Solving Combinational Optimization Problems With Evolutionary Single-Pixel Imaging

Wei Huang, Jiaxiang Li, Shuming Jiao, Member, IEEE, and Zibang Zhang

Abstract—In this paper, an evolutionary single-pixel imaging (SPI) scheme is proposed to solve combinational optimization problems. SPI is a unique optical imaging technique by replacing the pixelated sensor array in a conventional camera with a single-pixel detector. SPI is conventionally employed for capturing object images or performing image processing tasks. It is a novel attempt to leverage SPI for processing other types of data in addition to images. An Ising machine model is implemented optically with SPI for solving two combinational optimization problems including number partition and graph maximum cut. The binary illumination patterns are encoded based on the spinning states of all the elements and the object image pixel values are encoded to be proportional to the pairwise weighting factors. As the mathematical inner product between the object and illumination pattern, the recorded single-pixel intensity values simulate the Hamiltonian function values. In each iteration, the feedback of single-pixel values is used to update the illumination patterns by selection, crossover and mutation. As the illumination patterns are evolving, an optimal solution of the Hamiltonian function can be finally obtained by our proposed optoelectronic Ising machine scheme.

Index Terms—Single-pixel imaging, Ising machine, evolutionary algorithm, combinational optimization, optical computing.

I. INTRODUCTION

A novel optical imaging technique, single-pixel imaging (SPI) [1], [2] has received much attention in recent years. A pixelated sensor array is usually required for a conventional camera. However, as a unique feature, the sensor only has one single pixel in single-pixel imaging (SPI), shown in Fig. 1. A set of different illumination patterns are sequentially projected onto the object image. For each illumination, the single-pixel detector will collect the total light intensity of the object scene, which can be mathematically modelled as the inner product between the object image and one illumination pattern. Finally, a single-pixel intensity sequence is recorded after many illuminations. The object image can be computationally reconstructed when the illumination patterns and the data sequence are both known. Random illumination patterns and orthogonal transform basis patterns are commonly used in SPI. The object image can be computationally reconstructed by correlation, inverse transform, compressive sensing, deep learning and other algorithms. The illumination patterns in SPI can be considered as the optical (or physical) implementation of measurement matrix in compressive sensing.

SPI is inherently designed for capturing an object image and most previous works about SPI are related to image acquisition and processing. Examples include image denoising [3], [4], image classification [5], [6], image recognition [7], [8], object tracking in an image [9], [10], [11], [12] and people counting [13]. In fact, a SPI system can also be considered as an optical machine learning system (or optical information processing system) for performing calculation tasks for other types of data, in addition to images. In this work, we employ SPI for solving combinational optimization problems based on an Ising machine model and evolutionary illumination patterns.

An Ising machine model [14], [15], [16], [17] refers to a system consisting of discrete variables that represent magnetic dipole moments of atomic “spins” that can be in one of two states (+1 or −1). The total system energy depends on the states of all the spins. Some physical phenomena such as phase transition of two-dimensional square-lattice can be simulated by an Ising machine model. Many other combinational optimization problems in biology, telecommunications, logistics, economy and social networks can also be potentially modeled by an Ising machine. An Ising machine can be implemented optically in various ways such as a network of degenerate optical parametric oscillators [14], encoding of diffractive light field [15], [16], and a mesh of cascaded Mach–Zehnder interferometer (MZIs) [17]. As far
as the author knows, the Ising machine model has been seldom combined with SPI in previous works. Compared with the optical systems above [14], [15], [16], [17], SPI has a simple and low-cost experiment setup under incoherent lighting conditions. The objective of an Ising machine system is to achieve the ground state of an Ising Hamiltonian, which is an optimization process. In a recent work, the image reconstruction of SPI is formulated as an iterative evolutionary optimization of illumination patterns [18]. In this work, an Ising machine model is implemented by SPI to solve combinational optimization problems.

It is assumed that there are totally \( N \) elements in an Ising machine model and the binary state of each element is indicated by \( \sigma_i = 1 \) or \( \sigma_i = -1 \) (\( 1 \leq i \leq N \)). There is a weighting factor between the \( i \)th element and the \( j \)th element. The system energy can be defined by the following Hamiltonian function \( H \), given by (1). It depends on both the binary states of all the elements and all the values of weighting factors. The optimal solution can be obtained by minimizing \( H \). The Hamiltonian function in the Ising machine model is equivalent to the cost function in other evolving optimization methods.

\[
H = -\sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_i \sigma_j w_{ij} \tag{1}
\]

Two typical combinational optimization problems that can be solved by an Ising machine model are number partition problem and graph maximum cut problem. In the number partition problem, a set of \( N \) numbers, e.g., \{1,2,5,6,7,9\}, are divided into two groups and the objective is that the summations of the numbers in each group are as close as possible (e.g., \{1,5,9\} and \{2,6,7\}). This task will be equivalent to an Ising model with the following settings: 1) It is indicated by \( \sigma_i = 1 \) or \( \sigma_i = -1 \) (\( 1 \leq i \leq N \)) for whether each number belongs to the first group or the second group; 2) For any two pairs of numbers (ith one and jth one), the weighting factor \( w_{ij} \) is defined as their multiplication product. When the system energy (or Hamiltonian function) is minimized, the optimal number partition arrangement will be reached.

In the graph maximum cut problem, a number of \( N \) nodes are interconnected as a network and there is a pairwise weighting \( w_{ij} \) between the \( i \)th node and \( j \)th node. The objective is to divide the nodes into two sub-networks and the total connection strengths between two sub-networks are maximized. At the same time, the total connection strength within each sub-network will be minimized. Fig. 2 shows an example of final solution of a graph maximum cut problem. There are 6 nodes, and the connection strength between two sub-networks is much higher than that within each sub-network. Similarly, it is indicated by \( \sigma_i = 1 \) or \( \sigma_i = -1 \) (\( 1 \leq i \leq N \)) in an Ising machine model for whether each node belongs to the first or second sub-network. The connection strength between nodes are equivalent to the weighting factors. An optimal solution of the Ising machine model will yield a maximum cut configuration of the network.

Both the number partition problem and graph maximum cut problem are essentially identical to an Ising machine model. For brevity, the mathematical proofs are skipped in this paper.

II. PROPOSED ISING MACHINE SCHEME WITH EVOLUTIONARY SINGLE-PIXEL IMAGING

Our proposed optoelectronic Ising machine scheme with evolutionary SPI for solving combinational mathematical optimization problems is described as follows, shown in Fig. 3. In SPI, the Hamiltonian function can be considered as an inner product between a map of weighting factors \( w_{ij} \) and a correlation map.
of binary spinning states. The former one is represented by a grayscale object image with \( N \times N \) pixels (pixel intensities proportional to weighting factor values \( w_{ij} \)) and the latter one is represented by a programmable binary illumination pattern with \( N \times N \) pixels (pixel values \( \sigma_i \sigma_j \)). The illumination pattern is generated from a corresponding one-dimensional vector \([\sigma_1 \sigma_2 \ldots \sigma_N]\) with length \( N \) representing the binary spinning states of all the elements. The recorded single-pixel light intensity value is equivalent to the opposite of the function value since it is mathematically the inner product between the object image and one illumination pattern. The computation of Hamiltonian function (or cost function) is optically implemented in this way.

In order to finally obtain one optimal solution, a SPI scheme with evolutionary illumination patterns can be leveraged. It is assumed that there are totally \( K \) illumination patterns in each iteration, where \( K \) is referred to as the population size. We first randomly generate \( K \) different initial binary state vectors \([\sigma_1 \sigma_2 \ldots \sigma_N]\) and each vector is referred to as an individual. One two-dimensional \( N \times N \) pixelated illumination pattern representing the product of each pair of \( \sigma_i \) and \( \sigma_j \) elements in one vector can be generated. The pixels in an illumination pattern are still binary and their values depend on the same or opposite signs of \( \sigma_i \) and \( \sigma_j \). Consequently, \( K \) illumination patterns can be generated from \( K \) corresponding vectors. After that, these \( K \) illumination patterns are sequentially projected onto the weight factor object image. Then \( K \) single-pixel light intensity values can be recorded by the detector and they are ranked from smallest to largest in a sequence. The state vectors (or illumination patterns) corresponding to first \( J \) largest single-pixel values in the sequence will be preserved in the next iteration and the remaining \((K-J)\) individuals will be replaced by crossover results of the \( J \) ones. Crossover refers to the generation of a new individual by swapping vector elements at randomly selected positions between two parent individuals. Moreover, a certain percentage of random mutation is introduced and the binary values of some vector elements are randomly modified (e.g., “1” becomes “−1” or “−1” becomes “1”) as well. The individuals in a population (i.e., \( K \) state vectors or illumination patterns) are gradually evolving as the number of iterations increases and the largest recorded single-pixel value in the sequence will gradually increase. A converged optimal solution (or at least near-optimal solution) representing the minimum Hamiltonian function value (or maximum possible single-pixel value) can be usually obtained, revealed by the optimal state vector, after an adequate number of iterations. In fact, the evolutionary process of illumination patterns in our proposed scheme simulates the natural evolution of biological species [18].

III. RESULTS AND DISCUSSIONS

Our proposed scheme is first verified by computer simulation. Three simulation tests are implemented for both number partition problem and graph maximum cut problem. In the number partition problem, three random number sets \{2,4,5,6,9\}, \{1,2,3,4,5,6,7\} and \{1,2,5,7,9,11,15,16,18\} are selected to generate the corresponding weight maps by multiplying any two elements as the object images, shown in Fig. 4(a),(c) and (e) respectively. It is evident that the number of elements in these three sets are 5, 7 and 9 respectively. The number of individuals is set to be \( K = 6 \) and the number of iterations is set to be 6 in the evolution of illumination patterns. Using evolutionary SPI described above, the optimal solution can be obtained iteratively, and the error curves are shown in Fig. 4(b), (d) and (f). It can
be observed that the error between the ground truth and the optimal solution already obtained is gradually decreasing as the number of iterations increases. Meanwhile, the optimal number partition results (1: first group; -1: second group) at each iteration are shown in Fig. 5(a)–(c), and the green ones indicate the correct classification results. For example, for the first number set, the result is \{2,4,5\}–\{6,9\} in the third iteration (not optimal) but it becomes \{2,5,6\}–\{4,9\} in the fifth iteration (optimal). The results above show that evolutionary SPI can obtain the optimal state vector after a few iterations for the number partition problem.

Similarly, for the graph maximum cut problem, three networks with random connection strengths are selected to generate weight images, shown in Fig. 6(a), (c) and (e) respectively. The objective is to divide all the nodes in each network into two sub-networks so that the total cut between two sub-networks is maximum. The number of individuals is set to be \(K = 6\) and the number of iterations is set to be 6 in the evolution of illumination patterns. The final node grouping results obtained by our proposed scheme are \{1,3,5\}–\{2,4,6\}, \{1,2,3,4\}–\{5,6,7,8\} and \{1,3,5,7\}–\{2,4,6,8,9\} respectively. They agree with the theoretically correct results exactly. The total cut (overall connection strength between two group of nodes) results at each iteration are shown in Fig. 6(b), (d) and (f) respectively and it is evident that they are gradually rising. The network node grouping results at each iteration are shown in Fig. 7(a)–(c). These indicate that our proposed scheme is capable of solving the graph maximum cut problem as well.

Further simulation tests are implemented by increasing the number of elements from \(N = 10\) to \(N = 400\) with intervals 10 for the number partition problem. In each simulation test, the number of iterations is recorded until the correct partition result is obtained, shown in Fig. 8. It can be seen that even if the number of elements increases to 400 and the problem is at a large scale, our proposed scheme can still successfully perform the number partition task but require more iterations.

The experimental SPI setup is shown in Fig. 9. The binary illumination patterns are projected onto the printed weighting factor pattern on a paper sheet by a Canon REALiS SX7 commercial projector. A Thorlabs 100A2 PDA (photo-diode array) is used as the single-pixel detector and its recorded data are collected by a NI-USB-6216 data acquisition card connected.
Fig. 9. Experimental SPI setup in this work.

Fig. 10. Experimental results of number partition and graph maximum cut problems: (a) Weight factor image for number partition; (b) weight factor image for graph maximum cut; (c) error value curve at each iteration for (a); (d) cut value curve at each iteration for (b); (e) optimal state vectors (number partition results) at each iteration for (a); (f) optimal state vectors (node grouping results) at each iteration for (b).

to a computer. In the experiment, one example for the number partition problem and one example for the graph maximum cut problem are tested (N = 6). The total iteration number is 7 in both two experiments. The results are shown in Fig. 10. It shall be noted that the error curve of Fig. 10(d) has a slight rise in the middle part and it is caused by the signal acquisition error of the single-pixel detector. But the proposed scheme can still give the final correct result. After 7 iterations, a converged optimal solution emerges in both two experiments.

The feasibility of our proposed scheme is verified by optical experimental results. In practical application scenarios, an optoelectronic processor for finding the solution of an Ising machine model requires a more compact system size, a better computing capability with more parameters and higher accuracy, and lower power consumption. In future works, our proposed SPI system can be further improved with the advance of various optical technologies such as using metasurface for SPI [19].

IV. CONCLUSION

In previous works, SPI is usually used for capturing object images or performing image processing tasks. In this work, it is a novel attempt to leverage SPI for processing other types of data in addition to images. We propose an optoelectronic Ising machine scheme with evolutionary SPI that is capable of solving combinational optimization problems such as number partition and graph maximum cut optically. The simulated and experimental results show that our proposed scheme can optimize the Hamiltonian function effectively and obtain the optimal Ising machine model solutions for real problems.

REFERENCES

[1] G. M. Gibson, S. D. Johnson, and M. J. Padgett, “Single-pixel imaging 12 years on: A review,” Opt. Exp., vol. 28, no. 19, pp. 28190–28208, Sep. 2020.
[2] M.-J. Sun and J.-M. Zhang, “Single-pixel imaging and its application in three-dimensional reconstruction: A brief review,” Sensors, vol. 19, no. 3, Feb. 2019, Art no. 732.
[3] S. Rizvi, J. Cao, K. Zhang, and Q. Hao, “Deringing and denoising in extremely under-sampled Fourier single pixel imaging,” Opt. Exp., vol. 28, no. 5, pp. 7360–7374, Mar. 2020.
[4] H. Wu et al., “Deep-learning denoising computational ghost imaging,” Opt. Lasers Eng., vol. 134, Nov. 2020, Art. no. 106183.
[5] S. Jiao et al., “Optical machine learning with incoherent light and a single-pixel detector,” Opt. Lett., vol. 44, no. 21, pp. 5186–5189, Nov. 2019.
[6] Z. Zhang et al., “Image-free classification of fast-moving objects using ‘learned’ structured illumination and single-pixel detection,” Opt. Exp., vol. 28, no. 9, pp. 13269–13278, Apr. 2020.
[7] T. Bu, S. Kumar, H. Zhang, I. Huang, and Y. Huang, “Single-pixel pattern recognition with coherent nonlinear optics,” Opt. Lett., vol. 45, no. 24, pp. 6771–6774, Jun. 2021.
[8] L. Bian, H. Wang, C. Zhu, and J. Zhang, “Image-free multi-character recognition,” Opt. Lett., vol. 47, no. 6, pp. 1343–1346, Mar. 2022.
[9] D. Shi et al., “Fast tracking of moving objects using single-pixel imaging,” Opt. Commun., vol. 440, pp. 155–162, Jun. 2019.
[10] S. Sun et al., “Simultaneously tracking and imaging a moving object under photon crisis,” Phys. Rev. Appl., vol. 17, no. 2, Feb. 2022, Art. no. 024050.
[11] Z. Zhang, J. Ye, Q. Deng, and J. Zhong, “Image-free real-time detection and tracking of fast moving object using a single-pixel detector,” Opt. Exp., vol. 27, no. 24, pp. 35394–35401, Nov. 2019.
[12] Q. Deng, Z. Zhang, and J. Zhong, “Image-free real-time 3-D tracking of a fast-moving object using dual-pixel detection,” Opt. Lett., vol. 45, no. 17, pp. 4734–4737, Sep. 2020.
[13] E. Hagenaars, A. Pandharipande, A. Murthy, and G. Leus, “Single-pixel thermopile infrared sensing for people counting,” IEEE Sensors J., vol. 21, no. 4, pp. 4866–4873, Feb. 2021.

[14] A. Marandi, Z. Wang, K. Takata, R. L. Byer, and Y. Yamamoto, “Network of time-multiplexed optical parametric oscillators as a coherent Ising machine,” Nature Photon., vol. 8, no. 12, pp. 937–942, Dec. 2014.

[15] D. Pierangeli, G. Marcucci, and C. Conti, “Large-scale photonic Ising machine by spatial light modulation,” Phys. Rev. Lett., vol. 122, no. 21, May 2019, Art. no. 213902.

[16] Y. Fang, J. Huang, and Z. Ruan, “Experimental observation of phase transitions in spatial photonic Ising machine,” Phys. Rev. Lett., vol. 127, no. 4, Jul. 2021, Art. no. 043902.

[17] M. Prabhu et al., “Accelerating recurrent Ising machines in photonic integrated circuits,” Optica, vol. 7, no. 5, pp. 551–558, May 2020.

[18] B. Liu, F. Wang, C. Chen, F. Dong, and D. McGloin, “Self-evolving ghost imaging,” Optica, vol. 8, no. 10, pp. 1340–1349, Oct. 2021.

[19] P. Georgi et al., “Optical secret sharing with cascaded metasurface holography,” Sci. Adv., vol. 7, no. 16, Apr. 2021, Art. no. eabf9718.

Wei Huang was born in Heze, China, in 1998. He is currently working toward the Ph.D. degree with Hunan University, Hunan, China. His research interests include single-pixel imaging, 3D reconstruction, and deflectometry.

Jiaxiang Li was born in Guangdong, China, in 1999. He is currently working toward the master’s degree with Jinan University, Guangzhou, China. His research focuses on single-pixel imaging.

Shuming Jiao (Member, IEEE) received the Ph.D. degree in electronic engineering from the City University of Hong Kong, Hong Kong, in 2016. He is currently an Assistant Researcher with Peng Cheng Laboratory, Shenzhen, China. His research interests include holographic imaging and display, single-pixel imaging, optical computing, optical security, image processing, and machine learning.

Zibang Zhang received the Ph.D. degree from Jinan University, Guangzhou, China, in 2017. He is currently an Associate Professor with the Department of Optoelectronic Engineering, Jinan University. His research interests include computational imaging, specifically, single-pixel imaging, fringe projection profilometry, and lessless microscopy.