Remarkable long-range-systematic in the binding energies of α-nuclei II

D.H.E. Gross
Hahn-Meitner Institute and Freie Universität Berlin,
Fachbereich Physik.
Glienickerstr. 100
14109 Berlin, Germany

March 30, 2022

Abstract
In this letter I present further data that show the remarkable evidence for the existence of an α-cluster structure in the ground states of even-even $N = Z$ nuclei. Such a remarkable systematic was observed 20 years ago in [1] for these nuclei at $A \leq 72$ and is extended here up to $A = 100$.

1 Introduction
Twenty years ago we published a remarkable systematic of nuclear binding energies [1]. The two-nucleon separation energies of α-nuclei where found to be approximately constant and equal to the 2-proton, 2-neutron and PN-separation energies of the $^4\text{He}$-nucleus. At these days this systematics could only be followed up to $^{72}\text{Kr}$. In the past 20 years the table of nuclear binding energies was considerably extended[1]: It is now interesting how far this systematic can be followed also in these new data. This is indeed well possible as will be shown in this publication.

The nuclear separation energies are defined by the difference of the binding energies of initial nucleus minus the sum of that of the final nuclei. I correct it in the most simple way for the different Coulomb energies according to the trivial formula[1]:

\begin{align}
S_{2N}^{\text{nuclear}} &= (B_i + U_i^C) - (B_f + U_f^C) - U_{2N}^C, \quad (1) \\
U^C &= \frac{3}{5} (e^2/r_0) Z(Z-1)/A^{1/3}, \quad r_0 = 1.25 \text{ fm}, \quad (2)
\end{align}

where $B_i$ and $B_f$ are the experimental binding energies of the initial and final nucleus respectively, $2N$ corresponds to any pair of nucleons [proton-proton

[1] Nuclear Structure and Decay Data, National Nuclear Data Center, Brookhaven National Laboratory, Upton, NY 11973-5000; [http://www.nndc.bnl.gov/nndc/nndcnsdd.html](http://www.nndc.bnl.gov/nndc/nndcnsdd.html)
Figure 1: Nuclear two-nucleon separation energies $S_{\text{nuclear}}^{2N}$, corrected for Coulomb energies, in keV, following eq.[2] with $2N = \pi, \nu$ (neutron-proton pair), $2N = \nu, \nu$ (neutron-neutron pair), and $2N = \pi, \pi$ (proton-proton pair) as function of the original number $n_\alpha$ of $\alpha$’s. The upper curve is for an $n_\alpha$-mother-nucleus, the lower one is for a mother-nucleus with $n_\alpha$ $\alpha$-particles plus one additional nucleon pair which is then removed.

\[(\pi, \pi), \text{neutron-proton } (\nu, \pi), \text{and neutron-neutron } (\nu, \nu) \text{ pair}\]. As can be clearly seen in figure 1, the removal of a 2N-pair costs about the same energy as for a bare $\alpha$-particle $\approx 30$MeV. In contrast, the removal of an additional $\pi\pi$, $\nu\nu$ or $\pi\nu$ pair costs much less energy. Moreover, this strongly depends on the size the mother nucleus, resp. the number of $\alpha$-particles surrounding this pair.

Figure 2 show the separation energies of an $\alpha$-particle out of a $n_\alpha$-nucleus. The peaks at $n_\alpha = 4, 7, 14$, and the disappearance of peaks above $n_\alpha = 14$, remind of the special stable 3-dim clusters of closed packed spheres at $n = 4, 7, 13, 55$. The latter ones with icosaeder structure are observed also for atomic clusters of the heavier noble gases like $\text{Xe}^n$. For the lighter ones like $\text{Ar}$ clusters the shift from 13 to 14 is well known.

2 Acknowledgement

I am grateful to W.v.Oertzen for pointing at the new and far more extended table of nuclear binding energies.
Figure 2: The nuclear $\alpha$-separation energies $S_{\alpha}^{\text{nuclear}}$ for $n_\alpha$-nuclei, corrected for Coulomb energies following eq. 2 in keV. There are peaks at $n_\alpha = 4, 7, 14$ which are followed by breaks.

References

[1] D.H.E. Gross and M.C. Nemes. Remarkable long-range systematics in the binding energies of $\alpha$ nuclei. *Phys. Lett.*, B 130:131, 1983.
Remarkable long-range-systematic in the binding energies of α-nuclei II

D.H.E. Gross
Hahn-Meitner Institute and Freie Universität Berlin,
Fachbereich Physik.
Glienickerstr. 100
14109 Berlin, Germany

May 19, 2003

Abstract
In this letter I present further data that show the remarkable evidence for the existence of an α-cluster structure in the ground states of even-even $N = Z$ nuclei. Such a remarkable systematic was observed 20 years ago in [1] for these nuclei at $A \leq 72$ and is extended here up to $A = 100$.

1 Introduction
Twenty years ago we published a remarkable systematic of nuclear binding energies [1]. The two-nucleon separation energies of α-nuclei were found to be approximately constant and equal to the 2-proton, 2-neutron and PN-separation energies of the $^4$He-nucleus. At these days this systematics could only be followed up to $^{72}$Kr. In the past 20 years the table of nuclear binding energies was considerably extended\footnote{Nuclear Structure and Decay Data, National Nuclear Data Center, Brookhaven National Laboratory, Upton, NY 11973-5000; http://www.nndc.bnl.gov/nndc/nndcsdd.html.}: It is now interesting how far this systematic can be followed also in these new data. This is indeed well possible as will be shown in this publication.

The nuclear separation energies are defined by the difference of the binding energies of initial nucleus minus the sum of that of the final nuclei. I correct it in the most simple way for the different Coulomb energies according to the trivial formula 2 [1]:

$$ S_{2N}^{\text{nuclear}} = (B_i + U_i^C) - (B_f + U_f^C) - U_{2N}^C $$

$$ U^C = \frac{3}{5} (e^2/r_0) Z(Z-1)/A^{1/3}, \quad r_0 = 1.25 \text{ fm}, $$ (1)

where $B_i$ and $B_f$ are the experimental binding energies of the initial and final nucleus respectively, $2N$ corresponds to any pair of nucleons [proton-proton...
Figure 1: Nuclear two-nucleon separation energies $S_{2N}^{\text{nuclear}}$, corrected for Coulomb energies, in keV, following eq. 2, with $2N = \pi, \nu$ (neutron-proton pair), $2N = \nu, \nu$ (neutron-neutron pair), and $2N = \pi, \pi$ (proton-proton pair) as function of the original number $n_\alpha$ of $\alpha$-s. The upper curve is for an $n_\alpha$-mother-nucleus, the lower one is for a mother-nucleus with $n_\alpha$ $\alpha$-particles plus one additional nucleon pair which is then removed.

As can be clearly seen in figure 1, the removal of a 2N-pair costs about the same energy as for a bare $\alpha$-particle $\approx 30$MeV. In contrast, the removal of an additional $\pi\nu$, $\nu\nu$ or $\pi\nu$ pair costs much less energy. Moreover, this strongly depends on the size the mother nucleus, resp. the number of $\alpha$-particles surrounding this pair.

Figure 2 show the separation energies of an $\alpha$-particle out of a $n_\alpha$-nucleus. The peaks at $n_\alpha = 4, 7, 14$, and the disappearance of peaks above $n_\alpha = 14$, remind of the special stable 3-dim clusters of closed packed spheres at $n = 4, 7, 13, 55$. The latter ones with icosahedron structure are observed also for atomic clusters of the heavier noble gases like $\text{Xe}^n$. For the lighter ones like $\text{Ar}$ clusters the shift from 13 to 14 is well known.

2Acknowledgement

I am grateful to W.v.Oertzen for pointing at the new and far more extended table of nuclear binding energies.
Figure 2: The nuclear α-separation energies $S_{\alpha}^{\text{nuclear}}$ for $n_\alpha$-nuclei, corrected for Coulomb energies following eq.2, in keV. There are peaks at $n_\alpha = 4, 7, 14$ which are followed by breaks.

References

[1] D.H.E. Gross and M.C. Nemes. Remarkable long-range systematics in the binding energies of α nuclei. *Phys. Lett.*, B 130:131, 1983.