Understanding Photovoltaic Cell Dynamic Resistance Behavior with Changing Incident Light Intensity

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Abstract—We present a theoretical examination of the general behavior that should be expected to be displayed by the magnitude of the dynamic resistance of a conventional illuminated photovoltaic device within the power-generating quadrant of its $I$-$V$ characteristics, when measured in quasi-static conditions from the short-circuit point to the open-circuit point, at various incident illumination intensities. The analysis is based on assuming that the photovoltaic device in question may be adequately described by a simple conventional d-c lumped-element single-diode equivalent circuit solar cell model, which includes significant constant series and shunt resistive losses, but lacks any other secondary effects. Using explicit analytic expressions for the dynamic resistance, we elucidate how its magnitude changes as a function of the terminal variables, the incident illumination intensity and the model’s equivalent circuit elements’ parameters.

Index Terms—Photovoltaic device, solar cell model, dynamic resistance, Lambert $W$ function.

I. INTRODUCTION

The negative value of the reciprocal slope of the $I$-$V$ characteristics of an illuminated photovoltaic cell is usually called its dynamic resistance, $r$. This is a very useful parameter to measure when analyzing and optimizing the cell’s performance capabilities [1]. It is also essential when defining a solar cell’s maximum power point (MPP). Here we will look, from a basic theoretical point of view, into some fundamental issues regarding the dynamic resistance of conventional photovoltaic cells, with the intention of better understanding its expected general behavior as incident light intensity changes. The questions to be addressed are:

1) Does the magnitude of the dynamic resistance $r$ of conventional photovoltaic cells depend on the intensity of the incident illumination? Or, equivalently, is $r$ dependent on the magnitude of the photo-current, $I_{ph}$, produced by the separation and collection of photo-generated charge carriers?

2) Does the magnitude of $r$ measured under short-circuit or open-circuit conditions depend on incident light intensity?

3) If so, how are these dependencies?

To obtain theoretical answers to the above fundamental queries, we will start from the mathematical equations of the simplest model that can be used to describe the most significant physical phenomena responsible for the typical shape of conventional photovoltaic cells’ static $I$-$V$ characteristics under illumination. That simplest realistic model is an equivalent condensed-element (a.k.a. lumped-parameter) d-c electric circuit [1], whose configuration is that shown in Fig. 1.

The electric elements of this circuit consist of a parallel-connected combination of: a single diode (characterized by the linked pair of lumped parameter values $n$ and $I_0$, which merge the most significant charge carrier transport mechanisms at the cell’s junction); a photo-generated current source (with magnitude defined by its lumped parameter $I_{ph}$) oriented in the direction of the diode’s reverse current, which accounts for the photo-current produced by all collected photo-generated carriers; and a shunt resistor (specified by the value of its lumped parameter $R_s$) that includes all resistive losses across the cell’s junction. As shown in Fig. 1, this parallel combination is further connected in series to a second resistor (specified by the value of its lumped parameter $R_p$), which combines all series resistive losses caused by the cell’s contacts and terminals. Both resistive elements are assumed to stay approximately constant with respect to the circuit’ variables and illumination intensity.

We will assume that the magnitude of the dynamic resistance, $r$, embodied by the value of the illuminated photovoltaic cell $I$-$V$ characteristics’ reciprocal slope, can be experimentally determined at quasi-static conditions, either by performing a small-signal ac measurement at very low frequency, or by numerical calculation from two or more consecutive data points taken around the location of interest on the measured static $I$-$V$ characteristic [2].
II. METHODS

The first step to theoretically study the behavior of the dynamic resistance is to write the equation that describes the simplest equivalent circuit model capable of adequately representing the conventional non-ideal photovoltaic cell with series and parallel resistive losses \((R_s\neq0, G_p\neq0)\). We have chosen to use the single-diode model whose lumped-element equivalent circuit \([1]\) is shown in Fig. 1. The relationship between its terminals’ current and voltage is mathematically expressed using Kirchoff’s Laws and Shockley’s diode equation \([3]\), which results in the following implicit transcendental linear-exponential equation:

\[
I = I_{ph} - I_0 \left[ \exp\left(\frac{VR_s}{nV_{th}}\right) - 1 \right] - (IR_s + V)G_p . \tag{1}
\]

Notice that in writing \((1)\) we have defined the current as coming out of the positive terminal, that is, in the opposite direction (sign) as that shown in Fig. 1. We do so for convenience and without loss of generality, in order to adhere to the sign convention normally used by photovoltaic power engineers, where the power generating quadrant is presented as the first (instead of as the fourth) quadrant of the illuminated cell’s I-V characteristics (see Fig. 2).

A. Explicit solutions

Closed-form solutions of the cell’s terminal current and voltage equations as explicit functions of each other are preferred over \((1)\) from simulation and analysis points of views. Such explicit solutions are possible to be obtained from \((1)\) thanks to the use of the Lambert \(W\) function \([4]\). The solution of \((1)\) for the terminal current as an explicit function of the terminal voltage is \([5]\):

\[
I(V) = -\frac{nV_{th}}{R_s} W_0 \left[ \frac{I_0 R_s \exp\left(\frac{(I_0+I_{ph})R_s}{nV_{th}(G_p R_s + 1)}\right)}{nV_{th}(G_p R_s + 1)} \right] - \frac{I_0 + I_{ph}}{G_p R_s + I_{ph} - I_0} . \tag{2}
\]

where \(W_0\) represents the principal branch of the Lambert \(W\) function \([4]\). To find the corresponding expression for the terminal current, \(I_i\), of an ideal cell, i.e., a cell with negligible series and parallel resistive losses, substituting \(R_s=0\) and \(G_p=0\) directly into \((2)\) will not work. Strictly, we would have to find the limit of \((2)\) as \(R_s\rightarrow 0\) and \(G_p\rightarrow 0\). A much more direct route is to simply substitute \(R_s=0\) and \(G_p=0\) into \((1)\):

\[
I_i(V_i) = I_{ph} - I_0 \left[ \exp\left(\frac{V_i}{nV_{th}}\right) - 1 \right] . \tag{2i}
\]

Likewise, the solution of \((1)\) for the terminal voltage as an explicit function of the terminal current can be written also in terms of the Lambert \(W\) function \([5]\):

\[
V(I) = -nV_{th} W_0 \left[ \frac{I_0 \exp\left(\frac{I_0 + I_{ph}}{G_p nV_{th}}\right)}{G_p nV_{th}} \right] + \frac{I_0 + I_{ph}}{G_p} - IR_s . \tag{3}
\]

The corresponding expression for the terminal voltage, \(V_i\), of the ideal cell is quickly found by inverting \((2i)\):

\[
V_i(I_i) = nV_{th} \ln\left(\frac{I_{ph} + I_0 - I_i}{I_0}\right) . \tag{3i}
\]

B. Short-circuit and open-circuit expressions

An expression for the short circuit current \(I_{sc}\) can be written by evaluating \((1)\) at \(V=0\):

\[
I_{sc} = -\frac{nV_{th}}{R_s} \left[ \frac{I_0 R_s \exp\left(\frac{(I_0+I_{ph})R_s}{nV_{th}(G_p R_s + 1)}\right)}{nV_{th}(G_p R_s + 1)} \right] + \frac{I_0 + I_{ph}}{G_p R_s + I_{ph}} . \tag{4}
\]

To find the corresponding expression for the short circuit current, \(I_{sc}\), of an ideal cell, substituting \(R_s=0\) and \(G_p=0\) directly into \((4)\) will not work. Strictly, we would have to find the limit of \((4)\) as \(R_s\rightarrow 0\) and \(G_p\rightarrow 0\). A much more direct route is to simply evaluate \((2i)\) and at \(V=0\):

\[
I_{isc} = I_{ph} . \tag{4i}
\]

The expression for the open circuit voltage \(V_{oc}\) is found by evaluating \((2)\) at \(I=0\):

\[
V_{oc} = -nV_{th} \left[ \frac{I_0 \exp\left(\frac{I_0 + I_{ph}}{G_p nV_{th}}\right)}{G_p nV_{th}} \right] + \frac{I_0 + I_{ph}}{G_p} . \tag{5}
\]

Again, to find the corresponding expressions for the open-circuit voltage, \(V_{oc}\), of an ideal cell, substituting \(R_s=0\) and \(G_p=0\) directly into \((5)\) will not work. We would need to find the limit of \((5)\) as \(R_s\rightarrow 0\) and \(G_p\rightarrow 0\). However, a simpler direct route is to evaluate \((3)\) and at \(I=0\):

\[
V_{ioc} = nV_{th} \ln\left(\frac{I_{ph} + I_0}{I_0}\right) . \tag{5i}
\]

The above equations clearly indicate, that the short-circuit current and the open-circuit voltage of an ideal or real photovoltaic cell are both functions of incident light intensity (represented by the value of the photo-current \(I_{ph}\)). They also depend on the equivalent circuit’s lumped elements parameter values, except that \(V_{oc}\) does not depend on the value of \(R_s\), since the voltage drop across \(R_s\) is zero when \(I=0\).

C. A hypothetical photovoltaic cell example

We will use a hypothetical photovoltaic cell, with known equivalent circuit lumped-elements’ parameter values, as an example to generate synthetic I-V characteristics that will help to analyze and graphically illustrate the behavior of the dynamic resistance. The following parameters values of the elements of the circuit model shown in Fig. 1 will be used as example: diode’s junction quality factor \(n=1.5\), diode’s reverse saturation
current $I_p=10^{-7} \text{A}$, shunt resistive loss given by the stated value of $R_p=1/G_p$, and series resistive loss given by the stated value of $R_s$. We assume room temperature operation, i.e. a thermal voltage $V_{th}=kT/q=0.02586 \text{V}$.

Three synthetic $I$-$V$ characteristics of this hypothetical cell, calculated using the explicit solution for the current given by (2), are presented in Fig. 2, for three intensities of incident light which generate within the cell corresponding photo-current magnitudes of $I_{ph}=0.02$, 0.03, and 0.04A. Matching $I$-$V$ curves of an ideal cell, alike but without shunt or series resistive losses ($R_p=G_p=0$, $R_s=0$), are also shown for visual comparison.

The numeric values of short circuit current and open circuit voltage, calculated with (4), (4i), (5) and (5i) for this particular example, at three given incident light intensities, are presented in Table I. They correspond to the axes intercepts of the three $I$-$V$ curves shown in Fig. 2. It is worthwhile to point out that sometimes hastily stated assumption that $I_{sc}=I_{ph}$ is not always a valid approximation, as (4) clearly implies. Its validity depends on the relative magnitudes of the cell’s parallel and series resistive losses.

III. RESULTS

A. Behavior of the dynamic resistance along the $I$-$V$ curves

The magnitude of the dynamic resistance, $r$, was already defined in Section I as the negative of the reciprocal slope of the synthetic $I$-$V$ characteristics. Explicit analytic expressions for $r$ as a function of the terminal current, and of its reciprocal the dynamic conductance, $g$, are obtained by taking the corresponding derivatives of the explicit solutions (2) and (1) with respect to current and voltage, respectively. This can be accomplished without difficulty because the Lambert $W$ function is readily differentiable [4]. The resulting explicit expressions for $r$ and $g$ are:

![Fig. 2. Power-generating quadrant of synthetic current-voltage characteristics of a hypothetical illuminated real and ideal photovoltaic cell, with (solid lines) and without (dashed lines) resistive losses, respectively. The cell is modeled at room temperature by the equivalent circuit of Fig. 1, using the diode’s parameters given in the text. Solid lines: $1/G_p=R_p=100 \Omega$ and $R_s=10 \Omega$; Dashed lines: $G_p=0$ and $R_s=0$ (ideal cell). Curves calculated for three indicated values of photo-current. The negative of the curves’ reciprocal slope at any point is called the dynamic resistance.](image)

![Fig. 3. Magnitude of the dynamic resistance $r$ of the hypothetical illuminated cell with $R_p=100 \Omega$, and $R_s=10 \Omega$, calculated with either (6) or (7) at three values of photo-current intensity $I_{ph}$, from the short-circuit point ($V=0$) to the open-circuit point ($V=V_{oc}$). These values of $r$ correspond to the negative reciprocals of the $I$-$V$ curves’ slopes shown in Fig. 2.](image)
1/r(V, I) = \frac{dt}{dv} = -1 + \frac{dt}{dv}R_s \left[ G_p + \frac{l_0}{n v_{th}} \exp\left(\frac{R_s + V}{n v_{th}}\right)\right]. \quad (8)

Solving (8) for \( \frac{dl}{dv} \) and rearranging yields \( r(V, I) \):

\[
r(V, I) = -R_s - \frac{n v_{th}}{G_p n v_{th} + l_0 \exp\left(\frac{R_s + V}{n v_{th}}\right)}. \quad (9)
\]

Substitution of \( R_s=0 \) and \( G_p=0 \) into (9) yields the dynamic resistance \( r_i \) of an ideal photovoltaic cell:

\[
r_i(V) = -\frac{n v_{th}}{l_0 \exp\left(\frac{V}{n v_{th}}\right)}. \quad (9i)
\]

Another expression for the dynamic resistance \( r(V, I) \) as a function of both terminal variables can be obtained by substitution of (1) into (9):

\[
r(V, I) = \frac{dv}{di} = -R_s - \frac{n v_{th}}{G_p n v_{th} + l_0 \exp\left(\frac{R_s + V}{n v_{th}}\right)}. \quad (10)
\]

Substitution of \( R_s=0 \) and \( G_p=0 \) into (10) produces another form of dynamic resistance, \( r_i \), of an ideal cell equivalent to (9i):

\[
r_i(I) = -\frac{n v_{th}}{l_0 + l_0 \exp\left(\frac{V}{n v_{th}}\right)}. \quad (10i)
\]

B. Dynamic resistance dependence on incident light intensity

The most important general conclusion that can be drawn from Fig. 3 is that the magnitude of the dynamic resistance \( r \), at any voltage in the power-generating quadrant, always decreases as incident light intensity increases. Another conclusion from Fig. 3 worth of analysis is that for any incident light intensity, \( r \) decreases as voltage increases from the short-circuit point \( (V=0) \) to the open-circuit point \( (I=0) \). Furthermore, although it might not be fully obvious from Fig. 3, because of the insufficient variation of light intensities used (not wide enough range of values of \( I_p \)), it seems to suggest that the high and low limiting values of \( r \) at any voltage are approximately given by \( R_p + R_s \) and \( R_s \). This behavior of the dynamic resistance, \( r \), with incident light intensity can be more conveniently visualized by examining the magnitude of \( r \) at the open-circuit point and at the short-circuit points of the \( I-V \) curves as a function of photo-generated current \( I_p \).

C. Dynamic resistance at the open-circuit point

To calculate the magnitude of the dynamic resistance at the open-circuit point, we evaluate (9) at \( V=V_{oc} \) and \( I=0 \): 

\[
r_{oc}(V_{oc}) = -R_s - \frac{n v_{th}}{G_p n v_{th} + l_0 \exp\left(\frac{V_{oc}}{n v_{th}}\right)}. \quad (11)
\]

For the case of an ideal cell, evaluation of (9i) at \( V=V_{oc} \), or substituting \( R_s=0 \) and \( G_p=0 \) into (11) yields:

\[
r_{oc}(V_{oc}) = -\frac{n v_{th}}{l_0 \exp\left(\frac{V_{oc}}{n v_{th}}\right)}. \quad (11i)
\]

Alternatively, evaluating (10) at \( V=V_{oc} \) and \( I=0 \) yields an expression equivalent to (11):

\[
r_{oc}(V_{oc}, I_{ph}) = -R_s - \frac{n v_{th}}{G_p n v_{th} + I_{ph} + l_0 \exp\left(\frac{R_s + V_{oc} - V}{n v_{th}}\right)}. \quad (12)
\]

For the case of an ideal cell, evaluation of (10i) at \( I=0 \), or substitution of \( R_s=0 \) and \( G_p=0 \) into (12) yields an expression equivalent to (11i):

\[
r_{oc}(I_{ph}) = -\frac{n v_{th}}{I_{ph} + l_0}. \quad (12i)
\]

Table II presents three values of \( |r_{oc}| \) and \( |roc| \) as calculated with (12i) and (12), respectively, for three levels of incident light intensity, represented by the given three values of \( I_p \).

The variation of the dynamic resistance’s magnitude, \( roc \), calculated with (11) or (12) at the open-circuit point \((V=V_{oc}, I=0)\), is presented in Fig. 4 as a function of photo-generated current intensity, \( I_p \), which is a direct consequence of the incident light intensity, for \( R_p=100 \Omega \) and three values of series resistance: \( R_s=1, 4, \) and \( 10 \Omega \).

Fig. 4 indicates that as \( I_p \) becomes high (e.g., \( I_p > 0.04 \text{A} \) in this example), the curves start to flatten, tending the magnitude of \( roc \) to values near those of \( R_s = 1, 4, \) and \( 10 \Omega \) in this example). This observation is consistent with Fig. 3, which shows the case for \( R_s=10 \Omega \), where we see that as \( V \rightarrow V_{oc} \approx 0.5 \text{V} \) (open-circuit point) all three curves of \( r \) (calculated at three values of \( I_p \approx 0.02, 0.03, \) and \( 0.04 \text{A} \)) approach the value of \( R_s = 10 \Omega \).

Fig. 4 also reveals that the magnitudes of \( roc \) increase as \( I_p \) decreases until they saturate to constant values around \( R_p + R_s = 101, 104, \) and \( 110 \Omega \) in the present example), as \( I_p \) reaches very low values (about an order of magnitude lower in the present example). Notice that the saturation condition (\( roc \approx R_p + R_s \approx 110 \Omega \) in this example) cannot be observed at the open-circuit point of curves shown in Fig.3, since none of those three \( roc \) curves correspond to a low enough value of \( I_p \). The lowest value of \( I_p \) in Fig. 3 is \( 0.02 \text{A} \), and according to the curve for \( R_s=10 \Omega \) in Fig. 4, the value of \( roc \) at \( I_p=0.02 \text{A} \) is only slightly larger than \( R_s \) still far from being able to reach saturation with \( roc \approx R_p + R_s \).

| Incident light intensity, expressed as photo-current \( I_p \) (mA) | Ideal \( |r_{oc}| \) (Ω) | Real \( |ro| \) (Ω) |
|-----------------|-----------------|-----------------|
| \( G_0=0, R_s=0 \) | \( R_p=100 \Omega, R_s=10 \Omega \) |
| 20 | 1.939 | 12.462 |
| 30 | 1.293 | 11.517 |
| 40 | 0.970 | 11.095 |

Table II: Magnitude of the dynamic resistance at open-circuit conditions \((V=V_{oc}, I=0)\) of the hypothetical ideal and real solar cells, for three values of incident light intensity.
The three curves of $|\Delta R|$ shown in the upper pane of Fig. 4 for different values of $R_s$, may be made to collapse into a single curve by subtracting from $|\Delta R|$ the corresponding value of $R_s$, as (11) and (12) suggest, and the lower pane of Fig.4 illustrates.

We do not present plots of $|\Delta R|$ vs $I_{sc}$ because the difference with the plots of $|\Delta R|$ vs $I_{ph}$ shown in Fig. 4 would be almost imperceptible, since only the horizontal axis would need to be shifted by the small difference between $I_{sc}$ and $I_{ph}$ given by (4). In the case of the ideal cell that difference does not exist since (4i) compels $I_{sc}=I_{ph}$. Instead, we present in Fig. 5 the magnitude of the dynamic resistance $|\Delta r|$ at the open-circuit point, as a function of the reciprocal short-circuit current $1/I_{sc}$, calculated with (11) or (12) for the same three values of $R_s$ and $R_p$. The value of $I_{sc}$ is calculated using (4). Also shown in Fig. 5 for comparison is the $|\Delta r|$ vs $1/I_{sc}$ of the ideal cell ($G_p=0$ and $R_s=0$), calculated with (11i) or (12i), using $I_{sc}=I_{ph}$ according to (4i).

The variation of the dynamic resistance’s magnitude, $|\Delta r|$, at the open-circuit point is shown in Fig. 6 also as a function of open-circuit voltage, $V_{oc}$, corresponding to increasing values of $I_{ph}$, calculated using (11) or (12) for three values of $R_s$ and $R_p=100\,\Omega$. The value of $V_{oc}$ is calculated using (5). Also shown for comparison as a black dash straight line is the dynamic resistance $|\Delta r|$ of the ideal cell ($G_p=0$ and $R_s=0$), calculated with (11i).

Fig. 4. Log-log plot of the magnitude of the dynamic resistance $|\Delta R|$, calculated with (11) or (12) at the open-circuit point as a function of photocurrent $I_{ph}$ (upper plot), and the value of $|\Delta R|-R_s$ (lower plot), for three values of $R_s$ and $R_p=100\,\Omega$. Also shown for comparison as a black dash straight line is the magnitude of the dynamic resistance $|\Delta r|$ for the corresponding ideal cell ($G_p=0$ and $R_s=0$) as calculated with (12i).

Fig. 5. Linear plots of the magnitude of the dynamic resistance $|\Delta R|$ as a function of reciprocal short-circuit current $1/I_{sc}$, calculated at the open-circuit point with (11) or (12) for three values of $R_s$ and $R_p=100\,\Omega$. The value of $1/I_{sc}$ is calculated from $I_{ph}$ using (4). Also shown as a black dash straight line, is the $|\Delta r|$ of the ideal cell ($G_p=0$ and $R_s=0$) vs $1/I_{sc}$, calculated with (11i) or (12i), using $I_{sc}=I_{ph}$ according to (4i).

Fig. 6. Semi-logarithmic plot of the magnitude of the dynamic resistance $|\Delta r|$ at the open-circuit point, as a function of open-circuit voltage, $V_{oc}$, corresponding to increasing values of $I_{ph}$, calculated using (11) or (12) for three values of $R_s$ and $R_p=100\,\Omega$. The value of $V_{oc}$ is calculated using (5). Also shown for comparison as a black dash straight line is the dynamic resistance $|\Delta r|$ of the ideal cell ($G_p=0$ and $R_s=0$), calculated with (11i).
It is easy to visualize in Fig. 6 how all three curves of \(|roc|\) with \(Gp\neq 0\) and \(Rs\neq 0\) would transform into the single straight line representing \(|rioc|\) if their resistive losses \(Gp\rightarrow 0\) and \(Rs\rightarrow 0\).

The semi-logarithmic nature of Fig. 6 indicates that since the plot of the magnitude of the ideal cell’s \(|rioc|\) as a function of \(V_{ioc}\) shows up as a straight line with negative slope, the value of \(|rioc|\) must vary (decrease) as an exponential function of negative open-circuit voltage, \(V_{ioc}\), which in fact does according to (11i).

On the other hand, Fig. 6 also indicates that, in the case of a real cell \((Gp\neq 0\) and \(Rs\neq 0\)), the dependence of \(|roc|\) on \(V_{oc}\) happens to be in general considerably different from such purely exponential behavior. How much it differs depends on how significant the magnitudes of \(Rs\) and \(Gp\) are, as clearly implied by (11), and illustrated in Fig. 6 by way of the observed saturations to \(|roc|\rightarrow Rp+Rs\) at low \(V_{oc}\) and to \(|roc|\rightarrow Rs\) at high \(V_{oc}\). Obviously, the smaller the values of \(Rs\) and \(Gp\) are, the nearer (11) becomes to (11i), and thus, the closer the behavior of \(|roc|\) would be to that of \(|rioc|\), which behaves as a decreasing function of the open-circuit voltage.

**D. Dynamic resistance at the sort-circuit point**

Let us now look at the other end of the power-generating quadrant, i.e., the short-circuit point, defined by the coordinates \((V=0, I=I_{sc})\). The magnitude of the dynamic resistance \(|rsc|\) at the sort-circuit point is also a useful parameter to look at. To calculate it we need to evaluate (9) at \(V=0\) and \(I=I_{sc}\):

\[
|rsc(I_{sc})| = -R_s - \frac{n v_{th}}{G_p n v_{th} + I_0 \exp \left( \frac{I_{sc} R_s}{n v_{th}} \right)} . \tag{13}
\]

In the case of an ideal cell, substitution of \(Rs=0\) and \(Gp=0\) into (13) yields a constant value of \(|risc|\):

\[
|risc| = -\frac{n v_{th}}{I_0} . \tag{13i}
\]

Alternatively, evaluation of (10) at \(V=0\) and \(I=I_{sc}\) yields an expression equivalent to (13):

\[
|rsc| = |r(I_{ph}, I_{sc})| = -R_s - \frac{n v_{th}}{G_p n v_{th} + I_0 \exp \left( \frac{I_{sc} R_s}{n v_{th}} \right)} . \tag{14}
\]

For the case of an ideal cell, substitution of \(Rs=0\) and \(Gp=0\) into (14) yields the same constant as in (13i):

\[
|risc| = -\frac{n v_{th}}{I_0} . \tag{14i}
\]

The light intensity \(I_{ph}\) dependence of the magnitude of dynamic resistance \(|rsc|\) calculated at the sort-circuit point is presented in Fig. 7, for the same values of \(Rp=100\Omega\) and \(Rs=1, 4, 10\Omega\). A quick look reveals that \(|rsc|\) is clearly a function of light intensity \(I_{ph}\), in a way that in general qualitatively resembles that of \(|roc|\) shown in Fig.4. But this similarity refers only the fact that when \(I_{ph}\) reaches very large values, \(|rsc|\) exhibits saturated values near those of \(Rs\) (≈1, 4, and 10Ω in this example); and that as the value of \(I_{ph}\) starts to decrease, \(|rsc|\) starts to increase until at low values of \(I_{ph}\), \(|rsc|\) saturates to constant values around those of \(Rp + Rs\) (≈101, 104, and 110Ω in the present example).

Table III presents values of \(|rsc|\) and \(|risc|\) for three values of \(I_{ph}\). Notice that in the case of the ideal cell, the value of \(|risc|\) is very large and constant at any incident light intensity (represented by \(I_{ph}\)).

Fig. 7 is perfectly compatible with the behavior observed in Fig. 3. Notice, e.g., that when \(I_{ph}=0.02A\), the \(rsc\) curve corresponding to \(Rs=10\Omega\) (solid red line in Fig. 7) reaches a constant value of \(\approx Rp + Rs\) (≈107Ω in this example). This is the same value that the \(|r|\) curve that corresponds to \(I_{ph}=0.02A\) (dashed green line in Fig.3) reaches when \(V\rightarrow 0\) (short-circuit point) in Fig. 3.

**IV. DISCUSSION**

We have just established that the magnitudes of the dynamic resistance at the short-circuit and the open-circuit points, \(|rsc|\) and \(|risc|\), both experience a transition when \(I_{ph}\) changes as a consequence of a change of the incident light intensity. They both decrease from \(Rp + Rs\) at very low incident light intensities (very low \(I_{ph}\)) to \(-Rs\) at very high incident light

| TABLE III |
|----------------------|-----------------|-----------------|
| MAGNITUDE OF THE DYNAMIC RESISTANCE AT SHORT-CIRCUIT CONDITIONS \((V=0, I=I_{sc})\) OF THE ILLUMINATED IDEAL AND REAL SOLAR CELL, FOR THREE VALUES OF INCIDENT LIGHT INTENSITY |
| Incident light intensity, expressed as photo-current \(I_{ph}\) (mA) | Ideal \(|rsc|\) (kΩ) | Real \(|rsc|\) (kΩ) |
|----------------------|-----------------|-----------------|
| 30                   | 387.9           | 107.28          |
| 30                   | 387.9           | 87.87           |
| 40                   | 387.9           | 39.11           |

Equivalent circuit model diode element’s parameters:

\(v_s=0.02586V, n=1.5, I_0=10^{-5}A.\)
intensities (very high $I_{ph}$). Although the observed behavior of
these two magnitudes appear to follow similar sigmoid-function
type of behavior, a more careful comparison of Figs. 4 and 7
clearly reveals that their transitions from high to low values as
the light intensity increases differ from each other. Several
differences can be visualized in the present example. The first
observation is that the transition of $|roc|$ (Fig. 4) occurs at lower
values of $I_{ph}$ than the transition of $|rsc|$ (Fig. 7), and this is so
for all values of $R_s$.

The second observation is that a similar change of the value of
$R_s$ causes a much larger shift on the $I_{ph}$ axis (wider spread)
of the transition of $|rsc|$ (Fig. 7) than that of $|roc|$ (Fig. 4). The
shift produced by increasing the value of $R_s$ by the same amount
is smaller in the case of $|roc|$ (Fig. 4) than in the case of $|rsc|$ (Fig. 7).

The third and most obvious observation refers to how the
magnitudes of $|roc|$ and $|rsc|$ vary in different ways. In the case
of $|rsc|$ that difference is a consequence of the crossovers of the
curves, which can mean a total reversal of the dependence
direction, as can be seen in the central region of Fig. 7. For
example, assuming a value of $I_{ph}$ of 0.08A, the curves
presented in Fig. 7 for $|rsc|$ indicate that if the value $R_s$ increases
from 1, to 4, and then to 10Ω, the corresponding magnitude of
$|rsc|$ experiences a direct reduction from $\sim$101, to $\sim$62, and then
to $\sim$12Ω. This is the opposite dependence that occurs at very
high and low values of $I_{ph}$, shown at the right and left sides,
respectively, of Fig. 7. Even stranger behavior of $|rsc|$ can be
expected when $R_s$ changes at certain values of $I_{ph}$, e.g. at
$I_{ph}$=0.2A in Fig. 7. This type of effect does not exist in the case
of $|roc|$, whose value always increases as $R_s$ increases at any
value of $I_{ph}$, as shown Fig. 4.

A common mistake is falling to the temptation of a priori
assuming that $I_{ph}$=$I_{sc}$ as a good approximation regardless of
any considerations about the presence of significant parasitic
resistive losses; an assumption that is in general disavowed by
(4). Another approximation mistake that unfortunately is often
made about the dynamic resistance of illuminated solar cells is
to naively assume that is always approximately equal to
$RS$ when measured at the open-circuit point, or on the end of the
power-producing quadrant, assuming that the dynamic resistance
measured at the short-circuit point is always approximately equal to $R_P$ [6]. While these two approximations could be perfectly acceptable in many instances, the hazard is, as we have shown here, to assume that these approximations are always justified, regardless of the magnitude of parasitic resistive losses and the intensity of the incident light, as Figs. 4 and 7 clearly indicate. The results of this analysis, therefore, call
attention to the need to handle these assumptions about the
dynamic resistance with care, always checking their validity
before proceeding to draw conclusions that depend on them.

V. CONCLUSIONS

The following conclusions may be drawn about the expected
theoretical behavior of the dynamic resistance of illuminated
solar cells:

1) In general, the magnitude of the dynamic resistance of an
illuminated photovoltaic cell measured (or calculated) at
any point of its static $I-V$ characteristic, depends on the
illumination intensity incident upon the cell's surface. This
includes, of course, the dynamic resistance measured at the
points of the $I-V$ curve corresponding to short circuit, open
circuit, and MPP conditions. The incident light dependence
of the dynamic resistance, therefore, can affect the ability
to effectively track the MPP of a solar cell or panel as the
intensity of the solar illumination changes hourly, or
because of cloud shadowing, during the course of the day.

2) If the intensity of incoming illumination is allowed to
increase, e.g. through light concentration, so that the photo-
current can reach high enough levels, the value of the
dynamic resistance can eventually become approximately
equal to the value of the lumped series resistive loss ($R_s$) at
any operating point along the $I-V$ characteristic, including
the short-circuit, the open-circuit, and the MPP points.

3) Likewise, if the level of illumination decreases drastically,
e.g. at night or under cloud cover in the case of terrestrial
solar cells, so that the intensity of the photocurrent
becomes low enough, the value of the dynamic resistance
will approach and eventually will saturate to a value
approximately equal to the sum of lumped series and
parallel resistive losses ($R_s+R_P$) at any operating point along
the $I-V$ characteristic, including the short-circuit, the open-
circuit, and the MPP points.

4) Although a trivial fact, it is important to keep in mind that
changing the illumination level might significantly change
the values of both $I_{sc}$ and $V_{oc}$, which behave as linear and
logarithmic functions of the photo-generated current,
respectively.

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