Abstract: This paper demonstrates the need for the structural analysis of information as a distinctive methodological tool for two reasons. The first is that this form of inquiry is underrepresented and inadequate in the study of information. The second, a closely related reason, is that there are many misconceptions about the distinction between different forms of inquiry reduced to only two, characterized as qualitative and quantitative, with surprisingly little attention paid to the role of structural analysis, not only in the study of information, but in all scientific and intellectual domains. Since the first step towards this structural analysis is in defining concepts in structural terms, the established structural methodology may also contribute to choosing a satisfactory-for-all definition of information and to develop its uniform theory.

Keywords: information; structural methodology; qualitative methodology; quantitative methodology; measure of information; structure of information

1. Introduction

The title of this contribution refers to the structural analysis of information as a distinctive methodological tool for two reasons. The first is that this form of inquiry is underrepresented and inadequate in the study of information. The second, a closely related reason, is that there are many misconceptions about the distinction between different forms of inquiry with surprisingly little attention paid to the role of structural analysis, not only in the study of information, but in all scientific and intellectual domains.

The concept of information does not have a unique, commonly accepted definition. Instead, there are, dispersed in the literature, multiple attempts to define it in a large variety of conceptual frameworks and with diverse, but frequently insufficient levels of logical correctness. Naturally, the most popular are open-ended metaphorical expressions of very intuitive but vague associations generated by the word “information”. The closest to what could be considered a definition with a very high tolerance for ambiguity is Gregory Bateson’s view of information as “a difference that makes a difference” [1]. After all, if asked, everybody would claim to understand what a difference is, and on the other hand, the use of the idiomatic expression of “making difference” as a qualifier accommodates various preferences by allowing the unlimited freedom of interpretation.

This lack of clarity regarding the meaning of information may raise doubt that the task to develop a methodology for the structural study of an object lacking a clear identity is unrealistic. However, the absence of the definition does not preclude surprisingly strong agreement regarding the identification of the instances of the information in more specific disciplines of its inquiry. We can expect that the difficulty in finding a commonly accepted definition of information is a result of the negligence of its structural analysis in the specific instances that could serve the purpose of finding appropriate generalizations. This is why the development of structural methodology for the study of information should start from establishing a more general structural methodology independent from disciplinary diversification.
Moreover, if we want to maintain the identity and integrity of the study of information, carried out in its further development in the absence of the uniform definition, we have to establish methodological tools not necessarily identical for all forms of inquiry of informational phenomena, but at least being consistent and preferably allowing comparisons of the results of inquiries. There is an example of such a fundamental methodological uniformity traditionally but rather confusingly referred to as “the scientific method.” The confusion is likely if someone expects that scientific inquiries follow a unique, clearly defined, and unchanging method. However, even if scientific inquiries diverged into separate disciplines and they have developed their separate traditions and methods there is a common methodological conceptual framework that evolves and which is being reinterpreted to eliminate the domination of disciplines, which has secured prominent positions for themselves by their early, successful, technological applications.

2. Fallacy of the Distinct and Exclusive Qualitative and Quantitative Methodologies

This work has as its main objective to demonstrate the need for structural methodological tools for the study of information with a sufficient level of universality to relate studies of information within different disciplines. However, even with this restricted objective, it will be necessary to clarify some misunderstandings present in methodological analyses of practically all scientific disciplines and all contexts.

Let us begin with the issue of the relationship between quantitative, qualitative, and structural methodologies. My main claim in this matter is that the popular conviction of the apparent complementary, exclusive, and dichotomic opposition of the first two methodologies is based on the misconception perpetuated in virtually all scientific inquiries of the role of mathematics in general and of the numbers in particular [2,3]. Moreover, this mistaken view of the two methodologies, their exclusive and universal role in all inquiries obscures the fact that they both are just instances of structural analysis, in which mathematics can offer methodological tools going well beyond the present toolkit.

The fallacy of the opposition and complementarity of the quantitative and qualitative methodologies has its source in the hidden assumptions that are very rarely recognized and explicitly stated in the scientific practice. Another source is in the overextension of the mathematical concepts which have very specific and restricted meaning in mathematics to the applications in science where the conditions of their definitions are not satisfied or even not considered.

An outstanding example of this type of confusion is in the use of the concept of measure, which frequently in scientific applications is understood as a rather arbitrary assignment of real numbers to some set of not always clearly defined objects. This use of the term measure as a synonym of magnitude is very far from the understanding of the concept of a measure in mathematics. It would have been just a terminological inconsistency with mathematics, not an error if this non-mathematical concept of a measure was not mixed up with the mathematical concept in making conclusions regarding the results of the inquiry. Very often the meaning of the term measure is simply not clarified. Sometimes the intention of the use of the term is consistent with the measure theory, but there is nothing about the related concepts of the theory whose absence makes the central concept meaningless. The reference to a measure without any clarification of how it is defined has as its consequence the hidden import of the structure on which it has to be defined to retain its mathematical meaning (a sigma ortho-algebra of measurable subsets). Thus, there is a hidden structure associated with the subject of our study which serves as a tool for inquiry, but which is excluded from overt considerations.

To understand why the underlying structure is important, let us consider similar, but slightly different confusion in some explanations of the quantum theory in physics as a probability theory on non-Boolean orthomodular lattices. The problem is that this apparently true explanatory statement does not explain anything since the familiar expression “probability theory” is the probability theory on Boolean lattices (or algebras) and the transition from classical to quantum physics is moving away from Boolean lattices. Once
we consider probability measures on non-Boolean lattices the theory is fundamentally different. Thus, this confusing explanation of the quantum theory is essentially a meaningless statement that quantum theory is a probability theory, which is not a probability theory, but a theory that only seems similar in some respects. If we change the underlying structure to which we normally do not pay much attention (here we replace a Boolean lattice with a non-Boolean lattice), we make a transition to a very different theory with only some superficial similarities.

The case of using an altered concept of a measure is more dangerous because there is no warning about the alteration of the underlying structure, while the hidden structures which are prerequisites for the meaning of scientific concepts such as measure are very important. We may be tempted sometimes to omit their description (wrongly) assuming that its lack is a sufficient indication that we make a conventional choice and that the convention is obvious. Even if sometimes it is easy to guess the conventional choice of the underlying structure, this does not mean that this structure is absent or irrelevant.

If we decide to disregard the conditions in the mathematical concept of a measure and consider it simply as a magnitude, i.e., a real-valued function \( \varphi \) on some set \( S \) (i.e., \( \varphi : S \to \mathbb{R} \)), then we define for \( S \) an equivalence relation ~ defined by the partition of \( S \) into subsets of elements with equal values of \( \varphi \) (\( x \sim y \) if \( \varphi(x) = \varphi(y) \)). However, in this case, we have a pure case of the qualitative methodology based on partitions of a set into equivalence classes, which can be identified with qualities or properties of the elements of \( S \), but which equally well can be identified with numerical values. This shows that the distinction between the quantitative and qualitative methodologies is fuzzy. In both methodologies, we assume overtly, or most frequently covertly an essentially the same structure of an equivalence relation imposed on the universe of our study. More importantly, in both cases by imposing hidden mathematical structures on the subjects of our study we carry out a structural analysis involving equivalence relations. As long as the concept of a measure is not the one from measure theory and a measure is simply an assignment of a numerical value, the distinction between the two methodologies is rather conventional and is based on the way how equivalence relations are presented.

3. Structural Methodology as an Extension beyond Qualitative and Quantitative Methods

If this was the only confusion, then we could say that this is just a matter of different terminology or that we have structural methodology divided into quantitative and qualitative types based on some conventional criterion and there is no further need to refer to structural analysis. However, there is some additional danger in uncontrolled use of numerical functions as ways to introduce partitions of equivalence relation when we project properties of real numbers back to set \( S \). The set of real numbers has specific ordinal and topological structures which frequently are projected on set \( S \) in a hidden way. This does not create any problem if this projection produces an ordinal or topological structure consistent with the existing structures of \( S \). However, this requires a prior structural analysis of \( S \) that is rarely considered.

There are important examples of this confusion even in the most celebrated attempts to introduce measures of information and consciousness. The former is Shannon’s entropy as a measure of “information, choice and uncertainty” [4] (p. 20). How can be entropy a measure of information and at the same time its negation uncertainty? How can the same non-negative number be entropy and its opposite negentropy [5]? These curious contradictions are the result of the lack of rigorous structural analysis of information.

Even more fallacious is the reasoning behind the supposed measure \( \Phi \) of consciousness understood as integrated information in humans as well as elementary particles introduced by Giulio Tononi and used by him to promote panpsychism [6]. The function \( \Phi \) was derived from purely statistical analysis of simultaneous firing of neurons without any attempt to provide a structural analysis of consciousness or information integration. The mysterious non-zero value of \( \Phi \) for objects such as elementary particles (actually in the original text
“nuclei of hydrogen”) instead of being used as the evidence for the error in interpreting $\Phi$ as a measure of information integration became the argument for the attribution of consciousness to everything.

This reminds a cosmological claim that there exists ubiquitous dark matter because we can estimate its mass necessary to explain the deviation of phenomena from their theoretical description. This is of course a legitimate heuristic hypothesis. However, this does not constitute any evidence for the actual existence of dark matter. Similarly, the fact that we provide numerical value for consciousness of elementary particles does not constitute evidence that they are conscious, unless we can provide a structural analysis of the mechanism of consciousness. Here the analogy with heuristic of dark matter breaks. We have a very advanced theory describing the structure of universe in the cosmic scale and the hypothesis of dark matter was introduced to save this theory when it is confronted with empirical results. In the case of consciousness, we don’t have its structural theory and there is nothing to be saved.

Another example demonstrating the need for structural analysis going beyond the structures hidden within quantitative and qualitative methodologies can be found in the fallacious belief that we have only three possible types of topology. This is expressed in the distinction between continuous and discrete topological characteristics of sets with the latter including as a special case of finite sets. No other possibilities are considered whereas there are infinitely many possible topological structures on every infinite set and finite but large variety on a finite set. This confusion is a consequence of the false but popular assumption that the topology has to be defined by a metric and that the distance (metric) has to be represented uniquely by real numbers (continuous case) or by natural numbers (discrete case). Even more fallacious is the claim that discrete topology is one and trivial ignoring the large variety of non-$T_1$ topologies.

In the finite case, every topology can be identified uniquely with a quasiorder and every $T_0$ topology with a partial order [7]. Of course, every order on a set defines distinct topology, so even on the finite set, there is a large variety of topologies. This is an additional argument against the restriction of structural methodology to the structures generated by functions with real number values of quantitative methodology. Instead, we should consider diverse structural methodologies based on the wide range of relational, algebraic, metric, topological, or other structures.

4. Conclusions

The initial step towards structural analysis, or rather its prerequisite is in the definition of the object of inquiry in terms of structural concepts, i.e., concepts referring to its components, their mutual relations, and the way how they constitute its identity. However, important as it is, this condition does not provide any specific analytical methodology.

The task to develop such a general methodology is highly nontrivial because the general concept of a structure is far from being uniquely defined. Of course, a multitude of structures has been very clearly defined in mathematics and their theories constitute the entire mathematics itself. The question is what concept could serve as a genus for all of them. There are many different approaches [8]. Someone could argue that category theory with its concept of a category is the right answer, but there are many strong arguments against this view. Another possible approach is the use of structuralist methodology focusing on the concept of symmetry [9,10].

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