Analyses of $g_{D_s^*DK}$ and $g_{B_s^*BK}$ vertices in QCD sum rules

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Abstract. The coupling constants $g_{D_s^*DK}$ and $g_{B_s^*BK}$ are calculated in the framework of QCD sum rules. We evaluate the correlation functions of these vertices considering both $D(B)$ and $K$ mesons off-shell and obtain the results as $g_{D_s^*DK} = (2.89 \pm 0.25)$ and $g_{B_s^*BK} = (3.01 \pm 0.28)$.

1. Introduction

A precise determination of the coupling constants is a vital task to get knowledge about cross sections and the nature and structure of the encountered particles. Heavy-light pseudoscalar mesons strong coupling constants are of great importance especially in evaluating charmonium cross sections. Experimentally, it is believed that in the production of the charmonium states like $J/\psi$ and $\psi'$ from the $B_c$ or newly discovered charmonium $X$, $Y$ and $Z$ states by the BaBar and BELLE collaborations, there are intermediate two body states containing $D$, $D_s$, $D^*$ and $D_{s}^{*}$ mesons (for example, the kaon can annihilate the charmonium in a nuclear medium to give $D$ and $D_s$ mesons), which decay to the final $J/\psi$ and $\psi'$ states exchanging one or more virtual mesons. A similar story would happen in decays of heavy bottomonium. To exactly follow and analyze the procedure in the experiment, we need to have knowledge about the coupling constants among the particles involved.

In the literature, there have been series of works on coupling constants such as $D^*D\pi$ [1, 2], $DD\rho$ [3], $DDJ/\psi$ [4], $D^*DJ/\psi$ [5], $D^*D^*\pi$ [6, 7], $D^*D^*J/\psi$ [8], $D_s^*D^*K$, $D_s^*DK$ [9, 10], $D_s^0D_sK$, $D_s^0DK$[10], $DD\omega$ [11], $D^*D^*\rho$ [12], $D^*D\rho$ [13], $B_s^0BK$, $B_s^{*}B^*K$ [14], $D_s^*DK^*$ (892), and $B_s^*BK^*$ (892) [15] in the framework of QCD sum rules (QCDSR) technique [16].

In this work, we calculate the $D_s^*DK$ and $B_s^*BK$ vertices using QCDSR (for details see [17]). These coupling constants belong to the low energy sector of QCD, which is far from perturbative regime. Therefore, for calculation of these coupling constants some nonperturbative methods are needed. QCDSR is one of the most promising and predictive one among all existing non-perturbing methods in studying the properties of hadrons.

The paper is organized as follows. In section II, the model is shortly described. In this section, we calculate the correlation function when both the $D(B)$ and $K$ mesons are off-shell. Then we obtain QCD sum rules for the strong coupling constants of the $D_s^* - D - K$ and $B_s^* - B - K$ vertices. Finally in section III, we numerically analyze the obtained strong coupling constant sum rules for the considered vertices. We will obtain the numerical values for each coupling constant when both the $D(B)$ and $K$ states are off-shell. Then taking the average of the two
off-shell cases, we will obtain final numerical values for each coupling constant. In this section, we also compare our result on $g_{D_sDK}$ with existing predictions in the literature [9].

2. QCD sum rules for the coupling constants

The aim of this section is to calculate the coupling constants $g_{D_sDK}$ and $g_{B_sBK}$ which characterize the $D_s^*DK$ and $B_s^*BK$ decays, respectively. We start by considering the three-point correlation function. The three-point function associated to $D_s^*DK(B_s^*BK)$ vertex for both D meson off-shell and $K$ meson off-shell states is given respectively by

\[
\Pi^{D(B)}_\mu(p', q) = i^2 \int d^4x \ e^{ip' \cdot x} \ e^{iq' \cdot y} \langle 0 | T \left( \eta^K \right) \eta^{D(B)}_\mu(y) \eta^{D_s(B_s^*)}_\mu(0) | 0 \rangle,
\]

\[
\Pi^K(p', q) = i^2 \int d^4x \ e^{ip' \cdot x} \ e^{iq' \cdot y} \langle 0 | T \left( \eta^{D(B)}(x) \eta^K(y) \eta^{D_s^*(B_s^*)}_\mu(0) | 0 \rangle.
\]

Here $T$ is the time ordering product. Each meson interpolating field can be written in terms of the quark field operators as following:

\[
\eta^K(x) = \bar{\psi}(x)\gamma_5 u(x),
\]

\[
\eta^{D(B)}_\mu(x) = \bar{\psi}(x)\gamma_\mu c(b)(x),
\]

\[
\eta^{D_s^*(B_s^*)}_\mu(x) = \bar{\psi}(x)\gamma_\mu c(b)(x).
\]

where $u$, $s$, $c$ and $b$ are the up, strange, charm and bottom quark field, respectively. Each current has the same quantum numbers of the associated meson.

According to the idea of the QCDSR, we should calculate this correlator both in terms of hadrons and in quark-gluon language, and then equate these representations. The first side, called phenomenological or physical side, is obtained using hadronic degrees of freedom. The second, so called QCD or theoretical side is calculated using quark and gluon degrees of freedom by the help of the operator product expansion (OPE) in deep Euclidean region.

Firstly, let us focus on the calculation of the physical side of the correlation function Eq.(1) for an off-shell D(B) meson. The physical part can be obtained by saturating Eq.(1) with the appropriate $D(B)$, $D_s^*(B_s^*)$ and $K$ states. After some straightforward calculation, we obtain:

\[
\Pi^{D(B)}_\mu(p', p) = \frac{\langle 0 | \eta^K | K(p') \rangle \langle 0 | \eta^{D(B)}_\mu | D(B)(q) \rangle \langle K(p')D(B)(q) | D_s^*(B_s^*)_\mu(p, \epsilon) \rangle \langle D_s^*(B_s^*)_\mu(p, \epsilon) | \eta^{D_s^*(B_s^*)}_\mu | 0 \rangle}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s^*(B_s^*)}^2)} + ...
\]

where .... represents the contribution of the higher states and the continuum.

The phenomenological side of the sum rule is defined in terms of meson masses, meson decay constants and coupling constants. We introduce the meson decay constants $f_K$, $f_{D_s^*(B_s^*)}$ and $f_{D(B)}$ defined by the following matrix elements.

\[
\langle 0 | \eta^K | K(p') \rangle = i \frac{m_K f_K}{m_u + m_s},
\]

\[
\langle 0 | \eta^{D(B)}_\mu | D(B)(q) \rangle = i \frac{m_{D(B)}^2 f_{D(B)}}{m_{c(b)} + m_u},
\]

\[
\langle D_s^*(B_s^*)_\mu(p, \epsilon) | \eta^{D_s^*(B_s^*)}_\mu | 0 \rangle = m_{D_s^*(B_s^*)} f_{D_s^*(B_s^*)} \epsilon^*_\mu,
\]

\[
\langle K(p')D(B)(q) | D_s^*(B_s^*)_\mu(p, \epsilon') \rangle = g_{D_sDK(B_sBK)}(p' - q) \cdot \epsilon,
\]

(5)
where $\epsilon$ are the polarization vectors associated with the $D_s^*(B_s^*)$.

Finally, using Eqs.(4)-(5), the physical side of the correlation function for an off-shell D(B) meson can be written as:

$$\Pi^D(B)(p',p) = \frac{\Pi^D_B(p,B)}{g^2_D(B^*B)}(q^2) \frac{f_{D^*_B}(B^*)f_{D^*_B}^2f_{D^*_B}m_D^2m_D^2}(q^2 - m_D^2)(p^2 - m_D^2)(p^2 - m_K^2)(m_B + m_u)(m_s + m_u)$$

$$\times \left\{ \left[ 1 + \frac{m_D^2 - q^2}{m_D^2} \right] p_\mu - 2p_\mu \right\}. \quad (6)$$

Similarly, we get the final expression of the physical side of the correlation function for an off-shell K meson as:

$$\Pi^K(p,p') = \frac{\Pi^K_B(p,K)}{g^2_D(B^*K)}(q^2) \frac{f_{D^*_B}(K^*)f_{D^*_B}^2f_{D^*_B}m_K^2m_D^2m_D^2}(q^2 - m_K^2)(p^2 - m_K^2)(p^2 - m_B^2)(m_B + m_u)(m_s + m_u)$$

$$\times \left\{ \left[ 1 + \frac{m_D^2 - q^2}{m_D^2} \right] p_\mu - 2p_\mu \right\}. \quad (7)$$

To calculate the coupling constant, we will choose the structure, $p_\mu$ from both sides of the correlation functions. Now, we concentrate on the QCD side, the correlation function is calculated at deep Euclidean space, where $p^2 \to -\infty$ and $p'^2 \to -\infty$ in terms of the operator product expansion. For this aim, each correlation function, $\Pi^QCD_i(p,p')$, where $i$ stands for $D(B)$ or $K$, can be written in terms of perturbative and non-perturbative parts as:

$$\Pi^QCD = \Pi_{per} + \Pi_{nonper}. \quad (8)$$

where the perturbative part is defined in terms of double dispersion integral as:

$$\Pi_{per} = \frac{1}{4\pi^2} \int ds' \int ds \rho(s,s',q^2)(s - p^2)(s' - p'^2) + \text{subtraction terms}, \quad (9)$$

where $\rho(s,s',q^2)$ is called spectral density. In order to obtain the spectral density, we need to calculate the bare loop diagram (a) and (d) in Fig.1 for $D(B)$ and $K$ off-shell, respectively. We calculate these diagrams in terms of the usual Feynman integration technique by the help of the Cutkosky rules, i.e., by replacing the quark propagators with Dirac delta function: $\frac{1}{q^2 - m^2} \to (-2\pi i)\delta(q^2 - m^2)$. After some straightforward calculations, we obtain the spectral densities as follows:

$$\rho^{D(B)}(s,s',q^2) = \frac{N_c}{2\sqrt{s}2(s,s',q^2)} \left\{ (m_u - m_s)(q^2 - s)(m_{c(b)} m_s^2 + m_u (s - m_s^2 - q^2)) - s' \left( - m_s^3 m_u + 2 m_{c(b)}^3 (m_u - m_s) - 2 m_s q^2 + m_{c(b)}^2 (2m_s m_u + q^2 - s) \right) + q^2 (s - q^2) + m_s m_u (s + q^2) + m_{c(b)} (m_s - m_u) (m_s^2 + q^2 + s) - s' (m_{c(b)}^2 - m_{c(b)} m_s + m_{c(b)} m_u + q^2) \right\}, \quad (10)$$

$$\rho^K(s,s',q^2) = \frac{N_c}{2\sqrt{s}2(s,s',q^2)} \left\{ (m_{c(b)} - m_u)(q^2 - s)(m_{c(b)} (m_{c(b)} - m_u) + m_u (m_s - m_u - q^2)) + s' \right\} + \left\{ m_{c(b)}^3 (m_s - m_u) + 2 m_{c(b)}^2 m_u + m_{c(b)} (-m_s m_u - 2 q^2) + m_{c(b)}^2 (q^2 - s) + q^2 (s - q^2) - m_s m_u (q^2 + s) + m_{c(b)}(-2 m_s^3 + 2 m_s^2 m_u + m_u (q^2 + s) + m_s (q^2 + s)) \right\} s' + (-m_{c(b)} m_s + m_s^2 + m_s m_u + q^2) s', \quad (11)$$
for the $D_s^*DK$ and $B_s^*BK$ vertex associated with the off-shell D and K meson, respectively. Here $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ac - 2bc - 2ab$ and $N_c = 3$ is the color number.

To calculate the nonperturbative contributions in QCD side, we consider the quark condensate diagrams presented in (b), (c), (e), (f), (g), (h), (i), (j), (k), (l), (m) and (n) parts of Fig. (1). There is also a numerically negligible contribution from the heavy quark condensates, which we will not take into account in this calculation. Therefore, for the nonperturbative part, we only encounter contributions coming from light quark condensates. Contributions of the diagrams (c), (e), (f), (g), (i), (j), (k), (l), (m) and (n) in Fig. (1) are zero since applying double Borel transformation with respect to both of the variables $p^2$ and $p'^2$ will kill them because only one variable appears in the denominator in these cases. Hence, we calculate the diagrams (b), (h) and (j) in Fig. (1) for the off-shell D(B) meson. As a result, we obtain:

$$\Pi^{D(B)}_{\text{nonper}} = -\langle \mathcal{M} \rangle \left\{ \frac{m_u}{4r} \frac{m_u}{2}, \frac{m_0^2}{r} + \frac{m_0}{r} \right\},$$

for the off-shell D and B meson and

$$\Pi^{K}_{\text{nonper}} = 0,$$

for the off-shell K meson. Here $r = p^2 - m_{c(b)}^2$ and $r' = p'^2 - m_u^2$.

Now, it is time to apply the double Borel transformations with respect to $p^2(p^2 \rightarrow M^2)$ and $p'^2(p'^2 \rightarrow M'^2)$ to the physical as well as the QCD sides and equate the coefficient of the
selected structure \( p_\mu \) from two representations. Finally, we get the following sum rules for the corresponding coupling constants:

\[
\begin{align*}
\frac{g_{D(B)}^{D(B')}}{f_{D(B')}^2 f_D f_B} & = \frac{(q^2 - m_{D(B)}^2)(m_{c(b)} + m_u)(m_s + m_u)}{1 + \frac{m_{D(B)}^2}{m_{B(B')}^2}} e^{-\frac{m_{D(B)}^2}{M^2}} \text{e}^{\frac{m_{B(B')}^2}{M^2}}, \\
\frac{g_{D(K)}^{D(K)}}{f_{D(K')}^2 f_D f_K} & = \frac{(q^2 - m_{D(K)}^2)(m_{c(b)} + m_u)(m_s + m_u)}{1 + \frac{m_{D(K)}^2}{m_{B(K')}^2}} e^{-\frac{m_{D(K)}^2}{M^2}} \text{e}^{\frac{m_{B(K')}^2}{M^2}}.
\end{align*}
\]

for the off-shell \( D(B) \) and \( K \) meson associated with the \( D^*_sDK(B^*_sBK) \) vertex, respectively.

The integration regions for the perturbative part in Eqs.(14)-(15) are determined requiring that the arguments of the three \( \delta \) functions coming from Cutkosky rule vanish simultaneously. So, the physical region of the \( s \) and \( s' \) planes are described by the following non-equalities:

\[
-1 \leq f_{D(B)}(s, s') = \frac{2 s (m_{c(b)}^2 - m_s^2 + s') + (m_{c(b)}^2 - m_{s}^2 - s)(-q^2 + s + s')}{\lambda^{1/2}(m_{c(b)}^2, m_{s}^2, s)\lambda^{1/2}(s, s', q^2)} \leq 1
\]

\[
-1 \leq f_{D(K)}(s, s') = \frac{2 s (-m_{c(b)}^2 + m_s^2 - s') + (m_{c(b)}^2 - m_{s}^2 + s)(-q^2 + s + s')}{\lambda^{1/2}(m_{c(b)}^2, m_{s}^2, s)\lambda^{1/2}(s, s', q^2)} \leq 1
\]

for the \( D(B) \) and \( K \) off-shell meson associated with the \( D^*_sDK(B^*_sBK) \) vertex, respectively. These physical regions are imposed by the limits on the integrals and step functions in the integrands of the sum rules. In order to subtract the contributions of the higher states and continuum, the quark-hadron duality assumption is used, i.e., it is assumed that,

\[
\rho_{\text{higher states}}(s, s') = \rho^{\text{PE}}(s, s')\theta(s - s_0)\theta(s' - s'_0)
\]

where \( s_0 \) and \( s'_0 \) are the continuum thresholds.

Note that, the double Borel transformation used in the calculations is written as:

\[
\hat{B} \frac{1}{(p^2 - m_1^2)^m} \frac{1}{(p^2 - m_2^2)^n} \rightarrow (-1)^{m+n} \frac{1}{\Gamma(m) \Gamma(n)} e^{-m_1^2/M^2} e^{-m_2^2/M^2} \frac{1}{(M_1^2)^m-1(M_2^2)^n-1}.
\]

3. Numerical analysis
This section is devoted to the numerical analysis of the sum rules for the coupling constant. To obtain numerical values of the considered coupling constants, the following input parameters are used in calculations: \( m_K = (0.493677 \pm 0.000016) \text{ GeV}, m_D = (1.86480 \pm 0.00014) \text{ GeV}, m_{D^*_s} = (2.1123 \pm 0.0006) \text{ GeV}, m_B = (5.2792 \pm 0.0003) \text{ GeV}, m_{B^*_s} = (5.4154 \pm 0.0014) \text{ GeV} \)
states with the same quantum numbers. Our numerical calculations show that in the regions
thresholds with respect to the Borel mass parameters in the working regions. The continuum thresholds,
are not completely arbitrary but they are correlated to the energy of the first excited states and continuum are sufficiently suppressed and the contributions coming from higher dimensions are small. As a result, we can show that D off-shell stabilizes for 
\( M^2 \leq 14 \text{ GeV}^2 \) and \( 4 \text{ GeV}^2 \leq M^2 \leq 15 \text{ GeV}^2 \) associated with the \( D^*_1DK \) vertex. Similarly, the regions, \( 14 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2 \) and \( 5 \text{ GeV}^2 \leq M^2 \leq 20 \text{ GeV}^2 \) for B off-shell, and \( 6 \text{ GeV}^2 \leq M^2 \leq 20 \text{ GeV}^2 \) and \( 5 \text{ GeV}^2 \leq M^2 \leq 15 \text{ GeV}^2 \) for K off-shell are obtained for the \( B^*_1BK \) vertex. The dependence of considered coupling constants on Borel parameters for different cases are shown in Figs. (2)-(9). From these figures, we see a good stability of the results with respect to the Borel mass parameters in the working regions. The continuum thresholds, \( s_0 \) and \( s'_0 \) are determined requiring that both the contributions of the higher states and continuum are sufficiently suppressed and the contributions coming from higher dimensions are small. As a result, we can show that D off-shell stabilizes for \( 8 \text{ GeV}^2 \leq M^2 \leq 25 \text{ GeV}^2 \) and \( 5 \text{ GeV}^2 \leq M^2 \leq 15 \text{ GeV}^2 \) and K off-shell for \( 6 \text{ GeV}^2 \leq M^2 \leq 15 \text{ GeV}^2 \) and \( 4 \text{ GeV}^2 \leq M^2 \leq 12 \text{ GeV}^2 \) associated with the \( D^*_1DK \) vertex. Similarly, the regions, \( 14 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2 \) and \( 5 \text{ GeV}^2 \leq M^2 \leq 20 \text{ GeV}^2 \) for B off-shell, and \( 6 \text{ GeV}^2 \leq M^2 \leq 20 \text{ GeV}^2 \) and \( 5 \text{ GeV}^2 \leq M^2 \leq 15 \text{ GeV}^2 \) for K off-shell are obtained for the \( B^*_1BK \) vertex. The dependence of considered coupling constants on Borel parameters for different cases are shown in Figs. (2)-(9). From these figures, we see a good stability of the results with respect to the Borel mass parameters in the working regions. The continuum thresholds, \( s_0 \) and \( s'_0 \) are not completely arbitrary but they are correlated to the energy of the first excited states with the same quantum numbers. Our numerical calculations show that in the regions \( (m_f+0.3)^2 \leq s_0 \leq (m_f+0.7)^2 \) and \( (m_f+0.3)^2 \leq s'_0 \leq (m_f+0.7)^2 \), respectively for the continuum thresholds \( s \) and \( s' \), our results have weak dependence on these parameters. Here, \( m_i \) is the mass of initial particle and the \( m_f \) stands for the mass of the final on-shell state.

Now, using the working region for auxiliary parameters and other input parameters, we would like to discuss the behavior of the strong coupling constants in terms of \( q^2 \). In the case of off-shell D meson related to the \( D^*_1DK \) vertex, our numerical result is described well by the following mono-polar fit parametrization

\[
\frac{g^{(D)}_{D^*_1DK}(Q^2)}{g^{(D)}_{D^*_1DK}(Q^2)} = \frac{8.76(\text{GeV}^2)}{Q^2 + 7.12(\text{GeV}^2)}
\]

where \( Q^2 = -q^2 \). The coupling constants are defined as the values of the form factors at \( Q^2 = -m^2_{\text{meson}} \) (see also [13]), where \( m_{\text{meson}} \) is the mass of the off-shell meson. Using \( Q^2 = -m^2_D \)

\[ g_{D^*_1DK}(Q^2) = 8.76(\text{GeV}^2) \]

\[ Q^2 + 7.12(\text{GeV}^2) \]

\[ g^{(D)}_{D^*_1DK}(Q^2) = \frac{8.76(\text{GeV}^2)}{Q^2 + 7.12(\text{GeV}^2)} \]

\[ g^{(D)}_{D^*_1DK}(Q^2) = \frac{8.76(\text{GeV}^2)}{Q^2 + 7.12(\text{GeV}^2)} \]

\[ g^{(D)}_{D^*_1DK}(Q^2) = \frac{8.76(\text{GeV}^2)}{Q^2 + 7.12(\text{GeV}^2)} \]
This result is consistent with the result obtained in \cite{9} as

\begin{equation}
    \frac{g^{(K)}_{D^*_s DK}}{g^{(B)}_{B^*_s BK}}(Q^2) = 3.55 \ e^{\frac{-Q^2}{2(\text{GeV})^2}} - 0.88.
\end{equation}

Using $Q^2 = -m_K^2$ in Eq.(21), $g^{K}_{D^*_s DK} = 2.79 \pm 0.24$ is obtained. Taking the average of two above obtained values, finally we get the value of the $g_{D^*_s DK}$ coupling constant as:

\begin{equation}
    g_{D^*_s DK}(Q^2) = (2.89 \pm 0.25).
\end{equation}

This result is consistent with the result obtained in \cite{9} as $g_{D^*_s DK} = 2.84 \pm 0.31$. 

**Figure 4.** $g^{K}_{D^*_s DK}(Q^2 = 1 \text{GeV}^2)$ as a function of the Borel mass $M^2$. The continuum thresholds, $s_0 = 6.83 \text{ GeV}^2$, $s_0' = 5.97 \text{ GeV}^2$ and $M^2 = 7 \text{ GeV}^2$ have been used.

**Figure 5.** $g^{K}_{D^*_s DK}(Q^2 = 1 \text{GeV}^2)$ as a function of the Borel mass $M^2$. The continuum thresholds, $s_0 = 6.83 \text{ GeV}^2$, $s_0' = 5.59 \text{ GeV}^2$ and $M^2 = 5 \text{ GeV}^2$ have been used.

**Figure 6.** $g^{B}_{B^*_s BK}(Q^2 = 1 \text{GeV}^2)$ as a function of the Borel mass $M^2$. The continuum thresholds, $s_0 = 34.99 \text{ GeV}^2$, $s_0' = 0.99 \text{ GeV}^2$ and $M^2 = 15 \text{ GeV}^2$ have been used.

**Figure 7.** $g^{B}_{B^*_s BK}(Q^2 = 1 \text{GeV}^2)$ as a function of the Borel mass $M^2$. The continuum thresholds, $s_0 = 34.99 \text{ GeV}^2$, $s_0' = 0.99 \text{ GeV}^2$ and $M^2 = 5 \text{ GeV}^2$ have been used.
Similarly, for $B_2^*BK$ vertex, our result for B off-shell is better extrapolated by the exponential fit parametrization,

$$g_{B_2^*BK}^{(B)}(Q^2) = 0.66 \ e^{-\frac{Q^2}{23.34(GeV^2)}} + 0.23$$

(23)

and for K off-shell case, the parametrization is

$$g_{B_2^*BK}^{(K)}(Q^2) = 4.39 \ e^{-\frac{Q^2}{4.02(GeV^2)}} - 1.03.$$  (24)

Using $Q^2 = -m_B^2$ in Eq.(23), the coupling constant is obtained as $g_{B_2^*BK}^{(B)} = 2.40 \pm 0.22$. Also $g_{B_2^*BK}^{(K)} = 3.62 \pm 0.34$ is obtained at $Q^2 = -m_K^2$ in Eq.(24). Taking the average of these results, we get the following result

$$g_{B_2^*BK}(Q^2) = (3.01 \pm 0.28).$$  (25)

The errors in the results are due to the uncertainties in determination of the working regions for the auxiliary parameters as well as the errors in the input parameters.

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