Inverse design-momentum, a method for the preliminary design of horizontal axis wind turbines

L Battisti¹,³, G Soraperra², R Fedrizzi¹ and L Zanne¹

¹ DIMS - Dept. of Mechanical and Structural Engineering, University of Trento, via Mesiano, 77, 38050 Trento, Italy
² TOZZI NORD wind turbines, via S. Sebastian s.n.c. 38100 Trento, Italy
E-mail: lorenzo.battisti@ing.unitn.it

Abstract. Wind turbine rotor prediction methods based on generalized momentum theory BEM routinely used in industry and vortex wake methods demand the use of airfoil tabulated data and geometrical specifications such as the blade spanwise chord distribution. They belong to the category of “direct design” methods. When, on the other hand, the geometry is deduced from some design objective, we refer to “inverse design” methods. This paper presents a method for the preliminary design of wind turbine rotors based on an inverse design approach. For this purpose, a generalized theory was developed without using classical tools such as BEM. Instead, it uses a simplified meridional flow analysis of axial turbomachines and is based on the assumption that knowing the vortex distribution and appropriate boundary conditions is tantamount to knowing the velocity distribution. The simple conservation properties of the vortex components consistently cope with the forces and specific work exchange expressions through the rotor. The method allows for rotor arbitrarily radial load distribution and includes the wake rotation and expansion. Radial pressure gradient is considered in the wake.

The capability of the model is demonstrated first by a comparison with the classical actuator disk theory in investigating the consistency of the flow field, then the model is used to predict the blade planform of a commercial wind turbine. Based on these validations, the authors postulate the use of a different vortex distribution (i.e. not-uniform loading) for blade design and discuss the effect of such choices on blade chord and twist, force distribution and power coefficient. In addition to the method's straightforward application to the pre-design phase, the model clearly shows the link between blade geometry and performance allowing quick preliminary evaluation of non uniform loading on blade structural characteristics.

1. Introduction
The design of wind turbine rotors and the aerodynamic prediction of horizontal axis wind turbine (HAWT) performance are done mainly by using a synthesis of two separate models, one to compute the features of the flow field extending far upstream and far downstream from the turbine (the far flow field), and the other (the rotor’s near flow field) to analyze the rotor's interaction with said field. Either the momentum method (in which the rotor is considered as an actuator disk) or vortex methods are used to calculate the global flow field [1-8]. The actuator disc concept relies on the principle that

³ Corresponding author
rotors can be represented with equivalent forces distributed over a permeable disc of zero thickness in a flow domain. The flow conditions just upstream from the rotor disk plane will form the ‘inflow’ condition for the rotor, determining the aerodynamic forces on the blades. The calculation is conjugate in the sense that the magnitude of these forces (e.g. in terms of pressure drop across the actuator disk) provides the input required for the analysis of the global flow field. The methods reveals some weaknesses due to the inherent assumptions (most of them based on the original Glauert hypothesis [9]) which can penalize the reliability and accuracy of the solution, in particular the prediction of the flow in the inboard part of the rotor. Glauert, by considering the wake rotation as a loss, implicitly disregards the dependence of the radial pressure variation on the wake rotation, as a consequence that reduced $C_P$ are obtained for decreasing tip speed ratios. This problem was analyzed by de Vries [10], who concluded that for the uniformly loaded disk the cancellation of the power loss due to the wake rotation had the effect of increasing the power conversion. Also application of vortex methods [11] indicates that the local non-dimensional thrust is increased towards the root section.

In a recent paper, Sharpe [12] presented a general momentum theory based on an actuator disk modeling of a rotor with blades having radially uniform circulation and including the effect of wake rotation and expansion. The work emphasizes the fact that the angular velocity of an air particle in the wake will cause a centrifugal force, which has to be balanced by a radial pressure gradient imparted to the particle and hence to the streamline. This effect is apparently low for high tip speed ratios, but becomes very evident for low tip speed ratios, determining higher power coefficients than those predicted by the simple actuator disk momentum theory, which assumes that the pressure in the wake is uniformly atmospheric. A wide analysis of the problem of including the pressure gradient associated to wake rotation is included in a section of a recent Risø report of research in aeroelasticity [13]. The non-uniform loading of rotor disk is approached analytically by Conway [14-15], who developed a method able to treat any load distribution with bounded radial gradient.

The inclusion of the effect of pressure gradient is particularly important when the loading expressed as tangential velocity or vortex structures can be specified from the blade designer. Since different tangential velocity distributions will give rise to different radial pressure distributions in the rotating wake, the mass flow through the disk can be varied.

The paper investigates the effect of different non-uniform loading specifications on the rotor. These loads cases will determine different geometries of the blades to accomplish the vortex structure associated with such prescribed loads. To perform the investigation a model of rotating and expanding wake has been developed for the far flow field, while an adaptation of the isentropic radial equilibrium methods used for axial rotor design in turbomachinery has been implemented. The model can easily be extended to include the effect of viscous losses (non-isentropic radial equilibrium). The model uses neither actuator disk theory to model the flow field, nor classical BEM theory to model the loads and power output. To investigate the effect of not-uniform loading on blade geometry, instead of the usual direct or problem analysis methods, where the fluid-dynamic performance is obtained from a turbine of known blade geometry, we take the opposite approach, specifying a design goal for the blade, such as the loading distribution along the blade span, with a view to arriving at a synthesis of the blade planform (chord, twist angles) to meet this specification. Tangential forces, axial forces, momenta and power are readily evaluated. A given specification will naturally give rise to a particular blade planform. This approach for the ‘inverse’ or ‘design’ problem method for wind turbine rotor design has been named by the authors Inverse Design–Momentum (ID-M) Method. A general theoretical explanation of the model is given below, also presenting some preliminary results.

2. The general feature of the model
The inverse problem method generates the calculation procedure shown in table 1, where the main steps are compared with those adopted for conventional direct design problem methods to clarify the basic difference of the two approaches. In the latter the blade's aerodynamic features and geometry are assumed a priori, and the rotor's performance is analyzed. The aerodynamic performance of the
profiles $C_L$ and $C_D$ are drawn from tables, once the profile and the chord have been chosen. Alternatively, given the blade's geometry, either panel methods or CFD tools are used to obtain the aerodynamic performance [16]. The early selection of the profile determines the aerodynamic and structural behavior of the blade and thus the whole rotor and turbine architecture. Using this approach means the designer must be fully aware of the implications for the rotor's design of selecting a given profile and chord distribution. This approach is not very suitable for pre-design purposes, or for adapting the rotor design to some non-standard design objective (such as a low tip speed ratio).

From the flow field point of view, the flow field is divided in two macro regions. The first is called turbine’s near field and it consists on the control volume enveloping the rotor (and described by the model of paragraph 3). This volume is confined by sections 1 and 2 of the streamtube of figure 1. Within this volume the prescribed vortex distribution structure determines the flow deflection and the velocity triangles virtually imparted by the blades for a tentative inflow velocity. The far flow field (described in paragraph 4) envelops the rotor upstream and downstream flow field and is confined by sections 0 and 1 and 2 and 3 of the streamtube respectively. Some properties of the latter flow field (far upstream inlet flow areas and downstream outlet velocity) are revised until it accommodates the specified flow structure past the blades (near flow field). After obtaining the match, the local flow deflection and solidity determine the length of the chord according to a minimum loss criterion (see paragraph 5). The calculation flow path is reported in Appendix A. Blade masses and stresses can be estimated reasonably accurately using simple models [17,18]. A further step along the design path (not described here - it will be considered in the future completion of the model) involves selecting the blade profiles that satisfy the specified flow deflection. At this stage, real mass distributions and losses are considered and an optimization procedure is required, possibly ending with a new profile.

**Table 1.** Comparison between main steps for inverse (left) and direct (right) design.

| INVERSE METHOD | DIRECT (CONVENTIONAL) METHOD |
|----------------|-------------------------------|
| Assign $\Omega, V_0, Z, D$ | Assign $\Omega, V_0, Z, D$ |
| Assign $W(r)$ or a given objective function | Assign $C_L(AOA, Re, \frac{l}{c}), C_D(AOA, Re, \frac{l}{c}), c(r), \theta(r)$ |
| Calculate local flow field (rotor close field) Calculate global field (rotor far field) | Equate momentum and blade element forces |
| Predict $P, C_P, C_T, T$ | Predict $P, C_P, C_T, T$ |
| Calculate blade layout ($c(r), \theta(r)$) | Predict masses and loads |
| Predict masses and loads | Predict masses and loads |
| Create profiles | |
Assuming an axially symmetrical flow in an incompressible fluid of negligible viscosity, the mass conservation and the equations of motion in the radial circumferential and axial directions are (with reference to figure 1):

\[ \frac{dG}{dz} = 0 \]  
(1)

\[ \frac{1}{\rho} \frac{dp^0}{dr} = F_r + \frac{V_\theta}{r} \frac{d(V_\theta)}{dr} + V_z \frac{d(V_z)}{dr} + V_z \frac{d(V_r)}{dz} \]  
(2a)

\[ 0 = F_\theta + \frac{1}{r} \left( V_r \frac{d(V_\theta)}{dr} + V_z \frac{d(V_\theta)}{dz} \right) \]  
(2b)

\[ \frac{1}{\rho} \frac{dp^0}{dz} = F_z + \frac{V_\theta}{r} \frac{d(V_\theta)}{dz} - V_r \frac{d(V_z)}{dr} + V_r \frac{d(V_r)}{dz} \]  
(2c)

When the equations are applied to axial stations outside the rotor disk, the blade forces are zero. We further assume that in any arbitrary rotor upstream and downstream station the radial motion is small, and the pressure gradient is essentially due to the whirling motion alone. Given such assumptions, equations (2a, b, c) give for any arbitrary station:

\[ \frac{1}{\rho} \frac{dp^0}{dr} = V_\theta \frac{d(V_\theta)}{dr} + V_z \frac{d(V_z)}{dr} \]  
(3a)

\[ \frac{d(V_\theta)}{dz} = 0 \]  
(3b)

\[ \frac{1}{\rho} \frac{dp^0}{dz} = r V_\theta \frac{d(V_\theta)}{dz} \]  
(3c)

Equation (3a) is known as the equation of isentropic radial equilibrium: this condition establishes the radial pressure gradient needed to provide the centripetal force for maintaining a curved flow path outside the rotor. Equation (3b) states that the momentum of the tangential velocity is conserved in the axial direction. Equation (3c), with the help of equation (3b), becomes:
\[
\frac{1}{\rho} \frac{dp}{dz} = 0 \quad (4c)
\]

which establishes the stagnation pressure conservation along a streamline

\[
\frac{p}{\rho} + \frac{1}{2} V^2 + \frac{1}{2} V_\theta^2 = \text{const} \quad (4d)
\]

This equation reflects the loss of pressure associated with wake rotation. The model thus includes the effect of wake rotation and expansion.

3. The turbine's near field (the vortex model of the turbine)

At the rotor, a vortex flow distribution can be set as:

\[
V_\theta = ar^n + \frac{b}{r} \quad (5)
\]

where \(a\), \(b\) and \(n\) are constants. This vortex distribution should be imparted by the blade planform. From the radial equilibrium equation it is clear that the vortex flow distribution, and thus also the radial distribution of specific work (loading), can be controlled by an educated selection of the constants \(a\), \(b\) and \(n\). Some relevant vortex distributions are the free vortex (\(n = -1\)) and the forced vortex (\(n = 0\)).

The free-vortex flow distribution is characterized by a uniform radial loading of the rotor and the axial velocity \(V_z\) is constant for all radial streamlines. When other kinds of vortex are specified, the axial velocity \(V_z\) will vary from the hub to the tip of the blade, with consequent changes in the velocity triangles.

Let us move on now to consider the problem of how to analyze the three-dimensional flow past a wind turbine rotor. Assuming that the cascade and meridional flow structures can provide a manageable design framework, the wind turbine can be converted into a cascade row as in figure 2. The full three-dimensional flow can be treated for practical purposes as an axial symmetrical or circumferentially averaged 'meridional flow' and a series of superimposed 'cascade' flows to define the blade profiles at selected sections from hub to tip. The physical domain is divided into annular streamtubes, each characterized by a velocity triangle at any axial station.

In figure 2, the deflection of the current line due to interaction with the rotor is represented by means of inlet/outlet velocity triangles taken close to the leading and trailing edges planes. All angles measured from the axial direction are taken as positive. Absolute and relative angles (and tangential velocities) ahead of the rotor are measured as positive when measured in the direction of rotation. Absolute and relative angles (and tangential velocities) downstream of the rotor are measured as positive when measured against the direction of rotation. The tangential force exerted by the blades at a generic radius \(r\) equals the change in the tangential moment and we can write:

\[
F_\theta(r) = G(V_{\theta,1} + V_{\theta,2}) \quad (6)
\]

and, by expanding the mass flow, we can write:

\[
F_\theta(r) = \rho \cdot V_{z,m} \cdot s \cdot (V_{\theta,1} + V_{\theta,2}) = \rho \cdot V^2_{z,m} \cdot s \cdot (\tan \alpha_1 + \tan \alpha_1) = \rho \cdot V_{z,m} \Gamma \quad (7)
\]
the blade spacing (see figure 2) being:

\[
s = \frac{2\pi r}{Z}
\]  

(8)

Mean velocities in the axial and tangential plane are given by:

\[
V_{z,m} = \frac{V_{z,1} + V_{z,2}}{2}
\]
\[
V_{\theta,m} = \frac{V_{\theta,2} - V_{\theta,1}}{2}
\]

and the mean angle of the flow past the profile by:

\[
\alpha_m = \tan^{-1} \left( \frac{V_{\theta,m}}{V_{z,m}} \right)
\]  

(9)

Since the blade is moving at a peripheral speed \( \Omega r \), the specific work done at the radius of the generic blade element is written as:

\[
W(r) = \Omega r \cdot (V_{\theta,1} + V_{\theta,2}) = \Omega r \cdot V_{z,1} \cdot (\tan \alpha_1 + \tan \alpha_2) = \Omega r \cdot V_{z,1} \cdot (\tan \beta_2 - \tan \beta_1)
\]  

(10)
It also holds:

\[ W(r) = \frac{\Omega r \cdot \Gamma}{s} \]

The velocity triangles at each radius are calculated by composing the velocity vectors.

\[ \vec{\omega} = \vec{\omega}_r + \vec{V} \]

The flow angles are given by

\[ \beta_i = \tan^{-1}\left( \frac{\Omega r - V_{\theta,1}}{V_{z,1}} \right) \]

\[ \beta_m = \tan^{-1}\left( \frac{\Omega r + V_{\theta,m}}{V_{z,D}} \right) \]

\[ \beta_2 = \tan^{-1}\left( \frac{\Omega r + V_{\theta,2}}{V_{z,2}} \right) \]

For an incompressible flow at radius \( r \) with zero pre-whirl upstream from the rotor \((V_{\theta,1} = 0, \alpha_i = 0)\) and frictionless flow we can write:

\[ W(r) = \Omega r \cdot V_{\theta,2} \]

The swirl angle is:

\[ \alpha_2 = \tan^{-1}\left( \frac{W(r)}{\Omega r \cdot V_{z,1}} \right) \]

With the aid of the expressions in equation (5), the relationship between the vortex flow structure and the specific work is established as follows:

\[ W(r) = \Omega r \cdot \left( ar^n + \frac{b}{r} \right) \]

From now on, the required specific work or vortex structure can be considered as equivalent. The axial force is given by applying the momentum equation in the direction of \( z \):

\[ F_z(r) = (p_1 - p_2) \cdot s + \rho \cdot V_{z,m} \cdot s \cdot (V_{z,1} - V_{z,2}) = (p_1 - p_2) \cdot s \]

The static pressure is given as a function of the total pressure:

\[ (p_1 - p_2) = \frac{1}{2} \rho \cdot \left( V_{1}^2 - V_{2}^2 \right) + p_1^0 - p_2^0 = \frac{1}{2} \rho \cdot \left( V_{1}^2 - V_{2}^2 \right) + \Delta \rho \]

so we can write:

\[ F_z(r) = (p_1^0 - p_2^0) \cdot s + V_{z,m}^2 \tan^2 \alpha_2 \]
and we ultimately obtain:

$$F_z(r) = (p_1^0 - p_2^0) \cdot s + \frac{1}{2} \rho \cdot V_{θ,2}^2 \cdot s$$  \hspace{1cm} (19)

4. Far flow field
The velocity $V_{z,1}$ appearing in the equations describing the turbine's near field is not known because it depends on the far upstream velocity distribution $V_0$ and the specific work extracted from the rotor $W$. Near and far flow fields have to be matched and, to do so, we apply the energy conservation equation to each elementary streamtube. Each station is discretized in concentric rings. Along each streamline, the conservation equation for the upstream and downstream regions is given by:

$$\left( \frac{p_1}{\rho} + \frac{1}{2} V_1^2 \right) = const$$ \hspace{1cm} (20)

while, for the streamlines crossing the region enveloping the rotor, the following holds:

$$\left( \frac{p_1}{\rho} + \frac{1}{2} V_1^2 \right) - \left( \frac{p_2}{\rho} + \frac{1}{2} V_2^2 \right) = W$$ \hspace{1cm} (21)

The momentum equation applied again for the whole streamtube gives:

$$\int_A (p_1 - p_2) dA = \int_B \left[ \rho V_{z,3} (V_{z,0} - V_{z,3}) + (p_0 - p_3) \right] dA_3$$ \hspace{1cm} (22)

5. Blade architecture
We define the tangential force coefficient as:

$$C_y = \frac{F_0}{F_{θ,max}} = 2 \cdot \frac{s}{c_z} (\tan \beta_2 - \tan \beta_1) \cos^2 \beta_2$$ \hspace{1cm} (23)

We assume that $c_z$ is approximately related to the actual chord by

$$c_z = c \cdot \cos \beta_m$$ \hspace{1cm} (24)

Although not usually applied for applications where the $s/c$ ratio is far higher than 1, the results of Zweifel [19] to turbine rotors can be used to select an initial estimate of $s/c$ and thus deduce the chord distribution for the local spanwise solidity. Due to the physics onto which the correlation has been deduced, this approach can be satisfactory for the preliminary design phase and $C_s = 0.8$ was adopted here. Since the velocity triangles are at this stage known at each station, the chord distribution is readily generated. The lift coefficient is given by:

$$C_L = 2 \frac{s}{c} (\tan \beta_2 - \tan \beta_1) \cos \beta_m + C_D \tan \beta_m$$ \hspace{1cm} (25)

6. Results and discussion
The model presented here was implemented with a Matlab® code. The program converges rapidly and in a robust way with a few iterations down to tip speed ratios of about 1.
The far flow field was solved both with ID-M method and the simple actuator disk momentum theory for constant loading and disregarding tip losses, and the results compared. In figure 3, the dotted line indicates the ID-M solution while the full line describes the actuator disk. The specific work distribution was set at 8/9 along the non-dimensional disk radius, and is adimensionalized to the far upstream kinetic energy. The tip speed ratio used for the simulation was \( \lambda = 3 \). The constant specific work (or constant total pressure) distribution corresponds to the free vortex solution. The ID-M predicts an increase in the pressure gradient at the rotor hub (fig. 3a), which deviates from the constant gradient predicted by the actuator disk method. This gradient is due to the tangential velocity becoming higher and higher as the root is approached (fig. 3b). As a consequence, the axial velocity also increases towards the hub (fig. 3b), leading to an increase in the air mass ingested by the rotor, which results in a higher thrust coefficient (fig. 3c) and power coefficient (fig. 3d), which exceeds the Lanchester-Betz limit. As the tip speed ratio increases, the lesser magnitude of the tangential velocity attenuates the effect of the pressure gradient in the wake and the power coefficient asymptotically approaches the solution of the actuator disk. Sharpe [12] presented similar results. Our ID-M model completes his analysis by showing that these power coefficients exceeding the Lanchester-Betz limit are obtained at the expense of a dramatic increase in the chords, which far exceed the disk's diameter at the hub leading to unrealistic solutions.

![Figure 3](image)

**Figure 3.** Comparison between the actuator disk and the ID-M model for \( \lambda = 3.0 \); dotted line - ID-M solution full line - actuator disk; 3a - pressure gradient; 3b - velocities; 3c - \( C_T \); 3d - \( C_P \).

The ID-M model was validated using data for a known wind turbine, the GAIA WIND [20], which is a two-bladed downstream wind turbine, the main characteristics of which are listed in table 2.

| Table 2. Main characteristics of the GAIA wind turbine. |
|-------------------------------------------------------|
| **Rated power** | 11 kW |
| **Rotor diameter** | 13 m |
| **Tower high** | 18.2 m |
| **Rotors speed** | 56 rpm |
| **Profiles** | NACA 44XX |
The geometrical data for the rotor (not shown in table 2) were input in a conventional direct design method employing the BEM code, where the $C_D$ was set to zero. The output of the BEM in terms of specific work distribution along the blade was successively used as input for the ID-M model. The chord distribution generated is shown in figure 4a. A satisfactory consistency was found between the virtual and the real chord, the maximum overestimation being of the order of 15%. The twist angle, shown in figure 4b, was calculated on the assumption that the lift produced by the virtual deflection calculated using the ID-M method is the same as that given by a flat plate. The following equation was used:

$$\theta = \frac{\pi}{2} - \beta_m - \sin^{-1}\left(\frac{C_{L,ID-M}}{2\pi}\right)$$ (26)

The data reveal a good match with the real distribution, indicating that the ID-M method affords sufficiently accurate results for NACA profiles for preliminary design purposes.

**Figure 4.** Comparison of the real GAIA wind turbine chord (4a) and the twist angle (4b) distribution with the virtual ID-M method results along the blade span normalized to the values taken at the hub; full line – GAIA real data, dotted line – ID-M simulation data

After comparing the model with the classical actuator disk-BEM method and validating it for a real case, further numerical tests were carried out to show how competitive designs may be postulated for different types of vortex flow structure. In particular, we compared the free vortex design distribution with other than free vortex swirl distributions. Three distributions were selected, with the free vortex ($n = -1$), taken for reference, the forced vortex, $n = 0$ and a mixed vortex ($a/b=3; n=-1/3$). A three-bladed turbine at $\lambda = 6$ was used for the simulation. Figure 5 shows the effect of these choices on some of the relevant parameters, e.g. the specific work distribution, the axial and tangential velocity, the cumulative $C_p$, the static pressure difference across the disk, the tangential and normal forces, the flow angle and the flow deflection ($\beta_1 - \beta_2$) at the rotor disk, and the chord (equation 24) and twist (equation 26) distribution.

The three vortex distributions determine very different specific work distributions along the blade; the forced vortex implies a quadratic increase in the specific work with radius. This causes the axial velocity to increase towards the hub. The pressure difference is quite small at the hub, partly because of the low specific work demand and partly because of the low tangential velocity at the hub. The flow deflection remains small, and this blade appears to be less loaded in the flapwise direction. The $C_p$ is much lower than with the free vortex distribution, and does not exceed 0.4, because the increase in the mass flow towards the hub cannot compensate for the reduction in specific work imparted by the vortex structure. Looking at the blade planform, the forced vortex determines an increasing chord from hub to tip, accompanied by a very slender shape, and poses the conflicting design option between a less aerodynamically performing blade and a lightweight structure. The mixed vortex structure entails
a specific work distribution that increases towards the tip. This solution also leads to a greater axial velocity towards the hub, with a more pronounced flow deflection. The axial and tangential forces are much the same, unless the specific work produced at the inner sections is smaller than with the free vortex distribution, and the $C_p$ is correspondingly similar, but a little lower. Mixed vortex structures can generate almost constant chord distributions, for segments of the blade at least, and the small loss in power could be offset by simple technological solutions in the blade's manufacture.

Figure 5. Effect of different choice of the vortex distribution on flow, forces and power, for $\lambda = 6; Z = 3$; full line – free vortex, dashed line – forced vortex, dash/dotted line – mixed vortex; 5a – work distribution; 5b – axial velocity; 5c – $C_p$; 5d – pressure gradient; 5e - tangential force; 5f – normal forces; 5g – mean relative flow angle; 5h – flow deflection; 5i – chord distribution; 5j – twist distribution.
As a final consideration, vortex structures that are not free seem to determine a worse rotor performance in ideal conditions, but with lower forces in the spanwise and flapwise direction, and a lower flow deflection, especially in the hub regions. The blade planform is quite specific, moreover, and is characterized by much more twisted, smaller chord blades, with a different taper.

**Conclusion**

The ID-M method presented here is adapted from the usual formulation of the inverse design method for axial turbomachinery stage design for not confined flows which includes the wake rotation and expansion. It uses the vortex flow distribution to model the flow field past a wind turbine blade, while the inflow conditions are obtained by applying the energy and momentum conservation to the global flow field. This innovative approach to the design of wind turbine rotors has been shown to produce reliable flow fields and a realistic blade layout, starting from an arbitrarily specific work distribution. It also clearly links the effects of the required rotor performance on the rotor geometry and the global flow field.

The design characteristics of a conventional wind turbine rotor of known performance, obtained using a conventional Actuator disk-BEM model, were used as input in the ID-M method to validate the suitability of the approach.

Another relevant point to make is that, if we were to depart from the initial design constraint of the same specific work at all radii, we could select a wide range of vortex designs to obtain a specific design goal, to control the geometry and thus the spanwise mass distribution, and the loads on the rotor. A trade-off between performance and rotor architecture is clearly evident. Moreover, different vortex structures will have a different impact on the wake deficit in the downstream flow field.

These preliminary results demonstrate that the method has potential. The next steps will be taken to improve the theory in order to include the radial velocity component and the tip losses.

**Nomenclature**

\[a = \text{vortex constant}\]
\[A = \text{area}\]
\[\text{AOA} = \text{angle of attack}\]
\[b = \text{vortex constant}\]
\[c = \text{blade chord}\]
\[C_D = \text{drag coefficient}\]
\[C_L = \text{lift coefficient}\]
\[C_P = \text{power coefficient}\]
\[C_T = \text{thrust coefficient}\]
\[C_y = \text{tangential force coefficient}\]
\[D = \text{diameter}\]
\[F = \text{force}\]
\[G = \text{mass flow}\]
\[n = \text{vortex constant}\]
\[p = \text{pressure}\]
\[r = \text{radius}\]
\[\text{Re} = \text{Reynolds number}\]
\[s = \text{blade spacing}\]
\[t = \text{blade thickness}\]
\[T = \text{thrust}\]
\[V = \text{absolute velocity}\]
\[w = \text{relative velocity}\]
\[W = \text{specific work}\]
\[Z = \text{number of blades}\]

\[\beta = \text{angle of relative velocity to the axial direction}\]
\[\lambda = \text{tip speed ratio}\]
\[\theta = \text{blade twist}\]
\[\rho = \text{air density}\]
\[\Gamma = \text{circulation, single blade}\]
\[\Omega = \text{angular velocity}\]

**Subscripts**

0 = far upstream station
1 = rotor inlet station
2 = rotor outlet station
3 = far downstream station
D = middle rotor station
hub = root
ID-M = inverse design momentum method
m = mean
max = maximum
r = radial component
z = axial component
0 = tangential component
gaia = referring to GAIA WIND turbine

**Superscripts**

0 = total conditions
Acknowledgments
The authors thank Mr. Johnny Andringa, managing director of GAIA wind turbine for the kind collaboration in supplying data of the GAIA 11 kW grid connected wind turbine.

References
[1] Hansen M O L, Sørensen J N, Voutsinas S, Sørensen N and Madsen H Aa 2006 State of the art in wind turbine aerodynamics and aeroelasticity Progress in Aerospace Sciences 42 285-330
[2] Snel H 1998 Review of the present status of rotor aerodynamics Wind Energy 1 46-69
[3] Snel H, van Holten Th 1995 Review of recent aerodynamical research on wind turbines with relevance to rotorcraft Aerodynamics and Aerocoustics of Rotorcraft AGARD CP 552 pp 7-11
[4] Hansen C and Butterfield C P 1993 Aerodynamics of horizontal axis wind turbines Ann. Rev. Fluid Mech. 25 115-149
[5] Sørensen J N, Shen W Z and Munduate X 1998 Analysis of wake states by a full-field actuator disc model Wind Energy 1 73-88
[6] Sørensen J N and Kock C W 1995 A model for unsteady rotor aerodynamics J. Wind Eng. Ind. Aerodyn. 58 259-275
[7] Shen W Z, Mikkelsen R., Sørensen J N and Bak C 2005 Tip loss corrections for wind turbine computations Wind Energy 8 457-475
[8] Mikkelsen R 2003 Actuator disc methods applied to wind turbines MEK-FM-PHD 2003-02
[9] Glaeert H 1963 Airplane Propellers Volume IV in Aerodynamics Theory edited by Williams Frederick Durand, The Dover Edition
[10] de Vries O 1979 Fluid dynamic aspects of wind energy conversion AGARDograph AG-243
[11] Øye S 1990 A simple vortex model. In Proc. of the third IEA Symposium on the Aerodynamics of Wind Turbines, ETSU, Harwell, p. 4.1-5.15.
[12] Sharpe D J 2004 A general momentum theory applied to an energy extracting actuator disc Wind Energy 7 177-188
[13] Risø-R-1559(EN), 2006 Research in Aeroelasticity EFP-2005 Wind Energy Department, Edited by Christian Bak
[14] Conway J T 1995 Analytical Solutions for the Actuator Disk with Variable Radial Distribution of Load J. Fluid Mech., Vol. 297, pp. 327-355.
[15] Conway J T 1998 Exact Actuator Disk Solutions for Non-uniform Heavy Loading and Slipstream Contraction*, J. Fluid Mech., Vol. 365, pp. 235-267.
[16] Sørensen J N 1993 A survey of CFD methods in rotor aerodynamics 7th IEA Symp. On Aerodynamics of Wind Turbines (Lyngby, November 1993)
[17] Milborrow D J 1982 Performance, blade loads and size limits for horizontal axis wind turbines 4th BWEA Wind Energy Conversion (Cranfield: BHRA)
[18] Battisti L and Soraperra G 2007 Analysis and application of pre-design methods for HAWT rotors Proc. of EWEC (Milan, 7-10 May 2007)
[19] Zweifel O 1945 The spacing of turbomachine blading, especially with large angular deflections Brown Boweri Rev. Dec. 436-44
[20] www.gaia-wind.com
Appendix A

START

INPUT: $\rho_0, p_0, V_{0,0} = V_{s,0} = V_{r,0}$; $D, Z, \Omega, \psi, \rho_{s,0} = p_0, C_2$

$V_{s,2}$ Swirl velocity calculated at the station 2

First calculation of the flow field with the simple momentum theory: $P_1; P_2; V_3; V_{s,1,2}$

$r_2; r_3$ from conservation of mass

$V_{s,3}$ the conservation of the quantity $V_{s,2} \cdot r$ along a streamline

$p_3$ from the radial equilibrium in the far wake

$1 \frac{dp_3}{dz} = \frac{V_{s,3}^2}{\rho}$

$V_3; V_{s,3}$ conservation of energy in 0-3

$\int_A (p_1 - p_2) dA = \int_A \left[ \rho V_{s,3}^2 (V_{s,0} - V_{s,3}) + (p_0 - p_3) \right] dA$

$P_1; P_2; V_3, r_2/r_3$ conservation of energy and mass

Convergence?

Yes

$V_{s,3}: \alpha_2: \alpha_2; \psi; \psi; \beta_2; \beta_2$ velocities and angles in the absolute and relative frame

$F_2; J_2; P; T; C_p; C_T$ forces, power and thrust

$c; C_T$ chord distribution deduced from Zweifel correlation

$\theta$ twist distribution calculated from the approximation of a profile with a flat plate

END