Thermal fracture behavior of functionally graded materials – a brief review and some prospects

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Abstract. Functionally graded material (FGM) is a new type of material whose composition and structure vary smoothly in the spatial position, making the properties and functions change continuously. In this paper, the research status on the thermal fracture of FGMs is briefly reviewed, and some future work is also suggested.

1. Introduction

With the roaring growth of modern technology, the demand for material properties in all walks of life is becoming more and more stringent, e.g., materials are required to have high strength and toughness while possessing great heat and corrosion resistance. It is difficult for ordinary single materials to meet these requirements at the same time. To this end, many composite materials emerge. The composite materials made up of two or more materials with different properties by physical or chemical ways can make full use of the advantages of different components to cope with various complex conditions. However, an obvious interface exists in the conventional laminated composites. Therefore, stress concentration is easy to occur in service, resulting in interfacial debonding or even material failure.

Accordingly, to better propel the development of aerospace technology, the functionally graded material (FGM) was put forward by Japanese scientists in 1984 [1]. The design idea of FGM is to employ advanced material composite technology to combine the metal materials of high mechanical strength with the excellent heat-resistant ceramic materials which can withstand thousands of degrees of elevated temperature, making a smooth gradation in the composition and structure of the material; thus, a new type of composite material with graded properties and functions is obtained. In aerospace engineering, FGM is often used as a thermal coating and can effectively reduce residual stress and thermal stress.

Originally exploited as a super-heat-resistant material by astronauts, FGMs have been gradually extended in nuclear industry, energy engineering, biomedicine, optoelectronic engineering, microelectronics and so on, because of their novel designability and excellent performance. Even if FGMs have superior properties, there also inevitably exist various defects (i.e., dislocations, voids, inclusions and cracks) during the process of fabrication or service. The nucleated macro cracks may further propagate or drive the initiation of new cracks, which may reduce the reliability or even cause the failure of the associated structures. Simultaneously, considering that FGMs are widely applied under extreme thermal loads, the investigation of the thermal fracture behaviors of FGMs is of great significance.
2. Selected studies on thermal fracture of FGMs

2.1 Analytical studies

In past years, many efforts have been made using the analytical methods with regard to the thermal fracture behavior of FGMs. Noda and Wang [2] used the singular integral method to investigate the influence of crack spacing on the thermal stress intensity factors (TSIFs) in FGMs. Rangaraj and Kokini [3] adopted an analytical model to look into the thermal fracture behavior of functionally graded (FG) thermal barrier coatings. Guo et al. [4] developed a piecewise-exponential model to explore the TSIFs of an FG plate with a crack normal to the surfaces. Feng and Jin [5] applied a singular integral equation to examine the fracture behavior of an FG plate with parallel surface cracks subjected to a thermal shock. Zhou and Lee [6] analyzed a partial adiabatic interface crack problem in an FGM coating-substrate structure under thermal-mechanical loading based on the singular integral method. Petrova and Schmauder [7] discovered an interface crack problem in an FG/homogeneous bimaterial with an integral equation. Eshraghi et al. [8] obtained the transient TSIFs of FG cylinders using the weight function method. Due to the physical complexities of FGMs and the difficulties in mathematical algorithm resulting from analytical method, it is usually assumed that the gradient distribution of material properties is a specified function (e.g., exponential function), and only simple problems (e.g., infinite or semi-infinite plates) can be considered.

2.2 Numerical studies

Compared with the analytical methods, the numerical tools have stronger adaptability and wide application ranges. Typical computational approaches such as the finite element method (FEM), the boundary element method (BEM), the meshless methods, the extended finite element method (XFEM) and the numerical manifold method (NMM), have been employed to focus on the thermal fracture of FGMs.

The FEM is the most well-known method in computational mechanics and can be traced back to 1940s. The FEM not only has high accuracy, but also can deal with various complex shapes, so it has become an important means of engineering analysis. Ueda [9] calculated the transient TSIFs of an FG divertor plate by the FEM. Yildirim and Erdogan [10] tackled the thermal fracture problem of an FG coating via the FEM. Dag [11] observed the thermal fracture of orthotropic FGMs with the FEM. Kc and Kim [12] utilized the FEM and an interaction integral to compute the T-stress and mixed SIFs in the cracked FGMs. Zamani and Eslami [13] inspected the cracked FGMs problem under thermal shock through the FEM. Anandakumar and Kim [14] reported the mixed-mode crack propagation of 2D FGMs under thermo-mechanical loading using the FEM. Burlayenko et al. [15] conducted the fracture analysis of an FG plate under thermal shock by the FEM. Moghaddam and Alfano [16] integrated the interaction energy integral into the FEM to solve the thermal fracture problem of FG hollow cylinders with surface cracks. When simulating the crack problems using the FEM, the crack surface must coincide with the edge of the element, the crack tip must be arranged with nodes, and the high-density mesh should be adopted in the vicinity of the crack tip. In addition, totally or locally remeshing is required during the crack propagation process.

The BEM is a numerical method developed after FEM. Unlike the FEM discretization, the BEM just needs to discretize the boundary, thus reducing the dimension of the problem and improving the efficiency of solution. Via the BEM, Ekhlakov et al. researched the effect of thermal shock on dynamic SIFs of 2D FGMs [17]. Zheng et al. [18] applied the radial integration BEM to analyze the dynamic fracture mechanics of FGMs under the thermal shock loading. To use the BEM, the fundamental solutions of the problem must be obtained first, but for the nonlinear problem, the acquisition of ones is pretty complicated.

In the meshless methods, a set of points are used to discretize the problem domain, and do not require the mesh to be consistent with the crack in fracture analysis, nor do they involve the remeshing trouble. The most representative methods in meshless family are the element-free Galerkin method (EFG) and the meshless local Petrov-Galerkin method (MLPG). Chen [19] computed the TSIFs of
interface cracks in an orthotropic FGM coating-substrate structure with the EFG and an interaction integral. Feng et al. [20] extended the MLPG to dispose of the fracture problem of magneto-electro-thermo-elastic FGMs under dynamic thermal loading. Garg and Pant [21] executed an optimized EFG to examine the thermoelastic fracture of FGMs. Memari and Khoshravan [22] implemented the MLPG to gain the SIFs of an FG plate with an edge crack under steady thermal loading. However, such methods have bottlenecks in numerical integration, boundary condition processing and solving efficiency.

The XFEM enriched the FEM field approximation via the crack tip asymptotic basis function and the generalized Heaviside function, which characterizes the singularity of the crack tip field and the discontinuity of the crack tip displacement, respectively. The mesh is not required to be conformed to the crack, so there is no need to update the mesh information for modeling the crack propagation problem. Guo et al. [23] incorporated the modified interaction energy integral into the XFEM to dig the thermal fracture of FGMs containing material interfaces. By virtue of the XFEM, Rokhi and Shariati [24] studied the dynamic thermoelastic fracture problem. Combining the XFEM and the interaction integral, Hosseini et al. [25] implemented the thermal fracture analysis of isotropic and orthotropic FGMs. Bhattacharya and Sharma [26] simulated the fatigue crack growth of an FG plate under cyclic thermal loading with the XFEM. Elahi and Rokhi [27] probed the thermal fracture behavior of FGM thick-walled cylinders by means of the XFEM. Nevertheless, the drawbacks of XFEM are the great limitations in describing the jump of field functions cross the crack surface for complex cracks (e.g., intersected and multi-branch cracks).

The NMM [28], proposed by Shi, is a new numerical method originally used to solve mechanical problems in rock engineering. The core of NMM lies in its unique dual cover systems, that is, mathematical cover and physical cover. The primary virtues of the method are as follows [29]: (1) the mathematical cover system can be inconsistent with the internal and external boundaries of the physical domain; (2) the discontinuity of the field function (e.g., the displacement field and the temperature field across the crack surface) can be described naturally; (3) The local properties near the crack tip can be accurately characterized by choosing the appropriate cover function. Since its advent, the NMM has been developed to solve many fracture problems, e.g., [30-34]. As for its use in the concerned topic, Zhang et al. [35] extended the NMM to delve the thermal fracture problem of 2D FGMs based on their previous work around the isothermal fracture of FGMs [36] and thermal fracture of homogenous materials [37]. In NMM, since the mathematical cover can be inconsistent with the internal and external boundaries of the physical domain, it is difficult to straightforwardly apply essential boundary conditions like FEM; therefore, indirect methods such as penalty method and Lagrange multiplier method are frequently adopted.

3. Concluding remarks
As a new functional material, FGM has a wide range of applications and excellent properties, gradually turning into a hot topic in the material science. In light of the significance in engineering application, lots of efforts have been put on the thermal fracture behavior study of FGMs. In this paper, some previous work on this topic was briefly summarized from both analytical and numerical aspects. Accordingly, some further work is also suggested:

(1) Regarding the thermo-mechanical fracture of FGMs, the assumption of one-way coupling is frequently adopted, so more efforts on the fully coupled condition are awaited.

(2) Most of the present work is devoted to discovering the thermal fracture behavior of 2D FGMs; the investigation of complex 3D cases is in urgency.

(3) Recent years, functionally graded piezoelectric/piezomagnetic smart materials (FGPMs) have been widely used as sensors or actuators due to their excellent performances, and so more intensive researches on their fracture properties under coupled thermo-magneto-electro-mechanical loadings are also expected.

(4) Most of the existing papers focus on the linear elastic fracture behavior of FGMs subjected to thermal loading; therefore, more nonlinear studies such as the elastoplastic or viscoplastic fracture
analysis should be taken into account.

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