Supplement of

Rediscovering Robert E. Horton’s lake evaporation formulae: new directions for evaporation physics

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Supplementary Material

A. A useful derivation of lake equilibrium levels drawn from Horton (1927)

The following derivation complements Eqn. (5) of the main text. Though it does not represent the same physical problem, it does provide an analogous solution of the same skeleton, from which the derivation steps of Horton for Eqn. (5) can be inferred. Our inference of the same is provided below the derivation.

Time taken for lake levels to stabilize after channel has been modified resembles the formulation of the $x_c$.

Stage-discharge relationships for inflow and outflow of the lake are given by

$$Q = c + kh$$  \hspace{1cm} (A1)

Inflow, outflow and storage are related by,

$$ldt - Adh = Qdt$$  \hspace{1cm} (A2)

where

$$I = c + kh2$$  \hspace{1cm} (A3)

Rearranging

$$(I - Q)dt = Adh$$  \hspace{1cm} (A4)

$$(c + kh2 - c - kh)dt = Adh$$  \hspace{1cm} (A5)

Reducing

$$k(h2 - h1)dt = Adh$$  \hspace{1cm} (A6)

$$dt = \frac{A}{k h2 - h} dh$$  \hspace{1cm} (A7)
The time it takes for lake level to reach a new mean equilibrium level from the time the change to the channel is made \((h = h_1, \text{when } t = 0)\) is given by integrating the above,

\[
\int dt = \int \frac{A}{k} \frac{dh}{h_2 - h} \Rightarrow t = -\frac{A}{k} \log_e \frac{h_2 - h_1}{h_2 - h}
\]  

(A8)

where \(h_1\): water surface height at original mean stage (above improved channel bottom); \(h_2\): water surface height at original stage (above original channel bottom); \(A\): lake surface area.

Q: mean outflow rate; \(h\): depth at time \(t\) referred to new control sill elevation; \(t\): time taken for lake level to reach a new mean equilibrium level, time of change of channel \(t_0=0\).

**Inference of derivation vapor blanket horizontal distance considering Eqns. (A1-8):** Analogous to the above derivation, Horton’s derivation of Eqn. (5) must have been as follows: change of distance \((dx)\) of vapor blanket disturbance is directly related to horizontal rate of change of vapor pressure and inversely related to the amount of vapor transported horizontally \((m)\), and a constant related to elemental area \((C)\) from which vapor is transported, as well as the VVPD. Rearranging and integrating by parts, and taking limits from \(x=0\) to \(x_c\), we get ratio of evaporation from windward to leeward sides, i.e. fringe of the lake where it is maximum to where it approaches a constant value at a distance \(x_c\).

**B. Fitting a function to Horton’s wind velocity correction factor \((w_0)\)**

Several methods were tested in order to estimate the wind height at ground level as a function of measurement height and velocity at the given height, including: 1) Monkey Saddle; 2) shifted divergence in measurement height and root like behavior in velocity at that height; 3) multi-linear regression; and 4) polynomial regression (with and without log).

Monkey Saddle is given by,

\[
z = ax^3 - bxy^2 + c
\]  

(B1)

Root and shifted divergence is given by the shape,

\[
z = ax^p (b + (y - cx - d)^{-q})
\]  

(B2)

where, \(x\): velocity measured at height \(H\) \(w_H\); \(y\): Height \(H\); \(z\): velocity at ground (at 1 foot height from surface, \(w_{h=0}\)); values of \(a, b, c, p,\) and \(q\) are 14.555, 0.05, 16.644, -68.614, 1.617, 0.65.

Substituting, the values of coefficients, the final equation is given by

\[
w_0 = 14.555 w_H^{1.617} (0.05 + (H - 16.614 w_H + 68.614)^{-0.65}
\]  

(B3)
Dr. Nikolai Mikuszeit, on Stack Overflow, provided a solution with an $R^2$ value of 0.999, which is given by

$$w_0 = \frac{1.874}{(H + 13.83)^{0.162}} w_H \left( \frac{0.949}{(H + 1.228)^{0.052}} \right)$$

(B4)

C. Horton’s updated bibliography – most comprehensive to our knowledge: titles include those of papers, reports, books, and technical discussions

| S. No. | Year | Title |
|--------|------|-------|
| 1      | 1896 | A report for the New York State Engineer and Surveyor |
| 2      | 1900 | Computational works connected with hydraulic tests |
| 3      | 1900 | Report on the measurement of the volume of streams and the flow of water in the State of New York |
| 4      | 1901 | Available water power of Michigan and its economical development |
| 5      | 1901 | American canal problems with special reference to the state of New York |
| 6      | 1902 | The law of water as applied to paper mills |
| 7      | 1903 | Annual Report of the State Engineer and Surveyor of New York |
| 8      | 1905 | The drainage of ponds into drilled wells |
| 9      | 1905 | Snowfalls, freshets, and the winter flow of streams in the state of New York |
| 10     | 1905 | Progress of Stream Measurements for the Calendar Year 1904, Part 2, Hudson, Passaic, Raritan and Delaware River Drainages |
| 11     | 1905 | Report of progress of stream measurements for the calendar year 1904; Part VI, Great Lakes and St. Lawrence River drainage |
| 12     | 1906 | Surface drainage of land by tile |
| 13     | 1906 | Weir experiments, coefficients, and formulas |
| 14     | 1906 | Turbine water-wheel tests and power tables |
| 15     | 1906 | Underground water resources of Long Island, New York |
| 16     | 1906 | Report of progress of stream measurements for the calendar year 1905, Part II, Hudson, Passaic, Raritan, and Delaware River drainages |
| 17     | 1906 | Report of progress of stream measurements for the calendar year 1905; Part VI, Great Lakes and St. Lawrence River drainages |
| 18     | 1906 | Hudson, Passaic, Raritan, and Delaware River Drainages |
| 19     | 1906 | Great Lakes and St. Lawrence River drainages |
| 20     | 1907 | The Adirondack rainfall summit |
| 21     | 1907 | Weir experiments, coefficients, and formulas |
| Page | Year | Title |
|------|------|-------|
| 22   | 1907 | Determination of stream flow during the frozen season |
| 23   | 1908 | Deforestation, drainage and tillage with special reference to their effect on Michigan streams |
| 24   | 1910 | The Turbine Water Wheel as a Prime Mover |
| 25   | 1911 | Ebermayer’s experiments on forest meteorology |
| 26   | 1913 | Effects of recent flood on New York streams; study of rainfall and stream discharge, with hydrographs for fourteen rivers |
| 27   | 1913 | Flood frequency and flood control |
| 28   | 1914 | Evaporation from snow and errors of rain gage when used to catch snowfall |
| 29   | 1914 | Discussion of Report of Committee on Yield of Drainage Areas |
| 30   | 1914 | Derivation of runoff from rainfall data. Discussion |
| 31   | 1915 | Idiosyncrasies of Underground Water |
| 32   | 1915 | The melting of snow |
| 33   | 1915 | Discussion of paper by A. F. Meyer on Computing Runoff from Rainfall and other Physical data |
| 34   | 1915 | Discussion: Yield of Underground Reservoirs |
| 35   | 1916 | Standing-wave experiment |
| 36   | 1916 | Some better Kutter’s formula coefficients |
| 37   | 1916 | Diagram for full comparison of hydraulic turbines |
| 38   | 1916 | A study of the depth of annual evaporation from Lake Conchos, Mexico. Discussion by M. Hegly, Robert E. Horton, and J. W. Ledoux. p. |
| 39   | 1917 | A new evaporation formula |
| 40   | 1917 | A new evaporation formula developed |
| 41   | 1917 | Rational study of rainfall data makes possible better estimates of water yield |
| 42   | 1917 | Failure of hydraulic projects from lack of water prevented by better hydrology |
| 43   | 1917 | Determining the regulating effect of a storage reservoir |
| 44   | 1917 | Drainage Basin and Crop Studies Aid Water-Supply Estimates |
| 45   | 1918 | Air chimneys of ice below a waterfall |
| 46   | 1918 | Additional data needed by engineers |
| 47   | 1918 | Discussion on "obstruction to flow by bridge piers" |
| 48   | 1919 | Watershed Leakage in Relation to Gravity Water Supplies |
| 49   | 1919 | Additional meteorological data needed by engineers |
| 50   | 1919 | Evaporative capacity |
| 51   | 1919 | Device for obtaining maximum and minimum water surface temperatures |
| Page | Year | Title |
|------|------|-------|
| 52   | 1919 | Rainfall interception |
| 53   | 1919 | Some broader aspects of rain intensities in relation to storm-sewer design |
| 54   | 1919 | The measurement of rainfall and snow |
| 55   | 1919 | Discussion on The Duty of Water In the Pacific Northwest |
| 56   | 1920 | From the Committees: Hydrological Meteorology |
| 57   | 1920 | Modern hydraulic turbine design |
| 58   | 1920 | Comparison of snow-board and raingage-can measurements of snowfall |
| 59   | 1920 | Weather and literature |
| 60   | 1921 | Vapor pressure and humidity diagram |
| 61   | 1921 | Results of Evaporation Observations |
| 62   | 1921 | Correlation of maximum rain intensities for long and short time-intervals |
| 63   | 1921 | Discussion of the probable variation in yearly precipitation |
| 64   | 1921 | Cloudburst rainfall at Tarborton |
| 65   | 1921 | Unusual lightning |
| 66   | 1921 | Thunderstorm-breeding spots |
| 67   | 1921 | The beginning of a thunderstorm |
| 68   | 1921 | The depletion of ground-water supplies |
| 69   | 1922 | Discussion of “The American Mixed-Flow Turbine” |
| 70   | 1922 | Discussion of "Siphon Spillways" |
| 71   | 1923 | Group distribution and periodicity of annual rainfall amounts |
| 72   | 1923 | Transpiration by forest trees |
| 73   | 1923 | Rainfall interpolation |
| 74   | 1923 | Accuracy of areal rainfall estimates |
| 75   | 1923 | Rainfall duration and intensity in India |
| 76   | 1923 | Discussion |
| 77   | 1923 | Engineering Meteorology and Hydrology |
| 78   | 1924 | Determining mean precipitation on a drainage basin |
| 79   | 1924 | Discussion on Distribution of Intense Rainfall |
| 80   | 1924 | The Distribution of intense Rainfall and some other Factors in the Design of Storm Water Drains |
| 81   | 1924 | Flood reduction by reservoirs |
| 82   | 1924 | Discussion of paper by C. S. Jarvis on flood flow characteristics |
| 83   | 1927 | Hydrology of the Great Lakes |
| Page | Year | Title                                                                 |
|------|------|----------------------------------------------------------------------|
| 84   | 1927 | Report on the lake lowering controversy and a program of remedial measures |
| 85   | 1928 | Report on proposed tri-state compact [to] Board of Commissioners       |
| 86   | 1931 | The field, scope and status of the science of hydrology               |
| 87   | 1931 | Field, scope and status of hydrology, Water and Water Engineering     |
| 88   | 1931 | New gravity water-supply system of Albany, N. Y.                       |
| 89   | 1931 | Discussion of "Horton on Regulation of Niagara River"                 |
| 90   | 1932 | Water diversion between drainage basins                                |
| 91   | 1932 | Drainage basin characteristics                                        |
| 92   | 1932 | Discussion of the report of the committee on floods                   |
| 93   | 1933 | Slope table for fully controlled hydraulic experiments in open channels |
| 94   | 1933 | The relation of hydrology to the botanical sciences                   |
| 95   | 1933 | The role of infiltration in the hydrologic cycle                       |
| 96   | 1933 | Separate roughness coefficients for channel bottom and sides          |
| 97   | 1933 | Storm-flow prediction                                                 |
| 98   | 1933 | Columnar Vapor Drift                                                  |
| 99   | 1933 | Primary Rainfall Types                                                |
| 100  | 1934 | Water-losses in high latitudes and at high elevations                 |
| 101  | 1934 | Compilation and summary of the evaporation records of the Bureau of Plant Industry, U.S. Department of Agriculture, 1921-32 |
| 102  | 1934 | Snow-surface temperature                                              |
| 103  | 1934 | Laminar sheet flow                                                    |
| 104  | 1934 | Recent tendencies in relation to valuation of water rights            |
| 105  | 1934 | Discharge coefficients for tainter gates                               |
| 106  | 1934 | Composite roughness in channels                                      |
| 107  | 1935 | Surface runoff phenomena : Part I, Analysis of the hydrograph         |
| 108  | 1936 | Natural stream-channel storage                                        |
| 109  | 1936 | Maximum groundwater levels                                            |
| 110  | 1936 | Surface-runoff control, Headwaters Control and Use, Chapter II        |
| 111  | 1936 | Historical development of ideas regarding the origin of springs and ground water |
| 112  | 1936 | Relation of Hydraulic and Laboratory Research to Physical and Economic Geography |
| 113  | 1937 | Hydrologic Interrelations of Water and Soils                          |
| 114  | 1937 | Determination of infiltration capacity for large drainage basins      |
| Page | Year | Title                                                                 |
|------|------|----------------------------------------------------------------------|
| 115  | 1937 | Hydrologic aspects of stream-flow stabilization                      |
| 116  | 1937 | Natural stream channel-storage (Second paper)                        |
| 117  | 1937 | Hydrologic aspects of stream-flow stabilization                      |
| 118  | 1937 | Hydrologic research                                                  |
| 119  | 1938 | Analysis of simulated rainfall experiments                            |
| 120  | 1938 | Channel waves subject chiefly to momentum control                     |
| 121  | 1938 | Phenomena of the contact zone between the ground surface and a layer of melting snow |
| 122  | 1938 | Rain wave-trains                                                     |
| 123  | 1938 | Seddon's and Forchheimer's formulas for crest velocity of flood-waves subject to channel-friction control |
| 124  | 1938 | Report on Soil Conservation Service special advisory committee, 1937-1938 |
| 125  | 1938 | Definitions and classification of flood waves                        |
| 126  | 1938 | The interpretation and application of runoff experiments with reference to soil erosion problems |
| 127  | 1938 | Apples from Eden and other short stories                             |
| 128  | 1939 | Memorandum regarding purpose and procedure for research project on infiltration [in Delaware River] |
| 129  | 1939 | Analysis of runoff-plat experiments with varying infiltration capacity |
| 130  | 1939 | Hydrologic advisory committee to the Research Division of the United States Soil Conservation Service, 1938-1939 |
| 131  | 1939 | What Can We Do About the Weather?                                    |
| 132  | 1940 | Hydrologic advisory committee to the Research Division of the United States Soil Conservation Service, 1939-1940 |
| 133  | 1940 | The infiltration-theory of surface-runoff                             |
| 134  | 1940 | Hydrophysical approach to quantitative morphology of drainage basins |
| 135  | 1940 | An approach toward a physical interpretation of infiltration capacity |
| 136  | 1940 | Suggestion for a comprehensive research program on runoff phenomena  |
| 137  | 1940 | Delaware River Basin Flood Volumes, n. 1                              |
| 138  | 1940 | Determination of areal average infiltration-capacity from rainfall and runoff data |
| 139  | 1940 | Sprinkled Plat Runoff and Infiltration Experiments on Arizona Desert Soils |
| 140  | 1940 | Sprinkled Plat Runoff and Infiltration Experiments on Arizona Desert Soils |
| 141  | 1941 | The Role of Snow, Ice and Frost in the Hydrologic Cycle               |
| 142  | 1941 | Flood-crest reduction by channel storage                              |
| 143  | 1941 | Sheet erosion: past and present                                       |
| Page | Year | Title |
|------|------|-------|
| 144  | 1941 | Virtual channel-inflow graphs |
| 145  | 1941 | Hydrologic advisory committee to the Research Division of the United States Soil Conservation Service |
| 146  | 1941 | Discussion (in response to N. E. Edlefsens, Report of the committee on the physics of soil-moisture, 1940-1941, pp. 917-926) |
| 147  | 1941 | Discussion (in response to M. R. Huberty and A. F. Pillsbury, Factors influencing infiltration-rates into some California soils, pp. 686-693) |
| 148  | 1942 | Discussion (in response to A. B. C. Anderson, J. E. Fletcher, and N. E. Edlefsen, Soil-moisture conditions and phenomena in frozen soils, pp. 356-364) |
| 149  | 1942 | Derivation of infiltration-capacity curve from infiltrometer experiments |
| 150  | 1942 | Hydrologic advisory committee to the Research Division of the United States Soil Conservation Service, 1941-1942 |
| 151  | 1942 | Remarks on hydrologic terminology |
| 152  | 1942 | An experiment on flow through a capillary tube |
| 153  | 1942 | Closure to discussion (in response to Horton, R. E., An experiment on flow through a capillary tube, pp. 534-538) |
| 154  | 1942 | Simplified method of determining an infiltration-capacity curve from an infiltrometer-experiment |
| 155  | 1942 | A simplified method of determining the constants of the infiltration-capacity equation |
| 156  | 1942 | Some effects of rain erosion and sedimentation on infiltration-capacity |
| 157  | 1943 | Evaporation—Maps of the United States |
| 158  | 1943 | Hydrologic interrelations between lands and oceans |
| 159  | 1943 | On the relation of soil conservation to air and ground-water pollution |
| 160  | 1943 | A discussion of the relation of soil conservation to air and ground-water pollution |
| 161  | 1944 | Report on proposed improvement and extension of Hemlock Lake water supply system, Rochester, N.Y |
| 162  | 1944 | Some Hydrologic Characteristics of the United States, Part 1 |
| 163  | 1945 | Infiltration and runoff during the snow-melting season, with forest-cover |
| 164  | 1945 | Erosional development of streams and their drainage basins, hydrophysical approach to quantitative morphology |
| 165  | 1947 | Preliminary outline for a comprehensive research on runoff phenomena |
| 166  | 1948 | The physics of thunderstorms |
| 167  | 1948 | Statistical distribution of drop sizes and the occurrence of dominant drop sizes in rain |
| 168  | 1949 | Convectional vortex rings – hail |
D. Tips to find Horton’s papers and full bibliography:

1. Using data from the table provided above, perform a Google search term as follows: “$title + $year + “Robert E. Horton” (side note: we found it easy to save the full citation using Zotero’s plugin for browsers);

2. Search in AGU’s Virtual Hydrology bibliography list maintained at the website - https://connect.agu.org/hydrology/vhp-scope/roberthorton (accessed Nov. 1, 2021);

3. Check the online archive of Albion College (Horton’s Alma Mater, see Accavitti, 2019);

4. Contact the corresponding author (Solomon Vimal) by email to check in his personal bibliography collection (access can be granted to an in-progress Google Sheet where notes on bibliography and the content and working website/download link are curated);

5. Go to the U.S. National Archives in Maryland and dig into the 94 boxes (see list of boxes in Beven, 2004a).

One of these 5 approaches, in the order presented, should help simplify the search for the full paper and citation.

E. Guide to use U.S. and metric equivalent equations for pan, small and large lakes

| Required steps                         | Units and notes                  | Pan evaporation | Small lakes & ponds | Large lakes |
|----------------------------------------|----------------------------------|-----------------|---------------------|-------------|
| **Near ground wind velocity corrections** | Horton’s original formulation    | Equation 3b     | Equation 3b         | Equation 3b |
|                                        | Metric equivalents               | Equation 3c     | Equation 3c         | Equation 3c |
| **Wind Factor**                        | Horton’s original formulation    | Equation 2b     | Equation 2b         | Equation 2b |
| **Evaporation Formula**                | NA                               | 1a              | 1b                  | 1c          |
| **Correction for vapor blanket influence?** | NA                               | Yes, Eq. 2b is essential | Yes, Eq. 2b if the size is within ~10 m in radius | No |
| **Area factor correction?**            | NA                               | Yes, Eq. 4a-d   | Yes, Eq. 4a-d       | No          |
| **Correction for pan geometry**        | Horton’s units                   | Yes, Eq. 2c     | No                  | No          |
| **Convection correction**              | Calibration is necessary         | Yes, Eq. 2b     | Yes, Eq. 2b         | Yes, Eq. 2b |
| **Barometric pressure correction**     | NA, it is a ratio                | Yes, Eq. 8c     | Yes, Eq. 8c         | Yes, Eq. 8c |