Doubly Heavy Baryons from QCD Spectral Sum Rules

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Abstract

We consider the ratios of doubly heavy baryon masses using Double Ratios of Sum Rules (DRSR), which are more accurate than the usual simple ratios used for getting hadron masses. Our results are comparable with the ones from potential models. In our approach, the corrections induced by the anomalous dimensions of the correlators are the main sources of the $\Xi_{70}^0 - \Xi_{60}^0$ mass-splittings, which seem to indicate a $1/M_Q$ behaviour and can only allow the electromagnetic decay $\Xi_{70}^0 \rightarrow \Xi_{60}^0 + \gamma$ but not to $\Xi_{70}^0 + \pi$. Our results also show that the SU(3) mass-splittings are (almost) independent of the spin of the baryons and behave approximately like $1/M_Q$, which could be understood from the QCD expressions of the corresponding two-point correlator. Our results can improved by including radiative corrections to the SU(3) breaking terms and can be tested, in the near future, at Tevatron and LHCb.

Keywords: QCD spectral sum rules, baryon spectroscopy, heavy quarks.

1. Introduction

In a previous paper \cite{1}, we have considered, using Double Ratios of Sum Rules (QSSR) \cite{2,3}, the splittings due to SU(3) breakings of the baryons made with one heavy quark. This project has been pursued in the case of doubly heavy baryons in \cite{5}, which will be reviewed in this talk.

The absolute values of the doubly heavy baryon masses of spin 1/2 ($\Xi_{70}^0 \equiv QQu$) and spin 3/2 ($\Xi_{60}^0 \equiv QQu$) have been obtained using QCD spectral sum rules (QSSR) (for the first time) in \cite{6} with the results in GeV:

\[ M_{\Xi_{70}^0} = 3.58(5) \quad , \quad M_{\Xi_{60}^0} = 10.33(1.09) , \]

\[ M_{\Xi_{70}^0} = 3.48(6) \quad , \quad M_{\Xi_{60}^0} = 9.94(91) , \]

and in \cite{5}:

\[ M_{\Xi_{70}^0} = 6.86(28) . \]

More recently \cite{8,9}, some results have been obtained using some particular choices of the interpolating currents. The predictions for $M_{\Xi_{70}^0}$ and $M_{\Xi_{60}^0}$ are in good agreement with the experimental candidate $M_{\Xi_{70}^0} = 3518.9$ MeV \cite{10}. In the following, we shall improve these previous predictions using the DRSR for estimating the ratio of the 3/2 over the 1/2 baryon masses as well as their splittings due to SU(3) breakings, which we shall compare with some potential model predictions \cite{7,11,13}.

2. The Interpolating Currents

For the spin 1/2 ($QQu$) baryons, and following Ref. \cite{6}, we work with the lowest dimension currents:

\[ J_{\Xi_{70}^0} = \epsilon_{qdb} \left[ Q^T_q C \gamma_5 q_d b \right] Q_{t} , \]

where $q \equiv d, s$ are light quark fields, $Q \equiv c, b$ are heavy quark fields, $b$ is a priori an arbitrary mixing parameter. Its value has been found to be: $b = -1/5$, in the case of light baryons \cite{14} and in the range \cite{11,15,16}:

\[-0.5 \leq b \leq 0.5 , \]

for non-strange heavy baryons. The corresponding two-point correlator reads:

\[ S(q) = i \int d^4 x \, e^{iqx} \langle 0 | \bar{T} J_{\Xi_{70}^0}(x) J_{\Xi_{70}^0}(0) | 0 \rangle \]

\[ \equiv \hat{q} F_1 + F_2 , \]
where $F_1$ and $F_2$ are two invariant functions.

For the spin 3/2 (QQq) baryons, we also follow Ref. [6] and work with the interpolating currents:

$$J_{Z_q}^J = \frac{1}{3} \epsilon_{abc} \left[ 2 (Q_a T^c Y^d d_b) Q_1 + (Q_a T^c Y^d q_b) q_1 \right] \tag{6}$$

The corresponding two-point correlator reads:

$$S^{\mu\nu} (q) = i \int d^4 x \, e^{i q x} \langle 0 | T \, J_{Z_q}^J (x) J_{Z_q}^J (0) \rangle = g^{\mu\nu} \langle \tilde{q} F_1 + F_2 \rangle + \ldots \tag{7}$$

### 3. The Two-Point Correlator in QCD

The expressions of the two-point correlator using the previous interpolating currents have been obtained in the chiral limit $m_q = 0$ and including the mixed condensate contributions by [6]. In this work, we extend these results by including the linear strange quark mass corrections to the perturbative and $\langle \bar{s}s \rangle$ condensate contributions. The explicit expressions can be found in Ref. [8].

### 4. Form of the Sum Rules

We parametrize the spectral function using the standard duality ansatz: “one resonance” + “QCD continuum”. The QCD continuum starts from a threshold $t_c$ and comes from the discontinuity of the QCD diagrams. Transferring its contribution to the QCD side of the sum rule, one obtains the finite energy inverse Laplace sum rules [8, 17, 18]. Consistently, we also take into account the SU(3) breaking at the continuum threshold $t_c$:

$$\sqrt{\bar{c}SU(3)} \approx \left( \sqrt{\bar{c}SU(2)} + \bar{m}_s \right) + \bar{m}_s \tag{8}$$

where $\bar{m}_s$ is the running strange quark mass. As we do an expansion in $m_s$, we take the threshold $t_c = 4m_Q^2$ for consistency, where $m_Q$ is the heavy quark mass, which we shall take in the range covered by the running and on-shell mass because of its ambiguous definition when working to lowest order (LO). As usually done in the sum rule literature, one can estimate the baryon masses from the following ratios ($i = 1, 2$):

$$R_i^3 = \frac{\int_{t_c}^{\tau} dt \, t \, e^{-\tau t} \text{Im} F_1 (t)}{\int_{t_c}^{\tau} dt \, e^{-\tau t} \text{Im} F_1 (t)} \quad R_{21i}^3 = \frac{\int_{t_c}^{\tau} dt \, t \, e^{-\tau t} \text{Im} F_2 (t)}{\int_{t_c}^{\tau} dt \, e^{-\tau t} \text{Im} F_1 (t)} \tag{9}$$

where at the $\tau$-stability point:

$$M_{R_i}^3 \approx \sqrt{R_i^3} \approx R_{21i}^3 \quad (i = 1, 2) \tag{10}$$

These predictions lead to a typical uncertainty of 10-15% [8, 17, 18], which are not competitive compared with predictions from some other approaches, especially from potential models [7, 11]. In order to improve the QSSR predictions, we work with the double ratio of sum rules (DRSR):

$$r_i^{sd} \equiv \frac{\sqrt{R_i^3}}{R_i^2} \quad (i = 1, 2) ; \quad r_{21}^{sd} \equiv \frac{R_{21}^3}{R_{21}^2} \tag{11}$$

which take directly into account the SU(3) breaking effects. These quantities are obviously less sensitive to the choice of the heavy quark masses and to the value of the $t_c$ than the simple ratios $R_i$ and $R_{21}$. 

### 5. The $\Xi_{qq}^+$ mass ratio

We extract the mass ratios using the DRSR analogue of the one in Eq. (11) which we denote by:

$$r_i^{3/1} \equiv \frac{\sqrt{R_i^3}}{R_i^2} \quad (i = 1, 2) ; \quad r_{21}^{3/1} \equiv \frac{R_{21}^3}{R_{21}^2} \tag{12}$$

where the upper indices 3 and 1 correspond respectively to the spin 3/2 and 1/2 channels. We use the QCD expressions of the two-point correlators given by [6] which we have checked. In our analysis, we truncate the QCD series at the dimension 4 condensates until which we have calculated the $m_c$ corrections. We shall only include the effect of the mixed condensate (if necessary) for controlling the accuracy of the approach or for improving the $\tau$ or $t_c$ stability of the analysis.

#### The charm quark channel

Fixing $t_c = 25$ GeV$^2$ and $\tau = 0.8$ GeV$^{-2}$, which are inside the $t_c$ and $\tau$-stability regions (see Fig. [2] and Fig. [2b]), we show in Fig. [1] the $b$-behaviour of $r_i^{3/1}$ which shows that $t_i^{3/1}$ and $r_{21}^{3/1}$ are very stable but not $r_{12}^{3/1}$. We then eliminate $t_i^{3/1}$, where one can notice some common solutions for:

$$b \simeq -0.35 \quad \text{and} \quad b \simeq +0.2 \tag{13}$$

which are inside the range given in Eq. (4). For definiteness, we fix $b = -0.35$ and study the $\tau$-dependence of the result in Fig. [2] and its $t_c$-dependence in Fig. [2b]. The large stability in $t_c$ confirms our expectation for the weak $t_c$-dependence of the DRSR. In these figures, we have used $m_c = 1.26$ GeV and have checked that the results are insensitive to the change of mass to $m_c = 1.47$ GeV. We have also checked that the inclusion of the mixed condensate contribution does not affect the present result (within the high-accuracy obtained here) obtained by retaining the dimension-4 condensates.

\[2\]
can only decay electromagnetically but not to
six. The expressions can be obtained from the one of the two-point correlator in [6,7], while the new quark mass corrections can be found in [5]. The analysis for the charm quark is shown in Fig.3 from which we can deduce the result given in Table I. A similar analysis for the bottom quark is also given in Table I. We deduce from the ratios (in units of MeV):

\[ M_{\Omega_c} - M_{\Xi_c} = 92(24), \quad M_{\Omega_b} - M_{\Xi_b} = 49(13). \]  

Our results indicate an approximate decrease like \(1/m_Q\) of the mass splittings from the c to the b quark channels. This behaviour can be qualitatively understood from the QCD expressions of the corresponding correlator, where the non-perturbative corrections enter like \(m_c/m_Q\), and which can be checked using alternative methods.

7. The \(\Omega_{QQ}^*/\Xi_{QQ}^*\) mass ratio

We pursue our analysis for the spin 3/2 baryons. We deduce, for the charm quark, at the stability regions, the ratios given in Table I and the corresponding mass-splittings (in units of MeV):

\[ M_{\Omega_c} - M_{\Xi_c} = 94(27), \quad M_{\Omega_b} - M_{\Xi_b} = 50(15). \]  

which agree with the potential model results given in [7] (see Table I). Again, like in the case of spin 1/2 baryons, the SU(3) mass-differences appear to behave like \(1/M_Q\), which can be inspected from the QCD expressions of the two-point correlator.

One can also observe that the mass-splittings are almost the same for the spin 1/2 and 3/2 baryons.

6. The \(\Omega_{QQ}/\Xi_{QQ}\) mass ratio

We use the DRSR in Eq. [11] where their QCD expressions can be obtained from the one of the two-point

\[ M_{\Xi} = \frac{1}{2} \left[ m_Q + m_c + M_{\Xi} \right], \quad M_{\Xi} = 10019(3), \]  

which would correspond to the mass-splittings in MeV:

\[ M_{\Xi} - M_{\Xi} = 59(7), \quad M_{\Xi} - M_{\Xi} = 19(3), \]  

comparable with standard potential models [7,11] but not with the one of about 24 MeV obtained in [13] for the charm (see Table I). Our result excludes the possibility that \(M_{\Xi_{QQ}} \geq M_{\Xi} + m_c\), indicating that the \(\Xi_{QQ}^*\) can only decay electromagnetically but not to \(\Sigma_Q^* + \pi\).
8. The $\Omega_{bc}/\Xi_{bc}$ mass ratio

The $\Xi_{bc}$ and the $\Omega_{bc}$ spin 1/2 baryons can be described by the corresponding currents \[6, 7]\: 

$$
J_{\Delta c} = \frac{\epsilon_{ijk}}{2} \left[ (c_i^\dagger C y_3 d_j) + k(c_i^\dagger C d_j) y_3 \right] \gamma_5 b_i,
$$

$$
J_{\Omega_{bc}} = J_{\Xi_{bc}} (d \rightarrow s),
$$

(18)

where $d, s$ are light quark fields, $c, b$ are heavy quark fields and $k$ is $a$ priori an arbitrary mixing parameter.

Like in previous sections, we study the different ratio of moments for this case. The $b$-stability is obtained for $k \approx 0.05$ while the $\tau$ and $t_s$ behaviours are also very stable at which we deduce the DSR in Table 1 and the corresponding splitting:

$$
M_{\Omega_{bc}} - M_{\Xi_{bc}} = 41(7) \text{ MeV}.
$$

(19)

Table 1: QSSR predictions for the doubly heavy baryons mass ratios and splittings, which we compare with the Potential Model (PM) range of results in \[6,13\]. The PM prediction for the spin 3/2 is an average with the one for spin 1/2. The mass inputs are in GeV and the mass splittings are in MeV.

| Mass ratios | Mass inputs | Mass splittings | PM |
|-------------|-------------|-----------------|-----|
| $\Xi_{bc}/\Xi_{bc}$ | $\Xi_{bc} = 1.0166(19)$ | $\Xi_{bc} = 3.52(10)$ | 70-93 |
| $\Xi_{bc}/\Xi_{bc}$ | $\Xi_{bc} = 1.0069(3)$ | $\Xi_{bc} = 9.94(6)$ | 30-38 |
| $\Omega_{bc}/\Omega_{bc}$ | $\Omega_{bc} = 1.0260(70)$ | $\Omega_{bc} = 3.52(10)$ | 90-102 |
| $\Omega_{bc}/\Omega_{bc}$ | $\Omega_{bc} = 1.0049(13)$ | $\Omega_{bc} = 0.99(6)$ | 49(13) |
| $\Omega_{bc}/\Omega_{bc}$ | $\Omega_{bc} = 1.0260(75)$ | $\Omega_{bc} = 3.58(7)$ | 91-100 |
| $\Omega_{bc}/\Xi_{bc}$ | $\Omega_{bc} = 1.0050(15)$ | $\Omega_{bc} = 9.96(9)$ | 50(15) |
| $\Omega_{bc}/\Xi_{bc}$ | $\Omega_{bc} = 1.0060(17)$ | $\Omega_{bc} = 6.88(7)$ | 70-89 |

* We have combined your results for the mass-splittings with the experimental value of $M_{\Xi_{bc}}$ and with the central value of $M_{\Omega_{bc}}$ in Eq. 14.

9. Conclusions

Our different results are summarized in Table 1 and agree in most cases with the potential model predictions given in \[6,13\]:

- The mass-splittings between the spin 3/2 and 1/2 baryons, derived in Eqs. \[14\] and \[15\] are essentially due to the radiative corrections induced by the anomalous dimensions of the two-point correlator and seems to behave like $1/M_Q$.

- For the SU(3) mass-splittings, our results derived in Eq. \[16\] for the spin 1/2 and in Eq. \[17\] for the spin 3/2 indicate that the splittings due to the SU(3) breaking are almost independent on the spin of the heavy baryons and approximately behave like $1/M_Q$. These mass-behaviours can be qualitatively understood from the QCD expressions of the corresponding correlators where the leading mass corrections behave like $m_{cc}/M_Q$.

- Finally, we obtain, in Eq. \[19\], the SU(3) mass-splittings between the $\Omega_{bc}(s)$ and $\Xi_{bc}(cd)$, which is about 1/2 of the potential model prediction.

Our previous predictions can be improved by including radiative corrections to the SU(3) breaking terms and can be tested, in the near future, at Tevatron and LHCb.

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