Relative Performance of Linear Quadratic Regulator and Pole Placement Technique for Active Seismic Control of Structures

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Abstract. This paper presents the study conducted on the comparison of the performance of the two popular active control algorithms viz. Linear Quadratic Regulator (LQR) and Pole Placement technique in terms of their capability to alleviate the response of a structure to an earthquake. To compare the performance of the two control algorithms, a ten story regular shear building frame with three active tendons located in 1\(^{st}\), 4\(^{th}\) and 5\(^{th}\) Storys and subjected to an El Centro earthquake has been considered. A comparison of the numerical results shows that the reductions of maximum top floor displacement, maximum story drift and base shear obtained using LQR control algorithm are 71.48%, 59.70% and 62.7% while the reductions obtained using Pole Placement technique are 73.77%, 73.14% and 77% respectively. The percentage reduction of the three performance parameters per unit control force obtained using the two methods have also been estimated and compared. Pole placement technique gives higher reduction while the LQR algorithm gives more reduction per unit control force.

1. Introduction

In last fifty years, significant attention has been paid to develop passive and active control devices for response reduction of structures under wind and seismic forces [1]. Many of these devices have been installed in real buildings like ATC Tower Delhi Airport in New Delhi, India, Statue of Unity Ahmedabad, India, and Kyobashi Center Building (office, Tokyo) , Applause Tower (hotel, office & theatre, Osaka), Long Term Credit Bank (office, Tokyo) J City Tower (office, Tokyo) etc. Passive control systems include base isolation, and different energy dissipating devices like tuned mass damper, tuned liquid damper etc. Despite certain advantages like cost effectiveness, reliability, passive control systems have many demerits including limited capacity, lack of self-adaptability to the change in nature of vibration, loss of effectiveness in modes other than dominant mode etc. This created the need to develop active control systems. The concept of the active structural control was given by Yao in 1972 [2]. These systems adapt themselves with the changing environment and have vast capacity. Different types of the active control systems have been developed and executed as well. These include Active Mass Driver system, Active Tendon System, Active Bracing System etc. Different active control algorithms and controllers have also been developed and implemented. These include Linear Quadratic Regulator, Pole placement technique, proportional–integral–derivative PID control, Linear Quadratic Gaussian control etc. Among them, the most popular and practical is LQR controller [3]. LQR controller has been found effective and practical control algorithms for solving control problems. Ali Alavinasab
et al., introduced an energy concept in traditional LQR and implemented it in a three story building with tendons installed at different floors. It proved more efficient than traditional LQR and found applicability to inelastic structures as well [4]. Mohamed Abdel-Rohman et al., studied active control of tall building subjected to lateral loads. In this study active tendon system and active tuned mass damper were used as control devices. Results proved the active tendon system more efficient in reducing the response of the structure [5]. S.M Nigdeli, M.H. Boduroglu did research on seismic control of buildings using active tendons employing proportional integral derivation (PID) type controllers. The results obtained were encouraging [6]. Ebrahim Nazarimofrad et al., carried out research on optimal placement of active tendon in the building including soil–structure interaction effect using genetic algorithm. In this paper, multi-objective genetic algorithm was used as an optimization technique to find the optimal location of the tendons [7]. Makola M. Abdullah et al., studied placement of sensor and actuators on civil structures using genetic algorithms. A performance index with top floor displacement of a forty story structure and control force as parameters was proposed and optimized for the position of the actuators where the reduction of response was maximum and control force required was least[8]. S. Pourzanali et al., investigated active control of high rise building using fuzzy logic controller and genetic algorithm. In this study active tuned mass damper (ATMD) was used to control the response. The optimal parameters of the ATMD were obtained through genetic algorithm and the active control signal was generated through fuzzy logic control. The results were implemented on 11 story realistic building in Iran to control its response [9]. Mohamed Abdel-Rohman in his research on active control of large structures used pole placement to determine the optimal control law and generate the gain matrix. The optimal control law was applied to a two story building and was found successful in suppressing the response of the building [10]. Nikos Pnevmatikos studied the application of pole placement technique to both single and multi-degree of freedom systems subjected to earthquakes. The algorithm was found to be efficient and economical in terms of control forces [11].

From the literature available on active seismic control of structures, it is found that lot of work has been carried out on LQR and pole placement techniques. But the study on relative performance of the two control algorithms is missing in the literature. This research has been done to study this aspect of comparative performance of the two control algorithms. For this purpose a ten story shear building frame with three tendons placed each in 1st, 4th and 5th story and subjected to El Centro earthquake is taken for numerical simulation. This combination of tendons is taken from reference [12]. The relative performance of the two algorithms are studied on the basis of different parameters like reductions in floor displacements, story drifts, bases shear and these reductions per unit control force generated.

2. Control Methodology

The structural model of the ten story shear building frame model considered in this study and the free body diagram of the ith floor are shown in fig 1(a, b) respectively. The three active tendons each placed in 1st, 4th and 5th Stories of the frame are used. The story height is 3m and width is 4m. The stiffness of all story’s (ki, 1≤ i ≤ 10) is 26800000 N/m. Mass of all the floors (mi, 1≤ i ≤ 10) is 17100 kg. The stiffness (kt) of 12.7 mm, 7-ply tendon used in the modeling is 2300000 N/m and tendons are inclined at an angle (Φ) of 36.87°. The size of all columns and beams is same and equal to 300 x300 mm² and the damping ratio is taken as 5%.

The equation of motion of this model can be written as:

\[ M \ddot{X} + C \dot{X} + KX = \delta I \dot{X}_g + \gamma U \] (1)

The 1st, 2nd and 3rd terms on the left side of equation 1 denote the inertial force, damping force and restoring force of the model respectively and is subjected to an earthquake with M, C, and K being its mass, damping and stiffness matrices. Whereas the 1st and 2nd terms on right hand side give the earthquake and the control forces respectively. \( \delta \) and \( \gamma \) represent the coefficient vector for earthquake acceleration and location of tendons respectively.

The vectors X, \( \dot{X}_g \) and U are the floor displacements, ground acceleration and the control forces respectively. The values of all these parameters are given below.
Equation (1) is written in state space as

$$Z = A \dot{Z} + B_r \ddot{X}_g + B_u U$$

Where, $Z = \begin{bmatrix} X \\ \dot{X} \end{bmatrix}$, $A = \begin{bmatrix} \begin{bmatrix} 0_{10x10} & I_{10x10} \end{bmatrix} & \begin{bmatrix} -K \\ -C \end{bmatrix} \end{bmatrix}_{20x20}$, $B_r = \begin{bmatrix} \{0\} \\ \{g\} \end{bmatrix}_{2x1}$, $B_u = \begin{bmatrix} \{0\} \\ \{\gamma \} \end{bmatrix}_{2x3}$

$Z$ is a vector of displacements and the velocities of different floors, and matrix $A$ is known as uncontrolled plant matrix.

Here, the control force ‘$U$’ is assumed linearly proportional to state vector ‘$Z$’, i.e. $U = -GZ$. $G$ is the gain matrix. This gain matrix can be obtained using different control algorithms. Substituting the value of ‘$U$’ in equation (6), we get:
\[ \dot{Z} = \tilde{A}Z + B_u \tilde{x}_g, \]  

(8)

Where \( \tilde{A} = A - B_uG \), \( \tilde{A} \) is known as controlled plant matrix.

In linear quadratic regulator, the gain matrix is obtained by minimising the total energy associated with the model using different optimisation techniques. The equation of motion of the structure is considered as constraint in this optimisation. The total energy is given by a quadratic function below:

\[ J = \int_0^{t_f} (Z^T(t) QZ(t) + U^T(t)RU(t))dt \]  

(9)

Where, \( t_f \) denotes the duration of the earthquake excitation, \( Q \) is a \( 2n \times 2n \) semi-definite matrix, and \( R \) is an \( n \times r \) positive definite matrix. degrees of freedom and \( r \) is the number of control devices used. The effectiveness of this strategy is decided by the accuracy of selection of weighting matrices \( Q \) and \( R \).

Since this study is a comparative study of two algorithms, the parameters of these two algorithms are selected keeping in mind that the reduction in response is more and control force required does not exceed 100 kN. Here the \( Q \) and \( R \) matrices are chosen as:

\[ Q = q \begin{bmatrix} 1 & \ldots & 0 \\ \vdots & & \vdots \\ 0 & \ldots & 1 \end{bmatrix} \begin{bmatrix} 0 & \ldots & 0 \\ \vdots & & \vdots \\ 0 & \ldots & 0 \end{bmatrix}, \quad R = \text{eye}(3) \]  

(10)

The value of \( q \) is selected using a trial approach wherein the reduction in responses like maximum floor displacement, maximum base shear and maximum story drift are plotted against the control force as shown in fig (2) and that value of \( q \) is selected where the reduction in these responses are more and control force generated is less. In the figure

\[ p_1 = \frac{\text{maximum controlled floor displacement}}{\text{maximum Uncontrolled floor displacement}}; \quad p_2 = \frac{\text{maximum controlled story drift}}{\text{maximum uncontrolled story drift}} \]

\[ p_3 = \frac{\text{maximum controlled base shear}}{\text{maximum uncontrolled base shear}}, p_4 = \frac{\text{control force}}{\text{weight of the frame}} \times 100 \]

The value of \( q \) where the above criteria is satisfied is selected. Thus value of \( q \) is different for different responses. The gain matrix after minimisation of equation 9 is given by

\[ U(t) = \left( -\frac{1}{2} \right) R^{-1} B_u^T P Z(t) = -GZ(t) \]  

(11)

Matrix \( P \) is the solution of matrix Riccati equation shown below.

\[ \left[ P A - \left( \frac{1}{2} \right) P BR^{-1} B^T + A^T P + 2Q \right] = 0 \]  

(12)

In Pole Placement technique, the poles of the controlled plant matrix \( \tilde{A} \) are also assumed on trial basis by shifting the poles of the uncontrolled plant matrix \( A \) towards left of the complex plane and keeping in mind the above criteria. The gain matrix is obtained from the assumed poles by using Ackerman’s formula.

3. Results and Discussion

In this study, the ability of the two active control algorithms viz. Linear Quadratic Regulator and Pole Placement to reduce the responses of the ten story regular shear building frame subjected to El Centro earthquake is examined.
earthquake is investigated. The control parameters of the two algorithms are obtained using a trial approach discussed in section (2). The frame is equipped with active tendons each in 1st, 4th and 5th story. The performance of the of the two control algorithms with respect to maximum top floor displacement, maximum story drift and base shear are discussed first and subsequently their relative performance have been compared. The percentage reduction of responses per unit control force have also been computed in each case and compared.

LQR control algorithm: The percentage reductions in floor displacements and story drifts of the frame using LQR are shown in tables (1) and (2). As per table (1), the displacement of each of the floors reduces by more than 60%. The maximum uncontrolled displacement is at roof level. This displacement reduces by 71.48%. The percentage reduction per unit control force is 0.88 (table 1). The significant reduction in each of the story drift has been obtained. The uncontrolled drift of 1st story being the maximum is reduced by 59.70% and the reduction per unit control force is 0.73.

Pole Placement technique: The responses of the frame obtained using Pole Placement are given in tables (3) and (4). Results presented in table (3) indicate that this control algorithm reduces displacement of each of the floor by more than 71%. The uncontrolled maximum displacement at roof level is 139 mm, gets reduced by 73.77% to 36.7 mm. The percentage reduction of roof level displacement per unit control force is 0.59. It can also be visualized from table 4, that reduction in the story drift of each of the story is significant. The maximum reduction has taken place in 1st story drift i.e. 73.13% and this percentage reduction per unit control force is 0.63.
The comparison of the performance of LQR and Pole Placement techniques is shown in fig (3). Fig 3(a) shows the variation of floor displacements with respect to height above base under three cases i.e. when there is no control used (UC), when LQR is used, and when Pole Placement technique is used (PP). The figure shows that both LQR and Pole Placement algorithms has caused significant reduction in the floor displacements. Pole placement technique gives higher reductions than the LQR. The performance of LQR and Pole Placement techniques in terms of reduction in story drifts are compared in fig 3 (b). Both the algorithms are almost equally significant in reducing the story drift in most of the Stories. However, Pole Placement method is more effective to reduce drift in 1st story as compared to LQR. The comparison of the reduction in base shear is shown in figure 3(e). The Pole Placement technique performs better to reduce base shear found to be 77% which is more than 60% in case of LQR. Computed percentage reduction of the floor displacements, story drifts and base shear per unit control force are presented in figure 3(c, d, f). The figure shows that the reduction per unit control force is more in LQR control algorithm than in Pole Placement technique.

| Table 1. Comparison of Uncontrolled and Controlled floor displacements using LQR controller |
|---------------------------------------------|-----------------|-----------------|-----------------|-----------------|
| Floor no | uncontrolled floor displacement (m) | controlled floor displacement (m) | Percentage reduction in floor displacements | Percentage reduction in floor displacements per unit control force |
| 1 | 0.0201 | 0.0073 | 63.68 | 0.78 |
| 2 | 0.0398 | 0.0154 | 61.31 | 0.75 |
| 3 | 0.0586 | 0.0177 | 69.8 | 0.85 |
| 4 | 0.076 | 0.0204 | 73.16 | 0.90 |
| 5 | 0.0921 | 0.0241 | 73.83 | 0.90 |
| 6 | 0.1066 | 0.0282 | 73.55 | 0.90 |
| 7 | 0.1192 | 0.0327 | 72.57 | 0.89 |
| 8 | 0.1293 | 0.0363 | 71.93 | 0.88 |
| 9 | 0.1364 | 0.0387 | 71.63 | 0.88 |
| 10 | 0.1399 | 0.0399 | 71.48 | 0.88 |

| Table 2. Comparison of Uncontrolled and Controlled story drifts using LQR controller |
|---------------------------------------------|-----------------|-----------------|-----------------|-----------------|
| Story no | uncontrolled story drift (m) | controlled story drift (m) | Percentage reduction in peak story drifts | Percentage reduction in story drifts per unit control force |
| 1 | 0.0201 | 0.0081 | 59.7 | 0.73 |
| 2 | 0.0198 | 0.0102 | 48.48 | 0.59 |
| 3 | 0.019 | 0.0084 | 55.79 | 0.68 |
| 4 | 0.0181 | 0.0094 | 48.07 | 0.59 |
| 5 | 0.0172 | 0.0088 | 48.84 | 0.60 |
| 6 | 0.0159 | 0.0085 | 46.54 | 0.57 |
| 7 | 0.0138 | 0.0078 | 43.48 | 0.53 |
| 8 | 0.0112 | 0.0067 | 40.18 | 0.49 |
| 9 | 0.0079 | 0.0054 | 31.65 | 0.39 |
| 10 | 0.0041 | 0.0031 | 24.39 | 0.30 |
Figure 3. Comparison of the results obtained using LQR and Pole Placement techniques

Table 3. Comparison of Uncontrolled and Controlled floor displacements using Pole Placement technique

| Floor no | uncontrolled floor displacement (m) | controlled floor displacement (m) | Percentage reduction in floor displacements (%) | Percentage reduction in floor displacements per unit control force |
|----------|------------------------------------|-----------------------------------|-----------------------------------------------|---------------------------------------------------------------|
| 1        | 0.0201                             | 0.0043                            | 78.61                                          | 0.63                                                          |
| 2        | 0.0398                             | 0.0112                            | 71.86                                          | 0.58                                                          |
| 3        | 0.0586                             | 0.0076                            | 87.03                                          | 0.70                                                          |
| 4        | 0.076                              | 0.0181                            | 76.18                                          | 0.61                                                          |
| 5        | 0.0921                             | 0.0196                            | 78.72                                          | 0.63                                                          |
| 6        | 0.1066                             | 0.024                             | 77.49                                          | 0.62                                                          |
| 7        | 0.1192                             | 0.0283                            | 76.26                                          | 0.61                                                          |
| 8        | 0.1293                             | 0.032                             | 75.25                                          | 0.60                                                          |
| 9        | 0.1364                             | 0.0351                            | 74.27                                          | 0.60                                                          |
| 10       | 0.1399                             | 0.0367                            | 73.77                                          | 0.59                                                          |
4. Conclusions
In the present work, LQR algorithm has been used to control the response of a ten story shear building frame subjected to El Centro earthquake with optimal placement of the active tendons in 1st, 4th and 5th Storys. The computed responses have been compared with the response of the same frame obtained using Pole Placement technique. The performances of the two algorithms are compared in terms of reductions in maximum floor displacement, maximum story drift, base shear and control force generated. Percentage reduction of the responses per unit control force also has been computed, compared and concluded as under:

- The reduction in maximum floor displacement, maximum story drift and base shear is 71.48%, 59.70% and 62.7% in LQR and 73.77%, 73.14% and 77% in Pole Placement technique. Thus, Pole Placement technique gives considerably higher reductions than given by LQR algorithm and appears to be a superior algorithm. However, the percentage reduction per unit control force of maximum floor displacement, maximum story drift and base shear using LQR method respectively are 0.88, 0.73 and 0.78 all to be more than obtained using Pole Placement technique. To design an economical active control system for a building, the designer has to tradeoff between percentage reduction of the responses and their per unit control force values as per requirement of the structure.

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