Multi-operation machine with tool redundancy as a two-component recoverable system with instantly replenished time reserve

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Abstract. The object of the research is the technical system of a multi-operation machine tool, the tools of which can fail and be restored. The failed tool remains functional for some time due to a temporary reserve until a parametric failure occurs, the magnitude of which is random. All random variables describing the system have general distributions. The apparatus for constructing a mathematical model of the described system is a semi-Markov process with a discrete-continuous phase space of states. The stationary distribution of the embedded Markov chain is defined explicitly. For systems with parallel connection, the stationary time between failures of the system, the stationary time spent in the state of failure and the stationary system availability factor are found by the series of connections with disconnection and without disconnection of elements. The dependence of the stationary characteristics of the system on the size of the time reserve is demonstrated with a numerical example.

1. Introduction
One of the methods for increasing the reliability of technical systems, including multi-operational machines, is temporary redundancy. Consider a system consisting of two unreliable recoverable elements. Suppose that at the initial moment of time, both elements of the system are operational and begin to function. The uptime of the ith element is a random variable (RV) $\alpha_i$ with a distribution function (DF). $F_i(t), i = 1,2$ Immediately after the element fails, its recovery begins, which lasts for a random time $\beta_i$, with DF $G_i(t), i = 1,2$. In this case, the element continues to function due to a temporary reserve, which is instantly replenished at the moment of the functional loss of the element and is described by the RV $\gamma_i$ with DF $R_i(t), i = 1,2$. This condition is not considered a failure. If the malfunction of the element is eliminated during the temporary reserve, then it is put back into operation. An element falls into a failure state only when the temporary reserve is completely consumed and its recovery is not completed. After the completion of the repair work, the reliability characteristics of the element are fully restored.

Suppose that RV $\alpha_i, \beta_i$ and $\gamma_i$ are independent, have finite mathematical expectations $E\alpha_i, E\beta_i, E\gamma_i$ and dispersions; there are distribution densities $f_i(t), g_i(t), r_i(t)$ and $0 < P(\beta_i < \gamma_i) < 1$. 

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The time diagram of the functioning of the system under the assumption that at the moment of transition to the state of failure of one of the elements the other is not disactivated is shown in Fig. 1.

The concept of a system failure depends on its structure. When the elements are connected in parallel the system (duplicated with a loaded reserve) is considered to be in failure when the temporary reserves of both elements are exhausted and they are being restored. With a serial connection, the system failure occurs at the time of spending the temporary reserve of one of the elements.

The purpose of the article is to construct a semi-Markov model of the functioning of the described two-component system and, on its basis, to determine the following indicators of the quality of the system operation: the average stationary time between failures $T(E_+)$, the average stationary time in the state of failure $T(E_-)$, and the stationary availability factor $K_v$.

2. Materials and Methods

The presence of a temporary reserve can be considered as the existence of another element in the system, which is put into operation for the period of failure of the main element (reserve time is being expended) and is idle during the normal operation of the main element (reserve time is not being spent) [1-4]. Models of an unreliable one-component system with an instantly replenished reserve of time are presented in the works [4-7]. In this paper, stationary reliability characteristics for a two-component recoverable system are determined using the example of a multi-operation CNC machine with turret-type tool heads [8, 9], containing the main and backup (redundant) tools (Figure 2).

![Figure 1. Time diagram of system functioning.](image1)

![Figure 2. Version of a multi-stage CNC lathe with two turret-type tool systems for cutters and axial tools](image2)
The redundant tools replace the main tools in the case of a parametric failure (detected by a cutting force or torque sensor) for the period of their replacement or restoration. The considered recoverable system uses a model with a time reserve.

3. Result and Discussion

Construction of a semi-Markov model of the system. We describe the functioning of the system using a semi-Markov process $S(t)$ with a discrete-continuous set of states [10, 11]. We start by constructing the space of $E$ phase states of the system. Each of its elements can be in three different physical states: $1$ – function in an efficient state; $0$ – recover and function due to a temporary reserve; $2$ – recover and be in a failure state. The physical state of the system will be encoded by a two-dimensional vector, the components of which indicate the physical state codes of the first and second elements, respectively. Let us expand the states of the system to semi-Markov ones. To do this, before the system state code, we indicate the number of the element that changed its physical state last. After the code of the physical state of the $k^{th}$ element, which has not changed its state, we add continuous components, namely: $1x_k$ — the time $x_k$, remaining until the element fails; $0z_ku_k$ — the time $z_k$ of the remaining reserve, the time $u_k$, remaining until the end of the restoration of the element; $2u_k$ — the time $u_k$ until the end of recovery. For example, the state $(1,0,2u_2)$ means that the first element is out of order, its restoration has begun, and it continues to function at the expense of a reserve. In this case, the second element is in a state of failure, there is time $u_2$ left until the end of its recovery. The status $(2,0z_1u_1, 1)$ indicates that the second element was recovered last and started to work. At this moment, the first element functions at the expense of a reserve, until the use of which there is time $z_1$ left, and there is time $u_1$ until the end of the restoration of the element. The state $(2,2u_1,2u_2)$ means that the time reserve of the second element is over, and it went into a failure state there is time $u_2$ left until the end of its recovery. In this case, the first element continues to recover, until the end of its recovery there is time $u_1$ left.

The times $\theta$ of the system being in states are determined by the minimum of random and deterministic values depending on the state. For example, $\theta_{(1,0,2u_2)} = \beta_1 \wedge \gamma_1 \wedge u_2$, $\theta_{(2,0z_1u_1, 1)} = z_1 \wedge u_1 \wedge \theta_{(2,2u_1, 2u_2)} = u_1 \wedge u_2$.

Let us determine the probabilities and probability densities of transitions of the embedded Markov chain (EMC) $\{S_n, n \geq 0\}$. Let us take into account that from physical state 1 an element goes to state 0, from state 0 to states 1 or 2, from state 2 to state 1.

Let us describe, for example, the probabilities of transitions of state $(2,0z_1u_1, 1)$. Depending on the value of the minimum of $z_1$ $\wedge u_1$ $\wedge \alpha_2$ values a transition to one of the following states is possible:

1) if $z_1 < u_1, \alpha_2 \in z_1 + dx_2$, then
\[
\frac{d}{dx_2}P(2,0z_1u_1, 1) \rightarrow (1,2(u_1 - z_1), 1x_2) = f_2(z_1 + x_2, x_2) > 0;
\]
representation $\alpha_2 \in z_1 + dx_2$ means that $z_1 + x_2 < \alpha_2 \leq z_1 + x_2 + dx_2$;
2) if $u_1 < z_1, \alpha_2 \in u_1 + dx_2$, then $\frac{d}{dx_2}P(2,0z_1u_1, 1) \rightarrow (1,1,1x_2) = f_2(u_1 + x_2), x_2 > 0$;
3) if $\alpha_2 < u_1 \wedge z_1 = z_1, \alpha_2 \in z_1 - dy, \text{then}$
\[
\frac{d}{dy}P(2,0z_1u_1, 1) \rightarrow (2,0y(u_1 - z_1 + y), 0) = f_2(z_1 - y), y < z_1;
\]
4) if $\alpha_2 < u_1 \wedge z_1 = u_1, \alpha_2 \in u_1 - dy, \text{then}$
\[
\frac{d}{dy}P(2,0z_1u_1, 1) \rightarrow (2,0(z_1 - u_1 + y)y, 0) = f_2(u_1 - y), y < u_1.
\]

The probabilities and probability densities of the transitions of EMC $\{S_n, n \geq 0\}$ for the remaining states are written out in a similar way.

The stationary distribution of EMC is a solution to the system of integral equations, which in matrix form has the form [10]
\[
\rho(B) = \int_B \rho(dz)P(z, B),
\]
where \( P(z,B) \) — the probability of transition from the state \( z \) to a set of states \( B \). For the considered semi-Markov process, the following holds.

Theorem. The stationary distribution of EMC \( \{S_n, n \geq 0\} \) is determined by the formulas

\[
\rho(1, d_1(u_1), d_2(x_2, z_2, u_2)) = \rho_0 a_1(u_1) a_2(x_2, z_2, u_2),
\]

\[
\rho(2, d_1(x_1, z_1, u_1), d_2(u_2)) = \rho_0 a_1(x_1, z_1, u_1) a_2(u_2).
\]

\( \forall \) \( d_1(u_i) \in \{1,0,2u_i\}; d_1(x_i, z_i, u_i) \in \{1x_i, 0z_i, 2u_i\}; \rho_0 = \text{const}; \)

\[
a_1(u_i) = \int_0^\infty r_i(s) g_i(s + u_i) ds, d_i(u_i) = 2u_i;
\]

\[
a_i(x_i, z_i, u_i) = \begin{cases} 
\int_0^\infty r_i(s + z_i) g_i(s + u_i) ds, d_i(x_i, z_i, u_i) = 0z_i u_i, \\
\int_0^\infty r_i(s) g_i(s + u_i) ds, d_i(x_i, z_i, u_i) = 2u_i.
\end{cases}
\]

Proof of the theorem. The validity of the statement of the theorem is verified by direct substitution of expressions (2) into the system of equations (1). Let us verify this by the example of one of the equations of the system. Substitute in the right side of the equation

\[
\rho(1,2u_1, 0z_2 u_2) = \int_0^\infty r_1(t) g_1(t + u_1) \rho(1,0,0(t + z_2)(t + u_2)) dt + \\
+ \int_0^\infty r_2(t + z_2) g_2(t + u_2) \rho(2,0(t + u_1),0) dt
\]

expression

\[
\rho(1,0,0(t + z_2)(t + u_2)) = \rho_0 \int_0^\infty r_2(s + t + z_2) g_2(s + t + u_2) ds,
\]

\[
\rho(2,0t(t + u_1),0) = \rho_0 \int_0^\infty r_1(s + t) g_1(s + t + u_1) ds.
\]

As a result of the transformations we get

\[
\int_0^\infty r_1(t) g_1(t + u_1) \rho(1,0,0(t + z_2)(t + u_2)) dt + \int_0^\infty r_2(t + z_2) g_2(t + u_2) \rho(2,0(t + u_1),0) dt = \\
= -\rho_0 \int_0^\infty \frac{\partial}{\partial t} \left\{ \int_0^\infty r_1(s) g_1(s + u_1) ds \int_0^\infty r_2(s + z_2) g_2(s + u_2) ds \right\} dt = \\
= \rho_0 \int_0^\infty r_1(s) g_1(s + u_1) ds \int_0^\infty r_2(s + z_2) g_2(s + u_2) ds = \rho(1,2u_1, 0z_2 u_2).
\]

The constant \( \rho_0 \) is found from the normalization requirement and takes the value

\[
\rho_0 = \left[ \Sigma_{i=1}^2 (2 + P(t_{3-i} < \beta_{3-i})(Ea_i + E\beta_i)) \right]^{-1}.
\]

Stationary characteristics of the system. Let the phase space of the \( E \) be represented as a union of two disjoint subsets \( E_+ \) and \( E_- \), i.e. \( E = E_+ \cup E_-, E_+ \cap E_- = \emptyset \). The subset \( E_+ \) includes the states in which the system operates, and the subset \( E_- \) includes the states of failure.

It is known [10,11], that in the case of the existence of a single stationary distribution of the EMC \( \{S_n, n \geq 0\} \), the average stationary times \( T(E_+) \) and \( T(E_-) \) of the system working and failure states respectively, as well as the system availability factor \( K_a \), are found by the formulas:

\[
T(E_+) = \frac{\int_{E_+} m(x) \rho(dx)}{\int_{E_+} P(x,E) \rho(dx)}, \quad T(E_-) = \frac{\int_{E_-} m(x) \rho(dx)}{\int_{E_-} P(x,E) \rho(dx)}, \quad K_a = \frac{\int_{E_+} m(x) \rho(dx)}{\int_{E_+} m(x) \rho(dx)},
\]

(3)
where \( m(x) \) – average time in state \( x \in E \).

Partitioning a set \( E \) into a subset of functionally efficient states \( E_+ \) and failure states \( E_- \) depends on the structure of the system. Consider the case of parallel connection of elements (duplicated system with loaded reserve). In this case, we have, \( E_- = \{(1,2u_1,2u_2),(2,2u_1,2u_2)\} \), and all other states of the system belong to the subset \( E_+ \). Let us find expressions for the functionals included in formulas (3). We have

\[
\int_{E_+} m(x)\rho(dx) = \int_0^\infty \int_0^\infty \rho(1,2u_1,2u_2)du_1du_2 \int_0^\infty u_1\rho du_2 + \int_0^\infty \int_0^\infty \rho(1,2u_1,2u_2)du_1du_2 \int_0^\infty u_2\rho du_1 = \\
= \int_0^\infty dt \int_0^\infty r_1(s)\overrightarrow{G}_1(s+t)ds \int_0^\infty du \int_0^\infty r_2(y)\overrightarrow{G}_2(y+u)dy + \int_0^\infty dt \int_0^\infty du \int_0^\infty r_1(s)\overrightarrow{G}_1(s+u)ds \int_0^\infty r_2(y)\overrightarrow{G}_2(y+u)dy = \\
= -\int_0^\infty \frac{d}{dt} \left\{ \int_0^\infty du \int_0^\infty r_1(s)\overrightarrow{G}_1(s+u)ds \int_0^\infty r_2(y)\overrightarrow{G}_2(y+u)dy \right\} = \int_0^\infty \frac{d}{dt} \left( \int_0^\infty r_1(s)\overrightarrow{G}_1(s)ds \right) \int_0^\infty r_2(y)\overrightarrow{G}_2(y)dy = (E\beta_1 - E(\beta_1 \wedge \gamma_1))(E\beta_2 - E(\beta_2 \wedge \gamma_2)).
\]

Carrying out similar transformations for the remaining functionals, we obtain

\[
\int_{E_-} m(x)\rho(dx) = (E\alpha_1 + E(\beta_1 \wedge \gamma_1))(E\alpha_2 + E(\beta_2 \wedge \gamma_2)) + (E\alpha_1 + E(\beta_1 \wedge \gamma_1)) \times (E\beta_2 - E(\beta_2 \wedge \gamma_2)) + (E\beta_1 - E(\beta_1 \wedge \gamma_1))(E\alpha_1 + E(\beta_1 \wedge \gamma_1));
\]

\[
\int_{E_+} P(x, E_-)\rho(dx) = \int_{E_+} P(x, E_+)\rho(dx) = \int_0^\infty \int_0^\infty \rho(1,2u_1,2u_2) + \rho(2,2u_1,2u_2)du_1du_2 = \\
= \int_0^\infty r_1(s)\overrightarrow{G}_1(s)ds \int_0^\infty du \int_0^\infty r_2(y)\overrightarrow{G}_2(y+u)dy + \int_0^\infty r_1(s)\overrightarrow{G}_1(s)ds \int_0^\infty du \int_0^\infty r_2(y)\overrightarrow{G}_2(y+u)dy = \\
= P(\gamma_1 < \beta_2)(E\beta_2 - E(\beta_2 \wedge \gamma_2)) + P(\gamma_2 < \beta_2)(E\beta_1 - E(\beta_1 \wedge \gamma_1)).
\]

Let us denote \( T_1^{(10)} = E\alpha_1 + E(\beta_1 \wedge \gamma_1) \) and \( T_1^{(2)} = E\beta_1 - E(\beta_1 \wedge \gamma_1) \) — average stationary times of stay of the \( i^{th} \) element of the system in functionally efficient and failure states, respectively, during the regeneration period, i.e. between two adjacent moments of getting it into a working state after recovery. Then the average stationary residence times of the system in functionally efficient and failure states, the stationary system availability factor are determined by the formulas

\[
T(E_+) = \frac{T_1^{(10)} + T_1^{(2)}}{P(\gamma_1 < \beta_1)T_2^{(2)}} - T_1^{(2)}, T(E_-) = \frac{T_1^{(2)}T_2^{(2)}}{P(\gamma_1 < \beta_1)T_2^{(2)} + P(\gamma_2 < \beta_2)T_1^{(2)}},
\]

\[
K_i = 1 - \prod_{i=1}^2 T_1^{(2)}_{i} + \prod_{i=1}^2 T_1^{(10)}_{i} = 1 - \prod_{i=1}^2 (1 - K_i),
\]

where \( K_i = \frac{T_1^{(10)}_{i}}{T_1^{(10)}_{i} + T_1^{(2)}_{i}} \) — stationary availability factor of the \( i^{th} \) element of the system.

Suppose that the elements of the system are connected in series, and at the moment of transition of one of them to the failure state, the second element is not disconnected. In this case, the subset of functionally efficient states of the system has the form

\( E_+ = \{(1,1,x_2),(1,1,0z_2u_2),(1,0,1x_2),(1,0,0z_2u_2),(2,1x_1,1),(2,1x_1,0),(2,0z_1u_1,1),(2,0z_1u_1,0)\} \)

stationary characteristics of the system are calculated by the formulas

\[
T(E_+) = \frac{T_1^{(10)} + T_2^{(10)}}{P(\gamma_1 < \beta_1)T_2^{(2)}} + T(E_-) = \frac{T_1^{(2)} + T_2^{(2)}}{P(\gamma_1 < \beta_1)T_2^{(2)} + P(\gamma_2 < \beta_2)T_1^{(2)}},
\]

\[
E_- = \{(1,3,0z_2u_2),(1,3,0z_2u_2),(1,2,1x_2),(1,2,0z_2u_2),(2,1x_1,1),(2,1x_1,0),(2,0z_1u_1,1),(2,0z_1u_1,0)\}
\]


\[ K_r = \prod_{i=1}^{2} \frac{T_i^{(10)}}{T_i^{(10)} + T_i^{(2)}} = \prod_{i=1}^{2} K_i. \]  

Now let us consider a system with a series connection of elements, provided that at the moment of failure of one of the elements, the other is disconnected. We will assume that after the restoration of the failed element, the disabled element is put into operation with the same level of efficiency at which it was disconnected. In this case, the subset of functional elements \( E_+^* \) is the same as the equivalent subset \( E_+ \) for a system without outages. The number of states of the subset \( E_+^* \) decreases:

\[ E_+^* = \{(1,2u_1,1x_2),(1,2u_1,0z_2u_2),(2,1x_1,2u_2),(2,0z_1u_1,2u_2)\}, \]

and the time in these states is determined by the formulas \( \theta_{(1,2u_1,1x_2)} = \theta_{(1,2u_1,0z_2u_2)} = u_1 \), \( \theta_{(2,1x_1,2u_2)} = \theta_{(2,0z_1u_1,2u_2)} = u_2 \). As a result of transformations of expressions (3), we find

\[ T(E_+^*) = \frac{T_1^{(10)}_1}{T_1^{(10)} + T_1^{(2)} + T_2^{(2)}}, \]

\[ T(E_+^*) = \frac{T_2^{(10)}_1T_2^{(10)}_2}{T_2^{(10)} + T_2^{(2)} + T_1^{(2)}}, \]

\[ K_r = \frac{T_1^{(10)}_1T_2^{(10)}_2}{T_1^{(10)} + T_1^{(2)} + T_2^{(2)} + T_1^{(2)} + T_2^{(2)}}. \]  

If the time reserve in the system is constant and for the \( i^\text{th} \) element is equal to a deterministic value \( h_i \), then when calculating the stationary characteristics of the system in formulas (4) – (6), one should assume

\[ T_i^{(10)} = E\alpha_i + \int_0^{h_i} \overline{g_i}(t) dt, T_i^{(2)} = \int_{h_i}^{\infty} \overline{g_i}(t) dt, i = 1,2. \]

**Example.** Consider a system in which the average uptime of the elements equals \( E\alpha_1=10\) hour and \( E\alpha_2=12\) hour respectively. The recovery time of the first element obeys the third order Erlang law with an average value \( E\beta_1=1.5\) hour. The recovery time of the second has a Rayleigh distribution with density \( g_2(t) = 0.5te^{-t^2/4} \) and mean value \( E\beta_2=1.77\) hour. Table 1 shows the results of calculations by formulas (4)-(6) of the characteristics of the system: the stationary residence time \( T(E_+) \) in a functionally efficient state, the average stationary residence time \( T(E_-) \) of the system in a failure state, and the stationary system availability factor \( K_r \). Each element of the system has a non-random reserve of time, which varies from 0 hours to 1.0 hours at 0.2 hours intervals.

### 4. Conclusion

In this work, using the example of a multi-operation CNC machine with turret-type tool heads containing main and redundant tools, using the apparatus of semi-Markov processes with a discrete-continuous phase space of states, a model of the functioning of a two-component system with an instantly replenished time reserve is constructed. On its basis, calculation formulas have been obtained that make it possible to determine the stationary characteristics of the system and establish their dependence on the size of the time reserve.
Table 1. Influence of the time reserve on the stationary characteristics of the system

| $h_1$ | $h_2$ | $K_r$ | Change of characteristics, % | $T(E_+)$ | Change of characteristics, % | $T(E_-)$ | Change of characteristics, % |
|-------|-------|-------|-------------------------------|----------|-------------------------------|----------|-------------------------------|
|       |       |       |                               |          |                               |          |                               |
| Parallel system |       |       |                               |          |                               |          |                               |
| 0     | 0     | 0.983 | –                             | 47,586   | –                             | 0.812    | –                             |
| 0.2   | 0.2   | 0.987 | 0.3                           | 54,891   | 15.4                          | 0.718    | –11.6                        |
| 0.4   | 0.4   | 0.990 | 0.7                           | 66,063   | 38.8                          | 0.641    | –21.1                        |
| 0.6   | 0.6   | 0.993 | 1.0                           | 83,269   | 75.0                          | 0.581    | –28.5                        |
| 0.8   | 0.8   | 0.995 | 1.2                           | 109,678  | 130.5                         | 0.532    | –34.5                        |
| 1.0   | 1.0   | 0.997 | 1.4                           | 150,434  | 216.1                         | 0.493    | –39.3                        |
|       |       |       |                               |          |                               |          |                               |
| Series system without disconnection of elements |       |       |                               |          |                               |          |                               |
| 0     | 0     | 0.758 | –                             | 5,455    | –                             | 1.745    | –                             |
| 0.2   | 0.2   | 0.786 | 3.70                          | 5,605    | 2.80                          | 1.529    | –12.3                        |
| 0.4   | 0.4   | 0.813 | 7.40                          | 5,912    | 8.40                          | 1.356    | –22.3                        |
| 0.6   | 0.6   | 0.841 | 10.9                          | 6,641    | 17.7                          | 1.219    | –30.1                        |
| 0.8   | 0.8   | 0.866 | 14.2                          | 7,159    | 31.2                          | 1.111    | –36.3                        |
| 1.0   | 1.0   | 0.889 | 17.3                          | 8,170    | 49.8                          | 1.024    | –41.3                        |
|       |       |       |                               |          |                               |          |                               |
| Series system with disconnection of elements |       |       |                               |          |                               |          |                               |
| 0     | 0     | 0.771 | –                             | 5,455    | –                             | 1.624    | –                             |
| 0.2   | 0.2   | 0.796 | 3.30                          | 5,605    | 2.80                          | 1.437    | –11.5                        |
| 0.4   | 0.4   | 0.821 | 6.60                          | 5,912    | 8.40                          | 1.286    | –20.8                        |
| 0.6   | 0.6   | 0.846 | 9.80                          | 6,419    | 17.7                          | 1.166    | –28.2                        |
| 0.8   | 0.8   | 0.870 | 12.9                          | 7,159    | 31.2                          | 1.072    | –34.0                        |
| 1.0   | 1.0   | 0.892 | 15.7                          | 8,170    | 49.8                          | 0.994    | –38.8                        |

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