Properties of an equilibrium hadron gas subjected to the adiabatic longitudinal expansion *

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Abstract

We consider an ideal gas of massive hadrons in thermal and chemical equilibrium. The gas expands longitudinally in an adiabatic way. This evolution for a baryonless gas reduces to a hydrodynamic expansion. Cooling process is parametrized by the sound velocity. The sound velocity is temperature dependent and is strongly influenced by hadron mass spectrum.

*Work partially supported by the Polish Committee for Scientific Research under contract KBN-200579101
1 Introduction

In our last paper [1], there have been presented results for $J/\Psi$ suppression in an equilibrium hadron gas. The gas expands longitudinally according to the Bjorken pattern [2] and its initial conditions have been estimated from the data [3,4]. Since our purpose in Ref.1. was to study $J/\Psi$ suppression patterns, we left aside more detailed description of properties of the hadron gas subjected to the longitudinal cooling. Now, we would like to discuss this subject more carefully.

In the case of hadron-hadron collisions we shall assume an existence of a "central region” in the rapidity variable [2]. We are leaving aside here a problem of an appearance of a quark-gluon plasma phase. We begin our analysis with the formation of an equilibrium hadron gas, subjected to the longitudinal expansion. This gas consists of different species of hadrons and it has a non-vanishing baryon-number. This fact corresponds to present experimental conditions. A neutral central region is expected for higher then CERN collision energies of heavy ions.

2 Equation of state

An equation of state is given by an assumption of a thermal and chemical equilibrium of an ideal hadronic gas. Particle ratios are given by the temperature and chemical potentials related to conserved quantum numbers – strangeness and baryon-number. In our model we take into account all species of hadrons up to $\Omega^-$ baryon.

For an ideal hadron gas in thermal and chemical equilibrium, which consists of $l$ species of particles, energy density $\epsilon$, baryon-number density $n_B$, strangeness density $n_S$ and entropy density $s$ read (\(\hbar = c = 1\) always)

$$\epsilon = \frac{1}{2\pi^2} \sum_{i=1}^{l} (2s_i + 1) \int_0^\infty \frac{dpp^2 E_i}{\exp \left\{ \frac{E_i - \mu_i}{T} \right\} + g_i}, \quad (1a)$$

$$n_B = \frac{1}{2\pi^2} \sum_{i=1}^{l} (2s_i + 1) \int_0^\infty \frac{dpp^2 B_i}{\exp \left\{ \frac{E_i - \mu_i}{T} \right\} + g_i}, \quad (1b)$$

$$n_S = \frac{1}{2\pi^2} \sum_{i=1}^{l} (2s_i + 1) \int_0^\infty \frac{dpp^2 S_i}{\exp \left\{ \frac{E_i - \mu_i}{T} \right\} + g_i}, \quad (1c)$$

$$s = \frac{1}{6\pi^2 T^2} \sum_{i=1}^{l} (2s_i + 1) \int_0^\infty \frac{dpp^4 (E_i - \mu_i) \exp \left\{ \frac{E_i - \mu_i}{T} \right\}}{E_i \left( \exp \left\{ \frac{E_i - \mu_i}{T} \right\} + g_i \right)^2}, \quad (1d)$$
where $E_i = (m_i^2 + p^2)^{1/2}$ and $m_i$, $B_i$, $S_i$, $\mu_i$, $s_i$ and $g_i$ are the mass, baryon-number, strangeness, chemical potential, spin and a statistical factor of specie $i$ respectively (we treat an antiparticle as a different specie).

And $\mu_i = B_i \mu_B + S_i \mu_S$, where $\mu_B$ and $\mu_S$ are overall baryon-number and strangeness chemical potentials respectively.

These equations can be supplemented by a similar equation related to the electric charge density. A chemical potential $\mu_Q$ would be introduced due to the electric charge conservation. However, it is easy to see that for the ratio $A/Z = 2$ this chemical potential is always equal to zero. So we can omit an equation for the electric charge density at present experiments.

### 3 Time evolution

In general, all densities of the left sides of eqs.1a-d are functions of time during the cooling of the hadron gas. This means that gas parameters such as temperature and chemical potentials should be functions of time. We would like to obtain these functions explicite. This can be done by solving numerically the system of equations for entropy density $s$, baryon-number density $n_B$ and strangeness density $n_S$, with $s$, $n_B$ and $n_S$ given as time dependent quantities. Strangeness density is equal to zero during all the evolution process, but for other quantities some dynamical assumption is needed. In the following we shall assume a longitudinal expansion and we shall neglect transversal degrees of freedom.

From the baryon-number conservation we have then

$$n_B(t) = \frac{n_B^0 t_0}{t}, \quad (2a)$$

From the Bjorken model with vanishing baryon-number density we have the following solution for the longitudinal expansion [1,5]

$$s(t) = \frac{s_0 t_0}{t} \quad (2b)$$

where $s_0$ and $n_B^0$ are initial densities of the entropy and the baryon-number respectively. This gives an entropy conservation during the expansion stage. We shall assume an adiabatic cooling process, so eq.2a will be taken for granted also with nonvanishing baryon-number density. This means a deviation from a simple hydrodynamical model.
To solve (1b,c,d) with \( s \) and \( n_B \) given by (2) and \( n_S = 0 \), we need to know initial values \( s_0 \) and \( n^0_B \). To estimate initial baryon-number density \( n^0_B \) we use experimental results of [3]. These results are for S-S collisions, but since there are no data on baryon multiplicities for heavier nuclei we have to evaluate them in some way.

We assume that the baryon multiplicity per unit rapidity in the CRR is proportional to the number of participating nucleons. For a sulphur-sulphur collision we have \( dN_B/dy \cong 6 \) [3] and 64 participating nucleons. For an O-U collision we can roughly estimate the number of participating nucleons at \( 16 + 58 = 74 \).

The second factor of the sum has been obtained by the following assumption: since an oxygen nucleon is much smaller than an uranium one, we can approximate the part of the uranium, through which the oxygen passes, by the cylinder of the volume equal to \( \pi R^2_O \cdot 2R_U \). The same procedure can be applied to the S-U case. Here we obtain \( 32 + 93 = 125 \) participants.

Therefore, we have \( dN_B/dy \cong 7 \) and \( dN_B/dy \cong 11.7 \) for O-U and S-U collisions respectively. Having taken the initial volume in the CRR equal to \( \pi R^2_A \cdot 1 \text{ fm} \), we arrive at \( n^0_B \cong 0.25 \text{ fm}^{-3} \) for both cases.

To find \( s_0 \), first we have to solve (1a,b,c) with respect to \( T, \mu_S \) and \( \mu_B \), where we put \( \epsilon = \epsilon_0 \) and so on. For \( \epsilon_0 \) we have taken estimates given in [4]. As a result, we have obtained \( T_0 \cong 212 \text{ MeV}, \mu^0_S \cong 34.6 \text{ MeV} \) and \( \mu^0_B \cong 133 \text{ MeV} \) for the O-U collision \( (\epsilon_0 = 2.5 \text{ GeV/fm}^3) \) and \( T_0 \cong 209.1 \text{ MeV}, \mu^0_S \cong 36.7 \text{ MeV} \) and \( \mu^0_B \cong 143.3 \text{ MeV} \) for the S-U collision \( (\epsilon_0 = 2.3 \text{ GeV/fm}^3) \). Then, from (1d) we have \( s_0 \cong 13.68 \text{ fm}^{-3} \) for O-U and \( s_0 \cong 12.74 \text{ fm}^{-3} \) for S-U.

Now, having put (2) and \( n_S = 0 \) into (1b,c,d), we can solve them numerically to obtain \( T, \mu_S \) and \( \mu_B \) as functions of time.

4 Results

Our results are presented in fig.1, 2, where solid, long-dashed and dashed lines mean the temperature, the strangeness chemical potential and the baryon chemical potential respectively. Fig.1 shows results for O-U collision initial conditions and fig.2 for S-U ones. The time scale is chosen in a way which enables the temperature to reach the freez-out at 140 MeV (figs.1a and 2) or 100 MeV (fig.1b). This corresponds to the freez-out time equal to \( t_{f.o.} \cong 10.4 \text{ fm} \) or \( t_{f.o.} \cong 58.6 \text{ fm} \) respectively. The most interesting feature of our results is the behaviour of the temperature. For figs.1a,
1b, and 2 the following temperature approximations hold respectively:

\[ T(t) \approx 212.4 \cdot \left( \frac{1}{t} \right)^{0.178} \]  
\[ T(t) \approx 217.2 \cdot \left( \frac{1}{t} \right)^{0.180} \]  
\[ T(t) \approx 209.7 \cdot \left( \frac{1}{t} \right)^{0.179} \]

We can see that all above expressions have the form known from the solution for the longitudinal expansion of a baryonless gas with the sound velocity constant, namely [5]

\[ T(t) = T_0 \cdot \left( \frac{1}{t} \right)^{c_s^2}, \]  
where \( c_s \) is the sound velocity and we put the initial time \( t_0 \) equal to 1 fm.

We have checked that for \( n_B = 0 \) results for the temperature function are very similar to those in eq.3. This is shown in figs.3 and 4. Fig.3 has the same initial conditions as fig.1b and fig.4 the same as fig.2. The following approximations of the temperature function hold

\[ T(t) = 215.2 \cdot \left( \frac{1}{t} \right)^{0.180} \]  
\[ T(t) = 208.9 \cdot \left( \frac{1}{t} \right)^{0.172} \]

We can see that the power in eqs.3 and 4 is around 0.18. Therefore a question arises: is this value connected in any way with the sound velocity of the hadron gas? For the baryonless case the answer is positive. We have checked this computing straightforward

\[ c_s^2 = s/(T \frac{\partial s}{\partial T}) \]

which is the value of the sound velocity squared for a baryonless gas [5]. The results are presented in fig.5. For comparison, the square of the sound velocity of a pion gas is also depicted (dashed lines). We can see that in the range of the temperature 200-140 MeV the square of the speed of sound equals 0.17-0.18 indeed. We have also found a region of the temperature where the sound velocity decreases as the temperature increases.

Nevertheless, the condition for the stability of the expansion [6], \( d/dT(s/s_c/T) > 0 \) is still valid because \( s_c/T \) is an increasing function of temperature in the whole temperature region. This function is presented in fig.6.
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Figure Captions

Fig.1a. Dependence of the temperature (solid line), the strangeness chemical potential (long-dashed line) and the baryon chemical potential (dashed line) on time. Initial conditions are chosen for O-U collision with freeze-out at 140MeV.

Fig.1b. Same as fig.1a but for freeze-out at 100 MeV.

Fig.2. Same as fig.1a but for S-U collision.

Fig.3. Dependence of the temperature on time for hadron gas with non-vanishing (solid line) and vanishing (dashed line) baryon number. Initial conditions are chosen for O-U collision with freeze-out at 100MeV.

Fig.4. Dependence of the temperature on time for hadron gas with non-vanishing (solid line) and vanishing (dashed line) baryon number. Initial conditions are chosen for S-U collision with freeze-out at 140MeV.

Fig.5. Baryonless gas: dependence of the sound velocity squared on time. Solid line: all hadrons up to Ω− resonance. Dashed line: pion gas.

Fig.6. Stability factor $s c_s / T$ for a baryonless hadronic gas.
FIG. 1a
FIG. 1b
FIG. 3
FIG. 4
FIG. 5

square of sound velocity \([c^2]\)

temperature \([\text{MeV}]\)
FIG. 6
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