$Z'$, new fermions and flavor changing processes.

Constraints on $E_6$ models from $\mu \to eee$.

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Abstract

We study a new class of flavor changing interactions, which can arise in models based on extended gauge groups (rank >4) when new charged fermions are present together with a new neutral gauge boson. We discuss the cases in which the flavor changing couplings in the new neutral current coupled to the $Z'$ are theoretically expected to be large, implying that the observed suppression of neutral flavor changing transitions must be provided by heavy $Z'$ masses together with small $Z$-$Z'$ mixing angles. Concentrating on $E_6$ models, we show how the tight experimental limit on $\mu \to eee$ implies serious constraints on the $Z'$ mass and mixing angle. We conclude that if the value of the flavor changing parameters is assumed to lie in a theoretically natural range, in most cases the presence of a $Z'$ much lighter than 1 TeV is unlikely.

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I. Introduction

The large set of accurate measurements performed during the last few years has established that the standard electroweak theory provides an excellent description of the particle physics phenomena up to about 100 GeV. It should also be stressed that the present theory accommodates in a satisfactory way the whole spectrum of known particles, and only two states, the top–quark and the Higgs scalar that are necessary for the consistency of the model, have not been discovered yet. Another feature of the standard model (SM) that explains in a satisfactory way a large set of experimental limits on rare processes, is the Glashow-Iliopoulos-Maiani suppression of flavor changing neutral currents (FCNC) in the quark sector, together with the absence of lepton flavor violating (LFV) currents. All these features are quite peculiar of the SM, and in general most of its possible extensions predict a larger spectrum of states as well as larger rates for FCNC processes. Clearly the direct detection of any new unconventional particle would be a major breakthrough towards the identification of a new and more fundamental theory, but at the same time it is not unlikely that some hints on the existence of new physics will come from the observation of rare processes at rates larger than what expected in the standard theory.

The aim of this paper is to analyze a new class of FCNC interactions that are generally present in most of the extensions of the SM which predict one (or more) additional neutral gauge boson $Z_1$ together with new charged fermions, and that are induced by $Z_1$ interactions. In general the $Z_1$ is expected to be mixed with the standard $Z_0$, and as a consequence both the resulting mass eigenstates, that we will denote as $Z$ and $Z'$, will have flavor changing couplings to the known charged fermions.
In particular we will concentrate on LFV interactions, which are strictly forbidden in the SM, and we will show that they could provide a clear signature for this kind of new physics. In turn, the existing limits on LFV processes imply serious constraints for several interesting models. To stress the power of these constraints we will apply them to a class of E$_6$ grand unified theories (GUTs), which are a well known example of theories where additional fermions and new neutral gauge bosons are simultaneously present, and provide a good frame for illustrating the kind of effects that we want to explore. In an attempt of quantifying the constraints, we will assume for the relevant E$_6$ LFV couplings a range of values that we believe is theoretically natural, and we will then turn the existing limits on LFV decays into limits on the $Z'$ parameters. Though model dependent, our bounds turn out to be much tighter than the present limits obtained from direct searches at colliders [1] or as derived from the analysis of other $Z'$ indirect effects [2–4].

The interesting possibility of violating the conservation of lepton family number as a consequence of $Z_1$ interactions was already briefly analyzed in [5] for a general class of extended electroweak models. It was shown that LFV decays of the standard neutral gauge boson like $Z \to \ell_i \ell_j$, ($i \neq j$, hereafter understood) can be induced by the presence of a $Z_1$ if a sizeable mixing with the $Z_1$ is present, and ii) if the new neutral gauge boson do not couple universally to the fermion generations. While the first condition is a natural feature of extended gauge models, it could seem that the second one is somewhat more difficult to realize. However, we will show that both these conditions are naturally satisfied in extended gauge models that, like E$_6$, predict also new charged fermions. This was already noted in a recent paper [2], where the consequences of the simultaneous presence of new neutral gauge bosons and new fermions were analyzed, and a general formalism for taking into account their combined effects was outlined.

In models like E$_6$, where several new neutral leptons are present and a quite general form for the corresponding mass matrices and mixing patterns is possible, LFV processes like $Z \to \ell_i \ell_j$ arise naturally due to the contributions of
loop diagrams [6]. A $Z_0\ell_i\ell_j$ vertex for the gauge eigenstate $Z_0$ boson can however appear already at the tree level, due to the presence of new exotic charged leptons that could be mixed with the standard ones. A general discussion of this kind of flavor changing vertices is given e.g. in [7], while the particular case of $E_6$ models is analyzed in [8]. The interest of considering flavor changing $Z_1$ interactions stems from the fact that while in general the $Z_0$ flavor changing vertices are strongly suppressed as the ratio between the masses of the known and of the new fermions, in some class of models no suppression factors are expected for the $Z_1 f_i f_j$ vertices. Then, in the context of these models, the new interactions due to the additional gauge boson could well represent the main source of flavor changing transitions.

In Sec. II we will first review the essential formalism for dealing with gauge boson and fermion mixing effects [2] and we will also discuss the theoretical expectations for the various flavor changing parameters. In Sec. III we will concentrate on $E_6$ models. We will confront the theoretical expression for the decay $\mu \rightarrow eee$ with the extremely stringent experimental limit $Br(\mu^+ \rightarrow e^+ e^+ e^-) < 1 \times 10^{-12}$ obtained by the SINDRUM collaboration [9], and we will derive constraints on the mass of the new gauge boson and on the $Z_0-Z_1$ mixing angle as a function of the LFV parameters. Finally, in Sec. IV we will draw our conclusions.

**II. $Z'$ and new fermion effects. Formalism**

We will now review the formalism for describing the combined effects due to the presence of a new neutral gauge boson, and of new fermions that could be mixed with the known ones. We will follow closely the presentation given in [2] concentrating mainly on the charged fermions sector and on flavor changing effects, and we refer to [2] for a more general discussion. We will assume a low energy
gauge group of the form $G_{SM} \times U(1)$, where $G_{SM} = SU(2)_L \times U(1)_Y \times SU(3)_C$ is the usual SM gauge group. Then in the gauge basis, the corresponding neutral current Lagrangian reads

$$-\mathcal{L}_{NC} = e J_{em}^\mu A_\mu + g_0 J_0^\mu Z_0 \mu + g_1 J_1^\mu Z_1 \mu.$$  \hspace{1cm} (2.1)

In (2.1) $Z_0$ is the SM neutral gauge boson, which couples with strength

$$g_0 = (4\sqrt{2} G_F M_{Z_0}^2)^{1/2}$$ \hspace{1cm} (2.2)

to the usual combination of the neutral isospin and electromagnetic currents

$$J_0^\mu = J_3^\mu - s_W^2 J_{em}^\mu$$ \hspace{1cm} (2.3)

where $s_W^2 = \sin^2 \theta_W$ is the weak mixing angle. The new $Z_1$ corresponds to the additional $U(1)$ factor, and couples to the new $J_1$ current with strength $g_1$. In particular, if we assume that the $U(1)$ originates at low energy from a GUT based on a simple group, then the $q_1(f)$ charges of the fermions are fixed by the gauge group. Normalizing the new generator $Q_1$ to the hypercharge axis $Y/2$ and assuming a similar renormalization group evolution for the two abelian couplings $g_1$ and $g_Y$ down to the electroweak scale, the coupling strength of the new interaction can be written as

$$g_1 \simeq g_0 s_W.$$ \hspace{1cm} (2.4)

In general, after spontaneous symmetry breaking the $Z_0 - Z_1$ mass matrix turns out to be non diagonal. The $Z$ and $Z'$ gauge bosons mass eigenstates then correspond to two orthogonal combinations of $Z_0$ and $Z_1$ that we will parametrize in term of an angle $\phi$. As a result, the currents that couple with strength $g_0$ to the physical $Z$ and $Z'$ are:

$$
\begin{pmatrix}
J_Z^\mu \\
J_{Z'}^\mu
\end{pmatrix} =
\begin{pmatrix}
c_\phi & s_\phi \\
-s_\phi & c_\phi
\end{pmatrix}
\begin{pmatrix}
J_0^\mu \\
J_1^\mu
\end{pmatrix}.
\hspace{1cm} (2.5)
$$

We see that the main effects of the presence of the new gauge boson are the additional contribution to NC amplitudes, described by the third term in the
Lagrangian (2.1), and the mixing between the \( J_0 \) and \( J_1 \) currents in (2.5). We will not discuss additional indirect effects, as e.g. the shifts induced by \( Z_0 - Z_1 \) mixing in the value of the weak mixing angle \( s_W \) and in the overall coupling strength \( g_0 \) when expressed as a function of the \( Z \) mass [2-4], since they are irrelevant for the present analysis.

We now discuss the form of the two currents \( J_0 \) and \( J_1 \), and in particular the effects that could originate from the presence of additional charged fermions. We will henceforth denote as new, possible degrees of freedom in addition to the standard 15 known fermions per generation. Since no new fermions have been directly observed yet, the new states must be rather heavy, and \( m_{\text{new}} \sim M_Z/2 \) can be taken as the present model independent limit on the new masses. Accordingly, we will denote the corresponding mass eigenstates as heavy, while the known mass eigenstates will be labeled as light. In the presence of additional fermions, the light mass eigenstates will correspond to superpositions of the known and new states. Conservation of the electric and color charges forbids a mixing between gauge eigenstates with different \( U(1)_{\text{em}} \) and \( SU(3)_c \) quantum numbers, and, in turn, this implies that the electromagnetic and color currents of the light mass eigenstates are not modified in the presence of the new states. However, the neutral isospin generator \( T_3 \) and the new generator \( Q_1 \) are spontaneously broken, then a mixing between gauge eigenstates with different \( t_3 \) and \( q_1 \) eigenvalues is allowed, and as a result the couplings of the light mass eigenstates to the \( Z_0 \) and \( Z_1 \) will be affected.

Since in the gauge currents chirality is conserved too, it is convenient to group the fermions with the same electric charge and chirality \( \alpha = L, R \) in a column vector of the known (\( \mathcal{K} \)) and new (\( \mathcal{N} \)) gauge eigenstates \( \Psi^\alpha = (\Psi^K, \Psi^N)^T \). The relation between the gauge eigenstates \( \Psi^\alpha \) and the corresponding light and heavy mass eigenstates \( \Psi_\alpha = (\Psi^l, \Psi^h)^T \) is then given by a unitary transformation

\[
\begin{pmatrix}
\Psi^K \\
\Psi^N
\end{pmatrix}_\alpha = U_\alpha 
\begin{pmatrix}
\Psi^l \\
\Psi^h
\end{pmatrix}_\alpha, \quad \text{where} \quad U_\alpha = \begin{pmatrix} A & G \\ F & H \end{pmatrix}_\alpha, \quad \alpha = L, R. \quad (2.6)
\]

The submatrices \( A \) and \( F \) describe the overlap of the light eigenstates with the
known and the new states respectively, and from the unitarity of $U$ we have
\[ A^\dagger A + F^\dagger F = AA^\dagger + GG^\dagger = I. \] (2.7)

Note that we have not introduced an extra index to label the electric charge; nevertheless we will treat $\Psi_0^\alpha$ and $\Psi_\alpha$ as vectors corresponding to a definite value of $q_{\text{em}}$.

In terms of the fermion mass eigenstates the neutral current corresponding to a (broken) generator $Q$ reads
\[ J_\mu^Q = \sum_{\alpha = L,R} \bar{\Psi}_{\alpha} \gamma^\mu U_\alpha^\dagger Q_\alpha U_\alpha \Psi_\alpha, \] (2.8)

where $Q_\alpha$ represents a generic diagonal matrix of the charges for the chiral fermions. Here we have to consider only the mixing effects in $J_3$ appearing in (2.3) and in $J_{\text{em}}$, since the term proportional to $J_{\text{em}}$ in $J_0$ is not modified by fermion mixing. Hence in (2.8) $Q = T_3, Q_1$ and the elements of the corresponding matrices are given by the eigenvalues $t_3$ and $q_1$.

From (2.8) we see that if in one subspace of states with equal electric charge and chirality the matrix $Q_\alpha$ is proportional to the identity, then $U_\alpha^\dagger Q_\alpha U_\alpha = Q_\alpha$ and for these fermions the corresponding current is not modified in going to the base of the mass eigenstates. In the SM for example, for a given electric charge and chirality the $t_3$ eigenvalues of the fermions are indeed the same, and this implies in particular the absence (at the tree level) of FCNC. In models with new fermions in contrast, the diagonal matrices $Q_\alpha$ have the general form $Q_\alpha = \text{diag}(Q^K_\alpha, Q^N_\alpha)$ and do not commute with $U$.

To put in evidence the indirect effects of fermion mixings in the couplings of the light mass eigenstates, we now project (2.8) on the light components $\Psi_l$, obtaining
\[ J_\mu^Q = \sum_{\alpha = L,R} \bar{\Psi}_{l\alpha} \gamma^\mu \left[ A^\dagger_\alpha Q^K_\alpha A_\alpha + F^\dagger_\alpha Q^N_\alpha F_\alpha \right] \Psi_{l\alpha}. \] (2.9)

This equation is quite general, and describes the effects of fermion mixings in the neutral–currents of light–states for a wide class of models. If the gauge group is
generation independent, all the known states appearing in one vector $\Psi_\alpha^\sigma$ have the same eigenvalues with respect to the generators of the gauge symmetry, and hence we have $Q_\alpha^\sigma = q_\alpha^\sigma I$ with $q_\alpha^\sigma = t_3(f_\alpha^\kappa)$, $q_1(f_\alpha^\kappa)$. The same is not true in general for the new states; however, if we consider the particular case when the mixing is with only one type of new fermions with the same $q_\alpha^N$ charges, then we have $Q_\alpha^N = q_\alpha^N I$ as well. Under these conditions, and by means of the unitarity relation (2.7), (2.9) reduces to the simple form

$$J_{iQ}^\mu = \sum_{\alpha=L,R} \bar{\Psi}_{i\alpha} \gamma^\mu \left[ q_\alpha^\kappa I + (q_\alpha^N - q_\alpha^\kappa) F_\alpha^\dagger F_\alpha \right] \Psi_{i\alpha}. \quad (2.10)$$

We will restrict ourself to this equation, which is general enough for describing the mixing with the additional charged fermions in $E_6$, and we refer to [2] for a discussion of more general cases.

A few considerations are now in order. In (2.10) the first term $q_\alpha^\kappa I$ inside the square brackets gives the couplings of a particular light fermion in the absence of mixing effects. The second term represents the modifications due to fermion mixings. The matrix $F^\dagger F$ appearing in this term is in general not diagonal, and while the magnitude of the diagonal elements will affect the strength of the flavor diagonal couplings of the mass eigenstates, the off-diagonal terms will induce FCNC. Clearly whenever the coefficient $(q_\alpha^N - q_\alpha^\kappa)$ vanishes, the mixing effects are absent. When $t_3(f_\alpha^N) \neq t_3(f_\alpha^\kappa)$ the $J_0$ current is modified, and then the existing low-energy and on-resonance NC data as well as the current limits on rare processes, can be used to constrain directly the corresponding elements of $F^\dagger F$. Model independent limits on the diagonal elements $(F^\dagger F)_{ii}$ affecting $Z_0$ interactions were first given in [7] and subsequently updated in [10], and the most recent limits in the frame of $E_6$ models can be found in [2]. Bounds on the off diagonal terms $(F^\dagger F)_{i\neq j}$ have been given in [7] as well. All these limits turn out to be very stringent, usually at the level of 1% or better. In contrast, if $t_3(f_\alpha^N) = t_3(f_\alpha^\kappa)$ the $J_0$ current is not modified, but since in general we still have $q_1(f_\alpha^N) \neq q_1(f_\alpha^\kappa)$, sizable effects could indeed be present in $J_1$. We stress that
the present experimental data cannot be used to set limits on these mixings, since they only affect a new hypothetical interaction, and we can only rely on theoretical speculations to estimate their magnitude.

According to these considerations it is clear that from a phenomenological point of view it is convenient to classify possible new fermions in terms of their transformation properties under $SU(2)_L$. Since we are only interested in fermions with conventional electric charges, the new states must be singlets or doublets of weak–isospin. A rather heterodox exception is that of a gauge triplet of fermions [11], but we will not consider this possibility here. According to the nomenclature in use [2,7,10], we denote the particles with unconventional isospin assignments (left–handed singlets or right–handed doublets) as exotic fermions. All the standard fermions, as well as all the new states that have conventional $SU(2)_L$ assignments, are referred to as ordinary. For example mirror fermions, having opposite $SU(2)_L$ assignments from those of the known fermions, are exotic. Sequential fermions are simply repetition of the new fermions. They could be present in a complete new family or as components of large fermion representations and are clearly classified as ordinary. In this paper we will mainly concentrate on vector multiplets of new fermions for which the L and R components have the same $SU(2)_L$ transformation properties, and hence always contain both ordinary and exotic states. From the previous discussion, we see that while ordinary–exotic fermion mixings are tightly constrained due to the effects induced in the $J_0$ current, no limits can be given for the ordinary–ordinary mixings, since they affect only $J_1$.

This classification is also very convenient for discussing the possible form of the fermion mass matrices and the expected magnitude of the mixings between the known and the new fermions. To give an example, let us introduce for each fermion family a vector gauge singlet of new fermions $(X^o_{EL}, X^o_{OR})_i (E = \text{exotic}, O = \text{ordinary}, i = 1, 2, 3)$ with the same electric and color charges than the known
fermions \((f^o_{OL}, f^o_{OR})_i\). Then in the gauge eigenstate basis the mass term reads

\[
\mathcal{L}_{\text{mass}} = (\bar{f}^o_X e^o_L) \mathcal{M} \begin{pmatrix} f^o_X e^o_R \end{pmatrix}^\dagger + \text{h.c.}
\]

where e.g. \(f^o = (f^o_1, f^o_2, f^o_3)^T\) etc.. The non diagonal mass matrix \(\mathcal{M}\) takes the form

\[
\mathcal{M} = \begin{pmatrix} D & D' \\
S' & S \end{pmatrix},
\]

where \(D\) and \(D'\) are \(3 \times 3\) matrices generated by vacuum expectation values (vevs) of doublets multiplied by Yukawa couplings, while \(S\) and \(S'\) are generated by vevs of singlets. As a general rule, while the mass terms which couple ordinary L-fermions to ordinary R-fermions (or exotic L-fermions to exotic R-fermions) arise from vevs of Higgs doublets, the entries which couple ordinary fermions to the exotic ones are generated by vevs of singlets. Then in general Higgs singlets are responsible for the large masses of new heavy fermions in vector multiplets and, in most cases, also contribute to the mass of the new heavy gauge boson; hence it is natural to assume \(S, S' \sim \Lambda \gg D, D'\).

The diagonal mass matrix \(\mathcal{M}\) is obtained via a biunitary transformation acting on the L and R sectors:

\[
\begin{align*}
\mathcal{M}^2 &= U_L^{O-E} (\mathcal{M} \mathcal{M}^\dagger) U_L^{O-E \dagger} \\
&= U_R^{O-O} (\mathcal{M}^\dagger \mathcal{M}) U_R^{O-O \dagger}.
\end{align*}
\]

Since \(D/\Lambda, D'/\Lambda \sim \varepsilon \ll 1\), the order of magnitude of the different entries in \(\mathcal{M} \mathcal{M}^\dagger\) and \(\mathcal{M}^\dagger \mathcal{M}\) is

\[
\begin{align*}
\mathcal{M} \mathcal{M}^\dagger &\sim \Lambda^2 \begin{pmatrix} \varepsilon^2 & \varepsilon \\ \varepsilon & 1 \end{pmatrix}, \\
\mathcal{M}^\dagger \mathcal{M} &\sim \begin{pmatrix} \Lambda^2 & \Lambda^2 \\ \Lambda^2 & \Lambda^2 \end{pmatrix}.
\end{align*}
\]

Given the form of \(\mathcal{M} \mathcal{M}^\dagger\) in (2.14) and keeping in mind the expression (2.6) for the matrices \(U\), we see that for the matrix describing the ordinary–exotic
mixings in (2.13) it is natural to expect that the submatrices $F$ and $G$ would acquire an overall suppression factor $\varepsilon$, of the order of the ratio of the light to heavy mass scale. In contrast, since all the entries in (2.15) are of the same order of magnitude, such a suppression is not present for the ordinary-ordinary $F$ and $G$ mixing terms. Now, since it is precisely $F^\dagger F$ in (2.10) which affects the flavor diagonal couplings and also induces FCNC, the suppression of the ordinary–exotic mixings explains in a natural way the non–observations of these effects in the $Z_0$ interactions. On the other hand for the ordinary–ordinary mixings there is no reason to expect the elements of $F^\dagger F$ to be particularly small, and accordingly flavor changing processes can be expected to occur at a sizeable rate in $Z_1$ interactions.

Written explicitly, the flavor diagonal chiral couplings of a light $f$ fermion to the $Z_0$ and $Z_1$ gauge bosons are

$$
\varepsilon_{0\alpha}^f = t_3(f_\alpha) - s_W^2 q_{em}(f) + [t_3(f_\alpha^N) - t_3(f_\alpha^K)] (F_\alpha^\dagger F_\alpha)_{ff}
$$

$$
\varepsilon_{1\alpha}^f = q_1(f_\alpha) + [q_1(f_\alpha^N) - q_1(f_\alpha^K)] (F_\alpha^\dagger F_\alpha)_{ff}, \quad \alpha = L, R.
$$

(2.16)

while the $f_i f_j$ flavor changing couplings read

$$
\kappa_{ij}^{0\alpha} = [t_3(f_\alpha^N) - t_3(f_\alpha^K)] (F_\alpha^\dagger F_\alpha)_{ij}
$$

$$
\kappa_{ij}^{1\alpha} = [q_1(f_\alpha^N) - q_1(f_\alpha^K)] (F_\alpha^\dagger F_\alpha)_{ij}, \quad \alpha = L, R.
$$

(2.17)

The corresponding couplings to the physical $Z$ and $Z'$ bosons, that we will denote as $\varepsilon_{\alpha}^f, \varepsilon_{\alpha}^{f'}$ and $\kappa_{ij}^{\alpha}, \kappa_{ij}^{\alpha'}$, can be readily obtained via the transformation (2.5). For the flavor changing couplings we have for example

$$
\kappa_{ij}^{\alpha} = c_\phi \kappa_{ij}^{0\alpha} + s_\phi s_w \kappa_{ij}^{1\alpha}
$$

$$
\kappa_{ij}^{\alpha'} = -s_\phi \kappa_{ij}^{0\alpha} + c_\phi s_w \kappa_{ij}^{1\alpha}, \quad \alpha = L, R.
$$

(2.18)

and analogous expressions hold for $\varepsilon_{\alpha}^f$ and $\varepsilon_{\alpha}^{f'}$ too. In terms of the flavor non–diagonal couplings (2.18), the FCNC Lagrangian for the light $f^i$ and $f^j$ fermions in the mass eigenstate basis finally reads

$$
-L_{FC}^{ij} = g_0 \sum_{\alpha=L,R} (\bar{f}_\alpha^i \gamma^\mu \kappa_{ij}^{\alpha} f_\alpha^j Z_\mu + \bar{f}_\alpha^i \gamma^\mu \kappa_{ij}^{\alpha'} f_\alpha^j Z'_\mu).
$$

(2.19)
From the first equation in (2.18) we see that even in the case when only ordinary–ordinary mixing effects are present and hence $\kappa^{ij}_{0\alpha} = 0$, the $Z$ boson can still mediate flavor changing transitions, suppressed now by a factor proportional to the $Z_0$–$Z_1$ mixing [5]. However, for several models the existing limits on $\phi$ are rather stringent: $|\phi| \lesssim 0.02$ [2-4] and then we can expect that if the $Z'$ is not too heavy, FCNC processes would be mainly induced by $Z'$ exchange.

III. Constraints on $E_6$ models from $\mu \rightarrow eee$.

In the previous section we have shown that in the presence of new charged fermions the new neutral current $J_1$ could induce sizeable flavor changing transitions either via $Z'$ interactions, or as a consequence of a non–vanishing $Z_0$–$Z_1$ mixing. This kind of new physics would manifest itself in affecting the rates for several processes which are forbidden or highly suppressed in the SM. In the quark sector it could enhance the branchings for the leptonic decays of mesons like $K^0$, $D^0$, $B^0 \rightarrow \ell^+\ell^-$, and it would also affect the neutral meson mixings and mass differences. In the lepton sector it would induce several LFV neutrinoless $\tau$ decay modes like $\tau \rightarrow eee$, $\mu\mu\mu$, $\mu ee$, $e\mu\mu$, $\mu\pi$, $\mu\rho$ which are all constrained at the level of $\lesssim \text{few} \times 10^{-5}$ [12], but in particular it would also give rise to the decay $\mu \rightarrow eee$ for which the existing limit [9] is much more stringent:

$$Br(\mu^+ \rightarrow e^+e^+e^-) < 1.0 \cdot 10^{-12} \quad (\text{at } 90\% \text{ c.l.}).$$

(3.1)

Given the LFV Lagrangian (2.19), the expression for this decay rate relative to the charged current decay $\mu \rightarrow \nu e \bar{\nu}$ is
\[
\frac{Br(\mu \to eee)}{Br(\mu \to \nu e \bar{\nu})} = 2 \left[ 3(\varepsilon^2_R + \varepsilon^2_L)(\kappa^2_R + \kappa^2_L) + (\varepsilon^2_R - \varepsilon^2_L)(\kappa^2_R - \kappa^2_L) \right] + 2 \left( \frac{M^2_Z}{M^2_{Z'}} \right)^2 \left[ 3(\varepsilon'^2_R + \varepsilon'^2_L)(\kappa'^2_R + \kappa'^2_L) + (\varepsilon'^2_R - \varepsilon'^2_L)(\kappa'^2_R - \kappa'^2_L) \right] + 4 \left( \frac{M^2_Z}{M^2_{Z'}} \right)^2 \left[ 3(\varepsilon_R \varepsilon'_R + \varepsilon_L \varepsilon'_L)(\kappa_R \kappa'_R + \kappa_L \kappa'_L) + (\varepsilon_R \varepsilon'_R - \varepsilon_L \varepsilon'_L)(\kappa_R \kappa'_R - \kappa_L \kappa'_R) \right]
\]

where for \( \kappa_{R,L} \) and \( \kappa'_{R,L} \) defined in (2.18) we have dropped the indices \( i = e \) and \( j = \mu \), and \( \varepsilon_{R,L} \) and \( \varepsilon'_{R,L} \) refer to the electron couplings.

As a first result, by confronting (3.1) and (3.2) we can derive the limits on the \( Z\mu e \) LFV vertices. Assuming that the \( Z' \) is completely decoupled from the low energy physics (\( M_{Z'} \to \infty \) and \( \phi \to 0 \)) and taking \( s^2_W = 0.23 \), we obtain:

\[
|\kappa^e_{L}| < 1.1 \cdot 10^{-6}
\]

\[
|\kappa^e_{R}| < 1.2 \cdot 10^{-6}.
\]

As we have discussed, if these couplings do originate from some kind of mixing of the electron and muon with heavy exotic leptons, we expect them to be strongly suppressed, e.g. by a factor of the order \( m^2_{\mu}/M^2_Z \sim 10^{-6} \) or smaller, and then we see that the existence of FCNC induced by ordinary–exotic mixings does not conflict with the stringent limits in (3.3).

Now, in order to study the effects of the \( Z' \) flavor changing vertices, we first need to fix the \( q_1 \) charges of the fermions, which in turn determine the coefficient of the flavor changing mixings. This can be done by choosing a specific GUT, and we will carry out our analysis in the frame of \( E_6 \). Since \( E_6 \) has rank 6, while the SM gauge group \( G_{SM} \) has rank 4, the breaking of \( E_6 \) to the SM will lead to extra \( Z' \)s. We will consider the possibility that either \( E_6 \) breaks directly to rank 5, or that one of the two extra \( Z' \)s is heavy enough so that its effects on the low energy physics are negligible, and in these cases the formalism developed in the previous section can be straightforwardly applied. We will choose the embedding of \( G_{SM} \) into \( E_6 \) through the maximal subalgebra chain \( E_6 \to U(1)_\psi \times SO(10) \to U(1)_\chi \times SU(5) \to G_{SM} \).
then an effective extra $U_1(1)$ could arise at low energy as a combination of the $U(1)_\psi$ and $U(1)_\chi$ factors. We will parametrize this combination in terms of an angle $\beta$, and this will define an entire class of $Z'$ models in which each fermion $f$ is coupled to the new boson through the effective charge

$$q_1(f) = q_\psi(f) \sin \beta + q_\chi(f) \cos \beta.$$ (3.4)

Particular cases that are commonly studied in the literature [2–4,13] correspond to $\sin \beta = -\sqrt{5/8}$, 0, 1 and are respectively denoted $Z_\eta$, $Z_\chi$ and $Z_\psi$ models. $Z_\psi$ occurs in $E_6 \to SO(10)$, while $Z_\eta$ occurs in superstring models when $E_6$ directly breaks down to rank 5. As we will see this model plays a peculiar role in the present analysis, since it evades completely the kind of constraints that we are investigating. Finally, a $Z_\chi$ boson occurs in $SO(10) \to SU(5)$ and couples to the conventional fermions in the same way than the $Z'$ present in $SO(10)$ GUTs, however since $SO(10)$ does not contain additional charged fermions, the kind of flavor changing effects that we are studying here is absent. In contrast, new charged quarks and leptons are present in $E_6$. In the GUTs based on this gauge group the fermions are assigned to the fundamental 27 representation that contains, beyond the standard 15 degrees of freedom, 12 additional states for each generation, among which we have a vector doublet of new leptons $(N^-)_L, (E^+ N^c)^T_L$ on which we will now concentrate.

The chiral couplings of the leptons to the $Z_1$ and the coefficient of the LFV term, are determined by the $q_\psi$ and $q_\chi$ charges of the new and known states, which are

$$q_\psi(E_L) = -q_\psi(E_R) = -\frac{1}{3} \sqrt{\frac{5}{2}} \quad q_\chi(E_L) = q_\chi(E_R) = -\frac{1}{3} \sqrt{\frac{3}{2}}
$$

$$q_\psi(e_L) = -q_\psi(e_R) = \frac{1}{6} \sqrt{\frac{5}{2}} \quad q_\chi(e_L) = 3q_\chi(e_R) = \frac{1}{2} \sqrt{\frac{3}{2}}. \quad (3.5)$$

With respect to the $SU(2)_L$ transformation properties, the $E^+_L$ heavy leptons are exotic, and than the mixing of their CP conjugate states $E^-_R$ with $e_R$, $\mu_R$ and $\tau_R$ are constrained by $Z_0$ interactions. From (3.3) we have for example

$$(F_R^\dagger F_R)_{e\mu} < 2.4 \cdot 10^{-6}, \quad (3.6)$$
while the 90% c.l. limits on the flavor diagonal mixings given in [2] are respectively

\[(F_R^\dagger F_R)_{ee} < 1.3 \cdot 10^{-2}\]  
\[(F_R^\dagger F_R)_{\mu\mu} < 1.1 \cdot 10^{-2}.\]  

Due to the tight bound (3.6) it is reasonable to neglect the LFV couplings in the R-sector and (conservatively) set \(\kappa_{e\mu}^R = \kappa_{e\mu}'^R = 0\) in (3.2). According to (3.7), it is also justified to neglect the effects of the fermion mixings in the flavor diagonal couplings of the R-electrons.

In contrast, the \(E_L^-\) leptons are ordinary, and no bounds exist on their mixing with the light leptons. We will still neglect the diagonal term \((F_L^\dagger F_L)_{ee}\) since it is reasonable to expect that if this term were so large as to spoil the approximation \(\varepsilon_{f_{1\alpha}} \simeq q_1(f_{\alpha})\) the value of the off-diagonal term \((F_L^\dagger F_L)_{e\mu}\) would also be large, possibly leading to even stronger limits than the ones derived here.

Due to the approximations made, for each value of the parameter \(\beta\) in (3.4) the branching ratio (3.2) depends on the values of \(M_{Z'}^2, \phi\) and \(F_{e\mu} \equiv (F_L^\dagger F_L)_{e\mu}\). However, it is easy to see that since the gauge boson mixing effects in the diagonal electron couplings are in any case very small, being \(|\phi| \lesssim 0.02 [2-4]\), the relevant variables are actually only two, namely \(F_{e\mu} \cdot (M_Z^2/M_{Z'}^2)\) and \(F_{e\mu} \cdot \phi\). Moreover once the Higgs sector of the model is specified, \(M_{Z'}\) and \(\phi\) are no more independent quantities. For example an approximate relation that holds for small mixings and when \(M_{Z'}(\gg M_Z)\) originates from a large Higgs singlet vev [3] reads

\[\phi \simeq -\frac{M_Z^2}{M_{Z'}^2} s_W \sum_i t_i^3 q_1^i |\langle \phi^i \rangle|^2 \sum_i t_i^2 |\langle \phi^i \rangle|^2,\]  

and in this case the branching ratio (3.2) is in practice only a function of \(F_{e\mu} \cdot (M_Z^2/M_{Z'}^2)\).

As in the SM for the Cabibbo-Kobayashi-Maskawa (CKM) matrix, also in \(E_6\) we don’t have a clue for predicting the values of the fermion mixing parameters. Without attempting to push too far an analogy between the mixings we are
interested in and the CKM matrix, we will merely note that both these cases involve mixings among ordinary fermions, and that we do not expect in the present case any additional suppression factor. We also note that all the CKM matrix elements are $> 10^{-3}$ and that in particular the mixing between the first and the second generation is rather large. We will then assume that the LFV term $F_{e\mu}$ lies in the range $10^{-2}-10^{-4}$. Under this assumption the presence of a too light $Z'$ as well as a too large amount of $Z_0-Z_1$ mixing will clearly conflict with the limit (3.1). The bounds that can be derived in this way are indeed very strong, but obviously they cannot replace the direct [1] or indirect [2-4] limits on the $Z'$ parameters, since for very small values of the LFV $Z'$ couplings ($F_{e\mu} \lesssim 10^{-6}$) they would in fact be weaker. We will nevertheless present our constraints in the form of numerical limits on $M_{Z'}$ and $\phi$, since, in doing so, the strength of the arguments that have been discussed here is put in clear evidence.

Our results are collected in Figs. 1 and 2. Figure 1 shows the bounds on $M_{Z'}$, the thick solid line depicts the bounds obtained for $F_{e\mu} = 10^{-2}$ and by setting the gauge boson mixing angle $\phi$ to zero. The decay $\mu \rightarrow eee$ is due only to $Z'$ exchange in this case. The limits for different values of $F_{e\mu}$ can be read off this line as well, by assuming for the vertical axis units of GeV $[100F_{e\mu}]^{1/2}$. The thick dashed line, drawn here for convenience, shows the bounds corresponding to $\phi = 0$ and $F_{e\mu} = 10^{-3}$, vertical units are again in GeV in this case. We see that for $F_{e\mu} > 10^{-3}$ a $Z'$ below 1 TeV would be excluded for most of the values of $\beta$. Also, it is clear that $F_{e\mu} \approx 10^{-4}$ still leads to significative bounds, being $M_{Z'}$ constrained to values $\gtrsim 400$GeV for large part of the $\sin \beta$ axis.

To study the possible effects on these results of a non vanishing mixing angle $\phi$, i.e. when both the $Z'$ and $Z$ bosons contribute to the decay, we have used (3.9) assuming two doublets of Higgs fields $h_{Nc}$ and $h_N$ with vevs $\bar{v}$ and $v$. Since $\bar{v}$ and $v$ give mass respectively to the $t$ and $b$ quarks, $\sigma \equiv \bar{v}^2/v^2 > 1$ is theoretically preferred. The bounds on $M_{Z'}$ obtained by allowing for a $Z_0-Z_1$ mixing consistent with this minimal Higgs sector are shown Fig. 1 by the dotted and dot-dashed lines, which correspond to $\sigma = 1$ and $\infty$ respectively. It is apparent
that by allowing for a non vanishing value of $\phi$, the limits on the $Z'$ mass are only slightly affected.

Figure 2 depicts the constraints on the $Z_0-Z_1$ mixing angle $\phi$. The solid line shows the bounds obtained by assuming $\mathcal{F}_{e\mu} = 10^{-2}$ and taking the limit $M_{Z'} \to \infty$. In this case the decay $\mu \to eee$ is mediated only by the $Z$ boson, and is due to the mixing between the $Z_0$ and the $Z_1$. The dotted line shows the bounds for $\mathcal{F}_{e\mu} = 10^{-3}$ in the same limit. The limits for different choices of $\mathcal{F}_{e\mu}$ are easily obtained from the solid [dashed] lines by rescaling the vertical units by $(10^2 \mathcal{F}_{e\mu})^{-1} \ [10^3 \mathcal{F}_{e\mu}]^{-1}$.

The dotted ($\sigma = 1$) and dot–dashed lines ($\sigma = \infty$) enclose the regions of the limits obtained assuming a minimal Higgs sector. In this case the value of $M_{Z'}$ is finite and consistent, according to (3.9), with the values of $\phi$ at the bound. We see that with this additional constraint significant limits are found for $\mathcal{F}_{e\mu} = 10^{-4}$ as well.

From Fig. 2 it is apparent that for a minimal Higgs sector the limits on $\phi$ are significantly tighter than in the $M_{Z'} \to \infty$ limit, showing that (3.1) in first place gives direct constraints on the $Z'$ mass, while the bounds on the $Z_0-Z_1$ mixing obtained independently of $M_{Z'}$ are weaker. We note that this behaviour is opposite to what is encountered in deriving limits on the $Z'$ parameters from precise electroweak data [2-4], where in fact the best bounds on the $Z'$ mass are obtained from the tight limits on $\phi$ implied by the LEP measurements.

From Figs. 1 and 2 it is apparent that for the $\eta$ model, corresponding to $\sin \beta = -\sqrt{5}/8$, both the $Z'$ mass and the $Z_0-Z_1$ mixing angle are not constrained by the present analysis. This is due to the fact that in this model, for the ordinary–ordinary fermion mixings besides $t_3^\kappa = t_3^\eta$ we also have $q_3^\eta = q_3^\eta$, implying that both the coefficients of the $F_L^1 F_L$ term in the $J_3$ and in the $J_1$ currents vanish. This happens also in the quark sector and in the neutral sector, hence the unsuppressed flavor changing vertices are completely absent for the $Z_\eta$ boson, and this insures that besides the decay $\mu \to eee$ no other processes can be found for implementing this kind of constraints for the $\eta$ model. The reason for this can be understood by
considering the decomposition $E_6 \rightarrow SU(6) \times SU(2)_I$, where $SU(6)$ contains the SM group, while the $SU(2)_I$ is “inert” in the sense that $I_{3I}$ does not contribute to the $Q_{em}$ generator [14,15]. $I_{3I}$ corresponds to $\beta = \arctan \sqrt{3/5}$ in (3.4) and is orthogonal to $Q_\eta [\beta = \arctan(-\sqrt{5/3})]$ which is then contained in the $SU(6)$ factor as well. The fermions in the 27 of $E_6$ with the same SM quantum numbers $(q_{em}, t_3, \text{color})$ form multiplets (singlets and doublets) of $SU(2)_I$ and clearly these multiplets also carry definite values of the $Q_\eta$ charge. All the ordinary fermions with the same color and electric charges, being members of the same $SU(2)_I$ multiplet, have also the same $q_\eta$, and this implies the absence of both the diagonal and the flavor changing ordinary–ordinary mixing effects. In contrast ordinary–exotic fermion mixing could still give rise to FCNC also in the $\eta$ model, but as already discussed the corresponding transitions are expected to be largely suppressed, and do not imply any useful constraint.

According to this discussion, if an additional $Z'$ with a mass of a few hundreds GeV is found together with new fermions that could fit in the 27 of $E_6$, the observed absence of unsuppressed FCNC would suggest that it could most probably be a $Z_\eta$.

IV. Conclusions

We have carried out an analysis of models that predict a new neutral gauge boson and new charged fermions from the point of view of FCNC processes. We have argued that in most of these models unsuppressed flavor changing couplings of the light fermions to the new $Z'$ can be present as a consequence of a mixing between the known and the new charged fermions. By assuming that these flavor changing vertices should not be unnaturally small, we have inferred that the observed suppression of FCNC processes can still be explained in a natural way if the new gauge boson is sufficiently heavy and almost unmixed with the standard $Z$. Also we have
attempted a semi-quantitative analysis of this kind of new physics in the frame of $E_6$ models, by confronting the theoretical expectations for the LFV effects with the extremely stringent limits on the $\mu \to eee$ decay mode. Our conclusions are that the existence of $Z'$ bosons from $E_6$ much lighter than 1 TeV is unlikely, with the noticeable exception of the superstring inspired $\eta$ model which evades completely our constraints. At the same time, our analysis suggests that the observation of FCNC processes at rates larger than the SM predictions could be interpreted as a hint for the simultaneous presence of additional gauge bosons and new charged fermions. Indeed these new states could manifest themselves indirectly via this kind of flavor changing effects well before they are directly produced.
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Figure captions

Fig. 1: Limits on $M_{Z'}$ for a general E$_6$ neutral gauge boson, as a function of
$\sin \beta$ and for different values of the lepton flavor violating term $F_{e\mu} \equiv (F^\dagger_L F_L)_{e\mu}$. The thick solid line is obtained by setting the $Z_0$–$Z_1$ mixing angle $\phi$ to zero, and assuming $F_{e\mu} = 10^{-2}$. The limits for different values of $F_{e\mu}$ can be read off this line by assuming for the vertical axis units of GeV·(100$F_{e\mu}$)$^{1/2}$. The thick dashed line depicts the limits corresponding to $F_{e\mu} = 10^{-3}$ (vertical units in GeV). The bounds obtained by allowing for a non-vanishing $Z_0$–$Z_1$ mixing, consistent with the values of $M_{Z'}$ when a minimal Higgs sector is assumed, are also shown. The dotted lines correspond to equal vevs of the two Higgs doublets present in the model, i.e. $\sigma \equiv \bar{v}/v = 1$ while the dot–dashed lines correspond to $\sigma = \infty$.

Fig. 2: Limits on the $Z_0$–$Z_1$ mixing angle $\phi$ for a general $Z_1$ from E$_6$, as a function of $\sin \beta$ and for different values of the lepton flavor violating term $F_{e\mu} \equiv (F^\dagger_L F_L)_{e\mu}$. The thick solid and dashed lines are obtained in the limit $M_{Z'} \to \infty$ assuming $F_{e\mu} = 10^{-2}$ and $F_{e\mu} = 10^{-3}$ respectively. Limits for different values of $F_{e\mu}$ can be obtained from these lines by rescaling the vertical units by $(10^2 F_{e\mu})^{-1}$ and $(10^3 F_{e\mu})^{-1}$ respectively. The dotted ($\sigma = 1$) and dot-dashed ($\sigma = \infty$) lines show the limits obtained for a finite $Z'$ mass and assuming a minimal Higgs sector. In this case the bounds are tighter and are essentially determined by the corresponding limits on $M_{Z'}$ through eq. (3.9). Also the limits corresponding to $F_{e\mu} = 10^{-4}$ are shown in this case.