A Fuzzy EPQ Model for Non-Instantaneous Deteriorating Items

Sahidul Islam
*University of Kalyani, Nadia, West Bengal, India*

Abhishek Kanti Biswas
*R K M Vivekananda Centenary College, Kolkata, West Bengal, India*

**Abstract**

The inventory system has been drawing more intrigue because this system deals with the decision that minimizes the total average cost or maximizes the total average profit. For any firm, the demand for any item depends upon population, selling price and frequency of advertisement, etc. In most of the models, it is assumed that deterioration of any item in inventory starts from the beginning of their production. But in reality, many goods maintain their good quality or original condition for some time. So, price discount is availed on defective items. Our target is to calculate the total optimal cost and the optimal inventory level for this inventory model in a crisp and fuzzy environment. Here holding cost is taken as constant and no-shortages are allowed. The cost parameters are considered as Triangular Fuzzy Numbers and to defuzzify the model, Signed Distance Method is applied. A numerical example of the optimal solution is given to clarify the model. The changes of different parameters effect on the optimal total cost are presented and sensitivity analysis is given.

**Keywords:** EPQ Inventory, Non-Instantaneous Deterioration, Demand dependent Production, Defuzzification, Signed Distance Method

**JEL Classification:** C44, Y80, C61

**Paper Classification:** Research Paper

**Introduction**

In an EPQ inventory, it is important to control quality. Most of the models of the inventory control system are formulated with the assumption that all produced items are of good quality. But in reality, for any production company to produce all good quality products is impossible. On the other hand, due to the different phenomenons, there are so many goods which deteriorate after their lifetime. In such a situation, price discount is a common practice by the supplier that encourages the customer to purchase defective and deteriorated items other than regular purchase. So the effect of deterioration and defective items cannot be ignored in inventory models.
Most inventory models considered the request rate to be either stock needy or consistent or time-subordinate. It has been observed that decrease in the cost of the item for the most part positively affects demand of the item. It becomes a necessity to make a proper strategy to maintain the inventory economically. Ghare et. al. (1963) developed an inventory model for the exponentially decaying inventory system. These types of models were extended and improved by Misra (1975). The investigators generally have taken the demand as constant. In reality, demand always depends on selling price of an item, population of that area, deterioration, the frequency of advertisement of the product, etc. As time advanced, a few researchers created inventory models with deteriorating items, shortage items, demand patterns, cost patterns, items order cycles and their combinations. Bhunia et.al. (2014) derived a deterministic inventory model where deteriorated items demand depends upon the selling price of items and the frequency of advertisement. On the other hand, to reduce the cost, an intelligent businessman or a production company always produces products depending on demand.

Without any ambiguity, many inventory model based on different kinds of vulnerabilities are classically modelled using the approaches from the likelihood hypothesis. Some of the business fit such conditions, yet applying these models as they may be, for the most part, prompts incorrect choices. Here fuzzy inventory models fulfil that gap and can get more exact outcomes for inventory problems, rather than the conventional likelihood hypothesis by using fuzzy set theory. It was presented by Zadeh (1965), whose research work has been receiving considerable attention from investigators in production and inventory system. Bellmann et. al. (1970) proposed a scientific model of decision making in fuzzy condition. Later, Dubois et. al. (1978) defined some operations on fuzzy numbers. Zimmermann (1985) made an attempt to use the fuzzy sets in operation research. Syed et. al. (2007) investigated a fuzzy inventory model without shortages using signed distance method. Dutta et. al. (2012) contributed to fuzzy inventory model without shortage using trapezoidal fuzzy number. Maragatham et. al. (2014) researched a fuzzy inventory model for deteriorating items with price-dependent demand.

**Motivation & Contribution of Study**

In the proposed model, a fuzzy deterministic stock model for non-instantaneous deteriorating things with production proportional to demand is shown. Variable demand pattern depends on population, selling price and frequency of advertisement which is variable or constant according to any real-life situation. Here they are treated as constants. So, production company produces items according to demand. On the other hand, defecation and deterioration occur for any production. In such a situation price discount is a common phenomenon. The inventory parameters are taken as the triangular fuzzy number. Signed distance method is used to defuzzify the model. The goal for finding the solution for minimizing the total cost has been derived. Such type of model has not yet been discussed in the inventory literature.

**Definitions and Fuzzy Preliminaries**

**Definition 2.1:** A fuzzy set  \( \tilde{A} \) is a universe of discourse \( X \) is defined as the following set of pairs \( \tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X \} \). Where \( \mu_{\tilde{A}}(x) \rightarrow [0,1] \) is a mapping called the membership function of the set \( \tilde{A} \) and \( \mu_{\tilde{A}}(x) \) is called the membership value or degree of membership of \( x \in X \) in the fuzzy set \( \tilde{A} \). The larger \( \mu_{\tilde{A}}(x) \) is stronger the grade of membership form in \( \tilde{A} \).

**Definition 2.2 :** A fuzzy set \( \tilde{A} \) of the universe of discourse \( X \) is convex if and only if for all \( x_1, x_2 \in X, \mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)] \) when \( 0 \leq \lambda \leq 1 \).
Definition 2.3: A fuzzy set $\tilde{A}$ of the universe of discourse $X$ is called normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

Definition 2.4: $\alpha$-level set: The $\alpha$-cut of $\tilde{A}$ is defined as a crisp set $A_{\alpha} = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X\}$ where $\alpha \in [0,1]$. $A_{\alpha}$ is a non-empty bounded closed interval contained in $X$ and it can be denoted by $A_{\alpha} = [A_L(\alpha), A_R(\alpha)]$. Where $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed interval respectively.

Definition 2.5: A fuzzy number is a fuzzy set in the universe of discourse $X$ that is both convex and normal. The following figure 1 shows a fuzzy number $\tilde{A}$.

![Fig. -1:- Fuzzy number $\tilde{A}$ With $\alpha$-cuts.](image)

Figure 1 shows a fuzzy number $\tilde{A}$ with $\alpha$-cuts $A_{\alpha 1} = [A_{L}(\alpha_1), A_{R}(\alpha_1)]$, $A_{\alpha 2} = [A_{L}(\alpha_2), A_{R}(\alpha_2)]$. It is seen that if $\alpha_2 \geq \alpha_1$ then $A_{L}(\alpha_2) \geq A_{L}(\alpha_1)$ and $A_{R}(\alpha_2) \geq A_{R}(\alpha_1)$.

Definition 2.6: The function principle is used for the operation for Addition, Subtraction, Multiplication and Division of fuzzy numbers. Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) Addition: $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$, where $a_1, a_2, a_3; b_1, b_2, b_3$ are any real numbers.

(ii) Subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$, where $a_1, a_2, a_3; b_1, b_2, b_3$ are any real numbers.

(iii) Multiplication: $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$, where $a_1, a_2, a_3; b_1, b_2, b_3$ are all non-zero positive real numbers.

(iv) Division: $\tilde{A} / \tilde{B} = (a_1/b_1, a_2/b_2, a_3/b_3)$, where $b_1, b_2, b_3$ are all non-zero positive real numbers.

(v) Scalar Multiplication: For any real number $K$,

$K\tilde{A} = (Ka_1, Ka_2, Ka_3)$, Where $K \geq 0$,

$K\tilde{A} = (Ka_2, Ka_3, Ka_1)$ Where $K < 0$.

Definition 2.7: The $\alpha$-cut of $\tilde{A}$ is defined by $A_{\alpha} = \{x : \mu_{\tilde{A}}(x) = \alpha, \alpha \geq 0\}$.

Definition 2.8: Among the various shapes of fuzzy number, triangular fuzzy number (TFN) is the most popular one. $\tilde{A}$ is represented by the triplet $(a_1, a_2, a_3)$ and is defined by its continuous membership function where $\mu_{\tilde{A}}(x) : X \rightarrow [0,1]$ is given by
Definition 2.9: The $\alpha$-level set of the triangular number $\tilde{A}=(a_1, a_2, a_3)$ is:

$$\tilde{A} = \{x : \mu_{\tilde{A}}(x) \geq \alpha\} = [A_{L}(\alpha), A_{R}(\alpha)].$$

Where $A_{L}(\alpha)=a_1+(a_2-a_1)\alpha, a, \alpha \in [0,1]$, And $A_{R}(\alpha)=a_3-(a_3-a_2)\alpha, a, \alpha \in [0,1]$.

We represent $\tilde{A}=(a_1, a_2, a_3) = \cup [A_{L}(\alpha), A_{R}(\alpha)]; 0 \leq \alpha \leq 1$.

Definition 2.10: Defuzzification of $\tilde{A}$ can be found by Signed Distance Method. If $\tilde{A}$ is a triangular fuzzy number then sign distance from $\tilde{A}$ to 0 is defined as:

$$d(\tilde{A},0) = \frac{1}{2} \int_{0}^{1} d([A_{L}(\alpha), A_{R}(\alpha)], 0) d\alpha$$

Where, $\tilde{A}=[A_{L}(\alpha), A_{R}(\alpha)]$ and $\tilde{A}=[a_1+(a_2-a_1)\alpha, a_3-(a_3-a_2)\alpha], a, \alpha \in [0,1]$ is $\alpha$-cut off fuzzy set $\tilde{A}$, which is a close interval.

Notations and Assumptions

This inventory model is produced based on the accompanying assumptions and notations which are utilized in Crisp and Fuzzy Environment.

Notations:
- $I(t)$: The inventory level at any time $t$, $t \geq 0$.
- $C_1$: The fixed operating cost of the inventory.
- $C_2$: The advertisement cost per advertisement.
- $lp$: The production cost per unit per unit time.
- $Tac$: The total average cost per unit per cycle.
- $(C_{1})$: The Fuzzy fixed operating cost of the inventory.
- $(C_{2})$: Fuzzy advertisement cost per advertisement.
- $(Tac)$: Fuzzy total average cost per unit per cycle.
- $t_1$: The production time when the quality of products in stock reaches maximum $L_{m}, t_1 > 0$.
- $t_2$: The time duration where there is no production but deteriorating and end of $t_2$ the inventory level diminished gradually to zero, $t_2 > 0$.
- $t_1+t_2$: The length of cycle time, $t_1+t_2 > 0$.

Assumptions:
- The rate of non-instantaneous decay whenever any time $t > 0$ is time proportional, $\theta(t) = \beta t$; where, $\beta (0 < \beta < 1)$ is the scale parameter.
- The demand rate $D(m,p,f) = mf/p$ is dependent on population $(m)$, selling price $(p)$ of an item and the frequency of advertisement $(f)$, where $m, p, f > 0$. 
Production Inventory Model in Crisp Environment is produced as follow

Let, the producer start to produce items at the start of each cycle when \( t = 0 \) to satisfy the arriving demands in the inventory system. At the end of time \( t_1 \), the production stopped where some produced items are defective. The inventory level is assumed to reach to its highest level \( L_m > 0 \) at end of \( t_1 \). During the time interval \( t_2 \), the inventory level diminishes owing to customer demand and deterioration and finally falls to zero at \( t = t_1 + t_2 \). Figure – 2 delineates the inventory level of the proposed model.

![Inventory Level](image)

The Inventory Level in \( t_1 (0 \leq t \leq t_1) \) : The produced items during \( t_1 \) would be depleted due to the instant demand as well as defective items. Under above assumption, during the period \( t_1 \), the inventory status of the system is given by the following differential equation:

\[
dI_1(t)/dt = kD(m,p,f) - D(m,p,f) - r, \quad \text{for } (0 \leq t \leq t_1)
\]

From the initial Condition \( I_1(0) = 0 \) and \( I_1(t_1) = L_m \) got from above equation (1), and

\[
I_1(t) = \left( k \frac{mf}{p} - \frac{mf}{p} - r \right) t, \quad \text{for } (0 \leq t \leq t_1)
\]

\[
L_m = \left( k \frac{mf}{p} - \frac{mf}{p} - r \right) t_1.
\]

The Inventory Level in \( t_2 (t_2 \leq t \leq t_1 + t_2) \) : In this time, the inventory declines due to customers’ demand and deterioration. Hence, the status of the inventory level during \( t_2 \) is governed by the following Differential Equation,

\[
dI_2(t)/dt + \beta I_2(t) = - D(m,p,f), \quad \text{for } (t_1 \leq t \leq t_1 + t_2)
\]
From the boundary condition I_2 (t_1 + t_2) = 0 and dismissing the higher intensity of β and taking initial two terms of the exponential series, we get,
\[ I_2(t) = \left( \frac{mf}{p} \right) \left[ t_1 + t_2 - t + \beta \frac{t^2}{3} + \frac{β(t_1 + t_2)^2}{6} \right] \]
(5)

Now the costs functions are:

1. The Operating cost during the period [0, t_1 + t_2] : \( C_1 \)

2. The Production cost during the period [0, t_1] : \( lpk \frac{mf}{p} t_1 = klmf t_1 \)

3. The Inventory Holding Cost during the period [0, t_1 + t_2] : \( \int_0^{t_1} h_1(t) \ dt + \int_{t_1}^{t_1 + t_2} h_2(t) \ dt \). Using equation (2) and (5), then integrating, becomes the Holding Cost,
\[ h \left( \frac{mf}{p} - \frac{mf}{p} - r \right) \frac{t^2}{2} + h \frac{mf}{p} \left[ \frac{(t_1 + t_2)^2}{2} + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 - \frac{t^2}{2} - \beta \frac{t_4}{12} - \frac{β(t_1 + t_2)^2}{6} t_1 + \frac{β(t_1 + t_2)^3}{6} t_1^2 \right] \]
(8)

4. The Deteriorating Cost during the period [t_1, t_1 + t_2] : \( lp \int_{t_1}^{t_1 + t_2} \beta t_1 \ dt \). Using equation (5) and integrating, the Deteriorating Cost becomes
\[ \beta \left( \frac{(t_1 + t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^3}{2} t_2 \right) \]
(9)

5. The Advertisement cost during the period [0, t_1 + t_2] : \( C_2 f \)

6. The Price Discount during the period [t_1, t_1 + t_2] : \( lpd \int_t^{t_1 + t_2} \frac{mf}{p} \ dt \) From above, Price Discount is \( ldmf t_2 \)

Therefore the total average cost function per cycle : \( 1/(t_1 + t_2) [ Operating Cost + Production Cost + Inventory Holding Cost + Deteriorating Cost + Advertisement Cost + Price Discount ] \).

Hence the average net cost function is
\[ \text{Tac}(t_1, t_2) = \frac{1}{(t_1 + t_2)} [C_1 + klmf t_1 + h \left( \frac{mf}{p} - \frac{mf}{p} - r \right) \frac{t^2}{2} + h \frac{mf}{p} \left[ \frac{(t_1 + t_2)^2}{2} + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 - \frac{t^2}{2} - \beta \frac{t_4}{12} - \frac{β(t_1 + t_2)^2}{6} t_1 + \frac{β(t_1 + t_2)^3}{6} t_1^2 \right] + \beta \left( \frac{(t_1 + t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^3}{2} t_2 \right) + C_2 f + ldmf t_2 ] \]
(12)

Now, the necessary condition for the total average cost function of the system is minimized if equation (12) is satisfying
\[ \frac{\partial \text{Tac} (t_1, t_2)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial \text{Tac} (t_1, t_2)}{\partial t_2} = 0 \]
(13)

The solution, which might be called feasible solution of the problem, of the conditions (13) and (14) give the optimal solutions of \( t_1 = t_1^* \) and \( t_2 = t_2^* \) which minimize \( \text{Tac}(t_1, t_2) = \text{Tac}(t_1^*, t_2^*) \) provide they satisfy the sufficient conditions-
\[ \frac{\partial^2 \text{Tac}(t_1, t_2)}{\partial t_1^2} \cdot \frac{\partial^2 \text{Tac}(t_1, t_2)}{\partial t_2^2} - \left( \frac{\partial^2 \text{Tac}(t_1, t_2)}{\partial t_1 \partial t_2} \right)^2 > 0 \]
(15)

\[ \frac{\partial^2 \text{Tac}(t_1, t_2)}{\partial t_1^2} > 0 \quad \text{or} \quad \frac{\partial^2 \text{Tac}(t_1, t_2)}{\partial t_2^2} > 0 \]
(16)
However, it's difficult to solve the problem by inferring an explicit equation of the solutions from conditions (13) and (14). Therefore, the optimal service level of $t_1 = t_1^*$ and $t_2 = t_2^*$ is solved by using the software LINGO 17.0. Moreover, it is also verified that the sufficient conditions of the optimality of the solutions of $t_1 = t_1^*$ and $t_2 = t_2^*$ are satisfied (i.e. inequalities (15) and (16)) under certain conditions.

The proposed Inventory Model in Fuzzy Environment is produced as follows

Presently the above model will be produced in fuzzy Environment. Due to uncertainty, it is difficult to characterize every one of the parameters definitely. It is assumed that, $C_1 = (C_1^1, C_1^2, C_1^3)$, $b = (b^1, b^2, b^3)$, $p = (p^1, p^2, p^3)$, $C_2 = (C_2^1, C_2^2, C_2^3)$ is Triangular Fuzzy Number in LR-form then the total average cost function of the system per unit time in fuzzy environment is given by:

$$
\text{Tac}(t_1, t_2) = \frac{1}{(t_1 + t_2)}[C_1 + klmf_t + h^2 \left( k \frac{mf}{p} - \frac{mf}{p} - r \right) t_1^2 + h \frac{mf}{p} \left( \frac{(t_1 + t_2)^2}{2} \right) + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 - \beta \frac{t_1^4}{12} - \frac{t_1^4}{6} t_1 + \beta (t_1 + t_2) \frac{t_2^3}{6}] + \beta lmf \left( \frac{(t_1 + t_2)^2}{6} - \frac{t_1^2}{6} t_2 \right) - \frac{t_1^2}{6} t_2 + \beta \frac{(t_1 + t_2)^3}{6} - \frac{t_1^4}{6} t_2 + \beta (t_1 + t_2) \frac{t_2^3}{6}]
$$

Or, $\text{Tac}(t_1, t_2) = \frac{1}{(t_1 + t_2)}[C_1^2 + klmf_t + h^2 \left( k \frac{mf}{p} - \frac{mf}{p} - r \right) t_1^2 + h \frac{mf}{p} \left( \frac{(t_1 + t_2)^2}{2} \right) + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 - \beta \frac{t_1^4}{12} - \frac{t_1^4}{6} t_1 + \beta (t_1 + t_2) \frac{t_2^3}{6}] + \beta lmf \left( \frac{(t_1 + t_2)^2}{6} - \frac{t_1^2}{6} t_2 \right) - \frac{t_1^2}{6} t_2 + \beta \frac{(t_1 + t_2)^3}{6} - \frac{t_1^4}{6} t_2 + \beta (t_1 + t_2) \frac{t_2^3}{6}]

Where, $U = \frac{1}{(t_1 + t_2)}[C_1 + klmf_t + h^2 \left( k \frac{mf}{p} - \frac{mf}{p} - r \right) t_1^2 + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 - \beta \frac{t_1^4}{12} - \frac{t_1^4}{6} t_1 + \beta (t_1 + t_2) \frac{t_2^3}{6}] + \beta lmf \left( \frac{(t_1 + t_2)^2}{6} - \frac{t_1^2}{6} t_2 \right) - \frac{t_1^2}{6} t_2 + \beta \frac{(t_1 + t_2)^3}{6} - \frac{t_1^4}{6} t_2 + \beta (t_1 + t_2) \frac{t_2^3}{6}]

And, $W = \frac{1}{(t_1 + t_2)}[C_2 + klmf_t + h^2 \left( k \frac{mf}{p} - \frac{mf}{p} - r \right) t_1^2 + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 - \beta \frac{t_1^4}{12} - \frac{t_1^4}{6} t_1 + \beta (t_1 + t_2) \frac{t_2^3}{6}] + \beta lmf \left( \frac{(t_1 + t_2)^2}{6} - \frac{t_1^2}{6} t_2 \right) - \frac{t_1^2}{6} t_2 + \beta \frac{(t_1 + t_2)^3}{6} - \frac{t_1^4}{6} t_2 + \beta (t_1 + t_2) \frac{t_2^3}{6}]

The $\alpha$-cuts, $A_L(\alpha)$ and $A_R(\alpha)$ of triangular fuzzy number $\text{Tac}(t_1, t_2)$ are given by:

$$
A_L(\alpha) = U + (V - U)\alpha = \frac{1}{(t_1 + t_2)}[C_1 + klmf_t + h^2 \left( k \frac{mf}{p} - \frac{mf}{p} - r \right) t_1^2 + h \frac{mf}{p} \left( \frac{(t_1 + t_2)^2}{2} \right) + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 - \beta \frac{t_1^4}{12} - \frac{t_1^4}{6} t_1 + \beta (t_1 + t_2) \frac{t_2^3}{6}] + \beta lmf \left( \frac{(t_1 + t_2)^2}{6} - \frac{t_1^2}{6} t_2 \right) + \beta \frac{(t_1 + t_2)^3}{6} - \frac{t_1^4}{6} t_2 + \beta (t_1 + t_2) \frac{t_2^3}{6}]
$$

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And \[ A_R(\alpha) = W - (W - V)\alpha = \frac{1}{(t_1 + t_2)} \left[ C_1^2 + \text{klmf}_1 + h^3 \left( \frac{mf}{p^3} - \frac{mf}{p^3} - r \right) \right]^{1/2} + h^3 \frac{mf}{p^3} \left[ \frac{(t_1 + t_2)^2}{2} + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 \right]^{1/2} - \frac{t_1 t_2}{2} - \beta \frac{t_1^4}{12} + \beta \frac{(t_1 + t_2)^4}{12} + \beta(t_1 + t_2) \frac{t_1}{6} \] + \beta lm f \left[ \frac{(t_1 + t_2)^2}{6} - \frac{t_1 t_2}{2} \right] + C_2^2 f + ld mf t_2 \] - \frac{1}{(t_1 + t_2)} \left[ ((C_1^2 - C_2^2) + h^3 \left( \frac{mf}{p^3} - \frac{mf}{p^3} - r \right) \right]^{1/2} - \frac{h^2 \left( \frac{mf}{p^3} - \frac{mf}{p^3} - r \right)}{2} + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 - \frac{t_1 t_2}{2} - \beta \frac{t_1^4}{12} - \frac{\beta(t_1 + t_2)^4}{6} + \frac{\beta(t_1 + t_2)^4}{6} + \beta \frac{(t_1 + t_2)^4}{6} \] + \beta(t_1 + t_2) \frac{t_1}{6} \right] \}

The fuzzy average total cost function (T\(\bar{a}_{sd}\)) is defuzzified by Signed Distance Method as follows,

\[
T\bar{a}_{sd}(t_1, t_2) = \frac{1}{2(t_1 + t_2)} \left[ C_1^2 + \text{klmf}_1 + h^3 \left( \frac{mf}{p^3} - \frac{mf}{p^3} - r \right) \right]^{1/2} + h^3 \frac{mf}{p^3} \left[ \frac{(t_1 + t_2)^2}{2} + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 - \frac{t_1 t_2}{2} - \beta \frac{t_1^4}{12} + \beta \frac{(t_1 + t_2)^4}{12} + \beta(t_1 + t_2) \frac{t_1}{6} \] + \beta lm f \left[ \frac{(t_1 + t_2)^2}{6} - \frac{t_1 t_2}{2} \right] + C_2^2 f + ld mf t_2 \] - \frac{1}{2(t_1 + t_2)} \left[ ((C_1^2 - C_2^2) + h^3 \left( \frac{mf}{p^3} - \frac{mf}{p^3} - r \right) \right]^{1/2} - \frac{h^2 \left( \frac{mf}{p^3} - \frac{mf}{p^3} - r \right)}{2} + \beta \frac{(t_1 + t_2)^4}{12} - t_1 t_2 - \frac{t_1 t_2}{2} - \beta \frac{t_1^4}{12} - \frac{\beta(t_1 + t_2)^4}{6} + \frac{\beta(t_1 + t_2)^4}{6} + \beta \frac{(t_1 + t_2)^4}{6} \] + \beta(t_1 + t_2) \frac{t_1}{6} \right] \}

Now, the necessary condition for the average total cost function of the system is minimized if equation (18)satisfies

\[
\frac{\partial T_{\overline{a}_{sd}}(t_1, t_2)}{\partial t_1} = 0, \quad (19)
\]

And \[ \frac{\partial T_{\overline{a}_{sd}}(t_1, t_2)}{\partial t_2} = 0. \quad (20) \]

The solution, which might be called feasible solution of the problem, of the conditions (19) and (20) gives the optimal solutions of \( t_1 = t_1^* \) and \( t_2 = t_2^* \) which minimize \( T_{\overline{a}_{sd}}(t_1, t_2) = T_{\overline{a}_{sd}}(t_1^*, t_2^*) \) provided they satisfy the sufficient conditions-

\[
\frac{\partial ^2 T_{\overline{a}_{sd}}(t_1, t_2)}{\partial t_1 ^2}, \quad \frac{\partial ^2 T_{\overline{a}_{sd}}(t_1, t_2)}{\partial t_2 ^2} - \left( \frac{\partial ^2 T_{\overline{a}_{sd}}(t_1, t_2)}{\partial t_1 \partial t_2} \right)^2 > 0 \quad (21)
\]

And \[ \frac{\partial ^2 T_{\overline{a}_{sd}}(t_1, t_2)}{\partial t_1 ^2} > 0 \quad \text{Or,} \quad \frac{\partial ^2 T_{\overline{a}_{sd}}(t_1, t_2)}{\partial t_2 ^2} > 0 \quad (22) \]

However, it’s difficult to solve the problem by inferring an explicit equation of the solutions from conditions (19) and (20). Therefore, the optimal service level \( t_1^* \) and the optimal cycle time...
$t_1^* + t_2^*$ is solved by using the software LINGO 16.0. Moreover, it is also verified that the sufficient conditions of the optimality of the solutions $t_1^*$ and $t_2^*$ are satisfied (i.e. inequalities (21) and (22)) under certain conditions.

Similarly, the highest inventory level per unit time in fuzzy environment is given by

$$
\tilde{I}_m = \left( k \frac{mf}{p} - \frac{mf}{p} - r \right) t_1
$$

$$
= \left( k \frac{mf}{(p_1, p_2, p_3)} - \frac{mf}{(p_1, p_2, p_3)} - r \right) t_1
$$

(23)

Defuzzified value of fuzzy number ($L_m^*$) by using Signed Distance Method is given by-

$$
\left( \frac{L_m^*}{sd} \right) = \frac{1}{2} \left( k \frac{mf}{p_1} - \frac{mf}{p_1} - r \right) t_1 + \frac{1}{4} \left( k \frac{mf}{p_2} - \frac{mf}{p_2} - r \right) t_1
$$

$$
+ \frac{1}{2} \left( k \frac{mf}{p_3} - \frac{mf}{p_3} - r \right) t_1
$$

(24)

Numerical Solution

VI-A: For crisp model: To show the proposed technique, let’s consider the accompanying input value in-

| Table-1: Input Data |
|---------------------|
| $C_1$ | $k$ | $C_2$ | $m$ | $f$ | $h$ | $p$ | $d$ | $l$ | $r$ | $\beta$ |
| 175 | 1.4 | 50 | 2565 | 6 | 1.2 | 11.6 | 1.3 | 0.15 | 2 | 0.01 |

The solution of the crisp-model is in Table-2

| Table-2 : Result |
|------------------|
| $t_1^*$ | $t_2^*$ | $\text{Tac}(t_1, t_2)^*$ | $L_m^*$ |
| 0.9168 | 0.5023 | 3816.822 | 484.7281 |

VI-B : For Fuzzy Model : Gives a chance to assume the parameters in fuzzy sense as : $\overline{C}_1 = (150, 175, 200)$, $\overline{p} = (8.6, 11.6, 14.6)$, $\overline{C}_2 = (25, 50, 75)$, $\overline{h} = (0.84, 1.2, 1.56)$, where other parameters are unchanged. The solution of fuzzy model by Signed Distance Method is,

When $\overline{C}_1$, $\overline{p}$, $\overline{C}_2$ and $\overline{h}$ are all Triangular fuzzy number then the solution is given in-

| Table-3: Result |
|------------------|
| $t_1^*$ | $t_2^*$ | $\text{Tac}(t_1, t_2)^*$ | $L_m^*$ |
| 0.9191 | 0.5040 | 3814.925 | 503.4081 |

(2) When $\overline{C}_1$, $\overline{p}$, and $\overline{C}_2$ are Triangular fuzzy number then the solution is given in-

| Table-4: Result |
|------------------|
| $t_1^*$ | $t_2^*$ | $\text{Tac}(t_1, t_2)^*$ | $L_m^*$ |
| 0.9031 | 0.4924 | 3828.560 | 494.6113 |

(3) When $\overline{C}_1$, and $\overline{p}$ are Triangular fuzzy numbers then the solution is given in-
| Parameters | Changed Value | *PCPV | $t_1^*$ | $t_2^*$ | Tac $(t_1, t_2)^*$ | $L_m^*$ |
|-----------|--------------|-------|------------|------------|-----------------|--------|
| $C_1=175$ | -30          | 122.5 | 0.8572     | 0.4795     | 3778.723        | 453.2304 |
|           | -15          | 148.75| 0.8875     | 0.4911     | 3798.057        | 469.2162 |
|           | 00           | 175   | 0.9168     | 0.5023     | 3816.822        | 484.7281 |
|           | +15          | 201.25| 0.9454     | 0.5132     | 3835.066        | 499.8058 |
|           | +30          | 227.5 | 0.9731     | 0.5238     | 3852.839        | 514.4838 |
| $k=1.4$   | -30          | 0.98  | 0.0000     | 0.7685     | 4233.764        | 0.0000  |
|           | -15          | 1.19  | 1.7367     | 0.1635     | 3268.603        | 434.3065 |
|           | 00           | 1.4   | 0.9168     | 0.5023     | 3816.822        | 484.7281 |
|           | +15          | 1.61  | 0.4117     | 0.6943     | 4121.946        | 332.3405 |
|           | +30          | 1.82  | 0.0184     | 0.7681     | 4233.477        | 19.0962 |
| $C_2=50$  | -30          | 35    | 0.8123     | 0.4622     | 3750.002        | 429.4780 |
|           | -15          | 42.5  | 0.8660     | 0.4828     | 3784.308        | 457.8487 |
|           | 00           | 50    | 0.9168     | 0.5023     | 3816.822        | 484.7281 |
|           | +15          | 57.5  | 0.9653     | 0.5208     | 3847.801        | 510.3290 |
|           | +30          | 65    | 1.0115     | 0.5385     | 3877.443        | 534.8182 |
| Parameter | Value | Change | Percentage Change | Value | Change | Percentage Change |
|-----------|-------|--------|------------------|-------|--------|------------------|
| m=2565    | 1795.5| -30    | 1.1219           | 0.5797|        | 2762.855         | 414.5397 |
|           | 2080.25| -15    | 1.0329           | 0.5462|        | 3155.231         | 442.4768 |
|           | 2565   | 00     | 0.9168           | 0.5023|        | 3816.822         | 484.7281 |
|           | 2949.75| +15    | 0.8459           | 0.4753|        | 4337.446         | 514.5294 |
|           | 3334.5 | +30    | 0.7875           | 0.4529|        | 4854.907         | 541.6986 |
| f=6       | 4.2    | -30    | 0.9975           | 0.5325|        | 2707.156         | 308.5458 |
|           | 5.1    | -15    | 0.9508           | 0.5150|        | 3262.388         | 426.9689 |
|           | 6      | 00     | 0.9168           | 0.5023|        | 3816.822         | 484.7281 |
|           | 6.9    | +15    | 0.8911           | 0.4926|        | 4370.720         | 542.0407 |
|           | 7.8    | +30    | 0.8708           | 0.4850|        | 4924.238         | 599.0377 |
| h=1.2     | 0.84   | -30    | 1.0638           | 0.6151|        | 3709.147         | 562.4372 |
|           | 1.02   | -15    | 0.9820           | 0.5509|        | 3765.363         | 519.1984 |
|           | 1.2    | 00     | 0.9168           | 0.5023|        | 3816.822         | 484.7281 |
|           | 1.38   | +15    | 0.8633           | 0.4640|        | 3864.551         | 456.4212 |
|           | 1.56   | +30    | 0.8183           | 0.4328|        | 3909.253         | 432.6353 |
| p=11.6    | 8.12   | -30    | 0.7844           | 0.4104|        | 3945.814         | 593.0900 |
|           | 9.86   | -15    | 0.8545           | 0.4581|        | 3872.760         | 531.7821 |
|           | 11.6   | 00     | 0.9168           | 0.5023|        | 3816.822         | 484.7281 |
|           | 13.35  | +15    | 0.9743           | 0.5447|        | 3771.299         | 446.3356 |
|           | 15.08  | +30    | 1.0244           | 0.5829|        | 3735.587         | 416.1339 |
| d=1.3     | 0.91   | -30    | 0.1356           | 0.7619|        | 3325.834         | 71.7164  |
|           | 1.105  | -15    | 0.6054           | 0.6614|        | 3621.825         | 320.0902 |
|           | 1.3    | 00     | 0.9168           | 0.5023|        | 3816.822         | 484.7281 |
|           | 1.425  | +15    | 1.1212           | 0.3035|        | 3944.362         | 592.7702 |
|           | 1.69   | +30    | 1.2193           | 0.0642|        | 4005.503         | 644.6292 |
| l=0.15    | 0.105  | -30    | 0.9551           | 0.4762|        | 2870.387         | 504.9487 |
|           | 0.1275 | -15    | 0.9363           | 0.4894|        | 3343.782         | 495.0048 |
|           | 0.15   | 00     | 0.9168           | 0.5023|        | 3816.822         | 484.7281 |
|           | 0.1725 | +15    | 0.8968           | 0.5150|        | 4289.504         | 474.1139 |
|           | 0.195  | +30    | 0.8760           | 0.5274|        | 4761.822         | 463.1572 |
| r=2       | 1.4    | -30    | 0.9161           | 0.5024|        | 3817.035         | 484.9041 |
|           | 1.7    | -15    | 0.9165           | 0.5024|        | 3816.929         | 484.8161 |
|           | 2      | 00     | 0.9168           | 0.5023|        | 3816.822         | 484.7281 |
|           | 2.3    | +15    | 0.9172           | 0.5022|        | 3816.716         | 484.600 |
|           | 2.6    | +30    | 0.9176           | 0.5022|        | 3816.609         | 484.5518 |
| β=0.01    | 0.0070 | -30    | 0.9172           | 0.5048|        | 3816.068         | 484.8918 |
|           | 0.0085 | -15    | 0.9170           | 0.5038|        | 3816.446         | 484.8086 |
|           | 0.0100 | 00     | 0.9168           | 0.5023|        | 3816.822         | 484.7281 |
|           | 0.0115 | +15    | 0.9167           | 0.5011|        | 3817.197         | 484.6501 |
|           | 0.0130 | +30    | 0.9153           | 0.4865|        | 3821.674         | 483.8837 |

*PCPV = Percentage Change in Parameter Values.*
Observations

From the Table 9 the following can be closed:

(1) From the Table 9, for increasing of \( C_1 \), the optimal value of \( t_1^* \) and \( t_2^* \) increases slowly. With this effect, the total average cost \( Tac (t_1, t_2) \) and the highest inventory level \( L_m^* \) increases slowly.

(2) From the Table, when \( k<1 \) i.e. 0.98, the optimal value of \( t_1^* \) and the highest inventory level \( L_m^* \) becomes zero where the optimal value of \( t_2^* \) increases. With this effect the total average cost \( Tac (t_1, t_2) \) increases. Apart from this, for increasing \( k \), the optimal value of \( t_1^* \) decreases and the optimal value of \( t_2^* \) increases rapidly. With this effect, the total average cost \( Tac (t_1, t_2) \) increases and the highest inventory level \( L_m^* \) decreases rapidly.

(3) From the Table, for increase in \( C_2 \), the optimal value of \( t_1^* \) and \( t_2^* \) increases slowly. By this effect, the total average cost \( Tac (t_1, t_2) \) increases slowly and the highest inventory level \( L_m^* \) increases rapidly.

(4) From the Table, for increase in \( m \), the optimal value of \( t_1^* \) and \( t_2^* \) decreases slowly. With this effect, the total average cost \( Tac (t_1, t_2) \) and the highest inventory level \( L_m^* \) increases rapidly.

(5) From the Table, for increase in \( f \), the optimal value of \( t_1^* \) and \( t_2^* \) decreases slowly. With this effect, the total average cost \( Tac (t_1, t_2) \) and the highest inventory level \( L_m^* \) increases rapidly.

(6) From the Table, for increase in \( h \), the optimal value of \( t_1^* \) and \( t_2^* \) decreases slowly. With this effect, the total average cost \( Tac (t_1, t_2) \) increases rapidly and the highest inventory level \( L_m^* \) decreases slowly.

(7) From the Table, for increase in \( p \), the optimal value of \( t_1^* \) increases rapidly and the optimal value of \( t_2^* \) increases slowly. With this effect, the total average cost \( Tac (t_1, t_2) \) decreases slowly and the highest inventory level \( L_m^* \) decreases rapidly.

(8) From the Table, for increase in \( d \), the optimal value of \( t_1^* \) increases and the optimal value of \( t_2^* \) decreases rapidly. With this effect, the total average cost \( Tac (t_1, t_2) \) increases slowly and the highest inventory level \( L_m^* \) increases rapidly.

(9) From the Table, for increase in \( l \), the optimal value of \( t_1^* \) decreases and \( t_2^* \) increases slowly. With this effect, the total average cost \( Tac (t_1, t_2) \) increases rapidly and the highest inventory level \( L_m^* \) decreases slowly.

(10) From the Table, for increase in \( r \), the optimal value of \( t_1^* \) increases and the optimal value of \( t_2^* \) decreases slightly. With this effect, the increment of the total average cost \( Tac (t_1, t_2) \) and the decrement of the highest inventory level \( L_m^* \) is negligible.

(11) From the Table, for increase in \( \beta \), the decrement of the optimal value of \( t_1^* \) and \( t_2^* \) is negligible. With this effect, the increment of the total average cost \( Tac (t_1, t_2) \) is very slow and the decrement of the highest inventory level \( L_m^* \) is negligible.

Conclusions

Here a genuine E. P. Q. Inventory Model is proposed and solutions are provided along affectability examination approach. Table-9 indicates when deterioration, production cost, holding cost is lesser, average cost function of the system decreases. Whereas it is also observed that lesser the population, lesser the demand and lesser the selling price, greater the demand. Here, a crisp model is produced then it changed to fuzzy model taking triangular fuzzy number and illuminated by Signed Distance Method. Decision maker may get the ideal outcomes as per his desire utilizing the result of this model. In future, the other sort of membership functions,
for example, Parabolic Fuzzy Number (pFN), Generalised Fuzzy Numbers, Piecewise Linear Hyperbolic Fuzzy Number, Parabolic level Fuzzy Number (PfFN), Pentagonal Fuzzy Number and so forth can be considered to build the membership function and afterward that model can be effectively solved by Werner’s Approach, Nearest Interval Approximation, Geometric Programming (GP) strategy, Nearest Symmetric Triangular Defuzzification (NSTD) technique, and so forth.

**Limitations of the Study**

In this proposed model of the inventory system, there were a few constraints, which are as follows:

- The inventory system includes just a single item and one stocking point.
- The proposed model is restricted here on the grounds that shortages are not permitted.

**Future Scope**

In future, researchers can extend this model by taking allowable shortages, two warehouses, stock dependent demand, permissible delay in payment, stochastic demand and inflation. Further, other sort of membership functions, for example, Parabolic Fuzzy Number (pFN), Generalised Fuzzy Numbers, Piecewise Linear Hyperbolic Fuzzy Number, Parabolic level Fuzzy Number (PfFN), Pentagonal Fuzzy Number and so forth can be considered to build the membership function and afterward that model can be effectively solved by Werner’s Approach, Nearest Interval Approximation, Geometric Programming (GP) strategy, Nearest Symmetric Triangular Defuzzification (NSTD) technique, and so forth.

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**Authors’ Profile**

**Sahidul Islam** is working as Assistant Professor in Department of Mathematics, University of Kalyani, Nadia, West Bengal, India. He earned his Ph.D from the Indian Institute of Engineering Science and Technology, Shibpur, India. He received the Junior Research Fellowship and Senior Research Fellowship from the Council of Scientific and Industrial Research (CSIR), Indian in 2003 and 2005, respectively. He has published two books and over 52 papers in various reputed journals. His main research interests are information theory, fuzzy optimizations, operations research and optimization techniques.

**Abhishek Kanti Biswas**, M.Sc., is working as Assistant Professor, Department of Mathematics, R K M Vivekananda Centenary College, Kolkata, West Bengal, India.