Symmetry-Protected Topological Phase for Spin-Tensor-Momentum-Coupled Ultracold Atoms

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We propose a readily implementable experimental scheme to realize a one-dimensional synthetic magnetic flux lattice with constructing spin-tensor-momentum couplings in ultracold spin-1 atoms. Different from the Altland-Zirnbauer classification, we show that our system hosts a new symmetry-protected phase described by a \( Z_2 \) topological invariant. In single-particle spectra, we show that the topological nontrivial phase supports two kinds of edge states, which include two (four) zero-energy edge modes in the absent (presence) of two-photon detuning. We further study the bulk-edge correspondence in a non-Hermitian model by taking into account the particle dissipation. It is shown that the non-Hermitian system preserves the bulk-edge correspondence under the magnetic group symmetry (\( M \)) and exhibits the non-Hermitian skin effect while the \( M \) symmetry is broken down via nonzero magnetic fluxes. Our proposal may provide new insights of understanding and facilitating the experimental explorations for non-Hermitian topological quantum matters.

Introduction.—Topological quantum matters, which are characterized by gapped bulk states and symmetry-protected gapless edge states, drives much of the fundamental research ranging from condensed matter physics [1–3] to quantum information processing [4, 5]. The paradigms of topological states include integer quantum Hall effects [6–8], quantum spin-Hall effects [9–11], Majorana fermions [12–14], and Weyl semimetals [15–19]. Besides the topologically ordered phases with long-range quantum entanglement, symmetry-protected topological (SPT) phases whose edge states are only robust against local perturbations have attracted much attention in recent years [20–22]. However, the realization of SPT phases remains a challenging task, despite many SPT phases have been predicted in various theoretical proposals [23–26] and few experimental observation in solid-state material [27–29]. In addition, non-Hermitian systems could host striking exceptions beyond the paradigmatic bulk-edge correspondence in Hermitian system [30, 31]. Remarkably, due to the non-Hermiticity, it has been shown that the bulk spectra qualitatively depend on the boundary conditions, which is called non-Hermitian skin effects [32–37].

Meanwhile, the experimental breakthroughs of spin-orbit (SO) couplings in ultracold atoms have provided a new paradigm for exploring topological states in a clean environment and controllable way [38, 39]. The SO couplings have been realized via different schemes such as Raman transitions [40–43] and Raman-assisted tunneling in optical lattices [44, 45]. In particular, the Raman-assisted tunneling in optical lattices could realize the strong synthetic magnetic fields and establish the tunable magnetic flux lattices [46–49]. Up to now, the relativized SO couplings and the observed SPT phases are mainly focusing on the spin-1/2 quantum gases. Unlike the Rashba spin-vector-momentum coupling, a various of spin-tensor-momentum coupling (STMC) could be constructed for high-spin system, which provides exotic quantum phenomena [50–52]. It is of great interest to explore whether new SPT phases can be emerged via STMC in both Hermitian and non-Hermitian systems of ultracold atoms. An affirmative answer will significantly facilitate the experimental explorations of exotic topological quantum matters in high-spin system [53–55].

In this Letter, we show how to realize STMC using Raman-assisted staggered spin-flip hoppings in pseudospin-1 ultracold atoms. We find that, due to the interplay of synthetic magnetic flux and two-photon detuning, the system exhibits a new symmetry-protected topological phase which is beyond the Altland-Zirnbauer (AZ) classification. Without loss of generality, we consider the non-Hermitian spin-1 models with in-plane (\( B_x \)) and out-of-plane (\( B_z \)) imaging magnetic fields (dissipation loss), respectively. Strikingly, the magnetic group symmetry (\( M \)) may guarantee the bulk-edge correspondence even for non-Hermitian system in the presence of nonzero \( B_z \) fields. Remarkably, this is different from the recent results in non-Hermitian spin-1/2 models [56–63], where the bulk-edge correspondence is broken down when the \( B_x \) field is applied. In addition, the non-Hermitian skin effects in which the bulk eigenstates are localized near the boundary of system is predicted ascribe to the \( M \) symmetry-breaking for nonzero \( B_z \) fields.

Model.—We consider a quantum gas of noninteracting ultracold atoms subjected to a bias magnetic field \( \mathbf{B} \) along the quantization \( z \) axis. The three atomic ground
states form a pseudospin-1 manifold and their corresponding linear (quadratic) Zeeman shifts are \( h\omega_L \) (\( h\omega_q \)). In Fig. 1(a), we illustrate the atomic level structure and laser configuration. The atomic transition \( |\sigma\rangle \leftrightarrow |e_\sigma\rangle \) is coupled by a \( \pi \)-polarized standing-wave laser with frequency \( \omega_L \) and Rabi frequency \( \Omega_L(y) = \Omega_s \cos(kL_Ly) \), where \( k_L \) is the wave-vector and \( \sigma = \{\uparrow, \downarrow, 0\} \). To achieve Raman transitions, the atomic transition \(|0\rangle \leftrightarrow |e_\uparrow\rangle \) (\(|1\rangle \leftrightarrow |e_\downarrow\rangle \)) is driven by \( \sigma \)-polarized plane-wave lasers with frequencies \( \omega_L + \Delta \omega_L \) (\( \omega_L - \Delta \omega_L \)) for matching the Zeeman shifts and Raman selection rules. Here the Rabi frequency for the two plane-wave lasers is given as \( \Omega_p(y) = \Omega_p e^{-ik_L y} \) with \( \kappa = k_L \cos \theta \). We should emphasis that, for sufficiently large quadratic Zeeman shifts \([40]\), the off-resonant Raman processes are suppressed, where the linear Zeeman shift is compensated by the frequency difference of the Raman fields.

For a large light-atom detuning, i.e. \( |\Omega_s p/\Delta| \ll 1 \) and \( \Delta = \omega_L - \omega_q \), one can adiabatically eliminate the excited states \( |e_\sigma\rangle \). Therefore the light-atom interaction results a one-dimensional spin-independent optical lattice \( \hat{U}_{\text{ol}}(y) = U_s \cos^2(kL_Ly) \hat{I} \) with Stark shift \( \hat{U}_s = -\Omega_s^2/\Delta \) and lattice constant \( d = \pi/k_L \). By applying a analogous gauge transform \([64]\): \( \{|\uparrow\rangle \rightarrow e^{i\phi/2}|\uparrow\rangle \}, \{|\downarrow\rangle \rightarrow e^{i\phi/2}|\downarrow\rangle \}, \{|0\rangle \rightarrow e^{i\phi/2}|0\rangle \} \), the single-particle Hamiltonian reads

\[
\hat{h}_0 = \frac{(p - A)^2}{2M} + \Omega \cos(kL_y) \hat{F}_x + \delta \hat{F}_z + \hat{U}_{\text{ol}}(x) \hat{I},
\]

where \( M \) is the atomic mass, \( \hat{I} \) is the identity matrix, \( \hat{F}_{x,y,z} \) is the spin-1 matrix, \( \Omega = -\sqrt{2}\Omega_p, \Omega_p/\Delta \) is the Raman coupling strength, and \( \delta = \omega_L + 2\Omega_p^2/\Delta - \Delta \omega_L - \omega_q \) is the effective two-photon detuning under the condition of \( \Delta \omega_L = \Delta \omega_L - 2(\omega_q - 2\Omega_p^2/\Delta) \). In particular, \( A = h\kappa e^{-i\phi} \hat{F}_x/2 = -h\kappa [\hat{F}_x^2 - \hat{I}]/2 \) is the vector potential, which denotes the STMC with the spin-orbit coupling strength \( \kappa \).

For a sufficiently strong lattice potential with blue detunings (i.e. \( U_s > 0 \)), the tight-binding Hamiltonian with considering the lowest orbit and the nearest-neighbor hoppings takes the form

\[
\hat{H}_0 = \sum_{\sigma = \uparrow, \downarrow} \sum_n \left[ (-1)^n t_y (\hat{a}_{n,\sigma}^\dagger \hat{a}_{n+1,0} - \hat{a}_{n,\sigma}^\dagger \hat{a}_{n-1,0}) - t (\hat{a}_{n,\sigma}^\dagger \hat{a}_{n+1,0} e^{i\phi/2} + \hat{a}_{n,\sigma}^\dagger \hat{a}_{n+1,0} e^{-i\phi/2} + \text{H.c.}) \right] + \delta \sum_n (\hat{a}_{n,\uparrow}^\dagger \hat{a}_{n,\uparrow} - \hat{a}_{n,\downarrow}^\dagger \hat{a}_{n,\downarrow}),
\]

where \( \hat{a}_{n,\sigma}^\dagger \rightarrow \hat{a}_{n,\sigma}^\dagger \) is the atomic annihilation operator for the \( n \)-th site, \( t \) is the nearest-neighbor spin-independent hopping, \( t_y \) is the Raman-assisted nearest-neighbor spin-flip hopping, and \( \phi = \kappa d \) is the Peierls phase. With our laser configuration shown in Fig. 1(a), the synthetic magnetic flux \((-1 < \phi/\pi < 1\) can be easily tuned by changing the angle \( \theta \) with respect to the \( y \)-axis. In addition, the on-site spin-flip hopping is zero for the blue lattice potential \([64]\).

To gain more insights into the magnetic flux, we introduce the gauge transformation \( \hat{a}_{n,0} \rightarrow (-1)^n \hat{a}_{n,0}^\dagger \) to eliminate the staggering factor in the spin-flip hopping. Then the lattice Hamiltonian becomes as

\[
\hat{H}_0 = \sum_{\sigma = \uparrow, \downarrow} \sum_n \left[ t_y (\hat{a}_{n,\sigma}^\dagger \hat{a}_{n+1,0} - \hat{a}_{n,\sigma}^\dagger \hat{a}_{n-1,0}) - t (\hat{a}_{n,\sigma}^\dagger \hat{a}_{n+1,0} e^{i\phi/2} - \hat{a}_{n,\sigma}^\dagger \hat{a}_{n+1,0} e^{-i\phi/2} + \text{H.c.}) \right] + \delta \sum_n (\hat{a}_{n,\uparrow}^\dagger \hat{a}_{n,\uparrow} - \hat{a}_{n,\downarrow}^\dagger \hat{a}_{n,\downarrow}),
\]

whose corresponding schematic is shown in Fig. 1(b). In contrast to nonzero fluxes for spin-half model \([64]\), the net synthetic magnetic flux per plaquette is zero due to the STMC for our spin-1 system.

Under the periodic boundary condition (PBC) satisfying translational invariance, the Hamiltonian in the momentum space is given as

\[
\hat{h}_0(k) = \epsilon(k) \hat{I} + \delta \hat{F}_z + \hat{d}_1(k) \hat{Q}_{y,z} + \hat{d}_2(k) \hat{F}_z^2,
\]

where \( \epsilon(k) = 2t \cos(kd + \phi/2) \), \( \hat{d}_1(k) = 2t \epsilon(k) \sin(kd) \), \( \hat{d}_2(k) = 4t \cos(kd) \phi/2 \), and \( \hat{Q}_{y,z} \) is the generator of SU(3) Lie algebra \([65]\). Interestingly, the last two terms of \( \hat{Q}_{y,z} \) and \( \hat{F}_z^2 \) in Eq. (4) represent two different types of spin-tensor-momentum coupling (STMC), which play essential roles for forming topological states.
To explore the symmetry classes, we introduce three symmetry operators for spin-1 systems: the time reversal symmetry $T = e^{-i\hat{F}_y K}$, the particle-hole symmetry $C = ie^{-i\hat{F}_z K}$, and the chiral symmetry $S = C \otimes T = ie^{-i\hat{F}_z}$ (defined as the product of $T$ and $C$), where $K$ is a complex conjugate operator. Explicitly, the gauge-potential-induced dispersion $\epsilon(k)$ simultaneously breaks $T$, $C$, and $S$ symmetries for nonzero magnetic fluxes. Due to the STMC, the term of $d$ breaks both $T$ and $C$ symmetries, while the term of $d_2(k)\hat{F}_z^2$ breaks both $C$ and $S$ symmetries. Therefore, the Hamiltonian (4) is beyond the conventional AZ classification for the one-dimensional (1D) system in the absence of the particle-hold or chiral symmetry protection [66]. However, we find that the Hamiltonian $h_0(k)$ satisfies a magnetic group symmetry $\mathcal{M}h_0(k)\mathcal{M}^{-1} = h_0(-k)$ with $\mathcal{M} = e^{-i\hat{F}_z K} \otimes R_y$, which is a combination of $T$ and the mirror symmetry $M_y = e^{-i\hat{F}_z}R_y$ with $R_y$ representing the spatial reflection along the $y$ axis. In addition, the introduced magnetic group symmetry satisfies $[\mathcal{M}, h_0(k)] = 0$ and $\mathcal{M} = \mathcal{M}^{-1}$ and so that it brings a new SPT phase hosting a topological nontrivial phase characterized by the 1D $Z_2$ invariant [21, 67].

**Topology.**—To further characterize the SPT phase, we calculate the energy spectrum in $k$-space via $h_0(k)\{|u_0(k)\rangle\} = E_0(k)\{|u_0(k)\rangle\}$, where $E_0(k)\{|u_0(k)\rangle\}$ denotes the eigenenergies (eigenstates) with $\alpha = \{-, 0, +\}$ indexing the {lowest, middle, highest}-helicity branches, respectively. Due to the $Z_2$ invariant, the system topology can be described by the Zak phase $\varphi_{\text{Zak}} = \int_{-\pi/d}^{\pi/d} (\mu_-(k)) d\phi/k$ for the lowest branch. The associated two distinct phases are characterized by the gauge-dependent Zak phase with $\varphi_{\text{Zak}} = 0$ or $\pi$ representing the topological trivial (or nontrivial) state. Besides the topological property can also be characterized by the Wilson loop. For the first step, we ignore the gauge potential $\epsilon(k)$ which does not affect topological invariant of the helicity branches and topological phase transition of the system. As a result, the Hamiltonian with satisfying the magnetic group symmetry $\mathcal{M}$ will ensure an inversion symmetry: $Ph(k)\mathcal{P} = h(-k)$, where $P = e^{-i\hat{F}_z}$ is the inversion operator. The Bloch states at the two higher symmetric momenta $\{k = 0, k = \pi/d\}$ are eigenstates of $\mathcal{P}$, corresponding $\mathcal{P}|u_1(k = 0)\rangle = P_1|u_1(k = 0)\rangle$ and $\mathcal{P}|u_1(k = \pi/d)\rangle = P_2|u_1(k = \pi/d)\rangle$. Thus the winding number for the lowest helicity branch can be further experimentally extracted by measurement the topological invariant $\nu = -\text{Im}\langle\ln(P_1 * P_2)/\pi$ [68], where the $Z_2$ index is equivalent to the Zak phase with $\nu = \varphi_{\text{Zak}}/\pi$ corresponding to topological nontrivial ($\nu = 1$) and trivial ($\nu = 0$) states.

By diagonalizing Hamiltonian (4), we verify that the highest and lowest helicity branches possess identical topological invariant $\nu$ while the middle helicity branch is always topological trivial with $\nu = 0$. Figure 2(a) shows the phase diagram in the $\phi$-$\delta$ parameter plane, where $\nu = 1$ (0) corresponds to topological nontrivial (trivial) phase for the lowest helicity branch. The analytic $t_y$-independent phase boundary satisfies $\delta/t = \pm 4 \cos(\phi/2)$. As expected, the topological nontrivial phase $\nu = 1$ exhibits the topologically protected edge states due to the bulk-edge correspondence by imposing a hard-wall confinement along $y$-axis, as shown in Fig. 2(b).

![Figure 2](image-url)

Interestingly, the properties of the edge states under open boundary condition (OBC) depend on the strength of $\delta$. For absence $\delta$, the eigensystem reveals a dark stat: $|u_0\rangle = (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2}$, corresponds the spin-0 component decoupled from the spin-$\uparrow$ and $\downarrow$ components, where the band spectra is gapless with touching point at higher symmetric momenta. Fig. 2(d) displays the twofold degenerate zero-energy edge modes for $\delta = 0$. To understand this, we find that the system Hamiltonian (3) satisfies $TH_0T^{-1} = -H_0$, where $T$ is the gauge transformation operator $\hat{a}_n^\dagger \rightarrow (-1)^{n+1} \hat{a}_n^\dagger \sigma_x, \hat{a}_n \rightarrow (-1)^{n+1} \hat{a}_n \sigma_x$. Therefore, the edge states at left hand ($\langle \psi_\nu \rangle$ and right-hand ($\langle \psi_R \rangle$) boundaries are protected by the $\mathcal{M}$ and $T$ symmetries. As the definition, the two edge states $\langle \psi_\nu \rangle$ and $\langle \psi_R \rangle$ could transform to each other under the magnetic group symmetry:
\[ \mathcal{M}|\psi_L\rangle = |\psi_R\rangle \] and \[ \mathcal{M}|\psi_R\rangle = |\psi_L\rangle, \] which guarantees a pair of edge states with twofold degeneracies. In addition, the anticommutation relation between \( H_0 \) and \( T \) imply two edge states with opposite energy. Combination both \( \mathcal{M} \) and \( T \) symmetries, the left (\( |\psi_L\rangle \)) and right (\( |\psi_R\rangle \)) states with zero-energy edge modes are consistent with the numerical result in Fig. 2(d). Remarkably, the \( T \) symmetry forces the spatial distribution of the edge state \( |\psi_L\rangle \) only along the odd lattice site.

As to the nonzero \( \delta \), the two pairs of edge states with nonzero opposite energy are observed for topological nontrivial phase [Fig. 2(g)], which results can also be well understood by analysed symmetries of the system. In contrast to preserve \( T \) symmetry for \( \delta = 0 \), the Hamiltonian (3) breaks \( T \) symmetry when \( \delta \neq 0 \). Thus \( \mathcal{M} \) symmetry ensures each pair of degenerate edge modes exhibiting nonzero energy, whereas the \( \delta \) term fails to split out the degeneracy of the pair edge modes. Moreover, the system hosts a new gauge transformation symmetry defined as \( T' = e^{-i\delta F_z} \otimes T \) with \( T'H_0T'^{-1} = -H_0 \) for arbitrary \( \delta \). As a result, the symmetries of \( \mathcal{M} \) and \( T' \) commutation and anticommutation relations with \( H_0 \) support two pairs of edge states with opposite nonzero energy for topological nontrivial phase, leading to the four edge states in Fig. 2(f). Meanwhile, we verify that the wavefunction of the edge modes displays spatial distribution continuously along lattice site due to breaks \( T \) symmetry.

\[ \begin{align*}
(\text{a}) & \quad \text{gapless} \\
(\text{b}) & \quad \text{Re}(E/t) \\
(\text{c}) & \quad \text{Im}(E/t) \\
(\text{d}) & \quad |\psi_k|^2
\end{align*} \]

Figure 3. (color online). (a) The phase diagram of non-Hermitian Hamiltonian on the \( \delta-B_z \) plane. (b) The real part of spectra as functions of \( \delta \) with fixing \( B_z/t = 0.2 \). The bulk spectra under PBC (gray lines) is the same as the result based on OBC (blue lines), which demonstrates the bulk-edge correspondence. (c) The typical spectra for \( \delta/t = 2 \). The red dots in (b) and (c) represent the edge states. (d) The density distribution of bulk state for the topological nontrivial at \( \delta/t = 2 \). The other parameters are \( \phi/\pi = 0.1 \) and \( t_y/t = 1 \).

**Bulk-edge correspondence.**—Now we turn to study the bulk-edge correspondence for STMC spin-1 system including the dissipation loss \( \hbar(k) = h_0(k) + iB_zF_x + iB_zF_z \), where \( B_x \) and \( B_z \) is the non-Hermitian parameter of in-plane and out-of-plane imaginary magnetic field [69], respectively. To gain more insights, we first consider the atoms subjected to a \( B_x \) field. The non-Hermitian term of \( iB_zF_z \) preserves the \( \mathcal{M} \) symmetry, which ensures the topology of the non-Hermitian system remains characterized by the 1D \( Z_2 \) invariant. Fig. 3(a) summarize the phase diagram on the \( \delta-B_z \) plane with fixing \( \phi/\pi = 0.1 \). When the bulk gap closed at the exceptional points, i.e., \( E_z(k) = E_0(k) \), therefore exists two phase transition boundaries by tuning \( \delta \). The red solid line denotes the phase transition between the gapped and gapless topological nontrivial phases. While the topological trivial (\( \nu = 0 \)) to nontrivial (\( \nu = 1 \)) phase transition is characterized by the blue dashed-line. As can be seen, the region of topological nontrivial phase is roughly linear growing with increasing \( B_x \). Moreover, we find that the system exhibits a gapless phase in the topological nontrivial region, where the real part of complex band gap is closed but the imaginary part is opened. The gapless phase region is largely enhanced for the large \( B_x \).

Figure 3(b) plots the real part of \( E \) as a function of \( \delta \) under PBC (gray lines), which consist with bulk spectra under OBC (blue dots) excluding the expected two pairs edge states (red dots) for topological nontrivial phases. We find that the bulk-edge correspondence for our non-Hermitian spin-1 model is protected by the \( \mathcal{M} \) symmetry, which also ensures the real Bloch energy spectra for the gapped phases with satisfying \( E(k) = E^*(k) \) [Fig. 3(c)]. Fig. 3(d) shows the density distribution of the bulk state along the lattice site. We find that the randomly bulk eigenstate for the gapped phase under OBC is a Bloch (extended) state due to the \( \mathcal{M} \) protected bulk-edge correspondence. To understand this, we assume an ansatz that the bulk eigenstate with eigenenergy \( E(k) \) takes the form as [61]: \( |\psi\rangle = |\mu(k)\rangle \bigotimes \sum_n (re^{ikd}n)|n\rangle \), where \( r \) is real and positive decay index and \( |n\rangle = [a_{n,\uparrow}, a_{n,\downarrow}]^T \) is the atomic state for \( n \)th lattice site. Then we have \( \mathcal{M}|\psi\rangle = [\mathcal{M}|\mu(k)\rangle] \bigotimes \sum_n r^{-n}e^{ikd}\mu(n) \) corresponding the eigenenergy \( E^*(k) \). By utilized the commutation relation \( [\mathcal{M}, h(k)] = 0 \) and \( |\mu(k)\rangle = \mathcal{M}|\mu(k)\rangle \), we have the decay index \( r = 1 \), i.e., without the non-Hermitian skin effect. Notably, the non-Hermiticity with preserving bulk-edge correspondence is different to the non-Hermitian models proposed in Ref. [70–76] that possess the parity-time symmetry.

To further exploration the bulk-edge correspondence for high-spin system, we assure the system is illuminated by a \( B_z \) field. In contrast to the \( iB_zF_z \) term, the non-Hermitian term of \( iB_zF_z \) breaks the \( \mathcal{M} \) symmetry, which indicates that the complex band spectra could exist even for the gapped phases. However, we find that the real part of Bloch band gap close and reopen occurs at \( \delta/t = \pm 4 \cos(\phi/2) \), which condition is the same as the
Figure 4. (color online). (a) The $\delta$ dependence of the real part of spectra. The grey lines denote the periodic Bloch spectra under PBC, while the blue lines (red dots) represent the bulk (edge) spectra under OBC. (b) The typical density distribution of bulk state for the topological nontrivial phase with $\delta/t = 2$, which indicates the non-Hermitian skin effect with breaking bulk-edge correspondence. The other parameters are $\phi/\pi = 0.3$, $t_y/t = 1$, and $B_z/t = 1.5$.

 phase boundary for Hermitian Hamiltonian of Eq. (4) and is also independent of the value of $B_z$. However, the imaginary part of Bloch band gap is equal to the non-Hermitian strength $B_z$ at the boundary of topological phase transition.

 Figure 4(a) shows the real part of energy spectra of the non-Hermitian for $\phi/\pi = 0.3$ under the PBC with gray lines and OBC with black dots (bulk states) and red dots (edge states), respectively. As can be seen, the Bloch band spectra is quantitative different from the open-boundary spectra, where the significant divergence is ascribed to the bulk-edge correspondence breaking when $B_z \neq 0$ and $\phi \neq 0$. In particular, the non-Hermitian skin effect [57] is observed, as shown in Fig. 4(b), where the the bulk eigenstate localized near the boundary of system instead of the extended Bloch state. Interestingly, we should emphasize that the non-Hermitian system in present of nonzero $B_z$ can also host the bulk-edge correspondence with absent the magnetic flux. In fact, the bulk-edge correspondence is protected by the inversion symmetry ($P$) even without the magnetic group symmetry ($M$).

 **Conclusion.**—A spin-tensor-momentum-coupled spin-1 lattice model with tunable synthetic magnetic flux lattice and possessing the new symmetry protected phase with satisfying the magnetic group symmetry is proposed for ultracold quantum gases. The band topology which beyond the conventional AZ classification corresponding the chiral symmetry protection has been studied, which gives rise to two-type edge states under different symmetries. Subsequently, we have investigated the bulk-edge correspondence in a non-Hermitian spin-1 model possessing the imaginary Zeeman field. In particular, the preserved (destroyed) bulk-edge correspondence under satisfying (breaking) $M$ symmetry without (with) hosting non-Hermitian skin effect is achieved. Our model can be extended to study the spin-tensor-momentum-coupled exotic topological quantum matters [50–52] and higher-order topological phase transitions in non-Hermitian systems [62, 77, 78].

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