Checkerboard-like Bound States around a Vortex in High-Tc Cuprate Superconductors

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Abstract. We have studied the bound states around a single vortex in high-Tc cuprate superconductors, using the Bogoliubov-de Gennes equation. Experimentally, the bound states around the vortex in high-Tc cuprate superconductors are very different from those in the conventional superconductors. There are no peaks in the local density of states (LDOS) around the Fermi energy and the bound states shows checkerboard-like oscillations. These feature can not be explained in the frameworks of the theory of d-wave superconductivity (dSC). Therefore, we study the effect of pseudogap states, which coexist with d-wave superconductivity. We have adopted the d-wave spin-density-wave (dSDW) theory for the pseudogap state. After deriving the Bogoliubov-de Gennes equation for dSDW+dSC, we have solved it numerically, and obtained the LDOS, which shows bound states with checkerboard-like oscillations. This result shows that the dSDW states is a good candidate for the pseudogap state.

1. Introduction

The quasi-particle bound state structures around a single vortex in the high-Tc cuprate superconductors are much different from those for conventional s-wave superconductors [1-6]. By the scanning tunneling spectroscopy (STS), Pan et al. showed small quasi-particle bound states around a single vortex in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ [3]. They locate at $\pm 7$ mV when superconducting order parameter is $\Delta = 32$ mV. This structure is different from that of conventional s-wave superconductor [6]. But also it is different from that for pure d-wave superconductors, which is predicted theoretically [7,8]. In pure d-wave superconductors, the bound states peak appears around zero energy as in the s-wave superconductors [9,10]. Furthermore, the spatial distribution of quasi-particle bound states in the high-Tc superconductor shows a checkerboard pattern [11].

Recently, Matsuba et al. showed modulation periods of the checkerboard pattern are along one Cu-O bond and $(4/3)\alpha_0$ along another bond. Also anti-phase spatial variation between positive and negative energy states appears, by the high-spatial resolution STS experiment [12].

In the high-Tc superconductor, the d-wave superconductivity coexists with the pseudogap state, of which nature is still unknown. Therefore, in order to analyze these vortex states, we must take into
account the effect of the pseudo gap state to the vortex core states. Won, Morita and Maki showed that theory of the d-wave spin density wave (dSDW) states can describe the pseudo-gap states well and they call the coexistent states with dSDW and d-wave superconductivity (dSC) a gossamer superconductivity [13]. And we adopt this gossamer superconductivity as a model of high-Tc superconductivity.

In this study, we first derive the Bogoliubov-de Gennes equation for the dSDW+dSC state and using this equation, we analyze the vortex core states in high-Tc cuprates.

2. Numerical Methods

The Bogoliubov-de Gennes equation for the coexistence states with d-wave superconductivity and d-wave spin density wave states, or gossamer superconductivity is derived as,

\[
E_n u_n(r) = \left[ \frac{1}{2m} \left( p - e \frac{A}{c} \right)^2 - \mu \right] u_n(r) + \left( \partial_x \Delta_x \partial_x - \partial_y \Delta_y \partial_y \right) u_n(r)e^{iQr},
\]

\[
E_n v_n(r) = \left[ \frac{1}{2m} \left( p + e \frac{A}{c} \right)^2 - \mu \right] v_n(r) + \left( \partial_x \Delta_x \partial_x - \partial_y \Delta_y \partial_y \right) v_n(r)e^{iQr},
\]

\[
E_n u_n(r) = \left[ \frac{1}{2m} \left( p - e \frac{A}{c} \right)^2 - \mu \right] u_n(r) - \left( \partial_x \Delta_x \partial_x - \partial_y \Delta_y \partial_y \right) u_n(r) e^{iQr},
\]

\[
E_n v_n(r) = \left[ \frac{1}{2m} \left( p + e \frac{A}{c} \right)^2 - \mu \right] v_n(r) - \left( \partial_x \Delta_x \partial_x - \partial_y \Delta_y \partial_y \right) v_n(r) e^{iQr},
\]

Here \( u_{n\sigma}(r) \) and \( u_{n\sigma}(r) \) are the electron and hole parts of the quasi-particle wave function with spin \( \sigma \). \( \Delta_1 \) is the dSDW order parameter, \( \Delta_2 \) is the d-SC order parameter, and \( Q \) is the nesting vector for dSDW. Because of the nature of the SDW, the wavefunctions for up spin and down spin quasi-particle obey different equations.

Using the above equations, in order to obtain the quasi-particle bound state spectrum around a single vortex, we consider a disk and put the vortex at the center of the disk. Then we solve above equations for this geometry, using the Fourier-Bessel equation, where the basis is given as,

\[
\phi_{n\sigma}(r) = \frac{\sqrt{2}}{R j_{n+1}(\alpha_{n\sigma})} J_n\left( \frac{\alpha_{n\sigma}}{R} r \right)
\]

Then we obtain the local density of states for the quasi-particle bound states as follows,
\[ N(r,E) = \sum_{\alpha, \sigma} \left[ u_{\alpha \sigma}(r)^2 f(E - E_{\alpha \sigma}) + v_{\alpha \sigma}(r)^2 f(E + E_{\alpha \sigma}) \right]. \]  

3. Results

Numerical calculation shows that the sharp peak around zero energy becomes weak and shoulder peaks appear. Here we show spatial distribution of the local density of states of these bound states for \( \Delta_1 / \Delta_2 = 2.0 \) and \( k_F \xi = 3 \) in Fig. 1 (\( E/\Delta_2 = 0.2 \)) and Fig. 2 (\( E/\Delta_2 = -0.2 \)).

Figure 1. Spatial distribution of the local density of states at \( E/\Delta_2 = 0.2 \).

Figure 2. Spatial distribution of the local density of states at \( E/\Delta_2 = -0.2 \).
In this case we take $Q = (\pi, \pi)$. Both of LDOS show the checkerboard like oscillation. And such modulation is pronounced for positive energy case (Fig. 1) and there is anti-phase relation between positive and negative energy distribution. These features agree with the recent experiment [12]. Such modulation may come from the interference of the pseudo-gap state.

4. Summary
Using the Bogoliubov-de Gennes equation for the dSC+dSDW (Gossamer) model, we have analysed the bound states around a single vortex in the high-Tc cuprate superconductors. We show in our model there appear checkerboard like bound states inside vortex core as was observed using the STS. This is because dSDW order causes the modulation of the electron system inside of the vortex core.

Further studies, such as self-consistent calculation and analysis of the incommensurate structure of the checkerboard pattern, are future problems.

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