Coordinating charging request allocation between self-interested navigation service platforms

Marianne Guillet\textsuperscript{1,2} and Maximilian Schiffer\textsuperscript{1,3}

\textsuperscript{1}TUM School of Management, Technical University of Munich, 80333 Munich, Germany
\textsuperscript{2} TomTom Location Technology Germany GmbH, 12435 Berlin, Germany
\textsuperscript{3} Munich Data Science Institute, Technical University of Munich, 80333 Munich, Germany, marianne.guillet@tomtom.com, schiffer@tum.de

Abstract

Current electric vehicle market trends indicate an increasing adoption rate across several countries. To meet the expected growing charging demand, it is necessary to scale up the current charging infrastructure and to mitigate current reliability deficiencies, e.g., due to broken connectors or misreported charging station availability status. However, even within a properly dimensioned charging infrastructure, a risk for local bottlenecks remains if several drivers cannot coordinate their charging station visit decisions. Here, navigation service platforms can optimally balance charging demand over available stations to reduce possible station visit conflicts and increase user satisfaction. While such fleet-optimized charging station visit recommendations may alleviate local bottlenecks, they can also harm the system if self-interested navigation service platforms seek to maximize their own customers’ satisfaction. To study these dynamics, we model fleet-optimized charging station allocation as a resource allocation game in which navigation platforms constitute players and assign potentially free charging stations to drivers. We show that no pure Nash equilibrium guarantee exists for this game, which motivates us to study VCG mechanisms both in offline and online settings, to coordinate players’ strategies toward a better social outcome. Extensive numerical studies for the city of Berlin show that when coordinating players through VCG mechanisms, the social cost decreases on average by 42\% in the online setting and by 52\% in the offline setting.

\textbf{Keywords:} electric vehicles, mechanism design, resource allocation, congestion game
1. Introduction

Battery-powered vehicles support the shift towards more sustainable mobility systems, especially if coupled with the overall increased use of public transportation systems. After a slow start for EV adoption, the private electric vehicles market recently showed an encouraging growth, with global adoption doubling in 2020 and 2021, and car manufacturers planning to launch more than 100 new electric vehicle models by 2024. However, studies reveal strong adoption heterogeneity, at national and regional scales. In 2020, Norway had a (BH)EV market share of more than 70% [cleantechnica 2021], whereas Romania’s EV market share was less than 10% [AVER 2021]. In the latter case, the adoption is thus far hindered by poor or missing charging infrastructure. In general, charging-related anxieties caused by limited infrastructure availability dissuade conventional vehicle drivers from switching to electric vehicles. Drivers may experience so-called range anxiety, i.e., the fear of running out of battery, or charge anxiety, i.e., the fear of not finding any available and non-broken public charging station. While building new charging stations may alleviate these anxieties, operating over-dimensional charging infrastructure in low charging demand areas can be highly cost-inefficient [Nelder & Rogers 2019]. Overcoming this chicken-and-egg dilemma, i.e., deciding whether to pursue EV adoption or charging infrastructure first, can be mitigated with appropriate infrastructure planning and operation (cf. Enlit 2022).

From a short-term perspective, drivers’ anxieties related to charging in an undersized infrastructure can be addressed via advanced operational planning, e.g., a navigation system that reliably guides EV drivers until they find an available station, anticipating possibly blocked, unreachable, or out-of-order stations. To foster EV adoption from a long-term perspective, regulatory measures, e.g., a ban on conventional vehicles in city centers or financial incentivization are necessary, but must be complemented by planning solutions. Such solutions include developing the charging infrastructure with new equipment, consolidating existing urban infrastructure (e.g., street lamp charging), or increasing the reliability of real-time availability information through detection sensors or cameras.

The expected growing charging demand can nevertheless create new bottlenecks, i.e., charging station congestion, if demand is misaligned with charging infrastructure capacities. Even with an increased charging infrastructure coverage, several EV drivers may receive identical charging station recommendations from navigation services platforms, which creates station utilization conflicts due to long recharging times. Here, fleet-optimized recommendations – enabled by requests centralization – can better distribute the charging demand over the available charging stations and increase the overall users’ satisfaction (cf. Guillet & Schiffer 2022).

Such a fleet-optimization may work very well in presence of a single platform but finds its limits with the presence of several self-interested navigation platforms, each seeking to maximize their own drivers’ utilities. In this case, the overall outcome of the drivers’ assignment to stations may be far from optimal due to conflicting charging station allocations in between platforms. Accordingly, solely improving the infrastructure with no further charging demand coordination at the system level may worsen the EV charging experience by decreasing the likelihood of finding an available station within a minimum amount of time. If a system composed of multiple navigation services platforms cannot reach any good equilibrium, a centralized non-profit-driven authority,
e.g., a municipality, may aim to improve the system towards a socially efficient outcome, using financial incentives or penalties.

To understand the dynamics of such a setting, we introduce the fleet charging station allocation (FCSA) game in which several self-interested navigation platforms aim to individually optimize the assignment of available stations to their EV drivers, i.e., their charging demand. In this context, the scope of this paper is twofold. First, we use a game-theoretical framework to analyze the dynamics and bottlenecks that may arise in this setting due to non-cooperative driver assignments. We show that no guarantee for a pure Nash equilibrium (PNE) exists, which motivates our second scope: deriving mechanisms to stabilize and coordinate the charging station allocation, studying the impact of aligning platforms’ interests from a platform but also from a driver perspective.

1.1. Related Work

Our work relates to indivisible resource allocation problems for electric vehicles (EVs) and other related applications, e.g., parking utilization, which we review in the following. We first discuss resource allocation games, particularly with a focus on equilibrium analyses, before we review works that apply mechanism design to resource allocation problems.

Congestion games, initially introduced in Rosenthal (1973), model non-cooperative atomic resource allocation between homogeneous agents. Here, a player’s strategy is defined as a subset of the available resources, whose utilization cost depends on the total number of players selecting the resource. This class of games naturally applies to load-balancing (see, e.g., Even-Dar et al., 2003; Goemans et al., 2006), network design (see, e.g., Anshelevich et al., 2008), or internet routing (see, e.g., Secci et al., 2011). Congestion games fall into the category of potential games (cf. Monderer & Shapley, 1996), i.e., games that admit a potential function, which describes the payoff variation between two different strategies independent of the players. Such games possess a guaranteed PNE existence. Milchtaich (1996)’s work extends Rosenthal (1973)’s work by showing the existence of a PNE in congestion games with player-specific payoff functions or weighted players, but restricted singleton strategies. Ackermann et al. (2009) later show that strategies that consist of the bases of a matroid defined on the resources set still guarantee the PNE existence for games analyzed in Milchtaich (1996).

Most EV-related games deal with charging capacity allocation in grid networks, focused on optimizing grid operators’ revenues (Tushar et al., 2012), reducing energy peak load through load and capacity-dependent energy prices (Sheikhim et al., 2013), or mitigating the impact of charging vehicles on transportation networks (Sohet et al., 2021). Ayala et al. (2011) analyze related competitive parking slot assignment problems, and show the existence of a PNE when each player, i.e., driver, must select at most one parking spot for which a successful utilization depends on her and other players’ driving distances to all parking spots. He et al. (2015) further extend the result of Ayala et al. (2011) by analytically characterizing equilibrium parking slot assignments.

While the FCSA game studied in this paper resembles a congestion game, its payoff functions are user-specific and depend on the number of other players selecting an identical resource only under some conditions: while all drivers competing for the same resource affect each other’s
payoff in a classical congestion game, in the FCSA game only drivers that are closer to utilizing the resource affect the payoff function of drivers that are further away from utilizing the resource. Moreover, congestion games have not yet been studied in the context of charging station allocation via intermediate self-interested participants.

To deal with inefficient equilibria, mechanism design (MD) theory aims to design games in such a way that a socially desirable outcome is reached \cite{Nisan2007}. A third party, a so-called principal, centrally allocates goods or resources to its participants, who must reveal their preferences on the resources before paying a price for the received resource. In this context, the well-known VCG mechanism \cite{Vickrey1961,Clarke1971, Groves1973} defines a pricing scheme that ensures revelation truthfulness by aligning participants and the overall system interests in an offline setting. As an offline allocation may be hard to realize in practice, \cite{Parkes2003} propose two online VCG mechanisms, namely delayed and online VCG, that utilize a Markov decision process (MDP) to derive the minimal expected cost allocation, while ensuring that expected participant utilities align with the expected system interest. They show the Bayesian-Nash truthfulness of these mechanisms and apply them successfully to WiFi pricing \cite{Friedman2003}. As the pricing scheme relies on an optimal policy argument, the authors later discuss approximately efficient online MD using \(\epsilon\)-efficient policies. More recently, \cite{Stein2020} proposed a reinforcement learning-based mechanism that guarantees strategy-proofness when the resource allocation problem is solved online. Focusing on EV charging, \cite{Rigas2020} apply standard VCG pricing schemes to achieve efficient charge scheduling of self-interested EV drivers by minimizing the charging demand imbalance at a station. \cite{Gerding2011} design a mechanism to solve the online charge scheduling problem, blind to future charging demand. Several works have studied mechanisms in the related context of parking slot allocation. \cite{Xu2016} derive a trading mechanism to share private parking slots during office hours in big cities between self-interested owners, to remedy limited public parking spot availability. \cite{Zou2015} extend the parking slot assignment game of \cite{Ayala2011} by applying VCG mechanisms to a publicly-owned parking facility that aims to maximize the social welfare, both in static and dynamic settings. \cite{Wang2020} derive an optimal allocation pricing scheme based on a Demange-Gale-Sotomayor-based mechanism, for reservable and non-reservable parking resources in a city.

In summary, most EV-related settings focus on charge scheduling problems with homogeneous self-interested EV drivers, while this work focuses on navigation platforms as self-interested participants. Similarly, work on parking slot allocation problems focus on directly incentivizing drivers to truthfully report their valuations, but not on intermediaries that could balance aggregated parking demand over available slots.

1.2. Aims and Scope

To close the research gaps outlined above, we introduce a game-theoretical setting to study the dynamics of self-interested navigation platforms that aim to best balance their own clients’ charging requests among available charging stations, focusing first on equilibrium analysis, and second on mechanism design. We formalize the resulting charging station allocation problem as a novel game, and apply both offline and online VCG mechanisms to ensure socially desirable
outcomes in idealized but also in practical settings. Specifically, our contribution is three-fold. First, we define the FCSA game in a perfect-information setting as a new resource allocation game, and show that this game does not possess a guaranteed PNE, but also no approximated PNE with a sufficiently small approximation factor. Second, we coordinate players by applying VCG pricing in both an offline and an online setting, accounting for weighted players. We extend the delayed VCG mechanism to a weighted delayed VCG mechanism, and implement a data-driven online assignment policy. Finally, we conduct extensive simulation-based experiments to analyze the benefits of our coordinated allocation mechanism. Our results show that players’ coordination via the VCG mechanism can decrease the social cost on average by 52% in an offline setting and by 42% in an online setting when using our data-driven assignment policy. Our results further show that a player with a small share of drivers has a greater interest in participating in the VCG mechanism than a player with a larger share of drivers. A player’s payoff relative to its number of drivers decreases when its share of managed drivers increases.

1.3. Organization

The remainder of this paper is structured as follows. Section 2 details our FCSA game setting and corresponding equilibrium analyses. In Section 3, we introduce the VCG mechanisms, which we apply in both offline and online settings. Section 4 describes our experimental design, while we discuss our numerical results in Section 5. Section 6 concludes this paper and provides an outlook for future research. For the sake of conciseness, we shift all proofs to the Appendix.

2. Problem Setting

We focus on a non-cooperative offline resource allocation problem, where several navigation platforms aim to optimally assign their EV drivers to available charging stations, such that no visit conflict arises for their drivers and the total travel time for drivers to assigned stations is minimal. In the following, we consider a perfect-information setting, in which each platform is aware of overall charging demand, and of accurately reported real-time charging station availabilities. Note that such a perfect-information setting is unrealistic in practice but allows us to analyze whether stable players’ strategies exist at least in an idealized setting. Each platform optimally assigns free stations to its navigated EV drivers, who accordingly drive to the stations to recharge their vehicles. To focus on the dynamics between the navigation platforms, we assume that electric drivers do not deviate from their assigned stations. In case of a conflicting assignment, i.e., if two or more platforms assign one of their drivers to the same station, the station availability is guaranteed for the driver with the earliest arrival time. The other assigned drivers fail their search, which induces a penalty cost for the respective platform.

In the following, we formalize our problem setting as a resource allocation game, show that a PNE guarantee does not exist, and discuss the limits of refinement equilibrium concepts within this context.
2.1. Fleet Charging Station Allocation game

We now define the FCSA game, in which navigation service platforms constitute the players. A player’s strategy is an assignment of charging stations to its drivers, while its payoff describes the cost of the assignment, corresponding to the sum of the time needed by each driver to travel to its assigned station. We assume that if multiple drivers are assigned to the same station, all non-closest drivers must pay an extra penalty cost that represents the discomfort for failing to find a free charging station. We further restrict the set of reachable stations for a driver by setting a maximal driving time, and assume a deterministic driver behavior as reasoned in the section’s introduction.

Formally, we consider a set of players $\mathcal{N}$, a set of drivers $\mathcal{D}$, and a set of available charging stations $v \in \mathcal{V}$. We denote with $\mathcal{V}' = \mathcal{V} \cup \{v_0\}$ the set of charging stations extended with an artificial station $v_0$, which represents a non-feasible assignment of a driver to any physical station. Such non-feasible assignments may occur when the number of drivers exceeds the number of available stations. We let $t_{k,v}$ be the driving distance for driver $k$ to charging station $v$. Each player must serve a charging demand, that corresponds to a set of driver locations, denoted by $\mathcal{D}_i = \{k_i \in \mathcal{D} \mid \forall i \in \mathcal{N}\}$. A player’s pure strategy $s_i$ is an allocation function of charging stations to drivers, with $s_i : \mathcal{D} \mapsto \mathcal{V}'$. We restrict the stations that a driver $k$ may visit to the stations reachable within $\bar{S}$ meters, i.e., $s_i(k) \in \{v : v \in \mathcal{V}', \gamma(k, v) \leq \bar{S}\}$, with $\gamma(k, v)$ being the distance from driver $k$ to station $v$. A player can assign a physical station at most once to its drivers, but can assign an artificial vertex $v_0$ as many times as needed, formally

$$\forall k, k' \in \mathcal{D}_i, (s_i(k) = s_i(k')) \land (s_i(k) \neq v_0) \Rightarrow (k = k').$$

A player must further assign a real station to each of its drivers if possible, i.e., at most $|\mathcal{D}_i| - |\mathcal{V}|$ drivers can be assigned to the artificial station $v_0$. We let the strategy profile $s = (s_i)_{i \in \mathcal{N}}$ be the vector of strategies of all players, $s_{-i}$ be the vector of strategies for all players but $i$, and $S_i$ be the strategy space for player $i$. We denote with $u_i(s)$ the payoff for player $i$ when strategy profile $s$ is played, which corresponds to the assignment cost of $s_i$, resulting from the sum of its individual driver’s payoff functions $c_i$, given the other competing players’ strategies $s_{-i}$. Formally, we define $u_i(s)$ as

$$u_i(s) = \sum_{k \in \mathcal{D}_i} c_i(k, s),$$

with

$$c_i(k, s) = \begin{cases} \bar{\beta}, & \text{if } s_i(k) = v_0 \\ t_{k, s_i(k)}, & \text{if } \forall k' \in \mathcal{D}_j \forall j \in \mathcal{N}, j \neq i \ (s_j(k') = s_i(k)) \Rightarrow (t_{k', v} \geq t_{k,v}) \\ t_{k, s_i(k)} + \bar{\beta} & \text{otherwise} \end{cases}$$

An individual driver $k$’s payoff $c_i(k, s)$ ensures that, in case of conflicting driver to station assignments, all but the earliest arriving drivers assigned to a station $v$ must pay a penalty cost $\bar{\beta}$. The penalty $\bar{\beta}$ represents the discomfort for failing the charging station search. Then, the goal of each player $i$ is to find a strategy $s_i$ that minimizes $u_i(s_i, s_{-i})$, given that the assignment cost depends on other players’ selected strategies.

We consider the pure-strategy profile $s^*$ to be a PNE when it holds that

$$\forall i \in \mathcal{N}, s^*_i \in \arg \min_{s_i \in S_i} u_i(s_i, s^*_{-i}),$$
i.e., no player can unilaterally decrease its payoff by changing its strategy. The sum of all players’ payoffs defines the social cost.

From a system-perspective, i.e., in an idealized setting with collaborative players, the goal is to find a strategy profile that minimizes the social cost. We refer to the minimum total cost as the social optimum, defined as

$$S_g = \min_{s \in S} \sum_{i \in N} \sum_{k \in D_i} c_i(k, s).$$  \hspace{1cm} (2.5)

With $A_v$ being the assignment of drivers to station $v$, $s(k)$ being the station assigned to driver $k$, and strategy profile $s$, we can reformulate the social optimum as

$$S_g = \min_{s \in S} \sum_{v \in V} \sum_{k \in A_v} t_{k,s(k)} + (|A_v| - 1) \cdot \bar{\beta}.$$  \hspace{1cm} (2.6)

2.2. Equilibrium analysis

To analyze possible stable outcomes of the FCSA game, we discuss the FCSA game’s equilibrium properties in the following. Here, we note that a Nash equilibrium corresponds to a strategy profile such that no player has an incentive to unilaterally deviate from its current strategy. Moreover, there always exists a Nash equilibrium for a strategy profile of mixed strategies, i.e., a profile that corresponds to a probability distribution over pure strategies. However, mixed strategies do not reflect the behavior of a player in practice, as a player assigns a set of physical stations to its drivers once, which makes the interpretation of mixed strategy Nash Equilibria difficult for real-world analyses. In contrast, a PNE allows for more interpretable outcomes but is not guaranteed to exist in the FCSA game.

**Proposition 1.** The FCSA game does not always possess a PNE.

**Proof.** We refer to Appendix A for a proof of Proposition 1.

For games with non-guaranteed PNE existence, $\rho$-equilibria with pure strategies constitute an interesting alternative to interpret the outcome of a game. Specifically, a $\rho$-equilibrium corresponds to a near-stable state in which each player cannot decrease its payoff by more than the absolute factor $\rho \geq 0$. In practice, such an equilibrium corresponds to each player accepting to slightly deviate from the best solution it may obtain if its deviation stabilizes the game outcome. Formally, this corresponds to a profile $s$, such that

$$\forall i \in N, \forall s'_i \in \mathcal{S}_i, u_i(s'_i, s_{-i}) > u_i(s) - \rho.$$  \hspace{1cm} (2.7)

However, we can assume that such equilibrium refinement is only of practical interest if a player’s payoff marginally increases when the player deviates from its best response, i.e., if $\rho$ is sufficiently small. In Proposition 2 we show that the existence of an equilibrium with $\rho$ being sufficiently small cannot be guaranteed, as we can construct instances in which for any set of strategy profiles at least one player can significantly decrease its payoff by deviating from its current strategy.

**Proposition 2.** The existence of a $\rho$-PNE with $\rho < \bar{\beta} - \Delta_t$, with $\Delta_t = t_{\text{max}} - t_{\text{min}}$ being the difference between the largest and the lowest driving times between a driver and a station, cannot be guaranteed for the FCSA game.

**Proof.** We refer to Appendix A for a proof of Proposition 2.
3. Charging station allocation mechanism

In Section 2, we show that no pure PNE guarantee exists for the FCSA game, which motivates our interest to better coordinate the different players via MD to steer the system towards more socially desirable outcomes. To realize such a mechanism, we consider the existence of a white label operator (WLO) who decides on the system-optimal allocation of stations to drivers across several players, considering station preferences reported by all players. In practice, the WLO might be an inter-charge operator or a municipality that allocates, e.g., via a charging slot booking system, a subset of stations to the navigation platforms, who then assign the stations to their drivers.

Formally, we consider \( m = |V| \) charging stations constituting \( m \) indivisible resources to be concurrently allocated among \( |N| \) players. Each player has a cost valuation for each bundle of stations, that corresponds to the minimum cost assignment between its drivers and the bundle’s stations. Valuations are non-additive, e.g., if a bundle contains more stations than drivers, removing the non-assigned station from the bundle preserves the valuation. While the valuation is expressed in units of time in the following, it can be transposed to monetary units in practice.

**Observation 1.** As the principal already knows where stations are located, it can compute a player’s cost valuation if it also knows where a player’s drivers are located.

Accordingly, it suffices that a player reports the locations of its drivers to the principal rather than explicitly communicating its cost valuation for every bundle.

**Observation 2.** Reporting drivers’ locations is linear in the number of drivers owned by the player.

Accordingly, reporting drivers’ locations requires much less information-sharing between a player and the principal, than if a player would report its cost valuation for any possible combination of stations.

The principal computes the allocation of stations to drivers that minimizes the sum of all players’ bundle cost valuation and requires in return that each player pays a price for the station assignment. Each player aims to minimize its payoff, which corresponds to the sum of its cost valuation for the received station allocation and the price decided by the principal. We assume that players may lie about their preferences if lying can decrease their payoff compared to truthfully reporting their preferences. The goal of the principal is then to design a pricing rule, such that all players have an incentive to truthfully report their drivers’ locations. To ensure players’ truth-telling behavior, we apply the well known VCG pricing scheme which ensures that it is in a player’s best interest to reveal its true preference.

In the following, we first analyze the VCG pricing scheme from an offline perspective in Section 3.1 to derive an upper bound on the system efficiency improvement that can be reached. We then study how to design an online VCG mechanism for a practical implementation in Section 3.2.
3.1. Offline charging demand

We now develop an offline mechanism to ensure truthful reporting of the players’ information, i.e., the locations of their drivers. Focusing on an offline setting, we assume all players’ information to be simultaneously reported. The principal allocates stations based on the revealed information, and each player accordingly receives a subset of stations and the corresponding drivers assignment for each of these stations. In Section 3.1.1, we discuss the mechanism for unweighted players, whereas we discuss how to account for weighted players in Section 3.1.2.

3.1.1. Unweighted VCG Mechanism

In the following, we adapt the VCG mechanism for payoff minimization and social cost minimization. We consider a setting in which each player \( i \in \mathcal{N} \) has some hidden information \( \theta_i \), representing the set of its drivers’ locations that it chooses to accurately reveal or not. We denote with \( \theta = (\theta_i)_{i \in \mathcal{N}} \) the vector that contains all players’ information. Let \( a \in A \) be the set of alternatives that correspond to assignments of stations to drivers, which the principal computes and communicates to the players. Here, a real station can be assigned only once to a driver and a driver may only be assigned to a station if the driving distance is less than its maximum search radius \( \bar{S} \). The artificial station vertex \( v_0 \) can be assigned to multiple drivers and induces a penalty cost for each assigned driver.

We introduce the function \( f(\theta) \) that minimizes the sum of all players’ valuations based on their reported information \( \theta \), and refer to it as the social choice function

\[
f(\theta_1, ..., \theta_n) = \arg \min_{a \in A} \sum_{i} v_i(\theta_i, a) \ .
\] (3.1)

The valuation \( v_i(\theta_i, a) \) of player \( i \) for alternative \( a \) corresponds to the sum of assignment costs of drivers to stations given the player’s information \( \theta_i \), formally

\[
v_i(\theta_i, a) = \sum_{k \in D_i} c(k, a) ,
\] (3.2)

with \( c(k, a) \) being the cost of assigning driver \( k \) to station \( a(k) \)

\[
c(k, a) = \begin{cases} \bar{\beta}, & \text{if } a(k) = v_0 \\ t_{k,a(k)}, & \text{else} \end{cases}.
\] (3.3)

Within an allocation \( a \), each driver \( k \in D \) receives an assigned station \( a(k) \in \mathcal{V} \cup v_0 \). Using Equation 3.2, we can further express the social choice function as

\[
f(\theta_1, ..., \theta_n) = \arg \min_{a \in A} \sum_{k \in D} c(k, a) .
\] (3.4)

Note that the social choice function minimizes the social cost as defined in Equation 2.6, with the additional constraint that no more than one driver can be assigned to each physical station.

Adapting the VCG mechanisms for payoff minimization leads to non-positive VCG prices, which implies a positive transfer of money from the player to the system. Accordingly, the VCG pricing rule is given by

\[
p(\theta_{-i}) = h_{-i} - \sum_{j \neq i} v_j(\theta_j, a) ,
\] (3.5)
with
\[ h_{-i} = \min_{b \in A} \sum_{j \neq i} v_j(\theta_j, b) , \tag{3.6} \]
and with \( a = f(\theta) \), such that each player’s payoff function results to
\[ u_i = v_i(\theta_i, a) - p(\theta_{-i}) = \sum_{j \in N} v_j(\theta_j, a) - h_{-i} . \tag{3.7} \]
Using VCG prices, the principal aligns each player’s interest with the overall system interest as the term \( \sum_{j \neq i} v_j(\theta_j, a) \) in Equation 3.7 aligns the player payoff with the social cost (cf. Equation 2.6). Thus, a player minimizes its payoff when telling the truth, i.e., the pricing scheme guarantees incentive-compatibility (cf. Nisan et al. 2007). VCG prices further ensure that a player does not have an incentive to assign different drivers to each of the stations contained in its received subset (see Proposition 3).

**Proposition 3.** A Player has no incentive to deviate from the drivers assignment to stations prescribed by the allocation a chosen by the principal, when telling the truth.

**Proof.** We refer to Appendix A for a proof of Proposition 3. \( \square \)

Moreover, VCG prices ensure that truth-telling minimizes a player’s payoff even when it has the possibility to deviate from the received assignment (see Proposition 4). A player that misreports information on its drivers may receive a suboptimal stations assignment, which it may improve by modifying the assignment of its drivers to the stations contained in the received subset of stations. However, the resulting assignment will still yield higher payoff than when truthfully reporting its information and not deviating from the prescribed assignment.

**Proposition 4.** A Player has no incentive to lie on its revealed information, even when it can deviate from the realized assignment by the principal

**Proof.** We refer to Appendix A for a proof of Proposition 4. \( \square \)

### 3.1.2. Weighted VCG Mechanism

The presented VCG mechanism leads to a minimal social cost but does not ensure solution fairness for a single player, e.g., it may lead to an outcome with one player having none of its drivers assigned at all, while another player may have all of its drivers assigned. Accordingly, a player might be better off in some scenarios by not participating in the system-based allocation unless it gets prioritized. To mitigate these issues, we now study a VCG mechanism, in which players are weighted to better balance the resulting individual player’s assignments. We note that a VCG mechanism with weighted players guarantees truthfulness (cf. Roberts 1979).

To formalize this setting, we detail
\[ f(\theta_1, \ldots, \theta_n) = \arg \min_{a \in A} \sum_i w_i \cdot v_i(a) , \tag{3.8} \]
with \( w_i \) being the weight associated to player \( i \).

In this setting, the weighted pricing rule is
\[ p^w(\theta_{-i}) = h_{-i} - \sum_{j \neq i} \frac{w_j}{w_i} v_j(\theta_j, a) , \tag{3.9} \]
with \( h_{-i} \) being a cost independent of the allocation obtained by \( i \), and defined as

\[
h_{-i} = \min_{b \in A} \sum_{j \neq i} \frac{w_j}{w_i} v_j(\theta_i, b)
\]

(3.10)

to ensure positive money transfer from the players to the system.

**Weights optimization:** Assuming a weighted players VCG mechanism, the weights can be defined a-priori based on the players’ characteristics, e.g., the number of controlled drivers. We can also optimally derive weight values such that each player benefits from participating in the centrally optimized system. Deciding whether to participate in the system or not relates to a game, where each player can \{opt-in, opt-out\} of the system. By opting in, the player receives the VCG allocation and pays the related price, whereas by opting out, the player selfishly assigns stations to drivers, possibly conflicting with other players’ assignments. We assume that if one player opts out, then all players are forced to opt out and selfishly assign their charging demand, as the noise created by the opted-out participant prevents computing an optimal system assignment for the remaining participants. Thus, a player has only an incentive to opt in if its resulting payoff with VCG is lower than its payoff without VCG. Accordingly, our goal is to derive weights such that all players profit from participating in the VCG mechanism.

The resulting weights optimization problem can be formalized as follows: let \( b = (b_i)_{i \in N} \) be the selfish strategy profile for all players, and let \( u_i(b) \) be the respective payoffs for each player \( i \) with strategy profile \( b_i \). As we seek to derive the best possible weights, we adapt the pivot rule such that the minimization part of \( h_{-i} \) for player \( i \) is independent of the a-priori undefined weights for all players but \( i \), and let

\[
h_{-i} = \min_{b \in A} \sum_{j \neq i} \frac{1}{w_i} v_j(\theta_i, b) .
\]

(3.11)

Equation (3.11) still guarantees the price to be non-positive such that the mechanism does not transfer a positive amount of money to its players. We denote with \( \text{OPT}_{-i} \) the social optimum realized when \( i \) does not exist, as

\[
\text{OPT}_{-i} = \min_{b \in A} \sum_{j \neq i} v_j(\theta_i, b) .
\]

(3.12)

We let \( w = (w_i)_{i \in N} \), with \( 1 \leq w_i \leq W \), and \( W \) being the maximal weight value; and define the weights optimization problem as

\[
\min_{x \in A, w} \sum_{i \in N} w_i \cdot \sum_{k \in D_i} \sum_{v \in V \cup V_0} x_{kv} t_{kv}
\]

(3.13)

\[
\sum_{k \in D} x_{kv} \leq 1 \ \forall v \in V \cup V_0
\]

(3.14)

\[
\sum_{v \in V \cup V_0} x_{kv} = 1 \ \forall k \in D
\]

(3.15)

\[
\sum_{j \in N} w_j \cdot \sum_{k \in D_j} \sum_{v \in V \cup V_0} x_{kv} t_{kv} - \text{OPT}_{-i} \leq w_i \cdot u_i(b) \ \forall i \in N
\]

(3.16)

The objective (3.13) minimizes the weighted social cost. The first two constraints (3.14, 3.15) are assignment constraints, while the last constraint (3.16) ensures that the resulting payoff within the mechanism, i.e., \( u_i(\theta_i, x) - p_i^w(\theta_{-i}) \), is better for each player compared to the payoff
when acting selfishly.

3.2. Online charging demand

An offline setting as described in Section 3.1 is not applicable in practice, because it is not possible to delay the station assignment when charging requests arrive in the system. Against this background, we extend the offline VCG mechanism to an online setting to allow for immediate assignment decisions by implementing a delayed VCG mechanism similar to Parkes et al. (2004). Section 3.2.1 introduces the mechanism for both unweighted and weighted players, while Section 3.2.2 describes our online allocation policies.

3.2.1. Delayed VCG Mechanism

In this online setting, different platforms interact with the principal and sequentially reveal their drivers’ locations to the principal during the planning horizon. We assume that requests must be served immediately. Then, the goal of the principal is to make decisions over time that minimize the expected total cost assignment of all drivers in the system. In this setting, each player makes a delayed payment at the end of the planning horizon, that depends on the realized assignment.

The information vector \( \theta_i \) of each player is sequentially revealed, such that \( \theta_i = (\theta^k_i)_{k \in D_i} \), with \( k \) being the \( k \)th driver belonging to player \( i \). We let \( \theta^t_i := \theta^t_i \) be the information revealed at time \( t \), identifying the location of the requesting driver \( k \) and the player \( i \) it belongs to. We denote with \( \Theta = \{ \theta_i : i \in \mathcal{N} \} \) the set of player’s information vectors and we define a state \( x_t \) as the vector that describes the history of decisions and of revealed information such that \( x_t = (\theta^0_i, ..., \theta^t_i, a_0, ..., a_{t-1}) \). We define policy \( \pi = (\pi_1, ..., \pi_t) \) as the sequence of decisions made in each epoch, with \( a_t := \pi_t(x_t) \), being the station assigned to driver \( \theta^t_i \) by the principal. Each assignment decision induces an immediate cost corresponding to the time required by the associated driver to reach the station, \( t_{\theta^t_i, \pi_t(x_t)} \). At time \( t \), a player’s cost is the sum of all of its driver’s travel times or penalties. A player’s cost sequentially increases until the planning horizon ends.

From the principal’s perspective, the sequential station assignment problem can be modeled as an MDP, whose solution policy corresponds to the policy \( \pi \). Accordingly, we introduce an MDP defined by a policy \( \pi \), the state space \( X \), and transition functions \( p(x'|x, a) \) that describe the probability that the system in state \( x \) will transition to post-decision state \( x' \) after having taken action \( a \). As previously introduced, a state \( x \) describes the current stations’ assignment of existing drivers in the system and an action \( a \) represents the assignment decisions realized by the principal.

We define the immediate cost \( d^t_i(x_t, a_t) \) for each player \( i \) as

\[
d^t_i(x_t, a_t) = \begin{cases} c(\theta^t_i, a_t), & \text{if } i = \hat{i} \\ 0, & \text{else} \end{cases}
\]  

(3.17)

with \( \hat{i} \) being the player associated to request \( \theta^t_i \), using the station assignment cost \( c(\theta^t_i, a_t) \) as defined in (3.3). Let the total immediate cost be \( d_t(x_t, a_t) = \sum_{i \in \mathcal{N}} d^t_i(x_t, a_t) \). Further, we denote
by
\[
\hat{d}^i_{<T}(\theta, \pi) = \sum_{t=0}^{T} d^i_t(x_t = (\theta^0, ..., \theta^t, a_0, ..., a_{t-1}), \pi_t(x_t))
\] (3.18)

the accumulated cost for player \(i\) from \(t = 0\) until \(T\), with reported information \(\theta\). We can then define the MDP value function \(V^\pi\) as the expected value of the summed costs over all decision epochs

\[
V^\pi(x_t) = E[x_t \sum_{t=0}^{T} d_t(x_{\tau}, \pi_{\tau}(x_{\tau}))],
\] (3.19)

which we can also recursively express as

\[
V^\pi(x_t) = d_t(x_t, \pi_t(x_t)) + \sum_{x_{t+1}} p(x_{t+1} | x_t, \pi_t(x_t))V^\pi(x_{t+1}).
\] (3.20)

The objective of the principal is then to find a policy \(\pi\) that minimizes the expected cost value \(V^\pi(x_0)\) over the planning horizon, with \(x_0\) being the initial system state. The realized cost for each individual player corresponds to the summed costs over the entire planning horizon for its drivers, formally

\[
v^i(\theta^i, a_{<T}) = \sum_{\tau=0}^{T} d^i_{\tau}(x_{\tau}, a_\tau).
\] (3.21)

Similar to the offline setting in Section 3.1, we assume that a player \(i\) may misreport her information \(\theta^i\) and we refer to the reported information as \(\hat{\theta}\), used by the planner to decide on the next action \(\hat{a}_t\). Then, the immediate cost induced by action \(\hat{a}_t\) corresponds to the distance from the actual driver location to the next decided station, and depends on the actual location of the driver \(\theta^t\) and the action chosen. Accordingly, \(d^i_t((\theta^0, ..., \theta^t, \pi), \hat{a}_t) = c(\theta^t, \hat{a}_t)\) when \(i\) reports information \(\hat{\theta}\) with actual information \(\theta^t\).

Analogous to Parkes et al. (2004), we define the mechanism’s prices as the difference between the sum of realized costs of all players but \(i\) and the optimal realized cost with perfect information without player \(i\). Each player pays the price at the end of a given planning horizon. Formally, the pricing rule for player \(i\) is

\[
p_i(\theta, \pi) = -\sum_{j \neq i} \sum_{t=0}^{T} d^j_t((\theta_{<t}, a_{<t-1}), \pi_t(x_t)) + OPT_{\theta_{-i}},
\] (3.22)

with \(OPT_{\theta_{-i}}\) being the optimal assignment cost that can be obtained under full information without \(i\) being in the system, and \(\pi\) being the optimal MDP-policy, such that

\[
OPT_{\theta_{-i}} = \sum_{j \neq i} d^j_{<T}(\theta_{-i}, \pi).
\] (3.23)

Then, the mechanism \(\mathcal{M}_{on} = (\Theta, p, \pi)\) defined by the players’ information \(\Theta\), the pricing rule \(p = (p_i)_{i \in \mathcal{N}}\) and the assignment decisions policy \(\pi\) constitutes the delayed VCG mechanism. The pricing rule defined in Equation 3.22 makes \(\mathcal{M}_{on}\) Bayesian-Nash Incentive-compatible, i.e.,

\[
E_{\tau > t}[v^i(\theta^i, a_{<\tau}) - p_i(\theta, \pi)] \leq E_{\tau > t}[v^i(\hat{\theta}^i, \hat{a}_{<\tau}) - p_i(\hat{\theta}, \pi)] \forall \theta \forall t
\] (3.24)

with \(\hat{a}_{<\tau} = (a_0(x_0), ..., \pi_t(\hat{x}_t), ..., \pi_{\tau}(\hat{x}_{\tau}))\). Here, \(\hat{x}_\tau = (\hat{\theta}_{<\tau}, \hat{a}_{<\tau})\) with \(\hat{\theta}_{<\tau} = (\theta^0, ..., \theta^{t-1}, ..., \theta^\tau)\) \(\forall \tau \in [t, T]\). We assume that \(\hat{\theta}^\tau = \theta^\tau\) if player \(j \neq i\) reports in epoch \(\tau\), i.e., all players but \(i\) truthfully report their information. Equation 3.24 ensures that the expected player’s payoff cannot be better when the player reports false information \(\hat{\theta}\), instead of \(\theta^t\).
Proposition 5. Mechanism $\tilde{M}_{\text{on}}$ is Bayesian-Nash incentive-compatible.

Proof. We refer to Appendix A for a proof of Proposition 5.

Similar to the VCG mechanism (see Section 3), we show that the delayed VCG mechanism $M$ can be extended to account for weighted players. We can adapt the immediate cost by weighting players, and introducing the new immediate cost

$$\tilde{d}_t(x_t, a_t) = \sum_{i \in \mathcal{N}} w_i \cdot d_i^t(x_t, a_t) \quad (3.25)$$

with $w_i \geq 1 \ \forall i \in \mathcal{N}$. Note that $v^i(\theta^i, a_{\leq T})$ remains unchanged. We accordingly update the pricing rule as

$$\tilde{p}^i(\theta, \pi) = -\sum_{j \neq i} \sum_{t=0}^{T} \frac{w_j}{w_i} d_j^t((\theta_{<t}, a_{<t-1}), \pi_t(x_t)) + \tilde{OPT}_{\theta_{-i}} \quad (3.26)$$

where $\tilde{OPT}_{\theta_{-i}} = \sum_{j \neq i} \frac{w_j}{w_i} d_j^T(\theta_{-i}, \pi)$. We define $\tilde{M}_{\text{on}} = (\Theta, \tilde{p}, \pi)$ as the weighted delayed VCG mechanism, and show that $\tilde{M}_{\text{on}}$ is still in-expectation incentive-compatible.

Proposition 6. Mechanism $\tilde{M}_{\text{on}}$ is Bayesian-Nash incentive-compatible.

Proof. We refer to Appendix A for a proof of Proposition 6.

3.2.2. Online allocation policy

In an online setting, each decision stage is triggered by a new driver’s request, and the principal assigns a charging station to the driver reported by a player based on the online allocation policy. Finally, the mechanism computes prices at the end of the planning horizon, which terminates after a fixed number of considered requests. In the following, we derive two online allocation policies to solve the underlying MDP of the delayed VCG mechanism $M_{\text{on}}$. The first policy greedy allocates the closest available station to the requesting driver. The second policy data-driven bases on a data-driven algorithm (cf. Garatti & Campi 2022), and learns the policy parameterization based on historical charging requests. As both policies are suboptimal in practice, the truthfulness of the mechanism cannot be guaranteed in theory. We notice that if one wants to formally ensure truthfulness, one could extend the mechanism similar to the second chance mechanism as introduced in Nisan & Ronen (2007). In this case, players have a chance to report a different information only if it improves their utility, and accordingly the expected social cost.

**Greedy policy:** The greedy policy is a deterministic policy that assigns a requesting driver to the closest available station, such that $\pi_t(x_t) = \arg \min_{v \in \bar{V}} t_{\theta^t,v}$, with $\theta^t$ being the location of the requesting driver in state $x_t$, and $\bar{V}$ being the set of stations that have not been already assigned to preceding drivers.

**Data-driven policy:** The data-driven policy is a probabilistic policy, i.e., a policy that chooses its action based on a probability distribution. To sample actions, we use a parametric data-driven online algorithm to determine the action taken at each decision stage. Specifically, the
algorithm parameters denote with which probability to take a specific action. We learn the parameterization of this algorithm offline, based on a large set of training input sequences, each consisting of charging requests of a fixed length.

We derive the data-driven algorithm $A$ as follows. Formally, we let $I$ be the set of all possible scenarios $I$ where $I = [\theta^0, ..., \theta^t, ..., \theta^T]$, with $T + 1$ being the input sequence length and $\theta^t \in \Theta$ being the location of a driver in position $t$, such that $D_I$ corresponds to the set of drivers $k$ contained in $I$. Our goal is to find an algorithm $A$ that minimizes the competitive ratio $\alpha$ for all possible input sequences $I \in \Delta$, such that

$$A^* = \arg\min_A \alpha$$

$$ON(I_l, A) \leq \alpha OPT(I_l) \forall I \in \Delta,$$

$$\sum_{v \in V \cup V_0} x_{kv} = 1 \forall k \in D_I, \forall I \in \Delta,$$

$$\sum_{k \in D_I} x_{kv} \leq 1 \forall v \in V, \forall I \in \Delta$$

$$x_{kv} \in [0, 1] \forall k \in D_I, \forall v \in V \forall I \in \Delta,$$

where $ON(I, A)$ corresponds to the online solution obtained with $A$ for a given scenario $l$ and $OPT(I)$ corresponds to the offline solution obtained for $l$. The offline solution $OPT(I)$ corresponds to the minimum cost assignment of the drivers in $I$ to charging stations in $v \in V \cup V_0$, as

$$OPT(I) = \min_{x_{kv} \forall k \in D_I, v \in V \cup V_0} \sum_{v \in V \cup V_0} \sum_{k \in D_I} x_{kv} \cdot c(k, v)$$

$$\sum_{v \in V \cup V_0} x_{kv} = 1 \forall k \in D_I$$

$$\sum_{k \in D_I} x_{kv} \leq 1 \forall v \in V$$

$$x_{kv} \in \{0, 1\} \forall k \in D_I, \forall v \in V$$

$$c(k, v) = \begin{cases} \tilde{\beta}, & \text{if } v = v_0 \lor \gamma(k, v) \geq \tilde{S} \\ t_{k,v}, & \text{else}. \end{cases}$$

Here, the objective (3.28) is to find an assignment of drivers to stations that yields minimal cost. Each driver must be assigned to at least one physical or virtual station (3.29), and at most one driver can be assigned to a physical station (3.30). A driver assignment cost corresponds to the driving time from its current location to the physical station or to a penalty cost if the station is further away than $\tilde{S}$ or if the station is virtual (3.32).

To mitigate the computational burden, we chose $A$ to lie on a parametric class and relax the integer constraint on the decision variables $x$. We then learn the algorithm $A^*$ based on $L$ i.i.d. sampled training input sequences $I_0, ..., I_L$ of the uncertainty set $\mathcal{I} \subset \Delta$. Accordingly, the
The learning objective becomes

\[ A^* = \arg\min_A \alpha \]

\[ \text{ON}(I_l, A) \leq \alpha \text{OPT}(I_l) \forall I_l \in \mathcal{I}, \; l \in [L] \]

\[ \sum_{v \in V \cup \emptyset} x_{kv} = 1 \; \forall k \in \mathcal{D}_{I_l}, \; \forall I_l \in \mathcal{I}, \; l \in [L] \]

\[ \sum_{k \in \mathcal{D}_{I_l}} x_{kv} \leq 1 \; \forall v \in \mathcal{V}, \; \forall I_l \in \mathcal{I}, \; l \in [L] \]

\[ x_{kv} \geq 0 \; \forall k \in \mathcal{D}_{I_l}, \; \forall v \in \mathcal{V}. \] (3.33)

We introduce the parameterization vector of the algorithm \( A \), as \( p_{\theta,t} \in \mathbb{R}^{|V|} \) for each \((\theta,t)\)-pair, such that \( 0 \leq p_{\theta,t,v} \leq 1 \) represents the probability for a driver in location \( \theta \) with position index \( t \) in the input sequence to be assigned to station \( v \). Thus, we derive the algorithm \( A \) with parameterization \( p_{\theta,t}^* \) for all \((\theta,t)\)-pairs, as follows

\[ A = \arg\min_{p_{\theta,t,v} \forall \theta \in \Theta, t \in [0,...,T], v \in \mathcal{V}} \alpha \] (3.34)

\[ \sum_{t=0}^{T} \sum_{v \in \mathcal{V}} p_{\theta,t,v} \cdot c(\theta^t, v) \leq \alpha \cdot \text{OPT}(I_l) \; \forall l \in [L] \; \theta_t \in I_l \] (3.35)

\[ \sum_{v \in \mathcal{V}} p_{\theta,t,v} = 1 \; \forall \theta \in \Theta, \forall t \in [0, ..., T] \] (3.36)

\[ \sum_{t=0}^{T} \sum_{\theta \in \Theta} p_{\theta,t,v} \cdot n(\theta, t, I) \leq 1 \; \forall v \in V, \] (3.37)

\[ \alpha \in \mathbb{R}^+ \] (3.38)

with \( n(\theta, t, I) \) being the likelihood that a driver is located in \( \theta \) with position index \( t \), estimated based on the occurrence of such combination \((\theta,t)\) among all input sequences \( I_l, l \in [L] \). The Objective (3.34) is to find a parameterization that minimizes the competitive ratio \( \alpha \), i.e., the ratio between the optimal cost and the cost obtained with the parametric algorithm as introduced in (3.27). Constraint (3.35) ensures that the ratio between the expected online solution and the optimal offline solution is lower than the minimized competitive ratio \( \alpha \). Constraint (3.36) enforces that we compute a discrete probability distribution, while Constraint (3.37) ensures that we do not assign (in-expectation) more than one driver to a station.

We apply this data-driven policy in state \( x_t = (\theta^0, ..., \theta^t, a_0, ..., a_{t-1}) \) as follows. First, we exclude already allocated stations, i.e., \( v \in \{a_0, ..., a_{t-1}\} \), from the possible stations assignment by setting the probability to be selected to 0, such that \( p_{\theta,t,v} = 0 \), \( \forall \theta \) st. \( \theta = \theta^t \). Then, we scale the assignment probabilities of the remaining candidate stations to preserve the discrete probability distribution as

\[ p'_{\theta,t,v} := \frac{p_{\theta,t,v}}{\sum_{v \in V} \delta_v} \forall v \in V, \]

with \( \delta_v = 0 \) if \( v \in \{a_0, ..., a_{t-1}\} \). Finally, we select a station \( v \) for the driver in location \( \theta \) based on the probabilities \( p'_{\theta,t,v} \).

In a setting with weighted players, optimizing weights requires to know each player’s payoff resulting from a selfish behavior. However, as requests are unseen, we do not know a player’s
selfish payoff ahead of time and thus cannot optimize weights a-priori. To remedy this issue in practice, one could derive the weights a-posteriori, i.e., after a planning horizon has terminated, and apply them for the next planning horizon. In this case, one needs to additionally ensure that the assignment costs account for the players’ weights, as \( c(k, v, i) = w_i \cdot c(k, v) \). Consequently, the parameterization needs to be indexed on the players, such that \( p_{\theta, t, v, i} \) represents the probability that a driver in location \( \theta \) at requesting time \( t \) and belonging to player \( i \) should be assigned to station \( v \).

4. Experimental design

To analyze the impact of coordinating the charging demand at drivers’ and platforms’ levels, we derive real-world test instances based on the charging station network for the city of Berlin, Germany (cf. Figure 1).

We consider three unweighted navigation platforms, i.e., players. The ratio of available stations and requesting drivers is the main factor impacting our results. We accordingly vary the total number of drivers in the system \( N \in [2, \ldots, 40] \), the search radius \( (\bar{S} \in \{1000, 2000\} \text{ meters}) \) and the radius of the circular area in which all drivers depart \( (s' \in \{300, 700, 1100\} \text{ meters}) \). We set the penalty \( \bar{\beta} = 120 \text{ min} \), such that failing the search for a driver corresponds to a delay of two hours.

To account for the navigation platforms’ heterogeneity, we vary the players’ imbalance, with respect to the number of drivers managed by each platform as shown in Table 1. We compare three driver distribution scenarios. In the small scenario, one player accounts for 20% of the total demand, whereas in the big scenario, one single player accounts for 50% of the demand. In both scenarios, the other two players share the remaining demand equally. The equal scenario represents a homogeneous player scenario, with equal distribution of drivers between players.

For both offline and online charging demand, we benchmark the VCG-based assignment against two naive assignment strategies. In D-SELF, each EV driver visits the closest station in its vicinity, whereas in P-SELF, navigation platforms compute the cost-minimum assignment of stations to drivers with respect to their own charging demand. We compare all three settings,
i.e., VCG, D-SELF, and P-SELF, with respect to the social cost $S_g$, i.e., the sum of each player’s drivers to stations assignment cost, which reflects end-users’ satisfaction. To analyze the benefits of VCG pricing, we further compare each player’s payoff, including VCG prices when applicable. To complement the offline analyses, we analyze the impact of optimally weighting players, such that all players benefit from VCG when applying the weighted VCG pricing scheme. Here, we solve the optimization problem introduced in Equations (3.13)-(3.16), and set the maximal weight value to $W = 10$.

For the online charging demand setting, we assume that for both D-SELF and P-SELF, there is a latency between the time when the drivers stop at the station and the time the station’s availability status is up-to-date for the succeeding drivers. We consider a latency of $\tau = 3$ minutes, i.e., the time for the driver to stop and start charging and for the system to update the availability information. For P-SELF the latency only concerns the other player’s drivers, as the player knows which stations were assigned to its drivers. We further assume that requests arise every $\Delta_t$ minute with $\Delta_t \in \{0.5, 1.5, 2.5\}$ minutes.

For the data-driven parameterization, we limit the computational burden by identifying a set of at maximum 40 possible locations a driver can depart from ($|\theta| \leq 40$). In most instances, candidate departing locations cover all vertices in the corresponding road network, i.e., roads junctions. To evaluate the stochastic data-driven online policy on a test instance with $N$ drivers, we randomly sample $n = 500$ start locations for each driver, and compute the simulated estimates of the social cost and each player’s individual payoff.

5. Results

In the following, we first detail our results for an offline charging demand setting in Section 5.1 to obtain an upper bound on the system improvement as well as to understand the interactions between navigation platforms with different driver shares. We further discuss the impact of optimally weighted platforms in this setting in Section 5.2. We then discuss our results for an online charging demand setting in Section 5.3 and compare it to the offline benchmark.

5.1. Offline allocation results

Figure 2 shows the distribution of the social cost for all three strategies (P-SELF, D-SELF, VCG) depending on the number of drivers in the system for both small ($S = 1000m$) and large ($S = 2000m$) search radii. As can be seen, selfish navigation service platforms can increase the overall user satisfaction by optimally balancing their own charging demand (P-SELF) compared to selfish EV drivers taking greedy visit decisions (D-SELF). However, VCG pricing further reduces the social cost significantly. Our results show that the benefit of platform coordination increases when the ratio between the total number of charging stations and the total number of drivers in the system increases. In contrast, when the likelihood of visit conflicts is high, i.e., many drivers depart within small distance or with small search radius, the coordination improvement decreases. However, coordination remains necessary in these cases as selfish platform optimization becomes as bad as individual greedy driver decisions.
Figure 2.: Benefit of VCG pricing on the social cost

Results are averaged over all values of $s^r$ with $s^r \{300, 700, 110\}$ m.

**Result 1.** Coordinating platforms with VCG pricing decreases the social cost by up to 52% compared to an allocation obtained with selfish platforms.

Figure 3 shows all three platforms’ mean payoffs depending on the number of drivers and for each driver distribution scenario. To compare platforms with a heterogeneous driver share, we normalize each platform’s payoff by its number of managed drivers. As can be seen, VCG pricing significantly decreases a platform’s payoff on average by 27% (equal), respectively 21% (big), and 26% (small) compared to the payoff obtained for platform-optimized allocations (P-SELF). In case of a high charging demand, i.e., for more than 20 drivers departing in a small vicinity and with lower search times, the induced VCG prices increase a platform’s payoff compared to an uncoordinated setting. Focusing on the impact of the driver distribution, our results highlight two effects. Utilizing VCG pricing, a platform’s payoff decreases if its share of drivers in the system increases (see e.g., Platform C, in Figure 3). Contrarily, the improvement obtained with VCG pricing compared to a selfish assignment is larger for a platform with proportionally less drivers. In some cases, e.g., for $\bar{S} = 1000$ m and a very large number of drivers, VCG does not outperform P-SELF for a platform that manages the highest share of drivers. If a platform manages most drivers, then the subsystem composed of other platforms’ drivers is smaller such that the platform’s impact on it is low, which decreases the large platform’s price. In this case, the impact of other platforms on the platform with the largest drivers share decreases as well, such that the benefits of coordination do not always compensate the additional VCG prices induced by coordination.

**Result 2.** A navigation platform with a low share of drivers (nearly) always benefits from VCG pricing compared to P-SELF. In contrast, a platform with a higher share of drivers is sometimes better off not participating in a VCG setting.

Figure 4 compares the mean payoffs obtained for each platform and distribution scenario depending on the number of drivers. In line with Figure 3 results show that a platform with more drivers gets, proportionally to the number of drivers managed, a lower payoff compared to platforms with less drivers. As can be further seen, the impact of a heterogeneous driver share is...
Figure 3: Impact of the distribution scenario on each platform’s payoff for VCG, P-SELF, and D-SELF

bigger for a large search radius ($\bar{S} = 2000$ meters) compared to a small search radius ($\bar{S} = 1000$ meters). In summary, if a platform participates in a VCG setting, its payoff will inversely decrease with the number of drivers it manages in the system, but the benefit of participating in VCG compared to P-SELF with a lot of drivers will decrease, too. If a platform has few drivers, it is better off participating in the system.

**Result 3.** If participating in a VCG coordinated system, a navigation platform can decrease its cost by increasing its drivers share. However, the relative improvement obtained with VCG compared to P-SELF decreases in this case.

Figure 5a shows the average driving time to an available station for any driver in the system, while Figure 5b shows the average station assignment success rate for a platform for both $\bar{S} = 1000$ m and $\bar{S} = 2000$ m. Analyzing Figures 5a & 5b, our results show that coordination through VCG pricing slightly increases the average driven search time needed by its drivers to reach an available station for each platform compared to a selfish or a greedy behavior. However, lower driver search times in selfish settings (P-SELF & D-SELF) come at the expense
Figure 4.: Impact of the distribution scenario on each platform’s payoff

Figure 5.: Impact of VCG pricing on average drivers search times and success rates
of significantly lower success rates, which highlights the need of coordination to decrease station visits conflicts.

**Result 4.** Platform coordination mildly increases the average driving time of drivers in the system by 32 seconds on average, compared to local coordination (P-SELF), and by 47 seconds, compared to uncoordinated drivers (D-SELF). These longer searches significantly reduce the number of station visit conflicts, leading to a success rate increase of 33% compared to P-SELF, and of 49% compared to D-SELF.

5.2. A-priori weights optimization

In the following, we analyze the impact of optimizing weights prior to applying offline weighted VCG pricing by solving the optimization problem described in Section 3.1.2. Table 2 summarizes the number of instances in which unweighted VCG pricing ("VCG beneficial") benefits all platforms, i.e., each platform obtains lower cost with VCG than without, as well as the number of instances in which VCG pricing benefits all platforms only if players are optimally weighted ("Weighted VCG beneficial"). Furthermore, the table shows the number of instances for which optimal weights could not be derived ("Infeasible weights"), and the number of instances for which the weights optimization problem could not be solved within the computational time limit ("Not solved") of 120 minutes. We differentiate results between low ($\bar{S} = 1000$ m) and high ($\bar{S} = 2000$ m) search radius.

As can be seen, weighted VCG pricing increases the total number of instances in which all platforms benefit from VCG participation from 43% to 49%. There remain 31% of test instances that yield no feasible weights and 20% of instances for which the weights optimization problem could not be solved within the limited computational time. Focusing on test instances that yield infeasible weights, either unweighted VCG already benefits all platforms, or the ratio of available stations per driver is too low to distribute all drivers across stations. In the latter case, prices will be high for any weights values for at least one platform, which accordingly results in higher payoff with VCG than with P-SELF. In a limited number of cases (6% of the total number of test instances), the ratio of stations per driver is low enough such that unweighted VCG does not benefit all platforms, but large enough such that a minor reallocation of stations to platforms ensures that all platforms benefit from VCG over P-SELF. Here, platforms who initially did not benefit from VCG have larger weights than other platforms. Moreover, we observed that the range of improvement through weights optimization depends on the search radius, with 60% of the instances benefiting from weighted VCG in a low search radius ($\bar{S} = 1000$ m) setting against

| $\bar{S}$ | VCG beneficial | Weighted VCG beneficial | Infeasible weights | Not solved |
|-----------|----------------|-------------------------|--------------------|------------|
| 1000      | 23             | 14                      | 83                 | 51         |
| 2000      | 125            | 5                       | 24                 | 17         |
| total     | 148 (43%)      | 19 (6%)                 | 107 (31%)          | 68 (20%)   |
only 4% benefiting in a large search radius ($S = 2000 \text{ m}$) setting. In the latter case, initial benefits of VCG pricing are higher for a larger search radius ($S = 2000 \text{ m}$) due to the higher number of available stations per driver, which in turn decreases the improvement potential.

**Result 5.** Optimizing platforms’ weights increases the number of instances in which all platforms benefit from participating in VCG from 43% to 49%.

To further detail the impact of weighting players, Figures 10a-10c in Appendix B show the averaged payoff per platform and scenario in both unweighted and weighted cases, depending on the total number of drivers. Platform C appears to benefit the most from the weight optimization in the small scenario, when managing only 20% of the total number of drivers. Weighting platforms slightly benefits the system by decreasing a platform’s payoff. However, results show little differences between weighted platforms’ payoffs and unweighted platforms’ payoffs, and no clear trend on the weights values depending on the scenario. Accordingly, we focus on the unweighted platforms setting in the following analyses.

### 5.3. Online allocation results

The following section analyzes the benefit of VCG coordination with online charging requests. Preliminary studies (cf. Appendix B) show a similar effect of driver imbalance on a platform’s payoff in an online compared to an offline setting. Accordingly, we focus the results discussion on the balanced platforms scenario equal to evaluate the performances of VCG and compare the benefits with respect to a perfect-information benchmark (OFF).

Figure 6 shows the social cost deviation between the data-driven online policy (VCG-dd) and the greedy online policy (VCG-greedy), computed as follows $\Delta(\hat{\alpha}) = (\hat{\alpha}_{\text{dd}} - \hat{\alpha}_{\text{greedy}})/\hat{\alpha}_{\text{greedy}}$, with $\hat{\alpha}_{\text{dd}}$, respectively $\hat{\alpha}_{\text{greedy}}$, being the realized social cost for the data-driven, respectively greedy, online policy. Figure 7 compares the search cost distribution obtained with VCG-dd and VCG-greedy online policies against the perfect-information benchmark (OFF), per number of drivers, for a small ($S = 1000 \text{ meters}$) and large search radius ($S = 2000 \text{ meters}$).

Our results (cf. Figure 6) show an improvement of the data-driven policy over the greedy allocation policy in an online setting, due to a better anticipation of possible visit conflicts of the data-driven policy. For $S = 1000$, the averaged improvement over all values of $s^r$ is 23% in the best case ($N = 10$ drivers), whereas for $S = 2000$ it increases to 34% ($N = 22$ drivers). As
can be seen, the benefit of a data-driven policy is highest for a small number of drivers in case of a small search radius ($\bar{S} = 1000$ meters) but highest for a large number of drivers in case of a large search radius ($\bar{S} = 2000$ meters). In the former case, the social outcome for a higher number of drivers is generally bad due to a high number of unavoidable station visit conflicts. Here, the bottleneck significantly limits the improvement potential, such that the two online policies and the perfect-information setting ($OFF$) yield equally bad outcomes (cf. Figure 7). In contrast, the likelihood of station visit conflicts is low for a small number of drivers in the latter case, such that a greedy policy performs already nearly as good as a perfect-information setting allocation ($OFF$). In this case, there will always be enough available stations to allocate to incoming drivers, which will not increase the social cost with penalty costs.

**Result 6.** A data-driven online assignment policy improves the social outcome when the improvement potential is highest, i.e., for a small number of drivers with small search radius or for a large number of drivers with large search radius.

**Result 7.** A data-driven online assignment policy decreases the social cost on average by 14% given a large search radius, and yields a maximum reduction of 55% for $s' = 700$ meters and 32 drivers, compared to a greedy online assignment policy.

Table 3 shows the deviation of the realized social cost obtained with $P-SELF$, $D-SELF$, and $VCG$-greedy compared to the $VCG$-dd online policy. For $P-SELF$ and $D-SELF$, we detail results for a small time span between two driver requests ($\Delta_t = 0.5$ minutes), an average time span ($\Delta_t = 1.5$ minutes) and a large time span ($\Delta_t = 2.5$ minutes). For both $VCG$-greedy and
Table 3.: Social cost deviation for P-SELF, D-SELF, and VCG-greedy compared to VCG-dd

| S   | s'  | VCG-greedy | P-SELF | D-SELF | P-SELF | D-SELF |
|-----|-----|------------|--------|--------|--------|--------|
|     |     | ∆t = 0.5  | ∆t = 1.5 | ∆t = 2.5 | ∆t = 0.5 | ∆t = 1.5 | ∆t = 2.5 |
| 300 | 700 | -0.17%     | 15.6%    | 12.2%    | 10.7%    | 34.4%    | 11.9%    | 6.20%    |
| 1000| 700 | 9.95%      | 13.3%    | 9.23%    | 7.63%    | 19.0%    | 8.81%    | 5.31%    |
| 1100| 700 | 4.23%      | 37.6%    | 25.0%    | 20.5%    | 51.9%    | 24.3%    | 15.1%    |
| 300 | 700 | -0.19%     | 193%     | 167%     | 155%     | 259%     | 171%     | 114%     |
| 1100| 700 | 77.5%      | 994%     | 778%     | 670%     | 1199%    | 780%     | 536%     |

The table shows the search cost deviation computed as $\Delta(\hat{\alpha}) = (\hat{\alpha}_{op} - \hat{\alpha}_{dd})/\hat{\alpha}_{dd}$, with $\hat{\alpha}_{dd}$ being the realized social cost for the data-driven online policy and $\hat{\alpha}_{op}$ being the realized social cost for P-SELF, D-SELF, and VCG-greedy respectively.

VCG-dd online policies, results are independent of $\Delta_t$ as the central decision-maker is aware of charging station availability. As can be seen, both coordinated online assignment policies significantly outperform the two naive benchmarks. In line with Figure 6, results further show the superiority of the online VCG-dd policy over the online VCG-greedy policy, particularly in cases with widespread drivers, i.e., for a large search radius ($\bar{S} \geq 2000$ m) or for a large departing area ($s_r \geq 700$ m). Our results further show that the benefit of coordinating platforms increases with the requests frequency and are highest for $\Delta_t = 0.5$ minutes. We note that for a lower request frequency, i.e., $\Delta_t = 2.5$ minutes, a platform does not benefit from locally coordinating its charging demand. Here, the realized social outcome is better when EV drivers greedily decide on their station visits (D-SELF) compared to when navigation platforms locally coordinate their charging demand (P-SELF).

**Result 8.** Online coordination with VCG pricing yields a significant system benefit compared to an uncoordinated setting by decreasing the social cost by 42% when platforms act selfishly (P-SELF), and 44% when drivers act selfishly (D-SELF).

**Result 9.** Local charging demand coordination (P-SELF) can yield a higher social cost than without any coordination at all (D-SELF).

Finally, Figure 8 compares the realized platform payoff, averaged over all demand realizations and platforms, normalized by the number of navigated drivers per platform for each online setting, against the offline benchmark (OFF). These results highlight the trade-off that exists between the additional cost induced by coordination via VCG pricing and the cost reduction due to a system-optimized allocation. As can be seen, VCG pricing mostly benefits a platform when the ratio of stations compared to the number drivers in the system is large enough, as otherwise the price paid by a platform increases due to the platform’s negative impact on the system. For a large search radius ($\bar{S} = 2000$ meters), the online VCG-dd policy outperforms any naive allocation strategy, independent of the total number of drivers in the system.

**Result 10.** VCG pricing significantly outperforms any naive strategy (P-SELF, D-SELF) for larger search areas ($\bar{S} = 2000$ m) by decreasing a platform’s payoff on average by 58% compared to P-SELF and by 61% compared to D-SELF.
Result 11. In some cases, VCG pricing may slightly increase a platform’s payoff, when the room for improving the social outcome with coordination is small, e.g. for a large number of drivers in a small search area.

6. Conclusion

In this paper, we analyzed the dynamics between several self-interested navigation service platforms that seek to best allocate charging stations to EV drivers. In this context, we studied the problem of conflicting charging station assignments realized by independent platforms from a game-theoretical perspective and introduced the FCSA game. We showed that the game can neither be represented as a congestion game, nor admits a guaranteed PNE. To steer the system towards a stable and optimal social outcome, we studied the VCG mechanism in both offline and online settings, such that coordinated platforms’ assignment decisions benefit the overall system. We further discussed how to extend the mechanism to account for heterogeneously weighted players in both settings. In the online setting, we introduced a data-driven online allocation policy. We analyzed the benefits of implementing our mechanisms by conducting a case study for the city of Berlin, and showed that coordination decreases the social cost by up to 52% in the offline setting and by up to 42% in the online setting. We further showed in an offline setting that optimized players’ weights may increase the likelihood that participants benefit from participating in the VCG mechanism. Finally, our results showed that the online data-driven allocation policy performs on average slightly better than a myopic policy, but may significantly improve the social outcome by up to 55% in congested scenarios.

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Online Appendix A  Proofs

Proof of Proposition 1. We show the non-guaranteed PNE existence by finding a game instance that does not possess any. Figure 9 visualizes such a game instance. The instance comprises three stations a, b, and c for two players p1 and p2: the first player has two drivers d1 and d2, the second player has only one driver c1. None of the possible game outcomes corresponds to a Nash equilibrium. If each player individually optimizes the assignment of its drivers to the available station, we obtain the following profile ((b, c), (c)), with c conflicting. From there, p2 should re-assign its unique driver to b, which is then conflicting. In this case, p1 is better off reassigning d1 to c and d2 to a, as the total cost (=17) for strategy (c, a) is lower than the total cost (=18) for strategy (a, c). However, p1 should now re-assign its unique driver to c such that p1’s best response is again its individually optimized assignment solution. The last strategy profile now equals the initial strategy profile, such that the path $P = ((b, c), (b, c), b), ((c, a), b), ((c, c), a), (b, c), c$ of length 5 is an improvement path. Additionally, starting from any other strategy profile not included in P, one may reach P within at most two best responses moves, such that there exists no sink in the best response dynamics graph and, accordingly, no PNE.

Proof of Proposition 2. We show the non-existence guarantee of a $\rho$-PNE with $\rho \leq \bar{\beta} - \Delta t$, by using an intermediate game $G$, defined with the same set of strategies than the FCSA game but with new payoff functions. We ignore travel times, such that assigning a station induces a penalty cost only for the non-closest drivers, i.e.,

$$c_i(k, s) = \begin{cases} 
\beta, & \text{if } s_i(k) = v_0 \\
0, & \text{if } \forall k' \in D_j, \forall j \in N, j \neq i \text{ and } s_j(k') = s_i(k) \Rightarrow (t_{k', c} \geq t_{k, c}) \\
\bar{\beta} & \text{otherwise}
\end{cases} \quad (A.1)$$

Accordingly, player $i$’s payoff can be expressed as $u_i(s) = k \cdot \bar{\beta}$, with $k \in N$. Similar to Proposition 1, $G$ has no guaranteed PNE, as we can find an instance of $G$ that admits none. For such instance,

$$\forall s \in S, \forall i \in N, \exists s'_i \in S, \text{ st. } u_i(s'_i, s_{-i}) < u_i(s) \quad (A.2)$$

When a player can strictly decrease its payoff by switching from strategy $s_i$ to $s'_i$, then its payoff decreases by at least $\bar{\beta}$, such that $u_i(s) - u_i(s'_i, s_{-i}) \geq \bar{\beta}$, which corresponds to one additional player being successfully assigned to a station.
Figure 9.: G1’ instance with no PNE

By assumption, each feasible profile \( s \) in \( G \) is feasible in the FCSA game and the previous condition implies that for some FCSA game instances, a player can decrease its cost by obtaining at least one more driver successfully assigned to a station. In this case, assume for player \( i \), that \( l \) drivers are successfully assigned to a station in \( s_i \). For any alternative strategy improving \( s_i' \), this implies at least \( l + 1 \) successfully assigned drivers. Let us denote with \( t_{\text{max}} \) and \( t_{\text{min}} \) the highest and lowest driving times between stations and drivers. We let \( n_i = |D_i| \) be the number of drivers belonging to \( i \). Then, we have

\[
u_i(s_i, s_{-i}) \geq (n_i - l) \cdot t_{\text{min}} + l \cdot \bar{\beta}
\]

(A.3)

and

\[
u_i(s_i', s_{-i}) \leq (n_i - l - 1) \cdot t_{\text{max}} + (l - 1) \cdot \bar{\beta},
\]

(A.4)

such that \( i \) decreases its cost by switching from \( s_i \) to \( s_i' \) by at least \( \Delta = \bar{\beta} + (t_{\text{min}} - t_{\text{max}}) \). Finally, we showed that \( \rho \geq \Delta \).

Proof of Proposition 3. Assuming that all players tell the truth, the principal computes the cost-minimal allocation

\[
a = \arg \min_{b \in A} \sum_{k \in D} c(k, b),
\]

(A.5)

with \( c(k, a) \) as defined in Equation (3.3). The payoff of player \( i \) corresponds to \( \sum_{k \in D} c(k, a) - h_{-i} \).

We let \( a' \) be the modified allocation, when player \( i \) reassigns its drivers to its stations allocated by the principal. All drivers that do not belong to player \( i \) are still assigned to the same station.
in \( a' \). The payoff of player \( i \) in this case corresponds to

\[
\sum_{k \in D} c(k, a') - h_{-i} .
\]  

(A.6)

However, as allocation \( a \) minimizes the total cost, i.e., the assignment cost of drivers to stations, then

\[
\sum_{k \in D} c(k, a') \geq \sum_{k \in D} c(k, a) ,
\]  

(A.7)

such that deviating from the assigned allocation of stations to drivers increases the payoff of a player. Concluding, a player has no benefit from assigning its drivers differently to the stations allocated by the principal. \( \square \)

**Proof of Proposition 4.** First, let us assume that a driver cannot deviate from the received assignment of drivers to stations. Let \( \hat{\theta} \) be the reported information, and with a slight abuse of notation let \( \hat{k} \) be a reported driver information (i.e., its location), while \( k \) denotes the actual driver information. Let \( \hat{a} = f(\hat{\theta}) \). Then a player’s payoff corresponds to \( \sum_{k \in D} c(k, \hat{a}) - h_{-i} \), as the assignment cost \( c(k, \hat{a}) \) corresponds to the cost with actual driver information, but with allocation \( \hat{a} \) computed based on the driver’s reported information by the corresponding player, such that \( c(\hat{k}, \hat{a}) = c(k, \hat{a}) \). We differentiate two cases:

i) **Correct number of reported drivers:** If a single player correctly reports its number of drivers but misreports its drivers’ location, such that \( \hat{\theta} \neq \theta \), then

\[
\sum_{k \in D} c(k, \hat{a}) \geq \sum_{k \in D} c(k, f(\theta)) ,
\]  

(A.8)

as \( f(\theta) = \arg \min_{b \in A} \sum_{k \in D} c(k, b) \), and the player’s payoff increases when not telling the truth.

ii) **Incorrect number of reported drivers:** If a single player lies on the number of reported drivers, then each non-reported driver \( \hat{k} \) does not get any assigned station, which yields a penalty cost for each of them, such that \( c(k, \hat{a}) = \hat{\beta} \geq c(k, a) \forall a \neq \hat{a} \). Then, for each extra reported driver \( \hat{k} \) the actual driver assignment cost becomes \( c(k, \hat{a}) = 0 \). Accordingly, the principal optimizes the allocation of actual drivers on a reduced subset of stations \( V' \subset V \), i.e., stations that are not assigned to virtual reported drivers, such that

\[
\min_{b \in A, V'} \sum_{k \in D} c(k, b) \geq \min_{b \in A, V} \sum_{k \in D, V} c(k, a) .
\]  

(A.9)

In these two cases of misreported drivers’ numbers, a player’s payoff is at least smaller when all drivers are correctly reported. Furthermore, following i), a player’s payoff is at least smaller when all players’ information are correctly reported.

Second, let us assume that a driver can deviate from the received assignment of drivers to stations. If a player lies, then its payoff is larger than when telling the truth as shown above,
such that
\[
\sum_{k \in D} c(k, \hat{\alpha}) + h_{-i} \geq \sum_{k \in D} c(k, f(\theta)) + h_{-i},
\] (A.10)
as \(f(\theta) = \arg \min_{\theta \in A} \sum_{k \in D} c(k, b)\). As its payoff is not minimized when lying, it can deviate from the allocation \(\hat{\alpha}\) by reassigning other drivers to its received stations, such that the new allocation \(\alpha'\) decreases its payoff obtained with allocation \(\hat{\alpha}\). However, the newly obtained allocation \(\hat{\alpha}\) cost is at least as large as the minimum cost allocation obtained when telling the truth, such that a player is better off reporting its true information and not reassigning its drivers to different charging stations.

\[\square\]

\textbf{Proof of Proposition 5.}\ We recall the proof exposed in Theorem 1 from Parkes et al. (2004), that transposes the regular VCG incentive-compatibility to an-in-expectation incentive-compatibility.

Let \(t\) be the last decision epoch before the first epoch, where \(i\) misreports information \(\theta^{t+1}\) (and may further misreport her information), such that \(\hat{\theta}^{t} = \theta^{t}\). By expliciting payment \(p_{i}\) in Equation (3.24) omitting \(OPT_{\theta_{-i}}\) that does not depend on player \(i\), and omitting \(\sum_{i \in N} \sum_{\tau = t}^{T-1} d_{i}^{t}(x_{\tau}, \pi_{\tau}(x_{\tau}))\) that does not depend on realizations later than \(t\), we need to show that
\[
E_{\tau > t}[\sum_{i \in N} \sum_{\tau = t}^{T} d_{i}^{t}(x_{\tau}, \pi_{\tau}(x_{\tau}))] \leq E_{\tau > t}[\sum_{i \in N} \sum_{\tau = t}^{T} d_{i}^{t}(\hat{x}_{\tau}, \pi_{\tau}(\hat{x}_{\tau}))] \quad \forall \hat{\theta}^{t} \forall t \quad (A.11)
\]
\[
\iff E_{\tau > t}[\sum_{\tau = t}^{T} d_{i}^{t}(x_{\tau}, \pi_{\tau}(x_{\tau}))] \leq E_{\tau > t}[\sum_{\tau = t}^{T} d_{i}^{t}(\hat{x}_{\tau}, \pi_{\tau}(\hat{x}_{\tau}))] \quad \forall \hat{\theta}^{t} \forall t.
\]

The immediate cost for a misreported information depends on the driver’s true information, i.e., actual location, and on the decision taken upon the misreported information, such that
\[
\sum_{i \in N} \sum_{\tau = t}^{T} d_{i}^{t}(\hat{x}_{\tau}, \pi_{\tau}(\hat{x}_{\tau})) = \sum_{i \in N} \sum_{\tau = t}^{T} d_{i}^{t}((\theta_{< \tau}, \hat{a}_{< \tau-1}), \pi_{\tau}(\hat{x}_{\tau})).
\]
The right part of (A.11) corresponds to the expected cost from \(x_{t}\) (as \(\hat{x}_{t} = x_{t}\) by assumption) following policy \(\hat{\pi}\), with \(\hat{\pi}(x_{\tau}) = \pi(\hat{x}_{\tau}) \forall \tau \in [t, ..., T]\), i.e., \(V^{\hat{\pi}}(x_{t})\). The left part of (A.11) corresponds to the expected cost from \(x_{t}\) following policy \(\pi\), \(V^{\pi}(x_{t})\), such that Equation (A.11) can be rewritten as
\[
V^{\pi}(x_{t}) \leq V^{\hat{\pi}}(x_{t}).
\]
Since \(\pi\) is the optimal MDP-policy by assumption, Equation (A.11) holds; otherwise this contradicts \(\pi\)’s optimality. Finally, we justified that a player has no incentive to reveal false information.

\[\square\]

\textbf{Proof of Proposition 6.}\ We need to show that the following inequality
\[
E_{\tau > t}[v^{i}(\theta^{t}, a_{< \tau}) - \tilde{p}^{i}(\theta, \pi)] \leq E_{\tau > t}[v^{i}(\theta^{t}, \hat{a}_{< \tau}) - \tilde{p}^{i}(\hat{\theta}, \pi)] \quad \forall \hat{\theta}^{t} \forall t \quad (A.12)
\]
\[
\iff w_{i} \cdot E_{\tau > t}[v^{i}(\theta^{t}, a_{< \tau}) - \tilde{p}^{i}(\theta, \pi)] \leq w_{i} \cdot E_{\tau > t}[v^{i}(\theta^{t}, \hat{a}_{< \tau}) - \tilde{p}^{i}(\hat{\theta}, \pi)] \quad \forall \hat{\theta}^{t} \forall t
\]
holds. By omitting the same terms in (A.12) than in (3.24) (see Proposition 5), we can show
that the following inequality holds

\[
\begin{align*}
&\iff E_{\tau>t}[\sum_{i \in \mathcal{N}} \sum_{\tau=t}^{T} w_i \cdot d_{\tau}(x_{\tau}, \pi_{\tau}(x_{\tau}))] \leq E_{\tau>t}[\sum_{i \in \mathcal{N}} \sum_{\tau=t}^{T} w_i \cdot d_{\tau}(\hat{x}_{\tau}, \pi_{\tau}(\hat{x}_{\tau}))] \forall \hat{\theta}^t \forall t \\
&\iff E_{\tau>t}[\sum_{\tau=t}^{T} \tilde{d}_{\tau}(x_{\tau}, \pi_{\tau}(x_{\tau}))] \leq E_{\tau>t}[\sum_{\tau=t}^{T} \tilde{d}_{\tau}(\hat{x}_{\tau}, \pi_{\tau}(\hat{x}_{\tau}))] \forall \hat{\theta}^t \forall t.
\end{align*}
\] (A.13)

Using the same argument as previously, with \(\pi\) being in this case the optimal policy solving the MDP with updated immediate cost \(\tilde{d}\), we justified inequality (A.13). Accordingly, a player has no incentive to reveal false information when players are weighted, if the payment rule is weighted too.

\[\square\]

Online Appendix B  Additional Numerical results

Weights optimization: Figures 10a-10c show the averaged payoff per platform per scenario, depending on the total number of drivers. Payoffs are shown for both weighted and unweighted VCG. For instances that could not solve the weights optimization, unweighted payoffs are shown.

Online averaged platform payoffs: Similar to Figure 4, Figure 11 compares the normalized payoffs obtained for each platform depending on the number of drivers, for each distribution scenario in an online setting. Analogous to offline results, online results show that a platform with a lower share of drivers (platform B in small) obtains a higher payoff than a platform with a higher share of managed drivers. Compared to the offline setting, platforms’ payoffs are slightly higher due to the approximation induced by the online allocation policy compared to a perfect-information allocation (cf. Figure 4).
Figure 10.: Impact of weighted VCG on each platform’s payoff in an online setting
Figure 11.: Impact of the distribution scenario on each platform’s payoff in an online setting