Research of light diffraction on multilayer PPM-LC diffraction structures under conditions of external electrical influence

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Abstract. The paper presents a theoretical model of light diffraction on multilayer PPM-LC diffraction structures. A numerical calculation of the diffraction characteristics of four-layer PPM-LC diffraction structures under the influence of external electrical action is also given. The possibility of controlling the type of selective response under the influence of an external electric field on each layer of the structure is shown.

1. Introduction
In the last few years, various teams of researchers from different countries have been studying multilayer inhomogeneous holographic diffraction structures (MIHDS) [1-15]. MIHDS are several volumetric lattices separated by optically homogeneous intermediate layers [1-4]. Such structures are characterized by special properties due to the interference of waves from each lattice [1].

MIHDS have the perspective to be widely popular as elements of spectral filters, sensors, interconnects, multiplexers/demultiplexers in optical communication lines, as well as for generating femtosecond laser pulses [3].

The research considers the possibility of dynamic control of the type of selective response for the MIHDS formed in the PPM-LC. The implementation of dynamic control is possible in the presence of liquid crystals (LC) in the composition of the PPM [3]. The director of the LC is sensitive to electrical influences, thereby it can change its spatial orientation under the influence of an external electric field, and as a result, the conditions for the propagation of electromagnetic waves change.

In that way, the research of light diffraction on multilayer inhomogeneous holographic PPM-LC diffraction structures become relevant.

2. Theoretical part
In this research, we consider the diffraction of light on a multilayer holographic diffraction structure in PPM-LC (Fig. 1). We will assume that polymer-dispersed liquid crystals (PDLC), which are characterized by the combination of LC molecules into capsules. Light diffraction occurs only on those structures on which all recording processes have ended. The aperture of the reading beam is much larger than the thickness of the PPM-LC layer. Consequently, the processes of light diffraction will be described in a geometro optical approximation. Diffraction studies were carried out only on the main spatial harmonic of the refractive index of the diffraction structure.
PPM-LC is an anisotropic environment, therefore, the light beam inside the sample splits into two waves: ordinary and extraordinary (Figures 1 and 2).

**Figure 1.** Geometry of diffraction on MIHDS

**Figure 2.** Vector diagram

Amplitude profiles of beams $E_j^{n,m}$ for each solution of systems of equations of coupled waves (SECW) are in partial derivatives [4, 5]:

$$N_{v0}^{n,m}(r, E) \cdot \nabla E_0^{n,m}(r, E) = -iC_{1}^{n,m}(r, E)E_1^{n,m}(r, E)\eta_{n}^{n,m} \cdot (r) \exp \left( i\Delta K_{n}^{n,m} \cdot r \right), \quad (1)$$

$$N_{v1}^{n,m}(r, E) \cdot \nabla E_1^{n,m}(r, E) = -iC_{0}^{n,m}(r, E)E_0^{n,m}(r, E) \cdot \eta_{n}^{n,m}(r) \exp \left( -i\Delta K_{n}^{n,m} \cdot r \right), \quad (2)$$

where $E_j^{n,m}(r)$ – Amplitude profiles of beams, $N_{v0}^{n,m}$ – group normals, $C_{v}^{n,m}(r, E)$ – amplitude connection coefficients, $\eta_{n}^{n,m}(r)$ – normalized amplitude profile of the refractive index of the structure, $\Delta K_{n}^{n,m}$ – phase detuning vector.

The coupling coefficients included in expressions (1) and (2) are determined by [4]:

$$C_{0}^{n,m}(r, E) = \frac{1}{4} \frac{\omega}{c N_{0}^{n,m}(r, E)} e_0^{n,m}(r, E) \cdot \varepsilon(E) \cdot e_0^{n,m}(r, E), \quad (3)$$
\[ C_{i_{n,m}}(r,E) = \frac{1}{4} \epsilon^0 \epsilon_{n_{i_{n,m}}(r,E)} \cdot e_{i_{n,m}}(r,E) \cdot \hat{e}(E) \cdot e_{i_{n,m}}^*(r,E), \] (4)

where:
\[
\begin{align*}
n_{0_{i_{n,m}}}(r,E) &= \left[ n_{0_i} \epsilon_{i_{n_m}}^0 \right] / \left[ n_{0_i}^2 \sin^2 \left( \varphi_{E_{i_{n_m}}}(r,E) - \theta_{0_i}^m \right) + n_{0_i}^0 \cos^2 \left( \varphi_{E_{i_{n_m}}}(r,E) - \theta_{0_i}^m \right) \right]^{0.5}, \\
n_{1_{i_{n,m}}}(r,E) &= \left[ n_{1_i} \epsilon_{i_{n_m}}^0 \right] / \left[ n_{1_i}^2 \sin^2 \left( \varphi_{E_{i_{n_m}}}(r,E) - \theta_{1_i}^m \right) + n_{1_i}^0 \cos^2 \left( \varphi_{E_{i_{n_m}}}(r,E) - \theta_{1_i}^m \right) \right]^{0.5},
\end{align*}
\]

\( \hat{e} \) – the amplitude of the fundamental harmonic perturbation of the dielectric constant tensor, \( n_{0_i} \) and \( n_{1_i} \) – extraordinary and ordinary refractive indices of LC, \( c_i \) – the light speed in vacuum, \( \omega \) – angular frequency of light waves, \( \varphi_{E_{i_{n_m}}}(E) \) – the angle of rotation of the LC capsules, which can be found as [5]:
\[
\varphi_{E_{i_{n_m}}}(E) = \frac{1}{2} \arctg \left[ \frac{\cos(2\varphi_{0})}{e_{i_{n_m}}^2 + \sin(2\varphi_{0})} \right].
\] (5)

where \( \varphi_{0} \) – the angle between the electric field intensity vector and the capsule director at \( E = 0 \),
\[ e_{i_{n_m}}^2 = E(r) R_{K \Delta_{i_{n_m}}} \sqrt{5.7\delta_{K_{i_{n_m}}}^2 + 2.1\lambda_{a}} \] – parameter characterizing the effect of the electric field on the bipolar capsule of the LC, \( R \) – the drop radius, \( \delta_{LC} \) – capsule eccentricity, \( \lambda_{a} = R W_{a} / K_{33} \) – surface adhesion parameter, \( W_{a} \) – azimuthal surface coupling coefficient, \( \Delta_{E} \) – dielectric anisotropy of the bipolar capsule. Critical electric field strength the photoinduced Fredericks transition, in which the rotation of the LC director occurs, is defined as [4]:
\[
E_{c} = \frac{\pi}{d} \sqrt{\frac{8\pi}{K_{33} \cdot \Delta_{E}}}.
\] (6)

where \( e_{i_{n_m}}^e = \left( n_{0_i}^e \right)^2 \) and \( e_{i_{n_m}}^o = \left( n_{1_i}^o \right)^2 \) – tensor components measured at the longitudinal and transverse orientation of the LC director. At the same time, the dielectric constant tensor of each layer depends on the orientation of the LC director in it:
\[
\hat{\epsilon}_{n_{i_{n,m}}} = (1-p) \left( e_{p}^e \cdot \hat{1} + \sum_{m=0,e} \Delta\hat{\epsilon}_{n_{i_{n,m}}}^e \right) + p \left( \hat{e}_{i_{n,m}} + \sum_{m=0,e} \Delta\hat{\epsilon}_{n_{i_{n,m}}}^o \right),
\] (7)

where \( p \) – the share of the LC in the composition, \( e_{p} = n_{p}^e \) – dielectric constant of the polymer, \( \hat{1} \) – the unit tensor. And the change in the tensor (7) is defined as the sum of the spatial harmonics of the dielectric constant:
\[
\Delta\hat{\epsilon}_{n_{i_{n,m}}}^e\bigg|_{p} = \sum_{j=0}^{H} \Delta\hat{\epsilon}_{n_{i_{n,m}}}^e\bigg|_{j} \cdot \cos(i \cdot K_{n_{i_{n,m}}} \cdot r),
\]
\[
\Delta\hat{\epsilon}_{n_{i_{n,m}}}^o\bigg|_{p} = 2n_{p} \cdot n_{i_{n,m}} \bigg|_{p} \cdot \hat{1},
\]
\[
\Delta\hat{\epsilon}_{n_{i_{n,m}}}^o\bigg|_{c} = \sum_{j=0}^{H} \Delta\hat{\epsilon}_{n_{i_{n,m}}}^o\bigg|_{j} \cdot \cos(i \cdot K_{n_{i_{n,m}}} \cdot r),
\]
where $\alpha, \phi$ – average values; $\sigma_\alpha, \sigma_\phi$ – standard deviations

$$\Delta \hat{n}^{e,0} \bigg|_{lc} = 2n_i^{e,0} \cdot n_i^{e,0} \bigg|_{lc} \cdot \hat{I},$$

$$p(\alpha) = A \exp \left[ -\frac{(\alpha - \bar{\alpha})^2}{2\sigma_\alpha^2} \right],$$

$$q(\alpha) = B \exp \left[ -\frac{(\phi - \bar{\phi})^2}{2\sigma_\phi^2} \right],$$

where $p(\alpha)$ and $q(\phi)$ – Gauss distribution functions of LC molecules in an ellipsoidal capsule [4], $C(r,E)$ – the orientation of the director of the LC, which is expressed in terms of the angle of rotation described in expression (5):

$$C(r,E) = \begin{bmatrix}
\sin \left( \frac{\pi}{2} - \varphi_\alpha^m (r,E) \right) \\
\sin(\varphi_\phi) \\
\cos \left( \frac{\pi}{2} - \varphi_\phi^m (r,E) \right)
\end{bmatrix}.$$
$$T^{n,m}_{10}(E, δK) = -\frac{b^{n,m}_{p}}{2} \int_{-1}^{1} \frac{\exp \left[ -i \cdot \frac{1}{2} (1 - q) \right]}{c \cdot s \cdot \cosh \left[ c \cdot s \cdot \frac{1}{2} (1 - q) - t \right]} \cdot \frac{\exp \left[ \frac{-i \cdot \frac{1}{2} (1 - q)}{c \cdot s \cdot \cosh \left[ c \cdot s \cdot \frac{1}{2} (1 - q) - t \right]} \right]}{2F_1} \cdot w(q) \, dq,$$

$$T^{n,m}_{11}(E, δK) = 1 - \frac{b^{n,m}_{p}}{2} A \left[ \frac{\exp \left[ i \cdot \frac{1}{2} \deltaK(1 - q) / (1 + q) \right]}{c \cdot s \cdot \cosh \left[ c \cdot s \cdot \frac{1}{2} (1 - q) - t \right]} \right] \cdot \frac{\exp \left[ \frac{-i \cdot \frac{1}{2} (1 - q)}{c \cdot s \cdot \cosh \left[ c \cdot s \cdot \frac{1}{2} (1 - q) - t \right]} \right]}{2F_1} \cdot \frac{1 - \frac{q}{c \cdot s \cdot \cosh \left[ c \cdot s \cdot \frac{1}{2} (1 - q) - t \right]} + 1 + \frac{q}{c \cdot s \cdot \cosh \left[ c \cdot s \cdot \frac{1}{2} (1 - q) - t \right]} \cdot 2w(q)}{dq},$$

where $2F_1$ is the hypergeometric Gauss function; $w(q) = \frac{\sinh (c \cdot s \cdot (1 - q) / 2) \cdot \sinh (c \cdot s \cdot (1 + q) / 2)}{\cosh (c \cdot s \cdot \cosh (c \cdot s \cdot (s - t)))}$;

$A = \left[ c \cdot s \cdot \cosh (c \cdot t) \cdot \cosh (c \cdot (s - t)) \right]^{-1}$;

$b^{n,m}_{p} = \left[ d_n \cdot C^{n,(m)}_{n,m}(E) \right] / \sqrt{v_0 \cdot v_1}$; $d_n$ - thickness of the $n$ layer; $v_0 = \cos (\theta_{y}^{m,n})$; $\theta_{y}^{m,n}$ - angles between the group of normal $N_{0}^{n,m}$ and the $y$ axis.

The $c, s, t$ parameters are found separately for each layer by approximating the normalized spatial profile of the amplitude of the first harmonic of the refractive index $n_1(\nu)$. The holographic diffraction structures (HDS) obtained when recording the MIHDS, by the function $n_1(\nu, c, s, t) = \frac{1}{c^2} \left[ c(sy - t) \right]$. The $c, s, t$ parameters determine the degree of heterogeneity, asymmetry and displacement $n_1(\nu)$.

Consequently, the mathematical basis for calculating the selective properties of MIHDS is described through transfer functions. The selective properties of MIHDS directly depend on the applied voltage, the angle of incidence and the central frequency of the reading radiation. At the same time, the dependence of the modulus of the phase detuning vector also depends on the applied voltage:

$$\Delta K^{n,m} = \Delta K^{n,m}(\theta) + \Delta K^{n,m}(\omega) + \Delta K^{n,m}(E)$$

where $\Delta K^{n,m}(\theta) = (D / B) \theta$ and $\Delta K^{n,m}(\omega) = (C - AD / B) \omega$ - reflect the deviation from the conditions of phase synchronism, coefficients $A$, $B$, $C$, $D$ defined in [4],

$$\Delta K^{n,m}(E) = \frac{\partial}{\partial c} \left[ n_0^{n,m}(E) \cdot \left( N_{0}^{n,m} \cdot y_0 \right) - n_0^{n,m}(E) \cdot \left( N_{0}^{n,m} \cdot y_0 \right) \right] + \left( K^{n,m} \cdot y_0 \right).$$

The intermediate layer thickness gives a phase foray. We assume that the refractive index of the intermediate layer is equal to the refractive index of the hologram. Then the transition matrix $A^{n,m}$ will look like [5–7]:

$$A^{n,m} = \exp \left[ -i \left( k_{n^{m,m}}^{n,m} \cdot y_0 \right) t_n \right] \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] \cdot \exp \left[ -i \cdot \frac{1}{d_n} \Delta K^{n,m} \cdot t_n \right] \cdot \left[ \begin{array}{c} 0 \\ 1 \end{array} \right].$$

The connection between the input $E_0$ and diffraction $E^{N,m}$ a field from expression (8) at the output of MIHDS:

$$E^{N,m} = T^{m} \cdot E_0,$$

Where $T^{m} = T^{N,m} \cdot A^{N-1,m} \cdot T^{N-1,m} \cdot ... \cdot A^{n,m} \cdot A^{m,m} \cdot A^{l,m} \cdot A^{1,m}$ - matrix transfer function of the entire MIHDS.

In numerical simulation, we will consider the case of interaction of only plane quasi-monochromatic light beams with a single amplitude. In this case $E_0 = \delta(\omega, \theta)$, $\int (E_0 \cdot E_0^*) d\omega d\theta = 1$. In this case, the
polarization of the incident radiation coincides with the polarization of its own extraordinary waves in each layer. Thus, the diffraction efficiency (DE) at the output of MIHDS can be determined as \[5–10\]:

\[
\eta^{m}_{i}(E, \Delta K) = \left| \frac{E^{N,m}_{i}(E, \Delta K)}{E_{i}} \right|^2.
\]  

Expression (11) served as the basis for the numerical simulation of the diffraction characteristics of the MIHDS. The MIHDS with homogeneous and inhomogeneous profiles was used for modeling. MIHDS consisted of four layers of PPM-LC with a thickness \( d_n = 45 \) microns, separated by an intermediate layer \( t_n = 180 \) microns. The following parameters were also used: \( \lambda = 633 \text{ nm} \) – radiation wavelength; \( n^0_{lc} = 1.535; n^e_{lc} = 1.680 \) – ordinary and unusual refractive index of LC; \( n_p = 1.535 \) – refractive index of the polymer \( a; \) \( \theta_b = 50 \text{ degrees} \) – The Braggs angle; \( K_{33} = 7.45 \times 10^{-2}; \) \( c = 1.167;1.584;1.995;1.995;\) \( s = 1.11;0.875;0.759;0.759; t = 0.6;0.546;0.504;0.504 \) – parameters for uniform grating profiles obtained by recording with an optimized composition for each layer, and \( s = 1.723;0.521;0.069;0.069;\) \( t = 0.237;−1.665;−2.105;−2.105; \) – parameters for inhomogeneous grating profiles obtained by recording with the same composition for each layer.

By expression (11), the dependences of the diffraction efficiency of a four-layer diffraction structure on the change in the phase mismatch and the applied electric field on different layers (Figures 3–6) at extraordinary natural waves with homogeneous and inhomogeneous profiles are obtained.

For a four-layer HDS with inhomogeneous and homogeneous profiles, under the action of an applied electric field on the third and fourth layers, the selective response transforms (Figures 3, 4). In this case, the diffraction characteristic and the type of the selective response correspond to a two-layer HDS without external influence (Figures 3, 4). It should be noted that, with homogeneous profiles, there is no distortion of the diffraction characteristics (Figure 3), the minima of the local maxima of the selectivity contour reach zero, in contrast to the diffraction of light on inhomogeneous profiles (Figure 4).

\[4st: 2.16Ec on 3.4 layer\]
\[2st without Ec\]
\[4st without Ec\]
\[4st 1.24Ec on 3.4 layer\]

**Figure 3.** Dependence of DE for a four-layer HDS with homogeneous profiles on the applied electric voltage on the 3rd and 4th layers.
In a similar way, DEs were obtained numerically for a four-layer HDS with homogeneous and inhomogeneous profiles under the action of an external electric field on the 2nd and 3rd layers. In this case, there is a transformation of the selective response corresponding to a two-layer HDS with an increased thickness of the intermediate layer (Figures 5, 6). By "turning off" the diffraction on the 2nd and 3rd diffraction layers, these layers will be like additional buffer layers with a thickness of $d_n$. Consequently, the appearance of additional local maxima of the selectivity contour is observed (Figures 5, 6) due to an increase in the phase raid.

It should also be noted that in the case of inhomogeneous of the profiles, the selective response is greatly distorted, at which the side maxima begin to overlap with each other (Figure 6), which will have a negative effect if such structures are used as spectral filters. In addition, with inhomogeneous profiles, the total DE is lower in comparison with the diffraction of light by homogeneous ones, which is due to the difference in the amplitude of the first harmonic of the refractive index for subsequent layers.

**Figure 4.** Dependence of DE for a four-layer HDS with inhomogeneous profiles on the applied electric voltage on the 3rd and 4th layers.

**Figure 5.** Dependence of the DE for a four-layer HDS with homogeneous profiles on the applied electric voltage on the 2nd and 3rd layers.
Figure 6. Dependence of DE for a four-layer HDS with inhomogeneous profiles on the applied electric voltage on the 2\textsuperscript{nd} and 3\textsuperscript{rd} layers.

4. Conclusion

Thus, a dynamic change in the diffraction characteristics, in particular the type of selective response, is possible by external action of an electric field on certain diffraction layers of MIHDS with PPM-LC. Consequently, by turning off diffraction on the selected layers with holograms, it is possible to transform the selective response to a given one.

At the same time, homogeneous grating profiles obtained by optimizing the composition for each layer make it possible to reduce distortions of the selective response, in particular, the level of minima of local maxima of the selectivity contour does not increase, and the total DE also increases. Thus, optimization of the composition for the diffraction layers will reduce cross-talk, which will improve the performance of spectral filters designed on the basis of MIHDS.

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