Sensitivity Study of Extra Dimensions at TeV $e^+e^-$ colliders

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We study the sensitivity reach of 0.5 – 2 TeV $e^+e^-$ linear colliders in the context of the ADD model, in which gravity becomes strong at TeV scale. We include the real emission channel $e^+e^- \rightarrow \gamma G$, as well as the virtual-graviton-exchange channels $e^+e^- \rightarrow \gamma \gamma, q\bar{q}, \mu^+\mu^-, \tau^+\tau^-, e^+e^-$. Assuming no excess of events over the standard model predictions, we obtain the lower 95% C.L. limits on the effective Planck scale. These limits are better than those obtained in the Run II’s of the Tevatron and comparable to those of the LHC.

I. INTRODUCTION

Recent advances in string theory have stimulated many activities in particle phenomenology. The previously unreachable Planck, string, and grand unification scales ($M_{Pl}$, $M_{st}$, and $M_{GUT}$, respectively) can be brought down to a TeV range. In this case, one expects low energy phenomenology testable at current and future collider experiments.

An attractive realization of the above idea was proposed by Arkani-Hamed, Dimopoulos, and Dvali (ADD) [1]. In their model, the Standard Model (SM) particles live on a D3-brane, which is a topological object in higher dimensional space, where open strings end. On the other hand, gravity is allowed to propagate in the extra dimensions. In order to bring the Planck scale ($10^{19}$ GeV) to TeV range, the size of these compactified dimensions is made very large compared to $(M_{\text{weak}})^{-1}$. The relation among the Planck scale $M_{Pl}$, size $R$ of the extra dimensions, and the effective Planck scale $M_S$ is:

$$M_{Pl}^2 \sim M_S^{n+2} R^n,$$

where $n$ is the number of extra (compactified) dimensions. Assuming that the effective Planck scale $M_S$ is in the TeV range, it gives a very large $R$ of the size of a solar system for $n = 1$, which is obviously ruled out by experiments. However, for all $n \geq 2$ the expected $R$ is less than 1 mm, and therefore do not contradict existing gravitational experiments.

A graviton in the extra dimensions is equivalent, in the 4D-point of view, to a tower of infinite number of Kaluza-Klein (KK) states with masses $M_k = 2\pi k/R$ ($k = 0, 1, 2, ..., \infty$). The coupling of the SM particles residing on the brane to each of these KK states is of order $1/M_{Pl}$. The overall coupling is, however, obtained by summing over all the KK states, and so scales as $1/M_S$. Since the $M_S$ is in the TeV range, the gravitational interaction is as strong as electroweak interactions, and thus can give rise to many testable consequences in both accelerator and non-accelerator experiments. In general, present collider experiments can constrain the effective Planck scale at around 1 TeV. For a mini-review of the collider phenomenology please see Ref. [2].

In this study, we estimate the sensitivity reach of the $e^+e^-$ linear colliders (LC) of 0.5 – 2 TeV center-of-mass energies in the context of the ADD model. We include the following processes in our consideration:

- $e^+e^- \rightarrow \gamma G$
- $e^+e^- \rightarrow ZG$
- $e^+e^- \rightarrow \gamma\gamma$

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where $G$ denotes the graviton. The first two processes involve real emission of the graviton, which escapes into the extra dimensions and thus gives rise to missing-momentum events. The other processes involve virtual-graviton exchanges in addition to the usual electroweak exchanges. The interference between the SM amplitude and the graviton-exchange amplitude affects the total cross section and various distributions. If experimentally these distributions can be measured to a high precision, strong constraints can be placed on the new interactions of the graviton. In this study, we assume that the SM is correct and experiments give cross sections and distributions consistent with the SM, then we use the data to constrain the effective Planck scale.

II. PROCEDURES

A. Experimental Acceptance

We use typical kinematic and geometrical acceptance cuts for the detector in the LC:

$$|\cos \theta_i| < 0.9, \quad E_i > 20 \text{ GeV},$$

where $i = e, \mu, \tau, q, \text{or} \gamma$. The center-of-mass energies are 0.5, 1, 1.5, and 2 TeV and the integrated luminosities used in our study are 10 to 200 fb$^{-1}$. We employ an overall efficiency of 0.8 for all the events.

B. Monte Carlo data generation

In order to estimate the sensitivity of collider experiments to the ADD model, we need to generate some “realistic” data sets. We use the SM cross section to generate a smooth single or double differential distribution. We divide the distribution into bins: in each bin $i$ the expected number of events $S_{i}^{\text{SM}}$ is obtained by multiplying the cross section in this bin by the known integrated luminosity and efficiency. We further proceed with a Monte Carlo (MC) experiment. For each bin $i$ we generate a random number of events $n_{ij}$ using Poisson statistics with the mean $S_{i}^{\text{SM}}$. We use the MC data sets generated in this way to obtain the best fit and the limits on the gravity scale $M_{S}$ for the distributions.

As a result of the fit we obtain the 95% C.L. upper limit on $\eta$. We then repeat the above procedures many times. The limits obtained in these repeating experiments are histogrammed. The sensitivity to the scale $M_{S}$ is defined as the median of this histogram, i.e. the point on the sensitivity curve which 50% of the future experiments will exceed.

C. Fitting procedure

We extract the lower limit on the gravity scale $M_{S}$ by fitting each of the random MC experiments with a sum of the SM background and Kaluza-Klein graviton contribution. We employ the maximum likelihood method with the fitting parameter $\eta = \mathcal{F}/M_{S}^{4}$.

We, therefore, generate three templates that describe the cross section in the case of large extra dimensions. The first one describes the SM cross section. The other two describe terms proportional to $\eta$ (interference term) and to $\eta^{2}$ (Kaluza-Klein term), respectively. We then parameterize production cross section in each bin as

$$\sigma = \sigma_{\text{SM}} + \sigma_{4}\eta + \sigma_{8}\eta^{2},$$

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where $\sigma_{SM}$, $\sigma_4$, and $\sigma_8$ are the three templates described above. We combine channels by adding their likelihoods.

III. VARIOUS PROCESSES AND RESULTS

A. Real Emission: $e^+e^- \rightarrow \gamma G$

This is one of the earliest processes in the discussions of collider signatures for the ADD model [3,4]. The graviton escapes to the extra dimensions, which gives rise to a missing-energy signature. The differential cross section versus the cosine of the scattering angle is given by

$$
\frac{d\sigma}{d\cos\theta} = \frac{\pi G_N}{4 \left(1 - \frac{m^2}{s}\right)} \left[ (1 + \cos^2\theta) \left(1 + \left(\frac{m^2}{s}\right)^4\right) + \frac{3 - 3 \cos^2\theta + 4 \cos^4\theta}{1 - \cos^2\theta} \frac{m^2}{s} \left(1 + \left(\frac{m^2}{s}\right)^2\right) + 6 \cos^2\theta \left(\frac{m^2}{s}\right)^2 \right],
$$

where $s$ is the square of the center-of-mass energy and $m$ is the mass of the graviton. Since the spectrum of graviton states behaves like a continuous one at the energy scale that we are considering, the mass $m$ is to be integrated from 0 up to the center-of-mass energy of the $e^+e^-$ collider. The relation of the energy of the photon to the mass of the graviton is

$$
E_\gamma = \frac{s - m^2}{2\sqrt{s}},
$$

by which we can obtain the double differential cross section $d^2\sigma/dE_\gamma d\cos\theta_\gamma$.

The irreducible background comes from $e^+e^- \rightarrow \gamma \nu\bar{\nu}$. The main difference between the signal and background is the $Z$ peak of the background in the recoil-mass distribution. We used the following cuts to reduce the background [4]:

$$
M_{\text{recoil}} > 200 \text{ GeV}, \quad E_\gamma > 20 \text{ GeV}, \quad |\cos\theta_\gamma| < 0.9.
$$

The process $e^+e^- \rightarrow ZG$ is similar to the previous one with the $\gamma$ replaced by $Z$. The $Z$ boson decays into a lepton pair or a quark pair. The irreducible background comes from $e^+e^- \rightarrow Z\nu\bar{\nu}$. The cuts to reduce the background are similar to those in Eq. (4). The formula for this process can be found in Ref. [4]. Since the limits that can be obtained from this channel is inferior to the $e^+e^- \rightarrow \gamma G$ channel, we shall ignore this channel in obtaining the combined limits.

In the estimation of the sensitivity reach, we used the two-dimensional distribution of $d^2\sigma/dE_\gamma d\cos\theta_\gamma$. The reach by this channel is shown in Table I.

| $\sqrt{s}$ (TeV) | $\mathcal{L} = 10 \text{ fb}^{-1}$ | $\mathcal{L} = 50 \text{ fb}^{-1}$ | $\mathcal{L} = 10 \text{ fb}^{-1}$ | $\mathcal{L} = 50 \text{ fb}^{-1}$ |
|------------------|-----------------|-----------------|-----------------|-----------------|
| 2                | 4.7             | 5.7             | 7.2             | 8.8             |
| 3                | 2.8             | 3.4             | 4.6             | 5.4             |
| 4                | 2.1             | 2.4             | 3.5             | 4.0             |
| 5                | 1.7             | 1.9             | 2.9             | 3.3             |
| 6                | 1.4             | 1.6             | 2.5             | 2.8             |

TABLE I. 95% C.L. limit on $M_S$ (TeV) using the real graviton-emission channel $e^+e^- \rightarrow \gamma G$
B. Virtual Graviton Exchange

1. $e^+e^- \rightarrow \gamma\gamma$

The differential cross section versus $z = |\cos \theta|$ ($\theta$ is the polar angle of the outgoing photon) is given by [4]

$$\frac{d\sigma(e^+e^- \rightarrow \gamma\gamma)}{dz} = \frac{2\pi}{s} \left( \frac{1}{1-z^2} + \frac{\alpha}{4} s^2 (1+z^2) \eta + \frac{1}{64} s^4 (1-z^4) \eta^2 \right)$$  \hspace{1cm} (5)

where $\eta$ ranges from 0 to 1, and $\eta = F/M_S^4$. The factor $F$ is

$$F = \begin{cases} \ln \left( \frac{M_Z^2}{s} \right) & \text{for } n = 2, \\ \frac{1}{\eta^2} & \text{for } n > 2. \end{cases}$$  \hspace{1cm} (6)

where $n$ is the number of extra dimensions.

The four LEP collaborations have measured the diphoton production and using the data to constrain the deviation from QED by parameterizing the possible deviation from QED with a cutoff parameter $\Lambda_\perp$ in the angular distribution:

$$\frac{d\sigma}{dz} = \frac{2\pi\alpha^2}{s} \frac{1+z^2}{1-z^2} \left( 1 \pm \frac{s^2}{2\Lambda_\perp^2} (1-z^2) \right).$$  \hspace{1cm} (7)

The QED cutoff parameter $\Lambda_\perp$ can be related to $M_S$ by

$$\frac{M_S^4}{F} = \frac{\Lambda_\perp^4}{2\alpha}. \hspace{1cm} (8)$$

We convert the limit on $\Lambda_\perp$ to $M_S$ and $M_S$ is at most about 1.4 TeV for $n = 2$ and about 1 TeV for $n = 4$ [4].

The behavior of the new gravity interactions at higher $\sqrt{s}$ can be easily deduced from Eq. (7). The new interaction gives rise to terms proportional to $s^2/M_S^4$ and $s^4/M_S^8$, which get substantial enhancement at large $\sqrt{s}$ [4]. The angular distribution also becomes flatter because in the SM the distribution scales as $(1+z^2)/(1-z^2)$ whereas the terms arising from the new gravity interactions scale as $(1+z^2)$ and $(1-z^4)$.

2. $e^+e^- \rightarrow f\bar{f}$

The differential cross section is given by [4]

$$\frac{d\sigma(e^+e^- \rightarrow f\bar{f})}{dz} = \frac{N_f s}{128\pi} \left\{ (1+z)^2 (|M_{LL}(s)|^2 + |M_{RR}(s)|^2) + (1-z)^2 (|M_{RL}(s)|^2 + |M_{LR}(s)|^2) \right\}$$

$$+ \pi^2 s^2 (1-3z^2 + 4z^4) \eta \, s + \frac{8\pi e^2 Q_e Q_f z^3 \eta + \frac{8\pi e^2}{\sqrt{s} c_\theta} \left( \frac{1}{2} - g_a g_a' - g_c g_c' z^3 \right)}{s - M_Z^2} \, \eta$$

$$+ \frac{\delta_{f,s}}{128\pi} \left\{ (1+z)^2 (|M_{LL}(t)|^2 + |M_{RR}(t)|^2) + 2M_{LL}(s) M_{LL}(t) + 2M_{RR}(s) M_{RR}(t) \right\}$$

$$+4(|M_{LR}(t)|^2 + |M_{RL}(t)|^2) + \frac{\pi^2 s^2}{8} (121 + 228z + 198z^2 + 84z^3 + 9z^4) \eta^2$$

$$- \frac{\pi s}{2} \eta (M_{LL}(t) + M_{RR}(t) + M_{LL}(s) + M_{RR}(s))(1+z)^2(7+z)$$

$$+\eta (M_{LL}(t) + M_{RR}(t))(1+z)^2(1-2z) - 2\pi s \eta (M_{LR}(t) + M_{RL}(t))(5+3z) \right\}$$  \hspace{1cm} (9)

where
\[ M_{\alpha\beta}^f(s) = \frac{e^2 Q_e Q_f}{s} + \frac{e^2}{s^2 c_\theta^2} \frac{g_5^f g_5^f}{s - M_Z^2}, \tag{10} \]

\( N_f = 3(1) \) for quark (lepton) and \( z = \cos \theta \), where \( \theta \) is the scattering angle of the outgoing fermion. The latter part of the above equation is for the Bhabha scattering. The other channels have been studied in Ref. [6].

The results on the sensitivity reach are obtained by combining the likelihood functions of all these virtual-graviton exchange channels. The results are shown in Table II. The sensitivity is, in fact, dominated by the Bhabha scattering.

It is clear that the limits obtained by the virtual-graviton exchange channels are better than those by the real emission channel. This is because of the suppression factor \( 1/M_S^{n+2} \) in the real emission, especially severe for large \( n \), while no such a suppression factor appears in the virtual exchange processes.

Next, we compare the sensitivity reach of LC to that at the Tevatron and the LHC. The diphoton production at hadronic colliders has been shown very sensitive to the scale \( M_S \). The sensitivity reach at hadronic machines was estimated using the diphoton, dilepton channels and their combination. At the detector level, a photon behaves like an electron. The limits are summarized in the Table III.

The sensitivity reach of a mere 0.5 TeV LC with an integrated luminosity of 10–50 fb\(^{-1}\) is far better than the Run II’s of the Tevatron. An 1 TeV LC with an integrated luminosity of 200 fb\(^{-1}\) reaches a sensitivity as good as the LHC. Other studies of large extra dimensions at \( e^+e^- \) colliders can be found in Refs. [9].

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**TABLE II.** The 95% C.L. lower limits on the effective Planck scale \( M_S \) by combining all the graviton-exchange processes.

| \( n = 3 \) | \( n = 4 \) | \( n = 5 \) | \( n = 6 \) |
|---|---|---|---|
| \( \sqrt{s} = 0.5 \text{ TeV} \) | \( \sqrt{s} = 1 \text{ TeV} \) | \( \sqrt{s} = 1 \text{ TeV} \) | \( \sqrt{s} = 2 \text{ TeV} \) |
| \( \mathcal{L} = 10 \text{ fb}^{-1} \) | 4.6 | 3.9 | 3.5 | 3.3 |
| 50 | 5.7 | 4.8 | 4.3 | 4.0 |
| 10 | 7.8 | 6.5 | 5.9 | 5.5 |
| 50 | 9.4 | 7.9 | 7.2 | 6.7 |
| 100 | 10.5 | 8.9 | 8.0 | 7.5 |
| 200 | 11.2 | 9.4 | 8.5 | 7.9 |

| \( \sqrt{s} = 1.5 \text{ TeV} \) | \( \mathcal{L} = 100 \text{ fb}^{-1} \) | 200 | 100 | 200 |
|---|---|---|---|---|
| \( n = 3 \) | 14.3 | 15.4 | 17.8 | 19.3 |
| \( n = 4 \) | 12.0 | 13.0 | 15.0 | 16.2 |
| \( n = 5 \) | 10.8 | 11.7 | 13.6 | 14.7 |
| \( n = 6 \) | 10.1 | 10.9 | 12.6 | 13.6 |
TABLE III. Sensitivity to the ADD model parameter $\eta = \mathcal{F}/M_5^2$ in Run I, Run II of the Tevatron and at the LHC, using the dilepton, diphoton production, and their combination.

|               | $\eta_{95}$ (TeV$^{-4}$) | $n = 2$  | $n = 3$  | $n = 4$  | $n = 5$  | $n = 6$  | $n = 7$  |
|---------------|--------------------------|----------|----------|----------|----------|----------|----------|
| Run I (130 pb$^{-1}$) |                          |          |          |          |          |          |          |
| Dilepton      | 0.66                     | 1.21     | 1.32     | 1.11     | 1.00     | 0.93     | 0.88     |
| Diphoton      | 0.44                     | 1.39     | 1.46     | 1.23     | 1.11     | 1.03     | 0.98     |
| Combined      | 0.37                     | 1.48     | 1.53     | 1.29     | 1.16     | 1.08     | 1.02     |
| Run IIa (2 fb$^{-1}$) |                          |          |          |          |          |          |          |
| Dilepton      | 0.163                    | 1.92     | 1.87     | 1.57     | 1.42     | 1.32     | 1.25     |
| Diphoton      | 0.077                    | 2.40     | 2.26     | 1.90     | 1.71     | 1.60     | 1.51     |
| Combined      | 0.072                    | 2.46     | 2.30     | 1.93     | 1.74     | 1.62     | 1.54     |
| Run IIb (20 fb$^{-1}$) |                          |          |          |          |          |          |          |
| Dilepton      | 0.054                    | 2.70     | 2.47     | 2.08     | 1.88     | 1.75     | 1.65     |
| Diphoton      | 0.025                    | 3.40     | 3.00     | 2.53     | 2.28     | 2.12     | 2.01     |
| Combined      | 0.021                    | 3.54     | 3.11     | 2.61     | 2.36     | 2.20     | 2.08     |
| LHC (14 TeV, 100 fb$^{-1}$) |                          |          |          |          |          |          |          |
| Dilepton      | $2.20 \times 10^{-4}$    | 10.2     | 9.76     | 8.21     | 7.42     | 6.90     | 6.53     |
| Diphoton      | $1.24 \times 10^{-4}$    | 12.1     | 11.3     | 9.47     | 8.56     | 7.97     | 7.53     |
| Combined      | $1.05 \times 10^{-4}$    | 12.8     | 11.7     | 9.87     | 8.92     | 8.30     | 7.85     |

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