Electron Decay

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Abstract

The electron would decay into a photon and neutrino if the law of electric charge conservation is not respected. Such a decay would cause vacancy in closed shells of atoms giving rise to emission of x-rays and Auger electrons. Experimental searches for such very rare decay have given an estimate for the lifetime to be greater than $2.7 \times 10^{23}$ years. The simplest theoretical model which would give rise to such a decay is one where the electron is regarded as the first excited state and neutrino as the ground state of a fundamental spin $1/2$ particle bound to a scalar particle by a super strong force and the photon is considered as a bound state of a fundamental spin $1/2$ fermion-antifermion pair. The fine structure constant of the super strong coupling is found to be unity from the masslessness of the neutrino and the lower bound of the mass of the fundamental particles is estimated by using quantum mechanical formula for photon emission by atoms and found to be $10^{22}$ GeV from the bound for electron decay time indicating thereby that the composite nature of electron, neutrino and the photon would be revealed in the Planckian energy regime. A model based on extension of $SU(2) \otimes SU(2)$ symmetry of Dirac equation to $SU(3) \otimes SU(3)$ gives a lower bound for the mass of the gauge boson mediating the decay to be $10^9$ GeV which is the geometric mean of the masses of the electron and the fundamental particles.

I. ELECTRON DECAY

The stability of the proton constituting the nucleus of the lightest and most abundant element, the hydrogen atom, has been the topic of extensive theoretical and experimental investigations in the last three decades [1]. However, not much attention has been paid to the stability of its partner, the electron. Experiments give a lower bound of $2.7 \times 10^{23}$ years for its life-time [2] which is less than that of the proton by several orders of magnitude.

Just as proton would decay, among other particles, to a positron and a neutral pion if the constraint imposed by baryon number conservation is removed, so would the electron decay into a photon and a neutrino (also to a neutrino and neutrino-antineutrino pair) if electric charge conservation is not respected. Such decay of the electron in closed shells of atoms would cause vacancy giving rise to emission of x-rays and Auger electrons. der Mateosian
and Goldhaber looked for such emission from an Iodine atom located in a NaI(T1) crystal and deduced a lower limit of $10^{18}$ years on the electron life time. This limit has been revised to $10^{22}$ years for $e \rightarrow \gamma \nu$ by Moe and Reines [3] by reducing the detector background. The most recent limit given by the Particle Data Group [2] is $2.7 \times 10^{23}$ yrs.

The simplest model which would give rise to $e \rightarrow \gamma \nu$ decay is one where the electron is regarded as the first excited state and the neutrino as the ground state of a fundamental spin 1/2 particle bound to a scalar particle by a super strong force. Since electric charge is not to be conserved, and therefore not a good quantum number, the fundamental particles of the model should not possess electric charge. Consequently the photon can not couple to them directly. It can do so through its constituents if it too is regarded as a composite particle. It may be recalled in this connection that long ago Fermi and Yang [4] constructed a model for the composite pion and eta particles according to which

\[ \phi_i = N^+ \tau_i N \]
\[ \eta = N^+ N \]  

where

\[ N = \left( \begin{array}{c} p \\ n \end{array} \right) \]

and $\tau_i$ are isotopic spin matrices. The pion field $\phi_i$ is a vector in isospin or SU(2) space; the eta ($\eta$) the nucleon (N) are scalar and spinors respectively in this space. In terms isospin up (p), isospin down (n), the pion field as defined, in (1) can be written as

\[ \phi^{(+)} = n^+ p \]
\[ \phi^{(-)} = p^+ n \]
\[ \phi^{(0)} = \frac{1}{\sqrt{2}} (p^+ p - n^+ n) \]
\[ \eta = \frac{1}{\sqrt{2}} (p^+ p + n^+ n) \]  

In the case of photon we have to consider four dimensional space-time instead of two-dimensional isospin space and construct composite photon states from spin up and down states of fundamental spin 1/2 particles, which span a $SU(2) \otimes SU(2)$ space. The Dirac equation for these particles can be written in a symmetrical from using spinor representation of Dirac matrices : 

\[ (p_0 - \vec{\sigma} \cdot \vec{p}) \xi_f = m \eta_f \]
\[ (p_0 + \vec{\sigma} \cdot \vec{p}) \eta_f = m \xi_f \]  

Here $\sigma_i$ are Pauli spin matrices and $\xi_f$ and $\eta_f$ are two-component spinors in terms of which the four component Dirac field $\psi_f$ can be written as

\[ \psi_f = \begin{pmatrix} \xi_f \\ \eta_f \end{pmatrix} \]  

The spinors $\xi_f$ and $\eta_f$ span the $SU_2(\xi) \otimes SU_2(\eta)$ space. Analogous to the Fermi-Yang model for pion in isospin spin the photon can be represented in $SU_2(\xi) \otimes SU_2(\eta)$ space as [5]

\[ A_R = \psi_f^{(d)+} \psi_f^{(u)} \]
\[ A_L = \psi_f^{(u)+} \psi_f^{(d)} \]
\[ A_3 = \frac{1}{\sqrt{2}} (\psi_f^{(u)+} \psi_f^{(d)} - \psi_f^{(d)+} \psi_f^{(u)}) \]
\[ A_0 = \frac{1}{\sqrt{2}} (\psi_f^{(u)+} \psi_f^{(u)} + \psi_f^{(d)+} \psi_f^{(d)}) \]
where \( u \) and \( d \) stand for spin s up and down, \( A_R \) and \( A_L \) represent right and left circularly polarized photon, and \( A_3 \) and \( A_0 \) represent longitudinal and time-like photon fields. It will be noticed that the \( A_R \) and \( A_L \) are composed of fundamental fermions with parallel spins, \( A_3 \) and \( A_0 \) are composed of those with antiparallel spins. Just as parallel electric currents attract and antiparallel ones repel, so would parallel spins attract and antiparallel spins repel [6] since the force is of gauge origin as in quantum electrodynamics. For this reason \( A_R \) and \( A_L \) will be bound states while \( A_3 \) and \( A_0 \) remain unbound. This explains why there only two physical photon states.

Since the electron and neutrino have spin 1/2 and our fundamental fermions have also have spin 1/2, they former should be of composites of such a fermion and a fundamental spinless particle which is the counter part of the strange quark in \( SU(3) \) space, electron and neutrino being counterparts of K-mesons. In order to accommodate the spinless field we have to extend the \( SU_2 \otimes SU_2 \) model to \( SU_3 \otimes SU_3 \) and use extended Dirac equation:

\[
(p_0 - \lambda_a p_a) \xi = m\eta \\
(p_0 + \lambda_a p_a) \eta = m\xi
\]

where

\[
\xi_1 \xi_2 \xi_s = \eta_1 \eta_2 \eta_s
\]

and \( \lambda_a (a=1,2,...,8) \) are \( (3 \times 3) \) \( SU_3 \) matrices. These two equations can be combined into a single one:

\[
(\Gamma_0 p_0 - \Gamma_a p_a) \psi = m\psi
\]

where

\[
\psi = \begin{pmatrix} \xi \\ \eta \end{pmatrix} \Gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Gamma_a = \begin{pmatrix} 0 & -\lambda_a \\ \lambda_a & 0 \end{pmatrix}
\]

which can be derived from the Lagrangian density

\[
L = \bar{\psi} (\Gamma_0 p_0 - \Gamma_a p_a) \psi - m\bar{\psi}\psi
\]

The gauge interaction incorporated by replacing \( p_0 \) by \( (p_0 - g G_0) \) and \( (p_a - g G_a) \) in (11) gives

\[
L_{int} = -g \bar{\psi} \Gamma_0 G_0 \psi - g \bar{\psi} \Gamma_a p_a \psi
\]

The interaction between the fundamental spin 1/2 and spin 0 fields relevant for the formation and decay of the electron and neutrino contained in this comes out to be

\[
L_{fs} = -g \bar{\psi}_f \gamma_5 \psi_s G_k
\]

where \( \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \) and \( G_k = \begin{pmatrix} G_4 - i G_5 \\ G_6 + i G_7 \end{pmatrix} \)

\[
\psi_f = \begin{pmatrix} \xi_f \\ \eta_f \end{pmatrix} \psi_s = \begin{pmatrix} \xi_s \\ \eta_s \end{pmatrix}
\]

The electron and neutrino are first excited and ground states of the composite

\[
e^{(u)} = \psi_f^+ \psi_f^{(u)} \\
e^{(d)} = \psi_s^+ \psi_f^{(d)} \\
\sigma^{(u)} = \psi_f^{(u)} \psi_s^+ \\
\sigma^{(d)} = \psi_f^{(d)} \psi_s^+
\]

The interaction (13) leading to electron decay can be represented by the Feynman diagram
from which the life-time of the decay (as in proton decay) comes out to be

\[ \frac{1}{\tau} = \alpha g^2 m^5 \]  

Taking \( \alpha g = 1 \) and using the experimental lower bound for \( \tau \) as

\[ \tau > 2.7 \times 10^{21} \text{ years} \]  

This gives the bound for \( m_k \) as

\[ m_k > 10^9 \text{ Gev} \]  

A lower bound for the mass of the fundamental particles can be obtained by making use of the formula

\[ \frac{1}{\tau} = \frac{4\omega^3}{3} |<\nu\gamma | M | e>|^2 \]  

used for atomic transitions since the electron and neutrino have been taken to be excited and ground state of spin 1/2 composites of fundamental spin 1/2 and spin zero particles. In this formula, in that case, \( \omega \) would equal the electron mass. For obtaining an estimate, we take

\[ <\nu\gamma | M | e> = \alpha g/m \]  

Taking \( \alpha g = 1 \) we get from (19) and (20)

\[ \tau m^2/m_e^3 \]  

This gives the bound on \( m \) as

\[ m > 10^{22} \text{ Gev} \]
Combining (21) and (16) gives

\[ m_e m = m_k^2 \]  

(23)

Having considered electron decay we take up the problem of formation of electron through exchange of massive gauge bosons \( G_k \) between \( \psi_s \& \psi_f \). The second order Dirac equation for this can be written

\[
\left[ \frac{d^2}{dr^2} + \frac{2}{r} - \frac{\Lambda^2}{r^2} + 2\alpha g \frac{e^{im_k r}}{r} - \left( \frac{m^2}{4} - E^2 \right) \right] R(r) = 0
\]

(24)

where

\[
\Lambda^2 = \lambda(\lambda + 1), \quad |\lambda| = \sqrt{(j - \frac{1}{2})^2 - \alpha^4}
\]

In terms of scaled variables

\[
\rho = r/\beta \quad \epsilon = E\beta \quad \kappa = \frac{\beta m}{4}, \quad (\beta << 1)
\]

(25)

this equation reads

\[
\left[ \frac{\partial^2}{\partial \rho^2} + \frac{2}{\rho} \frac{d}{d\rho} + \frac{\Lambda^2}{\rho^2} - \frac{2\alpha g \epsilon e^{-m_k \beta \rho}}{\rho} + (4K^2 - \epsilon^2) \right] R = 0
\]

(26)

Since \( \beta << 1 \) we can put \( e^{-m_k \beta \rho} = 1 \) in which case equation (24) becomes a second order Dirac equation with Coulomb potential whose solution for energy \( \epsilon \) is

\[
\epsilon = \frac{2m_e}{\sqrt{1 + \frac{\alpha^2}{(\sqrt{\kappa^2 - \alpha^2} + n_r)^2}}}
\]

(27)

where

\[
\kappa = \begin{cases} 
1 \text{ for } j = l + \frac{1}{2} \\
-(l + 1) \text{ for } j = l - \frac{1}{2}
\end{cases} \quad \text{and} \quad n_r = n - |\kappa|.
\]

For the ground state which we have taken as the neutrino, we get from this

\[
m_\nu = 2m_2 \sqrt{1 - \alpha^2_g}
\]

(28)
i.e.,

\[
\alpha_g = \sqrt{1 - \left( \frac{m_\nu}{2m_2} \right)^2}
\]

Similarly for the electron, which we have taken to be the first excited state, we get

\[
m_e = \kappa + \frac{m_\nu}{2}
\]

(29)

If we take \( m_\nu = 0, \alpha_g = 1, \kappa = m_e \) in which case \( \beta \sim 10^{-25} \) as assumed. It will be noted that while the mass and energy in eqn(23) are in the Planck regime, those in eqn(25) are in the atomic regime.

We thus see that a model where the electron and neutrino are taken to be excited and ground states of a composite of a fundamental spin 1/2 fermion and a spin zero particle and the photon as a composite of the fundamental fermion antifermion pair, leads to relation between electron decay time, mass of the fundamental particles and also the mass of the gauge boson which provides the binding. The experimental bound for electron life time gives bounds for these masses. While the mass of the fundamental particles lies in the Planckian regime, the mass of the gauge boson is the geometric mean of this mass and that of the electron.
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