Acceleration of cosmic expansion through huge cosmological constant progressively reduced by submicroscopic information transfer

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In a previous paper (Ref. [1]) the presence of dark energy in our universe was explained as the fingerprint of a comprehensive, much older and expanding multiverse with positive spatial curvature, whose space-time is spanned by this energy, and which was created out of nothing. This concept is expanded by the addition of a model for explaining the decay of the mass density $\rho$ of dark energy from its origin until now by a factor of approximately $10^{-120}$.

Elementary particles contain information about which laws of nature they obey, but not what exactly these are. Most likely, the laws are not followed by obedience to a categorical imperative. Rather, it is assumed, that from the very beginning the information about them is coded in submicroscopic patches of the space-time. The initial density $\rho_i$ is supposed to belong to the unimpaired cosmological constant obtained from elementary particle theory. Due to its huge value it causes an extremely fast spatial expansion by which continuously new space-time elements are created. To them, the information about the physical laws must be transmitted from the already present space-time. This process needs time which with ever-increasing expansion velocity is getting scarcer and scarcer. It is concluded that this impedes the expansion through a friction-like process which can be described by a term proportional to the expansion-velocity. This term is subtracted from the expansion-acceleration. It is shown that the solutions thus obtained are also solutions of the cosmological standard equations employing a scalar field $\Phi$. In consequence, the present model can be considered as a re-interpretation of results which can be obtained with acknowledged methods.

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I. INTRODUCTION

In this paper, the question is addressed how our universe manages, that in each point of its space-time the physical laws are obeyed. Elementary particles, the building bricks of matter, or the fields representing them are endowed with specific properties which enter the physical laws regulating their behavior. However, they do not determine that fully but only in an indicative way by declaring which laws apply to them. How they follow the laws will hardly be virtual obedience to a categorical imperative. Examples from other fields may serve as an illustration. 1. In human society the laws to be obeyed by the individuals are rooted in the brains of the latter. Without the verifiable presence of this information they cannot be followed. 2. In biology, molecules that work together to form a plant or an animal obtain the information about what to do by the genetic code written down in the DNA. In both cases, the information needed is physically present in all places where laws control processes.

It is an empirical finding that once nature has developed a successful recipe, it will employ it not only once but repeatedly. Therefore, it appears natural to assume, that the information about the physical laws to be followed by all material elements must somehow be contained in the neighborhood of each space-time point. For the way how this is accomplished, mainly two methods offer themselves: 1. The information is encoded in higher dimensions which are inaccessible to us but connected with each spot of our space-time. 2. The information entered the universe in the process of a creation out of nothing and is encoded in the primordial space-time emerging from this. Both methods are pretty well hidden, what they should actually be, because so far they have not been discovered. In our view, both methods entail the same consequences, and both of them can prove right or wrong. (Due to its particular straightforwardness at some instances the second possibility will be preferred.) However, the results derived from them have attractive properties which are classified and rated in detail later on.

As can be read from the title, in this paper, the process of information transfer gets closely linked to the cosmological constant problem [2]. The latter was even the starting point for this investigation. Indirectly this has turned up already in Ref. [1] whose ideas are further developed here. Before that, however, the most important results from there are briefly recapitulated.
The question addressed in the previous paper is: why does our universe contain dark energy, in view of the fact, that universes similar to ours could well do without it. The answer given there is: our universe is a subuniverse in an all-encompassing and continuously expanding multiverse, whose space-time is established by dark energy that also fills our universe. The dark energy currently observed in the latter thus represents a fingerprint of the multiverse containing it.

Measurements, from which the spatial curvature of our universe can be deduced, render a value so small, that the space is either completely uncurved and infinitely extended – we refrain from topologically more complex constellations like an uncurved space of finite extent –, or, in the case of non-vanishing positive curvature, that it extends far beyond the boundary of the observable universe [3, 4]. (The latter is in line with Ref. [6] and with NASA’s conclusion from WMAP measurements [5]: “We now know (as of 2013) that the universe is flat with only a 0.4 % margin of error. This suggests that the Universe is infinite in extent; however, .... All we can truly conclude is that the Universe is much larger than the volume we can directly observe.”) In both cases there is a large region outside the latter, most of which is not causally connected with it and therefore structurally quite different from it. This shows, that assuming the existence of a multiverse containing our universe is justified, although there is no direct proof for it. From the two above mentioned possibilities a positively curved (and thus finitely extended) multiverse was chosen for several reasons: 1. Only a positively curved multiverse can be created out of nothing, which is of crucial importance for the purposes of this paper. 2. Based on results from Ref. [7] it was shown in Ref. [1] that the expansion of a positively curved space has generic properties which do not pertain to an infinitely extended uncurved space.

In a creation out of nothing, the multiverse emerges continuously from a quantum mechanical tunneling process. For this, its expansion must start at zero velocity immediately after the latter, and the initial mass density $\varrho_{\Lambda}$ of dark energy is fixed to about $10^{120}$ times the value $\varrho_0$ measured presently in our universe [1]. In Ref. [1] solutions of the cosmological equations for a multiverse with dark energy as the primary ingredient were derived. They have the required property that the mass density $\varrho$ of the dark energy decays from a huge initial value to an about $10^{-120}$ times smaller present value. Our universe is one among other subuniverses, which are supposed to emerge in the multiverse due to fluctuations of separate inflation fields of their own, all of them permeated by the dark energy field of the
multiverse. In properly chosen coordinates – in particular with identical proper time intervals $dt$ – the mass density of the latter, measured in our universe, is the same as measured in the multiverse. The above factor $10^{-120}$ agrees more or less with the factor by which a cosmological constant $\Lambda_0$ with the presently observed mass density $\rho_0$ is smaller than predicted by elementary particle theory \[8\]. This suggests that there is a physical process which with increasing expansion of the multiverse increasingly reduces the effect of $\rho_\Lambda$. Complementary to the findings of Ref. \[1\] in this paper a possible mechanism for this is proposed and elaborated with respect to its consequences.

Technically, in place of a huge cosmological constant $\Lambda$ we employ the corresponding mass density $\rho_\Lambda$ and consider it as a contribution to the energy momentum tensor on the right hand side of the Einstein equations, since conceptually it arises from fluctuations of matter fields. We assume that it keeps its large initial value during the whole evolution of the multiverse. Without an additional reduction process it would cause an extremely accelerated expansion of the multiverse. The reduction process employed in this paper is the following: Firstly, we assume that the information about all physical laws and the physical agents to whom they relate, is transmitted through the creational tunneling process to the subsequent initial state of the multiverse. Within the latter, the information about the material agents is contained in these themselves. Concerning the physical laws and their practical implementation, we assume, borrowing from the biological DNA coding, that the information about them is somehow coded in all places of the primordial space-time within patches of submicroscopic but finite extent.\[1\] Details about their structure (discrete or quantized \[10\]-\[12\] respectively, a quantum foam \[13\]-\[15\], perhaps even a structure below the quantum level, or otherwise \[16\]) and about the kind of encoding (DNA-like, geometrically or otherwise) are not needed for the following and are therefore not discussed. The additional space created by the $\rho_\Lambda$-induced spatial expansion must be equipped with the information about the physical laws and physical properties of its own (e.g. vacuum fluctuations) via transfer from already exist-

\[1\] For the storage of information in physical storage media, given their mass or energy and the amount of information, there is a size limit called Bekenstein bound \[9\], which cannot be fallen below. In our case, while the space-time patches used for storage contain dark energy, it is not clear whether or not the latter is involved in the storage of information (later both options are considered). Furthermore, it is unclear whether the physics used to derive said boundary is applicable here at all. Finally, for its application one would also have to know the amount of information to be stored. Here, this not only concerns the information contained in all physical laws, but also the definition of the quantities contained therein, the mathematics used in them, the logic underlying the latter, and in addition also the methods for practical implementation. Furthermore, it should be ensured that the physical laws known today are complete. An order of magnitude estimate of this amount of information seems feasible but would go far beyond the scope of this investigation. As it turns out, it is not necessary for the essential results of this work.
ting space. It is plausible that this process takes some time which is not available, if the spatial expansion proceeds too fast. In this regard, we now assume, that for gaining time the transfer of information to the newly created patches of space-time impedes the spatial expansion, and that this can be cumulatively represented by introducing an appropriate friction term into the cosmological equations. Without it, the relevant cosmological equation would be

$$\dot{a}^2(t) = \frac{8\pi G}{3} \varrho \Lambda a^2 - c^2$$  \hspace{1cm} (1)$$

with the immediate consequence

$$\ddot{a}(t) = \frac{8\pi G}{3} \varrho \Lambda a.$$  \hspace{1cm} (2)$$

The simplest way to include a friction term is a modification of this equation in the form

$$\ddot{a}(t) = -f \dot{a}(t) + \frac{8\pi G}{3} \varrho \Lambda a$$  \hspace{1cm} (3)$$

where \(f > 0\) is a constant. Multiplying this equation with \(\dot{a}(t)\) and integrating it with respect to \(t\) yields

$$\dot{a}^2(t) = \frac{8\pi G}{3} \varrho(t) a^2 - c^2$$ \hspace{1cm} \text{with} \hspace{1cm} \varrho = \varrho \Lambda + \varrho_f$$  \hspace{1cm} (4)$$

where

$$\varrho_f = -\frac{3f}{4\pi Ga^2} \int_0^t \dot{a}^2(t') \, dt'$$  \hspace{1cm} (5)$$

and where an integration constant was chosen such that at the time \(t=0\) immediately after the tunneling process, the multiverse starts with the unaltered mass density \(\varrho = \varrho \Lambda\) and assumes the form of the usual Friedman-Lemaitre equation (for short FL-equation). The result \(\varrho_f < 0\) means, that the force field represented by \(\varrho_f\) or \(f \dot{a}(t)\) respectively and introduced to reduce the effect of \(\varrho \Lambda\) via friction, has negative mass or energy like the gravitational field. It therefore makes sense to interpret it as a component of the gravitational field described by the left side of the cosmological field equations. In paragraph (5) of Section V it is shown that the density decomposition of Eq. (4) with (5), caused by the introduction of the friction term \(-f \dot{a}(t)\) in Eq. (3), can be covariantly integrated into the Einstein field equations.

Introducing an additional term in the cosmological equations is a delicate matter that could easily meet with rejection. However, choosing the solutions of the modified equation \(\text{(3)}\) such, that at all times the usual FL-equation \(\text{(4)}\) holds, leads back to the cosmolog-

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2 This assumption is supported by the existence of an upper speed limit of information processing found by Bremermann [17, 18], which is \(1.36 \cdot 10^{50}\) bits/(kg s).

3 Modifications of the Einstein field equations have indeed a long-standing tradition, beginning with Einstein’s addition of the cosmological constant for obtaining a static universe (see e.g. page 613 of Ref. [19]) to the introduction of additional mass generation terms for a steady state universe by H. Bondi and T. Gold (page 459 of Ref. [19]) and independently by F. Hoyle (page 616 of Ref. [19]) up to very recent alterations as in Refs. [20]-[22].
tical standard theory in which it suffices to solve the latter. Therefore, it can be said that the newly introduced model leads to standard solutions which are only re-interpreted in an unusual way. In Sec. V it is shown that even the cosmological equations for a scalar field $\Phi$, driven by a potential $V(\Phi)$, are satisfied by our solutions.

Because of the information about particle-properties contained therein, actually at least a cumulative mass density $\varrho_m$ for matter should be included. However, for the sake of clarity, and since it would not disclose anything new compared to the results of Ref. [1], we refrain from this and come back to it just briefly in Sec. V.

II. BASIC EQUATIONS AND BOUNDARY CONDITIONS

All calculations are carried out in Friedman Robertson Walker coordinates. In order to make numerical evaluations more transparent, we use MSI units. The basic equations to be satisfied in a closed multiverse with positive spatial curvature are Eqs. (3)-(4), the dependent variables to be calculated being $a(t)$ and $\varrho(t)$. Since Eq. (3) contains only $a(t)$, this variable is already completely determined by it, if appropriate boundary conditions are imposed. As soon as $a(t)$ is known, Eq. (4) can be used for calculating $\varrho(t)$. For determining $\varrho_f(t)$, Eq. (4) is not even needed since it can be calculated from $\varrho_f = \varrho(t) - \varrho_\Lambda$. For later purposes we still need the equations

$$\varrho = \frac{\hbar^2 \dot{\Phi}^2(t)}{2\mu c^4} + \frac{V(\Phi)}{c^2}, \quad p = \frac{\hbar^2 \dot{\Phi}^2(t)}{2\mu c^2} - V(\Phi), \quad (6)$$

$$\ddot{\Phi}(t) + 3(\dot{a}(t)/a) \dot{\Phi}(t) + \frac{\mu c^2}{\hbar^2} V'(\Phi) = 0 \quad (7)$$

which hold when the expansion of the multiverse is driven by a scalar field $\Phi$ ($\mu$ is the mass parameter of the field $\Phi$).

As already mentioned in the Introduction, for $t=0$ the initial conditions

$$\varrho = \varrho_\Lambda, \quad a = a_i = l_P, \quad \dot{a}(t) = 0 \quad (8)$$

must be satisfied. With this and $l_P/t_P = c$, from Eq. (4) we obtain

$$\varrho_\Lambda = \frac{3c^2}{8\pi G a_i^2} \quad \text{or} \quad \frac{8\pi G t_P^2}{3} = \frac{1}{\varrho_\Lambda} \quad (9)$$
and
\[
\rho_\Lambda = \frac{3}{8\pi G} \quad \rho_P = 0.616 \cdot 10^{96} \text{ kg m}^{-3},
\]
where \(l_P\) and \(t_P\) are given immediately below and \(\rho_P = c^5/(\hbar G^2)\) is the Planck density. A further boundary condition concerning the mass density \(\rho_0\) at the present time \(t_0\) is dealt with in Sec. III B.

For the evaluation of Eqs. (3)-(7) it is useful to introduce dimensionless quantities
\[
\tau = \frac{t}{t_P} \quad \text{with} \quad t_P = \sqrt{\frac{\hbar G}{c^3}} = 5.39 \cdot 10^{-44} \text{ s}
\]
\[
x = \frac{a}{l_P} \quad \text{with} \quad l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \cdot 10^{-35} \text{ m},
\]
\[
\rho = \frac{\rho}{\rho_\Lambda}, \quad \hat{\pi} = \frac{p}{\rho_\Lambda c^2},
\]
\[
\varphi = \frac{\hbar}{c^2} \sqrt{\frac{8\pi G}{3\mu}} \Phi, \quad v(\varphi) = \frac{V(\Phi)}{\rho_\Lambda c^2},
\]
where \(t_P\) and \(l_P\) are the Planck time and the Planck length respectively. Using Eqs. (9) and (11)-(13), Eq. (4) becomes
\[
\dot{x}^2(\tau) = \rho x^2 - 1,
\]
and using Eqs. (14) in addition, Eqs. (6)-(7) become
\[
\rho = \frac{\dot{\varphi}^2(\tau)}{2} + v(\varphi), \quad \hat{\pi} = \frac{\dot{\varphi}^2(\tau)}{2} - v(\varphi)
\]
and
\[
\ddot{\varphi}(\tau) + 3 h \dot{\varphi}(\tau) + v'(\varphi) = 0 \quad \text{with} \quad h = \frac{\dot{x}(\tau)}{x}.
\]
Finally, Eq. (3) becomes
\[
\ddot{x}(\tau) + 2\sigma \dot{x}(\tau) - x = 0 \quad \text{with} \quad \sigma = \frac{ft_P}{\rho_\Lambda}.
\]
This equation must be solved with appropriate boundary conditions. In terms of the dimensionless variables (11)-(12), from the initial conditions (8) the relevant ones become
\[
x = 1, \quad \dot{x}(\tau) = 0 \quad \text{for} \quad \tau = 0.
\]
The mass density corresponding to the solution \(x(\tau)\) follows from Eq. (15) and is
\[
\rho(\tau) = \frac{1}{x^2(\tau)} + \frac{\dot{x}^2(\tau)}{x^2(\tau)}.
\]
To find out how the solutions of Eq. (18) compare to solutions of the usual scalar field theory, we express the equation
\[ \ddot{a}(t) = -\frac{4\pi G}{3} \rho (1 + 3w) a \quad \text{with} \quad w = \frac{p}{\rho c^2}, \]

obtained with use of Eqs. (6)-(7) from the time derivative of Eq. (4), in terms of the dimensionless variables (12)-(13) and obtain
\[ \ddot{x}(\tau) = -\frac{1 + 3w}{2} \rho x. \quad (21) \]

III. SOLUTIONS

A. Calculation of the expansion \( x(\tau) \)

Eq. (18) can be solved with the ansatz \( x(\tau) = e^{\gamma \tau} \). The general solution thus obtained is
\[ x(\tau) = \alpha e^{\gamma_1 \tau} + \beta e^{\gamma_2 \tau} \quad (22) \]

with
\[ \gamma_1 = \sqrt{1+\sigma^2} - \sigma, \quad \gamma_2 = \sqrt{1+\sigma^2} + \sigma. \quad (23) \]

Imposing on it the initial conditions (19) yields
\[ \alpha + \beta = 1, \quad \alpha \gamma_1 + \beta \gamma_2 = 0. \]

From this, with use of Eqs. (23) we get
\[ \alpha = \frac{\gamma_2}{2 \sqrt{1+\sigma^2}} = \frac{1}{1 + \gamma^2}, \quad \beta = \frac{\gamma_1}{2 \sqrt{1+\sigma^2}} = \frac{\gamma^2}{1 + \gamma^2}, \quad (24) \]

where the relations \( \gamma_1 \gamma_2 = 1 \) and \( \gamma_1 + \gamma_2 = 2 \sqrt{1+\sigma^2} \), following from Eqs. (23), were used and the redefinition
\[ \gamma = \gamma_1 = \sqrt{1+\sigma^2} - \sigma \quad (25) \]

was carried out. Inserting the results (24) in Eq. (22) leads to
\[ x(\tau) = \frac{e^{\gamma \tau} + \gamma^2 e^{-\tau/\gamma}}{1 + \gamma^2} \quad (26) \]
as the final form of our solution. Resolving Eq. (25) with respect to $\sigma$ yields

$$\sigma = \frac{1 - \gamma^2}{2\gamma}. \quad (27)$$

The result (26) can be rewritten in the form

$$x(\tau) = x_1(\tau) + x_2(\tau) \quad \text{with} \quad x_1(\tau) = e^{\gamma\tau}, \quad x_2(\tau) = -\frac{\gamma^2}{1+\gamma^2}(e^{\gamma\tau} - e^{-\tau/\gamma}).$$

For the ratio $q = |x_2(\tau)/x_1(\tau)|$ we get

$$q = \frac{\gamma^2}{1+\gamma^2}(1 - e^{-(\gamma+1/\gamma)\tau}) \leq \frac{\gamma^2}{1+\gamma^2} < \gamma^2 = 1.02 \cdot 10^{-122},$$

where in the last step the later result (37) was introduced. It follows that with just an extremely small error we can put

$$x(\tau) = e^{\gamma\tau} \quad \text{for all} \quad \tau \geq 0. \quad (28)$$

Note that by this approximation the first of the conditions (19) is satisfied exactly and the second approximately, but due to $\gamma \ll 1$ with an extremely small error only.

### B. Calculation of the mass density $\varrho$ and determination of $\gamma$

As in Ref. [1] we assume that the agent driving the expansion of the multiverse is also responsible for the acceleration of the expansion presently observed in our universe. There, it was shown that the mass density of the corresponding dark energy in our universe can be identified directly with the mass density $\varrho$ of the expansion field in the multiverse. Accordingly, besides the initial conditions (8) or (19) respectively, at a much later point of time $t_0$, representing the present age of our universe in terms of the time $t$ measured in the multiverse, the further boundary condition

$$\varrho_0 = \varrho(t_0) = 0.683 \varrho_c, \quad \text{with} \quad \varrho_c = \frac{3H_0^2}{8\pi G} = 9.20 \cdot 10^{-27} \text{kg m}^{-3} \quad (29)$$

must be met. Thereby $\varrho_c$ is the critical density and $H_0$ the present Hubble parameter of our universe. Furthermore, for the important ratio $\varrho_0/\varrho_c$ we obtain with Eq. (10) the more
accurate value
\[
\frac{\rho_0}{\rho_\Lambda} = 1.02 \cdot 10^{-122}.
\] (30)

For the evaluation of the boundary condition it is recommended to employ a different subset of dimensionless variables which is better adjusted to the macroscopic scales involved, namely
\[
X = \frac{a}{a_0} = \frac{a}{R \zeta} \quad \text{with} \quad \zeta = \frac{a_0}{R},
\] (31)

where
\[
R = a_0 r_{bo} = 23.5 \cdot 10^9 \text{ly} = 2.22 \cdot 10^{26} \text{m}
\] (32)
is the present metric radius of the boundary of our observable universe (at radial coordinate \(r_{bo}\) and for scale factor \(a_0=a(t_0)\)), and
\[
T = \frac{t}{t_{H_0}} \quad \text{with} \quad t_{H_0} = \sqrt{\frac{3}{8\pi G \rho_\Lambda}} = \frac{1}{H_0} = 14.0 \cdot 10^9 \text{a} = 4.41 \cdot 10^{17} \text{s},
\] (33)
where \(t_{H_0}\) is the present Hubble time.

Now we calculate the density \(\rho_0=\rho(t_0)\), employing the approximation (28) for \(x(\tau)\). With this and its consequence \(\dot{x}(\tau)=\gamma x\), from Eq. (20) we get
\[
\dot{\rho} = \rho_\Lambda \gamma^2 + \frac{\rho_\Lambda}{x^2}.
\] (34)

With the relations
\[
\tau = \frac{t_{H_0}}{t_P} T, \quad x = \frac{R \zeta X}{l_P}
\] (35)
following from Eqs. (11)-(12), (31) and (33), and inserting numbers by using Eqs. (10), (12), (29) as well as Eq. (32), Eq. (34) becomes
\[
\frac{\rho}{\rho_0} = \frac{\rho_\Lambda}{\rho_0} \gamma^2 + \frac{0.522}{\zeta^2 X^2}.
\] (36)

(Note that in the calculation of the second term, the exponents of the extremely large value \(\rho_\Lambda/\rho_0=0.98\cdot10^{122}\) and the extremely small value \((l_P/R)^2=0.53\cdot10^{-122}\) cancel each other out quite naturally.) From this, for \(T=T_0\) with \(X(T_0)=1\) according to Eq. (31), and restricting ourselves to sufficiently large values \(\zeta \gtrsim 10^2\) (such that \(1/\zeta^2 \lesssim 10^{-4}\)), we obtain
\[
\gamma = \sqrt{\frac{\rho_0}{\rho_\Lambda}} = 1.01 \cdot 10^{-61}.
\] (37)
Inserting this result in Eqs. (34) and (36) yields
\[ \varrho = \varrho_0 + \varrho_0 \frac{\Lambda}{x^2} = \left(1 + \frac{0.522}{\zeta^2 X^2}\right) \varrho_0. \] (38)

It follows immediately that \( \varrho'(x) < 0 \), a condition needed in Sec. IV. However, we have to investigate still more closely the immediate neighborhood of \( x=1 \) or \( \tau=0 \) respectively, since the approximation (28), used for deriving Eq. (34), does not satisfy \( \dot{x}(\tau)|_{\tau=0} = 0 \) as required. For this purpose, we must go back to the exact equation (20), and insert in it \( \dot{x}(\tau) \) as obtained from Eq. (26), getting

\[ \frac{\varrho}{\varrho_0} = \frac{1}{x^2} + \gamma^2 \left( \frac{1 - e^{-\gamma(\gamma+1)/\gamma}}{1 + \gamma^2 e^{-\gamma(\gamma+1)/\gamma}} \right)^2 = \frac{1}{x^2} + \mathcal{O}(10^{-122}). \] (39)

Accordingly, we have with high precision \( \varrho'(x) = -2 \varrho_0/x^3 < 0 \) at and around \( x=1 \).

Using Eqs. (13a), (28) and (34), the resolution of Eq. (21) with respect to \( w \) yields

\[ w = -1 + \frac{2}{3 (1 + \gamma^2 x^2)} = -1 + \frac{2}{3 (1 + 1.92 \zeta^2 X^2)}, \] (40)

where in the last step Eq. (35b) and \( (\gamma R/l_P)^2 = 1.92 \) according to Eqs. (12), (32) and (37) was used.

### C. Relation between \( T \) and \( \zeta \)

Using Eqs. (11), (12), (32), (33) and (37), we obtain from Eqs. (35) for \( \zeta > 10^2 \)

\[ \gamma \tau = 0.826 T \quad \text{and} \quad x = 1.37 \cdot 10^{61} \zeta X. \]

From this and Eq. (28) we get

\[ X(T) = \frac{e^{0.826 T}}{1.37 \zeta} \cdot 10^{-61} \quad \text{or} \quad T = 170.35 + 1.21 (\ln \zeta + \ln X). \] (41)

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4 Eq. (13a) denotes the first of the Eqs. (13), Eq. (13b) the second, etc.
D. Tunneling solution for \( t \leq 0 \)

We assumed that the mass density \( \varrho_{\Lambda} \) emerges unreduced from a creation out of nothing. The latter can be described approximately by a tunneling solution, which means that for \( t \leq 0 \) we can set \( \varrho \equiv \varrho_{\Lambda} \) and must only solve Eq. (1) with the boundary condition \( \dot{a}(t) = 0 \) for \( t = 0 \). In the dimensionless variables (11)-(12) this equation becomes

\[
\dot{x}^2(\tau) = x^2 - 1, \tag{42}
\]

and the boundary condition becomes \( x = 1 \) for \( \tau = 0 \). As in Ref. [1], for \( \tau \leq 0 \) we set

\[
\tau = -iu \tag{43}
\]

with what Eq. (42) becomes

\[
\dot{x}^2(u) = 1 - x^2. \tag{44}
\]

The solution to the boundary condition \( \dot{x}(u) = 0 \) is

\[
x(u) = \cos u. \tag{45}
\]

E. Properties of the solutions

1. Solution \( x(\tau) \)

First, we consider \( x(\tau) \) in the immediate neighborhood of \( \tau = 0 \). For \( \tau \geq 0 \) we must use Eq. (26) and for \( \tau \leq 0 \) Eq. (45), in both cases \( \gamma \) given by Eq. (37). Due to the extreme smallness of \( \gamma \) it is useful to introduce a new variable \( v \) by

\[
\tau = \gamma v \quad \text{for} \quad v \geq 0, \quad u = \gamma v \quad \text{for} \quad v \leq 0. \tag{46}
\]

Expanding Eqs. (26) and (45) with respect to \( \gamma^2 \), with \( e^{-\tau/\gamma} = e^{-v} \),

\[
e^{\gamma \tau} = 1 + \gamma^2 v + \mathcal{O}(\gamma^4) \quad \text{and} \quad \cos(\gamma v) = 1 - \frac{\gamma^2 v^2}{2} + \mathcal{O}(\gamma^4)
\]
FIG. 1. \(\frac{x-1}{\gamma^2}\) depending on \(v=\tau/\gamma\) in the immediate neighborhood of \(\tau=0\)

\[
\frac{x-1}{\gamma^2} = \begin{cases} 
v + e^{-v} - 1 + \mathcal{O}(\gamma^4) & \text{for } v \geq 0 \\
-v^2/2 + \mathcal{O}(\gamma^4) & \text{for } v \leq 0 \end{cases}
\]

This result is represented graphically in FIG. 1.

Plotting \(x\) over \(\tau\) or \(u\) respectively gives a totally different picture, which is shown in FIG. 2. According to Eqs. (26) and (45), we have

\[
x(u) = \begin{cases} 
0 & \text{for } u = -\pi/2 \\
1 & \text{for } u = 0
\end{cases}, \quad x(\tau) = \begin{cases} 
1 & \text{for } \tau = 0 \\
1+\mathcal{O}(10^{-61}) & \text{for } \tau = \pi/2
\end{cases},
\]

which means, that \(x(\tau)\) remains extremely close to its initial value \(x=1\). In order to determine for how long this will be, we calculate the time from when on \(x\) deviates more than 1 percent from \(x=1\). From Eqs. (11), (28), (33) and (37) we get

\[
\tau = \frac{\ln 1.01}{\gamma} = 0.985 \cdot 10^{59} \quad \text{or} \quad t = 5.31 \cdot 10^{15} \text{ s} = 1.20 \cdot 10^{-2} t_H = 1.69 \cdot 10^{8} \text{ y}.
\]
FIG. 3. \(x(\tau)\) depending on \(v=\gamma \tau=0.83 \mathcal{T}\) in the wider neighborhood of \(\tau=0\)

Only after this very long time \(x\) deviates notably from 1.

By plotting \(x\) over \(v=\gamma \tau\) for \(v\geq 0\) and \(v=\gamma u\) for \(v\leq 0\), still another picture is obtained, which is shown in FIG. 3. For \(v\leq 0\) we have \(x(v)=\cos(v/\gamma)\approx \cos(10^6 v)\) whence \(x=0\) for \(v\approx 10^{-61} \pi/2\) and \(x=1\) for \(v=0\), i.e. the interval \(-\pi/2\leq u\leq 0\) has shrunk to an invisible size. On the other hand for \(v\geq 0\) we have \(x(v)=e^{v}\).

2. Solutions \(\mathcal{T}_0(\zeta), \varrho(x)\) and \(w(x)\)

Using \(X=1\) for \(\mathcal{T}=\mathcal{T}_0\), Eq. (41) gives

\[
\mathcal{T}_0 = 170.35 + 1.21 \ln \zeta \quad \text{and} \quad \mathcal{T} = \mathcal{T}_0 + 1.21 \ln X. \tag{48}
\]

For \(\zeta \geq 10^2\), required for reasons of simplicity in the derivation of Eq. (37), we obtain from this that \(\mathcal{T}_0 \geq 176\); furthermore, \(\mathcal{T}_0=178.7\) for \(\zeta=10^3\) and \(\mathcal{T}_0=181.5\) for \(\zeta=10^4\). As in the case of the solutions obtained in Ref. [1], the condition that our universe should fit into the multiverse both time- and space-wise, can easily be satisfied.

For \(x \geq 1\), the density \(\varrho\) decays according to Eq. (39); for \(x \gg 1\), it decays according to Eq. (38) what is shown in FIG. 4 for \(X \lesssim 1\) and several values of \(\zeta\). It is seen that \(\varrho\) approaches its present value already for \(X\)-values significantly below 1. In particular we have

\[
\frac{\varrho}{\varrho_0} - 1 \leq 10^{-2} \quad \text{for} \quad X \geq \begin{cases} 0.72 \cdot 10^{-1} \equiv \mathcal{T} = \mathcal{T}_0-3.2 \quad \text{when} \quad \zeta = 10^2, \\ 0.72 \cdot 10^{-2} \equiv \mathcal{T} = \mathcal{T}_0-6 \quad \text{when} \quad \zeta = 10^3, \end{cases} \tag{49}
\]

where at last Eqs. (48) were used. During the lifetime of our universe, i.e. for \(\mathcal{T}_0-0.98\leq \mathcal{T} \leq \mathcal{T}_0\), the density \(\varrho\) changes still much less, according to Eqs. (38) and (41) only by \(\Delta \varrho \approx 10^{-4} \varrho_0\)
for $\zeta=10^2$ and $\Delta \rho \approx 10^{-6} \rho_0$ for $\zeta=10^3$.

According to Eqs. (40), $w$ starts essentially with $w=-1/3$ at $x=1$ and converges to $w=-1$ for $x \to \infty$. The present value, obtained for $X=1$, is given by

$$w_0 = -1 + \frac{2}{3(1+1.92\zeta^2)}.$$  

From this follows

$$|w_0 + 1| \leq \begin{cases} 
3.5 \cdot 10^{-5} & \text{for } \zeta = 10^2 \\
3.5 \cdot 10^{-7} & \text{for } \zeta = 10^3,
\end{cases}$$

i.e. up to a very small error we have $w_0=-1$ for all $\zeta$-values considered (FIG. 5). This is essentially the same value as the one obtained from the cosmological constant model, which according to Ref. [23] has ”so far the best performance in fitting the observational data” (for data see e.g. Ref. [4] or Ref. [24]). According to Eqs. (40) and (41), as time goes by, our model is approaching closer and closer to the cosmological constant model so that it is virtually indistinguishable from it during the lifetime of our universe.
3. Curvature parameter $\Omega_k$

As already stated in the Introduction, the spatial curvature measured in our universe is very small. The parameter commonly used for its representation (see e.g. p. 41 of Ref. [25] or p. 491 of Ref. [26]) is

$$\Omega_k = - \frac{k c^2}{a_0^2 H_0^2} = - \frac{c^2 t_0^2}{R^2 \zeta^2} = - 0.36 \zeta^2,$$

(50)

where in the last two steps $X_0=1$ and Eqs. (31)-(33) were used. According to the Planck 2015 results [4], deduced from observations of CMB radiation anisotropies, $|\Omega_k| < 0.5 \cdot 10^{-2}$ must apply. From this, with use of Eq. (50) we obtain the condition

$$\zeta > 8.4,$$

which is easily complied with by our assumption $\zeta \gtrsim 10^2$ made for Eq. (37).

4. Energy expenditure for the information transfer

It is interesting to calculate the energy expenditure resulting from the friction term in Eq. (3) and used for information transfer to newly created spatial patches. We do this in dimensionless variables. The total positive energy of the multiverse at the expansion $a(t)$ is

$$E = \varrho(a) V c^2 \quad \text{with} \quad V = \delta a^3,$$

where $V$ is the total volume and $\delta$ is a dimensionless parameter of no further interest here. Using Eq. (13a), we now define a dimensionless energy

$$\tilde{E} = \frac{E}{\varrho \Lambda \epsilon^2 l_p^3} = \rho \tilde{V} \quad \text{with} \quad \tilde{V} = \delta x^3.$$

(51)

At the transition from $x$ to $x+\Delta x$ the energy changes by

$$\Delta \tilde{E} = \delta (\rho'(x) x^3 + 3 \rho(x) x^2) \Delta x = 3 \delta (\gamma^2 x^2 + 1/3) \Delta x,$$

(52)
where at last Eqs. (13a) and (34) have been used. Simultaneously, the total energy $\tilde{E}_\Lambda = \delta x^3$ provided by $g \equiv g_\Lambda$ or $\rho \equiv 1$ respectively changes by

$$\Delta \tilde{E}_\Lambda = \Delta \tilde{V} = 3 \delta x^2 \Delta x. \quad (53)$$

The energy expenditure in question is

$$\Delta \tilde{E}_{ex} = \Delta \tilde{E}_\Lambda - \Delta \tilde{E} = 3 \delta (x^2 - 1/3 - \gamma^2 x^2) \Delta x \approx 3 \delta (x^2 - 1/3) \Delta x. \quad (54)$$

(Note that in the total energy balance $\Delta \tilde{E}_{ex}$ appears as a contribution to the negative energy of the gravitational field.) Set in relation to the total available energy $\Delta \tilde{E}_\Lambda$ or the energy $\Delta \tilde{E}$ supplied to the new spatial patches, we have

$$\frac{\Delta \tilde{E}_{ex}}{\Delta \tilde{E}_\Lambda} = 1 - \frac{1}{3 x^2} \quad (55)$$

or

$$\frac{\Delta \tilde{E}_{ex}}{\Delta \tilde{E}} = \frac{x^2 - 1/3}{\gamma^2 x^2 + 1/3} \approx \begin{cases} 3 x^2 - 1 & \text{for } x \ll 1/\gamma \\ 1/\gamma^2 & \text{for } x \gg 1/\gamma \end{cases} \quad (56)$$

respectively. According to Eq. (55), $\Delta \tilde{E}_{ex}$ differs from $\Delta \tilde{E}_\Lambda$ for $x=10$ by less than 1 percent, and with increasing $x$ by still much less. This means that almost all available energy is used for information transfer over almost the entire range of $x$. For the energy $\Delta \tilde{E}$, Eq. (56) gives $\Delta \tilde{E} \approx \Delta \tilde{E}_{ex}/2$ for $x=1$ and

$$\Delta \tilde{E} \geq \gamma^2 \Delta \tilde{E}_{ex} \approx \gamma^2 \Delta \tilde{E}_\Lambda$$

for larger $x$, where for increasing $x \gg 1/\gamma$ the lower boundary $\gamma^2 \Delta \tilde{E}_\Lambda$ is approached more and more closely.

5. Bekenstein-like bound

It can be expected that for the storage of the information contained in the physical laws etc. also some kind of Bekenstein bound exists. For its form of appearance certainly plays a role, whether or not the dark energy of the multiverse is involved in the information storage. In the case of no involvement, our simple model provides no possibility for any determination. We therefore turn immediately to the case of an existing involvement.
For the reasons given in the Introduction, it makes no sense to estimate the amount of information I to be stored. Assuming, however, that to all new space-time patches the same amount of information will be transmitted, we can set (immediately in dimensionless variables)

$$\Delta \tilde{I} \sim \Delta \tilde{V}. \quad (57)$$

The original Bekenstein bound is 

$$I \leq CR E$$

where I = stored information, C = constant, R = radius of a sphere into which the storage device fits completely, and E = mass of the storage device times c^2. Replacing I with \(\Delta \tilde{I}\), R with x, E with \(\Delta \tilde{E}\), and using Eq. (57) the Bekenstein bound is transferred into

$$\Delta \tilde{V} \leq \tilde{C} x \Delta \tilde{E},$$

where \(\tilde{C}\) is an unknown number. By substituting Eqs. (52) and (53), from this we get

$$\frac{\Delta \tilde{V}}{x \Delta \tilde{E}} = \frac{x}{\gamma^2 x^2 + 1/3} \leq \tilde{C}. \quad (58)$$

For \(x = 1/(\gamma \sqrt{3})\) the left side of this inequality assumes a maximum with the value \(\sqrt{3}/(2\gamma)\), which means, that \(\tilde{C}\) may not be smaller than this. Thus, we finally arrive at the result that a Bekenstein-like bound of the form (58) can be met, if \(\tilde{C}\) is correspondingly large, i.e.

$$\frac{\Delta \tilde{V}}{\Delta \tilde{E}} \leq \frac{\sqrt{3} x}{2\gamma}. \quad (59)$$

As discussed in paragraph (3) of Section IV, there is still some flexibility with respect to the parameter \(\gamma\) so that even a lower upper limit could be met.

A similar transfer of the Bremermann limit does not seem possible because the volume of the device for information storage would have to be constant over time. But that does not change the fact that, as assumed in the Introduction, there will certainly exist a speed limit to the information transfer considered by us.
IV. INTERPRETING $x(\tau)$ AS A SOLUTION OF THE FL-EQUATION FOR A SCALAR FIELD $\varphi$

In Ref. [1] it was shown, that to any prescribed density $\varrho(x)$ with $\varrho'(x) \leq 0$ (a condition necessitated by Eq. (60) below and shown to be satisfied by our present results at the end of Subsection III.B) functions $\varphi(x)$, $v(\varphi)$ and $x(\tau)$ exist such that Eqs. (15)-(17) are satisfied. To show that in the present case these are meaningful solutions, we derive them now explicitly for the density (34). Here, Eq. (23b) of Ref. [1] assumes the form

$$\dot{\varphi}(\tau) = \sqrt{-\frac{x \varrho'(x)}{3 \varrho_\Lambda}}$$

and can be obtained by differentiating Eq. (16a) with respect to $\tau$ and inserting Eq. (17). Combining Eqs. (15) and (60) we get

$$\frac{d\varphi}{dx} = \frac{\dot{\varphi}(\tau)}{\dot{x}(\tau)} = \sqrt{-\frac{x \varrho'(x)}{3 (\varrho x^2 - \varrho_\Lambda)}},$$

while resolving Eq. (16a) with respect to $v$ and inserting the square of Eq. (60) yields

$$v = \frac{\varrho}{\varrho_\Lambda} + \frac{x \varrho'(x)}{6 \varrho_\Lambda}.$$  

Inserting Eq. (34) in Eqs. (61) and (62), and using Eq. (37) gives

$$\frac{d\varphi}{dx} = \frac{1}{x^2} \sqrt{\frac{2 \varrho_\Lambda}{3 \varrho_0}}, \quad v = \frac{\varrho_0}{\varrho_\Lambda} + \frac{2}{3 x^2}$$

and

$$\varphi(x) = \sqrt{\frac{2 \varrho_\Lambda}{3 \varrho_0}} \left(1 - \frac{1}{x}\right) = \sqrt{\frac{2 \varrho_\Lambda}{3 \varrho_0}} \left(1 - e^{-\gamma x}\right).$$

Thereby, an integration constant was chosen such that $\varphi(1)=0$, and at last Eq. (28) was used. Using $h=\dot{\varphi}(\tau)/x=\gamma$ according to Eq. (28), from this we get

$$\left|\frac{\ddot{\varphi}(\tau)}{3h \dot{\varphi}(\tau)}\right| = \frac{1}{3},$$

which means, that $\varphi(x(\tau))$ is no slow roll solution.

Resolving Eq. (63) with respect to $x$ and inserting the function $x(\varphi)$ thus obtained in our
last result for \( v \), with use of Eq. (37) we finally obtain

\[
v(\varphi) = \frac{\rho_0}{\rho_\Lambda} + \frac{2}{3} \left( \sqrt{\frac{3}{2}} \frac{\rho_0}{\rho_\Lambda} \varphi - 1 \right)^2 = \gamma^2 + \frac{2}{3} \left( \frac{3}{2} \gamma \varphi - 1 \right)^2.
\] (64)

This is a parabolic potential assuming the extremely small minimum \( v_{\text{min}} = \gamma^2 \) at \( \varphi = \sqrt{2/3}/\gamma \). According to Eq. (63), \( \varphi \) starts with the value 0 at the time \( \tau = 0 \) and reaches the minimum only after an infinite time, \( \tau \to \infty \). This is due to \( v_{\text{min}} \neq 0 \) because in a parabolic potential with vanishing minimum, \( \varphi \) would reach it after a finite time and would oscillate around it. It is very appropriate that such oscillations are avoided because they would spoil our multiverse concept. Note that in our model \( v_{\text{min}} \neq 0 \) is no arbitrary assumption but comes about inevitably through the underlying concept.

V. SUPPLEMENTS

1. Let us return for a moment to the effect which the inclusion of matter with mass density \( \rho_m \) would have according to Ref. [1]. Here, too, by an appropriate mix of the initial densities \( \rho \) of dark energy and \( \rho_m \) of matter it could be achieved, that the initial state is an equilibrium – albeit an unstable one [27]. The multiverse could thus remain still longer close to the latter than it would do according to FIGs. [13]. When it finally moves away, \( \rho_m \sim x^{-n} \) becomes, very soon, much smaller than \( \rho \sim x^{-2} \) (see Eq. (38); \( \rho_m / \rho \leq 10^{-2} \) for \( x \geq 10 \) if \( n = 4 \) and for \( x \geq 100 \) if \( n = 3 \)). For the largest part of the evolution \( \rho_m \) can therefore be neglected.

2. During the whole lifetime of our universe, i.e. for \( T_0 - 0.98 \leq T \leq T_0 \), the equation \( \rho = \rho_0 = \text{const} \) applies with high precision. Furthermore, by choosing \( \zeta \) sufficiently large, according to Eq. (50) the spatial curvature can be made so small that within our universe there are no measurable differences to flat space. Added together, this leads to an evolution of our universe just as in the cosmological standard model with small cosmological constant. The proof of this is not trivial considering the differences in the underlying space-time. However, because there are no relevant differences from the case considered in Ref. [1], it suffices to refer to the proof given there in Section 3.2. The same holds for the repercussion of our universe on the multiverse (there
3. The value $\approx 10^{122}$ of the ratio $\varrho_\Lambda/\varrho_0$ used in this paper is not unalterably fixed but could be adjusted to other theoretical values within several orders of magnitude. This can be achieved by imposing the initial conditions [8], required for the continuous connection to a tunneling process, at a value $a_i \gg l_P$. Choosing for example $a_i = 10 l_P$ or $a_i = 10^{16} l_P$, according to Eq. [9] this would yield $\varrho_\Lambda/\varrho_0 \approx 10^{120}$ or $\varrho_\Lambda/\varrho_0 \approx 10^{90}$ respectively. Here, the initial state of the multiverse would still be far in the quantum regime, and the approximate treatment of the initial tunneling process by continuation of the classical solution for $\tau \geq 0$ to imaginary values $u = i \tau$ for $\tau \leq 0$ is then justified for an even wider range of $u$ values. A further aspect may call for $a_i \gg l_P$: For encoding all information about the physical laws, similar to the case of the DNA, a certain minimum initial volume $V = 2\pi^2 a_i^3$ could be required which for $a_i = l_P$ might be too small.

4. The ansatz [3] can be extended to contain nonlinear friction terms such as $-g \dot{a}^2(t)$ with constant $g$. The disadvantage involved is a non-linearity of the differential equation to be solved. An expansion with respect to $g$ would offer itself for handling this problem.

It can also be shown, that solutions $x(\tau), \varphi(\tau)$ of Eqs. [15]-[17] of the standard theory with given potential $v(\varphi)$ satisfy Eq. [18] with $2\dot{\sigma} \dot{x}(\tau)$ replaced by $f(x) \dot{x}(\tau)$, i.e.

$$\ddot{x}(\tau) + f(x) \dot{x}(\tau) - x = 0.$$  \hspace{1cm} (65)

From this equation we get

$$\dot{x}^2(\tau) = x^2 - 2 \int_0^\tau f(x(\tau')) \dot{x}^2(\tau') d\tau'$$  \hspace{1cm} (66)

in the same way as Eqs. [4]-[5] are obtained from Eq. [3]. This equation must coincide with Eq. [15], why we must have

$$2 \int_0^\tau f(x(\tau')) \dot{x}^2(\tau') d\tau' = - \left( \frac{\varrho}{\varrho_\Lambda} - 1 \right) x^2.$$  

Differentiating this equation with respect to $\tau$, dividing it by $\dot{x}(\tau)$ and resolving it
with respect to \( f(x) \) yields

\[
f(x) = -\frac{xg'(x)/2+g(x)-\varrho\Lambda}{\varrho\Lambda\sqrt{g(x)/\varrho\Lambda-1/x^2}} = -\frac{x\rho'(x)/2+\rho(x)-1}{\sqrt{\rho(x)-1/x^2}}.
\] (67)

Thereby we used Eq. (13a), (15) and \( \dot{\varrho}\Lambda=\varrho\Lambda\dot{x} \). The function \( \varrho(x) \) is obtained from Eq. (16a) by inserting in it \( \tau=x(\tau) \), the inverse function of the solution \( x(\tau) \). By resolving Eq. (15) or \( x^2\rho(x)=1+x^2(\tau) \) for \( x \) and inserting the result \( x=g(\dot{x}) \) in \( f(x) \), Eq. (65) can even be brought into the form

\[
\ddot{x}(\tau)+h(\dot{x}(\tau))-x=0 \quad \text{with} \quad h(\dot{x}(\tau))=\dot{x}(\tau)f(g(\dot{x})).
\] (68)

5. The results of Section IV enable a covariant generalization of Eq. (4) with (5) and thus also indirectly of the ansatz [3]. The covariant contribution of \( \varrho\Lambda \) to the field equations is \( \Lambda g_{\mu\nu} \), where \( \Lambda=8\pi G\varrho\Lambda/c^2 \) (see e.g. p. 394 of Ref. [26]). Concerning the total density \( \varrho=\varrho_f+\varrho\Lambda \) or rather \( \rho=\varrho_f/\varrho\Lambda+1 \), we take advantage of the fact that, using Eq. (16a), it can be represented by the scalar field \( \varphi(x) \) of Eq. (63) and the potential \( v(\varphi) \) of Eq. (64). (Eq. (18) follows from this by use of Eqs. (67), (65), (36) and (27).)

In the Einstein field equations a (dimensioned) scalar field \( \Phi \) is covariantly represented by the energy-momentum tensor (see e.g. p. 527 of Ref. [19] or p. 549 of Ref. [26])

\[
T_{\mu\nu} = \frac{h}{\mu}(\partial_{\mu}\Phi)(\partial_{\nu}\Phi) - \left[ \frac{h^2}{2\mu}(\partial^\Lambda\Phi)(\partial_\Lambda\Phi) - V(\Phi) \right] g_{\mu\nu}.
\]

Under the symmetries of the cosmological principle and in Robertson-Walker coordinates, from this follows

\[
\frac{T_{00}}{c^2} = \varrho,
\]

where \( \varrho \) is given by Eq. (6a). With this, \( g_{00}=1 \) according to the Robertson-Walker metric, and the definition \( T_{00}^{(f)}/c^2=\varrho_f \) we can write \( \varrho=\varrho_f+\varrho\Lambda \) in the form

\[
\frac{T_{00}}{c^2} = \frac{T_{00}^{(f)}}{c^2} + \varrho\Lambda g_{00}.
\] (69)

Now we define

\[
T_{\mu\nu}^{(f)} = T_{\mu\nu} - \varrho\Lambda c^2 g_{\mu\nu}.
\] (70)
Because $T_{\mu\nu}$ and $g_\Lambda c^2 g_{\mu\nu}$ are tensors, this is also true for their difference $T^{(f)}_{\mu\nu}$. Since under given circumstances Eq. (70) reduces to Eq. (69), it is the covariant generalization of the latter. The corresponding covariant field equations also include a generalization of Eq. (3), although the kind of representation is different. (In an analog representation, the place of $\dot{a}(t)$ should be taken by $g_{\mu\nu;\lambda}$ rather than by an expression involving $\Phi$.) It is, however, questionable whether under lower symmetries the corresponding field equations allow for a similar simple interpretation as that given by Eq. (18), (65), or (68).

VI. CONCLUSIONS

As already indicated in the Introduction, the main ideas of this paper are speculative. This means, that they may be far away from reality, contain at least some truth, or constitute a step in the right direction. In any case, the solutions following from them turn out particularly well suited for generating an all-embracing multiverse which provides the dark energy of our universe and thus explains its presence there. At the end of the Introduction and in Sec. IV it was shown that they are also valid solutions of the cosmological standard theory and may be considered as a re-interpretation of the latter. What is thus a vindication on the one hand is a disadvantage for detecting the value of the underlying concept on the other, since the solutions themselves are not appropriate for a discrimination between the two interpretations. To accept the conception of a dark energy field with huge mass density $\varrho_\Lambda$, increasingly weakened by some agent – information transfer in our model –, may thus be considered as a question of taste.

The above-mentioned special usefulness of our new solutions for the purposes pursued in Ref. [1] was not foreseeable at first, because their mathematical structure is already completely determined through the new conceptual assumptions. Within the standard theory they could not be derived without some rather artificial assumptions, what may be regarded as some support to the newly added ideas. The solutions are very simple, and through the relation $\gamma=\sqrt{\varrho_0/\varrho_\Lambda}$ they depend in a very transparent way on the enormous discrepancy between $\varrho_0$ and $\varrho_\Lambda$ on which our new concept is based. Furthermore, they are easy to handle in spite of their dependence on the extremely small parameter $\gamma$ which allows for the very simple and nevertheless exceedingly good approximation (28). In the framework of the
cosmological standard theory they avoid oscillations around the necessarily non-vanishing minimum of their quadratic potential \( v(\varphi) \). During the whole lifetime of our universe the equation \( w = p/(\varrho c^2) = -1 \) is almost exactly satisfied and thus provides the best possible fit to observational data. The reason is, that, in spite of marked differences in the distant past, the solutions cling from today’s perspective already for quite some time very tightly to those of a cosmological constant model with \( \varrho = \varrho_0 \).

In contrast to the cosmological standard model assuming flat space, our model employs a finite space with positive curvature. The latter can be chosen so small that within our universe there are no measurable differences. Furthermore, during the lifetime of our universe the changes of the dark energy density \( \varrho \) are immeasurably small. Therefore, our model provides the same internal evolution of our universe as the cosmological standard model with cosmological constant. On the other hand, further outside the observable region, there is a significant difference: While there is no reason for spatial expansion in the causally disconnected external regions of flat space, the corresponding regions in the positively curved space of our model experience a strong and continuously increasing spatial expansion. Aside from that, as in the models of Ref. [1] the dark energy density \( \varrho \) is a monotonic function of the spatial curvature and is different from zero at all finite times. In consequence, the space-time spanned by the multiverse is firmly bound to the presence of matter or energy and thus fulfills an important consequence of Mach’s principle [28], a fact which does not apply to a flat-space multiverse.

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