BIMODALITY AS A SIGNAL OF THE NUCLEAR LIQUID-GAS PHASE TRANSITION

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Here we present an explicit counterexample to a bimodality concept as the unique signal of first order phase transition. Using an exact solution of the simplified version of the statistical multifragmentation model we demonstrate that the bimodal distributions can naturally appear in infinite system without a phase transition in the regions of the negative values of the surface tension coefficient. Also we propose a new parameterization for the compressible nuclear liquid which is consistent with the L. van Hove axioms of statistical mechanics. As a result the proposed model does not lead to the irregular behaviour of the isotherms in the mixed phase region which is typical for mean-field models. Peculiarly, the suggested approach to account for the nuclear liquid compressibility automatically leads to an appearance of an additional state that in many respects resembles the physical antinuclear matter.

1 Introduction

At the present time the bimodality is often considered as a signal of the first order PT in finite systems. The authors of such schemes [1, 2] identify each local maximum of the bimodal distribution with a pure phase. Such an idea goes back to T. Hill book [3]. Hill justified this assumption on bimodality by stating that due to the fact that an interface between two pure phases 'costs' some additional energy, the probability of their coexisting in a finite system is less than for each of pure phases [3]. It was found [4], however, that such an assumption can be valid for infinite systems only. In order to demonstrate that Hill assumption can be incorrect even in thermodynamic limit, here we present a clear counterexample by considering an exact analytical solution of the constrained statistical multifragmentation model (CSMM) in thermodynamic limit which leads to the bimodal fragment size distributions inside of the cross-over region. For this purpose we consider a more realistic equation of state for the liquid phase which, in contrast to the original SMM formulation [5], is a compressible one [6]. The second important element of the present model is a more realistic parameterization of the temperature dependent surface tension based on an exact analytical solution of the partition function of surface deformations [7].

1.1 CSMM with compressible nuclear liquid in thermodynamic limit

The general solution of the CSMM partition function formulated in the grand canonical variables of volume V, temperature T and baryonic chemical potential μ is given by [8]

\[ Z(V,T,\mu) = \sum_{\{\lambda_n\}} e^{\lambda_n V} \left[ 1 - \frac{\partial F(V,\lambda_n)}{\partial \lambda_n} \right]^{-1}, \]  

where the set of \( \lambda_n \) (\( n = 0,1,2,3,.. \)) are all the complex roots of the equation

\[ \lambda_n = F(V,\lambda_n), \]  

ordered as Re(\( \lambda_n \)) > Re(\( \lambda_{n+1} \)) and Im(\( \lambda_0 \)) = 0. The function \( F(V,\lambda) \) is defined as

\[ F(V,\lambda) = \left( \frac{m T}{2\pi} \right)^{\frac{3}{2}} z_1 \exp \left\{ -\frac{\mu - \lambda T b}{T} \right\} + \sum_{k=2}^{K(V)} \phi_k(T) \exp \left\{ \frac{(p_k(T) - \lambda T) b k}{T} \right\}. \]

Here \( m \approx 940 \text{ MeV} \) is a nucleon mass, \( z_1 = 4 \) is an internal partition (the degeneracy factor) of nucleons, \( b = 1/\rho_0 \) is the eigen volume of one nucleon in a vacuum (\( \rho_0 \approx 0.17 \text{ fm}^3 \) is the normal nuclear density at \( T = 0 \) and zero pressure). The reduced distribution function of the \( k \)-nucleon fragment in \( \phi_{k>1}(T) \) is defined as

\[ \phi_{k>1}(T) = \left( \frac{m T}{2\pi} \right)^{\frac{3}{2}} \frac{1}{k^5} \exp \left[ -\frac{\sigma(T) k^5}{T} \right]. \]
where $\tau \approx 1.825$ is the Fisher topological exponent and $\sigma(T)$ is the $T$-dependent surface tension coefficient. Usually, the constant, parameterizing the dimension of surface in terms of the volume is $\varsigma = \frac{2}{3}$.

In [3] the exponentials $\exp(-\lambda_b k)$ ($k = 1, 2, 3, \ldots$) appear due to the hard-core repulsion between the nuclear fragments [8], while $p_l(T, \mu)$ is the pressure of the liquid phase. Here we consider the thermodynamic limit only, i.e. for $V \to \infty$ we have $K(V) \to \infty$. Then the treatment of the model is essentially simplified, since Eq. (2) can have only two kinds of solutions [8], either the gaseous pole $p_g(T, \mu) = T \lambda_0(T, \mu)$ for $\mathcal{F}(V, \lambda_0 - 0) < \infty$ or the liquid essential singularity $p_l(T, \mu) = T \lambda_0(T, \mu)$ for $\mathcal{F}(V, \lambda_0 - 0) \to \infty$. The mathematical reason why only the rightmost solution $\lambda_0(T, \mu) = \max\{\text{Re}(\lambda_n)\}$ of Eq. (2) defines the system pressure is evident from Eq. (1): in the limit $V \to \infty$ all the solutions of (2) other than the rightmost one are exponentially suppressed.

In the thermodynamic limit the model has a PT, when there occurs a change of the rightmost solution type, i.e. when the gaseous pole is changed by the liquid essential singularity or vice versa. The PT line $\mu = \mu_c(T)$ is a solution of the equation of ‘colliding singularities’ $p_g(T, \mu) = p_l(T, \mu)$, which is just the Gibbs criterion of phase equilibrium. The properties of a PT are defined only by the liquid phase pressure $p_l(T, \mu)$ and by the temperature dependence of surface tension $\sigma(T)$.

In order to consider the compressible nuclear liquid in [6] we suggested the following parameterization of its
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\[ p_l = \frac{W(T) + \mu + a_2(\mu - \mu_0)^2 + a_4(\mu - \mu_0)^4}{b}. \]  

Here \( W(T) = W_0 + \frac{T^2}{W_0} \) denotes the usual temperature dependent binding energy per nucleon with \( W_0 = 16 \text{ MeV} \) \[5\] and the constants \( \mu_0 = -W_0, a_2 \simeq 1.233 \cdot 10^{-2} \text{ MeV}^{-1} \text{ and } a_4 \simeq 4.099 \cdot 10^{-7} \text{ MeV}^{-3} \). These constants are fixed in order to reproduce the properties of normal nuclear matter, i.e. at vanishing temperature \( T = 0 \) and normal nuclear density \( \rho = \rho_0 \) the liquid pressure must be zero.

In addition to the new parameterization of the free energy of the \( k \)-nucleon fragment \((3)\) we consider a more general parameterization of the surface tension coefficient

\[ \sigma(T) = \sigma_0 \left| \frac{T_{cep} - T}{T_{cep}} \right|^\zeta \text{ sign}(T_{cep} - T), \]  

with \( \zeta = \text{ const} \geq 1, T_{cep} = 18 \text{ MeV} \) and \( \sigma_0 = 18 \text{ MeV} \) the SMM. In contrast to the Fisher droplet model \([9]\) and the usual SMM, the CSMM surface tension \((6)\) is negative above the critical temperature \( T_{cep} \). An extended discussion on the validity of such a parameterization can be found in \([8]\). The resulting phase diagrams of the present model in different variables are shown in Fig. 1.

In order to elucidate the role of the negative surface tension coefficient we study the fragment size distribution in two regions of the phase diagram. To demonstrate the pitfalls of the bimodal concept of Refs. \([1, 2, 3]\) we compare the gaseous phase fragment size distribution with that one in the supercritical temperature region, where there is no PT by construction. As one can see from Fig. 2 in the gaseous phase, even at the boundary with the mixed phase, the size distribution is a monotonically decreasing function of the number of nucleons in a fragment \( k \). However, for the supercritical temperatures one finds the typical bimodal fragment distribution for a variety of temperatures and chemical potentials as one can see from Fig. 2.

A sharp peak at low \( k \) values reflects a fast increase of the probability density of dimers compared to the monomers (nucleons), since the intermediate fragment sizes do not have the binding free energy and the surface free energy and, hence, the monomers are significantly suppressed in this region of thermodynamic parameters. On the other hand it is clear that the tail of fragment distributions in Fig. 2 decreases due to the dominance of the bulk free energy and, hence, the whole structure at intermediate fragment sizes is due a competition between the surface free energy and two other contributions into the fragment free energy, i.e. the bulk one and the Fisher one. It was also found that with temperature increasing the minimum and maximum of the distribution function grow wider and shallower and they shift towards the smaller number of nucleons in a fragment (see the right panel of Fig. 2).

1.2 Conclusions

In the present work we showed that the bimodal distributions can naturally appear in an infinite system without a PT. Our analysis of the fragment size distributions in the region of negative surface tension coefficient shows

Figure 2. Left panel: Fragment size distribution of the model is shown for a fixed baryonic chemical potential \( \mu = -27.5 \text{ MeV} \) and three values of the temperature \( T \). Right panel: Same as in the left panel, but for a fixed baryonic chemical potential and different temperatures located at the region of negative values of the surface tension coefficient and \( \nu = 2 \).
that these distributions have a saddle-like shape. Such a behavior closely resembles the fragment size distribution observed in dynamical simulations of nuclear multifragmentation [10]. The compressible nuclear liquid pressure parametrization which generates the tricritical endpoint at the one third of the normal nuclear density is worked out.

References

[1] Ph. Chomaz and F. Gulminelli, Preprint GANIL-02-19, (2002).
[2] F. Gulminelli, Nucl. Phys. A 791, 165, (2007).
[3] T. L. Hill, *Thermodynamics of small systems* Dover, New York, 1994.
[4] K. A. Bugaev, Phys. Part. Nucl. 38 447, (2007); arXiv:nucl-th/0511031.
[5] K. A. Bugaev, M. I. Gorenstein, I. N. Mishustin and W. Greiner, Phys. Rev. C 62, 044320, 2000; arXiv:nucl-th/0007062.
[6] K. A. Bugaev, A. I. Ivanytskyi, V. V. Sagun and D. R. Oliinichenko, Phys. Part. Nucl. Lett. 10, 832 (2013); arXiv:1306.2481 [nucl-th].
[7] K. A. Bugaev, L. Phair and J. B. Elliott, Phys. Rev. E 72, 047106 (2005); arXiv:nucl-th/0406034.
[8] K. A. Bugaev, Acta. Phys. Polon. B 36, 3083, (2005) and reference therein.
[9] M. E. Fisher, Physics 3, 255, (1967).
[10] X. Campi, H. Krivine E. Plagnol and N. Sator, Phys. Rev. C 67, 044610, (2003).