Correlations between fracture width and Reynolds number of 3-D single fractures

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Abstract: This study shows a numerical simulation on the relationship between fracture width (W) and the Reynolds number (Re) for fractures having different joint roughness coefficients (JRCs). Numerical simulations by solving the Navier-Stokes (NS) equations are performed to model fluid flow through single fractures. The results show that the Re of the theoretical solutions of smooth models agrees well with those of numerical simulation results, indicating that the numerical code is reliable. As W increases from 5 mm to 200 mm, the Re increases significantly and then slightly. When the W is large, the change in Re gradually becomes smaller. This indicates that there is a critical W (WC), in which the influence of W on the Re is negligible when W exceeds the WC. When JRC is 0, 5.84 and 19.20, the corresponding WC is 27.38 mm, 26.55 mm and 24.63 mm, respectively.

Keywords: Fracture width; Reynolds number; single fracture

1. Introduction
Hydraulic properties of fractures are critical for the success of items such as enhanced oil recovery, geothermal energy development, nuclear waste disposal (Pruess et al., 1990; Cvetkovic et al., 2004). A comprehensive understanding of the fluid flow through fractures is of considerable significance in addressing geophysical problems (e.g., water burst
and coal gas outburst disasters (Xue et al., 2015; Ju et al., 2019) and expanding geological engineering applications (e.g., geothermal energy extraction (Zhao et al., 2015; Pandey et al., 2018), natural gas and oil production (Ellsworth, 2013; Singh and Cai, 2018), CO2 sequestration (Noiriel et al., 2013), and grouting activities (Liu and Sun, 2019; Wu et al., 2019)). Fluid flow, heat transfer, and solute transport in fractures are significantly related to the surface roughness and fracture aperture (Liu et al., 2016; Luo et al., 2016; Vogler et al., 2017; Dou et al., 2019; Stoll et al., 2019), which is difficult to determine due to the lack of accurate characterization and measurement techniques, because fracture geometries are heterogeneous and invisible. The Reynolds number is typically utilized for detecting the onset of nonlinear flow in single fractures or single fracture intersections, which only involve single/several fracture segments (e.g., Zimmerman et al., 2004; Koyama et al., 2008b; Javadi et al., 2014).

However, in the previous studies for fluid flow through 3-D fracture networks, the flow was typically restricted to linear regime because it is still a challenging task to model nonlinear flow through 3-D fracture networks due to enormous difficulties in establishing models considering the roughness and aperture heterogeneity of single fractures and solving the NS equations composed of a set of coupled nonlinear partial derivatives of varying orders (Brush and Thomson, 2003; Javadi et al., 2010).

In the present study, first, 15 fracture models are established based on the Barton’s standard profiles. Then, the fluid flow through the models is simulated by solving the NS equations using the software FLUENT. The theoretical values of the smooth model are calculated and the results are compared with the numerical simulation results to verify the validity of the numerical code. Finally, the critical widths of fractures are calculated, exceeding which the effect of width is negligible and the 3D models can be simplified to 2D models.

2. Numerical simulation

2.1. Basic assumptions

Some assumptions are made to simplify the model. The fluid is assumed to be pure water at room temperature and its density is 998.2 kg/m³. The fluid flows through the fractures and the matrix is impermeable. Only a single inlet and a single outlet are taken into account. The pressure drop between the inlet and outlet is set to a constant value of 0.1 MPa to ensure that the flow rate of the fluid is small enough to ensure that the fluid flow is in the linear flow regime. As shown in Figure. 1, the fracture width \( W \) ranges from 5 to 200 mm and the joint roughness coefficient (JRC) changes from 0 to 19.20. After meshing, the numbers of nodes and cells for \( W = 200 \) mm are \( 1.65 \times 10^6 \) and \( 8.75 \times 10^6 \), respectively. In this study, a finite volume method (FVM) code ANSYS FLUENT module that solves the NS equations (Fluent, 2019), is employed to simulate the fluid flow and solute transport in the experimental models considering water as a viscous incompressible Newtonian fluid.
2.2 Theoretical calculation of $Re$ of smooth models

For the case of incompressible and steady Newtonian fluid, the flow rate $Q$ is usually calculated by the cubic law:

$$Q = \frac{WW_e^3 \Delta P}{12\mu \Delta L}$$

(1)

where $\rho$ is the fluid density, $W_e$ is the hydraulic aperture, $\mu$ is the dynamic viscosity, $P$ is the hydraulic pressure, and $L$ is the length.

For fluid flow in fractures, the $Re$ define as the ratio of inertial force to the viscous force is typically utilized to characterized fluid flow, which is expressed as (Zimmerman et al., 2004; Javadi et al., 2010; Sommerfeld., 1908):

$$Re = \frac{\rho V D}{\mu}$$

(2)

where $V$ is the flow velocity, and $D$ is the characteristic length (or hydraulic diameter) that can be calculated by the following equation for the rectangular duct (Aharwal et al., 2008):

$$D = 4A/\chi = 2we / (w + e)$$

(3)

where $A$ is the cross-sectional area of the fracture, $\chi = 2 (w + e)$ is the length of the wetted periphery.

3. Results and analysis

Figure. 2 shows the streamline distributions under different JRC and $W$. With increasing JRC, both the number tortuous streamlines and pronounced channelization increase significantly.
As shown in Figure 3(a) and Figure 3(b), for the parallel plate model of JRC = 0, the Re calculated by the numerical simulation is basically consistent with the results obtained by the theoretical, and has the same trend, which can prove the reliability of the numerical simulation. By applying different values of W to the models, the relationship between W and Q/W and the relationship between Re and W are obtained. It is observed that both Q/W and Re both have quadratic relations with W. And as W increases, both Q/W and Re increase significantly and keep a constant incline rate when exceeding a certain W.
Figure 3 (a) Relations between $W$ and $Q/W$, (b) Relations between $Re$ and $W$, (c) Relations between $\delta$ and $W$, (d) partial enlarged view of relations between $\delta$ and $W$.

Figure 3(b) represents the relationship between $Re$ and $W$ of fractures under different JRC conditions. The results show that the $Re$ of the theoretical solutions of smooth models agrees well with those of numerical simulations, indicating that the numerical code is reliable. At a certain $W$, the value of $Re$ calculated by the smooth model is larger than those calculated by the models with JRC = 5.84 and 19.20, because the rougher fracture surface gives rise to the longer flow path and greater frictional loss that tend to reduce the flow rate. The inertial effects in the rough model are stronger than those in the smooth model due to the enhanced variation of geometry of flow paths. Therefore, when $W$ is large (e.g., $W > 200$ mm), the change in $Re$ gradually becomes smaller. These results show that the value of $Re$ can be significantly enhanced by the fracture width increase of single fractures.

To quantitatively estimate the turning point and determine a critical width, $W_c$, the relative deviation rate of $Re$, $\delta$, is defined as follows:

$$\delta = \frac{Re_0 - Re}{Re_0} \times 100\%$$

where $Re_0$ is the $Re$ of the single fracture with $W = 200$ mm, and $Re$ is the Reynolds number of the single fracture with any given $W$ smaller than 200 mm.

As shown in Figure 3(c), with increasing $W$ from 5 mm to 200 mm, $\delta$ decreases significantly and then slightly. When the $W$ is large enough, $\delta$ tends to 0. This indicates that when the width is larger than the critical value, $W_c$, the influence of $W$ on the Reynolds number is negligible. Therefore, we assume that when $\delta$ is less than 10%, the influence of $W$ is considered negligible. Figure 3(d) presents the partial enlarged view of relations between $\delta$ and $W$. The results show that $W_c = 27.38$ mm, 26.55 mm and 24.63 mm when JRC = 0, 5.84 and 19.20, respectively.

4. Conclusions

The present study investigated the relationships between the fracture width ($W$) and Reynolds number ($Re$) for fractures having different JRC values. The theoretical value of the $Re$ of the smooth fracture is
calculated by Eq. (1), and the results are consistent with the numerical results, verifying the validity of the numerical code. Finally, the critical width of fractures \( W_c \) is calculated and analyzed.

The results show that with increasing \( W \) from 5 mm to 200 mm, the \( Re \) increases with a decreasing rate. When JRC is 0, 5.84, 19.20, the corresponding \( W_c \) is 27.38mm, 26.55mm, 24.63mm, respectively. Both \( W \) and JRC have significant on the hydraulic behaviors of single fractures.

Further development of the proposed model is required to estimate its aperture effects and hydraulic pressure distributions.

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