Spontaneous Coherence and Collective Modes in Double-Layer Quantum Dot Systems

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Abstract

We study the ground state and the collective excitations of parabolically-confined double-layer quantum dot systems in a strong magnetic field. We identify parameter regimes where electrons form maximum density droplet states, quantum-dot analogs of the incompressible states of the bulk integer quantum Hall effect. In these regimes the Hartree-Fock approximation and the time-dependent Hartree-Fock approximations can be used to describe the ground state and collective excitations respectively. We comment on the relationship between edge excitations of dots and edge magneto-plasmon excitations of bulk double-layer systems.
I. INTRODUCTION

In recent years, a large body of work has been denoted to the study of double-layer two-dimensional electron systems and also to the understanding of electronic properties of two-dimensional quantum dot systems. In both cases such a huge interest has been triggered by advances in nanofabrication technology which made possible the synthesis of these artificial systems. In the case of double-layer systems, some of the most interesting novel physics has been uncovered in the strong magnetic field limit of quantum Hall effect. Recent experimental and theoretical work in such strong magnetic fields has identified an unusual type of broken symmetry responsible for an unexpected quantum Hall effect which occurs at the Landau level filling factor of the individual layers \( \nu = \nu_T/2 = 1/2 \). This broken symmetry state has *spontaneous* interlayer phase coherence, *i.e.* phase coherence even in the absence of tunneling between the quantum wells. This effect is produced by the Coulomb interaction between electrons in different layers. For quantum dot systems it is also true that most of the research in this context has been focused on electronic properties in a strong magnetic field. Much of the work on quantum dots in this regime is related to the existence of *maximum-density-droplet* (MDD) states which are the quantum dot analogs of the incompressible states responsible for the quantum Hall effect in bulk systems, and to the edge reconstructions which occur when these states become unstable. The recent experimental realization of layered quantum dot systems adds to the motivation for theoretical studies of these systems. In this article we discuss MDD states of double-layer quantum dot systems. Complementary numerical exact diagonalization study of double-layer quantum dot systems in a strong magnetic field have appeared recently and some preliminary results from the present study have been reported earlier.

The paper is organized as follows. In Section II the model used here for two-dimensional double-layer quantum dot systems is presented and discussed. In Section III the stability limits for the \( \nu_T = 1 \) MDD state are analyzed. This state is the quantum dot analog of the phase-coherent incompressible state in the bulk limit. In Section IV we discuss
collective excitations of this state with an emphasis on the interplay between the gapless edge excitations, always associated with the quantum Hall effect, and the gapless Goldstone modes of the broken-symmetry ground state in the bulk. In Section V We turn our attention to the \( \nu_T = 2 \) MDD state, which corresponds to the bulk state with a filled Landau level in each layer. Our emphasis here is on the relationship between low energy excitations of the quantum dot system and coupled edge magneto-plasmon modes of bulk double-layer systems. Our results are briefly summarized in Section VI.

II. THE MODEL

We model a double-layer system made out of two-dimensional quantum dots by assuming identical parabolic potentials, \( V(r) = \frac{1}{2} m^* \Omega^2 r^2 \), in the two layers. (In Section V it is emphasized that many of our results apply equally well to systems with bulk two-dimensional electrons in each layer.) The system is placed in a strong magnetic field \( \vec{B} \) perpendicular to the layers. We restrict our attention to the strong magnetic field limit where \( \Omega/\omega_c << 1 \), and only the states in the lowest Landau level are important. (Here \( \omega_c = eB/m^*c \) is the cyclotron frequency.) The lowest-Landau-level single-particle eigenstates in the symmetric gauge are labeled by the angular momentum \( m \):

\[
\langle \vec{r}|m \rangle = \frac{1}{\sqrt{2\pi \ell^2 2^m m!}} \left( \frac{z}{\ell} \right)^m \exp \left( -\frac{|z|^2}{4\ell^2} \right)
\]

(1)

and

\[
\varepsilon_m = \frac{1}{2} \hbar \omega_c + \gamma (m + 1)
\]

(2)

where \( \gamma = m^* \Omega^2 \ell^2 = \hbar \omega_c (\Omega/\omega_c)^2 \), \( \ell^2 \equiv \hbar c/eB \), \( \vec{r} = (x, y) \), \( z = x + iy \) is the 2D electron coordinate expressed as a complex number, and the allowed values of the single-particle angular momentum within the lowest Landau level are \( m = 0, 1, 2, \ldots \). We will assume that the electron system is completely spin-polarized by the magnetic field.

Including a phenomenological term describing tunneling between the two quantum dots
and up to an irrelevant constant, the second-quantized Hamiltonian of the system is given by

\[ H = \sum_{m\sigma} m\gamma c_{m\sigma}^\dagger c_{m\sigma} - \sum_{m\sigma\sigma'} (1 - \delta_{\sigma\sigma'}) t c_{m\sigma}^\dagger c_{m\sigma'} + \frac{1}{2} \sum_{m_1 m_2 m_1' m_2'} V_{m_1 m_2 m_1' m_2'}^\sigma c_{m_1\sigma}^\dagger c_{m_2\sigma'}^\dagger c_{m_2\sigma'} c_{m_1\sigma} \]

where

\[ V_{m_1 m_2 m_1' m_2'}^\sigma = \langle m_1' m_2' | V_0 | m_1 m_2 \rangle + \sigma \sigma' \langle m_1' m_2' | V_z | m_1 m_2 \rangle, \]

and \( t \) is the hopping amplitude between the two layers. The layer index is \( \sigma = \uparrow, \downarrow \) where invoking a helpful analogy between the layer degree of freedom and the spin degree of freedom, \( \sigma = \uparrow \) corresponds to electrons in the right layer and \( \sigma = \downarrow \) to electrons in the left layer. In Eq. 4, \( V_0 = (V_A + V_E)/2 \) and \( V_z = (V_A - V_E)/2 \), are proportional to the sum and difference of intra-layer \( (V_A) \) and interlayer \( (V_E) \) Coulomb interactions.

For \( \nu_T = 1 \) the ground state of the quantum dot in the Hartree-Fock approximation has its pseudospin polarized in the \( \hat{x} - \hat{y} \) pseudospin plane. Physically, electrons in this state occupy states which are a coherent linear combination of states localized in the separate layers. In order to study this state it is convenient to transform to a new representation, carrying out a rotation in pseudospin space by defining

\[ c_{m\sigma}^\dagger = \frac{1}{\sqrt{2}} (\sigma \alpha_{m\sigma}^\dagger + \alpha_{m\sigma}^\dagger). \]

In this representation, a pseudospin up electron is in a symmetric double-layer state and a pseudospin down electron is in an antisymmetric state. The Hamiltonian can alternately be expressed in the form:

\[ H = \sum_{m\sigma} (m\gamma - \sigma t) \alpha_{m\sigma}^\dagger \alpha_{m\sigma} + \frac{1}{2} \sum_{m_1 m_2 m_3 m_4 \sigma \sigma'} \langle m_1 m_2 | V_0 | m_3 m_4 \rangle \alpha_{m_1\sigma}^\dagger \alpha_{m_2\sigma'}^\dagger \alpha_{m_3\sigma'} \alpha_{m_4\sigma} \]

\[ + \langle m_1 m_2 | V_z | m_3 m_4 \rangle \alpha_{m_1\sigma}^\dagger \alpha_{m_2\sigma'}^\dagger \alpha_{m_3\sigma'} \alpha_{m_4\sigma}, \]

where \( \sigma = \downarrow, \uparrow \) for \( \sigma = \uparrow, \downarrow \). In this rotated pseudospin representation the hopping parameter \( t \) simply acts as an external magnetic field. We will discuss here only the limit \( t \rightarrow 0 \).
III. THE STABILITY OF MDD STATE

For \( N \) non-interacting electrons in a single-layer quantum dot, the many-body ground state is a single Slater determinant in which the confinement energy is minimized by occupying the orbitals from \( m = 0 \) to \( m = N - 1 \). This state is an exact many-body eigenstate of the Hamiltonian even when electron-electron interactions are included.\(^1\) We refer to this state as the maximum density droplet (MDD) state. The MDD state is the analog for quantum dots of the bulk \( \nu = 1 \) state. For double layers a new type of MDD state can occur which is the analog of the bulk double-layer \( \nu_T = 1 \) broken-symmetry\(^2\) ground state.\(^3\),\(^4\),\(^5\),\(^6\),\(^8\),\(^9\) In this state electrons occupy symmetric states (pseudospin up) with \( m = 0 \) to \( m = N - 1 \) such that the total angular momentum is \( M_0 = N(N - 1)/2 \). Many-body states with smaller total angular momentum than this MDD state will have the advantage of smaller confinement energies since the electrons are closer to the minimum of the confinement potential, but the disadvantage of larger interaction energies since the electrons are closer to each other. For weak confinement the angular momentum of the ground state of an interacting-electron droplet will occur at \( M > M_0 \) in order to reduce the Coulomb energy while for sufficiently strong confinement the ground state will occur at \( M < M_0 \) in order to reduce the confinement energy.

A necessary condition for the MDD state to be the ground state is that all occupied orbitals have a Hartree-Fock quasiparticle energy which is lower than the quasiparticle energies of all unoccupied orbitals. Using Eq. (6) we have found that the Hartree-Fock quasiparticle energies for the symmetric MDD state are given by

\[
\varepsilon_{m\sigma}^{HF} = m\gamma - \sigma t + \sum_{m'} n_{m'\sigma} U_{mm'}^0 + \sum_{m'} n_{m'\sigma} U_{mm'}^z
\]  

(7)

where

\[
U_{mm'}^0 = \langle mm' | V_0 | mm' \rangle - \langle m'm | V_0 | mm' \rangle,
\]  

(8)

\[
U_{mm'}^z = \langle mm' | V_z | mm' \rangle - \langle m'm | V_z | mm' \rangle.
\]  

(9)

For the symmetric MDD state \( n_{m\sigma} \) is 1 for \( \sigma = \uparrow \) and \( m < N \), and it is zero otherwise.
Fig. 1a shows Hartree-Fock quasiparticle energies for symmetric and antisymmetric orbitals for $N = 20$, $t = 0$, and $d = \ell$ at three different confinement strengths. The exchange splitting between the occupied symmetric orbitals and the unoccupied antisymmetric orbitals is due to interlayer interactions and decreases in magnitude as $d$ increases or as $m$ increases toward the edge of the dot. For $\gamma/(e^2/\epsilon\ell) = 0.06$ the confinement potential is relatively weak and unoccupied symmetric orbitals near the edge of the dot are lower in energy than occupied symmetric orbitals near the center of the dot. For $\gamma/(e^2/\epsilon\ell) = 0.12$ the confinement potential is now strong and unoccupied antisymmetric orbitals near the center of the dot are lower in energy than occupied symmetric orbitals near the edge of the dot. These results show that there is a finite interval of confinement strengths, including the case $\gamma/(e^2/\epsilon\ell) = 0.10$, for which the MDD state is stable. Fig. 1b shows a phase diagram constructed for $N = 20$ using this quasiparticle energy stability criterion, which shows how the interval where the confinement potential strength renders the MDD state stable nar-
rows and shifts as a function of layer separation. (The actual stability region for the MDD state beyond the Hartree-Fock approximation will be slightly narrower because of excitonic corrections to particle-hole excitation energies). For \( d \to 0 \) interlayer and intralayer interactions become identical and these results would apply equally well to the \( \nu_T = 1 \) state for a single-layer system in the limit of vanishing Zeeman coupling (Note that we assume strong Zeeman coupling for the double-layer systems discussed in this paper). For larger \( N \) the region of stability of a parabolically confined MDD droplet becomes smaller; for very large \( N \) the electron density in a parabolic external potential must follow the semi-elliptic behavior preferred by electrostatics rather than the nearly constant charge density profile of the MDD state. For sufficiently large \( N \) the MDD state is never stable in a parabolic external potential and can be stabilized only by the potential from an approximately neutralizing positively charged background.

IV. COLLECTIVE EXCITATIONS

In the time-dependent Hartree-Fock approximation, elementary excitations of the MDD state are constructed by allowing its particle-hole excitations to couple. The Hamiltonian for the double-layer system does not mix states in which the number of particles in symmetric (pseudospin \( \uparrow \)) states differs by an odd integer; elementary excitations with even and odd numbers of pseudospin reversed states do not mix. In order to describe the odd excitations we introduce the operators

\[
\rho_{-M,m}^- = \alpha_{m+M,\downarrow}^+ \alpha_{m,\uparrow} \quad \text{and} \quad \rho_{-M,m}^+ = \alpha_{m,\uparrow} \alpha_{m-M,\downarrow}^+.
\]  

(10)

The equations of motion for these operators are readily derived and reduce to a closed system of equations after Hartree-Fock factorization:

\[
\begin{bmatrix}
[H, \rho_{-M,m}^-] \\
[H, \rho_{-M,m}^+]
\end{bmatrix} =
\begin{bmatrix}
E_{mm'}^+ & F_{mm'}^M \\
-F_{mm'}^{-M} & E_{mm'}^M
\end{bmatrix}
\begin{bmatrix}
\rho_{-M,m'}^- \\
\rho_{M,m'}^-
\end{bmatrix}
\]  

(11)

where
\[ E_{mm'}^M = \delta_{mm'}(\varepsilon_{m+M\downarrow}^{HF} - \varepsilon_{m\uparrow}^{HF}) + \]
\[ (\langle m, m' + M | V_z | m + M, m' \rangle - \langle m' + M, m | V_0 | m + M, m' \rangle) \]  \hspace{1cm} (12) \]

and
\[
F_{mm'}^M = \langle mm' | V_z | m + M, m' - M \rangle - \langle m'm | V_z | m + M, m' - M \rangle. \]  \hspace{1cm} (13) \]

Diagonalizing the above matrix will give us the time-dependent Hartree-Fock approximation (TDHFA) pseudospin-flip elementary excitations of the MDD state. If these operators evolved according to the mean-field Hartree-Fock Hamiltonian for the ground state, the excitation energies would simply equal the difference of occupied and unoccupied Hartree-Fock eigenvalues. The additional terms in Eq. (11) reflect the changes in the mean-field Hamiltonian in the excited state and, in a diagrammatic derivation of the TDHFA, would arise from vertex and repeated-bubble corrections to the single-loop approximation for the pseudospin-flip response function. Below we refer to the influence of these additional terms as vertex corrections.

The even elementary excitations are found by considering the equation of motion for the operator \( \rho^0_{M,m} = \alpha_{m+M,\uparrow}^\dagger \alpha_{m,\uparrow} \). Hartree-Fock factorization of the equation of motion of these operators again gives a closed set of equations:
\[
[H, \rho^0_{M,m}] = G^M_{mm'} \rho^0_{M,m'}. \]  \hspace{1cm} (14) \]

Diagonalizing the matrix \( G^M_{mm'} \) gives the even elementary excitations of the MDD state. In Eq. (14)
\[
G^M_{mm'} = \delta_{mm'}(\varepsilon_{m+M\uparrow}^{HF} - \varepsilon_{m\uparrow}^{HF}) + \]
\[ (\langle mm' + M | V_0 | m + Mm' \rangle - \langle m' + Mm | V_0 | m + Mm' \rangle). \]  \hspace{1cm} (15) \]

Again vertex corrections cause the excitation energies to differ from the difference of Hartree-Fock eigenvalues.
FIG. 2. Isospin-flip excitations at $\nu_T = 1$ with vertex correction and without vertex correction for a double-layer quantum dot system with $N = 20$ electrons, interlayer distance $d = \ell$, and confinement potential $\gamma = 0.10e^2/\ell$.

Fig. 2 shows the odd elementary excitations obtained for layer separation $d = \ell$, $N = 20$, and confinement potential $\gamma = 0.10(e^2/\ell)$ as a function of angular momentum $M$. Note that for each $M$ the excitation energy has a contribution $M\gamma$ from the confinement potential. The number of particle-hole excitations with angular momentum change $M$ is $N$ for $M \geq 0$ and $N + M$ for $M < 0$. (The minority-spin angular momentum cannot be negative.) When vertex corrections are neglected, the lowest excitation energies occur at the most negative angular momentum and correspond to transferring an electron from pseudospin up at the edge of the quantum dot to pseudospin down at the center of the quantum dot. Vertex corrections change the elementary excitation spectrum qualitatively. The most obvious modification
is the appearance of a zero-energy excitation for $M = 0$ which we discuss below. The $M = -N + 1$ excitation energy is also reduced by vertex corrections; with increasing $\gamma$ the energy of this excitation will become negative before the quasiparticle energy stability criterion is violated so that the region of stability of the MDD state is narrower than implied by this criterion. A similar reduction in the excitation energies occurs at positive $M$.

In bulk systems the broken symmetry ground state of the $\nu_T = 1$ double-layer system has a gapless Goldstone mode\cite{footnote} with linear dispersion (It can be shown that the $M = 0$ zero-energy elementary excitation just corresponds to a global rotation of the pseudospin). Some remnant of these Goldstone modes should be present in the excitation spectrum of finite-size quantum dot systems. In the limit of large dots, we expect that $M/R$ should act like the azimuthal component of a two-dimensional wave vector and that the many collective modes that occur at a given $M$ are approximately related to the discrete set of radial wavevectors geometrically defined by the finite radius $R_N \approx \sqrt{2N\ell}$ of the quantum dot which is related, at least approximately, to the zeroes of $J_0(qR_N)$. We have, however, not yet been able to find a completely satisfactory semiclassical interpretation of our microscopic TDHFA results in this way. For example the pronounced asymmetry between the low-energy TDHFA collective modes for $M < 0$ and $M > 0$.

The even elementary excitations represent charge-density-wave edge excitations of the incompressible MDD state and occur only for $M > 0$. In Fig. 3 we show TDHFA results calculated for $N = 20$, $\gamma = 0.1(e^2/\ell\epsilon)$, and $d = \ell$. Again the excitation energies are substantially reduced by vertex corrections. These results are identical to those that would be obtained for a single-layer system with an interaction which is the average of the double-layer intra-layer and inter-layer interactions. As discussed in detail previously for the single-layer case\cite{footnote} the $M = 1$ excitation energy is exactly equal to $\gamma$ and is not influenced by electron-electron interactions. These charge density wave excitations of the quantum dot are harbingers of the edge magneto-plasmon excitations of bulk systems which we discuss at greater length in the following section. With decreasing confinement strength a collective mode will become unstable slightly before the quasiparticle-energy stability criterion is vio-
lated, narrowing the region of stability of the MDD state on the weak confinement side just as it is narrowed on the strong confinement side. Very recently\textsuperscript{21} for the $d = 0$ case, Sondhi \textit{et al.} have identified topological edge instabilities which further limit the stability region of MDD states.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Edge Charge density-wave edge excitations with vertex correction and without vertex correction at $\nu_T = 1$ for a double-layer quantum dot system with $N = 20$ electrons and interlayer distance $d = \ell$ and confinement potential $\gamma = 0.10(e^2/\ell)$.}
\end{figure}

\textbf{V. EDGE MAGNETO-PLASMONS AND $\nu_T = 2$ DOUBLE-LAYER QUANTUM DOTS}

The quantum dot analog of the bulk $\nu = 2$ state in a double-layer system is the single Slater determinant state in which the single-particle angular momentum states from $m = 0$ to $m = N - 1$ are occupied in each layer. It can be shown that this state is a pseudospin
singlet. In quantum dot systems this state becomes unstable with increasing confinement strength only when it becomes energetically favorable to occupy higher Landau levels, and becomes unstable with decreasing confinement strengths when it becomes favorable to move electrons away from the center of the quantum dot, for example by edge reconstruction. For parabolic confinement, the region of stability depends on $d$ and narrows with increasing $N$. The excitation spectrum of this state is less subtle than for the $\nu = 1$ case discussed above because a gap $\sim \hbar \omega_c$ exists in the bulk for both pseudospin-flip and charge density excitations. The only low-energy excitations are localized at the edge of the system.

In this section we work in the representation where pseudospin eigenstates are localized in individual layers. We consider the equations of motion for the operators which create in phase and out-of-phase density wave excitations at the edges of the two-layers:

$$\rho_{m,M}^0 = \sum_{\sigma} c_{m+M,\sigma}^\dagger c_{m,\sigma}$$  \hspace{1cm} (16)

and

$$\rho_{m,M}^z = \sum_{\sigma} \sigma c_{m+M,\sigma}^\dagger c_{m,\sigma}$$  \hspace{1cm} (17)

where $m < N$ and $m + M \geq N$. After a Hartree-Fock factorization the equations of motion for these operators close and a calculation very similar to that detailed in Section IV gives the following result:

$$[\mathcal{H}, \rho_{m,M}^0] = (\epsilon_{m+M}^{HF} - \epsilon_{m}^{HF})\rho_{m,M}^0$$ 

$$+ \sum_{m'} \rho_{m',M}^0 [2\langle m, m' + M|V_0|m + M, m'\rangle - \langle m, m' + M|V_A|m', m + M\rangle]$$  \hspace{1cm} (19)

and

$$[\mathcal{H}, \rho_{m,M}^z] = (\epsilon_{m+M}^{HF} - \epsilon_{m}^{HF})\rho_{m,M}^z$$ 

$$+ \sum_{m'} \rho_{m',M}^z [2\langle m, m' + M|V_z|m + M, m'\rangle - \langle m, m' + M|V_A|m', m + M\rangle].$$  \hspace{1cm} (21)

In this case the single-particle Hartree-Fock energies,

$$\epsilon_{m}^{HF} = \gamma (m + 1) + \sum_{m'} [2\langle m, m'|V_0|m, m'\rangle - \langle m, m'|V_A|m', m\rangle],$$  \hspace{1cm} (22)
are pseudospin independent.

![Graph showing spin-density wave edge excitations at \( \nu_T = 2 \) for a double-layer system with \( N = 20 \), interlayer distance \( d = \ell \), and confinement potential \( \gamma = 0.20(e^2/\ell) \).](image)

**FIG. 4.** Spin-density wave edge excitations at \( \nu_T = 2 \) for a double-layer system with \( N = 20 \), interlayer distance \( d = \ell \), and confinement potential \( \gamma = 0.20(e^2/\ell) \).

Illustrative TDHFA quantum dot collective excitation energies at \( \nu_T = 2 \), evaluated using these expressions, are shown in Figs. 4 and 5. We comment primarily on the \( M = 1 \) results which we relate to edge magneto-plasmons of bulk double-layer systems below. For \( M = 1 \), \( m = N - 1 \) in both Eq. (16) and Eq. (17), explicit expressions can be given for the excitation energies:

\[
E_1^{(0)} = \gamma + \sum_{m'=0}^{N-1} \left[ 2 \langle N, m' | V_0 | N, m' \rangle - \langle N, m' | V_A | m', N \rangle \right] \\
- \sum_{m'=0}^{N-1} \left[ 2 \langle N-1, m' | V_0 | N-1, m' \rangle - \langle N-1, m' | V_A | m', N-1 \rangle \right] \\
+ 2 \langle N-1, N | V_0 | N-1 \rangle - \langle N-1, N | V_A | N-1 \rangle, 
\]  

(23)
and

$$E_1^{(z)} = \gamma + \sum_{m'=0}^{N-1} [2\langle N, m'|V_0|N, m'\rangle - \langle N, m'|V_A|m', N\rangle]$$

$$- \sum_{m'=0}^{N-1} [2\langle N - 1, m'|V_0|N - 1, m'\rangle - \langle N - 1, m'|V_A|m', N - 1\rangle]$$

$$+ 2\langle N - 1, N|V_z|N, N - 1\rangle - \langle N - 1, N|V_A|N - 1, N\rangle. \quad (24)$$

FIG. 5. Charge-density wave edge excitations at $\nu_T = 2$ for a double-layer system with $N = 20$, interlayer distance $d = \ell$, and confinement potential $\gamma = 0.20(e^2/\ell)$.

The in-phase mode has interaction corrections from the difference of the interaction contribution to the Hartree-Fock single-particle energies at $m = N$ and $m = N - 1$ and also from vertex corrections. In the case of single-layer quantum dots these two classes of corrections are known to cancel exactly. The easiest way to establish this property in the single layer case is to note that the $M = 1$ excitation operator simply changes the state
of the center-of-mass of all electrons without changing the relative motion state to which interactions are sensitive. For $E^{(0)}_1$ the same argument goes through in the double layer case and only the external potential contribution to the excitation energy survives:

$$E^{(0)}_1 = \gamma.$$  \hspace{1cm} (25)

This cancelation is verified numerically as seen in Fig. 4. Eq. (25) can also be derived from the following identities which are established for interaction matrix elements in the lowest Landau level:

$$\sum_{m'=0}^{N-1} \langle N, m' | V | N, m' \rangle - \sum_{m'=0}^{N-1} \langle N-1, m' | V | N-1, m' \rangle = -\langle N-1, N | V | N, N-1 \rangle \hspace{1cm} (26)$$

and

$$\sum_{m'=0}^{N-1} \langle N, m' | V | m', N \rangle - \sum_{m'=0}^{N-1} \langle N-1, m' | V | m', N-1 \rangle = -\langle N-1, N | V | N-1, N \rangle.\hspace{1cm} (27)$$

These identities hold for any interaction potential and in particular for $V = V_0$ or $V = V_A$.

It follows from Eq. (26) and Eq. (27) that

$$E^{(z)}_1 = \gamma - 2\langle N-1, N | V_E | N, N-1 \rangle.\hspace{1cm} (28)$$

This simple result has also been verified numerically.

These $M = 1$ edge excitations are the quantum dot precursors of the edge magneto-plasmon collective excitations of larger $N$ electronic systems in strong magnetic fields which give rise to far-infrared resonances in micron scale systems and to surprisingly sharp radio-frequency resonances in large area 2DEG’s. Some helpful remarks on edge magneto-plasmons in double-layer systems follow from the above analysis. As we have emphasized, larger $N$ MDD states are bulk quantum Hall states and they will occur only when the external potential is quite close to that from a neutralizing background of positive charge. For a background charge density which neutralizes the electron density in each layer it follows from Eq. (26) that the difference between the expectation values of the external potential for the states with angular momenta $N$ and $N - 1$ is...
\[ \tilde{\gamma} = \langle N - 1, N | V_A + V_E | N, N - 1 \rangle \approx e^2 \frac{\pi}{R} [\ln(R/\ell) + \ln(R/d)]. \]  

(29)

The final form for \( \tilde{\gamma} \) assumes an electron disk whose radius \( R \) is large compared to both the magnetic length \( \ell \) and the layer separation \( d \). (We assume that \( d \) is at least \( \sim \ell \).) The two terms in square brackets in Eq. (29) come respectively from intra-layer and inter-layer interactions with the background. As discussed above, the energy of the in-phase collective magnetoplasmon excitations of the double-layer system \( E_{mpl}^+ \) depends only on the external potential and is therefore given by \( E_{mpl}^+ = \tilde{\gamma} \). Since \( \langle N - 1, N | V_E | N, N - 1 \rangle \sim (e^2/\pi R) \ln(R/d) \) for \( R \gg d \) it follows that the energy of the out-of-phase mode is

\[ E_{mpl}^- = e^2 \frac{\pi}{R} [\ln(R/\ell) - \ln(R/d)]. \]  

(30)

For large 2DEG’s the splitting of the edge magnetoplasmon energies due to coupling between the layers persists to very large layer separations; for example \( E_{mpl}^+ = 3E_{mpl}^- \) for \( d = (R\ell)^{1/2} \).

When the separation between the two layers is comparable to \( \ell \), often a requirement for observable coupling effects in bulk systems, the edge magnetoplasmon coupling will typically make \( E_{mpl}^- \) unobservably small. We remark that these results have been derived for disorder-free systems. We expect that these results for edge magnetoplasmon energies will remain valid in disordered systems provided that the microscopic length scale \( \ell \), which describes the degree of localization of the edge wave in the clean limit, is replaced by the appropriate disorder-dependent length.

**VI. SUMMARY**

We have studied double-layer quantum dot systems focusing on the MDD droplet states corresponding to the bulk incompressible quantum Hall states at \( \nu = 1 \) and \( \nu = 2 \). Ground states and collective excitations have been approximated using Hartree-Fock and time-dependent-Hartree-Fock approximations. The Hartree-Fock \( \nu = 1 \) MDD ground states have only symmetric double-layer orbitals occupied and have spontaneous interlayer phase coherence. The regime of stability of this state has been estimated as a function of layer
separation and confinement potential strength. The broken symmetry in this state gives rise to low-energy excitations geometrically confined ‘Goldstone’ collective excitations of the quantum dot. The Hartree-Fock $\nu = 2$ MDD ground state has orbitals in both layers occupied. The only low-energy collective excitations of this system are localized near the edge of the system and correspond to coupled magnetoplasmon excitations of bulk double-layer systems. We have used our microscopic calculations for quantum dots to derive expressions for the magnetoplasmon energies of bulk systems.

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REFERENCES

1 See for example J. Smoliner, E. Gornik and G. Weimann, Appl. Phys. Lett. 52, 2136 (1988); T.J. Gramila, J.P. Eisenstein, A.H. MacDonald, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 66, 1216 (1991); Phys. Rev. B 47, 12957 (1993); U. Sivan, P.M. Solomon, and H. Shtrikman, Phys. Rev. Lett. 68, 1196 (1992); P.J. Price, Physica B 117, 750 (1983); H.C. Tso, P. Vasilopoulos, and F.M. Peeters, Phys. Rev. Lett. 68, 2516 (1992); A.-P. Jauho, and H. Smith, Phys. Rev. B 47, 4420 (1993); L. Zheng, and A.H. MacDonald, Phys. Rev. B 48, 8203 (1993); K. Flensberg and Ben Yu-Kuang Hu, Phys. Rev. Lett. 73, 3572 (1994); J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. B 50, 1760 (1994); J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 68, 674 (1992).

2 For reviews see U. Merkt, Advances in Solid State Physics, 30, 77 (1990); Tapash Chakraborty, Comments on Condensed Matter Physics 16, 35 (1992); M.A. Kastner, Rev. Mod. Phys. 64, 849 (1992).

3 S. M. Girvin and A. H. MacDonald, in Novel Quantum Liquids in Low-Dimensional Semiconductor Structures, edited by Sankar Das Sarma and Aron Pinxzuk (Wiley, New York, 1995); and references therein.

4 S.Q. Murphy, J.P. Eisenstein, G.S. Boebinger, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 72, 728 (1994).

5 H. A. Fertig, Phys. Rev. B 40, 1989 (1087).

6 X. G. Wen and A. Zee, Phys. Rev. Lett. 69 1992 (1811); X. G. Wen and A. Zee, Phys. Rev. B 47, 1993 (2265).

7 A. H. MacDonald, P. M. Platzman and G. S. Boebinger, Phys. Rev. Lett. 65, (1990) 775.

8 Kun Yang, K. Moon, L. Zheng, A.H. MacDonald, S.M. Girvin, D. Yoshioka, and Shou-Cheng Zhang, Phys. Rev. Lett. 72, 732 (1994).

9 K. Moon et al Phys. Rev. B 51, 1995 (5138).
10 P. L. McEuen et al., Phys. Rev. Lett. 66, 1926 (1991); R. C. Ashoori et al., Phys. Rev. Lett. 71, 613 (1993).

11 A. H. MacDonald, S. R. Eric Yang and M. D. Johnson, Aust. J. Phys. 46, (1993) 345.

12 Strictly speaking when we go beyond the Hartree-Fock approximation the broken symmetry occurs only in the $N \to \infty$ limit.

13 P. Hawrylak, Phys. Rev. Lett. 71, 3347 (1993); J. J. Palacios et al., Phys. Rev. B 50, 5760 (1994); S. R. Eric Yang, and M. D. Johnson, Phys. Rev. Lett. 71, 3194 (1993).

14 C. de Chamon and X.-G. Wen, Phys. Rev. B 49, 8227 (1994); O. Klein, C. de Chamon, D. Tang, D. M. Abusch-Magder, X.-G. Wen, and M. A. Kastner, Phys. Rev. Lett. 74, 785 (1995); J. Dempsey, B. Y. Gelfand, and B. I. Halperin, Phys. Rev. Lett. 70, 3639 (1993).

15 G. S. Solomon, J. A. Trezza, A. F. Marshall, and J. S. Harris Jr., Phys. Rev. Lett. 76, 952 (1996) and work cited therein.

16 J. J. Palacios and P. Hawrylak, Phys. Rev. B 51, 1995 (1769); Hiroshi Imamura, Peter A. Maksym, and Hideo Aoki, preprint [cond-mat/9602120] (1996).

17 J. Hu, E. Dagotto, and A. H. MacDonald, Physical Phenomena At High Magnetic Fields II, edited by Z. Fisk, L. P. Gor’kov, D. Meltzer and J. R. Schrieffer. (World Scientific, 1996)

18 S. S. Nazin and V. B. Shikin, Zh. Eksp. Fiz. 85, 530 (1983) [Sov. Phys. JETP 58, 210 (1983)]; V. B. Shikin, T. Demel’ and D. Heitmann, Zh. Eksp. Teor. Fiz. 96, 1406 (1989) [Sov. Phys. JETP 69, 797 (1989)]; D. B. Chklovskii, B. I. Shklovskii, and L. I. Glazman, Phys. Rev. B 46, 4026 (1992); M. M. Fogler, E. I. Levin, and B. I. Shklovskii, Phys. Rev. B 49, 13767 (1994)

19 R. B. Laughlin, Phys. Rev. B 23, 5632 (1981); B. I. Halperin, Phys. Rev. B 25, 2185 (1982); A. H. MacDonald and P. Streda, Phys. Rev. B 29, 1616 (1984); M. Buttiker, Phys. Rev.
B 38, 9375 (1988); X.G. Wen, Phys. Rev. B 41, 12838 (1990); D.H. Lee and X.G. Wen, Phys. Rev. Lett. 66, 1765 (1991); X.G. Wen, Phys. Rev. B 44, 5708 (1991). X.G. Wen, Int. J. Mod. Phys. B6, 1711 (1992); A.H. MacDonald, Phys. Rev. Lett. 64, 222 (1990).

20 A. H. MacDonald, *Surface Science* 229, (1990) 1.

21 S.L. Sondhi, A. Karlhede, and S.A. Kivelson, Bul. Am. Phys. Soc. 41, 482 (1996).

22 S.J. Allen, H.L. Störmer, and J.C.M. Hwang, Phys. Rev. B 28, 4875 (1983); D.B. Mast, A.J. Dahm, and A.L. Fetter, Phys. Rev. Lett. 54, 1706 (1985); S.A. Govorkow *et al*., Pis’ma Zh. Eksp. Teor. Fiz. 44, 380 (1986) [Sov. Phys. JETP Lett. 44, 487 (1986)]; M. Wassermeyer *et al*., Phys. Rev. B 41, 10287 (1990) and work cited therein.

23 See V.A. Volkov and S.A. Mikhailov, Zh. Eksp. Teor. Fiz 94, 217 (1988) [Sov. Phys. JETP 67, 1639 (1988)] and work cited therein.