Scrambling and the black hole atmosphere

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Abstract

We argue that the scrambling time is the same, up to a numerical factor in three or more spacetime dimensions, as the time for the atmosphere to fall across the horizon or escape, to be replaced by new atmosphere. We propose that these times agree because the physical scrambling process is part and parcel of the atmosphere refreshment process. We provide some support for this relation also in two dimensions, but the atmosphere is not as localized, so the argument is less justified.

1 Introduction

The notion that black holes are fast scramblers of quantum information was motivated by the assumption of unitarity of the black hole S-matrix [1]. Several distinct time scales that might be associated with black hole scrambling converged on the same value,

\[ t_* \sim \frac{\beta}{2\pi} \log S \]  

(1)
where $\beta$ is the inverse temperature and $S$ the entropy of the black hole. These include apparent spreading of charge that falls into a black hole, a no-cloning constraint, and a chaos timescale probed holographically with Planckian shockwave collisions [1, 2, 3, 4]. A notion similar to the last of these was introduced some three decades earlier [5], as the time after which “a pure state description of a black hole will be very difficult”. In the shockwave analysis of [4] and some later work, the timescale depends on the energy input, and is longest when that energy is minimized. Taking the minimal energy disturbance to be one thermal unit at the Hawking temperature yields again the result (1). For an asymptotically Anti-de Sitter black hole, according to AdS/CFT duality, black hole scrambling is dual to thermalizing interactions of the dual, strongly coupled conformal quantum field theory, and the scrambling time in one thermal cell of the CFT is again of the form (1), but now with $S \sim N^2$, the entropy per thermal cell. The nature of scrambling in various quantum systems, including holographic duals of black holes, is fairly clear, and has been studied in a number of models. However, the nature of scrambling in terms of black hole physics remains mysterious.

At the classical level, spacetime outside a large black hole is quiescent and gently curved, and presents no reason to suspect any kind of “scrambling” might be taking place when a disturbance falls in across the horizon. However, the black hole is a deformed quantum field vacuum, so there is in fact much quantum activity around a black hole. Indeed, the tidal acceleration acting on these vacuum fluctuations produces the Hawking radiation. Previous authors have suggested that collisions of infalling quanta with outgoing quantum field fluctuations are responsible for scrambling. This mechanism was first discussed in [5], and the idea was further explored in the setting of AdS/CFT [4]. It is an elusive notion, since the outgoing fluctuations are in their ground state. That is, infalling quanta encounter nothing but vacuum outside the black hole, so there seems to be nothing with which they might interact. Here we propose a picture of the nature of black hole scrambling that involves the outgoing quantum field fluctuations in a different way.

The near horizon quantum vacuum is a thermal “atmosphere”. Most of the entropy in this atmosphere comes from outgoing modes with high transverse momentum, which quickly reflect from the effective potential and fall into the black hole, while modes of sufficiently high frequency and low angular momentum escape. The thermal atmosphere of a black hole must therefore be continually resupplied. On a fixed classical spacetime background, with Lorentz invariant quantum fields, there would be no UV cutoff, and the atmosphere would be resupplied from a trans-Planckian reservoir of modes just outside the horizon. But that picture neglects the gravitational effect of the vacuum fluctuations, and must be far from the truth. In particular, the finiteness of black hole entropy implies that no such reservoir exists. In the local vacuum state the outside field modes are entangled with partners
inside the horizon, so a transPlanckian reservoir would carry an infinite entanglement entropy, exceeding the finite Bekenstein-Hawking entropy $S_{BH}$ of the black hole. Evidently quantum gravity somehow cuts off this would-be divergent entropy contribution, and it follows that the resupply mechanism for the atmosphere must also involve quantum gravity.

Our proposal is that scrambling is part of the atmosphere replacement mechanism. In particular, we provide evidence that the scrambling time and the atmosphere refresh time agree up to a numerical factor, if the cutoff is chosen so that the atmosphere entropy matches the black hole entropy minus the extremal value for the same charge. The refresh time shares aspects of some other definitions of the scrambling time, but it is conceptually distinct.

This interpretation of scrambling provides some insight into the causality puzzle posed by the rapidity of the scrambling [1]. Hayden and Preskill noted that, when viewed as thermalization on the “stretched horizon”, the scrambling process is superluminal in the transverse dimensions. If the stretched horizon were an ordinary thermal system, no local causal dynamics could scramble that quickly. But it is not an ordinary thermal system: the atmosphere continually falls into the black hole or escapes from the near horizon region, and is replaced by “fresh” atmosphere. That replacement is itself a mysterious process. How do outgoing modes emerge from the near horizon region, in the absence of a transPlanckian reservoir to supply them [9]? It seems that the process must be nonlocal, because locally the horizon is an unremarkable place in spacetime. Together with the agreement of the refresh time and scrambling times, this inherent nonlocality strongly suggests that the mechanism of scrambling is intimately related to the atmosphere resupply mechanism.

2 Atmosphere model

To estimate the refresh time, we model the atmosphere as a gas of massless field modes, and use the ray optics approximation to evaluate the mode propagation. This crude model captures the consequences of radial causality, which we presume are all that really matters. While most of the entropy resides in outgoing modes that fall back across the horizon more quickly, the longest lived modes are those of the smallest angular momentum and highest frequency, some of which escape from the near horizon region. Those are also the modes that are the most relevant for the Hawking radiation. We define the refresh time as the time for those modes to escape from the “Rindlersphere” i.e., the region where the Rindler (flat spacetime) approximation is valid. More precisely, for static, spherical black holes, we define

\footnote{For discussions of the possible role of quantum horizon fluctuations in this cutoff see [6, 7, 8].}
the Rindlersphere as the region where the norm of the Killing vector is approximately proportional to the proper radial distance (normal to the Killing vector) from the horizon.

While we lack a sharp reason for using the top of the Rindlersphere to demarcate the “tropopause” of the black hole atmosphere, it is a natural choice because, for an evaporating black hole, the Rindlersphere is the region that is in equilibrium. As for the “bottom” of the atmosphere, as discussed above, quantum gravity presumably imposes a cutoff on modes close to the horizon. If the atmosphere entropy contributed by a single species is to be of order the Bekenstein-Hawking entropy

\[ S_{BH} = \frac{A}{4l_P^2}, \]

we should exclude modes localized closer to the horizon than the Planck length \( l_P \), as measured on a spatial hypersurface orthogonal to the horizon generating Killing vector [10, 5]. (We consider here only spherical black holes.) For charged black holes we set the lower cutoff so as to match only the portion of the black hole entropy above the extremal value for the same charge. We make this choice in order to match the scrambling time found by shockwave analytics in [11], but it is not entirely unmotivated. The extremal portion of the entropy resides in a ground state degeneracy. It is a different kind of entropy, and is not carried by an atmosphere that is continually refreshed. (See below for more discussion of this point.)

### 3 Atmosphere refresh time

We define the refresh time as the Killing time for a radial null geodesic to climb from the bottom to the top of the Rindlersphere. This is well-defined in spherical symmetry if the Killing time coordinate is chosen so its level sets are orthogonal to the Killing vector.\(^2\) The spherically symmetric, static line element then takes the form

\[ ds^2 = -N^2 dt^2 + dy^2 + r^2 d\Omega^2, \]

where \( N = N(y) \) and \( r = r(y) \). The coordinate \( y \) is the proper radial distance orthogonal to the Killing vector \( \partial_y \), whose norm is \( N \). For a black hole we set \( y = 0 \) at the horizon, so in the Rindlersphere we have \( N(y) \approx \kappa y \), where \( \kappa \) is the surface gravity.

A radial light ray satisfies \( dt = dy/N \), so as it propagates from the bottom to

\(^2\)Alternatively, we could define the refresh time using a round trip that reflects at a centrifugal barrier (or an imaginary mirror) at the top of the Rindlersphere, and falls back to the initial Killing orbit. The elapsed Killing time along that orbit does not depend on which Killing time coordinate is employed, and is just twice the time defined above.
the top of the Rindlersphere the elapsed Killing time is

\[ \Delta t = \int_{y_b}^{y_t} \frac{dy}{N} \approx \kappa^{-1} \log(y_t/y_b), \]

where \( y_b \) and \( y_t \) are the radial distances from the horizon to the bottom and top of the Rindlersphere. For a Schwarzschild black hole in four spacetime dimensions, the lower cutoff is the Planck length, \( y_b \sim l_P \), and we find \( y_t \sim r_+ \), so the log in (3) is \( \sim \log(r_+/l_P) \sim \frac{1}{2} \log S_{\text{BH}} \). The refresh time (3) therefore agrees with (1), up to the prefactor \( \frac{1}{2} \), since \( \kappa = 2\pi T_H/h = 2\pi/\beta \).

To locate the top of the Rindlersphere we consider the Taylor expansion of the lapse \( N(y) = \kappa y + \ldots \). The Rindlersphere ends where the higher order terms compete with the linear one. The even terms in \( y \) vanish for the static metrics we are considering, because regularity at the horizon implies that \( N^2 \) must be even under \( y \to -y \) reflection. Assuming that \( (N_{g_{yy}})_+ \) (\( \equiv d^3 N/dy^3 \) \( |_{y=0} \)) is nonzero, the top of the Rindlersphere lies where \( \kappa y \sim \left| N_{g_{yy}} \right|_+ y^3 \), i.e. at \( y_t \sim \left[ \kappa/|N_{g_{yy}}|_+ \right]^{1/2} \). The metrics we consider all satisfy \( N = dr/dy \). A straightforward computation using this relation shows that \( (N_{g_{yy}})_+ = \frac{\kappa}{2}[(-N^2)_{rr}]_+ \), so a general expression for the location of the top of the Rindlersphere is

\[ y_t \sim \left| (N^2)_{rr} \right|^{1/2}_+. \]  

For Schwarzschild black holes this yields \( y_t \sim r_+ [(D-2)(D-3)]^{-1/2} \). For AdS-Schwarzschild black holes that are much larger than the AdS length \( L \), and for the planar case, we find \( y_t \sim L[(D-1)(D-4)]^{-1/2} \). For Reissner-Nordstrom (charged) black holes in \( D = 4 \), we find \( y_t \sim r_+ [1 - 2r_-/r_+]^{-1/2} \), which yields \( y_t \sim r_+ \) as long as \( r_- \) is not equal (or very close) to \( r_+ /2 \). In exceptional cases where \( (N_{g_{yy}})_+ \) vanishes, we find instead \( y_t \sim \kappa (N^2)_{rrr} \left. \right|^{1/4}_+ \). This happens in \( D = 4 \) for for planar Schwarzschild-AdS black holes and for Reissner-Nordstrom black holes with \( r_- = r_+/2 \). When using this higher order relation, we find that the dependence of \( y_t \) on the metric parameters is the same as in the generic cases.

To locate the bottom of the atmosphere, \( y_b \), we require that the atmosphere entropy accounts for the portion of the black hole entropy above the extremal value, as discussed above. The thermal entropy can be estimated as \( S \sim A/y_b^{D-2} \) [10, 5]. Setting this equal to the entropy above the extremal value, \( (A - A_0)/l_P^{D-2} \), where \( A_0 \) is the extremal area for the given charge, yields

\[ y_b = l_P [A/(A - A_0)]^{1/(D-2)}. \]

Close to extremality, this can differ significantly from \( l_P \), and it can even become greater than \( y_t \), the top of the Rindlersphere. However, the thermal approximation we are employing should not be trusted too close to extremality [12]. For
a Reissner-Nordstrom black hole in $D = 4$ spacetime dimensions, (5) yields $y_b = l_P[r_+/(r_+ - r_-)]^{1/2}$. The condition required for validity of the equilibrium treatment is $r_+ - r_- \gtrsim l_P^2/r_+$, and as long as this holds, we have $y_b \lesssim r_+ \sim y_t$. That is, the bottom of the atmosphere lies below the top of the Rindler region.

4 Examples

We are now ready to evaluate the refresh time (3) for several examples.

4.1 Schwarzschild in $D$ dimensions

For a Schwarzschild black hole in any dimension $D = O(1) > 2$ we have $y_t/y_b \sim r_+/l_P \sim S^{1/(D-2)}$, so (3) yields $\Delta t \sim (\kappa(D-2))^{-1} \log S$. Apart from the prefactor $(D-2)^{-1}$, this agrees with (1).

It is interesting to consider also the very large $D \gg 1$ limit, if we presume that there exists a consistent UV completion of large $D$ quantum gravity in which black holes have finite entropy. Keeping track of just the leading order $D$ dependence, we have $y_t \sim r_+/D$. As for $y_b$, to apply our criterion that the atmosphere entropy matches the black hole entropy, we need to consider the large $D$ limit of entropy density of massless fields in a thermal state. The temperature dependence is $T \sim D^{D-1}$, and for each polarization there is a leading order $D^{D/2}$ numerical factor (coming from the large phase space contributing to the high energy tail of the Planck distribution) [14]. In addition, there are $D(D-3)/2$ graviton polarizations, but this does not change the leading order $D$ dependence, so the leading $D$-dependence of the entropy density is $s \sim D^{D/2}T^{(D-1)}$. Integrating this with the local temperature $T(y) \sim 1/y$ down to $y_b$ we obtain an atmosphere entropy $S_{atm} \sim D^{D/2}A/y_b^{D-2}$, and equating this to the black hole entropy $S_{BH} \sim A/l_P^{D-2}$ yields $(y_b/l_P)^{D-2} \sim D^{D/2}$, i.e. $y_b/l_P \sim D^{1/2}$. Putting the results together we thus have $y_t/y_b \sim D^{-3/2}r_+/l_P$. If this is not greater than unity, the thermal atmosphere lies entirely above the tropopause of the Rindlersphere, which means the calculation is inconsistent. However, also in this case the Hawking evaporation timescale is shorter than the Planck time, so the black hole cannot be treated semiclassically [13]. If we assume that the black hole is large enough that $r_+ \gg D^{3/2}l_P$, then the atmosphere refresh time (3) is $\sim \kappa^{-1} \log(D^{-3/2}r_+/l_P) \sim (\kappa D)^{-1} \log S$. This indicates a $1/D$ suppression of the refresh time compared to (1), no different from what we found for $D \sim O(1)$.

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3See also [13] for an investigation of scrambling for large $D$ black holes.
4.2 Reissner-Nordstrom in $D = 4$

For the Reissner-Nordstrom case in $D = 4$ we have from the above results $y_t \sim r_+$ and $y_b/l_P \sim [S/(S - S_0)]^{1/2}$ (5). Since also $r_+/l_P \sim S^{1/2}$, this yields $y_t/y_b \sim (S - S_0)^{1/2}$, so the refresh time is $\Delta t \sim \frac{1}{2} \kappa^{-1} \log(S - S_0)$. Up to the factor 1/2, this agrees with the scrambling time found in Ref. [11] via shockwave analytics with the injected energy of order the temperature. This result suggests that the scrambling does not involve the degrees of freedom counted by the extremal entropy. This makes some sense, since the extremal entropy does not correspond to a thermal atmosphere that falls into the black hole, so does not need to be refreshed. Perhaps when a charged black hole relaxes to the extremal state, unitarity may not require the scrambling of all the degrees of freedom.

A different interpretation of the shortened scrambling for near-extremal charged black holes was proposed in Ref. [15]. The viewpoint adopted there was that the infalling energy behaves like a fundamental string, and thus exhibits an effective transverse spreading as its momentum relative to the local static frame increases during its fall through the long throat region between the Newtonian exosphere and the Rindlersphere. That paper argued that this earlier spreading decreases the time needed to complete scrambling at the stretched horizon, and reproduces the result of [11] while maintaining the hypothesis that all of the degrees of freedom counted by the black hole entropy are involved in the scrambling. Note that this argument assumes that the excitation falls freely into the black hole, which produces the effective transverse spread. But a particle need not fall freely. If it is lowered toward the horizon, and only dropped in from the top of the Rindlersphere, then the long throat should play no role.

4.3 Very large Schwarzschild-AdS in $D = O(1) > 2$ dimensions

For an AdS-Schwarzschild black hole with $r_+ \gg L$, in any dimension $D = O(1) > 2$, we have $y_t/y_b \sim L/l_P$, so Eq. (3) yields $\Delta t \sim \kappa^{-1} \log(L/l_P) \sim [\kappa(D - 2)]^{-1} \log N^2$, where $N^2 \sim (L/l_P)^{(D-2)}$ is the entropy per thermal cell of the dual $SU(N)$ gauge theory [16]. Apart from the $1/(D - 2)$ prefactor, this agrees with the time estimated from charge spreading in [2]. This time can also be inferred from the shockwave analytics of Ref. [4]. The time found there is (1), with $S$ the black hole entropy, if the (homogeneously) injected energy corresponds to one unit of entropy. To express this in terms of quantities associated with a single thermal cell, note that $S = N^2/(N^2/S)$. The numerator is the number of degrees of freedom in a thermal cell, while the denominator is the reciprocal of the number of thermal cells, which is equal to the entropy injected per cell, given that

\footnote{We thank Douglas Stanford for an explanation of this point.}
just one unit of entropy was injected globally. If one unit of entropy were injected per thermal cell, the shockwave scrambling time would have been $\sim \kappa^{-1} \log N^2$, in agreement with our definition of the atmosphere refresh time.

### 4.4 de Sitter spacetime

It was noticed in Ref. [3] that the charge spreading timescale implies that de Sitter space, too, is a fast scrambler, whose scrambling time is $t_\star \sim \kappa^{-1} \log(L/l_P)$, where $L$ is the de Sitter radius.\(^5\) Moreover, recent work [17, 18] has studied scrambling in de Sitter space via shockwave computations of chaos timescales. A de Sitter horizon also has a thermal atmosphere [19], to which we may apply our definition of the refresh time. The top of the Rindlersphere is or order $\sim L$, so we obtain $y_t/y_b \sim L/l_P$, and thus $\Delta t \sim \kappa^{-1} \log(L/l_P) \sim (\kappa(D - 2))^{-1} \log S$.

Despite the similarities with black hole horizons, the atmosphere refresh mechanism for de Sitter horizons may differ, since it must take place within each static patch, whereas for a black hole it could be connected to infinity. While the refresh time according to our criterion is the same as for black hole horizons, perhaps that criterion is not correct for de Sitter horizons. It was suggested recently in Ref. [20] that the difference with black hole horizons is more profound, and leads to a “hyperfast” scrambling time $\sim \kappa^{-1}$, with no dependence on the horizon entropy.

### 4.5 Black holes in $D = 2$ dimensions

In two spacetime dimensions our previous considerations for the black hole geometries and our estimate of the thermal entropy of the atmosphere do not apply, yet various dilaton gravity theories in two spacetime dimensions have black hole solutions [21]. A horizon cross section for such black holes is just a single point, but they have an entropy determined by the value of the dilaton at the horizon, and an associated scrambling time if the theory supports black hole dynamics and Hawking radiation. In this subsection we briefly consider whether the close relation between the scrambling time and the atmosphere refresh time may hold in such theories, despite the differences from the higher dimensional cases. Our conclusion will be “perhaps”.

The atmosphere in two spacetime dimensions is qualitatively different from that in higher dimensions. The entanglement entropy of a half-line in a gapped theory with correlation length $\xi$ and a UV fixed point with central charge $c$ is $\xi_c \log(\xi/\epsilon)$, where $\epsilon$ provides a UV cutoff [22]. Thus, unlike in higher dimensions,

\(^5\)In [3] the string length was used rather than the Planck length, but this makes little difference to the log. Also, the prefactor $R$ in Eq. (2.3) of that paper is a typographical error.
the entropy depends on the log of the cutoff rather than a power. Also, if the entangled field is gapless, there is an IR divergence. Our criterion for locating the bottom of the atmosphere by setting the entanglement entropy equal to the black hole entropy thus appears not to be well-justified. If we ignore this and postulate that \( \epsilon \) should be identified with the cutoff \( y_b \) at the bottom of the atmosphere, and if we identify the IR cutoff with the top of the atmosphere \( y_t \), the entanglement entropy would be \( \frac{C_6}{c} \log(y_t/y_b) \). If this is set equal to the black hole entropy \( S \), one obtains \( \log(y_t/y_b) = \frac{C_6}{c}S \), so the atmosphere refresh time (3) becomes \( \Delta t = \frac{C_6}{c}\kappa^{-1}S \). This strongly disagrees with the usual scrambling time \( \kappa^{-1}\log S \). However, since this calculation is not well-justified, we do not regard the disagreement as necessarily indicating a failure of the atmosphere refresh time interpretation of scrambling. It may rather indicate that in two dimensions the nature of the atmosphere and its entanglement entropy in an underlying fundamental theory with finite black hole entropy is qualitatively different from that in higher dimensions.

While it is beyond the scope of this paper to attempt to characterize the black hole atmosphere in 2d theories, we shall just note here one possible alternative to the estimate given above, making use of the UV cutoff that is implied by the matrix model dual to two-dimensional string theory [23, 24, 25]. According to the analysis of Ref. [25], the entropy of an interval of length \( L \) in the emergent linear dilaton vacuum, in a region where the string coupling \( g_s = e^\Phi \) is weak, is \( S = \frac{1}{3} \log(L \exp(-\Phi_{\text{tw}} - \Phi_{1}/2 - \Phi_{2}/2)) \), where \( \Phi \) is the dilaton, which appears in the semiclassical action in combination with the Ricci scalar as \( e^{-2\Phi}R \), and the string length is set to unity. The subscripts “tw, 1, 2” label the values at the “tachyon wall” and the two endpoints of the interval, and it is assumed here that the interval is located in a region where \( \Phi_{1,2} < \Phi_{w} \ll -1 \). This formula indicates that the effective cutoff at the interval endpoint 1 scales with \( \Phi_{1} \) as \( \epsilon \sim e^{\Phi_{1}/2} \), which is exponentially smaller than the string length. If we set \( y_b = e^{\Phi_{\text{hor}}}/2 \), then the refresh time (3) becomes \( \Delta t \sim \kappa^{-1}(-\frac{1}{2}\Phi_{\text{hor}} + \log y_t) \). If black holes existed in the emergent spacetime of the matrix model, their entropy would be determined by the coefficient \( e^{-2\Phi_{\text{hor}}} \) of the Ricci scalar in the action, in which case this refresh time would be \( \kappa^{-1}(-\frac{1}{2}\Phi_{\text{hor}} + \log y_t) \). And if the entropy term were to dominate over the \( y_t \) term, then up to the factor \( \frac{1}{4} \) this would agree with the scrambling time. However, this identification of \( y_b \) with the cutoff inferred from the matrix model entanglement entropy is admittedly only vaguely motivated. Moreover, that theory does not even support black hole formation [26]. We are only pointing out that perhaps a UV cutoff set by stringy effects could be essential to the log rather than linear dependence of the refresh time on the black

\[ \text{For a near-extremal black hole one would set it equal to the entropy } S - S_0 \text{ that exceeds the extremal value } S_0 \]
horo entropy in two dimensions.

A recently much studied alternative to the matrix model for emergent 2d string theory that does contain black holes is the SYK model, which is dual to JT gravity. In Ref. [27] the chaos time scale in the SYK model was studied via the Lyapunov exponent in correlation functions. It was observed there that, although “the bulk theory dual to SYK has a tower of light fields roughly similar to a string spectrum” with string length of order the AdS radius, the contribution of the duals of those fields to the Lyapunov exponent (and hence to the scrambling time) in the SYK model is suppressed relative to the “gravity” contribution for near-extremal black holes. Perhaps the nature of the black hole atmosphere can be understood in this model, allowing its refresh time to be found. Stringy effects would presumably impose the UV cutoff of the atmosphere.

5 Conclusion

We have shown that the black hole scrambling time is the same, up to a numerical factor in three or more spacetime dimensions, as our estimate of the time for the thermal atmosphere to be refreshed. We propose that these times agree because the physical scrambling process is part and parcel of the atmosphere refreshment process. The discrepancy of the numerical factor is worrisome, but it may not point to a fundamental flaw in the proposal, since our crude model of the atmosphere and kinematic criterion for its refresh time may just be too crude to accurately capture this factor. We provide some evidence supporting also a relation in two spacetime dimensions between the refresh time and the scrambling time, but the picture is not as clear because the atmosphere is apparently not as localized near the horizon.

The atmosphere refreshment process must be nonlocal, since locally nothing distinguishes the horizon. The picture of atmosphere refreshment thus goes some way toward illuminating why the scrambling is nonlocal in spacetime, and therefore how it can be so fast. We have said nothing, however, that elucidates the dynamics of atmosphere replacement. The origin of the outgoing black hole modes remains as obscure as ever [9]. We may hope that, by having linked it to scrambling, perhaps some light may be shined on it.

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