Solution of two-phase nonisothermal fluid flow problem with nonlinear filtration law on heterogeneous computing systems

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Abstract. The work is devoted to the methods of solving two-phase fluid flow problems in the reservoirs. It is assumed that in domain with high velocities Darcy’s law is violated and a nonlinear filtration law is used. To determine the temperature use the law of conservation of energy (first law of thermodynamics). The temperature of the fluid and of the skeleton are considered the same. The fluid viscosities are taken by the temperature functions. The methods for solving two-phase flow problems in porous medium with a nonlinear filtration law based on the decomposition methods are proposed. The proposed methods are implemented on the heterogeneous computing systems.

1. Introduction
Mathematical models of fluid flow in porous media are systems of coupled nonlinear unsteady partial differential equations [1], [2]. An important equation in these systems is the law of filtration - the dependence between fluid velocity and pressure. Most fluid flows are described by the Darcy’s filtration law, which expresses a linear dependence between velocity and pressure gradient. A lot of works are devoted to the verification and study of the limits of the applicability of the Darcy’s law. In the course of these studies, it is shown that there are two main reasons for the deviation from the Darcy’s law. The Darcy’s law is violated in cases when there are high flow velocities (the upper limit of the applicability of the Darcy’s law) or too low flow velocities caused by the emergence of non-Newtonian rheological properties of the liquid [3]. In this paper we consider the first case related to the action of inertial forces.

In the development of oil and gas reservoirs the fluids contained in them can reach a temperature different from the reservoir temperature. The temperature change in the reservoir can occur due to the action of thermodynamic effects during the movement of fluids in a porous medium [3]: the barothermal effect, the effect of phase transformations, the injection into the reservoir of various displacers with a temperature different from the initial (cold or hot water, steam), the implementation of various thermochemical oxidation processes. In this paper, the model problem of hot water injection into the reservoir was considered. To determine the temperature used the law of conservation of energy (first law of thermodynamics). The temperature of the fluid and of the skeleton was considered to be the same. The viscosity of the fluid was taken as functions of temperature. The solution algorithm based on the decomposition
2. Problem statement

We will consider the three-dimensional confined reservoir $D$. In case of violation of the Darcy’s law, the flow is subject to the nonlinear Forchheimer’s law \([3]\\): $q_\alpha = -(A_\alpha + Bq_\alpha)^{-1}\text{grad } p$, where $\alpha = \{o, w\}$ is the index corresponding to oil or water, $A_\alpha = 1/K_\alpha$, $K_\alpha = kf_\alpha/\mu_\alpha$ is the phase permeabilities, $f_\alpha$ is the relative permeabilities, $k$ is the absolute permeability, $\mu_\alpha$ is the viscosities, $q_\alpha$ is the module of the fluid velocity vector, $B$ is the constant of the porous medium.

For the solution domain $D$ introduce the following notations: $\bar{D} = \bar{D}_0 \cup \bigcup_{i=1}^{N} \bar{D}_i$, $D_i \cap D_j = \emptyset$, $i \neq j$, $\bar{D}_0 \cap \bar{D}_i = \gamma_i$, where $D_0$ is the domain outside of the well region, $D_i$ is the well region. It is required to determine the pressure $p$, the temperature $T$ and the saturations $S_\alpha$ from the system of equations:

$$\nabla \left( q_w + q_o \right),$$

$$q_\alpha = -(A_\alpha + Bq_\alpha)^{-1}\text{grad } p \quad \text{in} \quad G_{i\alpha}, \quad l = 1, \ldots, N, \quad \alpha = \{o, w\},$$

$$q_\alpha = -K_\alpha \text{grad } p \quad \text{in} \quad D \setminus \bigcup_{l=1}^{N} G_{i\alpha}, \quad \alpha = \{o, w\},$$

$$\frac{\partial}{\partial t} \left( \sum_{\alpha} \rho_\alpha S_\alpha U_\alpha + (1-m)\rho_s C_s T \right) + \nabla \left( \sum_{\alpha} \rho_\alpha q_\alpha H_\alpha \right) - \nabla (\kappa T \text{grad } T) = 0,$$

$$\sum_{\alpha} S_\alpha = 1,$$

$$\nabla (q_w) + m \frac{\partial (S_w)}{\partial t} = 0.$$ (6)

Boundary conditions:

$$p = p_T \quad \text{on} \quad \Gamma_1, \quad q_w = q_{\Gamma_w} \quad \text{on} \quad \Gamma_2, \quad p|_{\partial V_l} = P_l, \quad l = 1, \ldots, N,$$

$$T = T_{w_l}, \quad l = 1, \ldots, M, \quad T = T_T \quad \text{on} \quad \Gamma,$$

$$S = S_{w_l}, \quad l = 1, \ldots, M, \quad S = S_w \quad \text{on} \quad \Gamma_3.$$ (9)

Initial conditions:

$$T = T^0 \quad \text{in} \quad D,$$

$$S = S_w^0 \quad \text{in} \quad D.$$ (11)

Here $\rho_\alpha$ is the densities of phase $\alpha$, $U_\alpha = C_\alpha T$ is the internal energies of phase $\alpha$, $C_\alpha$ is the phases heat capacity, $\rho_s$ the density of the skeleton, $C_s$ is the heat capacity of the reservoir, $H_\alpha = U_\alpha + p/\rho_\alpha$ is the enthalpies of the phases, $\kappa_T$ is the thermal conductivity of the reservoir, $\Gamma_1 + \Gamma_2 = \Gamma$ is the external boundary surface of the domain $D$, $\Gamma_3$ is the part of the surface $\Gamma$ through which the liquid enters the reservoir, $\partial V_l$ is the surface interval of opening of the $l$-th well, $P_l$ is the set pressure on the well, $p_T$ is the set pressure on the boundary $\Gamma_1$, $G_{i\alpha} = \{x \in D_i | Re(q_{\alpha}) > Re_{cr}\}$ is the domains with high flow rates, $q_{\alpha n}$ is the set value of the normal component of the fluid velocity at the boundary $\Gamma_2$, $N$ is the number of the wells, $M$ is the number of the injection wells ($M < N$), $S_{w_l}$ is the set value saturation on the injection wells. The boundaries of domains $G_{i\alpha}$ are not known in advance and must be determined in the decision process. In the problem statement $G_{i\alpha} \subset D_i$. The problem will be solved numerically with a given error $\epsilon$ on a grid that condenses to the well opening intervals. The reservoir $D$ is represented by a multi-connected region, the inner surfaces of which are determined by the surfaces of wells in the intervals of reservoir opening.
3. The solution methods

At each time step the systems of equations to determine the pressure $p$, temperature $T$ and the systems of equations to determine saturation $S_{\alpha}$ are solved separately. The nonlinear problem (1)-(4), (7), (8), (10) by definition the pressure $p$ and the temperature $T$ is solved iteratively. In the first iteration, the functions $p^1$ and $T^1$ are defined from the system of equations:

$$
\begin{align*}
\text{div}(\mathbf{q}_w^0 + \mathbf{q}_d^0),
\mathbf{q}_w^0 &= -K_\alpha \text{grad } p^1, \\
\frac{\partial}{\partial t} \left( \sum_\alpha \rho_\alpha S_\alpha U_\alpha (T^1) + (1 - m) \rho_s C_s T^1 \right) + \text{div} \left( \sum_\alpha \rho_\alpha \mathbf{q}_\alpha^0 H_\alpha (p^1, T^1) \right) - \text{div}(\kappa T \text{grad } (T^1)) &= 0,
\end{align*}
$$

with the conditions (7), (8), (10). Next, the module of fluid velocity vector $q^1_\alpha$ on each element is calculated by the formula $q^1_\alpha = |K_\alpha \text{grad } p^1|$. Subdomains $G^1_\alpha \subset D_\alpha$ are defined by the elements on which the condition $Re(q^1_\alpha) > Re_{cr}$ is satisfied. Let the functions $p^1$, $q^1_\alpha$, $T^1$ and the subdomains $G^1_\alpha$ be known. The functions $p^{i+1}$ and $T^{i+1}$ are determined from the solution of the equations:

$$
\begin{align*}
\text{div}(F_\alpha^1 + F_d^1) \text{grad } p^{i+1} &= 0, \quad \text{in } D, \\
\mathbf{q}_d^1 &= -F_d^1 \text{grad } p^{i+1}, \quad \text{in } D, \quad \alpha = \{o, w\}, \\
\frac{\partial}{\partial t} \left( \sum_\alpha \rho_\alpha S_\alpha U_\alpha + (1 - m) \rho_s C_s T^{i+1} \right) + \text{div} \left( \sum_\alpha \rho_\alpha \mathbf{q}_\alpha^i H_\alpha \right) - \text{div}(\kappa T \text{grad } T^{i+1}) &= 0,
\end{align*}
$$

where $U = U(T^{i+1})$, $H = H(p^{i+1}, T^{i+1})$, $\mathbf{q}_d^1 = -K_\alpha \text{grad } p^{i+1}$ on the elements in the subdomains $D \setminus \bigcup l=1^{N} G^1_\alpha$ and $\mathbf{q}_d^1 = -(A_\alpha + B_d^1)^{-1} \text{grad } p^{i+1}$ on the elements in the subdomains $G^1_\alpha$. The module of the flow velocity vector $q^1_\alpha$ is defined by equality $q^1_\alpha = |K_\alpha \text{grad } p^{i+1}|$ in the domain $D \setminus \bigcup l=1^{N} G^1_\alpha$. On the elements of the grid of the domains $G^1_\alpha$ the module of the fluid velocity is defined from the condition $|\text{grad } p^{i+1}| = (A_\alpha + B_d^1)^{q^1_\alpha}. The subdomains $G'^1_\alpha$ are defined by the elements on which the condition $Re(q'^1_\alpha) > Re_{cr}$. The system of equations (1)-(4), (7), (8), (10) to determine the pressure and temperature is solved, if the system (12)-(14) with the conditions (7), (8), (10) is solved with a given tolerance $\epsilon$ and $G'^1_\alpha = G^1_\alpha$.

To determine the saturations $S_{\alpha}$ is solved a system consisting of equations (5), (6), (9), (11), in which the pressure $p$, the temperature $T$ and the domains with high flow rates $G^1_\alpha$ are considered constant values. Approximation of the system is carried out by the method of control volumes [7] with an implicit scheme of time “upflow”.

Two different decomposition methods were used to solve the problem of two-phase fluid flow without taking into account high flow rates in [4], [5]. The decomposition method for solving difference equations for determining the pressure based on the independent solution of systems of algebraic equations for the fine grid segments in the subdomains $D_\alpha$ and a new type of coupling these solutions with the solution on the coarse grid. To take into account the high flow rates in the proposed decomposition method, each system for the fine grid segments must be solved by the algorithm described above. The decomposition method for determining the saturation field is based on a combination of elements of explicit and implicit schemes [8].

4. Numerical results

The proposed algorithm was tested in solving of two-phase flow problems with a different number of vertical producing and injection wells. The reservoir consisted of 10 layers (1 km × 1 km × 0.018 km) with the layers thicknesses $d_1 = 1$ m, $d_2 = 1$ m, $d_3 = 3$ m, $d_4 = 1$ m, $d_5 = 1$ m, $d_6 = 1$ m, $d_7 = 2$ m, $d_8 = 1$ m, $d_9 = 2$ m, $d_{10} = 5$ m and absolute permeability’s $k_1 = 10^{-3}$ darcy, $k_2 = 10^{-2}$
darcy, \( k_3 = 25 \times 10^{-3} \) darcy, \( k_4 = 10^{-2} \) darcy, \( k_5 = 10^{-3} \) darcy, \( k_6 = 10^{-2} \) darcy, \( k_7 = 5 \times 10^{-2} \) darcy, \( k_8 = 10^{-2} \) darcy, \( k_9 = 10^{-3} \) darcy, \( k_{10} = 15 \times 10^{-5} \) darcy respectively. On the top of the reservoir was considered impermeable, saturation on the side surface \( S_w = 0 \), saturation on the bottom surface \( S_o = 0 \). The initial saturation \( S_o = 1 \). The viscosity of the oil at a given temperature can be estimated by knowing the viscosity at two other temperatures \( \mu_o = \mu_1 \exp\left[\left(1 - T_1 / T\right)/(T_2 / (T_2 - T_1))\right] \ln(\mu_2 / \mu_1) \). The water viscosity is given by the Bingham formula \( \mu_w = (0.021482(T - 8.435) + (80784 + (T - 8.435)^2)^{0.5})^{-1} \). Oil density \( \rho_o = 0.882 \) g/cm\(^3\), water density \( \rho_w = 1 \) g/cm\(^3\). Specific heat capacity \( C_w = 4.182 kJ/(kg K) \), \( C_o = 0.92 kJ/(kg K) \). Thermal conductivity of the reservoir \( \kappa_T = 1.7 W/(m K) \), initial temperature \( T^0 = 300 K \). Each autopsy interval was simulated in a circular cylinder with base radius \( r = 0.1 \) m and is closed at the top and bottom spherical surfaces of radius \( r = 0.1 \) m. Thus, for each point on the surface of the opening intervals of the normal vector uniquely determined. Due to the lack of experimental data, the values of the constant \( B \) were chosen from the condition of proximity of the modules of flow velocity \( q_o \), calculated using Darcy’s laws and the Forchheimer’s law in the transition from the linear to the nonlinear law. To calculate Reynolds numbers we used the formula [3] \( Re(q) = 4\sqrt{\kappa \rho q_0/(m^{1.5}) \mu} \), where \( \kappa = k \mu / (\rho g) \), \( \mu \) is the viscosity, \( \rho \) is the density, \( g \) is the acceleration of gravity, \( m \) is the porosity. For various porous media \( Re_{cr} \) according to [3] lies within 0.0085–3.4. Two types of solutions were considered: the solution of the system without domain decomposition and the solution with domain decomposition. As previously noted, the coefficient \( B \) was chosen in such a way that the velocities calculated using Darcy and Forchheimer laws were close at the boundary of the transition from the linear law to the nonlinear one. Numerical experiments were carried out in cases where these velocities differed by 1%, 0.1%, 0.01%. For each of these cases, table 1 shows the values of the coefficient \( B \), at which numerical testing was carried out.

| \( \Delta q(\%) \) | \( Re_{cr} = 0.3 \) | \( Re_{cr} = 0.01 \) |
|-------------------|----------------|------------------|
| 1%                | \( 2.07 \times 10^{-5} \) | \( 6.22 \times 10^{-4} \) |
| 0.1%              | \( 2.05 \times 10^{-6} \) | \( 6.17 \times 10^{-5} \) |
| 0.01%             | \( 2.05 \times 10^{-7} \) | \( 6.16 \times 10^{-6} \) |

In table 2 the time of solving problems with different number of wells by methods without decomposition and with decomposition with Reynolds number \( Re_{cr} = 0.3 \) is given. The results of the solution showed the superiority in time of the speed of the solution of decomposition methods in comparison with the methods without decomposition.

The proposed algorithm was tested on a cluster consisting of 4-core computing nodes with Intel Core i7 2600 processors and equipped with NVIDIA GTX 560 TI graphics accelerators when solving a model three-dimensional problem of three-phase fluid flow with vertical production and injection wells. Each well area contained about 25000 nodes. The total number of nodes for the gathering of 200 wells was \( 5 \times 10^6 \). Systems of nonlinear equations for determining the saturations were solved by the implicit Newton method [9]. The algorithms are parallelized using MPI processes, OpenMP and CUDA technologies. We used C++ language and Visual Studio 2008 application development environment. When solving the problem, each MPI process is allocated an equal number of thickening portions of the grid. To solve problems in the thickening areas
corresponding to a single MPI process, streams are generated using OpenMP technology, which distribute these tasks to the processor cores and graphics devices. In this case, the tasks are distributed dynamically, that is, as they are solved. The problem was solved on two computing nodes with a total number of 8 cores (4 cores per node) and 2 GPU accelerators. The tasks were started in the mode of one MPI process per node, each MPI process corresponds to one GPU device. The following options for running tasks were considered: 1) On all available cores using a single MPI process and without using GPU accelerators. 2) On two cores using two MPI processes and two GPU devices. In this case, each MPI process had one GPU device and one core. 3) On all available cores using MPI processes and GPU devices. In this case, all available cluster resources (2 GPUs and 8 cores) were used.

The tables 3–4 shows the results of the solution. The comparison was made with the solution of the problem on one core without using GPU devices.

Table 2. Solution time with decomposition and without decomposition.

| Number of fine grid segments | Number of nodes | Solution time without decomposition (min) | Solution time with decomposition (min) |
|-----------------------------|----------------|------------------------------------------|----------------------------------------|
| 1                           | 11744          | 7                                        | 6                                      |
| 50                          | 365524         | 301                                      | 147                                    |
| 100                         | 726721         | 722                                      | 523                                    |

Table 3. Solution time acceleration without GPU devices.

| Number of cores | Acceleration |
|-----------------|--------------|
| 1               | 1            |
| 2               | 1.3          |
| 3               | 1.9          |
| 4               | 2.3          |

Table 4. Solution time acceleration with GPU devices.

| Number of GPU devices | 1 MPI(1 core) | 1 MPI(4 core) | 2 MPI(2 core) | 2 MPI(8 core) |
|-----------------------|---------------|---------------|---------------|---------------|
| 1                     | 17.3          | 20.2          | –             | –             |
| 2                     | –             | –             | 30.1          | 36.3          |

The table 4 shows the acceleration of the problem solution time when GPU devices was used. The efficiency of use of GPU devices reaches 80%.
5. Conclusion
The methods to solve the two-phase nonisothermal fluid flow problem with fine grid segments based on the methods of decomposition are constructed. In the case of high flow velocities (the Reynolds number is greater than the critical value), the nonlinear Forchheimer law was applied. Algorithms for solving the problem on heterogeneous computing systems are constructed on the basis of the proposed decomposition methods. The high efficiency of using multiprocessor systems built on the basis of graphics processors is shown.

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