A STOCHASTIC GAME FORMULATION OF ENERGY-EFFICIENT POWER CONTROL: EQUILIBRIUM UTILITIES AND PRACTICAL STRATEGIES

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ABSTRACT

Frequency non-selective time-selective multiple access channels in which transmitters can freely choose their power control policy are considered. The individual objective of the transmitters is to maximize their averaged energy-efficiency. For this purpose, a transmitter has to choose a power control policy that is, a sequence of power levels adapted to the channel variations. This problem can be formulated as a stochastic game with discounting for which there exists a theorem characterizing all the equilibrium utilities (equilibrium utility region). As in its general formulation, this theorem relies on global channel state information (CSI), it is shown that some points of the utility region can be reached with individual CSI. Interestingly, time-sharing based solutions, which are usually considered for centralized policies, appear to be part of the equilibrium solutions. This analysis is illustrated by numerical results providing further insights to the problem under investigation.

Index Terms — Distributed power control, Stochastic games, Folk theorem, Nash equilibrium, Game Theory.

1. INTRODUCTION

In the past decade, new types of wireless networks have appeared. Just to name a few, when inspecting the wireless literature, we find ad hoc networks, networks of wireless devices operating in licensed bands, wireless networks with cognitive radios, small cells based cellular networks. The decentralized nature of such networks make game-theoretic analyses relevant [1]. Interestingly, game theory offers a large set of tools and concepts to better understand important problems such as resources allocation and power control in distributed networks. By distributed, it is meant that the allocation or/and control policy is left to the terminal itself. In this paper, the problem under consideration is precisely the one of distributed power control. The assumed network model is a multiple access channel (MAC), which, by definition, includes several transmitters and one common receiver. A brief overview of previous works about power allocation for MACs is presented in [2]. In our framework, based on a certain knowledge which includes his individual channel state information, each transmitter has to tune his power level for each data block. In particular, it can ignore some specified centralized policies. The assumed performance criterion is the one introduced by [3] that is to say, that transmitters aim at maximizing the energy efficiency associated with the transmit radio-frequency signal, which is measured as a number of correctly decoded information bits per Joule consumed at the transmitter. The assumed channel model is the same as [3] that is, links are assumed to be frequency non-selective but the channel gains can vary from block to block. The authors of [3] have formalized the energy-efficient power control problem as a one-shot/static non-cooperative game: on each block, the players (the transmitters) play a one-shot game by choosing their action/move (their power level) in order to maximize their utility (their individual energy-efficiency). The main drawback of such a formulation is that it leads to an outcome (Nash equilibrium) which is not efficient. Indeed, it can be checked that, for each block, there exists a vector of power levels (an action profile) which allows all the players to have better utilities than those obtained at the Nash equilibrium; the latter is said to be Pareto-dominated.

Motivated by the existence of power profiles which Pareto-dominates the one-shot game Nash equilibrium solution, several authors proposed solutions which are both efficient and compatible with the framework of decentralized decisions. For instance, pricing is proposed in [4], a Stackelberg formulation is introduced in [5], and a repeated game formulation is exploited in [6]. The framework adopted in this paper is a more general framework than the one chosen in [6]. Indeed, although the repeated game model in [6] takes into account the fact that transmitters interact several/many times, the work in [6] has an important weakness: this is the need for a normalized game which does not depend on the realization channels. By definition (see e.g., [7]), a repeated game consists in repeating the same one-shot game. The consequence of such a modeling choice is a loss in terms of optimality in terms of expected utilities (averaged over the channel realizations). The main purpose of this paper is to propose a more general dynamic game model namely, a stochastic game. Based on this choice, the contributions of this paper are essentially as follows: (i) after providing the signal model (Sec. 2) and reviewing the one-shot game model (Sec. 3), we present the stochastic game model used and show how to exploit the recent game-theoretic results by [8][9] to obtain a Folk theorem for the power control game under investigation: this Folk theorem allows one to fully characterize equilibrium utilities when public signals are available to the transmitters and channel stats are quantized (Sec. 4); (ii) some simple equilibrium power control strategies relying on individual channel state information and possibly a recommendation of the receiver are proposed (Sec. 5); (iii) a numerical study is conducted to assess the performance of the proposed strategies and give more insights on tuning the relevant parameters of the problem (Sec. 6).

2. SIGNAL MODEL

We consider a decentralized MAC with $K \geq 1$ transmitters. The network is said to be decentralized as the receiver (e.g., a base...
station) does not dictate to the transmitters (e.g., mobile stations) their power control policy. Rather, all the transmitters choose their policy by themselves and want to selfishly maximize their energy-efficiency; in particular, they can ignore some specified centralized policies. We assume that the users transmit their data over quasi-static channels, at the same time and frequency band and without any beamforming \cite{10}. Note that a block is defined as a sequence of $M \geq 1$ consecutive symbols which contains a training sequence: a specific symbols sequence used to estimate the channel (or other related quantities) associated with a given block. A block has therefore a duration less than the channel coherence time. The signal model used corresponds to the information-theoretic channel model used for studying MAC, see \cite{2} for more comments on the multiple access technique involved. This model is both simple to be presented and captures the different aspects of the problem. It can be readily applied to specific systems such as CDMA systems \cite{3,5} or multi-carrier CDMA systems \cite{11}. The equivalent baseband signal received by the base station can be written as

$$y(n) = \sum_{i=1}^{K} g_i(n)x_i(n) + z(n) \quad (1)$$

where $i \in K$, $K = \{1, ..., K\}$, $x_i(n)$ represents the symbol transmitted by transmitter $i$ at time $n$, $\mathbb{E}[|x_i|^2] = p_i$, the noise $z$ is assumed to be distributed according to the zero-mean Gaussian random variable with variance $\sigma^2$ and each channel gain $g_i$ varies over time but is assumed to be constant over each block. For each transmitter $i$, the channel gain modulus is assumed to lie in a compact set $[g_{\text{min}}^{i}, g_{\text{max}}^{i}]$. This assumption models the finite receiver sensitivity and the existence of a minimum distance between the transmitter and receiver. At last, the receiver is assumed to implement single-user decoding.

At a given instant, the SINR at receiver $i \in K$ writes as:

$$\text{SINR}_i = \frac{p_i \eta_i}{\sum_{j \neq i} p_j \eta_j + \sigma^2} \quad (2)$$

where $p_i$ is the power level for transmitter $i$ and $\eta_i = |g_i|^2$.

### 3. ONE-SHOT POWER CONTROL GAME

In this section, the one-shot game model of \cite{3} is reviewed since it both allows one to build the stochastic game model of Sec. 4 and serves as a reference for performance comparison. A useful (non-equilibrium) operating point in this game is also defined, as a basis for the proposed control strategies in the stochastic game model.

**Definition 1 (One-shot power control game)** The strategic form of the one-shot power control game is a triplet $\mathcal{G} = (K, \{A_i\}_{i \in K}, \{u_i\}_{i \in K})$ where $K$ is the set of players, $A_i$ and $u_i$ are the corresponding sets of actions, $A_i = [0, P_{i}^{\text{max}}]$, $P_{i}^{\text{max}}$ is the maximum transmit power for player $i$, and $u_i$ are the utilities of the different players which are defined by:

$$u_i(p_1, ..., p_K) = R_i f(\text{SINR}_i) \frac{[\text{bit}/J]}{p_i} \quad (3)$$

We denote by $R_i$ the transmission information rate (in bps) for user $i$ and $f$ an efficiency function representing the block success rate. The numerator of the utility is thus the rate of bits successfully received at the base station. $f$ is assumed to be sigmoidal and identical for all the users; the sigmoidness assumption is a reasonable assumption, which is well justified in \cite{12,13}. Recently, \cite{14} has shown that this assumption is also justified from an information-theoretic standpoint.

In this game with complete information ($\mathcal{G}$ is known to every player) and rational players (every player does the best for himself and knows the others do so and so on), a major game solution concept is the Nash equilibrium (i.e. a point from which no player has interest in unilaterally deviating). When it exists, the non-saturated Nash equilibrium of this game can be obtained by setting $\frac{\partial u_i}{\partial p_i}$ to zero $\forall i \in K$ which gives an equivalent condition on the SINR: the best SINR in terms of energy-efficiency for transmitter $i$ has to be a solution of $x f'(x) - f(x) = 0$ (this solution is independent of the player index since a common efficiency function is assumed, see \cite{13} for more details). This leads to:

$$\forall i \in \{1, ..., K\}, \quad p_i^* = \frac{\sigma^2}{\eta_i} \frac{\beta^*}{1 - (K - 1)\beta^*} \quad (4)$$

where $\beta^*$ is the unique solution of the equation $x f'(x) - f(x) = 0$. An important property of the Nash Equilibrium given by (4) is that transmitters only need to know their individual channel gain $\eta_i$ to play their equilibrium strategy. Another interesting property is that the product $p_i^* \eta_i$ (instantaneous received power) is constant for all the players.

Interestingly, the authors of \cite{6} propose to study the power profile obtained when imposing the received signals to have the same instantaneous power. The idea is to solve $\frac{\partial u_i}{\partial p_i}(p) = 0$ under the aforementioned constraint. This leads to the following system:

$$\forall (i,j) \in K^2, \begin{cases} p_i \eta_j = p_j \eta_i \\ \frac{\partial u_i}{\partial p_i}(p) = 0 \end{cases} \quad (5)$$

The unique solution of \cite{6}, called Operating point can be checked to be:

$$\forall i \in K, \quad \tilde{p}_i = \frac{\sigma^2}{\eta_i} \frac{\tilde{\gamma}_K}{1 - (K - 1)\tilde{\gamma}_K} \quad (6)$$

where $\tilde{\gamma}_K$ is the unique solution of $x[1 - (K - 1) \cdot x] f'(x) - f(x) = 0$.

The difference between this operating point and the Nash equilibrium can be explained by the fact that $\frac{\partial u_i}{\partial p_i}(p) = \frac{\partial u_i}{\partial p_i}(p_i^*)$ when adding the instantaneous power equality constraint. This operating point can be proved to always Pareto-dominate the Nash equilibrium and reach the Pareto frontier for each channel realization; additionally, only individual CSI is needed to operate at the corresponding power levels. This point will serve as a basis of the power control strategies proposed in Sec. 4.

### 4. STOCHASTIC POWER CONTROL GAME

With the one-shot non-cooperative game model, transmitters are assumed to play once for each block and independently from block to block. The goal here is to take into account the fact that transmitters generally interact over several blocks, which is likely to change their behavior w.r.t. the one-shot interaction model even if

\footnote{By using the term “non-saturated Nash equilibrium” we mean that the maximum total power can be applied for each user, denoted by $P_{i}^{\text{max}}$, is assumed to be sufficiently high not to be reached at the equilibrium i.e. each user maximizes his energy-efficiency for a value less than $P_{i}^{\text{max}}$ (see \cite{5} for more details about the saturated case).}
they are always assumed to be selfish. As channel gains are time-varying, the most natural model is the one of stochastic games [15]. In such a model, important differences w.r.t. the one-shot game model are that averaged utilities are considered, the channel state \(\eta(t) = (\eta_1(t), ..., \eta_K(t)) \in \Gamma\) and \(\Gamma = \Gamma_1 \times \Gamma_2 \times ... \times \Gamma_K\) may vary according to a certain evolution law (the i.i.d. block fading case is the most simple of them), and the state can depend on the played actions (in conventional wireless settings this is however not the case). Stochastic game stages correspond to instants at which players can choose their actions. From one stage to another, the channel state \(\eta(t)\) is assumed to be discrete (e.g., resulting from quantization effects) and stochastically varies according to the transition probability distribution \(\pi\). This distribution is said to be an irreducible transition probability if for any pair of channel states \(\eta\) and \(\eta'\) we have \(\pi(\eta'|\eta) > 0\). For example, this irreducibility condition is met for i.i.d. Channels. The second important assumption we do to obtain a Folk theorem is to assume that a public signal is available to all the transmitters. Two special cases of interest are: (a) Every transmitter knows the power of the received signal that is, \(\sum_{k=1}^{K} \eta_k p_k + \sigma^2\) is known; (b) Every transmitter has global CSI and perfectly observes the action profiles that is, \((\eta, \pi)\) is known.

The Game Course. The game starts at the first stage with a channel state \(\eta(1)\) known by the players. The transmitters simultaneously choose their power levels \(p(1) = (p_1(1), ..., p_K(1))\) and are assumed to receive a public signal, denoted \(\theta\). At stage \(t\), the channel states \(\eta(t)\) are drawn from the transition probability \(\pi(\cdot|\eta(t-1)) \in \Delta(1\times A)\) and the players observe the public signal

\[
\phi : \Gamma \times A \rightarrow \Theta \eta, p \rightarrow \theta
\]

where \(A = A_1 \times \cdot \times A_K\). The sequence of past signals \(\theta(t) = (\theta(1), ..., \theta(t-1), \eta(t))\) is the common history of the players.

**Definition 2 (Players’ strategies)** A strategy for player \(i \in K\) is a sequence of functions \((\tau_i, \lambda_i)\) with

\[
\tau_i, \lambda_i : \Theta \rightarrow A_i
\]

The strategy of player \(i\) will therefore be denoted by \(\tau_i\), while the vector of strategies \(\tau = (\tau_1, ..., \tau_K)\) will be referred to a joint strategy. A joint strategy \(\tau\) induces in a natural way a unique sequence of action plans \(p(t)\) and a unique sequence of public signals \(\theta(t)\). The averaged utility for player \(i\) is defined as follows.

**Definition 3 (Players’ utilities)** Let \(\tau = (\tau_1, ..., \tau_K)\) be a joint strategy. The utility for player \(i \in K\) if the initial channel state is \(\eta(1)\), is defined by

\[
u_i(\tau, \eta(1)) = \sum_{t \geq 1} \lambda(1 - \lambda)^{t-1} \mathbb{E}_{\eta} \left[ u_i(p(t), \theta(t)) \right] \]

where \(p(t)\) is the sequence of power profiles induced by the joint strategy \(\tau\).

The parameter \(\lambda\) is the discount factor, which can model various effects such as the probability the game stops, the fact that players evaluate short-term and long-term benefits differently, etc. We now present the definition of a stochastic game.

**Definition 4 (Stochastic game)** A stochastic game with public monitoring is defined as a tuple

\[\mathcal{G} = (K, (\mathcal{T})_{\tau \in \mathcal{T}}, (v_i)_{i \in K}, (\mathcal{T}_i)_{i \in K}, \pi, \Theta, \phi),\]

where \(\mathcal{T}_i\) is the set of strategies for player \(i\) and \(v_i\) his long-term utility function.

**Equilibrium concept.** Let us define the Nash equilibrium of a stochastic game starting with the channel state \(\eta(1)\).

**Definition 5 (Equilibrium Strategies)** A strategy \(\tau\) supports an equilibrium of the stochastic game with initial channel state \(\eta(1)\) if

\[\forall i \in K, \forall \tau_i', v_i(\tau, \eta(1)) \geq v_i(\tau_i', \eta(1))\]

where \(-i\) is the standard notation to refer to the set \(K \setminus \{i\}\). Denote \(E_\lambda(\eta(1))\) the set of equilibrium utilities with initial channel state \(\eta(1)\).

We now characterize the equilibrium utility region for the stochastic game under study. For this, define the min-max level \(\tilde{v}_i\) of player \(i \in K\) as the most severe punishment level for player \(i \in K\). The feasible utility region is denoted by \(F_\lambda(\eta(1))\). The result of Dutta [16] states that if the transition probability \(\pi\) is irreducible, then the min-max levels, the feasible utility region and the equilibrium utility region are independent of the initial channel state \(\eta(1)\).

\[
\lim_{\lambda \rightarrow 0} \min v_i(\tau, \eta(1)) = \tilde{v}_i, \forall i \in K
\]

**Theorem 7** Suppose that the players see the same public signal. Then, for each utility vector \(\mu \in F^*\), there exists a \(\lambda_0\) such that for all \(\lambda < \lambda_0\), there exists a perfect public equilibrium strategy in the stochastic power control game, such that the long-term utility equals \(\mu \in F^*\).

The proof is not given here and is based on Theorem 2 of [8]. Note that such a characterization is very powerful. Indeed, the brute-force technique to find the feasible utility region would be to look at all possible action plans for the players and compute the corresponding utilities. This would be intractable even when every player could only choose two power levels for a finite stochastic games with 100 stages (each player could then choose between \(2^{100}\) action plans). The Folk theorem characterizes the equilibrium utilities from quantities far much easier to evaluate (namely \(E, F, \tilde{v}_i\)). Additionally, as shown by Dutta [15], there is no loss of optimality by restricting the set of strategies to Markov strategies (which only depend on the current channel state). This will be exploited in Sec. 6.

### 5. STRATEGIES FOR K-PLAYER GAMES

#### 5.1. Best user selection (BUS)

The strategy we propose is based on the operating point presented in Sec. 3. When channels gains vary from stage to stage, if every transmitter plays at the operating point, the network is not socially
optimal (in contrast with the case where the channel state would be constant). It turns out that we get better results in terms of social welfare if the set of players playing the operating point at each stage is shrunk. We name this approach the best user selection scheme.

At each stage $t$ of the game, the receiver sets $K^t$ as the optimal set of players playing the operating point in terms of social welfare. For player $i \in K$: • If $i \in K^t$, he is recommended to play the operating point at stage $t$; • If $i \notin K^t$, he should not transmit at this stage. To ensure the equilibrium of this strategy, if one of the player deviates from the strategy, the other players punish him by playing the one-shot Nash equilibrium for the remaining of the game.

5.2. Threshold-based user selection (T-US)

Note that in the former strategy, the set of players playing the operating point is decided at each stage but one can also imagine a simpler strategy with a threshold $\alpha \in [0, 1]$ set for the entire game such that for player $i \in K$: • If $\eta_i \geq \alpha \eta_{\text{max}}^t$, he is recommended to play the operating point; • If $\eta_i < \alpha \eta_{\text{max}}^t$, he should not transmit at this stage, where $\eta_{\text{max}}$ is the best channel gain realization at stage $t$ and $\eta_i$ the channel gain realization of player $i$ at stage $t$. As before, if one of the players deviates from the strategy, the other players punish him by playing the one-shot Nash equilibrium for the remaining of the game.

5.3. Properties of Best User Selection Scheme

Although one might think that the BUS scheme requires complex computations at the base receiver, it happens that the former does not have to compare all possible combinations of players. Indeed, the BUS scheme always includes the player with the best channel gain and a given number of other players with the following channel gains in decreasing order. Thus, the base station just has to order the channels gains in decreasing order and decides which gain level is the minimum to separate players allowed to play from the others, which is far less complex, especially when the number of players is very large. In term of complexity, for $K$ players, instead of considering $2^K$ possible combinations, the base station has just to consider $K$ combinations.

**Theorem 8 (BUS scheme for $k$ players)** At equal transmitting rate, the best user selection of $k$ players, $k \in K$, playing together at the operating point is the set of the $k$ players with the best channels gains.

For the strategy to be an equilibrium of the stochastic game, it is needed that no player has interest in deviating from the given plan. This condition is expressed by the fact that the cost of the punishment must always be higher than what a player can get when deviating at one stage. It results in the following theorem

**Theorem 9 (Equilibrium strategy)** The BUS strategy is an equilibrium of the stochastic game if $\forall i \in K$

$$\lambda \leq \frac{E[u_i(p_{\text{bus}}^{\text{bus}}, \eta)] - E[u_i(p_{\text{bus}}^{\text{bus}}, \eta)]}{\frac{1}{\alpha^2} (E[u_i(p_{\text{bus}}^{\text{bus}}, \eta)] - E[u_i(p_{\text{bus}}^{\text{bus}}, \eta)])}$$

(16)

The expected utility of the BUS scheme can be compared to the strategy based on the one-shot Nash equilibrium, the strategy based on pure Time-sharing and the Operating point strategy.

5.4. Information assumptions

For the BUS and the TUS schemes, players adapt their transmit power at the Operating point if they are recommended to play by the Base station. Thus, they need to know whether they are recommended to play. If so, given $K$, they need to know the number of players transmitting with them and the state of their own channel. The recommendation and the number of recommended players are sent by the Base station whereas the channel state is observed by the transmitter, all these signals are modeled by $\theta$. The last necessary piece of side information is due to the equilibrium condition of the strategy: if one player deviates from the plan, he is punished by the other players for the remaining of the game. It implies that players must be able to detect a deviation, what can be done if they know their SINR at each stage (see [6] for more details). From one strategy to another, the amount of information required may vary considerably. For instance, if one wants players to play the Social Optimun strategy, players would have to know channels gains of every other player whereas if they play the one-shot Nash equilibrium they would only need to know their own channel gain and the total number of players. To have a clear view about the amount of side information needed to implement the discussed strategies, we provide a simplified comparison for the various strategies under study in Fig.1. By “deviation alarm” we mean a signal allowing players to detect single deviations from the cooperative action plan.

6. NUMERICAL RESULTS

6.1. Simulation Parameters

To obtain numerical results, we work with the efficiency function $f(\gamma) = e^{-a}$ with $a = 2^\alpha - 1$, see [2] for more details about this efficiency function. All our results are obtained from games with $10^5$ stages.

6.2. Two-state channel $K$-player game

Fig.3 is obtained for a 10-player game where channels gains can only reach two states $\eta_{\text{min}}$ and $\eta_{\text{max}}$ with probability $\{\frac{1}{2}, \frac{1}{2}\}$ for each player and $a = 0.1$. It is interesting in the sense that it clearly highlights the fact that the idea of the proposed strategy is interesting when channels gains of players are sufficiently different. Indeed, we can observe that for a channel gain ratio $\eta_{\text{max}} = 1$, the proposed strategy is equal to the classical Operating point strategy. For $\eta_{\text{max}} > 1$, the proposed strategy becomes more efficient. By working with random distributions for the channels gains, the case of realizations with very different channels gains often occurs, which is interesting for the proposed strategy.

**Theorem 10 (Dominance)** For equal transmission rates, for i.i.d. channel states among the players, we have

$$\forall i \in K, E[u_i(p_{\text{bus}}^{\text{bus}}, \eta)] \geq E[u_i(p_{\text{bus}}^{\text{bus}}, \eta)]$$

$$\forall i \in K, E[u_i(p_{\text{bus}}^{\text{bus}}, \eta)] \geq E[u_i(p_{\text{bus}}^{\text{bus}}, \eta)]$$

$$\forall i \in K, E[u_i(p_{\text{bus}}^{\text{bus}}, \eta)] \geq E[u_i(p_{\text{bus}}^{\text{bus}}, \eta)]$$

Simulations based on this strategy are discussed in Sec.6.3.
| CSI                          | Recommendation signal | Nb of players | Deviation alarm |
|------------------------------|------------------------|---------------|-----------------|
| Pure time-sharing            | individual             | not needed    | not needed      |
| One-shot Nash                | individual             | not needed    | needed          |
| Operating Point              | individual             | not needed    | SIR            |
| T-US scheme                  | individual             | needed        | SIR            |
| BUS scheme                   | individual             | needed        | SIR            |
| Social optimum               | global                 | not needed    | not needed      |

Fig. 1. Information required for various stochastic strategies

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Fig. 2. Comparison of BUS utilities versus Nash utilities depending on the ratio between the good channel state and the bad channel state for a 10-player game.

Fig. 3 shows the achievable utility region for a 2-player, 2-states game (with $\frac{\eta_{\text{max}}}{\eta_{\text{min}}} = 4$ and $\alpha = 0.5$) when considering all the possible strategies. The minmax line delimits the equilibrium region. The mean utilities of BUS, Operating point and static Nash equilibrium are also represented in this region. It is clear that BUS strategy is closer to the Pareto frontier.

Fig. 3. Achievable region with expected utilities of various strategies

6.3. Truncated Rayleigh distribution for $K$-player game

In Fig. 4, we consider a $K$-player game with $K$ varying from 1 to 10 and $\alpha$ fixed to 0.1. In each game, channels states follow the same truncated Rayleigh distribution law for every player. Four strategies are compared on this figure: one-shot Nash equilibrium (Sec. 3), pure Time-sharing, Operating point (equation 5), T-US with $\alpha = 0.5$ (Sec. 5.2) and BUS (Sec. 5.1). There are several points to notice. First, for all the studied strategies, as the number of players increases, the mean utility decreases for each player. This is due to the fact that players see each other signal as interference: the more players in the game, the more they have to share resources. Second, it is clear that the Operating point and BUS strategies are more efficient than one-shot Nash equilibrium or Time-sharing strategies. As the number of players increases, this gap becomes even larger.

Fig. 4. Comparison of strategies utilities depending on the number of players

Fig. 5 shows a graphic representation of the different configurations $H^1_i(k)$ and $H^2_i(k)$ a player $i$ can meet at each stage of the stochastic game (owing to the symmetry in the channels distribution law, the configurations probabilities are the same for every player). $H^1_i(k) \subset \Gamma$ is the set of channels realizations where $k$ players are recommended to play and player $i$ is part of these players. $H^2_i(k) \subset \Gamma$ is the set of channels realizations where player $i$ is not one of the $k$ players recommended to play. The simulation is made with 5 players and $\alpha = 0.2$.

Fig. 5. Partition of $H^1_i$ and $H^2_i$ for a 5-player game with BUS strategy

Fig. 6 refers to theorem 9. It represents the maximum value the discount factor $\lambda$ can have for the BUS strategy to be an equilibrium of the stochastic game.
Best combination strategy and the expected utility of the one-shot Nash equilibrium increases with the number of players, the maximum value of the discount factor increases as well.

![Graph](image_url)

**Fig. 6.** Maximum value of discount factor $\lambda$ for the BUS strategy to be an equilibrium.

### 7. CONCLUSION

Conventionally, for i.i.d channels, power control schemes are designed such that the power levels are chosen in an independent manner from block to block. In distributed networks with selfish transmitters, the point of view has to be re-considered even if the channels are i.i.d. due to the fact that long-term interaction may change the behavior of selfish transmitters. In order to take into account this effect and the fact that channel gains may vary from block to block, the model of stochastic games is proposed. When transmitters observe a public signal (e.g. the sum of received signals), a recent game-theoretic result allows one to fully characterize the equilibrium utility region. It is shown how to reach some points of this region by assuming individual CSI only. Both analytical and simulation results show potential gains in terms of energy-efficiency induced by the proposed model. In particular, because of long-term interaction, transmitters may have interest of shutting down for some blocks, leading therefore to legitimate time-sharing based control policies. Further investigations on the proposed approach are needed. In particular, it is relevant to characterize which part of the equilibrium utility region can be reached under the individual CSI assumption, which is relevant to include fairness issues. Additionally, typical features of modern wireless networks such as finite size buffers, Markovian evolution law for the channel state, should be accounted for to make the proposed framework more applicable.

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