Complex Ridgelets for the Extraction of Morphological Features on Engineering Surfaces

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Abstract. In this paper, a complex ridgelets transform, which provides approximate shift invariance analysis of line singularities, is proposed to extract morphological features with linear or directional/objective property, by taking a Dual Tree Complex Wavelet Transform (DT-CWT) on the projections of the Finite Radon Transform (FRAT). The Numerical experiments show the remarkable potential of this methodology to analyse engineering and bioengineering surfaces with liner scratches in comparison to wavelet-based methods developed in our previous work.

1. Introduction

In the last decade, a wide range of methods based on wavelet transform for surface characterisation have been investigated and proposed. The orthogonal wavelet has been used for analysis of multi-scalar surfaces in engineering by Chen and Raja, in which the phase distortion of an orthogonal wavelet was neglected [1]. The first and second generation biorthogonal wavelet filtration for extraction of morphological features was proposed by our previous work [2]. The main advantages of the biorthogonal wavelet are that it is a brick wall filter with linear phase (leading to real outputs without aliasing and phase distortion) and a traceable location property. The model for second generation biorthogonal wavelet has helped the steel industry to identify multi-scalar surfaces; the pump industry to diagnose pump failures; and the bioengineering industry to reconstruct isolated morphological features.

In spite of the development of wavelet technologies for surface characterization, there are still no available techniques to extract morphological features such as linear scratches and plateau with direction/objective properties. In this paper, we proposed a Complex Finite Ridgelet Transform (CFRIT) for the shift invariant extraction of line scratches from engineering and bioengineering surface topography by applying the Finite Radon Transform (FRAT) to a DT-CWT [3-5].

2. Complex Ridgelet Transform

A bivariate complex ridgelet \(\psi_{a,b,\theta}^{c}\) in \(R^2\) space can be defined as:

\[
\psi_{a,b,\theta}^{c}(x) = a^{-1/2} \psi^{c}((x_1 \cos \theta + x_2 \sin \theta - b)/a)
\]  

(1)
Here, $a > 0$ is a scale parameter, $\theta$ is an orientation parameter, and $b$ is a location scalar parameter. This function is constant along lines: $x_1 \cos \theta + x_2 \sin \theta = \text{const}$, while its transverse is a complex wavelet $\psi^c = \psi^r + \sqrt{-1} \psi^i$. Here, $\psi^r$ and $\psi^i$ are themselves real wavelets. If the real and imaginary part of the complex wavelet can be viewed as two ‘fat’ points, then the complex ridgelet can be interpreted as two ‘fat’ lines so that it is especially adaptive to analyse line ridges/valleys contained in surface topography.

The continuous Complex Ridgelet Transform (CRIT) for an integrable bivariate function $f(x) \in L^2(\mathbb{R}^2)$ is defined as:

$$\mathcal{R}_f(a, b, \theta) = \int_{\mathbb{R}^2} \psi_{a, b, \theta}(x) f(x) dx$$

(2)

The reconstruction formula is given as:

$$f(x) = \int_{\mathbb{R}} \int_{\mathbb{R}} \mathcal{R}_f(a, b, \theta) \psi_{a, b, \theta}(x) \frac{da}{a^3} db \frac{d\theta}{4\pi}$$

(3)

Point and line singularities are related by the Radon transform, so Eq. (2) can be deducted into the application of 1-D DT-CWT to the projections of the Radon transform:

$$\mathcal{R}_f(a, b, \theta) = \int_{\mathbb{R}} R_f(\theta, t) \psi^c((t - b)/a) dt$$

(4)

The Radon transform is denoted as

$$R_f(\theta, t) = \int f(x_1, x_2) \delta(x_1 \cos \theta + x_2 \sin \theta - t) dx_1 dx_2$$

(5)

Where, $\delta$ is the Dirac function, means the integral along the line with orientation $\theta$.

From Eq. (4) and (5), one can see that the basic strategy for computing the CRIT is to first calculate the Radon transform $R_f(\theta, t)$, then to calculate the complex wavelet transform of the projections $R_f(\theta, \cdot)$.

According to Eq. (4) and (5), a direct way to design a ridgelet with the shift invariant property is to choose the $\psi^c$ to be shift invariant. Due to the fact that the selected DT-CWT wavelet basis $\psi^c$ have the shift-invariant property, the CRIT are also shift-invariant.

For the calculation of the Radon transform, numerous digital methods have been devised. However, most of them were not designed to be invertible transforms for digital surfaces or images. Alternatively, the finite Radon transform theory provided an interesting solution for finite length signals. According to the practical requirements of surface characterization, we use the digital form of the CRIT based on the FRAT and DT-CWT.

2.1. Finite Radon Transform
Denote $Z_p = \{0, 1, \cdots, p - 1\}$ and $Z^*_p = \{1, \cdots, p\}$ where $p$ is a prime number, the FRAT of a real function on the finite grid $G = Z^*_p$ is defined as [6]:

$$FRAT_f(k, l) = r_k[l] = \frac{1}{\sqrt{p}} \sum_{(i, j) \in L_{k, i}} f[i, j]$$

(6)

$$= \frac{1}{\sqrt{p}} \sum_{(i, j) \in Z^*_p} f[i, j] \delta_{L_{k, i}}[i, j] = \left\{ f, \frac{1}{\sqrt{p}} \delta_{L_{k, i}} \right\}$$

here, $L_{k, i}$ denotes the set of points that make up a line on the lattice $Z^*_p$, or more precisely:

$$L_{k, i} = \{(i, j): j = ki + l \quad i \in Z_p \}, \quad 0 \leq k < p; \quad L_{p, i} = \{(l, j): j \in Z_p \}$$

(7)

From the property of the finite geometry, one can get the inverse FRAT (IFRAT) of a zero mean function $f$ with the relations as below:
IFRAT = \frac{1}{\sqrt{p}} \sum_{(k,j) \in P_{k,j}} r_k[l] = \frac{1}{p} \sum_{(k,j) \in P_{k,j}} \sum_{(i',j') \in L_{k,j}} f[i', j'] \tag{8}

= \frac{1}{p} \left( \sum_{(i',j') \in L_{k,j}} f[i', j'] + pf[i, j] \right) = f[i, j]

Thus, the algorithm for the IFRAT has the same structure and is symmetric with algorithm for the forward transform.

2.2. Digital Ridgelet Transform

When define the complex wavelet basis as:

$$\{w_m^i, m \in Z_p, i \in N\} \tag{9}$$

The digital FRIT can be integrated as

$$\text{CFRIT}_f(k) = \left\langle \text{FRAT}_f[k, \cdot], w_m^{(k)}[\cdot] \right\rangle = \sum_{l \in Z_p} w_m^{(k)}[l] \left\langle f, \frac{1}{\sqrt{p}} \delta_{k,j} \right\rangle \tag{10}$$

Therefore the basis functions of the discrete complex ridgelet transform can be written as

$$\rho_{k,m} = \frac{1}{\sqrt{p}} \sum_{l \in Z_p} w_m^{(k)}(l) \delta_{k,j} \tag{11}$$

3. Experiments

This first experiment demonstrates the shift invariance of the CFRIT by an artificial image with a stepped edge. Fig. 1 shows 16 shifted versions of the image (at the top) and their subspace reconstructed components in turn from the coefficients at levels $j \leq j_0 = 4$ using the CFRIT (left) and real FRIT (right). In order to see the effects clearly, only the center of the profiles of these images is shown. Each shift is displaced down a little to give a waterfall style of display. Good shift invariance is seen from the fact that the shape and amplitude of each of the reconstructed components by CFRIT hardly varies as the input is shifted. As opposite, the reconstructed components by FRIT vary considerable with each shift.

Figure 1: The reconstructed components (centre profiles) at levels 1 to 4 of 16 shifted image with a stepped edge using the FRIT-C (left) and real FRIT (right). Each shift is displaced down a little to give a waterfall style of display.

The second experiment shows the good performance of CFRIT for the extraction of linear features from a noised image. We consider an artificial image with a deep scratch that is contaminated by an additive zero-mean Gaussian white noise of variance $\sigma^2$. By using the hard-thresholding denoising
algorithm in the DWT domain, DT-CWT domain and CFRIT domain separately, the linear feature has been extracted from the noised image. It can be seen from Fig. 2 that the CFRIT is effective in recovering straight edges, as well as in term of the signal to noise ratio (SNR). The CFRIT reconstruction does not contain the undesirable artifacts along edge that one finds in the DWT reconstruction. The simple thresholding scheme for CFRIT is effective in denoising the piecewise smooth image with line singularities. This is because the linear singularities are represented by a few significant coefficients in the CFRIT domain, whereas random noisy singularities are unlikely to produce the similar amplitude coefficients. The DT-CWT is relatively superior to the DWT to reduce the artifacts due to its shift invariance and good directional selectively. From the view of the hybrid approach, the CFRIT just combines the multiresolution analysis of DT-CWT with the anisotropy of the Radon transform. Furthermore, the complex ridgelet transform can provide better phase information that is useful for future research.

4. Conclusion
CFRIT was proposed by taking DT-CWT on the projections of the FRAT. It brings together the ideas of complex wavelet pyramids and the geometric features of ridgelets to solve problems that exist in previous wavelet-based methods. By mapping a line singularity into a point singularity through the FRAT, and then using the DT-CWT on each projection in the Radon transform domain, the CFRIT can efficiently represent functions with shift-invariant property. Numerical experiments have verified the ability of the linear feature extraction of this method. Overall, it is clear that the CFRIT have great potential to extract the scratch from the surface topography.

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