Energy gaps in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ cuprate superconductors

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The relationship between the cuprate pseudogap ($\Delta_p$) and superconducting gap ($\Delta_s$) remains an unsolved mystery. Here, we present a temperature- and doping-dependent tunneling study of submicron Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ intrinsic Josephson junctions, which provides a clear evidence that $\Delta_s$ closes at a temperature $T_{c0}$ well above the superconducting transition temperature $T_c$ but far below the pseudogap opening temperature $T^*$. We show that the superconducting pairing first occurs predominantly on a limited Fermi surface near the node below $T_{c0}$, accompanied by a Fermi arc due to the lifetime effects of quasiparticles and Cooper pairs. The arc length has a linear temperature dependence, and as temperature decreases below $T_c$ it reduces to zero while pairing spreads to the antinodal region of the pseudogap leading to a $d$-wave superconducting gap on the entire Fermi surface at lower temperatures.

The properties of the pseudogap and its relation to the superconducting gap are among the central issues in the search for the cuprate pairing mechanism. A number of spectroscopic studies such as scanning tunneling microscopy (STM) and angle-resolved photoemission spectroscopy (ARPES) have been reported$^{1-14}$. Some experiments indicate that the pseudogap may arise fully from precursor superconductivity (single-gap picture)$^{1-5}$, while others suggest an origin that is unrelated to superconductivity (two-gap picture)$^{6-14}$. In the latter case, uncertainty exists as precursor pairing in certain temperature range above the superconducting transition temperature $T_c$ is reported in some experiments$^{8,13,14}$, which contrast with other experiments in which the superconducting gap $\Delta_s$ is found to close at $T_c$ $^{6,9,10,12}$. In this paper, we address the issue using the temperature- and doping-dependent tunneling spectroscopy of submicron Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ intrinsic Josephson junctions.

For conventional Bardeen-Cooper-Schrieffer (BCS) superconductors, Giaever’s planar-type tunnel junctions$^{15}$ provided decisive measurements of the superconducting gap, the electronic density of states (DOS), the quasiparticle scattering rate, and the effective spectrum of phonons that mediate pairing$^{16-18}$. Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ intrinsic Josephson junctions$^{19}$ are the similar planar-type junctions with the best quality one may have for cuprate superconductors. As is shown in the inset of Fig. 1, these junctions are formed within the crystal with CuO$_2$ double-layers as superconducting electrodes and BiO/SrO interlayers as the tunnel barrier. Such superconductor-insulator-superconductor (SIS) junctions avoid all kinds of extrinsic uncertainties during experiment and can offer stable and reproducible temperature-dependent measurements. Earlier spectroscopic studies using these junctions suffered from sample’s self-heating that severely distorts the tunneling spectra and many efforts were made to solve the problem$^{20-27}$. One effort involved optimizing the surface-layer contact and reducing the junction size well below 1 $\mu$m, which are shown to suppress heating sufficiently in the case of near optimally doped samples$^{23-26}$. The data presented below were based on these works and extended to samples with different doping strength.

The present work demonstrates that the superconducting gap $\Delta_s$ closes at a temperature $T_{c0}$ well above $T_c$, but far below the pseudogap opening temperature $T^*$, which supports a two-gap picture with superconducting pairing persisting up to $T_{c0}$. The pairing is found to occur first on a limited Fermi surface near the node below $T_{c0}$, accompanied by a Fermi arc due to finite quasiparticle scattering rate and pair decay rate. The arc length has a linear temperature dependence, and as temperature decreases below $T_c$ it reduces to zero while pairing spreads to the antinodal region of the pseudogap leading to a $d$-wave superconducting gap on the entire Fermi surface at lower temperatures.

Results
Experimental spectra. In Fig. 1, we show the tunneling conductance $\sigma(V, T)$ at typical temperatures for four samples from underdoped (UD) to overdoped (OD) with $T_c = 71, 80, 89$ and 79 K, respectively (see Methods and
Temperature dependence of the superconducting gap. A key feature one expects for superconductors is that \( \Delta \) follows the BCS-like gap equation and closes at a temperature, possibly higher than \( T_c \), where Cooper pairs vanish. To clarify the situation, we fitted our experimental spectra with a DOS that is widely used in tunneling experiment for both BCS superconductors\(^{8,30}\) and cuprates\(^{1,26,31}\):

\[
N_s(\theta, \omega) = \text{Re} \left[ \frac{\omega + i\gamma_s}{\sqrt{\left(\omega + i\gamma_s\right)^2 - \Delta^2 \cos^2(2\theta)}} \right]
\]

where a \( d \)-wave gap is considered and the subscript \( s \) denotes the superconducting part. \( \theta \) and \( \gamma_s \) are the angle of in-plane momentum measured from \( (\pi,0) \) (see Fig. 2e) and the parameter characterizing the lifetime effects, respectively. The DOS was first proposed by Dynes \textit{et al.}\(^{29}\) and recently shown\(^{26}\) to be related to a phenomenological self-energy developed for the pseudogap discussion in which \( \Delta \) extends to the precursor pairing regime above \( T_c \).\(^{26}\) Taking the UD89K data as an example, we replotted half the peak position below \( T_c \) in Fig. 2f, in which lines are the BCS \( d \)-wave gap that closes at \( T^* \) (dashed), \( T_c \) (dotted) and \( T_{c0} = 140 \) K to be discussed below (solid). In the single-gap picture with pairing starting at \( T^* \), \( \Delta \) should vary along the dashed line near and above \( T_c \) if the lifetime effects are taken into account. \( \Delta \) obtained from fit to the normalized spectra \( \sigma(V, T)/\sigma(V, T^*) \) using \( N_s(\theta, \omega) \) over the whole Fermi surface\(^{26}\) is shown in Fig. 2f as open squares (\( \gamma_s \) not shown for clarity). It is seen that the result deviates significantly from the dashed line, which means that the single-gap picture does not lead to an appropriate description.

In an STM experiment on Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_8\)+\(\delta\) superconductors, Boyer \textit{et al.}\(^4\) found that when seemingly irregular experimental spectra are normalized to the one slightly above \( T_c \), they reveal a homogenous superconducting gap that closes at \( T_c \). This treatment eliminates the effect of the pseudogap that already exists above \( T_c \). Such treatment was also applied for Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_8\)+\(\delta\) SIS-type junctions\(^{26}\) and BCS SIS-type junctions where effects unrelated to superconductivity are successfully removed\(^{26}\). In the related two-gap picture, one may view the two phases as coexisting and being anticorrelated on the Fermi surface with different spectral weights, and there is a boundary \( \theta_p \) below and above which they dominate respectively\(^{11}\). Fig. 2e shows a simple cut-off presentation as used in STM experiments\(^{26}\). In the present work, we fitted the normalized spectra \( \sigma(V, T)/\sigma(V, T^*) \), considering consistently \( N_s(\theta, \omega) \) for \( \theta > \theta_p \) only on the Fermi surface so the pseudogap-dominant region was excluded in the tunneling current calculation (see Methods). The resulting \( \Delta \), taking \( \theta_p = 12^\circ \), a value close to the STM observation\(^2\), is shown as open circles in Fig. 2f for the UD89K sample. It can be seen that the fit is again unsatisfactory when compared to the BCS curve (dotted line).

A satisfactory fit was nevertheless obtained when it was performed with respect to \( \sigma(V, T)/\sigma(V, T_{c0}) \) where \( T_c < T_{c0} < T^* \), for which pair formation starting at \( T_{c0} \) should be assumed. Precursor pairing above \( T_c \) has been suggested previously in some experiments\(^{4,8,5,6,7,12,13}\). We note that the half peak position in Fig. 2c (squares) shows an obvious turning near 140 K. In Fig. 2f (also in c), \( \Delta \) from the fit considering \( T_{c0} = 140 \) K and excluding the pseudogap region of \( \theta < \theta_p = 12^\circ \) is shown as solid up-triangles. We see that \( \Delta \) follows nicely the BCS
Temperature range from 4.2 K to 100 K, respectively. For the more underdoped UD89K sample, 
the lifetime effects are opened if we look with decreasing temperature) at Tc
(Tc = 0 as down-triangles in Fig. 2c, which shows a clear tendency of
approaching the BCS solid line below Tc and from Tc to Tc0, half the peak separation of Δ(kF, ω) is shown (green) with a Fermi arc as observed in ARPES experiment. (f) Δs of the UD89K sample obtained from fits to the normalized σ(V, T)/σ(V, Tc*) (open squares) and σ(V, T)/σ(V, Tc) (circles) tunneling spectra. Both deviate considerably from the BCS gap closing at respective temperatures (dashed and dotted lines), as compared to the result from fit to σ(V, T)/σ(V, Tc0) (up-triangles, also in c) that shows a good agreement with the BCS prediction (solid line) above Tc, (see text for more details).

Figure 2 | Measured and fitted quantities showing that the superconducting gap Δs closes at Tc0 (Tc < Tc0 < Tc*), above which a pseudogap already exists. (a–d) squares: half the conductance peak position in meV; up-triangles: superconducting gap Δs (solid) and lifetime parameter γs (open) obtained by fitting the normalized spectra σ(V, T)/σ(V, Tc0) using Ns(0, ω) excluding the pseudogap-dominant region from 0 to ωp on the Fermi surface. Above Tc (indicated by arrows), Δs is seen to follow nicely the BCS d-wave gap (lines) closing at Tc0 = 150, 130, 140, 100 K, respectively. In (c) Δs obtained from fit considering the entire Fermi surface is plotted as down-triangles for comparison. (e) symbols and schematic gap profiles on the Fermi surface (yellow) in the temperature ranges of well below Tc (blue) and from Tc0 to Tc* (red). From near Tc to Tc0, half the peak separation of Δ(kF, ω) is shown (green) with a Fermi arc as observed in ARPES experiment. (f) Δs of the UD89K sample obtained from fits to the normalized σ(V, T)/σ(V, Tc*) (open squares) and σ(V, T)/σ(V, Tc) (circles) tunneling spectra. Both deviate considerably from the BCS gap closing at respective temperatures (dashed and dotted lines), as compared to the result from fit to σ(V, T)/σ(V, Tc0) (up-triangles, also in c) that shows a good agreement with the BCS prediction (solid line) above Tc, (see text for more details).

Parameters of the superconducting and pseudogap phases. We emphasize that our fit based on σ(V, T)/σ(V, Tc0) assumes a temperature-independent pseudogap. As is discussed by Boyer et al. this should be a reasonable approximation. In many experiments such as STM1 the pseudogap peak position is found nearly temperature independent and it disappears by "filling-up" as temperature approaches Tc*. If we take the half peak position at Tc0 in Fig. 2 a–d to characterize the pseudogap Δp, it shows a distinct doping dependence as that of Δs. In Fig. 4a, we plot Tc*, Tc0 and Tc against the doping level p, while Δs and Δp are shown in Fig. 4b and the resulting 2Δs/kTc0 in the inset. In Fig. 4b, Δs is seen to have a fast increase as p reduces to the more underdoped level, as observed in ARPES experiments. On the overdoped side, it continues to decrease to a value below Δs.

Fermi arcs derived from lifetime parameters. The lifetime effects play an important role in the precursor pairing regime from around Tc up to Tc0 due to increasing γs. One of the consequences is the appearance of a Fermi arc near the node with θ > θp (see Fig. 2e), which is defined through the peak separation of the spectral function.
A(\(k, \omega\)) around Fermi surface in ARPES experiments\(^{36,37}\). The above-mentioned self-energy model\(^{32}\), from which \(N_s(\theta, \omega)\) can be derived\(^{26}\), contains three parameters: the quasiparticle scattering rate \(\Gamma\), the pair decay rate \(\Gamma_\Delta\), and \(\Delta_s\), with \(\gamma_s = (\Gamma + \Gamma_\Delta)/2\). Assuming a linear temperature dependence of \(\Gamma\), we inferred both \(\Gamma\) and \(\Gamma_\Delta\) from the fitted parameters \(\gamma_s\) in Fig. 2a-d. With known \(\Delta_s\), \(\Gamma\) and \(\Gamma_\Delta\), \(A(\mathbf{k}, \omega)\) was determined and the arc length \(l_{\text{arc}}\) was calculated\(^{36,37}\) (see Methods). In Fig. 5, we show the calculated \(l_{\text{arc}}\) versus temperature for the four samples. The results display an approximate linear temperature dependence, which is quite general as discussed in the ARPES data analysis using the same self-energy in a simplified situation of \(\Gamma = \Gamma_\Delta\).
**Discussion**

We have shown that for the four Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ crystals with different doping levels the superconducting gap $\Delta_s$ closes at a temperature $T_\text{cs}$ well above the superconducting transition temperature $T_c$, but far below the pseudogap opening temperature $T^*$, thus an extensive precursor pairing regime between $T_c$ and $T_\text{cs}$ is demonstrated. In the Methods section, we present an alternative fitting procedure considering both the superconducting part ($\Lambda_\alpha$, $\gamma_p$) and the pseudogap part ($\Lambda_\rho$, $\gamma_p$), which leads to the same conclusion as using the conventional approach of normalizing out the pseudogap contribution described above. It is shown that $\Delta_s$ is nearly constant from slightly below $T_c$ up to $T^*$ while $\gamma_p$ experiences a continuous increase, which is consistent with the filling-up character of the pseudogap as temperature approaches $T^*$ from below.

So far the STM and ARPES results supporting the two-gap scenario alone for the Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ materials are still diverse and controversial. Some results suggest that below $T_c$ the superconducting gap would coexist with the pseudogap at the antinode\(^{11,12}\) while others indicate that they reside at the nodal and antinodal regions separately\(^{11,12}\). Above $T_{cs}$, precursor pairing is demonstrated in some experiments\(^{1,11}\) whereas a superconducting gap closing at $T_c$ is also observed\(^{11}\). The present tunneling results clearly support the precursor pairing view in the temperature range from $T_c$ to $T_{cs}$, which is similar to the results in Refs. 8 and 13. In this temperature range, the superconducting gap and the pseudogap locate predominantly at the node and the antinode, respectively. We note that both the results of $\Lambda_s$ presented as solid up- and down-triangles in Fig. 2c are obtained by fitting to the spectra that are normalized to the one at $T_{cs}$. In this case, the pseudogap contribution is not considered in the fits but it still exists. Therefore the result that the superconducting gap spreads into the antinodal region below $T_{cs}$ means that the two components coexist at the antinode. Since all the data of $\Lambda_s$ in Fig. 2a–d (solid up- and down-triangles) show similar upturns as temperature decreases below $T_{cs}$, we believe the coexisting nature to be true for all samples.

On the other hand, for the UD71K sample we see from Fig. 2a (squares) that the pseudogap spectral peak quickly diminishes and switches to the superconducting peak below $T_{cs}$. This may indicate that the spectral weight of the pseudogap becomes small compared to that of the superconducting gap below $T_c$ for this sample which is still not deep enough into underdoping, or the pseudogap structure is obscured by the growth of the dip structure in the tunneling spectra. For higher doping samples, uncertainty arises from the fact that the superconducting gap and pseudogap scales becomes similar (see Fig. 2).

Fermi arcs in the ARPES experiments often show a relatively large size just above $T_{cs}$ which collapse as temperature decreases below $T_c$. In the two-gap scenario, the collapse results from the opening of the superconducting gap on the arc at $T_c$. Our results are similar to those in a sense that $T_{cs}$ is in the place of $T_c$ and the arc region is defined from $\theta_p$ to $\pi/2 - \theta_p$ in Fig. 2e in the pseudogap state. As mentioned above, the lifetime effects in the superconducting state can be successfully used to explain the linear temperature dependence of the arc length $l_{\text{arc}}$. It is interesting to note that in the single-gap picture $l_{\text{arc}}$ will exhibit a faster rise as temperature increases across $T_c$ and therefore has a larger value compared to those in Fig. 5 just above $T_c$, which is consistent with the results observed in ARPES experiments\(^{15,56}\). On the other hand, our present results, including the development of the superconducting gap at $T_{cs}$ > $T_c$ on an arc spanned in the pseudogap state and the temperature dependence of the arc resulting from the lifetime effects in the superconducting state, as depicted in Fig. 5, bear a close resemblance to the STM observations\(^{20}\). The differences and similarities in these ARPES and tunneling experiments remain to be explained in the future.

Open questions that are of further interest are the nature of the pseudogap and whether the superconducting and pseudogap phases are formed from the same underlying physics. Recent experiments suggest that the pseudogap phase can result from various density-wave and other states, which may compete\(^{11}\) or have an intimate relationship with the superconducting state\(^{44}\). Our results indicate that the pseudogap $\Delta_r$ has a distinct temperature and doping dependence compared to $\Delta_s$, which may not be in favor of the view that they have a common microscopic origin. In the classical BCS superconductors, the strong Coulomb and phonon interactions between electrons in the normal state lead to an average correlation energy in the order of eV, which is much larger than the pair-binding energy of meV. The strong interactions are later removed in Landau’s Fermi-liquid theory with quasiparticles replacing the bare electrons. Consideration of the interaction neglected in Landau’s approximation leads to the coupling between quasiparticles and formation of Cooper pairs\(^{17}\). In the present case of cuprate superconductors, however, the situation is different and is more complicated as we see that the pseudogap size can be larger, comparable, and smaller than the superconducting gap when doping increases.

**Methods**

**Experimental details.** Mesa-type intrinsic Josephson junctions (JJ)s\(^{3,5,37}\) were used in this work with their geometry shown schematically in Fig. S1. Details of the sample fabrication have been described elsewhere\(^{20,22}\). To reduce samples self-heating, a notorious problem in JJs studies, we took special care to reduce the contact resistance between Au films and Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ crystals which result in the surface layers with good properties\(^{3,36}\). In addition, mesa sizes were reduced well below 1 $\mu$m as it was demonstrated that heating can be largely neglected in this case\(^{3,5}\). Other methods to reduce heating include using JJJs made of HgBr$_2$ intercalated Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ crystals\(^{23,22}\) and adopting short-pulse measurements\(^{31}\), which are discussed extensively recently\(^{22}\). These studies demonstrate tunneling spectra with moderate sharpness of the conductance peak and clear presence of the dip feature after the reduction of heating, as achieved in the present experiment shown in Fig. 1. (See Supplementary Information for further details.)

**Spectra fit separating the pseudogap contribution.** The $I$-$V$ characteristics of a superconductor-insulator-superconductor (SIS) junction can be calculated from:\(^{56}\)

$$I(V) = \frac{1}{2R_0} \int \frac{\pi}{2\alpha} n(\alpha)(\alpha + eV)|f(\alpha) - f(\alpha + eV)|d\alpha$$

where $R_0$ is junction’s normal-state resistance, $n(\alpha)$ is the DOS of two identical $S$-electrodes and $f(\alpha)$ is the Fermi function. Our results were obtained by fitting the normalized experimental spectra using $\sigma = dI/dV$ from equation (1) with the following normalized DOS for $n(\alpha)$:

$$N_{JJ}(\alpha) = \frac{\pi}{4C} N_{JJ}(\theta,0) \cos^2(2\theta)d\theta,$$

where $\cos^2(2\theta)$ comes from the directional tunneling matrix element which is to improve the description for the intrinsic tunneling process within Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ crystals\(^{35,36}\). In equation (2), integration is performed from $\theta_0$ to $\pi/4$ with $\theta_0 \geq 0$ due to symmetry. If the superconducting phase is considered on the entire Fermi surface, we have $\theta_0 = 0$. As discussed in the paper, our central results in Fig. 2a–d were obtained from fitting $\sigma(V, T)/\sigma(V, T_{cs})$ with a nonzero $\theta_0$ to exclude the pseudogap-dominant region on the Fermi surface.

The $\theta_0$ parameters obtained from fits to the four samples used in this work are listed in Table S1. For samples from UD89K to OD79K, $\theta_0$ decreases from 12° to 10°. This trend is consistent with the STM observations\(^{11}\). However, for UD80K and UD71K samples, $\theta_0$ is 10° and 11°. The slight inconsistency could be caused by the fact that the oxygen doped UD89K and OD79K samples might have altered crystalline arrangement resulting in reduced pseudogap expansion in momentum space. The satisfactoriness of our fit using these parameters can be seen in Fig. S4.

Using normalized spectra to get rid of the effects unrelated to superconductivity is a common practice in tunneling experiments for both BCS superconductors\(^{22,23}\) and cuprates\(^{22,51}\) in both SIN (N being a normal metal)\(^{22,51}\) and SIS\(^{35,36}\) type tunnel junctions. For example, McMillan and Rabe studied the SIS type Pb junctions\(^{23}\). By normalizing the data below $T_{cs}$ to the one above $T_{cs}$, additional structures in the measured spectra resulting from tunnel barrier phonons are successfully removed. The phonon spectra extracted from the data are exactly the same as those obtained from the SIN type Pb junctions. Below we further justify this approach for the present experiment by considering both the superconducting and pseudogap contributions in the fitting procedure.

According to the two-gap scenario, from $T_{cs}$ up to $T^*$ there is only the pseudogap phase located predominantly near the antinode with $0 < \theta_p$ and one has $N_s(\theta, \alpha) = 1$ for $0 > \theta_p$. Below $T_{cs}$ down to at least $T_c$ the superconducting and pseudogap phases
exist predominantly above and below $\theta_c$, respectively (see Fig. 2c). If we use the same form of $d$-wave DOS to model the pseudogap phase that may come from various density-wave states, now denoted by $N_p(\theta, \omega)$ with two parameters $\Delta_p$ and $\gamma_p$, we can write the following DOS for $n(\omega)$ in equation (1) for $T > T_{c0}$

$$N_p(\omega) = \frac{8}{\pi} \int_{0}^{\pi} N_p(\theta, \omega) \cos^2(2\theta) d\theta + \frac{8}{\pi} \int_{\theta_c}^{\pi} \cos(2\theta) d\theta \equiv D_p(\omega) + C_p,$$  

(3)

where $C_p$ is a constant from the ungapped part on the Fermi surface. For $T < T_{c0}$ we have

$$N_p(\omega) = D_p(\omega) + \frac{8}{\pi} \int_{0}^{\pi} N_p(\theta, \omega) \cos^2(2\theta) d\theta \equiv D_p(\omega) + D_{p1}(\omega).$$

(4)

The $I(V)$ curve can be calculated above $T_{c0}$ from

$$I_0(V) = \frac{1}{eR_N} \left[ D_p(\omega) + C_p \right] \left[ D_p(\omega + eV) + C_p \right] \frac{f(\omega) - f(\omega + eV)}{\omega},$$

and below $T_{c0}$ from

$$I_c(V) = \frac{1}{eR_N} \left[ D_p(\omega) + D_{p1}(\omega) \right] \left[ D_p(\omega + eV) + D_{p1}(\omega + eV) \right] \frac{f(\omega) - f(\omega + eV)}{\omega}.$$  

(6)

In Fig. 2S, we show the results from fits using $I_0(V)$ and $I_c(V)$ to the normalized experimental spectral function $\sigma(V, \Gamma)$ (see Fig. 2a,b) (note that $\Gamma$ is used as normalization temperature instead of $T_{c0}$), taking also the UD989K Ill[2] as the example. Up-triangles are replotted as $\Delta_p$ and $\gamma_p$ from Fig. 2c and 2f. Above $T_{c0}$ only the pseudogap is concerned, the parameters $\Delta_p$ and $\gamma_p$ are thus directly determined using $I_0(V)$, which are shown as down-triangles above $T_{c0}$. The down-triangles shown in the figure below $T_{c0}$ are obtained using $I_0(V)$ and the replotted $\Delta_p$ and $\gamma_p$ parameters. In other words, if these $\Delta_p$ and $\gamma_p$, are used, the two fitting approaches would produce the same $\Delta_p$ and $\gamma_p$. For comparison, squares in Fig. 2S show the $\Delta_p$ and $\gamma_p$ when $\Delta_p$ and $\gamma_p$ values at 150 K are used for temperatures below $T_{c0}$. These data show nearly the same $\Delta_p$ but slightly different $\gamma_p$. These results confirm our central conclusion that the superconducting gap $\Delta_p$ closes at $T_{c0}$. We note that $\Delta_p$ in Fig. 2S is nearly constant while $\gamma_p$ increases with increasing temperature all the way up to $\Gamma$, which means that the pseudogap disappears by “filling-up” as temperature approaches $\Gamma$. Since a continuing decrease of $\gamma_p$ down to $T_{c0}$ seems reasonable, both $\Delta_p$ and $\gamma_p$ parameters obtained from the simple fitting approach using $\sigma(V, \Gamma)$ (see Fig. 2a,b) and equations (1) and (2) should be a good approximation.

**Fermi arc calculation.** For the discussion of the cuprate pseudogap in ARPES experiments, Norman et al. proposed a phenomenological self-energy taking account of the lifetime effects:

$$\Sigma(k, \omega) = -\Gamma + \frac{\Delta^2}{\omega - \epsilon_k + \Gamma \Delta^2},$$

(7)

where $\epsilon_k$ is the energy of bare electrons relative to the value at the Fermi surface. From equation (7) it can be shown that the Green’s function $G(k, \omega) = 1/(\omega - \epsilon_k - \Sigma(k, \omega))$ has the form

$$G(k, \omega) = \frac{1}{\omega - \epsilon_k + \Gamma \Delta^2}.$$  

(8)

The spectral function on the Fermi surface $A(\omega, k, \epsilon_k) = -i/(\omega - \epsilon_k - \Sigma(k, \omega))$, assuming $\Delta_0 = \Delta_n + \Delta_{n+1}$, is given by

$$A(\theta, \omega) = \frac{1}{\pi} \frac{\Gamma^2 A^2 \cos^2(2\theta) + \Gamma \Delta^2 + \omega^2 \Delta^2}{(\omega - \epsilon_k - \Gamma \Delta^2 \cos(2\theta)) \Gamma^2}.$$  

(9)

In the ARPES experiments, it is considered to be gapped if $A(\theta, \omega)$ is maximal when $\theta = \theta_c$, $\epsilon_k \neq 0$, while Fermi arc appears at places where $A(\theta, \omega)$ has maximum only at $\omega = 0$. Thus $\omega_p$ can be found by setting the first derivative of equation (9) to zero:

$$\omega_p^2 = \left(1 + \frac{\Gamma^2}{\Gamma \Delta^2}ight) \epsilon_k - \Delta_n \cos(2\theta) \sqrt{\omega - \epsilon_k - \Gamma \Delta^2 \theta_c / \Gamma}.$$  

(10)

where $\gamma = \Delta^2 \cos(2\theta) + \Gamma \Delta^2$. By setting the second derivative to zero, the angle $\theta_c$ at which the arc starts to be

$$\theta_c = 0.5 \cos^{-1} \left( \frac{\Gamma}{\Gamma + 2\Gamma^2} \Delta_n \right)$$

(11)

The relative arc length $l_{arcs}$ is defined by

$$l_{arcs} = 1 - \left( \frac{\Delta_n}{\epsilon_k} \right) \theta_c.$$  

(12)

In the present work, the quasiparticle scattering rate $\Gamma$ and pair decay rate $\Gamma_n$ were estimated from the experimentally fitted parameter $\gamma_k$ in Fig. 2a,d via the relation $\gamma_k = (\Gamma + \Gamma_n)/2$. We assumed a linear temperature dependence of $\Gamma$ and considered that $\Gamma$ is larger than $\Gamma_n$, which should be reasonable from the basic physical considerations. In Fig. 6 the results of $\Gamma$ and $\Gamma_n$ for the four samples are shown, which were determined considering that $\Gamma_n = 0$ near $T_c$ and the slope of $\Gamma$ set close to that of $\gamma_k$. The corresponding $\Gamma_{arcs}$ vs $T$ calculated are plotted in Fig. 5.
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Author contributions

J.K.R., X.B.Z., Y.F.R., H.F.Yu and Ye T. did the measurement. J.K.R. and S.P.Z. performed the data analysis. N.L.W. provided and prepared single crystals for the UD71K, UD80K and OD79K samples. S.P.Z. designed the experiment and wrote the manuscript.

Additional information

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