Compatible Graphs on Traffic Lights Waiting Time Optimization

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ABSTRACT

Congestion at a crossroads caused by improper traffic light waiting times, can create overcrowding in some branches of the road. Therefore, in this article, we will determine the optimal waiting times of traffic lights at a crossroads by using compatible graphs so that the congestion can be reduced. The traffic flows at an intersection can be modeled into a compatible graph. A vertex represents a traffic flow from/to a road connected to the crossroads. Two vertices in a compatible graph are connected by an edge, if and only if the two flows as representations of these two vertices can move simultaneously without causing crossing. By applying various manipulations on a compatible graph model, some systems with simpler traffic flows are obtained, where each system corresponds to a simpler compatible graph. The next process is to determine all spanning subgraph possibilities of the simpler compatible graph, then select spanning subgraphs containing the least number of cliques. By using 60 seconds assumption in a cycle, we obtain the time needed for each traffic flow to move. Waiting time is a product of the time needed for each flow to move and the number of flows contained in the system. By choosing the most effective waiting time of all the systems, the optimal waiting time is obtained. Pasteur crossroads in Bandung is taken as a case in this discussion. Based on secondary data from the Bandung City Department of Transportation, the total average waiting times at this crossroads every hour is about 282 seconds on working days. Meanwhile, according to the compatible graph application, an optimal total waiting time of 135 seconds is obtained.

Keywords: compatible graph, optimal waiting time, traffic light

1. INTRODUCTION

Some policies to overcome the problem of traffic congestion have been carried out, including installing Alat Pemberi Isyarat Lalu Lintas (APILL) and setting the waiting time for traffic lights at the intersection manually. However, policies in regulating traffic lights have not been optimal due to the increasing volume of vehicles at the intersection. This paper is providing a solution by the application of compatible graphs in determining the optimal waiting time for traffic lights to reduce congestion at a crossroads. The traffic flows at an intersection can be modeled into a compatible graph. A vertex represents a traffic flow from/to a road connected to the crossroads. Two vertices in a compatible graph are connected by an edge, if and only if the two flows as representations of these two vertices can move simultaneously without causing crossing. By manipulating traffic flow conditions and using 60 seconds assumption in a cycle, we obtain the time needed for each traffic flow to move. Waiting time is a product of the time needed for each flow to move and the number of flows contained in the system.

Pasteur crossroads in Bandung is taken as a case in this discussion. This research uses the waiting times secondary data from Bandung City Department of Transportation. In the statistical review by using the one-way ANOVA analysis, the normality and homogeneity tests of waiting times data from this department is normally distributed. It has also a homogeneous variance, since the probability value is greater than 0.05 (1). By these two tests, which can be seen in Tables 1 and 2, all of the Sig (probability) values on Shafiro-Wilk are greater than 0.05. So, the waiting times data can be seen from its average. The average waiting times at this crossroads every hour is about 282 seconds on working days. Meanwhile,
according to the compatible graph application, an optimal total waiting time of 135 seconds is obtained.

### Table 1 Normality Test of Waiting Time Data

| Street Name | Kolmogorov-Smirnov\(^a\) | Shapiro-Wilk |
|-------------|--------------------------|--------------|
| Waiting Time |                          |              |
| Toll-Pasteur | 0.209                    | 0.992        |
| Surya Sumantri | 0.246                    | 0.970        |
| Dr. Djundjunan | 0.243                    | 0.972        |
| Sukaraja    | 0.246                    | 0.970        |

\(^a\): Lilliefors Significance Correction

### Table 2 Homogeneity Test of Waiting Time Data

| Levene Statistic | df1 | df2 | Sig.   |
|------------------|-----|-----|--------|
| 0.013            | 3   | 8   | 0.998  |

The graph used in this study is a simple and finite graph. A complete graph with \(n\) vertices \(K_n\) is a simple undirected graph with \(n\) edges and each different pair of vertices connected by an edge. The maximum degree of \(G\) is denoted by \(\Delta(G)\), where \(\Delta(G) = \max\{d(v) | v \in V(G)\}\) (2). A graph \(H\) is a subgraph of a graph \(G\), denoted by \(H \subseteq G\), if \(V(H) \subseteq V(G)\) and \(E(H) \subseteq E(G)\). If \(H\) is a subgraph of \(G\), then \(G\) is called a supergraph of \(H\). If \(V(H) = V(G)\), then \(H\) is a spanning subgraph of \(G\) (3). A clique of graph \(G\) is a complete subgraph of \(G\) (4). A compatible graph is a graph that contains vertices as objects to be arranged and edges that represent pairs of compatible objects (5). The cliques of a compatible graph are the maximum complete subgraphs of the compatible graph (6).

Previous researchers have provided some results regarding the optimal waiting time for traffic lights. The optimal waiting time of the Kaligarang crossroads in Semarang is 30.8% of the existing waiting times (7) and the optimal waiting time of Tuanku Tambusai crossroads in Pekanbaru is 26.9% of the existing waiting times (8). By using a similar method, the optimal waiting time has been obtained for a crossroads in Surabaya (5). By considering the weight of the current flow, a mathematical model has been constructed to optimize the waiting times of the Darmo crossroads in Surabaya (9). However, the optimal waiting time at the Pasteur crossroads in Bandung has not been studied. The crossroads has usually heavy congestion because it is an access to go in and out of the city of Bandung. Therefore, this research has the novelty of data and location, as well as contributing to the community.

### 2. OBJECTIVES

The purpose of this study is to determine the optimal waiting time of traffic lights to reduce congestion at a crossroads. So far, the adjustment of waiting times or duration of red light and green light is only based on the estimation of the authorities in this case the transportation department, and is not based on the results of the study. Therefore, the results of this research can be used as a reference in controlling traffic flow using traffic lights, especially at Pasteur intersection.

### 3. METHODOLOGY

The research began with some observations on the flow of traffic at an intersection. Observed data include the number of existing traffic flows and the waiting time for traffic lights. The flow of traffic is identified which flows are crossing and which are not crossing if moving simultaneously. The waiting time is the sum of the duration of the red light and green light when it is on. To obtain the primary data, some observations are taken at least three times, morning, afternoon, and evening on weekdays and holidays. However, it is hard to obtain the primary data. Therefore, we use the secondary data. The secondary data is obtained from Bandung City Department of Transportation. They have hourly waiting times data for 17 hours a day, from 06:00 to 23:00. The average hourly waiting times on working days can be seen in Table 3.

### Table 3 Existing Waiting Time in Working Days

| Light       | Toll-Pasteur | Surya Sumantri | Dr. Djundjunan | Sukaraja |
|-------------|--------------|----------------|----------------|----------|
| Red Light   | 198          | 207            | 75             | 207      |
| Yellow Light| 3            | 3              | 3              | 3        |
| Green Light | 85           | 74             | 207            | 74       |

Department of Transportation data on February 2020

This traffic flows above can be modeled into a compatible graph, which can be seen in the following figure.
Traffic flows \(a, b,\) and \(c\) are the flows moving from Sukaraja road. Traffic flows \(d, e, f,\) and \(m\) are the flows moving from Dr. Djundjunan road. Traffic flows \(g, h,\) and \(i\) are the flows moving from Surya Sumantri road. Traffic flows \(j, k,\) and \(l\) are the flows moving from Toll Pasteur road. By applying manipulation traffic flow on the compatible graph in Figure 1, several other simpler compatible graphs can be obtained. A simpler graph corresponds to a simpler traffic flow system. The next step is to determine all spanning subgraph possibilities of the simpler compatible graph, then select spanning subgraphs containing the least number of cliques. By using 60 seconds assumption in a cycle, we obtain the time needed for each traffic flow to move. Waiting times is a product of the time needed for each flow to move and the number of flows contained in the system. By choosing the most effective waiting time of all the systems, the optimal waiting time is obtained.

We define a smooth flow is a flow that is compatible with all other flows. It can move at any time with zero waiting time. The number of vertices of the graf in Figure 1 is 13 and the maximum degree of this graf is 12. This graf has four vertices of degree 12. Therefore, Pasteur crossroads has four smooth flows, namely \(a, d, g,\) and \(j\). This graf is the most complex system. A simpler system will be obtained by deleting one to all smooth flows, without changing the characteristics of this system.

First, we delete a smooth flow, so that a simpler system is obtained which is shown as a simpler compatible graph in Figure 2A. Then, select spanning subgraphs containing the least number of cliques. For the graph in Figure 2A, the selected spanning subgraphs are shown in Figures 2B and 2C. There are four cliques in every selected spanning subgraph and can be written as a set of vertices, namely \{defgjm, bc, hi, kl\} and \{bcdgj, ef, mi, hj, kl\}. By using 60 seconds assumption in a cycle, the time needed for each traffic flow to move is 15 seconds. Then, the waiting time of this system is 15 times 12 flows (180 seconds).

Second step, we delete two smooth flows. Another simpler compatible graph is shown in Figure 3A, while the selected spanning subgraphs are shown in Figures 3B and 3C. There are also four cliques in every selected spanning subgraph. Then, the waiting time of this system is 15 times 11 flows (165 seconds). Third step, we delete three smooth flows. Another simpler compatible graph is shown in Figure 4A, while the selected spanning subgraphs are shown in Figures 4B and 4C. The waiting time of this system is 15 times 10 flows (150 seconds).

The last step, we delete four smooth flows. Another simpler compatible graph is shown in Figure 5A and the selected spanning subgraph with four cliques is shown in Figure 5B. The waiting time of this system is 15 times 9 flows (135 seconds).
Based on the waiting time obtained in the last four systems, the minimum waiting time is indicated by the last system. Therefore, the most effective (optimal) waiting time is given by the last system. In this system, the compatible graph does not have a smooth flow. It only has one selected spanning subgraph. This selected spanning subgraph contains four cliques, three graphs $P_2$ and a cycle $C_3$. Three graphs $P_2$ represent three pairs of flows $bc$, $hi$, and $kl$, while a cycle graph represents three flows $efm$. Because the selected spanning subgraph contains four cliques and using 60 seconds assumptions in a cycle, so the optimal waiting time is 15 times 9 flows (135 seconds).

4. RESULT

The optimal waiting time of the Pasteur crossroads is 135 seconds. The waiting time is the sum of the duration of the red light and green light when it is on. The traffic flows $bc$ must wait until all the other 7 flows stopped before they can move. Meanwhile, each flow takes 15 seconds to move. Thus, flows $b$ and $c$ cannot move for 15 times 7 seconds (105 seconds). In the other words, flows $b$ and $c$ have a red light duration of 105 seconds and a green light duration 30. Both traffic flows $b$ and $c$ move from Sukaraja road, so that this road has a red light duration of 105 seconds and a green light duration 30 seconds. With a similar explanation, it is obtained that the flows $hi$ and $kl$, as well as $bc$, have the same red light and green light duration. Because both traffic flows $h$ and $i$ move from Surya Sumantri road, while traffic flows $k$ and $l$ both move from Toll-Pasteur road, so that the two roads have a red light duration of 105 seconds and a green light duration 30 seconds. Now, the traffic flows $efm$ must wait until all the other 6 flows stopped. Thus, flows $e$, $f$, and $m$ have a red light duration of 15 times 6 (90 seconds). These traffic flows move from Dr. Djundjunan road, so this road has a red light duration of 90 seconds and a green light duration 45 seconds. The duration of the red light and green light can be seen in Table 4.

|                            | Toll-Pasteur | Surya Sumantri | Dr. Djundjunan | Sukaraja |
|-----------------------------|--------------|----------------|----------------|----------|
| Red Light                  | 105          | 105            | 90             | 105      |
| Yellow Light               | 3            | 3              | 3              | 3        |
| Green Light                | 30           | 30             | 45             | 30       |

5. CONCLUSION

By using the optimization to all traffic flow system possibilities, we obtain the optimal waiting time is 47.9% of the existing waiting time. The strength of this paper is that it uses proven methods for optimal waiting times at many other crossroads. Another strength is to use a system that is in accordance with the existing conditions, which only reduces all left turn flow. Then, the waiting time obtained is not only based on mathematical and statistical calculations, but it is also considering all existing conditions. On the other hand, the weakness of this paper is that it does not involve variables such as the volume of vehicles and pedestrians.

As an alternative, the development of Artificial Intelligence (AI) in solving transportation problems can also be used to reduce congestion. AI can be applied to solve transportation problems, especially in traffic management, traffic safety, public transportation, and urban mobility (10). Furthermore, fuzzy logic control has been implemented to provide intelligence attributes that are sent to the Peripheral Interface Controller (PIC) microcontroller to drive traffic signals as desired. The results obtained indicate a 26% improvement in the overall outcome of traffic management compared to conventional traffic controllers (11).

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