Study on Reliability Calculation of Repairable System Based on Monte-Carlo Simulation

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Abstract. Nowadays, the complexity of engineering system grows with the higher requirement of quality. Reliability, one of the most important quality characteristics, has been drawn a lot of attention. The level of reliability decides whether the system can accomplish mission successfully or not, as well as the safety of system and operator. The reliability calculation (prediction) is the most important item in design phrase, as an approach to measure the reliability. There is much study which focuses on the unrepairable system, such as aircraft and satellite. The study presents a reliability calculation method for repairable system based on monte-carlo simulation. The inputs of reliability calculation, which are different from unrepairable system, are analysed firstly. Then, the simulation principles are introduced aiming at various reliability models. The effectiveness is verified through a case of vessel in the end, which shows the simulation calculation of repairable system is valid. It can make a contribution to the improvement of system reliability in design phrase and aid the developer to locate the weakness parts.

1. Introduction

With the development of science and technology, the engineering system is becoming larger and more complex, which brings risks and challenge with more investment and longer research period. It is much harder to keep this kind of system in the same reliability level as well as previous ones. Meanwhile, the environment which the systems applied is unceasingly expanded and gets more critical. It puts forward more requirement of high reliability level. Reliability calculation (prediction) is one of the most important works in design to verify the existing state and is an effective approach to improve reliability level. The repairable systems' reliability calculations are much different from the unrepairable ones. Aiming at the characteristics of repairable systems, reliability calculation method must be upgraded [1-2].

The paper firstly analyses the inputs of reliability calculation of repairable system which differs from the unrepairable one, such as equipment attribute, mission profile, RMS indexes and so on. Maintenance margin, an important concept, is introduced in the calculation. The different reliability model is also introduced, as well as the advantages and disadvantages. The fault tree analysis (FTA) is chosen as the best way to establish the repairable system reliability model in order to conduct the monte-carlo simulation. The simulation strategies aiming at various models are also expatiated. The reliability of a certain vessel in the typical mission profile, as an example, is conducted, compared to the unrepairable algorithm. The consequence shows that the system's can be evaluated more accurately and is more
approximate to the real system's reliability level, which can aid the designers to make alternatives and optimization.

2. Input of Reliability Calculation of Repairable System

2.1. Equipment Attribute

The repairable attribute of system can be divided into two kinds: one is repairable without the system halting, the other is system can stop work in specified time, which can be also called maintenance margin (explained in 2.2). The operation mode and environment will have an effect on whether the system can be repaired or not during the mission. The aircraft is one of the unrepairable typical systems. Other aerospace systems, such as satellite, missile, are unrepairable, too. Whereas the vessel and vehicle is usually can be repaired. It is related to mission profiles. The vessel will be unrepairable during combat operation period. The support structure would also be influential. The support resources in the operational level (O-level) are decisive, including personnel, support equipments, tools, spares and technical manual. They are must be supplied immediately during the operation. If there is no maintenance support resources, the system's state (repairable or not) would switch. So the ISS (international space station) can be repairable if the support resources are stored in the station. In the end, the equipments (or called parts) must be designed to be easy to remove or maintenance by the operator or elementary maintenance person, which is usually in the O-level. The general arrangement of all parts should be reasonable. In other words, the maintenance accessibility of the system must be favourable, as shown in Figure 1.

2.2. Mission Profile

Mission profile is the description of all events and environment's in order which system would go through during specified tasks. It is the basic input of reliability calculation. The reliability of the same system varies with the change of different mission profile. The mission profile consists of system operation time, operation mode of constituent parts and duty cycle in common. Whereas, all above factors in mission profile is inadequate for repairable system reliability calculation. Maintenance factors should be added, such as maintenance margin. The maintenance margin refers to the maximum time which the system can halt and be repaired well. The maintenance margin usually divided into several parts according to the mission profile. Because the whole mission profile consists of several parts, and different sub-mission profile (parts or phases of the whole profile) has the corresponding limits. When taking vessel as an example, maintenance is not allowed during the operation phase. On the contrary, sailors can repair the breakdown parts with the equipments and spares on board when sailing.

![Figure 1. Influence factors for repairable system.](image-url)
2.3. RMS Index

The indexes of each equipments in unrepairable system reliability calculation consists of mean time between failures (MTBF), mean time between critical failures (MTBCF) or probability of mission success (P). Whereas, the other RMS (reliability, maintainability and supportability) indexes are not taken into consideration. The meantime to repair (MTTR) needs to add at least. If the calculation need to be more accurate, other RMS indexes, such as the number of spares (in operational level), the MTTR of each failure mode of equipments (from FMEA), should be also attached to the equipment's attribute [3].

2.4. Common System Reliability Model

2.4.1. Reliability Block Diagram. The reliability block diagram is one of the most popular modelling methods applied in practical project, which is also called RBD. The advantages of RBD method concludes that easy to read or learn and can reflect the work relationship among every single constituent part in the system, which is more or less like a kind of circuit diagram. As shown in Figure 2, the series, paralleling and sequentially model is built with the method of RBD, which is the most common relationship in the system. It is need to analyse carefully to confirm the system's state once one or more units are to be failure 3.

![Figure 2. Typical reliability model in RBD.](image)

2.4.2. Fault Tree Analysis. The fault tree analysis (FTA) is a kind of logic diagram which refers the parts or parts assembly that lead to the certain fault. Both qualitative and quantitative analysis can be conduct through FTA. The event and logic gate is the basic element of FTA. Event represents parts or failure modes and logic gate represents relationship. This kind of modelling is clear and easy to figure out which parts' failure would lead to the whole system's failure. And the relationship among the parts which causes fault is distinct. It is very convenient to analyse the complicated relationship among much kinds of events. The followings are the common modes of FTA (according to the RBDs in Figure 3). It refers that the two kinds of modes can be converted to each other. The block No.0 is called top event, which represents the most unbearable consequence of the system. In reliability calculation, it usually represents failure.

![Figure 3. Typical reliability model in FTA.](image)
3. Simulation calculation on Monte-Carlo

3.1. Monte-Carlo method
The Monte-Carlo method is also called computer random simulation method, which is based on "random number". It is oriented from the project of atomic bomb in World War II. The moderator of the project named it after the city of Monte Carlo, which is famous for gambling. The basic theory of the method is to determine whether an event would occur or not by the random number. And the occurrence probability obeys a certain kind of distribution, which is much like the failure. The occurrence of failure is also random and the distribution of failure is called failure rate.

3.2. Minimal cut sets of FTA
The minimal cut sets are several sets of events, and the top event would occur when minimal cut sets all occur. If any events are removed from a cut set, the group of events cannot be a cut set. From the definition, it can be inferred that once the top event occurs, at least one minimal cut set should occur. So the simulation mode of system is based on the parts of system. The system's operational state is simulated by the parts simulation with a certain relationship, which is reflected by FTA. On the other hand, the relationship of FTA can be summarized by its structure function as followings. It is not hard to draw a conclusion that the function is the union set of all minimal cut sets [4].

\[ \psi(X) = \prod_{i=1}^{K} M_i(X) \]  

(1)

3.3. Simulation procedure
Base on the analysis above, it comes to a conclusion that the earliest minimal cut sets occurred is the cut sets which lead to the system failure. So it is same to make a random sample of top events and the cut sets. And the certain cut set can be easily expressed with array. Thus, the system operation state can be simulated by the digital logic of cut sets based on the FTA. The basic simulate calculation procedures are shown in Figure 4. Often, the whole simulate calculation procedure are conducted in special software to aid designer get the final result.

| a) input of system and mission profile data | b) FTA mode establishment | c) minimal cut sets analysis | d) simulation set | e) simulation conduction and consequence |

Figure 4. Simulate calculation procedures based on FTA.

From the analysis, in can be easily inferred that the simulate calculate method based on Monte-Carlo is not only suitable to the repairable system (with or without maintenance margin) but also suitable to the unrepairable system. N

4. Case of repairable system reliability calculation

4.1. Basic hypothesis
In the study, a typical repairable system, vessel's ventilating system is taken as an example. Before calculating, some basic hypothesises are made as followings: a) only parts which would influence the function or performance of system are taken into consideration; b) all the parts have only two state: normal or failure; c) all the parts' failures are independent; d) all the parts' failure rates obey exponential distribution or are approximate to exponential distribution; e) operators are completely reliable and the interactive between system and person is no problem; f) the maintenance support resource on board is
adequate; g) all the failures are repairable on board unless special statement; h) all the parts' different operation modes which may influence duty cycles are not taken into consideration [5-6].

4.2. Input parameters of system
The total time of mission profile is set in 10h. And operation time is set in 8h, and maintenance margin is set in 2h. The ventilating system consists of six parts, with the preconditions of ignore of secondary equipments. The RBD model of the system is shown in Figure 5 and the specific RMS indexes are shown in table 1.

![Figure 5. RBD of ventilating system.](image)

**Table 1. RMS indexes of ventilating system.**

| Parts                        | Failure Rate (1/h) | MTTR (h) | Repairable or Not |
|------------------------------|--------------------|----------|-------------------|
| air-condition-1 (A1)         | 0.001              | 0.5      | yes               |
| air-condition-2 (A2)         | 0.001              | 0.5      | yes               |
| Electrical motor-1 (B1)      | 0.004              | 1.5      | yes               |
| Electrical motor-2 (B2)      | 0.004              | 1.5      | yes               |
| Air-purification device (E1) | 0.002              | 1        | yes               |
| Cooler (F1)                  | 0.002              | /        | no                |

4.3. FTA
The FTA model of the ventilating system is shown in Figure 6 according to the RBD. In the model, the top event represents system failure and the bottom blocks (A1-F1) represent the parts. The block A/B is the middle event, which represent the two-paralleling model in the system. The FTA can be more logical and distinct by this way. The RMS attribute of parts are attached to the model when the FTA is established.

![Figure 6. FTA model of the ventilating system.](image)
4.4. Simulate Calculation
In order to compare the repairable and unrepairable calculation, the study presents the three calculation consequence: unrepairable, repairable without maintenance margin and repairable with maintenance margin. According to the analysis above, the reliability of repairable with maintenance margin is the highest, the repairable without maintenance margin is second and the unrepairable is third. The unrepairable and repairable without maintenance margin calculation can be obtained by mature commercial software. In the study, "Relex" of PTC is selected to calculate and the repairable with maintenance margin calculation is conduct by the self-made programme. The calculation consequences are various, such as mission reliability (Rm), MTBF and failure rate. However, the latter two parameters are system's instantaneous value, which cannot reflect the whole mission. Rm is the common parameter. The calculation consequences with 3 times simulation (10000, 100000 and 1000000 times) are shown in table 2. It needs to point out that the consequences of every calculation would change a bit due to the simulation method. Its influence can be eliminated by the way of improving simulate times, repeated simulations and averaging values. The system reliability of repairable calculation is higher than the unrepairable [7]. And the reliability can be improved effectively once the maintenance margin considered.

| No. | Repairable mode                  | Rm 1 | Rm 2 | Rm 3 |
|-----|----------------------------------|------|------|------|
| 1   | unrepairable                     | 0.591| 0.589| 0.592|
| 2   | repairable without maintenance margin | 0.670| 0.671| 0.669|
| 3   | repairable with maintenance margin | 0.752| 0.749| 0.751|

5. Conclusion
This study presented a method of repairable system reliability calculation with maintenance margin analysis based on monte-carlo simulation. The differences of calculation inputs between repairable systems and unrepairable ones are analysed. And the simulation theory and procedures are introduced based on the FTA cut sets. Finally, the validity of the method is verified by an case of ventilating system on the vessel. The method can aid the developers to locate the weakness of system and take serviceable measures to improve the reliability. And it can also help developers to consider the maintainability of equipment as well as the operation mode, which can put forward operation advice to the final user.

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