Numerical study of the influence of relative parameters of the metamaterial structure on its mechanical behavior

L R Akhmetshin$^{1,2}$

$^1$Institute of Strength Physics and Materials Science of Siberian Branch of Russian Academy of Sciences, 2/4 Akademicheski pr., Tomsk, 634055, Russia
$^2$Tomsk State University, 36 Lenin ave., Tomsk, 634050, Russia

E-mail: akhmetshin.lr@gmail.com

Abstract. In this paper, attention is focused on the influence of changes in the variable parameters of the structure of tetrachiral metamaterial on its linear elastic behavior, in particular, on its twist. The parameters characterizing the structure of the metamaterial were chosen in a relative form with respect to the unit cell size and changed independently of each other in the investigation. The results of the mechanical behavior of the tetrachiral metamaterial in the event of changes in the structural parameters were obtained. The dependencies of the rotation angle when the relative parameters change were established and analyzed. The parameters of the chiral structure, the most affecting the unusual behavior of mechanical metamaterial—a twist under uniaxial loading, were determined.

1. Introduction
The mechanical metamaterials are a kind of metamaterials rapidly developing over the past ten years. Metamaterials are materials whose properties are determined by their artificially created structure but not by their chemical composition.

The interest in the development of mechanical metamaterials is partly due to advances in additive manufacturing technologies, which allowed the production of materials with an arbitrarily complex micro/nanostructure. The desire to create mechanical metamaterials is associated with the creation of structures with a certain set of physical and mechanical properties.

In comparison to the natural materials which have congenital mechanical properties at different scale levels, metamaterials are characterized by construction-specific properties at high length scales and, if we look at their behavior at the scale of the unit cell (micro and nanoscales), it corresponds to the behavior of the base material. However, in general, the average macroscale mechanical behavior of metamaterials should have some unusual effects [1].

Specific classes of mechanical metamaterials were known already a few decades ago. In recent years, metamaterials with a negative Poisson's ratio (auxetics) have been gaining popularity [2]. They are interesting because exhibiting unusual behavior—expand transversely in the tension and compression. Specific examples of such materials have been observed, created, tested, and reported elsewhere [3, 4].

One type of auxetic metamaterials is the materials with chirality of structure. Chiral metamaterials have several known advantages, such as an additional degree of freedom under the uniaxial loading—twist (rotation) [5], a large volume of pores, which depend on the parameters of the structure of the chiral metamaterial [6].
In this paper, we will pay special attention to the influence of changes in the variables of relative parameters of the structure of mechanical tetrachiral metamaterial on their elastic behavior, in particular, on the twist.

2. Mathematical stage

2.1. Geometry model

Since the emphasis in this paper is placed on the structure of the metamaterial, it is worth saying that the tetrachiral structure is a structure consisting of a ring and 4 ribs tangentially connected to the ring (figure 1a). The resulting structure is obtained in the plane, i.e. is a two-dimensional structure. To obtain the three-dimensional metamaterial it is necessary to build an elementary cell composed of the two-dimensional structures (figure 1b). Then a sample of metamaterial was made up of the elementary cells (figure 1c) by connecting the cells in three orthogonal axes according to the following numbers 3–3–9 in the corresponding axes X–Y–Z. The total number of cells in the sample equals $n = 91$.

![Figure 1](image1.png)

**Figure 1.** The structure of the mechanical tetrachiral metamaterial.

The parameters of the chiral structure were changed independently of each other. For example, when a specific parameter was changed, the others took the original values. The initial values of the parameters of the structure of mechanical tetrachiral metamaterial are presented in table 1, and their notions are shown in figure 1a.

| $l$ (mm) | $t$ (mm) | $h$ (mm) | $r_1$ (mm) | $r_2$ (mm) | $\theta$ (°) |
|---------|----------|----------|------------|------------|--------------|
| 50      | 5        | 5        | 12.5       | 17.5       | 25           |

**Table 1.** Initial parameters of the chiral metamaterial structure.
Here \( l/2 \) is the unit cell length, \( t \) is the rib width, \( h \) is the rib thickness, \( r_1 \) is the inner radius of the ring, \( r_2 \) is the outer radius of the ring, \( \theta \) is the angle of the inclination of the rib to the horizontal axis.

2.2. Mathematical model

The quasi-static problem of solid mechanics within the limits of the linear theory of elasticity was solved using the software package ANSYS. Hooke's law was chosen as the constitutive relation:

\[
\sigma_{ij} = \lambda \cdot \delta_{ij} \cdot \varepsilon_{kk} + 2 \cdot \mu \cdot \varepsilon_{ij},
\]

where \( \varepsilon_{kk} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \), \( \lambda = \frac{E \cdot \vartheta}{(1 + \vartheta)(1 - 2 \cdot \vartheta)} \), \( \mu = \frac{E}{2 \cdot (1 + \vartheta)} \) are the Lame coefficients, \( \delta_{ij} \) is the Kronecker symbol, \( \sigma_{ij} \) is the stress tensor, \( \varepsilon_{ij} \) is the strain tensor, \( E \) is the Young’s modulus, \( \vartheta \) is the Poisson's ratio. The following values of elastic moduli were used in this paper \( E = 200 \) GPa, \( \vartheta = 0.3 \).

2.3. Boundary conditions

To simulate the uniaxial loading of the sample, the corresponding boundary conditions were applied to the lower (rigid fixation) and upper (displacement constraint in the \( Y \)-axis and free motion in the \( X \) and \( Z \) axes) boundaries:

\[
U^\text{fix}_x = U^\text{fix}_y = U^\text{fix}_z = 0 \\
U^\text{dop}_x = U^\text{dop}_y = \text{free} \\
U^\text{dop}_y = -15 \approx -450 \cdot 3.33\%
\]

The negative displacement given at the top of the metamaterial sample corresponds to compression. The absence of the minus sign would indicate that the metamaterial is stretched. From the last record of the boundary conditions, it can be understood that the deformation of metamaterials does not exceed 4%.

3. Results and discussion

The results of the influence of variable parameters of the cell structure of mechanical tetrachiral metamaterials were described in detail in [6]. Now, we will focus on the invariant relationships of the geometric model to make the structure dimensionless. Thus, we solve the problem of scalability of the metamaterial sample.

The cell length is taken as the basic value of the unit cell. This parameter will be in the denominator and the relative values look like \( r_1/l, t/l, h/l, \theta/l \). The angle \( \theta \) characterizes the change of the outer ring radius \( r_2 \).

By the form of the dependencies and the range of the angle \( \alpha \) in figure 2a, it is clearly seen that the mechanical response we are interested in—a twist—is rather similar when we change the inner radius of the ring and the width of the ribs. The twist angle of the sample having the initial parameters of the structure is obviously the same. The corresponding results for the maximum change in each of the considered parameters result in almost identical twist of the sample.

If one compares the influence of the two parameters of the rib section, namely the width and the thickness, the great difference will be found in the plot in figure 2a. Changing in the rib width has almost no effect on the sample twist. It comes as a surprise because the corresponding changes in the structural parameters of the metamaterial have approximately identical impact on the volume of the base material. Probably, this difference is due to the effect of bending forces on the rib structure of the metamaterial. Rib width is an external parameter for the in-plane chiral structure and has much less effect on the bending forces than the rib thickness. As a consequence, the twisting of the metamaterial is scarcely affected by the changing in rib width. This effect can be attributed also to the fact that
uniaxial loading of the sample affects the bending in that plane in which the change in thickness is significant but not in the width.

The angle of inclination of the rib to the X-axis is the parameter that most significantly affects the twist of the metamaterial sample, as it is responsible for the twisting of the ring (figure 2b). Decrease of the twist angle with increasing the relative parameter of the rib angle passes on the parabolic law, to the absolute zero value of the twist angle which corresponds to the value of $\theta/l = 0.9^\circ/mm$.

The further increase in the described parameter will lead to a further parabolic decrease of the twist angle into the region of negative values. That means that the mechanical tetrachiral metamaterial sample will twist in the opposite direction.

![Figure 2](image.png)

**Figure 2.** Twist angle of the metamaterial as a function of the variable relative parameters.

4. Conclusion

In this work, the mechanical behavior of the tetrachiral mechanical metamaterial sample was studied under uniaxial loading. The problem was treated as a quasi-static problem in the framework of solid mechanics. Numerical simulation of the metamaterial mechanical behavior was carried out within the framework of the linear theory of elasticity. The deformation of the metamaterial did not exceed 4%.

The parameters characterizing the structure of the metamaterial were presented in a relative form and varied independently. The dependencies of the twist angle on the changing of the relative parameters were obtained. The parameter of the chiral structure most affecting the unusual behavior of the mechanical metamaterial—a twist under uniaxial loading, is identified.

When increasing some parameters, the twist angle decreases. This can be associated with an increase in relative volume by a trend: “the larger the relative volume of metamaterial, the smaller the twist angle”. This trend is not valid for all parameters.

Acknowledgments

The work was performed according to the Government research assignment for ISPMS SB RAS, project No. III.23.2.12.

References

[1] Zadpoor A A 2016 Mater Horizons 3(5) 371-381
[2] Evans K E, Nkansah M A, Hutchinson I J and Rogers S C 1991 Nature 353(6340) 124-125
[3] Goldstein R V, Gorodtsov V A, Lisovenko D S and Volkov M A 2014 Phys. Mesomech. 17(2) 97-115
[4] Goldstein R V, Gorodtsov V A, Lisovenko D S and Volkov M A 2019 Phys. Mesomech. 22(4) 261–268
[5] Frenzel T, Kadic M and Wegener M 2017 Science 358(6366) 1072-1074
[6] Akhmetshin L R and Smolin I Yu 2020 Nanosci. Technol. Int. J. 11(3) 265-273