Testing the isotropy of the Universe by using the JLA compilation of type-Ia supernovae

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Abstract

We probe the possible anisotropy in the accelerated expanding Universe by using the JLA compilation of type-Ia supernovae. We constrain the amplitude and direction of anisotropy in the anisotropic cosmological models. For the dipole-modulated ΛCDM model, the anisotropic amplitude has an upper bound $D < 1.04 \times 10^{-3}$ at the 68% confidence level. Similar results are found in the dipole-modulated $w$CDM and CPL models. Our studies show that there are no significant evidence for the anisotropic expansion of the Universe. Thus the Universe is still well compatible with the isotropy.
I. INTRODUCTION

The cosmological principle says that the Universe is homogeneous and isotropic at large enough scales, which is one of the foundations in modern cosmology [1]. It is well consistent with the present observational data, such as the data of cosmic microwave background (CMB) radiation from Wilkinson Microwave Anisotropy Probe (WMAP) [2, 3] and Planck satellite [4, 5]. Until now, the cosmological observations are still in accordance with the cosmological constant plus cold dark matter (ΛCDM) model which is based on the cosmological principle. Thus the ΛCDM model becomes the leading model in cosmology.

Despite the great successes it achieved, the ΛCDM model still faces certain challenges [6–8]. As the improvements of accuracy, it is found from a large amount of observations that the Universe might deviate from statistical isotropy. These include the alignment of low multipoles in the angular power spectrum of CMB temperature fluctuations [9–12], the hemispherical power asymmetry of CMB temperature anisotropy [2, 13], the spatial variation of the electromagnetic fine-structure constant [14–16], the large-scale alignment of the quasar polarization vectors [17, 18], the large-scale bulk flow beyond the prediction of ΛCDM model [19–21], just name a few. All of these phenomena arouse us to rethink the validity of the cosmological principle. If the cosmological principle is proven to be failed, the modern cosmology should be rewritten.

Due to their consistent absolute magnitudes, type-Ia supernovae (SNe Ia) are regarded as the ideal distance indicators to trace the accelerated expansion of the Universe. In fact, they have been widely used to search for the anisotropic signals in the accelerated expansion of the Universe [22–42]. Especially, a statistic based on the extreme value theory shows that the gold data set is consistent with the isotropy [22]. The study [23] on the angular covariance function of supernova magnitude fluctuations is consistent with zero dark energy fluctuations by using the Union2 compilation [43]. A “residual” statistic shows that the isotropic ΛCDM model is not consistent with the Union2 data with $z < 0.05$ at $2 - 3\sigma$ [24]. The anisotropic Bianchi type I cosmological model is allowed by the Union2 data [25, 26]. The Randers cosmology is compatible with the isotropy [27, 29]. The hemisphere comparison is used to study the Union2 data and shows certain preferred directions [30–35]. By dividing the Union2 supernovae into 12 subsets according to their galactic coordinates, a dipole of the deceleration parameter is preferred at more than $2\sigma$ level [36]. By combining...
the data of Union2 and gamma-ray bursts, the isotropic ΛCDM model is well permitted \[37\] while the anisotropic Finsler cosmology is preferred at around \(2\sigma\) \[38\]. By using the data of Union2.1 \[44\] and gamma-ray bursts, a model-independent way shows a dipolar anisotropy at more than \(2\sigma\) \[40\]. It has been found that there may be certain correlation between the fine structure dipole and the dark energy dipole \[41, 42\].

Recently, a new sample of SNe Ia was released by the SDSS collaboration, which is called the “joint light-curve analysis” (JLA) compilation \[45\]. Compared to previous compilations such as Union2 \[43\] and Union2.1 \[44\], the number of SNe Ia in the JLA compilation is highly enlarged and the systematic uncertainties are significantly reduced. Recently, the JLA SNe Ia have been used to probe the anisotropic Hubble diagram in Bianchi type I cosmology \[26\] and test the cosmological principle \[46\]. However, the work \[46\] did not consider the full covariance matrix between SNe Ia. In this paper, we use the JLA compilation to restudy the anisotropic Hubble diagram of the Universe. Unlike certain previous works which have neglected the correlations between any two SNe Ia, we make use of the full covariance matrix to construct the likelihood (or chi-square). It has been shown that the statistical significance of the previously claimed evidence for a preferred direction could be highly lowered if the full covariance matrix of SNe Ia is considered in the Union2 compilation \[47\]. Thus we want to see whether the anisotropic signals in the accelerated expansion of the Universe still exist in the newly released JLA compilation.

The rest of the paper is arranged as follows. In section II, we demonstrate the anisotropic cosmological models and the numerical method used in our analysis. The observational data sets are briefly listed in section III. In section IV, we give our constraints on the anisotropic amplitudes and directions for the anisotropic expansion of the Universe. Our conclusion will be given in section V.

**II. MODELS AND METHODOLOGY**

The anisotropic expansion of the Universe can be induced by assuming that the dark energy has anisotropic repulsive force \[48, 50\] or the background has certain preferred directions \[26, 28, 29, 42\], and so on. Thus we can obtain the anisotropic luminosity distance. In this paper, we assume a dipole modulation to describe the deviation from the isotropic background. Phenomenologically, the distance modulus deviating from the isotropy can be
\[
\mu_{th} = \bar{\mu}_{th} (1 + D(\hat{n} \cdot \hat{p})) ,
\]

where \( D \) denotes the magnitude of the dipole modulation, \( \hat{n} \) is the dipole direction and \( \hat{p} \) the unit 3-vector pointing towards a supernova. Here the anisotropic amplitude \( D \) is assumed as a constant over all redshifts, and \( \bar{\mu}_{th} \) denotes the theoretical distance modulus predicted by the isotropic \( \Lambda CDM, wCDM \) or CPL models, respectively. In the galactic coordinates system, the dipole direction \( \hat{n} \) can be parameterized as \( (\ell, b) \), namely, \( \hat{n} = \cos(b) \cos(\ell) \hat{i} + \cos(b) \sin(\ell) \hat{j} + \sin(b) \hat{k} \) where \( \hat{i}, \hat{j}, \hat{k} \) are the unit vectors along the axes of a Cartesian coordinates system. The position of \( i \)th supernova with the galactic coordinates \( (\ell_i, b_i) \) can be written as \( \hat{p}_i = \cos(b_i) \cos(\ell_i) \hat{i} + \cos(b_i) \sin(\ell_i) \hat{j} + \sin(b_i) \hat{k} \).

In the spatially-flat isotropic background spacetime, we can express the luminosity distance \( d_L(z) \) in terms of the redshift \( z \), i.e.,

\[
d_L(z) = \frac{1 + z}{H_0} \int_0^z \frac{dz'}{E(z')} , \tag{2}
\]

where \( H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \) denotes the Hubble constant. The isotropic distance modulus \( \bar{\mu}_{th} \) can be given by

\[
\bar{\mu}_{th} = 5 \log_{10} \frac{d_L}{10 \text{pc}} . \tag{3}
\]

It is computed for the fixed fiducial value of \( h = 0.7 \), which is approximately acceptable \[51\]. In (2), the quantity \( E(z) \) is a function of redshift \( z \). In the \( \Lambda CDM \) model, it can be expressed as

\[
E^2(z) = \Omega_m (1 + z)^3 + (1 - \Omega_m) , \tag{4}
\]

where \( \Omega_m \) is today’s energy density of the matter. In the \( wCDM \) model, it can be expressed as

\[
E^2(z) = \Omega_m (1 + z)^3 + (1 - \Omega_m)(1 + z)^{3(1+w)} , \tag{5}
\]

where \( w = p/\rho \) denotes the equation of state (EoS) of dark energy. In the Chevallier-Polarski-Linder (CPL) parameterization \[52, 53\], the EoS of dark energy is determined by \( w = w_0 + w_1 \frac{z}{1+z} \). In this case, \( E(z) \) can be expressed as

\[
E^2(z) = \Omega_m (1 + z)^3 + (1 - \Omega_m)(1 + z)^{3(1+w_0+w_1)} \exp \left(-3w_1 \frac{z}{1+z} \right) . \tag{6}
\]

In this paper, we employ the Markov Chain Monte Carlo (MCMC) approach and the least square method to estimate the model parameters, respectively. The log-likelihood \(-2 \ln \mathcal{L}\)
(or chi-square $\chi^2$) can be given by the summation of chi-squares of all data sets, i.e.,

$$-2\ln\mathcal{L} \equiv \chi^2 = \chi^2_{JLA} + \chi^2_{CMB} + \chi^2_{BAO},$$

where we will combine the JLA compilation with the CMB and BAO datasets in our analysis. These data sets and their chi-squares are briefly demonstrated in the next section. In our study, we modify the public Markov Chain Monte Carlo sampler (CosmoMC) \footnote{CosmoMC} to estimate the background parameters and anisotropic parameters.

### III. OBSERVATIONAL DATA

In this paper, we constrain the anisotropy of the Universe by using the JLA compilation of SNe Ia, the Planck2013 data of the CMB temperature anisotropy, the WMAP9 observations on the CMB polarizations, and the BAO data from the SDSS-III BOSS DR11. In the following, we will briefly describe these datasets.

#### A. The JLA data

The JLA compilation \footnote{JLA} consists of 740 well-calibrated SNe Ia in the redshift range of $z \in [0.01, 1.30]$. It is a collection of several low-redshift samples, all three seasons from the SDSS-II, three years from SNLS, and a few high-redshift samples from the Hubble Space Telescope (HST). All of the SNe Ia have light curves of high quality, so their distance moduli can be abstracted with high precision. The positions of SNe Ia in the sky of equatorial coordinates system can be found at the website of IAU Central Bureau for Astronomical Telegrams.\footnote{http://www.cbat.eps.harvard.edu/lists/Supernovae.html} To compare with others’ work, we transform the positions of SNe Ia into the galactic coordinates system in our analysis.

From the observational point of view, the distance modulus of a SN Ia can be abstracted from its light curve through the empirical linear relation \footnote{JLA}

$$\hat{\mu} = m^*_B - (M_B - \alpha \times X_1 + \beta \times C),$$

where $m^*_B$ is the observed peak magnitude in rest frame $B$ band, the absolute magnitude $M_B$ depends on the host galaxy properties complexly, $X_1$ is the time stretching of the light curve,
and $C$ is the supernova color at maximum brightness. The three light-curve parameters $m_B^*$, $X_1$ and $C$ are different from one supernova to other one and can be derived directly from the light curves. The two nuisance parameters $\alpha$ and $\beta$ are assumed to be constants for all the supernovae.

For the JLA samples, the luminosity distance $d_L$ of one supernova in (2) can be given as

$$d_L(z_{\text{hel}}, z_{\text{cmb}}) = \frac{1 + z_{\text{hel}}}{H_0} \int_0^{z_{\text{cmb}}} \frac{dz'}{E(z')} ,$$

where $z_{\text{cmb}}$ and $z_{\text{hel}}$ denote the CMB frame and heliocentric redshifts, respectively. Then we can obtain the theoretical distance modulus $\mu_{th}$ in (11). Using the observed distance modulus $\hat{\mu}$ in (8), the anisotropic cosmological models can be fitted to the JLA supernovae by using the chi-square $\chi^2_{JLA}$ in (7) as

$$\chi^2_{JLA} = (\hat{\mu} - \mu_{th})^\dagger C^{-1} (\hat{\mu} - \mu_{th}) ,$$

where $C$ is the covariance matrix of $\hat{\mu}$, and $\mu_{th}$ denotes the theoretical distance modulus of each supernova predicted by the anisotropic cosmological models as mentioned in last section. The covariance matrix $C$ can be found in [45].

**B. Other data**

The CMB data sets are used to constrain the background parameters, especially the parameters for the EoS of dark energy. They are irrelelative to the anisotropic expansion of the Universe, because the CMB observations are probably insensitive to the anisotropic dark energy models [55]. The CMB datasets include the data of temperature fluctuations released by Planck collaboration in 2013 and the data of low-$\ell$ polarizations from the nine-year WMAP release. The Planck likelihoods [56] consist of the high-$\ell$ temperature likelihood ($\ell = 50 - 2500$) and the low-$\ell$ temperature likelihood ($\ell < 50$). The WMAP polarization likelihood is used to analyze the data of CMB polarizations. In this paper, the pivot scale is chosen as $k_p = 0.05\text{Mpc}^{-1}$. Here we will denote these data by “CMB” and their chi-square by $\chi^2_{\text{CMB}}$ in (7).

We also apply the BAO data to further constrain the dark energy parameters by using the angular diameter distance vs. redshift relation. The BAO data used in this paper come from the clustering of galaxies in the SDSS-III BOSS DR11 [57]. There are two data points
| parameters | ΛCDM          | wCDM          | CPL             |
|------------|---------------|---------------|-----------------|
| \(D\)     | \(< 1.04 \times 10^{-3}\) | \(< 1.08 \times 10^{-3}\) | \(< 1.09 \times 10^{-3}\) |
| \(\ell[^\circ]\) | 185.5 ± 100.3 | 186.7 ± 98.9  | 190.6 ± 100.0   |
| \(b[^\circ]\) | -16.6 ± 54.2  | -16.6 ± 54.8  | -17.7 ± 54.8    |

Table I: The 68% limits on the anisotropic amplitudes \(D\) and directions \((\ell, b)\) are obtained by applying the MCMC approach.

in our analysis. They are BOSS LOWZ \(D_V(r_{s,fid}/r_s) = 1264 ± 25\) Mpc at \(z = 0.32\) and BOSS CMASS \(D_V(r_{s,fid}/r_s) = 2056 ± 20\) Mpc at \(z = 0.57\), respectively. Here \(r_s\) denotes the distance to the sound horizon at the drag epoch, and \(r_{s,fid} = 149.28\) Mpc in the fiducial cosmology. In this paper we denote the BAO data by “BAO” and their chi-square by \(\chi^2_{BAO}\) in (7).

IV. RESULTS

As was mentioned above, we study the anisotropic signals of the dipole-modulated ΛCDM, wCDM and CPL models by using the JLA sample in this paper. The priors in the MCMC sample approach are given by \(100D \in [0, 10]\), \(\ell \in [0, 360]\) and \(b \in [-90, 90]\). Our results can be found in Table I, where the models and the anisotropic amplitudes and directions are given. Here the errorbars just denote the standard deviations, while the dipole actually can point towards any direction at 1σ level. Similarly, the best-fit results from the least square method can be found in Table II. The likelihood distributions of the anisotropic parameters \(D\), \(\ell\) and \(b\) in three anisotropic models are illustrated in Figure 1. The marginalized 1σ contour plots of the dipole direction \((\ell, b)\) are illustrated in Figure 2, which correspond to the preferred directions as well as their 1σ uncertainties. The nuisance parameters such as \(\alpha\) and \(\beta\) are marginalized in this paper, since they are not model parameters with significant meanings. We just focus on studying the anisotropic signals, and then neglect the topic of the model comparison.
| parameters | ΛCDM | wCDM | CPL |
|------------|------|------|-----|
| $D$        | $0.50 \times 10^{-3}$ | $0.44 \times 10^{-3}$ | $0.70 \times 10^{-3}$ |
| $\ell[^\circ]$ | 210.7 | 134.4 | 268.7 |
| $b[^\circ]$ | -58.6 | -61.7 | -28.9 |

Table II: The best-fit central values of the anisotropic amplitudes $D$ and directions ($\ell, b$) are obtained by applying the least square method.

Figure 1: The likelihood distributions for the amplitude $D$ and direction ($\ell, b$) of the dipole modulation in three modulated models.

A. The ΛCDM case

In the modulated ΛCDM model, we can well constrain the isotropic background parameters, i.e., $(\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \ln(10^{10}A_s), n_s) = (0.02211 \pm 0.00024, 0.1186 \pm 0.0014, 1.04136 \pm 0.00054, 0.091 \pm 0.012, 3.089 \pm 0.024, 0.9622 \pm 0.0052)$ at the 68% confidence level (C.L.). Here, we denote the baryon density today $\Omega_b h^2$, the cold dark matter density

Figure 2: The marginalized 1σ contour plot for the direction ($\ell, b$) of dipole modulation in three modulated models. From left to right, they are the modulated ΛCDM, wCDM and CPL models.
today $\Omega_c h^2$, the angular scale of the sound horizon at last-scattering $\theta_{MC}$, the Thomson scattering optical depth due to the reionization $\tau$, the amplitude of scalar power spectrum $A_s$, and the spectral index of scalar power spectrum $n_s$.

By using the MCMC approach, the anisotropic amplitude is constrained as $D < 1.04 \times 10^{-3}$ at the 68% C.L., which is consistent with the isotropy within 1σ. In the galactic coordinate system, the direction of the dipole modulation lies towards $(\ell, b) = (185.6^\circ \pm 100.3^\circ, -16.6^\circ \pm 54.2^\circ)$ at the 68% C.L.. Thus the JLA compilation finds no significant evidence for the deviations from the isotropy. By contrast, the Union2 sample gives constraints on the dark energy dipole as $D = (1.3 \pm 0.6) \times 10^{-3}$ and $(\ell, b) = (309.4^\circ \pm 11.5^\circ, -15.1^\circ \pm 11.5^\circ)$ at the 68% C.L. The anisotropic expansion of the Universe is permitted at more than 2σ [41]. However, the statistical significance can be highly lowered if the full covariance matrix of SNe Ia is considered in the Union2 compilation [47]. Our constraints on the anisotropic parameters are listed at the second column in Table I. Their likelihood distributions are illustrated by the red curves in Figure 1 and the marginalized contour plot of $(\ell, b)$ is given by Figure 2(a).

By using the least-square method, we obtain the anisotropic amplitudes $D = 1.50 \times 10^{-3}$ and directions $(\ell, b) = (210.7^\circ, -58.6^\circ)$, which are listed in the second column in Table II. The above results are compatible with those from the MCMC approach, even though the central values for the anisotropic parameters are different. In addition, the best-fit $\chi^2$ is given by $\chi^2_{\Lambda CDM} = 10503.9$.

B. The $w$CDM case

In the modulated $w$CDM model, there is an extra background parameter, i.e., the EoS of dark energy $w$. At the 68% C.L., the seven isotropic background parameters are constrained as $(\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \ln(10^{10} A_s), n_s, w) = (0.02206 \pm 0.00025, 0.1192 \pm 0.0019, 1.04130 \pm 0.00056, 0.090 \pm 0.013, 3.088 \pm 0.025, 0.9607 \pm 0.0058, -1.026 \pm 0.053)$, which are consistent with the isotropic $\Lambda$CDM background. We use the MCMC approach to constrain the anisotropic parameters as $D < 1.08 \times 10^{-3}$ and $(\ell, b) = (186.7^\circ \pm 98.9^\circ, -16.5^\circ \pm 54.8^\circ)$ at the 68% C.L., which are listed at the third column in Table I. Their likelihood distributions are illustrated by the blue curves in Figure 1 and the marginalized contour plot of $(\ell, b)$ is illustrated in Figure 2(b). By using the least-square method, we obtain the anisotropic
amplitudes $D = 0.44 \times 10^{-3}$ and directions $(\ell, b) = (134.4^\circ, -61.7^\circ)$, which are listed in the third column in Table II. They are also compatible with the above constraints from the MCMC approach. The best-fit $\chi^2$ is given by $\chi^2_{wCDM} = 10504.5$.

C. The CPL case

In the modulated CPL parameterization, there are two extra background parameter, i.e., the EoS parameters of dark energy, i.e., $w_0$ and $w_1$. At the 68% C.L., the eight isotropic background parameters are constrained as $(\Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, \ln(10^{10} A_s), n_s, w_0, w_1) = (0.02200 \pm 0.00027, 0.1206 \pm 0.0026, 1.04113 \pm 0.00061, 0.088 \pm 0.013, 3.087 \pm 0.025, 0.9576 \pm 0.0070, -0.926 \pm 0.118, -0.485 \pm 0.517)$, which are also consistent with the isotropic $\Lambda$CDM background. The anisotropic parameters are constrained to be $D < 1.06 \times 10^{-3}$ and $(\ell, b) = (189.6^\circ \pm 100.1^\circ, -17.9^\circ \pm 54.4^\circ)$ at the 68% C.L., which are listed at the last column in Table I. Their likelihood distributions are illustrated by the black curves in Figure 1 and the marginalized contour plot of $(\ell, b)$ is illustrated in Figure 2(c). By using the least-square method, we obtain the anisotropic amplitudes $D = 0.77 \times 10^{-3}$ and directions $(\ell, b) = (292.6^\circ, -71.7^\circ)$, which are listed in the last column in Table II. They are also compatible with the above constraints from the MCMC approach. The best-fit $\chi^2$ is given by $\chi^2_{CPL} = 10504.6$.

V. CONCLUSION

In this paper, we probe the possibly anisotropic expansion of the Universe by using the recently released JLA compilation of SNe Ia. We consider the dipole-modulated deviation from the isotropy in three different dark energy models. We obtain similar constraints on the anisotropic amplitude and direction in three cases. This indicates that the preferred direction for the maximum anisotropy is insensitive to the isotropic background models. Especially, our MCMC studies show that the anisotropic amplitude has an upper bound $D < 1.04 \times 10^{-3}$ at the 68% C.L. and the dipole direction points towards $(\ell, b) = (185.6^\circ \pm 100.3^\circ, -16.6^\circ \pm 54.2^\circ)$ for the dipole-modulated $\Lambda$CDM model. By contrast to Union2 and Union2.1 samples, the JLA compilation indicates no significant evidence for the anisotropic expansion of the Universe. Our results show that the accelerated expansion of the Universe
is still consistent with the isotropy, and no significant evidence are found for the anisotropy. Thus the isotropic ΛCDM model is still well consistent with the astronomical observations at present.

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