Research Article
Timing Evaluation of Telecom Investment Decision under Demand Uncertainty

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Under uncertain conditions, the problem of selecting investment timing for telecom network optimization and expansion projects is common in the investment practice of telecom operators. By examining the characteristics of the demand and price of the telecommunications industry, combined with historical data of the telecommunications industry, an exponential function relationship of prices meeting demand is proposed, and then an investment timing decision model under the condition of uncertain demand is established on this basis. Studies have shown that when the value of the investment project is 1.5634 times the investment cost, the value of the investment option is the largest, and this is the optimal investment opportunity. The value of the investment opportunity $F(V)$, the value of the project $V^*$, the degree of demand uncertainty $\sigma$, and discount rate $\rho$ are positively correlated, and it is negatively correlated with the opportunity cost $\delta$ of delaying project investment and keeping investment options alive.

1. Introduction
With the advent of the 5G era, the requirements for enhancing network communication capabilities or related business needs continue to grow; network update, optimization, and expansion will inevitably become a major investment project in the telecommunications industry. A large amount of literature, such as Murto & Keoop 2002; Small 1999; Pindyck 2001; Qiu Xinping & Lu Tingjie 2006, believe that demand uncertainty is at the core of the risk analysis of project investment evaluation. Investment fluctuations in the telecommunications industry are positively correlated with fluctuations in telecommunications demand. Approximately 70% of changes in the telecommunication industry investment are related to changes in telecommunications demand. When telecommunication requirements change, telecommunication operators will first respond as quickly as possible through technological transformation and expansion on the original network and the expansion of communication networks. At the same time, the uncertainty of the demand for telecom services has led to uncertainty about the value of investment network expansion projects. Therefore, under the influence of random uncertain factors in demand, how to choose the investment opportunity to make the project put into production and how to maximize the profits of telecom operators are the urgent problems to be solved.

2. Basic Assumptions
In order to meet the growing demand for telecommunication services, a telecommunication operator needs to make a telecommunication network capacity expansion investment decision. It is considering that the cost of further adjustment will be huge, once the total designed network capacity is completed, although the telecom network expansion and construction can be completed in stages. Therefore, the research of this article is limited to such a situation that regardless of whether or not to invest and regardless of when to invest, the telecom operator is only allowed to invest once, that is to say, the telecom operator only has one expansion option. It can choose whether to
execute or not and when to execute this option, but once they choose to invest, it means that it will lose the opportunity and possibility to implement the option again. Telecommunications operators can exercise the option at any time, so their investment opportunities are equivalent to American call options.

3. Demand Uncertainty and Its Mathematical Expression

Geometric Brownian motion is suitable for describing random walk paths with growth or decay characteristics, which is in line with the demand characteristics of the telecommunications whose industry-initial demand is slow. After a period of time, due to the economic development levels, network effects, transfer costs, etc., its explosive growth of demand is far from its initial demand, and its fluctuations are random.

On the other hand, because various forms of asset replication portfolios can be obtained in an efficient market, regardless of whether it is delta hedging or measurement transformation, the drift of the random process can be eliminated, and the final result will not be affected [1]. Many empirical results studies also show, such as Pindyck 1998, 1999, 2001; Baker, Mayfield & Paragon, 1998; Schwartz & Smith, 2000, that the use of geometric Brownian motion is unlikely to lead to large decision errors. Therefore, this article follows the assumptions and derivations of McDoncl & Siegel (1986) and Dixit & Pindyck (1994), assuming that the demand for Q_t at time t obeys the geometric Brownian motion process given by the following differential equation:

$$dQ_t = aQ_t dt + \sigma Q_t dz.$$  (1)

In formula (1), a is the drift coefficient, which represents the expected return of Q_t, and σ is the variance parameter describing the degree of uncertainty of demand changes, which represents the volatility of demand Q_t. Dz is the increment of the standard Wiener process, which independently obeys a normal distribution with a mean value of 0 and a variance of dt, namely dz ~ N(0, dt).

4. The Optimal Investment Timing Decision

I (>0) is the sunk cost of network optimization and expansion project investment, that is, the cost of executing the option. Considering that the size of variable costs in the telecommunication industry is relatively small compared to fixed costs, variable costs are ignored, and I is a fixed value. The return from investment at any time t is the value of the project minus the cost, that is, \( F(V) \) represents the value of the investment opportunity of the network expansion project (that is, the value of the investment option of the network expansion project). Therefore, under the condition of random and uncertain demand, the investment decision faced by operators is to choose the investment timing to maximize the expected present value of their investment return:

$$F(V, t) = \max E[(V_t - I)e^{-\rho t}].$$  (2)

In formula (2), t is the (unknown) future time in which the investment is made and ρ is the discount rate per unit time.

Since the project value V is equal to the product of the price P and the demand Q, and generally speaking, the price is related to the demand to a certain extent, so we need to examine the inverse demand function. Much literature studies, such as Murto, 2000; Gencetal, 2003; Murphy & Smeers, 2002, express the price P as a linear function of demand Q (linear inverse demand function): \( P = a - bQ \).

This article believes that telecommunication has a real-time balance of supply and demand output characteristics, and its price demand has its own characteristics. According to the relationship between telecommunication service prices and demand over the years, it can be seen that telecommunication service prices P and demand Q show an exponential function relationship. Based on the above analysis, we assume that the telecommunication output price \( P_t \) at time t satisfies (inverse demand function):

$$P_t = aQ_t^{\epsilon}.$$  (3)

Among them, \( \epsilon \) is the price elasticity of demand and satisfies \( 0 < \epsilon < 1 \). It is because that through 40 years of development in my country’s telecommunication industry, voice services provided by telecom operators have now become a necessity in people’s lives. It can be concluded that the value of the project at time t is

$$V_t = P_tQ_t = aQ_t^{1-\epsilon}.$$  (4)

Using Ito’s lemma to expand d\( V_t \), we get

$$dV_t = \left( \frac{\partial V_t}{\partial t}aQ_t + \frac{1}{2} \frac{\partial^2 V_t}{\partial Q_t^2} \sigma^2 \right) dt + \frac{\partial V_t}{\partial Q_t} \sigma Q_t dz = (1 - \epsilon) \left( a - \frac{\sigma^2 \epsilon}{2} \right) V_t dt + \sigma V_t dz.$$  (5)

Therefore, it can be seen that the random characteristic of the item value \( V_t \) at time t also conforms to the geometric Brownian motion process. The drift coefficient is \( (1 - \epsilon) (a - \sigma^2 \epsilon / 2) \), and the fluctuation rate of \( V_t \) is \( (1 - \epsilon) \sigma \). The dynamic programming method is used to write the following Bellman optimization equation:

$$F(V, t) = P_tQ_t dt + \frac{1}{1 + \rho dt} E[F(V + dV)].$$  (6)

It should be noted here that \( \rho > (1 - \epsilon)(a - \sigma^2 \epsilon / 2) > 0 \), which is very important. If \( \delta = \rho - (1 - \epsilon)(a - \sigma^2 \epsilon / 2) \delta < 0 \), then choose a larger \( t \), the integral in (6) may be infinite, and it is always a better policy to wait longer, and telecommunication operators will never invest. Therefore, \( \delta \) can be regarded as the opportunity cost of delaying project investment and keeping investment options alive. This article implicitly assumes that this is a continuous time, infinite bound optimization problem, because it can choose to invest at any time to execute the option (American option).
To ensure the optimal existence, \( F(V, t) \) in formula (2) must satisfy the following marginal conditions:

\[
F(0) = 0, \quad (7)
\]

\[
F(V^*) = V^* - I = \frac{I}{\beta_0 - I}, \quad (8)
\]

\[
F'(V^*) = 1. \quad (9)
\]

Among the three marginal constraints, formula (7) indicates that when \( V = 0 \), the investment option will be worthless. Equation (8) is the value-matching condition, which only shows that the project investment operator can obtain the net return of \( V^* - I \). Equation (9) is the smooth-pasting condition, which requires that the values of the two functions are equal at the margins, and the derivatives or slopes of the two functions are also equal at the margins. The value-matching condition and smooth-pasting condition make \( F(V) \) not only continuous but also smooth at the critical point \( V^* \) [2]. \( V^* \) is called the critical value or trigger value of the random process.

Under the constraint of formula (7), the solution form of formula (6) must be

\[
F(V) = AV^\beta_0. \quad (10)
\]

Among them, \( A \) is an undetermined constant not less than 0. It is a known constant \( \beta_0 \) whose value depends on the root of the following formula (11). To satisfy the constraint of formula (7), it must be greater than 0 (negative roots are discarded):

\[
\frac{1}{2}(1 - \varepsilon)^2 \sigma^2 \beta(\beta - 1) + (1 - \varepsilon)\left(\alpha - \frac{\sigma^2 \varepsilon}{2}\right) \beta - \rho = 0. \quad (11)
\]

The solution is as follows:

\[
\beta_0 = \frac{1}{2} - \frac{\rho - \delta}{\sigma^2(1 - \varepsilon)^2} + \sqrt{\left(\frac{\rho - \delta}{\sigma^2(1 - \varepsilon)^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2(1 - \varepsilon)^2}}. \quad (12)
\]

Then, according to (8)–(10) equations, \( A \) can be solved and the critical value \( V^* \) at which the investment is optimal, and the value of the project’s optimal investment option \( F(V^*) \) is

\[
V^* = \frac{\beta_0}{\beta_0 - I}, \quad (13)
\]

\[
A = \frac{(\beta_0 - 1)^{\beta_0 - 1}}{(\beta_0)^{\beta_0}} I^{\beta_0 - 1}. \quad (14)
\]

\[
F(V^*) = \frac{(\beta_0 - 1)^{\beta_0 - 1}}{(\beta_0)^{\beta_0}} I^{\beta_0 - 1} \left(\frac{V^*}{\beta_0}\right)^{\beta_0} \left(\frac{\beta_0 - 1}{I}\right)^{\beta_0 - 1}. \quad (15)
\]

Therefore, the optimal investment time of the project operator is

\[
T^* = \inf\{t \geq 0 | V \geq V^*\}. \quad (16)
\]

In order to further illustrate the difference between the real option method and the traditional NPV rule, let us examine (13). Since \( \beta_0 > 1 > 1, \) so \( \beta_0/\beta_0 - 1 > 1 > 1, \) so \( V^* > I. \) So, the simple NPV rule is wrong. The uncertainty of demand and the irreversibility of investment make telecommunication services have a bonus factor greater than one between the critical value and investment [3].

When the demand uncertainty \( \sigma \) continues to increase, the uncertainty of the project value also increases, \( \beta_0 \longrightarrow 1 \longrightarrow 1, \) waiting for the value of the option to continue to increase, and telecom operators will delay investment. When the demand uncertainty disappears, namely \( \sigma \longrightarrow 0, \beta_0 \longrightarrow \rho/(1 - \varepsilon)\alpha, \) at this time, the value of the waiting option is 0, and its project value function will return to the NPV rule. From this, we can get the value function of the project:

\[
F(V) = \begin{cases} 
AV^\beta_0 = \left(\frac{V}{\beta_0}\right)^{\beta_0} \left(\frac{\beta_0 - 1}{I}\right)^{\beta_0 - 1}, & V < V^*, \\
V - I, & V \geq V^*. 
\end{cases} \quad (17)
\]

5. Numerical Calculation and Parameter Analysis

When telecom operators use the model in this chapter to make optimal investment timing decisions, they can know whether the option value of the project reaches the investment threshold \( V^* \), and the investment threshold \( V^* \) is obviously affected by the parameters in the model. The determination and discussion of these parameters help to explain how the numerical solution of the model depends on the changes of these parameters. This is also one of the key points in the application of real option theory and methods to telecommunication investment decision-making [4]. The following is a discussion of the optimal investment rules for the results of the model solution based on the statistical data of the telecommunication industry from 1990 to 2017.

5.1. Numerical Calculation. It can be known from formula (10) that the undetermined constant \( A \) in the expression of the project investment option value \( F(V) \) is given by formulas (13) and (14), and both are functions of the parameters \( \rho, \delta, \) and \( \sigma. \) The parameters \( \rho \) and \( \delta \) can be assumed in advance, and the parameter \( \sigma \) can be obtained according to the statistical data of the telecommunication industry.

Under the assumption that the random uncertainty of demand obeys the geometric Brownian motion process of (1), assuming that the initial demand is \( Q_0 \) at \( t = 0, \) the expected value and variance of \( Q_t \) are...
\[ E(Q_t) = Q_0e^{\alpha t}, \]  
\[ \text{Var}(Q_t) = Q_0e^{2\alpha t}(e^{\sigma^2} - 1). \]  

We square (15) and substitute it into (19) to obtain

\[ \sigma^2 = \frac{1}{t} \ln \left( \frac{E(Q_t^2)}{[E(Q_t)]^2} \right). \]  

(20)

According to the data of the telecom business demand from 1990 to 2017, we can get \( E(Q_t) = \theta \cdot \delta \), \( [E(Q_t)]^2 = 1.3708e + 07 \), \( E(Q_t^2) = 4.8446e + 07 \), where the annual span is 28 years, then \( t = 28 \), so we get

\[ \sigma^2 = \frac{1}{t} \ln \left( \frac{E(Q_t^2)}{[E(Q_t)]^2} \right) = \frac{1}{28} \ln \left( \frac{4.8446e + 07}{1.3708e + 07} \right) = 0.0451. \]  

(21)

So, we get \( \sigma = 0.2124 \). According to the statistical results over the years, \( \varepsilon = 0.4 \), so the growth rate of the project value \( V \) is \( (1 - \varepsilon) \sigma = 0.12744 \).

Without the loss of generality, suppose \( I = 1 \), \( \rho = 0.04 \), \( \delta = 0.04 \), substituting them into formulas (12), (13), and (15) calculations, the values of \( \beta_0 V^* \), \( F(V^*) \) are

\[ \beta_0 = \frac{1}{2} - \frac{\rho - \delta}{\sigma^2 (1 - \varepsilon)^2} \]

\[ + \sqrt{\left( \frac{\rho - \delta}{\sigma^2 (1 - \varepsilon)^2} - \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2 (1 - \varepsilon)^2}} = 2.7751 \]

\[ V^* = \frac{\beta_0}{\beta_0 I} I = \frac{2.7751}{1.7751} I = 1.5634 \]

\[ F(V^*) = \left( \frac{V^*}{\beta_0} \right)^{\frac{\beta_0 - 1}{I}} \beta_0^{-1} = 0.5634. \]  

(22)

Figure 1 intuitively expresses the relationship between the above-mentioned \( V \) and \( F(V) \). Through the above analysis and calculation, it can be seen that when using the real option method to analyze the investment decision of the telecommunication industry, the value of the investment option should be the largest when the value of the investment project is \( 1.5634 \) times the investment cost, which is the optimal investment time.

5.2. Parameter Analysis. The changes of parameters \( \rho, \delta, \) and \( \sigma \) in the model have a great impact on \( V^* \). Now, we need to examine how the changes of various parameters affect \( V \) and \( F(V) \), and then affect the optimal investment timing.

Figures 2 and 3 show the relationship between the degree of demand uncertainty and the value \( F(V) \) of the investment opportunity and the value \( V^* \) of the project when the option is exercised. Although the parameter \( \sigma \) has been calculated by substituting (21) into the statistical data of the
It can be observed from the figure that the increase in the discount rate \( \rho \) and \( V^* \) depend on changes. According to the principle of financial options, the value of options is directly proportional to the volatility of financial asset prices. In real options, the value of investment opportunities \( F(V) \) and the value of the project \( V^* \) when the option is executed are also positively correlated. This can be visually shown in Figures 2 and 3. When \( \sigma \) increases, the value \( F(V) \) of the investment opportunity increases, and the value \( V^* \) of the project when the option is executed also increases. Higher uncertainty in investment projects increases the value of investment opportunities so that the value \( V^* \) of the project when the option is executed also becomes larger, and telecom operators tend to delay investment [4].

Figure 4 shows how \( V^* \) changes with \( \rho \) changes. It can be observed from Figure 4 that an increase in the discount rate \( \rho \) will lead to an increase in \( V^* \), that is, the value \( F(V) \) of the investment opportunity of the telecom operator and the value \( V^* \) of the project when the investment is executed are positively correlated with \( \rho \). Under constant conditions, the greater the discount rate \( \rho \), the greater the \( V^* \). This is because the current value of the investment cost \( I \) of the investment made at time \( T \) in the future is \( I e^{\rho T} \). But as a return, on the investment expenditure, the current value of the investment project value is \( V e^{\delta T} \). Therefore, when \( \delta \) is constant, an increase in the discount rate \( \rho \) will reduce the present value of the investment cost, and it will not reduce the return of the investment project.

Figures 5 and 6 show how \( F(V) \) and \( V^* \) change with \( \delta \) changes. It can be observed from the figure that the increase of \( \delta \) will lead to a decrease of \( F(V) \), and thus the critical value \( V^* \) decreases. That is, the value \( F(V) \) of the investment opportunity of the telecom operator and the value \( V^* \) of the project when the investment project is executed are in a negative correlation. When \( \rho \) is fixed, the larger the value of \( \delta \), the smaller the value \( V^* \) of the investment project. It is because that if other conditions remain the same, when \( \delta \) increases, and the expected growth rate of the investment project value \( V \) will decrease. In this case, the cost of waiting instead of investment will become more expensive, and investment will now be better than the invest in the future.

From the above discussion, it is easy to know that the value of investment opportunities \( F(V) \) and the value of the project \( V^{**} \) when the investment is executed are positively correlated with \( \sigma \) and \( \rho \), and negatively correlated with \( \delta \). That is, \( F(V) \) and \( V^* \) increase with the increase of \( \sigma \) and \( \rho \), as well as decrease of \( \delta \). Waiting is better than investing now. Therefore, under the condition of uncertain demand, it is necessary to comprehensively consider the changes of sum and cause the changes of \( F(V) \) and \( V^* \), and then to make decisions on investment timing according to the optimal investment rules [5].

### 6. Conclusions

The question of the investment timing of telecom network optimization and expansion projects under uncertain conditions is a common problem in the investment practice of telecom operators. Under normal circumstances, the price is expressed as a linear function of demand, but this article examines the characteristics of demand and price in the...
telecommunication industry, combined with historical data in the telecommunication industry, and proposes an exponential function relationship in which price meets demand, and then establishes demand on this basis investment timing decision model under uncertain conditions. The research shows the following:

(1) Under conditions of uncertain demand, the optimal investment timing for network expansion projects is $T^* = \inf \{ t \geq 0 | V \geq V^* \}$, which should be based on historical data of telecom statistics from 1990 to 2017 and calculated and concluded that under conditions of uncertain demand. The value of the investment project is 1.5634 of the investment cost. At this time, the value of the investment option is the largest, and this is the best investment time.

(2) The value $F(V)$ of the investment opportunity and the value $V^*$ of the project when the option is executed have a positive correlation with $\sigma$. With the increase of $\sigma$, the value $F(V)$ of the investment opportunity also increases, and the value $V^*$ of the project when the option is exercised also increases accordingly, and the telecom operators tend to postpone investment.

(3) The value $F(V)$ of the investment opportunity and the value $V^*$ of the project when the option is executed have a positive correlation with $\rho$. Under constant conditions, the greater the discount rate $\rho$ is, the greater $V^*$ is, but it will not reduce the return of the investment project.

(4) The value $F(V)$ of the investment opportunity and the value $V^*$ of the project when the option is executed have a negative correlation with $\delta$. If other conditions remain the same, when $\delta$ increases, the expected growth of the value of the investment project $V$ will decrease. The waiting cost will become more expensive, and the current investment is better than future investment.

Data Availability

The figures used to support the findings of this study are included in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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