TETRAQUARKS AND PENTAQUARKS IN STRING MODELS

Yu. M. Shabelski and M. G. Ryskin

Petersburg Nuclear Physics Institute, Gatchina, St.Petersburg, Russia

E-mail: shabelsk@thd.pnpi.spb.ru
E-mail: ryskin@thd.pnpi.spb.ru

ABSTRACT
We consider the production and decay of multiquark systems in the framework of string models where the hadron structure is determined by valence quarks together with string junctions. We show that the low mass multiquark resonances can be very narrow.

PACS. 25.75.Dw Particle and resonance production
1 Structure of multiquark systems

In the string models baryons are considered as configurations consisting of three strings (related to three valence quarks) connected at the point called ”string junction” (SJ) \cite{1-3}. The string junction has a nonperturbative origin in QCD. Many phenomenological results (some of them will be discussed below) were obtained in this approach 25 years ago \cite{2}, \cite{3}, \cite{6}-\cite{8}.

In QCD hadrons are composite bound state configurations built up from the quark $\psi_i(x), i = 1, \ldots N_c$ and gluon $G_a^\mu(x), a = 1, \ldots, N_c^2 - 1$ fields. In the string models the meson wave function has the form of ”open string” \cite{1,3}, as it is shown in Fig. 1a.

![Figure 1: Composite structure of a meson (a) and a baryon (b) and (c) in string models. Quarks are shown by open points and antiquarks by crossed points.](image)

The meson wave function (here and below we present only its colour structure) reads as

$$M = \bar{\psi}^i(x_1)\Phi^i(x_1, x_2)\psi^i(x_2).$$  \hspace{1cm} (1)

$$\Phi^i(x_1, x_2) = \left[ T \exp \left( g \int_{P(x_1, x_2)} A^\mu(z)dz^\mu \right) \right]_i^i,$$  \hspace{1cm} (2)

where the field $A^\mu$ is the matrix in colour space:

$$A^\mu = \sum_a t_a A^a_\mu,$$  \hspace{1cm} (3)

In the last equation $P(x_1, x_2)$ represents a path from $x_1$ to $x_2$ which looks like an open string with ends at $x_1$ and $x_2$. 

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For the baryons there exist two possibilities, "triangle", or $\Delta$ connection shown in Fig. 1b and "star", or $\text{Y}$ connection shown in Fig. 1c. The last variant is considered as the most interesting. Here a baryon is considered as configurations consisting of three strings attached to three valence quarks and connected in a point called the "string junction" (SJ) \[1, 3\]. Such a picture is confirmed by lattice calculations \[9\]. The correspondent wave function can be written as

$$ B = \psi_i(x_1)\psi_j(x_2)\psi_k(x_3)J^{ijk}, \quad (4) $$

$$ J^{ijk} = \Phi_i(x_1, x)\Phi_j(x_2, x)\Phi_k(x_3, x)\epsilon^{ijk}, \quad (5) $$

Such baryon wave function can be defined as a "star" or "$\text{Y}$" shape and it is preferable \[1, 3\] in comparison with "triangle" ("ring") or "$\Delta$" shape\(^1\).

The wave function of an antibaryon has the form

$$ \bar{B} = \bar{\psi}_i(x_1)\bar{\psi}_j(x_2)\bar{\psi}_k(x_3)J_{ijk}. \quad (6) $$

The operators $J^{ijk}$ and $J_{ijk}$ differ by the position of colour indices that gives possibility of annihilation of $B\bar{B}$ pair into mesons.

The presented picture leads to several phenomenological predictions \[10]-[12\]. In particular, there exist the rooms for exotic states, such as glueboll, or gluonium ("closed string"), Fig. 2a, \[3, 13\].

Glueboll = $\text{Tr} \left[ T \exp \left( g \int_{P(\text{closed})} A_\mu(z)dz^\mu \right) \right]. \quad (7)$

The multiquark bound states, such as 4-quark meson, Fig. 2b, pentaquark, Fig. 2c, etc. also can exist. Without specified model it is impossible to say about the sign of the correspondent binding energy, i.e. are they the bound states or not. However we can expect that the part of a particle momentum carried out by gluons in the case of multiquark states should be larger than for usual particles, Figs. 1a, 1c due to the larger number of string junctions.

Similarly to Eqs. (4), (5), the meson $M_4$ (tetraquark) wave function can be written as

$$ M_4 = \psi_i(x_1)\psi_j(x_2)J^{ijm} \times \bar{\psi}^k(x_3)\bar{\psi}^l(x_4)J^{kmn} \times \Phi_m(x_0^{(1)}, x_0^{(2)}), \quad (8) $$

and the pentaquark wave function

$$ B_5 = \psi_i(x_1)\psi_j(x_2)J^{ijn} \times \psi_l(x_4)\psi_m(x_5)J^{lnq} \times \bar{\psi}^k(x_3)\Phi^n(x_3, x_0^{(2)})\epsilon_{nqk}. \quad (9)$$

\(^1\)Strictly speaking we can not build up the "$\Delta$" configuration with the help of a string Eq. (2). In this ("$\Delta$") case the colour flux produced by the quark is divided between two strings. That is we need the fractional colour factor (like $g/2$) in the power of the exponent in Eq. (2).
specified form of colour wave functions of \( M_4 \) and \( B_5 \) presented above should result in weak mixing of them with \( qq\bar{q} \) and \( qqqq\bar{q} \) states produced radiatively from usual mesons and baryons.

2 Tetraquark and pentaquark production

The probability of SJ pair production in small space-time region is rather small. In the case of pentaquark production an estimation can be taken from the ratio of \( \pi^-p \rightarrow \bar{p}d \) reaction cross section to the total inelastic one. Experimentally [14] it is of the order of \( 10^{-3} - 10^{-4} \) at \( \sqrt{s} = 2.9 - 3.2 \) GeV. Note that these energies are rather close to the \( \bar{p}d \) threshold. The momenta of the final nucleons are about 200 - 300 MeV. So the expected suppression (~ 0.01) due to the low probability to form the deuteron is not too strong and it looks reasonable to assume that an additional factor of (0.01 – 0.1) is caused by the SJ pair production.

One way to avoid the smallness is to use the initially prepared SJ. So we can produce a tetraquark in \( \bar{p}p \) collisions and a pentaquark in \( \bar{p}d \) collisions at rather small energy. In the first case we can expect the annihilation of one \( q\bar{q} \) pair that corresponds to the planar Dual Topological Unitarization diagram which is the leading in \( 1/N_c \) expansion. In the last case we expect the annihilation of two \( q\bar{q} \) pairs and the additional smallness about \( 10^{-2} \) which is connected with probability to find a proton and a neutron in the
deuteron at small distances.

In the case of, say, pentaquark production in $\pi^-p$ collisions the ratio of signal to background should be worse in comparison with $\bar{p}d$ interactions.

## 3 Tetraquark and pentaquark decay

Let us start from the case when the mass of a meson $M_4$ (tetraquark), Fig. 2b, is large enough. In the considered the simplest mode of a meson $M_4$ decay is the breaking of the string between the points $x_0^{(1)}$ and $x_0^{(2)}$ with production of a light $q\bar{q}$ pair that should result in decay of $M_4$ into $B\bar{B}$ state. The decay into two mesons should be lesser preferable, that is supported also by the triality analysis [6]. Similarly a baryon $B_5$ (pentaquark), Fig. 2c, with high enough mass should decay preferably into $BB\bar{B}$ state via breaking of two strings and production of two $q\bar{q}$ pairs.

The situation becomes more complicate in the case of low mass multiquark states [17, 18]. Due to the completeness condition we can consider only real hadrons in the intermediate states. So we can imagine that now tetraquark decays firstly into virtual $B\bar{B}$ pair with their subsequent annihilation into mesons. Similarly, in the case of pentaquark decay the intermediate $BB\bar{B}$ system will results in the system of a baryon together with several mesons. The important point is the suppression factor coming from every virtual baryon with 4-momentum $k$. In comparison with the normal width of $N^*$ resonances ($\Gamma \sim 150 - 200$ MeV) here we have the suppression due to the loop which provides the annihilation of the virtual $B\bar{B}$ pair into mesons. Besides the numerically small factor $1/4\pi$ the loop contains the baryons far off the mass-shell. This leads to the suppression of the order of $(2m_\pi)^2/(m_B^2 - k^2) \sim 1/10$. That is we expect the supression of the decay amplitude $\sim 1/100$, i.e. we expect the width of about $10^{-4} \cdot 200$ MeV $= 20$ KeV or even smaller, since the $B\bar{B}$ pair mainly annihilates into a rather high multiplicity pion states and there should be an additional suppression due a limited phase space available for the final pions. Small width of pentaquarks was also obtained [19, 20] in the framework of QCD sum rules approach.

There exist the possibility to search tetraquarks and pentaquarks in the decay modes with strange particles, for example $K^0_s + \Lambda + X$, where $s\bar{s}$ pair can be produced in the process of string breaking. The constituent mass of a strange quark is not much larger than that of a light $(u,d)$ quarks. The correspondent decay mode should be suppressed by the factor $10^{-1}$, but the background should be suppressed more significantly.

Moreover, since the events with the $K^0_s$ or $\Lambda$ decay are easy to select experimentally, the decay modes with strange particles or/and the reactions of a strange pentaquark (the one which contains strange quark) production look as an attractive way to search

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The probability of two SJ annihilation (or production) is assumed to be small [6, 7, 15]. It was argue in Ref. [16] that the smallness of SJ annihilation may be caused by the small size ($\sim 0.2 - 0.4$ fm) of the “junction”.

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for such a multiquark states.

The small width of the pentaquark (tetraquark) can explain the problems of their search, see, e.g. reviews [21, 22]. Indeed, it is hard to find a very narrow state performing the usual partial wave analysis. On the other hand the cross section of the inclusive pentaquark (or tetraquark) production is expected to be very small in the processes where there is no three SJ in the initial state and one needs to create two new SJ.

Therefore, it looks more perspective to search for the pentaquark in the $\bar{p}d$ annihilation.

We conclude that the existance of tetraquarks and pentaquarks is natural in the string models. The production probability can be enhanced in special combinations of a beam and a target. The light multiquark systems can have rather large mean life time (in the scale of strong interactions) that can produce an additional problems for their experimental search.

We are grateful to A. K. Likhoded, L. N. Lipatov and V. Yu. Petrov for useful discussions. This paper was supported by DFG grant GZ: 436 RUS 113/771/1-2 and, in part, by grants RSGSS-1124.2003.2 and PDD (CP) PST.CLG980287.
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