A Closer Look at Knowledge Distillation with Features, Logits, and Gradients

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Abstract

Knowledge distillation (KD) is a substantial strategy for transferring learned knowledge from one neural network model to another. A vast number of methods have been developed for this strategy. While most method designs a more efficient way to facilitate knowledge transfer, less attention has been on comparing the effect of knowledge sources such as features, logits, and gradients. This work provides a new perspective to motivate a set of knowledge distillation strategies by approximating the classical KL-divergence criteria with different knowledge sources, making a systematic comparison possible in model compression and incremental learning. Our analysis indicates that logits are generally a more efficient knowledge source and suggests that having sufficient feature dimensions is crucial for the model design, providing a practical guideline for effective KD-based transfer learning.

Introduction

Knowledge distillation transfers knowledge from one neural network model to another by matching different information sources from the models: logits (Hinton, Vinyals, and Dean 2014), features (Romero et al. 2015), or gradients (Srinivas and Fleurat 2018). It has found wide applications in such areas as model compression (Hinton, Vinyals, and Dean 2014), Wang and Yoon (2020), Ba and Caruana (2014), Urban et al. (2017), incremental learning (Li and Hoiem 2017), Lee et al. (2019), privileged learning (Lopez-Paz et al. 2016), adversarial defense (Papernot et al. 2016), and learning with noisy data (Li et al. 2017). This knowledge transfer usually follows the teacher-student scheme, in that a high-performing teacher model provides knowledge sources for a student model to match.

Which part of the teacher provides a more informative knowledge source for a student to distill? Heo et al. (2019) shows the features are more effective; Tian, Krishnan, and Isola (2020) observes that logits are generally better, but matching the pairwise correlation with features outperforms logits. Recently, Kim et al. (2021) reports that logits achieve a better result when using L2 loss instead of KL-divergence. This puzzling inconsistency is due to the differences of both optimization criteria and the knowledge sources used in different methods, making a fair comparison across knowledge sources impossible.

This work provides a systematic approach to analyzing the above puzzle, providing a closer look at transferring knowledge in features, logits, and gradients. Specifically, we propose a new perspective to reinterpret the classical KD objective, using Taylor expansion to approximate the KL-divergence with different knowledge sources. This novel perspective leads to a generalized divergence that allows us to use logits and features under the same optimization criteria, while the gradients provide information about the importance of each feature for distillation. We therefore based on the mathematical insight as a unified framework, instantiating varied methods to distill features, logits, and gradients. Interestingly, when we instantiate our variants with the simplest design choices, the resulting methods are similar to previous knowledge transfer techniques. The similarity demonstrates the generalizability of our framework. Nevertheless, there are still nuanced differences between our variants and prior methods to ensure a fair comparison across knowledge sources, discussed later.

We explore two aspects that have never been jointly investigated with knowledge sources. The first is observing whether the trend of effectiveness is consistent across different transfer learning tasks. The two most popular KD tasks are included: model compression and incremental learning. The second aspect is to investigate the impact of model design and identify the key factors affecting knowledge sources’ effectiveness. In summary, this work provides the following contributions and findings:

- We provide a new perspective to interpret the classical KD, showing that a single framework can describe the distillation with different knowledge sources. To the best of our knowledge, this perspective has not been presented.
- The new perspective leads to a new strategy for improving feature-based KD by weighing the importance of features based on gradients from the teacher.
- Our systematic comparison shows that logits generally is the more effective knowledge source, followed by the features weighted by gradients and the plain features. This trend is consistent in both model compression and incremental learning.
- We further use a new metric with a normalized basis to analyze the factors that affect the above trend, pointing
Rethinking Knowledge Distillation

Our observation starts with the generic knowledge distillation criteria $L_{KD}$ for classification (Hinton, Vinyals, and Dean 2014). The criteria includes cross-entropy loss $L_{CE}$ for a student model to learn from ground-truth label $y^*$, and $D_{KL}$, for minimizing the difference between the predicted class distribution $p^s$ (from the student) and $p^t$ (from the teacher). The latter term makes the knowledge transfer happen, for which the coefficient $\lambda$ controls the intensity:

$$L_{KD} = L_{CE}(p^s, y^*) + \lambda D_{KL}(p^t||p^s),$$  \hspace{1cm} (1)

$$D_{KL}(p^t||p^s) = \sum_y p^t_y \log p^t_y - \sum_y p^s_y \log p^s_y$$  \hspace{1cm} (2)

The next step is to expand $D_{KL}$. Here, we are more interested in how intermediate outputs $z$ from the teacher ($z^t = g^t(x)$) and student ($z^s = g^s(x)$) affect $D_{KL}$; therefore, we treat the $z$ as the only variable in $p^t_y = f(y; z^t)$ and $p^s_y = f(y; z^s)$, leaving the parameters (if any) of the softmax-based classifier $f$ as constants and the same $f$ is used for both the teacher and student. By taking the Taylor expansion around $z^s$ for the second term of equation (2) and using the notation $dz = z^s - z^t$, $D_{KL}$ becomes:

$$D_{KL}(p^t||p^s) = \sum_y p^t_y \log p^t_y - \sum_y p^s_y \log p^s_y$$
$$- dz^T \sum_y p^t_y \frac{d}{dz} \log p^t_y$$
$$- \frac{1}{2} dz^T \left( \sum_y p^t_y \frac{d^2}{dz^2} \log p^t_y \right) dz + \epsilon$$  \hspace{1cm} (3)

In equation (3) the first-order term is zero, while the second-order has a form of Fisher information matrix $F(z^t)$ at its middle. Details of derivation are available in the Supplementary. The above equations omit the $y$ for conciseness. Lastly, by ignoring the higher-order term $\epsilon$, $D_{KL}$ can be rewritten as:

$$D_{KL}(p^t||p^s) \approx \frac{1}{2} (z^s - z^t)^T F(z^t)(z^s - z^t)$$  \hspace{1cm} (4)

Although equation (4) has a simple quadratic form, it provides two key insights:

1. Minimizing the difference between the student’s and teacher’s intermediate representations reduces the KL-divergence.
2. The Fisher information $F(z^t)$, which leverages the gradients regarding the teacher’s intermediate representation, provides a weighting mechanism for the importance of features.

The Generalized Divergence

This section discusses the simplest design choices to turn equation (4) into a framework that is easy to implement and instantiate its variants. There are two empirical considerations in applying equation (4) both stemming from the challenges of computing $F(z^t)$. The first is its $O(|z|^2)$ complexity. The computation could be expensive when $z$ has a large dimension (e.g. the flattened feature map from a convolution neural network based on image inputs). Here we follow the common simplification used by EWC (Kirkpatrick et al. 2017) and Adam (Kingma and Ba 2015) in which only the diagonal of Fisher information matrix is considered, reducing the complexity to $O(|z|)$. Second, marginalizing over $y$ for accumulating the gradients could be time-consuming; therefore, there is a need to use an alternative loss function to collect gradients of $z^t$.

To adopt the above considerations, we define a generalized divergence $D_G$, making equation (4) a special case of the generalized form:

$$D_G(z^t, z^s) = \alpha(z^s - z^t)^T W(z^t) (z^s - z^t)$$  \hspace{1cm} (6)

The coefficient $\alpha$ is a scaling factor that can be absorbed by $\lambda$. The $W(z^t)$ is still an $n$-by-$n$ weighting matrix like $F(z^t)$, given $z^t$‘s dimension $n$. The calculation of $W$ is still based on gradients from the teacher, specifically:

$$W(z^t) = \text{diag}((\frac{d}{dz} L_s)(\frac{d}{dz} L_s)^T)$$  \hspace{1cm} (7)

The function diag casts all off-diagonal elements to be zero. The $L_s$ is $log p^t$ when computing the Fisher information. Here we consider two design choices for $L_s$ to avoid the need of marginalizing over $y$:

$$L_E = \log p^t$$  \hspace{1cm} (8)

$$L_H = \frac{1}{k} \sum_{y=1}^k (l^t_y)^2$$  \hspace{1cm} (9)

$L_E$ is related to the empirical Fisher which requires knowing the ground-truth class $y^*$. $L_H$ is a heuristic criteria by using the mean-squared logits ($l^t_y$) over $k$ classes. $L_H$ does not require labels, but captures the gradients that lead to a large change in logits. $L_H$ is useful when the student (and its training data) has a different set of classes from the teacher, which is a case that $L_E$ is not applicable.

Overall, our full criteria $L_{KD-G}$ with the generalized divergence has the form:

$$L_{KD-G} = L_{CE}(p^s, y^*) + \lambda D_G(z^t, z^s)$$  \hspace{1cm} (10)

Note $z$ can be the logits or features. Besides, the knowledge in the teacher’s gradients are transferred to the student via $W(z^t)$. Therefore $L_{KD-G}$ provides a unified framework for comparing the effectiveness of each knowledge source by instantiating it in different ways. This is one of the main contributions of this paper, as our formulation allows an explicit fair comparison across knowledge sources within a unified framework. Below we elaborate the cases when $z$ is features or logits, and additionally extend the discussion to a model’s parameters, which is a popular knowledge source in the incremental learning.
importance-weighted parameter regularization method:

\[ L_{EWC} = L_{CE}(p_s, y^*) + \frac{\lambda}{2}(\theta^s - \theta^t)^T F(\theta^t)(\theta^s - \theta^t) \tag{11} \]

Although \( L_{KD-G} \) and \( L_{EWC} \) share a similar form, they have four fundamental differences: (1) \( L_{KD-G} \) is for features or logits, while \( L_{EWC} \) is only for parameters. (2) Our derivation starts from KL-divergence between categorical distributions, while EWC starts from the normal approximation of the posterior for parameters. (3) EWC requires the teacher and student model to be the same. As a result, EWC is not considered as a KD method and is not applicable to model compression. (4) \( L_{KD-G} \) achieves significantly better results than \( L_{EWC} \) in the task-incremental learning (shown in the experiment section).

The Four Instantiations

Based on the above discussion, we create four \( L_{KD-G} \) instantiations to enable a systematic comparison. The first two (Weighted\( _E \) Features-SE and Weighted\( _H \) Features-SE) are novel KD methods derived from our framework, while the latter two (Features-SE and Logits-SE) have close alternatives in previous works. The illustration of the first three variants is shown in Figure 1. Their names are listed below with a description of how their \( W \) is implemented for the \( L_{KD-G} \) (Note: SE means squared error):

- Weighted\( _E \) Features-SE: \( W \) uses \( L_E \)
- Weighted\( _H \) Features-SE: \( W \) uses \( L_H \)
- Features-SE: \( W = I \)
- Logits-SE: \( W = I \)

Experiments

Model Compression (MC)

Model compression is the primary task where knowledge distillation techniques are applied heavily. In this case, the student model has a smaller capacity than the teacher yet is asked to match the teacher’s outputs. If the target for matching contains a great deal of information that is not crucial for a downstream task, the student is more likely to waste its limited capacity on matching unimportant information, crippling the student’s ability to reach the teacher’s performance. This argument gives an intuition of why one can expect the weighted features to perform better than an unweighted one. A similar argument may apply to the logits since the linear layer before the logits imposes weights on the features. This section provides empirical support for the arguments with our unified framework.

Applying our framework

Two empirical considerations need to be addressed for applying equation 10 to model compression. The first one is that a large numerical range can result from \( D_G \), potentially making the optimization unstable. In the KD training procedure, the student is initialized randomly and is directly optimized from scratch with the knowledge distillation criteria. The random student model could make the squared difference between \( z^s \) and \( z^t \)
unbounded. This issue can be addressed by normalization. Specifically, we make \( \hat{z}^s \) and \( \hat{z}^t \) unit vectors:
\[
\hat{z}^t = \frac{z^t}{||z^t||}, \quad \hat{z}^s = \frac{z^s}{||z^s||}
\]  

(12)

The second empirical consideration is the mismatched dimensions between \( z^t \) and \( z^s \) when they are features. This case happens when the student and teacher have different types of neural network architectures or when the student has a smaller model width. We add a linear transformation \( r \) on the outputs of \( g^t(x) \) to match the teacher’s dimension:
\[
z^s = r(g^s(x))
\]  

(13)

The parameters of \( r \) are also optimized by the customized \( D_G \) for this section:
\[
D^M_G = (\hat{z}^s - \hat{z}^t)^T W(z^t)(\hat{z}^s - \hat{z}^t)
\]  

(14)

Note that \( r \) is only involved during training and is removed from testing; thus, the student model’s design has no dependency on \( r \). Additionally, \( r \) is only used when \( z \) is features, since the logits layer always has the same dimensionality (number of classes) between the teacher and student.

**Implementation Details** The procedure of the experiments here closely follows the model compression benchmark (Tian, Krishnan, and Isola 2020). The experiments include a large number of combinations between the teacher and student models. The teacher-student pairs include the models from the same architectural family but with different depth or width, and the models from different architectures that result in different sizes of features. The list of neural network architectures includes ResNet (He et al. 2016), WideResNet (Zagoruyko and Komodakis 2016), VGG (Simonyan and Zisserman 2015), MobileNet (Howard et al. 2017), and ShuffleNet (Zhang et al. 2018). We use the feature map output of the last convolutional block for the features and the last linear layer’s outputs for the logits. When the student’s feature map is different from the teacher’s, the transformation function \( r \) resizes the feature maps spatially with PyTorch’s pooling operation (Paszke et al. 2019). Then, the student’s number of channels is linearly projected by a 1x1-conv layer (\( r \)) to match the teacher’s channel number. Lastly, the resulting feature map is flattened for \( z \). For consistency, we include \( r \) in all our variants that use features (but not logits), regardless of whether the feature maps have the same size or not.

For learning with the CIFAR100 dataset, the models have an initial learning rate of 0.05, decayed by 0.1 every 30 epochs after the first 150 epochs until it reaches 240. For MobileNetV2, ShuffleNetV1 and ShuffleNetV2, the initial learning rate is 0.01 as suggested by (Tian, Krishnan, and Isola 2020). All the methods use SGD with a momentum of 0.9 and a batch size of 64. In short, we follow the benchmark settings (Tian 2020) and use the same teacher models provided to conduct all the experiments, ensuring a fair comparison between all methods.

**Hyper-parameter selection** could have a profound effect on most knowledge distillation-based model compression. We follow the benchmark protocol (Tian, Krishnan, and Isola 2020), which selects the hyper-parameters based on only one teacher-student pair. (we use resnet32x4/resnet8x4), then apply it to all other cases. Therefore, a method has to be robust to the hyper-parameters choice to perform well in all cases.

We make a step further to align the hyperparameter \( \lambda \) used in Features-SE and WeightedG Features-SE. This can be achieved by normalizing the \( W(z^t) \)'s outputs to make its diagonal to have a mean of one (like the identity matrix \( I \)) and unit variance. This normalization makes the features have an expected importance of 1 no matter how \( W \) is computed, leaving the gradient-based weighting the only factor to affect the performance between the two cases. As a result, we can use the coefficient \( \lambda = \lambda_F = 3 \) in all cases for features. Additionally, \( \lambda = \lambda_L = 15 \) when we use logits. Lastly, we add an extra setting by combining features and logits. It leads to the case of “B+C” in Tables 1 and 2 with the customized \( D_G \):
\[
D^M_{G-BC} = \lambda_L(\hat{z}_s - \hat{I}_s)^T(\hat{I}_s - \hat{I}_s) + \lambda_F(\hat{z}_s - \hat{I}_s)^T W_E(z_i)(\hat{z}_s - \hat{z}_i)
\]  

(15)

Note that \( \hat{I} \) is the normalized logits and \( \hat{z} \) is the normalized features. We use \( \lambda = 1 \) for \( D^M_{G-BC} \).

**MC Benchmark Results** First of all, we emphasize that our focus is on evaluating our general formulation and keeping their instantiations in the simplest form, revealing the intrinsic trend of KD with different knowledge sources. Our goal here is not being state-of-the-art (e.g. CRD), although our methods (B and C in Tables 1 and 2) achieve comparable or better performance. In our comparison, Tables 1 and 2 statistically agree with the arguments made at the beginning of the MC section: First, the weighted features (B) performs better than the unweighted one (A) in 9 out of 11 cases. Second, the logits (C) performs better than features (A and B) in 9 out of 11 cases. This result suggests the rank of “logits > weighted features > plain features” in their KD efficiency. Besides, our variants (A, B, C) outperform most of the previous KD methods, showing that our methods are efficient in extracting out the knowledge, and making the observed ranks more representative.

**Experiment on ImageNet** We use a larger and harder dataset for replicating the experiments of Table 1 with the standard ResNet. Table 2 uses the subset of ImageNet images (TinyImageNet 2017), showing the same trend of “logits > weighted features > plain features”, and combining all (B+C) leads to the best result. Tiny-ImageNet has 200 classes sub-sampled from ImageNet. Each class has 500 training images and testing images with size 64x64. All the methods in Table 2 use the same hyperparameters as in Tables 1 and 2. The training configuration is similar to Tables 1 except that the initial learning rate is 0.1, and is decayed by the factor of 0.1 at 50% and 75% of total (100) epochs. The weight-decay is 0.005. The models (ResNet-18/34/50) are the default models defined in the PyTorch. All students are trained from scratch (with random initialization). The teacher’s weights were initialized with an ImageNet(full)-pretrained model in PyTorch model zoo, then is fine-tuned with Tiny-ImageNet.
inevitably changes the model’s capacity, and a larger capacity may affect the ranking of knowledge sources. The recovered performance ratio (RPR) is therefore, subtracting its accuracy (e.g., resnet32: 64 channels; WRN-40-1: 64; WRN-40-2: 128) than the students in Table 1 generally have a smaller feature size (e.g., resnet32: 64 channels; WRN-40-1: 64; WRN-40-2: 128) than the students in Table 1 generally have a smaller feature size (e.g., resnet32: 64 channels; WRN-40-1: 64; WRN-40-2: 128) than the students in Table 1. One possible factor is that the students in Table 1 generally have a smaller feature size (e.g., resnet32: 64 channels; WRN-40-1: 64; WRN-40-2: 128) than the students in Table 1 generally have a smaller feature size (e.g., resnet32: 64 channels; WRN-40-1: 64; WRN-40-2: 128) than the students in Table 1.

Table 1: Test accuracy (%) of student models on CIFAR100 with transfer across very different teacher and student architectures.

| Teacher model | Student model | resnet32x4 | resnet8x4 | vgg13 | WRN-40-2 | WRN-40-2 | WRN-40-2 | resnet110 | resnet32 |
|---------------|---------------|------------|----------|-------|----------|----------|----------|-----------|----------|
| Teacher acc.  | 79.42         | 74.64      | 75.61    | 75.61 | 75.61    | 74.31    |
| Student acc.  | 72.50         | 70.36      | 73.26    | 71.98 | 71.14    |

Table 2: Test accuracy (%) of student models on CIFAR100 with transfer across very different teacher and student architectures.

| Teacher model | Student model | resnet32x4 | resnet8x4 | vgg13 | WRN-40-2 | WRN-40-2 | WRN-40-2 | resnet110 | resnet32 |
|---------------|---------------|------------|----------|-------|----------|----------|----------|-----------|----------|
| Teacher acc.  | 79.42         | 74.64      | 75.61    | 75.61 | 75.61    | 74.31    |
| Student acc.  | 70.5          | 71.82      | 70.5     | 70.36 | 64.6     | 64.6     |

Key factor analysis This section investigates the factor that affects the ranking of knowledge sources. The clues come from Tables 1 and 2 in which Table 2 has a larger difference on (B-A) and (C-B) than in Table 1. One possible factor is that the students in Table 1 generally have a smaller feature size (e.g., resnet32: 64 channels; WRN-40-1: 64; WRN-40-2: 128) than the students in Table 1 generally have a smaller feature size (e.g., resnet32: 64 channels; WRN-40-1: 64; WRN-40-2: 128) than the students in Table 1 generally have a smaller feature size (e.g., resnet32: 64 channels; WRN-40-1: 64; WRN-40-2: 128) than the students in Table 1. To investigate whether the student’s feature size has a substantial impact, we vary the feature size within the same model architecture. However, changing feature size inevitably changes the model’s capacity, and a larger capacity helps the student more easily match the teacher. Thus, comparing the absolute accuracy can not make a conclusive analysis.

We therefore make an additional contribution in addressing the above issue for the analysis, applying two strategies to minimize the capacity effect: (1) only the number of channels for the last convolutional block is changed, and (2) we propose a recovered performance ratio to measure the relative performance gain by subtracting the performance of a vanilla student (trained without a teacher). The vanilla student has its performance increased along with the capacity; therefore, subtracting its accuracy (Acc\text{vanilla}) excludes the capacity effect. The recovered performance ratio (RPR) is...
Table 3: Test accuracy (%) of student models on Tiny-ImageNet. The type of knowledge sources is categorized by: logits (L), features (F), higher-order features (i.e., instance-wise correlation, F⁺), and gradients (G). The value is averaged with 5 repeats.

| Method       | Knowledge source | ResNet50 Top 1 | ResNet50 Top 5 | ResNet18 Top 1 | ResNet18 Top 5 |
|--------------|------------------|----------------|----------------|----------------|----------------|
| Teacher      | -                | 62.22          | 83.52          | 57.86          | 80.39          |
| Student (baseline) | -            | 53.15          | 75.97          | 53.15          | 75.97          |
| HKD (Hinton 2014) | L             | 58.26 ± 0.34  | 80.92 ± 0.25  | 58.54 ± 0.24  | 81.07 ± 0.12  |
| AT (Zagoruyko 2017) | F             | 55.67 ± 0.32  | 79.40 ± 0.22  | 56.04 ± 0.24  | 79.09 ± 0.35  |
| SP (Tung 2019) | F⁺              | 55.48 ± 0.33  | 78.34 ± 0.31  | 55.28 ± 0.32  | 78.46 ± 0.29  |
| PTK (Passalis 2018) | F⁺          | 54.34 ± 0.20  | 77.49 ± 0.24  | 54.45 ± 0.42  | 77.48 ± 0.21  |
| RKD (Park 2019) | F⁺              | 54.30 ± 0.44  | 77.31 ± 0.43  | 54.37 ± 0.22  | 77.40 ± 0.30  |
| (A) Features-SE | F              | 56.86 ± 0.21  | 79.44 ± 0.38  | 56.57 ± 0.40  | 79.43 ± 0.22  |
| (B) Weighted Features-SE | F + G  | 57.15 ± 0.29  | 79.55 ± 0.24  | 57.08 ± 0.17  | 79.54 ± 0.36  |
| (C) Logits-SE    | L               | 58.95 ± 0.34  | 81.14 ± 0.12  | 58.53 ± 0.32  | 80.80 ± 0.31  |
| (B+C)           | L + F + G       | 59.30 ± 0.20  | 81.46 ± 0.15  | 58.79 ± 0.15  | 81.09 ± 0.06  |

Figure 2: The analysis of feature size versus sources of knowledge. The performance gain due to model capacity has been subtracted. This figure highlights the changes in the ranking of knowledge sources. The raw value of each bar is averaged with 5 repeats and is available in Supplementary.

Figure 2 uses RPR to examine various student feature sizes. The teacher and student models are resnet32x4 and resnet8x4, correspondingly. The student model’s last convolutional block has its width (number of channels) configured to be between 64 to 1024. The result confirms feature size’s impact on the ranking:

- A larger student feature (e.g., 256) leads to a consistent ranking of "logits > weighted features > plain features".
- A smaller feature (e.g., 64) breaks the trend.

We additionally subtract a stronger baseline (HKD (Hinton, Vinyals, and Dean 2014)) by replacing equation (16)’s \( \text{Acc}_{\text{vanilla}} \) with \( \text{Acc}_{\text{HKD}} \) in Supplementary. Its trend is still the same as Figure 2 providing extra support for the observation. The result suggests the design guideline:

- Having a larger student feature size can benefit KD. Note that using larger features does not conflict with the goal of model compression. ShuffleNet (Zhang et al. 2018) in Table 2 is a positive case that has a relatively smaller model while still having a sufficient feature size.

Incremental Learning (IL)

Incremental learning is a problem setting where KD is often applied. The setting has its model exposed to a sequence of tasks. These tasks have differences in either their input distribution, label distribution, or both. The model has no access to the training data of previous tasks when learning a new task. The shift of distributions among tasks introduces a significant interference to the learned parameters, largely undermining previous tasks’ performance. This phenomenon is called catastrophic forgetting. A popular strategy is to regularize the model’s parameters to mitigate the forgetting, reducing drift from its previously learned parameters. However, when the regularization is too strong, the model will not have sufficient plasticity to learn a new task well. Thus, there is a trade-off between minimizing forgetting and maximizing plasticity. A good trade-off strategy keeps important knowledge while allowing the less important ones to be overwritten by the new tasks. The parameter-based regularization dominates this line of strategy. Previous works (Kirkpatrick et al. 2017; Zenke, Poole, and Ganguli 2017; Aljundi et al. 2018) select important parameters based on gradients and avoid those parameters from changing too much. It is a setting that complements model compression, providing an excellent opportunity to compare not only features, logits, and gradients, but also the model parameters for transferring the knowledge.

Applying our framework We consider the task-incremental learning (Hsu et al. 2018; van de Ven and Tolias 2019) setting for our experiments. This setting has exclusive sets of classes in a sequence of classification tasks. The model learns each classification task sequentially with only access to the training data of the current task. During the
the gradients. In this section, our Logits-SE uses the gradients. The only difference is that the current task’s labels are out-of-scope for the previous (teacher) model. Specifically, \( D_G \) is customized by:

\[
D^L_{G-\text{logits}} = \sum_j \left( l^s_{t,j} - l^t_{t,j} \right)^T \left( l^s_{t,j} - l^t_{t,j} \right) \tag{17}
\]

\[
D^L_{G-\text{features}} = \sum_j \left( z^s - z^t \right)^T W[j](z^t)(z^s - z^t) \tag{18}
\]

\[
W[j](z^t) = \text{diag}(\frac{d}{dz} k \sum_{y=1}^k (l^t_{t,j,y})^2)( \frac{d}{dz} k \sum_{y=1}^k (l^t_{t,j,y})^2)^T \tag{19}
\]

The \( l_{t,j} \) is the logits from the \( j \)th task. The regularization term sums over the tasks except the current task \( T_{\text{current}} \) (i.e., task index \( j = \{ 1 \ldots T_{\text{current}} - 1 \} \)). Note that when \( T_{\text{current}} = 2 \), everything here (equations 17 to 19) falls back to equations 6 and 7. The only difference is that the current task’s labels are out-of-scope for the previous (teacher) model. In other words, this is the case that \( y^s \) is not valid for the teacher; therefore, equation 19 uses \( L^H_t \) to collect the gradients. In this section, our Logits-SE uses \( D^L_{G-\text{logits}} \), Weighted\_SE Features-SE uses \( D^L_{G-\text{features}} \), and Features-SE has its \( W[j](z^t) = I \). We additionally add three EWC variants for the comparison. SL (Zenke, Poole, and Ganguli 2017) accumulates the gradients along the optimization trajectory to replace the \( F(\theta^t) \) in equation 11. MAS (Aljundi et al. 2018) uses \( L^H_t \) to compute the gradients for a weighting matrix similar to our \( W \). The L2 sets its \( F(\theta^t) = I \) in equation 11. All experiments here closely follow the implementation and evaluation protocol described in the popular benchmark (Hsu 2018). More details are in Supplementary.

**IL Benchmark Results** Table 4 shows that the effectiveness of knowledge sources is ranked: logits (L) > features (F) > parameters (P). Although the number of classes imposes a very different difficulty to the problem (2 classes per task in S-CIFAR10 versus 20 in S-CIFAR100), the methods noted with "P" performs significantly worse than "F" and "L" on both datasets, suggesting that regularizing the outputs generally strikes a better balance between forgetting and plasticity. Furthermore, the comparison between Weighted\_SE Features-SE versus Features-SE shows that having the squared error weighted by gradients is very helpful. Both above observations are consistent with the trends in model compression.

### Related Work

We categorize KD methods by their knowledge sources and discuss the most related works. First, the features-based methods generally make a small student match a large teacher’s features without selection. One exception is (Heo et al. 2019), which selects useful features by using margin ReLU with a per-feature threshold. However, its heuristic nature is significantly different from our gradient-driven approach (i.e., Weighted-SE ). It is also worth noting that our Features-SE is closely related FitNet (Romero et al. 2015) and FT (Kim, Park, and Kwak 2018). FitNet, FT, and our method align the dimension of the features between the teacher and student, then use squared error to match the features. However, FitNet adds a convolutional regressor (with non-linearity) for the student to do the matching, and trains the student with squared error loss and cross-entropy loss in two separate stages. In FT, it uses two small auto-encoders to transform the features from both the teacher and student. Therefore, both FitNet and FT have a more complicated design than our linear transformation function \( r \) and our one-stage training procedure. Second, in previous gradient-based methods (Srinivas and Fleuret 2018; Zagoruyko and Komodakis 2017), they directly match the teacher’s and student’s Jacobian, which requires double backpropagation to optimize its loss function. In contrast, our weighted features-SE does not use Jacobian, avoiding the heavy overhead in optimization. Lastly, the logits-based methods (Hinton, Vinyals, and Dean 2014; Li and Hoiem 2017) have been discussed in previous sections. Other logits-based strategies such as early stopping (Cho and Hariharan 2019) and teacher assistant (Mirzadeh et al. 2020) are orthogonal approaches to our work and can be applied jointly.

### Conclusion

We present a new perspective that can utilize different knowledge sources under a unified KD framework. This framework leads to a new KD method that prioritizes the distillation of important features based on gradients, and provides a new justification on how simple squared error approximates the classical KD criteria. We instantiate our framework based on the type of knowledge sources utilized, finding that logits is generally more efficient than features, while gradients can help the latter. Furthermore, our analysis points out that a student’s feature size is impactful to the KD and has a more complicated design than our linear transformation function \( r \) and our one-stage training procedure. Second, in previous gradient-based methods (Srinivas and Fleuret 2018; Zagoruyko and Komodakis 2017), they directly match the teacher’s and student’s Jacobian, which requires double backpropagation to optimize its loss function. In contrast, our weighted features-SE does not use Jacobian, avoiding the heavy overhead in optimization. Lastly, the logits-based methods (Hinton, Vinyals, and Dean 2014; Li and Hoiem 2017) have been discussed in previous sections. Other logits-based strategies such as early stopping (Cho and Hariharan 2019) and teacher assistant (Mirzadeh et al. 2020) are orthogonal approaches to our work and can be applied jointly.

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### Table 4: The averaged classification accuracy of task-incremental learning. The S-CIFAR10/100 datasets split their classes into 5 tasks for a model to learn sequentially.

| Method          | Reg | S-CIFAR10 | S-CIFAR100 |
|-----------------|-----|-----------|------------|
| Upper-bound     | -   | 97.6 ± 0.1| 84.3 ± 0.4 |
| Baseline        | -   | 63.5 ± 1.7| 30.5 ± 0.6 |
| L2              | P   | 74.2 ± 0.6| 51.7 ± 1.3 |
| EWC (Kirkpatrick 2017) | P  | 84.4 ± 2.1| 61.1 ± 1.4 |
| SI (Zenke 2017) | P   | 79.1 ± 1.3| 64.8 ± 1.0 |
| MAS (Aljundi 2018) | P | 78.3 ± 0.7| 64.8 ± 0.8 |
| Features-SE     | F   | 77.4 ± 3.6| 70.5 ± 0.6 |
| Weighted\_SE Features-SE | F | 93.3 ± 0.5| 73.7 ± 0.6 |
| Logits-SE       | L   | 95.3 ± 0.2| 78.3 ± 0.3 |

The findings will inspire more works in this field.
References

Aljundi, R.; Babiloni, F.; Elhoseiny, M.; Rohrbach, M.; and Tuytelaars, T. 2018. Memory Aware Synapses: Learning what (not) to forget. In *ECCV*.

Ba, J.; and Caruana, R. 2014. Do deep nets really need to be deep? In *Advances in neural information processing systems*, 2654–2662.

Cho, J. H.; and Harihara, B. 2019. On the efficacy of knowledge distillation. In *Proceedings of the IEEE International Conference on Computer Vision*, 4794–4802.

He, K.; Zhang, X.; Ren, S.; and Sun, J. 2016. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, 770–778.

Heo, B.; Kim, J.; Yun, S.; Park, H.; Kwak, N.; and Choi, J. Y. 2019a. A comprehensive overview of feature distillation. In *Proceedings of the IEEE International Conference on Computer Vision*, 1921–1930.

Heo, B.; Lee, M.; Yun, S.; and Choi, J. Y. 2019b. Knowledge transfer via distillation of activation boundaries formed by hidden neurons. In *Proceedings of the AAAI Conference on Artificial Intelligence*.

Hinton, G.; Vinyals, O.; and Dean, J. 2014. Distilling the knowledge in a neural network. *NIPS Deep Learning Workshop*.

Howard, A. G.; Zhu, M.; Chen, B.; Kalenichenko, D.; Wang, W.; Weyand, T.; Andreetto, M.; and Adam, H. 2017. Mobilenets: Efficient convolutional neural networks for mobile vision applications. *arXiv preprint arXiv:1704.04861*.

Hsu, Y.-C. 2018. [Continual-Learning-Benchmark](https://github.com/GT-RIPL/Continual-Learning-Benchmark)

Hsu, Y.-C.; Liu, Y.-C.; Ramasamy, A.; and Kira, Z. 2018. Re-evaluating Continual Learning Scenarios: A Categorization and Case for Strong Baselines. In *NeurIPS Continual learning Workshop*.

Kim, J.; Park, S.; and Kwak, N. 2018. Paraphrasing complex network: Network compression via factor transfer. In *Advances in neural information processing systems*.

Kim, T.; Oh, J.; Kim, N.; Cho, S.; and Yun, S.-Y. 2021. Comparing Kullback-Leibler Divergence and Mean Squared Error Loss in Knowledge Distillation. In *IJCAI*.

Kingma, D. P.; and Ba, J. 2015. Adam: A method for stochastic optimization. *ICLR*.

Kirkpatrick, J.; Pascanu, R.; Rabinowitz, N.; Veness, J.; Desjardins, G.; Rusu, A. A.; Milan, K.; Quan, J.; Ramalho, T.; Grabska-Barwinska, A.; et al. 2017. Overcoming catastrophic forgetting in neural networks. *Proceedings of the national academy of sciences*.

Lee, K.; Lee, K.; Shin, J.; and Lee, H. 2019. Overcoming Catastrophic Forgetting With Unlabeled Data in the Wild. In *Proceedings of the IEEE International Conference on Computer Vision*, 312–321.

Li, Y.; Yang, J.; Song, Y.; Cao, L.; Luo, J.; and Li, L.-J. 2017. Learning from noisy labels with distillation. In *Proceedings of the IEEE International Conference on Computer Vision*, 1910–1918.

Li, Z.; and Hoiem, D. 2017. Learning without forgetting. *IEEE transactions on pattern analysis and machine intelligence*, 40(12): 2935–2947.

Lopez-Paz, D.; Bottou, L.; Schölkopf, B.; and Vapnik, V. 2016. Unifying distillation and privileged information. *International Conference on Learning Representations*.

Mizraadeh, S. I.; Farajtabar, M.; Li, A.; Levine, N.; Matsukawa, A.; and Ghasemzadeh, H. 2020. Improved knowledge distillation via teacher assistant. In *Proceedings of the AAAI Conference on Artificial Intelligence*.

Papernot, N.; McDaniel, P.; Wu, X.; Jha, S.; and Swami, A. 2016. Distillation as a defense to adversarial perturbations against deep neural networks. In *2016 IEEE Symposium on Security and Privacy (SP)*, 582–597. IEEE.

Park, W.; Kim, D.; Lu, Y.; and Cho, M. 2019. Relational knowledge distillation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 3967–3976.

Passalis, N.; and Tefas, A. 2018. Learning Deep Representations with Probabilistic Knowledge Transfer. In *Proceedings of the European Conference on Computer Vision (ECCV)*.

Paszke, A.; Gross, S.; Massa, F.; Lerer, A.; Bradbury, J.; Chanan, G.; Killeen, T.; Lin, Z.; Gimelshein, N.; Antiga, L.; Desmaison, A.; Kopf, A.; Yang, E.; DeVito, Z.; Raison, M.; Tejani, A.; Chilamkurthy, S.; Steiner, B.; Fang, L.; Bai, J.; and Chintala, S. 2019. PyTorch: An Imperative Style, High-Performance Deep Learning Library. In Wallach, H.; Larochelle, H.; Beygelzimer, A.; d’Alché-Buc, F.; Fox, E.; and Garnett, R., eds., *Advances in Neural Information Processing Systems 32*, 8024–8035. Curran Associates, Inc.

Romero, A.; Ballas, N.; Kahou, S. E.; Chassang, A.; Gatta, C.; and Bengio, Y. 2015. Fitnets: Hints for thin deep nets. *ICLR*.

Simonyan, K.; and Zisserman, A. 2015. Very deep convolutional networks for large-scale image recognition. *ICLR*.

Srinivas, S.; and Fleuret, F. 2018. Knowledge Transfer with Jacobian Matching. In *International Conference on Machine Learning*, 4723–4731.

Tian, Y. 2020. [HobbitLong/RepDistiller](https://github.com/HobbitLong/RepDistiller)

Tian, Y.; Krishnan, D.; and Isola, P. 2020. Contrastive Representation Distillation. In *International Conference on Learning Representations*.

TinyImageNet. 2017. [https://www.kaggle.com/c/tiny-imagenet](https://www.kaggle.com/c/tiny-imagenet)

Tung, F.; and Mori, G. 2019. Similarity-preserving knowledge distillation. In *Proceedings of the IEEE International Conference on Computer Vision*.

Urban, G.; Geras, K. J.; Kahou, S. E.; Aslan, O.; Wang, S.; Caruana, R.; Mohamed, A.; Philipose, M.; and Richardson, M. 2017. Do deep convolutional nets really need to be deep and convolutional? *ICLR*.

van de Ven, G. M.; and Tolias, A. S. 2019. Three scenarios for continual learning. *arXiv preprint arXiv:1904.07734*.

Wang, L.; and Yoon, K.-J. 2020. Knowledge distillation and student-teacher learning for visual intelligence: A review and new outlooks. *arXiv preprint arXiv:2004.05937*. 

Zagoruyko, S.; and Komodakis, N. 2016. Wide residual networks. *arXiv preprint arXiv:1605.07146*.

Zagoruyko, S.; and Komodakis, N. 2017. Paying more attention to attention: Improving the performance of convolutional neural networks via attention transfer. *ICLR*.

Zenke, F.; Poole, B.; and Ganguli, S. 2017. Continual Learning Through Synaptic Intelligence. In *International Conference on Machine Learning*.

Zhang, X.; Zhou, X.; Lin, M.; and Sun, J. 2018. Shufflenet: An extremely efficient convolutional neural network for mobile devices. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, 6848–6856.
A Closer Look at Knowledge Distillation with Features, Logits, and Gradients
-Supplementary Materials-

Paper ID 2350

Detailed Derivations of Equation [3]
This section provides the detailed steps that are not included in the main text for deriving equation 3 from equation 5.

(1) The first-order term in equation 3 is zero:

\[-dz^T \sum_y p^t_y \frac{d}{dz} \log p^t_y = -dz^T \sum_y \frac{d}{dz} p^t_y \]

\[= -dz^T (\frac{d}{dz} \sum_y p^t_y) \]

\[= 0 \]

(2) The second-order term in equation 3 has a form of Fisher information matrix \(F(z^t)\) at its middle:

\[\frac{1}{2} dz^T (\sum_y p^t_y \frac{d^2}{dz^2} \log p^t_y) dz \]

\[= \frac{1}{2} dz^T \sum_y p^t_y \left[ \frac{1}{p^t_y} \frac{d^2 p^t_y}{dz^2} - \left( \frac{dp^t_y}{dz} \right)^2 \right] \]

\[= \frac{1}{2} dz^T \sum_y \frac{d^2 p^t_y}{dz^2} \sum_y p^t_y - \sum_y \frac{dp^t_y}{dz} \sum_y \frac{dp^t_y}{dz} \]

\[= \frac{1}{2} d^T z \sum_y \frac{d^2 p^t_y}{dz^2} \sum_y p^t_y - \sum_y \frac{dp^t_y}{dz} \sum_y \frac{dp^t_y}{dz} \]

\[= \frac{1}{2} d^T z \sum_y \frac{dp^t_y}{dz} \sum_y \frac{dp^t_y}{dz} \]

\[= \frac{1}{2} d^T z \sum_y \frac{d}{dz} \log p^t_y \frac{d}{dz} \log p^t_y \]

\[= \frac{1}{2} d^T F(z^t) d \]

Additional analysis of Figure [2]
Figure [A] has the same experiment as in Figure [2] but the performance gain due to model capacity has been more aggressively removed by subtracting the performance of HKD (Hinton, Vinyals, and Dean 2014) in the RPR (equation [16]). This figure has a trend similar to Figure [2], strengthening the argument that the student’s feature size has a crucial impact on knowledge distillation. Table [A] provides the raw accuracy used in both Figures [2] and [A].

Model Compression Size
Tables [B] and [C] provide the size of each model used in Tables [1] and [2].

Implementation Details of IL Experiment
Our task-incremental learning experiment follows the implementation and evaluation protocol described in a popular continual learning benchmark (Hsu et al. 2018). The benchmark splits the image datasets CIFAR10 and CIFAR100 into 5 tasks; thus, each task has 2 and 20 classes, correspondingly. The evaluation is performed at the end of the learning curriculum, and the averaged classification accuracy of all tasks is reported with testing data. The regularization coefficient \(\lambda\) of all methods (except the baseline and non-incremental learning) are selected by a grid search with 20% of the dataset. We follow all the default training and testing configurations provided by the public benchmark (Hsu 2018) to conduct the experiments. Lastly, in our methods, the features used in Weighted Feature-SE and Feature-SE are the flattened outputs from the last convolutional block of WideResNet-28-2 (Zagoruyko and Komodakis 2016), which has the output dimension of 2048 (CIFARH=128x4x4). Note that we do not use the linear transformation function \(r\) in Section since the teacher and student models always have the same feature dimensions.
### Table A: The raw accuracy (CIFAR100) used in Figures 2 and A. The baseline is the student model trained without knowledge distillation.

| #channel | Baseline | HKD | (A) Features-SE | (B) Weighted Feature-SE | (C) Logits-SE | (B+C) |
|----------|----------|-----|-----------------|------------------------|--------------|-------|
| 64       | 68.12    | 67.87 | 69.79 ± 0.18 | 68.96 ± 0.47 | 68.77 ± 0.14 | 68.67 ± 0.49 |
| 128      | 70.55    | 71.34 ± 0.43 | 72.52 ± 0.25 | 72.88 ± 0.12 | 73.82 ± 0.28 | 73.73 ± 0.24 |
| 256      | 72.50    | 73.33 ± 0.25 | 74.83 ± 0.15 | 75.20 ± 0.15 | 76.29 ± 0.16 | 76.66 ± 0.22 |
| 512      | 73.62    | 74.98 ± 0.17 | 76.22 ± 0.10 | 76.66 ± 0.13 | 77.47 ± 0.25 | 78.29 ± 0.20 |
| 1024     | 74.40    | 75.94 ± 0.24 | 77.38 ± 0.20 | 77.64 ± 0.19 | 78.82 ± 0.08 | 79.47 ± 0.25 |

### Table B: The size of models used in Table 1

| Teacher model | Student model | Teacher size (M) | Student size (M) | Compression rate |
|---------------|---------------|------------------|------------------|-----------------|
| resnet32x4    | resnet8x4     | 7.43             | 1.23             | 0.17            |
| WRN-40-2      | WRN-16-2      | 2.26             | 0.70             | 0.31            |
| resnet110     | resnet32      | 1.74             | 0.57             | 0.25            |
| vgg13         | vgg8          | 9.46             | 0.47             | 0.27            |

### Table C: The size of models used in Table 2

| Teacher model | Student model | Teacher size (M) | Student size (M) | Compression rate |
|---------------|---------------|------------------|------------------|-----------------|
| resnet32x4    | ShuffleNetV1  | 7.43             | 0.95             | 0.13            |
| resnet32x4    | ShuffleNetV2  | 7.43             | 1.36             | 0.18            |
| WRN-40-2      | WRN-16-2      | 2.26             | 0.95             | 0.42            |
| ResNet50      | vgg8          | 23.71            | 3.96             | 0.17            |
| ResNet50      | MobileNetV2   | 23.71            | 0.81             | 0.03            |
| vgg13         | MobileNetV2   | 9.46             | 0.81             | 0.09            |

### Additional discussion: Model Compression versus Incremental Learning

This section highlights three differences between Model Compression (MC) and Incremental Learning (IL) in their problem settings. First, MC has its teacher share the same training data with the student. In contrast, the student in task-incremental learning has no access to the teacher’s training data and is exposed to a new set of classes in the new task.

Second, MC only applies the KD process once, while the task-IL repeats the KD process multiple times based on the length of the task sequence.

Lastly, the task-incremental learning setting obeys closely the assumptions made in Section , while MC relaxes those a little bit. Although MC’s formulation (equation 14) has the same form as equation 10, it deviates from the assumptions made in equation 4 in three ways: (1) the dz might not be small, (2) the z is normalized and linearly transformed, and (3) the softmax-based classifier $f$ is not forced to be the same between the teacher and student. Note that (2) and (3) happens when z is features but not logits, and the impact of (1) has been reduced by the normalization (equation 12).