Design and optimization of stepped compound horn based on finite element method

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\textbf{Abstract.} A stepped composite horn structure with column holes at the front end is proposed for use in power ultrasonic fields. Based on the finite element method, the vibration characteristics and amplitude magnification of the horn at resonance frequency are calculated. Analyzed the influence of different structural parameters on its performance parameters, analyzed the influence of the aperture size on the resonance frequency of the horn, optimized the design of the proposed composite horn to obtain the maximum amplitude magnification. Performed simulation analysis on the horn through ANSYS Workbench, extracted the seventh-order modal results, and performed modal analysis. The simulation results show that: compared with the traditional horn, the maximum amplitude of the step compound-shaped horn is increased by 72\%. The frequency of longitudinal vibration of this open-hole horn is 26.92 KHZ. The performance parameters of the optimized horn have been significantly improved.

\textbf{Keywords:} Ultrasonic Horn, ANSYS, Modal Simulation, Optimal Design

\section{Introduction}

In recent years, power ultrasound has been widely used in environmental protection, medical treatment, and agriculture. The ultrasonic vibration system composed of ultrasonic transducer, horn and ultrasonic generator is an important part in the application of power ultrasonic. Among them, the ultrasonic horn, also known as the ultrasonic concentrator, is particularly important in the ultrasonic technology, especially in the vibration system of the high-sound intensity ultrasonic equipment [1]. The main function of the horn is to amplify the displacement and velocity of the particle of the mechanical vibration, while concentrating the ultrasonic energy on a smaller area. The amplitude of the radiating surface of the ultrasonic transducer is only a few microns in the range of 20 KHZ. In hypersonic applications, in order to meet the needs of ultrasonic processing, the amplitude of the ultrasonic horn connected to the end face of the ultrasonic transducer must reach tens of microns or even several hundred microns [2]. Commonly used methods for designing horns include analytical method, transmission matrix method, equivalent circuit method [3] and finite element method [4]. With the increasing application of power ultrasonic technology, in order to improve the efficiency of ultrasonic processing and ultrasonic processing, higher requirements are put forward on the amplitude and other
performance parameters. It is extremely important to design a large amplitude horn.

In the ultrasonic vibration system, the ultrasonic horn [5] generates reciprocating vibration under the excitation of the transducer, and its vibration types include four types: torsional vibration type, longitudinal vibration type, bending vibration type and compound vibration (exist two vibration modes). In the ultrasonic machining process, the longitudinal vibration mode is generally used; Under the conditions of the same working frequency, material of the horn and the diameter of the large and small ends, the stepped shape has the largest amplification factor. Therefore, researchers have carried out extensive research on the stepped horn [6-7]. Zhang QJ, [8] studied the influence of the size of the stepped transition arc on the stress and displacement distribution of the horn. By increasing the radius of the arc, the maximum stress moves to a small section, so as to improve the stress concentration of the traditional stepped horn in the transition section which is easy to cause fatigue fracture. Kumar Patel, L [9] conducted modal harmonic frequency analysis on eight different shapes of horns, and finally proposed a new type of composite horn, In order to improve the vibration performance, such as increasing the amplification factor and shape factor of the horn, various combined horns have been proposed. Compared with the traditional single horn, significantly improve the performance parameters of the horn [10-11].

In order to further improve the application efficiency of power ultrasound, this paper proposes a stepped composite horn with a cylindrical hole at the front end. The vibration characteristics of the horn are calculated based on the finite element method, and the influence of the hole size on the vibration performance parameters is analyzed and optimized. Using ANSYS to build the horn model, the modal simulation results are compared with the results of the traditional single stepped horn. The results show that this stepped compound horn is compared with the traditional single horn vibration characteristics such as rod amplitude have been greatly improved.

2. Theoretical calculation of the horn
There is an assumption that the horn is composed of uniform and isotropic materials, excluding the mechanical loss, Longitudinal waves propagate in the axial direction. The internal stress distribution of the horn is uniform. Based on this assumption, the horn will vibrate after receiving the sound energy transmitted by the transducer. Due to the elastic restoring force, the adjacent molecules inside the horn will oscillate and eventually return to the original position. The inside of the horn is composed of tiny volume elements, and the vibration of any one tiny volume element will cause other volume elements to vibrate together. Based on the analysis of Newton's law, the dynamic equation is:

\[
\frac{\partial}{\partial x} (S \cdot \sigma) + \frac{\partial}{\partial t} \left( S \cdot \rho \frac{\partial \xi}{\partial t} \right) = 0
\]\n
Among them, \(S = S(x)\) is the cross-sectional area function of the rod, \(\xi = \xi(x)\) is the mass point displacement function, \(\sigma(x) = E \frac{\partial \xi}{\partial x}\) is the stress function, \(\rho\) is the density of the rod material, \(t\) represents time, \(E\) is Young's modulus. When the rod makes a simple harmonic motion, the formula (1) can be expressed as:

\[
\frac{\partial^2 \xi}{\partial x^2} + \frac{1}{S} \frac{\partial S}{\partial x} \frac{\partial \xi}{\partial x} + k^2 \xi = 0
\]\n
Equation (2) is the wave equation when the shape and area of the rod changes and the longitudinal vibration occurs. Among them, \(k^2 = \frac{\omega^2}{c^2}\), \(k\) is the number of circular waves, \(\omega\) is circular frequency, and \(c = \left(\frac{E}{\rho}\right)^{1/2}\) is the speed of ultrasonic waves in rods with different cross-sections. For horns with uniform cross-section, so \(\frac{\partial S}{\partial x} = 0\), formula (2) can be simplified as:
Because by solving equation (3), we can get:

$$\xi = A \cos kx + B \sin kx$$  \hspace{1cm} (4)

The mass point displacement A and constant coefficient B in equation (4) can be solved by the following boundary conditions: (1) Set the force on both ends of the horn to be 0; (2) The force between the internal volume elements of the horn is the same and the volume is opposite; (3) The displacement of the mass point at the node connection is the same.

Therefore, the elastic force $$F(x)$$ and stress $$\sigma(x)$$ equations are

$$F(x) = \frac{E}{j\omega} \frac{\partial \xi}{\partial x} = -j\rho \nu S(A \cos kx - B \sin kx)$$  \hspace{1cm} (5)

$$\sigma(x) = \frac{E}{j\omega} \frac{\partial \xi}{\partial x} = -j\rho \nu (A \cos kx - B \sin kx)$$  \hspace{1cm} (6)

From the continuity of the vibration at the boundary of the horn and the continuity of the elastic force, the boundary conditions and equations (4) and (6) can be obtained:

Frequency equation:

$$kl = \pi$$  \hspace{1cm} (7)

Half-wavelength resonance length:

$$l = \lambda/2$$  \hspace{1cm} (8)

Displacement node length:

$$x_0 = \lambda/4$$  \hspace{1cm} (9)

3. Design of horn based on finite element method

3.1. Theoretical basis of vibration analysis based on the finite element method

The design methods of horns mainly include analytical method, equivalent circuit method and finite element method. Analytical method and the equivalent circuit method are only suitable for solving horns with simple design rules and a single shape. However, the general vibration characteristics of a single horn cannot meet the better parameters we propose for the horn. But the finite element method has shown great advantages in solving and designing irregularly shaped composite horns. Because of its fine mesh and accurate and reliable calculation results, it is known as one of the most widely used numerical analysis techniques.

The dynamic equation of the vibration system can be written as:

$$[M] \ddot{x} + [C] \dot{x} + [K] x = [Q]$$  \hspace{1cm} (10)

In formula (10), As the following matrix: $$[M]$$, $$[C]$$, $$[K]$$ represent mass, damping, and stiffness respectively; The following vectors: $$\ddot{x}$$, $$\dot{x}$$ and $$x$$ respectively represent acceleration, velocity, and displacement; $$[Q]$$ is the generalized force matrix.

The horn is equivalent to free vibration without damping in the finite element analysis, so the dynamic equation of equation (10) can be simplified as

$$[M] \ddot{x} + [K] x = \{0\}$$  \hspace{1cm} (11)

For a linear system, the dynamic equation of free vibration can be obtained as

$$\{x(t)\} = \{u\} \sin(\alpha t + \psi)$$  \hspace{1cm} (12)

From the above formulas (11) and (12), we can get:

$$\left( [K] - p^2 [M] \right) \{u\} = 0$$  \hspace{1cm} (13)
Due to free vibration, the amplitude \( \{u\} \) of each node in the structure is not all 0. So, 
\[
[K] - p^2 [M] = 0
\]  
(14)

Therefore, equation (14) is about the \( n \)th degree equation of \( p^2 \), the natural frequency of \( n \) can be obtained, and \( p^2 \) is the generalized eigenvalue. For each natural frequency, \( \{u\} \) can be obtained as the corresponding vibration shape.

3.2. Structural design and parameter calculation of horn

The material selection of the horn has a great influence on its vibration characteristics [12], so in the application of ultrasonic processing, the material of the horn should be selected: (1) High fatigue strength and low acoustic impedance; (2) Low material loss in its working frequency range; (3) Material is easy to be machined. The horn material designed in this paper is 45# steel. The sound velocity of longitudinal wave in the horn of this material is 5200 m/s, and its performance parameters are shown in Table 1.

| Table 1. 45# Steel Performance Parameters |
|------------------------------------------|
| Density/(kg/m3)  | Poisson’s ratio | Elastic Modulus/GPa |
| 7800            | 0.28           | 210               |

The shapes of traditional horns include catenary, stepped, conical and exponential shapes, this type of horn is called a single horn. Horn parameters: area coefficient \( N = D/d \), the resonance length of the horn is \( L \), so when the area coefficient \( N < 3.3 \), \( L_e > L_a > L_d > L_c \); when \( N \) is the same, amplification factor \( M_d > M_b > M_a > M_c \); And when the form factor \( N \) is the same, \( \phi_d > \phi_a > \phi_b > \phi_c \); Therefore, based on the comprehensive consideration of vibration parameters such as resonance length \( L \), amplification factor \( M \), and shape factor \( \phi \) This paper designs and proposes a stepped and tapered composite horn structure. As shown in Fig.1, a cylindrical hole is opened at the end of the stepped composite horn, which is the energy-gathering place. The length of the horn is \( L \), the diameter of the large end and the small end are \( D \) and \( d \) respectively, and the transition arc radius is \( R \), the width of the cylindrical opening is \( a \), the length of the opening is \( b \), and the diameter of the cylindrical opening is \( d_1 \).

Figure 1. Schematic diagram of stepped composite horn

When the natural frequency of the horn is the same as the frequency of the ultrasonic wave, the condition of resonance is achieved. In order to achieve a larger amplitude, the total length of the horn must meet an integer multiple of 1/2 of the wavelength. Ultrasonic Wavelength \( \lambda = c / f \). Given that \( c \) is 5200 m/s, Excitation frequency \( f = 20 \) KHZ, 
\[
\frac{n \lambda}{2}, n \text{ takes 1}, \text{Horn length } L = 13 \text{ cm. The large end diameter } D \text{ matches the selected transducer size, } D = 52 \text{ mm, } d = 16 \text{ mm, The area factor } N \text{ is calculated to be } 3.25;
\]

So \( \lambda \) is calculated as 26 cm. L is \( \frac{n \lambda}{2} \), n takes 1, Horn length L = 13 cm. The large end diameter D matches the selected transducer size, D = 52 mm, d = 16 mm, The area factor N is calculated to be 3.25;
3.3. Modal analysis of horn based on finite element method
Modal analysis is an important method used to research the dynamic characteristics of structures. It is usually used to determine the vibration characteristics of the designed structure. The modal analysis method based on the finite element method is also called computational modal analysis. Each order of calculation corresponds to a corresponding frequency and mode. The finite element method is used to establish a model of the ultrasonic horn and reasonably divide it into blocks. Using the modal analysis module and the post-processing module, the vibration mode of the horn can be extracted, the displacement distribution can be determined, and the harmonic response analysis can be performed.

Fig.2 is the three-dimensional structure model of the traditional stepped horn. After drawing the model through SolidWorks, it is imported into ANSYS Workbench for finite element calculation and analysis. The ANSYS calculation process mainly includes three parts: model building, meshing, loading solution and post-processing. The structure of the horn after grid division is shown in (b), and the grid quality is 0.83, which meets the requirements of high-quality grids. Fig.3 shows the extracted modal results. The fourth order is the longitudinal mode, the resonance frequency is 23.1 KHZ, and the maximum amplitude is 2.59 μm.

4. Optimization of stepped compound horn
4.1. Design of stepped compound horn
From the analysis results in the previous section, it can be seen that the traditional stepped horn has a small amplitude, and the maximum stress is concentrated in the transition position, which is prone to damage. Therefore, this paper proposes a stepped composite horn. Compared with the traditional horn, this article appropriately adjusts the length ratio of the large and small ends to analyze the performance parameters of the horn, and finally obtains an optimal structure.

The horn structure proposed in this paper is composed of a stepped part and a cone part, and a cylindrical hole is opened on the end of the horn. The four-dimensional structure of the horn is shown in Fig.4. The length ratio of the large and small ends is P, From (a) to (d), P are 0.36, 0.52, 0.73, 1 in order; the opening sizes of the four structures remain the same. After finite element calculation and analysis are performed in ANSYS Workbench, the modal extraction results are shown in Fig.5.(a)-(d) The four structures correspond to modes (c)-(h), and the corresponding resonance
frequencies of the longitudinal mode are: 32.72 KHZ, 30.78 KHZ, 30.05 KHZ and 26.92 KHZ, the maximum amplitudes all occur at the end of the horn, the amplitudes are respectively: 3.11 μm, 3.0 μm, 3.25 μm and 4.46 μm.

![Image](a) (b) (c) (d)

**Figure 4.** Three-dimensional models of four composite stepped horns

![Image](e) (f) (g) (h)

**Figure 5.** Schematic diagram of four composite stepped horn modes

Based on the performance parameters of the horn, the amplitude magnification of structure (c) reaches the maximum. The ratio P of the large and small ends of the stepped complex shape is analyzed with the resonance frequency and amplitude of the horn. The analysis result is shown in Fig.6. As P increases, the resonant frequency shows a downward trend, while the amplitude increases. The result shows that the ratio P of the large and small ends is inversely proportional to the resonant frequency and directly proportional to the amplitude.
4.2. Analysis of opening parameters of stepped compound horn

According to the optimized model above, carry out the influence analysis of the opening size, take the change range of the opening diameter $d_1$ is taken $[8-14]$, and extract the seven-order modes for modal analysis; study the relationship between the opening diameter and the amplification factor and resonance frequency. From Fig. 7, it can be seen that the amplification factor $M_p$ is directly proportional to the opening diameter $d_1$, and the amplification factor increases significantly with the increase of the opening diameter; the longitudinal resonance frequency $f$ decreases with the increase of the opening diameter $d_1$.

5. Conclusion

In this paper, a composite stepped horn with a hole in the front end is proposed and optimized. The amplitude amplification factor at the resonance frequency is calculated based on the finite element method, and the effect of the ratio $P$ of the large and small ends of the horn on the resonance frequency and amplitude is studied. Through comparative analysis with the traditional stepped horn, the
following conclusions are obtained:

1) The proposed composite stepped horn structure with a cylindrical hole at the front end has an amplitude increase of 72% compared to the traditional single-shaped stepped horn.

2) Four sets of simulation analysis were carried out to study the relationship between the ratio P of the large and small ends of the stepped composite deformed horn and the resonance frequency and amplitude. The results show that as the value of P increases, the resonance frequency gradually decreases, but the amplitude increases instead.

3) Optimized the design of the hole size of the column hole, and studied the effect of the hole size on the magnification and resonant frequency. The analysis results show that the hole diameter has a greater influence on the two parameters.

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