Finite-Time Asynchronous Stabilization for Nonlinear Hidden Markov Jump Systems with Parameter Varying in Continuous-Time Case

1. Introduction

In recent years, hybrid systems have attracted the great attention and research due to their wide application in the industrial control field. Moreover, with the continuous upgrading and changes in the industrial environment, how to model and control hybrid systems in complex environments has become a major research hotspot in the control field. As a kind of special hybrid systems, Markov jump systems (MJSs) have received researchers’ great attention and many constructive results have been made. It should be noted that the synchronous controllers are always considered in many existing results when it comes to the stabilization problem of Markov jump systems. That means the controller of the systems can always track the information of the model of the system in real time. However, such a situation is always impossible to achieve in a Markov jump system with a complex environment. Therefore, the asynchronous characteristic between the system modal and the controller has attracted the attention of many researchers and they began to focus on the various characteristics of the MJSs. The static output constrained control [1], the output feedback control [2], and the observer-based asynchronous fault detection [3] problems of the MJSs with the discrete-time state are studied by the authors, respectively. In [4], the resilient asynchronous $H_{\infty}$ control problem of the discrete-time MJSs with singularly perturbed was considered. For the nonlinear MJSs, the quantized control [5] and the finite-time $L_{2}$-gain asynchronous control [6] problems by using T-S fuzzy model approach are studied, respectively. Moreover, many researchers also have carried out some work for the continuous-time MJSs. The asynchronous $H_{2}$-controller design and the asynchronous filter design problems are studied for the continuous-time MJSs in [7, 8], respectively. For more particulars of MJSs, the interested readers can read references [9, 10].

It should be pointed out that many results of the MJSs assume that the parameter matrices are constant matrices or
uncertain matrices that satisfy some given known conditions. In actual control systems, the system parameters may have the characteristics of convex polygonal linear parameters varying due to the sensor and actuator failures. For the control systems with such a special situation, it is necessary to design the corresponding nonlinear parameter time-varying controller to stabilization of the systems. Thus, many researchers proposed the linear parameter-varying control methods which laid a solid theoretical foundation for the design of nonlinear parameter time-varying controller. For a kind of parameter-varying system [11], an adaptive algorithm was proposed to achieve the active vibration control by using a new online secondary path estimation method. For bounded parameter variation linear parameter-varying systems, the LMI-based filter design problem is studied in [12]. The robust fault estimation [13] and set-membership fault estimation [14] problem is studied for parameter time-varying systems. For more particulars of parameter-varying systems, the interested readers can read references [15–18].

In the abovementioned references, the system analysis and/or some control method were concerned only over an infinite-time interval, which portrayed the asymptotic properties of the HMJSs and parameter-varying systems. However, the transient characteristics in a given finite-time interval is significant in many control systems [19–21] and should be considered simultaneously. For a class of nonlinear systems, the authors studied the finite-time adaptive fuzzy control problem in [22, 23], respectively. In [24], a finite-time control approach was used to solve the problem of accurate trajectory tracking for disturbed surface vehicles. Moreover, many researchers also combine the finite-time control scheme with the networked switched systems [25], nonlinear systems [26, 27], and quadrotors control [28] and have carried out lots of excellent works. The authors studied the design and implementation of bounded finite-time control algorithm problem for the speed regulation of permanent magnet synchronous motor in [29]. By using the state-dependent switching method, the adaptive fuzzy finite-time control of switched nonlinear systems is studied in [30]. The problem of fuzzy finite-time control for switched systems via adding a barrier power integrator is considered in [31]. For more particulars of HMJSs, the interested readers can read references [32–34].

However, few results have been reported regarding research on finite-time asynchronous control of HMJSs based on the parameter varying, which is the motivation of this work. Different from some existing results on the finite-time control problem [35, 36], the problem of SFTB-$H_{\infty}$ asynchronous control is studied for continuous-time HMJSs via parameter varying in this paper. Compared with the existing results of asynchronous control for discrete-time HMJSs [37–40], this paper mainly consists of the following threefold contributions:

1. The asynchronous characteristic between the controller modes and the system modes is characterized by the hidden Markov dynamics. Moreover, we firstly consider the asynchronous stabilization problem for the continuous-time HMJSs with parameter varying models.

2. Some sufficient conditions will be given to solve the SFTB-$H_{\infty}$ observation-mode-based asynchronous controller by considering the SLKF methods and introducing some auxiliary variables.

3. Considering the parameter time-varying, the methods of gridding technique and approximate basis function will be used in order to change the infinite LMI into finite LMI which can be solved by MATLAB LMI toolbox to get the finite-time asynchronous controller gain directly.

The organization of this paper is made of five parts. In Section 1, the background of the HMJSs, the parameter time-varying systems, finite-time control scheme, and the notation meaning of this paper are introduced and given. Section 2 introduces the system description of the parameter varying hidden Markov jump systems (PV-HMJSs) and designs a suitable asynchronous controller for the studied systems. Moreover, the main definitions and lemmas are also given in this part. Section 3 gives some sufficient conditions to solve the SFTB-$H_{\infty}$ observation-mode-based asynchronous controller. In Section 4, the simulation experiment of HMJSs via parameter varying with two subsystems is carried out, only to find that the closed-loop HMJSs via parameter varying fulfill the condition of SFTB-$H_{\infty}$ under the action of the designed asynchronous controller. The conclusion and future research work follow in Section 5.

Notation: the notation meaning throughout this paper is shown in Table 1. Furthermore, we assume that the matrices and notations in this paper are standard with composable dimensions.

### 2. System Formulation

#### 2.1. System Description

We consider the nonlinear parameter varying hidden Markov jump systems (PV-HMJSs) defined on the probability space $(\Gamma, \Delta, \text{Prob}(\cdot))$:

\[
\begin{bmatrix}
\dot{x}(t) \\
y(t)
\end{bmatrix} = \begin{bmatrix}
f_H(t)(x(t), w(t)) + g_H(t)u(t), \\
0
\end{bmatrix},
\begin{array}{c}
x(t) = x_0, \\
p(t) = p_0, \\
H(t) = H_0,
\end{array}
\begin{array}{c}
t = 0,
\end{array}
\]

(1)

where $H(t) = (h_1(t), h_2(t))$ and $p(t) = [p_1(t), p_2(t), \ldots, p_N(t)]^T$. The notations of the PV-HMJSs (1) are given in Table 2.

The right-continuous hidden Markov chain $H(t)$ is composed of the hidden state $h_1(t)$ in the finite set $\mathcal{M} = \{1, 2, \ldots, M\}$ and the observation state $h_2(t)$ in the finite set $\mathcal{M} = \{1, 2, \ldots, N\}$. Furthermore, the hidden Markov chain $H(t) \in \mathcal{M} \times \mathcal{M}$ can be seen as a homogeneous Markov process which satisfies
Complexity

Table 1: Symbol notations.

| Notation | Denotes |
|----------|---------|
| $\mathbb{R}^n$ | n-dimensional Euclidean space |
| $\mathbb{R}^{m \times n}$ | $n \times m$ real matrices |
| $*$ | symmetric matrix |
| $\mathcal{S}$ | The weak infinitesimal operator of $V$ |
| $I$ | Unit matrix |
| $A^T$ | Matrix transpose |
| $A^{-1}$ | Matrix inverse |
| $0$ | Zero matrix |
| $\| \cdot \|$ | Euclidean vector norm |
| $\text{Her}(A)$ | $A + A^T$ |
| $\lambda_{\text{max}}(P)$ | Maximum eigenvalue of $P$ |
| $\lambda_{\text{min}}(P)$ | Minimum eigenvalue of $P$ |
| diag$(AB)$ | The block-diagonal matrix of $A$ and $B$ |
| $\Gamma$ | The sample space |
| $\Delta$ | The algebra of events |
| $\mathbb{P}(\cdot)$ | The probability measure which defined on $\Delta$ |

\[
\mathbb{P}(H(t+\tau) = (\mathcal{J}, \mathcal{L}) \mid H(t) = (\mathcal{J}, \mathcal{K})) = \begin{cases} 
\mathcal{R}_{(\mathcal{J}, \mathcal{K}), (\mathcal{J}, \mathcal{L})} \mathcal{T} + o(\tau), & (\mathcal{J}, \mathcal{K}) \neq (\mathcal{J}, \mathcal{L}) , \\
1 + \mathcal{R}_{(\mathcal{J}, \mathcal{K}), (\mathcal{J}, \mathcal{L})} \mathcal{T} + o(\tau), & (\mathcal{J}, \mathcal{K}) = (\mathcal{J}, \mathcal{L}) ,
\end{cases}
\]

where $\mathcal{R}_{(\mathcal{J}, \mathcal{K}), (\mathcal{J}, \mathcal{L})} \geq 0, \forall (\mathcal{J}, \mathcal{K}) \neq (\mathcal{J}, \mathcal{L})$ is the transition rate with $\sum_{\mathcal{J}, \mathcal{K}} \mathcal{R}_{(\mathcal{J}, \mathcal{K}), (\mathcal{J}, \mathcal{L})} = -r(\mathcal{J}, \mathcal{K})$ and for any $\mathcal{L}, \mathcal{K} \in \mathcal{M}, \mathcal{J} \in \mathcal{J}$, which satisfies

\[
\mathcal{R}_{(\mathcal{J}, \mathcal{K}), (\mathcal{J}, \mathcal{L})} = \begin{cases} 
\alpha_{\mathcal{J}\mathcal{K}} \pi_{\mathcal{J}\mathcal{K}}, & \mathcal{J} \neq \mathcal{K} , \\
q_{\mathcal{J}\mathcal{L}}, & \mathcal{J} + \mathcal{K} = \mathcal{J}, \mathcal{L} = \mathcal{L}, \\
\pi_{\mathcal{J}\mathcal{L}} + q_{\mathcal{J}\mathcal{L}}, & \mathcal{J} = \mathcal{J}, \mathcal{K} = \mathcal{L} , \\
0, & \text{otherwise},
\end{cases}
\]

in which $\sum_{\mathcal{J}, \mathcal{K}} \alpha_{\mathcal{J}\mathcal{K}} = 1, \Pi = [\pi_{\mathcal{J}\mathcal{K}}]$ is the transition rate matrix and satisfies $\pi_{\mathcal{J}\mathcal{K}} \geq 0, \forall \mathcal{J} \neq \mathcal{K}$. $q_{\mathcal{J}\mathcal{L}}$ is the $\Pi$-dependent conditional probability matrix and satisfies $q_{\mathcal{J}\mathcal{L}} \geq 0, \forall \mathcal{K} \neq \mathcal{L}$. $\pi_{\mathcal{J}\mathcal{L}} = -\sum_{\mathcal{K} \neq \mathcal{J}} \pi_{\mathcal{J}\mathcal{K}}$, and $q_{\mathcal{J}\mathcal{L}} = -\sum_{\mathcal{K} \neq \mathcal{J}} q_{\mathcal{J}\mathcal{K}}$.

Remark 1. From relation (3), we can summarize that the same state will be visited of the hidden state $h_1(t)$ and the observation state $h_2(t)$ if $\mathcal{M} = \mathcal{M}, \alpha_{\mathcal{J}\mathcal{L}} = 0$ with $\mathcal{J} \neq \mathcal{L}$, and $\alpha_{\mathcal{J}\mathcal{L}} = 1$ with $\mathcal{J} = \mathcal{L}$, in this case, the full information of the hidden state $h_1(t)$ will be provided for the detector. If $\alpha_{\mathcal{J}\mathcal{L}} = 1$ and $\alpha_{\mathcal{J}\mathcal{L}} = 0$ with $\mathcal{K} \neq \mathcal{L}$, only one jump will occur between the hidden state $h_1(t)$ and the observation state $h_1(t)$. Moreover, the observer will not provide any information if $\mathcal{M} = \{1\}$, $q_{\mathcal{J}\mathcal{L}} = 0$, and $\alpha_{\mathcal{J}\mathcal{K}} = 1$ and the observation state is $h_2(t) = 1$. For more specific details of the relationship between the hidden state $h_1(t)$ and the observation state $h_2(t)$, we can refer to [3, 6, 35, 36].

In actual engineering applications, we often encounter situations where the system mode information accessible by the controller/observer is usually inaccurate. In other words, the actual model of the system cannot be observed by the controller, which makes the information between the controller mode and the system mode asynchronous. Therefore, a new controller mode $h_2(t)$ related to the system mode $h_1(t)$ needs to be introduced.

The PV-HMJFs (1) can be rewritten as the following PV-HMJFs:

\[
\begin{bmatrix} 
\dot{x}(t) \\
\dot{y}(t)
\end{bmatrix} = F_{h_1(t)} G[x(t), u(t), \omega(t)], \quad x(t) = x_0, \\
p(t) = p_0, \\
h_1(t) = h_{10}, \\
t = 0, \\
h_1(t) \in \mathcal{M},
\]

where

\[
F_{h_1(t)} = \begin{bmatrix} 
A_{h_1(t)}^P & B_{h_1(t)}^P & C_{h_1(t)}^P & D_{h_1(t)}^P & 0 \\
A_{h_1(t)}^P & B_{h_1(t)}^P & C_{h_1(t)}^P & D_{h_1(t)}^P & 0
\end{bmatrix},
\]

and

\[
G[x(t), u(t), \omega(t)] = \text{col}[x(t), u(t), \omega(t), f(x(t), t)],
\]

$A_{h_{10}(t)}^P$ is the time-varying parameter-dependent matrix, and $f(x(t), t)$ is the unknown nonlinear function.

We use $A_{h_1(t)}^P, B_{h_1(t)}^P, W_{h_1(t)}^P, C_{h_1(t)}^P, D_{h_1(t)}^P$, and $W_{h_{10}(t)}^P$ to denote $A_{h_1(t)}^P, B_{h_1(t)}^P, W_{h_1(t)}^P, C_{h_1(t)}^P, D_{h_1(t)}^P$, and $W_{h_{10}(t)}^P$, when $h_1(t) = \mathcal{J}, \mathcal{J} \in \mathcal{M}$, respectively.

For any $n = 1, 2, \ldots, m$, the time-varying parameter matrix $p(t)$ and its variation rate $\dot{p}(t)$ are both assumed bounded, i.e., $p_n(t) \in [p_{n1}^{s1}, p_{n1}^{e1}]$ and $\dot{p}_n(t) \in [p_{n2}^{s2}, p_{n2}^{e2}]$. Moreover, the time-varying parameter matrix $p(t)$ is also measurable and affine parameter dependent in real time. That means the following equations hold:
The systems state
The controlled output
The controlled input
The disturbance input
The time-varying parameter matrix
A right-continuous hidden Markov chain
The initial state of the systems
The initial time-varying parameter matrix
The initial hidden Markov process

$$A^p_J = A_0 + \sum_{\mathcal{J}=1}^{m} A_{\mathcal{J}} P_\mathcal{J},$$

$$B^p_J = B_0 + \sum_{\mathcal{J}=1}^{m} B_{\mathcal{J}} P_\mathcal{J},$$

$$W^p_{1,\mathcal{J}} = W_{10} + \sum_{\mathcal{J}=1}^{m} W_{1,\mathcal{J}} P_\mathcal{J},$$

$$C^p_J = C_0 + \sum_{\mathcal{J}=1}^{m} C_{\mathcal{J}} P_\mathcal{J},$$

$$D^p_J = D_0 + \sum_{\mathcal{J}=1}^{m} D_{\mathcal{J}} P_\mathcal{J},$$

$$W^p_{2,\mathcal{J}} = W_{20} + \sum_{\mathcal{J}=1}^{m} W_{2,\mathcal{J}} P_\mathcal{J},$$

$$F, \mathcal{J} = F_0 + \sum_{\mathcal{J}=1}^{m} F_{\mathcal{J}} P_\mathcal{J},$$

where $A_{\mathcal{J}}, B_{\mathcal{J}}, W_{1,\mathcal{J}}, F_{\mathcal{J}}, C_{\mathcal{J}}, D_{\mathcal{J}},$ and $W_{2,\mathcal{J}}$ with $\mathcal{J} = 0, 1, \ldots, m$ are known matrices.

We suppose the system states of the PV-HMJSs (9) be available, we design the following $h_2(t)$-dependent asynchronous controller:

$$u(t) = K^{p(t)}_{h_2(t)} x(t), \quad h_2(t) \in \mathcal{M},$$

where $K^{p(t)}_{h_2(t)}$ is the $h_2$-dependent controller gain which will be solved in Theorem 3.

Submitting the $h_2(t)$-dependent controller (7) into the PV-HMJSs (5), we can get the following closed-loop PV-HMJSs if $h_1(t) = \mathcal{J}$ and $h_2(t) = \mathcal{X}$, $\mathcal{J} \in \mathcal{M}, \mathcal{X} \in \mathcal{M}$:

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \bar{A}^p_{\mathcal{J}, \mathcal{X}} & W^p_{1,\mathcal{J}} P_{\mathcal{J}} \\ C^p_{\mathcal{J}, \mathcal{X}} & W^p_{2,\mathcal{J}} \end{bmatrix} \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix} + f(x(t), t),$$

$$x(0) = x_0,$$

$$\mathcal{J}(0) = \mathcal{J}_0,$$

$$\mathcal{X}(0) = \mathcal{X}_0,$$

where $\bar{A}^p_{\mathcal{J}, \mathcal{X}} = A^p_{\mathcal{J}} + B^p_{\mathcal{J}} K^p_{\mathcal{X}}$ and $C^p_{\mathcal{J}, \mathcal{X}} = C^p_{\mathcal{J}} + D^p_{\mathcal{J}} K^p_{\mathcal{X}}$.

2.2. Main Definitions and Lemmas

Definition 1 (see [32]). Given two positive constants $0 < \mathcal{J}_1 < \mathcal{J}_2$, a weighting matrix $R_{\mathcal{J}_1} > 0$, and a finite-time interval $[0, T]$, the closed-loop PV-HMJSs (8) with $\omega(t) \equiv 0$ is SFTS within $(\mathcal{J}_1, \mathcal{J}_2, T, R_{\mathcal{J}_1})$ if, for any initial condition $x^0_{\mathcal{J}}, x^0_{\mathcal{X}} \leq \mathcal{J}_1$, we have

$$\mathbf{E}\left\{ x(t)^T R_{\mathcal{J}_1} x(t) \right\} \leq \mathcal{J}_2, \quad t \in [0, T].$$

Definition 2 (see [32]). Given two positive constants $0 < \mathcal{J}_1 < \mathcal{J}_2$, a weighting matrix $R_{\mathcal{J}_1} > 0$, and a finite-time interval $[0, T]$, the closed-loop PV-HMJSs (12) is stochastically finite-time bounded (SFTB) within $(\mathcal{J}_1, \mathcal{J}_2, T, R_{\mathcal{J}_1})$ if, for any initial condition $x^0_{\mathcal{J}}, x^0_{\mathcal{X}} \leq \mathcal{J}_1$, we have

$$\mathbf{E}\left\{ x(t)^T R_{\mathcal{J}_1} x(t) \right\} \leq \mathcal{J}_2, \quad t \in [0, T].$$

Definition 3 (see [6]). Under zero initial condition, the $h_2(t)$-dependent controller (9) is said to be a SFTB-$H_{\infty}$ controller of the PV-HMJSs (7) if there exists a $h_2(t)$-dependent controller gain $K^p_{\mathcal{X}}, \mathcal{X} \in \mathcal{M}$ such that the closed-loop PV-HMJSs (10) be SFTB and satisfies the following $H_{\infty}$-gain performance index:

$$\mathbf{E}\left\{ \int_0^T y^T(t) y(t) dt \right\} < \varphi \mathbf{E}\left\{ \int_0^T \omega^T(t) \omega(t) dt \right\}.$$
SFTB are shown in Definitions 1 and 2. For more details of the SFTS and SFTB, we can refer to [31–33].

3. Main Results

In this section, we will give some sufficient conditions to solve the SFTB-$H_{\infty}$ observation-mode-based asynchronous controller (11) and obtain the $h_1(t)$-dependent controller gain $K_{0}^{n}$, $\mathcal{K} \in \mathcal{M}$. We aim the closed-loop PV-HMJSs (12) to fulfill the SFTB-$H_{\infty}$ condition under the action of the observation-mode-based asynchronous controller (11).

Theorem 1. Given four positive constants $\omega > 0$, $\mathcal{J}_1 > 0$, $\omega > 0$, and $T > 0$, the closed-loop PV-HMJSs (12) is SFTB within $(\mathcal{J}_1 \mathcal{J}_2 T \mathcal{R}_2 \omega)$ if there exist positive constants $0<\mathcal{J}_1<\mathcal{J}_2$, $\lambda_{P,j}>0$, $\Delta_{P,j}>0$, $\varphi > 0$, and $H(t)$-dependent positive-definite symmetric matrices $P_{j,x}$ and $R_{j,x}$, where $(\mathcal{J}, \mathcal{R}) \in \mathcal{M} \times \mathcal{M}$, such that

$$
\begin{array}{l}
\begin{bmatrix}
Y_{P,j,x} P_{j,x} W_{P,j,x}^{T} F_{P,j,x}^{T} P_{j,x} \Theta_{j,x} \\
\ast -\varphi I & 0 & 0 \\
\ast & -\beta^{-1} I & 0 \\
\ast & \ast & -\Psi_{j,x}
\end{bmatrix}
< 0,
\end{array}
\tag{11}
$$

$$
(\mathcal{J} \lambda_{P,j} + \varphi \omega)e^{\omega T} < \mathcal{J} \lambda_{P,j},
\tag{12}
$$

where $\mathcal{R} = \begin{bmatrix}
\text{Her}(P_{j,x} \mathcal{R}_{j,x} P_{j,x}) + \sum_{(j,x), \mathcal{R}} R_{j,x}(j,x) P_{j,x}
\end{bmatrix}$.

Proof. Select a SLKF candidate as follows:

$$
V(x(t), H(t), t) = \lim_{\tau \to 0} \frac{1}{\tau} [E[V(x(t + \tau), H(t + \tau), t + \tau)] - V(x(t), H(t), t)]
$$

$$
= x^{T}(t) \mathcal{P} x(t) + \text{Her}(x^{T}(t) P_{j,x} W_{P,j,x}^{T} \omega(t)) + \text{Her}(x^{T}(t) P_{j,x} F_{P,j,x} x(t)),
\tag{15}
$$

Recalling to Lemma 1 and Assumption 2, we know that

$$
\mathcal{R} = \begin{bmatrix}
\text{Her}(P_{j,x} \mathcal{R}_{j,x} P_{j,x}) + \sum_{(j,x), \mathcal{R}} R_{j,x}(j,x) P_{j,x}
\end{bmatrix}.
\tag{16}
$$

in which

$$
\sum_{(j,x), \mathcal{R}} R_{j,x}(j,x) P_{j,x} = \sum_{(j,x), \mathcal{R}} R_{j,x}(j,x) P_{j,x}
$$

$$
= \begin{bmatrix}
\text{Her}(P_{j,x} \mathcal{R}_{j,x} P_{j,x}) + \sum_{(j,x), \mathcal{R}} R_{j,x}(j,x) P_{j,x}
\end{bmatrix},
\tag{17}
$$

with

$$
\mathcal{R} = \begin{bmatrix}
R_{j,x}(j,x) \omega_{j,x}(j,x) R_{j,x}(j,x) \mathcal{R}_{j,x}(j,x)
\end{bmatrix},
\tag{18}
$$

where

$$
\mathcal{R} = \begin{bmatrix}
\mathcal{R}_{j,x}(j,x) \mathcal{R}_{j,x}(j,x) \mathcal{R}_{j,x}(j,x)
\end{bmatrix},
\tag{19}
$$

$$
\mathcal{R} = \begin{bmatrix}
\mathcal{R}_{j,x}(j,x) \mathcal{R}_{j,x}(j,x) \mathcal{R}_{j,x}(j,x)
\end{bmatrix},
\tag{20}
$$

and

$$
\mathcal{R} = \begin{bmatrix}
\mathcal{R}_{j,x}(j,x) \mathcal{R}_{j,x}(j,x) \mathcal{R}_{j,x}(j,x)
\end{bmatrix},
\tag{21}
$$

Recalling to inequalities (11) and (18)–(21), we have the following inequality with positive constant $\omega > 0$:


\[
E[\mathbf{S}V(x(t), H(t), t)] < \omega V(x(t), H(t), t) + \varphi \omega^T(t)\omega(t). \tag{23}
\]

Then, we have the following relation by integrating both left and right sides of inequality (23) from 0 to \(t\) for \(\forall t \in [0, T] \):

\[
E[V(x(t), H(t), t)] - E[V(x(0), H(0))] < \varphi \int_0^t \omega^T(r)\omega(r)dr + \omega \int_0^t E[V(x(t), H(t), r)]dr. \tag{24}
\]

Thus,

\[
E[V(x(t), H(t), t)] < \left(\int_0^T \omega(r)\omega(r)dr\right) + \omega \int_0^t E[V(x(t), H(t), r)]dr. \tag{25}
\]

The above inequality can be rewritten as (26) by considering the Gronwall inequality:

\[
E[V(x(t), H(t), t)] < (\int_0^T \omega(r)\omega(r)dr) e^{\omega T}. \tag{26}
\]

From Rayleigh inequality, we know that \(E[V(x(t), H(t), t)] \geq \lambda_{\omega x}^E \{e^{\alpha T}R_{\omega x} x(t)\}\). Then, we have

\[
E[\omega x(t)R_{\omega x} x(t)] < \frac{\left(\int_0^T \omega(r)\omega(r)dr\right) e^{\omega T}}{\lambda_{\omega x}^E}. \tag{27}
\]

Thus, the \(E[\omega x(t)R_{\omega x} x(t)] < \mathcal{F}_2^T \) for \(\forall t \in [0, T] \) holds by inequality (12). This completes the proof.

In Theorem 1, we give some sufficient conditions to ensure the SFTB of the closed-loop PV-HMJSs (12). Then, the SFTB-\(H_{\infty}\) condition will be given in the following

\[\text{Theorem 2. Given four positive constants } \omega > 0, \mathcal{F}_1 > 0, \mathcal{T} > 0, \text{ and } \varphi > 0 \text{ with } \int_0^T \omega^T(r)\omega(r)dr \leq \omega, \text{ the SFTB-}\(H_{\infty}\) \text{ condition (10) of the closed-loop PV-HMJSs (8) will be satisfied within } (\mathcal{F}_1, \mathcal{F}_2, T R_{\omega x}, \omega) \text{ if there exist positive constants } 0 < \mathcal{F}_1 < \mathcal{F}_2, \lambda_{\omega x}^P > 0, \lambda_{\omega x}^P > 0, \varphi > 0, \text{ and } H(t)-\text{dependent positive-definite symmetric matrices } P_{\omega x}, \text{ and } R_{\omega x}, \text{ where } (\mathcal{F}, \mathcal{H}) \in \mathcal{M} \times \mathcal{M}, \text{ such that inequalities (12) and (28) hold:}
\]

\[
\begin{bmatrix} \kappa \quad P_{\omega x} W_{\omega x}^T + C_{\omega x}^T P_{\omega x} W_{\omega x}^T & \mathcal{F}_1 P_{\omega x} \Theta_{\omega x} \\ * & W_{\omega x}^T W_{\omega x}^P - \varphi I & 0 & 0 \\ * & * & -\beta^T I & 0 \\ * & * & * & \Psi_{\omega x} \end{bmatrix} < 0, \tag{28}
\]

where \(\kappa = Y + e^{\omega T} C_{\omega x}^T C_{\omega x}^P\).

\[\text{Proof. The same SLKF is selected as in Theorem 1 and we introduce}
\]

\[
E[\mathbf{S}V(x(t), H(t), t)] - \omega V(x(t), H(t), t) < \varphi \omega^T(t)\omega(t) - e^{\omega T} E[y^T(t)y(t)]. \tag{29}
\]

Inequality (28) guarantees that inequality (29) is established. Then, we use \(e^{-\omega T}\) to multiply inequality (29) and integrate such inequality from 0 to \(T\) under zero initial condition, and it yields

\[
e^{-\omega T}E[\mathbf{S}V(x(t), H(t), t)] < \int_0^T (\varphi e^{-\omega T} \omega(r)\omega(r) - y^T(r)y(r))dr. \tag{30}
\]

We can rewrite inequality (30) as the following inequality by considering \(E[V(x(t), H(t), t)] > 0:\)

\[
E\left\{\int_0^T y^T((r)y(r))dr\right\} < E\left\{\int_0^T \varphi e^{-\omega T} d^T(r)d(r)dr\right\} < \varphi E\left\{\int_0^T \omega^T(r)\omega(r)dr\right\}. \tag{31}
\]

Thus, we can obtain

\[
E\left\{\int_0^T y^T((r)y(r))dr\right\} < \varphi E\left\{\int_0^T \omega^T(r)\omega(r)dr\right\}. \tag{32}
\]

Recalling Definition 3, we can get the SFTB-\(H_{\infty}\) condition (10) of the closed-loop PV-HMJSs (8). This completes the proof.

Through the analysis of Theorem 2, we know that the state feedback controller gain matrix \(K_{h_1}^P(t)\) cannot be solved by Matlab LMI tools due to the nonlinear terms in inequality (28). In the following Theorem 3, some sufficient conditions will be given to obtain finite-time asynchronous controller gain \(K_{h_1}^P(t)\).

\[\text{Theorem 3. Given four positive constants } \omega > 0, \mathcal{F}_1 > 0, \mathcal{T} > 0, \text{ and } \varphi > 0 \text{ with } \int_0^T \omega^T(r)\omega(r)dr \leq \omega, \text{ there exist a finite-time } H_{\infty}\text{-gain asynchronous controller with } h_2\text{-dependent state feedback gain } K_{h_2}^P = Z_{\omega x}^P \text{ such that the SFTB-}\(H_{\infty}\) \text{ condition (10) of the closed-loop PV-HMJSs (8) will be satisfied within } (\mathcal{F}_1, \mathcal{F}_2, T R_{\omega x}, \omega) \text{ if there exist positive scalars } 0 < \mathcal{F}_1 < \mathcal{F}_2, \lambda_{\omega x}^P > 0, \varphi > 0, \text{ and } H(t)-\text{dependent positive-definite symmetric matrices } X_{\omega x}, \text{ and } R_{\omega x}, \text{ where } (\mathcal{F}, \mathcal{H}) \in \mathcal{M} \times \mathcal{M}, h_2\text{-dependent positive-definite symmetric matrix } S_{\omega x}, \text{ and } S_{\omega x} \text{ such that}
\]

\[
\begin{bmatrix} \mathcal{D}_{1,\omega x}^P + \mathcal{B}_{1,\omega x} & 0 \\ \lambda \mathcal{R}_{\omega x}^{-1} X_{\omega x} & 0 \end{bmatrix} < 0, \tag{33}
\]

\[
\lambda \mathcal{R}_{\omega x}^{-1} X_{\omega x} < R_{\omega x}^{-1}, \tag{34}
\]

\[
\begin{bmatrix} \varphi \varphi - \frac{\mathcal{F}_1 e^{-\omega T}}{\lambda_1} \sqrt{\mathcal{F}_1} & \sqrt{\mathcal{F}_1} \\ \sqrt{\mathcal{F}_1} & -\lambda_1 \end{bmatrix} < 0, \tag{35}
\]

where
\( \mathcal{D}_{1,F,X}^p = \text{Her} \left( \begin{pmatrix} A^p_{f,S} S_{f,X} + B^p_{f,Z} Z_{f,X}^p & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (C^p_{f,L_S} S_{f,X} + D^p_{f,Z} Z_{f,X}^p)^T & 0 & 0 & 0 \\ e^{\omega T} (C^p_{f,L_S} S_{f,X} + D^p_{f,Z} Z_{f,X}^p)^T & 0 & 0 & 0 \end{pmatrix} \right) \).

\[
\begin{bmatrix}
\mathcal{R}_{1,F,X} & \mathcal{R}_{2,F,X} \\
* & \mathcal{R}_{3,F,X}
\end{bmatrix} < 0,
\]

(36)

**Proof.** We substitute \( \overline{A}_{f,X}^p \) and \( \overline{C}_{f,X}^p \) into inequality (34) and yields

\[
\begin{bmatrix}
\Xi_{1,F,X} & \Xi_{2,F,X} & \Xi_{3,F,X} & F_{p,F_X} & W_{p,F_X} & X_{F,X} & \Theta_{F_X}
\end{bmatrix}
\begin{bmatrix}
* & * & * & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & 0 & 0 \\
\end{bmatrix} < 0,
\]

(37)

where

\[
\Xi_{1,F,X} = \Xi_{1,F,X} + \Xi_{2,F,X} + \Xi_{3,F,X},
\]

\[
\Xi_{2,F,X} = \text{Her} \left( P_{f,X} \left( A^p_{f,F_X} + B^p_{f,K} K_{f,X}^p \right) \right),
\]

\[
\Xi_{3,F,X} = e^{\omega T} \left( C^p_{f,S} + D^p_{f,K} K_{f,X}^p \right)^T \left( C^p_{f,S} + D^p_{f,K} K_{f,X}^p \right),
\]

\[
\Xi_{4,F,X} = \left( \mathcal{R}_{(F_X,F_X)} - \omega \right) P_{f,X} + \beta^{-1} I,
\]

\[
\Xi_{5,F,X} = W_{p,F_X} + \left( C^p_{f,S} + D^p_{f,K} K_{f,X}^p \right)^T W_{p,F_X}.
\]

(38)

We use \( \text{diag} \{ P_{1,F,X}^p, 1, I, I \} \) to pre- and postmultiply inequality (37), and let \( X_{F,X} = P_{f,X}^p Y_{f,X}^p = K_{f,X}^p X_{F,X} \), and \( Q_{F,X} = X_{F,X} X_{F,X} \), and we use the Schur complement lemma. It can be seen that the following inequality satisfies with positive scalar \( \xi > 0 \):
Considering the asynchronous of inequality (39) as follows:
\[ \mathcal{R}_{1,q} + \mathcal{R}_{2,q} < 0, \quad (41) \]
where
\[ \mathcal{R}_{1,q} = \begin{bmatrix} \mathcal{R}_{1,1} & \mathcal{R}_{1,2} \\ \mathcal{R}_{1,3} & \mathcal{R}_{1,4} \end{bmatrix} \]
\[ \mathcal{R}_{2,q} = \mathcal{H} \begin{bmatrix} \mathcal{R}_{2,1} \\ \mathcal{R}_{2,2} \end{bmatrix} \]
\[ \Theta_{1,qq} = \begin{bmatrix} 0 & W_q & X_{qq}F_q \\ 0 & 0 & 0 \end{bmatrix} \]
\[ \mathcal{B}_{1,qq} = \begin{bmatrix} 0 & 0 & X_{qq} \Theta_{1,qq} \\ 0 & 0 & 0 \end{bmatrix} \]
\[ \Theta_{1,qq} = \begin{bmatrix} (R_{1,qq} - \omega)X_{qq} + \beta^{-1}gQ_{qq} \end{bmatrix} \]

For \( \mathcal{B}_{2,qq} \), we define \( X_{qq} = \mu_q L_q S_{qq} \), where \( L_q \) is a nonsingular unit matrix for \( \forall i \in \mathcal{M} \). We can obtain
\[ \begin{align*}
A_q^p X_{qq} + B_q^p Y_{qq} & = \mu_q (A_q^p + B_q^p K_q^p)L_q S_{qq}, \\
C_q^p X_{qq} + D_q^p Y_{qq} & = \mu_q (C_q^p + D_q^p K_q^p)L_q S_{qq}.
\end{align*} \]

Since \( L_q \) is a nonsingular unit matrix, we can get inequality (41) by inequality (33) with \( Z_{qq}^p = K_q^p S_{qq} \). Moreover, we know that \( \mathcal{H} (L_q S_{qq}) > 0 \), i.e., \( \mathcal{H} (S_{qq}) > 0 \) from inequality (33). Thus, \( Z_{qq}^p \) is also a nonsingular matrix, which means that \( K_q^p = Z_{qq}^p S_{qq}^{-1} \) can be defined and solved.

Defining \( \bar{X}_{qq} = R_{1,qq}^{-1/2}X_{qq}R_{1,qq}^{-1/2} \), \( \bar{X}_{qq} = \max_{(i,j) \in \mathcal{M}}(X_{qq})_{i,j} \), and \( \bar{X}_{qq} = \min_{(i,j) \in \mathcal{M}}(X_{qq})_{i,j} \), letting \( \bar{X}_{qq} \leq \lambda_{X_{qq}} \) and \( \bar{X}_{qq} \leq \lambda_{X_{qq}} \), and considering \( \lambda_{X_{qq}} = 1/\lambda_{Y_{qq}} \) and \( \lambda_{X_{qq}} = 1/\lambda_{Y_{qq}} \), inequality (12) can be rewritten as
\[ \frac{\mathcal{J}_1}{\lambda_1} + \phi \omega < \frac{\mathcal{J}_2 e^{-\omega T}}{\lambda_1} \quad (44) \]

By considering the eigenvalue conversion method and using the Schur complement lemma, inequalities (34) and (35) can be obtained. This completes the proof.

However, we also cannot get the state feedback controller gain matrix \( K_{hi}(t) \) by Matlab LMI tools in Theorem 3 due to the dependence of the parameters. Next, the methods of approximate basis function and gridding technique will be used to decentralize the parameter-dependent matrices in Theorem 3. The specific parameterization process is as follows:
\[ \mathcal{L}_p = \mathcal{L}_p^0 + \sum_{j=1}^m \mathcal{L}_p \varphi_j < 0, \quad \forall p \in 3, \quad (45) \]
\[ 3 = \left\{ p(t) \in \mathbb{R}^p : p_{1,j} \leq p_{2,j}(t) \leq p_{3,j}, \quad \forall j \in 1, 2, \ldots, m \right\}, \quad (46) \]
where \( p(t) \) is the time-varying parameter. From inequalities (45) and (46), we know that \( 3 \) denotes an LMI and \( \mathcal{L}_p \) has infinite number of LMIs. Then, we select the following basis function \( \{ F_{p}^{(i)}(t) \}_{j=1}^n \) to solve inequality (45):
\[ \mathcal{L}_p = \sum_{j=1}^n F_{p}^{(i)}(t) \mathcal{L}_p. \quad (47) \]

Furthermore, the infinite LMIs of the \( \mathcal{L}_p \) can be transformed into the finite ones if we divide the space of parameter changes into finite-dimensional grids, which means the following relation satisfied for each set of parameters on the grid for \( \forall p(t) \in 3 \):
\[ \mathcal{L}_p = \sum_{j=1}^n F_{p}^{(i)}(t) \mathcal{L}_p < 0. \quad (48) \]

Remark 3. In Theorem 3, the difficulty of calculation will be increased because of the asynchronous characteristic between the controller modes and the system modes. In order to solve such difficulty, the controller modes \( h_1(t) \) is converted \( h_1(t) \). Moreover, some sufficient conditions are obtained to make the closed-loop HMJSs with linear parameter varying be SFTB-H∞ by introduced auxiliary variables.

Remark 4. In addition, we have introduced the stochastic Lyapunov–Krasovskii functional methods in Theorem 3, which will induce somewhat conservatism of the main results. In future work, we can reduce the impact of conservativeness through replacing the quasi-one-sided Lipschitz condition or one-sided Lipschitz condition with local Lipschitz condition in Assumption 2.

4. Numerical Example

In this section, we consider a class of PV-HMJSs with two subsystems.

Subsystem 1:
where \( \sin(t) \) and \( \cos(t) \) are time-varying parameters and bounded with \([-1, 1]\). We assume the weighted matrix \( K = I \), the initial consideration \( f_1 = 0.8 \), the external disturbance \( \omega(t) = \begin{bmatrix} 0.1\sin^2(2t) \\ 0.2\sin^2(t) \end{bmatrix} \), and the unknown state-dependent nonlinear function \( f(x(t), t) = \begin{bmatrix} 0.3x_1(t) \\ 0.1x_2(t) \end{bmatrix} \) in finite-time interval \( T = 5 \). The other values of the constants are given as \( \omega = 0.3, \lambda_1 = 5.089, \lambda_2 = 0.5, \mu_1 = 1.2, \mu_2 = 1.5, \omega = 0.3, \beta = 0.3, \theta = 1, \) and \( \xi = 0.2 \).

We assume the \( h_2(t) \)-dependent controller gain \( K^p_{P,2} \) with two control schemes for two different jump modes, i.e., \( \mathcal{M} \in \{1, 2\} \) and \( \mathcal{M} \in \{1, 2\} \). The \( \Pi \)-dependent conditional probability matrices are

\[
N^1 = \begin{bmatrix} -0.6 & 0.6 \\ 0.4 & -0.4 \end{bmatrix}
\]

\[
N^2 = \begin{bmatrix} 0.3 & -0.3 \\ -0.1 & 0.1 \end{bmatrix}
\]

The parameters of \( a^1_{\beta} \) are

\( a_1 = a^1_{\beta} = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix} \) and \( a_2 = a^2_{\beta} = \begin{bmatrix} 0.2 & 0.8 \\ 0.5 & 0.5 \end{bmatrix} \).

Solving LMIs (33)–(35), we can get the following finite-time state feedback gain matrices:

\[
K^p_{P,2} = \begin{bmatrix} 2.9251 & -1.1431 \\ -1.1633\sin(t) & 0.3333\sin(t) \end{bmatrix},
\]

\[
K^p_{P,2} = \begin{bmatrix} 3.0817 & -0.7694 \\ -0.5792\sin(t) & -0.171\cos(t) \end{bmatrix}.
\]

Meanwhile, the other relevant solutions are given as \( J_2 = 5.0894 \) and \( \phi = 9.8 \). Then, the simulation results are shown in Figures 1–3.

The jump modes of \( h_1(t) \) and \( h_2(t) \) are shown in Figure 1. The state trajectory \( x(t) \) and \( y(t) \) of the closed-loop PV-HMJSs (8) are shown in Figures 2 and 3. From Figures 2 and 3, we can see the SFTB-H(∞) condition is satisfied with \( E[x(t)^TR_{P,2}x(t)] < J_2 \) in which \( J_2 = 5.0894 \) and \( \phi = 9.8 \).

Remark 5. Similar to some results of reinforcement learning methods [41–44], this paper also considered the problem of stabilization of stochastic Markov jump systems. Moreover, we also use the hidden Markov model to denote the asynchronous characteristic between system modes and controller modes. Different from some existing results on the finite-time control problem [35, 36], the problem of SFTB-H(∞) asynchronous control is studied for continuous-time HMJSs via parameter varying in this paper. Compared with the existing results of asynchronous control for discrete-time HMJSs [37–40], this paper firstly considers the asynchronous stabilization problem for the continuous-time HMJSs with parameter varying models.

For the future work, we can learn from the reinforcement learning methods to study the problem of finite-time asynchronous online control for the continuous-time PV-HMJSs.
Figure 1: The jump mode $H(t)$ of the PV-HMJSs. (a) The jump mode of the hidden state $h_1(t)$. (b) The jump mode of the observation state $h_2(t)$.

Figure 2: The $x(t)$ of the closed-loop PV-HMJSs.

Figure 3: The $y(t)$ of the closed-loop PV-HMJSs.
5. Conclusion

This paper studied the SFTB-$H_{\infty}$ asynchronous control problem for continuous-time PV-HMJSs. Some sufficient conditions are given to solve the SFTB-$H_{\infty}$ asynchronous control gain by considering the methods of SLKF and LMIs. The designed SFTB-$H_{\infty}$ asynchronous controller makes the closed-loop PV-HMJSs satisfy the SFTB-$H_{\infty}$ condition. Finally, we use a numerical example to show the validity of the main results of this paper. For the future research work, the asynchronous control problem for fuzzy PV-HMJSs apply approximation method will be considered.

Data Availability

The data findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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