Formation of low-mass condensations in the molecular cloud cores via thermal instability

Mohsen Nejad-Asghar\textsuperscript{1,2}

\textsuperscript{1}Department of Physics, University of Mazandaran, Babolsar, Iran

\textsuperscript{2}Research Institute for Astronomy and Astrophysics of Maragha, Maragha, Iran

nejadasghar@umz.ac.ir

ABSTRACT

The low-mass condensations (LMCs) have been observed within the molecular cloud cores. In this research, we investigate the effect of isobaric thermal instability (TI) applied for forming these LMCs. For this purpose, at first we investigate the occurrence of TI in the molecular clouds. Then, for studying the significance of linear isobaric TI, we use a contracting axisymmetric cylindrical core with axial magnetic field. Consideration to cooling and heating mechanisms in the molecular clouds shows that including the heating due to ambipolar diffusion can lead to the occurrence of TI in a time-scale smaller than dynamical time-scale. Application of linear perturbation analysis shows that isobaric TI can take place in outer region of the molecular cloud cores. Furthermore, the results show that perturbations with wavelengths greater than few astronomical units are protected from destabilization property of thermal conduction, so they can grow to form LMCs. Thus, the results show that the mechanism of TI can be used to explain the formation of LMCs as the progenitors of collapsing proto-stellar entities, brown dwarfs, or proto-planets.

Subject headings: ISM: clouds – ISM: evolution – Hydrodynamics – ISM: magnetic fields – diffusion – stars: formation.

1. Introduction

The molecular gases have a hierarchical structure which extends from the scale of the cloud down to much smaller masses of unbound substructures. The terminology for structure
of molecular clouds is not fixed. The over-dense regions within giant molecular clouds are termed clumps. The massive clumps are parenting clouds for the clusters of proto-stellar cores. These dense cores are expected to undergo gravitational collapse evolving towards singular points and designated as proto-stars (e.g., Stahler and Palla 2004). Two important informations intended for the molecular cloud cores are their shapes and density-profiles. Unfortunately, the core shape cannot be easily concluded from the apparent observations of the-plane-of-sky because it is impossible to be directly de-projected. Although, some works indicate a preference in prolate cores (e.g., Ryden 1996), but there are also some studies that favour to a strong preference in oblate cores of finite thickness (e.g., Tassis 2007). But about the density-profile in the molecular cloud cores, the observations show the systematically certain prominent common feature: a close-to-constant density over the central region \( r < r_{in} \), followed by nearly a power-law decline over large distances \( r > r_{in} \) (e.g., Hung, Lai and Yan 2010). According to these observational results, here, we turn our attention to the molecular cloud cores with cylindrical shape and appropriate decreasing density-profile.

The observational surveys of the interior of the molecular clouds have been increased enormously in the last decade, thanks to the increase of resolution provided by new millimeter and sub-millimeter interferometers, and also they are due to the systematic combination of observations of dust and molecular tracers (e.g., Bergin and Tafalla 2007). In this way, many embedded condensed objects within each star-forming core have been revealed (e.g., Pirogov and Zinchenko 2008) which called low-mass condensations (LMCs). They may be converted to the star-forming gravitationally unstable proto-stellar cores, brown dwarfs, or proto-planets. Historically, we can refer to the work of Langer et al. (1995) who observed LMCs in the core D of Taurus Molecular Cloud 1 in the regime of \( 0.007 - 0.021 \) pc and \( 0.01 - 0.15 M_{\odot} \). For the recent observations, we can refer to the discovery of very low luminosity objects by the Spitzer ‘From Molecular Cores to Planet-Forming Discs’ (c2d) project (e.g. Lee et al. 2009), or the results gained by Launhardt et al. (2010) in which they found that at least two thirds of 32 isolated star-forming cores, which were studied there, are the evidence of forming multiple stars. Clearly, these observations can also be used as a witness for existence of LMCs in the parent core before conversion to the star-forming entities. Although, due to limitations in resolution and sensitivity of the available observations (Tafalla 2008), our understanding about substructures within the cores is still incomplete, but this is well established that the LMCs are ubiquitous within the molecular cloud cores.

The LMCs move in the parent core, thus, they may be suitable places for assembly the dusts and other molecules; they also may be merged with each other (e.g., Nejad-Asghar 2010). This topic is beyond the scope of this paper. Here, we want to investigate the theoretical aspects explaining the formation of LMCs within the cores. Since the molecular
cloud cores are characterized by subsonic levels of internal turbulence (e.g., Myers 1983, Goodman et al. 1998, Caselli et al. 2002), small rotational velocity gradients (e.g., Goodman et al. 1993, Caselli et al. 2002), and subsonic infall motions (e.g., Tafalla et al. 1998, Lee, Myers and Tafalla 2001), the turbulence alone cannot be responsible for the formation of LMCs. Furthermore, the associated thermal pressure in the cores dominates the turbulent component by a factor of several (e.g., Tafalla et al. 2004). As another mechanism for formation of LMCs, we can refer to the work of van Loo, Falle and Hartquist (2007) who examined the effect of MHD waves on dense cores. They found that short-wavelength waves can play an important role in the generation of LMCs within a core without breaking it. Here, we can encourage ourselves to authenticate the suggestion that the thermal instability (TI) may be a possible favorable scenario for formation of LMCs. This idea can also be supported by the probes in temperature of the molecular cores in which the observations show there may be a deviation from uniform isothermal case (e.g., Harju et al. 2008, Friesen et al. 2009).

After the pioneer work of Field (1965), there are many works on development of TI in the interstellar medium, in both numerical and analytical approaches. The main idea is that the rapid growth of TI leads to a strong density imbalance between the cold dense inhomogeneity and its low-density environment. In the molecular clouds, Gilden (1984) calculated the cooling function to show that molecular gas may be thermally unstable in environment where CO cooling dominates. The numerical calculations of Falle, Ager and Hartquist (2006) give the idea that TI may have an important role in formation of internal substructures in the molecular clouds. Fukue and Kamaya (2007) considered the effect of ion-neutral friction of two fluids on the growth of TI. They found the TI of the weakly ionized plasma in the magnetic field, even at the small length scales, could be grown up. Nejad-Asghar (2007) applied this idea for a one-dimensional self-gravitating magnetized molecular slab including the frictional heating due to ion-neutral drift. He found that the heating of ambipolar diffusion in outer regions of the slab is more significant than the average heating rates of cosmic rays and turbulent motions, thus, isobaric TI could take place in this area. Nejad-Asghar and Molteni (2008) used the two-fluid smoothed particle hydrodynamics to simulate a partially ionized one-dimensional cloud. Their gained results indicated that the TI can insist on the occurrence of density contrast at outer parts of the molecular cloud cores. Also, the two-dimensional simulations of Nejad-Asghar and Soltani (2009) confirmed this idea that heating of ambipolar diffusion in the molecular cloud cores may lead to TI and formation of LMCs.

In this paper, we try to make an extension to this idea that TI may be responsible for the formation of LMCs within the molecular cloud cores. For this purpose, we consider a cylindrical contracting cloud as a background unperturbed arrange, with a power-law drop in its density-profile. In section 2, the net cooling function in the molecular clouds is
calculated, and happening of TI is investigated too. In section 3, the linear perturbation analysis with Fourier decomposition of space, is applied on the contracting cylindrical core to attain the criterion of isobaric TI. The effect of thermal conduction on TI stabilization is also investigated in Section 3. Finally, Section 4 is devoted to a summary and conclusions.

2. The occurrence of TI

Two key parameters to examine the happening of TI are the net cooling function and its time-scale. Determination of the cooling rate for an optically thick, dusty molecular medium is a complex non-LTE radiative transfer problem. Reviewed article by Dalgarno and McCray (1972) presents some of the earlier estimates of cooling rate by excitation the rotational levels of diatomic molecules. Since at that time, numerous authors had discussed various aspects of this problem. For example, Goldsmith and Langer (1978) analyzed in detail the cooling produced by line emission from a variety of molecular and atomic species at temperatures of 10, 20, and 40K and for H$_2$ densities in the range of $10^8 < n$(H$_2$) $< 10^{12}$m$^{-3}$. Holenbach and McKee (1979) evaluated the cooling of most coolant molecules by using the escape probability approximation which is equivalent to assumption of constant source function. The more comprehensive study of radiative cooling rate and emission-line luminosity in dense molecular clouds carried out by Neufeld, Lepp and Melnick (1995). They considered the radiate cooling of fully shielded molecular astrophysical gas over a wide range of temperatures (10 $< T <$ 2500 K) and H$_2$ densities ($10^9 < n$(H$_2$) $< 10^{16}$m$^{-3}$). Goldsmith (2001) computed the cooling effects of molecular depletion from the gas phase on grain surfaces in dark clouds. He parameterized the cooling function as

$$\Lambda_{(n,T)} = \Lambda_{(n)} \left( \frac{T}{10K} \right)^{\beta_{(n)}},$$

and gave the value of parameters to different depletion runs. This equation was used by Nejad-Asghar (2007) to investigate the formation of fluctuations in a one-dimensional molecular cloud layer.

In this paper, we use the cooling function based on the work of Neufeld, Lepp and Melnick (1995) which allows us to include cooling from potentially important coolants of five molecules and two atomic species: CO, H$_2$, H$_2$O, O$_2$, HCl, C, and O. The results presented in figures 3a – 3d of Neufeld, Lepp and Melnick (1995) are convenient to do rough parametrization like the equation (1), specially for temperatures between 10K and 250K. The effects of chemistry upon the cooling rate are dramatically illustrated by the fraction of the cooling rate attributed to H$_2$O, which rises rapidly above temperature $\sim$ 300 K due to an increase in water abundance. Thus, we focus our attention to the molecular clouds with
temperature less than 200 K. For this temperature range, the values of parameters \( \Lambda(n) \) and \( \beta(n) \) are given in the Fig. [1].

As we know, the thermal process may be important if the dynamical time-scale is in excess of the cooling time-scale. We consider the contraction time-scale of a cylindrical molecular cloud as a multiple \( \eta \geq 1 \) of the free-fall time-scale,

\[
t_{ff} = \sqrt{\frac{3\pi}{32Gmn}} \approx 3.5 \times 10^4 \left( \frac{10^{12} \text{m}^{-3}}{n} \right)^{1/2} \text{ year},
\]

where the latter is written for the spherical uniform density distribution. The cooling time-scale \( t_c \equiv \frac{3kT}{2m\Lambda} \) and the free-fall time-scale (2) are shown in Fig. 2. In fast contraction \( (\eta \sim 1) \), TI is important in small densities (e.g., less than \( 10^{14} \text{m}^{-3} \) for \( T = 100 \text{K} \)), while in slow contraction \( (\eta >> 1) \) in which the cooling time-scale is much smaller than the contraction time-scale, the importance of TI as a trigger mechanism is much evident.

There are several different heating mechanisms in models of interstellar matters. Since the ultraviolet photons are mostly screened out in dense molecular clouds, heating by collisional de-excitation of \( \text{H}_2 \) molecules after radiative excitation of Lyman bands, photoemission from grains, radiative dissociation of \( \text{H}_2 \), and by chemical reactions is not important. In addition, because of the small neutral hydrogen abundance, heating due to ejection of newly formed \( \text{H}_2 \) molecules from grain surfaces is negligible. The heating due to cosmic rays with sufficient energies \( (\sim 100 \text{MeV}) \) to penetrate dense clouds is common about \( \Gamma_{CR} = 2.5 \times 10^{-8} \text{J.kg}^{-1}.\text{s}^{-1} \), with assumption of an ionization rate per \( \text{H}_2 \) molecule of \( 2 \times 10^{-17} \text{s}^{-1} \) and a mean energy gains per ionization of 19eV (e.g., Glassgold and Langer 1973). Following Black (1987) the turbulence dissipation heating rate can be estimated as

\[
\Gamma_{TR} \approx \frac{1}{7} \frac{m v_{\text{turb}}^2}{mn} \left( \frac{n}{1 \text{pc}} \right)^3 \left( \frac{1}{l} \right) \text{J.kg}^{-1}.\text{s}^{-1},
\]

where \( v_{\text{turb}} \) is the turbulent velocity and \( l \) is the eddy scale. With \( v_{\text{turb}} \sim 1 \text{ km.s}^{-1} \) and \( l \sim 1 \text{ pc} \), we obtain \( \Gamma_{TR} \sim 1.6 \times 10^{-8} \text{J.kg}^{-1}.\text{s}^{-1} \), comparable to half of the heating rate of cosmic rays. In this way, we collect the values of these two heating rates to obtain \( \sim 4.1 \times 10^{-8} \text{J.kg}^{-1}.\text{s}^{-1} \). Another important heating mechanism of self-gravitating contracting cloud is the heating produced by gravitational compression work. An estimation for this heating can be derived directly from the rate of compression work per particle, \( pd(n^{-1})/dt \),
and is given by

\[ \Gamma_{GR} \approx \frac{p}{n t_{cn}} \]

\[ \approx 3.9 \times 10^{-8} \left( \frac{1}{\eta} \right) \left( \frac{T}{10 \text{K}} \right) \left( \frac{n}{10^{12} \text{m}^{-3}} \right)^{1/2} \text{J.kg}^{-1}.\text{s}^{-1}, \]  

(4)

where we have taken \( dn/dt \approx n/t_{cn} \) appropriate for a uniform contracting spherical cloud with contraction time-scale \( t_{cn} = \eta t_{ff} \) where \( \eta > 1 \) is for slow contraction, and \( \eta \sim 1 \) is for fast one (i.e., free-fall case).

The dissipation of magnetic energy would be considered as another heating mechanism, if this energy is not simply radiated away by atoms, molecules, and grains. The major field dissipation mechanism in the dense clouds is almost certainly ambipolar diffusion, which was examined by Scalo (1977) for density dependency of magnetic field in a fragmenting molecular cloud. Padoan, Zweibel and Nordlund (2000) presented calculations of frictional heating by ion-neutral drift in three-dimensional simulations of turbulent, magnetized molecular clouds. They show that average value of ambipolar drift heating rate can be significantly larger than the average of cosmic-ray heating rate. In addition, Nejad-Asghar (2007) considered a molecular slab under the assumption of quasi-magnetohydrostatic equilibrium, concluded that ambipolar drift heating is inversely proportional to density and its value in some regions of the slab can be significantly larger than the average heating rates of cosmic rays and the dissipating turbulent motions. To gain an insight on the heating rate of ion-neutral friction, we assume that the pressure and gravitational force on the charged fluid component are insignificant compared to the Lorentz force because of low ionization fraction, thus, the drift velocity \( v_d \) is inversely proportional to the density and directly proportional to the gradient of magnetic pressure (see equation \[8\] below). Here we choose, in a general form, \( v_d \propto \kappa \rho^{-b} \)

where \( \kappa \equiv \Delta(B^2/2\mu_0)/\Delta x \) is the change of magnetic pressure in length-scale \( \Delta x \), and the value of \( b \) may be approximated near \( 3/2 \) (this value is from the assumption of ionization equilibrium with the ion density being power law of the neutral density with power \( 1/2 \)). Nejad-Asghar (2007) adopted the value of \( b \) in the range between 0.5 and 2.0 to examine the isobaric TI in the regions of a self-gravitating molecular slab. The heating due to ambipolar diffusion is

\[ \Gamma_{AD} = \frac{f_d v_d}{\rho} = \gamma_{AD} \epsilon \rho^{1/2} v_d^2, \]  

(5)

where \( f_d = \gamma_{AD} \epsilon \rho^{3/2} v_d \) is the drag force per unit volume exerted on the neutrals by ions, \( \gamma_{AD} \sim 3.5 \times 10^5 \text{m}^3.\text{kg}^{-1}.\text{s}^{-1} \) is the collision drag in molecular clouds, and we used the relation \( \rho_i = \epsilon \rho_n^{1/2} \) between ion and neutral densities in ionization equilibrium state with \( \epsilon \sim 9.5 \times 10^{-15} \text{kg}^{1/2}.\text{m}^{-3/2} \) (Shu 1992). Substituting the appropriate values for typical
molecular cloud cores, the ambipolar diffusion heating rate is as follows
\[ \Gamma_{AD} \approx 4.6 \times 10^{-8} \left( \frac{\kappa}{1 \text{nT}^2 \text{AU}^{-1}} \right)^2 \left( \frac{n}{10^{12} \text{m}^{-3}} \right)^{-2b+0.5} \text{J.kg}^{-1}\text{s}^{-1}, \]  
(6)
where the general form \( \nu_d = \kappa \rho^{-b}/\gamma_{AD} \varepsilon \) is used according to equation (3) below. The total heating and cooling rates of a magnetized slow contracting molecular cloud core with typical value \( \eta = 10 \) are shown in Fig. 3 for different values of parameters \( b, \kappa \) and temperature \( T \).

Now, we can turn our attention to the occurrence of TI in the molecular clouds. The standard criterion applicable to the interstellar gas, in the isobaric instability condition, can be written as
\[ \Omega_T - \left( \frac{\rho_0}{T_0} \right) \Omega_\rho < 0, \]  
(7)
where \( \Omega \equiv \Lambda - \Gamma \) is the net cooling function, \( \Omega_\rho \equiv (\partial\Omega/\partial\rho)_T \) and \( \Omega_T \equiv (\partial\Omega/\partial T)_\rho \) are evaluated in equilibrium state (see, e.g., the review by Balbus 1995). When the density along the net cooling curve increases so that a density fluctuation is formed, the above criterion states it will tend to be amplified because the mentioned fluctuation has a larger cooling than its heating rate, thus, the surrounding gas will compress it further. Fig. 3 suggests that without heating of ambipolar diffusion, there would not be any TI while with it, the instability criterion (7) can worthily be satisfied specially in attenuated region of a molecular cloud. Furthermore, this scenario can also be shown by the pressure-density plane which is an elegant method to describe the isobaric instability modes in thermal equilibrium. In this method, an equilibrium state is specified by the intersection of a thermal equilibrium curve and a constant pressure line. In this way, any perturbation from equilibrium state leads to split the gas in two phases, as the individual gas element transfer into related phase in correspondence with its initial sign of fluctuation (i.e., density increase or density decrease). Pressure-density diagram of thermal equilibrium and instability criterion are depicted in Fig. 4. This figure shows that including the heating rate due to ambipolar diffusion can produce a two-phase medium in outer regions (low density) of a typical molecular cloud core.

3. Linear perturbation analysis

For investigation the importance of TI in outer region of a molecular cloud core, a long axisymmetric cylindrical geometry is assumed to extend in \( z \)-direction with an axial magnetic field \( B = B_0 \hat{k} \), which directly coupled only to the charged particles.
3.1. Contracting cylindrical core

Principally, the ion velocity $v_i$ and the neutral velocity $v_n$ in the molecular clouds, should be determined by solving separate fluid equations of these species, include their coupling by collision processes. However, in the time-scale considered here (see, Fig. 2), two fluids of ion and neutral are approximately coupled together with a drift velocity given by

$$v_d = v_i - v_n \approx \frac{1}{\mu_0 \gamma AD \rho^{3/2}} (\nabla \times B) \times B,$$

(8)

which is obtained by assuming that the pressure and gravitational forces on the charged fluid component are negligible compared to the Lorentz force because of the low ionization fraction. Here, in a good approximation we choose $\rho = \rho_n + \rho_i \approx \rho_n$. In this way, we can use the basic equations as were given by Shu (1992):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} + \rho \nabla \cdot \mathbf{v} = 0,$$

(9)

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla (p + \frac{B^2}{2\mu_0}) - (\mathbf{B} \cdot \nabla) \frac{\mathbf{B}}{\mu_0} - \rho \mathbf{g} = 0,$$

(10)

$$\frac{3}{2} \frac{\partial p}{\partial t} + \frac{3}{2} \mathbf{v} \cdot \nabla p + \frac{5}{2} p \nabla \cdot \mathbf{v} + \rho \Omega - \nabla \cdot (K \nabla T) = 0,$$

(11)

$$\nabla \cdot \mathbf{g} = -4\pi G \rho,$$

(12)

$$p - \frac{R}{\mu} \rho T = 0,$$

(13)

where $K \approx 2.16 \times 10^{-2} T^{1/2} J \cdot s^{-1} K^{-1} m^{-1}$ is the thermal conduction coefficient in molecular clouds (Lang 1986), and other variables and parameters have their usual meanings. The energy equation (11) contains a source term $-\frac{5}{2} p \nabla \cdot \mathbf{v}$, which describes the work is done by expansion or contraction of the medium. Thus, we note that the self-gravitating heating term (4) must be excluded from the net cooling function $\Omega$. The time evolution of magnetic field itself may then be obtained from the approximation that it freezes only in the plasma of ions and electrons. In this way, we obtain a nonlinear diffusion equation as follows

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{B} + \mathbf{B} (\nabla \cdot \mathbf{v}) - (\mathbf{B} \cdot \nabla) \mathbf{v} = \nabla \times (v_d \times \mathbf{B}).$$

(14)

If the right-hand side of the magnetic induction equation (14) is negligible, the field is well coupled to the whole fluid.

As a basis for small-perturbation analysis, we consider an axisymmetric cylindrical background that is contracting in a quasi-hydrostatic balance between the self-gravity and the repulsive forces (i.e., thermal and magnetic pressure forces). The background quantities will
be denoted with the subscript "0". We consider a similarity solution for the bulk unperturbed fluid so that its velocity field depends linearly on the axial distance \( r \) as \( \mathbf{v}(r,t) = \frac{ds}{dt} r \), where \( s(t) \) is a non-dimensional parameter which presents axial contraction in the co-moving Lagrangian coordinate (\( \dot{s} < 0 \)). The basic equations \((8)-(14)\) lead to

\[
\rho_0(r,t) = \rho_c \left( \frac{r}{r_{in}} \right)^{-\frac{2}{3}} \left( \frac{1}{s} \right)^{\frac{4}{3}},
\]

\[
T_0(r,t) = T_c \left( \frac{1}{s} \right)^{\frac{4}{3}},
\]

\[
B_0(r,t) = B_c f \left( \frac{r}{r_{in}} \right) \left( \frac{1}{s} \right)^{\frac{4}{3}},
\]

where \( \rho_c, T_c, \) and \( B_c \) are respectively initial central density, temperature, and magnetic field at the inner radius \( r_{in} \), \( f \left( \frac{r}{r_{in}} \right) \) is only a function of radius, and \( s(t) \) follows the equation

\[
s = \left[ 1 - \frac{2}{3} \left( \frac{t}{t_0} \right) \right]^{\frac{3}{2}},
\]

where \( t_0 \) is a time-scale free parameter (\( t \leq t_0 \)). The equations \((15)-(17)\) are accurate for \( r > r_{in} \), and we assume the density is homogenous for \( r \leq r_{in} \). For details, the reader is referred to the appendix. At the end of this contraction process in which \( t \) is equal to \( t_0 \), the minimum value of \( s(t) \) takes place, thus the maximum effect of this contraction is an approximately tenfold over density in its profile.

### 3.2. Linear perturbation

For obtaining a linearized system of equations, we split each variable into unperturbed and perturbed components, the latter is indicated by subscript "1". Then, we carry out a spatial Fourier analysis with components proportional to \( \exp(ikr) \) where \( k \) is the component of perturbation wave-vector perpendicular to the magnetic field. Simplifying the linearized equations \((8)-(13)\) by repeated use of the unperturbed background equations \((15)-(18)\), we obtain

\[
\frac{d\rho_1}{dt} + (ikr + 1) \frac{\ddot{s}}{s} \rho_1 + \left( ikr - \frac{2}{3} \right) \rho_0 \frac{r}{s} \nu_1 = 0,
\]

\[
\frac{\rho_0}{dt} \left( \frac{s}{r} - g_0 + \frac{4\pi G \rho_0 r}{ikr + 1} \right) + \left( \frac{\ddot{s}}{s} \right) \rho_1 + \frac{\ddot{s}}{s} \rho_0 \nu_1 + ikp_1 + \left( \frac{ikB_0}{\mu_0} + \frac{1}{\mu_0} \frac{\partial B_0}{\partial r} \right) B_1 = 0,
\]

\[
\frac{3}{2} \frac{dp_1}{dt} + \left( \frac{3}{2} \frac{ikr}{r} + \frac{5}{2} \right) \frac{\ddot{s}}{s} p_1 + \left( \frac{5}{2} ikr - 1 \right) \rho_0 \frac{r}{s} \nu_1 + \rho_0 \Omega \rho_1 + \left( \rho_0 \Omega_T + k^2 K_0 \right) T_1 = 0,
\]
\[
\frac{dB_1}{dt} + \left[ (ikr + 1) \frac{s}{s} + ikv_d + \frac{1}{r} \frac{\partial (rv_d)}{\partial r} \right] B_1 + \left( ikB_0 + \frac{\partial B_0}{\partial r} \right) v_1 + \frac{1}{r} \frac{\partial}{\partial r} (rB_0v_d) = 0, \tag{22}
\]
\[
p_1 = \frac{R}{\mu} \rho_0 T_1 + \frac{R}{\mu} T_0 \rho_1, \tag{23}
\]

where
\[
v_{d1} = -\frac{1}{\mu_0 \gamma_{AD} \rho_0^{3/2}} \left[ \left( ikB_0 + \frac{\partial B_0}{\partial r} \right) B_1 - \frac{3}{2} \frac{B_0}{\rho_0} \frac{\partial B_0}{\partial r} \rho_1 \right]. \tag{24}
\]

The perturbed drift velocity due to the perturbed magnetic field changes the ambipolar diffusion heating rate. In fact, the net cooling function \( \Omega \) in equation (21) must contain also the ambipolar heating due to perturbed magnetic field. However, because we have expressed the ambipolar heating in equation (6) by parameters, in numerical calculations for drawing figures, we can involve this effect by the amount of parameters. Thus, for simplicity, we neglect the explicit calculations of the changes of ambipolar heating due to perturbed drift velocity.

Since the equilibrium is time dependent, the normal modes of the system are time dependent too, thus, we must apply some approximation techniques namely WKB approximation (e.g., Bora and Baruah 2008) to gain valuable insight into the nature of problem; this analysis may be a challenging task in a subsequent research. Here, we consider the isobaric TI, which is more realistic phenomena in the interstellar gases. Gathering the equations (23) and (21) with equation (19), in isobaric case \( (p_1 = 0) \), leads to an exponential growth as follows
\[
\rho_1 = \rho_{1(t=0)} \exp \left[ -\int_0^t \omega(r, t') dt' \right], \tag{25}
\]
where \( \omega(r, t') \) is a complex function with real and imaginary components given by
\[
\Re[\omega(r, t')] = \frac{s}{s} + \frac{1 + \frac{5}{2} \frac{k^2}{k_T^2} r^2 + \frac{k^2}{k_K}}{1 + \left( \frac{5}{2} \right)^2 k^2 r^2} \left( k_T - k_\rho + \frac{k^2}{k_K} \right) c_0, \tag{26}
\]
and
\[
\Im[\omega(r, t')] = \frac{k_T}{s} kr \left[ \frac{k_T}{1 + \left( \frac{5}{2} \right)^2 k^2 r^2} \right], \tag{27}
\]
respectively, where the notation of the pioneer work of Field (1965) is used as follows
\[
k_T \equiv \frac{2}{3} \frac{\mu}{R} \frac{\Omega_T}{c_0}, \quad k_\rho \equiv \frac{2}{3} \frac{\mu \rho_0}{R T_0 c_0}, \quad k_K \equiv \frac{3}{2} \frac{\mu \rho_0 c_0}{\mu K_0}, \tag{28}
\]
where \( c_0 \) is the speed of sound in the unperturbed medium. The Field’s isobaric instability criterion \( (k_T < k_\rho - k^2/k_K) \) can be clearly revived via the equation (26) by considering a stationary cloud \( (s = 0) \).
Manipulating the equation (26) leads to the isobaric TI criterion in a contracting axisymmetric cylindrical cloud core as follows

\[ I_T(r, t) - I_\rho(r, t) + k^2 r^2 I_K(r, t) + \frac{1 + \left(\frac{5}{2}\right)^2 k^2 r^2}{1 + \frac{5}{2} k^2 r^2} I_c(t) < 0, \quad (29) \]

where

\[
I_T(r, t) \equiv \frac{2 \mu}{3 R} \int_0^{t/t_0} \Omega_T d(t'/t_0),
\]

\[
I_\rho(r, t) \equiv \frac{2 \mu}{3 R} \int_0^{t/t_0} \frac{\rho_0 \Omega_\rho}{T_0} d(t'/t_0),
\]

\[
I_K(r, t) \equiv \frac{2 \mu}{3 R} \int_0^{t/t_0} \frac{K_0}{r^2 \rho_0} d(t'/t_0) = \frac{2 \mu K_c t_0}{3 R \rho c r_{in}^2} \left(\frac{r}{r_{in}}\right)^{-\frac{4}{3}} t \left(1 - \frac{1}{3} \frac{t}{t_0}\right),
\]

\[
I_c(t) \equiv \int_0^{t/t_0} \frac{\dot{s}}{s} d(t'/t_0) = \frac{3}{2} \ln \left(1 - \frac{2}{3} \frac{t}{t_0}\right), \quad (30)
\]

where the \( I_K \) and \( I_c \) are explicitly evaluated, using the background equations (15)-(18), and \( K_c \equiv 2.16 \times 10^{-2} T_c^{1/2} \text{J.s}^{-1} \text{K}^{-1} \text{m}^{-1} \) is the thermal conduction coefficient in the axis of the cylinder.

There are three notable remarks for \( I_K \) and \( I_c \) in the equation (29): (I) Since \( I_K \) has a positive value and \( I_c \) has a negative value, their effects on instability criterion are reciprocal. Several studies have demonstrated that thermal conduction can stabilize and even erase a density perturbation (e.g., Burkert and Lin 2000). This implies that in the opposite of thermal conduction, the contraction can fortify TI because the energy is transported inward by the contraction process. (II) During the growth of density perturbations in isobaric regime, the resulting temperature gradient induces the conductive heating in fluctuations. The temperature gradient is greater at the distant region of the axis because in isobaric perturbation \( (T_1 = -\frac{\rho_0}{\rho_0} \rho_1) \), \( \rho_0 \) is a decreasing function of the axial radius \( r \). Thus, the larger temperature gradient at outer regions leads to amplify the \( I_K \) for more suppression of perturbations. (III) Consideration of time dependencies of \( I_K \) and \( I_c \) shows that increasing of \( |I_c| \) is faster than \( I_K \) for \( t > 0.5 t_0 \). Thus, in general the formation of LMCs via isobaric TI may be suppressed by the thermal conduction process as long as it overcomes on the contraction effect.

Scrutiny in the isobaric TI criterion (29) requires to apply some numerical values from the typical molecular cloud cores. Here, we consider a typical core with a dimension \( r_{out} = 0.1 \text{pc} \) and central density \( n_c = 4.6 \times 10^{33} \text{m}^{-3} \), which its total mass is approximately close to the one solar mass. The central uniform region is assumed to spread out to \( r_{in} = 0.002 \text{AU} \). Thus, the density in the envelope will be \( n_{out} = 10^9 \text{m}^{-3} \), which is obtained from initial
Choosing $T_c = 15\text{K}$ and $b = 1.5$, provides a suitable situation for happening the TI as depicted in Fig. 4. The thermally unstable region of a contracting axisymmetric cylindrical core, according to criterion (29), is shown in Fig. 5 for $t = 0$ to $t = t_0$, where it was assumed that $t_0 = 1\text{Myr}$. In this figure, the effect of thermal conduction that is true in the limit of very long wavelength ($k^2 r^2 << 1$) is ignored. We can see from the shade region of Fig. 5 when the thermally unstable region of the cloud shifts to the outer parts. If we consider an inner marginal part of the core which is thermally unstable (for example $r = 10^{5.2} r_{in}$), it may be converted to a LMC via TI process. But, by the time, this part will be stable as is shown in Fig. 5, thus, this preformed LMC will be self-perpetuated.

As we know, the medium may be stabilized by allowing the thermal conduction, and its significance depends on the perturbation wavelength which is known as Field’s length (Field 1965). For this purpose, the area of unstable region in the $r - t$ plane of Fig. 5 is depicted in Fig. 6 for various values of $k^2 r^2$ (i.e., different perturbation wavelengths). According to this figure, we can realize two critical wavelengths that are $k_{c1} r \sim 10^{1.5}$ and $k_{c2} r \sim 10^{3}$. If we consider the maximum value of radius as $r \sim 10^{6.5} r_{in}$, the first critical wavelength will be $\lambda_{c1} \sim 10^{5} r_{in}$. Perturbations with wavelengths greater than the mentioned critical value, in outer region of the molecular cores are thermally unstable and may be considered as suitable places for the formation of LMCs. On the other hand, using the minimum value of the radius of unstable region, $r \sim 10^{5} r_{in}$, the second critical wavelength will be $\lambda_{c2} \sim 10^{2} r_{in}$. Perturbations with wavelengths between $\lambda_{c1}$ and $\lambda_{c2}$ are partially affected by thermal conduction while perturbations with wavelengths less than $\lambda_{c2}$ will entirely be suppressed. For $k_{c2}$ we used the maximum allowed $r$ in the unstable region and for $k_{c1}$ its minimum was used. These choices are because of obtaining the minimum value of wavelength for $\lambda_{c2}$ and the maximum value of it for $\lambda_{c1}$.

It is interesting to obtain the growth time-scale of the TI as follows

$$t_{growth} \approx \frac{\int_0^t \Re[\omega(r, t')] dt'}{t} = \frac{c_0}{t/t_0} [I_T - I_T + I_c]$$

where is written for the limiting case $k^2 r^2 << 1$. The result is approximately about 0.2Myr that is much smaller than Jeans and magnetic instability time-scales (e.g., Fiege and Pudritz 2000, Basu, Ciolek and Wurster 2009), as we also expected from the Fig. 2. Thus, the TI can justify the formation of LMCs from all perturbations greater than $\lambda_{c2}$ in outer region (low density) of the molecular cloud cores.
4. Summary and conclusions

In this paper, we attempted to represent importance of TI as a trigger process for configuration the hierarchical structure of molecular clouds, specially formation of the observed LMCs in the cloud cores. The cores are usually well thought-out as cool molecular gases in which the temperature and density are not completely uniform. Here, we considered the cooling rate of the molecular cloud (Fig. 1) and various mechanisms of heating rates. Investigation of the net cooling rate in the molecular clouds shows that the consideration of the heating due to ion-neutral drift can lead to the thermally unstable gas (Figs. 3 and 4). As we know, the significance of a physical process might be regarded by its time-scale. For this purpose, we compared the cooling time-scale and the contracting time-scale of the core, and we concluded that in fast contraction (free-fall), the TI is important in small densities, while in slow contraction, the cooling time-scale is even much shorter than the contraction time-scale (Fig. 2). Thus, the TI can be considered as a trigger mechanism to imprint the growth of inhomogeneities within the density and temperature of the molecular cloud cores.

For investigation the importance of the TI and finding where it will take place, we applied the linear perturbation analysis on a contracting axisymmetric cylindrical cloud core. Here, we turned our attention to the isobaric TI. The obtained instability criterion demonstrates that in opposite to the thermal conduction, which can suppress the linear density perturbations in the medium, the contraction can fortify it because the energy is transported inward. Furthermore, the large gradient of temperature at outer regions of the core leads to more suppression of perturbations therein via the thermal conduction process. Another remark is that the formation of condensations via isobaric TI may in general be suppressed by the thermal conduction process as long as it overcomes on the contraction effect. Applying the numerical values of a typical molecular cloud core gave us some insight about the places where instability may be arisen. Fig. 5 depicts the thermally unstable regions of a contracting axisymmetric cylindrical core in the limitation of very long wavelength (i.e., the effect of thermal conduction is ignorable). According to this figure, by the time, unstable region of the cloud shifts to the outer region, the preformed LMCs can be self-perpetuated.

The medium would be stabilized by consideration of thermal conduction which its significance depends on perturbation wavelength (Fig. 6). The thermal conduction can entirely suppress the instability of perturbations with wavelengths less than $\lambda_{c2} \sim 0.2\mathrm{AU}$, while its effect is completely ignorable at wavelengths greater than $\lambda_{c1} \sim 200\mathrm{AU}$. Thus, LMCs more massive than $m_{H_2} \pi \lambda_{c1}^3 \sim 10^{25} \mathrm{kg}$ can grow via TI without being undergone the thermal conduction, and those smaller than $m_{H_2} \pi \lambda_{c2}^3 \sim 10^{16} \mathrm{kg}$ are completely disrupted by the thermal conduction process ($\pi$ is chosen approximately $10^{10} \mathrm{m}^{-3}$ where TI occurs). Pertur-
bations with having wavelengths between $\lambda_{c1}$ and $\lambda_{c2}$ are partially affected by the thermal conduction. The relative mass which is contained within the thermally unstable region of the cylindrical core is approximately $\pi r_{\text{max}}^2 - \pi r_{\text{min}}^2 \approx 0.99$, where $r_{\text{min}} \approx 10^{5.15} r_{\text{in}}$ and $r_{\text{max}} \approx 10^{6.25} r_{\text{in}}$ are chosen from Fig. 6. Thus, since approximately 99% of the core mass is contained within the thermally unstable region, its mass is sufficient to explain the formation of LMCs. Although, the TI may justify the formation of LMCs more massive than $10^{-5} M_\odot$, however, the puzzle is highly incomplete. We might investigate the behavior and evolution of these LMCs because they move and accumulate ubiquitous dust and other molecules. Furthermore, they may collide and merge to form the larger LMCs. Finally, they may be converted to collapsing proto-stellar entities, brown dwarfs, or proto-planets.

Acknowledgments

I appreciate the careful reading and suggested improvements by Sven Van Loo, the reviewer. This work has been supported by Research Institute for Astronomy and Astrophysics of Maragha (RIAAM).

A. Background evolution

In order to solve the set of equations (9)-(14), we look for a similarity contraction using the co-moving system: $r = s(t) x$, where $x$ is the Lagrangian coordinate and $s(t)$ is the contraction parameter. In this way, the velocity field is given by $v(r,t) = \frac{ds/dt}{s} r$. From the equation of continuity (9), we have

$$\frac{\partial \rho_0}{\partial t} + \frac{s}{s} \frac{\partial \rho_0}{\partial r} + 2 \frac{s}{s} \rho_0 = 0. \quad (A1)$$

Using the separation of variables to solve the equation (A1), we obtain

$$\rho_0(r,t) = \rho_c \left( \frac{r}{r_{\text{in}}} \right)^{a-2} \left[ \frac{1}{s(t)} \right]^a, \quad (A2)$$

where $\rho_c$ is the initial central density and the constraint $0 < a < 2$ fulfills the decreasing of density versus the cylindrical radius and its increasing with time. The equation (A2) is accurate for $r > r_{\text{in}}$, and we assume the density is homogenous (i.e. $a = 2$) for $r \leq r_{\text{in}}$.

Substituting the equation (A2) into the poisson equation

$$\frac{1}{r} \frac{\partial}{\partial r} (r g_0) = -4\pi G \rho_0, \quad (A3)$$
we obtain, for \( r > r_{in} \),
\[
g_0(r,t) = -g_c \left( \frac{r}{r_{in}} \right)^{a-1} \left[ \frac{1}{s(t)} \right]^a,
\]
(A4)

where \( g_c \equiv 4\pi G\rho_c r_{in}/a \) is the initial gravitational acceleration at \( r_{in} \) for a long homogenous cylinder, and we set \( a = 2 \) for \( r \leq r_{in} \), as before. We assume that the temperature of the background is homogenous and it depends only on the time so that \( p(r,t) = \frac{2}{3}\rho(r,t)T(t) \).

From the energy equation (11) with the On-The-Spot equilibrium in which the energy-gain is locally equal to the energy-loss at that spot (i.e., \( \Omega_0 \approx 0 \)), we have
\[
\frac{3}{2} \frac{1}{T_0} \frac{dT_0}{dt} + \frac{3}{2} \frac{1}{\rho_0} \frac{\partial \rho_0}{\partial t} + \frac{3}{2} \frac{\dot{s}}{s} r \frac{\partial \rho_0}{\partial r} + 5 \frac{\dot{s}}{s} = 0,
\]
(A5)

which gives
\[
T_0(r,t) = T_c \left[ \frac{1}{s(t)} \right]^{\frac{4}{3}},
\]
(A6)

where \( T_c \) is the initial homogenous temperature.

Separating the time-variable from the equations of momentum (10) and magnetic induction (14) by desiring the magnetic field as
\[
B_0(r,t) = B_c f \left( \frac{r}{r_{in}} \right) \left[ \frac{1}{s(t)} \right]^b,
\]
(A7)

where \( B_c \) is the initial central magnetic field strength and \( f(\frac{r}{r_{in}}) \) is a function of radius, we obtain the constraint \( a = b = \frac{4}{3} \) and the contraction parameter as
\[
s(t) = \left[ 1 - \frac{2}{3} \left( \frac{t}{t_0} \right) \right]^{\frac{2}{3}},
\]
(A8)

where \( t_0 \) is a time-scale free parameter \((0 \leq t \leq t_0)\). It is worth notice that the equations (A2), (A6), and (A7) show explicitly that the ratio of magnetic pressure to gas pressure will be constant in duration of contraction. Clearly, this special case is emerged from our linear similarity assumption of the velocity field in the contracting cylinder. The separated \( r \)-variable of the momentum equation (10) reduces to
\[
f_1(r_{in}) = \left\{ f_0^2 - \frac{2\mu_0 3\rho_c r_{in}^2}{B_c^2 4t_0^2} \left( \frac{r}{r_{in}} \right)^{\frac{4}{3}} \left[ \frac{1}{3} \left( \frac{r}{r_{in}} \right)^{\frac{4}{3}} + \frac{4RTc t_0^2}{3\mu r_{in}^2} \left( \frac{r}{r_{in}} \right)^{-\frac{4}{3}} + \frac{2g_c t_0^2}{r_{in}} \right] \right\}^{\frac{1}{2}}
\]
(A9)

where the constant \( f_0 \) is chosen so that the \( f(\frac{r}{r_{in}}) \) be a real function. The equation (A9) may be inserted into the separated \( r \)-variable of the magnetic induction equation (14) to obtain \( \mathbf{v}_d \) via integral
\[
\mathbf{v}_d = -\dot{r} \left[ \frac{1}{s(t)} \right]^{\frac{2}{3}} \frac{1}{t_0 r f} \int \left( \frac{2}{3} f + r \frac{df}{dr} \right) r dr,
\]
(A10)

instead the approximate relation (8).
REFERENCES

Balbus, S.A., 1995, ASPC, 80, 328
Basu, S., Ciolek, G.E., Wurster, J., 2009, NewA, 14, 221
Bergin, E.A., Tafalla, M., 2007, ARA&A, 45, 339
Black, J.H., 1987, in Interstellar Processes, ed. Hollenbach, D.J., Thronsen, H.A., D. Reidel Publishing Company, p. 731
Bora, M.P., Baruah, M.B., 2008, PhPl, 15, 3702
Burkert, A., Lin, D.N.C., 2000, ApJ, 537, 270
Caselli, P., Benson, P.J., Myers, P.C., Tafalla, M., 2002, ApJ, 572, 238
Dalgarno, A., McCray, R.A., 1972, ARA&A, 10, 375
Falle, S.A.E.G., Ager, M., Hartquist, T.W., 2006, ASPC, 359, 137
Fiege, J.D., Pudritz, R.E., 2000, MNRAS, 311, 105
Field, G.B., 1965, ApJ, 142, 531
Friesen, R.K., Di Francesco, J., Shirley, Y.L., Myers, P.C., 2009, ApJ, 697, 1457
Fukue, T., Kamaya, H., 2007, ApJ, 669, 363
Gilden, D.L., 1984, ApJ, 283, 679
Glassgold, A.E., Langer, W.D., 1973, ApJ, 179, 147
Goldsmith, P.F., Langer, W.D., 1978, ApJ, 222, 881
Goldsmith, P.F, 2001, ApJ, 557, 736
Goodman, A.A., Benson, P.J., Fuller, G.A., Myers, P.C., 1993, ApJ, 406, 528
Goodman, A.A., Barranco, J.A., Wilner, D.J., Heyer, M.H., 1998, ApJ, 504, 223
Harju, J., Juvela, M., Schlemmer, S., Haikala, L.K., Lehtinen, K., Mattila, K., 2008, A&A, 482, 535
Hollenbach, D., McKee, C.F., 1979, ApJS, 41, 555
Hung, C.L., Lai, S.P., Yan, C.H., 2010, ApJ, 710, 207
Lang, K.R., 1986, *Astrophysical Formula*, 2nd edn, Springer-Verlag, Berlin, p. 320

Langer, W.D., Velusamy, T., Kuiper, T.B.H., Levin, S., Olsen, E., Migenes, V., 1995, ApJ, 453, 293

Launhardt, R., Nutter, D., Ward-Thompson, D., Bourke, T.L., Henning, Th., Khanzadyan, T., Schmalzl, M., Wolf, S., Zylka, R., 2010, ApJS, 188, 139

Lee, C.W., Myers, P.C., Tafalla, M., 2001, ApJS, 136, 703

Lee, C.W, Bourke, T.L., Myers, P.C., Dunham, M., Evans, N., Lee, Y., Huard, T., Wu, J., Gutermuth, R., Kim, M., Kang, H.W., 2009, ApJ, 693, 1290

Myers, P.C., 1983, ApJ, 270, 105

Nejad-Asghar, M., 2007, MNRAS, 379, 222

Nejad-Asghar, M., 2010, MNRAS, 406, 1253

Nejad-Asghar, M., Molteni, D., 2008, Ap&SS, 317, 153

Nejad-Asghar, M., Soltani, J., 2009, SerAJ, 179, 61

Neufeld, D.A., Lepp, S., Melnick, G.J., 1995, ApJS, 100, 132

Padoan, P., Zweibel, E., Nordlund, Å., 2000, ApJ, 540, 332

Pirogov, L.E., Zinchenko, I.I., 2008, ARep, 52, 963

Ryden, B.S., 1996, ApJ, 471, 822

Scalo, J.M., 1977, ApJ, 213, 705

Shu, F.H., 1992, *The Physics of Astrophysics: Gas Dynamics*, University Science Books, p. 360

Stahler, S.W., Palla, F., 2004, *The Formation of Stars*, WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

Tafalla, M., Mardones, D., Myers, P.C., Caselli, P., Bachiller, R., Benson, P.J., 1998, ApJ, 504, 900

Tafalla, M., Myers, P.C., Caselli, P., Walmsley, C.M., 2004, A&A, 416, 191

Tafalla, M., 2008, Ap&SS, 313, 123
Tassis, K., 2007, MNRAS, 379, 50

van Loo, S., Falle, S.A.E.G., Hartquist, T.W., 2007, MNRAS, 376, 779
Fig. 1.— The parameters of total cooling rate in the form of $\Lambda_{(n,T)} = \Lambda_{(n)}(T/10K)^{\beta_{(n)}}$ for molecular clouds with temperatures between $10 - 200$ K.
Fig. 2.— The free-fall (dash) and cooling (solid) time-scales of a spherical molecular cloud core for three values of temperatures.
Fig. 3.— The total cooling (solid) and heating (dash) rates by choosing $\eta = 10$ and $\kappa = 0.001$ for $b = 1.5$ and 2.0. Without heating of ambipolar diffusion (i.e., $\kappa = 0$), there would not be any TI, while considering this heating can worthily satisfy the Field’s instability criterion (7) in attenuated region of a molecular cloud.
Fig. 4.— The Field’s instability criterion (top panel) and the pressure-density diagram of thermal equilibrium (bottom panel) for two values of parameter $b$ with $\kappa = 0.001$, and without ambipolar diffusion heating (i.e., $\kappa = 0$). The pressure $p_0$ is evaluated at density $10^{12} \text{m}^{-3}$ and temperature 10K. Occurrence of two-phase medium (constant pressure line) in the thermally unstable region is exhibited in the bottom panel as its instability criterion is fulfilled in the top panel.
Fig. 5.— The thermally unstable region (shade) of contracting axisymmetric molecular cloud core including the ambipolar diffusion with the parameter $b = 1.5$, central density $n_c = 4.6 \times 10^{13} m^{-3}$, $r_{in} = 0.002 au$, and $t_0 = 1$ Myr. The $k^2 r^2$ is assumed to be small so that the effect of thermal conduction in instability criterion (29) is ignorable.
Fig. 6.— Area of the unstable region in the $r-t$ plane of Fig. 5 versus $k^2 r^2$. The thermal conduction can entirely suppress the TI for a perturbation with wavelength less than $\lambda_{c_2}$, while its effect is completely ignorable at wavelengths greater than $\lambda_{c_1}$.