Goos–Hänchen and Imbert–Fedorov shifts for astigmatic Gaussian beams

Marco Ornigotti\textsuperscript{1} and Andrea Aiello\textsuperscript{2,3}

\textsuperscript{1} Institute of Applied Physics, Friedrich-Schiller University, Jena, Max-Wien Platz 1, D-07743 Jena, Germany
\textsuperscript{2} Max Planck Institute for the Science of Light, Günther-Scharowsky-Strasse 1/Bau24, D-91058 Erlangen, Germany
\textsuperscript{3} Institute for Optics, Information and Photonics, University of Erlangen-Nuernberg, Staudtstrasse 7/B2, D-91058 Erlangen, Germany

E-mail: marco.ornigotti@uni-jena.de

Received 29 January 2015, revised 14 April 2015
Accepted for publication 17 April 2015
Published 27 May 2015

Abstract

In this work we investigate the role of the beam astigmatism in the Goos–Hänchen and Imbert–Fedorov shift. As a case study, we consider a Gaussian beam focused by an astigmatic lens and we calculate explicitly the corrections to the standard formulas for beam shifts due to the astigmatism induced by the lens. Our results show that the different focusing in the longitudinal and transverse direction introduced by an astigmatic lens may enhance the angular part of the shift.

Keywords: physical reflection, beam shifts, astigmatic beams

1. Introduction

Geometrical optics considers light fields as rays directed along the propagation direction of the field itself. Within this approximation, most of the phenomena that we witness daily regarding light can be easily explained. An example is given by the phenomenon of reflection from an interface, which for an optical ray happens in a specular way, following Snell’s law \cite{1}. When the wave properties of optical fields are taken into account, however, deviations from specular reflection can be observed. This is the case of optical beams, whose finite transverse sizes affect its reflection and refraction across interfaces. The most common manifestation of these effects is given by the so-called Goos–Hänchen \cite{2–4} and Imbert–Fedorov \cite{5–15} shifts, the former occurring in the plane of incidence and the latter in the plane perpendicular to the plane of incidence. These phenomena have been extensively studied in the past for a vast category of beam configurations \cite{16–20} and interfaces \cite{21–24}. A comprehensive review on beam shift phenomena can be found in \cite{25}. Recently, the analogy between beam shifts and the quantum mechanical weak measurements has been also pointed out \cite{26–30}, which resulted in the possibility of observing amplified beam shifts \cite{31,32}.

In this paper we theoretically investigate the effect of the beam astigmatism on both Goos–Hänchen and Imbert–Fedorov shifts. Our calculations show that the astigmatism affects, at the leading order, both the spatial and angular shifts, with its main action being the introduction of an enhancement factor in the angular part of the shifts. The results presented here address the simple case of an astigmatic Gaussian beam, that we take as paradigmatic example for studying the effect of astigmatism on the Goos–Hänchen and Imbert–Fedorov shifts.

2. Model and methods

We start our analysis by considering a monochromatic fundamental Gaussian beam of frequency $\omega = ck$ (being $k = 2\pi/\lambda$ the wave number) characterized by a beam waist $w_0$, which impinges upon an astigmatic lens with two different focal lengths, namely $f_l$ in the longitudinal ($x$-) direction and $f_t$ in the transverse ($y$-) direction, as sketched in figure 1. After the lens, the beam will be focused at different lengths in
the x and y direction. To calculate the beam waist in these directions, we make use of the well-known formula for paraxial beams [36]:

$$\frac{1}{q_{\mu}(s)} = \frac{1}{q(s)} - \frac{1}{f_{\mu}},$$

(1)

where $\mu = \{1, 2\}$, $f_{\mu}$ is the focal length, $s$ is the distance between the Gaussian beam’s waist and the lens, and $q(s) = s - iz_{R}$ is the complex beam parameter, being $z_{R} = kw_{0}^{2}/2$ the Rayleigh range of the beam [36] and $q_{\mu}(s + z) = q_{\mu}(s) + z$. Obtaining $q_{\mu}(s)$ from the previous equation and setting its real part to zero gives the distance $s_{\mu}$ from the lens to the new waist as

$$\frac{1}{s_{\mu}} = \frac{1}{f_{\mu}} - \frac{s - f_{\mu}}{s(s - f_{\mu}) + z_{R}^{2}},$$

(2)

while the imaginary part of $q_{\mu}$ allows us to write the longitudinal and transverse beam waist as follows:

$$\left(\begin{array}{c} w_{0\mu} \\ w_{0} \end{array}\right)^{2} = \left(\frac{f_{\mu}}{s - f_{\mu}}\right)^{2} + z_{R}^{2}.$$

(3)

According to the convention used in [25, 33], we describe the reflection process by means of three different reference frames, namely one attached to the incident beam (\{x, y,, z\}), one attached to the reflected beam (\{x', y', z\}) and a third reference frame \{x, y, z\}, called the laboratory frame, attached to the reflecting interface. These reference frames are linked together by a rotation of an angle $\theta$ around the y axis [37]. The complex electric field impinging on the dielectric interface can be written in terms of its Fourier components, in the incident reference frame, as follows:

$$E^{i}(r) = \sum_{j=1}^{2} \int d^{2}K A_{j}(U, V, \theta) e^{i\alpha [U_{j} + V_{j} + W_{j} + \text{Di}]} I_{j},$$

(4)

where $d^{2}K = dU dV$, $\theta$ is the angle of incidence and $U$, $V$ and $W$ are the components of the wave vector in the incident reference frame, defined by the relation

$$k = k (U \hat{x} + V \hat{y} + W \hat{z}) = k_{x} \hat{x} + k_{y} \hat{y} + k_{z} \hat{z}.$$

(5)

In equation (4), $A_{j}(U, V, \theta) = \alpha_{j}(U, V, \theta) A(U, V)$ is the vector spectral amplitude of the field, with $A(U, V, \theta)$ being the local reference frame attached to the incident field [35], $\alpha_{j}(U, V, \theta) = \hat{f} \cdot \hat{e}_{j}(U, V, \theta)$ are the projections of the initial beam polarization $\hat{f} = f_{\mu} \hat{x} + f_{\mu} \hat{y}$ (with $|f_{\mu}|^{2} + |f_{\mu}|^{2} = 1$) onto the local basis $\hat{e}_{1}(k)$, $\hat{e}_{2}(k)$ and $\hat{k}$ attached to the wave vector $k$, and $A(U, V)$ is the scalar spectral amplitude associated with the beam, that in our case assumes the form of an astigmatic Gaussian beam, i.e.,

$$A(U, V) = e^{-k^{2}(w_{0\mu}^{2}U^{2} + w_{0\mu}^{2} V^{2})},$$

(6)

where $w_{0\mu}$ and $w_{0}$ are defined according to equation (3).

If we assume the beam to be well collimated, the paraxial approximation holds, and the expressions for the local reference frames and the polarization functions can be written in the following simple form [35]:

$$\hat{e}_{1}(U, V, \theta) = \hat{x} + V \hat{y} - U \hat{z},$$

(7a)

$$\hat{e}_{2}(U, V, \theta) = -V \hat{x} + U \hat{y} + V \hat{z},$$

(7b)

for the local basis, and

$$\alpha_{1}(U, V, \theta) = f_{\mu} + f_{\mu} \hat{V} \theta,$$

(8a)

$$\alpha_{2}(U, V, \theta) = f_{\mu} - f_{\mu} \hat{V} \theta,$$

(8b)

for the polarization functions. Notice, moreover, that the expression for $A(U, V)$ as given by equation (6) has been written in correspondence of the beam waist after the lens. The beam, therefore, has propagated a distance $D = d - s - s_{c}$ (see figure 1) before reaching the reflection surface. This propagation factor is correctly taken into account by the $z$-dependent part of the angular spectrum of the incident field.

Upon reflection, the single plane wave components of the field described by equation (4) experience geometrical reflection, according to the Snell’s law [1]. The reflected electric field can be then represented by equation (4) with the substitution

$$\hat{e}_{j}(k) e^{ikr} = r_{\mu}(k) \hat{e}_{j}(\hat{k}) e^{ikr},$$

(9)

where $r_{\mu}(k)$ are the usual Fresnel coefficients for $p$ ($\lambda = 1$) and $s$ ($\lambda = 2$) polarization [1], and $K = k - 2\hat{z} (\hat{z} \cdot k) = -U \hat{x} + V \hat{y} + W \hat{z}$. The electric field in the reflected frame can now be defined as follows:

$$\mathbf{E}^{R}(r) = \sum_{j=1}^{2} \int d^{2}K A_{j}(U, V) e^{i(UX + VY + WZ)},$$

(10)
where \( X = k x, \ Y = k y, \ Z = k (z R + D) \) and the vector angular spectrum in the reflected frame is given by \( \hat{A}_k(U, V) = A(U, V) r(U, V) \alpha_z (-U, V, \pi - \theta) \hat{e}_z \) \((-U, V, \pi - \theta)\).

The beam centroid can therefore be calculated as the weighted average of the position vector \( \mathbf{R} = X \hat{x} + Y \hat{y} \), with respect to the total beam intensity in the reflected reference frame \( \mathbf{E}^R(x, y, z) \). By employing the quantum notation for optical beams developed in [38], its explicit expression reads

\[
\langle \mathbf{R} \rangle = \frac{\langle \mathbf{E}^R | \mathbf{R} | \mathbf{E}^R \rangle}{\langle \mathbf{E}^R | \mathbf{E}^R \rangle} \equiv \langle X \rangle \hat{x} + \langle Y \rangle \hat{y},
\]

(11)

where \( \langle \xi \rangle \equiv \langle \xi \rangle(z) \), with \( \xi = X, Y \). The spatial (\( \Delta \)) and the angular (\( \Theta \)) Goos–Hänchen and Imbert–Fedorov shifts are then given by the following relations:

\[
\Delta_{GH} = \langle X \rangle \bigg|_{z=0}, \quad \Theta_{GH} = \frac{\partial \langle X \rangle}{\partial z}, \quad \Delta_{IF} = \langle Y \rangle \bigg|_{z=0}, \quad \Theta_{IF} = \frac{\partial \langle Y \rangle}{\partial z}.
\]

(12)

(13)

By substituting into equation (11) the expression of the electric field in the reflected frame given by equation (10) with a first order accuracy in \( (U, V) \) for the numerator and a second order accuracy in the denominator, we arrive, after a lengthy but simple calculation, to the following results:

\[
\Delta_{GH} = \frac{1}{k} \left[ \left( W_p \phi_p + W_i \phi_i \right) + \Gamma_4 \left( W_p \rho_p + W_i \rho_i \right) \right],
\]

(14a)

\[
\Theta_{GH} = -\frac{1}{k z R} \left( \frac{w_0}{w_{0,l}} \right)^2 \left( W_p \rho_p + W_i \rho_i \right),
\]

(14b)

for the Goos–Hänchen shifts, and

\[
\Delta_{IF} = \frac{\sqrt{W_p W_i}}{k z R} \cot \theta \left[ \sin \xi + \Gamma_4 \left( R_p^2 - R_i^2 \right) \right],
\]

(15a)

\[
\Theta_{IF} = -\frac{\sqrt{W_p W_i}}{k z R} \left( \frac{w_0^2}{w_{0,l}} \right) \left( R_p^2 - R_i^2 \right) \frac{\rho_p}{R_p \rho_i} \cot \theta,
\]

(15b)

for the Imbert–Fedorov shifts. In the previous equations, \( \rho_p = \text{Re} \{ \partial \ln r_p / \partial \theta \} \), \( \phi_p = \text{Im} \{ \partial \ln r_p / \partial \theta \} \), \( R_d = |r_d| \) (with \( \lambda \in \{ p, s \} \)) and

\[
\Gamma_4 \equiv \frac{-D}{k w_0^2} = \frac{2 \text{Re} \left\{ q_p(d) \right\}}{\text{Im} \left\{ q_p(d) \right\}},
\]

(16)

where \( q_p(d) \) can be calculated from equation (1). Moreover, \( W_j = f_j^2 R_j^2 / \left( U_j^2 R_p^2 + f_j^2 R_p^2 + \delta \right) \) is the fractional power contained in each polarization corrected by the beam’s astigmatism factor \( \delta \), whose explicit expression is given by

\[
\delta = -\frac{1}{4 k^2 w_0^2} \left\{ f_p^2 \left( \frac{\partial^2 R_p^2}{\partial \theta^2} - 2 R_p^2 \right) + \cot^2 \theta \left( R_p^2 - R_i^2 \right) \right\} + \cot \theta R_p \frac{\partial R_p}{\partial \theta} + f_i^2 \left( \frac{w_0^2}{2 w_{0,l}^2} \frac{\partial^2 R_i}{\partial \theta^2} + \cot^2 \theta \left( R_p^2 - R_i^2 \right) \right) + \cot \theta R_i \frac{\partial R_i}{\partial \theta} - R_i^2 \right\}.
\]

(17)

Equations (14) and (15) are the first main result of this work.

3. Results and discussion

A closer inspection on equations (14) and (15) reveals that the effect of the astigmatism on the Goos–Hänchen and Imbert–Fedorov shifts is twofold. Firstly, the fractional power normalization factor \( W_j \) is changed by the quantity \( \delta \). This means that, in contrast to the case of a fundamental Gaussian beam [33], the astigmatism introduces a correction on the energy stored in each polarization, being this correction dependent on both polarizations, as it appears clear from equation (17). However, since this correction is of the second order into \( (U, V) \), it is very small and can be therefore neglected. By doing so, therefore, the spatial shifts corresponds exactly to the ones of a fundamental Gaussian beam described in [33].

A second, and more interesting, effect of the astigmatism can be observed in the angular shifts, where an extra multiplicative factor appears. By neglecting \( \delta \) in the expression of \( \hat{W}_j \), the angular shifts can be therefore rewritten in the following inspiring form:

\[
\Theta_{GH} = \frac{w_0^2}{w_{0,l}^2} \Theta_{GH}^{(0)},
\]

(18a)

\[
\Theta_{IF} = \frac{w_0^2}{w_{0,l}^2} \Theta_{IF}^{(0)},
\]

(18b)

where \( \Theta_{GH}^{(0)}, \Theta_{IF}^{(0)} \) refer to the angular Goos–Hänchen and Imbert–Fedorov shifts for a fundamental Gaussian beam [33], respectively, and their explicit expression, in our case, is given by

\[
\Theta_{GH}^{(0)} = \frac{1}{k z R} \left( W_p \rho_p + W_i \rho_i \right),
\]

(19a)

\[
\Theta_{IF}^{(0)} = -\frac{\sqrt{W_p W_i}}{k z R} \left( R_p^2 - R_i^2 \right) \frac{\rho_p}{R_p \rho_i} \cot \theta.
\]

(19b)

This is our second main result. The enhancement factor

\[
\Omega_\mu = \frac{w_0^2}{w_{0,p}^2} \left( s - f_p \right)^2 + z_k^2 \frac{f_p}{f_p^2},
\]

(20)

in the angular shifts depends essentially on the fact that the astigmatic lens introduces a different focusing length (and, therefore, a different beam waist) in the longitudinal and

J. Opt. 17 (2015) 065608
M Ornigotti and A Aiello
transverse direction, respectively. It is worth noticing, moreover, that as the only effect of astigmatism is, in this case, to introduce different focal lengths along different directions, its main effect on GH and IF shifts consists in introducing a selective amplification of the angular shifts, as it appears clear from equation (18a). According to those equations, in fact, while the amplification factor in front of the angular GH shift only contains the longitudinal focal length through the quantity $w_0 l$, the transverse focal length only appears (through the term $w_0 (\mu f z)$) in the angular IF shift.

In order to quantify this enhancement, in figure 2 $\Omega_\ell$ is plotted against the normalized focal length $\zeta = f_\ell / z_R$. As can be seen, for values of the focal length (either in the longitudinal or in the transversal direction, depending on which shift is one interested in) approaches zero, the enhancement factor diverges, thus giving a significant enhancement of the angular shift. Although a rigorous vectorial analysis of the focusing process for $f \to 0$ would reveal that the considered divergence is only an artifact of the paraxial equation, the essential feature of it, namely the amplification of the angular shifts due to a shorter focusing length of the lens, is still very well captured by our paraxial model.

4. Conclusions

In conclusion, we have shown that the main effect introduced by an astigmatic lens put in the path of the beam before reflection, is the introduction of a selective, namely different in the $x$ and $y$ directions, enhancement factor on the angular shifts. This factor is essentially depending on the astigmatism of the lens. We have also shown that for small values of the focal length (compared with the beam’s Rayleigh range $z_R$) of the lens we used to model the beam’s astigmatism, a drastic amplification of the angular shifts occur. We believe that this result could be used experimentally to have another available channel of amplification, that could contribute to easily realize gigantic beam shifts that could lead to practical applications of this phenomenon.

References

[1] Born M and Wolf E 1999 Principles of Optics 7th edn (New York: Cambridge University Press)
[2] Goos F and Hänchen H 1947 Ann. Phys., NY 1 333
[3] Artmann K 1948 Ann. Phys., NY 2 87
[4] McGuirk M and Carignlia C K 1977 J. Opt. Soc. Am. 67 103
[5] Fedorov F I 1955 Dokl. Akad. Nauk SSSR 105 465
[6] Imbert C 1972 Phys. Rev. D 5 787
[7] Pillow F, Gilles H and Girard S 2004 Appl. Opt. 43 1863
[8] Schilling H 1965 Ann. Phys., NY 16 122
[9] Player M A 1987 J. Phys. A: Math. Gen. 20 3667
[10] Fedoseyev V G 1988 J. Phys. A: Math. Gen. 21 2045
[11] Liberman V S and Zel'dovich B S 1992 Phys. Rev. A 46 5199
[12] Bliokh K Y and Bliokh Y P 2006 Phys. Rev. Lett. 96 073903
[13] Bliokh K Y and Bliokh Y P 2007 Phys. Rev. E 75 066609
[14] Aiello A and Woerdman J P 2008 Opt. Lett. 33 1437
[15] Bliokh K Y, Shadrivov I V and Kivshar Y S 2009 Opt. Lett. 34 389
[16] Leung P T, Chen C W and Chiang H P 2007 Opt. Commun. 276 206
[17] Merano M, Aiello A, ‘t Hooft G W, van Exter M P, Elfer E R and Woerdman J P 2007 Opt. Express 15 15928
[18] Tamir T 1986 J. Opt. Soc. Am. A 3 558
[19] Landry G D and Maldonado T A 1996 Appl. Opt. 35 5870
[20] Ornigotti M and Aiello A 2013 J. Opt. 15 014004
[21] Kozaki S and Sakurai H 1978 J. Phys. Soc. Jpn. 46 5199
[22] Bliokh K Y, Shadrivov I V and Kivshar Y S 2009 Opt. Lett. 34 389
[23] Merano M, Hermosa N, Woerdman J P and Aiello A 2010 Phys. Rev. A 82 023817
[24] Aiello A and Woerdman J P 2010 Opt. Lett. 36 543
[25] Bliokh K Y and Aiello A 2013 J. Opt. 15 014001
[26] Dennis M R and Gütte J B 2012 New J. Phys. 14 073013
[27] Töppel F, Ornigotti M and Aiello A 2013 New J. Phys. 15 113059
[28] Hosten O and Kwiat P 2008 Science 319 787
[29] Qin Y, Li Y, Feng X, Xiao Y F, Yong H and Gang Q 2011 Opt. Express 19 9636
[30] Gorodetski Y, Bliokh K Y, Stein B, Genet C, Shchitri N, Kleiner V, Hasman E and Ebbesen T W 2012 Phys. Rev. Lett. 109 013901
[31] Jayawarl G, Mistura G and Merano M 2013 Opt. Lett. 38 1232
[32] Jayawarl G, Mistura G and Merano M 2014 Opt. Lett. 39 2266
[33] Aiello A 2012 New. J. Phys. 14 013058
[34] Mandel L and Wolf E 1995 Optical Coherence and Quantum Optics (New York: Cambridge University Press)
[35] Aiello A and Woerdman J P 2001 arXiv: 0903.3730v2 [physics.optics]
[36] Svelto O 2010 Principles of Lasers (Berlin: Springer)
[37] Aiello A, Merano M and Woerdman J P 2009 Phys. Rev. A 80 061801 R
[38] Bliokh K Y, Alonso M A, Ostrovskaya E A and Aiello A 2010 Phys. Rev. A 82 063825