1. INTRODUCTION

Astrophysical disks are observed to transport angular momentum. It has been hypothesized that such disks are transporting angular momentum through a turbulent process (Shakura & Sunyaev 1973). Despite decades of theoretical research, we still lack a sufficient understanding of turbulence to make quantitative predictions. Several candidate processes exist to sustain turbulence in these disks: the magneto-rotational instability (MRI; e.g., Balbus & Hawley 1991), gravitational instability (e.g., Lodato & Rice 2004, 2005), the vertical shear instability (Nelson et al. 2013), and the baroclinic instability (e.g., Klahr & Bodenheimer 2003). The MRI is still the leading candidate for angular momentum transport in accretion disks (see Turner et al. 2014 for a recent review). However, at the low temperatures and ionization fractions expected in protoplanetary disks, non-ideal MHD effects become important, qualitatively changing the nature of the turbulence and its associated transport properties, or rendering it ineffective, resulting in “dead zones” (e.g., Gammie 1996).

A corollary to the angular momentum transport problem in protoplanetary disks is the transport of dust particles. There is a large amount of astrophysical (e.g., Bouwman et al. 2001; Dullemond et al. 2006; Hughes & Armitage 2010; Owen et al. 2011b) and cosmochemical (e.g., Gail 2001; Bockelée-Morvan et al. 2002; Jacquet et al. 2012; Jacquet & Robert 2013) evidence to suggest that large-scale radial transport of dust particles occurs. In particular, the level of diffusion that results from the turbulence is an unknown parameter. The turbulent diffusion coefficient ($D_{t}$) is often parameterized in terms of the turbulent kinematic viscosity ($\nu$) as $D_{t} = \nu/Sc$, where $Sc$ is the Schmidt number. Assumptions of isotropic, Kolmogorov-like turbulence lead to the inference that $Sc \approx 1$ (Youdin & Lithwick 2007); however, a large range of reported experimental/theoretical/numerical values exist that span approximately two orders of magnitude in the range $Sc = 0.1–10$ (Prinn 1990; Dubrulle & Frisch 1991; Lathrop et al. 1992; Carballido et al. 2005; Johansen et al. 2006; Youdin & Lithwick 2007; Zhu et al. 2014).

Direct observational measurements of the strength of turbulent diffusion in the continuum will be complicated by dust drag, grain growth/fragmentation, and optical depth effects. However, it is not only dust particles that will experience turbulent diffusion; any rare gas-phase species (tracer species) where changes in temperature result in a concentration gradient will also experience turbulent diffusion. If, for example, a tracer species is predominately produced at a given radius, the tracer will then diffuse away from this radius, with a distribution that is strongly dependent on the value of the Schmidt number (Clarke & Pringle 1988). Jacquet & Robert (2013) considered the case of deuterated water and argued that the D/H water distribution in Chondrites implies that the Schmidt number is smaller than unity.

Snow lines, where a gas phase tracer (for example, H$_2$O, CO$_2$, CO) condenses out of the gas to form ice below a given temperature, represent a scenario where a sharp concentration gradient can occur. Species such as H$_2$O, CO$_2$, and CO are relatively abundant, such that the surface layers can be optically thick. Additionally, model degeneracies make it difficult to constrain snow lines directly (even with optically thin isotopologues). However, if the production/destruction of yet rarer tracers is regulated by the presence or absence of gas phase H$_2$O, CO$_2$, or CO, then these rare tracers could be used as proxies to detect ice lines. For the CO snow line, it is expected that N$_2$H$^+$ and H$_2$CO will only be abundant when CO freezes out (Jørgensen et al. 2004; Walsh et al. 2012; Qi et al. 2013a, 2013b). Such an expectation has been borne out in observations of star-forming cores (Friesen et al. 2010), and similar results were obtained in the DISCS SMA survey of several nearby protoplanetary disks (Qi et al. 2013a). Recently, Qi et al. (2013b) imaged a hole in N$_2$H$^+$ using ALMA at a radius of $\sim$30 AU, coincident with the expected location of the CO snow line (based on a freeze-out temperature of $\sim$17 K). The sharpness of the inner edge will depend strongly on the strength of the turbulent diffusion, with weaker diffusion resulting in a sharper hole.

In this Letter, we demonstrate how the distribution of a tracer species that is only abundant outside a snow line (being
destroyed inside) is strongly dependent on the Schmidt number, and that ALMA observations could be able to constrain the Schmidt number in protoplanetary disks.

2. DISK DISC MODEL

We consider a one-dimensional (1D) axis-symmetric disk, where the evolution of the gas surface density ($\Sigma$) and the surface density of any (gas-phase) tracer species ($\Sigma_i$) is given by (e.g., Lynden-Bell & Pringle 1974; Clarke & Pringle 1988; Birnstiel et al. 2010; Owen et al. 2011a; Owen 2014)

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left( R^{1/2} \frac{\partial}{\partial R} (\nu \Sigma R^{1/2}) \right)$$  \hspace{1cm} (1)

$$\frac{\partial \Sigma_i}{\partial t} + \frac{1}{R} \frac{\partial}{\partial R} \left[ R \Sigma u_g - D_k^R R \Sigma \frac{\partial X_i}{\partial R} \right] = \sum_j S_j(\Sigma_g, \Sigma_j, R, t),$$  \hspace{1cm} (2)

where $X_i$ is the concentration of the tracer species, $u_g$ is the net radial gas velocity, $D_k^R$ is the radial gas turbulent diffusion coefficient, and $S_j(\Sigma_g, \Sigma_j, R, t)$ is a source/sink term that represents the production and destruction of the tracer species.

2.1. Conditions at the Snow Line

The following general model can be applied to any snow line ($H_2O$, CO$_2$, CO, etc.) where the chemical and transport timescales decouple. Here, we specifically consider the case of N$_2$H$^+$ destruction at the CO snow line as observed in TW Hya (Qi et al. 2013b). Inside the CO snow line, N$_2$H$^+$ is destroyed by gas-phase CO; outside the CO snow line, this destruction channel is no longer dominant and it instead is destroyed at a much slower rate by dissociative recombination (Jørgensen et al. 2004). Simulations without transport suggest that the N$_2$H$^+$ abundance drops by several orders of magnitude inside the CO snow line (e.g., Walsh et al. 2012) with an abundance that depends on the square of the gas-phase CO abundance (Jørgensen et al. 2004).

2.1.1. Relevant Timescales

In order for chemical tracers at the snow line to be useful in terms of probing the strength of the turbulent diffusion, we must decouple the chemical and dynamical timescales. Namely, the desorption timescale must be faster than the transport timescales in order to create a sharp snow line; furthermore, the destruction timescale of the tracer species inside the snow line must also be short. The transport timescale of interest is the time it takes to move a radial distance $H$ (where $H$ is the disk’s scale height, which is of the order of the radial scale length). Therefore, the advection timescale ($t_{adv}$) is

$$t_{adv} \approx \frac{H}{u_g} = \frac{2 \alpha^{-1} \left( \frac{R}{H} \right) \Omega^{-1}}{3},$$

$$\approx 2 \times 10^4 \text{ yr} \left( \frac{\alpha}{0.01} \right)^{-1} \left( \frac{H}{R} \right)^{-1} \left( \frac{R}{0.1} \right),$$

$$\times \left( \frac{R_{\text{SL}}}{30 \text{ AU}} \right)^{3/2} \left( \frac{M_\odot}{1 M_\odot} \right)^{-1/2}.$$  \hspace{1cm} (3)

and the diffusive timescale ($t_{dif}$) is

$$t_{dif} \approx RH \frac{\left( \partial \log X \right)}{DH \partial \log R}.$$  \hspace{1cm} (4)

It is well known (e.g., Clarke & Pringle 1988; Jacquet et al. 2012; Jacquet & Robert 2013)2 that the logarithmic concentration gradient rapidly approaches $3\nu/2D_k^g$; thus the diffusive timescale is identical to the advection timescale (this is somewhat unsurprising as they are both driven by the same process). Thus, the relevant timescale for the movement of an individual tracer molecule over a radial scale $H$ is ($t_{adv+dif}$) $\approx 10^4$ yr.

We want to compare this transport timescale to the timescale for the desorption of the snow line species, along with the destruction of the tracer species inside the snow line. Considering our example of N$_2$H$^+$ and the CO snow line, the desorption timescale is obtained by balancing desorption with absorption (with rate constant $k_{g}(CO)$), so the desorption timescale becomes (Takahashi & Williams 2000)

$$t_{dorb} = \frac{1}{k_{g}(CO) n_{g}}$$

$$\approx 10 \text{ yr} \mu^{-2} \left( \frac{H}{1 \text{ mm}} \right) \left( \frac{X_d}{0.01} \right)^{-1} \left( \frac{X_{CO}}{10^{-4}} \right)^{-2} \times \left( \frac{\Sigma}{1 \text{ g cm}^{-3}} \right)^{-2} \left( \frac{H}{R} \right)^{2} \left( \frac{R}{0.1} \right)^{2}.$$  \hspace{1cm} (5)

where $a$ is the dust-grain size, $X_d$ is the dust-to-gas mass ratio, and $X_{CO}$ is the CO abundance. Additionally, the destruction timescale for N$_2$H$^+$ is (from the Jørgensen et al. 2004 simplified network)

$$t_{des} = \frac{1}{k_{des} n_{g}}$$

$$\approx 100 \text{ yr} \mu^{-1} \left( \frac{X_{CO}}{10^{-4}} \right)^{-1} \left( \frac{\Sigma}{1 \text{ g cm}^{-3}} \right)^{-1} \times \left( \frac{H}{R} \right)^{-1} \left( \frac{R}{0.1} \right)^{-1} \left( \frac{R}{30 \text{ AU}} \right)^{-1},$$  \hspace{1cm} (6)

where $k_{des}$ is the destruction rate constant3.

Therefore, the CO/N$_2$H$^+$ system clearly satisfies $t_{adv+dif} \ll t_{adv+dif}$ for conditions experienced in protoplanetary disks. As such, we may ignore the details of the chemical rate equations and simply model the destruction of any remaining N$_2$H$^+$ to occur instantaneously at the snow line radius; however, we emphasize that the following analysis can be applied to any tracer species with similar destruction timescales. Therefore, in this situation the source function $\sum_j S_j(\Sigma_g, \Sigma_j, R, t)$ is drastically simplified to

$$S_j(R, t) = -\frac{\dot{M} X_{\infty} \delta(R - R_{\text{SL}}(t))}{2\pi R}.$$  \hspace{1cm} (7)

where $\dot{M}$ is the mass-accretion rate, $X_{\infty}$ is the concentration at large radius, $\delta(R)$ is the Dirac delta function, and $R_{\text{SL}}$ is the radius of the snow line. This source function represents the instantaneous destruction of any remaining tracer species at

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1 We note that since the snow line is sharp, the relevant length scale for computing timescales is $H$, not $R$.

2 We will derive this dependence in Section 2.2 for a steady disk model, but it is a more general result—see Clarke & Pringle (1988).

3 Estimated from Figure 16 of Jørgensen et al. (2004).
Using Equation (7), we can integrate Equation (9) to find the radial concentration distribution:

\[ X_i(R) = \begin{cases} \frac{1}{\Sigma_i} \left( \frac{R}{R_{SL}} \right)^{-3} & \text{if } R > R_{SL} \\ 0 & \text{if } R \leq R_{SL} \end{cases} \]

Therefore, we see that the radial profile of the concentration is strongly sensitive to the value of the Schmidt number. In Figure 1, we show how the concentration varies with radius and Schmidt number. It is important to emphasize that concentration distribution is independent of assumptions of the (unknown) viscosity, and mass-accretion rate and can in principle provide a “clean” measurement of the Schmidt number.

3. OBSERVABLE CHARACTERISTICS

Unfortunately, it is not possible to directly observe the concentration gradient. What is directly observed is the surface-brightness distribution of the relevant species. The surface-brightness distribution is sensitive to the surface density of the tracer species rather than its concentration. Thus, we must multiply the concentration of our tracer species by the gas surface density. Adopting a power-law gas distribution of the form \( \Sigma_g/\Sigma_{SL} = (R/R_{SL})^{-\gamma} \) with a cutoff radius \( R_{out} \), the surface density of the tracer species is

\[ \Sigma_t = X_\infty \Sigma_{SL} \left[ \left( \frac{R}{R_{SL}} \right)^{-\gamma} - \left( \frac{R}{R_{SL}} \right)^{-3/2} \right] \]

in the range \( R_{SL} < R < R_{out} \) and \( \Sigma_t = 0 \) elsewhere.

3.1. Brightness Distribution and Visibility Profiles

The actual brightness distribution from rotational emission lines (such as \( \text{N}_2\text{H}^+ J = 4–3 \)), in addition to being sensitive to the surface density distribution, is sensitive to the background gas temperature and excitation temperature of the molecule (which in turn is a function of density and temperature) along with the optical depth.

However, in the case that the tracer species is optically thin, and the gas density is far above the critical density \( n_{cr} \) for the rotational transition, then the molecular line is thermalized. Thus, if the temperature gradient is weak compared to the scales of interest and \( n_g \gg n_{cr} \), then we can approximate the surface brightness as being directly proportional to the surface density of the tracer species, provided the line remains optically thin.

For the gas at 30 AU, the gas density is typically

\[ n_g = 5.3 \times 10^9 \text{ cm}^{-3} \mu^{-1} \left( \frac{\Sigma}{1 \text{ g cm}^{-2}} \right) \]

\[ \times \left( \frac{H/R}{0.1} \right)^{\gamma} \left( \frac{R}{30 \text{ AU}} \right)^{\gamma} \]

where \( \mu \) is the mean molecular weight of the gas. Comparing this to the critical density of the \( J = 4–3 \) transition of \( \text{N}_2\text{H}^+ \), which has a critical density of \( n_{cr} \sim 10^7 \text{ cm}^{-3} \) (e.g., Friesen et al. 2010), we see that \( n_g \gg n_{cr} \). We note, since temperature and density are expected to be power laws with radius (e.g., Chiang & Goldreich 1997; Hartmann et al. 1998), that the small additional corrections due to the temperature and density effects of converting \( \Sigma_t \) to the surface brightness distribution \( B_t \) will manifest themselves as changes in the power-law index \( \gamma \) in Equation (14).

Assuming the disk to be axisymmetric and observed face-on, we may write the brightness distribution on the sky as

\[ B_t(\theta) \approx B_0 \left[ \left( \frac{\theta}{\theta_{SL}} \right)^{-\gamma} - \left( \frac{\theta}{\theta_{SL}} \right)^{-3/2} \right] \]

in the range \( \theta_{SL} \ll \theta < \theta_{out} \) and \( B_t = 0 \) elsewhere, where \( B_0 \) is a constant, \( \theta \) is the angular size on the sky, and \( \theta_{SL} \) is the angular size of the snow line given by

\[ \theta_{SL} = 0.2 \text{ arcsec} \left( \frac{R_{SL}}{30 \text{ AU}} \right) \left( \frac{d}{150 \text{ pc}} \right)^{-1} \]

Thus, we see that the brightness distribution still retains a strong sensitivity to the Schmidt number. Since the angular...
resolution required to probe the value of the Schmidt number is only available through millimeter interferometry, we can use our brightness distribution to calculate synthetic visibilities. The visibilities can obtained by a Hankel transform of the axisymmetric brightness distribution such that

$$V_t(\eta) = 2\pi \int_0^\infty d\theta \theta B_1(\theta) J_0(2\pi \eta \theta),$$

where $\eta$ is a radial baseline coordinate defined as $\eta = \sqrt{u^2 + v^2}$, where $u$ and $v$ are the usual baseline coordinates. Following the observed N$_2$H$^+$ $J = 4$–$3$ emission by Qi et al. (2013b), we calculate our synthetic observations at 372 GHz with our standard model, adopting the best-fit parameters from Qi et al. (2013b) of $R_{\text{SL}} = 30$ AU, $R_{\text{out}} = 150$ AU, and $\gamma = 2$.

We calculate our synthetic observations assuming the source is observed face-on at a distance of 150 pc. In Figure 2, we show our simulated visibility curves in the left-hand panels and the surface density profiles in the right-hand panel. In the top panels we vary the Schmidt number between 0.1 and 10, in the middle panels we vary $\gamma$ from 1 to 3, and in the bottom panels we vary $R_{\text{SL}}$ from 15 to 60 AU.

The visibility curves in Figure 2 clearly show that variations in the Schmidt number give rise to significant differences. Furthermore, comparisons when varying the index of the gas surface density $\gamma$, the snow line radius, and the Schmidt number are not too degenerate. The simplified model presented here contains four free parameters $\{R_{\text{SL}}, R_{\text{out}}, S_{\text{CG}}, \gamma\}$, which would all need to be constrained by fitting the visibilities. Inspection of Figure 2 suggests that sensitivities $\lesssim 10\%$ at baselines $\sim 0.1$–1 km would allow the theoretically suggested range of Schmidt numbers (0.1–10) to be observationally constrained.

### 4. DISCUSSION

We have shown that observations of snow lines in protoplanetary disks using a tracer species (for example, N$_2$H$^+$ in the case of the CO snow line; Qi et al. 2013a, 2013b) can be used to probe the Schmidt number, a unknown parameter in studies of turbulent transport in accretion disks, where current estimates span a range of two orders of magnitude ($S_{\text{CG}} = 0.1$–10).

#### 4.1. Detectability with ALMA

The ALMA telescope is a sub-millimeter/millimeter interferometer; once completed, it will have $\sim 50$ individual
antennas, offering $\gtrsim 1000$ baselines with separations up to 12 km. At 372 GHz ($\sim N_2H^+$ $J = 4\rightarrow 3$ line) this provides a maximum spatial resolution of $\sim 0.01$ arcsec. Figure 2 clearly shows that ALMA possesses the required number of baselines ($\gtrsim 50$) with separations in the range of 0.1–1 Km to constrain Schmidt values within the current range of uncertainty, provided the observations are sensitive enough. Taking the TW Hya $N_2H^+$ observations as a reference (Qi et al. 2013b)—a source brightness of $\sim 200$ mJy beam$^{-1}$ km s$^{-1}$ with an rms noise of 8.1 mJy beam$^{-1}$ km s$^{-1}$ and beam size $\sim 0.6$ arcsec made using 23–26 antennas$^4$—a similar level of sensitivity, but with a beam size of $\lesssim 0.3$ arcsec, could be obtained assuming a fully operational ALMA of $\sim 50$ antennas. Thus, such observations are feasible and are of sufficiently high resolution to allow constraints to be placed on the Schmidt number at the distance to TW Hya. Longer integration times and larger baselines would be required to reach similar levels of sensitivity at distances of 150 pc.

4.2. Uncovering Properties of the Turbulence

We have argued that the sharpness of the hole in tracer species at the ice line probes the value of the Schmidt number independent of the assumed properties of the turbulence (e.g., assumed value of the viscous “$\alpha$” parameter). Since several snow lines are expected to occur at different radii, then measurements at different snow lines would allow the radial dependence of the Schmidt number to be probed. In particular, recent simulations suggest that different non-ideal MHD effects (which dominate at different radii; Turner et al. 2014) lead to different Schmidt numbers (Zhu et al. 2014). Thus, comparing the simulation predictions of the Schmidt number for various turbulent driving mechanisms with the observed value would allow inferences to be made about the nature of the turbulence. Furthermore, independently measuring the rms turbulent velocity ($\langle v_R^2 \rangle$; e.g., Hughes et al. 2011) and combining it with a measurement of the Schmidt number would allow the viscosity (including estimates of $\alpha$, where $\nu = \alpha H^2 \Omega$) to be calculated, since $D_R = \alpha H^2 \Omega / \Sigma_R \sim \langle v_R^2 \rangle / \Omega$.

4.3. Caveats and Limitations

We have constructed an idealized model to investigate whether snow lines could begin to probe the strength of turbulent diffusion. As such, there are several model improvements that must be made before fitting to real data. Therefore, our model presented in this Letter is a “proof of concept” rather than a road map for observational modeling. For example, a real protoplanetary disk is not one-dimensional. As such, the vertical temperature structure is not constant, and passively heated disks cool as one approaches the midplane (Chiang & Goldreich 1997). Therefore, the snow line is unlikely to occur exactly at a fixed radius, but is more likely to be an extended structure with a scale variation of $\sim H$ with its time-varying position (Martin & Livio 2012, 2013, 2014). Additionally, the conversion of the gas phase to ice particles at the snow line will result in turbulent diffusion of the gas and ice particles away from the snow line (in a manner identical to that discussed for the snow line tracer discussed here). As such, there is unlikely to be a very sharp change in the gas abundance at the snow line, but rather a smoother change. Furthermore, the chemical timescales may not fully decouple from the transport timescales. In the $N_2H^+$ case considered here, we have argued that the timescales are likely to be decoupled; this may not be the case for all ice-line tracer species, thus dynamical modeling that includes turbulent diffusion is needed to determine the importance of this effect. The model presented here should provide a stringent upper limit of the Schmidt number, and good measurement if it is small ($< 1$); however, if it is large, and the sharpness of the passive tracer has a width of $\sim H$, then the 1D model would only provide an order of magnitude estimate and a better model is need to constrain the Schmidt number. Finally, if the snow line resides in a dead zone, where there is limited or no turbulence, then this method can likely be used cleanly to probe the Schmidt number; however, dead zones are not expected at the large radius of the CO/$N_2H^+$ system discussed here.

5. SUMMARY

In this Letter, we have shown that the recent observations of snow lines through tracer species (e.g., $N_2H^+$ or H$_2$CO in the case of the CO snow line; Qi et al. 2013a, 2013b) could allow direct observational measurements of the Schmidt number in astrophysical accretion disks. In the case that the chemical timescale is suitably decoupled from the transport timescales, then the concentration gradient of the tracer outside the snow line directly depends on the Schmidt number in a power-law fashion ($\sim R^{-1.5}$), independent of the choice of turbulent $\alpha$ parameter.

We argue that the effects of turbulent diffusion on the surface brightness distribution of such a snow line tracer are detectable with ALMA observations of disks in nearby star-forming regions, which can possess high enough angular resolution to constrain the current theoretically/numerically estimated values of the Schmidt number ($\Sigma R \sim 0.1–10$).

Observations of different snow lines (e.g., H$_2$O, CO$_2$, and CO) at different radii in the disks would allow the radial dependence of the Schmidt number to be probed. Coupling these snow line observations with observational estimates of the gas surface density and turbulent linewidths would allow direct estimates of the strength and nature of the turbulence in astrophysical accretion disks.

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REFERENCES

Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Birnstiel, T., Dullemond, C. P., & Brauer, F. 2010, A&A, 513, A79
Bockelée-Morvan, D., Gautier, D., Hersant, F., Huté, J-M., & Robert, F. 2002, A&A, 384, 1107
Bouwman, J., Meeus, G., de Koter, A., et al. 2001, A&A, 375, 950
Carballido, A., Stone, J. M., & Pringle, J. E. 2005, MNRAS, 358, 1055
Chiang, E. I., & Goldreich, P. 1997, ApJ, 490, 368
Clarke, C. J., & Pringle, J. E. 1988, MNRAS, 235, 365
Dubrulle, B., & Frisch, U. 1991, P&SS, 43, 5355
Dullemond, C. P., Apai, D., & Walch, S. 2006, ApJL, 640, L67
Friesen, R. K., Di Francesco, J., Shimajiri, Y., & Takakuwa, S. 2010, ApJ, 708, 1002
Gail, H.-P. 2001, A&A, 378, 192
Gammie, C. F. 1996, ApJ, 457, 355
Hartmann, L., Calvet, N., Guillering, E., & D’Alessio, P. 1998, ApJ, 495, 385
Hughes, A. L. H., & Armitage, P. J. 2010, ApJ, 719, 1633
Hughes, A. M., Wilner, D. J., Andrews, S. M., Qi, C., & Hogerheijde, M. R. 2011, ApJ, 727, 85

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$^4$ Since the beam size of the current TW Hya observation possesses a resolution similar to $\theta_{25}$, then using the current observation to constrain the Schmidt number seems unlikely; however, using the velocity channels separately can increase the effective resolution (e.g., Qi et al. 2013a).
Jacquet, E., Gounelle, M., & Fromang, S. 2012, Icar, 220, 162
Jacquet, E., & Robert, F. 2013, Icar, 223, 722
Johansen, A., Klahr, H., & Mee, A. J. 2006, MNRAS, 370, L71
Jørgensen, J. K., Schöier, F. L., & van Dishoeck, E. F. 2004, A&A, 416, 603
Klahr, H. H., & Bodenheimer, P. 2003, ApJ, 582, 869
Lathrop, D. P., Fineberg, J., & Swinney, H. L. 1992, PhRvA, 46, 6390
Lodato, G., & Rice, W. K. M. 2004, MNRAS, 351, 630
Lodato, G., & Rice, W. K. M. 2005, MNRAS, 358, 1489
Lynden-Bell, D., & Pringle, J. E. 1974, MNRAS, 168, 603
Martin, R. G., & Livio, M. 2012, MNRAS, 425, L6
Martin, R. G., & Livio, M. 2013, MNRAS, 434, 633
Martin, R. G., & Livio, M. 2014, ApJL, 783, L28
Nelson, R. P., Gressel, O., & Umurhan, O. M. 2013, MNRAS, 435, 2610
Owen, J. E. 2014, ApJ, 789, 59
Owen, J. E., Ercolano, B., & Clarke, C. J. 2011a, MNRAS, 412, 13
Owen, J. E., Ercolano, B., & Clarke, C. J. 2011b, MNRAS, 411, 1104
Prinn, R. G. 1990, ApJ, 348, 725
Qi, C., Öberg, K. I., & Wilner, D. J. 2013a, ApJ, 765, 34
Qi, C., Öberg, K. I., Wilner, D. J., et al. 2013b, Sci, 341, 630
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Takahashi, J., & Williams, D. A. 2000, MNRAS, 314, 273
Turner, N. J., Fromang, S., Gammie, C., et al. 2014, in Protostars and Planets VI, ed. H. Beuther, R. Klessen, C. Dullemond, & Th. Henning, arXiv:1401.7306
Walsh, C., Nomura, H., Millar, T. J., & Aikawa, Y. 2012, ApJ, 747, 114
Youdin, A. N., & Lithwick, Y. 2007, Icar, 192, 588
Zhu, Z., Stone, J. M., & Bai, X.-N. 2014, arXiv:1405.2778