Coherence distribution in multipartite systems

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Abstract
This paper examines the coherence in multipartite systems. We first discuss the distribution of total coherence in a given multipartite quantum state into discord between subsystems and coherent dissonance in each individual subsystem, using the relative entropy as a distance measure. Then we give some trade-off relations between various types of coherence and discord within a bipartite system, and extend these results to the multipartite setting. Finally, the change of coherence in entanglement distribution is studied.

Keywords: coherence distribution, quantum correlations, quantum coherence

(Some figures may appear in colour only in the online journal)

1. Introduction

Quantum coherence arising from quantum superposition plays a central role in quantum mechanics, and it is also a common necessary condition for entanglement and other types of quantum correlations such as Bell nonlocality and quantum discord. Recently, researchers have begun to develop a resource-theoretic framework for understanding quantum coherence [1, 2]; for more discussions we refer to [3, 4].

It is well-known that nonclassical correlation (e.g. entanglement, discord) between subsystems is a form of coherence, but the converse is not necessarily true: coherence may also exist in individual subsystems. As suggested by Modi et al [5], for a quantum state \( \rho \), its nonclassical correlation is measured by the relative entropy of discord, which indicates the distance of \( \rho \) to the nearest classical state \( \chi_\rho \). Since coherence is a basis-dependent quantity, the nearest classical state \( \chi_\rho \) may not be diagonalizable under the reference incoherent states. This means that \( \chi_\rho \) may possess some coherence located in the subsystems. More extremely, product states in which no nonclassical correlation exists at all can be maximally coherent. For example, the product state \( |+\rangle|+\rangle \) where \( |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \) on a two qubit system is a maximally
coherent state with the basis \{\ket{ij}\}_{i,j=0}^1. Despite the considerable attempts to understand this phenomenon [6–13], one of the most important questions remains unresolved: How can we quantify the collective coherence and local coherence, and characterize the relations between them and the nonclassical correlation in a multipartite system? The answer to this question is also of importance for the analysis of quantum algorithms where coherent superposition is often essential.

In this paper, we examine the distribution of total coherence in a given multipartite quantum state into discord and coherent dissonance using the relative entropy as a distance measure. Trade-off relations between various types of coherence and discord within both bipartite and multipartite systems are also considered. The paper is organized as follows. In section 2, we recall a unified measure to quantify different correlations like discord, coherence, and coherent dissonance of multipartite quantum states. In section 3, we give some relations between these correlations for bipartite systems, when local measurements on one party are considered. These results are then generalized to the multipartite setting. We discuss the change of coherence in entanglement distribution in sections 4 and 5 is devoted to a brief conclusion.

2. Additivity relations for discord and coherence

Following the framework in [5], we employ relative entropy as a measure of distance to characterize the separation of total coherence into discord and coherent dissonance. For an \(N\)-partite system \(\mathcal{H}\), let \(\{\ket{k_i^j} : 1 \leq j \leq n_i\}\) be a fixed basis for system \(i\) where \(1 \leq i \leq N\), and \(\mathcal{K} := \{\ket{\bar{k}}\}\) the product basis of \(\mathcal{H}\) induced by them. The set of incoherent states with respect to \(\mathcal{K}\), denoted \(\mathcal{I}_K\), contains all locally distinguishable states

\[
\sum_{\ket{\bar{k}} \in \mathcal{K}} p_{\bar{k}} \bra{\bar{k}} \rho \ket{\bar{k}},
\]

where \(\{p_{\bar{k}}\}\) is a joint probability distribution. Note that \(\mathcal{I}_K\) is a convex and compact set. Furthermore, a state is said to be classical, if it is incoherent with respect to some product basis of \(\mathcal{H}\). We denote the set of classical states by \(\mathcal{I}_N\). Obviously,

\[
\mathcal{I}_N := \bigcup_{\mathcal{K}} \mathcal{I}_K.
\]

Let \(\rho\) be an \(N\)-partite state, and \(\mathcal{K}\) a product basis of \(\mathcal{H}\). The coherence of \(\rho\) with respect to \(\mathcal{K}\) can be measured by the relative entropy of discord, which is the distance between \(\rho\) and the nearest classical state in \(\mathcal{I}_K\) [2, 3], that is,

\[
C^K(\rho) := \min_{\delta \in \mathcal{I}_K} S(\rho|\delta).
\]

We denote by \(\delta^K := \sum_{\ket{\bar{k}} \in \mathcal{K}} (\bra{\bar{k}} \rho \ket{\bar{k}})/\mathcal{I}_K\) this nearest state achieving the minimum in equation (3) [2, 3]. It is well known that coherence has a nice closed expression, given by the entropy change caused by the dephasing operation on the state:

\[
C^K(\rho) = S(\delta^K) - S(\rho).
\]

On the other hand, we define the discord of \(\rho\) as the nonclassical correlation between different subsystems, which is characterized by the smallest distance between \(\rho\) and any classical state. That is,

\[
Q(\rho) := \min_{\chi \in \mathcal{I}_N} S(\rho|\chi).
\]
Note that the set \( \mathcal{I}_K \) is not a convex set; mixing two classical states written in different bases can give rise to a nonclassical state. Since the minimization of the relative entropy over classical states is identical to the minimization of the entropy \( S(\chi) \) over the choice of local basis \( \{\tilde{k}\} \). Modi et al gave a useful expression [5]

\[
Q(\rho) = S(\chi_{\rho}) - S(\rho),
\]

where \( S(\chi_{\rho}) = \min_{\{\tilde{k}\}} S(\sum_k |\tilde{k}\rangle\langle \tilde{k}| \rho |\tilde{k}\rangle\langle \tilde{k}|). \) Note that discord is independent of the choice of basis, from the point of view of [8], it is intrinsic coherence.

We denote by \( \chi_{\rho} \) this nearest state achieving the minimum in equation (5). Note that \( \chi_{\rho} \) may not be diagonalizable under the basis \( \mathcal{K} \), thus it may still have some coherence (with respect to \( \mathcal{K} \)) inside the individual subsystems, which can be captured with the help of coherence just defined above:

\[
D^K(\rho) := C^K(\chi_{\rho}) = \min_{\delta \in \mathbb{Z}^+} S(\chi_{\rho}||\delta).
\]

We call it coherent dissonance, which is similar to the quantum dissonance defined in [5], where quantum dissonance is defined as nonclassical correlations which exclude entanglement. From equation (4), we have

\[
D^K(\rho) = S(\delta^K_{\chi_{\rho}}) - S(\chi_{\rho}).
\]

Note that the dissonance defined here is different from the local coherence in [8] which is the relative entropy between the nearest separable state and the nearest incoherent state of \( \rho \).

The results above give us a method to compute discord and coherence, for any \( N \)-partite quantum state \( \rho \), the following additivity relations holds, that is,

\[
Q(\rho) \leq C^K(\rho) \leq Q(\rho) + D^K(\rho).
\]

The first inequality is obvious. Now from equations (4), (6) and (8), we have

\[
C^K(\rho) = S(\delta^K_{\chi_{\rho}}) - S(\rho) \\
\leq S(\chi_{\rho}) - S(\rho) + S(\delta^K_{\chi_{\rho}}) - S(\chi_{\rho}) \\
= Q(\rho) + D^K(\rho),
\]

where the inequality comes from the definition of coherence.

We define the quantity \( L(\rho) := S(\delta^K_{\chi_{\rho}}) - S(\delta^K_{\chi_{\rho}}) \); it describes the entropic costs caused by the optimal dephasing measurement with respect to the discord. Clearly, we have \( L(\rho) \geq 0 \), and also

\[
C^K(\rho) + L(\rho) = Q(\rho) + D^K(\rho).
\]

This relation corresponds to the closed path in figure 1 and means that the sum of the nonclassical correlation and coherent dissonance is equal to the sum of the total coherence and the entropic costs.

To conclude this section, we provide two examples to illustrate the different contributions for the coherence. The first one is a maximally coherent state \(|+\rangle+\rangle\) on a two qubit system, where \(|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\) is a maximally coherent state on one qubit system. Obviously, we have \( C^K = D^K = 2 \), but \( Q = L = 0 \). This implies that the coherence comes solely from the coherence located in the individual qubits. The second example is a Bell state \(|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\). It is easy to check that \( C^K = Q = 1 \), and \( D^K = L = 0 \). This means that the coherence comes solely from the correlation between subsystems.
3. Additivity relations via local measurement

Most of nonclassical correlations measures are limited to studies of bipartite correlations only as the original concept of discord, which involves bipartite system with classicality for only one subsystem. In this section, we will discuss the connection between nonclassical correlations and coherence in the bipartite system by the act of the local measurement on one or both subsystems. Let \( \rho^{AB} \) be a quantum state in a bipartite system \( AB \), and \( \{ |a\rangle^A \} \) be a given basis of \( A \). The quantum incoherent relative entropy of \( \rho \) with respect to \( \{ |a\rangle^A \} \) is defined as

\[
C^{AB}(\rho^{AB}) := \min_{\sigma \in I^A} S(\rho^{AB} \| \sigma^{AB}).
\]  

(12)

Here \( I^A \) is the set of all quantum incoherent states of the form, i.e. \( \sum_i p_i |a_i\rangle^A \langle a_i| \otimes I^B \), where \( \rho_B^B \) are quantum states on \( B \), and \( \{ p_i \} \) is a probability distribution [7]. It has been proved that the quantum incoherent relative entropy can be written as

\[
C^{AB}(\rho^{AB}) = S(\sigma^{AB}) - S(\rho^{AB}),
\]  

(13)

where \( \sigma^{AB} = \sum_i (|a_i\rangle^A \langle a_i|^A \otimes I^B) \rho^{AB} (|a_i\rangle^A \langle a_i| \otimes I^B) \), and \( I^B \) is the identity operator on the subsystem \( B \).

Furthermore, when the basis \( \{ |a\rangle^A \} \) varies, we obtain the set of all classical-quantum states, denoted \( \mathcal{I} \), clearly, \( \mathcal{I}_2 \subset \mathcal{I} \). Then, the one-way quantum discord [14] is defined as

\[
Q^{AB}(\rho^{AB}) = \min_{\omega \in \mathcal{I}} S(\rho^{AB} \| \omega^{AB}).
\]  

(14)

Similar to equation (6), the one-way quantum discord has also a useful expression, i.e.

\[
Q^{AB}(\rho^{AB}) = S(\omega^{AB}) - S(\rho^{AB}),
\]  

(15)

where \( S(\omega^{AB}) = \min_{\{ |i\rangle \}} S(\sum_i (|i\rangle^A \langle i|^A \otimes I^B) \rho^{AB} (|i\rangle^A \langle i| \otimes I^B)) \), and the minimization is taken over all orthogonal bases of subsystem \( A \). Note that for the nearest classical-quantum state \( \omega^{AB}_\rho \), its reduced state on \( A \) may not be diagonalizable under the basis \( \{ |a\rangle \} \), thus it may still have some coherence, which can be captured with the help of coherence just defined above:

\[
D^{AB}(\rho^{AB}) := C^{AB}(\omega^{AB}_\rho) = \min_{\sigma \in I^A} S(\omega^{AB}_\rho \| \sigma^{AB}).
\]  

(16)

Similar to coherent dissonance, we call it the one-way coherence dissonance. Then equation (13) yields a closed expression for one-way coherence dissonance

\[
D^{AB}(\rho^{AB}) = S(\omega^{AB}_\rho) - S(\omega^{AB}).
\]  

(17)

Equipped with these relations, we are now in a position to show a close connection between the one-way quantum discord \( Q^{AB} \), quantum incoherent relative entropy \( C^{AB} \) and the one-way coherence dissonance \( D^{AB} \), i.e.

\[
Q^{AB}(\rho^{AB}) \leq C^{AB}(\rho^{AB}) \leq Q^{AB}(\rho^{AB}) + D^{AB}(\rho^{AB}).
\]  

This relation is similar to the additivity relations (9), and provides an upper bound of quantum incoherent relative entropy.

Next, we compare the coherence and the discord defined in [15]. From equation (13), after some simple algebraic computation, we obtain a fundamental inequality as follows:

\[
C^{AB}(\rho^{AB}) \leq C(\rho^A) - C(\rho^B).
\]  

(18)
This relation has a clear operational interpretation: the difference of coherence between the total system and one of its subsystems is no less than the quantum incoherent relative entropy via the other subsystem.

From (13), it is easy to see the following inequality:
\[
\Theta_{A|B}(\rho_{AB}) + C(\rho_A) \leq C(\rho_{AB}),
\]
where \(\Theta_{A|B}(\rho_{AB}) = S_{\rho_{AB}}(A : B) - \min_{\{i\}} S_{\rho_{iA}}(A : B)\) is another definition of discord in [15]. Here \(S_{\rho_{iA}}(A : B)\) is the quantum mutual information of \(\rho_{iA}\), and \(\rho_{iA} = \sum_i (|i\rangle \langle i| \otimes I_B)\rho_{AB}(i\langle i| \otimes I_B)\) for any orthonormal basis \(\{|i\rangle\}\) of A. Then, we have the following inequality, which is tighter than the subadditivity of coherence in [13].

**Theorem 1.** For any bipartite quantum state \(\rho_{AB}\), the following inequality holds:
\[
\Theta_{A|B}(\rho_{AB}) + C(\rho_A) + C(\rho_B) \leq C(\rho_{AB}).
\]

**Proof.** This is direct from equations (18) and (19). □

The above theorem shows that the total coherence of a quantum state includes genuine local coherence located in the individual subsystems and quantum correlation between them.

To illustrate the inequality presented in theorem 1, let us consider two simple examples. The first one is a separable state with the reduced states both being maximally mixed [17]:
\[
\rho_{AB} = \frac{1}{4} (|+\rangle\langle+| \otimes |0\rangle\langle0| + |−\rangle\langle−| \otimes |1\rangle\langle1| + |0\rangle\langle0| \otimes |−\rangle\langle−| + |1\rangle\langle1| \otimes |+\rangle\langle+|).
\]
It is easy to show that $\Theta^A_B(\rho_{AB}) \approx 0.311$, $C(\rho_{AB}) = 0.5$, and $C(\rho^A) = 0 = C(\rho^B)$. The second example is the Isotropic state

$$
\rho_{AB} = (1 - p)\frac{I}{4} + p\left|\psi\right\rangle\left\langle\psi\right|,
$$

where $\left|\psi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|11\right\rangle\right)$ is a Bell state, and $p \in [0, 1]$. We know that its discord is greater than 0 when $p > 0$ and it is separable when $p \leq \frac{1}{3}$. Clearly, we have $C(\rho^A) = C(\rho^B) = 0$, and from [18], the nearest classical state of $\rho_{AB}$ is the closet incoherent state, and then $\Theta^A_B(\rho_{AB}) = C(\rho_{AB})$. That is to say, for Bell-diagonal states, the equality holds in equation (20). However, the question remains open for general mixed states.

This discord above with respect to the subsystem $A$ is defined, one can also define the discord by performing local measurement over the subsystem $B$ [16, 19], we should get a result similar to the relation (20). In particular, we can also consider the joint measurement on both subsystems $A$ and $B$, we call this quantity symmetric discord [20]. In this situation, we can provide a tighter lower bound for the relation (20). Recall that the symmetric discord is defined as

$$
\Theta(\rho_{AB}) = S_{\rho_{AB}}(A : B) - \min_{\{|ij\rangle_{AB}\}} S_{\rho_{ij}}(A : B),
$$

where $\rho_{\{ij\}AB} = \sum_{ij} |ij\rangle_{AB} \langle ij| \rho_{AB} |ij\rangle_{AB} \langle ij|$, and the set $\{|ij\rangle_{AB}\}$ constitutes a product orthonormal basis of $AB$. Clearly, we have $\Theta^A_B(\rho_{AB}) \leq \Theta(\rho_{AB})$. The following theorem presents an improved lower bound for the coherence.

**Theorem 2.** For any bipartite quantum state $\rho_{AB}$, the following inequality holds:

$$
\Theta(\rho_{AB}) + C(\rho^A) + C(\rho^B) \leq C(\rho_{AB}).
$$

**Proof.** Without loss of generality, we assume that $\{|ij\rangle_{AB}\}$ is the fixed product basis on $AB$. Then, we perform a measurement on $AB$ with respect to this basis, from the result [13], we have

$$
C(\rho_{AB}) - C(\rho^A) - C(\rho^B) = S_{\rho_{AB}}(A : B) - S_{\rho_{ij}}(A : B).
$$

Taking the optimization over all the orthonormal measurements on $AB$ completes the proof of the theorem.

Note that the inherent property of the mutual information is non-negative, and is viewed as the total correlation in the bipartite state. But, it does not have unique generalizations to the multipartite case [21]. In this paper, we take Modi’s suggestion [5], for an $N$-partite quantum state, its quantum mutual information is defined by the relative entropy between it and the product state obtained from its reduced states, and bears the interpretation of total correlation between all the subsystems. Thus, we now extend our results equations (19) and (22) to the multipartite setting.

**Theorem 3.** For any $N$-partite quantum state $\rho^{12...N}$, we have

$$
\sum_{i=1}^{N-1} \Theta^{1i+1...N}(\rho^{1i+1...N}) + \sum_{i=1}^{N} C(\rho^i) \leq C(\rho^{12...N})
$$

(23)
and
\[
\Theta(\rho^{12\cdots N}) + \sum_{i=1}^{N} C(\rho^i) \leq C(\rho^{12\cdots N}),
\]
(24)

where \(\Theta_{i|N}^{i+1\cdots N}(\rho^{i+1\cdots N})\) is the quantum discord based the measurement on the subsystem \(i\), \(\Theta(\rho^{12\cdots N})\) is a generalization of quantum discord in equation (21), and \(C(\rho^i)\) is relative entropy of coherence of the reduced system \(i\).

From equations (20) and (22), combining the properties of the discord and coherence, we can easily prove this theorem. The result shows that the total coherence in the multipartite system contains not only the local genuine coherence located in the individual subsystems, but also the nonclassical correlations among the multipartite system. Here, the nonclassical correlations characterize the collective coherence among the subsystems. In other words, the sum of the collective coherence and the local coherence shall not exceed the total coherence.

4. Coherence in entanglement distribution

In the previous section, we have given a clear operational interpretation for equation (18). Here, we discuss the general case via entanglement distribution. The general scenario for entanglement distribution is consider in [22–24]. We assume that two agents, Alice and Bob, have access to a tripartite quantum state \(\rho_{ABR}\), with Alice holding \(A\) and \(R\), and Bob holding \(B\). The entanglement distribution is realized by sending the particle \(R\) from Alice to Bob. If the quantum channel used for the transmission is noiseless, the amount of entanglement distributed in this process is quantified by the difference between the final amount of entanglement \(E_{AB}^{R}\) and the initial amount of entanglement \(E_{AR}^{B}\). It has been proven in [23, 24] that the amount of distributed entanglement cannot exceed the quantum discord between \(R\) and the remaining systems \(AB\). Then, we will give a bound on the increase of coherence during the entanglement distribution.

**Theorem 4.** Given a tripartite state \(\rho := \rho_{ABR}\), the following inequality holds:
\[
C_{AB|R}(\rho) + C_{ABR}(\rho) \leq C_{R|AB}(\rho).
\]
(25)

**Proof.** Let \(\{|j\rangle^A\}\) and \(\{|i\rangle^R\}\) be the fixed basis on \(A\) and \(R\), respectively. Then, we denote the states
\[
\rho' = \sum_j (I_{AB} \otimes |j\rangle^R \langle j|) \rho (I_{AB} \otimes |j\rangle^R \langle j|)
\]
(26)
and
\[
\rho^* = \sum_{i,j} (|j\rangle^A \langle j| \otimes I^B \otimes |i\rangle^R \langle i|) \rho (|j\rangle^A \langle j| \otimes I^B \otimes |i\rangle^R \langle i|)
\]
(27)
to arise from \(\rho\) via the local orthonormal measurements \(\{|i\rangle^R \langle i|\}\) on the subsystem \(R\) and the local orthonormal measurement \(\{|j\rangle^A \langle j| \otimes |i\rangle^R \langle i|\}\) on the subsystem \(AR\), respectively. This implies that the states \(\rho'\) and \(\rho^*\) are the nearest quantum incoherent states for the sake of \(C_{R|AB}(\rho)\) and \(C_{AB|R}(\rho)\), respectively. Then we have
\[ C^{AR|B}(\rho) = [S(\rho') - S(\rho)] + [S(\rho^*) - S(\rho')] = C^{R|AB}(\rho) + C^{AB|R}(\rho') \leq C^{R|AB}(\rho) + C^{AB|R}(\rho), \] (28)

where the inequality comes from the fact that quantum incoherent relative entropy does not increase under the action of incoherent operation.

We will now consider the situation where the total state is pure. One applies equation (25) to a tripartite pure state \(|\psi\rangle^{ABR}\), it reduces to

\[ S(\tilde{\rho}^{AB}) \leq S(\tilde{\rho}^{A}) + S(\tilde{\rho}^{R}), \] (29)

where the dephased state \(\tilde{\rho}^{AB} = \sum_{j}(|j\rangle^{A} \langle i|^{R})(j \otimes i)\rho^{AB}(|j\rangle^{A} \langle i|^{R})\) with the reduced state \(\rho^{AR} = \text{Tr}_{B}(|\psi\rangle^{ABR}\langle \psi|)\). This is the additivity of entropy for subsystems A and R after the measurements.

Finally, we consider the more general situation in which the channel used for entanglement distribution is noisy. If Alice uses an incoherent channel \(\Lambda^{R}\) to send her particle \(R\) to Bob, they end up in the final state \(\rho_{f} = \Lambda^{R}(\rho_{i})\), where \(\rho_{i} = \rho^{ABR}\) is a tripartite initial state. In the following theorem we show the amount of coherence in the entanglement distribution.

**Theorem 5.** Given a quantum incoherent channel \(\Lambda^{R}\) and two states \(\rho_{i}\) and \(\rho_{f}\), the following inequality holds:

\[ C^{AR|B}(\rho_{f}) - C^{AB|R}(\rho_{f}) \leq C^{R|AB}(\rho_{i}). \] (30)

**Proof.** We first apply equation (25) to the state \(\rho_{f}\), deriving

\[ C^{AR|B}(\rho_{f}) - C^{AB|R}(\rho_{f}) \leq C^{R|AB}(\rho_{i}). \]

One then completes the proof by noting \(C^{AB|R}(\rho_{f}) \leq C^{AB|R}(\rho_{i})\), which follows from the fact that the quantum incoherent relative entropy does not increase under the action of incoherent operation.

5. Conclusions

We explored the distribution of coherence in multipartite systems. We studied the separation of total coherence in a given quantum state into quantum correlations and coherent dissonance using the relative entropy as a distance measure, and an additivity relation between them is given. Then, some trade-off relations between various types of coherence and the discord within the bipartite setting are given, and we extended our results to multipartite setting. We also discussed the amount of change of coherence in the entanglement distribution by having access to a noiseless and noisy quantum incoherent channel. Our results have direct importance in the theoretical description of coherence and practical application in quantum algorithms where the key steps involve the coherent superposition. We also hope our results can be used to find the optimal quantum resources in quantum communication tasks.

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Note added. During the completion of this paper we became aware of the closely related independent work by Bu et al [25].

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