A Critical Probability for Biclique Partition of $G_{n,p}$

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Abstract

The biclique partition number of a graph $G = (V, E)$, denoted $bp(G)$, is the minimum number of pairwise edge disjoint complete bipartite subgraphs of $G$ so that each edge of $G$ belongs to exactly one of them. It is easy to see that $bp(G) \leq n - \alpha(G)$, where $\alpha(G)$ is the maximum size of an independent set of $G$. Erdős conjectured in the 80's that for almost every graph $G$ equality holds; i.e., if $G = G_{n,1/2}$ then $bp(G) = n - \alpha(G)$ with high probability. Alon showed that this is false. We show that the conjecture of Erdős is true if we instead take $G = G_{n,p}$, where $p$ is constant and less than a certain threshold value $p_0 \approx 0.312$. This verifies a conjecture of Chung and Peng for these values of $p$. We also show that if $p_0 < p < 1/2$ then $bp(G_{n,p}) = n - (1 + \Theta(1))\alpha(G_{n,p})$ with high probability.

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