Forecasting the curvaton isocurvature scenario with CMB lensing information

Larissa Santos
Dipartimento di Fisica, Università di Roma "Tor Vergata"
E-mail: larissa.santos@roma2.infn.it

Abstract. Some inflationary models predict the existence of isocurvature primordial fluctuations, in addition to the well known adiabatic perturbations. Such mixed models are not yet ruled out by available data sets. The talk presented at the Young Research Meeting covered the possibility of obtaining better constraints on the isocurvature contribution from future astronomical data. I use Planck satellite experimental specifications together with SDSS galaxy survey to forecast for the best parameter error estimate by means of the Fisher information matrix formalism. In particular, I consider how CMB lensing information can improve this forecast. I focus specially in the curvaton inflationary scenario to show a substantial improvement in all the considered cosmological parameters. In the case of isocurvature amplitude the improvement is above 20% around its fiducial value. In this sense, CMB lensing information will be crucial in the analysis of future data.

1. Introduction
Since the early measurement of the first acoustic peak in the Cosmic Microwave Background (CMB) angular power spectrum [10, 14], a pure isocurvature model of primordial fluctuations was ruled out [13]. In addition, recent CMB data from the WMAP satellite found no evidence for non-adiabatic primordial fluctuations [17]. However, small contributions from isocurvature primordial fluctuations cannot be excluded by the current data.

The standard inflationary scenario driven by the inflaton cannot account for isocurvature fluctuations. We consider the alternative scenario where perturbations to the curvaton light field are responsible for curvature perturbations and may also generate isocurvature fluctuations [23, 25, 22]. In this case, the isocurvature component is completely correlated or anti-correlated with the adiabatic component. Following [1], the isocurvature contribution to the purely adiabatic CMB power spectra is written in the form:

$$C_l = (1 - \alpha)C_{l,\text{ad}}^0 + \alpha C_{l,\text{iso}}^0 + 2\beta\sqrt{\alpha(1 - \alpha)}C_{l,\text{cross}}^0,$$  (1)

where the parameter $\alpha$ accounts for the isocurvature amplitude, while $\beta$ stands for the isocurvature correlation phase, given by $\beta = \cos\theta$, $-1 \leq \cos\theta \leq 1$. In the same way, the matter power spectrum, $P(k)$, can be decomposed into purely adiabatic, purely isocurvature and their cross correlation contribution. I assume that $n_{\text{cross}} = \frac{1}{2}(n_{\text{ad}} + n_{\text{iso}})$ as stated by [1] and take the pivot value $k_0 = 0.002\,\text{Mpc}^{-1}$ as used by the WMAP team.

There are already many studies in the literature constraining the isocurvature contribution using different data sets (see, for example, [21, 6, 18, 17, 24, 1, 2, 8]). I intend to use the
well-known Fisher information matrix formalism to estimate whether better constraints to the isocurvature contribution can be obtained in the near future using measurements of the CMB temperature and polarization power spectra from the Planck satellite, also using CMB lensing information, as well as the large-scale matter distribution observed by the Sloan Digital Sky Survey (SDSS).

A small introduction on CMB lensing is given in Section 2. In Section 3, a brief review of the Fisher information matrix formalism for the CMB and for a galaxy survey is presented. Finally, results are shown in Section 4, followed by the discussion and conclusions in Section 5.

2. CMB lensing

As new experiments are developed, the precision in CMB measurements makes small effects distinguishable by observations. CMB lensing is one of this effects and it has important quantitative contribution that should be taken into account. It is known that the CMB photons are deflected by gravitational potentials during their travel between the last scattering surface and the observer. For CMB temperature anisotropy, this is quantitatively written as:

\[ \frac{\Delta \tilde{T}(\hat{n})}{T} = \frac{\Delta T(\hat{n}')}{T} = \frac{\Delta T(\hat{n} + d)}{T}, \]

where the temperature \( T \) of the lensed CMB in a direction \( \hat{n} \) is equal to the unlensed CMB in a different direction \( \hat{n}' \). Both these directions, \( \hat{n} \) and \( \hat{n}' \), differ by the deflection angle \( d \) as it can be seen in the third equality above. To first order, the deflection angle is simply the lensing potential gradient, \( d = \nabla \psi \) (for a review of CMB lensing see [19]). In the same way, the effect of lensing in CMB polarization is written in terms of the Stokes parameters \( Q(\hat{n}) \) and \( U(\hat{n}) \) (for a review in CMB polarization theory, see [5]):

\[ \left[ Q + iU \right](\hat{n}) = \left[ Q + iU \right](\hat{n} + d). \]

We can study CMB lensing properties through the lensing potential, and the temperature and polarization power spectra, as well as their cross-correlation.

The lensing signal was detected for the first time by cross-correlating WMAP data to radio galaxy counts in the NRAO VLA sky survey (NVSS) [28]. The detection of the gravitational lensing using CMB temperature maps alone and the measurement of the power spectrum of the projected gravitational potential were already done using the Atacama Cosmology Telescope and the South Pole Telescope [9, 31]. While waiting for more precise data sets, much work is being done to CMB lensing reconstruction techniques (e.g. [15, 26, 29, 4, 7]). Here, I use the CAMB software package [20] to obtain the numerical lensed and unlensed power spectra for the chosen cosmological model and then forecast how CMB lensing information can help constraining the cosmological parameters.

3. Method

In order to search how an isocurvature contribution would affect the measurements of cosmological parameters I apply the Fisher information matrix formalism to a Planck-like experiment [3], considering both temperature and polarization for the lensed and unlensed CMB spectra, and the matter power spectrum from the Sloan Digital Sky Survey (SDSS).

3.1. Information from CMB

The Fisher information matrix for the CMB temperature anisotropy and polarization is given by the approximation in [32]:

\[ F_{ij} = \sum_l \sum_{XY} \frac{\partial C^X_l}{\partial p_i} (\text{Cov}_l^{-1})_{XY} \frac{\partial C^Y_l}{\partial p_j}, \]
where \( C_l^X \) is the power in the \( l \)th multipole, \( X \) stands for \( TT \) (temperature), \( EE \) (E-mode polarization), \( BB \) (B-mode polarization) and \( TE \) (temperature and E-mode polarization cross-correlation), and \( p \) is a parameter of the fiducial model that we want to constraint. We will not include primordial B-modes in the analysis since the measurement of the primordial \( C_l^{BB} \) by Planck is expected to be noise dominated. The covariance matrix becomes therefore:

\[
\text{Cov}_l = \frac{2}{(2l+1)f_{\text{sky}}} \begin{bmatrix}
\Xi_{TTTT} & \Xi_{TTEE} & \Xi_{ETTE} & \Xi_{TTEE} & \Xi_{ETTE} \\
\Xi_{ETTE} & \Xi_{EEEE} & \Xi_{EEET} & \Xi_{EEE} & \Xi_{EEET} \\
\Xi_{ETTE} & \Xi_{EEE} & \Xi_{EEEE} & \Xi_{EEET} & \Xi_{EEET} \\
\Xi_{TTEE} & \Xi_{EETE} & \Xi_{EEE} & \Xi_{EEEE} & \Xi_{EEET} \\
\Xi_{TTEE} & \Xi_{EEE} & \Xi_{EEET} & \Xi_{EEET} & \Xi_{EEET}
\end{bmatrix},
\]

(5)

where \( f_{\text{sky}} \) is the observed sky fraction. For the lensed case we have to perform a correction in the covariance matrix elements taking into consideration the power spectrum of the deflection angle and its cross correlation with temperature, \( C_l^{Td} \). Note that in this case we are taking into consideration the B-mode polarization generated by the CMB gravitational lensing from the E-mode polarization. When these corrections are included, the covariance matrix becomes:

\[
\text{Cov}_l = \frac{2}{(2l+1)f_{\text{sky}}} \begin{bmatrix}
\xi_{TTTT} & \xi_{TTEE} & \xi_{ETTE} & \xi_{TTEE} & \xi_{ETTE} & 0 \\
\xi_{ETTE} & \xi_{EEEE} & \xi_{EEET} & \Xi_{EEE} & \Xi_{EEET} & 0 \\
\xi_{ETTE} & \xi_{EETE} & \Xi_{EEEE} & \Xi_{EEET} & 0 & 0 \\
0 & 0 & \xi_{TddT} & 0 & 0 & 0 \\
0 & 0 & 0 & \xi_{ddTT} & 0 & 0 \\
0 & 0 & 0 & 0 & \xi_{BBBB}
\end{bmatrix},
\]

(6)

Full expressions for the elements of both covariance matrices can be found in [11] and [27]. The zero elements in the covariance matrix are the ones related to the cross correlated B mode power spectra, since we assume \( C_l^{TB} = C_l^{EB} = C_l^{BB} = 0 \). I also consider \( C_{E}^{dd} = 0 \). Note that in this case the B-mode polarization generated by the CMB gravitational lensing from the E-mode polarization has being taken into consideration. In both cases, it is assumed that the experiment covers 65% of the sky.

### 3.2. Information from galaxy survey

The Fisher information matrix for the matter power spectrum obtained from galaxy surveys is given by [30]:

\[
F_{ij} = \int_{k_{\text{min}}}^{k_{\text{max}}} \frac{\partial \ln P(k)}{\partial p_i} \frac{\partial \ln P(k)}{\partial p_j} V_{\text{eff}} \frac{k^2 dk}{(2\pi)^2},
\]

(7)

\[
V_{\text{eff}}(k) = \int \left[ \frac{\bar{n}(r) P_g(k)}{1 + \bar{n}(r) P_g(k)} \right]^2 d^3 r.
\]

(8)

We know that \( P_g(k) = b^2 P(k) \) and using the specifications of the SDSS experiment for the Bright Red Galaxy (BRG) sample, called Luminous Red Galaxies (LRG) in more recent papers, we assume a linear and scalar independent bias \( b = 2 \) [30, 16]. It is assumed that the expected number density of galaxies, \( \bar{n}(r) \), is independent of \( r \), \( n = 10^{5}/V_s \), in a volume-limited sample to a depth of 10^8 Mpc. The survey has an angle of \( \pi \) steradians; therefore the survey volume becomes, \( V_s = 10^9 \pi/3 \) [30, 11, 12].
Figure 1. Fisher contours for the $\Lambda$CDM model plus a contribution of initial isocurvature fluctuations with fiducial amplitude of $\alpha = 0.003$ and correlation phase of $\beta = -1$ for a curvaton scenario. The red and blue contours represent 95.4% and 68% C.L. respectively for the unlesed CMB + SDSS (dashed lines) and for the lensed CMB + SDSS (solid lines).

4. Results
For the purpose of this analysis, I consider as free cosmological parameters the adimensional value of the Hubble constant, $h$, the density of baryons and cold dark matter, $\Omega_b h^2$ and $\Omega_c h^2$, the spectral index of scalar adiabatic perturbations, $n_{ad}$, the aforementioned isocurvature parameter, $\alpha$, and the equation of state of dark energy $w$, assumed as a constant. I first performed the forecast for Planck alone, with and without considering CMB lensing. Then, I introduced the forecast for SDSS, combining the results as:

$$F^\text{Total}_{ij} = F^\text{Planck}_{ij} + F^\text{SDSS}_{ij}. \quad (9)$$

The fiducial parameters’ values are set to be, $h = 0.745$, $\Omega_b = 0.02293 h^2$, $\Omega_c = 0.1058 h^2$, $n_{ad} = 0.984$ ($n_{iso} = 0.984$ fixed), $\alpha = 0.003$, $\beta = -1$ and $w = -1$. The best constraint is reached considering the CMB lensing effect in the analysis (see, Table 1 and Figure 1), improving Planck limit to $\alpha < 0.0054$ (95% CL). For Planck (including CMB lensing) + SDSS, $\alpha < 0.0039$ (95% CL) against $\alpha < 0.0037$ (95% CL) for WMAP + BAO + SN. Adding therefore other cosmological probes, such as BAO and SN to Planck with lensing information + SDSS, the error bars in the isocurvature amplitude can become even smaller, allowing to a very limited isocurvature contribution in the primordial fluctuations.

5. Discussion and conclusions
In this presentation I considered a possible contribution of isocurvature initial perturbations in the pure adiabatic fluctuations scenario from the well tested $\Lambda$CDM model. Using the Fisher formalism I obtained the best constraints possible for the isocurvature parameters using CMB and galaxy distribution information. The main goal of this work has been to quantify how CMB lensing information can provide better constraints in the cosmological parameters, specially in the ones related to the isocurvature contribution.

In the tested inflationary scenario, the CMB lensing information improves the constraints of all chosen parameters, including the ones related to the isocurvature modes. If we consider
Table 1. Marginalized errors for ΛCDM model plus the contribution of initial isocurvature fluctuation driven by the curvaton field.

| Parameter | CMB alone Planck T + P | CMB alone Planck T + P | P(k) alone SDSS T + P | CMB + P(k) Planck + SDSS T + P | CMB + P(k) Planck + SDSS T + P + lens |
|-----------|-------------------------|-------------------------|------------------------|-------------------------------|--------------------------------------|-------------------------------------|
| $h$       | 0.055                   | 0.0345                  | 0.31                   | 0.0092                        | 0.0083                               |
| $h^2\Omega_b$ | 0.00012                | 0.00011                 | 0.027                  | 0.00011                       | 0.00011                              |
| $h^2\Omega_c$ | 0.0010                 | 0.00088                 | 0.092                  | 0.00091                       | 0.00077                              |
| $n_a d$   | 0.0027                  | 0.0025                  | 0.24                   | 0.0026                        | 0.0024                               |
| $\alpha$  | 0.0019                  | 0.0012                  | 0.17                   | 0.00047                       | 0.00045                              |
| $w$       | 0.14                    | 0.088                   | 7.47                   | 0.016                         | 0.015                                |

Percentage of the parameters’ fiducial values for each error above

| Parameter | CMB alone Planck T + P | CMB alone Planck T + P + lens | P(k) alone SDSS T + P | CMB + P(k) Planck + SDSS T + P | CMB + P(k) Planck + SDSS T + P + lens |
|-----------|-------------------------|--------------------------------|------------------------|-------------------------------|--------------------------------------|
| $h$       | 7.38%                   | 4.63%                         | 41.61%                 | 1.23%                         | 1.11%                                |
| $h^2\Omega_b$ | 0.52%                   | 0.48%                         | 117.75%                | 0.48%                         | 0.48%                                |
| $h^2\Omega_c$ | 0.94%                   | 0.83%                         | 87.90%                 | 0.86%                         | 0.72%                                |
| $n_a d$   | 0.27%                   | 0.25%                         | 24%                    | 0.26%                         | 0.24%                                |
| $\alpha$  | 63.33%                  | 40%                           | not constrained        | 15.67%                        | 15%                                  |
| $w$       | 14%                     | 8.8%                          | not constrained        | 1.6%                          | 1.5%                                 |

Planck information alone the improvement on $\alpha$ reaches almost 25% around its fiducial value (see the lower part of Table 1).

When the combined Planck + SDSS forecast is done, the improvement with the use of lensing information is not so significant. This is due to the poor ability of SDSS to constrain the parameters compared to Planck, especially when CMB lensing information is included. For a CMB experiment alone, or combined with any other precise experiments on galaxies distribution, lensing is an important extra information in the attempt to know how well observations can constrain the presence of isocurvature contribution to the primordial fluctuations.

References
[1] Bean, R., et al., 2006, Physical Review D, 74, 063503
[2] Beltrán, M., et al., 2005, Physical Review D, 72, 103515
[3] http://www.rssd.esa.int/SA/PLANCK/docs/Bluebook-ESA-SCI(2005)1_V2.pdf
[4] Bucher, M., et al., 2010, MNRAS, 407, 2193-2206
[5] Cabella, P. & Kamionkowski, M., 2004, ArXiv Astrophysics e-prints, astro-ph/0403392
[6] Carbone, C., et al., 2011, Cosmology Astropart. Phys., 9, 28
[7] Carvalho, C. S., & Tenero, I., 2011, Physical Review D, 84, 063001
[8] Crotty, P., et al., 2003, Physical Review Letters, 91, 171301
[9] Das, S., et al., Phys. Rev. Lett., 107, 021301 (2011).
[10] de Bernardis, P., et al., 2000, Nature, 404, 955-959
[11] Eisenstein, D., et al., 1999, Physical Review D, 518,2-23
[12] Eisenstein, D., et al., AJ, 122, 2267-2280 (2001).
[13] Enqvist, K., et al., 2000, Physical Review D, 62, 103003
[14] Hanany, S., et al., 2000, Astrophysical Journal Letters, 545, L5-L9
[15] Hu, W., 2001, Astrophysical Journal, 557, L79
[16] Hiitts, G., 2006, Astronomy and Astrophysics, 449, 891-902
[17] Komatsu, E., et al., 2011, Astrophysical Journal Supplement, 192,18
[18] Larson, D., et al., 2011, Astrophysical Journal Supplement, 192, 16
[19] Lewis, A. & Challinor, A., 2006, Phys. Rep., 429, 1-65
[20] Lewis, A. et al., 2000, Astrophysical Journal, 538, 473-476
[21] Li, H. et al., 2011, Physical Review D, 83, 123517
[22] Lyth, D. et al., 2003, Physical Review D, 67, 023503
[23] Lyth, D. & Wands, D., 2002, Physics Letters B, 524, 5-14
[24] Mangilli, A. et al., 2010, Cosmology Astropart. Phys., 10, 9
[25] Moroi, T. & Takahashi, T., 2002, Physical Review D, 66, 063501
[26] Okamoto, T. & Hu, W., 2003, Physical Review D, 67, 083002
[27] Perotto, L., et al., 2006, Cosmology Astropart. Phys., 10,13
[28] Smith, K., M., et al., 2007, Physical Review D, 76,043510
[29] Smith, K., M., et al., 2010, ArXiv e-prints, astro-ph/1010.0048
[30] Tegmark, M., 1997, Physical Review Letter, 79, 3806
[31] van Engelen, A., et al., ArXiv: astro-ph/1202.0546.
[32] Zaldarriaga, M. & Seljak, U., 1997, Physical Review D, 55, 1830-1840