Research Article

Influence of Distribution Modulus on the Compressive Strength of Ultra-High-Performance Concrete with Coarse Aggregate (UHPC-CA)

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The Modified Andreasen and Andersen model (MAA model) is commonly chosen to realize condensed particle packing in ultra-high-performance concretes (UHPCs). In the MAA model, the \( q \) (distribution modulus) is a parameter that plays a critical role in the UHPC matrix design. The objective of this study is to figure out the influence of the \( q \) for the compressive strength of UHPC-containing coarse aggregate (UHPC-CA). Therefore, a series of investigations were conducted, covering the following aspects: first, the traditional design procedure, based on the MAA model, of UHPC-CA is revised by taking water and steel fibers into account in the particle packing system. The results show that the revised design process of UHPC-CA yielded more excellent products with better workability and compressive strength. Second, different \( q \) values which are 0.2, 0.21, 0.22, 0.23, and 0.25 are employed. The results show that when \( q \) is 0.25, the maximum compressive strength can be achieved. However, the flowability decreases with the increase of the \( q \) value. Third, an empirical calculation of the optimal coarse aggregate content in the light of different \( q \) values and maximum size of coarse aggregate are proposed.

1. Introduction

In recent years, ultra-high-performance concrete with coarse aggregate (UHPC-CA), as an advanced cementitious composite, has been rapidly developed both in theoretical research and engineering applications. The UHPC-CA has been widely applied to construction engineering, bridge engineering, and national defense engineering, due to its high toughness, distinguished mechanical performance, and outstanding durability [1, 2]. Table 1 summarizes the experimental research findings and the representative structural applications of UHPC-CA.

To sum up, UHPC-CA has emerged as a promising, better defined, and more versatile cementitious material with the potential to be an ideal material to meet the challenges of harsh environmental exposure areas. In order to obtain these outstanding performance, a series of design theories was developed to realize a dense accumulation of particles, such as Compressible Packing Model [12], Solid Suspension Model [13], Linear Packing Density Model [14], and the Modified Andreasen and Andersen model (MAA model) [15]. Especially, the MAA model has been extensively adopted to design the mix proportion of UHPC-CA owing to its rationality and applicability. Only distribution modulus \( q \) needs to be determined artificially. For instance, Li et al. [16] employed the MAA model to design UHPC contains basalt coarse aggregates and observed that the powder content required was lower to design UHPC when coarse aggregates were introduced. Besides, \( q \), recommended in this study, is 0.9. Besides, Cao et al. [17] tried to incorporate coarse aggregate into UHPC to improve the impact resistance by use of the MAA model and \( q \) was set as 0.22.
Table 1: The summary of structural applications of UHPC-CA.

| Structures                                                                 | Reference |
|---------------------------------------------------------------------------|-----------|
| The 5th Nanjing Yangtze River Bridge                                      | [3, 4]    |
| Precast segments made of UHPC in the Czech republic (A footbridge)        | [5]       |
| Composite prestressed T-Girders                                           | [6]       |
| Ultra-high-performance concrete sleeper                                   | [7, 8]    |
| Dovetail joint of composite bridges                                       | [9]       |
| Shear key of precast segments                                             | [10]      |
| Large-scale reinforcement-confined column specimens                       | [11]      |

Though the MAA model could optimize the particle size distribution (PSD) of the adopted raw materials in concrete, it only takes the granular particles into account under the dry condition, which means the influence of water on the matrix packing is ignored [18]. Actually, the mixing process of particles is carried out under wet conditions and there are a various of complex effects, such as water film effect, hydration reaction, and particle interaction [19–21]. Therefore, Wang et al. [20] proposed the optimized design of wet filler density for UHPC. In addition, for the purpose of obtaining high toughness and strain hardening behavior, steel fibers with a high volume were typically introduced into the UHPC-CA matrix to improve its energy absorption capacity and enhance its mechanical performance [22–24]. However, the complicated shape of steel fiber, unlike a ball or particle, tends the designers to simplify their designed UHPC-CA particle packing system by ignoring the claviform steel fiber. Optimized matrix would be severely disturbed by the steel fibers introduced latterly due to the wedging effect, wall effect, and loosening effect [25, 26]. Dingqiang et al. [25] optimized the theoretical design of hybrid fiber UHPC by the D-Optimal Design (DOD) method. The used steel fibers are equivalent to spherical particles and incorporated into the MAA model to participate in particle packing [27].

Furthermore, the crucial influence of \( q \) on the UHPC-CA properties were concluded, as it plays a principal role in the selection of the cementitious paste/aggregate volumetric ratio [28]. Here, the paste is a mixture of water, cement, and supplementary cementitious materials (SCMs). Hüskens et al. [29] pointed out that higher distribution modulus \( (q > 0.5) \) favors coarse-grained mixtures, but smaller distribution modulus \( (q < 0.25) \) favors fine-grained-rich mixtures. Due to a large amount of fine particles adopted in UHPC, the \( q \) value is usually set to 0.23, as suggested by Hunger [30]. Also, for ultra-high-performance concrete, the value of \( q \) was recommended to be between 0.21 and 0.25 [31]. Obviously, for purpose of obtaining the desired rheological property and mechanical performance of the UHPC-CA matrix, the \( q \) value should be chosen according to the composition [28]. However, there have been few studies stated the effect of \( q \) on the compressive performance of ultra-high-performance concrete [28, 32]. Additionally, by ensuring good workability of UHPC-CA, the objective of minimizing the number and size of defects caused by trapped air bubbles is achieved. Concrete with high flowability commonly requires a lower \( q \) resulting in introducing more fine particles to ensure sufficient cementitious material to encapsulate the aggregate. However, the influence of distribution modulus on the workability of UHPC-CA has not been investigated clearly.

Based on the views above, this research incorporates water and steel fibers into the MAA model for developing an advanced UHPC-CA. Better performance of UHPC-CA can be observed in the particle packing skeleton with water and steel fibers compared to that without water and steel fibers. By this design method, different distribution moduli are employed in the MAA model and the effect of \( q \) on the rheological property and compressive strength of UHPC-CA are investigated. At last, an empirical calculation of the optimal coarse aggregate content corresponding to different \( q \) values and the maximum size of coarse aggregate are given.

2. Materials and Methods

2.1. Materials. In this study, ground granulated blast furnace slag (GGBS), silica fume (SF), and ordinary portland cement (OPC, P-O 52.5) are adopted. Moreover, their chemical compositions, tested by X-ray Fluorescence (XRF), can be found in Table 2. Additionally, three types of fine aggregates (size range: 0.85–2.36 mm, 0.425–0.85 mm, 0.212–0.425 mm) and granite coarse aggregate with a size range of 5–10 mm are used. Besides, a high range water reducing admixture (HRWRA) with solid content of 40% is adopted to enhance the fresh performance of UHPC-CA. In addition, steel fibers with the length of 15 mm and the diameter of 0.2 mm, as can be seen in Table 3, are employed here. Figure 1 shows more information about the raw materials employed in this paper.

2.2. Experimental Methods

2.2.1. Mix Proportion Design of UHPC-CA. In this paper, the dense UHPC-CA skeleton is designed according to the MAA model, and the model is shown in

\[
P(D) = \frac{D^q - D_{\text{min}}^q}{D_{\text{max}}^q - D_{\text{min}}^q},
\]

where \( P(D) \) represents the volume fraction of all the particles which are smaller than \( D \), \( D \) represents the particle size of raw materials, \( D_{\text{min}} \) is the minimum particle sizes, \( D_{\text{max}} \) represents the maximum particle sizes, and \( q \) represents the distribution modulus. Besides, the residual sum of squares (RSS) at discrete values of \( D \) is employed to adjust the component of raw materials [33, 34], as can be seen in

Table 2: The summary of structural applications of UHPC-CA.
Table 2: The main chemical components (wt%) of cement, silica fume and GGBS.

| Materials  | SiO₂  | CaO   | Al₂O₃ | SO₃   | Fe₂O₃ | MgO   | Na₂O   | K₂O   | P₂O₅  | LOI* |
|------------|-------|-------|-------|-------|-------|-------|--------|-------|-------|------|
| Cement     | 20.86 | 56.77 | 5.90  | 2.43  | 3.61  | 3.50  | 0.30   | 0.42  | 0.09  | 1.16 |
| Silica fume| 94.62 | 0.40  | 0.36  | 0.20  | —     | 0.39  | 0.42   | 0.55  | 0.15  | 2.8  |
| GGBS       | 32.47 | 40.07 | 17.29 | 3.23  | 0.57  | 0.30  | 0.50   | —     | 0.08  | 0.88 |

Note: *Loss of ignition.
$RSS = \sum_{i=1}^{n} \left( P_{\text{mix}}(D_{i}^{+1}) - P_{\text{tar}}(D_{i}^{+1}) \right)^2 \rightarrow \min,$ 

(2)

where $P_{\text{tar}}$ represents the target mixture and $P_{\text{mix}}$ represents the actual composite mixture. The mix proportion of the ultra-high-performance concrete is considered as the best one when the RSS is minimized. Besides, the $R^2$ (coefficient of determination), as an assessment of the quality of the curve fit between the target curve and the actual composed mix curve, is also calculated:

$R^2 = 1 - \frac{\sum_{i=1}^{n} \left( P_{\text{mix}}(D_{i}^{+1}) - P_{\text{tar}}(D_{i}^{+1}) \right)^2}{\sum_{i=1}^{N} \left( P_{\text{mix}}(D_{i}^{+1}) - P_{\text{mix}} \right)^2},$ 

(3)

$P_{\text{mix}}^- = \frac{1}{n} \sum_{i=1}^{n} P_{\text{mix}} (D_{i}^{+1}).$ 

(4)

The PSD of the raw materials adopted in this study, the actual composed mix curve, and the target curves of UHPC-CA are depicted in Figure 2 and 3. It could be found that mixture curves fit target curves well, and all $R^2$ are greater than 0.975. Moreover, to figure out the impacts of $q$ on the performance of UHPC-CA, 5 mixtures are given by changing the value of $q$ (0.2, 0.21, 0.22, 0.23, and 0.25), as shown in Table 4. Additionally, water and steel fibers are treated as particles that participated in the process of particle packing. In this way, the UHPC-CA mixtures are designed.

The UHPC-CA mixtures in this research are generated on the grounds of the following procedures, as shown in Figure 4.
Figure 3: Continued.
Table 4: The mix proportion of UHPC-CA (kg/m$^3$).

| No.   | OPC   | SF   | GGBS | FS   | MS   | CS   | CA   | Water | HRWRA | Steel fibers |
|-------|-------|------|------|------|------|------|------|-------|-------|--------------|
| $q = 0.20$ | 739.1 | 205.3 | 205.3 | 221.7 | 147.8 | 369.5 | 295 | 184   | 22.9  | 156          |
| $q = 0.21$ | 710.4 | 197.4 | 197.4 | 203.0 | 135.4 | 338.4 | 307 | 177   | 22.1  | 156          |
| $q = 0.22$ | 648.0 | 161.8 | 196.8 | 169.7 | 254.6 | 424.3 | 321 | 161   | 20.1  | 156          |
| $q = 0.23$ | 620.0 | 166.6 | 188.5 | 160.3 | 240.5 | 400.8 | 331 | 156   | 19.5  | 156          |
| $q = 0.25$ | 605.0 | 120.6 | 179.8 | 150.6 | 249.4 | 401.2 | 356 | 145   | 18.1  | 156          |
| $q = 0.23–1$ | 620.0 | 166.5 | 188.5 | 160.3 | 240.5 | 400.8 | 331 | 156   | 19.5  | 156          |
| $q = 0.23–2$ | 620.0 | 166.6 | 188.5 | 160.3 | 240.5 | 400.8 | 331 | 156   | 19.5  | 156          |
| $q = 0.23–4$ | 620.0 | 166.6 | 188.5 | 160.3 | 240.5 | 400.8 | 331 | 156   | 19.5  | 156          |
| $q = 0.23^*$ | 645.2 | 129.1 | 129.1 | 209.7 | 209.7 | 419.4 | 550 | 145   | 18.1  | 156          |

$q = 0.23–1$, $q = 0.23–2$, and $q = 0.23–4$: the dosages of coarse aggregate are different from $q = 0.23$ and are 190 kg/m$^3$, 290 kg/m$^3$, and 490 kg/m$^3$ respectively; $q = 0.23^*$: this group did not consider water and steel fibers to participate in particle packing.

Figure 3: The mixture and target curve of UHPC-CAs: (a) the target curve corresponding different values of $q$, (b) $q = 0.20$, (c) $q = 0.21$, (d) $q = 0.22$, (e) $q = 0.23$, (f) $q = 0.25$, and (g) $q = 0.23^*$: $R^2$, in the figures, were calculated by equation (3) and (4).

Figure 4: The mixing process of UHPC-CA.
2.2.2. Flowability Test. This study refers to ASTM C143 [35] to evaluate the workability of the fresh mixture. First, the fresh mortar was put into the frustum cone-like mold. Then, the mold was lifted vertically and promptly. Finally, the two mutually perpendicular diameters are measured and the data are recorded.

2.2.3. Compressive Strength Test. After flowability test, the UHPC-CA was cast into a mold measuring 100 mm × 100 mm × 100 mm and then, the mold surface was covered with a plastic film to reduce water loss. After 24 hours of curing, the cubes were removed from the molds and then cured at approximately 20°C and 95 ± 5% relative humidity. After 7 and 28 days of curing, the specimen were tested for compressive strength according to EN 12390–3 [36], and test three specimen each group.

3. Results and Discussion

3.1. Traditional Design Procedure Improvement. In this study, the UHPC-CA matrix design procedure was revised by taking water and steel fibers into account in the particle packing system. Here, water is regarded as a particle of the smallest size (5 mm) of coarse aggregate during particle packing. In this way, the mixture curve fits the target curve...
more easily. In addition, it can control the volume fraction of coarse aggregate will not be excessively high, as too much coarse aggregate degrades the mechanical properties of the UHPC [37].

Furthermore, the addition of steel fibers will disturb and destroy the packing skeleton of concrete due to the connection effect, wall effect, loosening effect, and wedging effect [25], resulting in a decrease in the packing density of concrete; on the other hand, fibers (especially fine and short fibers) also have a filling effect, which can fill the pores between the aggregates and improve the packing density [25], as illustrated in Figure 5. It can be seen that the influence of steel fibers on the particle packing system cannot be ignored. For this reason, many researchers have tried to consider fibers of nonspherical particles as particles with equivalent diameters in order to investigate their effects on concrete packing structures, such as the Ferrara model [38], Yu model [39, 40], and artificial neural network (ANN) [27]. Fan et al. [27] demonstrated that the Yu model is applicable to UHPC systems because the Yu model is based on the constant principle of stacking density, which takes into account the effect of fibers on compaction. In addition, Yu et al. [39] mentioned that for particles with regular shape, it can be calculated by definition, as shown in equation (5).where $d_{eq}$ represents the steel fibers equivalent diameter, $d_v$ represents the volume diameter (the diameter of a sphere with the same volume as the particle), $\phi$ represents the parameter of sphericity (the ratio of the surface area of the sphere to the volume of the actual surface area of the particle), $d_f$ represents the fibers diameter, and $L_f$ represents the fibers length.

$$d_{eq} = \left(3.1787 - \frac{3.6821}{\phi} + \frac{1.5040}{\phi^2}\right) \times d_v$$

$$\phi = 2.624 \times \frac{(L_f/d_f)^{2/3}}{1 + 2 \times (L_f/d_f)}$$

$$d_v = 1.145 \times (L_f/d_f)^{1/3} \times d_f$$

The equivalent diameter of steel fibers employed in the paper calculated by the Yu model is 6.78 mm. By considering the control threshold of coarse aggregate content and the effects of steel fibers on compactness, water and steel fibers

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**Figure 10:** Schematic illustration of equivalent skeleton of coarse aggregate.

**Figure 11:** Failure mode observed through compressive test.
are regarded as particles with equivalent diameters and then participate in particle packing. Compared to cementitious materials and aggregates packing system, the revised design procedure yields better workability and higher compressive strength UHPC-CA, as shown in Figure 6. As more coarse aggregate was employed for mixture curve to be close to target curve when water and steel fibers were ignored; therefore, the aggregate/paste ratio of the \( q = 0.23 \) group is greater than that of the \( q = 0.23^* \) group. Additionally, the workability of fresh concrete decreased when the aggregate/paste ratio increased. Thus, the workability of the \( q = 0.23 \) group was much better compared to the \( q = 0.23^* \) group. Besides, the compressive strength of the \( q = 0.23 \) group was also higher than that of the \( q = 0.23^* \) group. The reasons are as follows: (1) fewer flaw is introduced when the paste of \( q = 0.23 \) was cast into mold due to better flowability; (2) trapped air bubbles in \( q = 0.23 ^* \) paste is more difficult to escape; (3) the steel fibers are more evenly dispersed in \( q = 0.23 \) matrix. It can be concluded that the production of the revised design procedure possesses more superior properties than that of MAA model. Thus, by using the revised design procedure model, the UHPC-CA mixture proportions corresponding to different \( q \) values were designed.

3.2 Workability. In this study, the workability of fresh mixture was characterized by slump extension as per ASTM C143 [35]. Test results of workability of UHPC-CAs are illustrated in Figure 7, each group was tested 3 times and test results were averaged. It can be found that the slump extension decreases when the \( q \) value increases. The maximum slump extension reaches 590 mm when \( q = 0.20 \) and the minimum slump extension only is 480 mm when \( q = 0.25 \). And the slump extension of group \( q = 0.21, 0.22 \) and \( 0.23 \) was 3.4%, 8.5% and 11.9% lower than that of group \( q = 0.20 \), respectively. This phenomenon can be explained by the literature [37, 41] that increasing the volume concentration

![Figure 12: Derivation of empirical equation for coarse aggregate (\( (D)_{\text{max}} = 10 \text{ mm} \)) admixture based on the MAA model: (a) principle of derivation; (b) fitting equation.](image-url)
of coarse aggregate would degrade the workability of UHPC-CA. As shown in Figure 3(a), the larger the $q$ value is, the smaller curvature of the target curve is, which indicates that the volume ratio of large and small particles depends on $q$ value. In other words, the cementitious materials (cm)/aggregate volumetric ratio decreases with the increase of distribution modulus [28], as depicted in Figure 8. Therefore, the thickness of the slurry used to wrap the aggregate decreases as $q$ increases, resulting in the decrease of matrix fluidity [42]. In addition, the higher content of coarse aggregate produces a higher internal obstruction between paste and coarse aggregate attribute to the interlocking effect of coarse aggregate [43]. Furthermore, steel fibers tend to be centered on the coarse aggregate with the increase of concentrations of coarse aggregate [33].

### 3.3. Compressive Strength

Figure 9 illustrates the 7-days compressive strength and 28-days compressive strength of UHPC-CAs with different $q$ values. It can be seen from Figure 9 that the 7-days compressive strength of all samples exceed 125 MPa, and it increased with $q$. However, when $q = 0.21$, the 28-days compressive strength is a little lower than that when $q = 0.20$. This can be explained by more air bubbles trapped in samples during casting. The compressive strengths of UHPC-CA mixes $q = 0.22, 0.23, 0.25$ were approximately 4.5%, 8.9%, and 13%, respectively, higher than that of $q = 0.20$ at 28 days. On the whole, the compressive strength of UHPC-CAs increased as $q$ increased. The main reason for the improvement is the increase of the dosage of coarse aggregate. Some researchers also observed that when the content of coarse aggregate does not exceed a certain threshold, the compressive strength of UHPC-CA increase when the coarse aggregate content increase [37, 44]. It was reported that adding coarse aggregate to the UHPC matrix results in forming an equivalent skeleton, due to the aggregate interlocking, to enhance the rigidity of concrete, thus, improving the overall compression performance of UHPC-CA [44], as depicted in Figure 10.

Figure 11 illustrates the compressive failure mode of concrete. It can be seen that the degree of specimen failure is also closely related to $q$, the larger $q$ is, the more serious the specimen is destroyed. The reasons cannot escape from the fact that the introducing of coarse aggregate can cause stress concentration at the contact between the aggregate particles [16, 45]. Cracks, during initiation, tend to propagate around the coarse aggregate rather than through it. In addition, as mentioned above, the dosage of coarse aggregate increased with $q$. Therefore, there are more cracks in the matrix at $q = 0.25$, causing more serious failure of samples.

### 3.4. Empirical Prediction of the Optimal Coarse Aggregate Content

As mentioned above, the influence of $q$ on workability and compressive strength of UHPC-CA is distinct and cannot be ignored. The key impact parameter are the dosage and maximum size of CA. Therefore, it is necessary to establish a connection between $q$ and coarse aggregate content ($V_{CA}$) and maximum size ($D_{max}$). In this research, the empirical calculation formula of coarse aggregate content can be seen in (6), and the derivation of empirical equation for CA ($D_{max} = 10\text{mm}$) admixture based on the MAA model are shown in Figure 12. From Figure 12(a), it can be found that there are differences in the MAA models corresponding to different $q$ values, which also implies that the volume fraction of coarse aggregate varies in the range of 5000–10000 $\mu\text{m}$, i.e., the range corresponding to the coarse aggregate particle size. Here, the volume fractions of coarse aggregates corresponding to different $q$ values are plotted in Figure 12(b), which shows that the volume fractions of coarse aggregates are linear with $q$ values.

$$V_{CA} = 51.87691q + 4.09803.$$  \hfill (6)

Figure 13 shows the empirical and practical values of coarse aggregate volumetric ratio. It turns out they are pretty close. That is why, when the curve coincidence degree is
approximately the same, the content of coarse aggregate changes linearly with \( q \). To prove the correctness and validity of (3) and (6) groups, \( q = 0.23–1 \), \( q = 0.23–2 \), and \( q = 0.23–4 \) were designed. The only difference between the 4 groups \((q = 0.23–1, q = 0.23–2, q = 0.23, \text{and } q = 0.23–4)\) is the dosage of coarse aggregate, as can be seen in Table 4. Then, the obtained results were employed to validate the optimal coarse aggregate content in theory. As illustrated in Figure 14, the compressive strength is maximum at \( q = 0.23 \), and the compressive strength of UHPC-CA first increases and then decreases with the increase of coarse aggregate content. It demonstrates that the volume fraction calculated according to (7) may be the optimal value or pretty close to it.

However, it should be pointed out that (6) is only proposed for coarse aggregate with a particle size of 5–10 mm, without taking other sizes of coarse aggregate into account, that will cause more complex effects on the packing system. Thus, it is necessary to investigate other sizes of coarse aggregate employed into the packing model. In previous studies, the upper limit of coarse aggregate particle size is usually 8, 10, 16, or 20 mm [37, 44, 46, 47]. For the

| \( D_{\text{max}} \) (mm) | \( q = 0.20 \) (%) | \( q = 0.21 \) (%) | \( q = 0.22 \) (%) | \( q = 0.23 \) (%) | \( q = 0.25 \) (%) | Empirical equation |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 8               | 10.51           | 10.88           | 11.27           | 11.66           | 12.43           | \( V_{\text{CA}} = 38.51351q + 2.8 \) |
| 10              | 14.71           | 15.24           | 15.86           | 16.3            | 17.36           | \( V_{\text{CA}} = 51.87691q + 4.09803 \) |
| 16              | 22.91           | 23.69           | 24.48           | 25.27           | 26.84           | \( V_{\text{CA}} = 78.66216q + 7.175 \) |
| 20              | 26.5            | 27.39           | 28.28           | 29.17           | 30.94           | \( V_{\text{CA}} = 88.81081q + 8.74 \) |

Figure 15: Influence of maximum size of CA and \( q \) on the volume fraction of CA.

Figure 16: The relationships between (a) \( k \) and \( D_{\text{max}} \) and (b) \( b \) and \( D_{\text{max}} \).
coarse aggregate with size range of 5–8 mm, 5–10 mm, 5–16 mm, 5–20 mm, follow the above method, the relationship between the volume fractions of coarse aggregates of different particle sizes and q values can be assumed to be linear, as shown in

\[ V_{CA} = kq + b, \quad (7) \]

where \( V_{CA} \) is the volume fraction of coarse aggregate in the matrix, q is the distribution modulus, k and b are coefficient associated with \( D_{max} \), and \( D_{max} \) is the maximum particle size of coarse aggregate.

The derivation of empirical equation for CA with different maximum particle sizes (8, 10, 16, or 20 mm) based on the MAA model is summarized in Table 5. The influence of maximum size of CA and q on the volume fraction of CA are plotted in Figure 15.

According to Table 4, the relationships between k and \( D_{max} \) and b and \( D_{max} \) are derived as shown in Figure 16 and equation.

\[
k = 10.41952D_{max} - 0.22504D_{max}^2 - 29.81429, \\
b = 0.88319D_{max} - 0.01401D_{max}^2 - 3.33287, \quad D_{max} \in (8, 20).
\]

(8)

Here, to further confirm the applicability of equations (7) and (8), some research results [37, 46] are employed. The comparison of equations (7) and (8) with test results in literature is shown in Figure 17. In [46], the density of diabase is about 3 g/cm³, while crushed basalt with an apparent density of 2.86 g/cm³ was employed in [37]. Convert mass fraction of coarse aggregate to volume fraction, as depicted in Figure 17. In addition, q was fixed at 0.23, then the optimal coarse aggregate content calculated according to equations (7) and (8) are exhibited by red dashed line in Figure 17. It can be found that the compressive strength of UHPC-CA tends to be higher as the volume fraction of coarse aggregate is closer to the predict optimal CA content, which indicating that equations (7) and (8) have good estimation accuracy for predicting optimal CA content.

4. Conclusion

This paper reported that how to implant water and steel fibers in the MAA model to improve traditional design procedures. Moreover, the influence of q on the workability and compressive strength of UHPC-CA were investigated. Finally, an empirical calculation of the optimal coarse aggregate content according to q and maximum particle size of coarse aggregate was given. The following basic conclusions can be addressed from this study:

(1) The conventional design procedure was revised by regarding water and steel fibers as particles to implant them into the MAA model. The results demonstrate that the revised design procedure generates more distinguishing UHPC-CA with better flowability and higher compressive strength. Specifically, the slump expansion of q = 0.23 paste was 10.6% higher than that of q = 0.23* paste, and the 7-day compressive strength of q = 0.23 specimens was 8.4% higher than that of q = 0.23* specimens and 2.7% higher at 28 days.

(2) Distribution modulus affects the workability and compressive strength of UHPC-CA mainly by affecting the cementitious materials/aggregates volumetric ratio. In the range of 0.20–0.25 of q, the flowability decreases when q increases. However, the compressive strength increases with the increase of q. Compared with q = 0.20, the slump expansions of q = 0.21, q = 0.22, q = 0.23, and q = 0.25 decreased by 3.7%, 8.7%, 11.8%, and 18.0%, respectively; while the 28-day compressive strengths increased by –1.9%, 4.5%, 8.9%, and 13.0%, respectively.

(3) For coarse aggregate with maximum particle size of 8–20 mm, the volume fraction of coarse aggregate calculated according to equations (7) and (8) may be the optimal value. The results concluded that the compressive strength of UHPC-CA is higher when the content of coarse aggregate is closer to predict optimal CA content, indicating that equations (7) and (8) are effective.
Data Availability

The information about the particle size distribution of raw materials can be found at https://figshare.com/s/27a87b9b9a9576325a5b. The information about the mix proportion of UHPC-CA can be found at https://figshare.com/s/e6604b92e472916ed43. The information about the test date can be found at https://figshare.com/s/e66084b92e472916ed43. The information about the particle size distribution of raw materials can be found at https://figshare.com/s/e6604b92e472916ed43.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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