On Einstein-Hilbert Type Action of Superon-Graviton Model (SGM)

Kazunari SHIMA and Motomu TSUDA

Laboratory of Physics, Saitama Institute of Technology
Okabe-machi, Saitama 369-0293, Japan

December 2001

Abstract

The fundamental action of superon-graviton model (SGM) of Einstein-Hilbert type for space-time and matter is written down explicitly in terms of the fields of the graviton and superons by using the affine connection formalism and the spin connection formalism. Some characteristic structures including some hidden symmetries of the gravitational coupling of superons are manifested (in two dimensional space-time) with some details of the calculations. SGM cosmology is discussed briefly.

PACS: 04.50.+h, 12.60.Jv, 12.60.Rc, 12.10.-g / Keywords: supersymmetry, graviton, Nambu-Goldstone fermion, unified theory

*e-mail: shima@sit.ac.jp, tsuda@sit.ac.jp
1. Introduction

The standard model (SM) is established as a unified model for the strong-electroweak interaction, although interestingly it is still unsatisfactory in many aspects, e.g., it can not explain the particle quantum numbers \((Q_e, I, Y, \text{color})\) and the three-generations structure and contains more than 28 arbitrary parameters (in the case of neutrino oscillations) even disregarding the mass generation mechanism for neutrino. It seems inevitable to introduce new particles and new (gauge) symmetries for exploring the new physics and the new framework for the unification of space-time and matter beyond SM.

Supersymmetry (SUSY)\(^\text{[1]-[3]}\) may be the most promising gauge symmetry beyond SM, especially for the unification of space-time and matter. In fact the theory of supergravity (SUGRA) is constructed based upon the local SUSY, which brings the breakthrough for the unification of space-time and matter\(^\text{[4]}\). Nambu-Goldstone (N-G) fermion \(^\text{[5]}\) would appear in the spontaneous SUSY breaking and plays essentially important roles in the unified model building.

Here it is useful to distinguish the qualitative differences of the origins of N-G fermion. In O’Raifeartaigh model\(^\text{[6]}\) N-G fermion stems from the symmetry of the dynamics (interaction) of the linear representation multiplet of SUSY, i.e., it corresponds to the coset space coordinates of \(G/H\) where \(G\) and \(H\) are expressed on the field operators. While in Volkov-Akulov model\(^\text{[3]}\) N-G fermion stems from the symmetry (breaking) of spacetime \(G/H\) in terms of the (supersymmetric) geometrical arguments and gives the nonlinear (NL) representation of SUSY\(^\text{[8]}\).

As demonstrated in supergravity (SUGRA) coupled with Volkov-Akulov model it is rather well understood in the linear realization of SUSY (L SUSY) that N-G fermion is converted to the longitudinal component of spin 3/2 gravitino field by the super-Higgs mechanism and breaks local linear SUSY spontaneously by giving mass to gravitino\(^\text{[4]}\). N-G fermion degrees of freedom become unphysical in the low energy. The SM and grand unified theory (GUT) equipped naively with supersymmetry (SUSY) have revealed the remarkable features, e.g., the unification of the gauge couplings at about \(10^{17}\), relatively stable proton (now threatened by experiments), etc., but they contain more than 100 arbitrary parameters and less predictive powers and the gravity is out of the scope. While, considering seriously the fact that SUSY is naturally connected to spacetime symmetry, it may be interesting to survey other possibilities concerning how SUSY is realized and where N-G fermion has gone (in the low energy).

In Ref.\(^\text{[7]}\), N-G fermion is considered as the fundamental constituents of quarks and leptons and the field theoretical description of the model is attempted. In the previous paper \(^\text{[10]}\) we have introduced a new fundamental constituent with spin 1/2 superon and proposed superon-graviton model (SGM) equipped with NL SUSY as a model for unity of space-time and matter. In SGM, the fundamental entities of nature are the graviton with spin-2 and a quintet of superons with spin-1/2. They
are the elementary gauge fields corresponding to the ordinary local GL(4,R) and the
global nonlinear supersymmetry(NL SUSY) with a global SO(10), i.e. N=10 V-A
model, respectively. Interestingly, the quantum numbers of the superon-quintet are
the same as those of the fundamental representation 5 of the matter multiplet of
SU(5) GUT[11]. All observed elementary particles including gravity are assigned to
a single irreducible massless representation of SO(10) super-Poincaré(SP) symmetry
and reveals a remarkable potential for the phenomenology, e.g. they may explain
naturally the three-generations structure of quarks and leptons, the stability of pro-
ton, various mixings, ..etc[10]. And in SGM except graviton they are supposed to
be the (massless) eigenstates of superons of SO(10) SP symmetry [12] of space-time
and matter. The uniqueness of N=10 among all SO(N) SP is pointed out. The
arguments are group theoretical so far.

In order to obtain the fundamental action of SGM which is invariant at least under
local GL(4,R), local Lorentz, global NL SUSY transformations and global SO(10),
we have performed the similar geometrical arguments to Einstein general relativity
theory(EGRT) in high symmetric SGM space-time, where the tangent (Riemann-
flat) space-time is specified by the coset space SL(2,C) coordinates (corresponding
to N-G fermion) of NL SUSY of Volkov-Akulov(V-A)[3] in addition to the ordinary
Lorentz SO(3,1) coordinates[10], which are locally homomorphic groups[13]. As
shown in Ref.[13] the SGM action for the unified SGM space-time is defined as the
geometrical invariant quantity and is naturally the analogue of Einstein-Hilbert(E-
H) action of general relativity(GR) which has the similar concise expression. And in-
terestingly it may be regarded as a kind of a generalization of Born-Infeld action[15].
(The similar systematic arguments are applicable to spin 3/2 N-G case.[14])

In this article which is an evolved version of Ref. [24], after a brief review of SGM
for the self contained arguments we write down SGM action in terms of the fields
of graviton and superons in order to see some characteristic structures of our model
and also show some details of the calculations.

For the sake of the comparison the expansion is performed by the affine connection
formalism and by the spin connection formalism.

Finally some hidden symmetries and a potential cosmology, especially the birth of
the universe are mentioned briefly.

2. Fundamental action of superon-graviton model(SGM)

In Ref.[13], SGM space-time is defined as the space-time whose tangent(flat) space-
time is specified by SO(1,3) Lorentz coordinates $x^a$ and the coset space SL(2,C)
coordinates $\psi$ of NL SUSY of Volkov-Akulov(V-A)[3]. The unified vierbein $w^a_{\mu}$
and the unified metric $s_{\mu\nu}(x) \equiv w^a_{\mu}(x)w_{\alpha\nu}(x)$ of SGM space-time are defined by
generalizing the NL SUSY invariant differential forms of V-A to the curved space-
time\[13\]. SGM action is given as follows\[13\]

\[
L_{SGM} = -\frac{c^3}{16\pi G}|w|(\Omega + \Lambda),
\]

(1)

where \( \kappa(= \kappa_{V-A}) \) is an arbitrary constant up now with the dimension of the fourth power of length, \( e^{\alpha}_{\mu}(x) \) and \( \psi^j(x)(j = 1, 2, ..., 10) \) are the fundamental elementary fields of SGM, i.e. the vierbein of Einstein general relativity theory (EGRT) and the superons of N-G fermion of NL SUSY of Volkov-Akulov\[3\], respectively. \( \Lambda \) is a cosmological constant which is necessary for SGM action to reduce to V-A model with the first order derivative terms of the superon in the Riemann-flat space-time. \( \Omega \) is a unified scalar curvature of SGM space-time analogous to the Ricci scalar curvature \( R \) of EGRT.

SGM action (1) is invariant under the following new SUSY transformations

\[
\delta \psi^j(x) = \zeta^i + i\kappa(\bar{\zeta}^j\gamma^\rho \psi^j(x))\partial_\rho \psi^i(x),
\]

(3)

\[
\delta e^a_{\mu}(x) = i\kappa(\bar{\zeta}^j\gamma^\rho \psi^j(x))D_\rho e^a_{\mu}(x),
\]

(4)

where \( \zeta^i, (i = 1, ..., 10) \) is a constant spinor parameter, \( D_\rho e^a_{\mu}(x) = D_\mu e^a_{\rho} - D_\rho e^a_{\mu} \) and \( D_\mu \) is a covariant derivative containing a symmetric affine connection. The explicit expression of \( \Omega \) is obtained by just replacing \( e^a_{\mu}(x) \) in Ricci scalar \( R \) of EGRT by the unified vierbein \( w^a_{\mu}(x) = e^a_{\mu} + t^a_{\mu} \) of the SGM curved space-time, which gives the gravitational interaction of \( \psi(x) \) invariant under (3) and (4). The invariance can be easily understood by observing that under (3) and (4) the new vierbein \( w^a_{\mu}(x) \) and the new metric \( s_{\mu\nu}(x) \) have general coordinate transformations\[13\].

\[
\delta \zeta w^a_{\mu} = \xi^\nu \partial_\nu w^a_{\mu} + \partial_\mu \xi^\nu w^a_{\nu},
\]

(5)

\[
\delta \zeta s_{\mu\nu} = \xi^\kappa \partial_\kappa s_{\mu\nu} + \partial_\mu \xi^\kappa s_{\kappa\nu} + \partial_\nu \xi^\kappa s_{\mu\kappa},
\]

(6)

where \( \xi^a = i\kappa(\bar{\zeta}^j\gamma^\rho \psi^j(x)) \). The overall factor of SGM action is fixed to \( -\frac{c^3}{16\pi G} \), which reproduces E-H action of GR in the absence of superons (matter). Also in the Riemann-flat space-time, i.e. \( e^a_{\mu}(x) \to \delta^a_{\mu} \), it reproduce V-A action of NL SUSY\[3\] with \( \kappa_{V-A}^{-1} = \frac{c^3}{16\pi G} \Lambda \) in the first order derivative terms of the superon.

\[\dagger\] We use in this paper the Minkowski tangent space metric \( \frac{1}{2}\{\gamma^a, \gamma^b\} = \eta^{ab} = (+, -, -, -) \) and \( \sigma^{ab} = \frac{1}{2}[\gamma^a, \gamma^b] \). Latin \((a, b, ..)\) and Greek \((\mu, \nu, ..)\) are the indices for local Lorentz and general coordinates, respectively.

4
Therefore our model (SGM) predicts a small non-zero cosmological constant, provided $\kappa \sim O(1)$, and possesses two mass scales. Furthermore it fixes the coupling constant of superon (N-G fermion) with the vacuum to \((\frac{a^2}{\text{Planck}} \Lambda)^{\frac{1}{2}}\) (from the low energy theorem viewpoint), which may be relevant to the birth (of the matter and Riemann space-time) of the universe.

It is interesting that our action is the vacuum (matter free) action in SGM space-time as read off from (1) but gives in ordinary Riemann space-time the E-H action with matter (superons) accompanying the spontaneous supersymmetry breaking.

The commutators of new SUSY transformations induces the generalized general coordinate transformations

\[
[\delta_{C_1}, \delta_{C_2}]\psi = \Xi^\mu \partial_\mu \psi, \quad [\delta_{C_1}, \delta_{C_2}] e^a_\mu = \Xi^\rho \partial_\rho e^a_\mu + e^a_\rho \partial_\mu \Xi^\rho,
\]

where $\Xi^\mu$ is defined by

\[
\Xi^\mu = 2i\kappa (\bar{\zeta}_2 \gamma^\mu \zeta_1) - \xi_1^a \xi_2^b (D_\mu e^a_\sigma). \quad (9)
\]

In addition, to embed simply the local Lorentz invariance we follow EGRT formally and require that the new vierbein $w^a_\mu(x)$ should also have formally a local Lorentz transformation, i.e.,

\[
\delta_L w^a_\mu = e^a_b w^b_\mu \quad (10)
\]

with the local Lorentz transformation parameter $\epsilon_{ab}(x) = (1/2)\epsilon_{[ab]}(x)$. Interestingly, we find that the following generalized new local Lorentz transformations on $\psi$ and $e^a_\mu$:

\[
\delta_L \psi(x) = -i \epsilon_{ab} \sigma^{ab} \psi, \quad \delta_L e^a_\mu(x) = e^a_\sigma e^b_\mu + \frac{\kappa}{4} \epsilon^{abcd} \bar{\psi} \gamma^5 \gamma^\sigma \gamma^\tau \psi (\partial_\mu \epsilon_{bc}), \quad (11)
\]

are compatible with (10). [ Note that the second term in $\delta_L e^a_\mu(x)$ is a new term and that the equation (11) reduces to the familiar form of the Lorentz transformations if the global transformations are considered, e.g., $\delta_L g^{\mu\nu} = 0$.] The local Lorentz transformation forms a closed algebra, for example, on $e^a_\mu(x)$

\[
[\delta_{L_1}, \delta_{L_2}] e^a_\mu = \beta^a e^b_\mu + \frac{\kappa}{4} \epsilon^{abcd} \bar{\psi} \gamma^5 \gamma^\sigma \gamma^\tau \psi (\partial_\mu \epsilon_{bc}), \quad (12)
\]

where $\beta_{ab} = -\beta_{ba}$ is defined by $\beta_{ab} = \epsilon_{2ac} \epsilon_{1b} - \epsilon_{2bc} \epsilon_{1a}$.

We have shown that our action is invariant at least under

\[
[\text{global NL SUSY}] \otimes [\text{local GL}(4, R)] \otimes [\text{local Lorentz}] \otimes [\text{global SO(N)}], \quad (13)
\]

which is isomorphic to N=10 extended (global SO(10)) SP symmetry through which SGM reveals the spectrum of all observed particles in the low energy. In contrast
with the ordinary SP SUSY, new SUSY may be regarded as a square root of a generalized GL(4,R). The usual local GL(4,R) invariance is obvious by the construction. The simple expression (1) invariant under the above symmetry may be universal for the gravitational coupling of Nambu-Goldstone(N-G) fermion, for by performing the parallel arguments we obtain the same expression for the gravitational interaction of the spin-3/2 N-G fermion\[14\].

Now to clarify the characteristic features of SGM we focus on N=1 SGM for simplicity without loss of generality and write down the action explicitly in terms of $t^a_\mu$(or $\psi$) and $g^{\mu\nu}$(or $e^a_\mu$). We will see that the graviton and superons(matter) are complementary in SGM and contribute equally to the curvature of SGM space-time. Contrary to its simple expression (1), it has rather complicated and rich structures. To obtain (1) we require that the unified action of SGM space-time should reduce to V-A in the flat space-time which is specified by $x^a$ and $\psi(x)$ and that the graviton and superons contribute equally to the unified curvature of SGM space-time. We have found that the unified vierbein $w^a_\mu(x)$ and the unified metric $s_{\mu\nu}(x)$ of unified SGM space-time are defined through the NL SUSY invariant differential forms $\omega^a$ of V-A\[3\] as follows:

$$\omega^a = w^a_\mu dx^\mu,$$

$$w^a_\mu(x) = e^a_\mu(x) + t^a_\mu(x),$$

where $e^a_\mu(x)$ is the vierbein of EGRT and $t^a_\mu(x)$ is defined by

$$t^a_\mu(x) = i\kappa\bar{\psi}\gamma^a \partial_\mu \psi,$$

where the first and the second indices of $t^a_\mu$ represent those of the $\gamma$ matrices and the general covariant derivatives, respectively. We can easily obtain the inverse $w^a_\mu$ of the vierbein $w^a_\mu$ in the power series of $t^a_\mu$ as follows, which terminates with $t^4$(for 4 dimensional space-time):

$$w^a_\mu = e^a_\mu - t^\mu a + t^a_\rho t^\rho_\mu - t^a_\sigma t^\sigma_\mu t^\mu_\rho + t^a_\delta t^\sigma_\rho t^\kappa_\sigma t^\mu_\kappa.$$  (17)

Similarly a new metric tensor $s_{\mu\nu}(x)$ and its inverse $s^{\mu\nu}(x)$ are introduced in SGM curved space-time as follows:

$$s_{\mu\nu}(x) \equiv w^a_\mu(x)w^a_\nu(x) = w^a_\mu(x)\eta_{ab}w^b_\nu(x)$$

$$= g_{\mu\nu} + t_{\mu\nu} + t_{\nu\mu} + t^a_\rho t^b_\mu t^b_\rho.$$  (18)

and

$$s^{\mu\nu}(x) \equiv w^a_\mu(x)w^{ab}(x)$$
We can easily show
\[
\begin{align*}
= g^{\mu\nu} \\
- t^{\mu\nu} - t^{\nu\mu} \\
+ t^{\mu\rho} t^{\nu}_{\rho} + t^{\rho\mu} t^{\nu}_{\rho} + t^{\mu\rho} t^{\nu}_{\rho} \\
- t^{\mu\nu} t^{\rho}_{\rho} - t^{\rho\nu} t^{\mu}_{\rho} - t^{\mu\nu} t^{\rho}_{\rho} - t^{\rho\nu} t^{\mu}_{\rho} - t^{\mu\rho} t^{\nu}_{\rho} \\
+ t^{\mu\nu} t^{\rho}_{\rho} t^{\sigma}_{\sigma} + t^{\rho\nu} t^{\mu}_{\rho} t^{\sigma}_{\sigma} + t^{\mu\nu} t^{\rho}_{\rho} t^{\sigma}_{\sigma} + t^{\rho\nu} t^{\mu}_{\rho} t^{\sigma}_{\sigma} + t^{\mu\nu} t^{\rho}_{\rho} t^{\sigma}_{\sigma} + t^{\rho\nu} t^{\mu}_{\rho} t^{\sigma}_{\sigma}.
\end{align*}
\]

It is obvious from the above general covariant arguments that (1) is invariant under \(\psi\) and \(e\) variant terms with the (second order) derivatives of the superon beyond V-A model.

By using (15), (17), (18) and (19) we can express SGM action (1) in terms of the ordinarily GL(4,R) and under (3) and (4).

The existence of (in the Riemann-flat space-time) NL SUSY expansions show the complementary relation of graviton and (the stress-energy tensors of GR).

Remarkably the first term can be regarded as a space-time dependent cosmological term and reduces to V-A action [3] containing up to \(O(t^4)\) and \(R\) and \(R_{\mu\nu}\) are the Ricci curvature tensors of GR.

Remarkably the first term can be regarded as a space-time dependent cosmological term and reduces to V-A action [3] containing up to \(O(t^4)\) and \(R\) and \(R_{\mu\nu}\) are the Ricci curvature tensors of GR.

By using (15), (17), (18) and (19) we can express SGM action (1) in terms of \(e^a_{\mu}(x)\) and \(\psi^a(x)\), which describes explicitly the fundamental interaction of graviton with superons. The expansion of the action in terms of the power series of \(\kappa\) (or \(t^a_{\mu}\)) can be carried out straightforwardly. After the lengthy calculations concerning the complicated structures of the indices we obtain

\[
L_{SGM} = - \frac{c^3 A}{16\pi G} e |w_{V-A}| - \frac{c^3}{16\pi G} e R
\]

\[
+ \frac{c^3}{16\pi G} e \left[ 2t^{(\mu\nu)} R_{\mu\nu} \\
+ \frac{1}{2} \left( g^{\mu\nu} \partial^\rho \partial_{(\mu\nu)} - t_{(\mu\nu)} \partial^\rho g^{\mu\nu} \\
+ t^{\nu\rho} \partial^\mu g^{\nu\rho} - 2g^{\mu\nu} \partial^\rho t_{(\mu\nu)} \partial^\sigma g_{\rho\sigma} - g^{\rho\sigma} \partial^\nu t_{(\rho\sigma)} \partial^\mu g_{\nu\sigma} \\
+ (t^{\mu\rho} t^{\nu\sigma} + t^{\nu\rho} t^{\mu\sigma} + t^{\mu\rho} t^{\nu\sigma} )R_{\beta\mu} - (2t^{(\mu\rho)} t^{(\nu\sigma)} R_{\mu\nu} + t^{(\mu\rho)} t^{(\nu\sigma)} R_{\mu\nu}) \\
+ \frac{1}{2} t^{(\mu\nu)} (g^{\rho\sigma} \partial^\mu t_{(\rho\sigma)} - g^{\rho\sigma} \partial^\mu t_{(\rho\sigma)} + \ldots ) \\
+ O(t^3) + \ldots + O(t^{10}) \right].
\]

where \(e = det e^\mu_{\mu}, t^{(\mu\nu)} = t^{\mu\nu} + t^{\nu\mu}, t_{(\mu\nu)} = t_{\mu\nu} + t_{\nu\mu}, \) and \(|w_{V-A}| = det w^a_{b}\) is the flat space V-A action [3] containing up to \(O(t^4)\) and \(R\) and \(R_{\mu\nu}\) are the Ricci curvature tensors of GR.

The existence of (in the Riemann-flat space-time) NL SUSY invariant terms with the (second order) derivatives of the superons beyond V-A model.
are manifested. For example, the lowest order of such terms appear in $O(t^2)$ and have the following expressions (up to the total derivative terms)

$$+ \epsilon^{abcd} \epsilon^e_{fg} \partial_c t_{(e)} \partial_d t_{(g)}.$$  \hspace{1cm} (22)

The existence of such derivative terms in addition to the original V-A model are already pointed out and exemplified in part in [18]. Note that (22) vanishes in 2 dimensional space-time.

Here we just mention that we can consider two types of the flat space in SGM, which are not equivalent. One is SGM-flat, i.e. $w_a^\mu(x) \rightarrow \delta_a^\mu$, space-time and the other is Riemann-flat, i.e. $e_a^\mu(x) \rightarrow \delta_a^\mu$, space-time, where SGM action reduces to $-\frac{c^3}{16\pi G}$ and $-\frac{c^3}{16\pi G}|w_{V-A}| - \frac{c^3}{16\pi G}(\text{derivative terms})$, respectively. Note that SGM-flat space-time allows non trivial Riemann space-time.

3. SGM in two dimensional space-time

It is well known that two dimensional GR has no physical degrees of freedom (due to the local GL(2,R)). SGM in SGM space-time is also the case. However the arguments with the general covariance shed light on the characteristic off-shell gauge structures of the theory in any space-time dimensions. Especially for SGM, it is also useful for linearizing the theory to see explicitly the superon-graviton coupling in (two dimensional) Riemann space-time. The result gives the correct expansion up to $O(t^2)$ in four dimensional space-time as well.

3.1 SGM in affine connection formalism

Now we go to two dimensional SGM space-time to simplify the arguments without loss of generality and demonstrate some details of the computations. We adopt firstly the affine connection formalism. The knowledge of the complete structure of SGM action including the surface terms is useful to linearize SGM into the equivalent linear theory and to find the symmetry breaking of the model.

Following EGRT the scalar curvature tensor $\Omega$ of SGM space-time is given as follows

$$\Omega = s^\beta \mu \Omega^\alpha_{\beta \mu \alpha}$$

$$= s^\beta \mu \{ \partial_\mu \Gamma^\lambda_{\beta \alpha} + \Gamma^\alpha_{\lambda \mu} \Gamma^\lambda_{\beta \alpha} \} - \{ \text{lower indices}(\mu \leftrightarrow \alpha) \}, \hspace{1cm} (23)$$

where the Christoffel symbol of the second kind of SGM space-time is

$$\Gamma^\alpha_{\beta \mu} = \frac{1}{2} s^{\alpha \rho} \{ \partial_\beta s_{\rho \mu} + \partial_\mu s_{\beta \rho} - \partial_\rho s_{\mu \beta} \}. \hspace{1cm} (24)$$

The straightforward expression of SGM action [11] in two dimensional space-time, (which is $3^6$ times more complicated than the two dimensional GR), is given as follows

$$L_{2dSGM} = -\frac{c^3}{16\pi G} e \{ 1 + \ell_a^t a + \frac{1}{2} (\ell_a^t b - \ell_a b t)^b \} (g^{\beta \mu} - \ell^{(\beta \mu)} + \ell^2 (\beta \mu)).$$
\[
\times \left\{ \frac{1}{2} \partial_{\mu} (g^{\alpha \sigma} - \tilde{t}^{(\alpha \sigma)} + \tilde{t}^2 (\alpha \sigma)) \partial_{\beta} (g_{\delta \dot{\alpha}} + \dot{t}_{(\delta \dot{\alpha})} + \dot{t}^2 (\delta \dot{\alpha})) \\
+ \frac{1}{2} (g^{\alpha \sigma} - \tilde{t}^{(\alpha \sigma)} + \tilde{t}^2 (\alpha \sigma)) \partial_{\mu} \partial_{\beta} (g_{\delta \dot{\alpha}} + \dot{t}_{(\delta \dot{\alpha})} + \dot{t}^2 (\delta \dot{\alpha})) \right\} \\
- \left\{ \text{lower indices} (\mu \leftrightarrow \alpha) \right\} \\
+ \left\{ \frac{1}{4} (g^{\alpha \sigma} - \tilde{t}^{(\alpha \sigma)} + \tilde{t}^2 (\alpha \sigma)) \partial_{\lambda} (g_{\delta \dot{\mu}} + \dot{t}_{(\delta \dot{\mu})} + \dot{t}^2 (\delta \dot{\mu})) \\
(\tilde{g}_{\lambda \rho} - \tilde{t} (\lambda \rho) + \tilde{t}^2 (\lambda \rho)) \partial_{\beta} (g_{\rho \dot{\nu}} + \dot{t}_{(\rho \dot{\nu})} + \dot{t}^2 (\rho \dot{\nu})) \right\} \\
- \left\{ \text{lower indices} (\mu \leftrightarrow \alpha) \right\} \\
- \frac{c^3 \Lambda}{16\pi G} e^{w_{V - A}},
\]

(25)

where we have put

\[
s_{\alpha \beta} = g_{\alpha \beta} + t_{(\alpha \beta)} + \dot{t}^2 (\alpha \beta), \quad s^{\alpha \beta} = g^{\alpha \beta} - \tilde{t}^{(\alpha \beta)} + \tilde{t}^2 (\alpha \beta),
\]

\[
t_{(\mu \nu)} = t_{\mu \nu} + t_{\nu \mu}, \quad \dot{t}^2_{(\mu \nu)} = t^\rho_{\mu} t_{\rho \nu},
\]

\[
\tilde{t}^{(\mu \nu)} = t^{\mu \nu} + \dot{t}^\rho_{\mu} t^\mu_{\rho \nu}, \quad \tilde{t}^2(\mu \nu) = t^\rho_{\mu} t^\mu_{\rho \nu} + t^\nu_{\mu} t^\mu_{\nu \rho} + t^\mu_{\rho \mu} t^\nu_{\nu \rho},
\]

(26)

and the Christoffel symbols of the first kind of SGM space-time contained in (24) are abbreviated as

\[
\partial_{\mu} g_{\sigma \nu} = \partial_{\mu} g_{\sigma \nu} + \partial_{\nu} g_{\mu \sigma} - \partial_{\sigma} g_{\mu \nu},
\]

\[
\partial_{\mu} t_{\delta \nu} = \partial_{\mu} t_{\delta \nu} + \partial_{\nu} t_{\mu \delta} - \partial_{\delta} t_{\mu \nu},
\]

\[
\partial_{\mu} t^2_{\sigma \nu} = \partial_{\mu} t^2_{\sigma \nu} + \partial_{\nu} t^2_{\mu \sigma} - \partial_{\sigma} t^2_{\mu \nu}.
\]

(27)

By expanding the scalar curvature $\Omega$ in the power series of $t$ which terminates with $t^4$, we have the following complete expression of two dimensional SGM,

\[
L_{2dSGM} = -\frac{c^3 \Lambda}{16\pi G} e^{w_{V - A}}
-\frac{c^3}{16\pi G} e^{w_{V - A}} [R]
-2\tilde{t}^{(\mu \nu)} R_{\mu \nu}
+\frac{1}{2} \left\{ g^{\mu \nu} \partial^\rho \partial_{\rho} t_{\mu \nu} - \tilde{t}^{(\mu \nu)} \partial^\rho \partial_{\rho} g_{\mu \nu} \\
+g^{\mu \nu} \partial^\rho t_{\rho \mu \nu} - 2g^{\mu \nu} \partial^\rho \partial_{\rho} t_{\mu \nu} - g^{\mu \nu} \partial^\rho g_{\rho \mu \nu} - g^{\mu \nu} g^{\rho \sigma} \partial_{\rho \sigma} t_{\mu \nu} \partial_{\nu} g_{\rho \mu} \\
+\dot{t}^2 t_{(\mu \nu)} R_{\beta \mu}
\right\}
\]

(28)
\[ + \tilde{\epsilon}^{(\beta \mu)} \tilde{\epsilon}^{(\alpha \sigma)} R_{\mu \alpha \sigma \beta} \]
\[ - \frac{1}{2} \tilde{\epsilon}^{(\beta \mu)} \{ g^{\alpha \sigma} \partial_\mu \partial_\beta \tilde{t}^{(\alpha \sigma)} - \partial^\sigma \partial_\beta t^{(\alpha \sigma)} + \partial_\mu \tilde{\epsilon}^{(\alpha \sigma)} \partial_\beta g_{\alpha \sigma} - \partial_\mu g^{\alpha \sigma} \partial_\beta t^{(\alpha \sigma)} \}
\[ + \partial_\alpha g^{\alpha \sigma} \partial_\beta t^{(\alpha \sigma)} - \partial_\alpha \tilde{t}^{(\alpha \sigma)} \partial_\beta g_{\alpha \sigma} + 2 \partial^\rho t^{(\alpha \sigma)} \partial_\beta g_{\alpha \rho} \]
\[ - 2 g^{\alpha \sigma} \partial_\lambda \tilde{t}^{(\alpha \sigma)} \partial_\beta g_{\alpha \beta} \]
\[ + g^{\alpha \sigma} g^{\lambda \rho} \partial_\mu \tilde{x}^{(\lambda \rho)} \partial_\beta g_{\alpha \sigma} - 2 g^{\alpha \sigma} \partial_\mu \tilde{t}^{(\alpha \sigma)} \partial_\beta g_{\mu \beta} + g^{\alpha \sigma} \partial_\lambda \tilde{t}^{(\alpha \sigma)} \partial_\beta g_{\alpha \beta} \}
\[ - g^{\beta \mu} \partial_\mu (g^{\alpha \sigma} \partial_\beta \tilde{t}^{(\alpha \sigma)} \partial_\sigma g_{\alpha \beta} - \tilde{\epsilon}^{(\alpha \sigma)} \partial_\beta \tilde{t}^{(\alpha \sigma)} - \tilde{\epsilon}^{(\alpha \sigma)} (2 \partial_\beta t^{(\alpha \sigma)} - \partial_\sigma t^{(\alpha \sigma)}))
\[ + g^{\beta \mu} \partial_\alpha \{ g^{\alpha \sigma} (2 \partial_\beta \tilde{t}^{(\alpha \sigma)} - \partial_\sigma t^{(\alpha \sigma)}) + \tilde{\epsilon}^{(\alpha \sigma)} (2 \partial_\beta g_{\alpha \beta} - \partial_\sigma g_{\alpha \beta}) \}
\[ + 2 \partial^\rho g_{\lambda \mu} g^{\beta \lambda} (2 \partial_\mu \tilde{t}^{(\alpha \sigma)} - \partial_\alpha t^{(\alpha \sigma)} g^{\beta \mu} + 2 \partial^\lambda \partial_\lambda g_{\alpha \mu} g^{\beta \mu} (2 \partial^\sigma g_{\beta \mu} - \partial_\rho g_{\mu \beta})
\[ - 2 \tilde{\epsilon}^{(\lambda \rho)} \partial_\lambda \tilde{t}^{(\lambda \rho)} g^{\beta \mu} (2 \partial^\sigma g_{\beta \mu} - \partial_\rho g_{\mu \beta}) \}
\[ - 2 \tilde{\epsilon}^{(\alpha \sigma)} \partial_\alpha \tilde{t}^{(\alpha \sigma)} g^{\beta \mu} (2 \partial^\sigma g_{\beta \mu} - \partial_\rho g_{\mu \beta}) \]
\[ - \partial_\rho g_{\alpha \sigma} g^{\alpha \sigma} (2 \partial_\mu \tilde{t}^{(\alpha \sigma)} - \partial_\alpha t^{(\alpha \sigma)} g^{\beta \mu}) - \partial^\rho \tilde{\epsilon}^{(\alpha \sigma)} (2 \partial^\mu g_{\rho \mu} g^{\alpha \sigma} - \partial_\lambda g_{\lambda \sigma} g^{\beta \mu})
\[ - \tilde{\epsilon}^{(\lambda \rho)} \partial_\lambda \tilde{t}^{(\lambda \rho)} g^{\alpha \sigma} (2 \partial^\mu g_{\rho \mu} - \partial_\rho g_{\mu \beta}) - \tilde{\epsilon}^{(\lambda \rho)} \partial_\lambda \tilde{t}^{(\lambda \rho)} g^{\alpha \sigma} (2 \partial^\mu g_{\rho \mu} - \partial_\rho g_{\mu \beta})
\[ + \tilde{\epsilon}^{(\alpha \sigma)} \tilde{\lambda}^{(\rho \mu \beta)} (2 \partial^\mu g_{\rho \mu} - \partial_\rho g_{\mu \beta}) \]
\[ + \tilde{\epsilon}^{(\alpha \sigma)} \tilde{\lambda}^{(\rho \mu \beta)} (2 \partial^\mu g_{\rho \mu} - \partial_\rho g_{\mu \beta}) \]
\[ + \frac{1}{2} \tilde{\epsilon}^{(\beta \mu)} \{ g^{\alpha \sigma} \partial_\mu \partial_\beta \tilde{t}^{(\alpha \sigma)} \}
\[ - \partial^\sigma \partial_\beta \tilde{t}^{(\alpha \sigma)} + \partial_\mu \tilde{\epsilon}^{(\alpha \sigma)} \partial_\beta g_{\alpha \sigma} - \partial_\mu g^{\alpha \sigma} \partial_\beta \tilde{t}^{(\alpha \sigma)} \]
\[ + \partial_\lambda g^{\alpha \sigma} \partial_\beta \tilde{t}^{(\alpha \sigma)} - \partial_\alpha \tilde{\epsilon}^{(\alpha \sigma)} \partial_\beta g_{\alpha \sigma} \]
\[ + 2 \partial^\rho t^{(\alpha \sigma)} \partial_\beta g_{\alpha \rho} - 2 g^{\alpha \sigma} \partial_\lambda \tilde{t}^{(\lambda \rho)} \partial_\beta g_{\alpha \beta} \]
\[ + g^{\alpha \sigma} g^{\lambda \rho} \partial_\mu \tilde{x}^{(\lambda \rho)} \partial_\beta g_{\alpha \beta} \}
\[ - \frac{1}{2} \tilde{\epsilon}^{(\beta \mu)} \{ \partial_\mu (g^{\alpha \sigma} \partial_\beta \tilde{t}^{(\alpha \sigma)} - \tilde{\epsilon}^{(\alpha \sigma)} \partial_\beta \tilde{t}^{(\alpha \sigma)} - \tilde{\epsilon}^{(\alpha \sigma)} (2 \partial_\beta t^{(\alpha \sigma)} - \partial_\sigma t^{(\alpha \sigma)}))
\[ + \partial_\alpha \{ g^{\alpha \sigma} (2 \partial_\beta \tilde{t}^{(\alpha \sigma)} - \partial_\sigma t^{(\alpha \sigma)}) - \tilde{\epsilon}^{(\alpha \sigma)} (2 \partial_\beta g_{\alpha \beta} - \partial_\sigma g_{\alpha \beta}) \}
\[ + 2 \partial^\sigma g_{\lambda \mu} g^{\beta \lambda} (2 \partial_\mu \tilde{t}^{(\alpha \sigma)} - \partial_\alpha t^{(\alpha \sigma)} g^{\beta \mu} + 2 \partial^\lambda \partial_\lambda g_{\alpha \mu} g^{\beta \mu} (2 \partial^\sigma g_{\beta \mu} - \partial_\rho g_{\mu \beta})
\[ - 2 \tilde{\epsilon}^{(\lambda \rho)} \partial_\lambda \tilde{t}^{(\lambda \rho)} g^{\alpha \sigma} (2 \partial^\mu t^{(\alpha \rho)} - \partial_\rho t^{(\alpha \rho)} g^{\beta \rho} + 2 \partial^\rho \tilde{\epsilon}^{(\alpha \sigma)} (2 \partial^\rho g_{\beta \rho} - \partial_\rho g_{\alpha \beta}) \]
\[ + \tilde{\epsilon}^{(\alpha \sigma)} \tilde{\lambda}^{(\rho \mu \beta)} (2 \partial^\rho g_{\beta \rho} - \partial_\rho g_{\alpha \beta}) \}}
\[-2\tilde{t}^{(\lambda\rho)} \partial_{\lambda} t_{(\sigma)} g^{\beta\mu} (2\partial^\sigma g_{\beta\rho} - \partial_\rho g_{\alpha\beta} g^{\alpha\sigma}) \]
\[-\partial^\rho g_{\sigma\alpha} g^{\sigma\alpha} (2\partial^\mu t^{(\mu)}_{(\rho)} - \partial_\rho t^{2}_{(\mu\beta)}) - \partial^\mu g^{\sigma\alpha} (2\partial^\mu t^{(\mu)}_{(\rho)} - \partial_\rho g_{\alpha\beta} g^{\alpha\beta}) \]
\[-\tilde{t}^{(\alpha\sigma)} \partial_\sigma g_{\alpha\sigma} g^{\alpha\sigma} (2\partial^\mu g_{\rho\mu} - \partial_\rho g_{\mu\beta} g^{\beta\mu}) - \tilde{t}^{(\alpha\sigma)} \partial_\sigma g_{\alpha\sigma} g^{\alpha\sigma} (2\partial^\mu t^{(\mu)}_{(\rho)} - \partial_\rho g_{\mu\beta} g^{\beta\mu}) \]
\[-\tilde{t}^{(\alpha\sigma)} \partial_\sigma g_{\alpha\sigma} g^{\alpha\sigma} (2\partial^\mu g_{\rho\mu} - \partial_\rho g_{\mu\beta} g^{\beta\mu}) - \tilde{t}^{(\alpha\sigma)} \partial_\sigma g_{\alpha\sigma} g^{\alpha\sigma} (2\partial^\mu t^{(\mu)}_{(\rho)} - \partial_\rho g_{\mu\beta} g^{\beta\mu}) \]
\[\]
\[
\begin{align*}
+ g^{\alpha\sigma} (\partial_\alpha t^2_{(\sigma\mu)} + \partial_\mu t^2_{(\sigma\lambda)} - \partial_\sigma t^2_{(\mu\lambda)}) g^{\lambda\rho} (\partial_\beta t_{(\rho\alpha)} + \partial_\alpha t_{(\beta\rho)} - \partial_\rho t_{(\alpha\beta)}) \\
- \bar{t}^{(\alpha\sigma)} (\partial_\alpha t_{(\sigma\mu)} + \partial_{\mu} t_{(\sigma\lambda)} - \partial_{\sigma} t_{(\mu\lambda)}) g^{\lambda\rho} (\partial_\beta t_{(\rho\alpha)} + \partial_{\alpha} t_{(\beta\rho)} - \partial_{\rho} t_{(\alpha\beta)}) \\
- g^{\alpha\sigma} \partial_\alpha t_{(\sigma\alpha)} g^{\lambda\rho} (2\beta t^2_{(\mu\mu)} - \partial_\rho t^2_{(\mu\beta)}) \\
+ g^{\alpha\sigma} \partial_\alpha t_{(\sigma\alpha)} \bar{t}^{(\lambda\rho)} (2\beta t_{(\mu\mu)} - \partial_\rho t_{(\mu\beta)}) \\
- g^{\alpha\sigma} \partial_\alpha t^2_{(\sigma\sigma)} g^{\lambda\rho} (2\beta t_{(\mu\mu)} - \partial_\rho t_{(\mu\beta)}) \\
+ \bar{t}^{(\alpha\alpha)} \partial_\alpha t_{(\sigma\alpha)} g^{\lambda\rho} (2\beta t_{(\mu\mu)} - \partial_\rho t_{(\mu\beta)}) \} \\
+ g^{\beta\mu} \left\{ \frac{1}{2} (\partial_\mu \bar{t}^{2(\alpha\sigma)} \partial_\beta t^2_{(\sigma\alpha)} + \bar{t}^{2(\alpha\alpha)} \partial_\alpha (2\beta t^2_{(\sigma\mu)} - \partial_\sigma t^2_{(\mu\beta)})) \\
- \partial_\alpha \bar{t}^{2(\alpha\sigma)} (2\beta t^2_{(\sigma\mu)} - \partial_\sigma t^2_{(\mu\beta)}) \\
- \bar{t}^{2(\alpha\alpha)} \partial_\alpha (2\beta t^2_{(\sigma\mu)} - \partial_\sigma t^2_{(\mu\beta)}) \right\} \\
+ \frac{1}{4} \left\{ g^{\alpha\sigma} \left( \partial_\alpha t^2_{(\sigma\mu)} + \partial_\mu t^2_{(\lambda\sigma)} - \partial_\sigma t^2_{(\mu\lambda)} \right) g^{\lambda\rho} \left( \partial_\beta t^2_{(\rho\alpha)} + \partial_\alpha t^2_{(\beta\rho)} - \partial_\rho t^2_{(\alpha\beta)} \right) - g^{\alpha\sigma} \left( \partial_\alpha t_{(\sigma\mu)} + \partial_{\mu} t_{(\sigma\lambda)} - \partial_{\sigma} t_{(\mu\lambda)} \right) \bar{t}^{(\lambda\rho)} \left( \partial_\beta t_{(\rho\alpha)} + \partial_{\alpha} t_{(\beta\rho)} - \partial_{\rho} t_{(\alpha\beta)} \right) \\
+ g^{\alpha\sigma} \left( \partial_\alpha t^2_{(\sigma\mu)} + \partial_\mu t^2_{(\lambda\sigma)} - \partial_\sigma t^2_{(\mu\lambda)} \right) \bar{t}^{2(\lambda\rho)} \left( \partial_\beta t_{(\rho\alpha)} + \partial_{\alpha} t_{(\beta\rho)} - \partial_{\rho} t_{(\alpha\beta)} \right) \\
- g^{\alpha\sigma} \left( \partial_\alpha t^2_{(\sigma\mu)} + \partial_\mu t^2_{(\lambda\sigma)} - \partial_\sigma t^2_{(\mu\lambda)} \right) \bar{t}^{(\lambda\rho)} \left( \partial_\beta t_{(\rho\alpha)} + \partial_{\alpha} t_{(\beta\rho)} - \partial_{\rho} t_{(\alpha\beta)} \right) \\
- \bar{t}^{(\alpha\alpha)} \left( \partial_\alpha t_{(\sigma\mu)} + \partial_{\mu} t_{(\sigma\lambda)} - \partial_{\sigma} t_{(\mu\lambda)} \right) \bar{t}^{(\lambda\rho)} \left( \partial_\beta t_{(\rho\alpha)} + \partial_{\alpha} t_{(\beta\rho)} - \partial_{\rho} t_{(\alpha\beta)} \right) \\
+ \bar{t}^{2(\alpha\alpha)} \left( \partial_\alpha t_{(\sigma\mu)} + \partial_{\mu} t_{(\sigma\lambda)} - \partial_{\sigma} t_{(\mu\lambda)} \right) \bar{t}^{(\lambda\rho)} \left( \partial_\beta t_{(\rho\alpha)} + \partial_{\alpha} t_{(\beta\rho)} - \partial_{\rho} t_{(\alpha\beta)} \right) \\
- g^{\alpha\sigma} \partial_\alpha t^2_{(\sigma\mu)} g^{\lambda\rho} (2\beta t^2_{(\mu\mu)} - \partial_\rho t^2_{(\mu\beta)}) \\
+ g^{\alpha\sigma} \partial_\alpha t_{(\sigma\sigma)} \bar{t}^{(\lambda\rho)} (2\beta t_{(\mu\mu)} - \partial_\rho t_{(\mu\beta)}) \\
- g^{\alpha\sigma} \partial_\alpha t^2_{(\sigma\sigma)} \bar{t}^{(\lambda\rho)} (2\beta t_{(\mu\mu)} - \partial_\rho t_{(\mu\beta)}) \\
+ \bar{t}^{(\alpha\alpha)} \partial_\alpha t_{(\sigma\sigma)} \bar{t}^{(\lambda\rho)} (2\beta t_{(\mu\mu)} - \partial_\rho t_{(\mu\beta)}) \\
- \bar{t}^{2(\alpha\alpha)} \partial_\alpha t_{(\sigma\sigma)} \bar{t}^{(\lambda\rho)} (2\beta t_{(\mu\mu)} - \partial_\rho t_{(\mu\beta)}) \} \right],
\end{align*}
\]

where $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ and $R$ are the curvature tensors of Riemann space-time and $|w_{V-A}| = \{1 + t^\alpha + \frac{1}{2}(t^a t^b - t^a t^b)\}$ is V-A model in two dimensional flat space. Note that the result is still preliminary, for the multiplication by $|w_{V-A}|$ factorized in (28) should be expanded in the power series in $t$. 

3.2 SGM in the spin connection formalism

Next we perform the similar arguments in the spin connection formalism for the sake of the comparison. The spin connection $Q^{ab\mu}$ and the curvature tensor $\Omega^{ab\mu\nu}$ in SGM space-time are as follows;

$$Q_{ab\mu} = \frac{1}{2}(w_{\alpha}^a \partial_{\mu} w_{\beta}^b - w_{\alpha}^a \partial_{\beta} w_{\mu}^b),$$

$$\Omega^{ab\mu\nu} = \partial_{[\mu}Q^{ab\nu]} + Q^a_{\nu c\mu}Q^{cb\nu],}$$

(29)

(30)

The scalar curvature $\Omega$ of SGM space-time is defined by $\Omega = w_{\alpha}^a w_{\beta}^b \Omega^{ab\mu\nu}$. Let us express the spin connection $Q^{ab\mu}$ in two dimensional space-time in terms of $e^\alpha_{\mu}$ and $t^a_{\mu}$ as

$$Q_{ab\mu} = \omega_{ab\mu}[c] + T_{ab\mu}^{(1)} + T_{ab\mu}^{(2)} + T_{ab\mu}^{(3)},$$

(31)

where $\omega_{ab\mu}[c]$ is the Ricci rotation coefficients of GR, and $T_{ab\mu}^{(1)}$, $T_{ab\mu}^{(2)}$ and $T_{ab\mu}^{(3)}$ are defined as

$$T_{ab\mu}^{(1)} = \frac{1}{2}(e_{[a}^\rho e_{b]}^\nu \partial_{[\mu} t_{\nu]_{\rho}} - e_{[a}^\rho e_{b]}^\nu \partial_{[\mu} t_{\rho]_{\nu}} - e_{[a}^\rho e_{b]}^\nu \partial_{\rho} e_{\mu}^\nu - e_{[a}^\rho e_{b]}^\nu t_{c\mu} e_{\rho}^\nu),$$

$$T_{ab\mu}^{(2)} = \frac{1}{2}(e_{[a}^\rho e_{b]}^\nu \partial_{\rho} e_{\mu}^\nu - e_{[a}^\rho e_{b]}^\nu t_{c\mu} e_{\rho}^\nu),$$

$$T_{ab\mu}^{(3)} = \frac{1}{2}(e_{[a}^\rho e_{b]}^\nu \partial_{\mu} t_{\rho]_{\nu}} - e_{[a}^\rho e_{b]}^\nu t_{c\mu} e_{\rho}^\nu),$$

(32)

(33)

(34)

where $t^\mu_{a} = e^\beta_{a} e^\nu_{b} t^b_{\mu}$. Note that $T_{ab\mu}^{(1)}$ and $T_{ab\mu}^{(2)}$ can be written by using the spin connection $\omega^{ab}_{\mu}[c]$ of GR as

$$T_{ab\mu}^{(1)} = e_{[a}^\rho e_{b]}^\nu \partial_{\mu} t_{\rho]_{\nu} \sigma},$$

$$T_{ab\mu}^{(2)} = -t^\sigma_{\sigma} e_{[a}^\rho e_{b]}^\nu \partial_{\mu} t_{\rho]_{\nu} \sigma},$$

$$-\frac{1}{2} e_{[a}^\rho e_{b]}^\nu \partial_{\mu} (t_{c\mu} t_{\rho]_{\nu} \sigma}) - \frac{1}{2} t^\sigma_{\sigma} e_{[a}^\rho e_{b]}^\nu \partial_{\mu} t_{\rho]_{\nu} \sigma},$$

(35)

(36)

where $\hat{D}_{\mu} t_{\alpha\nu} := \partial_{\mu} t_{\alpha\nu} + \omega_{ab\mu} t_{\alpha\nu}$ and $\partial_{\mu} t_{[\alpha\sigma]} := \partial_{\mu} t_{[\alpha\sigma]} + \partial_{\sigma} t_{(\mu\rho)} - \partial_{\rho} t_{(\sigma\mu)}$. Then we obtain straightforwardly the complete expression of 2 dimensional SGM action(N=1) in the spin connection formalism as follows; namely,
\[ L_{2dSGM} = -\frac{c^3}{16\pi G} e^{|w_{V\Lambda}|} \frac{3}{2} \] 

\[ + 2(\mu\nu)_\rho + (\nu\rho)_\nu + (\mu\nu\nu)_\rho) R_{\mu\nu} + \lambda(\mu\nu\nu) R_{\mu\nu\rho\sigma} \] 

\[ - (g^{\rho\mu} g^{\nu\nu}) g^{\sigma\lambda} + g^{\sigma\mu} g^{\nu\nu} g^{\rho\lambda} - g^{\sigma\mu} g^{\lambda\nu} g^{\rho\nu} e^a_{\sigma} e^b_{\sigma} (\hat{D}_{\mu t_{\alpha}}) \hat{D}_{\nu t_{\beta}} \] 

\[ + g^{\rho\mu} g^{\nu\nu} g^{\sigma\lambda} (\partial_{\mu t_{[\rho|\sigma]}}) \partial_{[\beta]} + \frac{1}{4} g^{\rho\mu} g^{\nu\nu} g^{\sigma\lambda} (\partial_{\mu t_{[\rho|\sigma]}}) \partial_{[\beta]} \] 

\[ - 2(g^{\rho\mu} e^{\nu}_{[\rho]} \hat{D}_{\mu} e^{\alpha\sigma} + e^{\nu}_{[\rho]} e^{\mu}_{\nu} \hat{D}_{\mu} e^{\alpha\rho} \] 

\[ + e^{\nu}_{[\rho]} e^{\mu}_{\nu} e^{\alpha\sigma} \hat{D}_{\mu} e^{\alpha\sigma} \hat{D}_{\nu t_{[\rho]}} \] 

\[ - e^{\nu}_{[\rho]} e^{\mu}_{\nu} (\hat{D}_{\mu} e^{\alpha\sigma} \hat{D}_{\nu t_{[\rho]}} - g^{\mu\nu} e^{\alpha\nu} e^{\rho\sigma} (\hat{D}_{\mu} e^{\alpha\nu} \hat{D}_{\nu} t_{[\rho]}) \] 

\[ - D_{\mu}(g^{\rho\mu} g^{\nu\nu} (\partial_{[\alpha}] t_{[\beta]}) + 2 g^{\rho\mu} (\partial_{[\alpha]} t_{[\beta]}) \] 

\[ - e^{\nu}_{[\rho]} (\hat{D}_{\mu} e^{\alpha\nu}) (\partial_{[\alpha]} t_{[\beta]}) + \frac{1}{2} e^{\nu}_{[\rho]} (\hat{D}_{\mu} e^{\alpha\nu}) \partial_{[\alpha]} t_{[\beta]} \] 

\[ + \frac{1}{2} e^{\nu}_{[\rho]} (\hat{D}_{\mu} e^{\alpha\nu}) \partial_{[\alpha]} t_{[\beta]} \] 

\[ + 2 g^{\rho\mu} g^{\nu\nu} t_{[\rho]} t_{[\beta]} (\hat{D}_{\mu} t_{[\alpha]} \hat{D}_{\nu} t_{[\lambda]} \] 

\[ - 2(g^{\rho\mu} e^{\nu}_{[\rho]} t_{[\lambda]} - e^{\nu}_{[\rho]} (\hat{D}_{\mu} e^{\alpha\nu}) e^{\nu}_{[\rho]} t_{[\lambda]} \] 

\[ - (g^{\rho\mu} e^{\nu}_{[\rho]} - e^{\nu}_{[\rho]} e^{\mu}_{\nu} e^{\nu}_{[\rho]} e^{\rho\sigma} (\hat{D}_{\mu} t_{[\alpha]} \hat{D}_{\nu} t_{[\lambda]} \] 

\[ + (g^{\rho\mu} e^{\nu}_{[\rho]} - e^{\nu}_{[\rho]} e^{\mu}_{\nu} e^{\nu}_{[\rho]} e^{\rho\sigma} (\hat{D}_{\mu} t_{[\alpha]} \hat{D}_{\nu} t_{[\lambda]} \] 

\[ + 2(g^{\rho\mu} e^{\nu}_{[\rho]} e^{\sigma\nu} - g^{\rho\mu} e^{\nu}_{[\rho]} e^{\sigma\nu} (\hat{D}_{\mu} t_{[\alpha]} \hat{D}_{\nu} t_{[\lambda]} \] 

\[ - \frac{1}{2} g^{\rho\mu} g^{\nu\nu} t_{[\rho]} t_{[\beta]} (\hat{D}_{\mu} t_{[\alpha]} \hat{D}_{\nu} t_{[\lambda]} \] 

\[ - \frac{1}{2} g^{\rho\mu} g^{\nu\nu} t_{[\rho]} t_{[\beta]} (\hat{D}_{\mu} t_{[\alpha]} \hat{D}_{\nu} t_{[\lambda]} \]
\[-\frac{1}{4}(g^{\rho\mu\nu\lambda\kappa} - g^{\rho|\mu|\nu|\lambda})g^{\sigma\lambda}(\partial_\mu t_{[\rho\sigma\kappa]})\partial_\nu t_{[\lambda\kappa]}\]
\[+\frac{1}{2}D_\mu\left\{e^{a\mu}e_{[b|\nu]}\{2t^a_\sigma^{\rho\lambda}\partial_{[\rho\sigma\kappa]} - (2e_a^\rho t^\kappa_{\lambda\sigma} + t^\rho_\sigma t^\lambda_{\rho\sigma} - 2e_a^\rho t^\sigma_{\rho\sigma})\partial_{[\rho\sigma\kappa]}\right\}\]
\[+D_\mu\left\{g^{\rho|\mu|\nu|\lambda}\partial_\rho(t_{[\mu|\nu|\lambda\sigma]} + g^{\lambda|\mu|\nu|\lambda})t^\rho_\lambda\partial_\nu t_{[\rho|\sigma]}\right\}\]
\[+\frac{1}{2}(e^{a\mu}e_{[b|\nu]}e^{b\kappa} - e^{b|\mu|\nu|\lambda})D_\mu t^d_\kappa\left\{2t^\rho_\sigma^{\rho\lambda}\partial_{[\rho\sigma\kappa]} - (2e_a^\rho t^\lambda_{\kappa\sigma} + t^\rho_\sigma t^\sigma_{\rho\sigma})\partial_{[\rho\sigma\kappa]}\right\}\]
\[-(2e_a^\rho t^\lambda_{\kappa\sigma} + t^\rho_\sigma t^\sigma_{\rho\sigma} - 2e_a^\rho t^\sigma_{\rho\sigma})\partial_{[\rho\sigma\kappa]}\}
\[+(e^{a\mu}|\mu|\nu|\rho)\partial_{[\rho\sigma\kappa]} - e^{b|\mu|\nu|\lambda})\partial_{[\rho\sigma\kappa]} + t_{[\mu|\nu|\lambda]}b\}
\times\{e^{c\mu}e_a^{\sigma}(\partial_\mu t^{c\sigma}) - \frac{1}{2}e^\mu c\nu e^b(\partial_\nu t^{e\sigma}) - c^\mu e_a^\lambda e^b(\partial_\nu t^{c\lambda})\partial_{[\rho\sigma\kappa]}\}
\[+g^{\sigma\nu|\mu|\nu|\lambda|\rho}}t^\kappa_\sigma(\partial_\mu g_{\nu|\lambda})\partial_\nu t^{c\lambda}\]
\[+(g^{\sigma\nu|\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}e_{[b|\nu]}t^\lambda_{\rho\sigma}) + e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[\times\{e_{[d\mu]}\partial_{[\rho\sigma\kappa]} - t_{[d\mu]}t^b_\kappa\}t^\lambda_\kappa(\partial_\mu t^{d_\kappa})\partial_{[\sigma\nu]}t^{d_\kappa}\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[\times\{e_{[d\mu]}\partial_{[\rho\sigma\kappa]} - t_{[d\mu]}t^b_\kappa\}t^\lambda_\kappa(\partial_\mu t^{d_\kappa})\partial_{[\sigma\nu]}t^{d_\kappa}\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}
\[+\frac{1}{2}(g^{\rho\mu|\nu|\lambda|\rho}}t^\kappa_\sigma - e^{a\mu}g^{\rho|\kappa|\nu|\lambda})t^\rho_\lambda(\partial_{[\rho\sigma\kappa]}\partial_{[\sigma\nu]}t^{d_\kappa})\}.
Interestingly, the higher order nonlinear terms in $t^\psi$ supertranslation \[3\]
metric properties of (formation \[17\] with a generalized parameter
invariant under (besides the ordinary local GL(4,R)) the general coordinate trans-
in the inverse $w_a$ action (1) written by the vierbein
of freedom can not be eliminated by the redefinition (variation) of the fields.

The final results after carrying out the multiplication of $|w_{\psi-A}|$ may be reinterpreted as those of SGM
in a simpler form up to total derivative terms. As mentioned above SGM action is
exact solutions of Einstein equation of GR may be reinterpreted as those of SGM( $w^a\mu(x)$ and metric $s_{\mu\nu}(x)$) and may have new physical meanings for EGRT.

4. Discussions
We have shown that contrary to its simple expression (1) in unified SGM space-
time the expansion of SGM action posses very complicated and rich structures
describing as a whole gauge invariant graviton-superon interactions.
The final results after carrying out the multiplication of $|w_{\psi-A}|$ may be rewritten in a simpler form up to total derivative terms. As mentioned above SGM action is
remarkably a free action of E-H type in unified SGM space-time, the various classical
exact solutions of Einstein equation of GR may be reinterpreted as those of SGM( $w^a\mu(x)$ and metric $s_{\mu\nu}(x)$) and may have new physical meanings for EGRT.

Here we just mention that SGM action in SGM space-time is a nontrivial generalization of E-H action in Riemann space-time despite the simple linear relation
$w^a\mu = e^a\mu + t^a\mu$. In fact, by the redefinition(variation) of the metric $e^a\mu \rightarrow e^a\mu - t^a\mu$ and the corresponding redefinition(variation) of the metric $e_a\mu$ defined by $\delta e_a\mu = -e_a\nu e_b\mu \delta e^b\nu$ for $e^a\mu \rightarrow e^a\mu + \delta e^a\mu$, the inverse $w_a\mu$ (17) does not reduce to $e_a\mu$, i.e. interestingly the higher order nonlinear terms in $t^a\mu(\neq t^a\mu)$ can not be eliminated in the inverse $w_a\mu$. Because $t^a\mu$ is not a vierbein. Such a redefinition breaks the metric properties of $(w^a\mu, w_a\mu)$ and is forbidden. This shows that superon degrees of freedom can not be eliminated by the redefinition (variation) of the fields.

Concerning the abovementioned two inequivalent flat-spaces (i.e. the vacuum of the gravitational energy) of SGM action we can interpret them as follows. SGM action (1) written by the vierbein $w_a\mu(x)$ and metric $s^{\mu\nu}(x)$ of SGM space-time is invariant under (besides the ordinary local GL(4,R)) the general coordinate transformation \[17\] with a generalized parameter $i\kappa\zeta^\mu\psi(x)$ (originating from the global supertranslation $\psi(x) \rightarrow \psi(x) + \zeta$ in SGM space-time). As proved for E-H action
\[
\begin{align*}
- (g^{\mu\nu} x^{\mu\nu} g_{\lambda\sigma}) & - (g^{\mu\nu} x^{\mu\nu} t_{\lambda\sigma}) t_{ab} \{ \partial_{\mu} t_{[\lambda\sigma]} \delta \partial_{\lambda\sigma} \} \\
+ \left\{ (g^{\mu\nu} x^{\mu\nu} g_{\lambda\sigma}) + t_{\lambda\sigma} \right\} (\partial_{\mu} t_{[\lambda\sigma]} \delta \partial_{\lambda\sigma} t_{ab}) \\
- (g^{\mu\nu} x^{\mu\nu} g_{\lambda\sigma}) - (g^{\mu\nu} x^{\mu\nu} t_{\lambda\sigma}) t_{ab} \{ \partial_{\mu} t_{[\lambda\sigma]} \delta \partial_{\lambda\sigma} \} \\
+ \left\{ (g^{\mu\nu} x^{\mu\nu} g_{\lambda\sigma}) + t_{\lambda\sigma} \right\} (\partial_{\mu} t_{[\lambda\sigma]} \delta \partial_{\lambda\sigma} t_{ab}) \\
- \frac{1}{4} (g^{\mu\nu} x^{\mu\nu} t_{\lambda\sigma}) t_{ab} \{ \partial_{\mu} t_{[\lambda\sigma]} \delta \partial_{\lambda\sigma} \} \\
\end{align*}
\]
where $\hat{D}_\mu T_{ab} := \partial_{\mu} T_{ab} + \omega_{a\mu} T_{cb} + \omega_{b\mu} T_{ac} e^c_{\nu}$ and $D_\mu e^\nu := \partial_{\mu} e^\nu + \Gamma^\nu_{\lambda\mu} e^\lambda$.

As in the affine connection case the result is still preliminary, for the multiplication by $|w_{\psi-A}|$ factorized in (37) should be expanded in the power series in $t$. 

16
of GR\[20\], the energy of SGM action of E-H type is expected to be positive (for positive $\Lambda$). Regarding the scalar curvature tensor $\Omega$ for the unified metric tensor $s^{\mu\nu}(x)$ as an analogue of the Higgs potential for the Higgs scalar, we can observe that (at least the vacuum of) SGM action, (i.e. SGM-flat $w_{\mu}^{a}(x) \rightarrow \delta_{\mu}^{a}$ space-time,) which allows Riemann space-time and has a positive energy density with the positive cosmological constant $\frac{\Lambda}{16\pi G}$ indicating the spontaneous SUSY breaking, is unstable (i.e. degenerates) against the supertransformation (3) and (4) with the global spinor parameter $\zeta$ in SGM space-time and breaks down spontaneously to Riemann space-time (15).

$$w_{\mu}^{a}(x) = e_{\mu}^{a}(x) + t_{\mu}^{a}(x), \quad (38)$$

with N-G fermions superons corresponding to

$$\frac{\text{super}GL(4, R)}{GL(4, R)}. \quad (39)$$

Remarkably the observed Riemann space-time of EGRT and matter(superons) appear simultaneously from (the vacuum of) SGM action by the spontaneous SUSY breaking.

The analysis of the structures of the vacuum of Riemann-flat space-time (described by N=10 V-A action with derivative terms similar to (22) ) plays an important role to linearlize SGM and to derive SM as the low energy effective theory, which remain to be challenged. The derivative terms can be rewritten in the tractable form (22) up to the total derivative terms.

The linearization of the flat-space N=1 V-A model was already carried out\[19\]. The linearization of N=2 V-A model is extremely important from the physical point of view, for it gives a new mechanism generating a (U(1)) gauge field of the linearlized (effective) theory \[21\]. In our case of SGM the algebra(gauge symmetry) should be changed to broken SO(10) SP(broken SUGRA \[2\])symmetry by the linearization which is isomorphic to the initial one (13). The systematic and generic arguments on the relation of linear and nonlinear SUSY are already investigated\[22\]. Recently we have shown that N=1 gauge vector multiplet action with SUSY breaking Feyet-Iliopoulos term is equivalent to N=1 flat space V-A action of NL SUSY\[23\]. A U(1) gauge field, though an axial vector field for N=1 case, is expressed by N-G field (and its highly nonlinear self interactions). It is remarkable that the renormalizable model is obtained systematically by the linearization of V-A model. These are the suggestive and favourable results to SGM.

Finally we just mention the hidden symmetries characteristic to SGM. It is natural to expect that SGM action may be invariant under a certain exchange between $e_{\mu}^{a}$ and $t_{\mu}^{a}$, for they contribute equally to the unified SGM vierbein $w_{\mu}^{a}$ as seen
in \[(15)\]. In fact we find, as a simple example, that \(w^a_\mu\) and \(w^a_\mu\), i.e. SGM action is invariant under the following exchange of \(e^a_\mu\) and \(t^a_\mu\) \[24\] (in 4 dimensional space-time).

\[
e^a_\mu \rightarrow 2t^a_\mu, \quad t^a_\mu \rightarrow e^a_\mu - t^a_\mu, \quad e_a^\mu \rightarrow e_a^\mu,
\]

or in terms of the metric it can be written as

\[
g_{\mu\nu} \rightarrow 4t^\rho_\mu t^\rho_\nu, \quad t_{\mu\nu} \rightarrow 2(t_{\nu\mu} - t_{\rho\mu} t^\rho_\nu),
\]

\[
g^{\mu\nu} \rightarrow g^{\mu\nu}, \quad t^{\mu\nu} \rightarrow g^{\mu\nu} - t^{\mu\nu}.
\]

This can be generalized to the following form with two real(one complex) global parameters \[24\],

\[
\begin{pmatrix}
e^a_\mu \\
t^a_\mu \\
t^b_\mu e_b^c t^c_\nu
\end{pmatrix} \rightarrow
\begin{pmatrix}
0 & 2(\alpha + 1) & -2(\alpha^2 - \beta) \\
1 & -(2\alpha + 1) & 2(\alpha^2 - \beta) \\
1 & -(3\alpha + 2) & 2\alpha(2\alpha + 1) - 3\beta + 1
\end{pmatrix}
\begin{pmatrix}
e^a_\mu \\
t^a_\mu \\
t^b_\mu e_b^c t^c_\nu
\end{pmatrix}.
\]

This can be generalized to the following form with two real(one complex) global parameters \[24\],

The physical meaning of such symmetries remains to be studied.
Also SGM action has \(Z_2\) symmetry \(\psi^j \rightarrow -\psi^j\).

Besides the composite picture of SGM it is interesting to consider (elementary field) SGM with the extra dimensions and their compactifications. The compactification of \(w^A_M = e^A_M + t^A_M\), \((A, M = 0, 1, .. D - 1)\) produces rich spectrum of particles and (hidden) internal symmetries and may give a new framework for the unification of space-time and matter.

SGM for spin \(\frac{3}{2}\) superon(N-G fermion) \[14\] is also in the same scope. SGM cosmology is open.

The authors would like to thank Y. Tanii, T. Shirafuji and K. Mizutani for useful discussions and the hospitality at Physics Department of Saitama University. The work of M. Tsuda is supported in part by High-Tech research program of Saitama Institute of Technology.
References

[1] J. Wess and B. Zumino, *Phys. Lett.* **B49**, 52 (1974).
SUSY was found independently from the different motivations.

[2] Y.A. Golfand and E.S. Likhtman, *JET. Lett.* **13**, 323 (1971).

[3] D.V. Volkov and V.P. Akulov, *Phys. Lett.* **B46**, 109 (1973).

[4] S. Deser and B. Zumino, *Phys. Lett.* **B62**, 335 (1976).
D. Z. Freedman, P. van Nieuwenhuisen and S. Ferrara, *Phys. Rev.* **D13**, 3214 (1976).

[5] A. Salam and J. Strathdee, *Phys. Lett.* **B49** (1974) 465.

[6] L. O’Raifeartaigh, *Nucl. Phys.* **B96**, 331 (1975).

[7] W. Bardeen and V. Visnjic *Nucl. Phys.* **B194**, 422 (1982).

[8] K. Shima, *Phys. Rev.* **D20**, 574 (1979).

[9] S. Deser and B. Zumino, *Phys. Rev. Lett.* **38**, 1433 (1977).

[10] K. Shima, *European Phys. J.* **C7**, 341 (1999).

[11] H. Georgi and S. L. Glashow, *Phys. Rev. Lett.*, **32**, 32 (1974).

[12] K. Shima, *Z. Phys.* **C18**, 25 (1983).

[13] K. Shima, *Phys. Lett.* **B501**, 237 (2001).

[14] K. Shima and M. Tsuda, *Phys. Lett.* **B521**, 67 (2001).

[15] M. Born and L. Infeld, *Proc. Roy. Soc.(London)* **A144**, 425 (1934).

[16] K. Shima and M. Tsuda, hep-th/0109042.

[17] K. Shima and M. Tsuda, *Phys. Lett.* **B507**, 260 (2001).

[18] S. Samuel and J. Wess, *Nucl. Phys.* **B221**, 153 (1983).

[19] M. Roček, *Phys. Rev. Lett.* **41**, 451 (1978).
E. A. Ivanov and A.A. Kapustnikov, *J. Phys.*, **A11**, 2375 (1978).
T. Uematsu and C.K. Zachos, *Nucl. Phys.* **B201**, 254 (1982).
J. Wess, Karlsruhe University preprint(Festschrift for J. Lopszanski, December,1982).
[20] The positive definiteness of Einstein-Hilbert action was proved by E. Witten, *Commun. Math. Phys.* **80**, 381 (1981).

[21] K. Shima, Plenary talk at the Fourth International Conference on Symmetry in Nonlinear Mathematical Physics, July 7-14, 2001, Kyiv, Ukraine. To appear in the Proceeding.

[22] J. Wess and J. Baggar, *Supersymmetry and Supergravity*, 2nd edition (Princeton University Press 1992).

[23] K. Shima, Y. Tani and M. Tsuda, *Phys. Lett.* **B** in press, hep-th/0110102.

[24] K. Shima and M. Tsuda, in preparation.