Logical operation of one-dimensional photonic crystal based on series and parallel connection

Xiang-Yao Wu a, Ji Ma a, Xiao-Jing Liu a
Xiang-Dong Meng a, Yu Liang a, Hong Li a and Si-Qi Zhang a

a. Institute of Physics, Jilin Normal University, Siping 136000 China

In this paper, we have proposed the compound structure of one-dimensional photonic crystal (PC), which includes series connection and parallel connection PC. We have studied the transmission characteristics of series connection and parallel connection PC, and obtained some new results. In addition, we have proved the series connection one-dimensional PC can realize the logical AND operation, and the parallel connection one-dimensional PC can realize the logical OR operation. The compound structure of one-dimensional PC can design more new type structure optical devices, and will provide the basic for designing quantum computer.

PACS: 41.20.Jb, 42.70.Qs, 78.20.Ci
Keywords: PC; series connection; parallel connection; logical operation; quantum computer

1. Introduction

E. Yablonovitch and S. John had pointed out that the behavior of photons in 1987. It can be changed when propagating in the material with periodical dielectric constant, and termed such material PC [1,2]. PC important characteristics are: photon band gap, defect states, Light localization and so on. These characteristics make it able to control photons, so it may be used to manufacture some high performance devices which have completely new principles or can not be manufactured before, such as high-efficiency semiconductor lasers, right emitting diodes, wave guides, optical filters, high-Q resonators, antennas, frequency-selective surface, optical wave guides and sharp bends [3-4], WDM-devices [5-6], splitters and combiners [7]. optical limiters and amplifiers [8-10]. The research on PC will promote its application and development on integrated photoelectron devices and optical communication. To investigate the structure and characteristics of band gap, there are many methods to analyze PC including the plane-wave expansion method [11], Greens function method, finite-difference time-domain method [12-14] and transfer matrix method [15-20].

In this paper, we have proposed the compound structure one-dimensional PC, which include series connection, parallel connection compound structure PC. We have studied their transmission characteristics and given the relation of logical operation. The work frequency of light signal is taken as the defect mode frequency of PC. When the PC has the defect mode, transmissivity $T = 1$ corresponds to logical 1. When the PC has no defect mode, transmissivity $T = 0$ corresponds to logical 0. We can find the series connection one-dimensional PC can realize the AND operation, and the parallel connection one-dimensional PC can realize the OR operation. The compound structure PC will help to design more new type structure optical devices, and provide the basic for designing quantum computer.

2. Transfer matrix and transmissivity of one-dimensional PC

For one-dimensional PC, the calculations are performed using the transfer matrix method [21], which is the most effective technique to analyze the transmission properties of PC. For the medium layer $i$, the transfer matrices $M_i$ for $TE$ wave is given by [21]:

$$M_i = \begin{pmatrix} \cos \delta_i & -i \sin \delta_i / \eta_i \\ -i \eta_i \sin \delta_i & \cos \delta_i \end{pmatrix},$$

(1)

where $\delta_i = \frac{2\pi n_i d_i \cos \theta_i}{c}$, $c$ is speed of light in vacuum, $\theta_i$ is the ray angle inside the layer $i$ with refractive
index $n_i = \sqrt{\varepsilon_i \mu_i}$, $\eta_i = \sqrt{\varepsilon_i / \mu_i \cos \theta_i}$, $\cos \theta_i = \sqrt{1 - (n_0^2 \sin^2 \theta_0 / n_i^2)}$, in which $n_0$ is the refractive index of the environment wherein the incidence wave tends to enter the structure, and $\theta_0$ is the incident angle.

The total transfer matrix $M$ for an $N$ period structure is given by:

$$
\begin{pmatrix}
E_1 \\
H_1
\end{pmatrix}
= M_B M_A M_B M_A \cdots M_B M_A \begin{pmatrix} E_{N+1} \\
H_{N+1}
\end{pmatrix}
= M \begin{pmatrix} E_{N+1} \\
H_{N+1}
\end{pmatrix}
= \begin{pmatrix} A & B \\
C & D
\end{pmatrix} \begin{pmatrix} E_{N+1} \\
H_{N+1}
\end{pmatrix},
$$

(2)

where

$$M = \begin{pmatrix} A & B \\
C & D
\end{pmatrix},$$

(3)

with the total transfer matrix $M$, we can obtain the transmissivity $T$, it is

$$T = \left| \frac{E_{N+1}}{E_1} \right|^2 = \frac{2\eta_0}{A\eta_0 + B\eta_0\eta_{N+1} + C + D\eta_{N+1}}^2.$$

(4)

Where $\eta_0 = \eta_{N+1} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cos \theta_0$. By the Eqs. (1) and (4), we can calculate the transmissivity of one-dimensional PC.

3. Series connection and parallel connection one-dimensional PC transmissivity

Two or multiple one-dimensional PC can be connected by optical fiber, which can be designed series connection, parallel connection compound structure one-dimensional PC. The FIGs. 1 and 2 are series connection and parallel connection one-dimensional PC structures, respectively. The $PC_1$ and $PC_2$ are the one-dimensional PC, $E_{in}$ is the input electric field intensity, and $E_{out1}$ and $E_{out2}$ are the output electric field intensity of $PC_1$ and $PC_2$.

(1) The total transmission coefficient $t$ and transmissivity $T$ of series connection one-dimensional PC are

$$
t = \frac{E_{out2}}{E_{in}}, \quad T = \frac{E_{out2}}{E_{out1}}, \quad \frac{E_{out1}}{E_{in}} = t_2 \cdot t_1,
$$

(5)
\[ T = |t|^2, \]  
\[ (6) \]

where \( t_2 = E_{out2}/E_{out1} \) and \( t_1 = E_{out1}/E_{in} \) for the transmission coefficients of \( PC_1 \) and \( PC_2 \).

Similarly, we can obtain the transmission coefficient for \( n \) series connection \( PC \), such as \( PC_1, PC_2, \ldots, \) \( PC_n \), series connection transmission coefficient is

\[ t = t_n \cdot t_{n-1} \cdots t_1. \]  
\[ (7) \]

(2) The total transmission coefficient \( t \) and transmissivity \( T \) of parallel connection one-dimensional PC are

\[ t_{\pm} = \frac{E_{out}}{E_{in}} = \frac{E_{out1} \pm E_{out2}}{E_{in}} = \frac{t_1E_{in1} \pm t_2E_{in2}}{E_{in}}, \]  
\[ (8) \]

when \( E_{out1} \) and \( E_{out2} \) phase are the same (opposite), the numerator of Eq. (8) takes \( + \) (\(-\)).

When \( t_1 = t_2 = t \), there are

\[ t_+ = t_1 = t_2 = t, \quad t_- = \frac{t(E_{in1} - E_{in2})}{E_{in}} = t(c_1 - c_2), \]  
\[ (9) \]

where \( c_1 = E_{in1}/E_{in} \) and \( c_2 = E_{in2}/E_{in} \).

When \( t_1 \neq t_2 \), there is

\[ t_+ = c_1t_1 + c_2t_2, \quad t_- = c_1t_1 - c_2t_2 \quad (c_1 + c_2 = 1). \]  
\[ (10) \]

Similarly, the total transmission coefficient of \( n \) PC \( PC_1, PC_2, \ldots, PC_n \) parallel connection is

\[ t_{\pm} = \frac{E_{out}}{E_{in}} = \frac{E_{out1} \pm E_{out2} \pm \cdots \pm E_{outn}}{E_{in}} = \frac{t_1E_{in1} \pm t_2E_{in2} \pm \cdots \pm t_nE_{inn}}{E_{in}} = c_1t_1 \pm c_2t_2 \pm \cdots \pm c_n t_n. \]  
\[ (11) \]

4. Numerical result

In this section, we report our numerical results of compound structure one-dimensional PC, the PC \( PC_1 \) and \( PC_2 \) are constituted by media A and B. The main parameters are: the medium A refractive indices \( n_a = 2.45 \), thickness \( a = 469nm \), the medium B refractive indices \( n_b = 1.35 \), thickness \( b = 890nm \), the center frequency \( \omega_0 = \frac{1}{(n_a a + n_b b)} = 4.01 \times 10^{14}Hz \), the incident angle \( \theta_0 = 0 \).

Firstly, we study the transmission characteristics of series connection and parallel connection structure PC, which are constituted by \( PC_1 \) and \( PC_2 \), their structure are \( (AB)^2 \). In \( PC_1 \), the medium B thickness \( b = 890nm \). In \( PC_2 \), the medium B thickness \( b = 560nm \), which are shown in FIGs. 1 and 2. The series connection and parallel connection structure are referred to as \( PC_1 \cdot PC_2 \) and \( PC_1 + PC_2 \). From Eqs. (5) and (6), we can calculate the series connection structure transmissivity. By Eq. (10), we can calculate the parallel connection structure transmissivity \( |t_\pm|^2 \).

The FIG. 3 (a), (b), (c) and (d) are the transmissivity corresponding to the structure \( PC_1, PC_2, PC_1 \cdot PC_2 \) and \( PC_1 + PC_2 \). From FIG. 3 (a), (b) and (c), we can obtain some results about the series connection PC: (1) The forbidden band width of series connection PC becomes more wider, and it is the union of corresponding forbidden band of part PC \( PC_1 \) and \( PC_2 \), which is similar as the series connection ohm law in circuit. (2) We can obtain the more wider forbidden band by PC series connection. (3) With the number of series connection PC increasing, the total width of forbidden band increase. From FIG. 3 (a), (b) and (d) \( (c_1 = 0.5) \). We can find when coefficient \( c_1 = 0.5 \) the forbidden band of parallel connection (FIG. 3 (d)) is the intersection of corresponding forbidden band of \( PC_1 \) and \( PC_2 \).

Secondly, we begin to study the AND logical operation. In FIGs. 4, 5 and 6, we study the AND operation by the series connection one-dimensional PC \( PC_1 \) and \( PC_2 \). The work frequency of light signal is taken as
FIG. 3: The series connection structure transmissivity. (a) $PC_1$ transmissivity, (b) $PC_2$ transmissivity, (c) series connection transmissivity, (d) parallel connection transmissivity.

the defect mode frequency of PC. When the PC has the defect mode, transmissivity $T = 1$ corresponds to logical 1. When the PC has no defect mode, transmissivity $T = 0$ corresponds to logical 0. The logical AND operation are: $0 \cdot 0 = 0$, $1 \cdot 0 = 0$ and $1 \cdot 1 = 1$. In FIG. 4, the structures of $PC_1$ and $PC_2$ both are $\left(AB\right)^{12}$ with no defect mode. FIGs. 4 (a), (b) and (c) show the transmissivity corresponding to the structure $PC_1$, $PC_2$ and $PC_1 \cdot PC_2$. $PC_1$ and $PC_2$ both correspond to logical 0. The series connection $PC_1 \cdot PC_2$ has transmissivity 0 corresponding to logic AND operation $0 \cdot 0 = 0$. In FIG. 5, the $PC_1$ is symmetrical structure $\left(AB\right)^6(BA)^6$ with the defect mode and $PC_2$ structure is $\left(AB\right)^{12}$ without the defect mode. FIGs. 5 (a), (b) and (c) show the transmissivity corresponding to the structure $PC_1$, $PC_2$ and series connection $PC_1 \cdot PC_2$. $PC_1$ corresponds to logical 1. $PC_2$ corresponds to logical 0. The series connection $PC_1 \cdot PC_2$ has transmissivity 0 corresponding to logic AND operation $1 \cdot 0 = 0$. In FIG. 6, $PC_1$ and $PC_2$ both are symmetrical structure $\left(AB\right)^6(BA)^6$. They are both have defect mode. FIGs. 6 (a), (b) and (c) show the transmissivity corresponding to the structure $PC_1$, $PC_2$ and $PC_1 \cdot PC_2$. At the working frequency, $PC_1$ and $PC_2$ both have transmissivity 1 corresponding to the logic 1. The series connection $PC_1 \cdot PC_2$ has transmissivity 1 corresponding to logic AND operation $1 \cdot 1 = 1$.

Finally, we will study the OR logical operation based on the parallel connection one-dimensional PC $PC_1$ and $PC_2$. The logical OR operation are: $0 + 0 = 0$, $1 + 0 = 1$ and $1 + 1 = 1$, which correspond to FIGs. 7, 8 and 9, respectively. In FIG. 7, the structures of $PC_1$ and $PC_2$ both are $\left(AB\right)^{12}$. FIGs. 7 (a), (b) and (c) show the transmissivity corresponding to the structure $PC_1$, $PC_2$ and $PC_1 + PC_2$ (the parameter $c_1 = 0.9$), they are all with no defect mode, correspond to logic 0, which realize the logical OR operation $0 + 0 = 0$. For FIG. 8 (a), the $PC_1$ is symmetrical structure $\left(AB\right)^6(BA)^6$ with the defect mode corresponds to logical
For FIG. 8 (b), the $PC_2$ structure is $(AB)^2$ without the defect mode corresponds to logical 0. The FIG. 8 (c) show the transmissivity corresponding to the parallel connection structure $PC_1 + PC_2$ with the defect mode corresponds to logical 1, which realize the logical OR operation $1 + 0 = 1$. In FIG. 9 (a) and (b), the $PC_1$ and $PC_2$ are symmetrical structure $(AB)^6(BA)^6$ with the defect mode corresponds to logical 1. FIG. 9 (c) is the transmissivity corresponding to the parallel connection structure $PC_1 + PC_2$ with the defect mode corresponds to logical 1, which realize the logical OR operation $1 + 1 = 1$.

5. Conclusion

In summary, we have proposed the compound structure one-dimensional PC, which include series connection and parallel connection compound structure. We have studied series connection and parallel connection transmission characteristics and obtained some new results. In addition, we have proved the series connection one-dimensional PC can realize the logical AND operation, and the parallel connection one-dimensional PC can realize the logical OR operation. We think the other logical operations can be achieved by other compound structure PC. The compound structure PC will help to design more new type structure optical devices, and provide the basic for designing quantum computer.

6. Acknowledgment

This work was supported by the National Natural Science Foundation of China (no. 61275047), the Research Project of Chinese Ministry of Education (no. 213009A) and the Scientific and Technological Development Foundation of Jilin Province (no.20130101031JC).

---

1. E. Yablonovitch, Phys. Rev. Lett. 58, 2059 (1987).
2. S. John, Phys. Rev. Lett. 58 2486 (1987).
3. A. Lavrinenko, P.I. Borel, L.H. Frandsen, M. Thorhauge, A. Harpeth, M. Kristensen, T. Niemi, Opt. Express 12 234 (2004).
4. J. Pu, Y. Yomogida, K. K. Liu, L. J. Li, Y. Iwasa, and T. Takenobu, Nano Lett., 12 4013, (2012).
5. S. Fan, P.R. Villeneuve, J.D. Joannopoulos, H.A. Haus, Phys. Rev. Lett. 80 960 (1998).
6. A. Ferreira, N. M. R. Peres, R. M. Ribeiro and T. Stauber, Phys. Rev. B, 85 115438, (2012).
7. S. Kim, I. Park, H. Lim, Proc. Design of PC splitters/combines SPIE 5597 129 (2004).
8. N. M. R. Peres and Yu. V. Bludov, EPL, 101 58002, (2013).
9. R. Martinez-Sala, J. Sancho, J. V. Sanchez, V. Gomez, J. Llinares and F. Meseguer, nature 378, 241 (1995).
10. T. Cai, R. Bose, G. S. Solomon, and E. Waks, Appl. Phys. Lett. 102, 141118 (2013).
11. J. J. Joannopoulos, R. D. Meade, J. N. Winn, PC: molding the flow of light (Princeton University Press, New Jersey, 1995).
12. M. Minkov and V. Savona, Phys. Rev. B 88, 081303R (2013).
13. K. K. Yee, IEEE Trans. Antennas Propag. 14, 302 (1966).
14. K. S. Choi, H. Deng, J. Laurat, and H. J. Kimble, Nature 452, 67 (2006).
15. J. B. Pendry, Phys. Rev. Lett. 69, 2772 (1992).
16. M. Jachura, M. Karpinski, C. Radziewicz, and K. Banaszek, Opt. Express 22, 8624 (2014).
17. D. A. Miller, Nature Photonics 4, 3 (2010).
18. Kamal, A., Clarke, J. Devoret, M. H, Nature Physics, 7, 311 (2011).
19. Fan, L., Science, 335, 447 (2012).
20. Yu Z., Fan S., Nature Photon. 3, 91 (2009).
21. J. Zi, J. wan and C. Zhang, Appl. Phys. Lett, 73, 2084 (1998).
FIG. 4: The series connection structure transmissivity. (a) \(PC_1\) transmissivity without defect model (logical 0), (b) \(PC_2\) transmissivity without defect model (logical 0), (c) series connection transmissivity without defect model (logical 0), realizing logical AND operation \(0 \cdot 0 = 0\).

FIG. 5: The series connection structure transmissivity. (a) \(PC_1\) transmissivity with defect model (logical 1), (b) \(PC_2\) transmissivity without defect model (logical 0), (c) series connection transmissivity without defect model (logical 0), realizing logical AND operation \(1 \cdot 0 = 0\).
FIG. 6: The series connection structure transmissivity. (a) $PC_1$ transmissivity (logical 1), (b) $PC_2$ transmissivity (logical 1), (c) series connection transmissivity (logical 1, realizing logical AND operation $1 \cdot 1 = 1$.

FIG. 7: The parallel connection structure transmissivity. (a) $PC_1$ transmissivity without defect model (logical 0), (b) $PC_2$ transmissivity without defect model (logical 0), (c) series connection transmissivity without defect model (logical 0), realizing logical OR operation $0 + 0 = 0$. 
FIG. 8: The parallel connection structure transmissivity. (a) $PC_1$ transmissivity with defect model (logical 1), (b) $PC_2$ transmissivity without defect model (logical 0), (c) series connection transmissivity with defect model (logical 1), realizing logical OR operation $1 + 0 = 1$.

FIG. 9: The parallel connection structure transmissivity. (a) $PC_1$ transmissivity with defect model (logical 1), (b) $PC_2$ transmissivity with defect model (logical 1), (c) series connection transmissivity with defect model (logical 1), realizing logical OR operation $1 + 1 = 1$. 