CHERN-SIMONS GRAVITY FROM 3+1-DIMENSIONAL GRAVITY

G. Grignani

Dipartimento di Fisica
Università degli Studi di Perugia
I-06100 Perugia – ITALY

and

G. Nardelli

Dipartimento di Fisica
Università degli Studi di Trento
I-38050 Povo (TN) – ITALY

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ABSTRACT

In the context of a Poincaré gauge theoretical formulation, pure gravity in 3 + 1-dimensions is dimensionally reduced to gravity in 2 + 1-dimensions with or without cosmological constant $\Lambda$. The dimensional reductions are consistent with the gauge symmetries, mapping $ISO(3,1)$ gauge transformations into $ISO(2,1)$ ones. One of the reductions leads to Chern-Simons-Witten gravity.

The solutions of 2 + 1-gravity with $\Lambda \leq 0$ (in particular the black-hole solution recently found by Banados, Teitelboim and Zanelli) and those of 1 + 1-dimensional Liouville gravity, are thus mapped into 3 + 1-dimensional vacuum solutions.
2 + 1-dimensional gravity\(^1\) has attracted a growing attention in the last decade, both as a good theoretical laboratory for the construction of a quantum theory of gravity\(^2,3\) and as a simplified model to study “physical” gravitational systems\(^4\). Such roles played by 2 + 1-dimensional gravity are well exemplified by the black-hole solution of Einstein equations with negative cosmological constant recently found by Banados et al.\(^5,6\). The key properties of this solution are similar to those of its 3 + 1-dimensional counterpart (apart from not being asymptotically flat), so that one can study black-hole physics in a much simpler setting.

In this context it is then interesting to establish a clear link between \(D = 4\) and \(D = 3\) gravities by a dimensional reduction. This is the aim of the present letter and it is achieved in the framework of a gauge theoretical formulation of both theories. In fact as we showed in Ref.[7] (see also [8]) gravity in three and four dimensions with all possible couplings to matter fields and particles can be formulated as a gauge theory of the Poincaré group. In three dimensions this formulation is especially attractive as the Einstein-Hilbert action becomes the Chern-Simons term of the \(ISO(2, 1)\) gauge group\(^2,9\). Such a Chern-Simons action with the correct \(ISO(2, 1)\) gauge transformations can then be derived by dimensionally reducing the 3 + 1-dimensional Einstein-Hilbert action in its gauge theoretical formulation.

The same dimensional reduction process was used in Refs.[10] (see also [11]) to obtain, starting from an \(ISO(2, 1)\) gauge invariant theory in 2 + 1-dimensions, the gauge theoretical formulations\(^12\) of the 1 + 1-dimensional Liouville\(^13\) and black-hole gravity\(^14\).

As we provide the link between 4 and 3 dimensions, 1 + 1-dimensional solutions of Liouville and black-hole gravity can then be directly connected to four dimensional solutions of Einstein’s equations.

We shall perform two different dimensional reductions corresponding to two different identifications of the \(ISO(3, 1)\) generators of the four dimensional theory, to the \(ISO(2, 1)\) generators of the 2 + 1-dimensional one. Whereas the first reduction leads to a theory with vanishing cosmological constant, the second will produce a theory in which a (negative) 3D-cosmological constant is related to the 4D-Newton constant. Therefore the 3D-black-hole solution of Ref.[5] will be easily read as the cross section of a 4D “black-string” solution of the Einstein equations in vacuo.

We shall not report here on the dimensional reduction from 4D to 3D gravity in the presence of matter fields and point-particles, because such a reduction can be easily achieved following the lines of Ref.[10] for the 3D \(\rightarrow\) 2D reduction and starting from the matter field actions provided in Ref.[7].

We shall begin by briefly reviewing our approach to gravity as a gauge theory of the Poincaré group and then we shall present the dimensional reductions.

Key ingredients of the formulation of gravity as a Poincaré gauge theory are the so called Poincaré coordinates \(q^a(x)\), Higgs type fields that behave as vectors under Poincaré gauge transformations,\(^7,8\) and that are involved in the definition of the vielbein \(V^a_\mu\). The \(q^a\) can be interpreted as coordinates of an internal Minkowskian space \(\mathcal{M}_q\) that can be locally made to coincide with the tangent space. Any choice of Poincaré coordinates is equivalent to a gauge choice leaving the theory invariant under residual (local) Lorentz transformations. In our formalism the vielbein \(V^a_\mu\) is not identified with the component \(e^a_\mu\) of the gauge potential \(A_\mu\) along the translation generators \(P^a\), but it will be given by the Poincaré covariant derivative of the coordinates \(q^a\), namely \(V^a_\mu = \mathcal{D}_\mu q^a = \partial_\mu q^a + \omega^{ab}_\mu q^b + e^a_\mu\), where \(\omega^{ab}_\mu\) is the spin connection. Only in the so called “physical” gauge, where \(q^a = 0\), \(V^a_\mu = e^a_\mu\).
But this interpretation for $e^a_\mu$ only holds in a particular gauge choice of the translations and, consequently, in the framework of a Lorentz gauge theory.

For a more detailed discussion on the Poincaré coordinates and on the necessity of their introduction in the context of a Poincaré gauge theory of gravity see Ref.[7,8]. It is important to know, however, that this approach to gravity as a Poincaré gauge theory allows to couple in a gauge invariant fashion particle and matter fields to gravity in any dimensions, obtaining equations that are equivalent to Einstein’s equation with matter sources, to field theory in curved space or to the geodesic equations in the case of particles. Gravity becomes in this way as close as possible to any ordinary non-Abelian gauge theory and, even if in 4D the Einstein-Hilbert action does not assume the Yang-Mills form, the gauge fields transform in the usual way.

The Lorentz and momentum generators† $J_{AB}$ and $P_A$ satisfy the Poincaré algebra

$$[P_A, P_B] = 0,$$
$$[P_A, J_{BC}] = \eta_{AC}P_B - \eta_{AB}P_C ,$$
$$[J_{AB}, J_{CD}] = \eta_{AC}J_{BD} - \eta_{BC}J_{AD} + \eta_{BD}J_{AC} - \eta_{AD}J_{BC} .$$

As we mentioned, in order to have gauge fields transforming in the usual way under non-Abelian gauge transformations, the vierbein is defined through the covariant derivative of a Poincaré vector $q^A$, namely of a quantity that under gauge transformations behaves as

$$\delta q^A(x) = \kappa^A_B(x)q^B(x) + \rho^A(x) ,$$

where $\kappa^{AB} = -\kappa^{BA}$ and $\rho^A$ are the infinitesimal parameters corresponding to Lorentz transformations and translations, respectively. The ISO(3,1) covariant derivative of the Poincaré coordinates $q^A$ will contain an homogeneous part, with ISO(3,1) gauge potentials $\omega^{AB}_\alpha$, and an inhomogeneous part with gauge potentials $e^A_\alpha$ associated to the translation generators $P_A$, namely

$$D_\alpha q^A = \partial_\alpha q^A + \omega^{AB}_\alpha q^B + e^A_\alpha .$$

We need to construct, in terms of $D_\alpha q^A = V^A_\alpha$, an ISO(3,1) gauge scalar that can serve as a metric on the space-time, namely a quantity of the type $g_{\alpha\beta} = \eta_{AB}V^A_\alpha V^B_\beta = \eta_{AB}D_\alpha q^A D_\beta q^B$. For this purpose $D_\alpha q^A$ has to transform as a Lorentz vector under the gauge transformations, i.e. as $\delta D_\alpha q^A = \kappa^A_B D_\alpha q^B$. As a consequence one has to impose that $e^A_\alpha$ and $\omega^{AB}_\alpha$ under gauge transformations change as

$$\delta \omega^{AB}_\alpha = -\partial_\alpha \kappa^{AB} - \omega^{AC}_\alpha \kappa^C_B + \omega^{BC}_\alpha \kappa^C_A ,$$
$$\delta e^A_\alpha = -\partial_\alpha \rho^A - \kappa^A_B e^B_\alpha + \omega^{AB}_\alpha \rho_B .$$

† In our notations, latin indices $a, b, c, ... = 0, 1, 2, 3$ and capital latin indices $A, B, C, ... = 0, 1, 2, 3$ denote ISO(2,1) and ISO(3,1) internal (gauge) indices, respectively. They are raised and lowered by the Minkowski metrics $\eta_{ab} = (1, -1, -1)$ and $\eta_{AB} = (1, -1, -1, -1)$. In the dimensional reduction the first 3 values of $A, B, C, ...$ will denote the corresponding ISO(2,1) internal indices $a, b, c, ..., i.e. A = (a, 3), B = (b, 3), C = (c, 3), ....$ The 3 + 1 dimensional space-time indices are denoted by the first greek letters $\alpha, \beta, \gamma, ... = 0, 1, 2, 3$ whose first three components denote the corresponding 2 + 1-dimensional space-time indices $\mu, \nu, \rho, ... = 0, 1, 2$. We shall use the antisymmetric symbol $\varepsilon^{ABCD}$ with $\varepsilon^{0123} = 1$ and in 2 + 1-dimensions $\varepsilon^{abc} = \varepsilon^{abc3}$, so that $\varepsilon^{012} = 1$. 
The covariant derivative (3) (as we showed in Ref. [7] by gauging the action of a free relativistic particle) can then be interpreted as the space-time vierbein $V^A_\alpha$.

The transformation laws (4), are those introduced by Witten$^2$ for the $ISO(2,1)$ Chern-Simons formulation of gravity in 3D. Consequently, in our framework, the dimensional reductions that will be consistent with the $ISO(2,1)$ gauge invariance in 3D, will naturally arise.

Introducing the Lie algebra valued gauge potential $A_\alpha = e^A_\alpha P_A - (1/2)\omega^{AB}_\alpha J_{AB}$ and gauge parameter $u = \rho^A P_A - (1/2)\kappa^{AB} J_{AB}$, the transformation laws (4) become those of any ordinary non Abelian gauge theory, i.e. $\delta A_\alpha = -\partial_\alpha u - [A_\alpha, u] \equiv -\Delta_\alpha u$.

The Lie-algebra valued field strength is

$$F_{\alpha\beta} = [\Delta_\alpha, \Delta_\beta] = P_A T^A_{\alpha\beta} - \frac{1}{2} J_{AB} R^{AB}_{\alpha\beta},$$

where

$$T^{\alpha\beta}_{\alpha\beta} = \partial_\alpha e^{A}_\beta - \partial_\beta e^{A}_\alpha + \omega^{AB}_{\alpha} e_{B\beta} - \omega^{AB}_{\beta} e_{B\alpha}$$
$$R^{AB}_{\alpha\beta} = \partial_\alpha \omega^{AB}_{\beta} - \partial_\beta \omega^{AB}_{\alpha} + \omega^{AC}_{\alpha} \omega^{B}_{C\beta} - \omega^{AC}_{\beta} \omega^{B}_{C\alpha}.$$

$F_{\alpha\beta}$ transforms covariantly under gauge transformations and whereas $R^{AB}_{\alpha\beta}$ can be interpreted as the Riemann curvature tensor, $T^{\alpha\beta}_{\alpha\beta}$ does not correspond to the space-time torsion, as the vierbein is not given by $e^A_\alpha$. The space-time torsion $T^{\alpha\beta}_{\alpha\beta}$ can be easily evaluated in term of the $ISO(3,1)$ field strength and Poincaré coordinates as

$$T^{\alpha\beta}_{\alpha\beta} = \partial_\alpha V^A_\beta - \partial_\beta V^A_\alpha + \omega^{AB}_{\alpha} V^B_\beta - \omega^{AB}_{\beta} V^B_\alpha$$
$$= T^{\alpha\beta}_{\alpha\beta} + R^{AB}_{\alpha\beta} q_B,$$

so that only in the physical gauge $T^{\alpha\beta}_{\alpha\beta} = T^{\alpha\beta}_{\alpha\beta}$. Within this formalism the Einstein-Hilbert action $S^{4D}_{EH} = (4\pi G)^{-1} \int d^4x \sqrt{-g} R$, where $R$ is the scalar curvature and $G$ the Newton constant, can be rewritten in the form of a $ISO(3,1)$ gauge invariant action according to

$$S^{4D}_{EH} = -\frac{1}{16\pi G} \int d^4x \varepsilon^{\alpha\beta\gamma\delta} \varepsilon_{ABCD} D_\alpha q^A D_\beta q^B R^{CD}_{\gamma\delta}.$$

Had we used $e^A_\alpha$ instead of $D_\alpha q^A$ the action would not be Poincaré gauge invariant.

The equations of motion obtained by varying (8) with respect to $e^A_\alpha$, $\omega^{AB}_\beta$ and $q^A$ give, respectively, (provided that the vierbein is invertible) the vanishing of the space-time torsion $T^{\alpha\beta}_{\alpha\beta}$, the Einstein equations in vacuo, and an equation that is automatically satisfied if the other two are used. In 2 + 1 dimensions something peculiar happens. Here in fact the Einstein-Hilbert action $S^{3D}_{EH} = (4\pi G_{3D})^{-1} \int d^3x \sqrt{g} R$, in its $ISO(2,1)$ gauge invariant form, reads

$$S^{3D}_{EH} = -\frac{1}{8\pi G_{3D}} \int d^3x \varepsilon^{\mu\nu\rho} \varepsilon_{abc} D_\mu q^a R^{bc}_{\nu\rho},$$

and by means of the Bianchi identity $\varepsilon^{\mu\nu\rho} D_\mu R^{bc}_{\nu\rho} = 0$, $S^{3D}_{EH}$ becomes

$$S^{3D}_{EH} = -\frac{1}{8\pi G_{3D}} \int d^3x \varepsilon^{\mu\nu\rho} \varepsilon_{abc} e^a_{\mu} R^{cd}_{\nu\rho},$$

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up to a surface term that can always be chosen to vanish. \((G_{3D} \text{ in three dimensions has the mass dimension } [M]^{-1})\). Therefore all the terms containing \(q^a\) disappear from the action \(S^{3D}_{\text{EH}}\) and as long as pure gravity is concerned, \(e^a_{\mu}\) can indeed be interpreted as the space-time \textit{dreibein} and yet the theory is Poincaré gauge invariant, contrary to what happens in 3 + 1-dimensions. The absence of the \(q\) variables in (10) and the interpretation of \(e\) and \(\omega\) as gauge fields makes of (10) an action of the form \(AdA + A^3\) that can be conceived as a Chern-Simons three form. Introducing the dual of the connection \(\omega\) and of the Lorentz generators, with a suitable non-degenerate and invariant inner product among the ISO(2,1) generators, \(S^{3D}_{\text{EH}}\) actually becomes the ISO(2,1) Chern-Simons three-form\(^2\).

If matter or a gravitational constant is included and one still looks for a Poincaré gauge theory, the \(q^a\) variables have to be reintroduced. So that for example in the presence of a cosmological constant \(\Lambda_{3D}\), the ISO(2,1) gauge invariant action reads

\[
S^{3D}_\Lambda = \int d^3x \varepsilon^{\mu \nu \rho} \varepsilon_{abc} \left( \frac{1}{8 \pi G_{3D}} e^a_{\mu} R^{bc}_{\nu \rho} - \frac{\Lambda_{3D}}{3!} \mathcal{D}_\mu q^a \mathcal{D}_\nu q^b \mathcal{D}_\rho q^c \right) .
\]

\((11)\)

We want to show that there are two suitable dimensional reductions of the Poincaré generators of the ISO(3,1) theory and correspondingly of the space-time dimensions that from the action (8) and the algebra (1) lead to the Poincaré gauge theories (10) and (11). With such reductions from the ISO(3,1) gauge transformations (2) and (4), we shall obtain the corresponding gauge transformations in 2 + 1-dimensions.

We shall eliminate in both cases the third spatial dimension (of coordinate \(x^3\)) that will be compactified to a unit length. Furthermore, we shall set \(\partial_3(\text{anything}) = 0\) so that the integral in \(x^3\) in (8) will only give an overall unit factor. The dimensional reduction leading from the 4-dimensional ISO(3,1) theory of Eq.(8) to ISO(2,1) Chern-Simons gravity is given in Table A.

\[
\text{TABLE A}
\]

\[
5
\]
One can check that the ISO(3, 1) gauge transformations (2), (4), with the identifications of Table A are mapped respectively into

\[
\begin{align*}
\delta q^a &= \kappa^a b q^b + \rho^a \\
\delta e^a_{\mu} &= -\partial_{\mu} \rho^a - \kappa^a b e^b_{\mu} - \omega^{ab}_{\mu} \rho_b \\
\delta \omega^{ab}_{\mu} &= -\partial_{\mu} \kappa^a + \kappa^a c \omega^{cb}_{\mu} - \kappa^b c \omega^{ca}_a \mu ,
\end{align*}
\]

\(i.e.\) into the correct ISO(2, 1) gauge transformations. In particular the quantities that are set to a constant in Table A consistently have vanishing gauge transformations. By substituting the content of Table A into the action (8) one gets

\[
S_{4D}^{EH} \rightarrow S_{3D}^{EH} = \frac{1}{8\pi G_{3D}} \int d^3x \varepsilon^{\mu\nu\rho} \varepsilon_{abc} e^a_{\mu} \left( \partial_{\nu} \omega^{bc}_{\rho} - \partial_{\rho} \omega^{bc}_{\nu} + \omega^b_{d\nu} \omega^{dc}_\rho - \omega^c_{d\nu} \omega^{db}_\rho \right) ,
\]

where the constant \((G_{3D})^{-1} = (G)^{-1} \int dx^3\) is positive and has the correct mass dimensions \([M]^{-1}\). The right hand side of (13) is precisely \(S_{3D}^{EH}\) given in Eq. (10).

Table A provides the most natural reduction induced by the compactification of the third spatial dimension. In fact in the internal space we retain only the generators of the

| Dimensional Reduction A | 
|-------------------------|
| 3+1 Dimensions          |
| 2+1 Dimensions          |
| \(e^3_3\)               | 1                        |
| \(e^a_\mu\)             | \(e^a_\mu\)              |
| \(e^a_3\)               | 0                        |
| \(e^3_\mu\)             | 0                        |
| \(\omega^{ab}_{\mu}\)   | \(\omega^{ab}_{\mu}\)   |
| \(\omega^{a3}_{\mu}\)   | 0                        |
| \(\omega^{AB}_3\)       | 0                        |
| \(q^a\)                 | \(q^a\)                  |
| \(q^3\)                 | 0                        |
| \(\rho^a\)              | \(\rho^a\)               |
| \(\rho^3\)              | 0                        |
| \(\kappa^{ab}\)         | \(\kappa^{ab}\)         |
| \(\kappa^{a3}\)         | 0                        |
translations in time and in the direction 1 and 2 \((\rho^a \neq 0 \text{ and } \rho^3 = 0)\) and the only possible Lorentz transformations: boost and rotations on the plane 1,2 whose generators are \(J_{ab}\) \((\kappa^{ab} \neq 0 \text{ and } \kappa^{a3} = 0)\).

Denoting by \(\hat{P}_A\) and \(\hat{J}_{AB}\) the \(ISO(3,1)\) generators (we introduce the hat in order to avoid confusion with the corresponding \(ISO(2,1)\) generators), the \(ISO(2,1)\) generators that we get from the dimensional reduction in Table A are \(P_a = \hat{P}_a\) and \(J_{ab} = \hat{J}_{ab}\).

Also \(S^{3D}_A\) can be obtained with the dimensional reduction of Table A from the 4-dimensional \(ISO(3,1)\) gauge invariant action with non-vanishing cosmological constant

\[
S^{4D}_A = -\int d^4x \varepsilon^{\alpha\beta\gamma\delta} \varepsilon_{ABCD} \left( \frac{1}{16\pi G} D_\alpha q^A D_\beta q^B R^{CD}_{\gamma\delta} - \frac{\Lambda}{4!} D_\alpha q^A D_\beta q^B D_\gamma q^C D_\delta q^D \right). \tag{14}
\]

An analogous dimensional reduction has been used in Ref.[10] to obtain, from \(S^{3D}_A\), the Poincaré gauge theoretical formulation of Liouville gravity in 2-dimensions. Thus the connection from the 4-dimensional Poincaré gauge theory (14) to the 2-dimensional one is established.

The second dimensional reduction we shall be concerned with, allows to connect pure gravity in 4D, Eq.(8), to 2 + 1 gravity with a negative cosmological constant, Eq.(11). The cosmological constant, will be related to the 4D Newton constant through \((\Lambda^{3D}/3!) = -(4\pi G^2)^{-1} \int dx^3 = -(4\pi G G^{3D})^{-1}\). The dimensional reduction is shown in Table B.

### TABLE B

| Dimensional | Reduction B |
|-------------|-------------|
| 3+1 Dimensions | 2+1 Dimensions |
| \(e^3_3\) | 1 |
| \(e^A_\mu\) | 0 |
| \(e^a_3\) | 0 |
| \(\omega_{ab}_\mu\) | \(\omega_{ab}_\mu\) |
| \(\omega^{a3}_\mu\) | \((G)^{-1/2} V^a_\mu\) |
| \(\omega^{AB}_\mu\) | 0 |
| \(q^a\) | 0 |
| \(q^3\) | \(\sqrt{G}\) |
| \(\rho^A\) | 0 |
| \(\kappa^{ab}\) | \(\kappa^{ab}\) |
| \(\kappa^{a3}\) | 0 |
The action $S^{4D}_{EH}$ becomes

$$S^{4D}_{EH} \rightarrow S^{3D}_\Lambda = \int d^3 x \varepsilon^{\mu\nu\rho} \varepsilon_{abc} \left( \frac{1}{8\pi G_{3D}} V^a_\mu R^{bc}_\nu - \frac{\Lambda_{3D}}{3!} V^a_\mu V^b_\nu V^c_\rho \right), \quad (15)$$

that is equivalent to Eq.(11) once the identification for the dreibein $V^a_\mu = D_\mu q^a$ and the Bianchi identity are taken into account.

The dimensional reduction $B$ leads from the ISO$(3,1)$ theory with generators $(\hat{P}^A, \hat{J}^{AB})$ to a Lorentz, SO$(2,1)$ theory with generators $J_{ab}$ in $2 + 1$ dimensions by setting to zero the generators $\hat{P}^A$ and $\hat{J}^{a3}$ ($\rho^A = 0$ and $\kappa^{a3} = 0$) and by identifying $\hat{J}_{ab} = J_{ab}$. The Lorentz theory is then transformed into a Poincaré one by the definition of the dreibein $V^a_\mu$ as $V^a_\mu = D_\mu q^a$.

It is remarkable that only the theory with a negative cosmological constant can be obtained, this is in fact the theory where the $2 + 1$-dimensional black hole solution was found. Such metric can be directly translated into a $3 + 1$-dimensional solution with cylindrical symmetry of Einstein’s equations in vacuo.

The $4$-dimensional vierbein given by $V^a_\mu = D_\mu q^a$ can be easily obtained from the $3$-dimensional one using the identification of Table B. Consequently the $4D$ vacuum solution corresponding to the black-hole metric in $3D$ is given by a line element that is the same of the $3$-dimensional one with the addition of a $dz^2$ term with coefficient $g_{33} = -1$ and the cosmological constant substituted by the Newton constant (with the suitable power to maintain mass dimensions). In a similar way, using the results of Ref.[10], one can connect the Liouville gravity solutions in $2D$ to vacuum solutions in $4D$.

The dimensional reduction we have illustrated can be performed also in the presence of matter fields and point-particles that can be coupled to ISO$(3,1)$ gravity in a gauge invariant fashion as in Ref.[7].

Such descent does not entail any special difficulty and can be realized along the line of Ref.[10]. In particular the solutions with fields and particles presented in Ref.[10] for Liouville gravity can again be read as solutions with fields and particles of gravity in $4D$ with $\Lambda = 0$.

The Poincaré gauge theoretical formulation of gravitational theories is a natural context to connect, preserving gauge invariance, such theories in different dimensions. The dimensional descent provides a confirm that the program of realizing gravity as a gauge theory of the Poincaré group [7,15] can give interesting insights on the structure of the Einstein’s equations and on their solutions. At the same time it shows that what we proposed in Ref.[7,10] is the natural generalization, to any dimensions and with any matter couplings, of Witten’s approach to pure gravity in $2 + 1$-dimensions as a ISO$(2,1)$ gauge theory.

The solutions of $2 + 1$-dimensional gravity with a negative or vanishing cosmological constant $\Lambda$ (in particular the $2 + 1$-dimensional black-hole), and those of Liouville gravity, can be directly interpreted as vacuum solutions of Einstein’s equation in $4D$. The study of their four dimensional properties and their physical relevance is under investigation.
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