Data-Driven Emergency Frequency Control for Multi-Infeed Hybrid AC-DC System

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Abstract—With the continuous development of large-scale complex hybrid AC-DC grids, the fast adjustability of HVDC systems is required by the grid to provide frequency regulation services. This article develops a fully data-driven linear quadratic regulator (LQR) for the HVDC to provide temporal frequency support. The main technical challenge is the complexity and the nonlinearity of multi-infeed hybrid AC-DC (MIDC) systems dynamics that make the LQR intractable. Based on Koopman operator (KO) theory, a Koopman eigenpairs construction method is developed to fit a global linear dynamic model of MIDC systems. Once globally linear representation of uncontrolled system dynamics is obtained offline, the control term is constituted by the gradient of the identified eigenfunctions and the control matrix $B$. In case that $B$ is unknown, we propose a method to identify it based on the verified Koopman eigenfunctions. The active power reference is optimized online for LCC-HVDC in a moving horizon fashion to provide frequency support, with only locally measurable frequency and transmission power. The robustness of the proposed control method against approximation errors of the linear representation in eigenfunction coordinates is analyzed. Simulation results show the effectiveness, robustness and adaptability of the proposed emergency control strategy.

Index Terms—Koopman theory, LCC-HVDC system, multi-infeed hybrid AC-DC system, optimal emergency frequency control.

I. INTRODUCTION

A. Motivation

The technical and economic advantages of HVDC transmission technologies have promoted the development of multi-infeed hybrid AC-DC (MIDC) systems [1], in which multiple line-commutated-converter-based HVDC (LCC-HVDC) systems are connected to one AC system. In recent years, the consequent asynchronous interconnected regional power grids, complicated system dynamics and possible emergency faults of MIDC systems pose serious threats to the frequency stability of the system [2]. Frequency stability issues are caused by power imbalance. To deal with the considerable power imbalance in MIDC systems, an emergency frequency control (EFC) strategy is indispensable. Typical faults that cause frequency stability issues in the power grid usually involve sudden large active power deficits, such as generator tripping or load surges. Apart from generator tripping or load shedding operations, effective EFC strategies could be designed by utilizing the fast adjustability of HVDC systems, which have the potential to improve the system frequency stability [3], [4].

The conventional second defense line approach, which prepares reliable solutions in advance, is deployed to guarantee system stability in rare events of severe faults. However, with increasing penetration of renewable energy, the use of time-domain simulations struggles to cope with complex and varying operating conditions in the presence of uncertainty, resulting in significant risk of strategy mismatch. Data-driven models allow for fast model updates with system responses, to a certain extent addressing the challenges of varying operating conditions and resolving difficulties in traditional modeling from a mechanistic standpoint.

Data-driven control can be implemented as a complementary approach to the second defense line. After the emergency control measures of the second defense line are activated, indicators indicating stability are calculated, which then determine whether data-driven methods for emergency control should be initiated.

Considering the EFC of hybrid AC-DC systems, emergency DC power support (EDCPS) is an effective approach. In this article, to design a decentralized approach for EFC with LCC-HVDC systems participating, a fully data-driven decentralized EFC strategy is proposed to regulate DC power reference. The implementation of this approach relies only on measurements, making it suitable for complicated MIDC systems.

B. Literature Review

Numerous model-based EFC strategies have been developed for hybrid AC-DC systems. Ref. [2] proposed a decentralized EFC strategy based on coordinated droop for MIDC systems. It designed the optimal droop for power allocation based on state model of the system. By studying the overload capacity based on transient IGBT thermal models, Ref. [5] investigated a frequency-power droop controller and a maximum power release controller in modular multilevel converter (MMC)-based VSC HVDC system. Ref. [6] developed a continuous under-frequency load shedding scheme and improved the scheme by analyzing an extended system frequency response (SFR) model including frequency threshold and time delay. A centralized
response-based AC-DC coordinated control strategy was proposed in Ref. [7] that combined the EDCPS strategy and load shedding operations. However, there are two main barriers hindering the application of these model-based methods. First, it is difficult to maintain the accurate models of MIDC systems. Second, the dynamic procedure of emergency faults of MIDC systems features strong nonlinearity, and the solution of a nonlinear optimization problem is not off-the-shelf.

Consequently, data-driven EFC based on measurements is showing great potential. Among the promising advances in theory and numerical approximation in data-driven control, Koopman spectral theory [8], [9] has emerged as a dominant perspective over the past decade. In Koopman spectral theory, nonlinear dynamics are represented in terms of a Koopman operator, which is an infinite dimensional linear operator acting on the space of all possible measurement functions of the system. Finding a coordinate system as finite-dimensional approximations of the Koopman operator is a one-time upfront cost for the use of highly efficient linear optimal control tools [10].

Dynamic mode decomposition (DMD) and its variants are one of the workhorse algorithms [11], [12], [13] to approximate the Koopman operator. By using the dynamic mode decomposition with control (DMDc) method, Ref. [14] designed a wide area damping controller using discrete linear quadratic regulator (DLQR) to enhance the overall damping of low-frequency power system oscillations. However, DMDc estimate system dynamics with linear observables, which fail to capture the nonlinear transients of the system [15]. Extended DMD (EDMD) [16], augmented with nonlinear functions of the measurements was recently used for model predictive control with promising results. Based on Koopman model predictive control (KMPC) in Ref. [17], a EDMD based stabilization controller was proposed in Ref. [18] for power grid transient stability. For other variants of DMD, Ref. [19] constructed a frequency predictor for the wind farm by a specialized DMD methods with specially designed Koopman observables. Powered by representation capabilities of the neural networks, Ref. [20] approximated the Koopman operator with the deep neural network and designed a energy storage unit controller to enhance transient stability. To real-ize distributed control with partial measurements, Ref. [21] designed Koopman observables in the form of time-delayed embeddings to damp frequency oscillations.

Despite the many applications of the Koopman operator in control, challenges and issues remain. For common DMD algorithms and their variants, there is no guarantee that the nonlinear functions of the measurements found will form a closed subspace under the Koopman operator [15]. To address these issues, methods such as Koopman reduced-order nonlinear identification and control (KRONIC) [15], [22] and Koopman canonical transform (KCT) [23] have been introduced to directly identify eigenpairs. However, when applied to frequency stability control in MIDC systems, these algorithms can be computationally burdensome under specific conditions. Moreover, the number of verified eigenpairs obtained is relatively small (this is proved in case studies in Section IV-B 2)). Therefore, an improved Koopman eigenpairs approximation algorithm needs to be proposed.

Secondly, in traditional power system frequency control problems, the System Frequency Response (SFR) model [24] is commonly used as the linearized frequency dynamics of power systems. However, solely relying on data-driven techniques to identify the parameters of the SFR makes strong assumptions and simplifications about the system’s dynamics, leading to limitations in accurately capturing the true dynamics of the system. Therefore, a method needs to be proposed to achieve the global precise linearization of the highly nonlinear dynamics of the power system, facilitating the use of mature linear optimizers.

Finally, there are gaps in understanding the impact of representation errors of Koopman eigenpairs on control effects, and therefore, an analysis of the impact of representation errors of Koopman eigenpairs on the closed-loop dynamics of the system is necessary.

C. Contribution

In this article, we develop an Ensembled-Koopman-Emergency-Frequency-Control (EKFC) strategy, which optimizes the active power reference for each LCC-HVDC in a moving horizon fashion to provide emergency frequency support. To deal with strong nonlinear system dynamics, the proposed strategy finds linear embeddings of nonlinear MIDC system dynamics based on the Koopman theory to facilitate the use of the mature optimizer LQR. EKFC is purely data-driven because the globally linear representation of system dynamics are generated directly from history data. Moreover, it requires only the local frequency and the DC transmission power measurements as inputs, making it possible for distributed implementation.

In summary, the contributions of this article are as follows:

1) To approximate Koopman operator for frequency dynamics in MIDC systems, a Koopman eigenpairs construction method is developed. Physical knowledge of MIDC systems and a library bagging technique are introduced to power the construction method. Thus, the nonlinear MIDC system dynamics are reformulated with global accuracy in Koopman eigenfunction coordinates.

2) A fully data-driven dynamic optimal control method, named as EKFC, for multi-infeed hybrid AC-DC system frequency support is proposed. By combining the global linear dynamic model of frequency dynamics in MIDC systems, a fully data-driven LQR is designed.

3) The robustness of EKFC against approximation errors of the linear representation in eigenfunction coordinates is analyzed. Specifically, we provide a sufficient condition that guarantees the stability of MIDC systems with EKFC when there are Koopman eigenpairs approximation errors. Furthermore, the error bound of the closed-loop dynamics with consideration of the approximation errors is estimated.

The rest of this article is organized as follows. Section II proposes the EKFC strategy for MIDC systems. Section III examines the effect of an error in the representation of Koopman eigenpairs and provides a sufficient condition for the stability of MIDC systems with EKFC. In Section IV, an MIDC system...
case is presented and the effectiveness of the proposed control strategy is verified. Section V provides the conclusion.

II. EMERGENCY FREQUENCY CONTROLLER DESIGN

In this section, the EKFC strategy for MIDC systems is proposed. Firstly, by introducing a library bagging technique, a data-driven modelling method is developed to reformulate nonlinear frequency dynamics in Koopman eigenfunction coordinates. Secondly, control strategies are formulated directly in the eigenfunction coordinates. In case that the control matrix is unknown, we propose to identify it from data by combing the identified Koopman eigenpairs. To realize distributed control, we further select a special set of eigenpairs and revise them to adapt to partial measurements.

A. Identifying Koopman Eigenfunctions for Frequency Dynamics in MIDC Systems

In the following we formulate a framework to identify Koopman eigenpairs for frequency dynamics in MIDC systems directly, which unifies and extends innovations of the KRONIC algorithm by leveraging prior knowledge of power system dynamics and the idea of ensemble learning. For a better understanding of identifying Koopman eigenfunctions for frequency dynamics in MIDC systems, Koopman Operator theory and the related KRONIC algorithm are briefly overviewed in Appendix A-A and Appendix A-B.

1) Library Construction: Selecting a proper library $\Theta$ is fundamental for identifying underlying eigenfunctions, while the wrong library functions can obscure the simplest model [25]. However, for MIDC systems, selecting the best library functions is an open problem.

Our strategy is to start with knowledge of the dynamics of MIDC systems, and to increase the complexity of the library by including more possible terms to compensate for inadequacies in modelling of system dynamics.

A general form of MIDC systems considered in this article are the same as that in Ref. [2]. Considering system dynamics given in Appendix A-C, nonlinearities are mainly introduced by sinusoidal terms. Hence, trigonometric terms of $x$ are included in $\Theta$ as basis functions to capture the intrinsic nonlinearities of the MIDC system. Moreover, trigonometric transform of the subtraction between any two angles $\delta_i - \delta_j$ ($i, j \in N_D \cup N_B$) are also included.

Furthermore, to compensate for nonlinearities ignored in state modelling, such as deadzone setting, more possible terms should be included in $\Theta$. Here, polynomials are considered, since they represent Taylor series approximations for a broad class of smooth functions.

2) Library Subsampling and Ensemble Learning for Eigenpairs: While KRONIC has been demonstrated on a number of examples, it faces the following problems when applied to frequency dynamics of MIDC systems.

1) To make sure that the Koopman eigenfunction can be well approximated, $\Theta$ is often chosen large enough. However, in each iteration of $\Lambda$ and $\varphi(x)$, a least square solution of $\Lambda$ should be calculated, of which the calculation complexity is $\mathcal{O}(mL^2)$. Therefore, a large $\Theta$ is computational unfriendly.

2) Many eigenfunctions are spurious even when $\Lambda$ converges, i.e., these eigenfunctions do not behave linearly as predicted by their corresponding eigenvalues. Therefore, verified eigenpairs should be further selected from the $S$ identified eigenpairs. A verified Koopman eigenpair $(\lambda_s, \varphi_s)$ is obtained when the evolution of the eigenfunction $\varphi_s$ on a trajectory $X$ corresponds to the linear prediction using the eigenvalue $\lambda_s$, i.e., $e^{\lambda_s t} \varphi_s(x_1)$. Denote the number of verified eigenpairs as $S'$. Although the number of identified eigenpairs could be very large, the number of verified eigenpairs $S'$ may still be small. A small set of verified eigenpairs with $S' < N$ is less likely to model the high dimensional nonlinear dynamics (this will be demonstrated in Section IV).

To alleviate the above problems, a library bagging method is leveraged in our Koopman eigenpairs construction method. Given the set of library of basis functions, it is possible to sample them to produce several different subsets $\Theta_d (d = 1, 2, \ldots, D)$ and then apply identification of eigenpairs for each subset.

After acquiring $A_d = \text{diag}(\lambda_d^1, \lambda_d^2, \ldots, \lambda_d^{|\Theta_d|})$ and $\varphi_d(x) = (\varphi_d^1, \varphi_d^2, \ldots, \varphi_d^{|\Theta_d|})^T$ for each sub-library $\Theta_d$, where $|\Theta_d|$ represents the size of $\Theta_d$, the ensemble candidate set of eigenvalues and eigenfunctions are formed by $\Lambda = \text{diag}(\Lambda_1, \Lambda_2, \ldots, \Lambda_D)$ and $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_D)$.

The prediction error defined in (1) is computed on a tested trajectory $X^#(t)$ to distinguish accurate eigenpairs.

$$Er = \left| \varphi(x^#(t)) - e^{\lambda t} \varphi(x^#(0)) \right| / |\varphi(x^#(t))|$$

Subsequently, identified eigenpairs can be ranked according to the error $Er$. All eigenpairs with the error below a threshold may then be used to construct the dynamic model in eigenfunction coordinates.

Remark 1: Various random subsampling approaches can be used to produce sub-libraries. In our subsampling method, we first classify basis functions in the library by five categories, namely polynomials, sinusoids and cosinusoids terms of $x$, sinusoids and cosinoids of $\{ \delta_i - \delta_j | i, j \in N \}$. Polynomials are included in all subsets. For other four categories, each of them has a 50% probability to be sampled out to form the subset in each sampling. Then we have $2L^{-1}$ subsets in total. The training of eigenpairs on different subsets is in parallel.

Remark 2: Due to the different basis functions included, the acquired eigenfunctions are potential to be quite different. Specifically, introducing the library bagging technique is to execute the KRONIC framework with different initial points and search directions. Therefore, the optimization problem given in (49) is potential to converge to different optimum solutions with different sub-libraries, leading to an increase in the number of accurately identified eigenpairs.

The advantages to introduce library bagging technique are as follows.

1) Smaller libraries can drastically speed up model identification, as the complexity of algorithm for each subset drops to
\(O(m|\Theta|d^2)\). Library bagging can therefore help counteract the increasing computational cost of solving multiple regression problems in the ensemble.

2) A large set of verified eigenpairs with \(S' < N\) is more likely to model the high dimensional nonlinear dynamics (this will be demonstrated in Section IV).

Up to this point, the Koopman eigenpairs construction method for frequency dynamics in MIDC systems is developed.

**B. Koopman-Operator-Based Emergency Frequency Control Strategy**

In this subsection, we consider a control-affine system as

\[
d\frac{dt}{dt} x(t) = f(x) + Bu,
\]

where \(u \in \mathbb{R}^q\) is the multi-channel control input vector with \(u_i (i \in \mathcal{N}_D)\) as entries, \(B \in \mathbb{R}^{n \times q}\) is the control matrix.

In case that the control matrix \(B\) is unknown, we first propose a method to identify it. Next, we derive how the control input affects the dynamics of these eigenfunction coordinates. Followed by this, the optimal control problem is formulated in these coordinates and a corresponding Koopman eigenfunctions feedback controller is developed. The optimal control problem yields a nonlinear control law in the original state variables. Finally, the control law is revised to adapt to local measurements.

1) **Discovering Control Matrix From Data:** We use \(\Lambda_E\) and \(\varphi_E\) to denote verified eigenvalues and eigenfunctions with prediction errors below \(E\). According to Ref. [10], [22], with the approximated Koopman operator for an autonomous system, the control terms in (2) affect the dynamics of Koopman eigenfunctions as (3).

\[
d\frac{dt}{dt} \varphi_E(x) = \Lambda_E \varphi_E(x) + \nabla \varphi_E(x) \cdot Bu
\]

with \(M(x) := \nabla \varphi_E(x) \cdot B\).

In DC power reference regulation problem for EFC, even if \(B\) can be expressed explicitly with the diagonal element corresponding to \(p_i^{dc}\) as \(1/T_i^P\), \(B\) may still be unknown since \(T_i^P\) of LCC-HVDC \(i\) is hard to obtain due to dependency on operating conditions and parameter uncertainty [26]. So it is of interest to discover it from data. Based on the identification of Koopman eigenpairs in Section II-A, \(B\) can be estimated from (4) with sampled pairs \(\{x_u^{m}, u_m\}_{m=1}^{M}\) where \(u_m\) are random control inputs which can be zero-mean white noise signals, e.g. a truncated Gaussian distribution [27]. By rearranging (3) in terms of the library of basis functions, the control matrix \(B\) can be calculated by

\[
\begin{bmatrix}
\n(\nabla \Theta (x_{1}^T) \otimes u_1^T) \\
(\nabla \Theta (x_{2}^T) \otimes u_2^T) \\
\vdots \\
(\nabla \Theta (x_{M}^T) \otimes u_M^T) \\
\n\end{bmatrix}
\n= 
\begin{bmatrix}
\n\n\Theta (x_{1}^T) \\
\n\Theta (x_{2}^T) \\
\vdots \\
\n\Theta (x_{M}^T) \\
\n\end{bmatrix} \\
\n- 
\begin{bmatrix}
\n\Lambda \Theta (x_{1}^T) \\
\n\Lambda \Theta (x_{2}^T) \\
\vdots \\
\n\Lambda \Theta (x_{M}^T) \\
\n\end{bmatrix}
\]

\[
\begin{bmatrix}
\n\otimes \Theta (x_u^{m}) \\
\n\otimes \Theta (x_u^{m}) \\
\vdots \\
\n\otimes \Theta (x_u^{m}) \\
\n\end{bmatrix}
\end{bmatrix}
\]

with \(b = [b_1, \ldots, b_n]^T\) where \(b_m\) represents the \(m\)th row of \(B\) and \(\otimes\) is the Kronecker product. Here, the subscript \(E\) is omitted for \(\Lambda\) and \(\Xi\), where \(\Lambda_E\) is defined in (3) and \(\Xi_E\) represents the coefficient matrix for \(\varphi_E\). While \(\Xi\) and \(\Lambda\) have been discovered in Section II-A, \(\Theta\) and \(\mathcal{T}\) have been evaluated on the sampled pairs \(\{x_u^{m}, u_m\}_{m=1}^{M}\). Therefore, a least-squares solution of \(b\) can be calculated by (4).

2) **Formulation of the Optimal Control Problem:** We now design the LQR controller based on verified Koopman eigenfunctions and the identified control matrix. The control objectives is formulated as a quadratic function of the system states (in the Koopman eigenfunctions space) and control inputs. The control objectives maintain system frequency near the expected frequency, i.e., 50 Hz and optimize emergency power adjustment within an acceptable range for LCC-HVDC. The system dynamic model in Koopman eigenfunctions coordinates are constraints of the optimal control problem to facilitate the use of LQR.

For brevity, the subscript \(E\) are omitted later in this section. The control objective is a quadratic cost functional:

\[
J(\varphi, u) = \int_0^\infty \varphi^T(x)Q\varphi(x) + u^T Ru \, dt.
\]

where the structure of \(Q\) is chosen such that it only minimizes the norm of frequency. If \(x = \{\delta_h, \omega_i, p_i^{dc} | h \in \mathcal{N}_G \cup \mathcal{N}_D, i \in \mathcal{N}_D\}\) are locally measurable when regulating power reference for LCC-HVDC \(i\), EFK can be formulated as an optimization problem with (5) as the control objective and (3) as constraints.

Note that the full access to \(x = \{\delta_h, \omega_i, p_i^{dc} | h \in \mathcal{N}_G \cup \mathcal{N}_D, i \in \mathcal{N}_D\}\) requires wide area measurements, which is usually not an option due to the cost prohibitive communication infrastructure requirements. Therefore, it’s realistic to design distributed control with partial measurements, in which only system frequency \(\omega_i\) and \(p_i^{dc} (i \in \mathcal{N}_D)\) are known. Define \(X'_i = \{x'_i | x'_i = (\omega_i, p_i^{dc}), s \in \mathcal{N}_D\}\) for LCC-HVDC \(i\). A specific set of eigenfunctions for LCC-HVDC \(i\) can be selected, in which only eigenfunctions explicitly expressed in terms of \(X'_i\) are included. We use \(\Lambda_i\) and \(\varphi_i\) to denote the specific set of eigenvalues and the corresponding set of eigenfunctions for LCC-HVDC \(i\). We make the following assumption.

**Assumption 1:** The frequency dynamics in any node or generator are the same. Namely, spatio-temporal distribution characteristics of frequency dynamics can be neglected.

Based on the above assumption, if only \(\omega_i\) and \(p_i^{dc}\) is measurable at LCC-HVDC \(i\), \(\omega_i\) \((s \in \mathcal{N}_D)\) in \(\varphi_i\) take the value of \(\omega_i\) for the optimal control strategy calculation for LCC-HVDC \(i\). Even though **Assumption 1** may weaken the control effect of EFK, the robustness of EFK and the upper bound for error in the eigenvalues of controlled system dynamics, which will be discussed in the next section, may alleviate this problem.

Up to this point, EFK of LCC-HVDC \(i\) to the regulate DC power reference can be expressed as

\[
\min \int_0^\infty (\varphi_i(X'_i) - \varphi_i,ref)^T Q_i(\varphi_i(X'_i) - \varphi_i,ref) \\
+ u_i^T Ru_i dt
\]

s.t. \[
\begin{bmatrix}
\n\Lambda_i \varphi_i(X'_i) + M_i(X'_i)u_i \\
\nM_i(X'_i) = \nabla \varphi_i(X'_i) \cdot B_i
\n\end{bmatrix}
\]

The frequency dynamics in any node or generator are the same. Namely, spatio-temporal distribution characteristics of frequency dynamics can be neglected.
where \( \varphi_{i,\text{ref}} \) is the reference of \( \varphi \) when the frequency reaches its nominal value, \( Q_i \) is a diagonal weight matrix for koopman eigenfunctions. It has non-zero diagonal entries when the corresponding eigenfunction only explicitly expressed in terms of frequencies, \( \mathbf{B}_i \) is the column in \( \mathbf{B} \) corresponding to LCC-HVDC \( i \). The optimum of (6) can be solved by a state-dependent Ricatti equation (SDRE) given as

\[
\mathbf{u}_i^* = -\mathbf{R}^{-1} \mathbf{M}_i^T(\mathbf{X}_i') \mathbf{H} \varphi_i(\mathbf{X}_i'),
\]

where \( \mathbf{u}_i \) is DC power reference regulation amount when LCC-HVDC provides the frequency support for the MIDC system, \( \mathbf{H} \) satisfies

\[
\mathbf{Q} + \mathbf{H} \Lambda_i + \Lambda_i \mathbf{H} - \mathbf{H} \mathbf{M}_i^T(\mathbf{X}_i') \mathbf{R}^{-1} \mathbf{M}_i^T(\mathbf{X}_i') \mathbf{H} = 0.
\]

The control input can be determined by solving (7) and (8) online every \( \Delta t_2 \) over which the applied control is kept constant. The optimum of (8) can be solved by a state-dependent Ricatti equation (SDRE) given as

\[
\mathbf{u}_i^* = -\mathbf{R}^{-1} \mathbf{M}_i^T(\mathbf{X}_i') \mathbf{H} \varphi_i(\mathbf{X}_i'),
\]

where \( \mathbf{u}_i \) is DC power reference regulation amount when LCC-HVDC provides the frequency support for the MIDC system, \( \mathbf{H} \) satisfies

\[
\mathbf{Q} + \mathbf{H} \Lambda_i + \Lambda_i \mathbf{H} - \mathbf{H} \mathbf{M}_i^T(\mathbf{X}_i') \mathbf{R}^{-1} \mathbf{M}_i^T(\mathbf{X}_i') \mathbf{H} = 0.
\]

The EKFC strategies can be applied online as follows. LCC-HVDC \( i \) measures \( \mathbf{x}_i' = (\omega_i, p_i^{dc}) \) at bus \( i (i \in \mathcal{N}_D) \) periodically. When an emergency is detected and EKFC is enabled at \( t_1 \), the control input can be determined by solving (7) and (8) online every \( \Delta t_2 \) over which the applied control is kept constant. Denote the computation time of EKFC as \( \Delta t_2 \). After \( \mathbf{x}_i' \) is measured at \( t_k (k = 1, 2, \ldots) \), the optimal active power reference can be obtained at \( t_k + \Delta t_2 \). The optimal active power reference is then applied to LCC-HVDC \( i \) during \([t_k + \Delta t_2, t_{k+1} + \Delta t_2] \), where \( t_{k+1} = t_k + \Delta t_1 \). Note that \( \Delta t_1 \) should be set strictly larger than \( \Delta t_2 \).

In general, our data-driven EKFC framework is shown in Fig. 1.

One limitation of EKFC is that it requires specific datasets. To train Koopman eigenfunctions, a post-fault state trajectory with zero control inputs is necessary. To identify the control matrix \( \mathbf{B} \), a post-fault state trajectory with random control inputs, under the same fault, is required. Moreover, our proposed method does not avoid some of the inherent limitations of LQR, such as the need for adjusting the \( \mathbf{Q} \) and \( \mathbf{R} \) matrices in the cost function.

### III. Robustness Analysis and Error Estimation

In this section, we first establish the robustness properties of the proposed KEFC. Next, we examine the effect of an error in the representation of the Koopman operator on the close-loop dynamics and provide an error estimation method. For brevity, the subscript \( E \), which is the threshold for prediction errors, and the subscript \( i \), which represents LCC-HVDC \( i \) will be omitted in this section.

In Section II, the formulation of optimal control problem (6)-(8) in coordinates of eigenfunctions is based on the assumption that Koopman eigenpairs \( (\Lambda, \varphi(x)) \) are accurate. However, in real applications, there are inevitable representation errors in both \( \Lambda \) and \( \varphi(x) \), leading to gap between \( \varphi \) and \( \Lambda \varphi(x) \). There are three possible sources of the representation errors [28]. For the representation of eigenpairs for dynamics in MIDC systems, the three possible sources of the representation errors are listed in Appendix B.

These inevitable representation errors not only affect representation accuracy for eigenpairs, but impact the control effect of the feedback controller. Specifically, there are two questions we need to clarify when taking consideration of representation errors:

1. First, is the controlled system robust to representation errors in eigenpairs? In another word, would an arbitrarily small error destablize the close-loop system?
2. Second, if the closed-loop system is robust to representation errors in eigenpairs, how can we guarantee the dynamics of the controlled system under misrepresented Koopman operator is close to the ones under the accurate Koopman operator?

Before answering the two questions, we need to figure out how to model representation errors of the Koopman eigenpairs. According to the three sources of representation errors, the gap between \( \varphi \) and \( \Lambda \varphi(x) \) must be taken into consideration, as well as misrepresentation of both eigenvalues and eigenfunctions.

Assume eigenvalues and eigenfunctions with representation errors can be expressed as

\[
\hat{\Lambda} := \Lambda + \varepsilon \Lambda
\]

\[
\hat{\varphi}(x) := \varphi(x) + \varepsilon \varphi \psi(x)
\]
where $\epsilon_A \in \mathbb{C}^{n \times n}$ is a diagonal matrix representing the discrepancy to the true $\Lambda$; $\epsilon_\varphi \psi(x)$ is the discrepancy to the eigenfunctions $\varphi(x)$, where $\epsilon_\varphi$ is a matrix with very small numbers as diagonal entries and $\varphi(x) \neq \psi(x)$.

**Remark 3:** Even if accurate eigenpairs are obtained, if there are errors in measurements of $x$, such as time-delay measurements or noises, the control effects may still be weakened. If measurements with errors can be expressed in terms of $x$, then eigenpairs can be written as $(\Lambda, \varphi(kx))$, where $k(x)$ represents measurements with errors. In this case, $\varphi(kx)$ can be regarded as $\hat{\varphi}(x)$. In Section 7, $\omega_\epsilon(s \in \mathcal{N}_1)$ in $\varphi$, take the value of $\omega_i$. In this case, $k(x)$ is a $n$-dimensional vector where each entry equals to $1_i \cdot x = \omega_i$, where $1_i$ is a unit vector. Thus, the following analysis of misrepresented eigenpairs can also be used to analyse the effect of errors in measurements.

We examine the effect of misrepresentation of eigenpairs with $\beta(x)$ given as

$$\beta(x) = \hat{\dot{\varphi}}(x) - \tilde{\Lambda} \hat{\varphi}(x)$$  

where $(\tilde{\Lambda}, \hat{\varphi}(x))$ are eigenpairs learned from data. Accordingly, the dynamics of a misrepresented eigenfunction satisfies

$$\hat{\dot{\varphi}}(x) = \tilde{\Lambda} \hat{\varphi}(x) + \beta(x).$$  

Combining (12) and (45), $\beta(x)$ can be expressed as a function of $(\tilde{\Lambda}, \hat{\varphi}(x))$ given as

$$\beta(x) = \epsilon_\varphi \dot{\psi}(x) - \Lambda \epsilon_\varphi - \epsilon_A \epsilon_\varphi \psi(x).$$  

To answer the first question, we have the following proposition to guarantee the robustness of the closed-loop system with EKFC to errors in the representation of Koopman eigenpairs.

**Proposition 1:** Let the system dynamics in accurate eigenfunction coordinates be given by (3). Then, the closed-loop solution of SDRE with the system dynamics given by (12) is semiglobally asymptotically stable as long as (14) holds.

$$\dot{H} < H - Q - HMR^{-1}M^TH$$  

where $H$ satisfies a SDRE given as

$$Q + HA + \Lambda^T H - HMR^{-1}M^TH = 0,$$  

and

$$\dot{H} = H \epsilon_\varphi NR^{-1}N^T \epsilon_\varphi \epsilon_\varphi^T H$$  

with $N(x) := \nabla \psi(x) \cdot B(x)$.  

**Proof:** According to Ref. [29], $V = \varphi^T(x)H\varphi(x)$ is a candidate Lyapunov function of the system in accurate eigenfunction coordinates, where $H$ is the solution of the SDRE and $V > 0$. For control-affine system described by accurate eigenpairs, the derivation of $V$ is given as

$$\dot{V} = \varphi^T(x)\dot{H}\varphi(x) + \varphi^T(x)H\dot{\varphi}(x) + \dot{\varphi}^T(x)H\varphi(x)$$

$$\dot{\varphi}(x) = \Lambda \varphi(x) + \epsilon_\varphi \nabla \psi \cdot f(x)$$

Combined with (12), (18) leads to the relationship between $\epsilon_A, \epsilon_\varphi \psi(x)$ and $\beta(x)$

$$\epsilon_\varphi \nabla \psi \cdot f(x) = \beta(x) - \Lambda \varphi(x) + \hat{\Lambda} \hat{\varphi}(x)$$

If we augment the uncontrolled system (12) with a linear control term $Bu$, the dynamics of the eigenfunction in the corresponding control-affine system can be obtained as

$$\hat{\dot{\varphi}}(x) = \Lambda \hat{\varphi}(x) + M(x)u + \epsilon_\varphi \nabla \psi(x)(f(x) + Bu)$$

$$= \hat{\Lambda} \hat{\varphi}(x) + M(x)u + \beta(x) + \epsilon_\varphi N(x)u$$

$$= \left( \hat{\Lambda} + \frac{\beta(x)}{\varphi} \right) \hat{\varphi} + (M(x) + \epsilon_\varphi N(x))u$$

By applying (9) and (10), modelled system dynamics given in (20) can be transformed into $\varphi$ coordinates:

$$\hat{\varphi}(x) = \Lambda \varphi(x) + \hat{M}u$$

where

$$\hat{M} = M(x) + \epsilon_\varphi N(x).$$

A misrepresentation of the Koopman eigenpairs will affect the value of $V$. Specifically, by applying (21), we have

$$\dot{V} = \varphi^T(x)\dot{H}\varphi(x) + \varphi^T(x)H\dot{\varphi}(x) + \dot{\varphi}^T(x)H\varphi(x)$$

$$= \varphi^T(x)\dot{H}\varphi(x) + \varphi^T(x)H \left[ \Lambda \varphi(x) - M R^{-1} M^T H \varphi(x) \right]$$

$$+ \left[ \Lambda \varphi(x) - M R^{-1} M^T H \varphi(x) \right]^T H\varphi(x)$$

$$= \varphi^T(x) \left( \dot{H} - Q - HMR^{-1}M^TH + H\epsilon_\varphi NR^{-1}N^T \epsilon_\varphi \epsilon_\varphi^T H \right) \varphi(x)$$

According to Ref. [29], $\dot{H} - Q - HMR^{-1}M^TH < 0$. Note that though $\dot{H}$ is semi-positive, as long as $\dot{H} - Q - HMR^{-1}M^TH + \hat{H} < 0$ holds, the error in the representation of Koopman eigenpairs would not destabilize the closed-loop system. Note that the eigenvalues of a matrix depend continuously on its entries. Therefore, when eigenvalues of $\dot{H} - Q - HMR^{-1}M^TH$ are negative, eigenvalues of $\dot{H} - Q - HMR^{-1}M^TH + \hat{H}$ are still negative as long as $\|\epsilon_\varphi\|$ is sufficiently small.

In order to answer the second question, we examine the effect of misrepresented Koopman eigenpairs on the close-loop dynamics and provide an estimation of the upper error bound.

For accurate eigenpairs $(\Lambda, \varphi(x))$, the feedback control is given by

$$u = -R^{-1}M^T \varphi$$

and the resulting closed-loop dynamics can be obtained as

$$\dot{\varphi}(x) = \Lambda \varphi(x) + Mu$$

$$= (\Lambda - MR^{-1}M^T) \varphi(x).$$
Similarly, for eigenpairs with representation errors \((\Lambda, \hat{\varphi}(x))\), the control input is given by
\[
\bar{u} = -R^{-1}\bar{M}^T\bar{H}\varphi
\]  
(26)
where \(\bar{H}\) satisfies a SDRE with \(\bar{M}\) given as
\[
Q + \bar{H}\Lambda + \Lambda^T\bar{H} - \bar{H}MR^{-1}M^{-1}\bar{H} = 0,
\]  
(27)
and the resulting closed-loop dynamics can be obtained as
\[
\dot{\varphi}(x) = \Lambda\varphi(x) + M\bar{u} = \left(\Lambda - \bar{M}R^{-1}M^T\bar{H}\right)\varphi(x).
\]  
(28)
Define \(\mu := \Lambda - MR^{-1}M^T\) and \(\hat{\mu} := \Lambda - \bar{M}R^{-1}M^T\bar{H}\). The estimation error in \(\mu\) can be calculated as
\[
\|\mu - \hat{\mu}\|
= \left\| -MR^{-1}M^TH + MR^{-1}M^T\bar{H} \right\|
= \left\| -MR^{-1}M^TH + (M + \varphi N)R^{-1}(M + \varphi N)^T\bar{H} \right\|
= \left\| MR^{-1}M^T(\bar{H} - H) + \varphi (NR^{-1}M^T + MR^{-1}N^T)\bar{H} \right. \\
+ \varphi^2 NR^{-1}N^T\bar{H} \right\| \leq \left\| MR^{-1}M^T\right\| \left\| H - H \right\|
+ \|\varphi\| \left( \|NR^{-1}M^T\bar{H}\| + \|MR^{-1}M^T\bar{H}\| \right)
+ \|\varphi\|^2 \left\| NR^{-1}N^T\bar{H}\right\|, 
\]  
(29)
where \(\|\cdot\|\) represents the Frobenius norm.

The upper bound of \(\|\bar{H} - H\|\) on the right hand side is remained to be estimated explicitly in terms of representation errors of Koopman eigenpairs. Let \((\Lambda, Q, MR^{-1}M^T)\) be a triplet of so-called perturbed system matrices. When the continuity condition on the sequence of \((\Lambda, Q, MR^{-1}M^T)\) holds, where \(\pi \to \pi_0\) represents a mapping representing a deformation process from \((\Lambda, Q, MR^{-1}M^T)\) to \((\Lambda, Q, \bar{M}R^{-1}M^T)\). By Theorem 2.5 in Ref. [30], the upper bound of \(\|\bar{H} - H\|\) can be given as
\[
\|\bar{H} - H\| \leq \tau \chi
\]  
(30)
where \(\chi\) is a finite constant, and \(\tau\) satisfies
\[
\tau = \left\| \Lambda - \bar{\Lambda} \right\| + \left\| MR^{-1}M^T - \bar{M}R^{-1}\bar{M}^T \right\|
= \|\varphi\| + \|MR^{-1}M^T\|
- (M + \varphi N)R^{-1}(M + \varphi N)^T \right\|
\leq \|\varphi\| \left( \|MR^{-1}N^T\| + \|N^T R^{-1}M\| \right)
+ \|\varphi\|^2 \|NR^{-1}N^T\|, 
\]  
(31)
Therefore, the upper bound of \(\|\mu - \hat{\mu}\|\) can be further derived as
\[
\|\mu - \hat{\mu}\| \leq \|MR^{-1}M^T\| \tau \chi
\]
+ \|\varphi\| \left( \|NR^{-1}M^T\| + \|MR^{-1}M^T\| \right)
\leq \|\varphi\| \left( \|MR^{-1}M^T\| \chi \right)
\times \left( \|NR^{-1}M^T\| + \|MR^{-1}N^T\| \right)
\]  
(32)
From (32), it can be concluded that when \(\|\varphi\|\) and \(\|\varphi\|\) converge to 0, \(\|MR^{-1}M^T\|\) converges to 0. In other words, the error of the closed-loop dynamics is limited by the representation errors of Koopman eigenpairs.

Here, we further discuss when does the equality holds in the inequality (32).

Remark 4: In the deviation of the upper bound for \(\|\mu - \hat{\mu}\|\), (29), (31) and (32) are based on the triangle inequality and the sub-multiplicative inequality of the Frobenius norm. For any two arrays \(A\) and \(B\), equality for the triangle inequality holds when the two arrays are linearly dependent, while equality for the sub-multiplicative inequality holds if and only if each row of \(A\) and each column of \(B\) are linearly dependent.

Remark 5: By Theorem 2.5 in Ref. [30], a necessary condition for \(\|H - \bar{H}\|\) to reach the upper bound in (31) is the time of the Ricatti flow \(t \to \infty\).

Remark 6: \(\|MR^{-1}M^T\|\) is often strictly lower than the upper bound derived in (32). One of the reasons is that it’s unrealistic for an infinite control period in EFC. The gap between \(\|\mu - \hat{\mu}\|\) and its upper bound will be further illustrated in the simulations results in Section IV-D.

In conclusion, the robustness of EKFC and upper bound of \(\|MR^{-1}M^T\|\) promises the effectiveness of EKFC when there exist representation errors in eigenpairs. In practical engineering, it’s necessary for EKFC to adapt to complicated online operational contexts in EFC such as unknown time-delay measurements and the deadzone setting. The robustness of EKFC and the upper bound of \(\|MR^{-1}M^T\|\) help to address the problems of these uncertainties or perturbation in the system dynamics.

IV. CASE STUDY

In this section, the effectiveness of EKFC is illustrated by a case study on the CloudPSS platform [31], [32]. All of the following tests are conducted on PCs with Intel Xeon W-2255 processor, 3.70 GHz primary frequency, and 128 GB memory.

A. Test System and Datasets

The MIDC test system is a modified IEEE New England system combining the CIGRE HVDC benchmark systems [33] used in Ref. [2]. The full electromagnetic transient (EMT) model of the test system is built on the CloudPSS platform [34]. The main
AC system is connected with four ±660 kV monopolar 12-pulse LCC-HVDC systems. The system capacity is 4000 MW.

To identify eigenpairs for frequency dynamics in MIDC systems, we set generator G6 trip at the time of 20 s, which causes a 530 MW power imbalance. With the control input \( u = 0 \), data of \( x = \{ \theta_i, \omega_i, \phi_i^{dc} \mid h \in \mathcal{N}_g \cup \mathcal{N}_d, i \in \mathcal{N}_d \} \) were collected in the time span of 20 ~ 50 s with a rate of 100 Hz (3000 time points in total).

To identify the control matrix \( B \), data of \( x \) from the MIDC system with random control input should also be collected. Uniform-distributed numbers were generated as the control input in the time span of 20 ~ 50 s with a rate of 100 Hz. Since DC power reference regulation amount is constrained to be \( -20\% \) and \( +10\% \) of the nominal transmission power of each LCC-HVDC, the uniform distribution is limited on \([-0.2 \text{ p.u.,} +0.1 \text{ p.u.}]\). Data of \( x^c \) were collected in the time span of 20 ~ 50 s with a rate of 100 Hz. Based on the above settings, we obtain the following results.

**B. Obtained Linear Representations**

1) Library Construction: For library setting, a polynomial basis up to the second order, trigonometric terms of \( x \), trigonometric transform of subtraction between any two rotor angles are employed.

Considering the dynamics of a MIDC system given as (52)–(57), there are totally \( n = 22 \) state variables in the test system, of which 7 are the rotor angles of \( \mathcal{N}_g \), 11 are the frequencies deviation from the nominal frequency at \( \mathcal{N}_g \) and \( \mathcal{N}_d \), and 4 are the transmission power of LCC-HVDC. Based on library construction method in Section II-A1, there are 318 basis functions in total, of which 275 are polynomial terms, 22 are trigonometric transform of \( x \), and 21 are trigonometric transform of subtraction between any two rotor angles.

2) Library Subsampling and Ensemble Learning for Eigenfunctions: According to the library subsampling method in Section II-A2, \( D = 25^{-1} = 16 \) subsets are constructed in total.

Without the library bagging technique, the only verified eigenfunction with a prediction error under \( 1e^{-4} \) is \( \cos \omega_2 \) with an eigenvalue of \(-1.19e^{-4}\). Verified eigenpairs obtained by introducing the library bagging technique are listed in Table I in Appendix C. After the library bagging technique is introduced, the number of verified eigenfunctions with a prediction error under \( 1e^{-4} \) increases to 16.

Considering realistic communication infrastructure, we design distributed control with partial measurements, in which only system frequency \( \omega_i \) and \( \phi_i^{dc} \) \( (i \in \mathcal{N}_d) \) are known. Therefore, the eigenpairs #1-9 can be further selected to form \( \varphi_i \) in (6) for LCC-HVDC \( i \), since the others are expressed explicitly in terms of rotor angles.

3) Generality of Eigenpairs: Uncertain emergency faults are unavoidable conditions in practical online operational contexts. To identify eigenpairs for frequency dynamics in MIDC systems, trajectories of \( x \) after the trip of G6 are collected. To illustrate the generality of the learnt Koopman eigenpairs, we further calculate prediction errors of the verified Koopman eigenpairs on trajectories of other trip events. If the prediction errors are still small (under the threshold \( 1e^{-4} \)), then the generalization capability of the learnt Koopman eigenpairs can be verified. Other emergency faults considered include:

i) trip of another generator;

ii) trip of two generators at the same time.

We traverse all emergency frequency events in the above two cases in the MIDC test system. There are 7 generators in the system, thus \((7 - 1) + \sum_{i=1}^{7} 2 = 27 \) scenarios can be obtained. Data of \( x \) were collected in the time span of 20 ~ 50 s with a rate of 100 Hz in each scenario. Since EKFC is designed to improve the system frequency stability, we exclude the scenarios where voltage instability or angle instability occurs. We calculate prediction errors of the eigenpairs #1-9 on \( x \) collected in each scenario. The average prediction error of the eigenpairs in each scenario are given in Fig. 2. As shown in Fig. 2, prediction errors are still small even when different trip events occur. Therefore, the generality of the eigenpairs is verified, although the eigenpairs are obtained on limited datasets.

**C. Effectiveness of EKFC**

To demonstrated necessity to introduce data-driven methods for emergency control, we compare EKFC to a traditional offline strategy. Often, traditional offline strategies tune the control measures based on the acceptable minimum frequency of the power grid under typical operating conditions. When both G4 and G6 generators fail, the lowest frequency of the system is 49.2 Hz. Assuming that the minimum acceptable frequency of the system is 49.5 Hz, we set the DC power reference regulation amount to achieve a lowest frequency of 49.5 Hz. Suppose the synchronous generators in the system are replaced by renewable sources, and the system inertia decreases to 0.5 times of the original system. As illustrated in Fig. 5, when G4 and G6 fail simultaneously, the system frequency drops to around 49.0 Hz. We apply the same DC power reference regulation amount before renewable sources were integrated, and the frequency is restored to around 49.4 Hz. If EKFC is adopted, the frequency can be restored to around 49.53 Hz, demonstrating that the proposed data-driven method is more likely to be adaptable to changing system operating conditions. To demonstrate the effectiveness of EKFC, the active power reference of LCC-HVDC is determined adaptively to participate in frequency control by EKFC after generator G6 tripped at the time of 20 s. \( Q_1 \) is a diagonal weight matrix, which has 1 as the entry when the corresponding eigenfunction is expressed only in terms of frequencies, and 0 for other eigenfunctions. The weight matrix \( R = 2e^{-6} \). In our
To assess the performance of EKFC, it’s compared with P-f droop based EFC, a typical frequency control strategy, and KRONIC, which is a representative Koopman based control design method. Therefore, the following four subcases are compared: (1) The LCC-HVDCs have no control designed to provide frequency support for the system. (2) All LCC-HVDCs have EKFC as the EFC strategy. (3) All LCC-HVDCs have droop control with optimal coefficients calculated in Ref. [2]. (4) All LCC-HVDCs have KRONIC as the EFC strategy. (5) All LCC-HVDCs have DMDc as the EFC strategy. The results of frequencies of the AC main system are displayed in Fig. 3. The active powers of LCC-HVDCs are shown in Fig. 4.

As shown in Fig. 3, the system frequency of subcase (2) is higher than that in subcase (3). This is because eigenfunctions predict system dynamics globally and solve the open-loop optimization problem over an infinite time horizon. However, the droop control law is not able to predict frequency over time and is restricted to be proportional to frequency deviation. Moreover, the system frequency of subcase (2) is higher than that in subcase (4). This is because more verified eigenfunctions can be obtained to describe system dynamics in EKFC, so that more control objects can be included in the cost function in (5). Additionally, the system frequency of subcase (2) is higher than that in subcase (5). This is because DMDc estimate system dynamics with linear observables, which fail to capture the nonlinear transients of the system.

As shown in Fig. 4, in subcase (1), the emergency frequency regulation can only rely on the generators’ primary droop, but the power adjustment speed of generators is relatively slow. In subcases (2), (3) and (4), the fast power adjustability of the LCC-HVDC systems is utilized to provide considerable power support and relieve the frequency modulation pressure of the generators. By comparing Fig. 4(b) and (c), we also see that during 20 ~ 23 s, subcase (2) provides large power support at the moment emergency faults occur, while in subcase (3) the DC power gradually increases as the frequency decreases.

D. Error Bound Analysis With Eigenpairs Approximation Errors

In Section III, an expression for the upper error bound of $\mu$ is provided. To demonstrate the actual error of $\mu$ is strictly below the upper error bound, we consider an analytical example in Ref. [22] and the MIDC example in Section IV-A. In the analytical example, a closed and finite-dimensional Koopman approximation exists. $\varepsilon_{\Lambda}$ and $\varepsilon_{\phi}\psi(x)$ can be set to model possible errors in the representation of the Koopman operator. The results show that the actual errors along a given trajectory are strictly lower than the upper error bounds, validating our estimation for the error bound in (32). For more details, see Appendix C.

In MIDC examples, since the accurate Koopman operator is inaccessible, we assume that the verified eigenpairs obtained
in Section IV-B are the accurate ones. To demonstrate how the representation errors of eigenpairs influence the control effect of EKFC, $\epsilon_A$ and $\epsilon_{\psi}(x)$ defined in Section III can be artificially given. Then the system dynamics with EKFC of accurate eigenpairs and of eigenpairs with representation errors can be simulated. Subsequently, both sides of the inequality (32) can be calculated. If $\|\mu - \mu\|$ is strictly lower than the estimated upper error bound, then the inequality (32) can be verified.

To simulate representation errors of eigenpairs, we set $\epsilon_{\psi} = \epsilon L$ and $\psi(x) = I(\Xi) \cdot \Theta$, where $\epsilon$ is a real number, and $L$ represents an $L \times L$ identity matrix. $I(\Xi)$ is a $P \times L$ matrix in which $I_{ij}(\Xi) = 1$ if $\Xi_{ij} \neq 0$, else $I_{ij}(\Xi) = 0$.

Fig. 6 demonstrates two examples with different error settings. The actual errors and the upper error bounds are calculated on the trajectory of the closed-loop system in subcase (2) with accurate eigenpairs.

The results show that the actual errors along the given trajectory are strictly lower than the estimated upper error bounds, validating our estimation for the error bound in (32). As discussed in Section III, $\|\mu - \mu\|$ is often strictly lower than the upper bound derived in (32). Thus, we can conclude that when $\|\epsilon_A\|$ and $\|\epsilon_{\psi}\|$ are finite, $\|\mu - \mu\|$ is also finite.

E. Adaptability Test

Uncertain emergency faults, time-delay measurements are unavoidable in practical operation. Moreover, in engineering practice, a dead zone setting for EKFC is necessary. These realistic conditions may weaken the control effect. Therefore, the adaptability of EFKC to such realistic conditions is examined here. Note that the adaptability of EFKC to uncertain emergency faults benefits from the generality of the obtained eigenpairs on different faults, while adaptability of EFKC to time-delay measurements and the dead zone setting benefits from the robustness to eigenpairs with representation errors.

1) Unknown Emergency Events: In Section V-A, trajectories of $x$ after the trip of G6 are collected to construct system dynamics in eigenfunction coordinates. Here, varying trip events are considered to examine the adaptability of EKFC. The trip events considered are the same as in Section IV-B3.

Fig. 7 illustrates the adaptability of EKFC to unknown emergency frequency events. Note that EKFC still achieves satisfactory control performances in such totally unknown events. Results of more adaptability tests of EKFC to unknown emergency frequency events are given in Table II in Appendix C.

2) Partial Measurements: In Section II-B2, we assume that the frequency dynamics in any node or generator are the same. Therefore, we let $\omega_s$ ($s \in N_D$) in $\phi_i$ take the value of $\omega_i$ in the optimal control strategy calculation for LCC-HVDC $i$. The influence of neglecting spatial-temporal characteristics of frequencies is illustrated in Fig. 8(a). The results show that partial measurements causes the frequency nadir to be about 0.01 Hz lower but has no effect on the steady-state frequency.

3) Time-Delay of Measurements: Time-delay is unavoidable due to communication latency or control strategy computation. In practical grid, the time delay is often below 150 ms [35]. Here, to test the performance of EKFC, we assume that the measurements have an 1 s delay. The results are illustrated in Fig. 8(b).

The results show that time-delay of measurements causes the frequency nadir to be about 0.02 Hz lower but has no effect on the steady-state frequency.

4) A Deadzone Setting: In engineering practice, a dead zone setting for EKFC is necessary. When the system frequency changes due to some faults, the frequency limitation of the dead zone is utilized to determine whether there is an emergency and whether to enable EKFC. In this article, we assume that a frequency deviation limitation is used to set the deadzone. The influence of the deadzone setting of EKFC is illustrated in Fig. 8(c).

The results show that after EKFC is triggered by the dead zone setting, the frequency of the AC main system is stabilized soon. During the transient frequency process, the dead zone setting causes the frequency nadir to be about 0.2 Hz lower than the frequency nadir in subcase (2). However, the dead zone setting has no effect on the steady-state frequency.

The adaptability of EKFC benefits from generality of obtained Koopman eigenpairs and the robustness to eigenpairs approximation errors. As long as (14) is satisfied, EKFC can adapt to conditions not included in the initial dataset.

The above results imply that EKFC does not necessarily rely on access to massive datasets of different scenarios and is potential to enable the control of nonlinear systems even when limited scenarios are considered.

F. A Larger Test System

To further validate the effectiveness of our proposed method, we conducted simulation experiments on a test system adapted from a real provincial power grid in China. For the sake of brevity and clarity, we refer to this provincial-level power grid system as the PPG test system in subsequent discussions. It consists of 102
Fig. 8. Frequencies of the AC main system (a) when $\omega_s (s \in \mathbb{N})$ is measurable at LCC-HVDC i. (b) when measurements has 1 s time delay. (c) when EKFC has 49.80 Hz deadzone.

Fig. 9. Frequencies of the AC main system when G6 trip at 20 s.

500-kV buses, and possesses a load level of 2600 MW, with installed capacities of 2400 MW and 5400 MW for renewable and conventional energy sources, respectively. The system includes three HVDCs with active power of 800 MW.

As shown in Fig. 9, EKFC outperforms the compared control strategies. The results in Fig. 10(a) show that partial measurements causes the frequency nadir to be about 0.02 Hz lower but has no effect on the steady-state frequency. With regard to the scenario where time delay is present, Fig. 10(b) shows that time-delay of measurements causes the frequency nadir to be about 0.2 Hz lower but has no effect on the steady-state frequency. As for the scenario where a 49.80 Hz deadzone is present, Fig. 10(c) shows that after EKFC is triggered by the dead zone setting, the frequency of the AC main system is stabilized soon. During the transient frequency process, the dead zone setting causes the frequency nadir to be about 0.02 Hz lower than the frequency nadir in subcase (2). However, the dead zone setting has no effect on the steady-state frequency.

V. CONCLUSION

In this article, the discovery of linear representations of nonlinear MIDC system dynamics is developed based on Koopman operator theory. Applying the linear representations, a fully data-driven dynamic optimal control method EKFC is proposed for LCC-HVDCs to participate in the system frequency regulation service. Furthermore, an error bound of the closed-loop dynamics with consideration of Koopman operator approximation errors is estimated. The case study demonstrates the effectiveness of EKFC on providing frequency support. Moreover, actual errors are proved to be strictly lower than the upper bound estimated for the closed-loop dynamics. Simulation results show that EKFC adapts to practical conditions such as uncertain emergency faults and time-delay measurements. Furthermore, a dead zone setting for EKFC has no effect on the steady-state frequency. The above results indicate that EKFC does not necessarily rely on access to massive datasets of different scenarios and is potential to enable the control of nonlinear systems even when limited scenarios are considered.

APPENDIX A

A. Brief Overview of Koopman Operator Theory

Koopman operator is a linear but an infinite-dimensional operator that governs the time evolution of observables or outputs defined on the state space of a dynamical system [8]. In particular, we consider unactuated, autonomous dynamic systems of the form

$$\dot{x} = f(x),$$

with the state vector $x \in \mathbb{R}^n$.

The Koopman operator is a linear operator $K$ which advances a measurement function $g(x)$ of the state forward in time through the dynamics

$$(Kg)(x) = g(f(x))$$

For an eigenfunction $\varphi$ of $K$, corresponding to an eigenvalue $\lambda$, this becomes

$$(K\varphi)(x) = \lambda \varphi(x) = \varphi(f(x)),$$

where $(\lambda, \varphi)$ forms an eigenpair.

Here we provide more details about the koopman eigenvalues of the continuous-time system and discrete-time system.

The classical geometric theory of dynamical systems considers a set of coupled differential equations given as (33). In discrete time, the dynamics are given by

$$x_{k+1} = F(x_k)$$

where $F$ is the flow map of the dynamics in (35).

$$F(x_k) = x_k + \int_{k\Delta t}^{(k+1)\Delta t} f(x(\tau))d\tau$$

where $\Delta t$ is the data sampling time.
In discrete time, a Koopman eigenfunction $\psi(x)$ corresponding to a Koopman eigenvalue $\lambda$ satisfies

$$\psi(x_{k+1}) = \psi(F(x_k)).$$

(38)

In continuous time, a Koopman eigenfunction corresponding to a Koopman eigenvalue satisfies (39).

$$\dot{\varphi}(x) = \lambda \varphi(x).$$

(39)

Expressing (39) in a matrix form yields (6).

Expressing (39) in matrix form for discrete systems yields

$$\varphi(x_{k+1}) = e^{\lambda \Delta t} \varphi(x_k)$$

(40)

Comparing (38) and (40), we can know that Koopman eigenfunctions of continuous time are also Koopman eigenfunctions of discrete time and vice versa, i.e.

$$\psi(x) = \varphi(x).$$

(41)

For a Koopman eigenfunction, its corresponding Koopman eigenvalue $\lambda$ of continuous time and Koopman eigenvalue $\nu$ of discrete-time satisfies

$$\nu = e^{\lambda \Delta t}.$$  

(42)

### B. KRONIC Algorithm

The observable vector in intrinsic Koopman eigenfunction coordinates is defined as

$$\varphi(x) = [\varphi_1(x), \varphi_2(x), \ldots, \varphi_S(x)]^T,$$

(43)

where $\varphi = \{\varphi_s : \mathbb{R}^n \rightarrow \mathbb{C}, s = 1, 2, \ldots, S\}$ represents a nonlinear transformation of the state $x$ into eigenfunction coordinates. If $\varphi$ is differentiable at $x$, by applying the chain rule its evolution equation can be written as

$$\dot{\varphi}(x) = \nabla \varphi(x) \cdot f(x).$$

(44)

According to the definition of Koopman eigenfunctions, we obtain

$$\dot{\varphi}(x) = \Lambda \varphi(x),$$

(45)

where $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_S)$ is a matrix, with diagonal elements consisting of the eigenvalue $\lambda_s (s = 1, 2, \ldots, S)$ associated with the eigenfunction $\varphi_s$.

Combining (44) and (45), the following Koopman partial differential equation (PDE) should be satisfied by regression:

$$\nabla \varphi(x) f(x) = \Lambda \varphi(x).$$

(46)

According to Ref. [10], Koopman eigenpairs can be identified using the PDE (46) based on the sparse identification of nonlinear dynamics (SINDy) framework [36]. First, a library of candidate functions is chosen:

$$\Theta(x) = [\theta_1(x), \theta_2(x), \ldots, \theta_L(x)]^T.$$  

(47)

Note that $\Theta$ is often large so that Koopman eigenfunctions may be well approximated in this library:

$$\varphi(x) \approx \Xi \Theta(x),$$

(48)

where $\Xi \in \mathbb{C}^{P \times L}$.

Given $M$ snapshots of the state $X = [x_1, x_2, \ldots, x_M] \in \mathbb{R}^{n \times M}$ with the autonomous system dynamics (33), learning Koopman eigenpairs becomes finding an optimum solution for the optimization problem as given in

$$\min_{\Lambda, \Xi} \sum_{m=1}^{M} \| e^{\Lambda \Delta t} \varphi(x_1) - \varphi(x_m) \| + \alpha \| \Xi \|$$

(49a)

s.t. $\varphi(x) = \Xi \Theta(x)$

(49b)

where $\alpha$ is a thresholding parameter to balance between sparsity and prediction accuracy.

One of the leading algorithms to solve the problem is KRONIC [15], [22]. It proposed to identify each eigenpair separately based on the implicit formulation in (50). For the $s$th eigenvalue $\lambda_s(s = 1, 2, \ldots, S)$, the Koopman PDE in (46) yields

$$\xi_s(\lambda_s \Theta(X) - T(X)) = 0,$$

(50)

where $\xi_s$ represents the $s$th row of $\Xi$,

$$T(X) = [\nabla \theta_1(X) \cdot \dot{x}_1, \nabla \theta_2(X) \cdot \dot{x}_2, \ldots, \nabla \theta_L(X) \cdot \dot{x}_M]^T$$

(51)

where $T(X) \in \mathbb{C}^{L \times M}$. The time derivative $X = [\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_M]$ can be measured or approximated numerically by the total variation derivative [37].

KRONIC proposed to identify each eigenpair separately based on the implicit formulation in (50). The algorithm starts with an initial guess of the eigenvalues as $\Lambda^{\text{init}} = \ldots$
TABLE I
EIGENPAIRS LEARNT IN EKFC

| No. | Eigenvalue | Eigenfunction | Prediction error |
|-----|------------|---------------|-----------------|
| 1   | -1.26e-9  | 0.175P_{dc}e + 0.141 sin P_{dc}e + 1.00 cos P_{dc}e | 1.95e-7 |
| 2   | -6.23e-9  | 0.190P_{dc}e + 0.135 sin P_{dc}e + 1.00 cos P_{dc}e | 2.19e-7 |
| 3   | -1.58e-9  | 0.139P_{dc}e + 0.117 sin P_{dc}e + 1.00 cos P_{dc}e | 3.43e-7 |
| 4   | 2.91e-8   | 0.198P_{dc}e + 0.135 sin P_{dc}e + 1.00 cos P_{dc}e | 7.33e-7 |
| 5   | 3.28e-8   | 1.00 cos ω_{34} | 5.07e-6 |
| 6   | -2.26e-7  | 1.00 cos ω_{30} | 8.42e-6 |
| 7   | -1.39e-11 - 5.8e-10j | 0.017P_{dc}e + 1.00 cos ω_{38} | 1.09e-5 |
| 8   | -7.35e-7  | 1.00 cos ω_{32} | 1.15e-5 |
| 9   | -2.88e-6  | 1.00 cos ω_{36} | 4.25e-5 |
| 11  | -7.84e-8  | 1.00 cos ω_{34} | 5.27e-5 |
| 12  | 1.94e-10 - 1.13e-10j | 1.00 sin δ_{32} + (0.132 + 0.001j) cos δ_{32} | 6.94e-5 |
| 13  | -1.22e-10 - 9.14e-11j | (0.002 - 0.001j)δ_{32} + (0.008 - 0.008j)δ_{34} + 1.00 sin δ_{32} | 1.24e-4 |
| 14  | 1.40e-9   | 0.002δ_{34} + 1.00 cos δ_{36} + 0.0055 sin (δ_{34} - δ_{36}) | 4.76e-4 |
| 15  | 4.5e-9    | -0.03δ_{34} - 0.001 sin (δ_{34} - δ_{36}) + 1.00 cos (δ_{32} - δ_{34}) | 7.06e-4 |
| 16  | -6.68e-9  | -0.002δ_{32} + 0.065 sin (δ_{34} - δ_{36}) + 1.00 sin (δ_{38} - δ_{10}) | 7.57e-4 |

The (X)Ω(r)(X). For each λ, the technique subsequently alternates between an searching for the sparsest vector in the nullspace of λλ(λ(X))−r(X) and updating the eigenvalue λ, as (ξ, T(X)Ω(r)(X)ξ) and (ξ, Ω(X)ξ). The superscript † denotes the pseudoinverse operator. When Λ = diag(λ1, λ2, ..., λs) converges, Λ and φ(x) = (ϕ1, ϕ2, ..., ϕs)T are identified.

C. Dynamic Model of MIDC Systems

Considering the second-order dynamic models of generators and the first-order inertia models of a LCC-HVDC system [2], [38], [39], the dynamics of MIDC systems can be written as

\[
\dot{\delta}_i = \omega_i, i \in N_G \cup N_P
\]

\[
M_i \dot{\omega}_i + D_i \ddot{\omega}_i = P_i - \sum_{j \in N} B_{ij} \sin (\delta_i - \delta_j)
\]

\[
- k_{di} \omega_i, i \in N_G
\]

\[
0 = P_i + p_{dc}^i - \sum_{j \in N} B_{ij} \sin (\delta_i - \delta_j), i \in N_P
\]

\[
0 = P_i - \sum_{j \in N} B_{ij} \sin (\delta_i - \delta_j), i \in N_P
\]

\[
T_i \dot{p}_{dc}^i = p_{dc}^i + p_{dc}^i + u_i - k_{di} \omega_i, i \in N_P
\]

where three types of buses, i.e., generator buses, LCC-HVDC connected buses and passive load buses are denoted by N_G, N_P and N_P, respectively, N = N_G \cup N_P \cup N_P, \delta_i is the phase angle at bus i with reference to the synchronous rotation coordinate, \omega_i is the frequency deviation from the nominal frequency, M_i is the inertia constant of the generator i, D_i > 0 is the damping coefficient, P_i is the power injection (> 0) or demand (< 0), p_{dc}^i is the transmission power of LCC-HVDC, T_i is the nominal value of p_{dc}, u_i is DC power reference regulation amount when LCC-HVDC i provides the frequency support for the MIDC system, T^D is the inertia time constant of LCC-HVDC i, B_{ij} = B_{ij}V_iV_j is the effective susceptance of line (i, j), V_i is the voltage amplitude at bus i which is assumed to be constant due to its irrelevance to the frequency control, k_{di} > 0 is the droop coefficient of the generator i. When identifying eigenpairs for frequency dynamics in MIDC systems, we assume full access to the state x = {δh, ωi, p_{dc}^i | h \in N_G \cup N_P, i \in N_P}.

APPENDIX B

A. Possible Sources of the Representation Errors

For the representation of eigenpairs for dynamics in MIDC systems, the three possible sources of the representation errors are listed as follows.

1) The first source of representation errors is the reconstruction error of eigenfunctions. In our method, candidate functions in library are selected by integrating prior knowledge in dynamics of generators and LCC-HVDC system. However, we can hardly guarantee that the library is rich enough to include all categories of nonlinearities required for construction of eigenfunctions.

2) The second source of representation errors is the training error. In EKFC, the training of eigenvalues terminates when the training error is less than a specified tolerance. The training of eigenfunctions for each eigenvalue is to select an eigenfunction with minimum prediction error. Therefore, inadequate training leads to representation errors in Koopman eigenpairs.

3) The third source of representation errors is from sampling. The exact approximation of either eigenpairs or the control...
matrix requires that the approximation to be based on the entire state space and the entire control space. However, the approximation is based on sampling. Specifically, in practical online operational contexts in EKFC, there are numerous unpredicted conditions, such as uncertain emergency faults and time-delay measurements. However, it’s impossible for collected datasets to cover all the conditions above. Hence, there is an unavoidable bias error due to incomplete sampling of the state space.

### B. Simulation Results

For most practical dynamic systems, accurate Koopman eigenpairs are often inaccessible since representation errors of Koopman eigenpairs are often unavoidable. Here, to demonstrate the actual error is strictly bounded by the error bound estimated in (32), we consider an analytical example given as (58), where a closed and finite-dimensional Koopman approximation exists.

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} &= 
\begin{bmatrix} \mu_1 x_1 \\ \mu_2 (x_2 - x_1^2) \\ 0 \\ 0 \\ 2\mu_1 \\ 0 \end{bmatrix} + Bu. \\
&= \begin{bmatrix} 1 & 0 \\ 0 & -\mu_2 \\ 0 & 2\mu_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} Bu. \tag{59}
\end{align*}
\]

Koopman eigenfunctions of the unforced system are \( \varphi_{\mu_1} = x_1, \varphi_{\mu_2} = x_2 - bx_1^2 \) with \( b = \frac{\mu_2}{2\mu_1} \), corresponding to the eigenvalue \( \mu_1 \) and \( \mu_2 \), respectively.

Here, we set \( \mu_1 = -1 \) and \( \mu_2 = -1 \). Errors in the representation of \( \mu_1 \) and \( \mu_2 \) are simulated as \( \epsilon_{\mu_1} = -1e^{-3} \) and \( \epsilon_{\mu_2} = -1e^{-3} \). The errors in the representation of eigenfunctions are simulated as \( \epsilon_\varphi(x) = -0.2816I_3 \cdot (0, x_1^2, 0) \), where \( I_3 \) is a \( 3 \times 3 \) identity matrix. The results are illustrated in Fig. 11.

### APPENDIX C

#### TABLE II

| Event                      | G1                     | G2                     |
|----------------------------|------------------------|------------------------|
| LCC without control        | 49.78                  | 49.82                  |
| LCC with EKFC              | 49.91                  | 49.89                  |
| LCC without control        | 49.93                  | 49.96                  |
| LCC with EKFC              | 49.93                  | 49.96                  |

| LCC without control        | 49.95                  | 49.70                  |
| LCC with EKFC              | 49.90                  | 49.69                  |

| LCC without control        | 49.50                  | 49.73                  |
| LCC with EKFC              | 49.50                  | 49.73                  |

| LCC without control        | 49.72                  | 49.84                  |
| LCC with EKFC              | 49.70                  | 49.82                  |

| LCC without control        | 49.92                  | 49.79                  |
| LCC with EKFC              | 49.90                  | 49.93                  |

| LCC without control        | 49.50                  | 49.73                  |
| LCC with EKFC              | 49.76                  | 49.85                  |

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