Research on Error Modelling and Identification of 3 Axis NC Machine Tools Based on Cross Grid Encoder Measurement

Z C Du, C F Lv and M S Hong

School of Mechanical and Power Engineering, Shanghai Jiao Tong University, 1954, Huashan Road, Shanghai, 200030, China

E-mail: zcdu@sjtu.edu.cn

Abstract. A new error modelling and identification method based on the cross grid encoder is proposed in this paper. Generally, there are 21 error components in the geometric error of the 3 axis NC machine tools. However according our theoretical analysis, the squareness error among different guide ways affects not only the translation error component, but also the rotational ones. Therefore, a revised synthetic error model is developed. And the mapping relationship between the error component and radial motion error of round workpiece manufactured on the NC machine tools are deduced. This mapping relationship shows that the radial error of circular motion is the comprehensive function result of all the error components of link, worktable, sliding table and main spindle block. Aiming to overcome the solution singularity shortcoming of traditional error component identification method, a new multi-step identification method of error component by using the Cross Grid Encoder measurement technology is proposed based on the kinematic error model of NC machine tool. Firstly, the 12 translational error components of the NC machine tool are measured and identified by using the least square method (LSM) when the NC machine tools go linear motion in the three orthogonal planes: XOX plane, XOZ plane and YOZ plane. Secondly, the circular error tracks are measured when the NC machine tools go circular motion in the same above orthogonal planes by using the cross grid encoder Heidenhain KGM 182. Therefore 9 rotational errors can be identified by using LSM. Finally the experimental validation of the above modelling theory and identification method is carried out in the 3 axis CNC vertical machining centre Cincinnati 750 Arrow. The entire 21 error components have been successfully measured out by the above method. Research shows the multi-step modelling and identification method is very suitable for ‘on machine measurement’.

1. Introduction

Accuracy of machined components is one of the most critical considerations for any manufacturer. Many key factors like cutting tools and machining conditions, resolution of the machine tool, the type of workpiece etc., play an important role. Generally, there are three major types of error, which are known as geometric, thermal and cutting-force induced errors. However geometric errors make up the major part of the inaccuracy of a machine tool. Thus the modeling and identification of geometric errors are the key of the error measurement and compensation of machine tools. The style of error model and accuracy of error identification greatly affect the precision of error measurement and further compensation.
For NC machine tools, the radial error of circular motion is the comprehensive function result of all the error components of link, worktable, sliding table and main spindle block. How to identify each single error component from the radial error is the key to find the error sources. Therefore a great amount of work, over the last decade, has gone into this area. Studies on machine accuracy tests have been carried out by many authors. Weck [1] developed a method using a laser beam and a four-quadrant photodiode to measure the radial error-motion of a rotating table of a gear hobbing machine, and the parallelism between the rotating axis and a linear guide-way. Thus, Zhang et al. [2] developed a displacement method to assess the 21 error components. A hybrid on-line and off-line method for identifying machine geometric error components was presented by Ni and Wu [3]. The method utilizes a multi-degree-of-freedom optical laser system, to simultaneously measure multiple geometric errors. Bryan [4, 5] developed the magnetic Double-ball-bar (DBB) to obtain the position error of a machine at various points. More recent research is done by J. M. Lai and J. S. Liao[6] who proposed the identification method of 3-axis machine tool based on Double Ball Bar in 1997. Guiquan Chen, Jingxia Yuan, Jun Ni [7] and G.H.J. Florussen [8] give a displacement measurement approach for machine geometric error assessment in 2001 respectively. J.H. Lee, S.H [9] measured the geometric errors in a miniaturized machine tool using capacitance sensors in 2005. Usually the Least Square Method (LSM) is used to identify each error components [6, 10]. However the matrix of measurement equation is not full rank, which causes the solution not unique. In addition the complete mapping of all geometric error components is very time consuming. More than that, the installation, adjustment and operation of laser interferometer or Double Ball Bar is very difficult and time consuming.

Aiming to overcome the shortcoming of traditional error component identification method, a revised synthetic error model is developed. Further more, the mapping relationship between the error component and radial motion error of circular workpiece manufactured on the NC machine tools are deduced. According the mapping relationship, a new error component multi-step identification method by using the Cross Grid Encoder measurement technology based on the kinematic error model of NC machine tool.

2. Kinematic modelling of 3-axis NC machine tool
In the normal kinematic modeling process of NC machine tool [10-12], it is considered that the squarness errors only affect the transitional component of error matrices. However in the high precision motion error measurement of NC machine tool, we must consider the influence of the rotational error component which is affected by squarness error. The Homogeneous Transformation Matrix (HTM) method is widely used to describe relationship between two coordinate as shown in figure 1. Therefore, the HTM comes from the squarness errors $e_{xy}$, $e_{xz}$, $e_{yz}$ among the 3 slides can be represented by:

$$
T = T_{xy} + T_{yz} + T_{xz} =
\begin{bmatrix}
1 & -e_{xy} & e_{xz} & z \cdot e_{yz} \\
-e_{xy} & 1 & -e_{yz} & x \cdot e_{yz} - z \cdot e_{xz} \\
e_{xz} & e_{yz} & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(Number 1)

Figure 1. coordinate transform of two coordinate systems. Figure 2. 3-axis Vertical NC machine tool.
The typical 3-axis vertical NC machine tool is shown as figure 2, which include worktable (X-slide), sliding table(Y-slide) and the main spindle block (Z-slide), etc. Generally, there are 21 error components in the geometric error of the 3 axis NC machine tools: 3 translation errors and 3 rotational errors each of X-axis, Y-axis and Z-axis respectively, 3 squareness errors between each two Axis among X, Y, Z rotational Axis. Define the absolute reference coordinate system AS on the bed of machine tool, and define the relatively coordinate frame BS, CS and DS on the sliding table, worktable, and main spindle block respectively.

Suppose the error exist widely in the sliding table, worktable, and main spindle block. Therefore, when the sliding table goes along the Y-slide with moving distance $y$, the HTM $T_{A,B}$ represent the translation of sliding table coordinate frame with respect to the absolute reference coordinate system. Similarly, when the worktable and main spindle block go along the X-slide and Z-slide with moving distance $x$ and $z$ respectively, the HTM $T_{B,C}$ and $T_{A,D}$ correspondingly represent the error transformation matrixes between the two corresponding coordinate frames. The three HTMs are deduced and shown in below:

$$T_{A,B} = \begin{bmatrix}
1 & -\theta_x & \theta_y & \theta_z \\
\theta_x & 1 & \theta_y & \theta_z \\
-\theta_y & -\theta_x & 1 & \theta_z \\
0 & 0 & 0 & 1
\end{bmatrix} \quad T_{B,C} = \begin{bmatrix}
1 & -\varepsilon_x - \theta_x & x + \varepsilon_x + O_{BC} & \theta_z \\
\varepsilon_x + \theta_x & 1 & \theta_z & \theta_z \\
\theta_z & 1 & \delta_x + O_{BC} & 1 \\
0 & 0 & 0 & 1
\end{bmatrix} \quad T_{A,D} = \begin{bmatrix}
1 & -\theta_z & \theta_x & \delta_z + z \cdot e_x + O_{BC} \\
\theta_z & 1 & -\theta_x & \theta_x \\
-\theta_y & -\theta_x & 1 & \theta_z \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

Suppose that the theoretical coordinate vector of the workpiece in the worktable coordinate system CS is $W_C=[W_x, W_y, W_z]^T$, and the error coordinate vector is $\Delta W_C=[\Delta W_x, \Delta W_y, \Delta W_z]^T$. Meanwhile the coordinate vector of the tool tip in the main spindle block coordinate system DS is $U_C=[U_x, U_y, U_z]^T$. In the machining process, we suppose that the tool tip is always contacted with workpiece. Therefore the coordinate vector of cutting point on the workpiece $W_d$ and that of the tool tip must be equal in the absolute coordinate system AS. Then the following relationship between cutting point and tool tip can be expressed as:

$$T_{A,B} T_{B,C} (W_C + \Delta W_C) = W_d = U_d = T_{A,D} U_C$$

Therefore,

$$\Delta W_C = (T_{A,B})^{-1} (T_{B,C})^{-1} T_{A,D} U_C - W_C$$

In the actual modeling process, for computation simplicity, we set the projection point of the origin of the worktable coordinate system on the XOY plane is match together with that of the sliding table coordinate system. Therefore the error coordinate vectors of the NC machine tool can be got:

$$\Delta W_x = L \theta_x z + (z + O_{BC}) \theta_y y \theta_x x + \theta_x y + \theta_x z + \delta_x \cdot \delta_y + \delta_x \cdot \delta_y + z \cdot e_x$$

$$\Delta W_y = L \theta_x z + (z + O_{BC}) \theta_y y \theta_x x + \theta_x y + \theta_x z + \delta_x \cdot \delta_y + \delta_x \cdot \delta_y + z \cdot e_y$$

$$\Delta W_z = -x \theta_z x + y \theta_z x + \delta_z \cdot \delta_y + \delta_z \cdot \delta_y$$

Supposing the NC machine tool goes the circular motion, the coordinate of the center point of the circle is $P_0(x_0, y_0, z_0)$. And the coordinate of the sensor installed on the main spindle block is $P(x, y, z)$. Thus the nominal spatial distance of the $PP_0$ is defined as $R$. Therefore,

$$R^2 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$$

Process the differential operation to both sides of the equation (5), and change original equation style to increment style. Consequently, the mapping relationship between the error component and radial motion error of round workpiece manufactured on the NC machine tools was deduced and can be expressed as:

$$\Delta R = \frac{(x - x_0) \Delta W_x + (y - y_0) \Delta W_y + (z - z_0) \Delta W_z}{R}$$

It indicates that the mapping relationship is the function which variances are the error components of link, worktable, sliding table and spindle when NC machine tool goes the circular motion. Therefore the error component of the NC machine tool can be identified through the measurement of the radial motion error of the circular motion of NC machine tool.
3. Multi-Step identification method of error component

3.1. The traditional identification method of error component

For the 21 single error component of the high precision 3-axis machine tool, people generally use the polynomial with order 3 to fit the single error component of each axis [6, 10]. Normally we have known that the first order coefficient of the straightness error is 0 due to the definition and feature of the straightness error. Also the second order and third order coefficients of the positioning error of each axis are both 0 because of the homogeneous stretching and shrinking. Hence we can know that all the error components of the NC machine tool have 45 undetermined coefficients. Actually the process of the identification of 21 error components is coincident with that of determination of each coefficient.

For the ease of expression, the measuring equation can be rewritten from the above mapping relationship between the error component and radial motion error of round workpiece, shown as below:

\[ E = Q \times P \] (7)

In where, \( E \) is the column vector formed by radial error at different measuring point; \( Q \) is the fitting matrix of the coefficient; and \( P \) is the polynomial coefficient matrix of the geometric error. Therefore the polynomial coefficient matrix of the geometric error can be got by using the LSM on the measuring data.

\[ P = (Q^T \times Q)^{-1} \times Q^T \times E \] (8)

Some limitations of this study are singularity of the solution of the measuring equation. That is to say, when the column vectors of the fitting matrix \( Q \) of the coefficient are independent to each other, in other words, when the matrix of \( Q^T \times Q \) is full rank, the solution of the above equation is unique. Unfortunately, the actual measurement and computation seems to show the matrix \( Q^T \times Q \) is not always full rank on the absolute majority condition. As a result, the solution of the measuring equation becomes not unique any more.

3.2. The principle of Multi-Step error modelling and identification method

In order to overcome the shortcoming of traditional error component identification method, promote the measurement and computation efficiency, a new error component multi-step identification method by using the Cross Grid Encoder measurement technology based on the kinematic error model of NC machine tool and the above mapping relationship. Since the KGM 182 Cross Grid Encoder can measure not only the linear motion error but also the circular motion error in the area where its radius is less than 70mm from the center, the mentioned method can be divided into the following two main steps. Firstly, 12 error component including the straightness error components, positioning error and vertical squareness errors components of \( X \)-slide, \( Y \)-slide and \( Z \)-slide of the NC machine tool are measured and identified. Secondly, 9 rotational errors of the 3 rotational axes can be measured and identified by using the similar least square method.

3.2.1. Identification principle of translational error component, including positioning error, straightness error and squareness error.

When the NC machine tool goes linear motion along \( y=Y_0 \) and \( x=X_0 \) in the XOY plane, define \( s_0(x) \) and \( s_0(y) \) are separately the origin signal measured by the KGM 182 Cross Grid Encoder corresponding to the above two linear motion. Therefore two trend line \( \text{Trend}(x)=a_1x+b_1 \) and \( \text{Trend}(y)=a_2y+b_2 \) are included in the two set of measuring data, as shown in figure 3-A. The gradient coefficient \( a_1, b_1, a_2 \) and \( b_2 \) of the two trend lines can be fitted and given by LSM operation:

\[
\begin{bmatrix}
a_1 \\
b_1 \\
a_2 \\
b_2
\end{bmatrix}
= U^T \times X, \quad
\begin{bmatrix}
a_1 \\
b_1 \\
a_2 \\
b_2
\end{bmatrix}
= U^T \times Y
\]

\[
\begin{bmatrix}
a_1 \\
b_1 \\
a_2 \\
b_2
\end{bmatrix}
= U^T \times U
\]

Where \( X \) is the column vector formed by \( s_0(x) \) and \( Y \) is the column vector formed by \( s_0(y) \).

\[
U = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 & \ldots & 1 \\ 0 & 1 & 2 & \ldots & i & \ldots & N-1 \end{bmatrix}, \quad i=0,1,2,\ldots,N-1, \quad \text{Where } N \text{ is the number of sampling point.}
From figure 3-A, the squareness error between the X-slide and Y-slide can be got:
\[
\delta_{xy} = \frac{S}{2} - a_1 (10)
\]

Remove the error signal caused by the squareness error from the origin \(s_x(x)\) and \(s_y(y)\), shown as figure 3-B, the straightness error \(G_y\) of Y axis in the X direction and \(G_x\) of Y axis in the X direction can be achieved (After reference [13]):
\[
G_y = s_1(x) = s_0(x) - a_1 x (11)
\]
\[
G_x = s_1(y) = s_0(y) - a_2 y (12)
\]

Define \(Avg_{S_x}\) and \(Avg_{S_y}\) are the mean value of the two origin signal \(s_1(x)\) and \(s_1(y)\). Thus the two positioning error of Y axis and X axis can be got:
\[
\delta_{x} = Avg_{S_x} - Y_0 (13)
\]
\[
\delta_{y} = Avg_{S_y} - X_0 (14)
\]

Similar with the measurement of positioning errors, straightness errors and the squareness errors of NC machine tool in the XOY plane, the other positioning errors, straightness errors and the squareness errors in the other two orthogonal plane can also be measured and identified through the linear motion error measurement of NC machine tool in XOZ plane and YOZ plane by the same method.

### 3.2.2. Identification principle of rotational error component

Measurement motions of the rotational error include 3 circular motions which are in the XOY plane, XOZ plane and YOZ plane respectively. For example, considering the measurement circular motion of NC machine tool in the XOY plane, when the machine tool goes the circular motion with the center point \((x, y, z) = (0, 0, 0)\) in the XOY plane of \(Z=0\), the equation (4) can be rewritten as following:
\[
\Delta W_x = O_{BC} \cdot (a_{y_1} y + a_{y_2} y^2 + a_{y_3} y^3) - y \cdot (a_{y_1} y + a_{y_2} y^2 + a_{y_3} y^3)
\]
\[
- y \cdot (a_{x_1} x + a_{x_2} x^2 + a_{x_3} x^3) - k_{y_1} y^2 - k_{y_2} y^3 + k_{y_3} y
\]
\[
\Delta W_y = O_{BC} \cdot (a_{y_1} y + a_{y_2} y^2 + a_{y_3} y^3) + x \cdot (a_{x_1} x + a_{x_2} x^2 + a_{x_3} x^3)
\]
\[
- k_{x_1} y^2 - k_{x_2} y^3 + k_{x_3} y - b y x
\]

Therefore, after deduce from the circular motion error component described by the above equation, the measurement equation with respect to the circular motion error in the XOY plane can be got:
\[
E_1 = Q \times P_1 (17)
\]

Where:
\[
P_1 = [a_{y_1}, a_{y_2}, a_{y_3}, a_{x_1}, a_{x_2}, a_{x_3}, a_{x_1}, a_{x_2}, a_{x_3}, a_{y_1}, a_{y_2}, a_{y_3}]^T
\]
\[
Q = [O_{BC} \cos \theta, O_{BC} \cos \theta, O_{BC} \sin \theta, O_{BC} \sin \theta, O_{BC} y \cos \theta, O_{BC} \cos \theta, -y^2 \cos \theta, -y^2 \cos \theta, -y^2 \cos \theta, -y^2 \cos \theta, -y^2 \cos \theta, -y^2 \cos \theta, -y^2 \cos \theta, -y^2 \cos \theta]
\]
\[
E_1 = [\Delta R(\theta)+k_{y_2} y \cos \theta+k_{y_2} y^3 \cos \theta-k_{x_1} x \sin \theta+k_{x_2} x^3 \cos \theta+k_{x_3} x^3 \sin \theta-k_{y_3} x^2 \sin \theta-k_{y_3} x^2 \sin \theta-k_{y_3} x^2 \sin \theta]
\]

Fitting the equation (17) with LSM, the result can be got:
\[
P_1 = (Q_1^T \times Q_1)^{-1} \times Q_1^T \times E_1 (18)
\]
And, then coefficient of each order of the rotational error polynomial, which are of the roll error $\theta_y$, the yaw error $\theta_z$, the pitch error $\theta_x$ of the sliding table and the yaw error $\theta_{zX}$ of worktable, can be identified through the least square method (LSM).

By adopting the same method, when the machine tool goes the circular motion with the center point $(x, y, z) = (0, 0, 0)$ in the $XOZ$ plane of $Y=0$, the corresponding measuring equation can be deduced as (19). Similarly when the machine tool goes the circular motion with the center point $(x, y, z) = (c, 0, 0)$ in the $XOZ$ plane of $X=c$, the corresponding measuring equation can be deduced as shown in equation (20).

$$E_2=Q_2 \times P_2$$

$$E_3=Q_3 \times P_3$$

Using the same LSM method, the identifications of the polynomial coefficient of rotational error of the yaw error $\theta_Y$ of main spindle block, the pitch error $\theta_Y$ of worktable, the roll error $\theta_x$, the pitch error $\theta_x$ of main spindle block, the roll error $\theta_{zX}$ of the worktable can be processed.

4. Identification steps and results of new method

The experiment machine tool is a 3-axis Cincinnati 750 Arrow II vertical machining center, shown as figure 2. Its machining range is: $X \times Y \times Z = 762\text{mm} \times 510\text{mm} \times 510\text{mm}$. In order to examine all the 21 error component of 3-axis machining center, we adopt a new multiply step identification method of error component by using the Cross Grid Encoder measuring technology. The measurement of linear motion error and circular motion error was carried out in three orthogonal planes: $XOY$ plane, and $YOZ$ plane and $XOZ$ plane respectively. Also, the whole measuring device and installation methods in different plane are shown in the figure 4-a, 4-b and 4-c separately.

![Figure 4. KGM182 measuring the motion error trace in: (a) $XOY$ plane; (b) $XOZ$ plane; (c) $YOZ$ plane.](image)

The concrete measurement and identification steps are summarized in the following six parts:

1. Firstly, we let the machining centre go linear motion along $X$-axis and $Y$-axis in the $XOY$ plane, to identify the positioning errors $\delta_{xX}$, $\delta_{yY}$ of $X$ axis and $Y$ axis, then to identify the straightness error $\delta_{xX}$ of $X$-axis in the $Y$ direction and $\delta_{yY}$ of $Y$-axis in the $X$ direction, and

![Figure 5. Linear motion error traces of machine tool in $XOY$ plane: (a) along $Y$-axis; (b) along $X$-axis.](image)
subsequently to identify the squareness error $\varepsilon_{xy}$ between $X$ and $Y$ axis. Correspondingly the measuring data of linear motion errors in the $XOY$ plane of $Z=0$ are shown as figure 5.

(2) Afterward, we let the machining centre go linear motion along $X$ axis and $Z$ axis in the $XOZ$ plane of $Y=0$, to identify the positioning errors $\delta_{xz}$ of $Z$ axis, straightness error $\delta_{xZ}$ of $Z$ axis in the $X$ direction, $\delta_{zY}$ of $X$ axis in the $Z$ direction, and the squareness error $\varepsilon_{xz}$ between $X$ and $Z$ axis. Alternatively, the measuring data in the $XOZ$ plane of $Y=0$ are shown as figure 6.

(3) Thirdly, we let the machining centre go linear motion along $X$-axis and $Z$-axis in the $YOZ$ plane of $X=C$ in the same way, to identify the straightness error $\delta_{zY}$ of $Z$ axis in the $Y$ direction, $\delta_{zY}$ of $Y$ axis in the $Z$ direction, and the squareness error $\varepsilon_{yz}$ between $Y$ and $Z$ axis. Similarly, the measuring data in the $XOZ$ plane of $Y=0$ are shown as figure 7.

(4) Based on the above discussed multi-step error identification method, when the NC machine tool goes linear motion in the 3 orthogonal $XOY$, $XOZ$, and $YOZ$ plane, according the measuring data of machine tool error traces, the polynomial coefficient of the 12 translational errors such as positional

|          | X-slide | Y-slide | Z-slide |
|----------|---------|---------|---------|
| Uniform shrinking | -335.3  | Uniform shrinking | 462.1  | Uniform shrinking | 1101 |
| Y 2nd order Strai. Error | -23.24  | X Strai. 2nd order | 10.32  | X Strai. 2nd order | -10.19 |
| Z 3rd order Strai. Error | -1.057  | Y Strai. 3rd order | 0.0903 | Y Strai. 3rd order | -0.9836 |
| Z 2nd order Strai. Error | -12.05  | Z Strai. 2nd order | -21.64 | Z Strai. 2nd order | -15.51 |
| Z 3rd order Strai. Error | -0.529  | Y-Z squareness error | -1.602 | Y-Z squareness error | -1.422 |
| X-Y squareness error | -240.8  | -781.8  | -296.1  |

Figure 6. Linear motion error traces of machine tool in $XOZ$ plane: a) along $X$-axis; a) along $Z$-axis.

Figure 7. Linear motion error traces of machine tool in $YOZ$ plane: (a) along $Y$-axis; (b) along $Z$-axis.

Table 1. Each coefficient of the straightness error, positioning error and squareness error. ($\times 10^{-7}$)
errors, straightness errors, squareness errors can be calculated by LSM. The final results of translational error are shown in table 1.

(5) In the forth step, we let the machining centre go circular motion with feedrate speed \( F = 500 \text{mm/min} \) and radius \( R = 50 \) in the \( XOY \) plane of \( Z = 0 \), to separate and identify the rotational error component \( \theta_y, \theta_z, \theta_x, \theta_y \). In this step the error trace data of the circular motion is measured, as shown in Figure 8. Therein, figure 8-A is the error trace data of circular motion of worktable in Counter-clock-wise with roundness 4.6\( \mu \text{m} \), while figure 8-B is that of the circular motion in clock-wise with roundness 4.7\( \mu \text{m} \).

Table 2. Fitting coefficient of 3rd order polynomial of rotational error \( \theta_y, \theta_z, \theta_x \) and \( \theta_y \). (\( \times 10^{-8} \))

| \( \text{XOY plane} \) \( F = 500 \text{mm/min} \) | Roll error \( \theta_y \) of Y-slide | Yaw error \( \theta_y \) of Y-slide | Pitch error \( \theta_y \) of Y-slide | Yaw error \( \theta_x \) of X-slide |
|---|---|---|---|---|
| 1st order component | -53.64 | 1.522 | -1.021 | 1.229 |
| 2nd order component | 0.05558 | -0.00448 | -0.005841 | -0.01709 |
| 3rd order component | 0.01686 | 0.006446 | 0.0193 | 0.01544 |

Based on the identification principle mentioned above, the rotational error can be fitted by three order polynomial. Therefore, we can first substitute the translational error results solved above into equation (17). Hence the three order polynomial coefficients of the rotational errors \( \theta_y, \theta_z, \theta_x \) and \( \theta_y \) can be achieved through the Least Square Method. The final computational results of part rotational error are shown in table 2.
Next, we let the machining centre go circular motion with $R=50$ in the $XOZ$ plane of $Y=0$, to separate and identify the rotational error component $\theta_{zX}$ and $\theta_{zX}$. The corresponding error trace data of the circular motions in the $XOZ$ plane of $Y=0$ are measured, as shown in figure 9. Therein, the roundness error of the Counter-clock-wise circular motion trace and clock-wise circular motion trace in the $XOZ$ plane of $Y=0$ are 7.4 $\mu$m and 7.0 $\mu$m respectively.

In the final step, we let the machining centre go circular motion with $R=50$ in the $YOZ$ plane of $X=C$, to separate and identify the rotational error component $\theta_{xZ}$, $\theta_{xX}$ and $\theta_{zZ}$. Similarly, the error trace data of the circular motions in the $YOZ$ plane of $X=C$ are measured, as shown in figure 10. Meanwhile, the roundness error of the Counter-clock-wise circular motion trace and clock-wise circular motion trace in the $YOZ$ plane of $Y=0$ are 5.1 $\mu$m and 4.2 $\mu$m respectively.

Furthermore, we can also substitute the corresponding translational error results solved above into equation (19) and (20) respectively. Similarly, the three order polynomial coefficients of the rotational errors can be solved by using LSM. The final results are shown in table 3.

| $XOZ/YOZ$ plane | yaw error $\theta_{zX}$ of Z-slide | Pitch error $\theta_{xX}$ of X-slide | Pitch error $\theta_{zX}$ of Z-slide | Roll error $\theta_{xX}$ of X-slide | Roll error $\theta_{zX}$ of Z-slide |
|-----------------|-----------------------------------|-------------------------------------|-------------------------------------|-----------------------------------|-----------------------------------|
| $F=500$ mm/min  | 2.420                             | -46.27                              | 2.443                               | 2.746                             | 0.8702                            |
| 1st order component | -0.04448                          | -2.065                              | 0.3212                              | -0.4942                           | 0.03899                           |
| 2nd order component | -0.02678                          | -0.0006422                         | 0.1478                              | 0.02159                           | 0.005502                          |

Thus all the rotational error components of the 3-axis NC machine tool have been solved by LSM through 3 installations and 9 times of measurement in three orthogonal planes. The whole time of the examination and identification of 21 error component is only 1-2 hours because of easy installation, adjustment and operation. The measuring results indicate that all precision indexes of this machining center for inspection are in the normal range, which are accord with validation results by other measuring and identification method. However, the multi-step method presented herein can reveal the main error sources of the machine tool much more quickly. From the identification results shown in the table 1-3, the main error sources of the machining center are the positioning error and the squareness error of $X$-axis, $Y$-axis and $Z$-axis. On the contrary, the rotational error component is comparatively very small. This kind of measurement and identification method shows great feature to satisfy the request of ‘On machine’ measurement with the guarantee of enough measuring precision.
5. Conclusion
In the high precision measurement, the squareness error affects not only the translational error component of the error transform matrix of machine tool, but also the rotational error component. Based on this point, a revised synthetic error model is developed. Further more, the mapping relationship between the error component and radial motion error of circular workpiece manufactured on the NC machine tools are deduced. This mapping relationship shows that for NC machine tools, the radial error of circular motion is the comprehensive function result of all the error components of link, worktable, saddle and main spindle block.

The traditional error component identification method often exist an singularity problem, which mean the solution of the measuring equation is not always unique because the matrix is not full rank. Aiming to the shortcoming of traditional error component identification method, according the mapping relationship, a new error component multi-step identification method by using the Cross Grid Encoder measurement technology is proposed based on the kinematic error model of NC machine tool. The polynomial coefficients of 21 error component of a 3-axis machining center have been successfully identified by using the multi-step error component identification method. The theoretical and experiment research indicate that this method is of great efficiency and precision. And it is a very fast method to be suitable for the ‘On machine’ error measurement and identification of machine tools.

References
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