Reduced Hall carrier density in the overdoped strange metal regime of cuprate superconductors

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Efforts to understand the microscopic origin of superconductivity in the cuprates are dependent on knowledge of the normal state. The Hall number in the low-temperature, high-field limit $n_H(0)$ has a particular importance because, within conventional transport theory, it is simply related to the number of charge carriers, so its evolution with doping gives crucial information about the nature of the charge transport. Here we report a study of the high-field Hall coefficient of the single-layer cuprates Tl$_2$Ba$_2$CuO$_6$$\delta$ (Tl2201) and (Pb/La)-doped Bi$_2$Sr$_2$CuO$_{6+\delta}$ (Bi2201), which shows how $n_H(0)$ evolves in the overdoped—so-called strange metal—regime of cuprates. We find that $n_H(0)$ increases smoothly from $p$ to $1 + p$, where $p$ is the number of holes doped into the parent insulating state, over a wide range of doping. The evolution of $n_H$ correlates with the emergence of the anomalous linear-in-temperature term in the low-temperature in-plane resistivity. The results could suggest that quasiparticle decoherence extends to dopings well beyond the pseudogap regime.

In the search for the microscopic origin of high-temperature superconductivity in the cuprates, much effort has been directed to understanding their normal-state properties and how these are linked to superconductivity as a function of temperature $T$ and doping. The overdoped regime exhibits pseudogap phenomena as well as tendencies towards several types of order, or incipient order, including charge and spin density waves (CDW/SDW)1. In the regime where CDW order has been detected, at low temperature and high fields, the Hall number, $n_H$, changes sign, suggesting some form of Fermi surface reconstruction1. In some overdoped cuprates, beyond the doping level $p^*$ where the pseudogap disappears there do not appear to be any competing orders, so these materials potentially provide a simpler starting point from which to understand the emergence of high-temperature superconductivity.

In the far-overdoped regime, the normal-state behaviour of Tl$_2$Ba$_2$CuO$_6$$\delta$ (Tl2201) resembles, in many aspects, that of a conventional Fermi liquid, with coherent quasiparticles around the entire Fermi surface5-8, whose shape is found to be well described by conventional density functional theory9, albeit with a large (factor 3) renormalization in the effective mass that derives from a narrowing of the band5,6. One aspect of the overdoped regime that contrasts with that of conventional metals is the evolution of the in-plane resistivity $\rho_{xx}(T)$ with doping. $\rho_{xx}(T)$ evolves smoothly from linear, close to optimal doping, to quadratic in the far-overdoped regime10-12, leading to this being called the ‘strange metal’ regime11,12. The close resemblance of the cuprate phase diagram to that of other material families, such as heavy fermions and iron pnictides, where superconductivity occurs close to an antiferromagnetic quantum critical point (QCP)13, has led to speculation that the linear resistivity close to optimal doping in the cuprates may be a marker for a quantum critical transition to a hidden ordered phase14. The idea is that quantum fluctuations of the hidden phase provide the scattering mechanism that gives rise to both the linear resistivity and also the pairing mechanism for high-temperature superconductivity. Possible candidates for this order are the pseudogap or CDW, but the thermodynamic evidence for a true phase transition of any type at finite temperature is weak, and there is little evidence that either of these have a quantum critical end point15,16. For example, there are no observable anomalies in the specific heat as the material is cooled into the pseudogap or CDW regimes. On the other hand, a number of recent experiments might support the existence of a QCP close to optimal doping. First, quantum oscillations in YBa$_2$Cu$_3$O$_{6+x}$ (Y123) show that the quasiparticle mass $m^*$ increases with doping, beyond $p = 0.12$, with $1/m^*$ extrapolating to zero at $p \approx 0.18$ (ref. 17). Second, measurements suggest that, in Y123, the Hall number in the low-temperature, high-field limit, $n_H(0)$, undergoes a rapid increase from $p$ to $1 + p$ over a narrow doping range $0.16 < p < 0.20$ (ref. 18), revealing possible critical behaviour near the doping where the pseudogap is believed to end in this material.

Whether such features are really caused by a QCP and whether this QCP is relevant to superconductivity requires further study. Recently, a high-pressure study of YBa$_2$Cu$_3$O$_{6+x}$ (Y124)19 showed that, as the maximum superconducting critical temperature ($T_c$) is approached by pressure tuning (rather than by chemical doping), $m^*$ actually decreases. This suggests that, although the mass increase in Y123 near optimal doping may be linked to quantum CDW fluctuations, these fluctuations may not be the primary cause of the high $T_c$.

Interpreting $n_H(0)$ as a planar hole density may have implications in some of the systems studied so far. In Y123, the quasi-one-dimensional CuO chains layer increases the $b$-axis with doping.

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Fig. 1 | Field dependence of the Hall coefficient for Tl2201. The left panels show the raw data at different temperatures and the right panels show the same data with the field scaled by the estimated value of the resistivity $\rho_{xx}(T)$ at $H=0$. The doping value ($p$) is indicated in each panel. The dashed lines in the right panels show Hall coefficient ($R_H$) values corresponding to $1/n_e$, where $n_e=1+p$ or $n_e=p$. 
conductivity but does not contribute to the Hall conductivity ($\sigma_{\text{Hall}}$), thus increasing the measured $n_{\text{Hall}}(0)$ over that expected from the CuO planes alone (Supplementary Information). In La$_{2-x}$Sr$_x$CuO$_4$ (LSCO), it is found that $n_{\text{Hall}}(0)$ is nearly zero for $p < 0.08$ (refs. 11,12). At higher doping, $n_{\text{Hall}}$ increases well above $1+p$ (refs. 12,25), probably because the Fermi surface develops electron-like curvature for doping close to $p \approx 0.2$, where there is believed to be a Lifshitz transition32. In Nd-doped LSCO, the rise in $n_{\text{Hall}}$ with $p$ is sharper than in LSCO24, but again the interpretation may also be complicated by a change in Fermi-surface curvature25. A detailed discussion of these issues is provided in the Supplementary Information.

We have studied the evolution of $n_{\text{Hall}}(0)$ in two cuprate families, Tl2201 and Bi$_2$Sr$_2$CaCu$_2$O$_8$ (Bi2201), which have simple single-band Fermi surfaces and which may be overdoped to the edge of the superconducting dome or beyond without suffering Lifshitz transitions26–28. Single crystals spanning a wide range of doping from slightly underdoped to strongly overdoped were used in this study. We find that $n_{\text{Hall}}(0)$ evolves smoothly as a function of $p$ right across the overdoped regime so that $n_{\text{Hall}}(0)$ does not reach the value $1+p$ until close to the edge of the superconducting regime (note that our doping scales differ from some previous works, as described in the Supplementary Information). The behaviour correlates well with the evolution of the linear component of $\rho_{xx}(T)$, suggesting that the two have a common origin. Moreover, there does not appear to be a simple correlation between the evolution of $n_{\text{Hall}}(0)$ with $p$ and the closing of the pseudogap as previously conjectured19,24.

Tl2201 has unique properties for this study. It has a quasi-two-dimensional bandstructure with a single CuO$_2$ layer giving rise to one band crossing the Fermi level. Its maximum $T_c$ is 94 K and it may be sufficiently overdoped that it becomes non-superconducting yet remains electronically sufficiently clean for quantum oscillations to be observed4–6. For the overdoped compositions with $T_c \lesssim 26$ K, the Fermi surface geometry, scattering rate anisotropy and temperature dependence have all been accurately determined by quantum oscillation, angle-dependent magnetoresistance and angle-resolved photoemission (ARPES) measurements4–8,27,28.

Bi2201 is another single-layered cuprate with a substantially reduced maximum $T_c (=34$ K) and upper critical field ($H_{\text{c1}}$), allowing superconductivity to be suppressed over a wider range of field and temperature space and thereby reducing uncertainty in $n_{\text{Hall}}(0)$. Although quantum oscillations have not been observed in Bi2201, probably because it is more disordered than Tl2201, its electronic structure has nonetheless been well characterized via ARPES and scanning tunnelling microscopy50–52.

Figure 1 shows the field and temperature dependence of the Hall coefficient $R_{\text{Hall}}$ for five representative Tl2201 samples. The evolution of $R_{\text{Hall}}$ as a function of field and temperature at fixed doping can be understood, to some extent, in the overdoped regime using conventional Boltzmann transport theory. We have calculated the expected field and temperature dependence of $R_{\text{Hall}}$ for Tl2201 using the known Fermi surface geometry as well as the anisotropy and $T$-dependence of the scattering rate determined independently from $c$-axis magnetoresistance measurements50–52 (Supplementary Information). At low field, $R_{\text{Hall}}$ is enhanced with respect to $1/n_e$ (where $n_e$ is the carrier density determined by the Fermi surface volume and $e$ the electron charge) due to anisotropy in the Fermi velocity and scattering rate. At high field, this anisotropy is averaged out as the electrons complete increasing fractions of their cyclotron orbits before being scattered and $R_{\text{Hall}}$ consequently tends toward $1/n_e$ (Supplementary Fig. 7). The low field enhancement in $R_{\text{Hall}}$ is reduced at low temperature as scattering becomes dominated by isotropic impurity scattering and this, together with the smaller scattering rate, means that $R_{\text{Hall}}$ approaches $1/n_e$ at lower fields. Higher impurity scattering increases the field scale at which $R_{\text{Hall}}$ approaches its infinite field value, but it also considerably diminishes the enhancement of $R_{\text{Hall}}$ over $1/n_e$, so $R_{\text{Hall}} \approx 1/n_e$ at relatively low fields.

$R_{\text{Hall}}(H, T)$ for the most overdoped sample of Tl2201 in Fig. 1, with $p = 0.27$, follows the calculated behaviour well. $R_{\text{Hall}}$ at high temperature (120 K) decreases slowly with increasing field (~10% in 60 T), but, as the temperature is lowered, this field dependence becomes weaker and for $T \lesssim 10$ K it is essentially constant once superconductivity is suppressed ($\mu_e H > 20$ T at $T = 4.2$ K). The limiting value of $R_{\text{Hall}}$ at this doping is close to that expected for the Hall number $n_e \approx 1 + p$. This limiting behaviour is made clearer in the right-hand panels of Fig. 1, where $R_{\text{Hall}}$ is replotted against $H/\rho_{xx}^0$, which is proportional to $\omega_c$ and reflects the fraction of a cyclotron orbit traversed by an electron before it scatters (here, $\rho_{xx}^0$ is the extrapolated zero-field resistivity).
For the $p = 0.26$ sample, at elevated temperatures the $H$ dependence of $R_{ii}$ becomes larger, but again saturates in the high $H/T$ limit at a value consistent with $n_{ii} \approx 1 + p$. For $p = 0.22$, the irreversibility field has increased substantially. Nevertheless, 60 T still appears to be sufficient to reach the limiting value of $R_{ii}$, although, in this case, it is found to be notably higher than that expected from $n_{ii} \approx 1 + p$. Indeed, for the optimally doped sample ($p = 0.19$) and the slightly underdoped sample ($p = 0.14$), the values of $R_{ii}$ at the highest field and lowest temperature correspond more closely to $n_{ii} \approx p$ than to $n_{ii} \approx 1 + p$. For these higher $T_c$ samples, the temperature dependence of $R_{ii}$ is stronger than calculated from the estimated anisotropic scattering, although $R_{ii}$ still decreases with increasing $H$, as expected.

For Bi2201, the field dependence of $R_{ii}$ above $H_{c1}$ is much weaker than in Tl2201 (Supplementary Fig. 3), consistent with a much higher isotropic elastic scattering rate (residual resistivities are higher in Tl2201 (Supplementary Fig. 3), consistent with a much higher isotropic elastic scattering rate (residual resistivities are higher in Tl2201). In Bi2201, $n_{ii}(H)$ at fixed high fields for the different dopings are shown in Fig. 2 for both Tl2201 and Bi2201. Extrapolating $n_{ii}(T)$ to $T = 0$ for each composition gives an estimate of $n_{ii}(0)$ whose evolution with doping is plotted in the lower panels of Fig. 3. $n_{ii}(0)$ is found to evolve smoothly as a function of $p$ for both materials. For some doping values, multiple samples were measured and these gave consistent results, giving confidence that the error bars in $n_{ii}$ are accurate. For underdoped Tl2201 $n_{ii}(0) \approx p$, but it then increases over a broad doping range until it reaches the $n_{ii}(0) = 1 + p$ line at approximately $p = 0.25$ ($T_c = 40$ K). In Bi2201, previous measurements have shown that $n_{ii}(0)$ follows a non-monotonic behaviour below optimal doping, possibly due to the presence of a CDW similar to that found in Y123. In the overdoped regime however, $n_{ii}(0)$ in Bi2201 monotonically increases with increasing $p$, suggesting there are no regions of CDW order there. By taking into account the CuO chain conductivity, we show in the Supplementary Information (Supplementary Fig. 9) that the evolution of the planar contribution to $n_{ii}(0)$ in Y123 may show a very similar evolution of $n_{ii}(0)$ with $p$ to that found here for Tl2201 and Bi2201, although further measurements of the resistance anisotropy are needed to confirm this.

One potential interpretation of our results is that the evolution in $n_{ii}(0)$ evidences a slow closing of the pseudogap in the overdoped regime in these materials. Such behaviour has been suggested by a recent phenomenological model based on heterogeneous localization. One of the clearest experimental signatures of the pseudogap is a collapse of the size of the anomaly in the electronic specific heat $\gamma$ at $T_c (\Delta C(T))$ and a decrease in $\gamma(T)$ above $T_c$ (ref. 39). In Tl2201, $\Delta C(T)$ is largest at the lowest doping measured ($p \approx 0.20$), and $\gamma(T)$ is independent of temperature above $T_c$ (ref. 39). For a sample with $T_c = 85$ K, a small downturn in $\gamma(T)$ is evident that suggests $p^* < 0.20$ and so the $p$ to $1 + p$ transition in $n_{ii}(0)$ occurs in a regime where there is no pseudogap. This is consistent with NMR Knight shift measurements, which show that for Tl2201 the susceptibility $\gamma(T)$ is independent of temperature above $T_c$ for $p > 0.21$ (ref. 39).

For a sample with $T_c = 85$ K, a small downturn in $\gamma(T)$ is evident that suggests $p^* < 0.194$ (ref. 39). For Bi2201, analysis of the $\rho_{xx}(T)$ of our samples (Supplementary Information) suggests that $p^* < 0.215$, implying, again, that the transition in $n_{ii}(0)$ occurs at least partially in the region where there is no pseudogap. For Bi2201, other probes suggest a larger value of $p^*$. NMR results give $p^*$ in the range $0.23$–$0.25$ (ref. 40) and ARPES in the range $0.23$–$0.24$ (refs. 36,37; see Supplementary Information for a discussion). Nevertheless, these estimates are still in the range where $n_{ii}(0) < (1 + p)$.

It is possible, in principle, that a pseudogap energy scale may be below the zero-field $T_c$ and this may cause the reduction in $n_{ii}(0)$ we see. Such a pseudogap would not be manifest in $\gamma(T)$ above...
or at $T_c$, but instead there should be an anomaly below $T_c$ (ref. 42). Furthermore, this should be accompanied by an anomalous reduction in the growth of the superfluid density $(1/\rho_s)$ and $H_\text{c2}$ as the temperature is lowered, as both are known to be strongly reduced in the pseudogap regime43. None of these signatures are observed experimentally in Tl22017,15,16,44. Indeed, $1/\rho_s (T=0)$ and $\Delta C(T)$ are both found to be at a maximum at the lower doping measured $(p \approx 0.20)$, which seems to rule out this scenario in Tl2201.

In the cuprates, impurity scattering may be anisotropic45, arising, for example, from a region of the Fermi surface that lies close to a Van Hove singularity (vHs). This could reduce $\rho_{\text{in}}$ at low field, but not the high-field limit estimated here. Moreover, in both Tl2201 and Bi2201, all indications suggest that the vHs remains above the Fermi level at all doping levels studied28 (Supplementary Information). With decreasing doping, the Fermi level becomes ever further removed from the vHs. Thus, anisotropic scattering is unlikely to account for the decrease in $\rho_{\text{in}}(0)$ with decreasing doping.

The fall in $\rho_{\text{in}}(0)$ could also be interpreted as evidence of an as-yet-undetected reconstruction of the Fermi surface that begins in the far-overdoped regime. A reconstruction of the Fermi surface by a density wave could reduce $\rho_{\text{in}}(0)$46, however, this should also give rise to small Fermi pockets. As quantum oscillations (QOs) from the full $(1+p)$ Fermi surface are observed in Tl2201 for $p > 0.28$, the non-observation of such QOs from small pockets is evidence that they do not exist. However, it is possible that the QOs from these pockets could be damped if the density wave has poor coherence.

Previously, it was shown that $\rho_{\text{in}}(T)$ in both Tl2201 and LSCO could be modelled as the sum of $T$ and $T^2$ components. The $T^2$ (Fermi liquid like) component remains approximately independent of doping, whereas the anomalous linear-in-$T$ component $(\rho_{\text{in}})$ rises almost linearly with $p$ as $p$ is decreased from the edge of the superconducting dome14,15,46. Recently, $\rho_{\text{in}}$ has been associated with scattering at the so-called ‘Planckian limit’, which is the maximum allowed rate at which energy can be dissipated. Such strong scattering may derive from quasiparticle decoherence. As shown in Fig. 3, the reduction of $\rho_{\text{in}}(0)$ appears to correlate closely with the emergence of $\rho_{\text{in}}$ in both Tl2201 and Bi2201, and so an alternative interpretation of the reduction in $\rho_{\text{in}}$ is that it evidences a growth of quasiparticle decoherence on part of the Fermi surface. The Hall conductivity $\sigma_y/x$ is strongly weighted by parts of the Fermi surface with strong curvature. So, if decoherence developed on the flat sections of the Fermi surface but the quasiparticles remained coherent, this would lead to a decrease in $\sigma_y/x$, but would leave $\sigma_x$ relatively unchanged, resulting in a decrease of $\rho_{\text{in}} \approx \sigma_x^2/\rho_{\text{in}}$ as observed here. At lower doping, once the pseudogap has developed, $\rho_{\text{in}}(0) \approx p$ (refs. 19,21,22,49), a response that presumably comes solely from the remaining Fermi arcs, although the mechanism for this remains open to debate.

Intriguingly, for Tl2201, the region $(p > 0.275)$ where $\rho_{\text{in}}(0)$ merges with the $1+p$ line is the only region where QOs from the full $(1+p)$ Fermi-surface have been observed1. The quasi-classical model of angle-dependent, out-of-plane magnetoresistance, so successful in modelling the lower $T_c$ samples of Tl2201, also fails for doping less than this $(T_c < 20 K)^{31}$. Although there could be several reasons why QOs were not observed for higher $T_c$ samples, such as increased impurity scattering, in light of these new results it is plausible that a loss of quasiparticle coherence around the Fermi surface is preventing quantum oscillatory phenomena from being realized. The end point of the transition in $\rho_{\text{in}}(0)$, where $\rho_{\text{in}}(0) \approx p$ appears to occur approximately at optimal doping (Fig. 3), which is also where $\rho_{\text{in}}$ is maximum15,47, again shows the close correlation between the two properties. If this is indeed caused by decoherence, it appears that this onsets well before the pseudogap is evident in the resistivity or specific heat. Understanding exactly how decoherence affects the transport properties and superconductivity could prove to be a crucial part of the high-$T_c$ cuprate puzzle.

Online content

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References

1. Keimer, B., Kivelson, S. A., Norman, M. R., Uchida, S. & Zaanen, J. From quantum matter to high-temperature superconductivity in copper oxides. Science 349, 179–186 (2015).
2. LeBoeuf, D. et al. Electron pockets in the Fermi surface of hole-doped high- T superconductors. Nature 450, 533–536 (2007).
3. Sebastian, S. E., Harrison, N. & Lonzarich, G. G. Towards resolution of the Fermi surface in underdoped high- T superconductors. Rep. Prog. Phys. 75, 102501 (2012).
4. Vignolle, B. et al. Quantum oscillations in an overdoped high- T superconductor. Nature 455, 952–955 (2008).
5. Bangura, A. F. et al. Fermi surface and electronic homogeneity of the overdoped cuprate superconductor TlBa2CuO4+. as revealed by quantum oscillations. Phys. Rev. B 82, 140501 (2010).
6. Rourke, P. M. C. et al. A detailed de Haas-van Alphen effect study of the overdoped cuprate TlBa2CuO4+. New J. Phys. 12, 105009 (2010).
7. Hussey, N. E., Abdel-Jawad, M., Carrington, A., Mackenzie, A. P. & Balicas, L. A coherent three-dimensional Fermi surface in a high-transition-temperature superconductor. Nature 425, 814–817 (2003).
8. Plate, M. et al. Fermi surface and quasiparticle excitations of overdoped TlBa2CuO4+. Phys. Rev. Lett. 95, 077001 (2005).
9. Kubo, Y., Shimakawa, Y., Manako, T. & Igarashi, H. Transport and magnetic-properties of TlBa2CuO4+. showing a delta-gradient dependent transition from an 85-K superconductor to a nonsuperconducting metal. Phys. Rev. B 43, 7875–7882 (1991).
10. Manako, T., Kubo, Y. & Shimakawa, Y. Transport and structural study of TlBa2CuO4+. single-crystals prepared by the KCl flux method. Phys. Rev. B 46, 11019–11024 (1992).
11. Hussey, N. E. Phenomenology of the normal state in-plane transport properties of high- T cuprates. J. Phys. Condens. Matter 20, 123201 (2008).
12. Hussey, N. E., Gordon-Moys, H., Kokalj, J. & McKenzie, R. H. Generic strange-metal behaviour of overdoped cuprates. J. Phys. Conf. Ser. 449, 012004 (2013).
13. Shibata, T., Carrington, A. & Matsuda, Y. A quantum critical point lying below the superconducting dome in iron pnictides. Annu. Rev. Condens. Matter Phys. 5, 113–135 (2014).
14. Tüllerf, L. Scattering and pairing in cuprate superconductors. Annu. Rev. Condens. Matter Phys. 1, 51–80 (2010).
15. Blanco-Canosa, S. et al. Resonant X-ray scattering study of charge-density wavecorrelations in YBa2Cu3O7. Phys. Rev. B 90, 054513 (2014).
16. Tabis, W. et al. Synchrotron X-ray scattering study of charge-density-wave order in HgBa2CuO4+. Phys. Rev. B 96, 134510 (2017).
17. Cooper, J. R., Loram, J. W., Kankanovic, I., Storey, J. G. & Tallon, J. L. Pseudogap in YBa2Cu3O7 is not bounded by a line of phase transitions: thermodynamic evidence. Phys. Rev. B 89, 201104 (2014).
18. Ramshaw, B. J. et al. Quasiparticle mass enhancement approaching optimal doping in a high- T superconductor. Science 348, 317–320 (2015).
19. Radoux, S. et al. Change of carrier density at the pseudogap critical point of a cuprate superconductor. Nature 531, 210–214 (2016).
20. Putzke, C. et al. Inverse correlation between quasiparticle mass and Tc in a cuprate high-Tc superconductor. Sci. Adv. 2, e1501857 (2016).
21. Ando, Y., Kurita, Y., Komizu, S., Ono, S. & Segawa, K. Evolution of the Hall coefficient and the peculiar electronic structure of the cuprate superconductors. Phys. Rev. Lett. 92, 197001 (2004).
22. Balakirev, F. F. et al. Quantum phase transition in the magnetic-field-induced normal state of optimum-doped high-Tc cuprate superconductors at low temperatures. Phys. Rev. Lett. 102, 077004 (2009).
23. Horio, M. et al. Three-dimensional Fermi surface of overdoped La-based cuprates. Phys. Rev. Lett. 121, 077004 (2018).
24. Collignon, C. et al. Fermi-surface transformation across the pseudogap critical point of the cuprate superconductor La1-xNd_xSr2CuO4. Phys. Rev. B 95, 224517 (2017).
25. Matt, C. E. et al. Electron scattering, charge order, and pseudogap physics in La1-xNd_xSr2CuO4: an angle-resolved photoemission spectroscopy study. Phys. Rev. B 92, 134524 (2015).
26. Ding, Y. et al. Disappearance of superconductivity and a concomitant Lifshitz transition in heavily overdoped Bi$_2$Sr$_2$CuO$_6$ superconductor revealed by angle-resolved photoemission spectroscopy. *Chin. Phys. Lett.* **36**, 017402 (2019).

27. Abdel-Jawad, M. et al. Anisotropic scattering and anomalous normal-state transport in a high-temperature superconductor. *Nat. Phys.* **2**, 821–825 (2006).

28. Abdel-Jawad, M. et al. Correlation between the superconducting transition temperature and anisotropic quasiparticle scattering in Tl$_2$Ba$_2$CuO$_6$. *Phys. Rev. Lett.* **99**, 187002 (2007).

29. Kondo, T., Takeuchi, T., Kaminski, A., Tsuda, Y. & Shin, S. Evidence for two energy scales in the superconducting state of optimally doped (Ba,Pb)$_2$(Sr,La)$_2$CuO$_{4+x}$ *Phys. Rev. Lett.* **98**, 267004 (2007).

30. Wise, W. D. et al. Charge-density-wave origin of cuprate checkerboard visualized by scanning tunnelling microscopy. *Nat. Phys.* **4**, 696–699 (2008).

31. Wise, W. D. et al. Imaging nanoscale Fermi-surface variations in an inhomogeneous superconductor. *Nat. Phys.* **5**, 213–216 (2009).

32. Kondo, T., Khasanov, R., Takeuchi, T., Schmalian, J. & Kaminski, A. Competition between the pseudogap and superconductivity in the high-\( T_c \) copper oxides. *Nature* **457**, 296–300 (2009).

33. French, M. M. J., Analytis, J. G., Carrington, A., Balicas, L. & Hussey, N. E. Tracking anisotropic scattering in overdoped Tl$_2$Ba$_2$CuO$_{6+x}$ above 100 K. *New J. Phys.* **11**, 055057 (2009).

34. Balakirev, F. et al. Signature of optimal doping in Hall-effect measurements on a high-\( T_c \) superconducting cuprate Tl$_2$Ba$_2$CuO$_{6+x}$. *Science* **323**, 603–607 (2009).

35. Pecz, D., Popčević, P., Polek, M., Greven, M. & Barišić, N. Unusual behavior of cuprates explained by heterogeneous charge localization. *Sci. Adv.* **5**, eaau4538 (2019).

36. Tallon, J. M. & Loram, J. W. The doping dependence of \( T^* \)—what is the real high-\( T_c \) phase diagram. *Physica C* **349**, 53–68 (2001).

37. Wade, J. M., Loram, J. W., Mirza, K. A., Cooper, J. R. & Tallon, J. L. Electronic specific-heat of Tl$_2$Ba$_2$CuO$_{6+x}$, from 2 K to 300 K for \( 0 \geq x \geq 0.1 \). *J. Supercond. NMR* **26**, 261–264 (1994).

38. Fujiwara, K. et al. $^{63}$Cu knight shift study in high-\( T_c \) superconductor Tl$_2$Ba$_2$CuO$_{4+x}$, with a single CuO$_2$ layer. *J. Phys. Soc. Jpn* **59**, 3459–3462 (1990).

39. Kambe, S., Yasuoka, H., Hayashi, A. & Ueda, Y. NMR study of the spin dynamics in Tl$_2$Ba$_2$CuO$_{4+x}$ ( \( T_c = 85 \) K). *Phys. Rev. B* **47**, 2825–2834 (1993).

40. Kawasaki, S., Lin, C. T., Kuhns, P. L., Reyes, A. P. & Zheng, G. Q. Carrier-concentration dependence of the pseudogap ground state of superconducting Bi$_2$Sr$_2$La$_2$CuO$_{8+y}$ revealed by $^{63,65}$Cu-nuclear magnetic resonance in very high magnetic fields. *Phys. Rev. Lett.* **105**, 137002 (2010).

41. Kondo, T. et al. Disentangling Cooper-pair formation above the transition temperature from the pseudogap state in the cuprates. *Nat. Phys.* **7**, 21–25 (2011).

42. Tallon, J. M., Storey, I. G., Cooper, J. R. & Loram, J. W. Locating the pseudogap closing point in cuprate superconductors: absence of entrant or reentrant behavior. *Phys. Rev. B* **101**, 174512 (2020).

43. Umura, Y. et al. Magnetic-field penetration depth in Tl$_2$Ba$_2$CuO$_{6+x}$ in the overdoped regime. *Nature* **364**, 665–667 (1993).

44. Broun, D. M. et al. In-plane microwave conductivity of the single-layer cuprate Tl$_2$Ba$_2$CuO$_{6+x}$. *Phys. Rev. B* **56**, R11443–R11446 (1997).

45. Abrahams, E. & Varma, C. M. Hall effect in the marginal Fermi liquid regime of high-\( T_c \) superconductors. *Phys. Rev. B* **68**, 094502 (2003).

46. Eberlein, A., Metzner, W., Sachdev, S. & Yamase, H. Fermi surface reconstruction and drop in the Hall number due to spiral antiferromagnetism in high-\( T_c \) cuprates. *Phys. Rev. Lett.* **117**, 187001 (2016).

47. Cooper, R. A. et al. Anomalous criticality in the electrical resistivity of La$_{2-y}$Sr$_y$CuO$_4$. *Science* **332**, 603–607 (2009).

48. Legros, A. et al. Universal \( T \)-linear resistivity and Planckian dissipation in overdoped cuprates. *Nat. Phys.* **15**, 142–147 (2019).

49. Barišić, N. et al. Evidence for a universal Fermi-liquid scattering rate throughout the phase diagram of the copper-oxide superconductors. *New J. Phys.* **21**, 113007 (2019).

50. Mackenzie, A. P., Julian, S. R., Sinclair, D. C. & Lin, C. T. Normal-state magnetotransport in superconducting Tl$_2$Ba$_2$CuO$_{6+x}$ to millikelvin temperatures. *Phys. Rev. B* **53**, 5848–5855 (1996).

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Data availability
The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

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Author contributions
The project was conceived by A.C., C.P. and N.E.H. Pulsed field measurements on Tl2201 were performed by C.P. and Z.W. at HLD–Dresden and by C.P., S.B., W.T. and J.A. at LNCMI–Toulouse. J.L. and S.L. contributed to the Hall effect measurement on Bi2201 at HMFL–Nijmegen. Samples of Tl2201 were grown by L.M. and J.R.C. Samples of Bi2201 were grown by T.K. and T.T. A.C. performed the numerical simulations of $R_h(T, H)$. The manuscript was written by A.C. and N.E.H., with input from all the co-authors.

Competing interests
The authors declare no competing interests.

Additional information
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