Enabling Sparse Winograd Convolution by Native Pruning

Sheng Li * 1 † Jongsoo Park * 2 † Ping Tak Peter Tang * 3

Abstract

Sparse methods and the use of Winograd convolutions are two orthogonal approaches, each of which significantly accelerates convolution computations in modern CNNs. Sparse Winograd merges these two and thus has the potential to offer a combined performance benefit. Nevertheless, training convolution layers so that the resulting Winograd kernels are sparse has not hitherto been very successful. By introducing a Winograd layer in place of a standard convolution layer, we can learn and prune Winograd coefficients “natively” and obtain sparsity level beyond 90% with only 0.1% accuracy loss with AlexNet on ImageNet dataset. Furthermore, we present a sparse Winograd convolution algorithm and implementation that exploits the sparsity, achieving up to 31.7 effective TFLOP/s in 32-bit precision on a latest Intel Xeon CPU, which corresponds to a 5.4× speedup over a state-of-the-art dense convolution implementation.

1. Introduction

Convolution neural networks (CNN) have achieved undisputed success in many practical applications. These deep neural networks typically contain multiple layers, many (though not all) of which perform the namesake computation of convolution. A convolution layer is an architecture whose connection between an input and output tensor is via a number of convolution kernels, and the basic arithmetic operations are that of multiply-accumulate. Because over 90% of the computation during inference and training of recent CNN designs is in convolutions (Krizhevsky et al., 2012; Szegedy et al., 2015), different strategies have been devised to speed up this core operation. Sparse methods

is one such strategy. Here, many of the convolution kernel coefficients are made zero by some pruning or compression techniques (LeCun et al., 1989; Lebedev & Lempitsky, 2016; Liu et al., 2015). Sparsity exploiting convolution implementations are then employed in the actual inference process. (Park et al., 2017) reported a three-fold speed up on the convolution layers when an optimized sparse convolution implementation is applied on well pruned kernels.

Orthogonal to sparse methods, transform methods such as FFT (Mathieu et al., 2013; Vasilache et al., 2015) or Winograd transformation (Winograd, 1980; Lavin & Gray, 2016) have proved to be successful as well. For the typical small convolution sizes (e.g., 3×3) that arise in CNNs, the Winograd-kernel approach is more effective than FFT and has demonstrated more than twofold speed up over well-implemented spatial convolution approaches.

These recent advances beg the question of why not apply sparse methods on Winograd transform. The potential gain of this combination is obvious. To realize this potential, however, one must (1) be able to significantly prune away Winograd coefficients of a CNN with minimal impact to the CNN’s accuracy, and (2) develop computation implementations that can exploit the pruned Winograd parameters well enough that result in meaningful inference speedups. It turns out that pruning Winograd parameters is challenging for reasons we will explain shortly. The Winograd sparsity achieved reported so far is only moderate, which may explain why there is no published effort on optimized sparse Winograd convolution implementations to boost inference speed. This paper reports advances in both fronts, illustrated by pruning Winograd kernels to more than 90% sparsity while containing accuracy loss to within 0.1%; as well as an actual up to 5.4× and 2.1× speedups of our sparse Winograd convolution compared to dense direct and dense Winograd convolution, respectively.

Pruning the original convolution coefficients (the “spatial” domain) does not in general result in sparse Winograd kernels. More fundamentally, the linear transform that maps spatial to Winograd parameters is non-invertible as there are more Winograd than spatial coefficients. Thus any training and pruning method that needs to make use of both sets of parameters in conjunction will cause inconsistency, which in turn leads to major accuracy loss when achieving
Enabling Sparse Winograd Convolution by Native Pruning

Convolution Layers using Winograd Kernel

Compute $G W^T \rightarrow W F$

Winograd Layer

pointwise product

matrix vector product $B^T$

matrix vector product $A^T$

Figure 1. Schematic illustration for 1-D convolution and 1-D Winograd Convolution. The red schematic on the left illustrates a traditional convolution layer. This example takes the inputs $I_j$, $j = 1, 2, 3$, and kernel parameters $w_{T,j}$, $j = 1, 2$, and produces the output $O_j$, $j = 1, 2$. The blue schematic on the right illustrates a corresponding Winograd layer. Note that it takes three independent kernel inputs $w_{F,j}$, $j = 1, 2, 3$. Given any inputs $I_j$, the Winograd layer produces two outputs $O_j$. The standard Winograd-as-a-kernel implements a convolution layer using the Winograd computation method, and is illustrated by the green dotted line schematic. It takes the spatial convolution parameters (two coefficients) $w_T$ and the input $I$. Internally, it applies the (non-invertible) transform to $w_T$ that yields $w_F = G w_T$, which is then used in the Winograd computation. Consequently, the set of $w_F$ used here is at most two dimensional, instead of the Winograd layer that can exploit a three-dimensional input space for $w_F$.

acceptable sparsity.

We tackle this challenge and show that pruning Winograd parameters becomes successful when we replace the convolution layer in question by a Winograd layer, eliminating the need to use both the spatial and Winograd parameters in conjunction. Our effective pruning method then paves the road to our highly optimized sparse Winograd convolution to materialize the sparsity for high speed inference. Figure 1 illustrates the architecture of a Winograd layer using a simple 1D convolution of a 2-length spatial kernel with 4-length input. The corresponding Winograd kernel is 3-length. Replacing a convolution layer by a Winograd layer has several advantages. First, from a conceptual point of view, this new layer architecture reflects directly the relationship between the key parameters of the computation (which are the Winograd parameters) and the overall neural network. Second, as alluded to earlier, we have more Winograd than spatial parameters. Thus a Winograd layer in fact has a larger capacity as we are no longer restricted to use only those Winograd parameters that actually correspond to a set of spatial convolution parameters. Last but most important, steering of the training process such as pruning becomes possible and straightforward. We emphasize that this is not the case if one employs the spatial convolution layer: the non-invertible mapping between convolution and Winograd parameters is a major obstacle. While this obstacle may be overcome via approximation for small networks (LeNet) and datasets (MNIST) as reported in (Liu & Tumakulu, 2016), our experiments (Section 6.1) show that approximation does not work for larger networks (AlexNet) and datasets (ImageNet).

The main contributions of this paper are as follows:

1. We define and advocate the use of the Winograd layer as an architecture. Details of the forward and backward passes of this layer are derived and training methods aim to prune are devised (Sections 3-4).

2. We design and implement a highly optimized sparse Winograd convolution computation for fast inference by exploiting high sparsity obtained by our pruning methods (Section 5).

3. We demonstrate the effectiveness of training and pruning with the Winograd-layer architecture (Section 6) and fast inference with our sparse Winograd convolution. Particularly, we prune AlexNet (Krizhevsky et al., 2012) to eliminate its Winograd parameters by more than 90%, while maintaining its original accuracy. This leads to up to $2.1 \times$ speedups over an ideal dense Winograd convolution, one of the fastest convolution algorithms to date (Lavin & Gray, 2016).

2. Related Work

That convolution can be computed with less arithmetic complexity through the use of transformation is well understood. (Mathieu et al., 2013; Vasilache et al., 2015) appear to be the first ones that detailed the use of FFT as a compute kernel in Deep Learning. This kernel executes the “spatial-domain” convolution by element-wise multiplication in the “frequency-domain”. More recently, (Lavin & Gray, 2016) shows convincingly that Winograd transforms outperform FFTs in the common convolution use cases in Deep Learning. These works illustrate the use of transform methods (FFT or Winograd) as computation kernels to speed up a normal inference or training process.

Recently, there have been a few research attempts to further reduce compute requirements and memory footprint of
Winograd convolution by pruning. However, they have had limited success in pruning and not shown actual speedups in inference. (Liu & Turakhia, 2016) attempts to prune parameters in the transformed domain. For Winograd parameters, the overall network’s architecture is that of the original CNN, but the forward pass is performed using the Winograd kernel with some of the parameters set to zero. A backward pass is performed that updates the original “spatial-domain” parameters. We believe there is an inconsistency in this model caused by the fact that mapping between spatial-domain and Winograd convolution parameters is non-invertible. In general, there is no spatial-domain convolution parameters that correspond to the modified (masked off) Winograd parameters that are used to compute all the forward-pass intermediate quantities, while the backward pass computes the gradient using these intermediate quantities in conjunction with the spatial-convolution-kernel parameters. While they achieve reasonable results (Liu & Turakhia, 2016) with LeNet (LeCun et al., 1998a) on the MNIST dataset (LeCun et al., 1998b), our results on larger networks and datasets such as AlexNet (Krizhevsky et al., 2012) on ImageNet (Deng et al., 2009) as shown in Section 6.1 demonstrate that significant accuracy loss and/or low sparsity are inevitable for such approaches as in (Liu & Turakhia, 2016). Indeed, direct pruning of the parameters in our proposed Winograd layers is how we finally overcome this accuracy problem.

(Liu et al., 2017), a concurrent work to ours, similarly addresses the non-invertible issue by directly pruning in Winograd domain. Interestingly, they also move the ReLU operation into Winograd domain to obtain sparsity in activations as well. However, their sparsity is lower than ours (75% vs. 90+%) and is evaluated with smaller dataset (CIFAR-10 vs. ImageNet). Moreover, their work does not show actual inference speedups from sparsity in Winograd domain. Our work provides a highly optimized sparse Winograd convolution design and implementation for fast inference.

In a conceptual sense, (Rippel et al., 2015) relates closely to our current work. The authors advocate representing the convolution kernels in frequency domain and detailed the gradient calculation with respect to the frequency parameters. The network architecture, however, remains in the original form as outlined in Section 4 of the paper: When convolution is to be computed, the frequency parameters must first be transformed back to spatial domain in which a regular convolution is performed.

Since pruning in spatial domain does not provide a high enough sparsity in Winograd parameters to benefit from general sparse representations such as compressed sparse row (CSR), a hardware feature for zero-skipping has been proposed to take advantage of the low sparsity in Winograd parameters (Park et al., 2016). Our paper shows that, by directly pruning in Winograd domain, we can obtain high enough sparsity to speedup inference without such specialized hardware features.

3. Winograd Layers – Definition and Backpropagation

Consider a typical convolution layer where the input tensors with \( C \) channels of features maps each of dimension \( H_i \times W_i \) are transformed into \( K \) output channels via a simple unit-stride, unit-dilation linear convolution with kernels of size \( r \times s \):

\[
\mathcal{I} \in \mathbb{R}^{C \times H_i \times W_i} \rightarrow \mathcal{O} \in \mathbb{R}^{K \times H_o \times W_o}
\]

via

\[
\mathcal{O}(k, :, :) = \sum_{c=1}^{C} W(k, c, :) \ast \mathcal{I}(c, :, :)
\]

(1)

where \( H_o = H_i - r + 1 \) and \( W_o = W_i - s + 1 \) and \( \ast \) stands for 2D linear convolution.

The computation of Equation 1 can be performed using the Winograd transform which has a lower arithmetic complexity than Equation 1 suggests. The details of this computation we present now are crucial to our definition of Winograd layers and their training. A convolution \( W(k, c, :) \ast \mathcal{I}(c, :, :) \) of size \( H_i \times W_i \) can be broken down into many convolutions each involving smaller tiles of \( \mathcal{I} \). We illustrate this “overlapping” method by the following one-dimensional example, using self-explanatory Matlab-like array index notation.

\[
W(0 : 2) \ast I(0 : 5) \rightarrow O(0 : 3)
\]

by

\[
W(0 : 2) \ast \begin{bmatrix} I(0 : 3) \\ I(2 : 5) \end{bmatrix} \rightarrow \begin{bmatrix} O(0 : 1) \\ O(2 : 3) \end{bmatrix}
\]

Note that \( I \) is broken up into two tiles with some duplicating elements while \( O \) is partitioned (without duplication) into two tiles. More generally, given convolution kernels of size \( r \times s \) and (small) sizes \( m, n \) that divide\(^1\) \( H_o \) and \( W_o \), respectively, we reshape the input and output tensors \( \mathcal{I}, \mathcal{O} \) into \( \tilde{\mathcal{I}}, \tilde{\mathcal{O}} \).

\[
\begin{align*}
\tilde{\mathcal{I}} \in \mathbb{R}^{C \times H_i \times W_i} & \rightarrow \tilde{\mathcal{I}} \in \mathbb{R}^{C \times T \times (m+r-1) \times (n+s-1)} \\
and \ \tilde{\mathcal{O}} \in \mathbb{R}^{K \times H_o \times W_o} & \rightarrow \tilde{\mathcal{O}} \in \mathbb{R}^{K \times T \times m \times n}.
\end{align*}
\]

The value \( T \) is the number of resulting tiles of the reshaping, \( T = H_o W_o / (mn) \). The input tile size is \((m+r-1) \times (n+s-1)\), while the output tile size is \( m \times n \). We express the reshaping by two index mapping functions \( \phi \) and \( \psi \)

\[
\tilde{\mathcal{I}}(c, t, i, j) = \mathcal{I}(c, \phi(t, i, j)), \quad \tilde{\mathcal{O}}(k, i, j) = \mathcal{O}(k, \psi(i, j))
\]

\(^1\) This assumption simplifies the presentation and can be easily eliminated by for example zero padding.
where \( \phi \) is many-to-one and maps a 3-tuple to a 2-tuple while \( \psi \) is invertible and maps a 2-tuple to a 3-tuple. Using the overlapped form, we have

\[
\hat{O}(k, t, \cdot, \cdot) = \sum_{c=1}^{C} W(k, c, \cdot, \cdot) \ast \hat{I}(c, t, \cdot, \cdot). \tag{2}
\]

Computing Equation [2] with straightforward convolution takes \( mnrs \) multiplications in each of the summands. In contrast, Winograd’s method can possibly need only as few as \((m + r - 1)(n + s - 1)\) multiplications via:

\[
\hat{O}(k, t, \cdot, \cdot) = A_1^T \left[ \sum_{c=1}^{C} \left( G_1 W(k, c, \cdot, \cdot) G_2^T \right) \odot \left( B_1^T \hat{I}(c, t, \cdot, \cdot) B_2 \right) \right] A_2.
\tag{3}
\]

The six matrices \( A_j, G_j, B_j, j = 1, 2 \), (of consistent dimensions) are independent of \( W \) and \( \hat{I} \), and \( \odot \) is element-wise multiplication. In many instances the \( A_j, B_j, C_j \) matrices are so simple that applying them requires no multiplications. For example when \( r = s = 3 \) and \( m = n = 2 \), \( A_1 = A_2, B_1 = B_2 \) and \( G_1 = G_2 \) are simple matrices with 0, \( \pm 1 \), and \( \pm 1/2 \) are entries (see [Lavin & Gray 2016] for example).

Motivated by this, we define a Winograd layer to be a topology specified by a tensor \( W_F \in \mathbb{R}^{K \times C \times (m+r-1) \times (n+s-1)} \) that computes \( \hat{O} \) from \( I \) via

\[
\hat{I}(c, t, i, j) = I(c, \phi(t, i, j)),
\]

\[
\hat{O}(k, t, \cdot, \cdot) = A_1^T \left[ \sum_{c=1}^{C} \left( W_F(k, c, \cdot, \cdot) \odot \left( B_1^T \hat{I}(c, t, \cdot, \cdot) B_2 \right) \right) \right] A_2,
\]

\[
\hat{O}(k, i, j) = \hat{O}(k, \psi(i, j)). \tag{4}
\]

Since \( m, n > 1 \) in practice, \((m + r - 1)(n + s - 1) > rs \) and a Winograd layer (Equation [4]) has a higher capacity than a corresponding convolution layer (Equation [1]).

To incorporate a Winograd layer within a standard CNN framework (e.g., Caffe) so as to allow training and inference, it suffices to be able to compute the forward and backward passes. The forward pass is straightforward as it simply follows Equation [4] for which we note that [Lavin & Gray, 2016] details an optimized implementation. For the backward pass, we need to compute the partial derivatives of the scalar loss function \( L \) w.r.t. each of the variables \( I(c, i, j) \) and \( W_F(k, c, i, j) \) in terms of the known partial derivatives of \( L \) w.r.t. \( \hat{O}(k, i, j) \), or \( \partial L/\partial \hat{O} \) in short. We present the derivations here as they pertain to the Winograd layers and are thus unavailable elsewhere.\(^2\)

First, we use this key form of chain rule: Suppose the partial derivatives of a scalar function \( L \) w.r.t. an array of variables \( y_{ij} \), \( y \in \mathbb{R}^{u \times v} \), are known. Moreover, the variables \( Y \) are in fact dependent variables of an array of variables \( x_{ij} \), \( X \in \mathbb{R}^{r \times s} \) via \( Y = U^T X V \) where \( U, V \) are constant matrices of commensurable dimensions. The partial derivatives of \( L \) w.r.t. \( X \) are then given by \( \partial L/\partial X = U \partial L/\partial Y ^T V^T \).

Denote the intermediate variables in Equation [4] by \( \hat{I}_F \) and \( Z: \hat{I}_F(c, t, \cdot, \cdot) = B_1^T \hat{I}(c, t, \cdot, \cdot) B_2 \) and \( Z(k, t, \cdot, \cdot) = \sum_{c=1}^{C} W_F(k, c, \cdot, \cdot) \odot \hat{I}_F(c, t, \cdot, \cdot) \) for all applicable indices \( c, k, t \). Note in particular that \( Z(\cdot, i, j) = W_F(\cdot, i, j, \cdot) \hat{I}_F(\cdot, i, j) \), that is the 2-dimensional slice of \( Z \) with any fixed \( i, j \) is a matrix product. We can then compute \( \partial L/\partial W_F \) using the Chain Rule via

\[
\frac{\partial L}{\partial Z(k, t, \cdot, \cdot)} = A_1 \frac{\partial L}{\partial \hat{O}(k, t, \cdot, \cdot)} A_2^T \quad \frac{\partial L}{\partial W_F(\cdot, i, j)} = \frac{\partial L}{\partial Z(\cdot, i, j)} \left( \hat{I}_F(\cdot, i, j) \right)^T \tag{5}
\]

noting that \( \partial L/\partial \hat{O} \) is \( \partial L/\partial \hat{O} \) with a simple index mapping \( \psi^{-1} \).

Similarly, we use the Chain Rule to obtain \( \partial L/\partial I \), using \( \partial L/\partial Z \) computed above:

\[
\frac{\partial L}{\partial I_F(\cdot, i, j)} = (W_F(\cdot, i, j))^T \frac{\partial L}{\partial Z(\cdot, i, j)} \quad \frac{\partial L}{\partial I(c, t, \cdot, \cdot)} = B_1 \frac{\partial L}{\partial \hat{I}_F(c, t, \cdot, \cdot)} B_2^T \quad \frac{\partial L}{\partial \hat{I}(c, t, i, j)} = \sum_{(i', j')} \frac{\partial L}{\partial \hat{I}(c, t', j')} \tag{6}
\]

In summary, the backward propagation of Winograd layers is implemented by Equations [5] and [6].

4. Winograd Layers – Training and Pruning

Consider a \( L \)-layer network with a mixture of layers such as convolution, fully connected, and pooling. Let \( \sigma = [v_1, v_2, \ldots, v_L] \) be the parameters in the usual spatial domain, and \( \theta = [w_1, w_2, \ldots, w_L] \) be the parameters when the convolution layers are replaced by Winograd layers. Let \( L_s(\sigma) \) and \( L_w(\theta) \) be the respective loss functions. Training is typically done by minimizing an energy function that is the sum of the loss and a regularization penalty. [Lavin & Gray, 2016] does not consider Winograd as a layer architecture with full fledged backward propagation. They use Winograd solely to accelerate the convolution operations in both forward and backward propagation while all parameters reside in the spatial domain.

\(^2\)Note that the Winograd-kernel approach [Lavin & Gray, 2016] is utilized to efficiently compute the necessary Winograd matrices.
We aim to arrive at a $\theta^*$ such that many of its elements, including those within the Winograd layer, are zero. Pre-training, pruning, and fine-tuning (re-training) are the three major steps to achieve this goal.

4.1. Pre-training in spatial domain

In pre-training, we try to obtain a parameter $\theta^{(\text{start})}$ that makes the network accurate but is also amenable to pruning in the next stage. Because many open-source networks provide already-trained weights (in the spatial domain), the pre-training method we adopt here starts with these weights. We apply the standard SGD algorithm on the energy function of the form $E(\sigma) = L_\sigma(\sigma) + \lambda R(G(\sigma))$. Here $R$ is a regularization function applied on $G(\sigma)$, which converts “on-the-fly” the parameters related to the convolution layers to the Winograd coefficients using the $G_i$ matrices shown in Equation 4 in order to encourage sparsity in Winograd domain. We note that common frameworks such as Caffe allow the incorporation of various regularization penalties on different layers of a network in a straightforward manner. At the end of the SGD iterations, the obtained spatial parameter $\sigma^*$ is mapped into the Winograd domain by the same function $G$: $\theta^{(\text{start})} \leftarrow G(\sigma^*)$.

4.2. Pruning with Regularization and Gradient-based Thresholding

Pruning is done directly in Winograd domain, which is enabled by the Winograd layer. Regularization and gradient-based thresholding are the two techniques used for pruning. Regularization is a common and useful technique to attenuate over-fitting and induce sparsity. The energy function is

$$ E(\theta) = L_w(\theta) + R(\theta), \quad R(\theta) = \sum_{\ell=1}^{L} \lambda_\ell \|w(\ell)\|_{p_{\ell}}. \quad (7) $$

Common choices of norms are $p_{\ell} = 1$ or 2, and we use $L1$-norm for the layers to be pruned.

To achieve higher sparsity, thresholding is used in addition to regularization. The idea of thresholding is to set to zero particular entries within the parameter $\theta^{(k)}$ that are deemed inconsequential [Han et al., 2015; Wen et al., 2016; Guo et al., 2016]. Instead of using one uniform threshold to judge the significance, the threshold on a particular parameter is dependent on how significantly this parameter affects the underlying loss function $L$. The thresholding function $T_{\epsilon, \beta}$ is of the form

$$ T_{\epsilon, \beta}(w) = \begin{cases} 0 & \text{if } |w| \left( \frac{\partial L}{\partial w}(\theta) + \beta \right) < \epsilon \\ w & \text{otherwise} \end{cases} \quad (8) $$

We apply the thresholding function to a vector simply by applying it on each element, denoted with the slightly abused notation: $T_{\epsilon, \beta}(w)$. The numbers $\epsilon$ and $\beta$ are part of the “hyper-parameters” of the training procedure. This scheme uses smaller thresholds as the magnitude of gradient increases (threshold is $\epsilon/\beta$, $\epsilon$, and 0, when the magnitude of gradient is 0, 1 $- \beta$, and $\infty$, respectively). We find that the choice of $\epsilon=1e-4$ and $\beta=0.1$ works well. We emphasize that thresholding the parameters including Winograd parameters is now straightforward because of the Winograd parameters are the direct independent parameters of the Winograd layers.

In the Winograd domain, where the $L$-layer network contains both Winograd layers and other layers (e.g., pooling) with parameters: $\theta = [w_{(1)}, \ldots, w_{(L)}]$, the pruning step applies regularization and thresholding as follows where $\nabla_B$ is the familiar stochastic gradient with a mini batch $B$ and $\eta_k$ is the learning rate:

$$ E(\theta) = L_w(\theta) + \sum_{\ell=1}^{L} \lambda_\ell \|w(\ell)\|_{p_{\ell}}, $$

$$ \theta^{(k+1)} \leftarrow T_{\epsilon, \beta} \left( \theta^{(k)} - \eta_k \nabla_B E(\theta^{(k)}) \right) \quad (9) $$

4.3. Fine-Tuning: Recover Accuracy Loss

Similar to pruning in spatial domain, pruning Winograd parameters will cause accuracy loss, which necessitates a fine-tuning step to recover the loss. Same as pruning, fine-tuning is only done in the Winograd domain. During fine-tuning, the zero parameters obtained from the pruning step are fixed, while the network is trained to adjust the other non-zeros parameters to recover accuracy loss. We use L2 regularization during fine-tuning. The larger capacity of Winograd domain gives another benefit here. Even with high sparsity, the remaining degrees of freedom allow a better recovery of accuracy by the fine-tuning step. This further allows in general more aggressive regularization and thresholding during the pruning step.

5. Speeding Up Inference with Sparse Winograd Convolution

A highly optimized sparse Winograd Convolution implementation is paramount to turn the sparsity obtained into actual performance gain in inference computations. Winograd convolution consists of three steps: (1) input transformation that corresponds to multiplications of each tile with small matrices $B_1$ and $B_2$ in Equation 3 (2) element-wise multiplications in Winograd domain, and (3) output inverse transformation that corresponds to multiplications with $A_1$ and $A_2$. The bulk of computation is in the second step, which can be implemented as $(m + r - 1)(n + s - 1)$ independent multiplications of $K \times C$ matrices with $C \times T$ matrices, where we have number of $T$ tiles of $m \times n$ in size, $r \times s$ sized convolution kernels, $C$ input channels, and $K$
output channels. With a model pruned in Winograd domain, the $K \times C$ matrices are sparse and the matrix multiplications become sparse-matrix times dense-matrix multiplications (SpMDM).

We first parallelize all three steps over multiple images within a batch. When the batch is not large enough for each thread to have its own image, we further parallelize over input/output channels. This minimizes data exchanges among cores and also allows fusing the second and third steps for cache locality. Alternatively, the second step can be parallelized over $(m + r - 1)(n + s - 1)$ independent matrix multiplications. This approach has a benefit of increasing the multiplication size to $K \times C$ by $C \times (N \cdot T)$, where $N$ is the number of images per batch, exploiting more reuse out of the $K \times C$ matrices. However, this approach varies decomposition over the three steps, incurring significant data exchanges among the cores. Our scheme replicates the $K \times C$ matrices at each core, but this is mitigated by the fact that the $K \times C$ matrices are sparse and can be much smaller to fit L1 or L2 caches.

Libraries for dense convolution operations in CNN often layout the data interleaving multiple channels so that vectorization can be done over channels. However, since the sparsity pattern of parameters varies over channels, this layout and vectorization scheme is inefficient for SpMDM in the second step of sparse Winograd convolution. Therefore, we use a layout where the fastest moving dimension is tiles and vectorize over tiles. Our open source project will show more implementation details (link omitted for double blind review).

6. Experiments and Results

This section describes Winograd training/pruning and sparse Winograd inference results. We implement Winograd layer (forward and backward propagation for training/pruning) as in Section 3 and sparse Winograd convolution as in Section 5 in our branch of Caffe (Jia et al., 2014).

6.1. Winograd Convolution Pruning: Sparsity and Accuracy

We use pre-trained spatial domain model to save overall training time. Particularly, we start with the Caffe reference model from the Caffe model zoo (we call it AlexNet for simplicity even though it is a slight variation). We use the ImageNet ILSVRC-2012 dataset for pruning and test. Since the first convolution layer does not provide high enough sparsity to get speedups (Han et al., 2015, Wen et al., 2016), we do not attempt to prune that layer. We use the gradient-based thresholding described in Section 4 with $\epsilon=1e-4$ and $\beta=0.1$. We use learning rates of $5e-5$ and $1e-4$, and regularization factors of $5e-4$ and $5e-5$ in the pruning and fine-tuning steps, respectively. We use $200 \times$ smaller learning rates for the Winograd layers to ensure convergence.

Table 1 lists the accuracy and the sparsity of conv2-5 layers in AlexNet. Our method of training and pruning directly in Winograd domain (method A) results in 90.6–95.8% sparsity with only 0.1% top-1 accuracy drop from the reference model. Method B maintains convolution parameters both in spatial and Winograd domains and applies thresholding in 3 steps: (1) temporarily transform spatial parameters to Winograd, (2) threshold the Winograd parameters, and (3) find the least-square projection that maps the parameters back to the spatial domain. Since we have more parameters in Winograd domain, the Winograd parameters cannot be inversely transformed to the spatial domain exactly, hence the least-square projection. Due to this non-invertibility, method B either drops the accuracy by 8.1% (B1 in Table 1) or results in much lower sparsity (B2). Method C is from recent spatial domain pruning results (Park et al., 2017), which shows that, even when a model has ~90% high sparsity in spatial domain, the sparsity significantly degrades to 25–70% once converted to Winograd domain.

The results shown in Table 1 use gradient-based thresholding described in Section 4 where atop regularization the gradient-based thresholding further reduces the non-zero density of each layer by up to 1.3× without affecting accuracy. The fine-tuning step improves the accuracy from 56.5% to 57.3% in method A. When natively pruned in Winograd domain (method A), we frequently find sparsity patterns that have no counter parts in spatial domain such as a 6×6 kernel in Winograd domain with only one non-zero at the second row of the second column. This illustrates that CNN architectures with layers of directly trainable parameters in Winograd domain are more expressive and have more opportunity for pruning.

6.2. Sparse Winograd Convolution: Speedup Inference

Our sparse Winograd convolution inference is evaluated on a dual-socket server with Intel Xeon Platinum 8180 processors running at 2.5GHz, with total 56 cores and 77 MB last-level cache. This platform represents the latest server-class systems inside data centers. In Figure 2 SparseWinograd shows layer-wise performance of our sparse Winograd convolution design with the our obtain sparsity shown as method A in Table 1. DenseDirect is measured with libxsmm (Heinecke et al., 2016), a state-of-the-art open source dense convolution implementation. DenseWinograd_ideal is an ideally projected performance assuming that the speedup over DenseDirect is commensurate with the reduction in floating-point operations by using Winograd algorithm. We use this pro-
Table 1. The top-1 test accuracy and sparsity of $3 \times 3$ and $5 \times 5$ convolution layers of AlexNet resulted from different pruning methods. The accuracy of the original AlexNet is 57.4%. Method B uses spatial and Winograd parameters in conjunction similarly to [Liu & Turakhia, 2016]. B1 and B2 are with different hyper-parameters, with B1 to match the sparsity of method A and B2 to match the accuracy. For method C [Park et al., 2017], Winograd domain sparsity is obtained by transferring spatial domain kernels to Winograd domain kernels, with initial spatial domain sparsity shown inside parenthesis.

| Method   | Top-1 Accuracy | Sparsity of Convolution Layers in Winograd domain |
|----------|----------------|--------------------------------------------------|
|          | conv2 3 4 5    |                                                 |
| A (Ours) | 57.3%          | 90.6% 95.8% 94.3% 93.9%                        |
| B1       | 49.3%          | 92.8% 94.2% 93.2% 91.1%                        |
| B2       | 57.3%          | 54.1% 67.5% 62.4% 60.2%                        |
| C [Park et al., 2017] | 57.4% | 25.3% (85.6%) 69.3% (93.1%) 66.0% (91.8%) 61.3% (88.5%) |

Figure 2. Performance of sparse Winograd convolution compared with other methods, evaluated with $3 \times 3$ convolution layers of pruned AlexNet model shown in Table 1 and batch size 56 (i.e., 1 image per core). Except dense direct convolution, performance is evaluated as effective FLOP/s computed by (the number of floating-point operations that would have been executed by dense direct convolution)/(execution time).

7. Conclusion

As CNN has become pervasive, the relentless pursuit of fast convolution has inspired new algorithms and techniques for accelerating convolution. Transformation methods, especially Winograd convolution, and sparse methods are among the most successful approaches. This paper is the first to successfully combine them to construct sparse Winograd convolution. Moreover, we have demonstrated that our sparse Winograd can achieve 90+% sparsity without accuracy loss, leading to more than $5.4 \times$ speedup over dense direct convolution in $3 \times 3$ convolution layers of AlexNet on a latest Intel Xeon CPU. Looking ahead, our next step includes application to deeper networks like GoogLeNet [Szegedy et al., 2015] and ResNet [He et al., 2016] and to other platforms like FPGAs.

References

Deng, Jia, Dong, Wei, Socher, Richard, Li, Li-Jia, Li, Kai, and Fei-Fei, Li. ImageNet: A Large-Scale Hierarchical Image Database. In IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pp. 248–255, 2009.

Guo, Yiwen, Yao, Anbang, and Chen, Yurong. Dynamic Network Surgery for Efficient DNNs. In Proceedings of Advances in Neural Information Processing Systems (NIPS), pp. 1379–1387, 2016.

Han, Song, Pool, Jeff, Tran, John, and Dally, William J. Learning both Weights and Connections for Efficient Neural Networks. In Proceedings of Advances in Neural Information Processing Systems (NIPS), pp. 1135–1143, 2015.

He, Kaiming, Zhang, Xiangyu, Ren, Shaoqing, and Sun, Jian. Deep Residual Learning for Image Recognition. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pp. 770–778, 2016.
Enabling Sparse Winograd Convolution by Native Pruning

Heinecke, Alexander, Henry, Greg, Hutchinson, Maxwell, and Pabst, Hans. Libxsmm: accelerating small matrix multiplications by runtime code generation. In High Performance Computing, Networking, Storage and Analysis, SC16: International Conference for, pp. 981–991. IEEE, 2016. https://github.com/hfp/libxsmm.

Jia, Yangqing, Shelhamer, Evan, Donahue, Jeff, Karayev, Sergey, Long, Jonathan, Girshick, Ross, Guadarrama, Sergio, and Darrell, Trevor. Caffe: Convolutional Architecture for Fast Feature Embedding. In Proceedings of the ACM International Conference on Multimedia, pp. 675–678, 2014.

Krizhevsky, Alex, Sutskever, Ilya, and Hinton, Geoffrey E. ImageNet Classification with Deep Convolutional Neural Networks. In Proceedings of Advances in Neural Information Processing Systems (NIPS), pp. 1097–1105, 2012.

LeCun, Yann, Denker, John S., and Solla, Sara A. Optimal brain damage. In Proceedings of the 2Nd International Conference on Neural Information Processing Systems, pp. 598–605, 1989.

LeCun, Yann, Bottou, Léon, Bengio, Yoshua, and Haffner, Patrick. Gradient-Based Learning Applied to Document Recognition. Proceedings of the IEEE, 86(11):2278–2324, 1998a.

LeCun, Yann, Cortes, Corinna, and Burges, Christopher J. The MNIST database of handwritten digits. 1998b.

Liu, Baoyuan, Wang, Min, Foroosh, Hassan, Tappen, Marshall, and Penksy, Marianna. Sparse Convolutional Neural Networks. In IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2015.

Liu, Xingyu and Turakhia, Yatish. Pruning of Winograd and FFT Based Convolution Algorithm. 2016. http://cs231n.stanford.edu/reports2016/117_Report.pdf.

Liu, Xingyu, Han, Song, Mao, Huizi, and Dally, William J. Efficient Sparse-Winograd Convolutional Neural Networks. 2017. https://openreview.net/pdf?id=r1rqJyHKg.

Mathieu, Michael, Henaff, Mikael, and LeCun, Yann. Fast training of convolutional networks through iffts. arXiv preprint arXiv:1312.5851, 2013.

Park, Hyunsun, Kim, Dongyoung, Ahn, Junwhan, and Yoo, Sungjoo. Zero and data reuse-aware fast convolution for deep neural networks on GPU. In Hardware/Software Codesign and System Synthesis (CODES+ ISSS), 2016 International Conference on, pp. 1–10. IEEE, 2016.

Park, Jongsoo, Li, Sheng, Wen, Wei, Tang, Ping Tak Peter, Li, Hai, Chen, Yiran, and Dubey, Pradeep. Faster CNNs with Direct Sparse Convolutions and Guided Pruning. In International Conference on Learning Representations (ICLR), 2017.

Rippel, Oren, Snoek, Jasper, , and Adams, Ryan P. Spectral Representations for Convolutional Neural Networks. In Proceedings of Advances in Neural Information Processing Systems (NIPS), pp. 2449–2457, 2015.

Szegedy, Christian, Liu, Wei, Jia, Yangqing, Sermanet, Pierre, Reed, Scott, Anguelov, Dragomir, Erhan, Dumitru, Vanhoucke, Vincent, and Rabinovich, Andrew. Going Deeper with Convolutions. In IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pp. 1–9, 2015.

Vasilache, Nicolas, Johnson, Jeff, Mathieu, Michael, Chintala, Soumith, Piantino, Serkan, and LeCun, Yann. Fast Convolutional Nets with fbfft: A GPU Performance Evaluation. In International Conference on Learning Representations (ICLR), 2015.

Wen, Wei, Wu, Chunpeng, Wang, Yandan, Chen, Yiran, and Li, Hai. Learning Structured Sparsity in Deep Neural Networks. In Proceedings of Advances in Neural Information Processing Systems (NIPS), pp. 2074–2082, 2016.

Winograd, Shmuel. Arithmetic complexity of computations, volume 33. Siam, 1980.