Minimizing Multimodular Functions and Allocating Capacity in Bike-Sharing Systems

Daniel Freund, Shane G. Henderson and David B. Shmoys
Cornell University, Ithaca, New York 14853, USA, df365@cornell.edu, sgh9@cornell.edu, dbs10@cornell.edu

1. Introduction
Bike-sharing systems are now ubiquitous across the United States. Our industry partner, Motivate, operates the largest such systems: in New York City (Citi Bike), Chicago (Divvy), Boston (Blue Bikes), and the Bay Area (Ford GoBike). Together, these systems accounted for more than 70% of the 35 million bike-sharing trips that occurred in the United States in 2017 (NACTO Bike Share Initiative 2018); since 2010, the annual number of bike-sharing trips in the United States has grown a hundredfold. Motivate’s bike-sharing systems consist of a number of stations placed densely within each city; every station has a number of docks, each of which either holds a bike (is full) or is empty. In allowing users to rent a bike from any station with at least one bike and return it to any station with at least one empty dock, the systems provide commuters and tourists alike a sustainable transportation option. Yet, their success in increasing ridership goes hand in hand with a significant struggle to handle asymmetric demand due to “tidal” commuter flows. In bike-sharing, this asymmetry causes out-of-stock events for users who are unable to rent bikes at stations at which all docks are empty, or, arguably even worse, are unable to return a bike at stations where all docks are full. Attempting to alleviate these effects of asymmetric demand, operators try to rebalance the system by moving bikes from full to empty stations, typically by box truck or van. For operators, this constitutes one of the largest operational expenses.

2. Related Work
Given the growth in bike-sharing systems, it is unsurprising that the study of their operations has become a popular area within the field of OR. We refer the reader to extensive literature reviews in Freund et al. (2018), Freund (2018), and de Charbon et al. (2016) as well as the full version of this study.

3. Contribution
In this abstract, we describe how our work with Motivate has helped make the company an industry leader in the development and use of analytics to drive strategic, tactical, and operational decisions in bike-sharing systems. Our data-driven approach has informed the deployment of many operational levers to improve system performance. Beyond helping Motivate’s operations on a tactical level, our methodology has also informed their strategic vision for how to tackle imbalance. We describe a project in which our work introduced new elements to Motivate’s overall system design that reduce the need for (motorized) rebalancing and its associated financial and environmental costs. Essentially, not only did we help Motivate rebalance more efficiently, but we also guided Motivate toward system designs that require less rebalancing in the first place. The project formulated an optimization model to inform the allocation of docks across the system, and, moreover, characterized its mathematical structure, thereby enabling the development of computationally efficient algorithmic tools for its solution. The data-driven approach underlying the project has also been instrumental in driving the operational decisions in Motivate’s rebalancing operations, but we do not describe in detail those innovations here.

This high-impact work has been enabled through innovations in stochastic modeling; in data analysis to fit stochastic models; in optimization formulations, built on the stochastic models that capture key characteristics of strategic, tactical, and operational considerations; and in providing sufficient mathematical understanding of these formulations so as to facilitate the design and analysis of highly efficient optimization algorithms for them. Here, we summarize these contributions and detail their impact.

The foundation of our work with Motivate is the notion of user dissatisfaction functions (UDFs), as first defined by Raviv and Kolka (2013). These functions map a station’s capacity and inventory level at the beginning of a planning period to the expected
number of out-of-stock events over the course of the period. Computing UDFs requires selecting appropriate stochastic processes to model sequences of users wishing to rent and return bikes at stations, and, moreover, estimating the parameters of those stochastic processes. Demand data is, unfortunately, censored when stations are empty or full, due to out-of-stock events. We develop decensoring techniques to obtain time-dependent demand rates for both arrivals and returns at each location, which allows us to compute UDFs. We provide a nonlinear integer program (IP) to optimize over UDFs with flexible capacity at each location. We establish discrete convexity properties through coupling arguments, which then enables the development of an efficient algorithm that not only solves the IP to optimality, but also furthers our understanding of constrained optimization in the setting of discrete convexity. Based on solutions to the IP, Motivate carried out a pilot in NYC to reallocate dock capacity; we use subsequently collected usage data to provide an UDF-based method that estimates the pilot’s impact in reducing the need to rebalance. The pilot’s estimated impact, along with the powerful tools we developed, have induced Motivate to embrace the principle of moving dock capacity, with hundreds of docks moved to date in their systems nationwide.

3.1. Target-Levels for Bikes
The plots of the UDFs in Figure 1 suggest that the expected number of out-of-stock events is a convex function of the initial number of bikes at each station; it can be proven that this holds as a general result, and it has significant implications for Motivate’s operations. The minimizer of the convex function at each station provides a target-level at each point in time, that is, the number of bikes that minimizes out-of-stock events over a subsequent planning horizon.

3.2. System-Wide Optimization for Bikes
The UDFs not only provide target-levels for each station, they can also be used to find, at any given time, the optimal allocation of available bikes within the system. We can formulate this optimization problem, for a given time period, as an integer program: the decision variables are the number of bikes at each station, the constraints dictate that the total number of bikes is bounded by a budget, and that the number of bikes at each station should not exceed the number of docks at the station. The convexity of the UDFs allows us to efficiently solve the IP; in particular, the natural linear relaxation is known to have an optimal integer solution.

4. Allocating Capacity
As we incorporated analytics into Motivate’s rebalancing operations, we became more acquainted with demand patterns in Motivate’s systems and found significant potential in the idea of reallocating docks in the system. Specifically, we found that some locations made much better use of their allocated docks than others. To some extent, this was to be expected; the dock capacity of most stations in most of Motivate’s systems was set when they were first installed. Thus, the capacity was set before any demand was observed. We now describe a methodology, built upon UDFs, that ultimately led Motivate to move hundreds of docks in its systems.

5. Integer Programming Model
The integer program to optimize the system-wide allocation of bikes naturally extends to a formulation that allows for reallocated dock capacity; instead of treating the capacities as parameters that are part of the input, we treat them as additional decision variables. To ensure that the IP merely reallocates the dock capacity already present, we require that the total number of docks used by a solution be bounded above by the number of docks currently in use, though we can also explore the impact of increased dock budgets with the same formulation. Both technical and practical issues arise with this formulation. From a technical perspective, it is not clear how to optimally solve the resulting nonlinear IP; for example, the integrality property of the natural relaxation no longer holds. From a practical perspective, it turns out that the optimal allocation suggests moving more
docks than stakeholders (Motivate, Department of Transportation, etc.) are willing to countenance. Our algorithmic solution addressed both problems simultaneously, as explained next.

6. Gradient-Descent Search and from Local to Global Optima

The feasible solutions to the IP are given by allocations of bikes and docks across stations that fulfill the budget constraints (for both bikes and docks), the bounds on the potential capacity of each station. One can represent the search space of feasible solutions as an undirected graph in which each feasible solution corresponds to a node in the graph. Two nodes corresponding to two feasible solutions are adjacent if one can be obtained from the other by reallocation of at most one dock and at most one bike. With this definition of neighboring feasible solutions, one has a corresponding notion of a “local optimum” – a feasible solution with objective function as good or better than any of its neighbors. We can prove that, in this sense, any locally optimal solution is a global optimum; that is, any local optimum is an optimal solution to the integer program. This optimality characterization allows us to derive our optimization algorithm, which is an intuitive analogue to gradient-descent algorithms in continuous optimization: starting at any given feasible solution, the algorithm repeatedly updates the current solution to be the best solution within the neighborhood of the current solution. A careful implementation of the algorithm allows us to rapidly solve the IP on real instances, yielding optimal solutions within minutes.

7. Best in k Iterations

The discrete gradient-descent algorithm we derived has a desirable property beyond the fact that it quickly finds optimal solutions: starting with the solution to the system-wide optimization for bikes, k iterations of the algorithm yield the best allocation that can be obtained by moving at most k docks. This addresses the practical concern that the optimal solution moves more docks than stakeholders will countenance: by only running the algorithm for k iterations we obtain the optimal solution with the additional constraint that at most k docks may be moved (thereby advancing the scope of results obtainable by the tools of discrete convexity). As a result, we find that even though the optimal allocation may require moving thousands of docks (in NYC), significant impact can be had by moving only a few hundred docks.

8. Robustness

Docks are not as easy to move as bikes; they are heavy pieces of equipment that require a crane to lift them onto a flatbed truck. Consequently, in making a recommendation to move a substantial number of docks, we wanted to be sure that these recommendations were consistently supported by a number of different analyses. In particular, one should think of dock changes as being made on an annual basis, and therefore should be robust to any seasonality effects in the demand patterns. Furthermore, these changes are being made in the context of ongoing expansion to the footprint of the system, which could also affect the demand pattern at previously existing stations. For example, in NYC, a series of system expansions have increased the number of stations in the city from about 330 stations in 2015 to more than 700 stations in 2018 over a substantially increased geographic footprint. Such great changes to the system could easily change the demand structure in ways that would contradict the recommended changes. We have used the discrete gradient-descent algorithm to evaluate the impact of proposed changes on demand data stemming from many different months, including from different years and different seasons. Significantly, the improvement in the objective obtained by the proposed changes was robust with respect to these different demand estimates. Even in NYC, we found that the improvement due to optimization-determined reallocations based on 2015 data, is quite stable under 2018 data.

Beyond considering robustness with respect to demand patterns, Motivate expressed concern about an issue not captured by the optimization problem described above: effectively, the IP optimizes the service quality for a system with perfect bike allocations. Bike allocations are primarily influenced through overnight rebalancing, so the optimization model effectively assumes perfect overnight rebalancing. In practice, rebalancing is limited, and a goal of this project was to reduce the need for rebalancing. To account for this shortcoming, we defined an alternate objective: a long-run average of the user dissatisfaction function that computes the expected number of out-of-stock events not over the course of one day, but over the course of infinitely many days. In many ways, this is the diametrically opposite philosophy to the one we have adopted thus far – this models the expected number of out-of-stock events at a station with no rebalancing at all. By optimizing with the different objectives, we obtain two different “optimal” solutions. In principle, it is easy to construct examples of distributions over demand for individual stations such that when evaluating one optimal solution with the other objective, the performance is arbitrarily
worse than the optimal solution for the other. Surprisingly, we find that this did not occur on real data: optimizing for one objective gives a near-optimal solution for the other. It is worth emphasizing that this is a property of the data, not of the model. Even more surprisingly, this is not due to small L1 distance between the two “optimal” solutions or due to the objective being particularly flat near the optimum. Instead, it is a function of both objectives being flat in the direction of the other optimal solution. Phrased differently, it turns out that the solutions diverge on stations at which optimizing has little effect on the objective.

9. Implementation and Evaluation

After a long planning period, a pilot project in NYC entailing the movement of 34 docks went ahead in November 2017. The pilot allowed us to run a counterfactual analysis based on realized demand: for each of the three stations that had docks added, we computed, for every weekday in April 2018 (given the sequence of arrivals that occurred over the course of the day), the number of out-of-stock events that would have occurred with the same demand if the docks had not been added (and no additional rebalancing had been performed). This can be achieved in a purely data-driven manner, without the need to resort to stochastic process modeling and the attendant data-fitting issues, as we have proved. We also evaluated the increase in the number of out-of-stock events at the three stations that had their capacity reduced, though in that case, one must rely on fitted stochastic models. The resulting analysis indicates that the reduction in out-of-stock events per dock added to stations averaged about 1.5 per day, while the increase in out-of-stock events at stations with reduced capacity averaged about 0.08 per dock per day. As such, each dock that was moved reduced the need for rebalancing (with average service quality kept constant) by more than 1.42 bikes per weekday. Translating this into savings in reduced rebalancing needs, annual savings of tens of thousands of dollars were realized, even though the pilot moved only a tiny fraction of all docks. Other Motivate systems across the United States have since followed suit, with hundreds of docks moved to date.

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