The effect of diffusion on the Red Giant luminosity function ‘Bump’

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ABSTRACT

This paper investigates the effect of microscopic diffusion of helium and heavy elements on the location of the Red Giant Branch Luminosity Function Bump in Population II stellar models. To this aim updated evolutionary models taking into account diffusion from the Main Sequence until the Zero Age Horizontal Branch have been computed. The observational luminosity difference between the RGB bump and the ZAHB ($\Delta V_{\text{hb}}$), as collected for a sample of galactic globular clusters, has been compared with the corresponding theoretical values obtained by adopting both canonical and diffusive models. We find that the effect of diffusion, even if slightly improving the agreement between observations and theory, is negligible with respect to the observational uncertainties. In any case the theoretical predictions for $\Delta V_{\text{hb}}$ in models with and without diffusion appear in agreement with the observational results within the estimated errors. Thus canonical models can be still safely adopted, at least until much more accurate observational data will be available.

Key words: stars: evolution – stars: interiors – globular clusters: general

1 INTRODUCTION

From a theoretical point of view, one of the main features of the Red Giant Branch (RGB) of low mass stars is the existence of a region in which the stellar luminosity, instead of monotonically increasing, as usual, shows a significant drop before starting again to increase. As well known (see e.g. Thomas 1967, Iben 1968, Renzini & Fusi Pecchi 1988, Castellani, Chieffi & Norci 1989, Bono & Castellani 1992), this behavior is related to the passage of the thin H burning shell through the chemical discontinuity left by the convective envelope during the first dredge up. Such a temporary drop in luminosity produces a peculiar "bump" in the differential RGB luminosity function (LF), a feature that has been identified in the observed luminosity functions of galactic globular clusters (GCs) (see e.g. King et al. 1985, Fusi Pecchi et al. 1990, hereinafter FP90, Bergbush 1993, Sarajedini & Norris 1994, Sarajedini & Forrester 1995, Brocato et al. 1996a, Brocato Castellani & Ripepi 1995, 1996b, Sandquist et al. 1996).

For minimizing uncertainties due to the photometric data calibration and the cluster distance modulus, it is a common procedure, when comparing observational RGB bump luminosities with theoretical prescriptions, to consider the difference in the V magnitude between the RGB bump and the Zero Age Horizontal Branch (ZAHB), i.e. the parameter $\Delta V_{\text{hb}} = V_{\text{bump}} - V_{\text{hb}}$.

The first exhaustive comparison between theoretical and observed values of $\Delta V_{\text{hb}}$ for a large sample of GCs has been performed by FP90 by adopting the RGB models by Rood & Crocker (1989) and unpublished HB models by Rood. Since the bump luminosity depends on the age and both the bump and the ZAHB are affected by changes in the Helium abundance ($Y$) (FP90, Cassisi & Salaris 1997, hereinafter Paper I) FP90 explored various scenarios concerning the age and the initial Helium abundance of galactic globular clusters. Their conclusion was that, for constant $Y$, the run of the theoretical $\Delta V_{\text{hb}}$ values with the metallicity was in agreement with the observations, but the absolute values were too bright by $\approx 0.4$ mag.

After the work by FP90, some suggestions have been presented in order to provide an explanation of this disagreement between theoretical RGB stellar models and observations. Some authors interpreted the discrepancy as a proof of the intrinsic limitation of standard stellar models and, in order to solve it, undershooting from the formal boundary of the convective envelope (Alongi et al. 1991) has been invoked. On the contrary, other authors (Straniero, Chieffi & Salaris 1992, Ferraro 1992) suggested that the discrepancy could be partially reduced simply by computing stellar models accounting for the observed enhancement of the α-elements in the initial chemical composition of GC stars. However, for a long time this question remained unsettled.

In a recent paper (Paper I) Cassisi & Salaris investigated in detail this problem. They computed theoretical models with updated equation of state and opacity evaluations and collected a sample of galactic GCs with accurate photometric data and metallicity determinations. The dependence of the theoretical RGB bump and ZAHB luminos-
ity levels on different physical inputs adopted in evolutionary computations, as for example equation of state, opacity, mass loss, mixing length parameter, was analyzed. The final result was that the agreement between observational data and theoretical expectations was achieved within the observation uncertainties, in the conservative hypothesis of coeval GCs with the same initial Helium abundance. Moreover it was shown that for reasonable variations of the GCs ages and their original Helium content, the consistency with the observations is preserved. The discrepancy between the ages and their original Helium content, the consistency with the observational is preserved. The discrepancy between the results in Paper I and the conclusions of FP90 can be basically related to: i) the use of updated stellar models ii) the accuracy of new adoptive spectroscopical measurements for $[\alpha/Fe]$ and $[Fe/H]$ for GC stars iii) the accurate analysis of recent photometric data. The authors then concluded that there is no significant discrepancy between observations and canonical stellar models computed adopting updated input physics.

However, during the last years, it became evident that the introduction of the microscopic diffusion in standard solar models is fundamental to reach an agreement between the theoretical predictions of the models and the helioseismological observational results (see e.g. Christensen-Dalsgaard 1993, Guzik & Cox 1993, Bahcall & Pinsonneault 1995, Ciacio, Deigl'Inocenti & Ricci 1997, Bahcall et al. 1996). Moreover, the diffusion is well known to have important consequences also for the surface chemical composition of white dwarfs (Michaud, Fontaine & Charland 1984).

Stringfellow et al. (1983) were the first to calculate models of low mass, metal poor stars with Helium diffusion. After this pioneering work, further investigations on this matter have been performed (Proffitt & Vandenbergh 1991, Chaboyer, Sarajedini & Demarque 1992) with the aim to investigate the effect of He diffusion on GCs isochrones. More recently the effect of the diffusion of not only Helium but also of the heavy elements on low mass, metal poor stellar models has been taken into account by Castellani et al. (1997a).

However till now the influence of microscopic diffusion on the $\Delta V_{\text{hb}}$ has been never analyzed; in this paper we address this topic accounting for both He and heavy elements diffusion.

In Section 2 we present a brief description of the physical inputs and of the evolutionary models; the comparison with the observations and the discussion of the results follow in Section 3. Finally, the conclusions are presented in Section 4.

## 2 STELLAR MODELS.

All the theoretical stellar models have been computed by adopting the FRANEC evolutionary code (Chieffi & Straniero 1989) adapted to account for the diffusion of helium and heavy elements (Ciacio et al. 1997). The diffusion coefficients are calculated according to Thoul, Bahcall & Loeb (1994). The variation of the abundances of H, He, C, N, O and Fe due to atomic diffusion is followed along the structure and during all the evolutionary phases starting from the Zero Age Main Sequence to the RGB tip. Following the prescription by Thoul et al. (1994), all the other heavy elements are assumed to diffuse at the same rate as the fully ionized iron. To account for the effect of the heavy elements diffusion also on the opacity evaluations, we interpolated among opacity tables corrsponding to various metallicities (see Ciacio et al. 1997 for more details).

The OPAL opacity tables (Rogers & Iglesias 1995) for $T > 10000K$ and the Alexander & Ferguson (1994) opacities for $T \leq 10000K$ have been used. The high temperature opacity tables are computed adopting the solar heavy elements distribution by Grevesse & Noels (1993), also adopted in the nuclear reaction network, while the molecular opacity tables have been computed according to the Grevesse (1991) solar mixture. As for the equation of state (EOS) the OPAL EOS (Rogers, Swenson & Iglesias 1996) has been adopted (see also Paper I). For this work we updated the code with respect to Paper I by using more recent nuclear reaction rates for the H-burning (see Ciacio et al 1997 and Castellani et al 1997b), the improved prescription by Haft, Raffelt & Weiss (1994) for the plasma neutrino energy-loss rate (in Paper I we used the results by Munakata, Kohyama & Itoh 1985), and the value for the $3\alpha$ reaction rate from Caughlan & Fowler (1988 - in Paper I we used the rate from Caughlan et al 1985).

As for the calibration of the superadiabatic envelope convection, we have adopted a mixing length calibration obtained using the same procedure described in Salaris & Cassisi (1996). We have obtained that mixing length ($ml$) values $ml \approx 1.6$ at $Z=0.0002$, $ml \approx 1.7$ at $Z=0.001$ and $ml \approx 1.8$ at $Z=0.006$ (coincident with the ones obtained in Paper I for OPAL EOS models) have to be adopted in order to reproduce the empirical effective temperatures of GCs RGB as provided by Frogel et al. (1983).

We evolved models of $0.70M_{\odot}$, $0.80M_{\odot}$, $0.90M_{\odot}$ and $1.0M_{\odot}$ (fully covering the range of masses of stars presently evolving along the RGB of GCs) with $Z=0.0002$, $Z=0.001$ and $Z=0.006$ and an initial helium abundance equal to $Y=0.23$ from the Zero Age Main Sequence to the RGB tip. For each fixed metallicity, we interpolated among these models to obtain the value of $V_{\text{hb}}$ (as defined in Paper I) for an age of $t=12\text{Gyr}$.

This age (also adopted in Paper I) has been chosen following the recent results of different groups about the age of galactic GCs (Chaboyer & Kim 1995, Mazzitelli, D’Antona & Caloi 1995, Salaris, Deigl’Innocenti & Weiss 1997). As discussed by Castellani et al. (1997a), the combined effect of He and heavy elements diffusion reduces the estimated ages of GCs by only less than 1 Gyr.

Following the same procedure adopted in Paper I, for each chemical composition we computed also ZAHB models of low mass, metal poor stars with Helium diffusion. For the transformation from the theoretical to the observational plane we used, as in Paper I, the Kurucz (1992) transformations, adopting $M_{\text{bol},\odot} = 4.75$ mag. Then the theoretical value of $\Delta V_{\text{hb}}$ has been obtained by considering the difference between the $V$ magnitudes of the RGB

### Table 1.

Luminosities of the ZAHB at $\log T_{\text{eff}} = 3.85$, of the RGB bump and the values of $\Delta V_{\text{hb}}$ obtained by adopting (see text) $Y=0.23$, $t=12\text{Gyr}$ and using canonical stellar models.

| [M/H] | $M_{\text{V}}^{\text{ahb}}$ | $M_{\text{V}}^{\text{bump}}$ | $\Delta V_{\text{hb}}^{\text{bump}}$ |
|-------|------------------|------------------|------------------|
| -2.04 | 0.510            | -0.198           | -0.708           |
| -1.35 | 0.640            | 0.321            | -0.319           |
| -0.57 | 0.857            | 1.177            | 0.320            |
bump and of the ZAHB taken at $T_{eff} = 3.85$, which can be safely adopted as the average temperature of the RR Lyrae instability strip. This procedure has been followed for both canonical stellar models and for the models which take into account microscopic diffusion. The bump and the ZAHB V magnitudes and the value of $\Delta V_{hb}^{bump}$ which refer to the canonical stellar models (computed with the physical inputs discussed above), are shown in Table 1, while the same quantities for the models with He and heavy elements diffusion are reported in Table 2. As in Paper I, we define $\Delta V_{hb}$ for the models with He and heavy elements.

Table 1. As in Table 1, but in this case for the stellar models which include microscopic diffusion of helium and heavy elements.

| [M/H] | $M_{V}^{abh}$ | $M_{V}^{hb}$ | $\Delta V_{hb}^{bump}$ |
|-------|---------------|---------------|------------------------|
| -2.04 | 0.548         | -0.129        | -0.677                 |
| -1.35 | 0.661         | 0.392         | -0.269                 |
| -0.57 | 0.872         | 1.257         | 0.385                  |

Figure 1 shows the values of $\Delta V_{hb}^{bump}$ as a function of the global amount of heavy elements when the microscopic diffusion is properly included in stellar computations (the lines have been obtained by computing the parabolic function which crosses all points). In order to evaluate easily the effect of the diffusion, we have also plotted the theoretical relation obtained adopting the same physical inputs but canonical stellar models (hereinafter we refer to it as new canonical scenario). For the sake of comparison, the theoretical relation corresponding to the standard stellar models computed adopting the OPAL EOS, as presented in Paper I (corresponding to the same average age $t=12$ Gyr), is also displayed (old canonical scenario).

Before discussing the net effect on $\Delta V_{hb}^{bump}$ due to the inclusion of microscopic diffusion in stellar computations, it is important to notice the difference, in particular at low metallicities, between the 'old' and 'new' canonical scenario. Let us remind that the only differences between the two sets of canonical stellar models adopted for computing these theoretical relations are related to the nuclear reaction rates for the H-burning, the $3\alpha$ reaction rate and the plasma neutrino energy-loss rate. The main contribution to the changes of $\Delta V_{hb}^{bump}$ between the old and new canonical scenario comes from the different luminosity of the ZAHB, since the different core mass at the RGB tip due to the new plasma neutrino energy-loss rate and the $3\alpha$ reaction rate. This difference between the two standard scenarios can be assumed as a crude estimate of the uncertainty of the standard theoretical $\Delta V_{hb}^{bump}$ values related to uncertainties in neutrino energy losses and energy generation rates.

A more detailed discussion about the stellar models presented in this paper and the differences between the old and new canonical scenario, a comparison with previous computations and an analysis of the effects of the inclusion of He and heavy elements diffusion on both isochrones and He burning models can be found in Cassisi et al. (1997).

To investigate the effect of the diffusion, we have to compare the non-standard evolutionary scenario with the canonical one obtained using the same physical inputs (new canonical scenario). The result is that the proper inclusion of both He and heavy elements diffusion produces slightly larger $\Delta V_{hb}^{bump}$ value with respect to the new canonical scenario (see Table 2); the difference is on average by 0.05 mag and at maximum only by $\approx 0.065$ mag (at $[M/H]=-0.57$). This result is in agreement with the early suggestion provided at the end of Paper I and it can be understood when taking into account the following evidences:

i) for each fixed stellar mass and chemical composition, when diffusion is taken into account, the RGB bump is located at lower luminosity ($\Delta V_{hb}^{bump} \approx 0.07$ mag, the exact value depending on the chemical composition). This effect is related to the higher opacity at the base of the convective envelope, owing to the decrease of the He abundance (and the corresponding increase of the hydrogen abundance) in the envelope;

ii) the luminosity level of the ZAHB is also decreased ($\Delta V_{ZAHB}^{bump} \approx 0.02$ mag on average, the exact value depending again on the metallicity), due to the lower amount of Helium in the envelope when the stars settles on the Horizontal Branch, that causes a lower efficiency of the H burning shell;

iii) the second effect partially compensates the first one, so that the inclusion of microscopic diffusion in stellar computations does not modify significantly the theoretical scenario for the relation between $\Delta V_{hb}^{bump}$ and the metallicity.

3 COMPARISON WITH THE OBSERVATIONS.

As discussed in Paper I, for performing a meaningful comparison between theoretical and observed $\Delta V_{hb}^{bump}$ values one needs to select a sample of GCs for which high quality photometric data and accurate spectroscopical measurements of $[Fe/H]$ and $[\alpha/Fe]$ are available. Following this prescription we selected a sample of seven GCs with a quite
large range of heavy elements abundances: from a metal poor cluster as NGC6397 to the more metal rich ones as 47Tuc. We refer to Paper I for the description of the procedure adopted to obtain the ZAHB and RGB bump luminosity levels and for the sources of the photometric data and spectroscopical metallicities. We report in Table 3 only the data needed in order to compare models and observations. As for the relation between the global amount of heavy elements and the values of $[\text{Fe}/\text{H}]$ and $[\alpha/\text{Fe}]$ we adopt, as in Paper I, the relation (see Salaris et al. 1993 for more details):

$$[M/H] \equiv [\text{Fe}/\text{H}] + \log(0.638 \cdot f + 0.362)$$

where $\log(f) = [\alpha/\text{Fe}]$ is the enhancement factor of the $\alpha$ elements. This relation has been derived by using, as the reference solar heavy element distribution, the Ross & Aller (1976) mixture. However, we have verified that an analogous relation derived (following the prescriptions by Salaris et al. 1993) by using the Grevesse & Noel (1993) solar metal distribution provides $[M/H]$ values different by not more than 0.01 dex in the range of $[\alpha/\text{Fe}]$ we are dealing with. The global metallicities adopted in Paper I (from Salaris & Cassisi 1996) are reported in column 5 of Table 3.

It is also worth noting that recently Carretta & Gratton (1997) have provided a new compilation of $[\text{Fe}/\text{H}]$ values for a large sample of galactic GCs. For the clusters 47Tuc, NGC6752 and NGC6397, Carretta & Gratton have used new high resolution spectra, while for the other clusters in their sample, they have adopted high quality literature data (namely, equivalent widths from high dispersion spectra). All these data have been analyzed with an homogeneous and self-consistent procedure by adopting the model atmospheres by Kurucz (1992).

Since the $[\text{Fe}/\text{H}]$ values provided by Carretta & Gratton are systematically larger than the values given by Gratton and coworkers in previous works and largely adopted by Salaris & Cassisi (1996) in their compilation of galactic GC metallicities, it is interesting to test what is the effect of this new metallicity scale on the present investigation. For this reason, we report also in Table 3 (column 6), the value of $[M/H]$ obtained by adopting for each cluster the value of $[\text{Fe}/\text{H}]$ provided by Carretta & Gratton, but still relying on the $[\alpha/\text{Fe}]$ values reported in Salaris & Cassisi (1996).

The $[M/H]$ values from Paper I are between 0.05 and 0.16 dex lower than the ones derived using the Carretta & Gratton $[\text{Fe}/\text{H}]$ scale, with the exception of M5; in this case the Carretta & Gratton metallicity is larger by 0.29 dex with respect to the values from Paper I.

Figure 2a shows the comparison between the observational values of $\Delta V_{\text{hb}}$ for the selected clusters and the theoretical prescriptions for standard stellar models (new canonical scenario) and for models evolved accounting for He and heavy elements diffusion. The theoretical relations obtained adopting stellar models with microscopic diffusion for two different cluster ages are also plotted. The clusters metallicities are the ones used in Paper I (here, as in Figure 2b, an error bar on the $[M/H]$ values by $\pm 0.15$ dex is assumed). One notices that the agreement between the theoretical and observational values of $\Delta V_{\text{hb}}$, obtained by adopting the new canonical scenario, is only slightly improved by the introduction of the microscopic diffusion in the stellar models. As a matter of fact the effect of diffusion is not relevant with respect to the observational uncertainties. Moreover, as already found in Paper I, the present result is not significantly modified by relaxing the hypothesis of globular clusters coeval and 12 Gyr old. This is shown, in the same Figure 2a, by the location of the two theoretical

| NGC  | $V_{\text{zahb}}$ | $V_{\text{bump}}$ | $\Delta V_{\text{hb}}$ | $[M/H]_{\text{SC}}$ | $[M/H]_{\text{CG}}$ |
|------|------------------|-------------------|----------------------|---------------------|---------------------|
| 104  | 14.20            | 14.55             | 0.35 $\pm$ 0.18      | -0.70               | -0.60               |
| 1904 | (M79) 16.36      | 16.00             | -0.36 $\pm$ 0.12     | -1.27               | -1.22               |
| 5272 | (M3) 15.76       | 15.40             | -0.36 $\pm$ 0.07     | -1.31               | -1.16               |
| 5904 | (M5) 15.15       | 14.95             | -0.20 $\pm$ 0.07     | -1.19               | -0.90               |
| 6352 | 15.50            | 15.86             | 0.36 $\pm$ 0.12      | -0.70               | -0.54               |
| 6397 | 13.02            | 12.60             | -0.42 $\pm$ 0.14     | -1.70               | -1.64               |
| 6752 | 13.86            | 13.65             | -0.21 $\pm$ 0.12     | -1.28               | -1.20               |
lines corresponding to diffusive models for two different assumptions about the age of the stellar systems. In Figure 2b the same observational data and theoretical results are plotted, but this time the clusters \([M/H]\) are derived by adopting the Carretta & Gratton \([Fe/H]\) scale. The agreement between theory and observations is slightly worst, also if again theory and observations agree within the observational uncertainties and the indetermination on the GCs ages. There is only the observational point corresponding to M5 that is clearly not reproduced by the theoretical values and, it is important to note that M5 is also the cluster for which the Carretta & Gratton scale shows the largest difference with respect to previous \([Fe/H]\) estimates.

Since, as clearly stated by Carretta & Gratton (1997), the reason for the difference between their \([Fe/H]\) scale and previous determinations is mainly due to the use of the Kurucz (1992) model atmospheres, whose reliability should be tested by independent computations, we tend to consider the difference between the values of \([M/H]\) reported in columns 5 and 6 of Table 3 as an estimate of the uncertainty on the spectroscopic \([M/H]\) determinations presently available.

In conclusion, more numerous and much more precise observational data are needed before obtaining a clear evidence about the greater reality of a theoretical scenario in comparison with the other ones.

4 FINAL REMARKS.

To investigate the effect of microscopic diffusion on the RGB Luminosity Function Bump we computed updated stellar models of low mass, metal poor stars, in the canonical scenario and by including helium and heavy elements diffusion, from the Zero Age Main Sequence to the ZAHB phase. As a result, we found that the values of \(\Delta V_{hb}^{bump}\) obtained for stellar models with diffusion are only slightly larger than the canonical ones (the maximum difference is \(\approx 0.06\) mag), both being in general good agreement with the corresponding observational values.

Due to this very small influence of the diffusion on the \(\Delta V_{hb}^{bump}\) values, especially if compared with the typical uncertainties related to the observational determinations of \([M/H]\) and \(\Delta V_{hb}^{bump}\), stellar standard models can be still safely adopted in analyzing this important diagnostic of the inner chemical stratification in low mass stars, at least until much more accurate data for a larger sample of GCs will be available.

However, we point out that at present time the introduction of the Helium and heavy elements diffusion in stellar computations seems to be a "forced stage" in the analysis of helioseismological data or of the surface chemical abundances of white dwarfs and extremely hot horizontal branch stars (Moehler et al. 1995 and reference therein).

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This paper has been produced using the Blackwell Scientific Publications TeX macros.
solid – diffusion

long dash – canonical new scenario

short dash – canonical old scenario
solid – diffusion $t=12\text{Gyr}$
short dash – canonical new scenario
long dash – diffusion – $t=10\text{Gyr}$
dot – diffusion – $t=14\text{Gyr}$