Analysis of EC over Gamma Shadowed $\alpha$-$\eta$-$\mu$ Fading Channel

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Abstract - Composite fading channels are used to characterize the real time wireless communication systems, as it can account for multipath fading as well as shadowing effect on the channel. To analyse the performance of the wireless communication systems supporting real time applications effective capacity is a crucial performance measure. In this paper, analysis of effective capacity for the $\alpha$-$\eta$-$\mu$ fading channel under the shadowing effect is performed. To model shadowing gamma distribution is used. Moreover, asymptotic expressions are derived in the pursuance of obtaining insight behaviour of the channel.

Keywords - Delay constraint, effective capacity, generalized $\alpha$-$\eta$-$\mu$ fading channel, QoS provision

1. Introduction

Effective Capacity (EC) proposed by the authors in [1] is drawing a heavy attention as a key performance criteria to quantify the channel performance including Quality of Service (QoS) metric. The fading effect in the channel is one of the major reason for variation of the channel capacity. There is extensive literature available on the analysis of effective rate on to the various fading channels. EC is analysed over generalized and multiple input single output (MISO) system in [2] and [3] respectively. Authors have derived tractable expression in terms of well known mathematical function such as meijer G and fox-H. To analyse EC closed-form expressions for MISO system over the $\kappa$-$\mu$, $\eta$-$\mu$ and $\alpha$-$\mu$ fading channel is presented in [4], [5], [6]. In [7], EC of the generalized inverse gamma fading channel is studied by deriving a closed-form expression.

Enormous work has been devoted to the system EC analysis over several composite fading channels [8], [9], [10]. A generalized fading distribution $\alpha$-$\eta$-$\mu$ was introduced by the authors in [11]. The versatility of this distribution to account for the real world propagation aspects such as irregularity of the medium and number of multipath clusters have drawn keen attention to analyse the performance of communication system. Spectrum sensing study based energy detection for the gamma shadowed $\alpha$-$\eta$-$\mu$ and $\alpha$-$\kappa$-$\mu$ is presented [12]. In [13] simplified closed-form expression using fox H function over the lognormal(LN) shadowed $\alpha$-$\eta$-$\mu$ and $\alpha$-$\kappa$-$\mu$ fading channel is presented by the authors whereas ED performance is studied in [14]. On the extensive literature survey it has been found that no work has been presented on the study of the EC of composite $\alpha$-$\eta$-$\mu$ channel experiencing gamma shadowing. Motivated by this, authors here present a simple closed-form expression for the EC analysis of this composite fading channel.

This paper is systemized as follows. In section 2 channel model comprising of $\alpha$-$\eta$-$\mu$/gamma distribution is discussed. Generalized expression for EC of the mentioned composite fading channel is derived in section 3, followed section 4 comprising of asymptotic analysis in section 4. Section 5 puts light on the numerical and simulation results following with conclusion in section 6.

2. Channel Model

2.1. $\alpha$-$\eta$-$\mu$/gamma Distribution:
This distribution elucidates the non-homogeneity as well as non-linearity of the propagation channel. The composite PDF for received SNR $\gamma$ for the channel is given in [12] as

$$f_\gamma(\gamma) \approx C \gamma^{m-1} \sum_{k=1}^{K} w_k a_k e^{-b_k \gamma}$$

where $a_k$, $w_k$ and $K$ are the abscissas, weight factors, and number of Gaussian–Laguerre summation terms. So the composite PDF expression for gamma shadowed $\alpha$-$\eta$-$\mu$ channel can be given by (1) and other parameters of (1) are given as

$$C = \sqrt{\pi^2 \frac{2^m \mu^{\mu+1/2} e^{2m/\mu}}{\Gamma(\mu) \Gamma(\mu - 1/2)}}$$

The integral can be obtained by using equation 8.4.2.5 given in [15].

$$R^{\alpha-\eta-\mu/\text{gamma}} = -\frac{1}{A} \log_2 \left( C \sum_{k=1}^{K} w_k a_k \int_0^{\infty} (1 + \gamma)^{A} \gamma^{m-1} e^{-b_k \gamma} d\gamma \right)$$

The integral can be obtained by using equation 8.4.2.5 given in [15].

$$R^{\alpha-\eta-\mu/\text{gamma}} = -\frac{1}{A} \log_2 \left( C \sum_{k=1}^{K} w_k a_k \int_0^{\infty} \frac{1}{\gamma} \gamma^{m-1} e^{-b_k \gamma} G_{1,1}^{1,1} \left[ \gamma^1 - A \right] dy \right)$$

Transforming the exponential term by using equation (07.34.03.0228.01) of [16], (7) can be given as

$$R^{\alpha-\eta-\mu/\text{gamma}} = -\frac{1}{A} \log_2 \left( C \sum_{k=1}^{K} w_k a_k \int_0^{\infty} \frac{1}{\gamma} \gamma^{m-1} e^{-b_k \gamma} G_{1,1}^{1,1} \left[ \gamma^1 - A \right] dy \right)$$

Solving (8) using equation 07.34.21.0011.01 of [16] and rearranging the terms, the closed form expression for the EC of gamma shadowed $\alpha$-$\eta$-$\mu$ fading channel is given as

$$R^{\alpha-\eta-\mu/\text{gamma}} = -\frac{1}{A} \log_2 \left( C \sum_{k=1}^{K} w_k a_k \int_0^{\infty} \frac{1}{\gamma} \gamma^{m-1} e^{-b_k \gamma} G_{1,1}^{1,1} \left[ \gamma^1 - A \right] dy \right)$$

4. Asymptotic Analysis

4.1. High SNR Analysis
The closed-form expression explains limited physical significance of the quantitative effects of different parameters. So, high SNR analytical study is done to get the deeper understanding of the influence of the various parameters on the channel. Under high SNR regime where, \( \gamma \to \infty \), (2) can be reduced to

\[
R = -\frac{1}{A} \log_2 \left( \frac{\Gamma(m-A)}{b_k(m-A)} \right) \tag{10}
\]

At \( \gamma \to \infty \), EC for \( \alpha-\eta-\mu/\gamma \) is given by substituting (9) in (13) and Using equation 3.326.2 given in [17] we get

\[
R^\infty = -\frac{1}{A} \log_2 \left( C \sum_{k=1}^{\infty} w_k a_k \frac{\Gamma(m-A)}{b_k(m-A)} \right) \tag{11}
\]

4.2. Low SNR Analysis

In many wireless communication applications, power levels of the transmitted signals are considered to be very low so for the efficient design of such systems low SNR approximation provides a better insight of the system. In the low SNR sovereignty EC is approximated in the terms of transmitted normalized energy per information bit \( \frac{E_b}{N_0} \) [18] which is mathematically given as

\[
R(\frac{E_b}{N_0}) = S_0 \log_2 \left( \frac{E_b/N_0}{E_b/N_0_{min}} \right) \tag{12}
\]

where \( S_0 \) is the rate against the SNR slope and is expressed as \( E_b/N_0_{min} \) is the minimum transmit energy, needed to reliably convey non-zero transmission rate. Mathematically expressed as

\[
S_0 = -2 \left( \frac{R'(0,\theta)}{R''(0,\theta)} \right) \ln 2 \quad ; \quad E_b/N_0_{min} = \lim_{\rho \to 0} \frac{\rho}{R(\rho,\theta)} = \frac{1}{R''(0,\theta)} \tag{13}
\]

Calculating the 1st order and 2nd order partial derivatives of the EC function we get

\[
R'(0,\theta) = \frac{E(\gamma)}{\ln 2}, \quad R''(0,\theta) = \frac{A(\gamma)^2}{\ln 2} \tag{14}
\]

using equation 3.326.2 of [19] \( E(\cdot) \) is evaluated as

\[
E(\gamma^{-A}) = C \sum_{k=1}^{\infty} w_k a_k \frac{\Gamma(m-A)}{b_k(m-A)} \tag{15}
\]

By keeping \( A=-1 \) and \( A=-2 \), first and second moment of the received SNR can be obtained respectively.

\[
E(\gamma) = C \sum_{k=1}^{\infty} w_k a_k \frac{\Gamma(m+1)}{b_k(m+1)} \tag{16}
\]

\[
E(\gamma^2) = C \sum_{k=1}^{\infty} w_k a_k \frac{\Gamma(m+2)}{b_k(m+2)} \tag{17}
\]

Substituting (16) and (17) in (14) and then keeping the result in (13), \( S_0 \) can be calculated as

\[
S_0 = 2 \left( \frac{A(\gamma^2)}{C \sum_{k=1}^{\infty} w_k a_k \frac{\Gamma(m+2)}{b_k(m+2)}} \right) \tag{18}
\]

By substituting (18) in (12), the generalized expression for EC over \( \alpha-\eta-\mu/\gamma \) composite fading channels for low SNR can be given as

\[
R(\frac{E_b}{N_0}) \approx \frac{2 \log_2 \left( \frac{C \sum_{k=1}^{\infty} w_k a_k \frac{\Gamma(m+1)}{b_k(m+1)} E_b/N_0}{A} \right)}{2 \log_2 \left( \frac{C \sum_{k=1}^{\infty} w_k a_k \frac{\Gamma(m+2)}{b_k(m+2)}}{E_b/N_0} \right)} \tag{19}
\]

5. Results and Discussion

This section includes the comprehensive explanation of results obtained. The parameters \( h \) and \( H \) are evaluated using format 1 as mentioned in the analysis. The results are validated with Monte Carlo simulations using 10^6 samples. The analytical expressions obtained have been simulated using MATLAB 2015a. Fig. 1 depicts impact of fading parameter on the presented channel. It is clear from the graph that increment in the value of the fading parameters, enhances EC of the channel. This is followed by the fact
that, larger value $\alpha$ leads to larger fading gains, larger value of $\eta$ means higher power of in-phase component and $\mu$ represents the number of multipath cluster. Hence, as these parameters increases performance improves. The asymptotic curves tightly follow the analytical curves at high SNR. Fig 2 reveals profile of EC curve under the impact of the delay constraint $A$. Quantitatively, At $A=10$, the EC degrades by 1 bit/Hz/s as $\bar{\gamma}$ falls to 5 dB from 20 dB. At $\bar{\gamma} = 20$ dB as $A$ increases from 4 to 8 there is a decrement in the EC by 0.5 bits/Hz/s. On analysing the behaviour it can be clearly stated that as the delay increases the performance of the system degrades, however at a fixed delay value performance improves with increase in $\bar{\gamma}$.

Figure 3 explains how the shadowing parameter affects the channel performance. It can be observed that for the higher values of $m$ the EC high which indicates that shadowing is becoming less prominent. Also it is observed that as the $\bar{\gamma}$ is increasing the EC is also improving. Figure 4 reveals the EC behaviour in the under the low SNR condition. From the plots it can be stated that average transmit energy decreases as the shadowing index is increased. For example, as shadowing index increases to 1.5 from 1, average transmit SNR decreases by 1.3 dB also change in $A$ does not affects the EC performance in low SNR regime.

6. Conclusion

In this paper a closed-form analytical expression for the EC over the $\alpha$-$\eta$-$\mu$/gamma composite fading environment is derived. The profile of the EC for the shadowing index as well as fading parameters is comprehensively studied. The impact of the delay constraint parameter $A$ is shown in the results, which concludes that on increasing delay the EC performance of the channel ameliorates. Additionally, asymptotic analysis reveals the insight behaviour of the of the channel under the fading environment. Furthermore, results presented here can be elaborated to study over the techniques to maximize the capacity of the channel.
References

[1] Wung D and Negi R 2003 Effective capacity: A wireless link model for support of quality of service IEEE Trans. Wireless Commun. 2 630-643.

[2] Ji Z, Dong C, Wang Y and Lu J 2014 On the analysis of effective capacity over generalized fading channels IEEE International Conference on Communications (ICC), Sydney, NSW, Australia.

[3] You M, Sun H, Jiang J and Zhang J 2016 Effective rate analysis in Weibull fading channels IEEE Wireless Commun. Lett 5, 340-343.

[4] Zhang H, Tang Z, Wang H, Huang Q and Hanzo L 2014 Throughput of MISO systems over κ-μ fading channels, IEEE Trans Veh. Technol., 63, 943–947.

[5] Zhang J, Matthaiou M, Tan Z and Wang H 2013 Effective rate analysis of MISO η-μ fading channels, IEEE Int. Conf. Commun. (ICC), Budapest, Hungary.

[6] J. Zhang, L. Dai, Z. Wang, D. W. K. Ng and W. H. Gerstacker, 2015 Effective Rate Analysis of MISO Systems over α-μ Fading Channels, IEEE Global Communications Conference (GLOBECOM), San Diego, CA, USA, 6-10.

[7] You SK, Cotton SL, Sofotasios PC, Muhadait S and Karagiannidis GK, 2020 Effective Capacity Analysis Over Generalized Composite Fading Channels IEEE Access, 8, 123756-123764.

[8] Almehmadi FS, Badarneh OS. 2018 On the effective capacity of fisher–Snedecor F fading channels. Electronics Letters. 54 1068-1070.

[9] Zhang J and Gerstacker WH, “Effective capacity of communication systems over κ-μ shadowed fading channels,” Electronic Lett., vol. (19).51, p. 1540–1542, 2015.

[10] Al-Hmood H, Al-Raweshidy HS. 2019 Effective throughput of MISO systems over κ-μ shadowed fading channels: MGF based analysis. 2018 https://arxiv.org/abs/1804.10991.

[11] Fraidenraich G, Yacoub MD. The α-κ-μ and α-κ-μ fading distributions. Paper presented at: IEEE Ninth International Symposium on Spread Spectrum Techniques and Applications; August 28–31, 2006; Manaus-Amazon, Brazil: 16–20.
[12] Kumar S, Kaur M, Singh NK, Singh K, Chauhan PS. 2018 Energy detection based spectrum sensing for gamma shadowed $\alpha-\eta-\mu$ and $\alpha-\kappa-\mu$ fading channels. *AEU-Int J Electron Commun*. 93 26-31.

[13] Yadav P, Kumar R, Kumar S. 2020 Effective capacity analysis over generalized lognormal shadowed composite fading channels. *Internet Technology Letters*. e171. https://doi.org/10.1002/itl2.171

[14] Bhatt M and Soni SK 2018 A unified performance analysis of energy detector over $\alpha-\eta-\mu$/lognormal and $\alpha-\kappa-\mu$/lognormal composite fading channels with diversity and cooperative spectrum sensing *Int. J. Electron. Commun. (AEÜ)*, 367-376.

[15] Prudnikov AP, Brychkov YA, Marichev OL. 1990 *Integrals and series: more special functions*, 3. New York, NY: Gordon & Breach Sci. Publ.

[16] Wolfram. Wolfram function site. http://functions.wolfram.com/PDF/MeijerG.pdf. Accessed August 30, 2020.

[17] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed., Elsevier: Academic Press, 2007.

[18] Tulino AM, Verdú S, Lozano A. 2003 Multiple-antenna capacity in the low-power regime. *IEEE Trans Inf Theory*. 49 2527-2544.