ENERGY OF KERR-NEWMAN BLACK-HOLES
AND GRAVITOMAGNETISM

Marcelo Samuel Berman

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R. Candido Hartman,575 #17 Ed. Renoir-Champagnat

80730-440 Curitiba PR BRAZIL

ABSTRACT

New formulae are obtained for the energy of K.N. b.h.’s that point out a
gravitomagnetic energy effect. The results are valid for slowly or rapidly
rotating black-holes. The expression of the energy density of Kerr-Newman
back-holes in the slow rotation case, is obtained afterwards, and shown to
be essentially positive. Subsequently, we show how to attain a “repulsive”
gravitation (antigravitation) state identified with negative energy
distribution contents in a limited region of space, without violating the
Positive Energy Theorem.
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There are pretty standard pseudotensor calculations presented in the book by Adler et al.\textsuperscript{(1)}. The most general black hole is characterized by mass $M$, electric charge $Q$ and rotational parameter "$a$" and is given by Kerr-Newman’s metric, where in quasi-Cartesian form, is given by:

\[
ds^2 = dt^2 - dx^2 - dy^2 - dz^2 - \frac{2 [M - \frac{Q^2}{r_0}]}{r_0^2 + a^2 z^2} \cdot F^2
\]

\[
F = dt + \frac{Z}{r_0} dz + \frac{r_0}{(r_0^2 + a^2)} (xdx + ydy) + \frac{a(xdy - ydx)}{a^2 + r_0^2}
\]

\[
r_0^4 - (r^2 - a^2) r_0^2 - a^2 z^2 = 0
\]

and

\[
r^2 \equiv x^2 + y^2 + z^2
\]

The energy-momentum quadrivector $P_\mu$, the energy-tensor $T^V_\mu$ and the energy-momentum pseudo-tensor of the gravitational field $t^V_\mu$ obey the following relations:

\[
P_\mu = \int t \sqrt{-g} \left[ T^0_\mu + t^0_\mu \right] d^3x = \text{ constants.} \tag{5}
\]

\[
\sqrt{-g} t^V_\mu = \frac{1}{2G} \left[ U g^\nu_\mu - \frac{\partial U}{\partial g^\sigma_\nu} g^\pi_\mu \right] \tag{6}
\]

\[
U = \sqrt{-g} g^{\sigma \alpha} \left[ \begin{array}{c|c} \alpha & \beta \\ \hline \sigma \beta & \alpha \beta \end{array} \right] - \left[ \begin{array}{c|c} \alpha & \beta \\ \hline \alpha \beta & \beta \sigma \end{array} \right]
\]

\[
C = -\frac{8\pi G}{c^2} \tag{7}
\]

After a lengthy calculation, we find: (in G=c=1 units)
\[ P_0 = M - \left[ \frac{Q^2 + M^2}{4\rho^2} \right] \left[ 1 + \frac{(a^2 + \rho^2)}{a\rho} \text{arctgh} \left( \frac{a}{\rho} \right) \right] \] (9)

\[ P_1 = P_2 = P_3 = 0 \] (10)

By considering an expansion of the \( \text{arctgh}(\frac{a}{\rho}) \) function, in terms of increasing powers of the parameter "\( a \)", and by neglecting terms \( a^3 \simeq a^4 \simeq \ldots \simeq 0 \), we find the energy of a slowly rotating Kerr-Newman black-hole,

\[ E \simeq M - \left[ \frac{Q^2 + M^2}{R} \right] \left[ \frac{a^2}{R^2} + \frac{1}{2} \right] \] (11)

where \( \rho \to R \); this can be seen because the defining equation for \( \rho \) is:

\[ \frac{x^2 + y^2}{\rho^2 + a^2} + \frac{z^2}{\rho^2} = 1 \quad \text{and if } a \to 0, \; \rho \to R. \]

We can interpret the terms \( \frac{Q^2a^2}{3R^2} \) and \( \frac{M^2a^2}{3R^2} \) as the magnetic and gravitomagnetic energies caused by rotation. Virbhadra\(^{(2)}\) noticed the first of these effects in the year 1990, but since then it seems that he failed to recognize the existence of the gravitomagnetic energy due to \( M \), on an equal footing.

Furthermore, we can, from relation (11), find the energy density associated with the black-hole:

\[ \mu = \frac{1}{4\pi R^2} \frac{dE}{dR} = \frac{1}{4\pi R^2} \left[ Q^2 + M^2 \right] \left[ \frac{a^2}{R^2} + \frac{1}{2} \right] \] (12)

As expected, the energy density is essentially positive, so it obeys the weak energy condition.

We shall now show how to attain a "repulsive" gravitation (or antigravitation) state identified with negative energy contents in a limited region of space, without violation of the positivity of energy.

The total energy \( E \) of an isolated system should be positive in General Relativity, in accordance with the "Positive Energy Theorem", of Schoen and Yau, Choquet-Bruhat, Deser,
Teitelboim, Witten, York, etc\textsuperscript{(3)(12)}, as cited by Ciufolini and Wheeler\textsuperscript{(13)}. The exception is Minkowski space, where $E = 0$. (See also Weinberg\textsuperscript{(14)}). The condition usually stated for these theorems is that the dominance of energy condition be satisfied, which entails the weak energy condition $\mu \geq 0$, where $\mu$ stands for the energy density\textsuperscript{(13)}, while the Einstein field equations are taken as valid.

Even in popular books, like Hawking’s best-seller\textsuperscript{(15)}, it is stated that (total) energy in General Relativity is positive. The rule is broken by the energy of the Universe, $E = 0$, as stated in Hawking’s book\textsuperscript{(15)}, because zero is not positive, and the Universe’s metric is not Minkowskian (see Berman\textsuperscript{(16)}).

Ciufolini and Wheeler\textsuperscript{(13)} also comment that an electric charge circling in orbit creates magnetism and, likewise, a spinning mass creates gravitomagnetism; the gravitomagnetic effect due to the slow rotation of the Earth is measurable, by an orbiting gyroscope.\textsuperscript{(Lageos III; Gravity Probe B, etc.)}. However, we are far from a spinning black-hole\textsuperscript{(13)}.

When obtaining the central mass field, or Kerr metric, for a spinning massive object, one has to measure the energy contents of the distribution, encircled by a radial distance $R$, and we have shown that, for non-relativistic (slow) rotations, the energy contents is given by (within a radial distance $R$):

$$E = Mc^2 - \frac{GM^2}{2R} - \frac{a^2M^2}{3R^3}$$

(13).

where $M$, c G, R and ”$a$”, stand respectively for central mass, speed of light in vacuum, gravitational constant, radial distance, and angular momentum per unit mass, and $Q=0$.

The first rhs term above is the inertial energy, as known from Special Relativity; the second, is the relativistic equivalent to the self-gravitational energy, (in Newton’s theory $-\frac{3}{5} \left(\frac{GM^2}{R}\right)$).

The 3rd., stands for gravitomagnetic energy.

An embarrassing problem with (13), is that when $R$ becomes, for instance, $R_0 \leq \frac{GM}{2c^2}$, we have $E < 0$. This does not violate the positivity of ”total” energy theorem, but we interpret
as pertaining to a negative energy that represents a repulsive gravity region (anti-gravitation).

This effect could be used to motorize an antigravitational engine. The technological effort necessary to attain this objective may only succeed at about 20 years from now. We refer to the above equation for \( E \) in the rapidly spinning case,

\[
E = Mc^2 \frac{GM}{\rho^2} \left[ 1 + \left( \frac{a^2 + \rho^2}{a^2} \right) \text{arctgh} \left( \frac{a}{\rho} \right) \right]
\]

(14).

where \( \frac{x^2 + y^2}{a^2 + \rho^2} + \frac{z^2}{\rho^2} = 1 \) and \( \rho \geq 0 \), and \( E \) stands for the energy inside a surface of constant \( \rho \) values.

In fact, if we interpret the total energy, as the limit, of \( E \), when the radial distance goes to infinity, so that the "entire" space is involved (say, put observer at spatial infinity), the "positivity" of total energy will be preserved, for a positive mass \( M \), while we would still have, at certain "radial" distances \( \rho = \text{constant} \), the possibility for \( E < 0 \) (antigravitation or repulsive gravity). We must, indeed, not confuse the "total" energy (over all space), with the energy distribution contents at radial distance from the source of the field. The "total" energy is, of course, \( \lim_{R \to \infty} E = Mc^2 > 0 \). We conclude that antigravity can be obtained in practice.

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