Tensor correlations in nuclei and exclusive electron scattering

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The effect of tensor nucleon-nucleon correlations upon exclusive and semi-exclusive electronuclear reactions is studied. Differential cross sections for the semi-exclusive $^{16}$O(e, $e'$p) and exclusive $^1$H(e, $e'$pn) processes are computed by explicitly evaluating the dynamical electromagnetic coupling to a tensor correlated nucleon pair. In both reaction channels the tensor correlations contribute in a very substantial way. Tensor correlations are found to generate more electronuclear strength than central Jastrow correlations do.

In the history of nuclear physics, it has been notoriously difficult to detect signals that directly point towards phenomena beyond the scope of the effective mean-field theories. This holds in particular for the short-range correlations that reflect the remnants of the hard-core part of the nucleon-nucleon (NN) force in the medium. Recently, manifestations for strongly correlated proton pairs and the existence of Jastrow-like correlations emerged from the simultaneous detection of two protons upon absorption of one (virtual) photon by an atomic nucleus $^1$H. Nuclear many-body theories have produced vastly different predictions for the short-range behaviour of nuclei. The ongoing exclusive $A(e, e'pp)$ studies are expected to provide stringent tests of these theories. The correlations probed in proton-proton knockout are predominantly the state-independent scalar ones (often referred to as Jastrow correlations) related to the hard core part of the NN force. The tensor force, which is operative at intermediate internucleon distances ($\approx 1$-2.5 fm), is established to be an important ingredient of the NN force in the medium and is believed to be another source of important NN correlations which go beyond the mean-field level $^1$. High momentum components in the deuteron wave function, for example, are ascribed to a $D$-state admixture and are a direct manifestation of the presence of tensor correlations in the proton-neutron system. Since long, the tensor interaction has been established to be a rather weak but essential ingredient of the effective NN force. Despite intensive research a recent review $^2$ quoted its role as “elusive”. Earlier studies of the role played by tensor correlations in electron scattering concentrated on inclusive $A(e, e')$ response functions for which there are many competing effects and unambiguous information on the tensor correlations might be difficult to extract $^3$. In the near future, exclusive experiments that aim at probing both the proton-proton and proton-neutron correlations will be performed at MAMI and TJNAF. At MAMI, where central short-range correlations in nuclei are being studied with the aid of the $A(e, e'pp)$ reaction $^3$, high-resolution $A(e, e'pn)$ measurements have been scheduled for the target nuclei $^3$He $^4$ and $^{16}$O $^5$. These measurements will be performed at four-momentum transfers of the order $Q^2 \approx 0.05$ (GeV/c)$^2$. At TJNAF, on the other hand, the small distance structure of nuclei will be probed with the aid of the $^{12}$C(e, $e'pN$) ($N = p, n$) reaction at $Q^2 \geq 2$ (GeV/c)$^2$ $^6$. In the light of the upcoming experiments, we present model calculations that aim at exploring the possibility of using exclusive and semi-exclusive electronuclear reactions to elucidate the role of tensor correlations in the nuclear medium.

To establish a connection between the NN correlations and the measured electronuclear cross sections, we remark that the response of the target nucleus (with “correlated” ground-state wave function $|\Psi_i\rangle$) to the electromagnetic probe is determined by matrix elements of the form

$$\frac{\langle \Psi_f | J_{\mu}(q) | \Psi_i \rangle}{\langle \Psi_f | \Psi_f \rangle \langle \Psi_i | \Psi_i \rangle}.$$  \hspace{1cm} (1)

We introduce “correlated” nuclear wave functions in a standard manner

$$|\Psi\rangle = \hat{S} \prod_{i<j=1}^A \left( 1 - g_c(r_{ij}) + f_{t\tau}(r_{ij}) \hat{S}_{ij} \vec{\tau}_i \cdot \vec{\tau}_j \right) |\Psi\rangle,$$  \hspace{1cm} (2)

where $|\Psi\rangle$ is the uncorrelated and normalized (Slater determinant) wave function obtained from a mean-field calculation, $\hat{S}$ the symmetrizer, $g_c$ the central correlation function that accounts for (state-independent or Jastrow) short-range effects and $\hat{S}_{ij}$ the tensor operator that introduces state-dependent correlations for the $S = 1$ components in the nuclear wave functions. Strictly speaking the correlation operator also contains spin-isospin and spin-orbit terms. The $g_c$ and $f_{t\tau}$ are, however, the most important ones and without their presence finite nuclei would simply be unbound $^3$. A striking feature of the correlation functions $g_c$ and $f_{t\tau}$ is that they exhibit a modest $A$ dependence $^3$. This makes them an universal feature of atomic nuclei whose experimental determination is of the utmost importance. The hadronic current operator $J_{\mu}(q)$ in the above expression accounts for the
coupling of the electromagnetic field to the nuclear system. In our calculations, the one-body part \( J_{\mu}^{[1]} \) has been implemented in the standard Impulse Approximation (IA) fashion, whereas the two-body currents \( J_{\mu}^{[2]} \) include the conventional pion-exchange and intermediate \( \Delta_{33} \)-resonance terms \( \Delta_{33} \). The matrix element of Eq. \( (3) \) contains \( 2, 3, \ldots, A \)-body correlations and is usually calculated with the aid of an expansion that is cut at some order. It is worth stressing that most of the cluster expansions for correlated systems that are outlined in literature refer to calculations addressing the “inclusive” response of the correlated target nucleus to the electromagnetic probe. Hereby, closure properties can be exploited to help reducing the complexity of the calculations. In the \( (e, e') \) case, for example, closure can be applied when integrating over all excitation energies of the residual \( A-1 \) system \( \Delta_{33} \). Here, we aim at computing the effect of the correlations on some well defined parts of the phase space in exclusive and semi-exclusive electronuclear processes. Accordingly, no closure relations can be applied and most of the known cluster expansion techniques are not directly applicable. In inclusive \(^{12}\text{C}(e, e')\) \( ^{15}\text{O}(e, e') \) calculations the lowest-order approximation (LOA) was observed to account for the major \( NN \) correlations effects. Given that exclusive \( A(e, e'NN) \) processes are confined to two-nucleon phase space, three- and higher-body “correlations” are expected to produce even smaller corrections than in the inclusive case. For that reason, the transition matrix element of Eq. \( (3) \) was evaluated in the LOA. This procedure results in a clear separation between the contributions from the uncorrelated mean-field wave function that read

\[
\langle \Psi_f \mid \sum_{i=1}^{A} J_{\mu}^{[1]}(i; q) + \sum_{i<j=1}^{A} J_{\mu}^{[2]}(i, j; q) \mid \Psi_i \rangle ,
\]

and the ones that can be unambiguously attributed to the \( NN \) correlations

\[
\langle \Psi_f \mid \sum_{i<j=1}^{A} \left[ (J_{\mu}^{[1]}(i; q) + J_{\mu}^{[1]}(j; q) + J_{\mu}^{[2]}(i, j; q) \right] \times \left( -g_c(r_{ij}) + f_{t\tau}(r_{ij}) \hat{S}_{ij} \hat{r}_i \hat{r}_j + h.c. \right) \mid \Psi_i \rangle .
\]

The correlation functions \( g_c \) and \( f_{t\tau} \) establish the link between the measurements and the nuclear many-body theories and contain the information of the “beyond mean-field” structure of nuclei. In inclusive \( A(e, e') \) and exclusive \( A(e, e'p) \) reactions, signals from the “correlation” part \( \Delta_{33} \) in the transition matrix elements are frequently obscured by the dominant one-body term \( J_{\mu}^{[1]} \) in Eq. \( (3) \). For uncorrelated wave functions, two-nucleon knockout strength can solely be generated by the one-body current through final state interaction effects, e.g. \( A(e, e'p)A-1 \) followed by \( A-1(p, p'n) \). We will consider kinematical regimes in which the contaminant effect of rescattering processes is minimized. To that purpose we compute proton neutron knockout processes from a light nucleus \( ^{16}\text{O} \) with the condition that the residual nucleus \( ^{14}\text{N} \) is created at low excitation energies. Moreover, we will consider the situation that the two nucleons are ejected along the momentum transfer (“super-parallel kinematics”). Then, \( A-1(p, p'n) \) rescattering processes involve large momentum transfers and are heavily suppressed. This feature of heavily suppressed one-body current contributions to the uncorrelated matrix element of Eq. \( (3) \), opens up good opportunities to acquire a precise understanding of \( NN \) correlations by studying two-nucleon knockout processes.

![FIG. 1. The missing momentum dependence of the \(^{16}\text{O}(e, e'pn) \) differential cross section and recoil polarization observable \( P_t \) for excitation of the low lying two-hole states in \(^{14}\text{N} \) at \( \epsilon = 855 \text{ MeV}, \epsilon' = 640 \text{ MeV} \) and \( \theta_{\nu} = 18^\circ \). The calculations refer to the situation in which the proton (neutron) is detected parallel (anti-parallel) to the direction of the momentum transfer. The solid curve is calculated in the distorted-wave approximation including the two-body currents, central and tensor correlations. The dashed line is the contribution from the two-body currents. The dotted lines omit tensor correlations from the full calculation.](attachment:image.png)
rent contributions. In \(A(e,e'pp)\) and \(A(e,e'pm)\) studies, a powerful tool in the search for \(NN\) correlations is the selectivity of the final state with respect to the quantum numbers of the correlated nucleon pair that actively participates in the reaction process \[\text{[17]}\]. We have calculated \(16O(e,e'pm)14N\) cross sections for excitation of specific states in \(14N\). The results are displayed in Figure \[\text{[21]}\] and include apart from the central and tensor correlations, the “uncorrelated” contribution of competing meson-exchange and intermediate \(\Delta\)-resonance two-body current contributions. The model employed to calculate these cross sections includes a distorted wave description of the ejected and was extensively discussed in Reference \[\text{[11]}\]. Restricting ourselves to the (dominant) p-shell components, the following selectivity for the quantum numbers of the active proton-neutron pair emerges when considering \(16O(e,e'pm)\) decay to the different angular momentum states in \(14N\)

\[
\begin{align*}
J_R &= 0^+ : 1S_0(\Lambda = 0), \quad 3P_1(\Lambda = 1) \\
J_R &= 1^+ : 3S_1(\Lambda = 0, 2), \quad 3P_{0,1,2}(\Lambda = 1), \quad 3D_1(\Lambda = 0) \\
J_R &= 2^+ : 1S_0(\Lambda = 2), \quad 3S_1(\Lambda = 0, 2), \quad 3P_{1,2}(\Lambda = 1), \quad 3D_2(\Lambda = 0) \\
\end{align*}
\]

where the standard convention \((2S_{\pm 1}L_J)\) for the relative two-nucleon wave function is adopted and \(\Lambda\) is the orbital quantum number corresponding with the c.o.m. motion of the pair. We have calculated the differential cross section for these states in the low-energy spectrum of \(14N\) that are established to have a two-hole character relative to the ground state. In many respects the \((\pi^+, pp)\) reaction and our computed \(16O(e,e'pm)\) differential cross sections for the \(1^+\) states exhibit exactly the same qualitative features as those observed in the \(16O(\pi^+, pp)14N\) data of Ref. \[\text{[22]}\].

Also shown in Fig. \[\text{[18]}\] are the predictions for the double polarization observables \(P'_t\). This variable can be determined in \(A(e,e'pn)\) measurements and determines the recoil polarization along the direction \(\hat{t}\) which is in the reaction plane orthogonal to the direction of the ejected proton’s momentum. In superparallel kinematics, \(P'_t\) is uniquely determined by the \(W'_{LT}\) structure function. The background of two-body current contributions to proton-neutron knockout is almost exclusively transverse. With no ground-state correlations contributing, \(P'_t\) is extremely small. For these transitions with strong contributions from the tensor correlations, the calculations predict large values \(P'_t\), though. In that respect, the recoil polarization observable is a measure for the importance of tensor correlations in the medium. Double polarization observables have been frequently shown to be relatively free of ambiguities with respect to the final state interaction \[\text{[21]}\].

In general, triple coincidence measurements are challenging. An indirect way of accessing the ground-state correlations is the semi-exclusive \(A(e,e'p)\) process. Indeed, when probing higher missing energies \(E_n = \omega - T_p - T_{A-1}\), the residual nucleus is created at a high excitation energy \((E_x \gtrsim 30\text{ MeV})\). In this regime, exclusive single-nucleon knockout through the “uncorrelated” one-body current operator of Eq. \[\text{[8]}\] is heavily suppressed and besides the mesonic degrees of freedom, the “correlations” terms in Eq. \[\text{[9]}\] are expected to feed the \(A(e,e'p)\) channel. As the “correlations” are predominantly of two-body nature, the “correlated” \(A(e,e'p)\) strength will manifest itself as two-nucleon knockout. Often, in interpreting semi-exclusive \(A(e,e'p)\) and inclusive \(A(e,e')\) processes a factorization scheme is adopted. Hereby, the hadronic part of the \(A(e,e'p)\) cross section is written in terms of the probability to find a nucleon with a certain
momentum for a fixed missing energy of the $A-1$ spectators (the so-called spectral function). For the results presented here, an unfactorized approach is adopted. The method was outlined in Ref. [1] and is based on explicitly calculating the contribution from the operator $(J^{1}_\mu (i; q) + J^{2}_\mu (j; q)) \left( -\frac{\epsilon}{\epsilon'} (r_{ij}) + f(r_{ij}) \hat{S}_{ij} \vec{r}_i \cdot \vec{r}_j \right)$ to the two-nucleon knockout channels ($A(e, e'pn)$ and $A(e, e'pp)$) and integrating over the complete phase space of the undetected nucleon. The calculated missing energy spectrum of the $^{16}$O($e, e'p$) reaction at $\omega=300$ MeV and $|\vec{q}|=416$ MeV/c is shown in Figure 2 for a number of proton angles. It is worth stressing that in the absence of $NN$ correlations all computed strengths in Figure 2 would vanish identically. As the angle $\theta_{pq}$, which is the polar angle of the detected proton relative to the direction of the momentum transfer, grows higher initial proton momenta are probed. A striking feature of the results is that the tensor correlations generate a few times more “correlated” $^{16}$O($e, e'p$) strength than the central short-range correlations do. At $\theta_{pq}=40^\circ$, where the missing (or, initial proton) momentum ranges over $275 \leq p_m \leq 450$ MeV/c, inclusion of the tensor correlations increases the strength with a factor of five. As clearly illustrated in Figure 2 most of the “correlated” ($e, e'p$) strength can be ascribed to the proton-neutron knockout channel. With increasing polar angle $\theta_{pq}$ higher missing momenta are probed and the central correlations gain in relative importance. This can be explained by considering that central correlations refer to the hard part of the $NN$ force, whereas tensor correlations are of somewhat larger ranges ($1 \leq r \leq 3$ fm). Qualitatively, the central and tensor correlations exhibit a similar missing energy behaviour for all polar angles included in Figure 2. This feature reflects the fact that both the central- and tensor correlations are mostly affecting nucleon pairs in relative $S$ states, thus imposing stringent kinematical constraints on the $(E_m, p_m)$ regions where the strength attributed to the $NN$ correlations resides [24].

Summarizing, we have presented a framework permitting a systematic investigation of the effects of state-independent (central) and state-dependent (tensor) correlations upon $A(e, e'NN)$ and semi-exclusive $A(e, e'p)$ differential cross sections. The calculations reveal that the sensitivity of the $^{16}$O($e, e'pn$) cross sections to central correlations is rather modest. On the contrary, strong signals from the tensor correlations appear in these regions of the $d\Omega_{p}d\Omega_{n}d\Omega_{p}d\Omega_{n}$ phase space whenever the initial photoabsorption happens to occur on $S_1$ proton-neutron pairs. The tensor correlations are further predicted to produce substantially more semi-exclusive $^{16}$O($e, e'p$) strength than the central (Jastrow) correlations do. This peculiar feature of how the $NN$ correlations manifest themselves in electronuclear processes, makes the proton-neutron knockout channel to dominate the semi-exclusive ($e, e'p$) strength and could be considered as a microscopic confirmation of the suggested quasi-deuteron scaling of the “correlated” $A(e, e')$ strength [25].

Concluding, our results indicate that exclusive electronuclear reactions are a promising tool for providing insight into the role of tensor correlations in the nuclear medium.

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![FIG. 2. The missing-energy dependence of the computed contribution from $NN$ correlations to the semi-exclusive $^{16}$O($e, e'p$) differential cross section at various proton polar angles. Dashed curves include only the central correlations, solid curves both central and tensor correlations. The dotted curve is the calculated contribution from ($e, e'pn$) including tensor and central correlations. The electron kinematics is determined by $\epsilon = 1.2$ GeV, $\epsilon' = 0.9$ GeV and $\theta_e = 16^\circ$.](image)

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