Utilisation of the minimum paths in networks for transport terminals

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Abstract. The reason behind this material consisted in the lack of similarity between complementary issues in a field that includes graphs and networks. For example, a theoretical network problem pretends to determine the minimum number of nodes which must be excluded to remove all the grid links, a practical network problem claims to determine the shortest route between 2 nodes when it knows the length of each link. In a strange nod the first problem has a maximum solution, whereas the second problem has no maximum solution (only if the network is oriented and do not allow circuits, conditions that take out the issue from the general sphere, drastically customizing it). The first problem can also develop in the situation in which the issue is to exclude all arcs, less one and almost symptomatic: there was no procedure to offer the second minimum path in the graph. Here, it is proposed a modality to find the second minimum path avoiding the complete enumeration of paths between the 2 nodes taken in consideration. The study case presented in this paper is based on railway terminals – Gara de Nord’s traffic – after the first shortest way for stabling which will be the next, and also this type of application can be used on the efficiency of maritime berth. The utility of the idea concerning the second minimal path is found in the current exploitation of almost any transport system (for e.g. the review of the initial planning, when the established side-tracking in receiving board from the final station cannot be used for intensive traffic reasons, or receiving the train at to another line must be done so as all the future movement of the traffic circulation or train wagon marshal to mess as little as possible the set of movements necessary for further exploitation, receiving the bus to another platform because the first platform is busy with a defective bus).

1. Introduction. The problem of the shortest path

From the progresses in research operational fields, it’s said that the theory of networks is a concern of the simplest, more elegant and with a wide variety of applications. Research on the mathematical modelling in the transports field was also presented in the papers [1, 4-7].

As a result, it has been recognized as being one of the most important unitary concepts that binds many separate and independent applications. From the beginning, a distinction should be made between graph theory and network theory [2]. While a graph defines purely structural links between nodes, a network also contains the qualitative and quantitative qualities of the nodes and arcs. For example, a theoretical graph problem is as follows: in a connected graph with n nodes and m arches, what is the minimum number of nodes that would be excluded for remove all arches from the graph? This is the well-known problem of "minimal coverage" (it is obvious the dependence only on the structure of the graph). On the other hand, the formulation for the network problem is the following:
given the graph in the \( n \) nodes and \( m \) arcs and knowing that the arc \((i, j)\) has the length \(d_{ij}\), what is the shortest path between two given nodes \(s\) and \(t\)?

This is the well-known problem of “the shortest path” (here it is obvious that the answer does not depend only on the graph structure, but also the values of the lengths defined on the arcs as part of the graph.

The problems of the shortest path can be classified in four main categories.

Problem 1. Find the shortest path between the specified nodes \(s\) and \(t\)

Problem 2. Find the first \(m\) – paths, the shortest of the two specified nodes \(s\) and \(t\)

Problem 3 Find the shortest paths between an origins and all other nodes of the network, or, the same type of problem, find the shortest paths between all nodes and a terminus \(t\).

Problem 4. Find the shortest path between all the pairs of nodes

Problem 2 which is the subject of this paper is the generalization of problem 1 and needs some explanations. It is assumed that all paths from \(s\) to \(t\) are listed and their length is calculated; the \(j\)-th path in the enumeration is noted with \(\pi_j\), and his length with \(L(\pi_j)\). Supposedly, in continuation, that the paths are arranged in the ascending order of their lengths, so 
\[ L(\pi_1) \leq L(\pi_2) \leq \ldots \leq L(\pi_n). \]
The path \(\pi_1\), is named the shortest path between \(s\) and \(t\), path \(\pi_2\), the second shortest path etc.

In the problem it is required to determine the first \(m\) - the shortest path between \(s\) and \(t\). This is not a simple problem because it is desirable to avoid the complete enumeration of path from \(s\) and \(t\) [2, 8].

2. The algorithm for determining the first shortest path. The procedure for determining the second short path in a graph

The graphic representation is given below:

![Graph Diagram](image)

Fig. 1 The considered graph for the description of the proceeding (in the figure between two nodes we have on arches the distance )

The second minimal path between \(A\) and \(C\) is required. The solution can be obtain (one of the variants) after final matrix determination of the Shimbell algorithm, thus obtain the shortest path from \(A\) to \(C\): the shortest distance, of 5 (length units) is on the AEDGC route.
Tab. 1 The matrix of the minimal path for the graph above

|   | A  | B  | C  | D  | E  | F  | G  | H  |
|---|----|----|----|----|----|----|----|----|
| A | 0  | 4  | 5  | 2  | 1  | 1  | 4  | 3  |
| B | 4  | 0  | 2  | 3  | 4  | 3  | 1  | 1  |
| C | 5  | 2  | 0  | 3  | 4  | 5  | 1  | 3  |
| D | 2  | 3  | 3  | 0  | 1  | 4  | 2  | 2  |
| E | 1  | 4  | 4  | 1  | 0  | 2  | 3  | 3  |
| F | 1  | 3  | 5  | 4  | 2  | 0  | 4  | 2  |
| G | 4  | 1  | 1  | 2  | 3  | 4  | 0  | 2  |
| H | 3  | 1  | 3  | 2  | 3  | 2  | 2  | 0  |

In this proposal, for determine the second shortest path, the initial graph is converted into an equivalent one, but the last arc of the route (GC) is noted separately from the rest of the structure; with other words, the last but one node from the absolute minimal path will be dissociated in more node who have to task of separating the path:

- A route that maintains the absolute minimum path AC (but for nor enter in the cycle- then when is recalculated the minimum path in the modified graph –the length of the last arch will change in the direction of the increase with two units of length. To underline: in the course of this procedure it is obligatory that the arc lengths be expressed in integer numbers (aspect that can be solved by a mathematical procedure indifferent of the real length of any arc);
- A path or more that keeps in the graph the variants to achieve the final node on other ways than that proven to be absolute minimum;
- A path or more that keeps in the graph the hesitation possibilities of the last node that is on the absolute minimum path.

In the below figure:
- g1 = necessary node for prevent the solution of the absolute minimum path if would give up at the supplement of the 2 length units
- g2 = necessary node for the route from the initial node A forward final node C
- g3 = necessary node for the route from the initial node A backwards to another nodes without C node

Fig. 2. The modified graph so that it can be able to avoid the absolute minimum path (C end) (in the figure between two nodes we have on arches the distance)
The shortest distance, of 6 (length units) can be obtain on route A F H B g2 C and is resumed the procedure with the exemplification for the other end of the path.

Tab. 2 The matrix of minimum path for the graph from figure 2

|     | A | B | C | D | E | F | g1 | g2 | g3 | H |
|-----|---|---|---|---|---|---|----|----|----|---|
| A   | 0 | 4 | 6 | 2 | 1 | 1 | 4  | 5  | 4  | 3 |
| B   | 4 | 0 | 2 | 3 | 4 | 3 | 1  | 1  | 1  | 1 |
| C   | 6 | 2 | 0 | 5 | 6 | 5 | 3  | 1  | 3  | 3 |
| D   | 2 | 3 | 5 | 0 | 1 | 4 | 2  | 5  | 2  | 2 |
| E   | 1 | 4 | 6 | 1 | 0 | 2 | 3  | 6  | 3  | 3 |
| F   | 1 | 3 | 5 | 4 | 2 | 0 | 5  | 4  | 4  | 2 |
| g1  | 4 | 5 | 3 | 2 | 3 | 5 | 0  | 4  | 4  | 4 |
| g2  | 5 | 1 | 1 | 5 | 6 | 4 | 4  | 0  | 2  | 2 |
| g3  | 4 | 1 | 3 | 2 | 3 | 4 | 2  | 0  | 2  | 2 |
| H   | 3 | 1 | 3 | 2 | 3 | 2 | 4  | 2  | 2  | 0 |

Fig. 3 The modified graph so that it can be able to avoid the absolute minimum path (A end) (in the figure between two nodes we have on arches the distance)

e1 = necessary node for prevent the solution of the absolute minimum path if would give up at the supplement of the 2 length units

e2 = necessary node for the route from the initial node C forward final node A
e3 = necessary node for the route from the initial node C backwards to another nodes without A node

The shortest distance, of 6 (length units) is obtain on the routes
- C G B H F A.
- C G D A.
Tab. 3. The matrix of the minimal path for the graph from fig 3

|   | A  | B  | C  | D  | E1 | E2 | E3 | F  | G  | H  |
|---|----|----|----|----|----|----|----|----|----|----|
| A | 0  | 4  | 6  | 3  | 3  | 1  | 4  | 1  | 5  | 3  |
| B | 4  | 0  | 2  | 3  | 4  | 3  | 4  | 3  | 1  | 1  |
| C | 6  | 2  | 0  | 3  | 4  | 7  | 4  | 5  | 1  | 3  |
| D | 3  | 3  | 3  | 0  | 1  | 4  | 1  | 4  | 2  | 2  |
| E1| 3  | 4  | 4  | 1  | 0  | 4  | 2  | 4  | 4  | 3  |
| E2| 1  | 3  | 7  | 4  | 4  | 0  | 5  | 2  | 7  | 4  |
| E3| 4  | 4  | 4  | 1  | 2  | 5  | 0  | 3  | 3  | 3  |
| F | 1  | 3  | 5  | 4  | 4  | 2  | 3  | 0  | 4  | 2  |
| G | 5  | 1  | 1  | 2  | 4  | 7  | 3  | 4  | 0  | 2  |
| H | 3  | 1  | 3  | 2  | 3  | 4  | 3  | 2  | 2  | 0  |

3. Comments about the method proposed for determine the second minimal path

For what is lucrative the second minimal path:

a. Because following the exploitation flexibility of transport networks, the method provides to the deciders, alternatives of differentiated routes through only a few km

b. Because the method is a logical continuation of the link idea of origin-destination with minim length

c. Because the last minimum path in chronology is the longest path – objective still untouched from operational research till now

d. Because the known algorithm until now are applied to some math construction which are, in fact, simplifications of the networks reality (and also them are simplified mathematical constructions of the reality represented by the infrastructure)

Critics could say that mathematical approach can turn any graph into a more comprehensive one, what could make the determination of the second short past an attempt perhaps theoretically useful, but from practical view useless. Basically, adjusting in increase or in reduction the arches length can be assimilated special features of the nodes and / or arcs into the graph structure, e.g.

- The passing through a nodes can be assimilated by introducing an additional distance in graph
- The saturation level (with traffic) of an arch increases the arch length with a value that could be estimated
- The precarious condition of the contact surface with the land increase, in fact, the length of the path

But, for e.g., the stress caused by crossing a certain intersection or crossing a certain terrestrial communication path is difficult to be involved by the physical elements of the graph; and more significantly, some organizational measures

- Only vehicles with certain tonnage can access certain path [3]
- It is not allowed making manoeuvre at left
- It is not allowed the circulation than between certain hours, it introduces restrictions that are difficult to represent in graph construction

4. Review. The sense of this paper followed a practical problem: to determine in graphs the minimal paths, others than the absolute minimal – for which exist known. But along the way, result that the principal idea is that the complete procedure of the representation of the networks proprieties through the graphs is relative limited (for certain cases), even common particularities cannot be transposed in
graphs structure - usually used instead networks. Concrete, the links of a network can be characterized from several points of view, so:

- Some arches are short, but do not allow speeds that can ensure small time to go through, respective
- Other arches are long, but crossing them could be done with speeds that allow small time for going through

In this particular situation, a problem could be that of the determination not of the shortest path and not of the fastest path, but of the shortest and fastest path. In the reader professional papers wasn’t found a methods which offers o path with the both particularities (a path that is not the shortest and not the fastest is not keeping a reasonable distance to the both implicit

The solution that was discerned could be touched in two ways:

- One that introduces on arch a conventional value which contains both kilometre length so crossing time – in favourable parts for the objective – so as to after determination of the shortest path can be said that the path is short and fast;
- Other that would leave on the arch only one propriety (natural) and would focus on determination of the shortest path but also the next chronologically path (finally choose the convenient path)

The exemplification was possible on the structure of the first graph that was analysed in this paper. In the below figure some arches have insert two proprieties: kilometre and crossing time. So we can find:

- The shortest path from distance regard have 5 km, but it can be crossed in 4.5 minutes
- The shortest second path from distance regard have 6 km, but could be crossed in 5 minutes
- The next path regarding distance is 6 km length, but can be crossed in 4 minutes

The perception between the last two alternatives is clear: it is preferable the last route that needs only 4 minutes for the same number of kilometres. And finally: if the weight of the distance parameter in for e.g. 0.3, and the weight of the rapidity parameter is 0.7, then the 5 km shortest route is not preferred, but the second is 6 km because

$$6*0.3 + 4*0.7 < 5*0.3 + 4.5*0.7$$

It could be proved that some values of the parameters (length 1, 2, 3 respective time 1,2,3) leads to choose a path than only the shortest, only the fastest, e.g. for 3 path

- 80 km, but 150 min
- 130 km, but 110 min
- 100 km, but 120 min

And even at the equal weight given to the importance of length and rapidity is preferable to the third that is neither the shortest nor the fastest.
5. Study case

The bundle of lines from Gara de Nord Station – the picture below
Principal activity—regarding railways, not commercial standpoint – is not represented the receiving and the dispatching of the trains, in fact the principal activity is represented by the bringing and taking over of the trains by the Basarab and Grivita stations which compose-decompose them. This manoeuvring activity is so laborious that it can often disturb the major traffic activity itself, the idea is the following: introducing a train on arrival at an inadequate line can generate inappropriate times for the following movement and maneuverer that obliges:

- Or to stop some trains in the entry signal pending change these in line with the needs of these trains
- Or the introduction of trains on lines from which the following movements are again affected by hostile paths
- and the phenomenon of adjusting the exploitation activity for deviate from the conditions previously set as being appropriate

Fig. 4 The false optimum to use the absolute minimum path for apply in exploitation (in the figure between two nodes we have on arches the distance and the travel time for the 2 points)

Fig. 5 A very simplified sketch of the Nord - Basarab - Grivita Complex (the number of the trakers are noted: 1, 2, 3, 4, 5, 7, 8, 9, 10, 11)
6. Conclusion
The shortest path for the necessary exploitation succession in case of certain forming plan is direction $b$ ... lines 3 ... Giuvița ... line 3 ... direction $c$
Which involve passing over a number of 15 switch paths 3.4.5.8 then 8.7.10.11 then 9.6.7.8 then 8.7.6

![Diagram of train station layout]

Fig. 6. The minimum path of the route for the chronology considered (the number of the trakers are noted: 1, 2, 3, 4, 5, 7, 8, 9, 10, 11)

If line 3 is not available - for the same succession, then preferring line 1, obtain a number of 19 switch paths crossed 3.4.5.8 then 8.7.10.11 then 9.6.3.4.1.2 then 2.1.4.7.6.
If however the second minimal path would have been determined which involve the line 2 utilisation another way of solving would have been found namely with only 17 switch paths crossed: 3.4.5.8 then 8.7.10.11 then 9.6.3.4.5 then 5.4.3.6.
The authors consider that this example shows the utility of the second shortest path. It can be use in transport field as base of the determination of minimum path.

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