Decoherence of quantum states and its suppression in ensemble large-scale solid state NMR quantum computers

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Abstract

It is discussed the decoherence problems in ensemble large-scale solid state NMR quantum computer based on the array of $^{31}$P donor atoms having nuclear spin $I = 1/2$. It is considered here, as main mechanisms of decoherence for low temperature ($< 0.1\, \textrm{K}$), the adiabatic processes of random modulation of qubit resonance frequency determined by secular part of nuclear spin interaction with electron spin of the basic atoms, with impurity paramagnetic atoms and also with nuclear spins of impurity diamagnetic atoms. It was made estimations of allowed concentrations of magnetic impurities and of spin temperature whereby the required decoherence suppression is obtained. It is discussed the random phase error suppression in the ensemble quantum register basic states.

keywords: decoherence, ensemble, large-scale, nuclear magnetic resonance, qubit, quantum computer.

Introduction

There are five basic criteria for realization of a large-scale quantum computer, which can outperform all traditional classical computers [1]. Let us take a brief look at these criteria in respect to the large-scale NMR quantum computers.

1. For any physical system, which presents large-scale quantum register, it is required of a sufficiently large qubit number ($L > 10^3$) and each qubit should be separately identifiable.
   
   One such example of this register is solid-state homonuclear system, in which nuclear spin containing identical atoms are housed at regular intervals in a natural or an artificial solid-state structure.

2. There is a need to provide the conditions for preparation of initial and basic quantum register state. For a many-qubit NMR quantum computer the quantum register state initializing can be obtained by going to extra-low nuclear spin temperature ($< 1\, \textrm{mK}$ at fields of order of several Tesla).

3. The decoherence time of qubit states $T_d$ should be at least up to $10^4$ time longer than the "clock time". It has value of order of several seconds for NMR quantum computers. The decoherence suppression is one of the important problems in realization of a large-scale quantum computers.

4. There is a need to provide during a decoherence time the implementing of a set of quantum logic operations determined by a logic unitary transformation. This set should contain certain set of the one-qubit and two-qubits operations are shielded from random errors. The electromagnetic pulses that control the quantum operation should be performed with an accuracy of better than $10^{-4}$–$10^{-5}$.

5. There is a need to provide the accurate and sensitive read-out projective measurements of individual qubit states. This is another of the important and the most hard problems.

One of the pioneering schemes of large-scale NMR quantum computer with individual access to qubits was proposed by B.Kane [2, 3].

There are four peculiar difficulties in implementing of such quantum computer:

1. First of all, signal from the spin of an individual atom is very small and it is required of a highly sensitive single-spin measurements.

2. For initialization of nuclear spin states it is required to use very low nuclear spin temperature ($\sim \textrm{mK}$).

3. It is required to use the regular donors and gates arrangement with high precision in nanometer scale.

4. It is necessary to suppress the decoherence mechanism defined by thermal fluctuations of gate voltage (this mechanism will be denoted as external).

As an alternative, we have proposed the variant of an ensemble silicon-based quantum computer [4, 5, 6]. One would expect that with the ensemble approach when many independent "molecules" of Kane’s type, like in the bulk-ensemble liquid quantum computers, work simultaneously, the qubit state measurements seem to be greatly simplified. Furthermore, it seems to be solved certain problems associated with possible noncontrolled continuous random phases of individual qubits.
This article discusses only the decoherence mechanisms of one qubit quantum states in ensemble quantum register based on the regular array of $^{31}\text{P}$ atoms having nuclear spin $I = 1/2$. It is considered, as main mechanisms of decoherence for the very low temperature, the adiabatic processes of random modulation of qubit resonance frequency without spin flips produced by fluctuating local magnetic fields which are determined by secular parts of interactions of nuclear spins with electron spin of the basic atoms, with impurity paramagnetic atoms and also with nuclear spins of impurity diamagnetic atoms. We have named this mechanisms as internal. It was made estimations of allowed concentrations of magnetic impurities that are necessary for decoherence suppression. It is discussed also the random phase error suppression in the initializing process for ensemble quantum register states.

1 Mechanisms of qubit state decoherence in NMR solid-state quantum registers

The full electron spin polarization will be practically achieved at

$$\gamma_S \hbar B/kT \gg 1,$$

where $\gamma_S = 176 \text{rad GHz}/T$ is gyromagnetic ratio for electron spin, $\hbar = 1.05 \cdot 10^{-34} \text{J s/rad}$, $B$ is induction of external magnetic field, $T$ is temperature of environment (of electron spins). Thus for $B = 2 \text{T}$, $T = 0.1 \text{K}$ ($B/T = 20 \text{T/K}$) we obtain $\gamma_S \hbar B/kT = 27$.

The electron and nuclear longitudinal relaxation processes, that are followed by spin flips in four energy level system of phosphorus atoms doped in silicon, have been previously investigated experimentally in [7]. The electron longitudinal relaxation times at low temperatures were found to be extremely long (hours) and independent of phosphorus concentration below $\sim 10^{16} \text{cm}^{-3}$. The nuclear longitudinal relaxation time $T_\parallel$ were found to be equal to 10 hours [7].

The extremely long longitudinal relaxation times of the electron and nuclear spins imply the possibility to produce a long-lived nuclear nonequilibrium initialized state for lattice temperatures (say, for $T = 0.1 \text{K}$) when only the electron spins are polarized. The required initializing of nuclear quantum states (near-full nuclear polarizations) can be obtained by a short duration deep cooling to $T_\parallel \leq 1 \text{mK}$ only for nuclear spin system without deep cooling of the whole sample. There is the possibility to achieve it by indirect cooling of nuclear spin system using dynamic nuclear spin polarization techniques (such as the known Abragam’s solid-state effect) [8].

The relaxation of nonequilibrium state of the nuclear spin system represented by the product of independent (nontangled) one-qubit states, owing to the interaction with isotropic environment, shows two processes. One is a slow establishment of equilibrium state associated with dissipation of energy. For it the diagonal elements of density matrix decay with characteristic longitudinal (spin-lattice) relaxation time $T_\parallel$. The decay of non-diagonal matrix elements called decoherence of quantum states is characterized by a decoherence time $T_d$ or transverse (spin-spin) relaxation time $T_\perp$. The longitudinal relaxation times $T_\parallel$ in the case of nuclear spin of $^{31}\text{P}$ atoms as qubits is defined mainly by thermal modulation of hyperfine interaction accompanied by spin flips. It is usual that for solids $T_\perp \ll T_\parallel$.

We will not consider here the external decoherence process due to gate voltage noise. It was made in [3, 5, 6].

The internal adiabatic decoherence mechanisms due to a random modulation of qubit resonance frequency, produced by local fluctuating magnetic fields without spin flips, seem to be the leading.

2 Semiclassical model of adiabatic decoherence of one-qubit state

We will consider a long-lived non-equilibrium qubit state when diagonal elements of density matrix may be treated as a constant.

The random modulation of resonance frequency $\Delta \omega(t)$ that causes the dephasing of a qubit state are determined by the random phase shifts

$$\varphi(t) = \int_0^t \Delta \omega(t) dt.$$

The one-qubit density matrix of pure state in rotating frame with non perturbed resonance circular frequency will be

$$\rho(t) = 1/2 \begin{pmatrix} 1 + P_z & P_- \exp(i \varphi(t)) \\ P_+ \exp(-i \varphi(t)) & 1 - P_z \end{pmatrix},$$

where $P_\pm = \cos(\theta/2) \mp i \sin(\theta/2)$, $P_z = \cos(\theta)$, $\theta = \omega \Delta t/2 \equiv \gamma \Delta \omega \Delta t$.
where $P_x = P_z \pm iP_y$, $P_x, P_y, P_z$ are Bloch vector components of length $P = \sqrt{P_x^2 + P_y^2 + P_z^2} = 1$.

By treating the resonance frequency modulation as Gaussian random process after averaging (3) over phase distribution with $\langle \varphi(t) \rangle = 0$ we obtain
\[
\langle \rho(t) \rangle = 1/2 \left[ \begin{array}{cc} 1 + P_z & P_- \exp(-\Gamma(t)) \\ P_+ \exp(-\Gamma(t)) & 1 - P_z \end{array} \right],
\]  
where
\[
\Gamma(t) = 1/2 \cdot \left( t \int_0^t \Delta\omega(t) dt \right) = \int_0^t (t - \tau) \langle \Delta\omega(\tau) \Delta\omega(0) \rangle d\tau,
\]  
\[f(t) = \langle \Delta\omega(t) \Delta\omega(0) \rangle \]
is the frequency correlation function of a random process, which is characterized by variance $\langle \Delta\omega(0)^2 \rangle$ and correlation time $\tau_C$ such that for $t > \tau_C \langle \Delta\omega(t) \Delta\omega(0) \rangle \approx 0$. For $\Gamma(t) > 0$ the averaged density matrix presents a mixed quantum state with two non-zero eigen states
\[1/2 \cdot \left( 1 \pm \sqrt{1 - (P_x^2 + P_y^2)}(1 - \exp(-2\Gamma(t))) \right)
\]  
and the populations of states $p_{\pm} = 1/2(1 \pm P_z(0))$ at $\Gamma(t) \to \infty$.

Thus, the adiabatic decoherence problem is reduced to the determination of the function $\Gamma(t)$ or the correlation function of random frequency modulation.

In the case of an ensemble quantum register there is a need to average the one-qubit density matrix and correlation function over ensemble of independent equivalent spins-qubits.

3 The nuclear spin states decoherence due to hyperfine interaction of nuclear and electron spins

In this case the modulation of nuclear spin resonance frequency $\Delta\omega(t)$ is determined by the secular part of hyperfine interaction:
\[\Delta\omega(t) = A(t)S_z(t) - A_0 \langle S_z \rangle .\]  
The most studied exactly solvable one-boson models of adiabatic decoherence that is not followed by spin flips are not adequate [10]. The decoherence that is described by the direct one-phonon process with spin flips is non-adiabatic slow relaxation process. It is characterized by a rate comparable to $1/T_1$. To describe the dephasing or pure adiabatic decoherence we will take in to account one of the Raman two-phonon processes, which describes the hyperfine interaction constant modulation $A(t)$ due to the phonon scattering without of spin flip. In interaction representation we will have
\[A(t) = A_0[1 + \sum_{m\neq l} g_{m1}b_m^+b_l \exp(i(\omega_m - \omega_l)t) + g_{1m}^*b_l^+b_m \exp(-i(\omega_m - \omega_l)t)],\]  
where $A_0 = 725$ rad MHz is hyperfine interaction constant for atoms $^{31}P$ at $T = 0$, $b_l^+$ and $b_m$ are the emission and absorption of phonon operators with frequencies $\omega_l$ and $\omega_m$, $g_{m1} \sim \xi(\hbar/2MvN)(\omega_m\omega_l)^{1/2}$ is the coupling constants of hyperfine parameter with acoustic phonons, dimensionless parameter $\xi$ in the worse case is $\approx 1$, $v$ is the velocity of sound, $N$ is the total number of atoms in crystal (for simplicity we assume the simple cubic lattice), $M$ is the atom mass.

From [6] we obtain the sum of two independent terms, which correspond to the two mechanisms of adiabatic decoherence due to hyperfine interaction:
\[\Delta\omega(t) = \Delta\omega_S(t) + \Delta\omega_L(t) = A_0(\langle S_z(t) \rangle - \langle S_z \rangle) + +1/2 \cdot A_0 \sum_{m\neq l} g_{m1}b_m^+b_l \exp(i\omega_m t) + g_{1m}^*b_l^+b_m \exp(-i\omega_m t), \quad \omega_m = \omega_m - \omega_l .
\]  

At first, let us consider the first term in (3). The correlation function is determined by the fluctuations of electron spin polarization and depends on electron resonance frequency $\omega_S$, longitudinal $\tau_1$ (hours) and transverse $\tau_2$ relaxation times. In adiabatic case $\omega_S = \gamma_S B > 1/\tau_1 \gg 1/\tau_2$ and we will obtain:
\[\langle \Delta\omega_S(t) \Delta\omega_S(0) \rangle = \langle \Delta\omega_S^2 \rangle \cdot \exp(-t/\tau_1),
\]
\[ \langle \Delta \omega_2^2 \rangle = A_0^2 (\langle S_z^2 \rangle - \langle S_z \rangle^2) = A_0^2 (1 - \tanh^2 (\gamma g B/kT))/4. \] (11)

Now
\[ \Gamma(t) = \langle \Delta \omega_2^2 \rangle \tau_1^2 (t/\tau_1 - 1 + \exp(-t/\tau_1)). \] (12)

For \( \tau_1 \approx 10^4 \text{s} \) and \( t \sim T_d = 1 \text{s} \), we have the non-Markovian random process (slow damping fluctuations). In this case
\[ \Gamma(t) = \langle \Delta \omega_2^2 \rangle t^2/2, \quad \langle \Delta \omega_2^2 \rangle \approx 1/T_d^2. \] (13)

Let us write the requirement for the decoherence time for \( \gamma g B/kT \gg 1 \) in the form
\[ 1/T_d^2 \approx A_0^2 (1 - \tanh^2 (\gamma g B/kT))/4 \approx A_0^2 \exp(-\gamma g B/kT) < 1 \text{s}^{-2}, \] (14)

from which we find that the required decoherence suppression will be achieved at sufficiently large ratio \( B/T > 30 \text{T}/\text{K} \). For \( B/T = 20 \text{T}/\text{K} \), we have \( T_d \sim 10^{-3} \text{s} \).

For decoherence description due to the second term in (11), it is convenient to use directly the function \( \Gamma(t) \).

With
\[ \int_0^t \Delta \omega_b(t) dt = 1/2 \cdot A_0 \sum_{m,l} (\xi_{ml}(t)b_m^+ b_l + \xi_{ml}^*(t)b_l^+ b_m), \]
\[ \xi_{ml}(t) = g_{ml}(1 - \exp(i \omega_m t)/\omega_m), \quad \langle b_m^+ b_m \rangle = n(\omega_m, T) = (\exp(\omega_m/T - 1)^{-1}, \]
we will obtain
\[ \Gamma(t) = 1/4 \cdot \left( \int_0^t \Delta \omega_b(t) dt \right)^2 = \]
\[ = 1/4 \cdot A_0^2 \sum_{m \neq l} (\hbar/2 M v^2 N)^2 \cdot (\omega_{m\omega_l})(2n(\omega_m, T)n(\omega_l, T) + n(\omega_m, T) + n(\omega_l, T)) \frac{\sin^2 (\omega_{ml} t)}{\omega_{ml}^2}. \] (15)

Let us go now from sums to integrals:
\[ \sum_m \ldots \Rightarrow \frac{9N}{4\pi^3} \int_0^\Omega \ldots \omega_m^2 d\omega_m, \] (16)

where \( \Omega = v(6\pi)^{1/2}/a = k\Theta/h \) is Debye frequency, \( a \) is the lattice constant.

Taking into account that for \( \omega_m \gg 0 \) (near-elastic scattering)
\[ \frac{\sin^2 (\omega_{ml} t)}{\omega_{ml}^2} \approx \pi |t| \delta(\omega_{ml}), \] (17)

we now obtain
\[ \Gamma(t) = \frac{81\pi}{8} A_0^2 (\hbar/M v^2)^2 (T/\Theta)^7 (k\Theta/h) \int_0^{\Theta/T} \frac{x^6 \exp x}{(\exp x - 1)^2} dx |t|/T_d. \] (18)

For silicon: \( \Theta = 625 \text{K}, a = 5.4 \cdot 10^{-8} \text{cm}, v \approx 5 \cdot 10^8 \text{cm/s}, M = 0.46 \cdot 10^{-29} \text{Js/cm}^2. \)

At low temperature \( T/\Theta < 10^{-3} \) for \( T_d \) estimation we obtain:
\[ 1/T_d \approx \frac{81\pi}{8} A_0^2 (\hbar/M v^2)^2 (k\Theta/h)(T/\Theta)^7 \sim 3/4 \cdot 10^4 (T/\Theta)^7 \text{s}^{-1} \ll 1 \text{s}^{-1}. \] (19)

Thus, we see, that at low temperatures the phonon mechanism have an insignificant effect as compared with the mechanism of electron spin fluctuations.
4 The nuclear spin states decoherence due to interaction with magnetic impurity atoms.

An alternative reason for the modulation of individual nuclear spin resonance frequency is the secular part of their dipole-dipole interaction with random distributed paramagnetic center (impurity atom and defect) in the substrate, which for the magnetic impurity atoms in S-state is of the form:

$$\Delta \omega(t) = \gamma B_s(t) = \frac{\mu_0}{4\pi} \cdot \gamma \gamma_S \hbar \cdot \sum_i \frac{1 - 3z_i^2/r_i^2}{r_i^3} \cdot (S_z(i,t) - \langle S_z(i) \rangle),$$  \hspace{1cm} (20)

where $\mu_0 = 0.4\pi T^2 \text{cm}^3/J$, $r_i$ is radius-vector of the impurity atom distance.

In this case, once again, we have

$$\langle \Delta \omega(t) \Delta \omega(0) \rangle = \langle \Delta \omega^2 \rangle \cdot \exp(-t/\tau_{\text{imp}}),$$  \hspace{1cm} (21)

where for non-correlated homogeneous spatial distribution of impurity atoms

$$1/T^2_{S} \approx \langle \Delta \omega^2 \rangle \approx C_{S,\text{imp}}((\mu_0/4\pi) \cdot \gamma \gamma_S \hbar)^2 \cdot \frac{16\pi}{15a^3} \cdot (\langle S_z^2 \rangle - \langle S_z \rangle^2),$$  \hspace{1cm} (22)

where

$$\langle S_z^2 \rangle - \langle S_z \rangle^2 \approx \exp(-\gamma \gamma_S \hbar B/kT),$$  \hspace{1cm} (23)

$C_{S,\text{imp}}$ is concentration of magnetic atoms in cm$^{-3}$, $\gamma_S = 176$ rad GHz/T, $\gamma_{I}^{(31)P} = 108$ rad MHz/T, $a$ is minimal distance of order of lattice constant, for Si $a^{-3} \approx 5.0 \cdot 10^{22}$ cm$^{-3}$,

$$\langle \Delta \omega^2 \rangle \approx 33.4 \cdot 10^{-13} \cdot (C_{S,\text{imp}}a^3) \cdot \exp(-\gamma \gamma_S \hbar B/kT).$$  \hspace{1cm} (24)

It follows that a direct suppression of decoherence due to interaction with magnetic impurity atoms may be achieved by

- increase of relation $(B/T)$ and
- decrease of impurity concentration $C_{S,\text{imp}}$.

To estimate the allowed concentration of paramagnetic centers in the silicon substrate $C_{S,\text{imp}}$ we will consider as previously the case slow frequency modulation and the decoherence time $T_{\text{dec}} \sim 1 \text{s}$. For $B/T > 20 \text{T/K}$ we obtain

$$1/T^2_S \approx \langle \Delta \omega^2 \rangle \approx 0.74 \cdot 10^3 \cdot (C_{S,\text{imp}}a^3)^2 < 1 \text{s}^{-2},$$  \hspace{1cm} (25)

or

$$C_{S,\text{imp}} < 1.4 \cdot 10^{-3} \cdot a^{-3} \approx 0.7 \cdot 10^{29} \text{cm}^{-3}.$$  \hspace{1cm} (26)

Thus, the value of allowed concentration of magnetic impurity is practically unbounded at large enough value $\gamma \gamma_S \hbar B/kT$.

Another mechanism of individual qubit state decoherence is dipole-dipole interaction with nuclear spins $I \neq 0$ of impurity diamagnetic atoms having concentration $C_{I,\text{imp}}$. Isotope $^{29}\text{Si}$ with $\gamma_{I,\text{imp}} = -53$ rad MHz/T is one of the such atoms.

In this case correlation function take form

$$\langle \Delta S_z(\tau) \Delta S_z(0) \rangle = \langle \Delta \omega^2 \rangle \cdot \exp(-t/T_{||,\text{imp}}), \quad \tau_C = T_{||,\text{imp}},$$  \hspace{1cm} (27)

where $T_{||,\text{imp}} \approx 10^3 \text{s}$—impurity nuclear spin longitudinal relaxation time of isotope $^{29}\text{P}$ at low temperature [8].

$$1/T^2_{||} \approx C_{I,\text{imp}}((\mu_0/4\pi) \cdot \gamma \gamma_{I,\text{imp}} \hbar)^2 \cdot \frac{4\pi}{15a^3} \cdot (1 - \tanh^2(|\gamma_{I,\text{imp}}| B/kT_1))$$  \hspace{1cm} (28)

For $B = 2 \text{T}$ and for spin temperature $T_1$ at which there are near-full polarization of impurity nuclear spins

$$|\gamma_{I,\text{imp}} B/kT_1| < 1$$  \hspace{1cm} (29)

or for $T_1 < 0.8 \text{mK}$, we will obtain that the allowed concentration of the isotope $^{29}\text{Si}$ in % is

$$C_{I,\text{imp}} < 4.5 \cdot 10^{-2}\%$$  \hspace{1cm} (30)

This value may be increased provided a further decrease of spin temperature $T_1$.

For comparison, natural abundance of isotope $^{29}\text{Si}$ in natural silicon is 4.7%.

Hence the main reasons for the internal decoherence of qubit states are the modulation of resonance qubit frequency due to hyperfine interaction with fluctuating electron spin and due to interaction with randomly distributed impurity diamagnetic atoms containing nuclear spins.
The random phase errors in ensemble quantum register

The preparation of both the initialized and other basic states of quantum register is followed by error generation through interaction of qubits with the environment. Any one-qubit error in either of the basic state is defined as a superposition basic states $|0\rangle$ and $|1\rangle$.

As an example of one-qubit error action we consider the following unitary (it is not necessary unitary) transformation

$$U_\varepsilon = \frac{1}{\sqrt{1 + \varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2}} \begin{bmatrix} 1 + i\varepsilon_x & i\varepsilon_x + \varepsilon_y \\ i\varepsilon_x - \varepsilon_y & 1 - i\varepsilon_x \end{bmatrix},$$

(31)

where $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$ are independent random error function of time.

The action of this transformation on initialized state $\rho(0) = |0\rangle \langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ leads to the modified pure state

$$\rho_\varepsilon(0) = U_\varepsilon |0\rangle \langle 0| U_\varepsilon^{-1} = 1/2 \begin{bmatrix} 1 + P_+\varepsilon, & P_-\varepsilon \\ P_+\varepsilon, & 1 - P_-\varepsilon \end{bmatrix},$$

(32)

where the components of Bloch vector are

$$P_+\varepsilon = 1 - 2p_\varepsilon(0), \quad P_-\varepsilon = 2\sqrt{\frac{(1 + \varepsilon_x^2)(\varepsilon_x^2 + \varepsilon_y^2)}{1 + \varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2}} \cdot \exp(\pm i\varphi_\varepsilon),$$

(33)

$$p_\varepsilon(0) = (\varepsilon_x^2 + \varepsilon_y^2)/\left(1 + \varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2\right), \quad \tan \varphi_\varepsilon = (\varepsilon_x\varepsilon_y + \varepsilon_z)/(\varepsilon_x\varepsilon_y - \varepsilon_z).$$

(34)

The fidelity of initial state $|0\rangle$ is determined by

$$F(0) = \text{Sp}\rho_\varepsilon(0)\rho(0) = 1 - p_\varepsilon(0),$$

(35)

where $p_\varepsilon(0)$ is the random error probability. The diagonal elements of the perturbed density matrix $\rho_\varepsilon(0)$ depend only on the value of $p_\varepsilon(0)$ and have no random phase factor.

The random phase factors with arbitrary continuous random phase $\varphi_\varepsilon$ have only non-diagonal elements of density matrix $\rho_\varepsilon(0)$ or the Bloch components $P_+\varepsilon, P_-\varepsilon$.

All $2^k$ basic state vectors of quantum register $|n\rangle$ are generated by one-qubit unitary transformations iNOT which transform the initial basis states of certain qubits $|0\rangle$ in state $|1\rangle$. The containing error iNOT$\varepsilon$ operations leads also to appropriate random phase factors.

The existence of the such uncontrollable phase factors can involve a problem, which mainly lies, as was noted by S.Kak [1], in the inability of the quantum error correction codes (QECC) to correct this analog-type quantum phase errors as "they can potentially correct only bit flips and phase flips and some combination thereof, which errors represent a small subset of all the error that can corrupt a quantum state".

We will advance here some arguments in support of the ensemble approach use in connection with the generation of uncontrollable phase factors.

The producing of random phase factors (known as fasors) may be considered as a stationary random process realized by random functions $\varepsilon_x(t)$, $\varepsilon_y(t)$ and $\varepsilon_z(t)$ possessed by a statistical ensemble. A time-average of density matrix is practically unfeasible as it requires prolonged state measurements of an individual quantum register. But if it is assumed that random process in ensemble quantum registers is ergodic random process, it may be suggested to substitute the time average by the ensemble average. Practically the ensemble averaging will occur by natural way, when unitary transformations and ensemble quantum register states measurements are produced at the same time for the whole ensemble by the same controlling pulse, much as in bulk-ensemble liquid quantum computer prototype. The ensemble-averaged reduced density matrix will describe now same mixed state whose non-diagonal matrix elements will decay due to decoherence process and have no analog-type random values. Such states involve no special problems for QECC.

The decay of non-diagonal matrix elements comes much rapidly the more dimension of non-diagonal matrix element blocks. Thus during the process of preparing of many-qubit quantum register basis states a rapid decoherence can play the constructive role. The rapid decoherence may be produced, among other processes, by means of a sharp decrease, for the short time, of external magnetic field.

As the result mixing degree of reduced density matrix $\langle p_\varepsilon \rangle$ may be treat as temperature analog in a liquid quantum computers. For suited conditions the value $\langle p_\varepsilon \rangle \ll 1$. Analogously to the low temperature many-qubit
mixed state the considered state may be transformed to pseudpure state, but in so doing the entanglement property of large-scaled state at low temperature will not be violate [12]. In this case we will have the fully quantum computing instead of it imitation.

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Summary

The main mechanisms of decoherence for low temperature are the adiabatic processes of random modulation of qubit resonance frequency without spin flips produced by hyperfine interaction of nuclear spin with electron spin of the basic atoms and dipole-dipole interaction with nuclear spins of impurity diamagnetic atoms. It is estimated the allowed concentration of nuclear spin containing isotopes $^{30}$P. It was shown that the random phase error of quantum register state in the ensemble approach are averaged and the pure basic state transforms to the mixed state, analogical to mixed state at nonzero temperatures.

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