Charm-quark pole mass from HERA Combined and LHCb charm production data

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Abstract

Because of color confinement hypothesis, which states that colored objects are always confined to color singlet states, there are several definitions for charm-quark mass, which one of the most popular definition is charm-quark pole mass $m_c^{\text{pole}}$. This Quantum Chromo Dynamics (QCD) analysis attempts to extract charm quark-pole mass from H1-ZEUS combined (H1Z), LHCb and H1Z+LHCb charm production cross section data and then investigate its pure impact on proton parton distribution functions (PDFs). To reach this goal we makes several fits based on the very recently updated fixed flavor number scheme from ABM (FF ABM) at the next-to-leading order (NLO). We show that the pure contribution of charm-quark pole mass in the improvement of the fit quality, when it is considered as an extra fit parameter in perturbative Quantum Chromo Dynamics (pQCD) level is $\sim 0\%$, $\sim 25\%$ and $\sim 21\%$ corresponding to H1-ZEUS combined, LHCb and H1Z+LHCb charm production cross section data, respectively.

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I. INTRODUCTION

The HERA machine as a powerful electron-proton collider study simultaneously neutral current (NC) and charged current (CC) $e^\pm p$ collisions and their electroweak unification process. On the other hand, the LHCb detector study the charm-quark production at the Large Hadron Collider (LHC) in $pp$ reactions at $\sqrt{s} = 7$ TeV.

At the pQCD level the internal structure of the proton is probed by the experiments known as deep inelastic scattering (DIS) measurements. The DIS experiments serve the central data to determine the nucleon structure in terms of parton distribution functions. Contributions from all active quarks and anti quarks are included by the inclusive neutral NC and CC deep inelastic $e^\pm p$ scattering cross sections.

At the DIS measurements level the ratio of the virtual photon couplings corresponding to a heavy quark $Q_h$, $h = b, c$ approximated by $f(h) \sim \frac{Q_h^2}{\sum Q_q^2}$, where $Q_h = \frac{1}{3}$, $\frac{2}{3}$ are the $b$-quark and $c$-quark electric charges, respectively and $Q_q$, with $q = u, d, s, c, b$, represent the kinematically accessible quark flavors [1]. Accordingly for $c$-quark we have $f(c) \sim \frac{Q_c^2}{\sum Q_q^2} = \frac{4}{11} \approx 0.36$ and this means, more than one third or approximately 36 percent of the cross sections are coming from charm quarks in the final state [2]. This significant contribution of $c$-quark at the HERA events is our main motivation to determine charm-quark pole mass based on the H1-ZEUS combined [3], LHCb [4] and H1Z+LHCb charm production cross section data and then investigate the pure impact of this QCD parameter on the uncertainty bands of gluon and some of its ratios. The impact of H1-ZEUS charm production combined data only on PDFs and determination of the strong coupling, $\alpha_s(M_Z^2)$ has recently been published in Ref. [2].

In this QCD analysis we make several fits to exactly separate the role and influence of charm-quark pole mass from other phenomenological parameters on the uncertainty bands of PDFs and fit-quality, based on H1-ZEUS combined (H1Z), LHCb and H1Z+LHCb charm production cross section data within the pQCD framework.

From the pQCD point of view, DIS measurements depend on the various phenomenological input data and knowledge of the PDFs [2-9]. For this reason in addition of H1-ZEUS charm production HERA combined data the full five LHCb charm production cross section data sets at $\sqrt{s} = 7$ TeV are included in this QCD analysis to show the sensitivity of the gluon distribution and some of its ratios at low values of $x$, the fraction of proton momentum
carried by a parton. Since this kinematic range does not currently covered by other data set, inclusion of the LHCb charm production data at $\sqrt{s} = 7$ TeV dramatically improve the gluon distribution uncertainties and fit quality [10–16].

The outline of this paper is as follows. In Sec. (II) we describe the theoretical framework of our study and discuss about the inclusive differential cross section of charm-quark production. We introduce the charm-quark mass in the pQCD approach in Sec. (III). In Sec. (IV), data set and methodology is discussed. We present our results in Sec. (V) and then we conclude with a summary in Sec. (VI).

II. THEORY OF CHARM-QUARK PRODUCTION

The NC and CC deep inelastic $e^\pm p$ scattering at the centre-of-mass energies up to $\sqrt{s} \approx 320$ GeV are expressed in terms of the generalized structure functions:

$$\sigma^\pm_{r,NC} = \frac{d^2\sigma^\pm_{NC}}{dx dQ^2} \frac{Q^4x}{2\pi\alpha^2Y_+} = \tilde{F}_2 \mp \frac{Y_-}{Y_+} x \tilde{F}_3 - \frac{y^2}{Y_+} \tilde{F}_L ,$$

(1)

$$\sigma^\pm_{r,CC} = \frac{2\pi x}{G_F^2} \left[ \frac{M_Z^2 + Q^2}{M_W^2} \right]^2 \frac{d^2\sigma^\pm_{CC}}{dx dQ^2} = \frac{Y_+}{2} W^+_2 \mp \frac{Y_-}{2} x W^+_3 - \frac{y^2}{2} W^+_L ,$$

(2)

where $Y_\pm = 1 \pm (1 - y)^2$, $\alpha$ is the fine-structure constant which is defined at zero momentum transfer and $G_F$ is the Fermi constant [17].

Similarly, the inclusive differential cross section of charm production in DIS is expressed in terms of the dimensionless reduced cross sections:

$$\sigma^{CC}_{red} = \frac{d\sigma^{CC}(e^\pm p)}{dx dQ^2} \frac{Q^4x}{2\pi\alpha^2Y_+} = F^C_2 \mp \frac{Y_-}{Y_+} x F^C_3 - \frac{y^2}{Y_+} F^C_L .$$

(3)

At the low-value regions of $Q^2$ where $Q^2 \ll M_Z^2$, the parity-violating structure function, $x F_3$ is neglected and the reduced differential cross section of charm production may be expressed by

$$\sigma^{CC}_{red} = \frac{d\sigma^{CC}(e^\pm p)}{dx dQ^2} \frac{Q^4x}{2\pi\alpha^2Y_+} = F^C_2 \mp \frac{y^2}{Y_+} F^C_L .$$

(4)

A detailed study of the inclusive deep inelastic $e^\pm p$ scattering cross sections, charm production reduced cross section, generalized structure functions for NC and CC deep inelastic $e^\pm p$ scattering and other related parameters can be found in Refs. [2, 17].
III. CHARM QUARK MASS IN THE PQCD APPROACH

From the theoretical point of view, the reduced charm production cross section is obtained by convolution of matrix elements with PDFs. On the other hand, PDFs are extracted from inclusive deep inelastic $e^\pm p$ scattering cross sections. Accordingly, both matrix elements and proton PDFs strictly depend on the $c$-quark mass $[18, 19]$.

The non-observation of free quarks is explained by the hypothesis of color confinement, which states that colored objects are always confined to color singlet states and that no objects with non-zero color charge can propagate as free particles $[20–23]$. Accordingly, different definitions of the charm quark mass $m_c$ such as the pole mass $m_c^{\text{pole}}$ and $\overline{\text{MS}}$ mass $m_c(\mu_r)$ are available. In pQCD the pole mass is defined as the mass at the position of the pole in the $c$-quark propagator and $\overline{\text{MS}}$ mass is charm mass which is evaluated at the renormalization scale $\mu_r$. Each definition has own advantages and disadvantages. The pole mass $m_c^{\text{pole}}$ is a gauge invariant quantity and is well defined in any finite order of pQCD but has an intrinsic uncertainty of order $\frac{\Lambda_{\text{QCD}}}{m_c}$, where $\Lambda_{\text{QCD}} \sim 0.25$ MeV is the QCD scale $[2]$. The $\overline{\text{MS}}$ mass avoid this problem and its relation with pole mass $m_c^{\text{pole}}$ is given by

$$m_c^{\text{pole}} = m_c(m_c) \left( 1 + \frac{4\alpha_s(m_c)}{3\pi} \right), \quad (5)$$

where $m_c(m_c)$ is the $\overline{\text{MS}}$ running mass evaluated at the scale $\mu_r = m_c$.

IV. DATA SET AND METHODOLOGY

In this QCD analysis, we use three different data sets: the HERA run I and II combined NC and CC deep $e^\pm p$ scattering cross sections $[24]$, H1-ZEUS combined charm production reduced cross section data $[3]$ and the full five LHCb charm production cross section data sets at $\sqrt{s} = 7$ TeV $[4]$.

To determine and study the pure impact of charm-quark pole mass on PDFs and fit-quality we make six different fits in two separate steps as follow:

At the first step we fixed the charm quark mass to $m_c = 1.257$ GeV and make three different fits with 13 free parameters based on H1-ZEUS combined (H1Z), LHCb and H1Z+LHCb charm production cross section data to investigate the pure impact of these data sets on the full HERA run I and II combined NC and CC deep $e^\pm p$ scattering data.
At the second step we consider the charm-quark mass as an extra free parameter and refit the above fit procedures but this time with 14 free parameters to determine both charm-quark pole mass and pure impact of c-mass on the PDFs and fit-quality at the pQCD level.

Depending on the initial set-up of a QCD analysis, different approaches can be taken for treatment of the heavy quarks contribution [25–33].

In this QCD analysis to include the charm-quark contribution we use very recently ABM variant of the FFN scheme (FF ABM) as implemented in the xFitter package as a powerful QCD framework [34–38].

According to our QCD analysis, FF ABM scheme provides most reliable results and best fit-quality in the phase space of LHCb charm production data. In the fixed flavor number scheme heavy quarks are considered as massive at all scales but they do not considered as partons within the proton. The number of active flavors is fixed to three for c-quark and is fixed to four for b-quark. Updated variants of FFN scheme govern both charm-quark pole mass and $\overline{\text{MS}}$ running mass, however the calculations of this QCD analysis use FF ABM variant and is developed based on the charm-quark pole mass $m_{c}^{\text{pole}}$.

This QCD analysis is based on the HERAPDF functional form at the initial scale of the QCD evolution $Q_{0}^{2} = 1.9$ GeV$^2$ as:

$$xf(x) = Ax^{B}(1-x)^{C}(1+Dx+Ex^{2})$$

with 13 central free parameters and $m_{c}$ as an one extra free parameter. A detailed review of HERAPDF functional form and its related parameters has been reported in Ref. [17].

The initial set up of this QCD analysis is based on the following additional parameters: The strong coupling constant is fixed to $\alpha_{s}^{\text{NLO}}(M_{Z}^{2}) = 0.1176$ [17], the strangeness suppression factor is fixed to $f_{s} = 0.31$ [24], the initial value of charm-quark pole mass is set to $m_{c}^{\text{pole}} = 1.257$ GeV and then varied in steps of 0.001 [34] and finally the theory type based on the DGLAP collinear evolution mode and set the evolution starting scale to $Q_{0}^{2} = 1.9$ GeV$^2$ [39, 40].

V. RESULTS

In Table [1] we labeled the H1-ZEUS combined charm data, LHCb charm production data and H1-ZEUS combined charm data plus LHCb charm production data with BH1Z,
BLHCb and BH1Z+BLHCb, respectively which the letter “B” stands the charm-quark mass is fixed to $m_c = 1.257$ GeV. Table I shows the experimental data with $\chi^2_{\text{total}}$ and partial $\chi^2$ per data points for each experiment corresponding to three different BH1Z, BLHCb and BH1Z+BLHCb data sets.

| Scheme | charm-quark mass is fixed to $m_c = 1.257$ GeV |
|--------|---------------------------------------------|
| Experiment | BH1Z | BLHCb | BH1Z+BLHCb |
| HERA I+II CC $e^+p$ [24] | 57 / 39 | 63 / 39 | 63 / 39 |
| HERA I+II CC $e^-p$ [24] | 51 / 42 | 50 / 42 | 50 / 42 |
| HERA I+II NC $e^-p$ [24] | 218 / 159 | 223 / 159 | 224 / 159 |
| HERA I+II NC $e^+p$ 460 [24] | 214 / 204 | 211 / 204 | 211 / 204 |
| HERA I+II NC $e^+p$ 575 [24] | 213 / 254 | 210 / 254 | 211 / 254 |
| HERA I+II NC $e^+p$ 820 [24] | 63 / 70 | 61 / 70 | 62 / 70 |
| HERA I+II NC $e^+p$ 920 [24] | 426 / 377 | 423 / 377 | 426 / 377 |
| Charm H1-ZEUS [3] | 40 / 47 | - | 39 / 47 |
| LHCb 7TeV Dzero pT-y cross section [4] | - | 393 / 38 | 391 / 38 |
| LHCb 7TeV Dch pT-y cross section [4] | - | 117 / 37 | 118 / 37 |
| LHCb 7TeV Dstar pT-y cross section [4] | - | 86 / 31 | 86 / 31 |
| LHCb 7TeV Ds pT-y cross section [4] | - | 26 / 28 | 26 / 28 |
| LHCb 7TeV Lambdac pT cross section [4] | - | 5.1 / 6 | 5.2 / 6 |
| Correlated $\chi^2$ | 129 | 159 | 167 |

$\chi^2_{\text{total}}$ and $\chi^2$ per data points for each experiment corresponding to three different BH1Z, BLHCb and BH1Z+BLHCb data sets.

Table I: Experiments with $\chi^2_{\text{total}}$ and partial $\chi^2$ per data points for each experiment corresponding to three different BH1Z, BLHCb and BH1Z+BLHCb data sets.

In Table II we compare the pure impact of the BH1Z, BLHCb and BH1Z+BLHCb data on the fit quality, when charm quark mass is fixed to $m_c = 1.257$ GeV. As we can see from numerical results of Table II the best fit quality is corresponding to BH1Z combined charm production data.

In Table III we present NLO numerical values of 13 free central parameters and their uncertainties for the $xu_e$, $xd_u$, sea and gluon PDFs at the input scale of $Q_0^2 = 1.9$ GeV$^2$
charm-quark mass is fixed to $m_c = 1.257$ GeV

| Experiment   | $\chi^2_{\text{Total}}/\text{dof}$ | fit quality |
|--------------|------------------------------------|-------------|
| BH1Z         | 1410/1179                          | 1.19        |
| BLHCb        | 2029/1272                          | 1.59        |
| BH1Z+BLHCb   | 2078/1319                          | 1.57        |

Table II: Comparison the pure impact of the BH1Z, BLHCb and BH1Z+BLHCb data on the fit quality, when charm-quark mass is fixed to $m_c = 1.257$ GeV.

for three different BH1Z, BLHCb and BH1Z+BLHCb data sets, which as we previously mentioned the letter “B” stands for the fixed charm-quark mass scheme.

According to numerical values of Table III and three different fit qualities from Table III we expect to see dramatically impact of the BH1Z, BLHCb and BH1Z+BLHCb data on the shape of the gluon distribution and some of its ratios.

In Fig. 1 we show the gluon PDFs (four upper), the partial of gluon PDFs (four middle) and the partial ratio of gluon distributions over Σ-PDFs (four lower) as extracted for three different BH1Z (blue), BLHCb (yellow) and BH1Z+BLHCb (green) data sets. As we can see from Fig. 1 the best improvement of uncertainty error bands is corresponding to BH1Z+BLHCb data with green color. Also, the pure impact of BLHCb charm production data in improvement of the gluon distribution and some of its ratios (yellow color) is better than the pure impact of BH1Z combined charm data (blue color).
charm-quark mass is fixed to $m_c = 1.257$ GeV

| Parameter | BH1Z      | BLHCb     | BH1Z+BLHCb |
|-----------|-----------|-----------|------------|
| $B_{u_v}$ | $0.870 \pm 0.039$ | $0.840 \pm 0.027$ | $0.823 \pm 0.026$ |
| $C_{u_v}$ | $4.383 \pm 0.081$ | $4.405 \pm 0.087$ | $4.413 \pm 0.085$ |
| $E_{u_v}$ | $9.1 \pm 1.5$ | $9.3 \pm 1.3$ | $9.9 \pm 1.3$ |
| $B_{d_v}$ | $1.06 \pm 0.10$ | $0.996 \pm 0.080$ | $0.969 \pm 0.078$ |
| $C_{d_v}$ | $4.51 \pm 0.38$ | $4.70 \pm 0.37$ | $4.63 \pm 0.36$ |
| $C_{U}$   | $3.69 \pm 0.54$ | $2.40 \pm 0.32$ | $2.39 \pm 0.32$ |
| $A_{D}$   | $0.1764 \pm 0.0091$ | $0.1642 \pm 0.0072$ | $0.1653 \pm 0.0072$ |
| $B_{D}$   | $-0.1723 \pm 0.0065$ | $-0.1795 \pm 0.0056$ | $-0.1784 \pm 0.0055$ |
| $C_{D}$   | $5.8 \pm 1.5$ | $5.8 \pm 1.2$ | $5.8 \pm 1.2$ |
| $B_{g}$   | $0.23 \pm 0.26$ | $-0.171 \pm 0.024$ | $-0.196 \pm 0.021$ |
| $C_{g}$   | $4.96 \pm 0.97$ | $3.50 \pm 0.41$ | $3.09 \pm 0.36$ |
| $A'_{g}$  | $2.6 \pm 1.4$ | $1.98 \pm 0.26$ | $1.68 \pm 0.20$ |
| $B'_{g}$  | $0.091 \pm 0.061$ | $-0.134 \pm 0.024$ | $-0.161 \pm 0.021$ |
| $\alpha_s(M_Z^2)$ | $0.1176$ | $0.1176$ | $0.1176$ |
| $m_c$     | $1.257$ | $1.257$ | $1.257$ |

Table III: The next-to-leading order numerical values of 13 fit parameters and their uncertainties for the $xu_v$, $xd_v$, $x\bar{u}$, $x\bar{d}$, $x\bar{s}$ and $xg$ PDFs at the initial scale of $Q_0^2 = 1.9$ GeV$^2$ when charm-quark mass is fixed to $m_c = 1.257$ GeV.
Figure 1: The gluon PDFs (four upper), the partial of gluon PDFs (four middle) and the partial ratio of gluon distributions over $\Sigma$-PDFs (four lower) as extracted for three different BH1Z (blue), BLHCb (yellow) and BH1Z+BLHCb (green) data sets.
Now in the second step we consider the charm-quark mass as an extra free parameter and refit our previous fit procedures but this time with 14 free parameters to determine both charm-quark mass and pure impact of $c$-mass on the shape of the PDFs and fit quality at the pQCD level.

| Scheme                          | $m_c$ is taken as an extra free parameter |
|--------------------------------|-----------------------------------------|
| Experiment                     | H1Z     | LHCb    | H1Z+LHCb |
| HERA I+II CC $e^+p$ [24]       | 56 / 39 | 57 / 39 | 57 / 39   |
| HERA I+II CC $e^-p$ [24]       | 51 / 42 | 51 / 42 | 51 / 42   |
| HERA I+II NC $e^-p$ [24]       | 218 / 159 | 223 / 159 | 225 / 159 |
| HERA I+II NC $e^+p$ 460 [24]  | 214 / 204 | 209 / 204 | 208 / 204 |
| HERA I+II NC $e^+p$ 575 [24]  | 213 / 254 | 211 / 254 | 211 / 254 |
| HERA I+II NC $e^+p$ 820 [24]  | 63 / 70  | 63 / 70  | 63 / 70   |
| HERA I+II NC $e^+p$ 920 [24]  | 424 / 377 | 418 / 377 | 419 / 377 |
| Charm H1-ZEUS [3]              | 43 / 47  | -       | 61 / 47   |
| LHCb 7TeV Dzero pT-y cross section [4] | - | 108 / 38 | 119 / 38 |
| LHCb 7TeV Dch pT-y cross section [4] | - | 69 / 37  | 71 / 37   |
| LHCb 7TeV Dstar pT-y cross section [4] | - | 50 / 31  | 52 / 31   |
| LHCb 7TeV Ds pT-y cross section [4] | - | 30 / 28  | 28 / 28   |
| LHCb 7TeV Lambdac pT cross section [4] | - | 7.0 / 6  | 6.6 / 6   |
| Correlated $\chi^2$           | 127      | 215      | 226      |
| $\chi^2_{total \atop dof}$    | 1409     | 1710     | 1798     |
|                                | 1178     | 1271     | 1318     |

Table IV: Experiments with $\chi^2_{total \atop dof}$ and partial $\chi^2$ per data points for each experiment corresponding to three different H1Z, LHCb and H1Z+LHCb data sets.

In the Table IV which we consider the charm-quark mass as an extra free parameter the H1Z, LHCb and H1Z+LHCb, refer to the H1-ZEUS combined charm data, LHCb charm production data and H1-ZEUS combined charm data plus LHCb charm production data, respectively. Table IV shows experimental data with $\chi^2_{total \atop dof}$ and partial $\chi^2$ per data points for each experiment corresponding to three different H1Z, LHCb and H1Z+LHCb data sets.

In Table IV we compare the pure impact of the H1Z, LHCb and H1Z+LHCb charm
production data on the fit-quality, when the charm-quark mass is taken as an extra free fit parameter. As we can see from the numerical results of Table V, the best fit quality is corresponding to H1Z combined charm production data.

| Experiment     | $\chi^2_{\text{total}}$ | fit quality |
|----------------|--------------------------|-------------|
| H1Z            | 1409/1178                | 1.19        |
| LHCb           | 1710/1271                | 1.34        |
| H1Z+LHCb       | 1798/1318                | 1.36        |

Table V: Comparison the impact of the H1Z, LHCb and H1Z+LHCb data on the fit quality, when the charm-quark mass is taken as an extra free fit parameter.

In Fig. 2 we show the gluon PDFs (four upper), the partial of gluon PDFs (four middle) and the partial ratio of gluon distributions over $\Sigma$-PDFs (four lower) as extracted for three different H1Z (blue), LHCb (red) and H1Z+LHCb (yellow) data sets. As we can see from Fig. 2, the best improvement of the uncertainty bands of PDFs is corresponding to H1Z+LHCb data with yellow color. Also the pure impact of LHCb charm production data in the improvement of the gluon PDFs and some of its ratios (red color) is better than the impact of H1Z combined charm production data (blue color).

Now if we compare the numerical values of Tables II and V and PDFs shape in Figs. 1 and 2, we may conclude the following results:

- Although the PDFs shape is strongly sensitive to H1-ZEUS combined data but the quality of the fit does not change without and with considering the charm quark mass as an extra free parameter, when we use H1-ZEUS charm production combined data only.

- Both the gluon distribution shape and fit-quality are sensitive to the LHCb charm production data, so that according to the relative change of $\chi^2$ function which is defined by $\frac{\chi^2_{\text{final}}-\chi^2_{\text{initial}}}{\chi^2_{\text{initial}}}$, we obtain up to $\frac{1.59-1.34}{1.59} \sim 16 \%$ relative improvement in the fit quality, without and with charm-quark mass is considered as an extra free parameter along with the LHCb charm production data.

- Considering the H1-ZEUS combined charm data plus LHCb charm production data
Figure 2: The gluon PDFs (four upper), the partial of gluon PDFs (four middle) and the partial ratio of gluon distributions over Σ-PDFs (four lower) as extracted for three different H1Z (blue), LHCb (red) and H1Z+LHCb (yellow) data sets.

(H1Z+LHCb), the relative quality of the fit improves up to 1.57−1.36 \sim 13 \% without and with the c-quark mass is taken as an extra free parameter.

Table VI uses the numerical results from Tables II and V to present the pure impact of charm-quark pole mass on the fit quality.

| Experiment       | data only | data and $m_c$ | pure impact of charm-quark pole mass on the fit-quality |
|------------------|-----------|----------------|--------------------------------------------------------|
| H1Z              | 1.19      | 1.19           | $|1.19 - 1.19| \sim 0 \%$                                           |
| LHCb             | 1.59      | 1.34           | $|1.59 - 1.34| \sim 25 \%$                                          |
| H1Z + LHCb       | 1.57      | 1.36           | $|1.57 - 1.36| \sim 21 \%$                                          |

Table VI: pure contribution of charm-quark mass in improvement of the fit-quality.

Determination of the 14 free fit parameters, including charm-quark mass as an extra free
parameter are presented in Table VII. As can be seen from Table VII, we determine the charm-quark pole mass $m_c^{\text{pole}}$ using H1Z, LHCb and H1Z+LHCb charm production data. The best uncertainty improvement from the central value of $c$-quark mass is corresponding to use of H1Z+LHCb charm production data, $m_c = 1.691 \pm 0.028$.

The comparison of these results with the measurements from the PDG world average [41] shows a very good agreement with the expected charm-quark pole mass.

| Parameter | H1Z     | LHCb    | H1Z+LHCb |
|-----------|---------|---------|----------|
| $B_{u,v}$ | 0.869 ± 0.038 | 0.860 ± 0.028 | 0.841 ± 0.027 |
| $C_{u,v}$ | 4.378 ± 0.082 | 4.39 ± 0.10 | 4.433 ± 0.099 |
| $E_{u,v}$ | 9.0 ± 1.5 | 8.4 ± 1.3 | 9.2 ± 1.4 |
| $B_{d,c}$ | 1.06 ± 0.10 | 1.012 ± 0.089 | 0.987 ± 0.085 |
| $C_{d,c}$ | 4.51 ± 0.38 | 4.79 ± 0.39 | 4.73 ± 0.39 |
| $C_{\bar{U}}$ | 3.54 ± 0.51 | 2.35 ± 0.30 | 2.26 ± 0.28 |
| $A_D$     | 0.1809 ± 0.0094 | 0.1936 ± 0.0079 | 0.1907 ± 0.0075 |
| $B_D$     | $-0.1691 \pm 0.0065$ | $-0.1589 \pm 0.0049$ | $-0.1600 \pm 0.0047$ |
| $C_D$     | 5.7 ± 1.5 | 5.4 ± 1.1 | 5.4 ± 1.1 |
| $B_g$     | 0.24 ± 0.25 | 0.21 ± 0.13 | 0.162 ± 0.097 |
| $C_g$     | 5.4 ± 1.0 | 8.1 ± 1.1 | 6.96 ± 0.81 |
| $A'_g$    | 2.9 ± 1.5 | 5.8 ± 1.7 | 4.21 ± 0.95 |
| $B'_g$    | 0.104 ± 0.066 | 0.162 ± 0.083 | 0.111 ± 0.056 |
| $\alpha_s(M_Z^2)$ | 0.1176 | 0.1176 | 0.1176 |
| $m_c$     | 1.347 ± 0.049 | 1.760 ± 0.030 | 1.691 ± 0.028 |

Table VII: The next-to-leading order numerical values of 14 fit parameters and their uncertainties for the $xu_v$, $xd_v$, $x\bar{u}$, $x\bar{d}$, $x\bar{s}$, $xg$ PDFs and $m_c$ at the initial scale of $Q_0^2 = 1.9$ GeV$^2$, when the charm-quark mass is taken as an extra free parameter.

In Fig. 3, we show some of our QCD fits and illustrations of the consistency of the HERA run I and II combined NC and CC deep $e^\pm p$ scattering cross sections data [24], H1-ZEUS combined charm production reduced cross section data [3] and the LHCb charm production cross section data sets at $\sqrt{s} = 7$ TeV [4] with theoretical predictions as a function of $x$ for
different values of $Q^2$.

The gluon PDFs, the partial of gluon PDFs and the partial ratio of gluon distributions over $\Sigma$-PDFs as a function of $x$ are shown in Fig. 4 for fix and free charm-quark mass. The best improvement in the uncertainty bands is corresponding to H1Z+LHCb charm production data, when charm-quark mass is considered as an extra free parameter.
Figure 3: Some of our QCD fits and illustrations of the consistency of the HERA run I and II combined NC and CC deep $e^\pm p$ scattering cross sections data [24], H1-ZEUS combined charm production reduced cross section data [3] and the LHCb charm production cross section data sets at $\sqrt{s} = 7$ TeV [4] with the theoretical predictions as a function of $x$ for different values of $Q^2$. 

15
Figure 4: The gluon PDFs (four upper), the partial of gluon PDFs (four middle) and the partial ratio of gluon distributions over $\Sigma$-PDFs (four lower) as a function of $x$, without and with the charm-quark mass is taken as an extra free parameter. The best improvement in the uncertainty bands is corresponding to H1Z+LHCb charm production data, when the charm-quark mass is considered as an extra free parameter.
VI. SUMMARY

In this NLO QCD analysis, using three different H1Z, LHCb and H1Z+LHCb charm production data sets, we simultaneously determine PDFs and charm-quark pole mass $m_c^\text{pole}$.

We study the pure impact of the three different H1Z, LHCb and H1Z+LHCb charm production data sets and also pure contribution of charm-quark mass on the uncertainty bands of PDFs and fit-quality in two separate steps with following results:

• The best fit quality is corresponding to H1-ZEUS combined charm data (H1Z), without and with the charm-quark mass contribution.

• According to Figs. 1 and 2 the best improvement of PDFs uncertainties is corresponding to H1Z+LHCb charm production data without (green) and with (yellow) charm-quark mass contribution, respectively.

• According to Table VII, the best uncertainty improvement from the central value of $c$-quark mass is corresponding to use of H1Z+LHCb charm production data, $m_c = 1.691 \pm 0.028$.

• According to the numerical values of Table VI, the pure contribution of the charm-quark mass in the improvement of the fit-quality when it is considered as an extra fit parameter in pQCD level is $\sim 0\%$, $\sim 25\%$ and $\sim 21\%$ corresponding to H1Z, LHCb and H1Z+LHCb charm production data sets, respectively.

• According to numerical results from Table VII up to 16\% relative improvement occur in the fit-quality, without and with charm-quark mass is considered as an extra free parameter along with the LHCb charm production data.

In this next-to-leading order QCD analysis we show the central role of charm-quark mass in the improvement of uncertainty band of gluon distribution and QCD fit-quality, when it is considered as an extra free parameter at the pQCD level.

Standard LHAPDF library files of all fit processes in this QCD analysis are available and can be obtained via e-mail from the author.
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