Reliability Modeling Using an Adaptive Neuro-Fuzzy Inference System: Gas Turbine Application

Nadji Hadrouga, Ahmed Hafaifa, Abdelhamid Iratni and Mouloud Guemana

ABSTRACT
Recently, the development of the industry requires monitoring and follow-up of the working conditions of the facilities, to determine the reliability, availability, and durability of these systems, for objectively estimating the service life of these installations with reduced maintenance costs. In this sense, this work proposes a novel approach to reliability modeling, to determine failure assessment indicators based on an adaptive neuro-fuzzy inference system applied on a gas turbine. This is in order to describe the behavior of this rotating machine and to estimate their operating safety parameters, to improve its performance in terms of maintainability, availability, and operational safety with effective durability. The application of fuzzy rules to reliability estimation with practical implementations is innovative, making it possible to provide solutions to problems of reliable identification of gas turbines in their complex operating environments.

KEYWORDS
Reliability analyzes; reliability modeling; adaptive neuro-fuzzy inference system; gas turbine system

1. Introduction
The rapid evolution of technology has increased the complexity of industrial facilities and requires major challenges in terms of the safety of their operation. Hence, the maintenance of industrial installations has become an essential activity seeking better performance in terms of reliability, maintainability, availability, and safety in the monitoring systems of these industrial processes. In this context, this work proposes to develop a reliability modeling approach, based on a new development of fuzzy inference systems and artificial neural networks, applied to the study of a gas turbine system, installed at the gas compressor station SC3 SONATRACH of Moudjehara in the Wilaya of Djelfa in Algeria. To prevent unplanned outages, reduce the time required for maintenance, and optimize the operating time of each unit by deciding to intervene just in time. This fuzzy reliability approach guided by the operating data of this rotating machine is based on the modeling of adaptive neuro-fuzzy inference system, optimizes the reliability of the examined system, and materialized by their implementation to optimize maintenance and provide engineering advice and for improving the productivity of the gas turbine system.
Indeed, several works have been carried out in the industrial literature to develop robust reliability methods in industrial installations. Recently, in 2021 Chen et al. [1] have proposed a sequential approach based on a model for the diagnosis of gas turbine performance, for the modeling of progressive degradation of gas turbine components. This improves their availability, reliability and operating cost. This method effectively deals with the presence of random noise in the measurements and supports a significantly lower computational load compared to existing methods, to improve the reliability and energy efficiency of gas turbines. And Li et al. [2] estimated the fatigue reliability of a turbine using collaborative multi-agent modeling, to improve the calculation precision and the efficiency of the simulation of fatigue reliability estimation for the turbine rotor, this modeling approach makes it possible to have high efficiency and high precision for estimating the fatigue reliability of the studied turbine rotor. Also, Kiaee and Tousi [3] determined the deterioration indices based on a modeling of the prognosis of the gas path of gas turbines, this study allows to improve knowledge on the monitoring of the state of the studied turbine with an increase of 3.29%, and the average remaining useful life of this turbine.

Okpokparoro et al. [4] carried out an uncertainty modeling in the reliability analysis of the support structures of floating wind turbines, to test the load effects such as stresses and strains. This helps to underline the importance of integrating uncertainty and ensuring constant levels of reliability leading to cost reductions. And Wilkie and Galasso [5] have proposed a fatigue reliability analysis of offshore wind turbines, using Gaussian process regression, for the assessment of fatigue damage over the expected design lifetime of this system. This improves the sensitivity of various goodness-of-fit measures of the model and further reduces the computational effort required to perform regressions / predictions. Although, Carapellucci et al. [6] have improved the performance of regenerative gas turbines and steam injection with combined cycle, to improve the capacity of these systems, this allows a maximum power increase over 70%, while offering remarkable energy and economic performance.

In 2020, Lu et al. [7] proposed a substitution modeling strategy of the mobile extremum for the estimation of the dynamic reliability of turbine with multi-physical fields, to improve the dynamic reliability analysis of complex structures, this modeling strategy allows to manage the input and output parameters of this dynamic process, to improve their efficiency with an accuracy of performing the estimation of this dynamic reliability. And Dong et al. [8] analyzed the structural reliability of the contact fatigue design of gears in wind turbine drive trains, to understand their performance. Hence, the sensitivity of these reliability indices is used for the fatigue analysis of the contacts of gears based on estimated reliability. Also, Ivanhoe et al. [9] evaluated the reliability of offshore wind turbine windmill support structures under stochastic and time-dependent variables, in order to systematically consider these uncertainties in order to optimize these structures based on the studied reliability. And Gao et al. [10] proposed an integrated reliability approach with importance sampling, to improve the prediction of low cycle fatigue damage of turbine disks. This helps to reduce the computational load and improve the accuracy of the turbine disk damage reliability model. Hence, Velarde et al. [11] investigated the fatigue reliability of large single-stacks for offshore wind turbines, for scaling traditional single-stacks to support larger offshore wind turbines presents fatigue-related design challenges. This helps to identify the technical limits and the economic viability of moving upmarket mono-batteries.

These methods are based on the different information available to describe the behavior of the systems. The essential principle in this carried out work consists in detecting and
locating the internal faults and the external faults of the system operations, then in estimating the reliability characteristics in these industrial processes. This is to increase the ability to withstand failures and loads experienced during the operation of these processes.

This work concerns the study of a modern approach of reliability modeling based on the combination of neural networks and fuzzy systems, which adapts to the determination of reliability indicators, such as; mean time between failures (MTBF), failure rate and equipment availability, where the primary aim is the estimation of the availability in the studied turbine. To deal with these degradation problems, several tests were carried out to validate the proposed approach using the Weibull distribution. The obtained results are satisfactory and prove that the approach used can be a promising field of research in future reliability work.

2. Two Shafts gas Turbine Solar TITAN 130

Reliability analysis has become an important topic on production quality and safety in modern industrial facilities. Hence, the traditional theory of reliability conceives of two possible states for the system, the first is the operating state and the second is the failure state, in a binary coherence structure [12–16]. But in practice, it is often the case that the studied systems present several states (normal operation, operation in degraded mode, weighted operation, shutdown, emergency stop and other operating situations). In this sense, fuzzy inference systems are essential to quantify these situations, from experimental data, and to approximate the reliability indicators of this system, by integrating the imprecisions of human reasoning in the form of linguistic rules and variables. This work proposes a new approach to determine reliability indicators based on an adaptive neuro-fuzzy inference system applied to describe the behavior of a complex gas turbine system. However, several models have been developed in several applications, to identify the gas turbine behaviors, where the complexity and behavior of gas turbine systems increase the difficulty in obtaining a reliable model for this type of machine and get the time to have data and use the experience feedback. To perform reliable monitoring or to define indicators for monitoring by relevant states of this machine, it is not enough to have equipment suitable for this type of survey; the skills necessary for implementation must be acquired.

In industrial reality, the analysis and evaluation of gas turbine reliability approaches is used to avoid failures and damage, with the aim of increasing their availability and productivity [17–21]. These types of machines are used in many gas compressor stations in the oil and gas industry. They are always subject to degradation due to their harsh operating conditions and environment; these degradations are induced by several parameters which can lead to the failure of these machines. The objective of this work is to increase the efficiency of a gas compression plant, by preventing gas turbine failures, to ensure optimum availability of the turbine under consideration. The established reliability assessment models are essential to optimize decision-making and the implementation of maintenance policies in the studied gas turbine. The examined turbine in this work is a twin-shaft Solar TITAN 130 turbine, is designed for mechanical drive applications with a wide range of operating speed to meet the operating conditions of the most common driven equipment, such as centrifugal compressors. This turbine is used as a hard gas turbocharger in the Medjebara gas compressor station, located in the Djelfa region of Algeria. As shown in Figure 1, the characteristics of this gas turbine are shown in Table 1.
Figure 1. Gas turbine Solar TITAN 130.

Table 1. Gas turbine Solar TITAN 130 characteristics.

| Quantity             | Value                                      |
|----------------------|--------------------------------------------|
| Output Power         | 15,290 kW (20,500 hp)                      |
| Heat Rate            | 9940 kJ/kW-hr (7025 Btu/hp-hr)             |
| Exhaust Flow         | 180,050 kg/hr (396,940 lb/hr)              |
| Exhaust Temperature  | 505°C (940°F)                              |
| Max Speed            | 8855 rpm                                   |

The model parameters of the gas-examined turbine were identified with a series of operational data during start-up, shutdown and during phases of normal operation. This identification is necessary for the overall breakdown of the gas turbine system into several subsystems including the axial compressor, high pressure (HP) and base pressure (LP) turbine and the exhaust system. Hence, the reliability tests were carried out on the basis of the historical gas turbine data of failure events and overhauls, given in Table A1 in the appendix, occurring during the operation of this turbine during two years of operation (2018 and 2019). This provides operating data to quantify the reliability of this turbine. As a result of this work, the reliability modeling of twin-shaft Solar TITAN 130, using an approach based on an adaptive neuro-fuzzy inference system will be performed, compared with the usual reliability laws, using real data collected from the operation of this machine.

2.1. Gas Turbine TITAN 130 Performance Indices

In practice, reliability modeling consists of bringing together the knowledge available on turbine behavior, from experiments and/or theoretical analysis of the physical phenomena
Figure 2. Monitoring structure of the TITAN 130 gas turbine.

involved in the operation of this machine. This knowledge leads to approximate the reliability variables and to make these factors exploitable for an effective maintenance or monitoring strategy. Knowledge models are based on system input / output variables, from which a representation of failure states can be adopted for this type of reliability model. Indeed, the complexity of modeling their reliability increases further if the gas turbine operating life increases. Practically, gas turbines have a long service life and many changes occur during the life of these machines, these operating conditions motivate specialists to analyze and study the reliability of this type of machines, to prevention against operational failures.

In order to model the behavior of the TITAN 130 gas turbine by applying fuzzy techniques, the proposed monitoring structure is shown in Figure 2. In this study, a two-year history dataset derived from real data operating parameters are used for learning and identifying the reliability parameters and are used to validate the performance of the obtained fuzzy reliability model.

From turbine operating data, performance indicators provide a clear view of the situation in terms of turbine efficiency and productivity, to improve their availability for good overall efficiency of the equipment. However, the overall equipment efficiency (OEE) determined for the TITAN 130 turbine on the operating history of this machine, within the station from 1 January 2018 until 31 March 2019, is shown in Figure 3. During this period, the OEE was found to be low between 0.8672 and 0.9171, which explains the average of 0.8922, this is mainly due to loss of settings, breakdowns and other shutdowns which negatively influence the machine performance. With the Pareto failure analysis given in Figure 4, the
relationship between zones A, B and C shows that most of the problems in zone A are problems related to the lubrication circuit and the air circuit, zone B shows the problems related to the communication module and the pump and cleaning and for zone C shows the shutdown problems (change of sensor and flame detector, tightening of bolts, change of batteries, change of bottles (CO2), chimney welding).

The overall equipment efficiency (OEE) determined for the TITAN 130 turbine can be used to provide the parameters of reliability laws, such as failure rate, predicted reliability, average uptime and instantaneous failure rate for this turbine system. This justifies the
development of a new reliability approach based on adaptive neuro-fuzzy inference system functions, compared by a Weibull distribution.

2.2. Monitoring of TITAN 130 Turbine based on Failure Mode Strategy

Given the complexity of the TITAN 130 turbine and their severe operating constraints increase, due to operating conditions and/or the environment; Wear, fatigue, aging, with their use of added values on their monitoring strategy. In this context, a modeling and identification approach integrating artificial intelligence tools based on neuro-fuzzy approach is proposed, as shown in Figure 5, with the aim of implementing preventive maintenance intended for the monitoring of the failure modes of this turbine. To avoid deterioration of their performance, guarantee the supervision of these examined gas turbines. This made it possible to synchronize the monitoring actions of this rotating machine and to analyze their behavior based on the obtained failure rates for this machine.

According to the historical data of interventions on the TITAN 130 gas turbine, given in Table 2, relating to events (failures) that occurred during the operation of this equipment. The realized approach consists in observing for a certain operating time, under real conditions of use, a gas turbine in which we are interested and in listing all the failures that arise and the information relating to them (TBF, TTR). The basic data are thus obtained which make it possible to quantify the reliability of the gas turbine in question.

According to the test of Kolmogorov Smirnov, given by [18]:

\[ D_{N \text{-} \text{max}} < D_{n \alpha} \]  \hspace{1cm} (1)

This means that the Weibull model is accepted, and that the maximum value is taken into account by:

\[ D_{N \text{-} \text{max}} = |F(i) - F(t)| = 0.1204 \text{ while } D_{n\alpha} = D_{15.20} = 0.266 \]  \hspace{1cm} (2)

Therefore, the hypothesis of the Weibull model is acceptable and the law is validated for this study case.

| Rank | TBF(h) | N | \( \sum n_i \) | \( F_i \) | \( F(t) \) | \( D_{N \text{-} \text{max}} \) |
|------|--------|---|--------------|------|------|------------------|
| 1    | 93     | 1 | 1            | 0.0454 | 0.0993 | 0.0539           |
| 2    | 138    | 1 | 2            | 0.1103 | 0.1486 | 0.0383           |
| 3    | 164    | 1 | 3            | 0.1753 | 0.1764 | 0.0011           |
| 4    | 184    | 1 | 4            | 0.2402 | 0.1975 | 0.0427           |
| 5    | 188    | 1 | 5            | 0.3051 | 0.2017 | 0.1034           |
| 6    | 235    | 1 | 6            | \textbf{0.3701} | \textbf{0.2497} | \textbf{0.1204} |
| 7    | 449    | 1 | 7            | 0.4350 | 0.4412 | 0.0062           |
| 8    | 476    | 1 | 8            | 0.5   | 0.4621 | 0.0379           |
| 9    | 507    | 1 | 9            | 0.5649 | 0.4854 | 0.0795           |
| 10   | 641    | 1 | 10           | 0.6298 | 0.5759 | 0.0539           |
| 11   | 714    | 1 | 11           | 0.6948 | 0.6189 | 0.0759           |
| 12   | 1166   | 1 | 12           | 0.7597 | 0.8073 | 0.0476           |
| 13   | 1573   | 1 | 13           | 0.8246 | 0.8979 | 0.0733           |
| 14   | 1980   | 1 | 14           | 0.8896 | 0.9467 | 0.0571           |
| 15   | 2102   | 1 | 15           | 0.9545 | 0.9563 | 0.0018           |
3. Turbine Reliability Modeling based on Adaptive Neuro-fuzzy Inference Systems

Maintaining industrial facilities is an essential activity in the search for performance in terms of reliability, maintainability, availability and safety. Faced with this growing complexity of industrial installations and the problems encountered in operational safety and supervision, this section offers a modern approach to modeling gas turbine reliability based on
adaptive neuro-fuzzy inference systems, to determine the reliability parameters; the reliability function, the failure rate, the indicators (MTBF, MTTR). Then validation analyzes on the different reliability indicators will be done by estimating the Weibull distribution with the use of the least squares’ method. This is because the adaptive neuro-fuzzy inference system combines the concepts of fuzzy logic and neural networks to form a hybrid intelligent system, that automatically increases the capacity for learning and adaptation. In this work, to simulate the dysfunctional behaviors of a gas turbine system, the capitalization of the knowledge obtained by the neuro-fuzzy system of each of the components of the system is used, to evaluate and validate their reliability using the Weibull distribution. In order to get to the examined gas turbine reliability data analysis-based operational safety analysis, the proposed adaptive neuro-fuzzy inference system structure is shown in Figure 6. This structure is composed of five layers, for which each layer corresponds to the realization of a step of a fuzzy inference system of the Takagi–Sugeno type, to determine the parameters (η, β and y), which define the indicators of reliability and will be used to identify the associated distribution of Weibull to the examined turbine.

3.1. Adaptive Neuro-fuzzy Inference System (ANFIS)

The adaptive neuro-fuzzy inference system approach seeks to optimize the reliability of the gas turbine system for the development of maintenance actions associated with this facility.

**Figure 6.** Structure of proposed adaptive neuro-fuzzy inference system.
However, reliability is dependent on the operating interactions between the various process use events, hence, the reliability study attempts to limit the adverse effects of these factors by improving the understanding of these phenomena and events. Accordingly, the combination of neural networks and fuzzy logic can be drawn from the advantages of both methods, by the learning capacities of neural networks and the readability and flexibility of fuzzy logic [22–26]. The proposed adaptive neuro-fuzzy inference system is based on the automatic generation of fuzzy rules based on the Takagi–Sugeno inference model. This refines the fuzzy rules already established by human experts and adjusts the overlap between different fuzzy sets to describe the input-output behavior of the turbine system under consideration.

The reliability algorithm is based on a collection of rules $R_i$ with an input $x$ and output $f$, of the form If–Then, given by:

$$R_i: \text{If } X \text{ is } A_i \text{ Then } f(x) = p_i x + q_i y + r_i$$  \hspace{1cm} (3)

To demonstrate the basic architecture of the adaptive neuro-fuzzy model, it is considered to be a kind of first order Takagi–Sugeno fuzzy inference system [27–29]. Assume that there are two linguistic variables input $x_1$ and $x_2$ with output, and assume that the rule base contains two types of rules:

Rule 1: if $x_1$ is $A_1$ and $x_2$ is $B_1$ then $f_1 = p_1 x_1 + q_1 x_2 + r_1$

Rule 2: if $x_1$ is $A_2$ and $x_2$ is $B_2$ then $f_2 = p_2 x_1 + q_2 x_2 + r_2$  \hspace{1cm} (4)

where $x_1$ and $x_2$ are the input variables, $A_1, A_2, B_1$ and $B_2$ are the fuzzy sets, $y_i$ are the outputs of all neurons of defuzzification, $p_i, q_i$ and $r_i$ are the parameters of the rule consequent determined during the learning process.

The adaptive neuro-fuzzy inference system is a multi-layered network whose connections are not weighted, or all have a weight of 1. On the other side, there are two different types of nodes depending on their functionality and are shown in Figure 7 with two

![Figure 7. Structure of the adaptive neuro-fuzzy inference system (ANFIS).](image-url)
different shapes (square and circle). The square nodes contain the parameters and the circular node is empty, the output $o_{ik}^k$ of the layer $i$ in the node $k$ (called nodes $(i, k)$) depends on the parameters of the signals input from the layer $k - 1$ and node $(i, k)$, it can be expressed as follows:

$$o_{ik}^k = f(o_{ik-1}^{k-1}, \ldots, o_{i_n-1}^{k-1}, a, b, c, \ldots)$$  \hspace{1cm} (5)

where $n_{k-1}$ is the number of nodes in the layer $k - 1$.

**3.1.1. The First Layer of Fuzzification**

The first layer of the adaptive neuro-fuzzy inference system architecture expresses the conditions of membership, allows to perform the role of fuzzification. Each node is more or less activated depending on the degree of membership in a given fuzzy subset (the number of nodes associated with each entry is the number of fuzzy subsets that define this fuzzy variable). Hence, the activation function $f_i$ of the neurons $i$ of the first layer is given by:

$$f_i = \mu_{Ai}(X)$$  \hspace{1cm} (6)

The membership function $\mu_{Ai}(X)$ is chosen Gaussian type with the maximum equal to 1 and the minimum equal to 0, where this generalized function is of the following form:

$$\mu_{Ai}(X) = \exp \left[ -\frac{(X - c)^2}{\sigma^2} \right]$$  \hspace{1cm} (7)

Each node of this layer is a function of the input variables $x_1$ or $x_2$, given by:

$$\begin{cases} 
O_1^i = \mu_{Ai}(x_1) & \text{for } i = 1, 2 \\
O_1^i = \mu_{Bi-2}(x_2) & \text{for } i = 3, 4 
\end{cases}$$  \hspace{1cm} (8)

where $x_1$ and $x_2$ are the inputs of the nodes respectively $\{1, 2\}$ and $\{3, 4\}$, $A_i$ and $B_{i-2}$ are the linguistic terms associated with the membership functions $\mu_{Ai}$ and $\mu_{Bi-2}$. The outputs $O_1^i$ of the first layer represent the degrees of membership of the input variables $x_1$ and $x_2$ to the fuzzy set $A_i$ and $B_{i-2}$. In the Jang model given in [30], the membership functions are Gaussian, as shown in Figure 8, are expressed by:

$$\mu_{Ai}(x_1) \text{ or } \mu_{Bi}(x_1) = \frac{1}{1 + \left( \frac{x_1 - c_i}{a_i} \right)^2 o_j}$$  \hspace{1cm} (9)

where $\{a_i, b_i, c_i\}$ is the set of parameters.

The parameters of this layer are called premise parameters and the value of the corresponding membership function changes depending on the changes in values, in the studied case of gas turbine, the Gaussian membership function is used for the identification of these settings.

**3.1.2. The Second Layer of Fuzzy Rules**

The second layer is formed by a node for each fuzzy rule and generates the synaptic weights, these types of fixed nodes are noted $\Pi$. And each node among them generates an output
Figure 8. Gaussian membership function.

presented by the product (operator AND fuzzy logic) of all its inputs, which corresponds to the degree of membership of the rule concerned, given by:

$$O_i^2 = w_i = \mu_{A_i}(x_1) \cdot \mu_{B_i}(x_1) \text{ for } i = 1, 2$$  \hspace{1cm} (10)

3.1.3. The Third Layer of Normalization

The nodes of this layer are fixed nodes, its role is to ensure the normalization of the weights of the fuzzy rules according to the following relation:

$$O_i^3 = \bar{w}_i = \frac{w_i}{w_1 + w_2} \text{ for } i = 1, 2$$  \hspace{1cm} (11)

Each $i$ node in this layer is a circular node and noted $N$, where the output $i$ of the node is the degree of activation of the standard rule $i$.

3.1.4. The Fourth Layer of Defuzzification

Each node in this layer is adaptive and calculates the rule outputs based on the following node function:

$$O_i^4 = \bar{w}_i f_i = \bar{w}_i (p_i x_1 + q_i x_2 + r_i) \text{ for } i = 1, 2$$  \hspace{1cm} (12)

The parameters $\{p_i, q_i, r_i\}$ are the output parameters of the rule and are called consequential parameters.

3.1.5. The Fifth Layer of the Sum

The fifth layer has a single neuron that provides the adaptive fuzzy-neuro inference system (ANFIS) output by calculating the sum of the outputs from the previous layer that present
all incoming signals to the considered node. Its output is calculated based on the following relationship:

\[ O_i^S = y = \sum_{i=1}^{\infty} \bar{w}_i f_i = \frac{\sum w_i f_i}{\sum w_i} \text{ for } i = 1, 2 \]  

(13)

The adaptive neuro-fuzzy inference system (ANFIS) architecture shows the existence of two adaptive layers, the first and the fourth; the first layer has three adjustable parameters \( \{ a_i, b_i, c_i \} \) related to the functions of the input affiliations, called premise parameters and the fourth layer also contains three editable parameters \( \{ p_i, q_i, r_i \} \) called consequent parameters.

As a synthesis of the second, third and fourth hidden layer and the adaptive neuro-fuzzy inference system (ANFIS) output layer is simplified as follows:

\[
\begin{align*}
\text{Step 2: } W_k &= \mu_{A_i}(X) \\
\text{Step 3: } \bar{W}_k &= \frac{W_k}{\sum_{i} W_i} \\
\text{Step 4: } f^4_k &= \bar{W}_k \times f_k = \bar{W}_k \times (p_k x + q_k y + r_k) \\
\text{Step 5: } f^5_k &= \sum_{k} \bar{W}_k \times f^4_k
\end{align*}
\]

(14)

### 3.2. Intelligent Reliability Modeling Mechanism

For the identification of turbine system reliability, using the mechanism of learning neural networks on fuzzy inference techniques, two variables are used; the TBF uptime and reliability density \( F(t) \), their obtained membership functions are shown in Figures 9 and 10. As well as two output variables (Xaxis and Yaxis in cm), are used to determine the selected

![Figure 9. Gaussian-type ANFIS_1 membership functions.](image-url)
Figure 10. Gaussian-type ANFIS_2 membership functions.

Figure 11. Xaxis output variation using ANFIS_1.

dimensions \((x, y)\), their variations are shown in Figures 11 and 12. These variables are sufficient to adjust the architecture of the studied gas turbine reliability modeling systems, the system of two equations describing the input-output relationships is expressed by:

\[
\begin{align*}
Xaxis &= ANFIS_1(TBF) \\
Yaxis &= ANFIS_2(F(t))
\end{align*}
\]  
(15)
Subsequently, the least squares regression line technique is used to plot the mean line of the turbine reliability data points. Given a collection of pairs \((x, y)\) of numbers, which all \(x\) values are not the same, there is a row \(\hat{y} = \hat{\delta}_1 \times x + \hat{\delta}_0\) that best matches the data in the sense of minimizing the sum of squared errors, its slope \(\hat{\delta}_1\), and its \(y\)-intercept \(\hat{\delta}_0\) are calculated using the following formula:

\[
\begin{align*}
\hat{\delta}_1 &= \frac{QQ_{xy}}{QQ_{xx}} \\
\text{and} \quad \hat{\delta}_0 &= \bar{y} - \hat{\delta}_1 \bar{x}
\end{align*}
\]

Hence \(QQ_{xy}\) and \(QQ_{xx}\) are generalized functions, given as follows:

\[
\begin{align*}
QQ_{xy} &= \sum xy - \frac{1}{n} \left( \sum x \right) \left( \sum y \right) \\
\text{and} \quad QQ_{xx} &= \sum x^2 - \frac{1}{n} \left( \sum x \right)^2
\end{align*}
\]

where \(n\) is the number of pairs in the reliability data set.

The least squares regression line (Lsline) is calculated for the dataset have 15 points, the Figure 13 clearly shows the obtained regression line, using the ANFIS_3 model, shown in Figure 14. Using this model of the Weibull distribution, to determine the shape parameter \(\beta\) of the model, this variation is shown in Figure 15, assuming that \(\beta = 1.0888\) is the best-obtained value for this model. This is for the purpose of determining exactly the shape parameter value \(\beta\) with the use of the angle as input from ANFIS system of the Weibull distribution, given by:

\[
\beta = ANFIS_3(\theta)
\]
To determine the scale parameter $\eta$ of Weibull distribution, an ANFIS_4 system is proposed, as shown in Figure 16, which indicates the order of magnitude of the mean life from the time scale of the axis of $Xaxis$, the least squares regression line and the angle $\theta$ is used, to
Figure 15. Shape parameter variation of Weibull distribution.

determine the position of 63.2% reliability density $F(t)$, as follows:

\[
\eta = \text{ANFIS}_4 \left( \left( \frac{(19.4 - Y_{\text{min}})}{\tan(\theta)} \right) + X_{\text{min}} \right) \\
\text{with} \\
F(t) = 1 - e^{\left( \frac{\eta}{\beta} \right)^\beta} = 1 - e^{-1} = 63.2\% \rightarrow Y_{\text{axis}} = 19.4\text{cm}
\]

As a comparison, the two experimental and model responses are plotted at the same time, shown in Figure 17, defining the error represented by the deviation between the observed behavior of the system and the reference behavior expected by the ANFIS_4 model. Hence, the scale parameter value $\eta = 737.799$ is obtained, its variation is shown in Figure 18.

3.3. Learning Algorithm of the ANFIS System

Learning from a set of data is all about identifying the premises and parameters of consequences, where the network structure is supposed to be fixed. The learning algorithm starts by building an initial network, then an error back-propagation learning method is applied. Jang proposed a hybrid learning rule that combines a gradient descent algorithm with least squares estimation, which can be expressed as follows [30,31]:

\[
f = \tilde{w}_1 f_1 + \tilde{w}_2 f_2
\]

Based on the above definition of $f_1$ and $f_2$, this expression can be rewritten as:

\[
f = (\tilde{w}_1 x_1) p_1 + (\tilde{w}_1 x_2) q_1 + \tilde{w}_1 r_1 + (\tilde{w}_2 x_1) p_2 + (\tilde{w}_2 x_2) q_2 + \tilde{w}_2 r_2
\]

where $f$ is the activation function.
It is a linear combination of substantial changeable parameters \( \{p_1, q_1, r_1, p_2, q_2 \text{ and } r_2 \} \).

Note that in this algorithm the premise and the consequent parameters are optimized, this proposed structure, shown in Figure 19, uses two types of passes to modify these parameters.

### 3.3.1. Forward Pass

To go forward, the method of least squares is applied for the identification of the consequent linear parameters. For all local (nonlinear) parameters, training data can be used, so a system of equations is obtained:

\[
AX = B
\]  

(22)

where \( x \) is the matrix containing the unknown parameters in the set of consequent parameters, it is about a linear problem, the optimal solution \( X \) corresponds to the minimum of \( \|AX - B\|^2 \), given by:

\[
X^* = (A^T A)^{-1} A^T B
\]  

(23)

with \( S_1 \) is fixed and \( S_2 \) is calculated using the least squares error algorithm (LSE: least squares estimate), \( S_1 \) is the premise parameters and \( S_2 \) represents the consequent parameters.
Figure 17. Difference between the observed behavior and the reference behavior of the system.

Table 3. Adjustment of the ANFIS parameters.

|                  | Passage Forward | Back Passage  |
|------------------|-----------------|---------------|
| Premise parameters | Fixed           | Retro-propagation |
| Consistent parameters | Least Squares   | Fixed          |

3.3.2. Back Pass
In the back pass, error signals propagate backwards, the premises parameters are adjusted by the descent method. Hence $S_2$ is fixed and $S_1$ is calculated using the back-propagation algorithm based on the adjustment parameters for the proposed ANFIS as shown in Table 3.

4. Investigation Results
To determine the failure rate of examined gas turbine system in this work, adopting a hybrid neuro-fuzzy approach, with a view to its application for the reliability assessment of this rotating machine, using their inputs / outputs operating variables. Using the structure of the first model of adaptive neuro-fuzzy system ANFIS_1, the learning of the ANFIS system developed, where is a step which consists of network training with the adjustment of the premises and the consequent obtained parameters for the analysis of turbine reliability parameters.
The obtained reliability results make it possible to carry out the operational evaluation of the reliability of the studied turbine in operation, the estimation of reliability is made from the operating data of this machine. This practically allows their malfunctions to be corrected, after the collection of validated and developed data after the expertise and treatment process, relating to feedback, to develop a knowledge base on turbine operation. The proposed reliability process makes it possible to exploit three input variables ($\eta$, TBF and $\beta$) to generate the reliability parameters of the examined turbine as output with their associated errors; The distribution function or the probability of failure $F(t)$ shown in Figure 20, the reliability function $R(t)$ shown in Figure 21, the failure rate $\lambda(t)$ shown in Figure 22 and the probability density function $f(t)$ shown in Figure 23.

Modeling and estimating turbine reliability from a neuro-fuzzy system, makes it possible to build a database which serves as a decision-making in the maintenance of this machine. Thus, that these obtained reliability results are more precise and realistic estimates.

4.1. Validation of Obtained Results

The Weibull model is applied to the examined gas turbine to validate and confirm the results of the neuro-fuzzy approach proposed previously. Indeed, the Weibull distribution
Figure 19. Learning algorithm of the ANFIS system.

Figure 20. Obtained probability of failure $F(t)$ and their associated error.
Figure 21. Obtained reliability function $R(t)$ and their associated error.

is often used in several industrial applications; It characterizes well the behavior of equipment in the three phases of life, presented independently, according to the value of the shape parameter $\beta$, for the period of youth $\beta < 1$, period of useful life $\beta = 1$ and for the period of wear or aging $\beta > 1$. However, the Weibull law is defined by two parameters $\eta$ and $\beta$, it is characterized by the failure rate, given by [32]:

$$\lambda(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1}$$ \hspace{1cm} (24)

The general form of the reliability function $R(t)$ representing the probability of the times between failures over time $t$, given by [33]:

$$R(t) = e^{-\left( \frac{t}{\eta} \right)^{\beta}}$$ \hspace{1cm} (25)

The distribution function $F(t)$ is the probability of failure, expressed by:

$$F(t) = 1 - R(t) = 1 - e^{-\left( \frac{t-\gamma}{\eta} \right)^{\beta}}$$ \hspace{1cm} (26)

Hence, the probability density function $f(t)$ is calculated by the following expression [21,34]:

$$f(t) = \lambda(t) \times R(t) = \frac{\beta}{\eta} \left( \frac{t-\gamma}{\eta} \right)^{\beta-1} \cdot e^{\left( \frac{t-\gamma}{\eta} \right)}$$ \hspace{1cm} (27)
Figure 22. Obtained failure rate $\lambda(t)$ and their associated error.

Figure 23. Obtained density function $f(t)$ and their associated error.
where $\eta$ is the scale parameter, $y$ is the location parameter and $\beta$ is the shape parameter of the Weibull distribution.

The Weibull paper plot, shown in Figure 24, allows to graphically read the Weibull distribution parameters, in the case where the parameter $y$ is zero. This makes it possible to determine the Weibull distribution function associated with the parameters $\eta$, $\beta$ and $y = 0$, defined by:

$$ F(t) = 1 - e^{\left(\frac{t}{\eta}\right)^{\beta}} \Rightarrow \ln(1 - F(t)) = -\left(\frac{t}{\eta}\right)^{\beta} $$

$$ \Rightarrow -\ln(1 - F(t)) = \left(\frac{t}{\eta}\right)^{\beta} \Rightarrow \ln(-\ln(1 - F(t))) = \beta \ln \frac{t}{\eta} \quad (28) $$

$$ \Rightarrow \ln(-\ln(1 - F(t))) = \beta \ln t - \beta \ln \eta $$

$$ \Rightarrow Y = \beta X - \beta \ln \eta $$
4.2. Validation Tests

To justify that the operating time of the gas turbine follows a Weibull distribution, the model fit test via the least squares’ method is applied, the actual distribution function if the number of failures \( n \preceq 20 \) is calculated based on the following equation [8,35]:

\[
F_e(t_i) = \frac{\sum n_i - 0.3}{N + 0.4}
\]  

(29)

If the number of failures is \( 20 \leq n \leq 50 \), the distribution function used is:

\[
F_e(t_i) = \frac{\sum n_i}{N + 1}
\]  

(30)

Finally, if the failure count is \( n > 50 \), the following equation is used:

\[
F_e(t_i) = \frac{\sum n_i}{N}
\]  

(31)

However, the method of least squares estimation of the parameters is used, exploiting the following regression equations:

\[
\begin{align*}
\hat{a} &= \frac{\sum_{i=1}^{N} y_i}{N} - \hat{b} \frac{\sum_{i=1}^{N} x_i}{N} = \bar{y} - \hat{b} \bar{x} \\
\hat{b} &= \frac{\sum_{i=1}^{N} x_i y_i - \frac{\sum_{i=1}^{N} x_i \sum_{j=1}^{N} y_j}{N}}{\sum_{i=1}^{N} x_i^2 - \frac{(\sum_{i=1}^{N} x_i)^2}{N}}
\end{align*}
\]  

(32)

In this case, \( y_i \) and \( x_i \) can be presented as follows:

\[
\begin{align*}
y_i &= \ln(-\ln(1 - F(t_i))) \\
x_i &= \ln(t_i)
\end{align*}
\]  

(33)

The obtained results using the applied Weibull model to the examined gas turbine are shown in Table 4 and the results of the Kolmogorov Smirnov test are given in Table 5.

The values of \( F(T_i) \) are estimated from the median ranks, \( y_i \) and \( x_i \) are obtained and \( \hat{b} \) with \( \hat{\eta} \) can be easily calculated from the equations above, based on the values of the least squares analysis given in Table 5, and can be calculated using Equations (32) and (33), then, \( \hat{\beta} = \hat{b} = 1.089 \) and \( \hat{\eta} = e^{-\hat{a}/\hat{b}} = 738.01 \) hours, obtained as follows:

\[
\begin{align*}
\hat{a} &= \frac{\sum_{i=1}^{15} y_i}{15} - \hat{b} \frac{\sum_{i=1}^{15} \ln t_i}{15} = \bar{y} - \hat{b} \bar{x} = -7.1982 \\
\hat{b} &= \frac{\sum_{i=1}^{15} (\ln t_i) y_i - \frac{\sum_{i=1}^{15} \ln t_i \sum_{j=1}^{15} y_j}{15}}{\sum_{i=1}^{15} (\ln t_i)^2 - \frac{(\sum_{i=1}^{15} \ln t_i)^2}{15}} = 1.089
\end{align*}
\]  

(34)

The calculation of the variance is given by Equation (35), as follows [36]:

\[
\sigma^2 = \eta^2 \left[ \Gamma \left( \frac{2}{\beta} + 1 \right) - \Gamma^2 \left( \frac{1 + \beta}{\beta} \right) \right]
\]  

\[
\sigma = \eta \cdot B
\]  

(35)
### Table 4. Least squares analysis results.

| N  | \( T_i \) | \( \ln(T_i) \) | \( F(T_i) \) | \( y_i \) | \( (\ln(T_i))^2 \) | \( y_i^2 \) | \( \ln(T_i) \cdot y_i \) |
|----|----------|----------------|--------------|--------|----------------|--------|-------------------|
| 01 | 93       | 4.532          | 0.0993       | −2.257 | 20.54          | 5.097  | −20.3056          |
| 02 | 138      | 4.927          | 0.1486       | −1.827 | 24.27          | 3.338  | −16.7120          |
| 03 | 164      | 5.099          | 0.1764       | −1.639 | 27.195         | 2.292  | −14.3758          |
| 04 | 184      | 5.214          | 0.1975       | −1.514 | 27.420         | 2.221  | −13.1309          |
| 05 | 188      | 5.236          | 0.2017       | −1.490 | 27.420         | 2.221  | −13.1309          |
| 06 | 235      | 5.459          | 0.2497       | −1.247 | 29.807         | 1.555  | −11.9376          |
| 07 | 449      | 6.107          | 0.4412       | −0.541 | 37.295         | 0.293  | −10.0503          |
| 08 | 476      | 6.165          | 0.4621       | −0.477 | 38.012         | 0.228  | −10.0503          |
| 09 | 507      | 6.228          | 0.4854       | −0.408 | 38.794         | 0.1672 | −9.09776          |
| 10 | 641      | 6.463          | 0.5759       | −0.153 | 41.770         | 0.0235 | −8.04667          |
| 11 | 714      | 6.570          | 0.6189       | −0.035 | 43.176         | 0.0012 | −7.07326          |
| 12 | 1166     | 7.061          | 0.8073       | 0.498  | 49.862         | 0.248  | −6.07083          |
| 13 | 1573     | 7.360          | 0.8979       | 0.824  | 54.180         | 0.680  | −6.62610          |
| 14 | 1980     | 7.590          | 0.9467       | 1.075  | 57.621         | 1.156  | −6.2411           |
| 15 | 2102     | 7.650          | 0.9563       | 1.141  | 58.532         | 1.302  | −5.6654           |
| ∑  | 82,104   | 294.1906       | /            | −21.8262 | 2189,1262    | 64.0239 | −109.1789         |

### Table 5. Comparative study of Weibull distribution parameters of the examined turbine.

| System                          | Weibull distribution parameters |
|---------------------------------|---------------------------------|
| Weibull paper                   | \( \beta = 1.09 \), \( \eta = 737.88 \) |
| ANFIS expert system             | \( \beta = 1.0888 \), \( \eta = 737.799 \) |
| Least squares estimation        | \( \beta = 1.089 \), \( \eta = 738.01 \) |

The average time between failures

\[
MTBF = \int_0^\infty t \cdot F(t) = \gamma + \eta A = 711.9804 \text{ h}
\]

Root-mean-square error

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (\hat{F}(t_i) - F(t_i))^2}{n}} = 0.3060
\]

**Performance Parameters of TITAN 130 Gas Turbine**

- Coefficient of variation: \( CV(RMSE) = \frac{RMSE}{\bar{y}} = 0.5698 \)
- Mean time failure: \( TMP = 6 \text{ h} \)
- Average reliability: \( \frac{MTBF}{(TMP + MTBF)} = 99.16\% \)

While the mean time between failures (MTBF) is calculated on the basis of the following expression [37]:

\[
MTBF = \int_0^\infty tF(t) = \gamma + \eta A \tag{36}
\]

The functioning of the model established for the Weibull distribution depends on three parameters: \( \beta > 0 \) is the shape parameter, \( \gamma \geq 0 \) is the location parameter and \( \eta > 0 \) is the scale parameter.

The determined model allows the estimation of the reliability of the considered gas turbine is validated in adequacy with the test model, to validate the developed model, it is recommended to apply the Kolmogorov–Smirnov test, for this test if \( D_{N, \alpha} < D_{n, \alpha} \) the model is accepted and if \( D_{N, \alpha} > D_{n, \alpha} \) the model is rejected. It is clear that for the
developed model of studied gas turbine $D_{N.\text{max}} = 0.1204$ and $D_{nr} = 0.266$, which means that it is acceptable.

Indeed, the growing complexity of industrial systems has a major impact on the proper operation of these gas turbine installations, the developed fuzzy approach in this work has made it possible to model the effects of failures, to predict optimal operating performance of the examined gas turbine.

5. Conclusion

The assessment of the reliability of industrial equipment is essential for the design of increasingly efficient systems. Hence, the choice of the method to be applied is based on the objectives set and the tools available. In this work, modeling and evaluation of the reliability of an industrial turbine system based on a neuro-fuzzy system are proposed. This work proposes a method of modeling the reliability of a gas turbine based on a neuro-fuzzy system, in order to respond to the problems and challenges of the quality of safety of operation of these industrial installations. This approach, developed using a fuzzy inference system, makes it possible to deduce the reliability model to estimate the operating time of the studied gas turbine. While the primary aim is to reduce response costs and maximize service life to provide the best performance of the selected equipment under study. In order to construct this system, data collected from the history of the examined turbine were used. This neuro-fuzzy approach was performed and validated using standard Weibull three-parameter reliability approaches to determine the reliability parameters of the gas turbine under consideration.

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Disclosure statement

No potential conflicts of interest have been reported by the authors.

Notes on contributors

Dr Nadji Hadroug was born in Djelfa, Algeria. He is an associate professor with the faculty of Sciences and Technology at Djelfa university in Algeria. He is a member of Applied Automation and Industrial Diagnostics Laboratory, University Djelfa. He received his Ph.D. in neuro-fuzzy fault-tolerant control of a gas turbine for exploiting their reliability to improve their availability from University of Ziane Achour, Djelfa, Algeria. His work focuses on the development of new methods and tools in control fault-tolerant for industrial systems. I am an author and co-author of several publications and conference papers, my current research interests are in the monitoring and control of industrial gas turbine systems.
Prof. Ahmed Hafaifa was born in Algeria in 1974. He is a Ph.D. and Full Professor in Industrial Process: Automation/Diagnosis and Reliability Engineering at the Science and Technology Faculty of the University of Djelfa, Algeria, where he is actually the Dean of the Science and Technology Faculty since April 2018, after serving as the leader of Science and Technology Filed for five years since October 2014. Currently he is the Director of the Applied Automation and Industrial Diagnostic Laboratory of the University of Djelfa and the leader of the Gas Turbine Joint Research Team, where he is the founder of these research entities, where he is initiated and supported several international research projects in collaboration and innovation activities with the industrial sector.

Dr Abdelhamid Iratni is an Associate Professor of control engineering and automation systems at the Electrical Engineering Department of Faculty of Science and Technology at University of Bordj Bou Arreridj, Algeria, where he has been a faculty member since 2004. He is the Chair of the Scientific Committee of the Electrical Engineering department since 2019 after serving as the head of Automatic control bachelor and master’s programs for three years since 2015. He received his BSc and MSc degrees in electrical engineering from the University of Boumerdes, Algeria, in 1999 and 2003, respectively. He received his Ph.D. from the University of Setif, Algeria, in 2013. His current research areas include nonlinear filtering, estimation and control, biomedical and bioprocess engineering, and automation and reliability of industrial systems.

Dr Mouloud Guemana was born in Algeria in 1975 and received the graduated engineer of the national institute of hydrocarbons and chemistry INH, Mechanical Engineering Department of the University of Boumerdès, Algeria, inside 1998. From May 1999 to January 2003, he was associated postdoctoral research at the University of Boumerdès, Algeria, and associated professor from July 2004 to the present. He received in 2012 the Ph.D. degree in Industrial Maintenance and he is author and co-author of many scientific papers and research projects. His research interests include industrial maintenance, reliability system, dynamical systems, diagnostic and reliability optimization.

References

[1] Chen Y-Z, Zhao X-D, Xiang H-C, et al. A sequential model-based approach for gas turbine performance diagnostics. Energy. 2021;220:119657.
[2] Li X-Q, Bai G-C, Song L-K, et al. Fatigue reliability estimation framework for turbine rotor using multi-agent collaborative modeling. Structures. 2021;29:1967–1978.
[3] Kiaee M, Tousi AM. Vector-based deterioration index for gas turbine gas-path prognostics modeling framework. Energy. 2021;216:119198.
[4] Okpokparoro S, Sriramula S. Uncertainty modeling in reliability analysis of floating wind turbine support structures. Renew Energy. 2021;165(Part 1):88–108.
[5] Wilkie D, Galasso C. Gaussian process regression for fatigue reliability analysis of offshore wind turbines. Struct Saf. 2021;88:102020.
[6] Carapellucci R, Giordano L. Regenerative gas turbines and steam injection for repowering combined cycle power plants: design and part-load performance. Energy Convers Manage. 2021;227:113519.
[7] Lu C, Fei C-W, Liu H-T, et al. Moving extremum surrogate modeling strategy for dynamic reliability estimation of turbine blisk with multi-physics fields. Aerosp Sci Technol. 2020;106:106112.
[8] Dong W, Nejad AR, Moan T, et al. Structural reliability analysis of contact fatigue design of gears in wind turbine drivetrains. J Loss Prev Process Ind. 2020;65:104115.
[9] Ivanhoe RO, Wang L, Kolios A. Generic framework for reliability assessment of offshore wind turbine jacket support structures under stochastic and time dependent variables. Ocean Eng. 2020;216:107691.
[10] Gao H, Wang A, Zio E, et al. An integrated reliability approach with improved importance sampling for low-cycle fatigue damage prediction of turbine disks. Reliab Eng Syst Saf. 2020;199:106819.
[11] Velarde J, Kramhøft C, Sorensen JD, et al. Fatigue reliability of large monopiles for offshore wind turbines. Int J Fatigue. 2020;134:105487.
[12] Djeddi AZ, Hafaifa A, Salam A. Operational reliability analysis applied to a gas turbine based on three parameter Weibull distribution. Mechanics. 2015;21(3):187–192.
[13] Djeddi AZ, Hafaifa A, Kouzou A, et al. Exploration of reliability algorithms using modified Weibull distribution: application on gas turbine. Int J Syst Assur Eng Manag. 2017;8:1885–1894.
[14] Halimi D, Hafaifa A, Boua E. Maintenance actions planning in industrial centrifugal compressor based on failure analysis. Quart J Maint Reliab. 2014;16(1):17–21.
[15] Lin HT, Ferber MK. Mechanical reliability evaluation of silicon nitride ceramic components after exposure in industrial gas turbines. J Eur Ceram Soc. 2002;22(14/15):2789–2797.
[16] Guemana M, Hafaifa A, Rahmoune MB. Reliability study of gas turbines for improving their availability by ensuring optimal exploitation. OIL GAS European Magazine. 2015 June; vol. 2, p. 88–91.
[17] Djeddi AZ, Hafaifa A, Salam A. Gas turbine reliability model based on tangent hyperbolic reliability function. J Theor Appl Mech. 2015;53(3):723–730.
[18] Djeddi AZ, Hafaifa A, Guemana M, et al. Gas turbine reliability modelling based on a bath shaped rate failure function: modified Weibull distribution validation. Life Cycle Reliab Saf Eng. 2020;9:437–448.
[19] Kulor F, Markus ED, Kanzumba K. Design and control challenges of hybrid, dual nozzle gas turbine power generating plant: a critical review. Energy Rep. 2021;7:324–335.
[20] Hadroug N, Hafaifa A, Kouzou A, et al. Improvement of gas turbine availability using reliability modeling based on fuzzy System. Chapter in Applied condition monitoring book series, Advances in technical diagnostics Cham (Switzerland): Springer Nature Switzerland AG; 2018. vol. 10, p. 15–28.
[21] Park S, Shin J, Morishita M, et al. Validation of measured data on F/A ratio and turbine inlet temperature with optimal estimation to enhance the reliability on a full-scale gas turbine combustion test for IGCC. Int J Hydrogen Energy. 2019;44(26):13999–14011.
[22] Azar AT. Adaptive network based on fuzzy inference system for equilibrated urea concentration prediction. Comput Methods Programs Biomed. 2013;111(3):578–591.
[23] Fuh C-F, Jea R, Su J-S. Fuzzy system reliability analysis based on level (λ, 1) interval-valued fuzzy numbers. Inf Sci. 2014;272:185–197.
[24] Wu H-C. Fuzzy Bayesian system reliability assessment based on exponential distribution. Appl Math Model. 2006;30(6):509–530.
[25] Svalina I, Galzina V, Lujić R, et al. An adaptive network-based fuzzy inference system (ANFIS) for the forecasting: the case of close price indices. Expert Syst Appl. 2013;40(15):6055–6063.
[26] Dong M, Wang N. Adaptive network-based fuzzy inference system with leave-one-out cross-validation approach for prediction of surface roughness. Appl Math Model. 2011;35(3):1024–1035.
[27] Camargos MO, Bessa I, D’Angelo MFSV, et al. Data-driven prognostics of rolling element bearings using a novel error based evolving Takagi–Sugeno fuzzy model. Appl Soft Comput. 2020;96:106628.
[28] Wang X-I, Xie W-x, Li L-q. Interacting T-S fuzzy particle filter algorithm for transfer probability matrix of adaptive online estimation model. Digit Signal Process. 2021;110:102944.
[29] Chen Y, Liu Z, Philip Chen CL, et al. Adaptive fuzzy control of switched nonlinear systems with uncertain dead-zone: a mode-dependent fuzzy dead-zone model. Neurocomputing. 2021;432:133–144.
[30] Jang JSR. ANFIS: adaptive network based fuzzy inference systems. IEEE Trans Syst Man Cybern. 1993;23(5):665–685.
[31] Zhai Y, Lv Z, Zhao J, et al. Data-driven inference modeling based on an on-line Wang-Mendel fuzzy approach. Inf Sci. 2021;551:113–127.
[32] Zhang J, Zhao Y, Xiao Bing M. Reliability modeling methods for load-sharing k-out-of-n system subject to discrete external load. J Reliab Eng Syst Saf. 2019;193:106603.
[33] Tryon RG, Cruse TA, Mahadevan S. Development of a reliability-based fatigue life model for gas turbine engine structures. Eng Fract Mech. 1996;53(5):807–828.

[34] Gulshan T, Prasad J. Reliability and profit analysis of a power generating system with effect of ambient temperature and priority for repair to the gas turbine over steam turbine on system failure. In: Decision analytics applications in industry. Singapore: Springer; 2020. p. 309–330.

[35] Song Y, Basu B, Zhang Z, et al. Dynamic reliability analysis of a floating offshore wind turbine under wind-wave joint excitations via probability density evolution method. Renew Energy. 2021;168:991–1014.

[36] Verma M, Kumar A. A novel general approach to evaluating the reliability of gas turbine system. Eng Appl Artif Intell. 2014;28:13–21.

[37] Chang KD, Lee S-Y. Fuzzy reliability analysis of dual-fuel steam turbine propulsion system in LNG carriers considering data uncertainty. J Nat Gas Sci Eng. 2015;23:148–164.

Appendix

Table A1. Failures history of the turbine TITAN 130.

| No | Starting date   | Stop date   | TTR(h) | TBF(h) | DT(h) | The cause                                                                 | The action                                      |
|----|-----------------|-------------|--------|--------|-------|---------------------------------------------------------------------------|-------------------------------------------------|
| 01 | 09/01/2018      | 06/04/2018  | 5      | 2102   | 24    | - Check the fire operating system battery                                | - Change the battery                            |
| 02 | 07/04/2018      | 09/04/2018  | 12     | 93     | 48    | - Control and tightening bolts                                            | - Tighten the bolts                              |
| 03 | 11/04/2018      | 07/05/2018  | 3      | 641    | 24    | - Disassembly of a bottle (CO2)                                           | - Change of bottle of (CO2)                      |
| 04 | 08/05/2018      | 05/06/2018  | 6      | 714    | 48    | - The control of the fire protection system                                | - Sensor change (flame detector)                 |
| 05 | 07/06/2018      | 14/06/2018  | 4      | 184    | 24    | - Oil level adjustment for the gas turbine                                | - Addition of oil                                |
| 06 | 15/06/2018      | 22/06/2018  | 3      | 235    | 72    | - Oil cooler cleaning- Leakage at the drainage pump                       | - Cleaning- Pump change                          |
| 07 | 25/06/2018      | 10/09/2018  | 17     | 1980   | 144   | - Field-bus fault setting- Control defect position sensor I.G.V            | - Change of communication module-Setting the positioner parameters |
| 08 | 16/09/2018      | 19/09/2018  | 9      | 138    | 72    | Control and verification of air fan Oil level adjustment for the gas turbine | Fan change Addition of oil                       |
| 09 | 22/09/2018      | 10/10/2018  | 4      | 476    | 48    | TG3 Cleaning                                                               |                                                 |
| 10 | 12/10/2018      | 17/10/2018  | 3      | 188    | 72    | - Elimination of air leakage distributor- The extra oil                   | Cleaning                                         |
| 11 | 20/10/2018      | 24/10/2018  | 4      | 164    | 72    | - Cleaning behavior of airekison and combustion air                       | - Air hose attachment- Adjust oil                 |
| 12 | 27/10/2018      | 30/12/2018  | 2      | 1513   | 48    | - Cleaning behavior of airekison and combustion air                       | -Cleaning air filter                              |
| 13 | 01/01/2019      | 17/01/2019  | 7      | 449    | 72    | - Problem starting diesel- Oil leak at oil cooler level                   | -Air tube change- Oil cooler welding             |
| 14 | 20/01/2019      | 07/03/2019  | 9      | 1166   | 72    | - Check the operation of the solenoid valve MV2253                       | - Module change (valve MV2253)                   |
| 15 | 10/03/2019      | 31/03/2019  | 2      | 507    | 10    | - Check the exhaust of the turbine leakage at the flue level              | -Welding or chimney level                        |