Axial charges of the nucleon and $N^*$ resonances

Ki-Seok Choi, W. Plessas, and R. F. Wagenbrunn

Theoretische Physik, Institut für Physik, Karl-Franzens-Universität, Universitätsplatz 5, A-8010 Graz, Austria

The axial charges of the nucleon and the well-established $N^*$ resonances are studied within a consistent framework. For the first time the axial charges of the $N^*$ resonances are produced for the relativistic constituent quark model. The axial charge of the nucleon is predicted close to experiment, and the ones of $N^*(1535)$ and $N^*(1650)$, the only cases where such a comparison is possible, agree well with results from quantum chromodynamics on the lattice that have recently become available. The relevance of the magnitudes of the $N^*$ axial charges for the low-energy behavior of quantum chromodynamics is discussed.

PACS numbers: 11.30.Rd, 12.39.Ki, 14.20.Gk,
Keywords: Axial charge, Relativistic quark model, Nucleon resonance

The axial charge $g_A$ of the nucleon ($N$) is an essential quantity in understanding the electroweak and strong interactions within the Standard Model of elementary particles. In the first instance it is directly related to the neutron $\beta$ decay, and its experimental value can be derived from the ratio of the axial to the vector coupling constants $g_A/g_V = 1.2695 \pm 0.0029$ [1]; usually this is done under the assumption of conserved vector currents (CVC), which implies $g_V = 1$. The deviation of $g_A$ from 1, the axial charge of a point-like particle, can be attributed, according to the Adler-Weisberger sum rule [2] [3], to the differences between the $\pi^+N$ and $\pi^-N$ cross sections in pion-nucleon scattering. Through the Goldberger-Treiman relation, $g_A = f_\pi g_{\pi NN}/M_N$, the axial charge is connected with the $\pi$ decay constant $f_\pi$, the $\pi NN$ coupling constant $g_{\pi NN}$, and the nucleon mass $M_N$. Thus the axial charge of the $N$ plays a key role for the spontaneous breaking of chiral symmetry ($SB\chi S$) of quantum chromodynamics (QCD) in low-energy hadron physics, a phenomenon that is manifested by the non-vanishing value of the light-flavor chiral condensate $\langle \bar{q}q \rangle/\sqrt{2}$ $\approx -235$ MeV.

There have been a number of theoretical attempts to produce the axial charge of the $N$ ground state with many different methods. We mention only the more novel approaches via the relativistic constituent quark model (RCQM) [4] [5], by chiral perturbation theory [6], and within lattice QCD [7] [10] [11] [12] [13] [14] [15]. In general, the theoretical results come close to the experimental value of roughly 1.27, with the lattice-QCD predictions scattering over a range of approximately 1.10–1.40, depending on the various actions employed and a series of technical details entering the calculations by the different groups.

Recently, also the axial charges of the $N^*$ resonances have come into the focus of interest, as it was suggested that their values should become small or even vanishing for excited states that could be parity partners in a scenario of chiral-symmetry restoration higher in the hadron spectra [16] [17]. As the $g_A$ values of $N^*$ resonances can hardly be measured experimentally, this remains a highly theoretical question. However, the problem can be explored by ab-initio calculations of QCD on the lattice. Corresponding first results have become available lately for just two of the $N^*$ resonances, namely, $N^*(1535)$ and $N^*(1650)$ [15]. Both of them have total angular momentum (intrinsic spin) $J = \frac{3}{2}$ and parity $P = -1$. Unfortunately, there is not yet any lattice-QCD result for positive-parity states, and the above issue relating to parity-doubling remains unresolved on this basis.

It is thus most interesting to get insight into the $N$ and $N^*$ axial charges from other approaches. Especially by the RCQM we can investigate the problem in a comprehensive manner, as all the ground and resonant states are readily accessible. Here we report theoretical predictions of $g_A$ for positive- as well as negative-parity $N^*$ resonances up to $J = \frac{3}{2}$. The calculations are performed employing a RCQM with the quark-quark hyperfine interaction deduced from Goldstone-boson-exchange (GBE) dynamics. In particular, we use both existing versions of GBE RCQMs, the one with pseudoscalar (ps) spin-spin forces only [18] and the extended GBE (EGBE) RCQM with all the central, spin-spin, tensor, and spin-orbit force components included [19]. For the sake of comparison with another type of hyperfine interaction we employ also the RCQM with one-gluon-exchange (OGE) dynamics [20].

The calculations are performed in the framework of Poincaré-invariant quantum mechanics. In order to keep the numerical computations manageable, we have to restrict the axial current operator to the so-called spectator model (SM). It means that the weak-interaction gauge boson couples only to one of the constituent quarks in the baryon. This approximation has turned out to be very reasonable already in a previous study of the axial and induced pseudoscalar form factors of the nucleon [4], where the SM was employed specifically in the point form (PF) of relativistic quantum mechanics [21]. It has also been used in studies of the electromagnetic structure of the $N$, reproducing both the proton and neutron form factors in close agreement with the experimental data [6] [22] [23] [24].

The axial charge is defined through the value of the axial form factor $G_A(Q^2)$ at $Q^2 = 0$, where $Q^2 = -q^2$ is the four-momentum transfer. The axial form factor $G_A(Q^2)$ can be deduced from the invariant matrix element of
the axial current operator $\hat{A}_a^\mu(Q^2)$, with flavor index $a$, sandwiched between the eigenstates of $N$ or $N^*$. We denote the latter generally by $|P, J, \Sigma\rangle$, i.e. as eigenstates of the four-momentum operator $\hat{P}^{\mu}$, the intrinsic-spin operator $\hat{J}$ and its $z$-projection $\hat{\Sigma}$. Since $\hat{P}^{\mu}$ and the invariant mass operator $\hat{M}$ commute, these eigenstates can be obtained by solving the eigenvalue equation of $\hat{M}$

$$
\hat{M} |P, J, \Sigma\rangle = M |P, J, \Sigma\rangle ,
$$

where $M$ is the mass of $N$ or $N^*$. For the various $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ states considered here, the axial charges $g_A$ are thus computed from the matrix elements of the axial current operator $\hat{A}_a^\mu$ for zero momentum transfer

$$
\langle P, \frac{1}{2}, \Sigma' | \hat{A}_a^\mu | P, \frac{1}{2}, \Sigma \rangle = \bar{U}(P, \Sigma') g_A \gamma^\mu \frac{\tau_a}{2} U(P, \Sigma),
$$

$$
\langle P, \frac{3}{2}, \Sigma' | \hat{A}_a^\mu | P, \frac{3}{2}, \Sigma \rangle = \bar{U}^{\nu}(P, \Sigma') g_A \gamma^\mu \frac{\tau_a}{2} U_{\nu}(P, \Sigma),
$$

$$
\langle P, \frac{5}{2}, \Sigma' | \hat{A}_a^\mu | P, \frac{5}{2}, \Sigma \rangle = \bar{U}^{\nu}(P, \Sigma') g_A \gamma^\mu \frac{\tau_a}{2} U_{\nu\lambda}(P, \Sigma).
$$

(2)

Here $U(P, \Sigma)$ are the Dirac spinors for spin-$\frac{1}{2}$ and $U^{\mu}(P, \Sigma)$ the Rarita-Schwinger spinors for spin-$\frac{3}{2}$ and spin-$\frac{5}{2}$ particles, respectively. $\gamma^\mu$ and $\gamma_5$ are the usual Dirac matrices and $\tau_a$ the isospin matrix with Cartesian index $a$.

The matrix elements of $\hat{A}_a^\mu$ for any $N$ or $N^*$ read

$$
\langle P, J, \Sigma' | \hat{A}_a^\mu(Q^2 = 0) | P, J, \Sigma \rangle =
2M \sum_{\sigma, \sigma'} \int d^3\vec{k}_1 d^3\vec{k}_2 d^3\vec{k}_3 \frac{\delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)}{2\omega_1 2\omega_2 2\omega_3} \Psi_{P, J, \Sigma'} \left( \vec{k}_1, \vec{k}_2, \vec{k}_3; \sigma_1', \sigma_2', \sigma_3' \right) \times \langle k_1, k_2, k_3; \sigma_1', \sigma_2', \sigma_3' | \hat{A}_a^\mu | k_1, k_2, k_3; \sigma_1, \sigma_2, \sigma_3 \rangle \Psi_{P, J, \Sigma} \left( \vec{k}_1, \vec{k}_2, \vec{k}_3; \sigma_1, \sigma_2, \sigma_3 \right).
$$

(3)

The $\Psi$'s are the rest-frame wave functions of the $N$ or $N^*$ with corresponding mass $M$ and total angular momentum $J$ with $z$-projections $\Sigma$ and $\Sigma'$. Here they are represented as functions of the individual quark three-momenta $\vec{k}_i$, which sum up to $\vec{P} = \vec{k}_1 + \vec{k}_2 + \vec{k}_3 = \vec{0}$; $\omega_i = \sqrt{m_i^2 + \vec{k}_i^2}$ is the energy of quark $i$ with mass $m_i$, and the individual-quark spin orientations are denoted by $\sigma_i$.

The SM means that the matrix element of the axial current operator $\hat{A}_a^\mu$ between (free) three-particle states $|k_1, k_2, k_3; \sigma_1, \sigma_2, \sigma_3 \rangle$ is assumed in the form

$$
\langle k_1, k_2, k_3; \sigma_1', \sigma_2', \sigma_3' | \hat{A}_a^\mu | k_1, k_2, k_3; \sigma_1, \sigma_2, \sigma_3 \rangle = 3 \langle k_1; \sigma_1' | \hat{A}_a^\mu_{SM} | k_1; \sigma_1 \rangle 2\omega_2 2\omega_3 \delta_{\sigma_2 \sigma_2'} \delta_{\sigma_3 \sigma_3'}.
$$

(4)

For point-like quarks this matrix element involves the axial current operator of the active quark 1 (with quarks 2 and 3 being the spectators) in the form

$$
\langle k_1; \sigma_1' | \hat{A}_a^\mu_{SM} | k_1; \sigma_1 \rangle = \bar{u} (k_1, \sigma_1') g_A \gamma^\mu \frac{\tau_a}{2} u (k_1, \sigma_1),
$$

(5)

where $u (k_1, \sigma_1)$ is the quark spinor and $g_A^3 = 1$ the quark axial charge. A pseudovector-type current analogous to the one in Eq. (3) was recently also used in the calculation of $g_{\sigma NN}$ and the strong $\pi NN$ vertex form factor in ref. [22].

If we are interested only in the axial charges $g_A$, the expression (5) specifies to $\mu = i, j, k$ and can further be evaluated to give

$$
\bar{u} (k_1, \sigma_1') \gamma^i \gamma_5 \frac{\tau_a}{2} u (k_1, \sigma_1) = 2\omega_1 \chi_{\bar{\sigma}_i}^* \left[ 1 - \frac{2}{3} (1 - \kappa) \right] \sigma^i + \sqrt{\frac{5}{3}} \frac{\kappa^2}{1 + \kappa} \left[ [\bar{v}_i \otimes \bar{v}_j] \otimes \tilde{\sigma}^i \right] \frac{\tau_a}{2} \chi_{\sigma_i}.
$$

(6)

where $\kappa = 1/\sqrt{1 + \nu_{11}^2}$ and $\bar{v}_i = \vec{k}_i / m_1$. Herein $\sigma^i$ is the $i$-th component of the usual Pauli matrix $\sigma$ and $v_1$ the magnitude of the three-velocity $\vec{v}_1$. The symbol $[., \otimes .]_k$ denotes the $i$-th component of a tensor product $[., \otimes .]_k$ of rank $k$. We note that a similar formula was already published before by Dannbom et al. [20], however, restricted to the case of total orbital angular momentum $L = 0$. Our expression holds for any $L$, thus allowing to calculate $g_A$ for the most general wave function of a baryon specified by $J^P$. 
In Table I we give the predictions of the EGBE RCQM for the \( N \) ground state and the first two \( N^* \) excitations of \( J = \frac{1}{2} \) with positive as well as negative parity \( P \). It is seen that the result for \( g_A \) of the \( N \) comes close to the experimental value and is also congruent with the lattice-QCD results. The latter is also true with respect to the other cases, where lattice-QCD results are already available, the \( J^P = \frac{1}{2}^- \) resonances \( N^*(1535) \) and \( N^*(1650) \). The simple \( SU(6) \times O(3) \) nonrelativistic quark model used by Glozman and Nefediev cannot reproduce the \( g_A \) of the \( N \) and it yields exactly the same results for \( N \) and \( N^*(1440) \). The corresponding axial charge of \( N^*(1535) \) is non-zero but negative, while the results for \( N^*(1710) \) and \( N^*(1650) \) are similar to the ones of the EGBE RCQM. In the last column of Table I we also quote the results obtained in the nonrelativistic limit of the axial current operator of Eq. \( 6 \).

By comparing with the figures in the first column, one can see that relativistic effects related to the current operator are considerable in all instances.

**TABLE I: Predictions for axial charges \( g_A \) of the EGBE RCQM in comparison to available lattice QCD results** \cite{9, 11, 11, 12, 13, 14, 15}, the values calculated by Glozman and Nefediev (GN) within the \( SU(6) \times O(3) \) nonrelativistic quark model, and the nonrelativistic (NR) limit from the EGBE RCQM.

| State     | \( J^P \) | EGBE | Lattice QCD | GN | NR |
|-----------|-----------|------|-------------|----|----|
| \( N(939) \) | \( \frac{1}{2}^+ \) | 1.15 | 1.10~1.40   | 1.66 | 1.65 |
| \( N^*(1440) \) | \( \frac{1}{2}^+ \) | 1.16 | –           | 1.66 | 1.61 |
| \( N^*(1535) \) | \( \frac{1}{2}^- \) | 0.02 | ~0.00       | -0.11 | -0.20 |
| \( N^*(1710) \) | \( \frac{1}{2}^+ \) | 0.35 | –           | 0.33 | 0.42 |
| \( N^*(1650) \) | \( \frac{1}{2}^- \) | 0.51 | ~0.55       | 0.55 | 0.64 |

In Tables II and III we present the relativistic predictions of \( g_A \) for the \( N \) ground state and all positive- as well as negative-parity \( N^* \) excitations with masses below ~1.9 GeV for the three types of RCQMs considered here. We regard the EGBE result as the most advanced one, as this particular RCQM includes all force components in the hyperfine interaction and presumably produces the most realistic \( N \) and \( N^* \) wave functions. Still, the psGBE RCQM, which relies only on a spin-spin hyperfine interaction, performs similarly well for all positive-parity resonances and for most of the negative-parity resonances; only for the \( J^P = \frac{3}{2}^- \) states \( N^*(1520) \) and \( N^*(1700) \) there occur differences, which have evidently to be attributed to tensor and/or spin-orbit forces. Except for these two cases there are also no big differences to the results with the OGE RCQM, even though the theoretical resonance masses show sometimes considerable differences \cite{20, 27}. It will be very interesting to confront these predictions by the RCQMs with results by other approaches and in particular with further data from lattice QCD.

**TABLE II: Mass eigenvalues and axial charges \( g_A \) of the \( N \) ground state and the positive-parity \( N^* \) resonances as predicted by the EGBE, the psGBE, and the OGE RCQMs.**

| State     | \( J^P \) | EGBE | psGBE | OGE |
|-----------|-----------|------|-------|-----|
| \( N(939) \) | \( \frac{1}{2}^+ \) | 939  | 1.15  | 939  | 1.15  | 939  | 1.11  |
| \( N^*(1440) \) | \( \frac{1}{2}^+ \) | 1464 | 1.16  | 1459 | 1.13  | 1578 | 1.10  |
| \( N^*(1710) \) | \( \frac{3}{2}^+ \) | 1757 | 0.35  | 1776 | 0.37  | 1860 | 0.32  |
| \( N^*(1720) \) | \( \frac{3}{2}^+ \) | 1746 | 0.35  | 1728 | 0.34  | 1858 | 0.25  |
| \( N^*(1680) \) | \( \frac{5}{2}^+ \) | 1689 | 0.89  | 1728 | 0.83  | 1858 | 0.70  |

While the \( g_A \) of the nucleon as predicted by the RCQM falls well into the interval of values reported from lattice QCD, it is nevertheless smaller than the experimental value of ~1.27, supposed under the conjecture of CVC. It is not yet clear what is the reason for this undershooting of the nucleon’s \( g_A \) by the RCQM. For example, it could well be that CVC is violated up to 10% or the assumption of \( g_A^q = 1 \) for the constituent quarks is not justified. Further investigations are necessary to clarify these questions.

It is particularly satisfying to find the RCQM predictions for the axial charges not only of the nucleon \( N \) but also of the \( N^*(1535) \) and \( N^*(1650) \) resonances in agreement with the lattice-QCD results. We may thus be confident that
at least in the limit of zero momentum-transfer processes the mass eigenstates of the $N$ and the above $N^*$ excitations, especially as produced with the EGBE RCQM, are quite reasonable. It should be recalled that in this particular model the mutual interaction between constituent quarks is furnished by a linear confinement, whose strength is consistent with the string tension of QCD as well as the slopes of Regge trajectories \cite{18}, and by (all components of) a hyperfine interaction derived from GBE dynamics. The latter is supposed to model the SBχS property of low-energy QCD. This type of hyperfine interaction, which also introduces an explicit flavor dependence, has been remarkably successful in describing a number of phenomena in low-energy baryon physics. Most prominently, it produces the correct level orderings of the positive- and negative-parity $N^*$ resonances and simultaneously the ones in the other hyperon spectra, notably the Λ spectrum \cite{28}. The RCQM with GBE dynamics does not have any mechanism for chiral-symmetry restoration built in. As such it cannot be expected to produce parity doublets due to this reason. Nevertheless the GBE RCQM describes the pattern of $N^*$ resonance masses in good agreement with the experimental data (mostly within the experimental error bars or at most exceeding them by 4%). This is due to the refined interplay of different force components in the effective interaction between constituent quarks. In view of these findings it will be most interesting if the present results for $N^*$ axial charges derived within the RCQM will in the future be confirmed by lattice-QCD calculations.

Acknowledgments

This work was supported by the Austrian Science Fund (through the Doctoral Program on Hadrons in Vacuum, Nuclei, and Stars; FWF DK W1203-N08). The authors are grateful to L. Glozman for suggestive ideas and a number of clarifying discussions.

\cite{1} C. Amsler \textit{et al.} [Particle Data Group], Phys. Lett. B 667, 1 (2008).
\cite{2} S. L. Adler, Phys. Rev. Lett. 14, 1051 (1965).
\cite{3} W. I. Weisberger, Phys. Rev. Lett. 14, 1047 (1965).
\cite{4} M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).
\cite{5} L. Y. Glozman, M. Radici, R. F. Wagenbrunn, S. Boffi, W. Klink and W. Plessas, Phys. Lett. B 516, 183 (2001).
\cite{6} S. Boffi, L. Y. Glozman, W. Klink, W. Plessas, M. Radici and R. F. Wagenbrunn, Eur. Phys. J. A 14, 17 (2002).
\cite{7} D. Merten, U. Loring, K. Kretschmar, B. Metsch and H. R. Petry, Eur. Phys. J. A 14, 477 (2002).
\cite{8} V. Bernard, Prog. Part. Nucl. Phys. 60, 82 (2008).
\cite{9} D. Dolgov \textit{et al.} [LHPC collaboration and TXL Collaboration], Phys. Rev. D 66, 034506 (2002).
\cite{10} R. G. Edwards \textit{et al.} [LHPC Collaboration], Phys. Rev. Lett. 96, 052001 (2006).
\cite{11} A. A. Khan \textit{et al.}, Phys. Rev. D 74, 094508 (2006).
\cite{12} T. Yamazaki \textit{et al.} [RBC+UKQCD Collaboration], Phys. Rev. Lett. 100, 171602 (2008).
\cite{13} H. W. Lin, T. Blum, S. Ohta, S. Sasaki and T. Yamazaki, Phys. Rev. D 78, 014505 (2008).
\cite{14} C. Alexandrou \textit{et al.} PoS(LATTICE 2008), 139 (2008); \texttt{arXiv:0811.0724} [hep-lat].
\cite{15} T. T. Takahashi and T. Kunihiro, Phys. Rev. D 78, 011503 (2008).
\cite{16} L. Y. Glozman, Phys. Rept. 444, 1 (2007).
\cite{17} L. Y. Glozman and A. V. Nefediev, Nucl. Phys. A 807, 38 (2008).
\cite{18} L. Y. Glozman, W. Plessas, K. Varga and R. F. Wagenbrunn, Phys. Rev. D 58, 094030 (1998).
\cite{19} K. Glantschnig, R. Kainhofer, W. Plessas, B. Sengl and R. F. Wagenbrunn, Eur. Phys. J. A 23, 507 (2005).
[20] L. Theussl, R. F. Wagenbrunn, B. Desplanques and W. Plessas, Eur. Phys. J. A 12, 91 (2001).
[21] T. Melde, L. Canton, W. Plessas and R. F. Wagenbrunn, Eur. Phys. J. A 25, 97 (2005).
[22] R. F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas and M. Radici, Phys. Lett. B 511, 33 (2001).
[23] K. Berger, R. F. Wagenbrunn and W. Plessas, Phys. Rev. D 70, 094027 (2004).
[24] T. Melde, K. Berger, L. Canton, W. Plessas and R. F. Wagenbrunn, Phys. Rev. D 76, 074020 (2007).
[25] T. Melde, L. Canton and W. Plessas, Phys. Rev. Lett. 102, 132002 (2009).
[26] K. Dannbom, L. Y. Glozman, C. Helminen and D. O. Riska, Nucl. Phys. A 616, 555 (1997).
[27] W. Plessas, Few-Body Syst. Suppl. 15, 139 (2003).
[28] T. Melde, W. Plessas and B. Sengl, Phys. Rev. D 77, 114002 (2008).