Interpretation of the expansion law of planetary nebulae

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Abstract

We reproduce the expansion velocity–radius ($V_{\text{exp}} - R_n$) relation in planetary nebulae by considering a simple dynamical model, in order to investigate the dynamical evolution and formation of planetary nebulae. In our model, the planetary nebula is formed and evolving by interaction of a fast wind from the central star with a slow wind from its progenitor, the AGB star. In particular, taking account of the mass loss history of the AGB star makes us succeed in the reproduction of the observed $V_{\text{exp}} - R_n$ sequence. As a result, examining the ensemble of the observational and theoretical evolution models of PNe, we find that if the AGB star pulsates and its mass loss rate changes with time (from $\sim 10^{-6.4} M_\odot \text{yr}^{-1}$ to $\sim 10^{-5} M_\odot \text{yr}^{-1}$), the model agrees with the observations. In terms of observation, we suggest that there are few planetary nebulae with larger expansion velocity and smaller radius because the evolutionary time-scale of such nebulae is so short and the size of nebulae is so compact that it is difficult for us to observe them.

Key words: ISM: evolution — ISM: planetary nebulae: general — hydrodynamics

1. Introduction

Planetary nebulae (PNe) are the objects apart from the asymptotic giant branch (AGB), evolving to white dwarfs. It is thought that a stellar wind is important for the formation and evolution of PNe. To explain the features of PNe, Kwok et al. (1978) propose the interacting stellar wind (ISW) theory. After that many authors have studied PNe theoretically and observationally based on the ISW theory.

Observationally, before the ISW theory was presented, a trend of the expansion velocity ($V_{\text{exp}}$) of the gas of PNe was found (Kwok et al. 1978), and it has been interpreted in two different ways: one is in terms of a dynamical evolution of PN (Smith 1969; Bohuski and Smith 1974; Sabbadin and Hamzaoglu 1982; Robinson et al. 1982), another is in terms of the evolution of a central star (Renzini 1979).

Based on such studies, Sabbadin et al. (1984; hereinafter SGBO) proposed two models...
to interpret that trend, the “two-wind” model and the “two-phase” model. The former means that the planetary nebula is the region of interaction of a fast wind emitted by the nucleus with a slow wind expelled by its progenitor. They added the effect of the momentum transfer from radiation field to the model suggested by Kwok et al. (1978) and Kwok (1982). The latter means that the planetary nebula is a Strömgren sphere evolving in an expanding nebula which results from a sudden ejection by the AGB star. They conclude only the “two-phase” model agrees with the observational data because the “two-wind” model can not reproduce the observed mass ($M_n$) and radius ($R_n$) relation. In the two-wind model the resulting mass of nebula is much smaller than that derived from observations (the mass decrement problem), while it matches the $V_{\text{exp}}-R_n$ relation. The mass decrement problem of the two-wind model is also stressed and examined in Schmidt-Voigt & Köppen (1987a,b).

The “two-wind” model had another fault. SGBO didn’t take account of the effect of shock sufficiently. As Marten & Schönberner (1991) stated, the simplification of SGBO corresponds to the fact that the width of the shell is infinitesimally thin. To resolve the mass decrement problem, it is essential to examine the width of the shell (Marten & Schönberner 1991). Then, we confirm in this paper that it is easy to resolve the mass decrement problem if the width of the shell is determined as the effect of the shock. In this standing point, we try to study the two-wind model to satisfy $M_n-R_n$ and $V_{\text{exp}}-R_n$ relations. We would like to comment that after SGBO, they developed their study on PNe furthermore (e.g. Sabbadin et al. 2005).

We also examine the effect of the time-dependent mass loss rate of the AGB star, then we succeed in reproduction of the observed mass-radius relation simultaneously with the observed $V_{\text{exp}}-R_n$ relation. The importance of the mass-loss to explain the evolution of PNe has been stressed by Marten & Schönberner (1991). In this paper, we insist furthermore that our modified “two-wind” model reproduces the ensemble of observational data adequately if the radial density profile of AGB matter has a sudden decrement at about 0.1pc from the central star. This decrement is the result of the time-dependent mass-loss history of the AGB star. Thus, we reformulate the model concerning the mass-loss history of the AGB star.

Thus, to explore the evolution and formation of PNe, we revisit the “two-wind” model in this paper. In §2 we describe our model. In §3 the results of the calculation compared with the observational data are presented. In §4, we discuss the results, and summarize our study in §5.

2. Model description

According to SGBO, there are two problems in the classical “two-wind” model: (a) the resulting mass of the PN is much smaller than that determined from observations [hereinafter problem (a)], (b) the “two-wind” model can’t reproduce the observed $R_n-M_n$ relation [See Fig.6 and Fig.7 of SGBO and hereinafter problem (b)]. Then, we reformulate the problem here. According to Frank (1994), it is important for us to study the evolution of PNe as a problem
of radiation hydrodynamics. We admit it. However, if we are concerning the ensemble of the evolution of PNe, some simple models of PN evolution is useful, since the overall property of evolution of PNe is depicted as a combination among simple physics. Then, we try to reconstruct the two-wind model and explore its possibility as a tool to study PN evolution.

2.1. General description

Our model is based on the “two-wind” model which is proposed by SGBO. In the “two-wind” model, the authors assume that PN is the region of interaction of a fast wind from the central star with a slow wind from the AGB star, considering the effect of a momentum transfer from the radiation field to the nebula motion. We assume a shell of PN is a shock region and the nebula evolution obeys the following equations:

\[ \frac{dM_n}{dt} = \frac{M_{\text{agb}}(t)}{V_{\text{agb}}} v_n \]

and

\[ \frac{dv_n}{dt} = \frac{1}{M_n} \left[ \frac{L}{c} - \frac{M_{\text{agb}}(t)}{V_{\text{agb}}} (v_n - V_{\text{agb}})^2 + \frac{\dot{m}}{v} (v - v_n)^2 \right] \]

where \( M_{\text{agb}}(t) \) and \( V_{\text{agb}} \) are the mass loss rate and the velocity of the wind from the AGB star; \( L, \dot{m}, \) and \( v \) are the luminosity, the mass loss rate, and the velocity of the wind from the central star; \( M_n, R_n, \) and \( v_n \) are the mass, the radius, and the expansion velocity of the PN; \( c \) is the light velocity; \( \alpha \) is the efficiency factor of the transferred momentum.

Eq. (1) means that the mass of PN is consisted of the total matter included in the radius \( R_n \) thanks to the shock. \( M_{\text{agb}}(t) \) denotes the mass loss rate measured at the position of the expanding shell, and then it is represented of time explicitly.
In Eq.(2), the first term in the right hand side represents the contribution of the transfer of momentum from the radiation field to the nebula, the second term is the interaction with the AGB slow wind, and the third term corresponds to the interaction with the fast wind from the central star.

2.2. Models of the central star

Adopting Eqs.(1) and (2), we take into account the evolution of luminosity $L$ and the mass-loss rate $\dot{m}$ of the central star more realistically than SGBO adopted, while those evolution models are simplified as presented below.

We assume that the mass of the central star is $0.605 M_\odot$ because most nuclei of PNe are about $0.6 M_\odot$. Then we approximate the evolutionary track proposed by Blöcker (1995b) shown in Fig.1. Furthermore, we compute the mass-loss rate shown in Fig.2, according to Martin & Schönberner (1991).

2.3. Initial Conditions

As stated, SGBO insist there are two problems (mass decrement problem and $R_n-M_n$ relation problem) for the “two-wind” model. In order to solve these problems, we adopt Eq.(1) because the shock condition has to be satisfied there and the variation of the mass-loss rate of the AGB star to the “two-wind” model should be considered. Problem (a) is solved easily by only considering the shock condition. The previous model did not count the total mass reasonably.

Following Blöcker (1995a), we adopt the time-depending simplified mass-loss rate from the AGB star described in Fig.3 to reproduce the $R_n-M_n$ relation. In Fig.3, the horizontal axis represents the time before the star enters post-AGB phase, i.e. the right edge of it is 1000 yr.
Fig. 3. The mass-loss rate of the AGB star. Horizontal axis denotes time before a star enters PN phase, and vertical one is mass loss rate of the AGB star.

before the post-AGB phase and the left edge corresponds $10^{5.5}\text{yr}$ before it. For $-t < 1000$, $\dot{M}_{\text{agb}}$ decreases slowly and connects $\dot{m}$ in Fig.2.

On these bases, we set, as initial conditions, $R_n=0.01\text{pc}$, $M_n=10^{-2.5}\,M_\odot$, $v_n=V_{\text{agb}}+C_s$ ($C_s$: the sound speed in pre-shock matter) at $t=0$, and $V_{\text{agb}}=10\text{ km s}^{-1}$ which is kept during all the evolutionary time. These are chosen simply since PN emerges as a shocked layer. From $t=0$ to $t=100000\text{ yr}$, we calculate some models with different wind velocities from the central star: $500\text{ km s}^{-1} \leq v \leq 10000\text{ km s}^{-1}$ at the intervals of $500\text{ km s}^{-1}$. The initial set of the parameters of $R_n$ and $M_n$ are very consistent to those expected in a model calculation by Mellema (1994). According to the estimates of the H$_\alpha$ surface brightness in Fig.10 and the ionized mass in Fig.19 of Mellema (1994), when the PN shell is formed and fully ionized, the radius of the shell is less than $0.05\text{ pc}$ and its mass is less than $10^{-2}\,M_\odot$. Thus, we confirm that we choose appropriate values as the initial conditions from the study of Mellema.

2.4. Observational sample selection

In our analysis, we use the [OIII] data listed in SGBO (See their table.1 and Fig.1). We adopt this classical set of data because number of sample sets, in which both the size and expansion velocity are observed directly, does not increase even in recent years very much. Indeed, for our purpose, we must remove some objects whose expansion velocities are uncertain (small points in Fig.1 of SGBO).

According to Schönberner, Jacob, & Steffen (2005), Schönberner, et al. (2005), and so on, not only there are different sizes correspond to different components, optical imaging usually measures the size of the ionization front, not the actual size of the shell. Their statement is important if we are concerning the detailed structure of PNe. Fortunately, however, we are interested in the ensemble of the evolution of PNe, and do not examine the detailed structure.
of PNe. Furthermore, the shell can be almost ionized after about 1000 years (e.g. Perinotto et al. 2004). Then, we expect that as long as the overall trend of PNe is concerned, our simple approach is still useful.

3. **Results**

3.1. **Characteristic cases**

The results of our calculation in the case of $C_s=5\text{km s}^{-1}$ and $\alpha=0.5$ are shown in Fig.4, Fig.5, and Fig.6.

![Fig. 4. Rn-Mn relation. Horizontal axis is radius in the unit of pc, vertical one is mass in the unit of solar mass. The bottom line is $v = 500 \text{ km s}^{-1}$, and the top line is $v = 10000 \text{ km s}^{-1}$.](image1)

![Fig. 5. Resulting mass. Horizontal axis corresponds evolutionary time [yr], vertical axis is mass [$M_\odot$]. The bottom line is in the case of $v=500 \text{ km s}^{-1}$, and the top one is $v=10000 \text{ km s}^{-1}$.](image2)

Fig. 4 represents $R_n-M_n$ relation of our calculation. From this figure, because of the
time-depending AGB mass-loss rate, the increasing rate of $M_n$ decreases after PN has size of $R_n \sim 0.1$ pc, then the increase rate turns to be small. Thus, we can succeed in the reproduction of the observed tendency, and resolve the problem (b).

Around the right top of Fig.4, $M_n$ increases rapidly again. This is because the pulsation of the AGB star finishes (See Fig.3, it corresponds $\log(-t) \sim 4.5$). However it doesn’t affect the value of expansion velocity up to $R_n=0.3$ pc. It influences only the resulting mass of PN. Fig.5 is time-mass plane. The bottom line is the model $v=500$ km s$^{-1}$, the top is the model $v=10000$ km s$^{-1}$. From our simple models, it takes over 6000 yr that the radius of model nebula reaches 0.3 pc (See Fig.6), and the resulting mass is over $0.1M_\odot$ when $R_n \sim 0.3$ pc, thus we can solve the problem (a) in the same time.

In Fig.6, the relation of $2V_{\text{exp}}$-$R_n$ is presented as one of main results in our study. We can almost re-produce the distribution of data points by changing only the fast wind speed. The most left curve is the model $v=500$ km s$^{-1}$ and the most right one corresponds $v=10000$ km s$^{-1}$. From Eq.(2), if $v$ has large value, the shell of PN is accelerated strongly in the early phase then has a large expansion velocity. The expansion velocity is decelerated as the PN evolves, because of the decrease of the luminosity and mass loss rate of the central star. We also over-plot the evolutionary time as asterisks from $t=2000$ yr at steps of 2000 yr. One can comprehend from it that most of data points exist in the region of $t=2000$ yr-4000 yr. Star which mass is $0.605M_\odot$ enters the realm of white dwarf at $t \sim 7000$ yr (See Fig.1), so PN phase must be before 7000 yr. Our results are consistent in this meaning.
3.2. Parameter dependence

Setting the fast wind velocity \( v = 1000 \) km s\(^{-1}\), we inquire into the effects of other parameters: the sound speed \( C_s \), the transfer efficiency \( \alpha \), and the slow wind velocity \( V_{\text{agb}} \). The effect of other parameters to the results is essentially the same as examined in SGBO.

3.2.1. Sound speed

Fig. 7 shows the results of different sound speed in the pre-shock matter: \( C_s = 5-10 \) km s\(^{-1}\) at the intervals of 0.5 km s\(^{-1}\). Even if \( C_s \) changes, i.e. the temperature in the pre-shock matter changes, \( V_{\text{exp}} - R_n \) relation is not so different. The variation of \( C_s \) affects only \( R_n - M_n \) relation, as seen in Fig. 8. It is because we set that the mass of PN increases along Eq. (1) only as long as the shock occurs. The larger the sound speed is, the shorter the period during which the shock is occurring is and the smaller the final mass of PN is. If the shock doesn’t occur, the mass of PN has a constant value in our model.

3.2.2. Transfer efficiency

The sequences of different transfer efficiency are shown in Fig. 9 and Fig. 10. We vary \( \alpha = 0-1 \) at the steps of 0.1. As \( \alpha \) increases, the influence on the evolution of PN due to the variation of central star’s luminosity becomes large.

3.2.3. Slow wind velocity

The \( 2V_{\text{exp}} - R_n \) relations are drawn in Fig. 11, and the \( R_n - M_n \) relations are in Fig. 12 when we change the slow wind velocity from 5 km s\(^{-1}\) to 24 km s\(^{-1}\) at the intervals of 1 km s\(^{-1}\). In Fig. 11, the most left line corresponds \( V_{\text{agb}} = 5 \) km s\(^{-1}\) and the most right one does \( V_{\text{agb}} = 24 \) km s\(^{-1}\). Also in Fig. 12, the top curve is in the case of \( V_{\text{agb}} = 5 \) km s\(^{-1}\) and the bottom is \( V_{\text{agb}} = 24 \) km s\(^{-1}\). If we allow a turning point, at which the increasing rate of mass turns to be small, to have
Fig. 8. $\log R_n$-$\log M_n$: $C_s=5$-10 km s$^{-1}$. Horizontal axis corresponds radius, and vertical axis is mass. The bottom line is in the case of $C_s=10$ km s$^{-1}$.

Fig. 9. $2V_{\text{exp}}$-$R_n$: $\alpha=0$-1. Horizontal axis is expansion velocity [km s$^{-1}$], and vertical axis denotes radius [pc]. The most left dashed line is $\alpha=0$, the most right one is $\alpha=1$, and plus sign means observational data.

uncertainty of a factor 3 in $\log (R_n)$-$\log (M_n)$ plane, ones can find that the difference of AGB wind velocity also explains the distribution of observed data well. However, AGB wind velocity is not so large ($<30$ km s$^{-1}$) that the models varying only $V_{\text{agb}}$ does not cover the region with large expansion velocity. From this point of view, we conclude the central star’s wind velocity is more proper parameter which explains $2V_{\text{exp}}$-$R_n$ relation.

4. Discussion

In this section, we examine our model compared with the basic model basing on SGBO and discuss the physics of PN evolution.
Fig. 10. log Rn-log Mn:alpha=0-1. Horizontal axis is radius, and vertical axis is mass. The bottom line is in the case of $C_s=0$.

Fig. 11. $2V_{exp}$-Rn:Vagb=5-24 km s$^{-1}$. Horizontal axis denotes expansion velocity [km s$^{-1}$] and vertical axis is radius [pc]. Plus sign corresponds observed data, the most left dashed line is in the case of $V_{agb}=5$ km s$^{-1}$, and the most right one is the model with $V_{agb}=24$ km s$^{-1}$.

4.1. Difference from a basic model

As already mentioned, there are two problems in the classical two-wind model by SGBO: (a) the decrement of resulting mass, and (b) the discrepancy in the $R_n$-$M_n$ relation between observations and the model. Although SGBO’s model explains the $2V_{exp}$-$R_n$ relation, the authors abandon their idea because of the problems (a) and (b). On the other hand, we take account of the effect of shock and time-depending mass loss rate of the AGB star, then the problems (a) and (b) are solved easily. From this point, we convince for the evolution of PN to consider the influence of shock and variation of mass loss rate of the AGB star. That is, the AGB star is pulsating so its mass loss rate varies with time during AGB phase, and during the
Fig. 12. log Rn-log Mn: V_{agb}=5-24$\text{km s}^{-1}$. Horizontal axis is radius and vertical one is mass. The top line is V_{agb}=5$\text{km s}^{-1}$ and the bottom is V_{agb}=24$\text{km s}^{-1}$.

PN phase the fast wind speed is much larger than the slow wind speed to be a shock: $v \gg V_{agb}$.

4.2. Physical origin of the $V_{exp}$-$R_n$ relation

Fig. 13. $2V_{exp}$-$R_n$: $\dot{M}_{agb}=2\times10^{-5}M_\odot$ yr$^{-1}$. Horizontal axis is expansion velocity $[\text{km s}^{-1}]$, and vertical axis is radius $[\text{pc}]$. Plus sign means the data. $\dot{M}_{agb}=2\times10^{-5}M_\odot$ yr$^{-1}$, $\alpha=0.5$, $C_s=10$ km s$^{-1}$ and other parameter’s value are written in the figure.

Based on Sect.4.1, we discuss physical background of PN evolution in this subsection. In our model to explain the $2V_{exp}$-$R_n$ relation, the greatest point is considering the time-depending mass loss rate of the AGB star. We also calculate the models which $\dot{M}_{agb}$ value is a constant. Some results are presented in Fig.13, Fig.14, and Fig.15. The sequences of $\dot{M}_{agb}=2\times10^{-5}M_\odot$ yr$^{-1}$ are in Fig.13, those of $\dot{M}_{agb}=3\times10^{-5}M_\odot$ yr$^{-1}$ are in Fig.14, and those of $\dot{M}_{agb}=4\times10^{-5}M_\odot$ yr$^{-1}$ are in Fig.15. In these calculations, we set $\alpha=0.5$, $C_s=10$ km s$^{-1}$ and examine the range
Fig. 14. 2Vexp-Rn:$\dot{M}_{\text{agb}}=3\times10^{-5}M_\odot$ yr$^{-1}$. Same as Fig.13 but in this figure $\dot{M}_{\text{agb}}=3\times10^{-5}M_\odot$ yr$^{-1}$.

Fig. 15. 2Vexp-Rn:$\dot{M}_{\text{agb}}=4\times10^{-5}M_\odot$ yr$^{-1}$. Same as Fig.13 but in this figure $\dot{M}_{\text{agb}}=4\times10^{-5}M_\odot$ yr$^{-1}$.

of $v=500$-10000 km s$^{-1}$ at the steps of 500 km s$^{-1}$. The three cases of $V_{\text{agb}}=10,15,20$ km s$^{-1}$ are examined. In those figures, we plot only the models which satisfy the $R_n-M_n$ relation if the lines turn around the point of $\log(R_n) \sim -1.0$ and $\log(M_n) \sim -1.0$ in $\log(R_n)$-$\log(M_n)$ plane. The $R_n-M_n$ relations of the selected models are shown in Fig.16, Fig.17, and Fig.18.

From Figs.13, 14, and 15, when $\dot{M}_{\text{agb}}=\text{const}$, we can re-produce only the region of $2V_{\text{exp}} \leq 70$ km s$^{-1}$. The models which are able to trace the realm of larger expansion velocity do not match the $R_n-M_n$ data.

Comparing time-depending mass loss models (Fig.6) with steady mass loss models (Figs.13, 14, 15), one can know how important for the evolution of PNe to take account of the variation of AGB mass-loss rate. In other words, the radial density distribution of ambient matter around PN, $\rho(r)$, is not in proportion to $r^{-2}$. It needs to fall suddenly at $r\sim 0.1$pc due to the pulsation history during AGB phase. Furthermore, this indicates the difference between the
Fig. 16. log $R_n$-log $M_n$; $\dot{M}_{\text{agb}}=2 \times 10^{-5} M_\odot \text{yr}^{-1}$. Horizontal axis denotes radius and vertical axis is mass. $\dot{M}_{\text{agb}}=2 \times 10^{-5} M_\odot \text{yr}^{-1}$, $\alpha=0.5$, $C_s=10 \text{ km s}^{-1}$ and other parameter’s value are written in the figure.

Fig. 17. log $R_n$-log $M_n$; $\dot{M}_{\text{agb}}=3 \times 10^{-5} M_\odot \text{yr}^{-1}$. Same as Fig.16 but $\dot{M}_{\text{agb}}=3 \times 10^{-5} M_\odot \text{yr}^{-1}$.

dynamical age and evolutionary age of PNe since the expansion law is affected by the density profile around the proto-PNe. We expect the age-discrepancy problem of PNe (Mellema 1994) may be partially resolved if the origin of the velocity-radius relation is resolved.

By the way, the variety of $v$ contributes greatly to the explanation of data distribution in $2V_{\text{exp}}-R_n$ plane. In some models, $v$ has a very large value up to 10000 km s$^{-1}$ at $t=0$. However, $v$ may not attain such a large speed in the theory of line-driven wind. We deals with this problem in §§4.4 again.

4.3. Observational implications

In $2V_{\text{exp}}-R_n$ plane, most of observational data exist in the range of $2V_{\text{exp}}=40-80$ km s$^{-1}$ and $R_n=0.05-0.15$pc. We present by our calculations that these data are in the realm of $t=2000$-4000 yr(cf. Fig.6 in Sect.2) and it coincides with the fact that PN is an object between
the AGB star and white dwarf. Our results suggest the reason why only few data is in the smaller radius region. PNe evolve very quickly across this region, and then it is difficult for us to observe. In the larger radius region, it is also difficult to observe since PNe expand too large and diffuse. By a more highly accurate survey of PNe, in future, we will find that there are a lot of faint PNe and some of them have larger radius among them.

4.4. Velocity problem

As stated in subsection 4.2, we have a time gap problem concerning the fast wind velocity from the central star of PN. Along the line-driven wind theory, it takes about 8000 yr until \( v \) reaches 10000 km s\(^{-1}\), while in our calculation it needs to be attained at \( t=0 \). However we will solve this problem in two different simple ways: (1) accretion jet and (2) compactness of the central star. We shall examine the case of (1). In the line-driven wind it is assumed the PN field is spherical symmetry. The assumption is not so realistic and many PNe have an asymmetric jet due to its magnetic field. In this way, the gas of PN is accelerated so quickly that our supposition for \( v \) is good. By this way of thinking, we may say that the reason why few PNe are in the larger expansion velocity and smaller radius region in their early phase (<2000 yr) is that the total number of asymmetrically accelerated PNe is small in their early evolution.

The other explanation of the case of (2) is rather simple, although the acceleration mechanism can not be specified. If the central star has very small radius (\( \sim 1000 \) km), its gravity at the surface of the star is so large. To escape from the central star gravity field, a matter must have a very large velocity. In fact, a radius of typical white dwarf is in the range of 1000-10000 km. It is expected that the acceleration mechanism of PNe will be proved by future observations.

5. Summary

Constructing the modified “two-wind” model, we find that the observational data are reproduced adequately if the radial density of the AGB matter has a sudden fall at about 0.1pc from the central star. It corresponds the AGB star pulsates and its mass loss rate increases. When we set the start point of evolutionary track of PN is \( t=0 \) and the velocity of the AGB matter \( (V_{\text{agb}}) \) is 10 km s\(^{-1}\), we estimate \( \dot{M}_{\text{agb}}=10^{-6.4}M_\odot \) at \( t=-10^{4.5}\)yr to \( \dot{M}_{\text{agb}}=10^{-5}M_\odot \) at \( t=-10^{3.8}\)yr. It means that the features of each PN considerably depend on its progenitor’s mass loss history, and by only including the effect of time-dependent mass loss rate of the AGB star we can explain the observed data in the context of fluid dynamics. Also our model gives an observational suggestion that the PNe with larger \( V_{\text{exp}} \) (\( \geq 25 \) km s\(^{-1}\)) and smaller radius(\( \leq 0.1\)pc) are evolving so quickly that it is difficult to observe them. However, to completely account the observational trend, we should investigate the acceleration mechanism of the fast wind since as long as spherical symmetry model is adopted, we need the long time-scale to reach at 10000 km s\(^{-1}\). In order to prove the acceleration mechanism of the shell of PN, we
have to do a more accurate survey of PNe because observations of only very young PNe make us know the mechanism. We expect that such observations will be done in the near future.

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