Overview on Various Techniques for Reduction of cogging torque and torque ripples of Permanent Magnet motors of different rotor configurations

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Abstract. Different types of PM motors are finding applications substituting squirrel cage induction motors for its smooth control, energy efficient operations and high power density. Even though these motors are energy efficient, they are affected by ripples in the torque and cogging torque. Torque ripples are nothing but the oscillations leading to various mechanical issues like vibrations, rotor stress and noise. Factors like non sinusoidal distribution of flux, saturation, wrong selection of slots etc are creating ripples in developed torque. These ill effects are deteriorating PM motors performance in starting and steady state. Now a days reducing cogging torque and ripples in torque gained significance in the motor design. So in this work, various techniques to reduce ripples in torque and to reduce cogging torque of various PM motors has been investigated and reported.

Keyword. Permanent Magnet Synchronous Motor (PMSM), Surface mounted Permanent Magnet Synchronous Motor (SPMSM), Interior Mounted Permanent Magnet Synchronous Motor (IPMSM), Two Dimensional Finite Element Analysis (2D-FEM).

1. Introduction
Electric motors have become significant component of present industrial economy. In all industries, the electric motors consume nearly 70% of overall electricity. The electric motors are being used in compressor, crusher, crane, fan, pump, grinder, elevator and other various industrial applications[1]. Increased cost of fuels and awareness to reduce green house gas emissions have made the manufacturers to choose permanent magnet motors instead of induction motors to improve power factor and operational efficiency.

Construction wise the PM motors are classified into Interior mount permanent magnet synchronous motor(IPMSM), surface mount permanent magnet synchronous motor(SPMSM) and spoke type- permanent magnet synchronous motor. In all the Permanent magnet motors there are issues of ripples in the torque and cogging torque. Cogging torque is one of the torque components produced by interaction of angular variations in the magnetic flux of the rotor and magnetic reluctance of the stator.
Here, in this paper the various methods and techniques are proposed for the PM motors optimization. These methods and techniques are useful for the designers to reduce torque ripple and cogging torque during design stage to attain smoothness in output torque.

2. Permanent Magnet Synchronous Motor
PMSM are having advantages of high torque to volume ratio, high efficiency, fast dynamic response, good power factor and high torque to current ratio when compared to induction motor. So the PMSM are being used in servo drives, automatic drives, home appliances, electric vehicles and industrial tools. The important components of torque in PM motors are Reluctance torque, Mutual torque and cogging torque. Improvement in the quality of output torque can be achieved by undergoing reduction in ripples of mutual torque as it is related to back EMF.

The issues of ripples in the torque can be addressed from design side and control side. From design point of view, the effects of some design parameters like slot number combinations, poles combinations[2],skewing in slots, magnetization distribution, magnetic pole to pole pitch ratio of magnets, slot number, magnet thickness, air gap length, width of slot openings, pole number combinations have to be quantified. Many techniques such as genetic algorithm, bat algorithms, Maxwell equations, step skewing, limiting misalignment are used for optimization in initial design to improve torque quality.

2.1. Maxwell equation
Maxwell equations based analytical method to reduce torque ripple in PMSM gives reliable results for both initial design and optimal design. Finally it is found that the slight differences between FE calculations and analytical does not have impact on optimization results [3].

2.2. Genetic Algorithm
The genetic algorithm based FEM to optimize the PM shapes can reduce the maximum cogging torque to about 40.7% of actual maximum cogging torque produced by motor with general PM shapes and it is found that the computation time of this genetic algorithm based FEM is reduced to 0.017% of computation time required by general FEM[4].

2.3. Step Skewing
The motor design having fractional slot winding is found to have lower torque ripple and lower cogging torque with reduction of average torque. The step skewing technique reduces torque ripple along with reduction of average torque when compared to straight magnet motor. The step skewing magnet technique requires careful selection of magnet skewing angle and step number. If the skew angle is equal to half the optimum angle two steps are required for skewing. If skewing angle is equal to optimum skewing angle three steps are required for skewing to obtain reduced torque ripple. Here the increased number of steps leads to increased average output torque along with reduction in torque ripples[5].

2.4. Limiting manufacturing misalignment
Usually manufacturing of motors are carried out in bulk production. The manufacturing tools required for mass production has to be calibrated or replaced after particular period as prescribed by manufacturing tolerances. The expectation of accurate tolerance leads to increase in manufacturing cost and consumption of time. So the analysis of manufacturing tolerance has to be performed to maintain a predefined cogging torque range. Air gap eccentricity is found to be normal mechanical misalignment caused by manufacturing of end shields and centre rings of stator housing. Simulations are carried out for pole number ranges from 4 to 10 and slot number combinations ranges from 9 to 27 to find maximal allowable value for manufacturing misalignment to maintain prescribed cogging torque range[6].
3. Interior Permanent Magnet Synchronous Motor

IPMSM is having advantages of making use of magnetic torque along with reluctance torque due to presence of difference in magnetizing reactance of direct axis and quadrature axis. The presence of high value of quadrature axis inductance when compared to direct axis inductance makes the IPMSM to possess behaviours of both SPMSM and conventional synchronous motor. The IPMSM is used as power train in hybrid electric vehicles, compressors and in various home electrical equipments because of its wide speed range and high power density when compared to SMPMSM. The constant torque or average torque and pulsating torque producing ripples are two components of total resultant instantaneous torque of IPMSM[7]. The techniques to solve issues of IPMSM are

- Optimal shape design
- Design optimization method
- Topology optimization method
- Pole arc to pole pitch ratio
- Continuum sensitivity analysis
- Level Set method

3.1. Optimal shape design

The reduction of ripples in the torque of interior permanent magnet synchronous motor is achieved by approaching optimal shape design and it has been found that there are three origins responsible for the production of pulsating torque. They are

- Distribution of flux density in air gap is non-sinusoidal which results with production of field harmonic torque.
- Reluctance existing in between stator slots and permanent magnet produce detent torque.
- Variation in self-inductances of the phase windings produces reluctance torque under various load conditions.

![Interior Mounted Permanent Magnet Motor](image)

**Figure1.** Interior Mounted Permanent Magnet Motor

It is found that the radius of the holes and optimal location of holes suppress the torque pulsation for many loads. The optimal shape design is done by undergoing drilling to produce cycled type holes in the core of the rotor. In order to develop an accurate model, electrical equations and differential equations governing the stator windings and mechanical motion of rotor are related with field equations of magnet. Finally it is concluded that the optimized motor has low value of pulsating torque and high value of average torque.
The applications which require high level of accuracy in speed and position are disturbed due to torque ripples. In this IPMSM, d-axis current is utilized to make use of reluctance torque and it is concluded that when speed increases the requirement of d-axis current increases[8] and it is found that the average output torque is improved by 20% and ripples of torque is reduced from 84.6% to 48.1% by using harmonic current injection by making comparison with torque generated by sinusoidal current.

The two important approaches for reducing pulsating torque is based on current control and machine design[9]. Although the current control based approaches are found to be effective in many applications, the IPMSM is vulnerable to increase in current produced ripple in the output torque if the motor is not properly designed. So the importance is given to machine design approach in which coordination has been created in between numbers of slots per phase per pole of stator and rotor with many flux barriers layers for minimization of cogging torque on the basis of analytical insights.

Table 1. Summary of Issues and Techniques used for Interior Permanent Magnet Synchronous Motor.

| Motor types | ISSUES | TECHNIQUE USED | PERFORMANCE |
|-------------|--------|----------------|-------------|
| IPMSM       | Non sinusoidal distribution of air gap flux density | Drilling cycled type holes in the rotor core[7]. | Low pulsating torque and higher average torque. |
|             | Variation in self inductances |                  |             |
| IPMSM       | Back EMF of IPMSM contains much harmonics which leads to ripples in torque | Harmonics injected current[8] | Average torque increased by 20% and torque ripple decreased from 84.6%. |
| IPMSM       | Torque ripples | FE based Monte Carlo Optimization method [9] | Torque ripple reduced to less than 5% |
| IPMSM       | Cogging Torque | Notches[10] | Reduction of cogging torque is about 87% |
| IPMSM       | Magnetic Energy Variation | Topology Optimization Method[11] | Minimized energy variation without deterioration in back EMF |
| IPMSM       | Effects of back emf and torque ripples | Pole Arc to Pole Pitch ratio[12] | Improved Saliency |
| IPMSM       | Cogging Torque | Continuum sensitivity analysis[13] | Determined minimum possible level for cogging torque |
| IPMSM       | Torque Ripple | Level Set Method[15] | Less material usage |

3.2. Design Optimization Method
A shape design optimization method using unconstrained minimization technique in combination with FEM is used to reduce the cogging torque of IPMSM[10]. In this method, the holes of the rotor core are
chosen as design parameter for trial and error method which is time consuming one and it is not cogging torque analysis. Hence, to compensate these drawbacks, the analysis method is preferred. In this analysis method, the analysis of cogging torque has been done by space harmonic analysis in combination with calculation methods of cogging torque. So this method is called novel cogging reduction method in which the design parameter pole face of rotor core is designed and optimized using several notches avoiding other design parameters such as pole ratio, teeth shape etc. Finally, the FEM is used to optimize the position and location of notches in rotor core because the rotor core is having extreme saturation to achieve the reduction in cogging torque.

3.3. Topology optimization method
A topology optimization method is applied to stator design using reaction diffusion equation considering two design objectives, one is to reduce cogging torque directly and another is to minimize the magnetic energy variation against rotor movement to achieve reduction in cogging torque without facing deterioration in back Emf\[11\].

3.4. Pole arc to Pole pitch ratio
By considering saliency ratio and pole arc to pitch ratio the torque characteristics has been analysed. By analysing the effects of back Emf waveform and torque ripple, the optimal value for pole arc to pole pitch ratio is decided. Due to insertion of PM in duct structure the saliency problem arises. So by designing PM width and duct shape, the IPMSM can be designed with high saliency to work with wide constant power region\[12\].

3.5. Continuum sensitivity analysis
The continuum sensitivity analysis concurrence with the FEM can be used to get an optimal nature for rotor to determine minimum feasible level for cogging torque. During this procedure of continuum sensitivity analysis, the ad-joint system equation is not required to be calculated to save CPU time and efficiency and back EMF of original model are preserved while suppressing cogging torque\[13\]. From mass production point of view, IPM rotor type has gained more advantages than rotor with surface magnets.

3.6. Level set method
By utilising level set method and sensitivity analysis, the arrangement of a stator with low volume of material usage can be designed to reduce ripples in the torque for a specified range of stator current\[15\].

4. Surface mounted permanent magnet Synchronous motor
In SPMSM, the surface outward PMSM and surface inset PMSM are two types of arrangements of permanent magnets. In surface mounted PMSM, the PMs are mounted on the rotor surface with non-uniform air gap and in surface inset PMSM, the PMs are surfaced inset into the rotor to get cylindrical construction.
The techniques available to fix the issues of SPMSM are
- Global stochastic minimization method
- Conformal mapping
- Skewing
- Fourier series form of analytical solutions
- Maxwell Stress Sensor
- Techniques based on graph
- Notching Grove of Magnets
- Fourier Expansion and Energy Methods
- Magnetic pole segmentation
- Domain elimination algorithm
- Non-uniformly distributed teeth method.

4.1. Global stochastic minimization technique
In SMPMSM, the presence of torque ripple creates vibrations and noises. The cogging torque and ripple component created by harmonics in air gap flux density are two important contribution in the torque pulsation of SMPMSM[16]. Without separating these two different contributions, an optimization design problem comprising two step procedure of design can be followed. In first step, global stochastic minimization technique using one dimensional analysis of field is introduced to define the machine geometry with respect to minimum value of electromagnetic force harmonic components and maximum value of electromagnetic fundamental components. The magnet position within the pole pitch, pole pitch, magnet width, winding pitch and skew angle are design optimization unknowns. By using the first step of global stochastic minimization technique, the design optimization unknowns are determined. In second step, a FEM is utilized to use 2-D field analysis for calculating flux density distribution and for the cogging torque reduction. By mean of this analysis the width of the magnet is adjusted to reduce the cogging torque.

Some general procedure proposed to determine the cogging torque analytically are[17]:
- Determining the distribution of flux density in the ideal slot-less air-gap,
- Consideration of slot effects to derive the equations for absolute permeance of the air-gap,
- By undergoing modifications in the relative air-gap permeance for various positions of rotor, the cogging torque obtained from the air gap flux density distribution are computed.
Table 2. Summary of Issues and Techniques used for Surface Mounted Permanent Magnet Synchronous Motor.

| Motor types | ISSUES | TECHNIQUE USED | PERFORMANCE |
|-------------|--------|----------------|-------------|
| SMPMSM      | Cogging Torque harmonics occurrences in air gap flux density distribution | Global Stochastic Minimization Technique using One dimensional field analysis [16] | Reduced Cogging torque |
| SMPMSM      | Cogging Torque | Conformal Transformation [17] | Coenergy of air gap is used for cogging torque prediction |
| SMPMSM      | Torque ripples | Computation of various machine configurations and various magnet shapes using two dimensional FEA [18] | Reduced torque ripple is achieved without undergoing significant reduction in average torque |
| SMPMSM      | Changes in Cogging Torque and ripples of Torque. | Variation of ripples in torque under field weakening operation and variation of cogging torque in various magnet shapes are analysed | Reduced torque ripple due to higher optimum skew angle |
| SMPMSM      | Cogging Torque | Maxwell stress Tensor [19] | Achieved minimum cogging torque |
| SMPMSM      | Cogging Torque | Graph based approach [20] | Obtained optimum value for pole embrace |
| SMPMSM      | Cogging Torque | Energy methods [21] | Derived formula for Cogging torque |
| SMPMSM      | Cogging torque | Fourier expansion and Energy method [22] | Prediction of cogging torque |

4.2. Conformal transformation
Due to presence of large air gap in the surface mounted permanent magnet motors. This motor has undergone changes in tangential direction distribution of air gap for different value of radii. When slot opening are wide, the fields of the slots are having significant influence on the cogging torque. Finally prediction of cogging torque may be difficult. So a novel analytical approach is preferred for cogging torque prediction. So the field solution based on analytical approach is obtained using conformal transformation based on which a new analytical approach is derived for prediction of the cogging torque. Then as second step the co energy of the air gap and co energy of the slot regions are used for deriving the cogging torque. Finally it is concluded that the cogging torque curves showed good agreements with the results of FEA.
4.3. Skewing
The ripples of torque can be minimized by perfect selection of shapes of magnet in permanent magnet synchronous machine using step skewed magnet. Machines having higher optimum skew angle can realize the effectiveness of torque ripple reduction using skewing. Even after undergoing skewing in the magnet, the ripples in torque of few motor may increase if the design of magnet shape is not done in proper way[18]. This analysis is cumbersome due to machine saturation creating non linearities at higher torque levels. The machines having higher optimum skew angle of 12 slots/8 pole and 9 slots/6 pole combination machines are found to be effective in torque ripple reduction using skewing if magnet shapes are selected carefully.

4.4. Fourier Series form of Analytical solutions
The Analytical field solution obtained in Fourier series form is the flexible tool to predict motor performance [19]. While applying this to optimization in machine design it becomes crucial as it needs several repetitive computations in getting optimal solution and it is time consuming one. Three basic ways in the analytical computations of cogging torque are
- Computation of torque in terms of derivation of co-energy from the air-gap.
- Integration of lateral forces on slot sides.
- Integration of Maxwell stress tensor from tangential component of the air gap.

4.5. Maxwell stress tensor
This analytical method of calculating the cogging torque using Maxwell stress tensor requires details regarding tangential components and radial components of air gap flux density. The concept of complex relative air-gap permeance makes both tangential components and radial components of the air-gap flux density to be calculated to get closed form solution. But the computation of complex relative air gap permeance needs presence of non-linear equations which do not give the solution to overall cogging torque analytically.

The relative air gap permeance which is very complex derived from conformal mapping for calculation of radial component and tangential component of air gap flux density has been used to determine the tangential component integration of Maxwell stress sensor. This analytical solution has been applied to four pole surface mounted permanent magnet synchronous motor and results are compared with FEA. Finally it is concluded that this analytical solution is capable of determining the optimal magnetic arc to pole pitch ratio which results lower value of cogging torque.

4.6. Technique based on graph
To find the optimum value for magnet pole embrace, an analytical based graph technique is preferred [20]. Existing solutions to find optimum embrace is considering only the normal fields. In graph based technique, the expression of Fourier coefficient of cogging torque helps to get optimum value for pole embrace by considering tangential component of magnetic field. This method can be applied only to SMPMSM having integer number of slots per pole. The important parameters to be taken into account to reduce cogging torque are number of poles (2p), pole embrace (β), number of slots (Q) and stator slot opening width (ξ). A graph based approach is proposed to determine optimum value of pole embrace to achieve minimum cogging torque.

4.7. Notching Grove of Magnets
The effect of notching grove of magnets on cogging torque has to be studied to improve the working accuracy. The energy methods are used to derive formula for cogging torque under arbitrary groove [21]. The formula for the torque under double rectangular grooves and single rectangular grooves are derived on the basis of Fourier expansion and equivalent structure of rectangular grooves. FEM is used to analyse the cogging torque of SMPMSM under double and single groove of having different sizes to get optimal solutions.
4.8. Fourier expansions and energy method
By considering Fourier expansions and energy method[22], the cogging torque of PM motors with skewed type of rotors is predicted and effects of factors like skewing, combination of slot number and pole number are investigated. Following are the assumptions made during analysis.
- The dimensions and performances are considered as same for all permanent magnets.
- Slots are considered to be rectangular.
- The iron permeability is infinite.
- The distribution of magnetic field in air gap is one dimensional.
- Radial magnetization is considered for all permanent magnets.
- Permanent magnet permeability is considered as same as air permeability.

Followings are list of conclusions drawn.
- The more the periods, the lower the maximum value of cogging torque.
- If the skewing of slots is equal to the pitch of tooth, the cogging torque is found to be reduced.

4.9. Magnetic pole segmentation
The arced magnet pole can be segmented in radial magnetization into several arced magnet blocks to achieve reduction in cogging torque along with reduction of eddy current loss[23].

4.10. Domain elimination algorithm
The analytical method can be used to determine feasible region for the pole arc coefficient. To find best pole arc coefficient value, the FEM combined with domain elimination algorithm(DEM) for optimization can be preferred to realise the computing time reduction and cogging torque reduction[24].

4.11. Non uniformly distributed teeth method
Novel method of non-uniformly distributed teeth can be used to reduce cogging torque in which the difference can be realised in between width of one tooth tip to width of another tooth tip. In this method, both analytical and numerical methods are used to determine width ratio of teeth for cogging torque reduction and the effectiveness of this method is proved by FEM[25].

5. Spoke type- permanent magnet synchronous motor
The spoke type PMSM is not familiar when compared to other types of PMSMs[26]. The spoke type IPM magnet has advantage of good performances in the form of high efficiency, high torque density and power density arising from concentrated flux of rotor magnets. The demerits of spoke type IPM motor is having high level of distortion in air gap flux density distribution, which leads to rise in the cogging torque, the rise of ripples in output torque and the distortion in back Emf [27].

![Figure 3: Spoke type permanent magnet rotor](image)
The torque pulsation in this type of PMSM produce acoustic noise, unacceptable vibrations, power and speed control, running failure and some performance degradation. Therefore many techniques are available to address issues of spoke type-PMSM. They are

- Skewing and notching.
- Pole design.
- Rotor structure utilizing reluctance and magnetic torque.
- Novel rotor structure.
- Calibration coefficient.
- Segmentations in servo applications.
- Combination of lumped magnetic circuit model and sub domain model.
- Maximum torque control.
- Goodness factor.

5.1. Skewing And Notching

To achieve torque pulsation reduction in spoke type interior PM motor, the skewing and notching configurations [28] can be used. The fundamental concept in skewing is to impact interaction between the magnets of rotor and the stator spaces. Skewing can be realized on either the slots or the magnets.

To improve skewing, the skewing is done in axial direction symmetrically in rotor PMs within one magnet pole pitch to minimize ripple pulsation in torque and to remove unbalanced axial electromagnetic forces maintaining maximum available torque intact. Another method is sinusoidal PM shaping method. Both methods are using stepped rotor PM schemes and effective in reducing cogging torque by above 60% and reducing ripples in torque by above 40% and it was found that selection of PM step number in both methods should be in such a manner to reduce magnetic saturation arising from axial interactions[29].

Disadvantages of skewing are[30]

- increasing complexity of motor dimensions,
- increasing leakage inductances and stray loss,
- Reduced torque output.

To improve back Emf and flux density, pushing assistant magnet and sub assistant magnet can be inserted in existing spoke type rotor in which expensive Nd-PM can be replaced with relatively cheap Fe-PM to achieve reduced production cost [31].

Notches are nothing but dummy slots in both stator and rotor. This technique reduces cogging torque effects by reducing flux in the air gap. By changing the grades of magnet, the magnet flux density can be lowered to reduce air gap flux. Notching in rotor pole can decrease the variations in amplitude and can increase the periods of variations to give reduced peak value of the cogging torque.

Table 3. Summary of Issues and Techniques used for Spoke Type Permanent Magnet Synchronous Motor.

| Motor types       | ISSUES          | TECHNIQUE USED                  | PERFORMANCE                              |
|-------------------|-----------------|---------------------------------|------------------------------------------|
| Spoke-type PMSM   | Torque pulsation| Rotor pole skewing and notching techniques[26] | Achieved minimization of pulsation in torque |
| Spoke-type PMSM   | Torque ripple   | Response surface method , kriging method and steepest descent method[27] | Minimization of torque tipple is realised |
5.2. Pole Design Method

In IPMSM having concentrated windings, the understanding of sinusoidal back EMF is complex. The pole design method combining response surface method, kriging model and steepest descent method are preferred to minimize ripples in the torque of the spoke-type IPMSM having concentrated windings.

In spoke type PM motors, variation of core shape and insertion of barriers in rotors are done to reduce external field that leads to partial demagnetization of permanent magnet and cogging torque effect. With this rotor shape, another part of rotor pole is varied to create better sinusoidal distributed air gap flux density[32]. In this process response surface method and steepest descent method are applied to improve the level of convergence.

5.3. Two different rotor structure utilizing reluctance and magnetic torque

Two different rotor structure of spoke type PMSM using ferrite PMs are being used in automotive applications. One rotor structure is utilizing magnetic torque and another one is using reluctance torque. Using 2D-FEA, it is revealed that the rotor utilizing reluctance torque is having superior performance in characteristics when compared to characteristics of rotor structure utilizing magnetic torque in various aspects like torque ripple, average maximum torque, resistant to irreversible demagnetization[33].

5.4. Novel rotor structure

Four iron cores along with flux barriers are constructed in the rotor to ensure flux path reducing air gap field harmonics. All the iron cores are having two fillets each to resist flux leakage. By doing casting using beryllium copper, the shaft sleeve is constructed. Beryllium copper combines the properties of both high strength and nonmagnetic to ensure good mechanical strength. The magnets have been kept in slots between four iron cores and the magnets in slots are hold by wedges. Finally key parameters optimized by Taguchi method revealed that this novel structure is superior when compared to conventional one in aspects of back EMF, flux leakage, mechanical strength and output torque[34].

5.5. Calibration coefficient

The magnetic leakage flux in axial direction produced due to flux concentration of rotor structure is affecting the performance of motor like torque, efficiency etc. Calibration coefficient is used to evaluate
magnetic leakage flux in axial direction using 2D-FEM avoiding 3D-FEM. Because the usage of 3D-FEM needs more time to carry out analysis[35].

5.6. Segmentations in servo applications
The effect of variation of mechanical air gap in rotor segment tip is analysed by undergoing segmentation with different rotors in servo applications and revealed that the issues like torque quality, good THD levels are successfully addressed which are not done by mathematical approaches[36].

5.7. Lumped magnetic circuit mode (LMCM) and subdomain model
The motor has been divided into various subdomains such as inner sector magnet, airgap, rotor slots, outer sector magnet, stator slot and shaft. The Laplace and poison equations are solved to get back Emf, torque, magnetic field considering boundary conditions. Here analytical model combining both LMCM and subdomain model with consideration of tooth saturation is used to predict the performance of spoke type PMSM with trapezoidal magnet[37].

5.8. Maximum torque control
A design process possessing genetic algorithm and radial basis function are used to evaluate cogging torque and revealed that the torque and efficiency increases when compared to initial model due to increase of saliency ratio. Here, the maximum torque control is used in constant power and constant torque area to reduce cogging torque. The maximum torque control is the combination of maximum torque per ampere control and flux weakening control in constant torque and constant power area respectively[38].

The field domain is subdivided into eight subdomains such as inner and outer magnetic bridge, fan shaped inner magnet, fan shaped outer magnet, shaft, slot, slot opening and airgap. It is revealed that field distribution prediction is possible for magnetic bridges and rectangular magnets[39].

5.9. Goodness factor
Goodness factor is nothing but ratio of total flux in between various motor configurations. It is used to analyse two different rotor arrangements with same diameter of rotor, stack length, materials, magnet usage and airgap length to increase accuracy of prediction of performances. Considering the effects of axial leakage and bridge leakage, the good factor is used to show the improvement of output density of the spoke type PMSM when compared to output density of conventional SPMSM[40].

6. Conclusion
This study describes the various aspects of analysis, prediction and reduction topologies for decreasing cogging torque and torque ripples in various Permanent magnet motors. Pole arc to pole pitch ratio adjustment in IPMSM, Non-uniformly distributed teeth method in SPMSM and pole design and notching in spoke type PMSM are found to be effective in reducing ripples in developed torque and minimizing the cogging torque. Design trade-offs are available in between cogging torque, torque ripple, efficiency and magnet volume for PM motor. Based on that design trade-offs, designer can choose appropriate rotor configuration and magnet volume to get optimum performance in PM motors.

7. Future Scope
In Future, it is necessary to design and implement an PMSM by incorporating aforementioned techniques to reduce ripples in developed torque and cogging torque to make motor as best competitor to induction motor.
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