W, Z AND HIGGS SCATTERING AT SSC ENERGIES

Suraj N. Gupta and James M. Johnson
Department of Physics, Wayne State University
Detroit, MI 48202, USA

and

Wayne W. Repko
Department of Physics and Astronomy, Michigan State University
East Lansing, MI 48824, USA

ABSTRACT

We examine the scattering of longitudinal $W$, $Z$ and Higgs bosons in the Standard Model using the equivalent Goldstone-boson Lagrangian. Our calculations include the full one-loop scattering matrix between the states $W^+_L W^-_L$, $Z_L Z_L$ and $HH$ with no restrictions on the relative sizes of $M_H$ and $\sqrt{s}$. In addition to deriving the perturbative eigen-amplitudes, we also obtain quite striking results by unitarizing the amplitudes with the use of the K-matrix and Padé techniques.

1. Introduction

There has been much recent interest in the scattering of longitudinal gauge bosons in the Standard Model. Theoretically, the symmetry breaking mechanism of the Electroweak theory is not well known, especially in the case of a strongly coupled symmetry breaking sector. Therefore it is natural to examine the most basic processes, such as the scattering of the longitudinal gauge bosons. Experimentally, the scattering of gauge bosons will be measured at future hadron colliders such as the SSC. For a sufficiently large Higgs mass, this process will be a main source for Higgs bosons. However, for large $M_H$ and therefore strong coupling, the Feynman amplitudes violate unitarity and must be unitarized. The various unitarization methods must include all possible open channels, and at energies above $2M_H$ the $HH$ channel must be included. Therefore the whole $3 \times 3$ matrix of amplitudes between the states $W^+_L W^-_L$, $Z_L Z_L$ and $HH$ must be calculated. Moreover, since we are interested in strong coupling, the tree amplitude will be insufficient, and we will need the amplitudes to at least one-loop.

2. Calculation

We performed this calculation with the aid of the Goldstone Boson Equivalence Theorem, which allows one to replace the longitudinal vector gauge bosons with the corresponding scalar goldstone bosons. Previously, the $3 \times 3$ scattering matrix has been calculated at the tree-level and the $2 \times 2$ submatrix of gauge boson

*Presented by James M. Johnson.
scattering to one-loop. The one-loop calculation of the $HH$ channels is considerably more difficult because of the larger number of diagrams and their mathematical complexity. The complexity of diagrams increases with the number of massive internal propagators, and the new diagrams contained more of these. For example, of a total of six box diagrams in the entire calculation, five were new boxes needed for the $HH$ channels. After adding the contributions of all of the diagrams, the amplitudes were then numerically integrated to yield the s-wave projections, shown in Figure 1.

3. Unitarization

Although the Feynman expansion is unitary as a whole, unitarity is violated order-by-order. This is especially noticeable at the Higgs pole, where the Feynman amplitude becomes infinite (see Figure 2a). One remedy is to add a finite width for the Higgs boson. However, this is an ad hoc solution which will not solve the problem of unitarity violation due to large coupling. In Figure 2b for a $M_H = 1000$ GeV, the absolute value of the Feynman amplitude is larger than one for all energies above the Higgs pole.

The solution is to consistently unitarize the amplitudes. We have considered two popular methods: the K-matrix and Padé unitarizations. If one starts with a Feynman expansion: $A_1 + A_2 + \ldots$, where $A_1$ is the matrix of tree amplitudes and $A_2$ is the matrix of one-loop corrections, then the K-matrix unitarization is given by $\Re (A_1 + A_2) [I + i \Re (A_1 + A_2)]^{-1}$ and the Padé by $A_1 [A_1 - A_2]^{-1} A_1$.

Since both techniques are given by matrix expressions, it is not surprising that the channels become mixed, and contributions of other channels can influence even the $W^+ L W^- L$ scattering amplitude. Figure 2 shows the effects of these unitarizations on the Feynman amplitudes for this process. It is apparent that both techniques nicely unitarize the Higgs resonance, without the need to put in a width by hand. In the case of $M_H = 1000$ GeV, they both also reduce the large, unitarity violating amplitude above resonance. However, there are differences between them. In the case of $M_H = 500$ GeV, an additional resonance appears at $\sqrt{s} = 2800$ GeV in the Padé unitarization, but not for the K-matrix. For $M_H = 1000$ GeV the Padé amplitude also shows the effect of the $HH$ threshold at $\sqrt{s} = 2000$ GeV, more so than for the K-matrix. How much physical significance to give to the interesting features from the Padé unitarization is unclear.

In Figure 3 we present cross-sections for $pp \rightarrow W^+_L W^-_L \rightarrow W^+_L W^-_L X$ at the SSC for the two unitarizations. For $M_H = 500$ GeV there is little difference at experimentally realizable energies, while for a larger Higgs mass there is a much larger difference between the unitarizations.

5. Acknowledgements

This work was supported in part by the U.S. Department of Energy under grant No. DE-FG02-85ER40209 and National Science Foundation grant 90-06117.
Figure 1: Absolute value of the s-wave Feynman amplitudes of $W_L^+W_L^- \to HH$ and $HH \to HH$ scattering for $M_H = 500, 750$ and $1000$ GeV.

Figure 2: Comparison of the Feynman amplitude for $W_L^+W_L^- \to W_L^+W_L^-$ with the K-Matrix and Padé unitarized amplitudes. Results are shown for $M_H = 500$ and $1000$ GeV.

Figure 3: Cross-sections for $pp \to W_L^+W_L^- \to W_L^+W_L^-X$ at the SSC for $M_H = 500$ and $1000$ GeV. The dashed line is the K-matrix unitarization and the dotted is the Padé unitarization.
$W_L^+W_L^- \rightarrow HH$
$M_H = 500$  
$M_H = 750$  
$M_H = 1000$  

$HH \rightarrow HH$  

$|a_0|$  

$\sqrt{s} \ (\text{GeV})$  

0.0  
0.2  
0.4  
0.6  
0.8  
1.0  

0  
1000  
2000  
3000  
4000
$W_L^+W_L^- \rightarrow W_L^+W_L^-$

$M_H = 500$ GeV

$|a_0|$ vs $\sqrt{s}$ (GeV)
$W_L^+W_L^- \rightarrow W_L^+W_L^-$

$M_R = 1000 \text{ GeV}$
