A hybrid ECT image reconstruction based on Tikhonov regularization theory and SIRT algorithm

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Abstract. Electrical Capacitance Tomography (ECT) image reconstruction is a key problem that is not well solved due to the influence of soft-field in the ECT system. In this paper, a new hybrid ECT image reconstruction algorithm is proposed by combining Tikhonov regularization theory and Simultaneous Reconstruction Technique (SIRT) algorithm. Tikhonov regularization theory is used to solve ill-posed problem of image reconstruction and to obtain an estimated solution in the region of the optimal solution aggregate. Then, SIRT algorithm is used to improve the quality of the final reconstructed image. In order to satisfy the industrial requirement of real-time computation, the proposed algorithm is further been modified to improve the calculation speed. Test results show that the quality of reconstructed image is better than that of the well-known Filter Linear Back Projection (FLBP) algorithm.

1. Introduction

Electrical capacitance tomography (ECT) is a process tomography technique that has been widely studied. ECT is fast, non-invasive, low cost, safe and applicable to wide areas. It has become one of important techniques for two-phase flow parameter measurement. However, the ECT image reconstruction is a key problem that has not been well solved due to the influence of soft-field in the ECT system. [1]-[3]

Currently, most of the image reconstruction algorithms are based on the classic back-projection. Simple models which are based on some unrealistic assumptions are used for image reconstruction, the accuracy and the reconstructed image quality of ECT measurement systems are often unsatisfactory. [4-5]

In order to improve the quality of ECT reconstructed image, a new hybrid ECT image reconstruction algorithm is proposed by combining Tikhonov regularization theory and Simultaneous Reconstruction Technique (SIRT) algorithm. Tikhonov regularization theory is used to solve ill-posed problem of image reconstruction and to obtain an estimated solution in the region of the optimal solution aggregate. SIRT algorithm is then used to improve the quality of the final reconstructed image. Test results show that the quality of reconstructed image is better than that of the well-known Filter Linear Back Projection (FLBP) algorithm.
2. Image reconstruction model

A 12 electrodes ECT system consists of capacitance sensor, data acquisition system and image reconstruction computer, shown in figure.1

There are 66 independent capacitance measurements between any two of the 12 electrodes in all possible combinations. The value of the capacitance can be expressed by

\[ C_i = \int\int_D \varepsilon(x,y)S_i(x,y,\varepsilon(x,y))dx\,dy \quad i = 1,2,\ldots,66 \]

where \( D \) is the cross-section of pipe (image area), \( \varepsilon(x,y) \) is the dielectric distribution function which corresponds to the phase component distribution of two-phase flow, because a different phase component distribution of two-phase flow results in a different dielectric distribution. \( S_i \) is the sensitivity distribution function of \( C_i \).\[6]-[8]

The aims of image reconstruction are to solve the inverse problem of (1) and determine \( \varepsilon(x,y) \) from a limited number of measurements. It should be noted that it is very difficult to solve the inverse problem with only 66 measurements, because the relationship between the capacitance and the dielectric distribution is complex and nonlinear. If the difference between the dielectric constants of the constituent materials of two-phase flow (such as gas/oil two-phase flow or most gas-solid two-phase flow) is relatively small, the effect of the dielectric distribution on the sensitivity distribution is insignificant.\[5-10\] Assuming the dielectric distribution has no effect on the sensitivity function, (1) than can be simplified to

\[ C_i = \int\int_D \varepsilon(x,y)S_i(x,y)dx\,dy \quad i = 1,2,\ldots,66 \]

Using the normalization technology in the image reconstruction, the measurement capacitance \( C_i \) is normalized as \( C_{ri} \):

\[ C_{ri} = \frac{\int\int_D \varepsilon(x,y)S_i(x,y)dx\,dy - \int\int_D \varepsilon_o S_i(x,y)dx\,dy}{\int\int_D \varepsilon_m S_i(x,y)dx\,dy - \int\int_D \varepsilon_o S_i(x,y)dx\,dy} \]

\[ = \frac{\int\int_D (\varepsilon(x,y) - \varepsilon_o)S_i(x,y)dx\,dy}{\int\int_D S_i(x,y)dx\,dy} \]

where \( \varepsilon_m \) is the dielectric constant (permittivity) of oil (or solid particles), \( \varepsilon_o \) is the dielectric constant (permittivity) of gas. Then,

\[ \left( \int\int_D S_i(x,y)dx\,dy \right) C_{ri} = \int\int_D f(x,y)S_i(x,y)dx\,dy \quad i = 1,2,\ldots,66 \]

where
\[ f(x, y) = \frac{\varepsilon(x, y) - \varepsilon_o}{\varepsilon_m - \varepsilon_o} \]  

Finally, the image reconstruction model is obtained by the linear approximation, denoted in equation (6)

\[ P = SF \]  

where \( P=[p_1,p_2,...,p_M]^T \) is the vector of measurement projection data, \( S=[s_{ij}] \) is a weight matrix whose entries are obtained by the finite-element method (FEM), and \( F=[f_1,f_2,...,f_M]^T \) is the vector of grey level.

3. **New hybrid ECT image reconstruction algorithm**

The hybrid algorithm consists of two steps. The first step uses Tikhonov regularization principle to obtain the original reconstructed image, and the second step uses SIRT algorithm to improve the quality of the final image.

Firstly, the image area is divided into 54 pixels to make sure that there are enough projection data to determine the grey levels of the original reconstructed image, as shown in figure. 2. In this stage, the main issue of image reconstruction is the stability of the solution. In order to obtain a stable solution for \( (6) \), the Tikhonov regularization principle is employed.[9-11]

Suppose the regularization parameter \( \alpha > 0 \), then, solving \( (6) \) could be attributed to the following optimization problem:

\[ J(F) = \|SF - P\|^2 + \alpha Q(F) = \|SF - P\|^2 + \alpha \|F\|^2 \rightarrow \min \]  

Consequently, the regularized solution of \( (7) \) (the estimated grey level) can be formulated as

\[ F^* = (S^T S + \alpha I)^{-1} S^T P = BP \]  

where \( I \) is the identity matrix and \( B=(S^T S+\alpha I)^{-1}S^T \) is the regularized generalized inverse matrix. The regularized solution is the pixel vector, which is not accurate for the parameter \( \alpha \).

Secondly, in order to improve the image reconstruction accuracy, Simultaneous Reconstruction Technique (SIRT) is used. This algorithm has the advantages of high applicability and high accuracy. The main drawback of SIRT algorithm is that it is very computational intensive.[13] However, by utilizing the results of Tikhonov regularization, the computational burden can be greatly reduced since the results of Tikhonov regularization are in the region of the optimal solution aggregate. SIRT algorithm is described as follow:

(1) \( F^0=[f_{i0}^0,f_{20}^0,...,f_M^0] \) is given by \( F^0=[f_1,f_2,...,f_M] \).

(2) The calculated projection value of the \( i \)-th capacitance is obtained from the \( (i-1) \)-th iteration results of the pixel grey level \( q_i \),

\[ q_i = \sum_{j=1}^{M} s_{ij} f_j^i \quad i = 1,2,\cdots,N, j = 1,2,\cdots,M; \]  

where \( f_j^i \) is the \( k \)-th iteration result of the \( f_j \) pixel grey level,

(3) The revised grey level error \( \Delta f_{ij}^k \) is obtained as

\[ \Delta f_{ij}^k = \frac{p_i - q_i}{\sum_{j=1}^{M} s_{ij}} s_{ij} \quad i = 1,2,\cdots,N, j = 1,2,\cdots,M \]  

(4) The average revised grey level error \( \Delta f_j^k \) is

\[ \Delta f_j^k = \frac{1}{N_j} \sum_{i=1}^{N_j} \Delta f_{ij}^k \quad j = 1,2,\cdots,M \]  

where \( N_j \) is the number related with \( j \)-th pixel,

(5) The revised grey level \( f_j^{k+1} \) is calculated by the following equation

\[ f_j^{k+1} = f_j^k + \Delta f_j^k \quad j = 1,2,\cdots,M \]  

(6) Calculate and judge the aggregate error,
where $\epsilon$ is the threshold. Iterative calculation is stopped if the (13) is satisfied, otherwise jump to equation (9). Note that the pixel grey level will be limited between 0 and 1.0.

4. Test results
The test is conducted with the Electrical Capacitance Tomography developed by Zhejiang University.[12] Polyethylene granule is used to simulate the classic flow regimes. The proposed algorithm is compared with the linear back projection (FLBP) algorithm. FLBP algorithm is a well known algorithm used to reconstruct an image from capacitance data, shown as equation (14)

$$G(k) = \sum_{i=1}^{11} \sum_{j=1}^{12} \left[ \frac{C_{ij}^{m} - C_{ij}^{empty}}{C_{ij}^{full} - C_{ij}^{empty}} \times \frac{s_{ij}(k)}{\sum_{m=1}^{11} \sum_{n=1}^{12} s_{mn}(k)} \right]$$

where $G(k)$ is the grey degree rector of sensor section, $C_{ij}^{m}$, $C_{ij}^{empty}$, $C_{ij}^{full}$ are the capacitance values of electrode pair $(i,j)$ when pipe is filled with part medium, full air and full polyethylene, respectively.[5] And the final grey level is

$$F_j = \begin{cases} 1 & f_j \geq 1 \\ f_j & 0 < f_j < 1 \\ 0 & f_j \leq \eta \end{cases}$$

The test results of proposed algorithm and FLBP algorithm with the same flow regimes are shown in table 1. The experimental results show that qualities of the proposed algorithm are better than that of the FLBP algorithm. The two-phase interface of the proposed method is more legible than that of FLBP algorithm.

Since the industry application requires real-time calculation, the proposed algorithm has made two modifications in this test in order to improve the calculation speed,

1) In the course of image reconstruction, many reconstructed image results show that the pixel grey levels calculated by Tikhonov regularization method can not be improved anymore by the SIRT algorithm. The pixels near the interface can be improved while in the other regions can not. A threshold after Tikhonov regularization method is given as:

$$F_j \in \{ f_j \geq 0.8; f_j \leq 0.2 \}$$

The pixel grey level is judged with condition (16) before it is improved by SIRT algorithm. When (16) is not hold, start the SIRT algorithm. Otherwise, stop the algorithm at this pixel. Simulation test results show that image reconstruction SIRT calculations are focus on the pixels by the two phase interface and the speed is faster than ever.

2) Since the iteration convergences of different flow regimes differ in speeds, the convergence speeds of stratified flow and annular flow are faster than that of core flow, we can select different thresholds according to different flow regimes in order to further reduce calculation burden. Through mass test data investigation, the threshold is selected as follows:

$$|f_j^k - f_j^{k-1}| \leq \Delta \epsilon$$

When iteration error is smaller than $\Delta \epsilon$, the calculation of the flow regimes image reconstruction can be stopped. The advantage of this threshold is that the iteration number of every flow regime is minimized while the image quality is satisfied. If we select $\Delta \epsilon = 1 \times 10^{-4}$, the time cost of image reconstruction is less than 0.1 second.

The optimum iteration number and the max iterative error of different flow regimes is shown as in figure. 3.
Table 1 Test results of image reconstruction

| Set flow regimes | Results of FLBP algorithm | Results of proposed algorithm |
|------------------|---------------------------|------------------------------|
| stratified flow   | ![Image 1]                | ![Image 2]                  |
| annular flow      | ![Image 3]                | ![Image 4]                  |
| core flow         | ![Image 5]                | ![Image 6]                  |

Figure 3: SIRT convergence speed of different flow regimes
5. Conclusions

In this paper, a new hybrid ECT image reconstruction algorithm was proposed based on the combination of Tikhonov regularization theory and Simultaneous Reconstruction Technique (SIRT) algorithm.

Tikhonov regularization theory was used to solve ill-posed image reconstruction problem and to obtain a stable solution in the region of the optimized solution aggregate, while SIRT algorithm was employed to improve the quality of the final reconstructed image.

To satisfy the industrial requirement of real-time computation, the proposed algorithm was further been modified in two aspects: 1) the image reconstruction SIRT calculation was made to focus on the pixels around the two-phase interface to reduce the computing burden; and 2) an optimum threshold selection principle was proposed to improve the image reconstruction speed.

Test results show that the reconstructed image qualities of the proposed algorithm are better than those of FLBP’s, and the time consumed by the hybrid algorithm is less than that of only SIRT algorithm. If we select $\Delta x=1\times10^{-4}$, the time required for the image reconstruction is less than 0.1 seconds that satisfy the industrial requirements.

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