Forecast estimator of surface machining completion

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Abstract. We consider the problem of estimating the possible outcomes of surface machining with specified output characteristics. The process of characteristics changing during machining is considered as a random walk with two absorbing levels corresponding to successful and unsuccessful outcomes. For its quantitative description, a Markov process with discrete time and continuous states corresponding to the selected characteristic is used. The transition probabilities for the next step for each of these quantities are given by distribution functions which can be obtained empirically.

1. Introduction

Machining of complex details cannot be considered as a fully deterministic process [1 - 3]. Criteria estimators of the manufactured product quality may include a number of factors that are varying during process of machining. The causes of uncertainty may comprise abrasive contamination while grinding, foreign particles on the tool while polishing, thermal distortion, etc. Nevertheless the condition of machining completion can be specified as sojourn of characteristics of the form, cleanliness and physical properties of resulted surface within specified limits. In some cases, this result is not achievable, in particular, if there are requirements both for the geometric dimensions of the detail and its surface cleanliness, then it is possible to irreversibly spoil the workpiece by spending one of the machining allowances and not eliminating the defects[4]. Hence it is actually to reduce the probability of such an outcome of the process. To do this the probability itself must be calculated on early stages of the process and then minimizing changes must be applied to the process.

Currently, this problem is solving in various ways, depending on the process. Modeling and simulation of magnetic abrasive finishing is considered in the paper [5]. Stochastic parameters modeling when abrasive flow machining is considered in the papers [6 - 12], the items concerning jet finishing – in the papers [13 - 16]. Finishing of glass and silicon surfaces with very high requirements for the surface obtained is optimizing in the papers [17 – 20]. Analysis of the above studies shows the possibility of their unified description in the framework of a single mathematical model of machining as a stochastic process.

Let us set the goal of forecast estimators obtaining for the machining process completion by one or another outcome given the criteria for matching of the resulting surface characteristics with the specification and given assumptions about the current state of these characteristics and their probabilistic properties.
2. Assumptions

The criterion for the machining termination is the transition of the selected characteristics $q_1, q_2, \ldots, q_n$ of the workpiece outside the specified area. Let us consider the characteristics as continuous random variables that change over time independently from each other. For each characteristic $q_i$, let us specify the upper level of acceptable values $x_i$ and the lower level of acceptable levels $y_i$. The machining dwells while all the characteristics sojourn each between its upper and lower levels.

The machining stops either if one of the characteristics reaches its lower level (then the workpiece is sent to the previous stage of the machining) or if all the characteristics reach their upper level (the detail is made).

It is required to determine the probabilities of each of these outcomes $P_x$ and $P_y$ and form the control action tends to $P_x$ increase.

3. Inference

Let us consider the upper and the lower levels of each characteristic as absorbing levels while its random walk in continuous phase space of states [22]. In accordance with accepted mutual independence of the characteristics connected with the same process let us give the next relations for the values to find:

$$P_x = p_{1x} \cdot p_{2x} \cdot \ldots \cdot p_{nx},$$

$$P_y = 1 - P_x,$$

where $p_{nx}$ is the probability of $q_i$ random walk termination by absorption by its upper level, $p_{ny}$ is the probability of $q_i$ random walk termination by the absorption by its lower level.

Let us consider the process of continuous characteristic $q_i$ changing over time as discrete time stochastic process. Let us split the time span into equal intervals, each of which let us describe by the maximum, the minimum, the initial and the final values of the selected characteristic. Subtracting the initial value from all of them let us consider three resulted differences as continuous independent random variables.

Let us assign the serial number $k$ to each interval of time and compare the next values of the characteristic $q_i$: $q_{fik}$ – value at the beginning of the interval, $q_{max_{ik}}$ – the maximum value within the interval, $q_{min_{ik}}$ – the minimum value within the interval.

Let us consider the differences $\alpha_i = q_{max_{ik}} - q_{min_{ik}}$; $\beta_i = q_{f_{i+1}} - q_{f_{ik}}$ as continuous random variables with distribution functions $F_i(s)$, $G_i(s)$ and $V_i(s)$ respectively with finite distribution densities $f_i(s) = dF_i(s)/ds$; $g_i(s) = dG_i(s)/ds$; $v_i(s) = dV_i(s)/ds$.

Variations of considered values $q_{min}$, $q_{max}$ and $q_{min}$ over time can be described by discrete time markovian processes [22,23] with continuous phase space of states. To define them completely we need initial distribution function $R_i(s)$, let us suppose that it also possesses a finite distribution density $\pi_i(s) = dR_i(s)/ds$.

Since the exact value of the $q_i$ characteristic at the initial moment of time is unknown, the initial distribution can create three events: $S_1$: the characteristic has exceeded the upper level (conditionally we consider that further it does not return to the area of machining dwelling, if this event occurs for all characteristics, then the detail is considered to be made), $S_2$: the characteristic was discovered to be less than the lower level (the workpiece is transferred to the previous stage of machining), $S_3$: the characteristic is between the levels (at the initial moment of time the absorption has not occurred and the process of Markov random walk starts).

The probabilities of machining process termination by the absorption by the upper or the lower level for the characteristic $q_i$ can be expressed through the probabilities of these events by the formulas

$$p_{ix} = P(S_1) + P(S_2, x),$$

$$p_{iy} = P(S_3) + P(S_2, y),$$

$$P_x = 1 - p_{iy},$$

$$P_y = 1 - p_{ix},$$

$$P_x = p_{1x} \cdot p_{2x} \cdot \ldots \cdot p_{nx},$$

$$P_y = 1 - P_x,$$
where \( P(S_2, x) \) is the probability of the joint occurrence of the events «at the initial moment of time the characteristic \( q_i \) is between the levels» and «the subsequent random walk was terminated by the absorption by the upper level», \( P(S_2, y) \) is the probability of the joint occurrence of the events «at the initial moment of time the characteristic \( q_i \) is between the levels» and «the subsequent random walk was terminated by the absorption by the lower level».

Let us explain the probabilities of the events \( S_1, S_2, S_3 \) through markovian process initial distribution

\[
P(S_1) = \int_{s_i}^{x_i} \pi_i(s) ds ;
\]
\[
P(S_2) = \int_{y_i}^{x_i} \pi_i(s) ds ;
\]
\[
P(S_3) = \int_{y_i}^{\infty} \pi_i(s) ds .
\]

Let us determine the probabilities \( P(S_2, x) \) and \( P(S_2, y) \). Applying the law of total probability [24] to the continuous distribution with density \( \pi_i(s) \) we can obtain

\[
P(S_2, x) = \int_{y_i}^{x_i} \pi_i(s) P_{is}(s) ds ;
\]
\[
P(S_2, y) = \int_{y_i}^{x_i} \pi_i(s) P_{iy}(s) ds ,
\]

where \( P_{is}(s) \) is the probability of the characteristic \( q_i \) random walk process termination by the absorption by the upper level provided that at the initial moment of time \( q_i = s \), \( P_{iy}(s) \) is the analogous probability for the absorption by the lower level. Let us determine these values.

Let the characteristic \( q_i \) be equal to the value \( s \) at the initial moment of time. Within the first time interval three events are possible: \( C_1 \): the characteristic had crossed the upper level, \( C_2 \): the characteristic had crossed the lower level, \( C_3 \): the characteristic had been being between the levels during the entire time interval (the process of machining is incomplete and is transferred to the next time interval). Let us explain these events through the events \( A \) and \( B \) of reaching the upper and the lower levels by the characteristic and through the events, inverse to them of not reaching the upper level \( \overline{A} \) and not reaching the lower level \( \overline{B} \) within the first time interval. Using this notation the event \( C_1 \) occurs in one of the next cases: 1) Within the first time interval the events \( A \) and \( \overline{B} \) occur; 2) Within the first time interval the events \( A \) and \( B \) occur under the condition that the event \( A \) occurs first. The event \( C_2 \) occurs in one of the cases: 1) Within the first time interval the events \( \overline{A} \) and \( B \) occur; 2) Within the first time interval the events \( A \) and \( B \) occur under additional condition, that the event \( B \) occurs first. The event \( C_2 \) occurs when within the first time interval the events \( \overline{A} \) and \( \overline{B} \) occurs.

Let us write the next probabilities of events to terminate the first time interval:

\[
P(C_1) = P(A, \overline{B}) + P(t_{A} < t_{B} | A, B) ;
\]
\[
P(C_2) = P(\overline{A}, B) + P(t_{A} > t_{B} | A, B) ;
\]
\[
P(C_3) = P(\overline{A}, \overline{B}) ,
\]

where \( t_{A} \) is the time of the event \( A \) first occurrence within the interval, \( t_{B} \) is the same for the event \( B \).

Let us explain the probabilities of the events \( A \) and \( B \) through distribution densities introduced above.
The probabilities of inverse events can be obtained by subtraction of these values from the unit. Considering the events $A$ and $B$ as independent let us obtain the formulas for the probabilities of (10 – 12) right parts:

\[
P(A, B) = P(A) \cdot P(B) = \int_{x_i-s}^{\infty} f_i(t)dt \int_{s-y_i}^{\infty} g_i(t)dt ; \quad (15)
\]

\[
P(A, \bar{B}) = P(A) \cdot P(\bar{B}) = \int_{x_i-s}^{\infty} f_i(t)dt \int_{0}^{s-y_i} g_i(t)dt ; \quad (16)
\]

\[
P(\bar{A}, B) = P(\bar{A}) \cdot P(B) = \int_{0}^{\infty} f_i(t)dt \int_{s-y_i}^{\infty} g_i(t)dt ; \quad (17)
\]

\[
P(\bar{A}, \bar{B}) = P(\bar{A}) \cdot P(\bar{B}) = \int_{0}^{\infty} f_i(t)dt \int_{0}^{s-y_i} g_i(t)dt . \quad (18)
\]

Let us suppose that probabilities of each direct event first occurrence provided that both direct events occur within the time interval are in proportion with the probabilities of the events themselves.

\[
\frac{P(t_A < t_B | A, B)}{P(t_A > t_B | A, B)} = \frac{P(A)}{P(B)} . \quad (19)
\]

Since the (19) left part numerator and the denominator are the probabilities of incompatible events and the events $A$ and $B$ are independent so that the next relation is valid

\[
P(t_A > t_B | A, B) + P(t_A < t_B | A, B) = P(A) \cdot P(B) , \quad (20)
\]

let us obtain the formulas for each of these probabilities

\[
P(t_A < t_B | A, B) = \frac{P^2(A) \cdot P(B)}{P(A) + P(B)} ; \quad (21)
\]

\[
P(t_A > t_B | A, B) = \frac{P^2(B) \cdot P(A)}{P(A) + P(B)} . \quad (22)
\]

In accordance with (10), (15), (16) and (21), the probability of the crossing of the upper level by the characteristic $q_i$ within the first time interval will be

\[
P(C_1) = P_{d0i} (x, y, s) = \int_{x-s}^{\infty} f_i(t)dt \int_{s-y}^{\infty} g_i(t)dt + \int_{x-s}^{\infty} f_i(t)dt \int_{s-y}^{\infty} g_i(t)dt \cdot \left( \int_{x-s}^{\infty} f_i(t)dt \int_{s-y}^{\infty} g_i(t)dt \right) . \quad (23)
\]

Using the formulas (11),(15),(17) and (22) let us obtain the similar formula for the probability of the lower level crossing
Using the formulas (12) and (18) let us obtain the probability of the transfer of the machining to the next time interval:

\[ P(C_2) = P_{10y}(x, y, s) = \int_{0}^{\infty} f_1(t)dt \int_{s-y}^{\infty} g_y(t)dt \cdot \left( \int_{s-x}^{\infty} f_i(t)dt + \int_{s-x}^{\infty} g_i(t)dt \right)^2 \]  

(24)

Using the formulas (12) and (18) let us obtain the probability of the transfer of the machining to the next time interval:

\[ P(C_3) = P_{10}(x, y, s) = \int_{0}^{\infty} f_1(t)dt \int_{0}^{\infty} g_y(t)dt . \]  

(25)

Proceeding to the next time interval let us notice that the sum of all possible outcomes probabilities for it must be equal to (25). Using above introduced distribution densities let us obtain:

\[ P(q_{f1} > x_i) + P(q_{f1} < y_i) + P(x_i \leq q_{f1} \leq y_i) = r_{10}(x_i, y_i, s) \]  

(26)

Using the formula (26) and the inference of (23) – (25) let us obtain the probabilities for the second time interval:

\[ P_{11x}(x, y, s) = r_{10}(x, y, s) \cdot \left( \int_{x-s}^{\infty} v_1(t)dt + \int_{y-s}^{\infty} v_1(a) \cdot P_{10x}(x-s-a, s-y-a, s)da \right) ; \]  

(27)

\[ P_{11y}(x, y, s) = r_{10}(x, y, s) \cdot \left( \int_{y-s}^{\infty} v_1(t)dt + \int_{x-s}^{\infty} v_1(a) \cdot P_{10y}(x-s-a, s-y-a, s)da \right) ; \]  

(28)

\[ r_1(x, y, s) = r_{10}(x, y, s) \cdot \int_{y-s}^{\infty} v(a) \cdot r_0(x-s-a, s-y-a, s)da . \]  

(29)

For the third time interval let us obtain

\[ P_{12x}(x, y, s) = r_{11}(x, y, s) \cdot \left( \int_{x-s}^{\infty} v_1(t)dt + \int_{y-s}^{\infty} v_1(a) \cdot P_{10x}(x-s-a, s-y-a, s)da \right) ; \]  

(30)

\[ P_{12y}(x, y, s) = r_{11}(x, y, s) \cdot \left( \int_{y-s}^{\infty} v_1(t)dt + \int_{x-s}^{\infty} v_1(a) \cdot P_{10y}(x-s-a, s-y-a, s)da \right) ; \]  

(31)

\[ r_2(x, y, s) = r_{11}(x, y, s) \cdot \left( \int_{y-s}^{\infty} v_1(a) \cdot r_0(x-s-a, s-y-a, s)da = \right. \]

\[ = r_{10}(x, y, s) \cdot \left( \int_{y-s}^{\infty} v_1(a) \cdot r_0(x-s-a, s-y-a, s)da \right)^2 . \]  

(32)

Generalizing the results, using the sum of probabilities law and summing up the resulting progression, let us obtain the probabilities to be found of reaching each of the levels by the characteristic \( q_i \) through infinite quantity of time intervals provided that at the initial moment of time the value of the characteristic was equal to \( s \):
where

\[ P_{ix}(s) = P_{i0x}(x_i, y_i, s) + \frac{r_{i0}(x_i, y_i, s) \left( \int_{x_i-s}^{\infty} v_i(t) dt + \int_{y_i-s}^{\infty} v_i(a) \cdot P_{i0x}(x_i - s - a, s - y_i - a, s) da \right)}{1 - \int_{y_i-s}^{\infty} v_i(a) \cdot r_{i0}(x_i - s - a, s - y_i - a, s) da} \] ; \quad (33)

\[ P_{iy}(s) = P_{i0y}(x_i, y_i, s) + \frac{r_{i0}(x_i, y_i, s) \left( \int_{y_i-s}^{\infty} v_i(t) dt + \int_{y_i-s}^{\infty} v_i(a) \cdot P_{i0y}(x_i - s - a, s - y_i - a, s) da \right)}{1 - \int_{x_i-s}^{\infty} v_i(a) \cdot r_{i0}(x_i - s - a, s - y_i - a, s) da} \] ; \quad (34)

Then for each characteristic we can obtain the probabilities of crossing the upper and the lower levels under infinite increasing of time

\[ p_{ix} = \int_{x_i}^{\infty} \pi_i(s) ds + \int_{x_i}^{y_i} \pi_i(s) P_{ix}(s) ds ; \quad (35) \]

\[ p_{iy} = \int_{y_i}^{-\infty} \pi_i(s) ds + \int_{x_i}^{y_i} \pi_i(s) P_{iy}(s) ds = 1 - p_{ix} \] , \quad (36)

where all used distribution densities can be determined by any available technique.

Control of all the process is seemed by obtaining the estimates by the formulas (35 - 36), determining by them the probability of successful process termination by the formula (3) and performing the control corrections by the characteristic with minimum probabilities (35).

To do this the mathematical model is needed connecting stochastic parameters and control actions

\[ u_1, \ldots, u_m \] (varying of the pressure between the instrument and the tool, angular speed of the drive, parameters of the instrument reciprocation movement), what is above the scope of the present paper.

4. Conclusion

We obtained the forecast estimators of technological process completion by successful (the detail is produced, formula (35)) and unsuccessful (the detail is impossible to produce, formula (36)) outcomes provided several assumptions about probabilistic properties of the characteristics defining the condition of completion.

The proposed method of estimates obtaining is applicable in the next cases:

- The control system should provide an opportunity to periodically obtain a set of characteristics of the geometry and roughness of the machined surface

- The quantity of measurements should be sufficient for construction of distribution functions for each characteristic.

- At high machining intensity, it is advisable to choose characteristics that allow control without stopping the equipment.

The direct use of formulas (35–36) requires the assignment of the distribution functions of the quantities used, while the result of measurements is only a sample of values. This restriction can be overcome either by replacement of distribution densities by histograms or by reconstruction of continuous distribution densities based on the samples of values using the maximum likelihood principle or other methods.

- The probabilities of successful and unsuccessful outcomes calculated by the proposed method allow performing a control action by the change of factors affecting the characteristic
with the least probability of successful completion of processing provided the presence of a 
mathematical model linking characteristics with controlled parameters of the technological 
process

The application of the considered approach seems to be expedient when finishing of surfaces 
of rotation of the second order, where compliance with the shape, specularity and purity of the 
resulting surface is required.

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