Cosmic rays density fluctuations with turbulence universal spectrum - 8/3

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Abstract

An exact turbulence universal scaling law - 8/3 for the density fluctuations of cosmic ray (CR) is obtained on the basis of a new analytical compressible turbulence theory and known two-fluid model of the CR dynamics. It is shown that the origin of this scaling law may be due to the breaking of the nonlinear simple waves in CR medium near the scale of their Larmor radii as for the space plasma of solar wind and magnetosheath.

1. Introduction

Cosmic rays (CR) are the most ubiquitous population of charged relativistic particles in the universe, which play a crucial role in understanding the problems of high-energy physics, plasma physics, as well as the fundamental and applied problems of the atmospheric electrivity and climate variability on the Earth [1]-[13]. CR are particles (with energies \( E > 1\, \text{GeV/nucleon} \)) or photons (with energies \( E > 1\, \text{MeV} \)) pervading space, having a non-thermal energy distribution typically of the power-law \( (dE/dN) \propto E^{-\gamma} \) rather than Maxwell-Boltzmann form [1]-[6]. Here the differential energy spectrum \( dN(E)/dE = Vn/4\pi \approx cn/4\pi \) is closely related to the particle volume density \( n(E) \) of CR, where \( V \) and \( c \) is the particle velocity and the speed of light in vacuum, respectively. According to the observational data, for protons (which make up the main part of CR) with energies \( 10\, \text{GeV} < E < 3 \times 10^6\, \text{GeV} \) a power dependence \( dN(E)/dE \propto E^{-2.67 \pm 0.02} \) takes place [3]-[6].

One of the known mechanism for the formation of this spectrum is associated with the repeated passage of particles through the shock wave front where the main acceleration site of CR is obtained [7]-[9]. It is generally believed that collisionless shock waves in SNRs (supernover remnants) are the dominant agent for acceleration. After the shock cannot contain particles anymore they diffuse through the interstellar medium (ISM), producing secondary particles. The CR particles distribution function at the shock is roughly a power-law [7]-[9]

\[
I(E) = dN/dE \propto E^{-\gamma}; \gamma = n-2; n = 3\delta/(\delta-1); \delta = \rho/\rho_0
\]  

(1)

Where \( \delta \) - compression and \( \rho, \rho_0 \) are the density of ISM behind and before the shock wave front. Thus to the observed value \( \gamma \approx 8/3 \) it is need the value of...
compression $\delta = 14/5 = 2.8$. When $\delta = 4$ from (1) it is possible to obtain value $\gamma = 2$ [9].

Moreover, from the theory of similarity, [10]-[13] the same exponent $\gamma = 8/3$ is derived. To date, however, the problem of the origin of cosmic rays and the problem of establishing the mechanism responsible for the appearance of a spectrum with an exponent of $-8/3$ remain unresolved [3]-[6].

At the same time, the question of whether there is a correlation between the observed differential energy spectrum of cosmic rays and the turbulent energy spectrum characterizing the shock waves themselves has not been considered until now. This issue is closely related to the consideration of turbulent spectra in a medium consisting of cosmic ray particles, for example, in the framework of the well-known two-liquid model of cosmic ray propagation [8]. An analog of such a consideration, however, was carried out in [14], where, based on obtaining a two-point correlation function from the observational data and the corresponding turbulent spectrum for fluctuations in the electron distribution density in the interstellar medium, a scaling similar to the Kolmogorov-Obukhov law in the inertial scale interval was established (see Fig.1 below and Fig.1 in [14]).
As can be seen from Fig.1, in the inertial scale interval, the three-dimensional turbulent spectrum of electron density fluctuations has a power-law character. At the same time, the slope corresponding to the energy spectrum of -8/3 for the three-dimensional spectrum gives an exponent -14/3 (a solid black line added to Fig.1 of the article [14]). It can be seen from Fig.1 that the observational data correspond to this slope to a greater extent than the exponent -11/3, following from the Kolmogorov-Obukhov law -5/3 (represented by dots).

Moreover, based on the PAMELA observations, various options for the formation of turbulence and its effect on the observed change in the energy spectrum of cosmic rays, leading precisely to an indicator close to -8/3 [15], [16].

At the same time, in [17], for the inertial subrange, a universal spectrum of strong turbulence with an index of -8/3 was obtained, corresponding to the moment of occurrence of the shock wave. In particular, this made it possible to explain the mechanism of formation of the -8/3 turbulent spectrum observed in the cosmic plasma of the solar wind and the magnetosphere [18], [19]. At the same time, the adequacy of the -8/3 energy spectrum of the Kadomtsev-Petviashvili theory of strong acoustic turbulence [20] was also established in [17], for which the applicability of the energy spectrum with the -2 index obtained for a statistical ensemble of shock waves randomly located in space was previously assumed [21], [22].

There is an analogy between the mode of inertia motion of solar wind particles and known solution [23] of the Boltzman-Vlasov equation for CR particles propagating with a constant energy due to the balance between Fermi’s type acceleration and dissipative forces.

In our paper the universal turbulence scaling law-8/3 is obtained for the CR density fluctuation spectrum on the basis of an exact analytical solution to the problem of compressible turbulence [17] by using specific solution of the Boltzmann-Vlasov equations which is obtained in [23]. On the base of balance between Fermi acceleration and dissipation of the CR particles energy the isothermal mode of propagation of CR particles is obtained in [23] (see (44) in [23]). In this case the momentum and energy of the CR particles are also conserved and the motion of a medium consisting of such particles can be described by Euler hydrodynamics equations for a compressible medium with specific value of the adiabatic exponent $\gamma = 1$, as a particular case of the known two-fluid dynamic (TFD) theory [8]. Indeed, in the two-fluid theory, based on kinetic equations, an analogue of the one-dimensional Euler equation is derived.
to describe a compressible polytropic medium consisting of cosmic ray particles (see equations (6.4) and (6.5) in [8]).

The next Chapter 2 provides a description of the conditions for the implementation of theory [23], where the motion of a medium consisting of CR particles can be described on the basis of Euler’s hydrodynamic equations for a compressible medium. In the Chapter 3, based on the theory [17], a turbulent spectrum of fluctuations in the density of CR particles with a universal exponent \(-8/3\) is obtained. In the section 4 discusses the comparison of the conclusions of the theory and observational data, as well as results of the similarity theory [10]-[13].

2. The inertial motion of cosmic ray particles

In [23] analysis is based on the collision-less Boltzmann-Vlasov equation, which expresses conservation of CR particles in phase space:

\[
\frac{\partial f}{\partial t} + \frac{\partial}{\partial x_i} (f V_i) + \frac{\partial}{\partial p_i} \left( f \frac{dp_i}{dt} \right) = 0; i = 1,2,3
\]

In (1) \(f(\vec{x}; \vec{p}; t)\) is the distribution function and \(dp_i / dt\) is the force on a cosmic ray particle. Since protons are the dominant CR species, it is taking \(f\) as referring to protons [24]. In [24] equation (1) is transformed to the so called “wave-frame” which gives the possibility to take into account the interactions between CR particles and Alfvén waves.

The regime obtained in [23], corresponding to the possibility of establishing a balance between the forces leading to the acceleration of charged particles of CR in random magnetic fields and dissipative factors.

For the ultra-relativistic particles, when spatial diffusion is negligible and the cosmic ray particles transfer only with the forward Alfvén wave in [23] is considered the case of negligible background plasma motion \(V_0 \ll V_A\) (where \(V_0\) is the velocity of background plasma, \(V_A\) is the Alfvén velocity) when \(f(\vec{x}; \vec{p}) \propto \rho^s(\vec{x}, t) p^{-4}; s > 4, a = s/(2s-5)\). Here \(\rho\) is the number density of CR particles in their convection with the Alfvén wave [15]. As the result, the equation for the CR particles momentum is obtained (see (26) and (27) in [23]):

\[
\frac{1}{p} \frac{dp}{dt} = -b \text{div} \vec{u};
\]

\[
p \propto \rho^b
\]

\[
b = \frac{1}{3} \left[ \frac{2s(s-4)}{(2s-5)(s-3)} \right]
\]
In (2) $\bar{u} \propto V_A \bar{n}$, where $\bar{n}$ - is the unit vector directed along the background magnetic field. In [23] the CR particles are considered as a politropic gas with pressure $P_{CR} \propto p^s f \propto \rho^{(s-1)+a}$, number density $n_{CR} \propto p^3 f \propto \rho^{a+b(3-s)}$ and adiabatic exponent:

$$\gamma = \frac{d \ln P_{CR}}{d \ln n_{CR}} = \frac{2s^2 - 13s + 25}{2s^2 - 11s + 15}$$

(3)

From (2) and (3) it is possible to obtain the condition for inertia motion of CR particles with invariant energy and momentum $p = \text{const}$ when the net energy loss from CR is small (see [23]):

$$s = 5; b = 0; a = 1$$

(4)

$$\gamma = 1$$

(5)

When conditions (4), (5) are met, the Euler equation for the velocity field of such a medium, the continuity equation for density and the isothermal equation of state, respectively, can be used to describe the isothermal mode of motion of a medium consisting of CR particles:

$$\frac{\partial V_i}{\partial t} + V_j \frac{\partial V_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P_{CR}}{\partial x_i}; i = 1,...,d$$

(6)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho V_i) = 0; \rho = n_{CR}$$

(7)

$$P_{CR} \rho^{-\gamma} = \text{const}; \quad \gamma = 1; P_{CR} = c_s^2 \rho$$

(8)

In (6), (7) by repeating indices, summation is implied, where $d$ is the dimension of the space. In (8) $c_s$ - is the constant isothermal sound speed of such exotic medium, consisting CR particles. Equations (6), (7) exactly coincide with the equations of the two-fluid model [8]) see (6.4) and (6.5) in [8]) under the conditions of conservation of cosmic ray energy, according to the theory [23]. At the same time, the more general system of equations considered in [8] admits decoupling and allows the description of a medium of cosmic ray particles based on (6)-(8), regardless of the equations describing the motion of the medium in which cosmic rays propagate.

In the limit of the large Mach number $Ma = |\vec{V}|/c_s \gg 1$ the right hand side of equation (6) has an order $O(Ma^{-2}) \to 0$. In this case, with the right-hand side zero, equation (6) is a d-dimensional Hopf equation and in this case an exact analytical solution in Euler variables is obtained for (6) and (7) in [25]-[27].
Note at the same time that equation (7) for the motion of particles by inertia with zero resultant force, according to the d-dimensional Hopf equation, exactly coincides with the Boltzmann-Vlasov equation (1) for the case of zero force \( \frac{dp}{dt} = 0 \) (see [23]) and with the generalized Liouville equation in the statistical mechanics of non-Hamiltonian systems [28].

Therefore, the exact solution of the Hopf equation and equation (7) obtained in [25]-[27] also gives an example of a new exact solution of the Boltzmann-Vlasov equation and the generalized Liouville equation in the Euler variables for the wide class of inertial motions that are usually realizing in nature and technical systems.

For arbitrary finite Mach numbers, the exact solution of the system (6), (7) was obtained by Riemann in 1860 only for the one-dimensional case and in the form of an implicit dependence of a nonlinear simple wave on the initial velocity and density fields [29]. The peculiarity of a simple wave is that it has a functional relation between the density of the medium and the wave velocity.

For a medium consisting of CR particles, however, it turns out to be characteristic of the special case discussed below, when the adiabatic index in (8) is equal to one. For the one-dimensional case d=1, the exact solution of the system (6)-(8) in the form of a nonlinear simple Riemann wave taking into account [17] has the explicit form:

\[
\rho(x,t) = \int_{-\infty}^{\infty} d\xi \rho_0(\xi) \left[ 1 + t \frac{dV}{d\xi} \right] \delta(\xi - x + (\pm c_s + V_0(\xi)) t)
\]

\[
\rho_0(x) = \rho(x,t = 0) = \rho_{oo} \exp \left( \frac{V_0(x)}{c_s} \right) ; V_0(x) = V(x; t = 0)
\]

In (9) \( \delta \) - is the Dirac delta-function, \( \rho_0(x), V_0(x) \) - are the initial density and velocity fields.

The solution for the velocity field has a similar form with (9), in which it only needs to be replaced by \( \rho_0(\xi) \to V_0(\xi) \) in (9). In (9), the value \( \rho_{oo} = const \) is determined taking into account the relationship of the initial conditions for the density of the medium and for the distribution of the velocity field in it, which is characteristic precisely when considering a nonlinear simple Riemann wave.

Note that the solution (9) of the one-dimensional system of equations (6)-(8) preserves smoothness only over a finite time interval \( 0 \leq t < t_0 \), where the collapse time of a simple Riemann wave is explicitly defined in [17] and depends only on
the initial distribution of the velocity field and the adiabatic exponent (see Eq. (33) in [17])
\[ t_0 = \frac{2}{(\gamma + 1) \left( \max_{x=x_0} \left| \frac{dV_0}{dx} \right| \right)} . \]

For the case considered in Eq. (8), where \( \gamma = 1 \) this minimal existence time of a
smooth solution has the form:
\[ t_0 = \frac{1}{\left( \max_{x=x_0} \left| \frac{dV_0}{dx} \right| \right)} . \] (10)

In Eq. (10) \( x = x_m \), this is the place where the nonlinear collapse of a simple
Riemann wave occurs for the first time. Previously, before obtaining an explicit
form of the solution in [17], analytical representations were known for the
collapse time of the solution only for some special cases of the initial velocity field
[29]. Indeed, in [29], the collapse time of the solution or the time of the shock
wave occurrence can be explicitly obtained only in cases when, for a function
describing the initial velocity field, it is possible to obtain an explicit analytical
representation for its inverse function.

In Eq. (10), this can be done for any smooth initial velocity field having a finite
value of the energy integral in an unbounded space. For example, for the initial
velocity field of the form \( V_0(x) = a \exp(-x^2/2x_0^2) \) from Eq. (10), we obtain an
estimate \( t_0 = x_0/V_0(x=x_0); V_0(x_0) = a/\sqrt{e} \), for the minimum collapse time (loss of
smoothness or collapse of a nonlinear wave) of the solution, which occurs for the
first time at the value \( x_m = x_0 \). We assume, according to [30], for the drift velocity
(in the coordinate system associated with the MHD wave, as in [23]) of cosmic ray
particles in the Galaxy, the magnitude is \( a \approx 36 \text{ km/s} \) and the corresponding
characteristic scale of spatial inhomogeneity is of the order of
\( x_0 \approx 10^{-5} \text{ pc} \approx 3.085 \times 10^8 \text{ km} \). Then, according to (10), we obtain an estimate
\( t_0 \approx 1.4 \times 10^7 \text{ sec} \approx 0.5 \text{ year} \) for the collapse time of a nonlinear wave describing a
medium consisting of cosmic ray particles. The characteristic scale of spatial
inhomogeneity used to estimate the collapse time of a nonlinear wave has a value
close to the Larmor radius of protons when they have an energy of about 100-200
GeV [4]. In turn, it is at such energies that the spectral law \((8/3)\) is established in
the observational data [3], [4]. A similar change in the index of the turbulence
spectrum in the solar wind plasma and the establishment of the spectral law \((8/3)\)
also takes place, starting from the scale of the Larmor radius of ions. Therefore, it
can be assumed that, as in the cosmic plasma of the solar wind, the establishment
of the spectral law -8/3 is due to the mechanism noted in [19] associated with the
collapse of a nonlinear simple Riemann wave in the medium of cosmic ray
particles. This mechanism can be considered as an alternative to the existence of
additional unknown sources of primary cosmic rays and other variants noted in
[4] in connection with the observation of the spectrum fracture and the
establishment of the -8/3 law.

The representation Eq. (10) exactly coincides with the time of collapse
occurrence for the solution of the one-dimensional Hopf equation [25]-[27]. This
coincidence takes place only for the case \( \gamma = 1 \), despite the difference in the form
of the solution of the Hopf equation and the solution of the Euler equations in the
form of Eq. (9) obtained at arbitrary Mach numbers.

3. Exact turbulence scaling law-8/3 for the cosmic ray density fluctuations

Based on the new exact solution of the Euler equations (6)-(8) in the form (9), we
obtain representations for the two-point correlation function of density

\[
R_\rho (r) = \frac{1}{L \rho_0^2} \int_{-\infty}^{\infty} dx \rho(x + r; t) \rho(x; t) \quad \text{and the corresponding spectrum}
\]

\[
E_\rho (k) = \frac{1}{2\pi} \int dr R(r) \exp(-ikr)
\]
of density fluctuations in a medium consisting of
cosmic ray particles. Here is used the external integral scale \( L \), where

\[
L = P_0^2 / E_0; P_0 = \int_{-\infty}^{\infty} dx V_0(x); E_0 = \int_{-\infty}^{\infty} dx V_0^2(x) \quad [17].
\]

Thus, to describe the turbulence
spectrum here, as in [17], an averaging over a spatial variable in the unbounded
case is used.

In the limit \( k >> L^{-1} \) it is possible to obtain asymptotic representation (see also
[17]):

\[
E_\rho (k) = C_\rho k^{-8/3} + O(k^{-10/3}) \quad (11)
\]
\[
C_\rho = \frac{2^{1/3}}{c_s^2 L} \left( \frac{dV_0}{dx} \right)^{8/3} \left( \frac{d^3V_0}{dx^3} \right)^{-2/3} \left[ \Phi^2(0) \exp \left( \pm \frac{2V_0(x_M)}{\rho} \right) \right]
\]

In (11) \( \Phi(z) = \sqrt{\pi} \text{Ai}(z) \) is the Airy function, where \( \Phi(0) = \frac{\sqrt{\pi}}{3^{2/3} \Gamma(2/3)} \approx 0.629 \). The representation (11) is obtained by using the method of stationary phase in the estimation of an integral \( E_\rho(k) \) at the time \( t = t_0 \), where \( t_0 \) is determined in (10). So the main asymptotic term in (11) is determined with precise of the next term which has an order \( O(k^{-10/3}) \) in the limit of large wave number \( k \gg 1/L \).

The representation for the energy spectrum can be obtained in a similar way in the form of a Fourier transform from the correlation function of the velocity field. Thus, turbulence energy spectrum coincides with (11) if in (11) the next replacement \( C_\rho \to C_E, C_E = C_\rho c_s^2 \exp(\mp 2V_0(x_M) / \rho) \) is provided [17].

The universality of the spectrum (11) is manifested in the independence of the exponent -8/3 from the type of the initial velocity field.

Thus, in a medium consisting of cosmic ray particles, regardless of the initial conditions, a turbulent spectrum of density fluctuations corresponding to the universal scaling law -8/3 should be observed. Indeed, according to the above estimate, the collapse time of a nonlinear wave in such an environment has a value of about six months. Therefore, such regularly occurring collapses seem to be realized continuously for an unlimited time, but always each such collapse of a nonlinear wave leads only to the marked spectral law -8/3 in (11), despite the
difference in the constant factor $C_\rho$ in (11), which is depended on the initial conditions.

4. Discussion and comparison with observational data.

The obtained exact solution (11) for the universal turbulent spectrum of fluctuations in the density of CR particles is characterized by the scaling law $-8/3$, which does not depend on the initial conditions and on any parameters characterizing the equation of state of the medium. At the same time, the multiplier $C_\rho$ does not have such a universality property and significantly depends on the initial distribution of the velocity field $V_0$ and its derivatives at the point $x = x_M$, where, for the first time at time (10), a nonlinear simple wave describing the density distribution of CR particles collapsed. In addition, in (11) there is a significant dependence of this multiplier on the constant value of the speed of sound $c_s$, characterizing the equation of state of a medium consisting of CR particles according to (8). In this regard, for the possibility of the most complete comparison with the obtained exact solution (11), it is of interest in future experiments to consider the data of observations of fluctuations in the density of CR particles carried out simultaneously at two different points in space along the direction of propagation of CR particles. At the same time, data on measuring the magnitude of the speed of sound $c_s$ in a medium consisting of CR particles would be of particular importance.
It should be noted that at present there are only measurement data of the dependence of the density of CR particles on their energy related to observations carried out on board only one spacecraft [3], [4]. For ultra-relativistic CR particles, the magnitude of their energy is practically proportional to the magnitude of their momentum, according to the well-known relativistic relation \( E = \sqrt{m_r^2c^4 + p^2c^2} \approx pc \). On the other hand, the momentum of a particle is related to its wave number by well-known quantum mechanics the relation \( p = \hbar k \). Therefore, the dependence of the proton density in CR on their energy obtained in observations has a form 

\[ n_{CR} \propto k^{-2.67\pm 0.02} \]

that is consistent with the scaling index-8/3 in the exact solution (11) for the spectrum of fluctuations in the density of CR particles. At the same time, there is also a correspondence with a similar estimate of the scaling index-8/3, obtained from the theory of dimension and similarity in [10]-[13]. In [10], [13] the corresponding similarity considerations are stated on the introduction of two parameters- the CR energy generation rate \( G_{CR} \propto 1 - 5 \times 10^{40} \text{erg/sec} \) and the energy volume densities of CR \( w_{CR} \propto 10^{-12} \text{erg/cm}^3 \) (see (13) in [10]). If we use these two parameters, supplementing them with such a dimensional parameter as the speed of light \( c \) in a vacuum, then, for dimensional reasons, we can obtain an analogue of scaling's law (11) in the form \( E_p \propto \left( \frac{w_{CR}c}{G_{CR}} \right)^{5/6} k^{-8/3} \). In this case, instead of the speed of light, the invariant isothermal value of the speed of sound \( c_s \) introduced above in a medium of CR particles can also be used. Similarly, other
fundamental constants, such as Planck’s constant $\hbar$ or the gravitational constant $G$, can be used instead of the speed of light. In this case, we get, respectively

$$E_\rho \propto \left( \frac{w_{CR}}{\sqrt{\hbar G_{CR}}} \right)^{5/9} k^{-8/3}$$

and

$$E_\rho \propto \left( \frac{\sqrt{w_{CR}^5 G}}{G_{CR}^2} \right)^{1/3} k^{-8/3}.$$ Using the exact solution (11) and the corresponding explicit representation for a constant value $C_\rho$, arbitrary constants in these scaling laws obtained from considerations of dimensionality and similarity can also be precisely established.

It should be noted that in [31] it was precisely for dimensional reasons that the scaling law -8/3 was also obtained for the three dimensional turbulence energy spectrum. At the same time, in [31], as in this paper and in [17], integration over an unlimited space is used instead of statistical averaging when using dimensional parameters (such as the average rate of dissipation of turbulent energy) by analogy with the Kolmogorov-Obukhov theory. As in [31], we will introduce into consideration the integral value of the rate of energy occurrence (attributed to the unit of mass) particles of $\text{CR}^* \varepsilon_{CR} = \frac{1}{m_{CR}} \int d^3 x G_{CR}; m_{CR} = \int d^3 x \rho$, as well as the integral value of the energy spectrum for the energy spectrum

$$\bar{E}(k) = \int d^3 x M E(k; x_M) = 4\pi \int_0^\infty dx_M x_M^2 E(k; x_M)$$
corresponding to the exact solution (11).

At the same time, for reasons of dimension and similarity, we obtain the scaling law -8/3, which coincides in form with the given spectral law in [31] and has the form:
\[
\tilde{E}(k) = C_G \varepsilon_{CR}^{2/3} k^{-8/3}
\]  
(12)

\[
C_G = \frac{I_0}{\varepsilon_{CR}^{2/3}}, \quad I_0 = \frac{2^{1/3} 4\pi \Phi^2(0)}{L} I_{0R};
\]

\[
I_{0R} = V.P. \int_{0}^{\infty} dxx^2 \left( \frac{dV_0}{dx} \right)^{8/3} \left( \frac{d^3V_0}{dx^3} \right)^{-2/3}
\]  
(13)

In integral (13), the index of the variable is omitted and the integral itself is understood in the sense of its main value in the presence of special points in the integrand.

Note that the representation (12), (13) could be obtained directly from the solution for the energy spectrum corresponding to the exact solution (11) using the method of randomization for the exact solutions of the hydrodynamics equations [32]-[34]. In this case the introduction of an appropriate probability measure for the magnitude \( x_M \), when the isotropy of the distribution of CR particles in space is taken into account, is used.

**Conclusions**

In this paper, the isothermal mode of propagation of cosmic ray particles corresponding to the conservation of their energy is considered. It is described above in the framework of the well-known two-liquid model [8] of the interaction of the medium of cosmic ray particles and the medium in which they propagate. In this case, a closed description of the dynamics of the cosmic ray medium is carried out on the basis of one-dimensional Euler equations for a compressible
medium. Taking into account the exact explicit solution of these equations in [17] an explicit analytical form of a two-point correlation function for the cosmic ray particle density and to the corresponding universal spectrum -8/3 for cosmic ray particle density fluctuations is obtained which is matched with known observation data [14] (see also Fig.1). In contrast to the consideration of the two-fluid model of cosmic ray particle dynamics in [8], due to the use of the isothermal regime obtained in [23], the Euler equations for the cosmic ray medium are uncoupled from the equations describing the medium in which cosmic rays propagate. This made it possible to find and use the corresponding exact solution of the Euler equations to describe the spectrum of fluctuations in the density of cosmic ray particles, with the universal scaling law -8/3.

It is necessary to carry out additional measurements of two point correlations for fluctuations in the density of cosmic ray particles and the corresponding spectrum as in [14]. This is not an easy task, however, based on the already available data on fluctuations in the density of cosmic ray particles measured in the PAMELA experiment, it is possible to obtain a frequency spectrum of density fluctuations and make appropriate estimates for the spatial spectrum exponent taking into account the Taylor hypothesis. This will make it possible to establish the mechanism of formation of such a spectrum, based on comparison with the exact solution obtained in this paper when determining the scaling exponent -8/3 for the turbulent spectrum of fluctuations in the density CR particles, based only
on the exact solution of the Euler equations for a compressible medium. Of interest is also the possibility of establishing a correlation between the scaling law $-8/3$ observed in the turbulent spectra of cosmic plasma in the solar wind and magnetosheath [17] and the similar exponent of the turbulent spectrum of fluctuations in the density of particles of cosmic rays obtained in this work, and also with a similar spectrum of $-8/3$, characteristic of fluctuations in the intensity of the surface earth electric field [35]. It is of interest to further use the obtained exact solutions in the theory of turbulence and in the new theory of Vavilov-Cherenkov radiation [36]-[38] to analyze the problems of describing such turbulence and to consider the mechanism of acceleration of cosmic ray particles and their origin.

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Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Literature

1. C. J. Cesarsky, Cosmic-ray confinement in the Galaxy, Ann. Rev. Astron. Astrophys. 18, 289 (1980)
2. K. S. Carslaw, R. G. Harrison, and J. Kirkby, Cosmic rays, clouds, and climate, Science, 298, 1732 (2002)
3. O. Adriani, et. al., PAMELA measurements of cosmic-ray proton and helium spectra, Science, 332, 69 (2011)
4. I. A. Grinier, J. H. Black, and A. W. Strong, The nine lives of cosmic rays in Galaxies, Annu. Rev. Astron. Astrophys., 53, 199 (2015)
5. M. Aguilar et.al. Properties of Neon, Magnesium and Silicon primary cosmic rays results from the Alpha Magnetic Spectrometer, Phys. Rev. Lett. 124, 211102 (2020); https://doi.org/10.1103/PhysRevLett.124.211102
6. J.-S. Niu, Origin of hardening in spectra of cosmic ray nuclei at a few hundred GeV, Chines Physics C 45, 041004 (2021)
7. R. Blandford, and D. Eichler, Particles acceleration at astrophysical shocks: A theory of cosmic ray origin, Phys. Rep., 154, 1, (1987); https://doi.org/10.1016/0370-1573/87/90134-7
8. M. A. Malkov, and L. O’C Drury, Nonlinear theory of diffusive acceleration of particles by shock waves, Rep. Prog. Phys. 64, 429 (2001)
9. P. Mertsch, and S. Sarkar, AMS-02 data confront acceleration of cosmic ray seconaries in nearby sources, Phys. Rev. D, 90, 061301 (R) (2014); https://doi.org/10.1103/PhysRevD.90.061301
10. G. S. Golitsyn, The spectrum of cosmic rays from the point of view of similarity theory, Astronomy Letters, 23, 127 (1997)
11. G. S. Golitsyn, Phenomenological explanation of the spectrum of cosmic rays with energies E>10GeV, Astronomy Letters, 31, 500 (2005)
12. G. S. Golitsyn, Similarity criteria, galactic scales and spectra, J. of Modern Physics, 3, 1523 (2012); http://dx.doi.org/10.4236/jmp.2012.310188
13. G. S. Golitsyn, Probabilistic structures of the macrocosm, Moscow, Fizmatlit, 2021, 171 p.
14. J. W. Armstrong, B. J. Rickett, and S. R. Spangler, Electron density power spectrum in the local interstellar medium, The Astrophys. J. 443, 209 (1995)
15. P. Blasi, E. Amato, and P. D. Serpico, Spectral breaks as a signature of cosmic ray induced turbulence, Phys. Rev. Lett. 109, 061101 (2012); https://doi.org/10.1103/PhysRevLett.109.061101
16. A. Neronov, D. V. Semikoz, and A. M. Taylor, Low-energy break in the spectrum of the Galactic cosmic rays, Phys. Rev. Lett. 108, 051105 (2012); https://doi.org/PhysRevLett.108.051105
17. S. G. Chefranov, and A. S. Chefranov, Exact solution to the main turbulence problem for a compressible medium and the universal -8/3 law turbulence
spectrum of breaking waves, Phys. Fluids, 33, 076108 (2021); https://doi.org/10.1063/5.0056291

18. T. Passot, and P.-L. Sulem, A model for the non-universal power law of the solar wind sub-scale magnetic spectrum, Astrophys. J. Lett., 812, L37 (2015); https://doi.org/10.1088/2041-8205/812/2/L37

19. V. David, and S. Galtier, $K^{-8/3}$ Spectrum in kinetic Alfvén turbulence: Implications for the solar wind, Astrophys. J. Lett., 880, L10 (2019); https://doi.org/10.3847/2041-8213/ab2fe6

20. B. B. Kadomtsev and V. I. Petviashvili, On acoustic turbulence, Sov. Phys. Dokl. 18, 115 (1973)

21. V. S. L’vov, Y. L’vov, A. S. Newell, and V. Zakharov, Statistical description of acoustic turbulence, Phys. Rev. E 56, 390 (1997)

22. E. A. Kuznetsov, Turbulence spectra generated by singularities, JETP Lett. 80, 83 (2004)

23. J. Skilling, Cosmic rays in the Galaxy: convection or diffusion, The Astrophys. J., 170, 265 (1971)

24. J. Skilling, Cosmic ray streaming-III: Self-consistent solutions, Mon. Not. R. astr. Soc., 173, 255 (1975)

25. S.G. Chefranov, Exact statistically closure description of vortex turbulence and admixture in compressible medium, Sov. Phys. Dokl. 36 (4), 286-289 (1991)

26. S. G. Chefranov and A. S. Chefranov, Exact solution of the compressible Euler-Helmholtz equation and the millennium prize problem generalization, Phys. Scr. 94, 054001 (2019)

27. S. G. Chefranov, A.S. Chefranov, The new exact solution of the compressible 3D Navier-Stokes equations, Commun. Nonlinear Sci. Numer. Simul. 83, 105118 (2020)

28. G. S. Ezra, On the statistical mechanics of non-Hamiltonian systems: the generalized Liouville equation, entropy, and time-dependent metrics, J. Math. Chem., 35, 29 (2004)

29. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, 2nd ed., Theoretical Physics Vol. 6 (Pergamon, Oxford, 1987).

30. D. G. Wentzel, Cosmic-ray propagation in the Galaxy: Collective effects, Annu. Rev. Astron. Astrophys. 12, 71 (1974)

31. P. L. Sulem, and U. Frisch, Bounds on energy flux for finite energy turbulence, J. Fluid Mech., 72, 417 (1975)
32. E. A. Novikov, Statistical regimes based on exact solutions of hydrodynamic equations, Proceedings of the USSR Academy of Sciences, series Physics of the Atmosphere and Ocean, 12, No. 7, 755-761 (1976)

33. E. A. Novikov, and S. G. Chefranov, Statistical instability and evolution of perturbations for stationary regimes of turbulent flows, Preprint of the Institute of Atmospheric Physics of the USSR Academy of Sciences, Moscow, 1978, 22 p

34. S. G. Chefranov, Investigation of the statistical stability and predictability of turbulent flows, Ph. D., Moscow, 1979, 117p

35. S. V. Anisimov, E. A. Mareev, N. M. Shikhova, and E. M. Dmitriev, Universal spectra of electric field pulsations in the atmosphere, Geophys. Res. Lett., vol. 29 (24), 2217 (2002); https://doi.org/10.1029/2002GL015765

36. S. G. Chefranov, Relativistic generalization of the Landau criterion as a new foundation of the Vavilov-Cherenkov radiation theory, Phys. Rev. Lett., 93, 254801 (2004); https://doi.org/10.1103/PhysRevLett.93.254801

37. S. G. Chefranov, New quantum theory of the Vavilov-Cherenkov radiation and its analogues, arXiv: 1205.3774 [physics.gen-ph] 15 May 2012

38. S. G. Chefranov, Vavilov-Cherenkov radiation when cosmic rays pass through the relict photon gas and when fast charged particles traverse an optical laser beam, JETP, 123, 12 (2016); https://doi.org/10.1134/S1063776116050046