Numerical simulation of electron cyclotron resonance phenomenon using an axisymmetric transverse electric field

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Abstract. As a contribution to the study of the resonant interaction between of an electron and the transverse electric field of a stationary electromagnetic wave in the presence of a static homogeneous magnetic field in this work we develop a numerical study of the acceleration of electrons by cylindrical mode TE_{011}. In order to get a better understanding of the particle-wave interaction, the electric field of the microwave mode is decomposed as the superposition of a left- and a right-hand circularly polarized standing wave because electrons interact effectively only with right-handed circular polarized wave. The trajectory, energy and phase-shift between the electron transverse velocity and the electric field are determined by the numerical solution of the relativistic Newton-Lorentz equation using a finite difference scheme. For an electron injected longitudinally with an energy of 5 keV and that starts at the radial midpoint of the cavity, it is accelerated up to an energy of about 90 keV using an electric field amplitude of 14 kV/cm and a frequency of 2.45 GHz. These results are compared with those obtained for another two points of injection located in different radial positions. This levels of energy can be used to produce soft X-ray which has some important medical applications like imageology. The results suggest that the particle-wave interaction using the cylindrical mode TE_{011} could be optimized through the application of an external magnetic field which is gradually growing in time to preserve the resonance condition and sustain the phase stability.

1. Introduction
The resonant interaction between the transverse electromagnetic wave (TE wave) that propagates along a homogeneous magnetic field and a charged particle has been studied since the year 1962 [1, 2]. This scheme constitutes a well known mechanism that allows to accelerate charged particles taking advantage of the correct configuration between the TE wave electric field and the particle transverse velocity. This principle is also used for plasma heating based on electron cyclotron resonance (ECR). In order to get a better understanding of this mechanism, the linear polarized electric field of the wave is decomposed through the superposition of two circular waves, right hand and left hand sense, as the electrons interact with the right circular component effectively [3, 4]. Some microwave modes has been used for this poerpose, for example TE_{111} and TE_{011} cylindrical modes, the first one can be approximated as a linear polarized wave and the second one has the appropriated shape and geometry that allows for its implementation [5]. The cylindrical mode TE_{011} has been used recently for conceptualization, design and development of a ECR light source using microwave plasma, this source produces
visible and ultraviolet radiation for various applications [6].

The charged particles in the resonant interaction gains energy which produces that the ECR condition can not be satisfied. On the expression (1) is presented the relation between \( \Omega \) and \( \gamma \) (the relativist factor) and as we known if energy grows, \( \gamma \) grows too, it produces that the ECR condition is lost, however the resonant interaction can be optimized using external mechanisms, this phenomena is known as auto-resonance [7]. The spatial auto-resonance acceleration (SARA) uses as optimization mechanism a no homogeneous external magnetic field who varying with a required rate to sustain the ECR condition along the path of the electrons [8, 9] from this idea arose the design of a compact source of X-ray [10, 11]. On the other hand, the gyro-resonant accelerator (gyrac) implements a magnetic field which is gradually growing in time like mechanism for the sustenance of ECR condition [12–15], which has some important applications as the electron cyclotron resonance ion proton accelerator (ECR-IPAC) for cancer therapy [16] and to control plasma bunches with relativistic electrons [17]. This research is dedicated to numerical study the resonant interaction between the TE\(_{011}\) cylindrical mode and electrons using single particle approximation through the analytical decomposition of the TE wave electric field as the superposition circular polarized waves [18–20].

2. Simulation model

The basic principle of ECR plasma heating consist in decompose a linear polarized microwave field excited on a cylindrical chamber, in two circular polarized wave, the first one with right-handed circular polarization (RHP) and the other one with left-handed circular polarization (LHP). The electric field vector of the RHP wave rotates in the direction of the right-hand around the magnetic field at a frequency \( \omega \), the electron rotates at the same direction around the homogeneous external magnetic field \( B_0 \) with a frequency \( \Omega \), denoted as cyclotron frequency, which is given by Equation (1).

\[
\Omega = \frac{eB_0}{m_e\gamma},
\]

where \( e \) is the electron charge, \( B_0 \) is the magnetic field value, \( m_e \) is the mass of the electron and \( \gamma \) is the relativistic factor. Considering \( \omega = \Omega \), that is, in exact resonance, the force exerted by the electric field RHP accelerates the electron along its helical orbit, which produces a continuous gain in the transverse energy, while for the LHP wave an oscillating force is produced that on average is zero, as a result there is no energy gain. In order to get a better understanding of the resonant interaction, the evolution of the phase-shift is studied (\( \varphi \)) between the phase of the electric field vector of the wave and that of the transverse velocity vector of the particle. The electron absorbs energy from the electromagnetic field if the phase-shift is in the range \( \pi/2 < \varphi < 3\pi/2 \) denoted as the acceleration band. For the case of exact resonance, the phase-shift is equal to \( \pi \) as we can see on the Figure 1.

![Figure 1. Electron resonant interaction using RHP component in different instants: (a) \( t = 0 \), (b) \( t = T/4 \), (c) \( t = T/2 \) and (d) \( t = 3T/4 \) being \( T = 2\pi/\omega \) the period of the electromagnetic wave.](image-url)
In this work we study the electrons acceleration due to the resonant interaction with an microwave field, cylindrical mode TE_{011}, and a homogeneous external magnetic field. For this case, the components of the microwave field is given by Equation (2) and Equation (3).

\[ \vec{E}_{hf}(\vec{r}, t) = E^0 J_1 (k_T r) \sin (k_z z) \cos (\beta) \hat{\theta}, \]  
\[ \vec{B}_{hf}(\vec{r}, t) = E^0 \left[ \frac{k_z}{\omega} J_1 (k_T r) \cos (k_z z) \cos \left( \beta - \frac{\pi}{2} \right) \hat{r} + \frac{k_T}{\omega} J_0 (k_T r) \sin (k_z z) \cos \left( \beta + \frac{\pi}{2} \right) \hat{k} \right], \]  

where \( E^0 \) is the amplitude of the electric field, \( k_T = q_{01}/R \) being \( q_{01} = 3.83171 \) and \( R \) the radius of the cylindrical cavity, \( k_z = \pi/L \) where \( L \) is the length of the chamber and \( \beta = \omega t + \theta_0 - \pi/2 \) where \( \theta_0 \) is a arbitrary phase. The electric field (see Figure 2b) can be expressed as the superposition of a RHP component (\( \vec{E}^r \)) and a LHP (\( \vec{E}^l \)). The mathematical expressions for \( \vec{E}^l \) and \( \vec{E}^r \) in Cartesian basis are Equation (4) and Equation (5).

\[ \vec{E}^l = \frac{E^0}{2} J_1 (k_T r) \sin (k_z z) \left[ \sin (\beta - \theta) \hat{i} + \cos (\beta - \theta) \hat{j} \right], \]  
\[ \vec{E}^r = \frac{E^0}{2} J_1 (k_T r) \sin (k_z z) \left[ -\sin (\beta + \theta) \hat{i} + \cos (\beta + \theta) \hat{j} \right] \]

It is evident that adding the RHP and LHP fields gives the total electric field, see Equation (2). The microwave mode is excited in the chamber with a frequency of 2.45 GHz and an amplitude of 14 kV/cm. The radius and the length of the cavity are 7.81 cm and 20.0 cm, respectively. The cavity radius is chosen greater than the maximum Larmor radius expected. The external magnetic field (\( \vec{B}_0 \)) is considered homogeneous in \( \hat{k} \) direction, its magnitude corresponds to the value to obtain classical resonance, which is obtained from the Equation (1). The azimuth angle (\( \theta \)) and the radial distance (\( r \)) are obtained indirectly using Cartesian coordinates through expressions \( \tan \theta = y/x \) and \( r = \sqrt{x^2 + y^2} \).

**Figure 2.** (a) Physical Scheme: (1) electron injection points, (2) cylindrical cavity and (3) longitudinal electric field profile, and (b) transverse electric field in the cross section \( z = L/2 \) in the instant \( t = 0 \).
The motion of the electron is described by the relativistic Newton-Lorentz equation, which is expressed in a dimensionless form as Equation (6).

\[
\frac{du}{d\tau} = \vec{g}_0 + \frac{u}{\gamma} \times \vec{b},
\]

where \( \vec{u} = \vec{p}/mc \) is the momentum of the electron, \( \vec{g}_0 = -\vec{E}/B_0c \) is the microwave electric field, \( \vec{b} = -\vec{B}/B_0 \) is the total magnetic field (\( \vec{B} = \vec{B}_{hf} + \vec{B}_0 \)), \( \tau = \omega t \) the time and \( \gamma = \sqrt{1 + u^2} \).

The Equation (6) in a finite difference form can be written as Equation (7).

\[
\frac{u^{n+1/2} - u^{n-1/2}}{\Delta \tau} = g_0^n + \frac{u^{n+1/2} + u^{n-1/2}}{2\gamma^n} \times b^n,
\]

where \( n \) is the index for the temporal step. The Equation (7) is solved numerically using the Boris scheme. The position of the particle is determined by Equation (8).

\[
r^{n+1} = r^n + \frac{u^{n+1/2}}{\gamma^{n+1/2}} \Delta \tau,
\]

with \( \gamma^{n+1/2} = \left[ 1 + (u^{n+1/2})^2 \right]^{1/2} \). All distances were normalized according to the relativistic Larmor radius.

### 3. Results and discussions

For numerical experiments, the electrons are injected into the cavity along the \( z \) axis through three radial points, \( P_1, P_2 \) and \( P_3 \) whose values are \( R/2, \) \( 3R/8 \) and \( 9R/16 \) respectively (See Figure 2a). The first set of tests has as purpose to study the electron dynamics under the effect of the configuration of electromagnetic field in two schemes: case (i) under the total electric field effect of the microwave mode which is defined analytically by the Equation (2) and case (ii) under the effect of the RHP and LHP components defined by the Equations (4) and (5) separately. A longitudinal injection energy of 9 keV was used (while the transverse injection energy is chosen zero) and the chosen injection point is \( P_1 \).

![Figure 3](image-url)

**Figure 3.** (a) Helical trajectory of an electron injected longitudinally and its projections onto (b) \( xy \) plane and (c) \( yz \) plane.

The Figure 3(a) shows the trajectory for an electron injected in the conditions previously presented. The projection of the trajectory onto transverse planes are like concentric rings around the injection point as can be seen in the Figure 3(b). The Larmor radius is determined by the energy of the electron and the local magnetic field. Additionally, the projection of the
trajectory onto the $YZ$ plane is presented, see Figure 3(b). It is possible to appreciate in Figure 3(c) that in the region between $5 < z < 15$ cm the radius of the rings grows which is associated with the gain of transverse energy. When the electron leaves this region, the value of the Larmor radius becomes almost constant. The Figure 3(c) also presents some local maximums values associated with the longitudinal motion and the growth of the rotation radius. The distance between these points is approximately the same along the entire trajectory so we can conclude that the longitudinal velocity is almost constant. The longitudinal evolution of the electron energy and the phase-shift for the cases (i) and (ii) are presented in the Figure 4(a). On the region $0 < z < 5$ cm the electron does not gain energy considerably even if the system enters at the acceleration band quickly as we can see on the Figure 4(b) (black line) due to the profile longitudinal of the electric field chosen because on that region the amplitude of the electric field is still small. The electron gain energy significantly on the region between $5 < z < 15$ cm with a maximum value of 53 keV. Between $15 < z < 20$ cm, the energy gain is suspended due to the loss of the resonance condition besides in this region the amplitude of the electric field tends to zero too. On the Figure 4(a), the black line (total electric field) presents some fluctuations that occurs due to the interaction between electron and the LHP component of the electric field. The electrons gain energy due to their interaction with the RHP component of the electric field (see Figure 4(a), red line) while the contribution produced by LHP component produces few energy gain and only in small space regions, which is in agreement with the phase-shift calculated for each case and presented in Figure 4(b) in which $\varphi$ enters and leaves the acceleration band quickly (see Figure 4(b), blue line). For this reason it is possible to concentrate only on study of the effect produced by RHP component of the field for the understanding of this mechanism of acceleration.

![Figure 4. (a) Evolution of the electron energy and (b) phase-shift on longitudinal coordinate using total transversal electric field and its decomposition.](image)

The Figure 5(a) and Figure 5(b) presents the longitudinal evolution of the energy and the phase-shift considering different injection points respectively. The study was realized in two stages: The first one consists in the study of the electron dynamics for injected particles with an energy of 5 keV through each one of the injection points using the total field given by Equation (2), denoted as system 1. For the second stage, the electromagnetic field configuration is modified slightly. The new configuration or system 2 includes the microwave field mode $TE_{011}$ coupled with a modified external magnetic field, denoted as $\vec{B}_{Mod}$. The new external magnetic field is defined as $\vec{B}_{Mod} = 1.05 \vec{B}_0$, where $\vec{B}_0$ is the homogeneous magnetic field whose magnitude
correspond to the value to obtain classical resonance. We can see in the Figure 5(a) that the energy reached by the electrons using system 2 is considerably greater than that obtained with system 1 for all the injection points. That happens because system 2 allows the preservation of the resonance condition better than system 1 as we can see in the Figure 2(b), the phase-shift ($\varphi$) in all the tests for system 2 stays closer to the exact resonance condition throughout up to $z \approx 13$ cm hence the microwave field transfers more energy to the electrons for this field configuration while for the system 1 from $z \approx 14$ cm the phase-shift go out the acceleration band. For system 2, this happens in $z \approx 17$ cm. The energy reaches a maximum value of 58 keV for system 1 in $z = 14$ cm using the injection point $P_2$ and for the system 2 reaches 91 keV in $z \approx 17$ cm using $P_1$. For both cases, the system 1 and 2, the final energy dispersion for different injection points does not exceed 4%.

![Figure 5](image)

**Figure 5.** (a) Evolution of the electron energy and (b) phase-shift along the longitudinal coordinate for different injection points using the external magnetic field of resonance and the modified external magnetic field.

4. Conclusions
The numerical experiments presented in this article evidence the feasibility of the electrons acceleration using resonant interaction of these with an standing electromagnetic field in a cylindrical mode $TE_{011}$ and an external homogeneous magnetic field. The decomposition of the electric field on the superposition of two circular polarized standing waves (RHP and LHP) allows to understand the process through which the electron gains energy. The level of energy that we get through this mechanism can be used to produce soft X-ray for medical imaging. The present study can be used as a starting point for the study the acceleration of electrons in a magnetic field changing slowly in time (gyrac) using cylindrical $TE_{011}$ mode.

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