\textbf{ABSTRACT} The \(n\)-intuitionistic polygonal fuzzy set (\(n\)-IPFS), combined by the intuitionistic fuzzy and polygonal fuzzy sets, is an extended form of the triangular intuitionistic fuzzy set (TIFS) and the trapezoidal intuitionistic fuzzy set (TrIFS). The aim of this paper is to develop some new aggregation operators for \(n\)-IPFSs and apply them to multi-attribute group decision making (MAGDM) problems. First, the operational properties and the score function of \(n\)-IPFSs are defined. Then, three kinds of \(n\)-intuitionistic polygonal fuzzy aggregation operators are investigated including \(n\)-intuitionistic polygonal fuzzy weighted averaging (\(n\)-IPFWA) operator, \(n\)-intuitionistic polygonal fuzzy ordered weighted averaging (\(n\)-IPFOWA) operator and \(n\)-intuitionistic polygonal fuzzy hybrid aggregation (\(n\)-IPFHA) operator. Finally, we propose an improved technique for order preference by similarity to an ideal solution (TOPSIS) approach with \(n\)-IPFSs and unknown attributes weights. The attributes weights are obtained by combining the entropy weights and the subjective weights, and the entropy weights are calculated based on the score function of \(n\)-IPFS. The spatial closeness reflected by the Hamming distance and the grey relationship with the positive/negative solution are both considered in getting the relative closeness degree to rank the alternatives. The example analysis of a location selection is given to verify the practicality and the effectiveness of the proposed approach in this paper.

\textbf{INDEX TERMS} \(n\)-intuitionistic polygonal fuzzy set, multi-attribute group decision making, aggregation operator, TOPSIS, grey relation analysis.

\section{I. INTRODUCTION}
Multi-attribute group decision making (MAGDM) problems exist widely in the fields of economy, management, and social science. Uncertain and fuzzy information is inevitable due to the complex evaluation objects and the lack of quantitative data. The theory of fuzzy sets (FSs) was first proposed by Zadeh [1] to deal with uncertain information. The FSs have restrictions in handling conflicting information concerning membership of objects. The intuitionistic fuzzy sets (IFSs) introduced by Atanassov [2] are the generalization of the FSs, which consider membership and non-membership of objects. The hesitant considered conforms to the nature of fuzziness better in the objective world. Therefore, the IFSs have advantages in describing ill-known quantity and can deal with the uncertainty of information more flexibly. However, the domain of the IFSs is still a discrete set, i.e. a set of fuzzy concepts, which is the same as that of the FSs. In order to overcome this limitation, relevant literatures have extended the discrete IFSs to the continuous IFSs. The continuous IFSs can process information from different dimensions, mainly include triangular intuitionistic fuzzy sets (TIFSs) and trapezoidal intuitionistic fuzzy sets (TrIFSs). For TIFSs and TrIFSs, the membership degrees and the non-membership degrees are no longer relative to fuzzy concepts, but the triangular fuzzy numbers (TFNs) and trapezoidal fuzzy numbers (TrFNs).

In another extending direction, the polygonal fuzzy number as a natural generalization of TFN and TrFN, is a particular case of the parametric representation of fuzzy number with...
linear interpolation [3]. Liu [4] first proposed the concept of the polygonal fuzzy numbers to overcome the obstacle that the arithmetic operations of FSs based on Zadeh’s extension principle do not satisfy the closeness. The main advantage of the polygonal fuzzy number is that it can approximate a fuzzy number to ensure the closeness and accuracy of the linear operation. Liu and Li [5] introduced the notion of symmetric polygonal fuzzy numbers and obtained some relevant results about the mathematical structure of this space. Báez-Sánchez et al. [6] introduced a new mathematical formalization of the concept of polygonal fuzzy numbers that can be extended naturally to consider the polygonal fuzzy sets (PFSs) in higher dimensions. The polygonal fuzzy numbers are applied successfully in building fuzzy neural network (FNN). The main idea of polygonal FNN is to replace the traditional Zadeh extension operation with the arithmetic operations of the polygonal fuzzy numbers. Liu [4] introduced the arithmetic operations of polygonal fuzzy numbers into the neural network. Li and Li [7] suggested the specific model of a polygonal fuzzy neural network using the extension operations of polygonal fuzzy numbers. Wang and Li [8] studied the universal approximation of four-layer regular polygonal fuzzy neural networks. Wang and Gao [9] gave a three-layer polygonal fuzzy neural network (PFNN) model based on \(n\)-polygonal fuzzy numbers. Wang and Suo [10] constructed the multi-input multi-output (MIMO) polygonal fuzzy neural network model.

As an extended form of IFSs, the PFSs are approximated to the general bounded FSs [6]. The PFS describes a type of fuzzy information with the aid of the orderly representation of real numbers and can describe the fuzzy information according to arbitrary accuracy. Wang et al. [11] introduced arithmetic operations of the PFSs and established a new technique for order preference by similarity to an ideal solution (TOPSIS) evaluation system. Based on an ordered representation of the PFSs, some fuzzy interpolative reasoning (FIR) methods aiming at the sparse fuzzy rule-based systems have been presented based on the PFSs. Chen and Adam [12] dealt with FIR based on the ranking values of PFSs. Chen and Barman [13] studied the adaptive fuzzy interpolative reasoning based on similarity measures of PFSs. Adam [14] presented an adaptive fuzzy interpolative reasoning (AFIR) based on interval type-2 PFSs. The \(n\)-intuitionistic polygonal fuzzy sets (\(n\)-IPFSs) were proposed based on IFSs and PFSs [15]. The \(n\)-IPFS can overcome the shortcomings of the incomplete or inaccurate description of decision information, and make the fuzzy information processing more accurate. Wang and Duan [15] presented the arithmetic operation and Hamming distance formula of the \(n\)-IPFSs, and provided the TOPSIS to solve the multi-attribute decision-making problem.

Information fusion is of vital importance in MAGDM problems. The aggregation operators are widely used fusion methods, including the weighted averaging operator, the ordered weighted averaging operator, the hybrid aggregation operator, etc. TIFSs and TrIFSs were widely used in the fields of multi-attribute decision making (MADM) and MAGDM. Some generalized aggregation operators for triangular intuitionistic fuzzy numbers (TIFNs) and trapezoidal intuitionistic fuzzy numbers (TrIFNs) were studied, as shown in TABLE 1.

The power average operator [18]–[20], the Bonferroni harmonic aggregation operator [21], the arithmetic averaging operator [22], [29], [30], the geometric averaging operator [22], [25], [30], the ordered weighted averaging operator [23], [26], [27] and the hybrid aggregation operator [23], [24] are applied successfully for TIFNs and TrIFNs. While the aggregation operators for \(n\)-IPFSs are barely studied. When there is multiple decision information in \(n\)-IPFSs given by several experts, the evaluation information is hardly to be fused. Therefore, we define three aggregation operators for \(n\)-IPFSs in this paper, including \(n\)-intuitionistic polygonal fuzzy weighted averaging (\(n\)-IPFWA) operator, \(n\)-intuitionistic polygonal fuzzy ordered weighted averaging (\(n\)-IPFWOA) operator and \(n\)-intuitionistic polygonal fuzzy hybrid aggregation (\(n\)-IPFHA) operator. The \(n\)-IPFWA operator only considers the weighting of \(n\)-intuitionistic polygonal fuzzy values. The \(n\)-IPFWOA operator only considers the weighting of the ordered positions of \(n\)-intuitionistic polygonal fuzzy instead of \(n\)-intuitionistic polygonal fuzzy values themselves. Motivated by the idea of combining the weighted average operator and the ordered weighted average operator, we develop an \(n\)-intuitionistic polygonal fuzzy hybrid aggregation (\(n\)-IPFHA) operator. The \(n\)-IPFHA operator can consider weighting both the given \(n\)-intuitionistic polygonal fuzzy values and their ordered positions. The three aggregation operators are used to fuse the decision information for solving MAGDM problems.

As a representative MAGDM method, TOPSIS has been widely applied in numerous practical situations [32]–[34]. TOPSIS selects the best choice having the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. The traditional TOPSIS only reflects the spatial closeness. The alternative with the same spatial closeness may have different advantages, but they cannot be distinguished. In this paper, an improved TOPSIS is proposed based on the integrated information by the aggregation operators for \(n\)-IPFSs. The Hamming distance reflecting the spatial distance combines with the grey relation degree to calculate the relative closeness degree. The main contributions of this paper are summarized as follows:

1. Three aggregation operators for \(n\)-IPFSs are proposed first, and they are \(n\)-IPFWA operator, \(n\)-IPFWOA operator and \(n\)-IPFHA operator.
2. The weight vector of \(n\)-IPFWOA is optimized based on the programming-based method. A balancing factor is introduced to coordinate the given \(n\)-intuitionistic polygonal fuzzy values and their ordered positions in the \(n\)-IPFHA operator.
3. The aggregation operators for \(n\)-IPFSs are applied in MAGDM, and an improved TOPSIS is given out. The spatial closeness and the grey relationship are both
TABLE 1. The Relative Researches on Aggregation Operators for TIFNs and TrIFNs.

| The referred works | Fuzzy information used | The research contribution |
|--------------------|-----------------------|---------------------------|
| [16]               | TIFNs                 | A new ranking approach was presented for comparing TIFNs. The weighted average operator of TIFNs was defined and a new ranking method of TIFNs considering the risk attitude was given. |
| [17]               | TIFNs                 | Four power average operators of TIFNs were proposed. Three kinds of triangular intuitionistic fuzzy Bonferroni harmonic aggregation operators were defined. |
| [18] [19] [20]     | TrIFNs                | The trapezoidal intuitionistic fuzzy weighted arithmetic averaging operator and weighted geometric averaging operator were presented. The trapezoidal intuitionistic fuzzy ordered weighted averaging operator and trapezoidal intuitionistic fuzzy hybrid aggregation operator were proposed. |
| [21]               | TIFNs                 | The intuitionistic trapezoidal fuzzy ordered weighted averaging operator and trapezoidal fuzzy hybrid aggregation operator were proposed. Some families of geometric aggregation operators with intuitionistic trapezoidal fuzzy numbers were presented. An algorithm for ranking alternatives under trapezoidal intuitionistic fuzzy environment based on trapezoidal intuitionistic fuzzy weighted averaging operator and OWA operator was developed. A generalized triangular intuitionistic fuzzy ordered weighted geometric averaging (GTIFOWGA) operator was presented. Several aggregation operators of trapezoidal intuitionistic fuzzy sets were provided. |
| [22]               | TrIFNs                | Some arithmetic aggregation operators for TAIFNs were defined. A novel concept called linguistic trapezoidal fuzzy intuitionistic fuzzy sets (LTIFIS) was introduced and several weighted arithmetic and weighted geometric aggregation operators were proposed. |
| [23]               | TrIFNs                | Some hesitant triangular intuitionistic fuzzy aggregation operators and standardized hesitant triangular intuitionistic fuzzy aggregation operators were developed. |

TABLE 2. All abbreviations and used in this study.

| Names                                                   | Abbreviations |
|---------------------------------------------------------|---------------|
| n-intuitionistic polygonal fuzzy sets                    | n-IPFSs       |
| triangular intuitionistic fuzzy sets                    | TIFs          |
| trapezoidal intuitionistic fuzzy sets                   | TrIFs         |
| multi-attribute group decision making                   | MAGDM         |
| n-intuitionistic polygonal fuzzy weighted averaging      | n-IPFWA       |
| n-intuitionistic polygonal fuzzy ordered weighted averaging | n-IPFOWA     |
| n-intuitionistic polygonal fuzzy hybrid aggregation      | n-IPFHA       |
| technique for order preference by similarity TOPSIS      |               |
| to an ideal solution                                    |               |
| fuzzy sets                                              | FSs           |
| intuitionistic fuzzy sets                               | IFSs          |
| triangle fuzzy numbers                                  | TIFs          |
| trapezoidal fuzzy numbers                               | TrIFs         |
| polygonal fuzzy sets                                    | PFSs          |
| fuzzy neural network                                    | FNN           |
| polygonal fuzzy neural network                          | PFNN          |
| multi-input multi-output                                | MIMO          |
| fuzzy interpolative reasoning                           | FIR           |
| adaptive fuzzy interpolative reasoning                  | AFIR          |
| multi-attribute decision making                         | MADM          |
| triangular intuitionistic fuzzy numbers                 | TrIFs         |
| trapezoidal intuitionistic fuzzy numbers                 | TrIFNs        |

Moreover, in order to better illustrate our study methods in detail, an analysis with all abbreviations used is made and summarized in Table 2.

The structure of this paper is as follows. In section II, we introduce the definition and the operational laws of n-IPFSs, and then put forward the score function and some operational properties. In section III, three kinds of aggregation operators of n-IPFSs are defined. In section IV, a MAGDM approach based on an improved TOPSIS is proposed. In section V, an example is given to verify the effectiveness and feasibility of the proposed method. Some solid comparative analyses are displayed. Finally, the conclusions of this paper are summarized in section VI.

II. PRELIMINARIES

In this section, we briefly introduce the definition of n-IPFS. Hamming distance between n-IPFSs and operational laws of n-IPFSs. Moreover, we give out the operation properties and the score function of n-IPFS.

A. n-INTUITIONISTIC POLYGONAL FUZZY SETS

Definition 1 [15]: Let A is an intuitionistic polygonal fuzzy set (IPFS) in the set of real numbers.
If there are \(2n + 2\) orderly real numbers, \(A = \{(a_0^1, a_1^1, \ldots, a_n^1, a_{n+1}^1, \ldots, a_0^2, a_1^2)\}; \lambda, \delta\) which is an \(n\)-intuitionistic polygonal fuzzy set (\(n\)-IFPS) on \(R\). \(A\) satisfies the conditions: \(n \in N\) and \(a_0^1 \leq a_1^1 \leq \ldots \leq a_n^1 \leq a_{n+1}^1 = \ldots = a_0^2 \leq a_1^2 \leq \ldots \leq a_n^2 \leq a_{n+1}^2\), whose membership function and non-membership function are defined as follows:

\[
\mu_A(x) = \frac{(i-1)\lambda + \lambda \left(x - a_{i-1}\right)}{n \left(a_i - a_{i-1}\right)}, \quad x \in \left[a_{i-1}, a_i\right]
\]

\[
\nu_A(x) = \frac{n - (1 - \delta) (i-1) + (1 - \delta) \left(x - a_{i-1}\right)}{n \left(a_i - a_{i-1}\right)}, \quad x \in \left[a_{i-1}, a_i\right]
\]

where \(0 \leq \lambda, \delta \leq 1, \lambda + \delta \leq 1, x_i = \frac{a_i}{n}\) and \(x_i = \delta + \frac{(1-\delta)}{n}\) are \(n\) equal division points of \([0, \lambda]\) and \([\delta, 1]\), respectively. \(\mu_A(a_i^j) = \frac{\lambda_i}{n}, \nu_A(a_i^j) = \delta + \frac{(1-\delta)}{n}, i = 1, 2, \ldots, n\). \(\lambda\) and \(\delta\) are the maximum membership degree and the minimum non-membership degree of \(A\), respectively. Here, we only investigate the special case of \(\mu_A = \nu_A = \{(a_0^1, a_1^1, \ldots, a_n^1, a_{n+1}^1, \ldots, a_0^2, a_1^2)\}; \lambda_\delta, \delta\) which is a two-\(n\)-IFPS on \(R\), respectively, given \(n \in N\), and the coefficients of the hesitation degree are \(g_{A_1} = \lambda_\delta + 1 - 6_{a_i}, g_{B_1} = \lambda_\delta + 1 - 6_{b_i}\), the Hamming distance between \(A\) and \(B\) is defined as follows:

\[
h(A, B) = \frac{1}{2n+2} \sum_{i=0}^{n} \left(\left|g_{A_i} - g_{B_i}\right| + \left|g_{A_i}^2 - g_{B_i}^2\right|\right)
\]

(1) \(h(A, B) \geq 0\), and \(A = B \Rightarrow h(A, B) = 0\).

(2) \(h(A, B) = h(B, A)\).

(3) \(h(A, B) \leq h(A, C) + h(C, B)\).

where \(C\) is an \(n\)-IFPS on \(R\).

In this work, we use the Hamming distance to calculate the distances in the TOPSIS method. 

**Definition 3 [15]:** Let \(A = \{(a_0^1, a_1^1, \ldots, a_n^1, a_{n+1}^1, \ldots, a_0^2, a_1^2)\}; \lambda_\delta, \delta\) and \(B = \{(b_0^1, b_1^1, \ldots, b_n^1, b_{n+1}^1, \ldots, b_0^2, b_1^2)\}; \lambda_\delta, \delta\) are two \(n\)-IFPSs on \(R\), respectively, given \(n \in N\), the operational laws of \(A\) and \(B\) are defined as follows:

\[
A + B = \left\{a_0^1 + b_0^1, a_1^1 + b_1^1, \ldots, a_n^1 + b_n^1, a_0^2 + b_0^2, \ldots, a_n^2 + b_n^2\right\}; \lambda_\delta \wedge \lambda_\delta, \delta \vee \delta
\]

\[
A - B = \left\{a_0^1 - b_0^1, a_1^1 - b_1^1, \ldots, a_n^1 - b_n^1, a_0^2 - b_0^2, \ldots, a_n^2 - b_n^2\right\}; \lambda_\delta \wedge \lambda_\delta, \delta \vee \delta
\]

\[
A \cdot B = \left\{f_0^1, f_1^1, \ldots, f_n^1, f_0^2, f_1^2, \ldots, f_n^2\right\}; \lambda_\delta \wedge \lambda_\delta, \delta \vee \delta
\]

where \(f_0^1 = a_0^1b_0^1 \cap a_1^1b_1^1 \cap a_2^1b_2^1 \cap a_1^2b_1^2 \cap a_2^2b_2^2, f_1^1 = a_0^1b_1^1 \cap a_1^1b_2^1 \cap a_2^1b_0^1 \cap a_1^2b_2^2 \cap a_2^2b_0^2, i = 1, 2, \ldots, n\). \(\lambda_\delta \wedge \lambda_\delta, \delta \vee \delta\)

**C. OPERATIONAL PROPERTIES AND SCORE FUNCTION FOR n-IPFSs**

**Theorem 1:** Let \(A_j = \{(a_0^1, a_1^1, \ldots, a_n^1, a_{n+1}^1, \ldots, a_0^2, a_1^2)\}; \lambda_j, \delta\) (\(j = 1, 2, 3\)) are three \(n\)-IFPSs on \(R\), respectively, given \(n \in N\). The operational properties of \(A_j\) are defined as follows:

(1) \(A_1 + A_2 = A_2 + A_1\).

(2) \(A_1 \cdot A_2 = A_2 \cdot A_1\).

(3) \(\xi(A_1 + A_2) = \xi A_1 + \xi A_2, \xi \geq 0\).

(4) \(\xi A_1 + \xi A_2 = (\xi_1 + \xi_2) A_1, \xi_1, \xi_2 \geq 0\).

**Proof:** From Definition 3, we can know the operational properties of (1) and (2) are valid, the following is to prove the operational properties of (3) and (4).

\[
\xi (A_1 + A_2) = \xi \left\{(a_0^{(1)} + a_0^{(2)}, a_1^{(1)} + a_1^{(2)}, \ldots, a_n^{(1)} + a_n^{(2)}), a_0^{(2)}, \ldots, a_n^{(2)}, a_0^{(1)}, \ldots, a_n^{(1)}\right\}; \\
\lambda_1 \wedge \lambda_2, \delta_1 \vee \delta_2
\]

\[
= \left\{\xi \left(a_0^{(1)} + a_0^{(2)}\right), \xi \left(a_1^{(1)} + a_1^{(2)}\right), \ldots, \xi \left(a_n^{(1)} + a_n^{(2)}\right)\right\}; \\
\xi \left(a_0^{(2)} + a_0^{(1)}\right), \ldots, \xi \left(a_n^{(2)} + a_n^{(1)}\right), \xi \left(a_0^{(1)} + a_0^{(2)}\right); \lambda_1 \wedge \lambda_2, \delta_1 \vee \delta_2
\]

\[
= \left\{\xi a_0^{(1)} + \xi a_0^{(2)}, \xi a_1^{(1)} + \xi a_1^{(2)}, \ldots, \xi a_n^{(1)} + \xi a_n^{(2)}\right\}; \\
\xi a_0^{(2)}, \xi a_1^{(2)}, \ldots, \xi a_n^{(2)}, \xi a_0^{(1)}, \ldots, \xi a_n^{(1)}; \lambda_1 \wedge \lambda_2, \delta_1 \vee \delta_2
\]

where \(\xi\) is the score function.
\[ \lambda_1, \delta_1 \} + \left\{ \xi a_1^{1,2}, \xi a_1^{1,3}, \ldots, \xi a_n^{1,2}, \xi a_n^{1,3}, \ldots \right\} \\
\xi a_1^{2,3}, \xi a_2^{2,3}; \lambda_2, \delta_2 \right\} \\
= \xi A_1 + \xi A_2 \]

Hereby, the proof of the operational property (3) has been completed. In the same way, we can prove the operational property (4) is valid.

This completes the proof of Theorem 1.

In this study, we develop the score function of n-IPFS by extending the score function of TrIFSs [36]. The score function of n-IPFS is used to calculate the attributes weights.

**Definition 4:** Let \( A = \{ (a_1^0, a_1^1, \ldots, a_n^0, a_n^1, \ldots, a_k^0, a_k^1) \}; \lambda_j, \delta_j \} \) is an n-intuitionistic polygonal fuzzy set on \( \mathbb{R} \), given \( n \in \mathbb{N} \), the score function of \( A \) is defined as follows:

\[ S(A) = \frac{a_0^1 + a_1^1 + \cdots + a_n^1 + a_n^1 + \cdots + a_k^1 + a_0^1}{2n + 2} (\lambda(a) - \delta(a)) \]

where \( S(A) \in [0, 1] \). The higher the value of \( S(A) \) is, the larger

**III. n-INTUITIONISTIC POLYNOMIAL FUZZY AGGREGATION OPERATORS**

In this section, we develop three kinds of n-intuitionistic polygonal fuzzy aggregation operators and give proofs of them.

**A. n-INTUITIONISTIC POLYNOMIAL FUZZY WEIGHTED AVERAGING OPERATOR**

**Theorem 2:** Let \( A_j = \{ (a_1^0, a_1^1, \ldots, a_n^0, a_n^1, \ldots, a_k^0, a_k^1) \}; \lambda_j, \delta_j \} \) is an n-IPFS on \( \mathbb{R} \), give \( n \in \mathbb{N} \), \( j = 1, 2, \ldots, n \). if n-IPFWA: \( \Omega^n \rightarrow \Omega \), let \( \Omega \) be the set of all n-IPFSs, and the n-IPFWA operator of \( A_j \) is defined as follows:

\[ n-IPFWA_w (A_1, A_2, \ldots, A_k) = \left\{ \sum_{j=1}^{k} w_j a_{1j}^0, \sum_{j=1}^{k} w_j a_{1j}^1, \ldots, \sum_{j=1}^{k} w_j a_{nj}^0, \sum_{j=1}^{k} w_j a_{nj}^1 \right\} \]

where \( w = (w_1, w_2, \ldots, w_n)^T \) is the weight vector associated with n-IPFWA, satisfying \( 0 \leq w_j \leq 1 \) \((j = 1, 2, \ldots, n)\), and \( \sum w_j = 1 \). Specifically, if \( w = \left( \frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n} \right)^T \), the n-IPFWA operator reduces to the n-intuitionistic polygonal fuzzy averaging (n-IPFA) operator.

**Proof:** The Eq. (2) can be proven by using mathematical induction for \( n \).

We first prove that Eq. (2) holds for \( n = 2 \).

Since

\[ w_1 A_1 = \left\{ a_1^0, a_1^1, \ldots, a_n^0, a_n^1 \right\} \]

\[ w_2 A_2 = \left\{ a_1^0, a_1^1, \ldots, a_a^0, a_a^1 \right\} \]

Then

\[ n-IPFWA_w (A_1, A_2) = \left\{ \sum_{j=1}^{k} w_j a_{1j}^0, \sum_{j=1}^{k} w_j a_{1j}^1, \ldots, \sum_{j=1}^{k} w_j a_{nj}^0, \sum_{j=1}^{k} w_j a_{nj}^1 \right\} \]

**Then when \( n = k + 1 \), by the operational laws in Definition 3, we have**

\[ n-IPFWA_w (A_1, A_2, \ldots, A_k) = \left\{ \sum_{j=1}^{k} w_j a_{1j}^0, \sum_{j=1}^{k} w_j a_{1j}^1, \ldots, \sum_{j=1}^{k} w_j a_{nj}^0, \sum_{j=1}^{k} w_j a_{nj}^1 \right\} \]

\[ = \left\{ \sum_{j=1}^{k} w_j a_{1j}^0 + \sum_{j=1}^{k} w_j a_{1j}^1, \ldots, \sum_{j=1}^{k} w_j a_{nj}^0 + \sum_{j=1}^{k} w_j a_{nj}^1 \right\} \]

**Proof:** The Eq. (2) can be proven by using mathematical induction for \( n \).
\[ \begin{align*}
&= \left\{ \left( \sum_{j=1}^{k+1} w_j a_{\theta(j)}^1, \sum_{j=1}^{k+1} w_j a_{\theta(j)}^2, \ldots, \sum_{j=1}^{k+1} w_j a_{\theta(j)}^n \right), \\
&\quad \sum_{j=1}^{k+1} w_j a_{\theta(j)}^2, \ldots, \sum_{j=1}^{k+1} w_j a_{\theta(j)}^2, \sum_{j=1}^{k+1} w_j a_{\theta(j)}^2 \right) \right\};
&\quad \lambda_1 \wedge \lambda_2 \wedge \ldots \wedge \lambda_k \wedge \lambda_{k+1}, \delta_1 \vee \delta_2 \vee \ldots \vee \delta_k \vee \delta_{k+1} \right\}
\end{align*} \]
i.e., Eq. (2) holds for \( n = k + 1 \).
Therefore, Eq. (2) holds for all \( n \), which completes the proof of Theorem 2.

**B. n-INTUITIONISTIC POLYGONAL FUZZY ORDERED WEIGHTED AVERAGING OPERATOR**

**Theorem 3:** Let \( A_j = \{(a^1_j, a^2_j, \ldots, a^n_j, a^{1}_{\theta(j)}, a^{2}_{\theta(j)}, \ldots, a^{n}_{\theta(j)}), \lambda_j, \delta_j \} \) is an n-IPFS on \( R \), given \( n \in N, j = 1, 2, \ldots, n \).

If n-IPFOWA: \( \Omega^n \rightarrow \Omega \), let \( \Omega \) be the set of all n-IPFSs, and the n-IPFOWA operator of \( A_j \) is defined as follows:

\[ n-IPFOWA_{n}(A_j) = \sum_{j=1}^{n} \omega_j A_{\theta(j)} \]

where \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) is the weight vector associated with n-IPFOWA, satisfying \( 0 \leq \omega_j \leq 1 \) for \( j = 1, 2, \ldots, n \)

and \( \sum_{j=1}^{n} \omega_j = 1 \). (\( \theta(1), \theta(2), \ldots, \theta(n) \)) is a permutation of \( \{(1), (2), \ldots, (n)\} \), such that \( A_{\theta(j)} \) is the \( j \)-th largest element in \( A_j \). Specifically, when \( \omega = \left( \frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n} \right)^T \), the n-IPFOWA operator reduces to n-IPFA operator.

Strategies for producing weights of OWA operators can be roughly divided into four ways: programming-based method, experience-based method, quantifier-based method, and weight functions-based method. The programming-based method is one of the most frequently used ways to determine the OWA weight vector. In the programming-based method, a weight optimization model is always established with an objective function based on entropy, disparity, deviation, or variance.

The weight vector for n-IPFOWA operator is generated by using the maximum entropy-based programming method [37], which is shown as follows.

Maximize: \( H(\omega) = -\sum_{j=1}^{n} \omega_j \ln(\omega_j) \)

subject to:
\[ \begin{align*}
&\sum_{j=1}^{n} \omega_j = 1, \\
&0 \leq \omega_j \leq 1
\end{align*} \]

where \( \omega_j \) is associated with \( A_{\theta(j)} \), which is the \( j \)-th largest element in \( A_j \). \( \alpha \) is the orness measure \( orness(\omega) \).

\[ orness(\omega) = \frac{1}{n-1} \sum_{j=1}^{n} [(n-j) \omega_j] \]

**Proof:** The Eq. (3) can be proven by using mathematical induction for \( n \).

We first prove that Eq. (3) holds for \( n = 2 \).

Since
\[ \omega_1 A_{\theta(1)} = \left\{ \left( \omega_1 a^{1}_{\theta(1)}, \omega_1 a^{2}_{\theta(1)}, \ldots, \omega_1 a^{n}_{\theta(1)} \right), \lambda_1, \delta_1 \right\} \]
\[ \omega_2 A_{\theta(2)} = \left\{ \left( \omega_2 a^{1}_{\theta(2)}, \omega_2 a^{2}_{\theta(2)}, \ldots, \omega_2 a^{n}_{\theta(2)} \right), \lambda_2, \delta_2 \right\} \]

Then
\[ n-IPFOWA_{n}(A_1, A_2) = \omega_1 A_{\theta(1)} \oplus \omega_2 A_{\theta(2)} \]

\[ = \left\{ \left( \omega_1 a^{1}_{\theta(1)}, \omega_1 a^{2}_{\theta(1)}, \ldots, \omega_1 a^{n}_{\theta(1)} \right), \lambda_1, \delta_1 \right\} \]
\[ \quad \oplus \left\{ \left( \omega_2 a^{1}_{\theta(1)}, \omega_2 a^{2}_{\theta(2)}, \ldots, \omega_2 a^{n}_{\theta(2)} \right), \lambda_2, \delta_2 \right\} \]

If Eq. (3) holds for \( n = k \), that is
\[ n-IPFOWA_{n}(A_1, A_2, \ldots, A_k) \]

Then when \( n = k + 1 \), by the operational laws in Definition 3, we have
\[ n-IPFOWA_{n}(A_1, A_2, \ldots, A_k) \]

\[ = n-IPFOWA_{n}(A_1, A_2, \ldots, A_k) \oplus \omega_{k+1} A_{\theta(k+1)} \]
\[ = \left\{ \left( \omega_{k+1} a^{1}_{\theta(k+1)}, \omega_{k+1} a^{2}_{\theta(k+1)}, \ldots, \omega_{k+1} a^{n}_{\theta(k+1)} \right), \lambda_{k+1}, \delta_{k+1} \right\} \]
\[ \quad \oplus \left\{ \left( \omega_{k+1} a^{1}_{\theta(k+1)}, \omega_{k+1} a^{2}_{\theta(k+1)}, \ldots, \omega_{k+1} a^{n}_{\theta(k+1)} \right), \lambda_{k+1}, \delta_{k+1} \right\} \]

\[ = \left\{ \left( \omega_{k+1} a^{1}_{\theta(k+1)}, \omega_{k+1} a^{2}_{\theta(k+1)}, \ldots, \omega_{k+1} a^{n}_{\theta(k+1)} \right), \lambda_{k+1}, \delta_{k+1} \right\} \]
In this section, we develop an AGGREGATION OPERATOR proof of Theorem 3. Therefore, Eq. (3) holds for all \( n \), which completes the proof of Theorem 3.

C. **n-INTUITIONISTIC POLYNOMIAL FUZZY HYBRID AGGREGATION OPERATOR**

In this section, we develop an n-IPFHA operator based on n-IPFWA operator and n-IPFOWA operator, which considers weighing both the given n-intuitionistic polygonal fuzzy value and its ordered position.

**Theorem 4:** Let \( A_j = \{(a_{1j}, a_{2j}, \ldots, a_{nj})\}; \lambda_j, \delta_j \) is an n-IPFS on \( R \), given \( n \in N, j = 1, 2, \ldots, n \), if n-IPFHA: \( \Omega^n \rightarrow \Omega \), let \( \Omega \) be the set of all n-IPFSs, and the n-IPFHA operator of \( A_j \) is defined as follows:

\[
\text{n-IPFHA}_w = \sum_{j=1}^{n} W_j A_{\theta(j)} \quad (6)
\]

where \( W_j = w_j \tau + \omega_j (1 - \tau) \), \( \tau \) is the balancing factor, \( W = (W_1, W_2, \ldots, W_n)^T \) is the weight vector associated with n-IPFHA, satisfying \( 0 \leq W_j \leq 1 \) \( (j = 1, 2, \ldots, n) \), and \( \sum_{j=1}^{n} W_j = 1 \). \( w = (w_1, w_2, \ldots, w_n)^T \) is the weighting vector associated with n-IPFHA, satisfying \( 0 \leq w_j \leq 1 \) \( (j = 1, 2, \ldots, n) \), and \( \sum_{j=1}^{n} w_j = 1 \). \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) is the weight vector associated with n-IPFOWA, satisfying \( 0 \leq \omega_j \leq 1 \) \( (j = 1, 2, \ldots, n) \), and \( \sum_{j=1}^{n} \omega_j = 1 \). \((\theta(1), \theta(2), \ldots, \theta(n))\) is a permutation of \((1, 2, \ldots, n)\), such that \( A_{\theta(j)} \geq A_{\theta(j-1)} \) for all \( j \). \( A_{\theta(j)} \) is the \( j \)th largest element in \( A_j \). Specifically, when \( \tau = 1 \), the n-IPFHA operator reduces to the n-IPFWA operator, when \( \tau = 0 \), the n-IPFHA operator reduces to the n-IPFOWA operator.

**Proof:** Based on the proof of theorem 2 and theorem 3, we can get Eq. (6) holds for all \( n \), which completes the proof of Theorem 4.

IV. **MAGDM WITH n-INTUITIONISTIC POLYNOMIAL FUZZY AGGREGATION OPERATORS**

In this section, we present a MAGDM problem with n-intuitionistic polygonal fuzzy aggregation operators, and then we develop an improved TOPSIS approach to solve the MAGDM problem.

In the multi-attribute decision process of n-intuitionistic polygonal fuzzy linguistic, the evaluation information of all decision makers needs to be aggregated. The proposed n-intuitionistic polygonal fuzzy aggregation operators are used to aggregate the evaluation information of all decision makers. How to determine the attributes weights is another question in MAGDM problem due to different attribute importance degrees. The entropy method based on the score function is used to determine the attributes weights. Finally, TOPSIS and grey relation analysis [38]–[40] are combined to select the optimal alternative.

Let \( Q_m \) \( (i = 1, 2, \ldots, m) \) be the set of alternatives, \( m \) represents the total number of alternatives, \( X_d \) \( (j = 1, 2, \ldots, d) \) be the set of attributes, \( d \) represents the number of attributes, \( O_k \) \( (k = 1, 2, \ldots, t) \) be the set of decision makers, and \( t \) represents the number of decision makers. The proposed MAGDM procedure based on n-IPFA operators is shown as follows:

**Step 1:** Construct the standard decision matrix \( H' \) of n-intuitionistic polygonal fuzzy sets.

\[
H' = \begin{bmatrix}
H_{11}' & \cdots & H_{1j}' & \cdots & H_{1d}' \\
H_{21}' & \cdots & H_{2j}' & \cdots & H_{2d}' \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
H_{m1}' & \cdots & H_{mj}' & \cdots & H_{md}'
\end{bmatrix}
\]

where the decision matrix \( H' \) of n-intuitionistic polygonal fuzzy sets should be standardized as follows:

\[
h_{ij} = \left\{ \begin{array}{l}
\left( \frac{y_0}{\beta_j}(ij) \times \frac{y_0}{\alpha_j}(ij), \ldots, \frac{y_0}{\beta_j}(ij) \times \frac{y_0}{\alpha_j}(ij) \right) ; \lambda_{h_{ij}}, \delta_{h_{ij}} \\
\lambda_{h_{ij}}, \delta_{h_{ij}} \\
\end{array} \right\}, \quad j \in F
\]

\[
h_{ij} = \left\{ \begin{array}{l}
\left( \frac{\beta_j}{\gamma_0}(ij) \times \frac{\beta_j}{\gamma_1}(ij), \ldots, \frac{\beta_j}{\gamma_0}(ij) \times \frac{\beta_j}{\gamma_1}(ij) \right) ; \lambda_{h_{ij}}, \delta_{h_{ij}} \\
\lambda_{h_{ij}}, \delta_{h_{ij}} \\
\end{array} \right\}, \quad j \in I
\]

where, \( 0 \leq \gamma_0(ij) \leq \cdots \leq \gamma_0(ij) \leq \gamma_1(ij) \leq \cdots \leq \gamma_1(ij) \). \( F \) is the benefit-type attribute set, and \( I \) is the cost-type attribute set.
with where and (vector associated with is a permutation of \( H \))

\[ h_i = n \text{-IPFWA} \left( h_{ij}^1, h_{ij}^2, \ldots, h_{ij}^n \right) = \sum_{k=1}^{t} w_k h_{ij}^{(k)} \]  

\[ h_i = n \text{-IPFOWA} \left( h_{ij}^1, h_{ij}^2, \ldots, h_{ij}^n \right) = \sum_{k=1}^{t} o_k h_{ij}^{(k)} \]  

\[ h_i = n \text{-IPFWA} \left( h_{ij}^1, h_{ij}^2, \ldots, h_{ij}^n \right) = \sum_{k=1}^{t} W_k h_{ij}^{(k)} \]

where \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, t, k = 1, 2, \ldots, t \)

\( w = (w_1, w_2, \ldots, w_t)^T \) is the weight vector associated with \( n \text{-IPFWA} \), satisfying \( 0 \leq w_k \leq 1 \) \((k=1, 2, \ldots, t)\)

and \( \sum_{k=1}^{t} w_k = 1. \omega = (\omega_1, \omega_2, \ldots, \omega_t)^T \) is the weight vector associated with \( n \text{-IPFOWA} \), satisfying \( 0 \leq \omega_k \leq 1 \) \((k=1, 2, \ldots, t)\)

\( \sum_{k=1}^{t} \omega_k = 1. (\omega^{(1)}, \omega^{(2)}, \ldots, \omega^{(t)}) \) is a permutation of \( (1, 2, \ldots, t) \), such that \( h_{ij}^{(k-1)} \geq h_{ij}^{(k)} \) for all \( k \). \( h_{ij}^{(k)} \) is the \( k \)th largest element in \( h_{ij} \).

\( W = (W_1, W_2, \ldots, W_n)^T \) is the weight vector associated with \( n \text{-IPFOWA} \), satisfying \( 0 \leq W_k \leq 1 \) \((k=1, 2, \ldots, t)\)

and \( \sum_{k=1}^{t} W_k = 1. \tau \) is the balancing factor.

**Step 3:** Obtain the attributes weights and the weighted group decision matrix.

1. Calculate the score function of \( n \text{-IPFS} \) based on the aggregation results with \( n \text{-IPFWA} \) (or \( n \text{-IPFOWA}, n \text{-IPFWA} \)) operator. The score value of \( h_{ij} \) = \( \{a_{ij1}, a_{ij2}, \ldots, a_{ijn}, a_{ij}^{\min}, a_{ij}^{\max}\} \); \( \lambda_{ij}, \delta_{ij} \) is expressed as \( S_{ij} \).

2. Use the entropy weight method to normalize the score values \( S_{ij} \).

\[ l_{ij} = \frac{S_{ij}}{\sum_{i=1}^{m} S_{ij}} \]  

\[ e_j = -\sum_{i=1}^{m} \left( \frac{l_{ij} \ln l_{ij}}{\ln m} \right) \]  

If \( l_{ij} = 0 \), then \( l_{ij} \ln l_{ij} = 0 \).

3. Calculate the entropy weight \( q_j \) of the \( j \)th attribute by using the information entropy method \[ q_j = 2 - e_j = 2 + \sum_{i=1}^{m} \frac{l_{ij} \ln l_{ij}}{\ln m} \]

5. Get the ultimate attribute weight \( r_j \) by integrating the entropy weight and the subjective weight.

\[ v_j = O_j \cdot q_j \]  

where \( O_j \) is the subjective weight of the \( j \)th attribute.

\[ r_j = \frac{v_j}{\sum_{j=1}^{n} v_j} \]  

**Step 4:** Calculate the positive and negative ideal solution based on the weighted group decision matrix. The positive and negative ideal solution of the benefit-type attribute are defined as follows:

\[ G_j^+ = \max_{1 \leq i \leq m} G_{ij} = \left\{ \left( \max_{1 \leq i \leq m} g_{ij1}^1, \ldots, \max_{1 \leq i \leq m} g_{ijm}^1 \right), \max_{1 \leq i \leq m} \lambda_{G_{ij}} \right\} \]  

\[ G_j^- = \min_{1 \leq i \leq m} G_{ij} = \left\{ \left( \min_{1 \leq i \leq m} g_{ij1}^1, \ldots, \min_{1 \leq i \leq m} g_{ijm}^1 \right), \min_{1 \leq i \leq m} \lambda_{G_{ij}} \right\} \]

The positive and negative ideal solution of the cost-type attribute are defined as follows:

\[ G_j^+ = \min_{1 \leq i \leq m} G_{ij} = \left\{ \left( \min_{1 \leq i \leq m} g_{ij1}^1, \ldots, \min_{1 \leq i \leq m} g_{ijm}^1 \right), \max_{1 \leq i \leq m} \lambda_{G_{ij}} \right\} \]

\[ G_j^- = \max_{1 \leq i \leq m} G_{ij} = \left\{ \left( \max_{1 \leq i \leq m} g_{ij1}^1, \ldots, \max_{1 \leq i \leq m} g_{ijm}^1 \right), \min_{1 \leq i \leq m} \lambda_{G_{ij}} \right\} \]

**Step 5:** Utilize the Hamming distance to calculate the distance between the alternative and the positive/ negative ideal solution to get \( k_j^+ \) and \( k_j^- \), respectively. The grey relation degree between the alternative and the positive/ negative ideal
solution are obtained as $z_{ij}^+$ and $z_{ij}^-$, respectively.

$$k_i^+ (G_i, G^+) = \sum_{j=1}^d r_j h \left( G_{ij}, G_{ij}^+ \right)$$  \hspace{1cm} (21)

$$k_i^- (G_i, G^-) = \sum_{j=1}^d r_j h \left( G_{ij}, G_{ij}^- \right)$$  \hspace{1cm} (22)

$$z_{ij}^+ = \min_{i,j} \min_{j} \left| G_{ij} - G_{ij}^+ \right| + \rho \max_{i,j} \max_{j} \left| G_{ij} - G_{ij}^- \right|$$  \hspace{1cm} (23)

$$z_{ij}^- = \min_{i,j} \min_{j} \left| G_{ij} - G_{ij}^- \right| + \rho \max_{i,j} \max_{j} \left| G_{ij} - G_{ij}^+ \right|$$  \hspace{1cm} (24)

where $\rho$ is the distinguishing coefficient.

$$z_i^+ = \frac{1}{2n+2} \sum_{j=1}^d z_{ij}^+$$  \hspace{1cm} (26)

$$z_i^- = \frac{1}{2n+2} \sum_{j=1}^d z_{ij}^-$$  \hspace{1cm} (27)

By normalizing the indicators of Hamming distance and the grey relation degree, we can get the results as follows:

$$K_i^+ = \frac{k_i^+}{\max_{i} k_i^+}, \quad K_i^- = \frac{k_i^-}{\max_{i} k_i^-}$$  \hspace{1cm} (28)

$$Z_i^+ = \frac{z_i^+}{\max_{i} z_i^+}, \quad Z_i^- = \frac{z_i^-}{\max_{i} z_i^-}$$  \hspace{1cm} (29)

Step 6: Fuse the two types of indicators based on the normalization results.

$$N_i^+ = \gamma K_i^- + (1-\gamma) Z_i^+$$  \hspace{1cm} (30)

$$N_i^- = \gamma K_i^+ + (1-\gamma) Z_i^-$$

where $\gamma$ is the degree of preference for spatial distance, 1- $\gamma$ is the degree of preference for the grey relation degrees. $N_i^+$ represents the degree of closeness to the positive ideal solution, and $N_i^-$ represents the degree of closeness to the negative ideal solution.

Step 7: Calculate the relative closeness degrees of all alternatives with the ideal solution.

$$CZ_i = \frac{N_i^+}{N_i^+ + N_i^-}$$  \hspace{1cm} (31)

The larger the $CZ_i$ is, the better the alternative $Q_i$ is. The compromise solution is a feasible solution that is the closest to the ideal solution by making an agreement based on mutual concessions.

V. ILLUSTRATIVE EXAMPLE

A. DECISION FINDINGS

Due to the global environmental pollution for decades, resource shortage and other social problems are becoming more and more serious. China is attaching great importance to the promotion of a low-carbon economy. Electric vehicles have developed rapidly in recent years. Although the major cities in China have accelerated the construction of charging stations, the shortage problem of charging stations is still serious. Planning the location of charging stations has become a key problem faced by the government and manufacturers in developing electric vehicles.

A new energy automobile manufacturer decided to build a charging station to promote sales, and there were four alternative locations $Q_i (i = 1, 2, 3, 4)$. According to various factors affecting the construction and operation of charging stations and the actual situation of charging stations, the evaluation criteria of site selection of charging stations were determined by consulting the relative experts. Five criteria were selected finally and shown in TABLE 3.

Among the selected criteria, the operating cost $(X_3)$ is a cost-type criterion, and the rest criteria are all benefit-type criteria. The automobile manufacturer invited three decision makers $O_k (k = 1, 2, 3)$ to evaluate the four alternatives. In this case, 3-intuitionistic polygonal fuzzy set is used to express the evaluation information. The transformation standard between linguistic variables and 3-intuitionistic polygonal fuzzy sets are shown in TABLE 4 [15]:

Experts’ linguistic decision information were transformed into 3-intuitionistic polygonal fuzzy sets according to TABLE 4. The decision information of each decision-maker on alternatives are shown in TABLE 5 - VII.

Step 1: Construct the standard decision matrix. We can get $\alpha_j$ and $\beta_j$ by using Eq. (9-10). The $\alpha$ and $\beta$ of decision maker $O_1$ are: $\alpha_1 = 67, \alpha_2 = 85, \beta_3 = 1, \alpha_4 = 99, \alpha_5 = 85$. The $\alpha$ and $\beta$ of decision maker $O_2$ are: $\alpha_1 = 85, \alpha_2 = 85, \beta_3 = 14, \alpha_4 = 85, \alpha_5 = 99$. The $\alpha$ and $\beta$ of decision maker $O_3$ are: $\alpha_1 = 85, \alpha_2 = 99, \beta_3 = 31, \alpha_4 = 62, \alpha_5 = 99$. The

| Criteria | Explanations |
|----------|--------------|
| Quantity demanded $(X_1)$ | The ability to satisfy the expected charging demand |
| Resource availability $(X_2)$ | The availability of electric power and other infrastructures |
| Operating costs $(X_3)$ | The cost of the electric charge, labor, and maintenance, etc. |
| Extension ability $(X_4)$ | The ability and convenience of capacity extension in the future |
| Convenience $(X_5)$ | The accessibility to users and the transportation convenience |

TABLE 3. The definitions of criteria.

| Criteria | Explanations |
|----------|--------------|
| Quantity demanded $(X_1)$ | The ability to satisfy the expected charging demand |
| Resource availability $(X_2)$ | The availability of electric power and other infrastructures |
| Operating costs $(X_3)$ | The cost of the electric charge, labor, and maintenance, etc. |
| Extension ability $(X_4)$ | The ability and convenience of capacity extension in the future |
| Convenience $(X_5)$ | The accessibility to users and the transportation convenience |
### TABLE 4. Transformation standard between linguistic variables and 3-IPFs.

| Linguistic variables | 3-IPFs |
|----------------------|--------|
| Very poor            | \( \eta_r = \{ (0.1, 0.1, 0.1, 0.1, 0.1) \} \) |
| Poor                 | \( \eta_r = \{ (0.1, 0.1, 0.1, 0.1, 0.1) \} \) |
| Relatively poor      | \( \eta_r = \{ (0.1, 0.1, 0.1, 0.1, 0.1) \} \) |
| Medium               | \( \eta_r = \{ (0.1, 0.1, 0.1, 0.1, 0.1) \} \) |
| Relatively good      | \( \eta_r = \{ (0.1, 0.1, 0.1, 0.1, 0.1) \} \) |
| Good                 | \( \eta_r = \{ (0.1, 0.1, 0.1, 0.1, 0.1) \} \) |
| Very good            | \( \eta_r = \{ (0.1, 0.1, 0.1, 0.1, 0.1) \} \) |

\[
\begin{align*}
    h_{22} &= \left[ \begin{array}{cccc}
        14 & 22 & 31 & 49 \\
        85 & 85 & 85 & 85 \\
    \end{array} \right] ; 0.80, 0.17 \\
    h_{23} &= \left[ \begin{array}{cccc}
        1 & 1 & 1 & 1 \\
        45 & 45 & 39 & 39 \\
    \end{array} \right] ; 0.91, 0.07 \\
    h_{24} &= \left[ \begin{array}{cccc}
        57 & 59 & 64 & 71 \\
        99 & 99 & 99 & 99 \\
    \end{array} \right] ; 0.64, 0.25 \\
    h_{25} &= \left[ \begin{array}{cccc}
        31 & 36 & 41 & 49 \\
        85 & 85 & 85 & 85 \\
    \end{array} \right] ; 0.91, 0.06 \\
    h_{31} &= \left[ \begin{array}{cccc}
        1 & 1 & 1 & 1 \\
        67 & 67 & 67 & 67 \\
    \end{array} \right] ; 0.91, 0.07 \\
    h_{32} &= \left[ \begin{array}{cccc}
        57 & 59 & 64 & 71 \\
        85 & 85 & 85 & 85 \\
    \end{array} \right] ; 0.64, 0.25 \\
    h_{33} &= \left[ \begin{array}{cccc}
        1 & 1 & 1 & 1 \\
        67 & 67 & 67 & 67 \\
    \end{array} \right] ; 0.91, 0.06 \\
    h_{34} &= \left[ \begin{array}{cccc}
        14 & 22 & 31 & 49 \\
        99 & 99 & 99 & 99 \\
    \end{array} \right] ; 0.80, 0.17 \\
    h_{35} &= \left[ \begin{array}{cccc}
        1 & 1 & 1 & 1 \\
        85 & 85 & 85 & 85 \\
    \end{array} \right] ; 0.91, 0.07 \\
    h_{41} &= \left[ \begin{array}{cccc}
        14 & 22 & 31 & 49 \\
        67 & 67 & 67 & 67 \\
    \end{array} \right] ; 0.80, 0.17 \\
    h_{42} &= \left[ \begin{array}{cccc}
        31 & 36 & 41 & 49 \\
        85 & 85 & 85 & 85 \\
    \end{array} \right] ; 0.91, 0.06 \\
    h_{43} &= \left[ \begin{array}{cccc}
        1 & 1 & 1 & 1 \\
        45 & 45 & 39 & 39 \\
    \end{array} \right] ; 0.91, 0.07 \\
    h_{44} &= \left[ \begin{array}{cccc}
        72 & 79 & 81 & 86 \\
        99 & 99 & 99 & 99 \\
    \end{array} \right] ; 0.81, 0.16 \\
    h_{45} &= \left[ \begin{array}{cccc}
        1 & 1 & 1 & 1 \\
        85 & 85 & 85 & 85 \\
    \end{array} \right] ; 0.64, 0.25 \\
\end{align*}
\]

### Step 2: Obtain the group decision matrix of four alternatives. We got the aggregation evaluation results using n-IPFWA, n-IPFOWA, and n-IPFHGA operator, respectively. In this case, the weights of three decision makers are: \( w_1 = 0.25, w_2 = 0.40, w_3 = 0.35 \). We set that the orness measure \( \alpha \) equals to 1.3. The ordered weights obtained by Eq. (4) are: \( \omega_1 = 0.4948, \omega_2 = 0.3104, \omega_3 = 0.1948 \). ris set as 0.5. The aggregation results using n-IPFWA operator are as follows. We don’t list the aggregation results using n-IPFOWA and n-IPFHGA operators due to the length restriction of this paper.

\[
\begin{align*}
    h_{11} &= \{ (0.21, 0.32, 0.40, 0.55, 0.61, 0.70, 0.73, 0.77) ; 0.80, 0.17 \} \\
    h_{12} &= \{ (0.14, 0.24, 0.28, 0.36, 0.41, 0.48, 0.53, 0.56) ; 0.91, 0.07 \} \\
    h_{13} &= \{ (0.15, 0.16, 0.16, 0.17, 0.18, 0.20, 0.21, 0.22) ; 0.64, 0.25 \} \\
    h_{14} &= \{ (0.15, 0.26, 0.36, 0.57, 0.61, 0.69, 0.72, 0.76) ; 0.80, 0.17 \} \\
    h_{15} &= \{ (0.18, 0.26, 0.33, 0.46, 0.51, 0.58, 0.61, 0.63) ; 0.80, 0.17 \} \\
\end{align*}
\]

The decision information of alternatives given by decision makers were standardized as follows. Due to the restriction of the paper length, the standardized results of decision-maker \( O_2 \) and \( O_3 \) are not listed.

| \( X_1 \) | \( X_2 \) | \( X_3 \) | \( X_4 \) | \( X_5 \) |
|---------|---------|---------|---------|---------|
| \( O_1 \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) |
| \( O_2 \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) |
| \( O_3 \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) |

| \( X_1 \) | \( X_2 \) | \( X_3 \) | \( X_4 \) | \( X_5 \) |
|---------|---------|---------|---------|---------|
| \( O_1 \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) |
| \( O_2 \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) |
| \( O_3 \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) |
| \( O_4 \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) | \( \eta_r \) |
Step 3: Obtain the comprehensive attributes weights under different aggregation operators. In this case, the subjective attribute weight vector is: \( \omega = (0.20, 0.10, 0.20, 0.30, 0.20)^T \), the parameter \( \rho = \gamma = 0.4 \). The comprehensive attribute weight under \( n \)-IPFWA operator is: \( r = (0.1949, 0.0974, 0.1943, 0.3180, 0.1954)^T \). The comprehensive attribute weight under \( n \)-IPFOWA operator is: \( r = (0.1957, 0.0982, 0.1966, 0.3135, 0.1959)^T \). The comprehensive attribute weight under \( n \)-IPFHFA operator is: \( r = (0.1964, 0.0974, 0.1951, 0.3149, 0.1962)^T \).

The comprehensive attribute weights were fused into the group decision matrix to obtain the weighted group decision matrix. The fusion results under \( n \)-IPFHFA operator are listed as follows.

\[
\begin{align*}
G_{11} & = \{0.04, 0.06, 0.08, 0.11, 0.12, 0.14, 0.14, 0.15\}; \\
G_{12} & = \{0.01, 0.02, 0.03, 0.03, 0.04, 0.05, 0.05, 0.05\}; \\
G_{13} & = \{0.03, 0.03, 0.03, 0.03, 0.04, 0.04, 0.04\}; \\
G_{14} & = \{0.05, 0.08, 0.11, 0.18, 0.19, 0.22, 0.23, 0.24\}; \\
G_{15} & = \{0.03, 0.05, 0.06, 0.09, 0.10, 0.11, 0.12, 0.12\}; \\
G_{21} & = \{0.07, 0.08, 0.10, 0.12, 0.14, 0.15, 0.16, 0.17\}; \\
G_{22} & = \{0.05, 0.05, 0.06, 0.07, 0.08, 0.08, 0.08, 0.09\}; \\
G_{23} & = \{0.04, 0.04, 0.05, 0.05, 0.06, 0.08, 0.10, 0.20\}; \\
G_{24} & = \{0.07, 0.11, 0.12, 0.15, 0.18, 0.20, 0.22, 0.23\}; \\
G_{25} & = \{0.09, 0.10, 0.11, 0.13, 0.14, 0.15, 0.15, 0.16\}; \\
G_{31} & = \{0.05, 0.07, 0.08, 0.10, 0.12, 0.13, 0.14, 0.15\}; \\
G_{32} & = \{0.05, 0.06, 0.07, 0.08, 0.08, 0.09, 0.09\}; \\
G_{33} & = \{0.04, 0.04, 0.04, 0.04, 0.04, 0.06, 0.07, 0.10\}; \\
G_{34} & = \{0.10, 0.12, 0.15, 0.19, 0.21, 0.23, 0.24, 0.25\}; \\
G_{35} & = \{0.06, 0.08, 0.09, 0.10, 0.12, 0.13, 0.13, 0.14\}; \\
G_{41} & = \{0.09, 0.10, 0.11, 0.14, 0.15, 0.16, 0.17, 0.18\}; \\
G_{42} & = \{0.01, 0.02, 0.03, 0.04, 0.04, 0.05, 0.06, 0.06\}; \\
G_{43} & = \{0.04, 0.04, 0.04, 0.05, 0.05, 0.06, 0.07, 0.14\}; \\
G_{44} & = \{0.15, 0.17, 0.19, 0.23, 0.25, 0.27, 0.28, 0.29\}; \\
G_{45} & = \{0.06, 0.08, 0.09, 0.11, 0.12, 0.13, 0.14, 0.14\}; \\
G_{46} & = \{0.64, 0.25\}.
\end{align*}
\]
TABLE 8. The positive and negative ideal solutions of the group.

\[
\begin{align*}
G^+ & \approx \{0.09,0.10,0.11,0.14,0.15,0.16,0.17,0.18\};0.80,0.17) \\
G^- & \approx \{0.05,0.06,0.07,0.08,0.09,0.09\};0.91,0.07) \\
G^+ & \approx \{0.03,0.03,0.03,0.04,0.04,0.04,0.04,0.04\};0.64,0.25) \\
G^- & \approx \{0.15,0.17,0.19,0.23,0.25,0.27,0.28,0.29\};0.80,0.17) \\
G^+ & \approx \{0.09,0.10,0.11,0.13,0.14,0.15,0.15,0.16\};0.80,0.17) \\
G^- & \approx \{0.04,0.06,0.08,0.10,0.12,0.13,0.14,0.15\};0.64,0.25) \\
G^+ & \approx \{0.01,0.02,0.03,0.03,0.04,0.05,0.05,0.05\};0.64,0.25) \\
G^- & \approx \{0.04,0.04,0.05,0.05,0.06,0.08,0.10,0.20\};0.80,0.17) \\
G^+ & \approx \{0.05,0.08,0.11,0.15,0.18,0.20,0.22,0.23\};0.64,0.25) \\
G^- & \approx \{0.03,0.05,0.06,0.09,0.10,0.11,0.12,0.12\};0.64,0.25)
\end{align*}
\]

TABLE 9. The positive and negative distance indicator and the grey correlation degree indicator.

| \(K^r\) | \(Q_1\) | \(Q_2\) | \(Q_3\) | \(Q_4\) |
|----|----|----|----|----|
| \(n\)-IPFWA | 0.55 | 0.25 | 0.31 | 0.09 |
| \(n\)-IPFOWA | 0.56 | 0.22 | 0.33 | 0.09 |
| \(n\)-IPFHA | 0.35 | 0.22 | 0.34 | 0.09 |
| \(n\)-IPFWA | 0.12 | 0.25 | 0.18 | 0.45 |
| \(n\)-IPFOWA | 0.11 | 0.29 | 0.15 | 0.45 |
| \(n\)-IPFHA | 0.10 | 0.29 | 0.12 | 0.48 |
| \(n\)-IPFWA | 0.21 | 0.26 | 0.25 | 0.28 |
| \(n\)-IPFOWA | 0.20 | 0.26 | 0.25 | 0.29 |
| \(n\)-IPFHA | 0.21 | 0.26 | 0.24 | 0.29 |
| \(n\)-IPFWA | 0.30 | 0.26 | 0.21 | 0.22 |
| \(n\)-IPFOWA | 0.32 | 0.25 | 0.21 | 0.22 |
| \(n\)-IPFHA | 0.31 | 0.25 | 0.23 | 0.22 |

TABLE 10. The result of fusing the positive and negative distance indicator and the grey correlation degree indicator.

| \(N^r\) | \(Q_1\) | \(Q_2\) | \(Q_3\) | \(Q_4\) |
|----|----|----|----|----|
| \(n\)-IPFWA | 0.17 | 0.26 | 0.22 | 0.35 |
| \(n\)-IPFOWA | 0.17 | 0.27 | 0.21 | 0.35 |
| \(n\)-IPFHA | 0.17 | 0.27 | 0.16 | 0.37 |
| \(n\)-IPFWA | 0.32 | 0.26 | 0.25 | 0.17 |
| \(n\)-IPFOWA | 0.33 | 0.24 | 0.26 | 0.17 |
| \(n\)-IPFHA | 0.33 | 0.24 | 0.27 | 0.17 |

**Step 7:** Rank the four alternatives according to the relative closeness degrees.

The relative closeness degrees of four alternatives under \(n\)-IPFWA operator are: \(CZ_1 = 0.3466, CZ_2 = 0.5003, CZ_3 = 0.4667, CZ_4 = 0.6748\). The ranking result is \(Q_4 > Q_2 > Q_3 > Q_1\).

The relative closeness degrees of four alternatives under \(n\)-IPFOWA operator are: \(CZ_1 = 0.3326, CZ_2 = 0.5333, CZ_3 = 0.4465, CZ_4 = 0.6757\). The ranking result is \(Q_4 > Q_2 > Q_3 > Q_1\).

The relative closeness degrees of four alternatives under \(n\)-IPFHA operator are: \(CZ_1 = 0.3383, CZ_2 = 0.5334, CZ_3 = 0.4188, CZ_4 = 0.6877\). The ranking result is \(Q_4 > Q_2 > Q_3 > Q_1\).

The ranking results under different aggregation operators are the same, and \(Q_4\) is always the best alternative. Fig. 1 shows the comparison of the relative closeness degree under different aggregation operators in the proposed TOPSIS.

We can see that the relative closeness degrees of \(Q_3\) and \(Q_2\) have larger fluctuations. That is because the relative closeness degree reflects the comprehensive distances between alternatives and the negative/positive ideal solution, and \(Q_1\) and \(Q_4\) are close to the negative and positive ideal solution, respectively. Therefore, when the aggregation result changes, the relative closeness degree varies less for \(Q_1\) and \(Q_4\).

**B. COMPARATIVE ANALYSIS WITH THE RANK METHODS**

In order to illustrate the effectiveness of the improved TOPSIS, the traditional TOPSIS is also taken to solve the case to do a comparative analysis.

The closeness degrees of the four alternatives under \(n\)-IPFWA operator are: \(V_1 = 0.2478, V_2 = 0.5045, V_3 = 0.3661, V_4 = 0.8304\). The ranking result is \(Q_4 > Q_2 > Q_3 > Q_1\).

The closeness degrees of the four alternatives under \(n\)-IPFOWA operator are: \(V_1 = 0.2406, V_2 = 0.5625, V_3 = 0.3155, V_4 = 0.8270\). The ranking result is \(Q_4 > Q_2 > Q_3 > Q_1\).

The closeness degrees of the four alternatives under \(n\)-IPFHA operator are: \(V_1 = 0.2278, V_2 = 0.5642, V_3 = 0.2698, V_4 = 0.8415\). The ranking result is \(Q_4 > Q_2 > Q_3 > Q_1\).

The ranking results under different aggregation operators are the same, and \(Q_4\) is always the best alternative. Fig. 2 shows the comparison of the closeness degree under different aggregation operators.

![Figure 1](image-url)
From Fig. 1 and Fig. 2, we can see that the fluctuation range of the traditional TOPSIS method is bigger than that of the improved TOPSIS method under different aggregation operators. The traditional TOPSIS method only considers the spatial closeness to the ideal solution. While the improved TOPSIS method not only considers the spatial closeness to the ideal solution but also considers the grey relation. The grey relation analysis takes the similarity of curve shape as a measurement scale which reduces the subjectivity of the decision-making process. In our ranking method, the experts can adjust the parameter \( \gamma \) reflecting the preference of the distance and the shape factor to make the ranking results more reasonable and accurate.

VI. CONCLUSION

As an extended form of TIFSs and TrIFSs, \( n \)-IPFS can overcome the shortcomings of the incomplete or inaccurate description of decision information. This paper proposes a new MAGDM approach with \( n \)-IPFSs. Some generalized aggregation operators for TIFSs and TrIFSs were studied and applied widely. While the aggregation operators have not been extended to \( n \)-IPFSs. Therefore, this paper proposed three aggregation operators for \( n \)-IPFSs, i.e., \( n \)-IPFWA operator, \( n \)-IPFOWA operator, and \( n \)-IPFHA operator. These operators can effectively make up for the lack of decision information fusion method in \( n \)-intuitionistic polygonal fuzzy environment, and make the decision process more effective and accurate. Based on the information fusion of \( n \)-IPFSs, an improved TOPSIS approach is given out to rank the alternatives in the MAGDM problems. The introduction of the grey relation makes the relative closeness degree more comprehensive and avoids the inaccuracy caused by considering the spatial distance only. Moreover, to show the feasibility and effectiveness of the proposed approach, we present a location selection example and solid comparative analysis. Additionally, there is still space for further development. Some power geometric aggregation operators for \( n \)-intuitionistic polygonal fuzzy sets and their application to multi-attribute group decision making can be further researched.

CONFLICT OF INTEREST STATEMENT

There are no other relationships or activities that could appear to have influenced the submitted work.

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