Nuclear Enthalpies

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We propose to benefit from a concept of the enthalpy in order to include volume corrections to a nucleon rest energy, which are proportional to pressure and absent in a standard Relativistic Mean Field (RMF) with point-like nucleons. As a result a nucleon mass can decrease with Nuclear Matter (NM) density, making an Equation of State (EoS) softer. It is shown, how the EoS depends from nucleon sizes inside NM. The course of the EoS in our RMF model agrees with a semi-empirical estimate and is close to results obtained from extensive DBHF calculations with a Bonn A potential, which produce the EoS stiff enough to describe neutron star properties (mass–radius constraint), especially the masses of “PSR J16142230” and “PSR J0348+0432”, most massive (∼2M⊙) known neutron stars. The presented model has proper saturation properties, including good values of a compressibility.

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Taking into account thermodynamic effects of pressure in finite volumes, we will describe how an energy per nucleon \( \varepsilon_A = M_A/A \) and pressure evolves with NM density \( \varrho \) in an RMF approach [1, 2]. The original Walecka version [1] of the linear RMF in introduces two potentials: a negative scalar \( g_S U_S \) and a positive vector \( U_V = g_V(U^0_V, \theta) \) fitted to a nuclear binding energy at the equilibrium density \( \varrho = \varrho_0 \). The EoS for this linear, scalar–vector \( (\sigma, \omega) \) RMF model [1, 2] match a saturation point with too large compressibility \( K^{-1} = \varrho^2 \frac{d^2}{d\varrho^2} \varepsilon_A \approx 550 \text{MeV} \) and is very stiff for higher densities, where the repulsive vector potential starts to predominate the attractive scalar part. Nevertheless RMF models produces, after the Foldy-Wouthuysen reduction, the good value of a spin-orbit strength at the saturation density [1, 2]. The dynamics of the potentials in the RMF approach are discussed e.g. in four specific mean-field models [1–4]. In the ZM model [3] a fermion wave function is re-scaled and interprets a new, density dependent nucleon mass. It starts to decrease from \( \varrho = 0 \) and at the saturation point \( \varrho = \varrho_0 \) reaches 85% of a nucleon mass \( M_N \). But the nucleon mass replaced at the saturation point by a smaller value would change the nucleon deeply inelastic Parton Distribution Function (PDF) [2], shifting the Bjorken \( x \propto (1/M_N) \). Such a shift means that nucleons will carry 15% less of the Longitudinal Momentum (LM), what is evidence for a such huge enhancement [1] in the EMC effect for small \( x \). Also the nuclear Drell-Yan experiments [6, 12], which measure the sea quark enhancement, we described [13] with a small 1% admixture of nucleon pions and the \( M_N \) unchanged. Thus the deep inelastic phenomenology indicates that a change of the nucleon mass at the saturation density is rather negligible. A nonlinear extension of the RMF model [1, 2] assumes self-interaction of the \( \sigma \)-field with the help of two additional parameters fitted to \( K^{-1} \sim 250 \text{MeV} \) and an effective mass \( M^*_N = M_N + g_S U_S \). These modifications of a scalar potential give a softening of EOS with a good value of compressibility. Modern RMF calculations [9, 14] have adjusted the EOS, fitting more mesons fields (\( \rho \) for an isospin dependence) and including the octet of baryons.

We propose to improve nuclear RMF models in a different way, namely by taking into account volume contributions to a nucleon rest energy instead of a constant nucleon mass, used so far in standard RMF models. Any extended object inside a compressed medium (like a submerged submarine) needs an extra energy to preserve its volume. Thus from the “deep” point of view, finite pressure correction should be taken into account in RMF calculations with point-like nucleons, but also in the Quark-Meson Coupling (QMC) model [16]. To describe that dependence of a nucleon rest energy in a compressed medium we will adopt a bag model. Considering a role of finite nucleon sizes in compressed NM, the simplest, original \( (\sigma, \omega) \) model [1, 2] with point-like nucleons, which is too stiff, will be extended to get clear conclusions.

Fixed pressure and a zero temperature it is easy to show (see a first paragraph in a next section), that definitions of a chemical potential \( \mu \) or a Fermi energy, have the same energy balance as an average, single particle enthalpy. An enthalpy contains in a homogenous medium an interesting term, a work of a nuclear pressure \( p_H \) in a nuclear/nucleon volume, which will be investigated. It is the argument for our choice of a Gibbs free energy with an internal energy (here an internal energy) with the volume, as an independent variable. Our results are independent [17] of that choice; like expressions on a chemical potential \( \mu \) in [2].

We will neglect nuclear pion contributions above the saturation point. Dirac-Brueckner calculations show that a pion effective cross section, in the reaction of two nucleons \( N + N = N + N + \pi \), is strongly reduced at higher nuclear densities above the threshold [15] (also with RPA insertions to a self energy of \( N \) and \( \Delta \) [16]). We restrict our degrees of freedom to interacting nucleons.

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I. NUCLEAR ENTHALPY

At the beginning, let us consider effects generated by a volume of compressed NM. Start with $A$ nucleons which occupy a volume $\Omega_A = A/\varrho$. They have to perform a necessary work $W_A = p_H\Omega_A$ to keep a space $\Omega_A$ inside compressed NM against nuclear pressure $p_H = -(\partial M_A/\partial \Omega_A)$.

Thus interacting nucleons should provide not only the nuclear mass $M_A$, but rather the nuclear enthalpy

$$H_A \doteq M_A + W_A = M_A + A\frac{p_H}{\varrho}$$

(1)

which contains, besides the nuclear mass as an internal energy, the necessary work. Taking appropriate thermodynamical derivatives with respect to $A$, we get following relations between chemical potential $\mu$ and the enthalpy,

$$\mu \doteq (\partial M_A/\partial A)_{\alpha_A} \equiv (\partial H_A/\partial A)_{p_H} = \varepsilon_A + \frac{p_H}{\varrho} = H_A/A$$

(2)

for $A \to \infty$. Please note that the same relation with pressure fulfills a nucleon Fermi energy

$$E_F \doteq P_N^0(P_F) = (\partial M_A/\partial A)_{\alpha_A} = \varepsilon_A + \frac{p_H}{\varrho} = \mu$$

(3)

of a nucleon with a Fermi momentum $P_F$; well-known as the Hugenholtz-van Hove (HvH) relation [17], also proven in the self-consistent RMF approach [3].

The relativistic nuclear dynamics of nucleons in a nucleus, described by “light cone” momenta ($P_N^+, P_N^-, P_N^0$), can be formulated [6, 20] in the target rest frame, where $P_A = 0$. In order to specify a total nuclear energy $P_A^0$ in compressed NM in a single particle approach, let us discuss a longitudinal Momentum Sum Rules (MSR). Let’s focus our attention on the LM components $P_N^0 = P_N^0 + P_N^-$ of $A$ nucleons. The question is: do they add up to the internal energy $M_A$ or rather to the $H_A$, greater then $M_A$ for positive pressure? To proceed our question let us look at a LM distribution

$$f_N(y) = \frac{d^4P_N}{(2\pi)^4}\delta\left( y - \frac{AP_N^+}{P_A^-} \right) Tr[\gamma^+ G(P_N, P_A)] ,$$

(4)

with $y = AP_N^+/P_A^-$, which gives a Lorentz invariant fraction of a nucleon LM $P_N^+$ in the NM with a LM $P_A^+ = P_A^0$. This distribution is manifestly covariant and is expressed by a single nucleon Green’s function $G(P_N, P_A)$ in the nuclear medium, given e.g. in [1, 6]. The trace is taken over the Dirac and isospin indices and finally [6, 21]

$$f_N(y) = \frac{4}{\theta} \int \frac{dS(P_N)}{(2\pi)^3} \delta(y - \frac{AP_N^+}{P_A^-});$$

(5)

where a nucleon spectral function

$$S_N = n(\mid P_N \mid)\delta(P_N^0 - \sqrt{M_N^2 + P_N^2 - g_V U_N^0})$$

is given in the impulse approximation and $n$ is the Fermi distribution. Such a LM distribution [5], derived from matrix elements containing lower components of a hadron wave function, includes a flux factor $\alpha = (1+P_N^2/E_N^0)$ and thanks to this is properly normalized to the number of nucleons [20]. After integration (5) the result is:

$$f(y) = (3/4)[(AP_F)/\mu F^0][(\mu F^0)^2 - (y - AE_F/P_A^0)^2],$$

where $y$ takes the values determined by the inequality $(E_F - P_F)/P_A^0 < (y/A) < (E_F + P_F)/P_A^0$. Integrating the LM fraction $y$ in NM

$$\int dy f_N(y) = \frac{AE_F}{P_A^0} = \frac{\varepsilon_A + p_H/\varrho}{P_A^0} = 1,$$

(6)

and using HvH relation (3) in a middle step we get the longitudinal MSR (6) which gives a fraction of the nuclear LM taken by all nucleons [6, 20]; therefore equal 1.

Let us check it with the usual “on mass shell” choice: $P_A^0 = M_A = \varepsilon_A$. Then the MSR (6) is satisfied only at the saturation point where $p_H = 0$ [6]. However, in the beginning we advocate to choose the enthalpy $P_A^0 = H_A = \varepsilon_A + p_H\Omega_A$ as a total nuclear energy. Taking $H_A = \mu$ we get

$$\int dy f_N(y) = \frac{AE_F}{P_A^0} = \frac{AE_F}{H_A} = \frac{E_F}{\mu} = 1.$$ 

Now the MSR (6) is always satisfied [6] thanks to the finite volume contribution $p_H\Omega_A$ to the nuclear energy. Thus we will use enthalpies, as compact forms for total rest energies of nuclear or nucleon (parton) system.

II. NUCLEON ENTHALPY

We will discuss in a bag model, whether the nucleon mass $M_N$ or rather a nucleon enthalpy $H_N$ should be, eventually, constant - independent from the density inside the compressed medium. Such a question is absent in the standard RMF, where nucleons are point-like with the constant mass $M_N$ independent of pressure inside NM. But nucleons themselves are extended. In a compressed nucleon, partons (quarks and gluons) have to do a work $W_N = p_H\Omega_N$ to keep a space $\Omega_N$ for a nucleon "bag". It will involve functional corrections to a nucleon rest energy, dependent from external pressure with a physical parameter - a nucleon radius $R$. Others modifications connected with finite volume of nucleons, like correlations of their volumes, will be neglected. The situation is similar to nucleons inside NM described in the previous section, where we found that the MSR (7) is satisfied by the total energy $P_A^0$ equal to the nuclear enthalpy $H_A$. Analogously, we introduce a nucleon enthalpy $H_N$ with the nucleon mass $M_{pr}$ modified in the compressed medium

$$H_N(\varrho) \doteq M_{pr}(\varrho) + p_H\Omega_N$$

(7)

with $H_N(\varrho_0) = M_N$, as a “useful” expression for the total rest energy of a nucleon “bag”. Please note, that “external” pressure $p_H$
used in (7) is, of course, identical with nuclear pressure appearing in (12). Our volume corrections will change a nucleon rest energy but also will diminish effectively a free space between nucleons for the given nuclear density, what modifies an available space \( \Omega_A = (\Omega_A - A \Omega_N) \) and so nuclear pressure. Now \( p_H = -(\partial M_A/\partial \Omega_{A-})_A \). A total enthalpy \( H_A^T = H_A + A(H_N - M_N) \) and using (1,2,7) we arrive to the HvH relation with extended nucleons.

\[
H_A^T/A = \varepsilon_A - (\partial M_A/\partial \Omega_{A-})_A/\varepsilon_A + p_H/\varepsilon_A = E_F; \quad (8)
\]

### A. The nucleon mass in the Bag model in NM

Describing nucleons as bags, pressure will influence their surfaces [16, 22, 23]. Finite pressure corrections to a mass can not be described clearly by a perturbative QCD [24]. Let us discuss the relation (7) in the simple bag model where the nucleon in the lowest state of three quarks is a sphere of a volume \( \Omega_N \). Its energy \( E_{Bag} \) is a function of the radius \( R_0 \) with phenomenological constants - \( \omega_0, Z_0, B_0 \) and a density dependent bag “constant” \( (\partial (\omega_0, Z_0, B_0))_A \). We have [27]

\[
E^0_{Bag}(R_0) = \frac{3\omega_0 - Z_0}{R_0} + \frac{4\pi}{3}B(\theta_0)R^3_0 \propto 1/R_0, \quad (9)
\]

The condition

\[
p_B = - (\partial E^0_{Bag}/\partial \Omega_A)_{surface} = 0 \quad (10)
\]

for pressure inside a bag in equilibrium, measured on a bag surface, gives the relation between \( R_0 \) and \( B \), used in the end of (10). \( E^0_{Bag} \) fits to the mass \( M_N \) at equilibrium \( p_H = p_B \) = 0. \( E^0_{Bag} \) differs from the \( M_N \) by the c.m. correction [24]). In a compressed medium, pressure generated by free quarks inside the bag is balanced at the bag surface not only by intrinsic confining “pressure” \( B(q) \) but also by nuclear pressure \( p_B \) generated e.g. by elastic collisions with other hadrons [22, 24] bags, also derived in QMC model in a medium [16]. In equilibrium internal parton pressure \( p_B \) inside the bag is equal (cf. (10)), on a bag surface, nuclear pressure

\[
p_H = p_B = \frac{3\omega_0 - Z_0}{4\pi R^4} \quad (11)
\]

and we get the radius depending from \( B + p_H \):

\[
R(\theta) = \left( \frac{3\omega_0 - Z_0}{4\pi (B(\theta) + p_H(\theta))} \right)^{1/4}. \quad (11)
\]

Thus, the pressure \( p_H(\theta) \) between the hadrons acts on the bag surface similarly to the bag “constant” \( B(q) \). A mass \( M_{pr} \) for finite \( p_H(q) \) can be obtained from (11):

\[
M_{pr}(q) = \frac{4}{3} \pi R^3 [4(B + p_H) - p_H] = E^0_{bag} R_0 / R = p_H \Omega_N. \quad (12)
\]

This simple radial dependence is now lost in (12) and responsible for that is the pressure dependent correction to the mass of a nucleon given by the product \( p_H \Omega_N \). This term is identical with the work \( W_N \) in (7) and disappear for the nucleon enthalpy

\[
H_N(q) = E^0_{Bag} \frac{R_0}{R(\theta)} \times 1/R(\theta). \quad (13)
\]

The nucleon radius \( R(\theta) \) reflects a scale of a confinement of partons. Generally, for increasing \( R(\theta) \), \( H_N(\theta) \) decreasing, thus part of the nucleon rest energy is transferred from a confined region \( \Omega_N \) to an remaining space \( \Omega_A \). For decreasing \( R \), the \( H_N \) increasing: this allows the constant or increasing mass \( M_{pr} \). Let us continue with a “conventional” nuclear case, when a nucleon interaction does not change an energy of partons confined inside nucleons; therefore the enthalpy \( H_N(q) = M_N \) is constant. Now, the constant \( R \) require the work \( W_N \) to keep the constant volume at the expense of the nucleon mass \( M_{pr} \). It is obtained (11) for the constant effective pressure \( B_{eff} = B(\theta) + p_H(\theta) = B(\theta_0) \). The \( B(\theta) = B(\theta_0) - p_H \) gradually decreasing and disappears with pressure in favor of strongly correlated colored quarks in the de-confinement phase for \( p_H = B(\theta_0) \sim 60 \text{ MeVfm}^{-3} \) [24], when \( p_H \approx (0.5 - 0.6) \text{ fm}^{-3} \) (see FIG.1).

The internal pressure \( B(q) \), just as the external pressure \( p_H(q) \) (generated by an effective meson exchanges), has the same origin [25] from an interaction of quarks. Therefore, increasing \( p_H(q) \) we can expect the corresponding decrease in \( B(q) \). Really, when pressure \( p_H \) in NM is not taken into account \( p_H = 0 \) in (11) the nucleon radius \( R \), in the QMC model [23], increases in NM. However the nucleon radius \( R \) is discussed in the updated QMC model, which takes into account \( p_H \) contributions [16] to the bag radius. They found this radius as a specific property of the EoS, which depends from the nuclear compressibility. In particular, for the ZM model [3], which has the realistic value of \( K^{-1} \approx 225 \) MeV, the nucleon radius remains almost constant up to the density \( \rho = 10\rho_0 \) (the volume corrections (12) to the nucleon mass are absent). However, for the stiff EOS of the \((\sigma, \omega) \) model, they observe a strong increase of the nucleon radius up to the density \( \rho = 2\rho_0 \). Such an increase of the radius would diminish the total rest energy \( H_N \) and the nucleon mass [12], making the EOS substantially softer - as a consistent feedback. Besides, in a Global Color Symmetry Model (GCM) [24], it has been shown that a decrease of the \( B(q) \) from the saturation density \( \rho \) up to \( 3\rho \) by \( 60 \text{ MeVfm}^{-3} \) is accompanied by a similar increase of pressure \( p_H \).

Summarizing, the sum \( B(q) + p_H(q) \) weakly depends on density in GCM or QMC models with a reasonable stiff EOS, thus the bag radius remains almost constant [11]. It justify our “conventional” choice of the total nucleon rest energy \( H_N \), unchanged by an increasing NN repulsion. Just opposite to the case with the constant nucleon mass \( M_{pr} = M_N \), which requires the increasing total energy \( H_N \) [13] and a decrease of the nucleon size.
In the previous section we argued for the constant total rest energy \( H_N = M_N \), thus the size of the nucleon is constant, regardless of pressure. We applied therefore following formulas (7,8) for nucleon mass \( M_{pr} \) inside NM:

\[
M_{pr}(\rho) = M_N - p_H(\rho)\Omega_N, \quad \rho \geq \rho_0 \tag{14}
\]

\[
p_H(\rho) = \rho^2 \frac{\varepsilon_A'(\rho)}{(1 - \rho\Omega_N)}.
\]

To carry out calculations we combine the \( M_{pr} \) dependence of pressure \( p_H \) at the constant nucleon radius \( R = R_0 \), with the following standard \((\sigma - \omega)\) RMF equations [1,2] for the energy \( \varepsilon_A \) in terms of the effective mass \( M^*_{pr} \):

\[
\varepsilon_A = C_1^f \rho + \frac{C_2^f}{\rho} (M_{pr} - M^*_{pr})^2 + \frac{\gamma}{\rho_0} \int_0^{\rho_0} d\rho \frac{d^3p}{(2\pi)^3} \sqrt{P^2 + M_{pr}^2}.
\]

\[
M_{pr}^* = M_{pr} - \frac{\gamma}{2C_2^f} \int_0^{\rho_0} d\rho \frac{d^3p}{(2\pi)^3} \frac{M_{pr}^*}{\sqrt{P^2 + M_{pr}^2}}.
\]

\( \gamma \) denotes a level degeneracy and there are two \((coupling)\) constants: a vector \( C_1^f \) and a scalar \( C_2^f \), which were fitted [1,2] at two different saturation points \((\rho_0 = 0.16, 0.19 \text{ fm}^{-3} \text{ -- see a figure caption})\) in NM. In a formula \( 2C_1^f = C_2^f/M_N^2, 2C_2^f = M_N^2/C_1^f \) with \( g_\sigma U_S = M_{pr} - M^*_{pr} \). Now the finite pressure corrections to \( M_{pr} \) convert the recursive equations (15) above the saturation density \( \rho_0 \) to a differential-recursive set of equations, taking the general form

\[
f(\varepsilon_A, \varepsilon_A'') = 0 \text{ for } \rho \geq \rho_0. \tag{16}
\]

Note that (16) is obtained from the energy–momentum tensor for the model Hamiltonian with a constant nucleon mass [1]. Here we assume that the same equation with the mass \( M_{pr} \) is satisfied in compressed NM. It should be a good approximation, at least not very far from the saturation density.

Linear \((\sigma - \omega)\) models [1,2], with the constant mass \( M_N \) produce too stiff EoS; see FIG.1. Our results, which take into account nucleon volumes, are compared with a semi-experimental estimate [29] from heavy ion collisions and indeed they correct the EOS, making it much softer. We have a good course of the EoS in NM for the \( \{R_0 = 0.7 \text{ fm, set } S_2\} \) up to the density \( \rho = 0.6 \text{ fm}^{-3} \). In fact, below this density, a (partial) de-confinement is expected, which will change the EoS above a phase transition [31]. For \( R_0 = 0.55 \text{ fm, set } S_2 \) the EoS is relatively stiffer. However, it is a good candidate to investigate closely compact stars [32] in a case when hyperons will ”soften” [2,33] the EoS further. We see in the FIG.1 that both results for the set \( S_2 \) are rather close the DBHF results, which produce the EoS able to describe [34] the mass of “PSR J16142230” or “PSR J0348+0432” stars [32] (for \( R_0 = 0.7 \text{ fm} \) slightly below the DBHF for higher densities). Alternatively, for an additional softening of the EOS the \( S_1 \) parametrization with our corrections (dashed line) can be consider. It is worth mentioning that in a DBHF method there are additional corrections [30] from the self-energy, which diminish the nuclear mass with density. In our model a volume part \( p_H\Omega_N \) (14) of the constant total rest energy \( H_N = M_N \) effectively diminishes the nuclear compressibility \( K^{-1} \), changing its value from the unrealistic \( K^{-1} = 560 \text{ MeV} \) (set \( S_2 \)) to the reasonable \( K^{-1} = 290 \text{ MeV} \) obtained in our model for \( R_0 = 0.55 \text{ fm, set } S_2 \). Other features of the Walecka model, including a good value of the spin–orbit strength [1] remain unchanged in our model.

The nucleon volume \( \Omega_N \) is an important physical factor which strongly reduces [8] the available space \( \Omega_A \). The relation [8], \( E_F = \varepsilon_A + p_H/\rho \), connects the Fermi energy

\( \text{FIG. 1: Dotted lines show the pressure for NM, as a function} \]

\( \text{of the density and the constant nucleon mass } M_N, \text{ for two} \]

\( \text{different parameterizations of the } (\sigma - \omega) \text{ RMF equations} \]

\( \text{[1,2] for the energy } \varepsilon_A \text{ in terms of the effective} \]

\( \text{mass } M^*_{pr} \).

\( \text{Note that (16) is obtained from the energy–momentum} \]

\( \text{tensor for the model Hamiltonian with a constant nucleon} \]

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\( \text{The nucleon volume } \Omega_N \text{ is an important physical factor which strongly reduces [8] the available space } \Omega_A \). \text{ The relation [8], } E_F = \varepsilon_A + p_H/\rho, \text{ connects the Fermi energy} \)
with nuclear pressure $p_H$ acting in the volume $\Omega_A = (\Omega_A - A\Omega_N)$ and is met with the (0.1 - 3)% numerical accuracy; worse for a higher density, ensuring fulfillment of the MSR [3]. This is a simple generalization of the HvH relationship [11, 17] with finite-size nucleons.

**IV. CONCLUSIONS**

We have shown, how nucleon volumes in compressed NM affect the nuclear compressibility at equilibrium, reducing the nucleon mass and stiffness of the EoS. The compressibility [36] is lowered in linear ($\sigma - \omega$) model to the acceptable value, giving the good course of EoS for higher densities. The nucleon mass $M_{pr}(\rho)$ [14] occurred to be a pressure functional, what complements the expression for a nuclear energy in our model. It effectively corresponds to nonlinear, pressure dependent modifications of a scalar potential. Not accidentally, in the widely used standard [14] RMF model with point-like nucleons the good compressibility is fit by nonlinear modifications of a scalar mean field with the help of two additional parameters. Thus, our results suggests to reconsider these mean field parameters.

Particularly, when a nucleon “confining” radius is constant in density, we have found that the total rest energy $H_N$ of the nucleon is independent of density [14], although the nucleon mass decreases with $\rho$. Such a weak dependence of $R$ from $\rho$ is consistent with the phenomenological EOS. The nuclear enalphy [14] as the total nuclear energy, satisfy the longitudinal MSR [3] in the RMF approach. The presented model is suitable for studying heavy ion collisions and neutron star properties (mass–radius constraint); especially the most massive known neutron stars [33] recently discover and we plan to include the octet of baryon, including strangeness, in a next work.

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