Superconducting phase transitions in frustrated Josephson-junction arrays on a dice lattice

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Transport measurements are carried out on dice Josephson-junction arrays with the frustration index $f = 1/3$ and $1/2$ which possess, within the limit of the XY model, an accidental degeneracy of the ground states as a consequence of the formation of zero-energy domain walls. The measurements demonstrate that both the systems undergo a phase transition to a superconducting vortex-ordered state at considerably high temperatures. The experimental findings are in apparent contradiction with the theoretical expectation that frustration effects in the $f = 1/3$ system are particularly strong enough to suppress a vortex-ordering transition down to near zero temperature. The data for $f = 1/2$ are more consistent with theoretical evaluations. The agreement between the experiments and the Monte Carlo simulations of a XY model for $f = 1/3$ suggests that the order-from-disorder mechanism for the removal of an accidental degeneracy may still be effective in the $f = 1/3$ system. The transport data also reveal that the dice arrays with zero-energy domain walls experience a much slower critical relaxation than other frustrated arrays only with finite-energy walls.

Vortex matters in Josephson-junction arrays (JJA’s) and superconducting wire networks exposed to a magnetic field have been extensively studied in recent years. Studies of the systems of a variety of geometries have been concentrated especially on the nature of ordering transitions and excitations in the ordered state. More recently, special attention is given to highly frustrated systems which are unable to simultaneously satisfy the competing interactions and consequently have the highly degenerate ground states. The common cause to lift the accidental degeneracy is thermal fluctuations in the continuous phase variable, that is, spin waves. The difference in free energy of the fluctuations forces many systems to select a particular vortex pattern at finite temperatures. This mechanism of lifting an accidental degeneracy is often referred to as order-from-disorder.

Frustrated JJA’s on the dice (or $T_3$) lattice have attracted recent attention because of the possibility that the order-from-disorder mechanism for the removal of an accidental degeneracy may not be effective in the systems. Theoretical evaluations on the frustrated XY model, which most closely represents a JJA in a magnetic field, have demonstrated that for the frustration index (defined as the magnetic flux through a rhombus tile in units of flux quantum) $f = 1/3$, frustration effects on a dice array are particularly large and the system fails to order down to zero temperatures. The stabilization of a specific vortex pattern was expected to appear at extremely low temperatures only when one goes beyond the limits of the XY model by including the magnetic interactions of currents present in a real JJA. The predicted vortex-ordering transition temperature for $f = 1/3$ is as low as 0.01 in units of $J/k_B$, where $J$ is a junction coupling strength and $k_B$ the Boltzmann constant. However, it was also expected that the dynamically quenched relaxation of the vortex patterns may prevent an observation of the vortex-pattern ordering in experiments. For the fully-frustrated dice array with $f = 1/2$, a commensurate vortex pattern was expected to be similarly stabilized at $T \lesssim 0.1 J/k_B$ by the magnetic interactions of currents in a JJA rather than by the fluctuations. It was argued for $f = 1/2$ that the order-from-disorder mechanism becomes effective only at extremely low temperatures due to the reduced contribution to free energy from the fluctuations as a consequence of a hidden gauge symmetry and thus the magnetic interactions of current is a more important source for the ordering of vortices.

The results of theoretical evaluations contradict the Monte Carlo (MC) simulations of a XY model exhibiting a periodic vortex pattern for $f = 1/3$ at $T \lesssim 0.2 J/k_B$ as a consequence of a phase transition. For $f = 1/2$, the simulations present a Berezinskii-Kosterlitz-Thouless-like (BKT-like) transition to a dynamic glass state at $T \sim 0.05 J/k_B$.

The Bitter decoration experiments on superconducting wire networks have also revealed a periodic vortex pattern in the $f = 1/3$ case and no trace of regular vortex ordering for $f = 1/2$. For superconducting wire networks, both the magnitude and the phase of superconducting order parameter can vary near the temperatures of interest. Hence, a quite different situation may develop in JJA’s where the phase fluctuations play a more important role.

This paper presents the results of transport measurements on dice JJA’s with $f = 1/3$ and $1/2$. The results show a substantial departure from the theoretical behavior for $f = 1/3$. The agreement between the experiments and the Monte Carlo simulations for $f = 1/3$ suggests
that the order-from-disorder mechanism may still be effective for ordering in the $f=1/3$ system. The data for $f=1/2$ are more consistent with theoretical evaluations.

The experiments were performed on a dice array of $404 \times 525 \text{Nb/Cu/Nb Josephson junctions. The photograph in Fig. 1}$ shows a portion of the JJA sample used in this work. Nb islands with a thickness of $0.2 \mu m$ and an arm width of $4 \mu m$ were disposed on a $0.3-\mu m$-thick Cu film periodically with the elementary side length of the rhombus cell of $16 \mu m$ and a separation between adjacent islands of $1.4 \mu m$. The variation of the junction separation in the sample was less than $0.1 \mu m$. Measurements of the resistance and the $IV$ characteristics were carried out by using the standard four-probe technique. The sample voltage was measured by a transformer-coupled lock-in amplifier with a square-wave current at $23 \text{ Hz}$. The single-junction critical current $i_c$ at low temperatures was obtained directly from the $IV$ curve. The $i_c$ and the junction coupling strength $J(=\hbar i_c/2e)$ at high temperatures were determined by extrapolating the $i_c$ vs $T$ data at low temperatures by the use of the de Gennes formula\cite{14,17,18,19} for a proximity-coupled junction in the dirty limit. Additional details of the measurements are described in Ref. 14.

Figure 1 displays the resistance of the sample with $30-\mu A$ excitation current plotted against $f$ at four different temperatures. Local minima are present at low order rationals $f=1/6$, $1/3$, $2/3$, and $5/6$. A novel feature appears at $f=1/2$. The anomalous positive cusp at $f=1/2$ does not disappear even at low temperatures close to the superconducting transition temperature $T_c \sim 3.85 \text{ K}$, determined from $IV$ characteristics measurements (to be described below). The $f-R$ curve semi-quantitatively reflects the increase in the mean-field superconducting transition temperature as a function of $f$. One can thus estimate from the curve the energy cost for different vortex configurations. In general, the ground state at simple rational $f$ has a simple periodicity\cite{14,17,18,19}. The ground states for other $f$’s were found to consist of domains of nearby simple rational $f$’s.\cite{15,16,17} A finite energy required for exciting a domain wall causes a local minimum of the system energy at the simple rational $f$’s. If the low $T_c$ and the formation of a dynamic vortex glass proposed for $f=1/3$ in theory were the case, a positive cusp should appear at $f=1/3$. The appearance of a distinct minimum at $f=1/3$ in the $f-R$ curve indicates, against the theoretical prediction\cite{12}, that the system with $f=1/3$ may have a vortex-ordered state with a locally minimal energy and a simple periodic structure at considerably high temperatures. The positive cusp at $f=1/2$ suggests that the system fails to order within mean-field approximations and that excitations of domain walls cost an arbitrarily small energy. Apart from the $f-R$ data, the $T-R$ data in Fig. 2 of the sample with a smaller excitation current of $1 \mu A$ demonstrate that the $f=1/2$ system as well as the $f=1/3$ system may eventually make a superconducting transition to an ordered state when the temperature is reduced below $\sim 3.9 \text{ K}$.

The superconductivity for $f=1/2$ is verified by the $IV$ data plotted in Fig. 3. The distinct activated character of the low-temperatures $IV$ curves reveals a superconducting state with a long-range phase order as the ground state. As displayed more plainly in the $d(\log V)/d(\log I)$ vs $I/I_c$ plot of the same data, the low-temperature activated character changes to the high-temperature resistive character at $T \sim 3.85 \text{ K}$, corresponding to 0.10 in units of $J/k_B$. The $IV$ characteristics are quite similar to what observed in frustrated square arrays experiencing a vortex-solid melting transition\cite{14,17,18,19}. The similarity suggests that the superconducting-to-resistive transition at $T \sim 3.85 \text{ K}$ is a melting transition of a vortex solid driven by domain walls. Even though the $f-R$ data are compatible with the MC simulations\cite{14} exhibiting the dynamic glass state at low temperatures, the $IV$ data appear to be more consistent with the theoretical evaluations\cite{12}. The positive cusp at $f=1/2$ can be related with the extreme smallness of the fluctuation-induced free energy of zero-energy walls.
The $IV$ data for $f = 1/3$ are shown in Fig. 3. The evolution of the $IV$ data with temperature is similar to that for $f = 1/2$. The $IV$ curves at low temperatures have an exponential form, indicative of a vortex-ordered state with a long-range superconducting phase coherence. With the temperature raised, the low-temperature activated behavior crosses over to a resistive form. The $IV$ data along with the $f-R$ data indicate that the $f = 1/3$ system undergoes a superconducting phase transition into a vortex-ordered state at $T \sim 4.15$ K ($= 0.2J/k_B$). The $T_c \sim 0.2J/k_B$ is comparable to those of other frustrated arrays without any accidental degeneracy. The occurrence of a superconducting phase transition into a vortex-ordered state at such a high temperature is in apparent contradiction with the theoretical argument that frustration effects for $f = 1/3$ are particularly large enough to prevent the system from developing any order down to near zero temperature. However, it is in agreement with the MC simulations of a $XY$ model and compatible also with the decoration experiments on wire networks. The agreement between the experiments and the simulations may imply that the removal of an accidental degeneracy for $f = 1/3$ can be explained within the frame of a $XY$ model or that the order-from-disorder mechanism may still be effective in the $f = 1/3$ system.

Another interesting feature of our data is that the dynamic critical exponent $z$ of the systems determined from the $IV$ data is unusually high. For a continuous superconducting transition, the $IV$ data satisfy a simple power-law $IV$ relation, $V \sim I^{z+1}$, at $T_c$. One can thus obtain $z$ directly from the straight log $I$ vs log $V$ isotherm at $T_c$. $z$ of the dice arrays determined from the $IV$ data is $\sim 1.7$ for both $f = 1/3$ and $1/2$, which is much larger than those ($z \lesssim 1$) of other frustrated arrays experiencing a vortex-lattice melting transition driven by finite-energy domain walls. The exponential $IV$ characteristics at $T < T_c$ do not allow one to relate the large $z$ to the possible occurrence of a BKT-like transition. The large $z$ demonstrates that the critical relaxation in the dice arrays with an accidental degeneracy is much
FIG. 4: Evolution of the IV curves with temperature for \( f = 1/3 \).