Test of the Porter-Thomas distribution with the total cross section autocorrelation function

E. D. Davis

Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202, USA
Triangle Universities Nuclear Laboratory, Durham, North Carolina 27708-0308, USA

Abstract

On the basis of experiments conducted in the 1970s and earlier, it is believed that, in a compound nucleus (CN), fluctuations in transition strengths are drawn from a Porter-Thomas distribution (PTD), a $\chi^2$ distribution of one degree of freedom. However, work done post-1990 at the Los Alamos Neutron Science Center (LANSCE) and Oak Ridge Electron Linear Accelerator (ORELA) facilities has yielded instances of neutron resonance data sets of superior quality that are inconsistent with the PTD. In view of the importance of the status of the PTD to the foundations of statistical reaction theory, the question arises whether other data acquired at these facilities can be mined for evidence of deviations from the PTD? To date, the focus has been on data taken in the regime of isolated resonances. In this letter, arguments are put forward in support of the analysis of measurements of the total cross section in the regime where resonances are weakly overlapping.

Keywords: Compound nucleus, Porter-Thomas distribution, cross section autocorrelation function

The Porter-Thomas distribution [1] for fluctuations in decay mode strengths of excited nuclei is an integral feature of conventional statistical models of the compound nucleus [2]. Nevertheless, experiments conducted at LANSCE and ORELA in the last two decades have identified resonance data sets that are almost certainly statistically inconsistent with the PTD [3-6]. Moreover, subsequent analyses [7, 8] of the reduced neutron widths in the nuclear data ensemble seem to overturn the long held belief that it furnishes persuasive evidence for the applicability of the Gaussian orthogonal ensemble (GOE) of Hamiltonian matrices [9] in the description of CN fluctuation properties. There have been several attempts to reconcile these new findings with the standard statistical models of CN processes [10-22], but there is a consensus [23-25] that more data is needed to guide theoretical considerations.

The analyses of the experiments cited above have been confined to the resolved resonance regime. However, the measurements performed extend into the unresolved resonance regime, and invariably include data on total cross sections [26]. This archived data could be used to test statistical reaction models via appropriate correlation functions. The suggested tool for the analysis of the data on the total cross section $\sigma_{\text{tot}}(E)$ is the autocorrelation function

$$R_{\text{tot}}(\varepsilon) = \frac{\langle \sigma_{\text{tot}}(E + \frac{1}{2}\varepsilon)\sigma_{\text{tot}}(E - \frac{1}{2}\varepsilon) \rangle}{\langle \sigma_{\text{tot}} \rangle^2} - 1,$$

where the angle brackets denote averages over the scattering energy. [Implicit in Eq. (1) is the assumption of stationarity of the averages, i.e., they are independent of the energy $E$.] With a few exceptions (viz., Ref. [27]), autocorrelation function studies of nuclear reactions have previously been confined to the regime of strongly overlapping resonances [2]. In the present work, it is advocated that $R_{\text{tot}}(\varepsilon)$ be determined in the weakly overlapping resonance regime.

Why consider a correlation function involving the total cross section as opposed to any other type of cross section? Via the optical theorem, the fluctuations probed by $R_{\text{tot}}(\varepsilon)$ can be related to the two-point measure

$$C_{ab}(\varepsilon) = \langle \sigma_{aa}^{\text{fl}}(E + \frac{1}{2}\varepsilon)S_{ab}^{\text{n}}(E - \frac{1}{2}\varepsilon) \rangle$$

of fluctuations in elastic elements of the $S$-matrix ($S_{ab}^{\text{n}} \equiv S - \langle S \rangle$). Of significance in the present context is that the contributions to $R_{\text{tot}}(\varepsilon)$ for which the channels $a$ and $b$ in $C_{ab}(\varepsilon)$ coincide involve the variance of partial widths for the entrance channel. Thus, there is a direct link between $R_{\text{tot}}(\varepsilon)$ for neutron-induced reactions and fluctuations in reduced neutron widths. This aspect of $R_{\text{tot}}(\varepsilon)$ was recognized by Ericson in his seminal treatment of cross section fluctuations [28].

Another important feature of $R_{\text{tot}}(\varepsilon)$ concerns its computation within models of compound nucleus reactions for medium-weight and heavy nuclei. At present, the gold standard for such models is a stochastic treatment of resonance reactions invoking the GOE of Hamiltonian matrices, which is outlined in Sec. IV.B of Ref. [2] and is hereafter referred to as the Heidelberg model. Since the
ground-breaking work of Verbaarschot et al. [29, 30] on the exact calculation of averages within the Heidelberg model in the limit of infinitely many resonances, it has been known that $C_{a b}(\varepsilon)$ and, hence, $R_{\text{tot}}(\varepsilon)$ can be expressed as three dimensional integrals. Not only do these integral representations hold for any number of open channels and in all resonance regimes (from isolated to strongly overlapping), the input required for their evaluation is limited to the average $S$-matrix itself and, when $\varepsilon \neq 0$, the average level spacing (for each spin). Numerical evaluation of the integrals is demanding but feasible, and has been carried out in a number of comparisons of various statistical approaches to low-energy compound nucleus reactions [31, 32].

In principle, then, parameter-free predictions for $R_{\text{tot}}(\varepsilon)$ are possible in the Heidelberg model, and any large differences between these predictions and data cannot be attributed to approximations in the evaluation of $R_{\text{tot}}(\varepsilon)$ within the model. It is significant that the quite different maximum entropy approach to statistical nuclear reactions [32, 33] is known to yield results for averages like those in $R_{\text{tot}}(0)$ which are in complete agreement with the corresponding results obtained within the Heidelberg model when the number of resonances is infinite [33]. Use of the Heidelberg model is tantamount to the adoption of the most probable unbiased distribution of $S$-matrix fluctuations consistent with unitarity and causality. Non-generic dynamical effects are not accommodated.

What is the potential size of any discrepancy between empirical values of $R_{\text{tot}}(\varepsilon)$ and theoretical estimates deduced from the Heidelberg model? To address this issue, it is advantageous to consider the approximation of $C_{a b}(\varepsilon)$ in the statistical Breit-Wigner (SBW) model using the scheme of calculation laid out in Ref. [31]. Recent work along these lines in the analysis of fluctuations in the $^{235}$U fission cross section has shown that the SBW model can yield results which account quantitatively for the features in the isolated resonance regime of the cross section autocorrelation function studied [27].

In the SBW model, it is possible to relax the assumption that partial widths are drawn from the PTD. Instead, guided by the empirical characterisation of data on partial widths in Ref. [3], it is assumed that partial widths are drawn from a $\chi^2$ distribution of $\nu$ degrees of freedom. In the weakly overlapping resonance regime, it is found that the dominant contribution to $C_{a b}(0)$ is

$$C_{a b}(0) = \left(1 + \frac{2}{\nu} \delta_{a b}\right) T_a T_b I_a^{(0)}, \quad \text{(3)}$$

where the transmission coefficients $T_c = 1 - |(S_{c c})|^2$, and

$$I_a^{(0)} = \int_0^\infty \frac{\prod_{\tau} \left(1 + \frac{2}{\nu} T_c \tau\right)^{-\nu/2}}{(1 + \frac{2}{\nu} T_b \tau)} d\tau, \quad \text{(4)}$$

(The product in the numerator of the integrand above is over all open channels.)

Figure 1: The relative change of the dominant contribution to $C_{a a}(0)$ in the SBW mode [i.e., $\delta$ in Eq. (5)] versus $\nu$ for different choices of the number $\Lambda$ of open channels: $\Lambda = 5$ (dotted line), $\Lambda = 10$ (dashed line), $\Lambda = 20$ (dot-dashed line), and $\Lambda = \infty$ (solid line).

The $\nu$ dependence in Eq. (3) is encouraging. Figure 1 displays the relative change

$$\delta \equiv \frac{C_{d a}^{(d)}(0) - C_{d a}^{(d)}(0)_{\nu = 1}}{C_{d a}^{(d)}(0)_{\nu = 1}} - 1 \quad \text{(5)}$$

in the dominant contribution to $C_{a a}(0)$ as $\nu$ ranges from its value in the Porter-Thomas limit ($\nu = 1$) through values implied by the analysis of Pt neutron width data ($\nu \approx \frac{1}{2}$). [In Eq. (5), the value of the dominant contribution to $C_{a a}(0)$ for arbitrary $\nu$ is divided by its value for $\nu = 1.$]

In generating Fig. 1, all transmission coefficients have, for simplicity, been taken to be equal in all $\Lambda$ open channels, which means that $\delta$ is a function of only $\nu$ and $\Lambda$.

For values of $\nu$ comparable to those found in the statistical analysis of reduced neutron widths in Ref. [3], Fig. 1 suggests that $R_{\text{tot}}(0)$ could deviate from its value in the Heidelberg model by more than 20%. Even allowing for uncertainties in the transmission coefficients needed to evaluate the theoretical expression for $R_{\text{tot}}(0)$ within the Heidelberg model, this should be a large enough signal to warrant determination of $R_{\text{tot}}(0)$ with high quality total cross section data for weakly overlapping resonances in the unresolved resonance regime.

This letter has discussed a test of the Heidelberg model involving fluctuations in neutron-induced CN reactions. For weakly overlapping resonances, $R_{\text{tot}}(0)$ is sensitive to fluctuations in reduced neutron widths but insensitive to correlations between levels (beyond level repulsion). These properties make the study of $R_{\text{tot}}(0)$ for weakly overlapping resonances in the unresolved resonance regime a test, in effect, of the Porter-Thomas distribution. To date, there have been no investigations of this kind, but chaotic two-dimensional microwave resonator data has been used to test the Verbaarschot et al. result for $C_{a b}(\varepsilon)$ in the weakly overlapping resonance regime, and good agreement was found [4]. More recently, in a tour de force (building on
the supersymmetric methodology of Refs. 41, 42), Kumar et al. have managed to derive, within the framework of the Heidelberg model, a four dimensional integral representation for the characteristic function of $\sigma_{\alpha\beta}(E) = |S^\alpha_{\alpha\beta}(E)|^2$ when $a \neq b$ [43], which, like the result of Verbaarschot et al. for $C_{\alpha\beta}(E)$, is exact in the limit of infinitely many resonances and holds for any number of open channels and in all resonance regimes. This result for the characteristic function was used in a comparison with an experimental cross distribution inferred from weakly overlapping resonance data [44] for the reaction $^3\text{Cl}(p,\alpha)^4\text{He}$. As observed in Ref. 43, it is possible to conclude that the characteristic function is more than capable of reproducing the experimental cross distribution but there are two shortcomings to the comparison. First, the quality of the data could be significantly improved upon; only 51 of an estimated 120 or so resonances were observed in the energy interval of interest, and, more worryingly, no attempt was made to identify the resonance spins: the disturbing conclusions drawn from reduced neutron width data rest on the careful identification of resonances of a given spin and parity. The other deficiency in the comparison is the fact that adjustments to the number of effective channels and the associated transmission coefficient are made to obtain agreement. For a test of the kind contemplated in the present paper, a comparison free of fit parameters should be performed.

Acknowledgements

This work was supported in part by the US Department of Energy under Grant No. DE-FG02-97ER41042. I would like to thank Dr. P. E. Kohler for his interest in this work.

References

References

[1] C. E. Porter, R. G. Thomas, Fluctuations of Nuclear Reaction Widths, Phys. Rev. 104 (1956) 483–491. doi:10.1103/PhysRev.104.483
[2] G. E. Mitchell, A. Richter, H. A. Weidenmüller, Random matrices and chaos in nuclear physics: Nuclear reactions, Rev. Mod. Phys. 82 (2010) 2845–2901. doi:10.1103/RevModPhys.82.2845
[3] P. E. Kohler, F. Bečvář, M. Krátká, J. A. Harvey, K. H. Guber, Anomalous Fluctuations of s-wave Reduced Neutron Widths of $^{192,194}\text{Pt}$ Resonances, Phys. Rev. Lett. 105 (2010) 072502. doi:10.1103/PhysRevLett.105.072502
[4] P. E. Kohler, R. Reifarth, J. L. Ullmann, T. A. Brevedeg, J. M. O’Donnell, R. S. Rundberg, D. J. Vieira, J. M. Wouters, Arupt Change in Radiation-Width Distribution for $^{147}\text{Sm}$ Neutron Resonances, Phys. Rev. Lett. 108 (2012) 142502. doi:10.1103/PhysRevLett.108.142502
[5] P. E. Kohler, F. Bečvář, M. Krátká, K. H. Guber, J. L. Ullmann, Neutron resonance data exclude random matrix theory, Fortschr. Phys. 61 (2013) 80–94. doi:10.1002/prop.201200067
[6] P. E. Kohler, A. C. Larsen, M. Gutormsen, S. Stem, K. H. Guber, Extreme nonstatistical effects in $\gamma$ decay of $^{95}\text{Mo}$ neutron resonances, Phys. Rev. C 88 (2013) 041305. doi:10.1103/PhysRevC.88.041305
[7] P. E. Kohler, Reduced neutron widths in the nuclear data ensemble: Experiment and theory do not agree, Phys. Rev. C 84 (2011) 034312. doi:10.1103/PhysRevC.84.034312
[8] J. F. Shriner Jr., H. A. Weidenmüller, G. E. Mitchell, Uncertainties in the analysis of neutron resonance data. arXiv:1205.2439v3[nucl-th]
[9] R. U. Haq, A. Pandey, O. Bohigas, Fluctuation properties of nuclear energy levels: Do theory and experiment agree?, Phys. Rev. Lett. 48 (1982) 1086–1089. doi:10.1103/PhysRevLett.48.1086
[10] H. A. Weidenmüller, Distribution of Partial Neutron Widths for Nuclei Close to a Maximum of the Neutron Strength Function, Phys. Rev. Lett. 105 (2010) 232501. doi:10.1103/PhysRevLett.105.232501
[11] P. E. Kohler, F. Bečvář, M. Krátká, J. A. Harvey, K. H. Guber, Comment on “Distribution of Partial Neutron Widths for Nuclei Close to a Maximum of the Neutron Strength Function”. arXiv:1101.4533[nucl-th]
[12] G. L. Celardo, N. Auerbach, F. M. Izrailev, V. G. Zelevinsky, Distribution of Resonance Widths and Dynamics of Continuum Coupling, Phys. Rev. Lett. 106 (2011) 042501. doi:10.1103/PhysRevLett.106.042501
[13] A. Volya, Porter-Thomas distribution in unstable many-body systems, Phys. Rev. C 83 (2011) 044312. doi:10.1103/PhysRevC.83.044312
[14] G. Shchedrin, V. Zelevinsky, Resonance width distribution for open quantum systems, Phys. Rev. C 86 (2012) 044602. doi:10.1103/PhysRevC.86.044602
[15] D. Muñihall, Open quantum systems and random matrix theory, Phys. Rev. C 91 (2015) 014305. doi:10.1103/PhysRevC.91.014305
[16] V. Y. Fyodorov, D. V. Savin, Resonance width distribution in RMT: Weak-coupling regime beyond Porter-Thomas, Europhys. Lett. 110 (2015) 40006. doi:10.1209/0295-5075/110/40006
[17] A. Volya, H. A. Weidenmüller, V. Zelevinsky, Neutron Resonance Widths and the Porter-Thomas Distribution, Phys. Rev. Lett. 115 (2015) 052501. doi:10.1103/PhysRevLett.115.052501
[18] S. Mizutani, H. Alba, Level and width statistics of the open many-body systems, EPJ Web Conf. 122 (2016) 09005. doi:10.1051/epjconf/201612209005
[19] E. Bogomolny, Modification of the Porter-Thomas Distribution by Rank-One Interaction, Phys. Rev. Lett. 118 (2017) 022501. doi:10.1103/PhysRevLett.118.022501
[20] V. Sokolov, Neutron Resonances: Widths Distribution versus the Porter-Thomas Law, Acta Physica Polonica A 132 (2017) 17041706. doi:10.12693/physphys.132.17041706
[21] H. A. Weidenmüller, Limitations of the Porter-Thomas distribution, AIP Conf. Proc. 1912 (2017) 020021. doi:10.1063/1.5016146
[22] P. Fanto, G. F. Bertsch, Y. Alhassid, Neutron width statistics in a realistic reaction-reaction model, Phys. Rev. C 98 (2018) 014604. doi:10.1103/PhysRevC.98.014604
[23] E. S. Reich, Nuclear theory nudged, Nature 466 (2010) 1034. doi:10.1038/4661034a
[24] H. A. Weidenmüller, Open problems in applying random-matrix theory to nuclear reactions, J. Phys. G: Nucl. Part. Phys. 41 (2014) 014611. doi:10.1088/0954-3899/41/1/014611
[25] N. Auerbach, Super-radiance in Nuclear Physics, J. Phys.: Conf. Ser. 639 (2015) 012010. doi:10.1088/1742-6596/639/1/012010
[26] P. E. Kohler, K. H. Guber, Improved $^{192,194,195}\text{Pt}(n,\gamma)$ and $^{192}\text{Ir}(n,\gamma)$ astrophysical reaction rates, Phys. Rev. C 88 (2013) 055802. doi:10.1103/PhysRevC.88.055802
[27] G. F. Bertsch, D. Brown, E. D. Davis, Fluctuations in the $^{235}\text{U}(n,\gamma)$ cross section, Phys. Rev. C 98 (2018) 014611. doi:10.1103/PhysRevC.98.014611
[28] T. Ericson, A theory of fluctuations in nuclear cross sections, Ann. Phys. (N.Y.) 23 (1963) 390–414. doi:10.1016/0003-4916(63)90261-0
J. J. M. Verbaarschot, H. A. Weidenmuller, M. R. Zirnbauer, Statistical Nuclear Theory as an Anderson Model of Dimensionality Zero, Phys. Rev. Lett. 52 (1984) 1597–1600. doi:10.1103/PhysRevLett.52.1597

J. J. M. Verbaarschot, H. A. Weidenmuller, M. R. Zirnbauer, Grassmann integration in stochastic quantum physics: The case of compound-nucleus scattering, Phys. Rep. 129 (1985) 367–438. doi:10.1016/0370-1573(85)90070-5

J. J. M. Verbaarschot, Investigation of the formula for the average of two S-matrix elements in compound nucleus reactions, Ann. Phys. (N.Y.) 168 (1986) 368–386. doi:10.1016/0003-4916(86)90036-9

E. Davis, D. Boosé, On the variance of the fluctuating cross section, Phys. Lett. B 211 (4) (1988) 379–383. doi:10.1016/0370-2693(88)91879-5

S. Hilaire, C. Lagrange, A. J. Koning, Comparisons between various width fluctuation correction factors for compound nucleus reactions, Ann. Phys. (N.Y.) 306 (2003) 209–231. doi:10.1016/S0003-4916(03)00076-9

B. Dietz, H. Harney, A. Richter, F. Schäfer, H. Weidenmüller, Cross-section fluctuations in chaotic scattering, Phys. Lett. B 685 (2010) 263–269. doi:10.1016/j.physletb.2010.01.074

T. Kawano, P. Talou, H. A. Weidenmüller, Random-matrix approach to the statistical compound nuclear reaction at low energies using the Monte Carlo technique, Phys. Rev. C 92 (2015) 044617. doi:10.1103/PhysRevC.92.044617

P. A. Mello, P. Pereyra, T. H. Seligman, Information theory and statistical nuclear reactions. I. General theory and applications to few-channel problems, Ann. Phys. (N.Y.) 161 (1985) 254–275. doi:10.1016/0003-4916(85)90080-6

W. A. Friedman, P. A. Mello, Information theory and statistical nuclear reactions II. Many-channel case and Hauser-Feshbach formula, Ann. Phys. (N.Y.) 161 (1985) 276–302. doi:10.1016/0003-4916(85)90081-8

F. W. Brouwé, Generalized circular ensemble of scattering matrices for a chaotic cavity with nonideal leads, Phys. Rev. B 51 (1995) 16878–16884. doi:10.1103/PhysRevB.51.16878

T. E. O. Ericson, B. Dietz, A. Richter, Cross-section fluctuations in chaotic scattering systems, Phys. Rev. E 94 (2016) 042207. doi:10.1103/PhysRevE.94.042207

B. Dietz, T. Friedrich, H. L. Harney, M. Miski-Oglu, A. Richter, F. Schäfer, H. A. Weidenmüller, Chaotic scattering in the regime of weakly overlapping resonances, Phys. Rev. E 78 (2008) 055204. doi:10.1103/PhysRevE.78.055204

S. Kumar, A. Nock, H.-J. Sommers, T. Guhr, B. Dietz, M. Miski-Oglu, A. Richter, F. Schäfer, Distribution of Scattering Matrix Elements in Quantum Chaotic scattering, Phys. Rev. Lett. 111 (2013) 630403. doi:10.1103/PhysRevLett.111.030403

A. Nock, S. Kumar, H.-J. Sommers, T. Guhr, Distributions of off-diagonal scattering matrix elements: Exact results, Ann. Phys. (N.Y.) 342 (2014) 103–132. doi:10.1016/j.aop.2013.11.006

S. Kumar, B. Dietz, T. Guhr, A. Richter, Distribution of Off-Diagonal Cross Sections in Quantum Chaotic Scattering: Exact Results and Data Comparison, Phys. Rev. Lett. 119 (2017) 244102. doi:10.1103/PhysRevLett.119.244102

R. Clarke, E. Almqvist, E. Paul, Properties of levels excited in (p,α) reactions on O18, P31, Cl35, Cl37, K39 and Kr41, Nucl. Phys. 14 (1960) 472–497. doi:10.1016/0029-5582(60)90466-1