Formation of the law of steering angle control to maintain a trajectory of the vehicle movement

Y Levenkov\textsuperscript{1,2}, I Chichekin\textsuperscript{1} and N Vol'skaya\textsuperscript{1}

\textsuperscript{1}Bauman Moscow State Technical University

\textsuperscript{2}E-mail: levenkov_yy@bmstu.ru

**Annotation.** The object of the study is a control system for changing and maintaining the trajectory of the vehicle. Methods of maintaining a trajectory of the vehicle are the subject of research. Multi-body dynamics (MBD) methods are used to solve this problem. The purpose of the study is development of management model change the trajectory of the vehicle. On the example of a two-axle four-wheel drive truck, a mathematical model of the vehicle includes subsystems-transmission, springing system and steering. The study of the influence of the transmission and suspension to change the trajectory of the car is not considered. The paper deals with the formation of the steering angle control law to maintain a given trajectory. A detailed description of the developed control model and analytical dependencies for setting the parameters of the model. An example of selection of coefficients for its setting is shown, recommendations for other types of calculation modes are offered. In this paper, the calculation of one calculation case corresponding to the rearrangement of the car, followed by a return to the original lane. This example proves the adequacy of the model. The developed mathematical model can be used in the systems of calculation of solid body dynamos in the simulation of driver-driven cars, unmanned vehicles and adaptive driver assistance systems. The vehicle was described as a system of solid bodies connected by hinges and force interactions from the library of typical elements of the software package. According to this description of the system, the software package automatically formed a system of equations of motion and connections. The equations were solved using an implicit numerical method with variable pitch and automatic accuracy control.

1. **Introduction**

Currently, the evaluation of the projected design of the car, providing the specified performance properties, in the early stages of development is carried out using computer modeling and calculations. With the help of modern computer systems, it is possible not only to implement the known classical methods of calculating individual units or systems of the car, but also to create complex models of systems, units and the car as a whole [1–12]. The use of complex mathematical models based on the solution of the equation of dynamics of solids allows to take into account the mutual influence of aggregates and systems in the calculation, which is difficult in classical computational schemes.

The trajectory of the car, in turn, depends on many parameters-tires, suspension, transmission and especially steering. Also, there is a growing interest in the development of algorithms for controlling unmanned vehicles and adaptive driver assistance systems [13–15].
To maintain a given trajectory of the vehicle when modeling the dynamics of solids, it is necessary to develop a model for controlling the change in the trajectory of its movement. Setting the trajectory is beyond the scope of this work and is given as an example of the simplest required trajectory.

2. Description of the dynamic model of the car
The general view of the dynamic model of the car developed in the system of calculation of dynamics of bodies is presented in figure 1. The mathematical model of the car includes (figure 2) subsystems - a model of front and rear suspensions, all-wheel drive transmission with front and rear axle differentials, center differential, rack-and-pinion steering mechanism, tie rods, drive and steering wheel models [16–20]. The general technical characteristics of the simulated vehicle are presented in table 1.

Following assumptions were used:
– the links of the car are absolutely rigid, non-deformable bodies;
– the vehicle moves at a constant speed of the transfer case differential;
– there is no friction in the joints;
– the center of gravity of the car does not change when driving;
– simulation was carried out for the total weight of the car;
– wheel deformation is taken into account in the model of interaction of the wheel with the road surface, the forces on the wheel are applied in the center of the wheel;
– the road surface is non-deformable, absolutely flat.

The x-axis of the global coordinate system (GCS) coincides with the longitudinal axis of the car, the Z-axis of the GCS is directed vertically upwards, the y-axis is directed to the left. The HSC origin is at the intersection of the front axle and the plane of symmetry of the vehicle. The gravity vector is parallel to the Z axis of the HSC and directed in the opposite direction.

The MF-Tire model was used for the motion simulation, which is described in detail in the paper [21, 22].

Figure 1. General view of the car model
Figure 2. Components of the mathematical model of the car

Table 1. Technical characteristics of the car

| The general technical characteristics of the car: |
|--------------------------------------------------|
| Gross weight                                     | 3500 kg |
| Curb weight                                      | 1840 kg |
| The payload of                                  | 1660 kg |
| Base                                            | 3565 mm |
| Vehicle width                                   | 1700 mm |
| Vehicle height 2500 mm                          | 2500 mm |
| Center of mass height                           | 1200 mm |
The general technical characteristics of the car:

| Characteristics                        | Value                      |
|----------------------------------------|----------------------------|
| Distribution of the total mass on the axes of the car: |                           |
| Front axle                             | 1460 kg, 41.7%             |
| Rear axle                              | 2040 kg, 58.3%             |
| Vehicle suspension:                    |                            |
| Type                                   | Independent, double-rod    |
| Elastic element compression            | Spring                     |
| Damping element                        | Telescopic shock absorber  |
| Tyres, 235/55 R17                      | Tyres, 235/55 R17          |
| Steering                              | Rack and Pinion steering mechanism |
|                                        | Driven front wheels        |

For the virtual model of the driver in the mathematical model of the car added an auxiliary link 1 (figure 3) — the point of predicting the position of the body of the car, connected to the body of the car hinge allows only forward movement along the longitudinal axis. In the hinge connected to the steering mechanism, the law of movement of the steering link is formed. In the developed model, the control law is formed in the steering wheel rotation hinge 3 (figure 3).

Figure 3. Front axle of the car

Figure 4 shows the turning scheme of the vehicle. The change in the distance of the prediction point of the position of the car body 1, on the trajectory of the car 2, relative to the frame (figure 4) is given in proportion to the speed of the car \( (V_a) \):

\[
L_{BT} = V_a \cdot k, m
\]

\( k \) — delay the movement of the vehicle body along the trajectory of the prediction point of its position.
Figure 4. Scheme of the car in a turn

The desired trajectory of the car 3 (figure 4) can be given by an analytical equation \( y = f(x) \), \( x \) — longitudinal coordinate, \( y \) — lateral coordinate. You can also define a spline—a set of segments and interpolate the data using the built-in functions of the program for calculating the dynamics of solids.

The steering angle (figure 4) is proportional to the difference between the lateral displacement of the virtual point and the ordinate of the desired trajectory at the same distance in GSK:

\[
\theta_{str} = K \cdot (Y_{TP}(X_{BT}) - Y_{BT})
\]

where:
- \( \theta_{str} \) — steering angle, rad;
- \( K \) — steering ratio, rad / m;
- \( Y_{TP} \) — ordinate the desired trajectory of the vehicle;
- \( X_{BT} \) and \( Y_{BT} \) — the abscissa and ordinate of a virtual point in the XY plane of the GSC relative to the origin 5 (figure 4).

To analyze the simulation results on the car body, a point located in the center of gravity of the car is selected. The trajectory 4 is shown in figure 4. The paper shows a comparison of the deviation of the selected point on the car body from the transverse ordinate of the given trajectory.

3. Description of the results of modeling the curvilinear motion of the car

To analyze the operation of the virtual driver model, the evasive maneuver simulation was performed, followed by the return to the initial lane. Main plot sizes are selected according to GOST 31507-2012 (figure 5). The length of the plot on which the permutation is equal to 16 m. the program of calculation of dynamics of rigid bodies set to the desired trajectory of the virtual point.

Figure 5. Main dimensions of the test area

On the initial section of the road the car was accelerated to a speed of 31.6 km/h. Acceleration was carried out kinematically-by setting the angular speed of rotation of the transfer case differential. All differentials used in the model are simple symmetric. There is no internal friction.

The simulation is performed for nine variants of the proportionality coefficient values (table 2) of the virtual point position \( k \) (1), as well as the coefficient of proportionality of the steering wheel \( K \) (2).
Table 2. The coefficients of proportionality

| № variant | \( k, s \) | \( K, \text{ rad/m} \) |
|-----------|-------------|------------------|
| variant 1 | 0.36        | -5               |
| variant 2 | 0.36        | -2.75            |
| variant 3 | 0.36        | -0.5             |
| variant 4 | 0.72        | -5               |
| variant 5 | 0.72        | -2.75            |
| variant 6 | 0.72        | -0.5             |
| variant 7 | 1.08        | -5               |
| variant 8 | 1.08        | -2.75            |
| variant 9 | 1.08        | -0.5             |

The simulation results for all variants are presented in figure 7. Since the selected point on the body is removed from the virtual point, the car will move along a predetermined path with a delay depending on the speed and distance \( L_{BT} \).

According to the calculation results, it can be seen that with an increase in the coefficient of proportionality of the steering wheel \( \text{K} \) increased lateral displacement of the body (variants 3, 6, 9).

![Figure 6. The specified trajectory of the vehicle](image)

When the ratio of the virtual point position decreases, the trajectory of the selected point on the car body more precisely coincides with the specified trajectory, but the steering wheel turns frequently. It is recommended that you select lower values if you frequently change direction in short sections. When driving over long stretches, such as when modeling rough road traffic, it is recommended to increase the ratio of the virtual point position.

Based on the calculation results, the following values of the proportionality coefficients corresponding to option 5 are selected for the car in question, \( k = 0.72, s \) and \( K = -2.75, \text{ rad/m} \). Five screenshots of the motion simulation movie are shown in figure 8. The pictures show that the car didn’t leave the specified corridor.
Figure 7. The trajectory of the selected point of the car body
Figure 8. Screenshots of vehicle movement, with proportionality coefficients corresponding to option 5
where, the simulation time a-10.3 s, b-12.2 s, b-16.9 s, g-18.5 s

4. Conclusions
1. A model of driving allows you to simulate the movement of cars as a driver-driven and unmanned vehicles, or to simulate adaptive driver assistance systems.
2. The quantitative values of the proportionality coefficients for adjusting the regulators for the car in question are proposed. The values of the coefficients are as follows $k = 0.72, s$ and $K = -2.75, \text{rad}/\text{m}$. For vehicles with other mass inertial parameters, geometric dimensions and kinematics of suspension and steering, the values of these coefficients should be obtained again.
3. To obtain numerical values of the proportionality coefficients, it is recommended to simulate additional computational cases, for example, a snake, movement on an uneven support surface, etc.
4. It is enough to set the desired trajectory approximately, the developed model of driving smoothes the trajectory itself. In this case, the trajectory of the selected point of the car body corresponds to the specified trajectory quite accurately.
5. The developed model of driving is suitable for other types of vehicles, with different from the method of rotation considered in the article.

References
[1] Hashemi E, Pirani M, Khajepour A, Kasaiezadeh A 2016. Vehicle System Dynamics 54 1736–61
[2] Wei Y, Liu Y, Li X, Oertel C 2016. Vehicle System Dynamics 54 463–73
[3] Taheri M, Ahmadian M 2016. Vehicle System Dynamics 54 653–66
[4] Rodriguez J, Freeman P T, Wagner J, Pidgeon P, Alexander K 2016. Int. J. of Automotive Technol. 17 71–81
[5] Xia X, Xiong L, Sun K, Yu Z P 2016. Int. J. of Automotive Technol. 17 991–1002
[6] Diakov A S, Kotiev G O 2018. MATEC Web of Conf. 224 02096
[7] Evseev K B, Kartashov A B, Dashtiev I Z and Pozdeev A V 2018. MATEC Web of Conf. 224 02039
[8] Kotiev G O, Padalkin B V, Kartashov A B, Diakov A S 2017. ARPN J. of Eng. and Appl. Sci. 12 1064–71.
[9] Sarach E B, Kotiev G O, Beketov S A 2018. MATEC Web of Conf. 224 04009
[10] Klubnichkin V E, Diakov A S, Klubnichkin E E, Zakharov A Y, Vakhidov U Sh, Suchenina A S and Basmanov I V 2019. J. of Phys.: Conf. Series 1177 012048
[11] Volskaya N S, Zhileykin M M and Zakharov A Y 2018. IOP Conf. Series: Materials Science and Engineering 315 012028
[12] Klubnichkin E E, Klubnichkin V E, Kotiev G O 2018. IOP Conf. Series: Materials Science and Engineering 386 012025
[13] Zhileykin M M, Kotiev G O, Nagatsev M V 2018. IOP Conf. Series: Materials Science and Engineering 315 012031
[14] Kotiev G O, Butarovich D O, Kositsyn B B 2018. IOP Conf. Series: Materials Science and Engineering 315 012014
[15] Skotnikov G I, Jileykin M M and Komissarov A.I 2018. IOP Conf. Series: Materials Science and Engineering 315 012027
[16] Gorelov V A, Komissarov A I 2016. Procedia Engineering 150 1322–28.
[17] Gorelov V A, Komissarov A I, Miroshnichenko A V 2015. Procedia Engineering 129 300–7
[18] Keller A V, Gorelov V A, Anchukov V V 2015. Procedia Engineering 129 280-7.
[19] Vdovin D, Chichekin I 2016. Procedia Engineering 150 1276–79
[20] Ejsmont J, Taryma S, Ronowski G, Swieczko-Zurek B 2016. Int. J. of Automotive Technol. 17 237
[21] Pacejka, H B 2006. Tyre and Vehicle Dynamics. Second Edition (Oxford: Butterworth-Heinemann)
[22] Pacejka, H B, Besselink I J M 1997. Supplement to Vehicle System Dynamics 27 234