Advice for New and Student Lecturers on Probability and Statistics

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Journal of Statistics Education Volume 14, Number 1 (2006), ww2.amstat.org/publications/jse/v14n1/zacharopoulou.html

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Key Words: Key Words: Active learning; Contrasts; Problem Solving; Statistical Reasoning; Student Participation; Teaching Methods.

Abstract

Lecture is a common presentation style that gives instructors a lot of control over topics and time allocation, but can limit active student participation and learning. This article presents some ideas to increase the level of student involvement in lecture. The examples and suggestions are based on the author’s experience as a senior lecturer for four years observing and mentoring graduate student instructors. The ideas can be used to modify or augment current plans and preparations to increase student participation. The ideas and examples will be useful as enhancements to current efforts to teach probability and statistics. Most suggestions will not take much class time and can be integrated smoothly into current preparations.

1. Introduction

Elements of probability and statistics are taught to students at all educational levels, from grade school (e.g., Siegel 2004) to high school (e.g., Peck 2002, College Entrance Examination Board 2004) to college (Loftsgaarden and Watkins 1998, and references therein). Reform of curriculum at the undergraduate level (Moore 1997, Garfield, Hogg, Schau, and Whittinghill 2002, and references therein) and development of programs in high schools emphasize active learning and experiential learning with data (Moore 2000). An ever-increasing number of high-quality resources, including textbooks, teacher guides, websites (e.g., Carver and Peters 2004, Larsen 2004c), and computational tools are available to enable active learning of the subject. Increasing active learning when teaching probability and statistics can be challenging for a number of reasons, including inertia, inexperience, personality, and planning.

Lecture, at all levels, is a very popular presentation style. There are reasons why it is so as will be elaborated later. Some styles of lecture, however, do not encourage student participation and critical thinking in class. This article provides some examples and suggestions for increasing active learning within the lecture format.

The comments in this article are based on the author’s experience as a Senior Lecturer from 1999 to 2003 at The University of Chicago during which time he observed numerous graduate student instructors in class and in practice sessions (microteaching; Derek Bok Center 2002) and wrote notes that were discussed with students. The author also has been influenced by his own teaching experiences at the master’s and undergraduate level, including supervising course assistants (1 to 12 at a time, enrollments 13 to 205 students) and teaching parallel sessions with graduate student and new faculty instructors. The author has interacted with high school instructors as an exam reader and consultant (1999-2003) and summer weeklong institute leader (2001, 2002) for the College Board’s AP statistics exam.

The article was written after reviewing notes from observations of and interactions with graduate students at The University of Chicago. The suggestions, therefore, should be particularly appropriate for new and adapting college instructors, including graduate student instructors. They should also be relevant for many high school teachers who deliver lectures. Hopefully middle school teachers will find the examples and ideas of some use, as will researchers planning experimental comparisons of teaching and learning styles. No true experiment was conducted. Evaluations of graduate instructors and practices are based on the author’s experience without outside confirmation. Grade distributions and student evaluations are confounded with instructor characteristics and are not used to rate instructors or practices.

Section 2 discusses active learning and lecturing. Section 3 presents examples involving problem contrasts and questions. Problem contrasts are created when a problem is asked in similar, but different ways. Section 4 illustrates the value of outlines and writing more details while lecturing. Section 5 focuses on pictures and diagrams for problem solving and comments on modes of presentation. Examples are presented throughout these sections. A summary is given in Section 6.

2. Lecturing and Active Learning

Not all lecture styles lead to active learning, and it may be that no lecture style can promote active learning as much as do other forms of presentation. However, by incorporating the suggestions in this paper, instructors can give lectures that offer enhanced opportunities for active
learning by students without significantly changing lesson plans or time allocation in class. Section 2.1 discusses the meaning of active learning. Section 2.2 reviews reasons for lecturing.

2.1. Active Learning

Active learning is concerned with what students do in and out of class and how they gain insight into main ideas, technical details, and intuition. It is contrasted to passive learning in which information is presented to students who are expected to absorb it through contemplation and memorization of notes and textbook material. Students can learn actively by working problems for themselves, thinking about concepts to form their own summaries and statements, and explaining and discussing ideas with others (see, e.g., Garfield 1993). Working in small groups, learning through interactive case studies, and in-class problem solving are examples of active learning strategies. Homework assignments that require students to provide explanation in their own words also encourage active learning by forcing students to do more than copy phrases. Schwartz and Martin (2004) evaluated active learning in a ninth-grade class and found pedagogical advantages to discovery-based instruction. Kvam (2000) found advantages in terms of retention for average students in a college-level engineering statistics course. Magel (1996) and Lackritz (1997), among others, have discussed participation in large introductory statistics classes.

A traditional lecture format in which students take notes based on a presentation by an instructor is considered passive. Although students write notes, it is believed that most students do not think critically while writing. Some students do not even capture whole ideas or explanations. Rather, they copy what is written on the blackboard, but omit many explanations given verbally. An instructor might ask a question now and then. If students expect the instructor to provide the answer, however, then the pause after a question mainly acts as a chance to catch up with writing or to rest. Thus, a student in a classroom situation in which an instructor “gives knowledge” to students through lecture generally is not in an active learning situation. Since active learning generally is a desirable goal due to its positive impacts on retention and understanding, why, one can ask, lecture?

2.2. Why Lecture?

Lecturing is a very common form of presentation (Garfield, et al. 2002), because the instructor strictly controls the organization of and time allocated to topics. The instructor can prepare everything ahead of class and include interesting (to the instructor) examples. The material can be technically challenging and detailed. A well-prepared lecture is unlikely to be judged unprofessional. Members of many groups, as evidenced by the frequency of lecturing, find the advantages of lecturing compelling. These groups, based on the author’s experience, include graduate students with little teaching experience and possibly English as a second language, high school teachers or college or university instructors (including mathematics instructors) new to teaching probability and statistics, experienced instructors who are used to a traditional lecture format, instructors short of time to prepare participatory activities, and others for whom interacting with but not necessarily lecturing to large groups is intimidating. In other words, lecturing is attractive to many people who teach. Undergraduate students generally have encountered lecture and it is not unexpected. Even if one minimizes the amount of time spent in traditional lecture-style presentation, it is necessary and convenient now and then to give demonstrations and explanations to a class.

The examples and ideas of Section 3, Section 4 and Section 5 were suggested to and tried by graduate student instructors who had the tendency to prefer a lecture organization for their classes. Most instructors who made an effort were able to successfully incorporate some of the ideas into their teaching preparations and usual presentations and maintain desired time allocations to topics. Other sources for general advice on lecturing can be found in Knight (2002), Derek Bok Center (2004), and Mosteller (1980).

3. Problem Contrasts and Asking Students Questions

For many graduate student instructors at The University of Chicago from 1999 to 2003, lecture was the default mode of presentation in classes and problem sessions. Certainly these students were not unique in this regard, and there were some exceptions to the rule. For most graduate student instructors, expanding the use of problem contrasts and planning the use of questions helped lecture be more effective and involve more student participation without requiring a major change in lecture planning and preparation. The author presented previously some of the examples below (Larsen 2003) and has discussed some of them with high school teachers of statistics.

3.1. Problem Contrasts

Problem contrast can be very effective in helping students learn how to use and interpret formulas by examining results of alternative scenarios. It also can provide students a basis in experience for making general statements about what influences results. In terms of time planning in a course, effective use of contrasts allows one to present more problems in a shorter period of time than otherwise might be possible. At a minimum, students rehearse computations based on formulas. Ideally, the instructor will engage the students in a discussion of how and why results change when the problem details change. The instructor also could allow students a small amount of time (3-5 minutes) to work individually or in small groups, perhaps based on where students are sitting, on multiple versions of problem contrasts. A representative of each group could report an answer to the class. The following examples in five areas of introductory statistics illustrate the idea of problem contrasts. Further examples can be found in Larsen (2004a) and Larsen (2003).

- **Contrast 1: Change counting conditions.**

  In learning permutations \( P^x_n = n! \frac{n}{(n-r)!} \) and combinations \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \), student instructors typically explain formulas, then work several numerical examples. For example, the number of ways to select 2 books from a shelf of 12 without replacement but in order is \( P^2_{12} = 12 \cdot 11 = 132 \), whereas the number ignoring order
of selection is \( C_2^{12} = \frac{12!}{2!(11)!} = 66 \). Contrast can be used to examine what influences these calculations. How do results change if you want to pick 3 or 4 or 5 books? Answers: \( P_3^{12} = 12(11)(10) = 1320 \), \( P_4^{12} = 12(11)(10)(9) = 11880 \), and \( P_5^{12} = 12(11)(10)(9)(8) = 95040 \); \( C_3^{12} = \frac{12!(1)!}{3!(2)(1)!} = 220 \), \( C_4^{12} = \frac{12!(1)!}{4!(3)(2)(1)!} = 495 \), and \( C_5^{12} = \frac{12!(1)!}{5!(4)(3)(2)(1)!} = 792 \).

How do results change if the shelf holds 10 or 15 books? Answers: \( P_2^{10} = 10(9) = 90 \) and \( P_2^{15} = 15(14) = 210 \); \( C_2^{10} = \frac{10!(1)!}{2!(1)!} = 45 \) and \( C_2^{15} = \frac{15!(4)!}{2!(1)!} = 105 \). Students can work independently, with a neighbor, or in groups to solve the additional problems. By asking students to compute results for combinations and permutations with different numbers of books and asking them to summarize the differences, students can realize for themselves how fast factorials \((k!)\) grow with \(k\) and how much of a difference paying attention to order makes \((\frac{P_n^k}{C_n^r} \neq k!)\). In the author’s experience, students generally are surprised by how fast factorials grow and the impact that considering order has.

The Hypergeometric distribution provides another opportunity for the use of contrasts regarding combinations. Suppose of 100 plots of nonfederal land in the U.S. excluding Alaska, 27 are forestland (see NRCS 2004 for other percentages). In a sample of 10 plots, what is the probability that only one plot is forested? The answer is 0.151, which is computed as \( \frac{C_{27}^1 C_{73}^{9}}{C_{100}^{10}} \), where \( N = 100, r = 27, n = 10, \) and \( y = 1 \). Students could be asked to find the probability of other events in this scenario and in a scenario with different values of \( r, n, \) and \( N \). Larsen and Marx (1986; page 94) provide some additional examples. The game of Scrabble has 54 consonants and 44 vowels on game tiles. Seven tiles are distributed initially. What is the chance of receiving all consonants? After this calculation, students can be asked to compute the chance of 1, 2, or 3 consonants and 5, 6, or 7 vowels. Suppose two blank tiles are included in the original population and are neither consonants nor vowels. How do the probabilities change? The table below gives the probabilities to three decimal places for the number of consonants. Further examples of the type of contrast are included in Larsen (2004b).

| Number of consonants | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|----------------------|---|---|---|---|---|---|---|---|
| Probability, 98 tiles| 0.013 | 0.082 | 0.216 | 0.303 | 0.243 | 0.112 | 0.028 | 0.003 |
| Probability, 100 tiles| 0.011 | 0.074 | 0.204 | 0.300 | 0.253 | 0.123 | 0.032 | 0.003 |

- **Contrast 2: Change sets.**

Elementary set theory is used in descriptions of sample spaces and basic probability questions. Suppose among a population of students 40% own a personal computer (PC) and 25% have a PC but not a stereo. What percent have both? Answer: 40-25=15%. Suppose 70% have a PC, a stereo, or both. What is the probability that a randomly selected student has either a PC or a stereo but not both? Answer: 70-15=55%. Are having a stereo and having a PC independent? Answer: No, 15% have both, but 0.45 (0.4) = Pr(scream) Pr(PC) = 0.18. This is the extent of problems usually presented by student lecturers. Even if students are taking notes, it is unlikely that they all are able to think about the problem fast enough to realize the answers. One possible contrast is to change one or more percentage and ask students to repeat the questions. Another contrast is to ask the questions about not owning a PC. Too often students know how to do one type of problem, but cannot adapt their knowledge to slightly changed circumstances (Chance 2002).

- **Contrast 3: Change probabilities.**

This example is similar to the last, but concerns conditional probability and Bayes’ theorem. Often a medical testing example is used: Pr (disease)=0.10, Pr(test positive given disease)=0.96, and Pr(test negative given no disease)=0.94. What percent of randomly selected patients test positive? Answer: 0.10(0.96) + 0.90(1-0.94) = 0.096 + 0.054 = 15, or 15%. What is the probability of having the disease given the test is positive? Answer: 0.10(0.96)/0.15 = 0.096/0.15 = 0.64, or 64%. Student lecturers often stop after presenting formulas and answering these two questions. Time pressure is partially to blame. Typically only one fifty-minute college class per quarter (besides a problem session) is available for this topic. A handout with the statement of formulas and word problems or a worksheet would help with efficiency. Important additional questions, such as the probability of having the disease given a negative test, should be asked. Important contrasts would involve changing the prevalence (Pr(disease)=0.01 or 0.40), sensitivity (Pr(test +|disease)=0.99 or 0.90), and specificity (Pr(test -|no disease)=0.80 or 0.999). Students could be asked to work one or more of these variants, alone or in small groups, to discover the role played by each of these factors.

- **Contrast 4: Change parameter values.**

The Binomial\((n, p)\), Geometric\((p)\), and Poisson\((\lambda)\) distributions are three examples of discrete distributions that involve parameters. The
Normal(μ, σ) distribution is the primary continuous example in introductory statistics that involves parameters. It is common for instructors to state assumptions underlying the definition of the random variable, motivate the formula for calculating probabilities, state the mean and variance of the random variable, and provide a numerical application. What can be contrasted? The parameter \( p \) (0 < \( p \) < 1) in the Binomial distribution is the probability of success on one trial. Suppose two baseball players have batting averages of 0.310 and 0.270, respectively. If each has ten attempts (at bats), assuming the attempts independent of one another and \( p \) is equated to the batting average, which player is more likely to get 0 hits? 1 hit? 2 hits? at least 5 hits? The table below gives the probabilities (to three decimal places) for a number of hits for both players.

| Hits | 0   | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|------|-----|----|----|----|----|----|----|----|----|----|----|
| \( p = 0.310 \) | 0.024 | 0.110 | 0.222 | 0.266 | 0.209 | 0.113 | 0.042 | 0.011 | 0.002 | 0.000 | 0.000 |
| \( p = 0.270 \) | 0.043 | 0.159 | 0.265 | 0.261 | 0.169 | 0.075 | 0.023 | 0.005 | 0.001 | 0.000 | 0.000 |

A lecturer could calculate one or two scenarios for one player and then ask the students to work on the others. The students then could quantify the anticipated difference in performance of the two players. If there are 8 at bats instead of 10, how do probabilities change? How about for 12 at bats? The idea here is that every type of calculation that is done can be repeated a few times in order to give students practice applying formulas and to develop an intuition as to how the parameter affects the answer. At the end of class, the instructor could ask students to state in their own words how the parameter \( p \) for a given \( n \) and the number of trials \( n \) for a given value of \( p \) affects the likely outcomes. Insight into how changes affect outcomes is an element of statistical reasoning. See Garfield (2002) for other suggestions ways to improve the teaching of statistical reasoning.

- **Contrast 5: Change values in inference formulas.**

The principle of examining the impact of various factors on results can be applied to statistical inference. Students can apply formulas once they are explained and determine for themselves what affects confidence intervals and hypothesis tests. Suppose twenty college-age men report an average score of 22 and a standard deviation (SD) of 4.5 points on a self-esteem scale of 0-30, whereas thirty women have an average of 20 and a SD of 5.7. Are the means between similar groups of men and women clearly different? The answer requires a confidence interval or hypothesis test. Answer: Assuming the populations have equal standard deviations, the pooled standard deviation estimate is \( \sqrt{\frac{(20 - 1)4.5^2 + (30 - 1)5.7^2}{20 + 30 - 2}} \) and the 95% confidence interval (CI) for the mean difference is \( (22 - 20) \pm 2.01(5.26) \sqrt{\frac{1}{20} + \frac{1}{30}} = (-1.03, 3.03) \), which includes zero. Therefore, the means are not statistically significantly different. What if the averages were 22.5 and 19? Answer: The 95% CI is \( (22.5 - 19) \pm 2.01(5.26) \sqrt{\frac{1}{20} + \frac{1}{30}} = (0.43, 6.33) \), which does not include zero. Therefore, these differences are statistically significantly different. What if the sample sizes were twice as big? Answer: The pooled standard deviation estimate is still 5.26 (\( \sqrt{\frac{(40 - 1)4.5^2 + (60 - 1)5.7^2}{40 + 60 - 2}} \)) and the 95% CI, using the original mean values, is \( (22 - 20) \pm 1.98(5.26) \sqrt{\frac{1}{40} + \frac{1}{60}} = (-0.13, 4.13) \), which barely includes zero. An instructor could conclude these series of calculations with a summary of how different factors (mean difference, sample size, and standard deviations) affect inference for the mean difference.

### 3.2. Asking Questions

The second area in which most student instructors can improve is in terms of asking questions of students. Depending on personality, some first-time instructors naturally ask many questions, but in the author’s experience most do not. The basic practice of asking if students have questions is often problematic, because not enough time is allowed for students to think about lecture material and formulate a question. The tendency to move quickly through material and to cover a lot before asking for a question also contributes to the failure to elicit questions from students. Planning questions means deciding when in lecture and what to ask students. Waiting too long before asking a first question in a class tends to increase the reluctance of the students to participate. Waiting a long time between questions tends to increase the inertia against speaking in class. At the same time, it is necessary to provide adequate background and introduction to an example so that the students can formulate reasonable answers. It also is necessary to accomplish the goals of a class period, so there is a limit to the number of questions to be asked. In the author’s experience, instructors usually ask too few questions. A general or very sophisticated question will elicit little response. A very easy question also will decrease motivation to participate. In order to prepare students for participation, the instructor might need to tell students that a question is going to be asked and they will be expected to think about it and answer. Below are four examples of questions that many student instructors were able to use in their lectures.

- **Question 1: Ask about problem contrasts.**
The use of problem contrasts provides ample opportunity for asking students to answer substantive questions. After seeing the solution to a problem, working on similar problems, and comparing answers to neighbors, some students are willing to state their answers in class. It sometimes is helpful to prompt students to explain the method of solution. What did you write in the numerator? What did you write in the denominator? What is the next step? If there is a general principle that is illustrated by the problem contrasts, then students can be asked to try to summarize the experience.

- **Question 2: Ask a series of questions.**

  Graduate student instructors tend to ask very few questions on a single example. In many examples, however, there is the possibility of asking follow-up or continuation questions. Students might be more willing to answer additional questions once they understand the problem context. If a new context is introduced, students might remain hesitant to answer. A somewhat redundant question might help a student better gauge understanding and gain confidence if he or she can solve the problem. For example, suppose the example concerns predicting first year grade point average (GPA). One student might say that high school GPA should be a good predictor. Instead of ending the discussion here, questions could be asked about the range and distribution of GPAs, the strength of the association with first year GPA, variables that might lessen the relationship or be better predictors, and issues of sample design for collecting data. Once students start talking, they can produce several relevant ideas in a short time span.

- **Question 3: Ask for a statement of conclusions.**

  The mean of a Geometric($p$) random variable is $1/p$. The proportion $p$ could be for example the probability that a salesperson makes a sale (e.g., 10% or 0.10). Based on experience students can see that a low probability of success (small $p$) means that one can expect to need more attempts before the first success. Graphs of probability histograms ($p = 0.1$ versus $p = 0.5$, for example), simulations, and intuition all suggest that the mean should increase as $p$ decreases. Can a student state, or restate if appropriate, a conclusion concerning the relationship of the mean (and variance) to the parameter $p$?

  Students can be asked to make and support conclusions concerning, for example, factors that affect width of confidence intervals and power of significance tests. After examining formulas, contrasting numerical scenarios, conducting simulations, and making graphs of sampling distributions, what has been learned? If students are reluctant to make broad conclusions at the end of class, then asking them to make a specific statement based on one piece of evidence from the class period might elicit reader response. *Rumsey (2002)* discusses the importance of communication, such as the conclusions and statements that students could be asked to produce, in statistics. In regards to confidence intervals, delMas (2002) presents an extended discussion of assessing learning about confidence intervals that could be used to inform questions asked of students.

- **Question 4: Ask for evaluation of assumptions and data summaries.**

  In a data analysis, often several things can be said about a sample or pair of samples: center, spread, shape, outliers, etc. Which feature dominates? What is important? There are more opportunities to comment when comparing two or more groups. Which assumption of a model is least tenable? Why would, for example, two exam results not be independent?

  The use of problem contrasts and planned questions was not really difficult for student instructors at The University of Chicago to incorporate into their lectures. Generally it meant that lecturers had to take a few moments when preparing class to reflect on the main points and what their examples illustrated. Time usage was not greatly different than for student lecturers not using contrasts and planned questions. Occasionally, instructors trying to push through in strict lecture format were confronted with questions from confused students; these questions often took a lot of time to answer. Some lecturers reported that after a few times of trying to involve students that students learned to expect some questions and some problem work in class. Of course, some lecturers provided a numerical contrast primarily to appease the author while he observed their classes. In such a case, the instructor asked what amounted to rhetorical questions; the instructors answered the questions and did not really engage students. Discussion, demonstrations by senior graduate student lecturers, and practice sessions were used to try to more clearly communicate the purpose and practice of engaging students.

### 4. Writing More Details and Making Outlines

What should one write on the blackboard (or whiteboard or overhead) in lecture? Perhaps the image of an instructor with back to the audience scribbling away talking to the blackboard comes to mind. Based on observing new graduate student instructors in probability and statistics, a more common image is a lecturer pontificating from a set of notes and writing little on the board while students have difficulty keeping focused. An active learning approach that asks students questions and involves them in calculations, explorations, and activities decreases the tendency toward the first image. The second image can be overcome by writing more details on the board. Writing more details and using outlines in introductory statistics lectures can increase student participation by making the purpose of lecture and examples clearer without taking up too much time in class. Using contrast and asking planned questions can be combined readily with these suggestions. In the author’s opinion, the graduate students want to appear knowledgeable and precise and therefore prepare notes. Planning what to write and being organized can enhance the students’ perception of the instructor.

#### 4.1. Writing More Details

- **Writing more, example 1: Label sets.**

  Say the set $A$ is identified with students who own computers, and set $B$ with students who score well on a preliminary computer science exam. Describe the sets $A$ and $B$, $A$ or $B$, $A$ but not $B$, $B$ but not $A$, and neither $A$ nor $B$. If the definitions are said verbally but not written down, then a student has to remember the initial definitions before describing the new events. If an instructor writes “$A = \text{set of students who own a computer}, B = \text{set of students passing prelim exam}”$ on the board, then students have an easier time answering. They also
have more details in their notes for review and a better model for approaching a new problem.

- **Writing more, example 2: Label probabilities.**

Say 40% of students own a computer. Suppose 50% of students who own a computer pass the preliminary computer science exam, but only 30% of students without a computer pass. What percentage passes overall? Imagine if an instructor wrote “A: 0.4 B|A: 0.5 B|notA: 0.3” on the board. The answer is .4(.5) + .6(.3) = .38. What will appear in students’ notebooks? Do the notes on the board help students think about the problem? Do the notes connect to the context? Several times the author observed that such a limited presentation on the board caused students to ask for clarification: what is A? B? What is the question? Lack of participation also can result. Here is an alternative:

- Pr(owns computer)=0.4
- Pr(passes prelim given that owns computer)=0.5
- Pr(passes prelim given that doesn’t own computer)=0.3
- What is Pr(B)?

The writing is still fairly compact, provides context and details, and the question is clear. In class, it could take less time to write a clear problem statement than to have to re-explain the context to a group of confused students. When students work additional problems in class or at home, they have a more useful template for listing the provided information.

- **Writing more, example 3: Assumptions and checking assumptions.**

The Binomial distribution is used to model the probabilities of 0 through n “successes” out of n trials when the trials are independent and the probability of success (p) is constant across trials. The Binomial probability distribution can be well-approximated by a normal density when n is large, e.g., np > 10, n(1 - p) > 10. Students should appreciate that models are based on assumptions and conditions have to be met for numerical approximations to be accurate. Instead of writing “Assumptions” it would be much more informative and better training to write the following:

- Independent trials stated in problem (or simply, stated)
- Constant probability assumed (or stated, p = 0.35)
- n trials n fixed (or stated, n = 40)
- Large n n = 40, p = 0.35, np = 14, n(1 - p) = 26.

In subsequent problems done by students, they might adopt a short cut to the first three lines and write something like “Binomial n = 40, p = 0.35, independent.” In either approach, their understanding of the assumptions would be much more clearly expressed.

- **Writing more, example 4: Hypotheses.**

In hypothesis testing, one states a null and an alternative hypothesis and tests whether or not the null is tenable given the data. Many student instructors state hypotheses in very abbreviated numeric or symbolic forms. A test of independence between the factors defining a two-way table can be stated in a very minimalist fashion as “H₀ : independent, H₁ : dependent.” The statement of hypotheses does not explicitly relate to a two-table or to any particular context. Here is a more explicit version:

- Null hypothesis H₀ : row (owning a computer) and column (passing the prelim) are independent (pᵢⱼ = pᵢ pⱼ, Pr(row i, column j) = Pr(row i)Pr(col j))
- Alternative hypothesis H₁ : row and column are dependent

Although the statement of the alternative is not explicit, it is clear in contrast to the null hypothesis. The null hypothesis contains enough detail to demonstrate that it pertains to the context, analysis of a two-way table, and independence in a statistical sense. In which version of hypotheses, after computing the test statistic and associated P-value, will students be more successful in stating a conclusion based on the analysis?

- **Writing more, example 5: Formulas and calculations.**

Student lecturers have a tendency to not recopy formulas when working problems and to not really finish computations. Say based on 18 observations, the observed mean and standard deviation are 99 and 10, respectively. If the hypothesized mean value is 95, the two-sided t-test statistic is

\[ t = \frac{\bar{x} - \mu_0}{\sqrt{\frac{s}{n}}} = \frac{99 - 95}{\sqrt{\frac{10}{18}}} = 1.697 \]

Rewriting the formula in the problem rather than just the numbers provides repetition of the formula and leads students to better notes for study purposes. The P-value for testing versus a two-sided alternative is 0.108 or about 0.11 and does not lead to rejection of 95 as a plausible mean value. How should an instructor write the P-value calculation on the board? Writing “P = 0.11” provides no details and is ambiguous. Writing “P-value = 2Pr(t > 1.697) = 2(0.054) = 0.108” is much more complete. When working additional problems in class (say with n = 28 or 48, or \( \mu_0 = 94 \) or 90), students then have a model with enough detail to be replicated.

- **Writing more, example 6: Conclusions.**


Writing more, example 7: Derivations.

If \( x_1, x_2, \ldots, x_n \) are \( n \) observations on variable \( X \) and \( Y = a + bX \) is a linear transformation (producing \( y_1, y_2, \ldots, y_n \)), then

\[
\bar{y} = a + b \bar{x} \quad \text{and} \quad s_y^2 = b^2 s_x^2.
\]

It is not uncommon for instructors to skip steps when deriving these results. Of course, students can (and should) realize these relationships through application to data and simulation, but it would be appropriate to present a derivation to a more mathematical crowd. For students encountering derivations and manipulating abstract symbols for perhaps the first time, omitting details can be confusing and elicit questions such as “How did you go from the second to the third line?” Graduate student instructors often are very comfortable with a level of mathematics well beyond that of the students in introductory statistics. It is not necessary to mention Borel sets or absolutely continuous measures in introductory statistics!

4.2. Making Outlines

In addition to writing less than could be profitably written in some examples, most student lecturers do not present an outline of topics at the beginning of a lecture. Student instructors are not alone in omitting outlines. Many lecturers also do not make the structure of the class topics explicit as lecture progresses. Both of these practices, an initial and an evolving outline, can increase clarity, help students take notes, and emphasize the purpose of discussing a particular topic. Outlines can be written on a black/whiteboard before class. Topics can be “checked off” as they are covered in class. Here are some sample outlines that could have been used to describe lectures observed at Chicago:

| Outline Example 1 | Outline Example 2 |
|-------------------|-------------------|
| 1. Two-way tables | I. \( \Pr(A \text{ or } B) \) |
| 2. Chi square statistic | II. \( \Pr(A \text{ and } B) \) |
| 3. Chi square test | III. Independent versus Mutually Exclusive |
| 4. Chi square distribution | |

| Outline Example 3 | Outline Example 4 |
|-------------------|-------------------|
| A. Student’s \( t \) distribution | i. Average of a sum |
| B. Confidence interval for the mean | ii. Standard error of a sum |
| C. Matched pairs study | |

The outlines are short, highlight main points, and do not present details. In the first example, one could combine lines 2-4 into either two lines or one line. All four connect to the purpose of the lecture. It would not take much time to write such an outline and also the relevant section number of the textbook on the board at the start of class.

An evolving outline provides verbal and visual punctuation that keep students focused on the current topic. It helps students relate what is happening in class to the larger outline, or bigger picture. Winston (1999) emphasizes the value of making the structure of a talk explicit. Students tend to write in their notes what is written on the board. As in any audience, students in a class do not pay attention the whole time. At the end of class, students should be able to accurately state the main focus of the lecture and remember some impression of what happened. Of course students have to study outside of class to better learn details, but an evolving outline coordinated with an initial outline can help lecture become a better learning experience.

Say the lecture topic is the Binomial probability distribution. A lecture title could be “The Binomial Distribution.” An initial outline might list three topics: A. probabilities; B. mean and standard deviation; and C. normal approximations. The actual topics discussed during class could be enumerated as they occur as follows: 1. model and assumptions; 2. probabilities; 3. expectation; 4. variance and standard deviation; 5. \( \hat{p} \) – the sample proportion; 6. normal approximations to probabilities; and 7. assumptions, revisited. Under 2., students could perform calculations under different values of \( n \) and \( p \) and summarize the contrasts. Under 4., in addition to the formulas for variance and standard deviation, students could construct bar charts of Binomial probabilities: for what values of \( p \) is the distribution skew and symmetric? Under 6., students can apply normal approximations in situations with large \( n \), with small \( n \) (the approximation is not good), with a correction for continuity, and for the sample proportion. The main purpose of the lecture, the Binomial distribution, remains in sight. The subtopics are given names, and students can appreciate that they are the topics of interest to a statistician.

Graduate student lecturers tended not to present outlines or a coherent outline throughout lectures. Some did upon encouragement provide labels to sections and emphasize main themes. In the author’s opinion, the outlines and section titles increased the sense of organization in their classes. The organization and punctuation is what is important here, rather than numbering in a particular style. Certainly the author observed students in the classes copying the outlines and topic headers into their notes.
The combination of writing a little more and providing some sort of outline do not have to make lecture substantially longer, but can greatly increase student understanding as lecture progresses. The use of contrast and planned questions within a clear structure can help students be more aware of what is happening and why and to participate more readily.

5. Pictures and Alternative Modes of Presentation

Pictures and diagrams can help students learn to solve problems effectively and should be standard tools for many types of problems. Graduate student lecturers tend to use few visual aids, especially when using modern technologies that make it difficult to write a lot and draw diagrams in class.

5.1. Pictures and Diagrams for Problem Solving

Pictures and diagrams can be very helpful in problem solving in introductory probability and statistics. Most lecturers introduce some standard pictures, because they realize they are useful for explaining solutions and current textbooks (e.g., Peck, Olsen, and Devore 2001; McCabe, Moore, and Yates 1999) include them. Based on the author’s observations, however, students are not consistently encouraged to use pictures as problem-solving tools. Diezmann and English (2001) also advocate the use of pictures and diagrams in teaching and reasoning about probability and statistics. The examples below suggest diagrams and pictures that can be produced quickly and effectively in lecture to help students learn to solve problems in class.

• Pictures, example 1: Venn diagrams.

Venn diagrams represent sets by circles or other shapes (see, e.g., www.cs.uni.edu/~campbell/stat/venn.html) and are tools for visualizing unions and intersections of sets. Venn diagrams usually are introduced in introductory probability, but beyond a brief example or two are not used to reason about sets. Example 2 of Section 4.1 is relevant here. Let \( A \) be the event that a student owns a computer. Let \( B \) be the event that a student passes the preliminary computer science exam. What are \( \Pr(A \text{ and } B) \), \( \Pr(\text{not } A \text{ and } B) \), and \( \Pr( A \mid B) \)? Are events \( A \) and \( B \) independent? Answers to this problem are easier if a Venn diagram is drawn. It also is useful to have a Venn diagram if one changes the sets in the problem to their opposites or to contrast results, e.g., \( \Pr(A) = 0.60 \). Other topics such as the law of total probability \( \Pr(A) = \Pr(A \mid B) \Pr(B) + \Pr(A \mid \text{not } B) \Pr(\text{not } B) \) and questions such as “If \( \Pr(A \mid B) > 0 \) and \( \Pr(B) > 0 \), then is \( \Pr(B \mid A) > 0? \)” can be addressed with Venn diagrams as well as with other representations.

• Pictures, example 2: Permutations and Combinations.

Suppose you have twelve books (labeled \( A, B, \ldots, L \)) and plan to read one per month for a year. How many different orders are there for your selection of books? How many orders are there in the first four months? How many sets of four books could you choose for the first four months? These questions are answered with permutations (first and second questions) and combinations (third question). A diagram relevant for this problem is given below:

| Books:  | A, B, C, D, E, F, G, H, I, J, K, L |
|--------|----------------------------------|
| Months: | 1 2 3 4 | 5 6 7 8 9 10 11 12 |

The answer to the first question is \( P_{12}^{12} = 12! = 12 \{1\} \{10\} \cdots 1 \) more than 479 million and is illustrated by considering placing the 12 books in the 12 slots representing months. The answer to the second question is \( P_{4}^{12} = 12 \{1\} \{10\} \{9\} = 11880 \), which involves 4 of the 12 books and the first four slots. The answer to the third question is \( C_{4}^{12} = \frac{12!}{4! \{2\} \{1\}} = 495 \). If order does not matter so that, for example, \( ABCD \) is the same as \( BCDA \), then the four books to the left of the divider can be reshuffled into \( 4! = 24 \) orders without moving any book across the dividing line. The third answer is equivalent to \( C_{4}^{12} = \frac{12!}{(4!) \{12-4\}} = \frac{12!}{(4!) \{8\}} \). How can this be seen? \( 12! \) is the number of ways to arrange all twelve books in order. The four to the left can be reordered \( 4! \) ways without moving a book across the line. Similarly the other eight can be reordered \( 8! \) ways. Students after seeing this presentation can consider other problems in class.

• Pictures, example 3: Probability trees.

Students often find conditional probability confusing. More specifically, they find solving story or word problems difficult. If 70% of Mars rovers land in former streambeds and 60% of the rovers in streambeds on Mars find historic evidence of water whereas 10% of the rovers not in streambeds do, what percent of Mars rovers find evidence of water? Answer: 0.7(0.6) + 0.3(0.1) = 0.45. What is the chance that a rover that has found evidence of water is in a streambed? Answer: 0.7(0.6)/(0.7(0.6) + 0.3(0.1)) = 0.42/0.45. What is the chance if the rover has not found water? Answer: 0.7(0.4)/(0.7(0.4)+0.3(0.9)) = 0.28/0.55. Although graduate student instructors provide technically correct solutions, they do not necessarily help students learn a method for solving problems. Even if an example of a probability tree is provided, it typically is not a tool used consistently. Here is a tree for the problem stated above.
Lesser (2001) presents several graphical representations related to Simpson’s paradox that could be useful in this context. Some of these ideas can be used more generally in probability problems.

- **Pictures, example 4: Normal densities.**

Probabilities relating to the standard normal distribution are read off tables or calculated on calculators and computers. Students sometimes report the wrong tail area associated with a problem. A simple picture of the normal density helps students identify the correct area to report. For example, suppose a car starting with a full tank of gas gets on average 26 miles per gallon (mpg) when driving at highway speed for one hour and has a standard deviation of 1.5 mpg under these conditions. What is the probability that the car gets at least 27 mpg, less than 25 mpg, and at most 28 mpg? Answers: 0.25, 0.25, and 0.91. What are the 40th and 90th percentiles of the distribution of mpg? Answers: 26 + 1.5(-0.25) = 25.625 and 26 + 1.5(1.28) = 27.920. Assuming the distribution of mpg is normal under these conditions, a plot such as those below should be an automatic response. The vertical dashed lines are at the means (also the medians) of the distributions. The graph on the left is on the mpg scale, whereas the one on the right is on the standardized (mean zero, variance 1) scale.
Based on the graphs and an understanding of the median it is clear that \( \Pr(\text{mpg} \geq 27) \) and \( \Pr(\text{mpg} < 25) \) are both less than 0.50, but \( \Pr(\text{mpg} < 28) \) is greater than 0.50. It should also be clear that the 40th percentile is below but the 90th percentile is above 26 mpg. What happens if the mean instead of 26 is 24? How do probabilities change if the standard deviation is 1.7? The initial response for most students should be to draw a new graph. To encourage student participation in lecture, one could provide to students a handout with several pre-drawn probability densities for their use in class. Students could be directed to shade-in appropriate areas under curves corresponding to desired probabilities. Such a handout would not have to be restricted to problems with normal densities, but rather could be used with arbitrary probability density functions.

- **Pictures, example 5: Probability histograms for discrete random variables.**

Probability histograms or bar charts for discrete random variables are drawn to illustrate probabilities, but often are not connected to means, standard deviations, and the degree of skew of random variables. Contrast 4 in Section 3.1 suggests contrasting distributions by altering their parameters, such as the Binomial probability \( p \). Graphs of probabilities illustrate differences in means and variances, as well as skew. Probability histograms of Geometric random variables for two different values of \( p \) can be used to think about means, variances, and skew.

### 5.2. Alternative Modes of Presentation

The use of pictures and other ideas for lecturing are influenced by the technology available for use in class. Different modes of presentation, including blackboard or chalkboard, whiteboard or marker board, overhead projection, and computer projection, have certain advantages that can be exploited and disadvantages that should be avoided. A few brief comments are made below based on observing graduate student lecturers and teaching assistants.

Lecturing in front of a **blackboard or chalkboard** allows one to write a lot (usually more than in other formats) and draw pictures. One can plan a lecture presentation to use the whole chalkboard surface and preserve formulas (strategic erasing) that will be useful more than once during a lecture. Student lecturers observed by the author often did not effectively plan board use throughout the lecture and as a result frequently duplicated formulas. Some also did not fully erase old work or move obstacles, such as unused projectors and wastebaskets, to enable full use of the writing surface. If blackboard space is sufficient, it should be possible to keep an outline of lecture visible for quite awhile (see Section 4.2).

Beyond planning the layout of work on the board, a device for saving a significant amount of time and increasing student participation during class is a **handout**. Handouts can be useful in any teaching situation. They can be used to deliver pictures, long word problems, definitions and theorems, and outlines. They also can include blanks for students to enter work, definitions, and pictures (Magel 1996).

A **whiteboard or marker board** generally provides less space than a blackboard. Sometimes writing on one is clearer than a blackboard, but glare from lights and empty markers can be a big problem. Most of the advantages of a blackboard apply to whiteboards as well. Some would point to color as an advantage of writing with markers, but some students have color deficient vision; distinguishing points with symbols as well as color is a good idea. Given the reduced space, planning how topics will be arranged is especially important, as is considering a handout for long problems and definitions. An outline could be included on the handout as well.

**Overhead projectors** including those that project images directly from paper are popular with graduate student lecturers, because overhead slides can be prepared (even typed and shared with other instructors) in advance. The cautions about making presentations with projectors have been stated many times, and they are real. Instructors usually provide fewer details in explanations and answers because there is little space to write on transparency slides. It is hard to reference formulas and previous work, because it is not possible to keep multiple pages visible at a single time, unless you have more than one projector and screen. Printing often is too small, instructors move too fast through material, and it is boring to watch someone read from a projected image. Donald (1999) presents amusing comments about how not to use a projector for a presentation. Projectors, however, are great for displaying data and figures. Based on the instructors observed by the author, recommendations include writing an outline and providing details of some examples and answers to questions on the chalkboard or marker board if possible, having plenty of blank transparencies and dark pens available for writing, and practicing to avoid obstructing the projected image. A handout helps overcome many difficulties of using projectors.

**PowerPoint presentations** magnify the problems of using transparencies: too fast, no writing space, technical difficulties. The major advantage of a projecting from a computer is that it is possible to present data and graphics directly from a computer statistical package. A handout of topics and examples, having some space available on a chalkboard or marker board for writing notes and drawing pictures, and asking oneself the question, “can students take notes on my presentation?”, are recommended. And definitely use a computer statistical package with graphics if possible. A relatively new technology option is the **TabletPC**. Faculty members have reported to the author that it is a technology medium that allows far more interaction than traditional technology does. No further comments will be made in this article, because it was not in use during the period that observations and notes were made.

This article has not considered the special needs of instructors and students involved with **distance education**. The reader is referred to Stephenson (2001) for comments on the technical and pedagogical difficulties of teaching probability and statistics in a distance education environment.

This section has presented ideas for using pictures and diagrams in teaching probability and statistics and commented on various technological options. Student instructors are quick to embrace new technologies, certainly quicker than many more experienced instructors, the author included. Technology can enable new presentation styles and add interest. It also can have negative effects, including time lost due to technological failure or difficulties. One has to consider whether the technology is helpful to students trying to learn probability and statistics in a lecture format.
6. Summary

Lecture is a common presentation style that gives instructors a lot of control over topics and time allocation. Some styles of lecture, however, can encourage active student participation and learning. Based on the author’s experiences, some suggestions have been made to increase the level of involvement of students in a lecture class. Many of the ideas can be used to modify current plans and preparations without requiring a great deal of additional time in preparation or in class to increase student participation. Graduate student instructors, teachers new to probability and statistics, and others have a lot to do to organize and lead courses. The ideas and examples will be useful as enhancements to current efforts. Hopefully, the active involvement of students in classes develops into a very pleasant learning experience for both student and instructor. Other methods, such as small group activities (Keeler and Steinhorst 1995), cooperative learning (Magel 1998, Giraud 1997), and case studies (Weinberg and Abramowitz 2000, Nolan and Speed 1999, Gnanadesikan, Scheaffer, Watkins and Witmer 1997), certainly also should be considered in order to increase active participation and effective learning of probability and statistics.

Acknowledgements

The author would like to thank numerous students in classes, graduate students lecturers, course assistants, faculty members, and high school and middle school teachers for discussions and suggestions. The author would also like to thank the editor, an associate editor, and two anonymous referees for their suggestions that lead to improvements in the article.

References

Carver, R., and Peters, S. (2004), AP Statistics Web Guide [Online].
apcentral.collegeboard.com/members/article/1,3046,151-165-0-21971,00.html

Chance, B.L. (2002), “Components of statistical thinking and implications for instruction and assessment,” Journal of Statistics Education [Online], 10(3).
ww2.amstat.org/publications/jse/v10n3/chance.html

College Entrance Examination Board (2004), 2005, 2006 Course Description for AP Statistics.
apcentral.collegeboard.com/repository/statistics_cd_0506_4328.pdf

Derek Bok Center for Teaching and Learning (2002), What is Microteaching? [Online], Harvard University.
bokcenter.fas.harvard.edu/docs/microteaching.html

Derek Bok Center for Teaching and Learning (2004), Tips for Teachers: Twenty Ways to Make Lecture More Participatory [Online], Harvard University.
bokcenter.fas.harvard.edu/docs/TFTlectures.html

Diezmann, C. M., and English, L.D. (2001), “Promoting the use of diagrams as tools for thinking,” The roles of representation in school mathematics, 2001 Yearbook, Editors A. A. Cuoco and F. R. Curcio. National Council of Teachers of Mathematics: Reston, Virginia, 77-89.
delMas, R.C. (2002). “Statistical literacy, reasoning, and learning: A commentary,” Journal of Statistics Education [Online], 10(3).
ww2.amstat.org/publications/jse/v10n3/delmas_discussion.html

Donald, B.R. (1999), “How to Give a Talk,” [Online].
www.cs.dartmouth.edu/~brd/Teaching/Giving-a-talk/giving-a-talk.html

Garfield, J. (1993), “Teaching statistics using small-group cooperative learning,” Journal of Statistics Education [Online], 1(1): 4-30.
ww2.amstat.org/publications/jse/v1n1/garfield.html

Garfield, J. (2002). “The challenge of developing statistical reasoning,” Journal of Statistics Education [Online], 10(3).
ww2.amstat.org/publications/jse/v10n3/garfield.html

Garfield, J., Hogg, B., Schau, C., and Whittinghill, D. (2002), “First Courses in Statistical Science: The Status of Educational Reform Efforts,” Journal of Statistics Education [Online], 10(2).
ww2.amstat.org/publications/jse/v10n2/garfield.html

Giraud, G. (1997), “Cooperative Learning and Statistics Instruction,” Journal of Statistics Education [Online], 5(3).
ww2.amstat.org/publications/jse/v5n3/giraud.html

Gnanadesikan, M., Scheaffer, R.L., Watkins, A.E., and Witmer, J.A. (1997), “An activity-based statistics course,” Journal of Statistics Education [Online], 5(2).
ww2.amstat.org/publications/jse/v5n2/gnanadesikan.html
Keeler, C.M., and Steinhorst, R.K. (1995), “Using Small Groups to Promote Active Learning in the Introductory Statistics Course: A Report from the Field,” *Journal of Statistics Education* [Online], 3(2).
ww2.amstat.org/publications/jse/v3n2/keeler.html

Knight, A.B. (2002), *Lectures: Organizing Them and Making Them Interesting*. University of Oklahoma, Instructional Development Program [Online].
www.ou.edu/idp/tips/ideas/lectures.html

Kvam, P.H. (2000), “The effect of active learning methods on student retention in engineering statistics,” *American Statistician*, 52(2), 136-140.

Lackritz, J.R. (1997), “Increasing student participation in large introductory statistics classes,” *American Statistician*, 51(2), 210.

Larsen, M.D. (2003), “Using Technology to Connect Topics and Contrast Scenarios,” *Beyond the Formula* [Online].
www.public.iastate.edu/~larsen/btf03/index.html

Larsen, M.D. (2004a), “Active Learning of Probability through Contrasts,” *Statistics Teachers Newsletter*, [Online], 65, 3-4.
ww2.amstat.org/education/stn/pdfs/STN65.pdf

Larsen, M.D. (2004b), “The Importance of the Sample Space for Teaching Probability;”

Larsen, M.D. (2004c), *Internet Companion for Statistics: A guide to websites and activities*, 1st Ed., Belmont, CA: Duxbury.
larsen.duxbury.com

Larsen, R. J., and Marx, M. L. (1986), *An Introduction to Mathematical Statistics and Its Applications*, 2nd Ed., Upper Saddle River, NJ: Prentice Hall.

Lesser, L. M. (2001), “Representations of reversal: An exploration of Simpson’s paradox,” *The roles of representation in school mathematics, 2001 Yearbook*, Editors A. A. Cuoco and F. R. Curcio, Reston, VA: National Council of Teachers of Mathematics, 129-145.

Loftsgaarden, D.O., and Watkins, A.E. (1998), “Statistics teaching in colleges and universities: Courses, instructors, and degrees in fall 1995,” *American Statistician*, 52(4), 308-314.

Magel, R.C. (1996). “Increasing student participation in large introductory statistics classes,” *American Statistician*, 50(1), 51-56.

Magel, R.C. (1998), “Using Cooperative Learning in a Large Introductory Statistics Class,” *Journal of Statistics Education* [Online], 6(3).
ww2.amstat.org/publications/jse/v6n3/magel.html

McCabe, G., Moore, D., and Yates, D. (1999), *The Practice of Statistics: TI-83 Graphing Calculator Enhanced*, New York: W.H. Freeman and Company.

Moore, D.S. (1997), “New Pedagogy and New Content: The Case for Statistics,” *International Statistical Review*, 65(2), 123-137.

Moore, T. L. (2000), *Teaching Statistics: Resources for Undergraduate Instructors*, MAA Notes #52.

Mosteller, F. (1980), “Classroom and platform performance,” *American Statistician*, 34(1), 11-17.

Nolan, D., and Speed, T.P. (1999), “Teaching Statistics Theory Through Applications,” *American Statistician*, 53(4), 370-375.

NRCS. (2004), *National Resources Inventory 2002: Annual NRI Land Use*, U.S. Department of Agriculture, National Resource Conservation Service [Online].
www.nrcs.usda.gov/technical/land/nri02/landuse.pdf

Peck, R. (2002). “AP statistics turns 5! A report on the 2001 exam: Questions, performance, and communication,” *Stats. The Magazine for Students of Statistics*, 34, 14-16.

Peck, R., Olsen, C., and Devore, J. (2001), *Introduction to Statistics and Data Analysis*, Pacific Grove, CA: Thomson Learning, Duxbury Press.

Rumsey, D.J. (2002), “Statistical literacy as a goal for introductory statistics courses,” *Journal of Statistics Education* [Online], 10(3).
ww2.amstat.org/publications/jse/v10n3/rumsey2.html

Schwartz, D.L., and Martin, T. (2004), “Inventing to prepare for future learning: The hidden efficiency of encouraging original student production in statistics instruction,” *Cognition and Instruction*, 22(2), 129-184.

Siegel, M. (2004), “Let Them Roll, Then Show Us Your Data,” *Statistics Teachers Newsletter* [Online], 64, 4.
Stephenson, W.R. (2001), “Statistics at a Distance,” *Journal of Statistics Education* [Online], 9(3).

Weinberg, S.L., and Abramowitz, S.K. (2000), “Making general principles come alive in the classroom using an active case studies approach,” *Journal of Statistics Education* [Online], 8(2).

Winston, P. H. (1999), “Some Lecturing Heuristics,” [Online].

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