Optically Thick Outflows of Supercritical Accretion Discs: Radiative Diffusion Approach

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Abstract

Highly supercritical accretion discs are probable sources of dense optically thick axisymmetric winds. We introduce a new approach based on diffusion approximation radiative transfer in a funnel geometry and obtain an analytical solution for the energy density distribution inside the wind assuming that all the mass, momentum and energy are injected well inside the spherization radius. This allows to derive the spectrum of emergent emission for various inclination angles. We show that self-irradiation effects play an important role altering the temperature of the outcoming radiation by about 20\% and the apparent X-ray luminosity by a factor of 2 – 3. The model has been successfully applied to two ULXs. The basic properties of the high ionization HII-regions found around some ULXs are also easily reproduced in our assumptions.

Key words: Physical Data and Processes: diffusion, accretion, accretion discs
X-rays: individual (SS433, ULXs)

1. Introduction

Processes of gas accretion onto compact objects are studied since 60s when first X-ray sources were discovered (Frank et al. 2002). Among several works describing the details of disc accretion in binary systems Shakura & Sunyaev (1973) was the most successful. Their “standard disc” is still widely used to describe the thermal component of X-ray spectra of X-ray binaries. Standard disc model was worked out in several considerations such as high optical thickness, low geometrical thickness of the disc, etc. Among these assumptions one was that the power released in the accretion process does not exceed the Eddington luminosity. This
case is usually called subcritical accretion.

However, considering various phenomena such as growth of supermassive black holes and nova outbursts leads to the problem of supercritical accretion. It has been extensively investigated since 1980s when Abramowicz proposed it as a power source for active galactic nuclei (Abramowicz et al. 1980). Presently super-Eddington accretion is often applied to explain the observational properties of Ultraluminous X-ray Sources (ULXs, see Roberts (2007) for observational review) that are believed to be high-mass X-ray binaries with black holes accreting on thermal time scale.

The first, outflow-dominated model of supercritical accretion, has been developed by Shakura and Sunyaev in the paper mentioned above. Authors assumed that all the inflowing gas above the critical accretion rate is being ejected from the disc in the form of a wind. Another version of supercritical regime is based on the relaxation of the locality condition in “standard model” which leads to advective slim discs (Abramowicz et al. 1980) or Polish doughnuts (Abramowicz et al. 1978; Kozłowski et al. 1978; Abramowicz 2004). In reality, both processes work simultaneously.

A comprehensive model taking into account both for advection and ejection was recently developed by Poutanen et al. (2007). Authors consider the structure of the disc in radial direction and estimate three characteristic temperatures relevant for the outcoming radiation (inner disc temperature, temperature at spherization radius and effective temperature of the wind photosphere) but do not study the processes of radiation transfer in the wind and do not calculate the outcoming spectra.

Obviously, at high accretion rates the observational properties of the accretion flow are governed mostly by radiative transfer in the outflowing wind. The observational appearance of the pseudo-photosphere of the optically-thick wind of a supercritical accretion disc was considered by Nishiyama et al. (2007). The authors however assumed the optically-thick part of the wind fully adiabatic not taking into account radiative energy transfer that is expected to be important in the flow (see section 2.1).

Currently, we have at least one example of a persistent supercritical accretor in our Galaxy – the SS433 system, where most of accreting gas is being lost in a wind. Numerical simulations of this system partially support the outflow-dominated scenario. However, Okuda (2002) states that his thorough 2D simulation fails to reproduce the outflow rate and jet collimation in SS433. In Ohsuga et al. (2005) a supercritical accretor accreting at $10^3$ Eddington rate appears a bright (about $10^{38}$ erg s$^{-1}$) hard X-ray source if viewed edge-on. Ohsuga et al. (2005) and Heinzeller et al. (2007) calculate the structure and emergent spectra considering only the inner parts of the flow ($R \leq 500 R_G$) not taking into account that the region considered is coated by accreting and outflowing matter both being optically thick. The outcoming spectra will strikingly differ from those calculated by Heinzeller et al. (2007).

Optical and radio (Blundell et al. 2001) observations allow to measure the mass ejection
rate in the relatively slow \((1000 - 2000 \text{km s}^{-1})\) accretion disc wind seen in optical emission and absorption lines (Fabrika 2004) as well as the mass loss rate in the collimated mildly-relativistic jets launched along the disc axis. Infrared excess was used by Shklovskii (1981) and Van den Heuvel (1981) to estimate the mass ejection rate from SS433. No direct mass transfer rate measurements were ever made though it is usually supposed that it could not be much higher than the mass ejection rate \(\dot{M}_{\text{ej}} \sim 10^{-4} \text{M}_\odot \text{yr}^{-1}\) (Fabrika 2004).

The details of the processes in the supercritical disc itself are unclear mainly due to the strong absorption in the wind. In fact even the photosphere of the wind is difficult to study due to high interstellar absorption (about 8\(^m\)) making it impossible with the contemporary instrumentation to trace the Spectral Energy Distribution (SED) in the far ultraviolet (UV) spectral range were most of the radiation is expected to be emitted. The only and yet crude estimate of the blackbody temperature of SS433 was made by Dolan et al. (1997) and probably corresponds to the photosphere of the wind. Their measurements are consistent with a \((2 - 7) \times 10^4 \text{K}\) blackbody source.

Both observations of SS433 (Fabrika 2004) and numerical simulations (Okuda 2002; Ohsuga et al. 2005) support the conception that is traditionally called “supercritical funnel”. It assumes that nearly all the matter accreted is being ejected in a form of a slow, roughly virial at spherization radius, dense wind. Due to centrifugal barrier two conical avoidance sectors with half-opening angles of \(\theta_f \sim 30^\circ\) (Eggum et al. 1988) are formed along the accretion disc symmetry axis, filled with rarefied hot gas, which may be accelerated and collimated to form relativistic jets. In the inner parts of the wind this gas also can form a pseudo-photosphere (“funnel bottom”) at \(R_{\text{in}} \sim 10^9\text{cm}\) (Fabrika 2004). This radius is calculated in the consideration all the material ejected in the relativistic jets is uniformly distributed with respect to the polar angle. In case of any inhomogeneity of the jet material the inner radius becomes lower. The visible part of the wind may be therefore divided into three parts: funnel wall photosphere (or “photocone”), the outer photosphere and the inner photosphere inside the funnel.

Another class of objects supposed to be supercritical accretors are extragalactic Ultraluminous X-ray sources (ULXs). Katz (1986) supposed that an object similar to SS433 seen at low inclination angles can appear a bright X-ray source with super-Eddington apparent luminosity. ULXs were discovered about that time by Einstein (see Fabbiano (1989) and references therein). Though the nature of these sources remains unclear they are good candidates for the objects predicted by Katz (Poutanen et al. 2007).

The question about how representative is SS433 among the binary systems in the supercritical accretion regime in the observed Universe is difficult to answer. If one considers mass transfer in thermal timescale in a black hole high mass X-ray binary, the most relevant is the mass of the secondary that determines the timescale and hence the scaling for the mass accretion rate. It may be shown that in assumption the radius of the secondary scales with its mass as \(R \propto M^{1/2}\) and the black hole mass is close to 10 \(M_\odot\) dimensionless thermal-timescale.
mass transfer rate depends on the secondary mass as:

\[ \dot{m} \approx \left( \frac{M_2}{M_\odot} \right)^{5/2} \]

For the case of SS433 \((M_2 \simeq 20 M_\odot)\) our estimate predicts \(\dot{m} \sim 2000\), similar to the observed value \(^1\). If the donor star is highly evolved, accretion rate is higher and less stable and the phase itself is shorter. Unfortunately, initial binary mass ratio distribution for massive stars is poorly known but there are indications that mass ratios close to 1 are much more probable (Lucy 2007; Kobulnicky & Fryer 2007). The thermal timescale mass transfer rates in high-mass X-ray binaries are therefore likely to be highly supercritical, \(\dot{m} \sim 100 - 10^4\). Lower yet supercritical rates may take place in case of wind accretion for example in WR+BH binaries (Bauer & Brandt 2004). Those are likely to form a certain sub-sample of evolved ULXs with moderately supercritical accretion rates.

The paper is organized as follows. In the next section we construct a simple analytical model describing the internal structure of an optically thick wind flow that is likely to develop in the case of very high mass ejection (and accretion) rate \(\dot{m}\). Section 3 is devoted to effects of self-irradiation that are likely to play very important role in our funnel solution. In section 4 we consider the emergent SEDs for arbitrary inclination taking into account self-occultation effects. In section 5 we test it for two sets of publicly available X-ray data. We discuss the implications of the model and its early testing in section 6.

2. Structure of Supercritical Disc Winds

In this section we study the spatial structure of the radiation density field inside the wind. We will use the following set of assumptions (their reliability will be discussed below in Section 6): (i) the flow is axisymmetric (also symmetric with respect to the disc plane) and stationary, (ii) the flow is optically thick to true absorption, (iii) the velocity and density fields are not affected by energy and entropy transfer (i. e. we consider the wind already accelerated or accelerating/decelerating with a given power-law dependence on radius), (iv) all the motions are purely radial and non-relativistic, (v) the inner surface of the funnel, the funnel bottom and the wind photosphere are considered locally blackbody sources and (vi) for certainty we suggest the temperature of the bottom equal to the starting temperature of the walls. We simplified the picture somehow suggesting the mass is loaded in the center of symmetry. This assumption is violated in the inner parts of the wind.

Effects of special relativity (namely, Doppler boosts) may be approximately accounted for during the calculation of outcoming spectra. We also discuss the possible influence of

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\(^1\) Actually, there are no direct measurements of the mass transfer rate for SS433. However, the observed stability of the orbital period and evolution-time considerations exclude significantly higher mass transfer rates (Fabrika 2004; Goranskij et al. 1998).
relativistic effects in sections 4 and 6.

We assume that fraction \( K \lesssim 1 \) of the accreting mass is ejected in the wind. Because mass ejection rate is more relevant (see below) we will use the dimensionless notation \( \dot{m} = \dot{M}/\dot{M}_{cr} \) for the **mass ejection rate** where

\[
\dot{M}_{cr} = \frac{48\pi GM_{\odot}}{c\kappa_T} \simeq 3 \times 10^{-7} M_{10} \text{ M}_{\odot}\text{yr}^{-1}
\]

is the critical Eddington accretion rate as introduced by Poutanen et al. (2007). \( M_{10} \) here is the accretor mass in \( 10 M_{\odot} \) units. Mass accretion is characterised by \( \dot{m}/K \sim \dot{m} \), we assume \( K = 1 \) everywhere below bearing in mind that \( K < 1 \) results in a hotter and more luminous source but requires higher accretion rate. The outcomes of spectra depend mostly on the mass ejection rate. Mass accretion rate affects only the luminosity injected at the inner boundary in a logarithmic way. To achieve a 50% change in luminosity (and about 12% change in temperature) for \( \dot{m} = 100 \) one should assume 90% of the accreting material accreted by the black hole (\( K = 0.1 \)).

There are physical differences between the regime of highly supercritical accretion that we consider here and moderately supercritical accretion (\( \dot{m} \sim 1 - 10 \)). At \( \dot{m} \sim 10 \) the flow becomes translucent and relativistic and some of our approximations are violated.

For radial coordinate normalization we will use the “spherization radius” defined as:

\[
R_{sph} = \frac{3\dot{M}_K}{\Omega_f c} = \frac{18}{\cos \theta_f} R_G \dot{m}.
\]

\( R_G = 2GM/c^2 \) here is the gravitational (Schwarzschild) radius of the accretor. This value is proportional to the spherization radii used by Shakura & Sunyaev (1973)

\[
R^{(SS)}_{sph} = \frac{3\dot{M}_K}{8\pi c}
\]

and by Poutanen et al. (2007)

\[
R^{(P)}_{sph} = \frac{5}{3\dot{m}}
\]

but has also non-trivial dependence on geometry. \( \Omega_f = 4\pi \cos \theta_f \) is the solid angle of the wind.

Normalized radial coordinates will be hereafter denoted by small letters \( r \) in contrast to capital \( R \) reserved for physical distances.

We assume fixed geometry for the wind funnel, i.e. fixed half-opening angle \( \theta_f \) independent of the accretion/ejection rate. Note the difference with funnels in Polish doughnuts (Abramowicz 2004) where

\[
\theta_f \propto \sqrt{r_{in}/r_{out}} \propto e^{-0.5L/L_{Edd}} \propto \dot{m}^{-0.3},
\]

defined by equipotential surfaces. In contrast, when the wind is launched from a thin but supercritical disk at a certain radius and its velocity in the frame comoving with the disk is at any given starting radius proportional to the virial velocity (\( v = \xi v_K \) where \( v_K \) is the Keplerian velocity) and normal to the disk surface, the half-opening angle will be a function of \( \xi \) only (Poutanen et al. 2007):
$$\theta_f = \arctan \left( \frac{1}{\sqrt{\xi^2 - 1}} \right).$$  \hfill (6)

Radial velocity of the wind may be uniform ($v = \text{const}$) in the case of large initial speed or virial ($v \propto R^{-1/2}$) in the parabolic case when the initial velocity is close to the escape velocity from the wind acceleration region. We adopt a generalized self-similar scaling

$$v = \frac{1}{6} \sqrt{\cos \theta_f \sin^{-1/2} r^\alpha}$$  \hfill (7)

with approximately virial value at the spherization radius. $\alpha = 0$ corresponds to a constant velocity wind with no acceleration. In the parabolic case, when the velocity is proportional to the virial at any given radius, $\alpha = -0.5$.

Outside the spherization radius the gas density in the wind scales as

$$n \propto r^{-(2+\alpha)}$$  \hfill (8)

and vanishes (or, at least, drops by several orders of magnitude) inside the funnel. Inside the spherisation radius deviations from this law are expected and mass ejection rate should depend on radius roughly as $\dot{m}_{\text{eff}} \propto r$, and hence

$$n \propto r^{-(1+\alpha)}.$$  \hfill (9)

We briefly analyse the consequences of this difference in density slope in section 6.

2.1. Diffusion Equation

The main equation governing energy transfer may be derived from the energy conservation and Fick’s laws for the thermal energy flux

$$\mathbf{q} = -D \nabla u,$$  \hfill (10)

where $D = c/(3\kappa \rho)$ is the diffusion coefficient and $u$ is energy density. The first law of thermodynamics

$$d \left( \frac{u}{n} \right) = T ds + \frac{p}{n^2} dn$$  \hfill (11)

$$du = n T ds + \frac{p + u}{n} dn$$  \hfill (12)

where

$$T ds = (\nabla \mathbf{q}) dt$$  \hfill (13)

$$du/dt = \partial_t u + (\mathbf{v} \nabla) u$$  \hfill (14)

$$dn/dt = -n(\nabla \mathbf{v})$$  \hfill (15)

may be rewritten in the generic form

$$\partial_t u + (\mathbf{v} \nabla) u - \nabla (D \nabla u) - \gamma u(\nabla \mathbf{v}) = 0.$$  \hfill (16)

If the specific heat ratio $\gamma$ does not depend heavily on other parameters, this may be rewritten for an axisymmetric system in terms of enthalpy density $h = \gamma u$ as
\[
\frac{1}{v} \partial_t u + \partial_r u - \frac{1}{r^{2+\alpha}} \partial_r (r^{4+\alpha} \partial_r u) - \left( L_\theta - \frac{\gamma(2+\alpha)}{r} \right) u = 0 \tag{17}
\]

where
\[
L_\theta = \frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta ) \tag{18}
\]
is the polar part of the Laplace operator, and the second term in brackets accounts for adiabatic losses. Equation (17) is linear hence one may separate the variables. Eigenfunctions will depend on \(\cos \theta\) as linear combinations of Legendre polynomials \(P_k\) and Legendre functions of the second kind \(Q_k\) (Jahnke & Emde 1960).

In the stationary case
\[
u_k = (P_k(\cos \theta) + a_k Q_k(\cos \theta)) R_k(r) \tag{19}
\]
where \(R_k\) is the solution of the equation
\[
- \frac{1}{r^{2+\alpha}} e^{-1/r} \partial_r \left( r^{4+\alpha} e^{1/r} \partial_r u \right) - \left( k(k+1) - \frac{\gamma(2+\alpha)}{r} \right) u = 0 \tag{20}
\]

For the zeroth-order \((k = 0)\) solution asymptotics may be derived both for large distances \(r \gg 1\) where diffusion dominates, \(e^{-1/r} \simeq 1\) and \(u \propto r^{-(3+\alpha)}\), and for very low radii where adiabatic losses prevail and \(u \propto r^{-\gamma(2+\alpha)}\). Figure 1 shows the exact numerical solution for the radial eigenfunction \(R_0\) for \(\alpha = 0\) and \(\gamma = 4/3\). It may be rather well (with accuracy better than 2\%) approximated by the following function
\[
Y(r) = \left( 1 - e^{-\frac{1}{2+\alpha}} \right)^{1-(\gamma-1)(2+\alpha)} r^{-\gamma(2+\alpha)} \tag{21}
\]

2.2. Boundary Conditions

For a system with equatorial plane symmetry the solution will contain only even eigenfunctions. If we choose the inner radius \(R_{in}\) well below the spherization radius, equation (20) becomes a first-order one, and is governed by a single parameter which we assume to be the initial energy density. The latter may be estimated from the total central source luminosity \(L\) for the advective flow as
\[
u_{in} \sim \frac{L}{\Omega R_{in}^2 v(r_{in}) Y(r_{in})} \tag{22}
\]

Luminosity of a supercritical disk in different models depends logarithmically on the dimensionless mass accretion rate (Poutanen et al. 2007; Shakura & Sunyaev 1973) as
\[
L = L_{Edd} (1 + 0.6 \ln(\dot{m}/K)) \simeq 1.5 \times 10^{39} \frac{1.7}{1+X} M_{10} (2.8 + 0.6 \ln \dot{m}_3) \ \text{ergs}^{-1}, \tag{23}
\]
where \(\dot{m}_3\) is the mass ejection rate in \(10^3 \dot{M}_{cr}\), \(X\) is hydrogen mass fraction in the accreting gas, \(M_{10}\) is the accretor mass in \(10 M_\odot\) units. Temperature of the flow at the spherization radius is
Fig. 1. Radial dependence for the zeroth-order solution for $\alpha = 0$ and $\gamma = 4/3$ (solid line) normalized by the low-$r$ asymptotics. High-$r$ power-law asymptotics $u \propto r^{-(2+\alpha)}$ is shown by a dashed curve.

$T \sim 0.1\dot{m}_3^{-3/8}$ keV while the gas density is $n \sim 4 \times 10^{16}\dot{m}_3^{-1/2}$ cm$^{-3}$, which leads to a rather high radiation to gas pressure ratio:

$$\beta \simeq \frac{\dot{a}T^3}{n} \simeq 500\dot{m}_3^{-5/8}$$

(24)

For accretion rates that appear during thermal-timescale mass transfer $10 < \dot{m} < 10^5$ (see also sections 1 and 6) radiation pressure dominates in the advective inner parts of the wind. That allows to equate $\gamma = 4/3$ anywhere in the flow because the wind is radiation-pressure dominated in the regions where $\gamma$ is relevant.

Energy flux is directed radially in the inner advective parts of the wind and is practically normal to the walls near the funnel wall surface in the outer parts. In figure 2 we show the two-dimensional distribution of energy density and energy flux vector field for our solution. The exact angular dependence of the energy influx at the inner boundary is not very important as it is mixed near the spherization radius, so we will restrict ourselves to the zeroth angular harmonic solution.
Funnel wall cools efficiently, therefore a simple boundary condition

\[ u(r, \theta_f) = 0 \]  

may be adopted. The following zeroth-order solution satisfies it and decays fast enough at infinity to account for the photon escape through the funnel walls

\[ u = u_0 \times \ln \left( \frac{1 - \cos \theta}{1 - \cos \theta_f} \times \frac{1 + \cos \theta_f}{1 + \cos \theta} \right) Y(r) \]  

\[ u_0 \] normalization may be derived from the total luminosity (more accurately than in (22)) of the disk as

\[ u_0 \simeq \frac{L}{\Omega' R_{in}^2 Y(r_{in})} \]  

Here we define \( \Omega' = -8\pi \ln \sin \theta_f \). \( Y(r_{in}) \simeq r_{in}^{-\gamma(2+\alpha)} \) for \( r_{in} \ll 1 \).

2.3. Surface Effective Temperature Distribution

Below we consider funnel walls, bottom and outer photospheres as blackbody sources with the temperature determined by the energy flux normal component:

\[ F_n = \sigma_B T^4. \]  

For the funnel walls it is equal to the latitudal component:

\[ F_\theta = \frac{2L}{\Omega' \sin^2 \theta_f R_{sph}^2 r_{in}^{2+\alpha} Y(r_{in})} r^{1+\alpha} Y(r), \]  

and in the case of outer photosphere

\[ F_r = -D \partial_r u = \frac{(3+\alpha)L}{4\Omega' R_{sph}^2 r_{in}^{2+\alpha} Y(r_{in})} r^{1+\alpha} Y(r) \times \]  

\[ \times \ln \left( \frac{1 - \cos \theta}{1 - \cos \theta_f} \times \frac{1 + \cos \theta_f}{1 + \cos \theta} \right). \]  

Here \( r = r_{out} \) should be substituted for the outer photosphere.

Note that the fluxes are of the same order and their ratio does not depend on the radial coordinate.

For the funnel walls one obtains:

\[ T_{wall} = 0.038 \left( \Omega' \tan^2 \theta_f r_{in}^{2+\alpha} Y(r_{in}) \right)^{-1/4} \times \]  

\[ \times (2.8 + 0.6 \ln m_3)^{1/4} m_3^{-1/2} M_{10}^{-1/4} (r^{1+\alpha} Y(r))^{1/4} \text{ keV} \]  

where \( M_{10} \) is the mass of the accretor in 10\( M_\odot \) units. Effective temperature scales with radius (for \( \alpha = 0 \)) as

\[ T_{wall} \propto r^{-5/12} \]  

at lower radii and as

\[ T_{wall} \propto r^{-1/2} \]  

in the outer parts of the funnel. Temperature slope depends very weakly on all the parameters and is similar to the value \( p = 1/2 \) (if \( T \propto r^{-p} \)) characteristic for slim discs (Abramowicz
Fig. 2. Two-dimensional distribution of energy density in the optically thick wind. Thin solid lines correspond to logarithmically spaced (by one order of magnitude) constant energy density levels. $\theta_f = 0.4 \text{ rad} \simeq 23^\circ$, $\alpha = 0$. Inner funnel wall is shown by a thick solid line. Arrows are oriented in the energy flux direction, their length proportional to the energy flux logarithm.
2004) and supercritical accretion discs with massive outflows. *p-free* multi-blackbody spectra are characterised by a power-law intermediate asymptotics with photon index $\Gamma = 2/p - 2$. Many ULXs have power-law spectra with $\Gamma \simeq 2$ close to the expected in all these models. The similarity makes it very difficult to distinguish between the various supercritical accretion disc models by their X-ray spectra only.

We assume the temperature of the funnel bottom equal to the effective temperature of the adjacent wall surface

$$T_{bot} = 0.038 (\Omega' \tan^2 \theta_f r_{in})^{-1/4} (2.8 + 0.6 \ln \dot{m}_3)^{1/4} \dot{m}_3^{-1/2} M_{10}^{-1/4} \text{keV}$$ (34)

Minimal inner radius $R_{in}$ may be estimated as the last stable orbit radius divided by $\cos \theta_f$ as $r_{in} = 1/6 \dot{m}$ in $R_{sph}$ units. Temperatures characteristic for the high-energy cut-offs in ULX spectra are about one keV and require $r_{in} \gtrsim \dot{m}^{-1}$. It is clear that in the framework of our model the inner parts of the funnel should be practically transparent. If we make another assumption that the soft excess in ULX spectra corresponds to $T \sim 0.1 - 0.2$ keV temperature at the spherisation radius than accretion rates required are $\dot{m} \sim 100$. In section 5 we fit real X-ray spectra with our model obtaining similar results ($r_{in} \sim 10^{-3}$ and $\dot{m} \sim 100$) for two ULXs.

Effective temperature of the photosphere may be found in a similar way. The outer photosphere radius may be estimated as

$$r_{out} \simeq \left( \frac{2 \Omega_f \sqrt{m}}{(1 + \alpha) \sqrt{\cos \theta_f}} \right)^{1/(1+\alpha)}$$ (35)

in $R_{sph}$ units. The photosphere is significantly non-plane-parallel, but its luminosity and spectral energy distribution may be roughly estimated as:

$$L_{ph} \simeq \frac{(3 + \alpha)(2 + \alpha)(2+\alpha)(\gamma-1)^{-1}}{r_{in}^{2+\alpha} Y(r_{in})} \cos^2 \theta_f L$$ (36)

$$T_{ph} \simeq 0.0048 f_\alpha \left( \frac{r_{in}^{2+\alpha} Y(r_{in})}{\dot{m}_3} \right)^{-1/4} \sqrt{\cos \theta_f} \times M_{10}^{-1/4} (2.8 + 0.6 \ln \dot{m}_3)^{1/4} \times \dot{m}_3^{-3/4(1+\alpha)} \left( \ln \left( \frac{1 - \cos \theta_f}{1 + \cos \theta_f} \times \frac{1 + \cos \theta_f}{1 + \cos \theta_f} \right) \right)^{1/4} \text{keV},$$ (37)

where

$$f_\alpha = (3(1 + \alpha))^{1/2(1+\alpha)} (2 + \alpha)^{(2+\alpha)(\gamma-1)-1/4} \times \left( \Omega' \right)^{-(3+\alpha)/4(1+\alpha)}.$$ 

Characteristic temperatures derived above are fairly consistent with those reported by Poutanen et al. (2007). For a reasonably high $\dot{m} = 10^3$ and $\theta_f = 0.4 \text{rad}$ the outer photosphere temperature is $\sim 10^5 \text{K}$. Together with a photospheric luminosity $\sim 10^{39} \text{ergs}^{-1}$ this makes the wind photosphere both a bright UV/optical and a bright extreme ultraviolet (EUV) source capable for ionizing large amounts of interstellar and circumstellar gas. We discuss the properties of HII-regions created by supercritical accretor wind photospheres in section 6.4. Very
shallow temperature decline makes the spectrum flat from $E \sim 0.005$ keV to $\sim 1$ keV. The highest temperature value is predicted by accretion disc theory and is much higher than the temperature at spherisation radius (Poutanen et al. 2007):

$$T_{\text{max}} = 1.27M_\odot^{-1/4} \text{keV.}$$  

(38)

3. Effects of Irradiation

In the standard disc model irradiation is a minor, often negligible effect as the disc is thin and the fraction of emitted luminosity absorbed by the outer parts of the disc is of order of $O(H/R)^2$. On the contrary, in the case of supercritical funnel most of the radiation emitted at any point on the wall surface will be absorbed again or reflected. The probability of re-absorption is of the order $\cos \theta_f$, hence the energy balance will be significantly affected by irradiation. As long as true absorption dominates over electron scattering, $\sigma \gg \sigma_T$, all the quanta may be considered absorbed and re-emitted by the walls remaining a locally blackbody source.

The energy input due to incident radiation is characterized by the flux $F'(R)$ normal to the wall surface and directed inward, which may originate both from the funnel bottom and inner parts of the walls. In general, the incident flux may be expressed as

$$F' = F \int \left( \frac{T_{\text{eff}}(R')}{T_{\text{eff}}(R)} \right)^4 \frac{|(n \cdot d)(n' \cdot d)|}{d^2} dS'$$

(39)

where the meaning of $n$ and $d$ vectors is illustrated in figure 3 for both wall and bottom irradiation.

3.1. Problem Formulation

In this section Cartesian and spherical coordinates are used. $z$ axis coincides with the symmetry axis of the disc, funnel and jets. The remaining degree of freedom is adjusted to set to zero the azimuthal coordinate of the point under consideration, where the incident flux is calculated. This point is set by a radius vector $R$,

$$R = R \begin{pmatrix} \sin \theta \\ 0 \\ \cos \theta \end{pmatrix}$$

In the same way in the following subsections we define the bottom radius vector $R_0$ and the radius vector $R'$ of the variable point on the funnel walls.

$$R_0 = R_0 \begin{pmatrix} 0 \\ 0 \\ \cos \theta \end{pmatrix} \quad R' = R' \begin{pmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{pmatrix}$$
The geometry of funnel self-irradiation is shown in figure 3. Variable infinitesimal plate with area $dS'$, radius vector $\mathbf{R}'$ and normal $\mathbf{n}'$ contributes to the incident flux in the current point by:

$$dF' = |(\mathbf{n} \cdot \mathbf{d})(\mathbf{n}' \cdot \mathbf{d})| \frac{dS'}{2\pi d^2},$$

where $\mathbf{d} = \mathbf{R}' - \mathbf{R}$, $d = |\mathbf{d}|$. Due to surface brightness invariance from distance it is convenient to use dilution factor $w$ depending on geometry only and to normalize the incident flux over the outcoming flux $F(\mathbf{R})$ in the current point.

$$w = \frac{|(\mathbf{n} \cdot \mathbf{d})(\mathbf{n}' \cdot \mathbf{d})|}{d^2}$$

Incident flux in the units of the outcoming integral flux ($f = F'/F$) can be expressed as an integral over the funnel inner surface:

$$f = \int \left(\frac{T_{\text{eff}}(\mathbf{R}')}{T_{\text{eff}}(\mathbf{R})}\right)^4 \frac{|(\mathbf{n} \cdot \mathbf{d})(\mathbf{n}' \cdot \mathbf{d})|}{d^2} dS'$$

(40)

In the case of the funnel bottom we avoid integration suggesting the bottom a point-like source emitting as a flat plate.

3.2. Irradiation by the Funnel Bottom

The first heating term is simpler to handle because the source is practically point-like and its temperature is not affected by the funnel walls. Due to this, one can do without integration. Funnel bottom can be considered a spherical surface with the surface area $S_{\text{sph}} = 2\pi(1 - \cos \theta)R_0^2$ or a flat circular plate of radius $R_0 \sin \theta$ (because $R_0$ is the starting radial coordinate along the wall, not along the $z$-axis) with the surface area $S = \pi R_0^2 \sin^2 \theta$. Both expressions give similar results when $\theta \ll 1$. Here we assume the bottom having flat surface (see figure 3a). The photon source coordinate and normal unit vector are:

$$\mathbf{R}_0 = R_0 \begin{pmatrix} 0 \\ 0 \\ \cos \theta \end{pmatrix}, \quad \mathbf{n}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Distance vector connecting emitting and receiving points:

$$\mathbf{d} = \mathbf{R} - \mathbf{R}_0 = R_0 \begin{pmatrix} r \sin \theta \\ 0 \\ (r - 1) \cos \theta \end{pmatrix}$$

Here $r = R/R_0$. Dilution factor in that case:

$$w(x, \theta, \varphi) = \frac{|(\mathbf{n} \cdot \mathbf{d})(\mathbf{n}_0 \cdot \mathbf{d})| |S|}{2\pi d^4} = \frac{r \sin \theta \cos \theta}{2(1 + r^2 - 2r \cos \theta)^2}$$

(41)
Fig. 3. Schemes explaining the geometry of irradiation effects. (a) irradiation by the bottom. (b) irradiation by the funnel walls.

Total normalized irradiating flux from the bottom can be therefore expressed as:

\[ f_{\text{bottom}} = \frac{1}{2} \frac{(r - 1) \sin^3 \theta \cos^2 \theta}{(1 + r^2 - 2r \cos \theta)^2} \tau^{-4} \]  

(42)

3.3. Self-Irradiation by the Funnel Walls

The energy input from the absorbed photons is directly calculated using equation (39). Surface element is an elementary area on a conical surface, \( dS' = 2\pi \sin \theta R'dR'd\phi \). Radius vectors and normals are defined. Distance vector:

\[
d = R' - R = R \begin{pmatrix} 
\sin \theta (1 - x \cos \varphi) \\
-x \sin \theta \sin \varphi \\
\cos \theta (1 - x)
\end{pmatrix}
\]

where \( x = R' / R \). Dilution factor in this cases takes the form:

\[
w(x, \theta, \varphi) = \frac{|(n \cdot d)(n' \cdot d)|}{2\pi x^2} = \frac{1}{2\pi R'^2} \frac{x^2 \sin^2 \theta \cos^2 \theta (1 - \cos \varphi)^2}{2((1-x)^2 + 2x(1-\cos \varphi) \sin^2 \theta)^2}
\]

(43)

Absorbed flux is given by an integral over \( x \):

\[
F'(R) = 2\pi \sin \theta \int_{R_{\text{in}}/R}^{R_{\text{out}}/R} F(Rx) I(x, \theta) x dx
\]

where \( I = \int_{-\pi}^{\pi} w(x, \theta, \varphi) d\varphi \). Integration over \( \varphi \) is straightforward however complicated, so we leave it for appendix 1. Finally, normalized absorbed flux can be expressed as:

\[
f_{\text{walls}}(R) = \frac{\cos^2 \theta}{4 \sin \theta} \int_{R_{\text{in}}/R}^{R_{\text{out}}/R} \left( \frac{T_{\text{eff}}(R')}{T_{\text{eff}}(R)} \right)^4 \times \\
\times \left( 1 - \frac{|1-x|}{\sqrt{1 - 2x \cos(2\theta) + x^2}} \frac{1 - 2x(1 - 3 \sin^2 \theta + x^2)}{1 - 2x \cos(2\theta) + x^2} \right) dx
\]

(44)
Fig. 4. Funnel wall temperature dependence on radius, normalized by $r^{-1/2}$. Thick solid line: without irradiation, thick dotted line: with irradiation effects included, three iterations. Thin solid lines represent the best power-law fits to the irradiated funnel wall temperature at larger $r$ ($>1$) and lower $r$ ($10^{-2} < r < 1$), $T \propto r^{-0.44}$ and $T \propto r^{-0.50}$, correspondingly. Dot-dashed line represents the $T \propto r^{-3/4}$ temperature slope characteristic for standard discs.

Irradiation is treated iteratively. Three to five iterations generally sufficient for convergence with accuracy better than 5%. As may be seen in figures 4 and 5, mostly the inner parts of the funnel walls (and subsequently mostly the X-ray range) are affected. Incident flux produced at large radii by the irradiating funnel bottom and inner parts of funnel walls changes as $F' \propto r^{-3}$ (see equation (42)), while the predicted temperature dependence on radius implies $F' \propto r^{-2}$ for the emergent flux, therefore the effect is expected.

4. Emergent Spectra and Spectral Variability

Here we calculate the outcoming spectra as functions of inclination and supercritical wind parameters. All the surfaces are considered locally blackbody sources and the observed spectrum (in terms of apparent isotropic luminosity $\nu L_\nu$) is calculated by integrating over all the visible surfaces applying visibility conditions. The most important effect is that at inclinations
Fig. 5. Irradiation effects on the outcoming spectrum. Solid lines show the spectral energy distribution of a face-on supercritical funnel for different number of iterations (0, 1, 2 and 3). Funnel walls without irradiation give the spectrum shown by dashed lines. Dotted lines correspond to the contributions from the bottom (at high energies) and the outer photosphere. Parameters are the same as in figure 2.

\[ \sim \theta \]

Doppler boosts are accounted for approximately. We consider funnel walls moving at an angle \( \theta_f \) with the line of sight (that is exactly true for a face-on object), and the material consisting the funnel bottom moving at an angle \( i \). The velocity is determined according to equation (7) and is considered equal for the funnel bottom and the wind at \( r_m \). Photon energies are modified by a factor:

\[ \delta = \frac{1}{\gamma(1 - \beta \cos \theta)} \]  

where \( \beta = v/c \) is the dimensionless velocity, \( \gamma = 1/\sqrt{1 - (v/c)^2} \) is the Lorentz factor, and \( \theta \) is \( \theta_f \) for the walls and \( i \) for the bottom. Photon numbers are modified by a factor \( \delta \), and the time compression factor is not relevant here because the flow is stationary. Apparent luminosity \( L_E \) changes proportionally to \( L_E(E) \propto \delta^2 L_E^{(o)}(E/\delta) \), where \( L_E^{(o)} \) is the apparent luminosity calculated without relativistic effects. Note that Doppler boosts become important only when

\[ \theta \gtrsim \theta_f \]
the velocities in the inner parts of the flow are relativistic (either when $\dot{m} \lesssim 10$ or for $\alpha \lesssim -0.5$). See also discussion in section 6.2.

In figures 4 and 5 we compare the temperature profiles and SEDs with and without irradiation. We adopt $\alpha = 0$, $\dot{m} = 10^2$, $r_{\text{in}} = 1/6\dot{m} = 1.7 \times 10^{-3}$, $\theta_f = 23^\circ$. The temperature dependence on radius is close to a broken power-law in both cases, and the net effect of irradiation is in altering the mean temperature in the innermost parts of the funnel by about 20%. In figure 5 SEDs are calculated for zero inclination and different number of iterations used to account for irradiation effects. Evidently, only the funnel walls is affected by irradiation that alters the flux by a factor of $2 - 3$. Spectrum is practically flat in the EUV/soft X-ray region but curves near the $T_{\text{sph}} \sim 0.1\text{keV}$. High-energy cut-off is present at several keV if the inner radius is $r_{\text{in}} \sim 1/\dot{m}$.

4.1. Self-Occultation

Let $i$ be the inclination of the funnel (angle between the symmetry axis and the line of sight). For any given radial coordinate at the funnel wall surface three cases are possible: the annulus is fully visible (this is possible only for $i < \theta_f$), invisible or partly visible. If the inclination is larger than $\theta_f$ for different radii either second or third case takes place for every annulus. Visibility may be quantitatively described by a factor $y$ defined as the visible part of the given annulus (having constant distance $r$ from the center). It may be determined by an analytical ray-tracing method.

$$r = R + sl$$

Here $l$ is a unit vector directed towards the observer (we use the same scheme as in the previous section), $R$ is the radius vector of the intersection point between the funnel surface and the ray starting from $r$ and directed towards the observer. Boundary case when the intersection point has the radial coordinate equal to the photosphere radius is of interest here. Normalizing over the outer radius ($x = r/r_{\text{out}}$), one can express the vectors as follows.

$$r = x \begin{pmatrix} \sin \theta_f \cos A \\ \sin \theta_f \sin A \\ \cos \theta_f \end{pmatrix}$$

$$R = \begin{pmatrix} \sin \theta_f \cos A_0 \\ \sin \theta_f \sin A_0 \\ \cos \theta_f \end{pmatrix}$$

$$l = \begin{pmatrix} \sin i \\ 0 \\ \cos i \end{pmatrix}$$

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It is easy to express two of the unknowns (s and $A_0$) from the others and obtain the solution for $A$ (azimuthal coordinate of the starting point):

$$\sin A = \frac{1}{2} \left( \frac{\tan \theta_f}{\tan i} + \frac{\tan i}{\tan \theta_f} \right) + \frac{1}{2x} \left( \frac{\tan \theta_f}{\tan i} - \frac{\tan i}{\tan \theta_f} \right)$$

If the annulus is partially visible $y$ may be calculated as the length of the arc (divided by $2\pi$) connecting the two points defined by the solutions of the equation above. Finally:

$$y = \begin{cases} 
0 & K < -1 \\
\frac{1}{2} + \frac{1}{\pi} \arcsin K & -1 \geq K < 1 \\
1 & K \geq 1
\end{cases} \quad (46)$$

where:

$$K = \frac{1}{2} \left( \frac{\tan \theta_f}{\tan i} + \frac{\tan i}{\tan \theta_f} \right) + \frac{1}{2x} \left( \frac{\tan \theta_f}{\tan i} - \frac{\tan i}{\tan \theta_f} \right)$$

4.2. Implications of Self-Occultation Effects

Watarai et al. (2005) studied self-occultation effects for fat accretion discs coming to an evident conclusion that the spectrum becomes softer at large inclinations. Heinzeller et al. (2007) comes to a similar conclusion considering the results of 2D radiative hydrodynamic simulations by Ohsuga et al. (2005). In our case this effect is even more profound (see figure 6): at inclinations $i \sim \theta_f$ the X-ray component abruptly disappears (because only the inner parts of the funnel contribute significantly to the X-ray range) and only the EUV component may be observed.

Due to that reason some supercritical accretors (those viewed at large inclinations) will not show the ULX phenomenon (see figure 7) but remain luminous UV sources. For $\theta_f = 0.4\text{rad}$ the number of orphan ultraviolet sources of similar nature is about $\cos \theta_f / (1 - \cos \theta_f) \sim 10$ times higher than the number of ULXs.

Observations of these sources in the EUV are complicated by neutral gas absorption. In Abolmasov et al. (2008) we discuss the possibility of far-ultraviolet observations with GALEX (Martin et al. 2005) and HST in the ultraviolet. Supercritical accretors prove to be difficult but still reachable targets for GALEX and HST, appropriate for pointing observations.

We do not know yet how much there is similarity between ULXs and SS433. SS433 exhibits jet/disc axis direction variations with a super-orbital precession period about $160^d$. Similar effects are observed for some other X-ray binaries (Clarkson et al. 2003). There are indications that the supercritical disc (and consequently the wind with the funnel) follows the motions of the jets.

A nearly face-on supercritical accretion disc may show periodical X-ray variability by one-two orders of magnitude when $\theta_f$ is less than the maximal and higher than the minimal value of $i$. Precessional variability should have a distinguished light curve with a flat maximum
**Fig. 6.** Apparent isotropic luminosity as a function of inclination. Solid line represents bolometric luminosity. Dashed, dot-dashed and dotted lines represent harder X-rays (0.4–20 keV), softer X-rays (0.1–0.4 keV) and EUV (0.01–0.1 keV). Vertical lines mark $\theta_f$ and $\pi/2 - \theta_f$.

and continuous flux change near the minimum light. In figure 8 we present the predicted X-ray spectra as functions of precessional phase for two sets of parameters. We assume here a simplified version of the kinematical model applied to SS433 (Abell & Margon 1979):

$$\cos i = \cos i_A \cos i_0 + \sin i_A \sin i_0 \cos(2\pi \psi),$$

where $i_0$ is the inclination with respect to the precession axis, $i_A$ is the amplitude of inclination variation, and $\psi$ is the phase of super-orbital period.

The only ULX exhibiting variability that may be considered super-orbital is X41.4+60 in M82 (Kaaret et al. 2006). *RXTE* X-ray flux varies by $\sim 50\%$ for this source with a $62^d$ period. Irregular variability on similar timescales is much more ordinary among ULXs (La Parola et al. 2001) possibly indicating that the funnel shape itself changes rapidly enough to
level the effects of precessional variability.

5. Comparison with Observations

5.1. Data on ULXs

*XMM-Newton* datasets on two X-ray sources, NGC4559 X-7 and NGC6946 ULX-1 were analysed. We used archival *XMM-Newton* EPIC (Turner et al. 2001) data (MOS1,2 and PN detectors) obtained on 8 October 2006 (observation ID 0152170501) and 25 June 2004 (observation ID 0200670401), correspondingly. All the data were reduced using standard XMM-Newton Science Analysis System (SAS) procedures. Response files were made using SAS tools *rmfgen* and *arfgen*. We set *flag*= 0 for PN data. We used *pattern* $\leq 4$ for PN and $\leq 12$ for MOS data.

NGC4559 X-7 is known as a “supersoft” bright ULX (Cropper et al. 2004). It does not coincide with a bright stellar or nebular optical counterpart. However, there are indications that the source is connected with $\sim 10$Myr old stellar population (Soria et al. 2005).

NGC6946 ULX1 (Roberts & Colbert 2003) is known to be a source of moderate luminos-
Fig. 8. Spectral variability of precessing supercritical accretion disc funnel. Lines of constant flux ($EF_E$) as a function of energy and precessional phase are given. Flux is normalized over its maximal value. $\theta_f = 22.9$ deg, $\dot{m} = 10^3$, $\alpha = 0$. Upper diagram corresponds to $i_0 = 20$ deg and $i_A = 20$ deg, lower has $i_0 = 45$ deg and $i_A = 20$ deg. In the latter case funnel bottom is never visible and funnel walls always remain at least partially occulted.
ity \( L_X \sim 3 \times 10^{39} \text{ ergs}^{-1} \) with an X-ray spectrum usual for ULXs. Stellar optical counterpart was detected by \( HST \) in the visible range (Blair et al. 2001) as a relatively bright star \((M_V \lesssim -7)\). As many other ULXs, NGC6946 ULX1 coincides with a bright optical nebula. Abolmasov et al. (2008) analysing the emission spectrum of the nebula conclude that the central source may be a hard EUV source with temperature \( T \sim 10^5 \text{K} \) and luminosity \( L \sim 10^{40} \text{ erg s}^{-1} \), roughly consistent with what one may expect from a \( \dot{m} \sim 10^2 - 10^3 \) supercritical accretor.

Three models were used to fit the data: standard disc (Mitsuda et al. 1984) + power law, p-free disc (Mineshige et al. 1994) and self-irradiated multi-color funnel model presented here. Efficiency of the p-free model for ULXs was already shown by Vierdayanti et al. (2006) and other works. Because the temperature of the funnel walls decays in a practically power-law manner (with \( p \sim 0.5 \), see figure 4) one should expect p-free model to produce SEDs close to the SEDs predicted by our model. However, the temperature distribution required is very flat, so we have shifted the lowest allowed \( p \) from 0.5 to 0.05. For each object all the three extracted spectra were fitted simultaneously. Spectral ranges \( 0.1 - 12 \text{ keV} \) for PN and \( 0.1 - 10 \text{ keV} \) for MOS data were used. Photoelectric absorption by a solar metallicity material was included as a free parameter in all the models.

For the supercritical funnel model used for X-ray data fitting we fixed the velocity exponent \((\alpha = -0.5)\), the outer radius \((r_{out} = 100)\) and mass ejection rate \((\text{to } \dot{m} = 10^4)\) and the inclination to \(0^\circ\). For fixed \( T_{in}, r_{in} \) and \( r_{out} \) values the only effect of the mass accretion rate is in changing the gas velocities. As long as \( v(r_{in}) \ll c \), the mass ejection/accretion rate \( \dot{m} \) does not affect the shape of the X-ray spectrum.

Low values of \( \alpha \sim 0.. -0.5 \) provide better fits to the data. However, we fix the parameter to avoid degeneracy with \( T_{in} \) and \( r_{in} \). \( \alpha = -0.5 \) is expected if the velocity is proportional to the virial velocity at any given radius. In ballistic approximation (when the particles of the wind are first rapidly accelerated and then move along hyperbolic trajectories in the gravitational field of the accretor) \( \alpha \simeq -0.5 \) in the inner parts of the flow and approaches 0 at larger radii.

In table 1 we present the results of spectral fitting. It may be seen that in both cases a two-component standard disc + power law model gives the best results because of being capable to fit the harder part of the spectrum that is difficult to handle using only thermal models with exponential high-energy cut-offs.

Because the best-fit temperature in both cases is rather high \( T_{in} \sim 1 - 2 \text{ keV} \), we expect the inner parts of the funnel to be practically transparent and \( R_{in} \) to be close to the last stable orbit radius. In this assumption, \( r_{in} \) may be directly converted to the mass accretion rate. If we equate the inner radius to the last stable orbit for a Schwarzschild black hole,

\[
\dot{m} = \frac{\cos \theta_f}{18} \frac{R_{sph}}{R_G} \approx \frac{1}{6r_{in}}
\]

The estimated dimensionless mass ejection rates are therefore about 100 for NGC4559 X-7 and about 300 for NGC6946 ULX-1, correspondingly. The latter value is consistent by the
order of magnitude with the mass accretion rate estimate for NGC6946 ULX-1 resulting from the optical spectroscopy of the nebula MF16 associated with this X-ray source (Abolmasov et al. 2008).

The well-known high-energy curvature (Dewangan et al. 2005) can be explained by $T_{\text{in}} \sim 1-2.5\text{keV}$. If a hardening factor $\sim 1-2$ is used (see also section 6), the inner temperature may be similar to the expected maximal temperature for a critical accretion flow around a $\sim 10M_{\odot}$ black hole given by equation (38) or even slightly lower. High inner temperatures like $10\text{keV}$ also result in acceptable fits. Model is insensitive to $r_{\text{out}}$ save for the cases when this parameter is $< 1$ therefore we fixed the parameter to 100.

Low values of $r_{\text{in}}$ appear to be a real feature of ULXs. Funnel interior is expected to be practically transparent to the X-rays down to several gravitational radii, having rather deep-lying bottom pseudo-photosphere (or none at all). Observations in a broader spectral range are needed to distinguish between the thermal radiation from the immediate vicinity of the compact object and comptonisation effects.

Though the model fits the data in a quite acceptable way its parameters are poorly constraint. The spectral shape does not depend very much on the actual accretion rate and velocity law. There are numerous ways of making the spectrum harder: taking into account the difference between mass accretion and ejection rates, applying comptonisation effects etc.

6. Discussion

6.1. Limitations of the Model

Our calculations do not account for radiation feedback on the dynamics. That is not only wind acceleration within the regions of interest but also evaporation from the funnel walls and development of instabilities in the outer parts of the funnel in the strong radiation field of the inner parts. The instability may resemble that of accretion discs with irradiation (see Mineshige (1993) and references therein) but the wind is unable to influence the irradiating source and its role in developing the instability is purely passive.

The structure of the inner parts of the flow may be much more complicated than we assumed. At $r \sim 1$ the source of the wind becomes spatially resolved and our radial approximation fails. Besides this, the angular distribution of the energy influx may be more complicated and higher angular harmonics may appear.

Qualitatively the effects of non-zero size mass loading region may be considered as follows: let us assume that $\dot{m} \propto r$. The main radial equation for $k = 0$ takes the form:

$$\partial_r u - \frac{1}{r^{2+\alpha}} \partial_r \left( r^{3+\alpha} \partial_r u \right) + \frac{\gamma(2+\alpha)}{r} u = 0$$

It is easy to check that the equation allows two power-law solutions ($u \propto r^\sigma$, where $\sigma = -0.5 \left( (1+\alpha) \pm \sqrt{(1+\alpha)^2 + 4\gamma(2+\alpha)} \right)$) corresponding to inward and outward diffusion. For
$r \ll 1$ and $\alpha = -0.5$ energy density decreases as $u \propto r^{-1.68}$ instead of $u \propto r^{-2}$. The difference increases for higher $\alpha$.

We do not account for the emission of the hot interior of the funnel as well as for thermal comptonisation effects in the wind and pure reflection. All these effects are likely to harden the outcoming spectra making inner temperature estimates from the observational data shifted towards higher values. Hardening factor $T_h \simeq 2.6$ measured for standard discs (Borozdin et al. 1999) together with relativistic effects may lead to the inner temperatures about 1 keV appear as several keV.

Reflection and absorption by moving partially ionized gas may simulate the soft-excess observed in many ULX spectra (Stobbart et al. 2006) in a way it was proposed by Gierliński & Done (2004) for AGNi and recently by Gonçalves & Soria (2006) for ULXs. Understanding the structure of the outer photosphere of the wind requires more complicated modelling taking into account both significant non-sphericity of the outflow and various opacity sources.

6.2. Relativistic Effects

Broadly speaking, relativistic effects are expected to change photon energies and fluxes by factors close to $\delta = (\gamma (1 - \beta \cos \theta_f))^{-1}$. Here, $\beta$ and $\gamma$ correspond to the local dimensionless velocity and Lorentz-factor. The temperature of the X-ray spectrum is additionally increased by:

$$\frac{\Delta T}{T} = \delta - 1 \simeq 5 \times 10^{-3} \cos^{3/2} \theta_f \dot{m}_3^{-1/2} r_m^{\alpha}$$

The effect becomes significant if the velocity is relativistic in the inner parts of the flow (that corresponds to $\alpha \lesssim -0.5$ in the above formula). The total observed luminosity is altered by a factor $\delta^3$ ($\delta^2$ factor from geometrical reasons and one $\delta$ from the increased energy of the photons). For $\dot{m} = 10^3$ the effect is about one percent if the velocity at the spherisation radius is considered. However, if the inner parts of the wind have mildly relativistic velocities, the X-ray part of the spectrum appears about 2–3 times brighter for $\beta \sim 0.3$, $\theta_f \sim 20^\circ$.

The funnel bottom is the most likely part of the flow to be affected by relativistic effects. The gas inside the funnel cone is supposed to move with mildly relativistic velocities (Eggum et al. 1988), approximately towards the observer at low inclination angles. The observed colour temperature changes by a factor of $\delta \simeq 1 + \beta \simeq 1.1 - 1.5$. The total observed luminosity becomes several times higher. Our model allows to include the relativistic Doppler effect in calculations accurately for $i = 0$ (see section 4). At non-zero inclinations the deviations from our approximation (that all the funnel wall material moves at $\theta_f$ angle with respect to the line of sight) are of the order $O(\beta \sin i)^2$.

Relativistic aberration is also able to dump irradiation effects. The X-ray part of the spectrum will be the most affected. Irradiation becomes considerably smaller if the radiation emitted by the moving gas becomes anisotropic enough. However, for mildly relativistic flows ($\beta \lesssim 0.5$) the effect is not so severe compared to the effects mentioned above. For example,
the irradiating flux from the funnel bottom changes for \( r \approx r_\text{m} \approx 1/6 \dot{m} \) and \( \alpha = -0.5 \) by about \( \gamma^3 - 1 \approx 20 - 30\% \). At large radii the effect is smaller and may change sign at certain radii.

6.3. Observational Predictions for ULXs

There are several effects expected if our estimates are correct: (i) supercritical accretors viewed at low inclinations \( (i \lesssim \theta_f) \) will be seen as ULXs \( (L \gtrsim 10^{39} \text{ erg s}^{-1}) \); (ii) independently of its inclination a supercritical accretor is a luminous UV source; (iii) predicted X-ray spectra of supercritical accretors viewed face-on are similar to p-free model spectra with \( p \sim 0.4 - 0.6 \); (iv) because of high EUV luminosities supercritical accretors should ionize the wind above the photosphere and establish Strömgren zones; (v) if the accretion disc precession characterising SS433 is usual for supercritical accretors viewed nearly face-on, strong X-ray flux modulation is expected for at least a number of sources with “super-orbital” periods like tens and hundreds of days.

Most of the observational properties of ULXs are naturally explained in our model. There is observational evidence that at least some ULX nebulae are powered by photoionisation from the central source having luminosity comparable with the apparent X-ray luminosity. IMBH binaries should have difficulties in ionizing the surrounding gas unless the mass of the IMBH is very high, \( \gtrsim 10^4 \text{M}_\odot \) (see discussion in Abolmasov et al. (2008)).

For very high accretion rates \( \dot{m} \gtrsim 10^3 \) the outer photosphere is very large, comparable to the probable size of the binary system. Applying \( \alpha = 0 \) and \( \theta_f = 0.4 \text{rad} \) results in physical radius values:

\[
R_{\text{out}} \simeq 4 \times 10^{13} \dot{m}^{3/2}_3 \text{cm}. \tag{48}
\]

However in a real high-mass binary conditions the structure of the outflow is perturbed by tidal forces and the outflowing gas itself rapidly recombines (see next section). Because the flow is essentially supersonic and perturbed by a strong non-axisymmetric potential it is likely to become highly inhomogeneous. The equatorial outflow of SS433 is an example of such kind. The photosphere size in the optical then saturates at a radius of the order 10^{12} \text{cm} preventing wind photospheres from becoming “red hypergiants” with very high infrared luminosities. The effect may become significant starting from \( \dot{m} \sim 100 \).

Further understanding of supercritical accretor winds will require methods used for stellar atmosphere calculations. Rosseland mean for \( n \lesssim 10^{10} \text{cm}^{-3} \) and \( T \sim 10^4 - 10^5 \text{K} \) is very close to \( \kappa_T \) (Iglesias & Rogers 1996) but in certain spectral ranges the wind should be less transparent. An edge-on ULX will mimick an OB-hypergiant with dense and fast wind or a low-temperature hydrogen-rich WR.

6.4. Photoionized Nebulae

In the case of highly supercritical accretion UV and EUV spectra may give much more information about the mass ejection rate than the X-ray properties. In figure 9 we show the
dependence of H and He\textsuperscript{+} ionising fluxes (1 – 1.5 and 4 – 20 Ry ranges, respectively) and corresponding luminosities of recombination emission lines on mass accretion/ejection rate. Here we suggest that the HII regions have fixed temperature $T = 10^4$ K and density $n = 100 \text{cm}^{-3}$. Atomic data were taken from Osterbrock & Ferland (2006). One may see in the figure that the high emission-line luminosities observed in ULX nebulae may be well explained by high EUV luminosities of the central sources. The HeII $\lambda 4686 / \text{H}\beta$ ratios predicted by the model are close to the high HeII $\lambda 4686 / \text{H}\beta$ ratios $\sim 0.2$ measured for some high-excitation ULX nebulae or the inner high-excitation parts observed in some of the ULX shells (Abolmasov et al. 2007).

The situation becomes more complicated if one takes into account absorption in the wind optically thin to electron scattering. Density at the wind electron-scattering photosphere is

$$n = \frac{\dot{M}}{\Omega_f R_{\text{out}}^2 v} \simeq 3 \times 10^9 \dot{\mathcal{m}}_3^{-3/2} \text{cm}^{-3}$$

(49)

Recombination will occur in a layer having thickness:

$$\Delta R = \frac{\Delta n}{a n} \simeq 2 \times 10^{11} \dot{\mathcal{m}}_3 T_4^{-1/2} \text{cm},$$

(50)

where $a \simeq 2.6 \times 10^{-13} T_4^{-1/2} \text{cm}^3 \text{s}^{-1}$ is the effective recombination rate and $T_4$ is gas temperature in $10^4$ K units. The size of the recombination region is close to or smaller than the size of the outer photosphere. Actually that means that the outer photosphere of the wind will be optically thick to the EUV radiation of the central source.

Hard radiation from the pseudo-photosphere may however ionize the gas. Because the density of the wind falls off rapidly two regimes appear: ionized and neutral wind. First case makes the photosphere of the wind a UV object with high bolometric correction and a compact HII-region coincident with the X-ray source. In the latter the photosphere is much cooler and mimicks a hypergiant with broad emission lines.

The ability of the central source to ionize the wind may be calculated as follows. Recombination rate integrated over the outer wind is

$$I = \int_{R_{\text{out}}}^{R_{\text{max}}} a \Omega_f n^2(R) R^2 dR$$

(51)

$R_{\text{max}}$ here is the radius of Strömgren zone ionized by the central source. $I$ may be used as an estimate for the number of hydrogen-ionizing quanta intercepted by the wind, if $R_{\text{max}}$ is set to infinity:

$$S \simeq \frac{2}{\cos^3 \theta_f} \frac{a R_G \dot{\mathcal{m}}_3^{3/2}}{\sigma_T^2} \simeq 1.2 \times 10^{47} \dot{\mathcal{m}}_3^{3/2} \text{ s}^{-1}$$

(52)

This recombination rate is usually lower than the quanta production rate, affecting only the highest accretion rates. In figure 9 we present the ionizing quanta production rates for different mass ejection rates. ULX nebulae may appear even brighter due to two additional effects:
Fig. 9. Numbers of H (solid curve) and He$^+$ (dotted) -ionizing photons produced by our supercritical wind model (upper panel) and the luminosities of relevant recombination lines (b): H$\alpha$ and H$\beta$ (solid) and HeII$\lambda$4686 (dotted). $\alpha = 0$, $r_{in} = 1/6\dot{m}$ and $\theta_f = 23^\circ$ is assumed. Thin straight solid line shows the number of quanta absorbed in the wind.
“stripping” of the outer wind photosphere (the effect proposed in previous section) and higher accretion power due to $K < 1$.

7. Conclusions

Optically thick wind with irradiation is capable to explain the SEDs of ULXs in the standard X-ray band and even the high-energy curvature that is difficult to explain in the framework of unsaturated comptonisation cool-disc IMBH model (Dewangan et al. 2005). High-energy cut-off is predicted to appear at several keV. Higher observed values of $T_{in} \sim 1 - 2$ keV may be explained by applying a hardening factor $\sim 1 - 3$ similar to those predicted for accretion discs in X-ray binaries.

Outcoming spectra observed at low inclinations are similar to the spectra of slim discs and resemble $p$-free model spectra with the $p$ parameter close to 0.5. X-ray spectra of known ULXs are approximated equally good by our model, $p$-free and standard disc + power law two component models. However the parameters are poorly constraint supporting the idea that the properties of the X-ray spectrum depend rather weakly on the accretion disc and wind parameters. Relatively high inner temperatures argue for the funnel interior to be transparent. In this assumption, $r_{in}$ parameter may be used to estimate the mass ejection rate that appears to be of the order $\dot{m} \sim 100 - 300$ for the two sources analysed.

We stress the extreme importance of irradiation effects providing mild geometrical col-
limation of the observed X-ray radiation. In a simple assumption of local absorption and re-radiation of the absorbed energy we show that the temperature of the funnel wall surface is altered by about 20% in the inner parts of the funnel, and the outcoming apparent X-ray luminosity becomes about 2 – 3 times higher.

Photoionized nebulae are likely to be formed around supercritical accretors. Ionizing quanta production rates suggest that in most cases a supercritical accretor is capable to produce a photoionized HII-region with bright optical emission line luminosities $\sim 10^{37}$ ergs$^{-1}$ but higher luminosities may appear as well.

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Table 1. Best fitting results for the two selected ULXs. Errors correspond to 90% confidence range.

|                             | NGC4559 X-7 | NGC6946 ULX-1 |
|-----------------------------|-------------|---------------|
| **standard disc + power law** |             |               |
| $N_H$, $10^{22}$cm$^{-2}$   | 0.219$^{+0.03}_{-0.02}$ | 0.44$^{+0.08}_{-0.05}$ |
| $T_{in}$, keV               | 0.155$^{+0.010}_{-0.009}$ | 0.156$^{+0.018}_{-0.019}$ |
| **standard disc normalization** | 150$^{+120}_{-60}$ | 620$^{+200}_{-200}$ |
| $\Gamma$                    | 2.24$^{+0.05}_{-0.05}$ | 2.46$^{+0.15}_{-0.09}$ |
| **power law normalization** | $2.3^{+0.14}_{-0.14} \times 10^{-4}$ | $3.3^{+0.6}_{-0.5} \times 10^{-4}$ |
| $\chi^2/DOF$                | 648/673      | 523/504       |
| $L_{model}, 10^{39}$ ergs$^{-1}$ | 9.6$^{+4.4}_{-1.6}$ | 3.1$^{+3.0}_{-1.0}$ |
| **p-free disc**             |             |               |
| $N_H$, $10^{22}$cm$^{-2}$   | 0.0407$^{+0.0011}_{-0.0009}$ | 0.293$^{+0.03}_{-0.016}$ |
| $T_{in}$, keV               | 0.64$^{+0.22}_{-0.23}$ | 2.3$^{+0.4}_{-0.4}$ |
| $p$                         | 0.466$^{+0.004}_{-0.005}$ | 0.404$^{+0.01}_{-0.01}$ |
| **p-free disc normalization** | $1.4^{+0.9}_{-0.5} \times 10^{-4}$ | $3.3^{+2}_{-0.4} \times 10^{-4}$ |
| $\chi^2/DOF$                | 734/674      | 665/509       |
| $L_{model}, 10^{39}$ ergs$^{-1}$ | 9$^{+6}_{-3}$ | $3 \pm 1$   |
| **self-irradiated multi-color funnel** |             |               |
| $N_H$, $10^{22}$cm$^{-2}$   | 0.167$^{+0.026}_{-0.015}$ | 0.293$^{+0.016}_{-0.011}$ |
| $T_{in}$, keV               | 1.41$^{+0.16}_{-0.12}$ | 1.8$^{+0.3}_{-0.2}$ |
| $r_{in}$                    | $1.6^{+0.9}_{-1.1} \times 10^{-3}$ | $6^{+5}_{-3} \times 10^{-4}$ |
| $\theta$, deg              | 12.2$^{+0.7}_{-0.6}$ | 19$^{+2}_{-2}$ |
| normalization               | $30^{+62}_{-21}$ | $74^{+50}_{-50}$ |
| $\chi^2/DOF$                | 660/673      | 604/504       |
| $L_{model}, 10^{39}$ ergs$^{-1}$ | $10.5^{+6}_{-4}$ | $3^{+2}_{-1}$ |
Appendix 1. Integration over \( \varphi \)

The integral over \( \varphi \) can be calculated as follows:

\[
I = R^2 \int_{-\pi}^{\pi} w(R'/R, \theta_f, \varphi) d\varphi = \frac{1}{2\pi} x \sin^2 \theta_f \cos^2 \theta_f \int_{-\pi}^{\pi} \frac{(1 - \cos \varphi)^2}{(a + b \cos \varphi)^2} d\varphi
\]

Where \( a = 1 - 2x \cos^2 \theta_f + x^2 \), \( b = -2x \sin^2 \theta_f \).

\[
I = \frac{1}{2\pi} x \sin^2 \theta_f \cos^2 \theta_f \left( \int_{-\pi}^{\pi} \frac{1}{(a + b \cos \theta_f)^2} d\varphi - 2 \int_{-\pi}^{\pi} \frac{\cos \varphi}{(a + b \cos \theta_f)^2} d\varphi + \int_{-\pi}^{\pi} \frac{\cos^2 \varphi}{(a + b \cos \theta_f)^2} d\varphi \right) = \\
= \frac{1}{2\pi} x \sin^2 \theta_f \cos^2 \theta_f \left( (1 - \frac{a^2}{b^2}) \int_{-\pi}^{\pi} \frac{1}{(a + b \cos \theta_f)^2} d\varphi - 2 \left( 1 + \frac{a}{b} \right) \int_{-\pi}^{\pi} \frac{\cos \varphi}{(a + b \cos \theta_f)^2} d\varphi + \frac{2\pi}{b^2} \right)
\]

The two integrals can be expressed as follows (for details see for example Dwight 1961 or any other table of integrals):

\[
\int_{-\pi}^{\pi} \frac{1}{(a + b \cos \theta_f)^2} d\varphi = \frac{a}{(a^2 - b^2)} \int_{-\pi}^{\pi} \frac{1}{a + b \cos \theta_f} d\varphi = \frac{2\pi a}{(a^2 - b^2)^{3/2}}
\]

\[
\int_{-\pi}^{\pi} \frac{\cos \varphi}{(a + b \cos \theta_f)^2} d\varphi = -\frac{b}{a^2 - b^2} \int_{-\pi}^{\pi} \frac{1}{a + b \cos \theta_f} d\varphi = -\frac{2\pi b}{(a^2 - b^2)^{3/2}}
\]

Finally, the integral value becomes:

\[
I = \frac{x \sin^2 \theta_f \cos^2 \theta_f}{b^2} \left( 1 - \frac{a^2 + a - 2b}{a - b - a - b} \right) = \\
\frac{2\pi x \cot^2 \theta_f}{4} \left( 1 - \frac{|1 - x|}{\sqrt{1 - 2x \cos(2\theta_f) + x^2} - 1 - 2x(1 - 3 \sin^2 \theta_f) + x^2} \right)
\]