Image Denoising Based on Wavelet Transform and Improved Threshold Function

Jin-ming FU¹ and Yuan-fan Li²*

¹College of Science, Huazhong Agricultural University, 430070, Wuhan, China
²School of Foreign Languages, Wuhan Textile University, 430073, Wuhan, China

*Corresponding author

Keywords: Wavelet transform, Image denoising, Threshold function.

Abstract. This paper studied the wavelet threshold denoising method for the image, it studied the evaluation of denoising performance standards, the principle of wavelet threshold denoising, threshold function and threshold structure, and it constructed a new threshold function, at last the validity of this method is proved by numerical experiments.

Introduction

In the production, preservation, compression and transmission process, digital image often becomes the noisy image by the internal and external factors. If \( f(x,y) \) is the image signal, \( u(x,y) \) is the noise signal, \( h(x,y) \) is the noisy image signal, then the mathematical model of the noisy image can be simply represented as

\[
h(x,y) = f(x,y) + u(x,y),
\]

this paper mainly studies the zero mean Gauss white noise.

The Denoising Performance Evaluation Criteria

Commonly, there are two factors of the image denoising performance evaluation criteria, one is signal-to-noise ratio gain, the other is mean square error gain. If \( h(n) \) is the the original signal, \( \hat{h}(n) \) is the estimated signal after denoising, \( \eta \) is the signal to noise ratio gain, \( \mu \) is the mean square error gain, then

\[
\eta = \frac{SNR_h}{SNR_a}
\]

\[
\mu = \frac{1}{N} \sum_{n=1}^{N} [h(n) - \hat{h}(n)]^2
\]

\[
\frac{1}{N} \sum_{n=1}^{N} h^2(n)
\]

among them, \( SNR_h \) is the signal-noise- ratio of the denoised image, \( SNR_a \) is represented by the signal- to-noise ratio after denoising. After the image has been denoised by some method, the value of \( \eta \) is larger, the denoising effect is better, on the other hand, the value of \( \mu \) is smaller, the denoising effect is better.

Wavelet Transform Threshold Denoising Principle

Firstly, it can obtain one low frequency component and three high frequency components after wavelet transform for the image; and then select the appropriate threshold, the appropriate choice of wavelet coefficients in high frequency components, finally using wavelet coefficients of choice after two-dimensional wavelet transform reconstruction image, to complete the image denoising.
process. Of course, according to the actual demand of low frequency component of each image after wavelet decomposition and discrete wavelet transform, decomposition to the appropriate scale until now (Figure 1), and then select the appropriate threshold, the wavelet coefficients of the noise signal on different scales of proper choice, Finally, the denoised image is reconstructed by using the wavelet coefficients.

![Figure 1. Wavelet decomposition for the image of the three scale.](image)

**Wavelet Transform Threshold Denoising Steps**

- Discrete wavelet transform is performed on discrete images at different scales.
- Threshold value method is used to process the wavelet coefficients at different scales to get the estimated coefficients.
- Using two-dimensional wavelet transform from high to low scale in order to reconstruct the image, finally get the denoised image.

**Threshold Function Selection**

This paper uses the following threshold function:

\[
\hat{W} = \begin{cases} 
0, & |W| \leq \lambda_1 \\
\frac{\lambda_2 e^{(\lambda_2 - \lambda_1)}}{\lambda_2 - \lambda_1} (W - \lambda_1)e^{(W + \lambda_1)}, & \lambda_1 < W < \lambda_2 \\
\frac{\lambda_2 e^{(\lambda_2 - \lambda_1)}}{\lambda_2 - \lambda_1} (W + \lambda_1)e^{(W - \lambda_1)}, & -\lambda_2 < W < -\lambda_1 \\
W, & |W| \geq \lambda_2 
\end{cases}
\]  

(4)

**Threshold Selection**

In general, the image contains noise will increase with the wavelet decomposition scale decreases gradually, and the same scale of different frequency band noise components are not the same, based on the feature of different scales and different bands if you are using the same threshold, it will be completely or not, denoising will filter out some useful signals are either, resulting in "off" phenomenon, in order to solve this problem, can be considered in the scale and frequency of the same decomposition scale respectively under different thresholds of different decomposition. This paper uses the following threshold definition:

\[
\lambda_j = \begin{cases} 
\lambda_j, & HH \\
\lambda_j / \sqrt{2}, & HL, LH \\
\lambda_j / 2, & LL 
\end{cases}
\]  

(5)

Among them, $j$ is the scale of the decomposition, and

\[
\lambda = \sigma_n \sqrt{2 \log(N)}.
\]  

(6)
Numerical Experiments and Results Analysis

The use of MATLAB two group containing Gauss white noise image denoising in the numerical experiments, the first set of experiments on images containing noise (noise variance 0.04) respectively with the median filter and wavelet threshold denoising (scale 2) four kinds of methods for denoising experiments, based on the obtained experimental data were analyzed. It can be found that the filtering effect of wavelet threshold denoising is better than median (denoising effect is shown in Figure II, the experimental data are shown in Table I). The second set of experiments is to contain a pair of image denoising methods using wavelet threshold noise variance is 0.04 (the Gauss decomposition scale 2) denoising experiments, in the selection of threshold function, respectively using threshold function soft threshold function and hard threshold function, and the comparative experiment by comparison you can find the threshold function and threshold expressions can get better denoising effect (denoising effect is shown in Figure III, the experimental data are shown in Table II).

Table 1. Comparison of denoising results of two different.

| Filtering Method              | Signal to Noise Ratio Gain ($\eta$) | Mean Square Error Gain ($\mu$) |
|------------------------------|------------------------------------|-------------------------------|
| Median Filtering             | 0.7236                             | 0.1643                        |
| Wavelet Threshold Denoising  | 0.8456                             | 0.0985                        |

Table 2. Comparison of different threshold function and threshold denoising effect in wavelet threshold denoising.

| Selection Of Threshold Function | Signal To Noise Ratio Gain ($\eta$) | Mean Square Error |
|---------------------------------|------------------------------------|-------------------|
| Soft Threshold                  | 0.8625                             | 0.0924            |
| Hard Threshold                  | 0.8448                             | 0.0920            |
| Threshold Of This Paper         | 0.8957                             | 0.0892            |

Figure 2. Different filtering method denoising effect.
Summary

This paper studies the wavelet threshold denoising method, constructs a new threshold function, and improves the setting method of the threshold.

References

[1] J. Morlet. Wave Propagation and Sampling Theory and Complex Wave [J]. Geophysics, 1982, 47(2): 222-236.

[2] S. Mallat. A Theory of Multiresolution Signal Decomposition: The Wavelet Representation [J]. IEEE Trans. PAMI, 1989, 11: 674-693.

[3] I. Daubechies. Ten Lectures on Wavelet [C]. CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM, Philadelphia, PA, 1992

[4] Kass M., Witkin A., Terzopoulos D. Snakes: Active Contour Models [A]. In: Proceedings of the 1st International Conference on Computer Vision [C]. London: IEEE Computer Society Press, 1987: 259-268.

[5] Cohen L.D.. On active contour models and balloons [A]. In: Graphics and Image Processing: Image Understanding [C]. Computer Vision, 1991, 53(2): 211-218.

[6] Xu C., Prince J.L., Generalized gradient vector flow external forces for active contour [J]. Signal Process, 1998, 71: 131-139.

[7] Siddiqui K., Lauriere Y.B., Tannenbaum A .et al. Area and length minimizing flows for shape segmentation [J]. IEEE Trans Image Processing, 1998, 7: 433-443.
[8] Kimmel R. Fast edge integration [A]. In: Osher S. and Paragios N. Geometric Level Set Methods in Imaging, Vision and Graphics [M]. Springer Verlag, 2003, 59-75.

[9] D. Mumford, J. Shah. Optimal approximations by piecewise smooth functions and variational problems [J]. Communications on Pure and Applied Mathematics, 1989, 42: 577-685.

[10] Chan T., Vese L. Active contours without edges [J]. IEEE Transactions on Image Processing, 2001, 10(2): 266—277.