Towards Precision Measurements of $\gamma$: CLEO-C’s Pivotal Role

Andrew S. Powell

University of Oxford, Denys Wilkinson Building, Oxford, OX1 3RH, UK

Strategies that utilise the interference effects within $B^{\pm} \rightarrow D K^{\pm}$ decays hold great potential for improving our sensitivity to the CKM angle $\gamma$. However, in order to exploit fully this potential, detailed knowledge of the D meson decay structure is required. This essential information can be obtained from the quantum correlated $\psi(3770)$ datasets at CLEO-c. Results of such analyses involving the decay mode $D \rightarrow K \pi \pi \pi$, and their importance in the context of LHCb, will be presented.

1. Introduction

A means of testing the internal consistency of the Cabbibo-Kobayashi-Maskawa (CKM) model, whilst simultaneously searching for signatures of New Physics, is to perform precision measurements of the angles that compose the unitarity triangle: $\alpha$, $\beta$ and $\gamma$. While $\beta$ has been measured with extremely high precision at the B-factories $(20.5 \pm 1.0^\circ)^1$, determination of the other two angles currently presents a considerable experimental challenge; most notably $\gamma$ which is only constrained by direct measurements with a precision of $\sim \pm 30^\circ$. One of the most promising ways of determining the angle $\gamma$ is through strategies that exploit the interference within $B^{\pm} \rightarrow D K^{\pm}$ decays.$^a$ The most straightforward of these strategies considers two-body final states of the D meson, however, an abundance of additional information can be gained from strategies that consider multi-body final states instead. In order to exploit fully the wealth of statistics soon to arrive at the LHC, the LHCb$^{2,3}$ experiment plans to utilise all such strategies in its analysis. Multi-body strategies, however, only have significant sensitivity when combined with detailed knowledge of the D meson decay structure. Fortunately, the parameters associated with the specific multi-body final states needed for these analyses can be extracted from correlations within CLEO-c$^4 \psi(3770)$ data.

$^a$Here and subsequently, D will denote a $D^0$ or $\bar{D}^0$. 
2. Determination of the CKM angle $\gamma$ from $B^\pm \to DK^\pm$

The interference between $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$ decays, and equally between their CP conjugate states, provides a clean mechanism for the extraction of the CKM angle $\gamma$ when the $D^0$ and $\bar{D}^0$ mesons decay to a common final state, $f_D$. Decay rates in these channels have the following amplitude ratio

$$\frac{A(B^- \to \bar{D}^0 K^-)}{A(B^- \to D^0 K^-)} = r_B e^{i(\delta_B - \gamma)},$$

which is a function of three quantities: the ratio of the amplitudes absolute magnitudes $r_B$, a CP invariant strong phase difference $\delta_B$, and the CKM weak phase $\gamma$. Generically, the amplitude for the complete decay $B^- \to D(f_D)K^-$, normalised to the favoured $B \to DK$ amplitude, is defined as

$$\frac{A(B^- \to D(f_D)K^-)}{A(B^- \to D^0 K^-)} = A_{D^0} + r_B e^{i(\delta_B - \gamma)} A_{\bar{D}^0},$$

where $A_{D^0}$ and $A_{\bar{D}^0}$ represent the amplitudes for the $D^0$ and $\bar{D}^0$ decays, respectively. Due to colour suppression $r_B < 0.13 @ 90\%$ CL; therefore, the interference is generally small. A variety of strategies exist, however, that attempt to resolve this and maximise the achievable sensitivity to $\gamma$. One such tactic is to consider multi-body final states of the D meson.

2.1. ADS Formalism

Atwood, Dunietz and Soni (ADS)\(^6\) have suggested considering $D$ decays to non-CP eigenstates as a way of maximising sensitivity to $\gamma$. Final states such as $K^-\pi^+$, which may arise from either a Cabibbo favoured $D^0$ decay or a doubly Cabibbo suppressed $\bar{D}^0$ decay, can lead to large interference effects and hence provide particular sensitivity to $\gamma$. This can be observed by considering the rates for the two possible $B^-$ processes:

$$\Gamma(B^- \to (K^-\pi^+)D K^-) \propto 1 + (r_B r^{K\pi}_D)^2 + 2 r_B r^{K\pi}_D \cos (\delta_B - \delta_D^{K\pi} - \gamma),$$

$$\Gamma(B^- \to (K^+\pi^-)D K^-) \propto r^2_B + (r^{K\pi}_D)^2 + 2 r_B r^{K\pi}_D \cos (\delta_B + \delta_D^{K\pi} - \gamma),$$

where $r^{K\pi}_D$, $[(61.3 \pm 0.7) \times 10^{-3}]^7$, parameterises the relative suppression between $A_{D^0}$ and $A_{\bar{D}^0}$, and $\delta_D^{K\pi}$, $[(22^{+14}_{-15})^8]$, the relative strong phase difference.
Since $r_B$ and $r_D^{K\pi}$ are expected to be similar in magnitude, it can be seen that whilst Eq. (4) is the more suppressed of the two rates, it provides greater sensitivity to $\gamma$ as a result of the interference term appearing at leading order. Through considering the other two rates associated with the $B^+$ decay, and combining this with information from decays to the CP-eigenstates $K^+K^-$ and $\pi^+\pi^-$, an unambiguous determination of $\gamma$ can be made. The expected one-year sensitivity to $\gamma$ from these six rates is estimated to be 8-10$^\circ$ at LHCb, depending on the value of $\delta_D^{K\pi}$.

2.2. Multi-body Extension to the ADS Method

A wealth of additional statistics can be gained from considering multi-body decays of the $D$ meson. In the case of the ADS method, these are states involving a charged kaon and some ensemble of pions, such as $D \to K^-\pi^+\pi^0$ and $D \to K^-\pi^+\pi^-\pi^+$. However, a complication comes from the fact that the multi-body $D$ decay-amplitude is potentially different at any point within the decay phase space, because of the contribution of intermediate resonances. It is shown in Ref. 10 how the rate equations for the two-body ADS method should be modified for use with multi-body final states. In the case of the $B^-$ rates, for some inclusive final state $f$, Eq. (4) becomes:

$$\Gamma(B^- \to (f)D \bar{K}^-) \propto A_f^2 + r_B^2 A_f^2 + 2r_B R_f A_f \bar{A}_f \cos \left(\delta_B + \delta_D^f - \gamma\right),$$

where $R_f$, the coherence factor, and $\delta_D^f$, the average strong phase difference, are defined as:

$$A_f^2 = \int |A_{D^0}(x)|^2 \, dx, \quad \bar{A}_f^2 = \int |A_{\bar{D}^0}(x)|^2 \, dx,$$

$$R_f e^{i\delta_D^f} = \frac{\int |A_{D^0}(x)||A_{\bar{D}^0}(x)| e^{i\zeta(x)} \, dx}{A_f \bar{A}_f} \quad \{R_f \in \mathbb{R} \mid 0 \leq R_f \leq 1\},$$

with $x$ representing a point in multi-body phase space and $\zeta(x)$ the corresponding strong phase difference.

3. Determining $R_f$ and $\delta_D^f$ at CLEO-c

Through exploiting the fact that meson pairs produced via quarkonium resonances at $e^+e^-$ machines are in quantum entangled states, it is possible to obtain observables that are dependent on parameters associated with multi-body decays. In particular, it has been shown in Ref. 10 that, double-tagged $D^0\bar{D}^0$ rates measured at the $\psi(3770)$ provide sensitivity to both the coherence factor, $R_f$, and the average strong phase difference, $\delta_D^f$. Starting
with the anti-symmetric wavefunction\(^{11}\) of the \(\psi(3770)\) and then calculating the matrix element for the general case of two inclusive final states, \(F\) and \(G\), the double-tagged rate is found to be proportional to:

\[
\Gamma(F|G) \propto |A_F A_G^*|^2 - 2 R_F R_G A_F A_G \cos(\delta^F_D - \delta^G_D). \quad (8)
\]

From this, one finds three separate cases of interest for accessing both the coherence factor and the average strong phase difference. These are summarised in Table 1 below, in the instance of \(F = K\pi\pi\).

Table 1. Double-tagged rates of interest and their dependence on the coherence factor, \(R\), and the average strong phase difference, \(\delta^F_D\). The background subtracted yields from the 818 pb\(^{-1}\) data sample are shown along with the corresponding result for each measurement.

| CP-Tags | Measurement | 818 pb\(^{-1}\) Yield |
|---------|-------------|------------------------|
| \(K^\pm \pi^\mp \pi^\mp \pi^-\) | \((R_{K3\pi})^2 = 0.00 \pm 0.16 \pm 0.07\) | 30 \pm 6 |
| CP-Tags | \(R_{K3\pi} \cos(\delta^{K3\pi}_D) = -0.60 \pm 0.19 \pm 0.24\) | 2,183 \pm 47 |
| \(K^\pm \pi^\mp\) | \(R_{K3\pi} \cos(\delta^{K3\pi}_D - \delta^{K3\pi}_D) = -0.20 \pm 0.23 \pm 0.09\) | 38 \pm 6 |

3.1. Event Selection

At present, only double-tagged samples for the determination of \(R_{K3\pi}\) and \(\delta^{K3\pi}_D\) have been analysed using CLEO-c’s complete \(\psi(3770)\) dataset, corresponding to an integrated luminosity of 818 pb\(^{-1}\). To maximise statistics, a total of nine distinct CP tags are reconstructed against \(K^\pm \pi^\mp \pi^\mp \pi^-\): \(K^+K^-, \pi^+\pi^-, K^0\pi^0\), \(K^0\omega/\eta(\pi^+\pi^-\pi^0)\), \(K^0\pi^0\pi^0\), \(K^0\phi\), \(K^0\eta(\gamma\gamma)\) and \(K^0\eta'(\pi^+\pi^-\eta)\). Backgrounds within these CP-tagged samples are typically low; in the range of ~ 1 to 7%. The flat contribution to this background is assessed from sidebands within the beam-constrained mass distribution for each selection, whilst peaking contributions are determined from Monte Carlo. Depending on the final state, the selection efficiency ranges from ~ 4 to 30%. The background subtracted yields obtained are quoted in Table 1.

3.2. Preliminary Results

From the background subtracted yields determined, central values have been calculated for \(R_{K3\pi} \cos(\delta^{K3\pi}_D)\) for each of the 9 CP-tags; for \(R_{K3\pi} \cos(\delta^{K\pi}_D - \delta^{K3\pi}_D)\) using the \(K^\pm \pi^\mp \pi^\mp \pi^-\) vs. \(K^\pm \pi^\mp\) sample; and for \((R_{K3\pi})^2\) using the observed number of \(K^\pm \pi^\mp \pi^\mp \pi^-\) vs. \(K^\pm \pi^\mp \pi^\mp \pi^-\) events. In addition, the results of the 9 separate CP-tags are used to form a combined result for \(R_{K3\pi} \cos(\delta^{K3\pi}_D)\), taking full account of correlations between
systematic uncertainties. The preliminary results are quoted in the 2\textsuperscript{nd} column of Table 1, where the first error is statistical and the second systematic. The resulting constraints on the parameters $R_{K3\pi}$ and $\delta_{D}^{K3\pi}$ from these measurements are shown in Fig. 1. It is apparent, from Fig. 1, that the coherence across all phase space is low, reflecting the fact that many out of phase resonances contribute to the $K\pi\pi\pi$ final state. An inclusive analysis of this final state with the ADS analysis will therefore have low sensitivity to the angle $\gamma$, although the structure of Eq. (5) makes it clear that such an analysis will allow for a determination of the amplitude ratio $r_{B}$, which is a very important auxiliary parameter in the $\gamma$ measurement.

\textbf{References}

1. J. Charles \textit{et al.}, CKMfitter Group, Eur. Phys. J. C41, 1 (2005), updated results and plots at \url{http://ckmfitter.in2p3.fr}.
2. LHCb Technical Proposal, LHCb Collaboration, CERN/LHCC 98-4 (1998).
3. LHCb Reoptimized Detector Design and Performance, LHCb Collaboration, CERN/LHCC 2003-040 (2003).
4. Y. Kubota \textit{et al.}, Nucl. Instrum. Meth. Phys. Res., Sect. A 320, 66 (1992); D. Peterson \textit{et al.}, Nucl. Instrum. Meth. Phys. Res., Sect. A 478, 142 (2002).
5. M. Gronau and D. London, Phys. Lett. B 253, 483 (1991).
6. D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. 78, 3257 (1997).
7. Particle Data Group, W.-M. Yao \textit{et al.}, J. Phys. G 33, 1 (2006).
8. D. M. Asner \textit{et al.}, arXiv:0802.2268v1 [hep-ex] (2008).
9. M. Patel, LHCb-2008-011 (2008).
10. D. Atwood and A. Soni, Phys Rev. D 68, 033003 (2003).
11. M. Goldhaber and J. Rosner, Phys Rev. D 15, 5 (1977).