Explaining anomalies of $B$-physics, muon $g − 2$ and $W$ mass in $R$-parity violating MSSM with seesaw mechanism

Min-Di Zheng$^{1,a}$, Feng-Zhi Chen$^{1,2,b}$, Hong-Hao Zhang$^{1,c}$

1 School of Physics, Sun Yat-Sen University, Guangzhou 510275, China
2 Key Laboratory of Quark and Lepton Physics (MOE), Central China Normal University, Wuhan 430079, China

Abstract The recent experimental results including $R_K^{(s)}$, $R_{D^{(*)}}$, $(g − 2)_\mu$ and $W$ mass show deviations from the standard model (SM) predictions, implying the clues of new physics (NP). In this work, we investigate the explanations of these anomalies in the $R$-parity violating minimal supersymmetric standard model (RPV-MSSM) extended with the inverse seesaw mechanism. The non-unitarity extent $\eta_{ee}$ and the loop corrections from the interaction $\lambda$ are utilized to raise the prediction of $W$ mass through muon decays. We also find that the interaction $\lambda' \hat{L} \hat{Q} \hat{D}$ involved with right-handed (RH)/singlet (s)neutrinos can explain the $R_K^{(s)}$ and $R_{D^{(*)}}$ anomalies simultaneously when considering nonzero $\lambda'_{1jk}$. For nonzero $\lambda'_{2jk}$, this model fulfills the whole $b \to s \ell^+ \ell^−$ fit but cannot be accordant with $R_{D^{(*)}}$ measurements. The explanations in both cases are also favored by $(g − 2)_\mu$ data, neutrino oscillation data and the relevant constraints we scrutinized. Furthermore, this model framework can be tested in future experiments covering, e.g., the predicted lepton flavor violations (LFV) at Belle II and the Future Circular Collider with $e^+ e^−$ beams (FCC-ee), as well as the heavy neutrinos at future colliders.

1 Introduction

Recently, the experimental measurements implying the lepton flavor universality violation (LFUV) within the semileptonic decays of $B$-meson show striking results. The measurement of the observable $R_K = \frac{B(\overline{B} \to K \mu^+ \mu^-)}{B(\overline{B} \to K e^+ e^-)}$, reported by the LHCb collaboration [1] with the value $R_K = 0.846^{+0.042+0.013}_{−0.039−0.012}$ in $q^2$ bin $[1.1, 6]$ GeV$^2$, deviates from the SM prediction by 3.1$\sigma$. The relevant $R_K^{(s)}$ measurements show deviations larger than $2\sigma$ [2] from the SM. Besides, some discrepancies larger than $2\sigma$ are also reported in several measurements of $b \to s \mu^+ \mu^−$ processes, including $P_\mu^s$ [3], the branching ratios $B(B_s \to \phi \mu^+ \mu^-)$ [4], $B(B_s \to \mu^+ \mu^-)$ [5–7], etc. As for the charged-current decays of $B$-meson, the combined result of the experimental observables $R_{D^{(*)}} = \frac{B(B \to D^{(*)} \tau \nu)}{B(B \to D^{(*)} \ell \nu)}$ ($\ell = e, \mu$), given by the Heavy Flavor Averaging Group (HFLAV), shows currently $3.3\sigma$ away from the SM prediction with a relative correlation $−0.38$ [8,9]. Thus, the $b \to s \ell^+ \ell^−$ anomalies, especially the striking $R_K$ deviation along with $R_{D^{(*)}}$ deviation, all imply the NP with LFUV in $B$-physics.

Apart from the $B$-physics anomalies, clues of NP can also be probed in the precision measurements. The latest result of the muon anomalous magnetic moment $a_\mu = (g − 2)_\mu$ in the E989 experiment is reported by the Fermilab National Accelerator Laboratory (FNAL) as $a_\mu^{\text{FNAL}} = 116592040(54) \times 10^{-11}$ [10], which agrees with the previous result from the E821 experiment at Brookhaven National Laboratory (BNL), $a_\mu^{\text{BNL}} = 116592080(63) \times 10^{-11}$ [11], but is $3.3\sigma$ away from the SM prediction $a_\mu^{\text{SM}} = 116591804(43) \times 10^{-11}$ [12]. Combining the two experimental results yields $4.2\sigma$ deviation from the SM. $^{1}$ Intriguingly and very recently, another anomaly has been revealed in high precision measurement on the $W$-boson mass by the Collider Detector at Fermilab (CDF) collaboration at Tevatron. The measured value is $m_W^{\text{CDF-II}} = 80.4335 \pm 0.0094$ GeV [22], which shows $7\sigma$ deviation from the SM prediction $m_W^{\text{SM}} = 80.357 \pm 0.006$ GeV [23]. This measurement, if confirmed by other measurements, would constitute the strongest evidence for new physics beyond the SM.

$^1$ Recent lattice calculations for the hadronic vacuum polarization [13–15] induce a weaker tension with this combined measurement result, than the preceding review of various SM predictions [12]. However, these lattice results show a tension with the $e^+ e^- \to$ hadrons data [13–16]. Readers can see Refs. [17–21] for relevant discussions.
experiments in the future, will profoundly change the trend of NP searches.

Combining this astonishing $m_W^{CDF-II}$ with the $B$-physics anomalies and $(g - 2)_\mu$ data, particular NP features are competitive for explanations. In this work, we focus on the supersymmetric (SUSY) models extended by the $R$-parity violation. As one knows, this framework with the superpotential term $\lambda' \hat{L} \hat{Q} \hat{D}$ among the RPV terms, can provide explanations for $B$-physics anomalies in the $b \rightarrow s \ell^+ \ell^-$ or and the $b \rightarrow c \tau \nu$ processes (see, e.g., Refs. [24–27]). In Ref. [28], two of us provide RPV-MSSM extended by the inverse see-saw mechanism (named RPV-MSSMIS) firstly to explain the $b \rightarrow s \ell^+ \ell^-$ anomalies through the RPV-interactions including RH/singlet $(s)$neutrinos and explain the $(g - 2)_\mu$ data through the chirality-flipping. The explanation also fulfills the recent neutrino-oscillation data. Here we ask this question: can this model further accommodate the $R_{D(\mu)}$ measurements and $m_W^{CDF-II}$? The simultaneous explanations of the $R_{K(\nu)}$ and $R_{D(\mu)}$ anomalies in RPV-MSSM have been studied recently in Refs. [26,27,29,30], while with the experiment data updated, it is straightforward to reconsider this topic in RPV-MSSMIS. Besides, this model has several distinctive effects on some observables, e.g., the branching fractions.

2 The anomaly explanations in RPV-MSSMIS

In this section, we begin to investigate the NP effects on the flavor anomalies, i.e., $b \rightarrow s \ell^+ \ell^-$ anomalies, $R_{D(\mu)}$ deviations, and the $(g - 2)_\mu$ problem, as well as the new measurement of $m_W^{CDF-II}$, in the framework of RPV-MSSMIS.

2.1 RPV-MSSM framework

First we make some introductions to RPV-MSSMIS [28]. The superpotential and the soft SUSY breaking Lagrangian are respectively given by

\[
\mathcal{W} = \mathcal{W}_{\text{MSSM}} + Y^{ij}_{L} \hat{R}_{i} \hat{L}_{j} \hat{u}_{R} + M^{ij}_{R} \hat{R}_{i} \hat{S}_{j} + \frac{1}{2} \mu^{ij}_{S} \hat{S}_{i} \hat{S}_{j} + \lambda^{i}_{ijk} \hat{L}_{i} \hat{Q}_{j} \hat{D}_{k},
\]

\[
-\mathcal{L}_{\text{soft}} = -\mathcal{L}_{\text{MSSM}}^{\text{soft}} + (m^{2}_{R})_{ij} \hat{R}_{i}^{*} \hat{R}_{j} + (m^{2}_{S})_{ij} \hat{S}_{i}^{*} \hat{S}_{j} + (A_{ij})_{\nu} \hat{L}_{i} \hat{H}_{u} + B^{ij}_{\mu S} \hat{R}_{i}^{*} \hat{S}_{j} + \frac{1}{2} B^{ij}_{\mu S} \hat{S}_{i}^{*} \hat{S}_{j},
\]

where the MSSM parts, $\mathcal{W}_{\text{MSSM}}$ and $\mathcal{L}_{\text{soft}}^{\text{MSSM}}$, can be referred to Refs [34,35], and the neutrino sector consists of pairs of SM singlet superfields, $\hat{R}_{i}$ and $\hat{S}_{j}$. The generation indices $i, j, k = 1, 2, 3$ while the colour ones are omitted, and all the repeated indices are deaulted to be summed over unless otherwise stated. Besides, squarks/sleptons are denoted by $\tilde{c}^{s}$ here. The neutral scalar fields of the two Higgs doublet superfields, $\hat{H}_{u} = (\hat{H}_{u}^{+}, \hat{H}_{u}^{0})^{T}$ and $\hat{H}_{d} = (\hat{H}_{d}^{+}, \hat{H}_{d}^{-})^{T}$, acquire the non-zero vacuum expectation value, i.e., $\langle \hat{H}_{u}^{0} \rangle = v_{u}$ and $\langle \hat{H}_{d}^{0} \rangle = v_{d}$, respectively, and their mixing is expressed by $\tan \beta = v_{d}/v_{u}$.

The additional neutrino sector in the superpotential $\mathcal{W}$ provides the neutrino mass spectrum at the tree level, and the $9 \times 9$ mass matrix in the basis $(v, R, S)$ is

\[
\mathcal{M}_{\nu} = \begin{pmatrix}
0 & m^{T}_{D} & 0 \\
m^{0}_{D} & 0 & m_{R} \\
m^{0}_{R} & m_{R}^{T} & \mu_{S}
\end{pmatrix}.
\]

where the Dirac mass matrix $m_{D} = \frac{1}{\sqrt{2}} v_{u} Y^{T}_{u}$. Then $\mathcal{M}_{\nu}$ can be diagonalized through $\mathcal{M}_{\nu}^{\text{diag}} = V_{\nu} \mathcal{M}_{\nu}^{\text{diag}} V^{T}_{\nu}$. As to the sneutrino mass square matrix in the basis $(\nu^{T}_{L}(R), \tilde{R}^{T}(R), \tilde{S}^{T}(R))$, it is expressed as

\[
\mathcal{M}^{2}_{\nu_{L}(R)} = \begin{pmatrix}
\frac{m^{2}_{L}}{m^{0}_{L}} & (A_{\nu} - \mu \cot \beta) m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D}
\end{pmatrix}
\approx \begin{pmatrix}
\frac{m^{2}_{L}}{m^{0}_{L}} & (A_{\nu} - \mu \cot \beta) m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D}
\end{pmatrix}
\end{pmatrix}.
\]

\[
\mathcal{M}_{\nu_{L}(R)}^{2} = \begin{pmatrix}
\frac{m^{2}_{L}}{m^{0}_{L}} & (A_{\nu} - \mu \cot \beta) m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D}
\end{pmatrix}
\]

\[
\approx \begin{pmatrix}
\frac{m^{2}_{L}}{m^{0}_{L}} & (A_{\nu} - \mu \cot \beta) m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D}
\end{pmatrix}
\end{pmatrix}.
\]

\[
\mathcal{M}_{\nu_{L}(R)}^{2} = \begin{pmatrix}
\frac{m^{2}_{L}}{m^{0}_{L}} & (A_{\nu} - \mu \cot \beta) m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D}
\end{pmatrix}
\]

\[
\approx \begin{pmatrix}
\frac{m^{2}_{L}}{m^{0}_{L}} & (A_{\nu} - \mu \cot \beta) m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D} \\
M^{0}_{R} m^{0}_{D} & m^{2}_{R} + M_{R} M^{T}_{R} + m_{D} m^{T}_{D}
\end{pmatrix}
\]
where “±” as well as “R(I)”, denotes the CP-even (odd), and the mass square $m_L^2 = m_R^2 + \frac{1}{2}m_Z^2\cos2\beta + m_D m_{D'}^T$ is regarded as a whole input with $m_D^2$ being the soft mass square of $\tilde{L}$. Given that the value of $\mu_S$ is tiny and $B_{\mu_S}$ can be also relatively small [36], we have obtained the approximate result in Eq. (2.3), which induces the CP-even and CP-odd masses to be nearly the same. With this approximation, the mixing matrices $\tilde{\Sigma}^R(\tilde{\Sigma}^I)$, which diagonalize sneutrino mass square matrices through $\tilde{\Sigma}^R(\tilde{\Sigma}^I)\mathcal{M}^2_{\tilde{E}^c,i}(R^I) = (\mathcal{M}^2_{\tilde{E}^c,i}(R^I))^{\text{diag}}$, can also be regarded as the same whether CP is even or odd. In the rest of the paper, we can replace $\tilde{\Sigma}^R$ and the physical mass $m_{\tilde{\nu}_R}$ with the corresponding notations of $\tilde{\Sigma}^I$ and $m_{\tilde{\nu}_I}$.

Afterwards, we introduce the trilinear RPV interaction in this model. With the superpotential term $\lambda'_{i,j,k}\tilde{L}_i\tilde{Q}_j\tilde{D}_k$, the relevant Lagrangian in the context of mass eigenstates for the down-type quarks and the charged leptons is given by

$$L_{\text{LQD}} = \lambda_{ijk}^{\tau\tau} \bar{e}_{iv} \bar{d}_{Rk} d_{Lj} + \lambda_{ijk}^{\nu} (\bar{d}_{Ri} e_{v} + \bar{d}_{Ri} \nu_{v} d_{Lj}) - \lambda_{ijk}^{\ell} (L_i \bar{d}_{Rk} u_{Lj} + \bar{u}_{Li} \bar{d}_{Rk} l_{Lj} + \bar{d}_{Ri} \nu_{v} l_{Lj}^*) + \text{h.c.},$$

where “e” indicates the charge conjugated fermions, and the fields $\bar{e}_{L}, \bar{e}_{v}, u_{L}$ (aligned with $\bar{u}_{L}$) in the flavor basis have been rotated into mass eigenstates by the mixing matrices $\tilde{\Sigma}$, $\Sigma$, and $K$, respectively. Concretely, the indices $v = 1, 2, \ldots 9$ denote the generation of the physical (s)neutrinos, and the three $\lambda'$ couplings are deduced as $\lambda'_{\tau\nu} = \lambda'_{ijk}^{\tau\nu} \bar{\nu}_{vi} \nu_{vi}$, $\lambda'_{\tau\ell} = \lambda'_{ijk}^{\tau\ell} \nu_{vi}$, and $\lambda'_{\ell\ell} = \lambda'_{ijk}^{\ell\ell} \nu_{vi}^* \nu_{vi}$. In the following, the interaction (diagram) involving these $\lambda'$ couplings is called the $\lambda'$ interaction (diagram).

By the end of this section, we mention the chargino mass matrix from the MSSM sector, which is [35]

$$\mathcal{M}_{\chi^\pm} = \begin{pmatrix} M_2 & (\mu - \frac{\tilde{m}_u}{\sqrt{2 \sin \theta_W}}) \\ \mu & \frac{\tilde{m}_u}{\sqrt{2 \sin \theta_W}} \end{pmatrix},$$

where $M_2$ being the wino mass, $\mu$ being the Higgsino mass in the flavor basis, and $\theta_W$ being the Weinberg angle. The mixing matrices $\mathcal{V}$ and $\mathcal{U}$ diagonalize $\mathcal{M}_{\chi^\pm}$ by $U^* \mathcal{M}_{\chi^\pm} V^T = \mathcal{M}_{\chi^\pm}^{\text{diag}}$. For a review on the neutralino matrix, readers are also referred to Ref. [35].

2.2 $b \rightarrow s\ell^+\ell^-$ anomalies

To study the NP effects on the $b \rightarrow s\ell^+\ell^-$ process, the relevant Lagrangian of the low energy effective field theory can be represented as

$$\mathcal{L}_{\text{eff}}^{b\ell\ell} = \frac{4G_F}{\sqrt{2}} \eta_1 \sum_i \mathcal{O}_i \mathcal{O}_i + \text{h.c.},$$

where $\eta_1 = K_{b\ell} K_{s\ell}^*$ is the Cabibbo–Kobayashi–Maskawa (CKM) factor. The most favored operators to explain the $b \rightarrow s\ell^+\ell^-$ anomalies are

$$\mathcal{O}_0 = \frac{e^2}{16\pi^2} (\tilde{\delta} \gamma_\mu P_L b)(\tilde{\ell} \gamma^\mu \ell),$$
$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\tilde{\delta} \gamma_\mu P_L b)(\tilde{\ell} \gamma^\mu \gamma_5 \ell),$$

where $P_L = (1 - \gamma_5)/2$ is the left-handed (LH) chirality projector, and the Wilson coefficients $C_{9(10)} = C_{9(10)}^\text{SM} + C_{9(10)}^\text{NP}$. Then with the results of the model-independent global fit [37–51], RPV-MSSMIS can provide box contributions that are restricted to the $\mu(ee)$ channel, i.e., $C_{9(10)}^\text{NP} = C_{9(10)}^\text{NP} |_{\mu(ee)} < 0$ or $C_{9(10)}^\text{NP} = C_{9(10)}^\text{NP} |_{\mu(ee)} > 0$ to explain the $R_{b\ell\ell}$ anomaly. Especially, the case $C_{9(10)}^\text{NP} = C_{9(10)}^\text{NP} |_{\mu(ee)} < 0$ can be further in accord with the deviations of $b \rightarrow s\mu^+\mu^-$ measurements from the SM predictions, e.g., measurements of $B_s \rightarrow \mu^+\mu^-$ [5–7] and $B_s \rightarrow \phi\mu^+\mu^-$ [4,52].

In RPV-MSSMIS, we set sleptons and winos with masses of several $10^2$ GeV while the masses of all colored spinartides as well as the heavy neutrinos are around TeV or above, and all the model parameters are set at the scale $\mu_{\text{NP}} = 0.5$ TeV. To eliminate the unfavored contributions to $C_{9(10)}^\text{NP}$, we replace $P_L$ with $P_R$ in $C_{9(10)}^\text{NP}$ which can emerge at the tree level, we assume the coupling $\lambda'_{ijk}$ is non-negligible only for the single value $k$ at $\mu_{\text{NP}}$ scale [28]. Then the dominant contributions to $C_{9\ell} = -C_{10\ell}$ in RPV-MSSMIS are given by

$$C_{9\ell} = \lambda_{ijk}^{\tau\nu} \bar{\nu}_{vi} \lambda_{ijk}^{\tau\nu} (g_2 V_{m_1} \bar{\nu}_{vi}^T - V_{m_2} V_{m_1} \bar{\nu}_{vi}^T - V_{m_2} Y_{\nu}^T D_2 [m_{\tilde{\nu}_v}^2, m_{\tilde{\nu}_v}^2, m_{\tilde{\nu}_v}^2, m_{\tilde{\nu}_v}^2]),$$

where the formula of Passarino-Veltman function [53] $D_2$ is collected in appendix A, and the coupling $Y_{\nu}^{T} \equiv (Y_{\nu})_{ij} \bar{\nu}_{vi}^T$. One can see that the light sneutrinos and winos will help provide considerable NP effects. The Wilson coefficient of Eq. (2.8) is proportional to the product $\lambda_{2(1)}^{\tau\nu} \lambda_{2(1)}^{\tau\nu}$, which is related to the $\mu(ee)$-channel contribution, assuming no flavor transitions within sneutrino sectors. In appendix A, we provide the whole list of formulas from the scrutinized one-loop box diagrams of the $b \rightarrow s\ell^+\ell^-$ process, which are adopted in the numerical calculations. Under the assumption of single value $k$, only the LH-quark-vector-current contributions $C_{9(10)}^\text{NP}(\mu_{\text{NP}})$ exist, and the RH-quark-vector-current contributions $C_{9(10)}^\text{NP}(\mu_{\text{NP}})$ vanish. Due to the approximate conservation of (axial-)vector currents, there is $C_{9(10)}^\text{NP}(\mu_B) = C_{9(10)}^\text{NP}(\mu_{\text{NP}})$, and $C_{9(10)}^\text{NP}(\mu_B)$ still vanish when the scale runs down to $\mu_B = m_b$ through QCD renormalization. Then we can adopt the results at $\mu_B$ from the model-independent global fit in Ref. [43] to constrain the model inputs. The fit results show $C_{9\elle} = \ldots$
\(-C_{10}^{NP} = 0.37 \pm 0.10\) as the best fit for the \(R_{K^{(*)}}\) explanations through NP in the \(ee\) channel. As for the \(\mu\mu\) channel, \(c_{9}^{NP} = -C_{9}^{NP} = -0.35 \pm 0.08\) and \(C_{10}^{NP} = -C_{10}^{NP} = -0.39 \pm 0.07\) are utilized to explain \(R_{K^{(*)}}\) and the whole \(b \to s \ell^+ \ell^-\) anomaly, respectively. In this work, we restrict \(k = 3\) for a benchmark and consider one of \((\lambda_{1j3}', \lambda_{2j3}')\) nonzero at a time, i.e., \(\lambda_{2(1)3}' = 0\) for Case A(B).

2.3 \(R_{D^{(*)}}\) anomalies

Next, we turn to \(R_{D^{(*)}}\), implying the LFUV anomalies in the charged current. For the generic charged current process \(d_j \to u_n l_i v\), the effective Lagrangian is

\[
\mathcal{L}^{d_{ij}lv}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} K_{nj}(\nu_i^T) \tilde{\nu}_i n_j \gamma_{\mu} P_L d_i \tilde{\nu}_j \gamma^\mu P_L v_i + \text{h.c.,} \tag{2.9}
\]

where the first term in the bracket following the CKM element \(K_{nj}\) gives the SM contribution combined with the neutrino-generation mixing, and the second term is related to the \(\lambda'\) interactions with the function

\[
C_{njli'} = \frac{\tilde{\lambda}^{\nu}_{li3} \lambda'_{ij3}}{4\sqrt{2}G_F K_{nj} m_{l_i}^2}. \tag{2.10}
\]

It is useful to define the ratio

\[
R_{njl} = \frac{B(d_j \to u_n l_i v)_{\text{SM}+\text{NP}}}{B(d_j \to u_n l_i v)_{\text{SM}}}
= \sum_{i=1}^{3} \left| K_{nj} \right|^2 \left| V_{lj}^T + V_{lj}^T C_{njli'} \right|^2
= \sum_{i,i'=1}^{3} \left| V_{lj}^T \right|^2 \left| \delta_{li'} + C_{njli'} \right|^2 \approx \sum_{i=1}^{3} \left| \delta_{li'} + C_{njli'} \right|^2.
\]

Thus, it approximates the ratio in ordinary RPV-MSSM (see Eq. (24) in Ref. [29]) under nearly the unitarity bound of \(\gamma_{3x3}\). Then we get the ratio,

\[
\frac{R_{D}}{R_{D^{SM}}} = \frac{R_{D^{*}}}{R_{D^{SM}}^{*}} = \frac{R_{233}}{R_{232} + R_{231}} \tag{2.11}
\]

\[
= \left\{ \begin{array}{ll}
\frac{\tilde{\lambda}^{\nu}_{123} \lambda'_{133}}{\lambda_{123}^{\nu} \lambda_{133}^{\nu}}^2 + \frac{\sqrt{2}G_F K_{23} m_{l_2}^2}{\lambda_{123}^{\nu} \lambda_{133}^{\nu}} + \frac{\tilde{\lambda}^{\nu}_{123} \lambda'_{133}}{\lambda_{123}^{\nu} \lambda_{133}^{\nu}}^2
& \text{in Case A}\,

\frac{\tilde{\lambda}^{\nu}_{123} \lambda'_{133}}{\lambda_{123}^{\nu} \lambda_{133}^{\nu}}^2 + \frac{\sqrt{2}G_F K_{23} m_{l_2}^2}{\lambda_{123}^{\nu} \lambda_{133}^{\nu}} + \frac{\tilde{\lambda}^{\nu}_{123} \lambda'_{133}}{\lambda_{123}^{\nu} \lambda_{133}^{\nu}}^2
& \text{in Case B},
\end{array} \right.
\]

which is named as \(R_{D^{NP/SM}}^{NP/SM}\) in this work. We utilize the \(R_{D^{(*)}}\) world average with the so-called correlation \(\rho_{D^{(*)}}\), which denotes the \(B \to D^{(*)} \ell^{\pm} \nu_{\ell}\) correlation structure across \(R_{D}\) and \(R_{D^{*}}\) measurements, as zero [54]. This result is similar to the one of HFLAV [8,9]. Then in this model, we get the fit value \(R_{D^{NP/SM}}^{NP/SM} = 1.140 \pm 0.045\).

2.4 \(m_W\) shift

Here we discuss the NP contributions to the \(W\)-boson mass from the \(W\ell\nu\)-vertex loop corrections involving \(\lambda'\) couplings and the non-unitarity of \(\gamma_{3x3}\) in this model.

The non-unitarity of \(\gamma_{3x3}\) can be shown in

\[
\left( \gamma_{3x3}^T \right)_{ij} = (\delta_{ik} + \eta_{ik}) U_{kji}, \tag{2.13}
\]

where \(U\) is unitary, and the Hermitian \(\eta\) describes the non-unitarity extent. In the inverse seesaw framework, one can figure out \(\eta \approx -\frac{1}{2} m_D^2 (M_D^*)^{-1} (M_D^*)^{-1} m_D\).

Then we turn to the \(W\ell\nu\)-vertex expressed in the Lagrangian,

\[
\mathcal{L}_{\text{eff}}^{Wlv} = \frac{e}{\sqrt{2} \sin \theta_W} \tilde{\nu}_i \gamma^\mu P_L (\nu_i^T + h_{li}) v_i W^-_{\mu} + \text{h.c.,} \tag{2.14}
\]

where the first part in the bracket shows the SM contribution combined with the neutrino-generation mixing. The term \(\gamma_{ij}^T\) can be replaced by \(\delta_{ij} + \eta_{ij}\) by dropping out of the matrix \(U\) due to the limit of vanishing \(m_{l_3}\) [55]. The one-loop correction part, \(h_{li}\), is dominated by the \(\lambda'\) contribution \(h'_{li}\). As the analogy to the formula in Ref. [56], this contribution is given by [31]

\[
h'_{li} = -\frac{3}{64\pi^2} x_{b_R} f_{W}(x_{b_R}) \tilde{\lambda}^{\nu}_{li3} \lambda'_{i33}. \tag{2.15}
\]

where \(x_{b_R} \equiv m_t^2 / m_{b_R}^2\) and the loop function \(f_{W}(x) \equiv x^{-1} - (x-2) \log x / (x-1)^2\), and other minor parts including the ones proportional to \(\eta h'\) product are eliminated. This dominant part is from the \(u_i d_i b_R\) loop diagram, in which the engaging top quark with large mass and couplings \(\tilde{\lambda}^{\nu}_{li3} \lambda'_{i33}\) make \(h'_{li}\) dominant.

Since \(h'\) and \(\eta\) can contribute to the \(W\ell\nu\)-vertex at the same level, they both affect the muon decay and induce (see similar formulas in Refs. [32,55])

\[
G_\mu = G_F (1 + \eta_{\ell\ell} + h'_{\ell\ell}). \tag{2.16}
\]

where \(G_\mu\) is the Fermi constant extracted from the muon lifetime, while \(G_F\) corresponds to the one in the SM. When \(m_t\) and \(M_R\) are set diagonal, matrix \(\eta_{3x3}\) can be diagonal. Since we consider the index \(i\) in the \(\lambda'_{ij3}\) as 1 or 2 at a time, the
submatrix $h'_{22}$ (without $\tau$) is also diagonal. Here the $W\ell\nu_\ell$-effect from pure NP is omitted because it has no interference with the SM contribution, and thus we only keep $h'_{\ell\ell}$ without $h'_{\ell\tau}$ in Eq. (2.16). Then the prediction of $m_W$ is given by
\begin{equation}
\frac{m_W^2}{m_Z^2} = \frac{1}{2} + \frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_fm_Z^2}(1 + \eta_{\ell\ell} + \Delta r_0 + h'_{\ell\ell}),
\end{equation}
where $\Delta r_0$ represents the loop corrections from the SM and pure MSSM. The MSSM part contributes mainly to the $W$ self-energy. This is not considered in this work by setting sufficiently heavy up-type squarks. One can see that the negative values of $h'_{\ell\ell}$ or $\eta_{\ell\ell}$ can raise the prediction of $m_W$ to approach the CDF-II measurement. Besides, these two variables are related to the CKM elements extraction described as
\begin{equation}
K_{ud}^B \equiv K_{ud}(1 - \eta_{\mu\mu} - h'_{\mu\mu}),
\end{equation}
where $K_{ud}^B$ is extracted from beta decay. The terms $\eta_{ee}$ and $h'_{e\mu}$ enter both the $W\ell\nu_\ell$ vertex and $G_\mu$, which induces the canceling in Eq. (2.18). With the extraction of $K_{ud}$ and the analogous one of $K_{us}$ from kaon decay, it is found that the positive values of $\eta_{\mu\mu} + h'_{\mu\mu}$ are needed to alleviate the Cabbibo anomaly showing around $3\sigma$ tension and besides, the simple RH-neutrino extension cannot fully explain the data [57]. Given that the inverse seesaw framework provides negative $\eta_{\ell\ell}$ and $h'_{\ell\ell}$ is also negative commonly, we can set model parameters to make both $|\eta_{\mu\mu}|$ and $|h'_{\mu\mu}|$ sufficiently small to avoid worse tension.

Thus, in this model, we utilize the $W\ell\nu_\ell$-vertex loop correction $h'_{ee}(m_{br}^{-}\hat{\lambda}_{133}^\mu)$ and the non-universality extent $\eta_{ee}(m_{D1}^{-1})$ to explain the CDF-II measurement. In Case A as defined in Sect. 2.2, we need relative large $|\eta_{ee} + h'_{ee}| \geq 2 \times 10^{-3}$ favored to raise $m_W$ with $|\eta_{\mu\mu}| \sim 10^{-4}$; while in Case B, there is $|\eta_{ee}| \geq 2 \times 10^{-3}$ both $|\eta_{\mu\mu}|$ and $|h'_{\mu\mu}|$ around $10^{-4}$. It is worth mentioning that other measurements of $\sin \theta_W$, which are in tension with $m_W^{CDF-II}$, also constrain $\eta_{ee}$ [58–60], e.g., $|\eta_{ee}| < 1.3 \times 10^{-3}$ [60]. In this work, this bound is not considered as the essential one.

2.5 $(g - 2)_{\mu}$

At the end of Sect. 2, we will mention the NP contributions to $a_\mu$ in RVP-MSSM. Here we mainly utilize the one-loop charging and neutralino diagrams to explain, and these contributions to $a_\ell$ are given by [28]
\begin{equation}
\delta a_\ell^\pm = \frac{m_\ell}{16\pi^2} \left[ \frac{m_\ell}{m_\nu} \left( |c_{R}^{\ell L}|^2 + |c_{R}^{\ell R}|^2 \right) F_1^C (m_{\tilde{\chi}_0^\pm}^2/m_{\tilde{\ell}^\pm}^2) + \frac{m_{\tilde{\nu}_\ell}^2}{m_{\nu}^2} \Re (c_{R}^{\ell L} c_{R}^{\ell R}) F_2^C (m_{\tilde{\chi}_0^0}^2/m_{\tilde{\ell}^0}^2) \right],
\end{equation}
where $\Delta r_0$ represents the loop corrections from the SM and pure MSSM. The MSSM part contributes mainly to the $W$ self-energy. This is not considered in this work by setting sufficiently heavy up-type squarks. One can see that the negative values of $h'_{\ell\ell}$ or $\eta_{\ell\ell}$ can raise the prediction of $m_W$ to approach the CDF-II measurement. Besides, these two variables are related to the CKM elements extraction described as
\begin{equation}
K_{ud}^B \equiv K_{ud}(1 - \eta_{\mu\mu} - h'_{\mu\mu}),
\end{equation}
where $K_{ud}^B$ is extracted from beta decay. The terms $\eta_{ee}$ and $h'_{e\mu}$ enter both the $W\ell\nu_\ell$ vertex and $G_\mu$, which induces the canceling in Eq. (2.18). With the extraction of $K_{ud}$ and the analogous one of $K_{us}$ from kaon decay, it is found that the positive values of $\eta_{\mu\mu} + h'_{\mu\mu}$ are needed to alleviate the Cabbibo anomaly showing around $3\sigma$ tension and besides, the simple RH-neutrino extension cannot fully explain the data [57]. Given that the inverse seesaw framework provides negative $\eta_{\ell\ell}$ and $h'_{\ell\ell}$ is also negative commonly, we can set model parameters to make both $|\eta_{\mu\mu}|$ and $|h'_{\mu\mu}|$ sufficiently small to avoid worse tension.

Thus, in this model, we utilize the $W\ell\nu_\ell$-vertex loop correction $h'_{ee}(m_{br}^{-}\hat{\lambda}_{133}^\mu)$ and the non-universality extent $\eta_{ee}(m_{D1}^{-1})$ to explain the CDF-II measurement. In Case A as defined in Sect. 2.2, we need relative large $|\eta_{ee} + h'_{ee}| \geq 2 \times 10^{-3}$ favored to raise $m_W$ with $|\eta_{\mu\mu}| \sim 10^{-4}$; while in Case B, there is $|\eta_{ee}| \geq 2 \times 10^{-3}$ both $|\eta_{\mu\mu}|$ and $|h'_{\mu\mu}|$ around $10^{-4}$. It is worth mentioning that other measurements of $\sin \theta_W$, which are in tension with $m_W^{CDF-II}$, also constrain $\eta_{ee}$ [58–60], e.g., $|\eta_{ee}| < 1.3 \times 10^{-3}$ [60]. In this work, this bound is not considered as the essential one.

2.5 $(g - 2)_{\mu}$

At the end of Sect. 2, we will mention the NP contributions to $a_\mu$ in RVP-MSSM. Here we mainly utilize the one-loop charging and neutralino diagrams to explain, and these contributions to $a_\ell$ are given by [28]
of 13 TeV, induces the stringent bounds on SUSY models. The allowed masses of colored sparticles, such as gluinos, the first-two generation squarks, stops and sbottoms have been excluded up to 1–2 TeV scale [61–67]. In this work, the masses of colored sparticles, except RH sbottoms $b_R$, are all set around 10 TeV, whereas the masses of $b_R$, sleptons, charginos, neutralinos, charged Higgs, and the heavy neutrinos are all around $10^2$–$10^3$ GeV. Some recent experiments have pushed the lower limit of slepton masses over TeV scale [68–70], however, these searches consider nonzero $\lambda$ in the superpotential $\lambda_{ijk} \bar{L}_i \bar{L}_j \tilde{E}_k$. Given that we only consider nonzero $\lambda'$ in the model, processes of purely leptonic decays of sleptons can be neglected, and one can focus on sleptons decaying to the lightest neutralino $\chi_1^0$ and leptons. Although the dijet resonance pairs can emerge from pair-produced sleptons through the $\lambda'$ interactions, this provides relatively weak bounds due to the large QCD background [71]. Thus, we take a compressed scenario, $m_{\tilde{e}_L} \gtrsim 300$ GeV as well as $m_{\tilde{\mu}_L} > 300$ GeV based on the recent searches [72,73].

3.2 Tree-level processes

As we set RH sbottom not decoupled, the tree-level processes exchanging sbottoms will make constraints on the model parameters and these relevant processes include $B \rightarrow K^{(s)} \bar{\nu}_\nu$, $B \rightarrow \pi \bar{\nu}_\nu$, $K^+ \rightarrow \pi^+ \bar{\nu}_\nu$, $D^0 \rightarrow \ell^+ \ell^-$, and $\tau \rightarrow \ell \bar{\nu}_\ell$, as well as the charged current processes $B \rightarrow \tau \nu$, $D_s \rightarrow \tau \nu$, $\tau \rightarrow K (\pi \nu)$, and $\pi \rightarrow \ell \nu(\gamma)$. The first to be introduced are the semileptonic decays $B \rightarrow K^{(s)} \bar{\nu}_\nu$, $B \rightarrow \pi \bar{\nu}_\nu$, and $K^+ \rightarrow \pi^+ \bar{\nu}_\nu$ involving $d_j \rightarrow d_m \bar{\nu}_\nu$. The related effective Lagrangian is defined by

$$L^{\text{eff}}_{\nu(\gamma)} = (C_{m_j}^{\nu(\gamma)} + C_{m_j}^{\nu(\gamma)}) \langle \bar{d}_m \gamma_\mu p L d_j \rangle (\bar{\nu}_\ell \gamma^\mu p L \nu_\ell),$$

h.c.,

(3.1)

where the SM contribution is

$$C_{m_j}^{\nu(\gamma)} = \frac{\sqrt{2} G_F e^2 c^2 K_{ij} K_{im}^*}{4 \pi^2 s_W^2 m^2_W} X(x_i)$$

and the loop function $X(x_i) \equiv x_i (x_i + 2) / 8 (x_i - 1) + 3 x_i (x_i - 2) / 8 (x_i - 1)^2 \log(x_i)$ with $x_i \equiv m^2_\tau / m^2_W$ [74]. The NP contributions are

$$C_{m_j}^{\nu(\gamma)} = \frac{\lambda_j^{\nu(\gamma)} \lambda_j^{\nu(\gamma)} \lambda_j^{\nu(\gamma)}}{2 m^2_{\tilde{b}_R}} = \frac{\nu_{i' a'} \nu_{i a} \nu_{i' a} \lambda_j^{\nu(\gamma)}}{2 m^2_{\tilde{b}_R}}.$$  

(3.2)

It is worth mentioning that the difference between Eq. (3.2) and the corresponding formula in the ordinary RPV-MSSM [75] is the neutrino-generation mixing in $\lambda'$ interactions. With Eq. (2.13), we can further get

$$C_{m_j}^{\nu(\gamma)} \approx \frac{U_{i' a'} U_{i a} \lambda_j^{\nu(\gamma)} \lambda_j^{\nu(\gamma)}}{2 m^2_{\tilde{b}_R}}.$$  

(3.3)

Given that $\mathcal{U}$ is not a diagonal-like matrix, the NP effects on this process are unique compared to RPV-MSSM.

The experimental measurement $B(K^+ \rightarrow \pi^+ \bar{\nu}_\nu)_{\text{exp}} = (1.7 \pm 1.1) \times 10^{-10}$ [76] combined with the SM prediction $B(K^+ \rightarrow \pi^+ \bar{\nu}_\nu)_{\text{SM}} = (9.24 \pm 0.83) \times 10^{-11}$ [77] induces the strong constraint that $|\lambda_j^{\nu(\gamma)} \lambda_j^{\nu(\gamma)} | < 7.4 \times 10^{-4} (m_{\tilde{b}_R} / 1\text{TeV})^2$ [29]. Thus, we assume negligible $\lambda_j^{\nu(\gamma)}$ to avoid the bound, and then the $B \rightarrow \pi \nu \bar{\nu}$ process will also make no bound.

Then we investigate the constraint from $B \rightarrow K^{(s)} \nu \bar{\nu}$. One can define the ratio

$$R^{\nu\bar{\nu}}_K = \frac{B(b \rightarrow s \nu \bar{\nu})_{\text{NP} + \text{SM}}}{B(b \rightarrow s \nu \bar{\nu})_{\text{SM}}}$$

$$= \frac{\sum_{i=1}^{3} \left| C_{23}^{\nu(\gamma)} \right|^2 + \sum_{i,j \neq i'} \left| C_{i j}^{\nu(\gamma)} \right|^2}{3 \left| C_{23}^{\nu(\gamma)} \right|^2}.$$  

(3.4)

The related experimental data [78,79] and SM predictions [74,80] provide $R^{\nu\bar{\nu}}_K = 2.4 \pm 0.9$ for $B \rightarrow K^+ \nu \bar{\nu}$ and the upper limit $R^{\nu\bar{\nu}}_K < 2.7$ at the 90% confidence level (CL) for $B \rightarrow K^+ \nu \bar{\nu}$.

We collect the experimental results and SM predictions of $D^0 \rightarrow \ell^+ \ell^-$, $\tau \rightarrow \ell \nu_\ell$, $B \rightarrow \tau \nu$, $D_s \rightarrow \tau \nu$, and $\tau \rightarrow K \nu$, as well as the processes discussed above in Table 1. Following the same/analogous numerical calculations in the ordinary RPV-MSSM (see Refs. [29,81]), the constraint from $B(D^0 \rightarrow \mu^+ \mu^-)$ gives $|\lambda_{223}^{\nu(\gamma)}|^2 < 0.31 (m_{\tilde{b}_R} / 1\text{TeV})^2$, while the one from $B(D^0 \rightarrow \mu^+ \mu^-)$ is negligible due to the small $m_{\tau}$, and the bound from $B(\tau \rightarrow \ell \nu_\ell)$ provides $|\lambda_{323}^{\nu(\gamma)}|^2 < 0.38 (m_{\tilde{b}_R} / 1\text{TeV})^2$. Besides, $R_{133}$, $R_{223}$, and $R_{123}$, expressing the ratios of the measurement values to the SM predictions for $B(\tau \rightarrow \ell \nu)$, $B(D_s \rightarrow \tau \nu)$, and $B(\tau \rightarrow K \nu)$ respectively, are also constrained. As for $\tau \rightarrow \ell \nu(\gamma)$ decay, similar to the formula in Ref. [55], the bound (including loop corrections $h'$) is given by

$$\frac{1 + \eta_{\mu \mu} + h'_{\mu \mu}}{1 + \eta_{ee} + h'_{ee}} = 1.0010(9),$$  

(3.5)

which can be translated into $|\eta_{ee} + h'_{ee}| \lesssim 0.0028$ within the 2σ level for negligible $\eta_{\mu \mu}$ and $h'_{\mu \mu}$. The $\nu$ matrix is also bounded by $\tau (\mu)$ decaying to charged leptons and neutralinos at the tree level, while both couplings $\nu$ and $\nu'$ are generally constrained by these decays as well as the charged lepton flavor violating (cLFV) decays at one-loop level, which will be addressed in Sect. 3.3.

3.3 Loop-level processes

Here we investigate the loop-level constraints and focus on cLFV processes at first. We stress that the non-$\lambda'$ NP contributions to these decays, $\tau \rightarrow \ell \nu$, $\mu \rightarrow e \nu$, $\tau \rightarrow e \ell \ell$
Table 1: Current status of the related processes, which can be affected by RPV-MSSM at tree level. The experimental upper limits are given at the 90% CL.

| Observations | SM predictions | Experimental data |
|--------------|----------------|------------------|
| $B(K^+ \to \pi^+ \nu \bar{\nu})$ | $(9.24 \pm 0.83) \times 10^{-11}$ [77] | $(1.7 \pm 1.1) \times 10^{-10}$ [76] |
| $B(B^+ \to K^+ \nu \bar{\nu})$ | $(4.6 \pm 0.5) \times 10^{-9}$ [80] | $(1.1 \pm 0.4) \times 10^{-5}$ [78] |
| $B(B \to K^{0}\nu \bar{\nu})$ | $(9.2 \pm 1.0) \times 10^{-6}$ [74] | $< 2.7 \times 10^{-5}$ [79] |
| $B(D^0 \to \mu^+ \mu^-)$ | $\lesssim 6 \times 10^{-11}$ [82] | $< 6.2 \times 10^{-9}$ [82] |
| $B(\tau \to \mu \nu \bar{\nu})$ | $< 1.2 \times 10^{-8}$ [76] | $< 1.2 \times 10^{-8}$ [76] |
| $B(B \to \tau \nu)$ | $(9.47 \pm 1.82) \times 10^{-5}$ [83] | $(1.09 \pm 0.24) \times 10^{-4}$ [76] |
| $B(D_s \to \tau \nu)$ | $(5.40 \pm 0.30)\%$ [29] | $(5.48 \pm 0.23)\%$ [76] |
| $B(\tau \to K \nu)$ | $(7.15 \pm 0.026) \times 10^{-3}$ [84] | $(6.96 \pm 0.10) \times 10^{-3}$ [76] |

($\ell' \neq \ell$), and $\mu \to eee$, can be eliminated for the particular structures of (s)neutrino mass matrices. That is, only chiral mixing but no flavor mixing exists for the sneutrino sector and the neutrino sector involving RH ones (see discussions in Ref. [28] and Sect. 4). Then, we focus on the $\lambda'$ contributions to the cLFV decays. The branching fraction of the $\tau \to \ell \ell' \gamma$ decay is given by [85]

$$B(\tau \to \ell \ell' \gamma) = \frac{\tau \alpha m_{\ell}^5}{4} (|A_{\ell\ell'}^2| + |A_{\ell\ell'}^R|^2),$$

(3.6)

where the effective couplings $A_{\ell\ell'}^2 = -\frac{\lambda'_{\ell\ell'} \lambda'_{\ell\ell'}}{64\pi^2 m_{\ell}^2}$ and $A_{\ell\ell'}^R = 0$.

The limit of $m_{\ell}^2 / m_{\ell'}^2 \to 0$ is adopted here and also for other cLFV processes. It is worth noting that the cLFV muon decay $\mu \to e \gamma$ constrains the coupling product $|\lambda'_{\ell\ell'} \lambda'_{\ell\ell'}|$, with a TeV scale $m_{\ell'}$, very strongly for the experimental upper limit $B(\mu \to e \gamma)_{\exp} < 4.2 \times 10^{-13}$ at the 90% CL [76]. That is, simultaneous non-negligible $\lambda'_{\ell\ell'}$ and $\lambda'_{\ell\ell'}$ are not favored, and hence it is also a reason that only non-negligible $\lambda'_{\ell\ell'}$ or $\lambda'_{\ell\ell'}$ are considered at a time in this work. Then $\mu \to e \gamma$, $\mu \to eee$, and $\tau \to \ell \ell' \ell'$ will not be taken into consideration.

The remaining cLFV processes to be considered are $\tau \to \mu \gamma$, $\tau \to e \gamma$, $\tau \to \mu \mu \mu$, and $\tau \to eee$ decays (see the relevant formulas in Ref. [29]), with the experimental upper limits $B(\tau \to \mu \gamma)_{\exp} < 4.2 \times 10^{-8}$, $B(\tau \to e \gamma)_{\exp} < 3.3 \times 10^{-8}$, $B(\tau \to \mu \mu \mu)_{\exp} < 2.1 \times 10^{-8}$ and $B(\tau \to eee)_{\exp} < 2.7 \times 10^{-8}$ at 90% CL, respectively [76].

Next, we investigate the $B_s - \bar{B}_s$ mixing, which is mastered by

$$\rho_{\text{eff}}^{\text{SM} + \text{NP}} = (C_{B_{f}}^{\text{SM}} + C_{B_{f}}^{\text{NP}})(\bar{\epsilon} \gamma_{\mu} P_L b)(\bar{\epsilon} \gamma_{\mu} P_L b) + \text{h.c.},$$

(3.7)

\footnote{In this work, in order to consider all the process constraints at the 2σ level, we get the experiment bound $B(\tau \to \mu \gamma)_{\exp} < 5.1 \times 10^{-8}$, $B(\tau \to e \gamma)_{\exp} < 4 \times 10^{-8}$, $B(\tau \to \mu \mu \mu)_{\exp} < 2.6 \times 10^{-8}$, $B(\tau \to eee)_{\exp} < 3.3 \times 10^{-8}$ as well as $R_{K^0}^{\text{SM}} < 3.3$ under the assumption that the uncertainties follow the Gaussian distribution [86].}

where the SM contribution is $C_{B_{f}}^{\text{SM}} = \frac{1}{4\pi^2} G_F^2 m_{\ell}^2 \bar{\epsilon} \gamma_{\mu} P_L b(\bar{\epsilon} \gamma_{\mu} P_L b)$ with the defined function $S(x_{\ell}) = \frac{\sigma_{\ell \ell}}{\sigma_{\ell \ell}} = \frac{2\lambda_{\ell \ell}^2}{3(2\lambda_{\ell \ell}^2 - 1)}$, and the non-negligible NP contribution is

$$C_{B_{f}}^{\text{NP}} = \frac{1}{8\pi} \left( \lambda_{\ell \ell}^{T, \ell} D_2[m_{\ell}, m_{\ell}, m_{\ell}] + \lambda_{\ell \ell}^{T, \ell} D_2[m_{\ell}, m_{\ell}, m_{\ell}, m_{\ell}] \right),$$

(3.8)

with $\lambda_{\ell \ell}^{T, \ell} = \lambda_{\ell \ell}^{D, \ell} = \lambda_{\ell \ell}^{D, \ell} + \lambda_{\ell \ell}^{D, \ell} + \lambda_{\ell \ell}^{D, \ell}$. The recently updated measurement by LHCb combined with previous ones induces $\Delta M_{B_s}^\text{LHCb} = (17.765 \pm 0.007)\text{ps}^{-1}$ [87], leading to the strong constraint along with the SM prediction $\Delta M_{B_s}^\text{SM} = (18.4_{-0.2}^{+0.7})\text{ps}^{-1}$ [88].

$$0.90 < |1 + C_{B_{f}}^{\text{NP}} / C_{B_{f}}^{\text{SM}}| < 1.11,$$

(3.9)

at the 2σ level.

Following the introduction of $B_s - \bar{B}_s$ mixing, we will mention the $B \to X_s \gamma$ decay, which is mastered by the electromagnetic dipole operator $\mathcal{O}_7 = \frac{m_b}{4\pi^2} (\bar{s} \sigma^{\mu \nu} P_R b) F_{\mu \nu}$.

The $\lambda'$ contribution is given by

$$C_{\gamma}^{\lambda'} = \frac{\lambda_{\ell \ell}^{T, \ell}}{144G_F m_{\ell}^2} \lambda_{\ell \ell}^{T, \ell},$$

(3.10)

The non-$\lambda'$ ones can be referred to Ref. [89] and are predicted to be negligible for the decoupled $\mu$. In order to fulfill the bound from the recent measured branching ratio $B(B \to X_s \gamma)_{\exp} \times 10^4 = 3.43 \pm 0.21 \pm 0.07$ [90] and the SM prediction $B(B \to X_s \gamma)_{\SM} \times 10^4 = 3.36 \pm 0.23$ [91], Eq. (3.10) implies a cancellation in $\lambda_{\ell \ell}^{T, \ell}$ for the (nearly) degenerate $m_{\ell}$, $m_{\ell}$. This cancellation is also beneficial for fulfilling the bounds of $B_s - \bar{B}_s$ mixing (see Eq. (3.8)).

\footnote{This combined result by LHCb is very close to the recent result averaged by HFLAV as $\Delta M_{B_s}^\text{HFLAV-2021} = (17.765 \pm 0.006)\text{ps}^{-1}$ [8], with the much improved precision compared to the previous average $\Delta M_{B_s}^\text{HFLAV-2018} = (17.757 \pm 0.021)\text{ps}^{-1}$ [9].}
Then we move on to the constraints from the purely leptonic decays of $Z$, $W$ bosons, and $\tau(\mu)$ leptons. The effective Lagrangian of $Z \to l_i^+ l_j^-$ decay is given by [56]

$$L^Z_{eff} = \frac{e}{\cos \theta_W \sin \theta_W} \bar{I}_i \gamma^\mu \left( g_{ij}^{\nu L} P_L + g_{ij}^{\nu R} P_R \right) I_j Z_{\mu i}, \tag{3.11}$$

where $g_{ij}^{\nu L} = \delta_{ij} g_{ij}^{SM} + \delta_{ij}^{lL}$ and $g_{ij}^{\nu R} = \delta_{ij} g_{ij}^{SM} + \delta_{ij}^{lR}$, with $g_{ij}^{SM} = -\frac{1}{2} + \sin^2 \theta_W$ and $g_{ij}^{SM} = \sin^2 \theta_W$. In the limit of $m_i / m_Z \to 0$, the corresponding branching fractions are

$$B(Z \to l_i^- l_j^+) = \frac{3 m_Z^2}{16 \pi^2 \Gamma_Z} \left| g_{ij}^{lL} \right|^2 + \left| g_{ij}^{lR} \right|^2 \tag{3.12}$$

with $Z$ width $\Gamma_Z = 2.495$ GeV [76]. For $i \neq j$, the branching ratio should be given by $\frac{1}{2} [B(Z \to l_i^- l_j^+) + B(Z \to l_j^- l_i^+)]$. The NP effective couplings contributed mainly by $\lambda'$ effects are expressed as $\delta g_{ij}^{lL} = \frac{1}{3 \sqrt{2}} \lambda' \delta g_{ij}^{lL} = 0$ here, and the formulas of $B_{ij}^{lL}$ functions are collected in appendix B. Then the measurements of the partial width ratios of $Z$ bosons, i.e., $\Gamma(Z \to \mu \mu) / \Gamma(Z \to ee) = 1.0001(24)$, $\Gamma(Z \to \tau \tau) / \Gamma(Z \to \mu \mu) = 1.0010(26)$, and $\Gamma(Z \to \tau \tau) / \Gamma(Z \to ee) = 1.0020(32)$ [76], induce $|B_{11}| < 0.36$ and $|B_{33}| < 0.32$ when $\lambda_{ij}^{lL} = 0$ (Case A), and $|B_{22}| < 0.35$ and $|B_{33}| < 0.31$ when $\lambda_{ij}^{lL} = 0$ (Case B). Furthermore, the experimental upper limits, $B(Z \to \ell \ell) < 9.8 \times 10^{-6}$ and $B(Z \to \mu \tau) < 1.2 \times 10^{-5}$ at the 95% CL [76], make the bound on $|B_{13}|^2 < 1.9^2$ and $|B_{23}|^2 + |B_{32}|^2 < 2.1^2$ in Case A and B, respectively. We have checked that the additional effect on the $\theta_W$ extraction by $\eta$ and $h'$, which are in the range for $m_W^{NP}$ explanations, induces the extra influence on the bounds above up to $10^{-3}$, so this effect can be omitted safely.

The constraints from the purely leptonic decays of $W$ boson can be covered by the stronger ones from $\mu \to e\nu\nu\mu$ and $\tau \to \ell\tilde{\nu}\nu\ell$ decays. The fraction ratios of these lepton decays, i.e., $B(\tau \to \mu e\nu\nu\mu) / B(\tau \to e\nu\nu\mu), B(\tau \to \tilde{\nu}\nu\ell\nu\ell) / B(\mu \to e\nu\nu\mu)$, and $B(\tau \to \mu \tilde{\nu}\nu\ell\nu\ell) / (\mu \to e\nu\nu\mu)$, make the bounds [55] on the model parameters, which can be expressed as

$$1 + \eta_{\mu\mu} + \delta B X_{\mu\mu} = 1.0018(14),$$

$$1 + \eta_{ee} + \delta A X_{ee} = 1.0001(14),$$

$$1 + \eta_{\tau\tau} + h'_\tau = 1.0010(14),$$

$$1 + \eta_{\mu\tau} + \delta B X_{\mu\tau} = 1.0029(14),$$

where $X$ is $\mu$ for Case A and $X$ is $\tau$ for Case B. We only consider the $Wl_i\nu_j$-vertex, which has the interference with the SM contribution, but neglect the LFV-vertex $Wl_i\nu_j$ and $Zl'_i$, which can be embedded in $1 \to l_i\tilde{\nu} l_j$ process. With the last two formulas of Eq. (3.13) combined with $|\eta_{\mu\mu}(h'_\mu)| \lesssim 10^{-4}$, we should keep $|\eta_{\tau\tau} + h'_\tau| \lesssim 0.0018$ and $|\eta_{\mu\tau} + h'_\mu| \lesssim |\eta_{ee} + \delta A X_{ee}|$ at the $2\sigma$ level.

4 Numerical analyses

In this section, we begin to study the numerical explanations for $B$-physics anomalies, $(g-2)_\mu$ and $m_W$ shift in RPV-MSSM. With the data of neutrino oscillation [92], we consider normal ordering and zero $\delta CP$. Then the three light neutrinos have masses $0, 0.008, 0.05$ eV with $m_{\nu_1} \approx 0 , \sqrt{\Delta m_{21}^2}, \sqrt{\Delta m_{31}^2}$ [93]. The sets of fixed model parameters are collected in Table 2.

The diagonal inputs of $Y_\nu, M_R, m_{l_i},$ and $B_{M_R}$ induce no flavor mixings in sneutrino and neutrino sectors (when RH neutrinos engage), which are beneficial for fulfilling the bounds of cLFV decays (see appendix C for the particular discussions). Besides, the input values shown in Table 2 can induce a diagonal $\eta = - \text{diag}(2.53, 0.18, 0.15) \times 10^{-2}$. The values of $m_{l_i}$ induce $m_{\nu_i}(0) = 348(352)$ GeV, which are larger than the masses of the lightest neutralino and chargino, as 307 GeV and 325 GeV, respectively, and they are in accord with the constraints discussed in Sect. 3.1. The remaining parameters, $m_{b_R}, \lambda'_{323}, \lambda'_{333}, \lambda'_{1(2)23}$, and $\lambda'_{1(2)33}$ in Case A (B), can vary freely in the ranges considered.

Next, with the input values given above and all $\lambda'$ couplings as real numbers, we can get some numerical results of Wilson coefficients and observables which show the possibilities for simultaneous explanations of the anomalies. They are given as follows,

$$C_{9e(\mu)/10e(\mu)} = -C_{10e(\mu)} \approx -1.604(-1.623)\lambda'_{1(2)23}\lambda'_{1(2)33},$$

$$C_{B_4} / C_{B_3} \approx 76(\lambda'_{23}\lambda'_{33} + \lambda'_{23}\lambda'_{33})^2,$$

$$B_{31} \approx \frac{1}{\sqrt{m_{b_R}}} \left[ 0.146 \log(m_{b_R}/1 \text{ TeV}) + 0.2 \right] \lambda'_{333}, \tag{4.1}$$

where only the dominant parts are kept. To be favored by $R_{K^{(*)}}$ data at the $2\sigma$ level, there are $-0.355 \lesssim \lambda'_{23}\lambda'_{33} \lesssim -0.106$ in Case A and $0.117 \lesssim \lambda'_{23}\lambda'_{33} \lesssim 0.314$ in Case B, through the first formula in Eq. (4.1), which is induced by Eq. (2.8). As for the second formula in Eq. (4.1), valid for $1.5 \leq m_{b_R} \leq 10$ TeV, we get $|\lambda'_{23}\lambda'_{33} + \lambda'_{33}^2\lambda'_{333}| \lesssim 0.038$ with Eq. (3.9). This result implies that the $B_3 - B_4$ mixing bound demands the canceling between $\lambda'_{23}\lambda'_{33}$ and $\lambda'_{23}\lambda'_{333}$. In the last formula, the edge value of $\lambda'_{333}$ near the $Z \rightarrow \tau^-\tau^+$ bound can be gotten. With the rough NP features above, we will study them in detail.

In Case A, we can see that the $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies can be simultaneously explained at the $2\sigma$ level, although the overlaps are narrow, as shown in Fig. 1. The dominant constraints are from $B \rightarrow K^{(*)}\nu\nu$, $Z$ leptonic decays,
Table 2  The sets of fixed model parameters, defined at $\mu_{NP}$ scale

| Parameters | Sets | Parameters | Sets |
|-----------|------|------------|------|
| $\tan \beta$ | 15   | $Y_\nu$ | diag(0.41, 0.11, 0.10) |
| $M_{1}$ | 320 GeV | $M_R$ | diag(1, 1, 1) TeV |
| $M_{2}$ | 350 GeV | $B_{M_{2}}$ | diag(0.5, 0.5, 0.5) TeV$^2$ |
| $\mu$ | 450 GeV | $m_{\tilde{L}'}$ | diag(360, 350, 350) GeV |

Fig. 1  The experimental allowed regions for explaining the $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies in Case A. The masses $m_{\tilde{b}_R}$ are given in units of TeV. The $2\sigma$ favored areas for $R_{K^{(*)}}$ and $R_{D^{(*)}}$ measurements are denoted by green and blue, respectively. The hatched areas filled with the black-horizontal, red-vertical, cyan-vertical, and blue-vertical lines are excluded at the $2\sigma$ level by $B \to K^{(*)}\nu\bar{\nu}$, $Z \to l^-_i l^+_j$, $\tau \to eee$ decays, and $B_s - \bar{B}_s$ mixing, respectively. The common areas are denoted by purple.
$\tau \to eee$ decays, and $B_s - \bar{B}_s$ mixing. In Fig. 1a, when $\lambda'$ couplings except $\lambda'_{133}$ are fixed as shown in the figure, the lower limit of $m_{\tilde{b}_R}$ is constrained by the $Z$ leptonic decays, i.e., $Z \to \tau^- \tau^+$ specifically, while $b_R$ should also be lighter than around 2.42 TeV here to explain $R_{D(\tau)}$ data. Accordingly, the favored region for $R_{K^{(*)}}$ explanations is broad and nearly independent of $m_{\tilde{b}_R}$, and it only demands $\lambda'_{133} \gtrsim 1.$ In Fig. 1b, we set $\lambda'_{133} = 1.12$ and decrease $|\lambda'_{233}|$ slightly. The contributions to $R_{K^{(*)}}$ by $m_{\tilde{b}_R}$ cannot be omitted here because the charhino-neutrino one is weakened slightly. The values of $(m_{\tilde{b}_R}, \lambda'_{133})$ in the overlap get smaller, and then we get $-h'_{ee} = 3.3 \times 10^{-4}$ for $m_{\tilde{b}_R} = 2.1$ TeV. The NP prediction $m_{W}^{NP}$ without pure-MSSM effects is given by $m_{W}^{NP} \approx m_{W}^{SM}[1 - 0.20(h_{ee} + \eta_{\mu\mu} + h'_{ee})]$ (similar to the formula in Ref. [60]), inducing the value $m_{W}^{NP} = 80.412$ GeV, which can explain the CDF-II measurement at the 2$\sigma$ level. Keeping the values of $m_{\tilde{b}_R} = 2.1$ TeV, relevant overlaps are also found in Fig. 1c,d.

Then we move on to Case B. In this case, we will show that, given the stringent bounds from the $B_s - \bar{B}_s$ mixing as well as the constraints of $Z \to \tau^- \tau^+$ decay, the anomalies of $R_{K^{(*)}}$ and $R_{D(\tau)}$ cannot be explained simultaneously at the 2$\sigma$ level. From Eq. (2.12), the data prediction favors large $|\lambda'_{333}|$, positive $\lambda'_{232}\lambda'_{333}$, and negative $\lambda'_{233}\lambda'_{333}$. However, positive $\lambda'_{233}\lambda'_{333}$ is needed for explaining $R_{K^{(*)}}$. Thus, with Eq. (4.1), we let $\lambda'_{232}$ be the edge value $0.117/\lambda'_{333}$ for $R_{K^{(*)}}$ explanation, and $\lambda'_{332} = (-0.117 + 0.038)/\lambda'_{333}$ for the bound edge of $B_s - \bar{B}_s$ mixing, using $|\lambda'_{333}| = \sqrt{0.31 (m_{\tilde{b}_R}/1 \text{ TeV}) / \left[0.146 \log (m_{\tilde{b}_R}/1 \text{ TeV}) + 0.2 \right]^2}$ as the edge value near the $Z \to \tau^- \tau^+$ bound. Then the prediction of $R_{D(\tau)}^{NP}$ can be given as the function of $(m_{\tilde{b}_R}, \lambda'_{333})$, as shown in Fig. 2.

For $0.5 \lesssim \lambda'_{333} \lesssim 1$, we find that the ratio increases with $m_{\tilde{b}_R}$ increasing up to around 6 TeV because the edge value of $\lambda'_{333}$ constrained by $Z$ decays gets relatively large for heavier $m_{\tilde{b}_R}$. Then the ratio decreases for $m_{\tilde{b}_R} \gtrsim 6$ TeV. From the right panel of Fig. 2, it is obvious that $R_{D(\tau)}^{NP/SM}$ cannot be raised higher than 1.02, with below the lower limit 1.05 of 2$\sigma$ fit. Thus, the simultaneous 2$\sigma$-level explanation of $R_{K^{(*)}}$ and $R_{D(\tau)}$ anomalies is impossible in Case B. While we can still explain $R_{K^{(*)}}$ and other anomalies in the $b \to s\mu^+\mu^-$ process, which will be shown in Table 3, with the benchmark point in Case A collected as well.

In Table 3, as mentioned before, for the point in Case A, both $R_{K^{(*)}}$ and $R_{D(\tau)}$ are in the 2$\sigma$ ranges of the experimental measurements. In Case B(I), we utilize the relevant point to fulfill $R_{K^{(*)}}$ data at 2$\sigma$, and raise the ratio $R_{D(\tau)}^{NP/SM}$. However, this increase cannot reach the 2$\sigma$ accuracy region as we predicted. The point in Case B(II) can explain the $b \to s\ell^+\ell^-$ anomalies at the 2$\sigma$ level without considering $R_{D(\tau)}$ data. Besides, in all the cases, $(g - 2)_{\mu}$ data and CDF-II $m_W$ shift favor the points, which are also allowed by relevant constraints shown in Sect. 3. Due to the contributions of $\lambda'$ diagrams, there are slight differences among the predicted $\delta a^{NP}_{\mu}$ in the three cases. In case the CDF-II result is not supported by future measurements, we can adjust the coupling $Y_i$ in Table 2 into, e.g., diag(0.12, 0.11, 0.10), which induces $\eta_{ee} = -2.2 \times 10^{-4}$ to fulfill the current global $m_W = 80.379(12)$ GeV [76] and relevant measurements of $\sin \theta_W$ mentioned in Sect. 2.4. We have checked that the explanations for the other anomalies in Case A, Case B(I), and Case B(II) are still applicable.

Before the end of this section, we discuss probes for RPV-MSSM in future experiments. The observation of LFV in $Z$ decays would provide indisputable evidence for NP, and especially, $Z \to e\tau$ and $Z \to \mu\tau$ decays can connect to the $\tau$ flavor, which can probe the leptoquark or RPV-SUSY models. With a TeV scale of $m_{\tilde{b}_R}$ and relevant $\lambda'_{333}$ values in Table 3, the branching ratios $B(Z \to e\tau)$ and $B(Z \to \mu\tau)$ are predicted as $O(10^{-7})$ scale, which can reach the sensitivities $O(10^{-9})$ at FCC-ee [94]. Besides, the model predicts the branching ratios of the eLFV processes, $\tau \to \ell\gamma$ and $\tau \to \ell\ell\ell$ with the scale of $O(10^{-9} - 10^{-8})$, and they can reach the future sensitivities by Belle II (50 ab$^{-1}$) [95] and FCC-ee [94]. Even in $b_R$-decoupled scenario, which is only favored by $b \to s\ell^+\ell^-$ and muon $g - 2$ explanations, the heavy neutrinos with TeV masses can also reach future collider searches [96, 97]. For instance, in the multi-lepton channel at future colliders ($\sqrt{s} = 27$ TeV), active-sterile mixing as small as $|\nu_{lN}|^2 \sim O(10^{-3} - 10^{-2})$ can be probed at the 95% CL for heavy neutrino masses in the range $700 < m_{\nu_N} < 3500$ GeV with 15 ab$^{-1}$ of data [98].

5 Conclusions

The recently reported anomalies in $R_{D(\tau)}$, $R_{K^{(*)}}$, with other $b \to s\ell^+\ell^-$ observables, e.g., $P'_{S}B(B_s \to \phi\mu^+\mu^-)$, as well as the enduring muon $g - 2$, have shown that LFUV effects beyond the SM may exist. While the more recent precision measurement of $W$ mass, if confirmed by other experiments, will profoundly change the situation of NP search. Interestingly, the LFUV NP can affect $m_W$ prediction through muon decays, so it is straightforward to make a simultaneous investigation on the $B$-physics anomalies, $(g - 2)_{\mu}$, and $m_W$ in NP models.

In this work, we study these anomalies mentioned above in RPV-MSSMIS, which is the framework providing $\lambda'\tilde{L}\tilde{Q}\tilde{D}$ interaction involved with the (s)neutrino chiral mixing for explaining $B$-physics anomalies and increasing $m_W$ prediction, which is also contributed by the non-unitarity $\eta_{ee}$ in the seesaw sector. We consider nonzero $\lambda'_{i(2)jk}$ at a time in Case A(B), and find that the deviations of $R_{K^{(*)}}$, $R_{D(\tau)}$, and $m_W$ from SM predictions can be reduced to the 2$\sigma$ level.
Fig. 2 The ratio $R_{NP/SM}^{D(*)}$ varies with $m_{bR}$ and $\lambda'_{233}$ in Case B. In the right panel, $\lambda'_{233}$ is set as 1.5 (dotted), 1 (dashed), and 0.5 (solid), respectively.

### Table 3

The benchmark points in Case A ($\ell = 1$ and $e$) and B ($\ell = 2$ and $\mu$). The Wilson coefficient $C_{9U}^{NP}$ provides the lepton flavor universal contribution (see Ref. [28]).

|                | Case A       | Case B(I)    | Case B(II)   |
|----------------|--------------|--------------|--------------|
| $\lambda'_{123}$ | -0.08        | 0.1          | 0.1          |
| $\lambda'_{333}$ | 1.12         | 1.35         | 2            |
| $\lambda'_{323}$ | 0.06         | -0.04        | -0.1         |
| $\lambda'_{333}$ | 2            | 2.6          | 2            |
| $m_{bR}/\text{TeV}$ | 2.1          | 3            | 3            |
| $c_{NP}^{W}$ | -0.172       | -0.196       | -0.295       |
| $c_{NP}^{10t}$ | -0.157       | -0.158       | -0.001       |
| $\mathcal{R}_{NP/SM}^{D(*)}$ | 1.055       | 1.006        | 0.983        |
| $m_{W}^{NP}/\text{GeV}$ | 80.412   | 80.411        | 80.416       |
| $\delta_{\mu}^{NP} \times 10^9$ | 1.361 | 1.369        | 1.377        |

simultaneously in Case A. In Case B, the combined 2σ-level explanation of $b \to s\ell^+\ell^-$ anomaly and $m_W$-shift can be achieved, while NP effects on $R_{D(*)}$ are weak. Moreover, $(g - 2)_\mu$ data can be explained in both cases, which also fulfill neutrino oscillation data, the relevant constraints at collider, and a series of flavor-physics bounds from $B \to K^{(*)}\nu\bar{\nu}$, $Z$ leptonic decays, cLFV decays, $B_{s} - \bar{B}_{s}$ mixing, etc. Moreover, the LFV effects on the $Z$-boson and $\tau$ decays as well as the TeV scale heavy neutrinos in this model, can be testable in future experiments.

**Acknowledgements** We thank Seishi Enomoto for valuable discussions. This work is supported in part by the National Natural Science Foundation of China under Grant No. 11875327, the Fundamental Research Funds for the Central Universities, and the Sun Yat-Sen University Science Foundation. F.C. is also supported by the CCNU-QIPL Innovation Fund (QLPL2021P01).

**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Authors’ comment: All data generated during this study are already contained in this published paper.]

**Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

Funded by SCOAP3. SCOAP3 supports the goals of the International Year of Basic Sciences for Sustainable Development.
A One-loop box contributions to $b \to s \ell^+ \ell^-$ in RPV-MSSM

In this section, the whole Wilson coefficients from the one-loop $b \to s \ell^+ \ell^-$ boxes in RPV-MSSM are listed.

The LH-quark-current contributions from chargino boxes to $b \to s \ell^+ \ell^-$ process are given by

$$C_{c_{\ell\ell}^{\pm}} = - C_{\ell\ell}^{c_{\ell\ell}^{\pm}}$$

$$= - \frac{\sqrt{2}\pi^2 i}{2 G_{F} H_{\ell\ell}} (g_{2}^{3} K_{13} K_{12}^{*} V_{m_{1}} V_{m_{2}} [g_{2} Y_{\ell_{1}^T} - V_{m_{2}} Y_{\ell_{1}^T}]) \times (g_{2} V_{m_{1}} Y_{\ell_{2}^T} - V_{m_{2}} Y_{\ell_{2}^T}) D_{2}[m_{\tilde{\ell}_{1}}, m_{\tilde{\ell}_{2}}, m_{\tilde{\ell}_{3}}]$$

$$+ g_{2}^{2} K_{13} K_{12}^{*} V_{m_{2}} [g_{2} Y_{\ell_{1}^T} - V_{m_{2}} Y_{\ell_{1}^T}] \times (g_{2} V_{m_{1}} Y_{\ell_{2}^T} - V_{m_{2}} Y_{\ell_{2}^T}) D_{2}[m_{\tilde{\ell}_{1}}, m_{\tilde{\ell}_{2}}, m_{\tilde{\ell}_{3}}]$$

$$+ g_{2}^{2} K_{13} K_{12}^{*} V_{m_{2}} Y_{\ell_{2}^T} \times (g_{2} V_{m_{1}} Y_{\ell_{1}^T} - V_{m_{2}} Y_{\ell_{1}^T}) D_{2}[m_{\tilde{\ell}_{1}}, m_{\tilde{\ell}_{2}}, m_{\tilde{\ell}_{3}}]$$

$$- g_{2}^{2} K_{13} K_{12}^{*} Y_{\ell_{2}^T} \times (g_{2} V_{m_{1}} Y_{\ell_{1}^T} - V_{m_{2}} Y_{\ell_{1}^T}) D_{2}[m_{\tilde{\ell}_{1}}, m_{\tilde{\ell}_{2}}, m_{\tilde{\ell}_{3}}]$$

$$\times D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}],$$

$$+ 2 m_{u} m_{d} y_{u} y_{d} K_{13} K_{12}^{*} Z_{H_{12}}^{2} \tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3} \lambda_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}}$$

$$\times D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}],$$

$$+ 2 m_{u} m_{d} y_{u} y_{d} K_{13} K_{12}^{*} Z_{H_{12}}^{2} \tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3} \lambda_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}}$$

$$\times D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}],$$

$$- g_{2}^{2} K_{13} K_{12}^{*} Y_{\ell_{2}^T} \lambda_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}} v_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}} D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}]$$

$$+ 2 m_{u} y_{u} K_{13} Z_{H_{12}}^{2} \tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3} \lambda_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}}$$

$$\times D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}],$$

$$+ 2 m_{u} y_{u} K_{13} Z_{H_{12}}^{2} \tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3} \lambda_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}}$$

$$\times D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}].$$

(A.3)

The contributions of $W/H^{\pm}$ (means $W$ with $W$ Goldstones or charged Higgs box involved) box diagrams to $b \to s \ell^+ \ell^-$ process are given by

$$C_{W_{\ell\ell}^{\pm}} = - C_{\ell\ell}^{W_{\ell\ell}^{\pm}}$$

$$= - \frac{\sqrt{2}\pi^2 i}{2 G_{F} H_{\ell\ell}^{2}} (g_{2}^{3} K_{13} K_{12}^{*} Z_{H_{12}}^{2} Z_{H_{12}}^{2} [Y_{\ell_{2}^T}]) \times D_{2}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}]$$

$$- g_{2}^{2} K_{13} K_{12}^{*} Y_{\ell_{1}^T} \lambda_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}} v_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}} D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}]$$

$$\times D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}].$$

(A.4)

The mixing matrix elements $Z_{H_{12}} = - \sin \beta$, $Z_{H_{13}} = - \cos \beta$ with Goldstone mass $m_{H_{1}} = m_{W}$ and charged Higgs mass $m_{H_{2}} = m_{H^{\pm}}$, and $Y_{\ell_{1}^T} \equiv (Y_{\nu}, y_{\ell}^T)(Y_{\nu}, y_{\ell}^T)^{T}$. The contributions of $4 Y_{\ell}^T$ box diagrams to $b \to s \ell^+ \ell^-$ process are given by

$$C_{Y_{\ell_{1}^T}^{\ell_{1}^T}} = - C_{\ell_{1}^T}^{Y_{\ell_{1}^T}^{\ell_{1}^T}}$$

$$= - \frac{\sqrt{2}\pi^2 i}{2 G_{F} H_{\ell\ell}^{2}} (g_{2}^{3} K_{13} K_{12}^{*} Z_{H_{12}}^{2} Z_{H_{12}}^{2} [Y_{\ell_{2}^T}]) \times D_{2}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}]$$

$$- g_{2}^{2} K_{13} K_{12}^{*} Y_{\ell_{1}^T} \lambda_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}} v_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}} D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}]$$

$$\times D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}].$$

(A.5)

$$C_{Y_{\ell_{1}^T}^{\ell_{1}^T}} = - C_{\ell_{1}^T}^{Y_{\ell_{1}^T}^{\ell_{1}^T}}$$

$$= - \frac{\sqrt{2}\pi^2 i}{2 G_{F} H_{\ell\ell}^{2}} (g_{2}^{3} K_{13} K_{12}^{*} Z_{H_{12}}^{2} Z_{H_{12}}^{2} [Y_{\ell_{2}^T}]) \times D_{2}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}]$$

$$- g_{2}^{2} K_{13} K_{12}^{*} Y_{\ell_{1}^T} \lambda_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}} v_{\tilde{\ell}_{1} \tilde{\ell}_{2} \tilde{\ell}_{3}} D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}]$$

$$\times D_{0}[m_{\nu}, m_{\nu}, m_{\nu}, m_{\tilde{\ell}_{3}}].$$

(A.6)
The contributions of neutralino box diagrams only contain RH-quark-current parts, which are given by

\[ C^\prime \chi^0 = -C_{10}^\prime \chi^0 = -\frac{\sqrt{2} \pi^2 i}{2G_F \eta_1 e^2} \left\{ \frac{1}{2} (g_1 N_{n1})^2 + g_2 N_{n2} \right\} D_2 \left[ m_{l_L}^2, m_{l_L}, \right. \]

\[ \left. + \frac{2}{9} g_1^2 |N_{n1}|^2 \chi^0 \right\} \times D_2 \left[ m_{l_L}^2, m_{l_L}^0, m_{l_R}^0, m_{l_R} \right] +
\]

\[ \times \left( D_2 \left[ m_{l_L}^2, m_{l_L}, m_{l_L}^0, m_{l_R}^0 \right] + D_2 \left[ m_{l_L}^2, m_{l_L}, m_{l_L}^0, m_{l_R}^0 \right] \right), \]

(A.7)

where \( m_{\chi^0} \) is the neutralino mass after the diagonalization \( N \lambda \rightarrow \lambda \chi^0 N^T = \lambda \).

In the formulas above, the Passarino-Veltman functions \[53\] \( D_0 \) and \( D_2 \) are defined as

\[ D_0[m_1, m_2, m_3, m_4] = \frac{1}{(2\pi)^4} \frac{1}{(k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)(k^2 - m_4^2)}, \]

\[ D_2[m_1, m_2, m_3, m_4] = \frac{k^2}{(2\pi)^4} \frac{1}{(k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)(k^2 - m_4^2)}. \]

(A.8)

\[ B^{ij} \text{ The coupling functions in } Z \rightarrow l_i^- l_j^+ \text{ process} \]

With the effective Lagrangian of \( Z \rightarrow l_i^- l_j^+ \) process in Eq. (3.11), the functions \( B^{ij} \equiv (32\pi^2) \delta g^{ij}_{l_L} \) are given by the following two parts \[81\],

\[ B_1^{ij} = 3 \bar{\chi}^j \tau_3 \chi^i \left\{ -x_{b_R} (1 + \log x_{b_R}) + \frac{m_Z^2}{18 m_{b_R}^2} \right\} \times \left( \frac{(11 - 10 \sin^2 \theta_W)}{6 - 8 \sin^2 \theta_W} + \frac{1}{\sin^2 \theta_W} \right), \]

\[ B_2^{ij} = \sum_{j=1}^2 \bar{\chi}^j \tau_3 \chi^i \left\{ \left( \frac{m_Z^2}{18 m_{b_R}^2} \right)^2 \left( \frac{1 + \frac{4}{3} \sin^2 \theta_W}{9} \right) + \right\}. \]

Then there are \( B^{ij} = B_1^{ij} + B_2^{ij} \). \( B_1^{ij} \) is the dominant part due to the involved top quark.

\[ C \text{ The numerical form of the (s)neutrino mixing matrix} \]

With the input set in Table 2, the numerical form of the neutrino mixing matrix is listed as

\[
\begin{pmatrix}
0.835 & 0.526 & -0.145 & 0.050i & 0 & 0 & -0.050 & 0 & 0 \\
-0.247 & 0.600 & 0.761 & 0 & 0.013i & 0 & 0 & 0.013 & 0 \\
0.488 & -0.601 & 0.633 & 0 & 0 & 0.012i & 0 & 0 & -0.012 \\
0 & 0 & 0 & 0.707i & 0 & 0 & -0.707 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.707i & 0 & 0 & 0.707 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.707i & 0 & 0 & -0.707 \\
-0.059 & -0.037 & 0.010 & 0.705i & 0 & 0 & -0.705 & 0 & 0 \\
0.005 & -0.011 & -0.015 & 0.707i & 0 & 0 & 0.707 & 0 & 0 \\
-0.008 & 0.010 & -0.011 & 0 & 0 & 0.707i & 0 & 0 & -0.707
\end{pmatrix}
\]

(C.1)
related to the neutrino mass spectrum around \{0,8 \times 10^{-15}, 5 \times 10^{-14}, 1, 1, 1, 1, 1\} TeV. And the sneutrino mixing matrix is given numerically by

\[
\begin{pmatrix}
0.991 & 0 & 0 & 0.067 & 0 & 0 & -0.118 & 0 & 0 \\
0 & 0.999 & 0 & 0 & 0.018 & 0 & 0 & -0.032 & 0 \\
0 & 0 & -0.999 & 0 & 0 & -0.017 & 0 & 0 & 0.029 \\
-0.131 & 0 & 0 & 0.704 & 0 & 0 & -0.698 & 0 & 0 \\
0 & 0.036 & 0 & 0 & -0.707 & 0 & 0 & 0.706 & 0 \\
0 & 0 & 0.032 & 0 & 0 & -0.707 & 0 & 0 & 0.707 \\
0.037 & 0 & 0 & 0.707 & 0 & 0 & 0.706 & 0 & 0 \\
0 & 0 & 0.010 & 0 & 0 & 0.707 & 0 & 0 & 0.707 \\
0 & 0 & 0 & -0.009 & 0 & 0 & -0.707 & 0 & 0 & -0.707
\end{pmatrix}
\]

(C.2)

related to the neutrino mass spectrum \(\{348, 349, 349, 714, 708, 707, 1227, 1225, 1225\}\) GeV.

Then one can find that all the chargino-sneutrino and the neutralino-slepton diagrams among the non-\(\lambda'\) diagrams in the cLFV decays of leptons make negligible contributions due to the vanishing of flavor mixing in sneutrino sector, as shown in Eq. (C.2), as well as the diagonal mass matrix of charged slepton for simplicity. As regards \(W/\bar{H}^\pm\) neutrino diagrams, they are always connected to terms \(\nu^T_{(\alpha+3)\nu} \nu^T_{(\beta+3)\nu} e^T_{(\alpha+3)v} \nu^T_{(\beta+3)v} \nu^T_{\alpha v} \nu^T_{\beta v}\), and their conjugate terms \((\alpha, \beta = e, \mu, \tau \text{ and } \alpha \neq \beta)\). Readers can see the calculations of these diagrams in Ref. [99]. With the numerical form of Eq. (C.1), the terms \(\nu^T_{(\alpha+3)\nu} \nu^T_{(\beta+3)\nu}\) and \(\nu^T_{(\alpha+3)v} \nu^T_{\beta v}\) vanish. The term \(\nu^T_{\alpha v} \nu^T_{\beta v}\) can be decomposed into two parts, \(\sum_{N=1}^3 \nu^T_{(\alpha)N} \nu^T_{(\beta)N}\) and \(\sum_{i=1}^3 \nu^T_{(\alpha)N} \nu^T_{(\beta)N}\), related to the nearly degenerate heavy neutrinos and light neutrinos, respectively [100]. Then one can also find that \(\nu^T_{\alpha v} \nu^T_{\beta v}\) makes no effective contribution to the cLFV decays. Thus, we conclude that the non-\(\lambda'\) diagrams provide negligible effects on the cLFV decays mentioned in Sect. 3.3 in our input sets.

References

1. LHCb Collaboration, R. Aaij et al. Test of lepton universality in beauty-quark decays. Nat. Phys. 18(3), 277–282 (2022). arXiv:2103.11769
2. LHCb Collaboration, R. Aaij et al. Test of lepton universality with \(B^0 \to K^{\pm}\ell^\mp\ell^-\) decays. JHEP 08(2017). arXiv:1705.05802
3. LHCb Collaboration, R. Aaij et al. Measurement of \(C P\)-averaged observables in the \(B^0 \to K^{\pm}\mu^+\mu^-\) decay. Phys. Rev. Lett. 125(1), 011802 (2020). arXiv:2003.04831
4. LHCb Collaboration, R. Aaij et al. Branching Fraction Measurements of the Rare \(B^0_s \to f^+1525\mu^+\mu^-\) and \(B^0 \to \phi\mu^+\mu^-\) decays. Phys. Rev. Lett. 127(15), 151801 (2021). arXiv:2105.14007
5. LHCb Collaboration, R. Aaij et al. Measurement of the \(B^0 \to \mu^+\mu^-\) decay properties and search for the \(B^0 \to \mu^+\mu^-\) and \(B^0_s \to \mu^+\mu^-\) decays. Phys. Rev. D 105(1), 012010 (2022). arXiv:2108.09283
6. LHCb Collaboration, R. Aaij et al. Analysis of neutral B-meson decays into two muons. Phys. Rev. Lett. 128(4), 041801 (2022). arXiv:2108.09284
7. C.M.S. Collaboration, A.M. Sirunyan et al. Measurement of properties of \(B^0 \to \mu^+\mu^-\) decays and search for \(B^0 \to \mu^+\mu^-\) with the CMS experiment. JHEP 04, 188 (2020). arXiv:1910.12127
8. HFLAV Collaboration, Y. Amhis et al. Averages of $b$-hadron, c-hadron, and $\tau$-lepton properties as of 2021. arXiv:2206.07501

9. HFLAV Collaboration, Y. S. Amhis et al. Averages of $b$-hadron, c-hadron, and $\tau$-lepton properties as of 2021. Eur. Phys. J. C 81(3), 226 (2021). arXiv:1909.12524

10. Muon g-2 Collaboration, B. Abi et al. Measurement of the positive muon anomalous magnetic moment to 0.46 ppm. Phys. Rev. Lett. 126(14), 141801 (2021). arXiv:2104.03281

11. Muon g-2 Collaboration, G. W. Bennett et al. Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL. Phys. Rev. D 73, 072003 (2006). arXiv:hep-ex/0602035

12. T. Aoyama et al. The anomalous magnetic moment of the muon in the Standard Model. Phys. Rept. 887, 1–166 (2020). arXiv:2006.04822

13. S. Borsanyi et al. Leading hadronic contribution to the muon magnetic moment from lattice QCD. Nature 593 (7857) 51–55 (2021). arXiv:2002.12347

14. C. Alexandrou et al. Lattice calculation of the short and intermediate-time-distance hadronic vacuum polarization contributions to the muon magnetic moment using twisted-mass fermions. arXiv:2206.15084

15. M. Če et al. Window observable for the hadronic vacuum polarization contribution to the muon $g - 2$ from lattice QCD. arXiv:2206.06582

16. G. Colangelo, A.X. El-Khadra, M. Hoferichter, A. Keshavarzi, C. Lehner, P. Stoffer, T. Teubner. Data-driven evaluations of Euclidean windows to scrutinize hadronic vacuum polarization. Phys. Lett. B 833, 137313 (2022). arXiv:2205.12963

17. A. Crivellin, M. Hoferichter, C.A. Manzari, M. Montull, Hadronic vacuum polarization: $(g - 2)_\mu$ versus global electroweak fits. Phys. Rev. Lett. 125(9), 091801 (2020). arXiv:2003.04886

18. A. Keshavarzi, W. J. Marciano, M. Passera, A. Sirlin, Muon $g - 2$ and $\Delta \alpha$ connection. Phys. Rev. D 102(3), 033002 (2020). arXiv:2006.12666

19. E. de Rafael, Constraints between $\Delta a(H_f^2)$ and $(g - 2)_{\text{HVP}}$. Phys. Rev. D 102(5), 056020 (2020). arXiv:2012.13880

20. B. Malaescu, Schott. Impact of correlations between $a_\mu$ and $\alpha_{\text{QED}}$ on the EW fit. Eur. Phys. J. C 81(1), 46 (2021). arXiv:2008.08107

21. G. Colangelo, M. Hoferichter, P. Stoffer, Constraints on the two-pion contribution to hadronic vacuum polarization. Phys. Lett. B 814, 136073 (2021). arXiv:2010.07943

22. C.D.F. Collaboration, T. Aaltonen et al. High-precision measurement of the W boson mass with the CDF II detector. Science 376(6589), 170–176 (2022)

23. M. Awramik, M. Czakon, A. Freitas, G. Weiglein, Precise prediction for the W boson mass in the standard model. Phys. Rev. D 69, 053006 (2004). arXiv:hep-ph/0311148

24. W. Altmannshofer, P. S. Bhupal Dev, A. Soni, $R_{D^{(*)}}$: anomaly: A possible hint for natural supersymmetry with $R$-parity violation Phys. Rev. D 96(9), 095010 (2017). arXiv:1704.06659

25. N. G. Deshpande, X.-G. He, Consequences of $R$-parity violating interactions for anomalies in $B \to D^{(*)}\tau \nu$ and $b \to s\mu^+\mu^-$. Eur. Phys. J. C 77(2), 134 (2017). arXiv:1608.04817

26. S. Trifinopoulos, Revisiting $R$-parity violating interactions as an explanation of the B-physics anomalies. Eur. Phys. J. C 78(10), 803 (2018). arXiv:1807.01638

27. W. Altmannshofer, P. B. Dev, A. Soni, Y. Sui, Addressing $R_{D^{(*)}}$, $R_{K^{(*)}}$, muon $g - 2$ and ANITA anomalies in a minimal $R$-parity violating supersymmetric framework. Phys. Rev. D102(1), 015031 (2020). arXiv:2002.12910

28. M.-D. Zheng, H.-H. Zhang, Studying the $b \to s\ell^+\ell^-$ anomalies and $(g - 2)_\mu$ in $R$-parity violating MSSM framework with the inverse seesaw mechanism. Phys. Rev. D 104(11), 115023 (2021). arXiv:2105.06954

29. Q.-Y. Hu, Y.-D. Yang, M.-D. Zheng, Revisiting the $B$-physics anomalies in $R$-parity violating MSSM. Eur. Phys. J. C 80(5), 365 (2020). arXiv:2002.09875

30. P.S. Bhupal Dev, A. Soni, F. Xu, Hints of natural supersymmetry in flavor anomalies. Phys. Rev. D 106, 015014 (2022). arXiv:2106.15647

31. M.-D. Zheng, F.-Z. Chen, H.-H. Zhang, The $W$-$\nu$-vertex corrections to $W$-boson mass in the $R$-parity violating MSSM. arXiv:2004.06541

32. M. Blennow, P. Coloma, E. Fernández-Martínez, M. González-López, Right-handed neutrinos and the CDF II anomaly. arXiv:2204.04559

33. F. Arias-Aragón, E. Fernández-Martínez, M. González-López, L. Merlo, Dynamical minimal flavour violating inverse seesaw. arXiv:2204.04672

34. J. Rosiek, Complete set of Feynman rules for the minimal supersymmetric extension of the standard model. Phys. Rev. D 41, 3464 (1990)

35. J. Rosiek, Complete set of Feynman rules for the MSSM:_Erratum. arXiv:hep-ph/9511250

36. P. Bhupal Dev, S. Mondal, B. Mukhopadhyaya, S. Roy, Phenomenology of Light Sneutrino Dark Matter in cMSSM/mSUGRA with Inverse Seesaw. JHEP 09, 110 (2012). arXiv:1207.6542

37. A.K. Alok, A. Dighe, S. Gangal, D. Kumar, Continuing search for new physics in $b \to s\mu\nu$ decays: two operators at a time. JHEP 06, 089 (2019). arXiv:1903.09617

38. J. Alda, J. Guasch, S. Penaranda, Anomalies in B mesons decays: a phenomenological approach. Eur. Phys. J. Plus 137(2), 217 (2022). arXiv:2012.14799

39. A. Carvunis, F. Dettori, S. Gangal, D. Guadagnoli, C. Normand, On the effective lifetime of $B_s \to \mu\mu$. JHEP 12, 078 (2021). arXiv:2102.13390

40. L.-S. Cheng, B. Grinstein, S. Jäger, S.-Y. Li, J. Martin Camalich, R.-X. Shi, Implications of new evidence for lepton-universality violation in $b \to s\ell^+\ell^-$ decays. Phys. Rev. D104(3), 035029 (2021). arXiv:2103.12738

41. S.-Y. Li, R.-X. Shi, L.-S. Geng, Discriminating 1D new physics solutions in $b \to s\ell\ell$ decays. Chin. Phys. C 46(6), 063108 (2022). arXiv:2105.06768

42. A. Angelescu, D. Bećirević, D. A. Faroughy, F. Jaffredo, O. Sumensari, Single leptoquark solutions to the B-physics anomalies. Phys. Rev. D 104(5), 055017 (2021). arXiv:2103.12504

43. W. Altmannshofer, P. Stangl, New physics in rare B decays after Moriond 2021. Eur. Phys. J. C 81(10), 952 (2021). arXiv:2103.13370

44. C. Corcella, D.A. Faroughy, J. Fuentes-Martín, G. Isidori, M. Neubert, Reading the footprints of the B-meson flavor anomalies. JHEP 08, 050 (2021). arXiv:2103.16558

45. J. Kriewald, C. Hati, J. Orloff, A. M. Teixeira, Leptoquarks facing flavour tests and $\Delta F = 3$ , $\Delta F = 2$ and $\Delta F = 1$ anomalies in a minimal $R$-parity violating MSSM. Eur. Phys. J. C 81(3), 035041 (2021). arXiv:2104.11229
50. J. Alda, J. Guasch, S. Penaranda, Anomalies in B mesons decays: Present status and future collider prospects. In *International Workshop on Future Linear Colliders*, 5, (2021). arXiv:2105.05095

51. R. Bause, H. Gisbert, M. Golz, G. Hiller, Interplay of dineutrino modes with semileptonic rare B-decays. JHEP 12, 061 (2021). arXiv:2109.01675

52. LHCh Collaboration, R. Aaij et al. Angular analysis and differential branching fraction of the decay $B_s^0 \rightarrow \phi \mu^+ \mu^-$. JHEP 09, 179 (2015). arXiv:1506.08777

53. G. Passarino, M.J.G. Veltman, One Loop Corrections for $e^+ e^-$ Annihilation Into mu+ mu- in the Weinberg Model. Nucl. Phys. B 160, 151–207 (1979)

54. F. U. Bernlochner, M. F. Sevilla, D. J. Robinson, G. Wormser, LHCb Collaboration, G. Aad et al. Searches for electroweak production of chargedinos and sleptons decaying into final states with two leptons and missing transverse momentum in $\sqrt{s} = 13 \text{ TeV} pp$ collisions using the ATLAS detector. Eur. Phys. J. C 80(2), 123 (2020). arXiv:1908.08215

55. ATLAS Collaboration, A. Aboub et al. Searches for electroweak production of supersymmetric particles with compressed mass spectra in $\sqrt{s} = 13 \text{ TeV} pp$ collisions with the ATLAS detector. Phys. Rev. D 101(5), 052005 (2020). arXiv:1911.12606

56. A.J. Buras, J. Girrbach-Noc, C. Niehoff, D.M. Straub, $B \rightarrow K^{(*)}\tau \nu_T$ decays in the Standard Model and beyond. JHEP 02, 184 (2015). [arXiv:1409.4557]

57. ATLAS Collaboration, A. Aboub et al. Search for lepton-flavor violating in different-flavor, high-mass final states in $pp$ collisions at $\sqrt{s} = 13 \text{ TeV}$ with the ATLAS detector. Phys. Rev. D 98(9), 092008 (2018). arXiv:1807.06573

58. CMS Collaboration, A. Aboub et al. Search for supersymmetry in events with four or more charged leptons in 139 fb$^{-1}$ of $\sqrt{s} = 13 \text{ TeV} pp$ collisions with the ATLAS detector. JHEP 07, 167 (2021). arXiv:2103.11684

59. ATLAS Collaboration, M. Aaboud et al. Search for lepton-flavor violation in different-flavor, high-mass final states in $pp$ collisions at $\sqrt{s} = 13 \text{ TeV}$ with the ATLAS detector. Phys. Rev. D 98(3), 032009 (2018). arXiv:1804.03602

60. ATLAS Collaboration, M. Aaboud et al. Search for lepton-flavor violation in different-flavor, high-mass final states in $pp$ collisions at $\sqrt{s} = 13 \text{ TeV}$ with the ATLAS detector. Phys. Rev. D 98(9), 092008 (2018). arXiv:1807.06573

61. ATLAS Collaboration, G. Aad et al. Search for supersymmetry in events with four or more charged leptons in 139 fb$^{-1}$ of $\sqrt{s} = 13 \text{ TeV} pp$ collisions using the ATLAS detector. Eur. Phys. J. C 80(2), 123 (2020). arXiv:1908.08215

62. ATLAS Collaboration, G. Aad et al. Searches for electroweak production of supersymmetric particles with compressed mass spectra in $\sqrt{s} = 13 \text{ TeV} pp$ collisions with the ATLAS detector. Phys. Rev. D 101(5), 052005 (2020). arXiv:1911.12606

63. ATLAS Collaboration, M. Aaboud et al. Search for B-L R-parity-violating decays in the Standard Model and beyond. JHEP 02, 184 (2015). [arXiv:1409.4557]

64. ATLAS Collaboration, G. Aad et al. Search for lepton-flavor violation in different-flavor, high-mass final states in $pp$ collisions at $\sqrt{s} = 13 \text{ TeV}$ with the ATLAS detector. Eur. Phys. J. C 80(2), 123 (2020). arXiv:1908.08215

65. ATLAS Collaboration, A. Aboub et al. Search for electroweak production of supersymmetric particles with compressed mass spectra in $\sqrt{s} = 13 \text{ TeV} pp$ collisions with the ATLAS detector. Phys. Rev. D 101(5), 052005 (2020). arXiv:2011.12066

66. ATLAS Collaboration, M. Aaboud et al. Search for lepton-flavor violation in different-flavor, high-mass final states in $pp$ collisions at $\sqrt{s} = 13 \text{ TeV}$ with the ATLAS detector. Phys. Rev. D 99(3), 032003 (2018). arXiv:1710.05544

67. CMS Collaboration, A.M. Sirunyan et al. Search for long-lived particles decaying into displaced jets in proton-proton collisions at $\sqrt{s} = 13 \text{ TeV}$. Phys. Rev. D 99(3), 032011 (2019). arXiv:1811.07991

68. ATLAS Collaboration, A. Aboub et al. Search for heavy charged long-lived particles in the ATLAS detector in 36.1 fb$^{-1}$ of proton-proton collision data at $\sqrt{s} = 13 \text{ TeV}$. Phys. Rev. D 99(9), 092007 (2019). arXiv:1902.01636

69. ATLAS Collaboration, G. Aad et al. Search for long-lived, massive particles in events with a displaced vertex and a muon with large impact parameter in $pp$ collisions at $\sqrt{s} = 13 \text{ TeV}$ with the ATLAS detector. Phys. Rev. D 102(3), 032006 (2020). arXiv:2003.11956

70. ATLAS Collaboration, G. Aad et al. Search for R-parity-violating supersymmetry in a final state containing leptons and many jets with the ATLAS experiment using $\sqrt{s} = 13 \text{ TeV} pp$ proton-proton collision data. Eur. Phys. J. C 81(11) 1023 (2021). arXiv:2106.09609

71. CMS Collaboration, A. M. Sirunyan et al. Search for top squark production in fully-hadronic final states in proton-proton collisions at $\sqrt{s} = 13 \text{ TeV}$. Phys. Rev. D 104(5), 052001 (2021). arXiv:2103.01290

72. CMS Collaboration, A. Tumasyan et al. Combined searches for the production of supersymmetric top quark partners in proton-proton collisions at $\sqrt{s} = 13 \text{ TeV}$. Eur. Phys. J. C 81(11), 970, (2021). arXiv:2107.10892

73. ATLAS Collaboration, M. Aaboud et al. Search for supersymmetry in events with four or more leptons in $\sqrt{s} = 13 \text{ TeV} pp$ collisions with ATLAS. Phys. Rev. D 98(3), 032009 (2018). arXiv:1804.03602

74. ATLAS Collaboration, G. Aad et al. Search for lepton-flavor violation in different-flavor, high-mass final states in $pp$ collisions at $\sqrt{s} = 13 \text{ TeV}$ with the ATLAS detector. Phys. Rev. D 98(9), 092008 (2018). arXiv:1807.06573
89. B. de Carlos, P.L. White, R-parity violation and quark flavor violation. Phys. Rev. D 55, 4222–4239 (1997). arXiv:hep-ph/9609443
90. HFLAV Collaboration, Y.S. Amhis et al. Averages of b-hadron, c-hadron, and τ-lepton properties as of 2018. Eur. Phys. J. C 81(3), 226 (2021). arXiv:1909.12524
91. M. Misiak et al. Updated NNLO QCD predictions for the weak radiative B-meson decays. Phys. Rev. Lett. 114(22), 221801 (2015). arXiv:1503.01789
92. I. Esteban, M. Gonzalez-Garcia, M. Maltoni, T. Schwetz, A. Zhou, The fate of hints: updated global analysis of three-flavor neutrino oscillations. JHEP 09, 178 (2020). arXiv:2007.14792
93. J. S. Alvarado, R. Martinez, PMNS matrix in a non-universal \( U(1)_X \) extension to the MSSM with one massless neutrino. arXiv:2007.14519
94. F.C.C. Collaboration, A. Abada et al. FCC physics opportunities: Future circular collider conceptual design report volume 1. Eur. Phys. J. C 79(6), 474 (2019)
95. Belle-II Collaboration, W. Altmannshofer et al. The Belle II Physics Book. PTEP 2019(12), 123C01 (2019). arXiv:1808.10567. [Erratum: PTEP 2020, 029201 (2020)]
96. A. M. Abdullahi et al. The Present and Future Status of Heavy Neutral Leptons. in 2022 Snowmass Summer Study 3 (2022). arXiv:2203.08039
97. C. A. Argüelles et al. Snowmass white paper: Beyond the standard model effects on neutrino flavor. in 2022 Snowmass Summer Study 3, (2022). arXiv:2203.10811
98. S. Pascoli, R. Ruiz, C. Weiland, Heavy neutrinos with dynamic jet vetoes: multilepton searches at \( \sqrt{s} = 14, 27, \) and 100 TeV. JHEP 06, 049 (2019). arXiv:1812.08750
99. A. Abada, M.E. Krauss, W. Porod, F. Staub, A. Vicente, C. Weiland, Lepton flavor violation in low-scale seesaw models: SUSY and non-SUSY contributions. JHEP 11, 048 (2014). [arXiv:1408.0138]
100. J. Chang, K. Cheung, H. Ishida, C.-T. Lu, M. Spinrath, Y.-L.S. Tsai, A supersymmetric electroweak scale seesaw model. JHEP 10, 039 (2017). arXiv:1707.04374