Resolved Power Corrections to the Inclusive Decay $\bar{B} \to X_s \ell^+ \ell^-$

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ABSTRACT

We identify the correct power counting of all the variables in the low-$q^2$ window of the inclusive decay $\bar{B} \to X_s \ell^+ \ell^-$ within the effective theory SCET if a hadronic mass cut is imposed. Furthermore we analyse the resolved power corrections at the order $1/m_b$ in a systematic way. As a special feature, the resolved contributions stay nonlocal when the hadronic mass cut is released. Therefore they represent an irreducible uncertainty independent of the hadronic mass cut.

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1 Introduction

As regards the theoretically clean modes of the indirect search for new physics by means of flavour observables, the inclusive decay mode $\bar{B} \to X_s \ell \ell$ plays a crucial role (for reviews see Refs. [1–3]). This inclusive decay mode provides a nontrivial crosscheck of the so-called LHCb anomalies within the recent LHCb data on the corresponding exclusive mode [5,6]. As demonstrated in Refs. [7,8], the future measurements of the inclusive mode will be capable to resolve these puzzles.

A comparison between the inclusive the $\bar{B} \to X_s \gamma$ and the inclusive $\bar{B} \to X_s \ell^+ \ell^-$ decay reveals that the latter is a complementary and a more complex test of the SM, given that different perturbative electroweak contributions add to the decay rate. As a three body decay process it also offers more observables. Because of the presence of the lepton-antilepton pair, more structures contribute to the decay rate and some subtleties in the formal theoretical description arise. This inclusive mode is generally assumed to be dominated by perturbative contributions like the inclusive $\bar{B} \to X_s \gamma$ decay if one eliminates $c\bar{c}$ resonances with the help of kinematic cuts. Research regarding these perturbative contributions has been undertaken extensively and has already reached a highly sophisticated level. The latest analysis of all angular observables in the $\bar{B} \to X_s \ell^+ \ell^-$ decay has been presented in Ref. [4]. It contains all available perturbative NNLO QCD, NLO QED corrections and also includes the known subleading power corrections.

For the inclusive modes $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_s \ell^+ \ell^-$, it is possible to demonstrate that, if only the leading operator in the effective Hamiltonian ($O_\ell$ for $\bar{B} \to X_s \gamma$, $O_0$ for $\bar{B} \to X_s \ell^+ \ell^-$) is taken into account, the heavy mass expansion (HME) allows for the calculation of the inclusive decay rates of a hadron containing a heavy quark, especially a $b$ quark [10–11]. In this case, one arrives at a local operator product expansion (OPE) based on the optical theorem. The free quark model represents the first term in the constructed expansion in powers of $1/m_b$ and is therefore the dominant contribution. In the applications to inclusive rare $B$ decays, one finds no correction of order $\Lambda/m_b$ to the free quark model approximation within this OPE because of the equations of motion. As a consequence, the corrections to the partonic decay rate begin with $1/m_b^2$ only. This implies that there is a small numerical impact of the nonperturbative corrections on the decay rate of inclusive modes.

However, there are more subtleties to be taken into account if other than the leading operators are considered. As already demonstrated in Ref. [9], there is no OPE for the inclusive decay $\bar{B} \to X_s \gamma$ if one analyses operators beyond the leading electromagnetic dipole operator $O_\ell$. Indeed, there are the so-called resolved photon contributions. These include subprocesses in which the photon connects to light partons instead of coupling directly to the effective weak-interaction vertex [12–13]. Within the inclusive decay $\bar{B} \to X_s \gamma$, a systematic analysis [14] of all resolved photon contributions related to other operators in the weak Hamiltonian establishes this breakdown of the local OPE within the hadronic power corrections as a generic result. Within soft-collinear effective theory (SCET), an analysis of such linear power corrections is possible. Clearly, one has to confront difficulties if estimating such nonlocal matrix elements. An irreducible theoretical uncertainty of $\pm(4 - 5)\%$ for the total $CP$ averaged decay rate, defined with a photon-energy cut of $E_\gamma = 1.6$ GeV, cannot be eliminated [14].

In the present letter the resolved contributions to the inclusive decay $\bar{B} \to X_s \ell^+ \ell^-$ are studied. Within the inclusive decay $\bar{B} \to X_s \ell^+ \ell^-$, the hadronic ($M_X$) and dilepton invariant $(q^2)$ masses are independent kinematical quantities. An invariant mass cut on the hadronic final state system ($M_X \lesssim 2$ GeV) is necessary in order to suppress potential huge backgrounds. This cut implies no additional constraints in the high-dilepton-mass region. In the low-dilepton region, the cut on the hadronic mass leads to a specific kinematics in which the soft-collinear OPE collapses and one has to introduce nonperturbative $b$-quark distributions, so-called shape functions. Given the specific kinematics of low dilepton masses $q^2$ and of small hadronic masses $M_X$, one has to deal with a multi-scale problem for which soft-collinear effective theory (SCET) is the appropriate tool.

A former SCET analysis made use of the universality of the leading shape function to show that the reduction resulting from the $M_X$-cut can be calculated for all angular observables of the inclusive decay $\bar{B} \to X_s \ell^+ \ell^-$. The effects of subleading shape functions imply an additional uncertainty of $5\%$ [16]. A later analysis [18] estimates the uncertainties due to subleading shape functions more conservatively. In the future it may be possible to decrease such uncertainties significantly by constraining both the leading and subleading shape functions using the combined $\bar{B} \to X_s \gamma$, $\bar{B} \to X_u \ell \bar{\nu}$ and $\bar{B} \to X_s \ell^+ \ell^-$ data [18].

All these former analyses, however, are based on the problematic assumption, that $q^2$ is a hard scale in the kinematical region of low $q^2$ and of small $M_X$. By contrast, our present SCET analysis will demonstrate explicitly that the hadronic cut implies the scaling of $q^2$ being not hard but (anti-) hard-collinear in the low-$q^2$ region.

Therefore it is the first and primary task to identify the correct power counting of all the variables in
the low-$q^2$ window of the inclusive decay $B \to X_s \ell^+ \ell^-$ within the effective theory SCET in case a hadronic mass cut is imposed. Moreover, the resolved power corrections have to be examined systematically. As already mentioned, in these contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex. There would be no such contribution, if $q^2$ was hard in SCET. Furthermore, we will show that the resolved contributions have the special feature that they stay nonlocal when the hadronic mass cut is released. In this sense they thus lead to an irreducible uncertainty that is not dependent on the hadronic mass cut.

2 SCET analysis of the resolved contributions

The effective operator basis for the underlying parton interaction of the semileptonic flavour changing neutral current decay $B \to X_s \ell^+ \ell^-$ is well-known [19]. Many higher-order calculations have led to the availability of NNLO precision and NNLL resummation in the strong coupling $\alpha_s$. At the relevant scale $m_b$ of the $b$-quark, all heavier fields are integrated out. The effective operator basis only includes only active flavours.

When calculating the inclusive decay mode $B \to X_s \ell^+ \ell^-$, one is confronted with two problems: The first problem stems from the fact that the integrated branching fraction is dominated by resonant $q\bar{q}$ background, in particular with $q = c$, i.e. resonant $J/\psi \to \ell^+ \ell^-$ intermediate states for the (virtual) photon, which exceeds the nonresonant charm-loop contribution by two orders of magnitude. This phenomenon should not be misinterpreted as a striking failure of global parton-hadron duality as shown in Ref. [20]. In any case, $c\bar{c}$ resonances appearing as large peaks in the dilepton invariant mass spectrum are eliminated by appropriate kinematic cuts – leading to so-called ‘perturbative $q^2$-windows’, namely the low-dilepton-mass region $1 \text{GeV}^2 < q^2 < 6 \text{GeV}^2$, and also the high-dilepton-mass region with $q^2 > 14.4 \text{GeV}^2$.

The second problem to be faced is related to the fact that in a realistic experimental environment one has to suppress potential huge backgrounds by an invariant mass cut on the hadronic final state system ($M_X \lesssim 2 \text{GeV}$). This cut does not involve any additional constraints in the high-dilepton-mass region. But in the low-dilepton mass region we have in the $B$ meson rest frame due to $q = p_B - p_X$:

$$2 M_B E_X = M_B^2 + M_X^2 - q^2.$$  

Thus, for low enough $q^2$ in combination with $M_X^2 \ll E_X^2$, the $X_s$ system is jet-like with $E_X \sim M_B$. This also means that $p_X$ is near the light cone.

Considering these kinematic constraints, soft-collinear-effective theory (SCET) [21] is the adequate tool to analyse the factorization properties of inclusive $B$-meson decays in this region and to examine the multi-scale problem. Thus, the cuts in the two independent kinematic variables, namely the hadronic and dilepton invariant masses, force us to study the process in the so-called shape function region with a large energy $E_X$ of order $M_B$ and low invariant mass $M_X \sim \sqrt{m_b \Lambda_{QCD}}$ of the hadronic system. SCET makes it possible to get systematically hold of a scaling law of the momentum components. In this set-up the scales $\Lambda_{QCD}$, $M_X$, $q^2$ and $M_B$ are relevant. One arrives at the following hierarchy for the ratio of the scales:

$$\Lambda_{QCD}/M_B \ll M_X/M_B \ll 1.$$  

Therefore it is of high relevance to resum logarithms between these scales. One can resum the logarithms of these scale ratios systematically. What is more, one can factorize the effects resulting from diverse regions. In this way, one can calculate the process in a consistent expansion and factorize off effects that can be calculated perturbatively. This reduces the non-perturbative quantities to a limited set of soft functions.

If one defines $\lambda = \Lambda_{QCD}/M_B$, one numerically finds that $M_X \lesssim \sqrt{M_B \Lambda_{QCD}} \sim M_B \sqrt{\lambda}$. This implies the power-counting scale for the possible momentum components in light-cone coordinates $n^\mu = (1, 0, 0, 1)$ and $\bar{n}^\mu = (1, 0, 0, -1)$. Any four-vector may be decomposed according to $a^\mu = n \cdot a \; \bar{n}^\mu/2 + \bar{n} \cdot a \; n^\mu/2 + a_\perp$. The short-hand notation is defined to be $a \sim (n \cdot a, \bar{n} \cdot a, a_\perp)$ in order to specify the scaling of the momentum components in powers of $\lambda$. Within the validity of SCET, we have a hard momentum region $p_{\text{hard}} \sim (1, 1, 1)$, a hard-collinear region $p_{\text{hc}} \sim (\lambda, 1, \sqrt{\lambda})$, an anti-hard-collinear region $p_{\text{ach}} \sim (1, \lambda, \sqrt{\lambda})$, and a soft region $p_{\text{soft}} \sim (\lambda, \lambda, \lambda)$.

Regarding the two-body radiative decay, the kinematics requires $q^2 = 0$ and $E_\gamma \sim m_b/2$. The scaling including the invariant mass and photon energy requirement is fixed to be a hard-collinear hadronic jet recoiling against an anti-hard collinear photon.

In the case of a lepton-antilepton pair in the final state, one has to pose a restriction on the momentum transfer to the leptons around the mass window of the $c\bar{c}$ resonances as described above. In Figure 4a
When making use of the kinematical relations, the leptonic light-cone variables are set by

\[ q^+ = n \cdot q = M_B - p_X^+ \]

\[ q^- = \bar{n} \cdot q = M_B - p_X^- = q^2/(M_B - p_X^+) \, . \]  

In Figure 2, we demonstrate that the scaling of the momentum components of the hadronic system \( p_X^+ = n \cdot p_X \) and \( p_X^- = \bar{n} \cdot p_X \) (left plot) and of the lepton system \( q^+ = n \cdot q \) and \( q^- = \bar{n} \cdot q \) (right plot) as function of \( q^2 \) for three different values of the hadronic mass cut. Here we rely on the assumption for the experimentally envoked cuts that the effective two-body decay system \( B \to X\gamma^* \) is aligned along the light-cone axis without a perp component. In this case, the hadronic system scales as hard-collinear, while the lepton system scales as anti-hard collinear. But one can also extract from the plots that a lower cut of \( q^2 \lesssim 5 \text{ GeV}^2 \) instead of \( q^2 \lesssim 6 \text{ GeV}^2 \) is preferred because a higher value of the \( q^2 \) cut pushes the small component to values slightly beyond our assumptions of the momentum component scaling and therefore neglected higher order terms may have a more sizable contribution. Nevertheless, the assumption of a
hard $q$ momentum as used in the calculations of Refs. [16, 18] is not appropriate. It also implies both a different scaling and also a different matching of the operators.

Therefore, we use SCET to describe the hadronic effects with SCET in correspondence to an expansion of the forward scattering amplitude in non-local operator matrix elements. A factorization formula can be derived. It is completely analogous to the radiative decay in [14]:

$$d\Gamma(\bar{B} \to X_s \ell^+ \ell^-) = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} +$$

$$+ \sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[ \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \right.$$

$$+ \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes J_i^{(n)} \right] . \tag{5}$$

The formula includes the so-called direct contributions in the first line, whereas the second and third line describe the resolved contributions which occur first only at the order $1/m_b$ in the heavy-quark expansion. $H_i^{(n)}$ are the hard functions describing physics at the high scale $m_b$. $J_i^{(n)}$ are so-called jet functions characterizing the physics of the hadronic final state $X_s$ with the invariant mass in the range described above. The hadronic physics associated with the scale $\Lambda_{\text{QCD}}$ is parameterized by the soft functions $S_i^{(n)}$. Similar to the radiative decay investigated in Ref. [14], we have in addition resolved virtual-photon contributions in the second line, whose effects are described by new jet functions $\bar{J}_i^{(n)}$. This occurs due to the coupling of virtual photons with energies of order $\sqrt{m_b \Lambda_{\text{QCD}}}$ to light partons instead of the weak vertex directly. Consequently, they probe the hadronic substructure at this scale. Resolved effects may occur as a single or double “resolved” contribution due to interference of the various operators, which also have the “direct virtual-photon” contribution. Finally, the soft or shape functions are defined in terms of forward matrix elements of non-local heavy-quark effective theory (HQET) operators. This limited set of shape functions cannot be calculated perturbatively. Nevertheless it leads to a systematic analysis of hadronic effects in this decay mode. We denote the convolution of the soft and jet function due to the occurrence of common variables with the symbol $\otimes$. Finally, we note that this factorization formula cannot be completely proven. This has already been discussed in Ref. [14]: There is one particular case in which a UV divergent convolution integral exists within the resolved contribution. The contribution from $\mathcal{O}_8 - \mathcal{O}_8$ includes a UV divergence canceling the $\mu$-dependence of the corresponding subleading jet function – a cancelation that is expected and required. But in order to arrive at a consistent description, one has to use a proper factorization of the anti-jet functions. The convolution of the two anti-jet functions with the soft-function solves the item. To arrive at the correct factorization result, the limit of the DimReg parameter $\epsilon$ must be taken after the convolution has been carried out. However, this contradicts the
assumptions in the factorization formula. We note that there are also divergent convolution integrals in SCET in power-suppressed contributions to hadronic $B$ meson decays. The important difference to our present case is that these divergences have an IR-origin.

It is necessary to combine QCD $\otimes$ QED in terms of SCET in order to describe the process mentioned. Kinematically, one has to consider the fact that the hadronic part has to be described in terms of SCET for a proper and consistent description. This is also true for QED. We have to describe the QED fields in terms of an SCET-like theory. Thus, we examine the matching of $O_7$ onto SCET fields, where we consider the (virtual) photon to be power-counted as well. Then the electromagnetic dipole operator can then be written as

$$O_7 = -\frac{e}{8\pi^2} m_b \bar{s}a^\mu(1 + \gamma_5)F^{\mu\nu}b$$

(6)

$O_7$ is matched onto the operators following the notation of [15] to the leading operator with $A$ being the Wilson line dressed gauge-invariant photon field. Here we suppress a factor $-\frac{em_b}{4\pi}\gamma_i e^{-im_b\cdot x}$. 

$$O^{(1)}_{7A} = \xi_{hc} \frac{\not{n}}{2} [\bar{n} \cdot \partial A_{-}] \gamma_5 (1 + \gamma_5)h.$$ 

(7)

The scaling of the photon field is given by $(n \cdot A^{em}, \bar{n} \cdot A_{-}^{em}, A_{-}^{em}) \sim (0, \lambda, \sqrt{\lambda})$. Here gauge invariance implies $n \cdot A^{em} = 0$ even though it is off-shell. The scaling of $O_7$ is $\lambda^2$. As regards the semi-leptonic operators,

$$O_9 = \frac{\alpha}{2\pi} (\bar{s}b)_{V-A}(\ell\ell)_{V}, \quad O_{10} = \frac{\alpha}{2\pi} (\bar{s}b)_{V-\lambda}(\ell\ell)_{A}$$

(8)

the matching results into the following SCET operators:

$$O^{(1)}_{91} = \frac{\alpha}{2\pi} (\xi_{hc}^s[1 + \gamma_5]h) (\xi_{hc}^T \frac{\not{n}}{2})_{A}, \quad O^{(1)}_{10} = \frac{\alpha}{2\pi} (\xi_{hc}^s[1 + \gamma_5]h) (\xi_{hc}^T \frac{\not{n}}{2})_{A}.$$

(9)

The two operators scale as $\lambda^{2+\frac{1}{2}+2\frac{1}{2}} = \lambda^3$, thus, they are suppressed by $\lambda^2$ against the contribution from $O_7$. This feature changes in the high $q^2$ region as in this case the leptons are hard and do not add a power suppression. Thus, the leading order reference is given by $O_7 - O_7$ at the order of $\lambda^5$. We consider all contributions up to order $1/m_b$ corrections, i.e. terms up to $\lambda^6$. This implies that we have to take into account only the leading part of $O_{9,10} - O_{9,10}$ and also the subleading part of $O_7 - O_7$. This includes subleading soft and jet functions.

We consider the resolved contributions to order $1/m_b$. This includes the computation of the resolved contributions from $O_1 - O_7$, $O_7 - O_8$ and $O_8 - O_8$. We emphasize the fact that the conversion of a photon to the lepton pair does not lead to a further power suppression. Figure 3 illustrates the various resolved contributions.

Let us discuss the structure of the $O_8 - O_8$ contribution in an exemplary mode. For the differential decay rate one finds:

$$\frac{d\Gamma}{dn \cdot q \cdot d\mathbf{q}} \propto \frac{e^2\alpha_s}{m_b^2} \int d\mathbf{\omega} \delta(\omega + m_b - n \cdot \mathbf{q}) \int \frac{d\omega_1}{\omega_1 + n \cdot \mathbf{q} + i \varepsilon} \int \frac{d\omega_2}{\omega_2 + n \cdot \mathbf{q} - i \varepsilon} g_{88}(\omega, \omega_1, \omega_2),$$

(10)

while the shape function has the following structure:

$$g_{88}(\omega, \omega_1, \omega_2) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{du}{2\pi} e^{-i\omega_2 u} \int \frac{dt}{2\pi} e^{-i\omega t} \frac{1}{M_B} (\mathcal{B}[h(tn)\ldots s(tn + u\delta)\delta(rn)\ldots h(0)||\mathcal{B}).$$

(11)

There are two remarks in order: First, all diagrams in Figure 3 reveal that if the lepton momenta were assumed to be hard, there would be no resolved contributions; The hard momentum of the leptons would
also imply a hard momentum of the intermediate parton. The latter would be integrated out at the hard
scale and the virtual photon would be connected directly to the effective electroweak interaction vertex.
Secondly, as demonstrated in Eq. (11), the shape function is nonlocal in both light cone directions.
Therefore, the resolved contributions are still nonlocal even when the hardronic mass cut is relaxed.
In this sense the resolved contributions represent an irreducible uncertainty within the inclusive decay
\( \bar{B} \to X_s \ell^+ \ell^- \).
A complete analysis of all resolved contributions to order \( O(1/m_b) \) and their phenomenological impact
within the inclusive decay \( \bar{B} \to X_s \ell^+ \ell^- \) can be found in Ref. [22].

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