Enhancement of cooperation by giving high-degree neighbors more help

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Abstract. In this paper, we study the effect of preferential assistance on cooperation in the donation game. Cooperators provide benefits to their neighbors at some costs. Defectors pay no cost and do not distribute any benefits. The total contribution of a cooperator is fixed and he/she distributes his/her contribution unevenly to his/her neighbors. Each individual is assigned a weight that is the power of its degree, where the exponent $\alpha$ is an adjustable parameter. The amount that cooperator $i$ contributes to a neighbor $j$ is proportional to $j$’s weight. Interestingly, we find that there exists an optimal value of $\alpha$ (which is positive), leading to the highest cooperation level. This phenomenon indicates that, to enhance cooperation, individuals could give high-degree neighbors more help, but only to a certain extent.

Keywords: evolutionary game theory, random graphs, networks
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1. Introduction

Evolutionary game theory has been frequently employed as the theoretical framework to explain the emergence of cooperation among selfish individuals [1]. One of the most used game model is the prisoner’s dilemma game (PDG) [2]. In PDG played by two individuals, each one simultaneously decides whether to cooperate or to defect. They both receive $R$ upon mutual cooperation and $P$ upon mutual defection. If one cooperates but the other defects, the defector gets the payoff $T$, while the cooperator gains the payoff $S$. The payoff rank for PDG is $T > R > P > S$. With the rapid development of network science, studies of PDG and other game models are implemented on various networks [3–8], including regular lattices [9–16], random graphs [17, 18], small-world networks [19], scale-free networks [20–25], multiplex networks [26, 27] and so on. For a given network, nodes represent individuals and links reflect social relationships. Individuals play the PDG with their direct neighbors.

An important special case of the PDG is the so-called donation game, where a cooperator provides a benefit $b$ to the other player at his/her cost $c$, with $0 < c < b$. A defector pays no cost and does not distribute any benefits. Thus, the payoff parameters in the donation game are $T = b$, $R = b - c$, $P = 0$, and $S = -c$. Ohtsuki et al discovered that natural selection favors cooperation if the benefit-to-cost $b/c$ exceeds the average number of neighbors [28]. Allen et al provided a solution for weak selection that applies to any network and found that cooperation flourishes most in societies which are based on strong pairwise ties [29]. Wu et al investigated impact of heterogeneous activity and community structure on the donation game [30]. Hilbe et al showed that in large, well-mixed populations, extortion strategies can play an important role, but only as catalyzers for cooperation and not as a long-term outcome [31]. Szolnoki and Perc found that extortion is evolutionarily stable in structured populations if the strategy updating is governed by a myopic best response rule [32]. Xu et al discovered that extortion strategies can act as catalysts to promote the emergence of cooperation in structured populations via different mechanisms [33].
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In previous studies of the spatial donation game, a cooperator treats all neighbors equally and contributes to each neighbor with the same cost. However, in real life, an individual usually has a preference for somebody and provides more benefit to him/her. An example is that altruistic act happens more frequently among genetic relatives [34]. In this paper, we propose a heterogeneous donation game in which a cooperator helps one of his/her neighbors with the cost proportional to the neighbor’s weight. We assign each individual \(i\) a weight \(k_i^\alpha\), where \(k_i\) is \(i\)’s degree and \(\alpha\) is an adjustable parameter. We have found that, the cooperation level can be maximized at an optimal value of \(\alpha\).

2. Model

A cooperator \(i\) provides a benefit \(rc_{ij}\) to a neighbor \(j\) at a cost \(c_{ij}\), where \(r\) is the benefit-to-cost ratio. For simplicity, we assume that \(r\) is the same for all pair interactions. The total cost of cooperator \(i\) is

\[
c_i = \sum_{j \in \Omega_i} c_{ij},
\]

(1)

where the sum runs over all the direct neighbors of \(i\) (this set is indicated by \(\Omega_i\)). A defector pays no cost and does not distribute any benefits.

Each individual \(i\) is assigned a weight \(k_i^\alpha\), where \(k_i\) is \(i\)’s degree and \(\alpha\) is an adjustable parameter. For a fixed \(c_i\), cooperator \(i\) helps one of its neighbors \(j\) with a cost \(c_{ij}\) proportional to \(j\)’s weight

\[
c_{ij} = c_i \frac{k_j^\alpha}{\sum_{l \in \Omega_i} k_l^\alpha}.
\]

(2)

For \(\alpha > 0\(< 0)\), cooperators give high-degree (low-degree) neighbors more help. In the case of \(\alpha = 0\), a cooperator provides the same benefit to each neighbor.

The payoff of individual \(i\) is given by

\[
M_i = -c_i s_i + r \sum_{j \in \Omega_i} c_{ji} s_j,
\]

(3)

where \(s_i = 1\) if \(i\) is a cooperator and \(s_i = 0\) if \(i\) is a defector. Note that the donation is not symmetric, i.e. \(c_{ij} \neq c_{ji}\).

After each time step, all individuals synchronously update their strategies as follows. Each individual \(i\) randomly chooses a neighbor \(j\) and adopts \(j\)’s strategy with the probability [35]

\[
W(s_i \leftarrow s_j) = \frac{1}{1 + \exp[(M_i - M_j)/\beta]},
\]

(4)

where the parameter \(\beta\) (>0) characterizes noise to permit irrational choices. As the noise \(\beta\) decreases, the individuals become more rational, i.e. they follow the strategies of neighbors who have obtained higher payoffs with greater probabilities.
3. Results and analysis

We carry out our model in a Barabási–Albert (BA) scale-free network [36] with size \(N = 5000\). Without loss of generality, we assume that each cooperator pays the same total cost (\(c_i = 1\) for any cooperator \(i\)). Initially, the two strategies, cooperation and defection, are randomly distributed among the individuals with the equal probability 1/2. The equilibrium fraction of cooperators \(\rho_c\) is obtained by averaging over the last \(10^4\) Monte Carlo time steps from a total of \(10^5\) steps. Each data point results from 20 different network realizations with 10 runs for each realization.

Figure 1 shows the fraction of cooperators \(\rho_c\) as a function of the benefit-to-cost ratio \(r\) for different values of \(\alpha\). One can see that for each value of \(\alpha\), \(\rho_c\) increases to 1 as \(r\) increases. For a small value of \(\alpha\) (e.g. \(\alpha = -0.1\)), cooperators die out when \(r\) is close to 1. However, for a large value of \(\alpha\) (e.g. \(\alpha = 0.5\) or \(\alpha = 1.5\)), cooperators can still survive, even if \(r = 1\).

Figure 2 shows the dependence of \(\rho_c\) on \(\alpha\). One can see that for fixed values of other parameters, there exists an optimal value of \(\alpha\) (denoted as \(\alpha_{\text{opt}}\)), leading to the maximal \(\rho_c\). The value of \(\alpha_{\text{opt}}\) is not fixed. From the insets of figure 2, one can see that \(\alpha_{\text{opt}}\) decreases as \(r\) increases, but increases as the average degree \(\langle k \rangle\) or the noise \(\beta\) increases. Moreover, \(\alpha_{\text{opt}}\) is positive (around 0.6), indicating that cooperation can be optimally enhanced if individuals give large-degree neighbors more help, but only to a certain extent.

To explain the nonmonotonic behavior displayed in figure 2, we study an individual’s payoff as a function of its degree. The theoretical analysis is provided as follows.

The cost that a cooperator \(j\) helps one of its neighbors \(i\) can be calculated as

\[
c_{ji} = \frac{k_i^\alpha}{\sum_{i \in \Omega_j} k_i^\alpha} \frac{\sum_{l \in \Omega_j} k_i^\alpha k_l^\alpha}{k_j \sum_{k_i = k_{\text{min}}}^{k_{\text{max}}} P(k_i | k_j) k_i^\alpha},
\]

where \(P(k_i | k_j)\) is the conditional probability that a node of degree \(k_j\) has a neighbor of degree \(k_i\), \(k_{\text{min}}\) and \(k_{\text{max}}\) are the minimum and maximum node degrees of the network. Since BA networks have negligible degree–degree correlation [37], we have approximately

\[
P(k_i | k_j) = k_i P(k_i) / \langle k \rangle,
\]

where \(P(k_i)\) is the degree distribution of BA networks. Substituting equation (6) into equation (5), we obtain

\[
c_{ji} = \frac{k_i^\alpha \langle k \rangle}{k_j \langle k^{\alpha+1} \rangle}.
\]

According to equation (3) and the mean-field theory, we can write the payoff \(M_i\) as

\[
M_i = -\rho_c + r \rho_c \sum_{j \in \Omega_i} c_{ji}
= -\rho_c + r \rho_c k_i \sum_{k_j = k_{\text{min}}}^{k_{\text{max}}} P(k_j | k_i) c_{ji}.
\]

Substituting equations (6) and (7) into equation (8), we obtain
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\[ M_i = -\rho_c + \frac{r \rho_c k_i^{\alpha+1}}{\langle k \rangle^{\alpha+1}}. \] (9)

From equation (9), one can find that for \( \alpha > -1 (< -1) \), higher-degree (lower-degree) individuals gain higher payoffs. In the case of \( \alpha = -1 \), individuals with different degree classes have the same payoffs.

Figure 1. The fraction of cooperators \( \rho_c \) as a function of the benefit-to-cost ratio \( r \) for different values of \( \alpha \). The average degree \( \langle k \rangle = 10 \) and the noise \( \beta = 0.5 \). For each \( \alpha \), \( \rho_c \) increases to 1 as \( r \) increases.

Figure 2. The fraction of cooperators \( \rho_c \) as a function of \( \alpha \) for different values of (a) the benefit-to-cost ratio \( r \), (b) the average degree \( \langle k \rangle \) and (c) the noise \( \beta \), respectively. For (a), \( \langle k \rangle = 10 \) and \( \beta = 0.5 \). For (b), \( r = 2.1 \) and \( \beta = 0.5 \). For (c), \( r = 1.9 \) and \( \langle k \rangle = 10 \). For fixed values of other parameters, there exists an optimal value of \( \alpha \) (denoted as \( \alpha_{\text{opt}} \)), leading to the maximal \( \rho_c \). The insets show the dependence of \( \alpha_{\text{opt}} \) on \( r, \langle k \rangle \) and \( \beta \) respectively.
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Figure 3. The payoff $M_k$ as a function of degree $k$ for different values of $\alpha$. The benefit-to-cost ratio $r = 40$ and the average degree $\langle k \rangle = 20$. All individuals are set to be cooperators. The data point are simulation results and the solid curves are theoretical results from equation (9).

Figure 4. Time series of the cooperator density $\rho_c$ for high-degree nodes (top panel) and low-degree nodes (bottom panel) respectively. The average degree $\langle k \rangle = 10$, the benefit-to-cost ratio $r = 2.1$ and the noise $\beta = 0.5$. Without loss of generality, nodes with $k > 40$ ($k \leq 40$) are divided into the high-degree (low-degree) class. For $\alpha = -1$, both kinds of nodes gradually become defectors. For $\alpha = 0.5$, with time all high-degree nodes become cooperators and most low-degree nodes choose cooperation. For $\alpha = 4$, although all high-degree nodes finally become cooperators, most low-degree nodes choose deflection in the equilibrium state.
The quantity $\langle k^{\alpha+1} \rangle$ can be calculated as $\langle k^{\alpha+1} \rangle = \int_{k_{\text{min}}}^{k_{\text{max}}} k^{\alpha+1} P(k) dk$, where the degree distribution of BA networks is $P(k) = 2k^2_{\text{min}} k^{-3}$ \cite{37} and the maximum node degree of the network is about $k_{\text{max}} = k_{\text{min}} \sqrt{N}$ \cite{38}. After calculating the integral, we obtain $\langle k^{2} \rangle = k_{\text{min}}^2 \ln N$ and $\langle k^{\alpha+1} \rangle = 2k_{\text{min}}^{\alpha+1} (N^{\frac{\alpha+1}{2}} - 1)/(\alpha - 1)$ (for $\alpha \neq 1$). From figure 3, one can see that the theoretical and numerical results are consistent.

For very small values of $\alpha$, lower-degree individuals gain more payoffs. Besides, the scale-free network is mainly composed of low-degree nodes. In this case, strategies of low-degree individuals play important roles in the evolution of cooperation. Note that cooperators gain less payoff than defectors in the same degree class. Thus, the whole network will gradually fall into the state of full defection due to the presence of abundant low-degree defectors. For large values of $\alpha$, high-degree individuals (so-called hubs) reap massive profits and gradually become cooperators \cite{20, 21}. These hubs and some of their neighbors will form a cooperator cluster \cite{39}. Within the cluster, cooperators can assist each other and the benefits of mutual cooperation outweigh the losses against the outside defectors. However, for very large $\alpha$, most individuals inside the cooperator cluster gain nothing since almost all benefits are allocated to large-degree individuals. In this case, low-degree cooperators have negative payoffs since they have to pay the cost of cooperation. On the contrary, the payoffs of defectors are positive. As a result, for very large $\alpha$, the cooperator cluster is vulnerable to the invasion of defectors and become difficult to expand. Combining the results of the two limits of $\alpha$, the highest cooperation level should be achieved for some intermediate values of $\alpha$.

To confirm the above analysis, we divide nodes into two classes: high-degree and low-degree ones. Then we study the time evolution of the cooperation density for high-degree and low-degree nodes respectively. From figure 4, one can see that, for a small
value of $\alpha$ (e.g. $\alpha = -1$), the cooperator density for both kinds of nodes decreases to 0 as time evolves. For a larger value of $\alpha$ (e.g. $\alpha = 0.5$ or $\alpha = 4$), the cooperator density for high-degree nodes increases to 1 while the cooperator density for low-degree nodes first decreases and then increases to a steady value. For $\alpha = 0.5$, low-degree nodes inside the cooperator cluster can gain enough payoffs to resist the invasion of defectors, leading to a high value of the cooperator density (about 0.8) in the stable state. For $\alpha = 4$, low-degree nodes get little benefit and are vulnerable to the attack of defectors, resulting in a low cooperation level (about 0.3) in the equilibrium state.

Next, we study the cooperator density $\rho_c(k)$ in the steady state as a function of degree $k$ for different values of $\alpha$. From figure 5, one can see that for a small value of $\alpha$ (e.g. $\alpha = -1$), $\rho_c(k)$ is almost the same for different values of $k$. For a larger value of $\alpha$ (e.g. $\alpha = -0.4$ or $\alpha = 4$), almost all high-degree individuals become cooperators while some low-degree individuals still choose defection. Here, we also find that $\rho_c(k)$ is minimized for medium-degree individuals. Such phenomenon has been observed in the weak PDG [40].

In the above studies, we assume that each cooperator contributes the same total cost $c_i = 1$. To validate the universality of the enhancement of cooperation by preferential assistance, we consider a case in which the total cost of a cooperator is not a constant but proportional to its degree, i.e. $c_i = k_i$. For each $r$, $\rho_c$ is maximized at an optimal value of $\alpha$.

![Figure 6. The fraction of cooperators $\rho_c$ as a function of $\alpha$ for different values of the benefit-to-cost ratio $r$. The average degree $\langle k \rangle = 6$ and the noise $\beta = 0.5$. The total cost of each cooperator is equal to its degree, i.e. $c_i = k_i$. For each $r$, $\rho_c$ is maximized at an optimal value of $\alpha$.](https://doi.org/10.1088/1742-5468/aac2fc)

4. Conclusions

In conclusion, we have found that cooperation can be promoted when cooperators contribute more to high-degree neighbors, but only to some extent. In this case, high-degree individuals are proved to have high payoffs and act as cooperators. These hubs and some of their neighbors form a cooperator cluster, within which cooperators can
assist each other and the benefits of mutual cooperation outweigh the losses against defectors. The above finding is robust with respect to different values of the benefit-to-cost ratio, different kinds of network structure, different levels of the noise to permit irrational choices, and different choices of the total cost of a cooperator.

The heterogeneous resource allocation has also been considered in other kinds of game models such as the public goods game. Huang et al found that cooperation can be enhanced if individuals invest more in smaller groups [41]. Meloni et al allowed individuals to redistribute their contribution according to what they earned from the given group in previous rounds [42]. Their results showed that not only a Pareto distribution for the payoffs naturally emerges but also that if players do not invest enough in one round they can act as defectors even if they are formally cooperators. Note that the donation game and the public goods game are based on pair interactions and group interactions respectively. Together [41, 42] and our work offer a deeper understanding of the impact of the heterogeneous resource allocation on the evolution of cooperation.

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