Adaptive internal model-based suppression of torque ripple in brushless DC motor drives

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Permanent-magnet synchronous motors (PMSMs) are widely used as high-performance variable-speed drives. Ripple in the electric torque of such motors is often a source of vibration and tracking errors, especially at low speeds. We study the torque characteristics of PMSMs and propose a method to minimize the torque ripple. First, we establish a detailed model for the motor and present a Fourier analysis of the torque ripple, caused by the non-sinusoidal back electro-motive force (BEMF) and the cogging torque, where the main conclusion is that the frequencies present in the torque disturbance are integer multiples of six times the electric frequency. The resulting model is highly nonlinear. We propose an adaptive controller based on the internal model principle, where the resonant frequencies of the controller and the associated gains change according to the motor speed. This is achieved by replacing the time variable by the motor angle, which simplifies the nonlinear model. Our approach is passivity based and will work also for complex mechanical loads and several resonant frequencies. Simulation and experimental results are given to verify the new controller. We compare the performance of our adaptive algorithm with the well-known one from [Canudas de Wit, C., & Praly, L. (2000). Adaptive eccentricity compensation. IEEE Trans. Control Syst. Technol. 8, 757–766]. We find that it performs similarly in simple configurations, and it works also when the motor is part of a more complex system, for example, when the motor is connected to a load via a very flexible shaft.

Keywords: brushless DC; mutual torque; cogging torque; Park transformation; internal model; adaptive control

1. Introduction

A brushless DC motor (BLDCM) is a permanent-magnet synchronous machine (PMSM) connected to an inverter, usually operated by pulse width modulation and controlled by a processor that receives position measurements from a sensor (encoder) mounted on the motor shaft. BLDCMs offer high power density, reliability and efficiency. However, typical motors, driven by either rectangular or sine wave currents, exhibit torque pulsations or ripple.

At high speeds, torque ripple is mostly filtered out by the rotor inertia. However, at low speeds, the torque ripple produces undesirable speed variations, and inaccuracies in motion control. There are applications (e.g. computer tomography (Lee, Park, & Kwon, 2004), optics, machine tools (Jahns & Soong, 1996)) where very high precision is needed at low motor speeds.

In general, torque in a PMSM is developed by

\begin{itemize}
  \item mutual torque,
  \item cogging torque and
  \item reluctance torque,
\end{itemize}

see Hanselman (1994) and Park, Park, Lee, and Harashima (2000). Imperfections of the motor geometry give rise to harmonics, which are usually multiples of six times the electric angular velocity, in the mutual torque (see also Degobert, Remy, Zeng, Barre, & Hautier, 2006; Hanselman, Hung, & Keshura, 1992). DC offset in the current sensors (and hence in the phase currents) leads to ripples in the mutual torque that are at the electric angular velocity, see the analysis in Gan and Qiu (2004).

There are two main control strategies for operating the inverter in a BLDCM: the “classical” one suits motors with trapezoidal back electro motive force (BEMF), and it produces rectangular current pulses lasting each for 60 electrical degrees. In this operation, only two phases are conducting at any time, but the torque will be close to constant, see Pillay and Krishnan (1989). Torque ripples will appear due to the imperfections of the current rectangles and of the BEMF – see Lu, Zhang, and Qu (2008) for methods to combat these ripples. The other strategy is by controlling the $q$ component in order to control the electromagnetic torque. This leads to greater flexibility and it is better suited for PMSM with close to sinusoidal BEMF. In this paper, we consider only BLDCM operated under the latter (continuous) switching strategy.

Two general approaches have been proposed to reduce the torque ripple. One is to improve the motor’s geometrical structure, see Hwang, Eom, Jung, Lee, and Kang (2001), Jahns and Soong (1996) and Ko and Kim (2004).
The second approach is to control the winding currents to overcome the disturbances, see for instance Ahn, Chen, and Dou (2005), Canudas de Wit and Praly (2000), Hung and Ding (1993), Le-Huy, Perret, and Feuillet (1986) and Malaize and Levine (2009). An interesting review of a wide range of design techniques for torque ripple minimization is in Jahns and Soong (1996). In Canudas de Wit and Praly (2000), an adaptive observer is designed to estimate the periodic torque disturbance. This estimate is then injected in order to cancel the disturbance. This algorithm needs a precise estimate of the friction in the system. Xu, Panda, Pan, Lee, and Lam (2004) use an iterative learning control (ILC) module in order to estimate the cyclic torque and record the reference current signals over one entire cycle. The ILC module then uses those signals to update the reference current for the next cycle. A gain-shaped sliding mode observer is used to estimate the torque ripple. In Gan and Qiu (2004), a gain scheduled robust two degree of freedom speed regulator, based on the internal model principle (IMP) and pole-zero placement, is developed to eliminate the torque ripples caused by DC offset (as discussed above). Their internal model changes its (single) resonant frequency according to the velocity reference. To prove local stability, the authors use linearization and the small gain theorem. Their proof uses the assumption that the disturbance frequency equals the reference electric velocity, which is debatable. In Hung and Ding (1993), a Fourier series decomposition is used to find a closed-form solution for the current harmonics that eliminate torque ripple and maximize efficiency simultaneously. In Park et al. (2000), the BEMF data, according to the rotor position, are measured and set up in a lookup table. Optimized reference current wave-forms are then obtained from the lookup table using the position and speed information from the shaft encoder. The motor currents are forced to track the reference currents. Petrović, Ortega, Stanković, and Tadmor (2000) design a passivity-based controller, relying on the principles of energy shaping and damping injection. The information about torque ripple harmonics used in the adaptation is extracted from the electrical subsystem, and a current controller is designed to achieve ripple minimization. Qian, Panda, and Xu (2004) implement an ILC scheme in the time domain to reduce periodic torque pulsations. A forgetting factor is introduced in this scheme to increase the robustness. However, this limits the extent to which torque pulsations can be suppressed. To eliminate this limitation, a modified ILC scheme is implemented in the frequency domain using Fourier series. Mattavelli, Tubiana, and Zigliotto (2005) propose the application of repetitive techniques to the current control in a field-oriented PMSM drive, where the constant torque reference has been modified to achieve constant torque. Ferretti, Magnani, and Rocco (1999), the technique of Ferretti et al. (1998) is extended to cope with variable motor speed using an adaptive compensator. Degobert et al. (2006), Ruderman, Ruderman, and Bertram (2013) and also Yepes et al. (2010) propose controllers that eliminate ripples of the mutual torque, using an IMP-based controller whose resonant frequencies are adjusted online according to the motor velocity.

In this paper, we propose a new type of controller to reduce the torque ripple caused by both the mutual torque and the cogging torque. The idea is to use a resonant controller on the $q$-axis current reference but using the rotor angle in place of the time. This controller may be called adaptive in the sense that the resonant frequencies (when regarded in the time domain) adjust themselves according to the motor speed.

In Section 2, we establish a model for the motor to cover all dynamics without any assumptions on the signals. In Section 3, we derive a formula for the electromagnetic torque and express the torque ripple caused by non-sinusoidal BEMF. In Section 4, the constant speed version of our controller is presented. The velocity loop, which uses the $q$-axis current, includes an internal model and a feedforward block. In Section 5, we introduce the adaptive version of the internal model, which works at variable speed. It is based on a transformation of linear differential equations, when time is replaced by motor angle. In Section 6, we give a short review of the Adaptive Eccentricity Algorithm from Canudas de Wit and Praly (2000) and Malaize and Levine (2009). In Section 7, we prove that, for the adaptive algorithm from Section 5, and under the restrictive assumption that the reference $\omega_{\text{ref}}$ is constant, the velocity error tends to zero. Simulation and experimental results are provided in Sections 8–10, with detailed comparisons with the controller of Canudas de Wit and Praly (2000). In particular, in Section 9 we consider a more complex system with a load connected to the motor via a flexible shaft.

2. Motor model – electrical part

We give the derivation of a mathematical model for a PMSM with one pair of poles per phase (similar to Zhong & Weiss, 2011).

Assume that the windings of a PMSM are connected in star, with each winding having the series resistance $r$. The basic voltage equations are

$$\ddot{\bar{e}}_x = \frac{d\Psi}{dr} = -ri_x + v_x - v_m, \quad (1)$$

where $x$ can be one of $a, b, c$, $\bar{e}_x$ are the induced voltages, $\Psi$ are the phase winding flux linkages, $i_x$ are the phase currents, and $v_x$ are the phase voltages. The voltage $v_m$ in the center of the star cannot be measured, but this is also
not needed. Clearly,
\[ i_a + i_b + i_c = 0. \]

We denote by \( \theta \) the angle of the rotor such that \( \theta = 0 \)
corresponds to the rotor, creating a flux parallel to the axis of winding \( a \) and in the direction of the flux created by \( i_a > 0 \).

Assuming a round (non-salient) rotor and a magnetically non-saturated machine, the flux linkage equations are
\[
\begin{align*}
\Psi_a &= L_i i_a + M i_b + M i_c + F(\theta), \\
\Psi_b &= M i_a + L_i i_b + M i_c + F\left(\theta - \frac{2\pi}{3}\right), \\
\Psi_c &= M i_a + M i_b + L_i i_c + F\left(\theta + \frac{2\pi}{3}\right),
\end{align*}
\]
where \( L \) is the phase winding self-inductance, \( M \) is the mutual inductance between two windings (usually \( M = -0.5L \)), \( F(\theta) \) is the flux linkage through phase \( a \) due to the rotor, \( F(\theta - 2\pi/3) \) is the flux linkage through phase \( b \) due to the rotor and similarly for phase \( c \). Substituting Equation (2) in Equation (3), we get:
\[
\begin{align*}
\Psi_a &= L_\theta i_a + F(\theta), \\
\Psi_b &= L_\theta i_b + F\left(\theta - \frac{2\pi}{3}\right), \\
\Psi_c &= L_\theta i_c + F\left(\theta + \frac{2\pi}{3}\right),
\end{align*}
\]
where \( L_\theta = L - M \) is the equivalent motor phase inductance. From electromagnetic field theory, we know that \( \begin{bmatrix} LM \\ ML \end{bmatrix} > 0 \). It follows that \( L_\theta > 0 \) (usually \( L_\theta = 1.5L \)). In this model, we assume that \( L_\theta \) is independent of \( \theta \).

Rewriting Equation (4) in vector form, we get:
\[ \Psi = L_\theta \dot{i} + E(\theta), \]
where \( E(\theta) = [F(\theta)F(\theta - 2\pi/3)F(\theta + 2\pi/3)]^T \). Due to the motor structure, \( F(\theta) \) is a periodic function with period \( 2\pi \), so that we can write it as a Fourier series:
\[ F(\theta) = \Psi_f \sum_{n=0}^{\infty} \gamma_n \cos(n\theta) + \delta_n \sin(n\theta), \]
where \( \Psi_f > 0 \) is a parameter that will be chosen later. If the rotor is symmetric, then \( F(-\theta) = F(\theta) \) and hence the terms with \( \sin(n\theta) \) disappear: \( \delta_n = 0 \).

We argue that \( F(\theta) \) contains only odd harmonics. The symmetry of the rotor (the shape remains unchanged after a rotation of \( \pi \)) implies that \( F(\theta + \pi) = -F(\theta) \). By subtracting \( F(\theta + \pi) \) from \( F(\theta) \), we get
\[
\begin{align*}
F(\theta) &= \Psi_f \sum_{k=0}^{\infty} \gamma_{2k} \cos(2k\theta) \\
+ \sum_{k=1}^{\infty} \gamma_{2k-1} \cos((2k-1)\theta), \\
F(\theta + \pi) &= \Psi_f \sum_{k=0}^{\infty} \gamma_{2k} \cos(2k\theta) \\
- \sum_{k=1}^{\infty} \gamma_{2k-1} \cos((2k-1)\theta)
\end{align*}
\]
we get
\[ 2F(\theta) = F(\theta) - F(\theta + \pi) \]
\[ = 2\Psi_f \sum_{k=1}^{\infty} \gamma_{2k-1} \cos((2k-1)\theta). \]
Therefore,
\[ F(\theta) = \Psi_f \sum_{n=1}^{\infty} \gamma_n \cos(n\theta). \]

We normalize the coefficients such that \( \sum_{n=1}^{\infty} \gamma_n = 1 \). Then, we see that \( \Psi_f \) is the maximal flux linkage due to the rotor. We remark that for a “perfectly built” machine, we have no harmonics:
\[ F(\theta) = \Psi_f \cos \theta. \]

By differentiating Equation (5), we get
\[ \ddot{\Psi} = L_\theta \ddot{i} + \dot{E}(\theta) \]
so that \( e_a, e_b, e_c \) are the BEMF on each phase due to the rotor movement. According to Equation (6), we have
\[ e_a = -\omega \Psi_f \sum_{n=0}^{\infty} \gamma_n n \sin(n\theta) \]
and similarly for \( e_b \) and \( e_c \). Here, \( \omega = \dot{\theta} \) is the mechanical angular speed. We remark that for a “perfectly built” motor,
\[ e_a = -\omega \Psi_f \sin(\theta) \]
and similarly for \( e_b \) (with \( \theta - 2\pi/3 \) in place of \( \theta \)) and for \( e_c \). Comparing Equation (1) with Equation (8) and eliminating \( \tilde{e} \) we get:
\[ L_\theta \dot{i} = v - r_\theta \dot{i} - \dot{\epsilon} - \omega \epsilon_m, \]
where \( v = [v_a \ v_b \ v_c]^T \) and \( \epsilon_m = [\epsilon_m \ \epsilon_m \ \epsilon_m]^T \). Recall the Park transformation introduced in Park (1929), a unitary matrix \( U \) which transforms a vector from the \( a, b, c \)
coordinate system to the rotating \(d, q\) coordinate system:

\[
\begin{bmatrix}
    f_d \\
    f_q \\
    f_0
\end{bmatrix} = U \cdot \begin{bmatrix}
    f_a \\
    f_b \\
    f_c
\end{bmatrix},
\]

where

\[
U = \sqrt{2/3} \begin{bmatrix}
    \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
    -\sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\
    \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}.
\]

Multiplying Equation (11) with the unitary matrix \(U\) from Equation (12), we get:

\[
L_s U \cdot \begin{bmatrix}
    i_a \\
    i_b \\
    i_c
\end{bmatrix} = \begin{bmatrix}
    v_d \\
    v_q \\
    v_0
\end{bmatrix} - r \begin{bmatrix}
    i_d \\
    i_q \\
    i_0
\end{bmatrix} - \begin{bmatrix}
    e_d \\
    e_q \\
    e_0
\end{bmatrix} - \begin{bmatrix}
    0 \\
    0 \\
    \sqrt{3}v_m
\end{bmatrix}.
\]

Since, according to a short computation,

\[
\begin{bmatrix}
    i_d \\
    i_q \\
    i_0
\end{bmatrix} = U \begin{bmatrix}
    i_a \\
    i_b \\
    i_c
\end{bmatrix} + \omega \begin{bmatrix}
    -i_d \\
    i_d \\
    0
\end{bmatrix},
\]

we get

\[
L_s \begin{bmatrix}
    i_a \\
    i_b \\
    i_c
\end{bmatrix} - L_s \omega \begin{bmatrix}
    -i_d \\
    i_d \\
    0
\end{bmatrix} = \begin{bmatrix}
    v_d \\
    v_q \\
    v_0
\end{bmatrix} - r \begin{bmatrix}
    i_d \\
    i_q \\
    i_0
\end{bmatrix} - \begin{bmatrix}
    0 \\
    0 \\
    \sqrt{3}v_m
\end{bmatrix}.
\]

Notice that \(i_c = 0\) (hence also \(i_b = 0\)), due to Equation (2). Hence, according to the third equation (the third line) in (13), \(v_0 = e_0 + \sqrt{3}v_m\).

Thus, the dynamic voltage equations in \(d, q\) coordinates are

\[
L_s \dot{i_d} = v_d - r i_d - e_d + L_s \omega i_q,
\]

\[
L_s \dot{i_q} = v_q - r i_q - e_q - L_s \omega i_d.
\]

For the case of a perfectly built motor as in Equation (7), using the Park transformation on Equation (10) we get, after some computation,

\[
e_d = 0, \quad e_q = \sqrt{3} \cdot \omega \Psi_f.
\]

3. The motor torque and BEMF

Torque production inside brushless PMSM is mainly due to the interaction between the permanent magnet field and the currents in the phase windings (mutual torque), and the interaction between the permanent magnets, located in the rotor, and the slotted iron structure of the stator (cogging torque). In this section, we derive formulas for these torques.

The total energy stored in a PMSM is

\[
E_{\text{total}} = E_{\text{mag}} + E_{\text{kin}} = E_{\text{mag}} + \frac{1}{2} J \omega^2,
\]

where \(E_{\text{mag}}\) is the energy stored in the magnetic field and \(E_{\text{kin}}\) is the kinetic energy of the rotating body with inertia \(J\) that contains the rotor. Differentiating Equation (15), we get:

\[
\dot{E}_{\text{total}} = \dot{E}_{\text{mag}} + J \omega \ddot{\omega}.
\]

We denote by \(T_e\) the electromagnetic torque generated inside the motor, and by \(T_l\) the mechanical load torque. Substituting Newton’s law, \(J \ddot{\omega} = T_e - T_l\), into Equation (16), we get:

\[
E_{\text{total}} = E_{\text{mag}} + \omega(T_e - T_l).
\]

On the other hand, \(\dot{E}_{\text{total}}\) is the flow of power to the system.

This power is equal to the ingoing power minus the power lost in the resistors minus the power drawn by the load,

\[
\dot{E}_{\text{total}} = \psi \cdot \dot{i} - ri \cdot \dot{i} - \omega T_l = (\psi - ri) \cdot \dot{i} - \omega T_l,
\]

where \(\cdot\) is the inner product in \(\mathbb{R}^3\). From Equations (17) and (18), we conclude:

\[
\dot{E}_{\text{mag}} = (\psi - ri) \cdot \dot{i} - \omega T_e.
\]

Substituting Equation (11) into the above formula, we get:

\[
\dot{E}_{\text{mag}} = (L_s \cdot \dot{i} + \xi + v_m) \cdot \dot{i} - \omega T_e.
\]

Since \(\psi \cdot \dot{i} = 0\), multiplying the above formula by \(dt\), we get:

\[
dE_{\text{mag}} = (L_s \cdot \dot{i} + \xi) \cdot \dot{i} \cdot dt - T_e \, d\theta.
\]

Integrating both sides of Equation (20), while assuming no motion of the rotor, to charge the magnetic field, we get (using that \(d\theta = 0, \xi = 0\))

\[
E_{\text{mag}} = E_0(\theta) + \int_0^T L_s \cdot \dot{i} \cdot \dot{i} \, dt = E_0(\theta) + \frac{1}{2} (L_s \cdot \dot{i}) \cdot \dot{i},
\]

where \(E_0(\theta)\) is the energy in the magnetic field due to the rotor magnet (while there are no currents) at the angle \(\theta\).
In the sequel, we consider again the moving rotor. Differentiating Equation (21), we get

\[ \dot{E}_{\text{mag}} = \frac{d}{d\theta} E_0(\theta) \cdot \omega + (L_s \cdot \dot{i}) \cdot \dot{i}. \]

From here and Equation (19), we conclude:

\[ T_e = \frac{e \circ i}{\omega} - \frac{d}{d\theta} E_0(\theta). \]  \hspace{1cm} (22)

The term \( T_{em} = e \circ i / \omega \) is called the mutual torque, while \( T_{ec} = -(d/d\theta)E_0(\theta) \) is called the cogging torque.

The mutual torque may be calculated using phase currents and back EMF instant values, both in \((a, b, c)\) and in \((d, q)\) coordinates (we use that \( i_0 = 0 \)):

\[ T_{em} = \frac{e \circ i}{\omega} = \frac{(U \cdot \vec{e}) \circ (U \cdot \vec{i})}{\omega} = \frac{i_q e_d + i_d e_q}{\omega}. \]  \hspace{1cm} (23)

For a “perfectly built” motor, we have from Equation (14)

\[ T_{em} = \frac{i_d \cdot 0 + i_q \cdot \sqrt{3/2} \Psi_f \omega}{\omega} = i_q \cdot \sqrt{3/2} \Psi_f. \]  \hspace{1cm} (24)

As mentioned earlier, cogging torque is created by the interaction between the permanent magnets in the rotor and the slotted iron structure of the stator. Usually, the stator has slots to hold the windings, see Figure 1. Due to these slots, the rotor has preferred directions and cogging is produced.

The following expression (25) for the electromagnetic cogging torque is taken from Hwang et al. (2001):

\[ T_{ec} = \sum_{n=0}^{\infty} K_n n \sin (nN_{L} \theta), \]  \hspace{1cm} (25)

where \( K_n \) are constants determined by the machine’s geometry and \( N_{L} \) is the least common multiple of the number of stator poles per phase, one slot per stator pole and one pair of rotor poles in the rotor), we have \( N_{L} = 6 \). More generally, \( N_{L} \) is a multiple of 6.

Recall that for a “non-perfectly built” motor, the BEMF due to the rotor movement can be described by Equation (9). Let us look at the third harmonic of the BEMF:

\[ e_{3\text{rd harmonic}} = -\Psi_f \omega 3 \gamma_3 \sin (3\theta), \]

\[ e_{3 \text{rd harmonic}} = -\Psi_f \omega 3 \gamma_3 \sin \left( 3 \left( \theta - \frac{2\pi}{3} \right) \right) \]

\[ = -\Psi_f \omega 3 \gamma_3 \sin (3\theta), \]

\[ e_{3 \text{rd harmonic}} = -\Psi_f \omega 3 \gamma_3 \sin \left( 3 \left( \theta + \frac{2\pi}{3} \right) \right) \]

\[ = -\Psi_f \omega 3 \gamma_3 \sin (3\theta). \]

Since \( i_d + i_b + i_c = 0 \), we have from Equation (23) that the contribution of the third harmonic to the torque is \( \left( e_{3 \text{rd harmonic}} \circ \omega / \omega \right) = 0 \). A similar result can be derived for all the odd multiples of 3.

Therefore, we conclude that the harmonics which contribute to the mutual torque, \( T_{em} \), are multiples of \( \sin (n\theta) \), where \( n = 6p + 1 \) for \( p = 0, 1, 2, \ldots \) and \( n = 6p - 1 \) for \( p = 1, 2, 3, \ldots \).

Recall from Equation (23) that in \( d, q \) coordinates, the electromagnetic torque is \( T_{em} = (i_q e_d + i_d e_q) / \omega \). Since \( i_d \) and \( i_q \) are inputs to be chosen by the user, it is useful to look at the expressions for \( e_d \) and \( e_q \). If we compute \( e_d \) and \( e_q \) from (9) and (12), we obtain

\[ e_d = \sqrt{3} \omega \Psi_f \left[ \sum_{p=1}^{\infty} \gamma_{6p+1} (6p + 1) \right. \]

\[ + \gamma_{6p-1} (6p - 1) \sin (6p\theta) \right], \]

\[ e_q = \sqrt{3} \frac{\omega}{2} \Psi_f \left[ \gamma_1 + \sum_{p=1}^{\infty} \gamma_{6p+1} (6p + 1) \right. \]

\[ - \gamma_{6p-1} (6p - 1) \cos (6p\theta) \right]. \]  \hspace{1cm} (26)

Denoting for \( p = 1, 2, 3, \ldots \)

\[ \eta_{dp} = -\sqrt{3/2} \Psi_f \left[ \gamma_{6p+1} (6p + 1) + \gamma_{6p-1} (6p - 1) \right], \]

\[ \eta_{qp} = \sqrt{3/2} \Psi_f \left[ \gamma_{6p+1} (6p + 1) - \gamma_{6p-1} (6p - 1) \right], \]

and \( \eta_{dq} = \sqrt{3/2} \Psi_f \gamma_1 \), we get from Equation (26)

\[ e_d = \omega \left[ \sum_{p=1}^{\infty} \eta_{dp} \sin (6p\theta) \right], \]

\[ e_q = \omega \left[ \eta_{dq} + \sum_{p=1}^{\infty} \eta_{qp} \cos (6p\theta) \right]. \]  \hspace{1cm} (27)
If the motor has $\Pi$ pairs of poles per phase (instead of just one), then in Equations (25) and (27), we have to replace $\theta$ with $\Pi\theta$.

4. Velocity control (non-adaptive)

Figure 2 shows the current and velocity control loops of a BLDCM. Notice that $i_{d\text{ref}} = 0$ and $i_{q\text{ref}}$ is the sum of the velocity controller’s output and a term from the feedforward path with transfer function $\alpha = (1/\eta_0)(Js + D)$, which has the role of making the response faster. Here, $D$ is the viscous friction coefficient of the motor, so that $T_l = T_L + D\omega$, where $T_L$ is the external load torque. It is recommended to include a saturation block in the feedforward path.

In the sequel, we allow the motor to have $\Pi$ pairs of poles per phase (until now we had $\Pi = 1$), so that we rely on the modified version of Equation (27). We will assume complete and instantaneous control of the currents, hence $\varepsilon_d = 0$ and $\varepsilon_q = 0$ in Figure 2. If we operate in the linear range of the converter and the motor (no saturation), then this is due to the fact that the current loop is much faster than the velocity loop (typical bandwidths are 1 kHz for the current loop and 100 Hz for the velocity loop, see also Gan and Qiu, (2004)). Substituting Equation (27) in Equation (23), while assuming $i_{d\text{ref}} = 0$ and complete current control, we get

$$T_{em} = i_{q\text{ref}} \left[ \frac{\eta_0}{\Pi} + \sum_{p=1}^{\infty} \eta_{qp} \cos(6p\Pi\theta) \right]. \quad (28)$$

Using Equation (28), the block diagram in Figure 2 simplifies to the one in Figure 3.

In the sequel, we consider a control loop as in Figure 3. The special structure of this feedback system and the fact that $i_{q\text{ref}}$ is a function of the feedback result in a steady-state output $\omega$ whose spectral contents contain harmonics

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**Figure 2.** Full block diagram of the velocity control system. Here, $T_L$ and $T_e$ are the external load torque and the electromagnetic torque.

**Figure 3.** Block diagram of the simplified control system that corresponds to an ideal current control loop (i.e. no current tracking errors). The electromagnetic torque $T_e$ from Figure 2 is $T_{em} + T_{ec}$, where $T_{em}$ is the mutual torque and $T_{ec}$ is the cogging torque.
whose frequency is a multiple of 6$\Pi\omega_{\text{ref}}$. Thus, in steady state, $\omega$ will be periodic, with the fundamental frequency 6$\Pi\omega_{\text{ref}}$. We will see this phenomenon in our simulations and experiments.

Based on the IMP, we propose for $M$ a PI controller plus additional internal model terms:

$$M(s) = k_p + \frac{k_i}{s} + \sum_{m=1}^{N} \frac{k_m s}{\omega_m^2 + \omega_m^2}, \quad (29)$$

where $k_p, k_i, k_m > 0$ and $\omega_m = 6m\Pi\omega_{\text{ref}}$ is a resonant frequency of $M$. Such internal models have been used for instance in Jayawardhana and Weiss (2009), Knobloch, Isidori, and Flockerzi (1993) or Spitsa, Kuperman, Weiss, and Rabinovici (2006). The idea of the IMP is that $M$ has infinite gain at the frequencies $\omega_m$, resulting in zero tracking errors at these frequencies, if the closed-loop system is stable. Since there are a possibly large number of resonant frequencies, and they are multiples of a fundamental frequency, another way to build $M$ would be using delay lines, as in repetitive control, see, for instance, Weiss (1997), but we have not explored this further.

We mention that in the terminology of van der Schaft (2000), the controller $M$ is strictly input passive.

5. An adaptive controller

In this section, we modify the controller in Equation (29) to accommodate a variable motor speed. Internal models with variable resonant frequencies have appeared in Canudas de Wit and Praly (2000), Esbrook, Tan, and Khalil (2011), Lu et al. (2008) and Spitsa et al. (2006).

Recall from Equations (22), (25) and (28) that the electromagnetic torque generated inside the motor is

$$T_e = T_{\text{em}} + T_{\text{ec}}$$

$$= iq_0 + \sum_{p=1}^{\infty} \eta_{ip} \cos(6\Pi p \theta)$$

$$+ \sum_{n=0}^{\infty} K_n n \sin(nN_L \theta),$$

where $N_L$ is a multiple of 6$\Pi$.

From the above, it is obvious that the disturbance torque is periodic as a function of the motor position $\theta$, with frequencies which are usually multiples of 6$\Pi$. However, other integer frequencies may also appear, such as the frequency $\Pi$ due to DC offsets (Gan & Qiu, 2004) (we mentioned this in the Introduction). Since the disturbance in the motor speed is a function of $\theta$, we suggest using in the controller an internal model which is also a function of $\theta$ (instead of time). In order to do that, we visualize the input and the output of the controller as being functions of $\theta$ (instead of time). In this case, an internal model term to reduce the periodical disturbance, with frequency $m$, would be $k_m s^2 + m^2$. Overall, the controller would look as in Equation (29) and $\omega_{\text{ref}} = 1$. However, this is now a transfer function in an unconventional frequency domain, corresponding to $\theta$ as the time variable. To express our controller in the conventional time domain, first we revert from the Laplace to the $\theta$ domain where the term corresponding to the frequency $m$ is described by the differential equation

$$p_{\theta\theta} + p \cdot m^2 = k_m \epsilon_\theta. \quad (30)$$

Here, $e$ is the input to the IMP controller and $p$ is the output.

Converting Equation (30) to the time domain, while using

$$p_\theta = \frac{dp}{d\theta}, \quad p_{\theta\theta} = \frac{d^2 p}{d\theta^2},$$

we get:

$$p_\theta = \frac{1}{\omega}, \quad p_{\theta\theta} = \frac{\dot{p}}{\omega} - \frac{p}{\omega^2} \frac{\dot{\omega}}{\omega},$$

and similarly $e_\theta = \frac{d e}{d\theta} = \frac{d e}{d\theta} = \frac{1}{\omega^2} \dot{\omega} \cdot \frac{1}{\omega}$. (31)

This is a linear differential equation with variable coefficients (that depend on $\omega, \dot{\omega}$). A digital controller will be able to implement a good approximation of such a subsystem. Since in Equation (31) we have a division by $\omega$, it is recommended to impose a lower bound on $\omega$ in this division: For some small $\varepsilon > 0$, if $\omega < \varepsilon$, then replace $\omega/\varepsilon$ with $\omega/\varepsilon$.

The term $k_i/s$ in Equation (29) has to be expressed in the normal time domain as well. This is much easier and we get:

$$\dot{p} = k_i \epsilon_\theta. \quad (32)$$

The block diagram of the control system with the adaptive internal model controller (AIMC) is the same as in Figure 2 except that $M$ is replaced with the AIMC block that contains a realization of both Equations (31) and (32).

6. The adaptive eccentricity compensation algorithm

In Canudas de Wit and Praly (2000), an adaptive observer, called adaptive eccentricity compensator (AEC), is proposed for the velocity control of a BLDCM that estimates the angle-dependent periodic torque disturbances acting on an electric motor. A proof for stability is given for a proportional controller acting on a very simplified motor model (an integrator from torque to angular velocity) using a Lyapunov function. A friction predictor is added to cancel the Coulomb friction, which is feedforward, so that it cannot react to changes in the load torque. In our implementation of the controller from Canudas de Wit and Praly (2000),
used for comparison with our AICM controller, we replace the friction predictor by an integral component in the controller, which can compensate any step load torque – this is in the PI block in the middle of Figure 4. Thus, we use a slightly better implementation of the AEC controller than the original one in Canudas de Wit and Praly (2000). The rest of the block diagram is as in Figure 2.

The controller of Canudas de Wit and Praly (2000) has been further developed in Malaize and Levine (2009) for the case of high precision positioning of a BLDCM (i.e. tracking of an angle reference signal). Thus, there is no need to filter out the noise originated in the discrete derivative of the encoder. On the other hand, tuning this new controller scheme is not that simple. The proof of stability of the encoder. On the other hand, tuning this new controller scheme is not that simple. The proof of stability of the encoder. On the other hand, tuning this new controller scheme is not that simple.

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The symbol $\circ\circ\circ$ indicates a connection with the remaining part of the system, which is as in Figure 2.

![Figure 4. The velocity control loop with the AEC controller.](image)

Figure 4. The velocity control loop with the AEC controller. The symbol $\circ\circ\circ$ indicates a connection with the remaining part of the system, which is as in Figure 2.

7. A proof of convergence

We give a proof of convergence to zero of the speed tracking error $e$ from Figure 3 under the restrictive assumption that the reference speed $\omega_{ref}$ is constant. We shall also assume that the system remains throughout in the reasonable operating region where the angular velocity $\omega$ and the total torque $u$ acting on the motor are positive. We emphasize that this is not the “standard” situation from internal model-based control, because the disturbance is a superposition of sinusoidal signals in the variable $\theta$, and not in the time variable.

**Lemma.** Consider the system $P$ with transfer function

$$P(s) = \frac{1}{Js + D}$$

with input $u$ and output $\omega$ and let $\omega_{ref} > 0$ be a constant. Introduce the speed error $e = \omega_{ref} - \omega$, the reference torque $t_{ref} = D\omega_{ref}$, the torque error $v = t_{ref} - u$ and the Hamiltonian $H = J e^2 \omega_{ref}/2$. If we regard $P$ with $\theta$ (the integral of $\omega$) as the time variable, with $v$ as input and with $e$ as output, then $H$ becomes a nonlinear time-invariant system $P_{NL}$. In the region where $u, \omega \geq 0$, $P_{NL}$ is passive, i.e. $H_\theta \leq ve$.

**Proof** Since $\dot{e} = (-De + v)/J$, we have

$$H_\theta = \frac{H}{\omega} = \frac{Je\dot{e}\omega_{ref}}{\omega} = e(-De + v) \frac{\omega_{ref}}{\omega}$$

$$= \left[ -De^2 + ev \left( 1 - \frac{\omega}{\omega_{ref}} \right) \right] \frac{\omega_{ref}}{\omega} + ev$$

$$= \frac{[D\omega_{ref} + v]^2}{\omega} + ev = \frac{ue^2}{\omega} + ev.$$  

We see that if $u, \omega \geq 0$ then $H_\theta \leq ve$.

**Proposition.** Consider the feedback system from Figure 3 but with a perfectly built motor, meaning that $\eta_{qp} = 0$ for all $p > 0$, and with only finitely many terms in the sum defining the cogging torque $T_c$. Assume that the adaptive internal model $M$ is built according to Equations (31) and (32), with the resonant frequencies (in the $\theta$ domain) matching the frequencies of the cogging torque. Assume that $\omega_{ref} > 0$ and $T_L$ are constant and the system remains in the region where $u, \omega \geq 0$. Then, $\lim_{t \to \infty} e(t) = 0$.

**Proof.** Using simple block diagram manipulations (moving the summation point for the feedforward signal further ahead until the right end of the diagram), we can see that Figure 3 is equivalent to the one shown in Figure 5.

Now, we regard this feedback system in the $\theta$ domain (i.e. we use $\theta$ in place of the time). Then, the feedback loop in the right upper corner of Figure 5 disappears, because $\theta$ is no longer a state variable, and instead $T_c - T_L$ is now a disturbance generated by a finite-dimensional linear exosystem that has simple eigenvalues on the imaginary axis ($T_L$ is generated by an eigenvalue at zero). The linear system with parameters $J$ and $D$ at the right end of Figure 5 becomes the nonlinear passive system $P_{NL}$ described in the lemma, with the state $e$ (passivity holds as long as $u, \omega \geq 0$). The block diagram becomes very simple, and we leave it to the reader to redraw it. This new block diagram is the same as Figure 2 in Jayawardhana and Weiss (2009), with $\eta_{q0}M$ playing the role of the controller. It can be verified...
that the assumptions in Theorem IV.3 from Jayawardhana and Weiss (2009) are satisfied, and this implies that indeed \( e(t) \to 0 \).

We remark that the conclusion of the proposition remains valid if we add an arbitrary \( L^2 \) component to the constant load torque \( T_L \), and this follows from the same theorem in Jayawardhana and Weiss (2009).

8. Simulations without load

In this section (and also in the experiments), \( \omega \) is measured by taking the discrete derivative from a high-resolution encoder with 20,000 counts per turn. Since the simulations and experiments are done at low motor speeds, a low-pass filter is used on these velocity measurements. We chose a second-order low-pass filter with natural frequency \( \omega_n = 2\pi \frac{200}{2} \text{ rad/sec} \) and damping \( \zeta = 0.7 \). The angular acceleration (needed in Equation (31)) is computed by taking the discrete derivative of \( \omega \).

In order to facilitate the comparison of the AEC and AIMC controllers, the following assumptions are made:

- Since most of the ripple is due to the cogging, we assume \( \eta_{q0} = \sqrt{3/2} \Psi_r \) and \( \eta_{qp} = 0 \) for \( p \neq 0 \).
- The motor has two pole pairs per phase, that is, \( \Pi = 2 \).
- \( D \) is assumed to be zero. Hence, comparing Figure 4 with Figure 2, we choose \( \alpha = J/\eta_{q0} \).
- There is static friction on the rotor, 0.01 Nm, with sign depending on the direction of motion, of course.

We have obtained simulation results for the system shown in Figure 2, with: \( \Psi_r = 0.0358 \text{ V sec} \), \( \eta_{q0} = 0.04384 \text{ N m/} \text{A} \), \( J = 332 \times 10^{-7} \text{ kg m}^2 \), \( \Pi = 2 \), \( K_1 = 0.006 \text{ N m} \), \( k_1 = 10 \text{ A} \), \( k_p = 0.3125 \text{ A sec} \), \( L_s = 0.65 \times 10^{-3} \text{ H} \), \( V_{bus} = 20 \text{ V} \), \( r = 0.5 \Omega \). The coefficients of the PI part of the controller are chosen so that if multiplied by 2 the system would become unstable.

In the next two figures, the reference input and the cogging torque are as in Figure 7. The velocity responses, while using the PI controller (alone) and the AIMC algorithm, are plotted in Figure 8. Figure 9 shows the

First, we simulate the velocity response for a “perfectly built” motor, that is, \( T_{ec} = 0 \). We see from Figure 6 that \( \omega \) converges to \( \omega_{ref} \). The large overshoot is caused by the low-pass filter mentioned earlier. Next, we simulate the step response for a PMSM with cogging torque \( T_{ec} = 0.006 \cdot \sin (12 \cdot \theta) \) (see Figure 7). We also show its steady-state frequency contents, that is, the absolute value of its fast Fourier transform (FFT) for the time interval \([1,10]\) (sec). As we can see from Figure 7, the steady-state velocity is periodic, with the fundamental frequency \( 12\omega_{ref} \).

In the remaining simulations, we compare the two adaptive algorithms (AEC and AIMC). The PI part of the controllers is chosen as before. For the AEC controller, we choose \( h_0, h_1 = 3 \). For the AIMC controller, \( k_1 = 1.8 \). In both controllers, the gains are chosen such that if multiplied by 2 the system would become unstable.

In the next two figures, the reference input and the cogging torque are as in Figure 7. The velocity responses, while using the PI controller (alone) and the AIMC algorithm, are plotted in Figure 8. Figure 9 shows the

![Figure 5. A block diagram equivalent to the one from Figure 3 when \( \eta_{qp} = 0 \) for all \( p > 0 \), and using an AIMC, as in the proposition.](image-url)
velocity response with AEC algorithm compared with that of the AIME algorithm.

To check the ability of both controllers to track a variable speed command, we changed the reference input. The new reference input and the velocity response using a simple PI controller are shown in Figure 8. The velocity responses, when using the adaptive algorithms, are shown in Figure 11. They are so close that we cannot distinguish them visually in Figure 11. A comparison between the velocity error signals of the two algorithms, after 3 s, is shown in Figure 12.

Next, we simulate a second harmonic for the cogging disturbance (though it does not exist in our experiment) and check both algorithms. We take $K_2 = 0.003 \text{ N m}$ in Equation (25) (in addition to $K_1 = 0.006 \text{ N m}$) [taken previously]. To deal with the second harmonic as well, we add another AIMC block with $k_2 = 0.6$. The velocity response is shown in Figure 13. A comparison between the AIMC and the AEC for cogging with two harmonics is shown in Figure 14.

9. Simulation results with a flexible shaft and load

The AEC controller from Canudas de Wit and Praly (2000) was designed for a motor model that is just an integrator (from torque to angular velocity) and their proof of stability refers to this model. This model corresponds to a motor with a very simple mechanical load (just friction). The simulations in Section 8 use such a model and they show that in this case, the behavior of the control system
In this section, we consider a motor that is connected to the load via a flexible shaft with internal viscous damping. The load has its own moment of inertia $J_L$ and there is viscous friction between the load and the inertial reference system, with coefficient $b$. We denote by $K$ and $C$ the stiffness constant and the damping constant of the flexible shaft. The schematic representation of the mechanical system is shown in Figure 15.

We choose on purpose a very flexible (very soft) shaft, so that there will be a significant difference between the velocities at its two ends. The parameters are: $C = 0.005 \text{ Nm sec/rad}$, $K = 0.1 \text{ Nm/rad}$, $J_L = 332 \times 10^{-6} \text{ kg m}^2$, $b = 0.01 \text{ Nm sec/rad}$.

For the simulations, we make the same assumptions as in Section 8 and we use the same motor parameters. The additions to the block diagram from Figure 2, in order to incorporate the flexible shaft and the load, are shown in Figure 16.
With this control system, we obtain very good tracking for the reference signal from Figure 10, as shown in Figure 17. Figure 18 is a zoom of Figure 17.

10. Experimental results

For the experimental work, we used a RP23–54 24 V motor with an encoder of 20,000 counts/turn, a driver including the current sensors designed in the company Rafael and a dSpace setup, as shown in Figure 19. The sampling frequency for the current loop was 16 kHz and for the velocity loop, 4 kHz. The motor’s nominal parameters are given in Section 8. The angular velocity was calculated inside the dSpace controller by taking the discrete derivative of the encoder output. We have closed the current loop in the dSpace controller.

10.1. A. Stability of the experimental system

The coefficients of the PI part of the controller have been chosen so that if we only use the PI part, we get a reasonable gain margin (8.5 dB) and phase margin (52°).
Figure 20. Velocity response and its steady-state frequency contents using a PI controller (0–9 s) and then an AIMC controller (from 9 to 20 s) for a velocity reference of 4 rad/sec. (The FFT was performed on the intervals 1–7 s for the PI, and 12–18 s for the AIMC).

Figure 21. Velocity response and its steady-state frequency contents using a PI controller (0 up to 9 sec) and then an AEC controller (from 9 to 20 s) for a velocity reference of 4 rad/sec. (The FFT was performed on the intervals 1–7 s for the PI, and 12–18 s for the AEC).

### 10.2. B. Testing the adaptive algorithms

Figure 20 shows the motor velocity $\omega$ for $\omega_{ref} = 4$ rad/sec. We have used a PI controller up to 9 s, and then added an AIMC controller until 20 s. Figure 21 shows $\omega$ for the same $\omega_{ref}$ using a PI controller up to 9 s, and then switching to an AEC controller until 20 s. Since the AIMC and the AEC controllers give very similar results during the experiments (as can be seen by comparing Figures 20 and 21), in the next figures, we only give the results for the AIMC algorithm.

To check the adaptive controllers, the $\omega_{ref}$ command from Figure 10 was chosen. The velocity response using a PI controller versus an AIMC controller is shown in Figure 22. In order to see the reduction in the velocity error, Figure 23 shows the velocity response using a PI versus an AIMC controller for a variable velocity reference, in the region marked “Zoom 1” in Figure 22.

Figure 22. Velocity responses using a PI controller and an AIMC controller, for the reference command of Figure 10.

Figure 23. Velocity response using a PI versus an AIMC controller for a variable velocity reference, in the region marked “Zoom 1” in Figure 22.

Figure 24. Velocity response using a PI versus an AIMC controller, for a constant reference command of 16 rad/sec, in the region marked as “Zoom 2” in Figure 22.
we view in Figures 23 and 24 two areas of the response, marked in Figure 22 as Zoom 1 and Zoom 2.

11. Conclusions

We have designed an internal model-based controller to suppress the torque ripple in a PMSM. The biggest challenge has been to get the controller to work at variable speed. To achieve this, we have used the rotor angle $\theta$ in place of the time variable in the formulation of the controller. We call this an adaptive internal model-based controller. Assuming a correct choice of resonant frequencies in the $\theta$ domain, we have proved convergence to zero of the tracking error under the restrictive assumption that $\omega_{\text{ref}}$ is constant. After giving a short review of the AEC from Canudas de Wit and Praly (2000), we have compared this controller with ours by simulations. The simulations show the ability of both controllers to track a variable $\omega_{\text{ref}}$ command while suppressing the effects of the cogging torque.

We have shown how to use our adaptive controller for a motor connected to a load via a flexible shaft and we have experimentally tested both controllers without load. The experiments confirm that both algorithms perform very well.

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