Pilot Assignment in Cell-Free Massive MIMO based on the Hungarian Algorithm

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Abstract—This letter focuses on the problem of pilot assignment in cell-free massive MIMO systems. Exploiting the well-known Hungarian algorithms, several algorithms are proposed, either maximizing the system throughput, or maximizing the system fairness. The algorithms operate based on the knowledge of large scale fading coefficients and of the positions of the mobile stations. However, the latter information is not really necessary, since the paper shows that large scale fading coefficients can be used as a proxy for the distances between the mobile users and the access points with a very limited performance loss. Numerical results will show that the proposed pilot assignment algorithms outperform several competing alternatives available in the literature.

Index Terms—cell-free, massive MIMO, pilot assignment, Hungarian algorithm, wireless networks, 5G, 6G

I. INTRODUCTION

Cell-free (CF) massive MIMO (mMIMO) is a wireless network deployment architecture credited to be a possible evolution of traditional multicell mMIMO systems [1], [2]. In CF mMIMO, a very large number of distributed single-antenna access-points (APs) serves several mobile stations (MSs) using the same time-frequency resource. All APs are connected to a central processing unit (CPU) and cooperate via a backhaul network, and time-division duplex (TDD) protocol is used. CF mMIMO systems have actually no cell boundaries and benefit from large-scale fading diversity. They are thus able to ensure an improved level of fairness across users when compared with multicell mMIMO systems [3]–[5].

Similarly to multicell mMIMO, the performance of CF mMIMO systems is critically affected by the lack of a sufficiently large number of orthogonal pilot sequences, which prevents the possibility of acquiring channel state information (CSI) with no interference. The use of properly designed pilot assignment (PA) algorithms, thus, is crucial in order to ensure good performance in highly loaded networks. One of the first papers dealing with the problem of PA is [1]: based on the knowledge of the large-scale fading (LSF) channel coefficients, the greedy algorithm in [1], starting from a random PA, iteratively updates the pilot of the worst performing MS in order to increase the system fairness. The authors [6], instead, propose to use the algorithm in [1] using as starting point an assignment based on the location of the MSs. Similarly, patent [7] proposed an iterative algorithm, based on consecutive updates of the pilots for the worst and best performing MSs, again aiming at the maximization of the system fairness. In [8] a PA algorithm based on the knowledge of the MSs’ positions is proposed. Finally, reference [9] neglects the PA problem and shows that the channel estimation error can be lowered also through the optimization of the powers used to transmit the pilots.

In this paper we focus on the PA problem for CF mMIMO systems, and, leveraging the well-known Hungarian algorithm [10], introduce four different algorithms aimed at the maximization of either the system throughput or the system fairness, exploiting either the location of the MSs or the knowledge of the LSF coefficients as a proxy of the distances between the MSs and the APs. The numerical results, provided in Section IV, will reveal the superiority of the newly proposed solutions with respect to competing alternatives.

II. SYSTEM MODEL AND PERFORMANCE MEASURES

We consider an area with $K$ single-antenna MSs and $M$ APs with $N_{AP}$ antennas connected, by means of a backhaul network, to a CPU wherein data-decoding is performed. We denote by $K_m$ and $M_k$ the set of MSs served by the $m$-th AP, and the set of APs serving the $k$-th MS, respectively. The symbol $g_{k,m}$ denotes the $N_{AP}$-dimensional vector representing the channel between the $k$-th MS and the $m$-th AP; we assume $g_{k,m} = \sqrt{\beta_{k,m}} h_{k,m}$, with $h_{k,m}$ an $N_{AP}$-dimensional vector whose entries are i.i.d $CN(0,1)$ random variables (RVs), modeling the fast fading, and $\beta_{k,m}$ the LSF coefficient.

At each AP, channel estimation is performed by the linear minimum-mean-square-error (MMSE) processing. Denoting by $\tau_p < \tau_c$ the length (in time-frequency samples) of the uplink training phase and by $\tau_c$ the length (in time-frequency samples) of the coherence interval, the $m$-th AP forms an MMSE estimate of $\{g_{k,m}\}_{k\in K_m}$ based on the $N_{AP}$-dimensional statistics $\tilde{y}_{k,m} = \sqrt{\eta_k} g_{k,m} + \sum_{i=1,i\neq k}^{K} \sqrt{\eta_i} g_{i,m} \phi_i^H \phi_k + \tilde{w}_{k,m}$, where $\eta_k$ is the power employed by the $k$-th MS during the training phase, $\phi_i$ is the $\tau_p$-dimensional column pilot sequence transmitted by the $i$-th MS and $\tilde{w}_{k,m}$ a $N_{AP}$-dimensional vector with i.i.d. $CN(0,\sigma_w^2)$ entries containing the thermal noise contribution. We assume that the pilot sequences transmitted by the MSs are chosen in...
a set of $\tau_p$ orthogonal sequences $P_{\tau_p} = \{\phi_1, \phi_2, \ldots, \phi_{\tau_p}\}$, where $\phi_i$ is the $i$-th $\tau_p$-dimensional column sequence and $\|\phi_i\|^2 = 1$, $\forall i = 1, \ldots, \tau_p$. The MMSE channel estimate of the channel $g_{k,m}$ can be written as

$$
\hat{g}_{k,m} = \frac{\sqrt{\eta_k} \beta_{k,m}}{\sum_{i=1}^{K} \eta_i \beta_{i,m}} \hat{y}_{k,m} = \alpha_{k,m} \hat{y}_{k,m}.
$$

On the downlink, the APs treat the channel estimates as the true channels and perform conjugate beamforming, while on the uplink, the generic $m$-th AP participates to the decoding of the data sent by the MSs in $K_m$, but data decoding takes place in the CPU [3, 5].

As performance measures used for the testing of the proposed PA algorithms we will consider the achievable rates in downlink and uplink. Applying the use-and-then-forget (UatF) bounding techniques in [11] a lower-bound to the $k$-th MS downlink achievable rate is reported in Eq. (2) at the top of the next page. Similarly, the same bounding technique leads to the $k$-th MS uplink achievable rate reported in (3), again at the top of the next page. In these expressions, the following notation has been used: $W$ is the system bandwidth, $\tau_d$ and $\tau_u$ are the lengths (in time-frequency samples) of the downlink and uplink data transmission phases in each coherence interval; $\eta_{DL,k}$, a scalar coefficient controlling the power transmitted by the $m$-th AP to the $k$-th MS; $\sigma_d^2$ is the AWGN noise variance at the generic MS receiver; $\eta_{UL,k}$ is the uplink transmit power used by the $k$-th MS in the data transmission phase; $\sigma_u^2$ is the AWGN noise variance at the generic AP receiver; finally, $\gamma_{k,m} = \mathbb{E}\left| \hat{g}_{k,m}^H g_{k,m} \right| = \sqrt{\eta_k} N_A P \alpha_{k,m} \beta_{k,m}$. Details on the UatF bound and on the derivations of Eqs. (2) and (3) can be found in [11, 5, 11] and are here omitted due to the lack of space.

III. PILOT ASSIGNMENT ALGORITHM

We are now ready to illustrate the proposed PA schemes. To this end, we assume that the number of MSs $K$ is larger than the number $\tau_p$ of available orthogonal pilots and, also, that the ratio $K/\tau_p$ is an integer.

The schemes that we propose are iterative, have a common structure, and start with a random PA. Basically, the steps of the algorithms can be stated as follows:

1) Assign each MS a pilot randomly picked from the set $P_{\tau_p}$ of orthogonal pilots.
2) Consider the generic $k$-th MS; pick the $\tau_p - 1$ MSs that are closest to the $k$-th MS $k$. The set of these MSs, including the $k$-th one, forms the set $S_k$, of cardinality $\tau_p$. The remaining $K - \tau_p$ MSs are grouped in the set $T_k$.
3) Use the Hungarian algorithm to assign pilots to the users in the set $S_k$ considering the PA of the users in the set $T_k$ as fixed.
4) Repeat steps 2) and 3) for all values of $k = 1, \ldots, K$.
5) Repeat steps from 2) to 4) until the performance measures have reached convergence and/or the maximum number of allowed iterations has been reached.

We now provide further details to better clarify the meaning of the above steps.

A. Defining the set $S_k$

To execute the above steps, we consider the $(\tau_p - 1)$ MSs that are closest to the $k$-th MS to be selected. One simple way of doing this is to rely on the knowledge of the MSs’ positions. Indeed, if this knowledge is available at the CPU, the set $S_k$ can be readily defined. We say that in this case we are using a location-based (LB) procedure.

If, instead, MSs’ location is not available, knowledge of the LSF coefficients can be exploited as indicators of the distance between MSs and APs. Precisely, we are not able to select the $(\tau_p - 1)$ MSs that are closest to the $k$-th MS, but only the $(\tau_p - 1)$ MSs that are closest to (i.e., have the largest LSF coefficients to) the AP that is closest to MS $k$. The two sets of course cannot be claimed to be coincident but with high likelihood will have several common elements. In this case, we say that we are using a location-agnostic (LA) procedure.

In particular, the LA procedure works as follows. For the $k$-th MS, the CPU first computes the index of its nearest AP as $m^* = \arg \max_m \beta_{k,m}$. Then, consider the set of the LSF coefficients $D_{k,m^*} = \{\beta_{j,m^*}\}_{j=1,j \neq k}^K$, sort the entries of $D_{k,m^*}$ in decreasing order, and denote by $O_{m^*,k}(\ell)$ the MS index associated with the LSF coefficient appearing in the $\ell$-th position in the ordered version of the set $D_{k,m^*}$. The set $S_k$ will thus contain the index (MS) $k$ and the indexes (MSs) associated to the $(\tau_p - 1)$ largest coefficients in $D_{k,m^*}$, i.e.: $S_k = \{k, O_{m^*,k}(1), O_{m^*,k}(2), \ldots, O_{m^*,k}(\tau_p - 1)\}$.

B. Running the Hungarian algorithm

Once the sets $S_k$ and $T_k$ have been defined, the set of $\tau_p$ available orthogonal pilots is to be assigned to the $\tau_p$ MSs in $S_k$ according to some optimality criterion. Denoting by $a_{k,q}$ a scalar quantity measuring the reward, to be specified in the following subsection, for the system if the $q$-th pilot in $P_{\tau_p}$ is assigned to the $\ell$-th MS in the set $S_k$, and letting $x_{k,q}$ be a binary $0-1$ variable indicating that the $q$-th pilot sequence is assigned to the $\ell$-th MS, we are formally faced with the following optimization problem:

$$
\max_{x_{k,q} \in \{0,1\}} \sum_{\ell=1}^{\tau_p} \sum_{q=1}^{\tau_p} x_{k,q}^k a_{k,q} \quad \text{s.t.} \quad \sum_{q=1}^{\tau_p} x_{k,q}^k = 1 \forall q, \quad \sum_{q=1}^{\tau_p} x_{k,q}^k = 1 \forall \ell.
$$

Problem (4) accepts has an input the coefficients $a_{k,q}$ for all $k$, and solving it entails providing the values of the optimization variables $x_{k,q}$ for all $k$. The constraints (4b) and (4c) are needed to ensure that each pilot is assigned to just one user and that all the pilots are used once, respectively. The above combinatorial optimization problem can be solved in polynomial time using the Hungarian method Algorithm
C. Defining the reward coefficients

Let us now define how the coefficients $a^{(k)}_{x,q}$ are computed. If the goal is to maximize the system downlink or uplink sum rate, then a reasonable choice is to assume that $a^{(k)}_{x,q}$ is equal to the $\ell$-th MS rate when it is assigned the $q$-th pilot; we denote the $\ell$-th MS rate under this assumption as $R^x_\ell(\{x_{\ell,q} = 1\})$, where $x$ can be DL or UL and we thus have $a^{(k)}_{x,q} = \max_{\ell} R^x_\ell(\{x_{\ell,q} = 1\})$. It is important to remark that the above rate does not depend on the assignments that are decided for the other MSs in $\mathcal{S}_k$, since these MSs are using orthogonal pilots; rather, the rate will depend on the locations of the MSs in $\mathcal{T}_k$ that are assigned the same $q$-th pilot as the MS $k$.

If, instead, the system designer goal is to maximize fairness across users, then a different choice is in order. Denoting by $\mathcal{T}_k(q)$ the set of MSs in $\mathcal{T}_k$ that are using the $q$-th pilot, the following choice is proposed $a^{(k)}_{x,q} = \min_{\ell \in \mathcal{T}_k(q): \ell(q) = \ell} \max_{\ell} R^x_\ell(\{x_{\ell,q} = 1\})$, where again $x$ can be DL or UL. Otherwise stated, $a^{(k)}_{x,q}$ is the smallest rate computed among all the MSs in the system that are using the $q$-th pilot, including the $\ell$-th MS.

As a final remark, we notice that to define the reward coefficients we could make different choices, i.e., using only the downlink/uplink rates or using a combination of the uplink and downlink rates.

IV. NUMERICAL RESULTS

In our simulation setup, we consider a communication bandwidth of $W = 20$ MHz centered over the carrier frequency $f_0 = 1.9$ GHz. The antenna height at the AP is 10 m and at the MS is 1.65 m. The additive thermal noise is assumed to have a power spectral density of $-174$ dBm/Hz, while the front-end receiver at the APs and at the MSs is assumed to have a noise figure of 9 dB. We assume $M = 100$, $N_{AP} = 4$, $K = 40$ and a MS-centric approach [8], [9], where each MS is served by the $N = 20$ APs with the highest LSF coefficients and $K_m$ and $M_k$ are defined accordingly. The APs and MSs are deployed at random positions on a square area of $1000 \times 1000$ (square meters). In order to avoid boundary effects, the square area is wrapped around [11], [12]. The LSF coefficient $\beta_{k,m}$ is modeled as in [13, Table B.1.2.1.1-1], i.e.: $\beta_{k,m}[\text{dB}] = -36.7 \log_{10}(d_{k,m}) - 22.7 - 26 \log_{10}(f) + z_{k,m}$, where $d_{k,m}$ is the distance between the $k$-th MS and the $m$-th AP and $z_{k,a} \sim \mathcal{N}(0, \sigma^2_{z_k})$ represents the shadow fading. The shadow fading coefficients from an AP to different MSs are correlated as in [14, Table B.1.2.1.1-4]. Instead, the shadow fading correlation among MSs is modeled as in [4]. The orthogonal pilot sequences in $\mathcal{P}_p$ have length $\tau_p = 10$; the downlink and uplink data transmission phases durations are $\tau_u = \tau_d = \frac{\tau_p}{2}$, with $\tau_c = 200$ samples. The uplink transmit power for channel estimation is $\eta_k = 10\text{mW}$, with $\eta_k = 100\text{mW}$, $\forall k = 1, \ldots, K$. For the downlink transmit power we have $\eta_{DL,k,m} = \gamma_{k,m}P_{DL}/(\sum_{k \in K(m)} \gamma_{k,m})$, where each AP transmits $P_{DL} = 200\text{mW}$, $\forall m = 1, \ldots, M$; for the uplink data transmission we have $\eta_{UL,k} = 100\text{mW}$, $\forall k = 1, \ldots, K$. We plot the performance of the following versions of the proposed algorithm: (a) Location-based, sum-rate maximizing Hungarian PA (LB-SHPA); (b) Location-based, minimum-rate maximizing Hungarian PA (LB-MHPA); (c) Location-agnostic, sum-rate maximizing Hungarian PA (LA-SHPA); and (d) Location-agnostic, minimum-rate maximizing Hungarian PA (LA-MHPA). The reward coefficients have been computed using downlink rates. These algorithms are compared with a random PA (RPA) and with the solutions in [11], [6].

Exploiting relations in Eqs. (2) and (3), Figs. 1 and 2 report the sum-rate CDF for the downlink and the uplink, respectively. Similarly, Figs. 3 and 4 report the minimum rate CDF both in downlink and uplink, respectively. Inspecting the figures, the following conclusions can be drawn. First of all, LA-based algorithms performance is almost indistinguishable from the performance of LB-algorithms, thus implying that our proposal to use LSF coefficients as indicators of the
AP-MS distances is effective and entails almost no loss in performance. Then, results clearly show that the proposed solutions outperform competing alternatives, with the largest performance gain on the downlink.

V. CONCLUSION

In this letter, the problem of PA in a CF mMIMO system has been considered. An iterative procedure based on the Hungarian algorithm has been proposed. The algorithm parameters can be tuned so as to maximize either the sum-rate or the fairness across users, and can be implemented based on the knowledge of the LSF coefficients. Simulation results have shown that the proposed procedures exhibit a significant advantage over several competing alternatives.

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