Particle Spectrum in the Minimal Supersymmetric Standard Model with non-universal Higgs masses

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ABSTRACT: We present semi-analytical solutions of the supersymmetric non-universal masses models for low tan $\beta$ regime. In addition to this, scale and tan $\beta$ dependencies of the soft $(\text{mass})^2$ terms are given in the form of numerical solutions. By using the constrained form of the semi-analytic results, particular attention is paid on the non-universality assumption of the Higgs mass values and their potential measurable effects on the mass spectra of the minimal supersymmetric standard model. It is observed that, certain measurables are almost insensitive to the initial mass choices of the Higgs fields, like the mass of the light $CP$-even Higgs boson. On the other hand, large deviations exist on the mass of the remaining physical Higgs bosons signal that the allowed parameter space of the model can be probed successfully. For this aim, in addition to the other physical Higgs bosons, imprints originating from the heavier chargino ($\tilde{\chi}^{\pm}_{2}$), heavy neutralinos ($\tilde{\chi}^{0}_{3,4}$) and the light scalar tau ($\tilde{\tau}_1$) are necessary and found to be promising.

KEYWORDS: Supersymmetry Phenomenology.
1. Introduction

There are a number of motivations for phenomenological studies of the Supersymmetric (SUSY) theories among which unification of the gauge couplings and natural suppression of the radiative corrections on the masses of Higgs bosons can be mentioned (see i.e. [1], for a comprehensive list of motivations). Among those theories, due to least number of particles, the Minimal Supersymmetric Standard Model (MSSM) occupies a special place. In the near future, forthcoming experiments may reveal that the incorporation of the Standard Model (SM) into a more effective theory turns out as the MSSM. Indeed, if low energy supersymmetry is realized in Nature, phenomenological studies related with the MSSM and its variants will be important to unravel the hidden model. Since it has certain problems like the famous $\mu$ problem [2], flavor problem [3], and the unknown mechanism of the supersymmetric symmetry breaking, studies related with the extensions of the MSSM may be expected to shed light on future measurement, especially if nontrivial data inconsistent with the minimal model occurs.

In this work, we study particle spectrum in the MSSM with non-universal Higgs mass terms (NUHM) [4]. We provide most general semi-analytic solutions of evolving terms, in terms of high scale boundary conditions, for a low $\tan \beta$ value. Additionally, different scale and $\tan \beta$ dependencies of the soft ($\text{mass}^2$) terms will be presented numerically. Actually, the exploration of solutions to the renormalization group equations (RGEs) of a supersymmetric model with NUHM is a subject that has been investigated (see e.g. [5] and [6]), but, the novel feature of our analysis is that semi-analytic solutions may facilitate the exploration of the phenomenology of the model (see [7] for phenomenology of NUHM). As is well known, weak scale observables and Grand Unified Theory (GUT) scale boundaries are connected via RGEs in a complicated manner [8] and they can be solved with the help of certain softwares. Taste of numerical solutions can not be compared with analytical ones though the former ones are very accurate. As an alternative to the numerical ones, semi-analytic expressions [6] and construction of certain RG invariant forms are useful for phenomenological analysis of the MSSM and its extensions [9].

The possibility of non-universality specific to Higgs masses was studied in a series of papers [10],[4], by noting constraints from $b \rightarrow s\gamma$, cosmology and anomalous magnetic moment of muon and it was stressed that relaxing the scalar-mass universality assumption for the MSSM Higgs multiplets opens up many phenomenological possibilities (see also [11] for $B_s \rightarrow \mu^+ \mu^-$ and cold dark matter issues related with the NUHM). One of the aims of the present work is to present the full
form of semi-analytical expressions explicitly, so that all weak scale observables can be expressed in terms of GUT inputs. The analytical form of the results can provide considerable insight for similar issues (we ignore CP-violation during the numerical analysis of the NUHM, however, the full form of our results cover this issue too). Indeed, due to the complicated structure of the renormalization group equations, it is appealing to handle issues analytically and the solutions presented in this work can be useful for such an analysis even if they are given to the one loop order. As we will see, to keep the analysis simple, there are certain ignorance made on most of the correction terms, however, in the low tan β regime they do not affect our conclusions sizably and can further be added on demand.

The outline of the rest of this work is as follows: In Section 2, we introduce our notation and conventions. In Section 3 we present the effects of non-universal Higgs masses terms on the supersymmetric mass spectra for varying tan β and scale values. A subsection of the same section is given to benchmark the semi-analytic results. Section 4 is devoted to our conclusions. The full form of the solutions of the RGEs can be found in the Appendix A.

2. Notation and Conventions

We define the basic parameters of the model as soft supersymmetry breaking scalar masses $m_0$, gaugino masses $M$, the trilinear couplings $A_0$, bilinear coupling $B_0$ and supersymmetric Higgs mass parameter $\mu_0$, at the GUT scale. We assume third family dominance model and solve RGEs at the one-loop order. In this effective approach, by solving the RGEs explicitly, weak scale predictions are expressed in terms of GUT boundaries. We express Bino, Wino and Gluino with $M_1, M_2, M_3$, respectively, with a common initial value $M$. By writing the GUT boundaries, $A_i = c_{A_i} A_0$, $M_j = c_{M_j} M$, $m_k = c_k m_0$ (2.1)

where $i = t, b, \tau$ and $j = 1, 2, 3$, and for (mass)$^2$ terms $k = H_u, H_d, \bar{t}_L, \bar{t}_R, \bar{b}_R, \bar{\tau}_L, \bar{\tau}_R$. We will express weak scale value of each quantity in terms of corresponding mSUGRA parameters $m_0, M, A_0$, and a positive $\mu$ to be determined by the electroweak breaking conditions. From the solutions of the RGEs, weak scale and GUT scale values are connected and the most important restriction, in this respect, is the mass of Z boson:

$$\frac{1}{2} M_Z^2 = -\mu^2 + \frac{m_{H_u}^2 - \tan^2 \beta m_{H_d}^2}{\tan^2 \beta - 1} + \Delta$$

(2.2)

where $\tan \beta$ is the ratio of vacuum expectation values ($v_u/v_d$), and $\Delta$ stands for corrections on Higgs masses. We are interested in low vacuum expectation value ($\tan \beta = 10$), for which complete list of semi-analytic solutions are given in the Appendix A. In addition to this, we will present graphical solutions for different $\tan \beta = 10$ values in the next section. Instead of purely numerical values, expressing weak scale predictions in terms of GUT inputs proves very useful and helps to differentiate the importance of each term. In order to show the relative weigh of each term, we will express evolution of any soft (mass)$^2$ term as in the following forms

$$(\text{mass})^2 = \gamma_1 A_0^2 + \gamma_2 A_0 M + \gamma_3 M^2 + \gamma_4 c_{H_u}^2 m_0^2 + \gamma_5 c_{H_d}^2 m_0^2 + \gamma_6 m_0^2.$$ (2.3)

This decomposition enables one to lay stress upon the effects of non-universal Higgs mass choices. As it can be extracted from the above equation, sensitivity of each term to the initial values of $m_{H_u}$ and $m_{H_d}$ will be different. Notice that, by using the $M_Z$ constraint given in (2.2) one can obtain $\mu$ and this can be expressed as

$$b = \frac{2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2}{\tan \beta + \cot \beta}$$

(2.4)
hence, tree level relations of mass of physical Higgs boson can be written as in the followings \[2\]

\[
\begin{align*}
    m_{A_0}^2 &= 2b / \sin 2\beta \\
    m_{H^\pm}^2 &= m_{A_0}^2 + m_W^2 \\
    m_{H_0, h_0}^2 &= \frac{1}{2} \left[ m_{A_0}^2 + m_Z^2 \pm \sqrt{(m_{A_0}^2 + m_Z^2)^2 - 4m_{A_0}^2 m_Z^2 \cos 2\beta} \right].
\end{align*}
\]

Those relations will be modified, largely, due to top-stop loop corrections and \(h_0\) is the most affected one. Indeed, since the mass of the lightest \(CP\)-even Higgs boson is larger than 114 GeV \[3\] this correction must be included in the analysis. We will consider this correction and omit others in our effective approach. Meanwhile, the price that should be paid for that aim is predicting the spectra with small certain errors as will be shown in the following section. But this does not affect our conclusions, since the reaction of the SUSY particles to the non-universal Higgs boundary conditions is important for the present study.

The necessary expression for the most important correction is

\[\Delta(m_{h_0}^2) = \frac{3}{4 \pi^2} v^2 y_t^4 \sin^4 \beta \ln \left( \frac{m_{t_1}m_{t_2}}{m_t^2} \right), \quad (2.8)\]

where \(m_{t_{1,2}}\) can be extracted from the following mass matrix

\[
\mathbf{m}_t^2 = \begin{pmatrix}
    m_t^2 + m_{t_{1L}}^2 + \left( \frac{1}{2} - \frac{2}{3} s_w^2 \right) M_Z^2 \cos 2\beta & m_t^2 (A_t - \mu \cot \beta) \\
    m_t^2 (A_t - \mu \cot \beta) & m_t^2 + m_{t_{1R}}^2 + \frac{2}{3} s_w^2 M_Z^2 \cos 2\beta
\end{pmatrix}. \quad (2.9)
\]

This \(2 \times 2\) matrix can easily be diagonalized to obtain eigenvalues of stop quark masses in terms of GUT inputs, similarly the same should be done for \(m_b^2, m_s^2\) using the solutions presented in the appendices in order to get the full sparticle spectrum as usual (i.e. see \[2\]). Indeed, having such analytic expressions is very useful to visualize the ingredients of sparticles to indirectly probe the allowed range of non-universality of Higgs bosons. As an example, for \((\tan \beta = 10)\)

\[
m_{t_{1,2}}^2 = -0.052 A_0^2 + 0.192 A_0 M + 3.74 M^2 + (0.642 - 0.0176 c_{H_u}^2 - 0.161 c_{H_u}^2) m_0^2 + m_t^2
\]

\[- 0.245 M_Z^2 + \Omega\]

(2.10)

where the exact expression of \(\Omega\) is a quite lengthy function of all terms appearing in the first line of \[2.10\]. Notice that, it can be obtained using the full forms of the solutions given in the appendix. Now, let us make a simplifying assumption \(\mu_0 \sim m_0 \sim A_0\) and \(m_t \sim 2 m_0\) on the \(\Omega\) part of \[2.10\] to approximately predict the composition of stop masses

\[
m_{t_{1,2}}^2 \simeq -0.052 A_0^2 + 0.192 A_0 M + 3.8 M^2 + (0.64 - 0.018 c_{H_u}^2 - 0.16 c_{H_u}^2) m_0^2 + m_t^2
\]

\[- 0.25 M_Z^2 \mp 3.45 m_0^2 .\]

(2.11)

Using this analytical expression, for instance, one can conclude that weigh of up Higgs fields is larger than weigh of down Higgs fields but their relative weigh is negligible compared to other soft mass terms. To be specific, we will consider the specific reference point SPS1a' \[14\] in the numerical analysis to benchmark the solutions provided. However, even under the above rough approximation we found \(m_{t_1} = 472\) GeV and \(m_{t_2} = 506\) GeV, to be compared with the exact results. For the mass spectra of SUSY particles, effects of up Higgs field can be dominant, however, as we will see in the next section this can not be generalized to other sectors.

### 3. Numerical Analysis

In this section \(\tan \beta\) and scale evolutions of (mass)\(^2\) terms will be presented. For this aim, solutions of RGEs are performed such that high scale is set equal to \(1.9 \times 10^{16}\) GeV and the supersymmetry
breaking scale is chosen as 1 TeV. With this choices, unification of the gauge couplings is satisfied at the GUT scale as \( g_1 = g_2 = g_3 = 0.718 \pm 0.001 \).

One can obtain the solutions of RGEs for any \( \tan \beta = 10 \). To be specific, for \( \tan \beta = 10 \), mass of the heavy SM fermions fix the Yukawa couplings at the same scale as \( Y_t = 0.551 \), \( Y_b = 0.0547 \), \( Y_\tau = 0.0685 \). As a brief summary of the semi-analytic solutions, this specific choice of \( \tan \beta \) yields the followings equations

\[
\begin{align*}
m_{H_u}^2 &= -0.102 A_0^2 + 0.375 A_0 M - 1.93 M^2 - 0.709 m_0^2 + 0.0331 \tilde{c}_{H_u} m_0^2 + 0.612 \tilde{c}_{H_u}^2 m_0^2 \\
m_{H_d}^2 &= -0.0107 A_0^2 + 0.0309 A_0 M + 0.143 M^2 - 0.0241 m_0^2 + 0.955 \tilde{c}_{H_d}^2 m_0^2 + 0.0333 \tilde{c}_{H_d} m_0^2 \\
m_{L_L}^2 &= -0.0367 A_0^2 + 0.134 A_0 M + 4.33 M^2 + 0.757 m_0^2 + 0.00768 \tilde{c}_{H_u}^2 m_0^2 - 0.129 \tilde{c}_{H_u} m_0^2 \\
m_{l_t}^2 &= -0.068 A_0^2 + 0.25 A_0 M + 3.15 M^2 + 0.527 m_0^2 - 0.0429 \tilde{c}_{H_d}^2 m_0^2 - 0.194 \tilde{c}_{H_u}^2 m_0^2 \\
m_{b_R}^2 &= -0.00534 A_0^2 + 0.0192 A_0 M + 4.67 M^2 + 0.988 m_0^2 + 0.0149 \tilde{c}_{H_d}^2 m_0^2 - 0.0211 \tilde{c}_{H_u}^2 m_0^2 \\
m_{l_L}^2 &= -0.00271 A_0^2 + 0.00216 A_0 M + 0.493 M^2 + 0.994 m_0^2 - 0.0353 \tilde{c}_{H_d}^2 m_0^2 + 0.0325 \tilde{c}_{H_u}^2 m_0^2 \\
m_{t_R}^2 &= -0.00542 A_0^2 + 0.00432 A_0 M + 0.143 M^2 + 0.989 m_0^2 + 0.0595 \tilde{c}_{H_d}^2 m_0^2 - 0.065 \tilde{c}_{H_u}^2 m_0^2.
\end{align*}
\]

Notice that the analytical expressions given in (3.1) are constrained forms of the solutions presented in the Appendix (here we set \( \Phi_{i,j} \rightarrow 0 \) and \( c_i \rightarrow 1 \), except for \( c_{H_u} \) and \( c_{H_d} \)). And they can be used at SPS1a’ point \([13] \). We will benchmark our solutions using this point in the following subsection.

Different scale and \( \tan \beta \) effects can be extracted from the following figures (Figs. 1–7). In Fig. 1, we show \( \tan \beta \) and scale dependencies of the composition of \( m_{H_u}^2 \). Normally, mass of up Higgs fields get contributions from any of the 28 terms given in (A.2). When we assume CP is conserved (\( \Phi_{i,j} \rightarrow 0 \)) and accept universality is in charge (except for Higgs fields), mass of the up Higgs field can be decomposed in a neat form as in (2.3)

\[
m_{H_u}^2 = \gamma_1^{(H_u)} A_0^2 + \gamma_2^{(H_u)} A_0 M + \gamma_3^{(H_u)} M^2 + \gamma_4^{(H_u)} c_{H_u}^2 m_0^2 + \gamma_5^{(H_u)} c_{H_d}^2 m_0^2 + \gamma_6^{(H_u)} m_0^2.
\]

As can be seen from both panels of the first figure, largest contribution to mass of up Higgs field comes from Gaugino sector (dashed-blue curves). Contribution of down Higgs field to up Higgs field is negligible, in other words, deviation of down Higgs from the universal choice can not yield a detectable effect on up Higgs field. In all Figs. 1–7, solid red (green) curves corresponds to contribution of \( m_{H_u}^2 (m_{H_d}^2) \) on the related (mass)\(^2\) terms, which are \( m_{H_u}^2, m_{H_d}^2, m_{L_L}^2, m_{l_t}^2, m_{b_R}^2, m_{l_L}^2 \) and finally \( m_{t_R}^2 \), respectively. In order to show the effects of scale variations fix \( \tan \beta = 10 \) (right panels) and for varying \( \tan \beta \) values scale is fixed around the weak scale (left panels). The Figs. 1–7 denote that the gauge/gaugino sector contributions to scalar mass sector evolution increases scalar mass parameters as we go to the weak scale. It is visible in Figs. 1–7 that a strong reaction can be detected in the slepton sector to non-universal Higgs mass terms and this is true for any \( \tan \beta \) value. Notice that this can be expected for Higgs bosons too (see Figs. 1–7). We observe from Figs. 1–7 that, scalar top quarks are sensitive to NUHM only for very small \( \tan \beta \) values (\( \sim 2 – 3 \)).

During the numerical analysis we observed that following the physical Higgs bosons (except the CP-even light Higgs boson), sleptons are very sensitive to NUHM terms. Hence, we present Fig. 8 to show a bird-eye picture of the reaction of stau mass eigenvalues to NUHM parameters. As can be inferred from the very figure, reaction of sparticles to the mentioned non-universality drifts the mass predictions, to some extend. This effect ranges from a few GeV to \( \sim 30 – 40 \) GeV for different sparticles and it can be detectable since the correct spectrum is well known for the MSSM. See Tab. 1 for the reaction of the particles of the MSSM to NUHM.

### 3.1 Benchmark of the solutions

The most practical solution in order to test the trustability of our results is to use certain benchmark points. Though a large set of benchmark points and parameter lines in the MSSM parameter space
Figure 1: Evolution of contributions for $m_{H_u}^2$ with $\tan \beta$ (left panel) and scale (right panel) in NUHM. Scale is shown with the dimensionless quantity $t$ such that $t = 0$ denotes the GUT scale and $t \sim -0.2$ corresponds to the Z scale, $\tan \beta$ varies from 2 to 60. In both of the panels solid red, green and blue lines correspond to $\gamma_4^{(H_u)}$, $\gamma_5^{(H_u)}$ and $\gamma_6^{(H_u)}$. Dashed red, green and blue lines correspond to $\gamma_1^{(H_u)}$, $\gamma_2^{(H_u)}$ and $\gamma_3^{(H_u)}$ as given in (3.1).

Figure 2: The same as Fig. 1 but for $m_{H_d}^2$

Figure 3: The same as Fig. 1 but for $m_{t_L}^2$

is established, we will use one the the most studied points (see [15] for Snowmass Points and Slopes). Since we ignored most of the corrections except that of on the mass of the lightest $CP$-even Higgs
boson, a strict comparison with the state of art programs like ISAJET [16] or SOFTSUSY [17] should not be expected (see also [18] and the given web page for online comparison). Nevertheless, resulting error should not be too high and there should be a visible correlation. We observed this is indeed the case for our semi-analytic solutions. To be definite, if $\tan \beta = 10$, $M = 250 \text{GeV}$, $m_0 = 70 \text{GeV}$, $A_0 = -300 \text{GeV}$, $\mu_{[\mu]} = 1$ (which is the SPS1a reference point) then at the weak
scale (at 1 TeV) we end up with Table I.

Comparison of these results with the reference point denotes that the errors in predicting \(m_h, m_{\tilde{\tau}_1,2}\) and \(m_{\tilde{\chi}_1^0}\) are negligible. For other mass terms errors are somewhat large especially in predicting mass of the lightest neutralino \(m_{\tilde{\chi}_1^0}\); here absolute error is \(\sim 8\%\) which could be reduced if calculation are performed at two loops, corrections are noticed for all terms etc. However, apparent correlation is sufficient for our aim since we are basically interested in the reaction of those particles to the non-universal choices of the Higgs masses. Of course, this is true as far as corrections do not alter the weight of \(c_{H_u}\) and \(c_{H_d}\) on the SUSY particles, which we assumed to be true since the emerging mass difference of the worst prediction is less than \(\sim 8\%\). Nevertheless, a numerical simulation including all families and known corrections would be more decisive, which is beyond the scope of this work.

4. Conclusions

Using the semi-analytic solutions presented in this work it is observed that deviation from the universality assumption of the Higgs fields does not induce serious problems as in the case of other soft (mass)\(^2\) terms (especially if \(c_{H_u} \sim c_{H_d} \sim 1\)). This can be inferred from Tab II in which coefficients of up and down Higgs fields are varied from 0 to 2 \(m_0\). For this range, a striking
| Particle | $SPS1a'$ [GeV] | This Work [GeV] | % Difference | $\delta^{NUHM}$ [GeV] |
|----------|----------------|----------------|--------------|----------------------|
| $h^0$    | 116.0          | 110.3          | -4.91        | 0.2                  |
| $H^0$    | 425.0          | 425.7          | -0.17        | 35.1                 |
| $A^0$    | 424.9          | 425.3          | -0.09        | 35.1                 |
| $H^\pm$  | 432.7          | 432.8          | -0.02        | 34.5                 |
| $\tilde{t}_1$ | 366.5      | 374.3          | -2.13        | 5.1                  |
| $\tilde{t}_2$ | 585.5      | 578.9          | 1.13         | 5.1                  |
| $\tilde{b}_1$ | 506.3      | 502.9          | 0.67         | 2.4                  |
| $\tilde{b}_2$ | 545.7      | 530.7          | 2.75         | 0.9                  |
| $\tilde{\tau}_1$ | 107.9      | 111.4          | -3.24        | 8.8                  |
| $\tilde{\tau}_2$ | 194.9      | 199.7          | -2.46        | 2.1                  |
| $\tilde{\chi}_0^1$ | 97.7       | 105.3          | -7.78        | 0.2                  |
| $\tilde{\chi}_0^2$ | 183.9     | 194.3          | -5.65        | 1.2                  |
| $\tilde{\chi}_0^3$ | 400.5     | 400.6          | -0.02        | 15.9                 |
| $\tilde{\chi}_0^4$ | 413.9     | 417.3          | -0.82        | 14.5                 |
| $\tilde{\chi}_1^\pm$ | 183.7     | 193.7          | -5.44        | 1.3                  |
| $\tilde{\chi}_2^\pm$ | 415.4     | 417.9          | -0.60        | 14.6                 |
| $\tilde{\nu}_\tau$ | 170.5     | 176.6          | -3.58        | 3.8                  |

Table 1:

Numerical values for the mass of some of the supersymmetric particles and Higgs bosons in the reference point $SPS1a'$ [14] and their comparison with our semi-analytical results. The third column is obtained by $(SPS1a' - \text{our results}) \times 100/SPS1a'$. The fourth column denotes the sensitivity of each particle to the NUHM model parameters. The difference between maximal and minimal mass values is obtained by varying $c_{H_u}$ and $c_{H_d}$ in the $[0,2]$ interval and the emerging difference is called as sensitivity ($\delta^{NUHM}$) for each term.

A difference can be observed on the mass of certain supersymmetric particles (like sleptons) while the others are insensitive to the mentioned phenomena (like lightest neutralino).

Expected discovery of low energy SUSY at the Large Hadron Collider (LHC) and the International Linear Collider (ILC) [19] will require reconstruction of the supersymmetric theory parameters from the experimental data. This is necessary not only for the minimal model but also for NUHM, especially if experimental data signalling deviations from the minimal supergravity model (mSUGRA) [2] occurs. For this aim, precise measurements of mass of the light stau $m_{\tau_1}$, which is probably among the first sparticles to be discovered due to lepton nature and a light mass very sensitive to non-universality of the Higgs bosons, will be very suitable to shoot the NUHM parameter space.

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A. Explicit Solutions for low tan $\beta$

In this part we present explicit form of our semi-analytic solutions which are obtained by solving the RGEs explicitly, to the one loop order. The Gut scale is $M_{GUT} = 1.9 \times 10^{16}$ GeV and $\tan \beta = 10$. 
At the GUT scale we found the following results for gauge and Yukawa couplings

\[ g_1 = 0.7179, \quad g_2 = 0.7187, \quad g_3 = 0.7195 \]
\[ Y_t = 0.5510, \quad Y_b = 0.0547, \quad Y_\tau = 0.0685. \]  

(A.1)

For the weak scale scale (\( \sim 1 \text{ TeV} \)), our results for soft (mass)\(^2\) terms read

\[
m_{H_u}^2 = 0.000619 A_0^2 c_{A_{\nu}}^2 - 7.8 \times 10^{-7} A_0^2 c_{A_{\tau}}^2 - 0.103 A_0^2 c_{A_t}^2 + 0.00473 c_{M_2}^2 M^2 + 0.206 c_{M_2}^2 M^2 - 1.94 c_{M_2}^2 M^2 + 0.033 l c_{H_d} m_0^2 + 0.612 c_{H_d} m_0^2 - 0.0319 c_{b_R}^2 m_0^2 + 0.0325 c_{L_\tau}^2 m_0^2 - 0.0325 c_{\tau_R}^2 m_0^2 - 0.387 c_{L_\tau}^2 m_0^2 - 0.29 c_{\tau_R}^2 m_0^2 - 0.00572 c_{M_1} c_{M_2} M^2 \cos \phi_{12} - 0.0252 c_{M_1} c_{M_2} M^2 \cos \phi_{13} - 0.0000612 A_0 c_{A_{\nu}} c_{M_1} M \cos \phi_{1b} + 2.13 \times 10^{-7} A_0 c_{A_{\tau}} c_{M_1} M \cos \phi_{1\tau} + 0.0122 A_0 c_{A_t} c_{M_1} M \cos \phi_{1t} - 0.168 c_{M_2} c_{M_3} M^2 \cos \phi_{23} - 0.000535 A_0 c_{A_{\nu}} c_{M_2} M \cos \phi_{2b} + 1.12 \times 10^{-6} A_0 c_{A_{\tau}} c_{M_2} M \cos \phi_{2\tau} + 0.0726 A_0 c_{A_t} c_{M_2} M \cos \phi_{2t} - 0.00215 A_0 c_{A_\nu} c_{M_3} M \cos \phi_{3b} + 3.48 \times 10^{-6} A_0 c_{A_\tau} c_{M_3} M \cos \phi_{3\tau} + 0.293 A_0 c_{A_t} c_{M_3} M \cos \phi_{3\tau} - 1.54 \times 10^{-6} A_0^2 c_{A_{\nu}} c_{A_{\tau}} \cos \phi_{b\tau} + 0.000285 A_0^2 c_{A_{\tau}} c_{A_t} \cos \phi_{b\tau} - 3.29 \times 10^{-7} A_0^2 c_{A_{\nu}} c_{A_{\tau}} \cos \phi_{b\tau}. 
\]

(A.2)

\[
m_{H_d}^2 = -0.00992 A_0^2 c_{A_{\nu}} + 0.00272 A_0^2 c_{A_{\tau}} + 0.000286 A_0^2 c_{A_t} + 0.0361 c_{M_2} M^2 + 0.449 c_{M_2} M^2 - 0.0613 c_{M_3} M^2 + 0.955 c_{H_d} m_0^2 + 0.033 c_{H_d} m_0^2 + 0.0224 c_{b_R}^2 m_0^2 - 0.0353 c_{L_{\tau}}^2 m_0^2 + 0.0298 c_{\tau_R}^2 m_0^2 + 0.0232 c_{L_{\tau}}^2 m_0^2 - 0.0642 c_{\tau_R}^2 m_0^2 - 0.000383 c_{M_1} c_{M_2} M^2 \cos \phi_{12} - 0.000749 c_{M_1} c_{M_2} M^2 \cos \phi_{13} + 0.000538 A_0 c_{A_{\nu}} c_{M_1} M \cos \phi_{1b} + 0.000586 A_0 c_{A_{\tau}} c_{M_1} M \cos \phi_{1\tau} - 0.0000792 A_0 c_{A_t} c_{M_1} M \cos \phi_{1t} - 0.0097 c_{M_2} c_{M_3} M^2 \cos \phi_{23} + 0.0064 A_0 c_{A_\nu} c_{M_2} M \cos \phi_{2b} + 0.0016 A_0 c_{A_\tau} c_{M_2} M \cos \phi_{2\tau} - 0.000762 A_0 c_{A_t} c_{M_2} M \cos \phi_{2t} - 0.0258 A_0 c_{A_\nu} c_{M_3} M \cos \phi_{3b} - 0.0000721 A_0 c_{A_\tau} c_{M_3} M \cos \phi_{3\tau} - 0.00307 A_0 c_{A_t} c_{M_3} M \cos \phi_{3\tau} + 0.0000554 A_0^2 c_{A_{\nu}} c_{A_{\tau}} \cos \phi_{b\tau} + 0.00159 A_0^2 c_{A_{\tau}} c_{A_t} \cos \phi_{b\tau} - 4.46 \times 10^{-6} A_0^2 c_{A_\nu} c_{A_{\tau}} \cos \phi_{b\tau}. 
\]

(A.3)

\[
m_{\tau_L}^2 = -0.0031 A_0^2 c_{A_{\nu}} + 5.35 \times 10^{-6} A_0^2 c_{A_{\tau}} - 0.0342 A_0^2 c_{A_t} - 0.00678 c_{M_1} M^2 + 0.372 e_{M_2} M^2 + 4.04 c_{M_2} M^2 + 0.00768 c_{H_d} m_0^2 - 0.129 c_{H_d} m_0^2 - 0.014 c_{b_R}^2 m_0^2 - 0.0184 c_{L_{\tau}}^2 m_0^2 - 0.0108 c_{\tau_R}^2 m_0^2 + 0.868 c_{L_{\tau}}^2 m_0^2 - 0.0965 c_{\tau_R}^2 m_0^2 - 0.0197 c_{M_1} c_{M_2} M^2 \cos \phi_{12} + 0.00865 c_{M_1} c_{M_2} M^2 \cos \phi_{13} + 0.00016 A_0 c_{A_\nu} c_{M_1} M \cos \phi_{1b} - 1.29 \times 10^{-6} A_0 c_{A_\tau} c_{M_1} M \cos \phi_{1\tau} + 0.0403 A_0 c_{A_t} c_{M_1} M \cos \phi_{1t} - 0.0592 c_{M_2} c_{M_3} M^2 \cos \phi_{23} + 0.00196 A_0 c_{A_\nu} c_{M_2} M \cos \phi_{2b} - 6.44 \times 10^{-6} A_0 c_{A_\tau} c_{M_2} M \cos \phi_{2\tau} + 0.0239 A_0 c_{A_t} c_{M_2} M \cos \phi_{2t} + 0.00789 A_0 c_{A_\nu} c_{M_3} M \cos \phi_{3b} - 0.000017 A_0 c_{A_\tau} c_{M_3} M \cos \phi_{3\tau} + 0.00965 A_0 c_{A_t} c_{M_3} M \cos \phi_{3\tau} + 0.000106 A_0^2 c_{A_\nu} c_{A_{\tau}} \cos \phi_{b\tau} + 0.000627 A_0^2 c_{A_{\tau}} c_{A_t} \cos \phi_{b\tau} - 1.17 \times 10^{-6} A_0^2 c_{A_\nu} c_{A_{\tau}} \cos \phi_{b\tau}. 
\]

(A.4)
\[ m_{tr}^2 = 0.000412 A_0^2 c_{A_r}^2 - 5.2 \times 10^{-7} A_0^2 c_{A_r}^2 - 0.0686 A_0^2 c_{A_r}^2 + 0.0443 c_{A_r}^2 + M^2 
- 0.168 c_{M}^2 M^2 + 3.41 c_{M}^2 M^2 - 0.0429 c_{H_d}^2 m_0^2 - 0.194 c_{H_u}^2 m_0^2 + 0.0438 c_{b_r}^2 m_0^2 
- 0.0434 c_{b_r}^2 m_0^2 + 0.043 c_{r}^2 m_0^2 - 0.193 c_{L}^2 m_0^2 + 0.676 c_{b_r}^2 m_0^2 
- 0.00381 c_{M_1} c_{M_2} M^2 \cos \Phi_{12} - 0.0168 c_{M_1} c_{M_2} M^2 \cos \Phi_{13} 
- 0.0000408 A_0 c_{A_r} c_{M_1} M \cos \Phi_{1b} + 1.42 \times 10^{-7} A_0 c_{A_r} c_{M_1} M \cos \Phi_{1r} 
+ 0.0081 A_0 c_{A_r} c_{M_1} M \cos \Phi_{1t} - 0.112 c_{M_2} c_{M_3} M^2 \cos \Phi_{23} 
- 0.000357 A_0 c_{A_r} c_{M_1} M \cos \Phi_{2b} + 7.45 \times 10^{-7} A_0 c_{A_r} c_{M_2} M \cos \Phi_{2r} 
+ 0.0484 A_0 c_{A_r} c_{M_2} M \cos \Phi_{2t} - 0.00144 A_0 c_{A_r} c_{M_3} M \cos \Phi_{3b} 
+ 2.32 \times 10^{-6} A_0 c_{A_r} c_{M_5} M \cos \Phi_{3r} + 0.195 A_0 c_{A_r} c_{M_3} M \cos \Phi_{3t} 
- 1.03 \times 10^{-6} A_0^2 c_{A_r} c_{A_r} \cos \Phi_{2m} + 0.00019 A_0^2 c_{A_r} c_{A_r} \cos \Phi_{2b} 
- 2.19 \times 10^{-7} A_0^2 c_{A_r} c_{A_r} \cos \Phi_{2r}, \] (A.5)

\[ m_{tr}^2 = -0.00662 A_0^2 c_{A_r}^2 + 0.0000112 A_0^2 c_{A_r}^2 + 0.000191 A_0^2 c_{A_r}^2 + 0.0162 c_{A_r}^2 \cdot M^2 
- 0.00483 c_{M_2} M^2 + 4.66 c_{M_3} M^2 + 0.0149 c_{H_d}^2 m_0^2 - 0.0211 c_{H_u}^2 m_0^2 + 0.972 c_{b_r}^2 m_0^2 
+ 0.0217 c_{r}^2 m_0^2 - 0.0217 c_{r}^2 m_0^2 - 0.0279 c_{L}^2 m_0^2 + 0.0439 c_{b_r}^2 m_0^2 
- 0.000119 c_{M_1} c_{M_2} M^2 \cos \Phi_{12} - 0.000501 c_{M_1} c_{M_3} M^2 \cos \Phi_{13} 
+ 0.000361 A_0 c_{A_r} c_{M_1} M \cos \Phi_{1b} - 2.71 \times 10^{-6} A_0 c_{A_r} c_{M_1} M \cos \Phi_{1r} 
- 0.0000533 A_0 c_{A_r} c_{M_1} M \cos \Phi_{1t} - 0.00647 c_{M_2} c_{M_3} M^2 \cos \Phi_{23} 
+ 0.00427 A_0 c_{A_r} c_{M_3} M \cos \Phi_{2b} - 0.0000136 A_0 c_{A_r} c_{M_3} M \cos \Phi_{2r} 
- 0.000509 A_0 c_{A_r} c_{M_3} M \cos \Phi_{2t} + 0.0172 A_0 c_{A_r} c_{M_3} M \cos \Phi_{3b} 
- 0.0000363 A_0 c_{A_r} c_{M_3} M \cos \Phi_{3r} - 0.00205 A_0 c_{A_r} c_{M_3} M \cos \Phi_{3t} 
+ 0.0000222 A_0^2 c_{A_r} c_{A_r} \cos \Phi_{2m} + 0.00106 A_0^2 c_{A_r} c_{A_r} \cos \Phi_{2b} 
- 2.11 \times 10^{-6} A_0^2 c_{A_r} c_{A_r} \cos \Phi_{2r}, \] (A.6)

\[ m_{tr}^2 = 0.000011 A_0^2 c_{A_r}^2 - 0.00274 A_0^2 c_{A_r}^2 - 3.11 \times 10^{-7} A_0^2 c_{A_r}^2 + 0.0365 c_{M_1} M^2 
+ 0.457 c_{M_2} M^2 + 0.000035 c_{M_3} M^2 - 0.0353 c_{H_d}^2 m_0^2 + 0.0325 c_{H_u}^2 m_0^2 + 0.0325 c_{b_r}^2 m_0^2 
+ 0.965 c_{r}^2 m_0^2 + 0.0297 c_{r}^2 m_0^2 + 0.0325 c_{L}^2 m_0^2 - 0.065 c_{b_r}^2 m_0^2 
- 0.000204 c_{M_1} c_{M_2} M^2 \cos \Phi_{12} + 3. \times 10^{-6} c_{M_1} c_{M_3} M^2 \cos \Phi_{13} 
- 3.46 \times 10^{-6} A_0 c_{A_r} c_{M_1} M \cos \Phi_{1b} + 0.00059 A_0 c_{A_r} c_{M_3} M \cos \Phi_{1r} 
+ 2.42 \times 10^{-7} A_0 c_{A_r} c_{M_1} M \cos \Phi_{1t} + 0.0000132 c_{M_2} c_{M_3} M^2 \cos \Phi_{23} 
- 0.000014 A_0 c_{A_r} c_{M_2} M \cos \Phi_{2b} + 0.00162 A_0 c_{A_r} c_{M_2} M \cos \Phi_{2r} 
+ 1.07 \times 10^{-6} A_0 c_{A_r} c_{M_2} M \cos \Phi_{2t} - 0.0000176 A_0 c_{A_r} c_{M_3} M \cos \Phi_{3b} 
- 0.0000178 A_0 c_{A_r} c_{M_3} M \cos \Phi_{3r} + 1.78 \times 10^{-6} A_0 c_{A_r} c_{M_3} M \cos \Phi_{3t} 
+ 0.000222 A_0^2 c_{A_r} c_{A_r} \cos \Phi_{2m} - 1.28 \times 10^{-6} A_0^2 c_{A_r} c_{A_r} \cos \Phi_{2b} 
- 1.29 \times 10^{-6} A_0^2 c_{A_r} c_{A_r} \cos \Phi_{2r}, \] (A.7)
\[ m_{\tau_R}^2 = 0.0000221 \ A_0^2 \ c_{A_\phi}^2 - 0.00548 \ A_0^2 \ c_{A_\phi} - 6.21 \times 10^{-7} \ A_0^2 \ c_{A_\phi} + 0.147 \ c_{M_1}^2 \ M^2 \\
+ 0.00366 \ c_{M_2}^2 \ M^2 + 0.0000699 \ c_{M_3}^2 \ M^2 + 0.0595 \ c_{H_u}^2 \ m_0^2 - 0.065 \ c_{H_u}^2 \ m_0^2 - 0.065 \ c_{B}^2 \ m_0^2 \\
+ 0.0595 \ c_{H_u}^2 \ m_0^2 + 0.929 \ c_{H_u}^2 \ m_0^2 - 0.065 \ c_{H_u}^2 \ m_0^2 + 0.13 \ c_{H_u}^2 \ m_0^2 \\
- 0.000499 \ c_{M_1}^2 \ c_{M_2}^2 \ M^2 \ cos \Phi_{12} + 5.99 \times 10^{-6} \ c_{M_1}^2 \ c_{M_3}^2 \ M^2 \ cos \Phi_{13} \\
- 6.93 \times 10^{-6} \ A_0 \ c_{A_\phi} \ c_{M_1} \ M \ cos \Phi_{1b} + 0.00118 \ A_0 \ c_{A_\phi} \ c_{M_2} \ M \ cos \Phi_{1c} \\
+ 4.83 \times 10^{-7} \ A_0 \ c_{A_\phi} \ c_{M_2} \ M \ cos \Phi_{1t} + 0.0000264 \ A_0 \ c_{A_\phi} \ c_{M_3} \ M \ cos \Phi_{1u} \\
- 0.000028 \ A_0 \ c_{A_\phi} \ c_{M_2} \ M \ cos \Phi_{2b} + 0.00324 \ A_0 \ c_{A_\phi} \ c_{M_2} \ M \ cos \Phi_{2c} \\
+ 2.13 \times 10^{-6} \ A_0 \ c_{A_\phi} \ c_{M_2} \ M \ cos \Phi_{2t} - 0.0000352 \ A_0 \ c_{A_\phi} \ c_{M_3} \ M \ cos \Phi_{2b} \\
- 0.0000355 \ A_0 \ c_{A_\phi} \ c_{M_3} \ M \ cos \Phi_{3b} + 3.57 \times 10^{-6} \ A_0 \ c_{A_\phi} \ c_{M_3} \ M \ cos \Phi_{3b} \\
+ 0.0000444 \ A_0^2 \ c_{A_\phi} \ c_{A_\phi} \ M \ cos \Phi_{4b} - 2.55 \times 10^{-6} \ A_0^2 \ c_{A_\phi} \ c_{A_\phi} \ M \ cos \Phi_{4b} \\
- 2.58 \times 10^{-6} \ A_0^2 \ c_{A_\phi} \ c_{A_\phi} \ M \ cos \Phi_{4b} . \quad (A.8) \]

For Gauginos we found the followings

\[ M_1 = 0.432 \ c_{M_1} \ M, \ M_2 = 0.833 \ c_{M_2} \ M, \ M_3 = 2.51 \ c_{M_3} \ M . \quad (A.9) \]

Similarly, for trilinear terms we have

\[ A_t = -0.00198 \ A_0 \ c_{A_\phi} + 3.81 \times 10^{-6} \ A_0 \ c_{A_\phi} + 0.27 \ A_0 \ c_{A_\phi} - 0.0303 \ c_{M_1} \ M - 0.231 \ c_{M_2} \ M \\
- 1.55 \ c_{M_3} \ M \quad (A.10) \]

\[ A_b = 0.147 \ A_0 \ c_{A_\phi} - 0.00041 \ A_0 \ c_{A_\phi} - 0.0175 \ A_0 \ c_{A_\phi} - 0.00484 \ c_{M_1} \ M - 0.0675 \ c_{M_2} \ M \\
- 0.372 \ c_{M_3} \quad (A.11) \]

\[ A_t = -0.00101 \ A_0 \ c_{A_\phi} + 0.0989 \ A_0 \ c_{A_\phi} + 0.0000811 \ A_0 \ c_{A_\phi} - 0.0153 \ c_{M_1} \ M - 0.0493 \ c_{M_2} \ M \\
+ 0.00131 \ c_{M_3} \ M \quad (A.12) \]

Our expression for \( B \) is

\[ B = B_0 - 0.0095 \ A_0 \ c_{A_\phi} - 0.00276 \ A_0 \ c_{A_\phi} - 0.354 \ A_0 \ c_{A_\phi} - 0.0301 \ c_{M_1} \ M - 0.371 \ c_{M_2} \ M \\
+ 0.518 \ c_{M_3} \ M \quad (A.13) \]

and, lastly, for the \( \mu \) parameter our result reads

\[ \mu = 0.995 \mu_0 . \quad (A.14) \]

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