Tomlinson-Harashima Precoded Rate-Splitting for Multiuser MIMO Systems

Andre R. Flores 1, Bruno Clerckx 2 and Rodrigo C. de Lamare 1,3
1 Centre for Telecommunications Studies, Pontifical Catholic University of Rio de Janeiro, Brazil
2 Imperial College London, United Kingdom
3 Department of Electronic Engineering, University of York, United Kingdom
Emails: andre.flores@cetuc.puc-rio.br, b.clerckx@imperial.ac.uk, delamare@cetuc.puc-rio.br

Abstract—In this work, we investigate the performance of Rate-Splitting (RS) based on Tomlinson-Harashima Precoding (THP) in a multiple-antenna broadcast channel with perfect and imperfect Channel State Information at the Transmitter (CSIT). In particular, we consider RS using centralized and decentralized THP structures, where only one user splits its message into a common and private part, and develop expressions to describe the signal-to-interference-plus-noise (SINR) ratio and the sum rates associated with these schemes. Furthermore, we also assess the performance achieved by RS combined with Dirty-Paper Coding (DPC). Simulations show that RS with THP outperforms existing standard THP and RS with linear precoding schemes.

Index Terms—Multiple-antenna systems, ergodic sum-rate, rate-splitting, Tomlinson-Harashima precoding (THP).

I. INTRODUCTION

Multiple-input multiple-output (MIMO) techniques exploit multipath propagation by using multiple transmit and receive antennas. MIMO has become a fundamental part of several communications standards, such as WiFi and LTE, due to its ability to greatly increase the capacity and reliability of wireless systems [1]. A major research focus over the last decade has been on multi-user MIMO (MU-MIMO) systems. However, MU-MIMO systems suffer from multi-user interference (MUI) that can be dealt in the downlink (DL) by preprocessing the transmit signal at the Base Station (BS) using precoding algorithms. The quality of the Channel State Information (CSI) affects the performance of precoding algorithms. However, the ability to obtain highly accurate CSI at the transmitter (CSIT) is questionable [2], [3].

Recently, Rate-Splitting (RS), originally developed for the 2-user SISO interference channel in [4], has been introduced for the design of MIMO wireless networks [3]. RS schemes split the data transmitted from BS to the users into a common message and private messages. The common message must be decoded by all users, whereas the private message is decoded only by its corresponding user. The benefit of RS lies in its ability to partially decode interference and partially treat interference as noise, which enables to softly bridge the two extremes of fully decoding interference and treating interference as noise. As a consequence, RS provides room for rate and QoS enhancements in a wide range of network loads (underloaded and overloaded regimes) and user deployments (with a diversity of channel directions, channel strengths and qualities of CSIT) over standard schemes such as MU-MIMO with linear precoding and power-domain Non-Orthogonal Multiple Access (NOMA) [3], [5].

RS has been considered with linear precoding [2], [3], [6] using both perfect and imperfect CSIT. In particular, the sum-rate (SR) maximization problem using RS and linear precoding in the DL has been investigated. In [7], the problem of achieving max-min fairness amongst multiple co-channel multicast groups has been studied. RS has also been considered for robust transmissions under bounded CSIT errors in [8]. Studies of massive MIMO and MISO networks using RS strategies have been reported in [9] and [10], respectively.

RS has so far been studied and optimized using a linear precoding framework. Interestingly, the potential benefits of RS using nonlinear precoding techniques remain unexplored in the literature and we aim at filling this gap in this paper.

The combination of RS and nonlinear precoding is particularly interesting in the imperfect CSIT setting. Indeed, we know that nonlinear precoding comes very close to the optimal performance (sum-rate capacity) of a multi-antenna Broadcast Channel (BC), achieved by DPC, in the perfect CSIT setting. The sum-rate capacity of a K-user multi-antenna BC with imperfect CSIT remains an open problem, even though we know that RS is the key building block to achieve the optimal Degrees-of-Freedom of a K-user multi-antenna BC with imperfect CSIT [2], [11], [12]. From a DoF perspective, linearly precoded RS is sufficient and any form of nonlinear precoding combined with RS would not further increase the DoF. However, from a rate perspective, nonlinear precoding is beneficial over linear precoding. Hence the combination of nonlinear precoding and RS is a promising avenue to improve the rate performance, especially in the imperfect CSIT setting.

In this paper, we consider the design of nonlinearly precoded RS based on Tomlinson-Harashima Precoding (THP), simply denoted as THP-RS. In particular, we consider RS with centralized and decentralized THP structures, where only one user splits its message, and develop expressions to describe the signal-to-interference-plus-noise (SINR) ratio and the sum rates associated with these schemes. Furthermore, we also examine the benefits of combining RS with Dirty-Paper Coding (DPC). We evaluate the performance of THP-RS and existing schemes using the sum-rate as a metric, in perfect and imperfect CSIT.

The rest of the paper is organized as follows. Section II describes the system model and reviews standard linear precoding, THP and RS for multiuser MISO systems. Section III details the proposed THP-RS schemes, whereas the simulations are presented in Section IV. Finally, Section V concludes the paper.
II. SYSTEM MODEL

We consider a multiple-input single-output (MISO) BC with \( K \) users. The transmitter delivers a total of \( K \) messages to the \( K \) users, with one message intended per user. The BS is equipped with a total of \( N_t \) antennas with \( N_t \geq K \geq 2 \), whereas the terminals of the users are equipped with a single antenna. The transmission takes place over a channel whose parameters remain fixed during a data packet. The channel matrix \( \mathbf{H} = [\mathbf{h}_1 \ldots \mathbf{h}_K] \) contains in the \( k \)th column the channel vector that connects the BS to user \( k \). From this model, we can express the received signal at the \( k \)th user by

\[
    r_k = \mathbf{h}_k^H \mathbf{x} + n_k,\tag{1}
\]

where \( \mathbf{x} \in \mathbb{C}^{N_t} \) represents the transmitted signal, \( n_k \sim \mathcal{CN}(0, \sigma^2_n) \) is the additive white Gaussian noise, \((\cdot)^H\) is the Hermitian transpose and \( \mathbf{h}_k \in \mathbb{C}^{N_t} \) is the channel vector for user \( k \). In this work, for simplicity we consider equal noise variance \( \sigma^2_n \) for all users. The SNR is defined as \( \text{SNR} = \frac{E_{tr}}{\sigma^2_n} \), where \( E_{tr} \) denotes the total transmitted power. We also consider that \( \sigma^2_n \) remains fixed and has a non-zero value in order to avoid indetermination. This means that a modification of the SNR depends only on the parameter \( E_{tr} \). The model satisfies the transmit power constraint \( \mathbb{E}[|\mathbf{x}|^2] \leq E_{tr} \). In what follows we will review a standard MISO BC with linear precoding and THP, as well as RS using linear precoding.

A. Standard Linear Precoding

In a standard MISO BC using linear precoding\,[13], [14], [15], we consider \( K \) messages encoded into \( K \) independent data streams, forming the vector \( \mathbf{s} = [s_1, s_2, \ldots, s_K]^T \), where the superscript \( ^T \) denotes transpose. Moreover, we assume that \( \mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{R}_s = \mathbf{I} \), with \( \mathbf{I} \) the identity matrix. The precoding matrix \( \mathbf{P} \in \mathbb{C}^{N_t \times K} \) maps the symbols to the transmit antennas. The \( k \)th column of \( \mathbf{P} \) contains the precoder for user \( k \), denoted by \( \mathbf{p}_k \). It turns out that the transmit vector is given by \( \mathbf{x} = \mathbf{p}_s = \sum_{k=1}^{K} \mathbf{p}_k s_k \). Taking into account the assumptions made so far, the power constraint is reduced to \( \text{tr}(\mathbf{P}\mathbf{R}_s\mathbf{P}^H) \leq E_{tr} \).

B. Linearly Precoded Rate-Splitting

RS splits a message into a common part and a private part\,[3], [4]. For simplicity, we consider that only one user splits its message. The common part is then encoded into one common stream and the private parts into \( K \) private streams. The receivers share a codebook since the common message has to be decoded by all the users with zero error probability. In contrast, each private stream is decoded only by its corresponding user. This means that each receiver must decode two data streams, namely the common stream (decoded by all but intended to only one user) and the private stream (decoded by its respective user). The common stream is first decoded and all private messages are considered as interference and treated as noise. Then we use successive interference cancellation (SIC) to subtract the contribution of the common stream from the received signal, enhancing the detection of the private stream. At the end, the message sent via the private stream is decoded. When a user decodes its private stream, it treats the other private streams as noise. The strength of RS is its ability to adjust the content and the power of the common message to control how much interference should be decoded by all users (through the common message) and how much interference is treated as noise.

RS can be viewed mathematically as a non-orthogonal unicast and multicast transmission strategy given the superimposed transmission of common and private messages. However, conventional multicast messages are intended and decoded by all the users while the common message of RS is decoded by all users but is intended to one (or a subset) of the users. Its presence enables the decoding of part of the MUI and treating the remaining part of the interference as noise.

Splitting one message creates \( K+1 \) streams, which modifies the vector of data symbols to \( \mathbf{s}_{RS} = [s_c, s_1, s_2, \ldots, s_K]^T \), where \( s_c \) is used to designate a symbol of the common stream. A common precoder \( \mathbf{p}_c \in \mathbb{C}^{N_t} \) is added to the first column of \( \mathbf{P} \), from which we obtain \( \mathbf{P}_{RS} = [\mathbf{p}_c, \mathbf{P}] \). The transmitted signal is expressed by

\[
    \mathbf{x} = \mathbf{p}_c s_c + \sum_{k=1}^{K} \mathbf{p}_k s_k. \tag{2}
\]

The total transmit power is allocated partially to the private precoders and the common precoder. For uncorrelated inputs, the transmit power constraint is given by \( \|\mathbf{p}_c\|^2 + \sum_{k=1}^{K} \|\mathbf{p}_k\|^2 \leq E_{tr} \). Setting \( \|\mathbf{p}_c\|^2 \) to zero is equivalent to allocating no power to the common stream, i.e., the system performs no RS and the transmit signal is reduced to the standard linear precoding.\(^1\) Power allocation can be carried out to satisfy several system requirements such as maximizing the SR or achieving a specific QoS. Given a channel state, the average receive power at the \( k \)th terminal can be written as

\[
    T_{r,k} \triangleq \mathbb{E}[|r_k|^2] = \|\mathbf{h}_k^H \mathbf{p}_c\|^2 + I_{c,k}, \tag{3}
\]

with

\[
    I_{c,k} = \|\mathbf{h}_k^H \mathbf{p}_k\|^2 + I_k, \quad I_k = \sum_{i \neq k}^{K} \|\mathbf{h}_i^H \mathbf{p}_i\|^2 + \sigma^2_n, \tag{4}
\]

where \( I_{c,k} \) and \( I_k \) correspond the interference-plus-noise power when decoding the common and the \( k \)th private message, respectively.

C. Tomlinson-Harashima Precoding

THP is a nonlinear preprocessing technique employed at the transmit side. A standard THP algorithm implements three filters, the feedback filter \( \mathbf{B} \in \mathbb{C}^{N_t \times K} \), the feedforward filter \( \mathbf{F} \in \mathbb{C}^{N_t \times N_t} \) and the scaling matrix \( \mathbf{G} \in \mathbb{C}^{N_t \times N_t} \). The feedback filter deals with the multiuser interference by successively subtracting the interference from the current symbol. The matrix \( \mathbf{B} \) has a lower triangular structure, whereas the feedforward filter enforces the spatial causality. The scaling filter assigns a coefficient or weight to each stream of data, which means \( \mathbf{G} \) is a diagonal matrix.

There are two general THP structures in the literature\,[16], [17], [18], [19], [20], namely the centralized THP (cTHP) and the decentralized THP (dTHP). The main difference between these structures is that the scaling matrix \( \mathbf{G} \) is placed at the

\(^1\) Power-domain NOMA is also a particular case of RS whenever the entire message of a given user is encoded into the common message.\[5\]
transmitter for the cTHP, whereas for dTHP the same matrix is located at the receiver. These THP algorithms are implemented by performing an LQ decomposition on the channel matrix, i.e., $H = LQ$. The THP filters are then defined as follows:

$$F = Q^H,$$
$$G = \text{diag} \left( l_{11}, l_{22}, \ldots, l_{KK} \right)^{-1},$$
$$B^{(d)} = GL \quad \text{and} \quad B^{(c)} = LG,$$

where $B_d$ and $B_c$ correspond to the feedback filter for dTHP and cTHP, respectively. The received signal vector for each structure is obtained by stacking up the received signal of each user $r_k$ and is given by

$$r^{(d)} = \frac{1}{\beta^{(d)}} G \left( H \beta^{(d)} F w + n \right),$$
$$r^{(c)} = \frac{1}{\beta^{(c)}} \left( H \beta^{(c)} FG w + n \right),$$

where $\beta^{(d)} \approx \sqrt{\frac{E_{tr}}{K}}$ and $\beta^{(c)} \approx \sqrt{\frac{E_{tr}}{\sum_{k=1}^{K} (1/l_{kk}^2) \cdot K}}$ are the scaling factors used to fulfill the transmit power constraint.

The transmitted symbol $w_k$ of each user is successively generated as

$$w_k = s_k - \sum_{i=1}^{k-1} b_{k,i} w_i.$$  

However, this process increases the amplitude of $w_k$. A modulo operation is therefore applied in order to reduce the amplitude of the symbol to the boundary of the modulation. The modulo processing is equivalent to adding a perturbation vector $d$ to the transmit data $s$, i.e., $v = s + d$. Mathematically, the feedback processing is equivalent to an inversion operation over the matrix $B$. Then, we have

$$w = B^{-1}v = B^{-1}(s + d).$$

We can simplify the received signal using the expressions of the filters, which leads us to

$$r^{(d)} = v + \frac{1}{\beta^{(d)}} G n,$$
$$r^{(c)} = v + \frac{1}{\beta^{(c)}} n.$$  

THP introduces a power loss and a modulo loss in the system. The former comes from the energy difference between the original constellation and the transmitted symbols after precoding. The latter is caused by the modulo operation. Both losses can be neglected for analysis purposes and for moderate and large modulation sizes so that the power of $v$ is approximated by that of $s$.

D. THP Rate Analysis

Here, we consider that the power loss is measured by the factor $1/\lambda > 1$, i.e., $R_v = E[vv^H] = \lambda^{-1}I\left[22\right]$. Now the scaling factors $\beta^{(d)}$ and $\beta^{(c)}$ have to be multiplied by $\sqrt{\lambda}$. Then, the SINR for the $k$th user is given by

$$\gamma_k^{(d)} = \frac{\lambda E_{tr} l_{kk}^2}{K \sigma_n^2}, \quad \gamma_k^{(c)} = \frac{\lambda E_{tr}}{\sigma_n^2 \sum_{i=1}^{K} (1/l_{ii}^2)}.$$  

Assuming Gaussian distributed codebooks, the corresponding instantaneous rates for the $k$th user are given by

$$R_k^{(d)} = \log_2 \left( 1 + \gamma_k^{(d)} \right), \quad R_k^{(c)} = \log_2 \left( 1 + \gamma_k^{(c)} \right).$$

Let us now consider the imperfect CSIT scenario. Due to the estimation errors, the channel can be written as

$$H = \hat{H} + H_e,$$  

where $\hat{H}$ represents the channel estimate and $H_e$ is a random matrix corresponding to the error for each link. The channel for user $k$ can be written as $h_k = \hat{h} + h_{e,k}$. Each coefficient of the error matrix follows a Gaussian distribution, i.e., $\sim \mathcal{CN}(0, \sigma_e^2)$. Because of the errors present in the CSIT, both algorithms, dTHP and cTHP, can no longer effectively subtract the interference from other users. Therefore, the received signal for the conventional THP with imperfect CSIT for both schemes can be expressed as

$$r^{(d)} = v + GH_e FB^{-1} v + \frac{1}{\beta^{(d)}} G n,$$
$$r^{(c)} = v + H_e FGB^{-1} v + \frac{1}{\beta^{(c)}} n.$$  

We can expand equations (17) and (18) to get the received signal of each user as described by

$$i_k^{(d)} = v_k + \frac{1}{l_{kk}} h_{e,k}^H p_k^{(d)} v_k + \frac{1}{l_{kk}} h_{e,k}^H \sum_{i \neq k} p_i^{(d)} v_i + \frac{n_k}{\beta^{(d)}} l_{kk},$$
$$i_k^{(c)} = v_k + h_{e,k}^H p_k^{(c)} v_k + h_{e,k}^H \sum_{i \neq k} p_i^{(c)} v_i + \frac{1}{\beta^{(c)}} n_k.$$  

Using (19) and (20), and substituting the value of $\beta$ we arrive at the following expressions for the SINR of the $k$th user:

$$\gamma_k^{(d)} = \frac{\left| 1 + \frac{1}{l_{kk}} h_{e,k}^H p_k^{(d)} \right|^2}{\sum_{i \neq k} \frac{1}{l_{kk}} \left| h_{e,k}^H p_i^{(d)} \right|^2 + \frac{K \sigma_n^2}{\lambda E_{tr} l_{kk}^2}},$$
$$\gamma_k^{(c)} = \frac{\left| 1 + h_{e,k}^H p_k^{(c)} \right|^2}{\sum_{i \neq k} \left| h_{e,k}^H p_i^{(c)} \right|^2 + \frac{\sigma_n^2 \sum_{i=1}^{K} (1/l_{ii}^2)}{\lambda E_{tr}}}.$$  

Finally, the respective instantaneous rates are found using (15). In order to assess the performance of the proposed
that the ASR is given by

\[ R_k = \mathbb{E}[R_k|\mathbf{H}] \approx \frac{1}{M} \sum_{m=1}^{M} R_{k}^{(m)}, \]  

The ergodic rate is taken from the expected value of the ASR over multiple channel estimates, leading to

\[ R_{ESR} = \mathbb{E} \left[ \sum_{k=1}^{K} \tilde{R}_k \right] \]  

III. TOMLINSON-HARASHIMA PRECODED RATE-SPLITTING

In this section, we present the proposed THP-RS schemes and develop expressions to compute the SINR and the sum-rate of these schemes. The main motivation of THP-RS is to improve the sum-rate performance beyond that achieved by RS with linear precoding and to exploit RS to make THP schemes more robust against imperfect CSIT. The latter is especially important because imperfect CSIT tends to affect more adversely THP than linear precoding due to the interference cancellation \[16\], \[19\]. Note that due to the power loss and the modulo loss, THP techniques do not achieve the performance of DPC \[21\]. However, THP is significantly less complex than DPC.

A. Proposed THP-RS with perfect CSIT

Let us now investigate whether RS can be combined with THP to further reduce the gap with DPC in the perfect CSIT setting. We split the message of one user, linearly precode the common stream and use THP to precode the private streams. Mathematically, the transmitted signal is given by

\[ x = [p_e, \mathbf{P}^{(THP)}] [s_e, \mathbf{v}]^T. \]  

Taking into account that cTHP and dTHP define the structure of \( \mathbf{P}^{(THP)} \), we get

\[ \mathbf{P}^{(d)} = \beta^{(d)} \mathbf{FB}^{(d)}^{-1}, \]

\[ \mathbf{P}^{(c)} = \beta^{(c)} \mathbf{FGB}^{(c)}^{-1}. \]

The transmitted signal is then rewritten as

\[ x = p_e s_e + \sum_{i=1}^{K} p_i v_i, \]

where \( p_i \) is the \( i \)th column of the precoder defined in \[25\] or \[26\], depending on the structure adopted. Note that both scaling factors \( \beta \) are modified since part of the power is assigned to the common precoder, leading to \( \beta (d) \approx \sqrt{\frac{N E_r - \|p_c\|^2}{\sum_{i=1}^{K} (1/l_i^2)}}. \)

and \( \beta (c) \approx \sqrt{\frac{N E_r - \|p_c\|^2}{\sum_{i=1}^{K} (1/l_i^2)}}. \)

Then, the received signals of the proposed THP-RS schemes are described by

\[ r^{(n-d)} = \frac{1}{\beta^{(d)}} G \mathbf{H} p_e s_e + \mathbf{v} + \frac{1}{\beta^{(d)}} \mathbf{G} n, \]

\[ r^{(n-c)} = \frac{1}{\beta^{(c)}} H \mathbf{p}_e s_e + \mathbf{v} + \frac{1}{\beta^{(c)}} \mathbf{n}. \]

At the \( k \)th user we have

\[ r^{(n-d)} = \frac{1}{\beta^{(d)}} h_k^H \mathbf{p}_e s_e + v_k + \frac{n_k}{\beta^{(d)}} l_{k,k}, \]

\[ r^{(n-c)} = \frac{1}{\beta^{(c)}} h_k^H \mathbf{p}_e s_e + v_k + \frac{n_k}{\beta^{(c)}} n_k. \]

From the last equation we obtain the SINR for the common message of the \( k \)th user:

\[ \gamma^{(d)}_{c,k} = \frac{K |h_k^H \mathbf{p}_c|^2}{\lambda E_{tr} - |p_c|^2 + K \sigma^2_n}, \]

\[ \gamma^{(c)}_{c,k} = \frac{\lambda E_{tr} + \sigma^2_n}{\sum_{i=1}^{K} (1/l_i^2)}. \]

The instantaneous rates for user \( k \) can be obtained by \( R^{(d)}_{c,k} = \log_2 (1 + \gamma^{(d)}_{c,k}) \) and \( R^{(c)}_{c,k} = \log_2 (1 + \gamma^{(c)}_{c,k}) \) for dTHP and cTHP, respectively. The common rate is set to \( R_c = \min R_{c,k} \) to ensure that all users can decode the message. After decoding the common message, the receiver subtracts it from the received signal. The SINR expressions for the private messages are

\[ \gamma^{(d)}_{p,k} = \frac{\lambda^2 E_{tr} - |p_c|^2}{K \sigma^2_n}, \]

\[ \gamma^{(c)}_{p,k} = \frac{\lambda E_{tr} - |p_c|^2}{\sigma^2_n \sum_{i=1}^{K} (1/l_i^2)}, \]

which are similar to \[13\]. However, the value of \( \gamma_k \) is reduced due to the power assigned to the common message. It follows that the instantaneous rate for the private message is given by \[15\]. At the end, the sum-rates for the RS system can be expressed as

\[ R^{(d)} = R^{(d)}_{c} + \sum_{k=1}^{K} R^{(d)}_{k}, \quad R^{(c)} = R^{(c)}_{c} + \sum_{k=1}^{K} R^{(c)}_{k}. \]

B. Proposed Rate-Splitting THP with imperfect CSIT

In this section, we consider THP-RS under imperfect CSIT. Using \[27\] we can express the received signal as follows:

\[ r^{(n-d)} = \frac{1}{\beta^{(d)}} G \mathbf{H} p_e s_e + \mathbf{v} + \mathbf{G} \mathbf{H} \mathbf{F}^{-1} \mathbf{v} + \frac{1}{\beta^{(d)}} \mathbf{G} n, \]

\[ r^{(n-c)} = \frac{1}{\beta^{(c)}} H \mathbf{p}_e s_e + \mathbf{v} + \mathbf{H} \mathbf{F}^{-1} \mathbf{v} + \frac{1}{\beta^{(c)}} \mathbf{n}. \]
From the last equation we can obtain the received signal at each user equipment, which is given by

\[ r_k^{(nsd)} = \frac{1}{l_{k,k}} h_k^H p_s c_k + v_k + \frac{1}{l_{k,k}} h_{e,k}^H \sum_{i=1}^{K} p_i^{(d)} v_i + \frac{1}{l_{k,k}} n_k \]

\[ r_k^{(nc)} = \frac{1}{\beta(c)^2} h_{e,k}^H p_s c_k + v_k + \frac{1}{\beta(c)^2} \sum_{i=1}^{K} p_i^{(c)} v_i + \frac{n_k}{\beta(c)^2} \]  \hspace{1cm} (39)

Then the SINR for the common message can be computed by the following:

\[ \gamma_{c,k}^{(d)} = \frac{|h_k^H p_s|^2 / \beta(d)}{|l_{k,k} + h_{e,k}^H p_k^{(d)}|^2 + \sum_{i=1, i \neq k}^{K} |h_{e,k}^H p_i^{(d)}|^2 + \sigma_n^2 / \beta(d)} \]

\[ \gamma_{c,k}^{(c)} = \frac{|h_k^H p_c|^2 / \beta(c)}{|1 + h_{e,k}^H p_k^{(c)}|^2 + \sum_{i=1, i \neq k}^{K} |h_{e,k}^H p_i^{(c)}|^2 + \sigma_n^2 / \beta(c)} \]

The rate of the private messages can be calculated with equations (22) and (21). The transmit power should be changed to \( E_{tr} - \| p_c \|^2 \), as explained before for the perfect CSIT case. The resulting sum rate is computed with (36). Then the ASR and ESR can be found with (23) and (24), respectively. The rates for ZF-DPC-RS approximation based on [23] with imperfect CSIT can be obtained by using the previous expressions and by neglecting the power loss and the modulo loss.

IV. SIMULATIONS

In this section we evaluate the performance of the proposed THP-RS schemes using zero-forcing (ZF) filters and compare them with existing techniques. We consider a MISO BC channel with 4 transmit antennas and 4 users, where each user is equipped with a single antenna. The inputs follow a Gaussian distribution with variance \( \sigma_n^2 = 1 \). We also consider additive white Gaussian noise and flat fading Rayleigh channels scenario, where all the users experience the same SNR. The ASR was calculated using a total of 100 error matrices for each estimated channel. Then the ESR was obtained averaging over 50 independent channel estimates. The precoder for the common message was obtained using a singular value decomposition (SVD) of the channel matrix (H = USV), i. e., \( p_c = V(:, 1) \). A percentage of the power from the private precoders has been assigned to the common precoder. The power assigned to the common precoder was found through an optimization procedure, while the remaining power was uniformly allocated across the private precoders.

Figs. 1 and 2 illustrate the results for the precoding algorithms with perfect and imperfect CSIT, respectively. We consider a power loss factor of \( \lambda = 0.75 \) for the THP structures. For imperfect CSIT we used a fixed error variance equal to 0.2. The results show that the proposed THP-RS schemes outperform previously reported THP and linear schemes. RS-based schemes only offer a small gain over standard schemes with perfect CSIT, as illustrated in Fig. 1 whereas those gains are substantial in the presence of imperfect CSIT, which corroborates the sum-rate results in the literature for linearly precoded RS scheme. The results also show that the ZF-DPC approximation obtains the highest sum rates, as expected, followed by THP and linear schemes. Specifically among RS-based schemes, the ZF-DPC-RS [23] obtains the best result followed by dTHP-RS, cTHP-RS and linearly precoded RS. Note that for the ZF-DPC implemented here, we considered uniform power allocation among the streams. The performance advantage of dTHP structures over cTHP ones is explained by the error covariance matrix previously presented, and by the use of more complex receive filters at the users, whereas cTHP only employs filters at the transmitter which translates into lower complexity [19]. The curves obtained for imperfect CSIT exhibit saturation due to the fact that the variance of the CSIT errors does not scale with the SNR. This is expected for THP schemes due to error propagation associated with imperfect CSIT.
and linearly precoded RS. The sum rates achieved by dTHP-RS can be up to 25% higher than cTHP-RS and RS, whereas they can be up to 100% higher than those achieved by non RS-based schemes. This highlights the robustness of RS schemes against imperfect CSIT for a wide range of scenarios.

In the last example, we consider that the variance of the error scales with the SNR \( \sigma^2 = E_r^{-\alpha} \). The curves obtained in Fig. 4 have been computed with \( \alpha = 0.6 \). The results indicate that THP-RS schemes are more robust than standard THP schemes and achieve higher sum rates than linear schemes. It can be noticed that the slope achieved by RS-based schemes is significantly higher than that associated with non RS-type approaches, which corroborates the robustness shown in Fig. 3 and the superiority of RS in terms of DoF [2, 11].

![Fig. 3. Sum rate performance versus channel error variance](image)

![Fig. 4. Sum rate performance with RS, imperfect CSIT and \( \alpha = 0.6 \).](image)

V. Conclusion

In this paper we have proposed THP-RS schemes and derived SINR and sum-rate expressions to evaluate their performance with perfect and imperfect CSIT. Moreover, we have also examined the sum-rate performance of ZF-DPC with and without RS for perfect and imperfect CSIT. Simulation results have shown that the proposed THP-RS schemes can achieve higher sum rates than those of existing THP and linear schemes, and are more robust against imperfect CSIT than standard THP schemes.

---

**REFERENCES**

[1] L. Lu, G. Y. Li, A. L. Swinderlust, A. Ashikhmin, and R. Zhang, “An overview of massive MIMO: Benefits and challenges,” *IEEE Journal of Selected Topics in Signal Processing*, 2014.

[2] H. Joudeh and B. Clerckx, “Sum-rate maximization for linearly precoded downlink multiuser MISO systems with partial CSIT: A rate-splitting approach,” *IEEE Transactions on Communications*, vol. 64, no. 11, pp. 4847–4861, 2016.

[3] B. Clerckx, H. Joudeh, C. Hao, M. Dai, and R. B., “Rate splitting for MIMO wireless networks: a promising PHY-layer strategy for LTE evolution,” *IEEE Communications Magazine*, vol. 54, no. 5, pp. 98–105, 2016.

[4] T. Han and K. Kobayashi, “A new achievable rate region for the interference channel,” *IEEE Transactions on Information Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.

[5] Y. Mao, B. Clerckx, and V. Li, “Rate-splitting multiple access for downlink communication systems: Bridging, generalizing and outperforming SDMA and NOMA,” *EURASIP Journal on Wireless Communications and Networking*, in press.

[6] C. Hao, Y. Wu, and B. Clerckx, “Rate analysis of two-receiver MISO broadcast channel with finite rate feedback: A rate-splitting approach,” *IEEE Transactions on Communications*, vol. 63, no. 9, pp. 3232–3246, July 2015.

[7] H. Joudeh and B. Clerckx, “Rate-splitting for max-min fair multigroup multicast beamforming in overloaded systems,” *IEEE Transactions on Wireless Communications*, vol. 16, no. 11, pp. 7276–7289, 2017.

[8] ——, “Robust transmission in downlink multiuser MISO systems: A rate-splitting approach,” *IEEE Transactions on Signal Processing*, vol. 64, no. 23, pp. 6227–6242, 2016.

[9] M. Dai, B. Clerckx, D. Gesber, and G. Caire, “A rate splitting strategy for massive MIMO with imperfect CSIT,” *IEEE Transactions on Wireless Communications*, vol. 15, no. 7, pp. 4611–1624, July 2016.

[10] C. Hao and B. Clerckx, “MISO networks with imperfect CSIT: A topological rate-splitting approach,” *IEEE Transactions on Communications*, vol. 65, no. 5, pp. 2164–2179, 2017.

[11] E. Piovano and B. Clerckx, “Optimal DoF region of the K-user MISO BC with partial CSIT,” *IEEE Commun. Letters*, vol. 21, no. 11, pp. 2368–2371, Nov. 2017.

[12] A. G. Davoodi and S. A. Jafar, “Aligned image sets under channel uncertainty: Settling conjectures on the collapse of degrees of freedom under finite precision CSIT,” *IEEE transactions on Information Theory*, vol. 62, no. 10, p. 56035618, Oct. 2016.

[13] M. Joham, W. Utschick, and J. A. Nossek, “Linear transmit processing in MIMO communications systems,” *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 2700–2712, Aug. 2005.

[14] K. Zu, R. C. de Lamiare, and M. Haardt, “Generalized design of Low-Complexity block diagonalization type precoding algorithms for multiuser MIMO systems,” *IEEE Transactions on Communications*, vol. 61, no. 10, pp. 4232–4242, October 2013.

[15] W. Zhang, R. C. de Lamiare, C. Pan, M. Chen, J. Dai, B. Wu, and X. Bao, “Widely linear precoding for large-scale mimo with iqi: Algorithms and performance analysis,” *IEEE Transactions on Wireless Communications*, vol. 16, no. 5, pp. 3298–3312, May 2017.

[16] M. Tomlinson, “New automatic equaliser employing modulo arithmetic,” *Electronics Letters*, vol. 7, no. 5, pp. 138–139, March 1971.

[17] H. Harashima and H. Miyakawa, “Matched-transmission technique for channels with intersymbol interference,” *IEEE Journal on Selected Topics in Communications*, vol. 20, no. 4, pp. 774–780, Aug. 1972.

[18] C. Windpassinger, R. F. H. Fischer, T. Vencel, and J. B. Huber, “Precoding in multiantenna and multiuser communications,” *IEEE Transactions on Wireless Communications*, vol. 3, no. 4, pp. 1305–1316, July 2004.

[19] K. Zu, R. C. de Lamiare, and M. Haardt, “Multi-branch Tomlinson-Harashima precoding design for MU-MIMO systems: Theory and algorithms,” *IEEE Transactions on Communications*, vol. 62, no. 3, pp. 939–951, March 2014.

[20] L. Zhang, Y. Cui, R. C. de Lamiare, and M. Zhao, “Robust multibranch Tomlinson-Harashima precoding design in amplify-and-forward MIMO relay systems,” *IEEE Transactions on Communications*, vol. 62, no. 10, pp. 3476–3490, Oct. 2014.

[21] L. Sung and M. McKay, “Tomlinson-Harashima precoding for multiuser MIMO systems with quantized CSI feedback and user scheduling,” *IEEE Transactions on Signal Processing*, vol. 62, no. 16, pp. 4077 – 4090, July 2014.

[22] S. A. et al., *Lectures on Stochastic Programming: Modeling and theory*, Philadelphia, PA, USA:SIAM, 2009.

[23] G. Caire and S. Shamai, “On the achievable throughput of a multiantenna gaussian broadcast channel,” *IEEE Transactions on Information Theory*, vol. 49, no. 7, pp. 1691 – 1706, June 2003 2003.