Aircraft Loading Optimization: MemComputing the 5th Airbus Problem

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(Dated: March 21, 2019)

On the January 22th 2019, Airbus launched a quantum computing challenge to solve a set of problems relevant for the aircraft life cycle (Airbus challenge web-page). The challenge consists of a set of 5 problems that ranges from design to deployment of aircraft. This work addresses the 5th problem. The formulation exploits an Integer programming framework with a linear objective function and the solution relies on the MemComputing paradigm. It is discussed how to use current MemComputing software MemCPU™ to solve efficiently the proposed problem and assess scaling properties, which turns out to be polynomial for meaningful solutions of the problem at hand. Also discussed are possible formulations of the problem utilizing non-linear objective functions, allowing for different optimization schemes implementable in modified MemCPU™ software, potentially useful for field operation purposes.

I. INTRODUCTION

Automated computation in the 21st century has reached a paramount role in industry, finance, consumer technology and much more [1, 2]. However, the more that computation becomes relevant, the greater the challenges presented to both industry and academia. Solutions to today’s computational limits are provided by more and more sophisticated and high-performing CPUs, GPUs etc. [3], as well as by the insurgence of distributed computing in cloud infrastructures [4].

Nevertheless, the unceasing growth of computing demand is pushing towards completely new architectures as well as new paradigms. This can mean not only “handling more computation,” but also making the computation more efficient. For example, neuromorphic computing based on diverse realizations of artificial neural networks [5–7] promises a paradigm shift in machine learning and artificial intelligence.

In this scenario, new computing architectures based on quantum physics and not strictly on algorithmic approaches promise to solve among the most challenging problems ranging from drug discovery to AI [8–11]. How- ever, it is still not clear if and when reliable quantum computers will show what currently goes under the name of quantum supremacy over classical computing architectures [12–15].

However, companies and academia are already looking at applications for future quantum computers [16–18]. I focus here on the challenge recently launched by Airbus [17] consisting of 5 problems related to the life cycle of an aircraft, i.e., ranging from the design to the deployment of an aircraft. The general requirements for the challenge are that for each problem a suitable algorithm implementable on a quantum computer should be developed, tested on a simulated quantum computer, and finally assessed in terms of the performance at scale.

I consider in this work the 5th Airbus problem [17]: the Aircraft Loading Optimization (ALO) problem. The ALO requires optimizing the placement of containers of different sizes and weights in an aircraft subject to limitations on maximum weight allowed, maximum tolerable shear and center of gravity. However, instead of developing an algorithm for future quantum computers, I present a formalization of the problem using integer programming (IP) framework [18]. IP is a problem formulation widely used in industry and academia consisting of an objective function to be minimized over a set of variables. Variables may be constrained to take binary, integer or continuous values. Moreover, the objective function may be subject to further constraints in the form of linear inequalities among variables. Finally, depending on the nature of the objective function, we can have different IP problems such as linear, quadratic or other non-linear type.

The formulation I consider here covers all requirement of the ALO statement and leads to an IP problem involving binary variables only and having two objective functions to be minimized, one linear and the other non-linear. I therefore propose two different solution strategies to solve this multi-objective IP problem.

The binary IP (BP) problem is NP-complete, sometimes also known as 0-1 linear programming, and is one of Karp's 21 NP-complete problems [19]. Therefore, it is not surprising that Airbus poses a challenge about a problem whose natural formulation leads to a special case of an NP-complete problem. Several general-purpose open source [20–21] and commercial solvers [22–25] have been developed to solve IP problems. However, these can fail when the problems are particularly hard and therefore specialized solvers optimized to exploit specific structures for ILP have also been developed [26–28].

IP problems can be approached employing different techniques. A classification of those includes heuristics [21–23] and exhaustive algorithms [18, 24–26], also called “complete algorithms.” The complete algorithm for ILP that is most commonly employed is a combination of cutting planes and branch-and-bound, also known as the branch-and-cut algorithm [18]. All these solvers have demonstrated varied degrees of success on a variety of IP problems [27–29], however scaling properties for BP

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FIG. 1: Sketch of the Airbus Aircraft Loading Optimization problem. The aircraft is to be loaded by selecting a portion of the available payload. The containers can have different shapes. For this ALO, 3 different shapes are considered numbered from 1 to 3 as in the figure. The maximum weight allowed is limited. The center of gravity $x_{cg}$ of the system aircraft + containers must remain within the limit $[x_{cg\min}, x_{cg\max}]$ and the shear curve $S(x)$ must also be limited to remain under a given shear limit curve $S_{max}(x)$. Problems such as the one proposed by Airbus can easily show exponential blowing-up when applying one of those methods [19].

In this work I discuss the general purpose approach to the solution of the Airbus ALO problem based on the memcomputing paradigm introduced in [44]. Memcomputing (computing with and in memory [45]) approach is neither based on stochastic search nor on trial and error strategy. In addition, memcomputing does not use a set of instructions recursively employed to find solutions to the problem at hand. Therefore, we can classify the memcomputing approach as non-algorithmic [45] .

The memcomputing approach relies on embedding a given problem into an electronic circuit, which represents a possible realization of a memcomputing machine (MM) [44][46]. These electronic circuits, when designed in order to satisfy a few properties [46], naturally relax to an equilibrium that represents the solution of the problems at hand [45][48].

In order to approach the Airbus ALO problem I use the MM realization described in [35] employing self-organizing algebraic gates (SOAGs). The SOAG-based MM can be efficiently simulated in software and in this work I use the software developed at MemComputing Inc. [49] named MemCPU™.

In section III the BP formulation of the ALO problem is discussed. In section IV a quick overview of memcomputing and MemCPU™ is provided, while in section V are discussed both numerical results and scaling properties of MemCPU™ applied to the ALO problem.

II. AIRBUS AIRCRAFT LOADING OPTIMIZATION PROBLEM

The ALO problem proposed by Airbus is sketched in Fig. 1. Following the problem statement we need to optimize the placement of a subset of containers picked from an available payload formed by $n$ different containers. Each container is described by the triplet $(k, m_k, s_k)$ with $k$ the identification number, $m_k$ the mass and $s_k$ the size of the $k-th$ container.

The aircraft has $N$ available positions for standard cargo as depicted in Fig. 1. The size can be of 3 different types: size 1 occupies one position; size 2 occupies half a position and may share the position with another size 2 container; size 3 occupies 2 positions.

The problem requires maximizing the mass of the carried freight for the flight such that:

(A) the containers must be placed consistently with size and positions available
(B) the mass of the carried freight does not exceed the maximum payload capacity of the aircraft \( W_p \).

(C) the center of gravity position of the carried freight + aircraft must be within the limits \([x_{cg}^{\min}, x_{cg}^{\max}]\).

(D) the shear curve \( S(x) \) defined by the container distribution must be bounded by a given maximal shear curve \( S^{\max}(x) \).

(E) while maximizing the mass, also the position of the center of gravity of the carried freight + aircraft should be optimized to be as close as possible to a target center of gravity \( x_{cg}^t \).

These constraints are sketched also in Fig. 1. While the constraints (A) and (B) do not need additional details or explanations, the others do. In (C) the center of gravity is defined by the the equilibrium of all forces (container + aircraft) in the gravity axis direction (see force \( F \) in Fig. 1). Each container is supposed to have uniform mass, therefore the weight force can be considered as the weight of a point located at the center of the container. Finally, the mass and location of the center of gravity of the empty aircraft are provided.

The constraint (D) requires a consistent definition of the shear curve. The shear curve is defined by

\[
S(x) = \int_{-\frac{L}{2}}^{x} m(x') dx' \quad \text{for } x < 0 \tag{1k}
\]

\[
S(x) = \int_{x}^{\frac{L}{2}} m(x') dx' \quad \text{for } x > 0 \tag{1j}
\]

where \( L \) is the length of the loading region of the aircraft and \( x \) is the position relative to the center of the loading zone (see Fig. 1). In the statement of the problem, the maximum shear curve \( S^{\max}(x) \) can be either linear and symmetric with respect to \( x = 0 \), or asymmetric or some non-linear function of \( x \).

Finally, (E) represents an extra optimization requirement and not really a constraint. In fact, from the Airbus statement, what should be primarily optimized is the mass of the carried freight, then optimize the distribution such that the resulting center of gravity is as close as possible to a target.

### III. INTEGER PROGRAMMING FORMULATION

A natural way to formulate mathematically the problem posed in section II is using Integer Programming framework [13].

Let us start assigning a binary variable \( y_{k,j} \) to each container \( k = 1, ..., n \) located in the position \( j = 1, ..., N \) of the aircraft. Since each container can be located in at most one position in the aircraft we constrain these variables requiring

\[
\sum_{j=1}^{N} y_{k,j} \leq 1 \quad \text{for each } k = 1, ..., n. \tag{2k}
\]

On the other hand, we have the constraint on the size and the number of available spots on the aircraft that can be also expressed as a non-linear inequality. In order to formalize this constraint, let us further characterize the variable \( y_{k,j} \). We consider here, for simplicity, only the 3 sizes defined in the Airbus statement. However, this formulation can be easily extended to more complicated sizes and bin distributions in the aircraft. Let us define \( K_1, K_2 \) and \( K_3 \) as three sets of indexes such that \( K_1 \cup K_2 \cup K_3 = \{1, ..., n\} \). Moreover, if \( k \in K_h \) then \( s_k = h \).

Therefore, \( K_1, K_2 \) and \( K_3 \) regroup the variables in sets of variables corresponding to the same container size.

Using this index characterization, the constraint (A) on the sizes and bins can be expressed as

\[
\sum_{k \in K_1} y_{k,1} + \frac{1}{2} \sum_{k \in K_2} y_{k,1} + \sum_{k \in K_3} y_{k,1} \leq 1 \tag{2k'}
\]

\[
\sum_{k \in K_1} y_{k,j} + \frac{1}{2} \sum_{k \in K_2} y_{k,j} + \sum_{k \in K_3} (y_{k,j-1} + y_{k,j}) \leq 1 \quad \text{for } j = 2, ..., N - 1 \tag{2k''}
\]

\[
\sum_{k \in K_1} y_{k,N} + \frac{1}{2} \sum_{k \in K_2} y_{k,N} + \sum_{k \in K_3} y_{k,N-1} \leq 1 \tag{2k'''}
\]

It is easy to prove that these inequalities guarantee that for each bin there is no overlapping of containers except for containers of size 2 for which two of them can occupy the same bin (the coefficient \( 1/2 \) allows the overlapping). The case of the size 3 (containers that occupy two consecutive bins) is enforced by the term \( \sum_{k \in K_3} (y_{k,j-1} + y_{k,j}) \) in which two consecutive containers of size 3 appear in the same constraint; therefore a container of size 3 occupies 2 bins if selected. It is also worth noticing that containers of size 3 have \( j \) that ranges only from 1 to \( N - 1 \) because, since they occupy two bins, there are only \( N - 1 \) possible locations for them.

The constraint (B) can be trivially described by

\[
\sum_{k,j} m_k y_{k,j} \leq W_p. \tag{2l}
\]

The constraint (C) on the center of gravity requires the definition of signed distance \( d_{s_k,j} \) from \( x = 0 \) for a container of size \( s_k \) located in the bin \( j \). Considering equally distributed bins around \( x = 0 \) (this assumption is the same reported in the Airbus statement but can be easily relaxed), we have that

\[
d_{s_1,j} = \frac{2j - N - 1}{2N} L \quad \text{for } s_k = 1, 2 \tag{3l}
\]

\[
d_{s_3,j} = \frac{2j - N}{2N} L \quad \text{for } s_k = 3 \tag{3l}
\]
Using this distance definition, the center of gravity of the loaded aircraft can be evaluated as
\[
x_{\text{cg}} = \frac{\sum_{k,j} m_k d_{k,j} y_{k,j} + W_e x_{\text{cg}}^e}{\sum_{k,j} m_k y_{k,j} + W_e}
\] (4)
where \(W_e\) and \(x_{\text{cg}}^e\) are respectively the mass and the center of gravity of the empty aircraft.

Eq. (4) allows the formalization of the constraint (C) through the inequalities
\[
\sum_{k,j} m_k (d_{k,j} - x_{\text{cg}}^\text{max}) y_{k,j} \leq W_e (x_{\text{cg}}^\text{max} - x_{\text{cg}}^e)
\] (4)
\[
\sum_{k,j} m_k (x_{\text{cg}}^\text{min} - d_{k,j}) y_{k,j} \leq W_e (x_{\text{cg}}^e - x_{\text{cg}}^\text{min})
\] (4)
that express \(x_{\text{cg}} \leq x_{\text{cg}}^\text{max}\) and \(x_{\text{cg}} \geq x_{\text{cg}}^\text{min}\) respectively.

Regarding the constraint (D), the shear curve can be easily calculated at each bin location and set smaller than \(S_{\text{max}}(x)\):
\[
\sum_{k \in K_1 \cup K_2} \sum_{j'} m_k y_{k,j'} + \frac{1}{2} \sum_{k \in K_3} \left( \sum_{j'=1}^{j-1} m_k y_{k,j'} + m_k y_{k,j} \right) \leq S_{\text{max}}(x_j) \text{ for } j \leq N/2
\] (4)
\[
\sum_{k \in K_1 \cup K_2} \sum_{j'=j}^N m_k y_{k,j'} + \frac{1}{2} \sum_{k \in K_3} \left( \sum_{j'=j-1}^{N-1} m_k y_{k,j'} + m_k y_{k,j} \right) \leq S_{\text{max}}(x_j) \text{ for } j \geq N/2
\] (4)
which expresses the shear curve defined in Eq. (1) at the centers of the bins \(x_j\) taking into account the different sizes of the containers.

Finally we can define the new objective function of the ALO problem as
\[
f(y) = -\sum_{k,j} m_k y_{k,j}.
\] (5)
whose minimization over \(y\) subject to the collection of constraints (2) provides the distribution and the maximum mass of the carried freight satisfying the constraints (A)-(D) of section II.

### IV. Memcomputing Approach

The memcomputing approach to IP problems is based on the concept of Self-Organizing Algebraic Gates (SOAGs) introduced in [46]. SOAG is a novel circuit design developed at MemComputing, Inc. [49] by the author of this work and is part of a class of self-organizing circuits as Self-Organizing Logic Gates (SOLGs) introduced in [50].

Both SOLGs and SOAGs are building blocks for practical realizations of Universal Memcomputing Machines (UMM) [41, 45, 51], with digital input-output (Digital MM, DMM) [46].

The SOAG is designed to self-organize toward an algebraic relation representing a linear inequality among variables. In this work, the SOAGs are designed to satisfy linear relations between binary variables (see Fig. 2) since the ALO problem formulation involves binary variables only.

By connecting together SOAGs we form a Self-Organizing Algebraic Circuit (SOAC), see Fig. 3. The SOAC collectively self-organizes in order to satisfy the relations embedded in the gates. In this way, it is trivial to embed an IP problem directly into the SOAC. Each inequality in (2) or (7) can be mapped directly into a SOAG. The objective functions (5) can be easily refor-
Self-Organizing Algebraic Gate

\[ \sum_j a_j x_j - b_j \]

Dynamic correction module

Inject Current

Read Voltages

FIG. 2: Reprint from [35]. Sketch of a Self-Organizing Algebraic Gate. All terminals allow a superposition of incoming and outgoing signals from the surrounding circuit. The central unit processes the signals in order to satisfy a linear algebraic relation consistent with the requirement of the “out” terminal. The self-organization is enforced by the Dynamic Correction Modules that read voltages from all terminals and inject a current to the appropriate terminal as long as the algebraic relation is not satisfied.

FIG. 3: Reprint from [35]. Sketch of a Self-Organizing Algebraic Circuit (SOAC). SOAGs are connected together in an architecture that directly maps the IP into the SOAC.

Self-Organizing Algebraic Gate

\[ A_{eq} x = b_{eq} \]

\[ A_{ineq} x \leq b_{ineq} \]

satisfy

satisfy

satisfy

satisfy

satisfy

satisfy

FIG. 3: Reprint from [35]. Sketch of a Self-Organizing Algebraic Circuit (SOAC). SOAGs are connected together in an architecture that directly maps the IP into the SOAC.

\[ \sum f_{k,j}^+ (\tilde{b}) y_{k,j} \leq g^+ (\tilde{b}) \quad (10) \]

\[ \sum f_{k,j}^- (\tilde{b}) y_{k,j} \leq g^- (\tilde{b}) \quad (11) \]

where \( \tilde{b} \) is an extra parameter that again can be dynamically changed to find solutions of increasing quality, each time closer to the global optimum. \( f^\pm \) and \( g^\pm \) are linear functions of \( \tilde{b} \) defined by substituting \( \tilde{b} \) in (5). This also leads to defining \( \tilde{b} \) as the threshold of the distance of the actual center of gravity from the target \( x_{cg} \).

The problem formulated and embedded in a SOAC as described can be efficiently handled by either actually building the electronic circuit or just by simulating it since it involves only standard (non-quantum) electronic components. Simulating means solving differential equations of the form

\[ \frac{dQ(i, v, x)}{dt} = F(i, v, x), \quad (12) \]

with appropriate initial conditions and where \( Q \) and \( F \) are non-linear functions of the voltages \( (v) \), currents \( (i) \) and extra internal state variables \( (x) \) characterizing the circuit. In this picture a configuration of the voltages \( (v) \) at a given time represents an actual assignment to the variables of the IP problem by reading the voltages though thresholds: if a voltage at a node of the circuit is above the threshold, then it corresponds to a logical 1, and otherwise corresponds to a logical 0. On the other hand, the transition function of these machines (namely the function that maps input to output) is physical (analog) and takes full advantage of the collective state of the system to process information [46, 51, 52]. However, despite its analog nature, the mapping of voltages into binary variable through thresholds allows efficient size scaling for these machines, avoiding the bottleneck related to the precision of writing and reading inputs and outputs. Finally, it is worth noticing here that the memcomputing approach used to solve binary as presented in [35] for the case of IP problems, does not provide proof of optimality for a given solution, nor does it detect the infeasibility of a problem.

In this work we consider the simulation of the SOAC implemented in the software MemCPU™ developed at MemComputing, Inc. [49] and available also as Software at a Service.

The working principle of SOAGs and SOACs, i.e., self-organization of voltages and currents of an electronic circuit in order to satisfy algebraic relations, is enforced by both active and passive electronic elements with and without memory [35, 45, 46]. The features of DMMs have been investigated, and interesting properties emerge from a correct design such as long-range order correlations and topological robustness [53, 54]. If mathematical requirements described in [46] are fulfilled, then persistent os-
Self-organizing digital circuits (i.e., both SOLCs and SOACs), being a realization of DMMs, can in principle solve efficiently (i.e., with polynomial resources) complex combinatorial optimization problems, given that mathematical features on the constitutive equation are satisfied. Self-organizing digital circuits (i.e., both SOLCs and SOACs) have also been proved to efficiently handle a variety of combinatorial optimization problems ranging from maximum satisfiability (MAXSAT) to quadratic unconstrained binary optimization (QUBO) and from IP to training of neural networks.

V. SCALING RESULTS

In order to assess the scaling properties of solving the Airbus ALO problem employing MemCPU\textsuperscript{TM} we need to generate a set of meaningful benchmarks at different values of $N$ and $n$. Since we have no information about the actual values of $N$ and $n$ and not even on the ratio between number of containers at different sizes or typical mass distribution of the containers available, we use the sample data set provided by Airbus (see Table I for the data set, or it can also be found from [17]) to extrapolate. Notice that the data set provided from Airbus is just “for illustration and for testing the algorithm” [17].

In order to generate benchmarks for different sizes we need to generate a distribution of containers. One of the possible ways is to try to produce a distribution similar to that from the sample data set. In Fig. 4 the distributions of container masses from the data set in Table I is reported. Even though there are not enough data to certainly determine the distribution shape, it is reasonable to assume that both distributions can be recreated from a bimodal Gaussian distribution, cut off at certain boundaries.

In Fig. 4 I have included Matlab code that generates ALO problems for any $n$ and $N$ (it is also available a converter from Matlab to .mps format at [59]). The distribution of containers is generated in the subfunction container\_distribution\_generator. A bimodal Gaussian distribution of containers is generated for each size type. The ranges are chosen in such a way that the distributions in Fig. 4 can be qualitatively reproduced. It is also worth noticing that the masses are scaled depending on $N$. In fact, maintaining the same weight of the airplane and the same limit on the maximum weight, but increasing the number of bins, requires for consistency that each bin be scaled in size, and therefore we

| $k$ | $s_k$ | $m_k$ (kg) |
|-----|------|-----------|
| 1   | 1    | 2134      |
| 2   | 1    | 3455      |
| 3   | 1    | 1866      |
| 4   | 1    | 1699      |
| 5   | 1    | 3500      |
| 6   | 1    | 3332      |
| 7   | 1    | 2578      |
| 8   | 1    | 2315      |
| 9   | 1    | 1888      |
| 10  | 1    | 1786      |
| 11  | 1    | 3277      |
| 12  | 1    | 2987      |
| 13  | 1    | 2534      |
| 14  | 1    | 2111      |
| 15  | 1    | 2607      |
| 16  | 1    | 1566      |
| 17  | 1    | 1765      |
| 18  | 1    | 1946      |
| 19  | 1    | 1732      |
| 20  | 1    | 1641      |
| 21  | 2    | 1800      |
| 22  | 2    | 986       |
| 23  | 2    | 873       |
| 24  | 2    | 1764      |
| 25  | 2    | 1239      |
| 26  | 2    | 1487      |
| 27  | 2    | 769       |
| 28  | 2    | 836       |
| 29  | 2    | 659       |
| 30  | 2    | 765       |

TABLE I: Airbus ALO data set [17].
scale all container masses accordingly.

### A. MemCPU$^TM$ results

As discussed in section IV, MemCPU$^TM$ is an emulator of an electronic circuit with a given IP problem embedded within. However, in order to be efficient in solving the problem at hand, MemCPU$^TM$ needs a proper set of parameters characterizing electronic elements such as resistors, capacitors, etc. These parameters do not depend on the size of the problem (they are scale free) but depend on the structure of the problem. MemComputing currently provides a Monte Carlo routine that evaluates the distribution of the objective function of the IP problem as a function of the parameters [60]; however future releases will provide a predictor routine for parameters that will avoid the Monte Carlo-based tuning [60]. The objective function distribution is reported in Fig. 5. This distribution has been computed for the problem generated using the code in Fig. 4 using as input the container distribution in Table I and $x_{cg}^{\min} = -0.006$. The choice of $x_{cg}^{\min}$ is only for tuning purposes because this restricts the possible feasible solutions of the problem making it “harder.” In fact, it is easy to estimate the limit of the center of gravity $x_{cg}^{\lim}$ for any selection of containers ($\tau = 0$) from the available payload. In Fig. 6 the forces in the center of gravity evaluation (Eq. 4) are summarized for the limit of containers of all equal weight corresponding to the maximum weight allowed by the maximum shear curve. The value is $x_{cg}^{\lim} = -1/284 \approx -0.0035$ and I chose $x_{cg}^{\min} = -0.006$ that largely reduces the number of feasible solutions. This choice for $x_{cg}^{\min}$ leads to minimal feasible solutions that have objective far from maximum allowed carried freight $W_p$, however, used for tuning is a good choice because largely reduces the number of feasible solutions. This choice helps to produce a sharper distribution of objective function values versus parameters which also is useful for solving the problem in section III A.

| Time Constants | Static Constants |
|----------------|------------------|
| Time Const. 1  | Static Const. 1  | 0.79293 | 0.86027 |
| Time Const. 2  | Static Const. 2  | 0.28953 | 0.43655 |
| Time Const. 3  | Static Const. 3  | 0.19784 | 0.24784 |
| Time Const. 4  | Static Const. 4  | 0.85076 | 0.68352 |
| Time Const. 5  | Static Const. 5  | 0.42802 | 0.37106 |
| Time Const. 6  | Static Const. 6  | 0.39473 | 0.44440 |
| Time Const. 7  | Static Const. 7  | 0.76838 | 0.26562 |
| Static Const. 8 | Static Const. 8  | 0.21645 | 1.00000 |
| Static Const. 9 | Static Const. 9  | 0.88580 |
| Static Const. 10 | Static Const. 10 |  |  |

**TABLE II:** Parameters for MemCPU$^TM$ [49] extracted from the distribution of Fig. 5.
FIG. 6: Configuration of forces to evaluate how close the center of gravity \( x_{cg}^{lim} \) can approach the target \( x_{cg}^t = 0.1 \) (see Table I), without restriction on the minimum selected payload (\( \tau = 0 \)) and with maximum selected payload (\( \tau = 1 \)). For the maximum selected payload, the maximum density of 80000/L (container mass per unit of aircraft length) is assumed, consistent with the data set in Table I and the subfunction container_distribution_generator in Fig. 11.

From the objective distribution of Fig. 5, a set of the parameters for MemCPU\(^T^M\) can be easily extracted and used for running any other instance of the ALO problem. However, extracting a good set of parameters from the distribution can be done in multiple ways. In this work, the following procedure has been followed: from each Markov chain the set of parameters returning the best objective is selected. If there is more than one set of parameters with the same objective for the same Markov chain, the choice is made based upon the one that has the best average of the objectives related to the closest 10 sets of parameters within the same Markov chain. This super-selection rule comes from the fact that each MemCPU\(^T^M\) run starts with random initial conditions for voltages and currents, and therefore there could be a fluctuation in the output objective due to the finite (and short) simulation time [60]. Once we have the best parameter choice per Markov chain, we can run the same problem multiple times with random initial conditions for the same set of parameters and select the set of parameters that statistically arrives earlier to the best objective in a given time out. In this way, I extracted the set of parameters reported in Table II.

FIG. 7: MemCPU\(^T^M\) run time versus \( N \) to find a configuration of containers with total mass larger that than 99.9% of the maximum mass allowed on the aircraft or of the total payload if smaller than the maximum mass allowed. Dots connected by solid lines are the average time for 100 different ALO instances generated using the code in Fig. 11. Dashed curves are the scaling relation (13). Runs have been carried out on an Intel(R) Xeon(R) Gold 6138 CPU @ 2.00GHz.

FIG. 8: MemCPU\(^T^M\) run time versus \( n_l \) to find a configuration of containers with total mass larger that than 99.9% of the maximum mass allowed on the aircraft or of the total payload if smaller than the maximum mass allowed. Dots connected by solid lines are the average time for 100 different ALO instances generated using the code in Fig. 11. Dashed curves are the scaling relation (13). Runs have been carried out on an Intel(R) Xeon(R) Gold 6138 CPU @ 2.00GHz.

Using the parameters in Table II, MemCPU\(^T^M\) has been tested on a set of ALO instances generated for different \( n \) and \( N \) using the code in Fig. 11. The created benchmark consists of 100 ALO instances for each pair \((n, N)\). Each instance has a different container mass distribution randomly generated by the subfunction container_distribution_generator in Fig. 11.
tainer_distribution_generator of Fig. 11. For each instance, the container sizes were \( n_1 = \text{dim}(K_1) = n/2 \), \( n_2 = \text{dim}(K_2) = n/3 \) and \( n_3 = \text{dim}(K_3) = n/6 \) with appropriate rounding, in order to have integer \( n_j \) and \( n_1 + n_2 + n_3 = n \).

Since the value of the objective related to the global minimum of the ALO problem is not known and MemCPU\(^TM\) does not provide proof of optimality, I have set a very tight threshold to assess the quality of the solution: accept the solution if the objective function is smaller than \(-\tau W_p\) if \( W_p \leq \sum m_k \) or smaller than \(-\tau \sum m_k \) otherwise, with \( \tau = 0.999 \). This is equivalent to requiring a container selection that is larger than 99.9% of either the maximum weight allowed on the aircraft or of the total payload if the latter is smaller than the former. In Fig. 7 and 8 the time to find the solution for \( \tau = 0.999 \) is reported as a function of \( N \) (Fig. 7) or as a function of number \( n_l \) of nonzero entries in the matrix defined by constraints (2) (Fig. 8) for different ratios \( r = n_l/N \).

The results as a function of \( n_l \) are useful for properly assessing the MemCPU\(^TM\) scaling. Indeed, MemCPU\(^TM\) associates to each constraint a SOAG with a number of terminals equal to the terms in the constraint. For each terminal there is a dynamic correcting module (DCM) to simulate \( K_1 \). Therefore, the number of DCMs that MemCPU\(^TM\) simulates is equal to \( n_l \), and it is natural to measure the complexity in terms of \( n_l \) that is at the same time also a measure of the size of an IP problem. The interpolation curve on log-log scale in the plane \( n_l\)-time provides the following relation:

\[
t = 10^{-0.65r-4.8} n_l^{0.11r+1.25}
\]

(13)

where the dependence on \( r \) is valid for \( 0.5 \leq r \leq 3 \). The equation (13) shows a sub-quadratic scaling of the time to solution versus \( n_l \). From the (2) it is easy to realize that \( n_l \propto nN^2 \) and, substituting this relation in equation (13), the scaling as a function of \( N \) and \( n \) can be recovered.

It is worth discussing the dependence of Eq. (13) on \( r \). The first thing that can be noticed is that the dependence of \( n_l \) on \( r \) is very weak. The coefficient 0.11 in (13) makes the exponent range from 1.26 to 1.58 for \( 0.5 \leq r \leq 3 \). On top of the weak dependence, there is compensation from the stronger and negative dependence of the prefactor \( 10^{-0.65r-4.8} \) on \( r \). However, increasing \( r \), an evident dependence of the exponent on \( r \) disappears, and it saturates for \( r > 3 \). This is not surprising behavior because for larger \( r \) the problem becomes “easier” since there are many more choices of container selections that respect the constraints, and the burden in the calculation depends only on the size of the ALO. We do not show here numerical experiments to support the latter claim because considering \( n \) much larger the \( N \) seems meaningless in practical cases.

Let us consider now the ALO problem described in (section III A). In order to handle this problem just using a non-modified version of MemCPU\(^TM\), I have created a sequence of linear IP problems that converges to the solution of the non linear profit of (section III A). This sequence of problems can be easily created making use of the code of Fig. 11 as follows:

a. Set \( x_{cg}^{\min} \) and \( x_{cg}^{\max} \) for some initial values, for example, the ones given in Table I.

b. Generate the problem for a given \( x_{cg}^{\min} \) and \( x_{cg}^{\max} \).

c. Add to the matrix of the constraints the objective function of the problem as an extra constraint. As the right hand side of this constraint set \( \tau W^{\max} \) where \( W^{\max} = W_p \) if \( W_p \leq \sum m_k \) or \( W^{\max} = \sum m_k \) otherwise.

d. Substitute the objective function with a null objective function and solve the problem.

e. If a solution to the problem is found, evaluate \( x_{cg} \) by means of \( J, VL, V, x_{cg}^{\min} \) and \( x_{cg}^{\max} \) as reported in the code of figure 11.

f. if \( x_{cg} < x_{cg}^{t} \) set \( x_{cg}^{\min} = x_{cg} + \epsilon \) otherwise set \( x_{cg}^{\max} = x_{cg} - \epsilon \) for some \( \epsilon > 0 \). Go back to b.

In Fig. 9 the time to solution is reported for selecting the payload that maximizes the mass (\( \tau_{u} = 0.998 \) has been used in this case) and at the same time provides the center of gravity as close as possible to the target, for different value of \( N \) and \( n \). The dashed curves that interpolate this version of the ALO results as a shift of FIG. 9: MemCPU\(^TM\) run time versus \( N \) to find a configuration of containers that a) has total mass larger that 99.9% of the maximum mass allowed on the aircraft or of the total payload if smaller than the maximum mass allowed and b) optimizes the center of gravity of the aircraft. Dots connected by solid lines are the average time for 100 different ALO instances generated using the code in Fig. 11. Dashed curves are the scaling relation (13). Runs have been carried out on an Intel(R) Xeon(R) Gold 6138 CPU @ 2.00GHz.
FIG. 10: Center of gravity of the system aircraft + selected payload with total carried mass larger than 99.8% of $W_{\text{max}} = \min(W_p, \sum m_k)$. In the inset, the mean of the total mass generated by the subfunction container_distribution_generator of Fig. 11 and $W_p$ versus $r = n/N$ is plotted.

the equation (13), where the coefficient $10^{-0.65r-4.8}$ is replaced by $10^{-0.65r-4.2}$. Finding the same scaling properties as in the other problem is not surprising, since solving this ALO problem follows the steps a.-f., and so is a cascade of problems similar to the ones solved in Fig. 7.

Finally, it is worth noticing where the optimized centers of gravity are located. In Fig. 10 we can see that for $n = N/2$ the centers of gravity approach the first limit of Fig. 6. This is not surprising since in this case, because of the distribution of containers, $W_{\text{max}} \approx W_p/2$ (see inset of Fig. 10). On the other hand, if $n \geq N$, $W_{\text{max}} > W_p$ (see inset of Fig. 10) and the optimized centers of gravity approach the second limit of Fig. 6.

B. Further improvements

We briefly discuss here a few improvements that can be made in order to have an even more efficient time-to-solution for the Airbus ALO problem computed through MemCPU$^TM$. However, since they do not qualitatively affect the scaling assessed in this work, the implementation of these is beyond the current scope.

The first improvement concerns the initial values of the objective function (5). MemCPU$^TM$ searches solutions that have an objective equal to or lower than a target. Once found it decreases the objective and searches for the next solution (see section IV and [35]). In this work I have used infinite as the initial objective. With this choice, MemCPU$^TM$ provides many solutions at lower and lower objectives, progressing from the larger to smaller values. Even though this convergence is usually fast, this can be improved even more by giving as a starting value for the objective function $\tau \min(W_p, \sum m_k)$.

Another improvement that scales the time to solution down by a few orders of magnitude is running MemCPU$^TM$ on GPUs rather than CPUs [61]. In fact, the computation performed by MemCPU$^TM$ is nothing else than a circuit simulation that can efficiently distributed on GPUs [61].

Finally, the search for maximal carried freight that at the same time optimizes the center of gravity can be accelerated by implementing directly the nonlinear objective function (8), avoiding the sequence of ALO problems discussed in section VA and solving only one ALO problem.

VI. CONCLUSIONS

In summary, I have shown how to employ MemComputing through the software MemCPU$^TM$ to tackle the 5th Airbus problem efficiently without a quantum computer.

In order to use MemCPU$^TM$ for the 5th Airbus problem, which is an aircraft loading optimization problem, an IP problem has been formulated that includes all constraints required by Airbus with no exception and no approximation.

The scaling properties assessed in this work show subquadratic scaling of MemCPU$^TM$ as a function of the size of the problem measured by the number of nonzero elements of the constraint matrix of the associated IP problem.

The scaling properties of MemCPU$^TM$ allow efficient solutions of large ALO problems and it represents a solution ready to be deployed on the field.

Finally, I have also discussed extra improvements that can be made in order to further (and substantially) accelerate the time to solution for the ALO problem.

VII. ACKNOWLEDGMENTS

I thank the MemComputing team for useful discussions and in particular Dr. Tristan Sharp for the revision of the manuscript. This work has been supported by MemComputing, Inc.

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APPENDIX

function [problem, J, V, VL, xe_cg, We, input] = ...  
    Airbus_problem_generator(xmin_cg,xmax_cg,n1,n2,n3,N)

% airbus 5th problem generator

n = n1+n2+n3; % total number of containers
Wp = 40000; % Max payload
We = 120000; % Aircraft weight
xe_cg = -0.05; % Center of gravity position of the aircraft without payload
Smax_0 = 22000; % Max shear

% generate distributions of container mass

input = container_distribution_generator(n1,n2,n3,N);

type1 = find(input(:,2)==1);
type2 = find(input(:,2)==2);
type3 = find(input(:,2)==3);

indexM = zeros(n,N);
var = 0;
for j=1:n1
    indexM(type1(j),:) = var+(1:N);
    var = var+N;
end
for j=1:n2
    indexM(type2(j),:) = var+(1:N);
    var = var+N;
end
for j=1:n3
    indexM(type3(j),1:end-1) = var+(1:N-1);
    var = var+N-1;
end

% distance defined in Eq. (3)

d = zeros(n,N);
d(type1,:) = repmat(linspace(-N/2+1/2,N/2-1/2,N)/N,n1,1);
d(type2,:) = repmat(linspace(-N/2+1/2,N/2-1/2,N)/N,n2,1);
d(type3,1:end-1) = repmat(linspace(-N/2+1,N/2-1,N-1)/N,n3,1);

% initialize matrices for IP

Aineq = [];  
bineq = [];

% constraints on containers Eq. (2)

Aineq = [Aineq; 
    sparse(repmat((1:n1).’),1,N),indexM(type1,:),1,n1,var); 
    sparse(repmat((1:n2).’),1,N),indexM(type2,:),1,n2,var); 
    sparse(repmat((1:n3).’),1,N-1),indexM(type3,1:end-1),1,n3,var)];

bineq = [bineq; ones(n1+n2+n3,1)];
Aineq = [Aineq;
    sparse(1, indexM([type1; type2; type3], 1), [ones(n1, 1); ones(n2, 1)/2; ones(n3, 1)], 1,...
    var);
    sparse(repmat(1:N-2, n1+n2+2*n3, 1), [indexM([type1; type2; type3], 2:end-1);
    indexM(type3, 1:end-2)], [ones(n1, N-2); ones(n2, N-2)/2; ones(2*n3, N-2)], N-2, var);
    sparse(1, [indexM(type1, end); indexM(type2, end); indexM(type3, end-1)], [ones(n1, 1);
    ones(n2, 1)/2; ones(n3, 1)], 1, var)];

bineq = [bineq; ones(N, 1)];

% constraint on max weight
Aineq = [Aineq;
    sparse(1, indexM([type1; type2], :), repmat(input([type1; type2], 3), 1, N), 1, var)+...
    sparse(1, indexM(type3, 1:end-1), repmat(input(type3, 3), 1, N-1), 1, var)];

bineq = [bineq; Wp];

% constraint on the center of gravity
Aineq = [Aineq;
    sparse(1, indexM([type1; type2], :), repmat(-input([type1; type2], 3), 1, N), 1, var)+...
    sparse(1, indexM(type3, 1:end-1), repmat(-input(type3, 3), 1, N-1), 1, var)];

bineq = [bineq; -(xe_cg-xmax_cg)*We];

Aineq = [Aineq; Smax_0*j/floor(N/2)];

bineq = [bineq; Smax_0*j/floor(N/2)];

% objective function
f = full(-sparse(1, indexM([type1; type2], :), repmat(input([type1; type2], 3), 1, N), 1,...
    var)-sparse(1, indexM(type3, 1:N-1), repmat(input(type3, 3), 1, N-1), 1, var));
% generate output problem structure can be tested in Matlab using intlinprog

problem.Aeq = [];
problem.beq = [];
problem.Aineq = Aineq;
problem.bineq = bineq;
problem.f = f;
problem.lb = zeros(var,1);
problem.ub = ones(var,1);
problem.solver = 'intlinprog';
problem.options = [];
problem.intcon = 1:var;

% generate indexes to evaluate the center of gravity as
% x_cg = (sparse(1,J,VL)*X+xe_cg*We)./(sparse(1,J,V)*X+We)

J = indexM([type1; type2],:);
dummy = indexM(type3,1:end-1);
J = [J(:); dummy(:)];

VL = repmat(input([type1; type2],3),1,N).*d([type1; type2],:);
dummy = repmat(input(type3,3),1,N-1).*d(type3,1:end-1);
VL = [VL(:); dummy(:)];

V = repmat(input([type1; type2],3),1,N);
dummy = repmat(input(type3,3),1,N-1);
V = [V(:); dummy(:)];

function input = container_distribution_generator(n1,n2,n3,N)

w1 = ((3500-1500)/3*randn(n1*1000,1)+1500; (3500-1500)/3*randn(n1*1000,1)+3500));
w1 = round(w1((w1>1300)&(w1<3700))*20/N);

w2 = ((1800-700)/3*randn(n2*1000,1)+700; (1800-700)/3*randn(n2*1000,1)+1800));
w2 = round(w2((w2>500)&(w2<2000))*20/N);

w3 = ((7000-3200)/3*randn(n3*1000,1)+3200; (7000-3200)/3*randn(n3*1000,1)+7000));
w3 = round(w3((w3>3000)&(w3<7200))*20/N);

input = [(1:n1).' ones(n1,1) w1(randperm(length(w1),n1));
n1+(1:n2).' 2*ones(n2,1) w2(randperm(length(w2),n2));
n1+n2+(1:n3).' 3*ones(n3,1) w3(randperm(length(w3),n3))];

FIG. 11: Matlab code to generate Airbus ALO problems of any size.