Joint Power Splitting and Antenna Selection in Energy Harvesting Relay Channels

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Abstract

The simultaneous wireless transfer of information and power with the help of a relay equipped with multiple antennas is considered in this letter, where a “harvest-and-forward” strategy is proposed. In particular, the relay harvests energy and obtains information from the source with the radio-frequent signals by jointly using the antenna selection (AS) and power splitting (PS) techniques, and then the processed information is amplified and forwarded to the destination relying on the harvested energy. This letter jointly optimizes AS and PS to maximize the achievable rate for the proposed strategy. Considering the joint optimization is according to the non-convex problem, a two-stage procedure is proposed to determine the optimal ratio of received signal power split for energy harvesting, and the optimized antenna set engaged in information forwarding. Simulation results confirm the accuracy of the two-stage procedure, and demonstrate that the proposed “harvest-and-forward” strategy outperforms the conventional amplify-and-forward (AF) relaying and the direct transmission.

Index Terms

Energy harvesting, antenna selection, power splitting.

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I. INTRODUCTION

Energy harvesting is a promising solution to increase the life cycle of wireless nodes and hence alleviates the energy bottleneck of green wireless networks. As an alternative to conventional energy harvesting techniques, simultaneous wireless information and power transfer (SWIPT), which relies on the usage of radio frequency (RF) signals, is expected to bring some fundamental changes to the design of wireless communication networks [1]. Considering that a wireless relay is not always convenient to be equipped with fixed power supplies, the energy harvesting relay with SWIPT has been presented recently, where power splitting (PS) and time switching (TS) are two advanced protocols [2]. PS splits the received signal power at the relay into two parts, one is for information processing, and the other is for energy harvesting to power the forwarding of the processed information. While the relay utilizes different time blocks to realize these two operations separately in the TS protocol. Further on, thanks to the spatial processing in wireless nodes with multiple antennas, the TS protocol has been extended and the information processing and energy harvesting can be separated at different antennas over the same time, and antennas switch between decoding and rectifying based on an antenna selection (AS) scheme [3].

Intuitively, in the multiple-antenna scenario, the joint PS and AS design can reach a flexible utilization of the received RF signals, which provides better performances than the separated PS or AS does. Through AS, partial antennas are selected out only for energy harvesting, and the remaining antennas are specified for both information processing and energy harvesting, which can be optimized further by PS. Unfortunately, few works have discussed the joint PS and AS design for the relay with multiple antennas. In this letter, a “harvest-and-forward” strategy is proposed to improve the achievable rate in energy harvesting relay channels with multiple antenna configurations. Further, the achievable rate maximization problem through the joint AS and PS optimization is formulated and derived.

II. SYSTEM MODEL

An energy harvesting relay channel consisting of a single-antenna source $S$, a single-antenna destination $D$, and a multiple-antenna relay $R$, is depicted in Fig. 1(a). Both $S$ and $D$ are devices without energy harvesting and have continuous supply of power. $R$ is an energy harvesting device and relies on its harvested energy to participate in signal transmission and processing. To
complete the information delivering from $S$ to $D$ with the help of $R$, a two-phase “harvest-and-forward” strategy is presented in this letter. Specifically, in the first phase, $R$ receives a signal from $S$, and harvests energy from a part of its signal power. In the second phase, relying on the harvested energy, $R$ amplifies and forwards remnant signals to $D$. For simplicity, the transmission duration of each phase is set to be normalized, thus the terms “energy” and “power” can be used equivalently [4]. Besides, the power consumption for the signal receiving is assumed to be negligible, and the harvested energy at $R$ is only used for signal forwarding [4].

![Fig. 1. System model and signal processing at the energy harvesting relay](image)

In the first phase, the received signal at $R$ with $N$ antennas can be denoted by

$$r = \sqrt{P_s}h_s + z_a,$$

where $r = [r_1, r_2, \cdots, r_N]^T$ is an $N \times 1$ vector, $s$ denotes the transmitted signal from $S$ with normalized power $E(|s|^2) = 1$, and $P_s$ is the transmit power at $S$ with $P_s \leq P$. Besides, $h = [h_1, h_2, \cdots, h_N]^T$ denotes the $N \times 1$ channel vector between $S$ and $R$, and $z_a \sim CN(0, \sigma^2_a I_N)$ is the $N \times 1$ additive noise vector introduced by the receiver antennas at $R$ [2].

For the realization of the harvest-and-forward strategy, $N$ antennas are divided into two sets via the AS technique, thus the received signal $r$ is split into two sub-signals (i.e., $r_1$ and $r_2$). The components in the first antenna set are used to forward signals and harvest energy via the PS technique. The ratio of sub-signal power split for the information processing is denoted as $\lambda \in [0, 1]$, and the energy harvesting is denoted as $(1 - \lambda)$. The components in the second
antenna set are used to harvest energy solely. Consequently, the harvested energy from \( r \) can be calculated as

\[
P_r = \eta (1 - \lambda) \| r_1 \|_2^2 + \eta \| r_2 \|_2^2 = \eta (1 - \lambda) \sum_{i=1}^{n} |r_{s_i}|^2 + \eta \sum_{i=n+1}^{N} |r_{s_i}|^2,
\]

(2)

where \( \eta \in (0, 1] \) denotes the energy conversion efficiency from signal power to circuit power, and \( r_1 = [r_{s_1}, r_{s_2} \cdots r_{s_n}]^T \) presents the sub-signal received by the first antenna set, where \( n \in [1, N] \) is the number of antennas therein, and \( s_1, s_2 \cdots s_n \) are labels of them. \( r_2 = [r_{s_{n+1}}, r_{s_{n+2}} \cdots r_{s_N}]^T \) describes the sub-signal received by the second antenna set, and \( s_{n+1}, s_{n+2} \cdots s_N \) are labels of antennas therein. Note that these variables and vector sets (i.e., \( n, \lambda, \) and \( r_1, r_2 \)) should be optimized for performance improvement. What’s more, \( P_r \) is limited, since \( P_r \leq \eta \| r \|_2^2 \leq \eta P_s \).

During the second phase, the remnant signals will be amplified and forwarded to \( D \), which is powered by the harvested energy in the first phase. Note that a distributed beamforming design is adopted at \( R \) as in [5], since the joint optimization of a centralized beamforming, PS and AS is too complex to achieve a tractable solution. In this way, we only focus on the joint optimization of PS and AS in this letter. The processed signal at \( R \) is formulated as

\[
x_r = e^{j\theta} \beta (\sqrt{\lambda} r_1 + z_b),
\]

(3)

where \( x_r, z_b \) and \( \sqrt{\lambda} r_1 \) are \( n \times 1 \) vectors, \( z_b \sim CN(0, \sigma_b^2 I_n) \) is the additive noise vector introduced by signal conversion from passband to baseband [2]. The harvested energy allocation is based on the strength of remnant signals at each antenna. Thus, the relay amplification gain is depicted by

\[
\beta = \sqrt{\frac{P_r}{\lambda \sum_{i=1}^{n} |r_{s_i}|^2 + n\sigma_b^2}}.
\]

(4)

Besides, \( e^{j\theta} \) is the \( n \times n \) distributed beamforming diagonal matrix [5], which has \( e^{j\theta_i}, (i = 1, 2 \cdots n) \)'s on the main diagonal and zeros elsewhere, and \( \theta_i = -(\arg h_{s_i} + \arg g_{s_i}) \). The above processes are illustrated in Fig. 1(b). The received signal at \( D \) is expressed as

\[
y = g^T x_r + z = \beta \sqrt{\lambda} \sqrt{P_s} \sum_{i=1}^{n} |g_{s_i} h_{s_i}| s + \beta g e^{j\theta} (\sqrt{\lambda} z_{a'} + z_b) + z,
\]

(5)

where \( g = [g_{s_1}, g_{s_2} \cdots g_{s_n}]^T \) is the \( n \times 1 \) channel vector between these forwarding antennas and \( D \), \( z_{a'} \) is the noise vector from these \( n \) antennas comprising of \( s_1 \)-th, \( s_2 \)-th \ldots \( s_n \)-th items from \( z_a \), and \( z \sim CN(0, \sigma^2) \) is the Gaussian noise at \( D \).
Accordingly, the received signal-to-noise-ratio (SNR) at $D$ can be given by

$$\text{SNR} = \frac{\beta^2 \lambda P_s \left( \sum_{i=1}^{n} |g_{s_i} h_{s_i}| \right)^2}{\beta^2 \sum_{i=1}^{n} |g_{s_i}|^2 (\lambda \sigma_a^2 + \sigma_b^2) + \sigma^2}.$$  \hfill (6)

Consequently, an optimization problem is formulated for the proposed harvest-and-forward strategy to maximize the achievable rate:

$$(P1): \max_{\lambda,n,s_1 \cdots s_n} R = W \log_2(1 + \text{SNR}),$$

s.t. $$0 \leq \lambda \leq 1,$$

$$P_s \leq P,$$

$$n, i, j, s_i, s_j \in [1 : N],$$

where $W$ is the channel bandwidth, $n, s_1 \cdots s_n$ are variables determined by the AS scheme, and $\lambda$ is the variable determined by the PS scheme. Note that $P_s = P$ must be satisfied for the achievable rate maximization.

### III. Achievable Rate Optimization

Note that Problem (P1) is equivalent to the SNR maximization problem [5]. Since the above two kinds of variables are coupled together for calculating SNR in (6), the SNR maximization problem is non-linear and non-convex. To make this problem tractable, a general two-stage optimization procedure is proposed, where the antenna set is fixed firstly to determine the optimal $\lambda$, followed by the optimal antenna selection configuration.

#### A. Power Splitting and Optimal $\lambda$

According to (4), (6) and (7), the corresponding SNR maximization problem with a given antenna set is reformulated as

$$(P2): \max_{\lambda} J_\lambda = \frac{\eta P B_{\Omega_n} \lambda (R_N - R_{\Omega_n} \lambda)}{\eta A_{\Omega_n} (\lambda \sigma_a^2 + \sigma_b^2) (R_N - R_{\Omega_n} \lambda) + \sigma^2 (n \sigma_b^2 + R_{\Omega_n} \lambda)},$$

s.t. $$0 \leq \lambda \leq 1,$$

where $\Omega_n = [s_1, s_2 \cdots s_n]$ denotes the set of $n$ forwarding antennas, $A_{\Omega_n} = \sum_{i=1}^{n} |g_{s_i}|^2$, $B_{\Omega_n} = \left( \sum_{i=1}^{n} |g_{s_i} h_{s_i}| \right)^2$, $R_{\Omega_n} = \sum_{i=1}^{n} |r_{s_i}|^2$, and $R_N = \sum_{i=1}^{N} |r_{s_i}|^2$. 
Lemma 1: With a given antenna set $\Omega_n$, the received SNR $J_\lambda$ is a concave function in terms of the power splitting ratio $\lambda$.

Proof: Please refer to Appendix A.

Based on Lemma 1, Problem (P2) can be regarded as a convex optimization problem. The Karush-Kuhn-Tucker (KKT) conditions are employed for achieving the optimal solution, which can be readily derived by using the following theorem.

Theorem 1: The optimal power splitting ratio $\lambda$ under condition of a given antenna set $\Omega_n$ to maximize SNR for the proposed harvest-and-forward strategy in the multiple-antenna relay channel can be deduced by

$$
\lambda_{\Omega_n}^{opt} = \min \left\{ \lambda_{\Omega_n}, 1 \right\},
$$

$$
\lambda_{\Omega_n}^* = \begin{cases}
\frac{R_N}{2R_{\Omega_n}} & \text{; } \eta \sigma_b^2 A_{\Omega_n} R_N + n \sigma^2 \sigma_b^2 \sqrt{\eta \sigma_b^2 A_{\Omega_n} R_N + n \sigma^2 \sigma_b^2} - \sqrt{\sigma^2 R_N + n \sigma^2 \sigma_b^2} \\
R_{\Omega_n} (\eta \sigma_b^2 A_{\Omega_n} - \sigma^2) & \text{; } \eta \sigma_b^2 A_{\Omega_n} \neq \sigma^2
\end{cases}
$$

Proof: Please refer to Appendix B.

B. Exhaustive Searching Based Antenna Selection

Similar to the derivation of (8), for each feasible antenna set $\Omega_n$, substituting (4) and (9) into (6), the SNR expression can be reformulated as

$$
J_{\Omega_n} = \frac{\eta P B_{\Omega_n} (R_N - \lambda_{\Omega_n}^{opt} R_{\Omega_n}) \lambda_{\Omega_n}^{opt}}{\eta A_{\Omega_n} (R_N - \lambda_{\Omega_n}^{opt} R_{\Omega_n}) (\lambda_{\Omega_n}^{opt} \sigma_a^2 + \sigma_b^2) + \sigma^2 (n \sigma_b^2 + \lambda_{\Omega_n}^{opt} R_{\Omega_n})}.
$$

(10)

Considering the fact that the number of $\Omega_n$’s is finite, the maximization of (10) can be solved through searching the optimal one among all feasible antenna sets with the determined PS ratio.

C. Greedy Antenna Selection

The exhaustive searching process for the optimal antenna set $\Omega^{opt}$ can be categorized as a Non-deterministic Polynomial-time (NP)-hard problem because the number of feasible antenna sets is $2^N - 1$. A greedy antenna selection scheme is presented to approach the optimal solution, which is of a complexity of $O(N^2)$, and thus easier to handle. We use $\Phi_n = \{i_n \in [1, N] | i_n \notin \Omega_n^{opt}\}$ to denote an antenna set with energy harvesting solely, and $i_n$’s are antennas therein. We use $\Omega_n$ to denote a feasible antenna set with power splitting, which also serves in signal forwarding. Note that $|\Omega_n| = n$ and thus $|\Phi_n| = N - n$, where $|\cdot\cdot\cdot|$ denotes the cardinality of a set. The key idea is to determine whether there are received SNR gains when an antenna
$i_n$ is switched from energy harvesting set (i.e., $\Phi_n$) to signal forwarding set (i.e., $\Omega_n$). Thus, $\Omega_n = \Omega^{opt} \cup [i_n]$, and $\Phi_n = \Phi_{n-1}/[i_n]$, where / denotes the subtraction of sets. As described in Algorithm 1, there is no forwarding antennas initially (i.e., $\Omega^{opt} = \emptyset$). The optimal power splitting ratio (i.e., $\lambda_{\Omega_n}^{opt}$) and the achieved SNR (i.e., $J_{\Omega_n}$) for each feasible antenna set $\Omega_n$ are calculated. Then the largest SNR with $n$ forwarding antennas is determined by

$$J'(n) = \max_{\Omega_n} J_{\Omega_n}.$$  \hspace{1cm} (11)

The optimal $\Omega_n$ derived from (11) is settled for signal forwarding, which updates $\Omega^{opt}$. This algorithm ends until the above procedure cannot increase the SNR performance or all antennas are settled for signal forwarding. Note that no iteration is included in our algorithm, since the greedy AS procedure is given one shot. What’s more, the algorithm can converge, since it’s a one-time non-decreasing SNR based searching, and the number of feasible solutions is finite. TABLE 1 shows the complexity comparison between the proposed strategy and the exhaustive searching method.

**TABLE I**

| Complexity Comparison |
|------------------------|
| Proposed strategy      | $O(N^2)$           |
| Exhaustive searching method | $2^N - 1$        |

**Algorithm 1** Joint optimization of the AS and PS

1: Set the stage as $n = 1$, and the optimal antenna set for signal forwarding as $\Omega^{opt} = \emptyset$.

2: For all $i_n \in \Phi_n$

   Set a feasible antenna set $\Omega_n = \Omega^{opt} \cup [i_n]$;
   Calculate $\lambda_{\Omega_n}^{opt}$ according to (9), and $J_{\Omega_n}$ according to (10).

3: Derive $J'(n)$ and the optimal $\Omega_n$ according to (11).

4: If $J'(n) \geq J'(n - 1)$, mark the optimal $\Omega_n$ as $\Omega^{opt}$, and set $n = n + 1$.

5: Stop if $J'(n) < J'(n - 1)$, or $n = N + 1$, otherwise go to step 2.
IV. Numerical Results

The path loss model for the energy harvesting relay channel is denoted by $|\rho_i|^2 d_i^{-2}$, where $i = 1, 2, 3$, $d_1(d_2)$ is the distance between $S(D)$ and $R$, and $d_3$ is the distance between $S$ and $D$. Besides, $|\rho_i|$ denotes the short-term channel fading, and is assumed to be Rayleigh distributed. $|\rho_i|^2$ follows the exponential distribution with unit mean. We set the energy conversion efficiency as $\eta = 0.2$, and the noises as $\sigma^2 = -50$ dBm, $\sigma_a^2 = \sigma_b^2 = \sigma^2/2$. In addition, the bandwidth is $W = 1$ MHz, and the number of antennas is $N = 10$.

Achievable rate performances for different transmission strategies are evaluated in Fig. 2. The conventional amplify-and-forward (AF) relaying and the direct transmission are traditional information transmissions without energy harvesting processes. The consumed power of the system is assumed as $P = 10dBW$. Thus, the transmit power is $P$ at the source node for the proposed strategy and the direct transmission, and is $P/2$ at both the source node and the relay node for the conventional AF relaying. The distance between the source and the relay is normalized as $d_1 = 1$ meter. The results imply that the proposal enjoys a better achievable rate than the direct transmission and the conventional AF relaying.

Fig. 2. Achievable rate versus distance between $S$ and $D$, $d_1 = 1$, $N = 10$, $P = 10dBW$, $\sigma^2 = -50dBm$.

Fig. 3 is given to demonstrate the efficiency of the proposed joint optimization, where $d_1 = 5$.
meters, $d_2 = 10$ meters, $d_3 = 15$ meters. The exhaustive searching method is to find the optimal joint AS and PS solution numerically with the help of Theorem 1. The energy harvesting strategy with pure PS is a special case of the proposed strategy, where all antennas are selected for signal forwarding (i.e., $n = N$). The pure AS strategy is another special case of the proposed strategy, where the sub-signal power at the selected transmitting antennas is used solely for information processing (i.e., $\lambda = 1$). It’s obvious that the performance of the proposed strategy approaches to be optimal, which indicates that the proposal is accurate and efficient. What’s more, the proposed strategy outperforms the pure AS strategy or the pure PS strategy in the achievable rate. It reveals that both AS and PS techniques are indispensable to optimize the achievable rate performance of the proposed harvest-and-forward strategy.

![Fig. 3](image.png)

**Fig. 3.** Achievable rate versus transmit power, $d_1 = 5$, $d_2 = 10$, $d_3 = 15$.

**V. CONCLUSIONS**

A harvest-and-forward strategy in relay channels with multiple antenna configurations has been proposed in this paper, where the optimization problem in terms of the achievable rate has been solved through jointly designing antenna selection and power splitting techniques. Simulation results have indicated that the proposed strategy and the corresponding solution have significant
achievable rate performance gains. The optimization of energy efficiency performance when jointly considering power splitting and antenna selection would be analyzed in the future.

**APPENDIX A**

**PROOF OF LEMMA 1**

From (8), the second-order derivative of $J_\lambda$ is derived as

\[
\frac{\partial^2 J}{\partial \lambda^2} = \eta \lambda \omega_n (R_N - R_{\lambda} \omega_n) \left( \sigma_b^2 \lambda + \sigma_b^2 \right) + \eta \lambda \omega_n n_s \sigma_b^2 \left[ \left( R_N - \frac{3}{2} R_{\lambda} \omega_n \lambda \right)^2 + \frac{3}{4} (R_{\lambda} \omega_n \lambda)^2 \right],
\]

where $C_{\lambda} = \eta P B_{\lambda} / (\eta \lambda \omega_n) \lambda^2 - (\eta \lambda \omega_n \lambda^2 R_N - \eta \lambda \omega_n \lambda^2 R_{\lambda} \omega_n + \sigma_b^2 R_{\lambda} \omega_n) \lambda - \sigma_b^2 (\eta \lambda \omega_n R_N + n_s \sigma_b^2) \geq 0$.

Since $\lambda \leq 1$, and $R_{\lambda} \omega_n \leq R_N$, we have $R_N - R_{\lambda} \omega_n \lambda \geq 0$. Therefore, $\frac{\partial^2 J}{\partial \lambda^2} \leq 0$, and $J_\lambda$ is a concave function of $\lambda$. This completes the proof of Lemma 1.

**APPENDIX B**

**PROOF OF THEOREM 1**

Since (P2) is concave, and the feasible set for $\lambda$ is convex, the KKT conditions are sufficient for achieving the optimal solution with the Lagrange function

\[
L(\lambda, \mu) = J_\lambda - \mu (\lambda - 1),
\]

where $\mu \geq 0$ is the Lagrange multiplier associated with the constraint $\lambda - 1 \leq 0$. The KKT conditions are stated by

\[
\begin{align*}
\frac{\partial L(\lambda, \mu)}{\partial \lambda} &= \frac{\partial J_\lambda}{\partial \lambda} - \mu = 0, \\
\mu (\lambda - 1) &= 0, \\
\lambda - 1 &\leq 0,
\end{align*}
\]

where $\frac{\partial J_\lambda}{\partial \lambda}$ is the first-order derivative of $J_\lambda$, and is given by

\[
\frac{\partial J_\lambda}{\partial \lambda} = C_{\lambda} \left[ (R_{\lambda} \omega_n)^2 (\eta \lambda \omega_n R_N - \sigma_b^2) \lambda^2 - 2R_{\lambda} \omega_n (\eta \lambda \omega_n R_N + n_s \sigma_b^2) \lambda \\
+ R_N (\eta \lambda \omega_n R_N + n_s \sigma_b^2) \right].
\]

There are two groups of solutions for the KKT conditions (14). First, $\lambda_1 = 1$, and $\mu = \frac{\partial J_\lambda}{\partial \lambda} \big|_{\lambda=1}$. Second, $0 \leq \lambda < 1$, and $\mu = 0$. When $\mu = 0$ and $(\eta \lambda \omega_n R_N - \sigma_b^2) \neq 0$, it’s derived that

\[
\begin{align*}
\lambda_2 &= \sqrt[3]{\eta \lambda \omega_n R_N + n_s \sigma_b^2 \frac{\sqrt{\lambda \omega_n \sigma_b^2 R_N + n_s \sigma_b^2 + \frac{\lambda \omega_n \sigma_b^2 R_N}{\eta \lambda \omega_n R_N - \sigma_b^2}}}{R_{\lambda} \omega_n (\eta \lambda \omega_n R_N - \sigma_b^2)}, \\
\lambda_3 &= \sqrt[3]{\eta \lambda \omega_n R_N + n_s \sigma_b^2 \frac{\sqrt{\lambda \omega_n \sigma_b^2 R_N + n_s \sigma_b^2 - \frac{\lambda \omega_n \sigma_b^2 R_N}{\eta \lambda \omega_n R_N - \sigma_b^2}}}{R_{\lambda} \omega_n (\eta \lambda \omega_n R_N - \sigma_b^2)}},
\end{align*}
\]

(16)
whose values depend on \((\eta\sigma_b^2A_{\Omega_n} - \sigma^2)\). It’s clear that \(\lambda_2 < 0\) or \(\lambda_2 > 1\), thus \(\lambda_2\) is not feasible. When \(\mu = 0\) and \((\eta\sigma_b^2A_{\Omega_n} - \sigma^2) = 0\), it’s derived from \((14)\) that \(\lambda_4 = R_N/2R_{\Omega_n}\). Next, the optimal power splitting ratio \(\lambda_{\Omega_n}^{\text{opt}}\) is determined through monotonicity analysis of the optimization object \(J_\lambda\). According to \((15)\), when \((\eta\sigma_b^2A_{\Omega_n} - \sigma^2) \neq 0\), the expression can be factorize as

\[
\frac{\partial J_\lambda}{\partial \lambda} = C_{\Omega_n} (R_{\Omega_n})^2 (\eta\sigma_b^2A_{\Omega_n} - \sigma^2) (\lambda - \lambda_2)(\lambda - \lambda_3).
\]  

(17)

Since \(0 \leq C_{\Omega_n} (R_{\Omega_n})^2\) and \((\eta\sigma_b^2A_{\Omega_n} - \sigma^2) (\lambda - \lambda_2) \leq 0\), when \(\lambda_3 < 1\), \(\frac{\partial J_\lambda}{\partial \lambda}\) is non-negative among \([0, \lambda_3]\), and is non-positive among \((\lambda_3, 1]\), i.e., \(\lambda_{\Omega_n}^{\text{opt}} = \lambda_3\). In the case of \(\lambda_3 \geq 1\), \(\frac{\partial J_\lambda}{\partial \lambda}\) is non-negative among \([0, 1]\), i.e., \(\lambda_{\Omega_n}^{\text{opt}} = 1\). In conclusion, \(\lambda_{\Omega_n}^{\text{opt}} = \min\{1, \lambda_3\}\). In a similar way, it can be derived that \(\lambda_{\Omega_n}^{\text{opt}} = \min\{1, \lambda_4\}\), when \((\eta\sigma_b^2A_{\Omega_n} - \sigma^2) = 0\). Consequently, the optimal \(\lambda\) is derived as in \((9)\). This proves Theorem 1.

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