n-p Interaction Effects on the Double Beta Decay Nuclear Matrix Elements for Medium Mass Nuclei

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Abstract

The quality of HFB wave functions are tested by comparing the theoretically calculated results with the available experimental data for a number of spectroscopic properties like yrast spectra, reduced B(E2) transition probabilities, quadrupole moments and g-factors for the nuclei involved in $2\nu\beta\beta$ decay. It is observed that the np interactions vis-à-vis the deformations of the intrinsic ground states of medium mass nuclei play a crucial role in the fine tuning of the nuclear matrix elements, $M_{2\nu}$.

It is well established by now that the implications of nuclear $\beta\beta$ decays are far reaching in nature in general. $0\nu\beta\beta$ decay in particular is one of the very rare promising processes to test the physics beyond the standard model (SM) of fundamental particles. These aspects of nuclear $\beta\beta$ decay have been excellently elaborated in a number of review articles over the past years [1-8].

The $2\nu\beta\beta$ decay, a second order process of weak interaction that conserves the lepton number exactly, is allowed in the SM. The half life of $2\nu\beta\beta$ decay is a product of accurately known phase space factor and appropriate nuclear transition matrix element $M_{2\nu}$. The half lives of $2\nu\beta\beta$ decay have been already measured for about ten nuclei and the values of $M_{2\nu}$ can be extracted directly. Consequently, the validity of different models employed for nuclear structure calculations can be tested by calculating the $M_{2\nu}$.

It is observed that in all cases the $2\nu\beta\beta$ decay matrix elements are sufficiently quenched. The main motive of all the theoretical calculations is to understand the physical mechanism responsible for the suppression of the $M_{2\nu}$. The $M_{2\nu}$ is calculated mainly in three types of models. One is the shell model and its variants. The second is the quasiparticle random phase approximation (QRPA) and extensions their of. The third type of models is classified as the alternative models. The details about these models - their advantages as well as shortcomings - have been discussed excellently by Suhonen and Civitarese [5] and Faessler and Sinkovic [6].

All the nuclei undergoing $\beta\beta$ decay are even-even type. Hence the pairing degrees of freedom play an important role. Moreover, it has been conjectured that the deformation can play a crucial role in $\beta\beta$ decay rates. Hence it is desirable to have a model that incorporates the pairing and deformation degrees of freedom on equal footing in its formalism. For this purpose the Projected Hartree-Fock-Bogoliubov (PHFB) model is one of the most natural choices. Coincidentally, most of the $\beta\beta$ decaying nuclei fall in medium mass region. The success of the PHFB model in explaining the observed experimental trends in this mass region has motivated us to apply the PHFB wave functions to the study of nuclear $\beta\beta$ decay as well.

The mass region $A\approx$100 provides us with a nice example of shape transitions [9], where, at one end, nuclei can be described in terms of shell model wave functions involving a small number of...
configurations and, at the other end of this region, we find good evidence of rotational collectivity. These nuclei lie between doubly magic $^{132}\text{Sn}$ and the strongly deformed $^{100}\text{Zr}$, near which the structural changes are rather rapid with the addition of protons and neutrons. In the past there have been many attempts, [10-15] to explore the factors responsible for the structural changes in this mass region.

Federman and Pittel [14] computed the deformation energy in the framework of Hartree-Fock-Bogoliubov (HFB) theory in conjunction with the surface delta interaction (SDI), suggesting that the neutron-proton (n-p) interaction in the spin-orbit partner (SOP) orbits-1g9/2 and 1g7/2 in this case may be instrumental vis-à-vis the onset of deformation in Mo isotopes with A>100. A systematic study of the behavior of the low-lying collective states of neutron-rich even Cd, Pd, Ru, and Mo isotopes has lead to the conclusion that these structural changes are related to the exceptionally strong n-p interaction in this region. It has also been observed that the n-p interactions among the SOP orbits have a deformation producing tendency and the systematics of low-lying states are intricately linked with the nature of n-p interaction.

The sensitivity of the yrast spectra and the transition charge densities (TCD) to the neutron-proton interaction strength has lead to the fixing of these strengths very accurately and has been demonstrated [15] through the examples of $^{110}\text{Cd}$ and $^{114}\text{Cd}$. We have adopted this method for fixing the p-n strength of QQ interaction by looking at the spectra of $2^+$ state of the nuclei involved in $\beta\beta$ decay.

A large number of theoretical as well as experimental studies of $2\nu\beta\beta$ decay have already been done for $\beta^-\beta^-\text{Zr}$, $^{100}\text{Mo}$, $^{106}\text{Pd}$, $^{128,130}\text{Te}$ nuclei and $e^+\beta^-\beta^-\text{Ru}$, $^{106}\text{Cd}$, $^{124}\text{Xe}$ and $^{130}\text{Ba}$ nuclei over the past few years with more emphasis on $^{100}\text{Mo}$ and $^{106}\text{Cd}$ cases. The $\beta\beta$ decay is not an isolated nuclear process. The availability of data permits a rigorous and detailed critique of the ingredients of the microscopic models used to provide a description of these nuclei.

We have studied the $2\nu\beta\beta$ decay not isolatedly but together with other observed nuclear phenomena. This is in accordance with the basic philosophy of nuclear many body theory, which is to explain all the observed properties of nuclei in a coherent manner. Hence as a test of the reliability of the wave functions, we have calculated the yrast spectra, reduced B(E2) transition probabilities, static quadrupole moments and g-factors and compared with the available experimental data.

The theoretical formalism to calculate the half life of $2\nu\beta\beta$ decay mode has been given by Haxton and Stephenson [1], Doi et al [2,3] and Tomoda [4]. Very brief outlines of the calculation of nuclear transition matrix elements of the $\beta\beta$ decay in the PHFB model are presented here. Details of expressions used in calculation of spectroscopic properties can be found in Dixit et al [16].

The half-life of $2\nu\beta\beta$ decay for $0^+ \rightarrow 0^+$ transition is given by

$$T_{1/2}^{2\nu}(0^+ \rightarrow 0^+)^{-1} = G_{2\nu} |M_{2\nu}|^2$$

(1)

where

$$M_{2\nu} = \sum_N \frac{\langle 0^+ \| \sigma \tau^+ \| 1_N^+ \rangle \langle 1_N^+ \| \sigma \tau^+ \| 0^+ \rangle}{E_N - (M_I + M_F)/2}$$

(2)

and the integrated kinematical factor $G_{2\nu}$ can be calculated with good accuracy [8]. If the $E_N$ of Eq.(11) is replaced by an average $\langle E_N \rangle$, the summation over intermediate states can be completed using the closure approximation one obtains

$$M_{2\nu} = -\frac{2M_{2\nu}^{GT}}{\langle E_N \rangle - (M_I + M_F)/2} = -\frac{2M_{2\nu}^{GT}}{E_d}$$

(3)

where the double Gamow-Teller matrix element (DGT) $M_{2\nu}^{GT}$ is defined as follows.
Employing the HFB wave functions, one obtains the following expression for the $\beta\beta$ decay nuclear transition matrix element.

$$
M_{GT}^{2\nu} = \frac{1}{2} \left( 0^+ \mid \sum_{n,m} \sigma_n \sigma_m \tau_n^+ \tau_m^+ \mid 0^+ \right)
$$

(4)

The $\pi$ ($\nu$) represents the proton (neutron). PHFB calculations are summarized by the coefficients $(U_{im}, V_{im})$ and $C_{ij,m}$ and their matrices $(F_{N,Z})_{\alpha \beta}$ and $(f_{N,Z})$. The details can be found in reference [16].

In the present calculations we treat the doubly even nucleus $^{76}$Sr (Z=N=38) as an inert core and the valence space is spanned by the orbits $1p_{1/2}, 2s_{1/2}, 1d_{3/2}, 1d_{5/2}, 0g_{7/2}, 0g_{9/2}$ and $0h_{11/2}$ for protons and neutrons. The set of single particle energies (SPE’s) but for the $\varepsilon(0h_{11/2})$ which is slightly lowered, employed here is same as used in a number of successful shell model as well as variational model [10-16] calculations for nuclear properties in the mass region $A=100$. The effective two-body interaction is the PPQQ type [17].

Table 1. Variation in excitation energies in MeV of $J^\pi=2^+, 4^+$ and $6^+$ yrast states for $^{100}$Mo and $^{100}$Ru nuclei with change in $\chi_{pn}$ keeping fixed $G_p = -0.30$ MeV, $G_n = -0.20$ MeV and $\varepsilon(0h_{11/2}) = 8.6$ MeV.

| Nucleus | $\chi_{pn}$ | $\chi_{pp}$ | $\chi_{nn}$ |
|---------|-------------|-------------|-------------|
| $^{100}$Mo | 0.01826 | 0.01866 | 0.01906 | 0.01946 | 0.01986 | Exp. [21] |
| $^{100}$E2+ | 0.6865 | 0.5851 | 0.5356 | 0.4493 | 0.3923 | 0.5355 |
| $^{100}$E4+ | 1.7028 | 1.5333 | 1.4719 | 1.3070 | 1.1861 | 1.1359 |
| $^{100}$E6+ | 2.9355 | 2.7213 | 2.6738 | 2.4560 | 2.2854 |
| $^{100}$Ru | 0.01758 | 0.01798 | 0.01838 | 0.01878 | 0.01918 |
| $^{100}$E2+ | 0.6597 | 0.5923 | 0.5395 | 0.4930 | 0.4445 | 0.5396 |
| $^{100}$E4+ | 1.8175 | 1.6733 | 1.5591 | 1.4531 | 1.3372 | 1.2265 |
| $^{100}$E6+ | 3.2746 | 3.0615 | 2.8940 | 2.7329 | 2.5519 | 2.0777 |

The strength of the pairing interaction is fixed through the relation $G_p = -30/A$ MeV and $G_n = -20/A$ MeV. These values of $G_p$ and $G_n$ have been used by Heestand et al [18] to successfully explain the experimental $g(2^+)$ data of some even-even Ge, Se, Mo, Ru, Pd, Cd and Te isotopes in Greiner’s collective model [19]. The strengths of the like particle components of the QQ interaction are taken as: $\chi_{pp} = \chi_{nn} = -0.0105$ MeV b$^{-4}$. These values for the strength of the interaction are
comparable to those suggested by Arima on the basis of an empirical analysis of the effective two-body interactions [20].

Table 2. Experimental half-lives $T_{1/2}^{2\nu}$, and corresponding nuclear matrix elements $M_{2\nu}$, along with the theoretical values in different models for $0^+ \rightarrow 0^+ 2\nu\beta\beta$ decay of $^{100}$Mo. The numbers corresponding to (a) and (b) are calculated for $g_A=1.25$ and 1.0 respectively.

| Experiment | Theory |
|------------|---------|
| Ref Projects | $T_{1/2}^{2\nu}$ ($10^{18}$ yrs) | $|M_{2\nu}|$ |
| [22] UC-Irvin | 6.82$^{+0.06}_{-0.05}$ ± 0.68 | a)0.125$^{+0.014}_{-0.009}$ b)0.195$^{+0.014}_{-0.008}$ |
| [23] NEMO | 9.5±0.4 ± 0.9 | a)0.106$^{+0.013}_{-0.007}$ b)0.165$^{+0.013}_{-0.010}$ |
| [8] Average | 8.0±0.7 | |

As an illustrative case we look into the details of calculations for double beta decay of $^{100}$Mo nucleus. The $\chi_{pn}$ is varied so as to obtain the spectra of $^{100}$Mo and $^{100}$Ru in optimum agreement with the experimental results. In Table 1, we have presented the theoretically calculated yrast energies for levels of $^{100}$Mo and $^{100}$Ru for different values of $\chi_{pn}$. It is clearly observed that as the $\chi_{pn}$ is varied by 0.0016 MeV b$^{-4}$, the $E_2$, decreases by 0.2942 MeV in case of $^{100}$Mo and 0.2152 MeV in case of $^{100}$Ru respectively. This is understandable as there is an enhancement in the collectivity of the intrinsic state with the increase of $|\chi_{pn}|$, hence the $E_2$ decreases. The optimum values of $\chi_{pn}$ corresponding to $^{100}$Mo and $^{100}$Ru are 0.01906 MeV b$^{-4}$ and -0.01838 MeV b$^{-4}$ respectively. Thus for a given model space, SPE’s, $G_p$, $G_n$ and $\chi_{pp}$, we have fixed $\chi_{pn}$ through the experimentally available energy spectra.

Table 3. Experimental limit on half-lives ($T_{1/2}^{2\nu}$) and corresponding extracted matrix elements $M_{2\nu}$ along with their theoretically calculated values for $2\nu\beta^+\beta^+/\beta^+EC/ECEC$ decay of $^{106}$Cd for $0^+ \rightarrow 0^+$ transition. The numbers corresponding to (a) and (b) are calculated for $g_A=1.25$ and 1.0 respectively.

| Decay Mode$^\dagger$ | Experiment | Theory |
|---------------------|------------|---------|
| Mode$^\dagger$ | Ref $T_{1/2}^{2\nu}$ (yrs) | Ref Model | $|M_{2\nu}|$ ($T_{1/2}^{2\nu}$)$^\ddagger$ |
| $\beta^+\beta^+$ | 10$^{25}$ yrs | 27 | $>2.4\times10^{20}$ | [31] PHFB | 0.238 | 35.42 | 89.56 |
| | | 28 | $>1.0\times10^{19}$ | [32] QRPA | 0.166 | 72.79 | 180# |
| | | 29 | $>9.2\times10^{17}$ | [28] QRPA | 1.226 | 1.33 | 3.3 |
| $\beta^+EC$ | 10$^{21}$ yrs | 27 | $>4.1\times10^{20}$ | [31] PHFB | 0.238 | 8.97 | 22.69 |
| | | 28 | $>0.66\times10^{19}$ | [32] QRPA | 0.169 | 17.79# | 44.0# |
| | | 29 | $>2.6\times10^{17}$ | [33] SU(4) | 0.198 | 13.00 | 32.15 |
| ECEC | 10$^{20}$ yrs | 30 | $>5.8\times10^{18}$ | [31] PHFB | 0.238 | 11.24 | 28.42 |
| | | 32 | QRPA | 0.169 | 22.24# | 55.0# |
| | | 33 | SU(4) | 0.193 | 17.00 | 42.04 |

$^*$ denotes half-life limit for $0\nu + 2\nu$ mode, $^{**}$ denotes half-life limit for $0\nu + 2\nu M$ mode, # shows half-life with WS potential, $^\dagger$ Number below the mode is multiplication factor of Half-life for theoretical values.

From the overall agreement [16] between the calculated and observed electromagnetic properties, it is clear that the PHFB wave functions of $^{106}$Mo and $^{100}$Ru generated by fixing $\chi_{pn}$ to reproduce the yrast spectra are quite reliable.

The double beta decay of $^{100}$Mo $\rightarrow^{100}$Ru for $0^+ \rightarrow 0^+$ transition has been investigated by many experimental groups [22,23] as well as theoreticians by employing different theoretical frameworks.
In Table 2, we have compiled some of the latest available experimental and the theoretical results along with our calculated $M_{2\nu}$ and the corresponding half-life $T^{2\nu}_{1/2}$. We have used a phase space factor $G_{2\nu} = 9.434 \times 10^{-18}$ yr$^{-1}$ given by Doi et al [2] and an energy denominator $E_d = 11.2$ MeV given by Haxton et al [1]. In column 4 of Table 2, we have presented the $M_{2\nu}$ extracted from the experimentally observed $T^{2\nu}_{1/2}$ using the phase space factor given above. The phase space integral has been evaluated for $g_A=1.25$ by Doi et al [2]. However in heavy nuclei it is more justified to use the nuclear matter value of $g_A$ around 1.0. Hence, the experimental $M_{2\nu}$ as well as the theoretical $T^{2\nu}_{1/2}$ are calculated for $g_A=1.0$ and 1.25. The present calculation and that of Hirsch et al using SU3(SPH) [25] give nearly identical value. They are close to the experimental result given by De Silva et al [22] for $g_A=1.25$ while for $g_A=1.0$, the above two $M_{2\nu}$ are in agreement with the results of NEMO. The calculated values given by Stoica using SRPA(WS) [24] are too low and those from Suhonen et al [26] are slightly on higher side. Further the value $M_{2\nu}$ given by Hirsch et al using SU3(DEF) [25] favors the results of NEMO [23] for $g_A=1.25$.

Another example we take for the $0^+ \rightarrow 0^+$ positron $\beta\beta$ ($\beta^+\beta^+$, $\beta^+\text{EC}$ and ECEC) decay of $^{106}\text{Cd} \rightarrow ^{106}\text{Pd}$. This transition has also been investigated by many experimental groups and in different theoretical frameworks. In Table 3, we have compiled some of the latest available experimental [27-30] and the theoretical results [31-33] along with our calculated $M_{2\nu}$ and corresponding half-lives $T^{2\nu}_{1/2}$. We have used phase space factors given by Doi and Kotani [3] and the average energy from Haxton and Stephenson [1]. Our calculated values are nearly half of the recently given QRPA results of Suhonen and Civitaresse [32] for all the three modes. The theoretical values of PHFB and SU(4) [33] are in better agreement (factor of roughly two third) for the $\beta^+\text{EC}$ and ECEC modes.

From the above discussions, it is clear that the validity of nuclear models presently employed to calculate the $M_{2\nu}$ cannot be uniquely established due to error bars in experimental results as well as uncertainty in $g_A$. Further work is necessary both on the experimental and theoretical front to judge the relative applicability, success and failure of various models used so far for the study of double beta decay processes.

As an example to see quantitatively the effect of deformation on $M_{2\nu}$ vis-a-vis the variation of the strength of pn part of the QQ interaction, the results are displayed in Fig. 1 for $^{100}\text{Mo}$ and $^{106}\text{Cd}$ cases. It is observed that the deformations of the HFB intrinsic states play an important role in the calculations of $M_{2\nu}$ and hence on the half life.

To summarize, we have first tested the quality of HFB wave functions by comparing the theoretically calculated results for a number of spectroscopic properties of nuclei involved in double beta decay. To be more specific we have computed the yrast spectra, reduced B(E2) transition probabilities, quadrupole moments and g-factors. Some of the results have been presented for two very widely studied cases of $\beta^-\beta^-$ decaying $^{100}\text{Mo}$ and $e^+\beta\beta$ ($\beta^+\beta^+$, $\beta^+\text{EC}$ and ECEC) decaying $^{106}\text{Cd}$ nuclei. Reliability of the intrinsic wave functions for calculation of $2\nu\beta\beta$ nuclear matrix elements, $M_{2\nu}$ , has been discussed. Further, we have shown that the np interactions viz a viz the deformations of the intrinsic ground states of $^{100}\text{Mo}$, $^{100}\text{Ru}$, $^{106}\text{Cd}$ and $^{106}\text{Pd}$ play important role in arriving at the appropriate nuclear matrix elements. A reasonable agreement between the calculated and observed spectroscopic properties as well as the $2\nu\beta\beta$ decay rate of most of the nuclei in medium mass region makes us confident to employ the same PHFB wave functions for the study of $0\nu\beta\beta$ decay.

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