Global cluster morphology and its evolution: X–ray data vs CDM, ΛCDM and CHDM models

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Abstract

The global structure of galaxy clusters and its evolution are tested within a large set of TREESPH simulations aimed to allow a systematic statistical comparison with available X–ray data. Structure tests are based on the so–called power ratios introduced by Buote & Tsai. The cosmological models considered are flat CDM, \(\Lambda\)CDM (\(\Omega_\Lambda = 0.7\)) and CHDM (\(\Omega_h = 0.2\), 1 massive \(\nu\)). All models are normalized so to provide a fair number density of clusters. For each model we perform a P3M cosmological simulation in a large box, where we select the volumes where the most massive 40 clusters form. Then we go back to the initial redshift and run a hydrodynamical TREESPH simulation for each of them. In this way we can perform a statistical comparison of the global morphology of clusters, expected in each cosmological model, with ROSAT data, using the Student t–test, the F–test and the Kolmogorov–Smirnov test. The last test and its generalization to 2–dimensional distributions are also used to compare the joint distributions of 2 or 3 power ratios. We find that using DM distribution, instead of gas, as was done in some of previous analyses, leads to systematically biased results, as the baryon distribution is substantially less structured than DM distribution. We also find that the cosmological models considered have different behaviours in respect to these tests: \(\Lambda\)CDM has the worst performance. CDM and the CHDM mixture considered here have similar scores. Although the general trend of power ratio distributions is already fit by these models, a further improvement is expected either from a different DM mix or a non–flat CDM model.

Key words: Galaxies: clusters: general – galaxies: evolution – X–ray: galaxies – cosmology: theory – cosmology: simulations.
1 Introduction

Clusters of galaxies are the largest bound structures in the Universe. Within the hierarchical clustering scenario they were assembled through the merging and fragmentation of smaller size objects, which had collapsed first, within regions \( \sim 10^{-30} \text{Mpc} \) wide. Accordingly, their masses are estimated in the range \( \sim 10^{14} - 10^{15} M_\odot \). Their gravitational growth is dominated by collisionless dark matter (DM) and the gas distribution follows that of DM.

The outputs of such growth depend on a number of cosmological parameters. First of all, if the global matter density parameter \( \Omega_m < 1 \), the growth of primeval fluctuations is drastically slowed down before the present epoch. In particular, if \( \Lambda = 0 \) (vacuum energy density \( \rho_\Lambda = 0 \)), a transition from decelerated to steady cosmological expansion occurs at a redshift slightly later than \( \Omega_m^{-1} \) and this redshift \( (z_{tr}) \) approximately sets an end to fluctuation growth. If \( \Lambda \neq 0 \), instead, a freeze of fluctuation growing takes place after vacuum energy density \( \rho_\Lambda \) begins to dominate on matter energy density \( \rho_m \). As \( \rho_m \propto (1 + z)^3 \), \( z_{tr} \sim \Omega_m^{-1/3} \). Also in this case, fluctuations not close enough to turn around by \( z_{tr} \) are doomed to quite a slow growing at later epochs. Furthermore, even apart of such late stops to fluctuation growth, a determinant role is played by the shape of the transferred fluctuation spectrum, which depends on other cosmological parameters, besides of \( \Lambda \) and \( \Omega_m \). Henceforth cosmological parameters shed their influence over the number and space distribution of clusters (see, e.g., White, Efstathiou & Frenk 1993; Eke, Cole & Frenk 1996), over their evolution (see, e.g., Jing & Fang 1994; Bahcall, Fan & Cen 1997; Henry 1997), as well as over their morphology.

However, in the last case, hydrodynamics governs gas distribution, gravitation itself is intrinsically non–linear and, a priori, it is hard to state which observed features shall be related to unavoidable kinematical effects and which else are due to initial conditions set up by primeval spectra or, however, to cosmological parameters. It is therefore clear why attempts to trace back cosmological parameters from data were often concentrated on super–cluster scales.

Finding a suitable parameter set to quantify the global cluster morphology is a non trivial task. In this work we shall do so using the power ratios \( \Pi^{(m)} \), introduced by Buote & Tsai (1995, hereafter BT95), which are essentially related to a multipole expansion accounting for the angle dependence of cluster surface brightness. We shall see that this is an effective and synthetic way to discriminate cluster features, as the \( \Pi^{(m)} \) do depend on the cosmological model and discriminate among different cosmologies. It is quite likely, however, that they do not exhaust the characteristics of clusters which depend on the model. Henceforth, both before and after their introduction, many attempts were performed to provide further useful statistical tools.

Early works in this field tried to find a relation between the radial density profile \( \rho_{cl}(r) \) and the value of \( \Omega_o \). Cen (1994) and Mohr et al. (1995) obtained a steeper density profile for open models than for critical density. However, later analyses (Jing et al. 1995; Crone et al. 1997; Eke, Cole & Frenk 1996; Thomas et al. 1998) showed that the radial density profile, when scaled to the cluster virial radius, is substantially independent from \( \Omega_o \).

There can be little doubts that these are significant quantities, but their quantitative analysis can be performed only using really wide observational and model samples, in order to span the whole functional space in an appropriate way.

A more promising way to constrain the cosmological model arises from the study of substructures in the inner mass distribution. This approach has observational support both from internal galaxy distribution (Geller & Beers 1982; Dressler & Shectman 1988; West & Bothun 1990;
Bird 1995) and from X-ray image brightness (Jones & Forman 1992; Böhringer 1993; Mohr, Fabricant & Geller 1993).

Observed substructures were first compared with those expected within a spherical growth approximation (Richstone, Loeb & Turner 1992), then with those expected within a Press–Schechter approach, to follow the merger history of subclumps (Kauffman & White 1993; Lacey & Cole 1993). Cosmological simulations, however, are the most natural method to compare substructure evolution in different models. Early simulations (Evrard et al. 1993; Mohr et al. 1995) showed that the fraction of substructure in galaxy clusters is negligible in low–density CDM models, thus favoring a cosmology with $\Omega_o = 1$. Jing et al. (1995), instead, found a large fraction of galaxy clusters with substructures in $\Lambda$CDM models with $\Omega_m \simeq 0.3$.

An attempt to find a more synthetic parameterization of the degree of inhomogeneity was made by Crone, Evrard & Richstone (1996), who used the displacement of the centre of mass as a function of the overdensity level to quantify the amount of cluster substructures in different cosmological models. Henceforth, they conclude that the center of mass analysis is a good test to discriminate among different models, while previous tests suggested by Fitchett & Webster (1987) and Dressler & Shectman (1988) do not perform so well.

Other statistical approaches, trying to quantify substructures, are the moment of the distributions (Dutta 1995) and the hierarchical clustering method (Serna & Gerbal 1996, Gurzadyan & Mazure 1998). They all confirm that the level of substructure of galaxy clusters is quite sensitive to the underlying cosmological models, although their capacity to discriminate among them is often rather ambiguous. In our opinion, as already outlined, the most promising method is based on the so–called power ratios and amounts to a multipole expansion of the two–dimensional potential generating the observed surface brightness.

Further detail on their definition and significance are given in the next section, where we shall also discuss the observational material on which their analysis can be based. As a matter of fact, complete maps of optical surface brightness or, let alone, projected mass density (obtained, e.g., using gravitational lensing) are unavailable. Henceforth Buote & Tsai (1996, hereafter BT96) used maps of X–ray surface brightness, which trace the squared baryon density distribution.

Previous work in this field, besides of using available observational material, made also use of outputs of simulations already performed for different aims. They amounted to 6 hydro simulation for CDM model, and to a set of N–body simulations for various cosmological models. The effectiveness of the approach, however, calls for better observational material and simulations. In this paper we try to fulfill such requirements for what concerns simulations, running three sets of 40 cluster models, for different cosmological models, with a TREESPH code. We compare their outputs with observations using, first of all, the statistical tests used in previous work. However, some statistical tests will also be suitably improved, obtaining a higher discriminatory power.

It is also important to stress that we compared our results, based on baryon distributions, with those obtainable from the same clusters, if DM distributions are used. We find that, systematically, structures in gas are less pronounced than in DM. Replacing N–body simulations with TREESPH simulations is therefore a substantial step forward and does lead to a different score for the cosmological models considered.

In our opinion, however, a final word on model rating, based on $\Pi^{(m)}$ analysis, cannot yet be said. This is not only due to the limits of available observational material, whose biases we try to reproduce in our model analysis. Rather, some of improved statistical tests considered show
no agreement between data and any of the cosmological model considered. In spite of that, we can safely state that, from this kind of analysis, ΛCDM models come out disfavored; standard CDM and the CHDM mix considered here (which, unlike CDM, fits also COBE quadrupole data and has a fair spectral shape parameter Γ) are certainly closer to observations. However, in our opinion a general fit of data can be expected for some different DM mix different and/or for open CDM models.

The plan of the paper will be given now. In sec. 2 the definition of power ratios will be reviewed and we will also outline the correlation between power ratios and cluster evolution. Some details about the different cosmological models considered in our analysis will be reported in sec. 3. In sec. 4 we give suitable details on the P3M N–body code for the cosmological simulations and the TREESPH for the hydrodynamical simulations. The statistical tools used for the comparison between clusters worked out from each cosmological model and observational data will be summarized in sec. 5, where the results will also be discussed. Finally, in sec. 6 we present our conclusions.

2 Power ratios: definition and evolution; observational sample

We assume that the X–ray intensity received by ROSAT PSPC (Pfeffermann et al. 1987) arises from thermal bremsstrahlung and, therefore, is $I_X = \Lambda \rho_b^2(r)$. In principle, the coefficient $\Lambda$ depends both on the temperature $T_X$ and on the geometry of the source. However, the number of photons collected by ROSAT PSPC does not depend on $T_X$, provided that $T_X > \sim 1$ keV (see, e.g., NRA 91–OSSA–3, Appendix F: ROSAT mission description). Furthermore, $I_X$ shall be used in quantities whose ratios will be compared with data and, therefore, the absolute value of $\Lambda$ does not appear in the final expressions.

Henceforth, the values of $\Pi^{(m)}$ from a model cluster are obtained as follows (here we shall report a procedure fully consistent with BT95, BT96, Tsai & Buote 1996, hereafter TB, and Buote & Xu 1997, hereafter BX): (i) $\rho_b^2(r)$ [which is $\propto I_X(r)$] is projected on a plane $\pi_r$, chosen at random, to yield the X–ray surface brightness $\Sigma(x, y)$ (the direction $\hat{n}$, orthogonal to $\pi_r$, defines the line of sight; $x$ and $y$ are cartesian coordinates on $\pi_r$). (ii) The mass center $O$ of $\Sigma(x, y)$ (centroid) is used as origin of plane polar coordinates on $\pi_r$, to obtain $\Sigma(R, \varphi)$. (iii) By solving the Poisson equation:

$$\nabla^2 \Phi = \Sigma(R, \varphi)$$

we obtain the pseudo–potential $\Phi(R, \varphi)$ (constant factors, in front of $\Sigma$, would again simplify). (iv) We expand $\Phi$ in plane harmonics; the coefficients of such expansion will be used to build the power ratios $\Pi^{(m)}(R)$ as follows:

$$\Pi^{(m)}(R) = \log_{10}(P_m/P_0)$$

where

$$P_m(R) = \frac{1}{2m^2}(\alpha_m^2 + \beta_m^2) , P_0 = [\alpha_0 \ln(R/kpc)]^2$$
\begin{align*}
\alpha_m &= \int_0^1 ds s^{m+1} \int_0^{2\pi} d\phi [\Sigma(sR, \phi)R^2] \cos(m\phi) \quad (4) \\
\beta_m &= \int_0^1 ds s^{m+1} \int_0^{2\pi} d\phi [\Sigma(sR, \phi)R^2] \sin(m\phi) \quad (5)
\end{align*}

Here one can directly see that constant factors – unspecified here above – cancel out when ratio of $P_m$’s are taken.

Owing to the definition of the centroid $O$, $P_1$ vanishes. Henceforth, also $\Pi^{(1)} \equiv 0$. Furthermore, we shall restrict our analysis to $m \leq 4$, to account for substructures on scales not much below $R$ itself, and this leaves us with only 3 significant $\Pi^{(m)}(m = 2, 3, 4)$.

Because of its evolution, a cluster moves along a curve of the 3–dimensional space spanned by such $\Pi^{(m)}$’s; this curve is called evolutionary track. We shall also consider its 2–dimensional projections. Quite in general, a cluster starts from a configuration away from the origin, corresponding to a large amount of internal structure and evolves towards isotropization and homogeneization. This motion, however, does not occur with a steady trend: sudden bursts of structure appear, when matter lumps approach the cluster potential well, and eventually fade as lumps are absorbed by it. Gradually, however, the evolutionary track approaches the origin, as can be easily appreciated by averaging over the contributions of several clusters.

Actual data, of course, do not follow the motion of a given cluster along the evolutionary track. Different clusters, however, lie at different redshifts and can be used to describe a succession of evolutionary stages. Power ratios for model clusters are to be set in the 3–dimensional space spanned by $\Pi^{(m)}$’s, taking each model cluster at redshifts distributed as for data clusters.

Our data set is the same used by BT96. Among X–ray cluster images, taken with ROSAT PSPC, and such that the PSPC central ring contains a cluster portion whose radius exceeds $400 \, h^{-1}\text{kpc}$, we shall use those contained in the HEASARC–legacy database and belonging to the Ebeling (1993) or Edge et al. (1990) samples. Out of the 59 objects selected in this way, one can estimate $\Pi^{(m)}$ for 44 of them at $R = 0.8 \, h^{-1}\text{Mpc}$ and for 27 of them at $R = 1.2 \, h^{-1}\text{Mpc}$. PSPC data give the X–ray surface brightness $\Sigma_X(R, \varphi)$, which is to be used in the same way as the $\Sigma$’s obtained from models, to work out $\Pi^{(m)}$ for $R = 0.4, 0.8, 1.2 \, h^{-1}\text{Mpc}$. According to TB, the resulting sample is partially incomplete, but, clusters were not selected for reasons related to their morphology and the missing clusters are expected to have a distribution of power ratios similar to the observed one. They also tested this point using results of the Imaging Proportional Counter of the Einstein satellite. We shall later report on the lack of any similar correlation for model clusters.

In Fig. 1 we report the redshift distribution of clusters in our data set, for the three values of $R$ we use. We took this into account in our comparison with simulations, at variance with previous analyses. As we shall better detail in the next section, for each cosmological model we consider, we have outputs for 40 simulated clusters, at a set of redshifts $z_{in}$ (see below). This enables us to select simulation outputs so to reproduce the $z$–distribution of data using each model cluster only once, i.e. at a single $z$ value. This can be done in a large number of ways. Among them we randomly select 50 cluster sequences. However, even at this point, we can still choose the line of sight in each model cluster in each sequence in an arbitrary way. This is
fixed at random, once for all, in each cluster of each sequence. The cluster sequence selection is independently made for each cosmological model.

3 The cosmological models

In this work we considered three spatially flat cosmological models, that we shall indicate CDM, ΛCDM and CHDM. The Hubble parameter, normalized to $100 \text{ km s}^{-1} \text{Mpc}^{-1}$, is $h = 0.5$ for CDM and CHDM, and $h = 0.7$ for ΛCDM; for all models the primeval spectral index $n = 1$ and the baryon density parameter is selected to give $\Omega_b h^2 = 0.015$. In CHDM we have 1 massive neutrino with mass $m_\nu = 4.65 \text{ eV}$, yielding a HDM density parameter $\Omega_h = 0.20$. In ΛCDM the vacuum contribution to the energy density is $\Omega_\Lambda = 0.7$. Accordingly, the global density parameter of matter $\Omega_m$ is 1 for CDM and CHDM, and 0.3 for ΛCDM.

All models were normalized using their transfer function, so to give 32 clusters of mass $M_c > 4.2 h^{-1} \cdot 10^{14} M_\odot$ in a box of side $L = 200 h^{-1} \text{Mpc}$ (White et al. 1993, Biviano et al. 1993, Eke et al. 1996, Girardi et al. 1998). For CHDM and ΛCDM this gives quadrupole values within $2\sigma$ from COBE 4–year data (Bennet et al. 1996). As is known, CDM fails this test, but is however an important reference model. For CDM, also the shape parameter $\Gamma = 7.13 \cdot 10^{-3} \left( \sigma_8 / \sigma_{25} \right)^{10/3}$ ($\sigma_{8,25}$ are mass variances on $8, 25 h^{-1} \text{Mpc}$ scales) widely exceeds the observational interval. For APM galaxies, in fact, Peacock and Dodds (1994) found $\Gamma = 0.23 \pm 0.04$; for the Abell/ACO sample, Borgani et al. (1997) found $\Gamma$ in the interval $0.18–0.25$; on the contrary CDM predicts $\Gamma \sim 0.4$. CHDM and ΛCDM, instead, yield $\Gamma \sim 0.2$, well inside the observational interval.

The selection of cosmological models was made in order to have simple models with significantly different characteristics. Both for CHDM and ΛCDM, different parameter choices are possible. In particular CHDM features are highly dependent on the DM mix considered. The mix taken here is selected both for its physical significance and to ease simulations. In fact, CHDM with light neutrinos is non trivial to simulate, as thermal neutrino velocities rapidly displace hot particles from their initial positions and the hot spectrum on small scales becomes dominated by shot noise. With the parameter choice performed here and in the simulation section, this effect is almost absent in this work.
Table 1
Cosmological simulation parameters

| models | Ω_m | h | a_0 | N_p | σ_8 |
|--------|------|---|-----|-----|-----|
| CDM    | 1    | 0.5 | 4.5 | 10^6 | 0.6 |
| CHDM   | 1    | 0.5 | 5.8 | 10^6 | 0.64|
| ΛCDM   | 0.3  | 0.7 | 11  | 84^3 | 1.1 |

Note: For CHDM, the density parameters are Ω_c = 0.8 and Ω_h = 0.2 (1 massive ν).

4 The simulations

In order to achieve a safe statistical basis for our analysis, we built 40 simulated clusters for each cosmological model considered. Herebelow we report the details of the simulations, which were a significant part of our work. The procedure starts from taking a large simulation box, whose side (200h^{-1}Mpc) is fixed in order to provide more than 40 clusters, with mass > 10^{14}h^{-1}M_{⊙}. In this box we run a (purely gravitational) N–body P3M simulation for each cosmological model, starting from a redshift z_{in}. Then, using a friend–of–friend (FoF) algorithm, we located the 40 most massive clusters for each model. For each of them, restarting from z_{in}, we performed a hydrodynamical TREESPH simulation in spheres of radius ~ 15–25 h^{-1}Mpc).

Let us first describe the cosmological N–body simulations. The PM part of the code makes use of 256\(^3\) cells. For the PP part of the code the Plummer–equivalent softening distance is 330kpc for all the models. The particle number is N_p = 10^6 for CDM and CHDM models with Ω_m = 1, while N_p = 84^3 for ΛCDM with Ω_m = 0.3 (see Table 1).

The simulations were run from an initial expansion factor (a_{in} = 1), when σ_8 ≃ 0.1, to a final expansion factor a_0, when the cumulative cluster number density n_c(> M_c) = 4 \cdot 10^{-6}h^3Mpc^{-3} for M_c = 4.2h^{-1}10^{14}M_{⊙}. In CDM and ΛCDM models this is expected for σ_8 = 0.52Ω_{m-0.52}+0.13Ω_{m} (White et al. 1993; Biviano et al. 1993; Eke et al. 1996) and in CHDM models for σ_8 = 0.57 (Valdarnini, Kahniashvili & Novosyadlyj 1998). This procedure yields z_{in} = a_0 − 1, which must fulfill two competing requests: (i) In order to apply the Zel’dovich approximation, the linear Lagrangian theory must still be a valid approximation; this sets a lower limit to z_{in}, which depends on the model. (ii) However, the average particle separation decreases as the scale factor a(t) and, at z_{in}, it risks to approach the gravitational softening length which, in TREESPH simulations, has a constant physical size. The values of a_0 given in Table 1 have been chosen in order to fulfill both constraints. Just for the sake of comparison, in their CDM simulations, Navarro, Frenk & White (1995, hereafter NFW) have a_0 = 4.74.

At variance from CDM and ΛCDM, in CHDM the substance is made of three components. However, for the P3M simulation, this bears scarce relevance. In fact, the neutrino Jeans mass at z_{in} is M_{\nu} = 2 \cdot 10^{13}M_{⊙}, just above particle mass. This allows us to use a global transfer function \( T(k) = \Omega_c T_c(k) + \Omega_h T_h(k) \), where \( T_{c,h}(k) \) are the transfer functions for the cold and hot components, with a suitable growing coefficient α (Klypin et al. 1993).

Let us also outline that the same random numbers were used to set the initial conditions for all three cosmological models.

The FoF algorithm considers as friends the particles at distances smaller than 0.2Ω_{m}^{0.2} times the mean particle separation. The linking parameter is scaled with Ω_m in order to detect overdensities ≃ 200Ω_{m}^{−0.6}. Clusters with mass > 4.9h^{-1}10^{14}M_{⊙} were taken; clusters whose centers–of–mass lie within an Abell radius (R_A = 1.5h^{-1}Mpc) are merged together. The three
models yielded a spectral amplitude slightly above linear on the $8\, h^{-1}\text{Mpc}$ scale; the values of $\sigma_8$ which gave the required $n(> M_c)$ are reported in Table 1, together with other parameters for the models considered. The cluster mass distributions are given in Fig. 2.

For the 40 most massive clusters of each cosmological model, a hydrodynamic simulation was performed in physical coordinates, using a TREESPH code (Hernquist & Katz 1989, hereafter HK). In order to do so, we first located the center of each cluster at $z = 0$ and found all particles within $r_{200}$ (where the cluster density is $\approx 200\Omega_m^{-0.6}$ times the background density). These cluster particles were then located at $z_{in}$, in the original simulation cube.

According to a top–hat model we expect that a cube centered on the cluster center, and enclosing all its particles at $z_{in}$, has a side $L_c \approx 12r_{200}\Omega_m^{-0.2}$. However, we found that this value is often too small to enclose all particles found within $r_{200}$. In most cases the discrepancy is $\approx 10 - 30\%$, but, sometimes, it is much greater than so. In all cases we set $L_c$ so to enclose all particles found within $r_{200}$. $L_c$ values defined in this way range from 15 to $25h^{-1}\text{Mpc}$.

A high resolution lattice of $N_L = 22^3$ grid points was set in such cubes. Each grid point corresponds to a mass $m_{grid} = 2.7 \cdot 10^{11}\Omega_m h^2 (L_c/\text{Mpc})^3 M_\odot/N_L$. Such mass is shared in two or three parts, to obtain the masses $m_i = \frac{\Omega_i}{\sum \Omega_i} m_{grid}$ ($i =$ cold, hot, baryon) of particles describing the cosmic substance (of course, $\sum \Omega_i = \Omega_m$). Different lattices were used for each substance component, separated by suitable fractions of particle spacings along each spatial coordinate. For CHDM simulations a small peculiar velocity is given to the hot particles, drawn at random in magnitude and direction from a Fermi–Dirac distribution with $v_0 = 5(1 + z_{in})(10 eV/m_\nu)\text{km s}^{-1}$. For the gas particles we set an initial temperature $T_i = 10^4\, ^0\text{K}$.

The particle positions given by the high resolution lattices were then perturbed, using the same initial conditions of the original cosmological simulations, implemented by additional waves to sample the increased Nyquist frequency. Baryon fluctuations are given by the same transfer function as CDM. In CHDM simulations, neutrinos are perturbed according to their transfer functions.

However, such cubes are too small to neglect the action of matter laying outside them. Henceforth, each cube of side $L_c$ is located inside a greater cube of side $2L_c$, with parallel sides and equal center, whereinside we set matter yielding gravitational action, but expected not to take part to the cluster collapse. Accordingly, gas needs not to be used there and collisionless par-
articles representing the cold spectrum will account also for baryons. Also a lesser resolution is sufficient and the greater cube is therefore filled with a lattice of $N_L$ grid points (their spacing is therefore double than that for the inner cube). Accordingly, the mass related to each grid point is $8m_{\text{grid}}$. Particles are set only in the part of the greater cube outside the smaller one and their positions and masses are determined with the same procedure used for the inner cube.

Here again, however, the particle positions were then perturbed, using the same initial conditions of the original cosmological simulations, again implemented by (a smaller number of) additional waves to sample the increased Nyquist frequency. Hence, on the boundaries of such greater box, we match the initial conditions of the cosmological P3M.

The high resolution TREESPH simulation will then be carried on in physical coordinates, using all particles which lie inside a sphere of radius $L_c$, and with the same center as cubes. One must therefore bear in mind that further gravitational actions, which might have been caused from outside such external sphere, are necessarily neglected. This technique is similar to the one adopted by Katz & White (1993) and NFW.

It must be however outlined that, in spite of the above prescriptions, after running the TREESPH simulation down to $z = 0$, we found a few clusters for which the mass distribution within $r_{200}$ was contaminated by particles of the external shell. This feature is to be avoided, in order to prevent an anomalous two–body heating; therefore, these particles are identified and a new integration is performed with each of these particles split into 8 hot or cold DM sub–particles. For the CDM model, the origin of the contamination is fairly clear; contaminating particles were those which, after perturbing their position at $z_{\text{in}}$, to account for the initial spectrum of perturbations, were displaced inside a radius $L_c/2$ from the cube center. Unfortunately such rule of thumb is not so reliable for CHDM and ΛCDM models, the difference is presumably related to the different shapes of power spectra.

The gravitational softening parameter $\varepsilon$ are the same, independently of the cluster mass, in simulations for a given cosmological model (apart of 5 simulations in ΛCDM; see below). They are different, instead, for the different substance components, according to a law $\varepsilon \propto m_p^{1/3}$ ($m_p$ is the particle mass of a given substance component), mostly in order to avoid 2–body heating between different substance components. This eases other dynamical problems and ensures a constant central density for all particle species (Farouki & Salpeter 1982).

For gas particles the softenings are $\varepsilon_g = 80, 100, 60$ kpc for CDM, CHDM and most ΛCDM models, respectively. For the highest mass 5 ΛCDM clusters, $\varepsilon_g$ was set up to 80 kpc. For the CDM component, henceforth, $\varepsilon_c = 200, 231, 125$ (or 160) kpc for CDM, CHDM and ΛCDM models, respectively. Accordingly, the ratio $\varepsilon_g/r_{200}$ never exceeds $\simeq 0.04$. The actual force resolution of our hydrodynamic simulations compares with $\varepsilon = 100$ kpc in NFW, although they do not specify the substance or $m$ dependence of the softening parameters.

The time integration is done using a standard leap–frog scheme, allowing each particle $p$ to vary its own time step. In general, it will amount to $\Delta t_p = \Delta t_0/2^{k_p} \ (k_p \geq 0$ is the time–bin
Reference values for the four clusters used in the numerical tests. \(N_g\): number of gas particles, \(\varepsilon_g\): value of the gas softening parameter in \(h^{-1}\) kpc, \(M_{200}\): cluster mass within \(r_{200}\) in \(h^{-1} M_\odot\), \(r_{200}\) in units of \(h^{-1}\) Mpc. These values are for the standard integrations.

| cluster     | \(N_g\) | \(\varepsilon_g\) | \(M_{200}\) | \(r_{200}\) |
|-------------|--------|------------------|-----------|----------|
| CHDM00      | 5551   | 50               | 1.4 \(\times\) 10^{15} | 1.83     |
| CHDM39      | 5575   | 50               | 4.4 \(\times\) 10^{14}  | 1.25     |
| ACtDM00     | 5503   | 56               | 1.2 \(\times\) 10^{15}  | 1.98     |
| ACtDM39     | 5551   | 42               | 4 \(\times\) 10^{14}    | 1.37     |

The value of \(k_p\) is selected according to the values taken by \(p\)-particle velocity \(v_p\), acceleration \(a_p\) and binding energy per unit mass \(E_p\), by requiring that

\[a_p v_p \Delta t_p \leq \eta E_p\]  \hspace{1cm} (6)

(see Katz 1991). Here \(\eta\) is a tolerance parameter that we set to 0.02. We ran several integrations with the improved criteria

\[\Delta t_p \leq 0.3 (\varepsilon_p / a_p)^{1/2}\]  \hspace{1cm} (7)

\[\Delta t_p \leq 0.3 (\varepsilon_p / v_p)\]  \hspace{1cm} (8)

but we did not notice appreciable differences in numerical outputs, presumably thanks to the small value of \(\eta\) we used.

In addition to the above criterions the timesteps for gas particles must also satisfy the Courant stability constraint, so that sound waves cannot change in a single timestep over the SPH smoothing scale. This constraint reads \(\Delta t_p \leq \Delta t_C\). Here \(\Delta t_C\) has a fairly complex expression, given, \(e.g.,\) in Eq.(2.37) of HK.

In average, each cluster simulation required \(\sim 6\) hours of CPU time to be evolved from \(z_{in}\) to \(z = 0\) on a RS6000 computer workstation. Outputs from TREESPH simulations were preserved at various redshifts. Among the smaller ones, we used \(z_i = 0.15, 0.10, 0.049\) and 0 (\(i = 1, ..., 4\)).

The 40 cluster models worked out according to the above recipes, for each of the three models, will be the basis of the statistical discussions of the next sections. However, in order to assess the reliability of numerical integrations we have studied how output variables are sensitive to changes in the numerical parameters, \(e.g.,\) the number of particles or the softening parameters.

To this aim, we have taken the most and the least massive clusters found for the CHDM and ACtDM models (labeled 00 and 39, respectively) and, for each of them, we have performed a battery of numerical tests. In Table 2 we report the values of \(\varepsilon_g\) and \(N_g\) used in the standard integrations as well as the values of \(M_{200}\) and \(r_{200}\).

The tests start from the same initial conditions, for the TREESPH integrations, as in the standard case. On the contrary, we vary the gas softening parameter \(\varepsilon_g\) and/or the number of gas particles \(N_g\). In accordance with them, we also scale the parameters related to different
Fig. 3. Density and temperature profile in different numerical tests, for the four clusters used as reference. Each panel is for a different cluster and in any panel the upper plot is for densities and the lower plot for gas temperatures. In each density plot lower curves are for the gas and upper curves for the dark matter. Different curves are for integrations with a different number of gas particles $N_g$, standard integrations have been performed with $N_g \simeq 5550$. The other particle species have their number changed, with respect the standard value, in proportion to the change in $N_g$. 
substance components. The final part of this section is devoted to a detailed analysis of the stability of results against such changes. This analysis allows to conclude that the numerical integrations are adequately sampled for numerical parameters adopted in Table 2.

In Fig. 3 we show the behaviour (at $z = 0$) of densities and temperatures vs the distance $r$ form the cluster center, for different values of $N_g$. In each panel the upper plot is for the gas and dark matter densities, the lower plot for the gas temperatures ($T(r)$). Different curves refer to $N_g = (2, 1.5, 1, 0.75, 0.5) \times \bar{N}_g$; here $\bar{N}_g = 5550$ is the standard value of particle number. For these integrations $\varepsilon_g$ is given in Table 2.

In Fig. 4 we show the same variables of Fig. 3, but the integrations have been done keeping $N_g$ constant and varying the softenings ($\varepsilon_g$ and, in proportion, those of the other species). We considered $\varepsilon_g = (4, 2, 1, 0.5, 0.25) \times \bar{\varepsilon}_g$. Here $\bar{\varepsilon}_g$ is the standard values of Table 2.

Fig.s 3 and 4 show two different behaviours for $r$ smaller or greater than the core radius $r_c$. At $r > r_c$ discrepancies are modest. However, Fig. 3 shows that, when $N_g \geq 5,000$, even the central value of the gas density converges to within 10%. Furthermore, regardless of $N_g$, we find that the gas core radius $r_c \simeq 0.15–0.20\ Mpc$. The run of $T(r)$, instead, shows some scatter, at $r \leq r_c$, for variations in $N_g$ (LCDM39 has the strongest variations), but, again, the central gas temperatures keep within 20%, when $N_g \geq 5,000$.

There is also a trend for the central values to become smaller when $N_g$ is greater. This arises because we kept $\varepsilon_g$ constant. Then, when $N_g$ becomes small, two–body heating effects produce an increase in $T(r)$ and the process is more evident for $r \lesssim r_c$. In accordance with that, Fig. 4 shows that the central gas temperatures tend to be systematically greater when $\varepsilon_g$ decreases.

In order to keep the characteristic time of the two–body heating process above the cosmological time, our choice of $\varepsilon_g$ turns out to be the best one, for the particle number we took. In fact, according to Binney & Tremaine (1987), the 2–body heating time

$$\tau_r = \frac{0.34 \sigma^3}{G^2 m \rho \ln \Lambda}.$$  \hspace{1cm} (9)

Here $m$ is the dark particle mass, $\rho$ is the cluster density, $\sigma$ is the one–dimensional velocity dispersion and $\Lambda = R/\varepsilon$ gives the Coulomb logarithm associated to the gravitational interaction ($R \simeq r_{200}$ ought to be the typical size of the system).

In CDM models we can estimate $\tau_r$ in a simplified way, but the arguments below can be easily translated to other cosmological models. First of all, according to NFW, the approximate scaling $\sigma \simeq 911\ \text{km sec}^{-1}(M_{200}/10^{15}M_\odot)^{1/3}$ holds. Furthermore, the CDM particle mass $m$ can be expressed as a function of the cube side $L_c$ and thus of $M_{200}$. For $N_L = 22$ it is then easy to find that $m \simeq 2.3 \cdot 10^{-4}M_{200}$. Accordingly, eq. 9 yields:

$$\tau_r \simeq \frac{0.7 \cdot 10^9 \text{Gyr}}{(\rho/\rho_c) \ln \Lambda}.$$ \hspace{1cm} (10)

For the most massive cluster we met in CDM (indicated as CDM00), $M_{200} \simeq 9.3 \cdot 10^{14}h^{-1}M_\odot$, with $r_{200} \simeq 1.5h^{-1}\ Mpc$. The heating time decreases for greater cluster masses and densities. Therefore, this cluster is the worst possible case we met. Furthermore the heating time decreases as the density gets higher, we evaluate $\tau_r$ at the core radius $r_c \simeq 0.05r_{200}$, approximately the resolution limit of our simulations. It also well known that, according to NFW, the the DM best–fit density profile gives $\rho/\rho_c = 2.2 \cdot 10^4$, at $r \simeq r_c$. 
Fig. 4. The same as in Fig. 3, but different curves are for different values of $\varepsilon_g$. The values of $\varepsilon_g$ given in the panels are in kpc, the small vertical lines show their values. In these integrations the value of $N_g$ is given in Table 2.
As, for CDM00, \( \varepsilon = 200 \text{kpc} \) and \( \ln \Lambda = 2.7 \), we shall have the 2-body heating time, for it, is \( \tau_r \simeq 12 \text{Gyr} \). This shows that, outside of the core radius, our simulations are safely free of 2-body heating, as is also shown by the way how \( T(r) \) changes in the numerical tests we performed.

An inspection of Fig. 4 shows several regularities that it is worth outlining. First of all, let us take as a reference curve \( \bar{T}_g(r) \), yielding the \( r \) dependence of the temperature for the softening \( \bar{\varepsilon}_g \). For \( r \gtrsim r_c \), varying \( \varepsilon_g \) has scarce effects on \( T_g(r) \). At \( r < r_c \) we are approaching the resolution of our simulations. This is even truer for greater values of \( \varepsilon_g \). However, even for \( \varepsilon_g = 400 \text{kpc} \), the indication of the core structure is maintained.

More in detail, all \( T_g(r) \) lie below \( \bar{T}_g(r) \), when \( \varepsilon > \varepsilon_g \) (presumably because of the reduced spatial resolution of these integrations), while \( T_g(r) > \bar{T}_g(r) \) when \( \varepsilon \ll \varepsilon_g \). But, as previously outlined, for \( r > r_c \), we have just quite a modest shift.

It is also important to outline that the gas densities do not show a strong dependence on \( \varepsilon_g \), even for \( r < r_c \); the lack of resolution, therefore, lowers temperatures but scarcely affects densities. We presume that this is caused because two-body encounters, when \( \varepsilon_g \ll \bar{\varepsilon}_g \), play a significant role in the core. Here they yield some depletion of the gas density, close to the center, and a more relevant increase of its temperature.

Accordingly, the gas density trend as a function of \( \varepsilon_g \) seems to suggest that \( \varepsilon_g \simeq \bar{\varepsilon}_g \) is the optimal choice, given the values of the other parameters.

Another way to check the consistency of our results amounts to test how the overall X-ray cluster luminosity \( L_X \) changes in the parameter space spanned by the numerical tests of Figs. 3 and 4. Let us outline that global \( L_X \) variations, by themselves, would not affect power ratios. Nevertheless \( L_X \) is quite sensitive to the cluster central density and a stable \( L_X \) output is a key test of the convergence of gas density and temperature, in this kind of numerical simulations.

For the parameter choices used in Figs. 3 and 4 we evaluated also \( L_X \) at \( z = 0 \), according to the expression given by NFW (Eq. (6)). Results are reported in Fig. 5, where each panel refers to a single cluster. Left plots give \( L_X \) vs \( N_g \) and right plots for \( L_X \) vs \( \varepsilon \). The former ones show that, within 10\%, \( L_X \) is stable for \( N_g \gtrsim 5000 \) (once again the behaviour of \( \Lambda \text{CDM39} \) is peculiar); \( r.:h.:s. \) plots, instead, deserve a more detailed discussion.

Let us again denote \( \bar{\varepsilon}_g \) the standard values of Table 2 and let \( \varepsilon_g \) be the other softening parameters considered in Fig. 4.

In the \( r.:h.:s. \) plots of Fig. 5, the \( L_X \) dependence on \( \varepsilon_g \) shows a bell-shaped behaviour, peaked for \( \varepsilon_g \simeq \bar{\varepsilon}_g \). In fact, for \( \varepsilon_g \gg \bar{\varepsilon}_g \), the gas spatial resolution is degraded in the central cluster regions and the bulk of the contribution to \( L_X \) is depressed. On the contrary, for \( \varepsilon_g \ll \bar{\varepsilon}_g \), two-body heating effects, as already outlined in the comments to previous figures, produce a depletion in \( \rho_g(r) \) and an increase of \( T_g(r) \) in the central regions. The net result is a drop in the value of \( L_X \), with respect the one given by \( \bar{\varepsilon}_g \).

A countercheck of this interpretation can be obtained by considering simulations for which \( \varepsilon_g \) and \( N_g \) are simultaneously set to \( \sim 1/4 \) and \( \sim 2 \) times the standard values, respectively. In this case the two-body heating time is almost the same as in the standard integration. We have run such simulation for \( \Lambda \text{CDM39} \) and the value found for \( L_X \) (reported in the open diamond of Fig. 5-d) is indeed close to the standard one.

Let us remind that two-body heating is an effect inversely proportional to the mass of the most
Fig. 5. Final cluster X-ray luminosities for the integrations of Figs. 3 and 4. Left plots are for $L_X$ as a function of $N_g$ and $\epsilon_g$ is constant (Table 2), right plots have $N_g \approx 5550$ and $\epsilon_g$ varied. The open diamond in panel (d) is an integration with $\epsilon_g = 15$ kpc and $N_g = 11,500$.

massive particle pairs. As all clusters, for a given cosmological model, are described by the same numbers of particles, independently of their total mass, we might expect stronger effects for top mass clusters. As Figs. 3, 4 and 5 show, some anomaly was found, instead, for the cluster $\Lambda$CDM39, which is the lightest of the whole ensemble. It is therefore reasonable to assess that the scatter seen in previous tests for such cluster does not arise from two-body heating, but is to be referred to a casually irregular distribution. This is confirmed, e.g., by the trend of $L_X$ vs. $N_g$, which is anomalous with respect to all other clusters (see Fig. 5d left).
We do not expect that relaxation effects can alter significantly the values of our estimated $\Pi^{(m)}$. We have estimated a relaxation time of about 12 Gyr at the smallest structure we can resolve ($\simeq 100\,Kpc$), which is about 10% of the minimum $R_{ap}$ that we considered when the $\Pi^{(m)}$'s have been evaluated. We have however tested how the power ratios change when $\varepsilon$ and $N_g$ are varied. Results of such tests are summarized in Table 3. Here we compare the variance of $\Pi^{(m)}$'s due to the change of line of sight, with average values and variances of the same $\Pi^{(m)}$'s, obtained by varying softening or particle numbers. In most cases, the latter variances are smaller than the former one and, in most cases, the latter average values are consistent with the line-of-sight average and error. In a large portion of cases, such consistency occurs at the 1–$\sigma$ level. In three cases (underlined in Table 3), consistency is not recovered, even at the 3–$\sigma$ level. Two of them correspond to an anomalously small variance for changes of the line–of–sight. In the third case, the anomalous value has also quite a large variance; different selections of lines–of–sight, in this case, cancel the anomaly; we however refrained from such ad–hoc replacement. All three anomalies concern the same cluster, which, once again, is the smallest of its set. However it is CHDM39, instead of $\Lambda$CDM39, which seemed to have a suspect behaviour.

Because of the argument outlined above we do not expect two-body effects to come into play in the evaluation of the power ratios. This is confirmed from the results reported in Table 3, where we find no evidence of any trend in the dependence of $\Pi^{(m)}$ on $\varepsilon$ or $N_p$. Altogether, it is licit to conclude that our results on $\Pi^{(m)}$'s are free of biases coming from two–body relaxation or other resolution effects.

5 Results

In this work we use power ratios to provide a quantitative characterization for the global morphology of clusters. In Section 2 we discussed how they are evaluated on a subsample of ROSAT data. Here we shall work out power ratios from simulations and discuss how a safe statistical comparison can be performed between simulations and data.

Previous work on the same direction has been performed by: BT95, who used just mock simulations to show that power ratios are an effective tool to distinguish among different kinds of global morphologies; TB, who used 6 hydro simulations by NFW for pure CDM, normalized in order that $\sigma_8 = 0.63$, and performed a comparison with power ratios of ROSAT data (BT96); BX, who used a set of N–body (non–hydro) simulations for various models, and did a systematic comparison between them and ROSAT data.

In this work, we use a large set of hydro simulation (120), for 3 different cosmological models, to perform a similar comparison. The need to widen the extension of the sample had been felt also by BX, who tried to satisfy it by using non–hydro simulations, although they are quite aware of the problems this implies. In full agreement with their reserves, we find that the use of non–hydro simulations, for this kind of problems, can be misleading. This conclusion follows a detailed comparison of results concerning the global morphology of the same clusters, using their gas and dark matter distributions.

Henceforth, previous work on power ratios was essential to show that this is an effective tool to study the global properties of clusters and to discriminate, through them, among cosmological models. In this work, instead, we are able to provide the first set of simulations adequate to make use of such tool and to try a to select the best cosmological model. In this context, a basic contribution concerns the use of statistical tools, which are discussed in this section.
### Table 3
Average values and variances of $\Pi^{(m)}$'s obtained varying line of sight, softening or particle numbers

| cluster | $R_{ap}$ | line of sight | softening | # of gas particles | $\Pi^{(2)} \pm \sigma^{(2)}$ | $\Pi^{(3)} \pm \sigma^{(3)}$ | $\Pi^{(4)} \pm \sigma^{(4)}$ |
|---------|--------|--------------|-----------|--------------------|-----------------|-----------------|-----------------|
| CHDM00  | 0.4    | -4.95 ± 0.40 | -5.16 ± 0.05 | -5.37 ± 0.05 | -7.35 ± 0.15 | -7.16 ± 0.17 | -7.12 ± 0.13 | -7.17 ± 0.45 | -7.32 ± 0.07 | -7.58 ± 0.16 |
|         | 0.8    | -5.76 ± 0.43 | -5.89 ± 0.05 | -6.27 ± 0.08 | -8.08 ± 0.28 | -8.06 ± 0.11 | -7.95 ± 0.07 | -8.59 ± 1.07 | -8.54 ± 0.23 | -8.76 ± 0.09 |
|         | 1.2    | -6.47 ± 0.37 | -6.61 ± 0.06 | -6.90 ± 0.04 | -8.96 ± 0.37 | -8.75 ± 0.01 | -8.55 ± 0.04 | -9.31 ± 0.28 | -9.39 ± 0.24 | -9.31 ± 0.02 |
| CHDM39  | 0.4    | -4.77 ± 0.03 | -4.99 ± 0.05 | -5.10 ± 0.06 | -7.26 ± 0.26 | -7.21 ± 0.10 | -7.16 ± 0.27 | -6.61 ± 0.07 | -7.00 ± 0.14 | -7.25 ± 0.32 |
|         | 0.8    | -4.89 ± 0.16 | -5.32 ± 0.10 | -5.48 ± 0.18 | -7.04 ± 0.82 | -7.69 ± 0.14 | -7.35 ± 0.29 | -6.55 ± 0.52 | -7.35 ± 0.29 | -7.46 ± 0.46 |
|         | 1.2    | -5.45 ± 0.16 | -5.81 ± 0.09 | -5.92 ± 0.17 | -8.85 ± 0.20 | -8.98 ± 0.16 | -8.18 ± 0.52 | -7.46 ± 0.37 | -7.99 ± 0.06 | -8.04 ± 0.29 |
| ACDM00  | 0.4    | -5.69 ± 0.15 | -5.36 ± 0.03 | -5.45 ± 0.03 | -7.55 ± 0.37 | -7.03 ± 0.17 | -7.78 ± 0.18 | -8.32 ± 0.23 | -7.97 ± 0.43 | -8.47 ± 0.19 |
|         | 0.8    | -6.31 ± 0.20 | -5.83 ± 0.10 | -5.98 ± 0.09 | -8.08 ± 0.44 | -7.52 ± 0.12 | -7.73 ± 0.08 | -9.09 ± 0.40 | -8.47 ± 0.43 | -8.69 ± 0.25 |
|         | 1.2    | -6.87 ± 0.28 | -6.37 ± 0.10 | -6.52 ± 0.08 | -8.30 ± 0.26 | -8.09 ± 0.07 | -8.39 ± 0.19 | -9.22 ± 0.47 | -8.87 ± 0.37 | -9.28 ± 0.34 |
| ACDM39  | 0.4    | -6.20 ± 0.21 | -6.06 ± 0.23 | -6.12 ± 0.33 | -8.17 ± 0.20 | -8.18 ± 0.63 | -8.15 ± 0.32 | -8.78 ± 0.46 | -8.69 ± 0.28 | -8.79 ± 0.36 |
|         | 0.8    | -6.75 ± 1.03 | -6.32 ± 0.28 | -6.56 ± 0.15 | -9.67 ± 1.76 | -8.25 ± 0.37 | -8.49 ± 0.59 | -9.45 ± 1.65 | -8.87 ± 0.04 | -9.02 ± 0.22 |
|         | 1.2    | -7.10 ± 1.94 | -6.71 ± 0.12 | -6.83 ± 0.19 | -9.42 ± 2.82 | -8.68 ± 0.04 | -8.49 ± 0.22 | -9.75 ± 2.93 | -9.56 ± 0.07 | -9.56 ± 0.70 |
### Table 4
Averages and variances of $\Pi^{(m)}$ for ROSAT and simulated clusters in the three cosmological models.

|        | $\Pi^{(2)} \pm \sigma^{(2)}$ | $\Pi^{(3)} \pm \sigma^{(3)}$ | $\Pi^{(4)} \pm \sigma^{(4)}$ |
|--------|-------------------------------|-------------------------------|-------------------------------|
|        | 0.4 Mpc/h                     | 0.8 Mpc/h                     | 1.2 Mpc/h                     |
| ROSAT  | -5.74 ± 0.53                  | -5.96 ± 0.66                  | -6.46 ± 0.81                  |
| CDM    | -5.47 ± 0.54                  | -6.17 ± 0.88                  | -6.12 ± 0.85                  |
| ΛCDM   | -5.63 ± 0.45                  | -6.36 ± 0.73                  | -6.79 ± 0.86                  |
| CHDM   | -5.40 ± 0.54                  | -6.16 ± 0.71                  | -6.63 ± 0.72                  |
|        | 0.4 Mpc/h                     | 0.8 Mpc/h                     | 1.2 Mpc/h                     |
| ROSAT  | -7.43 ± 0.77                  | -7.39 ± 0.73                  | -7.75 ± 0.72                  |
| CDM    | -7.16 ± 0.73                  | -7.55 ± 0.93                  | -7.48 ± 0.93                  |
| ΛCDM   | -7.66 ± 0.83                  | -8.33 ± 1.07                  | -8.67 ± 1.17                  |
| CHDM   | -7.07 ± 0.79                  | -7.68 ± 0.85                  | -7.96 ± 0.80                  |
|        | 0.4 Mpc/h                     | 0.8 Mpc/h                     | 1.2 Mpc/h                     |
| ROSAT  | -7.90 ± 0.63                  | -7.90 ± 0.84                  | -8.10 ± 0.83                  |
| CDM    | -7.69 ± 0.69                  | -8.21 ± 1.10                  | -7.98 ± 1.07                  |
| ΛCDM   | -8.02 ± 0.73                  | -8.82 ± 1.08                  | -9.22 ± 1.21                  |
| CHDM   | -7.56 ± 0.84                  | -8.31 ± 0.94                  | -8.60 ± 0.89                  |

### 5.1 Power ratios for different cosmological models

In this sub-section we report the power ratios $\Pi^{(m)}$ that we computed for our three cosmological models from the gas distributions. The average and dispersions of the $\Pi^{(m)}$ are given in Table 4, together with the corresponding ROSAT values. The procedure followed to evaluate them was outlined in section 2. They were obtained using a single sequence of random planes and averaging over different realization for the redshift distribution. The three different apertures are in units of $h^{-1}$Mpc and correspond to those given by BX in their Table 2.

In fig. 6 we report the $\Pi^{(m)}$ distribution for the CHDM model. Similar plots could be given for the other cosmological models. Heavy lines give the model cluster outputs, while light lines correspond to ROSAT cluster data. Each line correspond to the same $R_{ap}$, each column to the same $\Pi^{(m)}$. Such figure shows a basic agreement between model and data clusters, which can also be seen through the figures reported in Table 4.

More in detail, for $R_{ap}h$/Mpc = 0.4 and for all moments, ROSAT and simulation distributions mostly agree, regardless of the cosmological model. This may be an indication that, over small scales, initial conditions are mostly erased by non–linear dynamics and a relaxation regime, independent of the model, is attained. For $R_{ap}h$/Mpc = 0.8 and 1.2, instead, all simulated distributions are shifted towards smaller values. However, while CDM and CHDM are still
marginally consistent with data, ΛCDM is far below them. This is particularly relevant for Π^2, but also Π^3 and Π^4 show a similar trend.

In order to quantify these differences, we used the Student t-test, the F-test and the Kolgomorov–Smirnov (KS) test. Let us recall that: (i) The Student t-test allows to compare two data sets with different means and finds the probability p–t (reported in Table 5) that, owing to the distribution of the outputs about such means, the two sets are originated by the same process. (ii) The F-test, instead, compares two data sets with different variances and finds the probability p–F (reported in Table 5) that the two sets are originated by the same process. (iii) Finally, the KS-test also compares two data sets with different distributions and finds the probability p–KS (reported in Table 5) that such different distributions can arise from the same process.

Even apart from the smallest aperture, which seems not so significant, according to Table 5 p–t (p–F, p–KS) is roughly in the ranges 0.11–0.60 (0.11–0.80, 0.04–0.45) for CDM, 0.03–0.37 (0.47–0.71, 0.33–10^{-2}–0.52) for CHDM, 0.15·10^{-4}–0.87·10^{-2} (0.01–0.75, 0.4·10^{-6}–0.01) for ΛCDM. Let us recall that only figures below 5% can be taken at face-value. Greater values are to be considered underestimates of the consistency probabilities.
Table 5
Statistical tests applied to the power ratio distribution of ROSAT and simulated clusters: $p_t$ is the Student t–test applied to the means, $p_F$ is the F–test for variances and $p_{KS}$ is the KS statistics to discriminate two distribution.

| models       | 0.4 Mpc/h | 0.8 Mpc/h | 1.2 Mpc/h |
|--------------|-----------|-----------|-----------|
|              | $p_t$     | $p_F$     | $p_{KS}$  | $p_t$     | $p_F$     | $p_{KS}$  | $p_t$     | $p_F$     | $p_{KS}$  |
| ROSAT-CDM    | .16E–01   | .96E+00   | .37E–01   | .21E+00   | .62E+00   | .14E+00   | .11E+00   | .80E+00   | .39E–01   |
| ROSAT-ACDM   | .29E+00   | .27E+00   | .67E+00   | .87E+02   | .49E+00   | .31E+02   | .12E+00   | .75E+00   | .11E–01   |
| ROSAT-CHDM   | .32E–02   | .86E+00   | .85E–02   | .17E+00   | .60E+00   | .24E+00   | .37E+00   | .49E+00   | .52E+00   |
| CDM-ACDM     | .16E+00   | .24E+00   | .28E+00   | .29E+00   | .25E+00   | .16E+00   | .78E+03   | .94E+00   | .24E–03   |
| CDM-CHDM     | .60E+00   | .98E+00   | .91E+00   | .90E+00   | .19E+00   | .57E+00   | .40E–02   | .30E+00   | .55E–01   |
| ACMD-CHDM    | .49E–01   | .24E+00   | .97E–01   | .22E+00   | .87E+00   | .40E+00   | .30E+00   | .26E+00   | .26E+00   |

Such figures seem to exclude that ΛCDM can be considered a reasonable approximation to data. The best score belongs to CDM, but also CHDM is not fully excluded and different mixtures could certainly have better performance.

Table 5 provides also comparisons among different models. CDM and CHDM, again, do not show a marked disagreement. ΛCDM is significantly different, as follows also from the figures reported hereabove. A possible interpretation of such output is that the actual amount of substructures is governed by $\Omega_0$ rather than by the shape of power spectra. An inspection of the model clusters actually shows that the ACDM model does produce less substructures than the other models do.

As a further test we have considered a two–dimensional generalization of the KS–test. The test applies to two 2–dimensional distributions and gives the probability that they originate from the same process (Peacock 1983, Fasano and Franceschini 1987, Press et al. 1992).

The results (see Table 6) confirm the KS test and show that $\Pi^{(m)}$ values, for the smallest aperture $R_{ap} = 0.4 \, h^{-1}$Mpc, are not so discriminatory. On the contrary, at greater apertures, tests on ΛCDM often yield probabilities of a few thousands. CHDM, instead, is almost anywhere consistent with data, at least at the $\sim 2$–σ level, apart of a case, where, however, the probability is not much lower. CDM, perhaps, has a slightly worse performance, as it never overcomes a 10% probability and is out of $\sim 2$–σ’s in 2 cases.
Table 6
Probabilities worked out using the PF2–test, a 2–dimensional generalization of KS–test. Notice that, for ΛCDM models, they may be as low as a few parts on a thousand; the greatest values are met for CHDM, which, apart of a case, always agrees with data at least at 2–σ level; the performance of CDM is only slightly worse.

| $R_{ap}^h$/Mpc | CDM  | ΛCDM | CHDM |
|---------------|------|------|------|
| $\Pi_2 \rightarrow \Pi_3$ | 0.4  | 6.25% | 10.57% | 11.73% |
| $\Pi_2 \rightarrow \Pi_4$ | 0.4  | 5.44% | 17.26% | 4.89%  |
| $\Pi_3 \rightarrow \Pi_4$ | 0.4  | 8.03% | 8.35%  | 6.83%  |
| $\Pi_2 \rightarrow \Pi_3$ | 0.8  | 6.66% | 0.71%  | 3.45%  |
| $\Pi_2 \rightarrow \Pi_4$ | 0.8  | 9.25% | 1.81%  | 5.42%  |
| $\Pi_3 \rightarrow \Pi_4$ | 0.8  | 2.79% | 0.13%  | 1.41%  |
| $\Pi_2 \rightarrow \Pi_3$ | 1.2  | 4.35% | 2.46%  | 10.21% |
| $\Pi_2 \rightarrow \Pi_4$ | 1.2  | 3.67% | 0.35%  | 8.82%  |
| $\Pi_3 \rightarrow \Pi_4$ | 1.2  | 6.62% | 0.20%  | 11.60% |

5.2 Power ratio evolution

During its evolution, a cluster moves along a line (evolutionary track, see BT96 and TB) in the 3–dimensional space spanned by the $\Pi^{(m)}$’s. Starting from a configuration away from the origin, corresponding to a large amount of internal structure, it evolves towards isotropization and homogeneization. This motion does not occur with a steady trend, and bursts of structure appear when further matter lumps approach the cluster potential well, to be absorbed by it. However, the evolutionary track eventually approaches the origin, and this can be more easily appreciated by averaging over the contributions of several clusters.

Actual data, of course, do not show the motion of a single cluster along the evolutionary track. Different clusters, however, lie at different redshifts and, in average, can be expected to describe a succession of evolutionary moments.

In Fig. 7 we give 9 plots for the space containing the evolutionary tracks projected on 3 planes and at three different $R$’s. For the sake of example, in Fig. 7 we describe the behaviour of CHDM. Filled dots refer to a single selection of simulated clusters. Their distributions show a linear trend and a linear regression gives place to the straight lines reported in the plots. Crosses, instead, indicate the location of data clusters. Their distributions do not show such a trend as simulated points and are more scattered than CHDM clusters.

A quantitative way to compare the correlations of two 2–dimensional distributions on a plane $x, y$, starts from defining the averages $\bar{x}, \bar{y}$ of $x_i$ and $y_i$ values and the deviations $\xi_i = x_i - \bar{x}$ and $\eta_i = y_i - \bar{y}$. Then the linear correlation coefficients read

$$ r = \frac{\sum_i \xi_i \eta_i}{\sqrt{\sum_i \xi_i^2 \sum_i \eta_i^2}}. \quad (11) $$

The use of this tool indicates that the probability of obtaining such different correlation coefficients is quite small and would seem to exclude the validity of all models considered.

There may be a number of reasons for such discrepant behaviours. The time elapsed from $z = 0.2$
up to now is approximately a quarter of the life of the Universe. During such time, galaxies evolve and may acquire different relevance in the overall X–ray emission, the intrachannel gas is suitably enriched and/or undergoes complex cooling processes, environmental effects and other physical variables may have had suitable trends. When evolution is not our aim, measures on samples with similarly weighted contributions from different redshift ranges may keep full significance, while, on the contrary, separating contributions from different \( z \) values might be demanding too much. We should therefore refrain from concluding that none of the models inspected fits the (un–)observed evolutionary trend. We rather suggest that the evolutionary trend in data is (at least partially) hidden by other effects.

We seeked a confirm of the latter hypothesis by trying to use a softer statistical approach to compare the evolution shown by simulations with data. The slope of the straight line in Fig. 7, obtained through a linear regression, is, by definition, a suitably weighted average among the slopes of straight lines through any couple of points. Accordingly, a linear trend is visible as a peak in the slope distribution.

If a similar trend is not completely hidden in data points, a peak should appear also in the distribution of the slopes of lines through pairs of data points. In Fig. 8 we compare the distributions of slopes for data points (thin histograms) and for simulated points (thick histograms) for CDM and CHDM models. This is done starting from the points shown in Fig. 7 for CHDM and for a similar selections for CDM. Fig.s 8, as expected, show a set of peaks for model clusters, more relevant for greater aperture radii.

Let us however outline that some peak structure is also present in data points. Also an eye comparison of CDM and CHDM with data shows that CHDM is substantially favoured. The only plot showing an opposite trend is \( \Pi_1 \) vs. \( \Pi_2 \), with \( R_{ap}/h^{-1}\text{Mpc} = 0.4 \). In a few cases the distributions of CHDM and data seem quite consistent or however strongly favoured, e.g. for all plots at \( R_{ap}/h^{-1}\text{Mpc} = 0.8 \). The probability of agreement, measured through KS coefficients, keeps low (\( \sim 10^{-1}–10^{-3} \)), but the indication in favour of CHDM in respect to CDM is significant. We do not show analogous plots for \( \Lambda \text{CDM} \), which turns out to be disfavoured in this test, based on gas distribution.

It would be unwise to draw any final conclusions on the basis of this test. In our opinion, data need to be significantly improved and enriched before that this kind of analysis may become really discriminatory. A possible pattern would amount to adding noise to model data, seeking the level at which an agreement with real data is approached. Hopefully, at this level, it may still be possible to discriminate among different cosmologies.

5.3 Power ratios for gas and dark matter

One of the results of this work concerns the reliability of comparisons between data and simulations based on DM – rather than gas – distributions. The quantitative outputs, quite independently from the cosmological model, are that: (i) DM \( \Pi^{(m)} \) are systematically less correlated than gas \( \Pi^{(m)} \) (DM slopes are more scattered than gas). (ii) DM \( \Pi^{(m)} \) are systematically greater than gas \( \Pi^{(m)} \).

Both effects seem related to the increased complexity of the phase space distribution for a substance which is not constrained to be described by fluid variables. The first effect is however puzzling, in connection with the last point outlined in the previous subsection. As a matter of fact, DM correlation is closer to real data than gas correlation. In Fig. 9, for the sake of
Fig. 7. Power ratios and evolutionary tracks for data (crosses) and CHDM clusters (filled dots). The straight line is the best-fit to dots.

example, we compare the slope distributions for gas and DM.

In an attempt to find a reason for such finding, we note that, at present, galaxies may be expected to have a behaviour closer to DM particles rather than to gas. Although most of the X-ray emission is thought to come from baryons in the intracluster gas, elliptical galaxies could significantly contribute. As a matter of fact, at low redshifts, well resolved clusters show clearly a clumpy emission associated with galaxies. This should be taken into account, together with the fact that, in our SPH simulations, particle masses still exceed the average galaxy size, while gas cooling is not included.

These arguments, however, are only indicative. During their formation stages, galaxies cannot be approximated by DM points and, to account for their present distribution, we should consider a late DM input arising from gas suppression, associated with cooling effects. In our opinion, at this stage, the main conclusion is that this point is among those to be explored through more resolved simulations, surely including gas cooling and possibly other physical effects.

However, if cosmological models are compared with data on the basis of DM Π^{(m)}, the conclu-
Fig. 8. Histogram in the range \((-\pi/2, \pi/2)\) of slope distributions. The thin line is for data points; the thick line is for model (CDM or CHDM) clusters.

Solutions drawn on the basis of Student t–test, F–test and KS–test, are reversed. For the sake of example, \(p-t\), \(p-F\) and \(p-KS\), for the comparison ROSAT–CDM and for \(R_{ap} = 0.8 \, h^{-1}\text{Mpc}\), pass from 0.21 to 0.68 \(\cdot 10^{-3}\), 0.06 to 0.28 and 0.14 to 0.48 \(\cdot 10^{-2}\), for \(\Pi_2\); from 0.37 to 0.05, 0.11 to 0.97 and 0.18 to 0.22, for \(\Pi_3\). These shifts coherently indicate an increase in the amount of substructures for DM with respect to the gas.

Altogether, DM \(\Pi^{(m)}\), unlike gas \(\Pi^{(m)}\), scarcely feel dissipative processes; hence, using DM \(\Pi^{(m)}\), CDM and CHDM models keep too many substructures and are no longer consistent with data; on the contrary, the increase of substructures pushes \(\Lambda\text{CDM}\) to agree with ROSAT sample.
6 Conclusions

The global morphology of galaxy clusters is well described by a multipole expansion of a pseudo-potential generating its surface brightness, around its centroid. Such expansion leads to the power ratios $\Pi^{(m)}$, first introduced by BT95, who also evaluated them for a ROSAT subsample. In this work we have performed an extensive comparison of such observational $\Pi^{(m)}$, with $\Pi^{(m)}$ worked out from model clusters. Such clusters were obtained for three cosmological models, all normalized to present cluster number density. Such models are CDM (which, therefore, is not consistent with COBE quadrupole data), $\Lambda$CDM and CHDM with 20% of HDM given by a single neutrino flavour (the latter two models are consistent with COBE quadrupole data).

Previous comparisons with simulated clusters were either restricted to 6 clusters, obtained with a TREESPH code by NFW, or used N–body non–hydro simulations. Here, instead, we worked out 40 cluster models for each cosmological model using a TREESPH code. We checked whether our hydro simulations have a resolution adequate to yield safe $\Pi^{(m)}$ values for the aperture radii considered. We tested that for a few model clusters of the whole set, by changing softening and particle numbers. We found that $\Pi^{(m)}$ systematically show a greater dependence on the choice of the line of sight, by which a model cluster is assumed to be seen, than on variation of softening or particle numbers.

The hydro code does not take into account gas cooling or supernova heating. This is expected to induce no bias, on the scales we considered, provided that most of the observed X–ray flux originates from the intracluster gas.

Among the results of our work, we wish to stress soon that $\Pi^{(m)}$ evaluated from gas distributions...
turn out to be substantially different from $\Pi^{(m)}$ worked out from DM distributions. This is to be ascribed to the smoothing effects of the interactions among gas particles, which erase anisotropies and structures. The effect is fairly significant and finally leads to a different score of cosmological models in respect to data. In fact, while DM $\Pi^{(m)}$ worked out for $\Lambda$CDM model clusters are close to data, this is no longer true for gas $\Pi^{(m)}$ within the same models. On the contrary, while DM $\Pi^{(m)}$ for CDM and CHDM models show too much structure to fit data, gas $\Pi^{(m)}$ are in much better agreement with data, for these models. Mohr et al. (1995) reached similar conclusions. More precisely, they find that observed X–ray morphologies of clusters are inconsistent with those obtained from a set of simulated clusters for low–density CDM models. It is worth noticing that their simulations had been performed using a combined N–body/hydro code (P3MSHP), while the BX set is purely gravitational. It should be stressed that BX, after comparing ROSAT data with N–body simulations, also outlined the need to extend the comparison to a large set of hydrodynamical simulations.

Here, the comparison between data and simulation was performed through different steps. First of all, as BT96, TB and BX, we used the Student t–test, the F–test and the KS–test to compare $\Pi^{(m)}$ distributions. We also considered the cluster distribution in the 3–dimensional parameter space with axes given by $\Pi^{(m)} (m = 2, 3, 4)$, as well as projections of such distributions on planes. Taking into account that data involve clusters at various redshifts, such distribution provides an average evolutionary track for clusters. However, comparing such distributions for data and models, we find a significantly stronger correlation of $\Pi^{(m)}$ in models than in data. The degree of correlation depends on the model, but seems however in disagreement with data. Model clusters tend to indicate a significantly faster evolution than data. The cosmological model which seems closest to data is CHDM and it is possible that different CHDM mixtures can lead to further improvements. It is also possible that a slower evolution is an indication that the density of the Universe is below critical. We therefore plan, in a near future, to perform cluster simulations for CHDM models with lighter neutrinos and for 0CDM models.

Let us recall that, at present, the main evidences in favour of open models amount to the detections of large–scale matter concentrations at high redshift, either thanks to direct inspection (Bahcall & Fan 1998), or through the statistics of arcs arising from lensing (Bartelmann et al. 1998). The latter analysis, in particular, seems to exclude a significant contribution of $\Lambda$ to the cosmic density, in agreement with the findings of this work. Evidences in favour of $\Lambda \neq 0$, instead, are mostly related to recent improvements in the use of SNIa as standard candles. They seem to support $\Omega = 1$ models with a large $\Omega_\Lambda (>0.6$, see Perlmutter et al. 1998 and Reiss et al. 1998).

Both such evidences seem however to disagree with $\Omega$ estimates based on X–ray cluster surveys, which may be consistent with $\Omega = 1$, without vacuum contribution (see Sadat, Blanchard & Oukbir 1998 and references therein).

In this work, the low probability of $\Lambda$CDM models is also confirmed by the analysis of $\Pi^{(m)}$ correlations, which may become a discriminatory tool to test models. We have seen that separate distributions of $\Pi^{(m)}$’s can be in fair agreement with data, while their joint distribution is not.

The disagreement found for such joint distributions also calls for an improvement of the observational data set. In our work we used the same observational set of previous analyses. We also tried to reproduce its possible biases by reproducing the redshift distribution of data clusters in models clusters. TB stressed that data selection is not based in cluster structure. However, we have performed a further check to improve the safety of the use of data. This amounted to test whether there is any correlation between cluster luminosity and multipole structure. To do so, we shared our model clusters in 2 or 4 subsamples, ordered according to their intrinsi-
cal luminosity, and verified that no correlation exists between $\Pi(m)$ and luminosity. However, before reaching final conclusions on the cosmological model through global cluster features, an improvement of data is surely opportune.

Let us finally outline that the pseudo–potential, used to perform the multipole expansion, is somehow related to the potential yielding weak lensing; however, while lensing effects are related to the total density $\rho$, here we are referring to gas only, through its square density $\rho_g^2$. The relation between $\rho$ and $\rho_g$ can be strongly model dependent, namely if the cosmic substance contains fair amounts of late–derelativizing HDM, whose space and velocity distribution, in non–linear structures, might be substantially different from CDM itself. We therefore plan to do further work on model clusters to test relations between lensing effects and $\Pi(m)$ expansion, also aiming to devise precise tests on DM composition.

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**References**

[1] Bahcall, N., Fan, X., Cen, R. 1997, ApJ, 485, L53
[2] Bahcall N.A. and Fan X. 1998, astro–ph/9803277
[3] Bartelmann M., Huss A., Colberg J.M., Jenkins A., Pearce F.R., 1998, AA, 330, 1
[4] Bennet, C.L., Banday, A.J., Górska, K.M., Hinshaw, G., Jackson, P., Keegstra, P., Kogut, A., Smoot, G.F., Wilkinson, D.T. & Wright, E.L., 1996, ApJ, 464, L1
[5] Binney, J., Tremaine, S., 1987, *Galactic Dynamics*, Princeton: Princeton Univ. Press
[6] Bird, C.M., 1995, ApJ, 445, 81
[7] Biviano A., Girardi M., Giuricin G., Mardirossian F., Mezzetti M. 1993 ApJ, 411, L13
[8] Böhringer, H., 1993, eds. Silk, J., Vittorio, N., Proc. E.Fermi Summer School, Galaxy Formation
[9] Borgani S., Moscardini L., Plionis M., Górska K.M., Holzman J., Klypin A., Primack J.R., Smith C.C., Strompor R., 1997, NewA., 1, 321
[10] Buote D.A., Tsai J.C., 1995, ApJ, 452, 522
[11] Buote D.A., Tsai J.C., 1996, ApJ, 458, 27
[12] Buote D.A., Xu G., 1997, MNRAS, 284, 439
[13] Cen, R., 1994, ApJ, 437, 12
[14] Crone, M.M., Evrard, A., Richstone, D.O., 1996, ApJ, 467, 489
[15] Crone, M.M., Governato, F., Stadel, J., Quinn, T., 1997, ApJ, 477, L5
[16] Dressler, A., Shectman, S.A., 1988, AJ, 95, 985
[17] Dutta, S.N., 1995, MNRAS, 276, 1109
[18] Ebeling H., 1993, Ph.D Thesis, Ludwig–Maximilians-Univ. Munchen
[19] Edge A.C., Stewart G.C., Fabian A.C., Arnaud K.A., 1990 MNRAS, 245, 559
[20] Eke V.R., Cole S., Frenk C.S., 1996 MNRAS 282, 263
[21] Evrard, A.E., Mohr, J.J., Fabricant, D.G., Geller, M.J., 1993, ApJ, 419, 9
[22] Farouki, R.T., Salpeter, E.E., 1982, ApJ, 253, 512
[23] Fasano G., Franceschini A., 1987, MNRAS, 225, 155
[24] Fitchett, M., Webster, R., 1987, ApJ, 317, 65
[25] Geller, M., J., Beers, T.C., 1982, ApJ, PASP, 92, 421
[26] Girardi, M., Borgani, S., Giuricin, G., Mardirossian, F. & Mezzetti, M., 1998, astro-ph/9804188
[27] Gurzadyan, V.G., Mazure, A., 1998, MNRAS, 276, 417
[28] Henry, J.P, 1997, ApJ, 489, L1
[29] Hernquist L., Katz N., 1989, ApJS, 70, 419
[30] Jing, Y.P., Fang, L.Z., 1994, ApJ, 432, 438
[31] Jing, Y.P., Mo, H.J., Börner, G., Fang, L.Z., 1995, MNRAS, 276, 417
[32] Jones, C, Forman, W., 1992, in Clusters and Superclusters of Galaxies, ed. A.C. Fabian, (Dordrecht: Kluwer), 49
[33] Katz, N., 1991, ApJ, 368, 325
[34] Katz N., White S.D.M., 1993, ApJ, 412, 455
[35] Klypin, A., Holtzmann, J., Primack, J., Regös, E., 1993, ApJ, 416, 1
[36] Kauffman, G., White, S.D.M., 1993, MNRAS, 261, 921
[37] Lacey, C., Cole, S., 1993, MNRAS, 262, 627
[38] Mohr, J.J., Fabricant, D.G., Geller, M.J., 1993, ApJ, 413, 492
[39] Mohr, J.J., Evrard, A.E., Fabricant, D.G., Geller, M.J., 1995, ApJ, 447, 8
[40] Navarro J., Frenk C.S., White, S.D.M., 1995, MNRAS, 275, 720
[41] Peacock J.A., 1983, MNRAS, 202, 615
[42] Peacock J.A., Dodds S.J., 1994, MNRAS, 267, 1020
[43] Perlmutter S. et al. , 1998, Nature , 391, 51
[44] Pfeffermann E. et al., 1987, Proc. SPIE, 733, 519
[45] Press W.H., Teukolsky S.A., Vetterling W.T., Flannery B.P., 1992, Numerical Recipes, Cambridge University Press
[46] Richstone, D., Loeb, A., Turner, E.L., 1992, ApJ, 393, 477
[47] Riess A.G. et al., 1998, astro-ph/9805201
[48] Sadat R., Blanchard A. & Oukbir J., 1998, A&A 329, 21

[49] Thomas, P.A., Colberg, J.M., Couchman, H.M.P., Efstathiou, G.P., Frenk, C.S., Jenkins, A.R., Nelson, A.H., Hutchings, R.M., Peacock, J.A., Pearce, F.R. & White, S.D.M., 1998, MNRAS, 296, 1061

[50] Tsai J.C., Buote, D.A., 1996, MNRAS, 282, 77

[51] Serna, A., Gerbal, D., 1996, A&A, 309, 65

[52] Valdarnini, R., Kahlia, T., Novosyadlyj, B., 1998, A&A, 336, 11

[53] West, M.J., Bothun, G.D., 1990, ApJ, 350, 36

[54] White S.D.M., Efstathiou G., Frenk C.S., 1993, MNRAS 262, 1023