Novel ballistic to diffusive crossover in the dynamics of a one-dimensional Ising model with variable range of interaction

Soham Biswas and Parongama Sen
Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata 700009, India
E-mail: soham.physics@gmail.com

Received 2 December 2010, in final form 15 February 2011
Published 15 March 2011
Online at stacks.iop.org/JPhysA/44/145003

Abstract
The idea that the dynamics of a spin is determined by the size of its neighbouring domains was recently introduced (Biswas and Sen 2009 Phys. Rev. E 80 027101) in a Ising spin model (henceforth, referred to as model I). A parameter $p$ is now defined to modify the dynamics such that a spin can sense domain sizes up to $R = pL/2$ in a one-dimensional system of size $L$. For the cutoff factor $p \to 0$, the dynamics is Ising like and the domains grow with time $t$ diffusively as $t^{1/z}$ with $z = 2$, while for $p = 1$, the original model I showed ballistic dynamics with $z \simeq 1$. For intermediate values of $p$, the domain growth, magnetization and persistence show a model-I-like behaviour up to a macroscopic crossover time $t_1 \sim pL/2$. Beyond $t_1$, characteristic power-law variations of the dynamic quantities are no longer observed. The total time to reach equilibrium is found to be $t = apL + b(1 - p)^3L^2$, from which we conclude that the later time behaviour is diffusive. We also consider the case when a random but quenched value of $p$ is used for each spin for which ballistic behaviour is once again obtained.

PACS numbers: 05.40.Ca, 02.50.Ey, 03.65.Vf, 74.40.Gh

(Some figures in this article are in colour only in the electronic version)

1. Introduction

An important field of research in statistical physics is concerned with different types of dynamical phenomena. Physical quantities in self-organized and/or driven systems show a rich time-dependent behaviour in many cases. Some of the dynamical phenomena which have attracted a lot of attention are critical dynamics, quenching and coarsening, reaction diffusion systems, random walks, etc.
In most of these phenomena, we find that there is a single timescale leading to a uniform time-dependent behaviour which in many cases is a power-law decay or growth [2]. However, in some complex systems, it has been observed that the dynamics is governed by a distinct short-time behaviour followed by a different behaviour at long times. For example, in spin systems, at criticality, the order parameter is observed to grow for a macroscopically short time [3] while at longer times it decays in an expected power-law manner. For correlated random walks, e.g., the persistent random walk on the other hand, one finds a ballistic (i.e. when the root mean square (rms) displacement scales linearly with time) to diffusive (the rms displacement varying as the square root of time) crossover in the dynamics [4]. Random walks on small world networks show a completely opposite behaviour; the number of distinct sites visited by the walker has an initial diffusive scaling followed by a ballistic variation with time [5]. This is also true for a biased random walker.

In this paper, we study a dynamical model of Ising spins in one dimension which is governed by a single parameter. The system is a generalized version of a recently proposed model in [1] (which we refer to as model I henceforth) where the state of the spins ($S = \pm 1$) may change in two situations. First when its two neighbouring domains have opposite polarity, and in this case the spin orients itself along the spins of the neighbouring domain with the larger size. This case may arise only when the spin is at the boundary of the two domains. The neighbouring domain sizes are calculated excluding the spin itself; however, even if it is included, there is no change in the results. Second, a spin is also flipped when it is sandwiched between two domains of spins with the same sign. When the two neighbouring domains of the spin are of the same size but have opposite polarity, the spin will change its orientation with 50% probability. Except for this rare event, the dynamics in the above model is deterministic. This dynamics leads to a homogeneous state of either all spins up or all spins down. Such evolution to absorbing homogeneous states is known to occur in systems belonging to directed percolation (DP) processes, zero temperature Ising model, voter model, etc [6, 7].

Model I was introduced in the context of a social system where the binary opinions of individuals are represented by up and down spin states. In opinion dynamics models, such a representation of opinions by Ising or Potts spins is quite common [8]. The key feature is the interaction of the individuals which may lead to phase transitions between a homogeneous state and a heterogeneous state in many cases [9].

Model I showed the existence of novel dynamical behaviour in a coarsening process when compared to the dynamical behaviour of DP processes, voter model, Ising models, etc [10–14]. The domain sizes were observed to grow as $t^{1/z}$ with the exponent $z$ very close to unity. It may be noted that the dynamics of a domain wall can be visualized as the movement of a walker and therefore the value $z \approx 1$ indicated that the effective walks are ballistic. When stochasticity is introduced in this model, such that spin flips are dictated by a so-called temperature factor, it shows a robust behaviour in the sense that only for the temperature going to infinity there is a conventional Ising-model-like behaviour with $z = 2$, i.e. the domain wall dynamics becomes diffusive in nature [15].

In this work, we have introduced the parameter $p$, which we call the cutoff factor, such that the maximum size of the neighbouring domains a spin can sense is given by $R = pL/2$ in a one-dimensional system of $L$ spins with a periodic boundary condition. It may be noted that for $p = 1$, we recover the original model I, where there is effectively no restriction on the size sensitivity of the spins. $R = 1$ corresponds to the nearest neighbour Ising model where $p \to 0$ in the thermodynamic limit.

By the introduction of the parameter $p$, we have essentially defined a restricted neighbourhood of influence on a spin. Thus, here we have a finite neighbourhood to be considered, which is like having a model with finite long-range interaction. In addition, here
we impose the condition that within this restricted neighbourhood, the domain structure is also important in the same way it was in model I. If one considers opinion dynamics systems (by which model I was originally inspired), the domain sizes represent some kind of social pressure. A finite cutoff (i.e. $p < 1$) puts a restriction on the domain sizes which may correspond to geographical, political, cultural boundaries, etc. The case with a uniform cutoff signifies that all the individuals have the same kind of restriction; we have also considered the case with random cutoffs which is perhaps closer to reality.

In section 2, we describe the dynamical rule and quantities estimated. We present the results for the case when $p$ is the same for all spins in sections 3 and 4, and in section 5 we consider the case when the values of $p$ for each spin is random, lying between zero and unity and constant over time for each spin. In section 6, we end with concluding remarks.

2. Dynamical rule and quantities calculated

As mentioned before, only the spins at the boundary of a domain wall can change its state. When sandwiched between two domains of the same sign, it will be always flipped. On the other hand, for other boundary spins (termed the target spins henceforth), there will be two neighbouring domains of opposite signs. For such spins, we have the following dynamical scheme. Let $d_{\text{up}}$ and $d_{\text{down}}$ be the sizes of the two neighbouring domains of types up and down of a target spin (excluding itself), respectively. In model I, the dynamical rule was like this. If $d_{\text{up}}$ is greater (less) than $d_{\text{down}}$, the target spin will be up (down) and if $d_{\text{up}} = d_{\text{down}}$, the target spin is flipped with the probability 0.5. Now, with the introduction of $p$, the definition of $d_{\text{up}}$ and $d_{\text{down}}$ are modified: $d_{\text{up}} = \min\{R, d_{\text{up}}\}$ and similarly $d_{\text{down}} = \min\{R, d_{\text{down}}\}$ while the same dynamical rule applies.

As far as dynamics is concerned, we investigate primarily the time-dependent behaviour of the order parameter, fraction of domain walls, and the persistence probability. The order parameter is given by $m = |L_{\text{up}} - L_{\text{down}}| / L$, where $L_{\text{up}}$ ($L_{\text{down}}$) is the number of up (down) spins in the system and $L = L_{\text{up}} + L_{\text{down}}$ is the total number of spins. This is identical to the (absolute value of) magnetization in the Ising model.

The average fraction of domain walls $D_w$, which is the average number of domain walls divided by the system size $L$, is identical to the inverse of average domain size. Hence, the dynamical evolution of the order parameter and fraction of domain walls is expected to be governed by the dynamical exponent $\gamma$; $m \propto t^{1/(2\gamma)}$ and $D_w \propto t^{-1/\gamma}$ [2].

The persistence probability $P(t)$ that a spin remains in its original state up to time $t$ [14] is also estimated. $P(t)$ has been shown to have a power-law decay in many systems with an associated exponent $\theta$. The persistence probability in finite systems has been shown to obey the following scaling form [16, 17]:

$$P(t, L) \propto L^{-\alpha} f(t/L^z).$$  

The exponent $\alpha = \theta z$ is associated with the saturation value of the persistence probability at $t \to \infty$, when $P_{\text{sat}}(L) = P(t \to \infty, L) \propto L^{-\alpha}$ [16].

In the simulations, we have generated systems of size $L \leq 6000$ with a minimum of 2000 initial configurations for the maximum size in general. Depending on the system size and time to equilibrate, maximum iteration times have been set. A random updating process has been used to control the spin flips. In general, the error bars in the data are less than the size of the data points in the figures and therefore not shown.
Figure 1. A schematic picture to show the dynamics in the present model for a finite value of \( R \). Both the encircled spins will change their state with 50% probability for the nearest neighbour Ising model \((R = 1)\). For \( R = 2 \), the encircled spin on the left will flip with the probability 1/2, while the one on the right will flip with the probability 1. For \( R = 3 \), the left one will not flip but the right one will.

Figure 2. Decay of the fraction of domain walls \( D_w(t) \) with time for \( R = 3 \) and two different system sizes shown in a log–log plot. The dashed line has slope equal to 0.5. The inset shows growth of magnetization \( m(t) \) with time for \( R = 3 \); the dashed line here has a slope equal to 0.25.

3. Case with finite \( R (p \to 0) \)

In this section, we discuss the case when \( R \) is finite. Effectively, this means that \( R \) does not scale with \( L \) and is kept a constant for all system sizes. Since \( R \) is kept finite, expressing \( R = pL/2 \) implies \( p \to 0 \) in the thermodynamic limit. For \( R = 1 \), the model is the same as the Ising model as the dynamical rule is identical to the zero temperature Glauber dynamics. But it may be noted that making \( R > 1 \) will make the dynamical rules different from the case of \( R = 1 \); as an example we show in figure 1 how making \( R = 2 \) or 3 changes the dynamical rule compared to \( R = 1 \).

We have simulated systems with \( R = 2 \) and \( R = 3 \) which show that the dynamics leads to the equilibrium configuration of all spins up/down. Not only that, the dynamic exponents also turn out to be identical to those corresponding to the nearest neighbour Ising values (i.e. \( \theta = 0.375 \) and \( z = 2 \)). As \( R \) is increased, the finite size effects become stronger; however, it is indicated that the Ising exponents will prevail as the system size becomes larger. In an indirect way, we have shown later that \( z = 2 \) as \( p \to 0 \) using a general scaling argument. The behaviour of the different dynamic quantities for \( R = 3 \) is shown in figures 2 and 3.
4. Case with $p > 0$

In this section, we discuss the case when $p$ is finite. We also assume that $p$ is uniform, which means each spin experiences the same cutoff.

The equilibrium behaviour is the same for all $p$, i.e. starting from a random initial configuration, the dynamics again leads to a final state with $m = 1$, i.e. all spins up or all spins down. For $p = 1$, that is in model I, it was numerically obtained that $\theta \simeq 0.235$ and $z \simeq 1.0$ giving $\alpha \simeq 0.235$, while in the one-dimensional Ising model $\theta = 0.375$ and $z = 2.0$ (exact results) giving $\alpha = 0.75$. It is clearly indicated that though model I and the Ising model have an identical equilibrium behaviour, they belong to two different dynamical classes which correspond to the $p = 1$ and the $p \to 0$ limit respectively of the present model. It is therefore of interest to investigate the dynamics in the intermediate range of $p$.

4.1. Results for $0 < p < 1$

Drastic changes in the dynamics are noted for the finite values of $p < 1$. The behaviour of all the three quantities, $m(t)$, $D_w$ and $P(t)$ shows the common feature of a power-law growth or decay with time up to an initial time $t_1$ which increases with $p$. The power-law behaviour is followed by a very slow variation of the quantities over a much longer interval of time, before they attain the equilibrium values. The power-law behaviour in the early time occurs with exponents consistent with model I, i.e. $z \simeq 1$ and $\theta \simeq 0.235$. This early time behaviour accompanied by model I exponents is easy to explain: it occurs while the domain sizes are less than $pL/2$ such that the size sensitivity does not matter and the dynamics is identical to that in model I. As the domain size increases beyond this value, the sizes of the neighbouring domains as sensed by the boundary spin become equal making the dynamics stochastic rather than deterministic as a result of which the dynamics becomes much slower.

We thus argue that since the domain size $\sim t^{1/z}$, the time up to which the model I behaviour will be observed is $t_1 = (pL/2)^z$. Since $z$ for model I is 1, we expect that $t_1 = pL/2$. For a fixed size $L$, one can then consider the scaled time variable $t' = t/p$, and plot the relevant
Figure 4. The collapse of the scaled order parameter versus scaled time for different values of $p$ shows $z = 1$ for $t < t_1$. The inset shows unscaled data. The system size is $L = 3000$.

Figure 5. The collapse of scaled fraction of domain walls versus scaled time for different values of $p$ shows $z = 1$ for $t < t_1$. The inset shows unscaled data. The system size is $L = 3000$.

scaled quantities against $t'$ for different values of $p$ to get a data collapse up to $t'_1 = t_1/p$, independent of $p$. We indeed observe this; in figures 4–6, the scaling plots as well as the raw data are shown. From the raw data, $t_1$ is clearly seen to be different for different $p$.

Although the model I behaviour is confirmed up to $t_1$ and explained easily, beyond $t_1$, the raw data do not give any information about the dynamical exponents $z$ and $\theta$ as no straightforward power-law fittings are possible. While an alternative method to calculate $\theta$ is not known, one may have an estimate of $z$ using an indirect method. It has been shown recently that for various dynamical Ising models, the time $t_{\text{sat}}$ to reach saturation varies as $L^x$, where $x$ is identical to the dynamical exponent $z$ [15, 18]. One may attempt to do the same here.
Figure 6. Persistence probability versus time for different values of $p$; the straight line with the slope 0.375 shown for comparison. The power-law behaviour can be observed only at the initial time. The system size is $L = 3000$. The inset shows the collapse of the scaled persistence probability versus scaled time indicating $z = 1$ for $t < t_1$.

Figure 7. Snapshot for $p < 1.0$ ($p = 0.4$) for the system size $L = 100$.

Actually, it is possible to find out theoretically the form of $t_{sat}$ from the qualitative behaviour of the dynamical quantities described above and the snapshot of the system (figure 7) at times beyond $t_1$. At $t > t_1$, the domain sizes of the neighbours of any spin at the boundary appear equal such that the domain walls perform random walks slowing down the annihilation process. Domain walls annihilate only after one of the neighbouring domains shrinks to a size $< pL/2$ again. In a small system, one can see that the slow process continues with only two domain walls separating two domains remaining in the system at later times (figure 7). Even in larger systems, there will be only a few domain walls remaining making $D_w \propto 1/N$ at $t > t_1$ as we note from the inset of figure 5: $D_w$ remains close to $O(1/N)$ for a long time before going to zero.
Figure 8. Scaled saturation time ($t_{sat}/L^2$) against $(1 - p)$ for different $L$ shows collapse with $t_{sat}/L^2 \propto (1 - p)^3$.

Thus, $t_{sat}$ will have two components, $t_1$, already defined, and $t_2$, the time during which there is a slow variation of quantities over time and the last two domains remain. While $t_1 \propto pL$, one can argue that $t_2 \propto (1 - p)^3L^2$. The argument runs as follows. Let us for convenience consider the open boundary case. Here, the size sensitivity of the spins is $R^{open} = qL$, where $0 \leq q \leq 1$ with the system assuming the model I behaviour for $q \geq 0.5$. At very late times, there will remain only one domain boundary in the system separating two domains of sizes, say, $\gamma L$ and $\beta L$, such that $\gamma + \beta = 1$. With both $\gamma, \beta > q$ the domain wall will perform random walk until either of the domains shrinks to a size $qL$. (This picture is valid for $q < 0.5$ and otherwise the dynamics will be simple model I type.) Let us suppose that the domain with the initial size $\beta L$ shrinks to $qL$ in time $t_{2, open}^{open}$ such that the domain wall performs a random walk over a distance $s$, where $\beta L - s = qL$. This gives

$$t_{2, open}^{open}(\beta) \propto (\beta - q)^2L^2.$$ 

Or, the average value of $t_{2, open}^{open}$ is given by

$$t_{2, open}^{open} \propto \int_q^{1-q} (\beta - q)\beta^2 L^2 d\beta = \frac{(1 - 2q)^3L^2}{3}.$$ 

The result for the periodic boundary condition is obtained by putting $q = p/2$ such that

$$t_2 \propto (1 - p)^3L^2$$

and therefore

$$t_{sat} = apL + b(1 - p)^3L^2.$$ (2)

The above form is also consistent with the fact that $t_{sat} \propto L^2$ for $p = 0$ and $t_{sat} \propto L$ for $p = 1$.

For large $L$, the second term in the above equation will dominate making $t_{sat} \propto (1 - p)^3L^2$.

In order to verify this, we have numerically obtained $t_{sat}$ and plotted $t_{sat}/L^2$ against $(1 - p)$ for different $L$ and found a nice collapse and a fit compatible with equation (2) (figure 8) with $a \sim 1$ and $b \sim O(10^{-2})$. We conclude therefore that in the thermodynamic limit at later times, for any $p \neq 1$, $z = 2$, i.e. the dynamics is diffusive. This argument, in fact holds for $p \rightarrow 0$ as well as shows that for $R$ finite, $z = 2$, as discussed in the previous section.
Figure 9. Persistence probability as a function of time for \( p = 0.4 \) for different sizes. The inset shows that the saturation values of the persistence probability shows a variation \( L^{-\alpha} \) for the values of \( p = 0.8, 0.4, 0.2 \) (from top to bottom) with \( \alpha \simeq 0.230 \).

Figure 10. \( PL^\alpha \) versus \( t/L^z \) for \( p = 0.4 \) shows a nice collapse for initial times up to \( t_1 \) using \( z = 1 \) and \( \alpha = 0.233 \) (a), while using \( z = 2 \) and the same value of \( \alpha \), the data collapses over later times (b).

We have discussed so far the time-dependent behaviour and exponents only. But another exponent \( \alpha \) which appears at \( t \to \infty \) for the persistence probability can also be extracted here. The persistence probabilities show the conventional saturation at large times, with the saturation values depending on \( L \). The log–log plot of \( P(L, t \to \infty) \) against \( L \) shows that the power-law behaviour is obeyed here with the exponent \( \alpha \) once again coinciding with the model I value \( \sim 0.23 \) for any value of \( p \neq 0 \) (figure 9).

Having obtained \( \alpha \), we use equation (1) with the trial values of \( z \) to obtain a collapse of the data \( PL^\alpha \) versus \( t/L^z \) for any value of nonzero \( p < 1 \). As expected, a unique value of \( z \) does not exist for which data will collapse over all \( t/L^z \). However, we find that using \( z = 1 \), one has a nice collapse for initial times up to \( t_1 \) while with \( z = 2 \), data collapses over later times (figure 10). The significance of the result is that a unique value of \( \alpha \) is good for collapse for both time regimes. However, it is not possible to extract any value of \( \theta \) for later times as \( \theta \) is extracted from equation (1) in the limit \( t/L^z < 1 \) only.
4.2. Discussions on the results

At this juncture, several comments and discussions are necessary. We have obtained a crossover behaviour in this model where an initial ballistic behaviour for macroscopic time scales is followed by a diffusive late-time behaviour. However, the diffusive behaviour at later times is not apparent in the simple log–log plots of the variables and can be extracted only from the study of the total time to equilibrate. This is due to the fact that the initial ballistic dynamics leaves the system into a non-typical configuration which is evidently far from those on diffusion paths. In fact, in the diffusive regime, the coarsening process hardly continues in terms of domain growth as only few domain walls remain at $t > t_1$.

A consequence of this is evident in the behaviour of the persistence at later times. One may expect that the persistence exponent $3/8$ may be obtained at very late times as here one has independent random walkers, few in number, which annihilate each other as they meet much like in a reaction diffusion process. However, such an exponent is not observed from data (figure 6). Although with $z = 2$ we can obtain a collapse at later times, it is not possible to obtain a value of $\theta$. Since persistence is a non-Markovian phenomenon and it depends on the history, the exponent may not be apparent even if the phenomenon is reaction diffusion like. Therefore, to analyse the dynamical scenario further, we study the persistence in a different way. In order to study the persistence dynamics beyond $t = t_1$, we reset the zero of time at $t = t_1$. In case the number of domain walls left in the system at $t_1$ is of the order of the system size ($O(L)$), the behaviour of persistence should be as in the case of the Ising model, i.e. a power-law decay with exponent $3/8$. On the other hand, if the number of independent random walkers is finite (i.e. vanishes in the $L \to \infty$ limit) which cannot annihilate each other, the persistence probability is approximately

$$P_{\text{rand}}(t, L) = 1 - ct^{1/2}/L,$$

where we have assumed that the number of distinct sites visited by the walker is proportional to the distance travelled, which is $O(t^{1/2})$.

We find that in this case, resetting the zero of time at $t_1$, the persistence probability shows a decay before attaining a constant value. The decay for a large initial time interval can be fitted to a form $\tilde{P}(t) = 1 - ct^y$, where the exponent $y$ increases with $L$ and clearly tends to saturate at 0.5 as the system size is increased. This shows that the persistence probability is identical to (3) in form (figure 11). This signifies that at $t > t_1$, the dynamics only involves the motions of random walkers which do not meet and annihilate each other for a long time and explains the fact that domain walls remain constant over this interval. Only at very large times close to equilibration the domain walls meet and the persistence probability starts deviating from the behaviour given by (3). Actually, once one of the neighbouring domains becomes less than $pL/2$ in size, the random walk will cease to take place and will become ballistic, which finally leads to the annihilation within a very short time. Therefore, although we have at later times independent walkers performing random walk, the power-law behaviour with exponent $3/8$ will never be observed (even when the origin of the time is shifted) as the annihilation here is not taking place as in a usual reaction diffusion system but determined by the model-I-like dynamics. It may also be noted that beyond $t = t_1$, annihilations occur only when the system is very close to equilibration unlike in a reaction diffusion system where annihilations occur over all time scales.

The reason why a single value of $\alpha$ is valid for both $t > t_1$ and $t < t_1$ is also clear from the above study. We expect that at $t = t_1$, the number of persistent sites $\propto L^{-\alpha}$ with the value of $\alpha \simeq 0.235$ as in model I. The additional number of sites which become non-persistent beyond
t_1 \text{ is proportional to } (t - t_1)^y / L \text{ and therefore at } t = t_{\text{sat}}, \text{ the expected number of persistent sites is}
\begin{equation}
c_1 L^{-\alpha} - c_2(t_{\text{sat}} - t_1)^y / L = c_1 L^{-\alpha} - c_2 t_2^y / L,
\end{equation}
where c_1, c_2 \text{ are proportionality constants. Since in the thermodynamic limit } y \to 1/2 \text{ and } t_2 \propto L^2, \text{ the number of persistence sites remains } \propto L^{-\alpha}. \text{ Here, we have assumed } c_2 \text{ to be independent of } L; \text{ the assumption is justified by the result.}

5. The case with quenched randomness

In this section, we briefly report the behaviour of the system when each spin is assigned a value of p(0 < p \leq 1) randomly from a uniform distribution. The randomness is quenched as the value of p assumed by a spin is fixed for all times.

Here, we note that the equilibrium behaviour, all spins up or down, is once again achieved in the system. However, the time to reach equilibrium values is larger than the p = 1 case.

The entire dynamics of the system, once again, can be regarded as walks performed by the domain walls. For p = 1, for all sites, the walks are ballistic with the tendency of a domain wall being to move towards its nearest one. For 0 < p \leq 1, but the same for all sites, as discussed in the previous section, the walk is either ballistic (at initial times) or diffusive (at later times) but identical for all the walkers. When p is different for each site, one expects that when a site with a relatively large p is hit, the corresponding domain wall will move towards its nearest domain wall while when a site with relatively small p is hit, the dynamics of the domain wall will be diffusive.

It has been previously noted that model I with noise (of a different kind) which induces similar mixture of diffusive and ballistic motions shows an overall ballistic behaviour (for finite noise) with the value of the dynamic exponent equal to unity [15]. In the present model with quenched randomness also, we find by analysing the saturation times that z = 1. However, the variation of the magnetization, domain walls and persistence shows power-law scalings with exponents corresponding to model I only for an initial range of time (figure 12).
6. Summary and concluding remarks

In summary, we have proposed a model in which a cutoff is introduced in the size of the neighbouring domains as sensed by the spins. The cutoff $R$ is expressed in terms of a parameter $p$. At $p \to 0$ (finite $R$) and $p = 1$, it shows pure diffusive and ballistic behaviour, respectively. In the uniform case, where $p$ is the same for all spins, a ballistic to diffusive crossover occurs in time for any nonzero $p \neq 1$. Usually in a crossover phenomenon, where a power-law behaviour occurs with two different exponents, the crossover is evident from a simple log–log plot. In this case, however, the crossover phenomenon is not apparent as a change in exponents in simple log–log plots does not appear. The crossover occurs between two different types of phenomena. The first one is pure coarsening in which domain walls prefer to move towards their nearest neighbours as in model I and one gets the expected power-law behaviour. At $t_1$, as mentioned before, some special configurations are generated and therefore the second phenomenon involves pure diffusion of a few domain walls (density of domain walls going to zero in the thermodynamic limit) which remain non-interacting up to large times. Naturally, the only dynamic exponent in the diffusive regime is the diffusion exponent $z = 2$, which is distinct from the growth exponent $z = 1$. So the two dynamic exponents not only differ in magnitude, but they are also connected to distinct phenomena. This crossover behaviour is therefore a striking feature for the model. For $R$ finite ($p \to 0$), there is no crossover effect, as the time $t_1$ is too small to generate these special configurations and the usual reaction diffusion type of behaviour prevails.

The persistence probability, in whichever way one sets the zero of time, does not show any power-law behaviour in the second time regime. At the same time, a single value of $\alpha$ is required for the collapse in the two regimes.

Another point of interest is that while $z = 2$ is expected for nonzero $p \neq 1$ values at later times, the behaviour of the total time to equilibrate as a function of $p$ is not obvious. Our calculation shows that it is proportional to $(1 - p)^3$, which is another important result of this work.
We also found that by making $p$ a quenched random variable taken from an uniform distribution, one gets back the model-I-like behaviour to a large extent. However, choosing a different distribution might lead to different results. The fact that the model has a different behaviour with uniform $p$, and with a quenched random value of $p$ is reminiscent of the different behaviour observed in agent-based models with savings in econophysics [19].

Acknowledgments

Financial support from the DST project SR/S2/CMP-56/2007 is acknowledged. SB acknowledges inspiring discussions with D Dhar. Partial computational help has been provided by the UPE project.

References

[1] Biswas S and Sen P 2009 Phys. Rev. E 80 027101
[2] Bray A J 1994 Adv. Phys. 43 357 and the references therein
[3] Janssen H K, Schaub B and Schmittmann B 1989 Z. Phys. B 73 539
[4] Renshaw E and Henderson R 1981 J. Appl. Probab. 18 403
Szasz D and Toth B 1984 J. Stat. Phys. 37 27–38
Sokolov I M, Klafter J and Blumen A 2000 Phys. Rev. E 61 2717
[5] Almaas E, Kulkarni R V and Stroud D 2003 Phys. Rev. E 68 056105
Procacci A, Sanchis R and Scoppola B 2008 Lett. Math. Phys. 83 181
[6] Hinrichsen H 2000 Adv. Phys. 49 815
Marro J and Dickman R 1999 Nonequilibrium Phase Transitions in Lattice Models (Cambridge: Cambridge University Press)
Odor G 2004 Rev. Mod. Phys. 76 663
[7] Liggett T M 1999 Interacting Particle Systems: Contact, Voter and Exclusion Processes (Berlin: Springer)
[8] Stauffer D 2009 Encyclopedia of Complexity and Systems Science ed R A Meyers (New York: Springer)
Sznajd-Weron K and Sznajd J 2000 Int. J. Mod. Phys C 11 1157
Galam S 2008 Int. J. Mod. Phys. C 19 409
[9] Baronchelli A, Dall’Asta L, Barrat A and Loreto V 2007 Phys. Rev. E 76 051102
Castellano C, Marsili M and Vespignani A 2000 Phys. Rev. Lett. 85 3536
[10] Fuchs J, Schelter J. Ginelli F and Hinrichsen H 2008 J. Stat. Mech. P04015
[11] Stauffer D and de Oliveira P M C 2002 Eur. Phys. J B 30 587
[12] Sanchez J R 2004 arXiv:cond-mat/0408518v1
[13] Shukla P 2005 J. Phys. A: Math. Gen. 38 5441
[14] Derrida B, Bray A J and Godreche C 1994 J. Phys. A: Math. Gen. 27 L357
For a review on persistence see Majumdar S N 1999 Curr. Sci. 77 370
[15] Manoj G and Ray P 2000 Phys. Rev. E 62 7755
Manoj G and Ray P 2000 J. Phys. A: Math. Gen. 33 5489
[16] Chatterjee A, Chakrabarti B K and Manna S S 2004 Physica A 335 155
Chatterjee A, Chakrabarti B K and Manna S S 2003 Phys. Scr. T 106 36