Remarks on Bessel beams, signals and superluminality

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March 31, 2022

Abstract

We address the question about the velocity of signals carried by Bessel beams wave packets propagating in vacuum and having well defined wavefronts in time. We find that this problem is analogous to that of propagation of usual plane wave packets within dispersive media and conclude that the signal velocity can not be superluminal.

PACS numbers: 02.30.Nw; 03.50.De; 04.30.Nk

Keywords: Wave propagation; Bessel beams; signal velocity; superluminality.

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1 Introduction

It is generally accepted that the causality principle in special relativity imposes the speed $c$ of light in vacuum as the upper limit for the velocity of propagation of signals or interactions. By the way, Maxwell equations in material media allow, in special circumstances such as anomalous dispersion, plane wave solutions propagating with superluminal (faster than $c$) phase velocities $[1]$. Although these velocities cannot be readily identified with the signal velocity, the question about the possibility of building a superluminal signal by superimposing these solutions naturally arises.

The above problem was studied by Sommerfeld and Brillouin at the beginning of the last century $[2]$. In order to define precisely a velocity of propagation Sommerfeld considered only signals carried by wave packets which have well defined wavefronts (and possibly ends) in time, which we call SB signals. The signal itself, or its “main part”, independently of the way it is defined, is obviously confined to the region behind the wavefront. These packets were assumed to be normally incident on a semi-infinite dispersive medium, in which boundary the arrivals of both the wavefront and the signal were assumed to coincide. The Sommerfeld’s main result was that, irrespective the material medium, the wavefront of such a packet propagates always with the speed $c$. Consequently, the arrival of the signal at a given point inside the medium can occur only after that (or simultaneously, if the medium is nondispersive) this point is reached by the wavefront. Therefore, Sommerfeld result implies that the speed $c$ of light in vacuum is an upper bound to the velocity of propagation of SB signals. Brillouin studied in great detail the evolution of these wave packets within dispersive media, giving a precise
definition for the signal velocity, which agrees with the Sommerfeld result. In what concerns the group velocity, which can be greater than $c$ in the presence of anomalous dispersion, or even negative, Brillouin stated that “...the group velocity has a meaning only so long as it agrees with the signal velocity.”

Recently, there is a growing interest on the subject of superluminal wave motion in the literature. For example, we can quote experiences measuring superluminal velocities in the passage of wave packets through barriers (see, for example, [3]-[6] and references therein) and a considerable amount of works, both in theoretical and experimental views, concerning the superluminality of Bessel beams and X waves ([8]-[15] and references therein). These works raised questions about the interpretation of such superluminal velocities, specially in what concerns the meaning of signal velocity and its connection with the group velocity and the causality principle. So, this is still a very debated subject.

In this letter we are concerned to Bessel beams wave packets, from which X waves are a special case. These beams are (inhomogeneous) plane wave solutions of the homogeneous scalar wave equation in vacuum which propagate with superluminal phase velocities. X waves are localized waves built up as special superpositions of Bessel beams and propagate rigidly (without dispersion) in vacuum with superluminal velocities. From these nondispersive properties some authors suggested that this superluminality could also be associated with the signal velocity [10, 11, 12]. We analyze the question of the maximum velocity of signals carried by Bessel beams wave packets. To this aim we follow the approach of Sommerfeld and Brillouin cited above and consider only SB signals, i.e., those carried by Bessel beams wave packets.
having well defined wavefronts (and ends) in time. We first study the *chopped Bessel beam*, which have a finite duration at its source. From the analysis of this packet we identify a mathematical analogy between its propagation properties and those of usual (inhomogeneous) plane wave packets propagating within a dispersive medium, namely a tenuous electronic plasma. This is our main result, because such analogy makes possible a straightforward application of Sommerfeld result to conclude that *the wavefronts of these packets propagate with the speed c, while the wave packets distort while propagating.* This is a curious result because we are dealing with propagation of waves in vacuum. As a direct consequence, the velocity of the signals carried by these wave packets can never be superluminal, independently of the way it is defined.

We also consider briefly the experiment of Mugnai, Ranfagni and Ruggeri [11], which posed a question about the superluminality of signals carried by Bessel beams. An explanation for the measured velocities in this experiment was given in references [13, 15] in terms of interference phenomena, showing that the superluminal velocities of the peaks moving along z axis were not causally connected and, therefore, did not represent signal velocities. To be able to say something about the signal velocity in this experiment we argue that the waves produced in it can be viewed as a kind of *finite aperture approximation to chopped X waves*. The last ones are ideal waves (they need an infinite aperture to be produced) built up as linear superpositions of chopped Bessel beams. We show that chopped X waves have wavefronts moving with velocity $c$ and thus the SB signals carried by them can not have superluminal velocities. Also, we suggest that the observed superluminal
peaks can be qualitatively explained from the fact that the chopped X wave distorts along the propagation, showing a kind of *reshaping phenomenon*, as observed in [13].

The generalization of the analogy with dispersive media to a large class of superpositions of Bessel beams is straightforward and will be done in Section 4. In the last section we make our concluding remarks. In particular, we comment on some discrepancies between our results and others in the literature concerning the wavefront velocities of chopped Bessel beams and chopped X waves.

## 2 Bessel beams wave packets

Bessel beams are cylindrically symmetric solutions of the scalar homogeneous wave equation in vacuum. They are given by [7, 8, 10, 13]

\[
\Psi^{(k_\rho, \omega)}(\rho, z, t) = J_0(\rho k_\rho) \exp\{i(k_z z - \omega t)\},
\]

where \(J_0(x)\) is the Bessel function of the first kind and order zero and the parameters \(k_z, k_\rho\) and \(\omega\) satisfy the following relation

\[
k_z^2 = \frac{1}{c^2} \left[ \omega^2 - c^2 k_\rho^2 \right].
\]

Thus, from a mathematical point of view, *any two* parameters among \(k_z, k_\rho,\) and \(\omega\) can be chosen independently. For the purposes of this letter we choose \(k_\rho\) and \(\omega\) as the independent ones and assume they are real. Making so, \(k_z\) will be given by (2) and it can, in principle, be imaginary.

At this point we emphasize that the above relation is mathematically identical to the dispersion relation of a tenuous electronic plasma if we identify \(c^2 k_\rho^2\) with the square of the plasma frequency \(\omega_p^2\) [1].
If $\omega^2 \geq c^2 k^2$ the Bessel beam (1) represents an unidimensional wave motion propagating along the $z$ direction, whose propagation properties are governed by the real wave number $k_z$. The surfaces of constant phase are planes perpendicular to the $z$ axis which propagate with the phase velocity $v_p$, given by

$$v_p = \frac{\omega}{k_z},$$

such that $|v_p| \geq c$, i.e., the phase velocity in vacuum is superluminal. These waves are also inhomogeneous plane waves, due to the presence of Bessel function $J_0(\rho k)$ in (1). In the case $k_\rho = 0$ the Bessel beams degenerate to usual homogeneous plane waves, with $|v_p| = c$. On the other hand, if $\omega^2 < c^2 k^2$ the wave number $k_z$ is imaginary. In this case there is no wave propagation in the $z$ direction, but instead of a behavior analogous to absorption or attenuation.

The most general (complex) solution formed from superposition of Bessel beams (1) is given by

$$\Psi(\rho, z, t) = \Psi^+(\rho, z, t) + \Psi^-(\rho, z, t),$$

where

$$\Psi^\pm(\rho, z, t) = \int_0^\infty dk_\rho \int_{-\infty}^{\infty} d\omega A^\pm(k_\rho, \omega) J_0(\rho k_\rho) \exp\{i[\pm k_z(k_\rho, \omega)z - \omega t]\}. \tag{5}$$

In this expression $A^\pm(k_\rho, \omega)$ are spectral distributions and the dispersion relation is given by

$$k_z(k_\rho, \omega) = \begin{cases} \frac{\omega}{c} \sqrt{1 - \frac{c^2 k_\rho^2}{\omega^2}}, & \text{if } |\omega| \geq ck_\rho \\ \frac{i\omega}{c} \sqrt{\frac{c^2 k_\rho^2}{\omega^2} - 1}, & \text{if } |\omega| < ck_\rho \end{cases}. \tag{6}$$

With this choice of signals the terms $\Psi^+(\rho, z, t)$ and $\Psi^-(\rho, z, t)$ in the superposition (1) correspond, respectively, to right and left moving wave packets.
Specifying the boundary conditions at \( z = 0 \),
\[
\begin{align*}
\Psi(\rho, 0, t) &= \Psi^0(\rho, t) ; \\
\frac{\partial \Psi}{\partial z}(\rho, 0, t) &= \Psi'^0(\rho, t) ,
\end{align*}
\]
we can determine the spectral distributions \( A^\pm(k_\rho, \omega) \) in (5) :
\[
A^\pm(k_\rho, \omega) = \frac{k_\rho}{2\pi} \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{0}^{\infty} d\rho J_0(\rho k_\rho) \left\{ \Psi^0(\rho, t) \pm \frac{1}{ik_z(k_\rho, \omega)} \Psi'^0(\rho, t) \right\} .
\]

To end this section we consider the situation in which only propagating modes of Bessel beams (1) enter into superposition (4), i.e., we restrict \( c k_\rho < |\omega| \) in (5). In this case we can write
\[
k_\rho = \frac{|\omega|}{c} \sin \theta \quad \text{and} \quad k_z = \frac{\omega}{c} \sin \theta , \quad \text{with} \quad \theta \in \left[ 0, \frac{\pi}{2} \right) .
\]

Using these relations we can change the integration variables in (5) from \( \{ k_\rho, \omega \} \) to \( \{ \theta, \varpi \} \) according to
\[
\begin{align*}
\varpi &= \omega ; \\
\theta &= \arcsin \left( \frac{ck_\rho}{|\omega|} \right) .
\end{align*}
\]
In this way the two terms in superposition (4) can now be written as
\[
\Psi^\pm(\rho, z, t) = \int_{0}^{\frac{\pi}{2}} d\theta \int_{-\infty}^{\infty} d\varpi B^\pm(\theta, \varpi) J_0 \left( \frac{\rho \varpi}{c} \sin \theta \right) e^{i\varpi \left( \pm z \cos \theta - ct \right)} ,
\]
\footnote{To derive this result we make use of orthogonality of Bessel functions
\[\int_{0}^{\infty} d\rho \rho J_0(\rho k_\rho) J_0(\rho k'_\rho) = \frac{1}{k_\rho} \delta(k_\rho - k'_\rho) .\]}
\footnote{We are taking into account the fact that \( J_0 \) is an even function.}
where \( B^\pm (\theta, \varpi) \) are the spectral distributions with respect to the new parameters. The parameter \( \theta \) is called the axicon angle. Clearly the superposition (4-5) is more general and includes this last one.

When the axicon angle \( \theta \) is fixed to some value \( \theta_0 \), the last expression yields

\[
\Psi^\pm_{\theta_0}(\rho, z, t) = \int_{-\infty}^{\infty} d\varpi \ S^\pm(\varpi) J_0 \left( \frac{\varpi c}{\rho} \sin \theta_0 \right) e^{i [\pm z \cos \theta_0 - ct]}.
\]

This expression is frequently found in the literature and defines the so-called X waves. Superposition (12) can be viewed as a cylindrically symmetric superposition of plane wave packets tilted over the \( z \) axis by an angle \( \theta_0 \) [8, 9]. The particular form of these packets is given by the spectral distributions \( S^\pm(\varpi) \). As an example, if we choose

\[
S^\pm(\varpi) = \frac{-ie^{i(\varpi-\omega_c)T_0}}{4\pi(\varpi-\omega_c)} \left[ e^{i(\varpi-\omega_c)T_0} - 1 \right] \left[ 1 \pm \frac{\omega_c}{\varpi} \right],
\]

then the plane wave packets are given by a rectangular pulse modulation of a carrier of frequency \( \omega_c \). In this expression \( T \) is the time duration of the pulses and \( t_0 \) is the instant of time in which their wavefronts reach the origin.

At this point we observe that X waves (12) are infinitely extended along the direction of propagation \( z \). So, they do not have a well defined wavefront into this direction and thus do not define SB signals. In the next section we shall consider the superposition (4-5) which “chops” (perpendicularly to \( z \) direction) a Bessel beam characterized by a given frequency \( \omega_0 \) and axicon angle \( \theta_0 \). From these chopped Bessel beams we will construct the corresponding chopped X waves.

\[3\] If \( S^+(\varpi) = \delta(\varpi - \omega_0) \) then \( \Psi^+_{\theta_0} \) defines a Bessel beam (1) characterized by the axicon angle \( \theta_0 \) and frequency \( \omega_0 \) propagating to the right. The same occurs if \( S^- = \delta \), with motion to the left.
3 Chopped Bessel beams

We now consider a SB signal consisting of a Bessel beam modulated in time by a rectangular pulse at the plane \( z = 0 \), i.e., a chopped Bessel beam. This signal is introduced in [13] through the boundary conditions (4-5), with

\[
\Psi^0(\rho, t) = T(t)J_0 \left( \frac{\omega_0}{c} \sin \theta_0 \right) e^{-i\omega_0 t},
\]

(14)

\[
\Psi^0(\rho, t) = T(t) \left( i\frac{\omega_0}{c} \cos \theta_0 \right) J_0 \left( \frac{\omega_0}{c} \sin \theta_0 \right) e^{-i\omega_0 t},
\]

(15)

where \( \omega_0 \) and \( \theta_0 \) are fixed constants and the time modulation is given by \( T(t) = \Theta(t) - \Theta(t - \tau) \), with \( \Theta(t) \) being the Heaviside step function. To determine the spectral distributions in the general superposition (4-5) which correspond to these boundary conditions we substitute these last expressions into (9). Then we have

\[
A^\pm(k_\rho, \omega) = \delta \left( k_\rho - \frac{\omega_0}{c} \sin \theta_0 \right) C^\pm(\omega),
\]

(16)

where

\[
C^\pm(\omega) = \frac{1}{4\pi} \frac{e^{i(\omega - \omega_0)\tau}}{i(\omega - \omega_0)} \left\{ 1 \pm \frac{\omega_0}{k_z(\frac{\omega_0}{c} \sin \theta_0, \omega)} \right\}.
\]

(17)

We see that the spectral distributions \( A^\pm(k_\rho, \omega) \) split into a factor depending only on \( k_\rho \) and another factor depending only on \( \omega \). Then, we can write the right moving term in (4) as

\[
\Psi^+(\rho, z, t) = \int_{0}^{\infty} dk_\rho \delta \left( k_\rho - \frac{\omega_0}{c} \sin \theta_0 \right) \phi^+(k_\rho, \rho, z, t),
\]

(18)

where

\[
\phi^+(k_\rho, \rho, z, t) = \int_{-\infty}^{\infty} d\omega C^+(\omega)J_0(\rho k_\rho) \exp\{i[k_z(\omega, \omega)z - \omega t]\}.
\]

(19)
We can readily identify this last integral with that describing a wave packet formed from superposition of (inhomogeneous) usual plane waves and moving within a tenuous electronic plasma, with plasma frequency given by \( \omega_p = ck_p \).

As the spectral distribution \( C^+(\omega) \) is analytical in the upper half of the complex \( \omega \)-plane, this packet itself has a well defined wavefront in time at \( z = 0 \) \([1, 2]\). Therefore, we can readily apply the results of Sommerfeld and Brillouin theory to conclude that this wavefront moves with the velocity \( c \) and the wave packet distorts while propagating.

To completely describe the wave motion to the right, we must now consider the \( k_p \) integral in (18). So, we have

\[
\Psi^+(\rho, z, t) = \phi^+ \left( \frac{\omega^0_p}{c}, \rho, z, t \right), \tag{20}
\]

where in this expression \( \omega^0_p = \omega_0 \sin \theta_0 \). Thus, all the conclusions after equation (19) remains valid substituting the plasma frequency \( ck_p \) by \( \omega^0_p \). These conclusions also hold for the left moving part of the wave packet (4).

Superposing now the chopped Bessel beams above via spectral distributions \( S^{\pm}(\omega_0) \), we have

\[
\Psi_{\theta_0}^{\text{chop}}(\rho, z, t) = \Psi^{(+)}_{\theta_0}^{\text{chop}}(\rho, z, t) + \Psi^{(-)}_{\theta_0}^{\text{chop}}(\rho, z, t), \tag{21}
\]

where

\[
\Psi^{(\pm)}_{\theta_0}^{\text{chop}}(\rho, z, t) = \int_{-\infty}^{\infty} d\omega_0 S^{\pm}(\omega_0) \phi^\pm \left( \frac{\omega^0_p}{c}, \rho, z, t \right). \tag{22}
\]

It is easy to verify that in the absence of the time modulation \( T(t) \) this superposition would satisfy the same boundary conditions as the X wave (12) if we identify the spectral distributions in the two expressions. Accordingly, we call superposition (21-22) a chopped X wave. As this wave is a linear
superposition of chopped Bessel beams, which have well defined wavefronts propagating with the velocity $c$, also the chopped X waves will have this property. As a consequence, they cannot carry superluminal SB signals.

Now we consider briefly the experiment of Mugnai, Ranfagni, and Ruggeri, in which the authors measured superluminal velocities in the propagation of X waves produced experimentally by a finite aperture device [11]. They posed the question about the possibility of interpretation of these superluminal velocities as velocities of signals. Causal explanations for the results of this experiment were given in references [13, 17] based on simple models showing interference phenomena of waves produced outside the axis along which the superluminal velocities were measured. These explanations refute the possibility of interpreting the measured superluminal velocities as velocities of signals in that experiment. In the reference [13] the authors call the attention to the fact that this experiment shows a kind of generalized reshaping phenomenon occurring in free space, characterized by the fact that the peak travels along the symmetry axis faster than the wavefront.

In order to apply our analysis to this experiment we assume that each pulse of the experimentally produced waves can be viewed as a finite aperture approximation to an ideal chopped X wave. We consider the propagation in the region $z > 0$, where the plane $z = 0$ contains the borders of the mirror which produce the wave during the time interval $[0, \tau]$. To set the theoretical (infinite aperture) model to the experiment we consider only the right moving packet in (22), with the spectral distribution $S^+$ given by (13). Now we interpret the parameter $\omega_c$ as the frequency of the microwave carrier in the experiment, $T$ as the time duration of the rectangular modulation of
this carrier and \( t_0 = \frac{R \sin \theta_0}{c} \) as the instant in which the peak (which travels along the \( z \) axis) begins to be generated at the \( z = 0 \) plane. \( R \) is the radius of the aperture and \( \theta_0 \) is the axicon angle fixed by the experiment.\(^4\) In order to produce the appropriate boundary conditions in the region of finite aperture (\( \rho \leq R \)) at the plane \( z = 0 \), the time duration \( \tau \) of the chopped X wave must satisfy \( \tau \geq T + 2t_0 \).

Assuming this model, our results about propagation of a chopped X wave imply that its wavefront propagate with velocity \( c \) and can not carry a superluminal signal, a result that agrees with references \([13, 15]\). Also, we suggest that the cited reshaping phenomenon could be at least qualitatively explained from the fact that the wave packet distorts along its propagation.

4 General results

In the last section the essential feature allowing us to identify the analogy between the propagation of the chopped Bessel beam in vacuum and the propagation of usual plane wave packets within a dispersive medium was the factorization of the spectral distribution \( A^\pm(k_\rho, \omega) \) into a product of a distribution depending only on \( k_\rho \) and another depending only on \( \omega \). We can generalize our results for the class of all spectral distributions satisfying this property. For simplicity, we will concern us to the right moving part of the wave packet (\( \Psi^+ \)). All the conclusions will also hold for the left moving part.

Let \( A^+(k_\rho, \omega) = A_\rho(k_\rho)A_\omega(\omega) \). Then, from (5) we have

\[
\Psi^+(\rho, z, t) = \int_0^\infty dk_\rho A_\rho(k_\rho)\phi^+(k_\rho, \rho, z, t), \tag{23}
\]

\(^4\)The axicon angle is given by \( \theta_0 = \arctan \frac{d}{2f} \), where \( d \) is the mean diameter of the slit and \( f \) is the focal length of the mirror.
where
\[
\phi^+(k_{\rho}, \rho, z, t) = J_0(\rho k_{\rho}) \int_{-\infty}^{\infty} d\omega A_{\omega}(\omega) \exp\{i[k_z(k_{\rho}, \omega)z - \omega t]\}. \tag{24}
\]

Following the same lines of the last section, we identify this expression as representing an inhomogeneous usual plane wave packet propagating within a tenuous electronic plasma, with plasma frequency $c k_{\rho}$. Again, the $k_{\rho}$ integral in (23) denotes a superposition of these packets. If each component packet have a well defined wavefront in time at some plane perpendicular to the $z$ axis, the complete superposition will also have a time wavefront which, by our previous analysis, propagates with velocity $c$.

To summarize, the velocity $c$ is the upper bound for the velocity of any SB signal carried by a wave packet belonging to this class.

## 5 Concluding remarks

In this letter we studied the wavefront velocity of time limited (chopped) Bessel beams wave packets and showed that they propagate in vacuum in a way analogous to usual (inhomogeneous) plane waves propagating within a tenuous electronic plasma. From this analogy we were able to apply the Sommerfeld and Brillouin results, originally conceiving the propagation of usual plane wave packets within dispersive media, to conclude that the wavefronts of these wave packets move always with velocity $c$, while the waveform distorts along the propagation. These conclusions were generalized for a large class of wave packets having well defined wavefronts and described by factorizable spectral distributions.

The above results contradict some conclusions of references [13, 14], in
which the authors conclude that chopped Bessel beams propagate *without distortion* with superluminal velocity. By a careful computation of the spectral distributions in (3) *which correctly give the boundary conditions* (14-15) we found that the spectral distribution for the parameter \( k_{\rho} \) is a delta distribution which fixes this parameter at the value \( \frac{\omega_0}{c} \sin \theta_0 \), contrary to the claims of the authors of [13], which consider \( k_{\rho} \) as a parameter depending on the varying frequency \( \omega \) (not to be confused with the fixed parameter \( \omega_0 \), characterizing the boundary conditions).\(^5\)

From superposition of chopped Bessel beams we constructed chopped X waves and suggested that these waves could be viewed as *infinite aperture theoretical models* for the waves produced in the experiment of Mugnai *et al*, which posed a question about the superluminality of signals carried by Bessel beams. By our analysis the chopped X waves have wavefronts which move with velocity \( c \). Therefore, they can not carry superluminal signals. Also, from the fact that these packets distort along the propagation, we suggest that the *reshaping* phenomenon cited in [13] could be at least qualitatively explained. It would be interesting to develop the analogy with dispersive media further in order to compare also quantitatively the results arising from this model with the experimental data of Mugnai *et al* and with the models based on spherical waves presented in [13, 15].

Our analysis introduced the formal analogy with dispersive media as an alternative way to approach the problem of velocities of signals carried by Bessel beams. We hope it can help us to better understand this debated question.

\(^5\)\(k_{\rho}\), in our notation, corresponds to the separation constant \( \Omega \), in their notation.
6 Acknowledgements

I thank Prof. Dr. B.M. Pimentel and Dr. L.A. Manzoni for critical reading the manuscript and CAPES/PICDT for partial support. I also thank the anonymous referee for useful criticisms and suggestions.

References

[1] J. D. Jackson, *Classical Electrodynamics* 3rd ed. (J. Wiley and Sons, New York 1999).

[2] L. Brillouin, *Wave Propagation and Group Velocity* (Academic Press, New York, 1960).

[3] L. J. Wang, A. Kuzmich, and A. Dogariu, Nature, 406, 277 (2000).

[4] T. Sauter, Phys. Lett. A 282 (2001) 145.

[5] B. Segev, P. W. Milloni, J. F. Babb, and R. Y. Chiao, Phys. Rev. A 62 (2000) 022114.

[6] G. Nimtz and A. Haibel, Ann. Phys. 9 (2000) 1.

[7] J. Durnin, J.J. Miceli, Jr., and J.H. Eberly, Phys. Rev. Lett. 58 (1987) 1499.

[8] P. Saari and K. Reivelt, Phys. Rev. Lett. 79 (1997) 4135.

[9] P. Saari, “Superluminal Localized Waves of Electromagnetic Field in Vacuo”. To be published in: Proceedings of the Conference “Time’s
Arrows, Quantum Measurements and Superluminal Behaviour” (Naples, October 2-6, 2000) (Italian NCR).

[10] E. Recami, Physica A 252 (1998) 586.

[11] D. Mugnai, A. Ranfagni, and R. Ruggeri, Phys. Rev. Lett. 84 (2000) 4830.

[12] D. Mugnai, Phys. Lett. A 278, 6 (2000); Phys. Lett. A 284 (2001) 304.

[13] W. A. Rodrigues, Jr., D. S. Thober, and A. L. Xavier, Jr., Phys. Lett. A 284 (2001) 217.

[14] E. Capelas de Oliveira, W. A. Rodrigues, Jr., D. S. Thober, and A. L. Xavier, Jr., Phys. Lett. A 284 (2001) 296.

[15] T. Sauter and F. Paschke, Phys. Lett. A 285 (2001) 1.