Research Article

Simulation of Constant-Volume Removal Rate Machining of Middle-Convex and Varying Ellipse Piston

Yanwei Xu 1,2, Yinhao Wang, 1 and Tancheng Xie 1,2

1 School of Mechatronics Engineering, Henan University of Science and Technology, Luoyang 471003, China
2 Intelligent Numerical Control Equipment Engineering Laboratory of Henan Province, Luoyang 471003, China

Correspondence should be addressed to Yanwei Xu; xuyanweiluoyang@163.com

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One kind of constant-volume removal rate machining method of the middle-convex and varying ellipse piston is proposed in this paper. By analyzing the structure and movement relationship of the middle-convex and varying ellipse piston machine, the NC machining model is built. And, the constant-volume removal rate machining model is also built by superposing the variable rotation satisfying the dynamic performance constraints on the uniform rotation of the spindle of the CNC piston lathe. Then, the instantaneous position parameters of each axis of the CNC piston lathe are obtained and turned into NC code. The functional feasibility of the method finally is verified by simulation machining.

1. Introduction

In the reciprocating process of crankshaft output torque, the piston transmits the gas pressure in the cylinder to this process through the piston pin and connecting rod, but the clearance between the piston and the cylinder sleeve should be as small and uniform as possible to prevent the piston from being “roughened” or “bitten” in the cylinder. However, in the coupled working environment of high temperature, high pressure, and alternating mechanical and thermal loads, the force acting on the top of the piston leads to the deformation of the piston skirt along the axis of the pin seat, the deformation of the side of the piston skirt, and the uneven thermal expansion. These causes lead to serious and irregular deformation of the piston, which makes the cross section of the piston skirt that is originally a round cross section cannot keep round in the working environment, and then become oval, resulting in that the “cylindrical” piston processed at room temperature presents the “elliptical cylinder” shape. This deformation directly affects the uniformity of fit clearance between piston and cylinder liner. The piston with middle-convex and varying ellipse piston skirt is designed, which can keep the ideal geometry under working conditions [1], ensure the good fit between the piston and the cylinder wall, reduce the cylinder clearance and the impact of the piston on the cylinder wall, and reduce the specific pressure and noise of the piston skirt. Moreover, a wedge-shaped oil gap is formed between the middle-convex skirt of the piston and the cylinder wall, which ensures the good lubrication of the piston and reduces the wear of parts.

For a long time, many scholars have deeply studied the machining method of middle-convex and varying ellipse piston [2–12]. These achievements have effectively promoted the progress of machining technology of middle-convex and varying ellipse piston, but the noncircular section machining technology of piston skirt studied by these achievements is basically based on numerical control turning method. NC turning is an important method for machining the piston with middle-convex and varying ellipse piston, which can effectively improve the machining accuracy and efficiency of the piston with middle-convex and varying ellipse piston. However, in the process of cutting a circular cross-sectional workpiece into an elliptical cross-sectional workpiece on a CNC piston lathe, the workpiece will rotate at a constant speed, and the cutting tool will move with high frequency and reciprocation. Moreover, the cutting depth of the cutting tool will change continuously with the rotation position.
of the workpiece, and the cutting area of the cutting tool in
the same cross section will change with the processing
position. The material removal rate and cutting force of the
workpiece change periodically, which will form dynamic
excitation force and affect the machining accuracy of piston
skirt to a certain extent.

Based on the basic principle that the cutting force is ap-
proximately proportional to the cutting area, this paper studies
the numerical control machining technology and simulation of
the equal volume cutting rate of the middle-convex and
varying ellipse piston. On the premise that the rotation position
of the shaft section of the piston skirt and the cutting depth of
the tool tip conforms to the ellipse trajectory, the variable-speed
rotation meeting the dynamic performance constraint is
superimposed on the uniform speed rotation of the main shaft
of the piston machine tool. The rotation speed of the main shaft
of the machine tool is continuously regulated, and the material
removal rate and cutting force of the middle-convex and
varying ellipse piston skirt are basically constant.

2. Processing Principle of Middle-Convex and
Varying Ellipse Piston Skirt

The profile characteristics of the middle-convex and varying
ellipse piston skirt are as follows: the generatrix of the piston
skirt is a middle-convex curve which represents the variation
of the diameter of the long axis of the elliptic section along
the direction of the piston axis. The shape of cross section
of piston skirt is similar to an ellipse, and the ovality of different
sections is different. In any section, the diameter of long axis
of ellipse section is the largest diameter, the long axis is along
the axis of pin seat, the diameter of short axis of this section
is the smallest diameter, and the short axis is perpendicular
to the axis of pin seat. The ellipticity of the elliptic section is
the difference between the diameter of major axis and that of
minor axis.

2.1. Elliptical Profile of Skirt Cross Section of Middle-Convex
and Varying Ellipse Piston. The geometric characteristics of
the skirt cross section of the piston with middle-convex and
varying ellipse can be described as follows [2]:

\[ R(h, \theta) = R_1(h) - \frac{G(h)}{4} C. \]  

(1)

In the formula, \( R(h, \theta) \) is polar radius value; \( G(h) \) is
ellipticity; and \( R_1(h) \) is elliptical long half-axis.

\[ R_1(h) = R_1(0) - k_1(h - h_0)^m. \]  

(2)

In the formula, \( R_1(0) \) is long half-axis of section with
maximum ellipticity.

\[ G(h) = G(0) - k_2 h. \]  

(3)

In the formula, \( G(0) \) is the maximum ellipticity; \( \theta \) is the
relative rotation angle of polar radius to long axis; \( h \) is the
skirt height; \( h_0 \) is the skirt height of section with maximum
ellipticity; \( \beta \) is the dimensionless coefficient (\( \beta = 0 \) is an
ellipse; \( \beta = 1 \) is a quadratic ellipse); \( m \) is the profile shape
characteristic coefficient of longitudinal profile; \( k_1 \) is
the dimensionless coefficient; and \( k_2 \) is the ellipticity change
rate.

\[ C = 1 - k_3 \left\{ \cos 2 \theta - \frac{\beta}{25} [1 - \cos 4 \theta] \right\}. \]  

(4)

In the formula, \( k_3 \) is the dimensionless coefficient (when
\( k_3 = 0 \), the cross section is circular; when \( k_3 = 1 \), the cross
section is elliptical).

Formula (1) is the variation of polar radius, and formula
(2) is the radial variation of the middle-convex profile along
the piston axis. If the piston is an elliptical piston, the
formula of any elliptical cross section of the piston skirt can
be obtained by substituting formula (4) into (1).

\[ R(h, \theta) = R_1(h) - \frac{G(h)}{4} (1 - \cos 2 \theta), \]  

(5)

among which, \( \theta = \omega t = 2\pi nt/60 = 2\pi ft \), \( \omega \) is angular ve-
clocity, and

\[ R(h, \theta) = R_1(h) - \frac{G(h)}{4} [(1 - \cos 2 \theta) + \beta(1 - \cos 4 \theta)]. \]  

(6)

2.2. Middle-Convex Profile of Middle-Convex and Varying
Ellipse Piston. The middle-convex profile of the middle-
convex and varying ellipse piston skirt is usually given
discrete points in design. Figure 1 is the design parameters
of the middle-convex profile of the Perkins 240 piston
skirt.

Usually, the discrete points of the convex profile are
fitted to smooth curves by cubic spline interpolation,
and then the equation of the middle-convex profile is ob-
tained. In the process of fitting the discrete points of
middle-convex profile in piston skirt shown in Figure 2,
\( N \) discrete points are put into the XOZ coordinate
system in which the \( X \)-axis is parallel to the piston cross section
and the \( Z \)-axis is the piston axis (\( X \) represents the long
half-axis of elliptical cross section and \( Z \) represents
the piston skirt height). If the first derivatives of the curve
composed of discrete points are \( x'_n \) and \( x'_n \) at the
beginning and end points, respectively, the function value \( x(z) \)
of any point \( z_i < z < z_{i+1} \) on the \( Z \)-axis can be expressed
as follows [13]:

\[ x(z) = M_{i+1} \left( \frac{z - z_i}{6L_i} \right)^3 - \frac{(z - z_{i+1})^3}{6L_i} \]

\[ + \left( \frac{x_i}{L_i} M_{i+1} \right) \cdot (z - z_{i+1}) \]

\[ + \left( \frac{x_{i+1}}{L_i} - \frac{L_i M_{i+1}}{6} \right) \cdot (z_{i+1} - z). \]  

(7)

In the formula, \( M_i \) satisfies the equation
among which, \( L_i = z_i - z_{i-1} \), \( \lambda_i = (L_{i+1}/L_i + L_{i+1}) \), and \( U_i = 1 - \lambda_i \). Solution \( M_i \), the fitting equation \( x(z) \) of middle-convex profile in piston skirt can be obtained by substitution formula (1).

2.3. Analysis of Processing Process of Middle-Convex and Varying Ellipse Piston Skirt. The structure of CNC piston machine tool is shown in Figure 3. Machine motion consists of workpiece (spindle C-axis) rotation motion, tool holder slider linear motion along the Z-axis (parallel to workpiece rotary axis), tool holder linear motion along the X axis (perpendicular to workpiece rotary axis), and tool holder \( U \)-axis reciprocating linear motion (control tool high-frequency reciprocating microdisplacement linear motion), X-axis. The cutting depth is controlled by \( U \)-axis, and the travel of the tool in high-speed reciprocating linear motion depends on the ellipticity of elliptical cross section at different skirt heights of piston skirt.

The process of turning skirt profile of middle-convex and varying ellipse piston by motion synthesis method can be divided into two independent motions: (1) the motion of tool relative to workpiece forming middle-convex profile is

\[
\begin{align*}
2M_0 + M_1 = & 6 \left( \frac{x_0 - x_1}{L_1} - x_0 \right), \\
M_{n+1} + 2M_n = & 6 \left( \frac{x_n - x_{n-1}}{L_n} \right), \\
U_i + 2M_i + \lambda M_{i+1} = & 6 \left( \frac{x_{i+1} - x_i}{L_i} - \frac{x_i - x_{i-1}}{L_i} \right) / L_i + L_{i+1},
\end{align*}
\] (8)
realized by the joint motion of X-axis and Z-axis driven by servo motor; (2) the motion of tool relative to workpiece forming elliptical profile, the reciprocating linear motion of U-axis driven by linear motor, and the motion of spindle.

2.3.1. Middle-Convex Profile Processing. The motion of forming the middle-convex profile is synthesized by the motion of the tool in the piston axis (Z-axis) and the radial direction (X-axis). After the fitting equation of discrete points on the middle-convex profile is obtained, the middle-convex profile is interpolated, and the cubic spline curve is approximated and fitted by micro line segments. According to the requirements of piston skirt surface processing accuracy and tool feed, the number of interpolation points is determined, and the middle-convex profile is interpolated in the piston axis direction. In this paper, the interpolation points are divided by equal interval method. Each step of interpolation, the workpiece rotates one week to complete a micro-short elliptical cylinder processing. Then, the Z-axis position is calculated, and the Z-axis servo motor drives the trawler to the next interpolation point. Figure 4 is a schematic diagram of NC machining of middle-convex profile in piston skirt.

2.3.2. Elliptical Profile Processing. The forming motion of the elliptical section of the piston skirt can be decomposed into a high-speed rotational motion of the workpiece and a high-speed reciprocating linear feed motion of the tool in the radial direction of the piston. During the forming process, the piston rotates with the spindle every one revolution, and the tool sequentially processes the long axis of the elliptical section → the short axis → the long axis → the short axis → the long axis, and the tool feeds twice in rapid reciprocating direction. The higher the spindle speed, the higher the tool feed frequency; the greater the ellipticity of the elliptical section, the greater the turning radius change, the greater the displacement of the tool’s fast reciprocating linear feed, and the greater the speed and acceleration of the tool. The key to the elliptical section turning of the piston skirt is the control of the tool path: (1) the high-frequency reciprocating linear motion of the tool; (2) the micro-displacement of the tool reciprocating linear feed motion and the angular displacement of the workpiece high-speed rotation maintain a strict one-to-one correspondence. The schematic diagram of the elliptical section machining process of the piston is shown in Figure 5.

Assuming that the starting position of the tool tip is located at the apex B of the ellipse long axis, the expression of the motion displacement of the tool tip vertex can be described as follows:
During the conventional forming and finishing of the piston—volume resection rate of middle-convex and varying ellipse piston—the volume \( \rho V \) of the material removed per unit time is equal during the elliptical cross section of the middle-convex and varying ellipse piston during processing.

The projected area \( \Delta S \) of the volume \( \Delta V \) on the elliptical section is

\[
\Delta S = \frac{\Delta V}{f}
\]
The projected area $\Delta S_{\text{max}}$ of the maximum volume $\Delta V_{\text{max}}$ allowed to be cut per unit time in the elliptical section is

$$
\Delta S_{\text{max}} = \frac{\Delta V_{\text{max}}}{f} \quad (15)
$$

In the schematic diagram of the elliptical cross section machining of the middle-convex and varying ellipse piston shown in Figure 4, the total area cut-off in the first quadrant is equal to the difference between the quarter circle area $S_{\text{AOI}}$ and the quarter ellipse area $S_{\text{BOK}}$. Then, there is

$$
S_{\text{cut}} = S_{\text{AOI}} - S_{\text{BOK}} = \frac{1}{4} \pi (a + a_p)^2 - \frac{1}{4} \pi ab. \quad (16)
$$

The total area cut out in the first quadrant is divided into $n$ equal parts for processing, so that the time taken to cut each aliquot area $\Delta S$ is the same.

$$
\Delta S = \frac{S_{\text{cut}}}{n} = \frac{\pi [(a + a_p)^2 - ab]}{4n} \quad (17)
$$

In the formula, $n$ must meet the conditions

$$
n \geq \frac{S_{\text{cut}}}{\Delta S_{\text{max}}} = \frac{\pi [(a + a_p)^2 - ab]}{4\Delta S_{\text{max}}} \quad (18)
$$

### Table 1: Cutting depth and its variation and the area cut in the same corner.

| Angle (°) | $a_p$ (mm) | $\Delta a_p$ (µm) | Equal-angle cut area (mm²) |
|----------|-------------|-------------------|---------------------------|
| 3        | 0.10027     | 0.27              | 0.24134                   |
| 6        | 0.10109     | 0.82              | 0.24266                   |
| 9        | 0.10244     | 1.35              | 0.24529                   |
| 12       | 0.10432     | 1.87              | 0.24919                   |
| 15       | 0.10669     | 2.37              | 0.25433                   |
| 18       | 0.10954     | 2.85              | 0.26065                   |
| 21       | 0.11283     | 3.29              | 0.26807                   |
| 24       | 0.11653     | 3.70              | 0.27652                   |
| 27       | 0.12059     | 4.06              | 0.28590                   |
| 30       | 0.12498     | 4.39              | 0.29611                   |
| 33       | 0.12964     | 4.66              | 0.30703                   |
| 36       | 0.13452     | 4.88              | 0.31855                   |
| 39       | 0.13958     | 5.05              | 0.33054                   |
| 42       | 0.14475     | 5.17              | 0.34286                   |
| 45       | 0.14997     | 5.23              | 0.35538                   |
| 48       | 0.15520     | 5.23              | 0.36797                   |
| 51       | 0.16037     | 5.17              | 0.38048                   |
| 54       | 0.16543     | 5.06              | 0.39278                   |
| 57       | 0.17031     | 4.89              | 0.40473                   |
| 60       | 0.17498     | 4.67              | 0.41621                   |
| 63       | 0.17937     | 4.39              | 0.42709                   |
| 66       | 0.18344     | 4.07              | 0.43724                   |
| 69       | 0.18715     | 3.70              | 0.44657                   |
| 72       | 0.19044     | 3.30              | 0.45496                   |
| 75       | 0.19329     | 2.85              | 0.46233                   |
| 78       | 0.19567     | 2.38              | 0.46860                   |
| 81       | 0.19755     | 1.88              | 0.47370                   |
| 84       | 0.19989     | 1.36              | 0.47758                   |
| 87       | 0.20073     | 0.82              | 0.48018                   |
| 90       | 0.2         | 0.27              | 0.48149                   |

![Figure 6: Cutting depth $a_p$ change trend.](image)

![Figure 7: Change in cutting depth $a_p$ change trend.](image)

![Figure 8: Area cut in the same corner.](image)
In the first quadrant, the area cut-off from $0 \sim \theta_1, \theta_1 \sim \theta_2, \ldots, \theta_n \sim \theta_n$ is equal to $\Delta S$. Then, there is
\[ i \Delta S = \frac{1}{2} \theta (a + a_p)^2 - \frac{1}{2} ab \cdot \arctan \left( \frac{a \tan \theta}{b} \right). \]  
(19)

Combining (17) and (19) gives
\[ i \pi \left[ (a + a_p)^2 - ab \cdot \arctan ((a/b) \tan \theta) \right] = \theta (a + a_p)^2 - \frac{ab}{2} \cdot \arctan ((a/b) \tan \theta). \]  
(20)

Solving the equation yields angle values of $\theta_1, \theta_2, \ldots, \theta_n$.

When the elliptical section of the middle-convex and varying ellipse piston skirt is machined by the equal-volume resection rate method, the displacement relationship of the four axes of motion of the machine tool can be expressed as follows:
\[
\begin{align*}
C: & \quad \theta_i = F(i, a, b, a_p), \\
Z: & \quad z = f(t), \\
X: & \quad x = F(z) = F(f(t)), \\
U: & \quad u_i = a - \sqrt{a^2 \cos^2 \theta_i + b^2 \sin^2 \theta_i}.
\end{align*}
\]  
(21)

This is the numerical control machining model when machining the middle-convex and varying ellipse piston skirt in the same volume resection rate method.

### 4. Equal-Volume Resection Rate Simulation Processing and Experiment

The skirt height $H$ of the middle-convex and varying ellipse piston in Perkins 240 is shown in Figure 1 and its corresponding elliptical cross-sectional long axis value and ellipticity value are shown in Tables 2 and 3, respectively.

According to Table 2, using cubic spline interpolation, the fitting equation and fitting curve of the ellipse long axis value of the Perkins 240 piston skirt can be obtained by MATLAB program fitting (Figure 9).

\[ d = 2.73 \times 10^{-3} z - 4.23424 \times 10^{-5} z^2 - 9.93358 \times 10^{-7} z^3 + 91.97066. \]  
(22)

According to Table 3, using linear interpolation, the equation for the ellipticity of the elliptical section $G$ with the height of the skirt is
\[ G = \begin{cases} 
0.2, & (0 \leq z \leq 41), \\
4.7619 \times 10^{-3} z + 4.7619 \times 10^{-3}, & (41 \leq z \leq 62).
\end{cases} \]  
(23)

Take the elliptical cross-sectional long axis maximum value plus twice the depth of cut $a_p$, as the cylinder workpiece diameter $d_0$ before the piston skirt forming process, starting from the skirt height $H = 0$, according to the feed amount $f = 0.001$ [14], calculate the different ellipse values of the long semiaxis $a$, the ellipticity $G$, and the short semiaxis $b$ of the section, and calculate the feed of the tool from the skirt height $H = 0$, the tool is cut from $d_0/2$ to the long semiaxis of each different elliptical section. For each different elliptical section, divide the area cut-off in the first quadrant into $n$ equal parts, and obtain the angle corresponding to each aliquot and the corresponding tool feed amount; then, according to the principle of symmetry, the workpiece is rotated within one week. The angle corresponding to each aliquot and the tool feed. Then, the numerical values of workpiece rotation angle and tool feed per equal part are transformed into corresponding NC machining program. According to the NC machining model of equal volume removal rate for middle-convex and varying ellipse piston, the piston skirt can be machined with equal-volume removal rate.

Taking the maximum elliptical section of the Perkins 240 piston as an example, the skirt height $H = 20$, the long axis diameter, the ellipticity $G = 0.2$, the long half axis $a = 46$, the short half-axis $b = 45.9$, and the first quadrant 0-degree cutting allowance $a_p = 0.1$. According to these parameters,
the angle corresponding to the area of each aliquot and the actual rotation angle of the workpiece and the corresponding depth of cut $a_p$ and its variation $\Delta a_p$ can be obtained. Take $n = 30$ aliquots, and the calculation results are shown in Table 4.

According to Table 4, when the maximum elliptical cross section of the middle-convex and varying ellipse piston skirt in Perkins 240 is processed by equal-volume resection; in the first quadrant, the change trend of the workpiece turning angle of each aliquot is as shown in Figure 10. The trend of the actual turning angle is shown in Figure 11; the change trend of each cutting area corresponding to the cutting depth is shown in Figure 12, and the corresponding cutting depth is shown in Figure 13.

The obtained actual cutting angle and the tool feed amount corresponding to each aliquot area and the positional parameters of the corresponding X-axis and Z-axis are converted into a numerical control machining program, and the elliptical section of the middle-convex and varying ellipse piston skirt can be obtained. Perform equal-volume resection rate processing. Skirt height $H = 20$ elliptical section first image inner limit equal volume resection rate machine tool motion parameters when middle-convex and varying ellipse piston skirt is processed as shown in Table 5.

According to the machine motion parameters, based on the VERICUT CNC machining simulation platform, the equal-volume resection rate simulation was performed on

| No. | $a_p$ (mm) | $\Delta a_p$ (µm) | Every aliquot (°) | Actual corner (°) |
|-----|------------|-------------------|------------------|------------------|
| 1   | 0.1006     | 0.612             | 4.4897           | 4.4897           |
| 2   | 0.1024     | 1.793             | 4.4358           | 6.2225           |
| 3   | 0.1053     | 2.852             | 4.3361           | 8.0612           |
| 4   | 0.1090     | 3.74              | 4.2015           | 9.8528           |
| 5   | 0.1134     | 4.434             | 4.0463           | 11.6277          |
| 6   | 0.1184     | 4.939             | 3.8818           | 13.3705          |
| 7   | 0.1236     | 5.275             | 3.7174           | 15.1089          |
| 8   | 0.1291     | 5.467             | 3.5592           | 16.8630          |
| 9   | 0.1347     | 5.542             | 3.4102           | 18.6150          |
| 10  | 0.1402     | 5.525             | 3.2733           | 20.3690          |
| 11  | 0.1456     | 5.435             | 3.1478           | 22.1230          |
| 12  | 0.1509     | 5.289             | 3.0338           | 23.8771          |
| 13  | 0.1560     | 5.102             | 2.9313           | 25.6311          |
| 14  | 0.1609     | 4.882             | 2.8390           | 27.3851          |
| 15  | 0.1655     | 4.639             | 2.7565           | 29.1391          |
| 16  | 0.1699     | 4.377             | 2.6826           | 30.8931          |
| 17  | 0.1740     | 4.102             | 2.6167           | 32.6471          |
| 18  | 0.1778     | 3.817             | 2.5583           | 34.4011          |
| 19  | 0.1813     | 3.524             | 2.5061           | 36.1551          |
| 20  | 0.1846     | 3.227             | 2.4603           | 37.9091          |
| 21  | 0.1875     | 2.925             | 2.4196           | 39.6631          |
| 22  | 0.1901     | 2.621             | 2.3841           | 41.4171          |
| 23  | 0.1924     | 2.315             | 2.3537           | 43.1711          |
| 24  | 0.1944     | 2.007             | 2.3274           | 44.9251          |
| 25  | 0.1961     | 1.699             | 2.3056           | 46.6791          |
| 26  | 0.1975     | 1.391             | 2.2878           | 48.4331          |
| 27  | 0.1986     | 1.081             | 2.2735           | 50.1871          |
| 28  | 0.1994     | 0.772             | 2.2632           | 51.9411          |
| 29  | 0.1998     | 0.463             | 2.2557           | 53.6951          |
| 30  | 0.2        | 0.154             | 2.2458           | 55.4491          |

Figure 10: Each resection area corresponds to the workpiece rotation angle.

Figure 11: The cut area corresponds to the actual rotation angle of the workpiece.

Figure 12: Cut area corresponds to the depth of cut.
the Perkins 240 piston skirt in Figure 1 (as shown in Figure 14).

The ellipticity value of the geometric parameters of the middle-convex and varying ellipse piston skirt in Perkins 240 is magnified 30 times, the remaining parameters are unchanged, recalculated, and simulated, and the simulation processing effect of the middle-convex line and the variable elliptic section can be clearly seen (as shown in Figure 15).

### 5. Conclusion

This paper analyzes the forming principle of the oval section profile of the middle-convex and varying ellipse piston skirt.
The middle-convex profile and ellipse profile of the middle-convex and varying ellipse piston are described mathematically, and the NC machining model of turning piston skirt is established. The distribution of the cutting area in the process of the ellipse section is analyzed. The numerical control machining model of equal-volume cutting rate for middle-convex and varying ellipse piston is established, and the machining method of equal volume cutting rate for middle-convex and varying ellipse piston is verified by simulation.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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