THEORETICAL ORBITAL PERIOD DISTRIBUTIONS OF CATACLYSMIC VARIABLES ABOVE THE PERIOD GAP: EFFECTS OF CIRCUMBINARY DISKS

BART WILLEMS
Department of Physics and Astronomy, Northwestern University, Evanston, IL; b-willems@northwestern.edu

RONALD E. TAAM
Department of Physics and Astronomy, Northwestern University, Evanston, IL; and ASIAA/National Tsing Hua University, Theoretical Institute for Advanced Research in Astrophysics, Hsinchu, Taiwan; r-taam@northwestern.edu

ULRICH KOLB
Department of Physics and Astronomy, Open University, Milton Keynes, UK; u.c.kolb@open.ac.uk

GUILLAUME DUBUS
Ecole Polytechnique, Laboratoire Leprince-Ringuet, Palaiseau; and Institut d’Astrophysique de Paris, Paris, France; gd@poly.in2p3.fr

AND

ERIC L. SANDQUIST
Department of Astronomy, San Diego State University, San Diego, CA; erics@mintaka.sdsu.edu

Received 2006 August 25; accepted 2006 October 27

ABSTRACT

Population synthesis tools are used to investigate the population of nonmagnetic cataclysmic variables with unevolved main-sequence–like donors at orbital periods greater than 2.75 hr. In addition to the angular momentum losses associated with gravitational radiation, magnetic braking, and mass loss from the system, we also include the effects of circumbinary disks on the evolution. For a fractional mass input rate into the disk, corresponding to \( \frac{3}{10} \times 10^{-4} \) of the mass transfer rate, the model systems exhibit a bounce at orbital periods greater than 2.75 hr. The simulations reveal that (1) some systems can exist as dwarf novae throughout their lifetime, (2) dwarf novae can evolve into novalike systems, and (3) novalike systems can evolve back into dwarf novae during their postbounce evolution to longer orbital periods. Among these subclasses, novalike cataclysmic variables would be the best candidates to search for circumbinary disks at wavelengths \( \geq 10 \, \mu m \). The theoretical orbital period distribution is in reasonable accord with the combined population of dwarf novae and novalike systems above the period gap, suggesting the possibility that systems with unevolved donors need not detach and evolve below the period gap as in the disrupted magnetic braking model. The resulting population furthermore reveals the possible presence of systems with small mass ratios and a preference of O/Ne/Mg white dwarfs in dwarf nova systems in comparison to novalike systems. The novalike population furthermore shows a lack of systems with high-mass white dwarfs. The importance of observational bias in accounting for the differing populations is examined, and it is shown that an understanding of these effects is necessary in order to confront the theoretical distributions with the observed ones in a meaningful manner.

Subject headings: binaries: close — methods: statistical — novae, cataclysmic variables — stars: evolution

1. INTRODUCTION

Cataclysmic variable (CV) systems are a class of compact binaries in which a main-sequence–like donor star transfers mass via Roche lobe overflow to its white dwarf (WD) companion. A theoretical understanding of their observed orbital period distribution has long been sought, since it offers the possibility of probing the formation and evolution of such systems. Key features in the observed distribution include a dearth of systems between \(~2.2\) and \(~2.8\) hr (known as the period gap) and the existence of a critical period at 77 minutes below which no systems are observed (also known as the period minimum). Furthermore, the number of observed systems above the period gap is found to be comparable to that below the gap.

The majority of mass-losing donor stars in these systems are unevolved, and hence stellar expansion resulting from nuclear evolution is unimportant for the binary evolution of these CVs. As a result, an understanding of the observed orbital period distribution indirectly provides constraints on the rate of orbital angular momentum loss necessary to promote the mass transfer rate. In this regard, Shafter (1992) has suggested that since the observed CV period gap is only clearly defined for dwarf novae (DNe), an investigation of the period distribution of DNe rather than the overall CV period distribution should be used to constrain theories of mass transfer. In this picture, the outburst phenomena, interpreted in terms of thermal instabilities in the accretion disk, can be used to provide constraints on the mass transfer rate as a function of orbital period. As an alternative, Martin & Tout (2005) suggest that in the context of the hibernation picture the ratio of novalike systems (NLs) to DNe as a function of orbital period would provide insight into the relative time spent by a given system in such phases (Shara et al. 1986). These relative times in turn constrain the orbital angular momentum loss rate driving the mass transfer in DNe and NLs.

Observationally, it has been inferred that the mass transfer rates are correlated with the orbital period, with CVs at longer orbital periods characterized by higher rates (e.g., Patterson 1984). As a result, it is generally believed that systems above the period gap are driven primarily by the angular momentum losses associated with a magnetically coupled stellar wind, whereas systems below the gap are primarily driven by the emission of gravitational wave radiation. Thus, understanding the angular momentum loss mechanism
is essential for a proper interpretation of the observed period distribution in terms of CV formation and evolution.

Pioneering work on the application of the rotational evolution of slowly rotating G-type stars (see, e.g., Skumanich 1972) to angular momentum losses in CVs was first carried out by Verbunt & Zwaan (1981). However, it has become increasingly recognized that such a prescription, extrapolated to higher rotation rates, overestimates the angular momentum loss rate of single stars based on both observational (Andronov et al. 2003) and theoretical grounds (see, e.g., Ivanova & Taam 2003). On the other hand, mass transfer rates promoted by lower angular momentum loss rates are in conflict with the conventional model for the period gap in terms of a discontinuous change in magnetic braking when the donor star becomes fully convective (Spruit & Ritter 1983; Rappaport et al. 1983), since a much too narrow gap would be produced (e.g., McDermott & Taam 1989). Furthermore, the viability of this picture has recently been subject to additional uncertainty related to the lack of discontinuous change in magnetic activity in low-mass stars (see, e.g., Donati et al. 2006). In this case of continuous magnetic braking, no gap would be formed, in sharp conflict with the observed orbital period distribution.

In view of this unsatisfactory state, other orbital angular momentum loss mechanisms have been explored for CV evolution, including consequential angular momentum losses associated with mass transfer (Barker & Kolb 2003) and a circumbinary (CB) disk (Spruit & Taam 2001). With respect to the latter angular momentum loss mechanism, Willems et al. (2005, hereafter Paper I) have recently shown that the effect of a CB disk can enhance the angular momentum loss rate sufficiently to increase the theoretical period minimum by 160 minutes for gravitational radiation dominated evolution to the observed value of 77 minutes. In addition, it provides a natural explanation to smear out the number of systems at short orbital periods, thereby avoiding an accumulation of systems at the period minimum, which plagued previous theoretical models. We note that the recent detection of excess infrared emission from the polars EF Eri, V347 Pav, GG Leo, and RX J0154 (Howell et al. 2006) provides observational support for the presence of cool gas (and possibly a CB disk) surrounding CVs.

In this paper, we further examine the effect of CB disks on the CV population by carrying out a theoretical study of systems with orbital periods longer than 2.75 hr. This investigation extends earlier work by incorporating evolutionary sequences of CVs with CB disks guided by calculations reported in Taam et al. (2003) for population synthesis simulations. Thus, this study supplements our recent work on short-period CVs presented in Paper I. We focus on the nonmagnetic CV population, since the evolution of magnetic CVs likely involves processes that we leave out of consideration in our calculations, such as the interaction of the magnetic WD with the magnetosphere of the donor star. This interaction can affect the rate of orbital angular momentum loss driving the mass transfer (e.g., Li et al. 1994) and may be responsible for the absence of a period gap in the subpopulation of magnetic CVs. In §2, we describe the input physics and parameters underlying our population synthesis calculations. The observed period distribution is presented in §3, and our treatment of selection effects for treating DNes and NLSs is described in §4. The present-day CV birthrates and numbers of systems currently populating the Galaxy are discussed in §5. In §§6 and 7, the theoretical orbital period distributions and variations of system properties (mass transfer rate, donor mass, WD mass, and mass ratio) as a function of orbital period resulting from our calculations are presented and described. The issue of CB disk detectability of these systems is considered in §8. In §9, finally, we discuss the implications of our results and give some concluding remarks.

2. INPUT PHYSICS AND PARAMETERS

As outlined in detail in Paper I, we use a hybrid binary population synthesis technique in which the BiSEPS binary population synthesis code (see Willems & Kolb 2002, 2004) is used to construct a population of zero-age CVs, and a state-of-the-art stellar and binary evolution code is used to evolve the population up to the current epoch. For the construction of the zero-age population, the initial primary masses $M_1$ are assumed to be distributed according to an initial mass function similar to that of Kroupa et al. (1993).

The distribution of initial secondary masses $M_2$ is obtained from a power-law distribution for the initial mass ratio $q = M_2/M_1$ or from the same initial mass function as adopted for the primary masses. The distribution of initial orbital separations is assumed to be logarithmically flat. These distributions are supplemented with a constant star formation rate normalized so that one binary with $M_1 > 0.8 M_\odot$ is born each year. The age of the Galaxy is assumed to be 10 Gyr, and all stars are assumed to be formed in binaries. We also limit ourselves to Population I chemical compositions. The main evolutionary processes affecting the formation of CVs are the common envelope (CE) phase leading to the formation of the WD and the orbital angular momentum losses driving the post-CE binary to the CV stage.

As is customary, we model the CE phase by equating the binding energy of the envelope with the energy lost from the orbit due to frictional forces as (Tutukov & Yungelson 1979; Webbink 1984)

$$ G(M_1 + M_2)M_e \alpha_{CE} \frac{\lambda_{CE} R_{RL}}{2a_f} = G(M_1 M_2) \frac{M_2}{2a_i} - G(M_1 + M_2) \frac{M_2}{2a_i} \cdot (1) $$

Here $G$ is the Newtonian gravitational constant; $M_1$, $M_2$, and $R_{RL}$ are the core mass, envelope mass, and Roche lobe radius, respectively, of the WD progenitor at the start of the CE phase; $M_2$ is the mass of the companion star; and $a_i$ and $a_f$ are the initial and final orbital separations at the start and at the end of the CE phase, respectively. Moreover, $\lambda_{CE}$ determines the binding energy of the envelope and $\alpha_{CE}$ the fraction of the orbital energy transferred to the envelope. As in Paper I, we investigate the sensitivity of our results to the modeling of the CE phase by considering different prescriptions for the determination of $\alpha_{CE}$ and $\lambda_{CE}$. In our standard population synthesis model (model A), we set $\alpha_{CE} = 0.5$. The other prescriptions are summarized in Table 1 (with RLO denoting the Roche lobe overflow phase) and described in detail in Paper I.

After the CE phase, orbital angular momentum losses due to magnetic braking and/or gravitational radiation drive the systems toward the start of the CV phase. Since the primary aim of this paper is to investigate the effects of orbital angular momentum losses due to CB disks, we adopt a simplified prescription assuming a constant magnetic braking timescale of 10 Gyr for all
main-sequence stars with masses between 0.35 and 1.25 \( M_\odot \). For stars less massive than 0.35 \( M_\odot \) or more massive than 1.25 \( M_\odot \), magnetic braking is assumed to be ineffective. We note that once mass transfer starts in the majority of cases the CB disk dominates the angular momentum losses by the point in the evolution when the donor is less massive than 0.35 \( M_\odot \). To determine the sensitivity of the population synthesis to the magnetic braking timescale, we explore in §9 a simulation for which the timescale is reduced to \( 4 \times 10^9 \) yr.

The evolution of the binaries after the onset of the CV phase is followed using detailed evolutionary tracks obtained from a full binary evolution code described in Taam et al. (2003). The models adopted for the donor stars use solar abundances. The CV phases are furthermore modeled assuming fully nonconservative mass transfer as a result of efficient mass loss during nova explosions. The mass of the WD is then fixed throughout the evolution of the CVs. Based upon the work of Taam et al. (2003), we require the fractional amount of mass deposited in the CB disk, \( \delta \), to be higher for systems above the period gap than below the period gap. The evolution of the CB disk and its implementation into the binary evolutionary code is described in Dubus et al. (2002) and Taam et al. (2003). Hence, we adopt the same model as in Paper I except that we consider CVs with orbital periods longer than 2.75 hr and, guided by the results of Taam et al. (2003), \( \delta = 3 \times 10^{-4} \). For further details on the computational technique and the adopted input physics, we refer to Paper I.

3. THE OBSERVED ORBITAL PERIOD DISTRIBUTION ABOVE 2.75 HR

Before presenting the results of the theoretical population synthesis calculations, we briefly discuss the main features of the observed orbital period distribution of nonmagnetic CVs with periods longer than 2.75 hr. For this purpose, we extract all DNe and NLs from the 2006 January edition (RKCat 7.6) of the Ritter & Kolb (2003) catalog of cataclysmic binaries, low-mass X-ray binaries, and related objects. The resulting orbital period distribution of nonmagnetic CVs is shown in Figure 1a. Its subdivision into DNe and NLs is shown in Figures 1b and 1c, respectively. In each panel the number in parentheses indicates the total number of systems in the sample.

The distribution of nonmagnetic CVs decreases rapidly with decreasing orbital periods between 160 and 180 minutes, corresponding to the upper edge of the period gap. It also shows a broad maximum at periods of 190–240 minutes and a slowly decreasing tail at periods longer than 240 minutes. For the purpose of this paper, we restrict the observed CV population to systems with periods of less than 400 minutes. Systems with longer orbital periods typically have significantly evolved donor stars (see Baraffe et al. 1998). When DNe and NLs are considered separately, two considerably different distributions emerge. The DN population exhibits three peaks of decreasing height at periods of 240, 300, and 380 minutes. The number of systems contributing to the distribution is, however, rather limited. Whether these peaks are representative of the full population or a consequence of small number statistics therefore remains to be confirmed by future observations. For this reason, we focus the population synthesis calculations on explaining the global increase in the number of systems with decreasing orbital periods from 400 to 240 minutes and the subsequent decline in the number of systems for periods smaller than 240 minutes.

The NL population, on the other hand, shows a broad peak in the orbital period distribution at periods of 190–220 minutes. The distribution decreases rapidly on both sides of the peak and possibly shows a hint of a secondary peak at 340 minutes. The statistics of the sample near this period are, however, again not sufficient to conclusively confirm or refute the presence of the peak.

4. OBSERVATIONAL SELECTION EFFECTS

The inherent differences between the populations of DNe (transient systems spending most of their lifetime in quiescence) and NLs (persistently bright systems) likely subject them to considerably different observational selection effects. In order to cope with these, we separate the CVs in the population synthesis calculations into transient and persistent sources according to the critical mass transfer rate for optically thick accretion disks derived by Hameury et al. (1998):

\[
\dot{M}_{\text{crit}} = 9.5 \times 10^{15} \alpha^{0.01} \left( \frac{M_{\text{WD}}}{M_\odot} \right)^{-0.89} \left( \frac{r_{\text{disk}}}{10^{10} \text{ cm}} \right)^{2.68} \text{ g s}^{-1}.
\]

Here \( \alpha \) is the viscosity parameter, \( M_{\text{WD}} \) is the mass of the WD, and \( r_{\text{disk}} \) is the radius of the accretion disk. In our calculations, we set \( \alpha = 0.01 \) and \( r_{\text{disk}} = \frac{1}{2} R_{\text{RL,WD}} \), where \( R_{\text{RL,WD}} \) is the Roche lobe radius of the WD. Systems with mass transfer rates below the resulting \( \dot{M}_{\text{crit}} \) then correspond to DNe, while systems with mass transfer rates above \( \dot{M}_{\text{crit}} \) correspond to NLs.

In Paper I, we modeled the selection effects by biasing the detection of DNe and assuming an isotropic distribution of systems in the Galactic disk. For a bolometric luminosity limited sample, the total observable volume then scales as the accretion luminosity \( L_{\text{acc}} \) raised to the power 1.5. Dwarf novae above the period gap can, however, reach outburst luminosities high enough to make them visible at distances large enough for the finite scale height of

![Figure 1](image-url)
the Galactic disk to affect their observed spatial distribution. The assumption of an isotropic distribution is then more appropriately replaced by an axisymmetric distribution limited by a fixed scale height vertical to the Galactic plane, so that the total observable volume for a bolometric luminosity limited sample is linearly proportional to $L_{\text{acc}}$. For simplicity, we consider both possible observational selection factors and compare the theoretical orbital period distributions obtained by weighting the contribution of each system to the population according to $L_{\text{acc}}$ with the distributions obtained by weighting the contribution of each system according to $L_{\text{vis}}$. The DN accretion luminosity is determined as

$$L_{\text{acc}} = \frac{GM_{\text{WD}} M_{\text{out}}}{R_{\text{WD}}} ,$$

where $M_{\text{out}}$ is the mass accretion rate during outbursts, and $R_{\text{WD}}$ is the radius of the WD. We approximate $M_{\text{out}}$ as

$$M_{\text{out}} = \frac{\dot{M}_d}{d},$$

where $\dot{M}_d$ is the secular mean mass transfer rate, and $d = t_{\text{out}}/t_{\text{rec}}$ is the duty cycle determined as the ratio of the duration ($t_{\text{out}}$) and the recurrence time ($t_{\text{rec}}$) of the outbursts. Since the discovery of DNe likely depends on the duration and recurrence time of the outbursts in addition to the magnitude of the accretion luminosity, we also weight the contribution of each system to the population according to its duty cycle $d$. The total weighting factor applied to each system is thus given by $W = dL_{\text{acc}}$ or $W = dL_{\text{acc}}^{1.5}$. For simplicity, we adopt a constant duty cycle, $d = 0.1$, for the majority of the population synthesis models. To determine the sensitivity of this choice we also consider a limited number of models with variable duty cycle $d = M_d/M_{\text{crit}}$. In the case in which $W = dL_{\text{acc}}$ the weighting factor in these models reduces to $W = GM_{\text{WD}} M_d/R_{\text{WD}}$, which is independent of $d$ and varies continuously from transient to persistent systems (see below).

As an alternative weighting factor, we also explore the effect of replacing the accretion luminosity $L_{\text{acc}}$ with the absolute visual luminosity $L_{\text{vis}}$ determined from the empirical $L_{\text{vis}}$-$P_{\text{orb}}$ relations, where $P_{\text{orb}}$ is the orbital period, derived by Warner (1987) or Harrison et al. (2004).

Novalike CVs, on the other hand, are persistently bright and Novalike CVs, on the other hand, are persistently bright and thus always tend to be visible up to distances large enough for the finite scale height of the Galactic disk to affect their distribution. As argued above, the total observable volume for a bolometric luminosity limited sample then scales as $L_{\text{acc}}$, where $L_{\text{acc}}$ is now given by equations (3) and (4) with $d = 1$. We therefore model the selection effects operating on NLs by weighting the contribution of each system to the population by a factor proportional

---

**Table 2: Present-Day Birthrates of Galactic CVs Forming with Orbital Periods Longer than 2.75 hr**

| Model | He WD | C/O WD | O/Ne/Mg WD | Total | CV Birthrates below 2.75 hr |
|-------|-------|--------|------------|-------|-----------------------------|
| $n(q) \propto q, 0 < q \leq 1$ | | | | | |
| A………………… 2.9 × 10^5 (7.1%) | 3.8 × 10^4 (92.4%) | 1.9 × 10^3 (0.5%) | 4.1 × 10^6 | 4.1 × 10^6 |
| CE1……………. 4.5 × 10^5 (10.3%) | 3.9 × 10^4 (89.7%) | 2.1 × 10^3 (<0.1%) | 4.3 × 10^6 | 4.3 × 10^6 |
| CE8……………. 4.2 × 10^5 (0.2%) | 2.0 × 10^4 (99.0%) | 1.6 × 10^3 (0.8%) | 2.0 × 10^6 | 2.0 × 10^6 |
| DCE1………….. 2.9 × 10^5 (13.6%) | 1.9 × 10^4 (85.7%) | 1.4 × 10^3 (0.7%) | 2.2 × 10^6 | 2.2 × 10^6 |
| DCE5………….. 2.9 × 10^5 (9.2%) | 2.9 × 10^4 (90.1%) | 2.2 × 10^3 (0.7%) | 3.2 × 10^6 | 3.2 × 10^6 |

| $n(q) = 1, 0 < q \leq 1$ | | | | | |
| A………………… 5.9 × 10^5 (8.1%) | 6.7 × 10^4 (90.8%) | 8.6 × 10^3 (1.2%) | 7.3 × 10^6 | 7.3 × 10^6 |
| CE1……………. 6.8 × 10^5 (12.1%) | 4.9 × 10^4 (87.9%) | 6.3 × 10^3 (<0.1%) | 5.6 × 10^6 | 5.6 × 10^6 |
| CE8……………. 1.0 × 10^5 (0.3%) | 3.9 × 10^4 (98.0%) | 7.2 × 10^3 (1.8%) | 4.0 × 10^6 | 4.0 × 10^6 |
| DCE1………….. 5.9 × 10^5 (14.0%) | 3.6 × 10^4 (84.4%) | 6.5 × 10^3 (1.5%) | 4.2 × 10^6 | 4.2 × 10^6 |
| DCE5………….. 5.9 × 10^5 (9.4%) | 5.6 × 10^4 (89.0%) | 9.6 × 10^3 (1.8%) | 6.2 × 10^6 | 6.2 × 10^6 |

| $n(q) \propto q^{0.99}, 0 < q \leq 1$ | | | | | |
| A………………… 2.4 × 10^5 (8.0%) | 2.7 × 10^4 (89.1%) | 8.7 × 10^3 (2.9%) | 3.0 × 10^6 | 3.0 × 10^6 |
| CE1……………. 2.1 × 10^5 (13.2%) | 1.4 × 10^4 (86.8%) | 3.7 × 10^3 (<0.1%) | 1.6 × 10^6 | 1.6 × 10^6 |
| CE8……………. 4.8 × 10^4 (0.3%) | 1.8 × 10^4 (95.9%) | 7.1 × 10^3 (3.8%) | 1.9 × 10^6 | 1.9 × 10^6 |
| DCE1………….. 2.4 × 10^5 (12.6%) | 1.6 × 10^4 (83.9%) | 6.5 × 10^3 (3.5%) | 1.9 × 10^6 | 1.9 × 10^6 |
| DCE5………….. 2.4 × 10^5 (8.6%) | 2.4 × 10^4 (88.0%) | 9.6 × 10^3 (3.5%) | 2.8 × 10^6 | 2.8 × 10^6 |

$M_d$ from the Same IMF as $M_1$

| Model | He WD | C/O WD | O/Ne/Mg WD | Total | CV Birthrates below 2.75 hr |
|-------|-------|--------|------------|-------|-----------------------------|
| A………………… 9.4 × 10^5 (10.7%) | 7.6 × 10^4 (86.6%) | 2.4 × 10^5 (2.8%) | 8.8 × 10^6 | 8.8 × 10^6 |
| CE1……………. 8.1 × 10^5 (17.1%) | 3.9 × 10^4 (82.9%) | 5.0 × 10^3 (<0.1%) | 4.7 × 10^6 | 4.7 × 10^6 |
| CE8……………. 1.9 × 10^5 (0.4%) | 5.1 × 10^4 (96.2%) | 1.8 × 10^3 (3.5%) | 5.3 × 10^6 | 5.3 × 10^6 |
| DCE1………….. 9.4 × 10^5 (16.5%) | 4.6 × 10^4 (80.4%) | 1.8 × 10^3 (3.1%) | 5.7 × 10^6 | 5.7 × 10^6 |
| DCE5………….. 9.3 × 10^5 (11.1%) | 7.2 × 10^4 (85.7%) | 2.6 × 10^3 (3.1%) | 8.4 × 10^6 | 8.4 × 10^6 |

Notes.—Shown are the present-day birthrates (in units of numbers of systems per Gyr) of CVs forming at orbital periods longer than 2.75 hr and their decomposition according to the type of WD in the system. The fractions of systems forming with different types of WDs are indicated in parentheses. The rates can be converted into approximate local birthrates by dividing them by $5 \times 10^{11}$ pc$^{-3}$ (see, e.g., Willems & Kolb 2004). For ease of comparison the last column shows the birthrates of CVs forming with orbital periods shorter than 2.75 hr, taken from Table 2 of Paper I. (In Paper I, we artificially increased the birthrates of systems with C/O WDs forming at 2.25 hr by a factor of 100 in order to get a sufficiently steep lower edge of the period gap in the theoretical orbital period distributions. This increases the total birthrates of systems forming at periods shorter than 2.75 hr by less than a factor of 3 compared to the birthrates listed in the last column of the table.)
to \( L_{\text{acc}} = GM_{\text{WD}} \dot{M}_d / R_{\text{WD}} \). A similar weighting factor is obtained if one adopts a visual magnitude limited sample and uses power-law fits to the bolometric correction to relate the visual magnitude to the bolometric luminosity and the distance of the source from the Sun. (Dünhuber 1994; Kolb 1996).

5. PRESENT-DAY GALACTIC CV POPULATION

5.1. Birthrates

The characteristics of the present-day birthrate of zero-age CVs as a function of the orbital period have been discussed extensively in Paper I. The dependence of the birthrate on the orbital period was found to be in good agreement with that derived by de Kool (1992) and Politano (1996). Here we therefore limit ourselves to the presentation of the numbers of CVs currently forming at periods above 2.75 hr, as well as its subdivision into the numbers of CVs forming with He, C/O, and O/Ne/Mg WDs. These birthrates are listed in Table 2, which complements the present-day birthrates of CVs forming at periods below 2.75 hr listed in Table 2 of Paper I. For ease of comparison, the latter birthrates are repeated in the last column of Table 2.

The total number of CVs forming at orbital periods longer than 2.75 hr is of the order of \( 10^{6} \) to \( 10^{7} \) yr\(^{-1} \) for all initial mass ratio or initial secondary mass distributions considered, except for \( n(q) \propto q^{-0.99} \). In the latter case, the present-day birthrate of Galactic CVs is smaller by an order of magnitude because the \( n(q) \propto q^{-0.99} \) distribution favors small initial secondary masses for which survival of the CE phase is much more difficult than for higher initial secondary masses.

The relative contributions of systems containing He, C/O, and O/Ne/Mg WDs to the zero-age CV population above 2.75 hr depend strongly on the adopted CE model and only weakly on the adopted initial mass ratio or initial secondary mass distribution. The relative number of systems born with He WDs and periods longer than 2.75 hr is typically of the order of 10%–15%, except for model CE8, in which the post-CE orbital separation of systems containing He WDs is often too large for them to evolve into a CV within the lifetime of the Galaxy. For model CE8, the He WD systems therefore constitute only a few tenths of a percent of the zero-age CV population. Systems containing C/O WDs, on the other hand, make up 80%–90% (95%–99% in the case of population synthesis model CE8) of the population of newborn CVs above periods of 2.75 hr. Systems containing O/Ne/Mg WDs, finally, are generally the least abundant group, constituting at most a few percent of the zero-age population. Their contribution to the population is almost completely negligible for model CE1 in which \( \alpha_{\text{CE}} / \alpha_{\text{CE}} \) is too small to successfully eject the WD progenitor’s massive envelope from the system.

Adding the birthrates listed in Table 2 to the birthrates derived in Paper I for CVs forming at periods below 2.75 hr yields total birthrates of Galactic CVs that are in excellent agreement with those derived by de Kool (1992), Politano (1996), and Hurley et al. (2002).

5.2. Population Statistics

In Table 3, we list the total number of DNe and NLs with orbital periods longer than 2.75 hr currently populating the Galaxy, as well as their decomposition according to the type of the WD accretor. The number of DNe and NLs is typically of the order of \( 10^5 \) to \( 10^6 \), with the DNe being a factor of 2–3 more abundant than NLs. The total number of nonmagnetic CVs (DNe + NLs) with periods above 2.75 hr is furthermore about an order of magnitude smaller than the total number of nonmagnetic CVs with periods below 2.75 hr (\( \sim 10^6 \) to \( 10^7 \); see Table 3 in Paper I).

In accord with the CV birthrates, the majority (\( \sim 90\% \)) of the present-day population of DNe and NLs consists of systems containing C/O WDs. Systems containing He WDs typically comprise less than 10% of the population due to mass transfer stability. The largest differences between the DN and NL populations occur for the O/Ne/Mg WD systems, which are 1–2 orders of magnitude more abundant in the DN population than in the NL population. This is directly related to two factors. First, for a given mass donor, the secular mean mass transfer \( \dot{M}_d \) decreases with decreasing mass ratio \( q = M_d / M_{\text{WD}} \), where \( M_d \) is the mass of the donor and \( M_{\text{WD}} \) the mass of the WD. Second, the critical mass transfer rate \( \dot{M}_{\text{crit}} \) increases rapidly with the increasing size of the white dwarf Roche lobe and, therefore, with decreasing \( q \). It is also interesting to note that the contribution of He WD systems to the population of DNe shows a strong dependence on the adopted CE model and only a weak dependence on the adopted initial mass ratio or initial secondary mass distribution, while the contribution of O/Ne/Mg WD systems shows a strong dependence on the adopted initial mass ratio or initial secondary mass distribution but not on the adopted CE model (except for model CE1).

The typical evolution of a CV surrounded by a CB disk starts off with an evolution from longer to shorter orbital periods driven by orbital angular momentum losses, followed by an evolution from shorter to longer orbital periods as the donor star is significantly driven out of thermal equilibrium by the mass loss from its surface. The transition from decreasing to increasing orbital periods occurs when \( (\partial \ln R_d) / (\partial \ln M_d) = 1 \), where \( M_d \) and \( R_d \) are the mass and radius of the donor star (Taam et al. 2003). The evolution is illustrated in Figure 2 for CVs with \( M_{\text{WD}} = 0.6 \ M_\odot \) and a CB disk mass input rate equal to \( 3 \times 10^{-4} \) times the donor’s mean secular mass transfer rate. The different curves correspond to donor stars with initial masses \( M_d = 0.35, 0.55, 0.75, \) and 0.95 \( M_\odot \). The “bounce” period at which the systems start evolving toward longer orbital periods is seen to increase with increasing initial donor mass due to the dependence of the orbital angular momentum loss rate on the CB disk mass (which in turn depends on the initial donor mass).

Hence, when CB disks are incorporated in the evolution of CVs, the population of systems above the period gap can be divided into systems evolving from long to short orbital periods (prebounce systems) and systems evolving from short to long orbital periods (postbounce systems). The relative number of pre- and postbounce systems and their decomposition into systems containing He, C/O, and O/Ne/Mg WDs are listed as percentages in Table 4. In general, the DN population consists of more prebounce than postbounce systems, while the NL population consists of more postbounce than prebounce systems. This reflects the fact that during the early phases of mass transfer, all systems have mass transfer rates low enough to be transients, while during the later phases only some have mass transfer rates low enough to be transients. Our models furthermore predict that almost all DNe and NLs containing He WDs should be postbounce rather than prebounce systems. This follows from the narrow range of initial donor masses (\( \sim 0.35–0.45 \ M_\odot \)) giving rise to stable mass transfer in systems containing He WDs above the upper edge of the period gap and the bounce of these systems subsequent to the onset of mass transfer (see Fig. 2).

6. THEORETICAL ORBITAL PERIOD DISTRIBUTIONS

As in Paper I, we present the results of the CV population synthesis calculations in two steps: we discuss first the intrinsic present-day population and next the theoretically expected observed present-day population. Both steps focus on CVs forming at periods longer than 2.75 hr and neglect any systems formed below 2.75 hr that are evolving to longer orbital periods (based on the results of
Paper I, less than 2% of the CVs forming below 2.75 hr evolve to periods longer than 2.75 hr). For ease of comparison, all distributions presented in the following subsections are normalized to unity.

6.1. The Intrinsic Orbital Period Distribution

We first consider the present-day CV population obtained by evolving the zero-age population up to the current epoch. At this stage, we do not yet account for any observational selection effects that may be affecting the observed orbital period distribution. The selection effects are treated separately and discussed in detail in § 6.2.

The resulting probability distribution functions (PDFs) and cumulative distribution functions (CDFs) of the orbital periods of the combined population of DNe and NLs (i.e., all nonmagnetic CVs) evolving under the influence of a CB disk are shown in Figure 3. For comparison, the observed orbital period distribution is shown by means of a thick solid line. The degree of agreement or disagreement between the theoretical and observed orbital period distributions is indicated by the Kolmogorov-Smirnov (KS) significance level $\sigma_{KS}$ in the CDF panels of Figure 3 (right). Here $\sigma_{KS} = 0$ indicates large differences between the theoretical and observed distributions, while $\sigma_{KS} = 1$ indicates excellent agreement between the two distributions. The results for the various population synthesis models listed in Table 1 are all qualitatively similar to those of population synthesis model A, regardless of the adopted initial mass ratio distribution. An initial secondary mass distribution equal to the initial primary mass distribution furthermore yields orbital period distributions similar to those resulting from the initial mass ratio distribution $n(q) \propto q^{-0.99}$. In this and the following sections, we therefore restrict the presentation and discussion of the theoretical orbital period distributions to the PDFs and CDFs obtained for population synthesis model A and the initial mass ratio distributions $n(q) \propto q^{-0.99}$, $n(q) = 1$, and $n(q) \propto q$.

For a flat initial mass ratio distribution $n(q) = 1$, the theoretical orbital period distribution of nonmagnetic CVs increases smoothly with decreasing period from 400 to 240 minutes and shows a broad peak at periods of 180–240 minutes. The same applies to the theoretical orbital period distribution obtained for

### Table 3: Total Number of DNe and NLs with Orbital Periods Longer than 2.75 hr Currently Populating the Galactic Disk

| Model   | He WD | C/O WD | O/Ne/Mg WD | Total |
|---------|-------|--------|------------|-------|
| A....... | $1.6 \times 10^4 (2.2\%)$ | $6.9 \times 10^3 (96.7\%)$ | $8.2 \times 10^3 (1.1\%)$ | $7.2 \times 10^5$ |
| CE1     | $3.4 \times 10^4 (5.1\%)$ | $6.2 \times 10^3 (94.9\%)$ | $0.0 \times 10^0 (0.0\%)$ | $6.5 \times 10^5$ |
| CE5     | $<0.1\%$ | $3.9 \times 10^3 (98.8\%)$ | $6.1 \times 10^3 (1.6\%)$ | $3.9 \times 10^5$ |
| DCE1..... | $1.6 \times 10^4 (4.0\%)$ | $3.7 \times 10^3 (94.4\%)$ | $6.0 \times 10^3 (1.5\%)$ | $3.9 \times 10^5$ |
| DCE5..... | $1.6 \times 10^4 (2.4\%)$ | $6.2 \times 10^3 (96.2\%)$ | $9.0 \times 10^3 (1.4\%)$ | $6.4 \times 10^5$ |

$\sigma_{KS} \left( q, 0 < q \leq 1 \right)$

| Model   | He WD | C/O WD | O/Ne/Mg WD | Total |
|---------|-------|--------|------------|-------|
| A....... | $2.9 \times 10^4 (1.9\%)$ | $1.5 \times 10^5 (95.4\%)$ | $4.2 \times 10^4 (2.7\%)$ | $1.5 \times 10^6$ |
| CE1     | $5.1 \times 10^4 (5.3\%)$ | $9.1 \times 10^3 (94.7\%)$ | $0.0 \times 10^0 (0.0\%)$ | $9.6 \times 10^5$ |
| CE5     | $<0.1\%$ | $9.2 \times 10^3 (96.6\%)$ | $3.3 \times 10^4 (3.4\%)$ | $9.5 \times 10^5$ |
| DCE1..... | $2.9 \times 10^4 (3.2\%)$ | $8.6 \times 10^3 (93.4\%)$ | $3.1 \times 10^4 (3.9\%)$ | $9.2 \times 10^5$ |
| DCE5..... | $2.9 \times 10^4 (2.0\%)$ | $1.4 \times 10^5 (94.8\%)$ | $4.7 \times 10^4 (3.2\%)$ | $1.5 \times 10^5$ |

$\sigma_{KS} \left( q, 0 < q \leq 1 \right)$

| Model   | He WD | C/O WD | O/Ne/Mg WD | Total |
|---------|-------|--------|------------|-------|
| A....... | $1.1 \times 10^5 (1.4\%)$ | $7.1 \times 10^4 (92.5\%)$ | $4.7 \times 10^3 (6.1\%)$ | $7.7 \times 10^4$ |
| CE1     | $1.5 \times 10^5 (5.0\%)$ | $2.9 \times 10^4 (95.0\%)$ | $0.0 \times 10^0 (0.0\%)$ | $3.0 \times 10^4$ |
| CE5     | $<0.1\%$ | $4.9 \times 10^4 (93.1\%)$ | $3.6 \times 10^3 (6.9\%)$ | $5.3 \times 10^5$ |
| DCE1..... | $1.1 \times 10^5 (2.2\%)$ | $4.5 \times 10^4 (90.7\%)$ | $3.5 \times 10^3 (7.1\%)$ | $5.0 \times 10^4$ |
| DCE5..... | $1.1 \times 10^5 (1.4\%)$ | $7.0 \times 10^4 (91.9\%)$ | $5.1 \times 10^3 (6.7\%)$ | $7.6 \times 10^4$ |

$M_2$ from the same IMF as $M_1$

| Model   | He WD | C/O WD | O/Ne/Mg WD | Total |
|---------|-------|--------|------------|-------|
| A....... | $4.2 \times 10^4 (1.8\%)$ | $2.2 \times 10^5 (92.5\%)$ | $1.4 \times 10^5 (5.7\%)$ | $2.4 \times 10^6$ |
| CE1     | $5.8 \times 10^4 (6.1\%)$ | $9.0 \times 10^3 (93.9\%)$ | $0.0 \times 10^0 (0.0\%)$ | $9.6 \times 10^5$ |
| CE5     | $<0.1\%$ | $1.6 \times 10^5 (93.7\%)$ | $1.0 \times 10^3 (6.3\%)$ | $1.7 \times 10^6$ |
| DCE1..... | $4.2 \times 10^4 (2.7\%)$ | $1.4 \times 10^5 (90.9\%)$ | $1.0 \times 10^2 (6.5\%)$ | $1.6 \times 10^6$ |
| DCE5..... | $4.2 \times 10^4 (1.7\%)$ | $2.2 \times 10^5 (92.0\%)$ | $1.5 \times 10^2 (6.5\%)$ | $2.4 \times 10^6$ |

Notes.—Shown are the total number of DNe and NLs with orbital periods longer than 2.75 hr currently populating the Galactic disk. The numbers only reflect systems that were formed above 2.75 hr and thus do not account for systems evolving through the period gap from periods below 2.75 hr. The relative contributions of systems with He, C/O, and O/Ne/Mg WDs to the population are indicated in parentheses. Approximate local space densities can be obtained by dividing the absolute numbers of systems by $5 \times 10^{11}$ pc$^{-3}$ (see Willems & Kolb 2004). There is a slight mismatch between the criterion separating systems with evolved donor stars from systems with unevolved donor stars in the BiSEPS binary population synthesis code used to construct the zero-age CV populations and the full binary evolution code used to construct the CV period distributions is indicated by the Kolmogorov-Smirnov (KS) test.
n(q) \propto q$, except that the peak is replaced by a cutoff at 160 minutes. For $n(q) \propto q^{-0.99}$, the simulated PDF reaches a maximum at $\sim 170$ minutes. All three theoretical distributions furthermore show a local plateau at $P_{orb} \approx 300$ minutes.

In accord with the relative numbers of systems discussed in the previous section, the theoretical orbital period distributions are dominated by systems containing C/O WDs. Systems containing He WD accretors provide a small contribution at periods of 180–200 minutes. A small contribution from systems containing O/Ne/Mg WDs is also visible when $n(q) \propto q^{-0.99}$.

The PDFs and CDFs of the separate DN and NL CV populations are shown in the left and right panels of Figure 4, respectively. For $n(q) = 1$ and $n(q) \propto q^{-0.99}$, the orbital period distribution of DNe displays a rapid increase in the number of systems with decreasing period from 400 to 300 minutes, followed by a much slower increase from 300 to 160 minutes. For $n(q) \propto q$, the simulated orbital period distribution is almost flat between 180 and 300 minutes. The simulated orbital period distributions of Galactic NLs, on the other hand, show a broad peak at 230 minutes with a rapid decline toward shorter orbital periods and a slower decline toward longer orbital periods.

The decomposition of the orbital period distribution of nonmagnetic CVs into distributions for DNe and NLs depends strongly on the adopted critical mass transfer rate separating transient from persistent systems. In Figure 5 we therefore illustrate the effects of uncertainties in $M_{crit}$ on the theoretical DN and NL orbital period distributions.

### TABLE 4

**Intrinsic Fractions of DNe and NLs Forming above 2.75 hr That are Evolving toward Shorter and Longer Orbital Periods**

| MODEL  | He WD C/O WD O/Ne/Mg WD Total     | He WD C/O WD O/Ne/Mg WD Total     |
|--------|----------------------------------|----------------------------------|
|        | $n(q) \propto q, \ 0 < q \leq 1$ | $n(q) \propto q, \ 0 < q \leq 1$ |
| A…….. | <0.1/2.2 67.7/28.9 0.9/0.3 68.6/31.4 | <0.1/3.2 47.9/48.9 0.1/0.1 47.9/52.1 |
| CE1…… | 0.1/5.1 65.4/29.5 0.0/0.0 65.4/34.6 | 0.1/5.2 47.8/47.1 0.0/0.0 47.8/52.2 |
| CE8…… | 0.0/0.0 69.1/29.3 1.2/0.3 70.3/29.7 | 0.0/0.0 48.2/51.7 0.1/0.1 48.2/51.8 |
| DCE1… | 0.1/4.0 66.1/28.2 1.2/0.3 67.4/32.6 | 0.1/6.6 45.1/48.2 0.1/0.1 45.1/54.9 |
| DCE5… | <0.1/2.4 66.4/29.7 1.1/0.3 67.5/32.5 | <0.1/4.7 41.0/54.2 0.1/0.1 41.0/59.0 |
|        | $n(q) = 1, \ 0 < q \leq 1$       | $n(q) = 1, \ 0 < q \leq 1$       |
| A…….. | <0.1/1.9 62.0/33.4 1.9/0.8 63.9/36.1 | <0.1/3.8 43.4/52.7 0.1/0.1 43.4/56.6 |
| CE1…… | 0.1/5.3 61.2/33.4 0.0/0.0 61.3/38.7 | <0.1/6.5 43.3/50.2 0.0/0.0 43.3/56.7 |
| CE8…… | 0.0/0.0 62.3/34.3 2.5/1.0 64.8/35.2 | 0.0/0.0 43.7/56.0 0.1/0.2 43.8/56.2 |
| DCE1… | <0.1/3.2 60.0/33.3 2.4/1.0 62.5/37.5 | <0.1/7.2 40.6/52.0 0.1/0.2 40.7/59.3 |
| DCE5… | <0.1/2.0 60.7/34.2 2.2/1.0 62.9/37.1 | <0.1/5.1 37.3/57.5 0.1/0.1 37.3/62.7 |
|        | $n(q) \propto q^{-0.99}, \ 0 < q \leq 1$ | $n(q) \propto q^{-0.99}, \ 0 < q \leq 1$ |
| A…….. | <0.1/1.4 54.2/38.3 3.7/2.4 57.9/42.1 | <0.1/4.3 38.7/56.8 0.1/0.1 38.8/61.2 |
| CE1…… | 0.1/5.0 56.7/38.2 0.0/0.0 56.8/43.2 | <0.1/7.8 38.6/53.6 0.0/0.0 38.6/61.4 |
| CE8…… | 0.0/0.0 53.8/39.4 4.4/2.4 58.2/41.8 | 0.0/0.0 39.1/56.0 0.1/0.4 39.3/60.7 |
| DCE1… | <0.1/2.2 52.1/38.6 4.5/2.6 56.6/43.4 | <0.1/7.3 36.3/56.0 0.1/0.3 36.4/63.6 |
| DCE5… | <0.1/1.4 53.1/38.8 4.1/2.6 57.2/42.8 | <0.1/5.2 33.8/60.8 0.1/0.1 33.8/66.2 |

$M_2$ from the Same IMF as $M_1$

| A…….. | <0.1/1.7 48.1/44.4 3.0/2.7 51.4/48.9 | <0.1/6.5 33.1/60.3 0.1/0.1 33.1/66.9 |
| CE1…… | 0.1/6.1 50.6/43.2 0.0/0.0 50.7/49.3 | <0.1/11.1 32.4/56.5 0.0/0.0 32.5/67.5 |
| CE8…… | 0.0/0.0 48.0/45.8 3.5/2.7 51.5/48.5 | 0.0/0.0 34.3/56.3 0.1/0.3 34.5/65.5 |
| DCE1… | <0.1/2.6 46.3/44.6 3.6/2.9 49.9/50.1 | <0.1/10.7 30.7/58.2 0.1/0.3 30.8/69.2 |
| DCE5… | <0.1/1.7 47.1/44.9 3.3/2.9 50.4/49.6 | <0.1/7.5 28.6/63.8 0.1/0.1 28.7/71.3 |

**Notes.** Shown are the intrinsic fractions of CVs forming above 2.75 hr that are still evolving toward the period minimum (prebounce systems) and that are evolving away from the period minimum (postbounce systems), and their decomposition according to the type of WD in the system. The fractions are expressed in percent, with the first number in each column corresponding to prebounce systems and the second to postbounce systems.
period distributions by increasing and decreasing $\dot{M}_{\text{crit}}$ by a factor of 3 with respect to the $\dot{M}_{\text{crit}}$ given by equation (2). The solid lines correspond to the orbital period distributions shown in Figure 4 for population synthesis model A and the initial mass ratio distribution $n(q) = 1$. The dash-dotted and dotted lines correspond to the orbital period distributions obtained for critical mass transfer rates that are a factor of 3 smaller and larger, respectively.

Decreasing $\dot{M}_{\text{crit}}$ by a factor of 3 significantly flattens the intrinsic DNe orbital period distribution, while increasing $\dot{M}_{\text{crit}}$ by a factor of 3 broadens the cutoff at 160 minutes to a peak spanning periods from 160 to 240 minutes. For the NL population, decreasing $\dot{M}_{\text{crit}}$ by a factor of 3 shifts the peak at 230 minutes to slightly lower period. Increasing $\dot{M}_{\text{crit}}$ by a factor of 3, on the other hand, shifts the peak in the theoretical NL orbital period distribution to periods of 280–300 minutes.

6.2. The Orbital Period Distribution Corrected for Observational Selection Effects

The present-day orbital period distributions of DN and NL CVs accounting for observational selection effects are shown in Figure 6. The DN distribution is obtained by weighting the contribution of each CV in the population according to the accretion luminosity raised to the power 1.5 and a constant duty cycle $d = 0.1$ (top left), according to the accretion luminosity raised to the power 1.5 and the duty cycle $d = M_d/\dot{M}_{\text{crit}}$ (top right), or according to a weighting factor that is linearly proportional to the accretion luminosity and the duty cycle $d = \dot{M}_d/\dot{M}_{\text{crit}}$ (bottom left). The weighting factor for CVs in the NL population (bottom right) is taken to be linearly proportional to the accretion luminosity.

All three of the observational selection factors considered for DNe diminish the abrupt cutoff in the intrinsic PDFs at $P_{\text{orb}} \approx 160$ minutes and significantly increase the contribution of systems containing O/Ne/Mg WDs to the theoretical orbital period distribution. For $n(q) \propto q^{-0.99}$, the selection factors introduce a local maximum at 190 minutes, which is at too short of an orbital period to explain the peak in the observed orbital period distribution at 240 minutes, while for $n(q) \propto q$, the selection factors introduce a small peak at 300 minutes that coincides with the secondary peak in the observed distribution. The initial mass ratio distribution

Note that for normalized distributions, the actual value of the constant $d$ does not play a role.
The observational selection factors adopted for the NL population do not considerably alter the shape of the orbital period distribution with respect to that of the intrinsic population. The main effects of the selection factors are a narrowing of the peak at 230 minutes and a decrease of the relative contribution of systems containing He WDs. The peak at 230 minutes also shifts to slightly longer orbital periods, which is opposite the trend required to improve the agreement between theory and observations. The deficit of NLs at long orbital periods as compared to the observed distribution likely results from our lack of including nuclear-evolved systems in the population synthesis. As a result, the level of disagreement inferred from the KS measure should not be taken at face value for models in which the maximum deviation between the theoretical and observed orbital period distributions occurs in the long-period tail of the distributions. However, the relative variations of this measure can be used to differentiate between the different models. In this capacity, the KS measure favors the initial mass ratio distribution \( n(q) \propto q^{-0.99} \), which is opposite the initial mass ratio distribution favored by the KS measure for DN-type systems. This apparent disparity in the best-fitting initial mass ratio distributions can possibly be attributed to uncertainties in the binary evolution model.

7. DISTRIBUTIONS OF SYSTEM PROPERTIES AS A FUNCTION OF ORBITAL PERIOD

While the orbital period is by far the most accurately determined parameter of observed CVs and thus best suited to compare theory to observation, it is instructive to also consider the distribution of other system properties such as the mass transfer rate and component masses as a function of the orbital period. As before, a comparison of theoretical with observed distributions requires a model for the observational selection effects. However, as the latter is still subject to considerable uncertainties, we here focus on the properties of the intrinsic theoretical distributions. These distributions may be compared with observation once samples complete within a given volume of space become available.

In Figure 7, we show the intrinsic distributions of the mass transfer rates in the simulated DN and NL CV populations as a function of the orbital period, for population synthesis model A and a flat initial mass ratio distribution \( n(q) = 1 \). Typical mass transfer rates are of the order of \((1-2) \times 10^{-9} M_{\odot} \, yr^{-1} \) for DNe and \((2-6) \times 10^{-9} M_{\odot} \, yr^{-1} \) for NLs, which are comparable to the typical mass transfer rates found by Howell et al. (2001) based on the magnetic braking rate of Rappaport et al. (1983) with \( \gamma = 3 \). In both cases, the rates increase with increasing orbital period, although for the DNe there is also a nonnegligible number of prebounce systems for which the mass transfer rates increase with decreasing orbital period. Accounting for observational selection effects reduces the relative number of DNe with mass transfer rates below \( 10^{-9} M_{\odot} \, yr \) but otherwise does not significantly change the DN and NL (\( M_d, P_{\text{orb}} \)) distributions displayed in Figure 7.

The intrinsic distributions of DN and NL CV donor masses as a function of the orbital period for population synthesis model A and a flat initial mass ratio distribution \( n(q) = 1 \) are shown in Figure 8. The majority of DNe are prebounce systems with donor masses above \( 0.3 \, M_{\odot} \). The NL population on the other hand shows a large number of postbounce systems with donor masses.

\( \text{Note that the PDFs show some signs of the finite number (~1300) of tracks used in the calculations, especially during the early (prebounce) stages of mass transfer. This does not affect the numbers or orbital period distributions presented in the previous sections because the initial (\( M_1, M_2, P_{\text{orb}} \)) parameter space is very well sampled.} \)
below $0.3 \, M_\odot$. Because of the increase of the critical mass transfer rate separating transient from persistent sources with increasing orbital period, no NLs are found with donor masses below $0.1 \, M_\odot$. Due to the period bounce at the upper edge of the period gap, the shape of the distributions is also intrinsically different from the shape of the distributions obtained for CVs evolving under the influence of gravitational radiation, magnetic braking, and mass loss during nova explosions only (see Fig. 5 in Howell et al. 2001). The main effect of adopting observational selection factors is to boost the relative contribution of systems with donor masses $M_d \lesssim 0.3 \, M_\odot$ in the population of DNe. These systems are all postbounce systems and thus have high mass transfer rates and accretion luminosities favoring their detection.

In Figure 9, we show the intrinsic distributions of WD masses in DN and NL CVs as a function of the orbital period for population synthesis model A and a flat initial mass ratio distribution $n(q) = 1$. Due to the period bounce at the upper edge of the period gap, the shape of the distributions is also intrinsically different from the shape of the distributions obtained for CVs evolving under the influence of gravitational radiation, magnetic braking, and mass loss during nova explosions only (see Fig. 5 in Howell et al. 2001). The main effect of adopting observational selection factors is to boost the relative contribution of systems with donor masses $M_d \lesssim 0.3 \, M_\odot$ in the population of DNe. These systems are all postbounce systems and thus have high mass transfer rates and accretion luminosities favoring their detection.
right of the $M_{\text{WD}}-P_{\text{orb}}$ parameter space corresponds to the transition from persistent to transient behavior due to the increase of the critical mass transfer rate with increasing orbital period. Indeed, systems near this boundary are all postbounce systems, so longer orbital periods correspond to lower donor masses. Correspondingly, persistent behavior requires the WD mass to decrease with increasing orbital period.

Figure 10, finally, shows the intrinsic distributions of mass ratios $q = M_d/M_{\text{WD}}$ as a function of the orbital period, for population synthesis model A and a flat initial mass ratio distribution $n(q) = 1$. Typical mass ratios near the upper edge of the period gap are of the order of $q = 0.4-0.7$ for both DNe and NLs. The additional orbital angular momentum losses caused by CB disks and the associated bounce of systems at the upper edge of the period gap furthermore allow for significantly smaller mass ratios for systems above the period gap than the standard gravitational radiation and magnetic braking driven CV evolution (see Figs. 6 and 11 in Howell et al. 2001). The main effect of incorporating observational selection effects is to increase the relative contribution of DNe with mass ratios lower than $\sim 0.3$ to the DN CV population.

8. CIRCUMBINARY DISK DETECTABILITY

As in Paper I, the population synthesis calculations also allow us to predict the typical properties of possible CB disks in DN and NL CVs, such as the fraction $\dot{J}_{\text{CB}}/\dot{J}_{\text{tot}}$ of the total orbital angular momentum loss caused by the CB disk and the mass $M_{\text{CB}}$ contained in the CB disk. The distributions of these quantities as
functions of the orbital period are shown in Figures 11 and 12, for population synthesis model A and the initial mass ratio distribution \( n(q) = 1 \). Accounting for observational selection effects tends to increase the relative contribution of systems with higher CB disk orbital angular momentum loss rates and CB disk masses, but otherwise it does not significantly alter the distributions displayed in Figures 11 and 12.

The orbital angular momentum losses caused by the CB disk dominate the evolution of the systems close to and beyond the period bounce for both DNe and NLs, with systems near the upper edge of the period gap typically losing more than 60% of their orbital angular momentum to the CB disk. The mass contained in the CB disk tends to saturate at \( M_{\text{CB}} \approx (2-3) \times 10^{-4} M_\odot \), about 2 orders of magnitude larger than the typical CB disk masses found in Paper I for systems below the period gap. The total angular momentum \( (GMa)^{1/2} \) is higher for binaries above the period gap; hence, much higher mass transfers into the CB disk are needed to influence the evolution. With the adopted \( \dot{\Omega} = 3 \times 10^{-4} \), \( M_{\text{CB}} \) saturates when about a \( 1 M_\odot \) of material has been transferred from the donor star.

Novalike CVs are probably the best candidates to search for CB disks as most systems are postbounce with large disks. This is not surprising as high mass transfer rates are needed for the accretion disk to be stable, and high mass transfer rates are associated with large angular momentum losses. Therefore, typical CB disk masses tend to be somewhat heavier for NLs \( [M_{\text{CB}} \approx (6-20) \times 10^{-5} M_\odot] \) than for DNe \( [M_{\text{CB}} \approx (2-8) \times 10^{-5} M_\odot] \). The CB disks in NLs moreover have a minimum mass of a few \( \times 10^{-5} M_\odot \).

Circumbinary disks around NLs in the 3–5 hr period range should have optically thick sizes of a few AU, inner surface densities of a few \( \times 10^3 \) g cm\(^{-2}\), and inner effective temperatures.
of 1000–2000 K (Taam et al. 2003). Searches for IR excess (up to 10 μm) or UV absorption have found no evidence for circumbinary material in the NLs IX Vel, V592 Cas, QU Car, and RW Tri (Belle et al. 2004; Dubus et al. 2004). However, CB disks are cold and geometrically thin, so UV absorption requires very favorable lines of sight (high inclinations). Furthermore, the cold CB disk dominates the binary emission only at wavelengths longer than 10 μm, so mid-IR observations may not be sufficient to uncover them.

9. DISCUSSION AND CONCLUSIONS

The population synthesis of nonmagnetic CVs with periods less than 2.75 hr carried out in Paper I has been extended to non-magnetic CVs with periods longer than 2.75 hr. Populations of zero-age CVs have been constructed for a range of different input models, and the secular CV evolution has been calculated using an up-to-date stellar and binary evolution code incorporating orbital angular momentum losses due to gravitational radiation, magnetic braking, mass loss through nova explosions, and CB disks.

Adopting a mass input rate into the CB disk equal to $3 \times 10^{-4}$ times the mass transfer rate leads to mass transfer rates that can vary by more than an order of magnitude at a given orbital period without the introduction of mass transfer cycles (see King et al. 1995) or considering the effect of nova outbursts (Kolb et al. 2001), specifically hibernation (see Shara et al. 1986; Martin & Tout 2005). These rates are sufficiently high ($\sim 10^{-9} M_\odot \text{ yr}^{-1}$) such that the degree of the departure from thermal equilibrium in the donor star causes systems to undergo a bounce at periods of 3–4 hr, thus providing an alternative explanation for the upper edge of the period gap to the standard disrupted magnetic braking model. In this description, systems evolving from longer to shorter orbital periods may spend their entire lifetime as DNe or, after an initial DN phase, become NLs as the evolution accelerates with increasing

![Fig. 11.](image)

Fig. 11.—Normalized distribution of $\dot{J}_{\text{CB}}/\dot{J}_{\text{tot}}$ for DNe (left) and NLs (right) as a function of the orbital period for population synthesis model A and the initial mass ratio distribution $n(q) = 1$, without regard for observational bias. The “fan” of high- and low-density lines at $\dot{J}_{\text{CB}}/\dot{J}_{\text{tot}} \lesssim 0.6$ is due to the finite number of evolutionary tracks underlying the calculation.

![Fig. 12.](image)

Fig. 12.—Same as Fig. 11, but for the mass $M_{\text{CB}}$ contained in the CB disk. The “fan” of high- and low-density lines at $\log M_{\text{CB}}/M_\odot \lesssim -4.5$ in the DN population is due to the finite number of evolutionary tracks underlying the calculation.
CB disk mass. On the other hand, systems evolving from shorter to longer orbital periods eventually always become DNe due to the increase in the critical mass transfer rate separating transient from persistent CV systems. We note that almost all systems containing He WDs belong to this latter group. In addition to the difference in sign of the period derivative between the pre- and postbounce systems and the much smaller donor mass at a given orbital period, the effective temperature of the mass-losing donor could differentiate the former from the latter. Specifically, donors in prebounce systems are characterized by higher effective temperatures than the donors in postbounce systems.

The occurrence of a period bounce also allows binaries above the period gap to reach significantly smaller donor mass to accretor mass ratios than in the standard disrupted magnetic braking model. In particular, the mass ratios can be sufficiently low \( (M_d/M_{\text{WD}} \lesssim 0.3) \) that systems with donors that are not nuclear-evolved can exhibit the superhump phenomena at orbital periods above the period gap. In support of this result, Patterson et al. (2005) have recently reported that the success rate for searching for positive superhump periods in nonmagnetic CVs declines from nearly 100% for short-period systems \( (P_{\text{orb}} \lesssim 2.75 \text{ hr}) \) to about 50% for systems at \( 3.1 \pm 0.2 \text{ hr} \). Upon comparison with our theoretical models, we find about 24% of systems with \( P_{\text{orb}} = 3.1 \pm 0.2 \text{ hr} \) have \( q < 0.3 \). When observational selection effects are taken into account, this fraction increases to 49%–62%, depending on the adopted observational selection factors (see § 4). These bias-corrected fractions are in good agreement with the fractions derived by Patterson et al. (2005).

Additional observational support for the occurrence of superhumps at longer orbital periods in the context of the CB disk model would be provided by the determination of the spectral types of these donors, which should differ from nuclear-evolved stars that have lost mass. On the other hand, our model calculations also reveal that the tail of the distribution of systems characterized by low mass ratios can extend to \( 6–7 \text{ hr} \), whereas so far no positive superhumps were detected in nonmagnetic systems above 4 hr. In particular, our models for the intrinsic CV population predict 15% of the systems with \( P_{\text{orb}} > 4 \text{ hr} \) to have \( q < 0.3 \). However, taking into account observational selection effects increases this fraction to 40%–53%. It is possible that the nondetection of these systems reflects the low amplitudes of the luminosity modulations resulting from the reduced tidal dissipation in the outer accretion disk of long-period systems with very low mass donors.

Patterson et al. (2005) and Knigge (2006) furthermore presented mass and radius estimates of observed CV donor stars in eclipsing systems and systems exhibiting superhumps. In Figure 13, we compare the masses and radii of CV donor stars in the theoretical intrinsic population of DNe and NLs with those of the long-period \( (P_{\text{orb}} > 2.75 \text{ hr}) \) systems considered by Patterson et al. (2005) and Knigge (2006). For the theoretical model, we adopted population synthesis model A and the initial mass ratio distribution \( n(q) = 1 \). We furthermore do not distinguish between systems exhibiting superhumps and systems not exhibiting superhumps in the theoretical population. Considering the uncertainties in the observational determination of donor masses and radii and the omission of observational selection effects in the theoretical model, the theoretical distribution for \( M_d \leq 0.5 M_{\odot} \) is in satisfactory agreement with observation. For \( M_d \geq 0.5 M_{\odot} \), the agreement is less satisfactory, but these systems likely have evolved donor stars (Knigge 2006), which are not included in our present population synthesis model.

The birthrates of zero-age CVs born at periods less than 2.75 hr are typically of the order of \( 10^{-3} \text{ yr}^{-1} \), which, depending on the adopted initial mass ratio distribution and population synthesis model parameters, is up to an order of magnitude larger than the birthrates found in Paper I for CVs born at periods less than 2.75 hr. If the initial secondary mass is assumed to be distributed independently according to the same initial mass function as the primary mass, the number of CVs formed above 2.75 hr is up to a factor of 2 smaller than the number of CVs formed below 2.75 hr. Despite the generally larger birthrates of systems above 2.75 hr, their shorter lifetime due to the higher mass transfer rates results in a smaller number of systems above than below the gap in the theoretical present-day CV populations. In particular, our models predict the present-day population of Galactic CVs to consist of \( 10^5–10^6 \) systems with periods longer than 2.75 hr and \( 10^6–10^7 \) systems with period less than 2.75 hr (Paper I). The number of systems in the two populations thus differs by about an order of magnitude, which is considerably more than the difference in the number of nonmagnetic CVs below and above 2.75 hr in the observed population: the 2006 January edition (KKCat 7.6) of the Ritter & Kolb (2003) catalog contains \( \sim 200 \) nonmagnetic CVs below 2.75 hr and \( \sim 170 \) above 2.75 hr. As systems below and in the period gap have significantly lower mass transfer rates than systems above the period gap, this possibly reflects stronger observational selection effects operating on systems with periods less than 2.75 hr.

The calculations presented in the previous sections are all based on a constant magnetic braking timescale of 10 Gyr. Shorter magnetic braking timescales will decrease the time required for post-CE binaries to evolve into a CV, as well as increase the longest possible post-CE orbital period for which binaries are able to evolve into a CV within the lifetime of the Galaxy. Both of these effects increase the formation rates of zero-age CVs as a function of orbital period. The secular evolution during the CV phase is not only affected during the early stages of mass transfer but also during the phase when the CB disk dominates the evolution as it has been built up more rapidly. In order to assess the effects of a stronger magnetic braking rate on the theoretical DN and NL orbital period distributions without recalculating all \( \sim 1300 \) evolutionary tracks used in the population synthesis calculations, we rescaled the orbital period evolution of the existing tracks to a stronger magnetic braking rate with a characteristic timescale of
by accelerating the time evolution of the tracks by a factor of $1 + 0.5(1 - 0.5\dot{J}_{\text{CB}}/\dot{J}_{\text{tot}})$. The resulting orbital period evolution is in excellent agreement with the $\tau_{\text{MB}} = 10^{10}$ yr evolutionary track, with the minimum attained period differing by less than $\approx 1$ minute.

The population synthesis calculations used to construct the zero-age CV populations incorporate a stability test of mass transfer when the donor first fills its critical Roche lobe. Systems that do not satisfy the test of stability are not followed further and are removed from the population. The stability test is performed in accord with the fully nonconservative approximation adopted in the calculation of the binary evolutionary tracks. However, once mass transfer is initiated, it takes a finite amount of time to build up the accretion disk and evolve toward the first nova explosion. Thus, matter is transferred to the WD accretor and the conservative mass transfer stability criterion may therefore be more appropriate than the nonconservative one in the construction of the zero-age CV population. In order to illustrate the effects of the stability criterion, the theoretical orbital distribution of nonmagnetic CVs obtained assuming conservative mass transfer for the construction of the zero-age CV population is shown in Figure 16. From comparison with Figure 3, it follows that the main effect of the conservative mass transfer stability criterion is to introduce a small bump in the theoretical orbital period distribution that coincides with the secondary peak at $P_{\text{orb}} \approx 300$ minutes in the observed orbital period distribution. This is particularly striking for the initial mass ratio distribution $n(q) \propto q$. The conservative mass transfer stability criterion also increases the relative contribution of systems with O/Ne/Mg WDs for which stability is less of an issue (because the stability of conservative mass transfer requires

**Fig. 14.**—Time evolution of the orbital period of a CV evolving under the influence of a CB disk and a magnetic braking timescale of $\tau_{\text{MB}} = 4 \times 10^9$ yr (solid line) or $\tau_{\text{MB}} = 10^{10}$ yr (dotted line). The initial WD and donor masses are equal to $0.55 M_\odot$ in both cases. The dashed line corresponds to the time evolution of the $\tau_{\text{MB}} = 10^{10}$ yr evolutionary track multiplied by a factor of $1 + 0.5(1 - 0.5\dot{J}_{\text{CB}}/\dot{J}_{\text{tot}})$. The resulting orbital period evolution is in excellent agreement with the $\tau_{\text{MB}} = 4 \times 10^9$ yr evolutionary track, with the minimum attained period differing by less than $\approx 1$ minute.

**Fig. 15.**—Same as Fig. 4, but for a stronger magnetic braking rate with a characteristic timescale of $4 \times 10^9$ yr.
smaller $M_d/M_{\text{WD}}$ ratios than nonconservative mass transfer). Similar tendencies are observed when the DN population is considered separately. The main effect on the NL population is to increase the peak in the theoretical distribution at $P_{\text{orb}} \approx 220$ minutes and shift the cutoff near the upper edge of the period gap from 180 to 200 minutes, which is at somewhat too long of an orbital period compared to the cutoff in the observed orbital period distribution.

Lada (2006) has suggested that the fraction of stars formed in binaries may be substantially lower for stars of spectral type M and later than for stars of earlier spectral types. Such a reduction in the binary fraction can affect the contribution to the orbital period distribution of CVs with donors initially less massive than 0.5 $M_\odot$, which, in our models, spend most of their lifetime at periods shorter than 4 hr. Reducing the binary fraction for low-mass stars by a factor of 2 therefore removes the peak in the theoretical intrinsic DN orbital period distributions at $\approx 170$ minutes (see Fig. 4), rendering the distributions almost completely flat between 170 and 300 minutes. The bias-corrected distributions change to a much smaller extent, since the peak at 170 minutes is already suppressed for these distributions (see Fig. 6). The intrinsic and bias-corrected NL populations are even less affected as NLs typically have larger minimum periods than DNe.

In conclusion, the main features of the nonmagnetic CV orbital period distribution can be understood by incorporating the angular momentum loss mechanism associated with CB disks in the evolution of CVs. The simulations reported here reveal that systems with main-sequence–like donors can undergo a period bounce near the upper edge of the observed orbital period gap at 2.75 hr, producing good agreement with the orbital period distribution of Galactic DNe.

Fig. 16.—Same as Fig. 3, but based on the conservative mass transfer stability criterion instead of the nonconservative one.

Fig. 17.—Normalized orbital period distribution of DNe for population synthesis model A and the initial mass ratio distribution $n(q) = 1$. As in Paper I, the lower edge of the period gap is modeled by increasing the zero-age CV birthrate at 2.25 hr by a factor of 100. Selection effects are taken into account using a detection probability factor $W = dL_{\text{acc}}$. The light gray, dark gray, and black shading indicates the fractions of He, C/O, and O/Ne/Mg WD systems contributing to the population, and the thick solid line represents the observed orbital period distribution of Galactic DNe.
difference in the selection effects for systems above and below the period gap. This is particularly important for the full population, since the mass transfer rates of the dwarf novae vary by more than 1 order of magnitude for these systems.

Finally, we point out that an understanding of the observational selection effects of the DN and the NL populations is necessary in order to properly compare the observed distribution to our simulated theoretical distributions (e.g., Pretorius et al. 2007). A comparison with our intrinsic distributions can fruitfully be undertaken once future surveys complete to within a given volume of space have been attained.

We thank M. Politano for a critical reading of and useful comments on a preprint of this paper. This research was supported in part by the National Science Foundation under grants AST 02-00876, a David and Lucille Packard Foundation Fellowship in Science and Engineering grant, and NASA ATP grant NAG5-13236. Partial support is also acknowledged from the Theoretical Institute for Advanced Research in Astrophysics (TIARA) operated under Academia Sinica and the National Science Council Excellence Projects program in Taiwan administered through grant NSC 95-2752-M-007-006-PAE. Astronomy Research at the Open University is supported by a PPARC Rolling Grant.

REFERENCES

Andronov, N., Pinsonneault, M., & Sills, A. 2003, ApJ, 582, 358
Baraffe, I., & Kolb, U. 2000, MNRAS, 318, 354
Barker, J., & Kolb, U. 2003, MNRAS, 340, 623
Belle, K. E., Sanghi, N., Howell, S. B., Holberg, J. B., & Williams, P. T. 2004, AJ, 128, 448
de Kool, M. 1992, A&A, 261, 188
Donati, J., Forveille, T., Cameron, A., Barnes, J., Delfosse, X., Jardine, M., & Valenti, J. 2006, Science, 311, 633
Dubus, G., Campbell, R., Kern, B., Taam, R. E., & Spruit, H. C. 2004, MNRAS, 349, 869
Dubus, G., Taam, R. E., & Spruit, H. C. 2002, ApJ, 569, 395
Dünhuber, H. 1994, Ph.D. thesis, Ludwig-Maximilians-Univ. München
Hameury, J. M., Menou, K., Dubus, G., Lasota, J. P., & Hure, J. M. 1998, MNRAS, 298, 1048
Harrison, T. E., Johnson, J. J., McArthur, B. E., Benedict, G. F., Szkoły, P., Howell, S. B., & Gello, D. M. 2004, AJ, 127, 460
Howell, S. B., Nelson, L. A., & Rappaport, S. 2001, ApJ, 550, 897
Howell, S. B., et al. 2006, ApJ, 646, L65
Hurley, J. R., Tout, C. A., & Pols, O. R. 2002, MNRAS, 329, 897
Ivanov, N., & Taam, R. E. 2003, ApJ, 599, 516
King, A. R., Frank, J., Kolb, U., & Ritter, H. 1995, ApJ, 444, L37
Kouge, C. 2006, MNRAS, 373, 484
Kolb, U. 1996, in IAU Colloq. 158, Cataclysmic Variables and Related Objects, ed. A. Evans & J. H. Wood (Dordrecht: Kluwer), 433
Kolb, U., Rappaport, S., Shenker, K., & Howell, S. 2001, ApJ, 563, 958
Kolb, U., & Willems, B. 2005, in ASP Conf. Ser. 330, The Astrophysics of Cataclysmic Variables and Related Objects, ed. J.-M. Hameury & J.-P. Lasota (San Francisco: ASP), 17
Kroupa, P., Tout, C. A., & Gilmore, G. 1993, MNRAS, 262, 545
Lada, C. J. 2006, ApJ, 640, L63
Li, J. K., Wu, K. W., & Wickramasinghe, D. T. 1994, MNRAS, 268, 61
Martin, R. G., & Tout, C. A. 2005, MNRAS, 358, 1036
McDermott, P. N., & Taan, R. E. 1989, ApJ, 342, 1019
Mestel, L., & Spruit, H. C. 1987, MNRAS, 226, 57
Patterson, J. 1984, ApJS, 54, 443
Patterson, J., et al. 2005, PASP, 117, 1204
Politano, M. 1996, ApJ, 465, 338
Politano, M., & Weiler, K. P. 2006, ApJ, submitted
Pretorius, M. L., Knigge, C., & Kolb, U. 2007, MNRAS, in press (astro-ph/0610278)
Rappaport, S., Verbunt, F., & Joss, P. C. 1983, ApJ, 275, 713
Ritter, H., & Kolb, U. 2003, A&A, 404, 301 (update RKcat7.6)
Shafer, A. W. 1992, ApJ, 394, 268
Shara, M. M., Livio, M., Moffat, A. F. J., & Orio, M. 1986, ApJ, 311, 163
Skumanich, A. 1972, ApJ, 171, 565
Spruit, H. C., & Ritter, H. 1983, A&A, 124, 267
Spruit, H. C., & Taam, R. E. 2001, ApJ, 548, 900
Taam, R. E., Sandquist, E. L., & Dubus, G. 2003, ApJ, 592, 1124
Tutukov, A., & Yungelson, L. 1979, in IAU Symp. 83, Mass Loss and Evolution of O-Type Stars, ed. P. S. Conti & C. W. H. de Loore (Dordrecht: Dordrecht), 401
Verbunt, F., & Zwaan, C. 1981, A&A, 100, L7
Warner, B. 1987, MNRAS, 227, 23
Webbink, R. F. 1984, ApJ, 277, 355
Willems, B., & Kolb, U. 2002, MNRAS, 337, 1004
———. 2004, A&A, 419, 1057
Willems, B., Kolb, U., Sandquist, E. L., Taam, R. E., & Dubus, G. 2005, ApJ, 635, 1263 (Paper I)