It is well known that quantum fluctuations of massless scalar and tensor (gravitational) fields are very much amplified at inflationary stage and create considerable density inhomogeneities [1-4] or relic gravitational waves [5-12]. This is closely related to the fact that these fields are not conformally invariant even though they are massless. Conformal invariance of a massless scalar field $\phi$ can be only achieved by the nonminimal coupling to the curvature scalar $R\phi^2/6$ and in this case quantum fluctuations are not amplified in the Robertson-Walker background. Gravitational waves are also conformally noninvariant in the standard General Relativity [13].

The amplification of the quantum fluctuations in cosmological conditions can be understood as particle production by external gravitational field. Since the Robertson-Walker metric is known to be conformally flat the background gravitational field does not produce particles if the underlying theory is conformally invariant [14]. In particular electromagnetic waves are not produced in such conditions since the classical electrodynamics is conformally invariant in the limit of vanishing masses of fermions. Quantum corrections however are known to break the conformal invariance. There are three possible sources of its breaking: nonzero masses of charged particles, absence of conformal invariance for (even massless) scalar field as mentioned above, and quantum conformal anomaly due to the celebrated triangle diagrams [15]. In what follows we consider the last case because the other two produce generically much smaller effects.

It was shown in ref. [16] that production of photons in conformally flat cosmological background due to the trace anomaly can be considerable and that the Maxwell equations are modified by the anomaly in the following way:

$$\partial_{\mu}F_{\mu} + \kappa a^{-1} F_{\mu} = 0$$

(1)

where $a = a(\tau)$ is the scale factor, $\tau$ is the conformal time, the metric has the form $ds^2 = a^2(\tau)(d\tau^2 - dr^2)$, and the contraction of the indices is made with the metric tensor of the flat space-time. The numerical coefficient $\kappa$ in $SU(N)$-gauge theory with $N_f$ number of charged fermions is equal to

$$\kappa = \frac{\alpha}{\pi} \left(\frac{11N}{3} - \frac{2N_f}{3}\right)$$

(2)

Here $\alpha$ is the fine structure constant which is to be taken at the momentum transfer $p$ equal to the Hubble parameter during inflation, $p = H$. In the asymptotically free theory one would expect $\alpha \approx 0.02$.

It is convenient to chose the gauge condition $\partial_{\mu}A^\mu = 0$. In this gauge the time component of the vector potential $A_\tau$ satisfies the same equation as in the conformally invariant case while the space component are modified by the extra term in the equations of motion:

$$(\partial^2_{\tau} - \frac{1}{a^2}\partial^2_r + \kappa a' a A^\prime_{\tau})A_j(\tau, r) = 0$$

(3)

where prime means differentiation with respect to conformal time $\tau$ and we put $A_\tau = 0$ since this component is not amplified by the Robertson-Walker background.

Quantization of the electromagnetic field in this background metric with the account of the conformal invariance breaking can be made with the usual decomposition

$$A_j(\tau, r) = \int \frac{d^3k}{(2\pi)^{3/2}(2\omega)^{1/2}} [c_j(k)A(\tau, k)e^{-ikr} + h.c.]$$

(4)

where operators $c_j$ satisfy

$$\langle c_j(k)e^{ikl}\rangle_{\text{vac}} = \delta(k' - k)\delta_j^i - k_1k_l/k^2$$

(5)

The C-function $A(\tau, k)$ satisfies the equation

$$A'' + k^2 A + \kappa a' A' = 0$$

(6)

Note that the amplitudes of massless minimally coupled scalar field and of gravitational wave satisfy the same equation with $\kappa = 2$. 

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Equation (6) is solved as

\[ A(k, \tau) = (k\tau)^{1/2-\kappa}[C_1 J_{\kappa-1/2}(k\tau) + C_2 J_{1/2-\kappa}(k\tau)] \]  

The coefficients \( C_1 \) and \( C_2 \) can be found from the conditions that \( A(k, \tau) \) tends to \( \exp(i\omega t) \) in the limit of vanishing Hubble parameter. In the De Sitter stage \( \tau \approx (1/H - t) \), as \( H \to 0 \), where \( t \) is the physical time. Using this equation and the asymptotic expressions for the Bessel functions (as \( \kappa \to \infty \)) we find

\[ C_1 = \frac{\sqrt{2\pi}}{1 - \exp(2i\pi \nu)} \left( \frac{H}{k} \right)^{\kappa/2} \exp\left(\frac{ik}{H} + \frac{i\pi}{4} - \frac{i\pi \nu}{2}\right) \]

\[ c_2 = -\exp(-i\pi \nu) \]  

where \( \nu = (\kappa + 1)/2 \).

Near the end of inflation that is for \( \tau \to 0 \), \( A(k, \tau) \) is given by the expression

\[ |A(k, \tau)| \to \frac{2^\nu \sqrt{2\pi}}{|1 - \exp(2i\pi \nu)| \Gamma(\nu + 1)} \]  

Using this expression and the relation between the physical and conformal momenta \( p = a(t)k \) we can evaluate the energy density of electromagnetic field generated during inflation at the moment when its wave reenters the horizon as

\[ F_{\mu\nu}^2 \approx (Hl)^\kappa / t^4 \]  

If \( \kappa \ll 1 \) the corresponding field is negligibly small but for \( \kappa = O(1) \) the amplitude of the magnetic field can be large enough to seed the observed magnetic fields in galaxies. Indeed it is known from observations that the strength of the galactic magnetic fields is of the order of \( B = 10^{-6} \)G and correspondingly its energy density is close to that of the electromagnetic background radiation. The field given by eq. (10) is about \( 10^{40} \) below the necessary value if \( (Hl)^\kappa = O(1) \). However the factor \( Hl \) is extremely large. For \( H = 10^{12} \) GeV and \( l = L_{gal} \) it is about \( 10^{46} \). So for \( \kappa = O(1) \) the magnetic field generated during inflationary stage can be large enough to create the observed fields in galaxies even without the dynamo amplification [17,18]. The latter may amplify the seed magnetic field by about 10 orders of magnitude permitting a slightly smaller value of \( \kappa \). In contrast to magnetic field the electric field generated during inflationary stage not only is not amplified but vice versa is dumped down due to the large conductivity of the primeval plasma. One may argue that the conductivity is nonzero and large already during the De Sitter stage because of the thermal background with the temperature \( H/2\pi \) [19]. This background is associated with the horizon at \( 1/H \). However the particle polarization which could damp the electric field cannot compete with the universe expansion and hence is not able to screen the field. Electric field should be screened during the Friedman stage after the inflaton energy density is transformed into energy density of real particles in the cosmic plasma. The characteristic time scale of this process is of the order of the horizon and so one may expect generation of chaotic electric currents on cosmologically large scales. These chaotic electric currents may in turn generate cosmic magnetic fields.

Note that there are several other proposals in the literature for magnetic field generation in a theory with broken conformal invariance. In ref [20] it was assumed that there exists the nonminimal coupling of the curvature scalar to electromagnetic field of the form \( \xi R A_\mu^2 \) so that not only conformal but also gauge invariance of electromagnetism is broken. Another proposal [21] is the coupling of the electromagnetic field strength tensor to the dilaton field \( \exp(\phi) F_{\mu\nu}^2 \). In both these cases it was argued that it is possible to generate the seed magnetic field of the appropriate amplitude during inflation. Our model does not demand any extra coupling in addition to the standard electrodynamics but to get a large enough field the number of charged particles with \( m < H \) should be about 30. Reverting the arguments one can put a bound on \( \kappa \) from the absence of too strong cosmic electromagnetic fields.

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