Aeroelastic Stability of Idling Wind Turbines

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Abstract. Wind turbine rotors in idling operation mode can experience high angles of attack, within the post stall region that are capable of triggering stall-induced vibrations. In the present paper rotor stability in slow idling operation is assessed on the basis of non-linear time domain and linear eigenvalue analysis. Analysis is performed for a 10 MW conceptual wind turbine designed by DTU. First the flow conditions that are likely to favour stall induced instabilities are identified through non-linear time domain aeroelastic analysis. Next, for the above specified conditions, eigenvalue stability simulations are performed aiming at identifying the low damped modes of the turbine. Finally the results of the eigenvalue analysis are evaluated through computations of the work of the aerodynamic forces by imposing harmonic vibrations following the shape and frequency of the various modes. Eigenvalue analysis indicates that the asymmetric and symmetric out-of-plane modes have the lowest damping. The results of the eigenvalue analysis agree well with those of the time domain analysis.

1. Introduction

In idling mode, the angles of attack (AOA) experienced by the blades significantly vary over one revolution under the combined effect of inflow turbulence, flow inclination and nacelle tilt and yaw. The variation of the AOAs remains important even in small yaw misalignments within the range of +/-15°. It is noted that yaw errors in the above range are considered as normal idling conditions by wind turbine manufactures. Going to moderate yaw angles, the variations of the AOA can be such that the rotor enters stall both at positive and negative AOAs and thereby stall induced vibrations are likely to occur. In the past a lot of research effort has been directed in the analysis of stall induced vibrations in normal operation [1],[2],[3], however very little has been done for parked or idling rotors [4],[5].

In the present paper the stability behavior of the conceptual 10 MW reference wind turbine (3 bladed, pitch regulated-variable speed turbine with diameter D=178.3 m) of the INNWIND.EU project [6] in slow idling operation is assessed using the linear eigenvalue stability tool GAST_lin [3]. The objectives of the work are to first identify idling situations at moderate yaw misalignment angles, that favor stall induced vibrations and subsequently predict which of the turbine modes exhibit the lowest damping characteristics. The analysis is confined to yaw angles within the range [-60°, +60°]. This is the absolute upper limit that engineering dynamic stall models can be trusted. Outside this range deep stall conditions are encountered that cannot be properly addressed by engineering aerodynamic models. This is because engineering models lack the appropriate tuning in such deep stall conditions. Furthermore, in deep stall, vortex shedding phenomena take place that lead to additional periodic excitation of the rotor not included in the present engineering modeling framework.
The eigenvalue analysis results are compared against results of non linear time domain analysis as well as with results of aerodynamic work computations coming from forced harmonic oscillation simulations in which the imposed motion follows the shape and the frequency of the turbine modes. This is done in an attempt to prove that fast linear eigenvalue stability tools that have been widely employed by the industry for damping characterization in normal operation conditions can be also trusted for predicting damping in idling operation conditions.

The inflow conditions that favor stall induced instabilities are identified through turbulent wind non-linear time domain aeroelastic simulations at yaw angles in the range \([-60^\circ, +60^\circ]\) (moderate to high yaw angles) using the hGAST aeroelastic modeling platform [7]. As already mentioned the above range of yaw angles defines the validated envelope of engineering dynamic stall models. Rotor azimuth positions and corresponding sectional AOAs at which instabilities are favored are assessed. Moreover, the average rotor speeds for the different yaw angles are recorded in order to be used as input to the linear eigenvalue stability tool.

Based on the findings of the above time domain analysis, eigenvalue stability simulations are performed focusing on the conditions for which maximum edgewise loads are obtained with hGAST code. Stability analysis in the case of yaw misalignment requires application of Floquet’s theory [8]. Due to the essentially periodic character of the resulting dynamic system, Coleman’s multi-blade transformation can no longer reduce the system to a constant coefficient one. Besides the anyway high cost of Floquet analysis, in idling conditions the rotational speed attains very low values that become at least ten times smaller than the lowest natural frequency of the turbine which further increases the cost to a prohibitive level for the here intended analysis.

By noting that at very small rotational speeds (\(\approx 1\) RPM) low order harmonics (up to 6p) are not expected to interact strongly with the natural frequencies of the turbine, as an alternative, non-rotating (static) analysis can be performed at different azimuth angles within the sector \([0^\circ, 120^\circ]\). In order to approximate as closely as possible the rotating case a free-free drive train (free rotation boundary condition over the generator side) is simulated and the idling rotational speed (obtained through time domain simulations) is taken into account in formulating the local to the blade section velocity triangle.

Finally, using as input the aeroelastic mode shapes and frequencies calculated through the eigenvalue analysis, aerodynamic work computations are performed. The turbine is set to a prescribed small amplitude harmonic motion following the shape and frequency of the various aeroelastic modes. The work done by the aerodynamic loads acting on the blades as a result of this forced vibration, is computed within one oscillation cycle. This work is directly related to the damping of the corresponding mode [1]. Non-linear work computations are compared to eigenvalue analysis results.

The results of the analysis indicated that the asymmetric and symmetric out-of-plane modes exhibit the lowest damping values. Among them, the asymmetric edge horizontal/tilt mode obtains negative damping values at the yaw angle of 30\(^\circ\). The results of the eigenvalue analysis agree well with the results of the time domain analysis.

2. Background on the dynamics and aerodynamic loading of an idling turbine

In the present section some important features related to the dynamics and aerodynamic loading of an idling turbine are discussed. This background information is regarded essential in order to better understand the stability results presented in the following sections.

Figure 1 presents the variation of the natural frequencies of the idling Reference 10 MW wind turbine versus pitch angle as it increases from 0\(^\circ\) to 90\(^\circ\) (towards feather). This plot is typically obtained for any three bladed turbine of similar design philosophy. As expected the two first tower modes M1 and M2 are not affected at all by the pitch angle. The frequency of the first asymmetric flap modes M3 and M4 slightly increases while the frequency of the first asymmetric edgewise modes M6 and M7 significantly decreases as the pitch angle increases. As a result asymmetric flapwise and edgewise modes come closer in frequency to each other, despite their initial distinct separation at 0\(^\circ\) pitch angle. Bringing the asymmetric flap and edge modes closer will render the cross coupling
between the corresponding bending directions stronger when the blades are in feather position. The frequency of the first symmetric out of plane mode M5 will gradually increase and finally at high pitch angles will exceed the frequency of the asymmetric edgewise modes M6 and M7. The frequency of the first symmetric in-plane (drive train) mode M8 decreases with the pitch angle and as the pitch tends to 90° the frequency of M8 tends to coincide with the frequency of the symmetric out of plane mode M5. The second asymmetric flapwise modes M9 and M10 exhibit a similar behavior with the asymmetric first flapwise modes M3 and M4.

![Diagram of natural frequencies](image)

**Figure 1.** Natural frequencies of the 10 MW Reference Wind Turbine vs. pitch from 0° to 90° (towards feather).

As the blade pitches to feather, the asymmetric flapwise modes switch from out-of-plane to in-plane. Inversely the asymmetric edgewise modes switch from in-plane to out-of-plane. So, as the blade pitch changes from 0° to 90° the first asymmetric edgewise (in-plane) vertical mode M6 turns into a first asymmetric edgewise (out-of-plane) yawing mode and the first asymmetric edgewise (in-plane) horizontal mode M7 turns into a first asymmetric edgewise (out-of-plane) tilting mode. This transformation of the asymmetric edgewise modes is depicted in Figure 2 (a) and (b). Inversely the first asymmetric flapwise (out-of-plane) yawing mode M3 turns into a first asymmetric flapwise (in-plane) vertical mode and the first asymmetric flapwise (out-of-plane) tilting mode M4 turns into a first asymmetric flapwise (in-plane) horizontal mode. Similar transformations are obtained in the second flapwise modes M9 and M10.

On the other hand the symmetric modes seem to retain their original character. So, as shown in Figure 3 the out-of-plane (flapwise) collective mode (M5) remains a collective out-of-plane (edgewise) mode as the pitch changes from 0° to 90°. At 0° pitch the coupling with the in-plane direction in M5 is negligible. At 90°, as the frequency of M5 gets close to the frequency of the collective in-plane M8 mode a coupling with the in-plane direction is established.
Figure 2. Mode shape alternation from 0° to 90° pitch for asymmetric edgewise modes (a) M6 and (b) M7. Red symbols: undeformed state, Black symbols: deformed state.

Figure 3. Mode shape alternation from 0° to 90° pitch for symmetric out-of-plane mode M5. Red symbols: undeformed state, Black symbols: deformed state.

Also as shown in Figure 4 the in-plane (edgewise) collective mode M8 remains a collective in-plane (flapwise) mode as the pitch changes from 0° to 90°. Originally at 0° pitch a coupling with the second flapwise modes is seen in the shape with the mode driven by the fact that the collective in-plane mode lies closely to the second flapwise asymmetric modes. At 90°, as the frequency of the mode decreases and approaches the frequency of the first collective out-of-plane mode a strong coupling between the two modes is established.

Figure 4. Mode shape alternation from 0° to 90° pitch for symmetric inplane mode M8. Red symbols: undeformed state, Black symbols: deformed state.
As already discussed the AOAs experienced by the blades of an idling rotor vary significantly over the revolution. The velocity triangle of a section of an idling blade is illustrated in Figure 5 for the blade azimuth positions of 0°, 90°, 180° and 270°. It is seen that as a result of yaw error the AOA reaches a minimum/maximum at 0°/180° azimuth respectively (whether the AOA will be positive or negative depends on the direction of the incoming flow—positive or negative yaw) while as a result of tilt and inflow inclination angles the AOA attains a minimum at 90° and a maximum at 270°. An example of the AOA variation of the 75% section of the reference 10 MW is shown in Figure 6(a) for a yaw angle of 15° and a tilt angle of 5°. The mean wind speed and the pitch setting are taken equal to 42.5 m/s and 90° respectively; the blade section is considered to have zero twist and the linear speed component due to the idling rotation of the blade is not taken into account (assuming an almost zero idling speed). It is seen that as the yaw angle increases the range of variation of the AOA equally increases. The range of variation of the AOA is equal to the yaw error angle. If the yaw error is combined with the tilt of the rotor (or the inclination of the mean inflow) then the range of variation of the AOA increases further. The azimuth angle at which maximum (positive or negative) AOA is obtained is shifted away from 0° and 180° given the 90° phase difference of these two effects. Beyond a certain yaw angle the blade will definitely enter stall both in the positive and the negative AOAs regime.

As the pitch of the blade increases to feather the local AOAs “seen” by the blade are equally shifted to lower values and therefore idling speed decreases. On the other hand reduction of the idling speed leads to higher AOAs along the blade span. An example of the effect of the idling speed on the AOAs of the 75% section is shown in Figure 6(b). For an inflow velocity of 42.5 m/s an almost 10° increase of the AOAs is noted as the idling speed decreases from 1 RPM to almost 0 RPM. So, the pitch angle of the blade and the rotor idling speed are two interrelated but also competing parameters as concerns the mean level of the AOA variation.

Whether stall induced vibrations will be triggered, strongly depends on the post stall characteristics of the airfoil sections. Higher negative slopes of the $C_L$ curve and deeper drop of the $C_L$ in the post stall region deteriorates stability. As already mentioned for moderate to high yaw angles the idling blade enters stall both at positive and negative AOAs. The post stall characteristics of cambered sections differ significantly between positive and negative angles. Usually, airfoil sections exhibit smoother post stall behavior at negative AOAs compared to that at positive ones (there are of course exceptions to the above statement). This implies that selection of the appropriate pitch setting and thereby of the appropriate idling speed could be critical in several cases when targeting the avoidance of stall induced vibrations. Adjustment of the blade pitch may shift the AOAs to a desired region.

3. Description of the tools

Non-linear time domain aeroelastic simulations are performed using NTUA’s in-house servo-aeroelastic solver hGAST [7]. In hGAST the full wind turbine is considered as a multi-component dynamic system having as components the blades, the drive train and the tower; all approximated as beam structures. Assembly into the full system is carried out in the framework of the analytic dynamics also known as multi-body approach. It consists of considering each component separately from the others but subjected to specific free-body kinematic and loading conditions to be imposed at the connection points of the components. Rotor aerodynamics is simulated using a Blade Element Momentum (BEM) model that accounts for dynamic inflow, yaw misalignment, and dynamic stall effect through the ONERA dynamic stall model.

Stability analysis is performed using the eigenvalue stability tool GAST_lin [3]. GAST_lin is the linearized version of hGAST. In GAST_lin the unsteady aerodynamic equations, the corresponding aerodynamic states of the dynamic stall model (circulation parameters of the ONERA model) and the structural equations along with the corresponding DOFs are treated uniformly in one system following the so-called “Aeroelastic Beam Element” concept [3].

Stability simulations are performed for a static rotor at different azimuth angles within the sector [0°, 120°] for the reference 3 bladed rotor. Although the rotor is considered static, a free rotation
boundary condition is imposed at the generator side in order to approximate as closely as possible idling operation. Also the average idling rotational speed which has been obtained through the time domain simulations, is taken into account in forming the local to the blade section velocity triangle.

Figure 5. Velocity triangles of idling blade at different azimuth positions.

Figure 6. Variation of the angle of attack over one revolution for a wind speed of 42.5 m/s (a) effect of yaw and tilt angle - tilt angle shifts the curve horizontally, the rotational speed is almost zero), (b) effect of rotational speed - the rotational speed shifts the curve vertically.

4. Results and discussion

4.1. Time-domain analysis results
Time domain aeroelastic simulations with turbulent wind are performed for the Reference 10 MW wind turbine, at a mean wind speed of 42.5 m/s, with a turbulence intensity (TI) 11%, for various yaw misalignment angles in the range [-60°, +60°]. Six 10-minute simulations are performed in every yaw angle corresponding to different realizations of the wind (wind seeds). Figure 7 presents the min-max envelope of the edgewise (out-of-plane) bending moments at blade root as functions of the yaw misalignment angle. Load results of all three blades for the different wind seeds, as well as average
loads are provided in the plot. It is seen that ultimate loads attain both maximum and minimum value at the yaw angle of +30°. When loads are averaged the minimum is found at +22.5° yaw angle (very close though to the load of the +30° yaw) while the maximum is still obtained at +30° yaw angle.

**Figure 7.** Ultimate blade root edgewise bending moments. Loads from 6 wind seeds are averaged. Wind speed 42.5 m/s; TI=11%; yaw angles in the range [-60°, +60°].

For the yaw angle of 30° a pattern of the edgewise bending moments at the three blade roots is shown in Figure 8 along with the time series of the azimuth angle and the AOA for a 45 s duration. The average idling speed of the rotor at the above conditions (wind speed 42.5 m/s and yaw 30°) is about 0.8 RPM. Also in Figure 9, the obtained C\textsubscript{L} values at r/R=90% are collected and plotted with respect to the corresponding AOA. In this plot, the regions of unfavorable lift slope are marked in grey.

At t=107 s (marked with a dashed blue line), mild edgewise vibrations start to grow on blade 3 as a result of increasing AOA that push the blade into stall. At t=107 s blade 1 is at the azimuth of 90° (0° azimuth corresponds to a blade being in vertical position and pointing upwards). The AOA on blade 3 crosses the level of 15° so stall takes place at positive AOA. Blade 2 also operates in deep stall experiencing negative AOA in the range [-30°, -40°]. Finally, the flow remains attached on blade 1. The AOA experienced by blade 1 remain at -10° AOA for approximately 5 s and therefore no vibrations are expected.

Moving in time, the situation is changing. At t=116 s (marked by a red dashed line), mild edgewise vibrations start now to grow on blade 1 as a result of increasing AOA that push this blade into stall at the negative AOAs regime. As indicated in Figure 8, at t=116 s, blade 3 is at the azimuth of 20° and the AOA “seen” by blade 1 grows to about -30°. As time evolves (time period [116s,120s]), blade 1 goes deeper into negative AOA stall (about -40°) while over the same time period, blade 3 remains in deep stall receiving AOA in the range [20°,30°]. This explains why blade 3 exhibits large vibrations over the whole time period [107 s, 120 s]. Recovery to attached flow conditions is only obtained for blade 3 at t=120 s. Opposite to the other two blades, now blade 2 operates in attached flow conditions (AOA> -15°).

Finally, during the time interval [80s, 95s], blade 1 is experiencing similar flow conditions to those experienced by blade 3 in the time interval [107s, 115s]. Blade 2 is now lying in the azimuth range of 80°-100°. The only difference with respect to the time period [107s, 115s] is that now the rotor speed is lower and therefore blade 1 remains longer time within the stall region. As is the previous case high load amplitudes are obtained for blade 1 within this time interval that decrease when blade 1 moves away from stall conditions (AOA and loads of blade 1 decrease in the time interval [95s, 105s]).
Figure 8. Times series of blade root edgewise bending moment, blade azimuth angle and angle of attack at r/R=0.90 spanwise position. Wind speed 42.5 m/s, TI 11%, yaw angle 30°.

Figure 9. C\textsubscript{L} - AOA plot at r/R=0.90 spanwise position. Wind speed 42.5 m/s, TI 11%, yaw angle 30°.

4.2. Eigenvalue analysis results

The modal damping (in logarithmic decrement) and frequencies of the rotor modes (M3-M8) at 0° yaw are shown in Figure 10 and Figure 11 as functions of the azimuth angle in the range of [0°, 120°] for blade 1. In Figure 12, Figure 13 the PSDs of the flapwise and edgewise bending moments coming from the earlier presented time domain analysis are also given for the same yaw angle.

It follows that the three out-of-plane modes, M5, M6 and M7 are clearly the lowest damped ones (Figure 11). This is definitely in agreement with the time domain analysis results as shown in the PSD plots of the edgewise bending moment at the root of the three blades (Figure 13). In the PSD plot, the three most pronounced peaks indicate the modes that are mainly excited. The corresponding frequencies are found at 0.69 Hz, 0.81 Hz, 0.95 Hz. They match very well with the values obtained...
with the stability tool (Figure 10). A lower peak also appears at 0.25 Hz which corresponds to the first longitudinal tower bending mode M1. On the contrary there is no peak in the PSD of the blade root flapwise bending moment (Figure 12) indicating that the aerodynamic damping of the in-plane modes M3, M4 and M8 is high. This is also predicted by the stability tool (all values are above 100% in logarithmic decrement). Worth noticing is that both the level of damping and the frequencies are independent of the azimuth position in the 0° yaw case.

Similarly, in the 30° yaw case, Figure 14 and Figure 15 give the frequency and damping results while Figure 16 and Figure 17 give the PSDs of the flapwise and edgewise bending moments at the blade root of the three blades. It is reminded that according to section 3, at 30° yaw the edgewise loads reach their maximum values. Also as explained in section 2, for this high yaw angle the AOAs experienced by the rotating blades significantly vary with the azimuth angle. As a result the modal frequencies and damping also vary significantly over the revolution in contrast to the 0° yaw case. At very high yaw angles, the AOAs seen by the blades are expected to enter post stall on both sides of the polar (positive and negative AOAs).

Compared to the 0° yaw case, lower damping values are obtained for most modes (in-plane and out-of-plane) indicating that stall induced vibrations start to develop at the yaw angle of 30°. The damping of the in-plane modes M3, M4 and M8 significantly drops. The damping becomes even
locally negative for M4 at the 0° and 100° azimuth positions. Nevertheless, damping still remains high positive at most azimuths indicating that the overall damping of the in-plane modes will be eventually positive. On the other hand, modes M5, M6, M7 still have low damping throughout the entire range of azimuth angles. Especially for M7 (asymmetric edge horizontal/tilt mode) the damping is negative at all azimuth angles. In line with the observations made in the time domain analysis of the previous section higher negative damping values are noted over the intervals [0°, 20°] and [75°, 110°] of the azimuth angle.

Again four peaks dominate the PSD plot of the edgewise bending moment at 0.25 Hz, 0.7 Hz, 0.8 Hz and 0.95 Hz (Figure 17). The first one corresponds to the first tower longitudinal bending mode M1 while the other three correspond to the three edgewise (out-of-plane) modes M5, M6, M7. Again, the frequencies of the highly excited modes in the results of the time domain analysis agree well with the frequencies predicted by the stability tool (Figure 14) for the low damped rotor out-of-plane modes. The largest peak is seen at the frequency of 0.8 Hz (the frequency of M7) indicating that indeed this is the lowest damped mode of the rotor. In Figure 16 some high energy levels are noted in the frequency range of [0.5 Hz, 0.6 Hz]. This peak corresponds to the in-plane mode M4. As discussed earlier, according to the stability predictions this is the lowest damped flapwise mode.

![Figure 14. Modal frequency of rotor modes, at U=42.5 m/s and yaw=30°](image)

![Figure 15. Modal damping of rotor modes, at U=42.5 m/s and yaw=30°](image)

![Figure 16. PSD of the flapwise bending moment at the blade root, at U=42.5 m/s and yaw=30°](image)

![Figure 17. PSD of the edgewise bending moment at the blade root, at U=42.5 m/s and yaw=30°](image)
In Figure 18 and Figure 19 the damping results for the 45° and 60° yaw cases are shown. It can be deduced that as the yaw angle further increases, the damping of the out-of-plane rotor modes returns to positive values. This is in agreement with the results of the time domain analysis which showed that edgewise loads gradually decrease beyond 30° yaw angle. It is interesting to note that at the yaw angles of 45° and 60° the lowest damped mode appears to be M6 (asymmetric edge vertical/yaw) and not M7 (asymmetric edge horizontal/tilt) as found in the 30° yaw case. The damping of M7 gradually increases as the yaw angle increases while the damping of M6 remains almost constant.

4.3. Work computation results

Next, the results of the eigenvalue analysis of the previous section are further evaluated by computations of the work of the aerodynamic forces for imposed harmonic vibrations following the shape and frequency of the different modes. The blade is set to an externally imposed harmonic motion following the shape and frequency of the mode considered and the unsteady aerodynamic loads are calculated along its span at various stations. Then the work done by the aerodynamic loads is computed over one cycle of the blade oscillation. The above work is directly associated with the damping of the corresponding mode. The mode shapes and eigen-frequencies used in the present analysis are the aeroelastic ones obtained through the eigenvalue analysis. As opposed to the structural mode shapes (obtained for the free vibration problem in vacuum conditions), the aeroelastic mode shapes also include the phase shift between the various components of the blade motion and therefore provide more realistic prediction of the work distribution.

Work analysis is able to bring more insight into stability computations because it provides the stability characteristics separately for every blade. When eigenvalue analysis gives negative damping value for a specific mode, work analysis can identify which blade is responsible for the instability as well as the spanwise extent of negative damping contribution. The non-linear behavior of the aerodynamic loads can be also investigated, by changing the amplitude of the forced oscillation. Since one of the main objectives of the present work is the evaluation of the linear eigenvalue analysis predictions, modal amplitude was kept low and equal to 0.2 m at the blade tip. As previously the analysis focuses on the yaw angle of 30° where stall induced vibrations mainly take place and only on rotor modes.

In Figure 20 aerodynamic work results (integrated over the span and for the three blades) are compared against eigenvalue analysis results. In regard to the flap modes, in general the two sets correlate well except for M3 at some specific azimuths (for example 5° and 115°). In these particular positions the eigenvalue analysis predicts positive damping, while in the work analysis the damping is negative. Also, in regard to the edgewise modes, the two methods compare well. In line with the
eigenvalue analysis results, the work calculated for mode M7 remains negative over the whole azimuth range whereas the same calculation for M5 and M6 gives positive work values. Although relatively small amplitude of 0.2 m has been used in the work analysis the differences with the eigenvalue analysis results are big in some cases. The explanation for the above inconsistencies lies in the strong non-linearity of the ONERA equations, especially within the stall region. In ONERA model, dynamic stall characteristics are derived through the solution of a set of second order in time, variable coefficient (coefficients depend on steady-state polars) differential equations. In regions where the gradient of the steady-state polars changes rapidly the linearized set of equations fails to represent correctly the actual non-linear set. It is noted that similar analysis performed using steady-state characteristics (not shown in the paper) yielded a much better agreement between eigenvalue and work analysis results.

Figure 20. Comparison between aerodynamic work (0.2m modal amplitude) and damping results versus azimuth angle; U=42.5 m/s yaw=30°

In Figure 21-Figure 24, work distributions along the three blades and $C_L$ hysteresis loops of the three blades at $r/R=0.90$ are shown for M7 and for the azimuth angles of 20° and 100°. At these azimuth angles the work method predicts higher negative work values for the particular mode. In the abovementioned plots 1 m amplitude of the blade tip motion has been considered. This way, wider hysteresis loops are attained and thereafter the unsteady character of the flow is better illustrated. At 20° rotor azimuth, blade 2 (located at 140° azimuth) provides the highest negative work. As seen in the $C_L$-AOA plot blade 2 experiences AOA in the post stall region at negative angles. Negative work is also contributed by blade 1 (located at 20°). Blade 1 operates in the post stall region at positive angles. It is noted that the difference in the shape of the $C_L$-AOA loops of blade 1 and blade 2 (both in the post stall region) is due to the different motion (in terms of flap-edge coupling and phase difference between the two directions of motion) undergone by the two blades in M7. Blade’s 1 motion is dominated by vibrations in the edgewise direction while blade 2 exhibits a stronger flap-edge coupling. As a result of the higher flapwise component in blade’s 2 motion, the range of AOA seen by blade 2 is also higher. At 100° rotor azimuth negative work is almost evenly contributed by blades 2 and 3 (located at 220° and 340° respectively). Again, the shapes of the $C_L$-AOA loops of blade 1 on one hand, and blades 2 and 3 on the other, are quite different. This is mainly because of the different flow conditions encountered by the different blades. Blades 2 and 3 operate in deep stall (well within the negative $C_L$-AOA slope region) while blade 1 encounters light stall conditions (mainly positive slopes.
up to the $C_L_{\text{min}}$). By comparing the above results with the time domain results presented in section 4.1, the agreement is found to be very good in qualitative terms.

**Figure 21.** Distribution of aerodynamic work over the three blades for mode M7. Azimuth angle 20°.

**Figure 22.** $C_L$-aoa loops of the three blades at r/R=0.90 for mode M7. Azimuth angle 20°.

**Figure 23.** Distribution of aerodynamic work over the three blades for mode M7. Azimuth angle 100°.

**Figure 24.** $C_L$-aoa loops of the three blades at r/R=0.90 for mode M7. Azimuth angle 100°.

5. Conclusions
The stability characteristics of 10 MW reference wind turbine in slow idling operation is assessed using an eigenvalue stability tool. The results of the eigenvalue analysis are compared against time domain aeroelastic predictions of loads for the idling turbine. The above cross-comparison proved that fast linear eigenvalue stability tools can be used as a basis to characterize stability of a turbine in idling operation. The analysis showed that the lowest damped modes of the 10 MW idling rotor are the out-of-plane ones (symmetric and asymmetric). At a yaw misalignment of 30° the asymmetric out-of-plane horizontal/tilt mode obtains negative damping values throughout the entire range of azimuth angles. Another interesting outcome of the analysis is that when the blades are pitched to feather, asymmetric in-plane and out-of-plane modes come close is frequency which suggests strong coupling between
modes and directions. When the rotor operates in stall, the above coupling, depending also on the structural pitch of the blades, might affect the stability behavior of the turbine.

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