The two-qubit controlled-phase gate based on cross-phase modulation in GaAs/AlGaAs semiconductor quantum wells

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We present a realization of two-qubit controlled-phase gate, based on the linear and nonlinear properties of the probe and signal optical pulses in an asymmetric GaAs/AlGaAs double quantum wells. It is shown that, in the presence of cross-phase modulation, a giant cross-Kerr nonlinearity and mutually matched group velocities of the probe and signal optical pulses can be achieved while realizing the suppression of linear and self-Kerr optical absorption synchronously. These characteristics serve to exhibit an all-optical two-qubit controlled-phase gate within efficiently controllable photon-photon entanglement by semiconductor mediation. In addition, by using just polarizing beam splitters and half-wave plates, we propose a practical experimental scheme to discriminate the maximally entangled polarization state of two-qubit through distinguishing two out of the four Bell states. This proposal potentially enables the realization of solid states mediated all-optical quantum computation and information processing.

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I. INTRODUCTION

It is well known that photons are ideal carriers in all-optical quantum information processing and computation since of their potentially wide range of applications and that they suffer little from decoherence. The lack of effective coherent photon-photon interactions and strong optical nonlinearities previously result in a serious obstacle to perform quantum computation and communication in conventional optical systems [1]. Fortunately, it has already been realized that a strong enough optical nonlinearity, typically Kerr nonlinearity, which corresponds to the refractive part of third-order susceptibility in an optical medium and plays a pivotal role in the field of nonlinear optics, could be available to mediate a photon-photon interaction [2].

In the early days of nonlinear optics, in spite of the far-off resonance or resonant excitation schemes, the Kerr nonlinearity is very small or may include serious optical absorption. Simultaneously, a large third-order susceptibilities requires the linear susceptibility to be as small as possible for the sake of minimizing the absorption of all fields participating in the nonlinear process [3]. However, this difficult can be solved by introducing the electromagnetically induced transparency (EIT) in such systems, which is capable of modifying the linear and nonlinear optical properties of medium predominantly in the resonant atomic systems [3–5]. Namely, a large cross-Kerr nonlinearity with nonlinear coefficient, wide adjustable parameters, and flexibility. As a matter of fact, it was generally recognized that such a semiconductor QW structures also have inherent advantages such as large electric dipole moments, high nonlinear optical coefficients, wide adjustable parameters, and flexibility. So that it is more helpful for realizing high-quality quantum coherence and interference effects to the solid state quantum systems, including semiconductor quantum wells (QW) and quantum dots, of which the discrete energy levels and optical properties are extremely analogy to atomic systems. As a consequence, there have been numerous developments on the quantum coherence and interference effects in semiconductor QW systems, for example, EIT and double EIT [12–14], ultrafast all-optical switching [15], slow light solitons [16–17], tunneling-induced transparency and related phenomenon based on Fano interference [18–24], etc.

As a matter of fact, it was generally recognized that such a semiconductor QW structures also have inherent advantages such as large electric dipole moments, high nonlinear optical coefficients, wide adjustable parameters, and flexibility. As a consequence, there have been numerous developments on the quantum coherence and interference effects to the solid state quantum systems, including semiconductor quantum wells (QW) and quantum dots, of which the discrete energy levels and optical properties are extremely analogy to atomic systems. As a consequence, there have been numerous developments on the quantum coherence and interference effects in semiconductor QW systems, for example, EIT and double EIT [12–14], ultrafast all-optical switching [15], slow light solitons [16–17], tunneling-induced transparency and related phenomenon based on Fano interference [18–24], etc.

In our scheme, a giant cross-Kerr nonlinearity with nearly π-conditional nonlinear phase shifts and mutually matched group velocities (slowed) can be achieved due to cross-phase modulation effect in the context of EIT, accompanied by vanishing the linear and self-Kerr optical absorption. Based on these characteristics, the two-qubit polarization quantum phase gate can be implemented within long-time interaction and effective maximal entanglement. Since the polarization single qubit rotation gate can be easily realized, the universal quantum computation can thus be achieved [1]. In addition, we propose a practical experimental scheme to identify the maximally entangled optical polarization state of two-qubit with two out of the four Bell states.

This paper is organized as follows. We introduce a four-level asymmetrical coupled-double GaAs/AlGaAs semicon-
ductor QW system with intersubband transitions in Sec. II. The linear and nonlinear optical properties of this system are studied in Sec. III. In Sec. IV, within the group-velocity matching of the probe and signal optical fields, we demonstrate the implementation of the two-qubit controlled-phase gate as well as the creation of maximally entangled state and propose a practical experiment to discriminate the maximally entangled state of the two-qubit through discriminating two out of four Bell states. A summary of our main conclusions are given in the final section.

II. THE ASYMMETRIC COUPLE QUANTUM WELLS

We consider an asymmetric coupled double GaAs/AlGaAs QW composed of four subbands (electron states) in the conduction band as shown in Fig.1, which has been realized in the latest experiment [25]. The double QW structure consists of coupled GaAs QW of 9 and 12 nm width, separated by 2 nm thick Al$_0$.35Ga$_0$.65As [see Fig. 1(a)]. For simplicity, we consider a schematic energy diagram as Fig. 1(b) which can also be regarded as four-level N configuration system. The description of the possible transitions are dipole allowed in such a system which interacts with two weak linear-polarized (pulsed) probe and signal fields and a continuous-wave (cw) pump lasers as follows. The two weak probe and signal fields, with half Rabi frequency ($\Omega_p = \mu_{31}E_p/2\hbar$) and the center frequency $\omega_p$, and with half Rabi frequency $\Omega_s = \mu_{42}E_s/2\hbar$ and the center frequency $\omega_s$, drive the transition $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$, respectively. While the strong control field with half Rabi frequency ($\Omega_c = \mu_{32}E_c/2\hbar$) and the center frequency $\omega_c$ is acting on the transition $|2\rangle \leftrightarrow |3\rangle$. Here, the dipole moments of the transitions $|i\rangle \leftrightarrow |j\rangle$, $\mu_{ij}$($i,j = 1,2,3,4$), are the polarization unit vectors of the laser field. The electric-field vector of the system can be written as $E_l$ ($l=p,c,s$), likewise $+\sigma$ and $-\sigma$ are the unit vectors of the right-hand circularly and left-hand circularly polarized basis, respectively.

Since the semiconductor QW structure are low doping, the many body effects resulting from electron-electron interactions may be neglected in our system [26]. Working in the interaction picture, by utilizing the rotating wave approximation and electro-dipole approximation [5,9,14], the semi-classical Hamiltonian of the system can be written as,

$$H_{int}/\hbar = - \begin{pmatrix} 0 & 0 & \Omega^*_p & 0 \\ 0 & \Delta_p - \Delta_c & \Omega^*_c & 0 \\ \Omega_p & \Omega_c & \Delta_p & 0 \\ 0 & \Omega_s & 0 & \Delta_p - \Delta_c + \Delta_s \end{pmatrix},$$

where $\Delta_p = \omega_p - \omega_{31}$, $\Delta_c = \omega_c - \omega_{32}$ and $\Delta_s = \omega_s - \omega_{42}$ are the one-photon detunings which denote the frequency difference between the center and the intersubband transitions $\omega_{ij}$ ($i,j = 1,2,3,4$) of the $|i\rangle \leftrightarrow |j\rangle$. By applying the linear Schrödinger equation, $i\hbar \partial |\Psi>/\partial t = H_{int}|\Psi>$, with $|\Psi>$ being the electronic energy state, the evolution equations for the sub-

![FIG. 1: (Color online) (a) Schematic conduction band profile for a single period of the asymmetric double-coupled GaAs/AlGaAs quantum wells structure consists of four discrete subband levels. (b) Schematic energy diagram, which corresponds to (a), consist of a four-level quantum well system in N configuration. They are labeled as |1\rangle, |2\rangle, |3\rangle and |4\rangle, respectively. $\Omega_p$ (polarization) is the half Rabi frequency of the probe field acts on the subband transition |1\rangle \leftrightarrow |3\rangle, $\Omega_c$ and $\Omega_s$ (c polarization) are the half Rabi frequency of the control and signal fields which interact with the subband transitions |2\rangle \leftrightarrow |3\rangle and |2\rangle \leftrightarrow |4\rangle, respectively. $\Delta_j$ ($j=2,3,4$) is single-photon detunings.]

bands probability amplitudes are

$$\frac{\partial A_1}{\partial t} = i\Omega_p^*A_3,$$  \hspace{1cm} (1a)
$$\frac{\partial A_2}{\partial t} = i(\Delta_2 + i\gamma_2)A_2 + i\Omega_p^*A_3 + i\Omega_s^*A_4,$$  \hspace{1cm} (1b)
$$\frac{\partial A_3}{\partial t} = i(\Delta_3 + i\gamma_3)A_3 + i\Omega_pA_1 + i\Omega_cA_2,$$  \hspace{1cm} (1c)
$$\frac{\partial A_4}{\partial t} = i(\Delta_4 + i\gamma_4)A_4 + i\Omega_sA_2,$$  \hspace{1cm} (1d)

where $A_j$ is the probability of the subband state $|j\rangle$ ($j=1-4$) satisfying the conservation condition $\sum_{j=1}^4 |A_j|^2 = 1$. The detunings are defined by $\Delta_2 = \omega_p - \omega_{31}$, $\Delta_3 = \omega_p - \omega_{32}$ and $\Delta_4 = \omega_s - (\omega_p - \omega_{31})$ (see Fig. 1), respectively. $\gamma_j$ represents the decay rates of level $|j\rangle$ which results from the the effect of lifetime broadening contribution. It is primarily due to the longitudinal-optical phonons emission events at low temperature, and dephasing, which is mainly owing to the electron-electron scattering, phonons scattering processes and the elastic interface roughness in such a QW structure.

III. LINEAR AND NONLINEAR OPTICAL SUSCEPTIBILITIES

To obtain the propagating properties of the probe and signal fields, we suppose the electric field $E_{p(s)} = \varepsilon_{p(s)} \exp[i(k_{p(s)} - \omega_{p(s)}t)] + c.c.$ Under the slowly varying amplitude approximation, we have

$$i \left( \frac{\partial}{\partial z} + \frac{1}{v_p} \frac{\partial}{\partial t} \right) \varepsilon_j + \frac{\omega_j}{2c} \chi_j \varepsilon_j = 0, (j = p, s)$$  \hspace{1cm} (2)
where \( v_{j=\mu,s}(s) \) is the group velocity of the probe (signal) field, which are defined as \( v_{g}(s) = c / (1 + n_{g}(s)) \), with \( n_{g}(s) = \text{Re}(\chi_{g}(s))/2 + (\omega_{g}(s)/2)|\text{Re}\chi_{g}(s)|/\partial\omega|_{\omega=\omega_{g}(s)} \) being the index of refraction and \( \chi_{g}(s) \) being the susceptibilities of the probe (signal) field.

We here suppose that the electrons are initially populated in the subband level \([1]\) and the typical temporal duration of the probe and signal fields is long enough so that the equations can be solved adiabatically. Under these approximation, we can obtain the steady-state solutions of Eq. (1). With the slowly varying parts of the polarizations of the probe and signal fields being, i.e., \( P_{g} = \epsilon_{0}X_{g}E_{g} = N\mu_{13}A_{3}A_{1}^{*} \) and \( P_{s} = \epsilon_{0}\chi_{s}E_{s} = N\mu_{24}A_{4}A_{2}^{*} \) (with \( \epsilon_{0} \) being the permittivity in free space and \( N \) being the electron density in the conduction band of the quantum well structure), the expressions of the electric susceptibilities of the probe and signal fields are

\[
\chi(\omega_{p}) = \frac{N|\mu_{13}|^{2}}{2\epsilon_{0}\hbar\Omega_{p}}A_{3}A_{1}^{*} \\
\approx \chi^{(1)}_{p} + \chi^{(3)}_{ps} |\varepsilon_{p}|^{2} + \chi^{(3)}_{pc} |\varepsilon_{s}|^{2}, \tag{3}
\]

and

\[
\chi(\omega_{s}) = \frac{N|\mu_{24}|^{2}}{2\epsilon_{0}\hbar\Omega_{p}}A_{4}A_{2}^{*} \approx \chi^{(3)}_{sc} |\varepsilon_{p}|^{2}, \tag{4}
\]

respectively. Here \( \chi^{(1)}_{p} \) is the linear susceptibility; \( \chi^{(3)}_{ps} \) and \( \chi^{(3)}_{pc} \) respectively depicts the third-order self-Kerr and cross-Kerr nonlinear susceptibility of the probe field; \( \chi^{(3)}_{sc} \) denotes the cross-Kerr nonlinear susceptibility of the signal field. The specific expressions of the linear and nonlinear susceptibilities are shown in Table I, where \( d_{j} = \Delta_{j} + i\gamma_{j} \) (j=2-4).

From Table I, one finds that the probe and signal fields are identical cross-Kerr susceptibility expressions. If the intensity of the control field is larger than the detunings and the decay rate of the intersubband transition, i.e., \( |\Omega_{c}|^{2} \gg d_{j}d_{3} \), we obtain

\[
\chi^{(3)}_{pc} = \chi^{(3)}_{sc} = -\frac{N|\mu_{13}|^{2}|\mu_{24}|^{2}}{8\epsilon_{0}\hbar^{3}} \cdot \frac{1}{d_{4}|\Omega_{c}|^{2}}. \tag{5}
\]

In Fig. 2, we plot the linear absorption property [which bases on \( \chi^{(1)}_{p} \)] versus the various detunings of the probe field \( \Delta_{p} \) and the control field field \( \Omega_{c} \) for different values of the decay rate \( \gamma_{2} \). We can see from Fig. 2(a) that for relatively small decay rate \( \gamma_{2} = \gamma_{3} \), a resonant probe field can propagate with little absorption (solid curve) when the switch beam (control field \( \Omega_{c} \)) is on. Namely, in the case of \( \Delta_{p} = \Delta_{c} = 0 \), the probe and control fields form a “A configuration” EIT subsystem, which induces a dark state where the imaginary part of the linear susceptibility \( \chi^{(1)}_{p} \) vanish. When the decay rate \( \gamma_{2} \) becomes relatively important [that is, it is larger as shown in the dashed and dotted curves in Fig. 2(a)], it gives rise to a resonant probe field being gradually more absorbed. This means that, at present, the response of tunneling induced interference between the two wells start to become less observable. Meanwhile, we proceed to examine the linear absorption profiles corresponding to variational Rabi frequency of control field \( \Omega_{c} \) along with the influence of the decay rate \( \gamma_{2} \). We find that, in Fig. 2(b), the linear absorption coefficient \( \text{Im}\chi^{(1)}_{p} \) rapidly decrease with the increase of the intensity of the control field \( \Omega_{c} \) (see the solid curve). Simultaneously, with the slightly augment of the decay rate \( \gamma_{2} \), the decrease of the absorption coefficient \( \text{Im}\chi^{(1)}_{p} \) becomes more and more slow. Specially, under a fixed control field \( \Omega_{c} \), the absorption of the probe correspondingly enhances with increasing the decay rate \( \gamma_{2} \). It implies that, under an appropriate condition from the contribution of the control field \( \Omega_{c} \) and the decay rate \( \gamma_{2} \), the linear absorption of the probe field can be efficiently suppressed.

In Fig. 3 we show the third-order self-Kerr nonlinear susceptibility of the probe field [which bases on \( \chi^{(3)}_{ps} \)] versus the various detunings of the probe field \( \Delta_{p}/\gamma_{3} \) and the third-order cross-Kerr nonlinear susceptibilities of the probe and signal fields [which base on \( \chi^{(3)}_{pc} \)] versus the various detunings of the signal field \( \Delta_{s}/\gamma_{3} \), respectively. From Fig. 3(a), one can see that, under the condition \( \Delta_{p} = \Delta_{c} = 0 \), the self-Kerr nonlinear susceptibility of the probe field can also be effectively suppressed concomitant with a appreciably wide EIT transparency window, which is identical to the result we have obtained in Fig. 2(a). Next, in Fig. 3(b), within the EIT transparency window, we find that the real parts of the cross-Kerr nonlinear susceptibilities of the probe and signal fields decay much more slowly than the imaginary parts. Consequently, this result demonstrates that a considerable cross phase modulation with a negligible absorption can be created on demand by setting the signal field rather far off resonance. In addition to this, it can also be seen that the cross-Kerr susceptibilities

| TABLE I: The specific expressions of linear and nonlinear susceptibilities. |
|-----------------|------------------|-----------------|------------------|
| \( \chi^{0}_{p} \) | \( \chi^{(1)}_{p} \) | \( \chi^{(3)}_{ps} \) | \( \chi^{(3)}_{pc} \) |
| \( \frac{N|\mu_{13}|^{2}}{2\epsilon_{0}\hbar\Omega_{p}}A_{3}A_{1}^{*} \) | \( \chi^{(1)}_{p} + \chi^{(3)}_{ps} |\varepsilon_{p}|^{2} + \chi^{(3)}_{pc} |\varepsilon_{s}|^{2} \) | \( \frac{N|\mu_{24}|^{2}}{2\epsilon_{0}\hbar\Omega_{p}}A_{4}A_{2}^{*} \) | \( \chi^{(3)}_{sc} |\varepsilon_{p}|^{2} \) |
| \( \frac{N|\mu_{13}|^{2}|\mu_{24}|^{2}}{8\epsilon_{0}\hbar^{3}} \cdot \frac{1}{d_{4}|\Omega_{c}|^{2}} \) | | | |

Fig. 2: (Color online) The imaginary part of the linear susceptibility of (a) \( \Delta_{p}/\gamma_{3} \) and (b) \( \Omega_{c}/\gamma_{3} \) with different decay rate \( \gamma_{2} \), respectively. We here make \( N|\mu_{13}|^{2}/(2\epsilon_{0}\hbar) \) as unit in plotting. The parameters used are scaled by \( \gamma_{3} = \gamma_{4} = 0.5\gamma_{4} \) and the control field field \( \Omega_{c} = 6.0\gamma_{4} \).
of the probe and signal fields, as represented in Eq. (5), are of the same order of the magnitudes. Therefore, these results reveal that, due to the quantum coherence and interference between the lower subbands under the EIT condition, not only the self-Kerr interaction can be vanished but the two cross-Kerr susceptibilities can be enhanced predominantly.

It is clearly shown that, with suitable parameters, the corresponding cross-Kerr susceptibilities $\chi_{pc}^{(3)}$ and $\chi_{sc}^{(3)}$ can be predominantly enhanced in the context of EIT condition. This situation corresponds to the case of the probe and control fields are approximate resonance ($\Delta_p = \Delta_c = 0$). Then we find, by appropriate tuning the value of the control field $\Omega_c$ and the decay rate $\gamma_4$, the linear absorption and self-Kerr nonlinearity susceptibilities can be effectively suppressed, while the cross-Kerr nonlinearity susceptibility of the probe and signal fields can be significantly enhanced.

**IV. GROUP-VELOCITY MATCHING AND ENTANGLEMENT OF TWO-QUBIT CONTROLLED-PHASE GATE**

It should be worth noting that, as first emphasized by Lukin and Imamoglu [27], the group velocity matching of the probe and signal fields is another crucial requirement for achieving a large nonlinear mutual phase shift. Only in this way, the two optical pulses can interact in an EIT medium for a sufficiently long time to induce an effective cross phase modulation.

The expressions of the group velocities for the probe and signal fields can be given by

$$v_g^{p,s} \sim \frac{4\hbar c \rho_{pc}}{N_n^j |\mu_{13}|^2 (1 + \beta |\Omega_n^j|^2) \delta p + |\mu_{24}|^2 (1 - \beta |\Omega_n^j|^2) \delta s},$$

where $\beta = (\Delta_p^2 - \gamma_4^2)/(\Delta_c^2 + \gamma_4^2)^2$. And both group velocities should be small and equal by regulating the dipole matrix elements and the probe and signal Rabi frequencies together with the coefficient $\beta$. Fortunately, if the EIT-resonance condition is disturbed by a small amount, it remains under the common transparency condition and the absorption may still be considered as ignorable.

As we know, the emergence of the cross-phase modulation is a very essential condition for realizing the controlled quantum phase gate between two optical qubits. In our QW scheme (as the four-state system shown in Fig. 1), within the cross-Kerr effect, a cross-phase modulation could be implemented whereby an optical field achieves a nonlinear phase shift relied chiefly on the situation of another optical field to generate two-qubit quantum phase gate. The quantum phase gate operation is defined by the input-output relationship as

$$\frac{\hat{a}_j}{\sqrt{2}} \rightarrow \exp (i \phi_j) \frac{\hat{a}_j}{\sqrt{2}},$$

in which $i, j = 0, 1$ depict the qubit basis. In this case, a universal two-qubit gate (which enable to entangle two initially factorized qubits) can be actualized when the conditional phase shift $\phi = \phi_{00} + \phi_{11} - \phi_{01} - \phi_{10}$ becomes different from zero [1].

Notice that only the “right” polarization (i.e., $\sigma^+$ and $\sigma^-$ respectively corresponds to the probe and signal fields) of the probe and signal fields, a controlled quantum phase gate could be realized when a significant and nontrivial cross-phase modulation between them arises. When the probe field has a $\sigma^-$ polarization (as for a $\sigma^-$ polarization signal field), namely, the “wrong” polarization can not couple to any levels and thus the corresponding pulse will achieve a trivial phase shift $\phi_p^{(0)} = k_p L$ (where $L$ is the length of the medium and $k_p = \omega_p / c$ denotes the free space wave vector). Under the cosideration that both the probe and the signal fields are $\sigma^-$ polarization, the probe field undergoes a self-Kerr effect and achieves a nontrivial phase shift $\varphi_p^{(A)} = \varphi_p^{(0)} + \varphi_p^{(1)} + \varphi_p^{(3)}$ (which is subjected to the EIT condition constituted by the $A$ configuration: [1], [2] and [3] levels), with $\varphi_p^{(A)} = k_p L (1 + 2 \pi \chi_{pc}^{(1)}(0))$ being the linear phase shift and $\varphi_p^{(3)}$ being the nonlinear phase shift caused by the self-Kerr nonlinearity. At the same time, the signal field achieve the the vacuum shift $\varphi_s^{(0)}$. Similarly, for the “right” polarization condition, the probe and signal fields achieve the nontrivial phase shifts $\varphi_s^{(T)} = \varphi_s^{(A)} + \varphi_s^{(3)}$ and $\varphi_s^{(T)} = \varphi_s^{(0)} + \varphi_s^{(3)}$, respectively.

The input probe and signal polarized single-photon wave packets form a superposition of the circularly polarized states can be expressed as

$$|\psi\rangle_j = \alpha_j^+ |\sigma^+\rangle_j + \alpha_j^- |\sigma^−\rangle_j, (j = p, s)$$

where $|\sigma^+\rangle_j = \int d\omega \xi_j(\omega) |\sigma^+\rangle_j(\omega)|0\rangle$, with $\xi_j(\omega)$ being a Gaussian frequency distribution of incident wave packets, centered at frequency $\omega_j$. The photon field operators experience a transformation while propagating through the QW medium of length $L$, i.e., $\alpha_\pm(\omega) \rightarrow \alpha_\pm(\omega) \exp \left[\frac{i\omega}{c} \int_0^L d\zeta \pm (\omega, z) \right]$. The real part of the refractive index $n_\pm(\omega, z)$ can be assumed to $n_\pm(\omega_j, z)$, one obtains that,

$$|\sigma^\pm\rangle_j \rightarrow \exp[-i \varphi^{\pm}_j] |\sigma^\pm\rangle_j,$$
where \( \varphi^\pm = (\omega/c) \int_0^L dz n_{\pm}(\omega_j, z) \), and the cross-phase shift of the probe field is given by

\[
\varphi_{pc}^{(3)} = k_p L \frac{h^2 \pi^{3/2} |\Omega_s|^2 \text{erf}[\zeta_p]}{4|\mu_{13}|^2} \text{Re}[\chi_{pc}^{(3)}],
\]

where \( \zeta_p = (1-\eta_p/v_p^s) \sqrt{2} L/(v_p^s \tau_s) \), with \( \tau_s \) being the width of the pulse and erf[\zeta_p] depicts the error function. The cross-phase shift of the signal field is obtained on interchanging \( p \leftrightarrow s \) in the equation above, that is

\[
\varphi_{sc}^{(3)} = k_s L \frac{h^2 \pi^{3/2} |\Omega_p|^2 \text{erf}[\zeta_s]}{4|\mu_{13}|^2} \text{Re}[\chi_{sc}^{(3)}],
\]

where \( \zeta_s \) can also be obtained from \( \zeta_p \) upon interchanging the indices \( p \leftrightarrow s \). In case of meeting with the group velocity matching, so \( \zeta_{ps} \to 0 \), i.e., the value of \( \text{erf}[\zeta_{ps}] \) reaches the maximum value \( 2/\sqrt{\pi} \). Thus, after encoding \( |\sigma^\pm \rangle_i \to |0 \rangle_i \) and \( |\sigma^+ \rangle_i \to |1 \rangle_i \), the explicit form of the polarization two-qubit controlled quantum phase gate (QPG) using the present QW structure is given by

\[
|0 \rangle_p |0 \rangle_s \to \exp \left[ -i \left( \varphi_p^{(0)} + \varphi_s^{(0)} \right) \right] |0 \rangle_p |0 \rangle_s, \quad (11a)
\]

\[
|0 \rangle_p |1 \rangle_s \to \exp \left[ -i \left( \varphi_p^{(0)} + \varphi_s^{(0)} \right) \right] |0 \rangle_p |1 \rangle_s, \quad (11b)
\]

\[
|1 \rangle_p |1 \rangle_s \to \exp \left[ -i \left( \varphi_p^{(0)} + \varphi_s^{(0)} \right) \right] |1 \rangle_p |1 \rangle_s, \quad (11c)
\]

\[
|1 \rangle_p |0 \rangle_s \to \exp \left[ -i \left( \varphi_p^{(T)} + \varphi_s^{(T)} \right) \right] |1 \rangle_p |0 \rangle_s, \quad (11d)
\]

with a conditional phase shift \( \varphi_{\text{con}} = \varphi_p^{(T)} + \varphi_s^{(T)} - \varphi_p^{(0)} - \varphi_s^{(0)} \) is nonzero (on the basis of the EIT condition, the linear effect and self-phase modulation can be suppressed, so that \( \varphi_p^{(0)} = \varphi_s^{(0)} \), \( \varphi_{\text{con}} = \varphi_{pc}^{(3)} + \varphi_{sc}^{(3)} \), and thus a two-qubit polarization controlled quantum phase gate can be realized in the coupled QW structure.

To show explicitly the form of the controlled phase gate, we next study the entanglement of the two-qubit state after the processing of EIT. For our special two-qubit system, the state takes the form \( |\psi^{ps}\rangle \) constituting by signal and probe fields. Consider the input state is a factorized (separable) pure state \( |\psi^{ps}\rangle = \langle \psi_p | \psi_s \rangle \), as presented in Eq.(7), while the signal and probe fields \( \langle \psi_p(\phi) \rangle \) are superposition of \( |0 \rangle \) and \( |1 \rangle \) with equal amplitudes \( \psi = (|0 \rangle + |1 \rangle)/\sqrt{2} \).

Then by the transformation presented in Eqs.(11), the final state is still a pure state but with a non-trivial conditioned phase so that the final state is entangled. The entanglement can be measured by the von Neumann entropy of the reduced density operator of signal field or probe field

\[
E(|\psi^{ps}\rangle) = S(p) = -\text{tr}[\rho_p \log \rho_p] = S(s) = -\text{tr}[\rho_s \log \rho_s],
\]

here, \( \rho_p \) and \( \rho_s \) are the reduced density operators of the sub-systems \( S^p \) and \( S^s \). Simply, if \( \lambda_x \) are the eigenvalues of \( \rho_{p(s)} \) [1], the entanglement of two-qubit expressed as von Neumann entropy takes the form, \( E(|\psi^{ps}\rangle) = -\sum_x \lambda_x \log \lambda_x \).

As is shown in Fig. 4, we plot the result of the degree of entanglement [von Neumann entropy \( E(|\psi^{ps}\rangle) \)] versus the detunings of the signal field \( \Delta_4 / \gamma_3 \) with different \( \Omega_x \). For simplicity, we set the value of \( N |\mu_{13}|^2 \omega_p / 2 \varepsilon_0 c \hbar = N |\mu_{13}|^2 \omega_0 / 2 \varepsilon_0 c \hbar = 2.5 \times 10^6 \) as an example. For a certain intensity of control field, i.e., \( \omega_x = 4 \gamma \), we can obtain a nearly 100% degree of entanglement when \( \Delta_4 = \pm 0.5 \gamma_3 \) (which corresponds to the black solid curve in Fig. 4), which is due to the characteristic of nonabsorption in this system. In accordance with the case of the maximum entanglement, we simultaneously achieve an approximate \( \pi \) radians conditional nonlinear phase shift (that is \( \varphi_{\text{con}} \approx \pi \)). It is also essential to note that the degree of entanglement of our system can be affected by the fluctuations of light intensities and the detunings of the probe and signal fields in the experimental demonstration.

It is evidently shown that, a strong third-order cross-Kerr nonlinearity is satisfied so that one can be capable of constructing a controlled-\( \pi \), a specific case of the controlled phase gate for the probe and signal fields in the case of the “right” polarization configuration. As mentioned above, it shifts the phase \( |1 \rangle_p |0 \rangle_s \) by \( \pi \), leaving the other three basis states unchanged. Consequently by single qubit rotation of signal and probe fields, the controlled-\( \pi \) transformation realized by the output modes of Eqs.(11), can be represented as \( |0 \rangle_p |0 \rangle_s \to |0 \rangle_p |0 \rangle_s, \quad |0 \rangle_p |1 \rangle_s \to |0 \rangle_p |1 \rangle_s, \quad |1 \rangle_p |0 \rangle_s \to |1 \rangle_p |0 \rangle_s \) and \( |1 \rangle_p |1 \rangle_s \to -|1 \rangle_p |1 \rangle_s \) which is the standard form in quantum computation. To our knowledge, this result indicates that, when reasonable regulation parameters are selected, this controlled-phase gate can create a maximally entangled state from a separable pure input state, (100% entanglement as shown in Fig. 4).

Since the single qubit rotation in Eq. (7) can be realized easily, the combination of controlled phase gate provided by EIT in the semiconductor system and the single rotation gate constitutes a universal set of gates for quantum computation.

The experimental scheme to realize the controlled quantum phase gate in the QW in GaAs/AlGaAs semiconductor...
FIG. 5: (Color online) Scheme of the proposed experiment for a Bell-states analyzer: CPM is the cross-phase modulation of the probe and signal fields in our system with conditional nonlinear phase shift $\phi^{\text{cond}} = \pi$ under the condition of EIT effect; PBS1, PBS2 and PBS3 are three polarization beam splitters; and $|H\rangle$ and $|V\rangle$ denote the horizontally and vertically polarized signal-photon states.

system is thus finished. Next, it will be interesting to check whether this scheme can indeed work, in particular, whether a maximally entangled state can be created by this controlled-$\pi$ phase gate. Thus it is desirable to have a scheme for observing the maximally entangled state among the two-qubit polarization controlled quantum phase gate. We here propose a practical experiment implementation for optical Bell-state analyzer in the coincidence basis, due to which one can also observe the “right” polarization and maximally entangled qubit. In order to explain our projection, as shown in Fig. 5, we first examine that the left part of the scheme is comprised of quantum phase gate which has been described above. The right part of the scheme is comprised of three polarizing beam splitters (PBS), two half-wave plate and four detectors with single-photon sensitivity. We here encoded $|\sigma^+\rangle_i \rightarrow |H\rangle_i$ and $|\sigma^-\rangle_i \rightarrow |V\rangle_i$. Due to the PBS transmits only the horizontal polarization component and reflects the vertical component, and the input of PBS1 is the indistinguishable identical particles, we can directly achieve the incident state as

$$\psi_{in} = a|H_1\rangle|H_2\rangle + b|H_1\rangle|V_2\rangle + c|V_1\rangle|H_2\rangle + d|V_1\rangle|V_2\rangle,$$

(13)

where the tensor product of the single-photon polarization basis states, as usual $|H\rangle_i = \begin{pmatrix} 1 \\ i \end{pmatrix}$ and $|V\rangle_i = \begin{pmatrix} 0 \\ i \end{pmatrix}$. It is also well known that the four Bell states are, $|\phi^+\rangle = \frac{|H_1\rangle|V_2\rangle + |V_1\rangle|H_2\rangle}{\sqrt{2}}$ and $|\Phi^+\rangle = \frac{|H_1\rangle|H_2\rangle + |V_1\rangle|V_2\rangle}{\sqrt{2}}$. According to the spirit of reference [29], by using the coincidence between the four detectors. Meanwhile, by using the coincidence between the four detectors. We can readily identify $\psi_{in}$ in terms of Bell states

$$\psi_{in} = \frac{(a + d)\phi^+ + (a - d)\Phi^-}{\sqrt{2}},$$

(14)

where the states $\phi^+$ and $\Phi^-$ are ambiguously distinguishable (as reference [30], we can correspond $|\phi^+\rangle$ and $|\phi^-\rangle$ to $|H_1\rangle|H_2\rangle$ and $|V_1\rangle|H_2\rangle$, respectively), thus, a half-wave plate was oriented at $\pi/8$ rotated the polarization of the horizontally polarized single photon to $\pi/4$ polarization state just before PBS2 and PBS3, respectively. Namely, $|H_1\rangle \rightarrow \frac{|H_1\rangle + |V_1\rangle}{\sqrt{2}}$ and $|V_1\rangle \rightarrow \frac{|H_1\rangle - |V_1\rangle}{\sqrt{2}}$, where (i=1,2). Finally, each of the two polarizations are split by PBS2 and PBS3 into two single photon detectors, respectively. And thus $\phi^+$ and $\Phi^-$ will be transformed into

$$|\phi^+\rangle \rightarrow |\phi^+\rangle = \frac{|H_1\rangle|H_2\rangle + |V_1\rangle|V_2\rangle}{\sqrt{2}}, \quad (15)$$

$$|\phi^-\rangle \rightarrow |\Phi^+\rangle = \frac{|H_1\rangle|V_2\rangle + |V_1\rangle|H_2\rangle}{\sqrt{2}}. \quad (16)$$

(16)

This shows that, we are able to identify two of the four incident Bell states using the coincidence between the polarization state of the probe and signal fields. Correspondingly, for the other two incident Bell states, which will result in no coincidence between the four detectors and will be signified by the kind of superposition of $|H_1\rangle|V_2\rangle$ and $|V_1\rangle|H_2\rangle$. Specially, it is clear that we can also identify the maximally entangled state among the two-qubit polarization quantum phase gate, based on the Bell state analyzer in this QW structures.

V. DISCUSSION AND CONCLUSION

It is important to emphasize that the measurement of total nonlinear phase shift is crucial to the experimental demonstration of the quantum phase gate. The fluctuations of light intensities and frequency detunings of the probe and signal fields will induce the errors of the nonlinear phase shift. As a consequence, there is certainly the possibility of taking all lasers phase-locked to each other for minimizing the effect of relative detuning fluctuations. And then the light intensity with fluctuations of 1% will lead to an error less than 4% in the phase measurement. Besides, it should also be noted that, for the moderate density even at room temperature, the additional broadening effects (which are induced by the carrier-carrier and carrier-photon interactions) we have neglected are very small in comparison with the final broadening [26, 31]. Moreover, it should be pointed out that the cross-phase modulation is a very promising candidate for the design of deterministic optical controlled quantum phase gates and the probe and signal fields have been treated in a classical way. Therefore, it would be a clear indication for generating the entanglement of macroscopic, coherent states instead of single-photon states.

In summary, we have investigated the two-qubit controlled-phase gate on the basis of the linear and nonlinear properties of the probe and signal pulses in an asymmetric AlGaAs/GaAs coupled-double QW structure. In our scheme, a giant cross-Kerr nonlinearity (which corresponds to $\pi$ radians conditional nonlinear phase shifts) and mutually matched (which is also slow) group velocities can be achieved within the suppression of both the linear and self-Kerr nonlinear optical absorption susceptibilities. Such properties stem from a constructive quantum interference effect in nonlinear susceptibility of the probe and signal fields related to the cross-phase modulation induced by EIT effect in our system. Due to such novel features, we can be able to acquire two-qubit controlled-phase
gates and high entanglement between the weak probe and signal pulses in the given system. In the mean time, it is also likely to achieve the controlled-\(\pi\) gate through the optical realization of the circuit in quantum computation. Furthermore, by adding just polarizing beam splitters and half-wave plates, we have proposed a practical experimental scheme, by which it comes in handy to discriminate the maximally entangled state of the two-qubit including two out of four Bell states by using the coincidence between the four detectors. And that such version of the Bell state analyzer can possibly be extended to the three-particle or N-particle cases. Considering that an asymmetric AlGaAs/GaAs coupled-double QW structure on the basis of the intersubband transitions has already been realized experimentally [25], the results achieved in the presented work are helpful for facilitating actual applications of all-optical quantum computing and quantum information processing mediated by a solid-state system.

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