ZERO-BIAS TRANSPORT ANOMALY IN METALLIC NANOBRIDGES

Magnetic field dependence and universal conductance fluctuations

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Abstract

We present data of transport measurements through a metallic nano-bridge exhibiting diffusive electron transport. A logarithmic temperature dependence and a zero-bias anomaly in the differential conductance are observed, independent of magnetic field. The data can be described by a single scaling law. The theory of electron-electron interaction in disordered systems, adapted to the case of finite-size systems in non-equilibrium, yields quantitative agreement with experiment. Measurements of universal conductance fluctuations support the assumptions of the theory about the electronic phase coherence.

It is well known that in bulk metals and semiconductors with diffusive transport the electron-electron interaction causes an anomaly in the electronic density of states (DOS) at the Fermi level. As explained by Aronov and Al’tshuler (A-A) in the 1980s [1, 2], this correction is induced by the long-range, retarded character of the dynamically screened Coulomb interaction in a diffusive system. It has been observed in thermodynamic equilibrium by tunneling spectroscopy on disordered metals [3, 4]. In the present article we address the question how this anomaly is modified in a nanoscopic sample or metal bridge whose size \( L \) is smaller than the dephasing length \( L_\phi \) (and all inelastic relaxation lengths), in particular when it is driven out of equilibrium by a finite bias voltage \( U \) applied between the ends of the bridge. Since in this situation energy relaxation is negligible for electrons traversing the bridge, no local
equilibrium is reached at any point in the bridge. Rather, the electron liquids penetrating from the left and right leads into the bridge remain at their respective electrochemical potentials $\mu_L$ and $\mu_R$. Consequently, the quasiparticle distribution function is a linear superposition of Fermi distributions in the left and right leads and displays a double-step form (see Fig. 1). This non-equilibrium distribution has been suggested theoretically [5] and has recently been observed experimentally by tunneling spectroscopy [6] (where in addition the steps were rounded due to interactions in long wires). It should be distinguished from the hot electron regime [7], where local thermalization in a current-carrying system occurs.

In this paper we report on the observation of a zero-bias conductance anomaly (ZBA) in metallic nanobridges [8] which, by their special design, allow to establish the well-defined non-equilibrium described above. The ZBA is characterized by a logarithmic scaling law, independent of an applied magnetic field. We show that the ZBA, including the scaling behavior, can be explained in detail [8] via a Landauer-Büttiker formula [9] as arising from the A-A correction to the electronic DOS of the bridge in non-equilibrium. The independence of the data of magnetic field allows us to distinguish the A-A anomaly from various other effects, like weak localization (WL) [10] and magnetic impurities, which might cause a ZBA as well. We also present measurements of universal conductance fluctuations (UCF) in one of the nanobridges. They confirm that the phase coherence extends over the entire nanobridge, which is the criterion for the Landauer-Büttiker approach to be applicable.
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Figure 3  Zero-bias conductance at $B = 0$ and $B = 8.5$ T. The magnetic field is applied perpendicular to the film. The constant background value of the conductance changes due to universal conductance fluctuations.

The data shown in the following are obtained from one sample of a Cu$_{82}$Au$_{18}$ nanobridge. We have investigated other Cu$_{100-x}$Au$_x$ and Cu bridges as well, with very similar results. Experimental details of the fabrication and the measurement are described in Ref. [8]. The bridge is $L = 80$ nm long, about 80 nm wide and has a thickness of $d = 10$ nm. It is placed in good metallic contact between two bulk Cu leads, which are about 70 times thicker than the bridge (see Fig. 2) and extend over a large area of about 1 mm$^2$ each. Hence the voltage applied to the sample drops only along the bridge, and the Joule heating power is reliably conducted away by the leads. The mean free path in the sample is about $\ell = 6.5$ nm, corresponding to a diffusion constant of $D = 34$ cm$^2$/s, i.e. it is comparable to the thickness $d$, but much shorter than the lateral length $L$. Therefore, the electronic density modes in the bridge obey the rules of two-dimensional (2D) diffusive motion. The 2D design also allows to distinguish the A-A conductance anomaly from a possible two-channel Kondo (TCK) effect induced by two-level systems [11], which has been put forward as the origin of ZBAs observed in ultrasmall point contacts [12]: In 3D both the A-A and the TCK anomalies show square-root power-law behavior; in 2D the A-A correction is logarithmic, while the TCK singularity, as a local effect, is independent of dimension. We observe a logarithmic temperature $T$ dependence of the zero-bias conductance $G(0, T) = G_0 + A \cdot \ln(T/1K)$ in a range of $T = 100$ mK to 2.1 K, with an amplitude of $A = 0.49 \, e^2/h$, as shown in Fig. 3 for vanishing magnetic field, $B = 0$ (lower curve). Below 100 mK, the data deviate somewhat from this logarithmic behavior, a fact that we
attribute to incomplete thermalization. It is seen that the amplitude $A$ is independent of magnetic field. When applying a finite bias voltage $U$, a small, voltage-symmetric anomaly in the conductance was found in the differential conductance $G(U, T)$ at low bias. Experimental raw data are shown in Fig. 4. When the temperature is lowered, the anomaly gets more pronounced. It can be characterized by a striking scaling property: When the zero-bias conductance $G(0, T)$ at the respective temperature is subtracted from $G(U, T)$, the data displayed as a function of $eU/k_BT$, collapse onto one single scaling curve in a wide region around zero bias. Moreover, after normalizing the conductance with the amplitude $A$ of the $T$ dependent linear response signal (see above), $(G(U, T) - G(0, T))/A$ is nearly identical for all investigated samples, where the mean free path $\ell$ and the thickness $d$ were varied within a factor of two. Hence, all the conductance data $G(U, T)$ can be described by a single scaling law,

$$G(U, T) = G_0 + A \cdot \ln(T/1K) + A \cdot \Phi(eU/k_BT).$$

(1.1)

The scaling function $\Phi(x)$ obtained in this way is displayed in Fig. 5, where the asymptotic behavior $\Phi(x) = \ln x$ for $x \gg 1$ may be extracted.

When a perpendicular magnetic field $B = 8.5$ T is applied, the scaling behavior persists, with the amplitude $A$ (Fig. 3) and, moreover, the scaling function $\Phi(x)$ (Fig. 5) remaining unchanged. This is clear evidence that WL is not observed in our measurements, as might already have been expected from the shortness of our samples. Logarithmic behavior may also be caused by magnetic impurities or by non-magnetic TCK defects [11] above their respective Kondo temperatures $T_K$. Since
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Figure 5  Scaling plot of the conductance at magnetic field $B = 0$ (top) and at $B = 8.5$ T (bottom). Lines represent experimental data for various fixed temperatures as in Fig. 4, where measurements both at positive and at negative bias are included. In both plots, circles represent theoretical calculations of the leading A-A conductance correction following from Eq. (1.4), which is independent of magnetic field $B$. The inset shows raw data of the ZBA in a magnetic field of $B = 8.5$ T for temperatures $T = 100$ mK and $T = 2$ K, where the position of the Zeeman energy (see text) is marked by arrows. The $B$ independence of the experimental data is clearly seen.
an applied field of $B = 8.5$ T does not modify the ZBA, any magnetic impurities present in the sample must have $T_K \gg 8.5$ K. However, the logarithmic behavior of the zero-bias conductance observed down to the lowest $T$ (Fig. 3) puts an upper bound to the Kondo temperature, $T_K < 0.1$ K, thus ruling out magnetic impurities as the origin of the ZBA. In the TCK scenario, from point-contact spectroscopy on Cu one expects $T_K \approx 5$ to 10 K [12, 13]. Hence, it is unlikely that the ZBA is due to TCK defects for the same reason as in the magnetic case. The assumption that there is no sizable number of TCK defects present in our Cu$_{82}$Au$_{18}$ samples is consistent with the fact that in Cu point contacts investigated previously the TCK signal completely disappeared upon doping with 1% Au or more [12]. Because of the good metallic contact between bridge and leads, charging effects at the interfaces [15] may be regarded as negligible in our devices.

In order to understand the logarithmic ZBA theoretically, it is important to note that the length $L$ of our disordered nanobridges is small compared to the dephasing length $L_\varphi \sim \sqrt{\hbar D/k_B T}$ and all inelastic relaxation lengths, as will be verified below. Hence, the electrons occupy the exact single-particle eigenstates of the disordered bridge while traversing the system, i.e. the DC transport is ballistic (i.e. zero-dimensional), since it involves only zero-frequency modes, even when a finite bias voltage is applied. In this situation the Landauer-Büttiker approach is applicable, where the conductance is expressed in terms of the exact eigenstates or channels of the transmitting region, and which has been generalized to interacting systems by Meir and Wingreen [9]. The current through the bridge at bias $U$ thus reads,

$$I(U) = \frac{e}{h} \Gamma \int \left[ f^0(E - \frac{eU}{2}) - f^0(E + \frac{eU}{2}) \right] N(E, U) \, dE,$$

where $f^0(E) = 1/(e^{E/k_B T} + 1)$ is the Fermi function and, for simplicity, the effective lead-to-bridge coupling $\Gamma$ is taken to be energy independent and symmetrical for left and right leads (the more general case is treated in Ref. [8]). The energy $E$ is measured with respect to $\mu = (\mu_R + \mu_L)/2$. Since the bridge is phase coherent, the quasiparticle distribution function in the bridge is uniform in space and has the double-step form [5] (Fig. 1),

$$f(E) = \frac{1}{2} \left[ f^0\left(E - \frac{eU}{2}\right) + f^0\left(E + \frac{eU}{2}\right) \right].$$

According to Eq. (1.2) the current is expressed in terms of the DOS $N(E, U)$ in the bridge, which in the interacting case may be strongly affected by the non-equilibrium distribution. In fact, diffusive density modes exist at finite (2D) wave numbers $q$, $2\pi/L < q < 2\pi/\ell$, and at
frequencies \( \Omega \) up to the elastic scattering rate \( 1/\tau = v_F/\ell \approx 0.2 \text{ fs}^{-1} \), although the DC transport is ballistic. These diffusion modes couple to the electronic DOS via the dynamically screened Coulomb interaction and thus give a singular correction to the conductance, as described by A-A in equilibrium [1]. The corresponding DOS corrections are shown diagrammatically to leading order in the effective electron-electron interaction in Fig. 6. It seen that in the exchange diagram the dynamically screened Coulomb interaction \( \tilde{v}_q(\Omega) \) enters, while the Hartree diagram contains the statically screened Coulomb interaction \( \tilde{v}_q(0) \) because of energy conservation at the impurity vertices. In an infinite system, the dynamically screened Coulomb interaction, combined with the diffusive vertex corrections (shaded triangles in Fig. 6a)), exhibits a hydrodynamic divergence \( (q \to 0, \Omega \to 0) \), while the statically screened one remains finite. Therefore, for a long-range bare interaction like the Coulomb interaction, the exchange contribution is always more strongly singular than the Hartree term [2].

In an infinite 2D film, the exchange term has logarithmic divergences both in the integral over the frequency \( \Omega \) and over the wave number \( q \) transferred by interaction, leading to the well-known DOS correction \( \delta N(E) \propto -\ln(E/h\tau) \ln(E/h\kappa^2D) \), where \( \kappa \) is the inverse 2D screening length [1]. In our finite-size 2D bridge the divergence in \( q \) is cut off by the inverse system size both in the Hartree and in the exchange contribution. It is transformed into a constant term \( \ln(\max(d, \ell) / \ell) \) which stems from the crossover to 3D behavior at short distances [8]. However, in the exchange term the divergence of the \( \Omega \) integral persists and dominates the Hartree term even in a finite system. Consequently, near the Fermi step(s), i.e. for energies \( |E| \approx (2\pi)^2E_{Th} \), with \( E_{Th} \) the Thouless energy, simple log behavior instead of \( \log^2 \) behavior remains [8]. The corresponding DOS correction may be cast into the scaling form

\[
\delta N(y, T) = \frac{\ln(\max(d/\ell, 1))}{\pi^2E_{Th}} \left[ \ln(T\tau) + \int du \left( -\frac{d\tilde{f}(u-y)}{du} \right) \ln|u| \right], \quad (1.4)
\]

where \( \tilde{f}(u) = f(\hbar\Omega/k_B T) \) is the (non-equilibrium) distribution function in terms of the dimensionless energy, and \( y = E/k_B T \). Eq. (1.4) dis-
plays two logarithmic singularities corresponding to the Fermi steps at $y = 0$ and $y = -eU/k_B T$. It is characteristic for logarithmic behavior that the prefactor of the term depending on the dimensionless energy $y$ is independent of $T$ (in contrast to, e.g., power-law scaling) and is equal to the amplitude of the $T$-dependent term at $y = 0$. Obviously, the universality of $\delta N(y, T)$ is preserved when the differential conductance correction $\Phi(eU/k_B T) = [G(U, T) - G(0, T)]/A$ is calculated using Eqs. (1.2), (1.4) and (1.3). The resulting scaling curve is shown in Fig. 5 and agrees quantitatively with the experimental data, both without and with applied magnetic field. The magnetic field independence of the experimental scaling curves as well as of the amplitude $A$ (Fig. 3) is expected from the A-A anomaly [2, 16]: The dominating exchange contribution (Fig. 6a)) is independent of magnetic field since here diffusion modes enter only through density vertices (shaded triangles), which conserve spin. Zeeman splitting of the diffusion modes occurs only in the particle-hole vertices (shaded squares in Fig. 6b)) with opposite particle and hole spins appearing in the Hartree term, which is negligible (see above). The position of the Zeeman splitting energy $\hbar \omega_s = g\mu_B B$ is marked in the inset of Fig. 5, where the experimental data show no structure, as expected.

Direct insight in the coherence properties of the samples may be obtained by investigating the universal conductance fluctuations (UCF). The magnetoconductance of our sample at $T = 100$ mK is shown in Fig. 7. The conductance fluctuations are are reproducible and symmetric with respect to reversal of the magnetic field, and are, thus, identified as UCF. The statistics of the UCF can be analyzed in a standard way
by means of the autocorrelation function

\[
C(\Delta B) = \frac{1}{2B_0} \int_{-B_0}^{B_0} \delta G(B') \delta G(B' + \Delta B) dB'
\] (1.5)

with \( \delta G(B) = G(B) - \langle G \rangle \) and \( B_0 = 8.5 \) T. The rms amplitude \( \delta G_{\text{rms}} = \sqrt{C(0)} \) is \( \delta G \approx 0.22 \, e^2/h \) for the data shown in Fig. 7. From the HWHM of the autocorrelation function we obtain a correlation field of \( B_c = 640 \) mT. Another, similar sample of the same size yields \( \delta G \approx 0.31 \, e^2/h \) and \( B_c = 1300 \) mT. The scatter of these data is not surprising: The range of experimentally applied fields, \( 0 \leq B \leq 8.5 \) T is not very large compared to the correlation field \( B_c \), so that no complete averaging over the microscopic phase configurations is obtained, and fluctuations in \( B_c \) are expected to be sizable. The amplitude of the effect is compatible with other experimental data on diffusive metal bridges and theoretical predictions [18]. On the other hand, the correlation field \( B_c \) should be inversely proportional to the phase coherent area \( A_\Phi \): \( B_c = c \cdot \Phi_o / A_\Phi \) [19], \( \Phi_o \) being the flux quantum and \( c \) a constant of \( O(1) \). Indeed, for our sample, a coherence field of \( B_c = 610 \) mT (\( B_c = 500 \) mT for the second sample) results when \( A_\Phi \) is taken as the bridge area, and the reservoir-like leads are assumed not to contribute to the phase coherent area. This agrees well with the value of \( B_c \) obtained from the UCF analysis and, therefore, supports our analysis in terms of the Landauer-Büttiker approach: The bridge is phase coherent over its whole spatial extent and the bridge eigenstates are well separated from the leads.

In conclusion, we have shown measurements on a nanoscale, fully phase-coherent, metallic nanobridge placed between two reservoir-like leads. The particular design allows us to establish a well-defined electronic non-equilibrium when a finite bias voltage is applied, corresponding to a double-step in the electronic distribution function. We observed logarithmic \( T \) dependence of the zero-bias conduction and logarithmic \( U \) dependence of the differential conductance, which can be combined into a single scaling law. The theory of electron-electron interaction in diffusive systems was adapted to the constrained bridge geometry, taking the non-equilibrium situation fully into account. Instead of a single anomaly in the density of states at the Fermi level, well known in equilibrium, two anomalies evolve at the two Fermi steps. The theoretical scaling function \( \Phi(eU/k_B T) \) coincides quantitatively with the experimental data without adjustable parameter. We also presented the magnetic field dependence of the data, showing universal conductance fluctuations, but no change in the zero-bias anomaly, in full agreement with the theoretical description.
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