Hadronic absorption cross sections of $B_c$

M. A. K. Lodhi$^a$; Faisal Akram$^b$ and Shaheen Irfan$^b$

$^a$Department of Physics, MS 1051, Texas Tech University, Lubbock TX 79409, USA

$^b$Center for High Energy Physics, Punjab University, Lahore, PAKISTAN

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Abstract

The cross sections of $B_c$ absorption by $\pi$ mesons are calculated using hadronic Lagrangian based on SU(5) flavor symmetry. Calculated cross sections are found to be in range 2 to 7 mb and 0.2 to 2 mb for the processes $B_c^+ \pi \rightarrow DB$ and $B_c^+ \pi \rightarrow D^* B^*$ respectively, when the monopole form factor is included. These results could be useful in calculating production rate of $B_c$ meson in relativistic heavy ion collisions.

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1 Introduction

T. Matsui and H. Satz [1] postulated that $J/\psi$ would be dissociated due to color Debye screening in deconfined phase of hadronic matter, called Quark-Gluon Plasma (QGP). Thus suppression of $J/\psi$ could be regarded as a signal for the existence of QGP. NA50 experiment at CERN [2] has observed an anomalously large suppression of events with moderate to large transfer energy from the Pb + Pb collision at $P_{Lab} = 158$ GeV/c. However, this observed suppression may also occur due to absorption by comoving hadrons. It has been argued by many authors that this phenomenon could be significant if the absorption cross section is in the range of at least few mb [3, 4, 5, 6, 7, 8]. Extensive work has been done to calculate these cross sections using perturbative QCD [9], QCD sum-rule approach [10], quark potential models [11] and hadronic Lagrangian based on flavor symmetry [12, 13, 14, 15].

Bottomonium states analogous to charmonium are also subjected to dissociation due color screening [1], therefore their suppression is also expected in QGP. Recently the most striking observation from CMS (Compact Muon Solenoid experiment) is that weakly bound states of the b-quark are heavily suppressed in Pb+Pb collisions [16]. This phenomenon is important for understanding the properties of the QGP. Once again the knowledge of absorption cross section is required to interpret the observed signal [12, 17]. It has also been suggested that the production rate of heavy mixed flavor hadrons would also be affected in the presence of QGP [18, 19]. In order to calculate production rates one require complete knowledge of production mechanism in the presence of QGP and absorption cross sections by comoving hadrons. In this paper we have focused on $B_c$ meson. It is expected that $B_c$ production could be enhanced in the presence of QGP. Due to color Debye screening, QGP contains many unpaired $b(\overline{b})$ and $c(\overline{c})$ quarks, which upon encounter could form $B_c$ and probably survive in QGP due to relatively large binding energy [20]. However, observed production rate would also depend upon the absorption cross

$^a$a.lodhi@ttu.edu

$^b$faisal.chep@pu.edu.pk (corresponding author)

$^\dagger$shaheen.irfan@ciitlahore.edu.pk
section by hadronic comovers. $B_c$ absorption cross section by nucleons has been calculated in [20] using meson-baryon exchange model. This cross section is found to have value on the order of few mb. In this paper, we have calculated $B_c$ absorption cross sections by $\pi$ mesons using hadronic Lagrangian based on SU(5) flavor symmetry.

In Sec. II, we define hadronic Lagrangian and derive the interaction term relevant for $B_c$ absorption of $\pi$ mesons. In Sec. III, we calculate the absorption cross sections. In Sec. IV, we discuss the numerical values of different couplings used in the calculation. In Sec. V, we present numerical results of the cross sections with and without form factor. Finally, some concluding remarks are made in Sec. VI.

2 Interaction Lagrangian

The following processes are studied in this work using SU(5) flavor symmetric Lagrangian.

$$B_c^+ \pi \rightarrow DB, \quad B_c^- \pi \rightarrow DB^*, \quad B_c^+ \pi \rightarrow D^* B^*, \quad B_c^- \pi \rightarrow D^* B^*$$

(1)

First and second processes are charge conjugation of each other and hence have same cross sections. Similarly third and fourth processes are also charge conjugation of each other and have same cross sections.

To calculate cross sections of the above processes, we use SU(5) flavor symmetric Lagrangian density [12]. Free SU(5) Lagrangian density is given by,

$$L_0 = Tr(\partial_\mu P^\dagger \partial^\mu P) - \frac{1}{2} Tr(F^\dagger_{\mu\nu} F^{\mu\nu})$$

(2)

Where, $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $P$ and $V_\mu$ denote pseudo-scalar and vector mesons matrices as given in ref. [12].

The following minimal substitutions,

$$\partial_\mu P \rightarrow D_\mu P = \partial_\mu P - \frac{ig}{2} [V_\mu, P]$$

(3)

$$F_{\mu\nu} \rightarrow F_{\mu\nu} - \frac{ig}{2} [V_\mu, V_\nu]$$

(4)

produce the following interaction Lagrangian density.

$$\mathcal{L} = L_0 + ig Tr(\partial_\mu P^\dagger [P, V_\mu]) - \frac{g^2}{4} Tr([P, V_\mu]^2)$$

$$+ ig Tr(\partial_\mu V^\dagger [V_\mu, V_\nu]) + \frac{g^2}{8} Tr([V_\mu, V_\nu]^2)$$

(5)

All mass terms, which breaks SU(5) symmetry, are added directly in the above Lagrangian. The Lagrangian density terms relevant for $B_c$ absorption by $\pi$ mesons are given by,

$$\mathcal{L}_{\pi DD^*} = ig_{\pi DD^*} D^\mu \overrightarrow{\tau} \cdot (\overrightarrow{D_\mu P} - \overrightarrow{P_\mu D}) + hc$$

(6a)

$$\mathcal{L}_{\pi BB^*} = ig_{\pi BB^*} B^\mu \overrightarrow{\tau} \cdot (\overrightarrow{B_\mu B} - \overrightarrow{B_\mu P}) + hc$$

(6b)

$$\mathcal{L}_{B_c BD^*} = ig_{B_c BD^*} D^\mu (B_c^- \partial_\mu B - \partial_\mu B_c^- B) + hc$$

(6c)

$$\mathcal{L}_{B_c B^* D} = ig_{B_c B^* D} (B_c^+ \partial_\mu D - \partial_\mu B_c D^*) + hc$$

(6d)

$$\mathcal{L}_{B_c B^* B^*} = -g_{B_c B^* B^*} B_c^+ B^\mu \overrightarrow{\tau} \cdot \overrightarrow{D_\mu B^*} + hc$$

(6e)

Where,
\[ D = (D^0 \ D^+) \ , \overline{D} = \begin{pmatrix} D_0^0 \ D_{-}\end{pmatrix}^T, D_\mu^* = (D_{\mu 0}^0 \ D_{\mu +})^T, \]
\[ B = (B^+ \ B^0)^T, B_\mu^* = (B_{\mu +}^+ \ B_{\mu 0}^0)^T, \]
\[ \overline{\pi} = (\pi_1, \pi_2, \pi_3), \pi^\pm = \frac{1}{\sqrt{2}} (\pi_1 \mp i\pi_2) \]

Here we follow the convention of representing a field by the symbol of the particle which it absorbs. The coupling constants in Eq. (6) are expressed in terms of SU(5) universal coupling constant \( g \) as following.

\[ g_{\pi DD^*} = g_{\pi DD^*} = \frac{g}{4}, \quad g_{BcBD^*} = g_{BcBD^*} = \frac{g}{2\sqrt{2}}, \quad g_{\pi BcB^*D^*} = \frac{g^2}{4\sqrt{2}} \]  

(8)

It is also noted that SU(5) symmetry also implies the following relation between the couplings.

\[ g_{\pi BcB^*D^*} = 2g_{\pi DD^*}g_{BcBD^*} = 2g_{\pi BcB^*D^*} \]

(9)

### 3 B\(_c\) absorption cross section

Feynman diagrams of the process \( B_c^+ \pi \rightarrow DB \) are shown in Fig. [1]

![Feynman Diagrams for \( B_c^+ \pi \rightarrow DB \)](image)

Figure 1: Feynman Diagrams for \( B_c^+ \pi \rightarrow DB \).

Scattering amplitudes of these diagrams are given by,

\[ M_{1a} = g_{\pi DD^*}g_{BcBD^*}(p_1 + p_3)_\mu \frac{-i}{t - m_{D^*}^2} \left( g^{\mu\nu} - \frac{(p_1 - p_3)^\mu(p_1 - p_3)^\nu}{m_{D^*}^2} \right)(-p_4 - p_2)_\nu \]

(10a)

\[ M_{1b} = g_{\pi BB^*}g_{BcBD^*}(p_1 + p_4)_\mu \frac{-i}{u - m_{B^*}^2} \left( g^{\mu\nu} - \frac{(p_1 - p_4)^\mu(p_1 - p_4)^\nu}{m_{B^*}^2} \right)(-p_3 - p_2)_\nu \]

(10b)

Total amplitude is given by,

\[ M_1 = M_{1a} + M_{1b} \]

(11)

Feynman diagrams of the process \( B_c^+ \pi \rightarrow D^*B^* \) are shown in Fig. [2]
Scattering amplitudes of these diagrams are given by,

\[ M_{2a} = -g_{\pi DD^*} g_{Bc} D^* (2p_1 - p_3) \frac{i}{t - m_D^2} (p_2 - p_1 + p_3) \epsilon^\mu_{\nu} (p_3) \epsilon^\nu_{\nu} (p_4) \]  

(12a)

\[ M_{2b} = -g_{\pi BB^*} g_{Bc} B^* (2p_1 - p_4) \frac{i}{u - m_B^2} (p_2 - p_1 + p_4) \epsilon^\mu_{\nu} (p_3) \epsilon^\nu_{\nu} (p_4) \]  

(12b)

\[ M_{2c} = -ig_{\pi Bc} B^* D^* \gamma_{\mu} \epsilon^\mu_{\nu} (p_3) \epsilon^\nu_{\nu} (p_4) \]  

(12c)

And total amplitude is given by,

\[ M_2 = M_{2a} + M_{2b} + M_{2c} \]  

(13)

Using the total amplitudes given in Eqs. 11 and 13, we calculate unpolarized but not the isospin averaged cross sections. The isospin factor in this case is simply 2 for the both processes.

4 Numerical values of input parameters

Numerical values of all the masses are taken from Particle Data Group [21]. The coupling constant \( g_{\pi DD^*} = 4.4 \) is determined from \( D^* \) decay width [22, 23]. The coupling \( g_{\pi BB^*} \) can be fixed by two methods. Heavy quark symmetries [23, 24, 25] imply that \( g_{\pi BB^*} \approx g_{\pi DD^*} \frac{m_B}{m_D} = 12.4 \) and from light-cone QCD sum rule [23], we obtain \( g_{\pi BB^*} = 10.3 \). In this paper, we use the value obtained from the former method.

The values of the couplings \( g_{Bc, BD^*} \) and \( g_{Bc, B^* D} \) are fixed by using \( g_{YYBB} = 13.3 \), which is obtained using vector meson dominance (VMD) model in ref. [12] and SU(5) symmetry result \( g_{Bc, BD^*} = g_{Bc, B^* D} = \frac{2}{\sqrt{5}} g_{YYBB} \) [20]. In this way we obtain \( g_{Bc, BD^*} = g_{Bc, B^* D} = 11.9 \).

There is no empirically fitted value available for the four-point coupling \( g_{\pi Bc, B^* D} \), thus we use SU(5) symmetry, which implies \( g_{\pi Bc, B^* D} = 2g_{\pi DD^*} g_{Bc, B^* D} = 2g_{\pi BB^*} g_{Bc, BD^*} \). These two identities give two values of 105 and 295, whereas their mean values in 200. The values of coupling constants used in this paper and methods for obtaining them are summarized in Table 1.

5 Results and Discussion

Fig. 3 shows the \( B_c \) absorption cross sections of the process \( B_c^+ \pi \to DB \) as a function of total center of mass (c.m) energy \( \sqrt{s} \). Solid and dashed curves in this figure represent cross sections...
| Coupling constant | Value | Method of Derivation |
|-------------------|-------|----------------------|
| $g_{\pi DD^*}$    | 4.4   | $D^*$ decay width    |
| $g_{\pi BB^*}$    | 12.4  | Heavy quark symmetries |
| $g_{Bc,BD^*}$ and $g_{Bc,B^*D}$ | 11.9 | VMD, SU(5) symmetry |
| $g_{Bc,B^*D^*}$   | 105 to 295 | SU(5) symmetry |

Table 1: Values of coupling constants used in this paper

Without and with form factors. Form factors are included to account the finite size of interacting hadrons. We use following monopole form factor at three point vertices.

$$f_3 = \frac{\Lambda^2}{\Lambda^2 + q^2}$$  \hspace{1cm} \text{(14)}

Where, $\Lambda$ is cutoff parameter and $q^2$ is squared three momentum transfer in c.m frame. At four point vertex, we use the following form factor.

$$f_4 = \left(\frac{\Lambda^2}{\Lambda^2 + q^2}\right)^2$$  \hspace{1cm} \text{(15)}

Where, $q^2 = \frac{1}{2} \left[ (\not{p}_1 - \not{p}_3)^2 + (\not{p}_1 - \not{p}_4)^2 \right]_{c.m}$

Figure 3: $B_c$ absorption cross sections for the process $B_c^+ \pi \rightarrow DB$. Solid and dashed curves represent cross sections without and with form factor respectively. Lower and upper dashed curves are with cutoff parameter $\Lambda = 1$ and 2 GeV respectively. Threshold energy is 7.15 GeV.

In general, the value of cutoff parameter used in the form factor could have different values at different vertices. There is no direct way to calculate the values of these parameters. In some cases cutoff parameters can be fixed empirically by studying hadronic scattering data in meson or baryon exchange models. Such empirical fits put the cutoff parameters on the scale of 1 to 2 GeV for the vertices connecting light hadrons ($\pi$, $K$, $\rho$, $N$ etc) [26]. However, due to limited information about the scattering data of charmed and bottom hadrons, no empirical values of...
the related cutoff parameters are known. In this case we can estimate cutoff parameters by relating them with inverse (rms) size of hadrons. Cutoff parameter for meson-meson vertex is determined by the ratio of size of nucleon to pseudoscalar meson in ref. [27].

\[ \Lambda_D = \frac{r_N}{r_D} \Lambda_N, \quad \Lambda_B = \frac{r_N}{r_B} \Lambda_N \]  

The values of the ratios \( r_N/r_D = 1.35 \) and \( r_N/r_B = 1.29 \) are determined by the quark potential model for \( D \) and \( B \) mesons respectively [27]. Cutoff parameter \( \Lambda_N \) for nucleon-meson vertex can be determined from empirical data of nucleon-nucleon system. In ref. [27] \( \Lambda_N = 0.94 \) GeV, is fixed from the empirical value of the binding energy of deuterium. Where as, nucleon-nucleon scattering data gives \( \Lambda_{\pi NN} = 1.3 \) GeV and \( \Lambda_{\rho NN} = 1.4 \) GeV [28]. A variation of 0.9 to 1.4 GeV in \( \Lambda_N \) produces variation of 1.2 to 1.8 GeV in \( \Lambda_D \) and \( \Lambda_B \). Based on these results we take all the cutoff parameters same for simplicity and vary them on the scale 1 to 2 GeV to study the uncertainties in cross sections due to cutoff parameter.

Fig. 3 shows that for \( B_c^+ \pi \rightarrow DB \) process the cross section roughly varies from 2 to 7 mb, when the cutoff parameter is between 1 to 2 GeV. Suppression due to form factor at cutoff \( \Lambda = 1 \) and 2 GeV is roughly by factor 11 and 3 respectively.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{\( B_c \) absorption cross sections of the process \( B_c^+ \pi \rightarrow D^* B^* \) for three different values of four-point coupling, \( g_{\pi B_c B^* D^*} = 105, 200, 295 \) for dotted, solid and dashed curve respectively (a) without and (b) with form factor. Cutoff parameter is taken 1.5 GeV.}
\end{figure}

\( B_c \) absorption cross section of the process \( B_c^+ \pi \rightarrow D^* B^* \) depends upon the four point contact coupling \( g_{\pi B_c B^* D^*} \), whose values is fixed through SU(5) symmetry. It is noted in the previous section that although SU(5) symmetry uniquely fix it, but difference in the values of the couplings \( g_{\pi DD^*} \) and \( g_{\pi BB^*} \) produces two values 105 and 295 of the four point contact coupling. In this paper, we treat this variation as uncertainty in the coupling and study its effect on the cross section of the process. Fig. 4a, shows how the value of the four point coupling could affect the values of \( B_c \) absorption cross sections through the process \( B_c^+ \pi \rightarrow D^* B^* \) without form factor. Both of the cross sections increase very rapidly for the values 105 and 295, which are not realistic. However, if we use the value of 200, the average to two extreme values the variation in the cross section, denoted by solid line is some what a compromise. Fig. 4b, shows the effect of uncertainty in the four point contact coupling, on the cross section with form factor. This figure indicates that the value of the contact coupling significantly affects the cross section only near the threshold energy (7.34 GeV). It will be discussed later that this effect is further marginalized in the total absorption cross section.
Figure 5: $B_c$ absorption cross sections for the process $B_c^+ \pi \rightarrow D^* B^*$. Solid and dashed curves represent cross sections without and with form factor respectively. Lower and upper dashed curves are with cutoff parameter $\Lambda = 1$ and 2 GeV respectively and $g_{\pi B_c B^* D^*} = 200$. Threshold energy is 7.34 GeV.

Fig. 5 shows the $B_c$ absorption cross sections of the process $B_c^+ \pi \rightarrow D^* B^*$ as a function of total center of mass (c.m) energy $\sqrt{s}$. The cross section of the process roughly varies from 0.2 to 2 mb, when the cutoff parameter is between 1 to 2 GeV and $g_{\pi B_c B^* D^*} = 200$. Suppression due to form factor at cutoff $\Lambda = 1$ and 2 GeV is roughly by factor 45 and 7 respectively. Relatively high suppression in this process is mainly due to large values of mass of final particles $D^*$ and $B^*$. It is noted that these estimates of cross sections are highly dependent on the choice of form factor and the value of cutoff, as well as on the values of coupling constants. However, it is observed that the effect of uncertainty in the four point contact coupling $g_{\pi B_c B^* D^*}$ is marginal on the total cross section due to relatively small value of the cross section of the second process. This is shown in the Fig. 6, in which total absorption cross section for $B_c + \pi$ is plotted for three different values of $g_{\pi B_c B^* D^*} = 105, 200, 295$.

6 Concluding Remarks

In this paper, we have calculated $B_c$ absorption cross section by $\pi$ mesons using hadronic Lagrangian based on SU(5) flavor symmetry. This approach has already been used for calculating absorption cross sections of $J/\psi$ and $\Upsilon$ mesons by hadrons. In our study, all the coupling constants are preferably determined empirically using vector meson dominance model, heavy quark symmetries or QCD sum rules instead of using SU(5) symmetry. The hadronic Lagrangian based on SU(5) flavor symmetry is developed by imposing the gauge symmetry, but this symmetry is broken when the mass terms are added in the Lagrangian. Thus SU(5) gauge symmetry exists only in limit of zero hadronic masses. Broken SU(5) symmetry does not necessarily implies that the coupling constants of three or four-point vertices should be related through SU(5) universal coupling constant. It is, therefore, justified to empirically fix the couplings. It can also be seen that the empirical values of the couplings also violate SU(5) symmetry relations given in Eqs. 8
and 9. It is also noted that four-point coupling constant $g_{\pi B^* D^*}$ cannot be fixed empirically. Thus in this case we have no choice except to make a reasonable estimate using SU(5) symmetry as discussed above. Calculated cross sections are found to be in range 2 to 7 mb and 0.2 to 2 mb for the processes $B_c^+ \pi \rightarrow DB$ and $B_c^+ \pi \rightarrow D^* B^*$ respectively, when the form factor is included. These results could be useful in calculating production rate of $B_c$ meson in relativistic heavy ion collisions.

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