Numerical study of double-diffusive convection in a vertical annular enclosure with a baffle

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Abstract. This paper numerically examines the influence of a circular thin baffle on thermosolutal convection in a vertical annular enclosure. The inner and outer cylindrical walls, and the baffle are retained with different temperatures and concentrations, while the upper and lower boundaries are kept at adiabatic and impermeable. The model equations are solved using an implicit finite difference scheme consisting of ADI and SLOR methods. Numerical simulations are performed to understand the size and position effects of the baffle on the thermosolutal convection and are successfully captured through our results. It has been observed that the baffle size and location has very important role in controlling the thermosolutal convective flow and the corresponding heat and mass transport characteristics. Further, our results are in good agreement with the available benchmark results for limiting cases.

1. Introduction

Based on the past investigations on double-diffusive convection or thermosolutal convection, it is realized that the thermosolutal convective flows are basically examined in bounded and unbounded geometries. Each of the two geometries has many important applications in their own domain. For thermosolutal convection in an enclosure, the direction of two buoyant forces can either enhances or suppresses the thermal and solutal transport rates. In addition, the ratio of thermal and solutal buoyancies known as the buoyancy ratio, an important parameter for double-diffusive convection, has an important role in altering the flow fields and the corresponding thermal and solutal transport rates. Double-diffusive convection is of great interest among the researchers due to its wide applications in natural process and engineering such as dispersion of chemical contaminants through water saturated soil, grain storage installation, metallurgy and oceanography.

Amongst the bounded geometries, the cylindrical annulus designed by two vertical, concentric tubes attracted many researchers due to the sheer existence of this geometry in wide range of engineering and industrial applications. Beji et al. [1] used the control volume method to analyse the thermosolutal convection in a vertical porous annulus. Bennacer et al. [2] studied the influence Darcy number on thermosolutal convection in a porous annulus using Darcy-Brinkmann formulation. Benzeghiba et al. [3] reported the numerical results on heat and mass transport processes in a vertical
annular enclosure formed by two concentric cylinders filled with a fluid saturated porous medium. In a vertical annulus, Chen et al. [4] reported the numerical results on double-diffusive convection and found that for the lower buoyancy ratios, the thermal buoyancy effect dominates the flow and for higher buoyancy ratios the flow is dominated by solutal buoyancy effect. Sankar et al. [5, 6] reported the influence of a discrete thermal and solutal source on thermosolutal convection in a vertical porous annulus. The location of discrete heat and solute source found to control the heat and mass transport inside the porous enclosure in a significant manner.

In a differentially heated enclosure, it is found that the heat and mass transport processes could be controlled effectively by manipulating the flow regimes as the transport processes are highly sensitive to the different flow regimes. Among the available techniques, a simple way to alter the flow regimes in a cavity is by fixing baffles or fins to the heated or cooled walls. Shi and Khodadadi [7] discussed the influence of a fin on buoyancy-driven convection in a rectangular cavity. They found that the flow patterns are modified by the fin, and presence of the fin also acts as an extra heating to the fluid. Convection in a square enclosure with a baffle fixed at the hot wall has been numerically investigated by Tasnim and Collins [8] and found that the presence of baffle increases the heat transfer rate. Bilgen [9] investigated the influence of a thin fin on convective heat transfer in differentially heated square cavity and shown that the heat transfer rate enhances with Rayleigh number, but suppressed with fin length and conductivity ratio. Recently, Khanafer et al. [10] reported natural convection to analyse the effect of a porous baffle fixed to the hot wall of a square enclosure and reported that the presence of porous fin alters the thermal transport and optimum fin location and angle is identified to obtain higher heat transport rate. Based on the detailed literature survey, it has been found that the impact of a circular baffle on double-diffusive convective flow, the associated heat and mass transport characteristics have not been investigated in a vertical annulus. Hence, the influence of baffle length and baffle position on thermosolutal convection in an annular enclosure has been thoroughly investigated.

2. Physical model and basic equations
The physical configuration chosen for the investigation is the vertical annular enclosure designed by an inner cylinder and an outer cylinder with radii \( r_i \) and \( r_o \) as given in Fig.1. Here the inner and outer cylinders have the same center forming an annular region with \( D \) as the annulus width and \( H \) as the height of the annulus. We considered the flow to be two-dimensional and the aspect ratio of the annulus has been taken as unity (\( H = D \)). A highly conductive horizontal baffle of length \( l \) is attached to different locations \( h \) on the inner wall. The horizontal (top and bottom) walls of the annulus are assumed to be thermally insulated, whereas the inner (left) cylinder is preserved at a high temperature and concentration and the outer (right) cylinder is preserved at a lower temperature and concentration. In addition, the temperature and concentration of the baffle is maintained at the same temperature and concentration of the inner cylinder. Further, the laminar flow and constant thermophysical properties assumptions are invoked in this analysis. The thermal and solutal buoyancy forces in the momentum equation are incorporated through the Boussinesq approximation. In addition, the Newtonian fluid is chosen by neglecting the viscous dissipation with the gravity acting in the negative z-direction, and on all solid boundaries no-slip conditions are assumed to be valid. For these assumptions, the equations governing the mass, momentum, energy and species in the dimensionless form are (see Sankar et al. [5, 6]):

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial R} + W \frac{\partial T}{\partial Z} = \nabla^2 T \tag{1}
\]

\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial R} + W \frac{\partial C}{\partial Z} = \frac{1}{Le} \nabla^2 C \tag{2}
\]
\[
\frac{\partial \zeta}{\partial t} + U \frac{\partial \zeta}{\partial r} + W \frac{\partial \zeta}{\partial z} - \frac{U \zeta}{R} = Pr \left[ \nabla^2 \zeta - \frac{\zeta}{R^2} \right] - Pr Ra_i \left[ \frac{\partial T}{\partial R} + N \frac{\partial C}{\partial R} \right]
\]

(3)

\[
\zeta = \frac{1}{R} \left[ \frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R} \frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial Z^2} \right]
\]

(4)

\[
U = \frac{1}{R} \frac{\partial \psi}{\partial Z}, \quad W = -\frac{1}{R} \frac{\partial \psi}{\partial R}, \quad \text{where} \quad \nabla^2 = \frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} + \frac{\partial^2}{\partial Z^2}.
\]

(5)

Figure 1. Physical configuration, co-ordinate system

The following dimensionless variables used in the equations (1) – (5) :

\[
(R, Z) = (r, z)/D, \quad (U, W) = (u, w)/(\alpha / D), \quad t = t^*/(D^2 / \alpha), \quad T = \frac{\theta - \theta_h}{\theta_h - \theta_c}, \quad C = \frac{S - S_h}{S_h - S_c}
\]

\[
P = p/(\rho_0 \alpha^2 / D^2), \quad \zeta = \zeta^*/(\alpha / D^2), \quad \psi = \psi^*/(D \alpha), \quad \text{where} \quad D = r_o - r_i.
\]

The non-dimensional parameters occur in the present study are: \( \lambda = \frac{r_0}{r_i} \) Radius ratio, \( A = \frac{H}{D} \) Aspect ratio, \( \varepsilon = \frac{L}{D} \) Baffle length, \( L = \frac{h}{H} \) Baffle location, \( Le = \frac{\alpha}{\alpha_D} \) Lewis number, \( Pr = \frac{v}{\alpha} \)
Prandtl number, \( N = \frac{Ra_{\infty}}{Ra_T} \). Buoyancy ratio. The heat and mass transport rates are estimated in terms of local and average Nusselt and Sherwood numbers. The local Nusselt (\( Nu \)) and Sherwood (\( Sh \)) numbers are respectively given by \( Nu = -\frac{\partial T}{\partial R} \) and \( Sh = -\frac{\partial C}{\partial R} \). The average Nusselt and Sherwood numbers are then calculated from the relation \( \overline{Nu} = \frac{1}{A} \int_0^A Nu dZ \) and \( \overline{Sh} = \frac{1}{A} \int_0^A Sh dZ \). The initial and boundary conditions in dimensionless form are:

\[
\begin{align*}
t = 0: & \quad U = W = T = C = 0, \psi = \zeta = 0 \\
t > 0: & \quad \psi = \frac{\partial \psi}{\partial R} = 0, T = C = 1; \quad \text{at outer wall and baffle} \\
& \quad \psi = \frac{\partial \psi}{\partial R} = 0, T = C = 0; \quad \text{at inner wall} \\
& \quad \psi = \frac{\partial \psi}{\partial Z} = 0, \frac{\partial T}{\partial Z} = \frac{\partial C}{\partial Z} = 0; \quad Z = 0 \text{ and } Z = A.
\end{align*}
\]

3. Solution methodology
The model equations and the appropriate boundary conditions are numerically solved by developing an in-house code, which is also magnificently validated with the available benchmark solutions. In particular, the two-step Alternating Direction Implicit method is used for the vorticity, energy and species equations. The second-order accurate central differencing approximation has been used for second order derivatives and first order linear derivatives, the nonlinear advection terms are discretized with second upwind difference technique and the forward differencing scheme is used for time derivative. But, the numerical solution of stream function equation is obtained through the SOR method after carefully choosing the relaxation factor. After discretization, the partial differential equations are reduced to tridiagonal matrix form and are solved using the well-known Thomas algorithm. By using the central difference approximation, velocity components are evaluated. The overall Nusselt and Sherwood numbers are determined from the Simpson's rule.

4. Results and discussion
Numerical simulations are carried out to develop the basic information to understand the effects of buoyancy ratio, Lewis number on the thermosolutal convective flow in the annulus geometry with a thin baffle of different lengths placed at different locations. Since the detailed investigation of this problem by considering all parameter ranges is not the main interest, only five main governing parameters, namely the buoyancy ratio (\( N \)), Lewis number (\( Le \)), Rayleigh number (\( Ra \)), baffle length (\( \varepsilon \)) and baffle location (\( L \)) are varied and the remaining parameters are kept at constant. The simulations are discussed through the streamlines, isotherms and isoconcentrations. The rates of thermal and solutal transport in the enclosure are calculated in terms of the average Nusselt and Sherwood numbers.

The influence of buoyancy ratio on streamlines, isotherms and isoconcentrations for three values of \( N (N = -5, 0 \text{ and } 5) \) is presented in the figure 2 for fixed values of \( Ra_T = 10^6, Le = 1, \varepsilon = 0.3, \) and \( L = 0.5 \). These values of \( N \) constitutes three important regimes in the thermosolutal convection flow, namely concentration-driven opposing flow (\( N = -5 \)), thermal-driven flow (\( N = 0 \)) and temperature dominated aiding flow (\( N = 5 \)). For the opposing flow (\( N = -5 \)), the direction of flow, thermal and solutal fields are counter clockwise with a small rotating vortex observed below the baffle. The streamlines are almost parallel above the baffle and the counter rotating cells cover the entire region of the cavity. For the case of pure thermal-driven flow (\( N=0 \)), the flow and thermal fields resembles that
of buoyancy-driven flow and the direction of flow is changed with clockwise rotating vortex appearing near top wall. The isotherms and isoconcentrations appear similar as the influence of solutal buoyancy is absent. Further, as the value of $N$ is raised to 5 units ($N=5$), the flow intensity is very stronger from the collective thermal and solutal buoyancy acting in the same direction as indicated by the magnitude of maximum stream function. The isotherms and isoconcentrations reveal the strong stratified structure forming a thick thermal and solutal boundary layer along the hot and cold walls, and along the baffle.

![Streamlines, Isotherms, Isoconcentrations](image)

**Figure 2.** Effect of buoyancy ratio on streamlines (top), isotherms (middle) and isoconcentrations (bottom). (a) $N= -5$, $\psi_{max} = 46.7$, (b) $N= 0$, $\psi_{max} = 31.4$, (c) $N= 5$, $\psi_{max} = 48.5$.

The effect of buoyancy ratio on the heat and mass transfer rates is an important quantity in double-diffusive convection. Hence, figures 3-5 represents the effect of buoyancy ratio on the average Nusselt number and average Sherwood number for different values of $Ra_T$, baffle size and baffle location by
fixing the Lewis number as $Le = 1$. In particular, the values of thermal Rayleigh number, baffle length, and baffle location are varied respectively as $10^4 \leq Ra_T \leq 10^6$, $0.25 \leq \varepsilon \leq 0.75$ and $0.25 \leq L \leq 0.75$. For Lewis number equals to unity, the thermal and solutal diffusivities are in same proportion in the mixture. As a result the average Nusselt and Sherwood number profiles appear identical for all values of thermal Rayleigh number. However, the thermal and solutal transport rates are significantly varied with the thermal Rayleigh number. A general observation of the results reveals that the higher thermal Rayleigh number produces maximum thermal and solutal transport rates. Also, the total heat and mass transfer rates are higher in aiding flow regimes compared to opposing flow regimes.

**Figure 3.** Influence of Rayleigh number on the average Nusselt and Sherwood numbers for different buoyancy ratio with $Le = 1$, $\varepsilon = 0.5$ and $L = 0.5$.

**Figure 4.** Influence of baffle length on the average Nusselt and Sherwood numbers for different buoyancy ratio with $Le = 1$, $Ra_T = 10^6$ and $L = 0.5$. 
The influence of size and location of baffle on the thermosolutal convection is the main objective of our investigation. Hence, the effect of different combinations of baffle length and baffle position on the heat and mass transfer rates are presented in figures 4 and 5. A thorough analysis of the results reveals that, for all the values of the Ra_T, baffle length and baffle location, the heat and mass transfer rates decreases with the buoyancy ratio for N < - 1, while for N > - 1, the heat and mass transfer rates are increasing with buoyancy ratio. Also it is observed that the heat and mass transfer increases with thermal Rayleigh number and baffle location, however, decreases with baffle length. From the simulations of the present analysis, it is observed that the double-diffusive convective flow and the associated heat and mass transfer characteristics can be effectively controlled by placing baffle of different lengths at various locations of the inner wall. The influence of both Rayleigh number and Lewis number on the average Nusselt and Sherwood numbers is reported in the figure 6 by fixing the values of N, ε and L. It is observed that as the Lewis number increases, the heat transfer decreases and mass transfer increases, which is in good agreement with the predictions of the existing studies.

Figure 5. Influence of baffle position on the average Nusselt and Sherwood numbers for different buoyancy ratio with \( Le = 1 \), \( Ra_T = 10^6 \) and \( \varepsilon = 0.4 \).
5. Conclusions

Double-diffusive convection in a vertical annular cavity is numerically investigated by considering a thin baffle having the same temperature and concentration of the inner cylinder. Flow circulation, thermal and solutal distributions are considerably altered by the baffle size and location. The numerical results reveal that the heat transfer rate increases with the thermal Rayleigh number, but decreases with the Lewis number. However, the mass transfer increases with both the Rayleigh and Lewis numbers. The heat and solute transport increases with an increase in baffle location, and higher heat and mass transport is observed for the baffle location near top of the cavity. The heat and mass transport decreases with the increasing baffle length due to blockage caused by the baffle to the fluid movement. For all thermal Rayleigh numbers, baffle sizes and baffle locations, the heat and mass transfer rates are found to be higher in aiding flow regimes due to combined buoyancy effect. Further, it has been observed that the heat and mass transfer rates can be effectively controlled by varying the size and location of the baffle.

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