Abstract. In this paper is considered Erdős-Mordell’s inequality for the triangle $\triangle ABC$ in the Taxicab plane geometry. It is shown that for the Erdős-Mordell’s inequality $R_A + R_B + R_C \geq w(r_a + r_b + r_c)$ holds for triangles in appropriate positions, if $w = 3/2$.

1. Introduction

Let the distance between the two points in Euclidean plane, as well as distance between the line and the point, be defined. Then for the $\triangle ABC$, in such a plane, holds Erdős-Mordell’s inequality [3], [16]:

$$R_A + R_B + R_C \geq 2(r_a + r_b + r_c)$$

where $R_A$, $R_B$ and $R_C$ are distances between the point $M$ from the $\triangle ABC$ to the vertices $A$, $B$ and $C$ respectively, and $r_a$, $r_b$ and $r_c$ are the distances of the interior point $M$ of the triangle to the corresponding segments defined by the vertices of $\triangle ABC$ (Fig. 1).

Figure 1: Geometric illustration of the Erdős-Mordell inequality in $\triangle ABC$. 
Let there be given two points \(A(x_A, y_A)\) and \(B(x_B, y_B)\). The distance between them in Taxicab geometry is defined as:

\[
d_1(A, B) = |x_A - x_B| + |y_A - y_B|.
\]  

(2)

Stated distance is also called Manhattan or city block distance. This metric is the special case of Minkowski metric of order \(k\) where \((k \geq 1)\), which is defined by the following formula:

\[
d_k(A, B) = \left(|x_A - x_B|^k + |y_A - y_B|^k\right)^{\frac{1}{k}}
\]  

(3)

Metric of Minkowski contains in itself the Taxicab metric for value \(k = 1\) and Euclidean metric for \(k = 2\) [8]. Term Taxicab was first introduced by K. Menger [14]. Graphical representation of the distance between the points \(A\) and \(B\) is given on Fig. 2 in the Taxicab metric with \(d_1\) (short/long dished lines) and in the Euclidean metric with \(d_2\) (continuous line).

\[
\text{Figure 2: Minkowski distances.}
\]

Further in this paper is considered only the Taxicab distance.

Let the \(\triangle ABC\) be a triangle with vertices \(A(0, r), B(p, 0), C(q, 0), p \neq q, r \neq 0\). Without diminishing generality, let \(p < q\). We denote by \(M(x, y)\) an arbitrary point in the plane of the triangle \(\triangle ABC\) (Fig. 1). The Taxicab distance from the point \(M\) to the point \(A, B\) and \(C\), are given by functions:

\[
R_A = d_1(M, A) = |x| + |y - r|,
\]

\[
R_B = d_1(M, B) = |x - p| + |y|,
\]

\[
R_C = d_1(M, C) = |x - q| + |y|.
\]  

(4)

Mathematicians Kaya et all [6] determine the distance from point to the line in Taxicab geometry with following statement:

**Lemma 1.** Distance of point \(M(x_M, y_M)\) to the line \(\ell: ax + by + c = 0\) in the Taxicab plane is:

\[
d_1(M, \ell) = \frac{|ax_M + by_M + c|}{\max\{|a|, |b|\}}.
\]  

(5)
Let us notice that
\[ r_a = d_1(M, \ell_{BC}), \quad r_b = d_1(M, \ell_{AC}), \quad r_c = d_1(M, \ell_{AB}). \] (6)

Based on (4) and (6), ERDŐS-MORDELL’s inequality (1) for the \( \triangle ABC \) in the Taxicab metric is determined by the following relation:
\[ |x| + |y - r| + |x - p| + |x - q| + 2|y| \geq 2 \left( |y| + \frac{|qr - rx - qy|}{\max\{|r|,|q|\}} + \frac{|pr - rx - py|}{\max\{|r|,|p|\}} \right). \] (7)

Inequalities in the Taxicab geometry are topic of interest of some recent research, such as [7]. Let us emphasize that the topic of the ERDŐS-MORDELL inequality is current, as it has been shown in the papers [2], [4], [9]–[12], [20] and books [1] and [15]. V. PAMBUCIAN proved that, in the plane of absolute geometry, the ERDŐS-MORDELL inequality is an equivalent to the non-positive curvature [18]. In the paper [13] is given an extension of the ERDŐS-MORDELL inequality on the interior of the ERDŐS-MORDELL curve. In relation to the ERDŐS-MORDELL inequality N. DERGIADES proved in the paper [2] one extension of the ERDŐS-MORDELL type inequality. The ERDŐS-MORDELL inequality in Taxicab plane geometry is considered by N. SÖNMEZ who has shown that in case of (1) holds strict inequality: \( R_A + R_B + R_C > 2(r_a + r_b + r_c). \) [19]. In this paper is proven that conclusion stated by N. SÖNMEZ is not correct. That shall be shown through following example.

**Example 1.** (Counterexample) Let the vertices of triangle \( \triangle ABC \) be given with \( p = -20, q = 40, r = 30 \) and let the point \( M(0,m) \) be demirned by \( m = 2 \) (Fig. 3). The Taxicab distance from the point \( M \) to the vertices of the \( \triangle ABC \) is given by (4) and distance from the point \( M \) to the lines \( \ell_{bc} : y = 0, \ell_{ac} : -rx - qy + qr = 0 \) and \( \ell_{ab} : -rx - py + pr = 0 \) is given by (5):

\[
R_A = d_1(M, A) = 28, \quad R_B = d_1(M, B) = 22, \quad R_C = d_1(M, C) = 42, \\
r_a = d_1(M, \ell_{ac}) = 2, \quad r_b = d_1(M, \ell_{ac}) = 28, \quad r_c = d_1(M, \ell_{ab}) = \frac{56}{3}. \] (8)

![Figure 3: Geometric illustration of Counterexample.](image)
From (8) we obtain \( L = R_A + R_B + R_C = 92 \) and \( R = r_a + r_b + r_c = \frac{146}{3} \). In case of Erdős-Mordell inequality should hold that \( L \geq 2R \), that is \( 92 \geq 97.\frac{3}{4} \). From there follows that Erdős-Mordell inequality in Taxicab geometry \( R_A + R_B + R_C > 2(r_a + r_b + r_c) \) doesn’t hold for all points inside of triangle \( \triangle ABC \). □

Further in this paper is considered Erdős-Mordell inequality for Taxicab geometry in the following form:

\[
R_A + R_B + R_C \geq w(r_a + r_b + r_c),
\]

where positive real number \( w \) is determined in such a way that the previous inequality holds for all points inside of the triangle \( \triangle ABC \). Main goal of this paper is to determine the upper bound \( M \) for the cases of positive weight coefficients \( w \) such that for \( 0 < w \leq M \) Erdős-Mordell inequality (9) holds.

2. The Main Results

Erdős-Mordell inequality in Taxicab plane geometry has the following form:

\[
|x| + |y - r| + |x - p| + |x - q| + 2|y| \geq w \left( |y| + \frac{|qr - rx - qy|}{\max\{|r|,|q|\}} + \frac{|pr - rx - py|}{\max\{|r|,|p|\}} \right).
\]

Let us remark that Erdős-Mordell inequality in Taxicab plane geometry given in (10) refers to triangles \( \triangle ABC \) in appropriate position where \( A(0,r), B(p,0) \) and \( C(q,0) \) while \( p,q,r > 0 \) (case 1° (a)); \( p \geq 0, q,r > 0 \) (case 1° (b), (c)) and \( p < 0 \) and \( q,r > 0 \) (case 2°). At that section, we don’t consider general position of triangle \( \triangle ABC \) in the Taxicab plane.

1° Let us analyse \( \triangle ABC \) when \( p,q,r > 0 \) (see Fig. 4). Then inside of the triangle holds

\[
|x| = x, \quad |x - p| = \begin{cases} \ p - x : x < p, \\ x - p : x \geq p, \end{cases} \quad |x - q| = q - x, \quad |y| = y, \quad |y - r| = r - y, \quad |qr - rx - qy| = qr - rx - qy, \quad |pr - rx - py| = -pr + rx + py.
\]

Then Erdős-Mordell inequality (10) transforms to:

\[
\begin{align*}
q + r + y + p - x & \geq w \left( y + \frac{qr - rx - qy}{\max\{|r|,|q|\}} + \frac{-pr + rx + py}{\max\{|r|,|p|\}} \right) : x < p \\
q + r + y + x - p & \geq w \left( y + \frac{qr - rx - qy}{\max\{|r|,|q|\}} + \frac{-pr + rx + py}{\max\{|r|,|p|\}} \right) : x \geq p
\end{align*}
\]

Symmetric positions of triangle \( \triangle ABC \) relative to the coordinate axes can analogously be considered.
2° Let us analyse \( \triangle ABC \) when \( p < 0 \) and \( q, r > 0 \) (see Fig. 4). Inside of triangle then holds:

\[
|x| = \begin{cases} 
-x & : x < 0 \\
 x & : x \geq 0 
\end{cases}, \quad |x - p| = x - p, \quad |x - q| = q - x,
\]

\[
|y| = y, \quad |y - r| = r - y,
\]

\[
|qr - rx - qy| = qr - rx - qy, \quad |pr - rx - py| = -pr + rx + py.
\]

From there ERDÖS-MORDELL inequality (10) transforms to:

\[
\begin{cases} 
-p + q + r + y - x \geq \frac{y + \frac{qr - rx - qy}{\max\{r, q\}} + \frac{-pr + rx + py}{\max\{r, -p\}}}{\max\{r, q\}} & : x < 0 \\
-p + q + r + y + x \geq \frac{y + \frac{qr - rx - qy}{\max\{r, q\}} + \frac{-pr + rx + py}{\max\{r, -p\}}}{\max\{r, q\}} & : x \geq 0 
\end{cases}
\]

(14)

Symmetric positions of triangle \( \triangle ABC \) relative to the coordinate axes can analogously be considered.

Let us notice that for vertex \( A(0,r) \) can be made following choices:

1° \( \langle a \rangle \) \( 0 < r \leq p < q, \quad \langle b \rangle \) \( 0 \leq p < r \leq q, \quad \langle c \rangle \) \( 0 \leq p < q < r; \)

in this case see Figure 4/1° with: \( \langle a \rangle \) dished long-short line, \( \langle b \rangle \) dished line, \( \langle c \rangle \) continuous line;

2° \( \langle a \rangle \) \( 0 < r \leq -p < q, \quad \langle b \rangle \) \( 0 < -p \leq r \leq q, \quad \langle c \rangle \) \( 0 < -p \leq q < r; \)

in this case see Figure 4/2° with: \( \langle a \rangle \) dished long-short line, \( \langle b \rangle \) dished line, \( \langle c \rangle \) continuous line.

\[\text{Figure 4: Two types of the triangles } \triangle ABC.\]
In the formula (11), for the triangles of the first type \((i = 1)\), branching is achieved for \(x = p\) and \(p\) is marked with \(x_1\). In the formula (13), for the triangles of the second type \((i = 2)\), branching is achieved for \(x = 0\) and \(0\) is marked with \(x_2\). Then, the ERDÖS-MORDELL inequality (10), with the weight coefficient \(w > 0\), is considered through the following theorems:

**Theorem 1.** It is true that:

\[
R_A + R_B + R_C \geq w (r_a + r_b + r_c) \iff \begin{cases} 
\alpha_{i1} x + \beta_{i1} y + \gamma_{i1} \geq 0 : x < x_i \ [\Pi_{i1}] \\
\alpha_{i2} x + \beta_{i2} y + \gamma_{i2} \geq 0 : x \geq x_i \ [\Pi_{i2}]
\end{cases}
\]  

(15)

where coefficients \(\alpha_{ij}, \beta_{ij}, \gamma_{ij}\) \((j = 1, 2)\), for \(i = 1\) are given in Tab. 1 and for \(i = 2\) are given in Tab. 2.

| \(1^o\) | \(\Pi_{i1}: \alpha_{i1} x + \beta_{i1} y + \gamma_{i1} \geq 0\) | \(\Pi_{i2}: \alpha_{i2} x + \beta_{i2} y + \gamma_{i2} \geq 0\) |
|---|---|---|
| \(\langle a \rangle\) | \(\langle b \rangle\) | \(\langle c \rangle\) |
| \(\Pi_{11}\) | \(\Pi_{12}\) | \(\Pi_{21}\) |
| \(x < p\) | \(x \geq p\) | \(x < 0\) |
| \(\alpha_{11}\) | \(\alpha_{12}\) | \(\alpha_{21}\) |
| \(\beta_{11}\) | \(\beta_{12}\) | \(\beta_{21}\) |
| \(\gamma_{11}\) | \(\gamma_{12}\) | \(\gamma_{21}\) |

**Table 1: ERDÖS-MORDELL inequality in Taxicab plane geometry for case \(1^o\)**

| \(2^o\) | \(\Pi_{i1}: \alpha_{i1} x + \beta_{i1} y + \gamma_{i1} \geq 0\) | \(\Pi_{i2}: \alpha_{i2} x + \beta_{i2} y + \gamma_{i2} \geq 0\) |
|---|---|---|
| \(\langle a \rangle\) | \(\langle b \rangle\) | \(\langle c \rangle\) |
| \(\Pi_{21}\) | \(\Pi_{22}\) | \(\Pi_{21}\) |
| \(x < 0\) | \(x \geq 0\) | \(x < 0\) |
| \(\alpha_{21}\) | \(\alpha_{22}\) | \(\alpha_{21}\) |
| \(\beta_{21}\) | \(\beta_{22}\) | \(\beta_{21}\) |
| \(\gamma_{21}\) | \(\gamma_{22}\) | \(\gamma_{21}\) |

**Table 2: ERDÖS-MORDELL inequality in Taxicab plane geometry for case \(2^o\)**

Let us notice that ERDÖS-MORDELL inequality reduces to the problem of positivity of the linear function

\[
f_{ij}(x,y) = \alpha_{ij} x + \beta_{ij} y + \gamma_{ij} \geq 0,
\]
for some choice of point \((x,y)\) from inside of the triangle, and for some determined values of the parameters \(\alpha_{ij}, \beta_{ij}, \gamma_{ij}\) given in some previous tables. Problem of determining minimum and maximum of linear functions \(f_{ij}(x,y)\) is reduced to finding minimum and maximum in vertices of considered triangles, according to [5]. From there follows that it is enough to consider cases of the minimum and maximum of linear function \(f_{ij}(x,y)\) in the vertices of \(\triangle ABD\) and \(\triangle BCD\) for \(A(0,r), B(p,0), C(q,0)\) and \(D(p, \frac{r}{q}(q-p))\) when \(i = 1\), and in the vertices of \(\triangle ABO\) and \(\triangle ACO\) for \(A(0,r), B(p,0), C(q,0)\) and \(O(0,0)\) when \(i = 2\).

Following statements are true.

**Statement 1.** Let it be that \(A(0,r) \in \Pi_{11}\). If for \(A(0,r)\) the inequality (10) holds, then for the weight coefficient \(w\) also holds:

\[
\langle a \rangle 0 < r \leq p < q \lor \langle b \rangle 0 \leq p < r \leq q \lor \langle c \rangle 0 \leq p < q < r \implies w \leq 2 + \frac{p+q}{r}. \tag{16}
\]

**Proof.** From table 1:

\(\langle a \rangle\) By replacing the coordinates \(x = 0\) and \(y = r\) in \(f_{11}(x,y) = \alpha_{11}x + \beta_{11}y + \gamma_{11}\) is obtained

\[
f_{11}(0,r) \geq 0 \iff ((p-q)wr-pq) \cdot 0 - pq(w-1) \cdot r + pq(p+q+r) \geq 0
\]

\[
\iff -pq(w-1) \cdot r + pq(p+q+r) \geq 0
\]

\[
\iff wr + p + q + 2r \geq 0
\]

\[
w \leq 2 + \frac{p+q}{r};
\]

\(\langle b \rangle\) \(q(r-pw) \cdot r + qr(w(p-r) + p + q + r) \geq 0\), and from there \(w \leq 2 + \frac{p+q}{r}\);

\(\langle c \rangle\) \(w(q-p-r) \cdot r + r(w(p-q) + p + q + r) \geq 0\), and from there \(w \leq 2 + \frac{p+q}{r}\).

**Statement 2.** Let it be that \(A(0,r) \in \Pi_{22}\). If for \(A(0,r)\) the inequality (10) holds, then for the weight coefficient \(w\) also holds:

\[
\langle a \rangle 0 < r \leq -p \leq q \lor \langle b \rangle 0 < -p \leq r \leq q \lor \langle c \rangle 0 < -p \leq q < r \implies w \leq 2 + \frac{q-p}{r}. \tag{17}
\]

**Proof.** From table 2:

\(\langle a \rangle\) \(-pq(w+1) + pq(2rw + p - q - r) \geq 0\), and from there \(w \leq 2 + \frac{q-p}{r}\);

\(\langle b \rangle\) \(q(r-pw) + qr(w(p-r) - p + q + r) \geq 0\), and from there \(w \leq 2 + \frac{q-p}{r}\);

\(\langle c \rangle\) \((w(q-p-r) + r)(w(p-q) - p + q + r) \geq 0\), and from there \(w \leq 2 + \frac{q-p}{r}\). \(\square\)
Statement 3. Let it be that \( B(p, 0) \in \Pi_{12} \). If for \( B(p, 0) \) the inequality \((10)\) holds, then for the weight coefficient \( w \) also holds:

\[
\langle a \rangle \quad 0 < r \leq p < q \lor \langle b \rangle \quad 0 \leq p < r \leq q \implies w \leq 1 + \frac{q^2 + pr}{r(q - p)}; \quad (18)
\]

\[
\langle c \rangle \quad 0 \leq p < q < r \implies w \leq 1 + \frac{p + r}{q - p}. \quad (19)
\]

Proof. From table 1:

\[
\langle a \rangle \quad ((p-q)wr + pq)p + pq(-p + q + r) \geq 0, \text{ and from there } w \leq 1 + \frac{q^2 + pr}{r(q - p)};
\]

\[
\langle b \rangle \quad r(w(r - q) + q)p + qr(w(p - r) - p + q + r) \geq 0, \text{ and from there } w \leq 1 + \frac{q^2 + pr}{r(q - p)};
\]

\[
\langle c \rangle \quad rp + r(w(p - q) - p + q + r) \geq 0, \text{ and from there } w \leq 1 + \frac{p + r}{q - p}. \quad \square
\]

Statement 4. Let it be that \( B(p, 0) \in \Pi_{21} \). If for \( B(p, 0) \) the inequality \((10)\) holds, then for the weight coefficient \( w \) also holds:

\[
\langle a \rangle \quad 0 < r \leq -p \leq q \lor \langle b \rangle \quad 0 < -p \leq r \leq q \implies w \leq \frac{q}{r} \left( 1 + \frac{r - p}{q - p} \right); \quad (20)
\]

\[
\langle c \rangle \quad 0 < -p \leq q < r \implies w \leq 1 + \frac{r - p}{q - p}. \quad (21)
\]

Proof. From table 2:

\[
\langle a \rangle \quad (pq - (p+q)wr)p + pq(2rw + p - q - r) \geq 0, \text{ and from there } w \leq \frac{q}{r} \left( 1 + \frac{r - p}{q - p} \right);
\]

\[
\langle b \rangle \quad r(w(r - q) - q)p + qr(w(p - r) - p + q + r) \geq 0, \text{ and from there } w \leq \frac{q}{r} \left( 1 + \frac{r - p}{q - p} \right);
\]

\[
\langle c \rangle \quad -rp + r(w(p - q) - p + q + r) \geq 0, \text{ and from there } w \leq 1 + \frac{r - p}{q - p}. \quad \square
\]

Statement 5. Let it be that \( C(q, 0) \in \Pi_{12} \). If for \( C(q, 0) \) the inequality \((10)\) holds, then for the weight coefficient \( w \) also holds:

\[
\langle a \rangle \quad 0 < r \leq p \leq q \implies w \leq \frac{p}{r} \left( 1 + \frac{q + r}{q - p} \right); \quad (22)
\]

\[
\langle b \rangle \quad 0 \leq p < r \leq q \lor \langle c \rangle \quad 0 \leq p < q < r \implies w \leq 1 + \frac{q + r}{q - p}. \quad (23)
\]

Proof. From table 1:

\[
\langle a \rangle \quad ((p-q)wr + pq)q + pq(-p + q + r) \geq 0, \text{ and from there } w \leq \frac{p}{r} \left( 1 + \frac{q + r}{q - p} \right);
\]
holds, then for the weight coefficient \( w \) also holds:

\[
\langle a \rangle \quad 0 < r \leq -p \leq q \implies w \leq \frac{-p}{r} \left( 1 + \frac{q + r}{q - p} \right);
\]

\[
\langle b \rangle \quad 0 < -p \leq r \leq q \implies w \leq 1 + \frac{q + r}{q - p}.
\]

**Proof.** From table 2:

\[
\langle a \rangle \quad (-pq - (p+q)wr)q + pq(2rw + p - q - r) \geq 0, \text{ and from there } w \leq \frac{-p}{r} \left( 1 + \frac{q + r}{q - p} \right);
\]

\[
\langle b \rangle \quad r(w(r - q) + q)q + qr(w(p - r) - p + q + r) \geq 0, \text{ and from there } w \leq 1 + \frac{q + r}{q - p};
\]

\[
\langle c \rangle \quad rq + r(w(p - q) - p + q + r) \geq 0, \text{ and from there } w \leq 1 + \frac{q + r}{q - p}.
\]

**Statement 6.** Let it be that \( C(q, 0) \in [\Pi_{22}] \). If for \( C(q, 0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:

\[
\langle a \rangle \quad 0 < r \leq -p \leq q \implies w \leq \frac{-p}{r} \left( 1 + \frac{q + r}{q - p} \right);
\]

\[
\langle b \rangle \quad 0 < -p \leq r \leq q \implies w \leq 1 + \frac{q + r}{q - p};
\]

\[
\langle c \rangle \quad 0 < -p \leq q \implies w \leq 1 + \frac{q + r}{q - p}.
\]

**Proof.** From table 1:

\[
\langle a \rangle \quad ((p - q)wr + pq)p - pq(w - 1)w(q - p) + pq(-p + q + r) \geq 0,
\]

and from there \( w \leq 1 + \frac{q^2 + pr}{2r(q - p)}; \)

\[
\langle b \rangle \quad r(w(r - q) + q)p + q(r - pw)w(q - p) + qr(w(p - r) - p + q + r) \geq 0,
\]

and from there \( w \leq 1 + \frac{q - p}{r + p} + \frac{q}{q - p}; \)

\[
\langle c \rangle \quad rp + (w(q - p - r) + r)w(q - p) + r(w(p - q) - p + q + r) \geq 0,
\]

and from there \( w \leq 1 + \frac{q - p}{r + p} + \frac{q}{q - p}. \)
Statement 8. Let it be that \( O(0, 0) \in [\Pi_{22}] \). If for \( O(0, 0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:

\[
\langle a \rangle 0 < r \leq -p \leq q \implies w \leq \frac{1}{2} + \frac{q-p}{2r}; \tag{28}
\]

\[
\langle b \rangle 0 < -p \leq r \leq q \implies w \leq 1 + \frac{q}{r-p}; \tag{29}
\]

\[
\langle c \rangle 0 < -p \leq q < r \implies w \leq 1 + \frac{r}{q-p}. \tag{30}
\]

Proof. From table 2:

\[
\langle a \rangle pq(2rw + p - q - r) \geq 0, \text{ and from there } w \leq \frac{1}{2} + \frac{q-p}{2r};
\]

\[
\langle b \rangle qr(w(p-r) - p + q + r) \geq 0, \text{ and from there } w \leq 1 + \frac{q}{r-p};
\]

\[
\langle c \rangle r(w(p-q) - p + q + r) \geq 0, \text{ and from there } w \leq 1 + \frac{r}{q-p}. \quad \square
\]

Let there be given positions of the points \( B \) and \( C \). From there we consider positions of point \( A(0, r) \) in concrete cases of \( \langle a \rangle, \langle b \rangle, \langle c \rangle \) which are discussed in Statements 1–8. In those Statements appear functions \( \omega \) of the upper bounds for weight coefficient \( w \):

\[
w \leq \omega(p, q, r).
\]

Our goal is to find the values for functions \( \omega(p, q, r) \):

\[
\mathcal{M} = \inf\{ \omega(p, q, r) | \langle \theta \rangle \}, \tag{31}
\]

which depend on concrete cases of \( \langle \theta \rangle \), where \( \theta \in \{a, b, c\} \). In this way Erdős-Mordell inequality (9) holds for \( w = \mathcal{M} \) for every point in the area of the triangle \( \triangle ABC \). If \( \mathcal{M} \) is minimum in some area, then in (9) is possible equality in such area.

Determining value \( \mathcal{M} \) by areas

In this section of paper are determined values \( \mathcal{M} \) by areas of the triangle \( \triangle ABC \) depending of cases of \( \langle \theta \rangle \), where \( \theta \in \{a, b, c\} \).

The following three propositions are obtained based on the Statement 1.

Proposition 1. Let it be that \( A(0, r) \in [\Pi_{11}] \). If for \( A(0, r) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:

\[
\langle a \rangle 0 < r \leq p < q \implies w \leq \omega(p, q, r) = 2 + \frac{p+q}{r}; \tag{32}
\]

and from there follows that

\[
\omega(p, q, r) \in (\mathcal{M}, \infty) \text{ and } \mathcal{M} = 4. \tag{33}
\]
Proof. Let us consider  $\langle a \rangle$ $0 < r \leq p < q$. Let us notice that the following holds true:
\[ \omega(p, q, r) = 2 + \frac{p + q}{r} \geq 2 + \frac{p + q}{p} = 3 + \frac{q}{p} > 4 \implies M = 4. \]

Stated conclusion is true as value $\frac{q}{p}$ fulfills $\frac{q}{p} > 1$ and it is possible to choose value $\frac{q}{p}$ arbitrarily close to 1. \qed

**PROPOSITION 2.** Let it be that $A(0, r) \in \Pi_{11}$. If for $A(0, r)$ the inequality (10) holds, then for the weight coefficient $w$ also holds:
\[ \langle b \rangle 0 \leq p < r \leq q \implies w \leq \omega(p, q, r) = 2 + \frac{p + q}{r} \] and from there follows that
\[ \omega(p, q, r) \in [M, \infty) \text{ and } M = 3. \] \hspace{1cm} (34)

Proof. Let us consider $\langle b \rangle 0 \leq p < r \leq q$. Let us notice that the following holds true:
\[ \omega(p, q, r) = 2 + \frac{p + q}{r} \geq 2 + \frac{p + q}{q} = 3 + \frac{p}{q} \geq 3 \implies M = 3. \] \hspace{1cm} (35)

**PROPOSITION 3.** Let it be that $A(0, r) \in \Pi_{11}$. If for $A(0, r)$ the inequality (10) holds, then for the weight coefficient $w$ also holds:
\[ \langle c \rangle 0 \leq p < q < r \implies w \leq \omega(p, q, r) = 2 + \frac{p + q}{r} \] and from there follows that
\[ \omega(p, q, r) \in (M, \infty) \text{ and } M = 2. \] \hspace{1cm} (36)

Proof. Let us consider $\langle c \rangle 0 \leq p < q < r$. Let us notice that the following holds true:
\[ \omega(p, q, r) = 2 + \frac{p + q}{r} > 2 \implies M = 2. \] \hspace{1cm} \qed

The following three propositions are obtained based on the Statement 2.

**PROPOSITION 4.** Let it be that $A(0, r) \in \Pi_{22}$. If for $A(0, r)$ the inequality (10) holds, then for the weight coefficient $w$ also holds:
\[ \langle a \rangle 0 < r \leq -p \leq q \implies w \leq \omega(p, q, r) = 2 + \frac{q - p}{r} \] \hspace{1cm} (38)
and from there follows that
\[ \omega(p, q, r) \in [M, \infty) \text{ and } M = 4. \] \hspace{1cm} (39)
Proof. Let us consider \( \langle a \rangle \ 0 < r \leq -p \leq q \). Let us notice that the following holds true:
\[
\omega(p, q, r) = 2 + \frac{q - p}{r} \geq 2 + \frac{q - p}{-p} = 3 + \frac{q}{-p} \geq 4 \implies \mathcal{M} = 4.
\]
\[\square\]

**Proposition 5.** Let it be that \( A(0, r) \in [\Pi_{12}] \). If for \( A(0, r) \) the inequality \((10)\) holds, then for the weight coefficient \( w \) also holds:
\[
\langle b \rangle \ 0 < -p \leq r \leq q \implies w \leq \omega(p, q, r) = 2 + \frac{q - p}{r}
\]
and from there follows that
\[
\omega(p, q, r) \in (\mathcal{M}, \infty) \text{ and } \mathcal{M} = 3.
\]

**Proof.** Let us consider \( \langle b \rangle \ 0 < -p \leq r \leq q \). Let us notice that the following holds true:
\[
\omega(p, q, r) = 2 + \frac{q - p}{r} \geq 2 + \frac{q - p}{q} = 3 + \frac{-p}{q} \geq 3 \implies \mathcal{M} = 3.
\]
\[\square\]

**Proposition 6.** Let it be that \( A(0, r) \in [\Pi_{12}] \). If for \( A(0, r) \) the inequality \((10)\) holds, then for the weight coefficient \( w \) also holds:
\[
\langle c \rangle \ 0 < -p \leq q < r \implies w \leq \omega(p, q, r) = 2 + \frac{q - p}{r}
\]
and from there follows that
\[
\omega(p, q, r) \in (\mathcal{M}, \infty) \text{ and } \mathcal{M} = 2.
\]

**Proof.** Let us consider \( \langle c \rangle \ 0 < -p \leq q < r \). Let us notice that the following holds true:
\[
\omega(p, q, r) = 2 + \frac{q - p}{r} > 2 \implies \mathcal{M} = 2.
\]
\[\square\]

The following three propositions are obtained based on the Statement 3.

**Proposition 7.** Let it be that \( B(p, 0) \in [\Pi_{12}] \). If for \( B(p, 0) \) the inequality \((10)\) holds, then for the weight coefficient \( w \) also holds:
\[
\langle a \rangle \ 0 < r \leq p < q \implies w \leq \omega(p, q, r) = 1 + \frac{q^2}{r(q - p)} + \frac{p}{q - p}
\]
and from there follows that
\[
\omega(p, q, r) \in [\mathcal{M}, \infty) \text{ and } \mathcal{M} = 3 + 2\sqrt{2}.
\]
Proof. Let us consider \( \langle a \rangle 0 < r \leq p < q \). Let us notice that the following holds true:

\[
\omega(p, q, r) = 1 + \frac{q^2}{r(q-p)} + \frac{p}{q-p}
\]

\[
\geq 1 + \frac{q^2}{p(q-p)} + \frac{p}{q-p}
\]

\[
= 3 + \frac{2p}{q-p} + \frac{q-p}{p} \geq 3 + 2\sqrt{2} \implies \mathcal{M} = 3 + 2\sqrt{2},
\]

as for \( t = \frac{p}{q-p} > 0 \) holds \( 2t + \frac{1}{t} \geq 2\sqrt{2} \).

\[\blacksquare\]

PROPOSITION 8. Let it be that \( B(p, 0) \in [\Pi_{12}] \). If for \( B(p, 0) \) inequality (10) holds, then for the weight coefficient \( w \) also holds:

\[
\langle b \rangle 0 \leq p < r \leq q \implies w \leq \omega(p, q, r) = 1 + \frac{q^2}{r(q-p)} + \frac{p}{q-p}
\]

and from there follows that

\[
\omega(p, q, r) \in [\mathcal{M}, \infty) \quad \text{and} \quad \mathcal{M} = 2.
\]

Proof. Let us consider \( \langle b \rangle 0 \leq p < r \leq q \). Let us notice that the following holds true:

\[
\omega(p, q, r) = 1 + \frac{q^2}{r(q-p)} + \frac{p}{q-p}
\]

\[
\geq 1 + \frac{q^2}{p(q-p)} + \frac{p}{q-p}
\]

\[
= 2 + \frac{2p}{q-p} \geq 2 \implies \mathcal{M} = 2.
\]

\[\blacksquare\]

PROPOSITION 9. Let it be that \( B(p, 0) \in [\Pi_{12}] \). If for \( B(p, 0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:

\[
\langle c \rangle 0 \leq p < q < r \implies w \leq \omega(p, q, r) = 1 + \frac{p+r}{q-p}
\]

and from there follows that

\[
\omega(p, q, r) \in (\mathcal{M}, \infty) \quad \text{and} \quad \mathcal{M} = 2.
\]
Proof. Let us consider \( \langle c \rangle \leq p < q < r \). Let us notice that the following holds true:

\[
\omega(p, q, r) = 1 + \frac{p + r}{q - p} > 1 + \frac{p + q}{q - p}
\]

\[
= 2 + \frac{2p}{q - p} \geq 2 \implies M = 2.
\]

\[\blacksquare\]

The following three propositions are obtained based on the Statement 4.

**Proposition 10.** Let it be that \( B(p, 0) \in [\Pi_{21}] \). If for \( B(p, 0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:

\[
\langle a \rangle \ 0 < r \leq -p \leq q \implies w \leq \omega(p, q, r) = \frac{q}{r} \left( 1 + \frac{r - p}{q - p} \right)
\]

and from there follows that

\[
\omega(p, q, r) \in [M, \infty) \text{ and } M = 2.
\]

**Proof.** Let us consider \( \langle a \rangle \ 0 < r \leq -p \leq q \). Let us notice that the following holds true:

\[
\omega(p, q, r) = \frac{q}{r} \left( 1 + \frac{r - p}{q - p} \right)
\]

\[
\geq \frac{q}{r} \left( 1 + \frac{r - p}{2q} \right)
\]

\[
\geq \frac{q}{r} \left( 1 + \frac{2r}{2q} \right) = \frac{q}{r} + 1 \geq 2 \implies M = 2.
\]

\[\blacksquare\]

**Proposition 11.** Let it be that \( B(p, 0) \in [\Pi_{21}] \). If for \( B(p, 0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:

\[
\langle b \rangle \ 0 < -p \leq r \leq q \implies w \leq \omega(p, q, r) = \frac{q}{r} \left( 1 + \frac{r - p}{q - p} \right)
\]

and from there follows that

\[
\omega(p, q, r) \in [M, \infty) \text{ and } M = 2.
\]

**Proof.** Let us consider \( \langle b \rangle \ 0 < -p \leq r \leq q \). Let us notice that the following holds true:

\[
\omega(p, q, r) = \frac{q}{r} + \frac{q}{r} \frac{r - p}{q - p} \geq 1 + \frac{q}{r} \frac{r - p}{q - p} \geq 2 \implies M = 2,
\]

as \( q(r + (-p)) \geq r(q + (-p)) \iff q \geq r. \]
PROPOSITION 12. Let it be that $B(p,0) \in [\Pi_{21}]$. If for $B(p,0)$ the inequality \[ 10 \] holds, then for the weight coefficient $w$ also holds:

\[ \langle c \rangle 0 < -p \leq q < r \implies w \leq \omega(p,q,r) = 1 + \frac{r-p}{q-p} \] \hspace{1cm} (54)

and from there follows that

\[ \omega(p,q,r) \in (\mathcal{M}, \infty) \text{ and } \mathcal{M} = 2. \] \hspace{1cm} (55)

Proof. Let us consider $\langle c \rangle 0 < -p \leq q < r$. Let us notice that the following holds true:

\[ \omega(p,q,r) = 1 + \frac{r-p}{q-p} > 1 + \frac{q-p}{q-p} = 2 \implies \mathcal{M} = 2. \]

The following three propositions are obtained based on the Statement 5.

PROPOSITION 13. Let it be that $C(q,0) \in [\Pi_{12}]$. If for $C(q,0)$ the inequality \[ 10 \] holds, then for the weight coefficient $w$ also holds:

\[ \langle a \rangle 0 < r \leq p < q \implies w \leq \omega(p,q,r) = \frac{p}{r} \left( 1 + \frac{q+r}{q-p} \right) \] \hspace{1cm} (56)

and from there follows that

\[ \omega(p,q,r) \in (\mathcal{M}, \infty) \text{ and } \mathcal{M} = 2. \] \hspace{1cm} (57)

Proof. Let us consider $\langle a \rangle 0 < r \leq p < q$. Let us notice that the following holds true:

\[ \omega(p,q,r) = \frac{p}{r} \left( 1 + \frac{q+r}{q-p} \right) \]

\[ \geq 1 + \frac{q+r}{q-p} \]

\[ = \frac{2q-2p+r+p}{q-p} > 2 + \frac{r+p}{q-p} > 2 \implies \mathcal{M} = 2. \]

\[ \square \]

PROPOSITION 14. Let it be that $C(q,0) \in [\Pi_{12}]$. If for $C(q,0)$ the inequality \[ 10 \] holds, then for the weight coefficient $w$ also holds:

\[ \langle b \rangle 0 \leq p < r \leq q \implies w \leq \omega(p,q,r) = 1 + \frac{q+r}{q-p} \] \hspace{1cm} (58)

and from there follows that

\[ \omega(p,q,r) \in (\mathcal{M}, \infty) \text{ and } \mathcal{M} = 2. \] \hspace{1cm} (59)
Proof. Let us consider \( \langle b \rangle \) \( 0 \leq p < r \leq q \). Let us notice that the following holds true:
\[
\omega(p, q, r) = 1 + \frac{q + r}{q - p} > 1 + \frac{q + p}{q - p} \geq 2 \implies M = 2.
\]
\[
\blacksquare
\]

**Proposition 15.** Let it be that \( C(q, 0) \in [\Pi_{12}] \). If for \( C(q, 0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:
\[
\langle c \rangle \ 0 \leq p < q < r \implies w \leq \omega(p, q, r) = 1 + \frac{q + r}{q - p} \tag{60}
\]
and from there follows that
\[
\omega(p, q, r) \in (M, \infty) \quad \text{and} \quad M = 3. \tag{61}
\]

**Proof.** Let us consider \( \langle c \rangle \) \( 0 \leq p < q < r \). Let us notice that the following holds true:
\[
\omega(p, q, r) = 1 + \frac{q + r}{q - p} = 2 + \frac{r + p}{q - p} > 2 + \frac{q + p}{q - p} \geq 3 \implies M = 3.
\]
\[
\blacksquare
\]

The following three propositions are based on the Statement 6.

**Proposition 16.** Let it be that \( C(q, 0) \in [\Pi_{22}] \). If for \( C(q, 0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:
\[
\langle a \rangle \ 0 < r \leq -p \leq q \implies w \leq \omega(p, q, r) = \frac{-p}{r} \left( 1 + \frac{q + r}{q - p} \right) \tag{62}
\]
and from there follows that
\[
\omega(p, q, r) \in (M, \infty) \quad \text{and} \quad M = 2. \tag{63}
\]

**Proof.** Let us consider \( \langle a \rangle \) \( 0 < r \leq -p \leq q \). Let us notice that the following holds true:
\[
\omega(p, q, r) = \frac{-p}{r} + \frac{-p}{r} \frac{q + r}{q - p}
\]
\[
\geq 1 + \frac{-p}{r} \frac{q + r}{q - p}
\]
\[
= 1 + \frac{-pq + (-p) \cdot r}{rq + (-p)r} \geq 2 \implies M = 2,
\]
as \(-pq + (-p)r \geq rq + (-p)r \iff -p \geq r\). \[
\blacksquare
\]

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PROPPOSITION 17. Let it be that \( C(q,0) \in [\Pi_{22}] \). If for \( C(q,0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:

\[
\langle b \rangle \ 0 < -p \leq r \leq q \implies w \leq \omega(p,q,r) = 1 + \frac{q + r}{q - p}
\]

(64)

and from there follows that

\[
\omega(p,q,r) \in [\mathcal{M}, \infty) \text{ and } \mathcal{M} = 2.
\]

(65)

Proof. Let us consider \( \langle b \rangle \ 0 < -p \leq r \leq q \). Let us notice that the following holds true:

\[
\omega(p,q,r) = 1 + \frac{q + r}{q - p} \geq 1 + \frac{q - p}{q - p} = 2 \implies \mathcal{M} = 2.
\]

\( \square \)

PROPPOSITION 18. Let it be that \( C(q,0) \in [\Pi_{22}] \). If for \( C(q,0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:

\[
\langle c \rangle \ 0 < -p \leq q < r \implies w \leq \omega(p,q,r) = 1 + \frac{q + r}{q - p}
\]

(66)

and from there follows that

\[
\omega(p,q,r) \in (\mathcal{M}, \infty) \text{ and } \mathcal{M} = 2.
\]

(67)

Proof. Let us consider \( \langle c \rangle \ 0 < -p \leq q < r \). Let us notice that the following holds true:

\[
\omega(p,q,r) = 1 + \frac{q + r}{q - p} > 1 + \frac{2q}{q - p} \geq 2 \implies \mathcal{M} = 2,
\]

as \( 2q \geq q - p \iff q \geq -p \).

\( \square \)

The following three propositions are based on the Statement 7.

PROPPOSITION 19. Let it be that \( D(p, \frac{q}{2}(q - p)) \in [\Pi_{12}] \). If for \( D(p, \frac{q}{2}(q - p)) \in [\Pi_{12}] \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:

\[
\langle a \rangle \ 0 < r \leq p < q \implies w \leq \omega(p,q,r) = 1 + \frac{q^2 + pr}{2r(q - p)}
\]

(68)

and from there follows that

\[
\omega(p,q,r) \in [\mathcal{M}, \infty) \text{ and } \mathcal{M} = 2 + \sqrt{2}.
\]

(69)
Proof. Let us consider \( \langle a \rangle 0 < r \leq p < q \). Let us notice that the following holds true:

\[
\omega(p, q, r) = 1 + \frac{q^2}{2r(q-p)} + \frac{p}{2(q-p)} \\
\geq 1 + \frac{q^2}{2p(q-p)} + \frac{p}{2(q-p)} \\
= 2 + \frac{q-p}{2p} + \frac{p}{q-p} \geq 2 + \sqrt{2} \implies M = 2 + \sqrt{2},
\]

as \( t = \frac{p}{q-p} > 0 \) for \( \frac{1}{2t} + t \geq \sqrt{2} \).

PROPOSITION 20. Let it be that \( D(p, \frac{r}{q}(q-p)) \in [\Pi_{12}] \). If for \( D(p, \frac{r}{q}(q-p)) \in [\Pi_{12}] \) the inequality \([10]\) holds, then for the weight coefficient \( w \) also holds:

\[
\langle b \rangle 0 \leq p < r \leq q \implies w \leq \omega(p, q, r) = 1 + \frac{q-p}{r+p} + \frac{q}{q-p}
\]

and from there follows that

\[
\omega(p, q, r) \in [M, \infty) \text{ and } M = \frac{3}{2} + \sqrt{2}.
\]

Proof. Let us consider \( \langle b \rangle 0 \leq p < r \leq q \). Let us notice that the following holds true:

\[
2\omega(p, q, r) = 2 + 2\frac{q-p}{r+p} + \frac{2q}{q-p} \\
= 3 + 2\frac{q-p}{r+p} + \frac{q+p}{q-p} \\
\geq 3 + 2\frac{q-p}{q+p} + \frac{q+p}{q-p} \geq 3 + 2\sqrt{2} \implies M = \frac{3}{2} + \sqrt{2},
\]

as for \( t = \frac{q-p}{q+p} > 0 \) holds \( 2t + \frac{1}{t} \geq 2\sqrt{2} \).

PROPOSITION 21. Let it be that \( D(p, \frac{r}{q}(q-p)) \in [\Pi_{12}] \). If for \( D(p, \frac{r}{q}(q-p)) \in [\Pi_{12}] \) the inequality \([10]\) holds, then for the weight coefficient \( w \) also holds:

\[
\langle c \rangle 0 \leq p < q < r \implies w \leq \omega(p, q, r) = 1 + \frac{q-p}{r+p} + \frac{q}{q-p}
\]

and from there follows that

\[
\omega(p, q, r) \in (M, \infty) \text{ and } M = 2.
\]
Proof. Let us consider \( \langle c \rangle_0 \leq p < q < r \). Let us notice that the following holds true:
\[
\omega(p, q, r) = 1 + \frac{q-p}{r+p} + \frac{q}{q-p} \geq 2 + \frac{q-p}{r+p} > 2 \implies \mathcal{M} = 2,
\]
as \( \frac{q}{q-p} \geq 1 \).

The following three propositions are based on the Statement 8.

**Proposition 22.** Let it be that \( O(0,0) \in [\Pi_{22}] \). If for \( O(0,0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:
\[
\langle a \rangle \ 0 < r \leq -p \leq q \implies w \leq \omega(p, q, r) = \frac{1}{2} + \frac{q-p}{2r} \quad (74)
\]
and from there follows that
\[
\omega(p, q, r) \in [\mathcal{M}, \infty) \text{ and } \mathcal{M} = \frac{3}{2}. \quad (75)
\]

**Proof.** Let us consider \( \langle a \rangle \ 0 < r \leq -p \leq q \). Let us notice that the following holds true:
\[
\omega(p, q, r) = \frac{1}{2} + \frac{q}{2r} + \frac{-p}{2(-p)} \geq \frac{1}{2} + \frac{q}{2(-p)} + \frac{-p}{2(-p)} \geq \frac{3}{2} \implies \mathcal{M} = \frac{3}{2}.
\]

**Proposition 23.** Let it be that \( O(0,0) \in [\Pi_{22}] \). If for \( O(0,0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:
\[
\langle b \rangle \ 0 < -p \leq r \leq q \implies w \leq \omega(p, q, r) = 1 + \frac{q}{r-p} \quad (76)
\]
and from there follows that
\[
\omega(p, q, r) \in [\mathcal{M}, \infty) \text{ and } \mathcal{M} = \frac{3}{2}. \quad (77)
\]

**Proof.** Let us consider \( \langle b \rangle \ 0 < -p \leq r \leq q \). Let us notice that the following holds true:
\[
\omega(p, q, r) = 1 + \frac{q}{r-p} \geq 1 + \frac{q}{2r} \geq 1 + \frac{1}{2} = \frac{3}{2} \implies \mathcal{M} = \frac{3}{2}.
\]

**Proposition 24.** Let it be that \( O(0,0) \in [\Pi_{22}] \). If for \( O(0,0) \) the inequality (10) holds, then for the weight coefficient \( w \) also holds:
\[
\langle c \rangle \ 0 < -p \leq q \implies w \leq \omega(p, q, r) = 1 + \frac{r}{q-p} \quad (78)
\]
and from there follows that
\[
\omega(p, q, r) \in (\mathcal{M}, \infty) \text{ and } \mathcal{M} = \frac{3}{2}. \quad (79)
\]

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Proof. Let us consider $0 < -p \leq q < r$. Let us notice that the following holds true:

$$\omega(p, q, r) = 1 + \frac{r}{q - p} > 1 + \frac{q}{q - p} \geq 1 + \frac{q}{2q} = \frac{3}{2} \implies M = \frac{3}{2}.$$  

Let emphasize that results of the previous three Propositions for Erdős-Mordell’s inequality in Taxicab geometry give an improvement of some results from paper [4].

3. Summa summarum

Based on the previously stated follows

**Theorem 2.** In Taxicab geometry for points inside of the triangle $\triangle ABC$, in appropriate positions, holds Erdős-Mordell’s inequality

$$R_A + R_B + R_C \geq \frac{3}{2} (r_a + r_b + r_c).$$

It is well known that Taxicab distance depends on the rotation of coordinate system, but doesn’t depend on its reflection about a coordinate axis or its translation [17]. For the arbitrary triangle $\triangle ABC$ we set the following open problem (illustrated by Fig. 5.).

**Conjecture 1.** In Taxicab geometry for point inside of any triangle $\triangle ABC$ holds Erdős-Mordell’s inequality

$$R_A + R_B + R_C \geq \frac{3}{2} (r_a + r_b + r_c).$$

![Figure 5. Geometric illustration of Conjecture 1.](image)

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