Fundamental cosmic strings

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February 1, 2008

Cosmic strings are linear concentrations of energy that may be formed at phase transitions in the very early universe. At one time they were thought to provide a possible origin for the density inhomogeneities from which galaxies eventually develop, though this idea has been ruled out, primarily by observations of the cosmic microwave background (CMB). Fundamental strings are the supposed building blocks of all matter in superstring theory or its modern version, M-theory. These two concepts were originally very far apart, but recent developments have brought them closer. The ‘brane-world’ scenario in particular suggests the existence of macroscopic fundamental strings that could well play a role very similar to that of cosmic strings.

In this paper, we outline these new developments, and also analyze recent observational evidence, and prospects for the future.

1 Spontaneous symmetry breaking

This is a feature of many physical systems, often accompanying a phase transition. For example, there are no preferred directions in a liquid such as water — it has complete rotational symmetry. But when it freezes the resulting crystal does have a definite orientation — the full rotational symmetry is broken, leaving only a very restricted group of symmetries of the crystal. It would be difficult for a microscopic inhabitant of the crystal to infer the existence of the larger symmetry!

When a liquid is cooled through its freezing point, crystals of solid will start to form, often nucleated by small impurities, but one cannot predict what their orientation will be. The choice is random, and different choices may be made at different centres, so that when the entire liquid has frozen, there may be mismatches, leading to grain boundaries or more subtle defects where the crystal lattice is deformed. For instance, one can have an extra layer of atoms ending on a linear ‘edge defect’, where the crystal is strained. Similar ‘topological defects’ occur in many theories of fundamental particle interactions.

Spontaneous symmetry breaking is a ubiquitous feature of our theories of fundamental particle interactions. (See, for example, [1].) The famous electroweak
theory, for which Sheldon Glashow, Abdus Salam and Steven Weinberg won the 1979 Nobel Prize, exhibits an underlying symmetry between the carriers of the electromagnetic force and the weak force — the photon and the W and Z particles. When we only had access to low-energy experiments this symmetry was completely hidden, its existence as difficult to guess as would be the full symmetry of the crystal to its microscopic inhabitant. The symmetry is apparent in very high-energy experiments, at energy scales well above 100 GeV ($10^{11}$ electron volts), but we live in a low-temperature phase where it is spontaneously broken by the so-called Higgs mechanism which imparts masses to the W and Z bosons, while leaving the photon massless.

Following the success of the electroweak model, physicists started to ask if the strong interactions too could be brought within a single unified framework. There is some experimental support for this idea of ‘grand unification’. The strengths of the fundamental interactions are determined by three ‘coupling constants’ $g_1, g_2, g_3$, which, despite their name, depend weakly (logarithmically) on energy. If one extrapolates from the low energies where they are measured one finds that all three come together at an energy scale of about $10^{16}$ GeV, strongly suggesting that something interesting happens at that scale. Several different ‘grand unified theories’ (GUTs) have been proposed to embody this idea, the most successful involve a symmetry between bosons and fermions, called supersymmetry.

Unfortunately particle energies of $10^{16}$ GeV are far beyond any scale accessible to present or future laboratory experiments, so GUTs are hardly likely to get the kind of solid experimental support the electroweak theory received. There is however one place — or rather time — where we believe such energies did occur, the very early universe in the first fraction of a second after the Big Bang. We know that the temperature of the early universe, back to when it was a few minutes old (the time of primordial nucleosynthesis), was falling like the inverse square root of its age, $T \propto 1/\sqrt{t}$. If we trace its history back still further, our best guess is that its temperature would have been above the electroweak phase transition, at a temperature of around $10^{15}$ K, when it was less than a microsecond old. Before that the full electroweak symmetry would have been evident. Even further back, if the grand unification idea is correct, it would have gone through a GUT transition, at the unimaginably early age of $10^{-35}$ s.

Of course no observers were around to check these speculations. We have to rely instead on very indirect evidence of these early phase transitions. One clue could come from looking for the characteristic topological defects that might have formed in the process of spontaneous symmetry breaking. Such defects are often stable, so some at least of them could have survived to much later times, perhaps even to the present day. Many types of defects are possible, depending on the nature of the symmetry breaking — point defects (monopoles), linear defects (cosmic strings), planar defects (domain walls), as well as combinations of these. However, the most interesting have turned out to be cosmic strings. (Reviews can be found in [2, 3, 4].)
2 Cosmic strings

The simplest model that shows what cosmic strings might be like is one with a single complex scalar field $\phi$, which we can also think of as a pair of real fields $\phi_{1,2}$, with $\phi = \phi_1 + i\phi_2$. The symmetry in this case is a phase symmetry. We assume that the Hamiltonian which defines the dynamics of the field is invariant under the phase rotation $\phi \rightarrow \phi e^{i\alpha}$, or equivalently

$$
\begin{align*}
\phi_1 &\rightarrow \phi_1 \cos \alpha - \phi_2 \sin \alpha, \\
\phi_2 &\rightarrow \phi_1 \sin \alpha + \phi_2 \cos \alpha.
\end{align*}
$$

(1)

In particular, there is a potential energy term $V$ which is a function only of $|\phi|$, usually taken to be the ‘sombrero potential’

$$
V = \frac{1}{2\lambda}(|\phi|^2 - \eta^2)^2 = \frac{1}{2\lambda}(\phi_1^2 + \phi_2^2 - \eta^2)^2,
$$

(2)

where $\eta$ is a constant (see Fig. 1). The important thing to notice is that the minimum is not at $\phi = 0$ but around the circle $|\phi| = \eta$. There is a degenerate ground state: any of the points $\phi = \eta e^{i\alpha}$ around the circle defines a ground state.

At high temperatures, there are large fluctuations in $\phi$ and the central hump in the potential is unimportant. Then there is obvious symmetry: fluctuations of $\phi$ in any direction are equally likely. But as the temperature falls there comes a point where the energy is too low to permit fluctuations over the hump. Then the field tends to settle towards one of the ground states. Which point on the circle of minima is chosen is a matter of random choice, like the choice of direction of fall for a pencil balanced on its tip. The spontaneous choice then breaks the original symmetry.

When a large system goes through a phase transition like this, each part of it has to make this random choice of the phase angle $\alpha$, but as in the process

![Figure 1: The sombrero potential.](image-url)
of crystallization the choice may not be the same everywhere — one part of the system does not ‘know’ of the choice made in a distant part [5]. Because there are terms in the energy involving the gradient of $\phi$, the phase angle will tend to become more uniform as the system cools. But this process may be frustrated; the random choices may not fit together neatly. In particular, there may be linear defects — cosmic strings (see Fig. 2) — around which the phase angle varies by $2\pi$ (or a multiple of $2\pi$).

Figure 2: A cosmic string. The directions of the arrows indicate the values of $\alpha$. The field $\phi$ vanishes in the core of the string.

3 Strings in the early universe

The important thing about cosmic strings is their stability. Continuity of $\phi$ means that a string cannot simply come to an end; it must form a closed loop or extend to infinity (or at least beyond the region we can see). For this reason, strings, once formed, are hard to eliminate. In the core of the string, $\phi$ vanishes, so there is trapped potential energy (as well as gradient energy). So strings represent trapped energy. In fact, the core of the string may be regarded as a relic of the earlier high-temperature phase, and the energy density there is similar to what it was before the transition. To lower this energy, strings will tend to shrink, smoothing out kinks, though at the same time they are being stretched by the expansion of the universe. They may cross and exchange partners (see Fig. 3a) — a process known as ‘intercommuting’, which creates new kinks. A string may also cross itself, forming a closed loop (see Fig. 3b). Such loops may shrink and eventually disappear, but a long string cannot do so directly. If a random tangle of strings was formed in the early universe, there would always be some longer than the radius of the visible universe, so a few would remain even today.

Analogous defects are formed in many condensed-matter systems undergoing low-temperature phase transitions. Examples are vortex lines in superfluid helium,
magnetic flux tubes in some superconductors, and disclination lines in liquid crystals. A nematic liquid crystal, for example, consists of rod-shaped molecules that like to line up parallel to each other. Everywhere in the liquid there is a preferred orientation, but note that diametrically opposite directions are equivalent. Around a disclination line, the preferred orientation rotates by $180^\circ$ (see Fig. 4). Along the line, molecules do not know which way to turn, and there is excess trapped energy. It is easy to see in this case too that a disclination line cannot simply end.

Experiments on cosmic strings would be impossible, even if we knew for sure
that they existed. But because there are these various analogues of cosmic strings, many interesting experiments have been done testing various aspects of the cosmic string formation and evolution scenario \cite{6}, though of course none of this can tell us whether there really are cosmic strings in the universe. For that we have to turn to astronomical observations, to which we shall return later.

In the late 80s and early 90s, cosmic strings generated a lot of excitement among cosmologists because they seemed to offer a plausible explanation for the origin of the density inhomogeneities from which galaxies later developed. Because they represent a lot of trapped energy, cosmic strings thrashing about in the early universe would significantly perturb the matter distribution, and it is not hard to get at least a rough estimate of how big the effect would be. The key parameter is the energy per unit length of the string which, for reasons of relativistic invariance (the characteristic speed of waves on the string is the speed of light, $c$), is equal to the string tension $\mu$. The strings are exceedingly thin, but very massive. Typically, for strings produced at a possible GUT transition, the mass per unit length $\mu/c^2$ would be of order $10^{21}$ kg m$^{-1}$; a string of length equal to the solar diameter would be about as massive as the Sun itself. The gravitational effects of a string are governed by a dimensionless parameter, $G\mu/c^4$, where $G$ is Newton’s constant. In particular, strings in the early universe would create density perturbations in which the fractional change in density is

$$\frac{\delta \rho}{\rho} \sim \frac{G\mu}{c^4}. \quad (3)$$

It happens that for GUT-scale strings, this ratio would be roughly $10^{-6}$ or $10^{-7}$. This is just the right order of magnitude to seed galaxy formation. In what follows, we shall choose units in which $c = 1$ and talk of the parameter $G\mu$.

Unfortunately this nice idea, like so many others, succumbed to the harsh realities of observation, in particular the observations of the small anisotropies in the cosmic microwave background. Measurements made by the COBE (COsmic Background Explorer) satellite and more recently by WMAP (the Wilkinson Microwave Anisotropy Probe) have yielded very precise information about these anisotropies. In particular, the angular power spectrum shows a series of peaks, so-called ‘acoustic peaks’, representing particular scales on which the anisotropy is large (see Fig. 5). The cosmic string scenario has no explanation for these features, predicting instead a single broad, flattish hump. On the other hand, the peaks are exactly what was predicted by the rival theory of inflation, in which the origin of the density perturbations can be traced to quantum fluctuations during an early period of accelerated expansion.

So inflation won, and cosmic strings were relegated at best to a minor supporting role, responsible for no more than 10% of the density perturbation at most. Many people lost interest in the idea.
Figure 5: Angular power spectrum of CMB [7]. The solid (red) line corresponds to $B = 0$, the dotted (black) line $B = 0.05$, the short-dash (green) line $B = 0.1$, the long-dash (light blue) line $B = 0.15$ and the dot-dash (dark blue) line to $B = 1$, where $B$ is the fraction of the power due to defects.

4 Superstring theory

There has, however, been a revival of interest, stemming largely from developments in our understanding of a very different kind of string — the fundamental strings of superstring theory, or its more modern incarnation, M-theory. (For a recent review, see [8].)

Fundamental string theory also originated in a search for unification, in particular the unification of gravity with the other interactions. This has long been the holy grail of theoretical physics, but has proved remarkably elusive. A major obstacle to creating a quantum theory of gravity (unified or not) has been the appearance of infinities. All quantum field theories are plagued by infinities, but for the other interactions we have learned how to tame them, by the process of ‘renormalization’. This allows us to extract meaningful, finite answers to physically significant questions, hiding all the infinities in supposedly unmeasurable ‘renormalization constants’. This has never been a wholly satisfactory procedure, but in any case it fails entirely in the case of gravity. No quantum version of Einstein’s theory of general relativity is renormalizable; there are infinities that cannot be swept under the carpet.
The basic reason for the appearance of infinities is the fact that we are dealing with point particles. They appear even in classical electromagnetic theory, for example, in the ‘self-energy’ of a charged particle: the potential energy stored in a spherical distribution of charge goes to infinity as its radius tends to zero.

This observation led to a very intriguing proposal: perhaps the fundamental objects of our theory should not be point particles, but extended objects, in particular strings. The basic idea is that all the particles we commonly think of as elementary — electrons, quarks, photons, and the rest — can be regarded as different modes or oscillation states of a fundamental string.

Even with strings as the basic building blocks, it proved difficult to eliminate all the infinities. One feature that made it easier was to incorporate supersymmetry into the theory. This is a remarkable symmetry that relates bosons (particles of integer spin in units of $\hbar$) to fermions (with half-integral spin). In a perfectly supersymmetric world, every boson would be partnered by a corresponding fermion of equal mass, and vice versa. Such partners have never been seen, so if our world is fundamentally supersymmetric, this symmetry too must be broken. But the great virtue of supersymmetry is that it removes a lot of infinities. This is because bosons and fermions often make equal and opposite contributions, so that if they are exact partners the infinities cancel.

One other strange feature was needed. Superstring models were eventually constructed that did seem to be free of all infinities, but not in the familiar four dimensions (three of space and one of time). The models were only consistent in 10 dimensions (nine space and one time) — or even sometimes 26! So the suggestion emerged that our universe is fundamentally ten-dimensional, but six of the dimensions are curled up very small, so that from our macroscopic perspective it looks four-dimensional — just as a drinking straw looks one-dimensional on a large scale.

These fundamental strings, as originally envisaged, were very different in many ways from cosmic strings. Firstly, their characteristic energy scale was much larger. Gravity is strongly energy-dependent, but would become as strong as the other interactions at the so-called Planck scale, corresponding to an energy around $10^{19}$ GeV, at least a thousand times higher than the GUT scale. Thus the parameter $G\mu$ for a fundamental string was of order one. Moreover, the fundamental strings could never have extended to macroscopic size: if you try to expand a fundamental string it will simply break into several small pieces.

But this picture has changed in several ways. There are other mechanisms for reducing the dimension from 10 to 4, whose effects are rather different, in particular the brane-world idea. Strings are not the only localized objects in a superstring theory. There can be two-dimensional membranes or their higher-dimensional analogues, which have come to be known as ‘p-branes’ (of dimension $p$) — a particle is a 0-brane, a string a 1-brane, and so on. We have D-branes (or $D_p$-branes), where the D denotes Dirichlet boundary conditions (see [9] for a review). Essentially this means that in addition to closed loops of fundamental string there may be open strings whose ends are tied to D-branes. There are also anti-branes, $\overline{D}$-branes. A $D_p$ brane and a $\overline{D}_p$ have equal and opposite conserved
charges, which means that they attract each other. Open strings usually give rise to matter fields while gravity comes from closed loops. This means that matter may be trapped on a Dp-brane while gravity feels all the extra dimensions.

The brane-world concept has also emerged recently from M-theory. M-theory is a conjectured umbrella theory that, in certain limits, reduces to the five known string theories or to supergravity. Supergravity was another attempt to unify all the forces, including gravity and incorporating supersymmetry and was formulated in 11 dimensions! It was shown that string theory at low energy is described by eleven-dimensional supergravity with the eleventh dimension compactified on an interval with $\mathbb{Z}_2$ (mirror) symmetry. The two boundaries of space–time are 10-dimensional planes, on which matter is confined. The extra six dimensions are compactified, but these compact dimensions are substantially smaller than the space between the two boundary branes. Thus, viewed on a larger scale, space–time looks five-dimensional with four-dimensional boundary branes. This is an example of a brane world model (see [10] for a recent review).

More generally speaking, in brane-world models normal matter is confined on a hypersurface (called a brane) embedded in a higher-dimensional space (called the bulk). Our universe may be such a brane-like object. Within the brane-world scenario, constraints on the size of extra dimensions become weaker because the standard particles propagate only in three spatial dimensions, while gravity (and some other exotic matter) can propagate in the bulk. Newton’s law of gravity, however, is sensitive to the presence of extra dimensions, and deviations are expected on scales below the brane separation. Gravity has so far been tested only on scales larger than a tenth of a millimeter [11]; possible deviations below that scale can be envisaged.

5 Cosmic Superstrings

One of the main motivations behind the brane-world scenario was to try to explain the vast difference between the Planck scale of gravity of $10^{19}$ GeV and the electroweak scale, which mediates radioactive decay, of $10^2$ GeV. The idea was to introduce warped space-time. In special relativity we are used to the invariant distance being given by $ds^2 = dt^2 - dx^2$. When space-time is ‘warped’ this becomes

$$ds^2 = e^{-A(y)}(dt^2 - dx^2) - dy^2,$$

where $y$ represents the extra dimensions and $A(y)$, the ‘warp’ factor, is a known, positive function. The warp factor is essentially a gravitational red-shift in the compact directions. In five-dimensional brane worlds this ‘warping’ of space-time was used to generate a hierarchy of scales such that gravity, which propagates on both the brane and in the bulk, could be at the Planck scale whilst the usual physics, confined to the brane, could have an energy scale much less than this. However, the warping of space-time is more general than brane-worlds and arises in many string theory models where there are 6 compact extra dimensions. In some cases the compact dimensions form a simple space such as a sphere or a
torus, with constant warp factor. But there are also solutions in which the warp factor varies strongly as a function of the compact dimensions, $y$, with special regions known as *throats* where it falls sharply to very small values, as shown in Figure 6. Here the warp factor is essentially 1 in the central region and much less than 1 in the throat. This means that, for a four-dimensional observer the fundamental string mass per unit length would appear to be

$$\mu = e^{-A(y)}\mu_0,$$

where $\mu_0$ is the ten-dimensional scale. Consequently fundamental strings may not be at such high energies after all, even if $\mu_0$ is at the Planck scale.

Another recent development in string theory is the concept of *brane inflation*. The idea of *inflation* is that, in the very early Universe there was a period of exponential expansion such that the visible Universe today was exponentially small at very early times. Since string theory is meant to be a theory of everything it should also explain the Universe at very early times, and consequently inflation. Recently it was discovered that string theory could give rise to inflation (for a review see [12]). Consider a Universe containing an extra *brane* and *anti-brane*. The brane and anti-brane are attracted to each other in the same way as an electron and positron are. However, when they annihilate they give rise to lower-dimensional branes [13] rather than photons. If the early universe contained an extra brane and anti-brane separated in the compact dimension, then the distance between them plays the role of a scalar field, called the inflaton. The potential energy of these branes drives the exponential expansion and therefore inflation. As the branes approach the potential between them becomes steeper before they annihilate. Once they annihilate lower dimensional branes are formed. It was shown that D-strings were generically formed in brane inflation [14].

The most fully developed model of brane inflation involves a D3/$\overline{D3}$ at the bottom of a throat. The D3 branes wrap the usual three spatial dimensions, so they appear as points in the throat of the extra dimensions, as shown in Figure 6. The inflation and subsequent brane annihilation take place in the throat. After brane inflation lower dimensional D-branes are formed, also in the throat. In

![Figure 6: Space with throat. In the middle we have the compactified space, with the throat on the left of the figure. The D/$\overline{D}$ branes are in the throat.](image)

- $\mu = e^{-A(y)}\mu_0,$
- $\mu_0$ is the ten-dimensional scale.
- String inflation is a theory that explains the early Universe.
- Recent developments in string theory include brane inflation.
- Branes and anti-branes can give rise to lower-dimensional branes.
- D-strings were generically formed in brane inflation.
the above example D1 branes or D-strings are formed. In addition fundamental strings, called F-strings can also be formed. Since the brane annihilation was in the throat where space-time is highly warped, then the energy scale of these strings is no longer the Planck scale but at a much lower energy scale. Estimates give the range to be between $10^{-11} < G\mu < 10^{-6}$ depending on details of the theory (see [15] for a review).

The idea of cosmic superstrings is not new — they were proposed as long ago as 1985 [16] — but they were dismissed as at too high an energy scale and unstable. However, the recent developments in string theory mean that not merely is this a distinct possibility but also perhaps the best way of observing string theory. The cosmic superstrings can have a range of values of string mass per unit length, $\mu$, compatible with observations and also the throat essentially provides a stabilizing potential. These two features circumvent the previous problems with cosmic superstrings.

Whilst both D- and F-strings can be formed in brane inflation they are fundamentally different objects. D-strings are more similar to the usual cosmic strings discussed in section 2 and are essentially classical objects, though like all D-branes they have a conserved charge. On the other hand F-strings are quantum mechanical objects.

Finally it now appears that grand unified theories will almost inevitably give rise to cosmic strings. In the long road from M-theory to the low-energy physics we observe in the laboratory, a natural route is via grand unified theories at an intermediate stage. In grand unified theories the coupling constants for the weak, electromagnetic and strong interactions meet at a high energy scale of about $10^{16}$ GeV. However, the theory is only successful when it is supersymmetric. A recent study [17] considered all possible grand unified theories, up to a certain level of complexity, with all possible symmetry breaking schemes which gave rise to the electroweak theory at low energy. The theories they considered all had a period of inflation. The ones that were not in conflict with observations all predicted the formation of cosmic strings at the end of inflation. Consequently it seems that cosmic strings are almost inevitable.

Since the grand unified theories studied were supersymmetric it seems natural to study the nature of cosmic strings in such theories. Supersymmetric theories give rise to two sorts of strings, called D-term or F-term strings [18], where the D and F refer to the type of potential required to break the symmetry, and have nothing directly to do with the distinction between the D- and F-strings discussed above. A recent analysis of supersymmetric theories with a D-term suggests that D-term cosmic strings may well be D-strings [19]. However, F-term strings are classical objects, and not apparently related to D- or F-strings. Nevertheless, the subtle relationships between different string theories regarded as limits of M-theory may affect some of these distinctions.
6 Cosmology of D- and F-strings

The cosmology of D-strings and F-strings is a little different from that of ordinary cosmic strings. In section 3 it was explained that when two cosmic strings meet they intercommute and that loops are formed by a cosmic string self-intersecting. For ordinary cosmic strings, the probability of intercommutation is 1. This is not the case for D- and F-strings. For D-strings this is because they can ‘miss’ each other in the compact dimension and F-strings are fundamentally quantum objects so their scattering can only be computed with a quantum mechanical calculation. The probability of intercommuting has been estimated to be between $10^{-3} < P < 1$ for F-strings and $10^{-1} < P < 1$ for D-strings. Similarly the probability of a string self-intersecting is reduced. This means that a network of such strings could look different from that of cosmic strings. There are suggestions that such a network would be denser, with the distance between strings related to $P$, and slower [20, 21]. It is likely that the net result would be to increase the number of string loops, despite the reduction in string self-intersection. A network of D- or F-strings could also emit exotic particles as a result of the underlying superstring theory. Ordinary cosmic strings emit particles, but those coming from cosmic superstrings could have distinctive characteristics.

Another interesting possibility is that, because they are different strings, when D- and F-strings meet they are unable to intercommute, instead forming a three-string junction, with a composite DF-string, as shown in Fig. 7. If this were the case then they would not form loops very effectively. This could be a problem since a usual cosmic string network loses energy via loop production. It is also possible in string theory to have bound states of $p$ F-strings and $q$ D-strings! The evolution of such an exotic system would be different from that discussed in section 3. It is possible that a such a network would become frozen and just stretch with the expansion of the universe. Consequently, there is much to investigate in the cosmology of D- and F-strings.

7 Observation of cosmic strings

The most promising way of observing cosmic strings is by searching for their very characteristic gravitational effects, either as gravitational lenses or emitters of gravitational waves.
The gravitational field around a straight, static string is quite unusual. Particles in the vicinity feel no gravitational acceleration, because in general relativity tension is a negative source of gravity and, since tension equals energy per unit length, their effects cancel. Space-time around the string is locally flat, but not globally flat. In cross-section, the space is cone-shaped, with a \textit{deficit angle} \[ \delta = 8\pi G\mu, \] as though a wedge of angle \( \delta \) had been removed and the edges stuck together. The deficit angle is \( \delta = 5'2(G\mu/10^{-6}) \), so for GUT-scale strings it is a few seconds of arc.

Thus the string acts as a cylindrical gravitational lens, creating double images of sources behind the string, with a typical angular separation of order \( \delta \) (see Fig. 8). A string would yield a very special pattern of lensing. We should expect to see an approximately linear array of lensed pairs, each separated in the transverse direction. In most the two images would have essentially the same magnitude. (The exception would be if we see only part of one of the images.) This is a very unusual signature; most ordinary gravitational lenses produce images of substantially different magnitude, usually an odd number of them \cite{22}.

There are several factors that may complicate this picture. Cosmic strings are not generally either straight or static. Whenever strings exchange partners kinks are created that straighten out only very slowly, so we expect a lot of small-scale structure on the strings. Viewed from a large scale, the effective tension and energy per unit length will no longer be equal. Since the total length of a kinky string between two points is greater, it will have a larger effective energy per unit length, \( U \), while the effective tension \( T \), the average longitudinal component of the tension force, is reduced, so \( T < \mu < U \). This means that there is a non-zero gravitational acceleration towards the string, proportional to \( U - T \). Moreover, the strings acquire large velocities, generally a significant fraction of the speed of light. If the string is moving with velocity \( \mathbf{v} \) perpendicular to its direction, the expression for the angular separation of an image pair is

\[ \alpha = \frac{8\pi GU}{\gamma(1 - v_r)} \frac{D_{ls}}{D_s} \sin \theta, \]  

where \( D_s \) is the angular-diameter distance of the source, \( D_{ls} \) that of the source from the lensing string, \( \theta \) is the angle between the string and the line of sight, \( \gamma = (1 - v^2)^{-1/2} \), and \( v_r \) is the radial component of \( \mathbf{v} \).

Another very characteristic effect is the distortion of the CMB produced by a moving string. A string moving across our field of vision will cause a blue-shift of...
the radiation ahead of it, and a red-shift of that behind, leading to a discontinuity in temperature of magnitude \( \delta T / T = 8\pi G\gamma v_\perp \sin \theta \), where \( v_\perp \) is the component of the string velocity normal to the plane containing the string and the line of sight.

Accelerated cosmic strings are sources of gravitational radiation. The most important signal would come from special places where the strings are moving exceptionally fast. Loops of string undergo periodic oscillations, with a period related to the size of the loop. The dynamical equations predict that during each oscillation there will be a few points at which the string instantaneously forms a cusp, where it doubles back on itself (see Fig. 9). In the neighbourhood of the cusp,

\[ \text{Figure 9: Cosmic string loop with a cusp.} \]

the string velocity approaches the speed of light. Such an event would generate an intense pulse of gravitational radiation, strongly beamed in the direction of motion of the cusp. If massive cosmic strings do indeed exist, these pulses are likely to be among the most prominent signals seen by the gravitational-wave detectors now in operation or planned, in particular LIGO and LISA.

This effect has already provided a stringent, though indirect, limit on the value of \( G\mu \). This comes from observations of the timing of millisecond pulsars. Gravitational waves between us and a pulsar would distort the intervening space-time, and so cause random fluctuations in the pulsar timing. The fact that pulsar timings are extremely regular places an upper limit on the energy density in gravitational waves, and hence on \( G\mu \). The upper limit is of order \( 10^{-7} \), though there is considerable uncertainty because this depends on assumptions about the evolution of small-scale structure.

For cosmic superstrings the situation is similar. However, because the intercommutation probability is less than unity the evolution of the network is a little different, resulting in a denser, slower network of strings with more cusps on it. A recent study suggests this could enhance the gravitational radiation emitted \(^{23}\) and that such strings could be detectable with the gravitational-wave detectors in the near future. Seeing such cosmic strings could provide a window into superstring theory!
8 Recent observations

8.1 A possible example of lensing by a cosmic string

One exciting recent piece of evidence was the observation of a possible example of cosmic-string lensing by a Russian–Italian collaboration, between the Observatory of Capodimonte in Naples and the Sternberg Astronomical Institute in Moscow. What Sazhin et al. [24] saw was a pair of images of apparently very similar elliptical galaxies, both with a red-shift of 0.46, separated by 2′′. The images have the same magnitude and the same colour — the magnitudes in three separate wavelength bands are equal within the errors. They could of course be images of two distinct, but very similar galaxies that just happen to lie very close together. Close pairs are not unusual, but it would be a remarkable coincidence to find two so similar so close together. The images could also be due to lensing by some more conventional foreground object, but the authors show that it would have to be a giant galaxy, of which there is no sign.

They conclude that the most likely explanation is lensing by a cosmic string. If so, the observed separation requires that $G\mu > 4 \times 10^{-7}$, which is at least marginally in conflict with the upper limits from CMB anisotropy and pulsar timing observations. However, it should be remembered that this is actually a limit on the effective quantity $G\mu U$ rather than $G\mu$ so there is at least some room for manoeuvre.

Another important piece of evidence comes from a later study by the same authors of the surrounding region [25]. If the image pair is due to lensing by a cosmic string, one would expect other lensed pairs in the vicinity, along the line of the cosmic string. The authors searched for such pairs in a 16′ × 16′ patch of sky around the original image pair (which is called CSL-1, the Capodimonte–Sternberg Lens Candidate no. 1 — there are three others yet to be analyzed). Among the roughly 2200 galaxies within this patch, they found 11 very likely candidates for lensed pairs, based on separation and colour matching. They estimate that a string should produce somewhere between 9 and 200 lensed pairs (depending on its configuration), while they should expect no more than 2 due to conventional lensing. So this adds weight to their interpretation, though they emphasize that the identification needs to be confirmed by a spectroscopic analysis of the pairs.

We can learn a lot from the distribution and alignment of the candidate pairs. A straight string should produce a linear array of lensed pairs, with the pairs separated in the transverse direction. A picture of the six brightest pairs [26] (see Fig. 9) does not show such a sharp concentration, but nor do they seem to be randomly scattered. The position angles of the pairs nos. 2,3,5,6 do suggest that they could line up on a smooth curve of string [27]. The others could perhaps be fitted to a string with a couple of kinks. The important test of this idea will come from a spectroscopic analysis of the candidate pairs, to show whether they are indeed images of the same object.
8.2 Possible lensing by an oscillating loop

The other intriguing development is an analysis by Schild et al. of brightness fluctuations in a very well known gravitational lens system that has been studied extensively for 25 years. The system is famous because it has been used to provide an estimate of the Hubble constant, independent of other distance measurements. It consists of a pair of images, which are known to be images of a single quasar, because of the constant observed time delay: brightness fluctuations in image A are generally followed by similar fluctuations in B 417.1 days later. The lensing in this case is due to a clearly visible foreground galaxy, and the time delay occurs because one light path from the quasar is a little more than one light year longer than the other.

In addition to the correlated fluctuations there are independent fluctuations of the two images, primarily caused by microlensing by individual stars in the foreground galaxy, that is, lensing in which different images are not resolved. What Schild et al. have found in data from observations in 1994–95 is an apparent sequence of synchronous fluctuations in both images with an amplitude of about 4% and no time delay. They see a sequence of three or four oscillations with a period of about 80 days.

If these oscillations do have a common origin, they must be due to some object quite close to us. One possibility would be lensing by a binary star, but to get the right period and amplitude the stars would need be among our near neighbours and to have masses of around eighty solar masses. It is inconceivable that such massive stars so near us could have escaped detection.

Another possibility is an oscillating loop of cosmic string. Since the period is proportional to the length of the string loop, the required 80-day period would imply a length of 160 light-days. The loop would probably be moving with a substantial fraction of the speed of light, so it would only remain between us and
the source for a year or so; thus it is not surprising that only a few oscillation cycles were seen. The apparent angular size would need to be somewhat less than the separation between the images (otherwise there would be sharp spikes of intensity when the string actually crossed one of the paths). On the other hand the loop cannot be very far away, otherwise the required value of $G\mu$ would be impossibly large. In fact, it must be well inside our galaxy. Since we know from CMB and gravity-wave limits that loops of this size must be quite rare, to find one so near would be remarkably fortuitous.

Of course, the sequence of synchronous fluctuations might just be a coincidence, but the authors argue that the probability of seeing three or more synchronous oscillations by chance is quite low. Nevertheless, this may be the simplest explanation; we need a proper statistical analysis of how likely it is that this should have happened by chance at some time during the many years of observation.

9 Conclusions

As we have described, recent developments in fundamental string theory, especially the brane-world concept, have greatly extended the range of different kinds of cosmic strings that might have been formed in the very early history of the universe. There have been intriguing hints of observations that might be signatures of cosmic strings. Further work in the near future should clarify their status. Even if these observations turn out to have more prosaic explanations, the quest for evidence of cosmic strings will certainly continue, in particular via searches for the gravitational waves they emit. The very characteristic signature of emission from a cusp might be detectable even by the present generation of gravitational-wave detectors, and certainly by the next. This may well provide the first direct evidence for an underlying superstring or M-theory.

Acknowledgments

We are indebted for valuable comments and discussion to Ana Achúcarro, Levon Pogosian, Fernando Quevedo, Mairi Sakellariadou and Miguel Sazhin. The work reported here was supported in part by PPARC, and in part by the ESF through the COSLAB Programme.

References

[1] Weinberg, S., 1996, *The Quantum Theory of Fields, Vol. II: Modern Applications*, ch. 19, 21. Cambridge: Cambridge University Press.

[2] Hindmarsh, M.B. & Kibble, T.W.B., 1995, Cosmic strings. *Rep. Prog. Phys.*, 58, 477–562 (http://ArXiv.org/hep-ph/9411342).
[3] Vilenkin, A. & Shellard, E.P.S., 1994, *Cosmic Strings and Other Topological Defects*. Cambridge: Cambridge University Press.

[4] Rajantie, A.K., 2003, Defect formation in the early universe, *Contemp. Phys.*, 44, 485–502 (http://ArXiv.org/astro-ph/0307387).

[5] Kibble, T.W.B., 1976, Topology of cosmic domains and strings. *J. Phys. A: Math. & Gen.* 9, 1387–98.

[6] Kibble, T.W.B., 2002, Testing cosmological defect formation in the laboratory, *Physica C* 369, 87–92 (http://ArXiv.org/cond-mat/0111082).

[7] Pogosian, L., Tye, S.-H.H., Wasserman, I. & Wyman, M., 2003, Observational constraints on cosmic string production during brane inflation, *Phys. Rev. D* 68, 023506, (http://ArXiv.org/hep-th/0304188).

[8] Green, M.B., 2000, Superstrings and the unification of the physical forces, in *Mathematical Physics 2000*, ed. Fokas, A., Grigoryan, A., Kibble, T. & Zegarlinski, B, pp. 59–86. London: Imperial College Press.

[9] Gauntlett, J.P., 1998, M Theory: strings, duality and branes, *Contemp. Phys.*, 39 317–328.

[10] Brax, P., van de Bruck, C. & Davis, A.C., 2004, Brane World Cosmology, *Rep. Prog. Phys.* 67, 2183–2232 (http://ArXiv.org/hep-th/0404011).

[11] Hoyle, C.D. et al, 2001, Submillimetre tests of the gravitational inverse square law: a search for large extra dimensions, *Phys. Rev. Lett.* 86 1418–1421 (http://ArXiv.org/hep-ph/0011014).

[12] Quevedo, F., 2002, Lectures on String and Brane Cosmology, *Class. Quant. Grav* 19, 5721–5779 (http://ArXiv.org/hep-th/0210292).

[13] Majumdar, M. & Davis, A.C., 2002, Cosmological Creation of D-Branes and Anti-D-Branes, *JHEP* 0203, 056 (http://ArXiv.org/hep-th/0202148).

[14] Sarangi, S. & Tye, S.-H.H., 2002, Cosmic String Production Towards the End of Brane Inflation, *Phys. Lett. B* 536, 185–192 (http://ArXiv.org/hep-th/0204074).

[15] Polchinski, J., 2004, Introduction to Cosmic F- and D-Strings, (http://ArXiv.org/hep-th/0412244)

[16] Witten, E., 1985, Cosmic Superstrings, *Phys. Lett. B* 153, 243.

[17] Jeannerot, R., Rocher, J. & Sakellariadou, M., 2003, How Generic is Cosmic String Formation in SUSY GUTs?, *Phys. Rev. D* 68 103515, (http://ArXiv.org/hep-ph/0308134).
[18] Davis, S.C., Davis, A.C. & Trodden, M., 1997, N=1 Supersymmetric Cosmic Strings, *Phys. Lett. B* 405, 257-264 (http://ArXiv.org/hep-ph/9702360).

[19] Dvali, G., Kallosh, R. & van Proeyen, A., 2004, D-Term Strings, *JHEP* 0401, 035 (http://ArXiv.org/hep-th/0312005).

[20] Dvali, G. & Vilenkin, A., 2004, Formation and Evolution of Cosmic D Strings, *JCAP* 0403, 010 (http://ArXiv.org/hep-th/0312007).

[21] Sakellariadou, M., 2004, A Note on the Evolution of Cosmic String/Superstring Network, (http://ArXiv.org/hep-th/0410234).

[22] Burke, W.L., 1981, Multiple Gravitational Imaging by Distributed Masses, *Ap. J.* 244, L1; Schneider P., Ehlers J, Falco E., 1992, *Gravitational Lenses* Springer-Verlag, New York.

[23] Damour, T. & Vilenkin, A., 2005, Gravitational Radiation from Cosmic (Super)strings, *Phys. Rev. D* 71, 063510 (http://ArXiv.org/hep-th/0410222).

[24] Sazhin, M., Longo, G., Capaccioli, M., Alcalá, J.M., Silvotti, R., Covone, G., Khovanskaya, O., Pavlov, M., Pannella, M., Radovich, M. & Testa, V., 2003, CSL-1: chance projection effect or serendipitous discovery of a gravitational lens induced by a cosmic string?. *Mon. Not. Roy. Astro. Soc.* 343, 353 (http://ArXiv.org/astro-ph/0302547).

[25] Sazhin, M.V., Khovanskaya, O.S., Capaccioli, M., Longo, G., Alcalá, J.M., Silvotti, R. & Pavlov, M.V., 2004, Lens candidates in the Capodimonte Deep Field in vicinity of the CSL-1 object, http://ArXiv.org/astro-ph/0406516.

[26] Sazhin, M.V., Khovanskaya, O.S., Capaccioli, M. & Longo, G., 2004, Possible observation of a cosmic string, Talk at Quarks 2004, 13th International Seminar on High Energy Physics, Pushkinskie Gory, Russia, May 24-30, 2004.

[27] Sazhin, M.V., 2004, private communication.

[28] Schild, R., Masnyak, I.S., Hnatyk, B.I. & Zhdanov, V.I., 2004, Anomalous fluctuations in observations of Q0957+561 A:B: smoking gun of a cosmic string?, http://ArXiv.org/astro-ph/0406434.