Modelling adhesively-bonded T-joints by a meshless method

I J Sánchez*, L D C Ramalho¹, R D S G Campilho¹2 and J Belinha¹2

¹ INEGI, Instituto de Ciência e Inovação em Engenharia Mecânica e Engenharia Industrial. Porto, 4200-465, Portugal
² ISEP, Department of Mechanical Engineering, Instituto Superior de Engenharia do Porto (ISEP), Instituto Politécnico de Porto. Porto, 4249-015, Portugal

*Corresponding author: isidrodjsa@gmail.com

Abstract. Bonding of non-parallel substrates has many applications in the transport industry. Adhesively-bonded T-joints are employed for that purpose. However, geometrical dimensions, substrates’ shape and material choices are broad. The effect that varying upper substrate thickness has on the joint strength ($P_{\text{max}}$) was investigated in this work through numerical analyses. The numerical analysis was performed using a meshless method, the natural neighbour radial point interpolation method (NNRPIM), which has been proven accurate and robust in another adhesive joint configuration. Materials were considered elastic-plastic. A yield criterion developed for rubber-like materials, the Exponent Drucker-Prager, was used for the adhesive layer, while the metallic substrates were analysed with the von Mises yield criterion. $P_{\text{max}}$ was determined numerically using a strain-based continuum mechanics failure criterion. Normalised peel and shear stress distributions along the bond-line are presented. Effective strains, both elastic and plastic, were also obtained. The estimated $P_{\text{max}}$ was compared with experimental data; a good agreement was found. The stress distribution along the bond line becomes asymmetric in joints with unbalanced substrate thicknesses. At $P_{\text{max}}$, from 10 to 25% of the bond-line has entered into plastic regime. The results indicate that the proposed methodology is suitable to analyse adhesively-bonded joints under different load solicitations.

Keywords: Adhesive joints, Elastic-plastic analyses, Exponent Drucker-Prager, Meshless methods, NNRPIM.

1. Introduction

Union of non-parallel substrates is commonly found in engineering practice. In such cases, T-joints are employed. Adhesively-bonded T-joints are commonly employed in the manufacture of wind turbines’ blades, automotive components [1], bulk panel to fuselage unions for aircraft, and several panel-to-hull fixings in the maritime industry, as described in [2].

This type of joint is composed of at least two non-parallel substrates. The lower substrate, often flat, is placed at the bottom. The upper substrate is the substrate or substrates that intersect with the lower substrate, as shown in figure 1. There are various methods to connect the upper substrates with the lower one, and the method selected depends on the application, e.g., [1,3].

Numerical analyses of T-joints are frequently performed using the Finite Element Method (FEM). From these analyses, it has been found that reinforcing the joint increases the joint strength ($P_{\text{max}}$) [1]; consequently, $P_{\text{max}}$ is related to substrate stiffness. Similarly, the yielding of the substrates was also considered as a limiting factor to $P_{\text{max}}$ [1]. Peak stresses were reported at the bond line’s ends; the use
of a bi-adhesive bond line was proposed as a method to balance the stresses [4]. These studies analysed joints in which the upper and lower substrates had equal thickness. Moreover, stress distributions along the bond line were often omitted. Later, joints composed of substrates with different upper substrate thicknesses were studied both experimentally and numerically, first, through the Cohesive Zone Modelling (CZM) [3]. Afterwards, through the eXtended Finite Element Method (xFEM) [5]; different crack initiation criteria were explored. This type of joint, with metallic substrates, and the combination of adhesive systems, was found to be more suitable for stress-based crack initiation criteria [5]. Previous research has found that ductile adhesives do not yield following the von Mises criterion [6]. Dean and Crocker [6] evaluated other yield criteria and compared them against the von Mises. It was found that using the Exponent Drucker-Prager (EDP) yield criterion, the numerical behaviour obtained is closer to the experimental data than with other criteria. Despite this, the EDP is not well spread in the literature, existing only a few applications to adhesive joints using commercial FEM software (e.g., [6]). Recently, the EDP was used with a meshless method (MM) in the analysis of single-lap joints (SLJ) [7].

Although FEM is often used to analyse adhesive joints, it also possesses some limitations because of its dependency on a mesh [8]. Meshless methods (MM) are an alternative to the FEM, aiming to overcome some of the FEM’s limitations [8,9]. Unlike FEM, which is a single method and is widely available as commercial software, MMs’ spectrum is broad, where most of the methods are custom-written computer programs. Despite the variety of methods, they can be classified into two types, those based on the strong form solution and those based on the Galerkin weak form [9]. Detailed reviews of such MMs are provided in [8,10]. Amongst those methods is the Radial Point Interpolation Method (RPIM), an accurate and not computational resource-demanding method based on radial-based interpolation functions [10]. The RPIM possesses the Kronecker’s delta property; hence, the imposition of boundary conditions can be performed as it is done in FEM, making this method notable amongst the reviews. Recently, a MM was developed as an evolution to the RPIM, the Natural Neighbour Radial Point Interpolation Method (NNRPIM) [9]. The NNRPIM only requires a nodal distribution from which it creates integration cells based on Voronoï cells and Delaunay tessellation, hence is a true meshless method [9]. MMs are known to be computationally more demanding than FEM; nevertheless, in the NNRPIM’s case, the use of non-uniform nodal distributions reduces the computational cost without an effect on the method’s accuracy [9]. Application of MMs to analyse adhesive joints is limited, being employed to analyse SLJs, double cantilever beam tests, as reviewed by Ramalho et al., [11]. Recently, Sánchez-Arce et al., [2] used the NNRPIM meshless method to analyse double-L joints; three adhesive types were considered. Numerical strength estimations for brittle and ductile epoxy adhesive had a good agreement with the experimental data [2]. Although this is one of the few works presenting stress distributions, no strain distributions or analyses were reported. Moreover, no application of MMs to analyse other joint configurations was found in the literature.

A T-joint configuration, composed of metallic substrates bonded with a ductile adhesive system, is analysed in this work. It aims to investigate how substrate thickness affects the stress and strain distributions in the bond line, and consequently, joint strength through elastic-plastic analyses. The adhesive has proven to obey the EDP yield criterion [7]. Substrate thickness from 1 mm to 4 mm were considered; the thinnest substrate could highly deform before the adhesive’s failure. Consequently, a MM is considered to perform the analyses, the NNRPIM in particular, through expanding previous work [2,7].

2. The natural neighbour radial point interpolation method

In this section, the basis of the meshless method is briefly described; nevertheless, a full description of the method is in References [9,12]. The first step to analyse a geometry using the NNRPIM is to discretise the geometry as a set of nodes \( N = \{x_1, x_2, ..., x_n\} \in R^d \) which distribution in the surface or volume could be uniform or random (without affecting the accuracy); \( n \) corresponds to the number of nodes inside the domain \( \Omega \). Then, nodal connectivity is determined through a Voronoï diagram created from \( N \), resulting in the ‘backing’ mesh for the method. Then, a Delaunay tessellation provides an influence cell \( V_i \) [9]. Considering a node \( x_i \in \Omega \) as the point of interest, all the nodes inside \( V_i \) influence
Then, as in the RPIM, any variable related to $\mathbf{x}_I$ can be described as the sum of two shape functions, one radial based (RBF) and one polynomial (PBF), as shown in equation (1) [9,12].

$$u(\mathbf{x}_I) = \sum_{i=1}^{n} R_i(\mathbf{x}_I) a_i(\mathbf{x}_I) + \sum_{j=1}^{m} P_j(\mathbf{x}_I) b_j(\mathbf{x}_I)$$  

where $R_i(\mathbf{x}_I)$ is the *radial basis function* (RBF), and $a_i(\mathbf{x}_I)$ is a non-constant coefficient of $R_i(\mathbf{x}_I)$ [9]. Similarly, $P_j(\mathbf{x}_I)$ is a *polynomial basis function* and $b_j(\mathbf{x}_I)$ are their non-constant coefficients [9]. An RBF based on the *Euclidean norm*, $r_{II} = [(x_I - x_i)^2 + (y_I - y_i)^2 + (z_I - z_i)^2]^{1/2}$, was chosen for this work, the multi-quadratics radial basis functions (equation (2)).

$$R(r_{II}) = (r_{II}^2 + c^2)^p$$  

The parameters $c$ and $p$ have the values of $c \approx 0.0001$ and $p \approx 0.9999$, which were optimised for the NNRPIM by Dinis et al., [13] using a minimisation process for each parameter [13]. Then, the non-constant coefficients $a_i(\mathbf{x}_I)$ and $b_j(\mathbf{x}_I)$ can be obtained by applying the interpolation function (1) to all nodes (n) in $\Omega$ [9,12]. A matrix representation of equation (1) simplifies the process, as in equation (3).

$$u = Ra + Pb$$  

where $R$ and $B$ are the coefficient matrices corresponding to the RBF and the PBF, respectively, upon further mathematical re-arrangement, equation (1) can be expressed as a function of the shape function $\Phi_i(\mathbf{x}_I)$ at $\mathbf{x}_I$ [9], as in equation (4). Later, its partial derivatives are calculated.

$$u(\mathbf{x}_I) = \sum_{i=1}^{n} \Phi_i(\mathbf{x}_I) u_i.$$  

Then, natural and essential boundary conditions, and the material matrix, are imposed through the weak-Galerkin form [9]. The equilibrium condition is established from the stress tensor, $\boldsymbol{\sigma}$, and the body forces, $\mathbf{b}$, as follows $\nabla \boldsymbol{\sigma} + \mathbf{b} = 0$. The natural and essential boundary conditions are established as follows:

$$\begin{align*}
\sigma n &= \mathbf{t} \\
u &= \mathbf{u}
\end{align*}$$  

where $\mathbf{n}$ is a normal vector to the boundary surface, $\mathbf{t}$ is the tension on the natural boundary ($l_t$), and $\mathbf{u}$ is the displacement on the essential boundary ($l_u$). Through the weak-Galerkin form in matrix form (equation (6)), where $\mathbf{K}$ is the stiffness matrix, the problem follows Hooke’s law and can be solved using the same numerical techniques used for the FEM [9].

$$\delta u [\mathbf{Ku} - F] = 0$$  

In this work, the NNRPIM formulation was programmed into MATLAB® (MATLAB 9.4, The Mathworks Inc. Natick Massachusetts, USA.) as a set of subroutines. Similarly, the elastic-plastic formulations (described below) were also included as subroutines.

### 3. Elastic-plastic formulation

Here, two yield criteria were used, the von Mises yield criterion for the metallic substrates, the EDP for the ductile adhesive. In brief, the von Mises yield criterion considers a cylindrical yield surface; hence, no hydrostatic stresses are considered; further details of the von Mises formulation and its implementation into the NNRPIM are in [12]. On the other hand, the EDP yield criterion considers the hydrostatic pressure, $p = \frac{1}{3} \text{tr} (\boldsymbol{\sigma})$, which influences the yielding point, $\sigma_y$, at that particular stress state ($\boldsymbol{\sigma}$). For adhesives, the exponent for the EDP is two, as described in [6]. The yield surface ($f(\boldsymbol{\sigma})$) for the EDP is a paraboloid, as follows:
where \( q \) is the equivalent stress, which corresponds to the von Mises stress, \( \lambda_{DP} \) is a constant relating the material strength in two independent loading conditions, i.e., shear (\( \tau \)) and tensile (\( \sigma \)), as follows:

\[
\lambda_{DP} = \frac{\tau^2}{\sigma_y^2}
\]  

(8)

Moreover, from equation (7), when \( \lambda_{DP} = 1 \), takes the form of the well-known von Mises yield criterion [6]; consequently, the numerical implementation is like the one for the von Mises criterion, differing only in the number of variables to be input, and so it is suitable for multi-material domains, as adhesive joints are.

4. Numerical analysis of T-joints

The modelled geometry corresponds to that proposed by Carneiro and Campilho [3], which is a simplified representation of the frame-skin and longeron-skin joints found in small aircraft. The geometry dimensions are presented in figure 1, in which the 50 mm dimension represents the distance between the lower substrate and the beginning of the gripping zone.

The substrates were made of an aluminium alloy, the AW6082-T651, bonded with Araldite® 2015, a ductile adhesive. The Araldite 2015 adhesive was chosen because it has been experimentally characterised; second, this adhesive follows the EDP yield criterion, as reported in References [7]. Mechanical properties from the substrates and adhesive were experimentally determined and reported in previous work [3]; these properties are listed in table 1. Similarly, experimental \( P_{max} \) for this geometry and material was determined and reported in [3], which was used as a benchmark for comparing the numerical predictions developed in this work. The \( \lambda_{DP} \) value was calculated following the procedure described in previous work [7].

The boundary conditions replicate the experimental work from Carneiro and Campilho [3]. The ‘grip zones’ in figure 1 correspond to the areas where the Universal Testing Machine (UTM) held the specimens. The boundary conditions for the horizontal grip zones were \( U_x = U_y = U_z = 0 \), those corresponding to the vertical grip zones were \( U_x = U_z = 0, U_y = \text{free} \). Moreover, the geometry is symmetric, which was used to reduce computational costs; the corresponding boundary condition was \( U_x = U_z = 0, U_y = \text{free} \). The pull-out load exerted by the UTM was simulated by an imposed displacement, \( \delta \), on the upper-most horizontal edge (figure 1); the magnitude of \( \delta \) was larger than the observed experimentally; this is further explained below in Section 4.1. The geometry was modelled in two dimensions as a plane strain case. The geometry and nodal distributions for each \( f_{0} \) case were created in commercial software for simplicity, as it does not affect the methodology [10]. The nodal distributions were then imported
into MATLAB®. Subsequently, material properties were assigned to each region prior to running the NNRPIM. The assignment of mechanical properties and boundary conditions was performed using a custom-written MATLAB® script.

| Property                        | Araldite 2015 | Aluminium |
|---------------------------------|--------------|-----------|
| Young's modulus, $E$ (MPa)      | 1850 ± 0.21  | 70100 ± 800 |
| Poisson's ratio ($\nu$)         | 0.35         | 0.30      |
| Yield strength, $\sigma_y$ (MPa)| 12.63 ± 0.61 | 261.61 ± 7.7 |
| Tensile strength, $\sigma_f$ (MPa)| 21.63 ± 1.61 | 324.00 ± 0.10 |
| Failure strain, $\varepsilon_f$ (%)| 4.77 ± 0.15  | 0.10      |
| Shear modulus, $G$ (MPa)        | 560 ± 0.21   |           |
| Shear yield strength, $\tau_y$ (MPa)| 14.6 ± 1.3  |           |
| Shear strength, $\tau_u$ (MPa)  | 17.9 ± 1.8   |           |
| Shear failure strain, $\gamma_f$ (%)| 43.9 ± 3.4  |           |
| $\lambda_{DP}$                 | 1.46         |           |

4.1. Numerical joint strength

Joint strength was numerically determined through a continuum mechanics criterion applied to the strain distributions at the bond line. For this particular adhesive under peel loads, the failure criterion was the effective strain ($\varepsilon_{\text{eff}}$), as suggested in [2,7]. Once the strain along the bond line had reached $\varepsilon_{\text{eff}} = \varepsilon_f$, the joint is considered to have attained its maximum strength ($P_{\text{max}}$). The strains along the bond line were evaluated in all increments until $\delta$ was reached.

This process differs from previous work [4] where the experimental load was used as input for the models for obtaining stress distributions. In this work, the displacement imposed allows determining $P_{\text{max}}$ and stress-strain distributions without a predetermined load.

5. Results and discussion

5.1. Numerical joint strength

Experimentally and numerically, the joint strength ($P_{\text{max}}$) increased exponentially as a function of the substrate thickness. $P_{\text{max}}$ estimations from three of the four cases bonded with the Araldite® 2015 were found within experimental values; the fourth was around 10% below the experimental range. Numerical $P_{\text{max}}$ is reported in figure 2. The increase in $P_{\text{max}}$ due to $t_{p2}$ was, in fact, expected because the substrate stiffness depends on its thickness; hence, the adhesive layer is more subject to peel loads than to cleavage loads, as previously described in [3,5].

![Figure 2. Comparison between numerical and experimental $P_{\text{max}}$.](image-url)
5.2. Stress distributions in the bond lines

Stresses along the bond lines were evaluated and normalised against the adhesive’s strength in the corresponding direction; those reported here correspond to the stresses at $P_{\text{max}}$. In the $t_{P2} = 1$ mm case, both normalised peel and shear stresses (figure 3) were approximately null except at the ends where they presented peaks, indicating that the substrates were highly deformed at the ends. Upon increasing $t_{P2}$, the stresses propagate towards the bond line’s centre, also indicating that the load is better distributed along the adhesive layer (figure 3).

![Figure 3](image.png)

**Figure 3** Normalised stress distributions along the bond line's centre at $P_{\text{max}}$. Corresponding to (a) peel stresses, (b) shear stresses. The increase in $t_{P2}$ improves the stress distribution along the bond line.

For the analysed configurations, similar peel stress distributions ($\sigma_{y}$) were observed. There is a high peak at the beginning of the bond line ($L/L_0 \approx 0$), which is closer to the load application point. There is, however, a second negative peak at the bond line’s end ($L/L_0 \approx 1$) where the joint is held. There is a difference between the left and right $\sigma_{y}/\sigma_u$ peaks, mainly in the thinner substrates, whilst it equalises for the thicker substrates ($t_{P2} = 3$ & 4 mm), as shown in figure 3a. This effect was caused for the difference between substrate thickness. The thinnest substrate deformed the most at the left, hence caused higher stresses in the adhesive, whilst the reaction force at the right was better distributed by the thicker lower substrate (3x thicker than the upper one). As $t_{P2}$ increased, the $\sigma_{y}/\sigma_u$ curve became symmetric.

In the $\tau_{xy}/\tau_u$ case, there is an increase in the shear stresses with $t_{P2}$, and sign changes in the stress distribution. The eccentric loading and supporting conditions cause flexion and shearing, which are transferred to the adhesive layer (figure 3). The negative shear is caused because the upper substrate is moving leftwards while the lower is constrained at its ends; this is more visible in the central area where the peel stresses are also low (figure 3). The changes of sign from negative to positive occur around the relative minimum and maximum $\sigma_{y}/\sigma_u$ stress points (figure 3a). In addition, for $L/L_0 \leq 0.2$ for $t_{P2} = 2$ to 4 mm, there is a clear change in slope; the point occurs in the transition between elastic and plastic deformation, as it can be observed in figure 4(b).

5.3. Strain distributions in the bond lines

Peel and shear strain distributions at the adhesive layer’s mid-thickness present a similar pattern as the stress distributions presented in figure 3. Nevertheless, from inspecting the normalised effective strain ($\varepsilon_{ef}/\varepsilon_u$), a high strain concentration is observed at $L/L_0 \approx 0$ (figure 4a). It is known that the bond line’s ends are stress raisers. Despite this, at $L/L_0 = 0$, the normalised strains were found $\varepsilon_{ef}/\varepsilon_u \leq 1$, indicating a peak in the proximities inside the bond line, which occurs in the vicinity where the shear strain changes sign. The effective plastic strain (figure 4b) show that at $P_{\text{max}}$ only 10% of the bond line had plasticised for $t_{P2}=1$ mm against 25% for $t_{P2}=4$ mm, indicating that the joint is still capable of supporting load prior to a catastrophic failure. The small portion of the bond line in plastic regime also supports the findings.
of Akpinar et al. [4], who proposed to use bi-adhesive (brittle + ductile) systems along the bond line to improve load transfer.

Upon analysing the strains in the whole adhesive layer, a few observations arose. Regardless of the upper substrate thickness, strains are concentrated at the left side of the adhesive layer (figure 5). For the case where \( t_{P2} = 1 \) mm, the strains across the adhesive thickness are symmetrical. Such effect reduces as \( t_{P2} \) increases, concentrating the strains closer to the lower interface, as shown in figure 5; this effect was previously described by Dean and Crocker [6] for other joint configurations. In addition, the location of the higher strain points indicates that lower substrate’s deformation has a large effect on the adhesive layer. Contrary to the upper substrate, which is not overly constrained, the lower substrate is highly constrained. Although the strains concentrate close to the lower interface, \( P_{max} \) was calculated through the stress or strain distributions at the adhesive layer’s mid-thickness because that is the common practice. It is worth mentioning that both stress and strain distributions for this type of joint, under an elastic-plastic regime, are not readily available in the literature.

6. Conclusions

In this work, a type of double-L joints was analysed using a meshless method. Materials were considered as bi-linear elastic-plastic. The adhesive was considered to yield following the Exponent Drucker-Prager criterion.

Numerical joint strength was obtained using strength of materials criteria. For this geometry and material combination, strain-based criteria provided the best results. For the models with upper substrate thickness from 1 to 3 mm, the numerical predictions were within the experimental limits. The cases of 2 and 3 mm provided the best estimations, being them below 5% different from the mean experimental data. Although the case with 1 mm thickness was within experimental limits, the value was close to the lower limit. Similarly, the case with thickness of 4 mm was underpredicted around 15% by the numerical analyses; nevertheless, with the choice of the peel strain as failure criterion, the numerical prediction was 2.3% below the mean experimental value. The closeness between experimental and numerical

![Figure 4](image1.png)

**Figure 4.** Effective strain distributions along the bond line at \( P_{max} \). (a) Normalised effective strain or von Mises strain. (b) Effective plastic strain.

![Figure 5](image2.png)

**Figure 5.** Peel and shear strain at \( P_{max} \) for the joint with \( t_{P2} = 3 \) mm. These contour plots at the first 5 mm of the adhesive layer.
predictions confirms that strain-based criteria are more suitable for ductile adhesives. Consequently, the chosen numerical methodology can be used to predict joint strength for this type of joint.

The stress and strain distributions are influenced by the stiffness of the upper substrate. Joints with thin substrates are not fully benefited from the adhesive properties because of the stress concentrations at the adhesive layer’s ends. A balanced joint distributes the stress along the adhesive layer instead than at its ends, as observed through the plasticisation of the adhesive. Stresses and strains were also found closer to the adhesive layer’s lower interface due to the constraining of the lower substrate. Although the stress and strain distributions cannot be compared with other sources, the distributions shown here were validated against other methods. Consequently, these can be used as a qualitative reference for future analyses. In summary, the use of meshless methods in the non-linear analysis of adhesive joints has been proven as a real alternative to the finite element analysis. Moreover, the appropriate selection of failure criteria, as well as the balancing of the substrates provides the strongest joints and the best correlation with experimental data.

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