ABSTRACT: If neutrinos are Dirac fermions, certain new physics beyond the standard model should exist to account for the smallness of neutrino mass. With two additional scalars and a heavy intermediate fermion, in this paper, we systematically study the general mechanism that can naturally generate the tiny Dirac neutrino mass at tree and in one-loop level. For tree level models, we focus on natural ones, in which the additional scalars develop small vacuum expectation values without fine-tuning. For one-loop level models, we explore those having dark matter candidates under $Z^D_2$ symmetry. In both cases, we concentrate on $SU(2)_L$ multiplet scalars no larger than quintuplet, and derive the complete sets of viable models. Phenomenologies, such as lepton flavor violation, leptogenesis, DM and LHC signatures are briefly discussed.

KEYWORDS: Dirac neutrino mass, dark matter, $SU(2)_L$ multiplets
### Contents

1 Introduction 1

2 Tree Level Models for Dirac Neutrino Mass 2

3 One-loop Models for Dirac Neutrino Mass 9

4 Phenomenology 13
   4.1 Flavor Constraints 13
   4.2 Leptogenesis 15
   4.3 Dark Matter 17
   4.4 LHC Signature 20

5 Conclusion 22

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**1 Introduction**

The mechanism responsible for tiny neutrino mass generation remains a puzzle. If the neutrinos are Majorana particles, the attractive scenario is to introduce Weinberg’s dimension five operator $\lambda LL\Phi\Phi/\Lambda$ [1], where $\Lambda$ is the typical high energy scale of underlying new physics. By adding new heavy intermediate states to the Standard Model (SM) particle content, there are three canonical mechanisms to realize above operator at tree level (referred to as type-I, II, III seesaw models [2–4]). The smallness of neutrino mass can also be achieved at low energy scale, either by pushing the mass operator beyond five dimension [5–14] or by attributing the mass term to purely radiative arising at loop-level (see Ref. [15–19] for classic examples). In these models, new physics may arise at TeV scale and thus be detectable at LHC or other planned collider machine [20]. In Ref. [13], the minimal realizations of the seesaw mechanisms at tree level are listed according to the nature of heavy intermediate $SU(2)_L$ multiplet fermions. In Ref. [21], the one-loop neutrino mass model proposed by Ma [16] is generalized to a class of related models with $SU(2)_L$ multiplet fields no larger than adjoint representation.

On the other hand, the experimental evidences establishing whether neutrinos are of Majorana or Dirac type are still missing. If neutrinos are Dirac particles and acquire their masses via direct coupling with SM Higgs boson, the Yukawa coupling constants have to be unnaturally small in comparison with other SM fermions. To solve the problem, some mechanism accounting for the smallness of Dirac neutrino mass have been proposed by many authors at tree(see Ref. [22–29] for earlier works and Ref. [30–36] for latest works) and loop level [37–46]. In Ref. [44], the generic topographies of diagrams with specific cases are presented.

In this work, we catalogue the related models that generate the tiny Dirac neutrino mass at tree and one-loop level. In Sec. 2, we focus on the minimal tree level realizations of Dirac seesaws with
at most two extra scalars $S_{1,2}$ and a heavy intermediated Dirac fermion $F$, see Fig. 1. As pointed out in Ref. [44], to obtain a naturally small Dirac neutrino mass, another symmetry is required to forbid the $\nu_L \nu_R \phi^0$ term, where $\phi^0$ denotes the SM Higgs field. Then the breaking of this symmetry induces the effective Dirac neutrino mass $m_D \nu_L \nu_R$. It naively appears that, by adding appropriate $SU(2)_L$ multiplet field variants to SM, there are infinite ways to realize tree level diagram in Fig. 1. However, we will see that, as the Majorana case [13], the number of candidate models is significantly reduced if only the models with non-tuning vacuum expectation values (VEVs) are considered.

At one-loop level, a typical diagram was proposed [37, 38, 44], in which the particle content includes two extra scalars and a gauge-singlet fermion being odd under $Z_2$ symmetry. As a result, the lightest beyond-SM field is stable and may be considered a dark matter (DM) candidate. In Sec. 3, we generalize the approach given in Ref. [21]. We list a class of models which generates the Dirac neutrino masses via the one-loop diagram in Fig. 2 and simultaneously includes a DM candidate. Without loss of generality, we mainly restrict our attention on the models with the $SU(2)_L$ multiplets fields no larger than adjoint while briefly list the models with larger multiplets in Appendix. For each model, we investigate its validity and the type of DM candidate which is compatible with direct detection experiments. We consider the phenomenology of the models in Sec. 4, discussing the issues of lepton number violation processes, leptogenesis and collider signals. A conclusion is given in Sec. 5.

2 Tree Level Models for Dirac Neutrino Mass

\[
\begin{array}{c}
\langle S_1 \rangle \\
\downarrow \\
\nu_L \\
\downarrow \\
\langle S_2 \rangle \\
\downarrow \\
F_R \times F_L \\
\downarrow \\
\nu_R
\end{array}
\]

Figure 1. Dirac neutrino mass at tree level.

Pathways to naturally small Dirac neutrino mass have been recently discussed in Ref. [44]. By adding an extra Dirac fermion singlet/doublet/triplet or scalar doublet, four tree-level seesaw models are found to realize the Dirac neutrino mass generation. However, the Dirac seesaw mechanism is more general when we move beyond the field content given in Ref. [44]. Following the spirit of Ref. [13, 14], we firstly discuss the tree-level realization of Dirac seesaw with at most two extra scalars $S_{1,2}$ and a heavy intermediated Dirac fermion $F^1$ (Fig. 1). Here, the global lepton number symmetry $U(1)_\ell$ is proposed to forbid the unwanted Majorana mass term $(m_N/2)\nu_L \nu_R$, meanwhile the discrete $Z_3$ [42, 45], $Z_4$ [35, 47, 48] and $\Delta(27)$ [49] symmetry are also optional.

\footnote{Here, we introduce three generations of heavy fermion $F$. For simplicity, we will not show the generation indices explicitly in the following discussion.}
Table 1. Cases of $Z_2$-charge assignments for relevant fields.

| Cases | $L_L$ | $S_1$ | $F_R$ | $F_L$ | $S_2$ | $\nu_R$ |
|-------|-------|-------|-------|-------|-------|---------|
| (A)   | +     | −     | −     | −     | +     | −       |
| (B)   | +     | +     | +     | +     | −     | −       |

In order to obtain a naturally small Dirac neutrino mass, another symmetry $S$ is required to forbid the $\bar{\nu}_L \nu_R \phi^0$ term. Then the broken of this symmetry $S$ induces the effective Dirac neutrino mass term $m_D \bar{\nu}_L \nu_R$ [44]. The choice of symmetry $S$ is model-dependent and here we take the $Z_2$ symmetry as an example. In Table 1, we show two possible cases of $Z_2$-charge assignment for relevant fields. Under the $Z_2$ symmetry, $\nu_R$ is $Z_2$-odd while other SM particles are $Z_2$-even in all cases, which is aiming to forbid the $\bar{\nu}_L \nu_R \phi^0$ term. Since $F_L$ carries same $Z_2$-charge as $F_R$, $M_F \bar{F}_L F_R$ is invariant under $Z_2$ as well as SM gauge symmetry. Therefore, $M_F$ could be assumed to be large. The $Z_2$ symmetry is broken explicitly by terms as $H S_1 S_2$, because of opposite $Z_2$-charge assignment of $S_1$ and $S_2$ for both case (A) and (B) in Table 1.

Some generic features are described from the general tree level diagram in Fig. 1:

- The heavy fermion $F$ is vector-like, which transforms as $F_{L,R} \sim (1, R_F, Y_F)$ under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry.
- The scalars $S_{1,2}$ transform as $S_{1,2} \sim (1, R_{1,2}, Y_{1,2})$, and they are necessarily distinct from each other, i.e., $R_1 \neq R_2$ or/and $Y_1 \neq Y_2$.
- The new particles $F$ and $S_{1,2}$ must contain a neutral component, which requires:

  $$|Y_i| \leq R_i - 1, \ (i = F, 1, 2). \quad (2.1)$$

  And $Y_i$ must be an integer to avoid fractionally charged particles as well.
- For isospin allowing to couple $F$ and $S_1(S_2)$ to $L_L(\nu_R)$, following relations should be satisfied:

  $$R_L \otimes R_{1} \supset R_F \quad \Rightarrow \quad |R_1 - R_F| = 1, \quad (2.2)$$
  $$R_{\nu} \otimes R_{2} \supset R_F \quad \Rightarrow \quad R_2 = R_F, \quad (2.3)$$

  where $R_L = 2$ and $R_{\nu} = 1$ are the isospin values for SM lepton doublet $L_L$ and neutrino singlet $\nu_R$, respectively.
- The neutrality of hyper charge $Y$ then requires that:

  $$-Y_F + Y_L + Y_1 = 0 \quad \Rightarrow \quad Y_1 = Y_F + 1, \quad (2.4)$$
  $$-Y_{\nu} + Y_F + Y_2 = 0 \quad \Rightarrow \quad Y_2 = -Y_F, \quad (2.5)$$

  where $Y_L = -1$ and $Y_{\nu} = 0$ are the hyper charges for SM lepton doublet $L_L$ and neutrino singlet $\nu_R$, respectively.
• Considering the above relations in Eq. 2.2–2.5 as well as the fact that the SM Higgs \( H \) has the quantum numbers as \( R_H = 2 = R_L \) and \( Y_H = 1 \), one can deduce the following relations:

\[
(R_H \otimes R_1) \otimes R_2 \ni 1, \quad (2.6)
\]
\[
Y_1 + Y_2 - Y_H = 0, \quad (2.7)
\]

which indicates that a trilinear term as \( \tilde{H} S_1 S_2 \) is always allowed in the scalar potential. Here, \( \tilde{H} = i \sigma_2 H^* \) is the conjugate of the SM Higgs doublet.

We arrive at the relevant terms to generate small Dirac neutrino masses as shown in Fig. 1:

\[
\mathcal{L} \ni y_1 \overline{F_R} L S_1 + y_2 \overline{F_R} F_L S_2 + M_F \overline{F_L} F_R + \mu \tilde{H} S_1 S_2 + \text{h.c.} \quad (2.8)
\]

Then the generic form of Dirac seesaw mechanism is realized, for which the neutrino mass from tree level contribution is given by

\[
m^\text{tree}_\nu \approx y_1 y_2 \langle S_1 \rangle \langle S_2 \rangle \frac{M_F}{M^2_F}, \quad (2.9)
\]

For \( m^\text{tree}_\nu \sim 0.1 \text{ eV} \), one can set \( y_1 \sim y_2 \sim 10^{-2} \), \( \langle S_1 \rangle \sim \langle S_2 \rangle \sim 10^{-2} \text{ GeV} \), and \( M_F \sim 10^2 \text{ GeV} \). It is an important issue on how \( S_1 \) and \( S_2 \) develop naturally small VEV comparing to \( H \), which will be discussed in the following. Before proceeding, one notes that the trilinear \( \mu \tilde{H} S_1 S_2 \) term also contributes to Dirac neutrino mass via the one-loop diagram in Fig. 2. Actually, if the VEVs of \( S_1 \) and \( S_2 \) are forbidden by an additional symmetry, e.g., \( Z^D_2 \) or \( U(1)_D \), only loop diagram can exist and contribute to the neutrino mass generation. In this case, it is possible to include dark matter candidates running in the loop, which is postponed for a more detail discussion in Sec. 3.

For the sake of simplicity, one assumes a degenerate mass spectrum for particles within \( F, S_1 \) and \( S_2 \), then the one-loop contribution to Dirac neutrino mass is given by

\[
m^\text{loop}_\nu = C_\nu \frac{\sin 2 \theta}{32 \pi^2} y_1 y_2 M_F \left[ \frac{M^2_{S_2}}{M^2_{S_2} - M^2_F} \ln \left( \frac{M^2_{S_2}}{M^2_F} \right) - \frac{M^2_{S_1}}{M^2_{S_1} - M^2_F} \ln \left( \frac{M^2_{S_1}}{M^2_F} \right) \right], \quad (2.10)
\]

where \( \theta \) is the mixing angle between \( S_1 \) and \( S_2 \). The coefficient \( C_\nu \) is determined by different particle sets running in the loop and the corresponding Clebsch-Gordan coefficients, thus is model
Considering the case of comparable masses with $M$ dependent. For example, in model (a) listed in Table 3 where $F \sim (1, 1, 0), S_1 \sim (1, 2, 1)$ and $S_2 \sim (1, 1, 0)$, we have $C_{\nu} = 1$. Depending on relative values between $M_F$ and $M_{S_{1,2}}$, the expression of $m_{\nu}^\text{loop}$ in Eq. 2.10 can be further simplified. In the heavy fermion limit with $M_F \gg M_{S_{1,2}},$

$$m_{\nu}^\text{loop} \sim C_{\nu} \frac{\sin 2\theta \ y_1 y_2}{32\pi^2} M_F \left[ M_{S_1}^2 \ln \left( \frac{M_{S_1}^2}{M_F^2} \right) - M_{S_2}^2 \ln \left( \frac{M_{S_2}^2}{M_F^2} \right) \right].$$

(2.11)

While in the opposite limit with $M_F \ll M_{S_{1,2}},$

$$m_{\nu}^\text{loop} \sim C_{\nu} \frac{\sin 2\theta \ y_1 y_2}{32\pi^2} M_F \ln \left( \frac{M_{S_2}^2}{M_{S_1}^2} \right).$$

(2.12)

And at last, when $M_F \approx M_{S_{1,2}},$

$$m_{\nu}^\text{loop} \sim C_{\nu} \frac{\sin 2\theta \ y_1 y_2}{32\pi^2} (M_{S_2}^2 - M_{S_1}^2).$$

(2.13)

Considering the case of comparable masses with $M_F \approx M_{S_{1,2}}$ around electroweak scale, we have

$$\frac{m_{\nu}^\text{loop}}{m_{\nu}^\text{tree}} \sim \frac{\sin 2\theta \ M_{S_2}^2 - M_{S_1}^2}{32\pi^2 \langle S_1 \rangle \langle S_2 \rangle}.$$  

(2.14)

Therefore, the tree level contribution might be dominant provided that $\langle S_{1,2} \rangle$ is not too small.

It seems that there could be an infinite number of models satisfying the above generic features. But when considering constraints from perturbative unitarity\(^2\) [50, 51], we will concentrate on $SU(2)_L$ scalar multiplet no larger than quintuplet in this paper. Meanwhile, the number of candidate models could be significantly reduced when we only consider the models with non-tuning VEVs for additional scalars $S_{1,2}$ [13]. Then, as we shall see, the viable models are finite and we would like to specify all of them.

First, we consider the simplest case when one of $S_1$ and $S_2$ is the SM Higgs doublet $H$. Following the conditions in Eq. 2.2–2.5, one can figure out four simplest models as: $S_1 = H \sim (1, 2, 1), F \sim (1, 2 \mp 1, 0), S_2 \sim (1, 2 \mp 1, 0)$ and $S_1 \sim (1, 2 \mp 1, 0), F \sim (1, 2, -1), S_2 = H \sim (1, 2, 1)$, which exactly correspond to the cases in Ref. [44]. At the same time, the trilinear term $\tilde{H} S_1 S_2$ becomes $\tilde{H} H S_{1/2}$ and induces a non-zero VEV of $S_{1/2}$

$$\langle S_{1/2} \rangle \approx \mu \frac{\langle H \rangle^2}{M_{S_{1/2}}^2}.\tag{2.15}$$

Notably, the trilinear term $\tilde{H} H S_{1/2}$ could be an explicit $Z_2$ breaking term as in the cases (A) and (B) shown in Table 1. Thus the small $\langle S_{1/2} \rangle$ is acquired in the technically natural limit of $\mu \ll \langle H \rangle$ even for $M_{S_{1/2}}$ around electroweak scale. Then the tree level neutrino mass in Eq. 2.9 is expressed as:

$$m_{\nu} \sim y_1 y_2 \frac{\mu}{M_F M_{S_{1/2}}^2} \langle H \rangle^3.$$  

(2.16)

\(^2\) The $2 \to 2$ tree-level processes of scalar multiplet pair annihilation into electroweak gauge bosons receive large contribution for scalar multiplet with large weak charge. So the upper limits on isospin and hypercharge of a scalar multiplet can be obtained by requiring that the zeroth partial wave amplitude satisfies the unitarity bound.
Typically, we can acquire $m_{\nu}^{\text{tree}} \sim 0.1$ eV by setting $g_{1,2} \sim 10^{-2}$, $\mu \sim 1$ MeV, and $M_F \sim M_{S_{1/2}} \sim 1$ TeV. Provided $M_{S_{1/2}} \sim M_F = M$, the tiny tree level Dirac neutrino mass is generated from a dimension $d = 6$ effective low-energy operator as $O_{\nu} = \mu \overline{\nu}_R L_L H^3 / M^3$.

Notably, a special case, the so-called neutrinoophilic two Higgs doublet model ($\nu$2HDM), appears in literature[34, 52, 53], where the new scalar doublet $\eta \sim (1, 2, 1)$ transforms the same as SM Higgs doublet under SM gauge group but carries some new charge, i.e., $Z_2$ or $U(1)$ [54–56]. In this model, the new Yukawa coupling $\overline{\nu}_R \eta^\dagger L_L$ is allowed and the small VEV of $\eta$ can be obtained by adding a soft $Z_2$ or $U(1)$ breaking term as $\eta^\dagger H$, leading to naturally small Dirac neutrino masses [44]. In this case, the tiny Dirac neutrino mass is generated from a dimension $d = 4$ effective low-energy operator as $O_{\nu} = \mu \overline{\nu}_R \tilde{H} L_L / M$.

Now we move beyond the simplest case and explore further generations with both $S_1$ and $S_2$ being new particles. After the electroweak symmetry breaking, VEVs of $S_{1,2}$ will usually contribute to the W and Z boson masses. Especially, for those scalars with $SU(2)_L$ representation $R_{1,2} > 2$, their VEVs $\langle S_{1,2}\rangle$ will affect the $\rho$ parameter away from the SM value $\rho = 1$ at tree-level, which then leads to tight bound on $\langle S_{1,2}\rangle \lesssim O(1)$ GeV [57]. The trilinear $H S_1 S_2$ term alone can not ensure that both $\langle S_{1,2}\rangle$ are naturally small in general case. Therefore, in order to produce non-tuning VEVs for $S_{1,2}$, the scalar potential $V(H, S_1, S_2)$ should contain linear $S_1$ or/and $S_2$ terms as $S_{1,2} H^n (n \leq 3)$. With these conditions in mind, we find the $S_1$ or/and $S_2$ with the quantum numbers as (see Ref. [13] for more details):

$$S_{1,2} \sim (1, 2, \pm 1), (1, 3, 0), (1, 3, \pm 2), (1, 4, \pm 1), (1, 4, \pm 3).$$  \tag{2.17}$$

On the other hand, if only $S_i$ obtains a naturally small VEV from the term $S_i H^n (n \leq 3)$, the trilinear $H S_i S_2$ term will induce a naturally suppressed VEV for $S_j$ as:

$$\langle S_j \rangle \simeq \mu \frac{\langle S_i \rangle \langle H \rangle}{M^2_{S_j}}, \text{ for } i \neq j.$$  \tag{2.18}$$

Thus we expect $\langle S_j \rangle \lesssim \langle S_i \rangle$, when $\mu \lesssim \langle H \rangle \lesssim M_{S_i}$.

Based on the above statement, the general strategy for determining a specific Dirac seesaw model is quite straight. First, one determines a scalar $S_i$ with quantum number in Eq. 2.17, and then the viable sets of quantum numbers for $F$ and $S_j$ can be obtained by Eq. 2.2–2.5. Following this procedure, we have listed all viable models in Table 2. Clearly from Table 2, naturally small Dirac neutrino mass arises from even number dimension effective operators as $O_{\nu} = \overline{\nu}_R L_L H^{2n+1} / \Lambda^{2n}$ and a higher scalar/fermion representation generally tends to a higher dimension effective operator.

Some comments on specific models are as following. Model (A) and (B) contains a scalar singlet $\phi \sim (1, 1, 0)$. In our consideration, VEV of $\phi$ is induced from the $Z_2$ breaking trilinear term $\mu \tilde{H} \phi$, thus $\langle \phi \rangle \simeq \mu \langle H \rangle^2 / M^2_{\phi}$ is naturally small when $\mu \ll \langle H \rangle \lesssim M_{\phi}$. Since the VEV of $\phi$ does not contribute to the $\rho$-parameters, it may be typically around electro-weak scale and originated from the spontaneous breaking of scalar potential [35, 58]. In this way, one needs the intermediate fermion $F$ with $O(10^{10})$ GeV mass scale to generate proper neutrino masses, just as the canonical type-I seesaw model.

Model (D) and (G) employ a scalar doublet $\eta \sim (1, 2, -1)$. Provided $\eta$ is $Z_2$-odd as $\nu_R$ (as case (B)), the Yukawa coupling $\overline{\nu}_R \eta^\dagger L_L$ is allowed. After $\eta$ develops a VEV from the soft term...
Then in addition to the four obvious minimal models—model (A), (B), (C) and (E)—we get two as

\[ \nu \text{ charge-conjugate field of } H \]

as Table 2

| Models | \( F \) | \( S_1 \) | \( S_2 \) | \([\mathcal{O}_\nu]\) | style |
|-------|------|------|------|---------|-----|
| (A)   | (1, 1, 0) | \( H(1, 2, 1) \) | \( \phi(1, 1, 0) \) | \( d = 6 \) | minimal |
| (B)   | (1, 2, -1) | \( \phi(1, 1, 0) \) | \( H(1, 2, 1) \) | \( d = 6 \) | minimal |
| (C)   | (1, 2, -1) | \( \Delta(1, 3, 0) \) | \( H(1, 2, 1) \) | \( d = 6 \) | minimal |
| (D)   | (1, 2, 1) | \( \Delta(1, 3, 2) \) | \( \eta(1, 2, -1) \) | \( d = (4)6 \) | (non-)minimal |
| (E)   | (1, 3, 0) | \( H(1, 2, 1) \) | \( \Delta(1, 3, 0) \) | \( d = 6 \) | minimal |
| (F)   | (1, 3, 0) | \( \chi(1, 4, 1) \) | \( \Delta(1, 3, 0) \) | \( d = 6, 8 \) | non-minimal |
| (G)   | (1, 3, -2) | \( \eta(1, 2, -1) \) | \( \Delta(1, 3, 2) \) | \( d = (4)6 \) | (non-)minimal |
| (H)   | (1, 3, -2) | \( \chi(1, 4, -1) \) | \( \Delta(1, 3, 2) \) | \( d = (6)8 \) | (non-)minimal |
| (I)   | (1, 3, 2) | \( \chi(1, 4, 3) \) | \( \Delta(1, 3, -2) \) | \( d = 8 \) | minimal |
| (J)   | (1, 4, 1) | \( \Delta(1, 3, 2) \) | \( \chi(1, 4, -1) \) | \( d = 8 \) | minimal |
| (K)   | (1, 4, 1) | \( \Phi(1, 5, 2) \) | \( \chi(1, 4, -1) \) | \( d = 10 \) | minimal |
| (L)   | (1, 4, -1) | \( \Delta(1, 3, 0) \) | \( \chi(1, 4, 1) \) | \( d = 8 \) | minimal |
| (M)   | (1, 4, -1) | \( \Phi(1, 5, 0) \) | \( \chi(1, 4, 1) \) | \( d = 10 \) | minimal |
| (N)   | (1, 4, 3) | \( \Phi(1, 5, 4) \) | \( \chi(1, 4, -3) \) | \( d = 10 \) | minimal |
| (O)   | (1, 4, -3) | \( \Delta(1, 3, -2) \) | \( \chi(1, 4, 3) \) | \( d = 10 \) | minimal |
| (P)   | (1, 4, -3) | \( \Phi(1, 5, -2) \) | \( \chi(1, 4, 3) \) | \( d = 10 \) | minimal |
| (Q)   | (1, 5, 0) | \( \chi(1, 4, 1) \) | \( \Phi(1, 5, 0) \) | \( d = 10 \) | minimal |
| (R)   | (1, 5, 2) | \( \chi(1, 4, 3) \) | \( \Phi(1, 5, -2) \) | \( d = 10 \) | minimal |
| (S)   | (1, 5, -2) | \( \chi(1, 4, -1) \) | \( \Phi(1, 5, 2) \) | \( d = 10 \) | minimal |
| (T)   | (1, 5, -4) | \( \chi(1, 4, -3) \) | \( \Phi(1, 5, 4) \) | \( d = 10 \) | minimal |

Table 2. Natural tree level seesaws for Dirac neutrinos. For simplicity, we denote new scalar singlet to quintuplet as \( \phi, \eta, \Delta, \chi \) and \( \Phi \), respectively.

\[ \eta H, \] the \( \bar{\nu} R \eta \dagger L_L \) term induces a Dirac mass term corresponding to dimension \( d = 4 \) effective operator as \( \mathcal{O}_\nu = \bar{\nu} R \tilde{H} \dagger L_L \). Meanwhile, the heavy intermediate fermion \( F \) together with \( \eta \) and another scalar triplet \( \Delta (1, 3, 2) \) generate Dirac neutrino mass from \( d = 6 \) effective operator as \( \mathcal{O}_\nu = \bar{\nu} R L_L H^3 / A^2 \). Thus, light Dirac neutrino mass have two contributions, which can be written as

\[ m_{\nu}^{\text{tree}} \simeq y_1 \frac{\langle H \rangle}{M_F} + y_2 \frac{\langle \eta \rangle \langle \Delta \rangle}{M_F}. \]  \hspace{1cm} (2.19)

Since \( \langle \Delta \rangle \ll M_F \), the Dirac neutrino mass is dominant by the first term in Eq. 2.19. Hence, model (D) and (G) are non-minimal, and can be regarded as just a more complicated extension of the \( \nu 2 \text{HDM} \) with new contributions to Dirac neutrino mass subdominant.

In contrast, if \( \eta \) is \( Z_2 \)-even as case (A) shown in Table 1, we might be able to treat \( \eta \) as the charge-conjugate field of \( H \), i.e., \( \eta = \tilde{H} \). In this case, the Yukawa coupling \( \bar{\nu} R \eta \dagger L_L = \bar{\nu} R \tilde{H} \dagger L_L \) is forbidden, so the light Dirac neutrino mass can only be induced by the heavy intermediate fermion \( F \) as

\[ m_{\nu}^{\text{tree}} \simeq y_2 \frac{\langle H \rangle \langle \Delta \rangle}{M_F}. \]  \hspace{1cm} (2.20)

Then in addition to the four obvious minimal models—model (A), (B), (C) and (E), we get two
more minimal models—model (D) and (G) with \( d = 6 \) effective operators as well. When counting on the heavy intermediate fermion, one more representation \( F \sim (1, 3, \pm 2) \) is employed in model (G), while we can regard \( F \sim (1, 2, 1) \) in model (D) as the charge-conjugate of \( F \sim (1, 2, -1) \) in model (B) and (C).

For the scalar triplet \( \Delta \sim (1, 3, \pm 2) \) which is involved in model (D), (G), (H), (I) and (J), the \( L_L L_L \Delta \) term is forbidden by the unbroken \( U(1)_L \) lepton symmetry in case of Dirac neutrino. Since for tree level models in this work, we can only assign lepton number \( L = 1 \) or \( L = 0 \) to new fermion \( F \) and scalars \( S_{1,2} \) (including \( \Delta \)), respectively.

Comparing with model (E) and (F), it is obvious that model (F) is essentially model (E) with an additional scalar quadruplet \( \chi \sim (1, 4, 1) \). As a result, model (F) is non-minimal, and neutrino mass is generated by two distinct tree level diagrams. And provided \( \eta = \tilde{H} \) in model (G), then model (H) is clearly also non-minimal. Under such circumstances, the light neutrino mass for model (F) and (H) is given by:

\[
 m^\text{tree}_\nu \simeq y_1^H \frac{\langle H \rangle \langle \Delta \rangle}{M_F} + y_2^\chi \frac{\langle \chi \rangle \langle \Delta \rangle}{M_F} ,
\]

which correspond to effective operators of dimension \( d = 6 \) and \( d = 8 \), respectively.

Now let’s look at a specific model, i.e., model (K) in Table 2, to show how to construct a complete model. First, we choose \( S_2 \sim (1, 4, -1) \) in Eq. 2.17. Then, the quantum number of \( F \sim (1, 4, 1) \) can be obtained from constraints in Eq. 2.3, 2.5 by the Yukawa coupling \( \nu_R F_L S_2 \). At last, inspection of constraints in Eq. 2.2, 2.4 by the other Yukawa coupling \( F_R L_L S_1 \) reveals that either \( S_1 \sim (1, 3, 2) \) or \( S_1 \sim (1, 5, 2) \) corresponding to model (J) and (K), respectively.

![Figure 3](image-url)  
*Figure 3.* Tree level Dirac seesaw of model (K) in Table 2.

In Fig. 3, we depict the tree level Dirac seesaw of model (K). The scalar quadruplet \( S_2 \sim (1, 4, 1) \) acquires a naturally small VEV from the quartic term \( \lambda S_2^4 H H^\dagger \tilde{H} \) as:

\[
 \langle S_2 \rangle \simeq \frac{\lambda \langle H \rangle^3}{M_{S_2}^2} .
\]

For the scalar quintuplet \( S_1 \sim (1, 5, 2) \), the trilinear \( \tilde{H} S_1 S_2 \) term ensures \( S_1 \) also develops a naturally small VEV as shown in Eq. 2.18:

\[
 \langle S_1 \rangle \simeq \mu \frac{\langle S_2 \rangle \langle H \rangle}{M_{S_1}^2} \simeq \lambda \frac{\mu \langle H \rangle^4}{M_{S_1}^2 M_{S_2}^2} .
\]
As a consequence, the tree level Dirac neutrino mass in model K is:

\[ m^{\text{tree}}_{\nu} \simeq y_1 y_2 \frac{\langle S_1 \rangle \langle S_2 \rangle}{M_F} \simeq y_1 y_2 \frac{\lambda_{\mu}}{M_F} \frac{\langle H \rangle^7}{M^6} \tag{2.24} \]

Supposing \( \mu \sim M_{S_1,2} \sim M_F = M \), then we have \( m^{\text{tree}}_{\nu} \propto \langle H \rangle^7 / M^6 \), which indicates that the tiny Dirac neutrino mass is induced by a dimension \( d = 10 \) effective operator as \( \mathcal{O}_\nu = \overline{\nu_R} L_L H^7 / M^6 \).

3 One-loop Models for Dirac Neutrino Mass

Now, we move forward to the purely radiative generation of Dirac neutrino mass. There were variant of models proposed in this direction. At one-loop level, the simplest model discussed in Ref.[59] is based on soft-broken \( Z_2 \) symmetry, where two new charged scalar singlets are employed. However, no DM candidate can be incorporated in this model. Another appealing way is introducing an additional symmetry, e.g., a dark \( Z_2 \) symmetry(\( Z_{D2} \)), under which \( S_1,2 \) and \( F \) carry \( Z_2 \)-odd charge while all SM fields transform trivially. In this way, the VEVs of \( S_1,2 \) is forbidden and neutrino mass can only be generated via the one-loop diagram shown in Fig. 2. Due to the \( Z_2 \) odd protection, the lightest neutral component within the inert fields \( S_1,2 \) or \( F \) is stable, and thus becomes a DM candidate. In this paper, we restrict our attention on the models involving neutral components and analyze their validity as a DM. We focus on models with representations no larger than the adjoint representation, and then briefly discuss larger multiplets with quadruplet and/or quintuplet of \( SU(2)_L \).

From Fig. 2, it reveals that, with slightly modification of statements around Eq. 2.1, they are still applicable for one-loop case. The difference comes from the fact that in loop models the neutral field does not have to propagate inside the loop, hence we only require that at least one of the new fields \( S_1,2 \) and \( F \) has a neutral content. The other constraints, i.e., Eq. 2.2–2.7, are directly coming or indirectly derived from the relevant Yukawa coupling, so they are still capable for loop models.

With the comments given above, a systematic analysis is made to exhaust the models that generate Dirac neutrino mass via Fig. 2. In Table 3, we list all possible distinct models with representations no larger than the adjoint representation. There are totally ten viable models, and the simplest three of them, i.e., model (a), (c) and (e), are already mentioned in Ref. [44]. For models with quadruplet and/or quintuplet, we depict them in Table 4.

The generic one-loop Dirac neutrino mass matrix has already been given in Eq. 2.10. The trilinear term \( \mu H S_1 S_2 \) still induces the mixing between inert scalars \( S_1 \) and \( S_2 \), meanwhile the newly employed \( Z_2^D \) symmetry forbids the mixing between \( S_{1,2} \) and SM Higgs doublet \( H \). To acquire \( m^{\text{loop}}_{\nu} \sim 0.1 \text{eV} \), we can set \( y_1 \sim y_2 \sim \theta \sim 10^{-3} \) with all inert particles around \( \mathcal{O}(\text{TeV}) \).

A detailed study on the DM phenomenology for all models presented in Table 3 and 4 is beyond the scope of this paper. First, we briefly discuss viable DM candidate in specific models in this section. Then in the next section, we choose model (a) as our benchmark model for a more detail study.

First, we turn our intention into fermion DM candidate. In model (a), an inert fermion singlet \( F \sim (1,1,0) \) is introduced. Possible annihilation channels are: 1), \( F \bar{F} \rightarrow \ell^+ \ell^-, \nu \bar{\nu} \) mediated by \( \eta \) via the Yukawa coupling \( y_1 \); 2), \( F \bar{F} \rightarrow \nu \bar{\nu} \) mediated by \( \phi \) via the Yukawa coupling \( y_2 \); 3),
coannihilation with $\eta(\phi)$, when $M_{\eta(\phi)}$ is close to $M_\phi$. For all these channels, not too small Yukawa couplings $y_1$ and/or $y_2$ of $\mathcal{O}(0.1)$ are required to generate the correct relic density [60, 61]. On the other hand, for $\eta$ involved channels, the Yukawa coupling $y_1$ receives tight constraints from lepton flavor violating processes [61]. Therefore, we expect that the Yukawa couplings satisfy the relation $y_1 \ll y_2$, which further indicates that the $\phi$ mediated process is dominant provided $M_\phi \approx M_\eta$.

Another notable model with viable fermion DM is model (h), where an inert fermion triplet $F \sim (1, 3, 0)$ is introduced. The neutral component $F^0$ can serve as a DM candidate. Due to its electroweak couplings to gauge bosons, the relic density of $F^0$ is dominantly determined by the annihilation and co-annihilation of itself and $F^\pm$, which requires that $M_{F^0}$ is around 2.6 TeV [62–64]. In this case, $S_1 \sim (1, 2, 1)$ and $S_2 \sim (1, 3, 0)$ should be heavier than 2.6 TeV, thus hardly being tested at LHC.

Fermion DM in other models with no larger than adjoint representation are excluded. Clearly, for model (b) and (g), $F \sim (1, 1, -2)$ and $F \sim (1, 2, -3)$ do not have neutral component, thus these two models do not have fermion DM candidate. On the other hand, model (c), (d), (e), (f) employ $F \sim (1, 2, \pm 1)$ and model (i), (j) employ $F \sim (1, 3, \pm 2)$, which contain neutral fermions. But all the neutral fermions in these model have non-zero hypercharge, which will lead to detectable DM-nucleon scattering cross section via $Z$-boson exchange. So they have already been excluded by direct detection experiments, such as, LUX [65] and PandaX-II [66].

Then we move onto scalar dark matter. Considering the constraints from LFV and tiny neutrino masses, it is better to set the Yukawa coupling $y_1 \sim y_2 \lesssim 10^{-2}$. In this way, the contribution of heavy fermion $F$ to scalar DM variables is negligible. Both model (a) and (c) introduce an inert scalar singlet $\phi \sim (1, 1, 0)$ [67] and an inert scalar doublet $\eta \sim (1, 2, 1)$ [68]. In principle, either of $\phi$ and $\eta$ can solely paly the role of dark matter candidate under the $Z_2^D$ symmetry. In these two models, the trilinear term $\mu \phi^\dagger \eta H/\sqrt{2}$ will induce the mixing between $\phi$ and $\eta_0^D$, and the allowed parameter space thus are expected enlarged. Detail phenomenological aspects for inert singlet-doublet scalar dark matter can be found in Ref. [69, 70]. From the result of Ref. [69, 70], we know that the mixing angle $\theta$ between $\phi$ and $\eta_0^D$ must be small enough to avoid too large DM-nucleon scattering cross section if DM is dominant by $\phi$ component. Notably, there exists a value of $\sin \theta$ for

| Models | $F$ | $S_1$ | $S_2$ | $Z_2^D$ DM |
|--------|-----|-------|-------|-------------|
| (a)    | $(1, 1, 0)$ | $\eta(1, 2, 1)$ | $\phi(1, 1, 0)$ | Inert Singlet or Doublet |
| (b)    | $(1, 1, -2)$ | $\eta(1, 2, -1)$ | $\phi(1, 1, 2)$ | Inert Doublet |
| (c)    | $(1, 2, -1)$ | $\phi(1, 1, 0)$ | $\eta(1, 2, 1)$ | Inert Singlet or Doublet |
| (d)    | $(1, 2, -1)$ | $\Delta(1, 3, 0)$ | $\eta(1, 2, 1)$ | Inert Doublet or Triplet |
| (e)    | $(1, 2, 1)$ | $\phi(1, 1, 2)$ | $\eta(1, 2, -1)$ | Inert Doublet |
| (f)    | $(1, 2, 1)$ | $\Delta(1, 3, 2)$ | $\eta(1, 2, -1)$ | Inert Doublet or Triplet |
| (g)    | $(1, 2, -3)$ | $\Delta(1, 3, -2)$ | $\eta(1, 2, 3)$ | Excluded |
| (h)    | $(1, 3, 0)$ | $\eta(1, 2, 1)$ | $\Delta(1, 3, 0)$ | Inert Doublet or Triplet |
| (i)    | $(1, 3, -2)$ | $\eta(1, 2, 1)$ | $\Delta(1, 3, 2)$ | Inert Doublet or Triplet |
| (j)    | $(1, 3, 2)$ | $\eta(1, 2, 3)$ | $\Delta(1, 3, -2)$ | Excluded |

Table 3. Radiative neutrino mass for Dirac neutrinos with DM candidate.
which the correct relic density is maintained only via the four-point gauge interactions when $M_\phi > M_W$. Around this point, the spin-independent detection cross section drops dramatically, since the only tree level contribution from the Higgs boson vanishes. Meanwhile, for $M_\phi < M_W$, the relic density is determined by the Higgs portal. And current direct detection experiments requires that $M_\phi \approx M_h/2$ should be satisfied for light DM [71]. On the other hand if the dark matter is dominant by $\eta$ component, either $\eta_R^0$ or $\eta_R^0$, then a mass splitting $\Delta M = |M_{\eta_R^0} - M_{\eta_L^0}| > 100$ keV between $\eta_R^0$ and $\eta_R^0$ is required to escape the direct detection bound. In these two models, the required mass splitting can be obtained by choosing certain values of $\mu$ in the trilinear term $\mu^2 \eta^3 H$ and $\kappa$ in the quartic term $\kappa (\eta^1 H)^2$ [37].

For model (b) and (e), the only DM candidate comes from the inert scalar doublet $\eta \sim (1, 2, -1) [68]$, since the other inert scalar $\phi \sim (1, 1, 2)$ is a charged scalar singlet. It is noted that in both models, the neutral components $\eta_R^0$ or $\eta_R^0$ does not contribute to radiative neutrino mass. Considering the fact that small mixing angle $\theta$ between $\phi^\pm$ and $\eta^\pm$ are favored by neutrino mass, the DM phenomenology of $\eta$ will be quite similar as a standard inert doublet model. Under constraints from relic density, direct detection and indirect detection, there are two mass region allowed for $M_{\eta_R^0}/\eta_R^0$: one is the low mass region with 50 GeV $\lesssim M_{\eta_R^0}/\eta_R^0$ $\lesssim 70$ GeV, and the other is the high mass region with 500 GeV $\lesssim M_{\eta_R^0}/\eta_R^0$ [68]. For the light mass region, pair and associated production processes as $\eta^+ \eta^-$ and $\eta^+\eta_R^0/\eta_R^0$ will lead to multi-lepton plus missing transverse energy $E_T$ signatures at LHC, which has been extensively studied in Ref. [72]. While for the high mass region, although hard to be test at LHC, most parameter space of this region is in the reach of CTA experiment [73].

For model (d) and (h), they employ an inert doublet $\eta \sim (1, 2, 1)$ and a real inert scalar triplet $\Delta \sim (1, 3, 0)[74–76]$. Alternatively, the DM candidate could be either $\eta_R^0/\eta_R^0$ in the inert doublet $\eta$ or $\Delta^0$ in the inert triplet $\Delta$ [77]. If the mass of inert triplet $\Delta$ is much heavier than the inert doublet $\eta$, we again arrive at the well studied inert doublet model [68] as just discussed above. Here, we consider the opposite case where $\Delta^0$ is lighter than the inert doublet $\eta_R^0/\eta_R^0$, and serve as the DM candidate. Determined by the DM relic density, $M_{\Delta^0}$ is found to be around 2.5 TeV if (co)-annihilation is via pure gauge coupling, meanwhile the scalar interactions could push $M_{\Delta^0}$ up to about 20 TeV due to the Sommerfeld effect[74]. Since $\Delta^0$ does not interact with $Z$-boson, the DM-nucleon scattering process through the exchange of SM Higgs $h$ at tree level and gauge bosons at one-loop level. And the spin-independent DM-nucleon scattering cross section at one-loop level is calculated as [77]

$$\sigma_{SI} = \frac{\pi}{256 \pi^3} \frac{g_N^2 M_N^4}{M_W^2} \left[ R_\Delta^2 - \frac{1}{8} \left( \frac{1}{M_W^2} + \frac{1}{M_\Delta^2} \right) - \frac{16 \pi \lambda_{h\Delta^0}}{g_2^2 M_W^2 M_{\Delta^0}} \right]^2 \quad (3.1)$$

Here, $g_2$ is the $SU(2)_L$ gauge coupling, $f_N = 0.3$ is the nucleon matrix element, $M_N = 939$ MeV is the average nucleon mass, $R_\Delta = 3$ is the dimension of inert triplet $\Delta$, and $\lambda_{h\Delta^0}$ is the coupling between the DM $\Delta^0$ and SM Higgs $h$. For vanishing DM-Higgs coupling $\lambda_{h\Delta^0} = 0$, the spin-independent cross section $\sigma_{SI}$ is $9 \times 10^{-10}$ pb, which is lower than current LUX bound [65]. Remarkably, for certain DM-Higgs coupling, i.e.,

$$\lambda_{h\Delta^0} = \frac{R_\Delta^2 - \frac{1}{8} \frac{g_2^2 M_{\Delta^0}}{16 \pi M_W} \left( 1 + \frac{M_W^2}{M_\Delta^2} \right)} \approx 0.4 \quad (3.2)$$
the spin independent cross section could be suppressed heavily \cite{77}, therefore $\Delta^0$ can easily escape
direct detection even in the future.

In model (f) and (i), an inert scalar doublet $\eta \sim (1, 2, 1)$ and a complex inert scalar triplet
$\Delta \sim (1, 3, 2)$ are added\cite{78}. Naively, we expect that the DM candidate is $\eta_R^0$ or $\eta_I^0$ in these two
models, since $\Delta_R^0$ or $\Delta_I^0$ cannot play the role of DM candidate solely if $\eta$ does not exist. However,
in these two models, a mass splitting $\Delta M = |M_{\Delta_R^0} - M_{\Delta_I^0}|$ between $\Delta_R^0$ and $\Delta_I^0$
exists due to the mixing between $\eta$ and $\Delta$ \cite{79}. Specifically speaking, the $\kappa(\eta^1H)^2$ term will induce the mass
splitting between $\eta_R^0$ and $\eta_I^0$. Then the trilinear term $\mu H^T i \sigma_2 \Delta^I \eta$ will induce the mixing between $\Delta_R^0$ and $\eta_R^0$ for the CP-even scalars, and mixing between $\Delta_I^0$ and $\eta_I^0$ for the CP-odd scalars, resulting
a mass splitting between $\Delta_R^0$ and $\Delta_I^0$. For instance, with $M_{\eta_R^0} = 5$ TeV, $M_{\eta_I^0}^2 = M_{\eta_R^0}^2 - 2\kappa v^2$,
$\kappa = 0.5, \mu = 1$ TeV, and $M_\Delta = 2.8$ TeV, the mass splitting $\Delta M = |M_{\Delta_R^0} - M_{\Delta_I^0}| \approx 1$ MeV.
Therefore this mass splitting $\Delta M$ is larger than the DM kinetic energy $O(100)$ keV, the tree level
DM-nucleon scattering via $Z$-boson is expected kinematically forbidden \cite{21}. In this way, $\Delta_R^0$ (or $\Delta_I^0$ when $\kappa < 0$) can escape the direct detection bound, thus becomes a viable DM candidate. And $M_{\Delta_R^0}/\Delta_I^0 \approx 2.8$ TeV is preferred to acquire the correct DM relic density \cite{76}.

In model (g) and (j), the only scalar DM candidate is $\Delta \sim (1, 3, -2)$, since the other scalar
$\eta \sim (1, 2, 3)$ does not have neutral component. But the scalar triplet $\Delta$ has already excluded by
the direct detection experiments \cite{76}. Therefore, these two models could not provide viable DM
candidate.

Note that the discrete $Z_2^D$ symmetry could be an accidental symmetry of a broken $U(1)_D$
symmetry \cite{80}. Usually, a SM scalar singlet $\sigma$ is introduced to break $U(1)_D \to Z_2^D$ spontaneously.
Under this extended $U(1)_D$ symmetry, the inert fermion $F$ as well as inert scalars $S_{1,2}$ carry certain
$U(1)_D$ charges. While all other ingredients could be the same as the $Z_2^D$ case, the quartic term
$\kappa(\eta^1H)^2$ for $\eta \sim (1, 2, 1)$ or $\kappa(\eta^T H)^2$ for $\eta \sim (1, 2, -1)$ is forbidden by the $U(1)_D$ symmetry.
The absent of this quartic term will lead to degenerate masses of $\eta_R^0$ and $\eta_I^0$ in model (b) and (e),
therefore they will be excluded by direct detection in the case of $U(1)_D$ symmetry. Similar for
model (f) and (i), $\eta_R^0$ and $\eta_I^0$ are degenerate, thus $\Delta_R^0$ and $\Delta_I^0$ are also degenerate. In this way,
model (f) and (i) are also excluded. Meanwhile for model (a), (c) or (d), (h), mixing between
$\phi \sim (1, 1, 0)/\Delta \sim (1, 3, 0)$ and doublet $\eta$ can also lead to a mass splitting between $\eta_R^0$ and $\eta_I^0$.
And $\eta_R^0$ is the DM candidate when $\phi/\Delta$ is heavier than $\eta$.

Last but not least, we give some comments on models with quadruplets or quintuplets in
Table 4. Obviously, model (m), (n), (q), (r), (s), (v), (w) and (s) have already excluded by direct
detection, since the neutral components in these models have non-zero hyper-charge and no mass
splitting between the real and imaginary part of the neutral fields could be induced. Model (k) and (t)
are the only two models with viable fermion DM and quadruplets or quintuplets. For scalar DM,
it could be inert triplet or quintuplet with $Y = 0$ as in model (k), (o) and (t). Note that the quartic
term $\kappa(\chi^1H)^2$ for $\chi \sim (1, 4, 1)$ or $\kappa(\chi^T H)^2$ for $\chi \sim (1, 4, -1)$ is allowed by the $Z_2^D$ symmetry.
Analogy to the inert doublet, this quartic will split the neutral components $\chi_R^0$ and $\chi_I^0$, which makes $\chi_R^0$ or $\chi_I^0$ a viable DM candidate. A mass splitting between real and imaginary part of the neutral
fields in triplet/quintuplet, i.e., model (l), (p) and (u), is also possible due to the mixing between
triplet/quintuplet and quadruplet. In this way, the corresponding $Y \neq 0$ triplet/quintuplet can avoid
the tight direct detection bounds as well in the present of $\chi \sim (1, 4, \pm 1)$.
4 Phenomenology

The natural Dirac seesaw models introduce two additional scalars and a heavy intermediate fermion, which would lead to rich phenomenology. In this section we choose model (B) for tree level models and model (a) for one-loop level models as our benchmark mark models to illustrate the relative phenomenon. We briefly highlight some important aspects, although a detailed research on phenomenology of other specific models is quite necessary.

4.1 Flavor Constraints

![Figure 4](image-url)

**Figure 4.** BR($\mu \rightarrow e\gamma$) as a function of mass of heavy intermediate particle for tree level model (B) and one-loop level model (a). Here, we assume an universal Yukawa coupling $|y_{ij1}| = y$ and degenerate masses for the three generation of heavy fermion $F$ for simplicity. We have $S_1 = \phi$ in model (B) and $S_1 = \eta$ in model (a), respectively.

The existence of Yukawa coupling $y_1F_RL_LS_1$ will induce lepton flavor violation (LFV) processes. Here, we take the current most stringent bound BR($\mu \rightarrow e\gamma$) < $4.2 \times 10^{-13}$ [81] and future limit BR($\mu \rightarrow e\gamma$) < $6 \times 10^{-14}$ [82] to illustrate, and more discussion on other LFV processes can be found in Refs. [83–85]. The general analytical expression for BR($\mu \rightarrow e\gamma$) is given by

$$\text{BR}(\mu \rightarrow e\gamma) = \frac{3\alpha}{64\pi G_F^2} \left| \sum_{i=1}^{3} \frac{y_{ei}^i y_{i1}^i}{M_{S_1}^2} \left[ Q_{F_i} F_1 \left( \frac{M_{F_i}^2}{M_{S_1}^2} \right) + Q_{S_1} F_2 \left( \frac{M_{F_i}^2}{M_{S_1}^2} \right) \right] \right|^2,$$

where the loop functions $F_i(x)$ are [86]

$$F_1(x) = \frac{2 + 3x - 6x^2 + x^3 + 6x \ln x}{6(1 - x)^3},$$

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1 - x)^4}.$$  

Here, $Q_{F_i}$ and $Q_{S_1}$ denote the electric charge of charged components in $F_i$ and $S_1$, respectively. More specifically, we have $Q_{F_i} = 1, Q_{S_1} = 0$ in model (B) and $Q_{F_i} = 0, Q_{S_1} = 1$ in model
EDM is about two to three orders of magnitude lower than current limit with the above parameters.

For tree level models, the tight upper bound on branching ratios of LFV could be transformed into a lower bound on $\langle S_1 \rangle [86]$. From the expression of neutrino mass in Eq. 2.9, it is estimated that $y_1 \simeq m_\nu/2 M_F^{1/2}/\langle S_1 \rangle$ by assuming $y_1 \simeq y_2$ and $\langle S_1 \rangle \simeq \langle S_2 \rangle$. Plugging this estimation into Eq. 4.1, one easily derives

$$M_{S_1} \langle S_1 \rangle \gtrsim \left( \frac{3\alpha m_\nu^2 M_F^2}{64\pi G_F^2 \text{BR}(\mu \rightarrow e\gamma)} \sum_{i=1}^{3} F_i \left( \frac{M_F^2}{M_{S_1}^2} \right) \right) \frac{1}{2} \sqrt{M_F^2} \frac{1}{M_{S_1}^2},$$

(4.4)

where we have assume $m_\nu \sim 0.1$ eV and $\sum F_i(x) \sim 0.1$ in the numerical estimation. For electroweak scale intermediate fermion $M_F \sim 200$ GeV, current limits on $\text{BR}(\mu \rightarrow e\gamma)$ requires that $M_{S_1} \langle S_1 \rangle \gtrsim 600$ GeV $\cdot$ MeV. Thus, the VEVs of scalars $S_{1,2}$ are expected to be larger than $O(\text{MeV})$ when the mass of $S_{1,2}$ is around electroweak scale as well.

The contribution to the anomalous magnetic moment of $\mu$ can be obtained as a by-product of the above calculation of LFV

$$\Delta a_\mu = \sum_{i} \frac{|y_1^{\mu}|^2}{16\pi^2 M_{S_1}^2} \left[ Q_F F_1 \left( \frac{M_F^2}{M_{S_1}^2} \right) \right. + \left. Q_{S_1} F_2 \left( \frac{M_F^2}{M_{S_1}^2} \right) \right].$$

(4.5)

Under constraints from LFV, the predicted value of $\Delta a_\mu$ is $4 \times 10^{-14}$ for an universal Yukawa coupling $y_1 \sim 0.01$ and both $F$ and $S_1$ around electroweak scale, which is clearly too small to interpret the observed discrepancy $\Delta a_\mu = (2.39 \pm 0.79) \times 10^{-9}$ [87].

Another tight constraint comes from electric dipole moments (EDM) of electron, which requires $|d_e| < 8.7 \times 10^{-20}$ e-cm [88]. In all the current Dirac neutrino models, the only new interactions for lepton doublet $L_L$ is the Yukawa coupling $y_1 F_R L_L S_1$, which can not give large contributions to EDM at one-loop level [89, 90]. Actually, the contribution of above Yukawa coupling $y_1 F_R L_L S_1$ to electron EDM first appears at two-loop level (see Fig. 4 of Ref. [43]). Considering constraints from LFV, a naive estimation for the order of magnitude gives [43]

$$d_e \sim \frac{M_e \text{Im}(y_1^2 \lambda)}{(16\pi^2)^2 M_{S_1}^2} \sim 10^{-31} \text{ e-cm},$$

(4.6)

with $M_{S_1} \sim 200$ GeV, $y_1 \sim 0.01$, and $\text{Im}(\lambda) \sim 0.1$. Here, $\lambda$ is the coefficient of the quartic coupling $S_1^H S_1^H H^1 H$. Therefore, the contribution of the new Yukawa coupling $y_1 F_R L_L S_1$ to electron EDM is about two to three orders of magnitude lower than current limit with the above parameters.
4.2 Leptogenesis

Within Majorana seesaw models, the observed baryon asymmetry can be explained via conventional leptogenesis [91], where the lepton number violation plays an essential role. Obviously, no lepton asymmetry is generated in Dirac seesaw models because the lepton number is conserved. However, the leptogenesis can still be accomplished in Dirac neutrino models [92], due to the fact that the sphaleron processes do not have direct effect on right-handed fields. Therefore, if an equal but opposite amount of lepton asymmetry in the left- and right-handed sectors is created, the lepton asymmetry in the left-handed sector can be converted into a net baryon asymmetry via sphaleron processes, as long as the effective Dirac Yukawa couplings are small enough to prevent the lepton asymmetry from equilibration before the electroweak phase transition. Detailed studies on Dirac leptogenesis can be found in Ref. [93]. For the models we discussed, the required lepton asymmetry in left- and right-handed sectors arises from the decays of the heavy intermediate fermion $F_i$ into $L_L S_1$ and $\nu_R S_2$.

For the tree-level model (B), it is possible to generate the baryon asymmetry via resonant leptogenesis with nearly degenerate $F_i$ around TeV-scale [94]. For simplicity, we consider the canonical thermal leptogenesis in the one-loop model (a), where very heavy $F_i$ is needed. The heavy Dirac fermion $F_i$ has two decay modes: $F_i \to L_L \eta$ and $F_i \to \nu_R \phi$, and the corresponding decay widths at tree level are

$$\Gamma(F_i \to L_L \eta) = \Gamma(F_i^C \to L_L^C \eta^*) = \frac{M_{F_i}}{16\pi} |y_1^I y_1^R|^2,$$

(4.7)

$$\Gamma(F_i \to \nu_R \phi) = \Gamma(F_i^C \to \nu_R^C \phi^*) = \frac{M_{F_i}}{32\pi} |y_2^I y_2^R|^2,$$

(4.8)

in the limit of $M_{\eta, \phi} \ll M_{F_i}$. As shown in Fig. 5, the required lepton asymmetry in the left-handed sector arise at one-loop level and is calculated as [37]

$$\epsilon_{F_i} = \frac{\Gamma(F_i \to L_L \eta) - \Gamma(F_i^C \to L_L^C \eta^*)}{\Gamma_{F_i}}$$

$$= \frac{1}{8\pi} \frac{1}{(y_1^I y_1^R)_{ii} + \frac{1}{2} (y_2^I y_2^R)_{ii}} \sum_{j \neq i} \text{Im} \left[(y_1^I y_1^R)_{ij} (y_2^I y_2^R)_{ji}\right] \frac{M_{F_i} M_{F_j}}{M_{F_i}^2 - M_{F_j}^2},$$

(4.9)

where the total decay width is $\Gamma_{F_i} = [(y_1^I y_1^R)_{ii} + (y_2^I y_2^R)_{ii}/2] M_{F_i}/(16\pi)$. Provided that $M_{F_i} \ll M_{F_{2,3}}$, then the final left-handed sector lepton asymmetry is dominantly determined by the decays of $F_1$:

$$\epsilon_{F_1} \approx \frac{1}{8\pi} \frac{1}{(y_1^I y_1^R)_{11} + \frac{1}{2} (y_2^I y_2^R)_{11}} \sum_{j \neq 1} \frac{M_{F_1} M_{F_j}}{M_{F_1}^2 - M_{F_j}^2} \text{Im} \left[(y_1^I y_1^R)_{1j} (y_2^I y_2^R)_{j1}\right],$$

(4.10)
Figure 6. $Y_B$ as a function of $M_\eta$ for $\theta = 0.01, 0.001$. The pink band corresponds to $1\sigma$ range of the observed value in Ref. [95].

We further take $y_1 = y_2$ for illustration, then the lepton asymmetry $\epsilon_{F_1}$ can be simplified as

$$\epsilon_{F_1} \simeq -\frac{1}{24\pi} \frac{1}{(y_1^\dagger y_1)_{11}} \sum_{j\neq 1} \frac{M_{F_1}^2}{M_{F_j}^2} \text{Im}\left[(y_1^\dagger y_1)_{1j}\right].$$

(4.11)

With the assumption $y_1 = y_2$, an upper bound on $\epsilon_L$ can be deduced after considering the radiative neutrino masses in Eq. 2.11 [37]

$$|\epsilon_{F_1}| \lesssim \frac{4\pi M_{F_1} m_3 |\sin \delta|}{3 \sin 2\theta \left| M_2^2 \ln \frac{M_2^2}{M_{F_1}^2} - M_1^2 \ln \frac{M_1^2}{M_{F_1}^2}\right|},$$

(4.12)

with $m_3$ the heaviest neutrino mass and $\delta$ the Dirac phase. Setting $M_{F_1} = 10^7$ GeV, $m_3 = 0.1$ eV, $M_\phi = 60$ GeV, $\theta = 0.01$ and $\sin \delta = -1$, we obtain $\epsilon_{F_1} \simeq -2.7 \times 10^{-7}$.

Then after the sphaleron processes, the desired baryon asymmetry

$$Y_B = \frac{n_B - n_\bar{B}}{s} = -\frac{28}{79} \frac{n_L}{s} \simeq -\frac{28}{79} \frac{n_{F_1}^{\text{eq}}}{s} \bigg|_{T = M_{F_1}} \simeq -\frac{\epsilon_{F_1}}{15 g_*} \approx 1.7 \times 10^{-10}$$

(4.13)

with $g_* = 106.75$ is obtained to explain the observed baryon asymmetry [95]. In Fig. 6, we show the value of $Y_B$ as a function of $M_\eta$ for $\theta = 0.01, 0.001$. With other parameters fixed, the larger $M_\eta$ is, the smaller the $\theta$ is required to obtain the observed value of $Y_B$. Meanwhile, the decays of $F_1$ should be out of equilibrium, which requires that

$$\Gamma_{F_1} \lesssim H(T)\big|_{T = M_{F_1}}, \quad \text{with} \quad H(T) = \left(\frac{8\pi^2 g_*}{90}\right)^{\frac{1}{4}} \frac{T^2}{M_{\text{Pl}}},$$

(4.14)

where $M_{\text{Pl}} = 1.22 \times 10^{19}$ GeV. With the assumption $y_1 = y_2$, $M_{F_1} \sim 10^7$ GeV and Eq. 4.7,4.8, the above condition indicates that the Yukawa coupling $y_1$ should satisfy

$$(y_1^\dagger y_1)_{11} \lesssim \left(\frac{210 \pi^5 g_*}{5 \pi^2}\right)^{\frac{1}{4}} \frac{M_{F_1}}{M_{\text{Pl}}} \sim 10^{-10}.$$
4.3 Dark Matter

In the two benchmark model we studied, there is no DM candidate in the tree level model (B). Meanwhile, for the one-loop model (a), there are viable DM candidate φ or η_{R,I}. In this work, we consider the case of $M_\phi < M_\eta$ with small mixing angle $\theta \lesssim 0.01$, thus the DM candidate is dominantly determined by $\phi$. The relic density of $\phi$ is mostly determined by the quartic coupling $\lambda_\phi \phi^2 H^\dagger H$, and the analytic expression is given by [96]

$$\Omega_\phi h^2 = \frac{1.07 \times 10^9 \text{GeV}^{-1}}{\sqrt{g_* M_{\text{Pl}}} J(x_f)},$$

where the function $J(x_f)$ is

$$J(x_f) = \int_{x_f}^{\infty} \frac{\langle \sigma v_{\text{rel}} \rangle(x)}{x^2} dx.$$  

And the freeze-out parameter $x_f = M_\phi/T_f$ is acquired by numerically solving

$$x_f = \ln \left( \frac{0.038 M_{\text{Pl}} (\sigma v_{\text{rel}})(x_f)}{\sqrt{g_* x_f}} \right).$$

As pointed out by Ref. [97], the QCD corrections for quarks in the final state, as well as three- and four-body final states from virtual gauge boson decays are important for total DM annihilation cross section. Following Ref. [67], we rewrite the annihilation cross section into all SM particles except $h$ as

$$\sigma v_{\text{rel}} = \frac{8 \lambda_\phi^2 v^2}{\sqrt{s}} \frac{\Gamma_h(\sqrt{s})}{(s - M_h^2)^2 + M_h^4 \Gamma_h^2(M_h)},$$

where $v = 246$ GeV and the tabulated accurate Higgs boson width as a function of invariant mass $\Gamma_h(\sqrt{s})$ can be found in Ref. [98]. For light DM $M_\phi < M_h/2$, the decay width $\Gamma_h(M_h)$ in the
denominator should add the contribution of Higgs invisible decay $h \rightarrow \phi\phi$. Meanwhile, for heavy DM $M_\phi > M_h$, the extra contribution from $\phi\phi \rightarrow hh$ has also to be supplemented. But above $M_\phi > 150$ GeV, we should use the tree-level expressions in Appendix B, since the loop corrections are overestimated [67]. The thermal average cross section is then carried out via

$$\langle \sigma v_{\text{rel}} \rangle = \frac{x}{16M_\phi^4 K_2^2(x)} \int_{4M_\phi^2}^{\infty} \sqrt{s - 4M_\phi^2} s K_1 \left( \frac{x\sqrt{s}}{M_\phi} \right) \sigma_{v_{\text{rel}}} ds,$$

where $K_{1,2}(x)$ are modified Bessel functions of the second kind. In Fig. 7, we show the relic density $\Omega_\phi h^2$ as a function of $M_\phi$ for $\lambda_\phi = 0.001, 0.01$, and 0.1. The correct relic density can be obtained in the low-mass region $M_\phi < M_h/2$ and high-mass region $M_\phi > M_h/2$ for fixed value of $\lambda_\phi$.

Then we consider possible constraints from DM direct detection. The cross section for spin independent DM-nucleon is

$$\sigma_{\text{SI}} = \frac{\lambda_\phi^2 f_N^2 \mu^2 m_N^2}{\pi M_h^2 M_\phi^2},$$

where $m_N = (m_p + m_n)/2 = 939$ MeV is the averaged nucleon mass, $f_N = 0.3$ is the matrix element, and $\mu = m_N M_\phi/(m_N + M_\phi)$ is the DM-nucleon reduced mass. Provided $\phi$ accounting for 100% of DM, the predicted value of $\sigma_{\text{SI}}$ is presented in Fig. 8. In the current simple scenario we considered, it is clear that the only possible region to escape tight direct detection constraints is around the Higgs mass resonance, i.e., $M_\phi \approx M_h/2$. Thus, the choice of $M_\phi = 60$ GeV in this work is safe to avoid direct detection constraints.

There are also possible constraints from indirect detection. In Fig. 9, we depict the predictions for $\langle \sigma v_{\text{rel}} \rangle_{\gamma\gamma, b\bar{b}}$, as well as the observed limits from Fermi-LAT [100] and H.E.S.S. [101]. In the $\gamma\gamma$ final state, only a tiny mass region $M_h/2 \lesssim M_\phi$ is excluded. Meanwhile, in the $b\bar{b}$ final state, two mass region $M_\phi < 51$ GeV and $M_h/2 \lesssim M_\phi < 70$ GeV are excluded.

**Figure 8.** The spin-independent DM-nucleon cross section $\sigma_{\text{SI}}$ as a function of $M_\phi$. The green and blue lines correspond to LUX2016 [65] and XENON1T [99] limits.
Figure 9. The velocity-averaged annihilation cross section times relative velocity $\langle \sigma v_{\text{rel}} \rangle$ into $\gamma\gamma$ (left) and $b\bar{b}$ (right).

Figure 10. Branching ratio of Higgs invisible decay $\text{BR}_{\text{inv}}$ as a function of $M_{\phi}$.

Last but not least, the SM Higgs $h$ will decay into DM pair in the low mass region $M_{\phi} < M_h/2$, which will induce Higgs invisible decay at colliders. The corresponding decay width is

$$\Gamma(h \rightarrow \phi\phi) = \frac{\lambda_{\phi}^2 v^2}{8\pi M_h^2} \sqrt{M_h^2 - 4M_{\phi}^2},$$ (4.22)

and the invisible branching ratio is $\text{BR}_{\text{inv}} = \Gamma(h \rightarrow \phi\phi)/(\Gamma(h \rightarrow \phi\phi) + \Gamma_{\text{SM}})$, where $\Gamma_{\text{SM}} = 4.07$ MeV for $M_h = 125$ GeV [98]. In Fig. 10, we show $\text{BR}_{\text{inv}}$ as a function of $M_{\phi}$ in the low mass region. The 8 TeV LHC limit, i.e., $\text{BR}_{\text{inv}} \lesssim 0.25$, comes from the fitting results of Higgs visible decay [102]. And according to Ref. [103], the HL-LHC might probe $\text{BR}_{\text{inv}} \sim 0.02$ in the
weak boson fusion channel. The 8 TeV LHC has excluded $M_\phi \lesssim 53$ GeV, which is less stringent than the LUX2016 limit. Meanwhile, the HL-LHC will be capable of excluding $M_\phi < 56$ GeV, which will be less stringent than the XENON1T.

In summary, we show the allowed parameter space in the $\lambda_\phi - M_\phi$ plane in Fig. 11, with the constraints from relic density, direct detection, indirect detection and Higgs invisible decay. Apparently, the only allowed mass region is a narrow one being close to $M_\phi \lesssim M_h/2$.

4.4 LHC Signature

Finally we briefly discuss possible LHC signatures. The newly introduced particles in $F, S_{1,2}$ can be pair/associated produced via Drell-Yan processes as long as they have non-zero gauge couplings. Then decays of new particles in $F, S_{1,2}$ will usually lead to multi-lepton signatures at LHC [104]. Since production cross section as well as the decay properties of new particles are model dependent, we take model (B) and model (a) for illustration here. Detailed study and simulation on specific models at LHC are highly encouraged to perform. First, for tree level model (B), a fermion doublet $F \equiv \Sigma = (\Sigma^0, \Sigma^-)^T \sim (1, 2, 1)$ and a scalar singlet $\phi \sim (1, 1, 0)$ are introduced. Hence, in model (B), only the fermion doublet $\Sigma$ can be largely produced at LHC via Drell-Yan processes $pp \to \Sigma^+\Sigma^-, \Sigma^0\Sigma^0, \Sigma^\pm\Sigma^0$.

The decay channels of the fermion doublet $F$ are, $\Sigma^0 \to \ell^- W^+, \nu Z, \nu h$ and $\Sigma^- \to \ell^- Z, \ell^- h, \nu W^-$. And if $M_\phi < M_\Sigma$, new decay channels as $\Sigma^0 \to \nu \phi$ and $\Sigma^- \to \ell^- \phi$ with $\phi \to W^+ W^-, ZZ, hh$ are also possible. Note that in Dirac neutrino mass models, there is no lepton number violation decays of $\Sigma^0$. Thus with $W^\pm, Z$ decaying leptonically, multilepton signatures can be generated. For $M_\Sigma < M_\phi$, ATLAS has performed an analysis on the signatures with three or more leptons based on $\Sigma^\pm \to \ell^\pm Z \to \ell^\pm \ell^+ \ell^-$, and $M_\Sigma$ in the range 114 – 176 GeV has been excluded [105]. The cross section of the inclusive trilepton signature $2\ell^\pm \ell^\mp + X$ is shown in left panel of Fig. 12. For $M_\Sigma > M_\phi$, the new decay channel $\Sigma^\pm \to \ell^\pm \phi \to \ell^\pm ZZ \to 3\ell^\pm 2\ell^\mp$ will lead to signatures with
Figure 12. Cross section of trilepton signature $2\ell^\pm\ell^\mp + X$ (left) and five-lepton signature $3\ell^\pm + 2\ell^\mp + X$ (right) at 14 TeV LHC.

five or more leptons. And the cross section of this inclusive five-lepton signature is shown in right panel of Fig. 12.

As for model (a), since both $F \sim (1, 1, 0)$ and $\phi \sim (1, 1, 0)$ are pure singlet, only the inert doublet $\eta = (\eta^+, (\eta^0_R + \eta^0_I)/\sqrt{2})^T \sim (1, 2, 1)$ can be pair produced at LHC

$$pp \rightarrow \eta^+\eta^-, \eta_R^0\eta_I^0, \eta^\pm_R\eta^\pm_I.$$  \hspace{1cm} (4.24)

Because there are always a pair of DM in the final states, the signatures will thus contains missing transverse energy $E_T$. Here, we consider multi-lepton plus $E_T$ signatures. For inert doublet DM $\eta_R^0/\eta_I^0$, multi-lepton plus $E_T$ signatures at LHC have been extensively studied in Ref. [72], thus we concentrate on $F$ or $\phi$ DM. For fermion singlet DM, the promising signature is

$$pp \rightarrow \eta^+\eta^- \rightarrow \ell^+ F + \ell^- F,$$  \hspace{1cm} (4.25)

which leads to $\ell^+\ell^- + E_T$ signature at LHC. Cross section of this dilepton signature is presented in left panel of Fig. 13. Searches for such dilepton signature has been performed by ATLAS [106] and CMS [107]. Assuming $\eta^\pm$ exclusive decays into $e^\pm F $ or $\mu^\pm F $, ATLAS has excluded the region with $M_{\eta^\pm} \lesssim 300$ GeV and $M_F \lesssim 150$ GeV [106], meanwhile the CMS limit is less stringent [107]. On the other hand, for scalar singlet DM, the promising signature is

$$pp \rightarrow \eta^+_R\eta^-_I \rightarrow W^\pm\phi + Z\phi \rightarrow 2\ell^\pm\ell^\mp + E_T.$$  \hspace{1cm} (4.26)

Cross section of this dilepton signature is presented in right panel of Fig. 13. Searches for such trilepton signature has also been performed by ATLAS [108] and CMS [107]. The more stringent limit is also set by ATLAS, with $M_{\eta^\pm} \lesssim 350$ GeV and $M_{\phi} \lesssim 120$ GeV being excluded [108]. Note that this exclusion limit is acquired in simplified SUSY model with chargino-neutralino associated production. The exclusion limit is expected weaker in model (a), mainly because the cross section of $\eta^+_R\eta^-_I$ is much smaller than the cross section of chargino-neutralino with same masses.
Before ending this section, we give one benchmark point for each of the two benchmark models by considering the constraints from the phenomenologies we just discussed above. First, for the tree level benchmark model (B), the benchmark point is

\[ y_1 = y_2 = 0.01, \quad M_F = M_\phi = 1 \text{ TeV}, \quad \langle \phi \rangle = 10 \text{ keV}. \]  

Then, for the one-loop level benchmark model (a), the benchmark point is

\[ y_1^{ij} = y_2^{ij} = 10^{-6}, \quad y_1^{ij} = y_2^{ij} = 10^{-2}, \quad \theta = 0.01, \]  

\[ M_\phi = 60 \text{ GeV}, \quad M_\eta = 200 \text{ GeV}, \quad M_F = 10^7 \text{ GeV}. \]  

5 Conclusion

With at most two additional scalars and a heavy intermediate fermion, we perform a systematical study on pathways that can naturally generate tiny Dirac neutrino masses at tree- and one-loop level. In both cases, we concentrate on the SU(2)_L scalar multiplet no larger than quintuplet, and derive the complete sets of viable models.

To realize tree level models in Fig. 1, the conservation of lepton number symmetry is assumed to forbid the unwanted Majorana mass term \( (m_N/2)\nu^\dagger_R \nu_R \). Then an extra \( Z_2 \) symmetry is employed to forbid direct \( \bar{\nu}_L \nu_R \bar{\phi} \) coupling. The breaking of this \( Z_2 \) symmetry will induce an effective small Dirac neutrino mass term \( m_D \bar{\nu}_L \nu_R \). For tree level models, a finite set of model is found by requiring the natural small VEVs of new scalars. If one of the added scalars is actually the SM Higgs fields \( H \) itself, there are four types of realizations, which correspond to \( d = 6 \) effective low-energy operators. On the other hand, if two new scalars are introduced, then they are usually triplets/quadruplets/qintuplets for minimal models, corresponding to \( d = 8 \) or \( d = 10 \) effective low-energy operators.
To realize purely radiative models in Fig. 2, we further impose $Z_2^D$ symmetry, under which $S_{1,2}$ and $F$ carry $Z_2^D$-odd charge while all SM fields transform trivially. The lightest particle within the inert fields $S_{1,2}$ and $F$ is stable, and thus becomes a dark matter candidate if it has no electric charge. We exhaust the list of viable models, and briefly discuss the possible DM candidate. Note that current direct detection limits have already excluded some models. Clearly, for fermion DM, it could be $F \sim (1,1,0), (1,3,0), (1,5,0)$, while for scalar DM, we have more option, e.g., the inert doublet $\eta \sim (1,2,1)$. The important fact is that if the DM candidate has non-zero hyper charge, a mixing between $S_1$ and $S_2$ and/or a quartic term as $(S_1 H)^2$ is required to induce a large enough mass splitting between the real and imaginary part of the neutral component.

As for the phenomenological issues, the Yukawa coupling $y_1 F_L L S_1$ will induce lepton flavor violation (LFV) processes. For tree level models, current limits on BR($\mu \rightarrow e\gamma$) denotes that $M_{S_1} \langle S_1 \rangle \gtrsim 600 \text{ GeV} \cdot \text{ MeV}$, with $y_1 = y_2$ and $\langle S_1 \rangle = \langle S_2 \rangle$ being assumed. Meanwhile for radiative models, tight constraints from LFV requires the Yukawa coupling $|y_1| \lesssim 0.01$ when both $F$ and $S_1$ are located around electroweak scale. On the other hand, if $F$ is heavy enough, i.e., $M_F \sim 10^7 \text{ GeV}$, which is also possible in radiative models, the leptogenesis is also possible. For the scalar singlet DM $\phi$ in model (a), we perform a brief discussion on relic density, direct detection, indirect detection and Higgs invisible decay. And we find the the only allowed region is $M_\phi \lesssim M_h/2$. To illustrate LHC signatures, we take model (B) and model (a) as an example. For tree level model (B), the promising signatures are trilepton signature $\Sigma^\pm \rightarrow \ell^\pm Z \rightarrow \ell^\pm \ell^+\ell^-$ when $M_\Sigma < M_\phi$. While for $M_\Sigma > M_\phi$, the five-lepton signature $\Sigma^\pm \rightarrow \ell^\pm \phi \rightarrow \ell^\pm ZZ \rightarrow 3\ell^\pm 2\ell^\mp$ might be promising. In case of loop level model (a), the promising signature is $\ell^+\ell^- + E_T$ if $F$ is DM, and $2\ell^\pm \ell^\mp + E_T$ if $\phi$ is DM.

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Appendix A: One-loop Neutrino Mass models with Larger Multiplets

| Models | $F$     | $S_1$    | $S_2$    | $Z_2^D$ DM                  |
|--------|---------|----------|----------|----------------------------|
| (k)    | (1, 3, 0) | (1, 4, 1) | (1, 3, 0) | Inert Triplet or Quadruplet |
| (l)    | (1, 3, −2) | (1, 4, −1) | (1, 3, 2) | Inert Triplet or Quadruplet |
| (m)    | (1, 3, 2)  | (1, 4, 3)  | (1, 3, −2) | Excluded                   |
| (n)    | (1, 3, −4) | (1, 4, −3) | (1, 3, 4)  | Excluded                   |
| (o)    | (1, 4, −1) | (1, 4 ± 1, 0) | (1, 4, 1) | Inert Triplet/Quintuplet or Quadruplet |
| (p)    | (1, 4, 1)  | (1, 4 ± 1, 2) | (1, 4, −1) | Inert Triplet/Quintuplet or Quadruplet |
| (q)    | (1, 4, −3) | (1, 4 ± 1, −2) | (1, 4, 3) | Excluded                   |
| (r)    | (1, 4, 3)  | (1, 4 ± 1, 4) | (1, 4, −3) | Excluded                   |
| (s)    | (1, 4, −5) | (1, 5, −4) | (1, 4, 5) | Excluded                   |
| (t)    | (1, 5, 0)  | (1, 4, 1)  | (1, 5, 0) | Inert Quadruplet or Quintuplet |
| (u)    | (1, 5, −2) | (1, 4, −1) | (1, 5, 2) | Inert Quadruplet or Quintuplet |
| (v)    | (1, 5, 2)  | (1, 4, 3)  | (1, 5, −2) | Excluded                   |
| (w)    | (1, 5, −4) | (1, 4, −3) | (1, 5, 4) | Excluded                   |
| (x)    | (1, 5, 4)  | (1, 4, 5)  | (1, 5, −4) | Excluded                   |

Table 4. Radiative neutrino mass for Dirac neutrinos with quadruplet or/and quintuplet and DM candidate.

Appendix B: Dark Matter Annihilation Cross Sections

Annihilation into SM fermions:

$$\sigma(\phi \phi \to f \bar{f})_{\text{rel}} = \frac{\lambda_\phi^2 M_\phi^2 N_c f (1 - 4 M_W^2 / s)^{3/2}}{2 \pi s [(s - M_h^2)^2 + M_h^2 \Gamma_h^2]}, \quad (5.1)$$

where $N_c f$ is the color factor for fermion $f$.

Annihilation into $W^+W^-$:

$$\sigma(\phi \phi \to W^+W^-)_{\text{rel}} = \frac{\lambda_\phi^2 (s^2 - 4 M_W^2 s + 12 M_W^4) \sqrt{1 - 4 M_W^2 / s}}{2 \pi s [(s - M_h^2)^2 + M_h^2 \Gamma_h^2]}, \quad (5.2)$$

Annihilation into $ZZ$:

$$\sigma(\phi \phi \to ZZ)_{\text{rel}} = \frac{\lambda_\phi^2 (s^2 - 4 M_Z^2 s + 12 M_Z^4) \sqrt{1 - 4 M_Z^2 / s}}{4 \pi s [(s - M_h^2)^2 + M_h^2 \Gamma_h^2]}, \quad (5.3)$$

Annihilation into $hh$ in the $s \to 4M_\phi^2$ limit:

$$\sigma(\phi \phi \to hh)_{\text{rel}} = \frac{\lambda_\phi^2 \left[ M_h^4 - 4 M_\phi^2 + 2 \lambda_\phi^2 (4 M_\phi^2 - M_h^2) \right]^2}{4 \pi M_\phi^2 \left( M_h^4 - 6 M_h^2 M_\phi^2 + 8 M_\phi^4 \right)^2} \sqrt{1 - \frac{M_h^2}{M_\phi^2}}, \quad (5.4)$$
Annihilation into $\gamma\gamma$:

$$\sigma(\phi\phi \rightarrow \gamma\gamma) v_{rel} = \frac{16\lambda_\phi^2 v^2 \Gamma_{\gamma\gamma}(s)}{\sqrt{s} (s - M_h^2)^2 + M_h^2 \Gamma_h^2}, \quad (5.5)$$

where the width $\Gamma_{\gamma\gamma}(s)$ is given by:

$$\Gamma_{\gamma\gamma}(s) = \frac{\alpha^2 s^{3/2}}{512\pi^3 v^2} \left| \sum_f N_i^f Q_f^2 A_{1/2}(\tau_f) + A_1(\tau_W) \right|^2, \quad (5.6)$$

with $\tau_i = s/(4M_i^2)$ and the form factor:

$$A_{1/2}(\tau) = 2[\tau + (\tau - 1)f(\tau)]\tau^{-2}, \quad (5.7)$$

$$A_1(\tau) = -(2\tau^2 + 3\tau(2\tau - 1)f(\tau))\tau^{-2}, \quad (5.8)$$

where $f(\tau)$ is

$$f(\tau) = \begin{cases} \arcsin^2 \sqrt{\tau} & \tau \leq 1 \\ -\frac{1}{4} \left[ \log \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right]^2 & \tau > 1 \end{cases} \quad (5.9)$$

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