Numerical simulation of hydro-mechanically coupled mud loss in fractured reservoirs

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Abstract. Mud loss is a common phenomenon in the drilling process with an important impact on the efficiency and safety of drilling. Mud loss in fractured formations is serious and previous methods have been based mostly on the single-fracture model, which had difficulty dealing with the loss of complex fracture networks in fractured reservoirs. A hydro-mechanically coupled model was constructed based on the poroelasticity. This model also coupled the fluid flow in fractures and matrix by fluid exchange term across fracture surface. The fully coupled model was solved using the extended finite element method (XFEM). The physical field characteristics were represented by enrichment functions, and grids were decoupled from fractures. Thus, the efficiency was improved greatly. A simple single-fracture model was used to verify this method, and a multi-fracture numerical model was then constructed to examine the influence of the fracture aperture, matrix permeability, fracture deformation, and mud viscosity on mud loss.

1. Introduction
The fluid flow in deformable porous media with fractures is a research hotspot in the engineering field, and research on mud loss in fractured formations is very important [1,2,3,4,5,6]. Dyke [7] was the first to study mud loss quantitatively, but his model was a simple single-fracture model. Lavrov and Tronvoll [8,9] considered the fracture aperture as a linear function of the fluid pressure in fractures and the influence of fracture deformation on mud loss for the first time. Tempone and Lavrov [10] examined the influence of fracture network deformation on mud loss. Xia [4,5] established a geological model of discrete fracture networks using the Monte Carlo stochastic simulation method and solved the mud loss problem in complex fracture networks. Xia [6] then established a general two-scale model of mudflow in naturally fractured formations, examined the mud loss problem of crossed-fracture networks, and used the extended finite element method (XFEM) for numerical discretization.

Several numerical methods have been used to solve related problems, but most required grids to conform with fractures, and it was often challenging to balance the calculation accuracy and efficiency. The XFEM [11,12] was introduced into related research to overcome inherent limitations for solving discontinuity problems. In the XFEM, the standard finite element approximation space is improved by enrichment functions based on an asymptotic analytical solution, and additional degrees of freedom are introduced. Based on the XFEM, Sukumar [13] and Moës [14] studied the three-dimensional fracture problem, and Réthoré [15] and Song [16] evaluated the dynamic fracture propagation problem. Salimzadeh and Khalili [3] used the Galerkin and finite difference methods to examine the single-fracture problem. de Borst [17] and Réthoré [18, 19] introduced the XFEM to study the pressure field. However, the above research did not construct better enrichment functions. Thus, the accuracy of the pressure field simulation is not as expected. Xia [20,21] improved the enrichment functions according
to the characteristics of the pressure field, which greatly improved the accuracy of the XFEM to solve fluid flow problems.

In this study, the discrete fracture model was used to describe the fractures explicitly. The hydro-mechanically coupled governing equations were derived based on the poroelasticity. The fluid flow in the fracture and rock matrix was coupled by fluid exchange through the fracture surface. Fully coupled equations were constructed and solved by the XFEM with improved enrichment functions. The effectiveness was verified using a single-fracture model, and a complex crossed-fracture networks model was then designed to examine the factors influencing mud loss. All these factors had a significant impact on mud loss and could be divided into two parts: one is controlled by rock and mud properties, and the other is controlled by hydro-mechanically coupled process.

2. Hydro-mechanically coupled governing equations

2.1. Governing equations of matrix

For the quasi-static process, the stress balance equation can be expressed as

$$\nabla \cdot \sigma + b = 0, \quad (1)$$

where $$\sigma$$ is the total stress tensor and $$\sigma'$$ is the effective stress, $$\alpha$$ is the Biot coefficient. $$p$$ is the pore pressure. $$I$$ is the identity matrix. The body force $$b$$ can be neglected, and the weak form can be constructed considering the boundary condition.

$$\int_{\Omega} (\nabla \cdot \delta u) : (C : \varepsilon - \alpha p I) d\Omega + \int_{\Gamma_f} \delta u p d\Gamma - \int_{\Gamma_f} \delta u \cdot \vec{t} d\Gamma = 0. \quad (3)$$

The local balance of the mass for the matrix is

$$\frac{\partial m_f}{\partial t} + \nabla \cdot \vec{w}_f = 0, \quad (4)$$

where $$m_f$$ is the fluid mass and $$\vec{w}_f$$ is the mass flux. The state equation for the fluid pressure $$p$$ can be expressed as [22]

$$p = Q\xi - \alpha Qe. \quad (5)$$

From Darcy’s formula,

$$\rho_f v_f = -\frac{k_m}{\mu} \nabla p, \quad (6)$$

where $$\rho_f$$ is the fluid density, and $$k_m$$ and $$\mu$$ are matrix permeability and mud viscosity, respectively. Substituting this and equation (5) into equation (4) allows the strong form of matrix flow equation to be obtained.

$$\alpha \nabla \cdot \vec{u} + \frac{1}{Q} \frac{\partial p}{\partial t} - \frac{k_m}{\mu} \nabla^2 p = 0 \quad (6)$$

The weak form of matrix flow can be expressed as

$$\int_{\Omega_m} \alpha \delta p \nabla \cdot \vec{u} d\Omega + \int_{\Omega_m} Q^{-1} \delta p \hat{d} d\Omega + \int_{\Gamma_f} \delta p \vec{q} d\Gamma + \int_{\Omega_m} k_m \nabla (\delta p) \nabla p d\Omega = \int_{\Gamma_f} \delta p \left[ \hat{q}_{f-m} \cdot \vec{n}_f \right] d\Gamma, \quad (7)$$

where $$\hat{q}_{f-m}$$ denote the mass transfer from the fracture into the matrix.

2.2. Governing equations of fractures

The mass balance equation for mudflow in the fracture can be expressed as

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \vec{v} = 0. \quad (8)$$
Consider the equation of state of a fluid $C_v = \frac{1}{\rho} \frac{d\rho}{dt}$, the weak form of equation (8) can be written as

$$\int_{\Omega} \delta p \mathbf{v} \cdot d\Omega = \int_{\partial \Omega} \mathbf{n} \cdot \mathbf{v} \, d\Gamma = \int_{\partial \Omega} \delta p \mathbf{n} \cdot d\Gamma = \int_{\partial \Omega} \delta p \mathbf{n} \cdot d\Gamma = \int_{\partial \Omega} \delta p \mathbf{n} \cdot d\Gamma = 0,$$

where $\mathbf{n}$ is normal to the crack surface. The fracture fluid is considered an incompressible Newtonian fluid. The balance of the momentum inside the two-dimensional fracture is as follows:

$$\mu \nabla^2 \mathbf{v} = \nabla p,$$

because the fracture aperture is much smaller compared to its length. Thus, the change in $p$ along the $n$ direction is negligible, and equation (10) can be expressed as

$$\mu \frac{\partial^2 \mathbf{v}}{\partial y^2} = \frac{\partial p}{\partial x}.$$

Integrating equation (11) twice along the direction of the fracture aperture, the velocity profile is

$$\mathbf{v}_f \cdot \mathbf{t}_f = v_f = \frac{1}{2\mu} \frac{\partial p}{\partial x} (y^2 - b^2) + v_t,$$

where $2b$ is the fracture aperture; $v_f$ is the essential boundary condition on two faces of the fracture.

$$\int_{\Omega} \delta \mathbf{v} \cdot \mathbf{v} \, d\Omega = \int_{\partial \Omega} \mathbf{n} \cdot \mathbf{v} \, d\Gamma = \int_{\partial \Omega} \delta p \mathbf{n} \cdot d\Gamma = \int_{\partial \Omega} \delta p \mathbf{n} \cdot d\Gamma = \int_{\partial \Omega} \delta p \mathbf{n} \cdot d\Gamma = 0,$$

Substituting (13) into equation (9) results in the fluid exchange term that includes $q_{f-m}$. Substituting the fluid exchange term into equation (7) and ignoring $v_f$ results in the weak form governing equation coupled with the fracture-matrix flow as follows:

$$\int_{\Omega} \mu \nabla \mathbf{v} \cdot \Delta \mathbf{v} \, d\Omega + \int_{\Gamma_f} \mathbf{Q} \cdot \mathbf{v} \, d\Gamma + \int_{\Gamma_v} \delta \mathbf{v} \cdot \mathbf{v} \, d\Gamma + \int_{\Gamma_f} \frac{k_m}{\mu} \nabla(p) \mathbf{v} \cdot \mathbf{v} \, d\Gamma + \int_{\Gamma_v} \frac{2b^2}{3\mu} \frac{\partial p}{\partial x} \mathbf{v} \cdot \mathbf{v} \, d\Gamma + \int_{\Gamma_f} C_i \delta \mathbf{v} \cdot \mathbf{v} \, d\Gamma = 0.$$

### 3. Extended finite element method

The fractures need to be described geometrically because the grids are independent of the fractures in the XFEM, which allows them to pass through the element. In this paper, level set functions $f(x)$ and $g(x)$ based on the signed distance function were used to describe the fractures [23].

#### 3.1. Displacement field approximate space

For the element penetrated by the fractures, the displacement field on both sides of the fractures jumps, and the displacement is discontinuous. The enrichment function based on the Heaviside function proposed by Moës [12] was adopted. For the element contained crack tips, the singularity of stress is represented by four singular functions. For the crossed fractures, the nodes were enriched by the junction function [24]. The displacement approximation was

$$u = \sum_{i \in \mathcal{N}_c} N_i(x) u_i + \sum_{n=1}^{n_f} \sum_{j \in \mathcal{N}_f} N_f_j(x)(H_f(x) - H(x)) u_f^j + \sum_{n=1}^{n_f} \sum_{j \in \mathcal{N}_j} N_j(x)(H_f(x) - H(x)) u_f^j + \sum_{n=1}^{n_f} \sum_{j \in \mathcal{N}_j} N_j(x)(H_f(x) - H(x)) u_f^j + \sum_{n=1}^{n_f} \sum_{j \in \mathcal{N}_j} N_j(x)(H_f(x) - H(x)) u_f^j + \sum_{n=1}^{n_f} \sum_{j \in \mathcal{N}_j} N_j(x)(H_f(x) - H(x)) u_f^j,$$

where $N_i$ is the set of all nodes in the discrete domain. $u_i$ is the displacement of the nodes. $N_i(x)$ is the standard finite element shape function. $n_f$, $n_f$, and $n_f$ are the number of fractures, fracture tips, and fracture junctions. $N_{fract}$ denotes the set of nodes in the element through which the $n$th crack passes completely, and they hold additional degrees of freedom, $a_f^j$. $N_{frac}$ denotes the set of nodes in the element that contains the $n$th tip, and they hold additional degrees of freedom $b_f^m$. $N_{frac}$ denotes the
set of nodes in the element that contains the \( nn \)th intersection point of crossed fractures, and they hold additional degrees of freedom, \( c_{nn}^{enr} \). \( H_n(x) \) is the Heaviside function.

\[
H_n(x) = H(f(x)) = \begin{cases} 1, & f(x) > 0 \\ -1, & f(x) < 0 \end{cases}
\]

The singular functions \( \xi_{nn}(r, \theta) \) are expressed as

\[
\xi_{nn}(r, \theta), nn = 1, 2, 3, 4 = \left[ \sqrt{r} \sin \frac{\theta}{2}, \sqrt{r} \cos \frac{\theta}{2}, \sqrt{r} \sin \frac{\theta}{2} \sin \theta, \sqrt{r} \cos \frac{\theta}{2} \sin \theta \right],
\]

where \( r \) and \( \theta \) are coordinates at the fracture tip in the local coordinate system.

\( f_j(x) \) and \( f_j(x) \) are the level set function value of the main and secondary fractures, respectively. The junction function for enriching the nodes in the elements where fractures cross can be defined as follows [25,26]:

\[
\begin{align*}
J_{nn}(x) &= H(f_j(x)) - H(f_j(x)) f_j(x) f_j(x) > 0 \\
J_{nn}(x) &= H(f_j(x)) - H(f_j(x)) f_j(x) f_j(x) < 0.
\end{align*}
\]

3.2. Pressure field approximate space enrichments

For pressure, the fracture is a weak discontinuity. The pressure on both sides of the fracture is continuous, but the pressure derivative is discontinuous. The improved absolute value function proposed by Moës [27] was adopted to characterize the weak discontinuity. For the nodes in the tip element, the singularity is characterized by the enrichment function based on the asymptotic analytical solution [21]. Hence, the pressure approximation is

\[
p = \sum_{i \in N} N_i p_i + \Psi(x) = \sum_{i \in N} N_i p_i + \sum_{j \in N_{enr}} N_j \phi_j(x) \hat{p}_j
\]

where \( N_{enr} \) is the set of nodes, which are enriched. \( \phi_j(x) \) denotes the set of enrichment functions corresponding to the additional degrees of freedom \( \hat{p}_j \). \( \phi_j(x) \) contains \( \phi_j^{enr}(x) \) and \( \phi_j^{enr}(x) \), which are written as follows:

\[
\phi_j^{enr}(x) = \sum_j \hat{f}(x) \xi_j(x) H_j(x) - \sum_j \hat{f}(x) H_j(x),
\]

\[
\phi_j^{enr}(x) = \sqrt{r} \cos \frac{\theta}{2}.
\]

3.3. Numerical scheme for the coupled governing equations

\( N \) denotes the shape function of the displacement field, and \( H \) denotes the shape function of the pressure field. The numerical scheme for the coupled governing equations can be obtained by substituting the approximate expressions of the displacement and pressure fields into the coupled governing equations (3) and (14).

\[
K_j = \int_{\Omega} \alpha H^T m^T Bd\Omega, \quad M_j = \int_{\Omega} b^C_{ij} H^T H d\Omega + \int_{\Gamma} \hat{b}C_{ij} H^T H d\Gamma,
\]

\[
F = \int_{\Gamma} N^T \hat{q} d\Gamma.
\]

where \( m = [1 \ 0 \ 0] \), \( B = \nabla N \), \( D = \nabla H \).
4. Numerical examples

4.1. Validation model

A two-dimensional 100 × 100-m model was designed to verify the correctness of the method in this paper, and the results were compared with those of COMSOL Multiphysics® version 5.6. The well was located in the center of the model, and there was a horizontal fracture, 40 m in length, passing through the well point. Figure 1 presents a schematic diagram and grids diagram of the model, and Table 1 lists the other parameters of the model. The time was 300 h, and Figure 2 shows the pressure field comparison diagram. Figure 3 presents the curve of the mud loss mass change with time. As shown in Figure 2, the pressure fields of the fully coupled model calculated by COMSOL and XFEM were similar, which shows the effectiveness of the method to a certain extent. As shown in Figure 3, the curve fitting effect of COMSOL and XFEM improved with time. The difference was relatively large in the early stages. They were much smaller in the later stages, which quantitatively proves the effectiveness and accuracy of the method in this paper.

In addition, the number of grids and the number of nodes in XFEM were obviously less than that of COMSOL. Therefore, the computational complexity was smaller. The calculation times of the XFEM and COMSOL were 9.8994 seconds and 13 seconds, respectively. Thus, the calculation efficiency was improved significantly.

![Figure 1. Schematic diagram of the validation model](image)

![Table 1. Parameters of the validation model](image)
4.2. Analysis of the influencing factors of mud loss

A multi-fracture model was designed to analyze the influence of various factors on mud loss. Table 2 lists the initial parameter design of the model, and Figure 4 shows the results under this condition.

| Parameters                                      | Value       | Unit  |
|-------------------------------------------------|-------------|-------|
| Initial reservoir pressure                      | 15          | MPa   |
| \( x \) horizontal in-situ stress              | 20          | MPa   |
| \( y \) horizontal in-situ stress              | 30          | MPa   |
| Bottom-hole pressure \( p_w \)                 | 20          | MPa   |
| Biot coefficient \( \alpha \)                  | 0.83        | /     |
| Compressibility coefficient of the mud          | \( 5 \times 10^{-10} \) | Pa    |
| Compressibility coefficient of the porous rock  | \( 2 \times 10^{-11} \) | Pa    |
| Young modulus \( E \)                          | 50          | GPa   |
| Poisson ratio \( \nu \)                        | 0.2         | /     |
| Initial matrix porosity                        | 0.08        | /     |
| Matrix permeability                             | \( 1 \times 10^{-17} \) | m²    |
| Mud viscosity                                   | 0.1         | mPa·s  |
| Mud density                                     | 1000        | kg/m³  |
| Fracture aperture                               | \( 2 \times 10^{-3} \) | m     |
1) **Influence of the fracture aperture on mud loss**

The fracture aperture is an important factor in determining mud loss. From equations (9)-(13), the fracture aperture can affect the matrix-fracture fluid exchange term. The mud loss under different fracture aperture conditions was compared, as shown in Figure 5. The mud loss volume is positively related to the fracture aperture. However, when the fracture aperture exceeds a certain value, the change in mud loss volume is not obvious with increasing fracture aperture.

2) **Influence of rock matrix permeability on mud loss**

Matrix permeability is one of the decisive factors of mud loss. The loss volume of different permeability values was determined. Figure 6 presents the comparative curves. When the permeability was less than 1 mD, a larger matrix permeability indicated more serious mud loss. However, with increasing permeability greater than 1 mD, the mud leaked quickly and became stable in a short time. The increase in permeability has no obvious effect on the increase in mud loss volume.

3) **Influence of the fracture deformation on mud loss**

Fracture deformation is an important factor affecting mud loss. This paper calculated the mud loss volume considering and not considering fracture deformation (Here, fracture deformation refers mainly to the change in fracture aperture). Figure 7 shows the comparison curve. The loss volume considering fracture deformation was significantly larger because the mud entered the fracture, and the fracture was opened further [6].

4) **Influence of mud viscosity on mud loss**

The viscosity of mud is an important parameter to characterize the loss capacity of mud. By changing the viscosity and comparing the mud loss volume (see Figure 8), the mud loss volume increased with decreasing mud viscosity.

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**Figure 4.** Results of the multi-fracture model after 100 h (the x-direction effective stress, the y-direction effective stress, and pressure)

**Figure 5.** Comparison of mud loss volume with different fracture apertures

**Figure 6.** Comparison of the mud loss volume with different matrix permeability

**Figure 7.** Comparison of mud loss volume with different fracture deformation conditions

**Figure 8.** Comparison of mud loss volume with different viscosities.
According to the study, mud loss is controlled by two factors. One is the properties of mud and rock, which are the decisive factor in fluid flow and mud loss. The other one is fluid–solid interaction, which mainly affects fluid and mud loss by controlling the deformation of the rock matrix and fractures. Therefore, the hydro-mechanically coupled mud loss model is essential for high-precision numerical simulation.

5. Conclusion
In this study, the fracture-matrix flow was coupled using the exchange term, fluid across the fracture surface, and the fluid–solid physical field was coupled with the poroelasticity. A fully coupled mud loss model was established and solved by XFEM. In this paper, the enrichment functions were selected according to the different physical characteristics of the displacement/stress field and pressure field, which can capture the physical characteristics accurately. The results of the validation model show that the method in this paper can achieve similar calculation accuracy to COMSOL while the computational complexity is lower, which reflects the advantages of the method in this paper. Finally, through a numerical example, the influence of four factors on mud loss was studied. The results showed that all four factors have a significant influence on mud loss. The analysis shows that factors can be divided into two types: mud loss controlled by mud and rock properties and mud loss controlled by fluid–solid interaction.

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