Quasi-one-dimensional topological-excitation liquid in Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ from tunneling spectroscopy

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Abstract. Tunneling measurements have been carried out on heavily underdoped, slightly overdoped and partially Ni-substituted Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ (Bi2212) single crystals by using a break-junction technique. We find that in-plane tunneling spectra below $T_c$ are the combination of incoherent part from the pseudogap and coherent quasiparticle peaks. There is a clear correlation between the magnitude of the pseudogap and the magnitude of the superconducting gap in Bi2212. The analysis of the data suggests that the tunneling pseudogap in Bi2212 is predominantly a charge-density-wave gap on dynamical charge stripes. The tunneling characteristics corresponding to the quasiparticle peaks are in excellent agreement with theoretical predictions made for a quasi-one dimensional topological-excitation liquid. In addition, the analysis of data measured by different techniques shows that the phase coherence along the $c$-axis is established at $T_c$ due to spin fluctuations in local antiferromagnetic domains of CuO$_2$ planes.

I. INTRODUCTION

Soon after the discovery of superconductivity in cuprates [1], it became clear that the concept of the Fermi liquid is not applicable to cuprates: the normal state properties of cuprates are markedly different from those of conventional metals [2]. The pseudogap (PG) which appears in electronic excitation spectra of cuprates above $T_c$ is one of the main features of high-$T_c$ superconductors (SCs). There is a consensus on doping dependence of the PG in hole-doped cuprates: the magnitude of the PG decreases with increase in hole concentration [3,4]. However, there is a clear discrepancy between the phase diagrams inferred from transport measurements, on the one hand, and from tunneling measurements, on the other hand: transport measurements [5-8] show that, in the overdoped region, the PG is absent above $T_c$, at the same time, in tunneling measurements [9-14] the PG is observed well above $T_c$.

The SC characteristics in cuprates have different doping dependences: the $T_c$ value has approximately the parabolic dependence on hole concentration, $p$, with the maximum near $p = 0.16$ [4], whereas the SC condensation energy has the maximum in the overdoped region near $p = 0.19$ [15].

Recent intrinsic $c$-axis tunneling data obtained in Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ (Bi2212) mesas [1] show that the pseudogap (PG) is the normal-state gap, and the PG and the superconducting gap (SG) coexist below $T_c$. High-resolution angle-resolved photoemission (ARPES) data [16] obtained in Bi2212 at momentum near the (0, $\pi$) also show that the quasiparticle (QP) peak and the gap have different origins, and they coexist below $T_c$. Thus, the PG in Bi2212 arises from either charge-density waves (CDW) or local antiferromagnetic (AF) correlations [or spin-density waves (SDW)], or from their combination [13,14,15,16]. It was proposed that, in order to develop further understanding of ARPES spectra, it is necessary to separate the coherent QP peak from the gap in ARPES spectra [17].

Tunneling spectroscopy is an unique probe of SC state in that it can, in principle, reveal the QP excitation density of states (DOS) directly with high energy resolution. In this paper we present tunneling measurements performed on heavily underdoped, slightly overdoped and partially Ni-substituted Bi2212 single crystals by using a break-junction technique. We find that in-plane tunneling spectra below $T_c$ are the combination of incoherent part from the PG and coherent QP peaks. There is a clear correlation between the magnitude of the PG and the magnitude of the SG in Bi2212. Analysis of the data suggests that the tunneling PG in Bi2212 is predominantly a CDW gap on dynamical charge stripes. The tunneling characteristics corresponding to the QP peaks are in excellent agreement with theoretical predictions made for a quasi-one dimensional (1D) topological-excitation liquid. In addition, analysis of data measured by different techniques shows that the phase coherence along the $c$-axis is established at $T_c$ due to spin fluctuations in local AF domains of CuO$_2$ planes. In the framework of the quasi-1D topological-excitation-liquid scenario in Bi2212, the discrepancy between the phase diagrams in the overdoped region, inferred from transport and tunneling measurements, becomes obvious: they measure two different PGs. Other data obtained in cuprates can be naturally understood in the framework of such a scenario for the SC in Bi2212. To our knowledge, the observation of a quasi-1D topological-excitation liquid in Bi2212 is presented in the literature for the first time.

II. THEORY AND MEASURED DATA

By performing tunneling measurements one can obtain the $I(V)$ and $dI/dV(V)$ characteristics. The $dI/dV(V)$ tunneling characteristic measured in a SC-insulator-
normal metal (SIN) junction corresponds directly to the DOS of QP excitations \([17]\). In this Section, first, we consider a theoretical \(I(V)\) tunneling characteristic in a SIN junction and a measured \(I(V)\) curve in a SC-insulator-SC (SIS) junction of Bi2212.

Figure 1(a) shows a theoretical \(I(V)\) characteristic in a SIN junction (Fig.6 in Ref. [18]). In the tunneling regime, it is expected that the \(I(V)\) curve at high positive (low negative) bias, depending on the normal resistance of junction, lies somewhat below (above) the normal-state curve [the dash line in Fig.1(a)] \([17]\). In conventional SCs, this prediction is verified by tunneling experiments \([17]\). However, we find that, in cuprates, the prediction is violated. Figure 1(b) shows the \(I(V)\) curve measured in a slightly underdoped Bi2212 single crystal with \(T_c = 83\) K (Fig.1 in Ref. [19]). In Fig.1(b), one can see that the \(I(V)\) curve at high positive (low negative) bias passes not below (above) the straight dash line but far above (below) the line. This fact cannot be explained by the d-wave symmetry of the order parameter. To our knowledge, this question has been never raised in the literature before. This finding is the main motivation of the present work.

Before we discuss our tunneling data, it is necessary to consider theoretical characteristics of a topological soliton and a bound state of two solitons. A topological soliton is an extremely stable nonlinear excitation which can be moving or entirely static \([20,21,22,23,24,25,26,27]\). The CDW and SDW instabilities in (spin-) Peierls systems are the consequence of lattice distortion. For example, in polyacetylene, the solitons are moving domain walls between the two degenerate dimerized phases \([27]\). Solitons (kinks) may carry a charge (which may be fractional) or spin of 1/2 or both. Figure 2(a) shows the characteristics of a kink. Since, in Fig.2, we consider the general solutions, the gap shown schematically in Fig.2 is either a CDW or SDW gap. So, the kinks occupy the midgap states. A kink (soliton) can form a bound state with another kink (soliton), which can be moving or stationary. Figure 2(b) shows the characteristics of a bound state of two solitons.

To our knowledge, a soliton SC was for the first time considered in Ref. [29] in order to explain the SC in quasi-1D organic conductors. In this model, two electron-solitons form a bisoliton having 2e charge and zero spin (a singlet state). The QPs surrounded by deformation move coherently as a unique entity along stacks of organic molecules without resistance. Later, Davydov \([22,23]\) applied the model of the bisoliton SC to cuprates. Recently, other authors proposed that the charge carriers in the normal state of cuprates reside on domain walls as a consequence of strong short-range electron-electron repulsion \([29]\).
The hole concentration in each of them estimated from empirical relation $T_c/T_{c,\text{max}} = 1 - 82.6(p - 0.16)^2$ is equal to 0.085, 0.19 and 0.2, respectively. The maximum $T_c$ value for the Ni-Bi2212 single crystals is estimated from $dT_c/dn_{Ni} \simeq -5$ K/at.%, where $n_{Ni}$ is the Ni content with respect to Cu. Initially, the tunneling measurements in the Ni-Bi2212 single crystals were motivated by the presence of magnetic impurities (Ni) in Bi2212. However, we use the data obtained in the Ni-Bi2212 as data in the overdoped region ($p = 0.2$) without emphasizing the presence of Ni.

Experimental details of our break-junction setup can be found elsewhere [21]. In short, many break-junctions were prepared by gluing a sample with epoxy on a flexible insulating substrate, and then were broken in the ab-plane by bending the substrate with a differential screw.

by EDAX. The Ni content with respect to Cu is approximately 1.5%. The $T_c$ value was determined by four-contact method. The transition width is less than 1 K in the overdoped Bi2212 crystals, and a few degrees in the Ni-Bi2212 single crystals. The underdoped samples were obtained from the overdoped single crystals by annealing them in vacuum.

The results presented here are obtained in three single crystals: one underdoped, one overdoped and one Ni-Bi2212, having the $T_c$ values of 51, 88 and 75, respectively. The hole concentration in each of them estimated from empirical relation $T_c/T_{c,\text{max}} = 1 - 82.6(p - 0.16)^2$ is equal to 0.085, 0.19 and 0.2, respectively. The maximum $T_c$ value for the Ni-Bi2212 single crystals is estimated from $dT_c/dn_{Ni} \simeq -5$ K/at.%, where $n_{Ni}$ is the Ni content with respect to Cu. Initially, the tunneling measurements in the Ni-Bi2212 single crystals were motivated by the presence of magnetic impurities (Ni) in Bi2212. However, we use the data obtained in the Ni-Bi2212 as data in the overdoped region ($p = 0.2$) without emphasizing the presence of Ni.

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IV. RESULTS

A. The underdoped region

Figure 3(a) shows the SIS $dI/dV(V)$ and $I(V)$ curves obtained in the underdoped Bi2212 single crystal, which look like usual tunneling spectra in Bi2212 [22]. In Fig.3(a), the Josephson $I_cR_n$ product is estimated to be 13.4 mV. The gap magnitude, $\Delta_{sc} = 64$ meV, is in good agreement with other tunneling measurements [22]. The $dI/dV(V)$ and $I(V)$ curves shown in Fig.3(b) are obtained within the same underdoped single crystal as those in Fig.3(a). In Fig.3(b), the wide humps resemble the humps in the $dI/dV(V)$ shown in Fig.3(a). It is suggestive that the spectra in Fig.3(b) correspond to the PG. The SC in Bi2212 is weak in the heavily underdoped region ($p < 0.1$) [22]. For example, in YBa$_2$Cu$_3$O$_{6+x}$ (YBCO), modest magnetic fields suppress the SC significantly in the heavily underdoped region [23]. This may explain why it is possible to observe separately the PG in the heavily underdoped Bi2212 by taking into account that tunneling spectroscopy probes the local DOS. The absence of the Josephson current in the spectra shown in Fig.3(b) indicates that the humps in the conductance are incoherent. The differences between the spectra shown in Figs 3(a) and 3(b) are presented in the inset of Fig.3(b), which correspond to a "pure SG". Some parts of the $dI/dV(V)$ in the inset of Fig.3(b) are slightly below zero because the spectra shown in Figs 3(a) and 3(b) are not taken under the exact same conditions. The small humps in $dI/dV(V)$ shown in the inset of Fig.3(b) are discussed below. The $dI/dV(V)$ and $I(V)$ spectra in the inset of Fig.3(b) resemble the characteristics of a bound state of

at low temperature in a He ambient. The electrical contacts (typically with the resistance of a few Ohms) were made by attaching gold wires to a crystal with silver paint. The $I(V)$ and $dI/dV(V)$ tunneling characteristics were determined by the four-terminal method by using a standard lock-in modulation technique. At low (constant) temperature, in one junction, we usually obtain a few tunneling spectra by changing the distance between broken parts of a crystal, going back and forth etc., and, every time, the tunneling occurs most likely in different places.

In addition to SIS-junction measurements, tunneling tests have been carried out in the overdoped Bi2212 single crystals by forming SIN junctions. Pt-Ir wires sharpened mechanically are used as normal tips.

The magnitude of the SG can, in fact, be derived directly from the tunneling spectrum. However, in the absence of a generally accepted model for the gap function and the DOS in cuprates, we calculate the gap magnitude $2\Delta$ as a half spacing between the coherent QP peaks in SIS conductance tunneling characteristics.
two solitons, shown in Fig.2(b). Thus, we find that the conductance in Fig.3(a) consists of two contributions: the incoherent humps which correspond to the PG (which is a normal-state gap $\Delta_0$), and the coherent QP peaks from condensed solitonlike excitations. In Fig.3(b), one can see that the PG is anisotropic.

We discuss now the PG. Quasi-1D topological excitations reside on quasi-1D "objects". Recently, quasi-1D charge stripes separated by 2D AF domains have been discovered in some cuprates $^{33,34,35}$. The presence of solitonlike excitations on stripes implies that the stripes are insulating. In other words, there is a charge gap along stripes, which is most likely a CDW gap. Then, the tunneling PG is predominantly a CDW gap. The same conclusion can be independently obtained from the analysis of the data: The magnitude of PG, $\Delta_{ps} = 130$ meV, shown in Fig.3(b), is too large to be explained by the development of local AF correlations, since the value of the superexchange energy, $J \approx 120$ meV $^{36}$, is not large enough to fit the data. Moreover, by taking into account that the PG increases with decrease in hole concentration, at lower doping ($p < 0.085$), the magnitude of the PG will become larger than 130 meV. Thus, one can conclude that the PG is too large to be the spin gap due to local AF correlations. For the second time, independently, we find that the tunneling PG is most likely a CDW gap.

It is possible that the subgap in the $dI/dV(V)$ shown in Fig.3(b) is due to the stripe excitations which were damped at higher bias by tunneling electrons. The SC is weak in the underdoped region $^{37,38}$.

**B. The overdoped region**

Figures 4(a) and 4(b) show the SIS $dI/dV(V)$ and $I(V)$ curves obtained in an overdoped Bi2212 single crystal. In Figs 4(a) and 4(b), the Josephson $I_cR_n$ product is estimated to be 6 and 7.5 mV, respectively. The $dI/dV(V)$ and $I(V)$ characteristics shown in Fig.4(a) look like usual tunneling spectra in Bi2212 $^{32}$. The $dI/dV(V)$ and $I(V)$ curves in Fig.4(b) resemble the spectra shown in the inset of Fig.3(b), and the characteristics of a bound state of two solitons in Fig.2(b). This means that, the contribution from the PG in the tunneling spectra shown in Fig.4(b) is small in comparison with the contribution from the QP peaks, at least, at low bias. This is most likely due to the fact that, in slightly overdoped cuprates, the SC is the strongest, and the "strength" of the PG is weak $^{31,32}$. At high bias, the contribution from the PG will be always predominant, even if the PG is weak.

In order to show that we observe not a SIS-junction effect but an intrinsic effect, we performed measurements in the overdoped Bi2212 single crystals by SIN junctions. The inset of Fig.4(a) shows the SIN $dI/dV(V)$ and $I(V)$ curves obtained in an overdoped Bi2212. In Fig.4(a), one can see that, basically, there is no difference between the $I(V)$ characteristics measured in SIS and SIN junctions [see the dash lines in Fig.4(a) and in the inset of Fig.4(a)].

The inset of Fig.4(b) shows the PG in the slightly overdoped Bi2212 at 122 K, which looks like the typical CDW gap in quasi-1D conductor NbSe$_3$ $^{37,38}$. The value of 122 K is chosen not by accident. Two independent tunneling studies show that, in slightly overdoped Bi2212, the onset of SC occurs at 110–116 K $^{39}$. Thus, the value of 122 K is above the onset of SC in slightly overdoped Bi2212. In the inset of Fig.4(b), one can see that the PG is anisotropic. The shape of the PG at low bias indicates that it has most likely an anisotropic $s$-wave rather than the $d$-wave shape.

We discuss now the tunneling data measured in Ni-Bi2212 single crystals which are also overdoped in oxygen. Figures 5(a) and 5(b) show the SIS $dI/dV(V)$ and $I(V)$ curves obtained within the same Ni-Bi2212 single crystal. In Figs 5(a) and 5(b), the Josephson $I_cR_n$ product is estimated to be 3.1 and 24.5 mV, respectively. The SIS $dI/dV(V)$ and $I(V)$ curves shown in Fig.5(a) look like usual tunneling spectra in Bi2212 $^{32}$. The $dI/dV(V)$ and $I(V)$ curves in Fig.5(b) resemble the spectra in the inset of Fig.3(b) and the characteristics of a bound state of two solitons shown in Fig.2(b). The effect of the absence of the contribution from the PG in the
tunneling spectra shown in Fig.5(b) is even stronger than that in Fig.4(b). It is important to emphasize that the spectra measured in underdoped Bi2212, shown in in the inset of Fig.3(b), and the spectra measured in overdoped Bi2212 and Ni-Bi2212, shown in Figs 4(b) and 5(b), are similar. So, the data obtained in underdoped and over-
doped Bi2212 are consistent with each other. However, in the overdoped region, the "strengths" of the PG and SG are reversed in comparison with those in the under-
doped region. In the overdoped region, the SC is the strongest, and the PG is weak [4,11]. In Fig.5(a), one can see that the humps in the conductance measured in Ni-doped Bi2212 are weaker than those in Figs 3(a) and 4(a). This means that Ni destroys the PG (CDW gap), so stripes become more metallic. This is true from another point of view: many of our attempts to measure the PG above $T_c$ in Ni-Bi2212 have failed.

Figure 6 displays temperature dependence of the con-
ductance shown in Fig.5(b). In Fig.6, the QP peaks dis-
appear below $T_c = 75$ K. This is due to the fact that

tunneling spectroscopy probes the local DOS; the $T_c$

is the macroscopic characteristic. Thus, in Fig.6, $T_{c,\text{local}} \approx 70$ K. Apparently, the data shown in Fig.6 are measured near an impurity (i.e. Ni). It seems that the small humps which appear in the $dI/dV(V)$ shown in Figs 4(b) and 5(b) at bias twice as large as bias of the QP peaks, relate to the QP peaks and not to the PG. In Fig.6, their temperature dependence resembles the temperature dependence of a SG. In the inset of Fig.3(b), a similar lump

![FIG. 5. SIS $dI/dV(V)$ and $I(V)$ measured at 15 K within the same Ni-Bi2212 single crystal with $T_c = 75$ K. Inset in the plot (b): $dI/dV(V)$ and $I(V)$ characteristics of the main plot at 70.3 K. The dash lines which are parallel to the $I(V)$ curves at high bias are guides to the eye.](image1)

![FIG. 6. Temperature dependence of the conductance peaks shown in Fig.5(b). The conductance scale corresponds to the 70.3 K spectrum, the other spectra are offset vertically for clarity.](image2)

![FIG. 7. Low temperature phase diagram of Bi2212 based on the present data: $\Delta_{sc}$ (squares), $\Delta_{ps}$ (diamonds), and $T_c$ (dots) The solid line corresponds to $T_c/T_{c,\text{max}} = 1 - 82.6(p - 0.16)^2$ [4]. The dash lines are guides to the eye.](image3)
C. The phase diagram

Figure 7 depicts the phase diagram of Bi2212 based on the present data. The magnitude of the SG is in good agreement with other tunneling measurements [32]. In Fig.7, one can see that $\Delta_{sc}$ and $T_c$ do not correlate with each other. As mentioned above, the tunneling PG shown in Fig.7 is different from the PG inferred from transport measurements [3,4,5].

V. THE MEASURED DATA AND THEORY

We compare now the tunneling data with theory. The characteristics of a kink and a bound state of two solitons are described by hyperbolic functions [20,21,22,23,24]. Since the conductance peaks of a bound state of two solitons, which are shown in Fig.2(b), look very similar to the conductance peaks not only of high-$T_c$ SCs but also of low-$T_c$ SCs (not the background), we rely here exclusively on the $I(V)$ characteristics which are conceptually different for the two models based on the Fermi liquid, in the first model, and on a quasi-1D topological-excitation liquid, in the second model.

A. $I(V)$ characteristics

Figure 8(a) shows the measured $I(V)$ curve from the inset of Fig.3(b). In Fig.8, for simplicity, we analyze the data only at positive bias. As shown in Fig.8(a), the data from the inset of Fig.3(b) can be fitted very well by the hyperbolic function $f(V) = A \times (\tanh(eV - 2\Delta)/eV_0 + \tanh(eV + 2\Delta)/eV_0)$, where $e$ is the electron charge; $V$ is the bias; $\Delta$ is the maximum SC energy gap, and $A$ and $V_0$ are the constants. In Fig.8(a), we also present the measured $I(V)$ curve of the PG from Fig.3(b). We find that any tunneling $I(V)$ curve in Bi2212 can be resolved into the two components shown in Fig.8(a). Thus, the $I(V)$ characteristics in Bi2212 (as well as the $dI/dV(V)$ characteristics) consist of two contributions: from the quasi-1D topological excitations and from the PG which is the normal-state gap $\Delta$. The "usual" $I(V)$ and $dI/dV(V)$ spectra in Bi2212 show the presence of both components [see Figs 3(a), 4(a) and 5(a)]. The absence [see Fig.3(b)] or weak contribution of one component [see Figs 4(b) and 5(b)] in tunneling spectra make the appearance of the spectra "unusual".

First, we analyze the tunneling data from Ref. [14]: Figure 8(b) shows the data from Fig.1(b), the $f(V)$ fit, and their difference. Figure 8(c) depicts the two components in the $I(V)$ curve from Fig.4(b). As shown in Fig.8(d), the contribution from the QP peaks in the SIN $I(V)$ from the inset of Fig.4(a) seems to be weaker than that in SIS junctions. As seen in Fig.8, all plots are similar. To fit the SIN $I(V)$, we use the same $f(V)$ function by substituting $2\Delta$ for $\Delta$. In Figs 8(b)–8(d), the amplitude, $A$, of the $f(V)$ fit can be changed, this only affects the scale but not the shape of the differences which correspond to the PG. The decomposition of the $I(V)$ curves in Figs 8(b)–8(d) into the two components explains why $I(V)$ tunneling characteristics in cuprates do not obey the theoretical predictions based on the Fermi-liquid model [18]. The $I(V)$ curves in Figs 4(a) and 5(a) and in Refs [14] and [22] can be resolved into the two components in the same manner. The data in Fig.3 are self-consistent.

FIG. 8. Measured $I(V)$ curves and the $f(V)$ fit (see text) in (a)-(b) underdoped and (c)-(d) overdoped Bi2212: (a) The data (diamonds) from the inset of Fig.3(b); the measured $I(V)$ of the PG from Fig.3(b), and the $f(V)$ fit. (b) The data from Ref. [19] shown in Fig.1(b); the $f(V)$ fit, and their difference. (c) The $I(V)$ from Fig.4(b); the $f(V)$ fit, and their difference. (d) The SIN $I(V)$ from the inset of Fig.4(a); the $f(V)$ fit [2\Delta \rightarrow \Delta$ in $f(V)$], and their difference. In the plots (b) and (c), the Josephson currents are removed.

FIG. 9. Measured $I(V)$ curves and the $f_n(V)$ fit (see text) in Ni-Bi2212: (a) The $I(V)$ from Fig.5(b); the $f(V)$ fit, and their difference. (b) The $I(V)$ from the inset of Fig.5(b) (diamonds) (the linear background is subtracted) and the $f_n(V)$ fit. In the plot (a), the Josephson current is removed.
However, we find that the $I(V)$ curve shown in Fig.5(b) consists of three components. Figure 9(a) shows the $I(V)$ from Fig.5(b), the $f(V)$ fit and their difference. In Fig.9(a), it is clear that the difference consists of two contributions: the contribution at high bias is predominantly from the PG, similar to those in Fig.8, and the contribution at low bias with the peak at 35 mV. By analyzing the data we show below that this peak corresponds to electron tunneling assisted by spin excitations [39]. In other words, this peak corresponds to the magnetic resonance peak observed in inelastic neutron scattering experiments [40].

Figure 9(b) shows the $I(V)$ curve from the inset of Fig.5(b), where the linear contribution [see the dash line in the inset of Fig.5(b)] is subtracted. As mentioned above, the $I(V)$ curve in Fig.9(b) is very similar to the $I(V)$ characteristic of a kink, shown in Fig.2(a). This means that, at this temperature, the quasi-1D topological excitations are already decoupled. We use the hyperbolic function $f_n(V) = A_1 \times \tanh(V/V_0)$ to fit the data in Fig.9(b), which gives a good interpolation. Thus, there is good agreement between the data below $T_c$ and above (local) $T_c$.

In conventional SCs, tunneling conductances measured in SIN and SIS junctions contain different information. If a SIN conductance corresponds directly to the SC DOS, a SIS conductance is the convolution of the DOS with itself [17]. In Fig.8, we use the same function to fit $I(V)$ characteristics measured in SIS and SIN junctions. Physically, it is incorrect. However, the main point of the fit is that the asymptotics of $I(V)$ characteristics corresponding to QP peaks, measured either in SIS or SIN junctions, are “flat” (constant). The “flat” asymptotics in tunneling spectra are a hallmark of the presence of one dimensionality in the system. Such asymptotics never appear in 2D or 3D systems.

So, we conclude that, in Bi2212, the $I(V)$ characteristics corresponding to the QP peaks definitely disagree with Blonder-Tinkham-Klapwijk’s predictions based on the Fermi-liquid model [13], and are in good agreement with the theoretical predictions made for a quasi-1D topological-excitation liquid [20,21,22,23,24].

B. $dI/dV(V)$ characteristics

As emphasized above, we do not rely on the $dI/dV(V)$ data in order to draw any conclusion about the origin of QPs in Bi2212. However, the analysis of the data would be not complete, if we did not consider the conductance spectra. The coherent QP peaks in the SIS tunneling spectra can be fitted very well by the derivative $[f(V)]' = A_2 \times (\text{sech}(eV - 2\Delta)/eV_0)^2 + (\text{sech}(eV + 2\Delta)/eV_0)^2$, as shown in Fig.10. The fit is applicable only to the coherent QP peaks, and not to the humps at high bias, which correspond to the PG, and neither to the Josephson current. The $dI/dV(V)$ curves in Figs 4(a) and 5(a) and in Figs 10(b) and 10(c) can be fitted in the same manner. For example, the conductance in an overdoped Bi2212 single crystal, shown in Fig.2 of Ref. [19], clearly is the combination of QP peaks and humps.

In Fig.10(c), the coherent QP peaks contain also a contribution from electron tunneling due to spin excitations [see Fig.9(a)]. As seen in Figs 5(b) and 6, the tops of the QP peaks have a composite structure which we attribute to the presence of the contribution from electron tunneling due to spin excitations: This contribution is literally developed on top of the QP peaks. In Fig.10(d), in order to fit the SIN $dI/dV(V)$ at low bias, we had to shift up the $[f(V)]'$ curve.

The conductance shown in the inset of Fig.5(b) can be fitted by $[f_n(V)]' = A_3 \times (\text{sech}(V/V_0))^2$.

C. Small humps

The small humps which appear in the $dI/dV(V)$ at bias twice larger than bias of the QP peaks, shown in Figs 4(b) and 5(b) as well as in Figs 10(b) and 10(c), can be understood in terms of a nanopteron soliton [2]. A nanopteron is a bound state resulting from the nonlinear interaction between the soliton (or kink) and the periodic wave (see Fig.6.27(c) in Ref. [20]). Instead of flat asymptotics, the characteristics of a nanopteron have oscillations. A bound state of two solitons can also exist in resonance with small amplitude linear waves (see Fig.1 in Ref. [11]). Since, in Figs 4(b) and 5(b), the predomi-
nant contribution in the conductances at high bias arises from the PG, it is impossible to follow the oscillations in the conductances at high bias. Fortunately, the oscillations can be also seen in the $dI/dV(V)$ characteristics shown in Figs 8(a) and 8(c). In Fig.8(a), the oscillations can be observed up to 480 mV. In Fig.8(c), only one hump is present in the $dI/dV$ characteristic at about 105 mV. The next question is to understand what periodic waves in Bi2212 can interact with the bound states of two solitons and cause these oscillations? Phonons are the primary candidate on this role. It is also possible that magnonlike excitations (spin 1, charge 0) which cause the appearance of the magnetic resonance peak in neutron spectra [40] correspond to these linear waves, which are in resonance with bound states of topological solitons.

It is well known that $dI/dV(V)$ characteristics measured in SIS junctions often have subgap structures [see, for example, Fig.4(a)]. These subgap structures can be also understood in terms of a nanopteron soliton. They most likely have the same origin as the oscillations at high bias, i.e. due to the nonlinear interaction between a bound state of two solitons and the periodic wave (see Fig.4 in Ref. [41]).

VI. STRIPES AND TOPOLOGICAL EXCITATIONS

Here we discuss the stripes in cuprates and stripes excitations.

Analysis of the data obtained in inelastic neutron scattering experiments suggests that the stripes in cuprates are of the $2k_F$ type [42]. The chains in YBCO also have the $2k_F$ charge modulation, as observed by tunneling spectroscopy [43]. The $2k_F$ ordering pattern on a stripe is shown schematically in Fig.11(a). Recent simulations of ARPES data in Bi2212 show that the stripes in Bi2212 are most likely site-centered [44]. So we consider only the $2k_F$ site-centered stripes.

The stripes in cuprates are dynamical [44]. Recently, the dynamics of insulating charge stripes was studied by Zaanen and co-workers [42, 45, 46]. Figures 11(b) and 11(c) show a soliton and a kink on $2k_F$ stripes, respectively. In the $2k_F$ charge modulation along stripes, a kink (soliton) carries an $e$ charge, consequently, a bi-soliton has $2e$ charge. Apparently, charge carriers in nanotubes are also topological solitons [14].

The soliton SC scenario, probably, is realized in quasi-1D organic metals [18], ladders [19], and on CuO chains in YBCO which have a CDW ground state [50]. The latter fact can be naturally understood since the stripes in CuO planes have also the CDW ground state. Recently, midgap solitons have been observed on CuO chains in YBCO by tunneling spectroscopy [51]. The spectrum averaged along a CuO chain shows that there is a weak bound state of solitons inside the CDW gap. The magnitude of the induced SG of the bound state of solitons is about 6 meV.

Charged solitons repel each other. In conventional SCs, two electrons which form a Cooper pair also repel each other. The occurrence of an attractive potential between electrons is central to the SC state. So, charged solitons will couple with each other if there is an attractive potential between them.

The next question is to understand how is the long-range phase coherence established? In CuO$_2$ planes, the phase coherence can occur, for example, due to the Josephson coupling between SC stripes. Indeed, the so-called Yamada plot suggests that $T_c$ increases if and only if stripes move closer together [52]. Since the SC in CuO$_2$ planes is fully 2D because the coupled excitations reside on stripes, the phase coherence along the c-axis is most likely established due to a different mechanism.

VII. C-AXIS PHASE COHERENCE AND SPIN FLUCTUATIONS IN CUPRATES

There is a consensus that the SC in cuprates is two-dimensional (2D). It is widely believed that the long-range phase coherence occurs at $T_c$ due to the Josephson coupling between SC CuO$_2$ (bi-, tri-, ...)layers. Recent measurements of the in-plane ($\rho_{ab}$) and out-of-plane ($\rho_{c}$) resistivities in Tl$_2$Ba$_2$CaCu$_2$O$_8$ as a function of applied pressure show that $\rho_{c}(T)$ shifts smoothly down with increase of pressure, however, $T_c$ first increases and then decreases [53]. This result can not be explained by the interlayer Josephson-coupling mechanism. The authors conclude [53]: “Any model that associates high-$T_c$ with the interplane Josephson coupling should therefore be revisited.”

Here we analyze data obtained in Andreev reflection, inelastic neutron scattering (INS), microwave, muon spin relaxation (µSR), tunneling and resistivity mea-

![Fig. 11. 2k_F half-filled stripes: (a) reference state; (b) soliton (S) and antisoliton, and (c) kink (K) and antikink. The full/empty circle denotes the presence/absence of the hole.](image-url)
measurements performed on different cuprates, mainly, on YBCO, Bi2212 and La2−xSrxCuO4+y (LSCO). Analysis of the data shows that the long-range phase coherence in the cuprates intimately relates to AF interactions along the c axis. We also analyze data measured in heavy fermions UPt3, UPd2Al3 and CeIrIn5, and in some layered non-SC compounds with ferromagnetic (FM) correlations.

A. Introduction

SC and magnetism were earlier considered as mutually exclusive phenomena. Recent research revealed a rich variety of extraordinary SC and magnetic states and phenomena in novel materials that are due to the interaction between SC and magnetism[14]. The coexistence of SC and long-range AF order was first discovered in RMoSes (R = Gd, Tb and Er), RRh4B4 (R = Nd, Sm and Tm), and RMgO2S8 (R = Gd, Tb, Dy and Er)[54]. Later, coexistence of SC and AF order was found in U-based heavy fermions (UPt3, URu2Si2, UNi2Al3, UPd2Al3, U6Co, and U6Fe), in heavy fermions RRu2Si2 (R = La and Y), Cr1−xRex, CeRu2, in borocarbides RNi2B2C (R = Tm, Er, Ho, Dy), in organic SCs[55,56] and in the new heavy fermion CeRh1−xIr1xIn5[53]. In CeRh0.5Ir0.5In5, the bulk SC coexists microscopically with small-moment magnetism (≤ 0.1μB)[57]. In all other heavy fermions, there are strong AF correlations present in the SC state[14,54,55]. Another class of materials in which SC and AF coexist are cuprates such as YBCO and LSCO compounds[54]. Coexistence of SC and FM order is found in the heavy fermion UGe2[54], and in Ru-based materials, for example, in RuSr2GdCu2O8[57].

In SC heavy-fermion systems, spin fluctuation (electron-electron interactions) are believed to mediate the electron pairing that leads to SC[55]. For the heavy fermions CeIr2, CePd2Si2[55], UPd2Al3[56] and UGe2[54], there is indirect evidence for spin-fluctuation mechanism of SC. This intimate relationship between the SC and magnetism also appears to be central to the SC cuprates[54], which inherited the magnetic properties from their parent compounds, AF Mott insulators. Many theoretical studies suggest that the SC in cuprates is mediated via the exchange of AF spin fluctuations[56].

In cuprates, there are two energy scales[52]: the pairing energy scale, Δp (see Δsc in Fig.7), and the phase-coherence scale, Δc, observed experimentally[52,53]. The two energy scales have different dependences on hole concentration, p, in CuO2 planes: Δp increases linearly with decrease in hole concentration, whereas Δc has approximately the parabolic dependence on p and scales with Tc as 2Δc ≃ 5.4kBTc[52].

FIG. 12. Temperature dependences of in-plane (dots) and c-axis tunneling QP peaks (squares) in slightly overdoped Bi2212 single crystals, ∆(T)/∆(Tmin). The BCS temperature dependence is shown by the solid line. The dashed lines are guides to the eye.

B. In-plane and c-axis tunneling in Bi2212

Figure 12 shows the temperature dependence of in-plane tunneling QP peaks, measured in slightly overdoped Bi2212 single crystals[52,53,54]. Tunneling measurements performed on slightly underdoped Bi2212 single crystals show similar temperature dependence[52]. So, the temperature dependence of in-plane QP peaks shown in Fig.12 can be considered as typical. In Fig.12, we also present the temperature dependence of c-axis QP peaks measured in slightly overdoped Bi2212 mesas[53]. Measurements performed on micron-size mesas present intrinsic properties of the material. In Fig.12, surprisingly, the temperature dependences along the ab planes and along the c axis are different. It is a clear hallmark of the coexistence of two different SC mechanisms in Bi2212: in-plane and along the c axis.

C. LSCO

In LSCO, there is evidence that the SC intimately relates to the establishment of AF order along the c axis. Recent μSR measurements performed on non-SC Eu-doped LSCO having different hole concentrations show that the SC phase of pure LSCO is replaced in Eu-doped LSCO by the second AF phase (see Fig.4 in Ref. [14]). Thus, the data show that it is possible to switch the entire hole concentration dependent phase diagram from SC to AF. It is a clear hallmark that the SC in LSCO intimately relates to the formation of AF order. We return to this important result later.

We turn now to the analysis of resistivity data measured in non-SC 2D layered compounds with AF or FM correlations. The data clearly show that, in all these layered compounds, the out-of-plane resistivity, ρc, has drastic changes either at Néel temperature, TN, or Curie
temperature, $T_C$, whereas the in-plane resistivity, $\rho_{ab}$, passes through $T_N$ or $T_C$ smoothly.

We start with YBCO. In AF undoped YBCO ($x = 0.35$; $0.33$; and $0.32$) having $T_N \approx 80$ K, $160$ K, and $210$ K, respectively, $\rho_c$ shows a sharp increase, by about 2 orders of magnitude, upon cooling through $T_N$ \[55\]. At the same time, $\rho_{ab}$ changes smoothly at $T_N$. The same effect has been observed in LuBCO ($x = 0.34$) \[66\]. So, the Néel ordering in undoped YBCO has remarkably different impact on the electron transport within CuO$_2$ planes and between them. The authors conclude \[55\]: “The Néel temperature actually corresponds to the establishment of AF order along $c$ axis.”

We now discuss the FM layered compounds. The structure of the FM compound Bi$_2$Sr$_3$Cu$_2$O$_y$ (BScoO) is similar to the structure of Bi$_2$2212, where CoO$_2$ planes are analogous with CuO$_2$ planes in Bi$_2$2212 \[67\]. BScoO becomes FM at $T_C = 3.2$ K. The resistivity data show that, at $T_C$, there is a cusp in $\rho_c$, but $\rho_{ab}$ changes smoothly. The authors conclude that the long-range magnetic order in BScoO develops at $T_C$ along the $c$ axis. In layered manganite La$_{1-x}$Sr$_x$Mn$_2$O$_7$ (LSMO) which is composed of the MnO$_2$ bilayers becomes FM at $T_C = 90$ K \[68\]. The resistivity data show that, at $T_C$, there are drastic changes in $\rho_c$ (a few orders of magnitude), but very small changes in $\rho_{ab}$. They conclude that, in LSMO, the long-range magnetic order develops at $T_C = 90$ K along the $c$ axis \[68\].

Neutron fermion URu$_2$Si$_2$ ($T_C = 1.2$ K) show that the AF order develops at $T_N = 17.5$ K along the $c$ axis \[64\]. So, it seems that, in all layered compounds, the long-range AF or FM order develops at $T_N$ or $T_C$ along the $c$ axis (the in-plane magnetic correlations exist above $T_N$ and $T_C$ \[8\]).

We return now to the analysis of the phase diagram of non-SC Eu-doped LSCO, where the SC phase of pure LSCO is replaced by the second AF phase \[54\]. The conclusion made in the previous paragraph signifies that either the main AF phase of Eu-doped LSCO or the second AF phase develops along the $c$ axis. Thus, the SC phase of pure LSCO is replaced in Eu-doped LSCO by the AF phase which develops along the $c$ axis. Consequently, the SC in LSCO intimately relates to the establishment of the long-range AF order along the $c$ axis.

In heavy fermion CeIrIn$_5$, $\mu$SR measurements discovered the onset of a small magnetic field ($\sim 0.4$ Gauss) which sets exactly at $T_s$ \[53\]. In YBCO ($x = 0.6$), recent INS measurements identified small magnetic moments directed along the $c$ axis, which increase in strength at and below $T_c$ \[70\].

### D. YBCO, Bi2212 and LSCO

Here we compare coherence SC and magnetic characteristics of YBCO, Bi2212 and LSCO. The comparison shows that the magnetic and coherence SC characteristics have similar temperature dependencies, and, at different dopings, their magnitudes are proportional to each other (and proportional to $T_c$). Thus, the coherence SC and magnetic properties of cuprates intimately relate to each other.

First, we describe the magnetic properties of cuprates. The low energy magnetic excitations in LSCO cuprate have been extensively studied, and the observed spin fluctuations are characterized by wave vector which is incommensurate with the lattice \[71\]. These modulated spin fluctuations in LSCO persist in both normal and SC states. The spin dynamics in YBCO and Bi2212 studied by INS exhibit below $T_c$ a sharp commensurate resonance peak which appears at well defined energy $E_r$ \[72,73,74,75,76,77,78,79,80,81,82\]. Incommensurability in YBCO has been also reported \[74\], and it is consistent with that in LSCO of the same hole doping, but, in YBCO, it occurs in the SC state. Now it is clear that the incommensurability and the commensurate resonance are inseparable parts of the general features of the spin dynamics in YBCO at all doping levels \[83\]. Thus, there is a clear evidence of coexistence of AF order and SC below $T_c$, at least, in LSCO and YBCO.

We now compare coherence SC and magnetic characteristics of the cuprates. Figure 13(a) shows the temperature dependences of the superfluid density in near optimally doped single crystals of Bi2212 \[43\] and YBCO \[85\], and in an overdoped LSCO ($x = 0.2$) single crystal \[86\], measured by microwave, $\mu$SR and ac-susceptibility techniques, respectively. The superfluid density is proportional to $1/\lambda^2(T)$, where $\lambda(T)$ is the magnetic penetration depth. Figure 13(b) shows the temperature dependences of Andreev-reflection gap measured in an overdoped Bi2212 thin film \[74\] and in overdoped single crystals of Bi2201 \[88\] and LSCO ($x = 0.2$) \[89\]. It is important to emphasize that Andreev reflections are exclusively sensitive to coherence properties of condensate. In Figs 13(a) and 13(b), one can see that there is good agreement among temperature dependences of coherence SC characteristics of different cuprates.

Figure 13(c) shows the temperature dependences of the peak intensity of the incommensurate elastic scattering in LSCO ($x = 0$) \[71\] and the intensity of the commensurate resonance peak measured by INS in near optimally doped Bi2212 \[72\] and YBCO \[70\].

In Fig.13, all temperature dependences of coherence SC and magnetic characteristics below $T_c$ exhibit a striking similarity. Since all temperature dependences shown in Fig.13 are similar to the temperature dependence of $c$-axis QP peaks in Bi2212, shown in Fig.12, and dif-
coincide with \( \Delta \) dopings, 

correlogram suggests that spin excitations are responsible to the magnetic interactions, at least, along the \( c \) axis.

Moreover, in YBCO \( (x = 0.6) \), recent INS measurements found small magnetic moments directed along the \( c \) axis, which increase in strength at and below \( T_c \) [70]. At the same time, as one can see in Figs 12 and 13, the in-plane mechanism of the SC in the cuprates has no or little relations to the magnetic interactions, at least, along the \( c \) axis.

Figure 14 shows the energy position of the magnetic resonance peak, \( E_r \), in Bi2212 [72], Bi2201 [73], YBCO [74,75,76,77,78,79,80,81,82] as a function of doping. The parabolic curve corresponds to the coherence energy scale, \( \Delta_c \), which is proportional to \( T_c \) as \( 2\Delta_c \simeq 5.4\kappa_B T_c \) [82]. At different doping levels, Andreev-reflection data coincide with \( \Delta_c \) [72,83,84]. In Fig.14, we present also the \( \Delta_c \) data from Fig.7, which correspond to the pairing energy scale [72]. In Fig.14, one can see that, at different dopings, \( E_r \) is proportional to \( \Delta_c \) as \( E_r \simeq 2\Delta_c \). This correlation suggests that spin excitations are responsible for establishing the phase coherence in YBCO and Bi2212 since the relation \( E_r \simeq 2\Delta_c \) is in good agreement with the theories in which the SC is mediated by spin fluctuations [81]. Recently, it was shown that applied magnetic fields suppress the magnetic resonance peak in YBCO, indicating that the resonance peak indeed measures the long-range phase coherence [85]. The strength of coupling between spin excitations and charge carriers is sufficient to account for the high \( T_c \) value in cuprates [90].

What is interesting is that all temperature dependences shown in Fig.13 are similar to the temperature dependence of the superfluid density in heavy fermion CeIrIn\(_5\), measured by \( \mu \)SR [75], and to the temperature dependence of the Andreev-reflection gap in heavy fermion UPt\(_3\) [71], which are shown in the insets of Figs 13(a) and 13(b), respectively. Spin fluctuations are believed to mediate the electron pairing in CeIrIn\(_5\) [52] and UPt\(_3\) [55] that leads to SC. The magnetic resonance peak has not yet been detected in CeIrIn\(_5\) or UPt\(_3\), however, the magnetic resonance peak has been observed by INS in another heavy fermion UPd\(_2\)Al\(_3\) [72] where spin fluctuations mediate the SC which coexists with the long-range AF order [41]. The latter facts also point to the presence of the spin-fluctuation coupling mechanism in cuprates.

The behavior of all temperature dependences shown in Fig.13 can be easily understood in terms of the spin-fluctuation SC mechanism (electron-electron interactions): crudely speaking, they exhibit the squared BCS temperature dependence.

E. Discussion

In spite of the unmistakable similarities among the magnetic and SC properties of YBCO, Bi2212 and LSCO...
(and some heavy fermions for which there is an indirect evidence for spin-fluctuation mechanism of SC), clearly, there is a difference between magnetic properties of LSCO and YBCO. If, in YBCO, \( T_c = T_{\text{com}} = T_{\text{inc}} \), where \( T_{\text{com}} (T_{\text{inc}}) \) is the onset temperature of the (in-)commensurate peak(s), in LSCO the situation is different. First, the commensurate peak has not been detected. Second, in LSCO, mainly, \( T_c < T_{\text{inc}} \) (Figure 13(c) shows the case when \( T_c = T_{\text{inc}} \)). So, the magnetic and SC properties of LSCO are similar to those of most heavy fermions: the commensurate peak has not been detected, and SC properties of LSCO are similar to those of most heavy fermions CeIrIn\(_5\), UPt\(_3\) and UPd\(_2\)Al\(_3\), on the one hand, and the cuprates, on the other hand. It is possible that, in all heavy-fermion and organic SCs, the long-range phase coherence is established due to spin fluctuations. The pairing mechanism may be different (as in Bi2212).

In LSCO, mainly, \( T_c < T_{\text{inc}} \) (in-)commensurate peak(s), in LSCO the situation is not yet been detected. Second, in LSCO, mainly, \( T_c < T_{\text{inc}} \) (Figure 13(c) shows the case when \( T_c = T_{\text{inc}} \)). In Bi2212, the situation looks more like that of YBCO, even if, the incommensurate peaks have not yet been detected.

In fact, the differences between magnetic and SC properties of YBCO and LSCO (and among some heavy fermions) can be understood in terms of the following chain of events: the formation of pairs at \( T_{\text{pair}} \) - the SC order parameter couples to the magnetic in-plane order parameter - the appearance of AF interactions along the c axis at \( T_m \) - the appearance of the long-range SC phase coherence. If there is no pairs, the SC is absent, even if, the AF order is established (as in the Eu-doped LSCO). If there is no pairs, the SC is absent, even if, the AF order is established (as in the Eu-doped LSCO).

There are common features in the SC state of the heavy fermions CeIrIn\(_5\), UPt\(_3\) and UPd\(_2\)Al\(_3\), on the one hand, and the cuprates, on the other hand. It is possible that, in all heavy-fermion and organic SCs, the long-range phase coherence is established due to spin fluctuations. The pairing mechanism may be different (as in Bi2212).

**F. Summary**

Analysis of the data obtained by different techniques in YBCO, Bi2212 and LSCO shows that the long-range phase coherence intimately relates to AF interactions along the c axis. Apparently, in cuprates, the magnetic and SC order parameters are coupled to each other, and the phase-coherence scale, \( \Delta_c \), has the magnetic origin (see Fig.14).

There are common features in the SC state of the heavy fermions CeIrIn\(_5\), UPt\(_3\) and UPd\(_2\)Al\(_3\), on the one hand, and the cuprates, on the other hand. It is possible that, in all heavy-fermion and organic SCs, the long-range phase coherence is established due to spin fluctuations. The pairing mechanism may be different (as in Bi2212).

**FIG. 14. Phase diagram: the energy position of the magnetic resonance peak, \( E_r \), in Bi2212 (squares) and in YBCO (dots) and the peak position in Fig.9(a), 35 mV, and the magnitude of SG in Fig.5(b), 2\( \Delta_{sc} \) = 34 meV, have similar values.**

In Fig.14, one can see that at \( p = 0.2 \), \( E_r \approx 2\Delta_{sc} \). Therefore, the tunneling in a SIS junction of Bi2212 having \( p \approx 0.2 \) can be assisted by spin excitations \( \Delta_{sc} \). Bearing this fact in mind, we interpret the peak at 35 mV in the difference shown in Fig.9(a) as the contribution from electron tunneling assisted by spin excitations. Indeed, the peak position in Fig.9(a), 35 mV, and the magnitude of SG in Fig.5(b), 2\( \Delta_{sc} \) = 34 meV, have similar values.

If the interpretation is correct, then the tunneling data in Fig.5(b) present additional evidence that spin fluctuations mediate the long-range phase coherence in Bi2212. First, this shows that charge carriers are strongly coupled to spin excitations. Second, by comparing the values of the Josephson \( J_cR_m \) product of the spectra shown in Figs 4(b) and 5(b) (7.5 mV and 24.5 mV, respectively), the high value of the Josephson product of the spectra in Fig.5(b) can only occur due to the contribution from the tunneling assisted by spin excitations because this contribution is the only difference between the spectra shown in Figs 4(b) and 5(b) [see Figs 8(c) and 9(a)]. Consequently, this signifies that the spin fluctuations mediate the phase coherence in Bi2212.

**IX. THE TWO ENERGY SCALES**

The two energy scales shown in Fig.14, both relate to the SC: the linear \( \Delta_{sc} \) (or \( \Delta_p \)) scale is the in-plane energy scale, whereas the scale proportional to \( T_c \) is the
c-axis energy scale, and they have different origins. It is important to emphasize that the $\Delta_{sc}$ energy scale is observed by tunneling spectroscopy and ARPES, whereas the $\Delta_c$ energy scale is measured in Andreev reflections [94]. However, in some cuprates, the $\Delta_c$ scale is also observed in tunneling measurements along the c axis [63, 93, 94]. For example, in near optimally doped YBCO single crystals, the maximum magnitudes of the tunneling gap into CuO$_2$ planes and along the c axis are 28 meV and 19 meV, respectively. The same values measured in electron-doped Nd$_{1.85}$Ce$_{0.15}$CuO$_{4-\delta}$ (NCCO) undoped in oxygen are 13 meV and 3.5 meV, respectively, and the magnitude of the Andreev gap is 3.5 meV [93]. So, in YBCO and NCCO, tunneling measurements along the c axis show the $\Delta_c$ scale. However, it is not the case in Bi2212, where tunneling measurements along the c-axis show the $\Delta_{sc}$ scale [93].

The symmetries of the two energy scales are discussed elsewhere [94]: the pairing energy scale, $\Delta_{sc}$ has most likely an anisotropic s-wave symmetry like the PG (CDW gap) which defines the magnitude of $\Delta_{sc}$, whereas the coherence energy scale, $\Delta_c$, has the d-wave symmetry. Since spin fluctuations mediate the phase coherence, they are responsible for the d-wave symmetry of $\Delta_c$ [94]. All phase-sensitive techniques, obviously, probe the symmetry of $\Delta_c$, and not of $\Delta_{sc}$, thus they detect the d-wave symmetry. At the same time, tunneling measurements show a s-wave symmetry of the condensate [94], i.e. the symmetry of $\Delta_{sc}$.

X. THE ANISOTROPY OF THE TUNNELING PG

Let us concentrate for a while on the tunneling PG. The analysis of the data (see above) suggests that the tunneling PG is predominantly a charge gap on charge stripes in CuO$_2$ planes. This charge gap is most likely a CDW gap. It is not difficult to show the shape of the CDW gap. As mentioned above, the shape of the PG shown in the inset of Fig.4(b) indicates that the PG has most likely an anisotropic s-wave symmetry. Second, from the data presented here and in Ref. [92], the maximum magnitudes of in-plane PG and $\Delta_{sc}$ in Bi2212 correlate with each other as $\Delta_{ps,in} \simeq 2\Delta_{sc,in}$. The intrinsic c-axis tunneling data in Bi2212 mesas show that, at low temperature, $\Delta_{ps,c} \simeq \Delta_{sc,c}$ (see Fig.1 in Ref. [92]). Since $\Delta_{sc,c} \simeq \Delta_{sc,in}$ (compare our data with the data in the overdoped sample in Ref. [92]), it is reasonable to assume that the value of $\Delta_{ps,c}$ corresponds to the minimum of the in-plane CDW gap because the tunneling along the c-axis is preferable from the minimum of in-plane CDW gap. Then, one can conclude that the in-plane CDW gap in Bi2212 is anisotropic with the anisotropy ratio of $\Delta_{ps,max}/\Delta_{ps,min} \simeq 2$.

From quasi-1D structure of the stripes, the CDW gap has either two- or four-fold symmetry. The maximum magnitude of the SG is most likely defined by the minimum of the charge gap on the stripes.

XI. CHANGES IN BI2212 WITH DECREASE OF TEMPERATURE

By synthesizing our findings we briefly describe here the changes occurring in Bi2212 with decrease in temperature.

Analysis of the data shows that (i) the tunneling characteristics corresponding to the QP peaks in Bi2212 are in excellent agreement with theoretical predictions made for a quasi-1D topological-excitation liquid, and (ii) the phase coherence along the c-axis in cuprates is established at $T_c$ due to spin fluctuations in local AF domains of CuO$_2$ planes. By taking into account (i), we assume that the quasi-1D topological excitations reside on quasi-1D charge stripes. The assumption is reasonable because AF domains which separate the quasi-1D charge stripes are truly 2D [34]. Consequently, the quasi-1D excitations cannot reside into the AF domains.

Depending on the compound, the charge stripes are formed in cuprates at $T_{charge} \sim 70$–300 K (or higher) (see references in Ref. [95]). The excitations along stripes appear with appearance of the stripes, thus, at $T_{charge}$. The stripe excitations couple with each other at $T_{onset} < T_{charge}$, which can be considered as an onset of SC in Bi2212. For example, in slightly overdoped Bi2212, two independent tunneling studies [96] show that the onset of SC occurs at 110–116 K. Finally, the long-range phase coherence is established at $T_c$ along the c-axis (i.e. between CuO$_2$ planes) due to spin fluctuations. Thus, there are 3 characteristic temperatures: $T_c \leq T_{onset} < T_{charge}$. In addition, between $T_{onset}$ and $T_{charge}$, there is a characteristic temperature [96] which was attributed to the spin ordering in AF domains [96].

The critical temperature, $T_c$, can be measured directly, and $T_{charge}$ and $T_{onset}$ are proportional to $\Delta_{ps}$ and $\Delta_{sc}$ (see Fig.7), respectively. It is interesting to note that, since $\Delta_{ps} \simeq 2\Delta_{sc}$, $T_{onset}$ is always about twice as small as $T_{charge}$. It is possible to estimate the relation between $T_{onset}$ and $\Delta_{sc}$. By using the data measured in slightly overdoped Bi2212 and presented in Refs [96], we have $T_{onset} \approx 0.4\Delta_{sc}/k_B$. Earlier, we proposed a magnetic coupling between stripes (MCS) model assuming that the Cooper pairs are formed along charge stripes, and the phase coherence is established due to magnetic coupling between SC stripes [94]. From the present work, the general idea of the MCS model is correct, however, the more precise name of the model of SC in cuprates should be the magnetic coupling between striped CuO$_2$ planes.
XII. COOPER PAIRS

As discussed in Section XI, the topological solitons condense at $T_{\text{onset}}$ by lowering their energy. Formally, the topological excitations shown in Fig.11 are “pure” charge excitations. The question is at what temperature is the spin sector condensed, at $T_{\text{onset}}$ or $T_c$? If Davydov’s bisoliton scenario $^{22,23}$ is realized than the spins condense together with the charges at $T_{\text{onset}}$ by creating singlet states. If the condensate consists of kink-kink bound states, then spins may condense at $T_c$. Since “pure” charge excitations have boson-like properties, generally speaking they do not need to create a bound state. So, it is most likely that the Cooper pairs in Bi2212 are Davydov’s bisolitons, and the phase coherence among the bisolitons is established at $T_c$ due to spin fluctuations (excitations).

It is important to emphasize that, in such a scenario, formally, the Cooper pairs exist above $T_c$, between $T_{\text{c}}$ and $T_{\text{onset}}$. However, de fuit, they are the local Cooper pairs. In order to jump from one stripe to another (above $T_c$), they have similar difficulties as a single electron.

One would be interested to know what is the glue between two electrons in the soliton scenario for the SC in Bi2212. It is phonons, and the electron-phonon interactions are strong and nonlinear $^{22,23}$.

XIII. DATA OBTAINED IN CUPRATES

The quasi-1D topological-excitation-liquid scenario is attractive, because by using it, it is possible to explain experimental data obtained in cuprates. Let’s briefly discuss some data.

The discrepancy for the presence of the PG in the overdoped region in transport and tunneling measurements can be easily understood: they measure two different PGs. In tunneling measurements, it is predominantly the charge gap, and, in transport measurements, it is the resistance to the propagation of stripe excitations along stripes which are located in AF environment. For example, if a molecular chain is embedded into a medium, a frictional force acts on the soliton $^{22,23}$.

In frameworks of the quasi-1D topological-excitation-liquid scenario in Bi2212, magnetic and non-magnetic impurities will affect the SC similarly, which is the case in cuprates $^{104}$. The oxygen isotope effect $^{101}$ can be naturally understood since the stripe dynamics above $T_c$ depends on the underlying lattice $^{102}$. At the same time, a very weak ”BCS isotope effect” in cuprates is well understood in terms of the soliton SC $^{22,23}$.

Mysterious vortex-like excitations found in the Nernst effect $^{103}$ correspond to the stripe excitations. The authors are correct by pointing out that the observed excitations are vortex-like because vortices themselves are solitons $^{24}$. Such scenario in Bi2212 can explain why the phenomenon of SC in hole- and electron-doped cuprates can be understood within a common scheme $^{13}$. The occurrence of sudden CDW ordering at low temperature in undoped cuprates $^{37}$ and the insulating behavior in a strong magnetic field $^{104}$ correspond to the disappearance of the stripe excitations.

By using the quasi-1D topological-excitation-liquid scenario in Bi2212 and the fact that the phase coherence along the $c$-axis is established due to spin fluctuations, it is possible to explain why $T_c$ and $\Delta_{sc}$ do not correlate with each other in cuprates: the magnitude of $\Delta_{sc}$ is defined by the magnitude of the charge gap on stripes, at the same time, the value of the bulk $T_c$ value depends on spin fluctuations into local AF domains.

It is possible that zero-bias conductance peak observed in tunneling measurements $^{38,44}$ corresponds not only to Andreev surface bound states $^{105,106}$ but also to the topological solitons (see Fig.2(a)).

The SC condensation energy has the maximum in the overdoped region near $p = 0.19$ $^{11}$. Why? The SC in Bi2212 is literally spread into the two channels which have different energy scales: in-plane and along the $c$-axis. From Fig.14, the two energy scales have the same values approximately at $p = 0.2$. Thus, the two energy scales emerge into one at $p = 0.19–0.2$.

Recently, a new energy scale is found in ARPES spectra of hole-doped cuprates: an abrupt change (a kink) has been observed in the electronic QP dispersion $^{107}$. The kink is associated either with phonons or with the magnetic resonance peak. In fact, the clue to the origin of the kink in ARPES spectra can be found in Ref. $^{108}$. Measurements in 2D layered 2H-TaSe$_2$ CDW compound found a similar energy scale in ARPES spectra $^{108}$. The authors conclude $^{108}$: ”A reduction in the scattering rates below this energy indicates the collapse of a major scattering channel with the formation of the CDW state.” In the CDW compound, it is clear that this bosonic mode corresponds to the appearance of solitons in the CDW state. Consequently, the origin of the kink in ARPES spectra of hole-doped cuprates is the same, i.e. due to solitons. In fact, it is also possible that this kink corresponds to the real Fermi surface in cuprates and in 2H-TaSe$_2$ (see Fig.2 in Ref. $^{20}$).

Lastly, it is important to note that, in the framework of the quasi-1D topological-excitation-liquid scenario in Bi2212, one can explain the violation of the sum rule, observed in infrared measurements $^{109}$: the stripe excitations do not obey the sum rule.
XIV. CONCLUSIONS

We would like to emphasize that we do not present here an evidence that the tunneling pseudogap in Bi2212 is a CDW gap. This conclusion is based on the analysis of the data. Here we found that the tunneling characteristics corresponding to quasiparticle peaks are in excellent agreement with theoretical predictions made for a quasi-1D topological-excitation liquid. In fact, this information is sufficient in order to conclude that the tunneling pseudogap in Bi2212 is predominantly a charge gap. Since solitonlike excitations, and not electrons (holes), play the role of quasiparticles in Bi2212, this implies that the stripes are insulating. In other words, there is a charge gap along stripes which is most likely a CDW gap. Then, in order to pull out an electron (a hole) from a stripe, it is necessary, first of all, to overcome the charge gap on the stripe. Consequently, the tunneling pseudogap is predominantly the charge gap. However, it is possible that the pseudogap shown in Fig.3(b) consists of two (or a few) contributions: the predominant contribution from the charge gap and another contribution, for example, from AF correlations.

In summary, tunneling measurements have been carried out on underdoped, overdoped and Ni-substituted Bi2212 single crystals. Tunneling spectra below $T_c$ are the combination of incoherent part from the pseudogap and coherent quasiparticle peaks. There is a clear correlation between the maximum magnitude of the pseudogap and the distance between the quasiparticle peaks. The tunneling characteristics corresponding to the quasiparticle peaks are in excellent agreement with theoretical predictions made for a quasi-1D topological-excitation liquid. So, the quasiparticle peaks in Bi2212 appear from the condensation of quasi-1D topological excitations which reside most likely on charge stripes. Analysis of the data suggests that (i) the tunneling pseudogap in Bi2212 is predominantly a charge gap (CDW gap) on dynamical charge stripes, which has most likely an anisotropic s-wave symmetry, and (ii) the Cooper pairs in Bi2212 are Davydov’s bisolitons. In addition, analysis of data measured by different techniques shows that the phase coherence is established at $T_c$ due to spin fluctuations in local antiferromagnetic domains of CuO$_2$ planes. It is possible that the magnon-like excitations which cause the appearance of the magnetic resonance peak in inelastic neutron scattering spectra are in resonance with the bound states of solitons

Many other data obtained in cuprates can be naturally understood in the framework of the quasi-1D topological-excitation-liquid scenario. In the phase diagram of cuprates, the topological-excitation-liquid conductor known as a ”strange metal” is located between an antiferromagnetic Mott insulator at low hole concentration and a Fermi-liquid metal at high hole concentration.

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