A “HORIZON-ADAPTED” APPROACH TO THE STUDY OF RELATIVISTIC ACCRETION FLOWS ONTO ROTATING BLACK HOLES

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ABSTRACT

We present a new geometrical approach to the study of accretion flows onto rotating black holes. Instead of Boyer-Lindquist coordinates, the standard choice in all existing numerical simulations in the literature, we employ the simplest example of a horizon-adapted coordinate system, the Kerr-Schild coordinates. This choice eliminates unphysical divergent behavior at the event horizon. Computations of Bondi-Hoyle accretion onto extreme Kerr black holes, performed here for the first time, demonstrate the key advantages of this procedure. We argue that it offers the best approach to the numerical study of the observationally increasingly, more accessible, relativistic inner region around black holes.

Subject headings: accretion, accretion disks — black hole physics — hydrodynamics — methods: numerical — relativity

1. MOTIVATION

Advances in satellite instrumentation, e.g., the Rossi X-Ray Timing Explorer and the Advanced Satellite for Cosmology and Astrophysics, are greatly stimulating and are guiding the theoretical research on accretion physics. The recent discovery of kilohertz quasi-periodic oscillations (QPOs) extends the frequency range over which these oscillations occur into timescales associated with the innermost regions of the accretion process (for a review, see van der Klis 1997). Stella & Vietri (1997) have proposed that observed low-frequency QPOs in neutron star X-ray binaries correspond to the precession of the accretion disk, i.e., the Lense-Thirring effect. This could be the first piece of evidence for a genuinely general relativistic effect, i.e., the dragging of inertial frames, in such systems. Morgan, Remillard, & Greiner (1997) identified a 67 Hz QPO in the black hole candidate GRS 1915+105 that may be associated with relativistic, trapped inner-disk oscillations (Nowak et al. 1997). Recently, Cui, Zhang, & Chen (1998) have extended the interpretation of Stella & Vietri to black hole binaries. Within this model, GRO J1655−40 and GRS 1915+105 are found to spin at a rate close to the maximum theoretical limit. Moreover, in extragalactic sources, spectroscopic evidence (broad iron emission lines) increasingly points to (rotating) black holes being the accreting central objects (Tanaka et al. 1995; Kormendy & Richstone 1995). Recently, Bromley, Miller, & Pariev (1998) placed a limit on the inner edge of the accretor giving rise to the iron Kα emission in MCG−6−30−15 at about 2.6 Schwarzschild radii. Their estimate is that the black hole is rotating at a rate that is about 23% ± 17% of the allowed maximum.

Early theoretical studies indicated that a rotating black hole in the presence of an accretion disk must be spinning at nearly the maximal rate (Thorne 1974). Rotation increases the available energy in the near-horizon region: the binding energy per unit mass of a test particle can reach up to 0.42c², and the innermost stable circular orbit approaches the horizon (Bardeen, Press, & Teukolsky 1972) and coincides with it (at least in areal coordinates) in the extreme case a = M (M is the mass of the hole, and a its specific angular momentum). In the rotating case, motions away from the equatorial plane are affected by the dragging of inertial frames, while motions in the ergoregion may (more speculatively) extract rotational energy from the hole.

Accretion theory is based primarily on the study of stationary flows and the linearized perturbations thereof. Establishing the nature of flow instabilities, though, will almost certainly require highly resolved and accurate, time-dependent, nonlinear numerical investigations. The possibility of establishing features of the accretion process that are reflecting the nature of the spacetime is especially intriguing. Such numerical probes rely crucially on adequate and consistent approximations of the geometry. For a wide range of accretion problems, a Newtonian theory of gravity is adequate for the description of the background gravitational forces. The extensive experience with Newtonian astrophysics suggests that the first explorations of the relativistic regime could benefit from the use of model potentials (Paczynski & Wiita 1980). This constitutes the Newtonian paradigm, which is still being developed (see, e.g., Nowak & Wagoner 1991 and Artemova, Björnsson, & Novikov 1996). For comprehensive numerical work, a full (i.e., three-dimensional) formalism is required, one that is able to cover the maximally rotating hole as well. In rotating spacetimes, the gravitational forces cannot be captured fully with scalar potential formalisms. A vivid example is provided by the wave systems examined in Chandrashekhar (1983), in which rotation introduces frequency-dependent potentials. Additionally, geometric regions such as the ergosphere would be very hard to model without a metric description. Whereas the bulk of emission occurs in regions with almost Newtonian fields, only the observable features attributed to the inner region may depend crucially on the nature of the spacetime.

Pioneering numerical efforts in the study of accretion onto black holes (Wilson 1972; Hawley, Smarr, & Wilson 1984; Hawley 1991) established the relativistic framework, the so-called frozen star paradigm of a black hole. In this, the time “slicing” of the spacetime is synchronized with that of asymptotic observers far from the hole. This is a powerful approach, leading to a very economical description of the geometry and, in principle, be used to capture all the interesting effects of the spacetime curvature. We focus here on the limitations of this approach, which further motivate this Letter. The short-
comings are due to the poor choice of coordinates near the black hole horizon and hence manifest themselves only in its neighborhood. We have argued, though, that this is precisely the region of most interest. A set of consistency problems arises from the need for correct boundary conditions at the horizon. Such conditions are easily imposed on simple supersonic inflows but become murkier for corotating accretion disks on rapidly rotating Kerr black holes. In addition, imposing boundary conditions on magnetic fields is problematic. Addressing this issue has led to the development of the so-called membrane paradigm. Starting with the description of the black hole processes in a nonsingular coordinate system (Damour 1978), this approach endows an approximate horizon (a timelike world tube) with special electric and magnetic properties and then reverts back to the frozen star picture for the description of the rest of the spacetime (Thorne & MacDonald 1986). However, the computation is still performed in the original singular system. Imposing boundary conditions near the horizon becomes a demanding practical task, since the singular coordinate coverage of the horizon leads to an unphysical blowup of coordinate-dependent quantities.

2. PROPOSAL

In Papadopoulos & Font (1998, hereafter PF), we put forward the idea of numerically computing accretion flows onto a black hole in a coordinate system adapted to the horizon. There we used simple, spherically symmetric flows to establish its basic feasibility. In addition, we computed axisymmetric, relativistic, hydrodynamic accretion onto Schwarzschild black holes. We noticed that even the stiff adiabatic index case was computable, illustrating one of the advantages of a regular coordinate system. In the remainder of this Letter, we demonstrate, using a model calculation, the functionality of our framework to study accretion flows onto rapidly rotating black holes.

Coordinate systems attached to physical observers are generically regular at the horizon but lack the important practical property of stationarity. If, besides regularity, we impose the additional requirements of a stationary metric and spacelike foliation, we obtain what we call horizon-adapted coordinates. A comprehensive family of those systems for the Kerr spacetime can be obtained with the following transformation from the standard Boyer-Lindquist (BL) $(t, r, \theta, \phi)$ to the new coordinates $d\tilde{t} = dt + [(1 + Y)/(1 + Y - Z)] dr$, where $Y = a^2 \sin^2 \theta q^2$, $Z = 2Mr/q^2$, $q^2 = r^2 + a^2 \cos \theta$, $\Delta = r^2 - 2Mr + a^2$, and $k$ is a nonnegative integer that parameterizes the family (natural units are used throughout, $G = c = 1$). All members of the family are regular at the horizon; hence, the algebraically simplest choice ($k = 1$) is preferred. This corresponds to the so-called Kerr-Schild (KS) form of the Kerr metric. With this choice, the line element becomes

$$ds^2 = -(1 - Z)dt^2 - 2aZ \sin^2 \theta d\tilde{t} d\tilde{\phi} + 2Z d\tilde{t} dr + (1 + Z) dr^2 - 2a(1 + Z) \sin^2 \theta dr d\tilde{\phi} + q^2 d\theta^2 + \sin^2 \theta [q^2 + a^2 (1 + Z) \sin^2 \theta] d\tilde{\phi}^2. \tag{1}$$

The regularity at the horizon, which is located at the largest root of the equation ($\Delta = 0$), is manifest. A more extensive discussion, in particular of the wider range of choices available in the nonrotating case, is given in PF.

The central aspect of our computational paradigm is that the integration domain includes, in a natural way, the event horizon of the black hole. In fact, it extends inside the horizon and can be truncated at an arbitrary inner radius. This prescription solves, at once, both conceptual and practical problems in the near-horizon region. Boundary conditions for fields can be imposed in an unambiguous manner, whereas there is a sharp reduction in the spatial gradients of the solution. The first benefit arises from the nature of the horizon as a one-way (ingoing) membrane. In the absence of good boundary conditions, even “poor” conditions would be acceptable, provided they allow a stable and converging computation. For example, unphysical reflections or numerical heating will never reemerge from the horizon, since the characteristic speeds for such processes are necessarily slower than $c$. The reduced resolution requirements arise from the fact that the projections of four vectors onto the three-dimensional integration space is everywhere regular, given the regular coordinate system.

The regularization of the horizon introduces two new non-zero metric elements. However, this additional algebraic complexity should not be much of a concern for relativistic integration algorithms that must be designed to handle a general metric. On the other hand, it is a fact that considerable intuition and mathematical tools have been obtained in the simpler frozen star form of the Kerr metric. This background work can still be used by transforming geometric quantities back and forth. For stationary accretion patterns, this process is entirely straightforward, and we include a demonstration of such transformations.

3. ACCRETION ONTO A RAPIDLY ROTATING BLACK HOLE

The generic astrophysical scenario in which matter is accreted, in a nonspherical way, by a compact object is the one suggested originally by Bondi & Hoyle (1944). The astrophysical importance of this process has led to its detailed numerical investigation (see, e.g., Benesochn, Lamb, & Taam 1997 and references therein). Recently, the relativistic version of the Bondi-Hoyle accretion has been extended to the non-axisymmetric, nonrotating black hole case (Font & Ibáñez 1998b). Its generalization to account for rotating black holes still remains unexplored. Here we take the first step in that direction, investigating the problem of adopting a simple geometrical scenario. At the same time, this scenario offers an adequate test-bed calculation to demonstrate the proposal of horizon-adapted coordinates for accretion onto rotating black holes.

In performing the computation, we adopt the “infinitesimally thin” accretion disk setup. This is motivated by simplicity considerations, before attempting three-dimensional studies. This initial setup has been used to some extent in Newtonian simulations of wind accretion in cylindrical and Cartesian coordinates (see, e.g., Matsuda et al, 1991, Benesochn et al. 1997, and references therein). In our setup, we use a restricted set of equations, where the vertical structure of the flow is assumed not to depend on the polar coordinate. This assumption requires that in the immediate neighborhood of the equator, vertical (polar) pressure gradients, velocities, and gravity (tidal) terms vanish. Those conditions are strictly correct at the equator for flows that are reflection symmetric there. Hence, despite the obvious reduction of generality, important insights into the accretion process can be obtained in a simplified framework. In particular, as will be apparent from the discussion of the flow morphology, our dimensional simplification still captures the
most demanding aspect of the Kerr background, which is encoded in the large azimuthal shift vector near the horizon. This is most relevant for the issues discussed in this work.

For the computation, we use the general relativistic hydrodynamic equations as presented in Banyuls et al. (1997). Hence, we take advantage of the explicit knowledge of the characteristic fields in order to build up a linearized Riemann solver to handle discontinuous solutions. The formulation of the equations in Schwarzschild coordinates and the numerical algorithm can be found in Font & Ibáñez (1998a). Tests of the code can be found in Font et al. (1994) (for the subclass of special relativistic flows) and Banyuls et al. (1997) (for general relativistic flows in Schwarzschild backgrounds). To perform the simulation that we present below, we specialize the equations to the KS form of the metric (eq. [1]). Specific details, including the description of the time-dependent variables, fluxes, and source terms of the equations in BL and KS coordinates, as well as a comparative study of the accretion flows in those systems will be reported in a forthcoming paper (Font, Ibáñez, & Papadopoulos 1998, hereafter FIP).

The initial state of the fluid is completely characterized by the asymptotic conditions upstream the accretor. We choose as free parameters the asymptotic velocity $v_a = 0.5$, the sound speed $c_s = 0.1$, and the adiabatic index $\gamma = 5/3$. In addition, the accretor is a rotating black hole, and there is an extra parameter, $\alpha$, its angular momentum per unit mass. We choose a rapidly rotating hole with $\alpha = 0.999\,M$. The simulation is performed in the equatorial plane ($\theta = \pi/2$) using an $(r, \phi)$ grid of 200 × 160 zones.

Representative results of our simulations are plotted in Figure 1, where we show isocontours of the logarithm of the density at the final time $t = 500\,M$. The hole is rotating counterclockwise. This simulation employs KS coordinates, with the innermost grid radius placed at 1.0$M$, i.e., inside the horizon, $r_s = M + \sqrt{M^2 - a^2}$, which, for our model, is 1.04$M$. The outermost radius corresponds to 50$M$. The dashed line indicates the position of the horizon. The simulation is characterized by the presence of a well-defined tail shock. The flow pattern reaches a steady state at around $t \approx 100$–200$M$, which was confirmed by computing the time evolution of the mass accretion rate as described in FIP.

The transformations to horizon-adapted coordinates are non-trivial, from a geometric point of view, in the sense that a new time coordinate is involved. There is no one-to-one correspondence between snapshots of evolutions in the two different systems. Still, for accretion flows that relax to a stationary state, the time dependence is, by definition, factored out, and a direct comparison is possible by transforming four-dimensional tensor quantities appropriately (details are given in FIP). Making use of the final stationarity of the flow, we plot in Figure 2 how the accretion pattern would look were the computation performed in BL coordinates. The transformation induces a noticeable wrapping of the shock around the central hole. The shock would wrap an infinite number of times before reaching the horizon, because of an additional coordinate pathology of the BL system. As a result, the computation in BL coordinates, although possible in principle, would be much more challenging than in KS coordinates, particularly near the horizon. Generic effects such as frame dragging are expected in the case of accretion that is not on the equatorial plane. Since the last stable orbit approaches the horizon closely in the case of maximal rotation, the interesting scenario of corotating extreme Kerr accretion would be severely affected by the strong gradients that develop in the strong field region. This will most certainly affect the accuracy, and also potentially the stability, of numerical codes. An example of instabilities that are possibly related to coordinate issues has been reported recently (Igumenshchev & Beloborodov 1997).

4. SUMMARY

We have shown the feasibility of a new geometrical approach to the numerical study of accretion flows onto rotating black
holes. Our procedure relies on the use of horizon-adapted coordinate systems in which all fields, i.e., metric, fluid, and electromagnetic fields, are free of coordinate singularities at the event horizon. Among the large family of those systems, we propose the use of the Kerr-Schild coordinate system—the simplest example of the class—as the natural framework to perform accurate numerical studies of relativistic accretion flows. We have discussed our approach in the context of the various paradigms in black hole astrophysics. Our proposal shares with the frozen star picture the exact representation of the relativistic geometry. It departs from it in significant ways in the crucial horizon region, in which a choice of different time coordinates allows the explicit use of the “one-way membrane” picture in the computations. We conclude that smooth coordinate systems at the horizon become an invaluable tool for the numerical study of accretion phenomena around extreme Kerr black holes. Coupled with high-resolution numerical methods, these systems may help clarify the basic dynamical processes around accreting black holes.

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