Nonclassicality of Thermal Radiation

Lars M. Johansen

Department of Technology, Buskerud University College, P.O. Box 251, N-3601 Kongsberg, Norway

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It is demonstrated that thermal radiation of small occupation number is strongly nonclassical. This includes most forms of naturally occurring radiation. Nonclassicality can be observed as a negative weak value of a positive observable. It is related to negative values of the Margenau-Hill quasi-probability distribution.

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The very signature of quantum mechanics, Planck’s constant $\hbar$, was measured for the first time in 1900 in blackbody radiation [1]. In order to derive the spectrum for blackbody radiation, Planck had to assume that the energy of the radiation field could only assume discrete values. This was the first indication that the classical electromagnetic theory of radiation did not provide a fully satisfactory model of nature. However, Planck himself maintained that the radiation field could be described by classical electromagnetism. For him, it was the emission from the material oscillators that was quantized [2]. More recent work has confirmed that the blackbody spectrum can be explained in terms of stochastic electromagnetism [3].

The invention of the laser around 1960 sparked the development of a whole new area of physics, quantum optics. It was found that coherent states, a non-thermal state of light, was a natural building block in a quantum theory of radiation [4]. Glauber and Sudarshan demonstrated that any density matrix could be expanded diagonally in terms of coherent states [4, 5];

$$\hat{\rho} = \int d^2 \alpha P(\alpha) | \alpha \rangle \langle \alpha |.$$  

(1)

The weight function $P(\alpha)$ is known as the Glauber-Sudarshan $P$-distribution. Glauber also proposed that nonclassical states, i.e., states that cannot be modelled by a classical theory, are those for which the $P$-distribution fails to be a probability distribution [4, 6]. Put differently, any state which cannot be expressed as a classical mixture of coherent states is nonclassical.

Among pure states, the coherent states are the only ones satisfying the Glauber classicality criterion [7]. However, it was recently shown that coherent states may display nonclassical properties in weak measurements [8]. This demonstrates that Glauber’s classicality criterion has not taken into account the possibility of weak measurements. Weak measurements were discovered by Aharonov, Albert and Vaidman (AAV) in 1988 [9].

Thermal radiation is almost the only form of naturally occurring radiation, emanating from any object of finite temperature. It can be seen in such diverse areas as the cosmic background radiation, the light from stars and from earthly sources such as a fire or a light bulb. The $P$-distribution for a thermal state is

$$P(\alpha) = \frac{1}{\pi \langle \hat{n} \rangle} e^{-|\alpha|^2/\langle \hat{n} \rangle},$$  

(2)

where $\langle \hat{n} \rangle$ is the expected photon number. This distribution is neither negative nor singular, hence this is essentially a classical state according to the Glauber criterion.

In this Letter, we will demonstrate that also thermal radiation may display nonclassical properties in weak measurements. The term “nonclassical” means, in this respect, that the effect cannot be reproduced by a model where the observables are stochastic $c$-numbers. We will also show that the effect is related to negativity of the Margenau-Hill quasi-probability distribution [10].

In a standard, projective measurement, the uncertainty of the pointer is initially very small so that the pointer can distinguish different eigenvalues of the observable [11]. In the weak measurement scheme of AAV, the pointer is assumed to be in a pure, gaussian state of large uncertainty. In this way, the pointer cannot discern the different eigenvalues of the observable. The theory of weak measurements has been generalized to arbitrary states of object and pointer in Ref. [12]. Generally, a weak measurement of an observable $\hat{c}$ conditioned on the outcome of projective measurement of an observable $\hat{d}$ yields the real part of the weak value

$$c_w(d) = \frac{\langle d | \hat{\rho} \hat{c} | d \rangle}{\langle d | \hat{\rho} | d \rangle}.$$  

(3)

Here $\hat{\rho}$ is the density operator of the object. $c_w$ is independent of the specific choice of pointer state. The pointer may be in an arbitrary state, pure or mixed, provided that the current density vanishes. Also, it is required that the interaction between the object and pointer should be sufficiently weak [12].

The Hamiltonian of the free radiation field is defined as

$$\hat{H} = \frac{1}{2}(\hat{p}^2 + \hat{q}^2).$$  

(4)

There are two contributions to the Hamiltonian from each of the quadratures $\hat{q}$ and $\hat{p}$. In this sense, the energy of the radiation field consists of one energy contribution...
from each quadrature. For a material oscillator, one term is kinetic and the other is potential energy. We shall consider weak measurements of the energy contribution from one quadrature postselected on the other. It was recently shown [8] that the weak value of $\hat{p}^m$ postselected on $\hat{q}$ is a conditional moment of the standard ordered distribution [13]

$$
(p^n)_w(q) = \frac{\int dp \, p^n \, S(q, p)}{\langle q | \hat{p} | q \rangle},
$$

where the standard ordered distribution is defined as [13]

$$
S(q, p) = \frac{1}{2\pi} \int dq \, y \, e^{-iyp}.
$$

The standard ordered distribution is the complex conjugate of the Kirkwood distribution [14].

By using Eq. (1), we may express the standard ordered distribution in terms of the $P$-distribution,

$$
S(q, p) = \frac{1}{2\pi} \int d^2\alpha P(\alpha) \int dq \, y \, |\alpha \rangle \langle q | e^{-iyp}.
$$

By using the quadrature representation of a coherent state $|\alpha\rangle$ [15]

$$
|q | \alpha \rangle = \pi^{-1/4} \exp \left[ -\frac{q^2}{2} + \sqrt{2} \alpha q - \frac{1}{2} \alpha^2 - \frac{1}{2} \alpha^2 \right],
$$

and by inserting the $P$-distribution for a thermal state in Eq. (7), it can be shown that the standard ordered distribution for a thermal state is

$$
S(q, p) = \frac{\exp \left[ -2\alpha^2 (q^2 + p^2 + 2ipq) \right]}{\pi \sqrt{1 + 4\sigma^2}},
$$

where

$$
\sigma^2 = \langle \hat{n} \rangle + \frac{1}{2}
$$

is the variance of each quadrature.

Thermal radiation is often well described by the spectral distribution of blackbody radiation. For blackbody radiation the expected occupation number is

$$
\langle \hat{n} \rangle = \frac{1}{\exp \left( \frac{\Delta p}{\Delta q} \right) - 1}.
$$

At maximal irradiance of the Planck distribution, as given by Wien’s displacement law, the occupation number is of the order $10^{-2}$. The Margenau-Hill distribution [10], which is the real part of the standard ordered distribution, has been plotted in Fig. 1 for an occupation number of $10^{-2}$, and in Fig. 2 at an occupation number of 1.

The standard ordered distribution yields correct marginals when integrated over. Thus, the marginal distribution for the quadrature $\hat{q}$ is

$$
\langle q | \hat{p} | q \rangle = \int dp \, S(q, p) = \frac{1}{\sqrt{2\pi} \sigma} \, e^{-q^2/(2\sigma^2)}.
$$

This is a standard gaussian distribution with variance $\sigma^2$. By inserting Eq. (9) in Eq. (5) we find that the weak value of $\hat{p}^2$ is

$$
(p^2)_w(q) = \frac{\sigma^2 + 4\sigma^2 q^2}{4\sigma^2}.
$$

This is an inverted parabolic curve which is negative for $|q| \geq \sqrt{\sigma^2 + 4\sigma^6}$. Thus, for sufficiently large $q$, the weak value of $\hat{p}^2$ is negative for any thermal state. A classical radiation model cannot reproduce this result. The theory of weak measurements can also be extended to classical theories. It can be shown that if observables are treated as classical $\epsilon$-numbers, the weak value of an observable
is just the conditional expectation value over a positive joint probability distribution [12]. Therefore, in a classical model, the weak value of a positive observable is always positive.

The probability of observing a negative weak value is

$$P = \int_{-\infty}^{-\sqrt{\sigma^2 + 4\sigma^6}} dq \langle q \mid \hat{\rho} \mid q \rangle + \int_{\sqrt{\sigma^2 + 4\sigma^6}}^{\infty} dq \langle q \mid \hat{\rho} \mid q \rangle. \quad (14)$$

The result is

$$P = \text{erfc} \left( \frac{1}{\sqrt{2}} + 2\sigma^4 \right). \quad (15)$$

This function has been plotted as a function of $\langle \hat{n} \rangle$ in Fig. 3. We see that the result is nonclassical essentially for $\langle \hat{n} \rangle < 1$. For occupation numbers typical of thermal radiation states there is a significant probability for observing a nonclassical negative weak value of $\hat{p}^2$.

In principle, weak measurements employ the same interaction Hamiltonians as strong (or projective) measurements (to this end, compare Refs. [12] and [17]). If a strong measurement can be performed with a specific interaction Hamiltonian, then a weak measurement can be performed with the same interaction type provided that the interaction strength is sufficiently weak.

It could also be possible to observe the nonclassical effect found in this Letter for a massive particle in thermal equilibrium in a harmonic oscillator potential. If a detector is placed in a distance exceeding $\sqrt{\sigma^2 + 4\sigma^6}$ from it’s equilibrium position, a weak measurement of kinetic energy should yield a negative value on average. However, this requires observation in the low temperature regime. For a temperature of 1 $\mu K$, the required oscillator frequency is of the order 100 kHz.

From Eq. (4) for the Hamiltonian, it can be shown that

$$\langle p^2 \rangle_\omega(q) = 2 \frac{\langle q \mid \hat{p} \hat{H} \mid q \rangle}{\langle q \mid \hat{\rho} \mid q \rangle} - q^2. \quad (16)$$

This suggests an alternative strategy for measuring the weak value of $\hat{p}^2$. It could also be done by performing a weak measurement of energy postselected on the quadrature $\hat{q}$, and subsequently subtracting the squared quadrature [18].

A comment can be made on the usage of pointer states. It has recently been shown that weak measurements can be performed with arbitrary pointer states, with the only restriction that the current density of the pointer state must vanish [12]. This means that a weak interaction between a thermal pointer state and another thermal object state may yield a pointer reading which cannot be explained in terms of a classical $c$-number model of radiation!

In conclusion, we have demonstrated that thermal radiation displays strong nonclassicality in weak measurements. The effect is present for low occupation numbers, as found in most naturally occurring forms of radiation. A weak measurement of the squared quadrature postselected on the second quadrature may yield a negative weak value. This effect cannot be reproduced by a classical model of radiation where observables are $c$-numbers. The nonclassical effect is related to negativity of the Margenau-Hill distribution for the thermal state.

* Electronic address: lars.m.johansen@hibu.no

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