Phenomenology of $n$–$\bar{n}$ oscillations revisited

S. Gardner* and E. Jafari

Department of Physics and Astronomy,
University of Kentucky, Lexington, Kentucky 40506-0055 USA

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Abstract

We revisit the phenomenology of $n$–$\bar{n}$ oscillations in the presence of external magnetic fields, highlighting the role of spin. We show, contrary to long-held belief, that the $n$–$\bar{n}$ transition rate is not suppressed, opening new opportunities for its empirical study.
1. Introduction. Searches for processes that violate standard model (SM) symmetries are of particular interest because their discovery would serve as unequivocal evidence for dynamics beyond the SM. The gauge symmetry and known particle content of the SM implies that its Lagrangian conserves baryon number $B$ and lepton number $L$, though it is the combination $B - L$ that survives at the quantum level. Thus the observation of neutron-antineutron ($n$-$\bar{n}$) oscillations, a $|\Delta B| = 2$ process, would show that $B - L$ symmetry is broken and ergo that dynamics beyond the SM exists. The current constraints on $|B| = 1$ operators from the non-observation of nucleon decay are severe, with the strongest limits coming from searches for proton decay to final states that respect $B - L$ symmetry, such as $p \rightarrow e^+ \pi^0$, for which the partial half-life exceeds $8.2 \times 10^{33}$ years at 90% C.L. [1]. Although particular $|\Delta B| = 1$ operators, such as those that mediate $n \rightarrow e^- \pi^+$, e.g., can also give rise to $n$-$\bar{n}$ oscillations, Mohapatra and others have emphasized that the origin of nucleon decay and $n$-$\bar{n}$ oscillations can be completely different [2–7]. Recently, moreover, simple models that give rise to $n$-$\bar{n}$ oscillations but not nucleon decay have been enumerated [6].

Phenomenological studies of meson mixing are typically realized in the context of a $2 \times 2$ effective Hamiltonian matrix [8]. The seminal papers on free $n$-$\bar{n}$ oscillations [9, 10] have also followed such a framework, and the existing experimental search [11] has, in turn, followed its guidance. Consequently we briefly review this work before turning to our generalization. The neutron magnetic moment is well-known, yielding an interaction with an external magnetic field $B$ of form $-\mu_n S_n \cdot B/S_n$, where $\mu_n$ is the magnitude of the magnetic moment and $S_n$ is the neutron spin. Nevertheless, the early papers [9, 10] analyze the effect of an external magnetic field in a $2 \times 2$ framework, explicitly suppressing the role of the neutron (and antineutron) spin. Supposing the neutron spin to be in the direction of the applied $B$-field and employing CPT invariance, the mass matrix $M$ takes the form [9]

$$M = \begin{pmatrix} M_n - \mu_n B & \delta \\ \delta & M_n + \mu_n B \end{pmatrix},$$

(1)

where we note that CPT invariance guarantees not only that the neutron and antineutron masses are equal but also that the projections of the neutron and antineutron magnetic moments on $B$ are equal in magnitude and of opposite sign. We work in units $\hbar = c = 1$ and ignore the finite neutron and antineutron lifetimes throughout. Diagonalizing $M$ yields
the mass eigenstates $|u_i\rangle$, namely,

$$
|u_1\rangle = \cos \theta |n\rangle + \sin \theta |\bar{n}\rangle ,
|u_2\rangle = - \sin \theta |n\rangle + \cos \theta |\bar{n}\rangle .
$$

(2)

Since the energy scale $\mu_n B$ naturally dwarfs that of $\delta$, for the latter is associated with $B$ violation, we note that the eigenvalue difference is $\Delta E \simeq 2\mu_n B$ and that $\theta$ is small: $\theta \simeq \delta / \Delta E$. The $n-\bar{n}$ transition probability becomes [12]

$$
P_{\pi}(t) \simeq 2\theta^2 [1 - \cos (\Delta E t)] .
$$

(3)

This result can be considered in two different limits: either (a) $\Delta E t \gg 1$ or (b) $\Delta E t \ll 1$. In case (a) the second term oscillates to zero, yielding $P_{\pi}(t) \simeq 2(\delta / \Delta E)^2$ whereas in case (b),

$$
P_{\pi}(t) \simeq \left( \frac{\delta}{\Delta E} \right)^2 (\Delta E t)^2 = (\delta t)^2 .
$$

(4)

Evidently unless $t \ll 1 / \Delta E$, the energy splitting of the neutron and antineutron in a magnetic field “quenches” the appearance of $n-\bar{n}$ oscillations. Thus the strategy in past and proposed searches for $n-\bar{n}$ oscillations has been to minimize the magnetic field [11–13], so that $t \ll 1 / \Delta E$, as well as to maintain a vacuum in the neutron flight volume [10], so that the neutrons are quasifree over the neutron observation time $t$.

Motivated by the realization that a neutron and an antineutron of opposite spin have the same energy in a magnetic field, we consider the possible spin-dependence of $n-\bar{n}$ oscillations explicitly. This thinking leads to new opportunities for their empirical study.

2. **Effective Hamiltonian for $n-\bar{n}$ transitions with spin.** Noting the low energies at which experimental searches operate, we can work in terms of neutron and antineutron degrees of freedom — and neglect their internal structure. Consequently we generalize the mass matrix of Eq. (1) to a $4 \times 4$ form, noting that the neutron and antineutron states can each also be in a spin-up or spin-down state, relative to a quantization axis $z$. We impose the constraint of Hermiticity, as well as those of charge-conjugation–parity ($\mathcal{CP}$) and time-reversal ($\mathcal{T}$) invariance, on the resulting mass matrix, to determine its most general form under these conditions.

Any $4 \times 4$ matrix has 32 real (16 complex) elements; with Hermiticity, $\mathcal{M} = \mathcal{M}^\dagger$, this reduces to 16 real elements. The discrete symmetries $\mathcal{CP}$ and $\mathcal{T}$ impose additional restrictions. We can implement the discrete symmetry transformations in relativistic quantum
field theory and translate them to quantum mechanics by noting [8]

\[
\begin{align*}
\mathbf{b}^\dagger(p, s) |0\rangle &= |n(p, s)\rangle \quad ; \quad \mathbf{d}^\dagger(p, s) |0\rangle &= |\bar{n}(p, s)\rangle,
\end{align*}
\]

where \(\mathbf{b}[\mathbf{b}^\dagger](p, s)\) and \(\mathbf{d}[\mathbf{d}^\dagger](p, s)\) denote annihilation [creation] operators for neutrons [antineutrons] of momentum \(p\) and spin projection \(s\), for which \(s = \pm 1 \equiv \pm \) with respect to the quantization axis \(z\). We determine the transformation properties of these operators under \(\text{CP}T\) recalling that \(\text{CP}T\) invariance of the ground (vacuum) state remains invariant under \(\text{CP}\) and \(T\) as follows. We work in the Dirac-Pauli representation for the \(\gamma^\mu\) matrices and note that the Dirac field operator \(\psi(x)\) has a plane wave expansion of form

\[
\psi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2E}} \sum_{s=\pm} \left\{ b(p, s)u(p, s)e^{-ip \cdot x} + d^\dagger(p, s)v(p, s)e^{ip \cdot x} \right\},
\]

with spinors defined as

\[
u(p, s) = \mathcal{N} \left( \begin{array}{c} \chi^{(s)} \\ \frac{\sigma \cdot p}{E+M} \chi^{(s)} \end{array} \right) ; \quad v(p, s) = \mathcal{N} \left( \begin{array}{c} \frac{\sigma \cdot p}{E+M} \chi^{(s)} \\ \chi^{(s)} \end{array} \right),
\]

noting \(\chi^{(s)} = \chi^{(-s)}\), \(\chi^+ = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\}\), \(\chi^- = \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right\}\), and \(\mathcal{N} = \sqrt{E+M}\). This yields

\[
\text{CP} \ b(p, s) (\text{CP})^\dagger = sd(-p, s) \quad ; \quad \text{CP} \ d(p, s) (\text{CP})^\dagger = -sb(-p, s)
\]

and

\[
T \ b(p, s) (T)^{-1} = sb(-p, -s) \quad ; \quad T \ d(p, s) (T)^{-1} = -sd(-p, -s)
\]

for the transformation properties under \(\text{CP}\) and \(T\), respectively. In what follows we assume that the ground (vacuum) state remains invariant under \(\text{CP}\) and \(T\): \(\text{CP}|0\rangle = |0\rangle\) and \(T|0\rangle = |0\rangle\).

Defining the mass matrix \(\mathcal{M}\) so that its entries \(\mathcal{M}_{ij}\) with \(i, j = 1, \ldots, 4\) correspond to bras and kets containing \(n(p, +), \bar{n}(p, +), n(p, -),\) and \(\bar{n}(p, -)\), respectively, we note that under an assumption of \(\text{CP}\) and \(T\) invariance relationships between the matrix elements follow. For example, under \(\text{CPT}\) invariance we have

\[
\langle n(p, s_1)|H|n(p, s_2)\rangle = \langle \bar{n}(p, -s_2)|H|\bar{n}(p, -s_1)\rangle,
\]

recalling that \(T\) is an anti-unitary operator. This yields \(\mathcal{M}_{11} = \mathcal{M}_{44}, \mathcal{M}_{22} = \mathcal{M}_{33}, \mathcal{M}_{13} = \mathcal{M}_{24}\), and \(\mathcal{M}_{31} = \mathcal{M}_{42}\). Analogously, under \(\text{CPT}\) invariance of the \(n \leftrightarrow \bar{n}\) matrix elements,

\[1\] These results differ from those in Ref. [8] because that work uses a different choice of antiparticle spinor, namely that \(v(0, s)\) has an angular momentum \(J_z = -s/2\) [14].
we recover $M_{12} = M_{34}$ as well. Thus under CPT and Hermiticity the mass matrix has ten parameters, and it is of form

$$
\begin{pmatrix}
A_1 & \delta & M_1 & \varepsilon_1 \\
\delta^* & A_2 & \varepsilon_2 & M_1 \\
M_1^* & \varepsilon_2^* & A_2 & \delta \\
\varepsilon_1^* & M_1^* & \delta^* & A_1
\end{pmatrix},
$$

(11)

where $A_1$ and $A_2$ are real constants. Under CP invariance we have: $\langle n(p, s_1)|H|n(p, s_2)\rangle = s_1 s_2 \langle \bar{n}(-p, s_1)|H|\bar{n}(-p, s_2)\rangle$, yielding $M_{11} = M_{22}$, $M_{33} = M_{44}$, $M_{13} = -M_{24}$, and $M_{31} = -M_{42}$ in the low energy limit, i.e., as $|p| \to 0$. Analogously for the $n \leftrightarrow \bar{n}$ matrix elements we have: $\langle n(p, s_1)|H|\bar{n}(p, s_2)\rangle = -s_1 s_2 \langle \bar{n}(-p, s_1)|H|n(-p, s_2)\rangle$, yielding, in the low-energy limit, $M_{12} = -M_{21}$, $M_{34} = -M_{43}$, $M_{14} = M_{23}$, and $M_{32} = M_{41}$. Thus under Hermiticity and CP and CPT invariance we have in this case

$$
\begin{pmatrix}
A_1 & i\delta & 0 & \varepsilon_1 \\
-i\delta & A_1 & \varepsilon_1 & 0 \\
0 & \varepsilon_1^* & A_1 & i\delta \\
\varepsilon_1^* & 0 & -i\delta & A_1
\end{pmatrix},
$$

(12)

where both $A_1$ and $\delta$ are real — and only four parameters suffice to characterize the mass matrix.

These parametrizations allow us to generalize our effective Hamiltonian framework to include external magnetic fields. For example, the interaction of an electrically neutral particle with an electromagnetic field is characterized at low energies by $-\mu \cdot B$ if T and P are not broken; this is the nonrelativistic limit of $\bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu}$, where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the usual electromagnetic field strength tensor. Under CP or T the fermion bilinear $\bar{\psi} \sigma^{\mu\nu} \psi$ transforms to $-\bar{\psi} \sigma^{\mu\nu} \psi$, and $F_{\mu\nu}$ transforms to $-F^{\mu\nu}$. Thus their scalar product is itself both CP and T invariant. However, the explicit CPT and CP constraints we have investigated operate on the fermion and antifermion degrees of freedom only; the terms in $H$ resulting from the overall minus sign associated with $\bar{\psi} \sigma^{\mu\nu} \psi$ under CP are revealed by comparing the parametrizations under Hermiticity and CPT with and without a CP constraint, Eqs. (11) and (12), yielding $A_1 \neq A_2$ and $M_1 \neq 0$.

Spin-dependent asymmetries can act as additional probes of the underlying operators that enter. Denoting $\Gamma(n(s_1) \to \bar{n}(s_2))$ as the $n(s_1) \to \bar{n}(s_2)$ transition rate for neutrons at
rest, we note

\[
A = \frac{\Gamma(n(+) \rightarrow \bar{n}(-)) + \Gamma(n(+) \rightarrow \bar{n}(+)) - \Gamma(n(-) \rightarrow \bar{n}(+)) - \Gamma(n(-) \rightarrow \bar{n}(-))}{\Gamma(n(+) \rightarrow \bar{n}(-)) + \Gamma(n(+) \rightarrow \bar{n}(+)) + \Gamma(n(-) \rightarrow \bar{n}(+)) + \Gamma(n(-) \rightarrow \bar{n}(-))}.
\]  

(13)

In the absence of external magnetic fields, Eq. (13) becomes a true test of CP since \(\Gamma(n(s_1) \rightarrow \bar{n}(s_2)) = \Gamma(\bar{n}(s_2) \rightarrow n(s_1))\) under Hermiticity.

3. Effective Hamiltonian for \(n-\bar{n}\) transitions in external magnetic fields. To start, we suppose the \(n\) and \(\bar{n}\) interactions in the presence of external magnetic fields, under CPT invariance, to be of form

\[
H_B = -\mu_n S_n \cdot B - \mu_{\bar{n}} S_{\bar{n}} \cdot B - \mu_{n\bar{n}} S_{n\bar{n}} \cdot B - \mu_{n\bar{n}} S_{\bar{n}n} \cdot B,
\]

(14)

where \(\mu_{n\bar{n}}\) is the \(n-\bar{n}\) transition magnetic moment and is assumed to be real. The spin operators each act in a \(2 \times 2\) subspace. We note that the states containing \(n(+)\) and \(\bar{n}(-)\) are associated with spinors of fixed polarization, as are the states with \(n(-)\) and \(\bar{n}(+)\). Thus we organize the spin operators so that the spinor polarizations are treated in a uniform way. With \((S_n)_{i,j}\) such that \((i, j) \in \{n(+), n(-)\}\), we choose \((S_{\bar{n}})_{i,j}\) with \((i, j) \in \{\bar{n}(-), \bar{n}(+)\}\), as well as \((S_{n\bar{n}})_{ij}\) and \((S_{\bar{n}n})_{ji}\) with \(i \in n(+)\), \(n(-)\) and \(j \in \bar{n}(-), \bar{n}(+)\). The diagonal matrix elements are arranged so that the underlying spinor polarization does not flip in the transition; thus these elements are associated with a longitudinal field \(B_0 = B_0 \hat{z}\). In the off-diagonal matrix elements the spinor polarization does flip, and thus in these elements a transverse magnetic field is present, namely, \(B_1 = B_1 \hat{x}\). Within a given subspace, we compute \(\mathbf{S} \cdot \mathbf{B} / S = \sigma \cdot \mathbf{B}\). Employing the usual Pauli matrices, this yields a form compatible with Eq. (11). Upon including the magnetic-field-independent Hamiltonian, Eq. (12), the Hamiltonian becomes

\[
H(t) = \begin{pmatrix}
M + \omega_0 & \delta_1 & \omega_1 & \varepsilon + \delta_0 \\
\delta_1 & M - \omega_0 & \varepsilon - \delta_0 & \omega_1 \\
\omega_1 & \varepsilon - \delta_0 & M - \omega_0 & \delta_1 \\
\varepsilon + \delta_0 & \omega_1 & \delta_1 & M + \omega_0
\end{pmatrix},
\]

(15)

where \(\delta_0 \equiv \mu_{n\bar{n}}B_0\), \(\delta_1 \equiv \mu_{n\bar{n}}B_1\), and the \(i\delta\) terms of Eq. (12) have been removed for want of an external agent to flip the spinor polarization. For fixed particle spin, Eq. (15) can be compared to the original phenomenological form, Eq. (1); the off-diagonal terms differ in structure and interpretation regardless of whether the \(i\delta\) terms are present. Moreover, we
see that CPT invariance guarantees that a neutron spin state and an antineutron spin state in vacuum are always degenerate irrespective of the size of the magnetic field: the presence of external magnetic fields cannot quench transitions between these states. The study of time-dependent fields can yield additional constraints on the $n$-$\bar{n}$ operators that can appear, particularly those associated with $n(\pm) \rightarrow \bar{n}(\pm)$ transitions [15].

4. Operator Analysis and Majorana Constraints. The leading mass dimension operators that yield $n$-$\bar{n}$ transitions have been analyzed in QCD [16, 17], and their associated $n$-$\bar{n}$ matrix elements have been computed in models [16, 18] and in lattice QCD [19]. They are of mass dimension nine, and they contain the quark-level building blocks $q_i^{T\alpha}Cq_i^\beta$, where $q_i^\alpha$ is a quark field of flavor $i \in u,d$ and color $\alpha$, whereas T denotes transpose. The final $n$-$\bar{n}$ operator is either symmetric or antisymmetric in the color indices. The building blocks are written in a chiral basis, $\chi \in L,R$, noting that $q_i^\alpha \equiv (1 \mp \gamma_5)q_i^\alpha$. The leading operators do not change the polarization of the low-energy spinors on which they act. The spinors have the same helicity, and thus the quark and antiquark have opposite helicity, as familiar from the analysis of $e^+e^-$ annihilation [14]. Explicitly, we have, e.g., $d_R^{T\alpha}Cd_R^\beta = -(d_R^\alpha)^*\gamma^2d_R^\beta$. Consequently, the operators describe the transition $n(s) \rightarrow \bar{n}(-s)$. This is also consistent with Dirac hole theory: the absence of a particle of spin $s$ is equivalent to the creation of a hole with spin $-s$. Thus the leading dimension operators map to $\text{Re}(\varepsilon_1)$ in Eq. (12). The other operators are of higher mass dimension.

The existence of $n$-$\bar{n}$ oscillations would connote that the neutron and antineutron can be rewritten in terms of Majorana states, and additional constraints on the form factors follow. A Majorana state $|\Psi_M\rangle$ transforms into itself under $C$, up to a global phase. Since $Cb(p,s)C^\dagger = sd(p,s)$,

$$|\Psi_M(p,s)\rangle = \frac{1}{\sqrt{2}} \left( s|\bar{n}(p,s)\rangle \pm |n(p,s)\rangle \right),$$

so that two distinct Majorana states, each with $s = \pm$, exist. The neutron and antineutron are distinct because they are distinguished by the sign of the lepton charge upon decay, and the Majorana basis must account for four degrees of freedom. In contrast, the Majorana neutrino is described by a two-component field. Nevertheless, additional constraints exist, irrespective of whether the Majorana state is pointlike or composite. Specifically, there are no $\gamma^\mu$, $\sigma^{\mu\nu}$, or $\sigma^{\mu\nu}\gamma_5$ form factors associated with a Majorana state [20–25]; thus the
constraint \(\langle \Psi_M^\pm (p, s)|H_B|\Psi_M^\pm (p, s)\rangle = 0\) implies that
\[
\langle \bar{n}(p, s)|H_B|n(p, s)\rangle + \langle n(p, s)|H_B|\bar{n}(p, s)\rangle = 0,
\]
or that \(\text{Re}(\delta_1) = 0\). The transition \(n-\bar{n}\) magnetic moment need not vanish, but the pertinent terms in Eq. (11) and Eq. (14) should be modified to take Eq. (17) into account. This bears analogy to studies in neutrino physics. The neutrino transition magnetic moment is associated exclusively with the transverse magnetic field and is flavor-changing; the transition magnetic moment for neutrinos of the same flavor vanishes by fermion anti-symmetry [21, 25]. The \(n-\bar{n}\) transition operators do not vanish for this reason because of their complex color-flavor structure; one can also show that the transition magnetic moment matrix elements yield those between distinct Majorana states and are thus nonzero. The possibility of resonant spin-flavor neutrino precession in matter, such as in the Sun, has also been studied [26–28].

5. Examples. To compute the transition probabilities, we must first find the normalized eigenvectors of the Hamiltonian matrix in terms of our chosen \(\{|n\pm\rangle, |n\mp\rangle, |\bar{n}\rangle\}\) basis; we denote a state of the latter by \(|n_i\rangle\) and a normalized eigenvector by \(|u_i\rangle\) with associated eigenvalue \(\lambda_i\), noting \(i \in 1, \ldots, 4\). The time evolution of a state of the Hamiltonian is thus given by
\[
|\psi(t)\rangle = \sum_{i=1}^4 e^{-i\lambda_i t}\langle u_i|\psi(0)\rangle |u_i\rangle.
\]
Letting \(|\psi(0)\rangle = |n_k\rangle\) and defining \(a_{ij} \equiv \langle n_j|u_i\rangle\), we find
\[
\mathcal{P}_{n_k \rightarrow n_j} = \left| \sum_{i=1}^4 e^{-i\lambda_i t}a_{ij}a_{ik}^* \right|^2.
\]
In our first example we assume no external matrix fields are present. Choosing \(H\) as
\[
\begin{pmatrix}
M & i\delta & 0 & \varepsilon \\
-i\delta & M & \varepsilon & 0 \\
0 & \varepsilon & M & i\delta \\
\varepsilon & 0 & -i\delta & M
\end{pmatrix},
\]
with \(\delta\) and \(\varepsilon\) real, we find \(\mathcal{P}_{n_+ \rightarrow n_+} = (\delta^2/(\delta^2 + \varepsilon^2)) \sin^2(t\sqrt{\delta^2 + \varepsilon^2})\), \(\mathcal{P}_{n_+ \rightarrow n_-} = 0\), and \(\mathcal{P}_{n_- \rightarrow n_-} = (\varepsilon^2/(\delta^2 + \varepsilon^2)) \sin^2(t\sqrt{\delta^2 + \varepsilon^2})\). The unpolarized \(n-\bar{n}\) transition probability is thus \(\mathcal{P}_{n \rightarrow \bar{n}} = \sin^2(t\sqrt{\delta^2 + \varepsilon^2})\). At small \(t\) its structure is just that of Eq. (4), so that the existing experimental limit [11] does indeed constrain the leading dimension \(n-\bar{n}\) transition operator.
As a second example, we consider the leading \( n-\bar{n} \) transition operator in the presence of an arbitrary magnetic field \( B \). The Hamiltonian with \( B_0 \) and \( B_1 \) is

\[
\begin{pmatrix}
M + \omega_0 & 0 & \omega_1 & \varepsilon \\
0 & M - \omega_0 & \varepsilon & \omega_1 \\
\omega_1 & \varepsilon & M - \omega_0 & 0 \\
\varepsilon & \omega_1 & 0 & M + \omega_0
\end{pmatrix},
\]

yielding

\[
P_{n\rightarrow n^+} = \frac{\omega_1^2 \sin^2(t\varepsilon) \sin^2(t\sqrt{\omega_0^2 + \omega_1^2})}{\omega_0^2 + \omega_1^2},
\]

\[
P_{n\rightarrow n^-} = \frac{\sin^2(t\varepsilon)(\omega_0^2 + \omega_1^2 \cos^2(t\sqrt{\omega_0^2 + \omega_1^2}))}{\omega_0^2 + \omega_1^2},
\]

and finally \( P_{n\rightarrow \bar{n}} = \sin^2(te) \) for the unpolarized \( n-\bar{n} \) transition rate. Thus we see that the figure of merit for a \( n-\bar{n} \) oscillation experiment is set by the interrogation time \( t \) and the number of available neutrons: the magnitude of the magnetic field is irrelevant. Although we have assumed \( B \) to be both static and uniform in this example, neither assumption is key to the final result because it derives from the equal energy associated with the \( n(s) \) and \( \bar{n}(-s) \) elements in Eq. (21), which persists even if \( \omega_0(t, x) \) as it is guaranteed by CPT invariance.

As a last example we include the subleading operator characterized by \( \delta_0 \), as per Eq. (15), in the Hamiltonian of Eq. (21). In this case the spin-dependent asymmetry of Eq. (13) is controlled by \( \delta_0 \varepsilon \), and to leading order in \( \delta_0 \) and \( \varepsilon \) we have

\[
A = \frac{\delta_0 \omega_1^4 \eta^2 (4t\omega_0^4 + 8t\omega_0^2 \omega_1^2 \sin(2t\eta) - 8t\omega_1^2 \omega_0^2 - 4t^2 \omega_0^2 \eta^2 + 2t\omega_1^2 \eta \sin(2t\eta))}{2t\varepsilon(\eta^2 - \omega_0\eta)^3(\eta^2 + \omega_0\eta)^2},
\]

with \( \eta \equiv \sqrt{\omega_0^2 + \omega_1^2} \), yielding \( A \sim 2\delta_0/\varepsilon \) for \( |\omega_0| \gg |\omega_1| \). The asymmetry in this limit grows linearly with \( |B_0| \). The noted enhancement of the \( n-\bar{n} \) oscillation probability with \( |B_0| \), though associated with a subleading operator, could help lead to its discovery.

6. New Experimental Prospects. An experimental limit on \( n-\bar{n} \) oscillations can be defined by writing the transition probability as \( P_{n\rightarrow \bar{n}} \sim (t/\tau_{nn})^2 \) and bounding \( \tau_{nn} \). The existing experimental limit of \( \tau_{nn} \geq 0.86 \times 10^8 \) s at 90% C.L. [11] was set using a cold neutron beam of intensity \( I = 10^{11} \) s\(^{-1} \) which was allowed to propagate for \( (t^2)^{1/2} \approx 0.1 \) s in a low pressure and magnetic field environment. No antineutrons (\( \bar{N} \leq 2.3 \) at 90% C.L.) were detected in a running time of \( t_{\text{run}} = 2.40 \times 10^7 \) s, to yield a limit on \( \tau_{nn} \), which we
capture crudely as $(\tau_{n\bar{n}})_{\text{beam}} \sim \left( H_{\text{run}}\langle t^2 \rangle / \bar{N} \right)^{1/2}$. Our work has shown that magnetic field mitigation is unnecessary; this implies that next-generation $n\bar{n}$ experiments [12, 13] can be realized at substantially reduced effort and cost. Concomitantly, applying a magnetic field cannot be used as a check of whether a claimed $n\bar{n}$ signal is genuine. Moreover, it becomes possible to study $n\bar{n}$ oscillations by confining neutrons in an external magnetic field. Such magnetic traps, or bottles, are under development for improved measurements of the neutron lifetime [29–31], though the optimal parameters for the current application would differ — here maximizing the number of neutrons in the trap is of greatest importance. A crude estimate of the oscillation lifetime is given by $(\tau_{n\bar{n}})_{\text{bottle}} \sim \left( N_{\text{fill}} N_{\text{trial}} \langle t^2 \rangle / \bar{N} \right)^{1/2}$, where $N_{\text{fill}}$ is the number of neutrons (i.e., $nV$ with $n$ the neutron number density and $V$ the volume of the trap) added to the bottle at one time, $N_{\text{trial}}$ is the number of times the trap is filled, and $\langle t^2 \rangle^{1/2}$ is the storage time in the trap. Estimating $N_{\text{fill}} \sim 10^7$, $N_{\text{trial}} \sim 10^5$, and $\langle t^2 \rangle^{1/2} \sim 400$ s yields $\tau_{n\bar{n}} \sim 2 \times 10^8$ s, so that the gain seems modest, though one can expect further improvements with bettered ultracold neutron sources.

7. Summary. Although many authors [32–36] have studied the impact of external magnetic fields on $n\bar{n}$ oscillations within the context of the $2 \times 2$ phenomenological framework [9], our work is the first to incorporate spin in a fundamental way. The results that emerge are remarkably different from earlier studies — in particular, magnetic field mitigation is not required to observe $n\bar{n}$ mixing, greatly enabling possible future experiments with cold neutron beams [12, 13]. Certain subleading $n\bar{n}$ operators can also be enhanced through the application of external magnetic fields, and their presence can be probed through a spin-dependent asymmetry. Magnetic field mitigation in this context yields a new probe of CP violation. In future work [15] we plan to study the role of matter effects and time-dependent magnetic fields, as well as to enumerate the various subleading operators that can appear.

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