Collisional Energy Loss in a Finite Size QCD Matter

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Abstract

Computation of collisional energy loss in a finite size QCD medium has become crucial to obtain reliable predictions for jet quenching in ultra-relativistic heavy ion collisions. We here compute this energy loss up to the zeroth order in opacity. Our approach consistently treats both soft and hard contributions to the collisional energy loss. Consequently, it removes the unphysical energy gain in a region of lower momenta obtained by previous computations. Most importantly, we show that for characteristic QCD medium scales, finite size effects on the collisional energy loss are not significant.

1 Introduction

The suppression pattern of high transverse momentum hadrons is a powerful tool to map out the density of a QCD plasma created in ultra-relativistic heavy ion collisions (URHIC) [1]-[3]. This suppression (called jet quenching) is considered to be mainly a consequence of medium induced radiative energy loss of high energy partons [4]-[7]. However, recent non-photonic single electron data [8, 9] (which present an indirect probe of heavy quark energy loss) showed large disagreement with the radiative energy loss predictions, as long as realistic values of parameters are assumed [10]. This raised the question of what is the cause for the observed discrepancy.

Recent studies [11, 12] suggested that one of the basic assumptions that pQCD collisional energy loss is negligible compared to radiative [13] may be incorrect. In [11, 12] it was shown that, for a range of parameters relevant for RHIC, radiative and collisional energy losses for heavy quarks were in fact comparable to each other, and therefore collisional energy loss can not be neglected in the computation of jet quenching. This result came as a surprise because from the earlier estimates [13]- [18], the typical collisional energy loss was erroneously considered to be small compared to the radiative energy loss. In [19] it was consequently suggested that the inclusion of collisional energy loss may help in solving the “single electron puzzle”. However, in that study (as well as [11]-[18]) finite size effects were not taken into account.

A recent paper by Peigne et al. [20] is the first study that made an attempt to include finite size effects in the collisional energy loss. This work suggested that collisional energy loss is large only in the ideal infinite medium case, while finite size effects lead to a significant reduction of the collisional energy loss. However, this paper did not completely separate collisional and radiative energy loss effects. Consequently, it remained unclear how important are the finite size effects on the collisional energy loss.

Therefore, it became necessary to consistently compute (only) the collisional energy loss in finite size QCD medium. Additionally, this paper aims to address whether -and to what extent- there is an over-counting between collisional and radiative energy loss computations.
The outline of the paper is as follows: In Section 2, we will compute the collisional energy loss in a finite size QCD medium. In Section 3, we will consider the special case when a particle is produced at \( x_0 = -\infty \) (infinite medium case). We will show that in special limits, our calculations recover previous results \[14, 15\]. However, contrary to \[14, 15\], our computation does not encounter unphysical energy gain in the low momentum region \[14, 15\]. In Section 4 we will give a numerical study of the collisional energy loss in both finite and infinite QCD medium. Contrary to the results obtained in Peigne et al. \[20\] we will show that finite size effects do not have a significant effect on the collisional energy loss. The conclusions and outlook are given in Section 5.

## 2 Collisional energy loss in finite size QCD medium

In this Section we will compute the collisional energy loss (up to the 0\(^{th}\) order in opacity) when the jet is produced in a finite size dielectric medium. The contribution to this energy loss comes from one Hard-Thermal Loop (HTL) gluon propagator (see Appendix A), which is the reason why we call it the 0\(^{th}\) order in opacity collisional energy loss (note the analogy with the 0\(^{th}\) order in opacity radiative energy loss \[21\]-\[23\], which is further discussed in Appendix A).

In this computation we use the most intuitive approach, i.e. we compute the diagram |\( M_{el} \)| shown in Fig. 1. Note that the "blob" represents the effective gluon propagator. A proof of the validity of this approach is given in Appendix A. This approach has already been used without proof in \[24, 25\], under the justification that it reproduces the same results as the imaginary time formalism.

![Figure 1: Feynman diagram for the amplitude that contributes to the collisional energy loss in QCD medium. The large dashed circle ("blob") represents the effective gluon propagator \[21\].](image)

Similarly as in \[23\] we introduce the finite size medium by starting from the approach described in \[26\]. We consider a static medium of size \( L \), and assume that collisional energy loss can occur only inside the medium. The Feynman diagram \( |M_{el}| \) (see Fig. 1) then represents the source \( J \), which at time \( x_0 \) produces an off-shell jet with momentum \( p \), and subsequently (at \( x_1 > x_0 \)) exchanges a virtual gluon with parton in the medium with momentum \( k \). The jet and the medium parton emerge with momentum \( p' \) and \( k' \) respectively. Since our focus is on heavy quark jets with mass \( M \), we here neglect the thermal shifts of the heavy quark mass.

We assume, as in \[27\], that \( J \) changes slowly with \( p' \), i.e. that \( J(p' + q) \approx J(p) \). Since we consider both soft and hard contributions, we take into account spin effects. The computation that we present in this paper is gauge invariant \[24\], but for simplicity we further use Coulomb gauge.

The effective gluon propagator shown in Fig. 1 has both transverse and longitudinal contributions \[28\]-\[32\]. In Coulomb gauge the gluon propagator has the following form:

\[
D^{\mu\nu}(\omega, \vec{q}) = -P^{\mu\nu}\Delta_T(\omega, \vec{q}) - Q^{\mu\nu}\Delta_L(\omega, \vec{q}),
\]

where \( q = (\omega, \vec{q}) \) is the 4-momentum of the gluon, while \( \Delta_T \) and \( \Delta_L \) are effective transverse and
longitudinal gluon propagators given by [32]:

\[ \Delta_T^{-1} = \omega^2 - \frac{\mu^2}{2} - \frac{(\omega^2 - \vec{q}^2)\mu^2}{2\vec{q}^2} (1 + \frac{\omega}{2\vec{q}} \ln \frac{\omega - |\vec{q}|}{\omega + |\vec{q}|}), \]  

\[ \Delta_L^{-1} = \vec{q}^2 + \mu^2(1 + \frac{\omega}{2\vec{q}} \ln \frac{\omega - |\vec{q}|}{\omega + |\vec{q}|}), \]  

where \( \mu_D^2 = g^2T^2(1 + \frac{N_c}{6}) \) is the Debye mass.

The only nonzero terms in transverse \( (P_{\mu\nu}) \) and longitudinal \( (Q_{\mu\nu}) \) projectors are the following:

\[ P^{ij} = \delta^{ij} - \frac{q^i q^j}{|q|^2}, \]  

\[ Q^{00} = 1. \]  

The matrix element for this 0th order in opacity collisional process can then be written in the following form (for simplicity we here omit color factors, whose contribution we will add in the end)

\[ iM_{el} = \int d^4x_0 \; J(x_0) \; d^4x_1 \; (-i) \; \int \frac{d^3\vec{p}}{(2\pi)^32E} \; \Theta(t_1 - t_0) \; \Theta(L/v - (t_1 - t_0)) \; \bar{u}(p', s') e^{ip'x_1} \; i\gamma^\mu u(p, s) e^{-ipx_1} \; \int d^4x_2 \; (-i) \; \int \frac{d^4q}{(2\pi)^4} \; D_{\mu\nu}(q) \; e^{-iq(x_2 - x_1)} \; \bar{u}(k', \lambda') e^{ik'x_2} \; i\gamma^\nu u(k, \lambda) e^{-ikx_2}. \]  

Here \( p, s, k \) and \( \lambda \) are the 4-momenta and spins of the incoming jet and medium parton, while the corresponding primed variables correspond to outgoing jet and medium parton (the medium parton can be quark, antiquark or gluon). The medium partons are considered to be massless, i.e. 4-momentum \( k (k') \) is assumed to be \( k = (|\vec{k}|, \vec{k}) \; (k' = (|\vec{k}'|, \vec{k}')). \) The condition that the interaction between jet and medium parton has to occur inside the QCD medium of finite size \( L \) is implemented through the second \( \theta \) function giving maximal interaction time of \( (t_1 - t_0)_{max} = L/v \). We will further define \( x \equiv x_1 - x_0 = (t, \vec{x}) \).

The Eq. (6) simplifies to

\[ iM_{el} = g^2 \int \frac{d^3\vec{p}}{(2\pi)^32E} \int \frac{d^4q}{(2\pi)^4} \; \int d^4x_0 \; J(x_0) e^{ip'x_1} \; \int d^3\vec{x} \; \int_0^{L/v} dt \; e^{-i(p-p'-q)x} \; (2\pi)^4\delta(k' - k - q) \; D_{\mu\nu}(q) \; \bar{u}(p', s') e^{ipx_1} \; \gamma^\mu u(p, s) \; \bar{u}(k', \lambda') e^{ik'x_2} \; i\gamma^\nu u(k, \lambda) \]  

\[ = J(p') \frac{1}{2E} \frac{1 - e^{-i(E-E'-\omega)L/v}}{E - E' - \omega} iM. \]  

where \( E = \sqrt{M^2 + \vec{p}^2} \), \( M \) is the jet mass, \( \vec{p} = p' - (\vec{k}' - \vec{k}) \) and \( \omega = |\vec{k}'| - |\vec{k}|. \)

In the last step we used \( J(p' + q) \approx J(p') \) [24] and defined

\[ M = g^2 D_{\mu\nu}(k' - k) \bar{u}(p', s') e^{ipx_1} \; \gamma^\mu u(p, s) \; \bar{u}(k', \lambda') e^{ik'x_2} \; i\gamma^\nu u(k, \lambda). \]  

In this paper we consider the case of highly energetic jets, where \( |\vec{q}| \ll E \). In this limit \( E' \) becomes \( E' \approx E - \vec{v} \cdot \vec{q} \). Here \( \vec{v} \) is the velocity of the initial jet, i.e. the jet 4-momentum \( p \) is equal to \( p = \left( \frac{M}{\sqrt{1-v^2}}, \frac{M\vec{v}}{\sqrt{1-v^2}} \right) \).
Further, the matrix element given in Eq. (7) has to be squared, averaged over initial spin s of the jet and summed over all other spins.

\[ \frac{1}{2} \sum_{\text{spins}} |M_{el}|^2 = |J(p')|^2 \frac{1}{E^2} \frac{\sin[(\omega - \vec{v} \cdot \vec{q}) \frac{L}{2n}]}{(\omega - \vec{v} \cdot \vec{q})^2} \frac{1}{2} \sum_{\text{spins}} |M|^2. \]  

(9)

where \( \frac{1}{2} \sum_{\text{spins}} |M|^2 \) is given in Appendix B.

The differential energy loss is equal to \( dE_{el} = \omega \, d\Gamma_{el} \), where collisional interaction rate \( d\Gamma_{el} \) can be extracted from Eq. (9) as (see [27])

\[ d^3N_J \, d\Gamma_{el} \approx \frac{1}{2} \sum_{\text{spins}} |M_{el}|^2 \frac{d^3p'}{(2\pi)^32E'} \frac{d^3k}{(2\pi)^32k} \frac{d^3k'}{(2\pi)^32k'} \sum_{\xi = q, g} n_{eq}^\xi(k)(1 \pm n_{eq}^\xi(k')). \]  

(10)

Here

\[ d^3N_J = d_R |J(p')|^2 \frac{d^3p'}{(2\pi)^32E'}, \]  

(11)

and \( d_R = 3 \) (for three dimensional representation of the quarks). In Eq. (10), \( n_{eq}^\xi(k) \) is the equilibrium momentum distribution at temperature \( T \) of the incoming parton \( \xi \), where \( \xi \) denotes quark, antiquark or gluon. \((1 \pm n_{eq}^\xi(k'))\) is a factor associated with the outgoing parton, where + corresponds to the gluon contribution, and – to (anti)quark contribution.

In this paper, we are interested only in the computation of the collisional energy loss. It is straightforward to show that in the collisional energy loss calculations, the \( \pm n_{eq}^\xi(k') \) part in \((1 \pm n_{eq}^\xi(k'))\) can be dropped because the corresponding term in the energy loss integrand is odd under the exchange of \( \vec{k} \) and \( \vec{k}' \), and integrates to zero (see also [24]). Therefore, for the purpose of computing the collisional energy loss, we can replace \((1 \pm n_{eq}^\xi(k'))\) by 1 in Eq. (10), which leads to

\[ d^3N_J \, d\Gamma_{el} \approx \frac{1}{2} \sum_{\text{spins}} |M_{el}|^2 \frac{d^3p'}{(2\pi)^32E'} \frac{d^3k}{(2\pi)^32k} \frac{d^3k'}{(2\pi)^32k'} n_{eq}(k). \]  

(12)

Here, \( n_{eq}(k) = \sum_{\xi = q, g} n_{eq}^\xi(k) \) is the equilibrium momentum distribution [24] at temperature \( T \) including quark, antiquark and gluon contributions (see Eq. (10)).

The collisional energy loss can now be obtained from Eqs. (9), (11) and (12), leading to

\[ \Delta E_{el} \approx C_R \frac{1}{E^2} \int d^3k \frac{n_{eq}(k)}{(2\pi)^32k} \int \frac{d^3k'}{(2\pi)^32k'} \frac{\omega}{(\omega - \vec{v} \cdot \vec{q})^2} \frac{\sin[(\omega - \vec{v} \cdot \vec{q}) \frac{L}{2n}]}{(\omega - \vec{v} \cdot \vec{q})^2} \frac{1}{2} \sum_{\text{spins}} |M|^2. \]  

(13)

Note that in Eq. (13) we added a color factor \( C_R \) and that \( \vec{q} = \vec{k}' - \vec{k} \).

Equation (13) can be further simplified by noting that, in a static medium, the collisional energy loss does not depend on the direction of \( \vec{v} \). After evaluating the \( \frac{1}{2} \sum |M|^2 \) and averaging the integrand over the directions of \( \vec{v} \), we obtain (see Appendix B)

\[ \Delta E_{el} = \frac{C_R g^4}{2 \pi^4} \int_0^\infty n_{eq}(|\vec{k}|) d|\vec{k}| \left( \int_0^{||\vec{q}||} |\vec{q}| d|\vec{q}| \int_0^{|\vec{q}|} \omega d\omega + \int_0^{|\vec{q}|_{\text{max}}} |\vec{q}| d|\vec{q}| \int_0^{|\vec{q}|} \omega d\omega \right) \left( |\Delta_L(q)|^2 \frac{2|\vec{k}| + \omega^2 - |\vec{q}|^2}{2} J_1 + |\Delta_T(q)|^2 \frac{|\vec{q}|^2 - \omega^2}{4|\vec{q}|^4} \left((\omega^2 |\vec{q}|^2 - \omega^2) J_1 + 2 \omega J_2 - J_3 \right) \right) \]  

(14)
where $J_1$, $J_2$ and $J_3$ are given by Eqs. (34, 35) in Appendix B, and $|\vec{q}|_{\text{max}}$ is given by the following formula [14]

$$|\vec{q}|_{\text{max}} = \min[E, \frac{2|\vec{k}|(1 - |\vec{k}|/E)}{1 - v + 2|\vec{k}|/E}], \quad (15)$$

Further numerical study of the collisional energy loss in a finite size QCD medium is given in Section 4.

## 3 Collisional energy loss in infinite QCD medium

Previous calculations of the collisional energy loss, in particular [11, 14, 15] were done for an infinite QCD medium. A problem with these calculations was that they produce unphysical energy gain in the lower momentum regions. Additionally, these computations led to different results and consequently provided quite a large uncertainty in the heavy quark (especially bottom) collisional energy loss.

In this Section and Appendix B.1 we present an improved calculation of collisional energy loss in an infinite QCD medium, with the goal of 1) removing the problems associated with previous calculations and 2) producing reliable infinite medium results which we will in Section 4 compare with the collisional energy loss in a finite medium.

In the case of an infinite QCD medium, the collisional energy loss per unit length $\frac{dE_{\text{el}}}{dL}$ is computed by assuming that the jet is produced at $x_0 = -\infty$. The energy loss for a finite size medium is than (simplistically) calculated by multiplying this $\frac{dE_{\text{el}}}{dL}$ with the thickness $L$ of the medium.

In Appendix B.1 we present an improved calculation of the collisional energy loss per unit length in an infinite QCD medium. The following result is obtained:

$$\frac{dE_{\text{el}}}{dL} = \frac{g^4}{6v^2\pi^3} \int_0^\infty n_{eq}(|\vec{k}|) d|\vec{k}| \left( \int_0^{|\vec{k}|/(1+v)} d|\vec{q}| \int_{-|\vec{q}|}^{v|\vec{q}|} \omega d\omega + \int_{|\vec{k}|/(1+v)}^{|\vec{q}|_{\text{max}}} d|\vec{q}| \int_{|\vec{q}|}^{v|\vec{q}|} \omega d\omega \right) \left( |\Delta_L(q)|^2 \frac{(2|\vec{k}| + \omega)^2 - |\vec{q}|^2}{2} + |\Delta_T(q)|^2 \left( (|\vec{q}|^2 - \omega^2) \frac{(2|\vec{k}| + \omega)^2 + |\vec{q}|^2}{4|\vec{q}|^4} (v^2|\vec{q}|^2 - \omega^2) \right) \right). \quad (16)$$

To compare our result with the computations done in [14, 15], we here introduce an arbitrary intermediate momentum scale $|\vec{q}|^*$. The contribution from $|\vec{q}| < |\vec{q}|^*$ is denoted soft, while contribution from $|\vec{q}| > |\vec{q}|^*$ is denoted hard [15].

The soft contribution is given by

$$\frac{dE_{\text{el}}^{\text{soft}}}{dL} = \frac{g^4}{6v^2\pi^3} \int_0^\infty n_{eq}(|\vec{k}|) d|\vec{k}| \int_0^{v|\vec{q}|} d|\vec{q}| \int_{-|\vec{q}|}^{v|\vec{q}|} \omega d\omega \left( |\Delta_L(q)|^2 \frac{(2|\vec{k}| + \omega)^2 - |\vec{q}|^2}{2} + |\Delta_T(q)|^2 \frac{(2|\vec{k}| + \omega)^2 + |\vec{q}|^2}{4|\vec{q}|^4} (v^2|\vec{q}|^2 - \omega^2) \right). \quad (17)$$

while the hard contribution is given by
by make a sharp transition from soft to hard limits, and consequently it does not require the introduction
tently treats the integration limits. Comparing to [15], the advantage of our result is that it does not
reproduces Eq.(4.1) in [14].

\[
\omega > \frac{(2|\vec{k}| + \omega)^2 - |\vec{q}|^2}{2} + \frac{|\Delta_T(q)|^2}{4|\vec{q}|^4} (2|\vec{k}| + \omega)^2 + \frac{|\vec{q}|^2}{4|\vec{q}|^4} (v^2|\vec{q}|^2 - \omega^2).
\] (18)

The soft contribution can be further simplified by keeping only the even contributions in the \( \omega \) integral (the odd contributions vanish under symmetric integration)

\[
\frac{dE_{\text{soft}}}{dL} = \frac{g^4}{3v^2\pi^3} \int_0^\infty |\vec{k}| n_{\text{eq}}(|\vec{k}|) d|\vec{k}| \int_{0}^{\infty} d|\vec{q}|^* \int_{-|\vec{q}|}^{+|\vec{q}|} \omega^2 d\omega \left(|\Delta_L(q)|^2 + \frac{1}{2}(1 - \frac{\omega^2}{|\vec{q}|^2})(v^2 - \frac{\omega^2}{|\vec{q}|^2})|\Delta_T(q)|^2\right)
= \frac{g^2}{6v^2\mu_D^2} \int_{0}^{\infty} d|\vec{q}|^* \int_{-|\vec{q}|}^{+|\vec{q}|} \omega^2 d\omega \left(|\Delta_L(q)|^2 + \frac{1}{2}(1 - \frac{\omega^2}{|\vec{q}|^2})(v^2 - \frac{\omega^2}{|\vec{q}|^2})|\Delta_T(q)|^2\right) \tag{19}
\]

where in the last step we used the fact that

\[
\int_0^\infty |\vec{k}| n_{\text{eq}}(|\vec{k}|) d|\vec{k}| = \frac{\pi^2T^2}{2} (1 + \frac{N_f}{6}). \tag{20}
\]

Equation (19) agrees with the Eq. (55) in [24].

For the hard contribution we use that, in the limit of large momentum transfer (|\vec{q}| \gg |\vec{q}|^*),

\(|\Delta_L(q)| \to \frac{1}{|\vec{q}|^2} \) and \(|\Delta_T(q)| \to \frac{1}{\omega^2 - |\vec{q}|^2} \). It is than straightforward to show that the hard contribution
reduces to the Eq. (17) in [24], i.e.

\[
\frac{dE_{\text{hard}}}{dL} = \frac{g^4}{6v^2\pi^3} \int_0^\infty n_{\text{eq}}(|\vec{k}|) d|\vec{k}| \left(\int_{|\vec{k}|^*}^{2|\vec{k}|/(1+v)} d|\vec{q}|^* \int_{-|\vec{q}|^*}^{+|\vec{q}|^*} \omega^2 d\omega + \int_{2|\vec{k}|/(1+v)}^{\infty} \frac{d|\vec{q}|^*}{|\vec{q}|^2} \int_{-|\vec{q}|}^{+|\vec{q}|} \omega d\omega \right)
= \frac{3 \omega^2}{4|\vec{q}|^2} - \frac{v^2}{2} \frac{|\vec{q}|^2}{|\vec{q}|^2 - \omega^2} + \frac{3|\vec{k}|(|\vec{k}| + \omega)}{|\vec{q}|^2} - (1 - \frac{v^2}{|\vec{q}|^2})|\vec{k}|(|\vec{k}| + \omega) \tag{21}
\]

Equations (17) and (55) from [24] were used in Ref. [15] to obtain their Eqs. (8) and (12). That is,
while our Eq. (16) is more general, in special cases (i.e. Eqs. (19) and (21)) it reproduces results from [15].

The computation in [14] considered only the soft gluon limit, and replaced |\vec{q}|^* by |\vec{q}|_{\text{max}}, where

|\vec{q}|_{\text{max}} is given by Eq. (15). Consequently, for the purpose of comparison with [14], we replaced |\vec{q}|^* by |\vec{q}|_{\text{max}} in Eq. (19). Additionally, the problem with this approach is that, in the high momentum
|\vec{q}| region, the method [14] is not able to treat the lower \( \omega \) bound properly (compare Eq. (19) with
Eq. (16)) where \( \omega \) bounds are properly treated). To overcome this problem, the calculation in [14]
was limited to the forward emission only (i.e. \( \omega > 0 \)). If this limit is also taken into account, our Eq. (19)
reproduces Eq.(4.1) in [14].

In summary, in the known limiting cases, our result (i.e. Eq. (16)) reduces to the results published in [14] [15]. The advantage of our result over [14] is that it includes the hard contribution and consistently treats the integration limits. Comparing to [15], the advantage of our result is that it does not make a sharp transition from soft to hard limits, and consequently it does not require the introduction of an unphysical momentum scale \( |\vec{q}|^* \) as in [15].
4 Numerical results

In this section we give a numerical study of the collisional energy loss in a QCD medium as presented in Sections 3 and 4. To do this, we further assume that the QCD plasma is characterized by $T = 0.225 \text{ GeV}$, $N_f = 2.5$ and $\alpha = 0.3$. For the light quark mass we take $M = \mu_D / \sqrt{6}$, where $\mu_D = g^2 T^2 (1 + N_f / 6) \approx 0.5 \text{ GeV}$ is the Debye mass. The charm mass is taken to be $M = 1.2 \text{ GeV}$, and the bottom mass is $M = 4.75 \text{ GeV}$.

4.1 Collisional energy loss in infinite QCD medium

![Graphs showing fractional collisional energy loss as a function of momentum for light, charm, and bottom quark jets.](image)

Figure 2: Fractional collisional energy loss is shown as a function of momentum for light, charm and bottom quark jets (left, center and right panels respectively). Dash-dotted curves are obtained by using Eq. (16) from this paper. Dashed curves correspond to Eqs. (8) and (12) from [15], while dotted curves are obtained by using Eq. (4.1) from [14]. Assumed thickness of the medium is $L = 5 \text{ fm}$.

In Fig. 2 we compare our collisional energy loss results in an infinite QCD medium (Eq. (16)) with previous computations by [14, 15]. We see that, while both BT [15] and TG [14] computations lead to unphysical negative energy loss results in the low momentum region, our computations give positive collisional energy loss in the whole jet momentum range. This is particularly important in the bottom quark case where the unphysical behavior persists up to 5 GeV in BT [15] case and up to 2 GeV in TG [14] case. The reason for this behavior is that only the leading logarithmic contribution was considered in the final steps of both BT and TG computations. Note that the problem of unphysical energy gain was addressed in Ref. [33], by including fully dressed gluon propagator in their calculations. However, that approach led to another problem, since the unphysical momentum scale $|\vec{q}|^*$ appeared in the collisional energy loss results [33]. Therefore, our results present a first complete solution to the unphysical energy gain problem.

Our numerical results agree with BT only in the asymptotic regions, which is likely the consequence of the following: 1) BT made a sharp (instead of continuous) transition from soft to hard limit and 2) they introduced a sharp boundary in the energy loss computations depending on whether the initial jet energy is much larger/smaller than $M^2 / T$.

Despite the fact that the BT computations are more up to date and treat the collisional energy loss more consistently than TG, we see that our results show better agreement with TG [14] computations. Particularly, in case of light and charm quark jets, there is a quite good agreement between our results and those of TG [14]. The good agreement is probably because the forward emission only (see Section 3) provides a quite plausible estimate for the collisional energy loss. However, for bottom quarks we see that neither BT nor TG computations provide a reasonable estimate for the collisional
energy loss. Therefore, in this case, the more accurate computation of collisional energy loss (i.e. our Eq. (16)) is needed.

4.2 Collisional energy loss in a finite QCD medium

![Graph of fractional collisional energy loss](image)

Figure 3: Fractional collisional energy loss is shown as a function of momentum for light, charm and bottom quark jets (left, center and right panels respectively). Full curves correspond to finite medium case (see Eq. (14)), while dash-dotted curves correspond to infinite medium case (see Eq. (16)). Assumed thickness of the medium is $L = 5$ fm.

![Graph of fractional collisional energy loss](image)

Figure 4: Fractional collisional energy loss is shown as a function of thickness of the medium for light, charm and bottom quark jets (upper, middle and lower set of curves respectively). Full curves correspond to finite medium case (see Eq. (14)), while dash-dotted curves correspond to infinite medium case (see Eq. (16)). Initial momentum of the jet is 10 GeV.

Figures 3 and 4 show the comparison between collisional energy loss in infinite and finite size QCD medium. Contrary to [20], we find that a finite size medium does not have a large effect on the collisional energy loss. The discrepancy between our results and those presented in [20] is due to the fact that what was called collisional energy loss in [20], is in fact combination of collisional and part of the 0th order radiative energy loss. Actually, the calculation in [20] does not present a complete 0th order energy loss either, since transition radiation [23] was not included in their computation.
Contrary to naive expectations, from Figs. 3 and 4 we found that collisional energy loss in a finite size medium can be somewhat larger than in an infinite medium. The reason is that in Eq. (16) there exists a term $2\omega J_2 - J_3$. If this term were equal to zero (as in the case of infinite medium), the energy loss in a finite medium case would always be smaller than in an infinite medium, as naively expected (compare dashed and dot-dashed curves in Fig. 5). However, in the finite medium case, the term $2\omega J_2 - J_3 \neq 0$, giving a noticeable positive contribution (see the dotted curve in Fig. 5) which will lead to somewhat larger (overall) energy loss in the finite medium case.

To further discuss this, let us look at the Eqs. (34-36) (Appendix A) in the finite medium case, and compare them to the corresponding Eqs. (52-54) in the infinite medium. The $\delta$ function in Eqs. (52-54) ensures energy conservation, which is satisfied when the jet is produced at $-\infty$. Consequently, in this case $2\omega J_2 - J_3 \equiv 0$ (see Eqs. (53) and (54)). However, when the jet is produced at finite time $x_0$, time translation invariance is broken, and therefore the energy of the collisional process is not conserved, leading to $2\omega J_2 - J_3 \neq 0$.

### 4.3 Comparison between collisional and radiative energy loss in a finite size QCD medium

In Appendix A we showed how to separate the contributions to the collisional and radiative energy loss. In this section we directly compare these two contributions in the case of finite size QCD medium.

To compute the net radiative energy loss, we note that there are three important effects that control this energy loss in a QCD medium. These effects are the Ter-Mikayelian effect [21], transition radiation [23] and medium induced radiation [34]. In [23], we combined these effects to compute the net radiative energy loss. We here use these results for the purpose of further comparison with the collisional energy loss. Note that in these computations, in order to simulate confinement in the vacuum, we introduce an effective gluon mass in the vacuum $m_{g,v} \approx \Lambda_{QCD} = 0.2$ GeV (for more details see [21]).
Figure 6: The comparison between collisional and radiative fractional energy loss is shown as a function of momentum for light, charm and bottom quark jets (left, center and right panels respectively). Full curves show the collisional energy loss, while dot-dashed curves show the net radiative energy loss. Assumed thickness of the medium is $L = 5$ fm and $\lambda = 1.2$ fm [19].

Figure 7: The comparison between collisional and radiative fractional energy loss is shown as a function of the thickness of the medium. Light, charm and bottom quark cases are shown on the left, center and right panels respectively. Full curves show the collisional energy loss, while dot-dashed curves show the net radiative energy loss. Mean free path is $\lambda = 1.2$ fm [19]. Initial momentum of the jet is $E = 10$ GeV.

In Figures 6 and 7 we show the collisional and radiative energy loss as a function of jet energy and thickness of the medium, respectively. We see that collisional energy loss is comparable with the net radiative energy loss in the medium. Therefore, the collisional energy loss contribution is significant and must be included in the computation of jet quenching in a QCD medium.

In particular, we note that in the lower momentum (i.e. $p < 10$ GeV) range, the collisional energy loss dominates the radiative one. At RHIC, jet suppression is mostly measured in this (lower) momentum range. Therefore, contrary to previous studies [4]-[7], our results indicate that it is collisional instead of radiative energy loss which gives the dominant contribution to the observed suppression values.
5 Conclusion

This paper addressed the 0th order contribution to the collisional energy loss in a finite size QCD medium. The interest in the collisional energy loss has been renewed by the recent studies \cite{11, 12}, particularly in the context of the single electron puzzle \cite{8-10}. In Refs. \cite{11, 12} it was claimed that, contrary to the previous beliefs, for the parameter ranges relevant in URHIC, radiative and collisional energy loss become comparable. However, a recent study by Peigne et al. \cite{20} suggested that collisional energy loss is large in the ideal infinite medium case, while the finite size medium effects lead to significant reduction of the collisional energy loss. The paper \cite{20}, however, did not completely separate collisional from radiative energy loss.

Additionally, even in the infinite medium case, the problem of collisional energy loss was not completely solved. Previous computations obtained unphysical results in the low momentum regions \cite{14, 15}, and an approach to solve this problem \cite{33} led to the results dependent on the unphysical momentum scales. In addition, these computations introduced quite a large uncertainty in the heavy quark (especially bottom) collisional energy loss, since they led to noticeably different numerical results.

The above reasons and the previously discussed single electron puzzle, motivated us to provide a detailed study of the 0th order collisional energy loss in a finite size QCD medium created in URHIC. First, in Appendix A we showed that, though 0th order collisional and radiative energy loss contributions come from the same one-loop HTL diagram, there is no overlap between collisional and radiative energy loss computations. More specifically, while 0th order collisional energy loss comes from the processes which have the same number of incoming and outgoing particles, the radiative energy loss has one gluon more as an outcome of the process. Additionally, we showed that in the 0th order calculations, there are no interference effects between collisional and radiative energy loss, which is different from a result in the recent paper \cite{35}. The absence of interference effects comes from the fact that, contrary to \cite{35}, in our study we consistently treat the gluon dispersion relation in the medium. This leads to the following conditions: 1) for the 0th order collisional energy loss contributions, the energy of the exchanged (virtual) gluon has to be smaller, or equal, to the gluon momentum, and 2) for the radiative energy loss contributions the energy of the radiated gluon has to be larger than its momentum. Therefore, these two contributions take non-zero values in non-overlapping regions, and consequently cannot interfere with each other.

In the case of infinite medium, our computation interpolates smoothly between soft to hard contributions and, contrary to \cite{15}, does not require the introduction of an arbitrary intermediate momentum scale. Additionally, our computation treats the lower momentum region consistently, removing the unphysical energy gain results obtained in previous computations \cite{14, 15}.

In the case of finite size QCD medium, contrary to the study by Peigne et al. \cite{20} we showed that finite size effects have small effect on the collisional energy loss. Therefore, consistently with the claims in Refs. \cite{11, 12} and our recent single electron suppression estimates \cite{19}, we here showed that collisional energy loss is important, and has to be included in the computation of jet quenching.

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A HTL contribution to the collisional energy loss

In this section we will derive the formula for the lowest order collisional interaction rate. The zeroth order contribution to both radiative and collisional rates comes from the diagram $M$ given in Fig. 8. We will below use this diagram as a starting point to separate contributions of collisional and radiative energy loss.

Diagram $M$ corresponds to $\sum M_n$, where $M_n$ is the amplitude of the diagram shown in Fig. 9.

The definition of “black circles” in Fig. 9 is shown in Fig. 10.

The diagram $M$ contains both collisional and radiative $0^{th}$ order contribution to the jet energy loss. It is useful to look at the simple $n = 1$ case (see Fig. 11) to better understand this.
The contribution to the collisional energy loss is obtained by “cutting” (i.e., putting on shell) the propagators of parton $k'$ and $p'$. On the other hand, the radiative contribution is obtained by putting the parton propagator $p'$ and the gluon propagator $q$ on shell. From this, it follows that collisional and radiative contributions come from different diagrams. Furthermore, from the conservation of energy and momentum it can be shown that cutting the gluon propagator $q$, leads to the condition $|\omega| > |\vec{q}|$, while cutting the propagator of parton $k'$ leads to the condition $|\omega| < |\vec{q}|$. Consequently, there is no interference (and over-counting) between collisional and radiative contributions\textsuperscript{1}. The computation of the radiative 0\textsuperscript{th} order energy loss has already been a subject of our previous work \cite{23, 21}. So, the contributions from the diagrams which give raise to the radiative energy loss, will not be further addressed here.

As we can see from the right side of the Fig. 11, there are two contributions from diagram $M_1$ to the collisional rate. These two contributions can be combined into one by allowing that the energy of the gluon can take both positive and negative values. Therefore, the contribution to the collisional rate from diagram $M_1$ ($d\Gamma_{M_1}$) is equal to

$$d^3N_J\, d\Gamma_{M_1} = \frac{d^3p'}{(2\pi)^32E'} \frac{d^3k}{(2\pi)^32k} \frac{d^3k'}{(2\pi)^32k'} n_{eq}(k) |M_{E_0}|^2,$$

where $M_{E_0}$ is the Feynman diagram shown in Fig. 12 and $\omega \in (-|\vec{q}|, |\vec{q}|)$. Note that in the above equation each of the terms in $n_{eq}(k) = \sum_{\xi=q,g} n_{eq}(k)$ should be multiplied by an extra factor $(1 \pm n_{eq}(k'))$ for the outgoing medium parton (see Section 2). Here $\xi$ corresponds to anti(quark) or gluon. The + sign is associated with gluon and the – sign with anti(quark) contributions. However, the $\pm n_{eq}(k')$ term in $(1 \pm n_{eq}(k'))$ does not contribute to the collisional energy loss (for more details see Section 2 and Ref. \cite{21}). Therefore, in Eq. (22) we keep only the contributions that give raise to the collisional energy loss.

\textbf{Figure 12:} Feynman diagram for the lowest order collisional energy loss.

1 Note that in this paper, we treat only the 0\textsuperscript{th} order contribution to collisional energy loss. It is, however, possible that interference effects and/or over-counting between collisional and radiative energy loss contributions would occur in the higher order computations. Higher order contributions are a separate non-trivial problem, which is not considered in this paper.
$M_n$ is equal to

$$d^3N_J \, d\Gamma_{M_n} = \frac{d^3\vec{p'}}{(2\pi)^3 2E'} \frac{d^3\vec{k}}{(2\pi)^3 2k} \frac{d^3\vec{k}'}{(2\pi)^3 2k'} \, n_{eq}(k) \sum_{i=0}^{n-1} M_{E_i} M_{E_{n-1-i}}^\dagger,$$

(23)

here the diagram $M_{E_n}$ is shown in Fig. 13.

Figure 13: Feynman diagram for the collisional energy loss with $n$ interactions with the medium.

Since $M = \sum_{n=0}^{\infty} M_n$, the contribution to the collisional energy loss from the diagram $M$ is equal to

$$d^3N_J \, d\Gamma = \frac{d^3\vec{p'}}{(2\pi)^3 2E'} \frac{d^3\vec{k}}{(2\pi)^3 2k} \frac{d^3\vec{k}'}{(2\pi)^3 2k'} \, n_{eq}(k) \sum_{i=0}^{\infty} \sum_{n=0}^{\infty} M_{E_i} M_{E_{n-1-i}}^\dagger,$$

(24)

We next want to prove that

$$d^3N_J \, d\Gamma = \frac{d^3\vec{p'}}{(2\pi)^3 2E'} \frac{d^3\vec{k}}{(2\pi)^3 2k} \frac{d^3\vec{k}'}{(2\pi)^3 2k'} \, n_{eq}(k) |M_{el}|^2,$$

where (see Fig. 14)

$$M_{el} = \sum_{n=0}^{\infty} M_{E_n}$$

(25)

Figure 14: Feynman diagram $M_{el}$ for the collisional energy loss in QCD medium. The large dashed circle ("blob") represents the effective gluon propagator [21].

To prove the above, we will first compute $|M_{el}|^2$

$$|M_{el}|^2 = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} M_{E_i} M_{E_j}^\dagger = \sum_{i=0}^{\infty} \sum_{n=1}^{\infty} M_{E_i} M_{E_{n-1-i}}^\dagger$$

(26)
where in the last step we defined \( n = i + j \rightarrow j = n - i \).

Since

\[
\sum_{i=0}^{\infty} \sum_{n=i}^{\infty} = \sum_{n=0}^{\infty} \sum_{i=0}^{n},
\]

we can conclude that

\[
d^3 N_f \ d\Gamma = \frac{d^3 \vec{p}'}{(2\pi)^3 2E'} \frac{d^3 \vec{k}}{(2\pi)^3 2k} \frac{d^3 \vec{k}'}{(2\pi)^3 2k'} \ n_{eq}(k) \sum_{n=0}^{\infty} |M_{el}|^2,
\]

which is what we wanted to prove. Therefore, the collisional interaction rate can be obtained by an intuitive approach of computing \( |M_{el}|^2 \) (see Fig. 14), where blob represents the effective gluon propagator.

## B  Collisional energy loss computations

In this Appendix we will derive the collisional energy formula given by Eq. (14). To do that we start from the Eq. (13), i.e.

\[
\Delta E_{el} = C_R \frac{1}{E^2} \int \frac{d^3 \vec{k}}{(2\pi)^3 2k} n_{eq}(k) \int \frac{d^3 \vec{k}'}{(2\pi)^3 2k'} \omega \sin[(\omega - \vec{v} \cdot \vec{q})^2 L_2 \mu \nu] \frac{1}{2} \sum_{spins} |\mathcal{M}|^2
\]

where \( n_{eq}(k) \) is the equilibrium momentum distribution \([24]\) at temperature \( T \) including quarks and gluons

\[
\sum_{spins} |\mathcal{M}|^2
\]

Here \( N \) is the number of colors and \( N_f \) is the number of flavors.

The matrix element \( \mathcal{M} \) (see Eq. (31)), has been already computed in \([24]\) (see Eqs. (45-46) in \([24]\)). However, for completeness and due to a typographical error in \([24]\), we here repeat the main steps in the computation of \( \frac{1}{2} \sum_{spins} |\mathcal{M}|^2 \).

\[
\mathcal{M} = g^2 D_{\mu\nu}(q)\bar{u}(p', s')\gamma^\mu u(p, s)\bar{u}(k', \lambda')\gamma^\nu u(k, \lambda).
\]

In Coulomb gauge, the only non-zero terms in the effective gluon propagator are given in Eqs. (4) and (5), which together with Eqs. (1-3) reduce the Eq. (31) to

\[
\mathcal{M} = g^2 \Delta_L(q)\bar{u}(p', s')\gamma^0 u(p, s)\bar{u}(k', \lambda')\gamma^0 u(k, \lambda)
\]

Here \( \bar{q}' \equiv q' / |\bar{q}| \).

The matrix element given in Eq. (32) has to be squared, averaged over initial spin \( s \) of the jet and summed over all other spins. After evaluating the Dirac traces, and applying the assumption that \( |\bar{q}| \ll E \) (highly energetic jet) we obtain similarly to \([24]\)
\[
\frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 = 16g^4 E^2 (|\Delta_L(q)|^2(|\mathbf{k}| + \mathbf{k}^\prime) + 2Re(\Delta_L(q)\Delta_T(q)^*) \left[ |\mathbf{k}|(\mathbf{v} \cdot \mathbf{q} \cdot \mathbf{q} \cdot \mathbf{k}^\prime - \mathbf{q} \cdot \mathbf{q} \cdot \mathbf{k}^\prime) + |\mathbf{k}^\prime|(\mathbf{v} \cdot \mathbf{k} - \mathbf{v} \cdot \mathbf{q} \cdot \mathbf{q} \cdot \mathbf{k}) \right] + |\Delta_T(q)|^2 \left[ 2(\mathbf{v} \cdot \mathbf{k} - \frac{\mathbf{v} \cdot \mathbf{q} \cdot \mathbf{q} \cdot \mathbf{k}}{|\mathbf{q}|^2})(\mathbf{v} \cdot \mathbf{k}^\prime - \frac{\mathbf{v} \cdot \mathbf{q} \cdot \mathbf{q} \cdot \mathbf{k}^\prime}{|\mathbf{q}|^2}) + (|\mathbf{k}| |\mathbf{k}^\prime| - \mathbf{k} \cdot \mathbf{k}^\prime)(v^2 - \frac{\mathbf{v} \cdot \mathbf{q} \cdot \mathbf{q} \cdot \mathbf{v}}{|\mathbf{q}|^2}) \right]. \tag{33}
\]

In a static medium, the collisional energy loss does not depend on the direction of \(\mathbf{v}\). Therefore, the Eq. (29) can be further simplified by averaging the integrand over the directions of \(\mathbf{v}\). The integrals that are required are

\[
\mathcal{J}_1 = \int \frac{d\Omega}{4\pi} \frac{\sin((\omega - \mathbf{v} \cdot \mathbf{q}) \frac{L}{\lambda v})}{(\omega - \mathbf{v} \cdot \mathbf{q})^2} = \frac{L}{4|\mathbf{q}|v^2} \left[ Si((v|\mathbf{q}| + \omega)\frac{L}{v}) + Si((v|\mathbf{q}| - \omega)\frac{L}{v}) \right] - \frac{1}{4v|\mathbf{q}|} \left[ 1 - \cos((v|\mathbf{q}| - \omega)\frac{L}{v}) \right] + \frac{1}{v|\mathbf{q}| + \omega} \right]. \tag{34}
\]

\[
\mathcal{J}_2 = \int \frac{d\Omega}{4\pi} \frac{\sin((\omega - \mathbf{v} \cdot \mathbf{q}) \frac{L}{\lambda v})}{(\omega - \mathbf{v} \cdot \mathbf{q})^2} (\omega - \mathbf{v} \cdot \mathbf{q}) = \frac{1}{4v|\mathbf{q}|} \left[ Ci((v|\mathbf{q}| - \omega)\frac{L}{v}) - Ci((v|\mathbf{q}| + \omega)\frac{L}{v}) + ln(v|\mathbf{q}| + \omega) \right] \tag{35}
\]

and

\[
\mathcal{J}_3 = \int \frac{d\Omega}{4\pi} \frac{\sin((\omega - \mathbf{v} \cdot \mathbf{q}) \frac{L}{\lambda v})}{(\omega - \mathbf{v} \cdot \mathbf{q})^2} (\omega - \mathbf{v} \cdot \mathbf{q})^2 = \frac{1}{2} \left( 1 - \frac{\cos(L\omega v)}{L|\mathbf{q}|} \frac{\sin(L|\mathbf{q}|)}{L|\mathbf{q}|} \right). \tag{36}
\]

By using Eqs. (34) (35), it can be shown that

\[
\int \frac{d\Omega}{4\pi} \frac{\sin((\omega - \mathbf{v} \cdot \mathbf{q}) \frac{L}{\lambda v})}{(\omega - \mathbf{v} \cdot \mathbf{q})^2} v^i = \mathcal{J}_1 q^i \frac{\omega}{|\mathbf{q}|} - \mathcal{J}_2 \mathbf{q}^i \tag{37}
\]

and

\[
\int \frac{d\Omega}{4\pi} \frac{\sin((\omega - \mathbf{v} \cdot \mathbf{q}) \frac{L}{\lambda v})}{(\omega - \mathbf{v} \cdot \mathbf{q})^2} v^i v^j = \mathcal{J}_1 \left( \frac{v^2|\mathbf{q}|^2 - \omega^2}{|\mathbf{q}|^2} \delta^{ij} + \frac{3\omega^2 - v^2|\mathbf{q}|^2}{2|\mathbf{q}|^2} q^i q^j \right) + \frac{2\omega \mathcal{J}_2 - \mathcal{J}_3 (\delta^{ij} - 3q^i q^j)}{2|\mathbf{q}|^2}. \tag{38}
\]

By using Eqs. (34) (36), it is straightforward to show that

\[
\int \frac{d\Omega}{4\pi} \frac{\sin((\omega - \mathbf{v} \cdot \mathbf{q}) \frac{L}{\lambda v})}{(\omega - \mathbf{v} \cdot \mathbf{q})^2} 16g^4 E^2 |\Delta_L(q)|^2(|\mathbf{k}| |\mathbf{k}^\prime| + \mathbf{k} \cdot \mathbf{k}^\prime) = 16g^4 E^2 |\Delta_L(q)|^2 \frac{2|\mathbf{k}| + \omega^2 - |\mathbf{q}|^2}{2} \mathcal{J}_1, \tag{39}
\]

16
\[ 
\int \frac{d\Omega}{4\pi} \sin\left[\frac{(\omega - \vec{v} \cdot \vec{q})^2}{\omega \cdot \vec{q}}\right] 32 g^4 \, E^2 \, Re(\Delta_L(q)\Delta_T(q)^*) \\
\times \left[ k' (\vec{v} \cdot \vec{k}' - \frac{\vec{v} \cdot \vec{q} \vec{q} \cdot \vec{k}'}{|\vec{q}|^2}) + \vec{k}' (\vec{v} \cdot \vec{k} - \frac{\vec{v} \cdot \vec{q} \vec{q} \cdot \vec{k}}{|\vec{q}|^2}) \right] = 0 \quad (40) 
\]

and

\[ 
\int \frac{d\Omega}{4\pi} \sin\left[\frac{(\omega - \vec{v} \cdot \vec{q})^2}{\omega \cdot \vec{q}}\right] 16 g^4 \, E^2 \, |\Delta_T(q)|^2 \\
\times \left[ 2 (\vec{v} \cdot \vec{k} - \frac{\vec{v} \cdot \vec{q} \vec{q} \cdot \vec{k}}{|\vec{q}|^2}) (\vec{v} \cdot \vec{k}' - \frac{\vec{v} \cdot \vec{q} \vec{q} \cdot \vec{k}'}{|\vec{q}|^2}) + \frac{(\vec{k}' \cdot \vec{k} - \vec{k} \cdot \vec{k}') (v^2 - \frac{\vec{v} \cdot \vec{q} \vec{q} \cdot \vec{v}}{|\vec{q}|^2})}{4|\vec{q}|^4} \right] |\Delta_T(q)|^2 \\
= 16 g^4 \, E^2 \, |\Delta_T(q)|^2 \frac{(|\vec{q}|^2 - \omega^2)(|2\vec{k}| + \omega)^2 + |\vec{q}|^2}{4|\vec{q}|^4} \left[ (v^2|\vec{q}|^2 - \omega^2)J_1 + 2\omega J_2 - J_3 \right]. \quad (41) 
\]

Therefore, averaging the \( \frac{\sin[\omega - \vec{v} \cdot \vec{q})^2]}{(\omega - \vec{v} \cdot \vec{q})^2} \frac{1}{2} \sum |\mathcal{M}|^2 \) over the directions of \( \vec{v} \) lead to

\[ 
\left\langle \frac{\sin[(\omega - \vec{v} \cdot \vec{q})^2]}{(\omega - \vec{v} \cdot \vec{q})^2} \frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 \right\rangle = 16 g^4 \, E^2 \bigl( |\Delta_L(q)|^2 \frac{(2|\vec{k}| + \omega)^2 + |\vec{q}|^2}{2} \bigr) J_1 \\
+ |\Delta_T(q)|^2 \frac{(|\vec{q}|^2 - \omega^2)(|2\vec{k}| + \omega)^2 + |\vec{q}|^2}{4|\vec{q}|^4} \left[ (v^2|\vec{q}|^2 - \omega^2)J_1 + 2\omega J_2 - J_3 \right]. \quad (42) 
\]

Since the collisional energy loss does not depend on the direction of \( \vec{v} \), the Eq. (40) can be written as

\[ 
\Delta E_{el} = C_R \frac{1}{E^2} \int \frac{d^3\vec{k}}{(2\pi)^3 2k} n_{eq}(k) \int \frac{d^3\vec{k}'}{(2\pi)^3 2k'} \omega \left\langle \frac{\sin[(\omega - \vec{v} \cdot \vec{q})^2]}{(\omega - \vec{v} \cdot \vec{q})^2} \frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 \right\rangle \\
= C_R \frac{1}{32\pi^4} \int |\vec{k}'| d|\vec{k}'| \int |\vec{k}| d|\vec{k}| \int \cos \theta n_{eq}(|\vec{k}|) \left\langle \frac{\sin[(\omega - \vec{v} \cdot \vec{q})^2]}{(\omega - \vec{v} \cdot \vec{q})^2} \frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 \right\rangle, \quad (43) 
\]

where \( \theta \) is the angle between vectors \( \vec{k} \) and \( \vec{k}' \). Using the fact that \( q = k' - k \), we obtain that the \( \cos \theta \) satisfies the following relation

\[ 
\cos \theta = 1 - \frac{|\vec{q}|^2 - \omega^2}{2|\vec{k}'||\vec{k}|}, \quad (44) 
\]

where \( |\vec{k}'| = |\vec{k}| + \omega \). We can now introduce the change of variables from \( |\vec{k}|, |\vec{k}'| \) and \( \cos \theta \), to \( |\vec{k}|, \omega \) and \( |\vec{q}| \), which reduces the Eq. (40) to the following form

\[ 
\Delta E_{el} = C_R \frac{1}{32\pi^4} \frac{1}{E^2} \int n_{eq}(|\vec{k}|) |\vec{k}| |\vec{q}| |d\omega \omega d|\vec{k}| d|\vec{q}| \left\langle \frac{\sin[(\omega - \vec{v} \cdot \vec{q})^2]}{(\omega - \vec{v} \cdot \vec{q})^2} \frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 \right\rangle \\
= C_R g^4 \frac{1}{2\pi^4} \int n_{eq}(|\vec{k}|) |\vec{k}| |\vec{q}| |d\omega \omega d|\vec{k}| d|\vec{q}| \left( |\Delta_L(q)|^2 \frac{(2|\vec{k}| + \omega)^2 - |\vec{q}|^2}{2} \right) J_1 \\
+ |\Delta_T(q)|^2 \frac{(|\vec{q}|^2 - \omega^2)(|2\vec{k}| + \omega)^2 + |\vec{q}|^2}{4|\vec{q}|^4} \left[ (v^2|\vec{q}|^2 - \omega^2)J_1 + 2\omega J_2 - J_3 \right]. \quad (45) 
\]
Limits of the integration can be obtained from Eq. (44), from which it follows that

$$0 < \frac{q^2 - \omega^2}{2|\mathbf{k}|(|\mathbf{k}| + \omega)} < 2,$$  \quad (46)
leading to the limits in energy transfer $\omega$

$$\text{Max}[|\mathbf{q}|, |\mathbf{q}| - 2|\mathbf{k}|] < \omega < |\mathbf{q}|.$$  \quad (47)

The limits on the momentum transfer $|\mathbf{q}|$ from collisional scattering off a thermal parton with energy $|\mathbf{k}|$ is (see [13])

$$0 < |\mathbf{q}| < \text{Min}[E, \frac{2|\mathbf{k}|(1 - |\mathbf{k}|/E)}{1 - v + 2|\mathbf{k}|/E}].$$  \quad (48)

Here $E$ and $v$ are the energy and velocity of the jet.

By using relations (47) and (48), the Eq. (45) finally reduces to

$$\Delta E_{el} = \frac{C_R g^4}{2\pi^2} \int_0^\infty n_{eq}(|\mathbf{k}|)d|\mathbf{k}| \left(\int_{|\mathbf{q}|}^{2|\mathbf{k}|+|\mathbf{q}|} |\mathbf{q}|d|\mathbf{q}| \int_{|\mathbf{q}|}^{2|\mathbf{k}|} \omega d\omega + \int_{|\mathbf{q}|}^{2|\mathbf{k}|} |\mathbf{q}|d|\mathbf{q}| \int_{|\mathbf{q}|}^{2|\mathbf{k}|+|\mathbf{q}|} \omega d\omega \right)$$

$$\left(|\Delta_L(q)|^2 (2|\mathbf{k}| + \omega)^2 - |\mathbf{q}|^2 \mathcal{J}_1 + |\Delta_T(q)|^2 \frac{(|\mathbf{q}|^2 - \omega^2)(2|\mathbf{k}| + \omega)^2 + |\mathbf{q}|^2}{4|\mathbf{q}|^4} (v^2|\mathbf{q}|^2 - \omega^2) \mathcal{J}_1 + 2\omega \mathcal{J}_2 - \mathcal{J}_3) \right),$$

where $|\mathbf{q}|_{max}$ is given in Eq. (48).

### B.1 Large $L$ limit

In this subsection we will consider the large $L$ limit case, and compute the collisional energy loss per unit length. To do that we multiply both sides of the Eq. (45) by $\frac{2v}{\pi L}$ i.e.

$$\frac{2v}{\pi L} \Delta E_{el} = \frac{C_R g^4}{2\pi^2} \int n_{eq}(|\mathbf{k}|)d|\mathbf{k}| \int_{|\mathbf{q}|}^{2|\mathbf{k}|+|\mathbf{q}|} |\mathbf{q}|d|\mathbf{q}| \omega d\omega \left(|\Delta_L(q)|^2 (2|\mathbf{k}| + \omega)^2 - |\mathbf{q}|^2 \mathcal{J}_1 + \frac{2v}{2\pi L} \mathcal{J}_2 \right)$$

$$\text{L} \rightarrow \infty \rightarrow$$

To compute $\frac{2v}{\pi L} \mathcal{J}_{1,2,3}$ in the limit when $L \rightarrow \infty$ we will use the following expression

$$\frac{2v \sin[(\omega - \vec{v} \cdot \hat{q}) \frac{L}{2v}]}{\pi L} \frac{L \rightarrow \infty}{(\omega - \vec{v} \cdot \hat{q})^2} \delta(\omega - \vec{v} \cdot \hat{q}).$$  \quad (51)

Then,

$$\frac{2v}{\pi L} \mathcal{J}_1 \frac{L \rightarrow \infty}{\frac{L}{4\pi}} \int \frac{d\Omega}{4\pi} \delta(\omega - \vec{v} \cdot \hat{q}) = \frac{1}{2v|\mathbf{q}|} \Theta(v^2|\mathbf{q}|^2 - \omega^2) \quad (52)$$

$$\frac{2v}{\pi L} \mathcal{J}_2 \frac{L \rightarrow \infty}{\frac{L}{4\pi}} \int \frac{d\Omega}{4\pi} \delta(\omega - \vec{v} \cdot \hat{q})(\omega - \vec{v} \cdot \hat{q}) = 0 \quad (53)$$

and
\[
\frac{2v}{\pi L} J_3 \xrightarrow{L \to \infty} \int \frac{d\Omega}{4\pi} \delta(\omega - \vec{v} \cdot \vec{q})(\omega - \vec{v} \cdot \vec{q})^2 = 0 ,
\]

leading to
\[
\frac{2v}{\pi L} \Delta E_{el} = \frac{C_R g^4}{2\pi^4} \int n_{eq}(|\vec{k}|) d|\vec{k}| |\vec{q}| d|\vec{q}| \omega d\omega \frac{1}{2v|\vec{q}|} \Theta(v^2|\vec{q}|^2 - \omega^2)
\]
\[
\left( |\Delta_L(q)|^2 \frac{(2|\vec{k}| + \omega)^2 - |\vec{q}|^2}{2} + |\Delta_T(q)|^2 \frac{(|\vec{q}|^2 - \omega^2)((2|\vec{k}| + \omega)^2 + |\vec{q}|^2)}{4|\vec{q}|^4} (v^2|\vec{q}|^2 - \omega^2) \right).
\]

Therefore, by using relations (47), (48) and \(v^2|\vec{q}|^2 > \omega^2\), the collisional energy loss per unit length in an infinite size QCD medium reduces the following expression \((C_R = 4/3)\)
\[
\frac{dE_{el}}{dL} = \frac{g^4}{6v^2\pi^3} \int_0^\infty n_{eq}(|\vec{k}|) d|\vec{k}| \left( \int_{0|\vec{k}|/(1+v)}^{v|\vec{q}|} d|\vec{q}| \omega d\omega + \int_{|\vec{k}|/(1+v)}^{|\vec{q}|_{max}} d|\vec{q}| \int_{|\vec{q}|-2|\vec{k}|}^{v|\vec{q}|} \omega d\omega \right)
\]
\[
\left( |\Delta_L(q)|^2 \frac{(2|\vec{k}| + \omega)^2 - |\vec{q}|^2}{2} + |\Delta_T(q)|^2 \frac{(|\vec{q}|^2 - \omega^2)((2|\vec{k}| + \omega)^2 + |\vec{q}|^2)}{4|\vec{q}|^4} (v^2|\vec{q}|^2 - \omega^2) \right).
\]

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