"$\lambda_\mu$"-CONNECTEDNESS IN GENERALIZED
TOPOLOGICAL SPACES

Pon Jeyanthi, Periyadurai Nalayini, and Takashi Noiri

Abstract. In this paper, we introduce the concept of $\lambda_\mu$-connectedness in
generalized topological spaces by means of $\lambda_\mu$-open sets and investigate their
properties.

1. Introduction

In 1997, Császár [2] introduced the concept of a generalization of topological
spaces, which is a generalized topological space. A generalized topology (briefly
GT) $\mu$ on a non-empty set $X$ is a collection of subsets of $X$ such that $\emptyset \in \mu$ and $\mu$
is closed under arbitrary union. Elements of $\mu$ are called $\mu$-open sets. A set $X$ with
a GT $\mu$ is called a generalized topological space (briefly GTS), denoted by $(X, \mu)$.
If $A$ is a subset of $(X, \mu)$, then $c_\mu(A)$ is the smallest $\mu$-closed set containing $A$ and
$i_\mu(A)$ is the largest $\mu$-open set contained in $A$. Clearly, $A$ is $\mu$-open if and only if
$A = i_\mu(A)$ and $A$ is $\mu$-closed if and only if $A = c_\mu(A)$ [4, 3]. A GTS $(X, \mu)$ is called
a strong generalized topological space if $X \in \mu$. The concept of $\gamma$-connectedness
was also introduced by Császár, further studied by several authors including Shen
[10] and Baskaran et al. [1]. In this paper, we introduce the concept of $\lambda_\mu$-
connectedness in generalized topological spaces and give some characterizations of
these spaces.

Definition 1.1. ([6]) Let $(X, \mu)$ be a GTS and $A \subseteq X$. Then the subsets
$\wedge_\mu(A)$ and $\vee_\mu(A)$ are defined as follows:

$$\wedge_\mu(A) = \begin{cases} 
\bigcap\{G : A \subseteq G, G \in \mu\} & \text{if there exists } G \in \mu \text{ such that } A \subseteq G; \\
X & \text{otherwise.}
\end{cases}$$

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are no non-empty disjoint sets $U; V$

$X$

A closed set is called a $A$ if $A = \cap \{G : A \subseteq G, G \in \lambda(\mathbb{X}, \mu)\}$.

Definition 1.3. (9) A subset $A$ of a GTS $(\mathbb{X}, \mu)$ is said to be $\lambda_{\mu}$-closed set if $A = T \cap C$, where $T$ is a $\wedge_{\mu}$-set and $C$ is a $\mu$-closed set. The complement of a $\lambda_{\mu}$-closed set is called a $\lambda_{\mu}$-open set.

For $A \subseteq X$, we denote by $c_{\lambda_{\mu}}(A)$ the intersection of all $\lambda_{\mu}$-closed subsets of $X$ containing $A$.

Definition 1.4. (8) Let $(\mathbb{X}, \mu)$ be a GTS. A subset $A$ of $X$ is called a $\wedge_{\mu}$-set if $A = \cap \wedge_{\mu}(A)$, where $\wedge_{\mu}(A) = \cap\{G : A \subseteq G, G \in \lambda(\mathbb{X}, \mu)\}$.

Definition 1.5. A subset $A$ of a GTS $(\mathbb{X}, \mu)$ is called a $\lambda_{\mu}$-closed set if $A = T \cap C$, where $T$ is a $\wedge_{\mu}$-set and $C$ is $\lambda_{\mu}$-closed. The complement of a $\lambda_{\mu}$-closed set is called a $\lambda_{\mu}$-open set.

We denote the collection of all $\lambda_{\mu}$-open (resp. $\lambda_{\mu}$-closed, $\lambda_{\mu}$-open, $\lambda_{\mu}$-closed) sets of $X$ by $\lambda_{\mu}(\mathbb{X}, \mu)$ (resp. $\lambda_{\mu}(\mathbb{X}, \mu)$, $\lambda_{\mu}(\mathbb{X}, \mu)$).

Definition 1.6. A GTS $(\mathbb{X}, \mu)$ is $\mu$-connected (5) if there are no non-empty disjoint sets $U, V \in \mu$ such that $U \cup V = \mathbb{X}$.

Definition 1.7. (1) Two subsets $A$ and $B$ in a GTS $(\mathbb{X}, \mu)$ are said to be $\mu$-separated if and only if $A \cap c_{\mu}(B) = \emptyset$ and $B \cap c_{\mu}(A) = \emptyset$.

Definition 1.8. (7) If $(\mathbb{X}, \mu)$ is a GTS and $Y$ is a subset of $X$, then the collection $\mu|_{Y} = \{U \cap Y : U \in \mu\}$ is a GT on $Y$ called the subspace generalized topology and $(Y, \mu|_{Y})$ is the subspace of $X$.

2. $\lambda_{\mu}$-Separateness

In this section, we introduce the notion of $\lambda_{\mu}$-separated sets and discuss its properties.

Definition 2.1. Two subsets $A$ and $B$ of a GTS $(\mathbb{X}, \mu)$ are said to be $\lambda_{\mu}$-separated if and only if $A \cap c_{\mu_{\mu}}(B) = \emptyset$ and $c_{\mu_{\mu}}(A) \cap B = \emptyset$.

From the fact that $c_{\mu_{\mu}}(A) \subseteq c_{\mu}(A)$, for every subset $A$ of $(\mathbb{X}, \mu)$, every $\mu$-separated set is $\lambda_{\mu}$-separated. But the converse may not be true as shown in the following example.

Example 2.1. Let $X = R$ and $\mu = \{\emptyset, Q\}$, where $R$ and $Q$ denote the set of all real numbers and rational numbers, respectively. The family of all $\lambda_{\mu}$-closed sets is $\{\emptyset, Q, R \setminus Q, R\}$. Then $Q \cap c_{\lambda_{\mu}}(R \setminus Q) = c_{\lambda_{\mu}}(Q) \cap (R \setminus Q) = \emptyset$ but $c_{\mu}(Q) \cap (R \setminus Q) \neq \emptyset$. Hence $Q$ and $R \setminus Q$ are $\lambda_{\mu}$-separated but not $\mu$-separated. \hfill $\square$
Remark 2.1. Since \( A \cap B \subseteq A \cap c_{\lambda_\mu}(B) \), \( \lambda_\mu \)-separated sets are always disjoint. The converse may not be true in general. \( \square \)

Example 2.2. Let \( X = R \) and \( \mu = \{0, Q\} \). The subsets \( \{\sqrt{2}, \sqrt{3}\} \), \( \{\sqrt{5}, \sqrt{7}\} \) are disjoint but not \( \lambda_\mu \)-separated.

\[ \square \]

Theorem 2.1. Let \( A \) and \( B \) be non-empty subsets in a GTS \((X, \mu)\). The following statements are hold:

(i) If \( A \) and \( B \) are \( \lambda_\mu \)-separated, \( A_1 \subseteq A \) and \( B_1 \subseteq B \), then \( A_1 \) and \( B_1 \) are also \( \lambda_\mu \)-separated.

(ii) If \( A \) and \( B \) are \( \lambda_\mu \)-closed sets such that \( A \cap B = \emptyset \), then \( A \) and \( B \) are \( \lambda_\mu \)-separated.

(iii) If \( A \) and \( B \) are \( \lambda_\mu \)-open, \( H = A \cap (X \setminus B) \) and \( G = B \cap (X \setminus A) \), then \( H \) and \( G \) are \( \lambda_\mu \)-separated.

Proof. (i) Since \( A_1 \subseteq A \), \( c_{\lambda_\mu}(A_1) \subseteq c_{\lambda_\mu}(A) \). Therefore \( B \cap c_{\lambda_\mu}(A) = \emptyset \) implies \( B_1 \cap c_{\lambda_\mu}(A) = \emptyset \) and \( B_1 \cap c_{\lambda_\mu}(A_1) = \emptyset \). Similarly \( A_1 \cap c_{\lambda_\mu}(B_1) = \emptyset \). Hence \( A_1 \) and \( B_1 \) are \( \lambda_\mu \)-separated.

(ii) Since \( A \) and \( B \) are \( \lambda_\mu \)-closed, \( A = c_{\lambda_\mu}(A) \) and \( B = c_{\lambda_\mu}(B) \). Now \( A \cap B = \emptyset \) implies \( c_{\lambda_\mu}(A) \cap B = \emptyset \) and \( c_{\lambda_\mu}(B) \cap A = \emptyset \). Hence \( A \) and \( B \) are \( \lambda_\mu \)-separated.

(iii) Since \( H \subseteq (X \setminus B) \), \( c_{\lambda_\mu}(H) \subseteq c_{\lambda_\mu}(X \setminus B) = X \setminus B \) and hence \( c_{\lambda_\mu}(H) \cap B = \emptyset \). Also \( G \subseteq B \) implies \( c_{\lambda_\mu}(H) \cap G = \emptyset \). Similarly \( H \cap c_{\lambda_\mu}(G) = \emptyset \). Hence \( H \) and \( G \) are \( \lambda_\mu \)-separated. \( \square \)

Corollary 2.1. Let \( A \) and \( B \) be non-empty sets in a GTS \((X, \mu)\). The following statements are hold:

(i) If \( A \) and \( B \) are \( \lambda_\mu \)-open sets such that \( A \cap B = \emptyset \), then \( A \) and \( B \) are \( \lambda_\mu \)-separated.

(ii) If \( A \) and \( B \) are \( \lambda_\mu \)-closed, \( H = A \cap (X \setminus B) \) and \( G = B \cap (X \setminus A) \), then \( H \) and \( G \) are \( \lambda_\mu \)-separated.

Theorem 2.2. The subsets \( A \) and \( B \) of a GTS \((X, \mu)\) are \( \lambda_\mu \)-separated if and only if there exist \( U, V \in \lambda_\mu O(X, \mu) \) such that \( A \subseteq U \), \( B \subseteq V \) and \( A \cap V = \emptyset \), \( B \cap U = \emptyset \).

Proof. Let \( A \) and \( B \) be \( \lambda_\mu \)-separated sets. Let \( V = X \setminus c_{\lambda_\mu}(A) \) and \( U = X \setminus c_{\lambda_\mu}(B) \). Then \( U, V \in \lambda_\mu O(X, \mu) \) such that \( A \subseteq U \), \( B \subseteq V \) and \( A \cap V = \emptyset \). Hence \( A \cap U = \emptyset \). On the other hand, let \( U, V \in \lambda_\mu O(X, \mu) \) such that \( A \subseteq U \), \( B \subseteq V \) and \( A \cap V = \emptyset \), \( B \cap U = \emptyset \). Since \( X \setminus V \) and \( X \setminus U \) are \( \lambda_\mu \)-closed, \( c_{\lambda_\mu}(A) \subseteq c_{\lambda_\mu}(X \setminus V) = X \setminus V \subseteq X \setminus B \). Thus, \( c_{\lambda_\mu}(A) \cap B = \emptyset \). Similarly \( A \cap c_{\lambda_\mu}(B) = \emptyset \). Hence \( A \) and \( B \) are \( \lambda_\mu \)-separated. \( \square \)

3. \( \lambda_\mu \)-Connectedness

In this section, we introduce the notion of \( \lambda_\mu \)-connectedness and discuss their properties.
DEFINITION 3.1. A subset $S$ of a GTS $(X, \mu)$ is said to be $\lambda_\mu$-connected if there exist no $\lambda_\mu$-separated subsets $A$ and $B$ and $S = A \cup B$. Otherwise $S$ is said to be $\lambda_\mu$-disconnected.

It is clear that each $\lambda_\mu$-connected set is $\mu$-connected. The converse may not be true in general as shown in the following example. In other words, each $\lambda_\mu$-disconnected is $\lambda_\mu$-disconnected.

EXAMPLE 3.1. Let $X = [1, 2]$ and $\mu = \{\emptyset, \{1\}, \{1, 2\}\}$. The family of all $\lambda_\mu$-closed sets is $\{\emptyset, \{1\}, \{1, 2\}, \{2\}, \{1, 2\} \}$, Thus, $\{1, 2\}$ is $\mu$-connected but not $\lambda_\mu$-separated.

\[ \square \]

THEOREM 3.1. A GTS $(X, \mu)$ is $\lambda_\mu$-disconnected if and only if there exists a non-empty proper $\lambda_\mu$-clopen subset.

PROOF. Assume that $(X, \mu)$ is $\lambda_\mu$-disconnected. There exist $\lambda_\mu$-separated sets $A$ and $B$ such that $A \cup B = X$, $A \cap B = \emptyset$. Hence $A = X \setminus B$ and $B = X \setminus A$. Since $A \cup B = X$ and $B \subseteq c_\lambda_A (B)$, $X \subseteq A \cup c_\lambda_A (B)$. But $A \cup c_\lambda_A (B) \subseteq X$. Thus, $A \cup c_\lambda_A (B) = X$. We have $A \cap c_\lambda_A (B) = \emptyset$ and $B \cap c_\lambda_A (A) = \emptyset$ which implies $A = X \setminus c_\lambda_A (B)$ and $B = X \setminus c_\lambda_A (A)$. Since $c_\lambda_A (A)$ and $c_\lambda_A (B)$ are $\lambda_\mu$-closed, $X \setminus c_\lambda_A (A)$ and $X \setminus c_\lambda_A (B)$ are $\lambda_\mu$-open. Thus, $A$ and $B$ are $\lambda_\mu$-open. Since $A = X \setminus B$ and $B = X \setminus A$, $A$ and $B$ are $\lambda_\mu$-closed. Conversely, assume that there exists non-empty proper $\lambda_\mu$-clopen subset $A$ of $X$. Let $B = X \setminus A$. Then $A \cap B = \emptyset$ and $A \cup B = X$. Since $A \cap B = \emptyset$, $c_\lambda_A (A) \cap B = \emptyset$ and $A \cap c_\lambda_A (B) = \emptyset$. Thus, $A$ and $B$ are $\lambda_\mu$-separated. Hence $(X, \mu)$ is $\lambda_\mu$-disconnected.

\[ \square \]

THEOREM 3.2. A GTS $(X, \mu)$ is $\lambda_\mu$-disconnected if and only if any one of the following statements holds:

(i) $X$ is the union of two non-empty disjoint $\lambda_\mu$-open sets.
(ii) $X$ is the union of two non-empty disjoint $\lambda_\mu$-closed sets.

PROOF. Assume that $(X, \mu)$ is $\lambda_\mu$-disconnected. By Theorem 3.1, there exists a non-empty proper $\lambda_\mu$-clopen subset $A$ of $X$. Also, $A \cup (X \setminus A) = X$. Hence $A$ and $X \setminus A$ satisfy the conditions (i) and (ii). Conversely, assume that $A \cup B = X$ and $A \cap B = \emptyset$, where $A$ and $B$ are non-empty $\lambda_\mu$-open sets. Then $A = X \setminus B$ is $\lambda_\mu$-closed. Since $B$ is non-empty, $A$ is a proper subset of $X$. Thus, $A$ is a non-empty proper $\lambda_\mu$-closed subset of $X$. By Theorem 3.1, $X$ is $\lambda_\mu$-disconnected. Let $X = C \cup D$ and $C \cap D = \emptyset$, where $C$ and $D$ are non-empty $\lambda_\mu$-closed sets. Then $C = X \setminus D$ so that $C$ is $\lambda_\mu$-open. Since $D$ is non-empty, $C$ is a proper $\lambda_\mu$-clopen subset of $X$. By Theorem 3.1, $X$ is $\lambda_\mu$-disconnected.

\[ \square \]

THEOREM 3.3. If $E$ is a $\lambda_\mu$-connected subset of a GTS $(X, \mu)$ such that $E \subseteq A \cup B$, where $A$ and $B$ are $\lambda_\mu$-separated sets, then either $E \subseteq A$ or $E \subseteq B$.

PROOF. Since $A$ and $B$ are $\lambda_\mu$-separated sets, $A \cap c_\lambda_B (B) = \emptyset$ and $B \cap c_\lambda_A (A) = \emptyset$. $E \subseteq A \cup B$ implies $E = E \cap (A \cup B) = (E \cap A) \cup (E \cap B)$. Suppose $E \cap A \neq \emptyset$ and $E \cap B \neq \emptyset$. Then $(E \cap A) \cap c_\lambda_B (E \cap B) \subseteq (E \cap A) \cap (c_\lambda_B (E) \cap c_\lambda_A (B)) = (E \cap c_\lambda_B (E)) \cap (A \cap c_\lambda_A (B)) = \emptyset$. Similarly, $(E \cap B) \cap c_\lambda_A (E \cap A) = \emptyset$. Therefore, $E \subseteq A$ or $E \subseteq B$.

\[ \square \]
Hence $E \cap A$ and $E \cap B$ are $\lambda_\mu$-separated. Thus, $E$ is $\lambda_\mu$-disconnected, which is a contradiction. Hence at least one of the sets $E \cap A$ and $E \cap B$ is empty. If $E \cap A = \emptyset$, then $E = E \cap B$ which implies that $E \subseteq B$. Similarly if $E \cap B = \emptyset$, then $E \subseteq A$. Therefore, either $E \subseteq A$ or $E \subseteq B$.

**Corollary 3.1.** If $E$ is a $\lambda_\mu$-connected subset of a GTS $(X, \mu)$ such that $E \subseteq A \cup B$, where $A$ and $B$ are disjoint $\lambda_\mu$-open (resp. $\lambda_\mu$-closed) subsets of $X$, then $A$ and $B$ are $\lambda_\mu$-separated.

**Proof.** Since $A \subseteq X \setminus B$, $c_\lambda(A) \subseteq c_\lambda(X \setminus B) = X \setminus B$. Thus, $B \cap c_\lambda(A) = \emptyset$. Similarly, $A \cap c_\lambda(B) = \emptyset$. Hence $A$ and $B$ are $\lambda_\mu$-separated.

**Theorem 3.4.** If $E$ is a $\lambda_\mu$-connected subset of a GTS $(X, \mu)$ and $C$ is a subset such that $E \subseteq C \subseteq c_\lambda(E)$, then $C$ is also $\lambda_\mu$-connected.

**Proof.** Suppose that $C$ is not $\lambda_\mu$-connected. There exist $\lambda_\mu$-separated sets $A$ and $B$ such that $C = A \cup B$. Since $E \subseteq C$, $E \subseteq A \cup B$. By Theorem 3.3, $E \subseteq A$ or $E \subseteq B$. Let $E \subseteq A$, then $c_\lambda(E) \subseteq c_\lambda(A)$ which implies $c_\lambda(E) \cap B \subseteq c_\lambda(A) \cap B = \emptyset$. Since $C \subseteq c_\lambda(E)$, $B \subseteq C \subseteq c_\lambda(E)$ and hence $c_\lambda(E) \cap B = B$. Thus, $c_\lambda(E) \cap B = \emptyset$ and $c_\lambda(E) \cap B$ implies $B = \emptyset$. Similarly, if we consider $E \subseteq B$, we obtain $A = \emptyset$, which contradicts $A$ and $B$ are non-empty. Therefore $C$ is $\lambda_\mu$-connected.

**Corollary 3.2.** If $E$ is a $\lambda_\mu$-connected subset of a GTS $(X, \mu)$, $c_\lambda(E)$ is also $\lambda_\mu$-connected.

**Proof.** This is obvious by Theorem 3.4.

**Theorem 3.5.** Let $E$ be a subset of a GTS $(X, \mu)$. If any two points of $E$ are contained in some $\lambda_\mu$-connected subset of $E$, $E$ is a $\lambda_\mu$-connected subset of $X$.

**Proof.** Suppose $E$ is not $\lambda_\mu$-connected. Then there exist non-empty subsets $A$ and $B$ of $X$ such that $A \cap c_\lambda(B) = \emptyset$, $B \cap c_\lambda(A) = \emptyset$ and $E = A \cup B$. Since $A$, $B$ are non-empty, there exists a point $a \in A$ and a point $b \in B$. By hypothesis, $a$ and $b$ must be contained in some $\lambda_\mu$-connected subset $F$ of $E$. Since $F \subseteq A \cup B$ and $F$ is $\lambda_\mu$-connected, either $F \subseteq A$ or $F \subseteq B$. It follows that either $a, b \in A$ or $a, b \in B$. Let $a, b \in A$. Then $A \cap B = \emptyset$, which is a contradiction. Hence $E$ is a $\lambda_\mu$-connected subset of $X$.

**Theorem 3.6.** The union of any family of $\lambda_\mu$-connected sets having a non-empty intersection is a $\lambda_\mu$-connected set.

**Proof.** Let $\{E_\alpha\}$ be any family of $\lambda_\mu$-connected sets such that $\cap \{E_\alpha\} \neq \emptyset$. Let $E = \cup \{E_\alpha\}$. Suppose $E$ is not $\lambda_\mu$-connected. Therefore, there exist $\lambda_\mu$-separated sets $A$ and $B$ such that $E = A \cup B$. Since $\cap \{E_\alpha\} \neq \emptyset$, $x \in \cap \{E_\alpha\}$. Then $x$ belongs to each $E_\alpha$ and so $x \in E$. Consequently, $x \in A$ or $x \in B$. Without loss of generality, assume that $x \in A$. Then $E_\alpha \subseteq A$ for each $\alpha$. Hence $\cup E_\alpha \subseteq A$ and so $E \subseteq A$. Thus, $A \cup B \subseteq A$. Therefore $A = E$ which implies $B = \emptyset$ which is a contradiction. Thus, $E$ is $\lambda_\mu$-connected.
Theorem 3.7. The union of any family of \(\ast\lambda_\mu\)-connected subsets of a GTS \((X,\mu)\) with the property that one of the members of the family, intersects every other members is a \(\ast\lambda_\mu\)-connected set.

Proof. Let \(\{E_\alpha\}\) be any family of \(\ast\lambda_\mu\)-connected sets of a GTS \((X,\mu)\) with the property that one of the member say, \(E_\alpha\), intersects every other members. By Theorem 3.6, \(E_\alpha\) is \(\ast\lambda_\mu\)-connected. Now, let \(E_\alpha\) and \(E_\alpha\) be any two members of the family. Then \(E_\alpha\cap E_\alpha\neq\emptyset\), \(E_\alpha\cap E_\alpha\neq\emptyset\) and hence \((E_\alpha\cap E_\alpha)\cup(E_\alpha\cap E_\alpha)\neq\emptyset\) by Theorem 3.6, \(\cup(E_\alpha\cap E_\alpha)\) for each \(\alpha\) is \(\ast\lambda_\mu\)-connected. Hence \(\cup E_\alpha\) is \(\ast\lambda_\mu\)-connected.

Theorem 3.8. If \(A \subseteq B \cup C\) such that \(A\) is a non-empty \(\ast\lambda_\mu\)-connected set in a GTS \((X,\mu)\) and \(B, C\) are \(\ast\lambda_\mu\)-separated, then one of the following conditions holds:

(i) \(A \subseteq B\) and \(A \cap C = \emptyset\).

(ii) \(A \subseteq C\) and \(A \cap B = \emptyset\).

Proof. This is obvious by Theorem 3.3.

Definition 3.2. Let \((X,\mu)\) and \((X,\mu’)\) be two GTS. A mapping \(f : (X,\mu) \rightarrow (Y,\mu’)\) is said to be \((\ast\lambda_\mu,\mu’\) continuous if for each \(\mu’\)-open set \(V, f^{-1}(V)\) is \(\ast\lambda_\mu\)-open.

Theorem 3.9. Let \(f : (X,\mu) \rightarrow (Y,\mu’)\) be a \((\ast\lambda_\mu,\mu’\) continuous function. If \(K\) is \(\ast\lambda_\mu\)-connected in \(X\), then \(f(K)\) is \(\mu’\)-connected in \(Y\).

Proof. Suppose that \(f(K)\) is \(\mu’\)-disconnected in \(Y\). There exist \(\mu’\)-separated sets \(G\) and \(H\) of \(Y\) such that \(f(K) = G \cup H\). Set \(A = K \cap f^{-1}(G)\) and \(B = K \cap f^{-1}(H)\). Since \(f(K) = G \cup H, K \cap f^{-1}(G) \neq \emptyset\) and hence \(A \neq \emptyset\). Similarly, \(B \neq \emptyset\). Now, \(A \cap B = (K \cap f^{-1}(G)) \cap (K \cap f^{-1}(H)) = K \cap (f^{-1}(G) \cap f^{-1}(H)) = K \cap (f^{-1}(G \cap H)) = \emptyset\). Thus, \(A \cap B = \emptyset\) and \(A \cup B = K\). Now, \(A \cap c_{\lambda,\mu}(B) \subseteq f^{-1}(G) \cap c_{\lambda,\mu}(f^{-1}(H))\). Since \(f\) is \((\ast\lambda_\mu,\mu’\)-continuous, \(A \cap c_{\lambda,\mu}(B) \subseteq f^{-1}(G) \cap c_{\lambda,\mu}(f^{-1}(H))\). Therefore, \(A \cap c_{\lambda,\mu}(B) = \emptyset\). Similarly, \(B \cap c_{\lambda,\mu}(A) = \emptyset\). Thus, \(A\) and \(B\) are \(\ast\lambda_\mu\)-separated in \(X\) which is a contradiction. Therefore \(f(K)\) is \(\mu\)-connected in \(Y\).

Corollary 3.3. Let \(f : (X,\mu) \rightarrow (Y,\mu’)\) be a \((\ast\lambda_\mu,\mu’\) continuous surjection. If \(K\) is \(\mu’\)-disconnected in \(Y\), then \(f^{-1}(K)\) is \(\ast\lambda_\mu\)-connected in \(X\).

Proof. Let \(f^{-1}(K)\) be not \(\ast\lambda_\mu\)-disconnected in \(X\). Then \(f^{-1}(K)\) is \(\ast\lambda_\mu\)-connected in \(X\) and by Theorem 3.10, \(f(f^{-1}(K)) = K\) is \(\mu’\)-connected. Hence \(K\) is not \(\mu’\)-disconnected in \(Y\). Therefore, the proof is completed.

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References

[1] R. Baskaran, M. Murugalingam and D. Sivaraj. Separated sets in generalized topological spaces. J. Adv. Res. Pure Math., 2(1)(2010), 74–83.
[2] Á. Császár. Generalized open sets. Acta Math.Hungar., 75(1-2) (1997), 65–87.
[3] Á. Császár. On the γ-interior and γ-closure of a set. Acta Math.Hungar., 80(1-2) (1998), 89–93.
[4] Á. Császár. Generalized open sets in generalized topologies. Acta Math.Hungar., 106(1-2)(2005), 53–66.
[5] Á. Császár. γ-connected spaces. Acta Math.Hungar., 101(4)(2003), 273–279.
[6] E. Ekici and B. Roy. New generalized topologies on generalized topological spaces due to Császár. Acta Math.Hungar., 132(1-2) (2011), 117–124.
[7] J. Li. Generalized topologies generated by subbases. Acta Math.Hungar., 114 (1-2)(2007), 1–12.
[8] P. Jeyanthi, P. Nalayini and T. Noiri. *λ_μ* sets and *ν_μ* sets in generalized topological spaces. Bol. Soc. Paran. Mat. (3s), 35(1)(2017), 33–41.
[9] B. Roy and E. Ekici. On (Λ,μ)-closed sets in generalized topological spaces. Methods Func. Anal. Top., 17(2)(2011), 174–179.
[10] R. X. Shen. A note on generalized connectedness. Acta Math.Hungar., 122(3)(2009), 231–235.

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PON Jeyanthi. Research Centre, Department of Mathematics, Govindammal Aditanar College for Women, Trichy, Tamil Nadu, India.
E-mail address: jeyajeyanthi@rediffmail.com

Periyadurai Nalayini. Research Scholar, Reg. No: 11769, Research Centre, Department of Mathematics, Govindammal Aditanar College for Women, Trichy, Tamil Nadu, India. Affiliated to Manonmaniam Sundaranar University, Abishekappatti, Tirunelveli 627012, Tamilnadu, India.
E-mail address: nalayini4@gmail.com

Takashi Noiri. Hinagu, Yatsushiro - shi, Kumamoto - ken, 869-5142 Japan
E-mail address: t.noiri@nifty.com