Symmetry energy within the BHF approach

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Abstract. We analyze the correlations of the slope and curvature parameters of the symmetry energy with the neutron skin thickness of neutron-rich isotopes, and the crust-core transition density in neutron stars. Microscopic Brueckner–Hartree–Fock results are compared with those obtained with several Skyrme and relativistic mean field models. Our results confirm that there is an inverse correlation between the neutron skin thickness and the transition density.

1. Introduction

Isospin asymmetric nuclear matter is present in nuclei, especially far away from the stability line, and in astrophysical systems, particularly in neutron stars. Therefore, a well-grounded understanding of the properties of isospin-rich nuclear matter is a necessary ingredient for the advancement of both nuclear physics and astrophysics. However, some of these properties are not well constrained yet. In particular, the density dependence of the symmetry energy $E_{\text{sym}}(\rho)$ is still an important source of uncertainties. Its value $J$ at saturation is more or less well established ($\sim 30$ MeV), and its behavior below saturation is now much better known [1]. However, for densities above $\rho_0$, $E_{\text{sym}}(\rho)$ is not well determined yet, and the predictions from different approaches strongly diverge. Why $E_{\text{sym}}(\rho)$ is so uncertain is still an open question whose answer is related to our limited knowledge of the nuclear force, and in particular of its spin and isospin dependence [2, 3, 4, 5, 6, 7, 8, 9]. Fortunately, a major effort is being carried out to study experimentally these properties, and experiments at CSR (China), FAIR (Germany), RIKEN (Japan), SPIRAL2/GANIL (France) and FRIB (USA), can probe the behavior of the symmetry energy close and above saturation density [10]. Additional information on $E_{\text{sym}}(\rho)$ can be extracted from the astrophysical observations of compact objects which open a window into both the bulk and the microscopic properties of nuclear matter at extreme isospin asymmetries [11]. In this work, we analyze the correlations of the slope, $L = 3\rho_0 \partial E_{\text{sym}} / \partial \rho|_{\rho_0}$, and curvature, $K_{\text{sym}} = 9\rho_0^2 \partial^2 E_{\text{sym}} / \partial \rho^2|_{\rho_0}$, parameters of the symmetry energy with the neutron skin thickness of neutron-rich isotopes, and the crust-core transition density in neutron stars. The results are
obtained within the microscopic Brueckner–Hartree–Fock (BHF) approach, and are compared
with those obtained with several Skyrme and relativistic mean field (RMF) models.

2. The BHF approach of asymmetric nuclear matter
Assuming charge symmetry of nuclear forces, the energy per particle of asymmetric nuclear
matter can be well approximated in terms of the isospin asymmetry parameter, \( \beta = (N – Z)/(N + Z) = (\rho_n – \rho_p)/\rho \), as

\[
\frac{E}{A}(\rho, \beta) \sim E_{SNM}(\rho) + E_{sym}(\rho)\beta^2 ,
\]

where \( E_{SNM}(\rho) \) is the energy per particle of symmetric nuclear matter, and \( E_{sym}(\rho) = E/A(\rho, \beta = 1) – E_{SNM}(\rho) \) is the so-called symmetry energy.

It is common to characterize the density dependence of the energy per particle of symmetric
matter around the saturation density \( \rho_0 \) in terms of a few bulk parameters by expanding it in a
Taylor series around \( \rho_0 \),

\[
E_{SNM}(\rho) = E_0 + \frac{K_0}{2} \left( \frac{\rho – \rho_0}{3\rho_0} \right)^2 + \frac{Q_0}{6} \left( \frac{\rho – \rho_0}{3\rho_0} \right)^3 + O(4) .
\]

The coefficients denote, respectively, the energy per particle, the incompressibility coefficient
and the third derivative of symmetric matter at saturation,

\[
E_0 = E_{SNM}(\rho = \rho_0) , \quad K_0 = 9\rho_0 \frac{\partial^2 E_{SNM}(\rho)}{\partial \rho^2} \bigg|_{\rho = \rho_0} , \quad Q_0 = 27\rho_0^3 \frac{\partial^3 E_{SNM}(\rho)}{\partial \rho^3} \bigg|_{\rho = \rho_0} .
\]

Similarly, the behaviour of the symmetry energy around saturation can be also characterized
in terms of a few bulk parameters,

\[
E_{sym}(\rho) = J + L \left( \frac{\rho – \rho_0}{3\rho_0} \right) + \frac{K_{sym}}{2} \left( \frac{\rho – \rho_0}{3\rho_0} \right)^2 + \frac{Q_{sym}}{6} \left( \frac{\rho – \rho_0}{3\rho_0} \right)^3 + O(4) ,
\]

where \( J \) is the value of the symmetry energy at saturation and the quantities \( L, K_{sym} \) and \( Q_{sym} \)
are related to its slope, curvature and third derivative, respectively, at such density,

\[
L = 3\rho_0 \frac{\partial E_{sym}(\rho)}{\partial \rho} \bigg|_{\rho = \rho_0} , \quad K_{sym} = 9\rho_0 \frac{\partial^2 E_{sym}(\rho)}{\partial \rho^2} \bigg|_{\rho = \rho_0} , \quad Q_{sym} = 27\rho_0^3 \frac{\partial^3 E_{sym}(\rho)}{\partial \rho^3} \bigg|_{\rho = \rho_0} .
\]

The BHF approach of asymmetric nuclear matter starts with the construction of all the
\( G \) matrices describing the effective interaction between two nucleons in the presence of a
surrounding medium. They are obtained by solving the Bethe–Goldstone equation

\[
G_{\tau_1 \tau_2; \tau_3 \tau_4}(\omega) = V_{\tau_1 \tau_2; \tau_3 \tau_4} + \sum_{ij} V_{\tau_1 \tau_2; \tau_i \tau_j} \frac{Q_{\tau_i \tau_j}}{\omega – \epsilon_i – \epsilon_j + i\eta} G_{\tau_i \tau_j; \tau_3 \tau_4}(\omega)
\]

where \( \tau = n, p \) indicates the isospin projection of the two nucleons in the initial, intermediate
and final states, \( V \) denotes the bare NN interaction, \( Q_{\tau_i \tau_j} \) the Pauli operator that allows
only intermediate states compatible with the Pauli principle, and \( \omega \), the so-called starting
energy, which corresponds to the sum of non-relativistic single-particle energies of the interacting
nucleons. The single-particle energy \( \epsilon_\tau \) of a nucleon with momentum \( \vec{k} \) is given by

\[
\epsilon_\tau(\vec{k}) = \frac{\hbar^2 k^2}{2m_\tau} + Re[U_\tau(\vec{k})] ,
\]
Figure 1. Neutron skin thickness for $^{208}$Pb (upper panels) and $^{132}$Sn (lower panels) versus $L$ (left panels) and $K_{sym}$ (right panels). The vertical dashed lines on the left panels denote the constraints on $L$ from isospin diffusion (ID) [10].

where the single-particle potential $U_{\tau}(\vec{k})$ represents the mean field “felt” by a nucleon due to its interaction with the other nucleons of the medium. In the BHF approximation, $U(\vec{k})$ is calculated through the on-shell energy $G$-matrix, and is given by

$$U_{\tau}(\vec{k}) = \sum_{\tau'} |\vec{k}'| < k_{F_{\tau'}} \langle \vec{k}' \vec{k} | G_{\tau\tau'}; \tau' (\omega = \epsilon_{\tau}(k) + \epsilon_{\tau'}(k')) | \vec{k}' \vec{k} \rangle_A$$

(8)

where the sum runs over all neutron and proton occupied states, and the matrix elements are properly antisymmetrized. Once a self-consistent solution of Eqs. (6) and (8) is achieved, the energy per particle of asymmetric matter (and consequently the symmetry energy) can be calculated as

$$\frac{E}{A}(\rho, \beta) = \frac{1}{A} \sum_{\tau} \sum_{|\vec{k}| < k_{F_{\tau}}} \left( \frac{k^2}{2m_{\tau}} + \frac{1}{2} Re[U_{\tau}(\vec{k})] \right).$$

(9)

We note here that the BHF calculation carried out in this work uses the realistic Argonne 18 [12] two-body potential and the Urbana IX [13] three-body force. For the use in the BHF calculation the three-body-force has been reduced to a two-body density dependent force by averaging over the coordinates of the third nucleon [14]. For further reading, and details on the Skyrme forces and the relativistic models considered in this work, the reader is referred to Ref. [15].

3. Results

It has been shown by Brown and Typel [16], and confirmed later by other authors, that the neutron skin thickness, $\delta R = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$, calculated in mean field models with either non-relativistic or relativistic effective interactions is very sensitive to the density dependence of the nuclear symmetry energy, and, in particular, to the slope parameter $L$ at the normal nuclear
saturation density. Using the Brueckner approach and the several Skyrme forces and relativistic models considered, we have made an estimation of the neutron skin thickness of $^{208}\text{Pb}$ and $^{132}\text{Sn}$ and we have studied its correlation with the slope $L$ and curvature $K_{\text{sym}}$ parameters. In Fig. 1 we show the correlation between $\delta R$ for $^{208}\text{Pb}$ and $^{132}\text{Sn}$ with the parameters $L$ (left panels) and $K_{\text{sym}}$ (right panels). It can be seen, as has already been shown by other authors, that both the Skyrme forces and the relativistic models predict values of $\delta R$ that exhibit a tight linear correlation with $L$. Note that the microscopic Brueckner calculation is in excellent agreement with this correlation. The linear increase of $\delta R$ with $L$ is not surprising since the thickness of the neutron skin in heavy nuclei is determined by the pressure difference between neutrons and protons, which is proportional to the parameter $L$, $P(\rho_0, \beta) \sim L\rho_0^{2/3}$ [17].

Another sensitive quantity to the symmetry energy is the transition density $\rho_t$ from non-uniform to uniform $\beta$-stable matter which may be estimated from the crossing of the $\beta$-equilibrium equation of state with the thermodynamical spinodal instability line. As it has been shown in Ref. [18] the predictions for the transition density from the thermodynamical spinodal are $\sim 15\%$ larger than the value obtained from a Thomas–Fermi calculation of the pasta phase. Therefore, we expect that our estimation of the transition density from the thermodynamical spinodal will define an upper bound to the true transition density [19]. We display in Fig. 2 $\rho_t$ as a function of the parameters $L$ and $K_{\text{sym}}$ for the BHF calculation together with the predictions of the several Skyrme forces and relativistic models. It is clear from the figure that $\rho_t$ is sensitive to the slope and curvature parameters $L$ and $K_{\text{sym}}$ of the symmetry energy, decreasing almost linearly with increasing $L$ and $K_{\text{sym}}$ in agreement with recent results [20, 21]. Using the experimental constraint on $L$ from isospin diffusion, we estimate the transition density to be between $0.063\,\text{fm}^{-3}$ and $0.083\,\text{fm}^{-3}$. This range is in reasonable agreement with the value of $\rho_t \approx 0.08\,\text{fm}^{-3}$ often used in the literature.

Finally, we show in Fig. 3 the transition density $\rho_t$ from non-uniform to $\beta$-stable matter as a function of the neutron skin thickness in $^{208}\text{Pb}$ (left panel) and $^{132}\text{Sn}$ (right panel) for our Brueckner calculation and the different Skyrme forces and relativistic models. The figure shows, as already pointed out by Horowitz and Piekarewicz [22], that there is an inverse correlation between the neutron skin thickness and $\rho_t$. In [22] a Walecka model with non-linear $\omega - \rho$ terms was used and the transition density was obtained with an RPA approach. We confirm the same trend for a larger set of nuclear models. Note that, again, our microscopic Brueckner results are in very good agreement with this correlation. As pointed out in Ref. [22], these

![Figure 2](image-url)
results suggest that an accurate measurement of the neutron radius in heavy nuclei like $^{208}$Pb or $^{132}$Sn is absolutely essential since it can provide considerable and valuable information on the properties of neutron star’s crust.

4. Summary
Summarizing, we have studied the correlation between the neutron skin thickness of neutron-rich isotopes and the parameters $L$ and $K_{sym}$. We have found that the BHF results are in very good agreement with the correlations already predicted by other authors using non-relativistic and relativistic effective models. We have also analyzed the correlations of $L$ and $K_{sym}$ with the transition density $\rho_t$ from the crust to the core in cold neutron stars. Using the experimental constraint on $L$ from isospin diffusion, we have estimated the value of $\rho_t$ to be between 0.063 fm$^{-3}$ and 0.083 fm$^{-3}$, a range in reasonable agreement with the the value of $\rho_t \approx 0.08$ fm$^{-3}$ often used in the literature. Finally, we have confirmed for a large set of nuclear models that there is an inverse correlation between the neutron skin thickness and the transition density $\rho_t$, a trend pointed out first by Horowitz and Piekarewicz in Ref. [22].

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