Consistency of the Hybrid Regularization with Higher Covariant Derivative and Infinitely Many Pauli-Villars

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Abstract

We study the regularization and renormalization of the Yang-Mills theory in the framework of the manifestly invariant formalism, which consists of a higher covariant derivative with an infinitely many Pauli-Villars fields. Unphysical logarithmic divergence, which is the problematic point on the Slavnov’s method, does not appear in our scheme, and the well-known value of the renormalization group functions are derived. The cancellation mechanism of the quadratic divergence is also demonstrated by calculating the vacuum polarization tensor of the order of $\Lambda^0$ and $\Lambda^{-4}$. These results are the evidence that our method is valid for intrinsically divergent theories and is expected to be available for the theory which contains the quantity depending on the space-time dimensions, like supersymmetric gauge theories.

1 Introduction

Manifestly gauge invariant regularization is useful for the gauge invariant renormalization, but there is not so many invariant regularization for non-Abelian gauge theories. The dimensional regularization is the most popular method, but is not manifestly invariant method for the theory which contains quantities depending on the space-time dimension. The chiral gauge theory is a good example of the theory which is not regularized by the dimensional regularization because the symmetry depending on $\gamma_5$ is relevant. Frolov...
and Slavnov proposed an invariant regularization method for this theory introducing an infinite number of Pauli-Villars (PV) fields \[1, 2\].

There is a similar problem in the three-dimensional Chern-Simons (CS) gauge theory, where the anti-symmetric symbol $\epsilon_{\mu\nu\rho}$ is defined by the space-time dimension. For this theory, we developed a parity-invariant PV regularization introducing an infinite number of PV fields \[3, 4\]. Using with the higher covariant derivative (HCD) method, we can construct a hybrid regularization of the HCD and the PV without breaking a parity invariance. By this regularization, we can classify the universality class of the CS coupling shift in the framework of the hybrid regularization \[3, 4\].

Another example of the theory depending on the space-time dimension is the supersymmetric (SUSY) gauge theory. For this theory, it is believed that a manifestly invariant regularization exists but there is few method preserving both the gauge symmetry and supersymmetry \[5\]. Since the PV regularization and the HCD regularization are defined independently of the space-time dimension, the hybrid regularization is expected to be one of the candidates for the invariant regularization for SUSY. It is indeed available for the three-dimensional SUSY gauge theory and gives the same value of the CS coupling shift which is predicted by Kao et al. \[6\] in the SUSY Yang-Mills-Chern-Simons (YMCS) theory \[7\].

When our method is extended to the four-dimensional SUSY gauge theory, it is not obvious whether our regularization works properly because the intrinsically divergence appears in four-dimensions though the three-dimensional theory is finite. So we have to confirm that our method is available not only the finite theory but also the divergent theory to construct a regularization scheme base on the hybrid regularization that works properly whenever the dimensional regularization fails. For this purpose, we consider the regularization and renormalization of the four-dimensional Yang-Mills (YM) theory in this paper, which is the simplest model of the divergent theory.

The hybrid regularization was originally proposed by Slavnov in 1970s \[8, 9, 10\]. When the HCD method is applied to the non-Abelian gauge theory, some one-loop diagrams are left unregularized because the HCD term renders the gauge propagator less divergent but vertices more divergent. To regularize these remaining diagrams, other regulator must be introduced. Slavnov employed the PV method for the regulator of the remaining diagrams.

Though the theory was formally regularized by his regularization, it was pointed out that his method leads wrong value of the renormalization group (RG) $\beta$- and $\gamma$-function for the four-dimensional YM theory \[11\]. Moreover, the unitarity breaking was confirmed when his method is applied to the quantum chromodynamics \[12\]. These problems were occurred by the
unphysical non-local logarithmic radiative corrections from the PV determinant. Namely, the Slavnov’s PV field was not the complete regulator for the remaining one-loop divergence.

To overcome this problem, two modifications have been proposed. One is to use the dimensional regularization instead of the Slavnov’s PV regularization [13]. This scheme does not lead the unphysical logarithmic corrections, but is not suitable for our aim to apply to the theory which is not regularized by the dimensional regularization. Another proposal is to modify the Slavnov’s PV fields not to lead the unphysical corrections. Introducing ‘gauge-fixing parameter’ for the PV fields, this task is accomplished with preserving the gauge symmetry [14].

On these two proposals, the extra regularization so-called ‘pre-regulator’ is needed to evaluate the divergence of the diagrams. Since it is inserted as a partial regulator, the regularization might be inconsistent in the scheme. If we use the dimensional regularization as the pre-regulator in the modified hybrid regularization, the scheme is not suitable for the regularization of the theory depending on the space-time dimension. The extra regulator is sometimes unwanted procedure for our aim so we have to develop a procedure without it.

On the other hand, the four-dimensional YM theory leads up to the quadratic divergence. Since the quadratic divergence is not ignored in the PV type of regularization though it does not appear in the dimensional regularization essentially, we also confirm whether it is canceled or not in the hybrid regularization scheme. The cancellation, however, is not verified so far, though the logarithmic divergence is shown to agree with the physical divergence derived from the other regularization scheme [15].

The reason why the cancellation does not confirmed and is not problematic in references [11, 12] is because they use the dimensional regularization as the ‘pre-regulator’ in addition to the fact that the quadratic divergence has no physical meanings. In their scheme, the quadratic divergence does not appear in principle though the HCD term leads the non-trivial contribution of quadratically divergent diagrams to the effective action. So the explicit cancellation of the quadratic divergence must be confirmed in our regularization scheme for the complete regularization.

The organization of this paper is follows. In Section 2, we apply our regularization scheme to the four-dimensional YM theory writing the explicit form of the action. It is confirmed that all the divergences are regularized by HCD terms except some diagrams at one-loop level by the superficial degree of divergence. A minor modification for the HCD term is given for the complete cancellation of the quadratic divergence. To treat the unregularized one-loop diagrams, the infinitely many PV fields are introduced in a similar
The regularization method. We consider the hybrid regularization of the YM theory in this section. The hybrid regularization consists of the following two steps. First we introduce HCD terms. They improve the behavior of propagators at large momentum, rendering the theory less divergent at the cost of irrelevant vertices. The theory is reduced to superrenormalizable one which has just a finite number of divergent loops. As see later, all the diagrams except one-, two-, three- and four-point functions at one-loop level are convergent with a suitable choice of these terms. Secondly, we deal with unregularized diagrams by a PV type of regularization. Since we are considering the gauge invariant regularization, the PV regulator must be constructed gauge invariant form and never lead any unphysical divergence.

The generating functional regularized by the hybrid regularization is written by

$$Z = \int D\alpha_D b D c \exp\left[-S_\Lambda\right] \prod_j \det -\alpha_j \lambda_j \prod_i \det \gamma_i \eta_i,$$  \hspace{1cm} (1)$$

where $S_\Lambda$ is an action regularized by HCD terms, $\det -\alpha_j \lambda_j$ and $\det \gamma_i \eta_i$ are PV determinants for the gauge and ghost respectively. In the following, we consider the regularization in detail and write the explicit form of the HCD terms and PV determinants.
In four-dimensional Euclidean space-time, the YM theory is given by the action

\[ S = S_{YM} + S_{GF}, \]  

where

\[ S_{YM} = \frac{1}{4} \int d^4 x F^a_{\mu\nu} F^{\mu\nu a}, \]  

\[ S_{GF} = \int d^4 x \, \frac{\xi_0}{2} b^a - b^a (\partial^\mu A^a_\mu)^a + \bar{c}^a (\partial_\mu D^{\mu} c)^a, \]

with the field strength \( F^a_{\mu\nu} = \partial_{\mu} A^a_\nu - \partial_{\nu} A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \) and the covariant derivative \( D^a_{\mu} = \delta^a_{\mu} - \partial^a_{\mu} + g f^{abc} A^b_\mu A^c_\nu \). Here \( A^a_\mu, c^a, \bar{c}^a \) and \( b^a \) denote the gauge field, ghost, anti-ghost and auxiliary field respectively, \( \xi_0 \) is the gauge-fixing parameter and \( f^{abc} \) is the structure constant of the gauge group \( SU(N) \). We abbreviate the color index in the following discussions.

### 2.1 Higher covariant derivative method

The basic idea of HCD method is to regularize the diagrams by improving the convergence of propagators with higher derivative terms. When we choose \( \Lambda \) as a cutoff parameter, the most general form of the HCD action is given by

\[ S_{HCD} = \frac{1}{4 \Lambda^{2n}} \int d^4 x \, D^n F_{\mu\nu} D^n F^{\mu\nu}. \]

Though the original propagator derived from (2) behaves \( \sim p^{-2} \) at large momentum \( p \), the large momentum behavior of the transverse part of the propagator is modified to \( \sim p^{-2-2n} \) after the insertion of (5), but the longitudinal part is not. So the whole of the propagator still behaves \( \sim p^{-2} \) at large momentum \( p \). To improve the convergence of the longitudinal part, we introduce a \textit{higher derivative} term \( H \) to the gauge-fixing action as follows [16]:

\[ S_{GF}^H = \int d^4 x \, \frac{\xi_0}{2} b^2 - b H \partial^\mu A_\mu + \bar{c} H \partial_\mu D^{\mu} c. \]

\( H \) is a dimensionless function of \( \partial^2 / \Lambda^2 \) and must contains a higher term than \( \partial^n / \Lambda^n \) to ensure the large momentum behavior of the propagator.

The most distinctive point of our regularized action is that the \textit{higher derivative} term for the ghost is introduced in (6), which is necessary for the BRST invariance on our method. There is another choice of the regularized action according to the references [11, 13] which does not need any higher
derivative term for the ghost. It seems simpler than our method and we prefer to use their action, but as we see later, the quadratic divergence is not completely canceled among the Λ-dependent terms \[16\]. We will come back to this problem in more detail later in Section 4.2.

So the regularized action in \[11\] is given by

\[
S_\Lambda = S_{YM} + S_{HCD} + S_{GF}^H, \tag{7}
\]

and invariant under the BRST transformations

\[
\delta_B A_\mu = (D_\mu c), \quad \delta_B b = 0, \quad \delta_B c = -c \times c, \quad \delta_B \bar{c} = b. \tag{8}
\]

Here \(\psi \times \phi\) means \(gf^{abc} \psi^b \phi^c\) and then the BRST operator \(\delta_B\) satisfies the usual nilpotency \(\delta_B^2 = 0\).

The superficial degree of divergence is calculated at

\[
\omega = 4 - 2n(L - 1) - E_A - \left(\frac{n}{2} + 1\right) E_c, \tag{9}
\]

where \(L, E_A\) and \(E_c\) are the number of loops, external line of the gauge and external line of the ghost, respectively. For all the diagrams higher than two-loop \((L \geq 2)\), \(n \geq 2\) always gives negative \(\omega\). This means that we may remove the higher loops by a suitable choice of \(n\). On the other hand for one-loop \((L = 1)\), \(\omega\) is not always negative by any \(n\). The most economical choice is \(n = 2\) and we adopt it.

The explicit form of \(H\) is determined by the behavior of the gauge propagator which is obtained as follows:

\[
\frac{\Lambda^4}{p^2(p^4 + \Lambda^4)(p^2 \delta_{\mu\nu} - p_\mu p_\nu) + \frac{\xi_0}{p^4 H^2(p^2/\Lambda^2)^2} p_\mu p_\nu}. \tag{10}
\]

The first term has the order of the momentum degree of \(-6\), the second term must be the same degree or less to ensure the convergence of the diagrams except one-, two-, three- and four-point functions at one-loop level; \(H^2\) behaves \(~ p^4\) at large \(p\). When we naively remove the higher derivative terms taking the limit of \(\Lambda \to \infty\), the propagator must recover familiar one; \(H^2\) converges to unity. So the simplest form of \(H^2\) in momentum space is

\[
H^2 \left(\frac{p^2}{\Lambda^2}\right) = 1 + \frac{p^4}{\Lambda^4}. \tag{11}
\]

It reduces the propagator to very simple form in which the first and the second term of \[10\] has the same denominator. Especially in Feynman gauge \((\xi_0 = 1)\) the propagator is reduced to

\[
\frac{\Lambda^4}{p^2(p^4 + \Lambda^4)} \delta_{\mu\nu}, \tag{12}
\]
which renders diagrams simpler so we choose the Feynman gauge in the following sections.

2.2 Regularization of one-loop divergence

All the diagrams are regularized except one-, two-, three- and four-point functions at one-loop level. We need an extra regularization to regularize the remaining one-loop diagrams. It was shown in reference [10] that the PV regularization does not break gauge invariance at one-loop level and therefore can be used to complete the regularization of the theory.

The PV determinant of (1) is introduced such as

$$\prod_j \det \frac{-\alpha_j}{2} A_j = \prod_{j=1}^{\infty} \det \frac{-\alpha_+}{2} A_{+j} \det \frac{-\alpha_-}{2} A_{-j}. \quad (13)$$

The basic idea of our PV regularization scheme is to regularize the theory by the pair of two PV determinants $\det \frac{-\alpha_+}{2} A_{+j}$ and $\det \frac{-\alpha_-}{2} A_{-j}$. These determinants are the generalization of ones in three dimensional CS gauge theory: where a pair is needed to make a parity-invariant PV regulator, because a part of the action breaks the invariance [3]. The same situation also arises in the regularization of the chiral gauge theory [1, 2], the regularization by the pair is regarded as a more general method than the usual PV regulator. So the YM theory ought to be regularized by the similar way of the pair.

An infinite number is needed to use such PV pairs. Introducing one pair corresponds to subtracting double the divergence, so we have to remedy the over subtraction by an insertion of another pair of opposite statistics. Then, to remedy the over addition we have to introduce the third pair. Such steps are repeated alternately until the divergence is removed. The divergence does not converged by finite steps but infinite, so we must introduce an infinite number of the PV pairs.

Such steps correspond to introducing fermionic PV fields ($\alpha_j = -1$) and bosonic PV fields ($\alpha_j = +1$) alternately. Following references [3, 4], we take the PV conditions for the gauge field such as

$$M_{\pm j} = M|j|, \quad \alpha_{\pm j} = (-1)^{|j|}. \quad (14)$$

$M_j$ denotes the mass parameter of the PV field $A_{j\mu}$, and the determinant is represented as

$$\det \frac{-\alpha_j}{2} A_j = \int \mathcal{D}A_{j\mu} \mathcal{D}b_j \exp[-S_{M_j} - S^{H}_b], \quad (15)$$
where

\[ S_{M_i} = \frac{1}{2} \int d^4x \, d^4y A_{j\mu}(x) \left[ \frac{\delta^2 S_A}{\delta A_{\mu}(x) \delta A_{\nu}(y)} - M^2_{j\mu} \delta(x - y) \right] A_{j\nu}(y), \quad (16) \]

\[ S_{b_j} = \int d^4x \left[ \frac{\xi_j}{2} b_j b_j - b_j \tilde{H} D^\mu A_{j\mu} \right]. \quad (17) \]

We do not take the summation with the index \( j \) in (16) and (17). \( b_j \) is an auxiliary field for \( A_{j\mu} \) and \( \xi_j \) a ‘gauge-fixing parameter’ for the PV field. In usual PV regularization, the ‘gauge-fixing parameter’ for the PV field is ordinary chosen in Landau gauge \( \xi_j = 0 \). In this gauge, however, PV determinants do not converge formally to a constant and then the anomalous divergence appears \[14\]. This anomalous divergence contributes to the renormalization and gives the wrong RG \( \beta \)- and \( \gamma \)-function \[11, 14\]. To resolve such a problem, we have to introduce \( \xi_j(\neq 0) \) in (17).

\( \tilde{H} = \tilde{H}(D^2/\Lambda^2) \) is a higher covariant derivative term which has the same effect as \( H(\partial^2/\Lambda^2) \) and satisfies the following two conditions. First, it must behave as \( \sim p^4 \) to improve the behavior of the propagator at large momentum and converges to unity at the limit \( \Lambda \to \infty \) to recover the usual PV fields. Secondly, it must be invariant under the BRST transformations which come from the change of the integration variables \( \phi_j \to \phi_j + \theta(\delta_B \phi_j) \), where \( \theta \) is an anti-commuting parameter and \( \phi_j \) denotes a PV field, then \( \delta_B \phi_j \) is given by

\[ \delta_B A_{j\mu} = A_{j\mu} \times c, \quad \delta_B b_j = b_j \times c. \quad (18) \]

On these conditions, \( \tilde{H}(D^2/\Lambda^2) \) must be the polynomial of the covariant derivative and the simplest form is given by

\[ \tilde{H}^2 \left( \frac{D^2}{\Lambda^2} \right) = 1 + \frac{D^4}{\Lambda^4}. \quad (19) \]

Consequently the PV determinant (15) is constructed to preserve the BRST invariance.

Similarly, the PV determinant for the ghost is written

\[ \prod_i \det^{\gamma_i} C_i = \prod_{i=1}^{\infty} \det^{\gamma_{+i}} C_{+i} \det^{\gamma_{-i}} C_{-i}, \quad (20) \]

\[ \det^{\gamma_i} C_i = \int D\bar{c}_i Dc_i \exp \left[ - \int d^4x \left( \bar{c}_i \tilde{H} D_{\mu} c_i - m^2 c_i \bar{c}_i c_i \right) \right], \quad (21) \]
where we do not take the summation with the index $i$ again. $\overline{c}_i$ and $c_i$ are PV fields for the ghost and anti-ghost of mass $m_i$. We can treat these PV fields under the PV conditions $m_{\pm i} = m_i$ and $\gamma_{\pm i} = (-1)^{|i|}$. The HCD term for the PV field is inserted as the function $\tilde{H}$ to preserve the BRST invariance of

$$\delta_B c_i = c_i \times c, \quad \delta_B \overline{c}_i = \overline{c}_i \times c.$$  \hspace{1cm} (22)

This HCD term is necessary for the cancellation of the quadratic divergence when the mass term is simply introduced in the usual form like $m_i^2 \overline{c}_i c_i$, as we see in Section 1.

In the following sections, we confirm that such PV fields completely regularize the theory by an explicit evaluation of the one-loop diagrams. In this calculation, one of the most important procedure is to summate an infinite number of the PV diagrams and derive a convergent function from them. Since this procedure is carried out before the momentum integration, all the parameters independent of the indices $i$ and $j$ must be chosen to be the same. So we assign the same momentum parameter to the internal line of each diagram when all the one-loop diagrams are drawn in the next section.

### 2.3 Feynman rules

The regularized action is decomposed into the kinetic part $K$ and the vertex part $V$ as follows \cite{15, 17}:

$$\int d^4x \Psi(x)(K + V + M^2)\Phi(x),$$  \hspace{1cm} (23)

where $\Psi(x)$ and $\Phi(x)$ denote arbitrary fields and $M$ their mass parameter. Since $K$ and $V$ consist of the original part from the YM term (denoting with suffix ‘0’) and $\Lambda$-dependent part from the HCD term (with suffix ‘$\Lambda$’), they are decomposed into

$$K = K_0 + \frac{1}{\Lambda^4} K_{\Lambda}, \quad V = V_0 + \frac{1}{\Lambda^4} V_{\Lambda}.$$  \hspace{1cm} (24)

Under this decomposition, the propagators are written in the form

$$\frac{1}{K + M^2} = \frac{1}{K_0 + M^2} \left(1 - \frac{K_{\Lambda}}{K_0 + M^2} \Lambda^{-4} + O(\Lambda^{-8})\right).$$  \hspace{1cm} (25)

So the Feynman rules are written by the order of $\Lambda^{-1}$, the quantum corrections are calculated order by order. The Feynman rules are listed in Appendix $\mathbb{A}$. 

9
3 One-Loop Contributions Independent of \( \Lambda \)

Now we check whether the PV fields cancel the unregularized divergence by an explicit calculation of the one-loop contributions. First we calculate the contribution which does not depend on \( \Lambda \).

All the diagrams are manipulated under the following three rules.

1. Take the same assignment for the internal momentum among graphically the same form.

2. Take the infinite sum of PV diagrams under the PV conditions adding a ‘virtual’ PV diagram constructed by taking the massless limit of the PV field.

3. Subtract the ‘virtual’ diagram to ensure the total contribution as a diagram from original fields.

As we mentioned in the previous section, Rule 1 is necessary to find a convergent function easily after the infinite sum of the diagrams. Rule 2 and 3 means to divide the contribution of the PV determinants into two parts, the part of the infinite sum and of the massless term.

In the ‘virtual’ PV diagram, the ‘zeroth’ field like \( A_0^\mu \) runs as the internal propagator. We call such a diagram as ‘zeroth’ diagram in the below. The same contribution must be subtracted to maintain the total contribution. This procedure is realized by the following equation:

\[
\prod_{j=1}^{\infty} \det -\frac{\partial}{\partial x_j} A_{+j} \det -\frac{\partial}{\partial x_j} A_{-j} = \prod_{j=-\infty}^{\infty} \det -\frac{\partial}{\partial x_j} A_j / \det -\frac{\partial}{\partial x_0} A_0, \tag{26}
\]

and so on. The series is defined to give an uniquely convergent function when we go to the r.h.s. of this equation.

In the below, we denote the contribution from the infinite sum by ‘massive’ contribution and the other by ‘massless’ contribution. The contribution from the diagrams of original fields and of massless zeroth PV fields belongs to the latter. This decomposition of the contribution clarifies the cancellation of the quadratic divergence as we see in the below.

3.1 Vacuum polarization tensor

All the diagrams contributing to the vacuum polarization tensor at \( \Lambda^0 \) order are listed in Figure [1]. We denote the quantum correction from the diagram (b) containing the internal field \( A_{j\mu} \) by \( \Pi_{j\mu}(p) \) and so on. The Feynman
Figure 1: All the diagrams contribute to the vacuum polarization tensor at $\Lambda^0$ order. The wavy line means the gauge field $A$, the curly line the PV field $A_j$, and the straight line the auxiliary field $b_j$. The rough- and fine-dotted line mean ghost $c$ and PV for ghost $c_i$, respectively. We use the same assignment of internal momenta, where $q = k - p$.

gauge $\xi_0 = 1$ is chosen for its simplicity in which the propagator is reduced to the simplest form as mentioned in the last section. The gauge fixing parameter for the PV field is taken as the same value, $\xi_j = 1$, for the same reason. All the diagrams are divided into three groups and the quantum corrections are calculated in each group.

3.1.1 Gauge field type (A-type) diagrams

The diagrams (a), (b), (c) and (d) in Figure 1 belong to this group. First we consider the diagram (b) that contains only $A_{j\mu}$ field in the internal line. To take the infinite sum following (26), the diagram of $'\Pi_{0\mu}'$ must be introduced to our calculation. Notice the diagram (a) has the same structure with the zeroth diagram of (b), both the diagrams are identifiable as $^{(a)}\Pi_{\mu\nu}(p) \equiv ^{(b)}\Pi_{0\mu\nu}(p)$. The infinite sum is taken without any extra virtual diagram. All the contribution from these diagrams are written in the form of the infinite
sum which is denoted by ‘massive’ contribution as follows:

\[
\Pi_{\mu\nu}^{(a)}(p) + \sum_{j=-\infty}^{\infty} \alpha_j \Pi_{j\mu\nu}^{(b)}(p) \bigg|_{A^0}^{\Lambda^0} = \sum_{j=-\infty}^{\infty} (-1)^j \Pi_{j\mu\nu}^{(b)}(p) \bigg|_{A^0}
\]

\[
= \frac{g^2 c_v}{32\pi^2} \left[ \frac{3M^2}{10} C_2 \delta_{\mu\nu} + \left( \frac{22}{3} \ln \left( \frac{\pi p}{2M} \right) - \frac{61}{9} - \frac{\pi^2 p^2}{9M^2} \right) p_{\mu} p_{\nu} \\
- \left( \frac{19}{3} \ln \left( \frac{\pi p}{2M} \right) - \frac{49}{9} - \frac{37\pi^2 p^2}{360M^2} \right) p^2 \delta_{\mu\nu} + O(M^{-4}) \right], \quad (27)
\]

where we use the PV conditions (14) and the formula (86). Under these conditions all the summations are calculated in the same manner with the matter field in the chiral gauge theory [1, 2] using the formula \[\sum_{j=-\infty}^{\infty} (-1)^j \frac{1}{A^2 + j^2} = \frac{\pi}{A \sinh \pi A} \]
and \( A^0 \). Then the massive contribution converges in the finite \( M \) as is shown in (89) in Appendix C. The first term of the second line, multiplied by \( C_2 \), expresses the quadratic divergence at \( M \to \infty \). \( C_2 \) is a dimensionless constant which originates from the integration with \( X = \frac{p^2}{M^2} \).

In the same way, we calculate the corrections from the diagrams (c) and (d) identifying \( \Pi_{\mu\nu}^{(c)}(p) = \Pi_{0\mu\nu}(p) \),

\[
\Pi_{\mu\nu}^{(c)}(p) + \sum_{j=-\infty}^{\infty} \alpha_j \Pi_{j\mu\nu}^{(d)}(p) = - \frac{g^2 c_v}{32\pi^2} \frac{M^2}{5} C_2 \delta_{\mu\nu} + O(M^{-4}). \quad (28)
\]

This contribution of (28) does not lead the logarithmic divergence, but give the quadratic divergence as well as (27).

### 3.1.2 Auxiliary field type (B-type) diagrams

For the diagrams (h), (i), (j) and (k), the zeroth diagram is needed to take the infinite sum of these diagrams. There is, however, no diagram which is identified as such a zeroth diagram for this group. So we have to add the virtual diagrams for (h), (i), (j) and (k) where \( b_0 \) or \( A_0 \) field runs as an internal propagator instead of \( b_j \) or \( A_j \). Then the contribution from the diagram (h) is calculated

\[
\sum_{j=-\infty}^{\infty} \alpha_j \Pi_{j\mu\nu}^{(h)}(p) = \sum_{j=-\infty}^{\infty} (-1)^j \Pi_{j\mu\nu}^{(h)}(p) - \Pi_{0\mu\nu}(p). \quad (29)
\]

The second term of the r.h.s. is a counter term for the zeroth diagram of (h) introduced to extract the massive contribution of the first term. Such
a counter term is classified into ‘massless’ contribution. We take the same
care for the other diagrams \((i), (j)\) and \((k)\). Then the total of massive
contributions is

\[
\sum_{j=-\infty}^{\infty} (-1)^j \left( \Pi_{j\mu\nu}^{(b)}(p) + \Pi_{j\mu\nu}^{(i)}(p) + \Pi_{j\mu\nu}^{(j)}(p) + \Pi_{j\mu\nu}^{(k)}(p) \right) \bigg|^{\Lambda^0} = -g^2 c_v \frac{M^2}{10} C_2 \delta_{\mu\nu} + \left( \frac{21}{3} \ln \left( \frac{\pi p}{2M} \right) - 7 - \frac{7\pi^2 p^2}{72M^2} \right) p^2 \delta_{\mu\nu} - \left( 6 \ln \left( \frac{\pi p}{2M} \right) - \frac{17}{3} - \frac{4\pi^2 p^2}{45M^2} \right) p_\mu p_\nu + O \left( M^{-4} \right),
\]

(30)

and the massless contributions,

\[
\left. - \Pi_{0\mu\nu}^{(b)}(p) - \Pi_{0\mu\nu}^{(i)}(p) - \Pi_{0\mu\nu}^{(j)}(p) - \Pi_{0\mu\nu}^{(k)}(p) \right\} \bigg|^{\Lambda^0} = g^2 c_v \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 q^2} \left[ (k^2 + q^2 - 2p^2) \delta_{\mu\nu} - k_\mu q_\nu - k_\nu q_\mu + 2p_\mu p_\nu \right].
\]

(31)

Here we use (86) to get the r.h.s. of (30).

3.1.3 Ghost field type (C-type) diagrams

The diagrams containing the ghosts or the PV for the ghosts are classified
into this group. Since there are some differences in the vertex functions
between the ghost and the PV field, \(\bar{c}\) and \(c\) do not play the role of \(c_0\) and \(\bar{c}_0\).
The diagram (e) cannot be identified as the zeroth diagram of (f) though the
both diagrams are the same in graphically. For the diagram (g), the situation
is the same as B-type diagrams. So we take the same care in the similar way
as (29) to extract the massive contribution. Then the massive contribution
of this group is

\[
\sum_{i=-\infty}^{\infty} (-1)^i \left( \Pi_{i\mu\nu}^{(e)}(p) + \Pi_{i\mu\nu}^{(g)}(p) \right) \bigg|^{\Lambda^0} = -g^2 c_v \frac{4}{3 \ln \left( \frac{\pi p}{2m} \right) - \frac{16}{9} - \frac{\pi^2 p^2}{90m^2}} \left( p^2 \delta_{\mu\nu} - p_\mu p_\nu \right) + O \left( m^{-4} \right),
\]

(32)

\(^{1}\text{The compensation term for the diagram (k), } \Pi_{0\mu\nu}^{(k)}(p), \text{ is zero because the mass parameter } M_j \text{ is multiplied in the numerator of the integrand all over when we write the contribution explicitly in the integral form. But we formally write it in the following discussions.}\)
and the massless contribution is

\[
\Pi^{(e)}_{\mu\nu}(p) - \Pi^{(f)}_{0\mu\nu}(p) - \Pi^{(g)}_{0\mu\nu}(p) \bigg|_{\Lambda^0} = -g^2 c_v \int \frac{d^4k}{(2\pi)^4} k^2 q^2 \left[ 2q^2 \delta_{\mu\nu} - k^2 q^2 q_\mu q_\nu - q_\mu k_\nu \right].
\]  

Both the massive contributions from (f) and (g) give the quadratic divergence proportional to $C_2$ after the use of (86). These contributions are canceled out because they have the same value except the sign, and then the quadratic divergence does not appear in (32).

### 3.1.4 Total contribution

We can get the total contribution of the vacuum-polarization tensor adding all the contributions calculated above. All the quadratic divergence is canceled out exactly and then only the logarithmic divergence remains as follows:

\[
\Pi^{\text{total}}_{\mu\nu}(p) \bigg|_{\Lambda^0} = -\frac{g^2 c_v}{8\pi^2} \left( \frac{10}{3} \ln \left( \frac{\pi p}{2M} \right) - \frac{28}{9} - \frac{\pi^2 p^2}{20M^2} \right) \left( p^2 \delta_{\mu\nu} - p_\mu p_\nu \right) + O(M^{-4})
\]

\[
-\frac{g^2 c_v}{8\pi^2} \left( \frac{1}{3} \ln \left( \frac{\pi p}{2m} \right) - \frac{4}{9} - \frac{\pi^2 p^2}{360m^2} \right) \left( p^2 \delta_{\mu\nu} - p_\mu p_\nu \right) + O(m^{-4})
\]

\[
+ \frac{g^2 c_v}{8\pi^2} \left( p^2 \delta_{\mu\nu} - p_\mu p_\nu \right) \left( \ln p^2 + C_1 \right),
\]

where the first line comes from the infinite sum (27), (28) and (30) and the second line from the infinite sum (32). We use (74) to derive the third line which is from the massless term (31) and (33). As we see in (76) it is estimated to have a logarithmic divergence such as $C_1 = -\ln \mathcal{M}^2$ where $\mathcal{M}$ is a parameter of the dimension of one. If we choose $M = m = \mathcal{M}$, the divergent part of the vacuum polarization tensor is calculated

\[
\Pi^{\text{total}}_{\mu\nu}(p) \bigg|_{\text{div}} = \frac{g^2 c_v}{8\pi^2} \frac{5}{3} \ln M \left( p^2 \delta_{\mu\nu} - p_\mu p_\nu \right).
\]

This result corresponds to the usual logarithmic divergence of the YM theory at the Feynman gauge.

Here we discuss how to cancel the quadratic divergence in the above calculation analyzing the contribution from each group of diagrams. We have divided the diagrams into the three types, A, B and C. The contribution from the massless part does not appear in A-type because the gauge field is
identified as the zeroth PV field and all the contribution from this type is calculated as the massive contribution. This situation is the same as the matter field in the chiral gauge theory. On the other hand, \( b, c \) and \( \overline{c} \) field is not identified as \( b_0, c_0 \) and \( \overline{c}_0 \) respectively, not only the massive contribution but the massless one arises in B- and C-type. In C-type, however, the quadratic divergence from the massive contribution is cancelled after the summation of the infinite diagrams and then the quadratic divergence only comes from the massless one. All the arising aspects of the quadratic divergence are listed in Table 1 of Section 4.

The cancellation of the massive contribution occurs between A- and B-type. Notice that B-type will be included in A-type after \( b \) and \( b_j \) are integrated out from the theory, we see this cancellation is essentially the same as the cancellation of massive contribution in C-type: the cancellation occurs in the same group and the quadratic divergence does not appear in outside.

On the other hand, the massless contribution is cancelled between B- and C-type. We remember that all the massless contribution essentially comes from lacking terms to take the infinite sum of the PV diagram, this cancellation means that the lacking term does not give the quadratic divergence in total although each diagram does.

### 3.2 Ghost self-energy and vertex correction

In the above subsection, we show that the vacuum polarization tensor is regularized except the logarithmic divergence. Our next task is to treat this divergence with the renormalization procedure. The simplest way to renormalize the YM theory is to calculate the contributions of the ghost self-energy \( \Omega(p) \) and of the gauge-ghost-ghost vertex \( (\Gamma_{A\overline{c}\overline{c}})_{\mu}(p) \). In this subsection, we calculate these contributions at one-loop level to give a renormalization in the following.

![Figure 2: The ghost self energy loop](image)

We cite the diagram of the ghost self-energy in Figure 2. Since there is no contribution from the PV fields we can easily calculate the diagram using
as follows:

\[ \Omega(p) \bigg|_{\text{div}}^{\Lambda^0} = -g^2 c_v \int \frac{d^4 k}{(2\pi)^4} \frac{k p}{k^2 q^2} = -\frac{g^2 c_v}{16\pi^2} p^2 \ln M. \]  

(36)

In the same way, the one-loop corrections to the gauge-ghost-ghost vertex in Figure 3 are calculated as follows:

\[ (\Gamma_{Ac})_{\mu} \bigg|_{\text{div}}^{\Lambda^0} = -\frac{i g^3 c_v}{32\pi^2} p_\mu \ln M^2. \]  

(37)

Here we use (74) again.

### 3.3 Renormalization

Now we give a renormalization procedure to absorb the logarithmic divergence which we calculate in the preceding subsections. Since we are only considering the one-loop corrections, the usual renormalization procedure of the YM theory can be used here. In that procedure, it is well known that all the divergences are renormalized by the three renormalization constants \( z_1, z_3 \) and \( z_c \) as follows:

\[
\begin{align*}
A_{\mu\text{bare}} &= z_3^\frac{1}{2} A_{\mu}, & \gamma_{\text{bare}} &= z_c \gamma, & \bar{\gamma}_{\text{bare}} &= z_c \bar{\gamma}, \\
b_{\text{bare}} &= z_3^{-\frac{1}{2}} b, & g_{\text{bare}} &= z_3^{-\frac{1}{2}} g, & \xi_{0\text{bare}} &= z_3 \xi_0.
\end{align*}
\]  

(38)

Here we denote the bare parameters with the index ‘bare’. Expanding the renormalization constants with \( \hbar \) such as \( z_i = \sum_n \hbar^n z_i^{(n)} = 1 + \hbar z_i^{(1)} + O(\hbar^2) \), the first order of \( \hbar \) corresponds to the one-loop corrections. Then we get the
following equations:
\[ \Pi_{\mu\nu}(p) \big|^{\Lambda^0}_{\text{div}} = z_3^{(1)} (p^2 \delta_{\mu\nu} - p_{\mu} p_{\nu}), \]  
(39a)
\[ \Omega(p) \big|^{\Lambda^0}_{\text{div}} = -z_c^{(1)} p^2, \]  
(39b)
\[ (\Gamma_{\Lambda c\sigma})_\mu(p) \big|^{\Lambda^0}_{\text{div}} = ig(z_1^{(1)} + z_c^{(1)} - z_3^{(1)}) p_\mu. \]  
(39c)

Comparing these equations with the results (35), (36) and (37), the renormalization constants are easily calculated as follows:

\[ z_1^{(1)} \big|_{\xi_0=1} = \frac{g^2 c_v}{16\pi^2} \frac{4}{3} \ln M, \quad z_3^{(1)} \big|_{\xi_0=1} = \frac{g^2 c_v}{16\pi^2} \frac{10}{3} \ln M, \quad z_c^{(1)} \big|_{\xi_0=1} = \frac{g^2 c_v}{16\pi^2} \ln M. \]  
(40)

From these results, the \( \beta \)- and \( \gamma \)-function at a renormalization point \( \mu \) are calculated

\[ \beta(g, \xi_0) \big|_{\xi_0=1} = \mu \frac{\partial g}{\partial \mu} = -\frac{g^3}{16\pi^2} \frac{11}{3} c_v + O(g^5), \]  
(41a)
\[ \gamma_A(g, \xi_0) \big|_{\xi_0=1} = \frac{\mu}{2} \frac{\partial \ln z_3}{\partial \mu} = -\frac{g^2}{16\pi^2} \frac{5}{3} c_v + O(g^4). \]  
(41b)

These values accord with the familiar value at Feynman gauge [18, 19, 20].

## 4 Contributions from HCD Terms

In the last section, we have seen that the quadratic divergence is completely canceled and the correct RG \( \beta \)- and \( \gamma \)-function are given with our regularization scheme by the calculation of the one-loop contributions in \( \Lambda^0 \) order. In this section, we mainly consider the contribution from \( \Lambda \)-dependent part.

We remind that the HCD terms render the renormalizable theory into a super-renormalizable one, it is reasonable to consider that all the divergence from higher than two-loop level in the renormalizable theory is translated into some divergence at one-loop level in the super-renormalizable one. So there is some divergence in \( \Lambda \)-dependent terms and it must be regularized by our regulators. We now check such divergence is regularized at least in \( \Lambda^{-4} \) order.

### 4.1 Vacuum polarization tensor in \( \Lambda^{-4} \) order

A new diagram listed in Figure 4 arises in this order in addition to the diagrams in Figure 4. This is from the irrelevant vertex which maintains the
BRST invariance. We classify this diagram into B-type because it contains $b_j$ field in the internal line. The contribution is calculated in a similar way with the case of $\Lambda^0$ order under the rules mentioned on the top of Section 3. Since the contribution is divided into two parts following the calculation rules, one is the infinite sums and the other is the massless terms, all our tasks are concentrated on the calculations of the infinite sums (arising from the PV fields like (30)) and of the massless terms (from the counter terms such as (31)).

For the contribution of the infinite sums, all the terms are written by the following formula:

$$
\sum_{j=-\infty}^{\infty} (-1)^j \frac{M_j^{2r}}{\Lambda^4} \int \frac{d^4k}{(2\pi)^4} \frac{k_{\mu_1} \cdots k_{\mu_{10-2r-1}} p_{\mu_{10-2r-s+1}} \cdots p_{\mu_{10-2r}}}{(k^2 + M_j^2)^2 (q^2 + M_j^2)^2} \\
\sim \frac{1}{\Lambda^4} \left( M_j^{2[3-\frac{r}{2}]} + O \left( M_j^{2[2-\frac{r}{2}]} \right) \right), \tag{42}
$$

where $r$ and $s$ denote the order of $M^2$ and $p_\mu$ in the integrand, and run in the region $0 \leq r \leq 5$ and $0 \leq s \leq 10 - 2r$ respectively. We use the computation rules discussed in Appendix C to get the second line and $[ \ ]$ denotes the Gauss’ notation. Its contribution depends on the order of the limit $M$ and $\Lambda$, but the difference does not affect renormalization group functions \cite{11}, we may restrict our calculations within the ratio $M/\Lambda = \text{constant}$. Then (42) behaves

$$
\sim M_j^{2[1-\frac{r}{2}]} + O \left( M_j^{2[\frac{r}{2}]} \right), \tag{43}
$$

and only the terms having $s = 0$ give the divergence in the limit $M \to \infty$, they are calculated in \cite{57}.

Similarly the contributions from the massless terms are written in the
following form using the formula (75) and (76),

\[
\frac{1}{\Lambda^4} \int \frac{d^4 k}{(2\pi)^4} \frac{k_{\mu_1} \cdots k_{\mu_6} \cdots p_{\mu_{6-s}} \cdots p_{\mu_6}}{k^2 q^2} \sim \frac{1}{\Lambda^4} \left( M^2[3-\frac{s}{2}] + p^2 M^2[2-\frac{s}{2}] + \cdots + p^2[3-\frac{s}{2}] \ln M^2 + \cdots \right) \quad \text{(44)}
\]

where \(0 \leq s \leq 6\). Also in this case, only \(s = 0\) terms give the divergence under the condition \(M/\Lambda = \text{constant}\) if we identify \(\mathcal{M}\) with \(M\).

Both in (42) and (44), the logarithmic divergence appears in the form of \(\ln M^2/\Lambda^4\). Since it vanishes in the ratio \(M/\Lambda = \text{constant}\) there is no logarithmic divergence and only the terms containing no external momentum \(p_\mu\) in the numerator give the quadratic divergence in this order. So we only consider \(s = 0\) terms and confirm whether the divergence cancels or not by calculating each diagrams.

Now we calculate the divergent contributions in the below. For the diagrams in A-type, since the diagram (a) and (c) play the massless diagram respectively we take the infinite sum without any extra diagram as in the \(\Lambda^0\) order. The divergent contributions from (a), (b), (c) and (d) are

\[
\sum_{j=-\infty}^{\infty} (-1)^j \left( \Pi_{j\mu\nu}(p) + \Pi_{j\mu\nu}(p) \right) \bigg|_{\text{div}}^{\Lambda^{-4}} = -\frac{g^2 c_v}{8\pi^2} \frac{9}{154} M^2 C_4 \delta_{\mu\nu}. \quad \text{(45)}
\]

Here we use (87) and \(C_4\) is a dimensionless constant.

For the diagrams in C- and B-type, we take the infinite sum adding the external diagrams as in (29)

\[
\Pi_{\mu\nu}(p) + \sum_{i=-\infty}^{\infty} (-1)^i \left( \Pi_{i\mu\nu}(p) + \Pi_{i\mu\nu}(p) \right) \bigg|_{\text{div}}^{\Lambda^{-4}} = -\frac{g^2 c_v}{\Lambda^4} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 q^2} \left( 2k^6 \delta_{\mu\nu} - 4k^4 k_\mu k_\nu \right), \quad \text{(46)}
\]

\[
\sum_{j=-\infty}^{\infty} (-1)^j \left( \Pi_{j\mu\nu}(p) + \Pi_{j\mu\nu}(p) + \Pi_{j\mu\nu}(p) + \Pi_{j\mu\nu}(p) + \Pi_{j\mu\nu}(p) + \Pi_{j\mu\nu}(p) \right) \bigg|_{\text{div}}^{\Lambda^{-4}}
\]

\[
= \frac{g^2 c_v}{8\pi^2} \frac{9}{154} M^2 C_4 \delta_{\mu\nu} + \frac{g^2 c_v}{\Lambda^4} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 q^2} \left( 2k^6 \delta_{\mu\nu} - 4k^4 k_\mu k_\nu \right). \quad \text{(47)}
\]

The first term of the r.h.s. in (47) arises from the infinite sums of PV diagrams, and the second term from the massless terms. In (46), we add
Table 1: Quadratic divergence at $\Lambda^0$ and $\Lambda^{-4}$ order. ‘massive’ and ‘massless’ means the contribution from the infinite sum such as (30) and the massless terms such as (31) respectively. The coefficient $\frac{g^2c^2}{32\pi^2}\delta_{\mu\nu}$ are abbreviated from explicit values.

| Diagram type | $\Lambda^0$ order | $\Lambda^{-4}$ order |
|--------------|-------------------|---------------------|
|              | massive | massless | massive | massless |
| Gauge        |          |          |          |          |
| A-type       | $+\frac{1}{10}M^2C_2$ | 0           | $-\frac{18}{77\Lambda^4}M^6C_4$ | 0           |
| B-type       | $-\frac{1}{10}M^2C_2$ | $+\frac{5}{6}M^2$ | $+\frac{18}{77\Lambda^4}M^2C_4$ | $+\frac{2}{105\Lambda^4}M^6$ |
| Ghost        | 0        | $-\frac{5}{6}M^2$ | 0        | $-\frac{2}{105\Lambda^4}M^6$ |

4.2 Inconsistent higher derivative action

There is another choice of gauge-fixing action but (6). Remembering that the higher derivative function for the gauge-fixing action is added to improve the convergence of the longitudinal part of the gauge propagator, we can choose the action with a higher derivative function $f$ according to the references [11, 20] without an explicit identification of the parameters $M = m = M$ which is necessary for the renormalization, because the cancellation is occurred between the divergence described by the same parameters: massive divergence is canceled by massive one and massless divergence by massless one.
as follows;

\[
S_{GF}^f = \int d^4 x \, b \frac{\xi_0}{2f} b - b \partial^\mu A_\mu + \varphi \partial_\mu D_\mu c. \tag{48}
\]

This action is extended to the Pauli-Villars field using a higher covariant derivative function \( \tilde{f} \),

\[
S_{b_j}^f = \int d^4 x \left[ b_j \frac{\xi_j}{2\tilde{f}^2} b_j - b_j D_\mu A_{j\mu} \right]. \tag{49}
\]

These actions give the same propagators for the gauge field and PV for gauge as we get with the actions (3) and (17) under the condition \( f = H \) and \( \tilde{f} = \tilde{H} \), but change some propagators and vertices. Following points are modified by the usage of the action (48) and (49):

1. The \( \Lambda \)-dependent parts of propagators \( \langle A_j b_j \rangle \) and \( \langle b_j b_j \rangle \) and vertices \( \langle Ab_j A_j \rangle \) and \( \langle AA b_j A_j \rangle \) are modified. Especially, these vertices do not give any \( \Lambda \)-dependent term;

2. New vertices \( \langle Ab_j b_j \rangle \) and \( \langle AA b_j b_j \rangle \) appear because of \( \tilde{f} \). These vertices give the new diagrams.

3. The ghost and its PV field have no \( \Lambda \)-dependence.

From the first fact, (14), which is the total contribution of the diagrams (h), (i), (j), (k) and (l), is changed. The diagram (l), especially, does not appear because of the absence of the vertex \( \langle AA b_j A_j \rangle \). Recalculating these diagrams with modified propagators and vertices, we get

\[
\sum_{j=-\infty}^{\infty} (-1)^j \left( \Pi^{(h)}_{j,\mu\nu}(p) + \Pi^{(i)}_{j,\mu\nu}(p) + \Pi^{(j)}_{j,\mu\nu}(p) + \Pi^{(k)}_{j,\mu\nu}(p) \right) \left|^{\Lambda^{-4}}_{\text{div}} \right.
\]

\[
= \frac{g^2 c_v}{8\pi^2} \frac{6}{154} M^2 C_4 \delta_{\mu\nu}. \tag{50}
\]

The second fact leads the new diagrams listed in Figure 5. Since the diagram (m) only gives the higher order of \( \Lambda^{-8} \) and the diagram (n) does not give any quadratic divergence, the diagrams we have to calculate here are the three diagrams of (o), (p) and (q). Each diagram gives the quadratic divergence after the infinite sum but all of them are canceled in total, the massive contribution does not appear from these new diagrams. Not only the massive contribution but massless one is not arising from these diagrams:
Figure 5: Diagrams generated by the new vertices appearing in (48) and (49). These diagrams only contribute to the \( \Lambda \) depending terms but do not give any quadratic divergence to \( \Lambda^{-4} \) in total.

All these diagrams contain mass parameter \( M_j \) in the numerator of integrand so their zeroth diagrams vanish taking the massless limit of them, like the diagram \((k)\) in Section 3.

Since the ghost and its PV do not give any \( \Lambda \)-dependent contribution in this case, the quadratic divergence in the \( \Lambda^{-4} \) order only comes from (45) and (50). Comparing these two contributions, we find the quadratic divergence does not cancel in this order.

The reason why the action (48) and (49) fail the cancellation of the quadratic divergence in \( \Lambda^{-4} \) comes from the fact that the function \( \tilde{f} \) gives a different effect to the mass term of the PV field for ghost, with the case of \( H \). Certainly, (48) ((49)) is translated into (6) ((17)) by the redefinition of \( \tau \rightarrow \tau H \) (\( \tau_i \rightarrow \tau_i H \)) and \( b \rightarrow bH \) (\( b_j \rightarrow b_j H \)) under the condition \( f = H \) (\( \tilde{f} = \tilde{H} \)), but the PV for ghost does not: the mass term gets the \( \Lambda \)-dependence and \( \det \gamma C_i \) does not coincide with (21). This fact says that as long as we construct PV fields by simply adding an usual mass term, (48) and (49) are not the correct higher derivative regulator. If we want to use these regulators, we must add a \( \Lambda \)-dependence to the mass term of the ghost PV field.

From another point of view, it is simply recognized that our actions are natural. The main purpose to introduce a higher (covariant) derivative function to the gauge-fixing action is to improve the convergence of the propagator of the gauge field as we explain in Section 2.1. So the function must regularize the gauge field. The function \( f \) (\( \tilde{f} \)) surely improves the convergence of the propagator by regularizing the gauge-fixing parameter \( \xi_0 \) (\( \xi_j \)),
but it is not the regulator for the gauge (PV) field. On the other hand, the higher (covariant) derivative function $H$ ($\tilde{H}$) regularizes $A_\mu$ ($A_{ij\mu}$) by multiplying to the cross term with the auxiliary field $b$ ($b_j$) and gives the complete cancellation of the quadratic divergence without any $\Lambda$-dependence in the mass term of the ghost PV field.

5 Conclusion and Discussion

In this paper, we check the consistency of the hybrid regularization in the four-dimensional Yang-Mills theory when we use the regularization scheme consists of the higher covariant derivative term and an infinitely many Pauli-Villars fields that we applied to the three-dimensional Chern-Simons gauge theory. By an explicit calculation of the vacuum polarization tensor, we get the correct factors of the renormalization group $\beta$- and $\gamma$-function. Furthermore, the cancellation of the quadratic divergence is also demonstrated in our method with an expansion of $\Lambda^{-1}$. These facts show that our regularization method is available for a divergent theory not only for a finite theory.

In our calculation, all the diagrams are classified into the three types and their quantum corrections are calculated separately in each type. In that calculation, all the contributions are separated into ‘massive’ and ‘massless’ contributions along the manipulation rules given in Section 3. As a result, we can clearly confirm the cancellation of the quadratic divergence: the ‘massive’ contribution is cancelled between A- and B-type diagrams and the ‘massless’ between B- and C-type. This clear cancellation mechanism works identically in both the order of $\Lambda^0$ and $\Lambda^{-4}$ as we listed in Table 1.

Such a mechanism is expected to occur in the quadratic divergence in the higher order of $\Lambda^{-1}$, e.g. $\Lambda^{-8}$, because the massless contribution of $A$-type is always absent. This argument is proved if it is shown that C-type diagrams do not give the massive contribution in any order of $\Lambda^{-1}$.

The higher (covariant) derivative functions for gauge-fixing terms, $H$ and $\tilde{H}$, especially, are important for the cancellation of $\Lambda$-depending contributions. The actions (48) and (49) sure give the cancellation in $\Lambda^0$ order, but do not in $\Lambda^{-4}$. The reason is that the mass term of the ghost PV does not accord with ours under the redefinition of fields. In other words, the functions $f$ and $\tilde{f}$ are only the higher (covariant) derivative for the gauge-fixing parameters and they do not regularize the gauge and its PV fields completely. Though the higher (covariant) derivative terms are originally introduced to improve the convergence of the gauge and PV field, such functions consequently must be inserted to regularize the gauge and PV field like the action (48) and (49), as far as we introduce an usual mass term like $m_i^2\bar{c}_i c_i$. 

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In usual case, some extra regularization so-called ‘pre-regulator’ is introduced to compare all the divergence, in our method, however, such a regularization is not needed. This is because that our method contains a scheme corresponding to a pre-regulator. In our calculation, all the divergence appears in the constants of integration which is derived from the differential equations of $X$ like (69) or (78). As a result, ‘massive’ divergence is characterized by $M$ or $m$ and ‘massless’ by $M$, and then we get the correct renormalization supposing $M = m = M$. This assumption will corresponds to the conventional pre-regulator in reference [21] where it is only needed to give a rigorous arguments. So our method is an alternative procedure which does not need any pre-regularization.

The very reason that we want to avoid the pre-regulator is that the procedure may break the invariance which we would like to preserve in the theory. But in our method, since a scheme corresponding to a pre-regularization is in the usual regularization method, we do not worry about the symmetry breaking.

Finally we comment on the problem of the overlapping divergences; how to treat the one-loop divergence with external PV fields [21, 22]. It is difficult to remove the divergence with our present method, however, the PV pair such as (13) will give the key to the problem. Since both the fields give the same diagram with each external field, if we can find a PV pair whose diagrams cancel each other like the parity-odd contributions in reference [3] where the pair is connected by the parity-transformation, all the one-loop divergence with external PV fields are canceled and the problem will be overcome. Unfortunately, such a PV pair is not found up to the present.

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I would like to thank Dr. T. Ebihara for many useful discussions on the subject of this paper.

I suddenly received the sad news of Mr. T. Sakuma’s early grave. His M.Sc. thesis entitled “Problems of regularization in non-Abelian gauge theory and a possibility of a new regularization” motivated me to do this work. I express my deep regret at his untimely death.

A Feynman Rules
A.1 Propagators

A.1.1 Propagators for original fields

\[
A^a_\mu \sim p^\nu \sim A^b_\nu = \delta^{ab} \left[ \frac{\Lambda^4}{p^4(p^4 + \Lambda^4)} (p^2 g_{\mu\nu} - p_\mu p_\nu) + \xi_0 \frac{p_\mu p_\nu}{p^2 H^2} \right] \\
\approx \frac{\delta^{ab}}{p^2} \left( p^2 g_{\mu\nu} + (\xi_0 - 1) \frac{p_\mu p_\nu}{p^2} \right) \left( 1 - \frac{p^4}{\Lambda^4} \right),
\]

(51)

\[
A^a_\mu \sim p \sim b^b = \frac{i \delta^{ab} p_\mu}{p^2 H} \approx \frac{i \delta^{ab} p_\mu}{p^2} \left( 1 - \frac{p^4}{2 \Lambda^4} \right),
\]

(52)

\[
c^a \sim p \sim \bar{c}^b = -\frac{\delta^{ab}}{p^2 H} \approx -\frac{\delta^{ab}}{p^2} \left( 1 - \frac{p^4}{2 \Lambda^4} \right),
\]

(53)

A.1.2 Propagators for PV fields

\[
A^a_{j\mu} \sim p \sim A^b_{j\nu} = \delta^{ab} \left[ \frac{\Lambda^4 (p^2 g_{\mu\nu} - p^4 p^\nu)}{(\Lambda^4 + p^4)^4 + M_j^2 \Lambda^4 p^2} + \xi_j \frac{p^\mu p^\nu}{p^2 (p^2 H^2 + \xi_j M_j^2)} \right] \\
\approx \frac{\delta^{ab}}{p^2} \left[ \frac{\xi_j}{p^2 + \xi_j M_j^2} \left( 1 - \frac{1}{\Lambda^4 p^2 + \xi_j M_j^2} \right) \right]
\]

(54)

\[
A^a_{j\mu} \sim p \sim b^b_j = \frac{i \delta^{ab} p_\mu H}{p^2 H^2 + \xi_j M_j^2} \approx \frac{i \delta^{ab} p_\mu}{p^2 + \xi_j M_j^2} \left( 1 - \frac{p^4}{2 \Lambda^4 p^2 + \xi_j M_j^2} \right),
\]

(55)

\[
b^a_j \sim p \sim b^b_j = \frac{-\delta^{ab} M_j^2}{p^2 H^2 + \xi_j M_j^2} \approx \frac{-\delta^{ab} M_j^2}{p^2 + \xi_j M_j^2} \left( 1 - \frac{1}{\Lambda^4 p^2 + \xi_j M_j^2} \right),
\]

(56)

\[
c^a_i \sim p \sim \bar{c}^b_i = \frac{-\delta^{ab} m_i^2}{p^2 H + m_i^2} \approx \frac{-\delta^{ab} m_i^2}{p^2 + m_i^2} \left( 1 - \frac{1}{2 \Lambda^4 p^2 + m_i^2} \right),
\]

(57)
A.2 Vertices

A.2.1 Three-point vertices

\[
\begin{align*}
A_{\mu_2}^{a_1} & \quad A_{\mu_3}^{a_2} & = & & A_{\mu_1}^{a_1} & \quad A_{\mu_3}^{a_2} \\
A_{\mu_2}^{a_3} & \quad p_1 & & & & & & & & A_{\mu_3}^{a_3} & \quad p_3 \\
A_{\mu_3}^{a_1} & \quad p_2 & & & & & & & & A_{\mu_3}^{a_1} & \quad p_3 \\
A_{\mu_3}^{a_2} & \quad p_1 & & & & & & & & A_{\mu_3}^{a_2} & \quad p_3 \\
\end{align*}
\]

\[
\begin{align*}
= \frac{ig}{\Lambda^4} f^{a_1 a_2 a_3} & \left[ -\Lambda^4 p_{1\mu_2} g_{\mu_3 \mu_1} - p_{1\mu_2} g_{\mu_3 \mu_1} + p_1^2 (p_3 - p_1)_{\mu_2} (p_1 \mu_3 p_3 \mu_1 - p_1 p_3 g_{\mu_3 \mu_1}) \right]_{\text{sym}} \\
= \frac{ig}{\Lambda^4} f^{a_1 a_2 a_3} & \left[ -\Lambda^4 \left\{ (p_1 - p_3)_{\mu_2} g_{\mu_3 \mu_1} + (p_3 - p_2)_{\mu_1} g_{\mu_2 \mu_3} + (p_2 - p_1)_{\mu_3} g_{\mu_1 \mu_2} \right. \\
& \left. - (p_1^2 p_1 - p_3^2 p_3)_{\mu_2} g_{\mu_3 \mu_1} - (p_3^2 p_3 - p_2^2 p_2)_{\mu_1} g_{\mu_2 \mu_3} - (p_2^2 p_2 - p_1^2 p_1)_{\mu_3} g_{\mu_1 \mu_2} \\
& \quad + (p_1^2 + p_3^2)(p_3 - p_1)_{\mu_2} (p_3 \mu_1 p_1 \mu_3 - p_3 p_1 g_{\mu_1 \mu_3}) \\
& \quad + (p_2^2 + p_3^2)(p_2 - p_3)_{\mu_1} (p_2 \mu_3 p_3 \mu_2 - p_2 p_3 g_{\mu_3 \mu_2}) \\
& \quad + (p_2^2 + p_3^2)(p_1 - p_2)_{\mu_3} (p_1 \mu_1 p_2 \mu_1 - p_1 p_2 g_{\mu_2 \mu_1}) \right] \quad (58)
\end{align*}
\]

where \([\quad]\)_{\text{sym}} implies that symmetrize all the indices.

\[
\begin{align*}
= g f^{a_1 a_2 a_3} & \left[ g_{\mu_1 \mu_3} + \frac{1}{2\Lambda^4} \left( p_2^4 g_{\mu_1 \mu_3} + (p_2^2 + p_3^2)(p_3 - p_2)_{\mu_1} p_3 \mu_3 \right) \right] \quad (59)
\end{align*}
\]

\[
\begin{align*}
= -ig f^{a_1 a_2 a_3} & p_3 \mu_1 \left( 1 + \frac{p_3^4}{2\Lambda^4} \right) \quad (60)
\end{align*}
\]
\[ A^{a_1}_{\mu_1} A^{a_4}_{\mu_4} = -ig f^{a_1 a_2 a_3} (p_2 + p_3)_{\mu_1} \left[ 1 + \frac{1}{2\Lambda^4} (p_3^4 + p_3^2 p_2 + p_2^4) \right] \text{, (61)} \]

### A.2.2 Four-point vertices

\[ A^{a_1}_{\mu_1} A^{a_4}_{\mu_4} = -g^2 f^{a_1 a_2 a_3} f^{a_3 a_4} \frac{\Lambda^4}{\Lambda} \left[ \Lambda^4 (g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} - g_{\mu_1 \mu_4} g_{\mu_2 \mu_3}) + (p_1 + p_2)^4 (g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} - g_{\mu_1 \mu_4} g_{\mu_2 \mu_3}) \right. \]
\[ + 8 (p_1 + p_2)_{\mu_1} \left\{ p_{4\mu_3} (p_2 p_4 g_{\mu_2 \mu_4} - p_2 p_4 g_{\mu_2 \mu_3}) - p_{3\mu_3} (p_2 p_3 g_{\mu_2 \mu_3} - p_2 p_3 g_{\mu_2 \mu_4}) \right\} \]
\[ + 4 p_1^2 \left\{ g_{\mu_2 \mu_3} (p_4 p_1 \mu_4 + p_4 p_1 \mu_3) - g_{\mu_2 \mu_4} (p_3 p_1 \mu_3 + p_3 p_1 \mu_4) \right\} \]
\[ - 4 p_1^2 (2p_1 + p_2)_{\mu_2} (g_{\mu_1 \mu_4} p_{1\mu_3} - g_{\mu_1 \mu_3} p_{1\mu_4}) \]
\[ + 2p_1_{\mu_1} \left\{ p_{3\mu_3} (p_2 p_4 g_{\mu_2 \mu_4} - p_2 p_4 g_{\mu_2 \mu_3}) - p_{4\mu_4} (p_2 p_3 g_{\mu_2 \mu_3} - p_2 p_3 g_{\mu_2 \mu_4}) \right\} \]
\[ + 4 (p_1 + p_2)^2 \left\{ g_{\mu_2 \mu_4} p_{4\mu_1} (p_3 + 2p_4)_{\mu_3} - g_{\mu_2 \mu_3} p_{3\mu_1} (p_4 + 2p_3)_{\mu_4} \right\} \] \text{sym} \text{, (62)}
Where \((1 \leftrightarrow 4)\) means the same expression exchanged all the index 1 with 4.

**B Momentum Integration**

All the momentum integrals that we consider in this paper are reduced to the general form of

\[
I_{\mu_1 \cdots \mu_n}(x_1, \ldots, x_N) \equiv \int \frac{d^4 k}{(2\pi)^4} \frac{k_{\mu_1} \cdots k_{\mu_n}}{\prod_{i=1}^{N} (k^2 + x_i)(q^2 + x_i)}. \tag{65}
\]

where \(q = k - p\) and \(p\) is the external momentum. This expression is written as follows using the Feynman parameterization and introducing \(z\) which is a source of the momentum:

\[
I_{\mu_1 \cdots \mu_n}(x_i) = \int \frac{d^4 k}{(2\pi)^4} \frac{\delta}{\delta z_{\mu_1}} \cdots \frac{\delta}{\delta z_{\mu_n}} \int_0^\infty \prod_{i=1}^{N} d\alpha_i d\beta_i \exp \left[ -k^2 \sum_{i=1}^{N} (\alpha_i + \beta_i) + k(2p \sum_{i=1}^{N} \beta_i + z) - \sum_{i=1}^{N} (\alpha_i + \beta_i) x_i - p^2 \sum_{i=1}^{N} \beta_i \right] \bigg|_{z=0}. \tag{66}
\]

First we take the momentum integration with \(k\) using the gaussian and the differential with \(z\). Inserting an unity \(1 = \int_{0}^{\infty} d\rho \delta (\rho - \sum \alpha_i - \sum \beta_i)\), and rescaling the parameters such as \(\alpha_i \rightarrow \rho \alpha_i\) and \(\beta_i \rightarrow \rho \beta_i\), the expression is
written as follows: for odd \( n = 2m - 1 \)

\[
I_{\mu_1 \cdots \mu_{2m-1}}(x_i) = \sum_{k=1}^{m} \frac{1}{2^{4 + m - k} \pi^2} \left[ p_{\mu_1} \cdots \cdot p_{\mu_{2k-1}} \delta_{\mu_{2k} \mu_{2k+1}} \cdots \delta_{\mu_{2m-2} \mu_{2m-1}} \right]_{\text{sym}}
\]

\[
\times \int_{0}^{1} \prod d\alpha_i d\beta_i \left( \sum \beta_i \right)^{2k-1} \delta \left( 1 - \sum \alpha - \sum \beta \right)
\]

\[
\times \int_{0}^{\infty} d\rho d\rho' \exp \left[ -\rho \left\{ p^2 \sum \alpha_i \sum \beta_i + \sum (\alpha_i + \beta_i) x_i \right\} \right], \quad (67a)
\]

and for even \( n = 2m \)

\[
I_{\mu_1 \cdots \mu_{2m}}(x_i) = \sum_{k=0}^{m} \frac{1}{2^{4 + m - k} \pi^2} \left[ p_{\mu_1} \cdots \cdot p_{\mu_{2k}} \delta_{\mu_{2k+1} \mu_{2k+2}} \cdots \delta_{\mu_{2m-1} \mu_{2m}} \right]_{\text{sym}}
\]

\[
\times \int_{0}^{1} \prod d\alpha_i d\beta_i \left( \sum \beta_i \right)^{2k} \delta \left( 1 - \sum \alpha - \sum \beta \right)
\]

\[
\times \int_{0}^{\infty} d\rho d\rho' \exp \left[ -\rho \left\{ p^2 \sum \alpha_i \sum \beta_i + \sum (\alpha_i + \beta_i) x_i \right\} \right], \quad (67b)
\]

where \( i \) runs from 0 to \( N \). We define \( l \equiv 2N + k - m - 3 \) and \( [ \quad ]_{\text{sym}} \) as the symmetrization about indices like \( [\delta_{\mu_1 \mu_2} p_{\mu_3}]_{\text{sym}} = \delta_{\mu_1 \mu_2} p_{\mu_3} + \delta_{\mu_2 \mu_3} p_{\mu_1} + \delta_{\mu_3 \mu_1} p_{\mu_2} \). We take \( \sum \alpha_i + \sum \beta_i = 1 \) because of the \( \delta \)-function.

To calculate the expression, we consider the \( \rho \)-integration previous to the parameter integration of \( \alpha_i \) and \( \beta_i \). First we consider the case when \( l \) is positive value. With an integration by parts we get,

\[
\int_{0}^{\infty} d\rho d\rho' \exp \left[ -\rho b \{ aX + Y \} \right]
\]

\[
= -\frac{\rho^l}{b \{ aX + Y \}} e^{-\rho b \{ aX + Y \}} \bigg|_{0}^{\infty} + \int_{0}^{\infty} d\rho \frac{l \rho^{l-1}}{b \{ aX + Y \}} e^{-\rho b \{ aX + Y \}} . \quad (68)
\]

The first term of r.h.s. vanishes under the condition \( b \{ aX + Y \} > 0 \). Integrating by parts recursively, we get the following result:

\[
\int_{0}^{\infty} d\rho d\rho' \exp \left[ -\rho b \{ aX + Y \} \right] = \frac{(-1)^l}{a^{l+1} (\partial X)^l} \left[ \frac{1}{aX + Y} \right]. \quad (69)
\]

Going to the last expression, we use the formula

\[
\int_{0}^{\infty} d\rho \exp \left[ -\rho b \{ aX + Y \} \right] = \frac{1}{b \{ aX + Y \}} . \quad (70)
\]
For negative \( l \) using the relation
\[
\frac{\partial|l|}{(\partial X)^{|l|}} \int_0^\infty \frac{d\rho}{\rho} e^{-\rho b(aX + Y)} = (-ab)^{|l|} \int_0^\infty d\rho e^{-\rho b(aX + Y)}
\] (71)
and (70), we get a differential equation about \( X \). The differential equation is solved by integrating with \( X \) recursively, and the solution is formally written by
\[
\int_0^\infty d\rho \rho e^{-\rho b(aX + Y)} = (-1)^{|l|} a |X| + \cdots
\] (72)
Here \( C_i \) arises from the \( i \)-th integral and represented using the parameter \( X \) which has the same dimension with \( X \) as
\[
C_1 = -\ln X, \quad C_2 = X, \quad C_3 = -\frac{X^2}{4}, \quad C_4 = \frac{X^3}{18}, \quad \cdots.
\] (73)
Using the results, we calculate the easiest formulae of \( N = 1 \) and \( x_i = 0 \) taking \( X = p^2 \) as follows:
\[
I(0) = -\frac{1}{16\pi^2} (\ln p^2 + C_1), \quad I_\mu(0) = -\frac{p_\mu}{32\pi^2} (\ln p^2 + C_1).
\] (74)
We can calculate the more general \( n \) of this case
\[
I_{\mu_1 \cdots \mu_n}(0) \sim C_{[1+\frac{n}{2}]} + p^2 C_{[\frac{n}{2}]} + \cdots + \frac{p^2 [\frac{n}{2}]}{[\frac{n}{2}]} C_1 + \text{const.}
\] (75)
where \([ \quad ]\) denotes the Gauss’ notation and \( C_i \) is represented using the parameter \( M \) of mass dimension of one by
\[
C_1 = -\ln M^2, \quad C_2 = M^2, \quad C_3 = -\frac{M^4}{4}, \quad C_4 = \frac{M^6}{18}, \quad \cdots.
\] (76)
The parameter \( M \) goes to \( \infty \) just like the cut-off parameter. Under this parametrization, (74) is read as the dimensionless divergence of \( \ln(M/p) \).

C Infinite Sum

Here we give a formula that we use the calculation of an infinite sum of PV fields. Under the Feynman gauge, all the expressions of the diagrams listed
in Figures 1 and 4 are reduced to the following forms;

\[ \sum_{j=-\infty}^{\infty} (-1)^j M_j^{2r} \int \frac{d^4k}{(2\pi)^4} \frac{k_{\mu_1} \cdots k_{\mu_n}}{(k^2 + M_j^2)^N (q^2 + M_j^2)^N}. \] (77)

At the beginning, we consider \( r = 0 \) case. Inserting (67) to this expression, we find that the most important calculation is the summation with \( j \) which is given after the \( \rho \)-integration,

\[ \sum_{j=-\infty}^{\infty} (-1)^j \int_{0}^{\infty} d\rho \rho^l \exp \left[-\rho M^2 b (aX + |j|^2)\right] \]

\[ = \frac{(-1)^l}{a^l (M^2 b)^l+1} \frac{\partial^l}{(\partial X)^l} \left[ \frac{1}{aX} - \frac{\pi^2}{6} + \frac{7\pi^4}{360} aX + \cdots \right], \] (78)

where we take \( M_j = M|j| \) and define \( a, b \) and \( X \) as follows:

\[ a \equiv \sum_{i=1}^{N} \alpha_i \sum_{i=1}^{N} \beta_i \sum_{i=1}^{N} \alpha_i + \beta_i, \quad b \equiv \sum_{i=1}^{N} \alpha_i + \beta_i, \quad X \equiv \frac{p^2}{M^2}. \] (79)

Going to the r.h.s. of (78) we use the formula \( \sum_{j \in \mathbb{Z}} (-1)^j A^{2+j^2} = \frac{\pi}{A \sinh \pi A} \) and expand it. The negative \( l \) means multiple integral of order \( l \) as mentioned Appendix B. In the case of \( l = -1 \), for instance, we get

\[ \sum_{j=-\infty}^{\infty} (-1)^j \int_{0}^{\infty} d\rho \rho^l \exp \left[-\rho M^2 b (aX + |j|^2)\right] \]

\[ = - \ln X + \frac{\pi^2}{6} aX - \frac{7\pi^4}{720} a^2 X^2 - aC_1 + \cdots, \] (80)

where \( C_1 \) is a dimension less constant and determined by the relation \( \frac{\pi}{\sqrt{aX \sinh \pi \sqrt{aX}}} = \frac{\partial}{\partial X} \left( \frac{2}{a} \tanh \frac{\sqrt{2a}}{a} \sqrt{aX} \right) \) such as

\[ C_1 = \frac{1}{a} \ln \left( \frac{\pi^2}{4a} \right). \] (81)

For \( r = 1 \) the key object is the following equation:

\[ \sum_{j=-\infty}^{\infty} (-1)^j M_j^2 \int_{0}^{\infty} d\rho \rho^l \exp \left[-\rho M^2 b (aX + |j|^2)\right]. \] (82)
Notice the relation
\[
\frac{\partial}{(\partial M^2)} \int_0^\infty d\rho \rho^{l-1} \exp \left[ -\rho M^2 b\{aX + |j|^2\} \right] = -b\{aX + |j|^2\} \int_0^\infty d\rho \rho^l \exp \left[ -\rho M^2 b\{aX + |j|^2\} \right],
\]
(83)

and using (78) we get
\[
\sum_{j=-\infty}^\infty (-1)^{l+r} M_j^2 \int_0^\infty d\rho \rho^l \exp \left[ -\rho M^2 b\{aX + |j|^2\} \right] = \frac{(-1)^{l+1}}{a^{l+1}(M^2)^{l+1}} \left[ X \left( \frac{\partial}{\partial X} \right)^{l+1} + l \left( \frac{\partial}{\partial X} \right)^{l-1} \right] \left[ \frac{1}{aX} - \frac{\pi^2}{6} + \frac{7\pi^2}{360} aX + \cdots \right].
\]
(84)

Using (78) and (84) iteratively, we get the formula for \( r \geq 1 \),
\[
\sum_{j=-\infty}^\infty (-1)^{j+r} M_j^2 \int_0^\infty d\rho \rho^l \exp \left[ -\rho M^2 b\{aX + |j|^2\} \right] = \frac{(-1)^{l+r}}{a^{l-r}(M^2)^{l+1-r}b^{l+1}} \sum_{q=0}^r \binom{r}{q} \frac{l!}{(l-q)!} \left( \frac{\partial}{\partial X} \right)^{l-q} \left[ \frac{1}{aX} - \frac{\pi^2}{6} + \frac{7\pi^2}{360} aX + \cdots \right].
\]
(85)

Here we give explicit calculations which we use in Section 3 and 4. For \( \Lambda^0 \) order, all the integrals are given by (77) with \( N = 1 \), we have to calculate up to \( n = 2 \) with \( r = 0 \) and \( n = 0 \) with \( r = 2 \). Since these integrals give the \( \rho \)-integration with \( l = -1 \) and \( l = -2 \) there appear dimension less constants

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\( C_1 \) and \( C_2 \) as follows:

\[
\sum_{j=\infty}^{\infty} (-1)^j I(M^2_j) = \frac{1}{16\pi^2} \left[ -2 \ln \left( \frac{\pi p}{2M} \right) + 2 + \frac{\pi^2 p^2}{36M^2} + O(M^{-4}) \right], \tag{86a}
\]

\[
\sum_{j=-\infty}^{\infty} (-1)^j I_{\mu_1}(M^2_j) = \frac{p_{\mu_1}}{16\pi^2} \left[ -\ln \left( \frac{\pi p}{2M} \right) + 1 + \frac{\pi^2 p^2}{72M^2} + O(M^{-4}) \right], \tag{86b}
\]

\[
\sum_{j=-\infty}^{\infty} (-1)^j I_{\mu_1\mu_2}(M^2_j) = \frac{p_{\mu_1}p_{\mu_2}}{16\pi^2} \left[ -\frac{2}{3} \ln \left( \frac{\pi p}{2M} \right) + \frac{13}{18} + \frac{\pi^2 p^2}{120M^2} + O(M^{-4}) \right]
+ \frac{p^2 \delta_{\mu_1\mu_2}}{32\pi^2} \left[ \frac{M^2}{30p^2}C_2 + \frac{1}{3} \ln \left( \frac{\pi p}{2M} \right) - \frac{4}{9} - \frac{\pi^2 p^2}{360M^2} + O(M^{-4}) \right], \tag{86c}
\]

\[
\sum_{j=-\infty}^{\infty} (-1)^j M^2_j I(M^2_j) = \frac{1}{16\pi^2} \left[ -\frac{M^2}{30}C_2 + \frac{p^2}{6} - \frac{\pi^2 p^4}{360M^2} + O(M^{-4}) \right]. \tag{86d}
\]

Similarly for \( \Lambda^{-4} \) order all the integrals are given by \( N = 2 \). The most divergent parts are calculated

\[
\sum_{j=-\infty}^{\infty} (-1)^j I_{\mu_1 \ldots \mu_{10}}(M^2_j, M^2_j) = \frac{[\delta_{\mu_1\mu_2} \ldots \delta_{\mu_9\mu_{10}}]_{\text{sym}}}{2^9\pi^2} \frac{M^6}{2772} C_4 + O(M^4), \tag{87a}
\]

\[
\sum_{j=-\infty}^{\infty} (-1)^j M^4_j I_{\mu_1 \ldots \mu_8}(M^2_j, M^2_j) = \frac{[\delta_{\mu_1\mu_2} \ldots \delta_{\mu_7\mu_8}]_{\text{sym}}}{2^8\pi^2} \frac{3M^6}{2772} C_4 + O(M^4), \tag{87b}
\]

\[
\sum_{j=-\infty}^{\infty} (-1)^j M^4_j I_{\mu_1 \ldots \mu_6}(M^2_j, M^2_j) = \frac{[\delta_{\mu_1\mu_2} \ldots \delta_{\mu_5\mu_6}]_{\text{sym}}}{2^7\pi^2} \frac{6M^6}{2772} C_4 + O(M^4), \tag{87c}
\]

\[
\sum_{j=-\infty}^{\infty} (-1)^j M^6_j I_{\mu_1 \ldots \mu_4}(M^2_j, M^2_j) = \frac{[\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}]_{\text{sym}}}{2^6\pi^2} \frac{6M^6}{2772} C_4 + O(M^4), \tag{87d}
\]

\[
\sum_{j=-\infty}^{\infty} (-1)^j M^8_j I_{\mu_1\mu_2}(M^2_j, M^2_j) = O(M^4), \tag{87e}
\]
where $C_4$ is the integration constant arising from the fourth order integration of $X$ which is caused by the $\rho$-integration with $l = -4$.

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