Analysis of harmonic oscillator mass parameters influence on multiband gap widening for Drude model acoustic metamaterial

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Abstract: Acoustic metamaterial is a kind of special microstructure of man-made composite material with vibration attenuation band gap. In view of the band gap widening methods, the Drude model with additional double harmonic oscillators five periodic spring-mass system is established, and then the steady-state displacement and velocity of each oscillator under different external excitation frequencies were calculated, the vibration transmission characteristic of the system is obtained. The band gap widening method is analyzed. The results show that, compared with the Drude model, a new high frequency band gap is developed by adding double harmonic oscillators, and with the increase of the additional oscillator mass, the total bandwidth of low frequency band gap is widen. The research provides theoretical basis and application basis to the design of vibration isolation structure implement specific frequency band of noise reduction.

1. Introduction
Under the external excitation, most of the natural materials respond in the same phase as the external excitation. However, a kind of composite materials can be realized by the careful design of the microstructure and the use of local resonance mechanism, so that the properties of the composite are anti-external excitation. This kind of materials with unusual response can be classified as metamaterials[1]. For example, in the field of acoustics, locally resonant phonon crystals [2] and periodically arranged Helmholtz cavity structures[3]. By adjusting the material microstructure, can realize the negative effective material parameters, such as local resonant phononic crystal can take advantage of the microstructure of local resonance, obtain the negative effective mass density near the resonant frequency.

The principle of negative quality can be revealed through a simple mass-spring system[4-6], Milton and Willis[4] studied the negative mass effect of the microstructure, Yao[7] realized with broadband acoustic metamaterial negative equivalent mass by experimental method, this kind of phenomenon analogy electromagnetic metamaterials intermediary electric constant meet the Drude model, got the wide band gap is lower than the resonant frequency.

The more demand for multiband wide band gap is strong in sound insulation and vibration isolation of engineering applications. In this paper, the Drude model is added with two harmonic oscillators, by changing the mass parameters the harmonic oscillators, the way to broaden the low-frequency band gap range is revealed, and the method of broadening the range of low frequency band gap is sought, and the feasibility of realizing multi-band band gap and wide-band band gap acoustic metamaterials is explored, so as to provide theoretical guidance for the application of vibration and noise reduction of acoustic
2. Dynamics equations for Drude model with series double harmonic oscillators acoustic metamaterial

The Drude model of finite-period acoustic metamaterials consists of two internal springs (elastic coefficient $G$) and a ring shell with mass $M$, each of which is attached to a fixed massless pillar inside the shell. Between each shell by a spring (elastic coefficient $k$), respectively. For the double harmonic oscillators with mass $m_1$ and $m_2$ in series outside the shell, the spring stiffness coefficients of the two harmonic oscillators are $k_1$, $k_2$, as shown in figure 1. Under the action of external excitation $F \sin \omega t$, the five-period oscillator model vibrates in one-dimensional range. Assuming that all springs at the initial moment are in the original long state, $x_1$–$x_5$ are 1–5 period shell displacements, $x_6$–$x_{10}$ are 1–5 period $m_1$ displacements, and $x_{11}$–$x_{15}$ are 1–5 period $m_2$ displacements respectively. Then the dynamic equations of each oscillators with five periods of $M$, $m_1$ and $m_2$ are obtained as follows:

$$
\begin{align*}
&M\ddot{x}_1 = F \sin \omega t + k(x_2 - x_1) - 2Gx_2 + c(\dot{x}_2 - 2\dot{x}_1) - k_1(x_1 - x_5) \\
&M\ddot{x}_2 = k(x_1 - 2x_2 + x_3) - 2Gx_3 + c(\dot{x}_3 - 2\dot{x}_1 + \dot{x}_2) - k_1(x_2 - x_6) \\
&M\ddot{x}_3 = k(x_2 - 2x_3 + x_4) - 2Gx_4 + c(\dot{x}_4 - 2\dot{x}_1 + \dot{x}_3) - k_1(x_3 - x_7) \\
&M\ddot{x}_4 = k(x_3 - 2x_4 + x_5) - 2Gx_5 + c(\dot{x}_5 - 2\dot{x}_1 + \dot{x}_4) - k_1(x_4 - x_8) \\
&M\ddot{x}_5 = k(x_4 - x_5) - 2Gx_4 + c(\dot{x}_4 - \dot{x}_5) - k_1(x_5 - x_9) \\
&m_1\ddot{x}_6 = k_1(x_1 - x_5) - k_1(x_6 - x_10) \\
&m_2\ddot{x}_7 = k_2(x_6 - x_5) \\
&m_2\ddot{x}_8 = k_1(x_5 - x_9) - k_2(x_7 - x_12) \\
&m_2\ddot{x}_9 = k_2(x_7 - x_12) \\
&m_2\ddot{x}_{10} = k_1(x_9 - x_8) - k_2(x_8 - x_{13}) \\
&m_2\ddot{x}_{11} = k_2(x_9 - x_{13}) \\
&m_2\ddot{x}_{12} = k_1(x_8 - x_7) - k_2(x_9 - x_{14}) \\
&m_2\ddot{x}_{13} = k_2(x_9 - x_{14}) \\
&m_2\ddot{x}_{14} = k_1(x_5 - x_{10}) - k_2(x_{10} - x_{13}) \\
&m_2\ddot{x}_{15} = k_2(x_{10} - x_{13})
\end{align*}
$$

(1)
3. Calculation and analysis of acoustic metamaterial band gap for Drude model with series double harmonic oscillators

For ease of comparison and analysis, selection the experimental parameters of references[7]are as follows:

\[ M=0.1011, m_1=0.1011, m_2=0.1011, k=117, k_1=37, k_2=37, G=37 \]

the units of the above physical quantities are SI.

In the first period, external excitation \( F \sin \omega t \) is applied to the shell, and in the fifth period, the steady-state response is picked up to calculate the vibration transmission characteristics of the system, as shown in Figure 2.

![Figure 2](image)

Figure 2. Transmission characteristics of five period Drude model with two harmonic oscillators.

In figure 2, 1.56 Hz, 1.78 Hz to 4.30 Hz and 4.86 Hz to 5.88 Hz, the vibration transmission characteristics is negative, the fifth cycle shell shell of steady-state vibration displacement is smaller than the first period steady-state vibration displacement, which show that the frequency of vibration can't pass from the first period to fifth period, that is vibration is suppressed, the corresponding frequency range belongs to the band gap.

When the external excitation frequency is 5.5Hz, the steady-state response of the five period shells are shown in Figure 3. Figure 3 shows that, after a period of time, the displacement-velocity phase curve of the shells is elliptical indicating that the vibration of the shell is the steady-state stage. From the first period to the fifth period, the steady-state amplitude of the casing decreases successively from 0.05m to 0.008m, and the vibration loaded on the shell in the first period is suppressed and cannot be effectively transmitted to the shell in the fifth period, which is a characteristic in the typical band gap range.

According to the conclusion of literature[7], Drude model band gap below \( \frac{\sqrt{2G}}{M} = 4.3 \) Hz and 0-4.3 Hz range is the single frequency band. The external series double harmonic oscillator will produce three band gaps shows in Figure2, of which the range of the first and second band gaps is almost the same as that of the Drude model, and the third band gap is the newly added high frequency band gap. The total band width of three band gap is increased, in another word, the multi-band band gap is widened.
4. Analysis of harmonic oscillator mass parameters influence on multiband gap widening

For analysis of double harmonic oscillator quality parameters on the band gap range, the influence of selected \( m_1, m_2 \) for different parameters of 8 kinds of calculating the vibration transfer characteristics, as shown in figure 4, the corresponding three calibration band gap range, as shown in table 4.

![Figure 3](image1.png)

(a) displacement time history curves  (b) phase diagrams

Figure 3. Displacement time history curves and phase diagrams of five periodic shells (5.5 Hz).

![Figure 4](image2.png)

Figure 4. Transmission characteristics with different \( m_1 \) and \( m_2 \).

According to the results in Figure 4 and Table 1, when the double resonators are added, the band gaps of the system change from a single band gap to three band gaps. The band gaps of the first band gap and the second band gap are basically the same as those of the Drude model without the oscillator. The third band gap appears in the high frequency range, which widens the overall band gap with additional oscillators.

With \( m_1 = 0.1011 \text{kg} \) unchanged, \( m_2 \) gradually increased from 0.05055-0.3033 kg, the band gap range of the three band gaps shifted to low frequency range, the total bandwidth of the three band gaps basically remained unchanged, and the band gap range of the third band gap gradually increased.
With \( m_2 = 0.1011 \text{kg} \) unchanged, \( m_1 \) gradually increases from 0.05055-0.4044kg, the band gap range of the three band gaps shifts to the low frequency band as a whole as well, the total bandwidth of the three band gaps basically remains unchanged, and the range of the third band gap gradually also increases.

| case | \( m_1 \), \( m_2 \) (kg) | First band (Hz) | Second band (Hz) | Third band (Hz) |
|------|------------------------|-----------------|-----------------|-----------------|
| 1    | \( m_1=0.1011, m_2=0.05055 \) | 0-1.87          | 2.23-4.93       | 5.57-6.09       |
| 2    | \( m_1=0.1011, m_2=0.1011 \) | 0-1.56          | 1.78-4.30       | 4.86-5.88       |
| 3    | \( m_1=0.1011, m_2=0.2022 \) | 0-1.21          | 1.36-3.93       | 4.50-5.83       |
| 4    | \( m_1=0.1011, m_2=0.3033 \) | 0-0.95          | 1.35-3.81       | 4.49-5.81       |
| 5    | \( m_1=0.05055, m_2=0.1011 \) | 0-1.67          | 2.07-4.67       | 6.32-7.02       |
| 6    | \( m_1=0.2022, m_2=0.1011 \) | 0-1.33          | 1.69-3.79       | 4.00-5.48       |
| 7    | \( m_1=0.3033, m_2=0.1011 \) | 0-1.19          | 1.38-3.55       | 3.65-5.40       |
| 8    | \( m_1=0.4044, m_2=0.1011 \) | 0-1.01          | 1.43-3.43       | 3.64-5.36       |

5. Conclusion

The additional series double resonant oscillator with the Drude model will expand the single band gap of the original Drude model into three band gaps, where the first and second band gaps are basically the same as the single band gap range of the Drude model, and the third band gap appears in the high frequency band of the single Drude model. Therefore, compared with the single Drude model, the additional series double resonant oscillator will broaden the overall band gap range, which is beneficial to vibration and noise reduction. The change of the mass of the additional oscillator will affect the range of the three band gaps. When \( m_1 \) remains unchanged, the band gap range of the third band gap gradually increases with the increase of \( m_2 \), the three band gaps will shift to the low frequency band, and the total bandwidth of the three band gaps will basically remain unchanged. Keeping \( m_2 \) constant, the result is similar as the value of \( m_1 \) increases. In general, increasing the mass of the additional vibrators is beneficial to low frequency vibration reduction.

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