The synergy between two threats: disinformation and Covid-19

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The breakdown of trusted sources of information is probably one of the most serious problems today, since in the absence of a common ground, it will be impossible to address the problems that trouble our contemporary world. The Covid-19 pandemic is just a recent situation where the lack of agreed stances has led to failure and hopelessness. In fact, disinformation surrounding the Covid-19 has been a distinctive feature of this pandemic since its very beginning and has hampered what is perhaps the most important initiative to prevent the spread of the coronavirus, viz., an effective communication between scientifically-minded health authorities and the general public. To investigate how disinformation threatens epistemic security, here we propose and solve analytically an evolutionary game-theoretic model where the individuals must accurately estimate some property of their hazardous environment. They can either explore the environment or copy the estimate from another individual, who may display a distorted version of its estimate. We find that the exploration-only strategy is optimal when the environment is relatively safe and the individuals are not reliable. In this doomsday scenario, disinformation erodes trust and suppresses the ability of the individuals to share information with one another.

I. INTRODUCTION

The Covid-19 pandemic has impacted our society in many different forms, some of which will probably only be known many years from now. In view of these broad repercussions, it is desirable that the mathematical and computational studies of the Covid-19 pandemic go beyond the analysis of the transmission of the physical infection through standard epidemiological models. The understanding of the more nuanced effects of the disease in the distinct age and socioeconomic segments of the population requires a multiscale and multifaceted approach.\textsuperscript{1-4, 5-7} This approach should consider also the impact of the measures to curb the spread of coronavirus (e.g., social distancing and lockdowns) on the mental health of the population,\textsuperscript{21, 22, 23} and on work productivity.\textsuperscript{22, 24, 26} This impact can be thought of as a second-order effect of the virus infection. Interestingly, the surge of loneliness associated to those prevention measures may be related to the widespread rising of authoritarianism seen today. In fact, as pointed out long ago by Hannah Arendt, “the chief characteristic of the mass man is not brutality and backwardness, but his isolation and lack of normal social relationships”.\textsuperscript{[3]} Thus, the present retrogress to authoritarianism could well be seen as a third-order effect of the coronavirus.

Perhaps one of the most important initiatives to prevent the spread of the coronavirus is an effective communication between the health authorities and the public, so as to guarantee universal access to scientific-based content about the disease. However, misinformation surrounding the Covid-19 has been a distinctive feature of this pandemic since its very beginning in Wuhan in late 2019.\textsuperscript{31} In a sample of statements about the coronavirus rated false or misleading by fact-checkers between January and March 2020, it was found that 59\% of misinformation was created by reconfiguring and recontextualizing information and 38\% was entirely fabricated.\textsuperscript{33} Here the term misinformation stands for inaccurate information that can can mislead and harm people, regardless of whether it results from an honest mistake, negligence, or intentional deception. Only in the latter case we use the term disinformation, which is then misinformation with the explicit intention to mislead.\textsuperscript{16, 17} In addition, disinformation can erode trust and inhibit our ability to share information with one another, i.e., disinformation is a threat to epistemic security. In fact, the breakdown of trusted sources of information is probably one of the most pressing problems today.\textsuperscript{18}

In this contribution we address quantitatively the threat of disinformation to epistemic security using an individual-based model where the individuals must accurately estimate a property of their surroundings in order to survive the environmental challenges. Only the survivors have a chance to contribute offspring to future generations. The individuals can either explore their surroundings or copy the estimate made by another individual. The hazardousness of the environment is determined by the parameter $\sigma$. The copying probability $w$, which we assume to be the same for all individuals, is a proxy for their credulity or, equivalently, for the degree of trust of the community. This is the leading parameter of the model and our aim is to determine the value $\hat{w}$ that maximizes the probability of survival of the individuals. In fact, the broad view of culture as something that we learn from each other\textsuperscript{8, 28} builds on copying (or sharing) of information, which suggests use of $\hat{w}$ as a measure of the degree of epistemic security of a community. Disinformation enters the model by allowing the individuals to behave deceitfully, which amounts to displaying a corrupted version of their environment’s estimates. This harmful behavior happens with probability $\gamma$.\textsuperscript{19, 40, 53, 63, 64, 70, 71}
Copying corrupted information reduces the chances of the copyist surviving an environmental challenge by a factor \(1/(1 - \eta/2)\) on average, where \(\eta \in [0, 1]\) is the cost of believing corrupted information.

Although the setup of our individual-based model is presented in a manner to stress its computer implementation, here we offer an analytical study of the model, instead. In particular, we derive an exact recursion equation for the probability of survival of the individuals in the limit of infinite population size and focus on the time-asymptotic (equilibrium) solutions. These solutions are characterized by the values of the optimal copying strategy \(\tilde{w}\).

Our main finding is that the exploration-only pure strategy \(\tilde{w} = 0\) is optimal in a large region of the space of model parameters, viz., in the regions where the probability of behaving deceitfully and the cost of believing corrupted information are moderate or large. Hence, by eroding trust, disinformation suppresses the copying or imitative behavior, which is vital for the success of our species\cite{9} and, less importantly, for the design of efficient problem-solving heuristics.\cite{20} This point is neatly expressed in the phrase\cite{10}: “Imitative learning acts like a synapse, allowing information to leap the gap from one creature to another”. Contrasting these ill-omened results, we find that the other pure strategy, viz., the copy-only strategy \(\tilde{w} = 1\) is optimal when the individuals are likely to behave honestly and the cost of copying corrupted information is high. This happens because the probability of survival of individuals in a totally credulous population (i.e., \(\tilde{w} = 1\)) does not depend on the cost \(\eta\). On the other hand, the population following a mixed strategy \(0 < \tilde{w} < 1\) is severely affected by the increase of \(\eta\). In fact, it is so badly affected that for large \(\eta\) only the pure strategies are optimal, in which case increase of \(\gamma\) leads to a discontinuous transition between the copy-only and the exploration-only strategies.

The rest of this paper is organized as follows. In Sec. \[II\] we present the rules that govern the interaction of the individuals with their environment, as well as with each other (e.g., through the copying and deceitful behaviors). We also introduce the game evolutionary dynamics that determines how the survivors of the environmental challenges will contribute to the composition of future generations. In Sec. \[III\] we derive a recursion equation for the mean number of individuals that survive the environmental challenges at generation \(t\), which we denote by \(\Lambda^{(t)}(w)\). This quantity gives a measure of the fitness of a population using the copying strategy \(w\) and its maximization with respect to \(w\) defines the optimal copying strategy \(\tilde{w}\) mentioned before. In Sec. \[IV\] we present the solutions of the recursion equation for \(\Lambda^{(t)}(w)\). We summarize our findings by presenting a phase diagram in the space of the model parameters that shows the regions where the pure and the mixed strategies are optimal. Finally, in Sec. \[V\] we review our main results and present some concluding remarks.

\section{THE MODEL}

We consider a population of \(N\) individuals that seek to assess some property of their environment, which is essential to their survival in some sense. In the context of animal behavior,\cite{14} this property could be the abundance of a key resource,\cite{13} the risk of predation in the group’s surrounding,\cite{42} or cues to migratory decision in seasonal migration.\cite{23} In the context of epistemic groups,\cite{27, 56, 37} which is the focus of the present contribution, that property could be the truthfulness of a certain stance, e.g., the efficacy of vaccines or face masks to prevent transmission of the coronavirus.

Our individual-based model builds on the following assumptions:

- Each individual \(i = 1, \ldots, N\) gathers information about the environment either through direct exploration, i.e., trial and error learning,\cite{41} or through the observation and copy of other individuals, i.e., social learning.\cite{11, 18} Accordingly, we denote the probability that a target individual copies another randomly chosen individual (the model) by \(w \in [0, 1]\). Hence, \(1 - w\) is the probability that the target individual explores the environment by itself.

- The clues about the key property of the environment are produced by a normal distribution of mean \(\mu\) and variance \(\sigma^2\). The true value of that property is the mean \(\mu\), which the individuals can estimate by sampling (i.e., exploring) the environment.\cite{2} The closeness of the estimate \(\xi \sim N(\mu, \sigma)\) to the true value \(\mu\) determines if an individual will or will not survive the environmental challenge. Examples of environmental challenge are encountering a predator or being exposed to the coronavirus.

- If individual \(i\) samples the value \(\xi_i \sim N(\mu, \sigma)\), then the probability \(S_i\) that it will survive the environmental challenge is given by

\[
S_i = \exp\left(-\frac{1}{2}(\xi_i - \mu)^2\right).
\]
We refer to the random variable $S_i$ as the viability of individual $i$. It is straightforward to derive an explicit expression for the probability distribution of $S_i$, viz.,

$$P(S) = \frac{1}{\sqrt{\pi\sigma^2}} \frac{S^{1/\sigma^2-1}}{\sqrt{-\ln S^{1/\sigma^2}}}$$

(2) for $S \in [0, 1]$. The moments of $P(S)$,

$$\mathbb{E}_S(S^n) = \int_0^1 S^n P(S) dS = \frac{1}{\sqrt{1+n\sigma^2}},$$

(3) will play a key role in the analytical study presented in Sec. III. We emphasize that since the distribution (2) does not depend on $\mu$, we can set $\mu = 0$ without loss of generality. We will refer to the variance $\sigma^2$ as the hazardousness of the environment, since the greater $\sigma^2$ is, the lesser the odds of an individual surviving the environmental challenge.

- Aside from exploring the environment by producing a sample $\xi \sim N(\mu, \sigma)$ or, equivalently, by sampling a new viability $S$ with the distribution (2), the individuals can face the environmental challenge by copying the estimate $\xi$ from another individual. Thus, it is implicit that the estimates of $\mu$ are publicly displayed by the individuals, which is a valid assumption in the epistemic context given the widespread use of social media to divulge personal viewpoints on practically any matter.

- An individual displays a corrupted version of its estimate of $\mu$ with probability $\gamma \in [0, 1]$, which results in the decrease of the viability of the copyist by a (random) factor $1/\epsilon$, where $\epsilon \sim \text{Uniform}(1-\eta, 1)$ with $\eta \in [0, 1]$. In other words, by copying an individual whose viability is $S$, the copyist has probability $\gamma$ of ending up with viability $\epsilon S$ and probability $1-\gamma$ of ending up with viability $S$. Hence the parameter $\gamma$ measures the degree of deceitfulness of the individuals, whereas $\eta$ measures the (detrimental) effect on viability of taking distorted information at face value (i.e., the cost of believing distorted information). We note that the maximum cost ($\eta = 1$) reduces the viability of the copyist by a factor 2 on average. The moments of the random variable $\epsilon$,

$$\mathbb{E}_\epsilon(\epsilon^n) = \int_{1-\eta}^1 \epsilon^n d\epsilon = \frac{1}{(n+1)\eta} \left[ 1 - (1-\eta)^{n+1} \right],$$

(4) will also be useful in the analytical study of Sec. III.

Our aim is to investigate whether there is an optimal information acquisition strategy $w$ that maximizes the chances of an individual surviving the environmental challenge in the hazardous and socially unreliable scenario introduced before. To do that we use an evolutionary game approach where the survivors contribute offspring to the next generation, who are then subjected to new challenges. In particular, survival to the environmental challenge is before. To do that we use an evolutionary game approach where the survivors contribute offspring to the next generation, who are then subjected to new challenges.

The game evolutionary dynamics proceeds as follows:

- At generation $t = 0$, each individual is assigned a random viability $S_i, i = 1, \ldots, N$ according to the probability distribution (2).

- Each individual decides independently whether to explore the environment or to copy another individual. These two alternatives occur with probability $1 - w$ and $w$, respectively. In the case individual $k$ decides to explore the environment, it generates a new sample of the viability $S_k$ using the distribution (2), which then replaces its previous viability. In the case individual $k$ decides to copy, it chooses randomly one of the $N-1$ individuals in the population, say, individual $l$. Then with probability $1 - \gamma$ individual $l$ displays its uncorrupted estimate of $\mu$, so that the viability of individual $k$ becomes $S_k = S_l$, and with probability $\gamma$ individual $l$ displays a distorted version of its estimate of $\mu$, so that $S_k = \epsilon S_l$. The update of the viabilities is done simultaneously for all individuals.

- Each individual is subjected to the environmental challenge that determines who will have a chance to contribute offspring to generation $t = 1$. Consider, for instance, individual $k$ whose viability is $S_k$ after the exploration/copying stage described in the previous item: it will survive the environmental challenge at $t = 0$ if $S_k > u_0$ where $u_0 \sim \text{Uniform}(0, 1)$. As before, all individuals are tested simultaneously and so at this point we can measure the fraction of individuals that survive the environmental challenge at generation $t = 0$, which we denote by $\Lambda^{(0)}$. 


• We form generation $t = 1$ by randomly selecting $N$ individuals with replacement from the $\Lambda(0)$ survivors of the environmental challenge at $t = 0$.

• Since we have now $N$ individuals characterized by the viabilities $S_i$, $i = 1, \ldots, N$, a situation similar to our starting point, we can repeat the procedure above to obtain the population composition at generations $t = 2, 3, \ldots$

Repetition of the game evolutionary dynamics using independent samples of the individuals’ viabilities in the initial population allows us to obtain the mean fraction of individuals that survive the environmental challenge $\langle \Lambda(0) \rangle$ at an arbitrary generation $t$. This is the main quantity we focus on in this paper, since it gives a measure of how well individuals using copying strategy $w$ are adapted to their environment. We recall that $w$ is a measure of the credulity of the individuals, $\gamma$ of their deceitfulness and $\eta$ of the cost of believing corrupted information.

Although we have described the model as an easy-to-implement individual-based simulation for finite $N$, in the next section we show how it can be solved analytically in the limit $N \to \infty$.

### III. ANALYTICAL SOLUTION FOR INFINITE POPULATION SIZE

Since all individuals are equivalent in the statistical sense (i.e., they differ only by the assignment of the random viabilities at generation $t = 0$, which are drawn from the same probability distribution), we can equate the mean fraction of individuals that survive the environmental challenge $\langle \Lambda(0) \rangle$ with the probability that an arbitrary individual survives that challenge at generation $t$. It turns out that this probability can be evaluated exactly in the limit of infinite population size $N \to \infty$, as described next.

We begin by calculating the probability that an arbitrary individual $k$ survives the environmental challenge at generation $t = 0$. There are three possible events. First, individual $k$ explores the environment by producing a new sample of its estimate of $\mu$ and, consequently, of the viability $S_k$, so the probability it survives is

$$ (1 - w) P(S_k > u_0) = (1 - w) S_k $$

since $u_0 \sim \text{Uniform}(0,1)$. Second, individual $k$ copies a randomly chosen individual, say individual $l$, who, in turn, chooses to exhibit its true estimate of $\mu$. In this event, the probability of survival of individual $k$ is

$$ \frac{w}{N-1} (1 - \gamma) P(S_l > u_0) = \frac{w}{N-1} (1 - \gamma) S_l, $$

where we have used that the probability of selecting a particular individual $l$ as a model is $1/(N-1)$. Third, individual $l$ chooses to exhibit a corrupted version of its estimate of $\mu$. Then individual $k$ survives the challenge with probability

$$ \frac{w}{N-1} \gamma P(\epsilon_{l,0} S_l > u_0) = \frac{w}{N-1} \gamma \epsilon_{l,0} S_l, $$

where $\epsilon_{l,0} \sim \text{Uniform}(1 - \eta, 1)$. Putting the contributions of these three events together, we can write down the (conditional) probability that individual $k$ survives the environmental challenge given $S_i$ and $\epsilon_{i,0}$ for $i = 1, \ldots, N$, viz.,

$$ \Lambda_k^{(0)} = (1 - w) S_k + \frac{w}{N-1} \sum_{l \neq k} S_l (1 - \gamma + \gamma \epsilon_{l,0}). $$

Hence the (conditional) mean fraction of the population that survive the environmental challenge at $t = 0$ is

$$ \Lambda^{(0)} = \frac{1}{N} \sum_{k=1}^{N} \left[ 1 \times \Lambda_k^{(0)} + 0 \times (1 - \Lambda_k^{(0)}) \right] = \frac{1}{N} \sum_{k=1}^{N} \Lambda_k^{(0)}. $$

At this point, it is important to note that the probability that an individual that survived the environmental challenge at generation $t = 0$ contributes an offspring to generation $t = 1$ is $1/N \Lambda^{(0)}$, so that each survivor contributes $1/\Lambda^{(0)}$ offspring to generation $t = 1$, on the average.

Finally, averaging Eq. (9) over the statistically independent random variables $S_i$ and $\epsilon_{i,0}$ and taking the limit $N \to \infty$ yield the unconditional mean fraction of the population that survive the environmental challenge at $t = 0$,

$$ \langle \Lambda^{(0)}(w) \rangle = (1 - w) \mathbb{E}_S(S) + w \mathbb{E}_S(S) [1 - \gamma + \gamma \mathbb{E}_\epsilon(\epsilon)] . $$

Here the moments of the random variables $S$ and $\epsilon$ are given in Eqs. (3) and (4), respectively.
The next step is to calculate the probability that target individual \( k \) survives the environmental challenge at generation \( t = 1 \). The simplest possibility is that individual \( k \) explores the environment, so that its probability of surviving the challenge at \( t = 1 \) is simply \((1 - w)S_k\), with \( S_k \) distributed according to the distribution \( \mathcal{L} \). The alternative is that individual \( k \) copies some individual \( l \) (the model), who may or may not exhibit its true estimate of \( \mu \). Clearly, the probability of choosing a model with viability \( S_l \) equals the probability of choosing an offspring of the survivor with viability \( S_l \), viz., \( 1/[(N - 1)\mathcal{L}(0)] \). Assuming that the survivor has explored the environment in the previous generation \( (t = 0, \text{in this case}), the probability that the copyist survives the challenge at \( t = 1 \) is

\[
\frac{w(1 - w)}{(N - 1)\mathcal{L}(0)} \sum_{l \neq k} P(S_l > u_0) \left[ (1 - \gamma)P(S_l > u) + \gamma P(\epsilon_{l,1}S_l > u) \right]
\]

\[
= \frac{w(1 - w)}{(N - 1)\mathcal{L}(0)} \sum_{l \neq k} S_l^2 \left[ 1 - \gamma + \gamma \epsilon_{l,1} \right],
\]

where the index of the uniform random variable \( u \) determines the generation at which the challenge was taken. Similarly, the second index of the random variables \( \epsilon \) determines the generation at which a corrupted estimate of \( \mu \) was produced. In sum, Eq. (11) yields the probability that target individual \( k \) survives the challenge by copying the offspring of an individual that has explored the environment in the previous generation. Let us assume now that the ancestor of model \( l \) has copied an authentic estimate of \( \mu \) from another individual \( m \) in the previous generation. In this case, the probability that target individual \( k \) survives the challenge at \( t = 1 \) is

\[
\frac{w^2(1 - \gamma)}{(N - 1)(N - 1)\mathcal{L}(0)} \sum_{l \neq k} \sum_{m \neq l} P(S_m > u_0) \left[ (1 - \gamma)P(S_m > u) + \gamma P(\epsilon_{m,1}S_m > u) \right]
\]

\[
= \frac{w^2(1 - \gamma)}{(N - 1)(N - 1)\mathcal{L}(0)} \sum_{l \neq k} \sum_{m \neq l} S_m^2 \left[ 1 - \gamma + \gamma \epsilon_{m,1} \right].
\]

We note that in this equation individuals \( k \) and \( l \) belong to generation \( t = 1 \), whereas individual \( m \) belongs to generation \( t = 0 \). Equation (12) yields the probability that individual \( k \) survives the challenge at \( t = 1 \) by copying the offspring of an individual that has copied an authentic estimate of \( \mu \) from another individual in generation \( t = 0 \). Lastly, we must consider the possibility that the ancestor of model \( l \) has copied a corrupted estimate of \( \mu \) from another individual \( m \) in the previous generation. In this case the probability that individual \( k \) survives the challenge at \( t = 1 \) is

\[
\frac{w^2\gamma}{(N - 1)(N - 1)\mathcal{L}(0)} \sum_{l \neq k} \sum_{m \neq l} P(\epsilon_{m,0}S_m > u_0) \left[ (1 - \gamma)P(\epsilon_{m,0}S_m > u) + \gamma P(\epsilon_{l,1}\epsilon_{m,0}S_m > u) \right]
\]

\[
= \frac{w^2\gamma}{(N - 1)(N - 1)\mathcal{L}(0)} \sum_{l \neq k} \sum_{m \neq l} \epsilon_{m,0}^2 S_m^2 \left[ 1 - \gamma + \gamma \epsilon_{l,1} \right].
\]

As before, this equation yields the probability that individual \( k \) survives the challenge at \( t = 1 \) by copying the offspring of an individual that has copied a corrupted estimate of \( \mu \) from another individual in generation \( t = 0 \).

The obstacle to proceed further, i.e., to carry out the averages over the statistically independent random variables \( S_l, \epsilon_{l,0} \) and \( \epsilon_{l,1} \), is the presence of the term \( \mathcal{L}(0) \) in the denominator of Eqs. (11), (12) and (13). To circumvent this difficulty, we will assume that the random variable

\[
\Lambda^{(t)} \equiv \frac{1}{N} \sum_{k=1}^{N} \Lambda_k^{(t)}
\]

is self-averaging in the limit \( N \to \infty \), i.e., \( \Lambda^{(t)} \to \langle \Lambda^{(t)} \rangle \) for all \( t \). The evidence we offer to support this assumption is the excellent agreement between our analytical predictions and the individual-based simulations for large \( N \) (see, e.g., Figs. 1 and 2). However, for \( t = 0 \), at least, the self-averaging assumption follows directly from the law of large numbers.

Finally, averaging over \( S_l, \epsilon_{l,0} \) and \( \epsilon_{l,1} \), and assuming that \( N \) is large so that we can write \( \Lambda^{(0)} \approx \langle \Lambda^{(0)} \rangle \) we obtain the probability that a randomly chosen individual survives the environmental challenge at generation \( t = 1 \),

\[
\langle \Lambda^{(1)}(w) \rangle = (1 - w) \left[ E_g(S) + \frac{w}{\langle \Lambda^{(0)} \rangle} b_1 E_g(S^2) \right] + \frac{w^2}{\langle \Lambda^{(0)} \rangle} b_2 E_g(S^2).
\]

(15)
where
\[ b_\tau = 1 - \gamma + \gamma E_s(\epsilon^\tau). \] (16)

The same reasoning leads to the probability that an individual survives the environmental challenge at generation \( t = 2 \),
\[ \langle \Lambda(2)(w) \rangle = (1 - w) \left[ E_S(S) + \frac{w}{\langle \Lambda(1) \rangle} b_1 E_S(S^2) + \frac{w^2}{\langle \Lambda(1) \rangle \langle \Lambda(0) \rangle} b_1 b_2 E_S(S^3) \right] + \frac{w^3}{\langle \Lambda(1) \rangle \langle \Lambda(0) \rangle} b_1 b_2 b_3 E_S(S^4), \] (17)

and at generation \( t = 3 \),
\[ \langle \Lambda(3)(w) \rangle = (1 - w) \left[ E_S(S) + \frac{w}{\langle \Lambda(2) \rangle} b_1 E_S(S^2) + \frac{w^2}{\langle \Lambda(2) \rangle \langle \Lambda(1) \rangle} b_1 b_2 E_S(S^3) \right] + \frac{w^3}{\langle \Lambda(2) \rangle \langle \Lambda(1) \rangle \langle \Lambda(0) \rangle} b_1 b_2 b_3 E_S(S^4). \] (18)

The idea behind the derivation of these expressions is to follow the ancestry of the target individual living at generation \( t \) until we find an ancestor that explored the environment. Here, ancestry is determined by who copy whom. Assume that this ancestor lived at generation 0 \( \leq \tau \leq t \) and that \( \gamma = 0 \) (hence \( b_i = 1 \)), for simplicity. Clearly, the probability of this happening is proportional to \( w^{t-\tau}(1 - w) \). In addition, the viability \( S \) drawn by the ancestor at generation \( \tau \) must pass \( t - \tau + 1 \) independent challenges and the descendants of the ancestor in the lineage of the target individual must be chosen in all the repopulation stages that take place from \( \tau \) to \( t - 1 \). The probability that this happens is
\[ \frac{P(S > u_\tau)}{\langle \Lambda(\tau) \rangle} \times \frac{P(S > u_{\tau+1})}{\langle \Lambda(\tau+1) \rangle} \times \ldots \times \frac{P(S > u_{t-1})}{\langle \Lambda(t-1) \rangle} \times P(S > u_t) \] (19)

which reduces to \( S^{t-\tau+1} \frac{\langle \Lambda(\tau) \rangle \langle \Lambda(\tau+1) \rangle \ldots \langle \Lambda(t-1) \rangle}{\langle \Lambda(t) \rangle} \) since \( u_i \sim \text{Uniform}(0, 1) \). Of course, there is also the possibility that all ancestors of the target individual down to generation \( t = 0 \) are copyists, which happens with probability proportional to \( w^{t+1} \), and is accounted for by the last terms in Eqs. (10), (15), (17) and (18). At this stage, we can readily write down the generalization of those recursion equations for arbitrary \( t \),
\[ \langle \Lambda(t)(w) \rangle = (1 - w) \sum_{\tau=0}^{t} a_{\tau,t} E_S(S^{\tau+1}) w^\tau + a_{t+1,t} E_S(S^{t+1}) w^{t+1} \] (20)

with \( a_{0,t} = 1 \),
\[ a_{\tau,t} = \frac{b_\tau}{\langle \Lambda(t-\tau) \rangle} a_{\tau-1,t} \] (21)

for \( \tau > 1 \), and we have defined \( \langle \Lambda(-1) \rangle = 1 \). In Sec. IV we offer explicit expressions for \( \langle \Lambda(t) \rangle \) in the limiting cases \( w = 0 \) and \( w = 1 \), as well as the first terms of its expansion in powers of \( w \).

IV. RESULTS

Figure 1 shows the effect of the population size \( N \) on the mean fraction of individuals that survive the environment challenge at generation \( t = 100 \). As pointed out, \( \langle \Lambda(t) \rangle \) is a proxy for the quality of the adaptation of the individuals to their environment, whereas the copying probability \( w \) is a proxy for the credulity of the individuals. In this figure, we set the deceitfulness of the individuals to \( \gamma = 0.5 \) and the cost of copying (or believing) corrupted information to \( \eta = 0.1 \). This means that the individuals have a 50% chance of behaving deceitfully and that the survival probability of a copyst is reduced by a factor of \( 1/(1 - \eta/2) = 100/95 \approx 1.05 \) on the average when copying a deceitful model.

The primary purpose of Fig. 1 is to show the excellence of the analytical prediction for large \( N \): already for \( N = 100 \) the individual-based simulations depart significantly from that prediction only in the region very close to
FIG. 1. Mean fraction of individuals that survive the environmental challenge at generation $t = 100$ against the copying probability $w$ for $N = 4$ (□), $N = 6$ (●), $N = 8$ (△) and $N = 100$ (○). The solid curve is the analytic prediction for $N \to \infty$. The other parameters are $\gamma = 0.5$, $\eta = 0.1$ and $\sigma^2 = 1$.

For $w = 1$. We note that for the small population sizes considered (viz., $N = 4, 6$ and $8$), all runs resulted in extinction for $w = 1$, whereas no extinction was observed for $N = 100$. This is expected since for $w = 1$ the individuals sample their environment at the initial generation $t = 0$ only and so their viabilities $S_k, k = 1, \ldots, N$ can never be greater than the maximum of $N$ samples drawn from the distribution $\mathcal{E}$. Of course, for sufficiently large $t$, the runs with $N = 100$ (and $w = 1$) should all result in extinction as well (see Fig. 2). This is not the case for $N \to \infty$, however. In fact, in this limit we can write an explicit equation for $\langle \Lambda(t)(1) \rangle$, viz.,

$$\langle \Lambda(t)(1) \rangle = \left[1 - \gamma + \gamma \mathbb{E}_\epsilon(\epsilon^{t+1}) \right] \frac{\mathbb{E}_S(S^{t+1})}{\mathbb{E}_S(S^t)} = \left[1 - \gamma + \frac{\gamma}{(t + 2)\eta} \right] \frac{1 + \eta^2}{1 + \eta^{t+2}}. \quad (22)$$

from where we obtain

$$\lim_{t \to \infty} \langle \Lambda(t)(1) \rangle = 1 - \gamma, \quad (23)$$

provided that $\eta > 0$. (For $\eta = 0$, Eq. 22 yields $\lim_{t \to \infty} \lim_{\eta \to 0} \langle \Lambda(t)(1) \rangle = 1$.) Thus, extinction is not certain if $\gamma < 1$, i.e., if there is a nonzero chance that the individuals will behave honestly. Nevertheless, a low value of $\langle \Lambda(t)(1) \rangle$ points to the maladaptation of a population of totally credulous individuals. Figure 2 summarizes these results. We stress that the observed slow convergence to the infinite population size limit as $N$ increases only for $w = 1$, because a finite population is certain to go extinct in this case. For $w < 1$, the convergence to the infinite population size limit is much faster, as illustrated in Fig. 1.

Another limiting case for which we can obtain an explicit solution of the recursion equation (20) is $w = 0$, which describes the situation where the individuals never copy their peers. In this case we have

$$\langle \Lambda(t)(0) \rangle = \mathbb{E}_S(S) = \frac{1}{\sqrt{1 + \sigma^2}}, \quad (24)$$

which does not depend on the generation $t$, as expected, since at each generation all individuals produce new estimates of $\mu$, as in the setup of the initial generation. It is interesting to note that in our model the popular notion of culture as information and experiences transferred from the older generations to the newer generation[9, 28] is intimately associated to a nonzero value of the copying probability $w$, which means that culture requires individuals that possess some degree of credulity.
FIG. 2. Mean fraction of individuals that survive the environmental challenge as function of the generation $t \leq 300$ for a population of totally credulous individuals ($w = 1$) of size $N = 10$ (□), $N = 10^2$ (○), $N = 10^3$ (△) and $N = 10^4$ (□). The solid curve is the analytic prediction (22) for $N \to \infty$. The other parameters are $\gamma = 0.5$, $\eta = 0.1$ and $\sigma^2 = 1$. In the limit $t \to \infty$ we find $\langle \Lambda^1(t) \rangle \to 0.5$. The generation index $t$ was shifted by one unit to the right to allow use of the logscale in the x-axis.

FIG. 3. Mean fraction of individuals that survive the environmental challenge for a population of infinite size at equilibrium against the copying probability $w$ for $\gamma = 0.5$, $\sigma^2 = 1$ and (top to bottom) $\eta = 0, 0.1, 0.3, 0.5, 0.7$ and 1. The dashed horizontal line is $\langle \Lambda^0(0) \rangle = 1/\sqrt{2} \approx 0.707$. For $\eta > 0$ we have $\langle \Lambda^0(1) \rangle = 0.5$, so that $\langle \Lambda^0(0) \rangle > \langle \Lambda^0(1) \rangle$. The critical cost of copying corrupted information beyond which epistemic security is lost is $\eta_c^0 = 4 - 2\sqrt{3} \approx 0.536$.

Henceforth we will consider only the limit of infinite population size $N \to \infty$, for which the recursion equation (20) holds true and can easily be iterated numerically to produce the mean fraction of individuals that survive the environmental challenge at any generation $t$. In particular, we will focus on the equilibrium regime, which we can assume to be reached at generation $t = 1000$ (see Fig. 2), so we can write $\langle \Lambda^\infty(w) \rangle \approx \langle \Lambda^{1000}(w) \rangle$ for all $w \in [0, 1]$. Although this is a safe assumption in general, it fails to produce high precision estimates of $\langle \Lambda^\infty(w) \rangle$ for $w \approx 1$ that are needed to determine the values of some critical parameters, as we will see later in this section. In that case we assume that the equilibrium is reached for $t = 10^4$. 
credulous individuals is $\eta$ (see Eqs. (23) and (24)) emerging since, as pointed out before, culture requires the copying of information produced by older generations.[9, 28]

The scenario where condition (25) is violated is illustrated in Fig. 4, which shows $\langle w \rangle$ which is satisfied when the probability of behaving deceitfully is high and the hazardousness of the environment is low. The remarkable result here is that above a critical cost $\eta_c$ we find $\bar{w} = 0$. In other words, the optimal strategy for survival is not copying at all, which means that culture (or an epistemic society) would never emerge since, as pointed out before, culture requires the copying of information produced by older generations.[0][23]

However, the scenario depicted in Fig. 3 happens only for $\langle \Lambda^{(\infty)}(1) \rangle < \langle \Lambda^{(\infty)}(0) \rangle$ that corresponds to the condition (see Eqs. (23) and (24))

$$\gamma > 1 - \frac{1}{\sqrt{1 + \sigma^2}},$$

which is satisfied when the probability of behaving deceitfully is high and the hazardousness of the environment is low. The scenario where condition (25) is violated is illustrated in Fig. 4, which shows $\langle \Lambda^{(\infty)} \rangle$ for $\gamma = 0.2$ and the same values of $\sigma^2$ and $\eta$ used in Fig. 3. As the cost $\eta$ increases from 0 to 1, the optimal copying probability $\bar{w}$ initially decreases indicating the advantage of some degree of skepticism, as expected, but then it starts to increase and at a critical value $\eta_c$ it reaches 1, remaining fixed at that value as $\eta$ increases further towards 1 (see curve for $\gamma = 0.2$ in Fig. 5). This apparently counter-intuitive result has a simple explanation: since $\langle \Lambda^{(\infty)}(1) \rangle$ does not depend on the cost $\eta$ (see Eq. (23)), increase of $\eta$ does not affect the survival of a population of totally credulous individuals ($w = 1$), while it is detrimental to a population of skeptical individuals ($w < 1$), as shown in Fig. 4. To understand why $\langle \Lambda^{(\infty)}(1) \rangle$ does not depend on $\eta$, we note that for $\gamma < 1$ there is always a chance that at least one individual behaves honestly, in which case the copied information is authentic. The fortunate copyst is very likely to get through the environmental challenge and, in the worst-case scenario that all the other individuals fail that challenge, repopulate the entire population with its clones.

Figure 5 illustrates the complex influence of the parameters $\gamma$ and $\eta$ on the optimal copying strategy $\bar{w}$. As pointed out before, there are two very distinct scenarios depending on whether $\langle \Lambda^{(\infty)}(0) \rangle$ is less or greater than $\langle \Lambda^{(\infty)}(1) \rangle$. In fact, given that $\bar{w}$ is obtained by maximizing the mean fraction of survivors $\langle \Lambda^{(\infty)}(w) \rangle$, it is clear that $\bar{w} < 1$ if $\langle \Lambda^{(\infty)}(0) \rangle > \langle \Lambda^{(\infty)}(1) \rangle$ (the trivial exception is for $\eta = 0$) and $\bar{w} > 0$, otherwise. In the first scenario, where the probability of behaving deceitfully is high, increasing the cost of copying corrupted information gives rise to a continuous phase transition to a regime characterized by individuals that solely explore their environment, because...
FIG. 5. Optimal copying probability $\tilde{w}$ against the cost of copying corrupted information $\eta$ for $\sigma^2 = 1$ and (top to bottom) $\gamma = 0.2, 0.28, 0.29, 0.3, 0.5$ and 1. For $\gamma > 1 - 1/\sqrt{2} \approx 0.293$ there is a continuous transition at $\eta_c^0$ to a regime where epistemic security is lost ($\tilde{w} = 0$), whereas for $\gamma < 1 - 1/\sqrt{2}$ there is a transition at $\eta_c^1$ to a regime where the individuals are totally credulous ($\tilde{w} = 1$). This transition is continuous for $\gamma = 0.2$ and discontinuous for $\gamma = 0.28$ and 0.29.

FIG. 6. Mean fraction of individuals that survive the environmental challenge for a population of infinite size at equilibrium against the copying probability $w$ for $\gamma = 0.2, \sigma^2 = 1$ and (top to bottom) $\eta = 0.45, 0.46, 0.47, 0.48$ and 0.49. The dashed horizontal line is $\langle \Lambda(\infty) \rangle(1) = 1 - \gamma = 0.8$. The point at which the continuous transition takes place is $\eta_c^1 \approx 0.480$.

copying is too risky. This doomsday scenario where culture is lost is characterized by the pure strategy $w = 0$. Surprisingly, in the second scenario, where the individuals are likely to behave honestly, increasing the cost $\eta$ prompts a transition to a regime where the population is composed of totally credulous individuals, i.e., the pure strategy $w = 1$ is optimal. The transition is continuous for small $\gamma$ but becomes discontinuous with increasing $\gamma$. In particular, for large $\eta$, say $\eta = 1$, $\tilde{w}$ jumps from 0 to 1 at $\gamma = 1 - 1/\sqrt{1 + \sigma^2}$.

In the case that condition (25) is satisfied, i.e., for $\gamma > 1 - 1/\sqrt{1 + \sigma^2}$, we can derive analytical expressions for the critical point $\eta_c^0$, as well as for the optimal copying probability $\tilde{w}$ near the critical point. In fact, examination of Fig. 5 reveals that the condition for $\tilde{w} > 0$ is that the derivative of $\langle \Lambda(\infty)(w) \rangle$ calculated at $w = 0$ be positive, so that the
critical cost $\eta^0$ can be determined by setting that derivative to zero. To take advantage of this observation, we need to expand $(\Lambda^{(2)})$ in powers of $w$. It is easy to see that to obtain the correct terms of order $w^k$ for $k = 0, \ldots, t$, it is enough to expand $(\Lambda^{(k)})$ to order $w^k$. In other words, the coefficients of the powers $w^0$, $w^1$ and $w^2$ are the same in the expansion of $(\Lambda^{(2)})$ as in the expansion of $(\Lambda^{(\infty)})$. We find

$$\langle \Lambda^{(\infty)} \rangle \approx E_S(S) - wE_S(S) \left[ 1 - b_1 \frac{E_S(S^2)}{E_S(S)} - w^2 b_1 \left( b_1 \frac{E_S(S^2)}{E_S(S)} - b_2 \frac{E_S(S^3)}{E_S(S)} \right) \right]$$

(26)

with $b_1$ defined in Eq. (16). Hence the critical or threshold parameter is determined by the equation

$$b_1 = \frac{E_S(S)}{E_S(S^2)}$$

(27)

that can be rewritten as

$$\eta^0_w = \frac{2}{\gamma} \left( 1 - \frac{\sqrt{1 + 2\sigma^2}}{1 + \sigma^2} \right).$$

(28)

We note that condition $\eta^0_w \geq 0$ is always satisfied, but condition $\eta^0_w \leq 1$ is satisfied only for

$$\gamma > 2 \left( 1 - \frac{\sqrt{1 + 2\sigma^2}}{1 + \sigma^2} \right).$$

(29)

Hence the maximum value that $\eta^0_w$ can take on is determined by the lower bound of $\gamma$, which is given either by inequality (25) or by inequality (29). Explicitly, for $\sigma^2 < 1.218$ inequality (25) holds and the maximum of $\eta^0_w$ is

$$\eta^0_w = 2 \left( 1 - \frac{\sqrt{1 + 2\sigma^2}}{1 + \sigma^2} \right) \left( 1 - \frac{1}{\sqrt{1 + \sigma^2}} \right)^{-1},$$

(30)

whereas for $\sigma^2 > 1.218$ inequality (29) holds and then $\eta^0_w = 1$. In Fig. 5 we have $\sigma^2 = 1$ so that $\eta^0_w = 2(1 - \sqrt{3}/2)/(1 - \sqrt{2}/2) \approx 0.915$.

Equation (26) also allows us to determine how $\bar{w}$, which can be seen as the order parameter of our model, vanishes when $\eta$ approaches $\eta^0_w$ from below, viz.,

$$\bar{w} \approx C (\eta^0_w - \eta)$$

(31)

where

$$C = \frac{\gamma}{4} \left[ \frac{E_S(S)}{E_S(S^2)} - b_2 \frac{E_S(S^3)E_S(S)}{E_S(S^2)} \right]^{-1}.$$
FIG. 7. Mean fraction of individuals that survive the environmental challenge for a population of infinite size at equilibrium against the copying probability $w$ for $\gamma = 0.29, \sigma^2 = 1$ and (top to bottom) $\eta = 0.85, 0.86, 0.8647, 0.87$ and 0.88. The dashed horizontal line is $\langle \Lambda^{(\infty)}(1) \rangle = 1 - \gamma = 0.71$. The point at which the discontinuous transition takes place is $\eta^c_1 \approx 0.8647$.

Figure 8 exhibits the rich phase diagram for $\sigma^2 = 1$ that summarizes the results presented up to now. In addition to the continuous and discontinuous phase transitions, we note the existence of a triple point at which the three regimes (or phases) coexist. The triple point is determined by the conditions $\langle \Lambda^{(\infty)}(0) \rangle = \langle \Lambda^{(\infty)}(\tilde{w}) \rangle = \langle \Lambda^{(\infty)}(1) \rangle$, which imply $\gamma_{\text{triple}} = 1 - 1/\sqrt{1 + \sigma^2}$ and $\eta_{\text{triple}} = \tilde{\eta}_0^c$ with $\tilde{\eta}_0^c$ given by Eq. (30).

To conclude our analysis, we stress that increase of the hazardousness of the environment $\sigma^2$ decreases the region of dominance of the unwelcome regime $\tilde{w} = 0$ in the parameter space $(\eta, \gamma)$. In particular, that regime disappears altogether when the minimum value of $\eta_{0c}$, which is obtained by setting $\gamma = 1$ in Eq. (28), is greater than 1 (see Fig. 8), i.e., for $\sigma^2 > 3 + 2\sqrt{3} \approx 6.46$. Hence, for highly hazardous environments, the exploration-only pure strategy is never optimal, regardless of the degree of deceitfulness of the individuals or of the cost of believing distorted information.

V. DISCUSSION

Our analytically solvable model for the effects of disinformation on epistemic security offers two main lessons:

- If the cost of believing corrupted information is not low, which happens to be the case for information regarding Covid-19 as the hospitalization and death rates among the unvaccinated are much higher than among the vaccinated,\[15\] then there is a maximum degree of deceitfulness the population can bear before trust is completely eroded and the optimal strategy for survival becomes to trust no one. In the context of the Covid-19 pandemic, this phenomenon is observed in the slow pace of vaccination rates among the elderly in China,\[44\] which may be in part because misinformation about mRNA technology led to mistrust of vaccines in general, but it may also reflect a broader distrust of the government.

- A completely credulous population can be surprisingly robust against the deceitful behavior of its members, provided the risk of exploring the environment (i.e., of finding the truth by oneself) is not too low (see Fig. 8). In fact, copying (uncorrupted) information is an almost sure bet, since the copyist acquires a viability that has already passed at least one environmental challenge. Hence our model predicts that the harsher the environment is, the greater the trust on the information exhibited by the older generations.

In order to study our model analytically we have opted for a population-centred approach,\[8\] in which the quantity to be maximized is the mean fitness of the population, with no reference to the interests of their members, which we choose to behave identically on the average, anyway. A more plausible scenario, however, is one where the individuals behave so as to maximize their own chances of survival and of passing their offspring to the next generation. Of course, the latter goal is best achieved by deceiving other individuals, so they are likely to fail the environmental challenge.
We have run extensive individual-based simulations of a scenario where the individuals are assigned random values of the copying probability \( w \), but the same degree of deceitfulness \( \gamma \), in the initial generation. The game evolutionary dynamics was followed until fixation occurs, i.e., until all individuals exhibit the same value of \( w \) – the winner of the contest – so they share a common ancestor at some previous generation.\[39\] We found that the average of the winner values of \( w \) over distinct runs exhibits a behavior that is qualitatively similar to that of the population-centred quantity \( \tilde{w} \) studied here. In particular, the critical point \( \eta_0 \) coincides with our theoretical estimate \( (28) \). These simulation results, together with those where the initial population is heterogeneous regarding both \( w \) and \( \gamma \), will be presented elsewhere.

To conclude, we note that use of evolutionary game theory to study the antagonism between truth-telling and lying has a long tradition both in the biological and the sociological contexts.\[40\] More recently, however, analogous problems have been successfully addressed by the active particles methods, which stand as an alternative general strategy toward the modeling of the collective dynamics of large systems of interacting living entities.\[5\]

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