Spatial autocorrelation study for laser beam quality estimation

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Abstract. High brightness electron beam is required by several applications in the accelerator physics field. In order to have a high brightness beam, that means a high current and a low emittance beam, it is important to study, among other things, the beams non uniformity due to the non perfect transverse laser beam uniformity. Regarding the transverse analysis of the beam, statistical tools as mean and standard deviation are usually used. In this work we will show how the autocorrelation function of a photocathode laser can be used for monitoring the spatial distribution of the beam non-uniformity, strictly connected with the high electron beam emittance: we will apply our analysis on the SPARC LAB data.

1. Introduction
A high brightness electron beam production by photoinjector is the primary requisite for a large number of applications such as Free Electron Laser (FEL) radiation sources [1], Plasma Wake Field Acceleration (PWFA) experiments [2, 3, 4, 5], generation of THz radiation [6] and Inverse Compton Scattering sources [7].

The brightness, $B$, is defined as:

$$B = \frac{Q}{\varepsilon_{nx}\varepsilon_{ny}\sigma_l\sigma_\gamma}$$

(1)

where $Q$ is the beam charge, $\varepsilon_{nx}$ and $\varepsilon_{ny}$ are respectively the normalized $xx'$ and $yy'$ transverse trace space emittances, $\sigma_l$ is the bunch length and $\sigma_\gamma$ is energy spread. A high brightness beam demands a high current and a low emittance beam, or rather is required for a large number of quasi-monochromatic electrons, concentrated in very short bunches, with small transverse size and divergence, that is high particles density $6D$ phase space. The total transverse beam emittance in the photoinjector can be written, assuming no correlation between each term, as the sum in quadrature of the different contributions, that are, the cathode intrinsic emittance, $\varepsilon_{int}$, the RF emittance, $\varepsilon_{RF}$, the space charge emittance, $\varepsilon_{SC}$ and the solenoid emittances ($\varepsilon_{chromatic}$, $\varepsilon_{geometric}$ and $\varepsilon_{Busch}$):

$$\varepsilon = \sqrt{\varepsilon_{int}^2 + \varepsilon_{RF}^2 + \varepsilon_{SC}^2 + \varepsilon_{chromatic}^2 + \varepsilon_{geometric}^2 + \varepsilon_{Busch}^2}$$

(2)

A lower total beam emittance value means a better RF photoinjector performance [8]. The emittance due to the space-charge effects arises from the repulsive coulomb forces between
electrons, comprising linear and nonlinear effects. Theoretical studies \[9\] show how it is possible to compensate, with an appropriate choice of a solenoid, placed outside of the gun, the linear space-charge forces. On the contrary the effects due to non linear space-charge forces can not be corrected. This last contribution to the emittance degradation comes from field’s non-linearity that can be reduced using a transversally and longitudinally uniform beam. Nevertheless the beam’s non uniformity is possible due to the non perfect laser uniformity and quantum efficiency variation on the cathode surface. Therefore, the estimation of the laser and the electron beam quality is very important. In this paper we present how to find an additional parameter, the \textit{spatial autocorrelation index}, \( \Lambda \), able to evaluate the transverse laser beam uniformity.

2. Analytical definition of spatial autocorrelation index

Quantities as mean, variance and standard deviation can be used as statistical tools to provide information about the uniformity of a set of data distributed on a surface, both for the transverse spot of a laser beam and for the extracted electron beam \[10\]. For both of them a matrix of pixels with a given intensity is fixed (each pixel represents the electrons or photons charge).

Given a beam transverse spot, in the matricial formalism is it represented by a 2D \( N \times M \) matrix:

\[
\begin{bmatrix}
  a_{11} & \cdots & a_{1j} & \cdots & a_{1M} \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  a_{i1} & a_{ij} & a_{iM} & & \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  a_{N1} & & a_{Nj} & \cdots & NM \\
\end{bmatrix}
\]

where \( a_{ij} \) is the generic sample, that is the pixel intensity.

Starting from the matrix definition, we can calculate the mean that describes the central value:

\[
\langle a \rangle = \frac{1}{T} \sum_{i=1}^{N} \sum_{j=1}^{M} a_{ij}; \quad (4)
\]

where \( N \) and \( M \) are the matrix dimensions and \( T = NM \) is the number of pixel involved. The samples can be considered in different ways depending also from the distance \( h \) from the generic sample \( a_{ij} \) as represented in figure 1.

\[
\begin{bmatrix}
  a_{i-hj-h} & \cdots & a_{i-hj} & \cdots & a_{i-hj+h} \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  a_{i-hj-h} & a_{ij} & a_{i-hj+h} \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  a_{i+hj-h} & a_{i+hj} & a_{i+hj+h} \\
\end{bmatrix}
\]

\textbf{Figure 1.} In this matrix the \( a_{ij} \) is the generic sample and \( h \) is the distance from other samples.

Given \( h \), the distance between the generic sample, \( a_{ij} \), and other samples of the matrix, the element \( a_{ijh} \) is the mean of the samples localized around the main sample \( a_{ij} \) and it is given by:

\[
a_{ijh} = \frac{1}{(2h + 1)^2 - 1} \sum_{l=-h}^{h} \sum_{m=-h}^{h} a_{i+l,j+m} - a_{ij}. \quad (5)
\]
From the matrix 3 we can evaluate the variance that represents the distance from the central value:

$$\text{var}(a) = \frac{1}{T} \sum_{i=1}^{N} \sum_{j=1}^{M} (a_{ij} - <a>)^2,$$

this quantity is always positive so that the standard deviation can be defined as:

$$\sigma_a = \sqrt{\text{var}(a)}.$$

The standard deviation, $\sigma_a$, describes the contrast between spots, in other words gives information about the non uniformity. For a perfectly uniformly beam cross section (photon or electrons beam), normalized to the higher sample, $<a>=1$ and $\sigma_a=0$. The parameters shown so far describe only non-uniformity but they don’t describe how this non-uniformity is distributed.

The spatial correlation concept describes this distribution. This notion comes from spatial statistics [11] and through the spatial correlation index, $\Lambda$, allows to find an additional parameter able to evaluate the transverse laser beam uniformity.

To obtain the right expression for the spatial correlation index, $\Lambda$, it is fundamental to introduce the covariance, defined as:

$$\text{cov}(a, h) = \frac{1}{T} \sum_{i=1}^{N} \sum_{j=1}^{M} (a_{ij} - <a>) \cdot (a_{ijh} - <a>),$$

where the element $(a_{ij} - <a>)(a_{ijh} - <a>)$ is named the covariance matrix. The covariance tells us if a sample and its neighbour are at the same time diverse or not from the mean value.

The spatial correlation index, $\Lambda$, is defined as the ratio of the covariance to the standard deviation, $\sigma_a$, squared:

$$\Lambda(a, h) = \frac{\text{cov}(a, h)}{\sigma_a^2}.$$

It is a quantity whose value is between -1 and 1, where the minus sign means that most samples are lower than the mean. The distance, $h$, and the matrix dimensions, $N$ and $M$ define the resolution of the spatial autocorrelation study.

3. Autocorrelation estimation and GPT electron beam emittance evaluation

With these tools we have estimated the autocorrelation index and, using the GPT code [12], we have evaluated the relation with the beam emittance growth [13], both for a theroretical distribution and for real laser spots.

3.1. Theoretical distribution

We have applied the spatial correlation concept to beam quality studies for a theoretical beam charge distribution. We have modeled the beam charge distribution, extracted from the cathode, as a sine and cosine function having a frequency $n$ and a charge intensity $\delta$:

$$\rho(i,j) = \rho_0 (1 + \delta \cos(k_n i))(1 + \delta \cos(k_n j)),$$

where $\rho_0$ is the normalization constant and $k_n = \frac{2\pi n}{R}$ with $R$ the beam radius. In figure 2 we report the matrix model of Eq.10. In images in figure 2, two points are correlated if they are placed at a certain distance, $(\frac{h}{\pi})\ast$ (that is an adimensional parameter), as it is possible to see in the figure 3. This distance coincides with the mean distance of the non homogeneity. We also report a plot of the spatial autocorrelation as a function of the distance $(\frac{h}{\pi})$, that
Figure 2. Matrix model of Eq.10 showing the distribution as function of spatial frequency $n$ with $\rho_0 = 1$, $R = 78$ pixel and $\delta = 0.8$ (better contrast between background and beam).

Figure 3. Three theoretical distribution of laser spot created from the Eq.10 at $n = 1$, $n = 5$, $n = 10$ and the corresponding correlogram.

is called correlogram of a given beam distribution. Concerning the case $n = 1$, we have estimated for the mean distance of the non homogeneity: $(\frac{h}{R})^* = 0.5$, with $R = 78$ pixel. If we want to know this value in \(\mu m\), with a camera pixel size= 6.45\(\mu m/pixel\), we obtain: 

$$h^*(\mu m) = 0.5 \cdot R \cdot (6.45\mu m/pixel) = 256\mu m.$$ 

These theroretical distributions have been studied, regarding the emittance degradation with the GPT code. For each distribution, including an ideal laser spot, we have simulated, reproducing a SPARC.LAB [14] working point, the electron beam dynamics from the photocathode to the first screen of photoinjector. The parameters used in the simulation are reported in table 1. In figure 4 we report, on the left, the evolution of the normalized beam emittance for each distribution, there is an emittance growth as the spatial frequency $n$ decreases. To estimate the emittance growth in function of the different laser spot distributions we have extrapolated from the GPT simulation the normalized beam emittance value at about 1 cm from the photocathode surface, and we have normalized these values to the emittance value for the ideal laser spot ($\epsilon_0 = 0.55$ mm mrad/mm), in other words to the normalized intrinsic emittance value. We report on the right of the figure 4 a plot representing this ratio in function of autocorrelation length for each laser spot distribution. We point out that since the correlation distance is evaluated as a global
Table 1. The SPARC_LAB parameters used in GPT simulation.

| Parameter                  | Value                             |
|----------------------------|-----------------------------------|
| $E_{RF}$                   | $115\, \text{MV/m}$              |
| RF phase                   | $30^\circ$                        |
| Laser pulse length         | 2ps-rms (Gaussian profile)        |
| Laser radius               | $500\, \mu\text{m}$ (Flat top profile) |
| Electron beam energy       | 5 MeV                             |
| Bunch charge               | 50 pC                             |
| Norm. intrinsic emittance, $\epsilon_0$ | $0.55\, \text{mm mrad/mm}$       |

statistical index, clearly a highly non symmetric transverse laser spot may requests a local index able to disentangle the different patterns, i.e. marginal covariances or more local statistical quantities.

Figure 4. Left: normalized electron beam emittance evolution from the photocathode to the first screen in the SPARC_LAB photoinjector. The solenoid is placed at $z = 0.089\, \text{m}$ from the photocathode. Each line corresponds to one distribution by varying the spatial frequency $n$. Also, an ideal laser spot is reported (red line). Right: the plot reports the ratio of the normalized electron beam emittance values to normalized intrinsic emittance value for the ideal laser spot in function of the autocorrelation length for each laser spot distribution.

The emittance increase, reported in figure 4, is due to the mutual repulsion of single beamlets, inside each distribution, in the radial direction from its centre. The space charge force reduces when the beamlets overlap, which in turn stops the emittance growth. This effect takes place near the cathode surface, typically during the first 10s picosecond after the emission, within 1 cm from the cathode [15].

3.2. Real laser spots
We have also performed the same analysis for 5 real laser spot. We have evaluated the spatial autocorrelation index as a function of the distance $(h/R)$ and performed the GPT simulation using the same parameters reported in table 1. The results are presented in figure 5. From the correlogram we have estimated a $(h/R)^*$ value of about 0.16 for the laser spots (2-5), whereas for the laser 1 a higher value of 0.22. From the GPT simulation we have extrapolated the beam emittance value at about 1 cm from the photocathode surface for each laser spot images. In using $(h/R)^*$ we assume the laser spots to exhibit a quite symmetric pattern in x/y; moreover,
Figure 5. Above: 5 real laser spot images used at SPARC_LAB. Bottom: On the left the corresponding correlogram, on the right the normalized beam emittance evolution from the photocathode to the first screen in the SPARC_LAB photoinjector for each laser spot. The solenoid is placed at \( z = 0.089 \text{m} \) from the photocathode. Also, an ideal laser spot is reported (brown line).

The reason why the curves dropping is different is due to the fact that, for real laser spots, the correlation index is the result of at least two effects: the residual gaussian shape of the laser and the higher frequency correlation length due to the non-homogeneities on the top of the main laser profile. This effect will be studied in details in future works. As is reported in table 2, for the laser 1 we have obtained a higher normalized emittance value compared to lasers 2-5. We have also normalized these values to the emittance value for the ideal laser spot \((\epsilon_0 = 0.55 \text{ mm mrad/mm})\). These results are in agreement with the laser spot images quality.

Table 2. The real laser spot analysis.

| Real laser spot | \(\epsilon\) (mm mrad) | \(\frac{\epsilon}{\epsilon_0}\) | \(\left(\frac{hR}{\pi}\right)^*\) |
|-----------------|-----------------|------------------|------------------|
| Laser1          | 0.62 ± 0.02     | 1.13 ± 0.06      | 0.218            |
| Laser2          | 0.59 ± 0.02     | 1.08 ± 0.06      | 0.166            |
| Laser3          | 0.58 ± 0.02     | 1.04 ± 0.06      | 0.168            |
| Laser4          | 0.58 ± 0.02     | 1.06 ± 0.06      | 0.166            |
| Laser5          | 0.59 ± 0.02     | 1.08 ± 0.06      | 0.166            |

4. Conclusions

Statistical tools as mean, standard deviation, variance and covariance give us information about non uniformity of laser and electron transverse beam. Whereas the spatial autocorrelation index, \(\Lambda\), determines how this non-uniformity is distributed. This concept describes the laser beam quality, concerning the uniformity, and it gives an idea of the emittance growth due to the laser beam degradation. The spatial autocorrelation index, \(\Lambda\) and the parameter \(\left(\frac{h}{R}\right)^*\) are good estimators of the beam quality since they are strictly correlated with beam emittance at the emission. Our next step will be experimental emittance measurements with real beam to settle the emittance growth trend and the systematic study with larger laser dataset.
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