The rare decays $B^+ \to D_s^{(*)} \phi$, $B^0 \to D_s^{(*)} K^+$ and $B^+ \to D_s^{(*)} \bar{K}^0$ can occur only via annihilation type diagrams in the standard model. We calculate these decays in perturbative QCD approach. We found that the calculated branching ratio of $B^0 \to D_s^- K^+$ agree with the data which had been observed in the KEK and SLAC B factories. The decay $B^+ \to D_s^{(*)} \bar{K}^0$ has a very small branching ratio at $O(10^{-8})$, due to the suppression from CKM matrix elements. The branching ratio of $B^+ \to D_s^{(*)+} \phi$ is of order $10^{-7}$ which may be measured in the near future by KEK and SLAC B factories. The small branching ratios predicted in the standard model make these channel sensitive to new physics contributions.

1 Introduction

The generalized factorization approach has been applied to the theoretical treatment of non-leptonic $B$ decays for years. It is a great success in explaining many decay branching ratios. The factorization approach (FA) is a rather simple method. Some efforts have been made to improve their theoretical application and to understand the reason why the FA has gone well. One of these methods is the perturbative QCD approach (PQCD), where we can calculate the annihilation diagrams as well as the factorizable and nonfactorizable diagrams.

The rare decays $B \to D_s^{(*)} K(\phi)$ are pure annihilation type decays. In the usual FA, this decay picture is described as $B$ meson annihilating into vacuum and the $D_s^{(*)}$ and $K(\phi)$ mesons produced from vacuum then afterwards. To calculate these decays in the FA, one needs the $D_s^{(*)} \to K(\phi)$ form factor at very large time like momentum transfer $O(M_B)$. However the form factor at such a large momentum transfer is not known in FA. The annihilation amplitude is a phenomenological parameter in QCD factorization approach (QCDF), and the QCDF calculation of these decays is also unreliable. Here, we will try to use the PQCD approach, to evaluate the $B \to D_s K(\phi)$ decays. By comparing the predictions with the experimental data, we can test the PQCD evaluation of the annihilation amplitude.

A $W$ boson exchange causes $\bar{b}d \to \bar{c}u$ or $\bar{b}u \to \bar{d}(\bar{s})c$, which is usually described by the effective four quark operators, and the additional $\bar{s}s$ quarks included in $D_s K(\phi)$ are produced from a gluon. This gluon attaches to any one of the quarks participating in the four quark operator. In the rest frame of $B$ meson, $s$ and $\bar{s}$ quarks included in $D_s K(\phi)$ each has $O(M_B^2/4)$ momenta, and the gluon producing them has \( q^2 = O(M_B^2/4) \). This is a hard gluon. One can perturbatively treat the process where the four quark operator exchanges a hard gluon with $\bar{s}s$ quark pair. Therefore the quark picture becomes six-quark interactions. The decay amplitude is then expressed as product of the hard six quark operators and the non-perturbative meson wave functions.
2 Framework

PQCD approach has been developed and applied in the non-leptonic $B$ meson decay for some time. In this approach, the decay amplitude is separated into soft($\Phi$), hard($H$), and harder($C$) dynamics characterized by different scales. It is conceptually written as the convolution,

$$A \sim \int d^4k_1d^4k_2d^4k_3 \, \text{Tr}[C(t)\Phi_B(k_1)\Phi_{D_s}(k_2)\Phi_K(k_3)H(k_1,k_2,k_3,t)e^{-S(t)}],$$

where $k_i$'s are momenta of light quarks included in each mesons, and $\text{Tr}$ denotes the trace over Dirac and color indices. $C(t)$ is Wilson coefficient of the four quark operator. In the above convolution, $C(t)$ includes the hard dynamics at larger scale than $M_B$ scale and describes the evolution of local 4-Fermi operators from $m_W$, down to the scale $t$, where $t = \mathcal{O}(\sqrt{\Lambda M_B})$. $H$ describes the four quark operator and the spectator quark connected by a hard gluon whose $q^2$ is at the order of $t$, and includes the hard dynamics characterized by the scale $t$. Therefore, this hard part $H$ can be perturbatively calculated, which is process dependent. $\Phi_M$ is the wave function which describes hadronization of the quark and anti-quark to the meson $M$. $\Phi_M$ is independent of the specific processes. Determining $\Phi_M$ in some other decays, we can make quantitative predictions here.

The large double logarithms ($\ln^2 x_i$) on the longitudinal direction are summed by the threshold resummation, and they lead to $S_t(x_i)$ which smears the end-point singularities on $x_i$. The last term, $e^{-S(t)}$, contains two kinds of logarithms. One of the large logarithms is due to the renormalization of ultra-violet divergence $\ln \tau b$, the other is double logarithm $\ln^2 b$ from the overlap of collinear and soft gluon corrections. This Sudakov form factor suppresses the soft dynamics effectively. Thus it makes perturbative calculation of the hard part $H$ applicable at intermediate scale, i.e., $M_B$ scale.

In general, $\Phi_{M,\alpha\beta}$ having Dirac indices $\alpha, \beta$ are decomposed into 16 independent components, $1_{\alpha\beta}$, $\gamma_\alpha^\mu$, $\gamma_\beta^\mu$, $(\gamma^\mu\sigma^{\mu\nu})_{\alpha\beta}$, $(\gamma^\mu\gamma_5)_{\alpha\beta}$, $\gamma_5\alpha\beta$. If the considered meson $M$ is $B$ or $D_s^*(s)$ meson, to be pseudo-scalar and heavy meson, the structure $(\gamma^\mu\gamma_5)_{\alpha\beta}$ and $\gamma_5\alpha\beta$ components remain as leading contributions. Then, $\Phi_{M,\alpha\beta}$ is written by

$$\Phi_{M,\alpha\beta} = \frac{i}{\sqrt{6}} \left\{ (\gamma_5^5)^{\alpha\beta} \phi^A_M + 5\gamma_5\alpha\beta \phi^P_M \right\}.$$  

(2)

As heavy quark effective theory leads to $\phi^P_M \simeq M_B^2 \phi^A_B$, we have only one independent distribution amplitude for B meson. The heavy $D_s$ meson’s wave function can also be derived similarly.

Figure 1: Diagrams for $B^+ \to D_s^+ \phi$ decay. The factorizable diagrams (a),(b), and non-factorizable (c), (d).

In contrast to the $B$ and $D_s$ mesons, for the $K$ meson, being light meson, the $\gamma_5\sigma^{\mu\nu}$ component remains. Then, $K$ meson’s wave function is parameterized as

$$\Phi_{K,ij}(x_3,b_3) = \frac{i\delta_{ij}}{\sqrt{2N_c}} \left[ 5 \gamma_5^3 P_3^A_K(x_3,b_3) + m_0 K^5 \phi^P_K(x_3,b_3) + m_0 K^5 (\phi^L - 1) \phi^T_K(x_3,b_3) \right],$$

(3)
where $m_{0K} = M_K^2/(m_u + m_s)$, $\nu = (0,1,0_T)$, $n = (1,0,0_T)$. In $B \to D_s\phi$ decay, only longitudinal polarization of the $\phi$ meson wave function is relevant, which is similar to $K$ meson.

There are four kinds of Feynman diagrams contributing to the six quark hard dynamics, which is shown in Fig.1. The calculation of the hard parts are tedious and channel dependent. The results are shown in Ref. 11, 12.

3 Numerical evaluation

In this section we show numerical results. First for the $B$ meson’s wave function, we use the same distribution amplitude as adopted in Ref. 11. This choice of $B$ meson’s wave function is almost a best fit from the $B \to K\pi\pi$ decays. For the $D_s^{(*)}$ meson’s wave function, we assume the same form as $D^{(*)}$ meson’s one. The wave functions $\phi_K^{A,P,T}$ of the $K$ meson are expanded by Gegenbauer polynomials, which are given in Ref. 12.

For the neutral decay $B^0 \to D_s^{(*)}K^+$, the dominant contribution is the nonfactorizable annihilation diagrams, which is proportional to the Wilson coefficient $C_2(t) \sim 1$. The factorizable annihilation diagram contribution is proportional to $a_2 = C_1 + C_2/3$, which is one order magnitude smaller. For the charged decay $B^+ \to D_s^{(*)+}K^{0}(\phi)$, it is the inverse situation.

The propagators of inner quark and gluon in FIG. 1 are usually proportional to $1/x_i$. One may suspect that these amplitudes are enhanced by the endpoint singularity around $x_i \sim 0$. However this is not the case in our calculation. First we introduce the transverse momentum of quark, such that the propagators become $1/(x_i x_j + k_T^2)$. Secondly, the Sudakov form factor $\exp[-S]$ suppresses the region of small $k_T^2$. Therefore there is no singularity in our calculation. The dominant contribution is not from the endpoint of the wave function. As a proof, in our numerical calculations, for example, an expectation value of $\alpha_s$ in the integration results in $\langle \alpha_s/\pi \rangle = 0.10$. Therefore, the perturbative calculations are self-consistent.

The predicted branching ratios are

\[
\begin{align*}
\text{Br}(B^0 \to D_s^- K^+) &= (3.1 \pm 1.0) \times 10^{-5}, \\
\text{Br}(B^+ \to D_s^{+} K^0) &= (1.7 \pm 0.4) \times 10^{-8}, \\
\text{Br}(B^0 \to D_s^{(*)-} K^+) &= (2.7 \pm 0.6) \times 10^{-5}, \\
\text{Br}(B^+ \to D_s^{(*)+} K^0) &= (4.0 \pm 0.8) \times 10^{-8},
\end{align*}
\]

for variation of the input parameters of wave functions. They agree with the experimental observation by Belle and Babar.

\[
\begin{align*}
\text{Br}(B^0 \to D_s^- K^+) &= (4.6^{+1.2}_{-1.1} \pm 1.3) \times 10^{-5}, \\
\text{Br}(B^0 \to D_s^- K^+) &= (3.2 \pm 1.0 \pm 1.0) \times 10^{-5},
\end{align*}
\]

and the experimental upper limit given at 90\% confidence level, $\text{Br}(B^+ \to D_s^{(*)+} K^0) < 1.1 \times 10^{-3}$. For $B^+ \to D_s^{(*)+}\phi$, the predicted branching ratio is $\text{Br}(B^+ \to D_s^{(*)+}\phi) = 3.0 \times 10^{-7}$, which is still far from the current experimental upper limit, $\text{Br}(B^+ \to D_s^{(*)+}\phi) < 3.2 \times 10^{-4}$.

Despite the calculated perturbative annihilation contributions, there is also hadronic picture for the $B^0 \to D_s^- K^+$ decay: $B^0 \to D^{-\pi^+}(\rho^+) \to D_s^- K^+$ through final state interaction. Our numerical results show that the PQCD contribution to this decay is already enough to account for the experimental measurement. It implies that the soft final state interaction is not important in the $B^0 \to D_s^- K^+$ decay. This is consistent with the argument in Ref. 16. We expect the same situation happens in other decay channels.

4 Conclusion

In two-body $B$ decays, the final state mesons are moving very fast, since each of them carry more than 2 GeV energy. There is not enough time for them to exchange soft gluons. The soft final
state interaction may not be important. This is consistent with the argument based on color-transparency. The PQCD with Sudakov form factor is a self-consistent approach to describe the two-body $B$ meson decays. Although the annihilation diagrams are suppressed comparing to other spectator diagrams, but their contributions are not negligible in PQCD approach.

We calculate the $B^0 \to D_s^{(*)} K^+$ and $B^+ \to D^{(*)+} \bar{K}^0(\phi)$ decays, which occur purely via annihilation type diagrams. The branching ratio of $B^0 \to D_s^- K^+$ decay is sizable, which has been observed in the $B$ factories. The predicted branching ratio is in good agreement with the data.

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