On the Transition from Confinement to Screening in QCD$_{1+1}$ Coupled to Adjoint Fermions at Finite $N$

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Abstract

We consider SU($N$) QCD$_{1+1}$ coupled to massless adjoint Majorana fermions, where $N$ is finite but arbitrary. We examine the spectrum for various values of $N$, paying particular attention to the formation of multi-particle states, which were recently identified by Gross, Hashimoto and Klebanov in the $N = \infty$ limit of the theory. It is believed that in the limit of vanishing fermion mass, there is a transition from confinement to screening in which string-like states made out of adjoint fermion bits dissociate into stable constituent “single particles”. In this work, we provide numerical evidence that such a transition into stable constituent particles occurs not only at large $N$, but for any finite value of $N$. In addition, we discuss certain issues concerning the “topological” properties exhibited by the DLCQ spectrum.
1 Introduction

Solving for the non-perturbative properties of physically realistic quantum gauge theories is typically an intractable problem. In order to gain some insights, however, a number of lower dimensional models have been investigated in the large \( N \) (or planar) approximation, with a plethora of examples emerging in the last few years (see reference \[1\] for an extensive review).

In this work, we will consider the 1 + 1 dimensional SU(\(N\)) gauge theory of QCD coupled to massless adjoint Majorana fermions, which is believed to exhibit the property of screening \[2, 10\]. A similar theory with complex adjoint fermions has also been considered \[11, 12\]. It will be advantageous to quantize the theory on the light-cone, and to adopt the light-cone gauge. It is then a straightforward task to extract numerical bound state solutions via an application of Discrete Light-Cone Quantization (DLCQ) \[3\].

Various non-perturbative studies of this model already exist in the literature \[4, 5, 6\], but in more recent work \[8\] by Gross, Hashimoto and Klebanov (hereafter ‘GHK’), it was suggested that in the massless fermion limit, the spectrum becomes continuous above a certain threshold. This was supported by the presence of states in the spectrum that have a mass consistent with the dynamics of two freely interacting stable particles. In this context, identifying either “single particle” or “multi-particle” states requires a careful analysis of the spectrum, since an analysis of the explicit Fock state content does not distinguish these states unambiguously.

It is worth clarifying this last remark to avoid possible confusion. Firstly, since the limit \( N = \infty \) is assumed in GHK, Fock states are single traces of fermion creation operators, and correspond intuitively to single closed strings of adjoint fermion bits. Multi-trace states (corresponding to multi-string states) do not appear in the analysis, since interactions with them are suppressed (the factor \( 1/N \) plays the role of a string coupling constant). Naively, one would view single-trace states as “single particles” and multi-trace states as “multi-particles”. Although the latter is expected to follow from the usual \( 1/N \) counting, GHK have shown that there may be exceptions to the former; namely, states that are superpositions of single-trace Fock states may exhibit masses that are consistent with the dynamics of two freely interacting stable particles.

This property of the large \( N \) spectrum was perhaps anticipated by the work of Kutasov and Schwimmer \[6, 7\]: namely, the presence of multi-particle states in the \( N = \infty \) theory becomes manifest after an appropriate rearrangement of the Hilbert space into
Kac Moody current blocks. At finite $N$, however, $1/N$ interactions prevent us from identifying obvious multi-particle candidates in the Hilbert space\footnote{For large $N$, multi-trace states are obvious candidates for multi-particles.} and we can no longer appeal to the correspondence proposed in\footnote{We consider $U(N)$ as well as $SU(N)$, since it was shown in earlier work\footnote{that the $SU(N)$ spectrum may be obtained by solving for the $U(N)$ spectrum, and then ‘factoring out’ $U(1)$ states. Numerically, this method is considerably more efficient than solving for the $SU(N)$ spectrum directly.} to argue for the existence of multi-particle states. Nevertheless, the formation of multi-particles in the finite $N$ spectrum may be inferred from the screening properties of the theory\footnote{It is therefore of interest to determine whether multi-particles actually appear in the finite $N$ spectrum or not, and we will devote ourselves towards answering this question via a numerical study of the DLCQ bound state equations. We also attempt to shed light on certain “topological” features exhibited by the DLCQ spectrum.}, since the theory screens for any finite $N$\footnote{Of course, working at finite $N$ has a price; the Fock space now admits multi-trace states, and the complexity of the bound state problem is dramatically increased. Nevertheless, we find that the numerical bound state problem is still tractable provided the discretization of the light-cone momentum $P^+$ is not too fine, but fine enough to resolve certain features of the theory we are interested in.}. It is therefore of interest to determine whether multi-particles actually appear in the finite $N$ spectrum or not, and we will devote ourselves towards answering this question via a numerical study of the DLCQ bound state equations. We also attempt to shed light on certain “topological” features exhibited by the DLCQ spectrum.

Of course, working at finite $N$ has a price; the Fock space now admits multi-trace states, and the complexity of the bound state problem is dramatically increased. Nevertheless, we find that the numerical bound state problem is still tractable provided the discretization of the light-cone momentum $P^+$ is not too fine, but fine enough to resolve certain features of the theory we are interested in.

The organization of the paper may be summarized as follows; in Section 2 we briefly review the $1 + 1$ $SU(N)$ gauge theory coupled to adjoint Majorana fermions, which we formulate in light-cone coordinates. We also discuss features of the DLCQ formulation at finite $N$ that differ from the large $N$ formulation originally discussed in\footnote{In Section 3 we tabulate the results of our finite $N$ numerical analysis, and compare spectra of candidate multi-particle states with mass predictions given by free-body kinematics. We conclude our investigation with a summary and discussion in Section 4.}. In Section 3 we tabulate the results of our finite $N$ numerical analysis, and compare spectra of candidate multi-particle states with mass predictions given by free-body kinematics. We conclude our investigation with a summary and discussion in Section 4.

## 2 Light-Cone Quantization and DLCQ at Finite $N$

The action for QCD$_{1+1}$ coupled to a single Majorana fermion transforming in the adjoint representation of $U(N)$ or $SU(N)$ is given by\footnote{We consider $U(N)$ as well as $SU(N)$, since it was shown in earlier work\footnote{that the $SU(N)$ spectrum may be obtained by solving for the $U(N)$ spectrum, and then ‘factoring out’ $U(1)$ states. Numerically, this method is considerably more efficient than solving for the $SU(N)$ spectrum directly.}}

$$S = \int d^2x \text{Tr} \left[ i\overline{\Psi} \gamma^\mu D_\mu \Psi - m\overline{\Psi} \Psi - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \right],$$

(1)

where the covariant derivative is defined by $D_\mu \Psi = \partial_\mu \Psi + i[A_\mu, \Psi]$, and the fermion field $\Psi = 2^{-1/4} \begin{pmatrix} \psi \\ \chi \end{pmatrix}$ is a two component spinor, each component representing an $N \times N$
Hermitian matrix of Grassmann variables. These matrices are traceless if the gauge group is $\text{SU}(N)$. The light-cone quantization of this theory in the light-cone gauge $A_- = 0$ was carried out in [4], and we refer the reader to that source for details and notation. In the present context, we simply note that the light-cone momentum $P^+$ and Hamiltonian $P^-$ may be expressed in terms of Fourier oscillator modes as follows:

\[ P^+ = \int_0^\infty dk \, k \, b_{ij}^\dagger(k)b_{ij}(k), \]

\[ P^- = \frac{1}{2} m^2 \int_0^\infty \frac{dk}{k} \, b_{ij}^\dagger(k)b_{ij}(k) + \frac{g^2 N}{\pi} \int_0^\infty \frac{dk}{k} \, D(k) \left( b_{ij}^\dagger(k)b_{ij}(k) - \frac{1}{N} b_{ii}^\dagger(k)b_{jj}(k) \right) + \frac{g^2}{2\pi} \int_0^\infty dk_1dk_2dk_3dk_4 \left\{ \delta(k_1 + k_2 - k_3 - k_4) \left[ A(k_i) \cdot b_{ij}^\dagger(k_3)b_{ij}^\dagger(k_4)b_{kl}(k_1)b_{li}(k_2) + B(k_i) \cdot b_{ij}^\dagger(k_3)b_{kl}(k_1)b_{ji}(k_2)b_{kj}(k_2) \right] + \delta(k_1 + k_2 + k_3 - k_4) \times C(k_i) \cdot \left[ b_{ij}^\dagger(k_3)b_{kl}(k_1)b_{li}(k_2)b_{ij}^\dagger(k_3) - b_{ij}^\dagger(k_1)b_{ij}^\dagger(k_2)b_{ij}^\dagger(k_3)b_{ki}(k_4) \right] \right\}, \]

where

\[ A(k_i) = \frac{1}{(k_4 - k_2)^2} - \frac{1}{(k_1 + k_2)^2}, \]

\[ B(k_i) = \frac{1}{2} \left( \frac{1}{(k_1 - k_4)^2} - \frac{1}{(k_2 - k_4)^2} \right), \]

\[ C(k_i) = \frac{1}{(k_2 + k_3)^2} - \frac{1}{(k_1 + k_2)^2}, \]

\[ D(k) = \int_0^k dp \, \frac{k}{(p - k)^2}. \]

For the gauge group $U(N)$, the fermionic oscillator modes above satisfy the following anti-commutation relations

\[ \{b_{ij}(k^+), b_{lk}^\dagger(\tilde{k}^+)\} = \delta(k^+ - \tilde{k}^+)\delta_{ii}\delta_{jk}. \]

If the gauge group is $\text{SU}(N)$ we need to adopt the following set of relations –

\[ \{b_{ij}(k^+), b_{lk}^\dagger(\tilde{k}^+)\} = \delta(k^+ - \tilde{k}^+)\delta_{ii}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}, \]

and discard the $\frac{1}{N} b_{ii}^\dagger(k)b_{jj}(k)$ term in $P^-$, since the $\text{SU}(N)$ fermion fields are traceless.

A number of comments are in order. If we compare the light-cone Hamiltonian $P^-$ of equation (3) – which is valid for any $N$ – to the large $N$ expression given in [4], we notice that there is an additional $bb \rightarrow bb$ term in (3) with amplitude $B(k_i)$. The color index
structure of this term implies that it is suppressed by $1/N$, since it splits a single-trace Fock state into multi-trace states. Nevertheless, the $Z_2$ symmetry $T : b_{ij} \rightarrow b_{ji}$ of the large $N$ theory is also manifest in the finite $N$ formulation, since this additional term is easily shown to be invariant under $T$.

The $D$ term in equation (3) is obtained by normal ordering quartic terms in $P^-$, and diverges linearly. However, this divergence is entirely absorbed by the Coulomb divergence generated by the $A$ term. So the theory is manifestly finite. We also note that the $D$ term used in the large $N$ expression for $P^-$ is unchanged if $N$ is finite – the $1/N$ contributions obtained from normal ordering quartic terms in accordance with the SU($N$) relations simply sum to zero.

In order to implement the DLCQ formulation of the bound state problem, we simply restrict the light-cone momentum variables $k_i$ appearing in equations (2),(3) for $P^\pm$ to the following set of discretized momenta: \( \{ \frac{P^-}{K}, \frac{3P^-}{K}, \frac{5P^-}{K}, \ldots \} \); i.e. only odd positive integer multiples of $P^-/K$ are allowed, which is equivalent to imposing anti-periodic boundary conditions for the fermion fields: $\psi_{ij}(x^-) = -\psi_{ij}(x^- + 2\pi R)$. The integer $K$ is called the harmonic resolution, and $1/K$ measures the coarseness of our discretization. Physically, $1/K$ represents the smallest unit of longitudinal momentum fraction allowed for each parton. As soon as we implement the DLCQ procedure, which is specified unambiguously by the harmonic resolution $K$, the integrals appearing in the definitions (2),(3) for $P^\pm$ are replaced by finite sums, and the eigen-equation $2P^+P^-|\Psi\rangle = M^2|\Psi\rangle$ is reduced to a finite matrix problem. Continuum values are obtained by extrapolating results to the $K = \infty$ limit. Typically, a computer program is used to generate and diagonalize the DLCQ matrix to solve for the mass eigenvalues $M^2$. In the present work, we are able to perform numerical diagonalizations for values of $K$ in the range $3 \leq K \leq 16$ with the help of Mathematica and a desktop PC. At $K = 16$, the DLCQ matrix has dimensions $375 \times 375$. A similar finite $N$ analysis of a two dimensional supersymmetric matrix model was performed recently by the authors.

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3 Repeated indices are always summed from 1 to $N$.
4 Periodic boundary conditions are also possible, but convergence in numerical calculations is slower.
3 DLCQ Bound State Solutions

In this section we present the results of our numerical diagonalizations of the DLCQ matrix for $M^2 = 2P^+P^-$ for values of $K$ in the range $3 \leq K \leq 16$, and for $N = 3, 10, 100$ and 1000. Strictly speaking, there is no upper limit on the size of $N$, since it appears as an algebraic variable in the DLCQ matrix. The mass $m$ of the fermion (see equation (3)) is set to zero. The results we obtain for $N = 1000$ agree with the large $N$ results presented in [4] to at least six significant figures.

As we stated earlier, our main objective is to determine whether the multi-particle states of the $N = \infty$ spectrum also persist at any finite $N$. Towards this end, we first calculate the masses of various “single particle” states – namely, the two lightest fermion and boson single-particle states – for various values of $K$ and $N$. The results are presented in Table 1 and Table 2.

Note that for small values of $K$ there is no dependence on $N$ (after expressing $M^2$ in units $g^2N/\pi$). Moreover, for larger values of $K$, the $N$ dependence is surprisingly small. This seems puzzling at first, since we know interactions between single and multi-trace states are governed by the ‘string coupling’ $1/N$, and so for $N = 3$, one expects considerable multi-trace contributions in a state such as $|F2\rangle$, which turns out to be a superposition of mainly five-parton single trace Fock states. Evidently a rather remarkable cancellation seems to be responsible for keeping the masses relatively independent of $N$. In this case, the limit $N = \infty$ provides an excellent approximation to the $N = 3$ case.

A crucial observation made by GHK [8] in their recent work suggested that many of the remaining states in the spectrum are adequately described as two or more freely interacting single particles: $|F1\rangle \otimes |F1\rangle$, $|F1\rangle \otimes |F2\rangle$, $|F1\rangle \otimes |B1\rangle$, and so on. This is why the states in Tables 1 and 2 are referred to as “single particles”, since they themselves are not seen to decompose into more fundamental stable particles.

The validity of this scheme may be tested in the DLCQ framework as follows [8]. For a given DLCQ resolution $K$, the mass $M_{F1\otimes F1}(K)$ of the composite state $|F1\rangle \otimes |F1\rangle$ is

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5 We exclude $N = 2$, since in this case one has to calculate the norm of each Fock state explicitly to avoid overlapping states, which is computationally very intensive.

6 We have also identified a number of other single particle states but they are not relevant for the discussion presented here.
Table 1: The masses $M^2$ (in units $g^2N/\pi$) of the two lightest “single particle” fermion states, $|F_1\rangle$ and $|F_2\rangle$ respectively.

Table 2: The masses $M^2$ (in units $g^2N/\pi$) of the two lightest “single particle” boson states, $|B_1\rangle$ and $|B_2\rangle$ respectively.

Given by free body kinematics according to the relation

$$M^2_{F_1\otimes F_1}(K) = \frac{M^2_{F_1}(n)}{K} + \frac{M^2_{F_1}(K-n)}{K-n},$$

where $n$ is any positive integer less than $K$, and where $M_{F_1}(n)$ and $M_{F_1}(K-n)$ are masses of the $|F_1\rangle$ particle carrying $n$ and $K-n$ momentum units respectively. In Table 3, we have applied equation (10) to determine the mass of $|F_1\rangle \otimes |F_1\rangle$ for various choices of $n$ and $K$. Since the mass of $|F_1\rangle$ for each $K$ is essentially independent of $N$ at five significant figures, the masses given in Table 3 are applicable for any $N$. In Table 4 we list the actual masses for bosons that are observed in the DLCQ spectrum for various choices of $K$ and $N$. Comparing Tables 3 and 4, we see for $N = 1000$ (i.e. essentially ‘large $N$’), that the masses corresponding to two free $|F_1\rangle$ particles appear in the actual DLCQ.
Calculated Masses for $|F1\rangle \otimes |F1\rangle$

| $K$  | $M^2_{F1\otimes F1}(K)$ |
|------|--------------------------|
| 8    | 20                        |
| 10   | (20.00) 22.461            |
| 12   | 20.953 25.127            |
| 14   | (20.891) 22.315 27.901 |
| 16   | 21.441 23.887 30.737 |

Table 3: Predicted values for $M^2$ (in units $g^2N/\pi$) for the composite particle $|F1\rangle \otimes |F1\rangle$ according to the two-free-body formula (10), for various choices of $n$ and $K$. The different numbers appearing in a given row correspond to varying the value of $n$ in equation (10). Numbers that are in parentheses correspond to a pair of identical $|F1\rangle$ particles (i.e. carrying the same momentum), and are therefore expected to be absent from the spectrum because of Fermi statistics.

| $K$  | $N$   | $M^2$ (in units $g^2N/\pi$) |
|------|-------|-----------------------------|
| 8    | 3     | 18.7312 20(2)               |
| 8    | 1000  | 18.7312 20(2)               |
| 10   | 3     | - 21.1213 22.461(2)         |
| 10   | 1000  | - 21.1213 22.461(2)         |
| 12   | 3     | 20.5603 20.9917(2) 23.954 25.0849(2) |
| 12   | 1000  | 20.5048 20.9532(2) 24.0033 25.1275(2) |
| 14   | 3     | - 21.7082 22.3756(2) 26.8465 27.8003(2) |
| 14   | 1000  | - 21.6346 22.3154(2) 26.9517 27.9010(2) |
| 16   | 3     | 21.2700 21.4620(2) 23.2368 23.9392(2) 29.7486 30.5640(2) |
| 16   | 1000  | 21.2303 21.4409(2) 23.1910 23.8869(2) 29.9211 30.7373(2) |

Table 4: Actual values for the mass squared $M^2$ (in units $g^2N/\pi$) of several bosonic states for different $N$ and $K$ (excluding single particle states). Numbers with a superscript $(n)$ correspond to an exact $n$-fold degeneracy in the spectrum.

spectrum with an exact two-fold degeneracy. In addition, the masses in Table 3 that correspond to two identical $|F1\rangle$ particles (in parentheses) are absent from the DLCQ spectrum, which is of course consistent from the Fermi statistics of two freely interacting identical fermions. The two-fold degeneracy we see here in the bosonic spectrum does not occur in the $N = \infty$ analysis of GHK [6], and this can be easily understood by analyzing the Fock state content of each doublet. What we find for any doublet at $N = 1000$ is that one state is a superposition of essentially single-trace Fock states, while the other is
mainly a superposition of two-trace Fock states. Since the Hilbert space for the $N = \infty$ theory is generated from single-trace Fock states only, the multi-trace states we see here will be absent in the $N = \infty$ spectrum.

One remarkable property of the theory, which can be observed from Table 4, is that for small values of $N$ – say, $N = 3$ (where $1/N$ interactions can no longer be neglected) – the exact two-fold degeneracy still survives, and the actual masses deviate only slightly (if at all) from the large $N$ results. Nevertheless, the Fock state content of each state in a doublet is drastically altered if we vary $N$; single-trace and multi-trace Fock states now contribute equally in each state for small $N$. Evidently, significant cancellations between $1/N$ contributions in the finite $N$ Hamiltonian must be occurring in order to keep masses relatively independent of $N$. We should point out that the additional term in equation (3) for $P^-$ that distinguishes it from the $N = \infty$ Hamiltonian is crucial in these calculations – no such mass degeneracy would be observed if it was omitted. It is still unclear at this point whether the very small deviations from the predicted masses in Table 3 and the mass of the doublets appearing in the finite $N$ DLCQ spectrum in Table 4 will remain in the continuum $K \to \infty$ limit. There is already evidence in Table 4 suggesting that the deviations are diminishing for larger values of $K$, but clarifying this issue will certainly require solving the DLCQ spectrum for larger values of $K$, and we leave this for future work.

As was remarked in the work of GHK [8], the remaining boson states listed in Table 4 agree with the masses presented in Table 3 up to $1/K$; i.e. as we increase the harmonic resolution, the agreement becomes more exact. These states are therefore multi-particle states that give rise to two-body continua in the continuum limit $K \to \infty$. We also note that this picture is not disturbed for relatively small values of $N$ (such as $N = 3$), and so the observations made by GHK in the context of the $N = \infty$ theory also appear to be valid at any finite $N$. In particular, for any finite $N$, our results are consistent with the interpretation that the spectrum of the (continuum) theory becomes continuous above the threshold mass $M^2 = 4M_{F1}^2$, where $M_{F1}$ is the mass of the lightest stable particle in the continuum theory.

An identical analysis may be performed for the composite state $|F1\rangle \otimes |F2\rangle$, and we find once again that the masses predicted by equation (14) emerge in the actual DLCQ spectrum as mass doublets for any finite $N$. Agreement is more precise for large values of $N$, but we nevertheless obtain good accuracy even for $N = 3$ as we did for $|F1\rangle \otimes |F1\rangle$ in Table 4. For composite particles with fermion statistics, such as $|F1\rangle \otimes |B1\rangle$, the
masses predicted by equation (10) also emerge in the DLCQ spectrum for any \(N\), but we no longer see the exact two-fold degeneracy that was observed in the bosonic sector. The fermion states that exactly satisfy equation (10) for large \(N\) are not observed in the \(N = \infty\) analysis of GHK, since these states are a superposition of two-trace Fock states (one fermion, and one boson) for large values of \(N\). For \(N = 3\), such states involve a superposition of both single and multi-trace states.

As was pointed out in [8], a large portion of the “multi-particle” spectrum in the fermionic sector can be summarized surprisingly well if we replace equation (10) with the modified formula

\[
\frac{M^2}{K-1} = \frac{M^2_{F_1}(n)}{n} + \frac{M^2_{F_1}(K-n-1)}{K-n-1}.
\]

In other words, some fermionic states at resolution \(K\) in the spectrum appear to be well approximated as two \(|F1\rangle\) particles with a combined momentum of \(K - 1\). There is still one unit of momentum unaccounted for, and one proposal [8] is to assume that this momentum is associated with a topological configuration of the light-cone vacuum that makes it distinct from the trivial light-cone vacuum. In fact, in order to get a fermion state overall, we need to assume that this topological vacuum carries fermion quantum numbers.

Although it is indeed possible to construct topologically distinct vacuum sectors in an SU(\(N\)) light-cone gauge theory with the properties cited above [13, 14], the empirical success of equation (11) lends itself to an alternative interpretation. Firstly, we know the theory has a supersymmetric point for a very specific value of the fermion mass [5]: \(m^2 = g^2 N/\pi\). As we reduce the fermion mass \(m\) to zero, one can show numerically that the mass degeneracy between the lightest boson-fermion supersymmetric pair is broken linearly as a function of \(m^2/g^2 N\) [3]. However, if we consider a heavier supersymmetric pair, this behavior is not necessarily observed. For example, at the supersymmetric point \((m^2 = g^2 N/\pi)\), the mass of the next-to-lightest fermion in the \(K = 15\) DLCQ spectrum is approximately \(M^2 = 43.5\), while its super-partner is identified in the \(K = 14\) DLCQ spectrum as a boson with mass \(M^2 = 40.7\) (in units \(g^2 N/\pi\)). The boson state is essentially a mixture of two-bit and four-bit single-trace states, while the fermion is observed to be essentially a mixture of three-bit single-trace states.

If we carefully track these states as the fermion mass \(m\) is decreased to zero, we find in the massless limit that the fermion state has mass \(M^2 = 20.86\), while its (former) mass 7

\footnote{The boson and fermion masses are expected to converge together in the continuum limit \(K \to \infty\).}
superpartner has the new mass $M^2 = 21.63$. Surprisingly, the masses are still approximately degenerate, so supersymmetry appears to be very weakly broken for these states. This suggests that the theory is in fact asymptotically supersymmetric (i.e. the spectrum of highly excited bound states is almost supersymmetric for any fermion mass $m$), an observation pointed out by Boorstein and Kutasov [7]. One can use formula (10) to show that the boson is in fact a composite state of two freely interacting $|F1\rangle$ particles [8]. We conclude therefore that the mass of particular fermionic states at DLCQ resolution $K$ may be estimated by an application of equation (10) with $K$ replaced with $K - 1$, since the composite boson at resolution $K - 1$ is still approximately equal in mass with the fermion state at resolution $K$. This scheme provides a natural derivation of equation (11) from (10).

At this point, we have made no reference to composite particles made from three or more single-particle states; to further validate our interpretation of the spectrum, we look for states of the form $|F1\rangle \otimes |F1\rangle \otimes |F1\rangle$. The mass $M^2_{F1\otimes F1\otimes F1}$ is now given by a simple generalization of equation (10):

$$\frac{M^2_{F1\otimes F1\otimes F1}}{K} = \frac{M^2_{F1}(K_1)}{K_1} + \frac{M^2_{F1}(K_2)}{K_2} + \frac{M^2_{F1}(K_3)}{K_3},$$

where $K_1 + K_2 + K_3 = K$. For example, setting $K_1 = 3, K_2 = 5,$ and $K_3 = 7$ gives $M^2_{F1\otimes F1\otimes F1} = 48.6915$. This is indeed observed in the DLCQ fermionic spectrum at $K = 15$, and we observe an exact two-fold degeneracy at any finite $N$. These states involve significant contributions from multi-trace Fock states, and are therefore only seen in a finite $N$ analysis.

4 Discussion

In this work we have implemented DLCQ to study the spectrum of QCD$_{1+1}$ coupled to a massless Majorana fermion transforming in the adjoint representation of the gauge group SU($N$), where $N$ is any integer $N \geq 3$. We observed in Tables 1 and 2 that the masses of the stable “single-particle” states in the spectrum are very weakly dependent on $N$. Evidently, considerable cancellations must be responsible for protecting masses from large $1/N$ corrections. It would be desirable to have a better understanding of this.

After comparing predicted masses for the composite particle $|F1\rangle \otimes |F1\rangle$ (Table 3) with the actual DLCQ spectrum for bosons (Table 4), we observed that the finite $N$ spectrum is consistently described as multi-particles with freely interacting stable constituents.
Therefore, the existence of a continuous spectrum above the threshold mass $M^2 = 4M_{F1}^2$ appears to be valid not only for $N = \infty$ [8], but also for any finite $N$. This behavior is compatible with the viewpoint that the finite $N$ theory is in a screening phase [2]. Clearly, probing larger values of $K$ will help clarify this issue, and we leave this challenge for future work.

We also attempted to explain the fermionic multi-particle spectrum by assuming the theory was asymptotically supersymmetric; i.e. excited states in the spectrum are almost supersymmetric for any fermion mass, including $m = 0$. An explicit example was given. This enables one to connect the mass of a (sufficiently heavy) fermion with its corresponding bosonic ‘super-partner’. Because the DLCQ resolution of a boson and fermion state differ by one unit of momentum (after imposing anti-periodic boundary conditions for fermions), we were able to derive naturally formula (11) from equation (10) describing the mass of two free particles.

Finally, we comment that the finite $N$ analysis we have adopted here affords interesting insights into large $N$ theories in general; solving a theory for increasing values of $N$ enables one to study the large $N$ limit in a way that is not accessible in the $N = \infty$ – or planar – approximation. In particular, subtleties can arise in the large $N$ limit where single and multi-trace states co-exist on an equal footing. This is especially true for theories that may be susceptible to a phase transition – such as the model studied here – and may play a crucial role in understanding the fundamental degrees of freedom of large $N$ gauge theories.

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