Spatial Statistics for Wireless Networks Research

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Abstract

Wireless networks have become an important part of our daily lives. However, while the networking research community has made great progress in developing the communication technologies themselves, the underlying dynamics of deployment and use of wireless technologies is still relatively poorly understood. This is especially true for user-deployed technologies such as Wi-Fi hotspots, as well as the large-scale use of radio spectrum. This situation is already starting to cause difficulties in the wireless networking research community due to the arising lack of network deployment models for performance evaluation of new wireless technologies. Also the governmental regulators planning for new policy frameworks are lacking models and hard data on how existing networks and devices use the radio spectrum made available to them by the current regulatory regime. These issues are very topical globally, and are being actively pursued by the Federal Communications Commission (FCC) in the USA, as well as its European and Asian counterparts. In this paper we discuss our work on applying spatial statistics techniques for constructing models for structure of wireless networks and the way they use the radio spectrum. We focus specifically on key research challenges that would be of particular interest to the wireless communications community.

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1. Introduction

Wireless communications systems and networks have become ubiquitous in our surroundings. Cellular networks cover already most of the inhabited areas of the world, and technologies such as Wi-Fi are widely used in practically all major population centers in developed countries. The engineering and computer science communities have made great progress over the past century on developing technologies for wireless communications, pushing the technology from the wireless telegraph to high-
speed data connections commonly available today. At the same time the actual deployment and use of wireless technologies remains relatively poorly understood. The economic principles controlling the deployment of large-scale wireless infrastructures are, of course, rather well known. However, the structure of the arising deployment is highly dependent on the distributions of population and often also of wealth over a region in question, which can have complex spatial structures. Further, more and more wireless communications systems are being deployed by users themselves, including Wi-Fi access points commonly purchased in conjunction with fixed-line Internet connections, and, more recently, femtocells which are effectively small cellular base stations. The dynamics and the arising spatial structures of such deployments can differ significantly from those of the large-scale infrastructure networks due to their emergent and local nature.

The reason why understanding the spatial structure of wireless networks is becoming increasingly important also for the networking community is that it plays a key role in the performance of those networks. As we shall discuss in the following sections, wireless communications systems deployed in a given region interact through the electromagnetic spectrum, and distances between transmitters and receivers dictate the quality of the experience the users have when using these systems. Also, until recently partitioning of the radio spectrum into discrete frequency bands by governmental regulators has been used as means to control these interactions between different networks and systems. However, recent research activities have shown that this can lead to an inflexible and inefficient use of the radio frequencies. Numerous solutions for so-called dynamic spectrum access (DSA) have been recently proposed to address this problem [1-3]. Systems utilizing DSA principles could “borrow” unused radio frequencies from other systems as long as harmful interactions, dominant component of which is often known as “interference”, are kept under control. This would potentially increase the efficiency of the systems involved, but use of DSA will also couple the dynamics of the various communications systems coexisting in same spatial region together in a manner that has not been considered in the literature until now. To explore the influence of such coupling in more detail, it is imperative to understand and to be able to model quantitatively the spatial structures of different networks and systems deployed in the same space.

Our objective in this paper is to give the reader with background in spatial statistics basic understanding of some of the key analysis and modeling problems related to spatial structure of wireless communication systems. We use selected case studies from our earlier work as illustrative examples, but our focus is on highlighting outstanding problems based on these intermediate results rather than on reporting new results. In order to enable the reader without prior background in wireless communications to benefit from the rest of the text, we provide in the following section a short introduction to some of the fundamental aspects relevant to spatial modeling problems. We shall then move on to discuss two classes of modeling and analysis problems for wireless communications, one amenable to treatment using point processes, and another resulting in the application of the theory of random fields. We focus in particular on the key research challenges in applying these modeling frameworks within the wireless communications domain.

2. A Primer on Wireless Communications

We begin by giving in this section a short introduction to fundamentals of wireless communications, specifically emphasizing the spatial aspects. It is, of course, impossible to give a comprehensive account due to space reasons, and we accordingly refer the interested reader to [4] for a more detailed discussion. In any wireless communication system a transmitter communicates with a receiver by means of modulated electromagnetic radiation. Only a certain range of frequencies, which we shall call a channel in the following, are actually used for conveying information. As the electromagnetic radiation propagates
from the transmitter, it gets attenuated, reflected and refracted by the environment before reaching the receiver, which then uses various estimation techniques to try to uncover the transmitted information from the received signal. For most communication technologies, the factor that roughly determines whether the information can be successfully recovered is the so-called signal-to-interference-and-noise-ratio (SINR), defined as $\text{SINR} = \frac{S}{I + N}$, where $S$ is the average power of the signal from the transmitter, $I$ is the sum of the average powers of signals originating from other ongoing transmissions not intended for the particular receiver considered, and $N$ denotes power of the noise in the system. This ratio also plays fundamental role in the theory of communications. From Shannon’s information theory [5] it is well known that the theoretical maximum rate of information exchange between the transmitter and the receiver, also called the capacity of the channel, is given by $C = W \log(1 + \text{SINR})$, where $W$ is the bandwidth of the channel, that is, the span of frequencies used for communication (see [4] for more detailed discussion esp. regarding the domain of validity for this bound).

The reason why the spatial relationships between the different transmitters and the receivers are key to determining the performance of the networks in question lies in the fact that the average powers $S$ and $I$ making up the SINR depend heavily on distances. In free space the power of electromagnetic waves decays quadratically over distance, but near to ground and when surrounded by obstacles the relationships between the average powers of the transmitted and received signals on the distances is more complex. Typically one measures both powers in logarithmic scale, usually in decibels (dB), and writes the relationship between $S$ and the average transmitted power $T$ in these units in the form

$$S = T - (A + B \log(d) + X),$$  \hspace{1cm} (1)

where $A$, $B$ and $C$ are constants depending on the type of the environment and on the channel being used, $d$ denotes the distance between the transmitter and the receiver, and $X$ is a spatially and temporally correlated random field used to model departures from the deterministic, distance-dependent part of the model. Similar equations then of course hold for the different components of the average power of the interference term $I$. Equations of type (1) are often called propagation models.

3. Spatial Characterization and Modeling of Wireless Networks

From the above discussion, several spatial modeling questions arise. First, since the distances between the nodes are featured in equation (1), it is natural to study the joint and marginal distributions of the inter-node distances. The natural framework for this is to characterize and model node locations as a point process. Second, one can treat both the average signal and interference powers $S$ and $I$ as random fields in a manner analogous to classical theory of shot noise. Third, assuming that node locations and other characteristics appearing in (1) are known, the random “shadowing” component $X$ in (1) is the key spatial object to be analyzed, which again can be accomplished using random field techniques. We shall focus here on the first two types of problems, as they are hitherto almost completely unexplored in the wireless communications and spatial statistics literature.

Figure 1 shows a typical example of a wireless network data set relevant to point process modeling, consisting of the base station locations for a major Spanish cellular network provider. From the figure it is clear that the distribution of network nodes is highly inhomogeneous, with the large-scale structure strongly driven by the underlying population distribution. Our earlier results [6] have indicated that for such large-scale data sets the scaling of the intensity of node locations typically follows at least roughly a power law, and this can be shown to be the case for this data set as well. For some network types the large-scale structure is in fact adequately represented in terms of second-order statistics by an inhomogeneous Poisson point process, with intensity function taken to follow smoothed population
distribution. This is not quite the case of this particular data set, due to the need for the cellular operator to provide coverage also in almost completely uninhabited regions.

The small-scale structure of the data set is quite distinct from the large-scale one also in urban regions, as can be seen in Figure 2, showing part of the data set corresponding to base station locations in Madrid. The node locations are clearly regular locally, although not forming an actual hard core process either. Figure 3 illustrates this quantitatively by giving the estimates of the J-function for the Madrid data sets computed using the spatstat package [7].
Fig. 3. The estimates of the J-function for the Madrid part of the Spanish operator data set for different wireless communication technologies, including suburbs (left) and for downtown data set only (right).

Addressing these kinds of inhomogeneities in modeling of wireless networks forms an interesting research challenge. In [8] we showed that locally in a region in which the intensity of nodes is roughly constant, rather simple Gibbs point process models can very well model the structure of a variety of different network types. Incorporating data on the population distribution as a covariate is, of course, one possible approach, and can indeed be in our experiences made to work quite accurately. However, for actual applications this is not entirely satisfactory solution since one would prefer an integrated model without need to specify a particular population distribution. Another interesting research challenge would be to make the mapping from the fitted Gibbs point process model to the distribution of distances occurring in equation (1) more analytically explicit. For wireless networking applications especially interesting would be the conditional distributions of 2nd, 3rd, etc. nearest neighbor distances conditioned on the distance to the nearest neighbor. Such distributions are often encountered when attempting to compute the SINR distribution for a given node location model.

We conclude by briefly discussing random field models for the average powers of $S$ and $I$. Two broad categories of approaches can be considered to construct such models. First, one can gather measurements of these quantities from operational networks, and treat the as samples of a random field without considering individual transmitter locations. Second, one can treat measurement results as samples from a point process shot noise field, with contributions from each of the transmitters being obtained from equation (1). The latter approach is obviously closely related to the point process modeling problem discussed above. Our early work reported on in [9] indicates that in regions of relative stationarity simple classical random field models, in particular Gaussian random fields with Matérn semivariogram model suffice to yield good results. Key challenge is again, however, the incorporation of the large-scale inhomogeneities into the models. Another challenge with highly interesting potential applications is the development of a suitable system identification framework, in which measurements of the average power field could be used to infer properties of the underlying transmitter distribution in the shot noise...
description. Such framework could be incorporated as a component into different wireless networking technologies to help enable optimization of the way radio frequencies are used for transmissions.

4. Conclusions

In this paper we have outlined some of the potential application areas and related research challenges for utilizing spatial statistics techniques in characterizing and modeling wireless communication systems. Our early results indicate that classical approach such as Gibbs point processes and Gaussian random fields can effectively be used as modeling framework locally. However, any large scale wireless network has significant inhomogeneities in part induced by the density fluctuations in underlying population distribution. Incorporating parsimonious models for these large-scale density fluctuations into the existing models is a highly interesting research challenge. Another key research challenge of importance to applications in run-time optimization of wireless networks is the development of theory of inference or system identification for the random field and point process models.

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References

[1] Q. Zhao and B. Sadler, “A survey of dynamic spectrum access,” IEEE Signal Processing Magazine, vol. 24, no. 3, pp. 79–89, 2007.
[2] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, “Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey,” Computer Networks: The International Journal of Computer and Telecommunications Networking, vol. 50, no. 13, pp. 2127 – 2159, 2006.
[3] S. Haykin, “Cognitive radio: brain-empowered wireless communications,” IEEE journal on selected areas in communications, vol. 23, no. 2, pp. 201–220, 2005.
[4] R. Gallager, “Principles of Digital Communication”, Cambridge University Press, March 2008.
[5] Claude E. Shannon: A Mathematical Theory of Communication, Bell System Technical Journal, Vol. 27, pp. 379–423, 623–656, 1948.
[6] J. Riihijärvı, P. Mähönen, M. Rubsamen, Characterizing Wireless Networks by Spatial Correlations, IEEE Communications Letters 11 (1) (2007) 37–39.
[7] A. Baddeley and R. Turner, “Spatstat: an R package for analyzing spatial point patterns,” Journal of Statistical Software, vol. 12, no. 6, pp. 1–42, 2005, ISSN 1548-7660. [Online]. Available: www.jstatsoft.org
[8] J. Riihijärvı and P. Mähönen, “Modeling Spatial Structure of Wireless Communication Networks,” in INFOCOM IEEE Conference on Com- puter Communications Workshops, 2010, 2010, pp. 1–6.
[9] M. Wellens, J. Riihijärvı, and P. Mähönen, “Spatial Statistics and Models of Spectrum Use”, Elsevier Computer Communications, vol. 32, no. 18, pp. 1998-2011, December 2009.