Abstract

By using an Ansatz for the pressure (measured in terms of the bag constant) of the hadronic gas in equilibrium, we have formulated a new bag-type model for the phase transition, i.e. the extended bag model. This allows one to take into account the nonperturbative vacuum effects from both sides of the equilibrium condition. We have explicitly shown that our model automatically provides an isentropic equilibrium deconfining phase transition from the quark-gluon plasma phase to the hadronic gas phase with a temperature and chemical potential dependent bag pressure at the phase boundary. This makes it possible to regulate the total entropy and baryon number in both phases, i.e. the flow of the specific entropy per baryon across the phase boundary. We have also determined the functional form of the temperature and chemical potential dependence of the bag pressure outside the phase boundary.

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I. The quark-gluon plasma (QGP) phase is a necessary step in the evolution of the Exit Matter from Big Bang to the present days. Apparently, the only way to study this phase of the expansion of the Universe is nuclear (heavy ion) collisions at high energies which makes it possible "to recreate conditions akin to the first moments of the Early Universe, the Big Bang, in the laboratory" [1]. Because of the confinement phenomenon, the nonperturbative vacuum structure must play a very important role in the transition from QGP to the formation of the hadronic particles (i. e., hadronization) and vice versa. As it was underlined in our papers [2], any correct model of the nonperturbative effects such as quark confinement or dynamical chiral symmetry breaking (DCSB) becomes a model of the true QCD ground state (i. e., the nonperturbative vacuum) and the other way around. Thus the difference between the perturbative (which is always normalizable to zero) and the nonperturbative vacua appears to be necessarily nonzero and finite so that it describes the above mentioned nonperturbative phenomena at zero temperature. The existence of the finite vacuum energy per unit volume - the bag constant [3, 4] - becomes important for a realistic calculation of the transition from hadronic to QGP phases and vice versa at nonzero temperature as well.

There are two main approaches to investigate QGP, namely the resummed finite-temperature perturbation theory (effective field theory method) [5-7] and the lattice one [8]. The former breaks down after the fifth order in the QCD coupling constant $g$ because of the severe infrared divergences in the Braaten-Pisarski-Kapusta (BPK) series in powers of $g^{m+2n}\ln^n g$, but it smoothly incorporates the case of nonzero chemical potential(s). The latter is a powerful nonperturbative tool to calculate equations of state for both phases. However, up to now, there are no realistic lattice data available for nonzero chemical potential(s) (for problems to introduce it on the lattice see, for example, recent paper [9]). As it was emphasized in Ref. [10], at present the phase transition at finite baryon chemical potential can be only studied within the phenomenological models. The most popular among them are, of course, the bag-type models [11], which differ from each other by modelling the equation of state of the hadronic phase [12]. The equation of state of the QGP phase is
usually approximated by the thermal perturbation theory as an ideal (noninteracting) gas consisting of gluons and massless quarks. The generalizations on massive quarks and interacting QGP are also possible though this is not a simple task because of the nonperturbative contributions to the thermodynamic potential, coming from the scales $gT$ and $g^2T$, which badly affect the above mentioned BPK series.

Within the framework of the bag-type models, a phase transition between the hadronic gas (HG) and the QGP phases is constructed via the Gibbs criteria for a phase equilibrium. So the phase transition is necessarily of the first order. This means that the thermodynamic quantities of interest are discontinuous across the critical curve. The entropy content of the QGP is much higher than in the hadronic phase, so the transition from QGP to HG phase is impossible at fixed temperature and chemical potential(s), since in physical processes like the heavy ion collisions the entropy cannot decrease. To make the phase transition reversible, it is necessary thus to change the values of temperature and chemical potential(s) in order to ensue the Gibbs conditions for the phase transition. In the above mentioned paper [10] another interesting way was proposed to construct a continuous transition which preserves the Gibbs criteria but leads to a temperature and chemical potential dependent bag constant. In particular, it is explicitly shown that the specific entropy per baryon, which is an important physical observable since it indicates the strangeness production in heavy ion collisions, becomes, in this case, continuous across the phase boundary.

We have recently proposed a new bag-type model, the so-called extended bag model [13]. It complements the standard bag model by an Ansatz for the pressure at the phase boundary (measured in terms of the bag constant) which allows one to correctly take into account the nonperturbative vacuum effects from both sides of the equilibrium condition. Our main goal in this paper is to show that our model automatically provides an isentropic equilibrium deconfining phase transition with a temperature and chemical potential dependent bag pressure at the phase boundary. Thus our model makes it feasible to regulate the total entropy and baryon number in both phases, i. e. the flow of the specific entropy per baryon across the phase boundary.
II. For the reader’s convinient and for the sake of future application, let us briefly discuss
the main features of our model in the case of the noninteracting QGP. The QGP state
equation determines the dependence of the QGP thermodynamical quantities such as the
energy density $\epsilon$ and pressure $P$ on the thermodynamical variables temperature $T$ and quark
chemical potentials $\mu_f$. There exist excellent reviews on the physics of the QGP (see, for
example, Refs. [14-16]), as well as on the phase transitions in it [17]. The QGP pressure (i.
e., the thermodynamic potential $\Omega$, apart from the sign) is [16]

$$P = \frac{1}{3} f_{SB} T^4 + \frac{N_f}{2} \mu_f^2 T^2 + \frac{N_f}{4\pi^2} \mu_f^4 - B, \quad (1)$$

where $B$ is the bag constant (see below), while $N_f$ is the number of different quark flavours.
In what follows we will consider the values $N_f = 0, 1, 2$ since the inclusion of the strange
(s) quark requires a special treatment [18]. The value $N_f = 0$ describes the case of the pure
gluon plasma. Note also that the state equation (1) is derived by neglecting quark current
masses.

The constant $f_{SB}$, entering the equation of state (1),

$$f_{SB} \equiv f_{SB}(N_f) = f_0(N_f) = \frac{\pi^2}{3} \left( \frac{8}{3} + \frac{7}{4} N_f \right) \quad (2)$$

is the Stefan-Boltzmann (SB) constant, which determines the ideal (noninteracting gluons
and massless quarks) gas limit. Obviously, for $N_f = 0$ it equals to the standard SB constant
of the ideal gluon gas. Note that the temperature dependence of the QGP equation of state
is dominant over the dependence on quark chemical potentials.

The energy density $\epsilon$ of the noninteracting QGP can be obtained from the thermodynamic
potential (1) as follows ($P = -\Omega$)

$$\epsilon = 3P + 4B, \quad (3)$$

so the bag constant determines, in general, deviation from the ideal gas relation between
pressure and energy density. Let us make a few remarks in advance. Our calculations for
the noninteracting QGP are not based on the bag model state equation (3) [11, 17]. The
constraint, determining the phase transition, will be obtained with the help of the Ansatz which is beyond the bag model and it is general (see below, part III and Ref. [13]). We will use numerical values for the bag constant which were obtained from a completely different source, namely they were calculated in the zero modes enhancement (ZME) model of the true QCD vacuum taking the instanton contributions into account as well [2, 13].

III. The Gibbs conditions for the phase equilibrium between hadronic gas (HG) and QGP phases at $T = T_c$ are formulated as follows [11]

\[ P_h = P_q = P_c; \quad T_h = T_q = T_c; \quad 3\mu_f = \mu_c, \quad (4) \]

where subscripts h, q and c refer to HG, QGP phases and transition (critical or crossover) region, respectively. At the same time, the difference between $\epsilon_q - \epsilon_h$ at $T = T_c$ can remain finite (nonzero) and determines the latent heat (LH), $\epsilon_{LH}$. Let us remind that $P_q$ and $\epsilon_q$ are determined by Eqs. (1) and (3), respectively.

Let us formulate our primary assumption (Ansatz) now. The state equation for the hadronic phase (the left hand side of the equilibrium condition (4)) is strongly model dependent [11, 12]. However, in any model the pressure $P_h$ at any values of temperature $T$ and baryonic chemical potential $\mu$, in particular at $T = T_c$ and $\mu = \mu_c$, can be measured in terms of the above mentioned bag constant, i.e. let us put

\[ P_h(T_c, \mu_c) = \frac{b_h}{a_h + N_f} B, \quad (5) \]

where the so-called parametric functions $b_h \equiv b_h(T_c, \mu_c)$ and $a_h \equiv a_h(T_c, \mu_c)$ describe the details of the HG phase at the phase boundary. Evidently, they may only depend on the set of independent dimensionless variables which characterize the HG phase. For example,

\[ a_h \equiv a_h(T_c, \mu_c) = a_h(x_c, t_c, y_c, z_c), \]

\[ b_h \equiv b_h(T_c, \mu_c) = b_h(x_c, t_c, y_c, z_c), \quad (6) \]

where

\[ x_c = \frac{\mu_c}{T_c}, \quad t_c = \frac{\tilde{B}^{1/4}}{T_c}, \quad y_c = \frac{\mu_c}{m}, \quad z_c = \mu_c R_0, \quad (7) \]
and \( m \) denotes the hadron mass while \( R_0 \) denotes the radius of the nucleon, so it allows one to take into account finite size effects due to hard core repulsion between nucleons (extended volume corrections) [19]. These variables are independent and all other possible dimensionless variables are obtained by combination of these, for example, \( R_0 T_c = z_c/x_c, \quad m/T_c = x_c/y_c, \quad \mu_c/\tilde{B}^{1/4} = x_c/t_c, \text{etc.} \) Also the set of independent dimensionless variables at the phase boundary (7) may be extended in order to treat the HG phase in a more sophisticated way. However, in any case, the parametric functions should be symmetric, i.e. \( a_h(T_c, \mu_c) = a_h(-T_c, -\mu_c) \) and \( b_h(T_c, \mu_c) = b_h(-T_c, -\mu_c) \).

From Eqs. (1) and (4-5) at \( T = T_c \) and \( \mu_f = \mu_c/3 \), one obtains

\[
 f_{SB}(N_f)T_c^4 + \frac{N_f}{6} \mu_c^2 T_c^2 + \frac{N_f}{108\pi^2} \mu_c^4 = \frac{3}{a_h + N_f} \tilde{B},
\]

where we introduced a new "physical" (effective) bag constant as follows

\[
 \tilde{B} = (b_h + a_h + N_f) B,
\]

and it linearly depends on \( N_f \), as it should be, at log-loop level (see our papers [2] and Ref. [13]). In connection with our Ansatz (5) a few remarks are in order. The alternative parametrization with respect to \( N_f \), namely \( P_h(T_c, \mu_c) = (b_h'/a_h'N_f + 1)B \) leads to the effective bag constant as \( \tilde{B} = (b_h' + a_h'N_f + 1)B \). The parameteric functions \( b_h \) and \( a_h \), as well as \( b_h' \) and \( a_h' \), as was mentioned above, may, in principle, arbitrarily depend on dimensionless variables (7). If, for example, \( a_h' \) vanishes at \( \mu_c = 0 \) then the linear dependence of \( \tilde{B} \) on \( N_f \) will be spoiled. In other words, the chosen parametrization guarantees the linear dependence of \( \tilde{B} \) on \( N_f \) and the alternative one does not.

From now on, \( T_c \) and \( \mu_c \) will be calculated in terms of \( \tilde{B} \) and not that of old \( B \), i.e. a definite numerical value will be assigned to \( \tilde{B} \). So we consider \( \tilde{B} \) as the physical bag constant, while \( B \) as an unphysical "bare" one. The bag constant is universal and it represents the complex nonperturbative structure of the QCD true vacuum. Thus the proposed Ansatz, allows one to treat nonperturbative vacuum effects (which are parametrized in terms of \( \tilde{B} \)) from both sides of the equilibrium condition (4). However, it still remains dependent on
the arbitrary parameter $a_h$. In order to eliminate this dependence, let us normalize the thermodynamic potential at the phase boundary in Eq. (8) to the standard SB constant (2). This yields

$$\tilde{f}_{SB}(N_f) = (N_f + a_h)f_{SB}(N_f) = f_{SB}(0) \quad \text{at} \quad N_f = 0,$$

and one immediately arrives at $a_h \equiv a_h(T_c, \mu_c) = 1$. This is our normalization condition and it leads to good numerical results for $T_c$ and $\mu_c$ (see Ref. [13]). So the general constraint (8) finally becomes uniquely determined, namely (Fig. 1)

$$f_{SB}T_c^4 + \frac{N_f}{6}\mu_c^2T_c^2 + \frac{N_f}{108\pi^2}\mu_c^4 = \frac{3}{N_f + 1}\tilde{B},$$

and consequently allows one to investigate the bulk thermodynamic quantities in the vicinity of a critical point. Let us emphasize the important observation that the numerical values of $T_c$ and $\mu_c$ calculated from the constraint (11) do not depend on the way how one approximates the equation of state of the hadronic phase. In contrast to the standard bag-type models (differed from each other by modelling the hadronic phase [12]), in the extended bag model their values depend only on $\tilde{B}$ which incorporates nonperturbative vacuum effects from both sides of the Gibbs equilibrium condition (4) as it has been already emphasized above.

The QGP energy density and pressure, however, remain undetermined with our Ansatz (5) at this stage. In terms of $\tilde{B}$ they become

$$P_h(T_c) = P_q(T_c) = \frac{b_h}{(N_f + 1)[(N_f + 1) + b_h]}\tilde{B},$$

and because of Eq. (3),

$$\epsilon_q(T_c) \equiv \epsilon_c = \frac{3b_h + 4(N_f + 1)}{(N_f + 1)[(N_f + 1) + b_h]}\tilde{B}.$$  

As mentioned above, the unknown $b_h$ reflects the fact that the hadronic phase state equation is strongly model dependent. The unknown $b_h$ is the price one pays to determine the above mentioned physical quantities with the help of our Ansatz. Precisely for this reason, the bag
model state equation (3) plays no role in our numerical investigation of the phase transition with the constraint (11) from which $T_c$, as well as $\mu_c$, can be derived. Concluding this part, let us note that our model certainly requires the coexistence regime between QGP and HG phases since $\epsilon_q(T_c)$, as given by Eq. (13), explicitly depends on $b_h$ which describes details of the HG phase at $T = T_c$.

IV. Let us discuss now some general features of the extended bag model. The bare bag constant $B$ in Eq. (9) as well as the hadronic gas pressure $P_h$ in Eq. (12) were determined at the phase boundary only, namely

$$B(T_c, \mu_c) = \frac{\tilde{B}}{b_h(T_c, \mu_c) + a_h(T_c, \mu_c) + N_f} \quad (14)$$

and

$$P_h(T_c, \mu_c) = \frac{b_h(T_c, \mu_c)}{(a_h(T_c, \mu_c) + N_f)[b_h(T_c, \mu_c) + a_h(T_c, \mu_c) + N_f]} \tilde{B}, \quad (15)$$

respectively. In both equations the normalization condition $a_h(T_c, \mu_c) = 1$ is to be used. Note that in these expressions (in comparison with Eqs. (9) and (12)) we have only restored the dependence on $T_c$ and $\mu_c$, i.e. using relations (6). Let us emphasize that $b_h$ in general is not constant. Only for a simple hadronic phase (consisting, for example, of only massless pion gas) it is constant [13]. The dependence of the QGP pressure $P_q$ (1) on $T$ and $\mu$ remains, obviously, in this case unchanged, since the derivatives of bare $B$ (14) with respect to $T$ and $\mu$ disappear. The phase transition with the bag pressure determined by Eq. (14) was investigated in our previous work [13].

One may go outside the phase boundary in our model as well by considering a $T$ and $\mu$ dependent parametric functions, $b_h(T, \mu)$ and $a_h(T, \mu)$, which, at the phase boundary, become $b_h$ and 1. So extrapolation of the bag pressure (14) outside the phase boundary is given by

$$B(T, \mu) = \frac{\tilde{B}}{b_h(T, \mu) + a_h(T, \mu) + N_f}. \quad (16)$$

Hence the HG pressure (15) also becomes dependent on $T$ and $\mu$ determining it outside the phase boundary as well, i.e.
\[ P_h(T, \mu) = \frac{b_h(T, \mu)}{(a_h(T, \mu) + N_f)} \frac{\tilde{B}}{[b_h(T, \mu) + a_h(T, \mu) + N_f]}, \]  

(17)

which, evidently, at the phase boundary becomes Eq. (12) or Eq. (15). The QGP pressure \( P_q \) (1) becomes

\[ P_q = \frac{1}{3} f_{SB}(N_f)T^4 + \frac{N_f}{2} \mu^2 T^2 + \frac{N_f}{4\pi^2} \mu^4 - \frac{\tilde{B}}{b_h(T, \mu) + a_h(T, \mu) + N_f}. \]  

(18)

A simple relation (3) between the energy density and pressure now does not take place. The QGP energy density should be determined from a general thermodynamic relation, namely

\[ \epsilon_q = T \left( \frac{\partial P_q}{\partial T} \right)_\mu + \mu \left( \frac{\partial P_q}{\partial \mu} \right)_T - P_q, \]  

(19)

where \( P_q \) is given by the previous equation. Then one obtains

\[ \epsilon_q = 3P_q + 4 \frac{\tilde{B}}{b_h(T, \mu) + a_h(T, \mu) + N_f} \]

\[ + \frac{\tilde{B}}{(b_h(T, \mu) + a_h(T, \mu) + N_f)^2} \left( T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} \right) (b_h(T, \mu) + a_h(T, \mu)), \]  

(20)

where we replaced \( \mu_f \) by \( \mu/3 \). From this equation one recovers the standard relation (3) when the bag constant \( B \) does not depend on \( T \) and \( \mu \). In the same way should be determined the hadronic energy density

\[ \epsilon_h = T \left( \frac{\partial P_h}{\partial T} \right)_\mu + \mu \left( \frac{\partial P_h}{\partial \mu} \right)_T - P_h, \]  

(21)

where \( P_h \) is now given by Eq. (17). One finally obtains

\[ \epsilon_h = \frac{\tilde{B}}{(b_h(T, \mu) + a_h(T, \mu) + N_f)^2} \left( T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} \right) b_h(T, \mu) \]

\[ - \frac{\tilde{B}b_h(T, \mu)[b_h(T, \mu) + 2a_h(T, \mu) + 2N_f]}{(a_h(T, \mu) + N_f)^2(b_h(T, \mu) + a_h(T, \mu) + N_f)^2} \left( T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} \right) a_h(T, \mu) - P_h. \]  

(22)

Note, Eq. (22) need not be multiplied by the overall factor which takes into account extended volume corrections [13, 19] since our parametric functions automatically incorporate them (see Ref. [13] and below). Combining this equation with Eq. (20) at the phase boundary, for the latent heat \( \epsilon_{LH} = \epsilon_c - \epsilon_h \) one obtains
\[
\epsilon_{LH} = \frac{4}{N_f + 1} \tilde{B} + \frac{\tilde{B}}{(N_f + 1)^2} \left( \frac{T}{\partial T} + \mu \frac{\partial}{\partial \mu} \right) a_h(T, \mu),
\]

(23)

where, after taking the derivatives, one needs to put \( T = T_c \) and \( \mu = \mu_c \). In derivation of Eq. (23), we have used \( P_h = P_q \) and \( \epsilon_q \equiv \epsilon_c \) at the phase boundary \( T = T_c \) and \( \mu = \mu_c \) and also the explicit expression (17) which becomes Eq. (15). Thus to leading order the latent heat, as given by Eq. (23), does not depend on the details of the hadronic phase which are described by the parametric functions \( b_h(T, \mu) \) and \( a_h(T, \mu) \). Note that the second (next-to-leading) term is suppressed approximately by one order of magnitude in comparison with the first (leading) term (especially for the physically relevant case of the two light quark species \( N_f = 2 \)). The parametric function \( a_h(T, \mu) \) is a slowly varying function (it is nearly constant in the vicinity of the phase transition due to its above mentioned normalization condition).

Using now our values of the effective bag constant \( \tilde{B} \) [13], one obtains the numerical values of the latent heat, up to leading order, in our model as follows

\[
\begin{align*}
\epsilon_{LH}(N_f = 0) &= 3.12 \text{ GeV}/\text{fm}^3, \\
\epsilon_{LH}(N_f = 1) &= 1.82 \text{ GeV}/\text{fm}^3, \\
\epsilon_{LH}(N_f = 2) &= 1.38 \text{ GeV}/\text{fm}^3.
\end{align*}
\]

(24)

At the same time, the QGP energy density \( \epsilon_q \) (20) depends explicitly on the parametric functions \( b_h(T, \mu) \) and \( a_h(T, \mu) \). Since they describe the details of the hadronic phase, so this requires the coexistence regime between QGP and HG phases in our model as it was already mentioned above.

Concluding this part of our work, let us remind the reader that the constraint condition (11), determining the critical curve, does not depend on how one goes outside the phase boundary. Thus the critical temperature \( T_c \) and the critical chemical potential \( \mu_c \) do not depend on the details of the hadronic phase which are described by the parametric functions \( b_h(T, \mu) \) and \( a_h(T, \mu) \). In our model their numerical values [13] are mainly determined by the vacuum effects in both phases.
V. It was proposed in Ref. [10] to construct the isentropic equilibrium phase transition from QGP to HG by considering a $T$ and $\mu$ dependent bag pressure. Due to the above general discussion, this can be easily incorporated in our model. The isentropic equilibrium phase transition from QGP to a hadronic gas at fixed $T$ and $\mu$ via the ratio specific entropy per baryon is determined as

$$\left(\frac{s}{n_B}\right)_q = \left(\frac{s}{n_B}\right)_h,$$

where $s$ and $n_B$ are the entropy and baryon number densities, respectively. For the QGP phase they are defined as

$$s_q = \left(\frac{\partial P_q}{\partial T}\right)_{\mu_f},$$

and

$$n_B = \frac{1}{3}\left(\frac{\partial P_q}{\partial \mu_f}\right)_T.$$  \hfill (26) \hfill (27)

In the HG phase these quantities are defined as follows

$$s_h = \left(\frac{\partial P_h}{\partial T}\right)_{\mu}.$$  \hfill (28)

and

$$n_B = \left(\frac{\partial P_h}{\partial \mu}\right)_T.$$  \hfill (29)

In the extended bag model the parametric functions $b_h(T, \mu)$ and $a_h(T, \mu)$ can only be the functions of the dimensionless variables, i.e.

$$a_h(T, \mu) = a_h(x, t, \tau, y, z, q),$$

$$b_h(T, \mu) = b_h(x, t, \tau, y, z, q),$$

where

$$x = \frac{\mu}{T}, \quad t = \frac{\tilde{B}^{1/4}}{T}, \quad \tau = \frac{T}{T_c}, \quad y = \frac{\mu}{m}, \quad z = \mu R_0, \quad q = \frac{\mu}{\mu_c}. \hfill (30) \hfill (31)$$
Evidently, this extrapolates the above introduced set of independent dimensionless variables (6-7) outside the phase boundary. These six variables are independent and all other possible dimensionless variables are obtained by a combination of these, for example, \( R_0T = z/x, \quad \frac{m}{T} = x/y, \quad \frac{\mu}{B^{1/4}} = x/t, \) etc. Note that the solutions should be symmetric, i.e. \( b_h(T, \mu) = b_h(-T, -\mu) \) and \( a_h(T, \mu) = a_h(-T, -\mu) \) because of the corresponding symmetry in the pressure. Also, the set of independent dimensionless variables (31) may be extended, treating the hadronic phase in a more sophisticated way, etc. However, for our main purpose in this work (isentropic equilibrium phase transition) this set of variables is completely sufficient.

The HG and QGP pressures are given by Eqs. (17) and (18), respectively. It is convenient to introduce the functions defined as

\[
\begin{align*}
    f &\equiv f(T, \mu) = b_h(T, \mu) + a_h(T, \mu) + N_f = f(x, t, \tau, y, z, q). \\
    g &\equiv g(T, \mu) = \frac{b_h(T, \mu)}{a_h(T, \mu) + N_f} = g(x, t, \tau, y, z, q).
\end{align*}
\]  

(32) and

(33)

Then from definitions (26) and (27), one obtains

\[
\begin{align*}
    s_q &= \frac{4}{3} f_{SB}(N_f) T^3 + \frac{N_f}{9} \mu^2 T - \frac{\partial}{\partial T} \frac{\tilde{B}}{f(T, \mu)}, \\
    n_B &= \frac{N_f}{9} \mu T^2 + \frac{N_f}{81 \pi^2} \mu^3 - \frac{\partial}{\partial \mu} \frac{\tilde{B}}{f(T, \mu)},
\end{align*}
\]

(34) and

(35)

respectively. Here we again replaced \( \mu_f \) by \( \mu/3 \).

The entropy and baryon number conservation condition (25), on account of relations (34-35) and Eq. (17) on account of definitions (32-33), finally becomes

\[
\frac{T^3 s_0 f^2 + \tilde{B} \frac{\partial f}{\partial T}}{T^3 n_0 f^2 + \tilde{B} \frac{\partial f}{\partial \mu}} = \frac{f \frac{\partial f}{\partial T} - g \frac{\partial f}{\partial T}}{f \frac{\partial f}{\partial \mu} - g \frac{\partial f}{\partial \mu}},
\]

(36)

where
\[ s_0 = \frac{4}{3} f_{SB}(N_f) + \frac{N_f}{9}(\mu/T)^2, \]
\[ n_0 = \frac{N_f}{9}(\mu/T) + \frac{N_f}{81\pi^2}(\mu/T)^3. \]  

(37)

This relation (36) itself is completely sufficient to guarantee the isentropic equilibrium transition (since it is only one relation for two unknown functions) but it is not sufficient to determine both functions \( f \) and \( g \) and hence the functional dependence of \( b_h \) and \( a_h \) on \( T \) and \( \mu \). One needs a second independent relation between them. We suggest the QGP fireball condition \((s/n_B)_q \approx 50 \) [20] as such. In terms of functions \( f \) and \( g \) this is

\[ \frac{T^3 s_0 f^2 + \tilde{B} \frac{\partial f}{\partial T} T^3 n_0 f^2 + \tilde{B} \frac{\partial f}{\partial \mu}}{T^3 n_0 f^2 + \tilde{B} \frac{\partial f}{\partial \mu}} = 50. \]  

(38)

Thus one obtains a system of two strongly coupled nonlinear differential equations in partial derivatives for \( f \) and \( g \) parametric functions which, of course, requires a separate consideration.

VI. It is instructive, however, to investigate a simplified case which allows us to compare our results with those obtained in Refs. [10] and [21]. As mentioned above, in Ref. [10] the isentropic equilibrium condition was constructed by extrapolating outside the phase boundary the bag pressure (16) only. In terms of our parametric functions \( f \) and \( g \) (32-33), this means that the dependence on the parametric function \( g \) in the isentropic equilibrium condition (36) is neglected. Then finally the condition (36) becomes

\[ \frac{s_0}{n_0} = \frac{\partial f}{\partial \mu}. \]  

(39)

In terms of the dimensionless variables (31), from Eq. (39) one obtains

\[ x n_0 \frac{\partial f}{\partial \tau} - x n_0 \frac{\partial f}{\partial t} - x(s_0 + x n_0 \frac{\partial f}{\partial x}) = s_0 \left( y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} + q \frac{\partial f}{\partial q} \right), \]  

(40)

where \( s_0 \) and \( n_0 \) are functions of variable \( x \) only and the explicit expressions can be easily obtained from relations (37) on account of the above mentioned substitution (31). It is easy to check that the general solution of this equation is

\[ f(x, t, \tau, y, z, q) = Ax^{-C} t^C y^C z^C q^C \left( 1 + a_1 x^2 + a_2 x^4 \right)^{\frac{C-1}{4}}, \]  

(41)
where
\[
a_1 = \frac{N_f}{6f_{SB}(N_f)}, \quad a_2 = \frac{N_f}{108\pi^2 f_{SB}(N_f)},
\]
(42)

Below, for simplicity, all dimensional as well as dimensionless constants, appearing in the solution (41), will be always included in the integration "constant" \( A \) and the same notation will be retained. Constants \( C's \) are the constants of separation. In terms of \( T \) and \( \mu \) it becomes
\[
f(T, \mu) = b_h(T, \mu) + a_h(T, \mu) + N_f = A \mu_c^{C_3+C_4-C_1} T_{c}^{C_1-C_2} T^{C-C_1} \left( 1 + a_1 x^2 + a_2 x^4 \right)^{\frac{C-C_1}{C_3}}.
\]
(43)

At the phase boundary \( x = x_c \), on account of the constraint (11), it finally becomes
\[
f(T_c, \mu_c) = b_h(T_c, \mu_c) + N_f + 1 = A \mu_c^{C_3+C_4-C_1} T_{c}^{C_1-C_2},
\]
where we used the normalization condition \( a_h(T_c, \mu_c) = 1 \). This expression should be finite at the end points \((T_c, 0)\) and \((0, \mu_c)\) of the phase diagram \((T_c, \mu_c)\) shown in Fig. 1, in order to make the hadronic pressure at the phase boundary also finite as it should be. Thus the solution (43) becomes
\[
f(T, \mu) = b_h(T, \mu) + a_h(T, \mu) + 1 = A T^{C-C_1} \left( 1 + a_1 x^2 + a_2 x^4 \right)^{\frac{C-C_1}{C_3}}.
\]
(45)

So the bag pressure (16) at the phase boundary becomes simply \( B = \tilde{B}/f(T_c, \mu_c) \), i.e. it remains finite but arbitrary (since the "constant" \( A \) may depend on \( T_c \) and \( \mu_c \)). Nevertheless the condition (25) remains continuous across the phase boundary since the ratio in the right hand side of the isentropic relation (39) remains, evidently, finite in the limit \( x \to x_c = \mu_c/T_c \). This is the result of that feature of our model that nonperturbative vacuum effects from both sides of the equilibrium condition are taken care of. Apparently, this makes it possible to regulate the total entropy and baryon number in both phases, i.e. the flow of the specific entropy per baryon across the phase boundary.

VII. It is easy to see that the asymptotics of the solution (45) outside the phase boundary are determined by the separation constant \( C-C_1 \) and they do not depend on the variables
Indeed, at $x \to 0$, which means hot HG, at fixed $T$ and low baryon density ($\mu \to 0$), from (45) one obtains that the value of $\tilde{B}$ in this limit is determined by fixed temperature and $C - C_1$, i. e. becomes constant, $A_0$. In the same way at $x \to \infty$, which means cold ($T \to 0$), baryon dense matter (at fixed $\mu$), from (45) one obtains that its value in this limit now is determined by fixed $\mu$ and again $C - C_1$, i. e. tends to constant, $A_\infty$. Then the qualitative behaviour of the bag constant $B$ (16) as a function of $x$ (or $T$ and $\mu$) is shown in Fig. 2.

Thus, in our model the bag pressure always remains finite and positive (for the simplified case this is certainly so and apparently this will be valid for the general case as well) in accordance with its definition as the difference between the perturbative (normalizable to zero) and the nonperturbative vacua. This definition remains valid also for a $T$ and $\mu$ dependent bag constant. In the relevant ranges of $T$ and $\mu$, the bag pressure, as obtained in Ref. [10], is slowly varying function, i. e. it is nearly constant. This can be seen from Figs. 1 and 2 presented in Ref. [21]. This behaviour of the bag pressure is in agreement with our model as shown in Fig. 2. In the above mentioned paper [21], this behaviour [10] was criticized since it increases with increasing chemical potential at a fixed $T$ and thus defies a physical interpretation. However, the finite zeros of the bag constant (especially as a function of $T$) obtained and shown in Figs. 1 and 2 of the above mentioned paper [21] are completely unsatisfactory since they strictly contradict the definition of the bag constant. As a function of $T$ and $\mu$, it can only tend to zero (in the worst case) but never assume zero, i. e. it must always remain positive. These unphysical finite zero points are artifacts of the inconsistent approximation used in that paper. Firstly, it is incorrect to directly compare results obtained at different scales of the QCD coupling constant, $\alpha_s = 0$ and the running $\alpha_s$. The former should be treated at an infinitely large scale while the latter one - at finite, fixed scale, so there is no smooth limit to $\alpha_s = 0$. That is why the noninteracting QGP should be investigated separately from the interacting one. Secondly, and more importantly, it is well known that the first nonperturbative contribution of order $\alpha_s^{3/2}$ to the thermodynamical potential is uncomfortably large even for small $\alpha_s$ [22]. So the result obtained on account
of only the first perturbative correction of order $\alpha_s$ will be completely distorted when the above mentioned nonperturbative contribution will be incorporated in the calculation of the thermodynamic potential. Thus to treat the noninteracting QGP from the very beginning is a selfconsistent approximation [10, 13] while it is not like that to treat the interacting QGP up to $\alpha_s$ order only [21]. In order to achieve that the approximation should be selfconsistent for a running $\alpha_s$ as well, it is necessary to consider at least the nonperturbative contributions from the scale $gT$. Finally, in Ref. [21], in comparison with Ref. [10] and present investigation, there is no phase boundary at all across which entropy per baryon may be continuous on account of a $T$- and $\mu$- dependent bag constant.

The extension of our model to the case of a running coupling constant (which has been already very tentatively treated in Ref. [23]) and the determination of the nontrivial fluctuations in the interacting QGP will be the subject of the subsequent paper.

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FIGURES

FIG. 1. The phase diagram in the plane \((T_c, \mu_c)\) measured in units of MeV. Here the curve is shown for the physically relevant case of the two light quarks \(N_f = 2\) and the value \(\tilde{B}^{1/4} = 300\) MeV was used (see Ref. [13]).

FIG. 2. The qualitative behaviour of the bag pressure (16) as a function of \(x = \mu/T\). The upper line corresponds to \(A_0 > A_\infty\), while the lower one to \(A_0 < A_\infty\).
