3-D Inversion of Crustal Structure in Frequency of Gravity

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Abstract. This work introduces a geologically based approach for calculation the gravity anomaly of 3-dimensional (3-D) earth model. The geologically driven earth model is decomposed into a 3-D grid and the gravity signature for different depth is calculated respectively. The results of gravity inversion are well, but when the boundary is near to 200Km, the results are very different. And as the depth increasing, the results are very similar. That is to say, the absolute depth is not sensitive for gravity inversion. In terms of the results of inversion, increasing the number of points and computing large-scale data can reduce the effect of boundary.

1. Introduction

Geophysics is the science that deals with the physical properties of the Earth. Gravitation is a part of geophysics dealing with the gravity field. From gravity anomalies measured on the earth’s surface one can deduce distribution of masses below it. In the gravity anomaly detection and the inversion of physical property, the parameters can reflect the characters and details of source, at the same time, it can enhance the resolution of the source. The determination of the geometry of a three-dimensional crustal structure from the gravity anomaly is a classical problem arising from many geophysical studies. In order to know the crustal structure, various inversion methods of gravity was introduced detailed by Bott[1].

Recently, many researchers try to use different inversion methods to inverse the structure of the underground and using them to deal with real data. William used a Backus-Gilbert approach to get the acceptable model “nearest” to initial estimate can be rapidly computed [2]. Methods based on a simple linear adjustment of model densities are showing in their application [3]. Other inversion methods devote to adjust the geometry of model anomalous bodies at depth [4]. The anomalous bodies are described by a set of discrete parameters [5,6]. For the fully nonlinear treatment, the exploration methods of space model can provide the best option [7], this detecting process maybe conducted randomly. Latterly, 3-D gravity inversion methods to inverse the underground structure are published Camacho et al [8,9] and Gottsmann et al [10]. Researching tectonic may determine the shape of material, the composition and distribution of crustal material and their interaction what is the researching basis of the earth dynamics problems.

The theory inversion methods of gravity data can be divided into spatial domain and frequency domain, in this paper, the 3-dimensional inversion method of frequency domain is introduced. And the theoretical model of tectonic is inversed. At the same time, several inversion profiles are given in this
article, the results show that the boundary effect will reduce after enhancing points when the distance of points is the same.

2. Theory
If the boundary of the earth’s crust and mantle (moho) is a 3-D curve \( f(x, y, z) = 0 \). When the interface is rolling, and the depth of the earth’s crust is about 30-35km, assuming that it is on feature depth \( D \) the average depth of the earth’s crust and mantle or the shallowest depth. Then it will form density distribution \( \sigma(x, y, D) \), for any unit section, we can get the follow equation:

\[
\sigma(x, y, D) = \Delta \rho \cdot \Delta H(x, y, D)
\]  

(1)

And \( \Delta \rho = \rho_1 - \rho_2 \), it is the density contrast of the earth’s crust and mantle, \( \Delta H(x, y, D) \) is height of the interface of the feature depth. The Laplace’s equation of gravity potential \( U \) will be satisfied out of the space of mass surface.

\[
\nabla^2 U = 0
\]  

(2)

In system of rectangular coordinate, the 3-D equation can be rewritten as following:

\[
\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0
\]  

(3)

So, the any anomaly of gravity potential can be written on the upwards of the mass surface:

\[
U(x, y, z) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} e^{-\sqrt{m^2 + n^2} z} \cdot \left\{ a_{mn} \cos mx \cos ny + b_{mn} \cos mx \sin ny \\
+ c_{mn} \sin mx \cos ny + d_{mn} \sin mx \sin ny \right\}
\]  

(4)

The anomaly of gravity can be written as follow equation,

\[
\Delta g(x, y, z) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} e^{-\sqrt{m^2 + n^2} z} \cdot \left\{ a_{mn} \cos mx \cos ny + b_{mn} \cos mx \sin ny \\
+ c_{mn} \sin mx \cos ny + d_{mn} \sin mx \sin ny \right\}
\]  

(5)

If the origin of coordinate is on the ground, then

\[
\Delta g(x, y, 0) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left\{ a_{mn} \cos mx \cos ny + b_{mn} \cos mx \sin ny \\
+ c_{mn} \sin mx \cos ny + d_{mn} \sin mx \sin ny \right\}
\]  

(6)

Therefore, anomaly of gravity can be rewritten above the mass surface space,

\[
\Delta g(x, y, D) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} e^{-\sqrt{m^2 + n^2} D} \cdot \left\{ a_{mn} \cos mx \cos ny + b_{mn} \cos mx \sin ny \\
+ c_{mn} \sin mx \cos ny + d_{mn} \sin mx \sin ny \right\}
\]  

(7)
In the equation, $a_{mn}$, $b_{mn}$, $c_{mn}$, $d_{mn}$ are coefficients of 2-dimensional Fourier. In addition, the quality of the gravitational field near a plane distribution can be shown as equation (8).

$$\Delta g(x, y, D) = 2\pi G \sigma(x, y, D)$$

(8)

In the equation, $G$ is gravitational constant. Then equation (8) can be rewritten,

$$\sigma(x, y, D) = \frac{\Delta g(x, y, D)}{2\pi G}$$

(9)

From equation (1) and (9), we can get the next equation:

$$\Delta H(x, y, D) = \frac{1}{\Delta \rho} \left[ \Delta g(x, y, D) / 2\pi G \right]$$

(10)

In order to improve the accuracy of inversion results, the number of interactional must be adjusted. In this paper, the modeling space is subdivided into a set of rectangular blocks of constant size to enable a linearization of the forward problem. Hence, this method is very well suited for development of objective and automatic inversion techniques.

3. Model calculation and analysis

In this paper, firstly, the inversion model must be built, then the observed Bouguer anomaly will be generated. A square column geological model is given. Its length and width are 200Km respectively. Its height is $H = 4$Km, the density contrast is $\Delta \rho = 0.43$g/cm. In computing, the grid step $16 \times 16$ and $32 \times 32$ are given for $2l = 40$Km grids. At the same time, assuming that the distance of the sampling sites is 20Km. The structure organization of the inversion is mainly in two independent parts: the first part is dedicated to the forward of Bouguer anomaly and the second part density is inverted.
Fig. 1. Inversion profile of anomaly depths are at different depths. The mesh sizes are $32 \times 32$ and $16 \times 16$, the depth is $25Km$ in Fig1 (a) and Fig1 (b), Fig1(c) and Fig1(d) are at depth $30Km$, the mesh sizes are $32 \times 32$ and $16 \times 16$ respectively.

Although Fast Fourier Transform Algorithm is fit to arbitrary measuring point, in this article, and mesh size are chosen, what is convenient for calculating of forward, at the same time, it will rest computing time. In order to research different factor, the inversion profiles of different anomaly depth are given in the paper. Fig. 1(a) and Fig.(c) are the mesh size inversion profiles of the anomaly depths at $25Km$ and $30Km$; and Fig. 1(b) and Fig.1(d) are the mesh size inversion profiles of the anomaly depths at $25Km$ and $30Km$ respectively. The unit of gravity anomaly is m gal.

From the above profiles, the results of gravity inversion are well, but when the boundary is near to 200km, the results are very different. And as the depth increasing, the results are very similar. That is to say, the absolute depth is not sensitive for gravity inversion. But, the absolute fitting error of the observed gravity may enhance. And as the number adding of measuring points, the profiles of inversion are no obvious improvement, the size of may be satisfy the accuracy of the inversion. But the results of inversion are improved at the corners.

4. Conclusion

An analytical expression for gravity of a 3-D rectangular prism with Fast Fourier Transform Algorithm is derived. Presented herein is a method to obtain the gravity signature from the given geological model. The geologically driven earth model is decomposed into a 3-D grid and the gravity signature for different depth is calculated respectively. In terms of the results of inversion, increasing the number of points and computing large-scale data can reduce the effect of boundary.

The difficult point of the 3-D is the velocity of calculation and the accuracy. The critical factor what affect the velocity is forward. In this paper, the equation (11) is used to forward, so the computation needs long time. And when the boundary is near to 200Km, the results are significant difference, therefore, how to improving the accuracy is very important for the actual data.

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