Comparative statics analysis of cubic mechanism

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Abstract. Most of current dust removal devices at construction site conduct the dust removal manually by coordinating the large-scale tools, and have low dust removal efficiency and small scope, with lower automation degree. For that reason, such paper designs a kind of self-induction dust removal system at construction site based on the charged spray. Such system includes the dust detection module, bus control module and electric spray device to guarantee the accuracy and timeliness for dust removal. Through experimental comparison and analysis, such system could improve the dust removal efficiency at construction site to some extent.

1. Introduction

Series and parallel mechanisms have their own advantages and disadvantages. These advantages and disadvantages have complementary relations. From the work space, Number ratio of CNC axes, performance-price ratio and other comprehensive effects, the full parallel robot is not necessarily the best, at present, the parallel and series compound robots are more valuable [1]. Serial and parallel robots can integrate the advantages of parallel and series mechanism, is a kind of very promising CNC machine [2]. In 2001, Zhao Wei [3] proposed a novel series parallel micro operation robot experimental system, the experimental results show that the composition, control and calibration method of the micro-operating robot system are effective for the completion of high-precision micro-operation. Xu Liju[4] realized the configuration scheme of a hybrid virtual axis machine tool with multi-coordinate NC machining by using the 4 degree of freedom spatial parallel mechanism as the main feeding mechanism and the bidirectional moving platform as an auxiliary. Liu Haitao[5] studies a new type of reconfigurable 5 degree of freedom manipulator composed of 2 degrees of freedom spherical parallel mechanism and two rotating and one mobile serial kinematic chain. In 2002, the robotics and Mechanical Design Laboratory of University of California presented a new type of 3 degree of freedom universal Cartesian coordinates series parallel robot UCR [6]. In this paper, a new hybrid cubic mechanism based on S (P) [T] output and P input is proposed (Among them, "(P)" is a passive telescopic joint, that is, an auxiliary joint. "[T]" is the output joint) [7], as shown in Figure 1, the motion of the space 6 degrees of freedom is realized through three-dimensional space linear driving control [8]. The position control and attitude control of the output platform of the mechanism achieve decoupling, the position control and attitude control of the output platform of the mechanism...
achieve decoupling, which simplifies the difficulty of control and compensation motion. The design of the new plane spherical hinge improves the traditional ball hinge's restriction on the swing angle. The symmetrical mechanism design makes the whole production, assembly and replaceability of the mechanism superior to the traditional Stewart platform mechanism. Under the premise of a given mechanism's position, attitude and output force screw, in this paper, the mechanism is further studied by using the theory of multi-rigid body screw theory to reverse the force screw of each hinge point. And combined with examples, we give intuitive analysis and calculation results for the research objects.

![Figure 1. The diagram of space cubic mechanism.](image1)

2. **Force screw analysis of mechanism force**

As shown in Figure 2, the position of each hinge point of the mechanism and the attitude of the output spindle in space have been known, the force screw at the end of the spindle output can be expressed as the force $F_m$ and the force moment $M_m$ with $F_m$ as the axis of rotation, because of the space force acting on a rigid body can be simplified as a force $f_i(S_i;S_0)$ and a couple $C_z(0;S_z)$, The force line vector and the couple vector can be any Angle, the two can be combined as a screw by using Screw Algebra.

$$f_iS_i = f_iS_i + \in f_iS_i^0$$

(1)

And $S_i$ is a unit screw, $S_i$ is a unit vector, $S_i \cdot S_i = 1$. Because $S_i$ is orthogonal to $S_i^0$, there is $S_i \cdot S_i^0 = 0$. Under the premise of the motion sliders, connecting rods, guideways and output spindles in the mechanism are rigid bodies, the space force and torque of the output spindle O1O2 and each input motion slider are analyzed.

![Figure 2. The diagram of mechanical force.](image2)

First of all, suppose that the distance between the upper and lower moving platform center is $h$, and the distance from the lower moving platform to the point $P$ of the tool is $h$, the azimuth angle of the
main axis in X, Y and Z axis is $\alpha$, $\beta$ and $\gamma$ respectively. In the following expressions, c and s before the Angle symbol represent cos and sin.

As shown in Figure 3, with the center point $O_1$ as the fulcrum, we list the torque balance equation (2) to (4).

$$F_w^x \cdot |P_0| \cdot |c\beta| + F_w^y \cdot |P_0| \cdot |c\gamma| + M_w^z = F_{\alpha_1}^x \cdot |O_1O_2| \cdot |c\gamma| + F_{\beta_1}^y \cdot |O_1O_2| \cdot |c\beta| \quad (2)$$

$$F_w^x \cdot |P_0| \cdot |c\alpha| + F_w^y \cdot |P_0| \cdot |c\gamma| + M_w^z = F_{\alpha_2}^x \cdot |O_1O_2| \cdot |c\gamma| + F_{\gamma_2}^y \cdot |O_1O_2| \cdot |c\alpha| \quad (3)$$

$$F_w^x \cdot |P_0| \cdot |c\beta| + F_w^y \cdot |P_0| \cdot |c\gamma| + M_w^z = F_{\beta_2}^x \cdot |O_1O_2| \cdot |c\alpha| + F_{\beta_2}^y \cdot |O_1O_2| \cdot |c\beta| \quad (4)$$

As shown in Figure 4, with the center point $O_2$ as the fulcrum, we list the torque balance equation (5) to (7).

$$F_w^x \cdot |P_0| \cdot |c\beta| + F_w^y \cdot |P_0| \cdot |c\gamma| + M_w^z = F_{\alpha_1}^x \cdot |O_1O_2| \cdot |c\gamma| + F_{\beta_1}^y \cdot |O_1O_2| \cdot |c\beta| \quad (5)$$

$$F_w^x \cdot |P_0| \cdot |c\alpha| + F_w^y \cdot |P_0| \cdot |c\gamma| + M_w^z = F_{\alpha_2}^x \cdot |O_1O_2| \cdot |c\gamma| + F_{\gamma_2}^y \cdot |O_1O_2| \cdot |c\alpha| \quad (6)$$

$$F_w^x \cdot |P_0| \cdot |c\beta| + F_w^y \cdot |P_0| \cdot |c\gamma| + M_w^z = F_{\beta_2}^x \cdot |O_1O_2| \cdot |c\alpha| + F_{\beta_2}^y \cdot |O_1O_2| \cdot |c\beta| \quad (7)$$
Figure 5. Force screw analysis of spindle with $O_2$ as a fulcrum under special circumstances.

As shown in Figure 5, when the external force and the torque direction of the output spindle are coincident with the spindle axis, since the lower moving platform is a hook hinge, the rotation of the spindle axis is limited, so it is only necessary to consider the force of the lower moving platform. Put the center point $O_2$ as the fulcrum and list the moment balance equation (8) to (10).

$$F^x_m = F^x_{O_1} \quad (8)$$

$$F^y_m = F^y_{O_1} \quad (9)$$

$$F^z_m = F^z_{O_1} \quad (10)$$

From the above formula (2) to formula (7), we can know that there are 6 unknown parameters, that is, the force components of each hinge point along the X, Y, and Z axes, because there are 6 equations in all, the equations can be solved.

Among them, $F^x_{O_1}, F^y_{O_1}, F^z_{O_1}, F^x_{O_2}, F^y_{O_2}, F^z_{O_2}$ are the force components of each hinge point along the X, Y, and Z directions on the moving platform. The force condition of each hinge point on the moving platform can be rewritten into equation (11).

$$[J][U]=[b] \quad (11)$$

Among them, $[J]$ is the Jacobi matrix, and $[U]$ is the unknown variable matrix that contains the hinge point force components of each moving platform, and $[b]$ is a known constant matrix, and $[U]=[F^x_{O_1} \ F^y_{O_1} \ F^z_{O_1} \ F^x_{O_2} \ F^y_{O_2} \ F^z_{O_2}]^T$. When $[J]^{-1}$ exists, the force $[U]$ at each hinge point of the moving platform can be calculated:

$$[U]=[J]^{-1}[b] \quad (12)$$

Through further analysis, it can be seen that the force moment of $O_1$ and $O_2$ at the center of the upper and lower moving platform of the mechanism is zero, so that the statics inverse solution of the moving platform hinge of the mechanism can be obtained.

Through the force screw equilibrium equation, the force and torque at the input slider $Ci(i=1,2,3,4,5,6)$ can be quickly obtained. Because of the special structure of the mechanism, the relationship between the three force components of each input motion slider along the three-dimensional axis of space (as shown in Figure 2) and the three force components along the three-dimensional coordinate axis (as shown in Figure 2) at the center point of the upper and lower moving platform is as follows: $F^z_{C_1} = F^z_{O_1}, F^z_{C_2} = F^z_{O_2}, F^z_{C_3} = F^z_{O_1}, F^z_{C_4} = F^z_{O_2}, F^z_{C_5} = F^z_{O_1}, F^z_{C_6} = F^z_{O_2}$. The residual force component of each input slider is 0. The three components of the torque at each input slider along the spatial coordinate axis (as shown in Figure 2) are as follows.
\[ M_{x_1} = F_{x_1}^z \times |O_1 C_1^z| \] (13)

\[ M_{x_1} = F_{x_1}^z \times |O_2 C_1^x| \] (14)

\[ M_{x_1} = F_{x_2}^z \times |O_1 C_2^y| + F_{x_3}^y \times |O_2 C_2^x| \] (15)

\[ M_{x_2} = F_{x_1}^z \times |O_1 C_2^y| \] (16)

\[ M_{x_2} = F_{x_2}^z \times |O_2 C_2^y| \] (17)

\[ M_{x_2} = F_{x_1}^x \times |O_1 C_2^y| + F_{x_2}^y \times |O_2 C_2^x| \] (18)

\[ M_{x_3} = F_{x_1}^z \times |O_1 C_3^z| \] (19)

\[ M_{x_3} = 0 \] (20)

\[ M_{x_3} = F_{x_1}^z \times |O_2 C_3^y| \] (21)

\[ M_{x_4} = F_{x_2}^z \times |O_1 C_4^z| \] (22)

\[ M_{x_4} = F_{x_2}^z \times |O_2 C_4^z| \] (23)

\[ M_{x_4} = F_{x_2}^z \times |O_1 C_4^z| + F_{x_3}^y \times |O_2 C_4^x| \] (24)

\[ M_{x_5} = F_{x_2}^z \times |O_2 C_5^z| \] (25)

\[ M_{x_5} = F_{x_2}^z \times |O_2 C_5^z| + F_{x_3}^y \times |O_2 C_5^y| \] (26)

\[ M_{x_6} = F_{x_2}^z \times |O_2 C_6^z| \] (27)

\[ M_{x_6} = F_{x_2}^z \times |O_2 C_6^y| \] (28)

\[ M_{x_6} = 0 \] (29)
In the form (12) to (30), $O_x i C O_1 , O_y i C O_1 , O_z i C O_1 (i = 1,2,3)$ are the spatial vector components of the lower moving platform center to the lower moving platform slide along the X, Y, and Z directions, $O_x j C O_2 , O_y j C O_2 , O_z j C O_2 (j = 4,5,6)$ are the spatial vector components of the upper moving platform center to the upper moving platform slider along the X, Y, and Z directions. As shown in Figure 5, the mechanism is in a special stress situation, the center of the upper platform of the mechanism is in a state of no force and torque, only the center of the lower moving platform is subjected to force and torque, so the formula (31) to (33) can get.

$$F_{C_1}^z = F_m \cdot c\gamma$$  \hspace{1cm} (31)

$$F_{C_2}^y = F_m \cdot c\beta$$  \hspace{1cm} (32)

$$F_{C_3}^x = F_m \cdot c\alpha$$  \hspace{1cm} (33)

The torque at each input slide of the lower moving platform is the same equation. (12) to (20), and the torque of each input slide of the upper moving platform is 0.

3. Calculation example analysis

Suppose that $a = 1.0 \text{m}$, $|PO_1| = 0.1 \text{ m}$, $|O_1O_2| = 0.2 \text{ m}$, $|O_1C_3| = 0.1 \text{ m}$, $|O_2C_6| = 0.1 \text{ m}$, the vertical height of Output spindle end point P in the cubic mechanism is 0.3m, In a plane parallel to the horizontal plane, Taking the spatial point S : (500,500,300) as the center of the circle, do a circular motion with a radius of 0.2m around the vertical axis $O_z$. The cosine Angle of the output spindle and the coordinate axis $O_x$ is always 30°, The forces acting on the organization are as follows: $F_m=100\text{N}, M_m=100\text{Nm}$, the direction of its space axis is [1 1 1]. The force and torque of each input motion slider can be obtained by Programming and drawing. As shown in Figure 6, When the stress and moment are in a special case (shown in Figure 5), the results of force and torque at each input slide of the lower moving platform are shown in Figure 7. (Note: The direction of the linear vector is the same as that of the corresponding coordinate axis, and its value is positive, otherwise it is negative. The vector that is rotating counterclockwise is positive, otherwise it is negative)

(a) Stress and torque at point $C_1$
(b) Stress and torque at point $C_2$

(c) Stress and torque at point $C_3$

(d) Stress and torque at point $C_4$

(e) Stress and torque at point $C_5$
Figure 6. Under normal circumstances, the stress and torque of input motion sliders at the upper and lower 

(f) Stress and torque at point $C_6$

Figure 7. Under special circumstances, the stress and torque of input motion sliders at the upper and lower 

(a) Stress and torque at point $C_1$

(b) Stress and torque at point $C_2$

(c) Stress and torque at point $C_3$
4. Conclusion
Due to the complex structure and workspace constraints of traditional parallel machine tools, there are a series of problems such as the coupling phenomenon and the difficulty of control algorithm, therefore, a new spatial cube structure with a new structure form is put forward to compensate for these shortcomings, and at the same time, it can also meet the actual needs of industrial production. The static analysis of the mechanism is carried out by the knowledge of the force helix theory. The analysis of the mechanism in the general and special stress conditions is carried out respectively, and the results of the derivation are calculated by computer programming. At the same time, the force and torque diagram of the input points under given conditions are presented, which lays a foundation for further research. Since this mechanism has not only some advantages of series mechanism, but also inherits some advantages of parallel mechanism, it is a kind of mechanism which has great research and practical use value. With the further research work, complex machining problems such as space surface processing will become a very good application area.

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