Thermalization component model of multiplicity distributions of charged hadrons measured at the BNL ($E_{NN}^{lab}=2-11.6$GeV), the CERN ($E_{NN}^{lab}=20-200$GeV), and the BNL ($\sqrt{s_{NN}}=19.6-200$GeV)

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We find that collective flow model which can successfully analyze charged particle distributions at AGS and lower SPS ($E_{NN}^{lab}$ less than 20GeV in the lab frame) but fails to analyze that of at RHIC. The tails of distribution of charged particle at RHIC has a jump from the collective flow model calculation as the energy increases. Thermalization Component Model is presented based on collective flow to study the multiplicity distributions at RHIC in this paper. It is realized that the region of phase space of collective flow can reflect that of thermalization region. By comparing the contributions of particle productions from thermalization region at different energies and different centralities, we can deepen our study on the feature of collective movement at RHIC.

Keywords: Thermalization, Thermalization component model

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I. INTRODUCTION

One of the central question at RHIC is the extent to which the quanta produced in the collision interact and thermalize $^1$. $^2$. Nuclear collisions generate enormous multiplicity and transverse energy, but in what extent does the collision generate matter in local equilibrium which can be characterized by the thermodynamic parameters temperature, pressure, and energy density? Only if thermalization has been established can more detailed questions be asked about the equation of state of the matter.

Recently, it is realized that the study of collective flow is one of the important tools to study multi-hadron production of relativistic heavy-ion collisions $^3$, $^4$, $^5$. This is because the longitudinal and transverse flow includes rich physics, and collective flow relates closely to early evolution and nuclear stopping. Collective flow is often utilized to express the thermalization degree of relativistic heavy-ion collisions system. Detailed studies of the observed final state flow pattern will deepen our understanding of dynamic mechanism of relativistic heavy-ion collisions. Collective Flow Model $^6$, $^7$ (CFM) was developed basing on the pure thermal model. It achieves success in the discussion of charged particle distribution at AGS and lower SPS energy (20GeV) and become an indicator for the existence of collective flow at AGS $^8$, $^9$, $^10$. But a detailed analysis of the experimental data at SPS and RHIC energy with CFM has shown that as the increase of collision energy, the two tails of the charged hadron distributions have a symmetric jump away from the calculation of CFM. These phenomena will make us to reconsider collective flow theory at higher collision energy.

As shown in Fig.1, CFM fails to analyze the charged hadron distribution at RHIC energy regions. As the increase of the collision energy, the tail of the distribution of experimental data jump from the calculation of CFM. The naive reason seems to be that experimental data on hadron yields are now available over a broad collision energy range as the increase of collision energy. As the increase of phase space of particle distribution, thermalization at whole phase space of particle production becomes more difficult. Detailed analysis of thermalization relation with centrality and energies at RHIC is needed.

How to simulate the data of charged hadron distribution at higher SPS and RHIC energy regions and what these results tell us are the main topic of the paper. PHOBOS has used three Gaussian distributions to simulate the distribution of charged hadron successfully $^8$, $^9$, $^{10}$, $^{11}$, $^{12}$. Georg Wolschin $^{14}$ et al also discussed the charged hadron distribution by using three component distribution functions to construct Fokker-Plank equation. Their models both assumed three random Gaussian distributions for central sources. The Three Gaussian sources represent target, projectile and central source in physics, respectively.

The main goal of this paper is to give a study of the thermalization features of multi-particle production in heavy ion collision at high energy in the framework of the collective flow theory. We restrict ourselves here to the basic features and essential results of the CFM approach. A complete survey of the assumptions and results, as well as of the relevant references, is available in Ref. $^{15}$, $^{16}$, $^{17}$, $^{18}$, $^{19}$.

The paper is organized as follows. The analysis details based on the Thermalization Component Model (TCM) are described in Sec.2. The comparisons of TCM calculations with experimental data and the related theoretical analysis with TCM are given in Sec.3. A summary is given in Sec. 4.
FIG. 1: (a) $\pi$ meson rapidity distribution of $E_{\text{lab}}^{NN} = 30$ GeV at SPS [7]; (b) charged hadron pseudo-rapidity distribution of $\sqrt{s_{NN}} = 200$ GeV

II. THERMALIZATION COMPONENT MODEL

The hot and dense matter produced in relativistic heavy ion collisions may evolve through the following scenario: pre-equilibrium, thermal (or chemical) equilibrium of partons, possible formation QGP or a QGP hadron gas mixed state, a gas of hot interacting hadrons, and finally, a freeze-out state when the produced hadrons no longer strongly interact with each other. Since the produced hadrons carry information about the collision dynamics and the entire space-time evolution of the system from the initial to the final stage of collisions, a precise analysis of the multiplicity distributions of charged hadrons is essential for the understanding of the dynamics and properties of the created matter.

A detailed analysis of the experimental data at SPS and RHIC energy with CFM has shown that as the increase of collision energy, the two tails of $\pi$ or the charged hadron distributions show a (symmetric) discrepancy between the data and the calculation. These phenomena will make us to reconsider Collective flow theory at higher collision energy. Detailed analysis of the relation with thermalization with centralities and energies at RHIC is needed. Let us first sketch our overall picture and detail our arguments subsequently. The model we considered contains three distinct assumptions some of which are rather different from those usually contained in other flow models.

(i) The size of phase space of the particle distribution increases with the increase of collision energy. It seems more difficult to realize thermalization at the whole phase space of particle production at SPS and RHIC data. It is assumed that the Gaussian distributions were fit to the distributions of the produced charged hadrons at the two fragmentation regions, and thermalization prefers to occur at the central rapidity region at SPS and RHIC.

(ii) The collective flow of central rapidity region carries information of the early time of heavy-ion collision. The system expands not only in the longitudinal direction, but also in the transverse direction. The two dimensional collective flow is used to study the thermalization process at RHIC.

(iii) The phase space is compartmentalized as the thermalization region and non-thermalization regions. The non-thermalization regions locate at the two fragmentation regions. The total multiplicity distributions are the summation of the contributions from the target fragmentation region, projectile fragmentation region and central region, respectively.

\[
\frac{dN}{dy} = N_1 F_1 + N_2 F_2 + N_3 F_3 = \sum_i N_i F_i \quad (1)
\]

Here $i = 1, 2, 3$ denotes target, projectile and central region, respectively. $N_i$ and $F_i$ are the particle numbers and the normalization functions of target, projectile and central regions, respectively.
As assumed before, the distributions of target and projectile fragmentation regions are given with Gaussian distributions:

\[ F_1 = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y+y_1)^2}{2\sigma^2}} \]  
\[ F_2 = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y+y_2)^2}{2\sigma^2}} \]  

Here \( \sigma \) is the distribution width of Gaussian, \( y_1, y_2 \) are the locations of central of target and projectile emitting source.

\( F_3 \) is the distribution of two dimensional flow, which is given by [4], [5]

\[ F_3 = \frac{g\tau R_f^2 K}{8\pi} \int_{m_{t}^{lo}}^{m_{t}^{hi}} dn_{t} m_{t} I_0(\alpha) \int_{-\eta_0}^{-\eta_f} d\eta \cosh(y-\eta) e^{\alpha/\beta} e^{-\eta/\eta_f} \cosh(y-\eta_f) \]  

Here \( m_{t}^{lo} \) and \( m_{t}^{hi} \) are the experimental limits in which the spectrum is measured. The freeze-out radius \( R_f \) and the longitudinal extend of the fireball is fixed via the finite interval \((-\eta_0, \eta_f)\), \( I_0 \) is modified Bessel function.

For the two dimensional flow theories, we should say a few words. The geometry of the freeze-out of two dimensional flow hyper-surface \( \sigma_f \) fixed as follows: in the time direction we take a surface of constant proper time. In \( \eta_f \) direction, the freeze-out volume extends only to a maximum space-time rapidity \( \eta_f \), which is required by the finite available total energy and breaks longitudinal boost-invariance proposed by Bjorken [16]. In the transverse direction the boundary is given by \( R_f \), which describes a cylindrical fireball in the \( \eta - r \) space. The detailed discussion was shown in Ref. [4, 5].

III. COMPARING WITH EXPERIMENTAL DATA

It is found that CFM describe experimental data of charged particle distribution very well when we discuss Au-Au center collisions at AGS energy region. The contribution of fragmentation regions can be ignored, so expressions (1) can be predigested:

\[ \frac{dN}{dy} = N_3 F_3 \]  

The results from CFM are consistent with experimental data in Au-Au collisions at AGS energy region, such as \( E_{NN}^{lab} = 2, 4, 8, 11.6 \text{ GeV} \) in the lab frame. This indicates that when at lower AGS energy region CFM can describe the charged particle distribution well, then our thermalization component model revert to collective flow model. The reason seems that phase space is small and the nucleus stopping power is very strong at AGS energy region, so particles can be almost completely thermalized in whole phase space. The same situation is true for that of SPS energy region below 20 GeV.

But with the collision energies increase (\( E_{NN}^{lab} \) above 30 GeV), the experimental points have a symmetric jump away from the calculation of CFM at two tails (as shown in Fig.1). This phenomenon can be explained by the nuclei’s penetrability. The higher the collision energies, the more transparent the nuclei, and the larger the extension of the phase space of the produced particle. Collective flow is formed at the central rapidity region after thermalization. The distributions of non-thermalization charged hadrons are presented by Gaussian. The thermalization area becomes one part of the whole phase space.

Since June 2000, the Relativistic Heavy-Ion Collider (RHIC) has opened a new energy region for the study of multi-hadron production. We have analyzed the experimental data of charged particle distribution in Au-Au center collisions in the RHIC energy region from 19.6 to 200 GeV of \( \sqrt{s_{NN}} \).

We can calculate the rapidity distribution of charged particles with expressions (1). It is known that we transfer rapidity distribution to pseudo-rapidity distribution just by multiplying a factor [21]:

\[ \frac{dN}{d\eta} = \frac{dN}{dy} \sqrt{1 - \left( \frac{m}{m_T \cosh y} \right)^2} \]
FIG. 4: The charged hadron pseudo-rapidity distribution at different centrality at $\sqrt{s_{NN}} = 62.4$ GeV and 200 GeV for Au+Au and Cu+Cu Collisions, respectively. Solid lines are the results from TCM. Experimental data are given by PHOBOS [8]-[13].

We fit the experimental data of RHIC energy region by HCM model by $\chi^2/dof$. The comparison of the measured and calculated distributions for the best fit ($\chi^2/dof$ minimization) is presented in Fig. 2. The TCM calculations are accordant with experimental data shown in Fig.2. The percentages of the charged hadron productions from the thermalization regions at AGS, SPS and RHIC energy region are presented in Fig.3 by TCM. It is found that most of the produced particles at AGS come from the thermalization region, and the percentage of produced particles from the thermalization region decreases as the energy increase. The reduction trend becomes weaker and seems to reach saturation as $\sqrt{s_{NN}}$ reaches 62.4 GeV at RHIC energy region. The detailed fit parameters of our TCM with experimental data are shown in Table 1.

TABLE I: The fit results of TCM with the experimental data at SPS and RHIC energy regions

| $E^*_{NN}$ | $\eta_0$ | $y_{1,2}$ | $n_1 + n_2$ | $n_3$ | $n_1/(n_1 + n_2 + n_3)$ |
|-----------|---------|-----------|-------------|------|------------------------|
| SPS       | 30      | 1.33 ± 2.1| 16          | 256  | 94.13%                 |
|           | 40      | 1.4 ± 2.05| 22          | 301  | 93.19%                 |
|           | 80      | 1.4 ± 2.0 | 64          | 392  | 85.91%                 |
|           | 158     | 1.38 ± 2.0| 100         | 507  | 83.52%                 |
| RHIC      | $\sqrt{s_{NN}} = 62.4$ GeV | 1.85 ± 2.6 | 370 | 1310 | 77.99% |
|           | 19.6    | 2.47 ± 3.15 | 670 | 2157 | 76.30% |
|           | 62.4    | 2.62 ± 3.45 | 1100 | 3016 | 73.25% |
|           | 200     | 2.8 ± 3.62 | 1320 | 3629 | 73.38% |

PHOBOS Collaboration Working at RHIC has presented many experimental data of different energy and different centrality including Au+Au collisions and Cu+Cu collisions at $\sqrt{s_{NN}} = 62.4$ and 200 GeV. It is found that the calculation results from TCM are consistent with that of the experimental data. The results are presented by Fig.4 and Table 2. The experimental data are taken from Ref. [8]-[13].

FIG. 5: The dependence of the percentage from the thermalization region on different centralities for $\sqrt{s_{NN}} = 62.4$, 200 GeV

It is shown from Fig.5 that the percentage ratios of the particle production from the thermalization regions increase with the increase of the centralities at RHIC. From Fig.5 (a), It is found that the contribution ratios from the thermalization region is appreciably larger for the smaller collision system (Cu +Cu) than that of larger collision system (Au +Au) at $\sqrt{s_{NN}} = 62.4$ GeV. But from Fig.5 (b), we find that the percentage ratios of particle production from thermalization regions is almost independent of the size of collision systems at $\sqrt{s_{NN}} = 200$ GeV.

In our TCM, the free parameters are the limitation of collective flow $\eta_0$ and the emission sources’ positions in fragmentation area $y_{1,2}$. We have $y_1 = -y_2$ in the case of symmetry collisions. The values of transverse flow and temperature of collective flow refer to Ref.[4, 8, 15, 20]. The values of $n_i (i = 1, 2, 3)$ are numbers of particles from the fragmentation and the thermalization regions, respectively.

A linear relationship is obtained between $\eta_0$ and $ln\sqrt{s_{NN}}$ by detailed study. The linear equations are...
given by fitting four data at SPS, RHIC energy regions as follows:

$$\eta_0 = 0.40 \ln \sqrt{s_{NN}} + 0.71$$  \hspace{1cm} (7)

Here $\eta_0$ is the extension of collective flow. From Eq.7, we can predict the extension of the thermalization region at LHC with the collision energy increase.

![Graph showing the relation between the limitation of thermalization region with $\ln \sqrt{s_{NN}}$.](image)

**FIG. 6:** The relation between the limitation of thermalization region with $\ln \sqrt{s_{NN}}$.

Here, we should mention that quite a few theoretical models can give equally good representation of the data of particle productions at AGS, SPS, and RHIC, such as these thermal models, based on the assumption of global thermal and chemical equilibrium, and hydrodynamic models based only on the assumption of local thermal equilibrium, to transport models that treat nonequilibrium dynamics explicitly. The thermal models have been very successful in accounting for the yield of various particles and their ratios, while the hydrodynamic models are particularly useful for understanding the collective behavior of low transverse momentum particles such as the elliptic flow. Since transport models treat chemical and thermal freeze-out dynamically, they are also natural and powerful tools for studying the Hanbury-Brown-Twiss interferometry of hadrons.

For hard processes that involve large momentum trans- fer, approaches based on the perturbative quantum chromodynamics (pQCD) using parton distribution functions in the colliding nuclei have been used. Also, the classical Yang-Mills theory has been developed to address the evolution of parton distribution functions in nuclei at ultra-relativistic energies and used to study the hadron rapidity distribution and its centrality dependence at RHIC. These problems have also been studied in the pQCD-based final-state saturation model. A multiphase transport (AMPT) model that includes both initial partonic and final hadronic interactions and the transition between these two phases of matter was constructed to describe nuclear collisions ranging from pA to AA systems at center-of-mass energies from about $\sqrt{s_{NN}} = 5$ to 5500 GeV at LHC.

**IV. SUMMARY AND CONCLUSIONS**

Hadron multiplicities and their distributions are observables which can provide information on the nature, composition, and size of the medium from which they are originating. Of particular interest is the extent to which the measured particle yields are showing thermalization. The feature of thermalization of high energy heavy ion collisions at RHIC has been analyzed in this paper.

CFM fails to analyze the charged particle distributions when the collision energies increase to above 30 GeV. The tail of distribution of the charged particle at RHIC has a jump from the CFM calculation with the energy increase. The naive reason seems to be that the experimental data on hadron yields are now available over a broad collision energy range with the increase of collision energy. It seems more difficult for thermalization at the whole phase space of particle production with the increase of the phase space of particle distribution.

On the other hand, the phenomena may suggest that something else happens, including interaction mechanism, such as the onset of de-confinement in the early stage of the reaction with the collision energy ($E_{lab}^{NN}$) above 30 GeV at the lab frame, which has been mentioned in Ref. 49. In Ref. 49, central Pb-Pb collisions were studied in the SPS energy range. At around $E_{lab}^{NN} = 30$ GeV the ratio of strangeness to pion production shows a sharp maximum, the rate of increase of the produced pion multiplicity per wounded nucleon increases and the effective temperature of pions and kaons levels to a constant value. These features are not reproduced by present hadronic models, however there is a natural explanation in a reaction scenario with the onset of deconfinement in the early stage of the reaction at SPS energy.

Collective flow in heavy-ion collisions is an unavoidable consequence of thermalization. The extension of the phase space of collective flow can reflect that of thermalization region. It is found that the TCM can fit the experimental data well for the particle production at the whole AGS, SPS, and RHIC energy regions. The percentage ratios of contributions of the particle production from the thermalization region are the largest at AGS, and decrease as collision energies increase at SPS and RHIC, but seem to reach saturation when $\sqrt{s_{NN}} = 62.4$-200 GeV at RHIC. It is also found that the extension of the flow shows a linear dependence on $\ln \sqrt{s_{NN}}$. From
that, we can predict the thermalization extension at future LHC experimental data.

It is shown from our study that the percentage ratios of particle production from thermalization regions increase with the increase of the centralities at RHIC. The contribution ratios from thermalization region are appreciably larger for the smaller collision system (Cu + Cu) at $\sqrt{s_{NN}}=62.4$ GeV, but independent of the collision system at $\sqrt{s_{NN}}=200$ GeV.

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