Leptons in Composite MFV

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Abstract

We study the lepton sector of composite Higgs models with partial compositeness. The standard anarchic scenario is in conflict with the absence of observable charged lepton flavor violation. This tension can be completely solved in MFV scenarios that require either left-handed or right-handed SM leptons to be equally composite. Constraints on this scenario are weak and the composite lepton partners could be as light as few hundreds GeVs with interesting LHC signatures. The contribution to the muon \((g - 2)\) in theories where the Higgs is a pseudo Nambu-Goldstone boson is also discussed.

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1 Introduction

The hope that new physics will be discovered at LHC relies on the existence of some special flavor structure of the new degrees of freedom that does not jeopardize the special features of the Standard Model (SM), in particular suppression of flavor changing neutral currents and charged lepton flavor conservation.

The simplest hypothesis is that the flavor structure of the new physics is the same as in the SM itself, where flavor symmetries are only broken by the Yukawa couplings. This possibility, known as Minimal Flavor Violation (MFV) [1], is for example automatically realized in supersymmetric theories with gauge or anomaly mediation. More recently it has been shown that MFV can also be elegantly realized in theories where the Higgs is a composite state of new strong dynamics and partially composite fermions [2, 3]. These models can also produce a different flavor structure, the one of “anarchic models” often considered in Randall-Sundrum scenarios, where a flavor protection against large new physics contributions is obtained because light generations are mostly elementary, suppressing their flavor transitions [4].

The flavor protection in anarchic models is not perfect but overall the picture is plausible in the quark sector. For leptons instead the absence of observable Charged Lepton Flavor Violation (CLFV) in present experiments poses a bigger challenge. In particular the bounds on the $\mu \to e\gamma$ transition strongly disfavor the anarchic hypothesis. With this motivation in mind, in this note we will extend the construction of Ref. [3], to realize MFV in the lepton sector of partially composite Higgs models.
Flavor problems are entirely solved for MFV leptons. Moreover the new fermions can be as light as a few hundred GeVs, a possibility certainly inconsistent with the anarchic hypothesis. This is exciting because in the range below TeV, the heavy leptons could be produced at the LHC and some bounds can already be extracted reinterpreting present searches. A correction to the $g - 2$ of the muon is also generated that could potentially explain the discrepancy with the SM prediction. We show however that this contribution is typically too small and only in extreme regions of parameters the required effect could perhaps be obtained.

The paper is organized as follows. In section 2, after reviewing the difficulties of anarchic models, we present the MFV composite leptons scenario. In 2.3 we extend the MFV construction to theories with $SU(2)$ flavor symmetries [5, 6]. Indirect and LHC bounds are considered in section 3. We discuss the contribution to the muon $(g - 2)$ in section 4. We summarize in 5. In the appendix we compute the contribution to $(g - 2)_\mu$ in models where the Higgs is a pseudo Nambu-Goldstone boson.

2 Partially Composite Leptons

We work within the framework of composite Higgs models with partial compositeness. Fields can be divided into elementary (SM) and composite. Each SM field mixes with composite states with identical quantum numbers under the SM gauge group that belong to a representation of the global symmetries of the composite sector. We have in mind in particular the case where the Higgs is pseudo Nambu-Goldstone Boson (NGB) of a strongly coupled sector with symmetry $G$ spontaneously broken to $H$, see [7] for the general framework. Many of the arguments concerning flavor however hold in general in scenarios with partial compositeness.

We will focus on the lepton sector. Assuming for simplicity the existence of a right-handed neutrino, the mixing terms are schematically

$$L_{mixing} = \lambda_L \bar{L} L + \lambda_{Re} \bar{E} L + \lambda_{Re} \bar{E} L + h.c. \tag{1}$$

where miniscule and capital letters refer to elementary and composite fermions and flavor indexes are implicit. The mass of the charged leptons is given by,

$$m_e \approx \frac{v}{\sqrt{2}} \epsilon_L \cdot Y_e \cdot \epsilon_{Re} \tag{2}$$

where $\epsilon \sim \lambda/m$ (with $m$ the mass of the composite partner), $v = 246$ GeV and $Y_e$ is the relevant composite sector coupling. Analogous formulae would hold for Dirac neutrinos. Within the logic of elementary-composite sectors it appears natural to realize Majorana neutrinos by adding a large mass for the elementary right-handed neutrino. In this case one finds,

$$m_\nu \approx v^2 \epsilon_L \cdot Y_e \cdot \epsilon_{Re} \cdot M^{-1} \cdot \epsilon_{Re} \cdot Y_\nu \cdot \epsilon_T \tag{3}$$

where $M$ is the Majorana mass matrix of $\nu_R$.\(^1\)

\(^1\)For other realizations of neutrino masses, see [8, 9]. Neutrinos will not play a role in what follows.
For several purposes the strong sector can be described truncating the theory to the lightest modes with the lagrangian \([10]\),

\[
\mathcal{L}_{\text{comp}} = m_L \bar{L}L + m_L \tilde{L}\tilde{L} + (\bar{L}H) \cdot Y_e \cdot \tilde{E} + (\tilde{L}H) \cdot Y_\nu \cdot \tilde{N} + \ldots
\]  

(4)

where

\[
L = \begin{pmatrix} N \\ E \end{pmatrix} \quad \tilde{L} = \begin{pmatrix} \tilde{N} \\ \tilde{E} \end{pmatrix}.
\]

(5)

are a vectorial copy of SM fermions. The dots stand for other fermions required by the global symmetries of the theory (minimally custodial symmetry \(SU(2)_L \times SU(2)_R\) and higher order terms suppressed by the compositeness scale. It should be kept in mind that the lagrangian \([4]\) can just be used to provide estimates. Moreover the interactions above do not take into account the NGB nature of the Higgs that is crucial for certain observables, in particular the \(g - 2\) of the muon as we will see.

### 2.1 Anarchic Leptons

We start reviewing the severe bounds that one obtains in anarchic scenarios from leptons \([13, 14, 15, 9]\). The assumption is that the composite sector has no hierarchies but is characterized by a typical mass scale \(m_\rho\) and a coupling \(g_\rho\) associated for example to the spin-1 resonances. In what follows we will treat fermionic parameters \(m_\psi\) and \(g_\psi \sim Y_{ij}\) as independent.

Let us first consider dipole operators. These are generated at 1-loop in the strong sector coupling with coefficient,

\[
\frac{e}{16\pi^2} \frac{v}{m_\psi^2} \left[ U_L^\dagger \cdot \epsilon_L \cdot \left( a_1 Y_e \cdot Y_e^\dagger + a_2 Y_\nu \cdot Y_\nu^\dagger \right) \cdot Y_e \cdot \epsilon_R \cdot U_R \right]_{ij} \epsilon^i_L \sigma^{\mu\nu} \epsilon^j_R F_{\mu\nu}
\]

(6)

where \(U_{L,R}\) are the rotation matrices to the mass basis determined by eq. \([2]\) and \(a_{1,2}\) are model dependent order one numbers. In general both electric and magnetic moments are generated of same size. The estimate parametrically agrees with the explicit computation using the lagrangian \([4]\) but robustly follows from the hypothesis of partial compositeness.

In anarchic scenarios one obtains the estimate for \(\mu \to e\gamma\) dipole operators,

\[
\left( \frac{g_\psi}{4\pi} \right)^2 \frac{e}{m_\psi^2} \left( \frac{e}{\epsilon_L} m_\psi \bar{\mu}_L \sigma^{\mu\nu} e_R + \frac{e}{\epsilon_L} m_\psi \bar{e}_L \sigma^{\mu\nu} \mu_R \right) F^{\mu\nu}
\]

(7)

The most favorable choice of mixings is \([14, 9]\),

\[
\epsilon^i_L \sim \epsilon^i_R \sim \sqrt{\frac{m_i}{g_\psi v}}
\]

(8)

from which it follows,

\[
\text{Br}(\mu \to e\gamma) \sim 5 \times \left( \frac{g_\psi}{3} \right)^4 \times \left( \frac{3\text{TeV}}{m_\psi} \right)^4 \times 10^{-8}.
\]

(9)
Table 1: CLFV branching fractions in anarchic scenarios. We choose $m_\rho = m_\psi = 3$ TeV and scan the couplings $Y_{ei} \sim g_\psi$ around the central value 3. $L_1$ corresponds to the “optimal choice”, while $L_2$ to $\epsilon_L = \text{Diag}[0.01, 0.02, 0.025]$. EXP is the experimental limit.

Comparing with the recent experimental bound $\text{Br}(\mu \rightarrow e\gamma) < 6 \times 10^{-13} @ 90$ C.L. [16], this result is orders of magnitude too large unless the coupling is extremely weak. This is however in tension with the strong sector logic.

Let us consider $\mu \rightarrow eee$. The leading contribution is due to the flavor violating $Z-$couplings. Without protected representations one finds,

$$\frac{\delta g_{L}}{g_{L}} \sim \left( \frac{g_\psi v^2}{m_\psi^2} + \frac{g_\rho v^2}{m_\rho^2} \right) \epsilon_L^i \epsilon_L^j$$

and similarly for the couplings to right-handed fermions. The two contributions correspond perturbatively to the mixing with heavy partners of fermions and vectors respectively, see for example [10]. From the first, for the optimal choice of mixings (8) one finds,

$$\text{Br}(\mu \rightarrow eee) \sim \left( \frac{g_\psi}{3} \right)^2 \times \left( \frac{3 \text{ TeV}}{m_\psi} \right)^4 \times 10^{-13}$$

while from the second,

$$\text{Br}(\mu \rightarrow eee) \sim \left( \frac{g_\rho}{3 g_\psi} \right)^2 \times \left( \frac{3 \text{ TeV}}{m_\rho} \right)^4 \times 10^{-13}.$$}

This last contribution scales differently with the fermionic coupling so that reducing $g_\psi$ as demanded by $\mu \rightarrow e\gamma$ leads to problems with $\mu \rightarrow eee$ [13]. A similar scaling is obtained for $\mu \rightarrow e$ transitions in nuclei, see [14].

These estimates can be validated performing a scan of parameters of the model (4) that reproduce the charged leptons masses. In the table we report the results for the optimal choice (L1) of mixing in eq. (8) and for left-handed mixings of same order (L2), to reproduce hierarchies of the neutrino mixing matrix. The results are in good agreement with the estimates.

\footnote{Strictly this is not necessary as the large neutrino mixing angles can originate from the neutrino sector.}
2.2 MFV Leptons

The severe tension from CLFV provides a strong motivation to realize MFV in partially composite Higgs models. We will follow the discussion for quarks in Refs. [3, 5]. We assume that the composite sector has an $SU(3)$ flavor symmetry with composite fermions transforming as triplets. This implies $Y_e = g_\psi \cdot \text{Id}$ and degenerate masses for different flavors. If the symmetry is respected by the left mixings,$$
abla L \propto \text{Id} \quad (13)$$
the known mass hierarchies are entirely due to the right mixings, that will be proportional to the SM Yukawa couplings
$$\epsilon_{Re} = \frac{y^e_{SM}}{\epsilon_L g_\psi}. \quad (14)$$

The mixings, or equivalently the Yukawas, are the only sources of breaking of flavor symmetries as in the SM, therefore realizing the MFV hypothesis. We call this scenario left-handed compositeness as the left-handed SM fermions have equal mixings.

MFV can also be realized with universal right-handed mixing, $\epsilon_{Re} \propto \text{Id}$. The simplest option for neutrino masses in this case is to introduce a composite scalar triplet [9]. This allows to write a bi-linear coupling of the elementary left-handed fields with the triplet that generates the dimension 5 operator responsible for neutrino masses. The right-handed neutrinos and their partners are not necessary and MFV requires an $SU(3)$ global symmetry of the strong sector. Alternatively if neutrino masses are also generated through linear couplings MFV demands and $SU(3)^2$ flavor symmetry in right-handed compositeness [3].

2.3 Beyond MFV

Realizing MFV in the lepton sector removes all new physics flavor effects at least as long as the see-saw scale $M \gg \text{TeV}$. A variation of this can be obtained in theories based on $SU(2)$ flavor symmetries [5, 6], that allow to treat the third generation independently, i.e. with different couplings and masses. This is motivated in the quark sector by the heaviness of the top. These theories share essentially the same success as MFV for what concerns flavor while they avoid the strong bounds from precision tests and compositeness on composite light quarks.

Once we assume $SU(2)$ to be a symmetry of the strong sector also composite leptons will be multiplets of this symmetry. A natural choice is that the first two generations are doublets and the third singlets. For the strong sector couplings this implies a structure,
$$Y_e = \text{Diag} \left[ g^1_\psi, g^1_\psi, g^2_\psi \right] \quad (15)$$
and similarly for the masses. As before we can assume that the flavor symmetry is respected by the mixings. One can choose the basis,
$$\epsilon_L = \text{Diag} \left[ \epsilon_1, \epsilon_1, \epsilon_2 \right]$$
$$\epsilon_{Re} = U \cdot \hat{\epsilon}_{Re} \quad (16)$$
Table 2: CLFV branching fractions in SU(2) models for $m_\rho = m_\psi = 3$ TeV. The couplings $g_1^\rho$ and $g_2^\rho$ are scanned independently around the central value 3.

| BR$(\mu \rightarrow e \gamma)$ | L$^{SU(2)}_1$ | L$^{SU(2)}_2$ | EXP |
|-------------------------------|---------------|---------------|-----|
| $10^{-9}$                     | $10^{-12}$    | $5 \cdot 10^{-13}$ |
| $10^{-8}$                     | $10^{-9}$     | $3 \cdot 10^{-8}$ |
| $10^{-8}$                     | $10^{-10}$    | $4 \cdot 10^{-8}$ |
| $Br(\mu \rightarrow 3e)$     | $10^{-12}$    | $10^{-15}$    | $10^{-12}$ |
| $10^{-10}$                    | $10^{-12}$    | $3 \cdot 10^{-8}$ |
| $10^{-10}$                    | $10^{-12}$    | $2 \cdot 10^{-8}$ |
| $Br(\mu \rightarrow e_T)$    | $10^{-11}$    | $10^{-14}$    | $5 \cdot 10^{-13}$ |

where $U$ is a unitary matrix and $\epsilon_{Re}$ is diagonal. The global symmetry of the theory in the charged sector is $SU(2)_L \times SU(3)_{e_R}$ broken by $\epsilon_{Re}$.

The reduced symmetry allows for new flavor structures and new physics flavor effects are not entirely decoupled. In particular, due to the non-degeneracy of third generation masses and couplings CLFV is generated but one still obtains a suppression of flavor transitions, see [28] for the analog in supersymmetry. We consider two representative examples,

| $L^{SU(2)}_1$ | $m_\rho$ (TeV) | $g_\rho$ | $\epsilon_L$ |
|---------------|----------------|----------|---------------|
| $3$           | $3$            | $(0.05, 0.05, 0.1)$ |

| $L^{SU(2)}_2$ | $m_\rho$ (TeV) | $g_\rho$ | $\epsilon_L$ |
|---------------|----------------|----------|---------------|
| $3$           | $3$            | $(0.01, 0.01, 0.1)$ |

Estimates for CLFV obtained scanning over parameters of the model are reported in Table 2. The most delicate observable is again $Br(\mu \rightarrow e \gamma)$ that is generated due to the difference between $y_1^e/m_\psi_1$ and $y_2^e/m_\psi_2$ in eq. (6). Flavor transitions are suppressed for hierarchical left mixings and can be in agreement with the data for $\epsilon_1/\epsilon_2 < 0.1$ at least for fermions around 3 TeV.

3 Bounds

In this section we discuss the bounds and experimental signatures of composite leptons in MFV scenarios. We focus on left-handed compositeness, similar arguments can be applied to right-handed compositeness.
3.1 Compositeness and Precision Tests

We start with the bounds on effective 4-Fermi operators. A list of experimental results can be found in [19]. With the normalization,

\[ \frac{2\pi}{\Lambda^2} (\bar{f} f)^2 \]

the bound on \( \Lambda \) can be up to 10 TeV. In our case these operators are generated by exchange of spin-1 resonances. The strongest constraint arises from the effective operator \((\bar{L}_L \gamma^\mu l_L)^2\) whose coefficient scales as,

\[ \frac{g^2_{\rho}}{m^2_{\rho}} \epsilon^4_L \]

From this we derive,

\[ \epsilon^2_L < \frac{1}{4 \frac{g_{\rho}}{m_{\rho}}} \times \frac{m_{\rho}}{\text{TeV}} \]

Searches of “excited leptons” place a limit on the operator,

\[ \frac{1}{2} \Lambda \bar{l}^R \sigma^{\mu\nu} \left[ \frac{g}{2} W^a_{\mu\nu} + g' Y^a B_{\mu\nu} \right] l_L \]

A conservative estimate in our scenario is [26],

\[ \frac{1}{\Lambda} \sim \epsilon_L \left( \frac{g_{\psi}}{4\pi} \right)^2 \frac{1}{m_{\psi}} \]

so that \( m_{\psi} \ll \Lambda \). Ref. [25] presents bounds in the plane \((m_{\psi}, \Lambda)\). For \( m_{\psi} < \Lambda \) an approximate bound is \( \Lambda > 11 \text{ TeV} - 5 m_{\psi} \) from which one derives,

\[ \epsilon_L < \frac{1}{10} \left( \frac{4\pi}{g_{\psi}} \right)^2 \times \frac{m_{\psi}}{\text{TeV}}. \]

The bounds above are rather weak. Stronger constraints on left-handed compositeness are found from precisions tests. In particular the coupling of the \( Z \) to muons is measured with per mille precision,

\[ R_{h} = \frac{\Gamma(Z \to q\bar{q})}{\Gamma(Z \to \mu^+\mu^-)} = 20.0767 \pm 0.25. \]

Barring cancellations this implies\(^3\)

\[ \frac{\delta g^L_{Z\mu^+\mu^-}}{g^L_{Z\mu^+\mu^-}} < 0.002 \]

---

\(^3\)Tree level contributions could actually be avoided using protected representations routinely used in the quark sector. In right-handed compositeness this is automatic as long as \( e^L_R \) couple to singlets [3].
From the modified couplings (10) one finds,

$$
\epsilon_L < \frac{1}{5 g_\psi} \times \frac{m_\psi}{\text{TeV}}
$$

(25)

where we assume that the fermion contribution dominates. This bound is similar to one on the mixing of the $b$ quark with the difference that the mixing can be small for leptons. Reproducing the $\tau$ mass indeed requires,

$$
\epsilon_L > \frac{1}{100 g_\psi}
$$

(26)

where the lower bound corresponds to a fully composite $\tau_R$.

Interestingly no bounds follow from lepton flavor universality also measured with per mille precision by LEP, because the shift of the couplings is universal. This is different in the theories based on $SU(2)$ flavor symmetries. The strongest bound follows from,

$$
\frac{\Gamma(Z \to e^+ e^-)}{\Gamma(Z \to \tau^+ \tau^-)} = 0.998 \pm 0.003.
$$

(27)

from which we derive,

$$
v^2 \Delta \left[ g_\psi^2 \epsilon_L^2 \right] < 5 \times 10^{-3}
$$

(28)

Assuming a smaller compositeness for the first two generations this places a bound on the compositeness of $\tau_L$.

Effects on the $T$–parameter are also generated but they are small unless leptons are strongly composite. For example if the the right-handed quarks couple to $SU(2)_R$ doublets the right-handed mixing breaks custodial symmetry and one finds [7],

$$
\hat{T} \sim \frac{g_\psi^4 \epsilon_R^4 \nu^2}{16 \pi^2 m_\psi^2}
$$

(29)

which could be relevant if $\tau_R$ is strongly composite.

Let us briefly discuss Higgs physics. In anarchic models where the Higgs is a pseudo-NGB the corrections to SM rates are typically small, of order $\nu^2/f^2$ where $f$ is the scale of the global symmetry breaking. In MFV one might expect larger effects to $h \to \gamma \gamma$ and other observables with light leptons. For example there is a correction to the coupling $h \gamma \gamma$ [20],

$$
\frac{\delta g_{h\gamma\gamma}}{g_{h\gamma\gamma}} \sim 3 \frac{g_\psi^2 \nu^2}{m_\psi^2} \epsilon_L^2
$$

(30)

where we have included the multiplicity factor for 3 generations. Using the bound from precision tests we get at most a per cent correction, irrelevant for LHC. Things could be different in the scenario with right-handed compositeness where $\epsilon_R$ could be larger, see [21].
3.2 LHC searches

In the anarchic scenario lepton partners are expected to be definitely out of reach for the LHC and indeed no specific searches have been performed so far. In models with MFV the fermions can be light and can be searched at the LHC.

We here consider the type of bounds that can be extracted from present searches. The detailed phenomenology depends on the symmetries of theory and the representations of the new fermions and is beyond the scope of this work. For the purpose of this section we will use the leading order lagrangian in eq. (4) with a vectorial generation of SM leptons. This is equivalent to the renormalizable models considered in [11, 12].

The most relevant LHC search is the one for type 3 see-saw models in Fig. 1 that can be reinterpreted for the lepton partners. In type III see-saw one introduces a fermion triplet of $SU(2)_L$. In Weyl notations, neglecting hyper-charge, the electro-weak interactions are,

$$gW^+_{\mu} \left[ \bar{N}_0 \sigma^\mu N_- - \bar{N}_+ \sigma^\mu N_0 \right] + h.c.$$

$$+ gW^3_{\mu} \left[ \bar{N}_+ \sigma^\mu N_+ - \bar{N}_- \sigma^\mu N_- \right]$$

(31)

For small mixing the mapping between the models in Refs. [11, 12] and eq. (4) is given by $Y_{\epsilon L} \approx \lambda L$, $Y_{\epsilon R} \approx \lambda^F$, $Y_\epsilon \approx \lambda$. In partial compositeness one parameter less is present because the SM Yukawa (2) is determined by the mixings and strong sector coupling.
and the decays widths satisfy [22],
\[ \Gamma[N_0 \rightarrow Wl] \approx 2\Gamma[N_0 \rightarrow Z\nu] \approx 2\Gamma[N_0 \rightarrow h\nu], \]
\[ \Gamma[N_+ \rightarrow W^+\nu] \approx 2\Gamma[N_+ \rightarrow Zl^+] \approx 2\Gamma[N_+ \rightarrow hl^+]. \] (32)

These formulae receive important corrections for light fermions but they will be sufficient for our estimates.

In our model, for the left-handed partners, the interactions are
\[ \frac{g}{\sqrt{2}} W^+_{\mu} \left( \tilde{N}_L \sigma^\mu E_L - \tilde{E}_R^c \sigma^\mu N_R^c \right) + h.c. \]
\[ + \frac{g}{2} W^3_{\mu} \left( \tilde{N}_L \sigma^\mu N_L - \tilde{E}_L^c \sigma^\mu E_L - \tilde{N}_R^c \sigma^\mu N_R^c + \tilde{E}_R^c \sigma^\mu E_R^c \right) \] (33)
The partners of right-handed electrons (\( \tilde{E} \)) only interact with hypercharge so we expect their production to be suppressed and we will not consider them. For the decay we have [10],
\[ \Gamma[E \rightarrow Zl] = \Gamma[E \rightarrow hl] = \frac{1}{2} \Gamma[N \rightarrow Wl] \approx \frac{g^2 e^2_R}{32\pi m_\psi} \] (34)
and we assume no other decay channels to be relevant (for example to \( \tilde{E} \) or decay through dipole interactions in eq. (20), see Ref. [26]). Note that in MFV the fermion partners are degenerate and will decay into the SM states of identical flavor.

The most sensitive search for our purposes is the one by ATLAS of type III see-saw models [27]. This search looks for \( pp \rightarrow N_\pm N_0 \rightarrow 4l \) where 3 leptons come from the decay of \( N_\pm \rightarrow Zl \rightarrow 3l \). This is sensitive to
\[ \sigma(pp \rightarrow N_\pm N_0) \times \text{Br}(N_\pm \rightarrow Zl) \times \text{Br}(N_0 \rightarrow Wl) \] (35)
In the type III see-saw model one finds,
\[ \text{Br}^{III}(N_\pm \rightarrow Zl) \times \text{Br}^{III}(N_0 \rightarrow Wl) \approx \frac{1}{4} \times \frac{1}{2} \] (36)
while with doublets,
\[ \text{Br}^{MFV}(E_{L,R} \rightarrow Zl) \times \text{Br}^{MFV}(N \rightarrow Wl) \approx \frac{1}{2} \times 1 \] (37)
From eqs. (31) and (32) and including a factor of 2 due to the degeneracy of electron and muon partners we find that \( \sigma \times \text{Br} \) is roughly four times as in type III see-saw. From the ATLAS exclusion plot in Fig. 1 a bound around 300 GeV will apply.

Let us mention that other searches could potentially improve the LHC sensitivity as suggested in [22, 23], where final states with lepton+jets were considered in the context of type III see-saw models. Supersymmetric searches with lepton+jets and missing energy could also be reinterpreted for composite leptons if the missing energy cuts are not too strong. The detailed collider phenomenology of light composite leptons will appear elsewhere.
4 Muon \(g−2\)

Both in anarchic and MFV scenarios flavor diagonal observables are generated. Of particular interest is the \(g−2\) of the muon whose experimental value is presently 3.5 \(\sigma\) away from the SM value. From eq. (6) follows the estimate,

\[
\Delta a_{\mu} \sim \left(\frac{g_\psi}{4\pi}\right)^2 \frac{m_\mu^2}{m_\psi^2}
\]

(38)

where as usual \(a_\mu = (g−2)_\mu/2\). A sharp prediction of the MFV scenario is the correlation between electron and muon contributions,

\[
\Delta a_e = \frac{m_e^2}{m_\mu^2} \Delta a_{\mu}
\]

(39)

that is just a general consequence of MFV if the leading operators are allowed but also holds approximately in anarchic models. This is of potential interest as it could be tested in future experiments \[24\].

At face value, to reproduce the muon anomaly, \(\Delta a_{\mu} \approx 2.8\cdot10^{-9}\), one finds

\[
m_\psi \sim g_\psi \times 150\,\text{GeV}
\]

(40)

This rough estimate suggests that the fermions should be very light to account for the \((g−2)_\mu\), in agreement with the result in Randall-Sundrum \[29\] and 4D renormalizable models \[11, 12\]. If we apply the MFV assumption to the latter a large effect for the \(g−2\) is correlated with a modified branching fraction of the Higgs into muons of order 10 times the SM value, that will be soon tested by the LHC. Within our MFV construction exactly the same correction will appear in the \(h\tau\bar{\tau}\) coupling. This is already grossly excluded by LHC data. The same conclusion is expected in composite models where the Higgs is a generic bound state. The required contribution to \(\Delta a_{\mu}\) could instead be consistent in models with \(SU(2)\) flavor symmetry, if the composite lepton partners associated to the third generation are heavier suppressing the modification of the \(\tau\) coupling to the Higgs.

When the Higgs is a NGB the situation is more subtle. These models are characterized by the global symmetry breaking scale \(f > v\) and strongly constrained by the symmetries. The modification of the Higgs couplings is of order \(v^2/f^2\) times a numerical factor that depends on the fermion representations and is small for phenomenologically plausible values of \(f\) (a relatively safe choice is \(f > 800\,\text{GeV}\)). Essentially for the same reason also the effect on the \(g−2\) is small, even for light fermion partners. Indeed parametrically one expects \[17\],

\[
m_\psi \sim g_\psi f
\]

(41)

that is in conflict with (40) since it would imply \(f\) around the electro-weak VEV.
An explicit computation in a model with Higgs NGB is presented in the appendix. The typical size of the contribution is,

\[
\Delta a_\mu \sim \frac{1}{16\pi^2} \times \frac{m_\mu^2}{f^2}
\]  

(42)

In Fig. 2 the result of a scan over parameters of the model is shown. We find that even for \( f \sim 500 \) GeV, a choice problematic for precision tests, Higgs physics and direct searches of top partners, the effect is too small. The sign is typically positive.

A somewhat larger effect can be obtained in certain regions of parameters. In the appendix we show that, for \( m_\psi \gg g_\psi f \), a refined version of the estimate that takes into account the NGB structure is,

\[
\Delta a_\mu \sim \frac{1}{16\pi^2} \times \frac{m_\psi}{g_\psi f} \times \frac{m_\mu^2}{f^2}
\]  

(43)

This indicates that the required effect could be obtained for \( m_\psi \sim 10 g_\psi f \) for \( f = 500 \) GeV. The dependence of these results on the composite fermion representations and models will be studied in [13].

Let us briefly discuss electric dipole moments. In anarchic models the coefficient of dipole operator \([39]\) is complex contributing both to electric and magnetic dipole moments. When the \((g - 2)_\mu\) anomaly is explained, due to \([39]\), the phase must be less than \(10^{-3}\) due to the constrains on the electron EDM. In the composite MFV scenario under consideration the one loop contribution from eq. \([6]\) is automatically aligned with the Yukawas itself so it only contributes to the magnetic dipole. One should still worry about other contributions from CP violating operators in the strong sector. These however are not present under the
minimal assumption that the strong sector respects CP. In this case the physical phases are as in the SM and higher order effects could be sufficiently small.

5 Summary

The hypothesis of partial compositeness allows new attractive realizations of flavor that are relevant for composite Higgs models. We have studied in this paper the leptonic sector.

Summarizing the various possibilities:

- In anarchic scenarios the most important bound arises from the absence of observable $\mu \to e\gamma$ transitions. If the strong sector coupling is large as intuitively expected, then the branching fraction is several orders of magnitude above the experimental limit. Other regions of parameters lead to problem with other observables such as $\mu \to eee$. Overall these bounds appear to invalidate the anarchic hypothesis for leptons.

- Realizing MFV completely solves the flavor problem in the lepton sector and contrary to quarks does not have strong constraints from precision tests or compositeness. The lepton partners could be as light as a few hundreds GeVs and within the reach of the LHC with signatures similar to type III see-saw models. The new fermions contribute to the $(g - 2)$ of the muon. We find that the contribution is most likely too small and only in extreme region of parameters it is conceivable to reproduce the long standing $(g - 2)_\mu$ anomaly. In this case a contribution to $(g - 2)_e$ is predicted and a modification of Higgs couplings should be visible.

- One can extend the MFV paradigm to theories where the third generation is split from the others. This is motivated in the quark sector by the heaviness of the top that suggests an approximate $SU(2)$ flavor symmetry. Practically this interpolates between MFV and anarchic scenarios. In this case, beside $(g - 2)_\mu$ and new fermions, charged lepton flavor violation could be visible in future experiments.

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\footnote{For other approaches see Refs. [15] [30].}
A \ (g-2)_\mu \ with \ Higgs \ NGB

In this appendix we briefly discuss the contributions to the muon \( g-2 \) in models with partial compositeness where the Higgs is a NGB. A detailed study will appear in [18].

We consider the minimal model based on \( SO(5)/SO(4) \) with composite leptons in the \( 5_{-1} \) rep of \( SO(5) \times U(1)_X \). To obtain our estimates we use a setup with a single \( SO(5) \) multiplet of composite fermions and follow the discussion in [20]. The heavy leptons decompose into a doublet of hypercharge \(-1/2 \) \((N, E)\) a doublet of hypercharge \(-3/2 \) \((E_{-1}, E_{-2})\) and a singlet \( \tilde{E} \) with hypercharge \(-1 \). In vector notation this is given by,

\[
F = \frac{1}{\sqrt{2}} \begin{pmatrix}
iE_{-2} - iN \\
E_{-2} + N \\
iE_{-1} + iE \\
E - E_{-1}
\end{pmatrix}
\]  

(44)

The action in the fermion sector reads,

\[
-\mathcal{L} = \lambda_L \bar{L} P_L F_R + \lambda_R \bar{F}_L P_R \mu_R + m \bar{F}_L F_R + g_\psi f (\bar{F}_L \Sigma)(\Sigma^T F_R) + \text{h.c.}
\]  

(45)

where \( P_{L,R} \) are projectors and \( \Sigma \) is a vector parametrizing the Higgs NGB. In the unitary gauge,

\[
\Sigma^T = (0, 0, 0, s_h, c_h)
\]  

(46)

where \( s_h \equiv \sin h/f \) and \( c_h \equiv \sin h/f \).

With a field redefinition the mass matrix in the charged sector takes the following form,

\[
M = \begin{pmatrix}
0 & \frac{\lambda_L (1+c_h)}{2} & \frac{\lambda_R s_h}{\sqrt{2}} \\
\frac{\lambda_R s_h}{\sqrt{2}} & m & 0 \\
\frac{\lambda_R s_h}{\sqrt{2}} & 0 & m
\end{pmatrix}
\]  

(47)

A mixing also appears for left-handed neutrinos. To leading order in the mixings,

\[
m_\mu \approx \frac{g_\psi f \lambda_L}{\sqrt{2}} \frac{\lambda_R}{m + g_\psi f} s_h c_h \approx \frac{v}{\sqrt{2}} \epsilon_L g_\psi \epsilon_{R\mu}
\]  

(48)

In the lepton sector where the mixings are naturally small this provides an excellent approximation. The masses of the heavy states are approximately \( m \) and \( m + g_\psi f \).

Note that the Higgs field appears only in the first line and row, a feature associated to the NGB nature of the Higgs. This differs from [11, 12] with important consequences. Given this mass matrix one can proceed as in those Refs. and compute the correction to the \( g-2 \).
To leading order in the mixings and in the limit $m \gg m_Z$ the result is:

$$\Delta a_\mu \approx \frac{1}{32\pi^2} \frac{m_\mu^2}{f^2} \left(1 + \frac{m}{g_\psi f}\right) + \frac{1}{16\pi^2} \frac{m_\mu^2}{f^2} 
= 4 \times 10^{-10} \times \left(\frac{500 \text{ GeV}}{f}\right)^2 \times \left(1 + \frac{m}{3g_\psi f}\right),$$

that reproduces to good accuracy the points in Fig. 2 obtained in a numerical scan. The effect becomes larger in the region $m \gg g_\psi f$ from which we extract the estimate (43). In the first line we have separated the $W + Z$ from the Higgs contribution, to emphasize that the latter is of similar size contrary to the SM where it is negligible.

References

[1] G. D’Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B 645, 155 (2002) [hep-ph/0207036].

[2] G. Cacciapaglia, C. Csaki, J. Galloway, G. Marandella, J. Terning and A. Weiler, JHEP 0804, 006 (2008) [arXiv:0709.1714 [hep-ph]]; R. Barbieri, G. Isidori and D. Pappadopulo, JHEP 0902, 029 (2009) [arXiv:0811.2888 [hep-ph]]; C. Delaunay, O. Gedalia, S. J. Lee, G. Perez and E. Ponton, Phys. Rev. D 83, 115003 (2011) [arXiv:1007.0243 [hep-ph]].

[3] M. Redi and A. Weiler, JHEP 1111, 108 (2011) [arXiv:1106.6357 [hep-ph]].

[4] K. Agashe, G. Perez and A. Soni, Phys. Rev. D 71, 016002 (2005) [hep-ph/0408134]; C. Csaki, A. Falkowski and A. Weiler, JHEP 0809, 008 (2008) [arXiv:0804.1954 [hep-ph]]; M. Blanke, A. J. Buras, B. Duling, S. Gori and A. Weiler, JHEP 0903, 001 (2009) [arXiv:0809.1073 [hep-ph]]; M. Bauer, S. Casagrande, U. Haisch and M. Neubert, JHEP 1009, 017 (2010) [arXiv:0912.1625 [hep-ph]].

[5] R. Barbieri, D. Buttazzo, F. Sala and D. M. Straub, JHEP 1207, 181 (2012) [arXiv:1203.4218 [hep-ph]].

[6] M. Redi, “Composite MFV and Beyond,” Eur. Phys. J. C 72, 2030 (2012) [arXiv:1203.4220 [hep-ph]].

[7] G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, JHEP 0706, 045 (2007) [hep-ph/0703164].

[8] K. Agashe, T. Okui and R. Sundrum, Phys. Rev. Lett. 102, 101801 (2009) [arXiv:0810.1277 [hep-ph]].
[9] B. Keren-Zur, P. Lodone, M. Nardecchia, D. Pappadopulo, R. Rattazzi and L. Vecchi, Nucl. Phys. B 867, 429 (2013) [arXiv:1205.5803 [hep-ph]].

[10] R. Contino, T. Kramer, M. Son and R. Sundrum, JHEP 0705, 074 (2007) [hep-ph/0612180].

[11] K. Kannike, M. Raidal, D. M. Straub and A. Strumia, JHEP 1202, 106 (2012) [Erratum-ibid. 1210, 136 (2012)] [arXiv:1111.2551 [hep-ph]].

[12] R. Dermisek and A. Raval, arXiv:1305.3522 [hep-ph].

[13] K. Agashe, A. E. Blechman and F. Petriello, Phys. Rev. D 74, 053011 (2006) [hep-ph/0606021].

[14] C. Csaki, Y. Grossman, P. Tanedo and Y. Tsai, Phys. Rev. D 83, 073002 (2011) [arXiv:1004.2037 [hep-ph]].

[15] K. Agashe, Phys. Rev. D 80, 115020 (2009) [arXiv:0902.2400 [hep-ph]].

[16] J. Adam et al. [MEG Collaboration], arXiv:1303.0754 [hep-ex].

[17] A. Simone, O. Matsedonskyi, R. Rattazzi and A. Wulzer, JHEP 1304, 004 (2013) [arXiv:1211.5663 [hep-ph]].

[18] M. Redi, “Dipoles in Composite Higgs models,” work in progress.

[19] http://pdg.lbl.gov/2011/listings/rpp2011-list-quark-lepton-compositeness.pdf

[20] A. Azatov and J. Galloway, Phys. Rev. D 85, 055013 (2012) [arXiv:1110.5646 [hep-ph]].

[21] Cd. Delaunay, C. Grojean and G. Perez, arXiv:1303.5701 [hep-ph].

[22] R. Franceschini, T. Hambye and A. Strumia, Phys. Rev. D 78, 033002 (2008) [arXiv:0805.1613 [hep-ph]].

[23] E. Del Nobile, R. Franceschini, D. Pappadopulo and A. Strumia, Nucl. Phys. B 826, 217 (2010) [arXiv:0908.1567 [hep-ph]].

[24] G. F. Giudice, P. Paradisi and M. Passera, JHEP 1211, 113 (2012) [arXiv:1208.6583 [hep-ph]].

[25] G. Aad et al. [ATLAS Collaboration], Phys. Rev. D 85, 072003 (2012) [arXiv:1201.3293 [hep-ex]].

[26] M. Redi, V. Sanz, M. de Vries and A. Weiler, arXiv:1305.3818 [hep-ph].

[27] ATLAS Collaboration, http://cds.cern.ch/record/1525526/files/ATLAS-CONF-2013-019.pdf
[28] G. Blankenburg, G. Isidori and J. Jones-Perez, Eur. Phys. J. C 72, 2126 (2012) [arXiv:1204.0688 [hep-ph]].

[29] M. Beneke, P. Dey and J. Rohrwild, arXiv:1209.5897 [hep-ph].

[30] G. Perez, L. Randall, JHEP 0901 (2009) 077. [arXiv:0805.4652 [hep-ph]]; M. -C. Chen, H. -B. Yu, Phys. Lett. B672 (2009) 253-256. [arXiv:0804.2503 [hep-ph]]; C. Csaki, C. Delau-nay, C. Grojean and Y. Grossman, JHEP 0810 (2008) 055 [arXiv:0806.0356 [hep-ph]]; F. del Aguila, A. Carmona and J. Santiago, JHEP 1008 (2010) 127 [arXiv:1001.5151 [hep-ph]]; A. Kadosh, E. Pallante, JHEP 1008 (2010) 115 [arXiv:1004.0321 [hep-ph]]; C. Hagedorn and M. Serone, JHEP 1110, 083 (2011) [arXiv:1106.4021 [hep-ph]]; A. Kadosh, JHEP 1306, 114 (2013) [arXiv:1303.2645 [hep-ph]].