On gravitomagnetic time-delay by extended lenses

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ABSTRACT
Gravitational lensing by rotating extended deflectors is discussed. The lens is modelled as a singular isothermal sphere. Because of spin, corrections on image positions, caustics and critical curves can be significant. Gravitomagnetic time-delays of ~0.2 d between different images of background sources can occur.

Key words: gravitation – gravitational lensing – relativity – astrometry.

1 INTRODUCTION
Mass–energy currents relative to other masses generate space–time curvature. This phenomenon, known as intrinsic gravitomagnetism, is a new feature of general theory of relativity and other conceivable post-Newtonian theories of gravity (see Ciufolini & Wheeler 1995, and references therein). The gravitomagnetic field has not been yet detected with high accuracy. Some results have been reported from laser-ranged satellites, when the Lense–Thirring precession, due to the Earth spin, was measured by studying the orbital perturbations of the LAGEOS and LAGEOS II satellites (Ciufolini & Pavlis 1998, 2004). According to the preliminary analysis, the predictions of the general theory of relativity were found to agree with the experimental values within ~10 per cent accuracy (Ciufolini & Pavlis 2004). The National Aeronautics and Space Administration (NASA) Gravity Probe B satellite should improve this measurement to an accuracy of 1 per cent. Gravitomagnetism might also play a relevant role in the dynamics of the accretion disc of a supermassive black hole or in the alignment of jets in active galactic nuclei and quasars (Ciufolini & Wheeler 1995).

Whereas the tests just mentioned are limited to the gravitational field outside a spinning body, the general theory of relativity predicts peculiar phenomena also inside a rotating shell (see Weinberg 1972, for example). Gravitational lensing can represent a tool to fully test the effects of the gravitomagnetic field (see Sereno 2003a, and reference therein). In this paper, we are interested in the gravitomagnetic time-delay induced in different images of the same source due to gravitational lensing. Whereas here we are mainly interested in intrinsic gravitomagnetism, i.e. with spinning deflectors, a translational motion of the lens can also induce interesting phenomena, which in the framework of general relativity are strictly connected to the Lorentz transformation properties of the gravitational field. The effect of the velocity of the deflector has been recently observed in the Jovian deflection experiment conducted at VLBI, which measured the time delay of light from a background quasar (Fomalont & Kopeikin 2003) but a controversy has emerged about the interpretation of this measurement (Samuel 2004). Gravitational time-delay by spinning deflectors has been addressed by several authors with very different approaches. Dymnikova (1986) discussed the additional time-delay due to rotation by integrating the light geodesics of the Kerr metric. Using the Lense–Thirring metric, Glicenstein (1999) considered the time-delay for light rays passing outside a spinning star. Kopeikin and collaborators (Kopeikin 1997; Kopeikin & Schäffer 1999; Kopeikin & Mashhoon 2002) analysed the gravitomagnetic effects in the propagation of light in the field of self-gravitating spinning bodies. The gravitational time-delay due to rotating masses was further discussed in Ciufolini et al. (2003) and Ciufolini & Ricci (2003), where the cases of light rays crossing a slowly rotating shell or propagating in the field of a distant source were analysed in the linear approximation of the general theory of relativity. Effects of an intrinsic gravitomagnetic field were further studied in the usual framework of gravitational lensing theory (Sereno 2002, 2003a,b; Sereno & Cardone 2002): (i) weak field and slow motion approximation for the lens; (ii) thin lens hypothesis (Schneider, Ehlers & Falco 1992; Petters, Levine & Wambsganss 2001). Expressions for bending and time-delay of electromagnetic waves were found for stationary spinning deflectors with general mass distributions (Sereno 2002).

The paper is organized as follows. In Section 2, the basics of gravitational lensing by a stationary extended deflector are reviewed. In Section 3, we introduce our reference model for the lens, i.e. a spinning singular isothermal sphere (SIS). Relevant lensing quantities for such a deflecting system are derived in Section 4. In Section 5, the lens equation is solved. A quantitative discussion of the gravitomagnetic time-delay is given in Section 6, where we also investigate the effect on the determination of the Hubble constant. Section 7 is devoted to some final considerations. Unless otherwise stated, throughout this paper we consider a flat cosmological model of the Universe with the cosmological constant with a Hubble constant $H_0 = 72$ km s$^{-1}$ Mpc$^{-1}$ and a pressureless cosmological density parameter $\Omega_{\Lambda} = 0.3$. 

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2 GRAVITOMAGNETIC DEFLECTION POTENTIAL

Gravitational lensing theory can be easily developed in the gravitational field of a rotating stationary source when the linear approximation of general relativity holds (Sereno 2002). The time-delay of a kinematically possible ray, with impact parameter $\xi$ in the lens plane, relative to the unlensed one, for a single lens plane, is (Sereno 2002)

$$\Delta t = \frac{1 + z_d}{c} \left[ \frac{1}{2} \frac{D_s D_l}{D_{ls}} \left( \frac{\xi}{D_s} \right)^2 - \bar{\psi}(\xi) \right]. \quad (1)$$

Here, $\bar{\psi}$ is the deflection potential, $D_s$, $D_l$, and $D_{ls}$ are the angular diameter distances between observer and source, observer and lens and lens and source, respectively, $z_d$ is the lens redshift and $\eta$ is the bi-dimensional vector position of the source in the source plane. We have neglected a constant term in equation (1), because it has no physical significance (Schneider et al. 1992).

The deflection potential can be expressed as the sum of two terms

$$\bar{\psi} \simeq \bar{\psi}_0 + \bar{\psi}_{\text{GRM}}. \quad (2)$$

The main contribution is

$$\bar{\psi}_0(\xi) = \frac{4G}{c^2} \int_{\xi_0}^{\xi} d\xi' \Sigma(\xi') \ln \left| \frac{\xi - \xi'}{\xi_0} \right|, \quad (3)$$

where $\xi_0$ is a length-scale in the lens plane and $\Sigma$ is the projected surface mass density of the deflector

$$\Sigma(\xi) = \int d\xi \rho(\xi, l) dl. \quad (4)$$

The gravitomagnetic correction to the deflection potential, up to the order $v/c$, can be expressed as (Sereno 2002)

$$\bar{\psi}_{\text{GRM}} \simeq \frac{8G}{c^3} \int_{\xi_0}^{\xi} d\xi' \Sigma(\xi')(v \cdot e_m)(\xi') \ln \left| \frac{\xi - \xi'}{\xi_0} \right|, \quad (5)$$

where $(v \cdot e_m)$ is the weighted average, along the line of sight $e_m$, of the component of the velocity $v$ along $e_m$

$$(v \cdot e_m)(\xi) = \frac{\int (v(\xi, l) \cdot e_m) \rho(\xi, l) dl}{\Sigma(\xi)}. \quad (6)$$

In the thin lens approximation, the only components of the velocities parallel to the line of sight enter the equations of gravitational lensing. We note that the time-delay function is not an observable, but the time-delay between two actual rays can be measured. Similar results, based on a multipolar description of the gravitational field of a stationary lens, can be found in Kopeikin (1997).

3 SINGULAR ISOTHERMAL SPHERE

In this section, we want to review the basics for the description of a rotating lens as a spinning isothermal sphere. Isothermal spheres are widely used in astrophysics to understand many properties of systems on very different scales, from galaxy haloes to clusters of galaxies (Schneider et al. 1992; Mo, Mao & White 1998). In particular, on the scale relevant to interpreting time-delays, isothermal models are favoured by data on early-type galaxies (Kochanek & Schechter 2004). The density profile of a SIS is

$$\rho(r) = \frac{\sigma_s^2}{2\pi G r^2}, \quad (7)$$

where $\sigma_s$ is the velocity dispersion. The corresponding projected mass density is

$$\Sigma(\xi) = \frac{\sigma_s^2}{2G} \frac{1}{\xi}. \quad (8)$$

Because the total mass is divergent, we introduce a cut-off radius $R_{\text{SIS}} \gg \xi$. The cut-off radius of the halo must be much larger than the relevant length-scale which characterizes the phenomenon, in order not to significantly affect the lensing behavior. A limiting radius can be defined as $r_n$, the radius within which the mean mass density is $n$ times the critical density of the Universe at the redshift of the galaxy $z_d$. For a SIS, it is

$$r_n = \frac{2\sigma_s}{\sqrt{nH(z_d)}}, \quad (9)$$

where $H$ is the time-dependent Hubble parameter.

The total angular momentum of a halo, $J$, can be expressed in terms of a dimensionless spin parameter $\lambda$, which represents the ratio between the actual angular velocity of the system and the hypothetical angular velocity that is needed to support the system (Padmanabhan 2002)

$$J = \lambda \frac{GM^5}{|E|^2 |\Sigma|}, \quad (10)$$

where $M$ and $E$ are the total mass and the total energy of the halo, respectively. In the hypothesis of the initial angular momentum acquired from tidal torquing, typical values of $\lambda$ can be obtained from the relation between energy and virial radius and the details of the spherical top-hat model (Padmanabhan 2002). As derived from numerical simulations, the distribution of $\lambda$ is nearly independent of the mass and the power spectrum. It can be approximated by a log-normal distribution (Vitvitska et al. 2002)

$$\rho(\lambda) d\lambda = \frac{1}{\sqrt{2\pi}\sigma_\lambda} \exp \left[ -\frac{\ln^2(\lambda/\bar{\lambda})}{2\sigma_\lambda^2} \right] d\lambda/\lambda, \quad (11)$$

with $\bar{\lambda} \simeq 0.05$ and $\sigma_\lambda \simeq 0.5$. The distribution peaks around $\lambda \simeq 0.04$ and has a width of $\sim 0.05$.

The total mass of a truncated SIS is

$$M_{\text{SIS}} = \frac{2\sigma_s^2}{G} R_{\text{SIS}}. \quad (12)$$

From the virial theorem, the total energy is easily obtained (Mo et al. 1998)

$$E_{\text{SIS}} = -M_{\text{SIS}}\sigma_s^2. \quad (13)$$

Finally, the total angular momentum of a truncated SIS can be written as

$$J_{\text{SIS}} = \lambda \frac{4\sigma_s^2 R_{\text{SIS}}^2}{G}. \quad (14)$$

In general, the angular velocity of a halo is not constant and a differential rotation should be considered (Capozziello et al. 2003). However, assuming a detailed rotation pattern does not significantly affect the results. In what follows, the case of constant angular velocity will be considered.

4 LENSING BY A ROTATING SIS

Let us consider gravitational lensing by a SIS in rigid rotation, i.e. with a constant angular velocity $\omega$, about an arbitrary axis passing through its centre. The deflection angle can be written as (Sereno & Cardone 2002)

$$\alpha_{\text{SIS}}(\xi, \theta) = 4\pi \left( \frac{\sigma_s}{c} \right)^2 \left[ \cos \theta + \frac{\omega \xi}{c} \frac{\sin 2\theta}{3} - \frac{\omega \xi}{c} \left( \frac{\cos 2\theta}{3} + 1 \right) - R_{\text{SIS}} \right], \quad (15)$$

where $R_{\text{SIS}}$ is the limiting radius of the halo.

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The parameter \( \equiv \omega L \) truncated SIS, \( \psi \) can be written as
\[
\psi = \frac{\alpha_1}{c} \left\{ \sin \theta + \frac{\omega_2}{c} \sin 2\theta \right\}
+ \frac{\alpha_2}{c} \left\{ \cos 2\theta \right\} + R_{\text{SIS}} \right\}.
\tag{16}
\]

where \((\xi, \theta)\) are polar coordinates in the lens plane and \(\omega_1\) and \(\omega_2\) are the components of \(\omega\) along the \(\xi_1\)-axis and \(\xi_2\)-axis, respectively. The parameter \(\omega\) has to be interpreted as an effective angular velocity \(\dot{\omega} \equiv J_{\text{SIS}}/I_{\text{SIS}}\) where \(I_{\text{SIS}}\) is the central momentum of inertia of a truncated SIS, \(I_{\text{SIS}} = 2/9M_{\text{SIS}}R_{\text{SIS}}^2\). In terms of the spin parameter \(\dot{\omega} = 9L \sigma_v/R_{\text{SIS}}\).

Equations (15) and (16) correct the result in equation (9) in Capozziello & Re (2001), which was obtained under the same assumptions but was affected by an error in the computation of the integrals. For a SIS, there are two main contributions to the gravitomagnetic correction to the deflection angle (Sereno & Cardone 2002): the first contribution comes from the projected momentum of inertia inside the radius \(\xi\); the second contribution is due to the mass outside \(\xi\) and can become significant in the case of a very extended lens, i.e. for a very large cut-off radius.

Let us consider a sphere rotating about the \(\xi_2\)-axis, \(\omega_1 = 0\), \(\omega_2 = \omega\). In order to change to dimensionless variables, we introduce a length-scale:
\[
\xi_0 = R_\theta = 4\pi \left( \frac{\sigma_v}{c} \right)^2 \frac{D_d D_\delta}{D_\delta}.
\tag{18}
\]

The dimensionless position vector in the lens plane is \(x \equiv \xi/\xi_0\). In what follows, \(x \equiv |x|\). The dimensionless deflection potential \(\psi\)
\[
\psi = \frac{D_d D_\delta}{D_\delta} \xi_0 \psi\text{SIS}(x_1, x_2) = x - \left( \frac{3}{2} r - x \right) L x_1.
\tag{19}
\]

where \(L \equiv (2/3)(\dot{\omega} R_\theta/c)\) is an estimate of the rotational velocity and \(r\) is the cut-off radius in units of \(R_\theta\). When \(L > 0\), the angular momentum of the lens is positively oriented along \(\xi_2\). The dimensionless Fermat potential \(\phi\) is defined as
\[
\phi(x, y) = \frac{1}{2} (x - y)^2 - \psi(x, y),
\tag{20}
\]

where \(y\) is the dimensionless source plane
\[
y \equiv y \left( \frac{D_d}{D_\delta} \xi_0 \right).
\]

The scaled deflection angle is related to the dimensionless gravitational potential \(\alpha\)
\[
\alpha(x) = \nabla \psi(x).
\tag{21}
\]

We obtain
\[
\alpha_1^\text{SIS}(x_1, x_2) = \frac{x_1}{x} + L \left( \frac{2x_1^2 + x_1^2}{x} - \frac{3}{2} x \right),
\tag{22}
\]
\[
\alpha_2^\text{SIS}(x_1, x_2) = \frac{x_2}{x} + L \frac{x_1 x_2}{x}.
\tag{23}
\]

The determinant of the Jacobian matrix reads
\[
A_{\text{SIS}}(x_1, x_2) \simeq 1 - \frac{1}{x} - L \frac{x_1}{x} \left( \frac{3}{2} - \frac{2}{x} \right).
\tag{24}
\]
and $r \simeq 15$. For some particular source positions, the shift in the image positions in the lens plane with respect to the static case can be as large as 0.1 arcsec. In Fig. 2, we plot the shift in the image positions for a source moving along the $y_1$-axis (i.e. for $y_2 = \text{const}$). The maximum variation occurs when the source nearly crosses the projected rotation axis.

The critical curve is slightly distorted. The solution of $\det A(x_1, x_2) = 0$, with respect to $x_2$, is

$$x_2(x_1) \simeq \pm \left( \sqrt{1 - x_1^2} + \frac{x_1}{\sqrt{1 - x_1^2}} L \right).$$  \hspace{1cm} (34)

Figure 3. A source (the filled dot) inside the central caustic of a rotating SIS is multiple imaged in a cross-shaped pattern; the four solid squares show the locations of the four images. The critical line is also plotted. It is $r = 15$ and $L = 3.5 \times 10^{-5}$.

The area of the critical curve slightly grows and its centre shifts of $L$ along the $x_1$-axis. The critical curve intersects the $x_1$-axis at $x_1 \simeq L \pm 1$. The changes in the width and in the height of the critical curve are of the order of $O(L^2)$.

By mapping the four extremal points of the critical curve on to the source plane through the lens equation, we can determine the corresponding cusps of the caustic. It is a diamond-shaped caustic with four cusps, centred at $(y_1, y_2) = \{ L[(3/2)r - 1], 0 \}$. The axes, of semiwidth $\sim L^2$, are parallel to the coordinate axes. The orientation and the position on the caustic depends on both the strength and orientation of the angular momentum and on the radius of the lens. When the source is inside the caustic, two additional images occur. Because the axial symmetry is broken by the gravitomagnetic field, the Einstein ring is no longer produced. A source, which is inside the central caustic, is imaged in a cross pattern (see Fig. 3).

6 TIME-DELAY

As we have seen in Section 2, the time-delay for a spinning SIS is made of three contributions: the geometrical time-delay, $\Delta t_{\text{geom}}$; the unperturbed gravitational time-delay by a static SIS, $\Delta t_0$; and finally the gravitomagnetic time-delay, $\Delta t_{\text{GRM}}$.

$$\Delta t \simeq \Delta t_{\text{geom}} + \Delta t_0 + \Delta t_{\text{GRM}}.$$  \hspace{1cm} (35)

The main contribution to the gravitational time-delay of a light ray with respect to the unperturbed path can be rewritten as

$$c \Delta t_0 = 4\pi(1 + z_d)D_d \left( \frac{\sigma_v}{c} \right)^2 \theta,$$  \hspace{1cm} (36)

where $\theta$ is the modulus of the angular position in the lens plane, $\theta = \xi/D_d$. The gravitomagnetic time-delay is

$$c \Delta t_{\text{GRM}} \simeq -24\pi(1 + z_d)D_d \left( \frac{3}{2} \frac{\theta}{\theta_{\text{SIS}}} \right) \theta_1 \left( \frac{\sigma_v}{c} \right)^3 \lambda.$$  \hspace{1cm} (37)
In general, it is $L \sim 36\pi D_d(1 + z_d) \left(\frac{\sigma_v}{c}\right)^3 \theta_1, \lambda$, \hfill (38)

where $\theta_1$ is the angular distance of an image from the projected rotation axis and $\theta_{\text{sys}} \equiv R_{\text{sys}}/D_d$. To obtain equation (38), we have used the relation $x \ll r$. An approximate relation holds between $\Delta t_0$ and the gravitomagnetic time-delay:

$$\Delta t_{\text{GRM}} \simeq -9 \cos \theta \left(\frac{\sigma_v}{c}\right) \lambda \Delta t_0.$$

\hfill (39)

Because $L \ll 1$, the angular separation between the two images is nearly

$$\Delta \theta \simeq 8\pi \left(\frac{\sigma_v}{c}\right)^2 D_b \frac{D_s}{D_d},$$

\hfill (40)

and the gravitomagnetic induced retardation between the two images, $\Delta T_{\text{GRM}} \equiv \Delta t_{\text{GRM}}(x_i) - \Delta t_{\text{GRM}}(x_b),$ can be approximated as

$$c \Delta T_{\text{GRM}} \simeq 288\pi^2 \cos \theta \left(\frac{\sigma_v}{c}\right)^3 \lambda \Delta t_0.$$

\hfill (41)

Because the two images are nearly collinear with the lens centre, the gravitomagnetic time-delay is nearly independent of the cut-off radius.

For a typical lensing galaxy at $z_d = 0.5$ with $\sigma_v \sim 250$ km s$^{-1}$, deflecting a background source at $z_i = 2.0$, the gravitomagnetic time-delay is $\sim 0.1–0.2$ d for $\lambda \sim 0.05–0.1$. In Fig. 4, the time-delay between the images is plotted as a function of the source position for a source moving perpendicularly to the projected rotation axis. The distance of the source from the $y_1$-axis determines the width of the transition between the extremal values.

\section*{6.1 Hubble constant}

Any gravitational lensing system can be used to determine the Hubble constant (Refsdal 1964). Neglecting the gravitomagnetic correction can induce an error in the estimate of the Hubble constant. In general, it is

$$H_0 \Delta t = \mathcal{F}(\sigma_v, \ldots, z_d, z_i, \Omega_0),$$

\hfill (42)

where the dimensionless function $\mathcal{F}$ depends on the lens parameters and on the cosmological density parameters, but this last dependence is not very strong. A lens model, which reproduces the positions and

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig4}
\caption{The gravitomagnetic time-delay (in units of days) for a source moving parallel to the $y_1$-axis. Thick and thin lines are for $y_2 = 0.01$ and 0.1, respectively. It is $L = 4 \times 10^{-5}, r = 15, z_d = 0.5, \sigma_v \sim 250$ km s$^{-1}$ and $z_i = 2.0$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig5}
\caption{Relative error in the estimate of the Hubble constant, due to neglecting the gravitomagnetic field, for a source moving with fixed $y_2 = 0.1$. It is $L = 2.5 \times 10^{-3}$ and $r = 15$.}
\end{figure}

magnifications of the images, provides the scaled time-delay $H_0 \Delta t$ between the images. Therefore, a measurement of $\Delta t$ will yield the Hubble constant. Let us consider a rotating galaxy, described by a SIS with known dispersion velocity and redshift, which forms multiple images of a background quasar at redshift $z_i$. An observer can measure the time-delay between the two images, $\Delta T_{\text{obs}}$, and their positions, $x_i$ and $x_b$. The unknown source position $y$ can be obtained by inverting the lens equation. In terms of the dimensionless Fermat potential $\phi$, the measured Hubble constant turns out to be

$$H_0 = \frac{1}{\Delta T_{\text{obs}}} F(z_d, z_i, \sigma_v) \phi(x_i, y) - \phi(x_b, y),$$

\hfill (43)

where

$$F(z_d, z_i, \sigma_v) \equiv (1 + z_d) \left[\frac{4\pi}{\sigma_v^2} \left(\frac{\sigma_v}{c}\right)^2 \frac{r_d D_d}{r_s} \right]$$

\hfill (44)

and $r$ is the angular diameter distance in units of $c/H_0$.

If we assume a static lens model to model the data, the estimated Hubble constant is

$$H_0^{\text{ST}} = \frac{1}{\Delta T_{\text{obs}}} F(z_d, z_i, \sigma_v) 2y^{\text{ST}},$$

\hfill (45)

where the ‘not correct’ estimated position of the source is

$$y^{\text{ST}} = \frac{1}{2} \left( \sum_{a,b} x_i - x_f / |x_f| \right).$$

\hfill (46)

The relative error in the determination of the Hubble constant is

$$\frac{\Delta H_0}{H_0} = \frac{2y^{\text{ST}} - |\phi(x_i, y) - \phi(x_b, y)|}{|\phi(x_i, y) - \phi(x_b, y)|}.$$ \hfill (47)

For a source at fixed $y_2$, the maximum error is $\sim (1/2) |L/v_2|$ (see Fig. 5). Because usually $L \lesssim 10^{-4}$, the induced relative error is really negligible.

\section*{7 DISCUSSION}

Because the physics of gravitational lensing is well understood, the gravitomagnetic time-delay may provide a new observable for the determination of the total mass and angular momentum of the lensing body (Ciufolini & Ricci 2003). Although a detailed model may be required to reproduce the overall mass distribution in the lens, interpretation of time-delay is based on a limited number of parameters. Provided the cluster where the deflector lies can be described by a simple expansion, the only parameters needed to model the time-delay are those needed to vary the average surface density of the lens
near the images and to change the ratio between the quadrupole moment of the lens and the environment (Kochanek & Schechter 2004). Furthermore, the presence of an observed Einstein ring can provide strong independent constraints on the mass distribution.

We have developed our treatment of the gravitomagnetic time-delay by modelling the lens as a SIS. Isothermal models are supported by both theoretical prejudices and estimates from observations of early-type galaxies, and the SIS turns out to be a surprisingly realistic starting point for modelling lens potentials. Gravitomagnetic time-delays of ~0.1–0.2 d can be produced in typical lensing systems. A broad range of methods for reliably determining time-delays from typical data and a deep understanding of the systematic problems have been developed in the last few years. Time-delay estimates are more and more accurate and an accuracy of 0.2 d has been already obtained in the case of B0218+357 (Biggs et al. 1999), so that the detection of the gravitomagnetic time-delay will be soon within the reach of astronomers. Observations with radio interferometers or the *Hubble Space Telescope* can measure the relative positions of the images and lenses to accuracies 0.005 arcsec. Shifts in the image positions due to a gravitomagnetic field are usually well above this limit.

Our results are nearly unaffected by the presence of low-mass satellites and stars. These substructures do not have any impact on time-delays and can only produce random perturbations of approximately 0.001 arcsec in image positions (Kochanek & Schechter 2004), quite below the gravitomagnetic effect. Deviations from circular symmetry due to either the ellipticity of the deflector or the local tidal gravity field from nearby objects should also be considered. However, for a singular isothermal model with arbitrary structure, the time-delays turn out to be independent of the angular structure (Kochanek & Schechter 2004). Other higher-order effects, such as the delay due to the quadrupole moment of the deflector, should be considered in addition to the gravitomagnetic time-delay. Unlike other effects, a gravitomagnetic field can break the circular symmetry of the lens, inducing characteristic features in lensing events (Sereno 2003b). At least in principle and for some configurations of the images, a suitable combination of the observable quantities can be used to remove additional effects, due to a quadrupole moment, from observational data (Ciufolini & Ricci 2002; Ciufolini et al. 2003). Furthermore, when the lensed images lie on opposite sides of the lens galaxy, the time-delay becomes nearly insensitive to the quadrupole structure of the deflector (Kochanek & Schechter 2004).

Measurements of gravitomagnetic time-delays could offer an interesting perspective to address the ‘angular momentum problem’. Cold dark matter models of the Universe with a substantial cosmological constant appear to fit large-scale structure observations well, but some areas of possible disagreement between theory and observations still persist. The most serious small-scale problem regards the origin and angular momentum in galaxies (Primack 2004). Two ‘angular momentum problems’ prevent the formation of realistic spiral galaxies in numerical simulations (Primack 2004): (i) overcooling in merging satellites with too much transfer of angular momentum to the dark halo; (ii) the wrong distribution of specific angular momentum in haloes, if the baryonic material has the same angular momentum distribution as the dark matter halo. Detection of gravitomagnetic effects in gravitational lensing systems due to the spin of the deflector, either through time-delay measurements, as discussed in this paper, or through observations of the rotation of the plane of polarization of light waves from the background source (Sereno 2005), could provide direct estimates of angular momentum and help in developing a better understanding of astrophysics in galaxies.

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