Dark energy and the mass of the Local Group

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ABSTRACT

Dark energy must be taken into account to estimate more reliably the amount of dark matter and how it is distributed in the local universe. For systems several Mpc across like the Local Group, we introduce three self-consistent independent mass estimators. These account for the antigravity effect of dark energy treated as Einstein’s cosmological constant \( \Lambda \). The first is a modified Kahn-Woltjer model which gives a value of the Local Group mass via the particular motions of the two largest members, the Milky Way and M31. Inclusion of dark energy in this model increases the minimum mass estimate by a factor of three compared to the classical estimate. The increase is less but still significant for different ways of using the timing argument. The second estimator is a modified virial theorem which also demonstrates how dark energy can ”hide” from detection a part of the gravitating mass of the system. The third is a new zero-gravity method which gives an upper limit to the group mass which we calculate with high precision HST observations. In combination, the estimators lead to a robust and rather narrow range for a group’s mass, \( M \). For the Local Group, \( 3.2 < M < 3.7 \times 10^{12} M_\odot \). Our result agrees well with the Millennium Simulation based on the \( \Lambda \)CDM cosmology.

Subject headings: galaxies: Local Group; cosmology: dark matter, dark energy

1. Introduction

It has long been known that galaxy groups and clusters would fly apart unless they were held together by the gravitational pull of much more material than what is actually seen. The rapid progress in dark matter studies stems from optical surveys of large areas and high redshifts, CMB fluctuation measurements, sharp X-ray images, gravitational lensing measurements, etc. Concurrently, conclusive observational indications have accumulated that dark matter and baryons contribute not more than 30% to the mass/energy content of the universe. The rest 70% is composed of a much more mysterious dark energy (Riess et al. 1998, Perlmutter et al. 1999). Dark energy does not cluster, and its simplest mathematical formulation goes back to 1917 when Einstein introduced the cosmological constant \( \Lambda \). This interpretation is assumed in the currently standard \( \Lambda \)CDM cosmology. A positive \( \Lambda \) corresponds to a positive constant energy density in all entire space. Such a medium with negative pressure \( (\rho_\nu > 0, \ p_\nu < 0) \) is characterized by the equation of state \( \rho_\nu = -p_\nu \ (\epsilon = 1) \), like that of vacuum. According to Einstein’s equations, gravity depends on pressure as well as density: the effective gravitating density \( \rho_{\text{eff}} = \rho + 3p \) is negative for a vacuum \( (\epsilon = -2p_\nu) \), leading to a repulsion, or antigravity. The dark energy antigravity revealed itself first in the accelerating cosmological expansion.

In this paper, we follow the \( \Lambda \)CDM cosmology and assume that the dark energy (DE) is a perfect fluid with a constant density every where and in any reference frame. From most recent CMB studies (Spergel et al. 2006), \( \rho_\nu = (0.72 \pm 0.03) \times 10^{-29} \text{ g cm}^{-3} \). We focus on relatively small spatial
scales, on the scale of groups of galaxies. We show that galaxies and their systems "lose" a part of their gravity due to the antigavity of the dark energy in their volumes. The dynamical methods of the mass estimation of dark matter (and baryons) should take into account this "lost-gravity" effect – otherwise the mass would systematically be underestimated.

2. Modified Kahn-Woltjer (MKW) estimator

Fifty years ago, Kahn & Woltjer (1959, KW) used simple linear two body dynamics to describe the relative motion of the Milky Way and M31 galaxies. The motion of the galaxies was described (in the reference frame of the binary’s center of mass) by the equation of motion

\[ \ddot{D}(t) = -\frac{GM}{D^2}, \]

where \( D \) is the distance between the galaxies and \( M \) is the total mass \( m_1 + m_2 \) of the binary. In the well-known first integral

\[ \frac{1}{2} \dot{D}(t)^2 = GM/D + E_0, \]

the constant total mechanical energy of the binary (per unit reduced mass) is negative for a gravitationally bound system, \( E_0 < 0 \). This fact and Eq.2 with the observed values \( D = 0.7 \) Mpc and \( \dot{D} = -120 \text{ km/s} \) lead to an absolute lower limit for the estimated binary mass: \( M > 1 \times 10^{12} M_\odot \).

The "timing argument" KW calculation integrates in reverse the equation of motion (1) from the above present-day separation and radial velocity requiring that the initial expansion of the two galaxies from one another started a time in the past equal to the age of the universe (13.7 billion years). Thus we calculate a mass for the pair of \( M \approx 4.0 \times 10^{12} M_\odot \). See Binney & Tremaine (1987) for an analytic discussion (for the 10 to 20 billion year age range suspected at that time).

With a minimal modification of the original KW method, including the dark energy background \( \rho_v \), the equation of motion and its first integral become:

\[ \ddot{D}(t) = -\frac{GM}{D^2} + \frac{4\pi}{3} \rho_vD, \]

\[ \frac{1}{2} \dot{D}(t)^2 = GM/D + \frac{4\pi}{3} \rho_vD^2 + E_0. \]

Now the total energy for a bound system embedded in the DE background is

\[ E_0 < -\frac{3}{2} \frac{GM^{2/3}}{D}(\frac{8\pi}{3} \rho_v)^{1/3}, \]

as may be easily seen from the gravitational potential \( U = -\frac{GM}{D} - \frac{4\pi}{3} \rho_vD^2 \) in the right side of Eq.4. With the same data as above for current separation and the relative velocity of the galaxies, Eqs. 4,5 lead to a new absolute lower mass limit: \( M > 3.2 \times 10^{12} M_\odot \), or 3 times the absolute lower limit of the original statement of the problem.

Again the MKW equations can be integrated back to the origin of the universe including the dark energy value given earlier. The same data lead to a larger mass: \( M = 4.5 \times 10^{12} M_\odot \).

In fact, the KW model can hardly be extrapolated to the origin of the universe, while it seems realistic for the stage of the binary collapse. In Fig. 1 we show the calculated mass as a function of the time back to the maximal separation of the Milky Way and M31; the lower and upper curves show the results without and with dark energy, respectively. In the ΛCDM cosmology gravitational instability is terminated in the linear regime about 7 Gyr ago with the start of the DE dominated epoch (e.g. Chernin et al. 2003). Taking this to be the minimal collapse time yields the maximal mass of the binary, and this is \( 3.3 \times 10^{12} M_\odot \) without dark energy and \( 4.1 \times 10^{12} M_\odot \) with dark energy.

These results demonstrate clearly the effect of the lost gravity in gravitationally bound systems embedded in the dark energy background. The relative motion of the bodies is controlled by the gravity (of dark matter and baryons) which is partly counterbalanced by the DE antigavity. Consequently, the mass estimate must be corrected: the mass of the Local Group given by the modified Kahn-Woltjer (MKW) estimator is significantly larger than that obtained without inclusion of dark energy. Here we have taken advantage of our position within the Local Group, the well known distance of M31 and the measured approach speed. We thus should modify conventional ways to estimate the masses of groups of galaxies in general.
3. Modified virial theorem (MVT)

Conventional virial mass estimators use the relation between the mean total kinetic \( < K > \) and potential \( < U > \) energies of a quasi-stationary gravitationally bound many-body system: \( < K > = 1/2 | < U > | \). The presence of dark energy modifies this relation. The total potential energy includes now not only the sum \( U_1 \) of the mutual potential energies of its member particles, but also the sum \( U_2 \) of the potential energy of the same particles in the force field of dark energy:

\[
U_1 = -\frac{1}{2} \sum \frac{G m_i m_j}{|r_i - r_j|} \quad \text{and} \quad U_2 = -\frac{4\pi \rho_v}{3} \sum m_i r_i^2
\]  

(6)

Here \( r_i \) is the radius-vector of a particle in the frame of the system’s barycenter; the summation in \( U_1 \) is over all particle pairs \( (i \neq j) \). The major contribution to the sum is from dark matter particles whatever their individual masses may be.

A hint to the structure of \( U_2 \) may be seen, e.g., from the second item in the right side of Eq.4; the summation in \( U_2 \) is over all the particles. Dark energy comes to the virial theorem via an extra contribution to the potential energy of the system.

A link to the kinetic energy is provided by the equation of motion of an individual particle:

\[
m_i \ddot{r}_i = \frac{\partial U}{\partial r_i} = -\frac{\partial U_1}{\partial r_i} + \frac{8\pi}{3} \rho_v m_i r_i \frac{\ddot{r}_i}{r_i}.
\]  

(7)

Averaging over time and using the Euler theorem on homogeneous functions (applied separately to the two functions that come from the two terms in the right side of Eq.7), we find

\[
< K > = -\frac{1}{2} < U_1 > + < U_2 > .
\]  

(8)

This is the new virial relation adapted to the universe with the dark energy background.

Eq.8 may be rewritten in terms of the total mass \( M \), a characteristic velocity \( \bar{V} \) and characteristic sizes \( \bar{R}_1, \bar{R}_2 \):

\[
M = \frac{\bar{V}^2 \bar{R}_1}{G} + \frac{8\pi}{3} \rho_v \bar{R}_2^3.
\]  

(9)

or in convenient units:

\[
\frac{M}{M_\odot} = 2.3 \times 10^8 \left( \frac{\bar{V}}{\text{km/s}} \right)^2 \left( \frac{\bar{R}_1}{\text{Mpc}} \right) + 0.9 \times 10^{12} \left( \frac{\bar{R}_2}{\text{Mpc}} \right)^3.
\]  

(10)

This new mass estimator – the modified virial theorem (MVT) estimator – includes an additional positive term which is equal to the absolute value of the effective (anti)gravitating mass of dark energy contained in the spherical volume of the radius \( \bar{R}_2 \): \( M_{eff} = -\frac{4\pi}{3} \rho_v \bar{R}_2^3 \). This term gives a quantitative measure of the lost-gravity effect.

One can get a physical sense of Eqs.9,10 by using representative values of the characteristic size \( \bar{R}_1, \bar{R}_2 \) and velocity \( \bar{V} \). In the simplest example of only one body orbiting a gravitating mass \( M \), the size \( \bar{R}_1 = \bar{R}_2 \) is the radius of the orbit, the velocity \( \bar{V} \) is the orbital velocity. In this case, the total effective gravitating mass within the orbit is the gravitating mass \( M \) plus the antigravitating (negative) mass \( M_{eff} \). A similar identification of the quantities is obvious as well when the dark mass of a galaxy is derived from its rotation curve. In both cases, the estimated mass is larger than that in conventional estimates: the additional mass is \( |M_{eff}| \).

Generally, the characteristic sizes and velocities for groups and clusters need more sophisticated analysis - even in the absence of dark energy (e.g. Peebles 1971). Here we merely point out the expected significance of this effect in small systems like the Local Group. The first (conventional) term in Eq.9 is estimated for the Local Group as \( 2.3 \times 10^{12} M_\odot \) (van den Bergh 1999) or \( 1.9 \times 10^{12} M_\odot \) (Karachentsev et al. 2009). Then the second term in Eq.9 increases the total mass by 30 to 50%, if the total size \( \bar{R}_2 \) is about 1 Mpc. Such an effect is compatible with the other methods here discussed.

Assuming \( \bar{R}_1 = \bar{R}_2 \), one may see that the relative contribution of the dark energy scales as the crossing time squared \( (\bar{R}/\bar{V})^2 \). If so, the lost-gravity effect is 10-30 times larger in a group like the Local Group than in a rich cluster like the Coma cluster.

4. Zero-gravity surface (ZGS) estimator

As it was mentioned elsewhere (Chernin 2001, 2008; Byrd, Chernin & Valtonen 2007; Teerikorpi et al. 2008), an isolated gravitationally bound galaxy group can exist on the dark energy background, only if gravity dominates over antigravity in the whole volume of the system. In a simple model, a group may be represented by a spheri-
cal mass $M$ of dark matter and baryons, embedded in the uniform dark energy background. Out of the mass at a distance $R$ from the mass center, the gravity force is given by Newton’s inverse square law in the reference frame related to the group barycenter. The antigravity force produced by the DE density $\rho_v$ is given by Einstein’s linear law (e.g. Chernin 2001, 2008, Chernin et al. 2006) in the same reference frame:

$$F_N = -\frac{GM}{R^2}, \quad F_E = +\frac{8\pi}{3}G\rho_vR. \quad (11)$$

Gravity and antigravity are exactly balanced at the zero-gravity surface of the radius $R = R_v$ (Chernin et al. 2000; Chernin 2001; Baryshev et al. 2001, Dolgachev et al. 2003):

$$R_v = \left(\frac{3M}{8\pi\rho_v}\right)^{1/3}, \quad R_v > R_0, \quad (12)$$

where $R_0$ is the radius of the group. Antigravity dominates at $R > R_v$, and gravity is stronger than antigravity at $R < R_v$. The lost-gravity effect is one hundred percent value on the zero-gravity surface: the total effective gravitating mass of the non-vacuum matter and vacuum energy contained within the surface is zero on the surface. No finite bound orbits are possible on the surface and further at distances $R > R_v$.

Since the zero-gravity radius $R_v$ depends on the gravitating mass of the group alone (for a fixed $\rho_v$), this mass may be found, if the zero-gravity radius is known from observations:

$$M = \frac{8\pi}{3}\rho_v R_v^3 \simeq 0.9 \times 10^{12}[R_v/(\text{Mpc})]^3 M_\odot. \quad (13)$$

The position of the zero-gravity surface can indeed be detected. The most complete high-precision data come from systematic observations of galaxies up to the distance 3 Mpc performed recently with the Hubble Space Telescope (Karachentsev 2005, 2007; Karachentsev et al. 2002, 2003, 2009). About 60 galaxies with good distances (8-10% accuracy) and velocities (5-10 km/s accuracy) are observed in the volume. Nearly half of them join the Milky Way and M31 galaxies forming the Local Group. In the velocity-distance diagram they occupy the distance range of about 1 Mpc; their radial velocities (relative to the group barycenter) are in the interval from -150 to +150 km/s. The other half of the observed galaxies (all of them are dwarfs) move away from the group and have only positive velocities in the diagram. It is important that their minimal distances are 1.1-1.6 Mpc from the group center (Karachentsev 2005, Chernin et al. 2007, Karachentsev et al. 2009). The flow of receding galaxies is rather regular: it follows mainly the linear velocity-distance relation, and its velocity dispersion is only about 25-30 km/s.

This radical difference in the phase-space structure of the group and the outflow around it is a most remarkable feature. We argue (Chernin et al. 2000, 2007, Chernin 2001, Teerikorpi et al. 2008) that the physics behind this feature might be due to the interplay between the gravity of the group and antigravity of the dark energy background, so that the group size $R_0$ is less than $R_v$ and the flow starts at the distances $R > R_v$. Therefore the zero-gravity surface is located somewhere in the gap between the group and the outflow, i.e. in the distance interval from 1.1 to 1.6 Mpc.

The double inequality $1.1 < R_v < 1.6$ Mpc and Eq.13 lead to the lower and upper limits for the non-vacuum mass of the group:

$$1.2 < M < 3.7 \times 10^{12}M_\odot. \quad (14)$$

This result is compatible with the estimations obtained in Secs.2,3. The ZGS-estimator uses the overall structure of the group as a whole, while the MKW- and MVT-estimators deal with the internal dynamics of the group. The new ZGS-estimator is possible only due to the existence of dark energy in the universe.

5. Conclusions and discussion

The local volume proves to be a prospective arena for the study of dark matter and dark energy (e.g. Chernin 2001, 2008; Byrd, Chernin & Valtonen 2007; Niemi et al. 2007; Teerikorpi et al. 2008). We have demonstrated above that, surprisingly enough, by taking the dark energy background into account enables one to learn more on the mass of dark matter in galaxies and their systems.

To summarize the results:

1. The minimally modified Kahn-Woltier method leads to a new absolute minimum of the Local Group mass: $M > 3.2 \times 10^{12}M_\odot$, and it is 3 times larger than the similar mass minimum obtained via the original KW method. The back
integration leads to a mass of the M31, Milky Way pair of $4.5 \times 10^{12} M_\odot$, significantly greater than the original method.

2. The modified virial theorem has the form: $< K > = -\frac{1}{2} < U_1 > + < U_2 >$. It accounts for the potential energy $U_2$ which is due to the antigravity force of dark energy. The theorem gives a clear measure of the lost-gravity effect.

3. The Local Group is located in the area of its self-gravity domination which makes it gravitationally bound, while the outflow around the system develops in the area where no bound orbits are possible. The zero-gravity surface separates these two areas, and its location identified with the observed velocity-distance diagram leads to the absolute upper limit of the group mass: $M < 3.7 \times 10^{12} M_\odot$.

Some additional remarks:

1. The linear binary model for the relative motion of the Milky Way and M31 galaxies (Sec.2) is an idealization. The real dynamics of the Local Group must be more complex (see the study by Valtonen et al. 1993). In particular, a transverse velocity might increase the mass value, but not more than by 10-15%. The earlier dynamical history of the binary (not even mentioning its formation) can hardly be described by this model. This process needs more studies, and cosmological N-body simulations are a real tool to clarify the matter. The most recent Millennium Simulation shows a mass range $(2 - 5) \times 10^{12} M_\odot$ for galaxy groups similar to the Local Group (Li and White 2008); it seems instructive that the interval found in Secs.2,4 is in a good agreement with the simulation result.

2. A study of the lost-gravity effect in groups in general is complicated by the fact that in existing group catalogues, the virial masses tend to be overestimated for other reasons (Niem et al. 2007); we will discuss this in detail elsewhere. We will also reexamine the large-scale mean dark matter and baryonic densities and study the related problem of the dark mass deficit in the local volume (Karachentsev et al. 2009).

3. The gravity potential of a galaxy group like the Local Group which contains a dominant binary of giant galaxies is obviously non-static and non-spherically symmetrical - contrary to the model of Sec.4. However non-sphericity and time-dependence are not significant when one considers relatively large distances from the group barycenter. Calculations show that the deviations of the zero-gravity surface from sphericity now and during the binary collapse are not larger than about 10% in radius (Dolgachev et al. 2003; Chernin et al. 2004). The deviations are even less in the area of the outflow around the group.

4. It is worthwhile to compare the zero-gravity method of Sec.4 with the classical method of the zero-velocity distance $R_0$ predicted by Lemaitre-Tolman solution without or with dark energy (e.g. Peirani & de Freitas Pacheco 2006, 2008, Karachentsev et al. 2009). In that approach the equation of motion for mass shells surrounding the central mass $M$ is solved by integrating from a moment near the Big Bang. The resulting present velocity-distance relation gives the turn-around radius $R_0$, to be compared with the observations. The mass turns out to be proportional to $h^2 R_0^3$ and the coefficient of proportionality is different for the solutions with or without the cosmological constant, so that for a given $R_0$ the inferred mass is again larger when dark energy is included (by about 30%; Peirani & de Freitas Pacheco 2006).

In contrast, our work on the early dynamics of the Local Group suggests that there is a finite time interval after the Big Bang where the outflow of dwarf galaxies originates (e.g. Chernin et al. 2004, 2007; Byrd, Chernin & Valtonen 2007). In this case the present zero-velocity distance will not be uniquely predicted, while, on the contrary, the zero-gravity distance has its near-constant value depending just on $M$ and $\rho_c$.

Peirani & de Freitas Pacheco (2008) did not use directly $R_0$, but fitted their LT-predictions through velocity-distance data points around the Local Group and some nearby groups. They did not get different mass estimates with or without the cosmological constant. A possible reason may be the assumption of the simultaneous origin of the outflow near the cosmological singularity, and their value $2.4 \times 10^{12} M_\odot$ should be regarded as a lower limit.

To conclude, the mass determination for galaxies and their systems is still a hard observational problem - even for the nearest well studied objects. The task needs also a reliable theory background. The lost-gravity effect is a new significant element of the underlying physics. It is included in three
different and self-consistent mass estimators introduced above. Their results are compatible with each other, and in combination they give a robust and rather narrow range of the Local Group non-vacuum mass: $3.2 < M < 3.7 \times 10^{12} M_\odot$. Even for the lower limit (from the MKW binary energy model), the individual dark halos of the Milky Way and M31 galaxies contain together at least 90% of the total dark matter of the group, and the rest might be distributed in the outer common halo of the group around its dominated binary.

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Fig. 1.— This diagram shows the difference between the Local Group mass predictions from the classical Kahn-Woltjer estimator (KW) and its modified form here introduced (MKW). The x-axis gives the look-back time of maximum separation and the y-axis gives the calculated total mass of the Milky Way & M31 binary.