Preliminary assessment of BeiDou Navigation Satellite System satellite orbit determination accuracy and positioning accuracy

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Abstract. Orbit determination accuracy and positioning accuracy are two key indexes to assess the performance of navigation system. With frontal positioning methods put forward, the effect of orbit determination accuracy on positioning accuracy is decreasing. In this paper, three main positioning methods provided by BDS is introduced, and the influence of BDS orbit determination accuracy on the results of single point positioning, differential positioning and precise point positioning is respectively deduced. After that, data from two receivers set at Beijing and Chengdu is analysed to evaluate orbit determination accuracy and positioning accuracy. Apart from several extreme values, BDS satellites radial orbit errors are within 1.5m, tangential orbit errors are within 2m and normal orbit errors are less than 5m. Accuracy of single point positioning differential positioning and precise point positioning is 1.9m 1.2m and 0.3m, respectively. Effect of orbit determination accuracy on single point positioning is significant. On the other hand, orbit determination accuracy has less effect on differential positioning accuracy and precise point positioning.

1. Introduction
The BeiDou Navigation Satellite System (BDS) has been used effectively in various geodetic and engineering works as an alternative tool for the traditional surveying techniques [1]. In order to get high accurate coordinates, it was necessary to maintain high accuracy in whole sections of positioning, especially in orbit determination. To obtain precise orbit, relative observation techniques and orbit determination methods were executed but brought little effect. Therefore, it was important to develop more advanced methods or strategies to fix orbit errors, and this was an easy task in the last periods especially with the rapid improvements in GNSS modern strategies and applications. As a result, three main positioning methods known as single point positioning, differential positioning and precise point positioning are provided to meet different demands.
In this research, we give a brief introduction to the methods of single point positioning, differential positioning and precise point positioning. While analysing effect of orbit determination accuracy on different positioning methods, strategies to fix orbit error within different method are discussed. To measure the actual BDS satellite orbit determination accuracy positioning accuracy and their mutual influence, we set two receivers at Beijing and Chengdu separately. After collecting data from May 26, 2020 to November 26, 2020 with receivers, we evaluated satellite ephemeris of 15 BDS satellites and analysed the orbit determination accuracy of each satellite. Secondly, we measured receiver coordinates using three different positioning methods and analysed the positioning accuracy from the data. Finally, we verified the effects of orbit determination accuracy on positioning accuracy and drew a conclusion. We hope that these results will provide helpful information for BDS users and give advices to application of different positioning methods.

2. Effects of orbit determination accuracy on positioning accuracy with different positioning methods

2.1. The effects on single point positioning
Satellite based positioning takes the satellite position as the known value to determine the position of the target. To achieve real-time-positioning, single point positioning simply applies satellite forecast orbit instead of using error correction strategies. Therefore, orbit error has a significant effect on the accuracy of single point positioning. According to the observation equation:

\[\rho_k = \sqrt{(X' - X_k)^2 + (Y' - Y_k)^2 + (Z' - Z_k)^2} + \delta t_{satclk} + \delta t_{revelk} + \delta \rho_{trop} + \delta \rho_{iono} + \delta \rho_{satant} + \delta \rho_{rcunt} + \delta \rho_{rel} + \delta \rho_{tide} + \delta \rho_{mul}\]  

(1)

where:
- \(\rho_k\) is the pseudo-range between \(j\)th satellite and \(k\)th receiver,
- \((X', Y', Z')\) is the position of the satellite,
- \((X_k, Y_k, Z_k)\) is the position of the receiver,
- \(\delta t_{satclk}\) and \(\delta t_{revelk}\) are satellite and receiver clock errors, respectively,
- \(\delta \rho_{trop}\) is the delay generated by the signal passing through the atmosphere,
- \(\delta \rho_{iono}\) is the delay generated by the signal passing through the ionosphere,
- \(\delta \rho_{satant}\) is the satellite antenna phase centre correction,
- \(\delta \rho_{pack}\) is the receiver antenna phase centre correction,
- \(\delta \rho_{rel}\) is the relativistic effect correction,
- \(\delta \rho_{tide}\) is tidal correction.

In the above formula, system errors are calculated by models and the ionospheric delay is eliminated by dual-frequency measurements combination. When satellite coordinate \((X', Y', Z')\) has an error of \((\delta X', \delta Y', \delta Z')\), it would cause a pseudo-range error:

\[\delta \rho'_k = \rho'_k - \rho_k = l'_k \delta X' + m'_k \delta Y' + n'_k \delta Z'\]

(2)

where \(l'_k, m'_k, n'_k\) are coefficients of \((X', Y', Z')\) project onto \(\rho'_k\). Therefore, \((l'_k)^2 + (m'_k)^2 + (n'_k)^2 = 1\).

Assume the mean square error of satellite coordinates is \(\sigma_{X_i}, \sigma_{Y_i}, \sigma_{Z_i}\), the mean square error of pseudo-range is:

\[\sigma_{\rho'_k} = \sqrt{(l'_k \sigma_{X_i})^2 + (m'_k \sigma_{Y_i})^2 + (n'_k \sigma_{Z_i})^2}\]

(3)

Based on the approximate hypothesis \(\sigma_{X_i} \approx \sigma_{Y_i} \approx \sigma_{Z_i} \approx \sigma_j\), we get a result that \(\sigma_{\rho'_k} \approx \sigma_j\).

To sum up, the pseudo-range error caused by orbit error is approximately equal to the average value of satellite orbit determination errors in radial, normal and tangential directions.

2.2. The effects on differential positioning
Within differential positioning method, common view time synchronization method is applied to compute station clocks first. The satellite clock errors are estimated basing on independent two-way
time synchronization observations. Then three-dimensional orbital errors could be solved. For the high stability of master station (Beijing) clock, it is taken as the common reference for the other reference stations. Common view time transfer is used to remove the station clock from pseudo-range residual. when combined with the master clock estimation, and we have all of the measurements in terms of synchronization. Finally, subtract the station clock error and orbit error from the station pseudo-range residual to achieve higher positioning accuracy.

The influence of orbit error $\Delta R$ on a baseline of length $l$, $\Delta x$ can be estimated by the following empirical formula:

$$\Delta x \approx \frac{l}{R} \cdot \Delta R$$  \hspace{1cm} (4)

where $R$ is the distance between satellite and the monitoring station. For GPS and BDS MEO satellites, $R \approx 20000$km, for BDS GEO and IGSO satellites, $R \approx 36000$km. Take BDS GEO and IGSO satellites as an example, Table 1 lists the positioning error caused by orbit error with different baseline. At present, the accuracy of BDS broadcast ephemeris is about 3.0 ~ 5.0m, and the accuracy of BDS precise orbit products obtained by overseas tracking stations is about 0.1m, and its impact on the baseline below 1000 km can be ignored (With 30 mainland stations, a baseline of 1000km could cover all over China and its neighbouring countries).

| Orbit error | Length of baseline | Differential positioning error |
|-------------|--------------------|------------------------------|
| 3.6m        | 1Km                | Ignorable                    |
| 3.6m        | 10Km               | 1mm                          |
| 3.6m        | 100Km              | 10mm                         |
| 3.6m        | 1000Km             | 100mm                        |
| 0.10m       | 1Km                | Ignorable                    |
| 0.10m       | 10Km               | Ignorable                    |
| 0.10m       | 100Km              | Ignorable                    |
| 0.10m       | 1000Km             | 2.8mm                        |

2.3. The effects on precise point positioning
Beidou wide area augmentation system puts forward a method named comprehensive zone correction to provide precise point positioning service. This method considers that errors from the user-side and error from the propagation path are related within a certain distance. Comprehensive zone is calculated using tracking data of 30 mainland stations, broadcast ephemeris, equivalent satellite clock and orbit corrections in the region [4]. The calculation flow and correcting method of comprehensive zone correction is proposed as follows.

2.3.1. Calculate the residual of dual-frequency ionosphere–free combined carrier-phase observation values from various station within the same region:

$$dL(i, t) = LC - \rho - \delta_{rec} + \delta_{sat} - \delta_{trop} - \delta_{rel} - \delta_{amb} - \delta_{ESC} - \delta_{orb} + \epsilon$$  \hspace{1cm} (5)

where:
- $L$ is the ionosphere-free combined carrier-phase value observed by station $i$ within the region,
- $\rho$ is the pseudo-range,
- $\delta_{rec}$ is approximate receiver clock error of monitoring station got by measuring pseudo-range,
- $\delta_{sat}$ is satellite clock error got from satellite ephemeris,
- $\delta_{amb}$ is approximate value of satellite ambiguity got by measuring pseudo-range and carrier-phase,
- $\delta_{ESC}$ is equivalent satellite clock correction,
- $\delta_{orb}$ is satellite orbit correction,
contains phase winding, receiver phase center correction, earth tide, sea tide and other errors and observation noise.

According to Eq. (5), apart from some common errors such as observation noise, satellite orbit error, satellite clock error and tropospheric model error, the carrier-phase observation value contains the residual term of receiver clock error and ambiguity.

2.3.2. Calculate the variation of carrier-phase observation values between epochs of each station in the same region:

\[
\Delta L(i, t, t - 1) = \begin{cases} 
0, t = 1 \\
\text{df}(i, t) - \text{df}(i, t - 1), t > 1
\end{cases}
\]  

(6)

2.3.3. Calculate comprehensive carrier-phase zone correction:

\[
dL(t) = f(\Delta L(t, t - 1)) + dL(t - 1)
\]  

(7)

where \( f \) is the synthesis function of comprehensive carrier-phase zone correction. Generally, \( f \) is the weighted average of each station, but when the satellites disappear or appear from different stations, it may lead to cycle slips. And some stations have clock slips, these cycle slips and clock slips need to be fixed or eliminated.

2.3.4. Correct errors using comprehensive zone correction:

After the comprehensive zone correction is obtained, users can use it with broadcast ephemeris, equivalent clock error and orbit correction to implement real-time precise point positioning.

\[
\begin{align*}
PC &= \rho + \delta_{rec} - \delta_{sat} + \delta_{trop} + \delta_{rela} + \delta_{ESC} + \delta_{orb} + \varepsilon_{PC} \\
LC &= \rho + \delta_{rec} - \delta_{sat} + \delta_{trop} + \delta_{rela} + \delta_{amb} + \delta_{ESC} + \delta_{orb} + dL + \varepsilon_{LC}
\end{align*}
\]  

(8)

In the equation, \( PC \) and \( LC \) are the ionosphere-free pseudo-range and carrier-phase observation values of B1B2 or B1B3 frequency points, and other symbols are the same as Eq. (5). By comparing Eq. (5) and (8), it can be found that although there are errors in the satellite orbit and clock error calculated from broadcast ephemeris, the residual error after the equivalent clock error and orbit correction is absorbed by comprehensive zone correction \( dL \). The monitoring station clock error included in the comprehensive zone correction can be absorbed by the user station clock error, and the satellite ambiguity residual of the monitoring station can be absorbed by the ambiguity of the user station. Since there is barely no variation of satellite orbit and error within a broadcast frequency time range (90s or 180s), when the broadcast ephemeris, equivalent clock error and orbit corrections used by the user station and comprehensive zone correction are the same, the UDRE of the user mainly shows the difference of observation noise and tropospheric model error caused by regional difference. In the case of pseudo-range, due to the ambiguity residuals contained in the comprehensive zone correction, it can only be corrected by the equivalent clock error and orbit correction. Therefore, compared with the unreal-time precise point positioning, the pseudo-range residuals include not only the observed value noise, but also the residual orbit and clock error after modifying the equivalent clock error and orbit correction.

3. Results & Discussion

To prove the theoretical analysis above, two receivers were set at Beijing and Chengdu to collect data from May 26, 2020 to November 26, 2020. Real-time satellite ephemeris of 15 BDS satellites in orbits is received and positioning results are calculated based on the satellite ephemeris. Measuring methods are shown as follow:

3.1. Orbit determination evaluation

The orbit determination accuracy is measured by comparing satellite ephemeris received to the precise satellite ephemeris provided by International GNSS Monitoring & Assessment System(iGMAS) and the orbit error is separated into three dimensions (radial, tangential, normal).
Figure 9. Orbit errors of IGSO-4 satellite

Figure 10. Orbit errors of IGSO-5 satellite

Figure 11. Orbit errors of MEO-3 satellite

Figure 12. Orbit errors of MEO-4 satellite

Figure 13. Orbit errors of IGSO-6 satellite

Figure 14. Orbit errors of MEO-6 satellite

Figure 15. Orbit errors of IGSO-7 satellite
The orbit errors of 15 satellites are pictured and ranked by their Sat SCID. The blue green and red dots represent radial tangential and normal orbit errors, respectively. As shown in the graphs, normal orbit errors far outweigh than radial orbit errors and tangential orbit errors in terms of GEO/IGSO satellites. One the other hand, normal and tangential orbit errors of MEO satellites are about the same. Radial errors play a minimum role for all satellites. Apart from several extreme values, radial orbit errors are within 1.5m, tangential orbit errors are within 2m and normal orbit errors are less than 5m.

Table 2. RMS of BDS satellites orbit errors

| Satellite | R(m) | T(m) | N(m) | 3D(m) |
|-----------|------|------|------|-------|
| GEO-8     | 0.148| 0.599| 1.561| 1.678 |
| GEO-6     | 0.164| 0.258| 0.636| 0.706 |
| GEO-7     | 0.173| 0.248| 0.673| 0.738 |
| GEO-4     | 0.384| 0.693| 1.546| 1.737 |
| GEO-5     | 0.168| 0.562| 0.845| 1.029 |
| Mean      | 0.207| 0.472| 1.052| 1.178 |

| IGSO-1    | 0.154| 0.286| 0.562| 0.650 |
| IGSO-2    | 0.132| 0.348| 0.699| 0.792 |
| IGSO-3    | 0.267| 0.383| 0.655| 0.804 |
| IGSO-4    | 0.149| 0.444| 0.623| 0.779 |
| IGSO-5    | 0.131| 0.382| 0.639| 0.756 |
| IGSO-6    | 0.246| 0.369| 0.577| 0.727 |
| IGSO-7    | 0.138| 0.290| 0.552| 0.639 |
| Mean      | 0.174| 0.358| 0.615| 0.735 |

| MEO-3     | 0.214| 0.743| 0.681| 1.030 |
| MEO-4     | 0.166| 0.654| 0.565| 0.880 |
| MEO-6     | 0.165| 0.674| 0.584| 0.907 |
| Mean      | 0.181| 0.690| 0.610| 0.939 |

Figure 16. 3D Orbit errors of all satellites
To further evaluate the orbit determination accuracy and compare orbit errors of satellites in different orbits, RMS of orbits errors is analysed. Statistics show that average RMSs of GEO orbit errors in radial-direction tangential-direction and normal-direction are 0.2m 0.5m and 1m, respectively. Meanwhile, RMSs of MEO orbit errors in radial-direction tangential-direction and normal-direction are 0.2m 0.7m and 0.6m, respectively. The RMSs of IGSO errors are relatively small, which are 0.2m 0.4m and 0.6m in radial-direction tangential-direction and normal-direction, respectively. According to the scatter diagram, the 3D orbit errors of GEO satellites are slightly large, which are about 6m. And the 3D orbit errors of IGSO satellites are similar to that of MEO satellites, which are under 4m.

3.2. Effects of orbit determination accuracy on positioning accuracy

In order to verify the effects of orbit determination accuracy on positioning accuracy, results of three positioning methods are analysed as follow:

Figure 17. Single point positioning accuracy measured by receiver at Beijing

Figure 18. Single point positioning accuracy measured by receiver at Chengdu

Figure 19. Differential positioning accuracy measured by receiver at Beijing

Figure 20. Differential positioning accuracy measured by receiver at Chengdu

Figure 21. Precise point positioning accuracy measured by receiver at Beijing

Figure 22. Precise point positioning accuracy measured by receiver at Chengdu
The orbit determination accuracy of BDS GEO satellites in radial tangential and normal directions are 0.2m 0.5m and 1m, respectively.

The orbit determination accuracy of BDS IGSO satellites in radial tangential and normal directions are about 1.5m 1.5m and 3m, respectively. The RMSs of IGSO orbit errors in radial tangential and normal directions are 0.2m 0.4m and 0.6m, respectively.

The orbit determination accuracy of BDS MEO satellites in radial tangential and normal directions are about 1m 2m and 2.5m, respectively. The RMSs of MEO orbit errors in radial tangential and normal directions are 0.2m 0.7m and 0.6m, respectively.

The accuracy of BDS single point positioning in north east and up components are about 3m 1.5m and 4m, respectively. The RMSs of single point positioning errors in north east and up components are about 0.5m 0.5m and 1m, respectively.

As for differential positioning, the satellite orbit errors are estimated basing on independent two-way time synchronization observations. So that the Differential positioning accuracy is slightly higher, which is about 2m. Precise point positioning method eliminates residual error after the equivalent clock error and orbit correction and reach the accuracy of about 1m.

4. Conclusions

BeiDou regional navigation satellite system has been providing navigation and positioning service with three methods. Based on the real data collected in Beijing and Chengdu, its orbit determination accuracy and positioning accuracy are evaluated preliminarily. We can conclude that:

- The orbit determination accuracy of BDS GEO satellites in radial tangential and normal directions are about 1.5m 2m and 5m, respectively. The RMSs of GEO orbit errors in radial tangential and normal directions are 0.2m 0.5m and 1m, respectively.
- The orbit determination accuracy of BDS IGSO satellites in radial tangential and normal directions are about 1.5m 1.5m and 3m, respectively. The RMSs of IGSO orbit errors in radial tangential and normal directions are 0.2m 0.4m and 0.6m, respectively.
- The orbit determination accuracy of BDS MEO satellites in radial tangential and normal directions are about 1m 2m and 2.5m, respectively. The RMSs of MEO orbit errors in radial tangential and normal directions are 0.2m 0.7m and 0.6m, respectively.
- The accuracy of BDS single point positioning in north east and up components are about 3m 1.5m and 4m, respectively. The RMSs of single point positioning errors in north east and up components are about 0.8m 0.6m and 1.6m, respectively.

| Position of Receiver | Positioning method | N(m)  | E(m)  | U(m)  | 3D(m)  |
|----------------------|--------------------|-------|-------|-------|--------|
| Beijing              | Single Point       | 0.902 | 0.628 | 1.514 | 1.871  |
|                      | Differential       | 0.517 | 0.277 | 1.296 | 1.423  |
|                      | Precise Point      | 0.127 | 0.142 | 0.348 | 0.397  |
| Chengdu              | Single Point       | 0.722 | 0.544 | 1.628 | 1.875  |
|                      | Differential       | 0.248 | 0.366 | 0.721 | 1.024  |
|                      | Precise Point      | 0.076 | 0.122 | 0.219 | 0.262  |
| Mean                 | Single Point       | 0.812 | 0.586 | 1.571 | 1.873  |
|                      | Differential       | 0.383 | 0.322 | 1.009 | 1.224  |
|                      | Precise Point      | 0.102 | 0.132 | 0.283 | 0.329  |

From the graphs above, positioning accuracy using different methods could be evaluated. The accuracy of BDS single point positioning in north east and up components are about 3m 1.5m and 4m, respectively. The accuracy of BDS differential point positioning in north east and up components are about 1.5m 1m and 3.5m, respectively. And the accuracy of BDS precise point positioning in north east and up components are about 0.5m 0.5m and 1m, respectively.
The accuracy of BDS differential point positioning in north east and up components are about 1.5m 1m and 3.5 m, respectively. The RMSs of differential point positioning errors in north east and up components are about 0.4m 0.3m and 1m, respectively.

The accuracy of BDS precise point positioning in north east and up components are about 0.5m 0.5m and 1m, respectively. The RMSs of precise point positioning errors in north east and up components are about 0.1m 0.1m and 0.3m, respectively.

The effect of orbit determination accuracy on single point positioning accuracy are approximately equal to the average value of satellite orbit determination error in radial, normal and tangential directions. The effect on differential positioning is relatively small. Orbits error barely has no effect on precise point positioning.

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