Adaptive notch filter based on Lorentz force magnetic bearing for the suppression of imbalance vibration

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Abstract. This study proposes an adaptive control approach based on Lorentz force magnetic bearing (LFMB) for the suppression of imbalance vibration in a magnetically suspended control and sensitive gyro (MSCSG). The mathematical model of the imbalance vibration in LFMB-rotor system with the proportional-integral-derivative (PID) plus filters cross feedback control law is firstly established. Due to the synchronous characteristics of the rotor imbalance disturbance and the linear characteristics of the LFMB, an adaptive notch filter based on LFMB is then proposed to plug into the LFMB-rotor controller to eliminate synchronous disturbance. Finally, experimental results demonstrate the capability of the proposed method to suppress rotor imbalance vibration.

1. Introduction
The magnetically suspended control and sensitive gyro (MSCSG) [1] is a kind of new-concept gyro. It combines the function of moment output in magnetically suspended control momentum gyroscope (MSCMG) and the function of attitude sensing in rotor-type rate gyroscope. MSCSG integrates control and sensing instruments into one device, reducing the volume, weight, power consumption and cost of the attitude control system significantly. Accurate attitude control is required to minimize disturbance for obtaining high-precision control and measurement. Besides, different from MSCMG that tilts rotor by indirect-drive gimballing torque [2-3], MSCSG uses Lorentz force magnetic bearing (LFMB) as tilting bearing to produce direct-drive tilting torque on the rotor system, which has the advantages of no contact, no delay and is helpful for torque output with high-precision and high bandwidth. The function of control and sensing in MSCSG requires the ability to suppress disturbances. However, the synchronous vibration caused by the residual imbalance of a high-speed rotor seriously degrades the performance of the MSCSG. So the vibration generated by rotary rotor should be as small as possible. The LFMB makes it possible to restrain the vibration in real time because of its linear characteristic and controllability.

There have been numerous ways proposed to deal with imbalance vibration problems of active magnetic bearing (AMB) rotor system. Matsumura uses state-space approach to achieve a rotation of the rotor around its axis of inertia [4]. Zhang realizes the rotation about the principal axis of inertia by identifying the imbalance parameters [5]. Chen constructed a double closed loop to suppress the imbalance vibration by adding a feedback from measured displacement to control current [6]. Similarly, Tang put forward an approach with the switches to rotor speed in the double closed loop [7]. Besides, an autobalancing control scheme using LMS adaptive feedforward compensation to suppress the synchronous vibration [8]. Modern intelligent control algorithms have also been imported into vibration control recently, such as sliding mode control [9], H∞ robust control [10], algebraic identification method [11], and so on, which need large allocations of computer resources and are hard...
to imported into practical application. In a word, most of the above works only focus on the AMB rotor system in MSCMG, which takes reluctance force-type magnetic bearing as torquer, and possess importance on compensating the imbalance vibration resulting from the current and the displacement stiffness at the same time. However, the above methods are not suitable for MSCSG, which takes LFMB as torquer and doesn’t suffer from displacement stiffness. Although several studies have been exploited to enhance the capabilities of generating torques for LFMB [12-13], few efforts have attempted to address imbalance vibration in the LFMB-rotor system.

In this letter, the solution is oriented to MSCSG, of which the rotor is tilted by LFMB. The present work proposes an adaptive notch filter that is based on LFMB to suppress the vibration, and the filter is incorporated with the controller to remove the synchronous components caused by rotor imbalance. Comparative experiments performed on the LFMB-rotor system in MSCSG show the capability of the proposed method to suppress rotor imbalance vibration.

2. Model of the LFMB-rotor system with rotor imbalance in the MSCSG

The MSCSG is designed and constructed for the first time in our lab [14]. The MSCSG consists of a rotor, an LFMB, radial magnetic bearings, axial magnetic bearings, sensors, a high-speed electric motor, and a gyro room, as is illustrated in Figure 1. Radial and axial magnetic bearings control the rotor translational motion along two radial degrees of freedom (DOFs) and the axial direction. The high-speed motor drives the rotor to spin around its axis at high speed. The LFMB functions as a stator and tilts the rotor to output a 2-DOF micro-gimbal moment. The sensors are used to detect the position of the rotor in 5 DOFs, and the gyro room is functioned as the foundation bed of the motor, bearings and sensors.

The tilting control of the MSCSG rotor is realized by the action of the LFMB. As shown in Figure 2, the LFMB is arranged at the outer rim of the MSCSG rotor. External and internal permanent magnets in the two layers generate close magnetic flux, which is represented by the dotted line in the figure. Ampere’s Law states that the linear force produced by LFMB is:

\[ F = 2nBL \]  \hspace{1cm} (1)

where, \( n \) is the number of coil turns, \( B \) is the magnetic strength, \( I \) is the current, and \( L \) is the coil length. Therefore, the linear torque generated by LFMB is represented as:

\[
\begin{align*}
  p_x &= 2 \times 2nBL_i_x \times l_x = 4nBL_i_x l_x \\
  p_y &= 2 \times 2nBL_i_y \times l_y = 4nBL_i_y l_y 
\end{align*}
\]  \hspace{1cm} (2)

where, \( l_x \) is the distance from the rotor center to LFMB, and \( i_x \) and \( i_y \) represent the excitation current components that tilt the rotor around the \( x \)- and \( y \)-axes, respectively.

![Figure 1 Configuration of the designed MSCSG](image)

Rotor imbalance results from the discrepancy between the geometric axis and the inertial axis of the rotor, as shown in Figure 2. When the mass distribution of the rotor is imbalanced, the relationship of the rotor tilting angles between the inertial and the geometric frame can be expressed as follows:
where, $\alpha_i$ and $\beta_i$ are two degree-of-freedom (DOF) radial tilting angles in the inertial frame; $\alpha$ and $\beta$ are the angles in the geometric frame; $\delta$ is the inclination angle between the two axes; and $\varphi$ is the initial phase of the dynamic imbalance.

Substituting (3) and (6) into (5) yields the following:

$$
\begin{align*}
J_x \ddot{\alpha}_i + J_x \Omega \dot{\beta}_i &= 4nBLl_i \alpha_t \\
J_y \ddot{\beta}_i - J_y \Omega \dot{\alpha}_i &= 4nBLl_i \beta_t 
\end{align*}
$$

where the control currents $i_a$ and $i_b$ satisfy:

$$
\begin{align*}
i_a &= -k_g g_w [g_{PID} \alpha(t) + g_{sa} \beta(t)] \\
i_b &= -k_g g_w [g_{PID} \beta(t) - g_{sa} \alpha(t)]
\end{align*}
$$

where, $k_g$ is the gain of the sensor. $g_w$, $g_{PID}$, and $g_{sa}$ are the transfer function transformation operator of the power amplifier, antialiasing filter, proportional–integral–differential(PID) controller, and cross-feedback controller, respectively.

Substituting (7) to Laplace transform yields:

$$
\begin{align*}
J_s^2 \alpha(s) + J_s \Omega \beta(s) &= -4nLl_k g_w \left\{g_{PID} \alpha(s) - g_w \Delta \alpha(s) \right\} \\
J_s^2 \beta(s) - J_s \Omega \alpha(s) &= -4nLl_k g_w \left\{g_{PID} \beta(s) - g_w \Delta \beta(s) \right\}
\end{align*}
$$

where, the PID controller is expressed as:

$$
\begin{align*}
i_a &= -k_g g_w \left\{g_{PID} \alpha(s) - g_w \Delta \alpha(s) \right\} \\
i_b &= -k_g g_w \left\{g_{PID} \beta(s) - g_w \Delta \beta(s) \right\}
\end{align*}
$$
\[ g_{\text{PID}}(s) = k_p + k_i \frac{1}{s} + k_d s \]
\[ g_n(s) = k_i \left( \frac{s}{s + \omega_i} \right)^2 - k_d \left( \frac{\omega_d}{s + \omega_i} \right)^2 \]

where \( k_p, k_i \) and \( k_d \) are the proportional, integral, differential coefficients of the PID controller, \( k_i \) and \( k_d \) are the cross coefficients of the cross-feedback controller, \( \omega_i \) and \( \omega_d \) are the cut-off frequency of the low-pass and high-pass filters in the cross-feedback controller.

In these equations, define \( \theta(s) = a(s) + j\beta(s) \), \( \Delta(s) = \Delta_\text{m}(s) + j\Delta_\text{r}(s) \), where \( j \) is a complex unit. Combining \( \Delta(s) \) and (12) yields:
\[ \Delta(s) = \frac{\delta e^{j\omega}}{s - j\Omega} \quad (9) \]

The following equation is obtained by accounting for the antisymmetric characteristics of (8), multiplying the second equation by \( j \), and then adding the result to the first equation yields the following equation:
\[ J\omega^2 \dot{\theta}(s) - jJ(I\omega \theta(s) = -4n\Omega \dot{\theta}(s) = g_n(s)g_\text{m}(s)[g_{\text{PID}}(s) + g_n(s)][\theta(s) - \Delta(s)] \quad (10) \]

Then, Equation (10) can be equivalent to a feedback control system, and its corresponding controlled plant can be respectively expressed as:
\[ G(s) = \frac{1}{s(JJ - jJ\Omega)} \quad (11) \]

So the real-coefficient two-input–two-output LFMB-rotor system is transferred into a complex-coefficient single-input–single-output system, as is shown in Figure 2.

**3. Suppression of imbalance vibration with an adaptive notch filter based on LFMB**

According to Equation (9), the rotor imbalance component acts as the periodic disturbance synchronized with rotational speed, thereby resulting in vibration in the radial tilting DOF. In addition, the vibration torque is generated by the control current of LFMB through the controller, power amplifier, and antialiasing filter. The synchronous component in the control current of LFMB should be eliminated so that the vibration can be suppressed.

The adaptive notch filter is a common technique to extract periodic signals. Therefore, advantageous features of the notch filter based on LFMB can be exploited as follows:

1. Because of the linear characteristics in Equation (2), the LFMB has the advantages of the high-precision and high-bandwidth;
2. The structure of the adaptive notch filter is simple, and this filter is easy for engineering realization;
3. The amount of parameters in the filter is small, and these parameters can be adjusted independently.

Therefore, an adaptive notch filter based on LFMB is proposed to obtain the synchronous components and to restrain the synchronous vibration torque, and the filter can be expressed as follows:
\[ g_n(s) = \frac{s^2 + \Omega^2}{s^2 + \alpha \Omega + \Omega^2} \quad (12) \]
where, \( \varepsilon \) is the damping coefficient. The total structure of the proposed LFMB-rotor control system that considers rotor imbalance vibration is shown in Figure 4.

Figure 4 Block diagram of the proposed LFMB-rotor control system for vibration suppression

4. Experimental results

Experiments are conducted on the LFMB-rotor system in MSCSG to verify that the proposed approach can suppress the imbalance vibration. As shown in Figure 4, the experimental setup is composed of the MSCSG, power source, TMS320C6713 controller, amplifier, and oscilloscope. The rotor of the MSCSG rotates at a steady speed of 4200 r/min. To validate the effectiveness of the proposed method, experimental results obtained with and without the vibration suppression method are compared. All the parameters of the LFMB-rotor system are listed in Table 1. Figure 4 shows the records printed by oscilloscopes, and the records include the time-domain tilt angle signals about two radial axes. In Figure 4, every vertical grid represents 0.008° for the time-domain signals, and every horizontal grid represents 0.1s. We can find that the peak–peak amplitude of the tilting angle vibration response is reduced from 0.144° to 0.032° when the LFMB-based adaptive notch filter is incorporated into the control loop, and 77.8% of the original vibration is removed. These results can be attributed to the removal of synchronous disturbance and the suppression of the imbalance vibration torque by the adaptive notch filter. The experiment results illustrate the validity of the proposed approach.

Table 1. Parameters of the rotor system

| Parameter | Value          | Parameter | Value          |
|-----------|----------------|-----------|----------------|
| \( J_z \) | 0.0238 kg·m²   | \( J_r \) | 0.0097 kg·m²   |
| \( k_p \) | 12.4           | \( k_i \) | 10.2           |
| \( k_d \) | 0.28           | \( B \)   | 0.4 T          |
| \( n \)  | 200            | \( l_o \) | 0.059 m        |
| \( L \)  | 0.1158 m       | \( k_c \) | 10300 V·m⁻¹    |
| \( l_r \) | 0.078 m        | \( k_a \) | 0.24           |
| \( \omega_k \) | 340 Hz     | \( \omega_p \) | 40 Hz       |
| \( \varepsilon \) | 0.12      | \( k_i \) | 0.09           |
| \( k_s \) | 0.11           | \( \Omega \) | 70 Hz         |
5. Conclusion
In this letter, an LFMB-based adaptive notch filter is proposed for the suppression of imbalance vibration in an MSCSG. When inserted into the control loop, the proposed adaptive notch filter can remove the synchronous disturbance components from the closed-loop control system. Experiments
show that the imbalance vibration is considerably suppressed by the proposed LFMB-based adaptive notch filter.

References

[1] C.F. Xia, Y.W. Cai, Y. Ren. Stability analysis method with extended double frequency Bode diagram for rotor of MSCSG. Chin. J. Aeronaut., 39, 2(2018)
[2] B. Han, S. Zheng S, Z. Wang Z, et al. Design, modeling, fabrication, and test of a large-scale single-gimbal magnetically suspended control moment gyro. IEEE Tran Ind Electron, 62, 12(2015)
[3] Y. Ren, D. Su, J. Fang. Whirling modes stability criterion for a magnetically suspended flywheel rotor with significant gyroscopic effects and bending modes. IEEE Trans. Power Electron. 28, 12(2013)
[4] T. Schuhmann, W. Hofmann, R. Werner: Improving operational performance of active magnetic bearings using kalman filter and state feedback control. IEEE Tran. Ind. Electron., 59, 2(2011)
[5] X. Zhang, T. Shin, L. Li, et al. Precision control for rotation about estimated center of inertia of spindle supported by radial magnetic bearing. MECH SY, 47, 1(2004).
[6] Q. Chen, G. Liu, S.Q. Zheng. Suppression of imbalance vibration for AMBs controlled driveline system using double-loop structure. J. Sound Vibr., 337, 2(2015)
[7] J. Tang, B. Liu, J. Fang, et al. Suppression of vibration caused by residual unbalance of rotor for magnetically suspended flywheel. J. Sound Vibr., 19, 13(2013)
[8] M. Xiang, T. Wei. ‘Autobalancing of high-speed rotors suspended by magnetic bearings using LMS adaptive feedforward compensation’. J. Sound Vibr., 20, 9(2014)
[9] S.Y. Chen, F.J. Lin. Robust nonsingular terminal sliding-mode control for nonlinear magnetic bearing system. IEEE Trans. Control Syst. Technol., 19, 3(2011)
[10] R. Jastrzebski, K. Hynynen, A. Smirnov. H∞ control of active magnetic suspension. Mech. Syst. Signal Process., 24, 4(2010)
[11] C. Beltran, F. Silva, G. Arias. Active unbalance control of rotor systems using on-line algebraic identification methods’, Asian J. Control, 15, 6(2013)
[12] J.C. Fang, X.B. Xu, J.J. Xie. Active vibration control of rotor imbalance in active magnetic bearing systems. J.Vibr. Control, 21, 4(2013)
[13] B. Xiang, J.Q. Tang. Suspension and titling of vernier-gimballing magnetically suspended flywheel with conical magnetic bearing and Lorentz magnetic bearing. Mechatronics, 28, 7(2015)
[14] Ren Y, Chen X, Cai Y, et al. Attitude-rate measurement and control integration using magnetically suspended control and sensitive gyroscopes. IEEE Tran Ind Electron, 65, 6(2018)
[15] S. Zheng, J. Xie, C. Ma, et al. Improving dynamic response of AMB systems in control moment gyros based on modified integral feedforward method. IEEE/ASME Trans. Mechatronics, 22, 5(2017)