Radiative Generation of Quark Masses and Mixing Angles in the Two Higgs Doublet Model

Alejandro Ibarra$^1$ and Ana Solaguren-Beascoa$^{1,2}$

$^1$ Physik-Department T30d, Technische Universität München, James-Franck-Straße, 85748 Garching, Germany
$^2$ Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany

March 12, 2014

Abstract

We present a framework to generate the quark mass hierarchies and mixing angles by extending the Standard Model with one extra Higgs doublet. The charm and strange quark masses are generated by small quantum effects, thus explaining the hierarchy between the second and third generation quark masses. All the mixing angles are also generated by small quantum effects: the Cabibbo angle is generated at zero-th order in perturbation theory, while the remaining off-diagonal entries of the Cabibbo-Kobayashi-Maskawa matrix are generated at first order, hence explaining the observed hierarchy $|V_{ub}|, |V_{cb}| \ll |V_{us}|$. The values of the radiatively generated parameters depend only logarithmically on the heavy Higgs mass, therefore this framework can be reconciled with the stringent limits on flavor violation by postulating a sufficiently large new physics scale.

1 Introduction

The quark masses and mixing angles are fundamental parameters in the Standard Model of Particle Physics which must be determined experimentally. While it is
generically expected that dimensionless parameters of the Lagrangian should be either $O(1)$ or zero, experiments have revealed hierarchies among the masses of quarks of different generations as well as hierarchies among the quark mixing angles, suggesting the existence of an underlying mechanism generating this structure.

Several ideas have been discussed in the literature to explain the observed pattern of quark masses and mixing angles. A very popular approach consists in postulating the existence of a “horizontal” $U(1)$ symmetry, under which the left- and right-handed quarks of different generations transform differently, and which is assumed to be spontaneously broken at an energy below a certain cut-off. The masses and mixing angles then arise as powers of the small ratio of the $U(1)$ symmetry breaking scale over the cut-off scale \[.\] This approach has been generalized to non-Abelian symmetries, e.g. in \[,\] or to discrete symmetries, e.g. in \[.\] A second approach consists in postulating tree level masses for the heavier generation quarks, while the lighter generations acquire masses by quantum effects, thus naturally explaining the observed hierarchy in the quark masses of different generations. Early attempts to radiatively generate fermion masses were presented in \[,\], based on a gauge group $SU(3)_L \times SU(3)_R$ with the leptons $e^-$, $\nu$ and $\mu^+$ forming a triplet. Since then, many authors have constructed radiative mass models by extending (without horizontal symmetries) the gauge sector, e.g. in \[,\], or by introducing supersymmetry, e.g. in \[,\].

In this letter we will present a mechanism to generate quark mass hierarchies and mixing angles in the framework of the general two Higgs doublet model. No new fermions nor new symmetries will be introduced. As is well known, this model generically leads to too large flavor violation, hence it is common to impose a discrete symmetry forbidding the simultaneous coupling of two Higgs bosons to the same fermion \[.\] However, the flavour violating effects can also be suppressed if the new physics arises at a sufficiently large energy scale. We will show that in this scheme the radiatively generated quark masses are only mildly dependent on the scale of new physics and therefore the same conclusions remain valid even in the decoupling limit.

---

\[^1\]A similar approach was pursued in \[,\] to generate a mild neutrino mass hierarchy.
2 Flavor structures in the 2HDM

The flavor dependent part of the general two Higgs doublet model has the following Lagrangian [15]:

$$-L_{\text{Yuk}} = (Y_u^{(a)})_{ij} \bar{q}_L^i u_R^j \Phi_a + (Y_d^{(a)})_{ij} \bar{q}_L^i d_R^j \tilde{\Phi}_a + \text{h.c.} \ ,$$

(1)

where $i, j = 1, 2, 3$ are flavor indices, $a = 1, 2$ is a Higgs index and $\tilde{\Phi}_a = i \tau_2 \Phi_a^*$. It will be convenient in what follows to work in the Higgs basis where one of the Higgs fields, say $\Phi_2$, does not acquire a vacuum expectation value. Therefore $\langle \Phi_0^1 \rangle = v/\sqrt{2}$, with $v = 246$ GeV, and $\langle \Phi_0^2 \rangle = 0$. In this basis, then, the Yukawa matrices $Y_{u,d}^{(1)}$ are proportional to the fermion mass matrices.

We will assume, in view of the large mass hierarchy between quarks of different generations, that all the Yukawa matrices have rank 1 at tree level. It can be checked that, by means of a basis transformation of the quark fields, the tree level Yukawa couplings to the Higgs $\Phi_1$ can be written in the form:

$$Y_u^{(1)}|_{\text{tree}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_u^{(1)} \end{pmatrix} , \quad Y_d^{(1)}|_{\text{tree}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon y_d^{(1)} \\ 0 & 0 & y_d^{(1)} \end{pmatrix} ,$$

(2)

which lead to

$$m_t^{\text{tree}} = y_u^{(1)} v/\sqrt{2} , \quad m_c^{\text{tree}} = m_t^{\text{tree}} = 0 ,$$

$$m_b^{\text{tree}} = y_d^{(1)} \sqrt{1 + \epsilon^2 v/\sqrt{2}} , \quad m_s^{\text{tree}} = m_d^{\text{tree}} = 0 .$$

(3)

Besides, the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix are $V_{ub} = 0$ and $V_{cb} = \epsilon$, while $V_{us}$ is not defined, since any rotation between the left-handed quarks of the first and second generation leaves the Lagrangian invariant. Experimentally $|V_{cb}| \ll 1$, hence we will assume in what follows that $\epsilon = 0$.

On the other hand, the Yukawa couplings to the Higgs $\Phi_2$ must take the most general form of a rank-1 matrix, namely:

$$Y_u^{(2)}|_{\text{tree}} = U_L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_u^{(2)} \end{pmatrix} U_R ,$$

$$Y_d^{(2)}|_{\text{tree}} = D_L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_d^{(2)} \end{pmatrix} D_R ,$$

(4)
where $U_{L,R}, D_{L,R}$ are $3 \times 3$ unitary matrices. The Yukawa matrix elements are $(Y_u^{(2)})_{ij} = y_u^{(2)}(U_L)_{3i}^*(U_R)_{3j}$, $(Y_d^{(2)})_{ij} = y_d^{(2)}(D_L)_{3i}^*(D_R)_{3j}$, hence only the last row of the unitary matrices is relevant, which we parametrize as:

$$
(U_L)_{31} = e^{i\rho_u} \sin \theta_u \sin \omega_u,
(U_L)_{32} = e^{i\xi_u} \sin \theta_u \cos \omega_u,
(U_L)_{33} = \cos \theta_u,
$$

and similarly for $U_R, D_L, D_R$. In what follows we will neglect the phases for simplicity.

### 3 Quantum effects on the quark masses and mixing angles

We calculate now the impact of the quantum effects on the Yukawa couplings leading to fermion masses, $Y_u^{(1)}$ and $Y_d^{(1)}$. The one loop corrected couplings approximately read:

$$
Y_u^{(1)}|_{1\text{-loop}} \simeq Y_u^{(1)}|_{\text{tree}} + \frac{1}{16\pi^2} \beta_u^{(1)} \log \frac{\Lambda}{M_H},
$$

$$
Y_d^{(1)}|_{1\text{-loop}} \simeq Y_d^{(1)}|_{\text{tree}} + \frac{1}{16\pi^2} \beta_d^{(1)} \log \frac{\Lambda}{M_H},
$$

where $\Lambda$ is the cut-off scale of the theory and $\beta_u^{(1)}, \beta_d^{(1)}$ are the beta-functions, which can be found in [16, 17].

We find that quantum effects generate a rank-2 matrix. The values of the Yukawa eigenvalues and the CKM matrix elements can be straightforwardly calculated from Eq. (6) using perturbation theory. Under the reasonable assumption $y_d^{(1)} \ll y_u^{(1)}, y_u^{(2)}$ (motivated by the empirical fact that $y_d^{(1)} \ll y_u^{(1)}$), the ratios between the Yukawa couplings of the second and third generation approximately read:

$$
\frac{y_c}{y_t} \simeq \left(\frac{1}{16\pi^2} \log \frac{\Lambda}{M_H}\right) \frac{3}{4} (y_u^{(2)})^2 \sin 2\theta_u \sin 2\theta_u,
$$

$$
\frac{y_s}{y_b} \simeq \left(\frac{1}{16\pi^2} \log \frac{\Lambda}{M_H}\right) \frac{y_u^{(1)} y_u^{(2)} y_d^{(2)}}{y_d^{(1)}} \cos \theta_u \sin \theta_d \cos N_d,
$$

where

$$
N_d = \left[9 \sin^2 \theta_d \cos^2 \theta_u + 4 \cos^2 \theta_d \sin^2 \theta_u - 3 \sin 2\theta_d \sin 2\theta_u \cos(\omega_d - \omega_u)\right]^{1/2},
$$
which are loop suppressed but enhanced by the large logarithm of the cut-off scale over the heavy Higgs mass. On the other hand, the first generation quarks remain massless in this simple scenario. They could be also generated radiatively if additional flavor structures were introduced in the model (e.g. by adding a third Higgs doublet or by postulating the existence of approximate rank-2 matrices at tree level). We also note that the same result arises if the tree level Yukawa matrix is rank-2 but with Yukawa eigenvalues displaying very large hierarchies. If this is the case, the one loop contributions to the strange and charm masses induced by the third generation quarks will be much larger than the corresponding tree level values and, consequently, the masses at the one loop level will still be well approximated by Eq. (7).

It is important to remark that the radiatively generated charm and strange masses depend logarithmically on the heavy Higgs mass, while flavor violating effects are suppressed by four powers of the latter. Therefore, by postulating a very large value for the heavy Higgs mass the predicted rates for the flavor violating processes will be within the experimental ranges, although deviations from the Standard Model values might be at the reach of future experiments, depending on the value of the heavy Higgs mass.

The 12 and 21 elements of the CKM matrix are also calculable and read:

\[
V_{us} \simeq -V_{cd} \simeq \frac{3 \sin \theta_{dL} \cos \theta_{uL} \sin(\omega_{dL} - \omega_{uL})}{N_d},
\]

while the 11 and 22 elements are \(V_{ud} \simeq V_{cs} \simeq \sqrt{1 - V_{us}^2}\). Notably, the Cabibbo angle is not loop suppressed. The reason lies in the ambiguity in the choice of the eigenvectors that diagonalize the tree level matrices \(Y_u^{(1)} Y_u^{(1)\dagger} \big|_{\text{tree}}\) and \(Y_d^{(1)} Y_d^{(1)\dagger} \big|_{\text{tree}}\) due to their two vanishing eigenvalues. When the perturbation is added, one non-vanishing eigenvalue is generated and the ambiguity is resolved, resulting in well defined eigenvectors which lead in turn to a well defined Cabibbo angle. In the perturbation theory language, the Cabibbo angle is generated at zero-th order. In the renormalization group language, this effect can be interpreted as an infrared quasi-fixed point for the Cabibbo angle, that depends on the value of the corresponding beta-function, but is independent of the value of the Cabibbo angle at the cut-off scale. This behavior was noted in [18, 19] and extensively discussed in [20] for the mixing angles in the neutrino sector in the presence of degenerate mass eigenvalues.
The remaining elements of the CKM matrix are:

\[
V_{ub} \simeq \left( \frac{1}{16\pi^2} \log \frac{\Lambda}{M_H} \right) \frac{3y_u^{(1)} y_d^{(2)}}{y_d^{(1)}} \sin \theta_{dL} \cos \theta_{dR} \cos \theta_{uL} \sin(\omega_{dL} - \omega_{uL}) ,
\]

\[
V_{cb} \simeq \left( \frac{1}{16\pi^2} \log \frac{\Lambda}{M_H} \right) \frac{y_u^{(1)} y_d^{(2)}}{y_u^{(1)} y_d^{(2)}} \left\{ \frac{1}{4} \frac{y_u^{(1)} y_d^{(2)}}{y_d^{(1)}} \sin 2\theta_{uL} (3 \cos 2\theta_{uR} + 2) + \cos \theta_{dR} \cos \theta_{uR} \left[ 2 \cos \theta_{dL} \sin \theta_{uL} - 3 \sin \theta_{dL} \cos \theta_{uL} \cos(\omega_{dL} - \omega_{uL}) \right] \right\} ,
\]

while \( V_{td} = -V_{ab}V_{cs} + V_{us}V_{cb} \) and \( V_{ts} = -V_{cb}V_{td} + V_{ub}V_{cd} \), as required by unitarity, and \( V_{tb} \approx 1 \). In contrast to the Cabibbo angle, all other off-diagonal entries of the CKM matrix are generated at first order of perturbation theory and are therefore expected to be much smaller than the 12 entry, in qualitative agreement with experiments.

The measured values of \( y_c/y_t, y_s/y_b \) and the CKM matrix can be accommodated within this framework by choosing appropriate model parameters. We note first that the right-handed angles \( \theta_{dR} \) and \( \theta_{uR} \) are univocally determined by the quark parameters:

\[
\frac{y_c}{y_t} \frac{V_{us}}{V_{ub}} \approx \tan \theta_{dR} ,
\]

\[
\frac{y_s}{y_b} \frac{V_{us}}{V_{ub}} \approx \frac{3 \sin 2\theta_{uR}}{2 + 3 \cos 2\theta_{uR}} ,
\]

which approximately give \( \theta_{uR} \approx 0.16, \theta_{dR} \approx 1.06 \). On the other hand, there are degeneracies among the remaining parameters. One possible choice is \( y_u^{(2)} \approx 1.04, y_d^{(2)} \approx 0.02, \theta_{dL} \approx 0.61, \theta_{uL} \approx 0.51, \omega_{dL} - \omega_{uL} \approx 0.10 \). It is notable that under the reasonable assumptions that the coupling \( y_u^{(2)} (y_d^{(2)}) \) is of the same order as \( y_u^{(1)} (y_d^{(1)}) \) and that the mixing angles are all \( \mathcal{O}(0.1) \) it is possible to naturally reproduce the measured masses of the second generation quarks and the mixing angles. A similar scheme could be responsible for the charged lepton masses in the presence of right-handed neutrinos, due to the quark-lepton symmetry in the type I see-saw mechanism. The implications for the neutrino masses and mixing angles will be discussed elsewhere [21].

The framework presented here contains a large number of free parameters and does not lead to any prediction. Nevertheless, the degeneracies could be broken by incorporating to the analysis other flavor observables, such as deviations from the Standard Model predictions in flavor changing neutral currents or the decay branching fractions of the heavy Higgs, which could be measured in future experiments.
4 Conclusions

The hierarchies among the quark masses of different generations, as well as the hier-
archies among the quark mixing angles, strongly suggest the existence of a dynamical
mechanism to generate this pattern. We have argued that a second Higgs doublet
added to the Standard Model particle content, with no additional fermions nor ad-
ditional symmetries, can be responsible for generating via quantum effects a mass
hierarchy between the second and third quark generations and a pattern of mixing
angles in qualitative agreement with observations. This scheme can reproduce the
measured values even in the decoupling limit of the heavy Higgs, therefore the strong
constraints on a second Higgs doublet from flavour changing neutral currents can be
easily avoided if the heavy Higgs mass is sufficiently large. On the other hand, if
the new physics scale is low enough, new phenomena could be observed in experi-
ments at the intensity and at the energy frontier, opening the possibility to test this
mechanism.

Acknowledgements

We are grateful to Camilo Garcia-Cely for useful discussions. This work was sup-
ported in part by the DFG cluster of excellence “Origin and Structure of the Uni-
verse” and by the ERC Advanced Grant project “FLAVOUR” (267104) (A.I.).

Note Added

While this work was being finalized, we learned of the work [22], where it is present-
a supersymmetric framework to radiatively generate quark masses and mixing angles.

References

[1] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B 147 (1979) 277.

[2] R. Barbieri, L. Giusti, L. J. Hall and A. Romanino, Nucl. Phys. B 550 (1999) 32.

[3] S. F. King and G. G. Ross, Phys. Lett. B 520 (2001) 243.

[4] E. Ma, Mod. Phys. Lett. A 17 (2002) 627.
[5] S. Weinberg, Phys. Rev. Lett. 29 (1972) 388.

[6] H. Georgi and S. L. Glashow, Phys. Rev. D 7 (1973) 2457.

[7] R. N. Mohapatra, Phys. Rev. D 9 (1974) 3461.

[8] B. S. Balakrishna, Phys. Rev. Lett. 60 (1988) 1602.

[9] B. S. Balakrishna, A. L. Kagan and R. N. Mohapatra, Phys. Lett. B 205 (1988) 345.

[10] N. Arkani-Hamed, H.-C. Cheng and L. J. Hall, Phys. Rev. D 54 (1996) 2242.

[11] F. Borzumati, G. R. Farrar, N. Polonsky and S. D. Thomas, Nucl. Phys. B 555 (1999) 53.

[12] A. Ibarra and C. Simonetto, JHEP 1111 (2011) 022.

[13] W. Grimus and H. Neufeld, Phys. Lett. B 486, 385 (2000).

[14] S. L. Glashow and S. Weinberg, Phys. Rev. D 15 (1977) 1958.

[15] For a review, see G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, Phys. Rept. 516 (2012) 1.

[16] W. Grimus and L. Lavoura, Eur. Phys. J. C 39 (2005) 219.

[17] T. P. Cheng, E. Eichten and L. -F. Li, Phys. Rev. D 9 (1974) 2259.

[18] J. R. Ellis and S. Lola, Phys. Lett. B 458, 310 (1999).

[19] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, Nucl. Phys. B 556, 3 (1999).

[20] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, Nucl. Phys. B 573 (2000) 652.

[21] In progress.

[22] W. Altmannshofer, C. Frugiule, R. Harnik, to appear.