Unitarity of the Standard Model at the higgs’ resonance

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Abstract

We compute a unitarity bound for higgs mass using one-loop corrected s-wave partial amplitude for $Z_LZ_L \rightarrow Z_LZ_L$ scattering. We use the equivalence theorem and show that the higgs mass has to be less than $\approx 600$ GeV in order to save the (perturbative) unitarity in the higgs’ resonance region. We also discuss about the validity of perturbation expansion in the symmetry breaking sector.

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The Higgs sector of the Standard Model is, unlike the other parts of it, still poorly understood. The minimal version of the Standard Model has one SU(2) Higgs doublet consisting of four real degrees of freedom. As a result of spontaneous symmetry breaking three of them become longitudinal components $W^\pm_L$ and $Z_L$, whereas one degree of freedom remains as a physical particle $H$. The vacuum expectation value of this particle is experimentally fixed, although the particle itself has not been observed: the mass of the Higgs boson $m_H$, or equivalently, the value of the self–interaction coupling constant $\lambda$ of the symmetry breaking sector is unknown.

By direct searches, the lower limit of the higgs mass has been able to push as high as $m_H > 60$ GeV \cite{1}. The combined fit of LEP data and low energy experiments favor the higgs mass not to be more than a couple of hundred GeV’s \cite{2} (assuming only the minimal Standard Model) but the upper limit is not very conclusive. If $m_H$ is large enough, perturbative calculations are not reliable any more because the symmetry breaking sector becomes strongly interacting. The scattering amplitudes of the would–be Goldstone bosons are related to the scattering amplitudes of the longitudinal weak gauge bosons $Z_L$ and $W^\pm_L$ according to so called equivalence theorem \cite{3,4}. It states that the scattering amplitudes of the longitudinal gauge bosons are, in leading order of $m_W/E$, where $E$ is the energy scale of the scattering, the same as for the corresponding would–be Goldstone boson scattering amplitudes calculated in the $R_\xi$–gauge.

Our aim is therefore to study, how large $m_H$ may be in order to consider perturbative calculations reliable. An upper bound for the higgs mass can be estimated by considering the limitations which the requirement of (perturbative) unitarity of the scattering matrix puts on its elements, in particular for the partial wave amplitudes. This approach was first introduced by Lee, Quigg and Thacker \cite{4} who found, using the tree level expression for the s–wave partial amplitude $a_{0}^{zz\rightarrow zz}(s)$ and a rough unitarity bound $|a_{0}^{zz\rightarrow zz}(s)| < 1$, that $m_H$ should be less than 1 TeV in order to save the tree level unitarity in the limit $s \rightarrow \infty$. Later the high energy unitarity bound was sharpened by Durand, Johnson and Lopez \cite{5} (see also \cite{6}) by including $O(g^2m_H^2/m_W^2)$ one–loop corrections and using the more restrictive unitarity condition

\begin{equation}
|a_{0}^{zz\rightarrow zz}(s) - i \frac{1}{2}| < \frac{1}{2}.
\end{equation}

Their result was that $m_H$ should be less than $\simeq 400$ GeV in order to save the unitarity.

Our approach is to test the bound \cite{4} near the resonance $s/m_H^2 = 1$. A good feature of this approach is that unlike the bound in \cite{5}, the bound we obtain to the higgs mass is independent of the effective energy scale up to which the Standard Model is considered to be a valid theory. It turns out that this approach gives the unitarity bound for the higgs mass which is of the same order of magnitude as the earlier bounds. Our aim is also to discuss, what is consistent way to perform the perturbative expansion of the physical quantities when the perturbation parameter $g^2m_H^2/(16\pi^2m_W^2)$ is rather large.

We calculate the neutral Goldstone boson $zz \rightarrow zz$ scattering amplitude at one–loop level and interpret the result as a large $m_H$ approximation for the $Z_LZ_L \rightarrow Z_LZ_L$ amplitude according to the equivalence theorem. In $Z_LZ_L \rightarrow Z_LZ_L$ scattering $s$, $t$ and $u$ channels are all open while in e.g. $w^+w^- \rightarrow w^+w^-$ scattering only $s$ and $t$ channels are open leading to a less stringent unitarity bound, in general \cite{4,5}. The interaction Lagrangian reads
\[ \mathcal{L} = -g_0^2 \frac{m_H^2}{8m_W^2} \left( w^+w^- + \frac{1}{2} z^2 + \frac{1}{2} H^2 + \frac{2m_W^2}{g_0} H + \frac{2m_W^2}{g_0 m_H^2} \delta T \right)^2, \]  

where subscript 0 refers to bare quantities. The quantity \( \delta T = \lambda_0 v_0 (v_0^2 - \mu_0^2 / \lambda_0) \), which vanishes at tree level, cancels the tadpole term generated in the loop expansion. It can be shown that \( \delta T \) is related to the Goldstone boson self-energy at zero momentum transfer \( \Pi(0) \) by \( \delta T = -v_0 \Pi(0) \). Therefore, our computational strategy is parallel to \( [3, 8] \): we ignore all tadpole terms and subtract the zero momentum Goldstone boson self-energy from all scalar self-energies, which guarantees that the Goldstone bosons remain massless in the presence of loop corrections, too. Furthermore, it is convenient to perform all calculations in the Landau gauge where bare \( w^\pm \) and \( z \) propagators have zero mass.

Our renormalization description is as follows: \( m_H \) is taken to be the physical higgs mass determined by the pole of the full propagator (\( \text{Im} \Pi \ll m_H^2 \))

\[ m_H^2 - m_{H0}^2 - \text{Re} \Pi(m_H^2) = 0, \]  

and the coupling constant is renormalized by defining

\[ \lambda = g^2 \frac{m_H^2}{8m_W^2}, \]  

which is formally similar to the tree level relation. The parameters \( g \) and \( m_W \) are renormalized so that only corrections proportional to \( m_H^2 / m_W^2 \) are taken into account in the counterterms \( \delta g \) and \( \delta m_W^2 \). As a consequence of this choice one obtains \( \delta g = 0 \) and

\[ \frac{\delta m_W^2}{m_W^2} = \frac{1}{8} \frac{g^2}{16\pi^2} \frac{m_H^2}{m_W^2}. \]  

Furthermore, we have to specify a wave function renormalization for the Goldstone bosons. We choose \( Z_z \) to be defined as the residue of the \( z \)-propagator at its pole \( s = 0 \), whence a calculation results

\[ Z_z = 1 - \frac{1}{8} \frac{g^2}{16\pi^2} \frac{m_H^2}{m_W^2}. \]  

Near the resonance \( s/m_H^2 = 1 \) the amplitude changes very rapidly so, that it is important to perform a full one loop calculation, in particular to include various imaginary parts coming from three and four point vertex functions. The complete one–loop s–wave partial amplitude reads

\[ a_0^{zz \rightarrow zz}(s) = \frac{1}{32\pi} \int_{-1}^1 d\cos\theta T(s, \cos\theta) \]  

\[ = \frac{Z_z^2}{32\pi} \int_{-1}^1 d\cos\theta \left[ \Gamma_3(s)G(s)\Gamma_3(s) + \Gamma_3(t)G(t)\Gamma_3(t) \right. \]  

\[ \left. + \Gamma_3(u)G(u)\Gamma_3(u) + \Gamma_4(s, t, u) \right], \]  

where \( G \) is the full, renormalized higgs propagator

\[ G(s) = \frac{i}{s - m_H^2 - (\Pi_H(s) - \text{Re} \Pi_H(m_H^2))}. \]
and $\Gamma_3$ and $\Gamma_4$ are the proper three and four point vertex functions, respectively. The scattering angle $\theta$ is related to the Mandelstam variables through the usual relations. Introducing dimensionless integration and dynamical variables $w = -t/m_H^2$ and $x = s/m_H^2$ the partial wave amplitude $a^{zz\rightarrow zz}_0$ reads

$$a^{zz\rightarrow zz}_0(x) = \frac{Z^2}{32\pi} \left( 2\Gamma_3^2(x) G(x) + \frac{4}{x} \int_0^x dw \Gamma_3^2(-w) G(-w) + \frac{2}{x} \int_0^x dw \Gamma_4(x, -w, w - x) \right).$$

(9)

To study the effects of the loop corrections we define the dimensionless functions $f_2$, $f_3$ and $f_4$ by

$$\Gamma_3(x) = -ig \frac{m_H^2}{2m_W^2} (1 + \alpha_H f_3(x)), \quad \Gamma_4(x, -w, w - x) = -\frac{3ig^2}{4} \frac{m_H^2}{m_W^2} (1 + \alpha_H f_4(x, -w, w - x)), \quad G(x) = \frac{i}{m_H^2} \frac{1}{x - x_0 - \alpha_H f_2(x)},$$

(10)

where $\alpha_H$ is the expansion parameter

$$\alpha_H = \frac{g^2}{16\pi^2} \frac{m_H^2}{m_W^2} = 0.42 \left( \frac{m_H}{\text{TeV}} \right)^2. \quad \text{(11)}$$

Note, that into the parameter $x_0 = 1 - \alpha_H 3\pi i/8$ are, actually, all $O(\alpha_H)$ corrections to the pole of the propagator $G$ included, because the function $f_2$ has the property $f_2(1) = 0$.

Our results are summarized in Figure 1. In the low energy region, $0 < x < 0.5$ where no resummation was made for the $s$ channel propagator $G(x)$, the partial wave amplitude reads in first order in $\alpha_H$

$$a^{zz\rightarrow zz}_0(x) = -\frac{g^2 m_H^2}{64\pi m_W^2} \left[ \left( 3 + \frac{1}{x - 1} - \frac{2}{x} \ln(1 + x) \right) \left( 1 - \frac{1}{4} \alpha_H \right) \right. \left. + \alpha_H \left( 2f_3(x) + \frac{f_2(x) - \frac{3\pi i}{8}}{x - 1} \right) + \frac{\alpha_H}{x} \int_0^x dw \left( 3f_4(x, -w, x - w) - \frac{4f_3(-w)}{w + 1} + \frac{2f_2(-w) - \frac{3\pi i}{4}}{(w + 1)^2} \right) \right].$$

(12)

Although both functions $f_3$ and $f_4$ can be in general evaluated only numerically, their asymptotic behaviour when their arguments approach zero is rather easy to obtain. One can check that logarithmic and constant terms cancel each other in accordance with the low energy theorem [10]. In the low energy regime, however, we can not interpret the Goldstone boson amplitudes as scattering amplitudes of the longitudinal gauge bosons because the condition $m_W/\sqrt{s} \ll 1$ of the equivalence theorem is not fulfilled.

In the resonance region, $0.5 < x < 2$, the curve with dashed line in Fig. 1 corresponds to $a^{zz\rightarrow zz}_0(x)$ evaluated in terms of a pole $x'_0$, a residue $Z$ and a remnant part $R(x)$:

$$a^{zz\rightarrow zz}_0(x) = \frac{Z}{x - x'_0} + R(x). \quad \text{(13)}$$
This form is adequate for the perturbation theory near a resonance where usual perturbation expansion would lead to a divergence. Therefore, to remove this singular behaviour, the leading $\alpha_H$ correction of the pole of the propagator has to be taken into account. The real part of the self energy contributes, however, only to the $\mathcal{O}(\alpha_H^2)$ correction whereas the correction to the imaginary part is $\mathcal{O}(\alpha_H)$. Thus in Eq. (13), the terms $Z$, $x_0$ and $R(x)$ are evaluated in first order in $\alpha_H$, whence $x_0' = x_0$. The curve with solid line in Figure 1 corresponds to $a^{zz \rightarrow zz}_0$ with proper $s$–channel higgs propagator:

$$a^{zz \rightarrow zz}_0(x) = -\frac{g^2}{64\pi m_W^2} \left[ \left( 3 - \frac{2}{x} \ln(1 + x) \right) \left( 1 - \frac{1}{4} \alpha_H \right) + \frac{1 + 2\alpha_H f_3(x) - \frac{1}{4} \alpha_H}{x - x_0 - \alpha_H f_2(x)} + \frac{\alpha_H}{x} \int_0^x dw \left( 3 f_4(x, -w, x, -w) - \frac{4 f_3(-w)}{w + 1} + \frac{2 f_2(-w) - \frac{3\pi i}{4}}{(w + 1)^2} \right) \right].$$

The $t$ and $u$ channel propagators need not to be resummed, because they are analytical in the $-w < 0$ region. In practise, the difference between these two forms is small in the resonance region, even when the parameter $\alpha_H$ is as large as $\alpha_H = 0.42 \cdot (0.6)^2 = 0.15$. When $x \gg 1$, situation is completely different because of the logarithmic terms of the function $f_2$. For large $x$ they are important, which is analogous to the concept of effective coupling constant which should be used as a perturbation parameter. One expects that in high energy region one should use Eq. (14) instead of Eq. (13).

Directly from the Fig. 1 we obtain the unitarity bound $m_H < 400 - 600$ GeV, though the unitarity is violated very softly. One also concludes from Fig. 1 that the point $x_c = s_c/m_H^2$ where the $s$–wave amplitude leaves the unitarity circle is a decreasing function of the higgs mass. For $m_H = 400$ GeV $x_c = 1.7$, while for $m_H = 600$ GeV $x_c = 1.4$. In the case of high energy unitarity limit one would interpret $\sqrt{s_c}$ to be the effective scale up to which the Standard Model is valid effective theory and above which new physics is expected to show up to save the perturbative unitarity. However, if the unitarity limit is near the higgs’ resonance, $\sqrt{s_c} \approx m_H$, this interpretation is not possible. The breaking of the unitarity is a sign of strong interaction physics and one expects that if the breaking scale $\sqrt{s_c}$ is not too far from $m_H$, the Higgs scalar is not a propagating state by itself but form a bound state opening a new channel. It pulls the $s$–wave amplitude inside the unitarity circle.

One should notice, that $\mathcal{O}(m_W/\sqrt{s})$ correction implied due the use of the equivalence theorem change the slope of the curves in Figures 1 by 20 % at most. One expects therefore that this correction does not change much the point where the partial wave amplitude curve leaves the unitarity circle, if the curve crosses the circle perpendicularly enough. To be on the safe side, the unitarity bound near the higgs’ resonance should be taken $m_H \leq 600$ GeV. Although the first order corrections lower the unitarity bound from that calculated in the tree approximation, the second order corrections are not expected to change first order results. When $m_H = 600$ GeV, the perturbation parameter $\alpha_H$ has the value $\alpha_H = 0.15$ which means that near the higgs’ resonance they are comparable to the corrections implied by the equivalence theorem. Furthermore, inclusion of the fermions, the $t$–quark in practice, affects the unitarity bound near the resonance ($m_H = 600$ GeV) about 20 %, too. The decay width of the higgs, in other words the imaginary part of the pole parameter $x_0$, is expected
to be the most sensitive to \( m_W/m_H \) corrections implied by the equivalence theorem and top–quark effects. The inclusion of them results \( x_c = 1.8 \) for \( m_H = 400 \) GeV and \( x_c = 1.5 \) for \( m_H = 600 \) GeV. One expects that these corrections to the parameters \( Z \) and \( R(x) \) in addition to \( x_0 \) tend to rise the value of \( s_c \) slightly but the unitarity bound for the higgs mass remains essentially unchanged. Thus we may conclude that the higgs mass should not exceed 600 GeV in order to save perturbative calculations in the Higgs sector of the Standard Model.
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Figure caption

Figure 1. The behaviour of the s-wave amplitude $a_0^{zz \to zz}$ for $0 < x < 2$. (a) $m_H = 300$ GeV, (b) $m_H = 400$ GeV and (c) $m_H = 600$ GeV. The tic–marks denote the values $x = 0.5, 1.0$ and 1.5. The curve with dashed line is Laurent–approximated s-wave amplitude (13) and that with solid line is the one from (14). The low energy part of the curve, $0 < x < 0.5$, is calculated from (12).