Quantum-Corrected Black Hole Thermodynamics in Extra Dimensions

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Abstract

Bekenstein-Hawking formalism of black hole thermodynamics should be modified to incorporate quantum gravitational effects. Generalized Uncertainty Principle (GUP) provides a suitable framework to perform such modifications. In this paper, we consider a general form of GUP to find black hole thermodynamics in a model universe with large extra dimensions. We will show that black holes radiate mainly in the four-dimensional brane. Existence of black holes remnants as a possible candidate for dark matter is discussed.

PACS: 04.70.-s, 04.70.Dy, 04.50.+h

Key Words: Quantum Gravity, Generalized Uncertainty Principle, Black Holes Thermodynamics, Large Extra Dimensions
1 Introduction

The idea of Large Extra Dimensions (LEDs) which recently has been proposed [1-5], might allow to study interactions at trans-Planckian energies in the next generation collider experiments. The ADD-model proposed by Arkani-Hamed, Dimopoulos and Dvali[1-3] adds $d$ extra spacelike dimensions without curvature, in general each of them compactified to the same radius $L$. In this scenario, all standard-model particles are confined to the observable 4-dimensional brane universe, whereas gravitons can access the whole $d$-dimensional bulk spacetime, being localized at the brane at low energies. In this scenario, the hierarchy problem is solved or at least reformulated in a geometric language. On the other hand, the setting of RS-model proposed by Randall and Sundrum[4,5] is a 5-dimensional spacetime with an non-factorizable geometry. The solution for the metric is found by analyzing the solution of Einstein’s field equations with an energy density on our brane, where the standard model particles live. In the type I model the extra dimension is compactified while in the type II model it is infinite.

The possibility of the existence of large extra dimensions has opened up new and exciting avenues of research in quantum gravity. In particular, a host of interesting work is being done on different aspects of low-energy scale quantum gravity phenomenology. One of the most significant sub-fields is the study of black hole production at particle colliders, such as the Large Hadronic Collider (LHC)[6] and the muon collider [7], as well as in ultrahigh energy cosmic ray (UHECR) airshowers [8,9]. Newly formed black holes first lose hair associated with multipole and angular momenta, then approach classically stable Schwarzschild solutions, and finally evaporate via Hawking radiation [10] up to possible Planck size remnants. Decay time and entropy completely determine the observables of the process. Black hole formation and decay can be described semiclassically, provided that the entropy is sufficiently large. The timescale for the complete decay of a black hole up to its supposed final Planck-sized remnant is expected to be of order of the $TeV^{-1}$.

Black Hole thermodynamical quantities depend on the Hawking temperature $T_H$ via the usual thermodynamical relations (for example Stefan-Boltzmann law). The Hawking temperature undergoes corrections from many sources, and these corrections are particularly relevant for black holes with mass of the order of the Planck mass. Therefore, the study of $TeV$-scale black holes in UHECR and particle colliders requires a careful investigation of how temperature corrections affect black hole thermodynamics. In this article, we concentrate on the corrections due to the generalized uncertainty principle (GUP)
in the framework of LEDs. These corrections are not tied down to any specific model of quantum gravity; since GUP can be derived using arguments from string theory [11] as well as other approaches to quantum gravity [12,13]. Black holes thermodynamics in four spacetime dimensions and in the framework of GUP, has been studied in several context[14-18]. Embedding a black hole in a space-time of higher dimensionality would seem, from the string theory point of view, to be a natural thing to do. Black holes in d extra dimensions have been studied in both compact [19] and infinitely extended [20] extra dimensions (see also [21] and references therein). Here we proceed one more step in this direction. Using a general form of GUP, we provide a perturbational framework to calculate temperature and entropy of a black hole in a model universe with large extra dimensions. Our approach will show that black holes decay mainly on the brane. We investigate also the possibility of having black holes remnants in extra dimensional scenarios. These remnants are good candidates for dark matter.

The paper is organized as follows: Section 2 gives our primary inputs for rest of the calculations. Section 3 is devoted to calculation of GUP-induced corrections of black hole thermodynamics. Section 4 considers the black holes remnants as a possible source of dark matter. The paper follows by conclusions in section 5.

2 GUP and LEDs

The canonical commutation relations between the momentum operator $p^\nu$ and position operator $x^\mu$, which in Minkowski space-time are $[x^\mu, p^\nu] = i\hbar \eta^{\mu\nu}$, in a curved space-time with metric $g_{\mu\nu}$ can be generalized as

$$[x^\mu, p^\nu] = i\hbar g^{\mu\nu}(x).$$

This equation contains gravitational effects of a particle in first quantization scheme. Its validity is confined to curved spacetime asymptotically flat so that the tensor metric can be decomposed as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu}$ is the local perturbation to the flat background[22]. We note that the usual commutation relations between position and momentum operators in Minkowsky spacetime are obtained by using the vierbein formalism, i.e. by projecting the commutator and the metric tensor on the tangent space. In which follows we consider another alternative: existence of a minimal observable length. As it is well known, a theory containing a fundamental length on the order of $l_P$ (which can
be related to the extension of particles) is string theory. It provides a consistent theory of quantum gravity and allows to avoid the above mentioned difficulties. In fact, unlike point particle theories, the existence of a fundamental length plays the role of natural cut-off. In such a way the ultraviolet divergencies are avoided without appealing to the renormalization and regularization schemes[23]. Besides, by studying string collisions at Planckian energies and through a renormalization group type analysis the emergence of a minimal observable distance yields to the generalized uncertainty principle

$$\delta x \geq \frac{\hbar}{2\delta p} + \text{const.} G \delta p,$$

(2)

At energy much below the Planck mass, \(m_p = \sqrt{\frac{\hbar c}{G}} \sim 10^{19}\text{GeV}/c^2\), the extra term in equation (2) is irrelevant and the Heisenberg uncertainty relation is recovered, while, as we approach the Planck energy, this term becomes relevant and is related to the minimal observable length on the order of Planck length, \(l_p = \sqrt{\frac{G\hbar}{c^3}} \sim 1.6 \times 10^{-35}\text{m}\). In terms of Planck length, equation (2) can be written as,

$$\delta x \geq \frac{\hbar}{2\delta p} \left( \frac{1}{\delta p} + \alpha^2 l_p^2 \frac{\delta p}{\hbar} \right).$$

(3)

where \(\alpha\) is a dimensionless constant of order one which depends on the details of the quantum gravity theory. Note that one should consider an extra term in the right hand side of relation (3) which depends to expectation values of \(x\) and \(p\). But since we like to dealing with absolutely smallest position uncertainty, this extra term is omitted.

It is important to note that various domains of modern physics lead to such a result. In addition to string theory, theories such as loop quantum gravity, black hole gedanken experiments and quantum geometry also give such generalized uncertainty principle. The matter which is very interesting is the fact that these generalized uncertainty principles can be obtained in the framework of classical Newtonian gravitational theory and classical general relativity[24]. In other words, although a full description of quantum gravity is not yet available, there are some general features that seem to go hand in hand with all promising candidates for such a theory where one of them is the existence of a minimal length scale. This minimal length scale gives an extreme quantum regime called Planck regime. In this scale the running couplings unify and quantum gravity era is likely to occur. At this scale the quantum effects of gravitation get as important as those of the electroweak and strong interactions. In this extreme conditions, the usual Heisenberg algebra should be modified regarding extra uncertainty induced by quantum gravitational
effects. This modified commutator algebra may be given as follows

$$[x, p] = i\hbar(1 + \beta'p^2), \quad (4)$$

It is possible to have more terms in the right hand side of (2). One can consider more generalized uncertainty relation such as[25],

$$\delta x\delta p \geq \frac{\hbar}{2} \left( 1 + \alpha'(\delta x)^2 + \beta'(\delta p)^2 + \gamma \right), \quad (5)$$

which leads to a nonzero minimal uncertainty in both position and momentum. This relation can lead us to following commutator relation

$$[x, p] = i\hbar \left( 1 + \alpha'x^2 + \beta'p^2 \right), \quad (6)$$

where $\gamma = \alpha'(x)^2 + \beta'(p)^2$. This statement shows that GUP itself has a perturbational expansion. We are going to consider the effects of GUP on black hole thermodynamics in model universes with extra dimensions. Therefore we need to representation of GUP in LEDs scenarios. As has been indicated, there are two main scenarios of extra dimensions:

- the Arkani-Hamed–Dimopoulos–Dvali (ADD) model[1-3], where the extra dimensions are compact and of size $L$;

and

- the Randall–Sundrum (RS) model[4,5], where the extra dimensions have an infinite extension but are warped by a non-vanishing cosmological constant.

A feature shared by (the original formulations of) both scenarios is that only gravity propagates along the extra dimensions, while Standard Model fields are confined on a four-dimensional sub-manifold usually referred to as the brane-world. In which follows we consider the ADD model as our LEDs scenario. In LEDs scenario, GUP can be written as follows

$$\delta x_i\delta p_i \geq \frac{\hbar}{2} \left( 1 + \frac{\alpha^2L_{Pl}^2}{\hbar^2}(\delta p_i)^2 + \frac{\beta^2}{L_{Pl}^2}(\delta x_i)^2 + \gamma \right). \quad (7)$$

Here $\alpha$, $\beta$ and $\gamma$ are dimensionless, positive and independent of $\delta x$ and $\delta p$ but may in general depend on the expectation values of $x$ and $p$. Planck length now is defined as $L_{Pl} = \left( \frac{\hbar G_d}{c^3} \right)^{\frac{1}{d-2}}$. Here $G_d$ is gravitational constant in $d$ dimensional spacetime which in ADD model is given by $G_d = G_4 L^n$ where $n$ is number of extra dimensions, $n = d - 4$. In which follows, we use this more general form of GUP as our primary input and construct a perturbational calculations to find thermodynamical properties of black hole and
its quantum gravitational corrections. It should be noted that since GUP is a model independent concept [26], the results which we obtain are consistent with any fundamental theory of quantum gravity.

3 Black Holes Thermodynamics

3.1 Black Holes Temperature

The Hawking temperature for a spherically symmetric black hole may be obtained in a heuristic way with the use of the standard uncertainty principle and general properties of black holes [27]. We picture the quantum vacuum as a fluctuating sea of virtual particles; the virtual particles cannot normally be directly observed without violating energy conservation. But near the surface of a black hole the effective potential energy can negate the rest energy of a particle and give it zero total energy, and the surface itself is a one-way membrane which can swallow particles so that they are henceforth not observable from outside. The net effect is that for a pair of photons one photon may be absorbed by the black hole with effective negative energy $-E$, and the other may be emitted to asymptotic distances with positive energy $+E$. The characteristic energy $E$ of the emitted photons may be estimated from the standard uncertainty principle. In the vicinity of the black hole surface there is an intrinsic uncertainty in the position of any particle of about the Schwarzschild radius, $r_s$, due to the behavior of its field lines [28], as well as on dimensional grounds. This leads to momentum uncertainty

$$\delta p \approx \frac{\hbar}{\delta x} = \frac{\hbar c^2}{2GM}, \quad \delta x \approx r_s = \frac{2GM}{c^2} \quad (8)$$

and to an energy uncertainty of $\delta pc = \frac{\hbar c^3}{2GM}$. We identify this as the characteristic energy of the emitted photon, and thus as a characteristic temperature; it agrees with the Hawking temperature up to a factor of $4\pi$, which we will henceforth include as a ”calibration factor” and write, with $k_B = 1$,

$$T_H \approx \frac{\hbar c^3}{8\pi GM}, \quad (9)$$

The related entropy is obtained by integration of $dS_B = \frac{c^2 dM}{T_H}$ which is the standard Bekenstein entropy,

$$S_B = \frac{4\pi GM^2}{\hbar c} = \frac{A}{4l_p^2} \quad (10)$$

6
where $A = 4\pi r_s^2$ (the area of event horizon). A $d$-dimensional spherically symmetric BH of mass $M$ (to which the collider BHs will settle into before radiating) is described by the metric,

$$ds^2 = -\left(1 - \frac{16\pi G_d M}{(d-2)\Omega_{d-2}c^2r^{d-3}}\right)c^2 dt^2 + \left(1 - \frac{16\pi G_d M}{(d-2)\Omega_{d-2}c^2r^{d-3}}\right)^{-1}dr^2 + r^2d\Omega_{d-2}^2$$ (11)

where $\Omega_{d-2}$ is the metric of the unit $S^{d-2}$ as $\Omega_{d-2} = \frac{2\pi^{d-1}}{\Gamma(\frac{d}{2})}$. Since the Hawking radiation is a quantum process, the emitted quanta should satisfy the generalized uncertainty principle (which has quantum gravitational nature) in its general form. Therefore, we consider equation (7), where $x_i$ and $p_i$ with $i = 1...d - 1$, are the spatial coordinates and momenta respectively. By modeling a BH as a $(d-1)$-dimensional cube of size equal to its Schwarzschild radius $r_s$, the uncertainty in the position of a Hawking particle at the emission is,

$$\delta x_i \approx r_s = \omega_d L_{Pl}m^{\frac{1}{d-3}}, \quad (12)$$

where

$$\omega_d = \left(\frac{16\pi}{(d-2)\Omega_{d-2}}\right)^{\frac{1}{d-3}},$$

$m = \frac{M}{M_{Pl}}$ and $M_{Pl} = \left(\frac{\hbar^{d-3}}{cd^{d-2}G_d}\right)^{\frac{1}{d-2}}$. Here $\omega_d$ is dimensionless area factor. A simple calculation based on equation (7) gives,

$$\delta x_i \approx \frac{L_{Pl}^2}{\beta^2\hbar}\left[1 \pm \sqrt{1 - \beta^2\left(\frac{\hbar^2(\gamma + 1)}{L_{Pl}^2(\delta p_i)^2}\right)}\right]. \quad (13)$$

Here, to achieve standard values (for example $\delta x_i; \delta p_i \geq \hbar$) in the limit of $\alpha = \beta = \gamma = 0$, we should consider the minus sign. One can minimize $\delta x$ to find

$$(\delta x_i)_{min} \simeq \pm\alpha L_{Pl}\sqrt{\frac{1 + \gamma}{1 - \alpha^2\beta^2}}. \quad (14)$$

This is minimal observable length on the order of Planck length. Here we should consider the plus sign in equation (14), whereas the negative sign has no evident physical meaning. Equation (7) gives also

$$\delta p_i \approx \frac{\hbar \delta x_i}{\alpha^2 L_{Pl}^2}\left[1 \pm \sqrt{1 - \alpha^2\left(\beta^2 + \frac{L_{Pl}^2(\gamma + 1)}{(\delta x_i)^2}\right)}\right]. \quad (15)$$
Here to achieve correct limiting results we should consider the minus sign in round bracket. From a heuristic argument based on Heisenberg uncertainty relation, one deduces the following equation for Hawking temperature of black holes\cite{14},

\[ T_H \approx \frac{(d-3)c\delta p_i}{4\pi} \]  

(16)

where we have set the constant of proportionality equal to \( \frac{(d-3)}{4\pi} \) in extra dimensional scenarios. Based on this viewpoint, but now using generalized uncertainty principle in its general form, modified black hole temperature in GUP is,

\[ T_H^{GUP} \approx \frac{(d-3)\hbar c\delta x_i}{4\pi\alpha^2L_{Pl}^2} \left[ 1 - \sqrt{1 - \alpha^2 \left( \beta^2 + \frac{L_{Pl}^2(\gamma + 1)}{\delta x_i^2} \right)} \right]. \]

(17)

Since \( \delta x_i \) is given by (12), this relation can be expressed in terms of black hole mass in any stage of its evaporation. Figure 1 shows the relation between temperature and mass of the black hole in different spacetime dimensions. Following results can be obtained from this analysis: In scenarios with extra dimensions, black hole temperature increases. This feature leads to faster decay and less classical behaviors for black holes. It is evident that in extra dimensional scenarios final stage of evaporation (black hole remnant) has mass more than its four dimensional counterpart. Therefore, in the framework of GUP, it seems that quantum black holes are hotter, shorter-lived and tend to evaporate less than classical black holes. Note that these results are applicable to both ADD and RS brane world scenarios.

### 3.2 Black Holes Entropy

Now consider a quantum particle that starts out in the vicinity of an event horizon and then ultimately absorbed by black hole. For a black hole absorbing such a particle with energy \( E \) and size \( l \), the minimal increase in the horizon area can be expressed as \cite{29}

\[ (\Delta A)_{\text{min}} \geq \frac{8\pi L_{Pl}^{d-2}El}{(d-3)\hbar c}, \]

(18)

then one can write

\[ (\Delta A)_{\text{min}} \geq \frac{8\pi L_{Pl}^{d-2}c\delta p_il}{(d-3)\hbar c}, \]

(19)
where \( E \sim c\delta p_i \) and \( l \sim \delta x_i \).

\[
(\Delta A)_{\min} \simeq \frac{8\pi L_P^d (\delta x_i)^2}{(d - 3)\alpha^2} \left[ 1 - \sqrt{1 - \alpha^2 \left( \beta^2 + \frac{L_P^2 (\gamma + 1)}{(\delta x_i)^2} \right)} \right],
\]  
(20)

Now we should determine \( \delta x_i \). Since our goal is to compute microcanonical entropy of a large black hole, near-horizon geometry considerations suggest the use of inverse surface gravity or simply the Schwarzschild radius for \( \delta x_i \). Therefore, \( \delta x_i \approx r_s \) and defining \( \Omega_{d-2} r_s^{d-2} = A \) or \( r_s^2 = \Omega_{d-2} \frac{A^2}{d-2} \) and \( (\Delta S)_{\min} = b \), then it is easy to show that,

\[
(\Delta A)_{\min} \simeq \frac{8\pi L_P^d \Omega_{d-2}^{\frac{d-2}{2}} A^{\frac{d-2}{2}}}{(d - 3)\alpha^2} \left[ 1 - \sqrt{1 - \alpha^2 \left( \beta^2 + \frac{L_P^2 (\gamma + 1)}{\Omega_{d-2}^{\frac{d-2}{2}} A^{\frac{d-2}{2}}} \right)} \right],
\]  
(21)

and,

\[
\frac{dS}{dA} \simeq \frac{(\Delta S)_{\min}}{(\Delta A)_{\min}} \simeq \frac{\Omega_{d-2}^{\frac{d-2}{2}} b \alpha^2 (d - 3)}{8\pi L_P^d A^{\frac{d-2}{2}}} \left[ 1 - \sqrt{1 - \alpha^2 \left( \beta^2 + \frac{\Omega_{d-2}^{\frac{d-2}{2}} L_P^2 (\gamma + 1)}{A^{\frac{d-2}{2}}} \right)} \right].
\]  
(22)

Two point should be considered here. First note that \( b \) can be considered as one bit of information since entropy is an extensive quantity. Secondly, in our approach we consider microcanonical ensemble since we are dealing with Schwarzschild black hole of fixed mass. Now we should perform integration. There are two possible choices for lower limit of integration, \( A = 0 \) and \( A = A_p \). Existence of a minimal observable length leads to existence of a minimum event horizon area, \( A_p = \Omega_{d-2} (\delta x_i)^{d-2} \). So it is physically reasonable to set \( A_p \) as lower limit of integration. This is in accordance with existing picture[14]. Based on these arguments, we can write

\[
S \simeq \varepsilon \int_{A_p}^{A} \frac{A^{-\frac{d-2}{2}}}{1 - \sqrt{\eta + \kappa A^{-\frac{d-2}{2}}}} dA
\]  
(23)

where,

\[
\varepsilon = \frac{\Omega_{d-2}^{\frac{d-2}{2}} b \alpha^2 (d - 3)}{8\pi L_P^d}, \quad \kappa = -\Omega_{d-2}^{\frac{d-2}{2}} 2 \alpha^2 L_P^2 (\gamma + 1), \quad \eta = 1 - \alpha^2 \beta^2,
\]

\[
A_p = \Omega_{d-2} (\alpha L_P)^{d-2} \left( \frac{1 + \gamma}{1 - \alpha^2 \beta^2} \right)^{(d-2)/2}
\]  
(24)
This integral can be solved numerically. The result is shown in figure 2. This figure shows that: In scenarios with extra dimensions, black hole entropy decreases. The classical picture breaks down since the degrees of freedom of the black hole, i.e. its entropy, is small. In this situation one can use the semiclassical entropy to measure the validity of the semiclassical approximation. It is evident that in extra dimensional scenarios final stage of evaporation (black hole remnant) has event horizon area more than its four dimensional counterpart. Therefore, higher dimensional black hole remnants have less classical features relative to their four dimensional counterparts. To obtain the relation between emission rate of black holes radiation and spacetime dimensions, we proceed as follows. As Emparan et al have shown [21], in $d$ dimensions, the energy radiated by a black body of temperature $T$ and surface area $A$ is given by

$$\frac{dE_d}{dt} = \sigma_d A T^d, \quad (25)$$

where $\sigma_d$ is $d$-dimensional Stefan-Boltzman constant,

$$\sigma_d = \frac{\Omega_{d-3}}{(2\pi)^{d-1}(d-2)} \Gamma(d)\zeta(d).$$

Now using equations (17) for modified Hawking temperature in the framework of GUP, equation (25) becomes

$$\frac{dE_d}{dt} = \frac{\Omega_{d-3}\Omega_{d-2}}{(2\pi)^{d-1}(d-2)} \Gamma(d)\zeta(d) \left( \frac{(d-3)hc}{4\pi^2L^2_P} \left[ 1 - \sqrt{1 - \alpha^2\left( \frac{\beta^2 + \frac{L^2_P(\gamma + 1)}{r_s^2}}{\alpha \frac{r_s^2}{(r_s^2)^2}} \right)} \right] \right)^d r_s^{2d-2}, \quad (26)$$

where we have set $\delta x_i \sim r_s$. This is a complicated relation. To compare emission rates of black holes in different $d$, note that $\sigma_n$ changes very little with dimension. This fact confirms that even though higher dimensional spacetimes have infinitely many more modes due to excitations in the extra dimensions, the rate at which energy is radiated by black body with radius $r_s$ and temperature $T \sim \frac{1}{r_s}$ is roughly independent of the dimension. Based on this argument, let us assume that $d = 4$, $d = 6$ and $d = 10$. Since $\sigma_4 = 0.08$, $\sigma_6 = 0.06$ and $\sigma_{10} = 0.097$, some numerical calculations give approximately

$$\frac{\left( \frac{dE_4}{dt} \right)}{\left( \frac{dE_6}{dt} \right)} \approx 11 \quad \text{and} \quad \frac{\left( \frac{dE_4}{dt} \right)}{\left( \frac{dE_{10}}{dt} \right)} \approx 12 \quad (27)$$

These results evidently show that black holes radiate mainly on the 4-dimensional brane. In fact, a higher-dimensional black hole emits radiation both in the bulk and on the brane.
Note that some corrections to equation (27) should be considered due to area appeared in (25). A detailed calculation shows that critical radius of black hole as an absorber is given by

$$r_c = \left( \frac{d - 1}{2} \right)^{1/3} \left( \frac{d - 1}{d - 3} \right)^{1/2} r_s.$$  

Therefore, equation (27) will change to

$$\left( \frac{dE_1}{dt} \right) \left( \frac{dE_6}{dt} \right) \approx 3.5 \quad \text{and} \quad \left( \frac{dE_4}{dt} \right) \left( \frac{dE_{10}}{dt} \right) \approx 1.5$$

According to the assumptions of the theory with Large Extra Dimensions, only gravitons, and possibly scalar fields, can propagate in the bulk and thus, these are the only types of fields allowed to be emitted in the bulk during the Hawking evaporation phase. On the other hand, the emission on the brane can take the form of scalar Higgs particles, fermions and gauge bosons. From the perspective of the brane observer, the radiation emitted in the bulk will be a missing energy signal, while radiation on the brane may lead to experimental detection of Hawking radiation and thus of the production of small black holes. In next section we discuss some of these experimental approaches.

4 Black Holes Remnants and Extra Dimensions

It is by now widely accepted that dark matter (DM) constitutes a substantial fraction of the present critical energy density in the Universe. However, the nature of DM remains an open problem. There exist many DM candidates, most of them are non-baryonic weakly interacting massive particles (WIMPs), or WIMP-like particles[30]. By far the DM candidates that have been more intensively studied are the lightest supersymmetric (SUSY) particles such as neutralinos or gravitinos, and the axions (as well as the axinos). There are additional particle physics inspired dark matter candidates[30]. A candidate which is not as closely related to particle physics is the relics of primordial black holes(Micro Black Holes)[31,32]. Certain inflation models naturally induce a large number of such a black holes. As a specific example, hybrid inflation can in principle yield the necessary abundance of primordial black hole remnants for them to be the primary source of dark matter[33,34]. Here we have shown that final stage of evaporation of a black hole is a remnant which has mass increasing with spacetime dimensions. One of the major problems with these remnants is the possibility of their detection. As interactions with black
hole remnants are purely gravitational, the cross section is extremely small, and direct observation of these remnants seems unlikely. One possible indirect signature may be associated with the cosmic gravitational wave background. Unlike photons, the gravitons radiated during evaporation would be instantly frozen. Since, according to our notion, the black hole evaporation would terminate when it reduces to a remnants, the graviton spectrum should have a cutoff at Planck mass. Such a cutoff would have by now been red-shifted to $\sim 10^{14}\text{GeV}$. Another possible gravitational wave-related signature may be the gravitational wave released during the gravitational collapse. The frequencies of such gravitational waves would by now be in the range of $\sim 10^7 - 10^8\text{Hz}$. It would be interesting to investigate whether these signals are in principle observable. Another possible signature may be some imprints on the cosmic microwave background (CMB) fluctuations due to the thermodynamics of black hole remnants-CMB interactions. Possible production of such remnants in Large Hadron Collider (LHC) and also in ultrahigh energy cosmic ray (UHECR) air showers are under investigation. If we consider hybrid inflation as our primary cosmological model, there will be some observational constraints on hybrid inflation parameters. For example a simple calculation based on hybrid inflation suggests that the time it took for black holes to reduce to remnants is about $10^{-10}\text{Sec}$. Thus primordial black holes have been produced before baryogenesis and subsequent epochs in the standard cosmology[35]. The events that can potentially lead to black hole production are essentially high-energy scattering in particle colliders and UHECR. The next generation of particle colliders are expected to reach energies above 10 $\text{TeV}$. LHC and Very Large Hadron Collider (VLHC)[36] are planned to reach a center-of-mass energy of 14 and 100 $\text{TeV}$. Therefore, if the fundamental Planck scale is of the order of few $\text{TeV}$, LHC and VLHC would copiously produce black holes. These black holes have masses on the order of $\text{TeV}$.

Black hole production by cosmic rays has also been recently investigated by a number of authors[37]. Cosmogenic neutrinos[38] with energies above the Greisen-Zatsepin-Kuzmin (GZK) cutoff[39] are expected to create black holes in the terrestrial atmosphere. The thermal decay of the black hole produces air showers which could be observed. The cross sections of these events are two or more orders of magnitude larger than the cross sections of standard model processes. Therefore, black holes are created uniformly at all atmospheric depths with the most promising signal given by quasi-horizontal showers which maximize the likelihood of interaction. This allows black hole events to be distinguished from other standard model events. Detecting $\text{TeV}$ black hole formation with UHECR
detectors may be possible through the decay of $\tau$-leptons generated by $\nu_\tau$'s that interact in the Earth or in mountain ranges close to the detectors. A secondary $\tau$ generated through the decay of a BH has much less energy than the standard model $\tau$ secondary. In addition, black holes may produce multiple $\tau$-leptons in their evaporation, a unique signature of $TeV$ gravity. Standard model processes that generate multiple $\tau$-leptons are highly unlikely, the detection of multiple $\tau$’s in earth-skimming and mountain crossing neutrinos will be a smoking gun for black hole formation.

5 Conclusion

In this paper, using generalized uncertainty principle in a general form as our primary input, we have calculated the temperature and microcanonical entropy of a black hole in the framework of large extra dimensional scenarios. Following results can be obtained from our analysis:

- In scenarios with extra dimensions, black hole temperature increases(figure 1). This feature leads to faster decay and less classical behaviors for black holes.

- It is evident that in extra dimensional scenarios final stage of evaporation( black hole remnant) has mass more than its four dimensional counterpart.

- In scenarios with extra dimensions, black hole entropy decreases(figure 2). The classical picture breaks down since the degrees of freedom of the black hole, i.e. its entropy, is small. In this situation one can use the semiclassical entropy to measure the validity of the semiclassical approximation.

- It is evident that in extra dimensional scenarios final stage of evaporation( black hole remnant) has event horizon area more than its four dimensional counterpart(figure 2).

- Black hole radiation is mainly on the brane. In other words, black holes decay by emitting radiation mainly on the brane. This is in accordance with the results of Emparan et al[21].

- Black hole production at the LHC and in cosmic rays may be one of the early signatures of $TeV$-scale quantum gravity. Large samples of black holes accessible by
the LHC and the next generation of colliders would allow for precision determination of the parameters of the bulk space and may even result in the discovery of new particles in the black hole evaporation. Limited samples of black hole events may be observed in ultra-high-energy cosmic ray experiments, even before the LHC era. If large extra dimensions are realized in nature, the production and detailed studies of black holes in the lab are just few years away. That would mark an exciting transition for astroparticle physics: its true unification with cosmology the Grand Unification to live for.

Therefore, in the framework of GUP, it seems that quantum black holes are hotter, shorter-lived and tend to evaporate less than classical black holes. Higher dimensional black hole remnants have less classical features than four dimensional black holes. It is evident from our calculations that black holes radiate mainly on the four-dimensional brane-world.

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429, 263-272 (1998) [arXiv:hep-ph/9803315].

[2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 436, 257-263 (1998) [arXiv:hep-ph/9804398].

[3] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. D 59, 086004 (1999) [arXiv:hep-ph/9807344].

[4] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690 [arXiv:hep-th/9906064].

[5] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370 [arXiv:hep-ph/9905221].

[6] http://lhc-new-homepage.web.cern.ch/lhc-new-homepage/

[7] http://www.fnal.gov/projects/muon-collider/

[8] J. L. Feng and A. D. Shapere, Phys. Rev. Lett. 88, 021303 (2002) [arXiv:hep-ph/0109106]; A. Ringwald and H. Tu, Phys. Lett. B 525, 135 (2002) [arXiv:hep-ph/0111042]; Phys. Lett. B 529, 1 (2002) [arXiv:hep-ph/0201139]; M. Ave, E. J. Ahn, M. Cavagli’a and A. V. Olinto, arXiv:astro-ph/0306344; Phys. Rev. D 68,
043004 (2003) [arXiv:hep-ph/0306008]; S. I. Dutta, M. H. Reno and I. Sarcevic, Phys. Rev. D 66, 033002 (2002) [arXiv:hep-ph/0204218]. R. Emparan, M. Masip and R. Rattazzi, Phys. Rev. D 65, 064023 (2002) [arXiv:hep-ph/0109287]; A. Mironov, A. Morozov and T. N. Tomaras, arXiv:hep-ph/0311318; P. Jain, S. Kar, S. Panda and J. P. Ralston, Int. J. Mod. Phys. D 12, 1593 (2003) [arXiv:hep-ph/0201232].

[9] M. Cavagli’a, Int. J. Mod. Phys. A 18, 1843 (2003) [arXiv:hep-ph/0210296]; G. Landsberg, arXiv:hep-ex/0310034; P. Kanti, arXiv:hep-ph/0402168.

[10] M. Cavaglia, Phys. Lett. B 569, 7 (2003) [arXiv:hep-ph/0305256]; R. Emparan, G. T. Horowitz and R. C. Myers, Phys. Rev. Lett. 85, 499 (2000) [arXiv:hep-th/0003118]; D. Ida, K. y. Oda and S. C. Park, Phys. Rev. D 67, 064025 (2003) [arXiv:hep-th/0212108]; C. M. Harris and P. Kanti, JHEP 0310, 014 (2003) [arXiv:hep-ph/0309054]; P. Kanti and J. March-Russell, Phys. Rev. D 66, 024023 (2002) [arXiv:hep-ph/0203223]; Phys. Rev. D 67, 104019 (2003) [arXiv:hep-ph/0212199].

[11] D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B 216, 41 (1989); D. Amati, M. Ciafaloni and G. Veneziano, Nucl. Phys. B 347, 550 (1990); D. Amati, M. Ciafaloni and G. Veneziano, Nucl. Phys. B 403, 707 (1993); K. Konishi, G. Paffuti and P. Provero, Phys. Lett. B 234, 276 (1990).

[12] M. Maggiore, Phys. Rev. D 49, 5182 (1994) [arXiv:hep-th/9305163]; M. Maggiore, Phys. Lett. B 319, 83 (1993) [arXiv:hep-th/9309034]; A. Kempf, G. Mangano and R. B. Mann, Phys. Rev. D 52, 1108 (1995) [arXiv:hep-th/9412167].

[13] M. Maggiore, Phys. Lett. B 304, 65 (1993) [arXiv:hep-th/9301067]; F. Scardigli, Phys. Lett. B 452, 39 (1999) [arXiv:hep-th/9904025]; F. Scardigli and R. Casadio, Class. Quant. Grav. 20, 3915 (2003) [arXiv:hep-th/0307174].

[14] R. J. Adler, P. Chen, D. I. Santiago, Gen. Rel. Grav. 33 (2001) 2101.

[15] B. Bolen and M. Cavaglia, arXiv: gr-qc/0411086

[16] A. J. M. Medved and E. C. Vagenas, Phys.Rev. D70 (2004) 1240

[17] A. J. M. Medved, Class.Quant.Grav. 22 (2005) 133-142

[18] G. Gour and A. J. M. Medved, Class. Quant. Grav. 20 (2003) 3307

15
[19] P.C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B 441, 96 (1998).

[20] A. Chamblin, S. Hawking and H.S. Reall, Phys. Rev. D 61, 0605007 (2000).

[21] R. Emparan, G. T. Horowitz and R. C. Myers, Phys. Rev. Lett. 85, 499 (2000) [arXiv:hep-th/0003118].

[22] A. Ashtekar, Proceedings of Banff Workshop of Gravitational Field, 1990.

[23] A. Kempf, Proc. 21st Intl. Coll. on Group Theor. Methods in Physics, Goslar, July 1996, arXiv: hep-th/9612082 Phys. Rev. D54 (1996) 5174-5178; Erratum-ibid. D55 (1997) 1114.

[24] R. Adler and D. Santiago, Mod. Phys. Lett. A14 (1999) 1371.

[25] A. Kempf, et al, Phys. Rev. D52 (1995) 1108.

[26] S. Hossenfelder et al, Phys. Lett. B 575 (2003) 85-99.

[27] An alternative heuristic derivation of the Hawking radiation is contained in H. Ohanian and R. Ruffini, Gravitation and Spacetime, 2nd ed., p. 481 (W. W. Norton, 1994).

[28] R. J. Adler and T. K. Das, Phys. Rev. D14, 2472(1976); R. S. Hanni, and R. Ruffini, ”Lines of Force of a Point Charge Near a Schwarzschild Black Hole”, in Black Holes, eds. C. DeWitt and B. S. Dewitt (Gordon Breach 1973).

[29] D. Christodoulou and R. Ruffini, Phys. Rev. D4 (1971) 3352.

[30] M. Cavaglià et al, Class. Quant. Grav. 20 (2003) L205-L212.

P. Gondolo, Lectures delivered at the NATO Advanced Study Institute ”Frontiers of the Universe”, 8-20 Sept 2003, Cargese, France; astro-ph/0403064.

[31] Ya. B. Zeldovich and I. D. Novikov, Sov. Astron. 10, 602 (1966).

[32] S. W. Hawking, Mon. Not. R. Astron. Soc. 152, 75 (1971).

[33] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, Phys. Rev. D 49, 6410 (1994).
[34] D. H. Lyth and A. Riotto, Phys. Rep. 314, 1 (1999); A. Linde, Phys. Rep. 333-334, 575 (2000).

[35] P. Chen, New Astron. Rev. 49 (2005) 233-239

[36] http://www.vlhc.org/

[37] M. Cavagli’a, Int. J. Mod. Phys. A 18, 1843 (2003) [arXiv:hep-ph/0210296]
G. Landsberg, arXiv:hep-ex/0310034; P. Kanti, arXiv:hep-ph/0402168.

[38] R. Engel, D. Seckel and T. Stanev, Phys. Rev. D 64, 093010 (2001) [arXiv:astro-ph/0101216].

[39] K. Greisen, Phys. Rev. Lett. 16, 748 (1966).
Figure 1: Temperature of black hole Versus its mass in different spacetime dimensions.
Figure 2: Entropy of black hole versus the area of its event Horizon in different spacetime dimensions.