Biased discrete symmetry and domain wall problem

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Abstract

We reconsider a cosmological evolution of domain walls produced by spontaneous breaking of an approximate discrete symmetry. We show, that domain walls may never collapse even if the standard bound on the vacuum energy asymmetry is satisfied. Instead of disappearing, these defects may form stable “bound states” - double wall systems. Possible stability of such a wall is a dynamical question and consequently restricts the allowed range of parameters. In particular, in the two Higgs doublet standard model with an anomalous $Z_2$ symmetry, the above restriction suggests the mass of the pseudoscalar Higgs (would be axion) being close to the mass of the scalar one.
1. Introduction

Spontaneously broken discrete symmetries often play a fundamental role in particle physics models. Unfortunately, in the cosmological context such theories exhibit serious problems [1] as they lead to the formation of the topologically stable domain walls [2].

Enormous energy stored in these structures very soon comes to dominate the universe, unless the scale of discrete symmetry breaking (or the effective self coupling constant of the corresponding Higgs field) is very small ($< 10^{-2} GeV$ or so) [1,2].

One famous solution for the domain wall problem is inflation [4]. However, normally inflation deals with scales as high as say $M_G \sim 10^{16} GeV$ (the GUT scale). In this situation, it is clear, that the topological defects formed below $M_G$, and in particular the ones attributed to the electroweak phase transition, will survive. Therefore, it seems that at least for the weak scale domain walls the inflation may not be a good solution.

Alternatively, cosmological troubles caused by the spontaneously broken discrete symmetry can be avoided if this symmetry is approximate, meaning that it is also explicitly broken by relatively small amount. From the naturalness point of view this ‘solution’ only makes sense if explicit breaking results dynamically from some underlying physics e.g. from the anomaly [4]. Another possibility [5] is that gravity may not respect ungauged discrete (or continuous) symmetries and explicit breaking can manifest itself through the higher dimensional Planck scale ($M_P$) suppressed operators in the scalar potential.
Whatever the source of the explicit violation is, below certain temperature it creates energy difference between initially degenerated vacua. Consequently one vacuum becomes energetically more favorable and corresponding domains start to expand pushing the walls away, whereas unfavorable domains tend to collapse and disappear. The necessary condition for the domain walls to disappear before dominating the universe has the following form [6]

$$\epsilon > \left(\frac{\sigma}{M_p}\right)^2$$  \hspace{0.5cm} (1)

where $\epsilon$ is the vacuum energy density difference and $\sigma$ is the wall energy per unit surface. In the simplest case, when the domain walls are formed by a single component real Higgs field $\Phi$ with vacuum expectation value (VEV) $<\Phi> = \pm V$, the condition (1) can be enough to guarantee that walls collapse without any trouble for cosmology. In the case of the real scalar field the expectation value $<\Phi>$ always vanishes across the wall and two neighbor walls (bounding collapsing domain from the opposite sides) sooner or later will annihilate each other. However, in the realistic theories the Higgs fields usually are represented by complex scalars and have several components. The elements of the discrete symmetry group can be mimicked by certain abelian (or even nonabelian) phase transformation (which of course is not a part of continuous gauge group, since we are not interested in walls bounded by strings [7]). This circumstance brings a qualitatively new point in the wall structure implying that there is no topological reason for the absolute value of the Higgs field $|\Phi|$ to vanish inside the wall. In other words, whether this will happen becomes a matter of the parameter choice in the scalar potential.
For a wide range of these parameters the energetically most favorable path in the field space, connecting two degenerated vacuums, never includes the point $|\Phi| = 0$. Thus, the Higgs field never vanishes inside the wall and we can define its “winding number” through the defect.

In the present paper we show that in such cases the domain walls may never collapse even if discrete symmetry is explicitly broken and (1) is satisfied. This has to do with the fact, that the walls of the collapsing domain can annihilate each other only if the Higgs field winds oppositely through this walls. Two walls with the same winding can never annihilate each other and disappear. Instead, they form classically stable ”bound states”-double wall systems. Expectation value of the Higgs field returns to its initial value after traversing such composite wall. If winding is parametrised by phase $\arg \Phi = \theta$, we will have $\Delta \theta = 2\pi$ through the wall. As far as we know, previously the $2\pi$-walls have been considered only in the context of the walls bounded by cosmic strings [8], which often appear in the axion cosmology. In such case initially there are no walls, but axionic string formed due to a global $U(1)_{PQ}$-symmetry breaking. $2\pi$-walls are formed later on when $U(1)_{PQ}$ gets explicitly broken (and only if there is no discrete subgroup left [9]). Each $2\pi$-wall is bounded by string. Existence of the string network crucially determines the cosmological evolution of the walls and makes them to decay without trace.

In contrast $2\pi$-walls discussed in the present paper can be cosmologically problematic, since there is no string network behind of their existence. They are formed as an bound states of would be topologically stable domain walls when later collapse due to an explicit breaking of discrete symmetry.
2. General mechanism

In this section we will study mechanism of the $2\pi$-wall formation on a simple model including one $SU(3)-$ color triplet quark $u_{L,R}$ transforming under discrete $Z_2$ symmetry:

$$u_L \rightarrow u_L, \quad u_R \rightarrow -u_R,$$  \hspace{1cm} (2)

This quark gets a mass from the following Yukawa coupling

$$g_u \Phi \bar{u}_L u_R + h.c$$  \hspace{1cm} (3)

were $\Phi = \Phi_R + i \Phi_I = |\Phi| e^{i\theta}$ is a complex Higgs scalar ($\Phi \rightarrow -\Phi$ under $Z_2$), whose VEV breaks $Z_2$ spontaneously.

We choose a tree level scalar potential in the form:

$$v = -\frac{M^2}{2} |\Phi|^2 + \frac{h}{4} |\Phi|^4 - \frac{m^2}{4} (\Phi^2 + h.c.) = -\frac{M^2}{2} |\Phi|^2 + \frac{h}{4} |\Phi|^4 - \frac{m^2}{2} |\Phi|^2 \cos 2\theta$$  \hspace{1cm} (4)

For definiteness we assume all parameters $m^2, M^2, h$ being real and positive. The possible quartic term $\Phi^4$ which is allowed by all symmetries we have excluded for the simplicity of the analysis. Note that first two terms in (4) respect global $U(1)$-symmetry $\Phi \rightarrow e^{i\alpha} \Phi$ which is explicitly broken by third one down to $Z_2 \subset U(1)$ and $m$ is the mass of would be goldstone boson. The absolute value of the Higgs VEV is given by $|\Phi| = V = (\frac{M^2 + m^2}{h})^{\frac{1}{2}}$. This vacuum is discretely degenerated, since the potential is minimized by any $\theta = \pi N$ (where $N$ is integer). This degeneracy results in the formation of the topologically stable domain walls. Topological constraint forces $\theta$ to
change by $|\Delta \theta| = \pi$ across the wall. However, this defects are not truly stable, because the $Z_2$-symmetry is anomalous [4].

The important point about the wall structure is whether the absolute value of the scalar field vanishes inside the wall. Certainly, this is the case if $m^2 > M^2$. In this situation the domain wall solution is given by kink (antikink)

$$\Phi_R = \pm V \tanh[xV(h/4)^{1/2}];$$

(5)

($x$ is a coordinate perpendicular to the wall) whereas the pseudoscalar component $\Phi_I$ is identically zero, since its effective \textit{mass} term $M_I^2 = h\Phi_R^2 + m^2 - M^2$, which is spatially dependent through the $\Phi_R$ VEV, is positive everywhere. In such a case, the wall network will evolve along the lines discussed in [4,5]. That is we expect that after $Z_2$ will be explicitly broken by anomaly, walls can collapse without any cosmological harm (provided (1) is satisfied).

The significant deviation from this scenario (to be discussed below) will occur if $|\Phi|$ stays nonzero everywhere including the vicinity of the domain wall. In this case the energetically most favorable trajectory connecting two neighbor vacuums $\theta = \pi N$ and $\theta = \pi (N \pm 1)$, goes through the saddle point $\Phi_I = \pm [(M^2 - m^2)/h]^{1/2}$ and $\Phi_R = 0$, whereas the point $|\Phi| = 0$ is maximum. Of course, the condition $M^2 > m^2$ is not yet sufficient to conclude that pseudoscalar will get a nonzero VEV inside the wall, since the gradient energy wants $\Phi_I$ to stay constant everywhere. So one has to check the stability of the small perturbation about $\Phi_I = 0$ in the background of kink. For example this can be done through the analysis of linearized equation for the pseudoscalar mode with the potential $M_I^2$ on the existence of the bound state along the
lines discussed by Witten [10] for the bosonically superconducting string. Such consideration shows that condensation of pseudoscalar inside the wall occurs at least for

\[ m^2 < M^2 \sim M^2 - m^2 \]  \hspace{1cm} (6)

In such a case it costs a lot of energy (\( \sim M^4 \) per volume) for \( |\Phi| \) to vanish somewhere in the space and in particular in the wall vicinity. To approximate the wall structure it is useful to put \( |\Phi| = V = constant \) for a moment. The equation of motion for \( \theta \) becomes then a well known sine-Gordon equation:

\[ 2 \frac{\partial^2 \theta}{\partial \xi^2} - \sin 2\theta = 0 \]  \hspace{1cm} (7)

(where \( \xi = mx \) and \( x \) is the coordinate transverse to the wall) which has a topologically stable solution (soliton)

\[ \theta(\xi) = 2 \cdot \tan^{-1}\exp(\xi). \]  \hspace{1cm} (8)

This is a reasonably good approximation for our purposes, although in reality \( |\Phi| \) can not stay constant across the wall due to a back reaction. In turn this will alter the shape of \( \theta(\xi) \), but whatever the exact form of \( \theta(\xi) \) is, it should change by \( |\Delta \theta| = \pi \) across the soliton.

At the QCD scale the color instanton effects become important and they explicitly break \( Z_2 \)-symmetry. The effective \( Z_2 \)-noninvariant term generated in the scalar potential by instanton vertex has the form

\[ V_{\text{inst}} = -m_u \Lambda^3 \cos \theta \]  \hspace{1cm} (9)
where $\Lambda$ is a typical scale factor of the order the QCD scale $\Lambda_{QCD} \sim 200\text{MeV}$ and $m_u$ is the quark mass.

This term creates energy density difference $\epsilon = 2m_u\Lambda^3$ between $\theta = 2N\pi$ and $\theta = (2N+1)\pi$ vacuums. This energy difference creates pressure difference which drives the domain walls. Assuming say $m_u\Lambda^3 > 0$, the preference is given to $\theta = 2N\pi$ domains which tend to expand, whereas thous with $\theta = (2N + 1)\pi$ will tend to collapse and disappear. The dynamics of the collapsing walls is defined by several factors. At early stages (when explicitly violating effects are negligible) this are mainly wall tension and the frictional force. Crudely the time dependences of this two factors are proportional to $\sim (\sigma M^2_P/t^3)^{1/2}$ and $\sim (M_P/t)^2$ respectively. At some point the system becomes dominated by pressure difference (with corresponding force per unit area $\sim \epsilon$) which forces the false vacuum to collapse. For walls to disappear, above has to happen before they dominate the universe and we are lead to the lower bound (1) for the vacuum energy asymmetry (for more details see [2,4,5,6]).

Due to above arguments in general one expects [4] the domain walls disappearing soon after the QCD phase transition, since for the weak scale walls (destabilized by QCD anomaly) condition (1) is satisfied $(\epsilon(M_P/\sigma)^2 \sim 10^{20}$ or so).

The similar conclusion was made in [5] for the case of the domain walls destabilized by "gravity induced" higher dimensional operators. However as it will be shown in the present paper, the resulting dynamics of the walls is more sensitive to the parameters than one can naively expect from (1). Namely for the wide rang of parameters (forming the subrange of (1)) the
walls may not collapse completely. In particular this can happen for (6) meaning, that the mass of the pseudoscalar mode (would be axion) is more than twice less the scalar one (radial mode). In such a case only the walls having opposite signs of $\Delta \theta$ can annihilate each other, whereas others can not. Instead the laters will be paired forming stable double wall systems across which $|\Delta \theta| = 2\pi$. Such a wall pairing can never occur if $|\Phi|$ vanishes across the wall. In this case the vacuas $\theta = 2\pi N$ (or $\theta = (2N + 1)\pi$) corresponding to different $N$ are identical. So in fact we have just two possible domains (say (+) and (−) respectively). This has to do with the fact that actually $\theta$ does not ”winds” through the wall, since it becomes ill defined at $|\Phi| = 0$. The planar domain wall solution separating (−) and (+) ((+) and (−)) domains is given by kink (antikink) (5) rather then by soliton (8). Now if we switch on the instanton effects the (−) domains will start to collapse and finally disappear, since the two neighbor domain walls (kink and antikink) will annihilate each other.

Situation will drastically change if nonzero $\Phi$ is energetically favored everywhere in the space. That is, for example, if we are in the range of parameters given in (6). Now $\theta$ is well defined for any spatial point $x$ (including thous in the vicinity of the domain wall) and as we show in the next section, this fact plays a crucial role in the subsequent evolution of the system.

3. Cosmological evolution of $|\Phi| \neq 0$ domain walls

Now let us follow the history of the early universe in our model. At high temperature ($T >> M, m$) the $\Phi$-dependent part of the potential receives correction (Yukawa interaction is neglected) [11]
\[ \Delta V(T) = |\Phi|^2 T^2 h / 6 \quad (10) \]

and we see that mass difference between the scalar and pseudoscalar fields \((m^2)\) never vanishes. Therefore, at the moment of the phase transition, when \(\Phi_R\) and \(\Phi_I\) pick up a nonzero VEVs, they already “know” about explicit breaking of global would be \(U(1)\)-symmetry. At this stage, probability of string-bounded wall formation is exponentially suppressed by the factor \(mR\), where \(R\) is a size of defect. Since the rang of our interest is \(m\) being not much (say within one order of magnitude) below \(M\), the probability of large string-wall formation is negligible. So we are left with a network of domain walls of very complicated geometry. Traversing each wall, \(\theta\) winds by \(\Delta \theta = \pm \pi\) continuously interpolating between two values that differ by \(\pi\). Difference among ”winding numbers” of neighbor walls becomes very important after explicit breaking of the discrete symmetry. Instanton induced term (9) in the scalar potential gives preference to a certain domains (say to \(\theta = 2\pi N\)) which tend to expand. Crucial point however is that not all \(\theta = (2N + 1)\pi\) domains will disappear. To see this consider collapsing domain \(\theta = \pi\) bounded by two infinite planar walls. It is clear that this boundaries can annihilate each other and disappear only if they correspond to the opposite windings of \(\theta\). That is two neighbor domains (of \(\theta = \pi\) one) must have both \(\theta = 0\) or both \(\theta = 2\pi\) (and never one 0 and another 2\(\pi\)). Contrastly, if \(\pi\)-domain was initialy created between 0 and 2\(\pi\) ones, the corresponding walls can never annihilate each other since they have both the same \(\Delta \theta\). Instead of annihilation such walls will create stable bound states. Double wall solution
can be approximated analytically if we assume $|\Phi| = constant$. In such case equation for $\theta$ is double sine-Gordon equation.

$$2\frac{d^2\theta}{d\xi^2} - \sin2\theta - a\sin\theta = 0$$ (11)

where $a = \frac{m\Lambda^3}{V^2m^2}$. The composite wall can be approximated by stable double-soliton solution

$$\theta = 2\arccos \frac{a/2 - Z^2}{2Z}, \quad Z^2 = 2 + a/2 + 2\sqrt{1 + a/2}\tanh(2\sqrt{1 + a/2}\xi).$$ (12)

Note that the usual sine-Gordon equation (7) also admits $2\pi$-soliton solution which is however unstable and tends to decay into two solitons with $\Delta\theta = \pi$ each. The last term in (11) opposes this decay and prevents two $\pi$-solitons from escaping each other, forming the bound state. The width of the bound state (typical distance between $\Delta\theta = \pi$ walls) is $\delta \simeq \frac{1}{m}\ln\frac{\Lambda}{\delta'}$. Since $\Lambda$ and $V(\sim m)$ correspond to QCD and weak scales respectively, this width is by order of magnitude larger the width of the constituent $\pi$-solitons $\delta' \sim \frac{1}{m}$. Infinite planar $2\pi$-wall is not topologically stable and can decay quantum mechanically. This decay goes through the tunneling process which creates a hole connecting $\theta = 0$ and $\theta = 2\pi$ vacuums with each other. Imagine a closed path that starts in one of the domains (say in $\theta = 0$), goes once through the hole and finally returns back to the base point by traversing the wall in some other place. The one who travels along the path, will find phase $\theta$ changing by $\Delta\theta = 2\pi$ at the end of the journey. Thus, $|\Phi|$ has to vanish at some point inside the region encircled by the path. By moving path around continuously (without crossing the edge of the hole), the point $|\Phi| = 0$ will
follow a closed line. So, there is a loop of the cosmic string enclosing the hole somewhere. Since, the space between two solitons is initially filled with $\theta = \pi$ phase, the formation of the hole is costly in the energy of its own edge. The later includes the energy of the (cylindric) wall surface that bounds the hole and the energy of the string loop that is somewhere in this boundary. Hole will start to expand classically when, this energy will be overcome by energy of the punched out piece of domain wall. The change in the energy caused by $R$-radius hole formation can be estimated to be:

$$\Delta E = \pi R[2(\mu^2 + \delta \sigma) - R(\delta \epsilon + 2\sigma)]$$

(13)

where $\mu^2 \simeq 2\pi V^2 \ln \frac{M}{m}$ and $\sigma \simeq 4mV^2$ are string and wall tensions respectively. Hole expands when its radius exceeds the critical value $R_c = (\delta + \mu^2/\sigma)/2$. The tunneling rate is exponentially suppressed by the factor $2\pi R_c^3 \sigma \sim \pi[\ln \frac{M}{m}]^3(V/m)^2$. So that even for $V/m \sim 1 - 10$ this wall can be stable for all practical purposes. Such $2\pi$-wall of the horizon size (if formed) can be disastrous cosmologically. To destabilize this structures we should increase parameter $m^2$ and convert $|\Phi| = 0$ into the saddle point. In the other words we have to increase the mass of angular pseudoscalar mode (would be goldstone boson) relative to scalar one (radial mode). Closer $|\Phi| = 0$ is to the saddle point, less costly is the string bounded hole formation in the wall sheet. In the limit when $|\Phi| = 0$ becomes a saddle point, $2\pi$-walls become classically unstable.

4. **Two doublet extension**

Above model is trivially extendable to the realistic example of ref [4].
This is the $SU(2) \otimes U(1)$ standard model with two Higgs doublets $\Phi_u, \Phi_d$ and anomalous discrete symmetry $Z_2$ needed for natural flavor conservation.

Under $Z_2$:

$$\Phi_u \rightarrow -\Phi_u, \quad \Phi_d \rightarrow \Phi_d, \quad Q_L \rightarrow Q_L, \quad U_R \rightarrow -U_R, \quad d_R \rightarrow d_R \quad (14)$$

where $Q_L = \left( \begin{array}{c} u \\ d \end{array} \right)$ is the left-handed quark doublet. $Z_2$-symmetry ensures that each Higgs doublet couples only to the fermions of the given charge. The scalar potential now takes the form

$$V = \sum_\alpha (M_\alpha^2|\Phi_\alpha|^2 + h_\alpha|\Phi_\alpha|^4) - h|\Phi_u^*\Phi_d|^2 - h'|\Phi_u|^2|\Phi_d|^2 + V_{phase} \quad (15)$$

$$V_{phase} = -\frac{\lambda}{2}(|\Phi_u^*\Phi_d|^2 + h.c) \quad (16)$$

where $\alpha = u, d$. If $h > 0$ the VEVs are pointing in the electrically neutral direction $\Phi_\alpha = \left( V_\alpha e^{i\theta_\alpha} \right)$. In this parameterization

$$V_{phase} = -\lambda V_u^2 V_d^2 \cos(2\theta) \quad \theta = \theta_u - \theta_d \quad (17)$$

For $\lambda > 0$, $V_{phase}$ is minimized by $\theta = \pi k$ ($k = 1, 2,...$) and there are topologically stable domain walls. If $h + h'$ is larger than $\lambda$, nonzero expectation values $V_u, V_d \neq 0$ will be energetically flavored everywhere in the space and even in the vicinity of the domain wall. So that as before, the winding of $\theta$ can be well defined. Explicit violation of $Z_2$-symmetry by QCD instanton vertices leads to the formation of "composite" $2\pi$-walls much in the way discussed in Sec.2. The source of energy difference induced by instanton vertex in scalar potential has the form (for simplicity we assume just one family of quarks):
\[ V_{\text{phase}} = -m_u m_d \Lambda^2 \cos \theta \]  

where as before $\Lambda \sim \Lambda_{QCD} \simeq 200\,\text{MeV}$ and $m_u, m_d$ are masses of $u$ and $d$-quarks respectively. The resulting double sine-Gordon equation for $\theta$ has the form (11), where now $a = \frac{m_u m_d \Lambda^2}{V^2 m^2}$. Once again, the resulting $2\pi$-walls can be practically stable and cause trouble for the cosmology, unless the parameter $\lambda$ is sufficiently large implying, that the mass of the would be axion is close to the Higgs mass.

4. Conclusions

We have shown that for the certain range of parameters, compatible with standard bound, the collapse of unstable domain walls may end up with formation of classically (and practically) stable double wall systems. If formed, this structures can be problematic cosmologically and therefore their nonexistence requires additional restriction of the parameter space. Of course, formation and subsequent evolution of this defects is very complicated process and requires much more detailed and perhaps numerical study. For example, collision of the domain walls may enhance the rate of hole formation. At this point our analysis can be considered as ‘first’ approximation, but nevertheless, it provides an interesting enough example how cosmological considerations may restrict parameters in the particle physics models.

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References

[1] Ya.B.Zel’dovich, I.Yu.Kobzarev and L.B.Okun, JETP 40 (1975) 1.

[2] For a review, see: T.W.B.Kibble, J.Phys. A9 (1976) 1387; Phys.Rep. 67 (1980) 183; A.Vilenkin, Phys.Rep. 121 (1985) 263.

[3] A.H.Guth, Phys.Rev.D23 (1981) 347; For a review, see e.g. A.D.Linde, Particle Physics and Inflationary Cosmology (Harwood Academic, Switzerland, 1990); K.A.Olive, Phys.Rep. 190 (1990) 307.

[4] J.Preskill, S.P.Trivedi, F.Wilczek and M.B.Wise, Nucl.Phys.B363 (1991) 207.

[5] B.Rai and G.Senjanovic, Phys.Rev. D49 (1994) 2729; For earlier work on the possible role of gravity, see B.Holdom, Phys.Rev. D28 (1983) 1419

[6] A.Vilenkin, Phys.Rev.D23 (1981) 852.

[7] T.W.B.Kibble, G.Lazarides and Q.Shafi, Phys.Rev D26 (1982) 435.

[8] A.Vilenkin and A.E.Everett, Phys.Rev.Lett. 48 (1982) 1867; For a review, see e.g. J.E.Kim, Phys.Rep. 150 (1987) 1.

[9] P.Sikivie, Phys.Rev.Lett 48 (1982) 1156.

[10] E.Witten, Nucl.Phys. B249 (1985) 557.

[11] S.Weinberg, Phys.Rev. D9 (1974) 3357; L.Dolan and R.Jackiw, Phys.Rev. D9 (1974) 3320; A.D.Linde, Rep.Prog.Phys. 42 (1979) 389.