Cosmological Signatures in Temporal and Spectral Characteristics of Gamma-Ray Bursts

Vahé Petrosian and Nicole Lloyd
Center for Space Science and Astrophysics, Varian 302c, Stanford University, Stanford, CA 94305-4060. 1

Andrew Lee
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309. 2

Abstract.
There have been several suggestions of the existence of cosmological redshift signatures in the temporal and spectral characteristics of gamma-ray bursts. However, recent discoveries of afterglows and redshift measurements indicate the presence of broad “luminosity functions” which may overwhelm such weaker cosmological signatures. The primary goal of this paper is to determine if the intrinsic and cosmological dispersions can be separated. We have expanded the search for cosmological signatures to several other temporal and spectral features, have determined correlations which could arise from the cosmological redshift of the sources and have carried out tests to determine if the observed correlations can be due to cosmology. We find that the intrinsic dispersions are the dominant factors and that detection of cosmological signatures must await accumulation of a much larger number of identification with galaxies and measurements of redshifts.

1. INTRODUCTION
Gamma-ray Bursts (GRBs) show a wide dispersion in their temporal and spectral characteristics and show a variety of correlations between these. Now that the association of some GRBs with external galaxies at high redshifts is firmly established, the question arises whether these dispersions and relations are the consequence of their redshift distribution or are intrinsic to the sources related to the physics of the emission processes. The first indication from a handful of sources with known redshifts is inconclusive in this regard. Figure 1 shows the Hubble diagram for GRBs and their afterglows with known redshifts: On the right we give the variation of the peak (or representative) fluxes, $f_p$ (or $f$), and on the left the fluences $F$ at different photon energies. There is no obvious

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Hubble relation for any of these measures of burst strength in most cosmological models. The small number of GRBs with known redshift $z$ makes it difficult to draw any significant conclusion except that the “luminosity functions” for all these measures are very broad and mask the weaker cosmological signature. More redshifts are needed to unravel these two effects from each other.

There have been several attempts to look for cosmological signatures in other characteristics of GRBs. Notable among these are the so-called time dilation effect as measured by the peak flux-duration correlation (see, e.g., Norris et al. 1994, 1995; J. Bonnell in these proceedings) and the spectral redshift effect as measured by the correlation between $f_p$ and $E_p$, the peak or break photon energy of the $\nu F_\nu$ spectrum (Mallozzi et al. 1996, 1998). The aim of this paper is to describe results from further explorations of these signatures and to examine whether or not we can shed more light on the question of intrinsic versus cosmological origins of these relations.

In §2 we describe the statistical methods we use for proper accounting of the selection biases and data truncations and how we determine these biases for any physical quantity measured by BATSE. In §3 we describe temporal relations based on Andrew Lee’s thesis and in §4 we discuss results from Nicole Lloyd’s thesis on spectral relations. In §5 we return to Figure 1 and give a brief summary and draw some conclusions.
2. THE STATISTICAL METHODS

2.1. Correlations and Distributions

The statistical problem at hand is to first determine the degree of the correlation between two measured quantities, say $y$ and $z$, and then determine their univariate distributions $\rho(z)$ and $\phi(y)$ from an observed bivariate distribution $\psi(y, z)$ which suffers from selection biases and is subject to multiple truncations. The left panel of Figure 2 shows some generic truncations. The distribution may be truncated parallel to the axis (dotted lines) which can be referred to as untruncated because there is no bias within the observed ranges. More interesting cases are when the truncations are not parallel to the axis. The data may suffer a one-sided truncation from below (solid curve) or above (dashed curve), truncated both from above and below. The most general truncation is when each data point, say $[y_i, z_i]$, has its individual upper and lower limits, $y_i^- < y_i < y_i^+$ and $z_i^- < z_i < z_i^+$, as shown by the large cross for one point. In several papers (Petrosian 1992, Efron & Petrosian 1992 and 1999) we have developed new methods for dealing with all of these situations. These are essentially non-parametric methods which avoid the usual arbitrary binning and the consequent loss of data. Here we give only a brief description of the methods. For further discussion and for examples of applications of these methods to quasar surveys and GRBs we refer the reader to Maloney & Petrosian (1999) and Lloyd & Petrosian (1999; hereafter LP99), respectively.

(a) Correlations: Suppose $y$ and $z$ are correlated such that any characteristic value of $y$, say its average value, varies with $z$ as $g(z)$. This would mean that we can write $\psi(y, z) = \phi(y/g(z))\rho(z)/g(z)$, where $\int_0^\infty \phi(x)dx = 1$. The determination of the correlation function $g(z)$ is based on the rank order $R_i$ of each source among its comparable or eligible set $J_i = \{j : x_j > x_i, \ x_j \in (x_i^-, x_i^+), x_i \in (x_i^-, x_i^+)\}$, where $x$ stands for either variable. In the absence of any correlation ($g(z) = \text{constant}$), these ranks will be distributed uniformly so that their average or expected values would be $E_i = (N_i + 1)/2$ and their variances $V_i = (N_i^2 - 1)/12$, where $N_i$ is the number of points in $J_i$. One then defines the test statistic $\tau = \sum_i (R_i - E_i)/\sqrt{\sum_i V_i}$. This statistic is equivalent to Kendall’s $\tau$ test and for independent variables its distribution should be a Gaussian with mean of zero and dispersion of unity. Thus, $y$ and $z$ will be considered uncorrelated or stochastically independent if $|\tau| < 1$, in which case one may assume that the correlation function is constant ($g(z) = 1$, say) and proceed with the determination of the univariate distributions $\phi(y)$ and $\rho(z)$ using the methods mentioned below. However, if $|\tau| > 1$ then - at the $1\sigma$ level - $y$ and $z$ cannot be considered independent and one may assume that the most likely explanation is the presence of some correlation ($g(z) \neq \text{constant}$). One can then determine the function $g(z)$ parametrically as follows.

Given a parametric form for the correlation function $g_k(z)$ one can perform the transformation $y_{o,i}(k) = y_i/g_k(z_i)$ and proceed with the determination of the test statistic $\tau(k)$ for the new variables $y_o$ and $z$ as a function of $k$. The most likely value of $k$ is that with $\tau(k) = 0$ and the range of $k$ for $1\sigma$ confidence level is $\{k : |\tau(k)| < 1\}$. The right panel of Figure 2 shows an example of our results using this procedure.
Figure 2. **Left Panel:** Demonstration of various types of data truncations: Parallel to axis (dotted lines), from below (the solid curve), from above (the dashed curve), and a general truncation when each data point has its specific observable range (shown by the cross for only one of the points). **Right Panel:** A determination of the correlation function parameter for the parametric form $g_k(z) = (z)^k$. The correlation statistic $\tau$ is shown as a function of $k$. The solid line at $\tau = 0$ gives the optimal value $k = 1.52$ and the dotted lines at $|\tau| = 1$ demonstrate the $1\sigma$ range $[1.40, 1.64]$.

**2.2. Determination of Truncations**

Before we can apply these methods we must accurately account for the observational selection biases and determine the exact form of the truncations. This task is generally easy for most astronomical samples of sources which are frequently only flux limited. However, this is not so simple for transient sources, in particular for GRBs as observed by BATSE, where many factors such as the trigger criterion, duration, light curve, spectral shape can introduce biases. In general, instead of a simple “luminosity function” one is dealing with a multivariate distribution of radiant energy, peak luminosity, and spectral and temporal parameters. Because of the interrelations between these variables, the selection biases truncate the BATSE data in a complex way. Fortunately BATSE has the well defined triggered criterion of peak count (on time scales $\Delta t = 1024, 256\ and$
64ms) $C_{\text{max}}$ being greater than some minimum value $C_{\text{min}}$. This can be used to determine the threshold(s) for other physical quantities measured by BATSE.

One way to carry out this task is by using extensive simulation for each specific measure (see e.g. Bloom et al. 1996; Pendleton et al. 1998). This could be very time consuming and computer intensive. We have developed a much simpler procedure to account for the selection biases and to determine the thresholds. Given the BATSE trigger criterion $C_{\text{max}} > C_{\text{min}}$, in the spirit of the $V/V_{\text{max}}$ test, we ask: What is the threshold for a given observable such that if the burst was as weak as this threshold (e.g., being farther away) its $C_{\text{max}}$ would fall below $C_{\text{min}}$?

It is easy to see that the threshold for any measure of burst strength, say the energy or photon fluences $F$ and $F_{\gamma}$, peak flux $f_p$ or the average flux $\bar{f}$, is obtained from the following relations:

$$\frac{C_{\text{max}}}{C_{\text{min}}} = \frac{f}{f_{\text{lim}}} = \frac{\bar{f}}{\bar{f}_{\text{lim}}} = \frac{F_{\gamma}}{F_{\gamma,\text{lim}}} = \frac{F}{F_{\text{lim}}}. \quad (1)$$

As shown by Lee & Petrosian (1996, 1997) the results from this simple but robust procedure when applied to the fluence $F$ agrees with that of the simulations by Bloom et al. (1996). This procedure need not be limited only to measures of burst strength, but can be used for any other measured quantity. Examples of this are temporal characteristics such as duration or other parameters describing the light curve, or spectral characteristics such as $E_p$, and low and high energy spectral indices $\alpha$ and $\beta$ in any broken power law spectral form. For these parameters the thresholds may be more complex; instead of a single lower limit there may be a lower and an upper threshold in which cases we use our procedure for two-sided truncated data. Given a burst of an observed bolometric (in the gamma-ray range) fluence and $E_p$, it is clear that such a burst would not trigger the BATSE detectors, i.e. $C_{\text{max}} < C_{\text{min}}$, if the spectrum was either two soft or two hard. In other words, $E_p$ must be confined in a range $E_{p,\text{min}} < E_p < E_{p,\text{max}}$ in order for that particular burst to be observed. For a preliminary application of this procedure see LP99. In essence the triggering criteria based on counts can be translated to thresholds on any other measured quantity. However, care is necessary when dealing with spectral parameters. The detector response nonlinearities, e.g. nondiagonal elements in the detector response matrix, DRM, can be important and must be taken into consideration. We discuss this further below.

## 3. TEMPORAL ANALYSES

Existence of some correlation between peak flux $f_p$ and some measures of burst duration or width is well established (see J. Bonnell and references cited there, in these proceedings). However whether this can be interpreted as a signature of the cosmological redshift is controversial. It appears that the time scale on the rising part of the bursts shows a much smaller or zero correlation than the decay time scale (Stern et al. 1997; Mirofanov 1998). Furthermore, as evident from Figure 1, the peak flux is not a good measure of distance or redshift. To shed some light on this controversy we have carried out several similar tests looking for correlations between other measures of burst strength and temporal...
characteristics. These are all outcome of Andrew Lee’s Ph.D. thesis. We now give a brief description of the procedures and some of the most relevant results from this thesis.

3.1. Data and Pulse Fitting Algorithm

We have used the BATSE Time-to-Spill (TTS) burst data, which records the times required to accumulate a fixed number of photons in each of four energy channels. The TTS data offer variable time resolution, usually finer than any other BATSE data types except for the time-tagged event (TTE) data, and usually can store complete time profiles of bright, long bursts to be stored in the limited memory on board the CGRO.

We have used the phenomenological pulse model of Norris et al. (1996) (see also Stern et al. 1997) to decompose gamma-ray burst time profiles into distinct pulses. In this model, each pulse is described by five parameters with the functional form

\[ C(t) = A \exp \left( -\frac{t - t_{\text{max}}}{\sigma t} \right), \]

where \( t_{\text{max}} \) is the time at which the pulse attains its maximum, \( \sigma t \) and \( \sigma d \) are the rise and decay times, respectively, \( A \) is the pulse amplitude, and \( \nu \) (the “peakiness” parameter) gives the sharpness or smoothness of the pulse at its peak. We have developed an interactive pulse-fitting program that can automatically find initial background level and pulse parameters using a Haar wavelet denoised time profile, and allows the user to add or delete pulses graphically. The program then finds the pulse parameters by using a maximum-likelihood fit for the gamma-distribution that the TTS spill times follow (for details see Lee et al. 1998).

3.2. Correlations from Burst to Burst

We first describe correlation between burst strengths and timescales for different bursts. This has direct bearing on the time dilation hypothesis.

We use the amplitude \( A \) (see eq. [2]) of the highest amplitude pulse in each burst as a measure of the peak flux, and the total counts \( C = \int C(t)dt = \frac{A}{\nu}(\sigma t + \sigma d)\Gamma(1/\nu) \) of the pulses as a measure of the photon fluence of each pulse and the sum of these for the fluence of each burst. The timescales that we use are the FWHM duration \((T_{5.5} = (\sigma t + \sigma d)(\ln 2)^{1/\nu})\) of the highest amplitude pulse in each burst, and the interval between the peak times of the two highest amplitude pulses in multiple-pulse bursts \((\Delta T_{1,2})\). The following are some of the results relevant to this paper.

(a) We find an inverse correlation between the amplitudes and the widths of the highest amplitude pulse of the bursts; Figure 3, left panel. This is similar to the “time dilation” trend observed by Norris et al. (1994, 1998) but the slope of the trend doesn’t appear to agree with the expected effects of cosmological time dilation alone; the variation in pulse width is greater than expected.

(b) We also find an inverse correlation between the highest pulse amplitude and the time interval between the peaks of the two highest pulses in each burst; Figure 3, right panel. This weaker variation may be consistent with the
expected results of cosmological time dilation. It is likely that this correlation is less affected by intrinsic properties of GRBs or by selection effects than the correlation in item (a) above. This agrees with Norris et al. (1996) and Deng & Schaefer (1998).

(c) On the other hand, we find a positive correlation between total count fluences of bursts and the widths $T_{5}$ of the highest amplitude pulse in each burst; Figure 4, left panel. This is in agreement with the fluence-total duration correlation described by Lee & Petrosian (1996, 1997). Clearly in both cases the cosmological effect (anticorrelation), if any, has been overwhelmed by the intrinsic correlations. (But we find no correlation between total burst count fluence and the interval between the two highest pulses in each burst; Figure 4, right panel.)

The simulations discussed in section 3.4 show that the observed correlations between the fluence and amplitude of the highest amplitude pulse and the two timescales in each burst do not appear to be strongly affected by the pulse-fitting procedure.

### 3.3. Correlations Among Pulses Within Bursts

For bursts with multiple pulses we can carry out the above tests among pulses within the bursts. Any correlation here must be intrinsic to the physical process of emission and not affected by the cosmological redshifts. We find trends qualitatively similar to those in items (a) to (c) above. The following are some of the results relevant to this question.

(a) We find that within individual bursts, higher amplitude pulses have a strong tendency to be narrower; the pulse amplitude-duration scatter diagrams tend to have more negative than positive slopes. This effect is even stronger...
for bursts with the stronger correlations probabilities between pulse amplitude and duration. The scatter diagrams with the individual linear least-squares fit to the logarithms of these quantities are shown in the left panel of Figure 5. Only one out of about thirty bursts shows a positive slope. The rest show the anticorrelation similar to the time-dilation seen among the bursts. Obviously the present anticorrelations cannot be due to cosmological effects and must result from intrinsic properties of the GRBs themselves, or from selection effects in the pulse-fitting procedure (see below).

(b) We also find that majority of bursts show a positive correlation between the count fluence and duration of their pulses. The right panel of Figure 5 shows the trends for bursts with significant correlation probabilities. This also is similar to the behavior among bursts which is opposite of what is expected from the refshift effect.

3.4. Biases and Simulation Results

There are a number of ways in which the pulse-fitting procedure may introduce biases into correlations between pulse characteristics. One is that the errors in the different fitted pulse parameters may be correlated. Another is that the pulse-fitting procedure may miss some pulses by not identifying them above the background noise. Still another cause of bias is that overlapping pulses may be identified as a single broader pulse.

We have tested for these selection effects by generating simulated burst time profiles using the pulse model in equation (2) with randomly generated parameters with distributions that are similar to those observed. We add appropriate noise to this data and then fit the simulated bursts exactly the same way as the
Figure 5. Pulse amplitudes versus pulse widths (left panel) and pulse count fluences versus pulse widths (right panel) within bursts for bursts with strongest correlations. The lines show the fitted power laws to pulses in individual bursts with high positive or negative correlation coefficients. Data from energy channel 2, 60-110 keV.

actual data. The following are results from comparing the simulated and fitted pulse characteristics.

For simulated bursts consisting of a single pulse in both the original simulation and in the fit, the identification of pulses between the simulation and the fit is unambiguous and unaffected by the effects of missing pulses. We find that when the fitted amplitude is larger than the original amplitude, the fitted width tends to be smaller than the original width, and vice versa (i.e. fluence roughly invariant). This could introduce a small bias in the sense of the observed anticorrelation between amplitude and duration. This tendency also indicates that there will be no bias introduced in the correlations between count fluences and pulse widths.

Figure 6 shows the amplitude-pulse width relations for the simulated bursts before (left panel) and after (right panel) our fitting procedure. As expected the left panel shows an almost random distributions of the fitted slopes while the right panel shows a greater tendency for negative than positive slopes (16 vs. 10). This means that the fitting procedure can create a weak inverse correlation between pulse amplitude and pulse width within bursts. We do not believe that this is sufficiently strong to explain the behavior observed in Figure 5 where only one out of thirty well correlated bursts shows positive slopes.

We also find that the fitting procedure slightly weakens the positive correlation between the pulse fluence and pulse width so that the positive correlation between these quantities found for the actual bursts (right panel Fig. 5) is a slight underestimation of the true correlation.
4. SPECTRAL ANALYSES

A similar cosmological signature has been claimed to be present in the correlation between the peak flux $f_p$ and the spectral hardness of GRBs, as measured by $E_p$ (Mallozzi et al. 1996, 1998; Mitrofanov et al. 1999). The question arises again whether this is caused by the redshift of the bursts or is intrinsic. This is because, as shown in Figure 1, it is not clear if the peak flux provides a good measure of source redshift. In order to clarify this, we have looked for correlations between $E_p$ and other measures of burst strength, in particular its total (bolometric) energy fluence, which (according to models with well defined total released energy, such as mergers or hypernovae) may be a better redshift indicator. We use the procedures described in §2 to determine the thresholds on these quantities and evaluate the degrees of their correlations. We then discuss whether the observed relations could be due solely to redshift effects.

4.1. Bias and Correlations

We have previously (LP99) used the burst spectral parameters (for a Band spectrum) obtained from four channel data. Here we use spectral parameters kindly provided by Dr. Robert Mallozzi, who used the software WINGSPAN to fit a Band spectrum to 16 channel CONT (continuous) data for a large sample of bursts. The above mentioned correlations were obtained from the latter data. The LP99 sample contains a complete sample of bursts with known $C_{max}$ and $C_{min}$ values and therefore has a well defined truncation. Unfortunately the Mallozzi sample, although more reliable in the values of the spectral parameters, does not have a well defined selection criterion. Because the brightest bursts give the best fits, this sample includes bright bursts with no known values of $C_{min}$; furthermore, most bursts in this sample with a known $C_{min}$ value have $C_{max}/C_{min} \gg 1$. We are currently collaborating with Dr. Mallozzi to determine
the biases in this sample in order to evaluate the significance of the correlations between \( f_p \) and \( E_p \) that he and his collaborators have reported. Without exact knowledge of the selection bias, we cannot account for data truncations properly or determine correlations accurately. To circumvent this situation, we chose a subsample with a better defined selection criterion as follows. We truncate the available data, parallel to the axes in the \( f_p – f_{p,lim} \) or \( F – F_{lim} \) plane, so that the above mentioned uncertainty is minimized. For the observed fluence in the range 50-300 keV we select a subsample with \( F_{obs} \geq 10^{-6} \text{ ergs/cm}^2 \), and for the total fluence (summed over all four LAD channels; 20keV-1.5MeV) we select sources with \( F_{sum} \geq 5 \times 10^{-6} \text{ ergs/cm}^2 \). (Note that \( F_{sum} \) is approximately equal to the total fluence of the burst, \( F_{tot} = \int_{E_{min}}^{\infty} F(E)dE \).) For peak flux, the cut was made at \( f_p \geq 3.0 \text{ ph/(cm}^2 \text{ s)} \). Hence, the results presented below are valid for a narrower range of the parameters near the bright end of the burst intensity distribution.

As discussed in §2 above, we must account for any truncation in the variables we are correlating. We use the method described in LP99 to get an estimate of the truncations on \( E_p \). In LP99 we pointed out that because BATSE triggers over a finite energy range, if the \( E_p \) is too far above or too far below the trigger range, the burst will not be detected. Hence, using the burst trigger condition, we can place both an upper and a lower limit on \( E_p \). However, it should be pointed out that this does not include effects of the DRM, which may play an important role in the significance of the truncation. This is less important for the present sample which has a narrow distribution of \( E_p \)'s and consequently requires a small correction due to instrumental biases (a delta function distribution requires zero correction). The truncations on the fluence and peak flux are straightforward, since, as described above, we have made a cut at some well determined value parallel in the axes in the \( F – F_{lim} \) and \( f – f_{lim} \) planes.

We have carried out the correlation test in the samples defined above. The results are shown in Table 1. The last quantity, \( f_{p, trig} \) is the peak photon flux between 50-300keV on the timescale in which the detector triggered (either 64ms, 256ms, or 1024ms). The fluences are correlated with the burst average \( E_p \), while the peak photon flux is correlated with the value of \( E_p \) at the time of the peak. The results are given in terms of the significance of the correlation using the Kendall’s \( \tau \) test mentioned in §2. The raw result shows the correlation without accounting for truncation in the variables, while the corrected result uses the techniques described in §2 to account for these truncations.

**Table 1**

| Correlation  | Raw result | Corrected result | Number of bursts |
|--------------|------------|------------------|-----------------|
| \( F_{obs} \) | (+) 5.6 \( \sigma \) | (+) 5.5\( \sigma \) | 147             |
| \( F_{sum} \) | (+) 6.5\( \sigma \) | (+) 5.8\( \sigma \) | 160             |
| \( f_{p, trig} \) | (+) 2.5\( \sigma \) | (+) 2.6\( \sigma \) | 101             |

As mentioned above, for our samples (obtained with more strict and rigorous selection criteria), the truncation effects are expected to be small. This is reflected in the relative values of the raw and corrected \( \tau \) values. We find that there is a strong correlation between the observed fluence and \( E_p \) as well as the
total fluence and $E_p$. Similar, but somewhat weaker, correlations are evident in other samples as well. However, we find only a moderate correlation between the peak photon flux and $E_p$ for the bright end of the intensity distribution of the bursts. This is not the case for the whole sample in Mallozzi’s list for which we find a correlation similar to the results reported previously (Mallozzi et al. 1996, 1998). However, the most significant correlation reported by Mallozzi and collaborators comes from bursts below our cutoff of 3 photons/(cm$^{-2}$ s) where the selection criterion is uncertain. These aspects of the problem will be addressed in future publications.

4.2. Cosmological Signature?

The above correlations between fluences and $E_p$ follow the same trend as that expected from cosmological effects. We would like to test if these correlations can be attributed fully to such effects. We will focus particularly on the $F_{\text{sum}} \approx F_{\text{tot}}$ results, because the total fluence can be related to the total radiated energy and the redshift of the burst, without any need for the so-called K-correction; $F_{\text{tot}} = E_{\text{rad}}/(\Omega_b[d_E(\Omega_i, z)]^2)$, where $E_{\text{rad}}$ and $\Omega_b$ are the total radiant energy (in the gamma-ray range) and the average beaming steradians, $\Omega_i$ denote the cosmological model parameters, and $d_E(\Omega_i, z) = d_L(\Omega_i, z)/\sqrt{1+z}$ with $d_L$ as the usual bolometric luminosity distance. For this task we need to specify a cosmological model and the distribution function of redshift and the intrinsic parameters, $\Psi(E_p, \Omega_b, E_{\text{rad}}, z)$. The beaming angle enters always in conjunction with $E_{\text{rad}}$. To simplify the matters, in what follows we assume a delta function distribution of $\Omega_b$ so that it can be eliminated from the distribution function. This amounts to replacing $E_{\text{rad}}$ with $E_{\text{rad}}/\Omega_b$. We also assume no evolution for the intrinsic parameters and no intrinsic correlation between $E_{\text{rad}}$ and $E_p$. These assumptions mean that the multivariate distribution function becomes separable as $\Psi(E_p, \Omega_b, E_{\text{rad}}, z) = \phi(E_{\text{rad}})\zeta(E_p)\rho(z)$. In this case the joint distribution of observed $E_p$ and $F_{\text{tot}}$ is given by

$$
\frac{d^2 N(E_p, F_{\text{tot}})}{dE_p dF_{\text{tot}}} = \int_0^\infty dz (dV/dz)\rho(z)[d_E(\Omega_i, z)]^2 \phi(F_{\text{tot}})[d_E(\Omega_i, z)]^2 (1+z)\zeta(E_p(1+z)),
$$

(3)

where $dV/dz$ is the differential of the comoving volume up to $z$. From this we can compute the individual distributions, the average value of $E_p$ as a function of $F_{\text{tot}}$ or vice versa. For example,

$$
\bar{E}_p(F_{\text{tot}}) = \frac{\int dE_p E_p [d^2 N(E_p, F_{\text{tot}})/dE_p dF_{\text{tot}}]}{dN(F_{\text{tot}})/dF_{\text{tot}}},
$$

(4)

where

$$
\frac{dN(F_{\text{tot}})/dF_{\text{tot}}}{dN(F_{\text{tot}})/dF_{\text{tot}}} = \int dE_p [d^2 N(E_p, F_{\text{tot}})/dE_p dF_{\text{tot}}].
$$

(5)

We try various plausible models for the functions $\zeta$, $\phi$, and $\rho$. We then compute the expected $\bar{E}_p(F_{\text{tot}})$, remove this cosmological correlation from the data by the transformation $E_p' = E_p/\bar{E}_p(F_{\text{tot}})$, and see if we are left with any correlation between the observed $F_{\text{tot}}$ and $E_p'$ distributions. Only if none remains,
Figure 7. **Left Panel:** The $\tau$ statistic as a function of $Q$, the mean intrinsic value of $E_p$ (assumed to have a Gaussian distribution), for two different values of the dispersion $\sigma_E$ and rate evolution $\rho(z)$. The distribution of $E_{rad}$ is assumed to be a delta function at $10^{53}$ ergs. Note that for a constant rate the observed correlation between $F_{tot}$ and $E_p$ can be due to cosmology if the mean and the dispersion values of $E_p$ are low. **Right Panel:** Same as the Left panel except for a power law distribution of $E_{rad}$ with spectral index $\beta$ and as function of the minimum value of this distribution. Here we assume $Q = 600$ keV, $\sigma_E = 500$ keV, $\rho$ proportional to the star formation rate and a Hubble constant of 60 km/(s Mpc).

then can we attribute the correlation between $F_{tot}$ and $E_p$ to cosmological effects alone. We assume that $\phi$ obeys either a delta function (standard candles) or a power law with spectral index $\beta$ in the radiated energy, $\zeta$ is a Gaussian in the intrinsic (i.e. rest frame) value of $E_p$ with a mean of $Q$ and dispersion $\sigma_E$, and $\rho$ is either a constant or follows the star formation rate. We present results for the Einstein de-Sitter cosmological model which are qualitatively similar to that of several other models that we have explored.

Our general conclusion is that, for plausible values of the parameters for this set of distributions, the correlation cannot be attributed to cosmological effects alone. This is illustrated in the two panels of Figure 7. The cosmological effects can account for all of the observed correlation for model parameters for which the statistic $\tau = 0$. As evident from these curves, for most of the plausible combinations of distribution parameters, $\tau$ remains well above one so that the correlation seen in the data cannot be accounted for by cosmological effects alone. After the removal of the cosmological contribution to the correlation, we are still left with a positive correlation except for a delta function distribution of the radiant energy and a narrow intrinsic distribution of $E_p$’s with a low mean value. As seen from the left panel of Figure 1 the first of these cannot be true and the other requirements are also unreasonable. One explanation of this is
that there is an intrinsic correlation between total fluence and $E_p$. The degree of this correlation, of course, depends on how we model the distribution of the burst parameters.

4.3. Distributions of $E_p$ and $F_{tot}$

Once the correlation between burst parameters is known, we can remove it and use the techniques described in §2 to get an accurate estimation of the true distributions of each parameter. In particular, we can use these methods to explore the truncation of the $E_p$ distribution. However, beginning with a narrow $E_p$ distribution for only the brightest bursts ($C_p \gg C_{lim}$), we find that truncation plays a small role. Not only do we need a more complete sample of spectral fits to really explore this problem, but we need to account for subtle effects of each burst’s detector response matrix when estimating the truncation. However, as shown in LP99 a more complete and well defined sample shows that the qualitative effects of the truncation is to produce an observed distribution that is significantly narrower than the actual distributions. This behavior has also been seen in the simulations by Brainerd et al. (1999), where a broad (power law) parent distribution gives rise to a Gaussian like observed distribution and a narrow parent distribution leads to a somewhat narrower observed distribution.

We can also use the available data from other instruments (sensitive to different energy ranges than BATSE) to better understand the true $E_p$ distribution. The left panel of Figure 8 superposes the BATSE, SMM, and GINGA $E_p$ distributions. The GINGA data (sensitive to lower energies than BATSE) and SMM data (sensitive to higher energies than BATSE) show that there are indeed
a significant number of bursts outside the BATSE trigger range as predicted by our methods. Again, a more complete sample of spectral fits along with the details of the bursts’ DRMs are needed to really get a handle on the distribution of $E_p$. However, the results from SMM and GINGA certainly indicate that a raw correlation analysis without accounting for truncation effects can lead to misleading results.

The right panel of Figure 8 shows the cumulative distribution (modulo the Euclidean part) of the total fluence and the peak flux with the redshifts of the known sources appropriately located. Significant deviations of these distributions from the Euclidean case are expected to begin at redshifts of less than one. The position of the redshifts along with the scatter diagram seen in Figure 1 once again point out the importance of the intrinsic dispersion vis-a-vis that expected from the cosmological effects.

5. SUMMARY AND CONCLUSIONS

Localization of GRBs by BeppoSAX, and the discovery of optical afterglows and underlying galaxies with measured redshifts has put the cosmological origin of GRBs on firm footing. The question which naturally arises is to what degree the redshift distribution of the sources affects the distributions and correlations between observed quantities, and to what extent one can deduce the redshift distribution from the latter. There have been several suggestions of the existence of cosmological redshift signatures in the temporal and spectral characteristics of GRBs. In this paper, we have expanded the search for the cosmological signatures to other temporal and spectral features in order to determine the validity of the claimed signatures, and the extent of the influence of the redshift distribution on observables.

We first point out that the observed redshifts when combined with the log$N$-log$S$ relation (Figs. 1 and 8) show the presence of a broad “luminosity function” not only at gamma-ray range (see, e.g. Stern et al. 1999) but at all photon energies. This should make the detection of cosmological signatures difficult, requiring a careful analysis of observational and data analysis selection biases. In §2, we discuss methods we have developed for determination of the selection biases and statistical procedures for accounting for the data truncations arising from these biases.

We then present results from our study of temporal characteristics of GRBs intended to investigate whether the reported anticorrelation between the peak fluxes and some measure of duration of the bursts (e.g. Norris et al 1994, Stern et al. 1997, Mitrafanov 1998, Deng & Schaefer 1999) is caused by cosmological “time dilation” or is an intrinsic property of the bursts. Using the TTS data and a different analysis than employed by the above authors, we find the following results:

- We confirm the above anticorrelation between the peak flux and several measures of duration of bursts. However, we point out that the observed anticorrelation among bursts is too strong compared to what is expected cosmologically, particularly if most bursts have high redshifts.
- Anticorrelations similar to the above among bursts are seen among pulses in individual bursts. We find a statistically significant excess of anticorrelations
over correlations between pulse amplitude (peak pulse counts) and pulse duration. This is especially significant for bursts when the correlation coefficient between the above two quantities is strong. Unlike the anticorrelation found among bursts, the anticorrelation among pulses of individual bursts cannot be attributed to cosmological time dilation and must be intrinsic. This raises the possibility that both effects are intrinsic.

- We find a strong correlation between various measures of fluence and duration among bursts, confirming the earlier results of Lee and Petrosian (1996, 1997). We also find a strong correlation between total counts or fluence and duration of pulses in individual bursts. Such correlations are not expected from redshift effects and must be intrinsic. The fact that amplitude, duration and fluences of pulses have complex correlations is an indication that neither the peak luminosity nor the total energy of pulses are standard candles. When this is extended to the whole burst, it can explain the absence of a Hubble Law in Figure 1.

Some or all of the above relations may be due to our methodology of defining pulses, and determining fitting parameters. For example, one would expect a more likely loss of short weak bursts than strong ones. To answer this question, we have simulated a representative sample of bursts and followed our pulse fitting and analysis procedure used for BATSE bursts. We do find some biases which can introduce anticorrelation between amplitude and duration when there is none, or weaken a strong correlation between fluence and duration of pulses. However, these effects are weaker than what is actually observed, especially when we compare simulated and actual bursts for which the trends are strongest and statistically more reliable.

Next we consider the claim of the presence of cosmological signatures, particularly the spectral redshift of the break energy, in the spectra of GRBs, most of which can be fitted to various forms approximating a broken power law (Mallozzi et al. 1996 and 1998, Mitrofanov et al. 1999). This claim is based on a correlation between the break energy and peak flux, which is assumed to be a good measure of distance or redshift. This requires a narrow distribution of peak luminosities of bursts, which - as discussed above - does not seem to be the case. We believe that selection effects can play an important role and possibly produce false correlations. To clarify this situation, we have selected a subsample of spectral fits to bursts (from a larger sample kindly provided by Dr. Mallozzi) which is nearly complete within well defined thresholds. Using the techniques described in §2, we have found the following results:

- We find a very strong correlation between several measures of burst fluence and spectral break energy (or $E_p$, the peak energy of the $\nu F_\nu$ spectrum), but only a weak correlation between peak flux and $E_p$. This apparent contradiction with the Mallozzi et al. result could arise from the fact that in obtaining a well defined sample, we are limited to the brightest burst, which - according to Mallozzi et al. (1996, 1998)- show only a weak correlation. That is, most of the claimed correlation between peak flux and $E_p$ comes from weaker bursts. Alternatively, it could be due to an improper accounting of the selection effects present in the data and analysis.

- A correlation between fluence and $E_p$ is expected from redshifts effects. We have quantitatively tested to see if the observed correlation could be at-
tributed to cosmological effects. Our tests show that this could be the case for standard candle radiated energy and for a narrow intrinsic distribution of $E_p$. Neither one of these requirements seem reasonable. For more reasonable models, e.g. a power law distribution in the radiated energy with a broad range (> two orders of magnitude as in Figure 1), the expected correlation from cosmological effects is weakened considerably. Hence, the observed correlations must be primarily intrinsic to the radiation processes at the source.

In summary, we see very little direct signs of cosmological redshift effects in the temporal and spectral properties. The bulk of the correlations we have described must be intrinsic to the source. A corollary of this is that the claimed hardness-duration relation (Kouveliotou et al. 1998) most likely is also intrinsic to the source. All of these intrinsic correlations must be explained by the energy release and radiation processes at the burst.

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