Flavor Anomalies Accommodated in A Flavor Gauged Two Higgs Doublet Model

Junmou Chen, Qiaoyi Wen, Fanrong Xu, Mengchao Zhang

Department of Physics and Siyuan Laboratory, Jinan University, Guangzhou 510632, P.R. China

E-mail: fanrongxu@jnu.edu.cn

ABSTRACT: The 3.1σ $R_K$ anomaly after Moriond 2021 and 3.3σ $\Delta a_\mu$ from Fermilab Muon g-2 experiment implicate that the lepton flavor universality violation (LFUV) may play a role in the exploration of new physics. A Flavor Gauged Two-Higgs Doublet Model (FG2HDM) is proposed and investigated in this work. To get rid of the redundancy in Yukawa coupling of 2HDM-III, a specific U(1) flavor symmetry is introduced. The charge difference between two scalar doublets forbid the appearance of pseudoscalar and hence there are only three particles (a charged and neutral heavy scalar together with a neutral gauge boson) adding to SM particle spectrum. The heavy neutral scalar-mediated flavor-changing interactions occur among down-type quarks. With obvious difference from 2HDM-II, the charged Higgs in FG2HDM can naturally explain $R_{D(\ast)}$ anomaly. The heavy neutral vector boson $Z'$, changing flavor for down-type quark uniquely as well, provides a solution to $R_{K(\ast)}$. The anomalous magnetic dipole moment (AMDM) of muon and electron, especially the new released $\Delta a_\mu$, can further discriminate $Z'$ parameter space.

KEYWORDS: 2HDM, FCNC, B anomalies, lepton non-universality, anomalous magnetic dipole moment
1 Introduction

So far the Standard Model (SM) is consistent with experiments well, except some anomalies. One type of anomalies occurs in B meson decays. In 2012 BaBar firstly measured the ratio between $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ and $\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell$ ($\ell = e, \mu$), which exceeded SM expectation by $2.0\sigma$ and $2.7\sigma$, respectively. This is the so-called $R_D$ and $R_{D^*}$ anomaly. Though there is a tension with Belle measurement in 2020, giving a more SM-like result, it is not the time to make a clear conclusion. The lepton flavor universality (LFU) is expected to be satisfied in SM. In recent years, however, a violation of LFU (or lepton non-universality) has been unfolded in semileptonic decay $B \to K^{(*)}\ell^+\ell^-$. In 2014, LHCb measured the ratio between $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$ and found a deviation from SM prediction $R_K^{SM} = 1 \pm 0.01$ by $2.6\sigma$. After the continuous updates in 2019 by LHCb and Belle, LHCb reported their latest result with full Run I and Run II data during Moriond 2021, $R_K = 0.846_{-0.039}^{+0.042} \pm 0.013$.
indicating the firm existence of $R_K$ anomaly. Parallel to the pseudoscalar mode, this non-universality also turns up in $B \to K^{*} \ell^{+} \ell^{-}$ process. The data from LHCb in 2017 implicated a 2.2 - 2.4$\sigma$ deviation at low $q^2$ and 2.4 - 2.5$\sigma$ at central $q^2$ region [7], while Belle in 2019 gave a measurement more close to SM [8]. Nevertheless, more precise results are anticipated in the near future with more data accumulated.

Though involving different types of interaction, the anomalies in $b \to c\ell
\nu\ell$ and $b \to s\ell\ell$ co-implicate that the opportunity for new physics may lie in lepton sector. In fact, there is a long-standing anomaly in muon anomalous magnetic dipole moment (AMDM). The SM calculation, including $O(\alpha^5)$ QED and electroweak correction, NNLO hadronic vacuum polarization as well as hadronic light-by-light scattering (HLbL) contribution (see the review [9]), differs the latest Fermilab measurement [10]

$$a^\text{FNAL}_\mu = (116592040 \pm 54) \times 10^{-11}$$

by $\Delta a_\mu = (230 \pm 69) \times 10^{-11}$, corresponding to a 3.3$\sigma$ discrepancy. For the electron AMDM, due to an improved measurement of fine-structure constant $\alpha$ [11] toward a deviation $\Delta a_e = -(8.7 \pm 3.6) \times 10^{-13}$ from theoretical prediction, corresponding to a negative 2$\sigma$ discrepancy. The opposite signs of AMDM for electron and muon provides an independent evidence for the violation of LFU. The latest attempt to connect B anomalies with muon AMDM can be found in [12] after the Fermilab Muon g-2 Experiment reported their first result.

In addition to the lepton sector, it is widely believed that Physics beyond the Standard Model (BSM) is partially encoded in the scalar sector as well. It is known that fermion mass, as well as the Yukawa interaction, is co-determined by scalar VEV and Yukawa couplings. Hence a natural consequence for extending scalar sector from its minimal model enlarges parameter space of Yukawa couplings, which can be further interpreted as one origins of LFU. Among various multiple Higgs models, the Two-Higgs-Doublet Model (2HDM) is one popular choice. In fact, 2HDM is contained naturally in the Minimal Supersymmetric Standard Model (MSSM), and also provides a possibility for a global U(1) symmetry leading to various axion models [13]. Moreover, the extra sources of CP violation in 2HDM can also generate sufficient baryon asymmetry of the universe (BAU) which is unable in SM. There are several variants of 2HDMs classified by the Yukawa interactions, among which Type I 2HDM is the simplest one as one doublet is decoupled with fermions. In the Type II model, up-type quarks and down-type quarks couple to different doublets while charged leptons couple to the same doublet as down-type quark. Comparing with Type II model, the Flipped 2HDM (or Type Y) is just to flip the doublet which the charged leptons couple to. In another popular 2HDM, Lepton-specific 2HDM (or Type X), quarks and charged leptons are assigned to different scalar doublets. (For more details on 2HDMs, one can refer to the review [14].)

As pointing out in [1], a tension between the 2HDM-II and $R_{D^{(*)}}$ anomaly indicates that simple 2HDMs are challenged by current experiments. On the other hand, if all the couplings to fermions are allowed generically, leading to Type III 2HDM [15, 16], the parameter space is too large to be determined. One effort is to impose the Cheng-Sher Ansatz [17] to narrow the parameter space, based on which some recent works can be
found in [18] and works hereafter. It is known that to open generic Yukawa coupling to all fermions bring in the dangerous flavor-changing neutral Higgs (FCNH). In a kind of 2HDM, BGL model, the scalar-mediated FCNC can be suppressed by small off-diagonal elements of CKM matrix under a global flavor symmetry[19]. Recently, a model to localize this flavor symmetry have been proposed [20] and developed [21, 22], in which a new gauge boson corresponding to the U(1) gauge group in charge of flavor symmetry and an extra scalar singlet are introduced in addition to the original 2 doublets in 2HDM.

We learnt some lessons from the above variant models of general 2HDM-III. On one hand, there are too many degrees of freedom in Yukawa coupling and scalar potential. On the other hand, to restrict the redundant parameters, more symmetries and hence extra model dependent parameters are required. One needs to keep a balance between the "restriction" and the "freedom". In this paper, we provide a more economic solution by proposing the flavor gauged 2HDM (FG2HDM). The degrees of freedom in Yukawa couplings are reduced by imposing a U(1) local flavor symmetry, similar as the BGL models, in the price of introducing a new neutral gauge boson with no other particles adding to particle spectrum. The new gauge boson together with exotic Higgs provides a source for lepton non-universality.

This paper is organized as follows. In Sec. 2 we present the main structure of FG2HDM. The FG2HDM contribution to $R_K$ and $R_{K^*}$, AMDM of charged leptons as well as $b \to c\tau\nu$ processes are calculated in Sec. 3. In Sec. 4 a combined numerical analysis is performed and solution space is given. The conclusion and outlook are made in Sec. 5. One can refer to Appendix A for more model details.

## 2 The Flavor Gauged Two-Higgs Doublet Model

The Flavor Gauged Two-Higgs Doublet Model (FG2HDM) is developed from (gauged) BGL model [19, 20]. Imposing the U(1) flavor symmetry on Yukawa interaction, Yukawa couplings have particular texture which further helps to tune the FCNC process mediated by neutral scalars. The U(1) charges of fermions and scalars are assigned with different charges to satisfy anomaly-free condition. Especially, we do not introduce more scalar fields in addition to the 2 doublets in 2HDM. With specific quantum numbers of U(1), some terms in the scalar potential is closed comparing with the most generic one.

### 2.1 Scalar sector

The scalar potential containing two scalar doublets in FG2HDM is of the form

$$V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2$$

$$+ \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1),$$

where $SU(2)_L$ scalar doublet is notated as $\Phi_i = \left( \phi_i^+, \frac{1}{\sqrt{2}}(\phi_i + i h_i + v_i) \right)^T$ and CP violating phases in VEV are not included. Comparing with a more generic potential, the vanishment
of $m_{12}$ and $\lambda_5$ terms is due to the different charges of new gauge group for the two doublets,\(^1\) leading the vanishing mass for the two pseudoscalars. (See Eq. (6) in [14].) The absence of physical pseudoscalars differs from other ordinary 2HDM in literatures. The mass terms for the remaining scalars are

\[
\mathcal{L}_{\phi^\pm} = -\frac{1}{2} \lambda_4 v_1 v_2 \left( \frac{\phi_1^+}{\sqrt{v_1}} \right) \left( \begin{array}{c} \phi_1^- \\ \phi_2^+ \end{array} \right) \left( \begin{array}{c} -1 \\ \frac{v_2}{\sqrt{v_1}} \end{array} \right) \left( \begin{array}{c} \phi_1^- \\ \phi_2^+ \end{array} \right)
\]

\[
\mathcal{L}_\rho = -\frac{1}{2} \left( \rho_1, \rho_2 \right) \left( \begin{array}{cc} \lambda_1 v_1^2 & \lambda_3 v_1 v_2 \\ \lambda_3 v_1 v_2 & \lambda_2 v_2^2 \end{array} \right) \left( \begin{array}{c} \rho_1 \\ \rho_2 \end{array} \right)
\]

where $\lambda_3 = \lambda_3 + \lambda_4$. After diagonalization, another massless charged scalar, together with the two massless neutral pseudoscalars, plays the role of Goldstone bosons which give masses to massive gauge bosons, $W^\pm, Z$ and $Z'$. The rotational matrices, transforming scalars from gauge eigenstates to mass eigenstates, are in the convention of

\[
\left( \begin{array}{c} h \\ H^0 \end{array} \right) = \left( \begin{array}{c} \cos \alpha \sin \alpha \\ -\sin \alpha \cos \alpha \end{array} \right) \left( \begin{array}{c} \rho_1 \\ \rho_2 \end{array} \right),
\]

\[
\left( \begin{array}{c} G^\pm \\ H^\pm \end{array} \right) = \left( \begin{array}{cc} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{array} \right) \left( \begin{array}{c} h \\ \rho \end{array} \right).
\]

with $\tan \beta = \frac{v_2}{v_1}$, vacuum expected value $v = \sqrt{v_1^2 + v_2^2} = \sqrt{2} G_F^{-\frac{1}{2}} \approx 246$ GeV and

\[
\tan \alpha = \frac{1}{2 \lambda_3 \sin \beta \cos \beta} \left[ \lambda_2 \sin^2 \beta - \lambda_1 \cos^2 \beta - \sqrt{(\lambda_1 \cos^2 \beta - \lambda_2 \sin^2 \beta)^2 + 4 \lambda_3 \sin^2 \beta \cos^2 \beta} \right],
\]

which is governed by Higgs coupling $\lambda_{1,2,3,4}$ and $\beta$. Note the angles $\alpha$ and $\beta$ are defined in the rotation of neutral scalar and charged scalar, respectively. In the limit of $\cos(\beta - \alpha) \rightarrow 1$ (or equivalently $\sin(\beta - \alpha) \rightarrow 0$), the mass basis (the basis we adopt here) is identical to Higgs basis. For the charged scalar $G^\pm$ absorbed by $W^\pm$, there remain three physical scalars: the discovered 125 GeV neutral scalar $h$, the undiscovered exotic heavy neutral scalar $H^0$ and heavy charged scalar $H^+$. Without loss of generality, the interactions among scalar eigenstates are given as

\[
-\mathcal{L}_{\text{scalar}} = \frac{\lambda_{h^3}}{3!} h^3 + \frac{\lambda_{h^2 H^0}}{2} h^2 H^0 + \frac{\lambda_{h H^0}}{2} h H^0 + \frac{\lambda_{H^0}}{3!} H^0^3
\]

\[
+ \lambda_{h H^+ H^-} h H^+ H^- + \lambda_{H H^+ H^-} H^+ H^- + \lambda_{H^0 H^+ H^-} \frac{H^0 H^+ H^-}{4} + \lambda_{H^+ H^-} H^0 H^0
\]

\[
+ \frac{\lambda_{h^4}}{4!} h^4 + \frac{\lambda_{h^3 H^0}}{3!} h^3 H^0 + \frac{\lambda_{h^2 H^0}}{4} h^2 H^0 + \frac{\lambda_{h H^0}}{3!} h H^0 + \frac{\lambda_{H^0}}{4!} H^0^4
\]

\[
+ \frac{\lambda_{h^2 H^+ H^-}}{2} h^2 H^+ H^- + \lambda_{h H^0 H^-} h H^0 H^- + \frac{\lambda_{H^0 H^+ H^-}}{2} H^0 H^+ H^- + \lambda_{H^0 H^+ H^-} H^0 H^+ H^- + \frac{\lambda_{H^0 H^+ H^-}}{2} H^0 H^+ H^-
\]

with $h_{[n]} H^0_{[m]} \equiv \underbrace{h \cdots h}_{n} H^0 \underbrace{\cdots H^0}_{m}$ and relevant couplings can be found in Appendix A.2.

\(^1\)For convenience let us adopt this scenario firstly, and later we will show how to realize this conjecture explicitly.
2.2 Yukawa interaction

The Yukawa interaction, including both quark and lepton sectors, are generally in the form of

\[ -\mathcal{L}_Y = \overline{Q} Y^0 (Y_d^1 \Phi_1 + Y_d^2 \Phi_2) d_R^0 + \overline{Q} Y^0 (Y_u^1 \Phi_1 + Y_u^2 \Phi_2) u_R^0 + \overline{L} Y^0 (Y_\ell^1 \Phi_1 + Y_\ell^2 \Phi_2) \ell_R^0 + \overline{L} Y^0 (Y_\nu^1 \Phi_1 + Y_\nu^2 \Phi_2) \nu_R^0 + h.c. \]  

(2.5)

in which the fermion fields with superscript 0 denotes the fields in gauge eigenstate. In total there are eight 3 \times 3 Yukawa matrices. In current work, we assume neutrino mass is Dirac type generated by the corresponding Yukawa matrices \( Y_\nu^i \).

Fermion mass

After spontaneous symmetry breaking, fermion mass terms can be written as

\[ -\mathcal{L}_m = \overline{u} M_u u_R + \overline{d} M_d d_R + \overline{\ell} M_\ell \ell_R + \overline{\nu} M_\nu \nu_R + h.c. , \]  

(2.6)

where \( M_f \) (\( f = u, d, \ell, \nu \)) is diagonal mass matrix of fermions, rotated from \( \tilde{M}_f \),

\[ M_f = U^\dagger fL \tilde{M}_f U^fR, \quad \tilde{M}_f = \frac{1}{\sqrt{2}} (v_1 Y_f^1 + v_2 Y_f^2) , \]  

(2.7)

and \( U_{fL(R)} \) are the rotation matrices connected the fermions in mass eigenstate and weak eigenstate via

\[ f^0_{L(R)} = U_{fL(R)} f_{L(R)} . \]  

(2.8)

The CKM and PMNS matrices then can be defined as

\[ V \equiv V_{\text{CKM}} = U^\dagger uL U^dL, \quad U \equiv V_{\text{PMNS}} = U^\dagger \nu L U^\ell L, \]  

(2.9)

which will play a role in gauge interactions and fermion scalar interactions.

The interactions among fermions and scalars

From the analysis in scalar potential sector, there are three physical scalar particles (\( h, H^0, H^+ \)) in the FG2HDM as the others are eaten by gauge bosons. After rotation and redefinition, one may write down the interaction among fermions and physical scalars in mass eigenstate as follows,

\[ -\mathcal{L} = \frac{\sqrt{2}}{v} H^+ \left[ \overline{u} \left( V_{\text{CKM}} N_d \overline{\Phi}_R - N_d^0 V_{\text{CKM}}^\dagger \overline{\Phi}_L \right) d + \overline{\nu} \left( V_{\text{PMNS}} N_\ell \overline{\Phi}_R - N_\ell^0 V_{\text{PMNS}}^\dagger \overline{\Phi}_L \right) \ell \right] + h.c. \]

\[ + \frac{1}{v} \left[ \cos(\beta - \alpha) H^0 - \sin(\beta - \alpha) h \right] [\overline{u} N_u u + \overline{d} N_d d + \overline{\ell} N_\ell \ell + \overline{\nu} N_\nu \nu] \]

\[ + \frac{1}{v} \left[ \sin(\beta - \alpha) H^0 + \cos(\beta - \alpha) h \right] [\overline{u} M_u u + \overline{d} M_d d + \overline{\ell} M_\ell \ell + \overline{\nu} M_\nu \nu] \]  

(2.10)

where the diagonal mass matrices are of the forms \( M_u = \text{diag}(m_u, m_c, m_t) \), \( M_d = \text{diag}(m_d, m_s, m_b) \), \( M_\ell = \text{diag}(m_e, m_\mu, m_\tau) \), \( M_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \), explicitly. Note in Eq. (2.10) all the fermion are in mass eigenstate, including neutrinos. Especially, according to the field in
original Lagrangian, the tree level flavor-changing current induced by scalar and controlled by $N_f$, given,

$$
N_f = \frac{1}{\sqrt{2}} U^\dagger_Y (v_1 Y_f^2 - v_2 Y_f^1) U_{fR}.
$$

(2.11)

Apparently, the form of $N_f$ is determined by the choice of $Y_f^i$, which further can be regarded as a result of new symmetry. Without loss of generality, $N_f$ can also be simplified to

$$
N_f = -\frac{v_2}{v_1} M_f + \frac{v_2}{\sqrt{2}} \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{fL}^\dagger Y_f^2 U_{fR}.
$$

(2.12)

The Yukawa coupling matrices, so far, have not received any restrictions and hence have the most general structure. We will show in below, under some particular symmetries, the special structure of Yukawa will bring in the good features: the controlled FCNC and the connection to CKM matrix.

**A specific texture for Yukawa coupling matrices**

Under the U(1) symmetry assigned with the particular quantum number shown in Appendix A.1, the Yukawa matrices are of special forms and the tree-level FCNC can be tuned by CKM matrix. Suppose the forms of quark Yukawa textures are

$$
Y_u^1 = \begin{pmatrix} * & 0 & 0 \\ * & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_u^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_1^d = \begin{pmatrix} * & * & * \\ * & * & * \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_2^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ * & * & * \end{pmatrix},
$$

(2.13)

in which ‘*’ denotes a non-zero arbitrary number in corresponding entry. Combing the definitions of $\tilde{M}_f$, $N_f$, one easily obtains the coupling matrix for quarks and scalar

$$
N_u = -\frac{v_2}{v_1} \text{diag}(m_u, m_c, 0) + \frac{v_1}{v_2} \text{diag}(0, 0, m_t),
$$

$$(N_d)_{ij} = -\frac{v_2}{v_1} (M_d)_{ij} + \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) V^\dagger_{i3} V_{3j} (M_d)_{jj},
$$

(2.14)

for the quark sector. The Yukawa texture for leptons are of the form

$$
Y_1^\ell = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}, \quad Y_2^\ell = \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_1^\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ * & * & * \end{pmatrix}, \quad Y_2^\nu = 0,
$$

(2.15)

thus the couplings among leptons and scalar are

$$
N_\nu = -\frac{v_2}{v_1} M_\nu,
$$

$$
N_\ell = -\frac{v_2}{v_1} \text{diag}(0, m_\mu, m_\tau) + \frac{v_1}{v_2} \text{diag}(m_e, 0, 0)
$$

(2.16)

for leptons.

Among all the fermions, only down-type quark receives FCNC mediated by neutral Higgs.
2.3 Gauge interaction

The kinematic terms for scalar fields in Lagrangian is

\[ \mathcal{L}_G = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2), \]  

(2.17)

in which the gauge derivative for scalar field is defined as

\[ D_\mu \Phi_j = \left( \partial_\mu - ig_1 Y_j B_\mu - ig' Q_j \tilde{Z}_\mu - ig_2 \frac{\tau^2}{2} \tilde{W}_\mu \right) \Phi_j, \]  

(2.18)

and hypercharge under \( U(1)_L \) is known as \( Y_1 = Y_2 = \frac{1}{2} \), \( Q_j \) is quantum number of \( \Phi_j \) under \( U(1)' \), given in Eq. (A.2).

Gauge boson mass

After spontaneous symmetry breaking, the mass terms for gauge bosons are

\[ \mathcal{L}_m^G = \begin{pmatrix} B & W^3 \end{pmatrix} \tilde{M} \begin{pmatrix} B \\ W^3 \end{pmatrix} = \begin{pmatrix} A & Z & Z' \end{pmatrix} M_d \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix}. \]  

(2.19)

Impose the condition between the charges of two scalar doublet

\[ Q_1 = -Q_2 \tan^2 \beta, \]  

(2.20)

the mixing of \( Z' \) and other two gauge bosons is decoupled, and the mass matrix can be simplified as

\[ \tilde{M} = \frac{1}{8} (g_1^2 + g_2^2) v^2 \begin{pmatrix} \sin^2 \theta_W & -\sin \theta_W \cos \theta_W & 0 \\ -\sin \theta_W \cos \theta_W & \cos^2 \theta_W & 0 \\ 0 & 0 & 4 \sin^2 \xi (Q_1^2 \cos^2 \beta + Q_2^2 \sin^2 \beta) \end{pmatrix} \]  

(2.21)

with \( \sin \theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}} \), \( \sin \xi = \frac{g'}{\sqrt{g_1^2 + g_2^2}} \). Then the diagonalized mass matrix is

\[ M_d = \frac{1}{8} (g_1^2 + g_2^2) v^2 \begin{pmatrix} 0 \\ 1 \\ 4Q_2^2 \sin^2 \xi \tan^2 \beta \end{pmatrix}. \]  

(2.22)

For more explicit, we obtain the relation of masses between \( Z' \) and \( Z \),

\[ \frac{m_{Z'}}{m_Z} = 2Q_2 \sin \xi \tan \beta, \]  

(2.23)

which is determined by \( Q_2, \xi \) and \( \beta \). And the rotation matrix connected mass eigenstate and gauge eigenstate is

\[ \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix} = U \begin{pmatrix} B \\ W^3 \\ \tilde{Z}' \end{pmatrix}, \quad U = \begin{pmatrix} \cos \theta_W & \sin \theta_W & 0 \\ -\sin \theta_W \cos \theta_W & \cos \theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]  

(2.24)

Note the kinetic mixing between two gauge bosons of SM \( U(1) \) and \( U(1)' \) is not forbidden in principle. For the convenience we choose to close the kinetic mixing in current work.
The interaction among gauge bosons and scalars

The three-point interaction among gauge boson and scalars can be extracted from full expansion of Eq. (2.17), giving

\[
\mathcal{L} = -i \frac{e}{2s_W c_W} (s_{2W} A_\mu + c_{2W} Z_\mu) (\partial^\mu H^- H^+ - \partial^\mu H^+ H^- )
- ig'(Q_1 \sin^2 \beta + Q_2 \cos^2 \beta) Z'_\mu (\partial^\mu H^- H^+ - \partial^\mu H^+ H^- )
- \frac{1}{2} g_2 \sin(\alpha - \beta) [\partial^\mu h(W^-_\mu H^+ - W^+_\mu H^-) + h(\partial^\mu H^- W^+_\mu - \partial^\mu H^+ W^-_\mu)]
- \frac{1}{2} g_2 \cos(\alpha - \beta) [\partial^\mu H^0 (W^-_\mu H^+ - W^+_\mu H^-) + H^0 (\partial^\mu H^- W^+_\mu - \partial^\mu H^+ W^-_\mu)]
+ \frac{1}{2} g_2 g' v \sin 2\beta (Q_2 - Q_1) Z''_\mu (W^-_\mu H^+ + W^+_\mu H^-) + \frac{1}{2} g_2^2 v W^+\mu W^-\mu [\cos(\beta - \alpha) h + \sin(\beta - \alpha) H^0 ]
+ g'^2 v (Q_1^2 \cos \beta \cos \alpha + Q_2^2 \sin \beta \sin \alpha) Z'_\mu Z''_\mu h + \frac{g' e v}{s_W c_W} (Q_1 \cos \beta \cos \alpha + Q_2 \sin \beta \sin \alpha) Z'_\mu Z''_\mu h
+ g'^2 v (Q_2^2 \sin \beta \cos \alpha - Q_1^2 \cos \beta \sin \alpha) Z'_\mu h + \frac{g' e v}{s_W c_W} (Q_2 \sin \beta \cos \alpha - Q_1 \cos \beta \sin \alpha) Z'_\mu h
+ \frac{e^2 v}{4s_W^2 c_W^2} [\cos(\beta - \alpha) h + \sin(\beta - \alpha) H^0 ] Z_\mu Z^\mu
\]

2.4 Fermion current under new gauge symmetry

By imposing the relation between two charges of scalar doublet Eq. (2.20), the mixing of \(Z'\) to other neutral gauge bosons is decoupled. Hence all the currents among fermions and gauge bosons in SM keep unchanged, while the new current brought by \(Z'\) is given as

\[
\mathcal{L}_{Z'} = g' \bar{f} \gamma^\mu \gamma_\mu L + Q_{fR} \gamma^\mu \gamma_\mu R f Z'_\mu
\]  
(2.25)

with \(f = u, d, \nu, \ell\). The coupling to fermions with different chirality is governed by different \(U(1)'\) charges, which in general are given as

\[
Q_{fL} = U_{fL}^\dagger X_{fL} U_{fL}, \quad Q_{fR} = U_{fR}^\dagger X_{fR} U_{fR}.
\]  
(2.26)
We can easily have the explicit expressions for different types of fermions,

\[
Q_{\mu L} = \frac{1}{2} \begin{pmatrix}
Q_{\mu R} + Q_{dR} & Q_{\mu R} + Q_{dR} \\
Q_{\mu R} + Q_{dR} & Q_{tR} + Q_{dR}
\end{pmatrix},
\]

\[
Q_{dL} = \frac{1}{2}(Q_{\mu R} + Q_{dR}) \begin{pmatrix}
1 & 1 \\
1 & 1
\end{pmatrix} + \frac{1}{2}(Q_{tR} - Q_{\mu R}) \begin{pmatrix}
|c_1|^2 & c_1^*c_2 & c_1^*c_3 \\
c_2c_1 & |c_2|^2 & c_2^*c_3 \\
c_3c_1 & c_3^*c_2 & |c_3|^2
\end{pmatrix},
\]

\[
Q_{\mu R} = \begin{pmatrix}
Q_{\mu R} & Q_{\mu R} \\
Q_{tR} & Q_{tR}
\end{pmatrix},
Q_{dR} = Q_{dR} \begin{pmatrix}
1 & 1 \\
1 & 1
\end{pmatrix},
\]

\[
Q_{\ell L} = \begin{pmatrix}
Q_{eL} & Q_{\mu L} \\
Q_{tL} & Q_{tL}
\end{pmatrix},
Q_{\ell R} = \begin{pmatrix}
Q_{eR} & Q_{\mu R} \\
Q_{tR} & Q_{tR}
\end{pmatrix},
\]

\[
(Q_{\nu L})_{ij} = Q_{eL}V_{e}^*V_{ej} + Q_{\mu L}V_{\mu}^*V_{ij} + Q_{tL}V_{t}^*V_{tj}, \quad i, j = 1, 2, 3,
\]

\[Q_{\nu R} = 0\]

in which we have defined \(c_i \equiv V_{ti}, i = d, s, b\). To obtain the above charges coupling to \(Z'\), we have made use of the structure of \(U_{uL}\) and \(U_{uR}\)

\[
U_{uL} = \begin{pmatrix}
* & * & 0 \\
* & * & 0 \\
0 & 0 & 1
\end{pmatrix},
U_{uR} = \begin{pmatrix}
* & * & 0 \\
* & * & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

and

\[
U_{\ell L} = U_{\ell R} = 1
\]

which are required by the form of mass matrices. Apparently, FCNC occurs only in down-type quark sector with left handed chirality.

3 Flavor Anomalies in FG2HDM

As illustrated in above section, there are only three additional particles besides SM particles. The FG2HDM provides a economic solution to the anomalies in \(B \to D^{(*)}\tau\nu\), \(B \to K^{(*)}\ell\bar{\ell}\) and anomalous magnetic dipole moments of muon and electron. In this section, we explicitly calculate the characterized quantities of \(R_{D^{(*)}}\) contributed from the charged Higgs, together with \(R_{K^{(*)}}\) and \(\Delta a_e\) originated from the exotic neutral gauge boson.

3.1 \(R_{K^{(*)}}\) and \(Z'\)

At tree-level, the FCNC process in FG2HDM can be mediated by both exotic neutral scalar \(H^0\) and \(Z'\) in down-type quark decays. It is known that scalar operators do not contribute to \(B \to K\ell\bar{\ell}\) process [23], hence we only consider the NP effect from \(Z'\).
We adopt the following convention to describe $b \rightarrow s \ell \ell$ transition, in which the effective Hamiltonian is in the form of
\[ \mathcal{H} = -\frac{4G_F}{\sqrt{2}vV_{tb}V_{ts}^* \frac{g^2}{16\pi^2}} \sum_i (C_i O_i + C'_i O'_i) + h.c. \] (3.1)
where the effective operators are defined as
\[ O_7 = \frac{m_b}{e} (\bar{s} \sigma_{\mu\nu} P_L b) F^{\mu\nu}, \quad O'_7 = \frac{m_b}{e} (\bar{s} \sigma_{\mu\nu} P_L b) F'^{\mu\nu}, \] (3.2)
\[ O_9 = (\bar{s} \gamma_{\mu} P_L b)(\bar{\ell} \gamma_{\mu} \ell), \quad O'_9 = (\bar{s} \gamma_{\mu} P_L b)(\bar{\ell} \gamma_{\mu} \ell), \]
\[ O_{10} = (\bar{s} \gamma_{\mu} P_L b)(\bar{\ell} \gamma_{\mu} \gamma_5 \ell), \quad O'_{10} = (\bar{s} \gamma_{\mu} P_L b)(\bar{\ell} \gamma_{\mu} \gamma_5 \ell), \]
\[ O_S = m_b (\bar{s} P_R b)(\bar{\ell} \ell), \quad O'_S = m_b (\bar{s} P_R b)(\bar{\ell} \ell), \]
\[ O_P = m_b (\bar{s} P_R b)(\bar{\ell} \gamma_5 \ell), \quad O'_P = m_b (\bar{s} P_R b)(\bar{\ell} \gamma_5 \ell). \]

The scattering amplitude for $b \rightarrow s \ell \ell$ from $Z'$-induced FCNC in FG2HDM, which occurs uniquely in down-type quark sector, is
\[ \mathcal{M} = \frac{g^2}{m_Z^2} \frac{1}{4} V_{tb} V_{ts}^* (Q_{LR} - Q_{UR}) \left[ (Q_{LR} + Q_{\ell L})(\bar{s} \gamma_{\mu} P_L b)(\bar{\ell} \gamma_{\mu} \ell) + (Q_{LR} - Q_{\ell L})(\bar{s} \gamma_{\mu} P_L b)(\bar{\ell} \gamma_{\mu} \gamma_5 \ell) \right]. \] (3.3)

Only the coefficients of $O_{9,10}$ are corrected, hence we extract the modification to Wilson coefficients, giving
\[ \Delta C^d_9 = \frac{g^2}{N m_Z^2} \frac{1}{4} V_{tb} V_{ts}^* (Q_{LR} - Q_{UR})(Q_{LR} + Q_{\ell L}), \] (3.4)
\[ \Delta C^d_{10} = \frac{g^2}{N m_Z^2} \frac{1}{4} V_{tb} V_{ts}^* (Q_{LR} - Q_{UR})(Q_{LR} - Q_{\ell L}), \]
with $N = \frac{4G_F}{\sqrt{2}vV_{tb}V_{ts}^* \frac{g^2}{16\pi^2}}$. We can see $\Delta C^d_{9,10}$ is indeed flavor dependent and corresponding factors can be found in Appendix A.1. It is worthy pointing out that the two degrees of freedom for U(1) charges in FG2HDM, formally giving $(\Delta C^d_9 \sim \frac{1}{b} Q_{LR} - Q_{LL}, \Delta C^d_{10} \sim \frac{1}{b} Q_{LR} + Q_{dL}, \Delta C^d_{9} \sim \frac{1}{2} Q_{LR} + 3Q_{dL}, \Delta C^d_{10} \sim \frac{1}{b} Q_{LR} + Q_{dL})$, provide a chance to explain lepton flavor dependent $R_K^{(\ell)}$ anomaly.

3.2 \( (g - 2)_\ell \) and $Z'$

The anomalous magnetic dipole moment (AMDM) of charged leptons, especially for muon and electron, are generally taken as a platform for checking new physics associated with lepton sector. The flavor-conserving interaction among leptons and exotic neutral gauge boson $Z'$ in FG2HDM, according to Eq. (2.25) and (2.27), indicates that AMDM of charged lepton can be generated via one-loop correction since $Z'$ decouples with photon and $Z$ in current scenario of FG2HDM.

The calculation for a general $Z'$ contribution to AMDM at one-loop level in Feynman gauge can be found in [24]. Ignoring the unphysical scalar contribution safely in the heavy mass limit of vector boson, we have $Z'$ contribution to charged lepton AMDM, denoted as
\[ \Delta a_{\ell} = -\frac{m_t^2 g^2}{8\pi^2 m_Z^2} \frac{2}{3} \left[ (Q_{\ell L}^2 + Q_{\ell R}^2) - 3Q_{\ell L} Q_{\ell R} \right], \] (3.5)
where \( \ell = e, \mu \) and the associated charges \( Q_{\ell_L}, Q_{\ell_R} \) are defined in Eq. (A.3). Comparing with these models with LFU (lepton flavor universality), the FG2HDM has a potential to explain the wrong sign \( \Delta a_\mu \) and \( \Delta a_e \) due to the charge differences among various fermions.

### 3.3 \( R_{D^{(*)}} \) and \( H^+ \)

The observable \( R_{D^{(*)}} \) occurred in \( B \rightarrow D^{(*)}\ell \nu \) decays are defined as

\[
R_{D^{(*)}} = \frac{\mathcal{B}(B \rightarrow D^{(*)}\ell \nu)}{\mathcal{B}(B \rightarrow D^{(*)}\ell \nu)} \bigg|_{\ell=e,\mu,\tau}.
\]

There are types of contributed new physics candidates, including exotic charged Higgs, charge gauge bosons, leptoquarks and so on. In FG2HDM, the exotic charged Higgs \( H^+ \) is naturally accomodated. By integrating out the heavy scalar, the quark level decay \( b \rightarrow c\ell \nu \) can be depicted by the following effective Hamiltonian

\[
\mathcal{H} = C_{RL}^\alpha O_{RL}^\alpha + C_{LL}^\alpha O_{LL}^\alpha + C_{LR}^\alpha O_{LR}^\alpha + C_{RR}^\alpha O_{RR}^\alpha
\]

with four effective operators

\[
O_{RL}^{\alpha k} = (\bar{c}_L \mathbb{P}_R b)(\bar{\ell}_k \mathbb{P}_L \nu_\alpha), \quad O_{LL}^{\alpha k} = (\bar{c}_L \mathbb{P}_L b)(\bar{\ell}_k \mathbb{P}_L \nu_\alpha),
\]

\[
O_{LR}^{\alpha k} = (\bar{c}_L \mathbb{P}_R b)(\bar{\ell}_k \mathbb{P}_R \nu_\alpha), \quad O_{RR}^{\alpha k} = (\bar{c}_R \mathbb{P}_R b)(\bar{\ell}_k \mathbb{P}_R \nu_\alpha),
\]

and their corresponding Wilson coefficients

\[
C_{RL}^{\alpha k} = -\frac{2}{m_H^2 v^2} (V N_d)_{23} (N_L)_{\alpha k}^\dagger, \quad C_{LL}^{\alpha k} = -\frac{2}{m_H^2 v^2} (N_u V)_{23} (N_L)_{\alpha k}^\dagger,
\]

\[
C_{LR}^{\alpha k} = -\frac{2}{m_H^2 v^2} (N_u V)_{23} (N_L)_{\alpha k}^\dagger, \quad C_{RR}^{\alpha k} = -\frac{2}{m_H^2 v^2} (V N_d)_{23} (N_L)_{\alpha k}^\dagger.
\]

where \( V \) is CKM matrix, \( m_H \) is the mass of charged Higgs \( H^+ \) and flavor index \( \alpha = e, \mu, \tau \) for neutrinos and flavor index \( k = 1, 2, 3 \) for charged leptons. The two coefficients \( C_{LR,RR}^{\alpha k} \) are negligible since their sizes are proportional to neutrino mass, leading to the two dominated contributions

\[
C_{RL}^{\tau 3} \approx -2\sqrt{2} G_F V_{cb} \frac{m_\mu m_\tau}{m_H^2} \left( \frac{2}{\tan^2 \beta} + 1 \right),
\]

\[
C_{LL}^{\tau 3} = -2\sqrt{2} G_F V_{cb} \frac{m_\mu m_\tau}{m_H^2}.
\]

In particular, one can see in \( C_{RL}^{\tau 3} \), the dependent behavior of \( \tan \beta \) and \( m_H \) changes dramatically comparing with 2HDM-II [25] and MSSM [26].

Based on [27], in FG2HDM, the charged Higgs contribution to \( R_D \) and \( R_{D^*} \) can be further parameterized as

\[
R_D = R_{D}^{SM} \left[ 1 + 1.5 \text{Re} \left( \frac{C_{RL}^{\tau 3} + C_{LL}^{\tau 3}}{C_{SM}^{\tau 3}} \right) + 1.0 \left| \frac{C_{RL}^{\tau 3} + C_{LL}^{\tau 3}}{C_{SM}^{\tau 3}} \right|^2 \right],
\]

\[
R_{D^*} = R_{D^*}^{SM} \left[ 1 + 0.12 \text{Re} \left( \frac{C_{RL}^{\tau 3} - C_{LL}^{\tau 3}}{C_{SM}^{\tau 3}} \right) + 0.05 \left| \frac{C_{RL}^{\tau 3} - C_{LL}^{\tau 3}}{C_{SM}^{\tau 3}} \right|^2 \right],
\]
where $C_{cb}^{SM} = 2 \sqrt{2} G_F V_{cb}$.

A correlated process to $B \to D^{(*)}\bar{\ell}\nu$ is $B_c \to \tau\bar{\nu}$. Incorporating scalar operator contribution, one obtains branching ratio of $B_c \to \tau\bar{\nu}$ [28, 29] in FG2HDM,

$$B(B_c \to \tau\bar{\nu}) = \frac{1}{8\pi} \tau_{B_c} G_F^2 |V_{cb}|^2 m_{B_c}^2 f_{B_c}^2 \left(1 - \frac{m_{\tau}^2}{m_{B_c}^2}\right)^2 \left[1 + \frac{m_{B_c}^2}{(m_b + m_c)m_{\tau}} C_P\right]^2$$  \hspace{1cm} (3.12)

where $C_P = (C_{RL}^{\tau^3} - C_{LL}^{\tau^3}) / C_{cb}^{SM}$. A numerical analysis to solution space will be carried on in the following Sec. 4.

4 Numerical Analysis

4.1 Experimental status and inputs

There have been continuous updates for the measurements of $R_K$ and $R_{K^*}$ by LHCb, Belle, CMS and ATLAS. The latest measurement, given by LHCb during Moriond 2021, shows the 3.1σ deviation in $R_K$ (see Eq. (1.1)) and confirms the tension between SM. To explore the dynamics in high energy, we make use of global fitting results, which rely on both the experimental data as well as the choices of fitting basis. The latest fitting results, including the LHCb new $R_K$ measurement, are presented in Moriond QCD 2021[30].

Based on 2019 data, the global fit works done by several independent groups [31–33] are consistent well in the following facts: i) large and negative $\delta C_9^\mu$(best fit $\sim -1$) , ii) relative small and positive $\Delta C_10^\mu$(best fit $\sim 0.5$). Two other parameters $\Delta C_{9,10}^c$ were only contained in the analysis of [32, 33], sharing the common features: i) positive and relative large $\Delta C_9^c$(best fit $\sim 0.8$) and ii) negative and relative large $\Delta C_{10}^c$(best fit $\sim -0.78$).

In the new fit of [30] (2D fit), $\Delta C_9^c$ is included compared with the previous work [33] and changes dramatically from [31, 32]: the sign of central value has been flipped. On the other hand, $\Delta C_{10}^c$ is still untouched. Hence a more complete global fit is highly anticipated. In current work, we mainly adopt the central values of 2D and 6D fits in [30], and conjecture the untouched $\Delta C_{10}^c$ combining the results in [31, 32] based on old data, giving

$$\begin{align*}
\Delta C_9^\mu &= -1.21 \pm 0.20, \\
\Delta C_{10}^\mu &= 0.15 \pm 0.20
\end{align*} \hspace{1cm} (4.1)$$

$$\begin{align*}
\Delta C_9^c &= -0.40 \pm 0.40, \\
\Delta C_{10}^c &= -0.78 \pm 0.40
\end{align*} \hspace{1cm} (4.2)$$

In particular, we have allowed more tolerant errors.

For the experimental values $R_D$ and $R_{D^*}$, we adopt world averages from the heavy flavor averaging group (HFLAV) [34]

$$R_D = 0.340 \pm 0.027 \pm 0.013, \quad R_{D^*} = 0.295 \pm 0.011 \pm 0.008, \hspace{1cm} (4.3)$$

which are based on measurements from BaBar, Belle and LHCb. The corresponding SM predictions are known with high precision, reads

$$R_D^{SM} = 0.299 \pm 0.003, \quad R_{D^*}^{SM} = 0.258 \pm 0.005,$$  \hspace{1cm} (4.3)

which is also quoted from HFLAV [34].
The lifetime of $B_c$ meson, we adopt the latest PDG value [35]

$$
\tau_{B_c} = 0.510 \pm 0.009 \text{ ps} \quad (4.4)
$$

The decay mode $B_c \to \tau^- \nu$ has not been measured. Here we take 3 conjectures (see also [29]),

$$
B(B_c \to \tau^- \bar{\nu}) \leq 30\%; \ 20\%; \ 10\%. \quad (4.5)
$$

for convenience.

Improvements of muon AMDM are made due to efforts from both the theoretical and experimental sides. The latest calculation in SM, including $O(\alpha^5)$ QED correction, electroweak correction, NNLO hadronic vacuum polarization (HPV) as well as Hadronic Light-by-Light (HLbL) contributions, is summarized in the review [9], giving

$$
a_{\mu}^{\text{SM}} = (116 591 810 \pm 43) \times 10^{-11} \quad (4.6)
$$

It differs the Brookhaven measurement [36] $a_{\mu}^{\text{BNL}} = (116 592 089 \pm 63) \times 10^{-11}$ by $\Delta a_{\mu} := a_{\mu}^{\text{BNL}} - a_{\mu}^{\text{SM}} = (279 \pm 76) \times 10^{-11}$, corresponding to a $3.7\sigma$ discrepancy. Recently, the Muon g-2 Experiment at Fermilab released their first result [10] after nearly 20 years from Brookhaven’s result, $a_{\mu}^{\text{FNAL}} = (116 592 040 \pm 54) \times 10^{-11}$ leading to the latest $\Delta a_{\mu}^{\text{FNAL}} = (230 \pm 69) \times 10^{-11}$, corresponding to a $3.3\sigma$ discrepancy, which confirms the existence of a tension and strengthens the evidence of new physics. Then the experimental average, by combining BNL and FNAL results together, is given [10] as

$$
a_{\mu}^{\text{exp}} = (116 592 061 \pm 41) \times 10^{-11} \quad (4.7)
$$

and hence the deviation is

$$
\Delta a_{\mu}^{2021} = (251 \pm 59) \times 10^{-11}, \quad (4.8)
$$

with a $4.2\sigma$ significance. In the following numerical calculation, we will take the new combined result Eq.(4.8) as the input.

Recently an improved measurement [11] of the fine-structure constant $\alpha$ toward a deviation in the electron AMDM from theoretical prediction

$$
\Delta a_e = -(8.7 \pm 3.6) \times 10^{-13}, \quad (4.9)
$$

corresponding to a negative $2.4\sigma$ discrepancy. It is worthy pointing out that the sign of $\Delta a_e$ differs the one of $\Delta a_{\mu}$.

Other input parameters are summarized in the in Table 1.

4.2 Numerical results

The priority here in FG2HDM is to find a solution space after introducing three additional particles in an economic way. We take two U(1) charges$^2$, $\tan \beta$, $m_{H^+}$ and $g'/m_{Z'}$ as

$^2$ Generally speaking, there are two free U(1) charges in FG2HDM. In the scenario shown in Eq.(2.20), imposing the decoupling limit of $Z'$, one degree of freedom can be eliminated.
Table 1. Input parameters used in the numerical analysis.

| Parameters | Values |
|------------|--------|
| $V_{cb}$   | $(42.2 \pm 0.8) \times 10^{-3}$ |
| $V_{ts}$   | $(39.4 \pm 2.3) \times 10^{-3}$ |
| $V_{tb}$   | $1.019 \pm 0.025$ |
| $G_F$      | $1.1663787 \times 10^{-5}$ GeV$^{-2}$ |
| $m_b$      | $4.18^{+0.03}_{-0.02}$ GeV |
| $m_{\tau}$ | $1776.86 \pm 0.12$ MeV |
| $m_c$      | $1.27 \pm 0.02$ GeV |
| $m_W$      | $80.379 \pm 0.012$ GeV |
| $m_Z^0$    | $91.1876 \pm 0.0021$ GeV |
| $m_e$      | $0.5109989461 \pm 3.1 \times 10^{-9}$ Mev |
| $m_{\mu}$  | $105.6583745 \pm 2.4 \times 10^{-6}$ Mev |
| $\alpha(m_W)$ | $\frac{1}{128}$ |
| $m_{B_c}$  | $6274.9 \pm 0.8$ MeV |
| $f_{B_c}$  | $0.434$ GeV |
| $\tau_{B_c}$ | $0.510 \pm 0.009$ ps |

Figure 1. The allowed parameter space from $b \rightarrow c\ell\nu$: (a) allowed regions for Wilson Coefficients of scalar operators by $R_D^{(*)}$ associated with conjectures of $B_c \rightarrow \tau\nu$; (b) the allowed parameter space purely from $R_D^{(*)}$ in FG2HDM.

free parameters in the following numerical calculation. Since the relying parameters are uncorrelated so far$^3$, we hence carry out the calculation of $R_D^{(*)}$ and $R_K^{(*)}$ separately. It is understandable that both anomalies can be accommodated in FG2HDM once their corresponding solution space is found.

$^3$A more comprehensive analysis of FG2HDM including more observables is in progress and to be shown shortly.
Figure 2. The allowed free U(1) charges by $R_K$ at $2\sigma$ with different choices of $\tan \beta$: (a) various $\Delta C^\ell_i$ allowed regions with $\tan^2 \beta = 10^2$; (b) various $\Delta C^\ell_i$ allowed regions with $\tan^2 \beta = 10^4$; (c) the allowed region combined all $\Delta C^\ell_i$ with $\tan^2 \beta = 10^2$ and (d) the allowed region combined all $\Delta C^\ell_i$ with $\tan^2 \beta = 10^4$.

The parameter space of $R_D$ associated with $B_c \to \tau \nu$ are presented Fig.1. In a more general model with scalar operators, $R_D$ has already put strong constraints, by shown the $1 \sim 3\sigma$ allowed regions in Fig.1(a), one can see 4 allowed areas at 1\sigma level in $C_{RL} - C_{LL}$ space ($C_{RL,LL}$ are the coefficients in Eq.(3.10). However, even with a very loose upper bound (say 30%), $B_c \to \tau \nu$ helps to exclude half of the regions. In the case of FG2HDM, the situation is quite friendly. As shown in Fig.1(b), most area in $\log(\tan \beta) - m_{H^+}$ space are allowed by $R_D$, especially for the large $\tan \beta$ region. The restriction from $B_c \to \tau \nu$ in $\log(\tan \beta) - m_{H^+}$ space is weak thus we do not show its effect in Fig.1(b). In fact, we have compared the the boundaries of 3\sigma $R_D$ and 30% upper bound of $B_c \to \tau \nu$ and find they are close to each other.

The physical regions of U(1) charges, constrained by $R_K$ are illustrated in Fig.2. As shown in Fig.1(b), large $\tan \beta$ is favored in $b \to c \ell \nu$. To show the $\tan \beta$ dependence in $b \to s \ell \bar{\ell}$, two scenarios with $\tan^2 \beta = 100$ and $\tan^2 \beta = 10000$ are chosen. Hence one may get the impression that the larger $\tan \beta$ is taken, the wider solution space can be found. We show in upper row the detailed regions allowed by four Wilson coefficients. Taking Fig.2(b) as an example, the constraints given by $\Delta C_{10}^e$ is loose. It is reasonable as we
Figure 3. The allowed free U(1) charges further discriminated by $\Delta a_{\ell}$: (a) $2\sigma$ allowed region by $\Delta a_e$ and $\Delta a_{\mu}$ with $\tan^2\beta = 10^2$; (b) $2\sigma$ allowed region by $\Delta a_e$ and $\Delta a_{\mu}$ with $\tan^2\beta = 10^4$.

have made a relatively loose conjecture on it based on old data. On the contrary, the region allowed by $\Delta C_{\mu}^\mu$ is narrowest as there are more fit results and hence more precise constraints putting on it. Then we show the survived space combined all the constraints to the 4 Wilson coefficients in Fig.2(c) and (d) in $2\sigma$ and $3\sigma$ level.

We show the behaviors of AMDM parameter space, especially making use of the new muon AMDM measurement, in Fig.3. Though the dependence of $\tan\beta$ is not linear, typically one may observe small $\tan\beta$ is somehow favored by AMDM. To extract the main features, we fix $\tan^2\beta = 100$ in Fig.3 as an illustration. From the $1\sigma$ allowed region by $\Delta a_{\mu}$ and $\Delta a_e$ shown in Fig.3(a), one may find: i) $\Delta a_{\mu}$ in general has a more narrow range than $\Delta a_e$; ii) the overlap range of $\Delta a_e$ and $\Delta a_{\mu}$ is largely reduced due to their opposite trend shown in Fig.3(a), originated from their sign difference; iii) the survived parameter space combing both AMDMs almost has no overlap with the allowed area from $R_K^{(*)}$, except the trivial solution around the origin. However, from Fig.3(b) the situation changes if $2\sigma$ error of AMDMs is allowed. In this case, $\Delta a_e$ gives almost no constraints while the constraint from $\Delta a_{\mu}$ is stronger, but still fills most of the presented area and entirely contains the $2\sigma$ allowed region from $R_K^{(*)}$.

Some points are summarize as follows:

- There is plenty of solution space for $R_D^{(*)}$.
- It is challenging to obtain $1\sigma$ allowed region from $R_K^{(*)}$, but there is rich $2\sigma$ solution, which is large $\tan\beta$ favorite.
- Though the solution exists purely from $1\sigma$ $\Delta a_{\mu}$ and $\Delta a_e$, but $2\sigma$ $R_K^{(*)}$ solution almost kills $1\sigma$ $\Delta a_{\ell}$ solution.
- The $2\sigma$ solution exists combing the latest $R_K^{(*)}$ and $\Delta a_{\ell}$ in FG2HDM.
5 Conclusion and Outlook

The Yukawa interaction and Higgs sector are naturally connected via spontaneously symmetry breaking. To get rid of the redundancy in Yukawa coupling of 2HDM-III, a specific U(1) flavor symmetry is introduced leading to FG2HDM. This symmetry brings different charges to the two Higgs doublets and hence forbids the $\lambda_5$ and $m_{12}$ terms in scalar potential. No physical pseudoscalar turns up and there are only three additional particles ($H^+, H^0$ and $Z'$) adding to SM particle spectrum.

The exotic neutral scalar and exotic gauge boson both can mediate flavor-changing current when they interact with down-type quark. In this work, we particularly investigate the role of $H^+$ and $Z'$ in the interpretation of the recent flavor anomalies. We make use of $Z'$ to generate new contribution to $b \to s\ell\bar{\ell}$, which helps to explain $R_{K^{(*)}}$ anomaly at 2$\sigma$ level. There is a tension between 2$\sigma$ $R_{K^{(*)}}$ and 1$\sigma$ $\Delta a_{\ell}$, combing the new released FNAL muon AMDM and recently improved electron AMDM. But the solution is safe if both $R_{K^{(*)}}$ and $\Delta a_{\ell}$ allow 2$\sigma$ error. There are plenty of rooms on $(m_{H^+}, \tan \beta)$ plane to provide solutions to $R_{D^{(*)}}$ anomalies. The rich parameter space, especially the one for explaining $R_{D^{(*)}}$ anomalies, will be confronted with more examinations in next step.

Nevertheless, currently we may conclude that in FG2HDM a solution indeed exists at 2$\sigma$ level for the tensions in $R_{D^{(*)}}, R_{K^{(*)}}$ and $\Delta a_{\mu}, \Delta a_{e}$.

Acknowledgments

This research is supported by NSFC under Grant No. U1932104.
A Some details of the model

A.1 Quantum numbers for flavor gauge symmetry

The nice texture of Yukawa matrices in Eq. (2.13) and Eq. (2.15) can be guaranteed by the subtile symmetry introduced by an extra U(1) transformation. Under this extended U(1) group, the behaviors of all the relevant fields are given as follows,

\[ \phi \rightarrow \phi' = e^{i\theta X_\phi} \phi, \quad (A.1) \]

where the U(1) charges to keep the above Yukawa structures are chosen to be

\[ X_{QL} = \frac{1}{2} \begin{pmatrix} Q_{uR} + Q_{dR} & Q_{uR} + Q_{dR} \\ Q_{lR} + Q_{dR} & Q_{lR} + Q_{dR} \end{pmatrix}, \]

\[ X_{uR} = \begin{pmatrix} Q_{uR} \\ Q_{uR} \\ Q_{lR} \end{pmatrix}, \quad X_{dR} = \begin{pmatrix} Q_{dR} \\ Q_{dR} \end{pmatrix}, \]

\[ X_\Phi = \frac{1}{2} \begin{pmatrix} Q_{uR} - Q_{dR} \\ Q_{lR} - Q_{dR} \end{pmatrix}, \]

\[ X_{\ell L} = \begin{pmatrix} Q_{\ell L} \\ Q_{\mu L} \\ Q_{\tau L} \end{pmatrix}, \]

\[ X_{\ell R} = \begin{pmatrix} Q_{\ell R} \\ Q_{\mu R} \\ Q_{\tau R} \end{pmatrix}, \quad X_{\nu R} = 0. \]

However, the charges are not that free as they should satisfy anomaly cancellation conditions. Imposing the anomaly cancellation, we established relations among these charges as

\[ Q_{uR} = -Q_{dR} - \frac{1}{3} Q_{\mu R}, \quad Q_{lR} = -4Q_{dR} + \frac{2}{3} Q_{\mu R}, \]

\[ Q_{\tau L} = Q_{dR} + \frac{1}{6} Q_{\mu R}, \quad Q_{\mu L} = -Q_{dR} + \frac{5}{6} Q_{\mu R}, \quad Q_{\ell L} = \frac{9}{2} Q_{dR} - Q_{\mu R}, \]

\[ Q_{\tau R} = 2Q_{dR} + \frac{1}{3} Q_{\mu R}, \quad Q_{\ell R} = 7Q_{dR} - \frac{4}{3} Q_{\mu R} \quad (A.3) \]

leaving only two degrees of freedom, denoted as \( Q_{dR}, Q_{\mu R} \). Our result here is consistent with the one in [20] by permuting \( e \) and \( \tau \).
A.2 Interactions among scalars

In Sec. 2.1, the detailed scalar interactions have been given. Here, we further provide the exact coefficients among them, giving

\begin{align}
\lambda_{h^3} &= 3 \left[ 2 \lambda_1 \cos \beta \cos^3 \alpha + 2 \lambda_2 \sin \beta \sin^3 \alpha + \lambda_{34} \sin 2 \alpha \sin(\alpha + \beta) \right] \\
\lambda_{h^2 H^0} &= \frac{1}{4} \left[ 2 \lambda_2 \sin \beta \cos \alpha \sin^2 \alpha - 12 \lambda_1 \cos \beta \sin \alpha \cos^2 \alpha + \lambda_{34} [\sin(\beta - \alpha) + 3 \sin(3 \alpha + \beta)] \right] \\
\lambda_{h H^0} &= \frac{1}{4} \left[ 12 \lambda_1 \cos \beta \cos \alpha \sin^2 \alpha + 12 \lambda_2 \sin \beta \sin \alpha \cos^2 \alpha + \lambda_{34} [\cos(\beta - \alpha) + 3 \cos(3 \alpha + \beta)] \right] \\
\lambda_{H^3} &= \frac{3}{2} \left[ 2 \lambda_2 \sin \beta \cos^3 \alpha - 2 \lambda_1 \cos \beta \sin^3 \alpha - \lambda_{34} \sin 2 \alpha \cos(\alpha + \beta) \right] \\
\lambda_{h H^+ H^-} &= \frac{1}{4} \left[ 2 \sin 2 \beta [\lambda_2 \cos \beta \sin \alpha + \lambda_1 \sin \beta \cos \alpha] + \lambda_{34} \cos(\alpha + 3 \beta) + (3 \lambda_3 - \lambda_4) \cos(\beta - \alpha) \right] \\
\lambda_{H^0 H^+ H^-} &= \frac{1}{4} \left[ 2 \sin 2 \beta [\lambda_2 \cos \beta \cos \alpha - \lambda_1 \sin \beta \sin \alpha] - \lambda_{34} \sin(\alpha + 3 \beta) + (3 \lambda_3 - \lambda_4) \sin(\beta - \alpha) \right] \\
\lambda_{h^4} &= \frac{3}{4} \sin^2 2 \alpha [\lambda_1 \cot^2 \alpha + \lambda_2 \tan^2 \alpha + 2 \lambda_{34}] \\
\lambda_{h^3 H^0} &= \frac{3}{4} \sin 2 \alpha [(\lambda_{34} - \lambda_1) \cos^2 \alpha + (\lambda_2 - \lambda_{34}) \sin^2 \alpha] \\
\lambda_{h^2 H^0} &= \frac{1}{4} \left[ (3 \lambda_1 + 3 \lambda_2 - 2 \lambda_{34}) \sin^2 2 \alpha + 4 \lambda_{34} \cos^2 2 \alpha \right] \\
\lambda_{h H^0} &= \frac{3}{4} \sin 2 \alpha [(\lambda_2 - \lambda_{34}) \cos^2 \alpha + (\lambda_{34} - \lambda_1) \sin^2 \alpha] \\
\lambda_{H^4} &= \frac{3}{4} \sin^2 2 \alpha [\lambda_1 \tan^2 \alpha + \lambda_2 \cot^2 \alpha + 2 \lambda_{34}] \\
\lambda_{h^2 H^+ H^-} &= \frac{1}{4} \sin 2 \alpha [\lambda_1 \tan \beta \cot \alpha + \lambda_2 \tan \alpha \cot \beta + \lambda_3 (\cot \alpha \cot \beta + \tan \alpha \tan \beta) - 2 \lambda_4] \\
\lambda_{h H^0 H^+ H^-} &= \frac{1}{4} \sin 2 \beta \sin 2 \alpha [(\lambda_3 - \lambda_1) \tan \beta + (\lambda_2 - \lambda_3) \cot \beta - 2 \lambda_4 \cot 2 \alpha] \\
\lambda_{H^0 H^+ H^-} &= \frac{1}{4} \sin 2 \alpha [\lambda_1 \tan \alpha \tan \beta + \lambda_2 \cot \alpha \cot \beta + \lambda_3 (\tan \beta \cot \alpha + \tan \alpha \cot \beta) + 2 \lambda_4] \\
\lambda_{H^+ H^-} &= \frac{1}{2} \sin^2 2 \beta [\lambda_1 \tan^2 \beta + \lambda_2 \cot^2 \beta + 2 \lambda_{34}],
\end{align}

which will be helpful in the Higgs phenomenology studies.

References

[1] BABAR collaboration, Evidence for an excess of $\bar{B} \to D^{(*)} \tau^+ \nu_\tau$ decays, Phys. Rev. Lett. 109 (2012) 101802 [1205.5442].

[2] BELLE collaboration, Measurement of $R(D)$ and $R(D^*)$ with a semileptonic tagging method, Phys. Rev. Lett. 124 (2020) 161803 [1910.05864].

[3] M. Bordone, G. Isidori and A. Pattori, On the Standard Model predictions for $R_K$ and $R_{K^*}$, Eur. Phys. J. C 76 (2016) 440 [1605.07633].

[4] LHCb collaboration, Search for lepton-universality violation in $B^+ \to K^+ \ell^+ \ell^-$ decays, Phys. Rev. Lett. 122 (2019) 191801 [1903.09252].
[5] Belle collaboration, Test of lepton flavor universality in \( B \to K\ell^+\ell^- \) decays, 1908.01848.

[6] LHCb collaboration, Test of lepton universality in beauty-quark decays, 2103.11769.

[7] LHCb collaboration, Test of lepton universality with \( B^0 \to K^{*0}\ell^+\ell^- \) decays, JHEP 08 (2017) 055 [1705.05802].

[8] Belle collaboration, Test of lepton flavor universality in \( B \to K^{*}\ell^+\ell^- \) decays at Belle, 1904.02440.

[9] T. Aoyama et al., The anomalous magnetic moment of the muon in the Standard Model, Phys. Rept. 887 (2020) 1 [2006.04822].

[10] Muon \( g - 2 \) Collaboration collaboration, Measurement of the positive muon anomalous magnetic moment to 0.46 ppm, Phys. Rev. Lett. 126 (2021) 141801.

[11] R. H. Parker, C. Yu, W. Zhong, B. Estey and H. Müller, Measurement of the fine-structure constant as a test of the Standard Model, Science 360 (2018) 191 [1812.04130].

[12] T. Nomura and H. Okada, Explanations for anomalies of muon anomalous magnetic dipole moment, \( b \to s\mu\bar{\mu} \) and radiative neutrino masses in a leptoquark model, 2104.03248.

[13] J. E. Kim, Light Pseudoscalars, Particle Physics and Cosmology, Phys. Rept. 150 (1987) 1.

[14] G. Branco, P. Ferreira, L. Lavoura, M. Rebelo, M. Sher and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models, Phys. Rept. 516 (2012) 1 [1106.0034].

[15] S. Davidson and H. E. Haber, Basis-independent methods for the two-Higgs-doublet model, Phys. Rev. D72 (2005) 035004 [hep-ph/0504050].

[16] H. E. Haber and D. O'Neil, Basis-independent methods for the two-Higgs-doublet model. II. The Significance of \( \tan \beta \), Phys. Rev. D74 (2006) 015018 [hep-ph/0602242].

[17] T. P. Cheng and M. Sher, Mass Matrix Ansatz and Flavor Nonconservation in Models with Multiple Higgs Doublets, Phys. Rev. D35 (1987) 3484.

[18] K.-F. Chen, W.-S. Hou, C. Kao and M. Kohda, When the Higgs meets the Top: Search for \( t \to c\ell^0 \) at the LHC, Phys. Lett. B725 (2013) 378 [1304.8037].

[19] G. C. Branco, W. Grimus and L. Lavoura, Relating the scalar flavor changing neutral couplings to the CKM matrix, Phys. Lett. B380 (1996) 119 [hep-ph/9601383].

[20] A. Celis, J. Fuentes-Martin, M. Jung and H. Serodio, Family nonuniversal \( Z' \) models with protected flavor-changing interactions, Phys. Rev. D92 (2015) 015007 [1505.03079].

[21] A. Ordell, R. Pasechnik, H. Serodio and F. Nottensteiner, Classification of anomaly-free 2HDMs with a gauged \( U(1)' \) symmetry, Phys. Rev. D100 (2019) 115038 [1909.05548].

[22] A. Ordell, R. Pasechnik and H. Serodio, Anomaly-free 2HDMs with a gauged abelian symmetry and two generations of right-handed neutrinos, Phys. Rev. D102 (2020) 035016 [2006.08676].

[23] C. Bobeth, G. Hiller and G. Piranishvili, Angular distributions of \( \bar{B} \to \bar{K}\ell^+\ell^- \) decays, JHEP 12 (2007) 040 [0709.4174].

[24] K. R. Lynch, Extended electroweak interactions and the muon \( g - 2 \), Phys. Rev. D65 (2002) 053006 [hep-ph/0108080].

[25] W.-S. Hou, Enhanced charged Higgs boson effects in \( B^- \to \gamma \) tau anti-neutrino, mu anti-neutrino and \( b \to \gamma \) tau anti-neutrino + X, Phys. Rev. D48 (1993) 2342.
[26] A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, $\Delta M_{d,s}, B^0 d, s \to \mu^+ \mu^-$ and $B \to X_s \gamma$ in supersymmetry at large $\tan \beta$, Nucl. Phys. B659 (2003) 3 [hep-ph/0210145].

[27] S. Fajfer, J. F. Kamenik and I. Nisandzic, On the $B \to D^* \tau \bar{\nu}_\tau$ Sensitivity to New Physics, Phys. Rev. D85 (2012) 094025 [1203.2654].

[28] Z.-R. Huang, Y. Li, C.-D. Lu, M. A. Paracha and C. Wang, Footprints of New Physics in $b \to c \tau \nu$ Transitions, Phys. Rev. D85 (2012) 094025 [1203.2654].

[29] A. G. Akeroyd and C.-H. Chen, Constraint on the branching ratio of $B_c \to \tau \bar{\nu}$ from LEP1 and consequences for $R(D^{(*)})$ anomaly, Phys. Rev. D96 (2017) 075011 [1708.04072].

[30] M. Alguero, Talk given in Moriond QCD 2021: Global Fits to $b \to s \ell^+ \ell^-$ data: State of the art, .

[31] A. Arbey, T. Hurth, F. Mahmoudi, D. M. Santos and S. Neshatpour, Update on the $b \to s$ anomalies, Phys. Rev. D100 (2019) 015045 [1904.08399].

[32] J. Aebischer, W. Altmannshofer, D. Guadagnoli, M. Reboud, P. Stangl and D. M. Straub, $B$-decay discrepancies after Moriond 2019, Eur. Phys. J. C80 (2020) 252 [1903.10434].

[33] M. Algueró, B. Capdevila, A. Crivellin, S. Descotes-Genon, P. Masjuan, J. Matias et al., Emerging patterns of New Physics with and without Lepton Flavour Universal contributions, Eur. Phys. J. C79 (2019) 714 [1903.09578].

[34] HFLAV collaboration, Averages of $b$-hadron, $c$-hadron, and $\tau$-lepton properties as of 2018, 1909.12524.

[35] Particle Data Group collaboration, Review of Particle Physics, PTEP 2020 (2020) 083C01.

[36] Muon g-2 collaboration, Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL, Phys. Rev. D73 (2006) 072003 [hep-ex/0602035].