Interference and Switching of Josephson current carried by nonlocal spin-entangled electrons in a SQUID-like system with quantum dots

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Josephson current of spin-entangled electrons through the two branches of a SQUID-like structure with two quantum dots exhibits a magnetic-flux response different from the conventional Josephson current. Due to their interference, the period of maximum Josephson current changes from $\hbar/2e$ to $\hbar/e$, which can be used for detecting the Cooper-pair splitting efficiency. The nonlocal spin entanglement provides a quantum mechanical functionale for switching on and off this novel Josephson current, and explicitly a switch is formulated by including a pilot junction. It is shown that the device can be used to measure the magnitude of split-tunneling Josephson current.

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Introduction – Nonlocal quantum entanglement becomes the focus of many investigations in recent years. The so-called Einstein-Podolsky-Rosen (EPR) pair is the hallmark of this phenomenon, which not only serves as the test case of violation of the Bell inequality, but also works as the medium for quantum communication and quantum computation. Photons are most intensively investigated, and their nonlocal entanglement has been successfully demonstrated. It would be interesting to observe this phenomenon in electron systems, where entanglement may arise in either spin or spatial degrees of freedom. However, the generation and detection of nonlocal entangled electrons in solid-state systems is still a challenge, since electrons interact with the macroscopic Fermi sea around them and it is hard to control a particular pair.

A possible approach to achieve nonlocal entanglement of an electron pair is to take advantage of the intrinsically spin entangled Cooper pairs in superconductivity. Based on this idea, three-terminal devices consisting of an s-wave superconductor coupled to two leads made of quantum dots (QDs), Luttinger liquid, and normal Fermi liquid have been theoretically proposed. It was shown that the Coulomb blockade effect may split a Cooper pair and force spin-entangled electrons tunnel into different leads. In a SQUID-like structure these spin-entangled electrons generate a new contribution to Josephson current. The anticipated Cooper pair splitting has then been explored experimentally in mesoscopic systems. The correlation between the resistances in the two leads due to the crossed Andreev effect (CAR) was used to measure the efficiency of Cooper pair splitting.

Generally speaking, however, the resistance correlation does not provide a direct evidence for the nonlocal spin entanglement, since processes without spin entanglement may be involved. In the present work, we propose to detect nonlocal spin entanglement based on the interference of Josephson current, which is immune to any process involving disentangled electrons. For this purpose, we adopt the system in Fig. 1 which was first discussed by Ref. 10. We notice that the novel Josephson current carried by spin-entangled electrons tunneling through the two paths in Fig. 1 responds to the magnetic flux differently from the conventional Josephson current for which a Cooper pair tunnels through one of the two leads. Due to their interference the maximum Josephson current exhibits a variation of period $\hbar/e$ responding to the magnetic flux, in contrast of the conventional $\hbar/2e$ known for SQUID, which can be used to detect the splitting efficiency of Cooper pair. The nonlocal spin entanglement is then shown to provide a quantum mechanical functionale for switching on and off this split tunneling process. The function of a switch based on a pilot Josephson junction is formulated explicitly. It is shown that the device can be used to measure directly the magnitude of the novel Josephson current.

Interference in presence of magnetic flux – It has been demonstrated experimentally that QD can work as a notch to control the Cooper pair splitting, since the
Coulomb blockade effect suppresses the tunneling of two electrons of a Cooper pair through the same QD. The system can be described with a tunneling Hamiltonian $H = H_0 + H_T$, with $H_0 = H_s + H_QD$, where $H_s = H_L + H_R$ is the BCS Hamiltonian of the two superconductors,

$$H_\alpha = \sum_{\sigma,k} \xi_{k\sigma} c_{\alpha \sigma, k} + \sum_{k} (\Delta_{c_{\alpha \uparrow, k}} c_{\alpha \downarrow, -k} + h.c.) \tag{1}$$

with $\alpha = L, R$ for the left and right superconductor, $\sigma = \uparrow, \downarrow$ for the electron spin; $H_QD = H_u + H_d$ is the Anderson-type Hamiltonian for the QDs with one localized spin-degenerate energy level

$$H_\eta = \epsilon_\eta \sum_{\sigma} a_{\eta \sigma}^\dagger a_{\eta \sigma} + U n_{\eta \uparrow} n_{\eta \downarrow} \tag{2}$$

with $\eta = u, d$ for the up and down QDs. In the presence of magnetic flux, the tunneling Hamiltonian is given by

$$H_{Tu} = e^{-i\frac{\Phi_0}{\hbar} \int_{-\pi}^{\pi} T a_{u \sigma} c_{R,\sigma}(r_{u,R}) + e^{-i\frac{\Phi_0}{\hbar} \int_{-\pi}^{\pi} T c_{L,\sigma}^\dagger(r_{u,L}) a_{u \sigma} + h.c.}$$

$$H_{Td} = e^{i\frac{\Phi_0}{\hbar} \int_{-\pi}^{\pi} T a_{d \sigma} c_{R,\sigma}(r_{d,R}) + e^{i\frac{\Phi_0}{\hbar} \int_{-\pi}^{\pi} T c_{L,\sigma}^\dagger r_{d,L}(r_{u,L}) a_{d \sigma} + h.c.} \tag{3}$$

with $r_{\eta,\sigma}$ denoting the positions where QDs are connected to superconductors, $T$ the tunneling matrix, $\Phi$ the magnetic flux, and $\Phi_0 = \hbar/2e$ the flux quantum.

The dc Josephson current can be evaluated using the Green function technique, and we arrive at the following three contributions $I_J = I_u + I_d + I_{ud}$:

$$I_u = \frac{4e^2 T^2}{\hbar} \text{Re} \int \frac{d\omega}{2\pi} n_F(\omega) \left| 1 + e^{i \frac{2\pi}{\hbar} \int_{-\pi}^{\pi} T a_{u \sigma} c_{R,\sigma}(r_{u,R}) + e^{-i \frac{2\pi}{\hbar} \int_{-\pi}^{\pi} T c_{L,\sigma}^\dagger(r_{u,L}) a_{u \sigma} + h.c.} \right| \tag{4}$$

$$I_d = \frac{4e^2 T^2}{\hbar} \text{Re} \int \frac{d\omega}{2\pi} n_F(\omega) \left| 1 + e^{-i \frac{2\pi}{\hbar} \int_{-\pi}^{\pi} T a_{d \sigma} c_{R,\sigma}(r_{d,R}) + e^{i \frac{2\pi}{\hbar} \int_{-\pi}^{\pi} T c_{L,\sigma}^\dagger(r_{d,L}) a_{d \sigma} + h.c.} \right| \tag{5}$$

from Cooper pairs tunneling through the up and down QD respectively, which are similar to the result for a single QD Josephson junction except the Peierls factors, and

$$I_{ud} = \frac{8e^2 T^2}{\hbar} \text{Re} \int \frac{d\omega}{2\pi} n_F(\omega) \left| 1 + e^{-i \frac{2\pi}{\hbar} \int_{-\pi}^{\pi} T a_{u \sigma} c_{R,\sigma}(r_{u,R}) + e^{i \frac{2\pi}{\hbar} \int_{-\pi}^{\pi} T c_{L,\sigma}^\dagger(r_{u,L}) a_{u \sigma} + h.c.} \right| \tag{6}$$

from Cooper pairs which are split and tunnel coherently through different QDs. Here $n_F(\omega)$ is the Fermi distribution function, $r_{u,a} = r_{u,R} - r_{d,a}$ is the terminal distance, $\mathcal{G}^R(\omega, \mathbf{r}) \equiv \langle \chi_{\uparrow R}(\mathbf{r}) \chi_{\uparrow L}(0) \rangle$ is the retarded anomalous Green function of the right superconductors and $\mathcal{G}^{\text{ret}}(\omega) \equiv \langle \chi_{\uparrow R}(\mathbf{r}) \chi_{\uparrow L}(0) \rangle$ is the retarded anomalous Green function of the QD. The superconductor is assumed to be a macroscopic system and thus its anomalous Green function is of the BCS form. The anomalous Matsubara Green function of the QD can be calculated with a contour integral method,

$$\mathcal{G}^{\text{ret}}(\omega) = \frac{1}{Z} \int d\omega' e^{-\beta \omega'} \text{Tr} \left[ \int \frac{d\omega}{2\pi} \mathcal{G}^R(\omega, \mathbf{r}) \mathcal{G}^R(\omega, \mathbf{p}) \right]$$

where $Z$ is the partition function and $\beta$ is the inverse temperature. Here we consider the regime $\epsilon_u = \epsilon_d = \epsilon > 0$ where the ground state of the QD is empty of electron, thus avoiding the complexity induced by the local electrons on the QDs. Evaluating the Green function by perturbation expansion in respect of the tunneling Hamiltonian, we arrive at the lowest order result of the Josephson current at low temperatures

$$I_J = |I_1 \sin(\phi_1 - \frac{\pi \Phi}{\Phi_0}) + I_1 \sin(\phi_1 + \frac{\pi \Phi}{\Phi_0}) + I_2 \sin \phi_1 | \tag{8}$$

where $\phi_1$ is the superconducting phase difference, and

$$I_2 = \frac{1}{\hbar} \sum_{k,p} \frac{4e^2 T^2 |\Delta_L| |\Delta_R|}{E_k E_p} \frac{1}{E_k + E_p + \epsilon} \frac{1}{2\epsilon + U} \tag{9}$$

with $E_k = \sqrt{\xi^2 + \Delta^2}$. It is diminished when the Coulomb interaction $U$ and the superconducting energy gap $\Delta$ are large, and

$$I_2 = \frac{1}{\hbar} \sum_{k,p} \frac{4e^2 T^2 |\Delta_L| |\Delta_R|}{E_k E_p} \frac{1}{E_k + E_p + \epsilon} \frac{1}{2\epsilon + U} \tag{10}$$

FIG. 2: (color online). Magnetic response of maximum dc Josephson current. The dash-double dotted, dashed, dash-dotted, dotted, solid line is for the case of splitting efficiency $\gamma = 0, 0.25, 0.5, 0.75, 1$ respectively. Each curve was normalized and shifted for clarity.
which is controlled by the distances between the terminals of QD channel\textsuperscript{24,25}. Here we note that the divergence of $I_2$ on $\epsilon$ is caused by the perturbation treatment, which can be eliminated by proper renormalization\textsuperscript{24,25}.

There are three contributions in the Josephson current in Eq. (8). The first two terms come from Cooper pairs co-tunneling from the up and down QD respectively. The interference between these two terms results in a period of $\Phi_0$ in the magnetic response of maximum Josephson current, identical to that of a conventional SQUID. The third term is unique for the present system, in which the two electrons of a Cooper pair are split and tunnel through different QDs, and the Peierls factors cancel each other exactly. As the result, the interference pattern of maximum dc Josephson current exhibits a period of $2\Phi_0$. It is clear that the behavior of the total Josephson current is significantly modulated by the process of Cooper pair splitting. In order to see this influence more clearly, we define the splitting efficiency $\gamma = I_2/2I_1$. In the regime of strong Coulomb interaction and single particle resonant tunneling with $\epsilon \ll \Delta \ll U$, one has $\gamma \approx \frac{\pi^2}{U} e^{-2\Phi_0/\pi} \sin^2 (k_F \delta r)/(k_F \delta r)^2$, where $\delta r = \delta r_L = \delta r_R$, where $\delta r = \delta r_L = \delta r_R$, $\xi$ is the coherence length and $k_F$ is the Fermi wave vector. Theorctically, the factor $\sin^2 (k_F \delta r)/(k_F \delta r)^2$ sets the common condition for observing the present quantity and the cross Andreev reflection\textsuperscript{26-28}. The variation of maximum Josephson current with the splitting efficiency is illustrated in Fig. 2. It is clear that with increasing splitting efficiency from zero to unity, the pattern evolves gradually, and the period changes from $\Phi_0$ to $2\Phi_0$. It is noticed that the splitting efficiency can be larger than unity, associated with a similar curve to $\gamma = 1$, except that the minima do not reach zero.

Switching of the novel Josephson current – In order to contribute to the split-tunneling Josephson current, two single electrons tunneling through the two paths should be in a spin singlet state. This strong spin entanglement can be used for switching on and off this novel Josephson current. Let us set $-U/2 \ll -\Delta < \epsilon < 0$ such that there is a localized electron on each QD\textsuperscript{40}. For the split-tunneling Josephson current, it is the "on"/"off" state when the spins of the two localized electrons are in a spin singlet/triplets state. The situation can be described by\textsuperscript{40}

\[ I = I_2 \left( \frac{1}{4} - \mathbf{S}_u \cdot \mathbf{S}_d \right) \sin \phi_s. \]  

(11)

Here we propose a way to control the two localized spins by introducing an additional pilot QD-junction as shown schematically in Fig. 3. The critical current $J$ of the pilot junction varies continuously when the gate voltage on the QD is tuned\textsuperscript{26-27}. The total Josephson current is then given by

\[ I_3 = [2I_1 + I_2 \left( \frac{1}{4} - \mathbf{S}_u \cdot \mathbf{S}_d \right) + J] \sin \phi_s \]  

(12)

with the co-tunneling current\textsuperscript{24}

\[ I_1 = -\frac{1}{\hbar} \sum_{k,p} \frac{2eT^4|\Delta_L\Delta_R|}{E_k E_p (E_k + E_p)(E_k - \epsilon)(E_p - \epsilon)} \]  

(13)

and the split-tunneling current\textsuperscript{10}

\[ I_2 = \frac{1}{\hbar} \sum_{k,p} \frac{4eT^4|\Delta_L\Delta_R| \cos(k \cdot \delta r_R) \cos(p \cdot \delta r_L)}{E_k E_p (E_k - \epsilon)(E_p - \epsilon)} \times \left[ \frac{1}{E_k + E_p} + \frac{1}{2|\epsilon|} \right] \]  

(14)

where only the zero order terms in $U$ are included for simplicity, since $U$ is the largest energy scale. Both $I_1$ and $I_2$ are similar to the previous section, except that $I_1$ is negative, which represents the $\pi$ junction nature of each QD for co-tunneling current. For most cases, $-I_1 > I_2$.

The critical current of the total system can be straightforwardly evaluated for the two spin configurations, which is plotted as function of the critical current $J$ of the pilot junction in Fig. 4. For $J < -2I_1 - I_2/2$, the critical current of the spin triplets state is larger than that of the singlet state, and vice versa.

When a current is injected into the system through one of the superconductors, the superconductivity phase difference will be adjusted in order to pass this current without dissipation. If the current is small, the localized spins can take either singlet or triplet state. For current in between the two critical currents, the system will adjust the localized spins to achieve the larger critical current in order to reduce dissipation\textsuperscript{10}. It is easy to figure out that for $J < -2I_1 - I_2/2$ the spin triplets, thus "off", state is realized, and for $J > -2I_1 - I_2/2$ the spin singlet, thus "on" state, is realized, as depicted in Fig. 4.

It is noticed that there is a gap $I_g$ in the maximal Josephson current of the total system when the critical

FIG. 3: (color online). Schematic setup of a switch of the novel Josephson current. The Cooper pair splitter displayed in Fig. 1 is connected in parallel with a pilot Josephson junction.

FIG. 4: (color online). Schematic drawing of a switching circuit of the novel Josephson current. Switching of the Cooper pair splitter displayed in Fig. 1 is connected in parallel with a pilot Josephson junction. The critical current is drawn as function of the gate voltage on the QD.
current of the pilot junction is swept, due to the split-tunneling Josephson current. By measuring the gap current we can evaluate the magnitude of the split-tunneling Josephson current since \( I_2 = 2I_g \). At the switching point, the Josephson current carried by co-tunneling processes is suppressed significantly down to \(-I_2/2\), and thus the splitting efficiency of Cooper pair is enhanced largely.

Before ending this section, we notice that the pilot QD can be replaced by a conventional SQUID, for which the critical current can be controlled by a magnetic field.

**Summary** – In summary, we analyze the tunneling processes in a SQUID-like device with a quantum dot embedded in each junction, where electrons with nonlocal spin entanglement are generated and tunnel through different junctions, which contribute to the Josephson current. In presence of a magnetic flux, they carry a zero Peierls phase due to the cancelation between the two tunneling paths, in contrast to the tunneling of Cooper pairs through one of the two junction. As the splitting efficiency increases, the period of the magnetic-flux response of maximum Josephson current changes from \( \Phi_0 = h/2e \) to \( 2\Phi_0 \). The nonlocal spin entanglement is then used to switch the split tunneling process. The "on" ("off") state of the switch corresponds to the spin singlet (triplet) of the two localized electrons in the quantum dots of the Cooper-pair splitter, which can be controlled by a pilot junction. It is shown that the device can be used to measure directly the magnitude of the Josephson current carried by single electrons with nonlocal spin entanglement.

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