Non-Supersymmetric Orbifolds

Anamaría Font\textsuperscript{*1} and Alexis Hernández\textsuperscript{†2}

\textsuperscript{*}Dept. de Física, Fac. de Ciencias, Universidad Central de Venezuela, A.P. 20513, Caracas 1020-A, Venezuela.

\textsuperscript{†}Dept. de Física, Universidad Simón Bolívar, A.P. 89000, Caracas 1080-A, Venezuela.

Abstract

We study compact non-supersymmetric $\mathbb{Z}_N$ orbifolds in various dimensions. We compute the spectrum of several tachyonic type II and heterotic examples and partially classify tachyon-free heterotic models. We also discuss the relation to compactification on K3 and Calabi-Yau manifolds.

\textsuperscript{1}afont@fisica.ciens.ucv.ve

\textsuperscript{2}ahernand@fis.usb.ve
1 Introduction

Orbifold compactifications provide a large class of exactly solvable string vacua in which the number of supersymmetries is reduced [1]. In this paper we will discuss type II and heterotic orbifolds in which supersymmetry is completely broken by the twisted boundary conditions. This generalizes the Harvey-Dixon construction [2] of the ten dimensional non-supersymmetric strings that can be found in various other ways [3].

To our knowledge, the study of non-supersymmetric compact orbifolds has not been carried out systematically until now. Compactifications of the $d=10$ non-supersymmetric theories were considered some time ago [4]. Non-supersymmetric closed string models in lower dimensions have been devised using asymmetric constructions [5], the Green-Schwarz formulation [6], Scherk-Schwarz compactifications [7] and compactifications on magnetic backgrounds [8]. The question of vanishing of the cosmological constant in non-supersymmetric strings has been addressed as well [9].

In the non-supersymmetric orbifolds that we discuss, supersymmetry is broken at the string scale. Thus, potential phenomenological applications in $d=4$ will face the hierarchy problem. Now, an approach to solve this problem in a similar scenario has actually been proposed [10] and other mechanisms could be conceived in the future. At any rate, since experimental evidence indicates that supersymmetry is broken, non-supersymmetric strings should be explored. Moreover, these theories have interesting features that are worth studying in more depth. Notably, such theories typically have tachyons and are thus unstable. It is then necessary to investigate processes of vacuum stabilization. Recently, condensation of closed type II tachyons has been analyzed in the case of non-compact orbifolds [11]. The fate of tachyons in compact orbifolds has however not been considered in detail. The case of heterotic tachyons has only been discussed in $d=10$ [12]. In this paper we will present several tachyonic examples in $d = 8, 6, 4$, that could serve as starting points to study type II and heterotic tachyon condensation in compact orbifolds. Furthermore, tachyons could be of relevance in electroweak symmetry breaking [13] and in inflation [14]. Yet another interesting issue is that of duality of string theories without supersymmetry. We have found several non-tachyonic heterotic models whose strong coupling duals could be described as in the $SO_{16} \times SO_{16}$ case [15].

This paper deals with type II and heterotic non-supersymmetric compactifications on $T^{10-d}/\mathbb{Z}_N$ for $d = 8, 6, 4$ and $N \leq 6$. Our main results consist of concrete examples. In particular, we have determined all tachyon-free heterotic models for $N \leq 5$. As a check on the massless spectra we have verified factorization of anomalies. We have also found...
relations to compactifications of the $d=10$ non-supersymmetric strings on smooth K3 and Calabi-Yau manifolds.

This paper is organized as follows. In section 2 we introduce the basic notation and determine the allowed $\mathbb{Z}_N$, $N \leq 6$, twists that break supersymmetry. In sections 3 and 4 we describe type II and heterotic orbifolds. In section 5 we present our final comments. For the sake of self-containedness we include an appendix with a review of tools needed to obtain the tachyonic and massless spectrum in compact orbifolds.

\section{Generalities}

We focus on $T^{10-d}/\mathbb{Z}_N$ orbifolds although the analysis can be easily extended to $\mathbb{Z}_N \times \mathbb{Z}_M$ actions and discrete torsion can also be included. In our standard notation the $\mathbb{Z}_N$ generator, denoted $\theta$, acts diagonally on internal complex coordinates as $\theta Y_i = e^{2\pi i v_i} Y_i$. Since $\theta^N = 1$, the $v_i$ are of the form $k/N$. The $v_i$ are restricted by the condition that $\theta$ must act crystallographically on the torus lattice. In particular this requires that the number of fixed points $\det(1 - \theta)$ be an integer. All such twists have been classified in ref.\[16\]. Spacetime supersymmetry imposes the extra condition

$$\pm v_1 \pm v_2 \pm v_3 = 0 \mod 2$$

(1)

Modular invariance gives the additional constraint

$$N \sum_i v_i = 0 \mod 2$$

(2)

Notice that (2) also ensures that $\theta$ is of order $N$ acting on the world-sheet fermionic degrees of freedom.

All crystallographic twists satisfying (1) and (2) were given in ref.\[1\]. If we relax (1) there will be no gravitini left in the untwisted sector so that supersymmetry is generically broken. In this case we can still use the results of ref.\[16\] to find the allowed $(v_1, v_2, v_3)$. The results for $N \leq 6$ are shown in Table \[1\]. For greater $N$ there are of course other solutions. Notice that in some $\mathbb{Z}_{2M}$ examples, the generator $\theta$ leaves the holomorphic two or three form invariant so that the resulting orbifold is either a singular K3 or a singular Calabi-Yau. Hence, we will be able to describe a class of non-supersymmetric compactifications in the orbifold limit of these manifolds.

It is also necessary to specify the torus lattice that allows a given action. We mostly consider products of two-dimensional sub-lattices. More precisely, for order two and order
Table 1: Twist vectors for non-supersymmetric $\mathbb{Z}_N$ actions, $N \leq 6$.

| $\mathbb{Z}_N$ | $(v_1, v_2, v_3)$ | $\mathbb{Z}_N$ | $(v_1, v_2, v_3)$ |
|---------------|-------------------|---------------|-------------------|
| $\mathbb{Z}_2$ | $(0,0,1)$         | $\mathbb{Z}_6$ | $(0,0,\frac{1}{7})$ |
| $\mathbb{Z}_3$ | $(0,0,\frac{2}{3})$ |               | $(0, \frac{1}{6}, \frac{1}{2})$ |
| $\mathbb{Z}_4$ | $(0, 0, \frac{1}{2})$ | $(0, 0, \frac{1}{2})$ | $(0, \frac{1}{3}, \frac{2}{3})$ |
|               | $(0, \frac{1}{4}, \frac{3}{4})$ |               | $(\frac{1}{6}, \frac{1}{6}, \frac{2}{3})$ |
|               | $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ |               | $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ |
| $\mathbb{Z}_5$ | $(0, \frac{1}{5}, \frac{3}{5})$ |               | $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ |

four rotations we take the $SO_4$ root lattice whereas for order three and order six rotations we take the $SU_3$ root lattice. The $\mathbb{Z}_6$ action is realized as the Coxeter rotation on the $SU_5$ root lattice.

Several of the twist vectors in Table 1 have been considered previously. The $\mathbb{Z}_2$ orbifold was originally analyzed in the context of non-supersymmetric 10-dimensional strings [2]. The $\mathbb{Z}_5$ has been considered in ref. [17] in relation to type IIB orbifolds of $AdS_5 \times S^5$. The $\mathbb{Z}_4$ with eigenvalues $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ has been used in [18] and [19] to discuss type 0B orbifolds and in [20] to construct non-supersymmetric tachyon-free orientifolds. The $\mathbb{Z}_4$ with $(0, 0, \frac{1}{2})$ has also been used to build a type IIB orbifold [19]. More recently, some of these generators have been considered in the case of non-compact orbifolds [11].

3 Type II

Symmetric type II orbifolds with the non-supersymmetric twists in Table 1 will have tachyons in some twisted sector. Notice that in some of the $\mathbb{Z}_{2M}$ examples, the $M$-th twisted sector is actually ‘untwisted’ in the sense that $\theta^M$ acts trivially in spacetime. In such cases the $\theta^M$ sector always includes a tachyon that could be referred to as untwisted. Since in these examples $\theta^M = (-1)^{F_S}$, where $F_S$ is the spacetime fermion number, there are altogether no spacetime fermions in the orbifold spectrum. In particular, the $\mathbb{Z}_2$ twist $(0, 0, 1)$ leads to the ten-dimensional type 0 closed strings. Below we will mostly describe examples with spacetime fermions.
3.1 \( d = 8 \)

In this case the little group \( SO_8 \) is broken to \( SO_6 \times SO_2 \), with \( 8_v = (6, 0) + (1, 1) + (1, -1) \) and \( 8_s = (4, -\frac{1}{2}) + (\bar{4}, \frac{1}{2}) \), where the second entry gives the \( U_1 \sim SO_2 \) charge.

As an example let us take the \( T^2 / \mathbb{Z}_3 \) with \( v = (0, 0, 0, 0, 2, 0, 0) \). Using the results shown in the appendix, we see that the massless states in the untwisted sector include the metric, dilaton and antisymmetric tensor. In the type IIB case, the remaining tachyonic and massless matter is given by

\[
\begin{align*}
\theta^0 & : \quad 1 + 1 + 15 + 4 + 4 \\
\theta & : \quad 3(1^-) + 3(1 + 1 + 15 + 4 + 4)
\end{align*}
\]

where states are labelled by their \( SO_6 \) representations and \( 1^- \) denotes a tachyon. The three tachyons (one per each fixed point) in the \( \theta \) sector have \( m_R^2 = m_L^2 = -\frac{1}{3} \). The massless states in the \( \theta \) sector all have oscillators acting on the twisted vacuum. To the above we must add the antiparticles.

In type IIA, in the untwisted sector there appears instead

\[ 1 + 6 + 10 + 4 + \bar{4} \]

and in the twisted sector the same with multiplicity three.

3.2 \( d = 6 \)

In this case we can classify the massless states according to the little group \( SO_4 \). We use \( SO_4 \sim SU_2 \times SU_2 \) so that the vector \((\pm 1, 0)\) becomes the \((\frac{1}{2}, \frac{1}{2})\) representation, the spinor \((\frac{1}{2}, -\frac{1}{2})\) becomes \((\frac{1}{2}, 0)\) and \(\pm (\frac{1}{2}, \frac{1}{2})\) is \((0, \frac{1}{2})\). In the untwisted sector there always appear massless states in the \( SU_2 \times SU_2 \) representations \((1, 1) + (0, 0) + (1, 0) + (0, 1)\) that correspond to the metric, dilaton and antisymmetric tensor. The remaining tachyonic and massless spectrum depends on the particular \( \mathbb{Z}_N \).

As a first example we take the \( T^4 / \mathbb{Z}_4 \) with \( v = (0, 0, \frac{1}{4}, \frac{3}{4}) \). In the \( \theta \) and \( \theta^3 \) sector there appear tachyons, denoted \((0, 0)^-\), with \( m_R^2 = m_L^2 = -\frac{1}{4} \). There are no tachyons in the \( \theta^2 \) sector since \( 2v \) is supersymmetric. However, the states in this sector do not fill supersymmetric multiplets. In type IIB, the tachyonic and massless matter includes

\[
\begin{align*}
\theta^0 & : \quad 14(0, 0) + 2(0, 1) + 4(1, 0) + 8(0, \frac{1}{2}) \\
\theta, \theta^3 & : \quad 8(0, 0)^- + 40(0, 0) + 8(1, 0) + 32(\frac{1}{2}, 0) \\
\theta^2 & : \quad 50(0, 0) + 10(0, 1) + 24(0, \frac{1}{2})
\end{align*}
\]
Notice that this massless spectrum is anomaly-free since the number of \((0,1)\) and \((1,0)\) tensors is the same and the number of \((0,\frac{1}{2})\) and \((\frac{1}{2},0)\) fermions is the same. We recall that in the supersymmetric \(T^4/\mathbb{Z}_4\), the massless spectrum consists of a gravity multiplet and 21 tensor multiplets, exactly as in type IIB compactification on a smooth K3. In the case at hand, the massless states in the \(\theta\) sector seem to fill tensor multiplets but the states differ in the twisted oscillators acting on them. In type IIA, the tachyonic and massless matter includes instead

\[
\begin{align*}
\theta^0 & : \quad 8(0,0) + 6\left(\frac{1}{2},\frac{1}{2}\right) + 4\left(0,\frac{1}{2}\right) + 4\left(\frac{1}{2},0\right) \\
\theta, \theta^3 & : \quad 8(0,0) - 32(0,0) + 8\left(\frac{1}{2},\frac{1}{2}\right) + 16\left(\frac{1}{2},0\right) + 16\left(0,\frac{1}{2}\right) \\
\theta^2 & : \quad 40(0,0) + 10\left(\frac{1}{2},\frac{1}{2}\right) + 12\left(0,\frac{1}{2}\right) + 12\left(\frac{1}{2},0\right)
\end{align*}
\] (6)

In the supersymmetric orbifold the massless spectrum has one gravity multiplet and 20 vector multiplets as in a K3 compactification.

Acting on the internal coordinates, the non-supersymmetric \(\mathbb{Z}_4\) has the same action as the supersymmetric one with \(v = (0,0,\frac{1}{4},-\frac{1}{4})\). We may thus suspect that the non-supersymmetric orbifold is related to a non-supersymmetric compactification on K3. The same observation applies to the \(T^4/\mathbb{Z}_6\) with \(v = (0,0,\frac{1}{6},\frac{5}{6})\).

Yet another example that can be related to a K3 compactification is the \(T^4/\mathbb{Z}_6\) with \(v = (0,0,\frac{1}{3},\frac{2}{3})\). In this case there are no fermions in the spectrum. Besides the untwisted tachyon, there are 18 twisted tachyons. The massless matter in the type IIB (type IIA) orbifold includes 128 (80) singlets and 24 antisymmetric tensors (48 vectors). Now, this \(\mathbb{Z}_6\) is of the form \(\mathbb{Z}_3 \times \mathbb{Z}_2\) where the \(\mathbb{Z}_3\) has the supersymmetric twist \(v = (0,0,\frac{1}{3},-\frac{1}{3})\) and the \(\mathbb{Z}_2\) has the \((-1)^F\) twist \(v = (0,0,0,1)\). Hence, we expect this orbifold to correspond to a K3 compactification of type 0 strings. Indeed, counting the zero modes in K3 of the massless type 0 fields we find the same massless matter as in the orbifold. However, it is not clear how the 18 twisted tachyons would appear in type 0 on K3 or disappear in the orbifold upon blowing up the singularities. In supersymmetric type II \(T^4/\mathbb{Z}_N\) orbifolds the massless spectrum coincides directly with the K3 spectrum.

Finally let us discuss the \(T^4/\mathbb{Z}_5\) that has no supersymmetric analog. In type IIB the tachyonic and massless matter is given by

\[
\begin{align*}
\theta^0 & : \quad 8(0,0) + 2(0,1) + 2(1,0) + 4(0,\frac{1}{2}) + 4\left(\frac{1}{2},0\right) \\
\theta, \theta^4 & : \quad 10(0,0) - 20(0,0) + 10(1,0) + 20\left(\frac{1}{2},0\right) \\
\theta^2, \theta^3 & : \quad 10(0,0) - 20(0,0) + 10(0,1) + 20(0,\frac{1}{2})
\end{align*}
\] (7)
In type IIA we instead find

\[ \theta^0 : \quad 4(0, 0) + 4(\frac{1}{2}, 0) + 4(0, \frac{1}{2}) + 4(\frac{1}{2}, \frac{1}{2}) \]
\[ \theta, \theta^4 : \quad 10(0, 0) + 10(0, 0) + 10(\frac{1}{2}, \frac{1}{2}) + 10(\frac{1}{2}, 0) + 10(0, \frac{1}{2}) \]
\[ \theta^2, \theta^3 : \quad 10(0, 0) + 10(0, 0) + 10(\frac{1}{2}, \frac{1}{2}) + 10(\frac{1}{2}, 0) + 10(0, \frac{1}{2}) \]

(8)

3.3 \quad d = 4

Now the massless states are classified by the little group $SO_2$, i.e. by helicity $\lambda$. For a $L \otimes R$ state, $\lambda = \lambda_r - \lambda_p$ where $\lambda_r$ can be read from the first component of the $SO_8$ weight $r$, and likewise for $\lambda_p$. In the untwisted sector, as usual, we find the metric, dilaton and antisymmetric tensor and now also a graviphoton. The remaining matter depends on the $\mathbb{Z}_N$.

As an example, we take the $\mathbb{Z}_6$ with $v = (0, \frac{1}{6}, \frac{1}{6}, \frac{2}{3})$. In the sectors $\theta^5$ and $\theta$ there appear tachyons with $m_R^2 = m_L^2 = -\frac{1}{6}$ and multiplicity three (one per each fixed point). There are no tachyons in other twisted sectors since $2v$ and $3v$ are supersymmetric. However, the matter in these sectors do not fill supersymmetric multiplets. Let us now consider type IIB more specifically. In the $\theta^3$ sector there further appear 5 massless vectors. We recall that in the supersymmetric case with $v = (0, \frac{1}{6}, \frac{1}{6}, -\frac{1}{3})$ there are 5 vector multiplets in this sector. The remaining massless states, labelled by helicity, are

\[ \theta^0 : \quad 11(0) + 8(\frac{1}{2}) \]
\[ \theta^5 : \quad 15(0) + 12(\frac{1}{2}) \]
\[ \theta^2 : \quad 30(0) + 24(\frac{1}{2}) \]
\[ \theta^3 : \quad 18(0) + 20(\frac{1}{2}) \]

(9)

In all the $d=4$ spectra we do not include the antiparticles. In type IIA we find extra massless vectors, five in the untwisted sector, 3 in the $\theta + \theta^5$, 15 in $\theta^2 + \theta^4$ and 6 in $\theta^3$. The remaining massless states are

\[ \theta^0 : \quad 6(0) + 4(-\frac{1}{2}) + 4(\frac{1}{2}) \]
\[ \theta^5 : \quad 12(0) + 6(-\frac{1}{2}) + 6(\frac{1}{2}) \]
\[ \theta^2 : \quad 15(0) + 12(-\frac{1}{2}) + 12(\frac{1}{2}) \]
\[ \theta^3 : \quad 17(0) + 10(-\frac{1}{2}) + 10(\frac{1}{2}) \]

(10)

This $T^6/\mathbb{Z}_6$ orbifold can be seen as a singular Calabi-Yau with $h_{11} = 29$ and $h_{12} = 5$. Besides the gravity multiplet and the dilaton hypermultiplet, the type IIA supersymmetric compactification has then 29 vector multiplets and 5 hypermultiplets.
We now discuss the \( T^6/Z_6 \) with \( v = (0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}) \) so that \( Z_6 \) is of the form \( Z_3 \times Z_2 \) where the \( Z_3 \) has the supersymmetric twist \( v = (0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}) \) and the \( Z_2 \) has the \((-1)^F\) twist \( v = (0, 0, 0, 1) \). Hence, we expect this orbifold to be related to type 0 compactification on the singular \( T^6/Z_3 \) Calabi-Yau with \( h_{11} = 36 \) and \( h_{12} = 0 \). In this orbifold there are no twisted tachyons, only the untwisted tachyon that appears in the \( \theta^3 \) sector. Besides the graviton, the massless spectrum in the type IIB (IIA) orbifold includes 222 (78) singlets and 2 (74) vectors. On the other hand, counting zero modes on a CY of the type 0B massless fields gives \((6 + 6h_{11} + 2h_{12})\) singlets and \((2 + 2h_{12})\) vectors. For type 0A there are instead \((6 + 6h_{12} + 2h_{11})\) singlets and \((2 + 2h_{11})\) vectors, as one might guess from mirror symmetry. Thus, the full spectrum of this non-supersymmetric \( T^6/Z_6 \) coincides with the type 0 compactification.

4 Heterotic

To study heterotic strings we need to describe the embedding in the gauge degrees of freedom. We use the bosonic formulation and realize the embedding by a shift vector \( V \) such that \( NV \in \Gamma \), where \( \Gamma \) is either the \( E_8 \times E_8 \) or the \( Spin(32)/\mathbb{Z}_2 \) lattice. Modular invariance of the partition function imposes \( N \sum_I V_I = 0 \mod 2 \) but this is trivially satisfied. Modular invariance further requires

\[
N(V^2 - v^2) = 0 \mod 2 \tag{11}
\]

For instance, in \( \mathbb{Z}_2 \) we can take \( V = 0 \) that leads back to the original heterotic string \([4]\). Indeed, in some cases supersymmetry can be recovered because missing partners appear in the extra twisted sectors. More generally, tachyons can be eliminated from the spectrum by level matching. The prime example is the non-supersymmetric non-tachyonic \( SO_{16} \times SO_{16} \) heterotic string in ten dimensions.

In the \( E_8 \times E_8 \) case, to determine the corresponding shift vectors \( V \), and the corresponding unbroken gauge group, we use the method of deleting nodes in the extended Dynkin diagram \([21]\). For \( Spin(32)/\mathbb{Z}_2 \) we use a similar procedure. All \( \mathbb{Z}_2 \) embeddings were given in ref. \([4]\) and are conveniently collected in Table 2 (the notation, say \( 1^n \), means that the entry 1 is repeated \( n \) times). The number of allowed \( V \)’s grows rapidly with \( N \) so that we will refrain from giving complete lists. In the following we will instead describe some selected examples briefly.

We have searched systematically for tachyon-free models for the \( Z_N \), with \( N \leq 5 \). By analyzing the mass formula for left movers \([13]\) we find that for some, typically larger
V^2, the putative tachyons cannot have \( m_R^2 = m_L^2 \) and hence disappear by level matching. Tachyons can also be eliminated by the generalized orbifold projection described in the appendix. Concrete examples are described below.

In the following we will also provide some tachyonic examples. In particular we will consider the standard embedding \( V = (v_1, v_2, v_3, 0, \ldots, 0) \) that always leads to models with tachyons.

### 4.1 \( d = 8 \)

We consider first the \( T^2/\mathbb{Z}_3 \) with \( v^2 = \frac{4}{9} \). In Table 3 we display the five \( E_8 \times E_8 \) allowed shifts. There are six embeddings in \( Spin(32)/\mathbb{Z}_2 \) that lead to gauge group of the form \( U_{3k+1} \times SO_{30-6k} \) and that have \( 3V \) of type \( (1^{3k+1}, 0^{15-3k}) \) or \( (1^{3k}, -2, 0^{15-3k}) \).

| 2V            | Gauge Group                          |
|---------------|--------------------------------------|
| \((2, 0^7) \times (0^8)\) | \(SO_{16} \times E_8\)               |
| \((1^2, 0^6) \times (1^2, 0^6)\) | \(E_7 \times SU_2 \times E_7 \times SU_2\) |
| \((2, 0^7) \times (2, 0^7)\) | \(SO_{16} \times O_{16}\)            |

| 3V            | Gauge Group                          |
|---------------|--------------------------------------|
| \((1^8, 0^8)\) | \(SO_{16} \times SO_{16}\)           |
| \((1^4, 0^{12})\) | \(SO_{24} \times SO_8\)             |
| \((2, 0^{15})\) | \(SO_{32}\)                          |
| \(((\frac{1}{2})^{16})\) | \(U_{16}\)                           |

Table 2: \( \mathbb{Z}_2 \) embeddings and groups.

| 3V            | Gauge Group                          |
|---------------|--------------------------------------|
| \((2, 0^7) \times (0^8)\) | \(SO_{14} \times U_1 \times E_8\)   |
| \((1^2, 0^6) \times (1^2, 0^6)\) | \(E_7 \times U_1 \times E_7 \times U_1\) |
| \((-2, 1^2, 0^6) \times (2, 0^7)\) | \(E_6 \times SU_3 \times SO_{14} \times U_1\) |
| \(\frac{1}{2}(-5, 1^7) \times (1^2, 0^6)\) | \(SU_9 \times E_7 \times U_1\)         |
| \(\frac{1}{2}(-5, 1^7) \times \frac{1}{2}(-5, 1^7)\) | \(SU_9 \times SU_9\)                   |

Table 3: \( \mathbb{Z}_3 \) embeddings and groups in \( E_8 \times E_8 \).
As an example we consider the standard embedding. In $E_8 \times E_8$ the resulting gauge group is $SO_{14} \times U_1 \times E_8$ and the matter spectrum, including tachyons, is

\[
\theta^0 : \quad (1 + 4)[(14, \frac{1}{\sqrt{2}}) + (64, -\frac{1}{2\sqrt{2}}) + (1, 0)]
\]

\[
\theta : \quad 3(1^-)[(14, -\frac{1}{3\sqrt{2}}) + (1, \frac{2}{3\sqrt{2}})] + 3(1 + 4)[(14, -\frac{1}{3\sqrt{2}}) + (64, \frac{1}{6\sqrt{2}}) + (1, -\frac{4}{3\sqrt{2}}) + 2(1, 2\sqrt{2})] \tag{12}
\]

Here $(1 + 4)$ indicates scalar plus fermion and $(1^-)$ tachyon. Together with the $SO_{14}$ representations we also give the $U_1$ charge $Q$. In $Spin(32)/\mathbb{Z}_2$ the gauge group is $SO_{30} \times U_1$ and the spectrum is

\[
\theta^0 : \quad (1 + 4)[(30, \frac{1}{\sqrt{2}}) + (1, 0)]
\]

\[
\theta : \quad 3(1^-)[(30, -\frac{1}{3\sqrt{2}}) + (1, \frac{2}{3\sqrt{2}})] + 3(1 + 4)[(30, -\frac{1}{3\sqrt{2}}) + (1, -\frac{4}{3\sqrt{2}}) + 2(1, 2\sqrt{2})] \tag{13}
\]

In all the $d=8$ heterotic spectra such as the above antiparticles are not included. Given the results in the appendix, it is straightforward to compute the spectrum for all $V$'s. We find that there is only one tachyon-free model with gauge group $SU_9 \times SU_9$ and massless matter given by

\[
\theta^0 : \quad (1 + 4)[(1, 1) + (84, 1) + (1, 84)]
\]

\[
\theta : \quad 3(1 + 4)(9, 9) \tag{14}
\]

For the $T^2/\mathbb{Z}_4$ with $v^2 = \frac{1}{4}$ there are 14 embeddings in $E_8 \times E_8$ and 17 in $Spin(32)/\mathbb{Z}_2$. There are only two tachyon-free models in which potential tachyons are eliminated by the generalized orbifold projection. One model, with $V = \frac{1}{8}(-7, 1^7) \times \frac{1}{8}(-7, 1^7)$, has group $(SU_8 \times SU_2)^2$ and massless matter given by

\[
\theta^0 : \quad (1)[(70, 1, 1, 1) + (1, 1, 70, 1) + 2(1, 1, 1, 1)] + (4)[(28, 2, 1, 1) + (1, 1, 28, 2)]
\]

\[
\theta : \quad 4(4)(8, 1, 8, 1)
\]

\[
\theta^2 : \quad (4)[(28, 1, 1, 2) + (1, 2, 28, 1)] \tag{15}
\]

The other non-tachyonic model has $V = \frac{1}{4}(1^{12}, 0^4)$, group $SU_{12} \times SO_8 \times U_1$ and the following massless matter

\[
\theta^0 : \quad (1)[(66, 1, \frac{1}{\sqrt{6}}) + (66, 1, -\frac{1}{\sqrt{6}}) + 2(1, 1, 0)] + (4)(12, 8_v, \frac{1}{2\sqrt{6}})
\]

\[
\theta : \quad 4(4)[(66, 1, -\frac{1}{2\sqrt{6}}) + (1, 8_s, \frac{3}{2\sqrt{6}}) + 2(1, 1, -\frac{3}{2\sqrt{6}})] \tag{16}
\]

\[
\theta^2 : \quad (4)(12, 8_c, \frac{1}{2\sqrt{6}})
\]

Notice that in the above $\text{Tr} \ Q = 0$. 

9
We have verified anomaly factorization in all models. To compute the anomaly polynomial we just need the contribution of a 4 fermion in a representation $R_i$ of the gauge group. This is given by

$$A_4 = \frac{1}{5} \text{Tr}_i F^5 - \frac{1}{12} \text{tr} R^2 \text{Tr}_i F^3 + \frac{1}{240} \text{tr} R^4 \text{Tr}_i F + \frac{1}{192} (\text{tr} R^2)^2 \text{Tr}_i F$$

(17)

We see then that factorization requires cancellation of the irreducible $\text{tr} F^5$ for all non-Abelian factors. Moreover it must be that $\text{Tr} Q = 0$ and $12 \text{Tr} Q^5 = 5 \text{Tr} Q^3$ for all $U_1$ factors. Summing the contribution of all massless fermions in each model we find that the anomaly polynomial factorizes as

$$A = \frac{1}{12} \text{Tr}_{a\ell} F^3 \left( \sum_a v_a \text{Tr}_a F^2 - \text{tr} R^2 \right)$$

(18)

Here $a$ runs over all gauge factors and $\text{tr} F^n_a$ stands for trace in the fundamental representation, whereas $\text{Tr}_{a\ell} F^3$ stands for trace of all gauge factors over all massless fermions. The coefficients $v_a$ depend only on the group. Using the conventions of ref. [23] they are $v_a = 2, 1, \frac{1}{3}, \frac{1}{6}, \frac{1}{30}$ for $SU_N$, $U_1$, $SO_N$, $E_6$, $E_7$ and $E_8$ respectively. To prove eq. (18) we use the results of ref. [23] together with identities such as

$$\text{Tr}_{a\ell} F^5 = (N - 16) \text{tr} F^5 + 10 \text{tr} F^2 \text{tr} F^3$$
$$\text{Tr}_{a\ell} F^5 = \frac{1}{2} (N - 6)(N - 27) \text{tr} F^5 + 10(N - 6) \text{tr} F^2 \text{tr} F^3$$

(19)

for antisymmetric $SU_N$ representations with $N \geq 5$.

### 4.2 $d = 6$

We study first the $T^4/Z_4$. The generator with $v = (0, 0, \frac{1}{4}, \frac{3}{4})$ has the same $V$’s that appear in the supersymmetric $v = (0, 0, \frac{1}{4}, -\frac{1}{4})$. There are 12 in $E_8 \times E_8$ and 14 in Spin$(32)/Z_2$. In the latter case the resulting gauge groups can be of the form $U_m \times SO_{2n} \times SO_{32-2m-2n}$, with $m+4n-2 = 0 \mod 8$. These arise from $4V$ with vector structure of type $(1^m, 2^n, 0^{16-m-n})$. It is also possible to obtain gauge group $U_{15-2k} \times U_{2k+1}$ that result from $4V$ without vector structure of type $\frac{1}{2}(1^{15-2k}, (-3)^{2k+1})$.

For the standard embedding in $E_8 \times E_8$ the resulting gauge group is $SO_{12} \times SU_2 \times U_1 \times E_8$
and the charged $m^2 \leq 0$ matter is

$$\begin{align*}
\theta^0 & : \quad [2(0,0) + (0, \frac{1}{2})][(12, 2, -\frac{1}{2}) + (32_c, 1, \frac{1}{2})] + 2(1, 1, 0)] + 2(\frac{1}{2}, 0)][(1, 1, 1) + (1, 1 - 1)] \\
\theta, \theta^3 & : \quad 4(0,0)^- [(12, 1, -\frac{1}{2}) + 2(1, 2, \frac{1}{2}) + c.c.]
+ 4[2(0,0) + (\frac{1}{2}, 0)][32_c, 1, \frac{1}{2})] + (1, 2, -\frac{3}{4}) + 2(12, 1, -\frac{1}{2}) + 3(1, 2, \frac{1}{2}) + c.c.
\theta^2 & : \quad (0, \frac{1}{2})[10(32_c, 1, 0) + 6(12, 2, 0) + 32(1, 1, \frac{1}{2}) + 32(1, 1, -\frac{1}{2})] + 2(0,0)[10(12, 0) + 6(32_c, 1, 0) + 32(1, 1, \frac{1}{2}) + 32(1, 1, -\frac{1}{2})]
\end{align*}$$

In the $SO_{32}$ heterotic the standard embedding has gauge group $SO_{28} \times SU_2 \times U_1$ and the following tachyonic plus massless matter

$$\begin{align*}
\theta^0 & : \quad [2(0,0) + (0, \frac{1}{2})][(28, 2, \frac{1}{2}) + 2(1, 1, 0)] + c.c.]
+ 2(\frac{1}{2}, 0)][(1, 1, 1) + (1, 1 - 1)] \\
\theta & : \quad 4(0,0)^- [(28, 1, \frac{1}{2}) + 2(1, 2, -\frac{1}{4}) + c.c.]
+ 4[2(0,0) + (\frac{1}{2}, 0)][2(28, 1, \frac{1}{2}) + 3(1, 2, -\frac{3}{4}) + (1, 2, \frac{3}{4}) + c.c.
\theta^2 & : \quad (0, \frac{1}{2})[6(28, 2, 0) + 32(1, 1, \frac{1}{2}) + 32(1, 1, -\frac{1}{2})] + 2(0,0)[10(28, 2, 0) + 32(1, 1, \frac{1}{2}) + 32(1, 1, -\frac{1}{2})]
\end{align*}$$

In this $Z_4$ we find several non-tachyonic examples. In both heterotic strings there are models with group $SO_{12} \times U_2 \times SO_{16}$. There is also a model with group $SO_{10} \times SO_{10} \times U_6$ and another with $SO_{10} \times SU_4 \times SU_8 \times SU_2$ and massless matter

$$\begin{align*}
\theta^0 & : \quad [2(0,0) + (0, \frac{1}{2})][(16, 4, 1, 1) + (1, 1, 28, 2)] + 2(1, 1, 1, 1) + c.c.]
+ 2(\frac{1}{2}, 0)][(10, 6, 1, 1)] + (1, 1, 70, 1)] \\
\theta, \theta^3 & : \quad 4[2(0,0) + (\frac{1}{2}, 0)][1, 4, 8, 1] + c.c.
\theta^2 & : \quad (0, \frac{1}{2})[10(10, 1, 1, 2) + 6(1, 6, 1, 2) + 2(0,0)[6(10, 1, 1, 2) + 10(1, 6, 1, 2)]
\end{align*}$$

This model has $V = \frac{1}{4}(-3, 1^3, 0^4 \times \frac{1}{8}(-7, 1^7)$.

We next discuss the $T^4/Z_6$ with $v = (0, 0, \frac{1}{3}, \frac{2}{3})$. There is a large number of allowed embeddings, a few of which with $3V^2 = \frac{2}{3} \mod 2$ and $3V \in \Gamma$ give back supersymmetric models. Although we have not searched systematically for tachyon-free models we have found a particular non-tachyonic example with group $SO_{12} \times SU_2 \times SO_{16} \times U_1$ that can be obtained in both heterotic strings. For instance, in the $SO_{32}$ heterotic with embedding
\[ V = \frac{1}{6}(3^6, 1^2, 0^8) \] the resulting spectrum is

\[
\begin{align*}
\theta^0 : & \quad 2(0, 0)\left[ (12, 2, 1, -\frac{1}{2}) + (1, 1, 1, 1) + 2(1, 1, 1, 0) + c.c. \right] + \\
& \quad (0, \frac{1}{2})\left[ (1, 2, 16, \frac{1}{2}) + c.c. \right] + 2(\frac{1}{2}, 0)\left[ (12, 1, 16, 0) \right] \\
\theta, \theta^5 : & \quad 9(\frac{1}{2}, 0)\left[ (32, 1, 1, \frac{1}{2}) + c.c. \right] \\
\theta^2, \theta^4 : & \quad 18(0, 0)\left[ (12, 2, 1, -\frac{1}{6}) + 2(1, 1, 1, -\frac{2}{3}) + 5(1, 1, 1, \frac{1}{3}) + c.c. \right] + \\
& \quad 9(0, \frac{1}{2})\left[ (1, 2, 16, -\frac{1}{6}) + c.c. \right] \\
\theta^3 : & \quad 2(0, \frac{1}{2})\left[ (32, 2, 1, 0) + (1, 1, 128, 0) \right] + \\
& \quad (\frac{1}{2}, 0)\left[ (32, 1, 1, -\frac{1}{2}) + (32, 1, 1, \frac{1}{2}) \right]
\end{align*}
\]

As in d=10, the \( E_8 \times E_8 \) spectrum follows from (23) upon exchanging the \( \theta^k \) and \( \theta^{\frac{N}{2}−k} \) fermions. As explained in section 3.2, in this case \( \mathbb{Z}_6 = \mathbb{Z}_2 \times \mathbb{Z}_2 \) where the \( \mathbb{Z}_3 \) respects supersymmetry and \( \mathbb{Z}_2 \) is equivalent to \((-1)^F_S\). Thus, this orbifold is arguably related to K3 compactification of the 10-dimensional non-supersymmetric heterotic theories. In fact, we claim that the model (23) can also be obtained from compactification of the tachyon-free \( SO_{16} \times SO_{16} \) theory on K3.

To support our claim we first observe that

\[ SO_{16} \supset SO_{12} \times SU_2 \times SU_2 \]  \( (24) \)

Thus, one \( SO_{16} \) can be broken to \( SO_{12} \times SU_2 \) by an \( SU_2 \) background with instanton number \( k = 24 \). This is the ‘standard embedding’ in the \( SO_{16} \times SO_{16} \) theory whose bosonic fields are \( (g_{\mu\nu}, B_{\mu\nu}, \phi) \) and \( A_\mu \) in the adjoint \( (120, 1) + (1, 120) \). The fermionic fields are \( 8_s \) spinors in the \( (16, 16) \) representation and \( 8_c \) spinors in \( (128, 1) + (1, 128) \). The number of zero modes of the massless d=10 fermions that transform in a representation \( \mathcal{R} \) of \( G_{10} = SO_{16} \) can be determined from an index theorem \( [24] \). One first decomposes \( \mathcal{R} \) under \( G_{10} \supset G \times H \) as

\[ \mathcal{R} = \sum_i (R_i, S_i) \]  \( (25) \)

where \( R_i \) and \( S_i \) denote representations of the unbroken group \( G \) and the bundle group \( H \). In the case at hand, \( G = SO_{12} \times SU_2 \) and \( H = SU_2 \) but many other examples can be worked out. The basic formula for the number of massless d=6 fermions transforming in the representation \( R_i \) is

\[ n_i = kT(S_i) − \dim S_i \]  \( (26) \)

where \( T(S_i) \) is \( \text{Tr}_{S_i} T^2 \). For \( SU_2 \), \( T(2) = \frac{1}{2} \) and \( T(3) = 2 \). We use conventions such that positive (negative) \( n_i \) corresponds to \( (0, \frac{1}{2}) \) (\( \frac{1}{2}, 0 \)) spinors. Also, recall that in
the supersymmetric case, (26) gives actually the number of hypermultiplets which have $2(0, \frac{1}{7})$. Hence there is an extra factor of two to be taken into account. To count the zero modes of the gauge field presumably we can use supersymmetry as a calculational tool and suppose that there exist gauginos in the adjoint. In the supersymmetric case, gaugino zero modes transforming as $(0, \frac{1}{7})$ spinors belong to hypermultiplets in which the scalars arise from the gauge field. Hence, the number of scalar zero modes is also given by (26) taking into account a multiplicity of four since hypermultiplets have four scalars. Recall further that zero modes of the metric and antisymmetric tensor give rise to 80 neutral scalars. We will need the following decompositions

\begin{align}
16 & = (12, 1, 1) + (1, 2, 2) \\
128 & = (32_c, 2, 1) + (32_s, 1, 2) \\
120 & = (66, 1, 1) + (1, 3, 1) + (1, 1, 3) + (12, 2, 2)
\end{align}  \hspace{1cm} (27)

Then, (26) with $k = 24$ implies that from the $(16, 16)$ fermions there arise 2 $(\frac{1}{2}, 0)$ fermions transforming as $(12, 1, 16)$ and 20 $(0, \frac{1}{2})$ fermions transforming as $(1, 2, 16)$. This agrees with (23) ignoring the $U_1$ charges, i.e. assuming that the $U_1$ is broken. Results for the number of fermions in the $(32_c, 2, 1)$ and $(32_s, 1, 1)$ representations also agree in this way. To obtain the massless scalars, we also use (26) as explained above. Then, from the decomposition of the $120$ we find 40 scalars transforming as $(12, 2, 1)$, same as in the orbifold, and 180 singlets. Adding the 80 moduli gives 260 singlets as compared to 264 in the orbifold. Presumably four scalars, corresponding to one hypermultiplet, disappear when the $U_1$ is Higgsed away as in the supersymmetric compactification.

We might as well consider compactification of the $d=10$ tachyonic models on K3. For example, we can start with the $SO_{16} \times E_8$ theory whose matter consists of tachyons transforming as $(16, 1)$ and massless fermions $8_s$ transforming as $(128, 1)$ plus $8_c$ transforming as $(T_{128}, 1)$ \cite{2}. The standard embedding is obtained by embedding an $SU_2$ instanton bundle with $k = 24$ in $SO_{16}$ so that the unbroken group is $SO_{12} \times SU_2 \times E_8$. Using eqs. (27) and (28) one can easily compute the resulting massless content. Now, we can compare the results with those of the standard embedding in the $T^4/\mathbb{Z}_6$ with $\mathbb{Z}_6 = \mathbb{Z}_3 \times \mathbb{Z}_2$. In this orbifold the gauge group is $SO_{12} \times SU_2 \times U_1 \times E_8$ and the massless states match those in the K3 compactification neglecting $U_1$ charges. As expected, in the orbifold there is an untwisted tachyon transforming as $(12, 1, 0)$ but there appear furthermore the following twisted tachyons

\begin{equation}
9 \left[ (12, 1, \frac{1}{3}) + 2(1, 2, -\frac{1}{6}) + c.c. \right] \hspace{1cm} (28)
\end{equation}
The states transforming as \((1, 2, -1/6)\) are of blowing-up type, i.e. they have left-moving oscillators acting on the twisted vacuum.

We finally consider the \(T^4/\mathbb{Z}_5\) orbifold that has no supersymmetric analog. Given \(v^2 = \frac{2}{5}\) we find 18 distinct embeddings in \(E_8 \times E_8\). In the \(SO_{32}\) heterotic there are instead 20 possibilities that lead to gauge groups of the form \(SO_{32-10n} \times U_{5n}\) and \(SO_{30-2j-4n} \times U_{n+j} \times U_{n+1}, j = 1, 6, 11\). There are groups that appear in both heterotic strings, namely \(SO_{12} \times U_5 \times U_5, SO_{14} \times U_7 \times U_2, U_8 \times U_8\) and \(SO_{10} \times U_3 \times U_5\). In this orbifold the shift \(V = 0\) is allowed. Hence, there are non-supersymmetric models in six dimensions with group \(E_8 \times E_8\) and \(SO_{32}\) whose tachyonic and massless spectrum is

\[
\begin{align*}
\theta^0 : & \quad 4(0, 0) + 2(0, 1/2) + 2(1/2, 0) \\
\theta, \theta^4 : & \quad 30(0, 0) - 50(0, 0) + 50(1/2, 0) \\
\theta^2, \theta^3 : & \quad 30(0, 0) - 50(0, 0) + 50(1/2, 0)
\end{align*}
\]

All states are neutral. In this \(\mathbb{Z}_5\) orbifold there is only one tachyon-free model with \(V = \frac{1}{5}(-4, 14, 0^3) \times \frac{1}{5}(-4, 14, 0^3)\), group \(SU_3^4\) and the following massless spectrum

\[
\begin{align*}
\theta^0 : & \quad [(0, 0) + (0, 1/2)]((5, 10, 1, 1) + (1, 1, 5, 10) + (1, 1, 1, 1) + c.c.) + \\
& \quad [(0, 0) + (1/2, 0)]((10, 5, 1, 1) + (1, 10, 5) + c.c.) \\
\theta, \theta^4 : & \quad 5[(0, 0) + (1/2, 0)]((5, 1, 5, 1) + c.c.) \\
\theta^2, \theta^3 : & \quad 5[(0, 0) + (0, 1/2)]((1, 5, 1, 5) + c.c.)
\end{align*}
\]

As a final example, we give the spectrum for the model with standard embedding in \(E_8 \times E_8\). The gauge group is \(SO_{12} \times U_1^2 \times E_8\) and the tachyonic plus massless matter turns out to be

\[
\begin{align*}
\theta^0 : & \quad [(0, 0) + (0, 1/2)]((12, 1/2, -1/2) + (32c, 0, 1/2) + (1, -1, 0) + (1, 0, 0) + c.c.) + \\
& \quad [(0, 0) + (1/2, 0)]((12, -1/2, -1/2) + (32c, 1/2, 0) + (1, 0, 1) + (1, 0, 0) + c.c] \\
\theta, \theta^4 : & \quad 5[(0, 0) -[(12, -1/10, -3/10) + (1, -3/5, 1/5) + 2(1, 2/5, 1/5) + c.c] + \\
& \quad 5[(0, 0) + (1/2, 0)][(32c, -1/10, 1/5) + (12, -1/10, -3/10) +] \\
& \quad 5[(0, 0) + (1/2, 0)][(32c, -3/10, 1/5) + (12, -3/10, -1/10) +] \\
& \quad 5[(0, 0) + (1, 3/5, -1/5) + 3(1, -1/5, 1/5) + c.c] \\
\theta^2, \theta^3 : & \quad 5[(0, 0) -[(12, 3/10, -1/10) + (1, 1/5, -3/5) + 2(1, -1/5, 2/5) + c.c] + \\
& \quad 5[(0, 0) + (0, 1/2)][(32c, -1/5, -1/10) + (12, 3/10, -1/10) +] \\
& \quad 5[(0, 0) + (1, 3/5, 2/5) + 2(1, -1/5, -3/5) + 3(1, -1/5, 2/5) + c.c]
\end{align*}
\]

All states are neutral under \(E_8\).

We have checked anomaly factorization in all models. The starting point is the contribution to the anomaly polynomial due to a \((1/2, 0)\) fermion in a representation \(R_i\) of the
gauge group. This is

$$A(\frac{1}{2}, 0) = \frac{\dim R_i}{240} (\text{tr } R^4 + \frac{5}{4}(\text{tr } R^2)^2) + \text{Tr}_i F^4 - \frac{1}{4} \text{tr } R^2 \text{Tr}_i F^2$$

(32)

For $(0, \frac{1}{2})$ fermions there is an overall minus sign. Cancellation of the irreducible $\text{tr } R^4$ requires equal number of $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ fermions which is a first easy check on the spectrum. Cancellation of the irreducible $\text{tr } F^4$ term can also be simply checked using the results of ref. [23] or ref. [25]. In fact, we have found that in all models the full anomaly polynomial factorizes as

$$A = \sum_a z_a \text{tr } F^2_a (\sum_a v_a \text{tr } F^2_a - \text{tr } R^2)$$

(33)

The coefficients $z_a$ depend on the spectrum of massless fermions and can be computed using the results in ref. [23] or in ref. [25] that employs slightly different conventions such that $v_a = 1$ for all groups.

4.3 $d = 4$

In the $T^6/Z_4$ with $v = (0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ we have looked systematically for tachyon-free models. We find that tachyons are eliminated by the orbifold projection in examples with groups $U_6 \times U_{10}, U_2 \times U_{14}$ and $SU_8 \times SU_2 \times E_6 \times U_2$. There are two more non-tachyonic examples in this orbifold that have groups $U_8 \times SO_6 \times SO_{10}$ and $SU_8 \times U_2^2 \times SO_{14}$ and can be constructed in both heterotic strings.

For other $T^6/Z_N$ we have not performed a systematic search of non-tachyonic models, although we have some examples. For instance, in the $Z_6$ with $v = (0, \frac{1}{3}, \frac{1}{3}, \frac{2}{3})$ there are models with group $SO_{16} \times SO_{10} \times SU_2 \times U_1^2$ in both heterotic strings.

In the $Z_6$ with $v = (0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ there are many allowed embeddings, including a few with $3V^2$ = even and $3V \in \Gamma$ that actually produce supersymmetric models. As explained in section 3.3, this orbifold can be related to compactifications of the $d=10$ non-supersymmetric theories on the singular $T^6/Z_3$ Calabi-Yau with $h_{11} = 36$ and $h_{12} = 0$. Thus, we expect to find non-tachyonic models arising from the $SO_{16} \times SO_{16}$ theory. Indeed, in both heterotic strings we obtain tachyon-free examples with group $SO_{16} \times U_6 \times SO_4$, $SO_{14} \times U_1 \times SO_{12} \times U_2$ and $SO_{16} \times SO_{10} \times SU_3 \times U_1$. In the latter the massless spectrum
turns out to be

\[\theta^0 : 3(0)[(1, 10, 3, \frac{1}{\sqrt{6}})] + (1, 1, 3, -\frac{2}{\sqrt{6}}) + 3(1, 1, 1, 0) + (\frac{1}{2})[128, 1, 1, 0] + 3(1, 16, 3, -\frac{1}{2\sqrt{6}}) + (1, 16, 1, -\frac{3}{2\sqrt{6}})\]
\[\theta : 27(\frac{1}{2})(16, 1, 1, -\frac{1}{\sqrt{6}})\]
\[\theta^2 : 27(0)[(1, 10, 1, -\frac{1}{\sqrt{6}})] + 3(1, 1, 3, 0) + (1, 1, 1, \frac{2}{\sqrt{6}}) + 27(\frac{1}{2})(1, 16, 1, -\frac{1}{2\sqrt{6}})\]
\[\theta^3 : (\frac{1}{2})[(16, 10, 1, 0) + 3(16, 1, \mathbf{3}, -\frac{1}{\sqrt{6}})]\]

This is obtained in \(E_8 \times E_8\) with \(V = (1, 0^7) \times \frac{1}{6}(-3, 1^3, 0^4)\). In the SO(32) heterotic with \(V = \frac{1}{6}(0^8, 3^5, 1^3)\) we obtain the same states.

The model in eq. (34) should correspond to the standard embedding in the SO_{16} × SO_{16} on the \(T^6/\mathbb{Z}_3\) Calabi-Yau orbifold. To support this claim we count the number of zero modes of the \(d=10\) fields as in the supersymmetric case. We start by decomposing representations under

\[SO_{16} \supset SO_{10} \times SU_3 \times U_1\] \hspace{1cm} (35)

The second SO_{16} is broken by a background in SU_3 equal to the spin connection. Neglecting U_1 charges to simplify and already including the unbroken SO_{16} we have

\[(1, 120) = (1, 45, 1) + (1, 1, 8) + (1, 1, 3) + (1, 1, \mathbf{3}) + (1, 10, 3) + (1, 10, \mathbf{3})\]
\[(16, 16) = (16, 10, 1) + (16, 1, 3) + (16, 1, \mathbf{3})\]
\[(1, 128) = (1, 16, 3) + (1, \mathbf{16}, \mathbf{3}) + (1, 16, 1) + (1, \mathbf{16}, 1)\]
\[(128, 1) = (128, 1, 1)\] \hspace{1cm} (36)

Since the (16, 10, 1) and the (128, 1, 1) are inert under SU_3, there is only one zero mode for each that transforms respectively as (16, 10) and (128, 1) under SO_{16} × SO_{10}. This agrees with the orbifold result. From (16, 1, 3) + (16, 1, \mathbf{3}) there follow instead \(h_{11} + h_{12}\) zero modes transforming as (16, 1). Now, \(h_{11} = 36\) and \(h_{12} = 0\) so that the total number coincides with the orbifold spectrum in eq. (34) if we count SU_3 dimensionality as multiplicity, i.e. assuming that SU_3 is broken. In fact, the 81 scalars transforming as (1, 1, 3, 0) in the \(\theta^2\) sector are precisely analogous to the blowing-up modes in the supersymmetric orbifold. Next, recalling that in \(d=10\) the (1, 128) has opposite chirality, we find \(h_{11} + 1\) zero modes transforming as (1, 16), and \(h_{12} + 1\) transforming as (1, \(\mathbf{16}\)). Notice that the number of families is \(|h_{12} - h_{11}|\). Again, when SU_3 is broken, this counting agrees with the orbifold result. Finally we can also deduce the number of charged scalars arising from the zero modes of the gauge field. From the adjoint decomposition we obtain
We can as well consider CY compactification of the tachyonic $SO_{16} \times E_8$ theory. The standard embedding leads to group $SO_{10} \times U_1 \times E_8$. Using (36) one can easily compute the number of charged zero modes that depend on $h_{11}$ and $h_{12}$. One can then compare for instance to the standard embedding in the non-supersymmetric $T^6/\mathbb{Z}_6$ with $v = (0, 1/3, 1/3, 1/3)$ which has group $SO_{10} \times U_1 \times SU_3 \times E_8$. The charged massless states agree assuming that $SU_3$ is broken upon repairing the orbifold singularities. Among the orbifold massless states there are indeed blowing-up scalars. In the orbifold there is only one untwisted charged tachyon, transforming in the $10$ of $SO_{10}$, that in the CY compactification presumably arises from the untwisted $d=10$ tachyon transforming as $16$ of $SO_{16}$.

To verify anomaly factorization we need the contribution to the anomaly polynomial due to a Weyl fermion in a representation $R_i$ of the gauge group. This is

$$A_+ = \text{Tr}_i F^3 - \frac{1}{8} \text{tr} R^2 \text{Tr}_i F$$

(37)

Cancellation of the irreducible $\text{tr} F^3$ term amounts to cancellation of non-Abelian cubic anomalies. We find that in all models the anomaly properly factorizes as

$$A = \frac{1}{8} \sum_b \text{Tr} F_b \left( \sum_a v_a \text{tr} F_a^2 - \text{tr} R^2 \right)$$

(38)

where $b$ runs only over the $U_1$ factors with $\text{Tr} Q_b \neq 0$. This factorization requires several non-trivial relations among the $U_1$ charges such as $\text{Tr} Q_b = 8 \text{Tr} Q_b^2$. It can be shown that there is at most one combination of $U_1$’s that is anomalous.

5 Final Comments

In this work we have used compact orbifolds to construct tachyonic and non-tachyonic non-supersymmetric models in various dimensions. Many more examples can be built. For instance, one can use $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds including discrete torsion [26]. Another interesting extension is to add Wilson lines to further break the gauge group [27]. Connections to non-perturbative orbifolds [28] and orbifolds of M-theory [25, 29] can be explored as well.

We have found that some $\mathbb{Z}_N$ non-supersymmetric orbifolds can be related to K3 and Calabi-Yau (CY) compactifications of the type 0 and non-supersymmetric heterotic strings in $d=10$. Indeed, there are tachyon-free orbifolds that match compactifications of the non-tachyonic $SO_{16} \times SO_{16}$ string. Since orbifolds are well defined string vacua...
and stable in the absence of tachyons, it then seems that even without supersymmetry compactification on K3 and CY manifolds gives consistent vacua at least at tree level. The standard embedding in K3 compactification that we discussed appears to be one example of a whole class of $d=6$ non-tachyonic models whose massless spectrum can be easily obtained and verified to satisfy anomaly factorization using basic results. We have also analyzed the standard embedding in CY and compared in a particular case to the corresponding orbifold.

We might also ask what happens when a $d=10$ tachyonic theory is naively compactified on K3 or a CY and how this relates to a corresponding orbifold in which the spectrum is exactly known. Concretely, in $d=6$ we can compare to the orbifold $T^4/Z_6$ with $v = (0, 0, \frac{1}{3}, \frac{2}{3})$ and in $d=4$ to $T^6/Z_6$ with $v = (0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. For type 0 on K3 we found that the massless states agree but there is a mismatch in the number of twisted tachyons. For type 0 in CY, results fit those of type II on the orbifold in which there is only one untwisted tachyon as in type 0. In the heterotic case we studied compactification of the tachyonic $SO_{16} \times E_8$ theory with standard embedding. In $d=6$, the orbifold has extra twisted charged tachyons that resemble blowing-up modes. In $d=4$, CY and orbifold results match.

Further orbifold examples associated to compactification of non-supersymmetric $d=10$ theories on K3 and CY can be worked out by taking the orbifold point group to be $\mathbb{Z}_N \times \mathbb{Z}_2$ where the $\mathbb{Z}_N$ is of supersymmetric type and $\mathbb{Z}_2$ is $(-1)^{F_S}$. In some orbifolds such as the $T^4/Z_4$ with $v = (0, 0, \frac{1}{4}, \frac{3}{4})$ or the $T^6/Z_6$ with $v = (0, \frac{1}{6}, \frac{1}{6}, \frac{2}{3})$, there should also be a relation to K3 or CY compactifications in which supersymmetry is broken. However, the interpretation is not as simple as in the $\mathbb{Z}_N \times \mathbb{Z}_2$ case.

To conclude we want to stress that there are many other non-supersymmetric orbifolds, such as $T^4/Z_5$, that are not of K3 or CY type. From the point of view of the orbifold construction all modular invariant choices of twist and embedding are on the same footing and can be equally analyzed. In particular, numerous examples can be built and used to study tachyon condensation. It would be specially interesting to examine heterotic models.

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6 Appendix : Orbifold Spectrum

To find the spectrum for each model, we follow the analysis in the supersymmetric case \cite{22}. There are $N$ sectors twisted by $\theta^k$, $k = 0, 1, \cdots, N - 1$. Each particle state is created by a product of left and right vertex operators $L \otimes R$. At a generic point in the torus moduli space, the tachyonic and massless right movers follow from

$$m_R^2 = N_R + \frac{1}{2} (r + k v)^2 + E_k - \frac{1}{2}$$  \hspace{1cm} (39)$$

In this formula $N_R$ is the occupation number of the right-moving oscillators and $E_k$ is the twisted oscillator contribution to the zero point energy given by

$$E_k = \sum_{i=1}^{3} \frac{k}{2} v_i (1 - kv_i)$$  \hspace{1cm} (40)$$

When $kv_i > 1$ we must substitute $kv_i \rightarrow (kv_i - 1)$ in (40). The vector $r$ is an $SO_8$ weight. When $r$ belongs to the scalar or vector class, $r$ takes the form $(n_0, n_1, n_2, n_3)$, with $n_i$ integer, this is the Neveu-Schwarz (NS) sector. When $r$ belongs to an spinorial class it takes the form $(n_0 + \frac{1}{2}, n_1 + \frac{1}{2}, n_2 + \frac{1}{2}, n_3 + \frac{1}{2})$, this is the Ramond (R) sector. The GSO projection turns out to be $\sum r_a = \text{odd}$. For example, in the untwisted sector, the condition $m_R^2 = 0$ implies $r^2 = 1$ and the possible solutions are

$$r = (\pm 1, 0, 0, 0) = 8_v \hspace{0.5cm}; \hspace{0.5cm} r = \pm (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) = 8_s$$  \hspace{1cm} (41)$$

where underlining means permutations. The vector $v$ in (39) is $(0, v_1, v_2, v_3)$, with the $v_i$ given in Table 1. For simplicity we are setting $\alpha' = 2$ everywhere.

The mass formula for left-movers depends on the type of string. For type II we have

$$m_L^2 = N_L + \frac{1}{2} (p + k v)^2 + E_k - \frac{1}{2}$$  \hspace{1cm} (42)$$

where $p$ is an $SO_8$ weight as well. In type IIB the GSO projection is also $\sum p_i = \text{odd}$ in both NS and R sectors. In type IIA one has instead $\sum p_i = \text{even}$ in the R sector. In the untwisted sector the spinor weights are then those of $8_v$.

For the heterotic string in the lattice formulation the left mass formula is instead

$$m_L^2 = N_L + \frac{1}{2} (P + k V)^2 + E_k - 1$$  \hspace{1cm} (43)$$
where $P$ belongs to the lattice $\Gamma$. The vectors $(P + kV)$ determine the representation of the state under the gauge group. $N_L$ is the occupation number for all left-moving oscillators.

States in a $\theta^k$-twisted sector with given $m_R^2 = m_L^2$ are characterized by $(N_R, r; N_L, p)$ in type II or by $(N_R, r; N_L, P)$ in the heterotic string. The degeneracy of these states follows from the generalized orbifold projection given by

$$D(\theta^k) = \frac{1}{N} \sum_{\ell=0}^{N-1} \chi(\theta^k, \theta^\ell) \Delta(k, \ell), \quad (44)$$

In the heterotic string the phase $\Delta$ is

$$\Delta(k, \ell) = \exp\left\{2\pi i [(r + kv) \cdot \ell v - (P + kV) \cdot \ell V + \frac{1}{2} k\ell (V^2 - v^2) + \ell (\rho_L + \rho_R)]\right\}, \quad (45)$$

In type II the phase $\Delta$ is analogous with $p, v$ instead of $P, V$. The factors $\rho_{R,L}$ appear only in the case of states with oscillation number due to the internal (complex) coordinates $Y_i$. The phase $e^{2\pi i \rho}$ indicates how the oscillator is rotated by $\theta$. In the $\theta$ sector, for example, the right-moving oscillators are of the form $\alpha^i_{-v_i}, \alpha^i_{1-v_i}, \cdots$ with $\rho_R = v_i$ and oscillators $\bar{\alpha}^i_{-1+v_i}, \bar{\alpha}^i_{-2+v_i}, \cdots$ with $\rho_R = -v_i$. Similarly, for left-movers there can be oscillators $\bar{\alpha}^i_{-v_i}, \bar{\alpha}^i_{1-v_i}, \cdots$ with $\rho_L = -v_i$ and oscillators $\bar{\alpha}^i_{-1+v_i}, \bar{\alpha}^i_{-2+v_i}, \cdots$ with $\rho_L = v_i$.

In (44) $\chi(\theta^m, \theta^m)$ is a numerical factor that counts the fixed point multiplicity. More concretely, $\chi(1, \theta^\ell) = 1$, so that in the untwisted sector $D(1)$ projects out precisely the states non-invariant under $\theta$. In twisted sectors $\chi(\theta^k, \theta^\ell)$ is the number of simultaneous fixed points of $\theta^k$ and $\theta^\ell$.

As an example, let us work out the type IIB string on $T^2/\mathbb{Z}_3$ with $v = (0, 0, 0, \frac{2}{3})$. In the untwisted sector there are no tachyons and the massless states are as follows. With $r \cdot v = p \cdot v = 0$ we have

$$r = \begin{pmatrix} \pm 1, 0, 0, 0 \end{pmatrix}, \quad p = \begin{pmatrix} \pm 1, 0, 0, 0 \end{pmatrix} \quad (46)$$

The first three entries in $r, p$ correspond to non-compact coordinates and give the Lorentz representations according to the little group $SO_6$. The NS-NS states in $(16)$ are thus

$$6 \otimes 6 = 20 + 15 + 1.$$  These are the metric, the antisymmetric tensor and the dilaton.

With $r \cdot v = p \cdot v = \frac{2}{3}$ mod int we have

$$r = \begin{pmatrix} 0, 0, 0, 1 \end{pmatrix}, \quad p = \begin{pmatrix} 0, 0, 0, 1 \end{pmatrix} \quad (47)$$

$$r = \begin{pmatrix} \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \end{pmatrix}, \quad p = \begin{pmatrix} \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \end{pmatrix} \quad (1, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}) \quad (47)$$
The four spinor weights form the $4$ of $SO_6$. Then, for instance when we combine right and left we obtain $1 + 15$ in the R-R sector. With $r \cdot v = p \cdot v = \frac{1}{3} \mod \text{int}$ the solutions are as in (47) with opposite sign. In the $\theta$-twisted sector the solutions with $m_R^2 \leq 0$ are

$$
\begin{array}{ccc}
  r & N_R & m_R^2 \\
  (0, 0, 0, -1) & 0 & -\frac{1}{3} \\
  \left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) & 0 & 0 \\
  \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right) & 0 & 0 \\
  (0, 0, 0, -1) & \frac{1}{3} & 0 \\
\end{array}
$$

The solutions with $m_L^2 \leq 0$ are similar. The solutions in the $\theta^2$-twisted sector are as in (48) with opposite sign and will lead therefore to the antiparticles of the states in the $\theta$ sector.

In type IIA we have to take the $p$’s arising instead from $8_c$. This implies for example, to change the $p$ spinor weights in (47) to the $\overline{4}$ of $SO_6$.

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