Entropy production and curvature perturbation from dissipative curvatons

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Abstract

Considering the curvaton field that follows dissipative slow-roll equation, we show that the field can lead to entropy production and generation of curvature perturbation after reheating. Spectral index is calculated to discriminate warm and thermal scenarios of dissipative curvatons from the standard curvaton model. In contrast to the original curvaton model, quadratic potential is not needed in the dissipative scenario, since the growth in the oscillating period is not essential for the model.
1 Introduction

Inflation and reheating are important cosmological events in the early Universe. Most models of inflation consider a scalar field that rolls slowly during inflation. The scalar field is called inflaton. The vacuum energy associated with the inflaton potential causes acceleration of the Universe expansion. The Universe during inflation is supposed to be cold because of the rapid red-shifting of the primeval radiation, except for the Hawking temperature that is intrinsic to de Sitter space. Reheating is thus required for the cold inflationary scenario to recover hot Universe after inflation. Since the reheating may cause thermal production of unwanted relics, such as gravitinos or moduli fields in the supergravity and the string theory, entropy production after reheating is sometimes very useful in ensuring successful nucleosynthesis. A significant example of such entropy production would be thermal inflation [1] in which the inflaton (this field should be discriminated from the inflaton of the primordial inflation that causes the first reheating) is trapped at the origin because of the symmetry restoration caused by the thermal background created by the primordial inflaton. Thermal inflation starts when the Universe cools down and eventually the vacuum energy of the thermal inflaton potential dominates the energy density, and ends when the symmetry breaking begins. During thermal inflation the temperature decreases rapidly due to the accelerated expansion. For the thermal inflation scenario small initial value of the field ($\phi < T_R$, where $T_R$ is the reheating temperature) is crucial for the thermal trapping, as the chaotic initial condition ($T_R \ll \phi \sim M_p$) may prevent the required symmetry restoration. In this paper, we consider a complementary scenario of thermal inflation in which the field fails to cause symmetry restoration. Considering the dissipative field motion after reheating, we show that the scenario may also cause generation of curvature perturbation. The situation is in contrast to the original thermal inflation scenario, in which the generation of the curvature perturbation is not significant. The mechanism for generating curvature perturbation is similar to the curva-

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2 This statement needs more explanations. Cosmological phase transitions may occur not simultaneous in space, but occur with time lags in different Hubble patches. The time lags may arise from the long-wavelength inhomogeneities caused by the light-field fluctuations that exit horizon during primordial inflation. Then the inhomogeneous phase transition may cause generation of the curvature perturbation at the end of thermal inflation. In Ref. [3], utilizing a fluctuating coupling of a light field...
tons, although there are crucial differences that discriminate our scenario from the original curvaton. In this paper the subscript “1I” is used for the first (primordial) inflation stage and “2I” for the dissipative curvaton.

2 Dissipative field in the Universe

In this section we first review the dissipative field motion in the warm inflation scenario. Then we consider the dissipative field motion of the curvaton field \( \phi_{2I} \) after reheating. It will be shown that the field \( \phi_{2I} \) may lead to secondary inflationary expansion and generation of the curvature perturbation after reheating.

2.1 Warm inflation (review)

The inflation scenario with the dissipating inflaton field is known as warm inflation \[4\], in which the dissipation sources radiation and keeps the temperature \( T > H \) during inflation. The condition \( T > H \) characterizes the warm inflation scenario, which includes weak dissipation (WD) scenario where the slow-roll is not due to the significant dissipation. One of the important points that discriminate our scenario from the warm inflation scenario is the source of the radiation. In the warm inflation scenario the radiation energy density \( \rho_R \) is continuously sourced by the dissipation of the inflaton field. The source mechanism leads to the situation that the time-derivative \( \dot{\rho}_R \) is negligible (\( \dot{\rho}_R \sim 0 \), or more precisely \( \dot{\rho}_R \ll 4H\rho_R \)) during inflation.\[4\] A confusing aspect of the warm inflation scenario may be that intuitively realizing significant radiation during inflation seems very difficult. On the other hand, if a field has time-dependent mass during inflation, one intuitively knows that excitation of such field is inevitable. The dissipation can be significant when the (flaton) with the fields in thermal bath, it has been shown that a mechanism of generating the curvature perturbation at the end of thermal inflation may liberate some inflation models and may cause significant non-gaussianity.

3 In contrast to the warm inflation scenario, our scenario considers the radiation-dominated Universe from the beginning, where the radiation is caused by the decay products of the primordial inflaton field. In the radiation-dominated Universe we have in mind the situation that the dissipation from the curvaton field may source radiation, but its amount may or may not be enough to sustain the radiation against red-shifting.
decay of the excitation causes energy loss. There may be some argument related to the thermalization of the decay product, but the thermalization is not essential for the dissipative curvaton model. Further details are discussed in Appendix A, with regard to thermalization and dissipation of the curvaton field.

Before discussing the slow-roll dynamics during the radiation-dominated Universe, we first review the scenario of warm inflation, in which the inflaton potential dominates the energy of the Universe and the dissipation from the inflaton field sources the radiation. Following the usual convention of warm inflation, the equation of motion when the inflaton is dissipative is given by

\[ \ddot{\phi}_{1l} + 3H(1+r)\dot{\phi}_{1l} + V(\phi_{1l}, T)\dot{\phi}_{1l} = 0, \]  

(2.1)

where the subscript denotes the derivative with respect to the field \( \phi_{1l} \). Understanding the dissipative dynamics from the quantum field theory is the challenge of realizing warm inflation. Much work has been devoted to this problem in terms of analytic approximations \([5, 6, 7]\) and with numerical based methods \([8]\). The quasiparticle approximation is considered in Ref. \([5, 6]\), which gives dissipative force in the zero-temperature background. And the equilibrium approximation, which gives dissipation in the background with non-zero temperature, has been considered by many authors. Obviously, the zero-temperature limit of the equilibrium approximation does not lead to the quasiparticle approximation. In these calculations the strength of the damping (i.e., the frictional force) is measured by the rate \( r \) given by the dissipation coefficient \( \Upsilon \) and the Hubble parameter \( H \);

\[ r \equiv \frac{\Upsilon}{3H}. \]  

(2.2)

A short note on the dissipation coefficient \( \Upsilon \) is added in the latter part of this section. The effective potential of the field \( \phi_{1l} \) is expressed as \( V(\phi_{1l}, T) \), which depends on the radiation temperature \( T \). The dissipation coefficient \( \Upsilon \) of the inflaton, which is related to the microscopic physics of the interactions, leads to the energy conservation equation;

\[ \dot{\rho}_R + 4H\rho_R = \Upsilon \dot{\phi}_{1l}^2, \]  

(2.3)

where \( \rho_R \) is the energy density of the radiation and the right hand side \((\Upsilon \dot{\phi}_{1l}^2)\) represents the source of the radiation from the dissipation.\(^4\)

\(^4\)The slow-roll equation leads to the equation for the source term; \( \Upsilon \dot{\phi}_{1l}^2 \simeq V_{\phi_{1l}} \dot{\phi}_{1l} \simeq V. \)
Universe, the radiation scales as $\rho_R \propto a^{-4}$ when the source term is negligible, but $\rho_R$ may behave like a constant ($\dot{\rho}_R \simeq 0$) when the source term is significant ($4H\rho_R \sim \Upsilon \dot{\phi}_I^2$) in the energy conservation equation. In this paper, the former will be denoted by the “thermal phase” and the latter will be denoted by the “warm phase”. The slow-roll equation in the thermal phase is the key topic in this paper.

Scenario with $r > 1$ is usually called strongly dissipating (SD) scenario. In this paper we consider only the SD motion with $r > 1$, because we have in mind the situation where the conventional slow-roll condition is already violated by the potential. With regard to the dissipative field equation, the effective slow-roll parameters are different from the conventional (non-dissipating) ones. They are given by

$$\epsilon_w \equiv \frac{\epsilon}{(1 + r)^2},$$
$$\eta_w \equiv \frac{\eta}{(1 + r)^2},$$

(2.4)

where the non-dissipating slow-roll parameters ($\epsilon$ and $\eta$) are defined by

$$\epsilon \equiv \frac{M_p^2}{2} \left( \frac{V_\phi}{\rho_{Tot}} \right)^2,$$
$$\eta \equiv \frac{M_p^2 V_{\phi \phi}}{\rho_{Tot}},$$

(2.5)

where $M_p \equiv (8\pi G)^{-1/2} \simeq 2.4 \times 10^{18}$GeV is the Planck mass and $\rho_{Tot}$ is the total energy density. Here the subscript under $V$ denotes the derivative with respect to the inflaton field $\phi$. The warm-phase condition $4H\rho_R \simeq \Upsilon \dot{\phi}_I^2$ combined with the slow-roll equation leads to more stringent conditions \[9\]

$$\epsilon < (1 + r)$$
$$\eta < (1 + r),$$

(2.6)

which is required for the slow-roll in the warm phase \[9\]. In the thermal phase, where the warm-phase condition is violated, we introduce an alternative $\dot{V} < 4HV$, which suggests that the potential decreases slower than the radiation. The slow-roll conditions in the thermal phase cannot be the same as in the warm phase.

To quantify the dissipation coefficient $\Upsilon$, it would be useful to consider past results for the typical superpotential \[6, 12\]. In the warm inflationary scenario, final product of the
dissipation is excitation of light fermions, which may enhance the loop corrections to the inflaton potential. Then the interactions related to dissipation may lead to the violation of the flatness conditions which inflation requires. Because of this concern, one needs to consider the loop corrections when the light fields thermalize during warm inflation. The light sector, where the decay products thermalize during inflation, typically have couplings representing self-interactions or interactions with other light fields. These interactions are implicitly assumed in addition to the basic interactions which are typically given by

\[ L_I \simeq g_1 \phi^2 \chi^2 - g_2 \chi^2 \psi^2, \]  

where the inflaton \( \phi \) couples to the intermediate field \( \chi \) and finally decay into light fermions \( \psi \). Therefore, the relaxation time of the radiation may be independent of the interactions related to the dissipation mechanism. We thus “assume” thermalization in this paper. For the adiabatic case, the dissipation coefficient usually depends on the temperature \( T \) and the expectation value of the field. Considering interactions given by the superpotential

\[ W = g_1 \Phi^2 X^2 + g_2 XY^2, \]  

where \( g_1 \) and \( g_2 \) are coupling constants, and \( \Phi, X, Y \) are superfields whose scalar components are given by \( \phi, \chi, y \). Fermionic partners of the scalar components \( \phi, \chi, y \) are \( \psi_\phi, \psi_\chi, \psi_y \). The supersymmetry is broken by the inflaton field, which leads to one-loop contribution to the inflaton potential. Usually, inflation is associated with a very weakly coupled inflaton in the inflation sector to protect the inflaton potential from large quantum corrections. However, in some inflation models, for instance when hybrid-type potential is considered, the interactions of the inflaton field may not be negligible[10]. In this case, supersymmetry can play a significant role in inflationary cosmology. In fact, in supersymmetric models the restrictions on the couplings are less severe due to the cancellations that may be still effective when supersymmetry is softly\(^5\) or thermally broken during inflation. As the result, the gradient of the inflaton potential can be small enough to allow inflation whilst the thermalization time for the radiation remains short, allowing warm inflation for moderate values of the coupling constants in these interactions.\(^6\) We consider the model in

\(^5\)”Soft breaking” does not mean that the energy scale of the supersymmetry breaking is small compared with the inflation scale. Soft breaking occurs when “soft terms” break supersymmetry. See textbook[11] for more details.

\(^6\)See Ref.[12] for more details.
which the field motion ($\dot{\phi} \neq 0$) causes excitation of the heavy intermediate field $\chi$ that has the mass proportional to $\phi$. Then the heavy intermediate field decays into light fermions to dissipate the energy from the field $\phi$. The situation (i.e., the heavy intermediate field + light fermions with $\dot{\phi} \neq 0$) is not peculiar for particle cosmology. During warm inflation, when $\phi$ is large, the field $y$ and its fermionic partner $\psi_y$ are massless in the global supersymmetric limit. The mediating field is $\chi$, which obtains large mass $m_\chi \sim g_1\phi$ from the interaction, and the dissipation is caused by the excitation of the mediating field $\chi$ that decays into massless fermions. Considering the equilibrium approximation, at high temperature ($m_\chi \ll T$) the dissipation coefficient is given by $\Upsilon \propto (g_1^2/g_2^3)T$, while at low temperature ($m_\chi \gg T$) the coefficient is given by $\Upsilon \propto g_1^2(T^3/\phi^2)$. The dissipation coefficient must include terms both from the equilibrium approximation and the quasiparticle approximation at the same time. However, in the conventional warm inflation scenario the less effective one can be disregarded, since during warm inflation the changes in the inflaton field $\phi$ and the temperature $T$ are not significant due to the slow-roll conditions. In contrast to the warm inflation scenario, the slow-roll equation for the curvaton in the thermal phase should be evaluated with the significant change in the temperature $T$. Therefore, for the dissipative curvaton scenario, the dissipation coefficient must include both terms from the low-temperature and the quasiparticle approximations. For instance, we consider $\Upsilon \simeq \Upsilon_1 + \Upsilon_2 \equiv C_1T^3/\phi^2 + C_2\phi$ for the dissipative motion after reheating, where $\Upsilon_1$ is obtained using the equilibrium approximation and is significant for $\phi < \phi_* \equiv T \times (C_1/C_2)^{1/3}$, while $\Upsilon_2$ is obtained using the quasiparticle approximation and is significant for $\phi > \phi_*$. See also Figure [1]. Considering $N_\chi$ and $N_\psi$ for the number of intermediate fields and the decay products, the coefficients $C_1$ and $C_2$ are typically given by $C_1 \sim 0.1 \times g_1^4N_\chi^2N_\psi^2$ [9] and $C_2 \sim 0.1 \times N_\chi/\sqrt{N_\psi g_1^3/g_2}$ [5, 6].

A natural expectation from the standard warm inflation scenario is that similar phenomenon (dissipative slow-roll) should be observed in the evolution of other scalar fields during and after inflation, either in the thermal or in the warm phases. In this paper we apply this idea to the field motion of the dissipative curvaton field $\phi_{2I}$ after reheating. Evolution of the field before reheating depends crucially on the inflation model, which may either be cold or warm. Because of the crucial model-dependence of the scenario, we will not discuss the evolution of the field before reheating. Instead, we simply assume
Figure 1: Υ₁ and Υ₂ are calculated using the equilibrium and the quasiparticle approximations, respectively. The critical point φ* is not a constant. It moves rapidly in the thermal phase because it depends crucially on the temperature.

that the curvaton stays at φ₂I > T_R before reheating. See Figure 2 for the evolution of the Universe.

2.2 Radiation and dissipation after reheating

To understand the evolution of the dissipative field after inflation, we consider V(φ₂I), which is the monomial potential given by

\[ V(φ₂I) = \lambda_n \frac{φ₂I^n}{M_p^{n-4}}, \quad (2.9) \]

where the less effective terms are disregarded for simplicity. The energy density of the Universe is given by

\[ 3H^2 M_p^2 \equiv ρ_{Tot} ≃ ρ_R + V(φ₂I). \quad (2.10) \]

The slow-roll field equation with r ≫ 1 leads to

\[ \Upsilon \dot{Φ}₂I \simeq \Upsilon \left( \frac{V_{φ₂I}}{3H} \right)^2 \]

\[ \simeq \begin{cases} \frac{(φ₂I V_{φ₂I})^2}{C₁ T^2} = \frac{n^2 V^2}{C₁ T^4} & (φ₂I < φ*) \\ \frac{(φ₂I V_{φ₂I})^2}{C₂ φ₂I^2} = \frac{n^2 V^2}{C₂ φ₂I^2} & (φ₂I > φ*), \end{cases} \quad (2.11) \]

where φ₂I V_{φ₂I} = nV(φ₂I) is used for the monomial potential. If the radiation satisfies the warm-phase condition \( \dot{ρ}_R < 4H ρ_R \), which leads to \( \Upsilon \dot{Φ}₂I \simeq 4H ρ_R \), the radiation of the
Figure 2: Evolution of the Universe is shown for the dissipative scenario. The potential of the dissipative field $\phi_2$ may start dominating the Universe either in the thermal phase or in the warm phase. Then the second inflation (dissipative inflation) starts for the dissipative field $\phi_2$. Dissipative inflation in the warm phase may be called warm inflation, but the spectral index is different from the original scenario. Therefore, we use “dissipative inflation” for the secondary inflation stage even if it is in the warm phase. Thermal phase is inevitable for the modest scenario with $T_R > T_{eq}$.

Universe is sourced by the dissipation and the temperature in the warm phase is given by

$$T^7 = \alpha_1 \frac{V(\phi_{2I})^2}{H} \quad (\phi_{2I} < \phi_*)$$  \hspace{1cm} (2.12)$$

$$T^4 = \alpha_2 \frac{V(\phi_{2I})^2}{H\phi_{2I}^3} \quad (\phi_{2I} > \phi_*)$$ \hspace{1cm} (2.13)$$

where we introduced a new parameter $\alpha_{1,2}$ defined by

$$\alpha_1 \equiv \frac{n^2}{4C_1C_T} \sim 10^{-6} \left( \frac{n}{4} \right)^2 \left( \frac{10^4}{C_1} \right) \left( \frac{10^2}{C_T} \right)$$  \hspace{1cm} (2.14)$$

$$\alpha_2 \equiv \frac{n^2}{4C_2C_T} \sim 10^{-2} \left( \frac{n}{4} \right)^2 \left( \frac{1}{C_2} \right) \left( \frac{10^2}{C_T} \right).$$  \hspace{1cm} (2.15)$$

Here $C_T$ is the constant that is defined as $\rho_R \equiv C_T T^4$. The warm phase must be an instantaneous phenomenon if it appears in the radiation-dominated Universe, where the relation $\rho_R > V(\phi_{2I})$ is crucial.
2.3 Slow-roll conditions for the dissipative field

Considering the dissipative field equation, the effective slow-roll parameters are given by

\[
\epsilon_w \equiv \frac{1}{(1+r)^2} \frac{M_p^2}{2} \left( \frac{V_{\phi}}{\rho_{Tot}} \right)^2 < 1
\]

\[
\eta_w \equiv \frac{1}{(1+r)^2} M_p^2 \frac{V_{\phi\phi}}{\rho_{Tot}} < 1,
\]

which are needed for the condition \( \ddot{\phi}_{2I} < (3H + \Upsilon)\dot{\phi}_{2I} \).

In the warm phase, \( \dot{\rho}_R < 4H\rho_R \) imposes more stringent conditions \( \epsilon < (1+r) \) and \( \eta < (1+r) \). In the thermal phase, the slow-roll conditions are \( \epsilon_w < 1 \) and \( \eta_w < 1 \), which result in \( \epsilon_w \leq \eta_w < 1 \) for the monomial potential. In addition to the conventional slow-roll conditions, there is an obvious condition \( \dot{V} < 4HV \) in the thermal phase, which is needed to ensure that the radiation decreases faster than the potential. During the radiation-dominated epoch, only the thermal phase is relevant for the dynamics of \( \phi_{2I} \).

2.4 After reheating (\( \rho_{Tot} \simeq \rho_R \))

After reheating the temperature decreases as \( T \propto a^{-1} \). If the thermal phase is connected to the warm phase, the condition \( \Upsilon \dot{\phi}_{2I}^2 \simeq 4H\rho_R \) is satisfied in the warm phase. However, as far as the radiation dominates the Universe, it is impossible to keep \( \Upsilon \dot{\phi}_{2I}^2 \simeq 4H\rho_R \) for a time period \( \Delta t \sim H^{-1} \). In the usual warm inflationary scenario (with monomial potential) the condition \( \Upsilon \dot{\phi}_{2I}^2 \simeq 4H\rho_R \) is equivalent to the slow-roll condition in the warm phase \( \eta/r \sim \rho_R/V_{\phi_{2I}} < 1 \), although the equivalence is obtained using the warm-phase condition combined with the slow-roll field equation. Warm inflation starts with \( \eta \ll r \) in the warm phase and ends at \( \eta \sim r \), where the slow-roll condition in the warm phase breaks. Unlike the conventional warm inflation scenario, which starts with the potential-dominated Universe in the warm phase, our scenario starts with the radiation-dominated Universe in the thermal phase, the warm-phase condition is violated. Natural conclusion is that the radiation-dominated Universe is always in the thermal phase because the radiation energy density is too large to be sustained by the dissipation caused by the potential. In fact, in the radiation-dominated Universe \( \Upsilon \dot{\phi}_{2I}^2 \simeq 4H\rho_R \) may appear for a short period of time \( \Delta t \ll H^{-1} \) but it is impossible to sustain the radiation against red-shifting. In our scenario we consider significant dissipation for the field motion, but
we may disregard the warm phase when the radiation dominates the Universe.

For the slow-roll in the thermal phase of the radiation-dominated Universe, we have to consider slow-roll conditions that are different from the usual warm inflation scenario. For the thermal phase we introduce the condition \( \dot{V} < 4HV \). Considering \( \dot{V} \simeq V_{\phi I} \dot{\phi}_{2I} \simeq \Upsilon \dot{\phi}_{2I}^2 \), it leads to

\[
\frac{\dot{V}}{4HV} \simeq \begin{cases} 
\frac{n^2 V_{\phi I} \sqrt{3} M_p}{4C_1 T^4 C_I^2 T^2} < 1 & (\Upsilon_1 > \Upsilon_2) \\
\frac{n^2 V_{\phi I} \sqrt{3} M_p}{4C_2 \dot{\phi}_{2I} C_I^2 T^2} < 1 & (\Upsilon_1 < \Upsilon_2).
\end{cases}
\tag{2.17}
\]

For the scenario with \( \Upsilon_1 > \Upsilon_2 \), \( T \sim V^{1/4} \) leads to the condition \( T > n^2 \sqrt{3} M_p/(4C_1 C_I^{1/2}) \), which means that \( T \sim V^{1/4} \) is consistent with the slow-roll condition only when \( T > n^2 \sqrt{3} M_p/(4C_1 C_I^{1/2}) \). Namely, if \( V < n^2 \sqrt{3} M_p/(4C_1 C_I^{1/2}) \), \( V(\phi_{2I}) \) starts decreasing faster than the radiation before it dominates the Universe. This scenario is not suitable for our argument and it will not be considered in this paper, because it can lead neither to the entropy production nor to the generation of the curvature perturbation.

The dynamics related to the time-dependent friction \( (\Upsilon \propto T^3) \) in the radiation-dominated Universe is similar to the usual curvaton dynamics before oscillation, where the friction scales as \( H \propto T^2 \). In this sense, one may say that what is considered in this section is nothing but the dynamics of the curvatons \([13]\) with the enhanced friction term \( \Upsilon \gg H \).

After reheating, the Universe is in the thermal phase, where the temperature decreases as \( T \propto a^{-1} \). Using \( H^2 = \rho_R/3M_p^2 \), we find for \( r > 1 \):

\[
\frac{1}{(1 + r)^2} \simeq \begin{cases} 
\frac{\rho_R \dot{\phi}_{2I}}{M_p^2 C_I^{1/4}} & (\phi_{2I} < \phi_*) \\
\frac{\dot{\phi}_{2I}}{M_p^2 C_2 \dot{\phi}_{2I}} & (\phi_{2I} > \phi_*).
\end{cases}
\tag{2.18}
\]

For the strong dissipative scenario, there are conditions for \( r > 1 \), which is given by

\[
\phi_{2I} < \frac{\sqrt{C_1}}{C_I^{1/4}} \sqrt{T M_p} \quad (\phi_{2I} < \phi_*) 
\tag{2.19}
\]

\[
\phi_{2I} > \frac{H}{C_2} \quad (\phi_{2I} > \phi_*),
\tag{2.20}
\]

which are trivial for the scenario. The slow-roll condition \( \epsilon_w \leq \eta_w < 1 \) leads to

\[
\eta_w \simeq \begin{cases} 
\frac{n(n-1) \rho_R}{C_I^2} \left( \frac{\dot{\phi}_{2I}}{V} \right)^2 \left( \frac{V}{\rho_R} \right) < 1 & (\phi_{2I} < \phi_*) \\\n\frac{n(n-1) V}{C_2^2 \dot{\phi}_{2I}} < 1 & (\phi_{2I} > \phi_*),
\end{cases}
\tag{2.21}
\]

\[\text{11}\]
which result in the conditions

\[
\phi_{2I} < \frac{C_1 T^3}{\sqrt{n(n-1)} V^{1/2}} \sim T \times 10^4 \left( \frac{C_1}{10^4} \right) \left( \frac{T}{V^{1/4}} \right)^2 (\phi_{2I} < \phi_*) \quad (2.22)
\]

\[
\phi_{2I} > \left[ n(n-1) \right]^{1/4} V(\phi_{2I})^{1/4} \left( \frac{1}{C_2} \right)^{1/2} \sim V(\phi_{2I})^{1/4} \left( \frac{1}{C_2} \right)^{1/2} (\phi_{2I} > \phi_*) \quad (2.23)
\]

Here \( \phi_* \) is estimated as

\[
\phi_* \simeq 10 \times T \left( \frac{C_1}{10^4} \right)^{1/3} \left( \frac{1}{C_2} \right)^{1/3}. \quad (2.24)
\]

For \( \phi_{2I} < \phi_* \), the condition \( \dot{V} < 4HV \) in the thermal phase puts a lower bound for the temperature. For \( \phi_{2I} > \phi_* \), it leads to the condition

\[
\phi_{2I} > \left( \frac{n^2 V \sqrt{3 M_p}}{4 C_2 C_T^{1/2} T^2} \right)^{1/3} \sim V^{1/4} \left( \frac{1}{C_T^{1/2} C_2} \right)^{1/3} \left( \frac{n^2 V^{1/4} M_p}{4 T^2} \right)^{1/3}, \quad (2.25)
\]

which will be more stringent than the conventional slow-roll condition given in Eq. \( (2.23) \).

These equations show that the slow-roll is conceivable for the field \( \phi_{2I} \) in the radiation-dominated Universe. The warm phase never appears during this period. As far as the slow-roll conditions are satisfied, the curvaton field \( \phi_{2I} \) rolls slowly due to the dissipation, and the radiation decreases as \( \rho_R \propto a^{-4} \). Then the potential \( V(\phi_{2I}) \) may eventually start to dominate the energy. We denote the temperature by \( T_{in} \), when the potential starts domination. The slow-roll conditions at \( T = T_{in} \simeq V^{1/4} \) give

\[
\phi_{2I} < 10^4 \times T_{in} \left( \frac{C_1}{10^4} \right) (\phi_{2I} < \phi_*) \quad (2.26)
\]

\[
\phi_{2I} > V^{1/4} \left( \frac{1}{C_T^{1/2} C_2} \right)^{1/3} \left( \frac{n^2 M_p}{4 V^{1/4}} \right)^{1/3} (\phi_{2I} > \phi_*). \quad (2.27)
\]

### 2.5 After \( V(\phi_{2I}) \) dominates the energy

The potential may start to dominate the energy of the Universe as the radiation decreases with time while the field \( \phi_{2I} \) rolls slowly. Considering the scenario in which the potential can start dominating the Universe, our question is whether the warm phase may appear for the dissipative curvaton \( \phi_{2I} \) before the end of the slow-roll. Domination by the potential starts in connection with the thermal phase of the radiation-dominated Universe, where \( \dot{\rho}_R \sim 4H \rho_R > \Upsilon \dot{\phi}_{2I}^2 \). Then the Universe may reach the warm phase \( (\dot{\rho}_R < 4H \rho_R \sim \Upsilon \dot{\phi}_{2I}^2) \) before the end of the slow-roll.
For the slow-roll in the thermal phase, the condition $\dot{V} < 4HV$ leads to

$$\frac{\dot{V}}{4HV} \simeq \begin{cases} \frac{n^2 \sqrt{3} M_p V^{1/2}}{4 C_1 T^3} < 1 & (\phi_{2I} < \phi_*) \\ \frac{n^2 \sqrt{3} M_p V^{1/2}}{4 C_2 \phi_{2I}^{3/2}} < 1 & (\phi_{2I} > \phi_*) \end{cases}$$

(2.28)

Conventional slow-roll conditions in the thermal phase lead to

$$\phi_{2I} < \phi_{c1} \equiv T \times 10^4 \left( \frac{C_1}{10^4} \right) \left( \frac{T}{V(\phi_{2I})^{1/4}} \right)^2 (\phi_{2I} < \phi_*)$$

(2.29)

$$\phi_{2I} > \phi_{c2} \equiv V(\phi_{2I})^{1/4} \left( \frac{1}{C_2} \right)^{1/2} (\phi_{2I} > \phi_*)$$

(2.30)

which are identical to the slow-roll conditions in the radiation-dominated Universe, except for the requirement $C_T^{1/4} T < V(\phi_{2I})^{1/4}$.

We will discuss how (and when) the warm phase appears in connection with the thermal phase of the potential-dominated Universe.

$\phi_{2I} < \phi_*$

The warm phase never appears if the slow-roll in the thermal phase ends before the warm phase begins. For $\phi_{2I} < \phi_*$, the thermal phase ends at

$$T_{\text{end}} = \left( \frac{n^2 \sqrt{3} M_p V(\phi_{2I})^{1/2}}{4 C_1} \right)^{1/3},$$

(2.31)

where the potential starts decreasing faster than the radiation. The above condition combined with $C_T^{1/4} T < V^{1/4}$ puts a severe bound for the curvaton potential. Moreover, the conventional slow-roll condition shows that the upper bound, which scales as $\phi_{c1} \propto T^3$, decreases fast in the thermal phase. If $\phi_{c1}$ reaches $\phi_{2I}$ before the warm phase begins, the thermal phase ends there and the warm phase never begins.\footnote{Equivalently, the slow-roll parameter increases rapidly in the thermal phase and eventually reach $\eta_w \sim 1$.}

The process may be delayed if the warm phase appears before the end of the thermal phase. In fact, the source term for the radiation is given by

$$\Upsilon \dot{\phi}_{2I}^2 \sim \frac{n^2 V(\phi_{2I})^2}{9 C_1 T^3},$$

(2.32)

which (in the thermal phase) increases as $\Upsilon \dot{\phi}_{2I}^2 \propto T^{-3}$. If $\Upsilon \dot{\phi}_{2I}^2$ reaches $\Upsilon \dot{\phi}_{2I}^2 \sim 4 H \rho_R$ in the thermal phase, warm inflation may start and the temperature will be sustained. However, the large potential energy required for the scenario may not be conceivable for
the curvaton scenario. Moreover, the chaotic initial condition for the field $\phi_{2I}$ may favor the scenario with $\phi_{2I} > \phi_*$. 

\[
\phi_{2I} > \phi_*
\]

In contrast to the scenario with $\phi_{2I} < \phi_*$, the slow-roll conditions combined with $\dot{V} < 4HV$ shows that for the scenario with $\Upsilon_2 > \Upsilon_1$ the thermal phase can be connected to the warm phase without introducing neither large potential nor the small (and fine-tuned) vacuum expectation value (VEV) for the curvaton field. Once the Universe goes into the warm phase, the dynamics related to the slow-roll is identical to the usual warm inflation scenario with the monomial;

\[
V(\phi_{2I}) = \lambda_n \frac{\dot{\phi}_{2I}^n}{M_p^{n-4}}. 
\tag{2.33}
\]

The field $\phi_{2I}$ can generate the cosmological perturbation once it dominates the Universe. The scenario for generating cosmological perturbation is different from the usual warm inflation scenario, because the field fluctuations that are relevant for the cosmological perturbations exit horizon during primordial inflation. The mechanism is rather similar to the curvaton scenario, although the amplification of the energy ratio is not due to the long-time oscillation of the curvaton field. The generation of the curvature perturbation will be discussed in Sec.2.6.

The scenario with $\phi_{2I} > \phi_{c2} > \phi_*$ allows dissipative inflation in the thermal phase that is connected to the warm phase. In the thermal phase, the lower boundary of the slow-roll region is determined by the condition $\dot{V} < 4HV$, which leads to

\[
\phi_{2I} > \left( \frac{n^2 \sqrt{3} M_p V^{1/2}}{4 C_2} \right)^{1/3}. 
\tag{2.34}
\]

The condition does not depend explicitly on the temperature $T$. The thermal phase can be connected to the warm phase if $\Upsilon \dot{\phi}_{2I}^2$ can reach $\Upsilon \dot{\phi}_{2I}^2 \sim 4H\rho_R$ before the slow-roll condition is violated. The warm phase appears at (i.e., the warm-phase condition $\Upsilon \dot{\phi}_{2I}^2 \sim 4H\rho_R$ is satisfied at)

\[
T_w \simeq \left( \frac{\alpha_2 M_p V(\phi_{2I})^{3/2}}{\phi_{2I}^3} \right)^{1/4} \sim 10^9 GeV \left( \frac{\alpha_2}{10^{-2}} \right)^{1/4} \left( \frac{V(\phi_{2I})^{1/4}}{10^{10} GeV} \right)^{3/2} \left( \frac{10^{13} GeV}{\phi_{2I}} \right)^{3/4},
\tag{2.35}
\]

\[\text{Here we have in mind the rapid change of } T \text{ in the thermal phase. In the warm phase there is the relation between } \phi_{2I} \text{ and } T, \text{ where both } \phi_{2I} \text{ and } T \text{ change slowly.}\]
which allows for $\alpha_2 \sim 10^{-2}$, $V(\phi_{2I})^{1/4} \sim 10^{10}\text{GeV}$ and $\phi_{2I} \sim 10^{13}\text{GeV}$ the number of e-foldings of $N^{th} \sim \ln(T_{in}/T_w) \sim 2$ in the thermal phase. Dissipative inflation in the thermal phase, which looks like thermal inflation without symmetry restoration, is indeed possible.

When the Universe in the thermal phase is connected to the warm phase, warm inflation starts with the dissipation coefficient $\Upsilon = C_2 \phi_{2I}$. Warm inflation ends when the radiation reaches $\rho_R \sim V(\phi_{2I})$ again, where the usual slow-roll condition for the warm inflation scenario is broken. For the monomial potential, warm inflation ends at $\eta \simeq r$, which leads to

$$\phi_{2I}^{(end)} \sim M_p \left( \frac{\sqrt{n} n(n-1)}{C_2} \right)^{2/(6-n)},$$

(2.36)

where $n = 6$ leads to a trivial result as far as the field rolls with strong dissipation. For $n = 6$, warm inflation may end when the strong dissipation ends. The number of e-foldings elapsed in the warm phase is

$$N^w \sim \int \frac{H}{\dot{\phi}_{2I}} d\phi_{2I} \sim \left\{ \begin{array}{ll} \frac{2C_2 n}{n(6-n)\sqrt{n}} \left[ \left( \frac{\phi_{2I}^{(ini)}}{M_p} \right)^{(6-n)/2} - \left( \frac{\phi_{2I}^{(end)}}{M_p} \right)^{(6-n)/2} \right] & (n \neq 6) \\ \frac{C_2}{n \sqrt{n}} \ln \frac{\phi_{2I}^{(ini)}}{\phi_{2I}^{(end)}} & (n = 6) \end{array} \right.$$  

(2.37)

where $\phi_{2I}^{(ini)}$ denotes the value of $\phi_{2I}$ when the warm phase starts. The number of e-foldings is crucially controlled by the initial condition of the field $\phi_{2I}$. More precisely, the field rolls slowly in the thermal phase from $T = T_R$ to $T = T_w$, but the value of $\phi_{2I}$ at the end of the thermal phase (it corresponds to $\phi_{2I}^{(ini)}$ at the beginning of the warm phase) is not determined by the model parameters. It depends on the initial value of $\phi_{2I}$ during the primary inflation. The situation may be in contrast to $\phi^{(end)}$, which is determined by the model parameters.

Finally, we will comment on the temperature at the beginning of the warm phase. In conventional warm inflation the temperature is already put at the equilibrium value from the beginning. In contrast to the usual scenario of warm inflation, our scenario starts with reheating after the primary inflation, which cannot lead to the same situation. Namely, the initial temperature (reheating temperature $T_R$) cannot be identical to the equilibrium
temperature of the warm phase($T_{eq}$). Therefore, we must consider the non-equilibrium evolution of the temperature before the warm phase. For $T_R > T_{eq}$, the evolution after reheating is very simple, as we have discussed for the thermal phase. However, for $T_R < T_{eq}$, it is not clear if the temperature can increase against red-shifting. We understand that this problem of the initial condition is one of the most confusing aspects of the warm inflation model. Intuitively it seems difficult to start warm inflation from the cold Universe. The expectation considered in Ref.[5] is that initially the dissipation related to the quasiparticle approximation with zero-temperature works to raise the radiation temperature from $T \sim 0$ to $T = T_{eq}$. The non-equilibrium phase will be very short if this expectation is true. In this paper we will not discuss the scenario with $T_R < T_{eq}$.

2.6 Curvature perturbation from the dissipative field

The field $\phi_{2I}$ that rolls slowly during primordial inflation will have the cosmological fluctuation $\delta \phi_{2I}$ that exits horizon during primordial inflation. If the slow-roll during primordial inflation is realized in the warm phase, $\delta \phi_{2I}$ may be enhanced by the temperature $T > H$. Moreover, if the potential of the field $\phi_{2I}$ dominates the energy after reheating, it can cause generation of the cosmological perturbation. If the amplitude of the cosmological perturbations created by the field $\phi_{2I}$ is larger than the one from the primordial inflation, it gives the new source of the cosmological perturbations.

Before discussing the scenario of the dissipative curvatons, it will be better to review the original scenario of the curvatons [14] to show clearly the differences that discriminate our scenario from the original. One of the crucial assumptions in the original curvaton model is that during inflation the Hubble expansion rate is larger than the effective mass of the curvaton field $\varphi$; $H_{1I} \gg m_\varphi$. Then the quantum fluctuations are created for the curvaton field forming a condensate with large VEV, $\varphi_0 \neq 0$. The potential of the curvaton field is negligible during inflation since it is much smaller than the inflaton potential. After inflation (and after reheating), the curvaton can stay at large VEV because of the large Hubble friction. During the radiation-dominated epoch the Hubble parameter scales as $H \propto T^2$. The curvaton follows the usual slow-roll equation of motion until

---

9Some other variations of the curvaton scenario are found in Ref. [15, 16, 17].
$H_{osc} \equiv m_\varphi$, when the curvaton starts oscillating around the origin. The initial amplitude of the oscillation is given by the VEV $\varphi_0$, which is due to the slow-roll assumption for the curvaton field. During oscillation the amplitude $|\varphi|$ is redshifted by the Hubble expansion. Since the oscillation with the quadratic potential behaves like matter with regard to the Hubble expansion rate $H(t)$, the energy of the oscillating curvaton may eventually start dominating over the inflaton decay products. In this scenario the longevity of the curvaton oscillation is crucial for its dominance. For the longevity of the curvaton, it is important that the curvaton does not evaporate due to interactions with the thermal plasma. In the presence of the interactions with the thermal plasma, the oscillation may lead to efficient decay. Therefore, contrary to our scenario, the curvaton in the original scenario must not have interactions with the inflaton decay products. The requirement for the longevity excludes many MSSM flat directions from the curvaton candidate.

According to the inflation paradigm, the curvature perturbation that is sourced by the inflaton perturbation is constant (i.e., inhomogeneous but time-independent) from the end of inflation. In this case the primordial curvature perturbation is given by

$$\zeta = \frac{H}{\phi_{1I}} \delta \phi_{1I}. \quad (2.38)$$

In the curvaton scenario the field responsible for the curvature perturbation can be any field different from $\phi_{1I}$. The curvature perturbation sourced by the curvaton $\varphi$ is given as a function of time by

$$\zeta(k, t)_\varphi = N_\varphi \delta \varphi(k), \quad (2.39)$$

where $N$ is the number of e-folds of expansion from the horizon exit at $k = aH$ to the time when $\zeta$ is evaluated. Here the subscript of $N$ denotes the derivative with respect to the field $\varphi$. For the original (cold) curvaton scenario the spectrum is given by

$$P_\zeta^{1/2} = \left| N_\varphi \frac{H_k}{2\pi} \right|. \quad (2.40)$$

\(^{10}\)In this respect the quadratic potential is crucial for the original curvaton model. The situation is in contrast to the dissipative curvaton model, in which $V \propto \phi_{2I}^n$ is possible.

\(^{11}\)Some MSSM directions can remain long lived \cite{18}. However, if the inflaton decay products have the MSSM quanta, their interactions with the MSSM curvaton may ruin the coherence and the longevity of the MSSM curvatons. In some cases the isocurvature perturbation puts crucial condition for the curvaton model \cite{19, 20}.
where $H_k$ is the Hubble parameter at horizon exit. The assumption of the curvaton scenario is that the curvaton oscillation lasts long in a background of radiation. Usually the curvaton energy density $\rho_\varphi$ is supposed to be negligible until long after the oscillation starts. Quantities associated with the curvaton energy density grows during oscillation since $\rho_\varphi/\rho_R$ is proportional to $a(t)$. This assumption is in contrast to the dissipative curvaton scenario, in which the curvaton energy density grows due to the dissipative slow-roll and decays fast after oscillation begins.\(^{12}\)

To see the growth of the perturbation associated with the curvaton field, it is useful to consider $\zeta_R \simeq 0$ for the radiation from the inflaton decay product and consider the curvature perturbation from the curvaton

$$\zeta_\varphi = \frac{1}{3} \frac{\delta \rho_\varphi}{\rho_\varphi} \simeq \frac{2}{3} \frac{\delta \varphi}{\varphi},$$

where the last equation is evaluated for the quadratic potential. The curvaton is supposed to decay before the cosmological scale enters the horizon. The curvature perturbation at the decay is given by

$$\zeta = \frac{1}{3} \frac{\delta \rho_\varphi}{\rho + P} \equiv r_\varphi \zeta_\varphi,$$  \hspace{1cm} (2.41)

where $\rho$ and $P$ are for the total energy density and the pressure of the Universe.

Next, we consider the curvature perturbation from the original warm inflation scenario. For the warm inflation scenario with $T > H$, the amplitude of the thermal fluctuation of a scalar field will be larger than the quantum fluctuation. The amplitude of the spectrum in the strong dissipative scenario of warm inflation is given by\(^{21}\)

$$P_\zeta \simeq \frac{H_k}{\phi_{1I}} \left[ \frac{4}{\pi} \right]^{1/4} \sqrt{T_k H_k} \simeq \frac{3}{4} \frac{H_k^2 r}{|V_{1I}|} \left[ \frac{4}{\pi} \right]^{1/4} \sqrt{T_k H_k},$$  \hspace{1cm} (2.42)

where $T_k$ is the temperature when the perturbation exit horizon. Again, it is useful to consider the curvaton scenario that is based on the idea that the field responsible for the curvature perturbation can be any field different from $\phi_{1I}$. Here we consider a dissipative field and denote the field by “dissipative curvaton field” $\phi_{2I}$. The dissipative curvaton $\phi_{2I}$ might be one of the fields in the (multi-field) warm inflation model, but in this paper $\phi_{2I}$ is supposed to be a field which has no significant effect on the dynamics of primordial inflation.\(^{13}\)

\(^{12}\)The efficient decay is supposed in the heavy curvaton scenario\(^{14}\). However, the heavy curvaton scenario requires a sudden growth of the curvaton mass (or a phase transition\(^{16,17}\)), which raises the curvaton energy density before the oscillation begins.

\(^{13}\)Evolution of the curvature perturbation in a multi-field warm inflation model is discussed in Ref.\(^{22}\).
curvaton is given as a function of time by

$$\zeta(k,t)_{2I} = N_{\phi_{2I}} \delta \phi(k)_{2I}. \quad (2.43)$$

The assumption of the dissipative curvaton scenario is that the curvaton slow-roll lasts long in a background of radiation. Usually the curvaton energy density $\rho_\phi$ is supposed to be negligible at reheating. The situation is the same in our model, in which the dissipative curvaton energy density $\rho_{\phi_{2I}}$ is supposed to be negligible at reheating. Quantities associated with the curvaton energy density grows during dissipative slow-roll epoch, where $\rho_{\phi_{2I}}/\rho_R$ is proportional to $a^4$. The situation related to the slow-roll is in contrast to the original curvaton scenario, as in the original curvaton model the energy of the curvaton cannot dominate the Universe during the slow-roll period, unless the VEV of the curvaton is very large ($\varphi_0 > M_p$). To see the growth of the perturbation associated with the curvaton field, it is useful to consider that the curvature perturbation associated with the primordial inflation is negligible ($\zeta_R \approx 0$), where $\zeta_R$ crucially depends on the primordial inflation model and can be adjusted without disturbing the curvaton scenario.

For the most significant case, where the curvaton is “warm” during primordial inflation (i.e., when the fluctuation of the curvaton is enhanced by the temperature $T > H$) and then the dissipative inflationary Universe in the thermal phase is connected to warm inflation, the total number of e-folds elapsed during the secondary inflation stage is given by

$$N \simeq N^{th} + N^w. \quad (2.44)$$

Since the temperature decreases rapidly in the thermal phase, we may expect that the variation of $\phi_{2I}$ during the dissipative inflation stage is negligible for the calculation. Then, we have $T_{in}/T_w \propto V^{1/4}\phi_{2I}^{3/4}/V^{3/8} \propto \phi_{2I}^{(6-n)/8}$ for the dissipative inflation period. Using these equations, $N_{\phi_{2I}}$ for the fluctuation of $\phi_{2I}^{(ini)}$ is evaluated as

$$N_{\phi_{2I}} \simeq \begin{bmatrix} 6 - n & 1 \\ 8 & \phi_{2I} \end{bmatrix} + \begin{bmatrix} C_2 \\ n \sqrt{\lambda_n} M_p \end{bmatrix} \left( \frac{\phi_{2I}}{M_p} \right)^{(4-n)/2}$$

$$\simeq \begin{bmatrix} 6 - n & 1 \\ 8 & \phi_{2I} \end{bmatrix} + \begin{bmatrix} C_2 \phi_{2I}^2 \\ n \sqrt{V} M_p \end{bmatrix}.$$ \quad (2.45)

Other applications and extensions of warm inflation scenario with regard to the multi-field inflationary model are found in Ref. [23].
which leads to the spectrum\textsuperscript{14}

\[ P_{\zeta}^{1/2}(k,t)_{2I} = \left[ \frac{6-n}{8} \frac{1}{\phi_{2I}} + \frac{C_2\phi_{2I}^2}{n\sqrt{V}M_p} \right] \left[ \left( \frac{\pi r}{4} \right)^{1/4} \sqrt{T_k H_k} \right]. \]  

(2.46)

Here significant interaction between the thermal plasma and $\phi_{2I}$ is assumed when we calculate the amplitude of the dissipative curvaton for $T_k > H_k$. Eq. (2.34) suggests that the contribution from the warm phase (the second term) is significant for the scenario.

### 2.7 Spectral index

Before calculating the spectral index of the model, it will be useful to show the basic equations for the rate of change of various parameters that are given by\textsuperscript{21}

\[
\frac{1}{H} \frac{d}{dt} \ln H = -\frac{1}{r} \epsilon, \quad (2.47)
\]

\[
\frac{1}{H} \frac{d}{dt} \ln T = -\frac{1}{4r} (2\eta - \beta - \epsilon), \quad (2.48)
\]

\[
\frac{1}{H} \frac{d}{dt} \ln \phi = -\frac{1}{r} (\eta - \beta), \quad (2.49)
\]

\[
\frac{1}{H} \frac{d}{dt} \ln \Upsilon = -\frac{1}{r \beta}, \quad (2.50)
\]

where $\beta \equiv \frac{1}{M_p^2} \frac{T_{eV}}{V}$ is the slow-roll parameter defined for the warm phase. Variation of $T$ is important when the perturbation $\delta\phi_{2I}$ that exits horizon is enhanced by the thermal effect. (i.e., when the amplitude of $\delta\phi_{2I}$ is given by $\left[ \left( \frac{\pi r}{4} \right)^{1/4} \sqrt{T_H} \right]$.) The enhancement occurs when the primary inflation is warm and the interactions of the field $\phi_{2I}$ with the radiation is significant. As in the original curvaton model, the equation for the perturbation $\delta\phi_{2I}$ gives

\[
\ddot{\delta\phi}_{2I} + 3H(1 + r_{2I})\dot{\delta\phi}_{2I} + V_{\phi_{2I}\phi_{2I}} = 0, \quad (2.51)
\]

where $r_{2I}$ is for the dissipative motion of the field $\phi_{2I}$. The equation leads to the variation of the perturbation

\[
\delta\phi_{2I} \propto \exp \left[ -\frac{V_{\phi_{2I}\phi_{2I}}}{3H^2(1 + r_{2I})} \Delta N \right]. \quad (2.52)
\]

\textsuperscript{14}See Ref.\textsuperscript{2} and Ref.\textsuperscript{3} for more discussions for generation of the curvature perturbation caused by the inhomogeneous phase transition. The idea of inhomogeneous phase transition is a generalization of precedent ideas of inhomogeneous preheating\textsuperscript{21}, inhomogeneous end of inflation\textsuperscript{25-28} and modulated scenarios of inflation\textsuperscript{25-29}.

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where $\Delta N$ is the number of e-foldings elapsed after horizon exit. Considering these equations, we find the spectral index for the curvature perturbation created in the thermal phase is given by

$$n_s - 1 = \left[ -\frac{1}{r_{1I}} \left( \frac{\epsilon_{1I}}{4} + \frac{\eta_{1I}}{2} + \frac{\beta_{1I}}{4} \right) + \frac{2}{r_{2I}} \eta_{2I} \right]_k ,$$

(2.53)

which is obtained with the assumption that primary inflation is warm and the interactions with $\phi_{2I}$ are significant. We also assumed that $\phi_{2I}$ is strongly dissipative ($r_{2I} > 1$) during primordial inflation. The subscript $k$ is for the value of the parameters at the horizon exit. If primary inflation is cold but the curvaton is dissipative, we find

$$n_s - 1 = \left[ -\frac{2}{r_{1I}} \epsilon_{1I} - \frac{2}{r_{2I}} \eta_{2I} \right]_k .$$

(2.54)

Note that the condition for the warm phase is not identical to the condition for the dissipative motion. The spectral index of the curvature perturbation created in the warm phase of dissipative inflation is different from the conventional warm inflation scenario. The crucial discrimination appears because the curvature perturbation is created by $\delta N^w \approx H \delta t \approx H \delta \phi_{2I}/\dot{\phi}_{2I}$, where $\delta \phi_{2I}$ is created at horizon exit during primordial inflation but $H/\dot{\phi}_{2I}$ is the value at the beginning of the warm phase. For the warm phase of dissipative inflation, the spectral index is thus given by

$$n_s - 1 = \left[ -\frac{1}{r_{1I}} \left( \frac{\epsilon_{1I}}{4} + \frac{\eta_{1I}}{2} + \frac{\beta_{1I}}{4} \right) + \frac{2}{r_{2I}} \eta_{2I} \right] - \frac{1}{r_{2I}} (2\epsilon_{2I} + 2\beta_{2I}) .$$

(2.55)

where the second term comes from $(H/\dot{\phi}_{2I})^2$ in the curvature perturbation and it is estimated at the beginning of the warm phase. Since the evolution of $\delta \phi_{2I}$ is considered separately in the above formula, we dropped $2\eta_{2I}$ in the second term. Results are shown in Table II for comparison, where both non-dissipative(ND) and dissipative cases are considered for $\phi_{2I}$. In Table 1 we considered the specific case in which $\phi_{2I}$ interacts with the background radiation sourced by $\phi_{1I}$ during primordial warm inflation. The criteria for significant dissipation ($r_{2I} > 1$) and enhancement of the amplitude $\delta \phi_{2I}$ during

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*Note however that for the evolution of the inflationary curvature perturbation the time-dependence is usually considered for $\zeta$. Eq.(2.52) is thus important for the curvaton scenario but it is not appropriate to consider separately the evolution of $\delta \phi_{1I}$ when one calculates the spectral index of $\zeta$. This distinguishes the curvaton scenario from the standard inflation model.*
primordial warm inflation may not be identical. Therefore, we considered as a possibility (New scenario 1) that there is a region where $\delta \phi_{2I}$ is enhanced by the radiation but the curvaton field is not dissipating ($r_{2I} < 1$) both during and after inflation. This scenario is not essential for the model.

### 3 Conclusions and discussions

In this paper we considered the cosmological scenario of the dissipative curvaton field. Strong dissipation ($r > 1$) for the curvaton field may lead to interesting possibility that the ratio of the curvaton energy density may grow without significant oscillation period after reheating. Moreover, the field fluctuations of the dissipative field may be enhanced by the warm background during primordial inflation. We investigated the possibility that these effects may alter the usual cosmological scenarios associated with the curvaton. The significant difference is that the dissipative curvaton can dominate the Universe before the curvaton begins oscillation. The dissipative slow-roll should be considered in two distinctive “phases” for the radiation; thermal phase where the radiation scales as $\rho_R \propto a^{-4}$ and the warm phase where the radiation is sustained by the dissipation. The inflationary expansion that is expected in the thermal phase is similar to thermal inflation, but the crucial difference is that symmetry restoration is not assumed for the dissipative inflation scenario.

Sometimes the (original) curvaton must be hidden from the inflaton decay products in order to avoid thermal evaporation. The situation is in contrast to the dissipative curvaton scenario, in which the interactions with the plasma causes strong dissipation for the curvaton and leads to the prolonged slow-roll that may be connected to the secondary

|          | Cold primary inflation | Warm primary inflation |
|----------|------------------------|------------------------|
| ND $\phi_{2I}$ | $[2\eta_{2I} - 2\epsilon_{1I}]_k$ | $[2\eta_{2I} - \frac{1}{r_{2I}} (\frac{\omega}{4} + \frac{m^2}{2} + \frac{2\beta_{2I}}{4})]_k$ |
| Standard Curvatons | | New scenario 1 |
| Dissipative $\phi_{2I}$ | $\left[\frac{2\eta_{2I}}{r_{2I}} - 2\epsilon_{1I}\right]_k - \frac{1}{r_{2I}} (2\epsilon_{2I} + 2\beta_{2I})$. | Eq. (2.55) |
| New scenario 2 | | New scenario 3 |

Table 1: Spectral index
We find that the spectral index is distinctive in the dissipative curvaton model. There is a benefit of the dissipative curvaton scenario. In contrast to the original curvaton model, quadratic potential is not needed for the curvaton field, since the growth in the oscillating period is not essential for the dissipative model.

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A Dissipative motion with $\gamma_2$

The most interesting aspect of the scenario may rely on the dissipation coefficient $\gamma_2$, which appears due to the zero-temperature excitation and the decay of the mass-changing heavy field $\chi$. It will therefore useful to illuminate the issue of warm inflation and the dissipative motion with regard to the coefficient $\gamma_2$. In this section we try to show intuitive and physical explanations of why the dissipative scenario is possible with $\gamma_2$.

The most important point that we should first mention in this appendix will be that basically the thermalization of the decay product is not essential for the dissipative motion with $\gamma_2$. This discriminates our scenario from the conventional warm inflation scenario, in which thermalization is necessary because the definition of the warm inflation scenario is given by $T > H$. In the scenario of dissipative curvatons the Universe is simply in the thermal phase if thermalization delays, and (in contrast to warm inflation) it does not mean that the scenario is not conceivable.

Perhaps the most confusing aspect of the warm inflation scenario would be that intuitively it seems difficult to realize significant radiation ($T > H$) during inflation. On the other hand, since the mass of the intermediate field $\chi$ is changing during inflation, one intuitively knows that excitation of the field is inevitable. However, it is not trivial to calculate the effect of the excitation and the decay with regard to the inflaton motion. Therefore, calculating the effect of the excitation mode and its decay will be the primary
challenge in this direction. For instance, much work has been devoted to this problem in terms of analytic approximations \[5, 6, 7\] to show that the dissipative equation motion is indeed appropriate for the consideration of the possible dissipation during inflation. There are works based on numerical methods \[8\], which show that the approximations considered in the analytic calculations will be correct for the model. To be specific of the quasiparticle approximation, the issue is considered in papers in Ref. \[5, 6\], which show that dissipative force in the zero-temperature background appears in the dissipative field motion with the form of Eq. (2.1). Therefore, the dissipative slow-roll in the thermal phase is compelling from the past analyses with regard to the dissipative motion in the zero-temperature and the thermal background. Then the difficult aspect of warm inflationary model would be in the realization of the warm phase, which depends crucially on the thermalization process in the light field sector that includes the light fermion $\psi$. In fact, the important “assumption” in the conventional warm inflation scenario would be that thermalization is so efficient that the radiation from the decay product sources continuously the radiation density to prevent the radiation from red-shifting. Here, what is important in discussing thermalization in the warm phase is the relaxation time of the radiation, which is basically independent of the interactions related to the dissipative motion. Therefore, thermalization is usually “assumed” in the warm inflation scenario without mentioning the specific interactions in the light-field sector, which can be changed without modifying the interactions related to the dissipative motion. For more arguments of the thermalization, see Ref. \[29\].

Although the relaxation time of the radiation is the important issue for realizing the warm phase, it would be modest if we assume in this paper that thermalization is so efficient that the dissipation can source radiation in the warm phase. If the thermalization delays, only the thermal phase appears for the dissipative curvaton. In this case the warm phase can be neglected from the discussions. The dissipative slow-roll ends at

$$\phi_{2I} \simeq \phi_{2I}^{(c)} \equiv \left( \frac{n^2 \sqrt{3M_p V^{1/2}}}{4C_2} \right)^{1/3},$$

(A.1)

which determines $\rho_{\phi_{2I}}$ and the temperature after the secondary reheating caused by the curvaton decay. Since the interactions of the dissipative curvaton are significant, it leads to the secondary reheating temperature $T_{R2} \simeq (\rho_{\phi_{2I}}/C_T)^{1/4}$. In this paper we did not
assume quadratic potential for the curvatons, but if we assume $V(\phi_{2I}) \approx m^2\phi_{2I}^2$, we find

$$\left(\phi_{2I}^{(c)}\right)^2 \approx \frac{n^2\sqrt{3}mM_p}{4C_2}$$

(A.2)

and

$$\rho_{\phi_{2I}} \approx \frac{n^2\sqrt{3}m^3M_p}{4C_2}$$

(A.3)

which leads to the secondary reheating temperature

$$T_{R2} \approx \left(\frac{n^2\sqrt{3}m^3M_p}{4C_2C_T}\right)^{1/4}$$

(A.4)

which is obtained for the specific case of the dissipative curvaton for $\Upsilon_2 > \Upsilon_1$ with quadratic potential and without thermalization.

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