\textbf{CP Violating B Decays with $R$-parity Violation}

Ji-Ho Jang $^*$ and Jae Sik Lee $^\dagger$

\textit{Department of Physics, Korea Advanced Institute of Science and Technology}
\textit{Taejon 305-701, Korea}

\textbf{Abstract}

We study $CP$ violating $B$ decays in the minimal supersymmetric standard model with $R$-parity violation. We estimate how much $R$-parity violation modifies the SM predictions for $CP$ asymmetries in $B$ decays within the present bounds. The effects of $R$-parity- and lepton-number-violating couplings on the ratio of the decay amplitude due to $R$-parity violation to that of the SM can differ by one or two orders of magnitudes depending on the models of the left-handed quark mixing. It is possible to disentangle the $R$-parity violating effects from those of the SM and $R$-parity-conserving supersymmetric models within the present bounds comparing different $CP$ violating decay amplitudes. We also study the effects of $R$-parity- and baryon-number-violating couplings and find that the effects could be large.

PACS Number: 11.30.Fs, 13.25.Hw

---

$^*$E-mail: jhjang@chep6.kaist.ac.kr

$^\dagger$E-mail: jslee@chep6.kaist.ac.kr

Typeset using REVTeX
I. INTRODUCTION

In the upcoming experiments at B factories, the large data samples will be acquired \[1\]. One of the most important objects of these experiments is a search for \(CP\) violation in \(B\) decays. The large data on \(B\) meson will enable us to probe the physics beyond the standard model (SM) via \(CP\) violating \(B\) decays. In a supersymmetric extension of the SM, there are many potential sources for \(CP\) violation in addition to the SM CKM phase. So, the SM predictions on \(CP\) asymmetries in \(B\) decays can be modified. Nondiagonality of the sfermion mass matrices in a basis where all the couplings of neutral gauginos to fermions and sfermions are flavor diagonal can change the SM predictions on \(CP\) violation \[2\]. The SM predictions can also be modified by the so-called \(R\)-parity-violating terms.

In supersymmetric models, there are gauge invariant interactions which violate the baryon number \(B\) and the lepton number \(L\) generically. To prevent presence of these \(B\) and \(L\) violating interactions in supersymmetric models, an additional global symmetry is required. This requirement leads to the consideration of the so-called \(R\)-parity. The \(R\)-parity is given by the relation \(R_p = (-1)^{(3B+L+2S)}\) where \(S\) is the intrinsic spin of a field. Even though the requirement of \(R_p\) conservation gives a theory consistent with present experimental searches, there is no good theoretical justification for this requirement. Therefore models with explicit \(R_p\) violation (\(R_p\)) have been considered by many authors \[3\].

In this paper, we wish to study \(CP\) violating \(B\) decays in the minimal supersymmetric standard model (MSSM) with \(R_p\). We investigate how much \(R_p\) modifies the SM predictions for \(CP\) asymmetries in \(B\) decays within the present bounds. We emphasize that the effects of \(R_p\) and \(L\) violation on the ratio of the decay amplitude due to \(R_p\) to that of the SM can differ by one or two orders of magnitudes depending on the models of the left-handed quark mixing. We also study the effects of \(R_p\) and \(B\) violation.

In the MSSM the most general \(R_p\) violating superpotential is given by

\[
W_{R_p} = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k. \tag{1}
\]

Here \(i, j, k\) are generation indices and we assume that possible bilinear terms \(\mu_i L_i H_2\) can be rotated away. \(L_i\) and \(Q_i\) are the \(SU(2)\)-doublet lepton and the quark superfields and \(E^c_i, U^c_i, D^c_i\) are the singlet superfields respectively. \(\lambda_{ijk}\) and \(\lambda'_{ijk}\) are antisymmetric under the interchange of the first two and the last two generation indices respectively; \(\lambda_{ijk} = -\lambda_{jik}\) and \(\lambda''_{ijk} = -\lambda''_{jik}\). So the number of couplings is 45 (9 of the \(\lambda\) type, 27 of the \(\lambda'\) type and 9 of the \(\lambda''\) type). Among these 45 couplings, 36 couplings are related with the lepton flavor violation.

There are upper bounds on a single \(L\)- and \(R_p\)-violating couplings from several different sources \[4\]. Among these, upper bounds from atomic parity violation and \(eD\) asymmetry \[5\], \(\nu\mu\) deep-inelastic scattering \[6\], neutrinoless double beta decay \[7\], \(\nu\) mass \[8\], \(K^+\), \(t\)-quark decays \[9\], and \(Z\) decay width \[10\] are strong. Neutrinoless double beta decay gives \(\lambda'_{111} < 3.5 \times 10^{-4}\). The bounds from \(\nu\) mass are \(\lambda_{133} < 3 \times 10^{-3}\) and \(\lambda'_{133} < 7 \times 10^{-4}\).

There are strong bounds on \(\lambda''_{ijk} < 0.012\) for \(j = 1\) and 2 from \(K^+\)-meson decays. But, these single bounds depend on the models of the left-handed quark mixing. The CKM matrix consists of the product of the mixing matrices of the left-handed up- and down-type quarks and we don’t know the mixings of the up- and down-type quarks separately. Therefore, in
this case, we need some assumptions about the mixings of the left-handed quarks to derive a single bound on $\lambda'$ coupling from the physical process. The bounds of $\lambda'_{(1,2)k} < 0.012$ are valid only when the mixing of the down-type quarks dominates the CKM matrix. On the contrary, if the mixing of the up-type quarks dominates the CKM matrix, the bounds on $\lambda'_{(1,2)k}$ are totally invalid. In general case where the CKM matrix has contributions from the up-quark sector as well as down-quark sector, the bounds from $K^+$-meson decays become invalid and the typical bounds on $\lambda'_{ijk}$ with $j = 2, 3$ and $\lambda'_{123,132}$ are $\mathcal{O}(0.1)$. We consider the general case as well as the case in which the single bounds from $K^+$-meson decays are valid. We find that the effects of $R_p$ violation can differ by one or two orders of magnitudes depending on the models of the left-handed quark mixing.

The upper bounds on $B$- and $R_p$-violating couplings are $\mathcal{O}(1)$ except $\lambda''_{112} < 10^{-6}$ and $\lambda''_{113} < 10^{-4}$ from the double nucleon decay and $n - \bar{n}$ oscillation respectively.

In this paper we assume that all masses of scalar partners which mediate the processes are 100 GeV. Extensive reviews of the limits on a single $R_p$ violating couplings can be found in [10].

There are more stringent bounds on some products of the $R_p$ violating couplings from the mixings of the neutral $K$- and $B$- mesons and the rare leptonic decays of the $K_L$-meson, the muon and the tau [8], $b\bar{b}$ productions at LEP [11], the rare leptonic and semileptonic $B^0$ decays [12-14], muon(ium) conversion, and $\tau$ and $\pi^0$ decays [15].

The $CP$ violating decays of $B$-meson can be induced by the baryon number violating couplings as well as by the lepton number violating ones. But, the baryon number and the lepton number violating couplings can not coexist in order to avoid too fast proton decays. So we will consider the baryon number violating case and the lepton number violating one separately.

About the baryon number violating coupling, there is a very strong upper bound on $\lambda''_{112} < 10^{-15}$ from the proton decay in gauge-mediated supersymmetry breaking models independently of the lepton number violating couplings [16]. Recently, the study of one-loop structure of the proton decay into very light gravitino or axino shows that all the baryon number violating couplings are constrained as $\lambda''_{any} < 10^{-6}$ even though these bounds depend on the precise value of the gravitino mass or the scale of spontaneous $U(1)_{PQ}$ breaking [17].

This paper is organized as follows. In section II, we introduce the general formalism for the $CP$ asymmetry in the case where the decay amplitude contains contributions from two terms. In section III, we consider the effects of $R_p$- and lepton-number-violating couplings on the $CP$ asymmetries of neutral $B$-meson. And the effects of $R_p$- and baryon-number-violating couplings on the $CP$ asymmetries are considered in section IV. We conclude in section V.

II. GENERAL FORMALISM

The time dependent $CP$ asymmetry is defined as

\[ \text{The single bounds on } \lambda'_{132}, \lambda'_{232}, \text{ and } \lambda'_{233} \text{ should be replaced with } 0.16 \text{ which are stronger bounds coming from Ref. [13].} \]
mixing phase as follows

\[ a_{f_{CP}}(t) \equiv \frac{\Gamma[B^0(t) \to f_{CP}] - \Gamma[\bar{B}^0(t) \to f_{CP}]}{\Gamma[B^0(t) \to f_{CP}] + \Gamma[\bar{B}^0(t) \to f_{CP}]], \]

where \( f_{CP} \) denotes the \( CP \) eigenstates into which the neutral \( B \) meson decay, and \( B^0(t) \) and \( \bar{B}^0(t) \) are the states that were tagged as pure \( B_d \) and \( \bar{B}_d \) at the production. This \( CP \) asymmetry can be rewritten by

\[ a_{f_{CP}}(t) = a_{f_{CP}}^\cos \cos(\Delta M t) + a_{f_{CP}}^\sin \sin(\Delta M t), \]

where \( \Delta M \) is the mass difference between the two physical states, and

\[ a_{f_{CP}}^\cos = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}, \quad a_{f_{CP}}^\sin = -\frac{2 \text{Im}\lambda}{1 + |\lambda|^2}. \]

Here \( \lambda \) is given by

\[ \lambda = \left[ \frac{\langle B^0|H_{\text{eff}}|B^0 \rangle > \langle f_{CP}|H_{\text{eff}}|B^0 \rangle >}{\langle B^0|H_{\text{eff}}|B^0 \rangle < \langle f_{CP}|H_{\text{eff}}|B^0 \rangle >} \right] = e^{-2i\phi_M} \frac{\bar{A}}{A}, \]

\[ < B^0|H_{\text{eff}}|B^0 > \equiv M_{12} - \frac{i}{2} \Gamma_{12} = \left| M_{12} - \frac{i}{2} \Gamma_{12} \right| e^{2i\phi_M}; \]

using \( M_{12} \gg \Gamma_{12}. \)

New Physics (NP) modifies the SM predictions on both \( \phi_M \) and \( A. \) NP affects \( B - \bar{B} \) mixing phase as follows

\[ \phi_M = \phi_M^{\text{SM}} + \delta \phi_M, \]

\[ \delta \phi_M = \frac{1}{2} \arctan \left( \frac{r_M \sin 2(\phi_M^{\text{NP}} - \phi_M^{\text{SM}})}{1 + r_M \cos 2(\phi_M^{\text{NP}} - \phi_M^{\text{SM}})} \right), \]

where \( \phi_M^{\text{NP}} \) and \( \phi_M^{\text{SM}} \) are defined by

\[ < B^0|H_{\text{eff}}^\text{full}|B^0 > = \left| M_{12}^{\text{SM}} \right| e^{2i\phi_M^{\text{SM}}} \left( 1 + r_M e^{2i(\phi_M^{\text{NP}} - \phi_M^{\text{SM}})} \right), \]

where \( r_M \equiv \frac{|M_{12}^{\text{NP}}|}{|M_{12}^{\text{SM}}|} \) and \( M_{12}^{\text{NP}} \gg \Gamma_{12}^{\text{NP}} \) is assumed. For \( r_{\text{NP}} \ll 1, \delta \phi_M \leq r_{\text{NP}}/2. \) However for \( r_{\text{NP}} \geq 1, \delta \phi_M \) can take any value. In the SM, the mixing phase \( \phi_M^{\text{SM}} \) is \( \beta \) and 0 for \( B_d - \bar{B}_d \) and \( B_s - \bar{B}_s \), respectively.

If NP contributions to \( A \) are dominated by one term and the size of the contribution is larger than that of the sub-leading SM corrections, \( A \) can be written as follows

\[ A = A_{\text{SM}} e^{i\delta_1} e^{i\delta_2} + A_{\text{NP}} e^{i\phi_2} e^{i\delta_2}, \quad \bar{A} = A_{\text{SM}} e^{-i\delta_1} e^{i\delta_2} + A_{\text{NP}} e^{-i\phi_2} e^{i\delta_2}, \]

where \( A_{\text{SM},\text{NP}} \) are real magnitudes, \( \phi_{1,2} \) are \( CP \) violating phases and \( \delta_{1,2} \) are \( CP \) conserving phases. For the sizes of the sub-leading SM corrections and the contributions of the \( R_{\beta} \)-conserving supersymmetric model, see Ref. [15].

With \( \phi_{12} = \phi_1 - \phi_2, \delta_{12} = \delta_1 - \delta_2, \) and \( r_D \equiv A_{\text{NP}}/A_{\text{SM}}, \)

\[ A = A_{\text{SM}} e^{i\phi_1} e^{i\delta_1} + A_{\text{NP}} e^{i\phi_2} e^{i\delta_2}, \]

where \( A_{\text{SM},\text{NP}} \) are real magnitudes, \( \phi_{1,2} \) are \( CP \) violating phases and \( \delta_{1,2} \) are \( CP \) conserving phases. For the sizes of the sub-leading SM corrections and the contributions of the \( R_{\beta} \)-conserving supersymmetric model, see Ref. [15].
\[ a_{fCP}^{\cos} = -\frac{2r_D \sin \phi_2 \sin \delta_2}{1 + 2r_D \sin \phi_2 \sin \delta_2} \approx -2r_D \sin \phi_2 \sin \delta_2, \]
\[ a_{fCP}^{\sin} = \frac{\sin(2\phi_M + \phi_1) - 2r_D \sin \phi_2 \cos(2\phi_M + 2\phi_1 + \delta_2)}{1 + 2r_D \sin \phi_2 \sin \delta_2} \approx \sin 2(\phi_M + \phi_1) - 2r_D \sin \phi_2 \cos 2(\phi_M + \phi_1) \cos \delta_2, \]

(9)
to the first order in \( r_D \).

For the rest of this paper, we concentrate on \( a_{fCP}^{\sin} \). To this end we write
\[ a_{fCP}^{\sin} \equiv \sin 2(\phi_M + \phi_1 + \delta\phi_D) \equiv \sin 2\phi, \]
(10)
For \( r_D \ll 1, \delta\phi_D \leq r_D \). However for \( r_D \geq 1, \delta\phi_D \) can take any value. In the following two sections, we will calculate \( r_D \) for several \( CP \) violating decay modes.

Note that NP contribution to the mixing phase \( \phi_M \) is universal for all kinds of decay modes. So, one can identify NP contributions to \( CP \) violating \( B \) decays independently of the NP contribution to the mixing by considering two different decay modes simultaneously.

### III. \( R_P \) AND \( L \) VIOLATION

In this section, we consider the effects of \( R_p \) and the lepton number violating couplings (\( \lambda' \)) assuming the baryon number violating couplings \( \lambda'' \)'s vanish.

Firstly we assume \( V_{CKM} \) is given by only down-type quark sector mixing. In this case, \( r_M \) and \( r_D(B_d \to \psi K_S, \phi K_S) \) are estimated in Ref. \[19\] as follows,

\[ r_M(B_d) \approx 10^8 |\lambda'_{13}\lambda'_{n31}| \left( \frac{100 \text{ GeV}}{M_{\nu}} \right)^2, \]
\[ r_D(B_d \to \psi K_S) < 0.02, \]
\[ r_D(B_d \to \phi K_S) < 0.8, \]
(11)
and \( |\phi(B_d \to \psi K_S) - \phi(B_d \to \phi K_S)| < \mathcal{O}(1) \). \( r_M(B_s) \) is given by replacing \( |\lambda'_{13}\lambda'_{n31}| \) with \( |\lambda'_{n23}\lambda'_{n32}| \) in \( r_M(B_d) \). In this section, we wish to investigate other decay modes and discuss how much the effects of \( R_p \) differ depending on the models of the left-handed quark mixings.

From Eq. (1), we obtain the following four-fermion effective Lagrangian due to the exchange of the sleptons

\[ \mathcal{L}_{\text{eff,2u-2d}}^{R_P} = \frac{4G_F}{\sqrt{2}} C_{ijkl}^L (\bar{d}_i P_L u_j)(\bar{u}_k P_R d_l), \]
\[ \mathcal{L}_{\text{eff,4d}}^{R_P} = \frac{4G_F}{\sqrt{2}} N_{ijkl}^R (\bar{d}_i P_L d_j)(\bar{d}_k P_R d_l), \]
(12)
where \( P_{L,R} = (1 \mp \gamma_5)/2 \) and the dimensionless couplings \( C_{ijkl}^L \) and \( N_{ijkl}^R \) are given by

\[ C_{ijkl}^L = \sqrt{2} \frac{3}{4G_F} \sum_{n,p,q=1} \frac{1}{M_{l_n}^2} V_{kq} V_{jp}^* \lambda'_{npi} \lambda'_{nqj}, \]
\[ N_{ijkl}^R = \sqrt{2} \frac{3}{4G_F} \sum_{n=1} \frac{1}{M_{l_n}^2} \lambda'_{i} \lambda'_{nkl}, \]
(13)
From the above effective Lagrangian, we calculate the amplitudes $A$ for the several decay modes under the factorization assumption and the results are shown in the Appendix.

In Table I, we show the R-parity- and lepton-number-violating product combinations which significantly contribute to each process assuming $V_{\text{CKM}}$ is given by only down-type quark sector mixing. For the decay mode $B_d \to \psi K_S$, there are four kinds of competitive contributions and the most significant one comes from $\lambda_{332}' \lambda_{333}'$ within present bounds. Typically, the constraints are order of $10^{-4}$ or $10^{-3}$. The decay modes with $10^{-3}$ constraint are $B_d \to \phi K_S$, $B_d \to \pi^0 K_S$, $B_s \to \phi K_S$, $B_d \to \phi \pi^0$, and $B_d \to \pi^0 \pi^0$. So these five decay modes are important ones in the presence of $R_P$ violation. See Table I for the estimated values of $r_D$.

The supersymmetric contributions to the decay modes $B_d \to \phi K_S$ and $B_d \to \pi^0 K_S$ are not dominated by only $R_p$ since there are comparable contributions from nondiagonal sfermion mass matrices to these decay modes, see the second paper of Ref. [18]. And the upcoming $B$ experiments will initially take data at $\Upsilon(4s)$ where only the $B_d$ can be studied and the mode $B_d \to \pi^0 \pi^0$ suffers from the large SM uncertainties. For the decay mode $B_d \to \phi \pi^0$, the SM prediction for the branching ratio of this decay mode is quite small: $\mathcal{B}_{\text{SM}}(B_d \to \phi \pi^0) = 1.9 \times 10^{-8}$ [20]. Consequently, it would be hard to measure $CP$ violation considering only one decay mode unless $R_p$ enhance the branching ratio of this mode significantly. But, the $R_{p\gamma}$- and $L$-violating effects can be disentangled from those of the SM or $R_p$-conserving supersymmetric models if we compare two or more decay modes. For example, let’s think about the decay modes of $B_d \to \psi K_S$ and $B_d \to \phi K_S$. The difference between $CP$ violating phases of these two decay modes vanishes in the SM or $R_p$-conserving supersymmetric models. But, it does not vanish in the $R_p$-violating model.

Now, let’s think the general case in which the down-type quark mixing does not dominates $V_{\text{CKM}}$. In this case, the strong bounds $|\lambda_{ijk}'| < 0.012$ with $j = 1, 2$ from $K^+$-meson decays becomes invalid. In this case, the typical bounds on $\lambda_{ijk}'$ with $i = 2, 3$ are $\mathcal{O}(0.1)$. This means that the constraints given in Table I can become weaker by one or two orders of magnitudes. For example, let us consider the contribution of $\lambda_{222}' \lambda_{223}'$ to the $CP$ asymmetry in the mode $B_d \to \psi K_S$. Neglecting the constraint from $K^+$ decays, the constraint on this combination is $3.2 \times 10^{-2}$ from $D$-decay [10]. Using this constraint, one can obtain $r_D(B_d \to \psi K_S) = 7.5$. Similarly, we find that the typical size of $r_D$ of all decay modes is $\mathcal{O}(1)$ if we neglect the constraint from $K^+$ decays. It means that it is possible to disentangle the $R$-parity violating effects from those of the SM and $R$-parity-violating supersymmetric models. In this case, one can also identify the NP effects independently of the NP contributions to the mixing by taking account of the differences between the angles $\phi$’s of the first five modes in Table I.

IV. $R_P$ AND $B$ VIOLATION

In this section, we consider the effects of $R_p$ and the baryon number violating couplings ($\lambda''$) assuming the lepton number violating couplings $\lambda$’s vanish.

---

2 In Ref. [10], only the contributions from $\lambda_{n22}' \lambda_{n23}'$ are considered.
From Eq. (1), we obtain the following four-fermion effective Lagrangian due to the exchange of the squarks

\[ L_{\text{eff}}^{2u-2d} = \frac{4G_F}{\sqrt{2}} C^B_{ijkl} \left[ (\bar{u}_i \gamma^\mu P_R u_j) (\bar{d}_k \gamma^\mu P_R d_l) - (\bar{d}_k \gamma^\mu P_R d_j) (\bar{u}_i \gamma^\mu P_R d_l) \right], \]

\[ L_{R_p}^{\text{eff,4d}} = \frac{4G_F}{\sqrt{2}} N^B_{ijkl} (\bar{d}_i \gamma^\mu P_R d_j) (\bar{d}_k \gamma^\mu P_R d_l), \]

where \( P_{L,R} = (1 \mp \gamma_5)/2 \) and the dimensionless couplings \( C^B_{ijkl} \) and \( N^B_{ijkl} \) are given by

\[ C^B_{ijkl} = \frac{\sqrt{2}}{4G_F} \sum_{n=1}^{3} \frac{2}{M_{\tilde{d}_n}^2} \lambda'' k_{kn} \lambda''_{jn}, \]

\[ N^B_{ijkl} = \frac{\sqrt{2}}{4G_F} \sum_{n=1}^{3} \frac{1}{M_{\tilde{u}_n}^2} \lambda'' n_{ik} \lambda''_{jl}. \]

From the above effective Lagrangian, we calculate the amplitudes for several decay modes using the factorization assumption and the results are shown in the Appendix.

By the inspection of \( N^B_{ijkl} \), one can easily see that \( R_p \)- and \( B \)-violating couplings does not contribute \( B \rightarrow \bar{B} \) mixing and \( B_d \rightarrow \phi K_S \) since \( \lambda''_{ijk} \) is antisymmetric under the exchange of the last two indices.

The present bounds on \( \lambda'' \) are so poor that \( r_D \)'s are generally quite large except \( B_d \rightarrow \pi \pi \) mode : see Table III. Large \( r_D \) means two things. One thing is that it is possible to have large \( CP \) violation completely different from the SM predictions. The other thing is that one can obtain more stringent bounds on the product combinations if the measured branching ratios of the decay modes are consistent with the SM predictions \[23\].

Note that one product combination contributes to two and more decay modes, see Table III. In this case, the differences of \( CP \) phases \( \phi \)'s of the decay modes are exactly the same as that of the SM.

In gauge-mediated supersymmetry breaking models, \( \lambda'' \) are severely constrained from the proton decay \[16,17\]. So, the contributions of \( R_p \)- and \( B \)-violating couplings to \( CP \) violating \( B \) decays can be safely ignored.

V. CONCLUSION

To conclude, we study \( CP \) violating \( B \) decays in the minimal supersymmetric standard model with \( R_p \). We estimate how much \( R_p \) modifies the SM predictions for \( CP \) asymmetries in \( B \) decays within the present bounds. The effects of \( R_p \) and \( L \) violation on the ratio of the decay amplitude due to \( R_p \) to that of the SM can differ by one or two orders of magnitudes depending on the models of the left-handed quark mixing. It is possible to disentangle the \( R \)-parity violating effects from those of the SM and \( R \)-parity-conserving supersymmetric models within the present bounds. We also study the effects of \( R_p \) and \( B \) violation and find that the effects could be large or the contributing product combinations can be strongly constrained by the near future experiments on \( B \) mesons. The effects of \( R_p \) and \( B \) violation can be ignored in gauge-mediated supersymmetric models.
ACKNOWLEDGEMENTS

We thank Y. G. Kim and P. Ko for their helpful remarks. This work was supported in part by KAIST Basic Science Research Program (J.S.L.).

APPENDIX

In this appendix, we present all decay amplitudes relevant to our analysis. We don’t need to know the exact values of the form factor since they are irrelevant in the calculation of $r_D$ for the most of cases. For the numerical calculation, we use the following values for the quark masses: $m_u = 4.2$ MeV, $m_d = 7.6$ MeV, $m_s = 122$ MeV, $m_c = 1.3$ GeV, $m_b = 4.88$ GeV and we take $N=3$.

A. SM

The amplitudes in SM are calculated using the effective Hamiltonian formalism. The short and long distance QCD effects in the nonleptonic decays are separated by means of the operator product expansion. For the numerical values of the Wilson coefficients (short distance effects), we use the values in Ref. [20]. The long distance contributions of the hadronic matrix elements are calculated under the factorization approximation.

\[ A(\bar{B}^0 \rightarrow \psi K_S) = \frac{G_F}{\sqrt{2}} [V_{cb} V_{cd}^* a_2 - V_{tb} V_{ts}^* (a_3 + a_5 + a_7 + a_9)] \times <K_S|\bar{s}b_\perp|\bar{B}^0> <\psi|\bar{c}c_\perp|0> \]  

\[ A(\bar{B}^0 \rightarrow \phi K_S) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* [a_2 + a_4 + a_5 + \frac{1}{2} (a_7 + a_9 + a_{10})] \times <K_S|\bar{s}b_\perp|\bar{B}^0> <\phi|\bar{s}s_\perp|0> \]  

\[ A(\bar{B}^0 \rightarrow \pi^0 K_S) = \frac{G_F}{\sqrt{2}} \left\{ \left[ V_{ub} V_{us}^* a_2 + \frac{3}{2} V_{tb} V_{ts}^* (a_7 - a_9) \right] <\pi^0|\bar{u}u_\perp|0> <K_S|\bar{s}b_\perp|\bar{B}^0> - V_{tb} V_{ts}^* \left\{ a_4 - \frac{1}{2} a_{10} + \frac{m_K^2 (2a_6 - a_8)}{(m_d + m_s)(m_b - m_d)} \right\} <K_S|\bar{s}d_\perp|0> <\pi^0|\bar{d}b_\perp|\bar{B}^0> \right\} \]  

\[ A(\bar{B}^0 \rightarrow D^+ D^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{cd}^* |a_2 - V_{tb} V_{td}^* \left\{ a_4 + a_{10} + \frac{2m_D^2 (a_6 + a_8)}{(m_c + m_d)(m_b - m_c)} \right\} \right\} \times <D^+|\bar{c}b_\perp|\bar{B}^0> <D^-|\bar{d}c_\perp|0> \]  

\[ A(\bar{B}^0 \rightarrow D_{CP} \pi^0) = \frac{G_F}{\sqrt{2}} \left( V_{cb} V_{cd}^* a_2 - \pi^0|\bar{d}b_\perp|\bar{B}^0> <D_{CP}|\bar{c}u_\perp|0> \right) \]  

\[ A(\bar{B}^0 \rightarrow D_{CP} \rho^0) = \frac{G_F}{\sqrt{2}} \left( V_{cb} V_{cd}^* a_2 - \rho^0|\bar{d}b_\perp|\bar{B}^0> <D_{CP}|\bar{c}u_\perp|0> \right) \]  

\[ A(\bar{B}_s \rightarrow \phi K_S) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) + \frac{m_s^2 (2a_6 - a_8)}{2m_s(m_b - m_s)} \right\} \times <K_S|\bar{d}b_\perp|\bar{B}_s> <\phi|\bar{s}s_\perp|0> \]
\[ A(\bar{B}^0 \to \phi \pi^0) = -\frac{G_F}{\sqrt{2}} V_{td}^* \left\{ a_3 + a_5 - \frac{1}{2}(a_7 + a_9) \right\} < \pi^0|\bar{d}b_-|\bar{B}^0 > < \phi|\bar{s}s_-|0 > \]  

\[ A(\bar{B}^0 \to \pi^+ \pi^-) = \frac{G_F}{\sqrt{2}} \left[ V_{td}^* a_1 - V_{td}^* \left\{ a_4 + a_{10} + \frac{2m^2(a_6 + a_8)}{(m_u + m_d)(m_b - m_u)} \right\} \right] \times < \pi^+|\bar{u}b_-|B^0 > < \pi^-|\bar{d}u_-|0 > \]  

\[ A(\bar{B}^0 \to \pi^0 \pi^0) = -\frac{2G_F}{\sqrt{2}} \left[ V_{td}^* a_2 + V_{td}^* \left\{ a_4 + \frac{3}{2}(a_7 - a_9) - \frac{1}{2}a_{10} + \frac{m^2(2a_6 - a_8)}{2m_d(m_b - m_u)} \right\} \right] \times < \pi^0|\bar{d}b_-|\bar{B}^0 > < \pi^0|\bar{u}u_-|0 > \]

The ± sign in \( \bar{B}^0 \to D_{CP} \pi^0(\rho^0) \) decay modes corresponds to the CP-even and CP-odd eigenstates of \( D_{CP} \) and the same convention is applied to the \( R_p \) violation case. In the numerical estimation of \( \bar{B}^0 \to \pi^0 K_S \) decay modes, we assume that \( |< \pi^0|\bar{u}u_-|0 > < K_S|\bar{s}b_-|\bar{B}^0 > | \approx |< K_S|\bar{s}d_-|0 > < \pi^0|\bar{d}b_-|\bar{B}^0 > | \).

**B. \( R_p \) and \( L \) violation**

In this case, the running effects of the \( R_p \) violating couplings are neglected. The hadronic matrix elements are also calculated under the factorization assumption.

\[ A(\bar{B}^0 \to \psi K_S) = \sum_{n,i,j} \frac{1}{M^2_{\text{in}}} \frac{1}{8N} \chi'_{n2} \chi'_{n3} V_{2j} V^*_{2i} < K_S|\bar{s}b_-|\bar{B}^0 > < \psi|\bar{c}c_-|0 > \]  

\[ A(\bar{B}^0 \to \phi K_S) = \sum_{n} \frac{1}{M^2_{\text{in}}} \frac{1}{8N} [\chi'_{n22} \chi'_{n23} + \chi'_{n32} \chi'_{n22}] < K_S|\bar{s}b_-|\bar{B}^0 > < \phi|\bar{s}s_-|0 > \]  

\[ A(\bar{B}^0 \to \pi^0 K_S) = \sum_{n} \frac{1}{M^2_{\text{in}}} \left\{ \left( \frac{1}{8N} \sum_{i,j} \chi'_{n2} \chi'_{n3} V_{1j} V^*_{1i} - \chi'_{n12} \chi'_{n13} + \chi'_{n31} \chi'_{n21} \right) \right. \]  

\[ + \frac{m^2_s}{8m_d(m_b - m_s)} (\chi'_{n12} \chi'_{n32} - \chi'_{n32} \chi'_{n11}) \} < \pi^0|\bar{u}u_-|0 > < K_S|\bar{s}b_-|\bar{B}^0 > \]  

\[ \left. + \left\{ \frac{1}{8N} (\chi'_{n11} \chi'_{n23} - \chi'_{n32} \chi'_{n11}) - \frac{m^2_{K_S}}{4(m_b - m_d)(m_d + m_s)} (\chi'_{n12} \chi'_{n13} - \chi'_{n31} \chi'_{n21}) \right\} \right] \times < K_S|\bar{s}d_-|0 > < \pi^0|\bar{d}b_-|\bar{B}^0 > \]  

\[ A(\bar{B}^0 \to D^+ D^-) = \sum_{n,i,j} \frac{1}{M^2_{\text{in}}} \frac{m^2_{D^-}}{4(m_d + m_c)(m_b - m_c)} \chi_{n1} \chi_{n3} V_{2j} V^*_{2i} \right\} \times < \bar{D}^+|\bar{b}_-|\bar{B}^0 > < \bar{D}^-|\bar{d}c_-|0 > \]  

\[ A(\bar{B}^0 \to D_{CP} \pi^0) = \sum_{n,i,j} \frac{1}{M^2_{\text{in}}} \chi_{n1} \chi_{n3} \frac{1}{8N} [V_{2j} V^*_{1i} \pm V_{1j} V^*_{2i}] < \pi^0|\bar{d}b_-|\bar{B}^0 > < D_{CP}|\bar{c}u_-|0 > \]  

\[ A(\bar{B}^0 \to D_{CP} \rho^0) = -\sum_{n,i,j} \frac{1}{M^2_{\text{in}}} \chi_{n1} \chi_{n3} \frac{1}{8N} [V_{2j} V^*_{1i} \pm V_{1j} V^*_{2i}] \]  

\[ \times < \rho^0|\bar{d}b_-|\bar{B}^0 > < D_{CP}|\bar{c}u_-|0 > \]
\[ A(\bar{B}_s \to \phi K_S) = -\sum_n \frac{1}{M^2_{i_n}} \left\{ \left[ \frac{1}{8N} (\lambda'_{n12} \lambda''_{n23} + \lambda'_{n32} \lambda''_{n21} + \lambda'_{n22} \lambda''_{n13} + \lambda'_{n31} \lambda''_{n22}) \right. \right. \\
\left. \left. \quad + \frac{m^2_{K_S}}{4(m_s + m_d)(m_s + m_b)} (\lambda'_{n12} \lambda''_{n23} + \lambda'_{n32} \lambda''_{n21} + \lambda'_{n22} \lambda''_{n13} + \lambda'_{n31} \lambda''_{n22}) \right) \times <\phi|\bar{s}b_-|\bar{B}_s> <\phi|\bar{s}s_-|0> \right. \\
\left. \quad - \frac{1}{8N} (\lambda'_{n12} \lambda''_{n23} + \lambda'_{n32} \lambda''_{n21}) <K_S|\bar{d}b_-|\bar{B}_s> <\phi|\bar{s}s_-|0> \right] \] 

\[ (32) \]

\[ A(\bar{B}^0 \to \phi \pi^0) = \sum_n \frac{1}{M^2_{i_n}} \frac{1}{8N} (\lambda'_{n21} \lambda''_{n23} + \lambda'_{n32} \lambda''_{n12}) <\pi^0|\bar{d}b_-|\bar{B}^0> <\phi|\bar{s}s_-|0> \] 

\[ (33) \]

\[ A(\bar{B}^0 \to \pi^0 \pi^-) = -\sum_{n,i,j} \frac{1}{M^2_{i_n}} \left[ \sum_{i,j} \frac{1}{4N} \lambda'_{ni1} \lambda''_{nj3} V_{ij} V^*_{1i} \right. \\
\left. \quad \times <\pi^+|\bar{u}b_-|\bar{B}^0> <\pi^-|\bar{d}u_-|0> \right. \\
\left. \quad - \left\{ \left[ \frac{1}{4N} - \frac{m^2_{\pi^0}}{4m_d}(m_b - m_d) \right] (\lambda'_{n11} \lambda''_{n13} - \lambda'_{n31} \lambda''_{n11}) \right\} <\pi^0|\bar{d}b_-|\bar{B}^0> <\pi^0|\bar{u}u_-|0> \right] \] 

\[ (34) \]

\[ A(\bar{B}^0 \to \pi^0 \pi^0) = \sum_n \frac{1}{M^2_{i_n}} \left[ \sum_{i,j} \frac{1}{4N} \lambda'_{ni1} \lambda''_{nj3} V_{ij} V^*_{1i} \right. \\
\left. \quad - \left\{ \left[ \frac{1}{4N} - \frac{m^2_{\pi^0}}{4m_d}(m_b - m_d) \right] (\lambda'_{n11} \lambda''_{n13} - \lambda'_{n31} \lambda''_{n11}) \right\} \times <\pi^0|\bar{d}b_-|\bar{B}^0> <\pi^0|\bar{u}u_-|0> \right] \] 

\[ (35) \]

In $B^0 \to \pi^0 K_S$ and $B^0 \to \phi K_S$ modes, we assume that the magnitudes of two form factors are approximately same.

**C. $R_p$ and $B$ violation**

The decay amplitudes for $R_p$ and $B$ violation are calculated in the similar way as the case of $R_p$ and $L$ violation.

\[ A(\bar{B}^0 \to \psi K_S) = -\sum_n \frac{1}{2M^2_{d_n}} (1 - \frac{1}{N}) \lambda''_{22n} \lambda''_{23n} <K_S|\bar{s}b_-|\bar{B}^0> <\psi|\bar{c}c_-|0> \] 

\[ (36) \]

\[ A(\bar{B}^0 \to \phi K_S) = 0 \] 

\[ (37) \]

\[ A(\bar{B}^0 \to \pi^0 K_S) = \sum_n \left[ \left\{ \frac{1}{2M^2_{d_n}} \lambda''_{21n} \lambda''_{13n} - \frac{1}{2M^2_{d_n}} \lambda''_{n12} \lambda''_{n13} \right\} (1 - \frac{1}{N}) \right. \right. \\
\left. \left. \quad \times <\pi^0|\bar{u}u_-|0> <K_S|\bar{s}b_-|\bar{B}^0> \\
\left. \quad - \frac{1}{2M^2_{d_n}} (1 - \frac{1}{N}) \lambda''_{n12} \lambda''_{n13} <K_S|\bar{s}d_-|0> <\pi^0|\bar{d}b_-|\bar{B}^0> \right] \] 

\[ (38) \]

\[ A(\bar{B}^0 \to D^+ D^-) = -\sum_n \frac{1}{2M^2_{d_n}} (1 - \frac{1}{N}) \lambda''_{21n} \lambda''_{23n} <D^+|\bar{c}b_-|\bar{B}^0> <D^-|\bar{d}c_-|0> \] 

\[ (39) \]

\[ A(\bar{B}^0 \to D_{CP} \pi^0) = \sum_n \frac{1}{2M^2_{d_n}} (1 - \frac{1}{N}) \left[ \lambda''_{21n} \lambda''_{13n} \pm \lambda''_{11n} \lambda''_{23n} \right] \times <\pi^0|\bar{d}b_-|\bar{B}^0> <D_{CP}|\bar{c}u_-|0> \] 

\[ (40) \]
\[ A(\bar{B}^0 \to D_{CP}\rho^0) = - \sum_n \frac{1}{2M^2_{d_n}} \left( 1 - \frac{1}{N} \right) [\lambda''_{21n}\lambda''_{13n} \pm \lambda''_{11n}\lambda''_{23n}] \]
\[ \times < \rho^0 | \bar{d}b_- | \bar{B}^0 > < D_{CP} | \bar{c}u_- | 0 > \]  
(41)

\[ A(\bar{B}_s \to \phi K_S) = - \sum_n \frac{1}{2M^2_{u_n}} \left( 1 - \frac{1}{N} \right) \lambda''_{n12}\lambda''_{n23} \]
\[ \times \left[ < \phi | \bar{s}b_- | \bar{B}_s > < K_S | \bar{d}s_- | 0 > - < K_S | \bar{d}b_- | \bar{B}_s > < \phi | \bar{s}s_- | 0 > \right] \]  
(42)

\[ A(\bar{B}^0 \to \phi \pi^0) = \sum_n \frac{1}{2M^2_{u_n}} \left( 1 - \frac{1}{N} \right) \lambda''_{n12}\lambda''_{n23} < \pi^0 | \bar{d}b_- | \bar{B}^0 > < \phi | \bar{s}s_- | 0 > \]  
(43)

\[ A(\bar{B}^0 \to \pi^+\pi^-) = - \sum_n \frac{1}{2M^2_{d_n}} \left( 1 - \frac{1}{N} \right) \lambda''_{11n}\lambda''_{13n} < \pi^+ | \bar{u}b_- | \bar{B}^0 > < \pi^- | \bar{d}u_- | 0 > \]  
(44)

\[ A(\bar{B}^0 \to \pi^0\pi^0) = \sum_n \frac{1}{M^2_{d_n}} \left( 1 - \frac{1}{N} \right) \lambda''_{11n}\lambda''_{13n} < \pi^0 | \bar{d}b_- | \bar{B}^0 > < \pi^0 | \bar{u}u_- | 0 > \]  
(45)

In $\bar{B}_s \to \phi K_S$ decay mode, we assume [21]
\[
\frac{< K_S | \bar{d}b_- | \bar{B}_s > < \phi | \bar{s}s_- | 0 > - < \phi | \bar{s}b_- | \bar{B}_s > < K_S | \bar{d}s_- | 0 >}{< K_S | \bar{d}b_- | \bar{B}_s > < \phi | \bar{s}s_- | 0 >} \approx O(1).
\]
REFERENCES

[1] T. Nakada, PSI-PR-96-22, [hep-ex/9609009], F. Grancagnolo, INFN-AE-90-07. CLEO Collaborations, CLNS-94-1277. J. N. Butler, FERMILAB-PUB-95-363, Nucl. Instrum. Meth. A368, 145 (1995), D. E. Kaplan, UW/PT 97-5, [hep-ph/9703347].

[2] N. Deshpande, B. Dutta, and S. Oh, Phys. Rev. Lett. 77, 4499 (1996); J. Silva and L. Wolfenstein, Phys. Rev. D53, 5331 (1997); A. Cohen, D. Kaplan, F. Leipentre and A. Nelson, Phys. Rev. Lett. 78, 2300 (1997); Y. Grossman and M. Worah, Phys. Lett. B395, 241 (1997); Y. Grossman, Y. Nir and R. Rattazzi, [hep-ph/9701231]; M. Ciuchini, E. Franco, G. Martinelli, A. Masiero and L. Silvestrini, Phys. Rev. Lett. 79, 978 (1997); Y. Grossman, Y. Nir and M. Worah, Phys. Lett. B407, 307 (1997); R. Barbieri and A. Strumia, Nucl. Phys. B508, 3 (1997).

[3] C.S. Aulakh, R.N. Mohapatra, Phys. Lett. B119, 136 (1982); F. Zwirner, Phys. Lett. B132, 103 (1983); I-Hsin Lee, Nucl. Phys. B246, 120 (1984); J. Ellis et al., Phys. Lett. B150, 142 (1985); G. G. Ross and J. W. F. Valle, Phys. Lett. B151, 375 (1985); S. Dawson, Nucl. Phys. B261, 297 (1985); R. Barbieri, A. Masiero, Nucl. Phys. B267, 679 (1986); S. Dimopoulos and L. Hall, Phys. Lett. B207, 210 (1987).

[4] V. Barger, G. F. Giudice and T. Han, Phys. Rev. D40, 2987 (1989).

[5] R. N. Mohapatra, Phys. Rev. D34, 3457 (1986); M. Hirsch, H. V. Klapdor–Kleingrothaus and S. G. Kovalenko, Phys. Rev. Lett. 75, 17 (1995).

[6] R. M. Godbole, P. Roy and X. Tata, Nucl. Phys. B401, 67 (1993).

[7] K. Agashe, M. Graesser, Phys. Rev. D54, 4445 (1996).

[8] D. Choudhury and P. Roy, Phys. Lett. B378, 153 (1996).

[9] G. Bhattacharyya, J. Ellis, and K. Sridhar, Mod. Phys. A10, 1583 (1995), G. Bhattacharyya, D. Choudhury and K. Sridhar, Phys. Lett. B355, 193 (1995).

[10] G. Bhattacharyya, [hep-ph/9709395] and references therein.

[11] J. Erler, J. L. Feng and N. Polonsky, Phys. Rev. Lett. 78, 3063 (1997).

[12] J. Jang, J. K. Kim and J. S. Lee, Phys. Rev. D55, 7296 (1997).

[13] J. Jang, Y. G. Kim and J. S. Lee, Phys. Lett. B408, 367 (1997).

[14] J. Jang, Y. G. Kim and J. S. Lee, Phys. Rev. D58, (1998) [hep-ph/9711504].

[15] J. E. Kim, P. Ko and D. Lee, Phys. Rev. D56, 100 (1997).

[16] K. Choi, E. J. Chun and J. S. Lee, Phys. Rev. D55, R3924 (1997).

[17] K. Choi, K. Hwang and J. S. Lee, Phys. Lett. B428, 129 (1998).

[18] M. P. Worah, [hep-ph/9711265]; M. Ciuchini, E. Franco, G. Martinelli, A. Masiero and L. Silvestrini in Ref. [2].

[19] D. Guetta, [hep-ph/9805274].

[20] A. Ali, G. Krammer, and C. Lü, [hep-ph/9804363].

[21] A. Deandrea, N. D. Bartolomeo, and R. Gatto, Phys. Lett. B318, 549 (1993).

[22] C. E. Carlson, P. Roy, and M. Sher, Phys. Lett. B357, 99 (1995).

[23] J. Jang and J. S. Lee, work in progress.
### TABLE I.

R-parity- and lepton-number-violating product combinations which significantly contribute within present bounds\(^\text{[10,13,14]}\) assuming \(V_{\text{CKM}}\) is given by only down-type quark sector mixing. Constraints on the magnitudes of the product combinations are also shown.

| Decay Modes | Dominating Combination | Constraint |
|-------------|------------------------|------------|
| \(B_d \to \psi K_S\) | \(\lambda'_{132}\lambda'_{121}, \lambda'_{132}\lambda'_{112}, \lambda'_{232}\lambda'_{211}, \lambda'_{332}\lambda'_{312}\) | \(1.9 \times 10^{-3}\) |
| \(B_d \to \phi K_S\) | \(\lambda'_{231}\lambda'_{221}, \lambda'_{232}\lambda'_{211}\) | \(1.1 \times 10^{-3}\) |
| \(B_d \to \phi K_S\) | \(\lambda'_{332}\lambda'_{322}\) | \(5.8 \times 10^{-3}\) |
| \(B_d \to \pi^0 K_S\) | \(\lambda'_{231}\lambda'_{221}, \lambda'_{232}\lambda'_{211}\) | \(1.9 \times 10^{-3}\) |
| \(B_d \to D^+ D^-\) | \(\lambda'_{331}\lambda'_{323} V_{22} V_{22}\) | \(1.4 \times 10^{-4}\) |
| \(B_d \to D_{CP} \pi^0 (\rho^0)\) | \(\lambda'_{331}\lambda'_{323} V_{22} V_{22}\) | \(1.4 \times 10^{-4}\) |
| \(B_s \to \phi K_S\) | \(\lambda'_{132}\lambda'_{121}, \lambda'_{132}\lambda'_{112}, \lambda'_{232}\lambda'_{211}, \lambda'_{332}\lambda'_{312}\) | \(1.9 \times 10^{-3}\) |
| \(B_d \to \phi \pi^0\) | \(\lambda'_{132}\lambda'_{121}, \lambda'_{232}\lambda'_{212}\) | \(5.8 \times 10^{-3}\) |
| \(B_d \to \pi^+ \pi^-\) | \(\lambda'_{211}\lambda'_{213} V_{11} V_{11}\) | \(1.4 \times 10^{-4}\) |
| \(B_d \to \pi^0 \pi^0\) | \(\lambda'_{331}\lambda'_{311}\) | \(5.8 \times 10^{-3}\) |
TABLE II. The maximum values of $r_D$ for $CP$ violating $B$ decays with $L$- and $R_p$-violating couplings assuming $V_{\text{CKM}}$ is given by only down-type quark sector mixing.

| Decay Mode Sub-quark process | $\phi_{\text{SM}}$ | $r_D$ |
|-----------------------------|--------------------|-------|
| $B_d \to \psi K_S$ $b \to c\bar{c}s$ | $\beta$ | 0.09 |
| $B_d \to \phi K_S$ $b \to s\bar{s}s$ | $\beta$ | 2.0 |
| $B_d \to \pi^0 K_S$ $b \to u\bar{u}s, b \to d\bar{d}s$ | $\beta$ | 2.8 |
| $B_d \to D^+ D^-$ $b \to c\bar{c}d$ | $\beta$ | 0.09 |
| $B_d \to D_{CP} \pi^0 (\rho^0)$ $b \to c\bar{u}d, b \to u\bar{c}d$ | $\beta$ | 0.06 |
| $B_s \to \phi K_S$ $b \to s\bar{s}d$ | $\beta$ | 8.0 |
| $B_d \to \phi\pi^0$ $b \to s\bar{s}d$ | $2\beta$ | 66 |
| $B_d \to \pi^+ \pi^-$ $b \to u\bar{u}d$ | $\alpha$ | 0.04 |
| $B_d \to \pi^0 \pi^0$ $b \to u\bar{u}d, b \to d\bar{d}d$ | $2\beta$ | 3.0 |

TABLE III. The product combinations which contribute to each decay mode and the maximum values of $r_D$ for $CP$ violating $B$ decays with $B$- and $R_p$-violating couplings. Present constraints on the magnitudes of the product combinations are also shown [10,22].

| Decay Mode | Combination | Constraint | $r_D$ |
|------------|-------------|------------|-------|
| $B_d \to \psi K_S$ | $\lambda_2^{212} \lambda_2^{113}$ | $6.4 \times 10^{-3}$ | 12 |
| $B_d \to \pi^0 K_S$ | $\lambda_2^{212} \lambda_2^{113}$ | $6.4 \times 10^{-3}$ | 7.2 |
| $B_d \to D^+ D^-$ | $\lambda_2^{212} \lambda_2^{223}$ | $7.8 \times 10^{-3}$ | 3.2 |
| $B_d \to D_{CP} \pi^0 (\rho^0)$ | $\lambda_2^{212} \lambda_2^{132}$ | 1.6 | 3000 |
| $B_s \to \phi K_S$ | $\lambda_2^{212} \lambda_2^{223}$ | $7.8 \times 10^{-3}$ | 25 |
| $B_d \to \phi\pi^0$ | $\lambda_2^{212} \lambda_2^{123}$ | $7.8 \times 10^{-3}$ | 680 |
| $B_d \to \pi^+ \pi^-$ | $\lambda_1^{122} \lambda_1^{123}$ | $1.3 \times 10^{-6}$ | 1.4 $\times 10^{-3}$ |
| $B_d \to \pi^0 \pi^0$ | $\lambda_1^{122} \lambda_1^{123}$ | $1.3 \times 10^{-6}$ | 0.01 |