Neural computations in control problems: aspects of computability and spatial-time characterization of cognitive functions

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Abstract. The thesis of the futurologist A. Clark that «sufficiently advanced technologies are indistinguishable from magic» is relevant nowadays as never before. Modern science, unlike magic, although it relies on logically consistent physical theories, formalisms of mathematics and the possibilities of computing, has not made the world less mysterious, especially with regard to the natural phenomena of consciousness, thinking and intelligence. From the point of view of computer science, a distinctive feature of these phenomena is their network-centric neuromorphic organization and the availability of complex memory resources - a non-casual computing mechanism for «transporting» processed data, the results of calculations and information interactions in time. Formalization of these features makes it possible to determine a model of cognitive processes on a multidimensional set of states, on which a strict relation between the time order and numerical equality is not introduced, therefore, contextually significant information, about past as well as current or forecast states, can be used for calculations at the same time states. For control systems operating under uncertainty, a distributed heterogeneous reconfigurable structure is proposed that implements the technology of «computation in memory» both for algorithms reflecting causal relationships and learning processes that require operational reconfiguration of the computation field.

Introduction

The paper is devoted to the formalization of the description of cognitive experience in the context of control problems based on the analysis of the computability properties of cognitive functions of «bounded rationality», represented by the operation of convolution of signals received at different points in time, in which the time argument is not a sequence of past, present and future states strictly ordered by «arrow of time» but a three-dimensional subspace of time, the integrity of which is based on the noncasual influence of the predicted «future» on decisions made in the «present».

A similar phenomenon, called retro causality, was described in quantum physics [1, 2] based on the effect of quantum nonlocality, which has an emphatically informational character. Modern approaches to computer modeling of the functions of consciousness have been developing for many years within the framework of the direction known as the Computational theory of cognition [3, 4], the essence of which is in the creation of computers capable of solving various mental tasks, such as reasoning,
recognition of situations, adaptation based on training, etc. However, until now, even relatively low-level natural cognitive processes, for example, such as perception, significantly exceed the capabilities of modern computer systems.

1. Computability and spatial-time characterization of cognitive functions

1.1. Background of the problem
The problem that limits the possibilities of creating «smart computers» is of a fundamental nature and is based on the uncertainty of the processed data [5, 6, 7].

Well-known methods of making decisions under uncertainty are based on the Bayesian approach, a standard mathematical inference model that is based on the concept of probability and strict equivalence (equality) of events based on their statistical characterization. The probabilistic approach, however, limits the ability to model many realistic scenarios based on causal event models. Obviously, the formalization of cognitive functions requires more subtle approaches linking the ideas of mathematics, computer and neurosciences based on the use of various equivalence relations, which differ in the amount of information used in the process of calculating the states of processes or objects.

It should be taken into consideration that the architecture of modern computers is designed to simulate physical processes of the Markov stochastic nature and, therefore, described by differential equations. As a result, computer modeling is reduced either to a «digital» solution of the Cauchy problem based on a priori given algorithms and initial conditions, or to the solution of an eigenvalue problem. The complexity of computer modeling of cognitive processes lies in the fact that their characteristic carrier set of states is not endowed with a strict temporal order relation reflecting their casual connectivity, therefore, from the point of view of the theory of computation, data on past, current and predicted future events should be simultaneously available for algorithmic processing.

1.2. Proposed approaches for tackling the problem
The approaches discussed in the paper are based on the methods of category theory, in which the set of processed events is characterized by both a set of metadata and a description of ways to compare one object of the considered category to another, including descriptors by which the objects of the category are considered equivalent.

The paper analyzes the possibility of using the topos category [3] as the basis for the architecture of a heterogeneous reconfigurable computer that implements cognitive functions using data-driven computation algorithms and taking into account the temporal connectivity of computation results with respect to their equivalence ratio with data stored in memory.

The description of any system as a kind of «computer» strongly assumes that the system is programmable; implements a specified sequence of data processing commands, the essence of which is the computation of a certain function that unambiguously connects the values of input and output variables with an equal sign.

This feature is associated with the notion of computability, which requires the existence of an algorithm that achieves the above exact equality in a finite number of steps. However, when using a digital representation of data, the accuracy from the formal mathematical concept of «equality» is parameterized and depends on the number of digits of representation of the processed numbers. Therefore, the characterization of cognitive functions from the point of view of the theory of programmed machines allows, when considering the properties of computability, to switching from «fixed bars of the equal sign» [4] to the equivalence relation introduced in the framework of category theory.

Although the cognitive functions of consciousness are still not a physical concept, i.e. part of physical theories, like force, mass or electric charge, such a function can be represented as a signal [3], combining data distinguishable by descriptors of past, present and future. In fact, this means that information within the framework of the cognitive description of the space of events can be transmitted not only by physical signals, but also by the form of a process that implements various
functions based on memory and predictive calculations. In this case, one should take into account the fundamental limitations of the theory of algorithms - enumerability, computability and solvability of sets, as well as the consequences of Gödel's theorems, applicable to the problems of modeling cognitive functions and improving AI systems. Obviously, instead of direct modeling of «electrophysiological» processes observed in the brain, other types of memory organization and computation processes can be used, which combine, on the one hand, digital and continuous, and, on the other, deterministic and probabilistic characteristics of cognitive processes. To do this, we define the cognitive function at time \( t \) by the attribute, which determines the amount of memory used to store information about past and predicted events. Such a function can be formally represented as a «signal» subject to convolution with the Dirac function. Such a sensory-flow signal reflects not only the spatial-time connectivity of perceived events, but also endows them with certain mental attributes, making it possible to calculate various equivalence relations on a multidimensional set of processed data. A computational structure that implements a control flow from a data flow is a model of an executive computation mechanism capable of obtaining a representation of a cognitive «signal» from the description of events, shown in Fig.1.

![Fig.1. Natural and Artificial computation mechanisms](image)

«Connectionism» postulates that a computer system should «resemble the mind», that is, consist of layers connected to each other. Such a computer system can not only be «programmed» with a set of instructions, it can also be «trained» with input data. Connectionist systems have so far been made based on artificial neural networks and have proven to be good at some tasks, such as pattern recognition, but much less good at other things, such as modeling and rule generation. It is clear to us that two very different problems, algorithmization and learning, are actually instances of the same problem. And we solve this problem as it is written in the article - by stratification into two connected sub-spaces - in one there is a solution of the direct modeling problem, in the other-the inverse problem of parametric identification.
In the Fig. 2, there are some levels of computational flow [8, 9].

Level 3 is an «edge» level with Edge RH HPC (Reconfigurable Heterogeneous High Performance Computer) units. It is a level of «Sensory» data processing. Purpose of the level is to provide an access to the «space» of big data, delivered by sensors and used to «extract» multimodal data.

Level 2 is Premises RH HPC units level. The level «Premises» is the action «I want» and is an analogue of the limbic system, implementation of the function of «instinctive / reflexive» behavior. Its purpose is - optimization of the specifications of the calculation field and data formats, «preparation of firmware» for RH HPC configuration, control of the adequacy of digital models.
Level 1 is DC-Cloud unit level. The level is the action «Must». It is an analogue of the neocortex, the implementation of the «social behavior» function. Purpose of the level is to do interpretation of the results of calculations and the solution of «inverse» problems of the synthesis of algorithms in the just in time mode or on schedule.

Fig. 3 shows that at all levels of the platform, each of the «Heterogeneous computing» nodes includes such computing units as: SIMD (Single Instruction Multiple Data) accelerators, MPU (Micro Processing Units) processors and RA (Reconfigurable Accelerator) based on FPGAs. It uses hyper converted memory resources grouped into an associative category of data related to past, current and predicted values.

The key constructive suggestion is related to the «fractality» and functional «stratification» of the computational space of cognitive functions. The basis of this bundle is the state space obtained using mathematical modeling algorithms (using CPU/GPU/FPGA) with a layer of neural networks «trained» on the basis of data obtained as a result of generalization of previous experience. Namely, «experience» (cognitive resources of memory) can be used as the basis for the regularization procedure for solving inverse problems (in particular, the problem of parametric identification of differential equations used for modeling using CPU/GPU). The «physics» here is that experience
«collects» only «stable» solutions to inverse problems, and everything that is «unstable» is simply filtered «in time and space» by physical reality itself.

It is clear that all calculations of functions that are used to express solutions to emerging «inverse» problems occur under conditions of lack of time, computational resources, and the use of only a part of the potentially available sensory data obtained at different points in time. In reality, the natural brain, and its artificial «twin» must retouch the details to highlight those that radically can affect the nature of the inverse problem decision. Considering that the brain plays both the role of semantic filter and hub of perception data flow we introduce the concept of «functions of bounded rationality», with the help of which we carry out the computational characterization of cognitive functions not from the point of view of classical algorithmic or Turing machine theory but as a functor in a sense of theory of category.

2. Discussion
Formalization is the foundation of all scientific methods. Therefore, the development of a model of cognitive processes that can be translated into various classes of computable functions is an important scientific task. A feature of such models is their hybrid nature, which allows expanding the boundaries of computer science, going beyond the boundaries of solvability of problems based on partially recursive functions (algorithmic rational description) in order to use the expressive capabilities of the function of «limited (partial) rationality», self-regulation and learning, which are based not only on past and current measurements, but also on forecast of future states. Such functions can be implemented using a distributed heterogeneous reconfigurable computational structure using calculation in memory and hardware reconfiguration technologies for both algorithms that reflect causal relationships and for learning processes that require the use of training samples and, in some cases, hardware reconfiguration of computational field.

The formalization of the considered models is based on the methods of category theory, which studies the properties of relations between various mathematical structures, regardless of their internal organization. In this case, the objects of category theory can be sets (objects of the category of sets), algebraic systems of a certain class (for example, rings are objects of the category of rings), topological spaces (objects of the category of topological gaps), schemes diagram (objects of the category of schemes), etc. The categories considered in the article are shown in Fig. 4 as a locally trivial stratification of the space of perceived unstructured «big data» E into the base space B - a set of perceived images with layers F that reflect the algorithmic model of the object localized in E in context of previous experience.

![Fig. 4. A commutative diagram of mapping π of the space of perceived and unstructured «big data» E to the space of the base B of images](image)

In this representation the essence of cognitive functions category is expressed by the way of stratifying the space of «big data» or the space E into the space of the base of images B - which is formed on the basis of solving inverse problems using regularization methods based on the choice of stable (robust) solutions.
Conclusion

From the point of view of neurophysiology, cognitive functions process not only operational information received from the senses, but also data based on past experience and stored in memory, as well as the results of predicting the consequences of not yet implemented control actions.

Therefore, in the space of states of cognitive functions, there is no linear ordering of time counts of events characteristic of physical processes, which endows this space with non-trivial topologies, which is reflected in the architecture of the proposed platform. The constellation of data based on various equivalence ratios of data stored in memory, information obtained through communication channels and by predicting the consequences of actions implemented «now» forms the basis of the proposed approach to computer modeling of cognitive functions. Convergent memory, considered as a multidimensional attribute of control processes, allows one to model two classes of cognitive processes - causally related events that form a causal sequence in which past events affect the present and the future, as well as events that form semantically related structures in which the potential possible future influences the control actions taken at the present time and changes the interpretation of past events in terms of the results achieved.

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