Propagation of spin waves

Jin Hu$^1$,* and Zhe Xu$^1$

$^1$Department of Physics, Tsinghua University, Beijing 100084, China

A mutilated model is constructed to approximate the collision term of spin Boltzmann equation that incorporates newly appearing collisional invariants i.e., the total angular momentum. With recourse to degenerate perturbation theory, the dispersion relations of hydrodynamic modes are formulated, among which spin modes are responsible for spin equilibration. We find that the non-locality does not change the sound speed but slows down the propagation of spin waves. The damping rates of spin modes are close to those of spinless modes over a reasonable parameter value range. The results reveal that both spin and momentum should be treated simultaneously in a unified transport framework. In the nonrelativistic limit, the short-wavelength behavior for normal modes is also explored and there exists a critical point for every distinct discrete mode over which only quasiparticle modes contribute.

Introduction: The experimental developments in measuring the spin-related observables of Λ hyperons [1–4] have raised extensive interests in global polarization [5–13] and local polarization [14, 15]. Theoretical calculations are not consistent with experiment results in local polarization, which is referred to as “spin sign problem” and originates from the fact that spin does not reach equilibrium as expected, see [16] for a recent review. In the past few years, different efforts are made to get insight into the spin puzzle along the line of spin hydrodynamics [17–28], and quantum kinetic theory [29–35], among which spin relaxation becomes essential because the relaxation rate of the spin density toward its equilibrium value is crucial in determining how the spin polarization evolves in time in theoretical simulations of QCD plasma.

Similar to the still unsettled question of how thermal equilibrium in relativistic heavy-ion collisions is reached, the questions of how the spin of quarks relaxes to equilibrium and whether it equilibrates faster than momentum or not remain under debates [28, 36–42]. When spin is considered, one must take into account the conservation of total angular momentum in hydrodynamic description. Accompanied by newly introduced degrees of freedom, there arises new physical phenomenon of the propagation of spin waves. Analogous to sound propagation in spinless (spin-averaged) fluids [43], the propagation of spin waves should be also fundamental in spin hydrodynamic theory and deserve a comprehensive exploration, which we expect to provide in this work.

As with the recent studies [26, 39, 44], we focus on a linear mode analysis in the current work and find that spin relaxation couples to the attenuation of spin modes. Therefore, spin relaxation time is identified as the lifetime of spin modes, based on which one can make a direct comparison of two typical time scales in relation with spin and momentum relaxation. Natural units $k_B = c = \hbar = 1$ are used. The metric tensor is given by $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, while $\Delta_{\mu\nu} = g_{\mu\nu} - u^\mu u^\nu$ is the projection tensor orthogonal to the four-vector fluid velocity $u^\mu$ and $\epsilon^{\mu\nu\alpha\beta}$ is Levi-civita tensor. In addition, we employ the antisymmetric shorthand, $X_{\mu\nu} \equiv (X^{\mu\nu} - X^{\nu\mu})/2$.

Mutilated collision operator: When it comes to spin-induced phenomena, the framework of transport theory must be extended to incorporate the information of spin evolution. To that end, the spin Boltzmann equation for massive spin-1/2 fermions with non-local collision effects was proposed, where the consistent interpretation for equilibrium state and collision invariants is elaborated [29].

Here we choose to work out the propagation of hydrodynamic modes by constructing a mutilated collision operator based on the collision invariants appearing in [29] instead of directly solving the complicated linearized integral equation derived in [26]. Note that the collision invariants are exactly eigenfunctions of linearized collision operator with zero as eigenvalues. If insisting on semipositive definiteness and self-adjointness of a linearized collision operator, we are led to treat it as an evolution Hamiltonian operator and obtain on homogeneous circumstance [26]

$$\chi(t, p, s) = \exp(-\frac{L}{u \cdot p} t) \chi(p, s),$$

where $\chi(t, p, s)$ represents the homogeneous deviation function from equilibrium distribution dependent on time $t$, particle momentum $p$ and spin $s$, and $L$ denotes the linearized collision operator. As time goes by, zero modes can survive long time while positive ones become damped exponentially and less dominant. Hereafter, we concentrate on the zero modes and see how they respond to the perturbation of non-uniformity.

The resulting linearized transport equation takes the form

$$p \cdot \partial \chi(x, p, s) = -L \chi(x, p, s),$$

where the deviation function $\chi(x, p, s)$ is dependent on space-time coordination $x$ when spatial non-uniformity is recovered. The linearized collision operator $L$ is now approximated by a mutilated operator

$$-L \sim \left(-\gamma + \gamma \sum_{n=1}^{11} |\lambda_n\rangle \langle \lambda_n| \right)$$

with $|\lambda_n\rangle$ being orthonormal eigenfunctions of $L$ and $\gamma$ being a representative positive eigenvalue. One can easily verify that $L$ inherit basic properties of what we require for a linearized collision operator such as

[1–4] have raised extensive interests in global polarization [5–13] and local polarization [14, 15].
semipositive definiteness, self-adjointness and \( L|\lambda_n\rangle = 0, n = 1, \ldots, 11 \) and \( L|\lambda_n\rangle = \gamma|\lambda_n\rangle, n > 11 \). Here the “mutated” means all positive eigenvalues collapse into one chosen positive eigenvalue (it is suggestive to take the smallest one). The zero modes with eleven-fold degeneracy exactly correspond to collision invariants \( J_\mu^{\nu} \) and \( J'^{\alpha\beta}_{\mu\nu} \), where the identification of \( J_\mu^{\nu} = 2\Delta^{\nu}p^\mu + \frac{1}{2}\Sigma_\mu^{\nu} \) as total angular momentum is seen in [29, 30] with

\[
\Delta^\mu \equiv \frac{1}{2m(p \cdot \hat{t} + m)} \epsilon^\mu_{\alpha\beta} p_\alpha \hat{t}_\beta
\]  

characterizing the non-locality in a collision, where \( \hat{t}^\mu \) is the time-like unit vector which is \((1, 0)\) in the frame where \( p^\mu \) is measured.

It may be observed that this is exactly a kind of relaxation time approximation (RTA) by identifying \( \gamma \) with the reciprocal of relaxation time \( \tau_{RT}^{-1} \). Compared to traditional RTA, the novel RTA or mutated operator is proved to reconcile the momentum dependence of the relaxation time with the macroscopic conservation laws. When the relaxation time has no momentum dependence, one can always argue that RTA is consistent with the conservation laws by imposing matching conditions but this is not the general case. Without elaboration, we refer to a recent letter [45]. From now on, \( \gamma \) is parameterized as energy dependent \( \gamma \equiv \gamma_T/\tau^\lambda \) with an energy-independent constant \( \gamma_T \).

In order to seek a solution of the form \( \chi \sim \hat{\chi} e^{-ik \cdot x} \), we substitute it into Eq.(2),

\[
\tau \omega \hat{\chi} + \hat{p}^\mu l_\mu \kappa \hat{\chi} = -iL \hat{\chi},
\]  

where \( L \hat{\chi} \equiv \gamma \tau \left( \hat{\chi} - \sum_{n=1}^{11} \tilde{\psi}_n \tau \tilde{\chi} \tilde{\psi}_n \right) \) with notations \( \tau \equiv \frac{p^\mu}{\hat{p}^\mu}, \hat{p} \equiv \frac{p}{\hat{p}}. \) Here we introduce dimensionless frequency and wave vector

\[
\omega \equiv \frac{u \cdot k}{n \sigma}, \quad \kappa^\alpha \equiv \frac{\Delta^\alpha_{\beta} k_\beta}{n \sigma},
\]  

and one unit vector

\[
l^\alpha \equiv \frac{\kappa^\alpha}{\kappa}, \quad \kappa \equiv \sqrt{-\kappa \cdot \kappa},
\]  

where \( n \) is density, \( T \) is temperature and \( \sigma \) is an arbitrary constant with the dimension of cross sections. On the other hand, the inner product is defined as

\[
(B, C) = \frac{1}{(2\pi)^3} \int d\Gamma \exp(-\beta \cdot p)B(p, s)C(p, s),
\]  

with the measure defined as \( d\Gamma \equiv d^4p \delta(p^2 - m^2)\sqrt{\frac{\hat{p}^\mu}{\Gamma}} d^4s \delta(s \cdot s + 3)\delta(p \cdot s) \), and the eigenfunctions \( |\lambda_n\rangle \) are also replaced by less-abstract functions \( \tilde{\psi}_n \) given by

\[
\tilde{\psi}_1 = \frac{1}{\sqrt{V_{1,1}^{(0)}}}, \quad \tilde{\psi}_2 = \beta \frac{u \cdot p - \frac{p}{p_{\mu} \Gamma_{\mu}}}{\sqrt{V_{2,2}^{(0)}}}, \quad \tilde{\psi}_3 = \beta l \cdot p \frac{1}{\sqrt{V_{3,3}^{(0)}}},
\]

\[
\tilde{\psi}_4 = \beta j \cdot p \sqrt{V_{3,3}^{(0)}}, \quad \tilde{\psi}_5 = \beta \frac{u \cdot p}{\sqrt{V_{3,3}^{(0)}}}, \quad \tilde{\psi}_6 = \frac{u_\mu j_{\mu} l_\nu}{\sqrt{V_{3,3}^{(0)}}},
\]

\[
\tilde{\psi}_7 = \frac{u_\mu j_{\mu} h_\nu}{\sqrt{V_{6,6}^{(0)}}}, \quad \tilde{\psi}_8 = \frac{u_\mu j_{\mu} h_\nu}{\sqrt{V_{6,6}^{(0)}}}, \quad \tilde{\psi}_9 = \frac{l_\mu j_{\mu} h_\nu}{\sqrt{V_{9,9}^{(0)}}},
\]

\[
\tilde{\psi}_{10} = \frac{l_\mu j_{\mu} h_\nu}{\sqrt{V_{9,9}^{(0)}}}, \quad \tilde{\psi}_{11} = \frac{j_\mu j_{\mu} h_\nu}{\sqrt{V_{9,9}^{(0)}}},
\]  

where

\[
I_{\kappa q} \equiv \frac{2}{(2q + 1)!!} \int \frac{d^4p}{(2\pi)^3} \delta(p^2 - m^2)(u \cdot p)^{n-2q},
\]

\[
V_{i,j}^{(\lambda)} \equiv \beta \int \frac{d\Gamma}{(2\pi)^3} \tau^\lambda \tilde{\psi}_i \kappa \cdot p \tilde{\psi}_j \exp(-\beta \cdot p).
\]  

Here \( \beta^{\mu} \equiv \beta u^{\mu} = \frac{u^{\mu}}{T} \) and we introduced two auxiliary unit vectors \( j, h \) to form an orthonormal triad with \( u \) and \( l \). In the remainder of this work, a quiescent background fluid i.e., \( u^{\mu} = (1, 0, 0, 0) \) is chosen, then the triad \( (u, j, h, l) \) stands for the projection to the directions of \( (t, x, y, z) \). Last but not the least, note the eigenfunctions Eq.(9) has been defined to fulfill the orthonormal condition

\[
(\tilde{\psi}_\alpha, \tau \tilde{\psi}_\beta) = \delta_{\alpha\beta}.
\]  

\( \text{Degenerate perturbation theory:} \) As a familiar problem in the perturbation theory, the solutions to Eq.(5) can be sought in the fashion as used in quantum mechanics by treating the spatial term \( \hat{p}^\mu \kappa \hat{\chi} \) as a perturbation with respect to \( -iL \hat{\chi} \), then the eigenfunctions and eigenvalues can be routinely expanded into

\[
\hat{\chi} = \hat{\chi}^{(0)} + \hat{\chi}^{(1)} + \cdots, \quad \omega = \omega^{(0)} + \omega^{(1)} + \omega^{(2)} + \cdots.
\]  

The dispersion relations, which are obtained from the secular equation for Eq.(5), are formulated up to first order in \( \kappa \) [26]

\[
\omega_1^{(1)} = -\omega_2^{(1)} = \sqrt{H_{2,2}^{(0)} + H_{1,1}^{(0)}}, \quad \omega_3^{(1)} = \omega_4^{(1)} = \omega_5^{(1)} = \omega_6^{(1)} = \omega_7^{(1)} = 0,
\]

\[
\omega_7^{(1)} = \frac{\omega_8^{(1)}}{H_{7,7}^{(0)}}, \quad \omega_9^{(1)} = \omega_1^{(1)} = -H_{7,7}^{(0)},
\]  

where

\[
H_{i,j}^{(\lambda)} \equiv \beta \int \frac{d\Gamma}{(2\pi)^3} \tau^\lambda \tilde{\psi}_i \kappa \cdot p \tilde{\psi}_j \exp(-\beta \cdot p).
\]  

One can readily verify that the results of the first five spinless modes are the same as those in [43] independent of the details of interactions involved. At the meantime, we find that only four transverse spin modes \((\omega_7, \omega_8, \omega_9, \omega_{10}) \) are propagating with the same speed of propagation \( c_{\text{spin}} = \omega_7^{(1)}/\kappa \) where the minus
When setting the non-locality $\Delta$ zero, conservation (SAC) is shown by the solid black line. Fig. 1. As a comparison, we also exhibit the sound speed are monotonically decreasing functions of $z$ and the inclusion of non-locality in collisions will slow down the propagation of spin waves (we call them spin waves in analogy with sound waves), while the propagation of sound wave is immune to that change. The non-locality does not alter the equation of state (EOS) in contrast to Enskog-type non-locality with hard-sphere exclusion potential. A quick inspection shows that 1 and $p^\mu$ are not collision invariants of Enskog collision term any more and the redefinition or modification to the static pressure is needed to retain the form of conservation laws [47].

\begin{equation}
\begin{align*}
\omega_1 &= c_s \kappa - i \Gamma_1, \\
\omega_2 &= -c_s \kappa - i \Gamma_1, \\
\omega_3 &= -i \Gamma_3, \\
\omega_4 &= \omega_5 = -i \Gamma_4, \\
\omega_6 &= -i \Gamma_6, \\
\omega_7 &= \omega_8 = c_{\text{spin}} \kappa - i \Gamma_7, \\
\omega_9 &= \omega_{10} = -c_{\text{spin}} \kappa - i \Gamma_7, \\
\omega_{11} &= -i \Gamma_{11},
\end{align*}
\end{equation}

with the damping coefficients defined as

\begin{equation}
\begin{align*}
\Gamma_1 &= \frac{1}{\gamma R} \left[ Q_{11}^{(\lambda)} + \frac{1}{2} \omega_1^{(1)} \left( \frac{H_{1,1}^{(0,2)}}{\omega_1^{(1)}} \right) \left( V_{11}^{(0)} \right)^2 + \frac{H_{2,2}^{(0,2)}}{\omega_2^{(1)}} \left( V_{22}^{(0)} \right)^2 \\
&\quad + \frac{V_3^{(1)}}{\omega_3^{(1)}} + \frac{2 H_{1,3}^{(0,2)} H_{2,3}^{(0,2)} \omega_1^{(2)}}{\omega_1^{(1)}} \omega_1^{(1)} \left( V_{11}^{(0)} \right)^2 \left( V_{22}^{(0)} \right)^2 - 2 \omega_1^{(1)} \left( H_{1,1}^{(0)} \right) \omega_1^{(1)} \left( V_{22}^{(0)} \right)^2 \\
&\quad + \frac{H_{2,3}^{(0)}}{\omega_3^{(1)} H_3^{(0)}} \right], \\
\Gamma_3 &= \frac{1}{\gamma R} Q_{3,3}^{(\lambda)}, \\
\Gamma_4 &= \frac{1}{\gamma R} Q_{4,4}^{(\lambda)}, \\
\Gamma_6 &= \frac{1}{\gamma R} Q_{6,6}^{(\lambda)}, \\
\Gamma_7 &= \frac{1}{\gamma R} \left[ Q_{7,7}^{(\lambda)} + \frac{1}{2} H_{7,7}^{(0,2)} \left( V_{6,6}^{(1)} \right) \left( V_{6,6}^{(1)} \right) + \frac{V_9^{(1)}}{\omega_9^{(1)}} + 2 H_{7,9}^{(0,2)} \right], \\
\Gamma_{11} &= \frac{1}{\gamma R} Q_{11,11}^{(\lambda)},
\end{align*}
\end{equation}

where

\begin{equation}
Q_{i,j}^{(\lambda)} = \beta \int \frac{d\vec{p}}{(2\pi)^3} \frac{\tau^{(\lambda) \left( p \cdot \kappa \right)^2}}{p \cdot u} \tilde{\chi}_i \exp(-\beta \cdot p),
\end{equation}

and $\tilde{\chi}_i$’s are given in Eq.(52) of [26].

Then all the damping rates are shown in Fig. 2 as functions of reduced mass $z$, which are crucial because the attenuation of spin modes couples to the dissipation of spin density [26] while other spinless modes are not responsible for it. Among the spin modes four propagating transverse modes are degenerate in the damping rates $\Gamma_7$, while the other two are non-propagating longitudinal modes which are purely decaying at their respective decaying rates $\Gamma_6$ and $\Gamma_{11}$. One can clearly see that the choice of $\lambda$ dramatically affects the attenuation of these discrete normal modes both in magnitude and in their dependence on reduced mass $z$.

The specific value of $\lambda$ relies on the dynamic details and corresponds to various physical scenarios. For example, $\lambda = 0$ corresponds to traditional RTA proposed firstly by Anderson and Witting (AW) [48], while $\lambda = 0.5$ is argued to well approximate the effective kinetic descriptions of quantum chromodynamics [45, 49–51]. When $\lambda$ is big enough (see $\lambda = 1.5$) and may be out of realistic range $0 \lesssim \lambda \lesssim 1$, the tendency even flips compared to AW case, all damping coefficients are monotonically increasing functions of $z$, and the spin modes are separated from spinless ones forming the hierarchy of $\Gamma_{\text{spin}} \lesssim \Gamma_{\text{nonspin}}$. On the other hand, when $\lambda$ is comparatively small $\lambda = 0$ or corresponds to relevant QCD scenario $\lambda = 0.5$, the damping is almost as slow as spinless ones, namely, the dynamic evolution of spin and momentum are twisted in this scenario over a wide

\begin{itemize}
\item[] 1 Strictly speaking, $\lambda = 0.38$ is the best fit for QCD scenario but here we take a comparatively close value 0.5. Besides, $\lambda = 0$ and $\lambda = 1$ are thought to be two extreme limits between which most theories lie. There are also exceptional cases such as four fermion interaction in the electroweak sector where the energy scale is far below the masses of gauge Bosons. In that case, $\sigma$ is shown to be proportional to energy square $E^2$ which gives an estimation of $\lambda \approx -1$ and is not within our range of consideration see [49, 50] for more details.
\end{itemize}
with the particle three velocity $v$, which implies $d\Gamma \to \frac{m^2}{2\sqrt{3}\pi} \int d^3v d^3s \delta(s^2 - 3)$, and
\[
\sqrt{\frac{z}{2}}(-i\omega\tilde{\chi} + i\kappa \cdot \vec{v}\tilde{\chi}) = -\gamma(\tilde{\chi} - \sum_{n=1}^{11}(\tilde{\psi}_n, \tilde{\chi})\tilde{\psi}_n), \tag{19}
\]
where a factor $\sqrt{\frac{z}{2}}$ has been absorbed into $\gamma$ and $\kappa = (0, 0, \kappa)$ without losing generality, then define fluctuation amplitudes
\[
\rho_n(\omega, \kappa) \equiv (\tilde{\psi}_n, (\omega, \kappa, p, s)). \tag{20}
\]
Note that the weight function in Eq. (8) is replaced by the equilibrium distribution $f_0 = (\frac{2}{\rho^2}) \exp(-\frac{1}{2}zv^2)$.

Introduce $c \equiv \sqrt{\frac{2}{z}}\omega, \tilde{c} \equiv \sqrt{\frac{2}{z}}\omega + i\gamma\kappa, \tilde{\gamma} \equiv \frac{\tilde{\gamma}}{z}$, and if $\tilde{c}$ is not real, we get
\[
\tilde{\chi} = \frac{\tilde{\gamma}}{\tilde{c}}\sum_{n=1}^{11}\rho_n\tilde{\psi}_n, \tag{21}
\]
then these amplitudes can be cast into
\[
\rho_T = \frac{\tilde{\gamma}}{2} \left( \frac{Z(\tilde{c})}{z^2} (1 + 2\tilde{c}^2) - \frac{2\tilde{c}}{z^2} \right) \rho_T + \frac{\tilde{\gamma}m}{4\sqrt{V_{9,9}}} \rho_T \times \left( (2\tilde{c} - \frac{2\tilde{c}}{z} - \frac{2\tilde{c}^3}{z})Z(\tilde{c}) + \frac{3}{z} + \frac{2\tilde{c}^2}{z} - 2 \right) \rho_9, \\
\rho_9 = \frac{\tilde{\gamma}m}{4\sqrt{V_{9,9}}} \left( (2\tilde{c} - \frac{2\tilde{c}}{z} - \frac{2\tilde{c}^3}{z})Z(\tilde{c}) + \frac{3}{z} + \frac{2\tilde{c}^2}{z} \right) - 2 \rho_T + \frac{\tilde{\gamma}m}{4\sqrt{V_{9,9}}} \left( (\frac{c^4}{z^2} + \frac{3\tilde{c}^2}{2z^2} - \frac{2\tilde{c}^2}{z^2} + 1) \right) \frac{1}{z} \rho_9, \\
+ 1)Z(\tilde{c}) - \frac{\tilde{c}^3}{z^2} - \frac{3\tilde{c}}{2z^2} - \frac{\tilde{c}}{2z^2} + \frac{2\tilde{c}^2}{z^2} \rho_9, \tag{22}
\]
with $V_{9,9} \equiv \frac{m^2}{4} (1 - \frac{2}{z} + \frac{5}{2z^2})$ and $Z(\tilde{c}) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \exp(-\frac{1}{2t^2}) \sqrt{\frac{z}{2}}e^{-t}$. For illustration we only display how to extract the encoding information from ($\rho_T, \rho_9$), which can be safely extended to other fluctuation amplitudes.

The determination of dispersion relations is equivalent to finding roots of
\[
\Phi_3(\tilde{c}, z) = \left[ \frac{\tilde{\gamma}z^2}{2} \left( \frac{Z(\tilde{c})}{z^2} (1 + 2\tilde{c}^2) - \frac{2\tilde{c}}{z^2} \right) - 1 \right] \frac{\tilde{\gamma}m^2}{4V_{9,9}} \times \left( (\tilde{c}^4 + \frac{3\tilde{c}^2}{z^2} - \frac{2\tilde{c}^2}{z^2} + \frac{1}{z} + 1)Z(\tilde{c}) \right) \\
- \frac{\tilde{c}^3}{z^2} - \frac{\tilde{c}}{2z^2} + \frac{2\tilde{c}^2}{z^2} - 1 \right] \frac{\tilde{\gamma}m^2}{16V_{9,9}} \times \left( (2\tilde{c} - \frac{2\tilde{c}}{z} - \frac{2\tilde{c}^3}{z})Z(\tilde{c}) + \frac{3}{z} + \frac{2\tilde{c}^2}{z} - 2 \right)^2. \tag{23}
\]

It is convenient to invoke the residue theorem that the number of zeros of $\Phi$ in a region of the complex $\tilde{c}$-plane in which $\Phi$ is an analytic function is equal to the times of the representative point $\Phi$ in the $\Phi$-plane encircles the origin. It is easy to verify that there is a critical value for $\kappa$ below which zeros exist, i.e. the dispersion relations of discrete normal modes hold. When $\kappa$ exceeds the critical value $\kappa_c$, there are no discrete modes.
TABLE I. The critical value $\kappa_o$ for longitudinal (L) and transverse (T) spin modes. In the last two columns, $a \pm b$ represents that $a$ is formulated with $z = 10$ and $b$ is the maximum discrepancy from $a$ when $z$ ranges from 5 to 20.

| $\kappa_o/\hbar \sigma \gamma$ | L(6th) | L(11th) | T |
|-------------------------------|--------|---------|----|
| 1.772                        | 1.762±0.040 | 1.754±0.079 |

The critical value $\kappa_o$ for various spin modes are numerically solved and displayed in TABLE I. The results are exact in the nonrelativistic limit.

Several comments are followed in order.

- The present calculation is limited to nonrelativistic situation, but the qualitative behavior of normal mode should be independent of whether we take this limit. As the concept of long or short wavelength concerns only the dynamics, it is thus irrelevant to kinematics. We expect the qualitative behavior of criticality is identical for both relativistic and nonrelativistic cases.

- The existence of the critical behavior of discrete normal modes reflects the transition from collision-dominated region to Knudsen region, through the realistic distinction may not be that clear. In simplified mutilated model, the transition region collapses into a critical point and the short or long wavelength dynamics can be uniformly described without extra changes.

- Discrete normal modes including spin modes are exactly the ways of ordered particle collective motion organized by collisions. In long wavelength limit, they return to zero modes. In free-flow dominated region, the initial fluctuation is carried away by disordered particles. Thus no discrete modes and dispersion relations are found, which happens while $\kappa$ exceeds $\kappa_o$. There is an exception that $\tilde{c}$ is real. The behavior of normal modes when $\tilde{c}$ is real is alike to that in Knudsen region. Two cases are deeply connected with each other by verifying that the fluctuation $\tilde{\chi}$ in both situations takes the form of single particle continuum spectrum. It makes sense that in the case of long wavelength there are hydrodynamic modes and quasiparticle modes and only quasiparticle modes are left in Knudsen region.

- As a supplement, the critical $\kappa_o$’s for two sound modes, two shear modes, and one heat mode are 1.853, 1.772 and 1.918 respectively [52]. Thus there exists an another hierarchy $\kappa_{o,\text{spin}} \approx \kappa_{o,\text{non}}$, which manifests that discrete spinless modes are more resistant to the “destruction” of non-uniformity. Nevertheless, this discrepancy is negligible.

Summary: In this paper, we constructed a mutilated model incorporating the collisional invariants in spin Boltzmann equation, where six new zero modes associated with total angular momentum arise from nontrivial dynamics of nonlocal collisions. The dispersion relations of all discrete normal modes, namely, the propagating speeds and attenuation rates are all computed up to second order perturbation. At the first order perturbation in spatial non-uniformity, the non-locality contributes nothing to sound speed but slows down the propagation of spin waves. At the second order perturbation, the damping rates of spin modes are close to those of spinless modes for various relevant energy dependence of $\gamma$. The results reveal that spin and momentum relax at comparable damping rates parameterized for modeling realistic physical scenarios and a unified transportation of both spin and momentum is necessary. In the nonrelativistic limit, we investigate the existence conditions for discrete normal modes and find there exists a critical point for every distinct discrete mode over which only quasiparticle modes contribute.

This work was supported by the NSFC Grant No.11890710, No.11890712 and No.12035006.

[1] L. Adamczyk et al. (STAR), Nature 548, 62 (2017), 1701.06657.
[2] E. Alpatov (), STAR (for the), J. Phys. Conf. Ser. 1690, 012120 (2020).
[3] J. Adam et al. (STAR), Phys. Rev. Lett. 123, 132301 (2019), 1905.11917.
[4] J. Adam et al. (STAR), Phys. Rev. C 98, 014910 (2018), 1805.04400.
[5] Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94, 102301 (2005), [Erratum: Phys.Rev.Lett. 96, 039901 (2006)], nucl-th/0410079.
[6] D.-X. Wei, W.-T. Deng, and X.-G. Huang, Phys. Rev. C 99, 014905 (2019), 1810.00151.
[7] I. Karpenko and F. Becattini, Eur. Phys. J. C 77, 213 (2017), 1610.04717.
[8] L. Csernai, J. Kapusta, and T. Welle, Phys. Rev. C 99, 021901 (2019), 1807.00151.
[9] A. Bzdak, Phys. Rev. D 96, 056011 (2017), 1703.03003.
[10] S. Shi, K. Li, and J. Liao, Phys. Lett. B 788, 409 (2019), 1712.00878.
[11] Y. Sun and C. M. Ko, Phys. Rev. C 96, 024906 (2017), 1706.09467.
[12] Y. B. Ivanov, V. D. Toneev, and A. A. Soldatov, Phys. Atom. Nucl. 83, 179 (2020), 1910.01332.
[13] Y. Xie, D. Wang, and L. P. Csernai, Phys. Rev. C 95, 031901 (2017), 1703.03770.
[14] F. Becattini and I. Karpenko, Phys. Rev. Lett. 120, 012302 (2018), 1707.07984.
[15] X.-L. Xia, H. Li, Z.-B. Tang, and Q. Wang, Phys. Rev. C 98, 024905 (2018), 1803.00867.
[16] F. Becattini (2022), 2204.01144.
[17] W. Florkowski, B. Friman, A. Jaiswal, and E. Spertanza, Phys. Rev. C 97, 041901 (2018), 1705.00587.
[18] H.-H. Peng, J.-J. Zhang, X.-L. Sheng, and Q. Wang (2021), 2107.00448.
[19] F. Becattini and L. Tinti, Annals Phys. 325, 1566 (2010), 0911.0864.
[20] W. Florkowski, A. Kumar, and R. Ryblewski, Prog. Part. Nucl. Phys. 108, 103709 (2019), 1811.04409.
[21] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, Phys. Lett. B 795, 100 (2019), 1901.06615.
[22] K. Fukushima and S. Pu, Phys. Lett. B 817, 136346
[23] J. Hu, Phys. Rev. D 103, 116015 (2021), 2101.08440.
[24] S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R. Ryblewski, Phys. Rev. D 103, 014030 (2021), 2008.10976.
[25] J. Hu, Phys. Rev. D 105, 076009 (2022), 2111.03571.
[26] J. Hu (2022), 2202.07373.
[27] J. Hu, Phys. Rev. D 105, 096021 (2022), 2204.12946.
[28] N. Weickgenannt, D. Wagner, E. Speranza, and D. Rischke (2022), 2203.04766.
[29] N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke, Phys. Rev. D 104, 016022 (2021), 2103.04896.
[30] N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke, Phys. Rev. Lett. 127, 052301 (2021), 2005.01506.
[31] D.-L. Yang, K. Hattori, and Y. Hidaka, JHEP 20, 070 (2020), 2002.02612.
[32] X.-L. Sheng, N. Weickgenannt, E. Speranza, D. H. Rischke, and Q. Wang, Phys. Rev. D 104, 016029 (2021), 2103.10636.
[33] Z. Chen and S. Lin, Phys. Rev. D 105, 014015 (2022), 2109.08440.
[34] Z. Wang and P. Zhuang (2021), 2105.00915.
[35] D.-L. Yang (2021), 2112.14392.
[36] S. Li and H.-U. Yee, Phys. Rev. D 100, 056022 (2019), 1905.10463.
[37] J. I. Kapusta, E. Rrapaj, and S. Rudaz, Phys. Rev. C 101, 024907 (2020), 1907.10750.
[38] J. I. Kapusta, E. Rrapaj, and S. Rudaz, Phys. Rev. C 102, 064911 (2020), 2004.14807.
[39] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee (2022), 2201.12390.
[40] J. I. Kapusta, E. Rrapaj, and S. Rudaz, Phys. Rev. C 101, 031901 (2020), 1910.12759.
[41] A. Ayala, D. De La Cruz, S. Hernández-Ortiz, L. A. Hernández, and J. Salinas, Phys. Lett. B 801, 135169 (2020), 1909.09274.
[42] A. Ayala, D. de la Cruz, L. A. Hernández, and J. Salinas, Phys. Rev. D 102, 056019 (2020), 2003.06545.
[43] S. R. De Groot, W. A. Van Leeuwen, and C. G. Van Weert, Relativistic Kinetic Theory. Principles and Applications (North-Holland, 1980).
[44] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee, JHEP 11, 150 (2021), 2107.14231.
[45] G. S. Rocha, G. S. Denicol, and J. Noronha, Phys. Rev. Lett. 127, 042301 (2021), 2103.07489.
[46] V. E. Ambrus, R. Ryblewski, and R. Singh (2022), 2202.03952.
[47] R. Mallfiet, Nucl. Phys. A 420, 621 (1984).
[48] J. L. Anderson and H. R. Witting, Physica 74, 466 (1974).
[49] K. Dusling, G. D. Moore, and D. Teaney, Phys. Rev. C 81, 034907 (2010), 0909.0754.
[50] K. Dusling and T. Schäfer, Phys. Rev. C 85, 044909 (2012), 1109.5181.
[51] A. Kurkela and U. A. Wiedemann, Eur. Phys. J. C 79, 776 (2019), 1712.04376.
[52] J. Boer and G. E. Uhlenbeck, Studies in the Statistical Mechanics (Orth-Holland Publishing Company, Amsterdam, 1970).