A Chiral D=4,N=1 String Vacuum
with a Finite Low Energy Effective Field Theory

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Abstract
Supersymmetric $N = 1$, $D = 4$ string vacua are known to be finite in perturbation theory. However, the effective low energy $D = 4$, $N = 1$ field theory lagrangian does not yield in general finite theories. In this note we present the first (to our knowledge) such an example. It may be constructed in three dual ways: i) as a $Z_3$, $SO(32)$ heterotic orbifold; ii) as a Type-IIB, $Z_3$ orientifold with only ninebranes and a Wilson line or iii) as a Type-IIB, $Z_6$ orientifold with only fivebranes. The gauge group is $SU(4)^3$ with three chiral generations. Although chiral, a subsector of the model is continuously connected to a model with global $N = 4$ supersymmetry. From the $Z_6$, Type IIB orientifold point of view the above connection may be understood as a transition of four dynamical fivebranes from a fixed point to the bulk. The $N = 1$ model is thus also expected to be S-dual. We also remark that, using the untwisted dilaton and moduli fields of these constructions as spurion fields, yields soft SUSY-breaking terms which preserve finiteness even for $N = 0$. 
Supersymmetric four dimensional string vacua are known to be finite theories of gauge and gravitational interactions. However a different issue is the finiteness properties of the effective low-energy field theory Lagrangian. Toroidal compactifications of e.g. heterotic strings on $T^6$ yield an $N = 4$ effective gauge theory Lagrangian which is finite. Other compactifications with fewer supersymmetries yield in general low energy field theories which are not finite.

If we compactify the heterotic string on $K^3 \times T^2$ we obtain an $N = 2$ theory which is in general not finite. However, it is very easy to obtain compactifications with $N = 2$ and a finite effective theory since for a given simple gauge group $G_\alpha$ finiteness is just controlled by the number of $N = 2$ hypermultiplets in the spectrum and the latter may be easily varied by appropriate changes in the compactification. In fact, it was shown in ref. [1] that in this class of theories one has $\beta_\alpha = 12(1 + \frac{\bar{V}_\alpha V_\alpha}{V_\alpha})$ where $\bar{V}_\alpha, V_\alpha$ are coefficients of gauge kinetic terms in the associated $D = 6$ theory obtained upon decompactification of the $T^2$ (see e.g. ref. [2] for a review and references). For the case of $E_8 \times E_8$, $K^3 \times T^2$ compactifications with instanton numbers in both $E_8$'s given by $(k_1, k_2)$, an $E_8$ with $k_1 = 10$ precisely has $\frac{\bar{V}_\alpha}{V_\alpha} = -1$ yielding $\beta = 0$ and a finite effective field theory.

It is natural to ask whether one can find $N = 1, D = 4$ string vacua with a finite effective field theory. In fact we are not aware of the existence in the literature of a single non-trivial example of this type. In order to have a one-loop finite $N = 1$ effective field theory the conditions

$$\beta_\alpha = 3C_\alpha(G) - \sum_i T(R_i)n_i = 0 \; ; \; \gamma_{il} = Y^{ijk}Y^{*}_{ljk} - 4g^2C(R)\delta^i_l = 0 \quad (1)$$

must be met. Here $i$ runs over the chiral multiplets of the model and $Y_{ijk}$ is a renormalizable Yukawa coupling. The first equation is the vanishing of the one-loop $\beta$-function for the gauge group and is controlled by the precise massless spectrum of each given compactification. The second is the vanishing of the wave function renormalization of chiral multiplets in the model. The first condition is not terribly difficult to meet. For example, it would be enough to construct a CY compactification of $E_8 \times E_8$ with 12 $E_6$ generations (or antigenerations). The second class of conditions are the hard ones since they depend on the Yukawa couplings of the model which are very model dependent and in general also depend on the moduli of the compactification. The constraint (1) also impose specific relationships between the Yukawa couplings and the gauge couplings which should also be of the same order of magnitude. It is this second issue which must first be addressed in looking for a finite effective theory.

In fact there is at least a class of $N = 1, D = 4$ heterotic vacua in which certain subsector of the theory has Yukawa couplings which are proportional to the gauge coupling constants. This is the untwisted sector of ordinary $Z_3, D = 4$ orbifold compactifications of heterotic string. Any such a compactification contains in that sector three replicas $\Psi^i, i = 1, 2, 3$ of the same set of chiral multiplets, one for each of the three complex compact dimensions. The Yukawa coupling constants $Y_{ijk}, i = 1, 2, 3$ among those fields verify at the tree level:

$$Y_{ijk} = a \, g \, \epsilon_{ijk} \quad (2)$$

where $g$ is the gauge coupling constant (inverse dilaton) and $a$ is some (model dependent) group theoretical factor. Notice that in this case indeed $\gamma_i = 0, i = 1, 2, 3$ for some specific $a$ factors.
Unfortunately this class of $Z_3$ orbifolds (see e.g. ref.[3] for a general discussion) come along also with extra chiral multiplets coming from twisted sectors which spoil this nice property. In order to find a finite theory we would like to find a $Z_3$ orbifold without the annoying properties of the twisted sector. Type I-Heterotic duality [4] gives us a hint in this search. Consider the related class of orientifold models \([5-15]\) obtained by starting with Type-IIB string and compactifying in a space-time toroidal orbifold. One accompanies the $Z_3$ twist with a certain gauge embedding on the Chan-Paton factors and a world-sheet parity reversal twist $\Omega$ which halves the number of supersymmetries and yields open strings as twisted sectors. This class of $Z_3$ Type-I ”orientifolds” in $D = 4$ where first constructed in ref.[11]. $Z_3$ orientifolds do not have five-branes and the charged chiral multiplets come all from open string states stretching between nine-branes (i.e., the usual Chan-Paton factors). These (9-9) chiral multiplets in $Z_3$, $D = 4$ orientifolds, like their $Z_3$ heterotic untwisted counterparts, come in three copies $\Psi^i$, $i = 1, 2, 3$ and have Yukawa couplings like the ones in eq.(2). This is nice because this is more or less what we were looking for. However, tadpole cancellation is a quite strong constraint in Type-IIB orientifolds and it turns out that there is a unique embedding on the Chan-Paton factors which leads to a consisten theory. The unique form for Chan-Paton matrices $\lambda_{ab}$, $a, b = 1, ..., 32$ in this $Z_3$, $D = 4$ case is [11, 13]:

$$\lambda = diag (\alpha I_{12 \times 12}, \alpha^2 I_{12 \times 12}, I_{8 \times 8})$$

(3)

where $\alpha = exp(i2\pi/3)$. The twisted tadpole cancellation conditions yield in the present case $Tr \lambda = -4$, which is indeed the case for the choice (3). This gives rise to a $D = 4$, $N = 1$ theory with gauge group $U(12) \times SO(8)$ with chiral multiplets coming in three copies of $(12, 8) + (66, 1)$. Unfortunately the $\beta$-functions of these groups are not vanishing and this is not what we were looking for.

We still have the option of adding discrete Wilson lines to this model. But again, tadpole cancellation is a very powerful constraint. Suppose we realize the discrete Wilson line as a matrix $W_{ab}$, with $W^3 = 1$, acting on the Chan-Paton factors. The 27 fixed points of the orbifold split now into three sets of nine fixed points feeling the gauge connections $\lambda$, $W\lambda$ and $W^2\lambda$ respectively. So tadpole cancelation conditions require simultaneously:

$$Tr(\lambda) = Tr(W\lambda) = Tr(W^2\lambda) = -4$$

(4)

These constraints are again very restrictive and force $W$ to be such that $(W\lambda)$ and $(W^2\lambda)$ have a similar form to that of $\lambda$ above, although with a different distribution of eigenvalues. In particular the following choice meets the above constraints:

$$W = diag (I_{4 \times 4}, \alpha I_{4 \times 4}, \alpha^2 I_{4 \times 4}, I_{4 \times 4}, \alpha I_{4 \times 4}, \alpha^2 I_{4 \times 4}, \alpha I_{8 \times 8})$$

(5)

This is the model we will concentrate on in the rest of the paper. It turns out that this model has a heterotic dual. In fact, the authors in ref.[11, 13] presented a heterotic candidate dual for the model without the Wilson line. It is easy to find out what would be the heterotic dual in our case. It is the $Spin(32)$ heterotic $Z_3$ orbifold obtained by embedding the twist through a shift $V$ and Wilson line $a$ acting on the $Spin(32)$ lattice as:

$$V = \frac{1}{3} (1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0)$$

$$a = \frac{1}{3} (0, 0, 1, 1, 2, 2, 0, 0, 1, 1, 2, 2, 1, 1, 1, 1)$$

(6)
Table 1: Charged Chiral multiplets in the Heterotic $\mathbb{Z}_3$ model.

It is easy to check that this embedding verifies the usual modular invariance constraints. The gauge group obtained is $U(4)^4$ and the charged particle spectrum is displayed in the table. It is equally easy to find out the charged particle spectrum of the Type-IIB orientifold dual defined by the Chan-Paton matrices $\lambda$ and $W$ (see table 2). It precisely yields charged fields identical to the untwisted sector of the heterotic orbifold but no trace of the other charged fields in the table. We will discuss this discrepancy below but notice first that, if we restrict to the orientifold spectrum, the field content with respect to $SU(4)_1 \times SU(4)_2 \times SU(4)_3$ is

$$3(1,4,4,1) + 3(4,1,4,1) + 3(4,4,1,1) + 3(1,1,1,6)$$

Thus the one-loop $\beta$-functions of $SU(4)^3$ indeed vanish. The fourth $SU(4)$ has a non-vanishing $\beta$-function but it does not affect the first $SU(4)^3$ since it only couples gravitationally to the other $SU(4)$s. The $U(1)$s do not spoil finiteness either since, as we are going to see momentarily, they are necessarily spontaneously broken.

Thus this $SU(4)^3$, $\mathbb{Z}_3$ orientifold model gives rise in its effective low-energy Lagrangian to a one-loop finite chiral $D = 4, N = 1$ field theory. This is the first string vacua of these characteristics that I am aware off. It is amusing that it has three quark-lepton generations under a Pati-Salam type of group $SU(4)_c \times SU(4)_L \times SU(4)_R$ since in fact we were not looking for three generations but only for one-loop finiteness.

This model can be constructed in yet another way which corresponds to performing a T-duality transformation to the above Type IIB orientifold along the first two compact complex dimensions. Doing this one obtains a new orientifold in which the IIB string compactified on $T^6$ is moded by the orientifold group $Z_6 = \mathbb{Z}_3 \times \mathbb{Z}_2$. Here $\mathbb{Z}_3$ is the standard $\mathbb{Z}_3$ action in $D = 4$ which involves $2\pi/3$ rotations on the three compact complex planes. The $\mathbb{Z}_2$ is generated by $\Omega R$, where $R$ is a reflection of the
first two complex coordinates. Since $\Omega$ is not a generator of the orientifold group, there are no ninebranes. The presence of the element $\Omega R$ and tadpole cancellation requires the presence of 32 Dirichlet fivebranes whose worldvolume lives in the four non-compact dimension plus the 3-d compact plane. Now we will chose a particular configuration of the fivebranes on the fixed points which obeys tadpole cancellation conditions. Eight fivebranes will be sitting at the origin. The corresponding CP matrix for these fivebranes have to verify $Tr\gamma_{5,\theta} = -4$ in order to cancell tadpoles. The open strings stretching among these fivebranes give rise to a $U(4)$ group with three 6-plets. The remaining 24 fivebranes will be sitting at some other fixed point away from the origin. Tadpole constraints imply in this case $Tr\gamma_{5,\theta} = 0$. The corresponding open strings give rise to a gauge group $U(4)^3$ and three copies of $(4,4,1) + (\bar{4},1,4) + (1,4,4)$. This is the finite model. In this T-dual orientifold version arises completely from 24 fivebranes sitting on a $Z_3$ fixed point. The chiral fields come in three copies because there are three different type of NS oscilators acting on the open string vacuum corresponding to the three compact complex coordinates. A similar finite (field theory) model may be constructed \[18\] as a theory in the worldvolume of certain configurations of branes \[19\] using techniques developed for the construction of chiral gauge theories in \[20, 21\].

Let us discuss now a number of points raised up by this string vacuum. The first point is the discrepancy between the massless spectrum of the orientifold and the heterotic duals. In particular, why some of the twisted particles do not appear in the orientifold spectrum. In fact 27 of the twisted singlets (those with $U(1)$ charges $\pm 4/3$) do have an orientifold counterpart: they correspond to the 27 singlets associated to the fixed points and coming from the Type-I unoriented twisted closed strings. The other singlets and the six-plets do not have a Type-I counterpart. In fact, you do not expect them to have perturbative Type-I counterparts since one can easily check that all those states corresponds to spinorial representations of $Spin(32)/Z_2$. Indeed, e.g. the sextets in the $V$ sector in the table correspond to heterotic states with shifted gauge momenta $(P + V) = \frac{1}{3}(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{3}{2}, \pm \frac{3}{2}, \pm \frac{3}{2}, \pm \frac{3}{2}, \pm \frac{3}{2})$, i.e., involve spinorial weights. Such spinorial representations are not expected to appear at the perturbative level in Type-I whose local gauge group is only $SO(32)$. Thus, if these two models are indeed dual to each other, they give us the interesting information that, at some point in the orientifold moduli space, non-perturbative spinorial states should become massless to precisely match the heterotic spectrum.

In fact, as noted in \[13\] for the case of the model without Wilson line, this is

| Sector             | SU(4) | $Q_x$ | $Q_1$ | $Q_2$ | $Q_3$ |
|--------------------|-------|-------|-------|-------|-------|
| Open Strings       | 3(1, 4, 4, 1) | 0     | 0     | 1     | 1     |
|                    | 3(4, 1, 4, 1) | 0     | 1     | 0     | -1    |
|                    | 3(4, 1, 4, 1) | -1    | -1    | 0     | 0     |
|                    | 3(1, 1, 1, 6) | -2    | 0     | 0     | 0     |
| Closed Twisted Strings | 27(1, 1, 1, 1) | 0     | 0     | 0     | 0     |
| Closed Untwisted Strings | 9(1, 1, 1, 1) | 0     | 0     | 0     | 0     |

Table 2: Chiral multiplets in the Type-I orientifold.
also related to the presence of an anomalous $U(1)$ in this type of vacua. Indeed, in the heterotic model the $U(1)_X$ with charges given in the table is anomalous. One can find that the mixed $U(1)_X$ anomaly $A_i$ with each of the $SU(4)_i$, $i = 1, 2, 3, 4$ is $A_i = -6$ and the mixed gravitational anomaly is $A_{grav} = -6 \times 24$, as it should in order for the anomaly to be cancelled by the usual $D = 4$ Green-Schwarz \cite{16} mechanism. This cancelation comes about from a shift in the imaginary partner $ImS$ of the heterotic dilaton. Now, in the presence of an anomalous $U(1)$ a dilaton-dependent Fayet-Iliopoulos term \cite{17} proportional to $TrQ_X$ is generated so that the gauge piece of the scalar potential corresponding to $U(1)_X$ becomes

$$V_g = \frac{g^2}{2} \left( \sum_i q_i |\phi_i|^2 + TrQ_x \frac{g}{192\pi^2} \right)^2. \quad (8)$$

In the case of the heterotic model one finds $TrQ_X = -144$ and that means that, as usual, some field with positive $q_X$ will be forced to acquire a vev. Looking at the table we see that the choice is essentially unique: the 27 twisted moduli are the unique fields with positive $q_X$ and those will be forced to get vevs. Notice that this will break spontaneously all of the $U(1)$s, not only $U(1)_X$. At the same time, since those have renormalizable Yukawa couplings with all the (spinorial) fields (the fields in the table with $q_X = -2/3$), all these fields disappear from the massless spectrum. Thus in the actual supersymmetric vacuum of the heterotic vacuum the spinorial fields are in fact not present, very much like in the dual orientifold model.

As we said above, the orientifold dual has only charged fields with quantum numbers like the ones of the untwisted heterotic dual. Then one observes that the $U(1)_X$ is again anomalous but now $TrQ_X = -36$. Since there are apparently no fields with the required positive $q_X$ in the massless spectrum one would be tempted to say that there is no supersymmetric (interacting) vacuum. However the heterotic discussion above tells us that this is not what is going on. The dual model suggests that what happens is that the 27 singlets from the Type-I twisted closed string spectrum will be participating in the cancellation of the D-term by getting non-linear transformations under the $U(1)$s. That this is so is also supported by the fact that, in order to cancel the $U(1)_X$ anomaly in the orientifold model, a generalized (non-universal) Green-Schwarz mechanism must be at work. Indeed, the mixed $U(1)_X$ anomaly with the first three $SU(4)$s is zero but the mixed anomaly with the last $SU(4)$ is $A = -6$. Thus not only the Type-I dilaton but twisted closed string singlets should be involved in this generalized GS mechanism.

This model has a couple of other interesting properties which deserve some discussion:

1) Although this model is chiral, one can see it is continuously connected to a model with $N = 4$ global supersymmetry. Indeed there is a flat direction (or, equivalently, a continuous Wilson line) by giving a vev to all the fields corresponding to the same untwisted complex plane. Then $SU(4)^3$ is broken to a diagonal $SU(4)_{diag}$ whose Kac-Moody algebra is realized at level 3. The spectrum includes 3 adjoints under this $SU(4)$ with the adequate Yukawa couplings (equal to the gauge coupling) so that the massless (charged) spectrum has global $N = 4$ supersymmetry. This $N = 4$ supersymmetry is only global since there is still only one gravitino. Furthermore there are a number of singlet chiral $N = 1$ multiplets which do not fall into $N = 4$ multiplets. So, in some sense, one can say that the $SU(4)^3$ charged sector of this model generically has $N = 4$ global supersymmetry and it is not chiral
but becomes chiral and only $N = 1$ supersymmetric in some points of the moduli space. For momenta smaller than the size of the vevs one has a subsector of the theory with global $N = 4$ supersymmetry and for momenta higher than those vevs the theory loses this extended symmetry and has only $N = 1$. Since $N = 4$ globally supersymmetric models are believed to be S-dual, the chiral $N = 1$ model continuously connected to it would also be expected to be S-dual.

It is interesting to notice that, looked from the point of view of the $Z_6$ orientifolds with fivebranes, the above transition to an $N = 4$, $SU(4)$ theory is nothing but the emission of the 24 Dirichlet fivebranes (which constitute 24/6 = 4 real dynamical fivebranes) from the fixed point to the bulk. Thus four dynamical fivebranes in the above geometry have $N = 4$ supersymmetry and an $SU(4)$ gauge group.

2) It is an interesting question whether the finiteness of the above $N = 1$ model survives even in the presence of some particular choice of SUSY-breaking soft terms. In fact that turns out to be the case under certain conditions. Assume, as in the approach of ref.[22] that the seed of supersymmetry breaking resides on the dilaton/untwisted moduli sector. This amounts to considering the dilaton and nine untwisted moduli chiral fields of the above model as spurion fields whose auxiliary components are generically non-vanishing. It was found in ref.[23] that for charged particles residing on the untwisted sector of a $Z_3$ orbifold, the soft SUSY-breaking masses $m_i$, $i = 1,2,3$ of the untwisted charged chiral fields associated to the $i$-th compact complex plane verify:

$$m_1^2 + m_2^2 + m_3^2 = |M|^2 \quad (9)$$

and besides

$$A_{ijk} = -\epsilon_{ijk} M \quad (10)$$

where $M$ is the gaugino mass and $A_{ijk}$ are the trilinear soft terms. This applies independently of the goldstino direction (see refs.[23] for details) but assumes that the cosmological constant is zero. The reason for raising this issue here is the following. It has been found in ref.[24] that a finite $N = 1$ field theory which has soft terms verifying boundary conditions as above, remain one-loop finite, even though supersymmetry is broken. Thus dilaton/moduli-induced SUSY-breaking will not spoil the finiteness properties of the model discussed in this note.

It would be interesting to understand the properties of this particular choice of soft terms from the construction of finite theories via branes as in ref.[19, 20, 21]. The further breaking from $N = 1$ to $N = 0$ could perhaps be accomplished [18] by a twisting of the brane configuration as in the elliptic models in [23]. The finiteness properties are however preserved in the process.

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