The effects of low anisotropy on non-canonical inflation

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Effects of anisotropy on the intermediate inflation in a non-canonical scalar field framework within a spatially homogeneous and anisotropic Bianchi type I universe are studied. It will be shown that in an anisotropy background, the non-canonical scalar model can significantly improve the compatibility with the observational data of plank 2015 of inflationary models. Also as an interesting result automatically, we obtain a steep potential which can manage inflation at the non-canonical in anisotropy metric setting. In addition, the related inflationary observable such as tensor to scalar ratio, the amplitude of scalar perturbation, scalar and tensor spectral indices, running spectral index and the e-folding number relation are obtained. Moreover, the results of our investigation are in a good agreement with CMB and Planck data. Additionally, the well known canonical inflation scenario in an anisotropic universe will be evaluated.

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I. INTRODUCTION

Based on over three decades of large-scale investigations [1–7], it is clear that inflationary paradigm is one of the important parts of modern cosmology. This theory can describe the early Universe evolutions successfully and also overcomes three vital problems which standard big bang theory was faced namely the horizon, the flatness and the relic problems [8–13]. In addition, inflation regime for obtain correct ratio tensor to scalar perturbations and correct behavior of primordial fluctuations is requisite[9, 14–20]. Despite introducing different inflationary models, the precise data from inspecting Cosmic Microwave Background, CMB, spectrum has limited the number of admissible inflationary models [21–23]. In the standard inflation, potential term of Lagrangian is dominated to canonical kinetic term, i.e. potential term dominated during inflation [8, 9, 12, 13]. However there exist inflationary models in which kinetic term has different form against canonical kinetic term, so called non-canonical models, such as Dirac-Born-Infeld (DBI) action, where non-canonical kinetic term is attributed to the scalar field. It could be realized DBI scalar field model can be assumed as a subset of k-inflation scenario [24–34]. Observational constraints on k-inflation and its perturbations have been considered in literature [25] and [26]. Additionally, some other noticeable researches have been worked out in the context of non-canonical inflationary scenario and we refer the reader to [27–33]. Mostly, in the slow-rolling inflationary scenario studies, one tries to obtain expressions for inflationary observable, as the tensor and scalar spectral indices, the running index and tensor to scalar ratio, there exist various ways to perform it. Amongst, introducing scale factor is an approach to think about inflationary scenario, where intermediate inflation is the most generic one, that first presented by [35]. In this procedure, the scale factor has an exponential function form of time, i.e. \( a(t) = \exp(\kappa t^f) \), \( \kappa > 0 \) and \( 0 < f < 1 \) [36, 37]. This scale factor leads to asymptotically negative power-law potential, albeit not exactly, for instance we can bring in the mind the steep potentials [38]. Considering this scale factor the expansion of Universe is faster than power-law one, i.e. \( a(t) = t^p, p > 1 \), and slower than de-Sitter inflation \( a(t) = \exp(HT), H = constant \). In Einstein gravity, intermediate inflation for \( \alpha = 2/3 \) creates a scale invariant perturbations [35–37, 39]. Due to intermediate proposal benefits in solving some problems in inflationary studies this scenario preserve an appropriate place in the scientists’ investigations and for more clarification we refer the reader to the literature [40–44]. In a clear term, one important
reason to consider the intermediate inflation is that, its results for tensor-to-scalar ratio and scalar spectrum index are in a good agreement in comparison with CMB data [45]. It is a well known fact that, the majority of investigations to find out dynamical evolution of inflationary models have been done in a homogenous and isotropic background, for instance Friedmann-Lemaître-Robertson-Walker, FLRW, metric. However, a bit deviation from isotropy at the level of $10^{-5}$, has also been proposed by Bennett et al., and subsequently this suggestion was verified by high resolution Wilkinson Microwave Anisotropy Probe, WMAP, data. Although, according to survey of the present Universe the anisotropy of Universe is small, but its imprints in large scale structure formation are important which the Bianchi type I (BI) models from different point of views have investigated the potential effects of anisotropy in the early Universe [46–52]. The Kasner-type is a specific case of BI model, in which cosmological scale factors evolve by a power-law function of time. In General Relativity, GR, the vacuum Kasner solutions [53] and their fluid filled counterparts, the BI models, were verified effective as a starting point for the investigation of the structure of anisotropic models. Barrow and Clifton [54, 55] have latterly indicated which is also conceivable to find out solutions of the Kasner type for $R^n$-gravity models. Newly, Hossienkhani et al., discussed the effects of low anisotropy on the interacting Dark Energy, DE, models and its comparison with the standard FLRW, A Cold Dark Matter, ΛCDM, and $w$CDM models results [56]. Additionally, they have depicted the anisotropy have a non-zero value at the present time albeit it is close to zero, i.e. the anisotropy will be very low after inflation. The BI model is a straightforward generality of flat FLRW, since it is simplest model of anisotropic and homogenous Universe with spatial flatness. Against the FLRW Universe with the same scale factor in three spatial directions, in BI Universe scale factor on any direction is different.

In this work, we focus on anisotropic version of intermediate non-canonical inflation [57–59]. In the standard canonical inflation, intermediate inflation has some problems. In Ref.[58], it was depicted the intermediate inflation expresses the scalar and tensor power spectral which are dislike in light of the observational results from the Cosmic Background Explorer, COBE, satellite. Beside, the intermediate inflation never cease without inquire any additional process [58]. At hand work, we would like to refine these problems by considering anisotropic intermediate inflation in a non-canonical framework.

This paper is organized as follows: In Sec. II, we express the main dynamic equations from
non-canonical intermediate inflation in anisotropic metric and evaluate the inflationary observable and analogy them with the Planck data 2015. Sec. III is related to canonical intermediate inflation in anisotropic metric and compare with the planck data 2015. At last, Sec. IV is devoted to conclusions and outlooks.

II. NON-CANONICAL INTERMEDIATE INFLATION IN ANISOTROPIC METRIC

We start with considering the following action

\[ S = \int d^4x \sqrt{-g} \mathcal{L}(X, \phi), \]  

(1)

where Lagrangian \( \mathcal{L} \), can be a function of scalar field \( \phi \) and the kinetic term \( X \equiv \partial_\mu \phi \partial^\mu \phi / 2 \). The equations of energy density \( \rho_\phi \) and pressure \( p_\phi \) of the scalar field for the action (1) are as follows [24–32]

\[ \rho_\phi = 2X \left( \frac{\partial \mathcal{L}}{\partial X} \right) - \mathcal{L}, \]

(2)

\[ p_\phi = \mathcal{L}. \]

(3)

The Bianchi cosmology refers to a spatially homogeneous background but not necessarily isotropic one. In this at hand work, we will consider BI cosmology. The metric of this model is given by

\[ ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2, \]

(4)

where the metric’s components \( A, B \) and \( C \) are merely functions of time. However, the underlying Lie algebra of the isometry group of the BI metrics is completely different [60]. The energy momentum tensor of the matter fields is given by

\[ T^\mu_\nu = \text{diag}[\rho, -p, -p, -p], \]

(5)

where \( \rho \) and \( p \) represent the energy density and pressure respectively. The field equations for the action (1) in the axially symmetric BI metric are [61–63]

\[ 3H^2 - \sigma^2 = \frac{1}{M_p^2}(\rho_\phi), \]

(6)

\[ 3H^2 + 2 \dot{H} + \sigma^2 = -\frac{1}{M_p^2}(p_\phi), \]

(7)
where \( M_p^2 = 1/(8\pi G) \) is the reduced Planck mass, and \( \sigma^2 = 1/2\sigma_{ij}\sigma^{ij} \) in which \( \sigma_{ij} = u_{ij} + \frac{1}{2}(u_{i;k}u_{j} + u_{j;k}u_{i}) + \frac{1}{3}\theta(g_{ij} + u_{i}u_{j}) \) is the shear tensor, that \( (\sigma_{ij}u^j = 0, \sigma^{i}_{i} = 0) \), describes the rate of distortion of the matter flow, and \( 3H = u^j_j \) is the scalar expansion, where \( u^j \) is 4-velocity, which in a comoving coordinate system, \( (u^i = \delta^i_0) \). The components of the Hubble parameter and the shear tensor are given by the metric Eq.(4) in the the comoving frame [61] as

\[
H = \frac{1}{3}(\dot{A}\dot{B} + \dot{B}\dot{C} + \dot{C}),
\]

\[
\sigma^2 = 3H^2 - (\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}}{AC}).
\]

If we take \( a = (ABC)^{1/3} = (B)^{(\lambda+2)/3} \), in which \( A = B^\lambda \) and \( A = C \), where \( \lambda \) is a real constant, then by assumption \( H_2 = \frac{B}{B} \), the Hubble parameter and the shear tensor are gotten as

\[
H = \frac{2+\lambda}{3}H_2,
\]

\[
\sigma^2 = \frac{(\lambda-1)^2H_2^2}{3}.
\]

The conservation equation of the energy density, is governed by

\[
\dot{\rho} + 3H (\rho + p) = 0.
\]

The slow-roll parameters, based on convention, are defined as follows

\[
\varepsilon = -\frac{\dot{H}}{H^2},
\]

\[
\eta = \frac{\dot{\varepsilon}}{H\varepsilon}.
\]

The requirement of \((\ddot{a} > 0)\) is satisfied if \( \varepsilon < 1 \). We know that to solve the horizon problem, inflation should to last for a sufficiently long time as well, in order to this condition persist, the acceleration of the scalar field has to be small. Hence, inflation occurs and persist if \( \varepsilon \ll 1 \) and \( |\eta| \ll 1 \), these assumptions are called the slow-roll approximation. The number of e-fold, \( N \) of inflationary expansion is defined as

\[
N = \int_{t_i}^{t} H dt = \int_{\phi_i}^{\phi} \frac{H}{\dot{\phi}} d\phi.
\]

In order to solve the horizon problem, we require at least e-fold number more than 60 e-folds [43].
In this study, we assume that the Lagrangian density of model has the following form

\[ L(X, \phi) = X \left( \frac{X}{M^4} \right)^{\alpha-1} - V(\phi), \quad (16) \]

where dimension of \( M \) is mass and \( \alpha \) is dimensionless parameter [30, 31]. If \( \alpha = 1 \), the Lagrangian (16) takes the general canonical Lagrangian form \( L(X, \phi) = X - V(\phi) \). Then, the Lagrangian (16) can be assumed as a generalized form of the usual canonical one. The above Lagrangian satisfies the requirements \( \frac{\partial L}{\partial X} \geq 0 \) and \( \frac{\partial^2 L}{\partial X^2} > 0 \) for the null-energy condition and the requirement of physical propagations of perturbations, respectively [28]. This Lagrangian has been taken into account prior to clarify some chaotic inflationary models and steep potentials [30], and also to reintroduce the power law inflation in light of Planck 2013 results also to indicate a sensible idea for the end of power law inflation [31].

Substituting the Lagrangian (16) into the Eqs. (2) and (3), the energy density and pressure of the scalar field \( \phi \) are given as

\[ \rho_\phi = (2\alpha - 1) X \left( \frac{X}{M^4} \right)^{\alpha-1} + V(\phi), \]

\[ p_\phi = X \left( \frac{X}{M^4} \right)^{\alpha-1} - V(\phi). \]

Including the Eqs. (17), (18) in the conservation equation (12), gives rise to the dynamic equation of the scalar field as

\[ \ddot{\phi} + \frac{3H}{2\alpha - 1} \dot{\phi} + \left( \frac{V'(\phi)}{\alpha(2\alpha - 1)} \right) \left( \frac{2M^4}{\dot{\phi}^2} \right)^{\alpha-1} = 0. \]

Inserting the Eqs. (17), (18) for the Lagrangian (16), into the Eqs. (13) and (14) these relations with respect to the potential \( V(\phi) \), gets

\[ \varepsilon_V = \sqrt{\frac{3(2\lambda + 1)}{2 + \lambda}} \left[ \frac{1}{2\alpha - 1} \left( \frac{3M^4}{V(\phi)} \right)^{\alpha-1} \left( \frac{M_P V'(\phi)}{\sqrt{2} V(\phi)} \right)^2 \right]^{\frac{1}{2\alpha - 1}}, \]

\[ \eta_V = \sqrt{\frac{3(2\lambda + 1)}{2 + \lambda}} \left( \frac{\alpha \varepsilon_V}{2\alpha - 1} \right) \left( \frac{2V(\phi)V''(\phi)}{V'(\phi)^2} - 1 \right). \]

The Eqs. (20) and (21), are called the first and second potential slow-roll parameters, respectively. In addition, the condition slow-roll approximation implies the potential energy more larger than the kinetic energy and thus the Friedmann equation (6) gives

\[ H^2(\phi) = \frac{(2 + \lambda)^2}{9(2\lambda + 1)} \frac{1}{M_P^2} V(\phi). \]

(22)
Meanwhile, in the slow-roll condition the dynamic equation of the scalar field, (19), takes
the form
\[ \dot{\phi} = -\theta \left\{ \frac{\sqrt{3}(2\lambda + 1)}{2 + \lambda} \left( \frac{M_P}{\sqrt{3}\alpha} \right) \left( \frac{\theta V'(\phi)}{\sqrt{V(\phi)}} \right) (2M^4)^{\alpha-1} \right\}^{\frac{1}{2\alpha - 1}}, \] (23)
where \( \theta = 1 \) when \( V'(\phi) > 0 \) and \( \theta = -1 \) when \( V'(\phi) < 0 \) \[30, 43\].

At present work, we want to study the intermediate inflation in anisotropic Universe, i.e, BI Universe, with the scale factor, \( a = (ABC)^{1/3} = (B)^{(\lambda + 2)/3} \) that for small deviations of the isotropic background the \( \lambda \) parameter is taken some different of unity, and \( B \) is as follow
\[ B(t) = a_i \exp \left[ \kappa^2 (M_P t)^f \right], \] \[24\]
where, \( a_i \) is the scale factor in \( y \) axis direction, i.e the \( g_{22} \) component of the metric tensor, at the initial time of inflation. Furthermore, one can define scale factor as \( a = (a_i \exp \left[ \kappa^2 (M_P t)^f \right])^{(\lambda + 2)/3} \). So based on this definition, we obtain the Hubble and the shear parameters as
\[ H = \frac{\kappa^2 f^2(M_P t)^{2f}(2 + \lambda)^2}{9t^2}, \] \[25\]
and
\[ \sigma^2 = \frac{\kappa^2 f^2(M_P t)^{2f}(-1 + \lambda)^2}{3t^2}, \] \[26\]
where \( \kappa > 0 \), and \( 0 < f < 1 \), \[57-59\]. In this work we take \( a_0 = 1 \), as the scale factor value at the present time. The reduced Planck mass is given for the non-dimensional argument of the exponential function. By using of Eqs.\((6)\) and \((7)\) for the intermediate scale factor \( (24) \) in anisotropic Universe, one receives
\[ \rho_\phi = \frac{\kappa^2 f^2 M_P^2 (M_P t)^{2f}(1 + 2\lambda)}{t^2}, \] \[27\]
\[ p_\phi = -\frac{\kappa f M_P^2 (M_P t)^{f}[2(-1 + f)(2 + \lambda) + \kappa f(M_P t)^{f}(5 + 2\lambda(1 + \lambda))]}{3t^2}. \] \[28\]
Considering the slow-roll conditions, i.e. \( \rho_\phi = V(\phi) \), and by virtue of the relation Eqs.\((27)\) we obtain
\[ V(\phi) = \frac{\kappa^2 f^2 M_P^2 (M_P t)^{2f}(1 + 2\lambda)}{t^2}. \] \[29\]
Substituting the Eq.\((29)\) into \((23)\) we obtain a differential equation for scalar field which its solution is as the following
\[ \dot{\phi}(t) = \left( -\frac{2^\alpha (-1 + f)(M^4)^{-1+\alpha} M_P \sqrt{1 + 2\lambda} \sqrt{\frac{\kappa^2 f^2 M_P^2 (M_P t)^{2f}(1+2\alpha)}{t^2}}}{\alpha (2 + \lambda) t} \right)^{\frac{1}{2\alpha}}, \] \[30\]
integrating the Eq. (30), the $t$ as function of $\phi$ is given

$$t(\phi) = 2^{-\frac{2\alpha}{2\lambda + 2f + 2\alpha} - \frac{(2 - f - 2\alpha) \left(2^\alpha (-1 + f) M_P^{-1+\alpha} M_P \sqrt{1 + 2\lambda} \sqrt{\kappa f^2 M_P^{2f+2f} (1 + 2\lambda)}\right)^{-\frac{1}{2}/\alpha}}{\alpha} \phi}.$$  

(31)

Now, we use the above solution in Eq. (29) and get

$$V(\phi) = V_0 \phi^s,$$  

(32)

where

$$V_0 = \kappa^2 f^2 M_P^4 (1 + 2\lambda) \times$$

$$\left(\frac{2\alpha \left(2^\alpha (-1 + f) M_P^{-1+\alpha} M_P \sqrt{1 + 2\lambda} \sqrt{\kappa f^2 M_P^{2f+2f} (1 + 2\lambda)}\right)^{-\frac{1}{2}/\alpha}}{2\alpha} \phi^2 \right)^{2\lambda + 2f} - 2 + 2f$$

is a constant and

$$s = \frac{2\alpha (-2 + 2f)}{-2 + f + 2\alpha}.$$  

(33)

It is clear that the achieved potential in the intermediate inflation in our non-canonical framework at anisotropy background is a power law function, such as the one in canonical inflation case [59]. Whereas, the value of $f$ parameter for the intermediate scale factor (24) should be between 0 and 1 [43, 57, 59], from Eq. (33) we conclude that the $s$ parameter in the potential (32) in terms of $\alpha$ parameter must be in the range $-2\alpha / (\alpha - 1) < s < 0$ to have intermediate non-canonical inflation at the anisotropic Universe. By the way, Since in the standard canonical setting ($\alpha = 1$), so the $s$ parameter is variable between $-\infty < s < 0$. Now, given the inverse power law potential as source inflation in the slow roll condition, we can obtain the relations required for determining the inflationary observable. The expression for the scalar and tensor power spectrum in the slow roll regime are given as [25]

$$P_s = \left(\frac{H^2}{2\pi c_s (\rho_\phi + p_\phi)^{1/2}}\right)^2_{aH = c_s k},$$  

(34)

$$P_t = \frac{8}{M_P^2} \left(\frac{H}{2\pi}\right)^2_{aH = k}.$$  

(35)

The above equation for lagrangian (16) and Eqs. (10, 20) in anisotropic Universe, given $H_{ani} = \frac{2+\lambda}{\sqrt{3(2\lambda + 1)}} H_{iso}$, where the $H_{ani}$ and $H_{iso}$ are Hubble parameters in anisotropic and
isotropic Universe respectively \[30, 31\]

\[
P_s = \frac{(2 + \lambda)^3}{(3(2\lambda + 1))^3/2} \frac{1}{72\pi^2 c_s} \left( \frac{6^\alpha V(\phi)^{5\alpha - 2}}{M_P^{4\alpha - 8} M^{4(\alpha - 1)} V'(\phi)^{2\alpha}} \right)^{\frac{1}{2\alpha - 1}}. \tag{36}
\]

\[
P_t = \frac{(2 + \lambda)^2}{3(2\lambda + 1)} \frac{2V(\phi)}{3\pi^2 M_p^4} a_{H=c_s k} \tag{37}
\]

The scalar power spectrum should be assessed at the sound horizon exit that specified by \(a_{H=c_s k}\) where \(k\) is the comoving wavenumber and \(c_s\) is the sound speed defined as \[24–32, 43\]

\[
c_s^2 \equiv \frac{\partial \rho_\phi}{\partial X} \frac{\partial \rho_\phi}{\partial X} = \frac{\partial \mathcal{L}(X, \phi)}{\partial X} \frac{\partial \mathcal{L}(X, \phi)}{\partial X^2} + \partial \mathcal{L}(X, \phi)/\partial X. \tag{38}
\]

at present model (16), it reduces to

\[
c_s = \frac{1}{\sqrt{2\alpha - 1}}, \tag{39}
\]

which is a constant quantity.

Replacing the potential (32) into Eqs.(36) and (37) after some manipulation, achieved

\[
P_s = \left( \frac{(2 + \lambda)}{\sqrt{3 (2\lambda + 1)}} \right)^3 \frac{\left( 6^\alpha M_P^{8-14\alpha} \alpha \mu^{4-4\alpha} \left( \frac{2\alpha(-2+2f)}{-2+f+2\alpha} \right)^{-2\alpha} (V_0)^{-2+3\alpha} \right)}{72\pi^2 c_s} \frac{1}{2\alpha - 1} a_{H=c_s k} \tag{40}
\]

\[
P_t = \left( \frac{(2 + \lambda)}{\sqrt{3 (2\lambda + 1)}} \right)^2 \frac{2V_0}{3M_p^4} \frac{\phi^{\frac{4(-1+f)}{2\alpha + f + 2\alpha}}}{a_{H=k}} a_{H=c_k}. \tag{41}
\]

From Eq.(40), it is clear that for the value of \(f = 2/3\), the scalar power spectrum is independent of the scalar field \(\phi\) and it expect a scale-invariant Harrison-Zel'dovich spectrum.

So for evolution power spectrum based on the e-fold number, \(N\), we need the scalar field with respect to e-fold number. Hence, we want to calculate the value of scalar field in the initial of inflation namely \(\phi_i\). to do this end, since slow-roll parameter in this work is as

\[
\varepsilon_V = \sqrt{3 (2\lambda + 1)} \frac{\alpha^{-1} 2^{-\frac{3}{2} - \alpha} M_P^{2\alpha} \left( \frac{2\alpha(-2+2f)}{-2+f+2\alpha} \right)^{2\alpha} \left( \frac{M_4}{V_0} \right)^{-1 + \alpha}} {2 + \lambda} \frac{1}{\phi^{\frac{1}{1+2\alpha}}} \tag{42}
\]

and in the initial time of inflation, \(\varepsilon_V = 1\) we obtain the value of initial scalar field as the following

\[
\phi_i = \left[ \left( \alpha^{-1} 2^{-\frac{3}{2} - \alpha} M_P^{2\alpha} v_0^{1-\alpha} M_P^{2\alpha} \right)^{\frac{1}{1+2\alpha}} \lambda \right]^{\frac{1}{2 + \alpha + 2\alpha}}. \tag{43}
\]
Using the Eq. (15), we get
\[ \phi = \left( \frac{\phi_i}{\Delta^{2.6+f+2\alpha}} + \frac{N}{\Lambda} \right)^{2\alpha-2f+2\alpha} \] (44)

Using Eqs. (40) and (44), the scalar power spectrum in terms of e-fold number is given as
\[ P_s = \frac{\left( \frac{2 + \lambda}{\sqrt{3(2\lambda+1)}} \right)^3 \left( \frac{6^\alpha M_P^{8-4\alpha}}{\alpha \mu^4} \right) \left( \frac{2\alpha(2+f)}{4(2+f+2\alpha)-(2+f+2\alpha)} \right)^{-2\alpha} (V_0)^{-2+3\alpha}}{72\pi^2 C_s} \times \left( \frac{\phi_i}{\Delta^{2.6+f+2\alpha}} + \frac{N}{\Lambda} \right)^{2\alpha-2f+2\alpha} \left( \frac{2 + \lambda}{\sqrt{3(2\lambda+1)}} \right)^{\alpha(6-4)} \] (45)

Whereas, \( H \) in the slow roll inflation and \( c_s \) are constant it ends up that at sound horizon exit \((aH = c_s k)\), [31]
\[ \frac{d}{d\ln k} \approx -\frac{d}{dN} \] (46)

The scalar spectral index is defined as
\[ n_s - 1 \equiv \frac{d\ln P_s}{d\ln k}. \] (47)

Therefore, using Eq. (45), we obtain
\[ n_s = 1 - \frac{(-4 + 6f)\alpha}{(2 - f)(1 - s) + 2\alpha s}\Lambda \times \frac{1}{\frac{N}{\Lambda} + \left( \frac{2\alpha-3-1+\alpha M^4(-1+\alpha)V_0^{1-\alpha}(M_p s)^{2\alpha}}{\alpha} \right)^{\frac{1}{1-2\alpha} \frac{1}{1-\alpha}} \left( \frac{1-2\alpha}{\alpha} \chi \right)^{s + \frac{2-f}{2+f+2\alpha}} \}] (48)

where \( \chi = \frac{2 + \lambda}{\sqrt{3(2\lambda+1)}} \). If we consider the running of the scalar spectral index, it is given as
\[ \alpha_s = \frac{dn_s}{d\ln k} = \frac{(-4 + 6f)\alpha}{\Lambda^2 (2 + f (-1 + s) + 2s (-1 + \alpha)) \times \left[ \frac{N}{\Lambda} + \left( \frac{2\alpha-3-1+\alpha M^4(-1+\alpha)V_0^{1-\alpha}(M_p s)^{2\alpha}}{\alpha} \right)^{\frac{1}{1-2\alpha} \frac{1}{1-\alpha}} \left( \frac{1-2\alpha}{\alpha} \chi \right)^{s + \frac{2-f}{2+f+2\alpha}} \right]^{-2} \] (49)

The tensor power spectrum for at hand non-canonical model Eqs. (16), (41) in terms of e-fold number is given by
\[ P_t = \frac{2V_0 \chi^2 \left[ \left( \frac{N}{\Lambda} + \left( \frac{2\alpha-3-1+\alpha M^4(-1+\alpha)V_0^{1-\alpha}(M_p s)^{2\alpha}}{\alpha} \right)^{\frac{1}{1-2\alpha} \frac{1}{1-\alpha}} \chi \right)^{s + \frac{2-f}{2+f+2\alpha}} \right]^{s + \frac{1}{2+f+2\alpha} \frac{1}{s+\alpha}}}{3M_p^4 \pi^2} \] (50)
The tensor spectral index is defined as
\[
n_t \equiv \frac{d \ln P_t}{d \ln k}.
\] (51)

Using (46), Eq. (50), the above equation gets the following expression
\[
n_t = \frac{-s(-2 + f + 2\alpha)}{((2 - f)(1 - s) + 2\alpha s)\Lambda} \times \frac{1}{\Lambda} + \left(\left(\frac{2\alpha M^4(1-\alpha) V_0^{1-\alpha} (M_p s)^{2\alpha}}{\alpha} \right)^{1-2\alpha} \left(\frac{1-2\alpha}{2\alpha} \chi \right) s + \frac{2-f}{-2+f+2\alpha} \right)^{s+ \frac{2-f}{-2+f+2\alpha}}
\] (52)

the tensor-to-scalar ratio is defined as
\[
r \equiv \frac{P_t}{P_s},
\] (53)

using Eqs. (45), (50) and (53), we find
\[
r \approx -8c_s n_t.
\] (55)

Substituting Eq. (52) into Eq. (56) give
\[
r \approx \frac{8c_s s(-2 + f + 2\alpha)}{((2 - f)(1 - s) + 2\alpha s)\Lambda} \times \frac{1}{\Lambda} + \left(\left(\frac{2\alpha M^4(1-\alpha) V_0^{1-\alpha} (M_p s)^{2\alpha}}{\alpha} \right)^{1-2\alpha} \left(\frac{1-2\alpha}{2\alpha} \chi \right) s + \frac{2-f}{-2+f+2\alpha} \right)^{s+ \frac{2-f}{-2+f+2\alpha}}
\] (56)

The consistency relation between observables \(r\) and \(n_t\) in the non-canonical inflation is as follows, that is different from the one for canonical case by coefficient \(c_s\), i.e. \(r = -8n_t\) [30, 31]
\[
r \approx -8c_s n_t.
\] (55)

Substituting Eq. (52) in to Eq. (56) give
\[
r \approx \frac{8c_s s(-2 + f + 2\alpha)}{((2 - f)(1 - s) + 2\alpha s)\Lambda} \times \frac{1}{\Lambda} + \left(\left(\frac{2\alpha M^4(1-\alpha) V_0^{1-\alpha} (M_p s)^{2\alpha}}{\alpha} \right)^{1-2\alpha} \left(\frac{1-2\alpha}{2\alpha} \chi \right) s + \frac{2-f}{-2+f+2\alpha} \right)^{s+ \frac{2-f}{-2+f+2\alpha}}
\] (56)

Now, we will to survey the consistency of model in light of observational data from Planck 2013 and 2015 data [64]. It is clear that the most important result of Planck data is \(r - n_s\) diagram, and reliability models depend on amount of consistent model with these observational data. Thus, we are going to depict the \(r - n_s\) diagram for our scenario by use of Eqs. (48) and (56). This diagram is shown in figure 1, in addition Confidence Levels (CLs)
68% and 95% CL allowed by Planck 2015 data TT, TE, EE+lowP data [64] are illustrated in the figure 1. Predictions of our model are specified by black lines for specified values of $\alpha = 10^3$, $\kappa = 10^{-6.5}$, $f = 0.42$ and $\lambda = 3$. In the figure 1 as it is clear, our model is in good agreement with observations. Then, it could be concluded that this scenario could be regard as a valid case for explaining the inflationary scenario. It should be notice that for the values of parameter $f$, $2/3 < f < 1$ we will have the power spectrum of order $10^{-22}$ and $n_s > 1$ which is not in agreement with the Planck 2015 data [64].

![Image](image.png)

FIG. 1: The $r-n_s$ diagram show Prediction of the non-canonical intermediate inflationary model at anisotropy background for specified values of $\alpha = 10^3$, $k = 10^{-6.5}$, $f = 0.42$ and $\lambda = 3$, $M_p = 10^{18}$, $M = 10^{14}$ and $\mu = 10^4$, in comparison with the observational results of Planck 2015. The thick black line indicate the predictions of our model.

The prediction of model for specified values of free parameter is presented in figure 1, which indicate that the results localized within 95% CL. It is clear that The grey, red and blue confidence levels 68% and 95% CL correspond to Planck 2013, Planck 2015 TT+lowP and Planck 2015 TT, TE, EE+lowP data [64], respectively. In Table I, we can see the prediction of model for perturbation parameters based on specified values of free parameters but different values of $\lambda$ parameter, it is clear that $\lambda$ parameter, i.e. the effect of low anisotropy only affects the scalar power spectrum. In addition, the $r-n_s$ diagram for $\alpha \geq 10^3$ and $\lambda$ in the range of $2 \leq \lambda \leq 3$ is more consistent with the Planck 2015 TT, TE, EE+lowP data [64].

In the following, we depict the prediction of our model in the $dn_s/dN - n_s$ diagram, i.e. the running spectral index $\alpha_s$ in comparison with the observational results of Planck 2015. For this reason, we regard as $\alpha = 10^3$, $\kappa = 10^{-6.5}$ and $\lambda = 3$. Hence, using Eqs. (48) and
TABLE I: The prediction of model for the perturbation parameters \( n_s, r, \mathcal{P}_s \) are prepared for different values of the free parameter \( \lambda \) and specified values of other free parameters, also \( M_p = 10^{18}, M = 10^{14} \) and \( \mu = 10^4 \).

\[
\begin{array}{cccccc}
\alpha & \kappa & f & \lambda & n_s & r & \mathcal{P}_s \\
10^2 & 10^{-6.5} & 0.42 & 1 & 0.9893 & 0.00298 & 7.188 \times 10^{-10} \\
10^2 & 10^{-6.5} & 0.42 & 0.1 & 0.9893 & 0.00298 & 3.09 \times 10^{-10} \\
10^2 & 10^{-6.5} & 0.42 & 3 & 0.9893 & 0.00298 & 2.40 \times 10^{-9} \\
10^2 & 10^{-6.5} & 0.42 & 10 & 0.9893 & 0.00298 & 1.89 \times 10^{-8} \\
10^2 & 10^{-6.5} & 0.42 & 2 & 0.9893 & 0.00298 & 1.41 \times 10^{-9} \\
\end{array}
\]

(49) we plot \( dn_s/dN \) versus \( n_s \). Fig. 2 shows the prediction of model can lie inside the joint 68% CL region of Planck 2015 TT, TE, EE+lowP data [64], and is in agreement satisfactory with observational.

FIG. 2: The \( dn_s/dN - n_s \) diagram show Prediction of the non-canonical intermediate inflationary model at anisotropy background for specified values of \( \alpha = 10^3, k = 10^{-6.5}, f = 0.42 \) and \( \lambda = 3, M_p = 10^{18}, M = 10^{14} \) and \( \mu = 10^4 \). in comparison with the observational results of Planck 2015. The thick black line depict the predictions of our model.

III. CANONICAL VERSION

In this section we are going to study, briefly, the behavior of anisotropic canonical inflationary model in comparison with above results in low anisotropic non-canonical formalism.
For this purpose, one could set $\alpha = 1$ in the Lagrangian, (16), and repeat the same procedure, hence different perturbative parameters, i.e. Eqs.(45, 48, 54), are reduced as follows

$$P_s = \frac{V_0 \left( \left( \frac{N}{X} + 2 \frac{\sqrt{f} - fs}{2f} \left( \frac{M_p s}{\sqrt{X}} \right)^{-1 + \frac{2}{f} f s} \right)^{\frac{6 - \frac{4}{f}}{2 + f (-1 + s)}} \right)^{\frac{6 - \frac{2}{f}}{2 + f s (1 + s)}}}{12M_p^6 \pi^2 s^2 c_s}$$

(57)

$$n_s = 1 - \frac{\left( 6 - \frac{4}{f} \right) f}{(2 + f (-1 + s)) \Lambda \left( \frac{N}{X} + 2 \frac{\sqrt{f} - fs}{2f} \left( \frac{M_p s}{\sqrt{X}} \right)^{-1 + \frac{2}{f} f s} \right)}$$

(58)

$$r = \frac{8 fs}{(2 + f (-1 + s)) \Lambda \left( \frac{N}{X} + 2 \frac{\sqrt{f} - fs}{2f} \left( \frac{M_p s}{\sqrt{X}} \right)^{-1 + \frac{2}{f} f s} \right)}$$

(59)

In Table (II), we will similar to the non canonical case in anisotropic back ground to consider the prediction of model for perturbation parameters based on specified values of free parameters but different values of $\lambda$ parameter for $\alpha = 1$, it is clear that $\lambda$ parameter, i.e. the effect of low anisotropy only affects the scalar power spectrum. In addition, the scalar power spectrum for $\lambda$ a bit different of $\lambda = 1$ is more consistent with the Planck 2015 TT,TE,EE+lowP data, that is, it confirm the early Universe is close to small deviation of isotropic Universe.

| $N_c$ | $\kappa$ | $f$ | $\lambda$ | $n_s$ | $r$ | $P_s$ |
|------|---------|-----|-----------|-------|-----|-------|
| 60   | $10^{-30}$ | 0.5 | 2        | 0.966 | 0.533 | $1.471 \times 10^{-8}$ |
| 60   | $10^{-30}$ | 0.5 | 1        | 0.966 | 0.533 | $3.09 \times 10^{-9}$ |
| 60   | $10^{-30}$ | 0.5 | 3        | 0.966 | 0.533 | $2.40 \times 10^{-8}$ |
| 60   | $10^{-30}$ | 0.5 | 10       | 0.966 | 0.533 | $8.27 \times 10^{-8}$ |
| 60   | $10^{-30}$ | 0.5 | 0.1      | 0.966 | 0.533 | $6.88 \times 10^{-10}$ |
| 60   | $10^{-30}$ | 0.5 | 0.5      | 0.966 | 0.533 | $2.4 \times 10^{-9}$ |

TABLE II: The prediction of model for the perturbation parameters $n_s$, $r$, $P_s$ are prepared for different values of the free parameter $\lambda$ and specified values of other free parameters and $M_p = 10^{18}$, $M = 10^{14}$ and $\mu = 10^4$.

By the way, from above table II, it would be concluded that the standard canonical intermediate inflation ($\alpha = 1$) in the anisotropic background is disfavored in light of Planck
2015 results, for sake of the values of $r$ and $n_s$ parameter. Beside, if we choose $\lambda$ a bit different of unity then prediction of canonical version for scalar power spectrum is improved.

IV. CONCLUSIONS

In this work we have investigated non-canonical intermediate inflation in the anisotropic background. The majority of inflationary models have been evaluated in the isotropic background, hence in anisotropic background a details and lengthy trend is required to provide inflationary observable to compare with data.

For the case of our model, the Hubble and shear parameters were obtained. Additionally by means of the data risen from Planck 2015, we have shown that the potential automatically takes a steep form, i.e. $V = V_0 \phi^s$ with $s < 0$ that can manage inflation in a better way for the non-canonical inflation in anisotropy metric setting. With this obtained potential, the tensor-to-scalar ratio, the scalar and tensor spectral indices have obtained based on e-folding number and other free parameters of the model. Then, since one of the most important results of Planck data is $r - n_s$ diagram, and the reliability of the models depends on amount of consistency of model with these observational data, hence in figure 1 for specified free parameters, comparing with 2013 and 2015 Planck results showed a very good agreement. In addition, Table. I. the predictions for effects of anisotropy, based on $\lambda$ parameter is that in the small deviation of isotropic, i.e. $2 \leq \lambda \leq 3$, the $r - n_s$ diagram is more consistent with data. Beside, in figure 2, the predictions for the running spectral index are in satisfactory agreement with observational data, also that is lie inside the joint 68% CL region of Planck 2015 TT, TE, EE+lowP data [64]. In addition, the tensor spectral index has been obtained as $n_t = -0.017$, it can precisely be measured in the future, that the constraint applied by the tensor-to-scalar ratio have been satisfied. For the case of canonical inflation, by set $\alpha = 1$ we have repeated the same procedure for different perturbative parameters, i.e. $r$, $P_s$ and $n_s$. Additionally, in Table. II, we have seen that effects of low anisotropy only affect the scalar power spectrum. In addition, the scalar power spectrum for $\lambda$ a bit different of unity is more consistent with the Planck 2015 TT, TE, EE+lowP data, that is, it confirms the early Universe is close to low anisotropic Universe instead of isotropic one. From Table. II, it was concluded that the standard canonical intermediate inflation ($\alpha = 1$) is disfavored in light of Planck 2015 results. But if we choose $\alpha$ enough large and $\lambda$ a bit different of unity then
result of our non-canonical intermediate inflationary model in the anisotropic background can be lied inside the regions favored according to Planck 2015 data.

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