Twisting Toroidal Magnetic Fields and the Seasonal Oscillation of the Solar Neutrino Flux II

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Abstract

An intriguing possibility is explored that the solar neutrino data could be used as a probe to the magnetic field structure in the sun. Various cyclic phenomena occurring on the surface of the sun have been accounted for by the so-called dynamo-mechanism. According to this, a self-generating mechanism of solar cyclic activities gives rise to twisting toroidal magnetic fields in the convective zone. Although its magnitude is not known, the orientation of the twist is certainly opposite in northern and southern hemispheres of the sun. We show by numerical calculation that the solar neutrino flux, being sensitive to the twist, could exhibit observable seasonal oscillation, provided that the twist is sizable in magnitude and the neutrinos have reasonably large magnetic moments. This oscillation is ascribed to the fact that the earth’s orbit is slightly inclined to the sun’s equator, and that solar neutrinos pass through the northern (southern) hemisphere of the sun around September (March). We also argue that similar seasonal oscillation could be exhibited in the azimuthal asymmetry of recoiling electrons scattered by the solar neutrinos which are expected to be observed in the super-Kamiokande detector.
1. INTRODUCTION

The deficit of the solar neutrinos observed in Homestake [1], Kamiokande II (KII) [2] and the gallium experiments [3,4] has been one of the strongest stimuli for theoretical ideas in elementary particle physics. Among many others, the most interesting mechanisms which attracted a lot of attention to solve the solar neutrino puzzles are: (1) resonant amplification of neutrino oscillation proposed by Mikheyev and Smirnov and Wolfenstein (MSW) [5], and (2) possibility of large magnetic moments of neutrinos, which convert solar neutrinos into sterile right-handed ones [6,7]. These seminal ideas were followed by further elaboration thereof [8] and a few review articles [9] on these subjects are now available.

These mechanisms have been expected to open new windows in neutrino physics, and to suggest what lies beyond the standard model. In the present paper, however, we would like to take a little bit different attitude of looking at the solar neutrino data: we will explore a possibility to make use of the solar neutrino data to investigate the magnetic structure of the sun, supposing that neutrinos are endowed with reasonably large magnetic moment.

The observation of solar activities has a long history of its own. Everybody now agrees that phenomena occurring on the surface of the sun are outer manifestations of inner dynamics which controls the magnetic structure of the so-called convective zones of the sun. A task set upon solar physicists has been to find a coherent explanation of self-generating mechanisms of the cyclic phenomena of the sun. The cycle is such that the toroidal magnetic force lines undergo periodical change in orientation in the convective zones, which leads to the 11-year cycle of the sunspots occurrence.

One of the important implications of the solar dynamo is that the toroidal magnetic field receives twisting in the course of its time-development. Interesting enough
is that the orientation of the twisting is very likely to be opposite in the northern and southern hemispheres of the sun. We will focus our eyes on this peculiar fact, and will investigate the possibility that the evidence in favor of opposite twisting in each hemisphere has already been encoded in the solar neutrino data.

The study of the neutrino propagation in the presence of twisting magnetic fields was started by Vidal and Wudka [10] and also by Aneziris and Schecter [11]. Smirnov [12] later clarified the implications of the twist and pointed out a new mechanism of resonant neutrino oscillations. Qualitatively we can easily speculate the following phenomena. First of all let us recall that the earth's orbit makes an angle of $7.25^\circ$ with the plane of the solar equator. In other words, neutrinos observed on the earth have passed through the northern (southern) hemisphere of the sun around September (March). This means that neutrinos would experience twisting magnetic fields in opposite directions depending on the season, i.e., effects of the twisting fields on the neutrino flux would exhibit semi-annual variation.

The purpose of the present paper is to put these qualitative considerations on a more quantitative basis. Preliminary results have been reported in ref. [13]. We will show here that, with reasonably large magnetic moments, solar magnetic fields and sizable twisting, the total amount of solar neutrino flux should go up and down every year. We will also argue that there should be seasonal change in the observation of the azimuthal asymmetry in the distribution of recoiling electrons scattered by neutrinos which are to be observed in the super-Kamiokande detectors.

The seasonal variation to be discussed in this paper reminds us of the similar one discussed several years ago by Okun, Voloshin, and Vysotsky [6]. They were aware of the fact that there is a small equatorial “slit” in the solar magnetic field where the magnetic strength is very weak. They noticed that there would be a maximum in the left-handed neutrino flux in June and December. Perhaps it should be born in our mind that, while their ideas are interesting in their own right, the origin of
their seasonal variation differs from ours. In their case the latitude dependence of the magnetic field strength is the source of the variation, while in our case the twist is playing the crucial role.

The present paper is organized as follows. First of all in sec. 2, we recapitulate the neutrino evolution equation to explain how the twisting magnetic field could affect the neutrino flux. In sec. 3, we describe the profile of the solar magnetic field, and set up a simple model for it which will be used in our numerical calculations in sec. 5. The azimuthal asymmetry of electron recoil in KII (and/or super-Kamiokande) detectors is discussed in sec. 4 on the basis of the model in sec. 3. Sec. 6 is devoted to discussions and summary.

2. EVOLUTION EQUATION AND TWIST

Let us begin with the evolution equation of neutrinos in the presence of twisting magnetic fields. The evolution equation is usually discussed in a parallel way with the neutral kaon system. In the presence of magnetic fields coupled to neutrino spin, the evolution equation is to be derived on the basis of relativistic wave equations. For a pedagogical derivation, readers are referred to Mannheim’s paper [14], where both Dirac and Majorana cases are discussed.

In our present investigation, we consider Majorana neutrinos, whose time evolution is described by the Schrödinger equation

\[
\frac{d\Psi}{dt} = H\Psi, \tag{1}
\]

where we include into our state vector \(\Psi\), electron and muon neutrinos and their charge conjugation (denoted by \(C\))

\[
\Psi^T = (\nu_{eL}, \nu_{\mu L}, (\nu_{eL})^C, (\nu_{\mu L})^C). \tag{2}
\]
Inclusion of tau neutrinos are rather straightforward. To give a definite form of the Hamiltonian in (1), we take henceforth the moving direction of neutrinos the $z$-axis. Thereby the Hamiltonian becomes.

$$H = \begin{pmatrix} H_+ & M \\ M^\dagger & H_- \end{pmatrix},$$

where

$$H_\pm = \begin{pmatrix} \pm V_e & \Delta m^2 \sin \theta/4E \\ \Delta m^2 \sin \theta/4E & \pm V_\mu + \Delta m^2 \cos 2\theta/2E \end{pmatrix},$$

$$M = \begin{pmatrix} 0 & -\mu(B_x + iB_y) \\ -\mu(B_x - iB_y) & 0 \end{pmatrix}. \tag{5}$$

The matter potentials perceived by electron and muon neutrinos are denoted by $V_e = G(2N_e - N_n)/\sqrt{2}$ and $V_\mu = -GN_n/\sqrt{2}$, respectively, where $N_e$ and $N_n$ are the number densities of electrons and neutrons, respectively. The lepton-number violating Majorana masses $m_{ee}$, $m_{\mu\mu}$ and $m_{e\mu}$ appear in the evolution equation only in the combination of $\Delta m^2 = (m_{ee}^2 - m_{\mu\mu}^2)^2 + 4m_{e\mu}^4$, together with the mixing angle $\theta$.

In the course of deriving eq. (3), it has been assumed that the neutrino energy $E$, neutrino (transition) magnetic moment $\mu$, magnetic field strength $B$, and neutrino masses satisfy $E \gg m_{ij}(i, j = e, \mu)$, $E \gg \mu B$. The reason for the presence of the term

$$B_x \pm iB_y \equiv B_T \exp(\pm i\phi) \tag{6}$$

is due to the fact that the off-diagonal elements in $M$ come from the spin-flipping terms of the magnetic moment interactions. Terms that contain the $z$-component of
the magnetic field $B_z$ would appear in the off-diagonal part of $H_\pm$. They are, however, suppressed by a power $\mu B_z/E$ and have been neglected in (3) in our approximation. The phase $\phi$ is often referred to as geometrical or topological phases, because it would give rise to Berry’s phase [15] if it were adiabatically time-dependent.

To have a physical insight into the phase $\phi$ in eq. (6), it is more sensible to truncate our $4 \times 4$ matrix into the following $2 \times 2$ matrix equation

$$i \frac{d}{dz} \begin{pmatrix} \nu_{eL} \\ (\nu_{\mu L})^C \end{pmatrix} = \begin{pmatrix} V/2 & -\mu B_T \exp(i\phi) \\ -\mu B_T \exp(-i\phi) & -V/2 \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ (\nu_{\mu L})^C \end{pmatrix}.$$  \hspace{1cm} (7)

The potentials $V_e$ and $V_\mu$ have been replaced in the above by $\pm V/2$, where

$$V \equiv V_e + V_\mu = \sqrt{2}G(N_e - N_n).$$  \hspace{1cm} (8)

This is allowed because we can always take out common diagonal phase. Note also that the time derivative has been replaced by the $z$-derivative in (7), since neutrinos travel nearly with the light velocity. The mass term $\Delta m^2$ has been neglected in (7) in order to make our argument as definite as possible. The combination of $\nu_{eL}$ and $(\nu_{\mu L})^C$ as a Dirac field has been considered by Zel’dovich and Konopinski and Mahmoud (ZKM) [16] and will be referred to as ZKM Dirac scheme.

The effect of geometrical phase was first discussed in the hope that the existence of the phase might be efficient enough to solve the solar neutrino problem even with very small neutrino magnetic moment. Further investigation, however, showed that this hope was an illusion and that the adiabatic approximation used often in connection with Berry’s phase was not applicable in this problem [11, 12, 17]. Although the original hope was turned down, Smirnov [12] realized that there exists a potentially interesting conspiracy between the geometrical phase and the resonant spin-flip as well as resonant spin-flavor transitions.
The best way to see the effect of the phase is, following Smirnov, to transfer to the new state basis
\[
\begin{pmatrix}
\tilde{\nu}_{eL} \\
(\tilde{\nu}_{\mu L})^C
\end{pmatrix} = \begin{pmatrix}
\exp(-i\phi/2) & 0 \\
0 & \exp(i\phi/2)
\end{pmatrix} \begin{pmatrix}
\nu_{eL} \\
(\nu_{\mu L})^C
\end{pmatrix}.
\]

(9)

The change of the basis transforms eq. (7) into
\[
\frac{d}{dz} \left( \begin{pmatrix}
\tilde{\nu}_{eL} \\
(\tilde{\nu}_{\mu L})^C
\end{pmatrix} \right) = \begin{pmatrix}
(V' + \phi')/2 & -\mu B_T \\
-\mu B_T & -(V' + \phi')/2
\end{pmatrix} \begin{pmatrix}
\tilde{\nu}_{eL} \\
(\tilde{\nu}_{\mu L})^C
\end{pmatrix},
\]

(10)

where \(\phi'\) is the space derivative along the direction of the neutrino path. It is now obvious from eq. (10) that spatial variation of the phase amounts to either increasing \((\phi' > 0)\) or decreasing \((\phi' < 0)\) the potential. The resonant spin-flavor transition is modified sensitively in accordance with the sign of \(\phi'\).

3. A SIMPLE MODEL OF TWISTING TOROIDAL MAGNETIC FIELDS IN THE SUN

The solar activities and in particular the solar magnetic field profiles seem to be extremely complicated at first sight. Observations over many years, however, revealed characteristic patterns of solar activities. One of the most familiar facts is the cyclic occurrence of sunspots in every 11 years. Observational investigations show that the sunspot zones drift equatorially both in northern and southern hemispheres. The magnetic nature of sunspots has also been studied. The sunspots show up always in pair and the polarity of the pair is opposite. Notably, alternations in magnetic-field polarity of bipolar sunspot groups in every 11 years have been confirmed and the oscillation period is now doubled to 22 years. The sunspots give us much information of the lower and middle latitude of the sun, while around the
poles the polarity of the magnetic fields are known to change at the maximum phase of the solar cycles.

These observations were extremely strong driving forces to figure out the origin of the self-sustaining mechanism of the cyclic phenomena. An important ingredient to study the solar magnetic fields is the differential rotation of the sun. In plasma fluid with high electric conductivities, magnetic force lines tend to go along the motion of the fluid. The differential rotation elucidates therefore how to generate toroidal magnetic fields from poloidal ones. Although the differential rotation is crucial to explain some of the solar phenomena, another mechanism converting the orientation of the toroidal magnetic fields had to be looked for elsewhere to explain the cyclic phenomena.

The global convection current has been called for to fill in the missing link of the cycle. The global convection is a global-scale fluid motion in the convective zone, extending over the sphere. Qualitatively, the Coriolis force is responsible for twisting the magnetic fields and the time-evolution processes of magnetic fields are illustrated in Fig. 1 schematically following the work of Yoshimura [18]. One can see in Fig. 1 that the toroidal magnetic fields are twisted in the course of development. The reversed toroidal magnetic fields locate deep inside the convective zone. The toroidal fields in the outer area are pushed toward the surface of the sun, spread and create sunspots, and then dissipate. Eventually they are replaced by the reversed toroidal fields, that is the polarity reversal at the maximum phase of the solar cycle.

A quantitative analysis of the solar cycle requires detailed study of the magneto-hydrodynamical (MHD) equations, which describe the time evolution of magnetic fields under given velocity fields of plasma media. To handle the MHD equations analytically, however, is nearly impossible and numerical integrations are the best means to solve the MHD equations.

Yoshimura [18] has been engaged in elaborating numerical solutions to the MHD
equations. He prescribed the velocity fields of differential rotation and global convections by introducing several adjustable parameters. His numerical computations reproduce the basic characteristics of the solar activities. It has been observed that magnetic field tubes (torus) encircling the rotational axis of the sun appear naturally. The reversal of orientation of toroidal magnetic fields takes place inside the convection zone. The reversed magnetic fields propagate towards the surface of the sun, which turns out to be solar activities of the sunspots. The parameter dependence of his simulation was also examined. Although the magnetic fields around the circling tube is twisting, the twisting parameter is not uniquely determined.

In the present paper, being motivated by Yoshimura’s computer simulation, we will use a simplified model of the solar magnetic fields. We consider toroidal magnetic fields winding along the tori which sit in the northern and southern convective areas (Fig. 2). Each torus is parametrized by its radius $a$, the latitude of its center $\Delta$, and the distance $b$ measured from the center of the sun. The latitude $\lambda$ in Fig. 2 shows the direction of the path of neutrinos which are assumed to have been produced at the center of the sun.

We take the magnetic fields to be distributed symmetrically around the rotating axis of the sun and the twisting proceeds at the constant rate along the torus. We introduce a parameter $X$ which parametrizes the degree of the twist and is defined as the distance along the torus to wind it once. (See Fig. 3.) The orientation of the twist varies in the first and second 11 years and is opposite in the northern and southern tori. The patterns are schematically shown in Fig. 4.

The direction of neutrino propagation is taken to be $z$-axis and the longitudinal (latitudinal) direction along the $x$ ($y$)-axis. In this coordinate system the twisting
magnetic fields are parametrized by

\[
\begin{pmatrix}
B_x \\
B_y \\
B_z
\end{pmatrix} = B \begin{pmatrix}
n_x \cos \theta(z) \\
n_y \sin \theta(z) \cos \alpha(z) \\
n_z \sin \theta(z) \sin \alpha(z)
\end{pmatrix}.
\]

(11)

Here angles \(\theta(z)\) and \(\alpha(z)\) shown in Fig. 5 are defined by

\[
\theta(z) = \tan^{-1}\left[\frac{2\pi R(z)}{X}\right],
\]

(12)

\[
\cos \alpha(z) = \frac{b \cos(\Delta - \lambda) - z}{R(z)},
\]

(13)

with

\[
R(z) = \sqrt{[b \sin(\Delta - \lambda)]^2 + [b \cos(\Delta - \lambda) - z]^2}.
\]

(14)

The parameters \(n_x, n_y\) and \(n_z\) in eq. (11) are sign factors specifying the twist which depends on seasons and either the first or the second periods of 11 years. They are given from Fig. 4 by

\[
(n_x, n_y, n_z) = \begin{cases}
(+, +, -) & \text{for NI: northern hemisphere, first 11 yr,} \\
(-, +, +) & \text{for SI: southern hemisphere, first 11 yr,} \\
(-, -, +) & \text{for NII: northern hemisphere, second 11 yr,} \\
(+, -, -) & \text{for SII: southern hemisphere, second 11 yr.}
\end{cases}
\]

(15)

So much for the parametrization of the magnetic field. We are now in a position to evaluate \(B_T\) and the phase \(\phi\) from eq. (11), and we find

\[
B_T = B \sqrt{\cos^2 \theta(z) + \sin^2 \theta(z) \cos^2 \alpha(z)},
\]

(16)
\[ \phi(z) = n \tan^{-1}\left[ \frac{2\pi}{X} (b \cos(\Delta - \lambda) - z) \right] + \delta. \] (17)

Here a sign factor \( n \) and \( \delta \) are given by \((n, \delta) = (+1, 0), (-1, \pi), (+1, \pi), (-1, 0)\) for NI, SI, NII, and SII, respectively. We are interested in the \( z \)-derivative of \( \phi \) in the evolution equation

\[
\phi'(z) = \begin{cases} 
-1 & 
\frac{2\pi/X}{1 + (2\pi/X)^2 [b \cos(\Delta - \lambda) - z]^2} \quad \text{for NI, NII, SI, SII.} 
\end{cases}
\] (18)

This shows clearly that the effect of the phase depends crucially on whether neutrinos pass through the northern or southern hemispheres of the sun.

Qualitatively, if neutrinos go through the northern hemisphere (around September), then the negative sign on the right hand side of eq. (18) indicates suppression of the diagonal element of the Hamiltonian in (3), resulting in an relative enhancement of the off-diagonal effect. We can expect large spin-flip conversion rate around September and considerable neutrino flux suppression. On the contrary if neutrinos are passing through southern hemisphere (around March), then the neutrino flux will be less suppressed. The seasonal contrast depends on the magnitude of the magnetic field. If the magnetic field strength is larger (e.g., in the maximum phase of the solar cycle), then the seasonal variation is rather veiled. This is qualitatively different from what Okun, Voloshin and Vysotsky [6] have considered. In their case, larger the magnetic field, more conspicuous the seasonal variation.

4. AZIMUTHAL ASYMMETRY OF THE ELECTRON RECOIL

Let us turn to another possibility to detect the twist of the solar magnetic field. A conceivable direct evidence of the twist could be found in the azimuthal asymmetry in the distribution of the electrons scattered by solar neutrinos. The origin of the asymmetry is the interference between electromagnetic scatterings due to
the neutrino magnetic moment and weak processes. Such an asymmetry has been discussed by Barbieri and Fiorentini [19] in their search for an evidence of helicity conversion. They considered Dirac neutrinos with magnetic moment.

Neutrinos produced at the center of the sun are purely in the left-handed state, whose probability amplitude is given by \( (\tilde{\nu}_e L, (\tilde{\nu}_\mu L)^C) = (1, 0) \). In the course of traversing the solar convective zones, the amplitudes acquire complex phases. Suppose that the neutrinos just entering the super-Kamiokande detector are described by the amplitudes

\[
(\tilde{\nu}_e L, (\tilde{\nu}_\mu L)^C) = (|a_L| \ e^{i\phi_L}, |a_R| \ e^{i\phi_R}).
\]  

The spin of the neutrinos then has an expectation value

\[
< S > = \begin{pmatrix}
|a_L a_R| \cos(\phi_L - \phi_R) \\
|a_L a_R| \sin(\phi_L - \phi_R) \\
(|a_R|^2 - |a_L|^2)/2.
\end{pmatrix}
\]  

The phases \( \phi_L \) and \( \phi_R \) of the left and right components are calculable numerically by use of (10). We immediately notice that the particular direction of the neutrino spin (20) may produce an asymmetry with reference to this direction. In fact, an asymmetry can be found if we look at the azimuthal distribution of the recoiling electrons. The phases \( \phi_L \) and \( \phi_R \) are sensitive to the twisting magnetic field in the sun, we might be able to expect a semi-annual variation in the azimuthal asymmetry data.

The differential cross sections relevant to our case becomes as follows:

\[
\frac{d\sigma}{dT d\phi} = \frac{d\sigma_{eL}^{\mu L}}{dT d\phi} + \frac{d\sigma_{em}}{dT d\phi} + \frac{d\sigma_{int}^{\mu L}}{dT d\phi} + \frac{d\sigma_{W}^{\bar{\nu}_R}}{dT d\phi} + \frac{d\sigma_{int}^{\bar{\nu}_R}}{dT d\phi}.
\]  

Here, \( T \) is the electron kinetic energy and \( \phi \) is the azimuthal angle of the recoiling electron. The weak gauge-boson exchange contributions for \( e\nu_{eL} \rightarrow e\nu_{eL} \ (e(\nu_{\mu L})^C \rightarrow \)
$e(\nu_{\mu L})^C$ and electromagnetic one are denoted by $d\sigma^\nu_W$, \(d\sigma^\bar{\nu}_W\) and $d\sigma_{em}$, respectively. These cross sections together with their interference ($d\sigma^\nu_{int}$ and $d\sigma^\bar{\nu}_{int}$) are explicitly given by

\[
\frac{d\sigma^\nu_W}{dT \ d\varphi} = |a_L|^2 \frac{G^2 m_e}{\pi^2} \left[ g_L^2 + g_R^2(1 - \frac{T}{E})^2 - g_L g_R \frac{m_e T}{E^2} \right],
\]

\[
\frac{d\sigma_{em}}{dT \ d\varphi} = \left( \frac{\mu}{\mu_B} \right)^2 \frac{\alpha^2}{2m_e^2} \left( \frac{1}{T} - \frac{1}{E}\nu \right),
\]

\[
\frac{d\sigma^\nu_{int}}{dT \ d\varphi} = -(\xi_T \cdot k) \left( \frac{\mu}{\mu_B} \right) \frac{G\alpha}{2\sqrt{2}\pi m_e T} \left[ g_L + g_R \left( 1 - \frac{T}{E}\nu \right) \right],
\]

\[
\frac{d\sigma^\bar{\nu}_W}{dT \ d\varphi} = |a_R|^2 \frac{G^2 m_e}{\pi^2} \left[ g_R^2 + g_L^2(1 - \frac{T}{E})^2 - g_L' g_R \frac{m_e T}{E^2} \right],
\]

\[
\frac{d\sigma^\bar{\nu}_{int}}{dT \ d\varphi} = (\xi_T \cdot k) \left( \frac{\mu}{\mu_B} \right) \frac{G\alpha}{2\sqrt{2}\pi m_e T} \left[ g_R + g_L' \left( 1 - \frac{T}{E}\nu \right) \right].
\]

The couplings are listed below.

\[ g_L = \frac{1}{2} + \sin^2 \theta_W, \quad g_L' = -\frac{1}{2} + \sin^2 \theta_W, \quad g_R = \sin^2 \theta_W. \]

The momentum of the recoiling electron is denoted by $k$. Since we have neglected neutrino masses, these cross sections are common for Majorana and ZKM neutrinos. Note that the cross sections (22), (23) and (24) were considered by Barbieri and Fiorentini [19] and eqs. (25) and (26) are our new addition.

We have introduced a notation $\xi_T$, the transverse component of the neutrino spin $<S>$ (times 2), namely,

\[
\xi_T = 2 \begin{pmatrix}
|a_L a_R| \cos(\varphi_L - \varphi_R) \\
|a_L a_R| \sin(\varphi_L - \varphi_R) \\
0
\end{pmatrix}.
\]

\[ 13 \]
The existence of $\xi_T$ in eq. (24) and (26) shows clearly that the azimuthal distribution of kicked electrons would exhibit asymmetry with respect to the direction (28). Barbieri and Fiorentini [19] have calculated $\varphi_R - \varphi_L$ as a function of $\mu B$ for untwisting magnetic field case. They pointed out that the asymmetry measurement appears to be within the reach of observation in future experiments. The point we would like to stress here is that the phases $\varphi_L$ and $\varphi_R$ are potentially sensitive to $\phi'$ as we see from the evolution equation (10) and we anticipate that the axis of the asymmetry varies from season to season.

Before concluding this section, we present a formula for the suppression rate of neutrino flux observed in KII experiment

$$R(\text{KII}) = \frac{\int dE_\nu \Phi(E_\nu; SSM) \int_{W}^{T_{\text{max}}} dT d\varphi \frac{d\sigma}{dT d\varphi} (|a_L|^2 = 1)}{\int dE_\nu \Phi(E_\nu; SSM) \int_{W}^{T_{\text{max}}} dT d\varphi \frac{d\sigma}{dT d\varphi}}.$$  \hspace{1cm} (29)

Here $\Phi(E_\nu; SSM)$ denotes the neutrino energy spectrum calculated in the standard solar model (SSM). The integration over $T$ is taken from the threshold energy $W$ of KII detector up to maximally possible value $T_{\text{max}} = T_{\text{max}}(E_\nu)$.

The suppression rate differs from $|a_L|^2$, since $(\nu_{\mu L})^C$ also contributes to the detection [20]. If the magnetic moment is on the order of $\left(10^{-10} \sim 10^{-11}\right) \mu_B$, the electromagnetic contribution is negligibly small and this becomes

$$R(\text{KII}) = |a_L|^2 + 0.144 |a_R|^2.$$  \hspace{1cm} (30)

In passing note that the suppression rate in the Homestake experiment is simply given by $R(\text{Cl}) = |a_L|^2$, since their detector is blind to $(\nu_{\mu L})^C$.

5. NUMERICAL ANALYSIS OF THE SOLAR NEUTRINO FLUX AND THE AZIMUTHAL ASYMMETRY
We have undertaken numerical analysis of eq. (10). The integration is performed from \( z = z_0 \) to \( z = z_1 \), where \( z_0 \) and \( z_1 \) are the entrance and exit coordinates of the magnetic zones

\[
z_{0,1} = b \cos(\Delta - \lambda) \pm \sqrt{a^2 - b^2 \sin^2(\Delta - \lambda)}.
\]  

(31)

The initial condition is set as \((\bar{\nu}_{eL}, (\bar{\nu}_{\mu L})^C) = (1, 0)\) at \( z = z_0 \) and various parameters are taken as follows:

\[
\lambda = \pm 7^\circ, \quad \Delta = 15^\circ, \quad a = 0.1694R_\odot, \quad b = 0.8813R_\odot.
\]  

(32)

The values \( \Delta, a, \) and \( b \) were read off from the Yoshimura’s computer simulation. The choice of the latitude of the neutrino’s path \( \lambda = 7^\circ \) \((-7^\circ)\) corresponds to the period from August to October (from February to April). In these latitude, the magnetic field strength varies only a little, and we have assumed in our calculations that \( B \) in eq. (11) is spatially constant. We take the matter potential [21]

\[
V(z) = 0.178R_\odot^{-1}\exp[10.84\sqrt{1 - z/R_\odot}]cm^{-1}
\]  

(33)

in the convective zone.

One might perhaps wonder how it could be that \( a + b \) is greater than the solar radius \( R_\odot \) as in (32). The reason for our choice of the values \( a \) and \( b \) in (32) is as follows. Although we said that magnetic field tubes were formed as in Fig. 2, the shape of the tubes is not an exact torus according to the Yoshimura’s simulation. Since we are thinking of neutrinos passing through in the latitude \( \lambda = \pm 7^\circ \), we have to take a model reproducing the magnetic fields at the edge of the tubes in lower latitudes. The choice of the values \( a \) and \( b \) are taken to realize these realistic magnetic fields along the neutrino path.
We are basically interested in the $\mu B$-dependence of the flux of the left-handed electron neutrinos $P(\nu_L) = |a_L|^2 = 1 - |a_R|^2$, where $a_L$ and $a_R$ are the probability amplitudes in (19). For a better understanding of numerical computation, we note that at $z = 0.85R_\odot$, for example,

$$V(z = 0.85R_\odot) = 1.70 \times 10^{-10} \text{cm}^{-1}, \quad (34)$$

$$\phi'(z = 0.85R_\odot) = \pm (1.44 \times 10^{-11}) \frac{(2\pi R_\odot/X)}{1 + 5.16 \times 10^{-4}(2\pi R_\odot/X)^2} \text{cm}^{-1}. \quad (35)$$

This is to be compared with

$$\mu B = 2.9 \times 10^{-11} \left( \frac{\mu}{10^{-10}\mu_B} \right) \left( \frac{B}{1\text{kG}} \right) \text{cm}^{-1}. \quad (36)$$

In other words, if the twist parameter $X$ is a fraction of $R_\odot$ and $\mu B$ is of the order of $(1 \sim 10) \times 10^{-10} \mu_B \text{kG}$, then the seasonal variation is likely to be seen in $P(\nu_L)$.

In Fig. 6, $P(\nu_L)$ is plotted as a function of $\mu B$, for various twist, $X = (\pi/3)R_\odot$, $X = \pi R_\odot$, $X = 2\pi R_\odot$, and $X = 3\pi R_\odot$. The solid (broken) curves correspond to the cases where neutrinos go through the $7^\circ$ latitude in the northern (southern) hemisphere, that is, around September (March). The solid curve denoted by “toroidal” corresponds to the $X = \infty$ case.

Characteristic features discussed qualitatively can be seen in Fig. 6. When $\phi'(z)$ is negative as in September, then the off-diagonal element of the Hamiltonian (3) is more effective and the spin-flip process occurs more frequently than the opposite case. In fact the neutrino flux in September is smaller than that without twist ($X = \infty$). When $\phi'(z)$ is positive as in March, things become opposite, i.e., the suppression of the flux is weakened. The seasonal differences are distinct for relatively weak magnetic field $\mu B = (2 \sim 6) \times 10^{-10} \mu_B \text{kG}$. For stronger magnetic fields, say, $\mu B = (8 \sim 10) \times 10^{-10} \mu_B \text{kG}$, the seasonal difference is obscured by the frequent
spin-flip processes and even opposite tendency arises in comparison with the weak magnetic field.

To confront these numerical computations with long-term observational data at Homestake and KII, we have to vary the magnetic field strength $B$ in accordance with the phase of the solar cycle. In the light of our poor knowledge on such long-term variation of $B$, we take here a phenomenological approach. Namely, we evaluate $B$ by assuming that it is proportional to $\sqrt{\text{sunspots number}}$. In Fig. 7, the sunspots number in the last two decades are displayed together with our fit. At the maximum phase of the solar cycle, we input a value $(\mu B)_{\text{max}}$, and the 11-year variation of $B$ is determined by making use of Fig. 7.

In Fig. 8, we compare our estimation of the seasonal variation with the $^{37}$Cl data in the last twenty or so years. Our numerical calculation was done for two cases

(a) $X = (\pi/3)R_\odot$, $(\mu B)_{\text{max}} = 10 \times 10^{-10}\mu BkG$,

(b) $X = \pi R_\odot$, $(\mu B)_{\text{max}} = 9 \times 10^{-10}\mu BkG$.

Broken lines connecting predicted values are drawn only for a guide. Our calculations show the seasonal variations more clearly in the quiet period of the sun than in the active period. Although the observational data are afflicted with large error, it is not unfair to say that our predicted fluxes reproduce the data rather well. One can see an up-down structure of the flux in the Cl data. Our claim is that this structure could be interpreted as a manifestation of the twisting magnetic field.

The KII data are also compared with our numerical calculations of (29) in Fig. 9. Our numerical estimate shows again an up-down structure in particular in the quiet period of the sun (i.e., 1986-1988). The available KII data are those in the maximum phase of the sun and the expected structure is not so conspicuous. It is also to be mentioned that the data are averaged over 7 to 9 months and the up-down structure,
if it were present, may have been washed away by the averaging procedure. It is more desirable to take data in a larger detector (like super Kamiokande) without taking average over long term.

The difference of the azimuthal angle $\varphi_R - \varphi_L$ is evaluated in Fig. 10 for the above two cases, (a) and (b). Black (white) triangles are those around September (March) for the case (a) and black (white) circles around September (March) for (b). It is obviously seen that the asymmetry axis swings between two completely different directions. As far as the asymmetry measurement is possible in a large volume detector as was remarked by Barbieri and Fiorentini, the measurement of the seasonal variation in so large angles should also be feasible. It is therefore a good proposal in future experiments to measure the seasonal variation of the asymmetry axis.

Finally we add a little comment on the adiabaticity. In our present calculations we have solved (10) numerically and there is no reason to worry about the adiabaticity. It is, however, more instructive to check the validity of the adiabatic approximation by using our explicit parametrization (18) and (33). The adiabaticity means that the mixing of states proceeds very slowly during the passage of neutrinos in a typical precession length. More explicitly, the adiabaticity is ensured (near the resonance point) provided that

$$2(\mu B)^2 \gg V'(z) + \phi''(z).$$

(37)

Numerically the parametrization (33) tells us $V'(z) = (2 \sim 3) \times 10^{-20}$cm$^{-2}$ for $0.85 < z/R_\odot < 0.90$. On the other hand eq. (18) gives $\phi''(z) = (-4 \sim 4) \times 10^{-20}$cm$^{-2}$ for $0.85 < z/R_\odot < 0.90$ and $X = (\pi/3)R_\odot$. These values should be compared with $(\mu B)^2 = 8.41 \times 10^{-22}(\mu/10^{-10}\mu_B)^2(B/1kG)^2$cm$^{-2}$. One can see therefore that for the parameter range of $\mu B$ adopted in our numerical calculations, the adiabaticity condition is not well satisfied. This is of course consistent with the observation made
previously in literatures [10, 12, 17].

6. SUMMARY AND DISCUSSION

In the present paper we have been seeking for a possibility of making use of the solar neutrino data to probe the solar magnetic structure. More specifically we have examined the effect of twisting magnetic field to the solar neutrino flux and the azimuthal asymmetry which could be measured in the super-Kamiokande experiment. We have pointed out that within a reach of observation there should be semi-annual variation in the solar neutrino flux and the azimuthal asymmetry axis, provided that the twist parameter is sizable and \( \mu B = (1 \sim 10) \times 10^{-10} \mu_B \text{kG} \).

There have been a lot of attempts of putting upper bounds on neutrino magnetic moments from both terrestrial and celestial observations. The Dirac-type as well as Majorana-type magnetic moments are constrained by antineutrino-electron scattering data [22] (\( |\mu| \leq 4 \times 10^{-10} \mu_B \)), and stellar energy loss through neutrino pair emission [23] (\( |\mu| \leq 1 \times 10^{-11} \mu_B \)). There is also a possibility of deriving a bound from a reactor experiment [24]. The synthesis of \(^4\text{He}\) in the big bang gives us a bound for a Dirac-type magnetic moment [25]. There are several studies of restricting the magnetic moment more severely from supernova 1987A [26]. This is, however, still under debate [27]. In any way, the Majorana-type moment is not constrained by supernova 1987A. Considering the ambiguity of the solar magnetic field strength, we believe that our choice of the parameter \( \mu B \) is within a reasonable range.

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Figure Captions

Fig. 1: Schematic pictures of the twist of toroidal magnetic fields in the convective zone. The twist is generated by the differential rotations of the sun and the global convection of the plasma fluid. These pictures are given from the work of Yoshimura [18].

Fig. 2: A model of twisting toroidal magnetic fields. For simplicity, we assume that magnetic fields are winding along the tori which are located in the northern and southern hemispheres. The parameter $\lambda$ represents the latitude of the path of neutrino which is assumed to be created at the center of the sun. The latitudes of neutrino paths are restricted by $|\lambda| \leq 7.25^\circ$.

Fig. 3: Parametrization of the twist by $X$ which is the distance along the torus to wind it once.

Fig. 4: Twists and the direction of toroidal magnetic fields. They change symmetrically, depending on the hemisphere and the period.

Fig. 5: Description of toroidal magnetic fields.

Fig. 6: The $\mu B$ dependence of the left handed neutrino flux for various magnitude of the twist parameter $X = (\pi/3)R_\odot$, $X = \pi R_\odot$, $X = 2\pi R_\odot$, $X = 3\pi R_\odot$. Solid lines represent cases where neutrinos pass through the northern hemisphere, while broken lines correspond to cases where neutrinos pass through the southern hemisphere. The curve denoted by “toroidal” corresponds to the $X = \infty$ case.

Fig. 7: Fit of sunspot numbers which we used to obtain the time variation of the magnitude of magnetic fields.

Fig. 8: Comparison between our predictions and the Cl-Ar data. Broken lines are drawn as a guide. The up-down structure of our predictions is the inevitable
consequences of the twist of toroidal magnetic fields. We have analyzed two cases: (a) \( X = (\pi/3)R_\odot, (\mu B)_{\text{max}} = 10 \times 10^{-10} \mu B \text{kG} \) and (b) \( X = \pi R_\odot, (\mu B)_{\text{max}} = 9 \times 10^{-10} \mu B \text{kG} \).

Fig. 9: Comparison between our predictions and the KII data. Our predictions show a structure, but the KII data do not. This is because their data are those averaged for 7 to 9 months which are too long to see the structure. Also their observation are almost in the maximum phase of the solar magnetic fields where seasonal difference is minimum.

Fig. 10: Theoretical prediction of the difference of the azimuthal angle \( \phi_R - \phi_L \) in the last two decades. Black (white) triangles are those around September (March) for (a) \( X = (\pi/3)R_\odot, (\mu B)_{\text{max}} = 10 \times 10^{-10} \mu B \text{kG} \). and black (white) circles those around September (March) for (b) \( X = \pi R_\odot, (\mu B)_{\text{max}} = 9 \times 10^{-10} \mu B \text{kG} \).