The CP violating asymmetry in $B^\pm \to M\bar{M}\pi^\pm$ decays

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ABSTRACT

We analyze the asymmetry in the partial widths for the decays $B^\pm \to M\bar{M}\pi^\pm$ ($M = \pi^+, K^+, \pi^0, \eta$), which results from the interference of the non-resonant decay amplitude with the resonant amplitude for $B^\pm \to \chi_c0\pi^\pm$ followed by the decay $\chi_c0 \to M\bar{M}$. The CP violating phase $\gamma$ can be extracted from the measured asymmetry. We find that the partial width asymmetry for $B^\pm \to \pi^+\pi^-\pi^\pm$ is about $0.33 \sin\gamma$, and about $0.45 \sin\gamma$ for $B^\pm \to K^+K^-\pi^\pm$, while it is somewhat smaller for $B^\pm \to \pi^0\pi^0\pi^\pm$ and $B^\pm \to \eta\eta\pi^\pm$. Potential sources of uncertainties in these results, primarily coming from poorly known input parameters, are discussed.
The measurement of CP asymmetries in charged B meson decays might provide the first demonstration of CP violation outside the K system [1, 2]. Among the usual three CP-odd phases $\alpha, \beta$ and $\gamma$, the phase $\gamma$ seems to be the most difficult to explore experimentally [3]. One possibility to measure this CP odd phase $\gamma = \arg(V^*_{ub})$ has been suggested in [4, 5]. In this approach the asymmetry appears as a result of the interference of the nonresonant $B^\pm \to M\bar{M}\pi^\pm$ decay amplitude and the resonant $B^\pm \to \chi_{c0}\pi^\pm \to M\bar{M}\pi^\pm$ amplitude, where $\chi_{c0}$ is the $0^{++}$ $c\bar{c}$ state at 3.415 GeV. The absorptive phase necessary to observe CP violation in the partial width asymmetry is provided by the $\chi_{c0}$ width. There have also been suggestions to determine the CP violating phase $\gamma$ by analyzing the Dalitz plots in the decays $B^\pm \to \pi^+\pi^-\pi^\pm$ [6] and $B^\pm \to \pi^-\pi^+K^\pm$ [7].

On the experimental side the CLEO collaboration has reported upper limits on some of the nonresonant decays of the type $B^+ \to h^+h^+h^-$ [8]. CLEO found the upper limits on the branching ratios $BR(B^+ \to \pi^+\pi^-\pi^\pm) \leq 4.1 \times 10^{-5}$ and $BR(B^+ \to K^+K^-\pi^\pm) \leq 7.5 \times 10^{-5}$. In [4] the branching ratio for $B^+ \to \pi^+\pi^-\pi^+$ was estimated to be in the range $1.5 \times 10^{-5}$ to $8.4 \times 10^{-5}$.

Motivated by this theoretical expectation and the CLEO experimental results, we further investigate the asymmetry in the decays $B^\pm \to M\bar{M}\pi^\pm$, where $M = \pi^+, K^+, \pi^0, \eta$, improving upon the calculation of the nonresonant part of the decay amplitude. We will assume, as in [3], the resonant decay amplitude in $B^\pm \to M\bar{M}\pi^\pm$ is due to the $c\bar{c}$ resonance $\chi_{c0}$ which subsequently decays into $M\bar{M}$, where $M = \pi^+, K^+, \pi^0, \eta$. Note that we are interested only in the kinematical region where the $M\bar{M}$ invariant mass is close to the $\chi_{c0}$ mass, as in [3]. Thus $M\bar{M}$ arising from other resonances such as the $\rho$ need not be considered. However, we will use the nonresonant $B^\pm \to M\bar{M}\pi^\pm$ decay amplitudes, calculated using techniques developed previously in our analysis of $D_{l4}$ decays [10]. In particular, we use the factorization approximation, in which the main contribution to the nonresonant $B^\pm \to M\bar{M}\pi^\pm$ amplitude comes from the product $<M\bar{M}|(\bar{u}b)V_{-A}|B^- > < \pi^-|(\bar{d}u)V_{-A}|0>$ where $(\bar{q}_1q_2)_{V-A}$ denotes $\bar{q}_1\gamma_{\mu}(1 - \gamma_5)q_2$. For the calculation of the matrix element $<M\bar{M}|(\bar{u}b)V_{-A}|B^- >$ we extend the results obtained in [10], where the nonresonant $D^+ \to K^-\pi^+\nu\bar{\nu}$ decay was analyzed. In this analysis the experimental result for the branching ratio of the nonresonant $D^+ \to K^-\pi^+\nu\bar{\nu}$ decay was successfully reproduced within a hybrid framework which combines the heavy quark effective theory (HQET) and the chiral Lagrangian (CHPT) approach.
The combination of heavy quark symmetry and chiral symmetry has been quite successful in the analysis of D meson semileptonic decays [11] - [16]. The heavy quark symmetry is expected to be even better for the heavier B mesons [14, 15]. However, CHPT could be worse in B decays due to the large energies of light mesons in the final state. It is really only known that the combination of HQET and CHPT is valid at small recoil momentum. In [16] we have modified the hybrid model of [11] - [14] to describe the semileptonic decays of D mesons to one light vector or pseudoscalar meson state. Our modification is quite straightforward: we retain the usual HQET Feynman rules for the vertices near and outside the zero-recoil region, as in [14, 15], but we include the complete propagators instead of using the usual HQET propagator [16]. This reasonable modification of the hybrid model enabled us to use it successfully over the entire kinematic region of the D meson weak decays [10, 16, 17]. The details of this approach can be found in [10, 16].

In the following we systematically use this model [10, 16] to calculate the nonresonant $B^\pm \to \bar{M}M\pi^\pm$ decay amplitude. We find there are important contributions, which were not taken into account previously [4].

The weak effective Lagrangian for the nonleptonic Cabibbo suppressed $B$ meson decays is given by [4]

$$\mathcal{L}_w = -\frac{G_F}{\sqrt{2}} V_{ud}^*V_{ub}(a_{1\text{eff}}O_1 + a_{2\text{eff}}O_2)$$  \hspace{1cm} (1)

where $O_1 = (\bar{u}b)V_{-A}(\bar{d}u)V_{-A}$ and $O_2 = (\bar{u}u)V_{-A}(\bar{d}b)V_{-A}$. We take the effective coefficients $a_{i\text{eff}}$ ($i = 1, 2$) from the phenomenological fit [18], which gives $a_{1\text{eff}} \simeq 1.08$ and $a_{2\text{eff}} \simeq 0.21$. These values are also in agreement with other analyses [19, 20]. We do not take into account the contributions arising from penguin operators, since these contributions are not expected to be important [4, 6, 21]. The quark currents required in the weak Lagrangian (1) can be expressed in terms of the meson fields, as previously described explicitly in [11, 16].

Using the factorization approximation, as in [4], we can analyze all possible contributions to the $B^\pm \to \bar{M}M\pi^\pm$ nonresonant amplitude. The various kinds of contributions are shown in Fig. 1. To illustrate the use of the factorization of the amplitude we consider the specific decay $B^- \to \pi^-\pi^-\pi^+$. It is easy to see, then, that the two contributions shown in Fig. 1a, which come
from the operator $O_1$, cancel each other in the chiral limit $m_π \to 0$, since in this limit

$$< \pi^- (p_1) π^+(p_2) π^−(p_3)|O_1|B^−(p_B) > = < \pi^- (p_1) π^+(p_2) π^−(p_3)|(\bar{d}u)_A|0 > \times$$
$$< 0|(\bar{u}b)_A|B^−(p_B) > +$$

$$< \pi^- (p_1) π^+(p_2) π^−(p_3)|\mathcal{L}_s|π^−(p_B) > \frac{-1}{m_B^2 - m_π^2} \times$$

$$< \pi^−(p_B)|(\bar{d}u)_A|0 > < 0|(\bar{u}b)_A|B^−(p_B) > = 0$$

(2)

From Fig. 1b there is a contribution from operator $O_2$:  

$$< \pi^- (p_1) π^+(p_2) π^−(p_3)|O_2|B^−(p_B) > =$$

$$< \pi^- (p_1) π^+(p_2)|(\bar{u}u)_V|0 > < \pi^- (p_3)|(\bar{d}b)_V|B^−(p_B) > +$$

$$< (p_1 \leftrightarrow p_3).$$

(3)

The matrix element $< \pi^- (p_1) π^+(p_2)|(\bar{u}u)_V|0 >$ can be calculated using the model developed previously [10]. In this model the matrix element (3) is determined by the pion form factor, which is dominated by the $ρ$ meson pole. However, the $ρ$ meson contribution is not relevant in the present calculation of the nonresonant decay amplitude. Therefore, the contributions coming from diagrams in Fig. 1b can be neglected in our calculation of the nonresonant decay amplitude.

The only important contributions to the nonresonant decay amplitude comes from the diagrams in Fig. 1c and is given by

$$< \pi^- (p_1) π^+(p_2) π^−(p_3)|O_1|B^−(p_B) > =$$

$$< \pi^- (p_3)|(\bar{d}u)_V−A|0 > < \pi^- (p_1) π^+(p_2)|(\bar{u}b)_V−A|B^−(p_B) > +$$

$$< (p_1 \leftrightarrow p_3).$$

(4)

For the matrix element of $< \pi^- (p_1) π^+(p_2)|(\bar{u}b)_V−A|B^−(p_B) >$ we use the results obtained in the analysis of the nonresonant $D^+ → π^+K^−lν_l$ decay width [11]. Following this analysis we write the matrix element $< \pi^- (p_1) π^+(p_2)|(\bar{u}b)_V−A|B^−(p_B) >$ in the general form

$$< \pi^- (p_1) π^+(p_2)|\bar{u}γ_μ(1 - γ_5)b|B^−(p_B) > = i\Gamma(p_B - p_2 - p_1)_μ$$

$$+ i\omega+(p_2 + p_1)_μ + i\omega−(p_2 - p_1)_μ - 2h_μαβγP_αP_βP_γ.$$  

(5)
In the present case only the nonresonant form factors $w_{nr}$ and $w_{nr}^r$ contribute. Note that the contribution proportional to $r$ is of order $m^2_\pi$ and therefore can be safely neglected. However, in [3], where a different parametrization of the form factors was used, in neglecting the contributions of the order $m^2_\pi$, the contributions proportional to $w_\pm$ were also dropped. In the notation of [3] the product $< M(p_1)M(p_2)|(\bar{u}b)_{V-A}|B(p_B) > < \pi(p_3)|(\bar{d}u)_{V-A}|0 >$ is proportional to $F_4 m^2_\pi$ and one can easily show that $F_4 m^2_\pi = -m^2_\pi r + m^2_\pi/2 (w_+ + w_-)(p_B+p_3)\cdot p_3 + m^2_\pi/m^2_\pi(w_+ - w_-)(p_B + p_1)\cdot p_3$, which explicitly demonstrates that the terms proportional to the form factors $w_\pm$ arising from the product $< MM|(\bar{u}b)_{V-A}|B > < \pi|(\bar{d}u)_{V-A}|0 >$ can not be neglected, but are important contributions to the nonresonant decay amplitude. Moreover, this contribution cannot even be treated as being constant [3, 22]; it depends significantly on the variables $s = (p_B - p_3)^2 = (p_2 + p_1)^2$, $t = (p_B - p_1)^2 = (p_2 + p_3)^2$ and $u = (p_B - p_2)^2 = (p_1 + p_3)^2$.

Using the preceding analysis we can write the amplitude for the nonresonant decay $B^\to \pi^+\pi^-\pi^-$

$$\mathcal{M}_{nr}(B^\to \pi^-(p_1)\pi^-(p_3)\pi^+(p_2)) = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{ub}$$

$$\{a^{eff}_1 \left[ \frac{f_\pi}{2} (m^2_B - s - m^2_\pi) w^nr_+ (s, t) + \frac{f_\pi}{2} (2t + s - m^2_B - 3m^2_\pi) w^{nr}_r (t) \right] + (s \leftrightarrow t), \quad (6)$$

where

$$w^nr_+ (s, t) = -\frac{g}{f^2_\pi} \frac{f_B m_\pi^{3/2} m_B^{1/2}}{t - m^2_B} \left[ 1 - \frac{1}{2m^2_B} (m^2_B - m^2_\pi - t) \right]$$

$$+ \frac{f_B}{2f^2_\pi} - \frac{\sqrt{m_B}}{2f^2_\pi} \frac{1}{m^2_B} (2t + s - m^2_B - 3m^2_\pi), \quad (7)$$

and

$$w^{nr}_r (t) = \frac{g}{f^2_\pi} \frac{f_B m_\pi^{3/2} m_B^{1/2}}{t - m^2_B} \left[ 1 + \frac{1}{2m^2_B} (m^2_B - m^2_\pi - t) \right] + \frac{\sqrt{m_B} \alpha_1}{f^2_\pi}. \quad (8)$$

The parameters $\alpha_{1,2}$ are explicitly defined in [3]. Note that both the $\alpha_1$ and $\alpha_2$ terms are important in [3] and [8], which was overlooked previously [3].

For the nonresonant decay amplitude $B^\to \pi^0(p_1)\pi^0(p_2)\pi^-(p_3)$ there are two contributions: one proportional to $a^{eff}_1 < \pi^-|(\bar{d}u)_{A}|0 > < \pi^0\pi^0|(\bar{u}b)_{A}|B^\to$
and another one proportional to $a_2^{eff} < \pi^0|\bar{u}u\rangle_A|0 > < \pi^0\pi^-|db\rangle_A B^- >$. The amplitude for $B^-(p_B) \to \pi^0(p_1)\pi^0(p_2)\pi^-(p_3)$ is then given by

$$\mathcal{M}_{nr}(B^- (p_B) \to \pi^0(p_1)\pi^0(p_2)\pi^-(p_3)) = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \frac{f_\pi}{4}$$

$$\{a_1^{eff}[w^r_+(s,t)(m^2_B - m^2_\pi - s) + w^r_-(t)(2t + s - m^2_B - 3m^2_\pi)]$$

$$+a_2^{eff}[w^r_+(u,t)(m^2_B - m^2_\pi - u) + w^r_-(t)(2t + u - m^2_B - 3m^2_\pi)]$$

$$+ (t \leftrightarrow u)\}. \quad (9)$$

The form factors $w^r_\pm$ appearing in the part of amplitude proportional to $a_2^{eff}$ are given by (7) and (8), including the terms proportional to $\alpha_{1,2}$, while in the part of amplitude proportional to $a_1^{eff}$ these terms depending on $\alpha_{1,2}$ are absent.

A similar analysis of the nonresonant amplitude for the decay $B^- (p_B) \to K^+(p_1)K^-(p_2)\pi^-(p_3)$ gives

$$\mathcal{M}_{nr}(B^- (p_B) \to K^+(p_1)K^-(p_2)\pi^-(p_3)) = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^*$$

$$\{a_1^{eff}[\frac{f_\pi}{2}(m^2_B - s - m^2_\pi)w^r_+(s,t) + \frac{f_\pi}{2}(2t + s - m^2_B - m^2_\pi)$$

$$- m^2_\pi - 2m^2_K)w^r_-(t)]\}, \quad (10)$$

where

$$w^r_+(s,t) = -\frac{g}{f^2_K} \frac{f_{Bs}m^{3/2}_B m^{1/2}_B}{t - m^2_{Bs}}[1 - \frac{1}{2m^2_{Bs}}(m^2_B - m^2_\pi - t)]$$

$$+ \frac{f_B}{2f^2_K} - \frac{\sqrt{m^2_{Bs}\alpha_2}}{2f^2_K} \frac{1}{m^2_B}(2t + s - m^2_B - m^2_\pi - 2m^2_K), \quad (11)$$

and

$$w^r_-(t) = \frac{g}{f^2_K} \frac{f_{Bs}m^{3/2}_B m^{1/2}_B}{t - m^2_{Bs}}[1 + \frac{1}{2m^2_{Bs}}(m^2_B - m^2_\pi - t)] + \frac{\sqrt{m^2_{Bs}\alpha_1}}{f^2_K}. \quad (12)$$

The analysis of the decay $B^- (p_B) \to \eta(p_1)\eta(p_2)\pi^-(p_3)$ is a little more complicated due to $\eta - \eta'$ mixing. The nonresonant decay amplitude is

$$\mathcal{M}_{nr}(B^- (p_B) \to \eta(p_1)\eta(p_2)\pi^-(p_3)) = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \frac{f^2_K}{8}[(1 + c_\eta^2)\frac{1}{f_\eta} + s_\theta c_\theta \frac{1}{f_\eta}]^2$$
\{a_1^{eff}[w_{+}^{nr}(s, t)(m_B^2 - m_\pi^2 - s) + w_{-}^{nr}(t)(2t + s - m_B^2 - m_\pi^2 - 2m_\eta^2)] \\
+ a_2^{eff}[w_{+}^{nr}(u, t)(m_B^2 - m_\eta^2 - u) + w_{-}^{nr}(t)(2t + u - m_B^2 - 2m_\eta^2 - m_\eta^2)] \\
+ (t \leftrightarrow u)\}. (13)

The form factors \(w_{+}^{nr}\) in (13) are given by equations (11) and (12), with \(m_K \to m_\eta\) and without the terms proportional to \(\alpha_1, \alpha_2\). The \(\eta - \eta'\) mixing is defined as usual with \(\eta = \eta_8 c_\theta - \eta_0 s_\theta\) and \(\eta' = \eta_8 s_\theta + \eta_0 c_\theta\), where \(c_\theta = \cos \theta\) and \(s_\theta = \sin \theta\) and \(\theta \simeq -20^\circ\) [24]. In the numerical calculations below we used the values \(f_\eta = 0.13\) GeV and \(f_{\eta'} = 0.11\) GeV determined in [24]. We will not analyze cases where the the \(\eta'\) meson is in the final state since one expects that in the nonleptonic decays of B mesons into final states \(\eta' X\) the gluonic penguin contributions are probably very important (see, for example [23, 29]).

The partial width for the nonresonant decay \(B^- \to M\bar{M}\pi^-\) is

\[\Gamma_{nr}(B^- \to M\bar{M}\pi^-) = \frac{1}{(2\pi)^3} \frac{1}{32m_B^3} \int |\mathcal{M}_{nr}|^2 \, ds \, dt.\] (14)

(There is an additional factor of 1/2 for the decays with two identical pseudoscalar mesons in the final state.) The lower and the upper bounds on \(s\) are \(s_{\text{min}} = (m_2 + m_3)^2\), \(s_{\text{max}} = (m_B - m_1)^2\), while for \(t\) they are given by

\[t_{\text{min, max}}(s) = m_1^2 + m_2^2 - \frac{1}{s}[(s - m_B^2 + m_1^2)(s + m_2^2 - m_3^2) \\
\mp \lambda(s, m_B^2, m_1^2)\lambda(s, m_2^2, m_3^2)],\] (15)

where \(\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + ab)\).

Unfortunately there is no experimental measurement of the B meson decay constant and therefore we will use the usual heavy quark symmetry relation [14]

\[\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}}\] (16)

to obtain \(f_B\) from the \(f_D\), even though it has not been experimentally measured either. But, there are some data on \(f_D\) [9], albeit with large uncertainty, and taking \(f_D \simeq 200\) MeV is reasonable [17, 27]. Then the B decay
constant is $f_B \simeq 128\ MeV$. We will use the value of $|V_{ub}| = 0.0031$, the latest average in [9]. The parameter $g$ in the form factors, determined from the $D^* \to D\pi$ decay, is $g = 0.3 \pm 0.1$ [28]. From $D^0 \to K^-\ell^+\bar{\nu}$ we found $g = 0.15 \pm 0.08$ [17]. In the present calculations we will consider the range $0.2 \leq g \leq 0.23$, the overlap between these two determinations of $g$. We have previously determined the numerical values of $\alpha_1$ and $\alpha_2$ analyzing the $D \to V\ell\nu$ decays [16, 17], and must extrapolate to B mesons from these D meson values. To this end we apply the soft scaling of the axial form factors as discussed in [14]. This scaling procedure has the virtue that it does not neglect the masses of the light vector mesons when the heavy quark symmetry is used. It is completely in the same spirit as the basic assumption underlying our simple modification of the HQET propagators developed previously [10, 16, 17] and used in the present analysis. Soft scaling [14] of the axial form factors means that

$$A_{HV}^{A_1}(q_{\text{max}}^2) = \text{const} \frac{\sqrt{m_H}}{m_H + m_V},$$

(17)

and

$$A_{HV}^{A_2}(q_{\text{max}}^2) = \text{const} \frac{m_H + m_V}{\sqrt{m_H}},$$

(18)

where $H$ and $V$ denote heavy pseudoscalar and light vector mesons, respectively. It is easy to see that this scaling leads to the following relations: $\alpha_{DK^*} = \alpha_{B\rho}^{D_0}$ and $\alpha_{DK^*}^{DK^*}/m_D = \alpha_{B\rho}^{D_0}/m_B$. Among all cases found in [16, 17], we select the values $\alpha_{DK^*} = -0.13\ \text{GeV}^{1/2}$, $\alpha_{DK^*}^{DK^*} = -0.13\ \text{GeV}^{1/2}$, since only this choice of parameters gives $3.4 \times 10^{-5} \leq BR(B^- \to \pi^-\pi^+\pi^+) \leq 3.8 \times 10^{-5}$, consistent with recent data [8]. All the other combinations of $\alpha_{1,2}$, found in [10], give a $B^- \to \pi^-\pi^+\pi^+$ branching ratio larger than the experimental upper limit [8]. And for this same set of parameters, we also find $1.4 \times 10^{-5} \leq BR(B^- \to K^-K^+\pi^-) \leq 1.5 \times 10^{-5}$, which is below the experimental upper limit [8]. We also note the following limits for the unmeasured branching ratios: $1.5 \times 10^{-5} \leq BR(B^- \to \pi^-\eta\pi^0) \leq 1.7 \times 10^{-5}$ and $1.0 \times 10^{-5} \leq BR(B^- \to \pi^-\eta\eta) \leq 1.1 \times 10^{-5}$. We note that the contributions to the branching ratios arising from $\alpha_{1,2}$ are very important in these numerical results.

In addition to the uncertainties in our results arising from the uncertainties in the values of the parameters discussed above, there is a potentially quite large error that could come from the uncertainty in the CKM matrix
element $V_{ub}$ and the decay constant $f_B$. For example, the range of values $0.0018 \leq V_{ub} \leq 0.0044$ could change the branching ratios by as much as a factor of 2, but the resulting uncertainty in the CP asymmetry, which we discuss next, is somewhat smaller.

In order to obtain the CP violating asymmetry, one also needs to calculate the resonant decay amplitude $B^+ \rightarrow \chi_{0c} \pi^- \rightarrow M \bar{M} \pi^-$. This amplitude can easily be determined in the narrow width approximation, as in [4]:

\[
\mathcal{M}_r(B^- \rightarrow \chi_{0c} \pi^- \rightarrow M \bar{M} \pi^-) = \frac{1}{s - m_{\chi_{0c}}^2 + i\Gamma_{\chi_{0c}} m_{\chi_{0c}}} M(\chi_{0c} \rightarrow M \bar{M}) + (s \leftrightarrow t). \quad (19)
\]

In our numerical calculations we will use the estimate $BR(B^+ \rightarrow \chi_{0c} \pi^+)/BR(\chi_{0c} \rightarrow \pi^+\pi^-) = 5 \times 10^{-7}$ derived in [5]. The $\chi_{0c}$ decay data [9] then fix the decay amplitudes for $\chi_{0c} \rightarrow M \bar{M}$, $(M = \pi^+, \pi^0, K^+, \eta)$.

Finally we can calculate the partial width asymmetry in the $B^\pm \rightarrow M \bar{M} \pi^\pm$ decays. We are only interested in the kinematical region where the $M \bar{M}$ invariant mass is close to the mass of the $\chi_{0c}$ meson, $m_{\chi_{0c}} = 3.415$ GeV. The partial decay width $\Gamma_p$ for $B^- \rightarrow M \bar{M} \pi^-$, which contains both the nonresonant and resonant contributions, is obtained then by integration from $s_{\text{min}} = (m_{\chi_{0c}} - 2\Gamma_{\chi_{0c}})^2$ to $s_{\text{max}} = (m_{\chi_{0c}} + 2\Gamma_{\chi_{0c}})^2$, where $\Gamma_{\chi_{0c}} = 0.014 \pm 0.005$ GeV is the width of the $\chi_{0c}$:

\[
\Gamma_p = \frac{1}{(2\pi)^3} \frac{1}{32m_B^3} \int_{s_{\text{min}}}^{s_{\text{max}}} ds \int_{t_{\text{min}}(s)}^{t_{\text{max}}(s)} dt |\mathcal{M}_{nr} + \mathcal{M}_r|^2. \quad (20)
\]

Similarly, $\Gamma_p$, the partial decay width for $B^+ \rightarrow M \bar{M} \pi^+$, also contains both the nonresonant and resonant contributions. The CP-violating asymmetry is defined by

\[
A = \frac{|\Gamma_p - \Gamma_{\bar{p}}|}{\Gamma_p + \Gamma_{\bar{p}}}. \quad (21)
\]

For the range of values of $g$ and selected $\alpha_{1,2}$ discussed above we obtain the ranges

\[
0.33 \sin \gamma \leq A(B^\pm \rightarrow \pi^\pm \pi^- \pi^\pm) \leq 0.34 \sin \gamma, \quad (22)
\]
\[ 0.44 \sin \gamma \leq A(B^\pm \to K^+K^-\pi^\pm) \leq 0.45 \sin \gamma, \quad (23) \]
\[ 0.23 \sin \gamma \leq A(B^\pm \to \pi^0\pi^0\pi^\pm) \leq 0.24 \sin \gamma, \quad (24) \]

and
\[ 0.17 \sin \gamma \leq A(B^\pm \to \eta\pi^\pm) \leq 0.20 \sin \gamma. \quad (25) \]

In [4] it was found that \( A(B^\pm \to \pi^+\pi^-\pi^\pm) = (0.44 - 0.49)\sin \gamma \), which differs from (22) due to the importance of the \( \alpha_{1,2} \) terms.

The uncertainties due to the experimental errors in the remaining input parameters have not been included here, but we can roughly estimate that the rather large current uncertainties in \( V_{ub}, \Gamma_{\chi c0} \) and \( \Gamma(B^- \to \chi_c\pi^-) \) could result in the error in the asymmetry being as large as even 100%.

To summarize, we have analyzed the partial width asymmetry in \( B^\pm \to M\bar{M}\pi^\pm \) decays \((M = \pi^+, K^+, \pi^0, \eta)\), which signals CP violation, and can potentially be used to determine \( \sin \gamma \). The asymmetry results from the interference of the nonresonant decay amplitude with the resonant decay amplitude \( B^\pm \to \chi_{cc}\pi^\pm \) followed by \( \chi_{c0} \to M\bar{M} \). The asymmetry, which is rather sensitive to the choice of parameters, was estimated to be 0.33 \( \sin \gamma \) for \( B^\pm \to \pi^+\pi^-\pi^\pm \) and 0.45 \( \sin \gamma \) for \( B^\pm \to K^-K^+\pi^\pm \), while it is smaller for \( B^\pm \to \pi^0\pi^0\pi^\pm \) and \( B^\pm \to \eta\eta\pi^\pm \) decays. The estimates of these partial width asymmetries, while perhaps uncertain by as much as a factor of 2, do provide useful guidance for the experimental searches for CP violation and a measurement of the phase \( \gamma \).

This work was supported in part by the Ministry of Science and Technology of the Republic of Slovenia (B.B., S.F., and S.P.), and by the U.S. Department of Energy, Division of High Energy Physics under grant No. DE-FG02-91-ER4086 (R.J.O.). S.F. thanks the Department of Physics and Astronomy at Northwestern University for warm hospitality.
**Figure Caption**

**Fig. 1.** Skeleton diagrams for the various contributions to the nonresonant $B^{-} \rightarrow M\bar{M}\pi^{-}$ amplitude. The square in each diagram denotes the weak transition due to the weak Lagrangean $L_{w}$ (II), while each dot denotes one of the two corresponding weak currents.
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Fig. 1