Sterile neutrinos in lepton number and lepton flavor violating decays

Juan Carlos Helo, Sergey Kovalenko, Ivan Schmidt

Universidad Técnica Federico Santa María,
Centro-Científico-Tecnológico de Valparaíso,
Casilla 110-V, Valparaíso, Chile

(Dated: May 11, 2010)

We study the contribution of massive dominantly sterile neutrinos, \( N \), to the Lepton Number and Lepton Flavor Violating (LNV, LFV) semileptonic decays of \( \tau \) and \( B, D, K \)-mesons. We focus on special domains of sterile neutrino masses \( m_N \) where it is close to its mass-shell. This leads to an enormous resonant enhancement of the decay rates of these processes. This allows us to derive stringent limits on the sterile neutrino mass \( m_N \) and its mixing matrix elements \( |U_{eN}|, |U_{\mu N}|, |U_{eN}U_{\tau N}|, |U_{\tau N}U_{\tau N}| \), by carrying out a joint analysis of the existing experimental bounds on the decay rates of the studied processes. In contrast to other approaches in the literature, our extraction of these limits is free from ad hoc assumptions on the relative size of the sterile neutrino mixing parameters. Special attention was paid to the limits on meson decays with \( e^\pm e^\pm \) in final state, derived from non-observation of \( 0\nu\beta\beta \)-decay. We point out that observation of these decays may, in particular, shed light on CP violation in the neutrino sector.

PACS numbers: 12.15.-y,12.60.-i,13.15.+g,13.20.-v,14.60.Pq
Keywords: bottom mesons, neutrino, decay rates
I. INTRODUCTION

Neutrino oscillation experiments have shown conclusively that neutrinos are massive, although very light, particles mixing with each other. Moreover, neutrino oscillations is the first and so far the only observed phenomenon of lepton flavor violation (LFV). Theoretically LFV is only possible if neutrinos are massive. Being electrically neutral neutrinos can be either Dirac or Majorana type particles. Majorana neutrino masses violate total lepton number conservation and, therefore, can induce lepton number violating (LNV) processes. Up to date such processes have not yet been observed experimentally. Searching for LNV processes is a challenging quest, pointed to probe the nature of neutrinos and answer the question about wether they are Majorana or Dirac particles. However it is well known that in Standard Model (SM) extensions with very light and very heavy neutrinos, the rates of both LFV and LNV processes are so small that their experimental observation turns out to be unrealistic. Perhaps the only lucky exception is neutrinoless double beta decay ($0\nu\beta\beta$). The experiments searching for this LNV process are believed to reach even such small neutrino masses $^{[1]}$ as those which are relevant for neutrino oscillations, and to probe the heavy Majorana neutrino sector up to the masses of several TeV $^{[3]}$.

On the other hand there are other extensions of the SM in which both LFV and LNV effects could be significant for various reasons. In particular there may contain new massive particles like moderately heavy Majorana neutrinos, as in the low-scale version of the see-saw scenario, or other new particles such as neutralinos, heavy Majorana particles mixing with neutrinos in the framework of SUSY models with R-parity violation. These new heavy particles may lead to observable effects beyond the light neutrino sector, since they could manifest themselves indirectly via their virtual contribution to processes involving ordinary particles. If these new particles are not very heavy lying within the mass reach of the running of forthcoming experiments, they can be searched for directly among the products of colliding particles. They may also contribute to some decays as virtual, but nearly on-mass-shell states, leading to huge resonant enhancement of the corresponding decay rates.

Here we consider an extended see-saw scenario, including $n$ species of SM singlet right-handed neutrinos $\nu_R^{ij} = \nu_R^{i1}, ..., \nu_R^{in}$, besides the three left-handed weak doublet neutrinos $\nu_L^{i} = (\nu_{L_e}^{i}, \nu_{L_\mu}^{i}, \nu_{L_\tau}^{i})$. The general mass term for this set of fields can be written as

$$\frac{1}{2} \bar{\nu}^{i} \mathcal{M}^{(\nu)}_{\nu} \nu^{i} + \text{h.c.} = - \frac{1}{2} \left( \begin{array}{c} \nu_L^{i} \varepsilon \\ \nu_R^{i} \end{array} \right) \left( \begin{array}{cc} \mathcal{M}_L & \mathcal{M}_D \\ \mathcal{M}_D^T & \mathcal{M}_R \end{array} \right) \left( \begin{array}{c} \nu_L^{i} \\ \nu_R^{i} \end{array} \right) + \text{h.c.}$$

$$= \frac{1}{2} \left( \sum_{i=1}^{3} m_{\nu_{i}} \bar{\nu}_{i}^{i} \nu_{i} + \sum_{j=1}^{n} m_{\nu_{j}} \bar{\nu}_{j}^{i} \nu_{j} \right) + \text{h.c.}$$

Here $\mathcal{M}_L, \mathcal{M}_R$ are $3 \times 3$ and $n \times n$ symmetric Majorana mass matrices, and $\mathcal{M}_D$ is a $3 \times n$ Dirac type matrix. Rotating the neutrino mass matrix to the diagonal form by a unitary transformation

$$U^T \mathcal{M}^{(\nu)} U = \text{Diag} \{ m_{\nu_{1}}, \cdots, m_{\nu_{3+n}} \}$$

we end up with $3 + n$ Majorana neutrinos with masses $m_{\nu_{1}}, \cdots, m_{\nu_{3+n}}$. In special cases there may appear among them pairs with masses degenerate in absolute values. Each of these pairs can be collected into a Dirac neutrino field. This situation corresponds to conservation of certain lepton numbers assigned to these Dirac fields.

Neutrino interactions are represented by the SM Charged (CC) and Neutral Current (NC) Lagrangian terms. In the mass eigenstate basis they read

$$\mathcal{L} = \frac{g_2}{\sqrt{2}} \sum_i U_{li} \bar{\nu}_{l}^{\gamma \mu} P_L \nu_{i} W_{\mu}^{\nu} + \frac{g_2}{2 \cos \theta_W} \sum_{\alpha, \alpha', j} U_{\alpha j}^* U_{\alpha' j} \bar{\nu}_L^{\gamma \mu} P_L \nu_{j} Z_{\mu},$$

where $l = e, \mu, \tau$ and $i = 1, ..., n + 3$.

In this scenario the light neutrinos have the mass scale $\mathcal{M}_D^2 / \mathcal{M}_R$, and as a result there should be also heavy Majorana neutrinos ($N$) with mass scale $\mathcal{M}_R$. In particular, they may include moderately heavy neutrinos $N$ in the MeV-GeV domain, which we are going to consider in the next sections. If they exist they may contribute to some LNV and LFV processes as intermediate on-mass-shell states. This would result in enormous resonant enhancement of their contribution to these processes and would allow one to set stringent limits on heavy neutrino masses and mixing from non-observation of the corresponding processes. The presence or absence of these neutrino states, which are conventionally called sterile neutrinos, is a subject to be revealed by experimental searches. Having these arguments in mind we study neutrino states $N$ with masses $m_N$ in this region of values and examine their phenomenological impact on the LFV and LNV decays of $\tau$ and some mesons. The phenomenology of sterile neutrinos have been studied in the literature in different contexts and from different points of view. Their resonant contributions to $\tau$ and meson decays have been studied in Refs. $^{[5]}$, $^{[12]}$, $^{[7]}$. Another potential process to look for sterile Majorana neutrinos is like-sign
dilepton production in hadron collisions [8], [9], [10], [11]. Possible implications of sterile neutrinos have been also studied in LFV muonium decay and high-energy muon-electron scattering [12]. (for a recent review of sterile neutrino phenomenology see [13]). The present paper contributes to some still uncovered aspects of the phenomenology of sterile neutrinos.

The paper is organized as follows. In the next section II we present decay rate formulas for $\tau$ and meson LFV and LNV decays and their reduced forms, in the resonant domains of neutrino masses. In sec. III we discuss theoretical uncertainties in the calculation of the total decay width of sterile neutrinos and present our “inclusive” approach based on Bloom-Gilman duality. This approach allows one to avoid large uncertainties related to the heavy meson decay constants, which are typical in the conventional channel-by-channel approach. In sec. IV we discuss the extraction of the limits on the active-sterile neutrino mixing matrix elements $|U_{eN}|, |U_{\mu N}|, |U_{eN}\tau N|, |U_{\mu N}\tau N|$ and present the corresponding exclusion plots. In sec. V we discuss the $0\nu\beta\beta$-decay constraints on sterile neutrino parameters and their possible implications for CP-violation in the neutrino sector. Here we also give predictions for the rates of some LNV and LFV decays of $K, D, D_s, B, B_c$-mesons. Sec. VI summarizes our main results.

## II. DECAY RATES

In what follows we analyze the above mentioned resonant contribution of heavy sterile neutrino to the semileptonic LNV and LFV decays of $\tau$ and the pseudoscalar mesons $M = K, D, B$:

$$\tau^- \rightarrow l^\mp \pi^\pm \pi^- , \quad M^+ \rightarrow \pi^\pm l^+_1 l^+_2 .$$

(5)

Lowest order diagrams for the case of meson decays are shown in Fig. 1. There is only one tree-level diagram with an intermediate Majorana or Dirac neutrino, shown in Fig. 1(a), contributing to LFV decays. For LNV decays mediated by Majorana neutrinos, in addition to the tree-level diagram Fig. 1(a), there appears a two-loop diagram shown in Fig. 1(b). The tree-level diagram is known [7] to dominate in these processes (5). As will be seen, in the studied domain of the heavy neutrino mass $m_N$, the two-loop diagram in Fig. 1(b) is absolutely negligible. An important point is that the calculation of the tree-level diagram, Fig. 1(a), does not require knowledge of the hadronic structure needed for the diagram in Fig. 1(b).

![Fig. 1: The lowest order diagrams contributing to the semileptonic meson decays.](image)

The decay rates of the studied processes are given by the expressions:

**LNV:** \( \Gamma(M^+ \rightarrow \pi^- l^+_1 l^+_2) = \)

$$= \kappa_{ij} \int ds \sum_k \left| U_{\mu k} U_{e k} m_{\nu k} \right|^2 G^j_M \left( \begin{array}{c} s \\ m^2_M \end{array} \right) + \kappa_{ij} \int ds \sum_k \left| U_{\mu k} U_{e k} m_{\nu k} \right|^2 G^i_M \left( \begin{array}{c} s \\ m^2_M \end{array} \right) +$$

$$+ 4 \kappa_{ij} \text{Re} \left[ \int ds_1 \left( \frac{U_{\mu k} U_{e k} m_{\nu k}}{s_1 - m^2_{\nu k}} \right)^* \int ds_2 \left( \frac{U_{\mu k} U_{e k} m_{\nu k}}{s_2 - m^2_{\nu k}} \right) H^j_M \left( \begin{array}{c} s_1 \\ m^2_M \end{array} \right) \left( \begin{array}{c} s_2 \\ m^2_M \end{array} \right) \right], \quad (6)$$

**LFV:** \( \Gamma(M^+ \rightarrow \pi^+ l^-_1 l^-_2) = \)
Here the unitary mixing matrix \( U_{\nu j} \) relates \( \nu^\prime = \nu_i \nu_j \), the weak \( \nu^\prime \) and mass \( \nu \) neutrino eigenstates. In Eq. (6) the factor \( \kappa_{ij} - \delta_{ij}/2 \) takes into account a combinatorial factor 1/2 for identical final leptons. In the above equations we introduced the following functions:

\[
G_{ij}^M(z) = e^M \phi_{ij}(z)\left[ h^2_{i}(z) - x^2_\pi h_i^+(z) \right]\left[ h^2_{j}(z) - h^2_{j-}(z) \right],
\]

\[
H_{ij}^M(z_1, z_2) = \frac{e^M}{m^2_M} \left\{ h^i_-(z_1)h^j_+(z_2) + x^2_\pi \left[ h^i_-(z_1)h^j_- (z_2) + x^2_i + x^2_j \right] - \frac{1}{2} \left[ h^i_-(z_1) + h^j_-(z_2) \right]\left[ h^i_-(z_1)h^j_+(z_2) + h^i_+(z_1)h^j_-(z_2) \right] \right\},
\]

with

\[
\phi_{ij}(z) = \lambda^{1/2}(z, x_\pi^2, x_\pi^2)\lambda^{1/2}(z, x_j^2, 1), \quad h^\pm_i(z) = z \pm x^2_i,
\]

\[
e^M = \frac{G_F^4}{128\pi^2} \rho_{ij}^2 m^5_M |V_{ud}|^2 |V_{us}|^2, \quad \lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz,
\]

and the dimensionless variables \( x_i = m_{l_i}/m_M, x_\pi = m_{\pi}/m_M \), where \( m_{l_i} \) and \( m_M \) are the masses of the charged leptons \( l_i = e, \mu, \tau \) and the initial meson \( M \) respectively. Numerical values of meson masses \( m_M \), decay constants \( f_M \) and the CKM factors \( V_M \) are specified in Table I. The integration limits in Eqs. (6), (7) are

\[
s_i^- = m^2_M(x_\pi + x_\pi^2), \quad s_i^+ = m^2_M(1 - x_\pi^2),
\]

\[
s_{l_{12}}^\pm = \frac{m^2_M}{2y} \left[ (x^2_1 - x^2_\pi)(x^2_2 - 1) + y(1 + x^2_1 + x^2_2 + x^2_\pi) - y^2 \pm \phi_{l_{12}}(y) \right],
\]

with \( y = s_1/m^2_M \).

If one assumes that neutrinos are separated into light \( \nu_k \) and heavy \( N_k \) states, with masses \( m_{\nu_k} << \sqrt{s_i^-} \) and \( \sqrt{s_i^+} << m_{N_k} \), then the branching ratios [5] turn out to be extremely small:

\[
R_{l_{12}} = \frac{\Gamma(M^+ \rightarrow l_1^+ l_2^- \pi^-)}{\Gamma(M^+ \rightarrow all)} \leq 10^{-30} \quad 3 \text{ light neutrino scenario.}
\]

\[
R_{l_{12}} \leq 10^{-19} \quad 3 \text{ light + 1 heavy neutrino scenario}
\]

These values are far beyond experimental reach.

On the other hand, if we assume that there exists at least one massive neutrino \( N \) with mass \( m_N \) in the range

\[
\min \left[ \sqrt{s_i^-}, \sqrt{s_j^-} \right] \leq m_N \leq \max \left[ \sqrt{s_i^+}, \sqrt{s_j^+} \right]
\]

the situation changes drastically. Then the s-channel M-decay diagram in Fig. 1(a) blows up because the integrands under the single integrals in Eqs. (6), (7) have a non-integrable singularity at \( s = m^2_N \) corresponding to an on-mass-shell intermediate neutrino. Therefore, in this resonant domain the total decay width of heavy neutrino \( \Gamma_N \) has to be taken into account. This can be done by the substitution \( m_N \rightarrow m_N - (i/2) \Gamma_N \). As will be seen in sec. [V] the heavy neutrino width \( \Gamma_N \) is very small in the resonant domain, \( \Gamma_N \approx 10^{-9} \text{ MeV} \). Therefore, the neutrino propagator in the single integrals of Eqs. (6), (7) has a very sharp maximum at \( s = m^2_N \). The double integrals, being finite in the limit \( \Gamma_{\nu_i} = 0 \), can be neglected in the considered case. Thus, with good precision we obtain from Eqs. (6), (7) the following decay rate formulas for a nearly on-mass-shell intermediate neutrino:

\[
LNV : \quad \Gamma^{\text{ex}}(M^+ \rightarrow \pi^- l_i^+ l_j^-) \approx \kappa_{ij} \pi(G_{ij}^M(z_M) + G_{ij}^{\dagger}(z_M)) m_N |U_{iN}|^2 |U_{jN}|^2 \frac{m_N}{\Gamma_N}
\]

\[
LFV : \quad \Gamma^{\text{ex}}(M^+ \rightarrow \pi^- l_i^+ l_j^-) \approx \pi(G_{ij}^M(z_M) + G_{ij}^{\dagger}(z_M)) m_N |U_{iN}|^2 |U_{jN}|^2 \frac{m_N}{\Gamma_N},
\]
with \( z_M = (m_N/m_M)^2 \). Similar arguments apply to tau decays \( \tau^- \to l^+ \pi^- \pi^+ \) [3], which in the neutrino mass domain \( m_{\pi} + m_l \leq m_N \leq m_{\tau} - m_{\pi} \) lead also to resonantly enhanced decay rates:

\[
\Gamma_{\text{res}}^{\tau^- \to l^+ \pi^- \pi^+} \approx \pi G^l(z_\tau) \frac{m_N |U_{\tau N}|^2 |U_{l N}|^2}{\Gamma_N}
\]

where

\[
G^l(z) = e^{\frac{\theta(z)}{z^2}} [(z - x_l^2)^2 - x^2 (z + x_l^2)] [(z - 1)^2 - x^2 (z + 1)],
\]

\[
\phi^l(z) = \lambda^{1/2} (z, x_l^2, x^2) \lambda^{1/2} (z, x^2, 1), \quad c^l = \frac{G_F^l}{128 \pi^3} (G_F^l/32)(\pi)^{-3} f_{n}^2 m_{\nu}^2 |V_{ud}|^4
\]

with \( z_\tau = (m_N/m_{\tau})^2 \). We will use the above decay rate formulas Eqs. (15)-(18) in our analysis of the sterile neutrino contribution to \( \tau \) and meson decays.

III. NEUTRINO DECAY WIDTH

Heavy sterile neutrinos \( N \), being mixed with active neutrino flavors, can decay in various final states depending on their mass \( m_N \). These decays are represented by purely leptonic \( N \to l_1 l_2 \nu_\ell \), 3\( \nu \) and semileptonic \( N \to l H \) modes, where \( H = M, B, ... \) are hadronic states represented by mesons and baryons. These decays proceed via CC and NC interactions of the SM given by the Lagrangian [4]. The total decay rate, \( \Gamma_N \), of heavy neutrinos is conventionally calculated in the literature in the channel-by-channel approach [3-13], in which one sums up the partial decay rates of all the leptonic and two-body semileptonic decay channels open for a given value of \( m_N \). In the present paper we apply another approach. This is the inclusive approach we proposed in Ref. [3], in which we approximate the semileptonic decays of the heavy neutrino \( N \) by its decays into quark-antiquark pairs \( N \to l(\nu)q_1q_2 \), as suggested by Bloom-Gilman duality [14]. This implies that in an average over sufficiently wide range of the invariant mass of the final hadronic state \( H \), the sum of all the open decay channels \( N \to l(\nu)H \) is approximately equal to the rate of \( N \to l(\nu)q_1q_2 \). In comparison with the former approach the latter does not require information on the parameters of the final mesons, such as masses and decay constants, some of which are poorly known for the meson states starting from \( \eta'(985) \) and heavier. The inclusive approach is supposed, according to the duality arguments, to take into account all the semileptonic channels and therefore, in this case \( \Gamma_N \) should be larger than in the channel-by-channel approach, in which some hadronic states are neglected. We apply a simplified version of the inclusive approach, neglecting perturbative and nonperturbative QCD corrections to the tree-level quark production diagram. This leading-order approximation is expected to be reasonable for \( m_N \gg \Lambda \approx 200 \text{ MeV} \). At lower masses a more viable approach would be to relate by dispersion relations the semileptonic \( N \) decay rate to the imaginary parts of the W and Z self-energies \( \Pi(s) \), in analogy to the approach applied in the literature for the \( \tau \to \nu + \text{hadrons} \) inclusive decay [15]. However for our rough estimations we do not need this more sophisticated treatment and will use the above mentioned leading-order approximation.

Here we summarize the partial decay rates for the inclusive approach [3], including leptonic and semileptonic decay modes of the heavy sterile neutrino \( N \). In the latter case the final hadronic states for low neutrino masses \( m_N \) is represented by the lightest mesons while for larger \( m_N \) by \( q\bar{q} \)-pairs. The list of decay rates is as follows:

\[
\Gamma(N \to l_1^- l_2^+ \nu_{\ell 1}) = |U_{l_1 N}|^2 \frac{G_F^l}{192 \pi^3} m_N^5 \frac{f_{n}^2}{|V_{ud}|^4} \left(I_1(y_{\ell v_2}, y_{\ell v_1}, y_{\ell 2}, y_{\ell 1})(1 - \delta_{l_1 l_2}) \equiv |U_{l_1 N}|^2 \Gamma^{(l_1 l_2 \nu)} \right), \quad (19)
\]

\[
\Gamma(N \to \nu_{l_1} l_2^+ l_2^-) = |U_{l_1 N}|^2 \frac{G_F^l}{96 \pi^3} m_N^5 \left[(g_{l_1}^l g_{l_2}^R + \delta_{l_1 l_2} g_{l_2}^R, I_2(y_{\ell v_1}, y_{\ell v_2}, y_{\ell 2}, y_{\ell 1}) + \left((g_{l_1}^l)^2 + (g_{l_2}^R)^2 + \delta_{l_1 l_2}(1 + 2g_{l_1}^l)) I_1(y_{\ell v_2}, y_{\ell v_1}, y_{\ell 2}, y_{\ell 1}) \right) \equiv |U_{l_1 N}|^2 \Gamma^{(l_1 l_2 \nu)} \right), \quad (20)
\]

\[
\sum_{l_2 = e, \mu, \tau} \Gamma(N \to \nu_{l_1} l_2^+ \bar{\nu}_{l_2}) = |U_{l_1 N}|^2 \frac{G_F^l}{96 \pi^3} m_N^5 \equiv |U_{l_1 N}|^2 \Gamma^{(3\nu)} \right), \quad (21)
\]

\[
\Gamma(N \to l_1^- P^+) = |U_{l_1 N}|^2 \frac{G_F^l}{16 \pi} m_N^3 f_P^2 |V_{P l}|^2 F_P^2(y_{l v_1}, y_{l v_2}) \equiv |U_{l_1 N}|^2 \Gamma^{(l P)} \right), \quad (22)
\]

\[
\Gamma(N \to \nu_{l 1} P^0) = |U_{l_1 N}|^2 \frac{G_F^l}{64 \pi} m_N^3 f_P^2 (1 - y_P^2)^2 \equiv |U_{l_1 N}|^2 \Gamma^{(\nu P)} \right), \quad (23)
\]
Table I: Masses $m_{P,V}$ and decay constants $f_{P,V}$ of pseudoscalar $P$ and vector $V$ mesons.

| $P$ | $m_P$ (MeV) | $f_P$ (MeV) | $V_{ud}$ | $V_{cd}$ | $V_{cs}$ |
|-----|-------------|-------------|---------|---------|---------|
| $\pi^+$ | 139.6 | 130.7 | $\rho^+$ | 775.8 | 220 |
| $K^+$ | 493.7 | 159.8 | $K^{*0}$ | 891.6 | 217 |
| $D^\pm$ | 1869.4 | 222.6 | $D^{*\pm}$ | 2010 | 310 |
| $D_s^\pm$ | 1968.3 | 266 | $D_s^{*\pm}$ | 2112.1 | 315 |
| $B^\pm$ | 5279 | 190 | $\rho^0$ | 776 | 220 |
| $B_s^\pm$ | 6277 | 399 | $\omega$ | 782.59 | 195 |
| $\pi^0$ | 135 | 130 | $K^{*0}$, $\bar{K}^{*0}$ | 896.1 | 217 |
| $K^{*0}$, $\bar{K}^{*0}$ | 497.6 | 159 | $\phi$ | 1019.456 | 229 |
| $\eta$ | 547.8 | 164.7 | $D^{*0}$, $\bar{D}^{*0}$ | 2006.7 | 310 |
| $\eta'$ | 957.8 | 152.9 | $J/\psi$ | 3096.916 | 459 |
| $\eta_c$ | 2979.6 | 355.0 | - | - |
| $B_s^0$ | 5367.5 | 216 | - | - |

\[ \Gamma(N \to l^-_i V^+) = |U_{i1}N|^2 \frac{G_F^2}{16\pi} m_N^3 |F_\nu|^2 |V_\nu|^2 (y_{l1}, y_{\nu}) \equiv |U_{i1}N|^2 \Gamma^{(W)}, \]  
\[ \Gamma(N \to \nu_i V^0) = |U_{i1}N|^2 \frac{G_F^2}{2\pi} m_N^3 f_{\nu}^2 \kappa_{\nu} (1 - y_{\nu}^2)^2 (1 + 2y_{\nu}^2) \equiv |U_{i1}N|^2 \Gamma^{(\nu)}, \]  
\[ \Gamma(N \to \nu_i u\bar{d}) = |U_{i1}N|^2 |V_{ud}|^2 \frac{G_F^2}{64\pi^3} m_N^5 I_1(y_{l1}, y_{u}, y_{d}) \equiv |U_{i1}N|^2 \Gamma^{(ud)}, \]  
\[ \Gamma(N \to \nu_i q\bar{q}) = |U_{i1}N|^2 \frac{G_F^2}{32\pi^3} m_N^5 F_Z(m_N) \left[ g_L^\nu g_R^\nu I_2(y_{v1}, y_{q}, y_{q}) + \right. \]  
\[ \left. + (g_L^\nu)^2 + (g_R^\nu)^2 \right] I_1(y_{v1}, y_{q}, y_{q}) \equiv |U_{i1}N|^2 \Gamma^{(\nu qq)} \]  

Here we denoted $y_i = m_i/m_N$ with $m_1 = m_i, m_P, m_V, m_q$. For the quark masses we use the values $m_u \approx m_d = 3.5$ MeV, $m_s = 105$ MeV, $m_c = 1.27$ GeV, $m_b = 4.2$ GeV. In Eqs. (24), (25) we denoted $u = u, c, t$; $d = d, s, b$ and $q = u, d, c, s, b, t$. The SM neutral current couplings of leptons and quarks are
\[ g_L^\nu = -1/2 + \sin^2 \theta_W, \quad g_R^\nu = 1/2 - (2/3) \sin^2 \theta_W, \quad g_L^d = -1/2 + (1/3) \sin^2 \theta_W, \quad g_R^d = (1/3) \sin^2 \theta_W, \]  
\[ \kappa_{\nu} = \sin^2 \theta_W/3 \quad \text{for} \quad \rho^0, \omega, \]  
\[ \kappa_{\nu} = -1/4 + \sin^2 \theta_W/3 \quad \text{for} \quad K^{*0}, \bar{K}^{*0}, \phi, \]  
\[ \kappa_{\nu} = 1/4 - 2 \sin^2 \theta_W/3 \quad \text{for} \quad D^{*0}, \bar{D}^{*0}, J/\psi \]

The kinematical functions are
\[ I_1(x, y, z) = 12 \int \frac{ds}{s} (s-x^2)(1+z^2-s)\lambda^{1/2}(s, x^2, y^2)\lambda^{1/2}(1, s, z^2), \]  
\[ I_2(x, y, z) = 24yz \int \frac{ds}{s} (1+x^2-s)\lambda^{1/2}(s, y^2, z^2)\lambda^{1/2}(1, s, x^2), \]  
\[ F_P(x, y) = \lambda^{1/2}(1, x^2, y^2)(1+x^2)(1+x^2-y^2) - 4x^2], \]  
\[ F_V(x, y) = \lambda^{1/2}(1, x^2, y^2)(1-x^2)^2 + (1+x^2)y^2 - 2y^2], \]
The total decay rate $\Gamma_N$ of the heavy neutrino $N$ is equal to the sum of the partial decay rates in Eqs. (19)-(27), which we write in the form:

$$\Gamma_N = \sum_{l_1, l_2, H} \left[ \eta_N \Gamma(N \to l_1^- H^+) + \eta_N \Gamma(N \to l_1^+ l_2^- \nu_{l_2}) + \Gamma(N \to l_2^- l_1^+ \nu_{l_1}) + \Gamma(N \to \nu_{l_1} \nu_{l_2} \bar{\nu}_{l_2}) \right],$$

where we denoted the hadronic states $H^+ = P^+, V^+$, $du, su, dc, sc$ and $H^0 = P^0, V^0, \bar{q}q$. We introduced the factor $\eta_N = 2$ for Majorana and $\eta_N = 1$ for Dirac neutrino $N$. Its value $\eta_N = 2$ is related with the fact that for Majorana neutrinos both charge conjugate final states are allowed: $N \to l_1^- l_2^+ \nu_{l_2}, l_1^+ l_2^- \bar{\nu}_{l_2}$ and $N \to l^+ H^\pm$. For convenience we write Eq. (31) in the form:

$$\Gamma_N = a_e(m_N) \cdot |U_{eN}|^2 + a_\mu(m_N) \cdot |U_{\mu N}|^2 + a_\tau(m_N) \cdot |U_{\tau N}|^2$$

(32)

where

$$a_l(m_N) = \Gamma^{(lH)} + \sum_{l_2} \left( \Gamma^{(l_2l_2\nu)} + \eta_N \Gamma^{(l_2l\nu)} \right),$$

(33)

and $l, l_2 = e, \mu, \tau$. In the inclusive approach the hadronic contribution is calculated as

$$\Gamma^{(lH)} = \theta(\mu_0 - m_N) \sum_{P,V} \left( \Gamma^{(\nu P)} + \Gamma^{(\nu V)} + \eta_N \Gamma^{(lP)} + \eta_N \Gamma^{(lV)} \right) + \theta(m_N - \mu_0) \sum_{u,d,q} \left( \eta_N \Gamma^{(lud)} + \Gamma^{(lqq)} \right)$$

(34)

The parameter $\mu_0$ denotes the mass threshold from which we start taking into account hadronic contributions via $q\bar{q}$ production. In our numerical study we use the mass $\mu_0 = m_K = 493.7$ MeV. Thus in the first term of Eq. (34) we take into account only $P = \pi^+, \pi^0$. At this rather low threshold ($\mu_0$) the QCD corrections become significant, but it is reasonable to expect that the tree-level contribution still dominates the semileptonic decay rates since $\mu_0 > \Lambda_{QCD}$. We believe that the accuracy of the above presented inclusive approach is sufficient for estimations of limits on the parameters of sterile neutrino in this range of masses. In this respect it is worth remembering that in the channel-by-channel approach uncertainties related to the heavy meson decay constants $f_M$ and other hadronic parameters are large and theoretically not well controllable.

IV. LIMITS ON STERILE NEUTRINO MASS AND MIXING

In the literature there are various limits on the mass $m_N$ and mixing $U_{\alpha N}$ (with $\alpha = e, \mu, \tau$) of a sterile neutrino $N$, extracted from direct and indirect experimental searches [16] for this particle, in a wide region of its mass. A recent summary of these limits, extracted from the corresponding experimental data, is given in Ref. [13]. In Figs. 2-3 we show the exclusion plots from Ref. [13] together with our exclusion curves, derived from a joint analysis of semileptonic LNV and LFV decays of $K, B, D, D_s$-mesons and $\tau$.

As seen from Eqs. (15), (17) and (32), the decay rates of these processes depend on all the three $U_{e,\mu,\tau N}$ mixing matrix elements. In the literature it is a common practice to adopt some ad hoc assumptions on their relative sign in order to extract limits on them from the experimental bounds on the corresponding decay rates. These assumptions reduce the reliability of the obtained limits. In what follows we carry out a joint analysis of the above mentioned processes, without any additional assumptions of this sort. We apply a numerical Monte Carlo sampling in a parametric space

$$|U_{eN}|, |U_{\mu N}|, |U_{\tau N}|, m_N, \text{ with } 0 \leq |U_{1N}| \leq 1, \quad m_N \in \text{Res}(M, \tau)$$

(35)

taking into account the existing experimental bounds on the branching ratios of LNV and LFV decays: $M \to ll\pi$, $\tau \to l\tau\pi$. Here $\text{Res}(M, \tau)$ denotes resonant regions of sterile neutrino mass corresponding to the studied meson $M = K, B, D, D_s$ and $\tau$ decays. The experimental limits on their branching ratios were taken from Ref. [16], and are shown in Table II. The obtained exclusion curves for $|U_{eN}|$ and $|U_{\mu N}|$ are shown in Figs. 2-3 together with other existing limits. Figs. 1 show the exclusion curves for $|U_{eN}|, |U_{\tau N}|$ and $|U_{\mu N}U_{\tau N}|$.

It is sometimes useful to have analytic expressions for the limits on sterile neutrino mixing. Such expressions for the case of $|U_{eN}|^2$ and $|U_{\mu N}|^2$, without ad hoc assumption on the relative size of different matrix elements $|U_{1N}|^2$, can be derived directly from Eqs. (15) - (17) and (32). Using the fact that $|U_{\mu N}|^2 < 1, |U_{\tau N}|^2 < 1$ one finds

$$|U_{eN}|^4 - |U_{eN}|^2 F_{e\alpha}(M)a_{\alpha} - F_{e\tau}(M)(a_{\mu} + a_{\tau}) < 0$$

(36)

$$|U_{\mu N}|^4 - |U_{\mu N}|^2 F_{\mu\alpha}(M)a_{\alpha} - F_{\mu\tau}(M)(a_{\mu} + a_{\tau}) < 0$$
Solving these inequalities one gets limits

$$|U_{eN}|^2 < \frac{F_{ee}(M) a_e}{2} + \sqrt{\left(\frac{F_{ee}(M) a_e}{2}\right)^2 + F_{ee}(a_{\mu} + a_\tau)}$$

(37)

$$|U_{\mu N}|^2 < \frac{F_{\mu\mu}(M) a_\mu}{2} + \sqrt{\left(\frac{F_{\mu\mu}(M) a_\mu}{2}\right)^2 + F_{\mu\mu}(a_e + a_\tau)}$$

(38)

valid in the resonant regions of decays $M \to eee$ and $M \to \mu\mu\pi$. We introduced the notation

$$F_{ee}(M) = \frac{\Gamma^{exp}(M \to \mu\pi\pi)}{c^2 m_N G^\mu(z_0) + G^\pi(z_0)}, \quad F_{\mu\mu}(M) = \frac{\Gamma^{exp}(M \to \mu\mu\pi)}{c^2 m_N G^\mu(z_0) + b G^{\pi\mu}(z_0)},$$

(39)

in agreement with Eqs. (15) - (18). Here the $\Gamma^{exp}$ are experimental upper bounds on the rate of the indicated decays. Eqs. (37), (38) allow setting limits on $|U_{eN}|^2$ and $|U_{\mu N}|^2$, considering different LNV and LFV decays separately, and using the existing experimental bounds on their rates.

Stronger limits can be extracted from a joint analysis of certain sets of LNV and LFV decays, if these decays have nontrivial intersection of their resonant regions in $m_N$. Then for values of $m_N$ in the intersection one can extract limits for $U_{eN}$ in an analytic form in the following way. Let us consider, for instance, the set of LNV and/or LFV decays: $M \to \pi\mu\mu$, $M \to \tau\pi\mu$ and $\tau \to \pi\pi\pi e$. Then from Eqs. (15) - (17) and (31) we obtain

$$F_{ee}(M) \times |U_{eN}|^2 < \frac{|U_{eN}|^4}{a_e |U_{eN}|^2 + a_{\mu} |U_{\mu N}|^2 + a_{\tau} |U_{\tau N}|^2}, \quad F_{\mu\mu}(M) \times |U_{\mu N}|^2 < \frac{|U_{\mu N}|^4 |U_{\mu N}|^2}{a_e |U_{\mu N}|^2 + a_{\mu} |U_{\mu N}|^2 + a_{\tau} |U_{\tau N}|^2}$$

(40)

$$F_{\tau}(\tau) < \frac{|U_{\tau N}|^2}{a_e |U_{\tau N}|^2 + a_{\mu} |U_{\mu N}|^2 + a_{\tau} |U_{\tau N}|^2}$$

(41)

combining these Eqs. we find

$$|U_{eN}|^2 < a_e F_{ee}(M) + a_{\mu} F_{\mu\mu}(M) + a_{\tau} F_{\tau}(\tau)$$

(42)

Analogously, from the set $M \to \pi\mu\mu$, $M \to \tau\pi\mu$ and $\tau \to \pi\pi\pi e$ we get the limit

$$|U_{\mu N}|^2 < a_e F_{ee}(M) + a_{\mu} F_{\mu\mu}(M) + a_{\tau} F_{\mu}(\tau)$$

(43)

Using in these formulas (42), (43), the experimental bounds on the branching rates of the corresponding decays [16], one can analyze all the sets of decays of the above mentioned type for $B$, $D$, $D_s$, $K$-mesons, and extract exclusion curves in $|U_{eN}|^2 - m_N$ and $|U_{\mu N}|^2 - m_N$ planes. The resulting exclusion curve for $|U_{eN}|^2$ is limited to the mass range $m_N = 141 - 1637$ MeV, while for $|U_{\mu N}|^2$ to the range $m_N = 246 - 1637$ MeV. We do not present these curves since they are not very different in these limited mass regions from those derived in the Monte Carlo approach and shown in Figs. 21. The same is true for Eqs. (37), based on the analysis of different decays separately. Thus the above presented analytical formulas (42), (43), or (37), considered in the corresponding regions of sterile neutrino mass $m_N$, can be treated as a reasonable alternative to a more involving Monte Carlo analysis.

V. LIMITS: COMPATIBILITY AND INTERPLAY

Let us study some implications of the interplay and compatibility of the constraints on the same combinations of parameters derived from different processes.

A. Impact of $0\nu\beta\beta$-decay limits.

The exclusion curve from neutrinoless double beta decay ($0\nu\beta\beta$) in Fig. 2, despite the presence of some uncertainty in nuclear matrix elements (of a factor $\sim 2$), is so stringent that within this uncertainty it overrides all the other constraints except for the narrow region where the exclusion curve of the beam dump PS 191 experiment [18] goes lower. However the PS 191 curve was obtained on the basis of various ad hoc assumptions, touching upon both data processing and their theoretical interpretation, making the corresponding limits not too firm to compete with the $0\nu\beta\beta$-limits. In particular, the PS 191 limits can be evaded if one admits that the sterile neutrinos have dominant decay channels into invisible particles [13] [19].
Table II: Experimental and theoretical upper bounds on the branching ratios of meson and τ decays. The experimental bounds are taken from Ref. [16]. The theoretical upper bounds are shown in the square and curly brackets and are discussed in sec. V.

| $Br(M \to \pi l_1 l_2)$ | $K$ | $D$ | $D_\alpha$ | $B$ | $B_\alpha$ |
|--------------------------|-----|-----|------------|-----|-----------|
| $M^+ \to \pi^+ e^+ e^+$ | 6.4 $\times 10^{-10}$ | [1.4 $\times 10^{-8}$] | 3.6 $\times 10^{-6}$ | 8.6 $\times 10^{-11}$ | 6.9 $\times 10^{-4}$ | [1.0 $\times 10^{-9}$] | 1.6 $\times 10^{-6}$ | [2.3 $\times 10^{-13}$] | {4.1 $\times 10^{-11}$} |
| $M^+ \to \pi^- \mu^+ \mu^+$ | 3 $\times 10^{-9}$ | 4.8 $\times 10^{-6}$ | 3.1 $\times 10^{-7}$ | 2.9 $\times 10^{-5}$ | 3.7 $\times 10^{-6}$ | 1.4 $\times 10^{-6}$ | [3.4 $\times 10^{-10}$] | 5.4 $\times 10^{-8}$ | |
| $M^\prime \to \pi^+ \mu^+ e^+$ | 1.3 $\times 10^{-11}$ | [7.2 $\times 10^{-8}$] | 3.4 $\times 10^{-5}$ | 2.5 $\times 10^{-10}$ | 6.1 $\times 10^{-4}$ | [3.1 $\times 10^{-9}$] | 1.7 $\times 10^{-7}$ | [5.7 $\times 10^{-13}$] | [1.0 $\times 10^{-10}$] |
| $M^\prime \to \pi^+ \mu^+ e^+$ | 5 $\times 10^{-10}$ | [6.9 $\times 10^{-8}$] | 5 $\times 10^{-5}$ | [1.7 $\times 10^{-10}$] | 7.3 $\times 10^{-4}$ | [2.2 $\times 10^{-9}$] | 1.3 $\times 10^{-6}$ | [4.6 $\times 10^{-13}$] | {8.3 $\times 10^{-11}$} |
| $M^+ \to \pi^+ \tau^+ e^+$ | - | - | - | - | - |
| $M^+ \to \pi^- \tau^+ e^+$ | - | - | - | - | - |
| $M^+ \to \pi^+ \tau^- \mu^+$ | - | - | - | - | - |
| $M^+ \to \pi^- \tau^- \mu^+$ | - | - | - | - | - |
| $M^+ \to K^- e^+ e^+$ | - | {4.0 $\times 10^{-12}$} | {5.3 $\times 10^{-11}$} | {1.6 $\times 10^{-14}$} | {3.1 $\times 10^{-12}$} |
| $M^+ \to K^- \mu^+ e^+$ | - | {9.8 $\times 10^{-12}$} | {1.3 $\times 10^{-10}$} | {4.2 $\times 10^{-14}$} | {7.8 $\times 10^{-12}$} |
| $M^+ \to K^- \mu^+ e^+$ | - | {8.0 $\times 10^{-12}$} | {1.0 $\times 10^{-10}$} | {3.4 $\times 10^{-14}$} | {6.3 $\times 10^{-12}$} |
| $M^+ \to K^- \mu^- \mu^+$ | - | {7.1 $\times 10^{-9}$} | {1.0 $\times 10^{-7}$} | {1.7 $\times 10^{-11}$} | {2.8 $\times 10^{-9}$} |
| $M^+ \to K^- \tau^- e^+$ | - | - | - | - | {9.2 $\times 10^{-14}$} | {1.6 $\times 10^{-11}$} |
| $M^+ \to K^- \tau^- e^+$ | - | - | - | - | {9.2 $\times 10^{-14}$} | {1.6 $\times 10^{-11}$} |
| $M^+ \to K^- \tau^- \mu^+$ | - | - | - | - | {1.4 $\times 10^{-10}$} | {2.4 $\times 10^{-8}$} |
| $M^+ \to K^- \tau^- \mu^+$ | - | - | - | - | {1.2 $\times 10^{-10}$} | {2.1 $\times 10^{-8}$} |

Let us survey some impacts of the $0\nu\beta\beta$ constraints. Presently the best experimental lower bound on the $0\nu\beta\beta$-decay half life is that obtained for $^{76}$Ge [2]:

$$T_{1/2}^{0\nu[76\text{Ge}]} \geq 1.9 \times 10^{25}\text{yrs.}$$

(44)

In Ref. [4] this bound was used to constrain the contribution of Majorana neutrinos of arbitrary mass. In a good approximation this constrain reads:

$$\sum_k \frac{|U_{e k}|^2 e^{i\alpha_k} m_{\nu k}}{m_{\nu k}^2 + q_0^2} \leq 5 \times 10^{-8} \text{GeV}^{-1}.$$  

(45)

with $q_0 = 105$ MeV. Here $\alpha_k$ denotes the Majorana CP-phase of the Majorana neutrino state $\nu_k$ of mass $m_{\nu k}$. For the light-heavy neutrino scenario one has:

$$\sum_{N=\text{heavy}} \frac{|U_{e N}|^2}{m_N} e^{i\alpha_{N}} + q_0^{-2} \sum_{i=\text{light}} |U_{ei}|^2 e^{i\alpha_{i}} m_{\nu i} \leq 5 \times 10^{-8} \text{GeV}^{-1}.$$  

(46)

where $m_N \gg q_0$ and $m_{\nu i} \ll q_0$. This upper limit, applied to each term separately, leads to a very stringent constraint on the heavy neutrino mass and mixing

$$\frac{|U_{e N}|^2}{m_N} \leq 5 \times 10^{-8} \text{GeV}^{-1}.$$  

(47)

One may use this limit to constraint LNV and LFV processes with $e^\pm$ in the final state. From Eq. (32), with the fact that $a_l(m_N) |U_{iN}|^2 \leq \Gamma_N$, we obtain a useful inequality

$$F_{l_1 l_2}(m_N) \equiv \left| \frac{|U_{i_1 N}|^2 |U_{i_2 N}|^2}{\Gamma_N} \right| \leq \left| \frac{|U_{i_1 N}|^2}{a_{l_2}} \right| l_1, l_2 = e, \mu, \tau$$

(48)

valid for all masses $m_N$. Combining Eqs. (48) and (47) we get

$$F_{e l} \leq \frac{(m_N/1\text{MeV})}{a_l} \cdot 5 \times 10^{-11} \text{ for } l = e, \mu, \tau.$$  

(49)
FIG. 2: Exclusion plots for the sterile neutrino mass $m_N$ and mixing matrix element $|U_{eN}|^2$ from various experimental searches (for a summary and references see Ref. [13]). The thick solid line is our exclusion curve, derived from the existing experimental upper bounds Ref. [16] on the rates of $\tau$ and $K,D,B$-meson semileptonic LNV and LFV decays, using the Monte Carlo sampling method. The inclusive approach is applied for the calculation of the total decay rate of a heavy Majorana neutrino $N$.

Using these limits in Eqs. [15]-[17] we can derive upper limits on some LNV and LFV decays imposed by non-observations of $0\nu\beta\beta$-decay [14]. Varying $m_N$ within the resonant regions [14] of the considered decays we determine the largest rates of these decays, which we present in Table II. These limits represent absolute upper bounds on given decays within the sterile neutrino extension of the SM, compatible with $0\nu\beta\beta$-constraints. Here an important assumption is implied. We assumed that CP is conserved in the neutrino sector. This case corresponds to no cancellation between different terms in Eq. (45), when the limit (47) is valid. If there are more than one heavy neutrino state $N_k$, with different CP violating phases $\alpha_N$, then different terms in the first sum of (46) may compensate each other, reducing the individual $0\nu\beta\beta$-constrain on each of them. Note that even with the presence of only one heavy neutrino there may happen this kind of compensation between the heavy $N$ and the three light neutrinos $\nu_i$, since they all coherently contribute to $0\nu\beta\beta$-decay. Thus we conclude that observation of some of the processes in Table II, at rates above the indicated limits from $0\nu\beta\beta$, shown in square brackets, may point to the following: (a) Presence of a sterile Majorana neutrino $N$ with the mass $m_N$ in the resonant range of these processes and that CP in the neutrino sector is violated; (b) Presence of a Dirac sterile neutrino in this mass range, if LFV decay is observed; (c) There is an exotic low mass scale new physics with light particles within the same resonant range of masses, but other than a sterile neutrino. So far most of the existing experimental limits shown in Table II are significantly weaker than the corresponding $0\nu\beta\beta$ limits, except for $K \to e\pi$, $\mu\pi$. We note, however, that the latter is not yet definite and requires more detailed analysis [13] of the corresponding experimental data. Uncertainties are related to the fact that for small mixing only part of the produced heavy sterile neutrinos decay in the detector and as a consequence the actual experimental bounds on $K \to e\pi$, $\mu\pi$ could be several orders of magnitude lower than indicated in Table II. Therefore more careful analysis of the existing experimental data and further searching for decays with $e^\pm$ in final states may provide an important insight into CP violation in the neutrino sector. The experimental bounds on branching ratios of the considered decays are expected to improve in the near future by an order of magnitude or more. This is, in particular, possible in LHCb experiment.
FIG. 3: Exclusion plots for the sterile neutrino mass $m_N$ and mixing matrix element $|U_{\mu N}|^2$ from various experimental searches (for a summary and references see Ref. [13]). The thick solid line is our exclusion curve derived from the existing experimental upper bounds Ref. [16] on the rates of $\tau$ and $K, D, B$-meson semileptonic LNV and LFV decays, using the Monte Carlo sampling method. The inclusive approach is applied for the calculation of the total decay rate of a heavy Majorana neutrino $N$.

B. Limits on as yet experimentally unconstrained decays.

Presently there are no experimental limits on semileptonic decays of mesons with $\tau$ in final states $M_1 \to \tau l_2 M_2$, and on any semileptonic decay of $B_c$ like $B_c \to l_1 l_2 M$. Below we derive theoretical upper limits for these decays within the considered sterile neutrino extension of the SM, on the basis of the fact that the maximal rates are reached in the resonant range for the sterile neutrino masses. Although such limits are linked to a specific scenario, they may have a more general meaning, considering the fact that the sterile neutrino extension of the SM is the only reasonable model having a light particle, sterile neutrino $N$, which can cause resonant enhancement of the processes in question. Other known beyond the SM scenarios are associated with larger mass scales and heavier particles outside the resonant regions of these processes and then their contribution is expected to be much smaller than that from the resonant mass sterile neutrino.

For derivation of the limits on as yet unconstrained decays we use the constraints (49) and

$$\mathcal{F}_{\tau l} \leq \frac{\Gamma_{\tau l}(l \to l_1 l_2 \pi)}{\pi G_F^2(z_{\tau})m_N}; \quad \mathcal{F}_{l_1l_2} \leq \frac{\Gamma_{l_1l_2}(M \to l_1l_2 \pi)}{\pi \kappa_{ij}(G_M^2(z_M)+G_M^2(z_M))m_N};$$

(50)

The first two constraints are derived from Eqs. (15)-(17) and are valid in the range $m_{\tau} + m_l \leq m_N \leq m_{\tau} - m_{\pi}$ for $\tau$-decays and for meson decays in the ranges indicated in Eqs. (14). The third constraint is a particular case of the inequality (48). The last one originates from precision electro-weak measurements [8, 20]. In order to derive upper limits on as yet unconstrained decays we scan their corresponding resonant ranges (14), selecting for each value of $m_N$ the largest value of their rates compatible with the constraints in Eqs. (49) and (50). In Eqs. (50) we use the experimental upper bounds $\Gamma_{\tau l}(l \to l_1 l_2 \pi)$, $\Gamma_{l_1l_2}(M \to l_1l_2 \pi)$ shown in Table II. For $|U_{\mu N}|^2$ in Eq. (50) we select the most stringent of the known (see Fig. 3) limits for a given value of $m_N$. The resulting upper limits on
FIG. 4: Exclusion curves for the sterile neutrino mass $m_N$ and mixing matrix elements $|U_{eN} U_{\tau N}|$ and $|U_{\mu N} U_{\tau N}|$, derived from the existing experimental upper bounds Ref. [16] on the rates of $\tau$ and $K, D, B$-meson semileptonic LNV and LFV decays using the Monte Carlo sampling method. The inclusive approach is applied for the calculation of the total decay rate of a heavy Majorana neutrino $N$.

the branching ratios for some LFV and LNV processes are listed in Table II in curly brackets. Note that the limits presented for the decays with $e^\pm$ in the final states imply CP conservation in the neutrino sector, discussed in the previous subsection. As seen from Table II only the experiments searching for the LNV and LFV decays of $D$ and $D_s$ are rather close to probe the rates of these processes not excluded by our analysis. Searches for other decays are far away from this perspective. As to the $K$-meson LFV and LNV decays, we refer to our comments at the end of the previous subsection.

VI. SUMMARY AND CONCLUSIONS

We have studied LFV and LNV decays of $\tau$ and $B, D, K$ mesons mediated by heavy Majorana neutrinos. We focussed on the dominant mechanism via resonant Majorana neutrino production.

On the basis of joint analysis of experimental limits on certain sets of LNV and LFV decays with intersecting resonant regions we extracted upper bounds on the heavy Majorana neutrino mass $m_N$ and mixing $|U_{eN}|, |U_{\mu N}|, |U_{eN} U_{\tau N}|, |U_{\mu N} U_{\tau N}|$ without additional assumptions on these parameters. Our limits derived from the above mentioned decays are shown in Figs. [24] As seen they are less stringent than the corresponding limits derived in the literature. To our best knowledge the latter typically involve various ad hoc assumptions on the relative size of the heavy neutrino mixing matrix elements. Therefore, it is natural that our limits, not referring to any kind of such assumptions, are less stringent. The mixing $U_{\tau N}$ is left unconstrained by the present experimental data, and requires searches for processes with $\tau \tau$ in the final state.

In our analysis we used the method of calculation of the total decay rate of heavy Majorana neutrino on the basis of an inclusive approach. Instead of a detailed calculation of the sum of all the possible decay channels with different hadrons in the final state, we approximated the hadronic final states by a $q\bar{q}$-pair, as suggested by Bloom-Gilman duality. This allows us to avoid large theoretical uncertainties in the decay constants of heavy mesons.
Special emphasis has been made on the stringent limits on the heavy Majorana neutrino from non-observation of $0\nu\beta\beta$-decay, from which we also derived indirect limits on various LNV and LFV processes. Our results, displayed in Fig. 2 and Table II, have been obtained assuming no CP-violation in the neutrino sector. As seen from Table II, our expectations for the prospects of observation of the decays with both $e^\pm e^\mp$ and $e^\pm e^\pm$ in the final states are pessimistic. One of the messages of the present paper is that despite of such a discouraging prediction these processes are worth searching. The point is that any observation of such decays above the limits set by the CP conserving $0\nu\beta\beta$ decay constraints would most likely point to the existence of heavy Majorana neutrinos in the resonant regions of these processes, and CP-violation in the neutrino sector, as explained in sec. V.

We applied our limits on heavy Majorana neutrino mass $m_N$ and its mixing $U_{lN}$ for a prediction of upper limits on the rates of as yet experimentally unconstrained LFV and LNV decays of $B$ and $D$ mesons. These limits are shown in Table II, and indicate that only the experiments searching for the LNV and LFV decays of $D$ and $D_s$ are rather close to probe the rates of these processes not excluded by our analysis. Searches for other decays are far away from this perspective.

Acknowledgements This work is supported by FONDECYT (Chile) under projects 1100582, 1100287 and Centro-Científico-Tecnológico de Valparaíso PBCT ACT-028.

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