Evaluation of fatigue crack initiation using accumulated entropy

M.A. Amooie, H. Ghadimi, B. Hajshirmohammadi and M. M. Khonsari
Department of Mechanical and Industrial Engineering
Louisiana State University
3283 Patrick Taylor Hall
Baton Rouge, LA 70803, USA

Abstract

The accumulated entropy is used to evaluate the initiation of crack in flat notched specimen made of stainless steel 304. Two notch radii have been studied to capture the thermography of crack initiation and measure accumulated heat dissipation. It has been observed that under pull-push loading with an R-ratio of -1, the accumulated entropy for crack initiation and crack length's growth up to a specific number, Fracture Fatigue Entropy remains nearly constant. Anaclastic deformation dissipation has been taken into account and a mathematical model based on Kelvin-Voigt type element has been discussed. The effect of anelastic deformation has been studied in terms of temperature evolution at the beginning of the test.

Nomenclature

\[ A_k \] Thermodynamic forces associated with internal variables
\[ a \] Specimen thickness
\[ b \] Width of the specimen (m)
\[ c_p \] Specific heat capacity (J/kgK)
\[ d_e S \] Entropy flow (J/K)
\[ d_i S \] Internal entropy generation (J/K)
\[ dS \] Entropy change (J/K)
\[ E \] Elastic modulus (GPa)
\[ f \] Frequency (Hz)
\[ F \] Fatigue fracture entropy (MJ/m^3K)
\[ h \] Convection coefficient (W/m^2K)
\[ k \] Thermal conductivity (W/mK)
\[ N_f \] Number of cycles to failure
\[ Q \] Rate of heat generation (J/s)
\[ Q_gen \] Rate of heat generation (J/s)
\[ Q_{in} \] Heat rate entering control volume (J/s)
\[ Q \] Rate of heat transfer from the system (J/s)
\[ \dot{Q} \] Rate of heat transfer from the system (J/s)
\[ \dot{S}_1 \] Entropy production rate (J/m^3K)
\[ T \] Absolute temperature (K)
\[ T_0 \] Absolute ambient temperature (K)
\[ T_s \] Absolute surface temperature (K)
\[ t_f \] Time to failure (s)
\[ t^* \] Time at which cooling begins (s)
\[ U \] Rate of internal energy change (J/s)
\[ V \] Control volume (m^3)
\[ \dot{V} \] Rate of internal energy change (J/s)
\[ \dot{\nu}_r \] Internal variables
\[ \dot{w}_p \] Plastic work dissipation (J/sm^3)
\[ W \] Specimen thickness (m)
\[ \dot{W} \] Work done on the system per second (J/s)
\[ \vartheta \] Temperature rise \( \vartheta = T - T_0 \)

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1 Corresponding author: Khonsari@lsu.edu
### 1. Introduction

Fatigue drastically reduces the life of machinery with costly results. The failure due to fatigue is mostly without warning, increasing the risk of catastrophic events. The complexity of studying fatigue of materials at each stage from crack initiation, crack propagation and failure has been a subject for researchers to address. Reliable methods are introduced to model fatigue behavior, especially with stress concentration which appears to be an inevitable part of industrial machinery components design. Stress-strain base characterization of fatigue life is conventionally used for predicting life[1-3], and more recently, energy-based methods are expanding [4-8]. Lately, the thermography of fatigue has been getting more attention due to the widespread use of infrared technology. Thermography, as a non-damaging technique, provides an opportunity to predict fatigue life according to energy-based models [9, 10].

Under fatigue loading, the stress concentration in components contributes to advanced failure [11]. Stress concentration can be caused by the geometry of the component in the form of non-uniform sections, component applications mandating to have holes for screws, and limitations in installation which can alter the geometry based on initial machinery design or welded parts [12]. Conventionally investigating the fatigue behavior in the presence of stress concentration is done by creating various shapes of relatively large notches on testing specimen. The stress concentration factor is controlled by the radius of notches [13]. Numerous models are presented to address the behavior of materials undergoing cyclic loadings with stress concentration, such as 'Frost's model [14]. Frost's model introduces a relationship between crack length and fatigue limit. In 1995, Yao et al. [15] studied the fatigue notch factor, and categorized various expressions for this parameter into three groups They compared experimental results with each model. The results showed that the stress field intensity (SFI) is the most sensible model. Ding et al. [16] offered an elastic-plastic FM model to describe fatigue crack growth on notched round 1070 steel. Various R-ratios were studied in this work and the results showed that for positive R-ratio, plasticity in the vicinity of the notch is the influential element in crack growth; however, for negative R-ratios, the plasticity was decreased near the tip of the crack due to the crack closure. In 2008, McEvily et al. [17] proposed an approach to estimate fatigue notch factor $K_F$ and its dependence on various parameters such as notch radius, material property, etc. In 2010 Ranganathan et al. [18] studied a short crack approach to observe the life of 2024 T351 alloy in which crack propagates to a detectable size. It was shown that this approach is in good agreement with the conventional local strain approach. Ewest et al. [19] proposed a model based on compliance model for a single edge notch to study crack growth.

| Symbol | Description                                      | Unit     |
|--------|--------------------------------------------------|----------|
| $Q_{out}$ | Heat rate exiting control volume (J/s)          |          |
| $q_{cv}$  | Convective heat transfer (J)                    |          |
| $Q$       | Thermal current (J/s)                           |          |
| $\rho$   | Density (kg/m$^3$)                              |          |
| $\sigma$ | Stress (MPa)                                    |          |
| $\gamma_i$ | Accumulated entropy for crack initiation       |          |
The model takes stiffness variations into account for each cycle and suggests a correlation between crack length and stiffness alterations. Eq. 1 is introduced to approximate average crack length.

\[ \bar{a} = b_1 \cdot \beta^5 + b_2 \cdot \beta^4 + b_3 \cdot \beta^3 + b_4 \cdot \beta^2 + b_5 \cdot \beta \]

Where \( \beta = 1 - \frac{K}{K_0} \), \( K \) is the stiffness for each cycle and \( K_0 \) is the stiffness for a plain unnotched material. \( b_i \)s are numerical constants. The influence of notch size based on the Weibull model on fatigue life of Al 2024-T351 was studied by Zhu et al. [20] in 2020. The experimental results were compared with the model outcome. It has been shown that the model could estimate the life of plain specimens and center hole plates (4 different radii) within the \( \pm 2 \) life scatter band. In 2022 Braun et al. [21] studied numerous affecting factors on fatigue life of specimens with notch or having welded regions. The results showed fracture mechanics and consideration of stress distribution around the notched area is a suitable method for predicting the total life, especially for notches with lower acuity; however, crack propagation had more complexity.

In recent years, methods based on the thermodynamics of fatigue behavior have gained attention. It is due to the development of thermography measurement equipment and ease of use. In 2007 Meneghetti [22] utilized an infrared camera to capture temperature in notched specimens with various radii and load amplitudes and measured dissipated heat per unit volume per cycle. It was shown that the dissipated heat per cycle is an independent parameter from the load sequence and only depends on the imposed stress amplitude. Wang et al. [23] used temperature increment as a fatigue indicator to predict the fatigue life of notched specimens through the thermographic technique. Meneghetti et al. [24] studied plain and notched AISI 304 L specimens under stress and strain control fatigue tests and used dissipated heat as a damage index. It was shown that the dissipated heat per unit volume per cycle is an independent parameter from the geometry or the boundary condition of the specimen. This parameter only settles in a linear band with respect to imposed stress. In 2018, Wang et al. [25] proposed a thermographic model based on energy dissipation on 316L steel to study fatigue crack growth near the crack tip zone. The test outcome showed a dependence between dissipated energy and stress ratio; however, the model was improved considering the crack closure effect. Karimian et al. [26] used volumetric thermodynamic entropy to study damage in the notched specimen. In their study, plastic deformation was measured by Digital Image Correlation (DIC) in the vicinity of the notch on the specimen. Sheridan et al. [27] compared the specimen-compliance approach with thermal and optical ones for detecting fatigue crack initiation in AM components under fatigue loading. Zhang et al. [28] found a relation between plastic deformation and thermal energy dissipation. They also investigated the relationships between temperature increase to damage and mechanical load in coupling beam dampers. Sieber et al. [29] used Lock-in-thermography to detect short cracks in riveted connections.
The literature discussed, presented a profusion of research conducted to predict fatigue life, crack initiation and crack propagation rate of materials concerning number of cycles in the presence of stress concentration in the form of notches. It should be noted that in most traditional models, there is a significant dependence on knowing stress concentration factors, stiffness alterations, etc. Recent efforts have resulted in the development of new thermographic models to facilitate life prediction of materials undergoing cyclic loadings. One recently introduced method is Fracture Fatigue Entropy (FFE), which uses accumulated entropy as an index to predict life [29]. FFE has been experimentally validated to be a property of material that does not depend on the type of loading or geometry and boundary conditions [30]. Applications of this index to predict life has been expanded to different materials such as composites [31] and various working conditions [32]. The present study utilizes accumulated entropy to predict crack initiation in notched specimens experiencing push-pull cyclic loading with different radii and stress amplitudes. The second section of this work describes the governing thermodynamic equations for a control volume in the vicinity of a notch. Section 3 gives a concise description of fatigue endurance limit and the role of internal friction in energy dissipation, and its influence as a non-damaging parameter. Sections 4 and 5 are devoted to material specifications and explaining the experimental process respectively. Section 6 presents obtained results and discuss applying thermodynamic laws to show the validity of FFE in predicting crack initiation. Conclusive remarks are stated in Section 7, and outcomes for FE simulations are presented in Appendix (section 8).

2. Theory and Formulation

Fig. 1 shows the notched specimen under cyclic axial loading. A control volume around the notch has been taken out and shown in this figure. According to the first thermodynamic law for this control volume, the rate of work done on the control volume \( W \), and the dissipated energy rate \( Q \), yields the rate of internal energy change in the control volume. The integration of these changes over a period of time (one cycle) for the control volume yields in Eq. 2.

\[
\int_V \Delta U \cdot dV = \int_V W \cdot dV - \int_V Q \cdot dV
\]  

Eq. 2 can be rewritten as follows in terms of mean power exchange.

\[
\int_V (\rho c_p \frac{\partial T_m}{\partial t} + \dot{E}_p) \cdot dV = \int_V (\dot{\phi} \sigma_{ij} \cdot d\varepsilon_{ij}) \cdot f \cdot dV - \int_V (q_{CD} + q_{CV} + q_{RD}) \cdot dV
\]  

Where \( \rho \) is the density, \( T_m \) is the temperature in one cycle, \( c_p \) is the specific heat, \( \dot{E}_p \) is the rate of damaging energy, and \( f \) is the test frequency. The thermoelastic coupling in one cycle does not contribute to irreversible heat dissipation or generation in one cycle, which means there is a zero-net amount for reversible energy. The heat generation side of equation 3 can be rewritten as follows.
\[ \int_{V} Q_{\text{gen}} \, dV = \int_{V} (\phi \sigma_{ij} \cdot d\varepsilon_{ij}) \cdot f \cdot dV - \int_{V} \dot{E}_{p} \cdot dV \] \quad (4)

Therefore, the left hand of Equation 3 can be redefined in terms of dissipated heat in three forms of conduction, convection and radiation, and change of internal energy.

\[ \int_{V} Q_{\text{gen}} \, dV = \int_{V} (q_{CD} + q_{CV} + q_{RD}) \cdot dV + \int_{V} (\rho c_{p} \frac{\partial T_{m}}{\partial t}) \cdot dV \] \quad (5)

The change in the system entropy is greater or equal to the heating rate divided by temperature, which is predicted by the second law of thermodynamics.

\[ dS \geq \frac{\partial Q_{\text{gen}}}{T} \] \quad (6)

One can add the extra term of internal energy generation, \( d_{i}S \), to inequality of Eq. 2 and find an equilibrium as:

\[ dS = d_{e}S + d_{i}S \] \quad (7)

where, \( \frac{\partial Q_{\text{gen}}}{T} \) in Eq. 5 is replaced with \( d_{e}S \), which is referred to as entropy flow.

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Figure 1 The notched specimen under cyclic axial loading and the control volume in the vicinity of the notch
A cyclic force $F$ acts on the specimen, the control volume around the notch root illustrating various forms of heat transfer. The Clausius–Duhem inequality characterizes the entropy generation by providing the following expression.

$$\dot{S}_i = \frac{1}{T} \sigma : \dot{\varepsilon}_p - \frac{1}{T} A_k \dot{v}_k - \frac{1}{T^2} q \nabla T \geq 0$$

(8)

$\sigma$ and $\dot{\varepsilon}_p$ are the stress tensor and plastic strain rate, respectively. $v_k$ are the internal variables and their associated conjugate thermodynamic forces denoted by $A_k$. $q$ is the heat flux, and $T$ is the absolute temperature. It has been shown that for many metals, the term $A_k \dot{v}_k$ is relatively small [33]. It is also a valid assumption that entropy generation due to heat production, $\frac{1}{T^2} q \nabla T$ is negligible [34].

$$\dot{S}_g = \frac{\dot{w}_p}{T}$$

(9)

Where heat generation per unit volume is given by $\dot{w}_p$, which is the rate of change in left hand side of Eq. 5.

The accumulated entropy, $\gamma_i$, from the start of the cyclic actuation $t = t_0$ to the onset of crack initiation, when $t = t_i$ is:

$$\gamma_i = \int_{t_0}^{t_i} \frac{\dot{w}_p}{T} dt$$

(10)

If $\dot{w}_p$ is assumed constant from begging of loading to the moment of crack initiation Eq. 10 can be written as Eq. 11.

$$\gamma_i = \dot{w}_p \int_{t_0}^{t_i} \frac{1}{T} dt$$

(11)

Eq. 11 can be written as Eq. 12 with respect to the number of cycles for crack initiation $N_i$.

$$\gamma_i = \frac{\dot{w}_p}{\bar{T}_i} t_i = \frac{N_i \dot{w}_p}{f \bar{T}_i}$$

(12)

In which $f$ is the frequency of loading and $\bar{T}_i$ is defined using Eq. 11.

$$\bar{T}_i = \frac{t_i}{\int_0^{t_i} \frac{1}{T_s} \, dt} = \frac{N_i}{\int_0^{N_i} \frac{1}{T_s} \, dt}$$

(13)

Mehdizadeh and Khonsari [35] showed that $\dot{w}_p$ is found as:
\[ \dot{\omega}_p = \rho c R_\theta \]  

(14)

Where \( \rho \) is density and \( c \) is specific heat capacity. \( R_\theta \) is the slope of temperature rise at the beginning of cyclic loading. The heterogeneities at microscopic scale are considered to evaluate heat dissipation due to cyclic loading at a local point near the notch; though, it should be considered that heat dissipation sources can be categorized into two various types. Not recoverable heat dissipation due to inelastic deformation and a recoverable dissipation due to anelastic deformation which occurs because of oscillatory movements of slip bands and formation of kinks in the lattice. The effect of latter on measurements of the initial slope will be further discussed in the next section. The following equations are to describe the mechanisms of these two categories in a thermodynamical framework.

Grain total local strain is formed of three various types including elastic, inelastic and anelastic parts which can be quantified according to Eq. 15 [36].

\[ \varepsilon = \varepsilon^e + \varepsilon^{in} + \varepsilon^{an} \]  

(15)

The external energy that is equal to the dissipated energy can then be described as follows [37].

\[ \Pi = (1 - f_v) \bar{\Sigma} \dot{\varepsilon} + f_v \sigma : \dot{\varepsilon}^e + f_v \sigma : \dot{\varepsilon}^{an} + f_v \sigma : \dot{\varepsilon}^{in} \]  

(16)

Where the first two terms are elastic parts in macroscopic and microscopic scale respectively. \( f_v \) is the fraction volume of the sites investigated, and \( f_v \sigma : \dot{\varepsilon}^{an} + f_v \sigma : \dot{\varepsilon}^{in} \) is the term for dissipated energy. Note that \( f_v \sigma : \dot{\varepsilon}^{in} \) contains the stored energy term, and \( \bar{\Sigma} \) and \( \bar{E} \) are defined below considering a heterogenous volume \( V \) in the vicinity of maximum imposed stress and local elastic moduli \( c(r) \). Let’s define a displacement \( u^d = \bar{E}. \chi \) inside the boundary \( \partial V \) which \( \bar{E} \) is uniform in the boundary, Averaging the local stress and strain and implementing of scale transition theory over the representative volume element (RVE), macroscopic stress and strain can be described [38].

\[ \bar{E} = \frac{1}{V} \int_V \varepsilon(r) dV \]  

(17)

\[ \bar{\Sigma} = \frac{1}{V} \int_V \sigma(r) dV \]  

(18)

As mentioned, the movements of dislocations in anelastic deformation are recoverable thermodynamic actions, and can be viewed as the dislocation of a spring with viscous damping. However, the deformation in inelastic movements will result in multiplication and grow in dislocations which further results in material’s fatigue degradation. The anelastic behavior can be
modeled using Kelvin-Voigt type element (Fig.2) which models the system based on a viscous damping and spring stiffness. Therefore, anelastic stress can be defined using Eq. 19 considering elastic spring like movement un which spring critical load is not exceeded [36].

\[ \sigma = k\varepsilon^an + \mu\dot{\varepsilon}^an \]  

(19)

\( k \) and \( \mu \) are the local elastic shear modulus and local viscosity respectively. Eq. 20 describes anelastic cyclic stress having mean stress and stress amplitude and frequency. Please note that in the current work R-ratio and mean stress are considered to be -1 and 0 respectively. However, Eq. 20 gives a generalized description of anelastic stress.

\[ \sigma = \sigma_m + \sigma_a \sin (2\pi ft + \varphi) \]  

(20)

In which \( \sigma_m \) and \( \sigma_a \) are mean stress and stress amplitude respectively. Taking Eq. 19 and Eq. 20 into account, one can integrate the anelastic energy dissipation over one cyclic loading (Eq.21). Eq. 22 gives a mathematical model for anelastic dissipation per cycle [36].

\[ \bar{d}_{an} = \frac{\mu}{T_s} \int_{t}^{t+T_s} \sigma : \varepsilon^an \, dt \]  

(21)

\[ \bar{d}_{an} = \frac{2\mu(\pi f \sigma_a)^2}{k^2 + 4\pi^2 f^2 \mu^2} \]  

(22)

Where \( k \) can be defined as \( \frac{E}{2(1+v)} \). Eq. 22 suggests that the dissipated energy due to anelastic behavior is related to the stress amplitude and frequency of the loading and is independent of the mean stress.
3. Fatigue Endurance Limit and Internal Friction

Conventionally, the Endurance limit or fatigue limit is defined as the stress below which cyclic loading will not result in failure of the material. In other words, stress level is not enough at fatigue limit for the initialization and propagation of crack due to fatigue loading. Therefore, the material can undergo an infinite number of cycles. Although no sign of damage is detected at this level, there is a measurable temperature rise due to cyclic loading. Studies have reported heat dissipation under the fatigue limit [39].

In general, dissipated heat under fatigue results from several reversible and irreversible phenomena in the material. So, only the portion of dissipated heat related to the irreversible plastic dissipation is responsible for crack initiation [40]. The remained part is dissipated to the environment without causing any damage.[41]. Damage accumulation theory propose that the accumulation of micro cracks and microplastic dissipation is the result of cyclic loading [4, 5, 42]. According to this theory, this accumulative damage is the consequence of the irreversible dissipated process. The non-damaging part of dissipated energy is referred to as internal friction. Since internal friction has no role in initiating crack, its value has to be deducted from the total dissipated energy. From the non-damaging phenomena involving internal friction, dislocation oscillation can be accounted [43].

Under the fatigue limit, internal friction is the reason for the dissipated heat, and since it is not responsible for crack initiation, it has to be deducted from the total plastic strain energy generated during fatigue [44]. The effect of internal friction has been studied in the fracture fatigue entropy by Mehdizadeh and Khonsari [35], and it was shown that for accurately measuring the accumulated entropy of fatigue, internal friction has to be eliminated from the total dissipated heat. In another research, Mehdizadeh et al. [45] confirmed the importance of excluding internal friction from the total dissipated heat when the material undergoes torsional loading. Guo et al. [4] also proposed a method to find the internal friction and found an approach for fatigue life prediction based on energy dissipation as an indicator of fatigue damage.

To eliminate the internal friction effect, Eq. 14 is rewritten as Eq. 23.

$$\dot{w}_p = \rho c (R_\theta - R_{\theta l})$$  \hspace{1cm} (23)

where $R_{\theta l}$ is the slope of temperature rise at beginning of cyclic loading at fatigue limit. Using Eq. 23 in Eq. 12 yields Eq. 24.

$$\gamma_i = \frac{\dot{w}_p}{\dot{T}_i} t_i = \frac{N_i \rho c (R_\theta - R_{\theta l})}{f \dot{T}_i}$$  \hspace{1cm} (24)
4. Material and Specimen

The utilized specimens for this study are made of stainless steel 304 bars cut by a wire EDM machine. All the test specimens are polished with sandpapers, starting from grade 800, followed by 1000 and 1500 grits and the surface roughness is to be lower than 2 microns. The thickness of the test specimen is 1.82mm. Fig. 3 shows the geometry of the specimen.

Table 1 Material properties of the test specimen (Stainless steel 304)

| Hardness                  | Elastic modulus (GPa) | $\sigma_0$ (MPa) | $k$ (W/mK) |
|---------------------------|-----------------------|-----------------|-------------|
| Rockwell B85 (Medium)     | 200                   | 207             | 16.2        |

Table 2 Composition of the test specimens' material (Stainless steel 304)

| Element | Percentage | Element | Percentage |
|---------|------------|---------|------------|
| Nickel  | 8-15%      | Sulfur  | 0-0.35%    |
| Carbon  | 0-0.08%    | Molybdenum | 0-2.5%    |
|       |          |       |       |
|-------|----------|-------|-------|
| **Manganese** | 0.30-0.90% | **Chromium** | 17.5-24% |
| **Phosphorous** | 0-0.2% | **Iron** | 53.48-74.5% |
| **Silicon** | 0-1% | **Iron** | **** |

5. Experimental Procedure

Crack initiation tests are performed using a servo-hydraulic test machine with a maximum axial load capacity of 50kN (Fig. 4). A Flir A615 records evolution temperature. The thermal image sizes are 640×480 pixels. To see details of surface temperature, a close-up IR lens (16mm×12mm) 1.5X (25μm) is mounted on the IR camera. The camera has a temperature sensitivity of 50 mK with a maximum capturing rate of 200 frames per second.

The specimen is monitored from the back surface by a 40-1000X digital microscope to detect the initiated crack. Once a crack is observed, several images are captured for the rest of the fatigue test, each related to several cycles passed from the beginning of the test. The IR camera captures 16-bit thermal images from the specimen's front surface, which is painted black for accurate temperature recording.
6. Results and Discussion

Fig. 5 shows the maximum surface temperature for the specimen of 0.5 and 3mm notch radius from the beginning to the final fracture of the specimen. As it can be observed the steady state temperature and life are affected by imposed stress and radius of the notch which affects the stress concentration ratio.
Figure 5 Temperature evolution for tests stress 170 and 190MPa for specimen with R=0.5 and 3 mm

Fig. 6 shows the thermal images of the specimen under cyclic load for mean stress of 170 and 190 MPa or specimen with 0.5 and 3 mm notch radii.
Figure 6 Thermal images of tests for 170 and 190MPa with R=0.5 and 3mm

Fig. 7 shows the microscope images of the specimen under cyclic load for mean stress of 170 and 190 MPa or sample with 0.5 and 3 mm notch radius. As illustrated, the crack is initiated from the tip of the notch. The stress concentration is calculated using ANSYS. The stress concentration value for specimen with R=3 mm is 3.25 and for R=0.5 mm is 6.31 (See appendix).

Figure 7 Microscope images of tests for 170 and 190MPa with R=0.5 and 3mm

In Fig. 8, The dissipated heat is depicted as the function of the longitudinal coordinate on the specimen. As observed, $Q$ increases by moving close to the notch root, rapidly dropping at locations near the notch tip. The maximum value of $Q$ is around 0.15mm from the notch's root.
Fig. 8 depicts the value of the accumulated entropy for crack initiation in different lives for two specimens with R=0.5 and 3mm notch root radii. To measure the accumulated entropy for crack initiation, \( \gamma_i \), Eq. 24 is used. Dissipated heat is the maximum value for each test based on Fig. 8. To find \( R_{\delta l} \), a series of experiments are carried out to determine the fatigue limit for both specimens with R=0.5mm and R=3mm. The fatigue limit was 132 MPa for the specimens of R=0.5mm and 165 for R=3mm. Dissipated heat related to the fatigue limit is 0.147 J/m³/cycle for R=0.5mm and 0.327 J/m³/cycle for R=3mm.
Figure 9 Accumulated entropy for crack initiation of the specimen with R= 0.5 and 3mm.

Fig. 10 shows the cycles to crack initiation for different stresses of specimen of 0.5 and 3mm notch root radius. It is seen that the difference between crack initiation cycles decreases with mean stress. For instance, the crack initiation cycle in specimens with R=3mm is around three times the same stress in specimens with R=0.5mm. While, in 160 MPa stress level, crack initiation cycle is almost ten times more in specimen with R=3mm.
It is necessary to find the stress limit for the specimen of different notch root radii to use Eq. 24. To this end, a series of fatigue tests are performed with reducing stress level. The fatigue limit is set for each specimen when no crack is initiated after running the test up to 20 million cycles. The stress limit for R= 0.5mm specimen is 120 MPa. For this stress limit $R_\theta$ is 0.147 K/ cycle. The fatigue limit is 160 MPa for R=3mm specimen and $R_\theta$ related to that stress level is 0.327 K/ cycle.

Figure 10 Stress-life of crack initiation for tests of the specimen with R= 0.5 and 3mm.
Fig. 11 shows the dissipated heat per volume for different mean stresses and comparison with Rigon [46]. The value for dissipated heat found in the present study is close to the reported ones by Rigon.

![Graph showing dissipated energy per volume for specimen with R=0.5 and 3mm and comparison with Rigon [46].](image)

Figure 11 Dissipated energy per volume for specimen with R=0.5 and 3mm and comparison with Rigon [46].

Fig. 12 shows $\bar{T}_i$ for two specimens with 0.5 and 3mm notch root radii. As expected, the temperature for crack initiation increases with cycles to initiation. Also, $\bar{T}_i$ is higher for R=3mm.
7. Conclusions

To find a relation between the accumulated entropy for crack initiation in the severely notched specimen, completely reversed, constant amplitude stress-controlled fatigue tests have been performed on cold rolled notched specimen made of stainless steel 304 with notched radii of 3 and 0.5mm. The dissipated heat $Q$ at the notch tip was measured via an infrared camera with 25μm/pixel to calculate the entropy generation. The initial temperature rise slope, $R_\theta$, was measured to find $Q$. The value of dissipated heat from the notched specimen was evaluated by available data in a study based on the reported value of mean stress and notch root radius. Several images were captured for each test from the back of the specimen after a visible macro crack was initiated using a digital microscope. The microscope pictures were used to find the number of cycles at which crack length reaches 500μm. The accumulated entropy from the beginning of the axial load fatigue test until the crack lengths are 500μm was calculated.

Anelastic deformation as a recoverable phenomenon that is responsible for dissipated energy due to internal friction, has been introduced and a mathematical model is presented to describe such deformations based on Kelvin-Voigt type element. The dissipated heat in the fatigue limit was deducted from the total heat dissipation of tests in different stress levels to find the entropy. Accumulated entropy for crack initiation is found for notched specimens with two different root
radii. The experimental results show that accumulated entropy for crack initiation is constant for different root radii and stress concentrations in different stress levels.

8. Appendix

The specimen is simulated in ANSYS Workbench 19 to find stress concentration for different notch root radiiuses. A series of mesh refinement is carried out to find the value of stress concentration. One end of the specimen is considered fixed with uniform axial force applied on the other end. The acting boundary condition and pressure location are the same as the grips' location in the experiment. (Fig. A.1) For finite element analysis (FEA) simulations and stress concentration calculations, the ANSYS Workbench software package is utilized. The presented specimens' geometry is created in SpaceClaim, and the static structural-mechanical ANSYS system is used to mesh the geometry, apply forces, and define the boundary conditions. SOLID186 is an ANSYS element type that is used for meshing. Mesh analysis is carried out, and the mesh quality is studied to ensure mesh independence. It's found that (for R=0.5mm case), a total of 5396 elements and 38675 nodes are suitable. A static (steady-state) structural analysis is performed on the model by fixing one end of the specimen and applying a uniformly distributed tension load to the other end. A path is specified as a construction geometry starting from the notches' roots' center point to the opposite side of the specimen. The normal stresses are mapped along the path and are then tabulated and utilized to calculate the stress concentrations.

Figure A.1. The normal stress distribution in a specimen simulated in ANSYS
9. References

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