Production of Spin-Two Gauge Bosons

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Abstract

We considered spin-two gauge boson production in the fermion pair annihilation process and calculated the polarized cross sections for each set of helicity orientations of initial and final particles. The angular dependence of these cross sections is compared with the similar annihilation cross sections in QED with two photons in the final state, with two gluons in QCD and W-pair in Electroweak theory.
Our intention in this article is to calculate leading-order differential cross section of spin-two tensor gauge boson production in the fermion pair annihilation process $f \bar{f} \rightarrow TT$ and to analyze the angular dependence of the polarized cross sections for each set of helicity orientations of initial and final particles. The process is illustrated in Fig.1 and receives contribution from three Feynman diagrams shown in Fig.3. These diagrams are similar to the QED and QCD diagrams for the annihilation processes with two photons or two gluons in the final state. The difference between these processes is in the actual expressions for the corresponding interaction vertices. The corresponding vertices for spin-two tensor bosons can be found through the extension of the gauge principle [11]. The extended gauge principle allows to define a gauge invariant Lagrangian $L$ for high-rank tensor gauge fields $A^a_{\mu}$, $A^a_{\mu\nu}$, ... and their cubic and quartic interaction vertices [11, 12, 13]:

$$L = L_{YM} + L_2 + L'_2 + ...$$

Not much is known about physical properties of similar gauge field theories with infinite tower of fundamental fields [1, 2, 3, 6, 7, 8, 9] and in the present article we shall ignore subtle aspects of functional integral quantization procedure because we limited ourselves to calculating only leading-order tree diagrams. Expanding the functional integral in perturbation theory, starting with the free Lagrangian, at $g = 0$, one can see that the theory contains tensor gauge bosons and fermions of different spins with cubic and quartic interaction vertices [11, 12, 13]. Explicit form of these vertices is presented in [13].

Below we shall present the Feynman diagrams for the given process, the expressions for the corresponding vertices and the transition amplitude. The transition amplitude is gauge invariant, because if we take the physical - transverse polarization - wave function for one of the tensor gauge bosons and unphysical - longitudinal polarization - for the second one, the transition amplitude vanishes [14]. That is unphysical - longitudinal polarization states are not produced in the scattering process. We shall calculate the polarized cross sections for each set of helicity orientations of the initial and final particles (16), (17) and shall compare them with the corresponding cross sections for photons and gluons in QED and QCD, as well as with the W-pair production in Electroweak theory.

The annihilation process is illustrated in Fig.1. Working in the center-of-mass frame, we make the following assignments: $p_- = (E_-, \vec{p}_-)$, $p_+ = (E_+, \vec{p}_+)$, $k_1 = (\omega_1, \vec{k}_1)$, $k_2 = (\omega_2, \vec{k}_2)$, where $p_{\pm}$ are momenta of the fermions $f \bar{f}$ and $k_{1,2}$ momenta of the tensor gauge
Figure 1: The annihilation reaction \( f \bar{f} \rightarrow TT \), shown in the center-of-mass frame. The \( p_\pm \) are momenta of the fermions \( f \bar{f} \), and \( k_{1,2} \) are momenta of the tensor gauge bosons \( TT \).

bosons \( TT \). All particles are massless \( p_+^2 = p_-^2 = k_1^2 = k_2^2 = 0 \). In the center-of-mass frame the momenta satisfy the relations \( p_+ = -p_- \), \( k_2 = -k_1 \) and \( E_+ = E_- = \omega_1 = \omega_2 = E \).

The invariant variables of the process are:

\[
\begin{align*}
s &= (p_+ + p_-)^2 = (k_1 + k_2)^2 = 2(p_+ \cdot p_-) = 2(k_1 \cdot k_2) \\
t &= (p_- - k_1)^2 = (p_+ - k_2)^2 = -s \frac{1}{2}(1 - \cos \theta) \\
u &= (p_- - k_2)^2 = (p_+ - k_1)^2 = -s \frac{1}{2}(1 + \cos \theta),
\end{align*}
\]

where \( s = (2E)^2 \) and \( \theta \) is the scattering angle.

The Feynman rules for the Lagrangian (11) can be derived from the functional integral over the fermion fields \( \psi_i, \bar{\psi}_j, \psi_i^\mu, \bar{\psi}_j^\mu, \ldots \) and over the gauge boson fields \( A_\mu, \ A_{\mu\nu}, \ldots \) [11, 12, 13]. The Dirac indices are not shown, the indices of the symmetry group \( G \) are \( i, j = 1, \ldots, d(r) \), where \( d(r) \) is the dimension of the representation \( r \) and \( a = 1, \ldots, d(G) \), where \( d(G) \) is the number of generators of the group \( G \).

In the momentum space the interaction vertex of vector gauge boson \( V \) with two tensor gauge bosons \( T \) - the VTT vertex - has the form [12, 13]

\[
V_{\alpha\beta\gamma\delta}^{abc}(k, p, q) = -gf^{abc}F_{\alpha\beta\gamma\delta}, \tag{2}
\]

*See formulas (62),(65) and (66) in [13].
Figure 2: The interaction vertex for vector gauge boson $V$ and two tensor gauge bosons $T$ - the VTT vertex - in non-Abelian tensor gauge field theory \[13\]. Vector gauge bosons are conventionally drawn as thin wave lines, tensor gauge bosons are thick wave lines. The Lorentz indices $\alpha\alpha'$ and momentum $k$ belong to the first tensor gauge boson, the $\gamma\gamma'$ and momentum $q$ belong to the second tensor gauge boson, and Lorentz index $\beta$ and momentum $p$ belong to the vector gauge boson.

where

$$F_{\alpha\beta\gamma\delta}(k, p, q) = [\eta_{\alpha\beta}(p - k)_{\gamma} + \eta_{\alpha\gamma}(k - q)_{\beta} + \eta_{\beta\gamma}(q - p)_{\alpha}]\eta_{\delta\gamma} - \frac{1}{2} \left\{ + (p - k)_{\gamma}(\eta_{\alpha\gamma}\eta_{\delta\beta} + \eta_{\alpha\delta}\eta_{\beta\gamma}) + (k - q)_{\beta}(\eta_{\alpha\gamma}\eta_{\delta\gamma} + \eta_{\alpha\delta}\eta_{\gamma\gamma}) + (q - p)_{\alpha}(\eta_{\delta\gamma}\eta_{\beta\gamma} + \eta_{\delta\beta}\eta_{\gamma\gamma}) + (p - k)_{\alpha}\eta_{\beta\gamma} + (p - k)_{\gamma}\eta_{\alpha\beta} + (k - q)_{\beta}\eta_{\gamma\gamma} + (k - q)_{\gamma}\eta_{\alpha\gamma} + (q - p)_{\alpha}\eta_{\beta\gamma} + (q - p)_{\gamma}\eta_{\alpha\beta} \right\}.$$ (3)

The Lorentz indices $\alpha\alpha'$ and momentum $k$ belong to the first tensor gauge boson, the $\gamma\gamma'$ and momentum $q$ belong to the second tensor gauge boson, and Lorentz index $\beta$ and momentum $p$ belong to the vector gauge boson. The vertex is shown in Fig.2. Vector gauge bosons are conventionally drawn as thin wave lines, tensor gauge bosons are thick wave lines.

It is convenient to write the differential cross section in the center-of-mass frame with tensor boson produced into the solid angle $d\Omega$ as

$$d\sigma = \frac{1}{2s}|M|^2 \frac{1}{32\pi^2} d\Omega,$$ (4)

where the final-state density is $d\Phi = \frac{1}{32\pi^2} d\Omega$. 
Figure 3: Diagrams contributing to fermion-antifermion annihilation to two tensor gauge bosons. Dirac fermions are conventionally drawn as thin solid lines, and Rarita-Schwinger spin-vector fermions by thick solid lines.

We shall calculate the polarized cross sections for this reaction, to lowest order in $\alpha = g^2/4\pi$. The lowest-order Feynman diagrams contributing to fermion-antifermion annihilation into a pair of tensor gauge bosons are shown in Fig. 3. In order $g^2$, there are three diagrams. Dirac fermions $\psi$ are conventionally drawn as thin solid lines, and Rarita-Schwinger spin-vector fermions $\psi^{\mu}$ by thick solid lines. These diagrams are similar to the QCD diagrams for fermion-antifermion annihilation into a pair of vector gauge bosons. The difference between these processes is in the actual expressions for the corresponding interaction vertices \cite{11, 12, 13}. The probability amplitude of the process can be written in the form

$$M_{\mu\alpha\nu\beta}e^*_{\mu\alpha}(k_1)e^*_{\nu\beta}(k_2) = (5)$$

where $u(p_-)$ is the wave function of spin $1/2$ fermion and $v(p_+)$ of antifermion, the final tensor gauge bosons wave functions are $e^*_{\mu\alpha}(k_1)$ and $e^*_{\nu\beta}(k_2)$. The Dirac and symmetry group indices are not shown.

This amplitude is gauge invariant, that is, if we take the physical - transverse polarization - wave function $e_T$ for one of the tensor gauge bosons and longitudinal polarization for the second one $e_L$, the transition amplitude vanishes: $M e_T e_L = 0$ \cite{14}. This Ward identity expresses the fact that the unphysical - longitudinal polarization - states are not produced in the scattering process.

Indeed, considering the last term in (5) and taking the polarization tensor $e^*_{\nu\beta}(k_2)$ to be longitudinal $e^*_{\nu\beta}(k_2) = k_{2\nu}\xi_\beta + k_{2\beta}\xi_{\nu}$, the polarization tensor $e^*_{\mu\beta}(k_1)$ to be transversal
and then using relations (8) for the wave function $e_{\mu\beta}^*(k_1)$, we shall get

$$i f^{abc} e^\mu \bar{v}(p_+) \gamma^\rho u(p_-) \frac{1}{4} e^*_{\rho\alpha}(k_1) \xi^\alpha. \quad (6)$$

Now let us consider the first two terms in (5). Taking again the polarization tensors $e_{\nu\beta}^*(k_2)$ to be longitudinal and using relations (8) for the wave function $e_{\mu\beta}^*(k_1)$ we shall get

$$\frac{1}{4} \bar{v}(p_+) \{-t^a t^b \gamma^\mu + t^b t^a \gamma^\mu\} u(p_-) e_{\mu\alpha}^*(k_1) g^\alpha\beta \xi^\beta =$$

$$= - \frac{1}{4} i f^{abc} e^\nu \bar{v}(p_+) \gamma^\mu u(p_-) e^*_{\nu\alpha}(k_1) \xi^\alpha. \quad (7)$$

This term precisely cancels the contribution coming from the last term of the amplitude (6). Thus the cross term matrix element between transverse and longitudinal polarizations vanishes: $M_{e T e L} = 0$. Our intention now is to calculate the physical matrix element $M_{e T e T}$ for each set of helicity orientations of initial and final particles.

Using the explicit form of the vertex operator $F^{\mu\nu\rho\sigma} (2), (3)$ and the orthogonality properties of the tensor gauge boson wave functions

$$k_1^\mu e_{\mu\alpha}(k_1) = k_2^\alpha e_{\mu\alpha}(k_1) = k_2^\sigma e_{\mu\alpha}(k_1) = k_1^\alpha e_{\mu\alpha}(k_1) = 0, \quad (8)$$

where the last relations follow from the fact that $\vec{k}_1 \parallel \vec{k}_2$ in the process of Fig.1 we shall get

$$M^{\mu\nu\rho\sigma} e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2) = (ig)^2 \bar{v}(p_+ \right. \left\{ \gamma^\mu \bar{\gamma}^\nu + \gamma^\nu \bar{\gamma}^\mu \right\} u(p_-) e_{\mu\alpha}^*(k_1) e_{\nu\beta}^*(k_2). \quad (9)$$

As the next step we shall calculate the above matrix element in the helicity basis for initial fermions and final tensor gauge bosons. This calculation of polarized cross sections is very similar to the one in QED [10]. The right- and left-handed spinors wave functions are:

$$u^R(p_-) = \sqrt{2E} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad u^L(p_+) = \sqrt{2E} \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} \quad (10)$$
and the tensor gauge boson wave functions for circular polarizations along the $k_1$ direction are

$$
\epsilon^{\mu\alpha}_R(k_1) = \frac{1}{2} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & \cos^2 \theta & i \cos \theta & -\cos \theta \sin \theta \\
i \cos \theta & -1 & -i \sin \theta & \\
0 & -\cos \theta \sin \theta & i \sin \theta & \sin^2 \theta
\end{pmatrix},
$$

and

$$
\epsilon^{\mu\alpha}_L(k_1) = \frac{1}{2} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & \cos^2 \theta & -i \cos \theta & -\cos \theta \sin \theta \\
i \cos \theta & -1 & i \sin \theta & \\
0 & -\cos \theta \sin \theta & -i \sin \theta & \sin^2 \theta
\end{pmatrix}.
$$

It is easy to check that the wave functions \((11)\) are orthonormal

$$
\epsilon^{*\mu\alpha}_R(k_1)\epsilon_L(k_1)_{\alpha\nu} = 0, \quad \epsilon^{*\mu\alpha}_R(k_1)\epsilon_R(k_1)_{\mu\alpha} = 1, \quad \epsilon^{*\mu\alpha}_L(k_1)\epsilon_L(k_1)_{\mu\alpha} = 1
$$

and fulfil the equations \((8)\). The helicity states for the second gauge boson are $\epsilon^{\mu\nu}_R(k_2) = \epsilon^{\mu\nu}_L(k_2) = \epsilon^{\mu\nu}_R(k_1)$, where $k_1^\mu = (E, E \sin \theta, 0, E \cos \theta)$ and $k_2^\mu = (E, -E \sin \theta, 0, -E \cos \theta)$.

Now we can calculate all sixteen matrix elements between states of definite helicities. Let us start with $f_R f_L \rightarrow T_R T_R$. The scattering amplitude \((9)\) for these particular helicities $M_{R,L}^{\mu_1\mu_2} \epsilon^{*\mu_1}_R(k_1)\epsilon^{*\mu_2}_R(k_2)$ contains three terms. By plugging explicit expressions for the helicity wave functions \((10)\), \((11)\) into the matrix element \((9)\) we can find the first term

$$(ig)^2 \bar{v}_L(p_+) \left\{ \gamma^\mu t^a \frac{1}{p_- - k_2^\mu} t^b \gamma^\nu \right\} u^R(p_-) \epsilon^{*\mu}_R(k_1)\epsilon^{*\nu}_R(k_2) = \frac{(ig)^2}{4} t^a t^b \sin \theta,$$

then the second one

$$(ig)^2 \bar{v}_L(p_+) \left\{ \gamma^\nu t^b \frac{1}{p_- - k_1^\nu} t^a \gamma^\mu \right\} u^R(p_-) \epsilon^{*\mu}_R(k_1)\epsilon^{*\nu}_R(k_2) = -\frac{(ig)^2}{4} t^b t^a \sin \theta,$$

and finally the third one

$$(ig)^2 \bar{v}_L(p_+) \{ i f^{abc} t_c \frac{k_2 - k_1}{k_3^2} (g^{\mu\nu} g^{\alpha\beta} - \frac{1}{2} g^{\mu\beta} g^{\nu\alpha}) \} u^R(p_-) \epsilon^{*\mu}_R(k_1)\epsilon^{*\nu}_R(k_2) = -i \frac{(ig)^2}{2} f^{abc} t_c \sin \theta,$$

so that all together they will give

$$M_{R,L}^{\mu_1\mu_2} \epsilon^{*\mu_1}_R(k_1)\epsilon^{*\mu_2}_R(k_2) = \frac{(ig)^2}{4} \left[ [t^a, t^b] - 2i f^{abc} t_c \right] \sin \theta = -i \frac{(ig)^2}{4} f^{abc} t_c \sin \theta. \quad (12)$$

To compute the cross section, we must square the matrix element \((12)\) and then average over the symmetries of the initial fermions and sum over the symmetries of the final tensor gauge bosons. This gives

$$\sum |M|^2_{R,L \rightarrow RR} = \frac{g^4}{16d^2(r)} tr(f^{abc} f^{abld} t^d) \sin^2 \theta = \frac{g^4 C_2(r) C_2(G)}{16 d(r)} \sin^2 \theta; \quad (13)$$
where the invariant operator $C_2$ is defined by the equation $t^a t^b = C_2$. Similarly, using the helicity wave functions (10) and (11), we can calculate the amplitude $f_R f_L \rightarrow T_L T_L$. This gives

$$
\sum |\mathcal{M}|_{R L \rightarrow LL}^2 = \frac{g^4}{16d^2(r)} t^r (f^{abc} f^{abd} t^c t^d) \sin^2 \theta = \frac{g^4}{16} \frac{C_2(r) C_2(G)}{d(r)} \sin^2 \theta.
$$

(14)

The amplitude $f_R f_L \rightarrow T_R T_L$ vanishes because the common factor to all three pieces of this amplitude - $g^2 \epsilon^\mu \epsilon_\nu (k_1) \epsilon^{* \mu \nu} (k_2)$ - is equal to zero. Thus only four amplitudes out of sixteen are nonzero:

$$
f_R f_L \rightarrow T_R T_R, \quad f_R f_L \rightarrow T_L T_L, \quad f_L f_R \rightarrow T_R T_R, \quad f_L f_R \rightarrow T_L T_L.
$$

(15)

From this analysis it follows that the total spin angular momentum of the final state is one unit less than that of the initial state, therefore a unit of spin angular momentum is converted to the orbital angular momentum and the final state is a P-wave.

We can calculate now the leading-order polarized cross sections for the tensor gauge boson production in the annihilation process. Plugging matrix elements (13) into our general cross-section formula in the center-of-mass frame (4) yields:

$$
d\sigma_{f_R f_L \rightarrow T_R T_R} = \frac{g^4}{16} \frac{C_2(r) C_2(G)}{d(r)} \sin^2 \theta \frac{1}{2s} \frac{1}{32\pi^2} d\Omega = \frac{\alpha^2}{s} \frac{C_2(r) C_2(G)}{64d(r)} \sin^2 \theta \ d\Omega,
$$

(16)

where $\alpha = \frac{\rho^2}{4\pi}$. For the rest of the helicities we shall get

$$
d\sigma_{f_R f_L \rightarrow T_R T_R} = d\sigma_{f_R f_L \rightarrow T_L T_L} = d\sigma_{f_L f_R \rightarrow T_R T_R} = d\sigma_{f_L f_R \rightarrow T_L T_L},
$$

(17)

where for the $SU(N)$ group we have $\frac{C_2(r) C_2(G)}{64d(r)} = \frac{(N^2-1)}{128N}$. Adding up all sixteen amplitudes and dividing by four, to average over the initial particle spins, we recover the unpolarized cross section (14).

This cross section should be compared with the analogous annihilation cross sections in QED and QCD. Indeed, let us compare this result with the electron-positron annihilation into two transversal photons. The $e^+ e^- \rightarrow \gamma \gamma$ annihilation cross section (15) in the high-energy limit is

$$
d\sigma_{\gamma \gamma} = \frac{\alpha^2}{s} \frac{1 + \cos^2 \theta}{\sin^2 \theta} d\Omega
$$

(18)
except very small angles of order $m_w/E$. The cross section has a minimum at $\theta = \pi/2$ and then increases for small angles \cite{16}. The quark pair annihilation cross section into two transversal gluons $q\bar{q} \rightarrow gg$ in the leading order of the strong coupling $\alpha_s$ is

$$d\sigma_{gg} = \frac{\alpha_s^2}{s} \frac{d\sigma}{d(r)} \left[ \frac{1 + \cos^2 \theta}{\sin^2 \theta} - \frac{C_2(G)}{4C_2(r)} (1 + \cos^2 \theta) \right] d\Omega,$$  \hspace{1cm} (19)$$

and also has a minimum at $\theta = \pi/2$ and increases for small scattering angles \cite{17}. The production cross section of spin-two gauge bosons \cite{16}, \cite{17} shows dramatically different behaviour - $\sin^2 \theta$ - with its maximum at $\theta = \pi/2$ and decrease for small angles.

It is also instructive to compare this result with the angular dependence of the $W$-pair production in Electroweak theory. The high energy production of longitudinal gauge bosons is \cite{18}

$$d\sigma_{e^+e^-\rightarrow W^0_W^-} = \frac{\alpha^2}{s} \left[ \frac{1}{256} \sin^4 \theta_w \cos^4 \theta_w \right] \sin^2 \theta d\Omega,$$ \hspace{1cm} (20)$$

where $\cos \theta_w = \frac{m_w}{m_z}$ and it is similar to the spin-two transversal gauge boson production \cite{16}. One can only speculate that at high enough energies, may be at LHS energies and above the threshold, we may observe the standard spin-one gauge bosons together with new spin-two gauge bosons \cite{12}. To predict the threshold energy one should first construct a massive theory and even in that case the corresponding Yukawa couplings most probably will be unknown.

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