ENERGY IMPLICATIONS OF TEMPERATURE FLUCTUATIONS IN PHOTOIONIZED PLASMA

Luc Binette and Valentina Luridiana
Instituto de Astronomía
Universidad Nacional Autónoma de México
Received March 19, 2022; accepted March 19, 2022

ABSTRACT

We quantify the energy radiated through all the collisionally excited lines in a photoionized nebula which is permeated by temperature fluctuations. We assume that these correspond to hot spots which are the results of an unknown heating process distinct from the photoelectric heating. We consider all the effects of using a higher mean temperature (as compared to the equilibrium temperature) due to the fluctuations not only on each emission line but also on the ionization state of the gas. If this yet unknown process was to radiate a fixed amount of energy, we find that the fluctuations should correlate with metallicity \( Z \) when it exceeds 0.7 solar. The excess energy radiated in the lines as a result of the fluctuations is found to scale proportionally to their amplitude \( t^2 \). When referred to the total energy absorbed through photoionization, the excess energy is comparable in magnitude to \( t^2 \).

Key Words: ISM – ABUNDANCES – PLANETARY NEBULAE – HII REGIONS

1. INTRODUCTION

The temperatures of photoionized nebulae are observed to be significantly lower when derived using recombination lines rather than from forbidden line ratios (e.g. Peimbert et al. 1995). This phenomenon has been ascribed to the existence of temperature fluctuations permeating the nebulae. Assuming that the fluctuations inferred by various authors (e.g. Peimbert et al. 1995, Esteban et al. 1998, Rola & Stasińska 1994) are caused by an additional albeit unknown heating agent (beside photoionization), we proceed to quantify the energy contribution which this unknown heating process must contribute to the total energy budget of the nebula in order to account for the much higher temperatures characterizing the collisionally excited lines as compared to those inferred from recombination lines (or nebular Balmer continuum). In order to study the effects of arbitrary temperature fluctuations, we first describe the modi-
fications made to the multipurpose photoionization-shock code Mappings ic (Ferruit et al. 1997). In Section 3 we present photoionization calculations in which we consider different levels of fluctuation amplitudes and quantify how they alter the global energy budget of the nebula. A brief discussion is presented in Section 4.

2. THE ENERGY EXPENSE CAUSED BY NEBULAR HOT SPOTS

We present a few definitions followed by the procedure we adopt to implement the effect of temperature fluctuations in the code Mappings ic.

2.1. Definitions of \( t^2 \) and mean temperature \( \overline{T}_0 \)

Following Peimbert (1967), we define the mean nebular temperature, \( \overline{T}_0 \), as follows

\[
\overline{T}_0 = \frac{\int_V n_e^2 T dV}{\int_V n_e^2 dV},
\]

in the case of an homogeneous metallicity nebula characterized by small temperature fluctuations; \( n_e \) is the electronic density, \( T \) the electronic temperature and \( V \) the the volume over which the integration is carried out. The rms amplitude \( t \) of the temperature fluctuations is given by

\[
t^2 = \frac{\int_V n_e^2 (T - \overline{T}_0)^2 dV}{\overline{T}_0^2 \int_V n_e^2 dV}.
\]

Note that we simplified the expression presented by Peimbert (1967) whose definition of \( t^2 \) differs in principle with each ionic species density \( n_i \) while in the above equations we implicitly consider only ionized H (by setting \( n_{H^+} = n_e \)). Since \( t^2 \) in this paper is not an observed datum but an a priori global property of the nebular model, such differences are not important.

The intensity of a recombination line is in general proportional to \( T^\alpha \) while for a collisionally excited line it is proportional to \( T^\beta \exp(-\Delta E/kT) \) where \( \Delta E \) is the energy separation of the two levels involved in the transition. In either case, \( \beta \) and \( \alpha \) typically lie in the range \(-0.5 \) to \(-1\).

2.2. Approximating fluctuations as hot spots above \( T_{eq} \)

Since the fluctuations’ amplitudes \( t^2 \) derived from observations are much larger that that predicted by photoionization models (cf. Pérez 1997), the solution to this inconsistency resides not in adding an additional uniform heating/cooling term to the thermal balance equations (like heating by dust grain photoionization) since this would simply result in a uniform raise of \( T \) and not in larger fluctuations. What is required to increase \( t^2 \) in models is that such process be operating in a non-uniform manner across the nebula and the picture which we propose is that of many hot spots created by this heating process.

It is beyond the scope of this paper to consider any particular physical process to account for the fluctuations nor to model them in details. Our aim is limited to study the effect of ad hoc temperature fluctuations a la Peimbert on the energy budget under the assumption that these arise from this unknown extra heating mechanism acting non-uniformly across the nebula.

In photoionization calculations, it is customary to define and use at every point in the nebula a local equilibrium temperature, \( T_{eq} \), which satisfies the condition that the cooling by radiative processes equals the heating due to the photoelectric effect.

In our hot spots scheme, by construction, \( T_{eq} \) corresponds to the temperature floor above which take place all the fluctuations. For illustrative purposes,
we show in Fig. 1 a possible rendition of fluctuations (the solid wavy line) characterized by an amplitude $t^2 = 0.005$ (from Eq. 1) and consisting of hot spots. Because $T_{eq}$ is a minimum in the distribution of $T$ fluctuations, it defines the null energy expense when calculating the extra energy emitted as result of the hot spots.

2.3. The determination of $\bar{T}_0$

The fluctuations are taken into account by MAPPINGS IC only in the statistical sense, that is by defining and using temperatures which are derived from $\bar{T}_0$. Given a certain amplitude of fluctuations $t^2$, we use everywhere a value for the mean temperature which is derived from the computed local equilibrium temperature ($T_{eq}$) and which takes into account that $T_{eq}$ is a lower extremum in the distribution of $T$ fluctuations (consisting of hot spots). To define $\bar{T}_0$, we found adequate using the following expression (based on the inverse of Eq. 1)

$$\bar{T}_0 \simeq T_{eq}[1 + \gamma(\gamma - 1)t^2/2]^{-1/\gamma} \quad \text{(3)}$$

but where the optimum value of $\gamma$ has been inferred using numerical simulations of the hot spots. One simulation shown in Fig. 1 is characterized by $T_{eq} = 9500$ K and $t^2 = 0.005$. By fitting $\gamma$ so that the value of $\bar{T}_0$ matches the numerically computed value of $9800$ K, we obtain that $\gamma \approx -15$. Within the deduced regime in which $\gamma \ll -1$, the above equation varies slowly with $\gamma$. The purpose of this expression is that it defines consistently $\bar{T}_0$ whatever the value of the equilibrium temperature calculated by MAPPINGS IC. Interestingly, $\gamma$ (or $\bar{T}_0$) is invariant to simultaneously scaling up and down of all the hot spot amplitudes (which is equivalent to increasing or decreasing $t^2$). On the other hand $\gamma = -15$ strictly characterizes a specific distribution of hot spot frequencies and widths. It is the correct value for fluctuations resembling those depicted in Fig. 1 but for other radically different distributions of hot spot widths and frequencies, $\gamma = -15$ would only provide a first order albeit acceptable estimate of $\bar{T}_0$. Despite this caveat, we estimate the uncertainties affecting the final determination of $\Gamma_{\text{heat}}$ to be $< 20\%$.

2.4. Recombination processes

For small fluctuations, the temperature can be expanded in a Taylor series about the mean $\bar{T}_0$. In the case of recombination lines the intensity, $I_{\text{rec}}$, of a given line is affected by a factor

$$I_{\text{rec}} \propto \bar{T}_0^\alpha (1 + \alpha(\alpha - 1)t^2/2) \quad \text{(4)}$$

This expression which can be used to compute individual recombination line intensities in the presence of small fluctuations, is equivalent to calculating the intensity (which is proportional to $T^\alpha$) using instead the effective temperature $\bar{T}_{\text{rec}}$

$$\bar{T}_{\text{rec}} = (T^\alpha)^{1/\alpha} \simeq T_0[1 + \alpha(\alpha - 1)t^2/2]^{1/\alpha} \quad \text{(5)}$$

As shown by Peimbert (1995 and references therein), the temperature fluctuations have in general much less impact on recombination than on collisional processes which are usually governed by the exponential factor. For this reason, we adopt the simplification of considering a single value of $\alpha = -0.83$ for all recombination processes (such $\alpha$ is the appropriate value for the H$\beta$ line at 10000K). This approximation will allow us to use a single temperature $\bar{T}_{\text{rec}}$ when solving for the ionization balance of H, He and all ions of metals (equations in which enter recombination rates).

If we consider the fluctuations drawn in Fig. 1 as example, we see that the mean recombination temperature $\bar{T}_{\text{rec}}$ derived using Eq. 1 lies slightly below $\bar{T}_0$.

Interestingly, in the case of the ionization-bounded dustfree models considered in this work, the adoption of one temperature or another when calculating recombination processes does not affect the global energy budget of the recombination lines. In effect, the total number of recombinations taking place across the whole nebula must equal the number of ionizing photons produced by the UV source, whatever the value and behaviour of the temperature. It is a self-regulating process: a hotter nebula [which results in slower recombinations rates and hence in a plasma containing less neutral H] of the type discussed below will simply turn out more massive in ionized gas in order that the number of recombinations remains equal to the same number of ionizing photons. The sum of all hydrogen recombination lines intensities will remain unchanged. Also constant is the total amount of heat deposited in the nebula through photoionization.

2.5. Collisional processes

To compute the forbidden line intensities, we solve for the population of each excited state of all ions of interest assuming a system of 5 or more levels according to the ion. (In the case of intercombination, fine structure and resonance lines, we treat those as simple 2 level systems.) More specifically, when evaluating the excitation ($\propto T^{3/2} \exp[-\Delta E_{ij}/kT]$) and deexcitation ($\propto T^{3/2}$)
rates of a given multi-level ion, each rate $ij$ (population) or $ji$ (depopulation) is calculated using $T_0$ (instead of $T_{\text{eq}}$) and then multiplied by the appropriate correction factor, either

$$c_{ij}^{\text{exc}} = 1 + \frac{t^2}{2} \left[ (\beta_{ij} - 1) \left( \frac{\Delta E_{ij}}{kT_0} \right) + \left( \frac{\Delta E_{ij}}{kT_0} \right)^2 \right]$$

in the case of excitation, or

$$c_{ji}^{\text{deexc}} = 1 + \beta_{ji} (\beta_{ji} - 1) \frac{t^2}{2},$$

in the case of deexcitation. These factors result in general in an enhancement of the collisional rates in the presence of temperature inhomoegeneities. They are adapted from the work of Peimbert et al. (1995) and were applied to all collisionally excited transitions.

For the case of the particular rendition of the fluctuations depicted in Fig. 3, the mean collisional temperature characterizing the [O III]$\lambda$4363 line (Peimbert et al. 1995) is shown by the horizontal dash line. Despite the large $\Delta E_{ij}$ involved in the emission of the line, the distance between this temperature and $T_0$ is nevertheless smaller than that separating $T_0$ from $T_{\text{eq}}$. Therefore, in some instances the extra energy radiated through hot spots might depend as much on the distance separating $T_0$ from $T_{\text{eq}}$ than on the above correction factors.

Inspection of the line ratios calculated with MAPPINGS ic using $T_0$ and the above correction factors confirms (as expected) that the higher is $\Delta E_{ij}$ the higher the line intensity enhancement (at constant $t^2$). It can be shown on the other hand that the far infrared lines (or any transition for which $\exp[-\Delta E_{ij}/kT] \approx 1$) are less affected by the fluctuations (similarly to the recombination lines) and can even become weaker as a result of the fluctuations, in contrast to most collisionally excited lines.

2.6. The energy radiated through hot spots

Our aim is to quantify the excess energy generated by temperature fluctuations under the assumption that these are caused by a putative heating mechanism which operates within small regions randomly distributed across the nebula. To calculate this energy we simply integrate over the nebular volume $V$ the luminosity of each line (or transition) $ij$ using the statistically determined local $T_0$ and the multiplicative correction factors of Eqs 6 and 7, and then subtract the corresponding luminosity obtained by using the equilibrium temperature $T_{\text{eq}}$ instead.

This excess energy radiated in the form of collisionally excited lines can be normalized respective to the total photoheating energy available. This defines the quantity $\Gamma_{\text{heat}}$.

$$\Gamma_{\text{heat}} = \frac{\sum_{ij}\int V[4\pi j_{ij}^{\text{fluc}} - 4\pi j_{ij}^{\text{eq}}] dV}{\sum_{ij}\int V 4\pi j_{ij}^{\text{eq}} dV + \sum_{k}\int V q_k^{\text{eq}} dV}$$

$$= \frac{L_{\text{fluc}} - L_{\text{eq}}}{L_{\text{eq}} + Q_{\text{eq}}},$$

where $j_{ij}^{\text{fluc}}$ corresponds to the local nebular emissivity of line $ij$ calculated using $T_0$ and taking into account the above correction factors while $j_{ij}^{\text{eq}}$ is the corresponding emissivity assuming equilibrium temperature everywhere. The term with $q_k^{\text{eq}}$ corresponds to various cooling rates from processes not involving line emission such as freefree emission, while $Q_{\text{eq}}$ represents the total volume integrated value of this term. $L_{\text{fluc}}$ and $L_{\text{eq}}$ correspond to the integrated energy loss across the whole nebula due to collisionally excited lines with fluctuations and without, respectively. Since $T_{\text{eq}}$ must satisfy the condition that the cooling rate equals everywhere the heating rate, we obtain that $L_{\text{eq}} + Q_{\text{eq}}$ is also equal to the total energy deposited into the electronic gas by the photoelectric effect.

A larger fraction of the ionizing radiation simply keeps the nebula ionized (resulting in recombination lines) but does not affect the nebular temperature. An alternative way therefore of expressing the importance of the excess cooling due to the fluctuations is to use as reference the total energy absorbed by the nebula including the energy emitted as recombination lines and nebular recombination continuum. We define $\Gamma_{\text{abs}}$ as follows

$$\Gamma_{\text{abs}} = \frac{L_{\text{fluc}} - L_{\text{eq}}}{L_{\text{total}}},$$

with $L_{\text{total}}$ the ionizing luminosity of the exciting star

$$L_{\text{total}} = \int_{\nu_0}^{\infty} L_\nu d\nu,$$

where $L_\nu$ is the energy luminosity distribution of the ionizing star and $\nu_0$ the frequency corresponding to the ionization threshold of H. To be consistent with this definition, we must consider only nebular models which are ionization-bounded and fully covering the ionizing source (over 4$\pi$ sterad). Depending on the hardness of the ionizing radiation, $\Gamma_{\text{abs}}$ turns out
to be 2–3 times smaller than $\Gamma_{\text{heat}}$, because of the larger fraction of the absorbed energy which goes into photoionizing rather than into heating the gas.

In summary, the modifications made to MAPPINGS ic to consider the effects of $T$ fluctuations not only include the calculation of the line intensities using the formalism described in Peimbert et al. (1995) but also considers their impact on the ionization balance across the nebula through the use of $T_{\text{rec}}$ (instead of $T_{\text{eq}}$) to derive the recombination rates. Since the nebula is substantially hotter on average when $t^2 > 0$, it will be more ionized since the recombination rates are slower at higher temperature. This in turn results in a lower photoionization and heating rates given that there is less neutral H in the nebula, therefore the equilibrium temperature $T_{\text{eq}}$ computed by MAPPINGS ic will be lower than without fluctuations. All these effects have been taken into account self consistently and do not affect the energy conservation principle in the case of ionization-bounded nebulae as discussed above and in in Sect. 2.4.

3. MODEL CALCULATIONS

We have explored the behaviour of $\Gamma_{\text{heat}}$ in photoionization models of different metallicity ($Z$), excitation ($U$) and different spectral energy distributions (hereafter SED). We will express the nebular metallicity with respect to the solar abundances (from Anders & Grevesse 1989) for which we take that $Z = 1$. To define other metallicities, we simply scale the abundances of all the metals respective to H by a constant multiplicative factor equal to $Z$. We summarize below our results under various model conditions.

For the photoionized H II regions, we have selected unblanketed LTE atmosphere models from Hummer & Mihalas (1970) of temperatures $T_{\text{eff}}$ of 40000 K, 45000 K and 50000 K [see Evans (1991) for a comparative study of nebular models using different metal atmospheres]. To represent planetary nebulae, we simply employed black bodies of 10$^5$ K and 10$^5.3$ K truncated at 54.4 eV. The geometry adopted in the calculation is plane-parallel with a gas density of $n_H = 10$ cm$^{-3}$ in all cases. The excitation of the nebula is defined by the excitation parameter $U$ as follows

$$U = \frac{1}{c n_H} \int_{\nu_0}^\infty \frac{L_\nu}{4 \pi r^2 h \nu} d\nu = \frac{\varphi_H}{c n_H},$$

where $c$ is the speed of light, $h$ the Planck constant and $r$ the distance of the slab from the ionizing star. $U$ is the ratio between the density of ionizing photons impinging on the slab ($\varphi_H/c$) and the total H density. All the calculations carried out were ionization-bounded.

Since hotter SEDs result on the other hand in much higher photoelectron energies and hence hotter nebulae at all metallicities, the maximum of the long-dashed curve corresponding to the 10$^5$ K blackbody in Fig. 2 is shifted towards higher $Z$ relative to the 45000 K SED.

3.1. Dependence of excess heating on metallicity

We have calculated nebular models of different metallicities covering the range 1% solar ($Z = 0.01$) to 4.7 times solar. In the models presented in Fig. 2 all other parameters are identical, namely, $U = 0.01$, $t^2 = 0.04$ and a spectral energy distribution (SED) having either $T_{\text{eff}} = 45000$ K or 100000 K. It can be seen that a maximum in $\Gamma_{\text{heat}}$ occurs within the range $Z \sim 0.2$–0.4. For the H II region models, the average values for $T_{\text{eq}}$ across the nebulae are $\sim 15000$ K, 9000 K, 5000 K and 1500 K for the $Z = 0.01, 0.7, 2.5$ and 4.7 models, respectively. Given that the nebular temperatures decrease monotonically with increasing $Z$, the curves’ behaviour can be understood as follows: at very low metallicities $Z \ll 0.2$, the forbidden lines of metals are not the main cooling agent and the fluctuations have
therefore a negligible impact on the total cooling. At higher Z values around solar, however, the cooling due to metals is very large and the nebula turns out much cooler, to the extent that many optical lines become now less intense despite the increase of the metal abundances. At even higher Z, the optical lines are progressively ‘switched off’ and cannot contribute to the cooling of the nebula. In this regime, these lines have $\Delta E \ll kT_0$ and become somewhat brighter as the temperature is further lowered, explaining why the fluctuations now cause $\Gamma_{heat}$ to become negative. The final rise at the upper Z end in the H II region model sequence (solid line) reflects the fact that at such low temperature the cooling can only be carried out by the low energy transitions (far infrared lines) where again $\Delta E > kT_0$.

The abrupt decrease of $\Gamma_{heat}$ above solar metallicity has interesting consequences if the unknown heating process responsible for the fluctuations needed to radiate a comparable amount of energy in different objects. This could arguably be the case for instance if this process was reconnection of magnetic field lines. All other factors being equal, nebulae three times solar would require much larger amplitude turbulences (larger $t^2$) to radiate the same amount of energy than a solar metallicity nebula. In this case we might expect to observe a correlation between $t^2$ and metallicity (beyond $Z \geq 0.7$).

3.2. Dependence of excess heating on $t^2$

In the following calculations, we adopt two representative metallicities of Z = 0.2 and Z = 1 (solar). We found no clear trends across different SEDs of how $\Gamma_{heat}$ varied with $U$ and therefore we only report results concerning a single ionization parameter of value $10^{-2}$. Campbell (1988) has shown that the range of $U$ for most H II galaxies lies in the range $10^{-2.6}$ to $10^{-1.8}$.

In Figs. 3 and 4, we show the behavior of both $\Gamma_{heat}$ and $\Gamma_{abs}$ as a function of increasing $t^2$ of the models. Each line corresponds to a given SED and Z. The increase in $\Gamma_{heat}$ is steeper at small values of $t^2 \lesssim 0.1$ followed by a more linear regime with a slope $\sim 2.5$ at large $t^2$. The radiated energy contribution from the fluctuations (compared to photoelectric heating) is of order $2.5t^2$ (Fig. 3) but with a wide dispersion when $Z = 0.2$ (curves labelled S). For $Z = 1$ (curves labelled S), the range of $t^2$ for these models is too small to allow labelling of each SED. $\Gamma_{heat}$ becomes approximately linear at larger $t^2$. For comparison we show a thick line representing $\Gamma_{heat} = 2.5t^2$.

If we consider the total energy budget (photoheating plus recombination energy, see Eq. 10) and not just the photoelectric heating part, we find that $\Gamma_{abs}$ is comparable in magnitude to $t^2$ (Fig. 4) but

Figure 3. Behaviour of $\Gamma_{heat}$ with increasing $t^2$. Two metallicities were used: solar (labelled S) and 0.2 solar (labelled L) and 4 SED: 40000 K, 45000 K, 50000 K, 100000 K and 200000 K labelled 40, 45, 50, 100 and 200, respectively (the L sequences are too close to allow labelling of each SED). $\Gamma_{heat}$ becomes approximately linear at larger $t^2$. For comparison we show a thick line representing $\Gamma_{heat} = 2.5t^2$.

Figure 4. Behaviour of $\Gamma_{abs}$ with increasing $t^2$. Same notation as in Fig. 3. For comparison we show thick lines representing $\Gamma_{abs} = t^2$ and $2.5t^2$, respectively.
again with a wide dispersion which result from differences in either $Z$ or the SED.

4. DISCUSSION

If the turbulences are the result of heating by a yet undiscovered process, we infer from Fig. 3 and 4 that the extra energy radiated via all the collisionally excited lines due to the turbulences is a substantial fraction ($\Gamma_{\text{abs.}} > 10\%$) of the total photoionization energy whenever $t^2 > 0.02$ for planetary nebulae and $t^2 > 0.08$ for H II regions, respectively. Taking instead as reference only the heating by photoionization ($\Gamma_{\text{abs.}}$), these limits are reduced by two. In this case almost all nebulae would radiate more than 10% ($\Gamma_{\text{abs.}}$) of their energy as a result of $T$ fluctuations. In effect, typical empirically determined $t^2$ values for galactic H II regions lie in the range 0.02–0.06 (Luridiana 1999). For extragalactic H II regions (Luridiana et al. 1999, González-Delgado et al. 1994) and planetary nebulae (Peimbert and Peimbert & Peña 1995), even larger values have been encountered, of order 0.1 or more. In those cases, the energy involved can be a large fraction of the energy budget especially if the exciting stellar temperature exceeds $10^4$ K.

The underlying assumption behind our calculations is that an external heating agent is at work to study in some details: 

1. Metallicity inhomogeneities Temperature fluctuations would result naturally from nebulae consisting of a multitude of condensations of greatly varying metallicities (see Torres-Peimbert, Peimbert & Peña 1990).

2. Gas expansion Fluctuations could arise from an outflowing wind generated at the surface of dense condensations (problyds) as a result of the champagne effect. Rapid adiabatic expansion would result in overcooled emission regions in the wake of the wind (resulting in negative $T$ fluctuations with respect to $T_{\text{eq}})$.

3. Ionizing source variability A rapidly varying ionizing field will generate a sequence of outwardly propagating ionization and recombination fronts. The basic asymmetry existing between ionization fronts (propagating at a speed limited by changes in opacity) and recombination fronts (non-propagating), would lead to temperature fluctuations.

To our knowledge, none of these mechanisms have yet been incorporated explicitly in any nebular model and we therefore cannot assess their particular merits. Mechanisms which are good candidates to explain the extra heating assumed in this work (see also discussion by Stasińska 1998) and which are not directly related to photoionization, have been proposed and include the following processes:

1. Shock heating Either as a result of stellar wind or from supernovae. In the case of giant H II regions, this later mechanism was favored by Luridiana et al. (1999).

2. Magnetic heating Turbulent dissipation from Alfvén waves and magnetic line reconnection has been proposed to explain the variations of the [N II]/H$\alpha$ ratio emitted by the warm diffuse ionized gas in the Galaxy (Reynolds et al. 2000). In the case of planetaries, the large values of $t^2$ found would then make sense in the light of the success of the magnetically accelerated wind models of García-Segura (2000). Furthermore, Peimbert et al. (1995) has shown that the nebulae with the highest gas velocity dispersion show the highest values of $t^2$, which would be consistent with an increasing role played by magnetic acceleration of the gas in the objects with the largest velocity dispersion.

In summary, the current work shows how $\Gamma_{\text{abs.}}$ and $\Gamma_{\text{heat}}$, vary with metallicity and $t^2$ as well as how these quantities are affected by the ionizing energy distribution. Our results can be translated into energy requirements (function of $t^2$) which competing explanations of the temperature fluctuations must satisfy and, hence, can be used to probe their respective viability.

The work of LB was supported by the CONACyT grant 32139-E.

REFERENCES

Anders, E., Grevesse, N. 1989, Geochim. Cosmochim. Acta 53, 197
Campbell, A. W. 1988, ApJ, 335, 644
Evans, I. 1991, ApJS, 76, 985
Ferruit, P., Binette, L., Sutherland, R. S. and Pécontal, E. 1997, A&A, 322, 73
González-Delgado, R. et al. 1994, ApJ, 437, 239

\[^{1}\] Measured values of $t^2$ in excess of 0.1 should be considered only as rough estimates since the assumed regime of small fluctuations does not apply anymore.
Hummer, D. G., & Mihalas, D. M. 1970, MNRAS, 147, 339
Luridiana, V., 1999, PhD thesis: Condiciones físicas en nebulosas gaseosas, (UNAM: Mexico city)
Luridiana, V., Peimbert, M. & Leitherer, C. 1999, ApJ, 527, 110
Peimbert, M. 1967, ApJ, 150, 825
Peimbert, M. 1995, in proc. of The Analysis of Emission Lines, ed. R. E. Williams and M. Livio (Cambridge: Cambridge University Press), 165
Peimbert, M., Luridiana, V., & Torres-Peimbert, S. 1995, RevMexAA, 31, 131
Pérez, E. 1997, MNRAS, 290, 465
Reynolds, R. J., Haffner, L. M., & Tufte, S. L., 2000, in proc. of Astrophysical plasmas: Codes, Models and Observations, RevMexAA conf. series, Eds. J. Franco and J. Arthur, in press
Stasińska, G., 1998, in proc. of Abundance Profiles: Diagnostic Tools for Galaxy History, eds. D. Friedli, M. Edmunds, C. Robert, and L. Drissen, ASP Conf. Ser., 147, 142.
Torres-Peimbert, S., Peimbert, M., Peña, M., 1990, A&A, 233, 540

L. Binette: Instituto de Astronomía, UNAM, Ap. 70-264, 04510 D.F., México (binette@astroscu.unam.mx)
V. Luridiana: Instituto de Astronomía, UNAM, Ap. 70-264, 04510 D.F., México (vale@astroscu.unam.mx)