An epidemic model with the closed management in Chinese universities for COVID-19 prevention

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Abstract. COVID-19 has deeply changed people’s way of life. While the anti-epidemic work in China has been successful, many Chinese universities still prohibit students from leaving the campus. In the meanwhile, faculty and staff can be on or off campus at will. This paper establishes an SI model to describe such a phenomenon. It is assumed that the latent population in society is a tiny constant, to which only a fraction of university population is exposed. Due to the closed management, a single individual cannot be exposed to the university and social population at the same time. A set of differential equations are proposed, and the standard solution is found. The results show that the entire closeness makes the virus not transmit in the university, and partial closeness leads the virus to spread on campus. By numerical simulation, the latent proportion as a function of different off-campus-allowed proportions is presented. It is found that to minimize the latent proportion, as many individuals should be allowed to be off-campus as possible when they are less likely to leave the campus, and there exists optimum allowed proportion when they are more likely to leave the campus. Furthermore, based on theoretical analysis, a quantitative formula describing the optimum allowed proportion in the general situations is given, providing the university administration with a method to calculate the optimum policy.

1. Introduction
The ongoing COVID-19 has had a profound impact on the world and our daily life [1]. The academic community has carried out timely research on the epidemic situation. Based on computer models, some studies have estimated the basic reproduction number and epidemic data in Wuhan [2] and other parts of the world [3], revealing the current situation of the virus transmission. Another study has illustrated that containment strategies based on non-pharmaceutical interventions (NPIs) are effective [4], providing a method for anti-epidemic. For the future development trend of the epidemic situation, Kissler et al. [1] predicted that COVID-19 would coexist with the human for a long time.

The classical SIR model that describes the spread of an epidemic was first proposed in 1927 by Kermack and McKendrick [5]. In the classical SIR model, the population is divided into three compartments: (i) Susceptible (S). (ii) Infected (I). (iii) Recovered (R). The virus spreading from the infected to the susceptible at the rate of $\alpha$ and $\alpha S I$ susceptible individuals become infected individuals in
the unit time. Simultaneously, the infected individuals recovering at the rate of $\mu$, $\mu I$ infected individuals become recovered individuals in the unit time. Since the recovered individuals with COVID-19 have no permanent immunity to it, this paper removes compartment R, and assumes the $\mu I$ infected individuals becoming susceptible individuals in the unit time, and the SIR model is simplified to an SI model. On the contrary, previous studies added various details to the classical epidemic model. For instance, a variety of nonlinear incidence functions are formulated to describe the transmission rate, such as non-monotone incidence rate [6], bilinear incidence rate [7], fractional incidence rate [8].

As an extension of the single-chain epidemic model, Meska et al. [9] analyzed the global asymptotic stability of a two-strain epidemic model. Combining the epidemic model with complex networks, Liu and Zhao [10] analyzed the spreading behavior of an SIQR under the small world network environment. The mechanism of epidemic models is universal and widely used to describe other phenomena (e.g., information propagation). From the spread of rumors [11-13], to the radicalization [14-16], and then to conflicting public opinions [17], the research of epidemic models has been crossed with different fields, and has made great progress.

The current work is inspired by the closed management policy of Chinese universities in epidemic prevention, which has aroused dissatisfaction and widespread concern among college students. On the premise that the anti-epidemic work in China has won great victory, some Chinese universities still prohibit students from leaving the campus. In the meanwhile, faculty, staff, and other social personnel can enter and leave at will. This paper proposes an SI model to describe this phenomenon, where only a part of the university population is allowed to be off-campus and exposed to the social population. Through this work, we try to answer the following question: does closed management only for a fraction of the university population always promote epidemic prevention? In addition, by studying the population evolution before any latent individual is diagnosed, we attempt to use the model to provide the university administration with scientific policy-making methods on closed management.

2. Model
We denote the population in the university as $N_0$, and assume no difference among students, faculty, and staff in a mathematical sense. In the unit time, the university population is allowed to be off-campus with a proportion of $\theta$, and the remaining individuals with a proportion of $(1-\theta)$ are circumscribed to campus. Intuitively, if only faculty and staff are allowed to leave the campus, then $\theta$ is a very small number. We divide the university population into two compartments: (i) Susceptible (S); (ii) Latent (I). The susceptible individuals are not infected, and the number of them at time $t$ is denoted by $S(t)$. The latent individuals have been infected, but not diagnosed. We denote the number of latent individuals at time $t$ as $I(t)$, where $S(t)+I(t)\equiv N_0$. Once a latent individual is found, the university administration will take emergency measures, thus this model no longer works. Therefore, we only study the population evolution before any latent individual is detected.

The social population, isolated from the remaining university population, is denoted by $N_1$. In the social population, there are $I_0$ number of latent individuals. Considering the government’s management, $I_0$ is assumed to be a tiny positive constant, and the evolution of the infected proportion in the social population is not taken into account.

We assume the university population is well-mixed (i.e., within the mobility allowed, people will be fully exposed to each other). The university administration allows $\theta$ proportion of the population to leave the campus, but different compartments (S and I) are indistinguishable for the university administration. Therefore, within the unit time, on average, there are $\theta S(t)$ susceptible individuals and $\theta I(t)$ latent individuals allowed to be off-campus and exposed to the social population. The departure of the remaining $(1-\theta)S(t)$ susceptible individuals and $(1-\theta)I(t)$ latent individuals is prohibited, who can be only exposed to the other university population. Below, we discuss the exposure of individuals in different compartments.

The number of susceptible individuals allowed to be off-campus is $\theta S(t)$. The population being well-mixed, these susceptible individuals are exposed to the social population with the possibility of
In the numerical simulation, we adopt the forward-Euler difference method:

\[
\begin{align*}
I(t_{k+1}) &= I(t_k) + \Delta t \left. \frac{dI(t)}{dt} \right|_{t=t_k} \\
S(t_{k+1}) &= S(t_k) + \Delta t \left. \frac{dS(t)}{dt} \right|_{t=t_k}
\end{align*}
\]
with step $\Delta t = 0.01$, and $t_{k+1} = t_k + \Delta t$, $t_0 = 0$.

First, according to common sense, we tentatively choose $\alpha = 0.1, N_0 = 1, N_1 = 10, I_0 = 0.0001$, $\mu = 0.05, \theta = 0.1$. Since the social population and the university population only play a proportional role in the model, the absolute size of which does not affect the results. Therefore, here they simply take 10 and 1. Obviously, the initial condition is $S(0) = 1, I(0) = 0$. The simulation being carried out to $t = 1000$, Figure 2 is obtained.

![Figure 2](image1.png)

**Figure 2.** The number of individuals in different compartments as a function of $t$. $\alpha = 0.1, N_0 = 1, N_1 = 10, I_0 = 0.0001, \mu = 0.05, \theta = 0.1. S(0) = 1, I(0) = 0. \Delta t = 0.01$.

Secondly, we choose $\theta = 0.2$ over $\theta = 0.1$ (i.e., the proportion of university individuals allowed to be off-campus is increased), with other parameters unchanged. The simulation being carried out to $t = 1000$, we obtain Figure 3.

![Figure 3](image2.png)

**Figure 3.** The number of individuals in different compartments as a function of $t$. $\alpha = 0.1, N_0 = 1, N_1 = 10, I_0 = 0.0001, \mu = 0.05, \theta = 0.2. S(0) = 1, I(0) = 0. \Delta t = 0.01$.

As seen in Figure 2 and 3, when $t = 0$, the number of susceptible individuals is 1, and the number of latent individuals is 0. The university population is uninfected initially. As time passes, the number of latent individuals $I(t)$ gradually increases. The susceptible individuals gradually become latent ones, and the number of susceptible individuals $S(t)$ decreases. Ultimately, the ratio of the two will achieve equilibrium, which does not change with time any longer until the latent individuals are diagnosed and the university administration takes emergency measures. The equilibrium of the ratio depends on the parameters.

Comparing Figure 2 and 3, we find that when more university individuals are allowed to be off-campus (Figure 3), there are less latent individuals compared with that when fewer university individuals are allowed to be off-campus (Figure 2). In order to study the influence of the allowed
university individual proportion on the number of latent individuals, we let \( \theta \) change from 0 to 1 with step 0.01, and take \( \theta \) as abscissa. For each \( \theta \), we simulate \( 10^4 \) unit time, and take the number of latent individuals at \( t = 10^4 \) as the approximated stable latent number \( I^* \approx I(10^4) \). We take the ratio of the stable latent number \( I^*/N_0 \) as the ordinate of the corresponding abscissa \( \theta \).

First, we choose \( \alpha = 0.1, N_0 = 1, N_1 = 1, I_0 = 0.0001, \mu = 0.05, S(0) = 1, I(0) = 0, \Delta t = 0.01 \). Here, \( N_0 = N_1 \), which means the allowed university individuals will be on campus and off-campus with the same probability. The simulation results show the stable latent proportion \( I^*/N_0 \) as a function of the fraction of university individuals allowed to be off-campus \( \theta \) in Figure 4.

**Figure 4.** The stable latent proportion \( I^*/N_0 \) as a function of \( \theta \). \( \alpha = 0.1, N_0 = 1, N_1 = 1, I_0 = 0.0001, \mu = 0.05, S(0) = 1, I(0) = 0, \Delta t = 0.01 \).

As seen in Figure 4, when \( \theta = 0 \), the proportion of latent individuals is 0, which is consistent with common sense. If there is no access, there will not be any transmission of the virus. However, when \( \theta \neq 0 \), the results are contrary to our intuition. When a fraction of the university population is allowed to be off-campus, a certain number of latent individuals will be generated in the university, but the more population is allowed to be off-campus, the less these latent individuals will be. When all individuals are allowed to be on or off the campus freely (\( \theta = 1 \)), the proportion of latent individuals will be reduced to 0, which is the same as completely circumscribing the whole population (\( \theta = 0 \)).

Secondly, we change the parameter. We choose \( N_1 = 10 \) over \( N_1 = 1 \) (i.e., the probability of the allowed individuals to be off-campus is increased by 10 times), with other parameters unchanged. The simulation results show the stable latent proportion \( I^*/N_0 \) as a function of the fraction of university individuals allowed to be off-campus \( \theta \) in Figure 5.

**Figure 5.** The stable latent proportion \( I^*/N_0 \) as a function of \( \theta \). \( \alpha = 0.1, N_0 = 1, N_1 = 10, I_0 = 0.0001, \mu = 0.05, S(0) = 1, I(0) = 0, \Delta t = 0.01 \).
Figure 5 reveals different conclusions. As seen in Figure 5, when the proportion of individuals allowed to be off-campus is small ($\theta < 0.55$), the conclusions remain the same. The more population is allowed to be off-campus, the less the latent individuals will be. However, when the proportion of individuals allowed to be off-campus is great ($\theta > 0.55$), allowing more population to be off-campus makes the latent individuals rebound. In other words, when $\theta \neq 0$, we have the optimum allowed proportion, with which the fraction of latent individuals will achieve the minimum. The optimum allowed proportion can be found in Figure 5. This result may provide guidance for the policy-making of the university administration.

In order to test the robustness of the results, we again change the parameter. We choose $N_1 = 1000$ over $N_1 = 1$ (i.e., the probability of the allowed individuals to be off-campus is increased by 1000 times), with other parameters unchanged. The simulation results show the stable latent proportion $I^*/N_0$ as a function of the fraction of university individuals allowed to be off-campus $\theta$ in Figure 6.

![Figure 6. The stable latent proportion $I^*/N_0$ as a function of $\theta$. $\alpha = 0.1, N_0 = 1, N_1 = 1000, I_0 = 0.0001, \mu = 0.05, S(0) = 1, I(0) = 0, \Delta t = 0.01$.](image)

The results in Figure 6 are not qualitatively different from those in Figure 5. In other words, the results of 10 times and 1000 times of the probability for the allowed individuals to be off-campus have little difference. Hence, relevant conclusions are robust.

In all events, only faculty and staff who account for a small number of the university population being allowed to be on or off the campus freely (i.e., $\theta \to 0^+$ but $\theta \neq 0$ in Figure 4 ~ 6) will make the situation the worst. This phenomenon can also offer an explanation of why “nursing homes” and “prisons” in many countries have become high incidence places of the epidemic. From the perspective of physics, it can be partially attributed to a small part of the population being exposed to the outside world, but most of the population flowing inside, forming a clustering infection. At present, due to the closed management, the physical structure of the population in some Chinese universities resembles that in “nursing homes” and “prisons”, making the students in these universities at risk.

Another discussion is that, the decrease of the latent individuals with the increase of the fraction of allowed individuals can be interpreted by common sense. Unlike a clustering infection, being on or off the campus freely is equivalent to integration into a large society, where there is less average contact between individuals, hence the smaller chance of infection. For example, we assume there is a latent student in a university. If the university administration prohibits students from being off-campus, then the latent student can only go to the canteen to eat, with all the students gathering in the canteen. The latent student will infect the whole university population within the time of a meal. On the contrary, if the university administration allows students to be off-campus, then the latent student may be out of school to eat. Although being off-campus will also infect others, but the infected number will be definitely less than the whole university population.
4. Theoretical analysis and validation

Some unusual results in the numerical simulation can also be explained and even validated by theoretical analysis of equation (1). To simplify the expression, we denote

\[ \lambda = \theta - \frac{N_i}{N_0 + N_i} \]

Thereby, the expression of \( \frac{dl(t)}{dt} \) in equation (1) can be rewritten as

\[ \frac{dl(t)}{dt} = \alpha_0 \lambda S(t) - \mu I(t) + \alpha(2\lambda^2 - 2\lambda + 1)S(t)I(t) \]

(4)

By eliminating \( S(t) \) from the constraint \( S(t) + I(t) = N_0 \), we can obtain

\[ \frac{dl(t)}{dt} = \alpha l^2(t) + bI(t) + c \]

(5)

where

\[
\begin{align*}
  a &= -\alpha(2\lambda^2 - 2\lambda + 1) \\
  b &= -\alpha l_0 \lambda - \mu + \alpha(2\lambda^2 - 2\lambda + 1)N_0 \\
  c &= \alpha l_0 \lambda N_0 
\end{align*}
\]

(6)

Simplifying \( b^2 - 4ac \), we get

\[ b^2 - 4ac = [\alpha(2\lambda^2 - 2\lambda + 1)N_0 - (\alpha l_0 \lambda + \mu)]^2 + 4\alpha l_0 \lambda \cdot \alpha(2\lambda^2 - 2\lambda + 1)N_0 \]

\[ = [\alpha(2\lambda^2 - 2\lambda + 1)N_0]^2 + (\alpha l_0 \lambda + \mu)^2 + 2\alpha(2\lambda^2 - 2\lambda + 1)N_0 \cdot (\alpha l_0 \lambda - \mu) \]

(7)

Since \( \alpha l_0 \lambda \geq 0 \), \( \mu > 0 \), it is easy to know \( \alpha l_0 \lambda + \mu > \alpha l_0 \lambda - \mu \). Thus, we can verify:

\[ b^2 - 4ac > [\alpha(2\lambda^2 - 2\lambda + 1)N_0]^2 + (\alpha l_0 \lambda - \mu)^2 + 2\alpha(2\lambda^2 - 2\lambda + 1)N_0 \cdot (\alpha l_0 \lambda - \mu) \]

\[ = [\alpha(2\lambda^2 - 2\lambda + 1)N_0 + \alpha l_0 \lambda - \mu]^2 \geq 0 \]

(8)

Equation (5) is a Riccati equation. To solve it, we can directly separate the variables [18] and integrate:

\[ \int \frac{1}{\alpha l^2(t) + bI(t) + c} dl(t) = \int dt \]

(9)

Considering \( 4ac - b^2 < 0 \) and the initial condition \( I(0) = 0 \), the time evolution function of the latent population is solved as:

\[ I(t) = -2c \frac{1 - e^{\sqrt{b^2 - 4ac}}}{{\sqrt{b^2 - 4ac}} + b + (\sqrt{b^2 - 4ac} - b)e^{\sqrt{b^2 - 4ac}}} \]

(10)

which can describe the time evolution of the latent population in Figure 2 and 3 exactly.

Furthermore, we substitute equation (5) into the stability condition:

\[
\begin{align*}
  \frac{dl(t)}{dt} &= 0 \\
  \frac{dS(t)}{dt} &= 0 
\end{align*}
\]

(11)

and the latent population \( I^* \) as \( t \to \infty \) can be solved as:
It is easy to know $c > 0$. Since $(2\lambda^2 - 2\lambda + 1) \geq \frac{1}{2} > 0$, we have $a < 0$. According to Vieta theorem, there is $I_1^* I_2^* = \frac{c}{a} < 0$. So, in $I_1^*$ and $I_2^*$, one is positive and the other is negative. Let go of the negative root, which has no pragmatic significance. The standard solution of the latent population $I^*$ is:

$$I^* = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

(13)

In order to facilitate the theoretical analysis, we will ignore some minima. According to the model postulate, $I_0 \to 0^+$, so we have approximation $b \approx -\mu + \alpha(2\lambda^2 - 2\lambda + 1)N_0$, $c \approx 0$, and equation (13) can be approximated as:

$$I^* \approx N_0 - \frac{\mu}{\alpha} \cdot (2\lambda^2 - 2\lambda + 1)^{-1}$$

(14)

In equation (14), when $\lambda = \lambda_0 = \frac{1}{2}$, the latent population $I^*$ achieves the minimum. Substituting it into equation (3), we finally obtain the optimum allowed university individual proportion:

$$\theta_0 = \frac{1}{2} \left( 1 + \frac{N_0}{N_1} \right)$$

(15)

We substitute the value of parameter $N_0$ and $N_1$ in Figure 4 ~ 6 into equation (15), and it is respectively given $\theta_0 = 1$, $\theta_0 = 0.5500$, $\theta_0 = 0.5005$, which is consistent with the results in Figure 4 ~ 6. Equation (15) not only validates the numerical results, but also provides a general formula to calculate the optimum proportion of the university population allowed to be off-campus. We find that such a proportion has nothing to do with the virus transmission rate $\alpha$ and the recovering rate $\mu$. As indicated in equation (15), with a larger university population (i.e., the allowed individuals are more likely exposed to the university population), the allowed university individual proportion should be increased to avoid a clustering infection. If there is a larger social population (i.e., the allowed individuals are more likely exposed to the social population), then the allowed university individual proportion should be reduced, to avoid bringing the latent virus into the campus. For the university administration, given the accessible university population $N_0$ and the accessible social population $N_1$, the optimum proportion of the university population allowed to be off-campus $\theta_0$ can be obtained.

5. Summary

Based on an SI model, this work studied the closed management of some Chinese universities in epidemic prevention. In the model, only a fraction of the university population is allowed to be off-campus and exposed to the social population. If an individual is off-campus, it is isolated from the university population, and if the individual is on campus, it is isolated from the social population. We assumed the population is well-mixed, so the probability of an individual to be on or off the campus depends on the university and social population. We show numerical results of the differentiation equations, and validate them by theoretical analysis. The results show that most of the time there exists latent individuals, which is not the case in reality. This is because the results are in the mean-field level. In effect, under the current anti-epidemic achievements in China, the probability of infection in the universities is slim. However, our model presented the average results including infection and non-infection, which is still instructive to the application.

Through the work of this paper, we have acquired the following conclusions:
(i) In the epidemic prevention, the university administration only allows a fraction of the university population to be on and off campus freely, who will be exposed to the social population and bring the virus into the campus with a certain probability. Before any latent individual is diagnosed, the proportion of infected and uninfected individuals will evolve to an equilibrium state, and the latent proportion in such a state depends on the fraction of the proportion of the university population allowed to be off-campus.

(ii) The numerical simulation indicates that, allowing no individual to be off-campus makes the virus not transmit in the university, and allowing some individuals to be off-campus leads the virus to spread in the university. On the one hand, when the allowed individuals are less likely to be off-campus, the ultimate latent proportion decreases with the allowed proportion. The outcome of the entire prohibition and permission is the same, and the result of only permitting a tiny fraction of individuals is the worst. On the other hand, when the allowed individuals are more likely to be off-campus, there exists an optimum allowed university individual proportion (under the parameters that we take, it is approximately 0.5), to reduce the latent individuals to the minimum.

(iii) The theoretical analysis gives the formula \( \theta_0 = \frac{1}{2} \left( \frac{N_0}{N_1} \right) \) to calculate the optimum allowed university individual proportion, which is in great agreement with the numerical results. Such an optimum proportion has nothing to do with the virus transmission rate and the recovering rate. For university administration, given the accessible university population and the accessible social population, the optimum proportion of the university population allowed to be off-campus can be calculated. This may offer help to relevant policy-making.

Future research can start from the following perspectives. First, the university individuals with different characteristics (e.g., students, faculty, and staff) can be distinguished, as such we can study the epidemic dynamics in specific complex networks where different crowds contact. Secondly, we can treat the relative size of parameter \( N_0 \) and \( N_1 \) as a single parameter that describes the subjective psychology whether to leave the campus or not of the university population, and distinguish it with the objective allowed proportion \( \theta \) and have further discussion.

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