Functional renormalization group approach to SU(N) Heisenberg models: 
Momentum-space RG for the large-N limit

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In frustrated magnetism, making a stringent connection between microscopic spin models and macroscopic properties of spin liquids remains an important challenge. A recent step towards this goal has been the development of the pseudofermion functional renormalization group approach (pf-FRG) which, building on a fermionic parton construction, enables the numerical detection of the onset of spin liquid states as temperature is lowered. In this work, focusing on the SU(N) Heisenberg model at large N, we extend this approach in a way that allows us to directly enter the low-temperature spin liquid phase, and to probe its character. Our approach proceeds in momentum space, making it possible to keep the truncation minimalistic, while also avoiding the bias introduced by an explicit decoupling of the fermionic parton interactions into a given channel. We benchmark our findings against exact mean-field results in the large-N limit, and show that even without prior knowledge the pf-FRG approach identifies the correct mean-field decoupling channel. On a technical level, we introduce an alternative finite temperature regularization scheme that is necessitated to access the spin liquid ordered phase. In a companion paper [1] we present a different set of modifications of the pf-FRG scheme that allow us to study SU(N) Heisenberg models (using a real-space RG approach) for arbitrary values of N, albeit only up to the phase transition towards spin liquid physics.

I. INTRODUCTION

In the field of quantum magnetism, the Heisenberg model has long served as one of the most elementary microscopic frameworks to capture a wide range of magnetic phenomena. Of particular interest are Heisenberg antiferromagnets that easily experience frustration effects by competing exchange interactions or geometric constraints which inhibit the formation of a conventional Néel ordered ground state. Instead, quantum spin liquid (QSL) states can form – collective quantum states, in which the elementary magnetic moments do not order and remain strongly fluctuating, albeit in a correlated manner. QSLs have caught the imagination of condensed matter physicists ever since their theoretical inception some four decades ago, though it has long remained hard to unambiguously identify these elusive states in candidate materials or microscopic model systems. Today, a quantum spin liquid state is best characterized and positively discriminated from more conventionally ordered ground states by (i) the formation of macroscopic entanglement and (ii) the simultaneous emergence of fractionalized degrees of freedom and gauge fields.

The connection between a particular spin liquid state and its parent spin Hamiltonian is still fragile, and it remains an open challenge to derive the above characteristic macroscopic phenomena related to spin liquid physics directly from microscopic Heisenberg spin models. To overcome this obstacle, one would like to explore the properties of candidate spin systems starting from their microscopic description over all length scales down to the very long-wavelength physics, where spin liquid behavior is presumed to occur. A natural framework for such a description is provided by renormalization group (RG) methods, where the low-energy phenomenology is systematically approached upon successively integrating out short-wavelength fluctuations by lowering the RG momentum scale \( \Lambda \). Specifically for spin models, a pseudofermion functional renormalization group (pf-FRG) has been developed, that applies the established functional renormalization group approach to fermion systems on the level of auxiliary (pseudo) fermions introduced in a spin decomposition (or parton construction). This purely fermionic RG approach treats all interaction channels on equal footing, and therefore avoids a bias which is introduced, for instance, by choosing a specific mean-field decoupling. The applications of this pf-FRG approach have provided indications for spin-liquid states in a number of frustrated quantum magnets. However, the evidence for spin liquid physics in the pf-FRG approach is typically an indirect one – such as the absence of a singularity in the RG flow of the spin susceptibility (which would otherwise indicate the onset of magnetic ordering). An unambiguous, positive identification of spin liquid physics as well as a detailed characterization of a putative spin liquid ground state has so far remained elusive within the pf-FRG approach. The reason for this is twofold:

(1) A number of spin liquids are not characterized by an order parameter that is bilinear in the fermion fields, and as such do not exhibit an obvious ordering transition. Instead their nature reveals itself in fermionic four-point correlation functions and their momentum structures, which however remain subtle and hard to detect. Examples of such spin liquids are gapless quantum spin liquids with spinon Fermi surfaces, Majorana Fermi surfaces, or Bose metal.

(2) Bilinearly ordered spin liquids, in contrast, are bound to exhibit a divergence in the fermion four-point function at some finite RG scale \( \Lambda_{\text{div}} \), as discussed in a companion paper of this manuscript. While the divergence itself indicates the onset of a new phase, it inhibits an analysis of the RG flow for scales \( \Lambda < \Lambda_{\text{div}} \) and as such a more detailed characterization of the emergent spin liquid.

A common way to circumvent the second problem is to introduce an order parameter field by means of a Hubbard-Stratonovich transformation (bosonization) and include its
correlation functions into the RG analysis. While this approach works very well if the structure of the order parameter is known, it introduces a questionable bias if this is not the case. Since spin liquid order parameters can be rather unconventional and often vary only from one another by subtle changes in their symmetry structure, it is much more desirable to have a computational framework that distills this subtle information in the RG flow than vice versa (probing potential order parameters). As such the plain Hubbard-Stratonovich strategy cannot be faithfully applied in the context of spin liquid physics.

In this manuscript, we will proceed to describe precisely such an unbiased computational approach and implement a pf-FRG scheme that allows to positively identify and characterize spin liquid physics in SU(N) Heisenberg models. To do so, we adapt an approach first applied in the context of fermionic FRG calculations and introduce an infinitesimal symmetry breaking initial condition. The explicit symmetry breaking regularizes the divergence of the fermion correlation function if it addresses the correct symmetry pattern and thus allows to compute infrared observables for scales \( \Lambda < \Lambda_{\text{div}} \). Conversely, if the chosen infinitesimal symmetry breaking term was not correct, it will remain infinitesimal throughout the entire flow. As such, our novel approach allows us to unite the advantages of both, the fermionic and bosonized strategies outlined above.

**Overview of results**

The main result of the manuscript at hand is an extension of the pseudofermion functional renormalization group (pf-FRG) formalism to SU(N) Heisenberg models, for which we demonstrate that, for the first time, we can run the RG flow deep into spin liquid phases. Example results for the spin-1/2 SU(N) Heisenberg antiferromagnet on the square lattice substantiate the applicability of this extended pf-FRG framework and make contact to exact results in the limit. So far, it has remained elusive to directly access the spin liquid phase within the pf-FRG framework, which in its original real-space formulation (adapted to SU(N) Heisenberg models in a companion manuscript) is valid only up to the onset of ordering (either in the magnetic or spin liquid sector) where the flow of the RG equations is rendered invalid by a divergence of the fermionic four-point function. Here, we devise a simplified truncation scheme that analytically reproduces this behavior in terms of a single running coupling in momentum space. To expand the validity of the pf-FRG description deep into the spin liquid phase, we introduce a small explicit symmetry breaking term in the ordering channel that allows to effectively regularize the divergence. Essential information about zero temperature observables, such as the gap and magnetic susceptibility, is then obtained from the solution of only two RG equations. Furthermore, we exhibit the absence of long-range magnetic ordering by showing that magnetic order parameters neither grow nor regularize the divergence. The bias introduced by this procedure is thus under systematic control. As opposed to the Hubbard-Stratonovich approach, no fluctuating boson field is introduced, and the description of the system is entirely in terms of fermionic fields and their fluctuations. We show explicitly that this implies a positive discrimination between possible ordering channels, since an inappropriate choice of the symmetry breaking pattern does not regularize the fermionic divergence and thus predicts its own unsuitability.

In previous pf-FRG approaches, a cutoff scheme has been proposed, where temperature and running scale parameter could be identified with each other via a simple linear relation. Here we show that this relation breaks down inside the ordered phase. Instead we propose to access the finite temperature regime by an alternative cutoff scheme. By this procedure, we are able to exactly reproduce mean-field observables for any given temperature. The application of this cutoff scheme furthermore opens up a strategy to exactly implement fermion number constraints by means of Popov-Fedotov chemical potentials.

The discussion of our results in the remainder of this manuscript is structured as follows. In Sec. II we introduce the staggered flux spin liquid, a well-understood and simple model system that will be used to benchmark our pf-FRG extension. Sec. III introduces a pf-FRG approach in momentum-space tailored for simplicity and quick analysis of symmetry breaking patterns. It is then benchmarked in the symmetric regime. In Sec. IV the symmetry broken regime is accessed within a zero-temperature formalism, which already partially reproduces the results from Sec. II. Furthermore, the unbiasedness of the method is demonstrated by analyzing a different symmetry breaking pattern. As a limitation of this \( T = 0 \) approach, we exhibit that the linear relation between the RG scale parameter and physical temperature breaks down inside of the spin liquid phase. Therefore, finite temperature observables cannot be extracted reliably. To overcome this limitation, Sec. V then addresses the problem of finite-temperature observables inside the symmetry broken regime by implementing a finite-temperature cutoff scheme. Finally, we demonstrate how the fermion number constraint, that provides the relation to the original spin model, is implemented exactly within our pf-FRG scheme by means of the Popov-Fedotov method. Conclusions are finally drawn in Sec. VI, followed by some appendices providing details of the computations.

**II. MODEL SYSTEM: STAGGERED-FLUX SPIN LIQUID**

To establish controlled access to spin liquid phases by means of pf-FRG, we study a well-understood realization of the Heisenberg model. Here, we introduce our model and briefly recapitulate known results. Our starting point is the Hamiltonian of the nearest-neighbor SU(N)-symmetric Heisenberg model on the two-dimensional square lattice

\[
\mathcal{H} = \frac{J}{N} \sum_{(ij)} \mathbf{S}_i \cdot \mathbf{S}_j \tag{1}
\]

where the components \( S_i^\mu \) of the SU(N) spin operators \( \mathbf{S}_i \) carry the site indices \( i, j \) and \( \mu \in \{1, \ldots, N^2 - 1\} \). The spins
interact via the antiferromagnetic exchange coupling \( J > 0 \).

For a concrete calculation, a symmetry group and representation for the spin operators \( S_i^\mu \) must be chosen. The generic SU(2) group in its fundamental representation is known to exhibit a (classical) antiferromagnetically ordered ground state at vanishing temperature. Several different symmetry groups and representations have been investigated in the past in order to suppress and enhance different types of fluctuations, eventually leading to different ground states as well. For SU(\( N \)) spins in their fundamental spin-\( 1 \) representation, it was found that the classical antiferromagnet can give way to a “staggered flux spin liquid” phase for \( N \rightarrow \infty \). In fact, this result is intimately tied to a representation of the spin operators by Abrikosov pseudofermions.

\[
S_i^\mu = f_i^\dagger \sigma^{\mu\nu} x_i^\nu, \quad f_i^\dagger = \frac{N}{2},
\]

with \( T^\mu \) being the \( N \)-dimensional representation of the generators. Summation over equal spin indices is implied. The number constraint in Eq. (2) ensures that the Hilbert space of the pseudofermions is not enlarged w.r.t. the one of the original spin operators.

The partition function of the model can now be written as \( Z = \int D[f, \chi] \exp \{-S[f] + S_c[\lambda]\} \) with the action

\[
S[f] = \int_\tau \left[ \sum_i f_i^\dagger f_i - J_\tau \sum_{ij} f_i^\dagger f_j^\dagger f_j f_i \right],
\]

where \( \int_\tau := \int_0^\beta d\tau \) and we used commutation relations to rearrange the pseudofermions in the four-fermion interaction coming from the Hamiltonian in Eq. (1). Further, we defined

\[
S_c[\lambda] = i \int_\tau \sum_i \lambda_i \left( f_i^\dagger f_i - \frac{1}{2} \right),
\]

to impose the fermion number constraint, cf. Eq. (2). The four-fermion interaction term can be decoupled by means of a Hubbard-Stratonovich transformation, introducing the spin-singlet site-nonlocal boson field \( Q_{ij} \sim f_i^\dagger f_j \). It can be seen as an order parameter field for the local U(1) symmetry that was introduced artificially when representing the Heisenberg Hamiltonian in terms of the pseudofermions.

After Hubbard-Stratonovich transformation, the partition function reads

\[
Z = \int D[f, Q_m, \chi] \exp \left\{ \int_\tau \left[ -\sum_i f_i^\dagger f_i + S_c[\lambda] \right] + S_c[\lambda] \right\}.
\]

Since the action is now quadratic in the fermion fields, the latter can be integrated out, resulting in a new action \( S_Q \) and the gap equations

\[
\frac{\delta S_Q}{\delta Q^\dagger} = \frac{\delta S_Q}{\delta Q} = \frac{\delta S_Q}{\delta \lambda} = 0.
\]
III. MOMENTUM-SPACE FRG: SYMMETRIC REGIME

Let us now set up a pf-FRG scheme which fully incorporates the exact large-\(N\) results from Sec. IIIB. Part of this scheme has already been realized in Ref. [1] where the symmetric regime was investigated. The goal of this work is to extend the pf-FRG into the symmetry broken regime by introducing symmetry breaking initial conditions. It is, of course, also possible to implement the latter in the position space pf-FRG scheme used in Ref. [1]. Here, we do not follow this path mainly for two reasons: Firstly, the numerical cost for even the simplest possible realization of a spatially non-local order parameter as required by the spin liquid description in Sec. II is very high. Secondly, in more realistic cases, it is in general not obvious what the particular structure of the prospective spin liquid order parameter is. One would therefore have to test different order parameter structures to find the physically realized one, further increasing the cost.

We therefore strive to develop a simplified framework which provides the opportunity to conduct comparatively quick and cheap tests. Furthermore, such a framework provides additional analytical insight as the reduction to only a few running couplings is bound to expose the decisive physical ingredients. To that end, we apply momentum space functional renormalization group for the flowing effective average action \(\Gamma_A\) as given by the Wetterich equation[11]

\[
\partial_A \Gamma_A = \frac{1}{2} \int \sum_i f_i^\dagger \partial_A \mathcal{P}_A f_i + \frac{1}{2} \text{STr} \left[ \frac{\partial \mathcal{P}_A}{\Gamma_A} \right].
\] 

(9)

Here, \(A\) is the RG scale parameter. For \(A \to A_{UV}\), it is identical to the microscopic action given in eq. (5). For \(A \to 0\), it becomes the full effective action of the model. \(\mathcal{P}_A\) is the free inverse propagator, including a (multiplicative) regularization function (see below). The FRG equation (9) itself is exact. In order to solve it, however, we have to employ approximations. Primarily, this means a restriction of \(\Gamma_A\) with respect to the infinite number of terms which are compatible with the symmetries of the model.

Before we implement symmetry breaking initial conditions, we consider a symmetric version of our simplified framework in order to introduce our simplifications and check, whether results from real space pf-FRG such as the critical temperature are reproduced correctly. Our ansatz for the effective average action follows the action defined in Eq. (3) and in frequency-momentum space is given by

\[
\Gamma_A^\text{sym} = \int_{1\in FZ} f_i^\dagger \mathcal{P}_A f_i - \frac{J_A}{N} \int_{1...4\in FZ} \left[ \cos(k_{2,x} - k_{3,x}) + \cos(k_{2,y} - k_{3,y}) \right] f_1^\dagger f_2 f_3^\dagger f_4 \delta_{1234},
\]

(10)

where \(1...4\) correspond to combined frequency and momentum variables \((\omega_1, k_1), ..., (\omega_4, k_4)\) and we have introduced the shorthand notations \(\int_{1\in FZ} := \int_{\omega_1} f_{1\in FZ}^\dagger f_{1\in FZ}\) for the frequency and momentum integration as well as \(\delta_{1234} := \delta(1-2+3-4)\). The momenta are integrated over the full first Brillouin zone (FZ) of the square lattice and we limit ourselves to \(T = 0\) for the time being. Furthermore, the free propagator

\[
\mathcal{P}_A = i \omega \cdot \partial_A^{-1}
\]

(11)

is equipped with the inverse multiplicative cutoff function \(\partial_A^{-1}\) which will be specified below.

Compared to the position-space ansatz of Ref. [1] drastic simplifications have been made: the four-point function is reduced to one single running coupling \(J_A\) independent of frequency and momentum. The running of the two-point function is neglected. As a result, the unique projection rule for the \(\beta\) function of the four-point function is particularly simple,

\[
\partial_A J_A (f_\alpha^\dagger f_\alpha f_\beta^\dagger f_\beta) \Omega = -\frac{N}{2} \partial_A \Gamma_A^\text{sym,4f}.
\]

(12)

Here, \(\Omega\) is the space-time volume and \(\Gamma_A^\text{sym,4f}\) is the four-fermion sector of the effective average action or the Wetterich equation, respectively. \(\partial_A J_A\) is then obtained by comparison of coefficients. Furthermore, the projection rule is evaluated for \(f_\alpha\) at vanishing frequencies and momenta, implemented by \(f_\alpha^{(1)} = f_\alpha^{(1)} \delta(\omega_\alpha)\delta(k_\alpha)\). This simplifies the non-trivial momentum structure of the four-fermion sector.

Employing a fluctuation expansion of the Wetterich equation, the large-\(N\) \(\beta\) function can be extracted as

\[
\partial_A J_A = -J_A^2 \partial_A \int_\omega \frac{\delta^2}{(i\omega)^2} = -\frac{J_A^2}{\pi\Lambda^2}.
\]

(13)

In order to arrive at the last identity in eq. (13), the cutoff function \(\partial_A\) had to be specified. Since we work at \(T = 0\), we employ a zero-temperature step function cutoff

\[
\partial_A = \Theta(|\omega| - \Lambda), \quad \partial_A \partial_A = -\delta(|\omega| - \Lambda).
\]

(14)

Eq. (13) can now be integrated directly, yielding

\[
J_A = \frac{\pi\Lambda}{\pi\Lambda - 1}.
\]

(15)

If not stated otherwise, the initial value will be set to \(J_A \to J_A \to \infty = 1\) for convenience. The breakdown of the flow at \(\Lambda_{ab} = \pi^{-1}\) indicates the onset of spontaneous symmetry breaking.

Since the cut-off frequency integral can be interpreted as an approximation to a Matsubara sum, it is possible to identify the symmetry breaking scale \(\Lambda_{ab}\) with a critical temperature[12]. This identification can, at least in the symmetric phase, be used to calculate observables such as the magnetic susceptibility. The precise conversion factor, however, may depend on the structure of the \(\beta\)-function and should therefore be determined case by case, see below.

The mean-field results, cf. Fig. [1] we want to compare to are obtained with the fermion number constraint released completely. Thus, we can also perform an FRG calculation at finite fermion Matsubara temperature. In order to do so, we...
The scale derivative of the cutoff, \( \Lambda \), has to be taken into account in the RG flow.

The r.h.s. of this equation is not continuous and we solve it directly for the square lattice antiferromagnet at hand. To that end, we introduce the term

\[
\Gamma_{\Lambda}^{\text{sb}} = \sum_{(ij)} Q_{\Lambda} \left[ f_{i\alpha} f_{j\gamma} + f_{i\gamma} f_{j\alpha} \right],
\]

into the ansatz for the effective average action \( \Gamma_{\Lambda} = \Gamma_{\Lambda}^{\text{sym}} + \Gamma_{\Lambda}^{\text{sb}} \) which breaks the local \( U(1) \) symmetry explicitly. Importantly, \( \Gamma_{\Lambda}^{\text{sb}} \) is not introduced by means of a Hubbard-Stratonovich transformation. The four-fermion interaction term \( J_\Lambda \) is present in \( \Gamma_{\Lambda}^{\text{sb}} \) and unaltered with respect to \( \Gamma_{\Lambda}^{\text{sym}} \). In the limit \( Q_{\Lambda} \to \infty = 0 \), the symmetric ansatz from Eq. (10) is recovered.

For finite \( Q_{\Lambda} \to \infty \), the term \( \Gamma_{\Lambda}^{\text{sb}} \) effectively becomes a contribution to the two-point correlation function or self-energy. Building on Ref. [3] it was shown in Ref. [1] that a truncation of \( \Gamma_{\Lambda} \) including terms up to fourth order in the fermion fields and involving Katin’s improvement scheme[29], the self-consistent gap equation for the order parameter can be reproduced exactly from the RG flow equations. Since \( Q_{\Lambda} \) is effectively nonlocal in space, the considerations from Ref. [1] are directly applicable. We can therefore expect, that \( Q_{\Lambda} \to 0 = Q_{\text{mf}}(T = 0) \), thus reproducing the mean-field result for the magnitude of the order parameter from Sec. [1]. Furthermore, the presence of a finite \( Q_{\Lambda} \) is expected to regularize the finite-\( \Lambda \) divergence of the running fermionic coupling(s) due to spontaneous symmetry breaking: since the symmetry is already broken explicitly, no non-analytic behavior can occur.

With respect to the quartic fermion sector of the simplified ansatz in Eq. (10), there is a subtlety to be considered. By introducing the self-energy term, Eq. (18), the fermion propagator becomes effectively nonlocal (bilocal) in position space. However, the simplified ansatz \( \Gamma_{\Lambda}^{\text{sym}} + \Gamma_{\Lambda}^{\text{sb}} \) does not fully account for this so far. In particular, the process depicted in Fig. [2] contributes to leading order at large \( N \) and generates the new vertex structure

\[
\Gamma_{\Lambda}^{I_{\Lambda}} = \frac{I_{\Lambda}}{N} \int \sum_{(ij)} f_{i\alpha}^\dagger f_{j\alpha} f_{i\beta}^\dagger f_{j\beta} \cos(k_{2,x} + k_{4,x}) + \cos(k_{2,y} + k_{4,y}) \delta_{1234}.
\]

Comparing the momentum space vertices in Eqs. (10) and (19), it becomes clear that their only difference is provided by the momentum structure itself. The projection onto specific momentum structures is rather cumbersome and we therefore follow a different path here: The lattice geometry and symmetry of the Heisenberg interaction, cf. Eq. (1), permit a partition of the original square lattice into two sublattices [33] \( A \) and \( B \). Making use of this partition explicit by introducing spinors \( \Psi_{1,\alpha} = \left( f_{1,\alpha}^\dagger f_{2,\alpha}^\dagger \right) \), a new \( 2 \times 2 \) sublattice space is created, parametrized by the matrices \( 1, \sigma_x/y/z \) or \( \sigma_x = \frac{1}{2}(\sigma_x \pm i\sigma_y) \), respectively. The full momentum space ansatz for the effective average action is then given by

\[
\Gamma_{\Lambda}^{I_{\Lambda}} = \sum_{(ij)} Q_{\Lambda} \left[ f_{i\alpha} f_{j\alpha} + f_{i\alpha}^\dagger f_{j\alpha} \right],
\]
\[
\Gamma_A = \int_{1 \in \mathbb{L}^2} \left\{ \text{const} \left( \Psi_{1 \alpha}^\dagger \Psi_{1 \alpha} + 2Q_\Lambda \left( \Psi_{1 \alpha}^\dagger \sigma_+ \Psi_{1 \alpha} \right) \right) \left[ \cos(k_x) + \cos(k_y) \right] \right\}
\]

\[
- \frac{2J_A}{N} \int_{1 \ldots 4 \in \mathbb{L}^2} \left( \Psi_{1 \alpha}^\dagger \sigma_+ \Psi_{2 \alpha} \right) \left( \Psi_{3 \beta}^\dagger \sigma_- \Psi_{4 \beta} \right) \left[ \cos(k_{2,x} - k_{3,x}) + \cos(k_{2,y} - k_{3,y}) \right] \delta_{1234}
\]

\[
- \frac{J_A}{N} \int_{1 \ldots 4 \in \mathbb{L}^2} \left( \Psi_{1 \alpha}^\dagger \sigma_+ \Psi_{2 \alpha} \right) \left( \Psi_{3 \alpha}^\dagger \sigma_- \Psi_{4 \alpha} \right) \left[ \cos(k_{2,x} + k_{4,x}) + \cos(k_{2,y} + k_{4,y}) \right] \delta_{1234}
\]

\[
- \frac{J_A}{N} \int_{1 \ldots 4 \in \mathbb{L}^2} \left( \Psi_{1 \alpha}^\dagger \sigma_- \Psi_{2 \alpha} \right) \left( \Psi_{3 \alpha}^\dagger \sigma_+ \Psi_{4 \alpha} \right) \left[ \cos(k_{1,x} + k_{3,x}) + \cos(k_{1,y} + k_{3,y}) \right] \delta_{1234},
\]

where LZ in Eq. (21) denotes the first Brillouin zone of either one of these two sublattices (“little zone”).

By means of the sublattice ansatz, the two vertices \( \sim J_k \) and \( \sim J_k \) now become algebraically distinguishable due to their different structure in sublattice space. The projection rules for the running couplings are

\[
\langle \partial_\Lambda Q_\Lambda \rangle \langle \Psi_{1 \alpha}^\dagger \sigma_+ \Psi_{1 \alpha} \rangle = \frac{1}{4 \Omega} \partial_\Lambda \Gamma_\Lambda^\sigma_{\alpha},
\]

\[
\langle \partial_\Lambda J_\Lambda \rangle \langle \Psi_{1 \alpha}^\dagger \sigma_+ \Psi_{1 \alpha} \rangle \langle \Psi_{\beta}^\dagger \sigma_- \Psi_{\beta} \rangle = -\frac{N}{4 \Omega} \partial_\Lambda \Gamma_\Lambda^\sigma_{\alpha} \sigma_{\beta},
\]

\[
\langle \partial_\Lambda I_\Lambda \rangle \left[ \langle \Psi_{1 \alpha}^\dagger \sigma_+ \Psi_{1 \alpha} \rangle^2 + \langle \Psi_{1 \alpha}^\dagger \sigma_- \Psi_{1 \alpha} \rangle^2 \right] = -\frac{N}{2 \Omega} \partial_\Lambda \Gamma_\Lambda^\sigma_{\alpha} \sigma_{\alpha}.
\]

The actual flow equations can, again, be extracted by comparison of coefficients.

A straightforward calculation, cf. App. A, now leads to the respective \( \beta \) functions. More conveniently, considering

\[
X_\Lambda = J_\Lambda + I_\Lambda, \quad Y_\Lambda = J_\Lambda - I_\Lambda,
\]

instead of the original couplings, the flow equations for \( X_\Lambda \) and \( Y_\Lambda \) decouple. This greatly simplifies further analyses, in particular, as the flow of \( Q_\Lambda \) is independent of \( Y_\Lambda \) as well. The \( \beta \) functions are finally given by

\[
\partial_\Lambda Q_\Lambda = -\frac{X_\Lambda}{\pi} \int_{p \in \mathbb{FZ}} \frac{Q_\Lambda \alpha^2}{4Q_\Lambda^2 \alpha^2 + \Lambda^2},
\]

\[
\partial_\Lambda X_\Lambda = \frac{X_\Lambda^2}{\pi} \int_{p \in \mathbb{FZ}} \left( \frac{4Q_\Lambda \alpha^2 - \Lambda \partial_\Lambda Q_\Lambda \alpha - \Lambda^2}{[4Q_\Lambda^2 \alpha^2 + \Lambda^2]^2} \right),
\]

and

\[
\partial_\Lambda Y_\Lambda = \frac{Y_\Lambda^2}{\pi} \int_{p \in \mathbb{FZ}} \left\{ \frac{\Lambda \alpha^2 (\partial_\Lambda Q_\Lambda)}{Q_\Lambda [4Q_\Lambda^2 \alpha^2 + \Lambda^2]} - \frac{4Q_\Lambda^2 \alpha^4 + \Lambda^2 \alpha^2}{[4Q_\Lambda^2 \alpha^2 + \Lambda^2]^2} \right\}
\]

\[
+ \alpha \partial_\Lambda Q_\Lambda \left\{ \frac{\arctan \left( \frac{\Lambda}{2Q_\Lambda \alpha} \right)}{2Q_\Lambda^2 \alpha^2} \right\},
\]

where \( \alpha \equiv |\cos px + \cos py| \). Note that Katanin corrections to the running of the four-fermion couplings have been included, cf. App. A, Eqs. (24) and (25), can now be solved by standard numerical techniques. In Fig. 3, the results for the order parameter \( Q_\Lambda \), converted into \( \tilde{Q}(T) \), are shown for different initial values \( Q_\Lambda \rightarrow \infty \) and compared to the mean-field result for \( \varphi = 0 \), cf. Fig. 1.

For \( Q_\Lambda \rightarrow \infty \rightarrow 0 \), the results for \( T_c \) as well as \( Q(T = 0) \) converge towards the mean-field result as expected. The reproduction of the critical temperature hereby serves as a sanity check, since the reduced system, Eq. (15), already provided this feature. The correct magnitude of the order parameter at vanishing temperature ensures that the interpretation of \( Q_\Lambda \) as order parameter is meaningful.

Interestingly, the simple linear correspondence between temperature and RG scale does not hold in the symmetry broken regime anymore. As apparent from Fig. 3, the approach of \( Q_\Lambda \rightarrow 0 \) follows a different law compared to \( Q_{\text{mf}}(T \rightarrow 0) \). A similar result has previously been found by the authors of Ref. [51] with respect to a charge density wave ordered system. In order to understand this behavior, we again have to think of the correspondence between \( \Lambda \) and \( T \) as being due to a simple integral approximation of the Matsubara sum. This works in the symmetric phase but breaks down in the symmetry broken regime. The relation may be rendered nonlinear inside the ordered phase due to the presence of the non-trivially \( \Lambda \)-dependent coupling \( Q_\Lambda \) in the fermionic propagator.

This finding is important for the interpretation of finite-temperature observables obtained from a zero temperature pF-FRG analysis. As long as symmetries are asymptotically preserved (i.e., for \( Q_\Lambda \rightarrow \infty \rightarrow 0 \)), a simple correspondence can be found. As soon as spontaneous symmetry breaking occurs, however, a linear interpretation of \( \Lambda \) as an effective tempera-
ture is not possible anymore. Reliable values for finite temperature observables such as, e.g., the magnetic susceptibility, can only be recovered by employing true Matsubara frequencies and adapted cutoff functions $\partial_\Lambda$ such as Eq. (16). We come back to this point in more detail in Sec. [VI].

There is another detail of the results presented in Fig. [V] that deserves attention. Comparing free energies, the mean field calculation shows that the phase with $\varphi = 0$ is not the true ground state, but rather the one with $\varphi = \pi/2$. Technically, our RG analysis has therefore not brought us into a true spin liquid phase but rather into a BZA resonating valence bond phase. This is understood as follows: Both BZA and staggered-flux spin liquid phases are characterized by spontaneous breaking of the local U(1) symmetry of the fermions. They even share the same critical temperature. The additional breakdown of lattice-translational invariance and/or isotropy associated with the spin liquid cannot be represented in the stripped-down ansatz, Eq. (21). The possibility of translation symmetry breaking could be implemented by the inclusion of nontrivial momentum dependencies of $Q_\Lambda$, $J_\Lambda$, $I_\Lambda$ into our scheme.

Besides the possibility to fine-grain the momentum structure of couplings in the present approach, once real space pf-FRG schemes are developed which are able to efficiently deal with explicitly broken local symmetries, this problem will automatically be overcome by the very nature of the position space ansatz and its inherent structural complexity. For now, we leave this to future work. The goal of the present approach is, instead, to efficiently probe, identify and characterize the mechanism driving the general behavior of the system with respect to its symmetries.

B. Magnetic channel

One of the main reasons to employ pf-FRG directly, instead of a bosonized flow is the unbiased nature of the former. In a fully momentum and frequency dependent pf-FRG scheme, the structure of the four-Fermi vertex can faithfully monitor the evolution of complicated and even competing ordering processes. In contrast, a Hubbard-Stratonovich transformation at the UV scale essentially focuses on one particular channel. Spontaneous symmetry breaking in a different one may go completely unnoticed and a possible competition between channels may be lost, at least partially, unless other Hubbard-Stratonovich fields are additionally considered.

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Let us begin with setting up a finite temperature description. Since the number constraint is effectively removed from the strong hint towards ordering in a different channel. One can then further explore the situation by including more/different symmetry breaking terms until the decisive one is found. In this sense, the unbiased nature of the fermionic field calculation shows that the phase with $\varphi = 0$ is not the true spin liquid phase but rather into a BZA resonating valence bond phase. This is understood as follows: Both BZA and staggered-flux spin liquid phases are characterized by spontaneous breaking of the local U(1) symmetry of the fermions. They even share the same critical temperature. The additional breakdown of lattice-translational invariance and/or isotropy associated with the spin liquid cannot be represented in the stripped-down ansatz, Eq. (21). The possibility of translation symmetry breaking could be implemented by the inclusion of nontrivial momentum dependencies of $Q_\Lambda$, $J_\Lambda$, $I_\Lambda$ into our scheme.

Besides the possibility to fine-grain the momentum structure of couplings in the present approach, once real space pf-FRG schemes are developed which are able to efficiently deal with explicitly broken local symmetries, this problem will automatically be overcome by the very nature of the position space ansatz and its inherent structural complexity. For now, we leave this to future work. The goal of the present approach is, instead, to efficiently probe, identify and characterize the mechanism driving the general behavior of the system with respect to its symmetries.
mean-field calculation of Sec. II, anyway, we will be able to compare results directly. On a formal level, the only thing that needs to be done is implementing the finite-temperature cutoff, Eq. (16), and compactifying the imaginary time axis with circumference $\beta = \frac{1}{T}$. Ignoring the flow of $Y_\Lambda$ as being of minor importance for the observables to be considered, the flow equations are then given by

$$
\partial_\Lambda Q_\Lambda = -\frac{X_\Lambda}{2} \int_{p \in FZ} \frac{\pi T^2 Q_\Lambda \alpha^2 \tilde{\omega}_n}{\left[ 4Q^2_\Lambda \alpha^2 \partial^2_\Lambda + \pi^2 T^2 \tilde{\omega}_n^2 \right]^2},
$$

(29a)

$$
\partial_\Lambda X_\Lambda = -X_\Lambda \int_{p \in FZ} \frac{\alpha^2}{\left[ 12\pi^2 T^2 \tilde{\omega}_n^2 \partial^2_\Lambda Q^2_\Lambda \alpha^2 - \pi^4 T^4 \tilde{\omega}_n^4 \partial^2_\Lambda Q_\Lambda \right]} \left[ 4\tilde{\omega}_n^2 Q^2_\Lambda \alpha^2 + \pi^2 T^2 \tilde{\omega}_n^2 \right]^3 
- \sum_n \frac{4Q_\Lambda \omega_n^2 \partial^2_\Lambda \alpha^2 - 16\partial^2_\Lambda \omega_n^2}{\left[ 4\partial^2_\Lambda Q_n^2 \alpha^2 + \omega_n^2 \right]^3} \partial_\Lambda Q_\Lambda \right],
$$

(29b)

For conciseness, we introduced $\tilde{\omega}_n = \left( 2 \left[ \frac{\Lambda}{2\pi T} \right] + 1 \right)$ and $\tilde{\Lambda} = \left( 1 + \frac{\Lambda}{\pi T} - \frac{\Lambda}{2\pi T} \right)$. For the largest part, the Matsubara sum was reduced to two terms and could thus be performed analytically. The summands of the Katain contribution which appear in the last summation in Eq. (29b) come with a factor $\partial_\Lambda Q_\Lambda$ instead of $\partial_\Lambda \tilde{\Lambda}$. Therefore, they do not experience this reduction and the Matsubara sum cannot be performed analytically. This statement also holds for a smoothed version of the cutoff in Eq. (16) due to its singularity structure in the complex plane. A numerical evaluation of Eqs. (29) is therefore much more costly as compared to the $T = 0$ system. Due to the overall simplicity, though, the absolute cost is still rather small. Moreover, in case momentum and frequency dependence of the vertex functions is to be included in future work, analytic integration cannot be done in the $T = 0$ case either. So, there is no true increase in numerical cost due to the use of the finite- $T$ cutoff.

As expected, the evaluation of Eqs. (29) yields agreement with the mean-field results at percent level already for $\mathcal{O}(100)$ Matsubara frequencies, as exhibited in Figs. 4 and 5. This demonstrates the viability of the approach.

### B. Popov-Fedotov constraint

One of the main reasons to employ $T = 0$ flows in previous pf-FRG approaches and to recover finite temperature results by the correspondence between $\Lambda$ and $T$ is the fact that the fermion number constraint is exactly fulfilled at vanishing temperature.

When introducing actual finite fermion temperatures, unphysical states are populated which are not part of the original spin system but rather of the auxiliary fermion description, cf. Eq. (2). Thus, the results from Sec. V A are not necessarily solutions of the original spin model. There are different possibilities to implement the number constraint on the level of the path integral:

1. A new field $\lambda$ is introduced and coupled to the fermion density. This generates a functional delta function enforcing the constraint, cf. Sec. II. However, the saddle-point solution $\lambda = 0$ puts this mechanism out of action. In the pf-FRG, Eq. 4 effectively represents a Yukawa interaction term between the constraint field $\lambda$ and the fermions. Higher-order contributions will be generated which would have to be taken into account complicating a systematic analysis. We therefore refrain from pursuing this approach.

2. Alternatively, the constraint can be implemented by means of an imaginary chemical potential $\mu_{ppv} = i \frac{\gamma}{2}$ for SU(2), according to Popov and Fedotov. It can be shown that the presence of such an object in the Hamiltonian cancels the unphysical states out of the partition function or any expectation values of (physical) observables. Generalizations to different representations of SU( $N$ ) and higher spin $S$ have been suggested in the literature. This construction justifies the ignorance of the fermion number constraint in the $T = 0$ case where $\mu_{ppv} = 0$. However, it also casts a shade of
doubt on the finite temperature interpretation of flows stopped at some \( \Lambda_{\text{stop}} \), as there is no way to implement the constraint for these.

An additional advantage of the finite-temperature regularization scheme, Eq. (16), is that the implementation of Popov-Fedotov-type constraints by means of an imaginary chemical potential becomes feasible. For any finite \( N \), the corresponding \( \mu_{\text{ppv}} \) may be introduced, explicitly. To illustrate the simplicity of their treatment, let us consider flow equations for the case \( N = 2 \). Provided a system of \( \beta \) functions for two- and four-point functions, \( \partial_\Lambda Q_A \) and \( \partial_\Lambda X_A \), respectively, the modified RG equations are given by

\[
\partial_\Lambda Q_A^{\text{ppv}} = \frac{\partial_\Lambda Q_A}{2} \left| \omega_k = \omega_{k,1}^\Lambda, \omega_{k,2}^\Lambda \right|, \quad (30a)
\]

\[
\partial_\Lambda X_A^{\text{ppv}} = \frac{\partial_\Lambda X_A}{2} \left| \omega_k = \omega_{k,1}^\Lambda, \omega_{k,2}^\Lambda \right|, \quad (30b)
\]

were \( \omega_{k,1} = \left( 2 \left| \frac{k}{2\pi T} \right| + \frac{1}{2} \right), \omega_{k,2} = \left( 2 \left| \frac{k}{2\pi T} \right| + \frac{1}{2} \right) \) and \( \omega_n^\Lambda = \omega_n + \mu_{\text{ppv}} \). At least for small and finite \( N \) an exact treatment of the number constraint is thus easily achieved. The limit \( N \to \infty \) is more involved as a continuous distribution of imaginary chemical potentials must be employed.\(^{[31]}\)

We therefore leave an application to the present system for future work.

VI. CONCLUSIONS

To summarize, we have introduced a generalized pf-FRG approach that allows us to access the low-temperature physics of SU(\( N \)) Heisenberg models, both in the symmetric and symmetry broken regimes. For the square lattice SU(\( N \)) Heisenberg model we calculated the order parameter and magnetic susceptibility in the large-\( N \) limit for arbitrary temperatures, which we carefully compared to established results. We found that the commonly employed linear correspondence between the RG parameter \( \Lambda \) and temperature \( T \) does not hold anymore in the symmetry broken regime. Introducing a finite-\( T \) regularization, we overcome the inability of the \( T = 0 \) pf-FRG approach to correctly predict finite temperature observables for any system that exhibits spontaneous symmetry breaking. An additional decisive feature of our approach is the introduction of a symmetry-breaking term which by itself induces only a minimal bias, as opposed to the common strategy of a single-channel Hubbard-Stratonovich transformation in the fermion interaction. In order to illustrate this, we analyzed another symmetry-breaking initial condition, breaking the global SU(\( N \)) instead of the local U(1) symmetry. Indeed, we found that the fermionic divergence is not regularized in this case, confirming that an inappropriate choice of symmetry breaking initial condition does not bias the investigation. We also devised a strategy to properly handle the fermion number constraint at finite temperatures, supplementing the action by a Popov-Fedotov-type\(^{[31]}\) imaginary chemical potential. Within this framework, fulfilling the constraint exactly is easy for \( N = 2 \) and we leave the generalization of this procedure to arbitrary \( N \) for future work.

The technical developments presented in this manuscript and its companion paper\(^{[1]}\) set the stage for future investigations of spin liquid physics within the pf-FRG approach. As a general strategy, we suggest a two-step scheme to investigate a model of interest: (1) Quick determination of the symmetry-breaking properties by the maximally stripped-down pf-FRG approach presented here. While this gives a strong hint on the leading physical instability, this simplified approach may not resolve more subtle aspects of the spin liquid phase (as we have seen for the spatio-temporal structure of the order parameter in the case study of the manuscript at hand). (2) In-depth analysis of the identified channel by means of fine grained position or momentum space pf-FRG, as developed in our companion manuscript, to resolve the fine-print of particular spin liquid phases.

A future challenge will be to fully incorporate gauge degrees of freedom into the pf-FRG approach. To do so, we expect to strongly benefit from the simplicity of the stripped-down, momentum-space approach developed in this manuscript. This will open up the possibility to directly link the emergent gauge theory description of spin liquid ground states to their parent lattice spin models.

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Appendix A: Flow equations for the symmetry-broken regime

In the following, we provide details of the computation leading to the flow Eqs. (24), (25) and (26). After having derived these, we consider the limit \( Q_{\Lambda \to \infty} \to 0 \) to reproduce the symmetric flow Eq. (13). The starting point is the ansatz in Eq. (21) for the effective average action. In order to be able to explicitly evaluate the projection rules, Eqs. (22), the r.h.s. of the Wetterich Eq. (9) will be expanded as

\[
\partial_\Lambda \Gamma_\Lambda = C + \frac{1}{2} \partial_\Lambda \text{STr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left[ P_\Lambda^{-1} F_\Lambda \right]^n. \quad (A1)
\]

Here, \( C \) comprises field-independent and bilinear terms that will not be important for our considerations and \( \partial_\Lambda \) is a scale derivative that acts on the \( \Lambda \) dependence of the cutoff function, only. While \( P_\Lambda^{-1} \) is the usual regulated propagator, \( F_\Lambda \) is the field-dependent contribution to the second functional derivative of \( \Gamma_\Lambda \). While the flow equation for \( Q_\Lambda \) is extracted from the \( n = 1 \) term of Eq. (A1), the flow of the quartic couplings \( I_k \) and \( J_k \) is encoded in the \( n = 2 \) contribution.
Performing the functional derivative and applying the single-mode projection $\Psi^{(1)}_a = \Psi^{(1)}_a(\omega_a)\delta(p_a)$ yields

$$P^{-1}_a = \frac{-\delta\delta(\omega - \nu)\delta(p - q)}{\omega + 4\theta^2 Q^2 \alpha^2} \begin{pmatrix} 0 & P_f \\ P_f & 0 \end{pmatrix},$$

(A2)

where $P_f = i\omega\theta + 2Q\tilde{\alpha}\theta\sigma_x$ and $\tilde{\alpha} = \cos(p_x) + \cos(p_y)$ and $\alpha = |\tilde{\alpha}|$. Further

$$F_{\Lambda} = \frac{2\tilde{\alpha}}{N} \delta(\omega - \nu)\delta(p - q) \begin{pmatrix} 0 & O_V \\ -O_V & 0 \end{pmatrix},$$

(A3)

with $O_V = O_I + O_J$ and

$$O_I = I_A [\sigma^+(\Psi^\dagger_\alpha \sigma^+ \Psi_\alpha) + \sigma^-(\Psi^\dagger_\alpha \sigma^- \Psi_\alpha)],$$

$$O_J = J_A [\sigma^+(\Psi^\dagger_\alpha \sigma^- \Psi_\alpha) + \sigma^-(\Psi^\dagger_\alpha \sigma^+ \Psi_\alpha)].$$

(A4)

For $\mathcal{F}_\Lambda$, only leading order terms in $N$ have been kept. Now straightforward matrix multiplication yields

$$\text{Str} \left[ \mathcal{P}^{-1}_a \mathcal{F}_\Lambda \right] = A_\Lambda \Psi^\dagger_\alpha \sigma_\alpha \Psi_\alpha,$$

(A5)

$$\text{Str} \left[ (\mathcal{P}^{-1}_a \mathcal{F}_\Lambda)^2 \right] = B_\Lambda \left[ (\Psi^\dagger_\alpha \sigma^+ \Psi_\alpha)^2 + (\Psi^\dagger_\alpha \sigma^- \Psi_\alpha)^2 \right] + C_\Lambda (\Psi^\dagger_\alpha \sigma^+ \Psi_\alpha)(\Psi^\dagger_\alpha \sigma^- \Psi_\alpha).$$

(A6)

Here, $A_\Lambda$, $B_\Lambda$ and $C_\Lambda$ are the coefficients to be used in the projection rules, Eqs. (22), to extract the flow equations. Applying the projection rules based on the expansion for $\Gamma^\Lambda$, Eq. (A1) directly results in flow equations that do not include the Katanin improvement yet. In order to achieve this, it is necessary to replace $\partial_{\Lambda} \rightarrow \partial_{\Lambda} + (\partial_{\Lambda} Q\Lambda)\partial_{Q\Lambda}$, for the projection on $\partial_{\Lambda} I_A$ and $\partial_{\Lambda} J_A$. Taking this modification into account the flow equations for the couplings are finally given by

$$\partial_{Q\Lambda} Q = -\frac{1}{2\pi} \int_{p \in \mathbf{FZ}} \frac{Q \alpha^2 (J + I_A)}{4\alpha^2 + \Lambda^2},$$

(A7a)

$$\partial_{I_A} I_A = \frac{1}{\pi} \int_{p \in \mathbf{FZ}} \frac{2Q^2 \alpha^4 (J^2 + I^2_A) - \Lambda^2 \alpha^2 J I_A}{[4\alpha^2 + \Lambda^2]^2} - \frac{\partial_{Q\Lambda} Q}{2\pi} \int_{p \in \mathbf{FZ}} \alpha \left\{ 4\Lambda Q \alpha^3 J^2 + \frac{J^2 + I^2_A - 2 J I_A}{4\alpha^2 + \Lambda^2} \left[ \text{arctan} \left( \frac{\Lambda}{2\alpha Q} \right) - \frac{\pi}{2} \right] \right\},$$

(A7b)

$$\partial_{J_A} J_A = \frac{1}{2\pi} \int_{p \in \mathbf{FZ}} \frac{8Q^2 \alpha^4 I_A - \Lambda^2 \alpha^2 (J^2 + I^2_A)}{[4\alpha^2 + \Lambda^2]^2} \left[ \text{arctan} \left( \frac{\Lambda}{2\alpha Q} \right) - \frac{\pi}{2} \right] \right\},$$

(A7c)

The peculiar structure of the occurrence of $I_A$ and $J_A$ is already indicative of the transformation $X_A = J_A + I_A$, $Y_A = I_A - J_A$. Indeed, performing this transformation, the decoupled flow Eqs. (24), (25) and (26) are obtained.

Let us now consider the limit $Q \Lambda \rightarrow 0$. Since $\partial_{Q\Lambda} Q \sim Q$, this yields $Q = 0$ throughout the flow. As the initial value $I_A \rightarrow 0$, we have $\partial_{I_A} I_A = 0$, as well because the corresponding $\beta$ function, Eq. (A7b), contains only components $\sim Q\Lambda$, $\sim \partial_{Q\Lambda} Q\Lambda$ and $\sim J_A$. Consequently, only

$$\partial_{J_A} J_A = -\frac{1}{2\pi} \int_{p \in \mathbf{FZ}} \frac{I^2_A}{\alpha^2 + \Lambda^2}$$

(A8)

persists, which is equivalent to the zero-temperature symmetric flow Eq. (13) as it should be.

Appendix B: Gap equation from the RG flow

It can be proven quite generally that the mean field gap equation can be derived from the Katanin-improved flow equation. Here, we will demonstrate this procedure for our system. The derivation is particularly simple since neither the fermionic couplings nor the self-energy contribution $Q\Lambda$ retain any dependencies on momenta or frequencies. While also serving as a sanity check for our results, this relation nicely illustrates the influence of the regularization scheme. Let us introduce a shorthand notation for the flow Eqs. (24) and (25) reading

$$\partial_{Q\Lambda} Q = \chi_\Lambda \tilde{\partial}_{Q\Lambda} \chi_\Lambda, \quad \partial_{I_A} \chi_\Lambda = -X^2_\Lambda \partial_{I_A} f_\Lambda.$$  

(B1)

For Eq. (B1), there is an exact (albeit implicit) solution,

$$X_\Lambda = X_{\infty} - X_{\infty} X_A f_\Lambda.$$  

(B2)

Plugging Eq. (B2) into the second identity in Eq. (B1),

$$\partial_{Q\Lambda} Q = X_\Lambda \tilde{\partial}_{Q\Lambda} X_\Lambda - X_\Lambda \tilde{\partial}_{Q\Lambda} \tilde{\partial}_{Q\Lambda} f_\Lambda$$

$$= X_\Lambda \tilde{\partial}_{Q\Lambda} X_\Lambda - X_\Lambda \tilde{\partial}_{Q\Lambda} \chi_\Lambda f_\Lambda.$$  

(B3)

Conveniently, it holds that $\partial_{Q\Lambda} \chi_\Lambda = -f_\Lambda$ and therefore

$$\tilde{\partial}_{Q\Lambda} \chi_\Lambda - (\partial_{Q\Lambda} \chi_\Lambda) f_\Lambda = \tilde{\partial}_{Q\Lambda} \chi_\Lambda + (\partial_{Q\Lambda} \chi_\Lambda) \partial_{Q\Lambda} \chi_\Lambda = \partial_{I_A} \chi_\Lambda.$$
Finally, we obtain

\[ \partial_\Lambda Q_{\Lambda} = X_\infty \partial_\Lambda g_{\Lambda}. \]  

(B4)

Since the left and right hand sides of Eq. (B4) are total derivatives, the differential equation can be integrated directly.

\[ Q_0 - Q_\infty = X_\infty (g_0 - g_\infty) \xrightarrow{Q\rightarrow0} Q_0 = X_\infty g_0. \]  

(B5)

For the last step, it has been assumed that the explicitly symmetry breaking initial condition \( Q_\Lambda \) has been sent to zero. Since \( g_\Lambda \sim Q_\Lambda \), it is \( g_\Lambda = 0 \) as well. Notice furthermore that \( \theta_0 = 1 \) almost everywhere. Eq. (B5) is the well-known mean-field gap equation.

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