Penta-quark in Anisotropic Lattice QCD

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Penta-quark (5Q) baryons are studied using anisotropic lattice QCD for high-precision measurement of temporal correlators. A non-$NK$-type interpolating field is employed to study the 5Q states with $J^P = 1/2^\pm$ and $I = 0$. In $J^P = 1/2^+$ channel, the lowest-lying state is found at $m_{5Q} \approx 2.25$ GeV, which is too massive to be identified as the $\Theta^+(1540)$. In $J^P = 1/2^-$ channel, the lowest-lying state is found at $m_{5Q} \approx 1.75$ GeV. To distinguish a compact 5Q resonance state from an $NK$ scattering state, a new method with “hybrid boundary condition (HBC)” is proposed. As a result of the HBC analysis, the observed state in the negative-parity channel turns out to be an $NK$ scattering state.

1. Introduction

The discovery of a narrow resonance $\Theta^+(1540)$ has an important impact on the studies of exotic hadrons [11]. This resonance is peaked at $1.54 \pm 0.01$ GeV with a decay width narrower than 25 MeV. It is confirmed to have baryon number $B = 1$, charge $Q = +1$ and strangeness $S = 1$. The quantum number itself implies that the $\Theta^+$ has to contain at least one $\bar{s}$. Hence, its simplest configuration is $uudd\bar{s}$, i.e., a manifestly exotic penta-quark (5Q). The experimental discovery was motivated by a theoretical prediction [2].

Since its discovery, there have been an enormous number of theoretical contributions [3, 4]. The determination of its parity is an important topic. Experimental determination of parity of the $\Theta^+$ is difficult, and the opinions are divided into two pieces in the theoretical side. There are several lattice QCD studies of 5Q available [5, 6, 7, 8, 9, 10], which, however, have not yet reached a consensus. Refs.[5, 6, 10] claim the existence of a bound negative-parity 5Q state, whereas Ref.[7] claims that the lowest-lying 5Q bound state has positive-parity. (Note that, below this positive-parity state, they actually obtained a negative-parity state, which they claim to be a scattering state.) Refs.[8, 9] have observed no evidence for narrow 5Q resonances on lattice. Ref.[11] studies the static 5Q potential to provide physical insights into effective models. Under these circumstances, our aim is to provide an accurate lattice QCD result using anisotropic lattice technique. Furthermore, we propose a new method with “hybrid boundary condition (HBC)”, which can raise the s-wave $NK$ threshold artificially. HBC is expected to serve as a convenient

\textsuperscript{*}The lattice QCD numerical calculation has been done on NEC SX-5 at Osaka University.
tool in the studies of negative-parity 5Q states.

2. Lattice QCD Calculation

2.1. Results with the standard periodic BC

We employ the standard Wilson gauge action at \( \beta = 5.75 \) \((a_\gamma^{-1} \simeq 1.1 \text{ GeV})\) on \( 12^3 \times 96 \) lattice with the renormalized anisotropy \( a_s/a_t = 4 \) to generate 504 gauge field configurations. The anisotropic lattice technique is known to serve as a powerful tool for high-precision measurements \[12\] \[13\] \[14\]. We use \( O(a) \)-improved Wilson quark (clover) action \[12\] \[13\] adopting four values of hopping parameter \( \kappa = 0.1210(0.0010)0.1240 \), which cover the quark mass region as \( m_s - 2m_u \). By keeping \( \kappa_s = 0.1240 \) fixed for \( s \) quark, we change \( \kappa = 0.1210 - 0.1240 \) for \( u \) and \( d \) quarks for chiral extrapolation. We adopt the spatially extended source to enhance the low-lying spectrum.

Assuming that the quantum number of the \( \Theta^+ \) is spin \( J = 1/2 \) and isospin \( I = 0 \), we employ a non-\( NK \) type interpolating field \[15\]

\[
\psi \equiv \epsilon_{abc}\epsilon_{def}\epsilon_{efg} \left( u_a^T C\gamma_5 d_b \right) \left( u_d^T C d_e \right) C s_g^T ,
\]

to construct two-point 5Q correlators. Here, \( a - g \) denote color indices, and \( C \equiv \gamma_4\gamma_2 \) denotes the charge conjugation matrix. The Dirac bispinor field \( \psi \) couples to baryon states of both parities. Since \( \psi \) transforms as \( \psi(t, \vec{x}) \rightarrow +\gamma_4\psi(t, -\vec{x}) \) under the spatial reflection, the projection matrix \( P_\pm \equiv (1 \pm \gamma_4)/2 \) can be used to project the intermediate states into positive and negative parities, respectively, in the “forward-propagation” region \( 0 \ll t \ll N_t/2 \).

In both the parity channels, the effective-mass plots have plateaus, where the single-exponential fit analysis is performed. In the positive-parity channel, we obtain \( m_{5Q} \simeq 2.25 \text{ GeV} \) after the chiral extrapolation, which is too massive to be identified with the experimentally observed \( \Theta^+(1540) \). In the negative-parity channel, we obtain \( m_{5Q} \simeq 1.75 \text{ GeV} \) after the chiral extrapolation, which is rather close to the empirical value.

2.2. Results with HBC

In order to identify this negative-parity state as the \( \Theta^+(1540) \), it is necessary to confirm that it is a compact 5Q resonance state rather than an \( NK \) scattering state. For this purpose, we propose a new method with “hybrid boundary condition (HBC)”. In the standard boundary condition, the spatially periodic BC is imposed on \( u, d \) and \( s \) quark fields. Hence, \( N \) and \( K \) are subject to the spatially periodic BC, and \( s \)-wave \( NK \) scattering spectrum starts at \( E_{\text{th}} \simeq m_N + m_K \) as a consequence of the negligibly weak \( NK \) interaction in comparison with \( m_N \) or \( m_K \). In HBC, the spatial BC is twisted in a flavor-dependent manner. The spatially periodic BC is imposed on \( s \) quark field, whereas the spatially anti-periodic BC is imposed on \( u \) and \( d \) quark fields. In this case, since \( N(\bar{u}d\bar{d}) \) and \( K(u\bar{s}, \bar{d}s) \) contain odd number of \( u \) and \( d \) quarks, they are subject to the spatially anti-periodic BC. Due to the finiteness of the spatial box, allowed spatial momenta of \( N \) and \( K \) are quantized as \( p_i = (2n_i + 1)\pi/L \), where \( n_i \in \mathbb{Z} \), and \( L \) denotes the spatial extension of the box. Note that the momentum is quantized in a different manner from the standard periodic BC case, and \( N \) and \( K \) have the non-zero minimum momentum as \( |\vec{p}_{\text{min}}| = \sqrt{3}\pi/L \). As a result, \( NK \) scattering spectrum in HBC starts at an artificially
Figure 1. Lattice QCD result for the negative-parity 5Q baryon. The results of standard periodic BC (l.h.s.) are compared with HBC (r.h.s.) for each hopping parameter $\kappa$. Circles denote results obtained from the single-exponential fit analysis. The solid lines denote the corresponding $NK$ thresholds $E_{\text{th}}$.

raised value as $E_{\text{th}} \simeq \sqrt{m_K^2 + \vec{p}_{\text{min}}^2} + \sqrt{m_K^2 + \vec{p}_{\text{min}}^2}$ depending on the spatial extension of the box. On the other hand, since the $\Theta^+(uudd\bar{s})$ contains even number of $u$ and $d$ quarks, the $\Theta^+$ is subject to the spatially periodic BC. Hence, it can have the zero-spatial momentum. If the spatial size of the $\Theta^+$ is sufficiently smaller than $L$, it is safely expected that this compact 5Q state should be less sensitive to the change of the spatial BC. In this case, its mass will stay at the same location.

In Fig. 1, we show the lattice QCD result for 5Q baryon. We compare the HBC results with the standard periodic BC results. The closed circles denote the results of the single-exponential fit analysis. The solid lines denotes the corresponding $NK$ thresholds. We see that the $NK$ thresholds are raised by 200–250 MeV due to HBC, and that the best fit masses are raised by consistent amount as the shifts of the $NK$ thresholds. This implies that the negative-parity states with the mass $m_{5Q} \simeq 1.75$ GeV observed in the present lattice QCD calculation is an $NK$ scattering state rather than a compact 5Q resonance.

3. Summary and Discussions

To summarize, we have performed the anisotropic lattice QCD studies of the 5Q state for spin $J^P = 1/2^\pm$ and isospin $I = 0$. We have obtained the lowest-lying positive-parity state at $m_{5Q} \simeq 2.25$ GeV after the chiral extrapolation, which is considered to be too massive to be identified with the $\Theta^+(1540)$. The lowest-lying negative-parity 5Q state has been found at $m_{5Q} \simeq 1.75$ GeV, which is rather close to the empirical value. It is necessary to clarify whether this state is a compact 5Q resonance state or an $NK$ scattering state in order to identify it as the $\Theta^+(1540)$. For this purpose, we have proposed a new method with "hybrid boundary condition (HBC)", which can raise the s-wave $NK$ threshold by a few hundred MeV depending on the extension of the spatial box. From the HBC analysis, it has turned out that the negative-parity state observed at $m_{5Q} \simeq 1.75$ GeV is actually an $NK$ scattering state.
In this way, we have observed no clear signals for the compact 5Q resonance $\Theta^+(1540)$ both in the negative and the positive-parity channels. Note that another null-result is reported by Ref.[8] in lattice QCD. Of course, one of the possible implications of these two negative results is that QCD may not accommodate the $\Theta^+(1540)$ as a resonance pole. Note that experimental existence of the $\Theta^+(1540)$ has not yet been established so far. As an interesting possibility, these null-results may be a consequence of a possibly complicated intrinsic structures of the $\Theta^+$ as suggested by Refs.[16] [17] [18] [19]. If this is the case, the $\Theta^+(1540)$ may not be easily reachable by the lattice QCD with a simple 5Q interpolating field, and it would be desirable to introduce series of new non-local interpolating fields, which can fit such non-trivial structures [20]. Another possible implication is that the $\Theta^+(1540)$ may have simply a different quantum number. Indeed, Refs.[21] [22] [23] [24] discuss the possibility of spin $J = 3/2$. In this respect, it would be interesting to perform lattice QCD calculation for spin 3/2 states [25]. Of course, it is necessary to perform more systematic studies in order to reveal the mysterious nature of the penta-quark $\Theta^+(1540)$.

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