Strong Enhancement of Superconducting $T_c$ in Ferromagnetic Phases

Theodore R. Kirkpatrick
Dietrich Belitz
Thomas Vojta
Missouri University of Science and Technology, vojtat@mst.edu
Rajesh S. Narayanan

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It is shown that the critical temperature for spin-triplet, \( p \)-wave superconductivity mediated by spin fluctuations is generally much higher in a Heisenberg ferromagnetic phase than in a paramagnetic one, due to the coupling of the magnons to the longitudinal magnetic susceptibility. Together with the tendency of the low-temperature ferromagnetic transition in very clean Heisenberg magnets to be of first order, this qualitatively explains the phase diagram recently observed in UGe\(_2\).

![Diagram](image)

**FIG. 1.** Schematic phase diagram showing the paramagnetic (PM), ferromagnetic (FM), and superconducting (SC) phases in a temperature (T)-control parameter (CP) plane. (a) The qualitative prediction of paramagnon theory [5], and (b) the qualitative phase diagram as observed in UGe\(_2\) [3] and explained by the theory presented here. In Ref. [3], hydrostatic pressure serves as CP.
has no analog in the PM phase. We will see that this produces a superconducting transition temperature which under reasonable assumptions can easily be 50 times larger than in the PM phase.

We have included this effect in a McMillan-type $T_c$ calculation similar to the one in Ref. [5]. A representative result of our analysis is shown in Fig. 2. The solid curve (left-hand scale) represents the superconducting $T_c$ as a function of the dimensionless distance $t$ from the FM critical point. Also shown is the magnetization $M$ in the FM phase, in units of the saturation magnetization $\mu_B n$, with $n$ the electron number density and $\mu_B$ the Bohr magneton. $T_c$ is measured in units of a characteristic temperature $T_0$ that is given by either the Fermi temperature or a bandwidth, depending on the model considered. The dashed curve shows the result in the PM phase scaled by a factor of 50 (right-hand scale), and the dotted curve in the FM phase (also scaled by a factor of 50, right-hand scale) represents the result that is obtained upon neglecting the mode-mode coupling effect. Notice that the maximum $T_c$ in the FM phase is more than 50 times higher than in the PM phase. This relative difference between $T_c$ in the two phases is the important result of our analysis. The absolute values should not be taken very seriously, as calculating $T_c$ is notoriously difficult and our simple mean-field treatment is certainly not adequate for this purpose. However, the relative comparison we expect to be reliable. It predicts a pronounced asymmetry between the PM and FM phases, which in the case of UGe$_2$ means that superconductivity in the PM phase should not be expected at temperatures above at most 10 mK, in agreement with the experiment.

Also of interest is the fact that at low temperatures the FM transition in very clean itinerant Heisenberg systems is generically of first order, as has been predicted theoretically [9] and is indeed observed in UGe$_2$ [3] as well as in MnSi [10]. For the purpose of our discussion this simply means that values of $|t|$ smaller than some minimum value are not experimentally accessible.

In the remainder of this Letter we sketch the theoretical analysis that has led to these results. For an order parameter (OP) field, we choose $\mathcal{F}(x, y) = \psi(x)\psi(y)$, with $\psi(x)$ an electronic field with spin index $\sigma$ and space-time index $x$ [11]. The OP, i.e., the expectation value $\langle \mathcal{F}(x, y) \rangle = F(x - y)$, is the anomalous Green function. The orbital symmetry of the OP is still unspecified; we will later choose the $p$-wave case.

We have derived coupled equations of motion for $F$ and the normal Green function, $G$, that lead to a loop expansion for the equation of state [12]. Our model is a microscopic action $S$ with a free-electron part, $S_0$, and spin-singlet and spin-triplet interaction terms,

$$S_{\text{int}} = \frac{\Gamma}{2} \int dx [n_s(x)]^2, \quad S_s = \frac{-\Gamma_s}{2} \int dx [n_s(x)]^2.$$

Here $n_s(x)$ and $n_c(x)$ are the electronic spin and charge density fields, respectively, and $\Gamma$ and $\Gamma_s$ are the spin-triplet and spin-singlet interaction amplitudes. We assume that screening has been built into the starting action, so the interaction amplitudes are simply numbers. By putting $\Gamma_s = \Gamma$, one obtains the Hubbard model considered in Ref. [5]. The magnetic equation of state we treat in zero-loop approximation. The superconducting equation of state needs to be calculated in one-loop approximation in order to capture the spin-fluctuation induced pairing. It takes the form of linearized strong-coupling equations that are similar to those in Ref. [5]. These equations can be rewritten as an eigenvalue problem, which can then be solved numerically, using some theory for the (para)magnon propagators as input. This is the established procedure to calculate the critical temperature for phonon-mediated superconductivity [13], and it has been employed in the case of magnetically induced superconductivity or superfluidity in Refs. [2,6].

Even with a complete numerical solution of the strong-coupling equations, the superconducting $T_c$ is notoriously hard to predict. This holds a fortiori in the case of magnetically mediated superconductivity since (1) there is much less experimental information about the paramagnon propagator that could be used as input than about phonon spectra, and (2) there is no analog of Migdal’s theorem. Our ambition here is therefore not to calculate $T_c$, but rather to perform a relative comparison of $T_c$ values in the PM and FM phases, respectively. For this purpose, a simple McMillan-type approximation for $T_c$ [13] suffices. We obtain

$$T_c = T_{0}(t) \exp \left[-(1 + d_0^L + 2d_0^L)/d_0^T \right].$$

Here $T_{0}(t)$ is a temperature scale that will be specified below. Specializing to the $p$-wave case, the $d_{0,1}^{L,T}$ read

$$d_0^L = \frac{\Gamma_s N_F}{(k_F)^2} \int_0^{2k_F} dk k \left(1 - \frac{k^2}{2(k_F)^2} \right) D_{L,k}(k,i0),$$

$$d_0^T = \frac{\Gamma s N_F}{(k_F)^2} \int_0^{2k_F} dk k D_{L,k}(k,i0),$$

![FIG. 2. Superconducting $T_c$ (solid curve, left scale) as a function of the distance from the critical point $t$, and the magnetization $M$. The dashed line (right scale) shows $T_c$ in the PM phase scaled by a factor of 50, and the dotted curve (right scale) is the result in the FM phase without the mode-mode coupling effect. See the text for further explanation.](image-url)
we obtain in the PM phase, and in the limit of small wave numbers,
\[ D_{L,T}(q, i0) = \frac{1}{[t + b_{L,T}(q/2k_f)^2]}. \] (4)

In the Gaussian approximation of Ref. [14], \( b_L = b_T = 1/3 \). However, there is no reason to prefer this Gaussian approximation over any other approximation scheme. The functional form of the long-wavelength expression, Eq. (4), on the other hand, is generic. We therefore adopt Eq. (4) with \( b_{L,T} \) arbitrary coefficients of \( O(1) \). By the same reasoning, we have in the FM phase, in the limit of long wavelengths and small frequencies,

\[ D_L(q, i0) = \frac{1}{[5|t|/4 + b_L(q/2k_f)^2]}, \] (5a)
\[ D_T(q, i\Omega) = \frac{\Delta/4e_F}{(1 - \tau)^2} \left( \frac{1}{i\Omega/4e_F + (\Delta/2e_F)b_T(q/2k_f)^2} - \frac{1}{i\Omega/4e_F - (\Delta/2e_F)b_T(q/2k_f)^2} \right), \] (5b)

with \( \Delta \) the Stoner band splitting. For \( 0 < \Delta < n\Gamma_i \), \( \Delta \) is related to the magnetization \( M \) by \( M = \mu_B\Delta/T_i \).

Two comments follow: (1) In a strict long-wavelength expansion of the propagators from Ref. [14] the \( b_L \) and \( b_T \) in Eqs. (5a) and (5b) become magnetization dependent. We ignore this effect and use the same values as in the PM phase. We have compared this approximation against using the full propagators from Ref. [14]; see below. (2) The factor of 5/4 in Eq. (5a) arises since we keep the particle number density fixed, as is the case experimentally, rather than the chemical potential; see Ref. [5].

We now consider the longitudinal magnetic propagator in the FM phase in more detail. In a Heisenberg ferromagnet, the transverse spin waves or magnons couple to the longitudinal susceptibility \( \chi_L \). This effect is most easily demonstrated within a nonlinear sigma-model description of the ferromagnet, which treats the order parameter \( M \) as a vector of fixed length \( M \), and parametrizes it as \( M = M(\sigma(x), \pi_1(x), \pi_2(x)) \) with \( \sigma^2 + \pi_1^2 + \pi_2^2 = 1 \), \( M \) the magnetization, and \( x \) a space-time index [15,16]. The diagonal part of the \( \pi_i - \pi_j \) propagator, \( \langle \pi_i(\pi_i) = (M^2/2N_F)D_T \), is proportional to the transverse propagator \( D_T \), and the off-diagonal part has been calculated in Ref. [16]. The longitudinal propagator, \( D_L = (M^2/2N_F)\langle \sigma(x)\sigma(y) \rangle \), can be expanded in a series of \( \pi \)-correlation functions,

\[ \langle \sigma(x)\sigma(y) \rangle = 1 - 2\langle \pi_1(x)\pi_1(y) \rangle + \langle \pi_1(x)\pi_2(y)\pi_2(y) \rangle + \ldots, \] (6)

where repeated indices are summed over. At one-loop order, the term of order \( \pi^4 \) yields the diagram shown in Fig. 3. Notice that the sigma model, which neglects all longitudinal fluctuations, replaces the external legs by constants. Power counting shows that at nonzero temperature, and for dimensions \( d < 4 \), this contribution causes the homogeneous longitudinal susceptibility to diverge everywhere in the FM phase [17]. More generally, this one-loop contribution, together with the zero-loop one, Eq. (5a), yields a functional form for \( D_L \) in the FM phase that is exact at small wave numbers. This diagram has no analog in the PM phase, while all other renormalizations of the propagators will give comparable contributions in both the PM and FM phases. It is therefore reasonable to calculate \( T_c \) based on a one-loop approximation in the FM phase, and compare it to a zero-loop calculation in the PM phase. We have used Eq. (5b) for the internal propagators in Fig. 3. Since the coupling constants involve a wave number integral, Eqs. (3), we also need to go beyond the sigma model and keep the wave number dependence of the external ones. For computational simplicity, we have modeled the external legs by replacing Eq. (5a) with a step function that cuts off the momentum integral at \( k/2k_f = \sqrt{5|t|/4b_L} \). With these approximations, the momentum integral in Fig. 3 can be done analytically, leaving the frequency sum to be performed numerically. The result and the resulting contribution to \( d^L \) and \( d^T \) depend on the temperature, so Eq. (2) now needs to be solved self-consistently for \( T_c \).

We still need to specify the temperature scale \( T_0(t) \). Following Ref. [5], we use the prefactor of \(|t| \) in Eqs. (4) and (5a) as a rough measure of the magnetic excitation energy,

\[ T_0(t) = T_0[\Theta(t)t + \Theta(-t)|t|/4], \] (7)

with \( T_0 \) a microscopic temperature scale that is related to the Fermi temperature (for free electrons) or a bandwidth (for band electrons). This qualitatively reflects the suppression of the superconducting \( T_c \) near the FM transition due to effective mass effects [2,5,6].

FIG. 3. Mode-mode coupling contribution to the longitudinal (L) propagator \( D_L \) from the transverse (T) ones.
We are now in a position to choose parameters and calculate explicit results. We put \( \Gamma_z = \Gamma_t \) [5]; other reasonable choices yield similar results. Let us first ignore the mode-mode coupling contribution to \( d^T_c \) and \( d^L_0 \). We have performed the calculation both with the full propagators from Ref. [14] and with the schematic Landau propagators, Eqs. (4) and (5a). With \( b_L = 0.23, b_T = 0.4 \) the two results are within 10% of one another, and also very similar to those obtained by Fay and Appel [5]. We then use these values of \( b_{LT} \) to calculate the mode-mode coupling contribution, and solve the \( T_c \) equation. The result is shown in Fig. 1 and has been discussed above. We have also explored the effect of varying the parameters \( b_{LT} \). With \( b_1 = b_2 = 1 \) we obtain the result shown in Fig. 4. The (unphysical) zero-loop result in the FM phase is very sensitive to the parameters, while the enhancement of the (physical) one-loop result over the \( T_c \) in the PM phase is rather robust. However, the position of the maximum of \( T_c \) changes compared to Fig. 1; it now occurs at the point where the magnetization reaches its saturation value.

The reason is as follows: As one approaches the magnetization saturation point from low magnetization values, the transverse coupling constant \( d^L_t \) vanishes, and remains zero in the saturated region. Effectively, the Heisenberg system turns into the Ising model discussed in Ref. [6]. If the longitudinal coupling constant \( d^L_0 \) still has a substantial value at that point, then this leads to an increase in \( T_c \). This is a very strong effect in the zero-loop contribution (see Fig. 4), and the effect qualitatively survives in the one-loop result. If, however, \( d^L_0 \) is already very small, then \( d^L_0 \) going to zero has only a small effect on \( T_c \), as is the case in Fig. 1. Which of these two cases is realized depends on the parameter values. We finally mention that the first order nature of the magnetic transition [3,9] adds another mechanism for suppressing \( T_c \) in the PM phase: For a sufficiently strong first order transition, and if the case shown in Fig. 4 is realized, then the effective \( t \) may be large enough everywhere for the system to miss the maximum of \( T_c \) in the PM phase, but not in the FM phase.

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![Figure 4](Image)

**FIG. 4.** Same as Fig. 2, but for different parameter values (see the text).