Longitudinal aerodynamic coefficients modeling from simulated flight data of fixed wing aircraft based on three methods

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Abstract. The equation error method, output error method, and neural network method are used to model the aircraft longitudinal aerodynamic coefficients from simulated flight data. The equation error method and the output error method are based on postulated mathematical models of the longitudinal aerodynamic coefficients, and the least square principle and the maximum likelihood principle are used to estimate the model parameters respectively. The neural network method uses the BP network output to fit the aerodynamic coefficients without assumptions of mathematical model of aerodynamic coefficients. Two sets of simulation tests were conducted, one set of data is used for model identification, and the other set is used for model validation.

1. Introduction

Accurate mathematical model is important in the process of aircraft design, and the foundation of flight control system and flight simulation system development. In recent years, many scholars are devoted to extracting the model of aerodynamic coefficients from flight test data [1-4].

The frequently-used time domain identification methods include equation error method (EEM), output error method (OEM) and filtering error method (FEM). EEM is a direct method, which is affected by measurement noise in the independent variable. OEM estimates parameters by minimizing the mean square error of measurements and model output[5]. FEM takes into account the process noise and can fit the measurements well. However, the modeling error in FEM may be regarded as the process noise, so the improvement of the model is not obvious [6]. Neural network [7,8] generates models by mapping outputs to the nonlinear functions of inputs, without requiring the physical mechanisms of inputs and outputs[9-11].

In this paper, three methods are used to extract the model of longitudinal aerodynamic coefficients from aircraft simulation data. The longitudinal motion of the aircraft is excited by the elevator input, and the measurements of the aircraft contain random noise.

2. Simulation flight test design

A simulation model is established based on the wind tunnel test data of a UAV. The longitudinal motion model of the aircraft is determined by state equation (1) and output equation (2), where $\bar{q}$ represents the dynamic pressure, $\bar{q}=\frac{1}{2} \rho V^2$. The force coefficients of X-axis, Z-axis and pitching moment coefficients of airframe are respectively expressed as $C_x, C_z, C_m$. In the simulation process, aerodynamic
coefficients are derived from the wind tunnel data interpolation table. The work of this paper is to
determine the relationship between aerodynamic coefficients $C_x, C_z, C_m$ and $\delta_s, \alpha, q$.
\[
\dot{u} = -q\omega - g \sin \theta + \frac{\omega^2 C_x + T}{m}, \quad \dot{w} = q\mu + g \cos \theta + \frac{\omega^2 C_z}{m}, \quad \dot{\phi} = \frac{\omega^2 C_m}{m}, \quad \dot{\theta} = q
\]
\[
V = \sqrt{\dot{u}^2 + \dot{w}^2}, \quad \alpha = \arctan \frac{\dot{w}}{\dot{u}}, \quad q = q, \quad \theta = \theta, \quad a_s = \frac{\omega^2 C_m}{m}
\]

(1) \hspace{1cm} (2)

The elevator excitation is applied on the trim state of aircraft to excite its longitudinal motion. The
double and multistep command signal generated by the computer is directly applied to the elevator. Two
groups of simulation maneuvers are conducted for modeling and verifying the longitudinal aerodynamic
coefficients, the simulated data is shown in figure 1 (a) and (b) respectively. For each measured variable,
the white noise signal is added to imitate the measurement noise.

In the identification process of EEM and OEM, the aerodynamic coefficients $C_x, C_z, C_m$ are assumed
as linear model of state variables $\alpha, q$ and control input $\delta_s$, as shown in equation (3).
\[
C_x(i) = C_x^0 + C_x^\alpha \alpha(i) + \frac{C_x^q(i)}{2V} + C_x^\delta \delta_s(i)
\]
\[
C_z(i) = C_z^0 + C_z^\alpha \alpha(i) + \frac{C_z^q(i)}{2V} + C_z^\delta \delta_s(i)
\]
\[
C_m(i) = C_m^0 + C_m^\alpha \alpha(i) + \frac{C_m^q(i)}{2V} + C_m^\delta \delta_s(i)
\]

(3)

The unknown parameters in the aerodynamic coefficients model (3) constitute the parameter vector
$\theta$ to be identified:
\[
\theta = [C_x^0, C_x^\alpha, C_x^q, C_x^\delta, C_z^0, C_z^\alpha, C_z^q, C_z^\delta, C_m^0, C_m^\alpha, C_m^q, C_m^\delta]^T
\]

(4)

3. Modeling methods
This section describes the principle of modeling aerodynamic coefficients by EEM FEM and BP
network.

3.1. Equation error method
In the state equation (1), except for the aerodynamic coefficients, the other quantities can be obtained
by measurements, and $\dot{\theta}$ is calculated by the numerical differentiation of $q$. Therefore, the aerodynamic
coefficients $C_x, C_z, C_m$ can be obtained by the inverse solution of the motion equation (1). The equation
error method (EEM) transforms the mathematical model (3) into a least square problem to estimate the
parameters. In the case of EEM, the parameters $\theta$ in model (3) are estimated separately. Take $C_x$ as an
example. The mathematical model of x-axis force coefficients in (3) is rewritten as the following
regression form (5)
\[
z = X\theta_x
\]

(5)

where, matrix of vectors of ones and regressors $X$, vector of depended variables $z$, vector of unknown
parameters $\theta_x$ are defined in (6-8) The least square solution of such a linear regression problem is (9)
\[
X = \begin{bmatrix}
1 & \alpha(1) & \frac{\omega^2 q(1)}{2V} & \delta_s(1)\\
1 & \alpha(2) & \frac{\omega^2 q(2)}{2V} & \delta_s(2)\\
M & M & M & M \\
1 & \alpha(N) & \frac{\omega^2 q(N)}{2V} & \delta_s(N)
\end{bmatrix}
\]
\[
z = [C_x(1) \ C_x(2) \ ... \ C_x(N)]^T
\]

(6) \hspace{1cm} (7)
Figure 1 Flight data from simulation test 1 for aerodynamic modeling (a), Flight data from simulation test 2 for model validation (b)

\[ \dot{\theta}_x = \begin{bmatrix} C_{x_1}^0 & C_{x_2}^0 & C_{x_3}^0 & C_{x_4}^0 \end{bmatrix}^T \]  
\[ \dot{\theta}_x = (X^T X)^{-1} X^T \]  

The similar treatment can be used to estimate the parameters in models of \( C_z \) and \( C_m \).

3.2. Output error method

The dynamic model is defined as (10), including state equation, output equation, and measurement equation.

\[ \dot{X}(t) = f[X(t), Y(t), \theta] \]
\[ Y(t) = h[X(t), U(t), \theta] \]
\[ Z(t) = Y(t) + \nu(t) \]  

- Where \( X(t) \) is the state variables vector, \( Y = [V, \alpha, q, \theta]^T \), \( Y(t) \) is the output variables vector, \( Z(t) \) is the discrete measurements vector of \( Y(t) \). The output error method estimates the model parameters by iteratively minimizing the squared errors between the measurements \( Z(t) \) and the model output \( Y(t) \). In equation (10), the definition of equation of state is consistent with equation (1), with the aerodynamic coefficients in equation (1) replaced by postulated model (3); and the output equation is consistent with equation (2). In the case of OEM, all longitudinal aerodynamic parameters are estimated simultaneously. The parameters to be estimated are defined in equation (4).
According to the maximum likelihood criterion, the cost function is defined as formula (11), and the output error identification method minimizes the cost function [6]. R is the covariance matrix of measurement noise, and the maximum likelihood estimation of R is as formula (12):

\[
J(\theta, R) = \frac{1}{2} \sum_{i=1}^{N} v_i^2 R^{-1} v_i + \frac{N}{2} \ln \det(R) + \frac{n_i N}{2} \ln 2\pi
\]

\[
R = \frac{1}{N} \sum_{i=1}^{N} [Z(i)-Y(i)][Z(i)-Y(i)]^T
\]

Parameter is updated according to G-N formula (13-15)

\[
\theta_{k+1} = \theta_k + \Delta \theta, \Delta \theta = -F^{-1}G
\]

\[
F = \sum_{i=1}^{N} \left[ \frac{\partial Y(k)}{\partial \theta} \right]^T R^{-1} \left[ \frac{\partial Y(k)}{\partial \theta} \right]
\]

\[
G = -\sum_{i=1}^{N} \left[ \frac{\partial Y(k)}{\partial \theta} \right]^T R^{-1} [Z(k)-Y(k)]
\]

Where F and G represent Hessian matrix and gradient vector respectively. The sensitivity matrix \( \frac{\partial Y(k)}{\partial \theta} \) is approximated by a numerical partial differential, where the element \( \frac{\partial Y_i}{\partial \theta_j} \) is calculated by (16), here \( Y_i \) is the response of model (10) with parameter \( \theta \); \( \Delta \theta_j \) means a small perturbation in the \( j \)th component of \( \theta \), and \( Y^\ast \) is the response of model (10) with the perturbed parameter \( \theta + \Delta \theta_j \).

\[
\frac{\partial Y_i}{\partial \theta_j} = \frac{Y_i^\ast (i) - Y_i (i)}{\Delta \theta_j}
\]

The best estimation of \( \theta \) is solved by iteratively executing (13) to (15), and the initial value of \( \theta \) is selected as the result of EEM.

![Figure 2: The structure of BP neural network model](image)

3.3. Neural network based modeling method

In the neural network identification method, back propagation neural network (BP) is used to approximate the relationship between aerodynamic coefficients and elevator deflection, angle of attack, pitching rate and velocity. The structure of BP neural network model is shown in the figure 2. The elevator deflection, angle of attack, pitch rate and velocity are taken as network input and aerodynamic coefficients as network output. The relationship is determined by the structure of network, the weight between network layers and the type of activation function. BP neural network includes input layer, hidden layer and output layer, and the number of neurons in the hidden layer is 20 in this paper. \( W_1 \) and \( W_2 \) are the weight and deviation matrices of input layer to hidden layer and hidden layer to output layer respectively, and \( f(x) \) is the nonlinear sigmoidal activation function (21). The propagation of the
network follows the equation (17-20), where $x$ represents the network input, $x_h$ and $x_o$ represent the output of the hidden layer and the output layer respectively. In the process of network training, the error between network output and measurements is minimized by continuously adjusting network parameters $W_1$ and $W_2$.

$$x_1 = W_1 x + W_{1h}$$
$$x_h = f(x_1)$$
$$x_2 = W_2 x_h + W_{2b}$$
$$x_o = f(x_2)$$

(17) (18) (19) (20)

Hyperbolic tangent function

$$f(x) = \tanh(\frac{x}{2})$$

(21)

At the $k$th discrete data point, the cost function is chosen as the local output-error (22),

$$E(k) = \frac{1}{2} [z(k) - x_o(k)]^T [z(k) - x_o(k)]$$

(22)

The network parameters are updated according to the steepest descent method to minimize the cost function (22), $W_2$ is updated according to equation (23) and $W_1$ is updated according to equation (24), $\lambda$ is the learning rate parameter.

$$W_2(k+1) = W_2(k) + \lambda f'(x_2)[z(k) - x_o(k)]x_h^T(k)$$
$$W_1(k+1) = W_1(k) + \lambda f'(x_1)W_2^T f'(x_2)[z(k) - x_o(k)]x_h^T$$

(23) (24)

The derivative of the activation function is as follows (25)

$$f'(x) = \frac{x}{2}[1 - \tanh^2(\frac{x}{2})]$$

(25)

The above network parameters are updated recursively from the first data point to the $N$th data point as an iteration, and network parameters need to be updated iteratively to achieve satisfactory fitting performance. The final fitting performance of the network is measured by mean square error (MSE).

$$MSE = \frac{1}{N} \sum_{k=1}^{N} \sum_{j=1}^{n} [z_j(k) - x_{o_j}(k)]^2$$

(26)

4. Identified results and model validation

For flight data from simulation 1, the above three methods are used to model the longitudinal aerodynamic coefficients. The EEM and OEM are based on the postulated model (3), and the identified parameters are stable derivative and control derivative, as shown in Table 1. It can be seen from table 1 that the identification results of EEM and OEM are relatively close, and the standard deviation of the OEM is lower. The network parameters $W_1$ and $W_2$ trained by steepest descent method is of no physical significance so they are not listed, and the MSE of network fitting is 0.001.

The motion equation (1) in which $C_x, C_z, C_m$ is replaced by three identified models, is excited by the elevator input of simulation test 2, and the outputs of the three identified models are obtained. By comparing the response of the identified model with the measurements of the aircraft in simulation test 2, the prediction performance of the identification model is verified, as shown in Figure 3. We can see that the model responses of OEM and neural network fit the measurements better than EEM.
Figure 3 Predicted output of the three models and measurements in simulation test 2

Table 1 Longitudinal control and stability derivatives of by EEM and OEM, with the standard deviation in the brackets

|                | EEM      | OEM      | EEM     | OEM      | EEM     | OEM     |
|----------------|----------|----------|---------|----------|---------|---------|
| \( \dot{C}_x \) | -0.0554  | -0.0514  | 0.0158  | -0.0154  | -0.0390 | -0.0430 |
| (0.0004)       | (0.0001) | (0.0048) | (0.0005)| (0.0006) | (0.0001)|        |
| \( \dot{C}_z \) | 0.5346   | 0.5238   | -4.0967 | -4.0179  | -0.1448 | -0.1358 |
| (0.0022)       | (0.0006) | (0.0224) | (0.0034)| (0.0031) | (0.0011)|        |
| \( \dot{C}_\alpha \) | 1.9298  | 2.3893   | -22.2506| -29.2490| -6.6116 | -7.4706 |
| (0.1031)       | (0.0221) | (1.0184) | (0.1012)| (0.1450) | (0.0065)|        |
| \( \dot{C}_{\alpha \delta} \) | 0.1108  | 0.1417   | -0.1308 | -0.3746  | -0.4652 | -0.4975 |
| (0.0036)       | (0.0007) | (0.0358) | (0.0030)| (0.0005) | (0.0004)|        |

5. Conclusion
In this paper, three methods are used to extract the model of longitudinal aerodynamic coefficients, including EEM, OEM and neural network modeling method. The data applied for model identification and model validation comes from two different simulation tests, and the measured data are disturbed by random noise. The results show that the three neural network methods and the output error method can better predict the aircraft state and output variables.
The EEM is based on the least square principle and is easy to operate, but the estimation results are affected by the measurement noise in the regression factors. Based on the maximum likelihood criterion, the OEM iteratively minimizes the squared error between the model output and the system measurements. The OEM accounts for the measurement noise, which is more commonly used in the actual system. However, the OEM needs to solve the aircraft state equation, so this method is affected by the initial values of the parameters to be identified. It needs to select the initial value of the parameters close to the true value or use the results of the EEM to initialize the output error step, as is done in this paper. The neural network modeling method applies the network to approximate the input-output relationship of the actual system. The neural network method does not require the specific model of the system to be identified and solving the state equation, so it is very convenient to implement. Moreover, the neural network can well approximate the nonlinear characteristics, so it can be used to model aerodynamic coefficients at high angles of attack.

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