On the Field-Aligned Particle Acceleration in the presence of Electron-Positron Pairs

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Abstract. We study self-consistency for the pulsar polar cap model. In general, there will remain an unscreened electric field at the pair production front (PPF). We have derived the condition of electric field screening by pairs in the presence of returning particles. A previous belief that pairs with a density higher than the Goldreich-Julian density immediately screen out the electric field beyond PPF is unjustified. Pairs have little contribution on screening the electric field, and the geometrical screening is the only known way of getting a finite potential drop.

1. Introduction

One of the promising models for the pulsar action is the polar cap model where the field-aligned electric field accelerates charged particles up to TeV energies, and causes an electromagnetic shower. The potential drop is normally thought to be confined in a small region: the field-aligned electric field is screened out at the both ends of the accelerator (Fig. 1). The polar cap potential drop is a part of the electromotive force produced by the rotating magnetic neutron star. The voltage drop is a few percent of the available voltage for young pulsars, while it becomes some important fraction for older pulsars. The localized potential is maintained by a pair of anode and cathode regions, the formation mechanism of which is the long issue of the polar cap accelerator.

2. Pair Production Front (PPF) and Screening

If the field-aligned electric field is formed to accelerate particles up to the Lorentz factor of $10^6 - 10^7$, then a number of curvature gamma-photons convert into electron-positron pairs under strong magnetic field $\sim 10^{12}$ G above a surface, which may be called the pair production front (PPF). The expected multiplicity — number of pairs produced by one primary particle — is typically $10^3$ within a distance $\sim 10^5$ cm.

In a number of literatures, it is assumed that the field-aligned electric field is screened out above PPF because the density of pairs is higher than the Goldreich-Julian (GJ) density. However, this screening process had not been studied in detail. Shibata, Miyazaki & Takahara (1998) showed that pair polarization is not efficient even if pair density is much higher than the GJ density. In their work, presence of the particles reflected backward by unscreened electric field was ignored. In this paper, taking the backward-moving particles into account, we derive an expression for the steady screening at PPF.

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\[ E = \phi \]

\[ E_{\text{eff}} = E + \frac{q}{\varepsilon_0} \frac{\partial \phi}{\partial r} \]

\[ \text{PPF} \]

\[ \text{stellar surface} \]

Fig. 1. localized potential drop
3. Model

Let us consider a model for the steady accelerator which has a finite potential drop along the magnetic field with the electric field screened at both ends. For definite sign of charge, let us assume electrons are accelerated outward, i.e., the electric field points toward the star. γ-rays emitted by the electrons convert into pairs beyond PPF continuously.

Schematic picture of the screening region is shown in Fig. 2. At the top panel, electrons are accelerated to the right, so the electric field points to the left. The γ-rays emitted by the electrons are converted in to pairs above PPF (the right side of PPF in the figure). Since the pairs are created almost continuously in a half space to the right of PPF, the pair-flux monotonically increases with the coordinate \( x \) along the magnetic lines of force, where curvature is supposed to be ignored since the scale considered here is very short.

The second panel indicates the pair flux \( F(x) \), which is defined as the number of pairs created below \( x \). The multiplication factor \( m(x) \) may be much convenient: \( F(x) = m(x) j_0 \) where \( j_0 \) is the primary flux of electrons coming up from a region below PPF.

Here, it must be reminded that why the accelerating electric field is formed below PPF. The electric field well within the light cylinder is conveniently decomposed by the co-rotational and non-corotational fields:

\[
E = -\frac{\Omega \times r}{c} \times B - \nabla \Phi, \tag{1}
\]

where \( \Phi \) is the non-corotational electric potential, and field aligned electric field is given by \( E_\parallel = -B \cdot \nabla \phi / B \). The Poisson equation for \( \Phi \) reads

\[
-\nabla^2 \phi = 4\pi (\rho - \rho_0), \tag{2}
\]

where \( \rho_0 \equiv \Omega B_z/2\pi c \) is the GJ charge density. From this expression it follows that the anode region is formed where \( \rho > \rho_0 \), and the cathode region is formed where \( \rho < \rho_0 \).

In the screened region, \( \rho = \rho_0 \).

In our sign convention, the GJ density is negative, and electronic flow with super-GJ density can produce cathode (negative) region, which is located above the stellar surface (see Fig. 1). There must be such a negative region below the PPF. Above PPF, created pair positrons will be returned back by the electric field, so they can destroy the cathode region. This fact sets a constraint to the flux of pair positrons. In the problem of electric field screening, one needs an anode region where the effective charge \( \rho - \rho_0 \) is positive. There are two possible ways of producing the anode region. One is the 'geometrical effect', and the other is pair polarization.

The geometrical effect is based by the fact that the GJ charge density is proportional to the magnetic component along the rotation axis \( B_z \) while the charge density of the electronic flow is proportional to the magnetic field strength, or equivalently inversely proportional to the cross sectional area of the magnetic flux tube if the flow velocity is \( \approx c \) (constant). Therefore, if the flow is curving toward the rotation axis, the GJ density decreases slower than the primary electronic charge density so that the electronic charge is insufficient to compensate the GJ density, and the anode is formed. This effect is effective, and the accelerating electric field can be terminated without pair creation. Although geometrical screening is effective, PPF can in general appear before geometrical screening. If this is the case, dynamics of pairs just after PPF will be important.

Our interest in this paper is not the geometrical screening but the pair effect. In general, at PPF, whatever values of the effective charge can be; it can even be negative so that anode formation rely only on pair polarization.

As shown in Fig. 2, pair electrons are accelerated while pair positrons are decelerated after their birth. As far as the speed of pair positron is relativistic, net space charge due to pairs are essentially zero. However, once the pair positron become non-relativistic, positive space charge appears owing to charge conservation. This is the effect which produces anode charge. If the electric field is strong enough, some positrons are reflected backward and weaken the cathode space charge. Assuming steadiness, one can relate the strength of the electric field to the pair creation rate, the returning positron flux, and the effective space charge before PPF. If the pair creation rate is more or less uniform, one can do this analytically as shown in the following section. Then, we have an screening condition.

4. Results

If the pair creation rate is more or less uniform, the screening condition with returning positron can be derived analytically. Detailed calculation will be published near future, but main results are discussed below.

The field-aligned electric field \( E_\parallel \) at PPF is screened if the following two conditions are satisfied:

\[
\frac{1}{2} E_\parallel^2 = j_\alpha [\phi_1 (2 - \phi_1) + \zeta] - (j - j_0 - j M_1) \phi_0. \tag{3}
\]

\[
\rho_0 = j_\alpha (1 - 2\phi_1) - (j - j_0 - M_1 j) > 0. \tag{4}
\]

(\ref{3}) is just the result of Gauss’ law: a condition for giving the column positive charge to screen the electric field. (\ref{4}) is the positiveness of the space charge density at the screening surface where \( E_\parallel \) vanishes.

\( E_t \) is the normalized electric field to be screened:

\[
E_t = (\epsilon \lambda_0 / mc^2) E_\parallel, \quad \lambda_0 = \sqrt{c^2 / 2\Omega B}, \quad B = 2.0 \text{ cm} \]

\( B_{12}^{-1/2} P_{1}\text{cm}^2 \) is the Debye length of typical GJ plasma. For \( 10^{13} \) Volt acceleration in \( 10^4 \) cm, \( E_t \sim 100 \). \( j \) is the primary electronic current density in units of the typical...
GJ density: $j = J/(−ΩB/2π)$. $α_φ$ is the pair multiplicity — number of pairs produced by one primary electron — in an unit breaking distance $λ_*$, with which a non-relativistic particle turns by the unscreened electric field, i.e., $eE_{∥}/λ_∗ = mc^2$. In non-dimensional form, this distance is $1/E_{∥}$. $ϕ_1$ is defined in such a way that a pair positrons produced in between $ϕ = 0$ (PPF) and $ϕ_1$ are reflected backward, and therefore $M_1 = α_ϕϕ_1$ gives the returning multiplication factor, i.e., $jM_1 = jα_ϕϕ_1$ gives the returning positron flux. $j_0$ is the normalized (local)
GJ charge density: $j_0 = B_z/B_\perp$. $\zeta$: a numerical factor determined by the Lorentz factor at the birth of pairs; e.g., for $\gamma_{\text{birth}} \sim 100$, $\zeta = 1.7$.

5. Application

Positive space charge by pair polarization is little. A previous belief that pair creation with a pair density higher than the Goldreich-Julian density immediately screen out the electric field is unjustified. This is because

(1) only the non-relativistic positrons can provide positive space charge,

(2) the non-relativistic positrons are reflected backward by the electric field and leave the pair electrons behind, so that a negative space charge is produced in the screening region where the positive charge is required.

In the case where geometrical screening is ineffective, more specifically, $j - j_0 - jM_1 > 0$, we have a condition on the multiplication factor produced in unit breaking distance $\lambda_*$ at PPF:

$$\Delta M_{\text{screen}} \geq \frac{E_\parallel^2}{8\pi mc^2 n_0 \zeta' j},$$

which indicates that if $j \sim 1$, a multiplication factor of $10^3$ is required in 0.01 cm. This condition will not be satisfied in the pulsar magnetosphere.

Thus the geometrical screening (toward curvature) is the only known way of screening, i.e., in (8), the positive effective charge $-j + j_0 + jM_1 > 0$ last in an appropriate distance (until the electric field is screened out or weakened enough to be screened by pair polarization). Even on toward curved field lines, if the current density is super-GJ, or if $E_\parallel$ at PPF is not so weakened, then there is no way of maintaining a steady potential drop along field lines.

Difficulty in pair screening is always true for field lines curving away from the rotation axis regardless of the strength of the current density.

In our analysis, we do not take interaction between the particle beams into account. Beam instability may produce some frictional force on returning positrons and enlarge positive space charge. Time dependent behavior such as repetition of acceleration and screening may be an alternative way for a self-consistent model.

References

Shibata S., Miyazaki J., Takahara F., 1998, MNRAS 295, L53