Naturally Light Dirac Neutrinos from

\[ SO(10) \times U(1)_\psi \]

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Abstract

A new solution is presented where the right-handed neutrino \( \nu_R \) in \( SO(10) \) pairs up with \( \nu_L \) to form a naturally light Dirac neutrino. It is based on the framework of \( E_6 \rightarrow SO(10) \times U(1)_\psi \), then \( SO(10) \rightarrow SU(5) \times U(1)_\chi \).
Introduction: Neutrinos (ν) are observed but not understood. They are not massless but very light [1]. They may be self-conjugate two-component spinors (Majorana) or four-component spinors (Dirac) with right-handed components (ν_R) which have no electroweak interactions. Neutrinoless double beta decay experiments have so far no conclusive evidence for them to be Majorana, which is the prevalent theoretical thinking.

Why is the Dirac option disfavored? There are two well-known answers. (1) To have a Dirac neutrino, ν_R must exist. However, under the standard-model (SM) gauge symmetry SU(3)_C × SU(2)_L × U(1)_Y, it is a trivial singlet and not mandatory. If it is added anyway, then it is allowed a Majorana mass. Together with the Dirac mass linking ν_L to ν_R through the one SM Higgs doublet Φ = (φ⁺,φ⁰), this forms the well-known 2 × 2 mass matrix

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}.$$  (1)

With the usual reasonable assumption m_D << m_R, the famous seesaw mechanism yields a very small Majorana neutrino mass m_ν ≃ m_D²/m_R. (2) To protect ν_R from having a Majorana mass, a symmetry has to be imposed. The obvious one is lepton number. In that case, m_R is forbidden, but m_D is allowed. However there is no understanding why the Yukawa coupling which generates m_D is so small, 10⁻¹¹ or less, since ⟨φ⁰⟩ = 174 GeV and m_ν < 1.1 eV [2]. To have a compelling case for Dirac neutrinos, two conditions must be met.

- (A) The existence of ν_R should not be ad hoc, but based on a well-motivated theoretical framework, within which it should not acquire a Majorana mass.

- (B) The smallness of the Dirac neutrino mass should be obtained naturally, without any extra symmetry.

In past studies [3], limited success has been achieved, but at the expense of extra symmetries, some of which are softly broken. Here a new solution based on E_6 → SO(10) × U(1)_ψ, then
SO(10) → SU(5) × U(1)χ is presented which satisfies for the first time both (A) and (B) in full measure.

**Essential Existence of νR:** To justify the presence of νR, it ought to transform under a symmetry related to those of the SM. It could be a global symmetry such as lepton number mentioned earlier. However, it would be more convincing and compelling if it were a gauge symmetry such as U(1)B−L [4] or SU(2)R × U(1)B−L [5]. A recently proposed alternative is a gauge U(1)D symmetry [6, 7] not related to the SM but essential for dark matter, with νR as the bridge between the two sectors.

The breaking of U(1)B−L [and SU(2)R] is usually assumed without hesitation to allow νR to obtain a large Majorana mass, thereby already not satisfying condition (A). To avoid this eventuality, there is a simple solution. This breaking does not have to be ∆L = 2. If it is ∆L = 3 for example, then neutrinos are Dirac. This was first pointed out [8] for a general U(1)X symmetry and applied [9] to U(1)L for Dirac neutrinos. However, this mechanism does not by itself explain why the neutrino Higgs Yukawa couplings are so small.

The existence of νR may also be justified in SU(6) as shown recently [10, 11]. However, it is best known as the missing link which allows the 15 fermions (per family) of the SM to form a complete 16 representation of SO(10). Whereas the usual study of SO(10) proceeds from its left-right decomposition SU(3)C × SU(2)L × SU(2)R × U(1)B−L, here the SU(5) × U(1)χ alternative is considered:

\[
16 = (5^*, 3) + (10, -1) + (1, -5),
\]

where

\[
(5^*, 3) = \begin{pmatrix} d^c \\ d^c \\ d^c \\ e \\ \nu \end{pmatrix}, \quad (10, -1) = \begin{pmatrix} 0 & u^c & u^c & -u & -d \\ -u^c & 0 & u^c & -u & -d \\ u^c & -u^c & 0 & -u & -d \\ u & u & u & 0 & -e^c \\ d & d & d & e^c & 0 \end{pmatrix}, \quad (1, -5) = \nu^c.
\]
It has been shown [12] that the $U(1)_\chi$ charge may be used as a marker for dark matter, with dark parity given by $(-1)^{Q_\chi+2j}$. Here it will be shown how a naturally small Dirac mass linking $\nu$ to $\nu^c$ is obtained instead.

**Naturally Small Dirac Neutrino Mass**: To obtain a naturally small Dirac neutrino mass, the mechanism of Ref. [13] is the simplest solution. Let there be two Higgs doublets, say $\Phi = (\phi^+, \phi^0)$ and $\eta = (\eta^+, \eta^0)$ which are distinguished by some symmetry, so that $\bar{\nu}_R \nu_L$ couples to $\eta^0$, but not $\phi^0$. This symmetry is then broken by the soft dimension-two $\mu^2 \Phi^\dagger \eta$ term, with $m_\Phi^2 < 0$ as usual, but $m_\eta^2 > 0$ and large. Consequently, the vacuum expectation value $\langle \eta^0 \rangle$ is given by $-\mu^2 \langle \phi^0 \rangle / m_\eta^2$, which is naturally small, implying thus a very small Dirac neutrino mass. In the original application [13], $\nu_R$ also has a large Majorana mass, hence the $\nu_L$ mass is doubly suppressed. In that case, $m_\eta$ could well be of order 1 TeV. On the other hand, if the symmetry and the particle content are such that $\nu_R$ is prevented from having a Majorana mass, then a much larger $m_\eta$ works just as well for a tiny Dirac neutrino mass.

Recently this mechanism has been used [6, 7] in the framework of a gauge $U(1)_D$ symmetry under which the SM particles do not transform, but $\nu_R$ and other fermion singlets do. The $U(1)_D$ symmetry is broken by singlet scalars which transform only under $U(1)_D$. The connection between the SM and this new sector is a set of Higgs doublets which transform under both, so that $\nu_R$ pairs up with $\nu_L$ to form a Dirac neutrino. The particle content is chosen such that global lepton number is conserved as well as a dark parity or dark number.

A more compelling case [11] is to identify $\nu_R$ as part of the fundamental representation of $SU(6)$ which breaks to $SU(5) \times U(1)_N$. Here a new solution is presented with $E_6 \rightarrow SO(10) \times U(1)_\psi$, then $SO(10) \rightarrow SU(5) \times U(1)_\chi$.

**Sources of Quark and Lepton Masses**: In the SM, there is only one Higgs doublet. All quark and lepton masses come from just this one source. In the conventional left-right model, where
\( \nu_R \) must appear as part of an \( SU(2)_R \) doublet, the common source of all quark and lepton masses is a Higgs \( SU(2)_L \times SU(2)_R \) scalar bidoublet. It is thus not possible to separate out the Dirac neutrino mass from the others. In the case of \( SO(10) \) which encompasses the left-right symmetry, consider how the fermions of the 16 acquire mass. They do so from coupling to the scalars

\[
\begin{align*}
\text{10} &= (5, 2) + (5^*, -2), \\
\text{120} &= (5, 2) + (5^*, -2) + (10, -6) + (10^*, 6) + (45, 2) + (45^*, -2), \\
\text{126}^+ &= (1, 10) + (5, 2) + (10^*, 6) + (15^*, -6) + (45^*, -2) + (50^*, 2),
\end{align*}
\]

under \( SU(5) \times U(1)_\chi \).

From the \( 16_F \times 16_F \times 10_S \) Yukawa term, it is seen that \( d^c d \) and \( ee^c \) couple to \( (5^*, -2) \), whereas \( u^c u \) and \( \nu \nu^c \) couple to \( (5, 2) \). All would acquire masses if the scalar doublets

\[
\Phi_1 = (\phi_1^0, \phi_1^-) \sim (1, 2, -1/2, -2), \quad \Phi_2 = (\phi_2^0, \phi_2^0) \sim (1, 2, 1/2, 2)
\]

under \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N \) develop nonzero vacuum expectation values. However,

\[
\tilde{\Phi}_1 = i\sigma_2 \Phi_1^* = (\phi_1^+, -\bar{\phi}_1^0), \quad \tilde{\Phi}_2 = i\sigma_2 \Phi_2^* = (\phi_2^0, -\phi_2^-)
\]

transform exactly as \( \Phi_{2,1} \) respectively. Hence both \( \Phi_{1,2} \) would couple to all quarks and leptons. This is the analog of the well-known property of the \( SU(2)_L \times SU(2)_R \) scalar bidoublet, i.e.

\[
\eta = \begin{pmatrix} \eta_1^0 & \eta_2^+ \\ \eta_1^- & \eta_2^0 \end{pmatrix},
\]

where

\[
\bar{\eta} = \sigma_2 \eta^* \sigma_2 = \begin{pmatrix} \bar{\eta}_2^0 & -\eta_1^+ \\ -\bar{\eta}_2^- & \bar{\eta}_1^0 \end{pmatrix},
\]

which transforms exactly as \( \eta \).
To distinguish $\Phi_{1,2}$ from $\Phi_{2,1}$, the origin of the fermion $16_F$ of Eq. (2) and the scalars of Eqs. (4),(5),(6) is assumed to be from $E_6 \rightarrow SO(10) \times U(1)_\psi$. Using

$$27 = (16, 1) + (10, -2) + (1, 4),$$

the scalars of Eqs. (4),(5),(6) must then have $Q_\psi = -2$. Thus $\Phi_{1,2}$ transform differently from $\Phi_{2,1}$ which have $Q_\psi = 2$. It is now possible to have two different $(10, -2)$ scalars, one with negative mass-squared which breaks along the $(5^*, -2, -2)$ component, so it gives mass to $d^c d$ and $e e^c$, and the other with large mass-squared which has an induced tiny vacuum expectation value along the $(5, 2, -2)$ component and gives mass to $\nu \nu^c$.

The next step is to find how the $u^c u$ term may be distinguished from $\nu \nu^c$. Under $SU(5) \times U(1)_\chi$, the former comes from $(10, -1) \times (10, -1)$ which couples to the scalars $(5, 2)$ and $(45, 2)$, whereas the latter comes from $(5^*, 3) \times (1, -5)$ which couples only to $(5, 2)$. Now it is well-known that $(45, 2)$ also contains the doublet $(1, 2, 1/2, 2)$. Let it be called $\Phi_4$. So if $\langle \phi_0^0 \rangle$ is large and $\langle \phi_0^2 \rangle$ is small, then the Dirac neutrino mass is guaranteed to be small. This is the key observation of the present study.

**Symmetry Breaking Details**: The relevant scalars of this model are listed in Table 1. The scalar $\zeta_0$ breaks $E_6$ to $SO(10) \times U(1)_\psi$; $\zeta_1$ breaks $U(1)_\psi$; $\zeta_2$ breaks $SO(10)$ to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi$. The corresponding vacuum expectation values (VEV) $v_{0,1,2}$ are all big. The scalar $\zeta_3$ breaks $SO(10) \times U(1)_\psi$ to $SU(5) \times U(1)_\chi$, but $v_3$ may be small because both $U(1)_\psi$ and $SO(10)$ have already been broken by $v_{1,2}$. Finally, $\zeta_4$ breaks $U(1)_\chi$, presumably at a lower scale. The doublets $\Phi_{1,2,3,4,5}$ all break $SU(2)_L \times U(1)_Y$ to $U(1)_Q$ as in the SM. There are of course other possible scalar doublets, but they are all assumed heavy enough, not to affect the outcome of the SM.

In the $27 \times 27^* \times 78$ trilinear scalar coupling, $v_0$ splits the $(10, -2)$ component from $(1, 4)$ and $(16, 1)$, but the two $SU(5)$ components of $(10, -2)$, i.e. $\Phi_{1,2}$, are degenerate. To split them with $\Phi_1(\Phi_2)$ having negative (positive) mass-squared, the $U(1)_\psi$ charge plays an
Table 1: Scalars for $E_6 \to SO(10) \times U(1)_\psi$, $SO(10) \to SU(5) \times U(1)_X$, and $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$.

important role. Whereas $(126^*, -2)$ comes from $351'$, $(126, -2)$ comes from $1728$. They may have different masses. This would not be possible in $SO(10)$ alone. Note also that $\Phi_5$ cannot couple to the SM fermions because it comes from $126$ not $126^*$. The choice of $(1050, 0)$ for $\zeta_2$ is because it couples to $10 \times 126^*$ and $120 \times 126^*$, but not $10 \times 120$. This allows $\Phi_2$ not to mix with $\Phi_4$, whereas $\Phi_{1,3,5}$ may mix in a $3 \times 3$ mass-squared matrix as shown below.

The trilinear scalar $\mu_{15} \Phi_1 \Phi_3^\dagger \zeta_2$ and $\mu_{35} \Phi_3 \Phi_5^\dagger \zeta_2$ couplings are allowed by $SU(5) (5^* \times 5 \times 24)$, $(45^* \times 5 \times 24)$, $SO(10) (10 \times 126^* \times 1050)$, $(120 \times 126^* \times 1050)$, and $E_6 (27 \times 1728^* \times 5824^*)$, $(351 \times 1728^* \times 5824^*)$. The $\Phi_1 \Phi_3^\dagger \zeta_2$ coupling is forbidden because $10 \times 120 \times 1050$ is not allowed under $SO(10)$. The resulting $3 \times 3$ mass-squared matrix for $\Phi_{1,3,5}$ is given by

$$M^2_{1,3,5} = \begin{pmatrix}
m_{10}^2 & 0 & \mu_{15} v_2 \\
0 & m_{120}^2 & \mu_{35} v_2 \\
\mu_{15} v_2 & \mu_{35} v_2 & m_{126}^2
\end{pmatrix}.$$  \hspace{1cm} (12)

The corresponding $2 \times 2$ mass-squared matrix for $\Phi_{2,4}$ is simply

$$M^2_{2,4} = \begin{pmatrix}
m_{10}^2 & 0 \\
0 & m_{120}^2
\end{pmatrix}.$$  \hspace{1cm} (13)

It is thus possible to keep $m_{10}^2 > 0$ and $m_{120}^2 < 0$, and find two negative eigenvalues in
\(M_{1,3,5}^2\), with eigenstates of linear combinations of \(\Phi_{1,3,5}\). They will contribute to the \(d^c d\) and \(ee^c\) masses, whereas \(\Phi_4\) is responsible for the \(u^c u\) masses, and since \(\Phi_2\) has no VEV at this stage, neutrino masses are zero.

The \(\Phi_{2,4}\) scalars are possibly connected to \(\Phi_{1,3,5}\) through \(\zeta_3\). The trilinear \(\mu_{21} \Phi_2 \Phi_1 \zeta_3\) coupling is allowed by \(SU(5) (5 \times 5^* \times 1)\), \(SO(10) (10 \times 10 \times 45)\), and \(E_6 (27 \times 27 \times 351)\). This results in

\[
\langle \phi^0_2 \rangle \simeq -\frac{\mu_{21} \langle \phi^0_1 \rangle v_3}{m_{10}^2}, \tag{14}
\]

as discussed earlier. This suppressed VEV is the source of the Dirac neutrino mass. As for the \(\Phi_2 \Phi_3\) and \(\Phi_2 \Phi_5\) terms, they are not allowed because the former does not obey \(SU(5)\) and the latter \(U(1)_\psi\). The only other allowed coupling is \(\Phi_3 \Phi_4 \zeta_3\), which is expected because it comes from \(351 \times 351 \times 351\). Finally \(\zeta_4\) transforms as \((1, 1, 0, -5, -3)\) under \(SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{\chi} \times U(1)_\psi\), whereas a Majorana neutrino mass for \(\nu^c\) would require a singlet transforming as \((1, 1, 0, 10, -2)\). The absence of the latter with a VEV means that lepton number remains a global symmetry in the SM.

**Concluding Remarks:** In the context of \(E_6 \to SO(10) \times U(1)_\psi\), then \(SO(10) \to SU(5) \times U(1)_\chi\), it is shown how the neutrino obtains a naturally small Dirac mass. This involves five scalar doublets: \(\Phi_1, \Phi_3, \Phi_5 \sim (1, 2, -1/2, -2)\) from the \((5^*, -2)\) of 10 in \(SO(10)\), the \((45^*, -2)\) of 120, and the \((5^*, -2)\) of 126, all of which have \(Q_\psi = -2\) under \(E_6 \to SO(10) \times U(1)_\psi\); and \(\Phi_2, \Phi_4, \sim (1, 2, 1/2, -2)\) from the \((5, 2)\) of 10 in \(SO(10)\), and the \((45, 2)\) of 120. The key is that whereas \(\Phi_{1,2}\) and \(\Phi_{3,4}\) are not split by themselves, the conjugate of \(\Phi_5\) lies in a different \(E_6\) multiplet, so the would-be \(\Phi_6\) may be assumed much heavier and be ignored.

With the addition of the five singlet scalars of Table 1, it is shown how \(\Phi_{1,3,4}\) may develop VEV for the \(d^c d\), \(ee^c\), and \(u^c u\) masses. The \(\nu^c\) masses come only from \(\Phi_2\), which keeps a large positive mass-squared, so its VEV is induced and suppressed by the mechanism of Ref. [13]. The breaking of \(U(1)_\chi\) comes from \(\zeta_4\) which allows global lepton number to remain.
The fermions of this model are the same as in the SM, except that the neutrino is Dirac. They form complete 16 representations of SO(10) which may have a remnant $U(1)_X$ symmetry \[12\] from breaking to $SU(5) \times U(1)_X$, instead through the usual left-right intermediate step. There are possibly three scalar doublets ($\Phi_{1,3,4}$) at the electroweak scale, together with a singlet $\zeta_4$ at the $U(1)_X$ breaking scale. The $Z_X$ gauge should also appear at this scale, with known couplings to all SM particles. From present collider data, its mass is greater than a few TeV.

Acknowledgement: This work was supported in part by the U. S. Department of Energy Grant No. DE-SC0008541.

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