The influence of number of grid points and radius increments in determining safety factor and estimated sliding volume on three-dimensional slope stability analysis

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Abstract. The existence of geometric complexity in the longitudinal direction makes 3D Limit Equilibrium Method (LEM) able to depict conditions in the field well and obtain Safety Factor (SF) that is more representative than the 2D limit equilibrium method. Safety factor calculations are not only influenced by the Direction of Sliding (DoS), but also by the location of the slip surface. Therefore, determining the location of the slip surface is important in the 3D limit equilibrium method. In this study, the method used for generating slip surfaces is a grid search where variations are made on the number of grid points and the radius increments to determine the effect of these two variables on the determination of safety factor and estimated sliding volume. Based on the results of the study, it can be stated that the greater the number of grid points and the radius increments, the estimated sliding volume will be more precise, while the calculation of safety factor is only affected by the number of grid points where the greater the number of grid points, the search of critical slip surface will be more accurate.

1. Introduction
PT Bukit Asam Tbk (PTBA) applies an open pit mining system. The production and operations of an open pit mine are largely determined by slope stability. Slope stability is influenced by many factors, both internal and external factors. One of the internal factors that influence slope stability is its geometry. Slope geometry must be designed with regard to geotechnical aspects to minimize the occurrence of landslides that could potentially interfere with mining activities or the occurrence of failure. Therefore, slope stability analysis is needed to evaluate the overall slope plan design. Slope stability is represented by the value of safety factor. One of the methods that can be used to calculate safety factor values is limit equilibrium method.

Slope stability analysis with limit equilibrium method is commonly carried out in two-dimensional analysis. But in modeling complex geometries, 2D analysis cannot simulate them properly. The 2D analysis assumes that the slope width is infinitely wide. This assumption is contrary to the original state where slope width is finite and there is a change in the longitudinal direction of the slope. Slope stability analysis with three-dimensional method can depict slope conditions including changes in longitudinal direction and also include spatial which is not calculated by 2D method. Therefore, the 3D analysis is considered to be able to describe the conditions in the field better than the 2D analysis.

Analysis of slope stability with 3D limit equilibrium method starts by assuming the geometry of the sliding mass. The results of the calculation of slope stability using limit equilibrium method can be expressed in safety factor. In this method, safety factor is not only influenced by the direction of
sliding, but also by the slip surface [1]. Naderi [2] states that safety factor is sensitive to critical slip surface locations. Therefore the determination of a critical slip surface is very important. Safety factor can be obtained correctly if the determination of critical slip surface is accurate. One of the methods that can be used to determine the critical slip surface is the grid search method. This method is used because based on the results of kinematic analysis conducted by Simatupang [3] and Murtejo [4] in the same research area, the sliding type most likely to occur in this area is circular sliding type. In the grid search method, the first thing to do is to determine the size of the grid box with dimensions x, y, and z. After the grid box is available, the user determines the number of grid points that you want to use in the x, y, and z directions. This point serves as the center of rotation. Each center of rotation can produce a number of circles that are used as slip surfaces. The number of circles produced at each center of rotation from the minimum radius to the maximum radius is called the radius increment. Illustration of the number of grid points and radius increment can be seen in Figure 1 and Figure 2

![Figure 1. Illustration of grid search method [5]](image1)

![Figure 2. Illustration of the radius increment in the grid search method](image2)

After assuming the field geomaterial collapsed, the next step is mass discretization of the sliding mass into a number of columns [6]. Square nets are applied to the sliding mass so that the sliding mass is divided into columns. There are two kinds of columns; the active column where the column is inside the sliding mass boundary line, and the inactive column where these columns are outside the sliding mass boundary line. In the calculation, the inactive columns are ignored so that the discrete sliding...
mass is determined only from the sum of the active columns. Figure 3 shows the illustration of the discretization of the sliding mass using a square grid.

![Discretization of the sliding mass using a square grid](image1.png)

**Figure 3.** Discretization of the sliding mass using a square grid [1]

After discretizing the sliding mass, internal and external forces in each column can be calculated based on moment equilibrium, force equilibrium, or both depending on what method of calculation is used. Figure 4 shows the internal and external forces acting on the column where Where $N_i$, $U_i$ = effective normal force and pore pressure force on column base; $S_i$ = mobilized shear force on column base; $\alpha_i$ = space shear angle; $a'$ = projected shear angle; $E_i$ = intercolumn normal force; $X_i$ = intercolumn vertical shear forces; $H_i$ = intercolumn horizontal shear forces; and $P_{vi}$ = vertical external force.

![Internal and external forces that work in a column](image2.png)

**Figure 4.** Internal and external forces that work in a column [7]

2. Methodology
The study was conducted in Pit 3 Timur, Banko Barat, PT Bukit Asam Tbk., South Sumatra, Indonesia. The data used in this study are secondary data owned by PT Bukit Asam. The data used in the study are: 2018 pit plan design, lithology maps, rocks unit weights, rock cohesions and friction angles. Then the data is processed using Slide 3 from Rocscience [5] to analyze slope stability with
limit equilibrium method. The grid search method is used to find the critical slip surface. The calculation method used in this study is Bishop’s simplified where this method assumes that the sliding mass has a circular slip surface. This method only fulfills moment equilibrium [8]. The Bishop’s simplified method in 3D analysis is developed from the Bishop’s simplified 2D analysis. By considering the overall moment equilibrium in the direction of x and the axis that crosses and parallels the y axis, the following equation is obtained:

\[
F_{my} = \frac{\sum[K_i f_{iy}R_i + f_{iy}R_i]}{\sum(W_i + P_{ci}) + \sum N_i \gamma (g_{1y}R_i - g_{1y}R_i) + \sum N_i \gamma (g_{2y}R_i - g_{3y}R_i)}
\]

\[
F_{mx} = \frac{\sum[K_i f_{ix}R_i + f_{ix}R_i]}{\sum(W_i + P_{ci}) + \sum N_i \gamma (g_{1x}R_i - g_{1x}R_i) + \sum N_i \gamma (g_{2x}R_i - g_{3x}R_i)}
\]

in which:

\[
K_{yx} = \frac{C \left[ \frac{(W_i + P_{ci}) \gamma}{\gamma} \right] \tan \phi_i}{1 + \left[ \frac{(f_{iy}) \tan \phi_i}{\gamma} \right]}
\]

\[
K_{zx} = \frac{C \left[ \frac{(W_i + P_{ci}) \gamma}{\gamma} \right] \tan \phi_i}{1 + \left[ \frac{(f_{iz}) \tan \phi_i}{\gamma} \right]}
\]

By considering the overall equilibrium moment passing and parallel to the z axis, the following equation is obtained:

\[
F_{mz} = \frac{\sum[K_i f_{iz}R_i + f_{iz}R_i]}{\sum N_i \gamma (g_{1z}R_i - g_{1z}R_i) + \sum N_i \gamma (g_{2z}R_i - g_{3z}R_i)}
\]

in which:

\[
K_{zx} = \frac{C \left[ \frac{(W_i + P_{ci}) \gamma}{\gamma} \right] \tan \phi_i}{1 + \left[ \frac{(f_{iz}) \tan \phi_i}{\gamma} \right]}
\]

Where \(F_{mx}, F_{my}, \) and \(F_{mz} \) = moment equilibrium in x, y and z directions; \(R_{xi}, R_{yi}, \) and \(R_{zi} \) = lever arm in the in x, y and z directions; \(W_i \) = column weight; \(C_i = c'A_i \) and \(c' \) and \(A_i \) = effective cohesive strength and the base area of the column, respectively; \(g_{1x}, g_{2x}, g_{3x} \) and \(f_{1x}, f_{2x}, f_{3x} \) = unit vectors adopted from Huang and Tsai [9] and Cheng and Yip [7].

Based on the moment equilibrium in this method, global safety factor \(F_m \) can be determined as follows:

\[
F_m = F_{mx} = F_{my} = F_{mz}
\]

Safety factor can be determined by changing the projected sliding direction at a certain angle interval so that \(F_{mx}, F_{my} \) dan \(F_{mz} \) generate the same value.

3. Result and discussion

A few factors that influence the slope stability are rock properties. Properties of rocks which include physical properties and mechanical properties are input parameters in analyzing slope stability and calculating safety factor in limit equilibrium method. There are four lithologies in the study area, which are claystone, siltstone, sandstone, and coal. The mechanical properties of claystone and coal used in this study are obtained from the line mapping so that the rock mass strength is obtained. The rock mass strength is used because it has taken into account the discontinuities that exist in rock masses, thus representing rock strength in the field better than intact rock. Meanwhile, since there is
no line mapping in siltstone and sandstone, the mechanical properties used for the two lithologies are intact rock. The intact rock strengths are obtained from direct shear tests conducted by PT Bukit Asam. Rock mass strength possessed by claystone and coal is considered sufficient to represent rock strength in the study area because Pit 3 Timur consists of 65% claystone and 10% coal [10]. Rock properties used in this study can be seen in Table 1.

3D limit equilibrium analysis is done with Slide3. Since there is no groundwater parameter, Ru coefficient is used as a substitute for it. Ru coefficient is able to model pore pressure as a fraction of vertical soil pressure for each column. The Ru coefficient used in this analysis is 0.7 as an assumption of slope conditions in a natural state. The output of 3D limit equilibrium analysis is safety factor and estimated sliding volume. Figure 6 shows the 3D model used for slope stability analysis. The green color of the model symbolizes claystone lithology, blue is siltstone, yellow is sandstone, while black is coal.

| Lithology | Properties | Value | Strength |
|-----------|------------|-------|----------|
| Claystone | C (KPa)    | 45.81 | Rock mass|
|           | φ (°)      | 34.65 |          |
|           | Unit weight (kN/m³) | 15.44 |          |
| Siltstone | C (KPa)    | 284.00| Intact rock|
|           | φ (°)      | 32.11 |          |
|           | Unit weight (kN/m³) | 17.37 |          |
| Sandstone | C (KPa)    | 346.44| Intact rock|
|           | φ (°)      | 44.95 |          |
|           | Unit weight (kN/m³) | 14.76 |          |
| Coal      | C (KPa)    | 161.60| Rock mass|
|           | φ (°)      | 58.73 |          |
|           | Unit weight (kN/m³) | 10.47 |          |

The grid search method is used to find critical slip surface. The grid search method starts by specifying the grid box dimension. The location and dimensions of the grid box must cover the entire study area so that the search for critical slip surface can be performed optimally. The location and dimensions of the grid box in the direction of x, y, and z can be seen in Table 2.
Table 2. Dimensions and location of the grid box in the analysis of 3D limit equilibrium method

|        | \( X_{\text{max}} \) | \( Y_{\text{max}} \) | \( Z_{\text{max}} \) |
|--------|----------------|----------------|----------------|
| \( X_{\text{min}} \) | 371808 | 9586510 | 500 |
| \( Y_{\text{min}} \) | 370696 | 9585570 | 50 |
| \( Z_{\text{min}} \) | 500 | 450 | 450 |

\( \Delta X = 1112 \) \( \Delta Y = 940 \) \( \Delta Z = 450 \)

To determine the effect of the number of grid points and the radius increments on the safety factor and the estimated sliding volume produced, an analysis of slope stability is carried out several times by varying the two variables. Variations in the number of grid points used in the direction of \( x \), \( y \), and \( z \) are 20x20x10, 30x30x15, and 40x40x20, while the radius increments used are 10, 20, 30, 40, and 50. Table 3 shows the distance between each grid point in the \( x \), \( y \), and \( z \) direction.

Table 3. The number of grid points used in slope stability analysis and the distance between each point

| Grid 20x20x10 | Grid 30x30x15 | Grid 40x40x20 |
|---------------|---------------|---------------|
| \( X(20) = \) | 59 m | 38 m | 29 m |
| \( Y(20) = \) | 49 m | 32 m | 24 m |
| \( Z(10) = \) | 50 m | 32 m | 24 m |

Examples of limit equilibrium analysis results with a grid number of 40x40x20 and a radius increment 40 can be seen in Figure 7, and the global minimum location can be seen in Figure 8. Figure 8 shows that the global minimum is located at the northern highwall. As we can see in Figure 7, the highwall is dominated by orange color which represents the range of safety factor values of 0.8 - 1.2. This is because the highwall slope is dominated by claystone which has a low cohesion and internal friction angle compared to the other lithologies. Recapitulation of slope stability analysis results with variations in the number of grid points and radius increments are shown in Table 4.

Figure 7. The results of 3D slope stability analysis with 40x40x20 grid points and 40 increment radius [10]
Figure 8. The global minimum with 40x40x20 grid points and 40 increment radius is located in the northern highwall [10]

Table 4. Recapitulation of the results of 3D slope stability analysis

| Radius Increment | Number of Grid Points | Safety Factor | Volume (m³) | Location | Direction of Sliding (°) | Center of Rotation | Running time (hour) |
|------------------|-----------------------|---------------|-------------|----------|--------------------------|--------------------|--------------------|
| 10               | 20x20x10              | 0.868         | 2,245,470   | North-HW | 246.7                    | 371070 9586190     | 470 0.84           |
|                  | 30x30x15              | 0.874         | 1,998,950   | North-HW | 246.6                    | 371069 9586190     | 446 2.54           |
|                  | 40x40x20              | 0.873         | 2,009,660   | North-HW | 246.5                    | 371068 9586190     | 443 6.90           |
| 20               | 20x20x10              | 0.868         | 2,245,470   | North-HW | 246.7                    | 371070 9586200     | 429 1.52           |
|                  | 30x30x15              | 0.874         | 2,028,600   | North-HW | 246.5                    | 371069 9586200     | 429 5.30           |
|                  | 40x40x20              | 0.872         | 1,934,080   | North-HW | 246.4                    | 371069 9586200     | 428 17.23          |
| 30               | 20x20x10              | 0.868         | 2,245,470   | North-HW | 246.7                    | 371070 9586170     | 470 2.29           |
|                  | 30x30x15              | 0.906         | 1,628,780   | North-HW | 246.2                    | 371092 9586210     | 352 8.83           |
|                  | 40x40x20              | 0.867         | 1,937,390   | North-HW | 246.1                    | 371076 9586170     | 459 31.94          |
| 40               | 20x20x10              | 0.868         | 2,245,470   | North-HW | 246.7                    | 371070 9586190     | 470 2.82           |
|                  | 30x30x15              | 0.876         | 1,756,790   | North-HW | 246.2                    | 371076 9586200     | 404 13.22          |
|                  | 40x40x20              | 0.872         | 1,934,080   | North-HW | 246.4                    | 371069 9586200     | 428 49.64          |
| 50               | 20x20x10              | 0.868         | 2,245,470   | North-HW | 246.7                    | 371070 9586190     | 470 3.53           |
|                  | 30x30x15              | 0.876         | 2,068,950   | North-HW | 246.3                    | 371077 9586180     | 459 17.77          |
|                  | 40x40x20              | 0.879         | 1,968,990   | North-HW | 246.3                    | 371078 9586190     | 421 72.72          |

Slope stability calculations with variations in the number of grid points and radius increments produce various safety factor values and estimated sliding volumes. Based on these results, the
The relationship between the number of grid points and safety factor for each increment radius can be graphed. The chart can be seen in Figure 9.

![Safety Factor vs Number of Grid Points](image)

**Figure 9.** The relationship between safety factor with the number of grid points for each increment radius

It can be observed in Figure 9 that the greater number of grid points used, the safety factor value will be more headed to the convergent point and more representatives. At the number of grid points of 20x20x10 in each radius increment produces the same safety factor because the distance between the grid points is still far apart from a distance of 50 - 59 meters per point. This far distance causes the point taking for the center of rotation to fall at the same point because each point has a wide area of influence. The resemblance of the safety factor on the number of grid points of 20x20x10 is also caused by the direction of sliding that have the same value.

However, there is an anomaly in the number of grid points 30x30x15 and radius increment 30 where there is a significant increase in safety factor compared to the other radius increments on the same number of grid points. The increase in safety factor value is not affected by the radius increment. As described by Naderi [2] that there is no relationship between the length of the sliding arc, which in this study is represented by the number of radius increments, to the value of safety factor. The increase in safety factor on the number of grid points 30x30x15 and radius increment 30 is affected by the location of the center of rotation where the center of rotation is at the lowest altitude (352 meters) compared to the other number of grid points and radius increment resulting a difference distance in the critical slip surface than the other hence the safety factor produced is too high. The relationship between sliding arc length and safety factor according to Naderi [2] can be seen in Figure 10.
In the number of grid points of 40x40x20, safety factor drops but not much different from the safety factor resulting from the number of 30x30x15 grid points. At the number of grid points of 40x40x20, there are two data that produce safety factor values that are not uniform compared to the others, which are safety factor at radius increment 30 and 50. Safety factor produced at radius increment 30 is too low because the center of rotation at the this radius increment lies in the highest elevation (459 meters) compared to other radius increments on the same number of grid points resulting in a low safety factor value. In contrast, the center of rotation at the radius increment 50 has the lowest elevation (421 meters) compared to the others so as to produce a high safety factor. The difference in elevation at the center of rotation at the radius increment 30 and 50 affects the location of the critical slip surface so that the safety factor produced is also different.

The output of 3D slope stability analysis is not only in the form of safety factor, but also estimated sliding volume. Just like determining the safety factor, the estimated sliding volume is done by varying the number of grid points and the radius increment to see the effect of the number of grid points and the radius increments on the estimated sliding volume. Figure 11 shows a chart of the relationship of the estimated sliding volume with the number of grid points for each increment radius.

![Estimated Sliding Volume vs Number of Grid Points](image)

**Figure 11.** Number of grid points vs. estimated sliding volume of 3D analysis for each radius increment
It can be observed from Figure 11 that at the number of grid points of 20x20x10 the estimated sliding volume produced is too high. This is because the distance between each grid point is still far apart. The distance between points on the x-axis is 59 meters, y 50 meters, and z 50 meters which means that the search for the critical slip surface has a less thorough coverage resulting in a less accurate estimation of sliding volumes. From Figure 11 it can also be seen that along with the increase in the number of grid points, it will go towards the convergent point. The distance between the grid points is close enough and can cover the entire area of the study thoroughly so that a more precise estimation of sliding volume is obtained.

Estimated sliding volume is not only affected by the number of grid points, but also influenced by the location of the center of rotation. It can be seen in Table 4 that the center of rotation in the radius increment of 10, 20, 30, 40, and 50 in the number of grid points of 20x20x10 is in the same location so that the same sliding volume estimation is obtained. Whereas in the radius increments 30 and 40 at the number of grid points 30x30x15 estimates a low sliding volume compared to the other radius increments. This is because in the radius increment of 30 and 40 in the 30x30x15 grid points, the rotation center is at a low elevation resulting in a low estimated sliding volume.

4. Conclusions
Based on the results of research and discussions in this study, the influence of the number of grid points and radius increments used in calculating safety factor and estimating sliding volume using Bishop’s simplified method on 3D limit equilibrium method can be concluded as follow:
1. Calculation of the safety factor in 3D analysis is only influenced by the number of grid points because it determines the location of the center of rotation. The greater number of grid points, the more precise the search for critical slip surface will be. On the other hand, the safety factor is not affected directly by the length of the sliding arc which represented by the radius increments in this study.
2. The greater number of grid points and the radius increment used in the 3D analysis, the more accurate the estimated sliding volume will be. Just like the safety factor calculation, the location of the center of rotation holds an important role in calculating the estimated sliding volume.

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