((A, B) = (0, 1/4)\) and another, TCP\(_2\), at \((A, B) = (1/4, -1/8)\). The triple point represented by \( T \) is at \((A, B) = (1/8, 0)\).

One can reduce Eq. (2.6) as well as an explicit breaking term to

\[
V(\sigma, \chi) = A\sigma^2 + B\chi^2 + \sigma^4 + \chi^4 - h\sigma + C\sigma^2\chi + D\chi^3 + F\sigma^2\chi^2. \tag{2.9}
\]

We will take \( C = -1 \) without loss of generality in the following calculations.

We start with the potential for \( D = F = 0 \) and \( h = 0 \),

\[
V = A\sigma^2 + B\chi^2 + \sigma^4 + \chi^4 - \sigma^2\chi. \tag{2.10}
\]

Phases from this potential can be classified by the coefficients \( A \) and \( B \). The expression of the phase boundaries is summarized in Appendix A. Here we discuss the obtained phase structure shown in Fig. 1. There are three distinct phases characterized by two order parameters: Phase I represents the system where both chiral symmetry and its center are spontaneously broken due to non-vanishing expectation values \( \chi_0 \) and \( \sigma_0 \). The center symmetry is restored when \( \sigma_0 \) becomes zero. However, chiral symmetry remains broken as long as one has non-vanishing \( \chi_0 \), indicated by phase II. The chiral symmetry restoration takes place under \( \chi_0 \to 0 \) which corresponds to phase III. The phases II and III are separated by a second-order line, while the broken phase I from II or from III is by both first- and second-order lines. Accordingly, there exist two tricritical points (TCPs) and one triple point. One of these TCP, TCP\(_2\) in Fig. 1, is associated with the center \( Z_2 \) symmetry restoration rather than the chiral transition.

Two phase transitions are characterized by susceptibilities of the corresponding order parameters. We introduce a 2-by-2 matrix composed of the second derivatives of \( V \) as

\[
\hat{C} = \begin{pmatrix} C_{\sigma\sigma} & C_{\sigma\chi} \\ C_{\chi\sigma} & C_{\chi\chi} \end{pmatrix}, \tag{2.11}
\]

with

\[
C_{\sigma\sigma} = \frac{\partial^2 V}{\partial \sigma^2}, \quad C_{\chi\chi} = \frac{\partial^2 V}{\partial \chi^2},
\]

\[
C_{\sigma\chi} = C_{\chi\sigma} = \frac{\partial^2 V}{\partial \sigma \partial \chi}, \tag{2.12}
\]

under the solutions of the gap equations, \( \sigma_0 \) and \( \chi_0 \). A set of susceptibilities is defined by the inverse of \( \hat{C} \):\(^{[15]}\)

\[
\hat{\chi} = \frac{1}{\det \hat{C}} \begin{pmatrix} C_{\chi\chi} & -C_{\sigma\chi} \\ -C_{\chi\sigma} & C_{\sigma\sigma} \end{pmatrix}. \tag{2.13}
\]

We identify the susceptibilities associated with 2-quark and 4-quark states as

\[
\chi_{2Q} = \hat{\chi}_{11}, \quad \chi_{4Q} = \hat{\chi}_{22}. \tag{2.14}
\]

The \( \chi_{2Q} \) is responsible to the \( Z_2 \) symmetry and the \( \chi_{4Q} \) to the chiral symmetry restoration.

We consider \( \chi_{2Q} \) and \( \chi_{4Q} \) around the TCP\(_1\) in Fig. 1 where the potential has zero curvature and thus \( \det \hat{C} = 0 \). When approaching the TCP\(_1\) from broken phase I by tuning \( A \) and \( B \) as \( A \to A_{\text{critical}} = 0 \) and \( B = 1/4 \), these susceptibilities diverge as

\[
\chi_{2Q} \sim t^{-1}, \quad \chi_{4Q} \sim t^{-2/3}, \tag{2.15}
\]

where \( A_{\text{critical}} - A \sim t \) with the reduced temperature or chemical potential, e.g., \( t = |\mu - \mu_c|/\mu_c \). The gap equations determine the scaling of 2-quark and 4-quark condensates as

\[
\sigma_0^2 \sim t^{1/3}, \quad \chi_0 \sim t^{1/3}. \tag{2.16}
\]

Consequently, the quark number susceptibility \( \chi_q = -\partial^2 V/\partial \mu^2 \) exhibits a singularity as

\[
\chi_q \sim \sigma_0^2 \cdot \chi_{2Q} \sim t^{-2/3}. \tag{2.17}
\]

This critical exponent is same as the one in the 3-d Ising model. The coincidence can be understood due to the same \( Z_2 \) symmetries

\( \#2 \).

The critical behavior near the TCP\(_2\) involves more: When the \( A \) is approached as \( 1/4 - t \) with \( B = -1/8 \) fixed, \( \chi_{2Q} \) and \( \chi_{4Q} \) diverge as

\[
\chi_{2Q} \sim t^{-1}, \quad \chi_{4Q} \sim t^{-1/2}, \tag{2.18}
\]

and only \( \sigma_0 \) vanishes as \( \sigma_0^2 \sim t^{1/2} \). As a result, the quark number susceptibility \( \chi_q \) diverges as

\[
\chi_q \sim t^{-1/2}. \tag{2.19}
\]

\( \#2 \) The \( Z_2 \) symmetry in the 3-d Ising system is not the center of two-flavor chiral group, but emerges in the direction of a linear combination of quark number and scalar densities\(^{[16]}\).
Note that the critical exponent 1/2 is different from the one near the TCP, which may reflect different symmetries possessed by the system at TCP, SU(2) ⊗ SU(2) and the center Z_2, from that at TCP, SU(2) ⊗ SU(2) including its center (Z_2) L × (Z_2) R. Those exponents at TCP are changed when D ≠ 0 (see below).

When the second-order phase transition separating phase I from II or from III is approached from the broken phase with a fixed B, we have

\[ \chi_2q \sim t^{-1}, \quad \chi_4q \sim \frac{1}{B}, \quad (2.20) \]

where B is a finite number, which thus gives no singularities in \( \chi_4q \). The 2-quark condensate scales as \( \sigma_0^2 \sim t^1 \) and the quark number susceptibility \( \chi_q \) is finite along the second-order phase transition line:

\[ \chi_q \sim \sigma_0^2 \cdot \chi_2q \sim t^0. \quad (2.21) \]

Nevertheless, \( \chi_q \) is enhanced toward the phase transition induced by \( \chi_2q \) and becomes small above the phase transition. Such abrupt changes in \( \chi_q \) indicate the phase transition, especially for a negative B which is driven by the center symmetry restoration rather than the chiral phase transition.

Near the second-order chiral transition between phase II and III, one obtains from \( B \sim t \)

\[ \chi_0^2 \sim t^1, \quad \chi_4q \sim t^{-1}. \quad (2.22) \]

Since the chiral symmetry including the center symmetry prohibits the Yukawa-type coupling of \( \chi \) to a fermion and an anti-fermion in the fundamental representation, the coupling of \( \chi \) to the baryon number current would be highly suppressed. Therefore, \( \chi_q \) shows less sensitivity around the chiral transition. \(^3\)

Once small \( h \) is turned on, chiral symmetry and its center are explicitly broken. Second-order phase boundaries are replaced with cross over and the two TCPs with two critical points. The singularity in \( \chi_q \) is now governed by the \( Z_2 \) universality class of 3-d Ising systems. Thus, the scaling of \( \chi_q \) at the critical points (CPs) will be given by

\[ \chi_q \sim t^{-2/3}. \quad (2.23) \]

A cubic term in \( \chi \) modifies the previous phase structure shown in Fig. I. The phase diagram from the potential,

\[ V = A\sigma^2 + B\chi^2 + \sigma^4 + \chi^4 - \sigma^2\chi + D\chi^3, \quad (2.24) \]

is classified by the following regions of \( D \): (i) \(-1 < D < 0\), (ii) \(D \leq -1\), (iii) \(0 < D < 1\) and (iv) \(1 \leq D\). One observes a deformation of the boundary lines depending on \( D \) as in Fig. [2]. The phase transition line separating phase II from phase III becomes of first order due to the presence of \( D\chi^3 \). For any negative \( D \), (i) and (ii), a critical point CP1 appears as a remnant of TCP, for \( D = 0 \). TCP2 remains on the phase diagram for \(-1 < D < 0\), (i), which eventually coincides with the triple point at \( D = -1\), (ii). For positive \( D \), (iii) and (iv), the transition line which separates phase I from phase II turns to be of first order everywhere. The triple point approaches the TCP1 and coincides when a positive \( D \) reaches unity. The different order of phase transition between phase I and phase II for \(-1 < D < 0\) to that for \(0 < D < 1\) can be understood as follows: For \( D = 0 \) (see Fig. [I]) the vacuum expectation value (VEV) \( \chi_0 \) is positive in phase I near the phase boundary between phase I and II due to the existence of the \(-\sigma^2\chi\) term in the potential. In phase II, on the other hand, when the positive \( \chi_0 \) provides a local minimum of the potential, \(-\chi_0\) also does, and both coincide with the global minima. These two vacua are physically equivalent, so that the phase transition from phase I to phase II can be of second order. When we add \( D\chi^3 \) term with negative \( D \) to the potential, the local minimum corresponding to the positive \( \chi_0 \) is only the global minimum in phase II. This can be smoothly connected to the vacuum in phase I where the VEV \( \chi_0 \) is positive. On the other hand, when \( D \) is positive, the negative \( \chi_0 \) gives the global minimum in phase II. Thus, there is a mismatch of \( \chi_0 \) along the phase boundary separating phase I from phase II, which indicates a first-order transition.

\[ D \] also affects the quark number susceptibility \( \chi_q \). As in the case of \( D = 0 \), the \( \chi_q \) exhibits a more relevant increase toward the \( Z_2 \) symmetry restoration than at the chiral phase transition. The critical exponents of \( \chi_q \) are summarized in Table I. One finds that the two regions, \( D \leq 0 \) and \( 0 < D \), corresponds to different universality. The cubic term plays a similar role to an explicit symmetry breaking term in the potential. This may be an origin for the appearance of a critical point.

For \(-1 < D < 0\), TCP2 for \( h = 0 \) becomes a critical point, CP2, for finite \( h \). When the value of \( h \) is increased, the CP2 approaches the triple point and coincides with it for a certain value of \( h, h_0 \). The topology of the phase diagram for larger \( h \geq h_0 \) agrees with that for \( D \leq -1 \). Similarly, the TCP1 in the \( 0 < D < 1 \) phase diagram becomes a critical point CP1 and disappears for a sufficiently large \( h \). On the other hand, the CP1 stays in the phase diagram Fig. [2] (i) and (ii) for any value of \( D \). The

|   | CP1 | TCP1 | TCP2 |
|---|-----|------|------|
| \( D < 0 \) | \( 2/3 \) | --- | \( 1/2 \) |
| \( D = 0 \) | --- | \( 2/3 \) | \( 1/2 \) |
| \( D > 0 \) | --- | \( 1/2 \) | --- |

\(^3\) As we will show below, the phase transition from phase II to phase III is of first order in a more general parameter choice. Thus, \( \chi_q \) exhibits a jump at the chiral phase transition point.
FIG. 2: Phase diagram for different values of $D$ under $F = 0$ and $h = 0$. The solid and dashed lines indicate first and second order phase boundaries, respectively.

scaling of $\chi_q$ there will be given by

$$\chi_q \sim t^{-2/3}. \quad (2.25)$$

We note that adding finite $F$ to the potential does not generate any essential differences from the above result with $F = 0$.

3. HYPOTHETICAL PHASE DIAGRAM AND QUARK NUMBER SUSCEPTIBILITY

From the above observations one would expect phase diagrams mapped onto $(T, \mu)$ plane. In the chiral limit a new phase where the center symmetry is unbroken but chiral symmetry remains broken might appear in dense matter since at $\mu = 0$ this phase is strictly forbidden by the no-go theorem. With an explicit breaking of chiral symmetry one would draw a phase diagram as in Fig. 3. The intermediate phase remains characterized by a small condensation $|\sigma_0| \ll |\chi_0|$. One would expect a new critical point associated with the restoration of the center symmetry, CP$_2$, rather than that of the chiral symmetry if dynamics prefers a negative coefficient of the cubit term in $\chi$. Multiple critical points in principle can be observed as singularities of the quark number susceptibility.

It has been suggested that a similar critical point in lower temperature could appear in the QCD phase diagram based on the two-flavored Nambu–Jona-Lasinio model with vector interaction [17] and a Ginzburg-Landau potential with the effect of axial anomaly [18]. There the interplay between the chiral (2-quark) condensate and BCS pairings plays an important role. In our framework without diquarks, the critical point discussed in Fig. 3 (left) is driven by the interplay between the 2-quark and 4-quark condensates, and is associated with restoration of the center symmetry where anomalies have nothing to do with its appearance. Nevertheless, the cross over in low temperatures may have a close connection to the quark-hadron continuity [19] and it is an interesting issue to explore a possibility of dynamical center symmetry breaking in microscopic calculations. The present potential (2.29) leads to a first-order transition of chiral symmetry even with an explicit breaking. This may be replaced with a cross over when one considers higher order terms in fields and other symmetry breaking terms as well as in-medium correlations to baryonic excitations, which is beyond the scope of this paper.
FIG. 3: Schematic phase diagram mapped onto \((T, \mu)\) plane with a negative \(D\) (left) and with a positive \(D\) (right). The solid lines indicate first order phase boundaries, and dashed lines correspond to cross over.

FIG. 4: The behavior of the baryon number susceptibility as a function of chemical potential assuming the phase diagram of Fig. 3 (left). The condensates and the susceptibility show a jump also at \(\mu_{z2}\) when the phase structure of Fig. 3 (right) is preferred.

 Appearance of the above intermediate phase seems to have a similarity to the notion of Quarkyonic Phase \cite{10, 13}, which is originally proposed as a phase of dense matter in large \(N_c\) limit. The transition from hadronic to quarkyonic world can be characterized by a rapid change in the net baryon number density. This feature is driven by the restoration of center symmetry and is due to the fact that the Yukawa coupling of \(\chi\) to baryons is not allowed by the \(Z_2\) invariance. Fig. 4 shows an expected behavior of the quark (baryon) number susceptibility which exhibits a maximum when across the \(Z_2\) cross over. This can be interpreted as the realization of the quarkyonic transition in \(N_c = 3\) world. How far \(\mu_{z2}\) from \(\mu_{\mathrm{chiral}}\) is depends crucially on its dynamical-model description. \(^4\)

It should be noticed that the critical point in low density region, indicated by CP\(_1\) in Fig. 3 (left), is different from a usually considered CP \cite{20} in the sense that the CP\(_1\) is not on the cross over line attached to the \(T = 0\) axis. When we take a path from the broken phase (both \(\sigma_0\) and \(\chi_0\) are large) to the symmetric phase (both \(\sigma_0\) and \(\chi_0\) are small) passing near the CP\(_1\), the \(\chi_{2Q}\) may exhibit two peaks; one is located near CP\(_1\) and another is on the cross over line. We show a schematic behavior of \(\chi_{2Q}\) as a function of temperature, together with \(\sigma_0\) and \(\chi_0\) in Fig. 5. The appearance of two peaks in \(\chi_{2Q}\) reflects the fact that \(\sigma_0\) becomes small across the CP\(_1\) and the cross over. The first decrease in \(\sigma_0\) near CP\(_1\) is caused by a dropping \(\chi_0\), while the second is by the chiral symmetry restoration.

\(^4\) Thus, the present analysis does not exclude the possibility that both transitions take place simultaneously and in such case enhancement of \(\chi_B\) is driven by chiral phase transition. The phase with \(\chi_0 \neq 0\) and \(\sigma_0 = 0\) does not seem to appear in the large \(N_c\) limit \cite{5, 6, 7}. It would be expected that including \(1/N_c\) corrections induce a phase with unbroken center symmetry.
4. HADRON MASS SPECTRA AND PION DECAY CONSTANT

In this section we derive meson mass spectra in a linear sigma model. The Lagrangian with the potential (4.10) is expressed in terms of the mesonic fields as

\[
\mathcal{L} = \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \bar{\phi} \cdot \partial^\mu \phi \right) + \frac{1}{2} \left( \partial_\mu \chi \partial^\mu \chi + \partial_\mu \bar{\psi} \cdot \partial^\mu \psi \right) - U(\sigma, \phi, \chi, \psi) \tag{4.11}
\]

with

\[
U = -\frac{m^2}{2} \left( \sigma^2 + \bar{\phi}^2 \right) + \frac{\lambda^2}{4} \left( \sigma^2 + \bar{\phi}^2 \right)^2 - \frac{m_0^2}{2} \left( \chi^2 + \bar{\psi}^2 \right) + \frac{\lambda^0}{4} \left[ \chi^2 + \bar{\psi}^2 + \frac{2}{3} \bar{\chi}^2 \right] + \frac{\lambda^2}{4} \left( \chi^2 + \bar{\psi}^2 \right)^2 - \frac{m^2}{2} \left( \sigma^2 + \bar{\phi}^2 \right)^2 \tag{4.12}
\]

where \( m_0 \equiv -g \left( g > 0 \right) \) and \( g_2 = 0 \) were taken. In addition, we set \( g_4 = 0 \) since the \( g_4 \)-term generates only a shift in \( m^2 \) for \( N_f = 2 \). We also set the explicit breaking being zero.

The condensate of the mesonic fields in the phase where both the chiral symmetry and its center \( Z_2 \) are broken are determined from the coupled gap equations given by

\[
\begin{align*}
\sigma_0 &= \frac{2}{\sqrt{3} g} \left( \frac{\lambda^2}{3} \chi_0 - m^2 + g_3 \sqrt{\chi_0} \right) \chi_0, \\
\chi_0 &= \frac{1}{\sqrt{3} g} \left( \chi^2 \sigma_0^2 - m^2 \right),
\end{align*}
\]

with \( \lambda^2 = \lambda^2_1 + 3 \lambda^2_2 \). Shifting the fields as

\[
\sigma \rightarrow \sigma + \sigma_0, \quad \chi \rightarrow \chi + \chi_0,
\]

the potential reads

\[
\mathcal{U} = \frac{1}{2} \sigma_0^2 + \frac{1}{2} \chi_0^2 \sigma_0^2 + \frac{1}{2} \psi_0^2 + \frac{1}{2} \sigma^2 \chi_0^2 + \frac{1}{2} \bar{\psi}^2 \tag{4.13}
\]

where ellipses stand for the terms including the fields more than three, and

\[
\begin{align*}
m_\sigma^2 &= 2 \lambda^2 \sigma_0^2, \\
m_\chi^2 &= \frac{\sqrt{3}}{2} \frac{g_3}{\chi_0} \sigma_0^2 + \frac{2}{3} \lambda^2 \chi_0^2 + \frac{1}{\sqrt{3}} g_3 \chi_0, \\
m_\psi^2 &= \frac{4}{\sqrt{3}} \frac{g_3}{\chi_0} \sigma_0^2,
\end{align*}
\]

The mass terms thus become

\[
\mathcal{U}^{(2)} = \frac{1}{2} \left( \sigma, \chi \right) \left( \begin{array}{cc}
m_\sigma^2 & -\sqrt{3} g_3 \sigma_0 \\
-\sqrt{3} g_3 \sigma_0 & m_\chi^2
\end{array} \right) \left( \begin{array}{c}
\sigma \\
\chi
\end{array} \right)
\]

\[
+ \frac{1}{2} \left( \psi, \bar{\psi} \right) \left( \begin{array}{cc}
m_\psi^2 & -\sqrt{2} g_3 \sigma_0 \\
-\sqrt{2} g_3 \sigma_0 & m_\psi^2
\end{array} \right) \left( \begin{array}{c}
\psi \\
\bar{\psi}
\end{array} \right). \tag{4.14}
\]

Obviously, the determinant of the above mass matrix for \( \phi \) and \( \psi \) is zero and thus massless pseudo-scalar fields are a mixture of 2-quark and 4-quark states.

The mass eigenstates are introduced with a rotation matrix as

\[
\begin{pmatrix}
S' \\
S
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\sigma \\
\chi
\end{pmatrix}, \tag{4.15}
\]

\[
\begin{pmatrix}
\bar{\rho}' \\
\bar{\rho}
\end{pmatrix} = \begin{pmatrix}
\cos \bar{\theta} & -\sin \bar{\theta} \\
\sin \bar{\theta} & \cos \bar{\theta}
\end{pmatrix}
\begin{pmatrix}
\bar{\sigma} \\
\bar{\psi}
\end{pmatrix}, \tag{4.16}
\]

with the angles

\[
\tan (2\bar{\theta}) = \frac{2 \sqrt{3} g_3 \sigma_0}{m_\chi^2 - m_\phi^2}, \quad \tan (2\bar{\theta}) = \frac{4 \sqrt{3} g_3 \chi_0}{3 \sigma_0^2 - 8 \chi_0}. \tag{4.17}
\]

The masses of scalar mesons are given by

\[
\begin{align*}
m_S^2 &= m_\phi^2 \cos^2 \theta + m_\chi^2 \sin^2 \theta - \sqrt{3} g_3 \sigma_0 \sin (2\theta), \\
m_{\bar{\phi}}^2 &= m_\phi^2 \cos^2 \theta + m_\chi^2 \sin^2 \theta + \sqrt{3} g_3 \sigma_0 \sin (2\theta), \tag{4.18}
\end{align*}
\]

and those of pseudo-scalar mesons by

\[
\begin{align*}
m_P &= 0, \\
m_{\bar{\rho}}^2 &= \frac{g(3\sigma_0^2 + 8\chi_0^2)}{2\sqrt{3} \chi_0}, \tag{4.19}
\end{align*}
\]

with

\[
\cos \bar{\theta} = \frac{\sqrt{3} \sigma_0}{\sqrt{3 \sigma_0^2 + 8 \chi_0}}, \quad \sin \bar{\theta} = \frac{2 \sqrt{2} \chi_0}{\sqrt{3 \sigma_0^2 + 8 \chi_0}}. \tag{4.20}
\]

The pion decay constant is read from the Noether current, \( J_3^\mu \sim \sigma_0 \partial^\mu \phi + 4 / \sqrt{3} \chi_0 \partial^\mu \psi \), as

\[
F_\pi = \sqrt{\frac{2}{3} \frac{g_3}{\chi_0}}. \tag{4.21}
\]

Since we consider a system in the chiral limit, the massive \( P' \) state is decoupled from the current and \( F_{P'} \), as it should be. It should be noted that, when \( |\sigma_0| \gg |\chi_0| \), the NG boson is dominantly the 2-quark state. The 4-quark component becomes more relevant for \( \sqrt{3} |\sigma_0| \leq \sqrt{8} |\chi_0| \), i.e., \( \bar{\theta} > \pi/4 \).

When the coupling \( g_3 \) is negative, which corresponds to \( D < 0 \) in the Ginzburg-Landau potential given in section 2, the phase transition from phase I (\( \sigma_0 \neq 0 \) and \( \chi_0 \neq 0 \)) to phase II (\( \sigma_0 = 0 \) and \( \chi_0 = 0 \)) can be of second order. In such a case, the restoration of the center \( Z_2 \) symmetry is characterized by vanishing \( \sigma_0 \). Approaching the restoration from broken phase, one finds the lowest scalar meson mass degenerate with the \( P \) state, while the pion decay constant remains finite due to \( \chi_0 \neq 0 \);

\[
\begin{align*}
m_S &\rightarrow m_P = 0, \\
F_\pi &\rightarrow \sqrt{\frac{8}{3} \chi_0}. \tag{4.22}
\end{align*}
\]

with

\[
\chi_0 = \sqrt{\frac{3m^2}{\lambda^2} + \left( \frac{\sqrt{3} g_3}{2 \lambda^2} \right)^2 - \frac{\sqrt{3} g_3}{2 \lambda^2}}. \tag{4.23}
\]
The vanishing $S$-state mass corresponds to a divergence of the susceptibility $\chi_2$, which is responsible to restoration of the center symmetry. The scalar $S$ and pseudo-scalar $P$ states thus become the chiral partners on the phase boundary. In the $Z_2$ symmetric phase the meson masses are found from the potential (4.2)

\[ m_\sigma^2 = -m^2 - \sqrt{3} g \chi_0, \quad m_\omega^2 = -m^2 + \frac{g}{\sqrt{3}} \chi_0, \quad m_\chi^2 = \frac{2}{3} A^2 \chi_0^2 + \frac{g}{\sqrt{3}} \chi_0, \quad m_\psi^2 = 0. \]  

There is no mixing in this phase, so that $\sigma, \phi, \chi, \psi$ are the mass eigenstates. This implies that the pure 4-quark state is massless. In the broken phase, $\chi_0 = 0$ will be of weak first-order. In this case, $\chi_0$ is the isospin 2 state will become very light near the phase transition. This may suggest that, when the $g_3\text{Det}_2$ term is small and the chiral phase transition is of weak first-order, a light exotic states with $I = 2$ might exist in dense baryonic matter.

When $|g_3/g| \ll 1$, the chiral phase transition from phase II ($\sigma_0 = 0$ and $\chi_0 \neq 0$) to phase III ($\sigma_0 = 0$ and $\chi_0 = 0$) will be of weak first-order. In this case, $\chi_0$ is the mass eigenstate. The vector and axial-vector states neither degenerate in mass $[6]$, since both vector and axial-vector currents are invariant under the $Z_2$ transformation but broken chiral symmetry does not dictate the same masses. The vector and axial-vector states are not degenerate in mass $[6]$, since both vector and axial-vector currents are invariant under the $Z_2$ transformation but broken chiral symmetry does not dictate the same masses.

It is conceivable that phase II and phase III in Fig.1 with two TCPs. When the cubic term $\sigma^3$ is included, it is conceivable that phase II and phase III in Fig.2 are separated by a second-order phase boundary, which will become a first-order one when we take quantum fluctuations into account $[23]$. The topologies are expected to be quite similar to those shown in Fig.2, so that we expect a strong enhancement of the quark number susceptibility at the $Z_3$ restoration point. Differently from the case for $N_f = 2$, the $\Sigma_{ab}$ field is allowed to couple to the octet baryon states as, e.g. $B_a \Sigma_{ab} B_b$, and the baryon number current couples to the $\chi$ state which becomes massless at the chiral restoration point. As a result, the quark number susceptibility might show another peak at the chiral restoration point. Hadron masses in the $Z_3$ symmetric phase are slightly different from those under $Z_2$ invariance: In the mesonic sector the party partners are degenerate and the degeneracy does not generally occur in the baryonic sector $[6]$. Following Ref. $[6]$, possible operators for the baryons are expressed as

\[ B_1^L = (q_L q_L q_R)_L, \quad B_2^L = (q_L q_R q_R)_L, \quad B_3^L = (q_L q_R q_L)_L, \quad B_1^R = (q_R q_R q_R)_R, \quad B_2^R = (q_R q_L q_R)_L, \quad B_3^R = (q_R q_R q_R)_R, \]  

where the color and flavor indices are omitted. For the octet baryons, the representations under the chiral $SU(3)_L \times SU(3)_R$ of these baryonic fields are assigned as

\[ B_1^L \sim (3, \bar{3}), \quad B_2^L \sim (3, 3), \quad B_3^L \sim (8, 1), \quad B_1^R \sim (3, \bar{3}), \quad B_2^R \sim (3, 3), \quad B_3^R \sim (1, 8). \]  

When the $B_3^R$ is the lightest octet baryon, which we call
the naive assignment, it is still massive in the $Z_3$ symmetric phase, since the Yukawa coupling of the 4-quark state $\Sigma_{ab}$ is possible as, e.g. $B_a \Sigma_{ab} B_b$. When the lightest baryons are described by a combination of $B^1$ and $B^2$, which we call the mirror assignment, they are degenerate with each other in the $Z_3$ symmetric phase. We summarize these features in Table III. The baryon masses crucially depend on a way of chirality assignment. It would be an interesting issue to clarify this within a more elaborated model.

The main assumption in this paper is a dynamical breaking of chiral symmetry $SU(N_f)_L \times SU(N_f)_R$ down to a non-standard $SU(N_f)_V \times (Z_{N_f})_A$ although this seems to be theoretically self-consistent. Calculations using the Swinger-Dyson equations or Nambu-Jona-Lasinio type models with careful treatment of the quartic operators may directly evaluate this reliability. Besides, anomalously light NG bosons, $m_\pi^2 \sim O(m_q^2)$, could lead to an s-wave pion condensation as discussed in [24]. A calculation using the Skyrme model shows a similar intermediate phase [3]. Although the above non-standard pattern of symmetry breaking was not imposed in the Skyrme Lagrangian, the result could suggest an emergent symmetry in dense medium. This intermediate phase would be an intriguing candidate of the quarkyonic phase if it could sustain in actual QCD at finite density and would lead to a new landscape of dense baryonic matter.

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APPENDIX A: PHASE BOUNDARIES FROM GINZBURG-LANDAU POTENTIAL

The relevant expressions for the phase boundaries obtained from the potential (2.9) are given below. We will take $D = F = 0$ and the chiral limit $\hbar = 0$.

- Second-order phase transition when $B \geq 1/4$:

$$A = 0 \quad (A.1)$$

The solutions for $\sigma$ and $\chi$ on this boundary are given by

$$\{\sigma_0, \chi_0\} = (0, 0) \quad (A.2)$$

- First-order phase transition when $0 \leq B < 1/4$ and $0 < A \leq 1/8$:

$$A = \left( \frac{3}{8} - \sqrt{\left( \frac{3}{8} \right)^2 + \frac{3}{2} \left( B - \frac{1}{4} \right)} \right) \sqrt{\left( \frac{3}{8} \right)^2 - 2 \left( B - \frac{1}{4} \right)} + \left( \frac{3}{8} \right)^2 + \frac{1}{2} \left( B - \frac{1}{4} \right) \quad (A.3)$$

\[\begin{array}{|c|c|}
\hline
\text{phase I: } \sigma_0 \neq 0, \chi_0 \neq 0 & \text{phase II: } \sigma_0 = 0, \chi_0 \neq 0 \\
\hline
SU(2)_V & SU(2)_V \times (Z_{2})_A \\
m_S \neq 0, m_P = 0 & m_S \neq m_P \neq 0, m_P' = 0 \\
m_V \neq m_A & m_V \neq m_A \\
F_s = \sqrt{\sigma_0^2 + (8/3)\chi_0^2} & F_s = \sqrt{8/3}\chi_0 \\
m_{N^+} \neq 0 & (i) \text{ naive: } m_{N^+} = m_{N^-} \neq 0 \\
& (\text{all states}) \\
& (i) \text{ mirror: } m_{N^+} = m_{N^-} \neq 0 \\
& \text{(all states)} \\
\hline
\end{array}\]
\[ \begin{array}{|c|c|}
\hline
\text{phase I: } \sigma_0 \neq 0, \chi_0 \neq 0 & \text{phase II: } \sigma_0 = 0, \chi_0 \neq 0 \\
\hline
SU(N_f)_V & SU(N_f)_V \times (Z_{N_f})_A \\
m_S \neq 0, m_P = 0 & m_S = m_P \neq 0, m_P' = 0 \\
m_V \neq m_A & m_V \neq m_A \\
m_{N^+} \neq 0 & (i) \text{ naive: } m_{N^+} \neq 0 \\
 & (ii) \text{ mirror: } m_{N^+} = m_{N^-} \neq 0 \\
\hline
\end{array} \]

**TABLE III**: Same as in Table II but for \( N_f = 3 \).

The solutions for \( \sigma \) and \( \chi \) on this boundary are given by

\[
(\sigma_0, \chi_0) = (0, 0),
\]

\[
\pm \left[ \frac{1}{2} \left( \frac{1}{8} + \sqrt{\left( \frac{3}{8} \right)^2 + \frac{1}{2} \left( B - \frac{1}{4} \right)} \right) \right] \sqrt{3/8 - 2 \left( B - \frac{1}{4} \right) + \sqrt{\left( \frac{3}{8} \right)^2 + \frac{1}{2} \left( B - \frac{1}{4} \right)}}^{1/2},
\]

\[
\frac{1}{2} \sqrt{-\frac{3}{8} - 2 \left( B - \frac{1}{4} \right) + \sqrt{\left( \frac{3}{8} \right)^2 + \frac{1}{2} \left( B - \frac{1}{4} \right)}}.
\] (A.4)

- **First-order phase transition** when \(-1/8 < B < 0\) and \(1/8 < A < 1/4\):

\[
A = \frac{1}{8} - B.
\] (A.5)

The solutions for \( \sigma \) and \( \chi \) on this boundary are given by

\[
(\sigma_0, \chi_0) = \left( 0, \pm \sqrt{-\frac{B}{2}} \right), \left( \pm \sqrt{\frac{1}{2} \left( B + \frac{1}{8} \right)}, \frac{1}{4} \right).
\] (A.6)

- **Second-order phase transition** when \( B \leq -1/8 \) and \( A \geq 1/4 \):

\[
A = \sqrt{-\frac{B}{2}}.
\] (A.7)

The solutions for \( \sigma \) and \( \chi \) on this boundary are given by

\[
(\sigma_0, \chi_0) = \left( 0, \pm \sqrt{-\frac{B}{2}} \right).
\] (A.8)

- **Second-order phase transition** when \( A > 1/8 \):

\[
B = 0.
\] (A.9)

The solutions for \( \sigma \) and \( \chi \) on this boundary are given by

\[
(\sigma_0, \chi_0) = (0, 0).
\] (A.10)

\[ [1] \text{For reviews, see e.g., T. Hatsuda and T. Kunihiro, Phys. Rept. 247, 221 (1994); R. D. Pisarski, hep-ph/9503330.} \]

F. Klingl, N. Kaiser and W. Weise, Nucl. Phys. 247, 221 (1994); R. D. Pisarski, [hep-ph/9503330].
[1] K. Rajagopal and F. Wilczek, Phys. Rev. Lett. 82, 527 (1999), A. 624, 527 (1997), K. Rajagopal and F. Wilczek, [arXiv:hep-ph/0011333], hep-ph/0003183, G. E. Brown and M. Rho, Phys. Rept. 363, 85 (2002), R. Rapp and J. Wambach, Adv. Nucl. Phys. 25, 1 (2000), M. Buballa, Phys. Rept. 407, 205 (2005), R. S. Hayano and T. Hatsuda, [arXiv:0812.1702 [nucl-ex]], R. Rapp, J. Wambach and H. van Hees, [arXiv:0901.3289 [hep-ph]].
[2] M. G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B 537, 443 (1999).
[3] M. Knecht and J. Stern, [arXiv:hep-ph/9411253], J. Stern, [arXiv:hep-ph/9712438], [arXiv:hep-ph/9801282].
[4] B. Holdom and G. Triantaphyllou, Phys. Rev. D 51, 7124 (1995), B. Holdom, Phys. Rev. D 54, 1068 (1996).
[5] P. Maris and Q. Wang, Phys. Rev. D 53, 4650 (1996), F. S. Roux, T. Torma and B. Holdom, Phys. Rev. D 61, 056009 (2000).
[6] I. I. Kogan, A. Kovner and M. A. Shifman, Phys. Rev. D 59, 016001 (1999).
[7] M. G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. A 538, 215 (1999), A. M. Halasz, A. D. Jackson, R. E. Shrock, M. A. Stephanov and J. J. M. Verbaarschot, Phys. Rev. D 58, 096007 (1998), Y. Hatta and T. Ikeda, Phys. Rev. D 67, 014028 (2003).
[8] S. Roessner, T. Hell, C. Ratti and W. Weise, Phys. Lett. B 591, 277 (2004), C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D 73, 041019 (2006), E. Megias, E. Ruiz Arriola and L. L. Salcedo, Phys. Rev. D 74, 065005 (2006), S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa and R. Ray, Phys. Rev. D 73, 114007 (2006), S. Roessner, C. Ratti and W. Weise, Phys. Rev. D 75, 034007 (2007), C. Ratti, S. Roessner and W. Weise, Phys. Lett. B 649, 57 (2007), C. Sasaki, B. Friman and K. Redlich, Phys. Rev. D 75, 074013 (2007), H. Hansen, W. M. Alberico, A. Beraudo, A. Molinari, M. Nardi and C. Ratti, Phys. Rev. D 75, 065004 (2007), Z. Zhang and Y. X. Liu, Phys. Rev. C 75, 064910 (2007), S. Roessner, T. Hell, C. Ratti and W. Weise, Nucl. Phys. A 814, 118 (2008), T. Hell, S. Roessner, M. Cristoforetti and W. Weise, Phys. Rev. D 79, 014022 (2009), K. Fukushima, Phys. Rev. D 77, 114028 (2008) [Erratum-ibid. D 78, 039902 (2008)]; [arXiv:0901.0783 [hep-ph]].
[9] L. McLerran, K. Redlich and C. Sasaki, Nucl. Phys. A 824, 86 (2009).
[10] R. D. Pisarski and F. Wilczek, Phys. Rev. Lett. 82, 3956 (1999).
[11] M. Asakawa and K. Yazaki, Nucl. Phys. A 504, 668 (1989), J. Berges and K. Rajagopal, Nucl. Phys. B 538, 215 (1999), A. M. Halasz, A. D. Jackson, R. E. Shrock, M. A. Stephanov and J. J. M. Verbaarschot, Phys. Rev. D 58, 096007 (1998), Y. Hatta and T. Ikeda, Phys. Rev. D 67, 014028 (2003).
[12] S. D. H. Hsu, F. Sannino and M. Schwetz, Mod. Phys. Lett. A 16, 1871 (2001).
[13] R. D. Pisarski and F. Wilczek Phys. Rev. D 29, 338 (1984).
[14] T. D. Cohen and W. Broniowski, Phys. Lett. B 342, 25 (1995).