IMPLICATIONS OF THE NARROW PERIOD DISTRIBUTION OF ANOMALOUS X-RAY PULSARS AND SOFT GAMMA-RAY REPEATERS

Dimitrios Psaltis\textsuperscript{1} and M. Coleman Miller\textsuperscript{2}

\textit{Draft version January 15, 2022}

ABSTRACT

The spin periods of the ten observed Anomalous X-ray Pulsars and Soft Gamma-ray Repeaters lie in the very narrow range $6 - 12$ s. We use a point likelihood technique to assess the constraints this clustering imposes on the birth period and on the final period of such systems. We consider a general law for their spin evolution described by a constant braking index. We find that, for positive values of the braking index, the observed clustering requires an upper cut-off period that is very close to the maximum observed period of $\approx 12$ s. We also show that the constraint on the birth period depends strongly on the assumed value of the braking index $n$, ranging from a few milliseconds for $n \gtrsim 2$ to a few seconds for $n \lesssim 2$. We discuss the possible ways of tightening these constraints based on similarities with the population of radio pulsars and with future observations of such sources with current X-ray observatories.

\textit{Subject headings:} stars: neutron — pulsars: general — X-ray: stars

1. INTRODUCTION

In the last few years, evidence for the existence of neutron stars with ultrastong magnetic fields, or magnetars, has become very compelling. The discovery of rapid spin down in the pulsations observed from soft gamma-ray repeaters (SGRs; e.g., Kouveliotou et al. 1998, 1999) gave support to the suggestion by Thompson & Duncan (1995) that the very energetic bursts observed from these sources require the presence of $10^{15}$ G magnetic fields. Furthermore, the lack of detectable companions to the Anomalous X-ray pulsars (AXPs; e.g., Mereghetti, Israel, & Stella 1998; Hulleman, van Kerkwijk, & Kulkarni 2000), combined with their rapid spin-down rates and spectral properties (see, e.g., Özel 2001), favor the magnetar interpretation.

A striking feature of all magnetar candidates (SGRs and AXPs) is that their periods lie in a relatively narrow range, between 6 s and 12 s. This property has been discussed since the original study of Mereghetti & Stella (1995) and was addressed by Colpi, Geppert, & Page (2000) in the context of models with magnetic field decay (see also Chatterjee & Hernquist 2000 for a discussion of the period clustering expected in a variant of accretion models for AXPs). However, a quantitative, statistical analysis of the constraints imposed on magnetar models by this period clustering is still lacking.

Here we quantify the tightness of the observed period distribution, using a point likelihood technique. In § 2 we review the detections of each of the 10 magnetar candidate sources and focus in particular on selection effects that could restrict the range of periods that can be discovered. In § 3 we perform a likelihood analysis to determine the allowed range of periods using a mathematical model for the period evolution of AXPs that has broad applicability and of which a dipole spin-down law is a special case. In § 4 we discuss the implications of these results and explore the potential for improving these constraints using future observations of AXPs and SGRs with current X-ray telescopes.

2. OBSERVATIONS AND SELECTION EFFECTS

In this section we describe briefly the observations that led to the discovery of the ten AXPs and SGRs with measured spin periods, in order to assess the observational selection effects that could have affected their period distribution. Even though no systematic searches for AXPs and SGRs have been performed to date, we argue that these discovery observations were not confined to periods of order $\sim 6 - 12$ s and hence no observational selection effects can account for the observed period clustering.

2.1. \textit{Anomalous X-ray Pulsars}

Two of the AXPs were known persistent X-ray sources before pulsations were detected in their X-ray emission. 4U 0142+61 ($P = 8.69$ s) was discovered as a persistent source in an early all-sky survey. Subsequent power-spectral analysis of \textit{pointed} observations of this source with \textit{EXOSAT} revealed the pulsations (Israel, Mereghetti, & Stella 1994). The search was performed using \textit{EXOSAT} ME data that had a timing resolution of 1 s, yielding a lower limit on the search period of only 2 s. The duration of the observation was 12 h and the upper limit on the search period was of order $10^4$ s.

1E 1048.1$-$5937 ($P = 6.44$ s) was serendipitously discovered as an X-ray source with Einstein. The pulsations were discovered by Seward, Charles, & Smale (1986), who searched in both Einstein and \textit{EXOSAT} data but did not report the period range of their searches.

The remaining three AXPs were discovered in searches for pulsed X-ray sources, and hence the strategy followed in their observations could have introduced significant se-
lection effects. 1E 2259+586 ($P = 6.98$ s) was discovered using Einstein data, following a systematic search for a pulsar in the supernova remnant G 109.1−1.0 (Fahlman & Gregory 1981). The range of periods searched was $0.1−200$ s. 1E 1841−045 ($P = 11.77$ s) was discovered with ASCA, following a systematic search for pulsations from all point sources within the supernova remnant Kes 73 (Vasisht & Gotthelf 1997). The search was performed with three timing resolutions: $488 \mu$s, $32$ ms, and $0.5$ s and power spectra were produced for each timing resolution. As a result, the minimum period at which pulsations could be detected was $\ll 1$ s. Moreover, the search was performed over timescales of $96$ min, $10$ min, and $1$ min and thus was sensitive to pulsations with periods $\gg 100$ s. 1RXS J170849.0−400910 ($P = 10.99$ s) was discovered by ASCA in a survey of the galactic plane and was later identified with a ROSAT source (Sugizaki et al. 1997). The observation was performed with a timing resolution of $62.5$ ms in the high-bit-rate mode and $0.5$ s in the medium-bit-rate mode. As a result, searches for pulsations were sensitive to periods $\ll 1$ s. The highest period searched was quoted as $\approx 600$ s. AX J1845−0258 ($P = 6.97$ s) was discovered with ASCA in the distant Milky Way (Gotthelf & Vasisht 1998) in the supernova remnant Kes 75. For computational efficiency, the search was performed only for long periods, i.e., for $P \ll 100$ s.

2.2. Soft Gamma-Ray Repeaters

SGRs are identified through their recurrent $\gamma$-ray bursts and not because of their pulsations. However, pulsations have been detected in all four, securely identified SGRs. In SGR 1900+14, pulsations have been observed both in the quiescent emission and during bursts at a period of $P = 5.16$ s (Hurley et al. 1999). In SGR 1806−20 ($P = 7.47$ s) and SGR 1627−41 ($P = 6.4$ s) pulsations have been detected only during the quiescent emission (Kouveliotou et al. 1998; Woods et al. 1999) whereas in SGR 0525−66 ($P = 8$ s) pulsations have been observed only during bursts (Barat et al. 1979). All these searches were performed with data obtained using ASCA or RXTE and, therefore, were not limited to periods only comparable to those observed.

3. ANALYSIS OF PERIOD CLUSTERING

The period clustering of AXPs and SGRs has often been attributed to a general prediction of a large class of spin-evolution models in which the spin period derivative decreases with increasing period. In these models, the objects evolve quickly through the small periods, making their detection improbable at these periods and their steady-state period distribution insensitive to the birth values. However, because the objects spend increasingly more time at long periods, the observed cutoff towards high periods can be used for placing a very strong constraint on the maximum period at which they are detectable. In this section we quantify the above statement using a point likelihood technique.

3.1. Analytical Setup

In order to model the period distribution of magnetar candidates, we assume a general braking law of the form

$$\dot{\Omega} = -\kappa \Omega^n,$$

where $\Omega$ is the spin frequency of the stars and $n$ is the braking index. For $n = 3$ and $\kappa \sim B^2$, equation (1) corresponds to the standard spin-down law for an inclined magnetic dipole of strength $B$. In order to avoid introducing unnecessary complications to our model, we assume that all systems are born with the same initial period $P_{in}$ and become undetectable when they reach period $P_f$.

The evolution of the period distribution function $f(P)$ between $P_{in}$ and $P_f$ is described by the conservation law

$$\frac{\partial f(P)}{\partial t} + \frac{\partial}{\partial P} \left[ \dot{P} f(P) \right] = 0,$$

where we have assumed that there is no evolution of the factor $\kappa$. In steady state, the distribution of systems over spin period is then $f(P) \sim \dot{P}^{-1}$, or

$$f(P) = C P_n^{-2},$$

where the constant $C$ is calculated from the requirement that $f(P)$ is normalized, or

$$C = \left\{ \begin{array}{cc} \frac{(n-1)(P_{in}^{-n-1} - P_f^{-n-1})^{-1}}{\ln (-P_f/P_{in})}, & n \neq 1 \\ \ln (-P_f/P_{in}), & n = 1 \end{array} \right..$$

We now use this period distribution to estimate the best values for the initial and final period, using a likelihood analysis and given the fact that $m$ systems have been detected with measured periods $P_j; j = 1, \ldots, m$. (We assume that all periods are measured to arbitrary precision.) To this end, we subdivide the available parameter space into infinitesimally small bins of width $\Delta P$, such that each bin has either zero or one data point in it. The likelihood of the data given the model is then simply

$$\mathcal{L}(P_{in}, P_f) = \prod_{j=1}^{m} f(P_j) \Delta P = C^m (\Delta P)^m \prod_{j=1}^{m} P_j^{-n-2}.$$  

If we assume two prior probability distributions $G(P_{in})$ and $G(P_f)$ for the parameters, then the posterior probability distribution is proportional to the above likelihood. Following the standard Bayesian approach, we then obtain the posterior probability distribution for an individual parameter (e.g., $P_{in}$) by integrating the full multidimensional posterior distribution over all parameters but the one of interest, i.e., “marginalizing” over the remaining parameters. In practice, the posterior distribution of, e.g., $P_{in}$ is

$$\mathcal{P}(P_{in}|P_f) = \frac{\int \mathcal{L}(P_{in}, P_f) G(P_{in}) G(P_f) dP_f}{\int \int \mathcal{L}(P_{in}, P_f) G(P_{in}) G(P_f) dP_{in} dP_f}.$$  

For the prior probability distribution over $P_{in}$ we assume a flat distribution in $\log P_{in}$ between $10^{-3}$ s and the minimum observed period of $P_{min} = 6.44$ s, which does not imply any particular period scale. We chose $10^{-3}$ s as our lowest acceptable initial spin period, because this is comparable to the fastest neutron-star spins allowed by
4. DISCUSSION

The narrow range of observed periods of magnetar candidates can be used to constrain their birth periods, the periods at which they cease to be active, or both, depending on the value of their braking index. For positive values of the braking index, we showed in §3 that the final periods must lie very close to the maximum observed period of $\sim 12$ s. At the same time, the braking index is $n > 2$ then the birth periods must be $\sim 5$ s, i.e., very slow. On the other hand, if $n > 2$, then the birth periods are largely unconstrained from the analysis of the observed clustering.

It is interesting to note that braking indices of young radio pulsars have been measured to be from as low as $1.81 \pm 0.07$ (PSR B0540–69; Zhang et al. 2001) to as high as $2.837 \pm 0.001$ (PSR B1509–58; Kaspi et al. 1994), bracketing the value of $n = 2$ that separates a strong constraint on $P_{\text{in}}$ from a very weak constraint. However, slow radio pulsars have second period derivatives (and hence braking indices) that are variable and larger by factors of $\sim 10^2$–$10^4$ than what is predicted by simple magnetic braking (see, e.g., Cordes & Helfand 1980). Such anomalously large braking indices are thought to be the result of glitches and of timing noise, both of which are known to occur at least in AXPs, which can have
Fig. 3.—The inferred equatorial dipole magnetic fields of radio pulsars, AXPs, and SGRs versus their spin periods. The short-dashed line shows the empirical death line for radio pulsars; the long-dashed line shows the upper limit in the AXP and SGR periods inferred in this paper.

instantaneous braking indices of order $\sim 10^3$ (Kaspi, Lackey, & Chakrabarty 2000; Kaspi et al. 2001).

Our analysis, however, and in particular the constraint on the birth periods, depends on the value of the average braking index, to the extent that the spin evolution of the magnetar candidates is described approximately by a law of the form (1), and not on any instantaneous index. For the case of twenty moderate-aged radio pulsars, Johnston & Galloway (1999) computed braking indices integrated over $\approx 5-20$ yr and found values in the wide range $-220 \lesssim n \lesssim 35$. This result is not surprising, given that the braking indices of such pulsars computed over shorter timescales vary rapidly and often change sign. It shows, however, that unfortunately little progress can be expected in constraining the braking indices of magnetar candidates and hence their birth periods.

On the other hand, the strong constraint derived here on the maximum period may provide some insight into the physical mechanism that might be causing it. For example, comparing the inferred equatorial dipole magnetic fields and spin periods of magnetar candidates with those of radio pulsars (see Fig. 3) indicates that the empirically drawn death line of radio pulsars, when extrapolated towards higher magnetic fields, appears to be unrelated to the maximum period of magnetar candidates.

An exponentially decaying magnetic field, as discussed by Colpi et al. (2000), provides a plausible explanation for both the period clustering and the young ages of magnetar candidates. It is worth noting, however, that neither the inferred luminosities of AXPs nor their pulsed fraction appear to be correlated with their periods, spin-down ages, or inferred field strengths, even though the values of the latter two span more than an order of magnitude (see, e.g., Özel, Psaltis, & Kaspi 2001).

Finally, we address the dependence of our results on the number of magnetar candidates that we considered. In our analysis, we chose to assume that both AXPs and SGRs are formed and evolve in the same way. We show in Figure 4, however, where we use the dipole spin-down law as an example, that the resulting constraints depend rather weakly on the number of systems that are known within the same period range. Indeed, even considering simply the six known AXPs would be enough to reach similar conclusions.

Figure 4 also shows that increasing the sample of magnetar candidates with similar periods even by a factor of two will only affect our results mildly: the constraint on the birth period will become as high as $\approx 2$ s, but only at the 68% level. However, the detection of even a single magnetar candidate with period larger than $\approx 12$ s will change the constraint on $P_1$. Such a detection does not require detectors with fast timing capabilities and is, therefore, possible, if such systems exist, with current X-ray observatories such as Chandra and XMM/Newton.

We thank Feryal Özel for many useful discussions and comments on the manuscript. D. P. thanks the Astronomy Department of the University of Maryland for its hospitality. M. C. M. acknowledges the support of NASA grant 5-9756 and NSF grant AST 0098436.

REFERENCES

Barat, C., Chambon, G., Hurley, K., Niel, M., Vedrenne, G., Estulin, I. V., Kurt, V. G., & Zenchenko, V. M. 1979, A&A, 79, L24
Chatterjee, P., & Hernquist, L. 2000, ApJ, 543, 368
Colpi, M., Geppert, U., & Page, D. 2000, ApJ, 529, L29
Cook, G. B., Shapiro, S. L., & Teukolsky, S. A. 1994, ApJ, 424, 823
Cordes, J. M., & Helfand, D. J. 1980, ApJ, 239, 640
Eshelman, G. G., & Gregory, P. C. 1981, Nature, 293, 202
Gotthelf, E. V., & Vasisht, G. 1998, New Astronomy, 3, 293
Hulleman, F., van Kerkwijk, M. H., & Kulkarni, S. R. 2000, Nature, 408, 689
Hurley, K. et al. 1999, ApJ, 510, L111
Israel, G. L., Mereghetti, S., & Stella, L. 1994, ApJ, 433, L25
Johnston, S., & Galloway, D. 1999, MNRAS, 306, L50
Kaspi, V. M., Gavriil, F. P., Chakrabarty, D., Lacyey, J. R., & Muno, M. P. 2001, ApJ, 558, 253
Kaspi, V. M., Lackey, J. R., & Chakrabarty, D. 2000, ApJ, 537, L31
Kaspi, V. M., Manchester, R. N., Siegmam, B., Johnston, S., & Lyne, A. G. 1994, ApJ, 422, L83
Kouveliotou, C. et al. 1998, Nature, 393, 235
Kouveliotou, C. et al. 1999, ApJ, 510, L115
Mereghetti, S., Israel, G. L., & Stella, L. 1998, MNRAS, 296, 689
Mereghetti, S., & Stella, L. 1995, ApJ, 442, L17
Özel, F. 2001, ApJ, 563, 276
Özel, F., Psaltis, D., & Kaspi, V. 2001, ApJ, 563, 255
Seward, F. D., Charles, P. A., & Smale, A. P. 1986, ApJ, 305, 814
Stella, L., Israel, G. L., & Mereghetti, S. 1998, in The Many Faces of Neutron Stars, ed. R. Buccheri, J. van Paradijs, & M. A. Alpar (Dordrecht: Kluwer), 397
Sugizaki, M., Nagase, F., Torii, K., Kinugasa, K., Asanuma, T., Matsuzaki, K., Koyama, K., & Yamauchi, S. 1997, PASJ, 49, L25
Thompson, C., & Duncan, R. C. 1995, MNRAS, 275, 255
van Paradijs, J., Taam, R. E., & van den Heuvel, E. P. J. 1995, A&A, 299, L41
Vasisht, G., & Gotthelf, E. V. 1997, ApJ, 486, L129
Woods, P. M. et al. 1999, ApJ, 519, L139
Zhang, W., Marshall, F. E., Gotthelf, E. V., Middleditch, J., & Wang, Q. D. 2001, ApJ, 554, L177