Timing information at HL-LHC: Complete determination of masses of Dark Matter and Long lived particle

Dong Woo Kang, a Seong Chan Park b

 a Department of Physics, Sungkyunkwan University, Suwon 440-746, Korea
 b Department of Physics and IPAP, Yonsei University, Seoul 03722, Republic of Korea

E-mail: kdwgod54@skku.edu, sc.park@yonsei.ac.kr

Abstract: A long standing problem in kinematics at the hadron colliders is to determine the mass of invisible particles. This issue is particularly important for the signals of dark matter, which becomes one of the prominent targets of future collider experiments. In this paper, we show that the additional information from the precise timing measurement, which will be available at the planned high-liminosity run of the LHC (HL-LHC), will shade new light on the kinematics study. As a concrete example, we focus on the signal of the pair produced long-lived particles \((LLP_{1,2})\), respectively leaving displaced vertex with visible \((V_{1,2})\) and invisible \((I_{1,2})\) final state, \(pp \rightarrow LLP_{1} + LLP_{2} \rightarrow (V_{1} + I_{1}) + (V_{2} + I_{2})\). We explicitly show that this system is completely solvable with timing information.
1 Introduction

After the Run-2 and Run-3 of the LHC, a significant upgrade is planned to go to the High Luminosity LHC (HL-LHC) to achieve instantaneous luminosities a factor of five times larger than the LHC nominal value. This enables the LHC experiments to collect their data by one order of magnitude larger compared with the LHC baseline program, but the multiple pile-up events would cause complications in the analysis which leads to limitations on the physics potentials in search for new physics beyond the standard model. To challenge this issue, both CMS [1] and ATLAS [2] have proposed to install the new timing detector, which is considered as one of the most noticeable components of the upgrade for the HL-LHC. The proposed sensitivity of the timing detector reaches the precision of about $\Delta t \simeq 30$ ps. Importantly, it is noticed that the timing detector would not only help the pile-up issue but also provide a unique physics potential in search for long-lived particles (LLPs) by directly measuring the Time-of-Flight (ToF) of LLP [3].

In this paper, we pursue this direction further and show the unique power of the timing information in dealing with the invisible particle (i.e. dark matter) as well as the mother LLP. In particular, we explicitly demonstrate how the timing information enables us to determine the masses of invisible particles and also the LLPs in the signal of the pair produced LLPs. Each LLP subsequently decays into visible and invisible particles: $pp \rightarrow LLP_a + LLP_b \rightarrow (V_a + I_a) + (V_b + I_b)$. This can happen in many well-motivated beyond the standard model (BSM)
Theories: the second lightest supersymmetric particle decays to the lightest supersymmetric particle (LSP) and the second lightest Kaluza-Klein particle decays to the lightest Kaluza-Klein particle (LKP), respectively [4–8]. In the case when a LLP decays only into visible particles, all kinematics are directly measurable. However, as in our interest, the involved invisible particles make the problem unbreakable without the timing information. Indeed this problem is known to be unsolved (see e.g. [7, 9–12]).

This paper is structured as follows: In section 2, we develop two reconstruction methods for the events with LLPs and their decays to dark matter particles and visible particles using the timing and the displaced vertex measurements of the LLPs. In section 3, we show the validity of our methods with the realistic Monte-Carlo event simulations. Finally, we conclude in section 4.

2 Reconstruction using the timing information

We begin by reviewing the proposed timing detector, in particular, for the CMS [1] shown schematically in Fig. 1. The picosecond timing detector will be installed between the tracker system (green) and the electromagnetic calorimeter (cyan) covering the barrel as well as the end-cap of the detector. In the case of ATLAS, the coverage is only the end-cap region [2]. The designed precision timing layer is capable of detecting minimum ionizing particles (MIPs) with excellent efficiency (nearly 100%) and time resolution of about 30 ps throughout the HL-LHC. The timing layer can be attached as a thin standalone detector with minimal impact on the neighboring sub-detectors. The fundamental detector unit consists of a LYSO: Cerium crystal tile read out with a silicon photomultiplier (SiPM). At the HL-LHC, the proton bunch...
crossing happens in every 25 ns and the exact timing of the collision, \( T_0 \approx T_{\text{collision}} \), with uncertainty, \( \Delta T \sim 30 \) ps, would be time-stamped by the timing layer when the initial state radiation (ISR) jet from the collision point passes through the detector [3]. The measurement of \( T_0 \) would help to sort out the pile-up events. We can apply the same technique for the events of our interest: the pair production of LLPs, \( pp \rightarrow LLP_a + LLP_b \).

An LLP with a suitable life time would decay inside the tracker, provided that the LLP flies around 0.1 cm - 1m before its decay into visible (V) and invisible (I) particles. Here V and I may be composed of several particles in general but we may treat V (and I) as a single object. Taking V as a massless particle, we can determine the timing of the decay by measuring the moment of the visible particle passing through the timing layer, \( T_V \approx T_{\text{decay}} \). The associated displaced vertex (DV) of the LLP can be also reconstructed using the information from the tracking system. Having all these informations, we now completely determine the 3-vector displacement of the LLP \( \Delta r_{LLP} \) and the associated TOF \( \Delta t_{\text{ToF}} = T_{\text{decay}} - T_0 \) or the 3-velocity \( \beta \) of LLP. In short, the precision timing detector could measure the 3-velocity of the LLP:

\[
\beta_{LLP} = \frac{\Delta r_{LLP}}{\Delta t_{\text{ToF}}}. \tag{2.1}
\]

The uncertainties of the designed detectors for the displaced vertex \( \Delta r \) and the timing \( \Delta t_{\text{ToF}} \) are impressive: \( |\delta(\Delta r_P)| \lesssim (10 - 30) \) \( \mu \)m and \( |\delta(\Delta t_{\text{ToF}})| \lesssim (30 - 300) \) ps [1]. We will take these uncertainties to figure out the feasibility of our method in the later section. Of course, the benefit of the timing information this far is rather universal provided that the decay products of the LLP include a visible (and light) particle independent of the presence of an invisible particle (or dark matter).

2.1 LLP decays to visible and invisible particles (\( LLP \rightarrow V + I \))

Before considering the pair production of LLPs, we now focus on the two body decay of a single LLP (\( LLP \)) to a visible (V) and an invisible (I) particles (\( LLP \rightarrow V + I \)). The visible particle (V), by definition here, is identified unambiguously with its 4-momentum in the lab frame: \( P_{V}^{\text{lab}} = (E_{V}^{\text{lab}}, \mathbf{p}_{V}^{\text{lab}}) \). The mass of V is easily calculable by \( m_{V}^2 = E_{V}^{\text{lab}} - \mathbf{p}_{V}^{\text{lab}} \). The 3-velocity of LLP in the lab frame \( \beta_{LLP}^{\text{lab}} \) is also measurable as we explained in the previous section. On the other hand, through the energy-momentum conservation, \( \mathbf{p}_{LLP}^{\mu} = \mathbf{p}_{V}^{\mu} + \mathbf{p}_{I}^{\mu} \) in the rest frame of the LLP, we can determine the 3 momentum of the invisible particle from the balance

\[
E_{V}^{LLP} = \gamma_{P}^{\text{lab}} \left( E_{V}^{\text{lab}} - \mathbf{p}_{V}^{\text{lab}} \cdot \beta_{LLP}^{\text{lab}} \right), \tag{2.2}
\]

where the relativistic gamma factor is introduced as \( \gamma_{P}^{\text{lab}} = 1/\sqrt{1 - (\beta_{P}^{\text{lab}})^2} \). On the other hand, through the energy-momentum conservation, \( \mathbf{p}_{LLP}^{\mu} = \mathbf{p}_{V}^{\mu} + \mathbf{p}_{I}^{\mu} = (m_P, 0) \) in the rest frame of the LLP, we can determine the 3 momentum of the invisible particle from the balance...
of the visible and invisible particle, \( p_I = -p_V \), thus the energy of the visible particle in the LLP rest frame as a function of the involved masses:

\[
E_{LLP}^V = \frac{m_{LLP}^2 - m_I^2 + m_V^2}{2m_{LLP}},
\]

(2.3)

where \( m_{LLP}, m_I \) and \( m_V \) are the masses of the LLP, invisible and visible particle, respectively. Combining Eq. 2.2 and Eq. 2.3, we get the expression for the mass of the LLP,

\[
m_{LLP} = E_{LLP}^V + \sqrt{(E_{LLP}^V)^2 + m_I^2 - m_V^2}.
\]

(2.4)

One should notice that \( E_{LLP}^V \) and \( m_V \) are measurable (as they are for the visible particle!) but we still needs the mass of the invisible particle to complete the calculation.\(^1\)

Even though we cannot measure the full 3 momentum of the invisible particle directly, we can still determine the transverse components of the momentum \( p_{I,T}^{\text{lab}} \) by the measured missing momentum. By equating the sum of the transverse momenta of visible and invisible particles and that of the LLP, we find the energy of the LLP in the lab-frame as

\[
p_{LLP,T}^{\text{lab}} = p_{I,T}^{\text{lab}} + p_{V,T}^{\text{lab}}
\]

(2.5)

\[
\Rightarrow E_{LLP}^{\text{lab}} = \frac{\beta_{LLP,T}^{\text{lab}} \cdot (p_{I,T}^{\text{lab}} + p_{V,T}^{\text{lab}})}{|\beta_{LLP,T}^{\text{lab}}|^2}.
\]

(2.6)

Here the quantities in the right handed side are all measurable so that \( E_{P}^{\text{lab}} \) is also completely determined. Therefore, the mass of LLP is determined as well:

\[
m_{LLP} = \left( \gamma_{LLP}^{\text{lab}} \right)^{-1} E_{LLP}^{\text{lab}}
\]

(2.7)

\[
= \sqrt{1 - (\beta_{LLP}^{\text{lab}})^2} \frac{\beta_{LLP,T}^{\text{lab}} \cdot \left( p_{I,T}^{\text{lab}} + p_{V,T}^{\text{lab}} \right)}{|\beta_{LLP,T}^{\text{lab}}|^2}.
\]

(2.8)

Finally, the mass of the invisible particle is obtained with \( m_{LLP} \) in Eq. 2.4. Eq. 2.8 gives the final result:

\[
m_I = \sqrt{m_{LLP}^2 - 2m_{LLP} E_{V}^{LLP} + m_V^2}.
\]

(2.9)

In summary, we can completely determine the masses of the LLP and invisible particle in the decay process \( LLP \rightarrow V + I \) by measuring the displaced vertex with the timing information!

\(^1\)Even though the relations of the masses is written in terms of the second order equation but there’s no sign ambiguity involved in Eq. 2.4. The sign in front of the square root is chosen by considering the massless limit, \( m_V \rightarrow 0 \) and \( m_I \rightarrow 0 \).
2.2 Production of two (different) LLP particles: \( LLP_a \neq LLP_b \)

Fully equipped with the method developed in the previous section, we now tackle our physics problem: production of two (different) LLPs. Let us call them as \( LLP_a \) and \( LLP_b \). Here we assume \( LLP_a \neq LLP_b \). The energies and momenta of the LLPs are \( E_a, E_b \) and \( p_a, p_b \) respectively.

For the pair produced LLPs and their decay products, Eq. 2.5 is now generalized as

\[
p_{a,T} + p_{b,T} = p_{I,T} + p_{V_a,T} + p_{V_b,T}
\]

\Rightarrow E_a \beta_{a,T} + E_b \beta_{b,T} = p_{I,T} + p_{V_a,T} + p_{V_b,T},
\]

(2.10)

where the total transverse momentum of the invisible particles is given as \( p_{I,T} = p_{I_a,T} + p_{I_b,T} \). Taking the two independent relations from the two vectors on the transverse plane in Eq. 2.10, we can analytically solve the two unknown energies \( (E_a \) and \( E_b) \) as the following:

\[
E_a = \left[ \frac{\beta_b \times (p_{T}^{\text{miss}} + p_{V_a} + p_{V_b}) \cdot \hat{k}}{\beta_b \times \beta_a \cdot \hat{k}} \right], \quad E_b = \left[ \frac{\beta_a \times (p_{T}^{\text{miss}} + p_{V_a} + p_{V_b}) \cdot \hat{k}}{\beta_a \times \beta_b \cdot \hat{k}} \right],
\]

(2.11)

where \( \hat{k} \) is beam direction unit vector. The detailed derivation is found in the Appendix.

With the solved \( E_a \) and \( E_b \), we now find the full 4-momenta of the system:

\[
p_a = (E_a, E_a \beta_a), \quad p_b = (E_b, E_b \beta_b), \quad p_{I_a} = p_a - p_{V_a}, \quad p_{I_b} = p_b - p_{V_b},
\]

(2.12)

where we assumed that \( \beta_{a,b} \) are directly measured with timing information. One should note that the derivation is completely generic thus can be applied in any system with the same event topology even though the accompanied uncertainties may depend on the kinematical and experimental details. Since all of the four-momenta of the system are known, the masses of LLPs and invisible (dark matter) particles are also completely determined. This is one of our main results in this paper.

One may wonder why these complete solutions are found when the timing informations are available. The reason is fairly simple: when we count the number of unknowns, we find 16 from the four 4-vectors of the two LLPs and two invisible particles. Without the timing information, the number of measurable quantities and the conservation conditions are \( 4 + 4 + 2 + 2 + 2 = 14 \) where the first two 4’s are from the two 4 momenta of visible particles, the next two 2’s are from the direction of the displaced vertices and the last 2 are from the total transverse momentum of the invisible particles. The timing information of the two ToF’s complete the sufficient condition (in total 16 conditions) to determine the whole system.

2.3 Production of two identical LLPs: \( LLP_a = LLP_b \)

When the pair produced LLPs are identical and their decay products are also identical, the symmetries of the system reduces the actual number of unknowns so that we can completely solve the system even without the additional timing information.
Let’s begin with the relation from the 4-momentum conservation in each branch of the decay processes \((LLP_i \rightarrow V_i + I_i)\) for \(i = a\) or \(b\), respectively:

\[
p_{I_a} = p_a - p_{V_a} \\
\Rightarrow m_{I_a}^2 = m_{a}^2 + m_{V_a}^2 - \left(2E_{V_a} \sqrt{m_{a}^2 + |p_a|^2} - 2p_{V_a} \cdot p_a\right), \quad (2.13)
\]

\[
p_{I_b} = p_b - p_{V_b} \\
\Rightarrow m_{I_b}^2 = m_{b}^2 + m_{V_b}^2 - \left(2E_{V_b} \sqrt{m_{b}^2 + |p_b|^2} - 2p_{V_b} \cdot p_b\right). \quad (2.14)
\]

Demanding \(m_a = m_b = m_{LLP}\) and \(m_{I_a} = m_{I_b} = m_I\) as our assumptions taken as a priori conditions, we can combine the two equations and get a combined 2nd order equation for \(\Delta \equiv m_{LLP}^2 - m_I^2 (> 0)\):

\[
A_a \Delta^2 + 2B_a \Delta + C_a = m_{LLP}^2 = A_b \Delta^2 + 2B_b \Delta + C_b, \quad (2.15)
\]

\[
\therefore (A_a - A_b) \Delta^2 + 2(B_a - B_b) \Delta + (C_a - C_b) = 0, \quad (2.16)
\]

where the coefficients for \(i = a, b\) are all measurable and explicitly written as

\[
A_i = \frac{1}{4E_{V_i}^2}, \quad B_i = A_i (m_{V_i}^2 + 2p_{V_i} \cdot p_i), \quad C_i = \frac{B_i^2}{A_i} - |p_i|^2. \quad (2.17)
\]

For each event, all coefficients (\(A’s\), \(B’s\) and \(C’s\)) are given modulo the experimental uncertainties then \(\Delta\) can be easily obtained numerically or analytically. The physical solution should satisfy the conditions \(m_{LLP} > m_V + m_I\) and \(m_P > 0\) and \(m_I > 0\). Knowing this physical solution of \(\Delta\), we can determine \(m_{LLP}\) and \(m_I\) by Eq. 2.15. This is also one of our results in our paper.

### 2.4 Summary of reconstruction methods

The two equations Eq. 2.13 and Eq. 2.14, which is obtainable with or without the timing information but with the measured displace vertices, provide two independent algebraic curves, \(C_a\) and \(C_b\), on the two dimensional plane with the coordinates \((m_{LLP}, m_I)\) for \(i = a\) or \(i = b\). The point giving the true values of the masses, \((m_{LLP}, m_I)\), should lie on the corresponding curve \(C_i\) but we still need additional information to determine the true point.

We point out two completely solvable cases:

- Case-1: The identical particles are produced \((LLP_a = LLP_b\) and \(I_b = I_a)\)
- Case-2: The timing information is available \((LLP_a \neq LLP_b\) and \(I_a \neq I_b)\)

For a special case with \(m_{LLP_a} = m_{LLP_b}\) and \(m_{I_a} = m_{I_b}\), the cross point of two curves would give the true solution thus we don’t rely on the timing information but on the displace vertex measurement. We call this displaced vertex based reconstruction without timing information simply “w/o timing reconstruction”. We also developed the method to reconstruct the 3
momenta of LLP’s and invisible particles without timing information and the details are shown in the Appendix. On the other hand, when the timing information is available, we can always find the solution and we call it “timing reconstruction”. We summarize all relevant cases in Table 1.

| Case   | w/o timing | timing |
|--------|-------------|--------|
| LLPa   | △           | ○      |
| LLPb   | △           | ○      |
| Ia     | △           | ○      |
| Ib     | △           | ○      |
| pLLPa  | ○           | ○      |
| pLLPb  | ○           | ○      |
| pIa    | ○           | ○      |
| pIb    | ○           | ○      |

Table 1: The summary of the reconstruction. The mark ○ (×) is for the case when we can (cannot) reconstruct the system. The triangle (△) means that we can reconstruct the system only with ambiguities.

3 Feasibility study with MC simulation

In the previous section we developed two new reconstruction methods: “w/o timing reconstruction” and “timing reconstruction”. To show the feasibility of both methods, we perform MC simulations and reconstruct the signals for two different cases.

- Case-1 LLPa = LLPb and Ia = Ib
- Case-2 LLPa ≠ LLPb and Ia ≠ Ib

The events are generated by MG5aMC [13] and the particle decays are simulated with pythia8 [14–16]. For realistic analysis, we smear the objects with gaussian error with the 30ps timing resolution and include 2% momentum resolution for visible particles and 12 µm as position resolution for DV at the HL-LHC. The typical event topology is shown in Fig. 2. For our signal events, no significant SM background is expected except for the potential misidentified SM events. We thus regard our signal almost background free and focus on signal event for the analysis. The event simulation is done with a toy model, which encapsulates the relevant features of any kind of new physics model which has the same event topology. Therefore, our method will be useful in analyzing a wide range of new physics models.

3.1 Case-1 LLPa = LLPb and Ia = Ib

Here we consider the case when long-lived particles are pair produced and each LLP decays into the same invisible particles. This case can be applied when the new particles in BSM models are pair produced then decay into a dominant decay channel with an invisible particle...
and visible particles: gluino decay in GMSB SUSY, slepton decay, gluball decay in hidden sector model and many others [17–22]. As a concrete example, we choose $M_{\text{LLP}} = 400$ GeV and $M_{\text{Inv}} = 200$ GeV. $c\tau \approx \mathcal{O}(100)$mm. Using both reconstruction methods, we reconstruct both $M_{\text{LLP}}$ and $M_{\text{Inv}}$. One noticeable advantage we can enjoy in our analysis here is that no significant combinatoric issue would be involved because the two distinguishable displaced vertices are independently identified and measured.

Assuming the identical particles ($\text{LLP}_a = \text{LLP}_b$ and $I_a = I_b$), the reconstructed mass distributions with “w/o timing reconstruction” method is shown in the Fig. 3. We find very clean peaks at the $M_{\text{LLP}}$ and $M_{\text{Inv}}$. However, to use the method, we have to assume that the two LLPs are identical and the invisible particles in the final state are also identical with no a priori justification. Of course, we knew the assumption is a right one and the fitted results are in good agreement with the true values. The fitted values are $M_{\text{LLP}} = 401.309 \pm 0.306$ GeV and $M_{\text{Inv}} = 202.248 \pm 0.468$ GeV. The errors are statistical errors that come from the repetitive fitting procedure with 10 data sets.

Now without assuming the identical particles, we can reconstruct $M_{\text{LLP}_a}$ and $M_{\text{LLP}_b}$ separately with timing reconstruction method. The result is shown in Fig. 4. The clear peaks near the true $M_{\text{LLP}}$ and $M_{\text{Inv}}$ values are prominently seen for each decay chain individually. The result is also in good agreement with the true values. The fitted masses are 404.506 GeV, 404.343 GeV for two LLPs and 206.347 GeV, 205.872 GeV for the two invisible particles. Here without the assumption of the identical particles, the results are slightly farther off from the true values mainly due to the additional source of error in the timing measurements compared with the results from the former analysis with the assumption.
Figure 3: The mass reconstruction without timing information for $M_{\text{LLP}} = 400$ GeV, $M_{\text{Inv}} = 200$ GeV. Left: LLP mass reconstruction; Middle: Invisivle particle mass reconstruction; Right: LLP vs invisible mass 2D histogram.

Figure 4: The mass reconstruction using timing information with 30 ps smeared result for $M_{\text{LLP}} = 400$ GeV, $M_{\text{Inv}} = 200$ GeV. Left: LLP mass reconstruction; Middle: Invisivle particle mass reconstruction. Red and blue color indicate each decay chain; Right: LLP vs invisible mass 2D histogram for each decay chain.

3.2 Case-2 $LLP_a \neq LLP_b$ and $I_a \neq I_b$

To consider a more general case, we take the different LLPS and different invisible particles. For the reference values, we take $M_{\text{LLP}} = 300$ GeV, $M_{\text{LLP}} = 600$ GeV, $M_{\text{Inv}} = 100$ GeV, and $M_{\text{Inv}} = 300$ GeV, $c\tau \approx O(100)\text{mm}$. Two different long-lived particle are produced and each LLP decays into different invisible particle. Here, the assumption that we used in the first method is wrong with all particles different from the previous section. Without the timing information, we only get a result which leads to a wrong conclusion. The reconstructed mass distributions are shown in the Fig. 5 for both methods. The clean peaks appear at the true mass values when we use timing reconstruction. However, the reconstructed masses appear as a broadly uncertain distribution from the “w/o timing reconstruction” with the wrong assumption of the identical particles as we expected.

In Table. 2 we summarize the determined masses for Case-1 and Case-2 using two different...
reconstruction methods (and different assumptions) and the reconstruction efficiencies. The reconstruction efficiency $\epsilon_{\text{reco}} = \frac{N_{\text{reco}}}{N_{\text{gen}}}$ is the ratio between the number of generated event $N_{\text{gen}}$ and the number of reconstructed events $N_{\text{reco}}$. In the realistic MC simulation results, the smearing effect would lead to some number of events having the imaginary solutions thus we cannot apply our reconstruction methods for them.

We could not get a set of reasonable values for Case-2. This failure is due to the wrong assumption about the involved particles. Otherwise, we conclude that our reconstruction methods are nicely working and could be used to provide direct measurement of the masses of LLPs and dark matter particles at HL-LHC.

| Case  | w/o timing | timing |
|-------|------------|--------|
| m_{LLP_a} | m_{LLP_b} | m_{I_a}  | m_{I_b}  | $\epsilon_{\text{reco}}$ |
| Case 1 | 401.309    | 401.309 | 202.248  | 202.248  | 0.86        |
|        | 404.506    | 404.343 | 206.347  | 205.872  | 0.72        |
| Case 2 | 319.307    | 633.317 | 132.094  | 341.503  | 0.51        |

**Table 2:** Reconstructed masses and reconstruction efficiencies.

### 4 Conclusion

The newly proposed ‘timing detector’ at the HL-LHC would open an entirely new window to study the kinematics of the produced particles. In particular, for the long lived particle (LLP) decaying to a dark matter particle (i.e. invisible particle) and a visible particle (i.e. the standard model particle), as we have shown in this paper, the new timing information allows us to completely determine the masses of the LLP as well as the dark matter particle, which was not possible without the timing information. In this paper, we have developed
two novel reconstruction methods: (i) the first method is based on the precision displaced vertex measurement and applicable when the pair produced LLPs are identical and each of the LLP decays to the same dark matter particle. (ii) the second method entirely relies on the timing information of the long lived particle(s) and applicable completely general cases of two different LLPs decaying to different invisible particles.

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A Determination of 3 momenta with displaced vertices

In this section, we will show that we can determine the 3-momenta of LLPs (but not the energies or equivalently masses of LLPs) even without the timing information under the following conditions:

- **Assumption-1**: We measure the displaced vertex of the LLP,
- **Assumption-2**: MET is only from \( I_a \) and \( I_b \),
- **Assumption-3**: We fully reconstruct the 3-momentum of \( V \) (with the known mass, \( m_V \)).

**Proof:**

We notice that the energy of LLP\(_a\) in lab frame is found to be related with 3-velocities, \( \beta_a \) and \( \beta_b \) as Eq. 2.11. Let us derive this relation first, Eq. 2.10 We can find the energy by cross producting \( \beta_{i,T} \) and dot producting to the beam axis \( \hat{k} \).

\[
E_a = \left( \frac{\beta_{b,T} \times (p_{I,T} + p_{V_a,T} + p_{V_b,T}) \cdot \hat{k}}{\beta_{b,T} \times \beta_{a,T} \cdot \hat{k}} \right) \beta_{b,T} \times \beta_{a,T} \cdot \hat{k}
\]

\[
E_b = \left( \frac{\beta_b \times (p_{I,T}^{miss} + p_{V_a} + p_{V_b}) \cdot \hat{k}}{\beta_b \times \beta_a \cdot \hat{k}} \right) \beta_b \times \beta_a \cdot \hat{k}
\]

In the second line we have used the **Assumption-2** and the vector identity, \( \hat{k} \times \vec{V} \cdot \hat{k} = 0 \) for an arbitrary vector \( \vec{V} \) after decomposing the vectors into longitudinal (\( \propto \hat{k} \)) and perpendicular components.
As the momentum is related with the energy by the relation $p_a = E_a \beta_a$, the 3-momenta of $LLP_a$ and similarly to the $LLP_b$ are obtained as

$$p_a = \left[ \beta_b \times \left( p_T^{\text{miss}} + p_V a + p_V b \right) \cdot \hat{k} \right] \beta_a \tag{A.3}$$

$$p_b = \left[ \beta_a \times \left( p_T^{\text{miss}} + p_V a + p_V b \right) \cdot \hat{k} \right] \beta_b \tag{A.4}$$

The above relations are independent of the magnitude of the velocity vectors $\beta_a$ and $\beta_b$ but dependent only on the direction vectors of them,

$$\hat{r}_a = \beta_a/|\beta_a|, \quad \hat{r}_b = \beta_b/|\beta_b|, \tag{A.5}$$

thus

$$p_a = \left( \frac{r_b \times \left( p_T^{\text{miss}} + p_V a + p_V b \right) \cdot \hat{k}}{r_b \times r_a \cdot \hat{k}} \right) r_a, \tag{A.6}$$

$$p_b = \left( \frac{r_a \times \left( p_T^{\text{miss}} + p_V a + p_V b \right) \cdot \hat{k}}{r_a \times r_b \cdot \hat{k}} \right) r_b. \tag{A.7}$$

This completes the proof.

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