INNER MAGNETOSPHERIC ACCELERATORS IN HIGH MAGNETIC FIELD PULSARS

BING ZHANG\textsuperscript{1}, ALICE K. HARDING\textsuperscript{2}
\textsuperscript{1}Astronomy \& Astrophysics Department, Pennsylvania State University, USA
\textsuperscript{2}Laboratory of High Energy Astrophysics, NASA Goddard Space Flight Center, USA

ABSTRACT. Photon splitting is a QED process that can potentially suppress pair production in the inner magnetosphere of a pulsar with a super-strong magnetic field, and hence, may quench radio emission from these objects. While it is unknown how many splitting modes really operate in super-critical fields, we derive gap parameters of a high magnetic field pulsar for both the vacuum-type gap and the space-charge-limited flow accelerator under the assumption that all three splitting modes permitted by QED operate. The competition between photon splitting and pair production depends on the gap parameters, and the “photon-splitting-dominant” line for both cases are derived. We discuss the implications of these results and the possible connection of the high magnetic field pulsars with the anomalous X-ray pulsars and soft gamma-ray repeaters which are conjectured to be magnetars.

1. Introduction

Growing evidence indicates that soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs) are very likely magnetars, isolated neutron stars with surface magnetic fields of the order of \(10^{14} - 10^{15}\) G (Duncan \& Thompson 1992), although consensus is not fully achieved (see, e.g., Zhang 2001). One prominent feature of these objects is that none of them has been firmly detected to have pulsed radio emission, making them a distinct category from the normal radio pulsars. Radio quiescence of magnetars has been attributed to the function of an exotic QED process, namely photon splitting, which, in a magnetic field in excess of roughly the critical field strength, \(B_\text{q} = 4.4 \times 10^{13}\) G, causes \(\gamma\)-ray photons to split into two lower energy photons before they are materialized to electron-positron pairs in reaction with the strong magnetic fields (Baring \& Harding 1998; 2001). This argument, however, is not certain since a complete pair-suppression requires that both photon polarizations split. This will be realized if all the three photon splitting modes permitted by charge-parity invariance in QED, i.e., \(\perp \rightarrow \parallel \parallel\), \(\perp \rightarrow \perp \perp\), and \(\parallel \rightarrow \perp \parallel\), operate. However, under the weak linear vacuum dispersion limit (which may not be true in a magnetar environment) only one splitting mode \((\perp \rightarrow \parallel \parallel\)) can fulfill the energy and momentum conservation requirements simultaneously (Adler 1971). Recently, three high magnetic field pulsars (HBPs) are detected to be above the “photon splitting deathline” for surface emission proposed by Baring \& Harding (1998), and one of them, PSR J1814-1744 resides very closely to an AXP, 1E 2259+586, in the \(P - \dot{P}\) (period - spindown rate) phase space, casting doubt on the effectiveness of photon splitting to suppress radio emission and the proximity in the nature of HBPs and magnetars.
(Camilo et al. 2000). Zhang & Harding (2000), after investigating the formation condition and the properties of the two types of inner accelerators (space-charge-limited flow, SCLF, or vacuum gap, VG) in HBPs, conjectured that the distinct observational difference between PSR J1814-1744 and 1E 2259+586 is due to a geometric effect, i.e., different orientations of the magnetic pole with respect to the rotational pole. In this interpretation and in our present paper, the assumption that all three photon splitting modes operate is still adopted. Here we investigate this idea in more detail by deriving explicit gap parameters of the HBPs and by re-investigating the competition between photon splitting and pair production for various cases.

2. Vacuum type models: deathline for anti-parallel rotators

If all three photon splitting modes operate, pair production is strongly suppressed when the magnetic field is strong enough. The condition for photon splitting dominating pair production is that the attenuation length for photon splitting gets shorter than that of pair production. We define a line in the \( B_p - P \) (or \( P - \dot{P} \)) diagram which satisfies this condition: the photon-splitting-dominant (PSD) line (Eq. [5]). Here \( B_p \) is the surface magnetic field at the pole for the dipolar field component. The adoptions for the \( \perp \) mode, the \( \parallel \) mode, or the polarization-averaged photons do not change the result significantly, and hereafter we adopt the polarization-averaged attenuation coefficients. A precise study of the competition between photon splitting and pair production can only be attained through numerical simulations (e.g. Baring & Harding 2001). However, present simulations have not included the pulsar inner accelerator physics, which we intend to study here. For this purpose, we derive some approximate analytic expressions for photon splitting.

The polarization-averaged photon splitting attenuation coefficient is (see e.g. Harding, Baring & Gonthier 1997)

\[
T_{sp}(\epsilon) \approx \frac{\alpha^3}{10\pi^2} \frac{1}{\lambda} \left( \frac{19}{315} \right)^2 F(B') B'^6 \epsilon^5 \sin^6 \theta \approx 0.37 F(B') B'^6 \epsilon^5 \sin^6 \theta, \tag{1}
\]

where \( \alpha \) is the fine-structure constant, \( \lambda \) is the Compton wavelength of the electron, and \( F(B') \) is a strong-field modification factor (see Harding et al. 1997 for an explicit expression). Here the photon energy \( \epsilon \) is in units of electron rest energy, the magnetic field \( B' \) is in units of the critical field \( B_{cr} \), and \( \theta \) denotes the angle between the photon momentum and the magnetic field. The function \( F(B') = 1 \) for \( B' << 1 \), but \( \approx 1.9 B'^{-6} \) for \( B' >> 1 \). In the regime \( B' \sim 1 \) in which the photon splitting starts to suppress pair production (which is the most interesting regime we are discussing in this paper), one expects \( F(B') \approx A B'^{-\alpha} \), where \( 0 < \alpha < 6 \), and \( A \) is a factor of the order of unity. Numerical simulations give \( \alpha \approx 9/4 \) (Baring & Harding 2001). The photon splitting attenuation length, \( \lambda_{sp} \), can be obtained by solving \( \int_0^{\lambda_{sp}} T_{sp}(s) ds = 1 \), where \( s \) is the trajectory length of the photon propagation which is approximately \( s \approx \rho \sin \theta \), and \( \rho \) is the curvature radius of the field line. This gives \( \lambda_{sp} \approx 1.5A^{-1/7} B'^{-15/28} \epsilon^{-5/7} \rho^{6/7} \). For a pure dipolar geometry, one has \( \rho = 9.2 \times 10^7 (\text{cm}) P^{1/2} r_{e,6}^{1/2} \xi^{-1} \), where \( r_e = 10^6 (\text{cm}) r_{e,6} \) is the emission altitude, \( P \) is the pulsar period, and \( 0 < \xi = \Theta_s/\Theta_{pc} < 1 \) is the ratio of
the field line magnetic colatitude at the surface to the polar cap angle at the surface. The splitting attenuation length is then

$$\lambda_{sp} \simeq 1.0 \times 10^7 \text{cm} A^{-1/7} B'^{-15/28} e^{-5/7} r_{e,6}^{-3/7} P^{3/7} \xi^{-6/7}. \quad (2)$$

Since $1 < A < 1.9$, $A^{-1/7}$ is approximately unity, and we will drop it out hereafter. For the magnetic field strength considered, pair production should occur as soon as the threshold condition is fulfilled, i.e., $\epsilon \sin \theta = 2$. Thus the pair production attenuation length can be approximated as

$$\lambda_{pp} = 2 \rho e^{-1} \simeq 1.84 \times 10^8 \text{cm} \epsilon^{-1} r_{e,6}^{1/2} P^{1/2} \xi^{-1}. \quad (3)$$

Photon splitting can cause a pulsar to “die” only when the inner accelerator of the pulsar is vacuum-like, formed under the condition of strong binding of charged particles in the neutron star surface, because pair formation must take place near the surface (Zhang & Harding 2000). Therefore the so-called “photon splitting deathline” is only relevant for VG accelerators. One remark is that the deathline of Baring & Harding (2001) is derived under several conditions: (a) emission comes from the surface; (b) field configuration is dipole; (c) photon energy is the “escape energy” for both pair production and photon splitting. We note that although both (a) and (b) can be still adopted, the condition (c) may be not of much physical meaning when taking into account the inner accelerators. This is because the typical energy of the photons produced by the primary particles in the gap is usually different from the escape energy, and more importantly, both $\lambda_{sp}$ and $\lambda_{pp}$ depend on $\epsilon$, but with different dependences. Furthermore, the characteristic $\epsilon$ from the gap also depends on pulsar parameters (e.g., $P$, $B_p$), which will alter the slope of the deathline.

Now we derive the PSD line for VG accelerators. This is physically a radio emission deathline for the anti-parallel rotators (APRs) (Zhang & Harding 2000). First, we neglect photon splitting and derive the gap parameters following the same procedure as Zhang, Harding & Muslimov (2000). The difference here is that we derive the photon mean free path by adopting the near-threshold condition for pair creation. Assuming dipolar configuration at the surface, the curvature radiation (CR) controlled gap has a height $h_{(CR - VG)} = 3.2 \times 10^3 \text{cm} P^{4/7} B'^{-3/7} \xi^{-2/7}$, and the resonant inverse Compton scattering (ICS) controlled gap has a height of $h_{(ICS - VG)} = 3.3 \times 10^3 \text{cm} P^{3/7} B'^{-1} \xi^{-3/7}$.

Notice that although both gaps have a similar height for the typical values, the ICS gap height has a much steeper inverse dependence on $B'$, which means that in slightly higher fields, ICS becomes the dominant mechanism for pair production. We therefore adopt ICS-VG parameters to define the photon splitting deathline. The characteristic photon energy from such an accelerator, which is the lowest photon energy produced within the gap by resonant inverse Compton scattering of the electrons off the thermal photons that can produce pairs if photon splitting is neglected, can be expressed as

$$\epsilon_c(ICS - VG) \simeq 5.5 \times 10^{4} P^{1/14} B' \xi^{-3/7}. \quad (4)$$

Plug Eq.(4) into Eqs. (2) and (3), and using

$$\lambda_{sp} \leq \lambda_{pp} \quad (5)$$
as the definition of photon splitting dominating pair production, the photon splitting deathline for the APRs is (see Fig.1)

\[ B_p \geq 1.0 \times 10^{14} \frac{G(P/1s)^{-10/49}}{\xi^{4/49}} , \]  

(6)

or by adopting \( B_p = 6.4 \times 10^{19} \sqrt{PP} \), \( \dot{P} \geq 2.44 \times 10^{-12}(P/1s)^{-69/49} \xi^{8/49} \). This line has a steeper slope than the one derived using the “escape energy” criterion (Baring & Harding 2001), and allows two HBPs with relatively short periods, i.e., PSR J1119-6127 and PSR J1726-3530, to be below the deathline, which means that they should not be dead even if their gaps are (but not necessary are) vacuum type. The reasons are that the faster pulsars tend to have larger acceleration potentials and hence, produce more energetic photons than the slower pulsars, and that for higher energy photons, even higher magnetic field strength is required for photon splitting to suppress pair production due to the different \( \xi \)-dependences of both attenuation lengths (see Eqs.[2] and [3]). Another remark is that the deathline has a very weak dependence on \( \xi \), thus the deathline is a very nice indicator regardless of the location of the sparks in the polar caps. We note that all known “magnetars” are above this deathline, so that their radio quiescence is understandable if their surface charges are bound to the surface so that the type of their inner gaps is expected to be vacuum-like if they exist.

3. Space-charge-limited flow models: are the gaps lengthened?

PSR J1814-1744 is above the deathline of the APRs, and thus must be a parallel rotator (PR) with a SCLF accelerator, if all three splitting modes operate (Zhang & Harding 2000). In such a case, we can also define a PSD line. But this line is no longer a “deathline”, but rather a line which determines whether the accelerator should be lengthened with respect to its normal height (that derived assuming that photon splitting plays no role). SCLF accelerators with frame-dragging effect taken into account have been explicitly studied by Muslimov & Tsygan (1992) and Harding & Muslimov (1998). Despite the complicated form of the parallel electric field in the gap, some approximations could be made in certain regimes. More specifically, for the solution with upper boundary condition, \( E_{\parallel} \) increases linearly with height in the beginning and gets saturated at a certain level when the gap height is comparable to the polar cap radius (Harding & Muslimov 1998). For young pulsars, the \( E_{\parallel} \) of the gaps have not reached the saturated values, and the gaps are pancake-shaped. This is defined as Regime I. For older pulsars, \( E_{\parallel} \) have attained the saturated values well below the pair formation front. The gap shape in this case is long and narrow, and we define it as the Regime II SCLF gap. For both CR or ICS controlled SCLF, we can have both the cases for regime I and regime II. We then have four types of accelerators. Using the near-threshold pair production condition which is applicable for HBPs, we have derived the gap parameters of all these types. We find that in the phase space where the three HBPs reside, the gaps are marginally in Regime II if the gap is controlled by CR, but are in regime I if the gap is controlled by ICS (PSR J1814-1744 is marginally in regime I). The CR-controlled gap has a height of \( h(CR - SCLF) = 2.4 \times 10^5 cm B^{r-3/4} P^{7/4} R_6^{-5/4} \xi^{-1/2} (\cos \chi)^{-3/4} \), while the ICS-controlled gap has a much
lower height of \( h(\text{ICS} - \text{SCLF}) = 8.1 \times 10^5 \text{cm} B'^{-14/15} P^{13/30} R_w^{3/10} \xi^{-8/15} (\cos \chi)^{-1/15} \)

and a lower potential, where \( \chi \) is the pulsar obliquity. Therefore ICS is the dominant pair production mechanism for the HBPs. Another argument is that in a strong field a stable ICS-controlled accelerator could be formed right above the surface (Harding & Muslimov 1998). The characteristic ICS photon energy produced from such a gap is

\[
\epsilon_c(\text{ICS} - \text{SCLF}) \approx 2.3 \times 10^4 B'^{14/15} P^{1/15} R_w^{1/5} \xi^{-7/15} (\cos \chi)^{1/15},
\]

which is smaller than \( \epsilon_c(\text{ICS} - \text{VG}) \) (Eq.[4]).

The PSD line is therefore expected to be lower. Notice that the dependence on \( \cos \chi \) is rather weak. Using a same criterion (Eq.[5]), the PSD line for the SCLF gaps is then (see Fig.1)

\[
B \geq 3.6 \times 10^{13} G (P/1\text{s})^{-22/113} \xi^{4/113},
\]

or \( \dot{P} \geq 3.16 \times 10^{-13} (P/1\text{s})^{-157/113} \xi^{8/113} \). Notice that this line is only a rough indication due to the analytic approximation adopted here, and detailed numerical calculations are necessary. Nevertheless, with Eq.[6] we find that all the three known HBPs as well as some more pulsars are above this line (Fig.1), which means that the delayed pair production conjectured by Zhang & Harding (2000) is necessary to interpret these pulsars. The gaps in these pulsars must be lengthened to allow pair formation front to occur at a higher altitude. The deathline for the HBPs (which should be PRs), is still the “SCLF deathline” defined according to the binding condition of the electrons, as discussed in Zhang & Harding (2000), which occurs around \( B_p \gtrsim 2 \times 10^{14} \text{G} \).

4. Possible implications: connection with the AXPs/SGRs

All the results presented here are based on the assumption that all three photon splitting modes operate. Therefore the consequences from this picture can be then regarded as the criteria to test the assumption itself. All known HBPs as well as some pulsars with lower fields do need to invoke “delayed pair production” if their gaps are SCLF types, as expected before (Zhang & Harding 2000). The particle luminosity from such an accelerator therefore should be enhanced with respect to that of a normal pulsar. This allows a direct distinction between the two splitting scenarios, since different scenarios may result in different \( \gamma \)-ray and X-ray luminosity predictions. Further work is necessary to reveal such differences. A caution is that the analytic approach adopted here may not be good enough to describe the phase space where PSR J1814-1744 is located, so that the “photon splitting dominant line” may be altered after numerical calculations. Nonetheless, the qualitative conclusion that “delayed pair production” is necessary will not be changed.

If it turns out that in a magnetar environment, still only one splitting mode can operate, some other ideas must be introduced to account for the radio quiescence of the AXPs and the SGRs. One such idea is that the particle flows induced by bursting activities may be able to short out the inner accelerator of a magnetar for a long period of time (Thompson 2000). In this picture, all AXPs must be assumed to have experienced some bursting behaviors recently. Alternatively, radio quiescence in AXPs/SGRs is natural if these objects are powered by accretion from their fossil disks.
Fig. 1. A $B_p - P$ diagram of some pulsars and magnetars, and different PSD lines. Line I is the PSD line for VGs, which is the deathline of the APRs; Line II is the PSD line for SCLFs, which is the critical line to define whether delayed pair formation is necessary; Line III is the Baring & Harding line, which is defined using the escape energy criterion.

Acknowledgements

We thank Matthew Baring for helpful discussions. B.Z. acknowledges NASA NAG5-9192 and NAG5-9193 for support.

References

Adler, S. L. 1971, Ann. Phys. 67, 599.
Baring, M. G., Harding, A. K.: 1998, Astrophys. J. 507, L55.
Baring, M. G., Harding, A. K.: 2001, Astrophys. J. 547, 929.
Camilo, F., et al.: 2000, Astrophys. J. 541, 367
Duncan, R. C., Thompson, C.: 1992, Astrophys. J. 392, L9.
Harding, A. K., Baring, M. G., Gonthier, P. L.: 1997, Astrophys. J. 476, 246.
Harding, A. K., Muslimov, A. G.: 1998, Astrophys. J. 508, 328.
Muslimov, A. G., Tsygan, A. I.: 1992, Mon. Not. R. Astr. Soc. 255, 61.
Thompson, C.: 2000, astro-ph/0010016
Zhang, B.: 2001, in these proceedings.
Zhang, B., Harding, A. K.: 2000, Astrophys. J. 535, L51.
Zhang, B., Harding, A. K., Muslimov, A. G.: 2000, Astrophys. J. 531, L135.