A Model of the Pulsating Extremely Low-mass White Dwarf Precursor WASP 0247−25B

A. G. Istrate1, G. Fontaine2, and C. Heuser3

1 Center for Gravitation, Cosmology, and Astrophysics, Department of Physics, University of Wisconsin-Milwaukee, P.O. Box 413, Milwaukee, WI 53201, USA; istrate@uwm.edu
2 Département de Physique, Université de Montréal, C.P. 6128, Succursale Centre-Ville, Montréal, QC H3C 3J7, Canada
3 Dr. Karl Remeis-Observatory & ECAP, Astronomical Institute, Friedrich-Alexander University Erlangen-Nürnberg, Sternwartstr. 7, D-96049 Bamberg, Germany

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Abstract

We present an analysis of the evolutionary and pulsation properties of the extremely low-mass white dwarf precursor (B) component of the double-lined eclipsing system WASP 0247−25. Given that the fundamental parameters of that star have been obtained previously at a unique level of precision, WASP 0247−25B represents the ideal case for testing evolutionary models of this newly found category of pulsators. Taking into account the known constraints on the mass, orbital period, effective temperature, surface gravity, and atmospheric composition, we present a model that is compatible with these constraints and show pulsation modes that have periods very close to the observed values. Importantly, these modes are predicted to be excited. Although the overall consistency remains perfectible, the observable properties of WASP 0247−25B are closely reproduced. A key ingredient of our binary evolutionary models is represented by rotational mixing as the main competitor against gravitational settling. Depending on assumptions made about the values of the degree index ℓ for the observed pulsation modes, we found three possible seismic solutions. We discuss two tests, rotational splitting and multicolor photometry, that should readily identify the modes and discriminate between these solutions. However, this will require improved temporal resolution and higher S/N observations, which are currently unavailable.

Key words: asteroseismology – close – stars: evolution – stars: low-mass – white dwarfs

1. Introduction

The discovery of multi-periodic pulsations in the extremely low-mass white dwarf precursor (ELM proto-WD) WASP 0247−25B by Maxted et al. (2013) has opened the way for the application of asteroseismic methods for testing and constraining evolutionary models of these intriguing stars believed to descend from stripped red giants through binary evolution. This was followed a year later by the report of Maxted et al. (2014b) on luminosity variations with comparable periods coming again from the ELM proto-WD component in another close binary system, WASP 1628+10, belonging also to the so-called EL CVn type. The EL CVn binaries are rare double-lined eclipsing systems characterized by an A-type primary that outshines the secondary, showed that the three new pulsators belong to the same region of the effective temperature-surface gravity plane as the two WASP stars discussed above (Gianninas et al. 2014a, 2015). Additionally, it has been shown that the atmospheres of these stars contain amounts of helium comparable to those of hydrogen.

It is known that He is the essential ingredient, through He II–He III ionization, for driving pulsation modes in the regime of effective temperature where the known pulsating ELM proto-WDs are found, i.e., in a region where H is totally ionized and cannot contribute to pulsational driving (Jeffery & Saio 2013; Van Grootel et al. 2015; Córtesco et al. 2016; Gianninas et al. 2016). In this context, Gianninas et al. (2016) provided the first empirical evidence that pulsations in ELM proto-WDs can only occur if a significant amount of He is present in the atmosphere and, by extension, in the driving region. We note, in this respect, that the atmospheric He content has not been determined for the two WASP systems studied by Maxted et al. (2013, 2014b), given the complicated nature of their double-lined spectra (but see below).

Along with He, the atmospheres of ELM proto-WDs, in general, often show traces of metals such as Mg and Ca, as revealed in the systematic investigation of the class properties carried out by Gianninas et al. (2014a), and in the few available detailed abundance analyses of individual objects that currently exist (Kaplan et al. 2013; Gianninas et al. 2014b; Hermes et al. 2014; Latour et al. 2016). Given that gravitational settling, if left unimpeded, leads to the formation of a pure H atmosphere on a very short timescale in such stars (Althaus et al. 2001, 2013), a competing mechanism must be at work to account for the presence of elements heavier than hydrogen.

Among the possible processes discussed by Gianninas et al.
data set provided by Maxted et al. (2013) for this eclipsing system.

2. Strategy

We seek binary evolutionary models that have component masses that agree with the obtained values of Maxted et al. (2013). Ideally, the evolutionary track describing the B component should pass through the inferred location of WASP 0247–25B in the effective temperature-surface gravity plane, and have a proto-WD age consistent with the age obtained by satisfying the orbital period constraint. The proto-WD age is defined as the time elapsed since the end of the mass-transfer phase (for more details, see Istrate et al. 2014). In addition, the model of the B component at that same point in the \( T_{\text{eff}} - \log g \) diagram should show pulsations in its period spectrum that agree with the three observed periods. In this, we restrict the search to degree values of \( \ell = 0, 1, \) and \( 2 \) on the basis of geometric arguments for mode visibility.5 In particular, the modes of interest in the model should be predicted excited, meaning that rotational mixing must have retained enough He in the envelope for pulsation driving to occur.

In this quest, we also exploit additional information about the atmospheric properties of WASP 0247–25B that can be obtained from the work of Maxted et al. (2013). In order to check on the values of the effective temperature of the two components derived from their detailed eclipse analysis, the authors report—in the supplementary online material associated with their paper—that they went through the difficult exercise of disentangling the combined spectrum. Their preliminary analysis of the spectrum of the B component indicates a value of \( T_{\text{eff}} = 10,300 \pm 200 \) K for a fixed value of \( \log g = 4.61 \) and a fixed metallicity of \( Z = 0.002 \). These numbers are to be compared with the values of \( T_{\text{eff}} = 11,380 \pm 400 \) K and \( \log g = 4.576 \pm 0.011 \) based on their primary analysis of the eclipsing light curve, which the authors preferred and reported in Table 1 of their main paper (Maxted et al. 2013). A more involved study of the disentangled spectrum of WASP 0247–25B was postponed to a later date.

Recently, one of us (C.H.) carried out a self-consistent analysis of that spectrum using the suite of tools available at Dr. Karl Remeis-Observatory. The details of that quantitative study will be reported elsewhere, but we use here the derived values of \( T_{\text{eff}} = 10,870 \pm 230 \) K and \( \log g = 4.70^{\pm 0.12} \), which, importantly in the present context, provide an independent (purely spectroscopic) estimate of the location of WASP 0247–25B in the \( T_{\text{eff}} - \log g \) plane. The reported uncertainties are the combined statistical errors (which are smaller than 0.02 dex) and the assumed systematical errors on \( \log g \) (0.1 dex) and \( T_{\text{eff}} \) (2%). We note that the two independent sets of estimates are not in direct conflict as they overlap at the \( 1 \sigma \) level in both \( T_{\text{eff}} \) and \( \log g \). In addition, this analysis provides estimates of, among others, the He, O, and Ca contents in the atmosphere of WASP 0247–25B, values that can ultimately be compared with predictions of our evolutionary models.

We have assembled, Table 1, the constraints available in the pursuit of our exercise. Those include the two independent sets of \( T_{\text{eff}} - \log g \) values discussed above, as well as the three detected periods in WASP 0247–25B, \( P_1 \) through \( P_3 \). In a first

5 We note in this context that Maxted et al. (2013) have already demonstrated that the spacings between the three observed periods in WASP 0247–25B are inconsistent with the idea of modes with the same value of the degree index \( \ell \). At least modes with degree values of \( \ell = 0 \) and 1, and perhaps 2, must be involved; see their Figure 5.
step, we seek evolutionary models that would best be compatible with the relevant entries listed in Table 1. We compute such models in a way similar to those described in Istrate et al. (2016a, 2016b), using the publicly available binary stellar evolution code MESA, version 7624 (Paxton et al. 2011, 2013, 2015), with rotational mixing and diffusion processes turned on. We specifically followed the evolution of H, He, C, O, and Ca in the calculations presented here. We next carry out a detailed nonadiabatic pulsation study of these models using the Montréal pulsation code (Brassard et al. 1992; Fontaine et al. 1994) in order to best fit the observed pulsation periods and verify if these modes are predicted to be excited.

3. Evolutionary Models

The evolutionary track followed by an ELM WD in the $T_{\text{eff}}$–log $g$ plane, especially in the proto-WD phase, is very sensitive to the total mass and the assumed metallicity of the models. This important point is discussed thoroughly in Istrate et al. (2016b). Using the very precise estimates of the final masses of the A and B components of the WASP 0247–25 system listed in Table 1 as primary anchor points, we considered many different evolutionary sequences with varying metallicity and initial masses for the binary components. A first result is that we could not find, for any reasonable metallicity, an evolutionary track for WASP 0247–25 in the narrow range of mass $M(B) = 0.186 \pm 0.002 \ M_{\odot}$ that would go through the preferred location of Maxted et al. (2013) in the $T_{\text{eff}}$–log $g$ diagram, i.e., $T_{\text{eff}} = 11,380 \pm 400 \ K$ and log $g = 4.576 \pm 0.011$\(^6\) and that satisfies in the same time the orbital constraints. A slightly higher mass is needed, but we then encounter the problem of a mismatch between the predicted and the observed orbital period of the system at that

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**Note.** (A) the primary A-type star component; (B) the secondary ELM proto-WD; (a) multicolor photometry and time-phased spectroscopy; (b) spectroscopic analysis of the disentangled time-averaged spectrum; (c) multicolor photometry.

**References.** (1) Maxted et al. (2013); (2) C. Heuser (2017, in preparation).

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\(^6\) The location of WASP 0247–25B in Figure 1 of Istrate et al. (2016a) has been obtained by averaging the value of $T_{\text{eff}} = 11,380 \pm 400 \ K$ of Table 1 of Maxted et al. (2013) and that derived from their preliminary analysis of the disentangled spectrum of the star, which leads to $T_{\text{eff}} = 10,300 \pm 200 \ K$, while keeping the log $g$ value at 4.576 $\pm$ 0.011. The track going through that new location shown in their figure shows pulsational instabilities in the correct range, but still corresponds to a mass of $0.189 \ M_{\odot}$, which is too large compared to the more stringent requirement of the present paper.
abundance, \( \log \text{Ca/H} = -6.69 \), is slightly lower than the lower limit on the spectroscopic estimate given in Table 1 (\( \log \text{Ca/H} = -6.63 \)). We note, in this context, that the relative metal composition that we used by default in our MESA calculations is that of Grevesse & Sauval (1998). Had we used instead the most recent evaluation of Asplund et al. (2009), the predicted \( \log \text{O/H} \) ratio would have been about 0.04 dex lower than found here, while the predicted \( \log \text{Ca/H} \) value would have been some 0.09 dex higher, thus reducing the slight discrepancies depicted in Figure 3.

The basic properties of our proposed evolutionary model at proto-WD age of 507 Myr are summarized in Table 2. By construction, the orbital period is equal to the observed value of Maxted et al. (2013) and the initial masses of the components were picked to lead to final masses very similar to their measured values. In addition, the atmospheric properties of our model at that age are entirely consistent with the estimates obtained in the spectroscopic analysis of C. Heuser (2017, in preparation). Those, we recall, provide completely independent constraints from the inferences made in the light-curve analysis of Maxted et al. (2013). In this respect, we would like to point out that the differences we find with the preferred values of Maxted et al. (2013) for \( T_{\text{eff}} \) and \( \log g \) remain small in an absolute sense. Those differences probably imply some small systematic effects between the light-curve analysis method of Maxted et al. (2013) and the way our binary evolutionary models are calculated. Overall, however, we conclude that our assumed evolutionary model for WASP 0247–25B provides a rather realistic description of the basic properties of that star. Further on, we check if the model can also pass the test of seismology.

### Table 2

| Quantity     | Value     |
|--------------|-----------|
| \( M(B) \)   | 0.186 \( M_\odot \) |
| \( M(A) \)   | 1.354 \( M_\odot \) |
| \( P_{\text{rotation}} \) | 16.027908 hr |
| \( T_{\text{eff}}(B) \) | K |
| \( \log g(B) \) | 4.685 cm s\(^{-2}\) |
| \( \log \text{He/H}(B) \) | -0.66 |
| \( \log \text{Ca/H}(B) \) | -6.69 |
| \( \log \text{O/H}(B) \) | -4.23 |

### 4. Pulsation Calculations

We carried out a nonadiabatic pulsation analysis of the individual equilibrium structures found along the retained evolutionary track described in the previous section. Figure 4 summarizes some of our results in the range of effective temperature of interest. Significantly, the figure indicates that the only acceptable seismic models are confined to the relatively narrow interval of 10,500 K \( \lesssim T_{\text{eff}} \lesssim 11,100 \) K. On the cool side, the three detected periods are globally shorter than the periods found in the band of excited modes, while, on the warm side, the three observed periods are longer than those predicted unstable. Given that the effective temperature of our “best” evolutionary model at age 507 Myr described in Table 2 is well within that interval, this constitutes a first consistency check with this important constraint coming out of our seismic analysis.

An examination of Figure 4 reveals that the pulsation periods are rather strongly dependent on the location of the equilibrium
structures along the evolutionary proto-WD track (measured here by the effective temperature). As the model evolves and gets hotter, the predicted periods (each defined by a given set of \( \ell - n \) values) get shorter and some of them cross the range where the three observed periods are found. This implies that there must exist one or more values of the effective temperature corresponding to a best simultaneous match between the three detected periods and three of the periods belonging to the theoretical spectrum. In order to quantify this search for a structure providing the best fit to the observed periods, we use a standard \( \chi^2 \) approach. Hence, we look for minima of the quantity,

\[
\chi^2 = \sum_{i=1}^{3} \left( \frac{P_{\text{obs}}^{(i)} - P_{\text{th}}^{(i)}}{\sigma_{\text{obs}}^{(i)}} \right)^2,
\]

where the \( P_{\text{obs}}^{(i)} \)'s and the \( \sigma_{\text{obs}}^{(i)} \)'s are the three detected periods and their associated 1\( \sigma \) uncertainties as reported in Table 1, while the \( P_{\text{th}}^{(i)} \)'s are the best-matching theoretical periods among the available values for a given location along the evolutionary track.

If, for simplicity’s sake, we assume that the detected pulsation modes belong to degree indices \( \ell = 0 \) and/or 1, then we find two \( \chi^2 \) minima in the range of effective temperature of interest. A simple look at the top and middle panels of Figure 4 is sufficient to anticipate such a minimum for an effective temperature just slightly higher than the value of \( T_{\text{eff}} = 10,585 \) K associated with one of the equilibrium structures along the evolutionary track. Likewise, corresponding to a change of \( \Delta n = -1 \) for all three modes (but keeping the same identification for \( \ell \)), a second minimum can be anticipated for an effective temperature somewhat higher than \( T_{\text{eff}} = 10,866 \) K associated with another, more evolved, equilibrium structure. Also, clearly to the eye, this second minimum can only provide a lower quality fit than the previous one.

On the other hand, if we allow for the possibility that the detected modes belong to degree indices \( \ell = 0, 1, \) and/or 2, then a quick examination of the middle and lower panels of Figure 4 reveals that a potentially very good fit could be obtained for an effective temperature in between those of the equilibrium structures at 10,723 K and 10,866 K, and closer to the former. Granted, the mode density is higher for the \( \ell = 2 \) modes, so this increases the probability that a better period match is obtained by chance. Our standard \( \chi^2 \) approach is blind to that possibility, but we revisit this issue below. At the same time, the idea that quadrupole modes could be driven to observable amplitudes in a close binary system, such as WASP 0247–25, has its own obvious appeal.

In summary, we find three possible seismic solutions in the sense of best matching the observed periods, while representing excited modes. In order to improve the “resolution” along the evolutionary track and more effectively pin down the optimal period fit, we used parabolic interpolation in effective temperature. The results of that operation are shown in Figure 5, which shows, in the upper panel, the behavior of \( \chi^2 \) as a function of \( T_{\text{eff}} \) for each of the three solutions. Taking into account the logarithmic scale used to depict \( \chi^2 \), each of the solutions corresponds to a very well-defined minimum.

This being said, a well-defined minimum in \( \chi^2 \) does not necessarily imply a good fit from a statistical point of view. Within the framework of our formal \( \chi^2 \) approach, it is possible to use the \( Q \)-probability formalism discussed in Press et al. (1986) to estimate the quality of the fit (and see Randall et al. 2005 for an astrophysical application similar to the current one for more details). The usual adopted value for a \( \chi^2 \) fit to qualify as “acceptable” is \( Q \geq 1 \times 10^{-3} \), which corresponds in the present case with 3 degrees of freedom (0 free parameter and 3 data points) to \( \chi^2 \leq 16.25 \). If we relax this definition of an acceptable fit to include the solutions for which the three theoretical periods fall simultaneously within the \( \pm 3\sigma \) ranges of the three detected periods, then the passing value is \( Q \geq 5.88 \times 10^{-6} \), which corresponds to \( \chi^2 \leq 27.0 \). It is the latter threshold value that is plotted as a dotted horizontal line in the top panel of Figure 5. The defining characteristics of each of our three possible seismic solutions are listed in Table 3. Details on the mode identification and period fits are provided in Table 4.

Formally speaking, only Solution 3 provides a statistically meaningful period match. That solution is also excellent by current seismic standards involving evolutionary models. Still, we think it unwise to dismiss the other two possibilities at this stage. On the first account, Solution 3 proposes a mode identification that involves two \( \ell = 2 \) modes out of the three
The quality of the formal \( \chi^2 \) observed periods are restricted to the modes differ in the mode identification with each solution. As illustrated by the curves in black, there are two possibilities of unequal quality as observed in WASP 0247–25B. As it is, we found three possible seismic solutions (one of which is qualitatively excellent) in the close vicinity of that best evolutionary model along the retained evolutionary track. The latter is defined by \( T_{\text{eff}} = 10,700 \) K along that track, while, in comparison, seismic Solutions 1, 2, and 3 are found, respectively, at \( T_{\text{eff}} = 10,598, 10,920, \) and 10788 K. We note, in the present context, that the theoretical period spectra of proto-WD models are very sensitive to the total mass (see, e.g., Istrate et al. 2016a), and this can be exploited to estimate what adjustment can be made to the total mass so that the best evolutionary model would coincide even better with a seismic solution than is the case here. For instance, on the basis of Solution 3 and using other sequences with different masses, we estimate by interpolation that a model with a mass reduced by the small amount of \( 0.0009 M_\odot \) from the current value of \( M(B) = 0.186 M_\odot \) would show pulsation periods very close to the observed ones at the epoch along its track when the predicted orbital period is equal to the measured one. We conclude from this discussion that our proposed model of WASP 0247–25B (Table 2) is consistent with the seismic information currently available.

5. Rotation Properties

Our binary evolutionary models make specific predictions about the state of the internal rotation profile of the ELM proto-WD component. This can be exploited further to calculate the effects of rotational splitting on the pulsation spectrum in Fourier space. Given seismic observations of high enough temporal resolution (not yet available for WASP 0247–25B, however), this can be used to test further the proposed seismic solutions and, in particular, to check the derived mode identification and discriminate between the various possibilities.

While the newly formed proto-WD is initially characterized by external layers rotating much faster than the inner regions (see Istrate et al. 2016b for details), differential rotation settles into solid-body rotation by the time a proto-WD has entered its final contraction phase toward its maximum effective temperature before turning into an ELM WD.

Table 5 lists the solid-body rotation period, \( P_{\text{rot}} \), for each of the three seismic solutions, along with basic information concerning the mode identification associated with each solution. The predicted internal rotation period is very close to the measured orbital period at that stage in the evolution, and decreases with passing time (increasing \( T_{\text{eff}} \)) as the orbital period.

For a given pulsation mode of frequency \( \nu_{\ell n} \) the inverse of the pulsation period, first-order perturbation theory predicts that solid-body rotation lifts the \( 2\ell + 1 \) degeneracy associated with spherical symmetry and leads to a set of equally spaced frequency components separated by

\[
\Delta \nu_{\ell n} = \frac{2\pi}{P_{\text{rot}}} (1 - C_{\ell n}),
\]

where \( C_{\ell n} \) is the first-order rigid rotation coefficient, which is specific to each degenerate mode specified by a set of indices \( \ell - n \). That quantity can be computed from the unperturbed (rotation-free) eigenfunctions of the mode of interest and is reported in Table 5. As is well known, to first order, the

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**Figure 5.** Seismic solutions for the retained evolutionary track. The top panel illustrates the three possible solutions in terms of well-defined minima in the formal \( \chi^2 \) along the track as measured by the effective temperature. If the three observed periods are restricted to the \( \ell = 0 \) and/or 1 identification, then there are two possibilities of unequal quality as illustrated by the curves in black. Those differ in the mode identification by a change of \( \Delta \ell = -1 \) in the radial order of the modes, from the lower to the higher \( T_{\text{eff}} \). If, instead, modes with \( \ell = 0, 1, \) or 2 are allowed, then the best-fit solution is given by the curve in red. The quality of the formal \( \chi^2 \) fit is estimated through the \( Q \)-probability formalism; values of \( \chi^2 \) below the dotted horizontal line are considered good fits. The lower panel illustrates the variance \( \sigma \) in units of seconds—associated with each solution.

observed ones, and this goes against the expected degree-amplitude correlation based on geometric cancellation effects (Dziemowsk 1977). Note, however, that sectorial \( \ell = 2 \) modes could be favored in a close binary system such as WASP 0247–25B. Second, as indicated above, our \( \chi^2 \) method is insensitive to the mode density, i.e., if more modes are available in a given period window (as is the case considered in Solution 3 when the quadrupole modes are also included in the best-fit search), the chances of good matches naturally increase. To take into account that characteristic, Brassard et al. (2001) proposed to introduce weights in a modified \( \chi^2 \) approach, which they used in their asteroseismic study of the pulsating hot subdwarf B star PG 0014+067. The weights correspond to the inverse of the mode density for each family of modes with a given value of the degree index \( \ell \) in a given period window. Translated into the present context, this means that such a modified \( \chi^2 \) for Solution 1 would improve by a factor of \( (24/15)^2 = 2.56 \) with respect to Solution 3 and perhaps become acceptable. These numbers are based on the observation that there are \( 7, 8, \) and \( 9 \) modes, respectively, with \( \ell = 0, 1, \) and 2 in the observed period band in Figure 4. Thus, we submit that Solution 1 and Solution 2 should not be completely eliminated at this point, especially the former one. It is unfortunate, of course, that only three periods were detected in WASP 0247–25B as other pulsations would have provided much welcomed additional constraints on the possible seismic models that we can currently build.

In an ideal situation, our best evolutionary model described in Table 2 would have shown three pulsation modes in its spectrum with periods nearly equal to the three detected periods in WASP 0247–25B. As it is, we found three possible seismic solutions (one of which is qualitatively excellent) in the close vicinity of that best evolutionary model along the retained evolutionary track. The latter is defined by \( T_{\text{eff}} = 10,700 \) K along that track, while, in comparison, seismic Solutions 1, 2, and 3 are found, respectively, at \( T_{\text{eff}} = 10,598, 10,920, \) and 10788 K. We note, in the present context, that the theoretical period spectra of proto-WD models are very sensitive to the total mass (see, e.g., Istrate et al. 2016a), and this can be exploited to estimate what adjustment can be made to the total mass so that the best evolutionary model would coincide even better with a seismic solution than is the case here. For instance, on the basis of Solution 3 and using other sequences with different masses, we estimate by interpolation that a model with a mass reduced by the small amount of \( 0.0009 M_\odot \) from the current value of \( M(B) = 0.186 M_\odot \) would show pulsation periods very close to the observed ones at the epoch along its track when the predicted orbital period is equal to the measured one. We conclude from this discussion that our proposed model of WASP 0247–25B (Table 2) is consistent with the seismic information currently available.

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where \( C_{\ell n} \) is the first-order rigid rotation coefficient, which is specific to each degenerate mode specified by a set of indices \( \ell - n \). That quantity can be computed from the unperturbed (rotation-free) eigenfunctions of the mode of interest and is reported in Table 5. As is well known, to first order, the
pulsation frequency of a radial mode is not affected by rotation and there is also no possible rotational splitting in that case. On the other hand, dipole modes split into triplets and quadrupole modes into quintuplets in Fourier space, with a spacing

\[ \Delta \nu_{\text{rot}} = 2 \times 16.0 \muHz \]

(381 s) and \( 2 \times 15.7 \muHz \) (406 s). At the same time, the 421 s pulsation—identified as a dipole mode—should show a doublet structure with a spacing given by \( \Delta \nu_{\text{rot}} = 2 \times 12.8 \muHz \). Of course, these predictions/diagnostics can only be of use if and when observations of sufficient temporal resolution become available in the future. This would imply a timebase covering several rotation/orbital periods.

### 6. Conclusion

The primary goal of the present investigation has been to test our ability, using new evolutionary models including rotational mixing, in reproducing the properties of WASP 0247–25B, the best-studied pulsating ELM proto-WD so far (Maxted et al. 2013). After investigating many possibilities, we isolated an evolutionary sequence featuring a final mass of 0.186 \( M_{\odot} \) for the B component of the system (and a final mass of 1.554 \( M_{\odot} \) for the A component) after mass transfer has ceased. Those values are, by full intent, perfectly compatible with the estimates listed in Table 1. Along that evolutionary track, we picked the structure corresponding to the epoch—some 507 Myr after the beginning of the proto-WD phase—when the orbital period of the system is exactly equal to the value reported in Table 1. Quite encouragingly, this “best” evolutionary model of ours is found rather close to the location of WASP 0247–25B in the \( T_{\text{eff}} \)-log g diagram inferred by Maxted et al. (2013), although not rigorously at the same place. On the other hand, our best evolutionary model is located right within the 1σ error box in that diagram associated with the independent spectroscopic analysis of C. Heuser (2017, in preparation). Moreover, the predicted atmospheric abundances of He, O, and Ca in that evolutionary model are quite close to the spectroscopic values, although they are formally outside the estimated 1σ uncertainty ranges (see Table 2 in comparison to Table 1).

On the pulsation front, we found that the only acceptable seismic models along the retained evolutionary track have to be within the relatively narrow interval 10,500 K \( \lesssim T_{\text{eff}} \lesssim 11,100 \) K, which encompasses, as it ideally should, our best evolutionary model at 10,700 K. This constraint comes from the requirement that the theoretical pulsation modes with periods falling within the range of the observed periods, 381–421 s, must be predicted to be excited in the model. In a perfect situation, the 10,700 K model would show three excited
modes in its period spectrum that would match perfectly the three observed periods in WASP 0247−25B. Instead, we found three possible seismic solutions in the close vicinity of our best evolutionary model along the retained evolutionary track. Those are located at $T_{\text{eff}} = 10,598$ K (Solution 1), 10,920 K (Solution 2), and 10,788 K (Solution 3) along the track. Solutions 1 and 2 follow from the a priori assumption that the observed modes are either radial and/or dipole modes, while Solution 3 admits the presence of modes with either $\ell = 0, 1$, and/or 2.

Solution 3 is definitely qualitatively and quantitatively superior and should be retained in principle, but, in a conservative spirit, we prefer to retain the other two possibilities at this stage. Given that our evolutionary approach also makes definite predictions about the state of internal rotation of WASP 0247−25B, we devised a test to identify the $\ell$ index of each observed mode through rotational splitting. As pointed out above, this will require follow-up observations with a temporal resolution significantly better than that achieved in the experiment carried out by Maxted et al. (2013). Given the eclipsing nature of the system, it will be hard work to improve on that past work, but the detection of rotational splitting would prove ideal for discriminating between the three proposed seismic solutions.

We note, in this context of mode identification, that the work of Maxted et al. (2013) is also the source of additional information that might help in this regard. Indeed, from Supplementary Table 3 of Maxted et al. (2013), we can compute amplitude ratios such that, for instance, the ratio of the amplitude in the $u'$ filter to that in the $r'$ band is $A(u')/A(r') = 1.25 \pm 0.08$, $1.00 \pm 0.40$, and $1.64 \pm 0.23$ for the observed modes with periods of $P_1$, $P_2$, and $P_3$ (see Table 1), respectively. To correctly interpret these numbers, it will be necessary to develop the appropriate modeling, following, for example, Randall et al. (2005). This involves detailed work using specific intensities from proper model atmospheres, as well as nonadiabatic eigenfunctions. In the absence of such modeling for ELM proto-WDs, we can still make the point that ratios of the sort bear a unique signature of the $\ell$ index. For instance, in Solutions 1 and 2, both the $P_1$ and $P_3$ modes are identified as radial modes (see Table 4). Hence, the amplitude ratio should be exactly the same for both modes. To the extent that the value of $A(u')/A(r') = 1.25 \pm 0.08$ for the $P_1$ mode is truly significantly different from the value of $A(u')/A(r') = 1.64 \pm 0.23$ for the $P_3$ mode, this would immediately eliminate both seismic solutions. In contrast, Solution 3 boosts a different $\ell$ identification for mode $P_1$ compared to mode $P_3$, and this leads to different expected amplitude ratios as possibly observed. In truth, before the amplitude-color method can be exploited properly, we need higher S/N observations to more effectively pin down the pulsation amplitudes and their uncertainties in various filters.

We thus plea for improved temporal resolution and higher S/N observations of the tantalizing eclipsing system WASP 0247−25. We note, with some irony, that we are aware of one such attempt with the SOAR telescope made in 2014, which, quite unfortunately, turned out to be unproductive. As it is, the pulsations seem to have “disappeared” below detectability on the one night when these follow-up observations were made (B. Dunlap 2017, private communication). Since it is difficult to imagine how the driving mechanism could be stopped over a timescale as short as a year or two, the more likely explanation could be amplitude modulation—either intrinsic or due to beating between closely spaced frequency components, the latter perhaps caused by rotational splitting—that would cause the observable pulsation amplitudes to drop below detectability from time to time or periodically. Given that WASP 0247−25B has the potential for becoming the “Rosetta Stone” of ELM proto-WD seismology, we feel that further efforts on the observational front would be worthwhile.

We conclude by pointing out that the comparison exercise that we carried out in this paper has turned out to be extremely encouraging in our quest for more realistic models of ELM proto-WDs. We indeed found it rather gratifying that the known properties of WASP 0247−25B could be closely reproduced—although, quite admittedly, not perfectly—with our current evolutionary models that include rotational mixing as the main competitor against gravitational settling. Taking into account the known constraints on the mass, orbital period, effective temperature, surface gravity, and atmospheric composition, there was no a priori guarantee that acceptable seismic solutions could be found for WASP 0247−25B. Yet, we found models along our retained evolutionary track that would both be compatible with these constraints and show pulsation modes that (1) have periods close to the observed values, and (2) are predicted excited. We hope that the required data become available some day, so that one can test our proposed seismic solutions either through detection of rotational splitting or through the application of the amplitude-color method. It is likely that such follow-up observations would also reveal additional pulsation modes, which would help in refining our seismic models.

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ORCID iDs
A. G. Istrate @ https://orcid.org/0000-0002-8811-8171
G. Fontaine @ https://orcid.org/0000-0002-2357-1012

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