Empirical frame hyper-surfaces as models of multi-parametrical technological systems

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Abstract. This paper is devoted to the mathematical and computer simulation of multi-parametrical technological systems. The method and algorithm can easily be used in spline-frame simulation of the systems. Simulation is based on experimental data and achieved by the variation of one-dimensional spline approximations. A set of variable one-dimensional cubic spline-frames generate the model of hyper-surface, which is a model of the process. Each of the spline-frames is the image of a section of input parameters area. Software realization is based on the single algorithm that is used repeatedly. The problem of building some model of hyper-surface is based on empirical irregular 0-dimensional frame. We consider the problem as an inverse problem of modeling. The method has been used in investigation some technological conditions of thread seams for sewing industry. We investigated the durability and harshness of the seam. Geometric models and parameter valuations were generated by special software. The equations and diagrams described our experiments with sufficient exactness and there were used for researching the process. The model together with software HYPER-DESCENT may be applied for simulation multi-parametrical systems or technological processes of light industry.

1. Introduction

Technological processes in various fields of industry are always considered as multi-parametrical systems. It is impossible to research such systems and processes without correct mathematical models. There are many mathematical and geometric methods of modeling, but it is usually difficult to find a proper model for given multi-parametrical process. Therefore, the quest for new models and development of existing ones are an actual problem at present. Some new promising resources are opened thanks to the development of computational technologies and computer visualization [1].

Traditional approaches to modeling the systems presuppose availability the following data:
1. Some multi-dimensional area of input parameter variations.
2. Allocation the nets of input parameters inside the area.
3. Selection principles for rather simple and adequate functions from some classes.
If all these functions circumscribe the output parameters with given precision, they may be considered as models of the system. The nets of input parameters are usually considered as point and regular nets. However, to find a proper model it is necessary to solve a lot of problems. For example, there exists the problem of selection. To solve this problem it is essential to have maximum information about technological essence of the process a priory. If the system or the process belongs to multi-parametrical one the problem of preliminary data analyses arises and it is rather hard to solve it formally. If several models describe the same system or the same process it is necessary to choose an optimal model by means a certain criterion [2 – 4]. These problems and difficulties force researchers to find other approaches. Approach which is proposed in the paper has the purpose to avoid partly all these difficulties.

The suggested method refers to frame modeling of multi-dimensional surfaces since it is based on theoretically developed spline approximation. However after introducing slight modifications it can be used for other geometric objects, too.
2. Object of research
As an object of investigation we chose thread seams which were produced by shuttle stitches on the sewing machine DDL – 8100e Juke. The pressure of working part and diameter of sewing needle (№ 90) were constant.

Patterns of stitch seams were executed by sewing threads 25LL, 35LL, 45LL, 70 LL. The seams were carried out along and across the base threats, and at 45° angles to the base threats. The numbers of stitches were 2 – 3, 3 – 4, 5 – 6 by 1 sm. Allowance size was equal to 10 mm by the threat.

Fabric characteristics were as follows:
1. Filamentous compositions were: NPF on the base and PR/cotton on the others.
2. Surface thickness was equal to 213 g/sm².
3. Weave was linen.
4. Durability (P) on the base was equal to 180 kg.
5. Harshness (EI) on the base was equal to 17764 mcN·sm².

All properties of sewing machine PT – 250M and harshness was examined by apparatus PT – 2 according to standards.

Geometric models and parameter valuations were generated by special software [5]. The equations and diagrams described our experiments with sufficient exactness and there were used for researching the process.

3. General considerations
In this paper we consider (n + k)-dimensional Euclidean space \( X^n \times Y^1 \times \ldots \times Y^k \) together with the following four objects:
1. The hyper-plane \( X^n = X^1 \times \ldots \times X^n \) of input parameters.
2. An area \( D = \bigcup D_i, D_i \subset X^1, D_i = \{ x_i \in R : x_{i, \text{min}} \leq x_i \leq x_{i, \text{max}} \} \), i = 1, ..., n of input parameter variations.
3. An area \( E_k \subset Y^1, E_k = \{ y_k \in R : y_{k, \text{min}} \leq y_k \leq y_{k, \text{max}} \} \) of output parameter variations.
4. Some hyper-surfaces \( y_k = f_k(x_1, \ldots, x_n) \).

The space of input parameters \( D \) is usually considered together with a discrete regular point nets \( x_{i, \text{min}} = x_{i, 1} < x_{i, 2} < \ldots < x_{i, m(i)} - 1 < x_{i, m(i)} = x_{i, \text{max}} \)

where all numerical values \( m(i) \) may be depend upon the values of i. The space of output parameters \( E \) is determined by means of experimental data and it is unknown beforehand. Extreme values \( y_{k, \text{min}} \) and \( y_{k, \text{max}} \) are determined as a result of decision an extreme problem.

The hyper-surfaces \( y_k = f_k(x_1, \ldots, x_n) \) are given as a discrete sets of points
\[
y(j_1), \ldots, j(n), k = f_k(x_1, j(1), \ldots, x_n, j(n)), 1 \leq j(i) \leq m(i).
\]

These points form irregular nets in \( D \times E_k \).

Analytical models of hyper-surfaces are usually generated by approximation of input data [6, 7]. In our case we may consider the problem of approximation as a problem of regularization and also as researching of some regular mapping \( D \rightarrow D \times E_k \). This mapping transforms the irregular net to a regular one.

4. Direct and inverse problems of modeling
Let’s assume that hyper-surface \( y = f(x_1, \ldots, x_n) \) is given analytically in the space \( D \). We may consider the hyper-surface as regular one. Then we can define it by means of some discrete p-dimensional frame, \( 0 \leq p \leq n - 1 \). We must have in mind that there is infinite number of modes to build the frame for each value of \( p \).

For example, if \( p = n - 1 \) we may build the following frames:
\[
y = f(x_1, \ldots, x_{n-1}, a_1(x_n), a_2(x_n), \ldots),
y = f(x_1, \ldots, x_{n-2}, x_n, a_1(x_{n-1}), a_2(x_{n-1}), \ldots),
\]
These frames are \((n-1)\)-dimensional level surfaces. If \(p = n - 2\) we may receive the following frames:

\[
y = f(x_1, \ldots, x_{n-2}, a_1(x_{n-1}, x_n), a_2(x_{n-1}, x_n), \ldots),
\]

\[
y = f(x_3, \ldots, x_n, a_1(x_1, x_2), a_2(x_1, x_2), \ldots).
\]

And so on.

We consider the simulation of some hyper-surface by means of \(p\)-dimensional frames as a direct problem. It is characterized by the following properties:
1. The property of regularity for \(p\)-dimensional frame, \(0 \leq p \leq n - 1\).
2. The frame may be tightened up to infinity.
3. There exists the possibility to diminish the space of input parameters.

If we can to describe \(p\)-dimensional frame as an \((n-p)\)-parametric family of \(p\)-surfaces of the same degree, we have a regular frame.

Let’s assume that hyper-surface \(y = f(x_1, \ldots, x_n)\) is given by irregular net of experimental points in the space \(D \times E\). This net may be considered as empirical 0-dimensional frame. Simulation of multi parametrical technological processes compels us to deal with precisely such frames. The empirical frames having the properties of regularity are not typical ones into practice. Fig. 1 shows the differences between some theoretical regular frame and empirical irregular one. All attempts to stretch some regular surface on an empirical frame or, in other words, to find some regular mapping lead to considerable complexity of the model or to the great loss of precision. Generally speaking, it leads to loss the sense of modeling, at all.

**Figure 1.** The mappings \(D \times E \rightarrow D\) of 1-dimensional frames of 2-surfaces:

a) regular and quadratic frame; b) empirical irregular frame

Inverse problem of creating some model of hyper-surface arises when we want to use an empirical irregular 0-dimensional frame. It is necessary to take into account the fact that irregularity of data may be revealed in any dimension from \(p = 1\) up to \(p = n - 1\). Here, we consider only one case when irregularity is appeared if \(p = 1\). It means that one can build the model of hyper-surface in form of some 1-dimensional frame, but it is impossible to build any regular 2-dimensional frame. Irregular 1 dimensional frame is usually built as a cubic spline net. Preliminary evaluation of regularity is fulfilled visually.
5. General algorithms of inverse problem

There exist three types of inverse problem:
1. To calculate the values of output parameters if the values of input parameters are given.
2. To find the set of input parameters values or to generate hyper-plane section of $D \times E$ and its projection onto the area $D$ if some value of output parameter is given.
3. To find the set of input parameters values if some set of output parameter values is given.

To solve the first inverse problem we may use the following simple algorithm:

Step 1: Hyper-plane sections of $D$: $x_1 = x_{1,0}$, $x_2 = x_{2,0}$, ..., $x_n = x_{n,0}$ are built.
Step 2: Points generated by intersection the hyper-plane sections and 1-dimensional irregular spline net are determined.
Step 3: Auxiliary cubic splines $S_1(x_{1,0}, x_{2,0}, ..., x_n)$, ..., $S_n(x_1, x_2, ..., x_{n-1}, x_{n,0})$ are generated by the points of intersection.
Step 4: The values of output parameters are calculated for all auxiliary cubic splines.
Step 5: The value of output parameters to be found is calculated as an average of all values.

General algorithm for the second problem is as follows:
Step 1: Points generated by intersection the hyper-plane $y = y_0$ and 1-dimensional irregular spline net are determined.
Step 2: Projections of these points onto regular net of area $D$ are built.
Step 3: Irregular spline net is built in $D$ by means of the points of intersection.
Step 4: Border-splines of the spline net are determined. These border-splines limit some sub-area $D_0 \subset D$ which is an area of admissible values. Each point of $D_0$ corresponds to given value of output parameter.

General algorithm for the third problem is $k$-repeated second algorithm together with some procedure of finding the area $D_0 = D_{1,0} \cap ... \cap D_{k,0}$.

6. Optimization of thread seam parameters

Our goal of investigation is to find input parameters of thread seams corresponding to given output parameters: durability of the seam $y_1 \geq P$, $N$; harshness of the seam $N_1 \leq y_2 \leq N_2$, mcN-sm$^2$.

In this case dimension of model space $D_1 \times E_2$ is equal 4. Since the number of input parameters is the same as the number of output ones, the area to be found is a point.

The model of thread seam was created by means of special author’s software HYPER-DESCENT based on above-mentioned algorithms [6].

Graphic model of the hyper-surface is shown in Fig. 2. The values of linear thickness of threads are on abscissa axis. Each curve shows the variation of output parameter by variation of stitch number and direction of seam. Influence of thread linear thickness on the durability of the seam is shown in upper part of Fig. 2. Influence of thread linear thickness on the harshness of the seam is shown in lower part of Fig. 2.

Technological conditions for thread seam by various values of durability, namely $P = 130$ kg and $P = 150$ kg, $EI = 250000$ mcN-sm$^2$, was determined. Level planes and the sections of hyper-surface were drawn through these points. The curves 1, 2 describe the durability functions and the curves 3, 4 describe the harshness functions (see Fig. 3). The area of input parameters for thread seam corresponding to given output parameters is only two points N and M.

Hence, stitch seam on jacket mixture material with given values of durability and harshness may be created by using the threads with linear thickness equal to 43.5 and with the number of stitches equal to 3.5 on one sm.

Durability $P = 150$ kg and harshness $EI = 250000$ mcN-sm$^2$ correspond to the point N. Linear thickness of the thread is equal to 65 and the number of stitches is equal to 4.5 by one sm.

These results may be used in sewing industry for production the goods from jacket mixture materials by means of machine $DDL – 8100e Juke$ and inside some chosen thickness range of threads.
Durability of the seam $P$, kg

Harshness of the seam $E_I$, mcN · cm²

Linear thickness of threads $h$, tex
Figure 2. Geometric models of thread seam durability and harshness are spline-frames of thread linear thickness for jacket mixture material

Figure 3. Thread seam optimal parameters for jacket mixture material

7. Conclusions
The mathematical simulator which is an empirical one-dimensional spline-frame of hyper-surface inside the area of parameters is described. Effectiveness of the model is confirmed by applications to sewing industry. The model allows us to solve some similar problems for the other fields of industry. The model together with software HYPER-DESCENT may be applied for simulation multi-parametrical systems or technological processes of light industry. Software HYPER-DESCENT uses only one-dimensional spline approximation. This approach simplifies the process of the software creation. Evaluation of regularity of the frames is realized visually by means of outputting appropriate diagrams.

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