NUCLEAR ATTENUATION of CHARGED MESONS
IN DEEP INELASTIC SCATTERING

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Abstract

We propose extended version of stationary string model to describe the nuclear attenuation. This model takes into account flavour content of particles and allows to include into consideration all hadrons created from string. The predictions of the model are compared with experimental results obtained by HERMES collaboration (DESY) on different nuclei (N and Kr).

1. Introduction

The study of the hadrons production in deep inelastic lepton-nucleus scattering offers the possibility to investigate quark propagation in dense nuclear matter and the space-time evolution of hadronization. In particular, the measurements of high energy hadrons attenuation in nuclear matter is a well-known tool to specify the parameters of models related to the early stage of particle production.

Hadron production in deep inelastic scattering (DIS) of leptons on nucleons goes through two stages. On the first stage the virtual photon knocks out the (anti)quark from nucleon, which on the second stage produce a observable hadron. The strong interactions theory - perturbative Quantum Chromo Dynamics (QCD) at present can not describe the process of quark hadronization because of in this case "soft" interactions play a greater role. Therefore, experimental investigation of quark hadronization and building of phenomenological models describing the most common characteristics of given process is of importance for development of QCD.

Let us consider the simplest possible state, a color-singlet quark-antiquark system. The lattice QCD studies lend support to a linear confinement picture, i.e. the energy stored in the color dipole field between a charge and an anticharge increases linearly with the separation between the charges. This is quite different from the behaviour in QED and is related to the presence of a triple-gluon vertex in QCD. If the tube is assumed to be uniform along its length, this automatically leads to a confinement picture with a linearly rising potential. From hadron spectroscopy the string tension, i.e. the amount of energy per unit length is deduced to be \( \kappa \simeq 1 \text{ GeV/fm} \) \cite{17}. The linear confinement provide a simple explanation for the existence of linearly rising Regge trajectories. In this scheme the string tension determined by the Regge trajectory slope: \( \kappa = (2\pi \alpha' R)^{-1} \), which is also \( \simeq 1 \text{ GeV/fm} \) \cite{22, 23}.

Hadronization process takes place at distances of few fm, right after the deep inelastic
interaction with nucleon. As this corresponds to the size of the nucleus, it is clear that answers to these questions can be obtained through investigating the processes of hadron production in DIS of leptons on nuclear targets. The available experimental results were obtained on different nuclear targets with (anti)neutrino [1, 2], muon [3, 4], electron [5] and positron [6] beams. The experiments showed the presence of nuclear attenuation effect for charged hadrons with \( x_F > 0.1 \) (\( x_F \) is Feynman’s variable), which is essential in the energy range of \( \sim 10 \) GeV and almost vanish at energies of more than 50 GeV. It means that the multiplicity of hadrons on nuclei is different from the multiplicity on deuterium (both per nucleon). The experiments showed also the strong dependence of this effect on the energy transfered by leptons to nucleus, for electroproduction processes it is energy of virtual photon.

In the meantime, experimental investigations let to development of phenomenological models. For description of nuclear attenuation, mainly the stationary string model (SSM) was used [7, 10]. It is supposed that after the DIS of lepton on internuclear nucleon, a color string is stretched between the knocked out quark and the nucleon remnant, a string which consists of gluons with constant tension over it’s length. The color field of the string creates quark-antiquark and diquark-antidiquark pairs, which lead to breaking of original string into many short strings. This process results in creation of many string-hadrons. The quark which is on the fast end of string, while passing through the nucleus, can exchange the color with one of the quarks of the nucleon, lying within it’s trajectory. As a result, the string breaks on two strings, the energy of the main string reduces and as a consequence, less fast hadrons will be produced in nucleus (per nucleon), than on free nucleon. It is also presumed, that energy and momentum of interacting quarks change insignificantly (color interaction). Some authors suppose that the color interaction cross-section is constant \( \sigma^c_q = \text{const} \) [8, 10], others believe that it depends on \( Q^2 \) [12] (\( Q^2 = -q^2 \), where \( q \) - is the 4-momentum of the virtual photon). We return to this question later.

The model supposes that the fast end of the string always contains a quark. It means that the validity of the model is limited by the valence region \( x_{Bj} > 0.2 \) (\( x_{Bj} \) is Bjorken’s variable), because in the ”sea” region, it is likely that an antiquark can be on the fast end of the string, and model does not describe the antiquark interaction. The quark may also deviate from it’s primary movement direction as a result of interaction with nucleon by means of Pomeron or other Reggeon exchange.

Let us also briefly consider other models for nuclear attenuation. In [13] authors propose a model of hadronization of highly virtual quark in nuclear environment by means of gluong bremsstrahlung and the deceleration of the quark as a result of radiative energy loss. This model practically transforms into the SSM at \( Q^2 < 3 \text{ GeV}^2 \). Recently in [14] the authors supposed that in deeply inelastic eA collisions in the framework of multiple parton scattering, the quark fragmentation functions are modified due to higher-twist effects. The hadronization of the quark takes place outside of the nucleus, which is correct in very high energy ranges. The model has lower limit of application over \( Q^2 \), which is not clearly stated by the authors. Also, unrealistic nuclear density distribution functions for middle and heavy nuclei is used. These questions are important in sense of application to the HERMES kinematics.

Conclusion is that in virtual photon energy region \( \nu = 5 - 25 \) GeV and \( <Q^2> \) equal 2 - 3 GeV\(^2\) which are interesting for us in connection with the HERMES experiment [6], most suitable phenomenological model which can be applied for quantitative calculations
is SSM. Therefore, let us outline those points of this model which are improved in present paper:
- description in final state only hadrons summed over kinds and charges;
- the simple description of nucleon structure without "sea" partons is used. It leads to the limitation of applicability of the SSM only for valence region \((x_{Bj}>0.2)\);
- using only part of the quark-nucleon cross section connected with color interaction;
- the problem connected with description of the \(Q^2\) dependence of quark-nucleon and "hadron"-nucleon cross-sections, where "hadron" means colorless quark-antiquark system on early stage of it creation without "sea".

We propose development of SSM, including in consideration the kind and charge of hadron in final state. For this goal we take into account flavours and flavour contents of (anti)quarks and diquark from initial nucleon and quark-antiquark pairs produced in color field of string and also of final hadron.

Second innovation is that in framework of some fragmentation scheme we calculate the partial energy \(z_h = E_h/\nu\) (where \(E_h\) and \(\nu\) are energy of hadron and virtual photon in target rest frame, respectively) and constituent formation length for any hadron produced from string. It allows to calculate the nuclear attenuation with high accuracy. To simplify the calculations we limited ourselves only a three fastest hadrons. In other hand because of all calculations in this work were performed with the cut on \(z_h > 0.1-0.2\), then one can take in account the limited number of fast hadrons in the string. This is reasonable because of according to multi-periferical model the parton energy during hadronization is divided between the final hadrons in the following proportion: \(z_{h1} \sim 1/2; z_{h2} \sim 1/4; z_{h3} \sim 1/8\) and etc.

We considered in final state only charged mesons, because the mechanism of (anti)protons has some peculiarities, consequently our model in present time does not claim to describe the nuclear attenuation of (anti)protons or \(h^+\) and \(h^-\). We hope to include them in consideration in near future.

1. Description of developed approach

The proposed approach is based on the SSM, in which it is supposed that into the nucleus at the point with longitudinal coordinate \(z\) and the impact vector \(b\) the DIS takes place on one of nucleons (proton or neutron). Between the knocked out (anti)quark and nucleon remnant the color string is stretched. The maximal length of string is \(L \approx \nu/k\) \((k\) is the string tension). Then the breaking of string takes place by means of quark-antiquark and diquark-antidiquark pair production in color field. The first constituent (anti)quark of the hadron with definite kind, charge and partial energy \(z_h\) is produced on a distance \(l_c\) from the point of deep inelastic interaction. The \(l_c\) is satisfied the condition \(0 \leq l_c \leq l_{cmax}\), where \(l_{cmax} = (1 - z_h)\nu/k\). This relation was derived in \([9]\) based on more general reasons than the string model. However this relation could be explained very simple in the framework of the string model if we suppose that hadron in own rest frame is the string with \(l_0 = m_h/k\) length, and in the system where the hadron energy is equal \(E_h\), \(l = l_0E_h/m_h\) and \(l_c = L - l\), thus the relation is realized. Usually to describe the nuclear attenuation effect the normalized (per nucleon) ratio of
hadrons multiplicity produced on nuclear and deuterium targets is used. This ratio could be expressed via the ratio of corresponding fragmentation functions:

\[ R_{hA}^{h}(z_h) = \frac{2 D_{hA}^{h}(z_h)}{AD_{hD}^{h}(z_h)} \]  

where \( D_{hA}^{h}(z_h) \) and \( D_{hD}^{h}(z_h) \) are the quark fragmentation functions in deuterium (assumed same as in vacuum) and in \( A \) nucleus respectively.

\[
D_{hA}^{h}(z_h) = E(x_{Bj}) \int d^2b \int_{-\infty}^{\infty} dz \rho(b,z) \int_0^{l_{c,max}} dl_c T_h(z + l_c, \infty) \sum_{i=1}^{n} C_{fi}^{h}(A, x_{Bj}, Q^2) D_{c}^{i}(z_h, l_c) \tag{2}
\]

\[
D_{hD}^{h}(z_h) = \int_0^{l_{c,max}} dl_c \sum_{i=1}^{n} C_{fi}^{h}(2, x_{Bj}, Q^2) D_{c}^{i}(z_h, l_c) F_i(z_h) \tag{3}
\]

\[
T_{q(h)}(a, b) = exp(-\int_a^b dz_1 \sigma_{q(h)}(z_1) \rho(b, z_1)) \tag{4}
\]

where:  
- \( b \) - is the impact parameter;  
- \((b, z)\) - is the coordinate of point in which the DIS takes place on one of internuclear nucleons;  
- \((b, z')\) - is the point at which the color interaction of the (anti)quark located at the end of string with one of the internuclear nucleons being on trajectory of movement of knocked out quark takes place;  
- \( E(x_{Bj}) \) - function, which takes into account the EMC-effect, the suppression of nuclear structure function at large values of the \( x_{Bj} \) \[12\];  
- \( \rho(r) \) - is nuclear density function. To parametrize this density the shell model \[15\] was used for light nuclei , and the Woods-Saxon parametrization is used \[16\] for middle and heavy nuclei. The normalization condition is \( \int dr \rho(r)=A \);  
- \( z_{h'}=z_h \nu/\nu(t) \), description of \( \nu(t) \) will be given after expression (6);  
- \( n \) - is the ordinal number of hadron produced from the string. The hadron created at the fast end of string (usually it is called the leading hadron) has the ordinal number \( n = 1 \), hadron created behind it has the ordinal number \( n =2 \) and etc. To simplify the numerical calculations the limited values \( n \leq 3 \) were used.

\[ C_{fi}^{h}(A, x_{Bj}, Q^2) \] - are the functions, which take into account the probability of hadron’s

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1 Strictly speaking the expression (2) is valid in valence region \( x_{Bj} > 0.2 \), since in this region leading parton in string is quark. In “sea” region leading parton may be also antiquark, which color interaction cross section can be different from it for quark \( \sigma_q \neq \sigma_{\bar{q}} \). However for the sake of simplicity we do not take into account this difference.
creation (with given type, charge and ordinal number \(i\) in string) from quarks and antiquarks being into the internuclear nucleon on which the DIS takes place and \(q\bar{q}\) pairs created in color field of the string. The detailed shape for this function will be presented in Appendix A.

\(D_i^c(z_h, l_c)\) - are constituent formation length \(l_c\) distribution functions for hadron \(i\) from string which carry away the partial energy \(z_h\). Detailed shape for these functions is given in Appendix B.

\(F_i(z_h)\) - are probabilities that \(i\)-th hadron produced from string carry away the part of the virtual photon energy \(z_h\). Detailed shape for this function is given in Appendix C.

\(\sigma_h(z)\) is the hadron-nucleon total cross section.

\(\sigma_q(z)\) is the total cross section of quark-nucleon interaction which could be presented as a sum of two components.

\[
\sigma_q(z) = \sigma^c_q(z) + \sigma^0_q(z) \tag{5}
\]

where \(\sigma^c_q(z)\) is part of the total cross section, which is connected with color interaction without essential change of energy and momentum of interacting partons. This part of cross section can be parametrize according to [12]:

\[
\sigma^c_q(t) = \frac{C}{Q^2(t)} \tag{6}
\]

\(t = z' - z;\quad Q^2(t) = \frac{\nu(\nu Q^2)}{\nu + iQ^2};\quad \nu(t) = \nu - kt;\quad C = 1.32\, \text{mbGeV}^2.\)

Parametrization (6) has bottom limit of validity which is equal \(Q^2 = Q^2_0 = 0.06\) GeV\(^2\). At \(Q^2 < Q^2_0\) \(\sigma^c_q(Q^2) = C/(QQ_0)\) is using. The shape of (6) is obtained from the idea of color transparency, therefore naturally suppose that other part of quark nucleon cross section has the the same shape:

\[
\sigma^0_q(t) = \frac{C'}{Q^2(t)} \tag{7}
\]

where \(C'\) is the constant generally speaking different from \(C\). However in order to escape the supplementary fit we assume in this work that \(C' = C\). Really in hadronization process we have deal not with isolate quark, but with quark which together with antiquark compose the string or piece of string (hadron). This system is colorless quark-antiquark dipole. Consequently, instead of (7) we can use the phenomenological dipole cross section. For example simple and convenient parametrization has been suggested in [24]. Easy to see that for the HERMES \(\nu\) and \(Q^2\) kinematic region the expression (9) from [24] turn in formula (7) of our work. Parameter \(C'\) which obtained in this manner from dipole cross section is weakly changing function over \(\nu\) and \(Q^2\), with the middle value of \(C' \sim 1\) mb-GeV\(^2\). However, taking into account, that in energy range \(W^2 = 4 \div 50\) GeV\(^2\) the value of \(\sigma^\text{tot}_{\pi N}\) from [24] seems to be underestimated, we are using \(\sigma^\text{tot}_{\pi N} = 25\) mb, and obtained for \(C'\) numerical value close to \(C\).

In fact to describe the attenuations measured experimentally as function of \(\nu, Q^2\), or
z_h, we are integrating the expression (2) over two of three mentioned above variables.

The calculations were performed using the following values for total cross section of hadron h with internuclear nucleon (we suppose that cross section for the interaction of hadron with proton and neutron are equal):

\[ \sigma_{\pi^+} = \sigma_{\pi^-} = 25 \text{ mb}; \quad \sigma_{k^+} = 17 \text{ mb}; \quad \sigma_{k^-} = 23 \text{ mb}. \]

Some authors are using the inelastic cross section instead of total ones [19]. Unfortunately, the available experimental information does not allow to make a choice between these two possibilities.

In present work the value of 1 GeV/fm is used for the string tension.

Also for all calculations the following kinematical conditions were used:

- \( P_h > 0.5 \text{ GeV} \),
- \( Q^2 > 1 \text{ GeV}^2 \),
- \( \nu > 4 \text{ GeV} \),
- \( W^2 > 4 \text{ GeV}^2 \),
- \( y < 0.85 \),
- \( z_h > 0.1 \)

where \( P_h \) is the hadron momentum, \( W^2 \) is the hadron invariant mass square, \( y = \frac{\nu}{E_0} \), \( E_0 \) is the incident lepton energy (\( E_0 = 27.5 \text{ GeV} \)).

With the presence of more data it will be possible to make a global fit to determine the optimal values of string tension, the parameter \( C \) in quark-nucleon cross section as well as to precise the value of \( \sigma_{\pi^\pm} \) and \( \sigma_{k^\pm} \) cross-sections.

2. Results and Discussion

The obtained results for the nuclear attenuation of charged pions and kaons \( (R_A^{\pi(k)}) \) calculated for four different targets (N, Ne, Kr, Xe) are presented on the figures 1-6. It is clearly seen that in case of pions attenuation as a function of \( \nu \) there is no difference for opposite charges of pions. In case of kaons this difference becomes noticeable at low values of \( \nu \) (\( \nu < 12 \text{ GeV} \)). Also one can see that the attenuation for \( \pi (K)^- \) in case of symmetric targets is larger than for \( \pi (K)^+ \). While in case of non-symmetric targets the situation is vice versa. For \( z_h \) dependence one can note that the difference between opposite charged hadrons is increased for pions, particularly for heavy targets. Because of mainly HERMES Collaboration [6, 21] has produced the data with different charged pion’s attenuations during last years, the calculated curves were tuned to the HERMES kinematics. The following additional cuts were applied: \( \nu > 7 \text{ GeV}, \quad z_h > 0.2, \quad 4.0 < P_h < 13.5 \text{ GeV} \) for N and Kr, respectively. One can see good enough agreement for the predicted \( \nu \)-dependence of \( \pi^\pm \), attenuations in case of N and Kr (Fig. 7a, c). For \( z_h \) dependence the sum of \( \pi^+ \) and \( \pi^- \) attenuations has been published and except one point there is also good agreement for calculated values and data (Fig. 7b).

In conclusion, we have to note that developed approach can provide much better description of data after the fitting procedure based on all existing data for different type of hadrons and targets. The parton distributions used for this approach [18] could be modified using the last results on parametrization for special case of nuclear parton distributions. As it was discussed above in the valence region the quark is knocked out from the nucleon, so the color string is stretched between this quark and nucleon remnant, and the final hadrons are produced from the string. In the "sea" region mainly the "sea" (anti)quark is knocked out from the nucleon and two strings are created. One of them with the knocked out (anti)quark takes away the main part of virtual photon energy
passing the next string an energy $\Delta E$ which is enough to produce 1-2 hadrons or one resonance. In the framework of Regge model such production of hadrons from the big and little strings is called "Undeveloped Pomeron". This mechanism could lead to some changes of the attenuation values (particularly at low $\nu$), as well as to relative changes for different hadrons and charges for the same hadron. As it is followed from the Regge model the $\Delta E$ value is estimated to be 1 - 3 GeV and this value also should be fitted. The preliminary estimations of possible influence of the mentioned above scheme with $\Delta E \sim$ 1 - 3 GeV showed that they expected changes in obtained results will be not essential. In generally we have to note that the attenuation values for different types of hadrons as well as for the oppositely charged mesons of the same kind are very close. For the most heavy target $^{131}$Xe the difference between the charged mesons is less than 5 %, and for different mesons is less than 10 %. In the same time one can see strong enough $\nu$ dependence for attenuations within the HERMES kinematics range.

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Appendix A: Functions $C^h_{fi}(A,x_{Bj})$

Notations: $i = 1,2,3$ means the ordinal number of three hadrons produced last in string (and the first number corresponds the last one)

a) $\pi^+(u\bar{d})$

$$C^\pi_{f1} = \left\{ \frac{Z}{A} \left[ \frac{4}{9} (u_v(x_{Bj},Q^2) + u_s(x_{Bj},Q^2)) + \frac{1}{9} \bar{d}_s(x_{Bj},Q^2) \right] + \frac{N}{A} \left[ \frac{4}{9} (d_v(x_{Bj},Q^2) + d_s(x_{Bj},Q^2)) + \frac{1}{9} \bar{u}_s(x_{Bj},Q^2) \right] \right\} \gamma_q$$

where $A$ - is atomic number; $Z$ and $N$ - numbers of protons and neutrons in nuclei; $\gamma_q$ - is the probability of light $qq$ pairs ($u\bar{u},d\bar{d}$) production in color field [17]; $u_v(x_{Bj},Q^2)$, $d_v(x_{Bj},Q^2)$, $u_s(x_{Bj},Q^2)$ etc. - parton distribution functions in proton. We used parametrization of NLO(\bar{MS}) parton distributions from [18].

$$C^\pi_{f2} = \left\{ \frac{Z}{A} \left[ \frac{1}{9} (d_v + d_s + \bar{d}_s) \right] + \frac{N}{A} \left[ \frac{1}{9} (u_v + u_s + \bar{u}_s) \right] \right\} \gamma^2_q$$

$$C^\pi_{f3} = C^\pi_{f2}$$

b) $\pi^-(d\bar{u})$

$$C^\pi_{f1} = \left\{ \frac{Z}{A} \left[ \frac{4}{9} (u_v + u_s + \bar{u}_s) \right] + \frac{1}{9} (d_v + d_s + \bar{d}_s + s_s + \bar{s}_s) \right\} \gamma_q$$

$$C^\pi_{f2} \text{ and } C^\pi_{f3} \text{ the same as in previous case.}$$

c) $k^+(u\bar{s})$

$$C^{k+}_{f1} = \left\{ \frac{Z}{A} \left[ \frac{4}{9} (u_v + u_s) \gamma_s + \frac{1}{9} \bar{s}_s \gamma_q \right] + \frac{N}{A} \left[ \frac{4}{9} (d_v + d_s) \gamma_s + \frac{1}{9} \bar{d}_s \gamma_q \right] \right\} \gamma_q$$

$$C^{k+}_{f2} = C^{k+}_{f3} = C^{\pi^+}_{f2} \gamma_q$$

where $\gamma_s$ - is the probability of $ss$ pairs production in color field [17].

d) $k^-(s\bar{u})$

$$C^{k^-}_{f1} = \left\{ \frac{Z}{A} \left[ \frac{1}{9} s_s \gamma_q + \frac{4}{9} \bar{u}_s \gamma_s \right] + \frac{N}{A} \left[ \frac{1}{9} s_s \gamma_q + \frac{4}{9} \bar{d}_s \gamma_s \right] \right\}$$

In further (for brevity) we omit arguments of parton distribution functions
\[ C_{f2}^k = C_{f3}^{k-} = C_{f2}^{k+} \]

For the all described here hadrons: \[ C_{f1}^h = C_{f2}^{h} \], where \( i = 3, 4, 5 \)

Appendix B: The distribution of constituent formation lengths in Field-Feynman fragmentation scheme

\( D_i^c(x, l_c) \) - is the distribution function of constituent formation lengths \( l_c \), for \( i \)-th hadron in string carry away part of energy \( x \) of virtual photon. To obtain these functions in [19] it was used the function \( f(x) \), which have the meaning of probability, that the first hierarchy (rank - 1) primary meson leaves the fraction of momentum \( x \) to the remaining cascade. We are using function \( f(x) \) obtained in [20]: \( f(x) = 1 - a + 3ax^2 \)

As it is well known the \( \pi \) and \( K \) mesons could be produced in direct way and also as a result of resonances decay. In this work only the direct production is taking in account. In case of pions it was estimated the contribution coming from the \( \rho \) meson decay and it was shown that the second mechanism (via resonances) did not change the values of calculated attenuations.

As noticed above, we are limited only with three hadrons on the fast end of string. For these hadrons the distribution functions are given as

\[
D_1^c(x, l_c) = f(1 - x)\delta[l_c - (1 - x)L] 
\]

\[
D_2^c(x, l_c) = \frac{1}{x + \frac{l_c}{L}} f\left(\frac{l_c}{x + \frac{l_c}{L}}\right) \frac{1}{L} f(x + \frac{l_c}{L}) \tag{14}
\]

\[
D_3^c(x, l_c) = \frac{1}{x + \frac{l_c}{L}} f\left(\frac{l_c}{x + \frac{l_c}{L}}\right) \frac{1}{L} \int_{u_{1\text{min}}}^{x} \frac{du_1}{u_1} f\left(\frac{Lc + xL}{Lu_1}\right) \tag{15}
\]

where \( u_{1\text{min}} = \frac{L + xL}{L} \); \( L \) - length of string (\( L = \nu/k \)).

The general formula for \( D_i^c(x, l_c) \) is given as

\[
D_i^c(x, l_c) = \frac{1}{x + \frac{l_c}{L}} f\left(\frac{l_c}{x + \frac{l_c}{L}}\right) \frac{1}{L} \int_{u_{1\text{min}}}^{x} \frac{du_1}{u_1} f\left(\frac{Lc + xL}{Lu_1}\right) \cdots \int_{u_{i-2\text{min}}}^{x} \frac{du_{i-2}}{u_{i-2}} f\left(\frac{Lc + xL}{Lu_1...u_{i-2}}\right) \tag{16}
\]

where \( i = 3, 4, 5, \ldots \), and \( u_{j\text{min}} = \frac{L_c + xL}{Lu_1...u_{j-1}} \)

Appendix C: The functions \( F_i(x) \)

\( F_i(x) \) - is the probability, that the \( i \)-th hadron in string carry away the fraction of momentum \( x \).

The general formula for \( F_i(x) \) is given as

\[
F_i(x) = \int_{\eta_1}^{\eta_{i-1}} \frac{d\eta_1}{\eta_1} f(\eta_1) \int_{\eta_{i-1}}^{\eta_{i-2}} \frac{d\eta_2}{\eta_2} f(\eta_2) \cdots \int_{\eta_{i-2}}^{\eta_{i-3}} \frac{d\eta_{i-1}}{\eta_{i-1}} f(\eta_{i-1}) f\left(1 - \frac{x}{\eta_1...\eta_{i-1}}\right) \tag{17}
\]
For cases of $i = 1, 2, 3$

$$F_1(x) = f(1 - x)$$ \hspace{1cm} (18)

where $f(x)$ was defined in Appendix B

$$F_2(x) = \int_x^1 \frac{d\eta}{\eta} f(\eta) f(1 - \frac{x}{\eta})$$ \hspace{1cm} (19)

$$F_3(x) = \int_x^1 \frac{d\eta}{\eta} f(\eta) \int_{x/\eta}^1 \frac{d\eta_1}{\eta_1} f(\eta_1) f(1 - \frac{x}{\eta\eta_1})$$ \hspace{1cm} (20)
References

[1] J. Berge et al. Phys. Rev. D18(1978) 3905
[2] W. Burkot et al. Z. Phys. C.70(1996) 47
[3] A. Arvidson et al. Nucl. Phys. B246(1984) 381
[4] J. Ashman et al. Z. Phys. C52(1991) 1
[5] L. Osborne et al. Phys. Rev. Lett. 40(1978) 1624
[6] A. Airapetian et al., Eur. Phys. J. C., DOI 10.1007/s100520100697 (2001).
[7] A. Bialas Acta Phys. Pol. B11(1980) 475
[8] M. Gyulassy et al. Nucl. Phys. B346(1990) 1
[9] B. Kopeliovich Phys. Lett. B243(1990) 141
[10] J. Czyzewski and P. Sawicki. Z.Phys. C 56(1992) 493
[11] B. Kopeliovich et al. preprint of JINR E2-91-150(1991), Dubna
[12] B. Kopeliovich et al. Hadron Structure '92. Proceedings Stara Lesna, Czechoslovakia, Sept. 6-11, 1992 P.164
[13] B. Kopeliovich, J. Nemchik, E. Predazzi. "Future Physics at Hera" DESY 96-235(1996) v.2 P.1038
[14] X. Guo et al. Phys. Rev. Lett. 85(2000) 3591; X.-N. Wang hep-ph/0111404; E. Wang et al. hep-ph/0202103
[15] L. Elton ”Nuclear Sizes” Oxford University Press, 1961, p.34
[16] A. Bialas et al. Phys. Lett. B133(1983) 241
[17] T. Sjöstrand preprint CERN-TH 7112/93(1993)
[18] M. Glück et al. Z. Phys. C67(1995) 433
[19] A. Bialas, M.Gyulassy Nucl. Phys. B291(1987) 793
[20] R. Field, R.Feynman Nucl. Phys. B136(1978) 1
[21] V. Muccifora hep-ex/0106088 (2001)
[22] A. Casher, H.Neuberger, S.Nussinov Phys. Rev. D20(1979) 179
[23] E. G. Gurvich Phys. Lett. B87(1979) 386
[24] B. Kopeliovich et al. Phys. Rev. C65(2002) 035201
Figure 1: The $\nu$-dependence of the ratio $R_A^{\pi}$ for the different targets: a) $^{14}$N; b) $^{20}$Ne; c) $^{84}$Kr; d) $^{131}$Xe. The dashed curves correspond to the case $\pi^-$ and dotted - $\pi^+$ mesons.
Kaon

Figure 2: The same as in Fig. 1, but calculated for the charged kaons
Figure 3: The $z_h$ - dependence of the ratio $R_A^\pi$ for the different targets: a) $^{14}$N; b) $^{20}$Ne; c) $^{84}$Kr; d) $^{131}$Xe. The dashed curves correspond to the case $\pi^-$ and dotted - $\pi^+$ mesons.
Figure 4: The same as in Fig. 3, but calculated for the charged kaons
Figure 5: The $Q^2$ - dependence of the ratio $R_A^\pi$ for the different targets: a) $^{14}$N; b) $^{20}$Ne; c) $^{84}$Kr; d) $^{131}$Xe. The dashed curves correspond to the case $\pi^-$ and dotted - $\pi^+$ mesons.
Figure 6: The same as in Fig. 5, but calculated for the charged kaons.
Figure 7: The comparison of predicted values for attenuation with HERMES data: a) $\pi^\pm$ vs $\nu$ for $^{14}$N [6]; b) $\pi^+ + \pi^-$ vs $z_h$ for $^{14}$N [6]; c) $\pi^\pm$ vs $\nu$ for $^{85}$Kr [21] (preliminary results)