Mathematical model for calculating the equilibrium point of the refrigerant circuit

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Abstract. Calculating the equilibrium point of the refrigerant circuit is one of the most important processes in the air treatment industry. Knowing the evaporation and condensation temperatures serves not only to obtain information on the performance and operating point of the compressor, but also to evaluate the performance of the machine under different climatic conditions. Often, the solution of the refrigerant circuit is accomplished by empirical methods or numerical methods such as the Newton – Raphson method or the medium method. Since the iteration time with these methods is unknown at the time of the iteration start, it is very important to implement algorithms that provide convergence information or not, and which can bring the system to a solution in the shortest possible time. The implementation of such a method would pave the way for simulating the behaviour of machines in the air treatment industry as an advanced verification process for real evaluation of machine performance. In this article, experimental results and mathematical model calculations will be presented and discussed.

1. Introduction

Calculating the equilibrium point of the refrigeration circuit is a critical operation in machines containing such a circuit, especially in the air treatment industry. We can say this because we often need to control parameters such as temperature, humidity or both simultaneously.

For different application purposes the refrigeration circuit may be in different configurations. Here we mention configurations with variable gas flow regulated by inverters, circuit with more than one compressor located in parallel or cascade or refrigeration circuit where the condensing or evaporating element can be composed of more than one part. In this paper we will refer to the simplest model of the refrigeration circuit, although the following reasons apply to each model of the refrigeration circuit. [1] Below we will schematically present the refrigeration circuit that will be studied.
In the figure 1 above we distinguish that the circuit is composed of 5 main elements that are:

- Compressor
- Condensing element
- Gas expansion valve
- Evaporative element
- Gas and liquid passage pipes

The compressor set in motion would compress the gas. The latter according to the law of ideal gas in an isohoric process would increase its temperature in a linear relationship with the ratio of temperature change as follows [2]:

$$T = \frac{P_2}{P_1} \cdot T_1$$  \hspace{1cm} (1)

Gas at high temperatures (about 70 °C), condenses having contact with air at much lower temperatures and according to the second law of thermodynamics the air temperature rises. This temperature exchange is determined by various factors such as the geometry of the exchanger, air and gas temperature, humidity and air flow.

Condensed gas passes through the gas expansion valve where its pressure decreases greatly. Once again referring to equation (1) we determine that the gas temperature decreases too much as the fluid passes into the gas expansion valve. Here we again have the thermal exchange of gas with the air where the latter has a much higher temperature value than the temperature of the fluid. Even in this case the amount of thermal exchange depends on the factors mentioned above.

2. Equilibrium of the refrigeration circuit

The process of condensation and evaporation both occur at certain temperatures. These temperatures are the product of several factors where we can mention the geometry of the exchangers, the climatic conditions of the air and the air flow. During the heat exchange in the condensing and evaporating phase each of the elements presents a certain power.

Thermal power will be called the full power developed by the condensing element, while the refrigeration power will be called the total power developed by the evaporating element.

If we accept all other invariant factors, the power of the condensing element can be modelled as a function of the condensing temperature, and the power of the evaporating element as a function of the
evaporation temperature [3]. At the same time, one of the most classic forms of calculating the working parameters of the compressor is the polynomial model as a function of two variables, which are the evaporation temperature and the condensation temperature. This model applies to the compressor refrigeration power, electric power, electric current and gas mass per hour. The model is presented as follows [1]:

\[
F(\text{tev},\text{tco}) = A + B \cdot \text{tev} + C \cdot \text{tco} + D \cdot \text{tev}^2 + E \cdot \text{tev} \cdot \text{tco} + F \cdot \text{tco}^2 + G \cdot \text{tev}^3 + \\
+ H \cdot \text{tco} \cdot \text{tev}^2 + I \cdot \text{tev} \cdot \text{tco}^2 + L \cdot \text{tco}^3
\] (2)

We say that the circuit is in equilibrium at the moment that the cooling power of the compressor is equal to the cooling power of the evaporating element and the thermal power of the compressor is equal to the thermal power of the condensing element. The thermal power of the compressor will be calculated as the sum of the cooling power and the electric power. In these conditions, finding the equilibrium conditions of the refrigeration circuit consists in detecting the evaporation and condensation temperature for which the circuit is in equilibrium. From the recognition of these temperatures it can be calculated the power at the output of each element and as a result also the climatic conditions at the output in the case of air treatment machines. There are several studies that treat this concept, but it is difficult to find algorithms that can be programmed or procedures that can help on finding the equilibrium point of refrigerant circuit in a non empirical way. In this paper we are presenting two different approaches (graphical and analytical) that can be implemented in a program or can be used to solve these kind of problems manually. We think that this work can also help on improving the intuition of finding equilibrium point of a refrigerant circuit.

3. Graphic solution
As a problem that cannot be specifically solved, the solution to this problem is iterative. As a two-dimensional function, the graphical representation of the compressor's refrigeration and thermal power would be presented as a network of possible values within which the compressor could operate. Below we present the graphical solution of this problem based on an air treatment machine which has a configuration of the refrigeration circuit as the one considered in this study.

![Figure 2. Graphical presentation of the equilibrium of the refrigeration circuit](image-url)
In the graph above we have graphically presented the two work surfaces of the compressor and the respective curves of the condensing and evaporating element. Reiterating that the thermal power of the compressor is the sum of the refrigeration power and the electric power, we can easily distinguish from the graph that the surface positioned below represents the refrigeration power, while the one above represents the thermal power. Graphically we say that we are in equilibrium when the curve of the condensing element pierces the surface of the thermal power in the pair of condensing and evaporating temperatures for which the curve of the evaporating element pierces the surface of the refrigeration power.

Starting from the refrigeration power, we first build the surface and the respective curve presented as follows.

![Graphical representation of the surface and curve of refrigeration power](image)

**Figure 3.** Graphical representation of the surface and curve of refrigeration power

By moving the evaporator curve towards the surface of the compressor's refrigeration power (according to the condensing temperature axis) until the curve touches the surface. At the cutting point we read the value of the condensation temperature from the surface. Further moving the condenser element curve to the evaporation temperature point where we have the point of intersection with the surface, we see if this curve intersects the surface of the thermal power at the value of the condensation temperature detected on the refrigeration surface. If not, this process is repeated with another point by moving the refrigeration power curve until the solution is reached as shown in figure 2.

This process, if performed graphically, would take a long time and would not be efficient. The first problem would be to build surfaces and curves for system components and then to perform iterative steps until a solution is reached. For this reason, we will look at the analytical solution and the problem-solving algorithm analytically.
4. Analytical solution

As stated above, if we accept that the climatic and geometric data of heat exchangers are constant, then the resultant power of each of these elements remains a function of the evaporation and condensation temperature. So we would have:

- Refrigerant power of the evaporating element: $Eva_{refr}(te)$
- Thermal power of the condensing element: $Con_{th}(tc)$
- Compressor refrigeration power: $Com_{refr}(te,tc)$
- Compressor thermal power: $Com_{th}(te,tc) = Com_{refr}(te,tc) + Com_{el}(te,tc)$

where with "el" we indexed the electrical power obtained from the network. Since the following solution will be a mathematical solution, we will physically accept the temperature values in a certain range of values.

If we accept that we are in equilibrium we would have:

$$Eva_{refr}(te) = Com_{refr}(te,tc)$$  \hspace{1cm} (3)

If we accept that the mathematical function for calculating the vaporizing and condensing element are known functions, the equation (3) would turn us into a third order equation with one variable. Thus, according to function (2) we would have:

$$Eva_{refr}(te) = A + B * te + D * te^2 + G * te^3 + tc * (C + E * te + H * te^2) +$$

$$+ tc^2 * (F + I * te) + tc^3 * L$$  \hspace{1cm} (4)

Recognizing that we are iterating with evaporation temperature it follows that:

$$A + B * te + D * te^2 + G * te^3 \rightarrow \text{const.} (C)$$
$$C + E * te + H * te^2 \rightarrow \text{const.} (C_{te}$$
$$F + I * te \rightarrow \text{const.} (C_{te3})$$
$$0 = (C_{te} - Eva_{refr}(te)) + tc * C_{te} + tc^2 * C_{te2} + tc^3 * L$$  \hspace{1cm} (5)

We see that the above equation has the general form of the third order equation with one variable. The solution of this equation can be achieved with different numerical methods such as Newton-Raphson method, Bisection method or referring to the general solution of the equation presented by N. Tartaglia, L. Ferrari and S. del Ferro [4].

From the equation solutions we will filter the resulting values in the range of acceptable values of condensation temperature. By substituting this value for the function that calculates the thermal power of the condensing element and for the function that calculates the thermal power of the compressor we would have 2 respective thermal powers. At this point the assessment of the relative difference of these powers would give us the information needed to judge whether we are in equilibrium or not. Hence, we have:

$$d = \left| \frac{Com_{th}(te,tc) - Con_{th}(tc)}{Com_{th}(te,tc)} \right| \times 100$$
If the value of the percentage deviation were less than the tolerance (usually 5%) we say that we are in equilibrium conditions, otherwise we would increment the value of the evaporation temperature. In this analytical solution we will need to define some important parameters that are:

- Initial value of evaporation temperature
- The step of iteration
- Tolerance
- Maximum number of iterations

Based on the experience and experiments below we will give some recommendations for each of these parameters. To determine the evaporation temperature, experience dictates that it is recommended to start from an expected value of equilibrium evaporation temperature or from a value calculated as the difference in air temperature at the inlet of the minus K, where K is located in the segment [15, 20]. This is a parameter for which it is difficult to suggest specific values, but it is recommended not to increment with whole numbers, but with steps that can bring as much convergence as possible such as 0.07, 0.12, 0.17, etc. The tolerance required in air treatment machines usually varies in the interval 5% - 10%. This can be increased in situations where we know that the compressor polynomial model carries errors in polynomial coefficients or when the power function for each of the exchanges does not represent high accuracy. The maximum number of iterations should usually be defined as a value in the range of 50-100 iterations.

5. Conclusions
- It was noticed that when we used the analytical approach, high accuracy levels were achieved, considering a maximum tolerance of 5%, this in confront with other commercial softwares which usually use algorithms that iterate both temperatures. Using these softwares we had to define a tolerance of 15 – 20% for iteration to converge.
- Using the graphical solution of the problem is simple. Nevertheless, it is more complex due to the fact that we need to calculate a wide range of values. This makes the solution less usable.
- Using the compressor model to find the condensation temperature when iterating with evaporating temperature with cooling power treated as a known variable, helps in increasing the accuracy levels and accelerates the iterative process.
- From the experiment, we came to the conclusion that this method grants a better performance in simulating the hvac/r machines, in comparison to the iterative bidirectional methods.

6. Appendices
Pseudocode:
Set numIter to 0
Set Success to 0
Set actualEvapTemp to (EvapCoilAirIn – Delta)
While numIter < 100 and not Success
    Set EvapCoil evaporation temperature to actualEvapTemp
    If EvapCoil calculation is valid
        //finding condensing temperature
        Set condensTemp to
        CondensCoilTemp = condensTemp
        If CondensCoil calculation is valid
            Calculate compressor thermal power for actualEvapTemp and condensTemp
            Calculate relative error
            If Error < tolerance
                Set Success to 1
Else
Increment actualEvapTemp
Increment numIter

Else
Increment actualEvapTemp
Increment numIter

7. References
[1] Santa R and Garbai L 2013 Annals of Faculty Engineering Hunedoara 271 – 80
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[4] https://mathworld.wolfram.com/CubicFormula.html