\( b \to s\mu^+\mu^- \) anomalies and related phenomenology

in \( U(1)_{B_3-x_\mu L_\mu-x_\tau L_\tau} \) flavor gauge models

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(Dated: February 19, 2019)

Abstract

We propose a generation dependent lepton/baryon gauge symmetry, \( U(1)_{B_3-x_\mu L_\mu-x_\tau L_\tau} \equiv U(1)_X \) (with \( x_\mu + x_\tau = 1 \) for anomaly cancellation), as a possible solution for the \( b \to s\mu^+\mu^- \) anomalies. By introducing two Higgs doublet fields, we can reproduce the observed CKM matrix, and generate flavor changing \( Z' \) interactions in the quark sector. Thus one can explain observed anomalies in \( b \to s\ell^+\ell^- \) decay with the lepton non-universal \( U(1)_X \) charge assignments. We show the minimal setup explaining \( b \to s\ell^+\ell^- \) anomalies, neutrino masses and mixings and dark matter candidate, taking into account experimental constraints of flavor physics such as charged lepton flavor violations and the \( B_s-\bar{B}_s \) mixing. Finally we discuss collider physics focusing on \( Z' \) production at the Large Hadron Collider and relic density of our dark matter candidate.

Keywords:

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I. INTRODUCTION

Although the standard model (SM) of particle physics is very successful we still do not have clear understanding of the physics regarding the flavors; namely the origin of fermion masses and mixing patterns. Then it is interesting to construct a model describing flavor physics with some symmetry as a guiding principle. One of the attractive possibility is an introduction of flavor dependent $U(1)$ gauge symmetry which can constrain structure of Yukawa couplings generating masses for quarks, charged leptons and neutrinos. In this kind of approaches to the flavor problem, these models may generate flavor changing neutral current (FCNC) processes through $Z'$ boson exchange, which will induce rich phenomenology.

Recently there have been some indication of anomalies in $B$ physics measurements for $b \rightarrow s \ell^+\ell^-$ process. The angular observable $P_5'$ in decay of $B$ meson, $B \rightarrow K^*\mu^+\mu^-$ [1], indicates $3.4\sigma$ deviations from the data with integrated luminosity of 3.0 fb$^{-1}$ at the LHCb [2], confirming an earlier observation with $3.7\sigma$ deviations [3]. In addition, $2.1\sigma$ deviations were reported for the same observable by Belle [4, 5]. Furthermore, an anomaly in the measurement of lepton flavor universality by the ratio $R_K = BR(B^+ \rightarrow K^+\mu^+\mu^-)/BR(B^+ \rightarrow K^+e^+e^-)$ [6, 7] at the LHCb shows $2.6\sigma$ deviations from the SM prediction [8]. Moreover the LHCb collaboration also reported an anomaly in the ratio $R_{K^*} = BR(B \rightarrow K^*\mu^+\mu^-)/BR(B \rightarrow K^*e^+e^-)$ where the observed values are deviated from the SM prediction by $\sim 2.4\sigma$ as $R_{K^*} = 0.660^{+0.110}_{-0.070} \pm 0.024(0.685^{+0.113}_{-0.069} \pm 0.047)$ for $(2m_{\mu}^2) < q^2 < 1.1$ GeV$^2$ ($1.1$ GeV$^2 < q^2 < 6$ GeV$^2$) [9].

These anomalies in the $b \rightarrow s\ell^+\ell^-$ channels (with $l = e, \mu$) can be explained by flavor dependent $Z'$ interactions inducing effective operator of $(\bar{b}\gamma^\alpha s)(\bar{\mu}\gamma_\alpha\mu)$, if new physics contribution to the corresponding Wilson coefficient $C_9^\mu$ is roughly $\Delta C_9^\mu \sim -1$ by global fits [10] [13]. Then many models have been proposed to explain the anomalies by $Z'$ interactions [14] [42].

In this paper, motivated by $b \rightarrow s\ell^+\ell^-$ anomalies, we propose a model based on flavor dependent Abelian gauge symmetry $U(1)_{B_3-x_\mu L_\mu-x_\tau L_\tau}$, which is anomaly-free for $x_\mu + x_\tau = 1$. In this model we introduce two Higgs doublet fields to generate the realistic CKM matrix, where small mixings associated with third generation quarks can be obtained naturally [14]. Then, $b \rightarrow s\ell^+\ell^-$ anomalies can be explained by the effective operator induced by $Z'$ exchange if $x_\mu$ has negative value to get $\Delta C_9^\mu \sim -1$. We then consider the minimal model explaining $b \rightarrow s\ell^+\ell^-$ anomalies and generating non-zero neutrino masses in which two SM
singlet scalar fields are introduced. Also we introduce Dirac fermionic dark matter (DM) candidate in order to account for the dark matter of the Universe. In addition to $\Delta C_9^\mu$, we formulate neutrino mass matrix, lepton flavor violations (LFVs) and $B_s-\overline{B}_s$ mixing, and experimental constraints from them are taken into account. Then we discuss collider physics regarding $Z'$ production at the Large Hadron Collider (LHC) and relic density of our DM candidate.

This paper is organized as follows. In Sec. II, we introduce our model and discuss quark mass, $\Delta C_9^\mu$ by $Z'$ and scalar masses in the minimal case. In Sec. III we discuss neutrino mass matrix, charged lepton flavor violations and $B_s-\overline{B}_s$ mixing taking into account experimental constraints. The numerical analysis is carried out in Sec. IV to discuss collider physics for $Z'$ production at the LHC and relic density of DM candidate showing allowed parameter region. Finally summary and discussion are given in Sec. V.

\section{Models and Formulas}

In this section we introduce our model based on flavor dependent $U(1)_{B_s-x_\mu L_\mu-x_\tau L_\tau}$ gauge symmetry that we denote simply $U(1)_X$ in the following \footnote{In our analysis we ignore kinetic mixing between $U(1)_Y$ and $U(1)_X$ assuming it is sufficiently small.}. The SM fermions with 3 right-handed (RH) neutrinos are charged under the $U(1)_X$ as shown in Table.\footnote{In our analysis we ignore kinetic mixing between $U(1)_Y$ and $U(1)_X$ assuming it is sufficiently small.} The gauge anomalies are cancelled when the $U(1)_X$ charges of fermions satisfy the condition

\begin{equation}
 x_\mu + x_\tau = 1, \quad (\text{II.1})
\end{equation}

| Fermions | $Q_{3L}$ | $u_{iR}$ | $d_{iR}$ | $Q_{3L}$ | $t_R$ | $b_R$ | $L_{1L}$ | $L_{2L}$ | $L_{3L}$ | $e_R$ | $\mu_R$ | $\tau_R$ | $\nu_1R$ | $\nu_2R$ | $\nu_3R$ |
|----------|----------|----------|----------|----------|-------|-------|----------|----------|----------|-------|-------|-------|----------|----------|----------|
| $SU(3)_C$ | 3        | 3        | 3        | 3        | 3     | 1     | 1        | 1        | 1        | 1     | 1     | 1     | 1        | 1        | 1        |
| $SU(2)_L$ | 2        | 1        | 1        | 2        | 1     | 2     | 2        | 1        | 1        | 1     | 1     | 1     | 1        | 1        | 1        |
| $U(1)_Y$  | 1/6      | 2/3      | $-1/3$   | 1/6      | 2/3   | $-1/3$| $-1/2$   | $-1/2$   | -1       | -1    | 0     | 0     | 0        | 0        | 0        |
| $U(1)_X$  | 0        | 0        | 0        | 1/3      | 1/3   | 1/3   | 0        | $-x_\mu$ | $-x_\tau$| 0     | $-x_\mu$| $-x_\tau$| 0        | $-x_\mu$ | $-x_\tau$|

\textbf{Table I:} Charge assignment for the SM fermions and right-handed neutrinos where the indices $i = 1, 2$ indicate the first and second generations.
which we will always assume in the following. In Sec. IIA, we first discuss the case with
general \( x_{\mu,\tau} \) and investigate an explanation of \( b \to s\ell^+\ell^- \) anomalies via flavor-changing
\( Z' \) interactions. Then the minimal model with \( x_{\mu} = -1/3 \) is constructed in Sec. II B,
taking into account the generation of active neutrino masses and mixings via Type-I seesaw mechanism.

A. Discussion for general \((x_{\mu}, x_{\tau})\) case

Firstly we consider quark sector which does not depend on our choice of \( x_{\mu} \) and \( x_{\tau} = 1 - x_{\mu} \). In this model we have to introduce at least two Higgs doublets in order to induce
the realistic CKM mixing matrix:

\[
\Phi_1 : (1, 2)(1/2, -1/3), \quad \Phi_2 : (1, 2)(1/2, 0), \quad (SU(3)_C, SU(2)_L)(U(1)_Y, U(1)_X) \quad (\text{II.2})
\]

Then the Yukawa couplings for quarks are given by

\[
-L_Q = \bar{Q}^i L \tilde{\Phi}_2^u u^i R + \bar{Q}^i L \Phi_2^d d^i R + \bar{Q}^3 L \Phi_2^t t^R + \bar{Q}^3 L \tilde{\Phi}_1^u u^3 R + \tilde{\Phi}_1^u u^i R + \tilde{\Phi}_1^d d^i R + \text{h.c.}, \quad (\text{II.3})
\]

where \( i = 1, 2 \) and \( \tilde{\Phi}_i = i\sigma_2 \Phi_i^* \). \( \Phi_2 \) is the Higgs doublet with vanishing \( U(1)_X \) charge, and is the SM-like Higgs doublet. After two Higgs doublet fields get the non-zero vacuum expectation values (VEVs) \( \langle \Phi_{1,2} \rangle = (0, v_{1,2}/\sqrt{2})^T \), we obtain the following forms of quark mass matrices:

\[
M^u = \frac{1}{\sqrt{2}} \begin{pmatrix}
    v_{2y_{11}}^u & v_{2y_{12}}^u & 0 \\
    v_{2y_{21}}^u & v_{2y_{22}}^u & 0 \\
    0 & 0 & v_{2y_{33}}^u
\end{pmatrix} + \begin{pmatrix}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    (\xi_u)_{31} & (\xi_u)_{32} & 0
\end{pmatrix},
\]

\[
M^d = \frac{1}{\sqrt{2}} \begin{pmatrix}
    v_{2y_{11}}^d & v_{2y_{12}}^d & 0 \\
    v_{2y_{21}}^d & v_{2y_{22}}^d & 0 \\
    0 & 0 & v_{2y_{33}}^d
\end{pmatrix} + \begin{pmatrix}
    0 & 0 & (\xi_d)_{13} \\
    0 & 0 & (\xi_d)_{23} \\
    0 & 0 & 0
\end{pmatrix}, \quad (\text{II.4})
\]

Note that the matrices \( (\xi_{u,d})_{ij} \equiv \tilde{y}_{ij}^u v_1/\sqrt{2} \) have the same structure as those discussed in Ref. [14]. We shall assume the second terms with \( \xi_{u,d} \) are small perturbation effects generating realistic \( 3 \times 3 \) CKM mixing matrix where the (33) elements are \( v_{2y_{33}}^{u(d)} \sim \sqrt{2}m_{t(b)} \).

As in the SM, the quark mass matrices are diagonalized by unitary matrices \( U_{L,R} \) and \( D_{L,R} \) which change quark fields from interaction basis to mass basis: \( u_{L,R} \to U_{L,R}^T u_{L,R} \) (\( d_{L,R} \to \ldots \))
where diagonal mass matrices are given by \( m_{d\text{diag}} = \text{diag}(m_d, m_s, m_b) \) and \( m_{u\text{diag}} = \text{diag}(m_u, m_c, m_t) \). These relations indicate that the off-diagonal elements associated with 3rd generations are more suppressed for \( M^u (M^u)^\dagger \) and \( (M^d)^\dagger M^d \) than those in \( (M^u)^\dagger M^u \) and \( M^d (M^d)^\dagger \). Then we can approximate \( U_L \) and \( D_R \) to be close to unity matrix since they are associated with diagonalization of \( M^u (M^u)^\dagger \) and \( (M^d)^\dagger M^d \), respectively. Therefore CKM matrix can be approximated as \( V_{CKM} \approx D_L \), and \( D_R \approx 1 \), as obtained \([14]\).

1. \( Z' \) interactions with SM fermions

The \( Z' \) couplings to the SM fermions are written as

\[
\mathcal{L} \supset -g_X \left( x_{\mu} \bar{u} \gamma^{\mu} u + x_{\tau} \bar{\tau} \gamma^{\mu} \tau + x_{\mu} \bar{p} \gamma^{\mu} p L \nu_{\mu} + x_{\tau} \bar{\nu}_{\tau} \gamma^{\mu} p L \nu_{\tau} + x_{\mu} \bar{\nu}_{2} \gamma^{\mu} p R \nu_{2} + x_{\tau} \bar{\nu}_{3} \gamma^{\mu} p R \nu_{3} \right) Z'_{\mu} + \frac{g_X}{3} \bar{t} \gamma^{\mu} t Z'_{\mu} + \frac{g_X}{3} \left( \bar{d}_{\alpha} \gamma^{\mu} p L d_{\beta} \Gamma_{\alpha \beta}^{d_L} + \bar{d}_{\alpha} \gamma^{\mu} p R d_{\beta} \Gamma_{\alpha \beta}^{d_R} \right) Z'_{\mu},
\]

where \( g_X \) is the gauge coupling constant associated with the \( U(1)_X \) and the lepton sector is given in the flavor basis here. The coupling matrices \( \Gamma_{\alpha \beta}^{d_R} \) and \( \Gamma_{\alpha \beta}^{d_L} \) for down-type quarks are given approximately by

\[
\Gamma_{\alpha \beta}^{d_L} \simeq \begin{pmatrix}
|V_{td}|^2 & V_{ts} V_{td}^* & V_{tb} V_{td}^* \\
V_{ts} V_{td}^* & |V_{ts}|^2 & V_{tb} V_{ts}^* \\
V_{td} V_{ts}^* & V_{ts} V_{tb}^* & |V_{tb}|^2
\end{pmatrix}, \quad \Gamma_{\alpha \beta}^{d_R} \simeq \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

where \( V_{qq}'s \) are the CKM matrix elements. We have applied the relation \( V_{CKM} \approx D_L \), as we discussed above. In our model the \( Z' \) mass, \( m_{Z'} \), is dominantly given by the VEV of SM singlet scalar field as discussed below.
FIG. 1: The contours showing $Z'$ contribution to $\Delta C^\mu_9$ on the $m_{Z'}-g_X$ plane with $x_\mu = -\frac{1}{3}$ where yellow(light-yellow) region corresponds to 1$\sigma$ (2$\sigma$) region from global fit in Ref. [11].

2. Effective interaction for $b \rightarrow s\mu^+\mu^-$

Gauge interactions in Eq. (II.6) induce the effective Hamiltonian for $b \rightarrow s\mu^+\mu^-$ process such that

$$\Delta H_{\text{eff}} = -\frac{x_\mu g_X^2}{3 m_{Z'}^2} (s\gamma^\mu P_L b) (\bar{\mu}\gamma_\mu \mu) + h.c.$$  

$$= \frac{x_\mu g_X^2}{3 m_{Z'}^2} \left( \frac{\sqrt{2}\pi}{G_F \alpha_{em}} \right) \left( -4 G_F \alpha_{em} V_{tb} V_{ts}^* \right) (s\gamma^\mu P_L b) (\bar{\mu}\gamma_\mu \mu) + h.c., \quad (\text{II.8})$$

where $G_F$ is the Fermi constant and $\alpha_{em}$ is the electromagnetic fine structure constant. We thus obtain the $Z'$ contribution to Wilson coefficient $\Delta C^\mu_9$ as

$$\Delta C^\mu_9 = \frac{x_\mu g_X^2}{3 m_{Z'}^2} \left( \frac{\sqrt{2}\pi}{G_F \alpha_{em}} \right) \simeq 0.174 \times x_\mu \left( \frac{g_X}{0.1} \right)^2 \left( \frac{1 \text{ TeV}}{m_{Z'}} \right)^2. \quad (\text{II.9})$$

In order to obtain $\Delta C^\mu_9 \sim -1$, $x_\mu$ should be negative and $g_X$ is required to be $\sim 0.42$ for $m_{Z'} = 1 \text{ TeV}$ and $x_\mu = -\frac{1}{3}$. Figure 1 shows the contour of $\Delta C^\mu_9$ in the $(m_{Z'}, g_X)$ plane where we took $x_\mu = -\frac{1}{3}$ where the yellow(light-yellow) region corresponds to 1$\sigma$ (2$\sigma$) region from global fit in Ref. [11].
TABLE II: Scalar fields and extra fermion $\chi$ in the minimal model and their representation under $SU(2) \times U(1)_Y \times U(1)_X$ where these fields are color singlet.

| Fields | $\Phi_1$ | $\Phi_2$ | $\varphi_1$ | $\varphi_2$ | $\chi$ |
|--------|----------|----------|-------------|-------------|--------|
| $SU(2)_L$ | 2 2 | 1 1 | 1 1 |
| $U(1)_Y$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 0 | 0 0 |
| $U(1)_X$ | $-\frac{1}{3}$ | 0 | $\frac{1}{3}$ | 1 | $\frac{5}{6}$ |

B. Minimal model

Here we consider the minimal cases for choosing $U(1)_X$ charges of leptons as

$$x_\mu = -\frac{1}{3}, \quad x_\tau = \frac{4}{3}. \quad (II.10)$$

In this case we add two $SU(2)_L$ singlet scalar fields:

$$\varphi_1 : (1, 1)(0, 1/3), \quad \varphi_2 : (1, 1)(0, 1), \quad (II.11)$$

where $\varphi_1$ is also necessary to induce $\Phi_1^\dagger \Phi_2$ terms, while $\varphi_2$ is added for generating the $23(32)$ element of Majorana mass matrix of right-handed neutrino. In addition we introduce additional Dirac fermion $\chi$ of mass $m_X$ with $U(1)_X$ charge $5/6$, which can be our DM candidate since its stability is guaranteed due to fractional charge assignment under $U(1)_X$.

Note that the stability of Dirac fermion DM $\chi$ is guaranteed by remnant $Z_2$ symmetry after $U(1)_X$ symmetry breaking: particles with $U(1)_X$ charge $2n/6$ ($n$ is integer) are $Z_2$ even and those with $U(1)_X$ charge $(2n + 1)/6$ are $Z_2$ odd, since $U(1)_X$ symmetry is broken by VEVs of scalar fields $\varphi_1, \varphi_2$ and $\Phi_1$ whose charges correspond to $2n/6$. We summarize the charge assignment of scalar fields and new fermion in Table II. In the later analysis, we will adopt this minimal setting.

In our set up, the full scalar potential for scalar fields in our model is given by

$$V = -\mu(\Phi_1^\dagger \Phi_2 \varphi_1^* + h.c.) + \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 - \mu_{\varphi_1}^2 |\varphi_1|^2 + \mu_{\varphi_2}^2 |\varphi_2|^2 + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \lambda_{\varphi_1}^2 |\varphi_1|^4 + \lambda_{\varphi_2}^2 |\varphi_2|^4 + \lambda_{\Phi_1 \varphi_1} |\Phi_1|^2 |\varphi_1|^2 + \lambda_{\Phi_2 \varphi_1} |\Phi_2|^2 |\varphi_1|^2 + \lambda_{\varphi_1 \varphi_2} |\Phi_1|^2 |\varphi_2|^2 + \lambda_{\varphi_2 \varphi_2} |\Phi_2|^2 |\varphi_2|^2 + \lambda_{\varphi_1 \varphi_2} |\varphi_1|^2 |\varphi_2|^2 - \lambda_X (\varphi_1^3 \varphi_2^* + h.c.), \quad (II.12)$$

Note that we need one more scalar singlet to generate neutrino mass when $x_\mu \neq -1/3$. 


where we assumed all the coupling constants are real for simplicity. The VEVs of singlet scalar fields are written by $\sqrt{2} \langle \varphi_1 \rangle = v_{\varphi_1}$ and $\sqrt{2} \langle \varphi_2 \rangle = v_{\varphi_2}$. In our scenario, we assume $v_{\varphi_1}^2 \gg v_{\varphi_2}^2 \gg v_{1,2}^2$ and $U(1)_X$ symmetry is spontaneously broken at a scale higher than the electroweak scale. We then approximately obtain VEVs of $\varphi_{1,2}$ from the condition $\partial V/\partial v_{\varphi_{1,2}} = 0$:

$$v_{\varphi_1} \simeq \sqrt{-\mu_{\varphi_1}^2/\lambda_{\varphi_1}}, \quad v_{\varphi_2} \simeq \lambda_X v_{\varphi_1}^3/4\mu_{\varphi_2}^2 + 2\lambda_{\varphi_1\varphi_2} v_{\varphi_1} v_{\varphi_2}^2,$$

where the above assumption for VEV hierarchy can be consistent requiring $\lambda_X v_{\varphi_1}^2 \ll \mu_{\varphi_2}^2$.

Then the mass of the $Z'$ boson is approximately given by

$$m_{Z'} \simeq \frac{1}{3} g_X v_{\varphi_1}.$$

Note that the $Z-Z'$ mass mixing is highly suppressed by $v_{\varphi_1}^2/v_{\varphi_2}^2$ factor and we will ignore this effect in our analysis.

After $U(1)_X$ symmetry breaking, we obtain two-Higgs doublet potential effectively:

$$V_{THDM}(\Phi_1, \Phi_2) = m_{1(2)}^2 |\Phi_1|^2 + m_{2(2)}^2 |\Phi_2|^2 - (m_{33}^2 \Phi_1^\dagger \Phi_2 + h.c.)$$

$$+ \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2,$$

$$m_{1(2)}^2 = \mu_{1(2)}^2 + \frac{1}{2} \lambda_{\varphi_1(2)\varphi_1} v_{\varphi_1}^2 + \frac{1}{2} \lambda_{\varphi_1(2)\varphi_2} v_{\varphi_2}^2, \quad m_{33}^2 = \frac{1}{\sqrt{2}} \mu_{\varphi_1} v_{\varphi_1}.$$  

Here we write $\Phi_i (i = 1, 2)$ as

$$\Phi_i = \begin{pmatrix} w_i^+ \\ \frac{1}{\sqrt{2}} (v_i + h_i + iz_i) \end{pmatrix},$$

As in the two-Higgs-doublet model (THDM), we obtain mass eigenstate $\{H, h, A, H^\pm\}$ in two Higgs doublet sector:

$$\begin{pmatrix} z_1(w_1^+) \\ z_2(w_2^+) \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G_Z(G^+) \\ A(H^+) \end{pmatrix},$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix},$$

---

3 Here we do not consider scalar bosons from $\varphi_{1,2}$ since they are assumed to be much heavier than those from Higgs doublets and mixing among singlet and doublet scalars will be small.
where \( \tan \beta = v_2/v_1 \), \( G_Z(G^+) \) is a Nambu-Goldstone boson (NG) absorbed by the \( Z(W^+) \) boson, and \( h \) is the SM-like Higgs boson. The masses of \( H^\pm \) and \( A \) are given as in THDM:

\[
m_{H^\pm}^2 = \frac{m_3^2}{\sin \beta \cos \beta} - \frac{v^2}{2} \lambda_4, \quad m_A^2 = \frac{m_3^2}{\sin \beta \cos \beta}. \tag{II.20}
\]

Mass eigenvalues of CP-even scalar bosons are also obtained by

\[
m_{H,h}^2 = \frac{1}{2} \left( M_1^2 + M_2^2 \pm \sqrt{(M_1^2 - M_2^2)^2 + 4M_{12}^4} \right), \tag{II.21}
\]

\[
M_1^2 = v^2(\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta) + \frac{v^2}{2}\bar{\lambda}\sin^2 2\beta, \tag{II.22}
\]

\[
M_2^2 = \frac{m_3^2}{\sin \beta \cos \beta} + v^2 \sin^2 \beta \cos^2 \beta(\lambda_1 + \lambda_2 - 2\bar{\lambda}), \tag{II.23}
\]

\[
M_{12}^2 = \frac{v^2}{2} \sin 2\beta(-\lambda_1 \cos^2 \beta + \lambda_2 \sin^2 \beta) + \frac{v^2}{2}\bar{\lambda}\sin 2\beta \cos 2\beta, \tag{II.24}
\]

where \( \bar{\lambda} = \lambda_3 + \lambda_4 \) and lighter mass eigenvalue \( m_h \) is identified as the SM-like Higgs mass.

Note that Higgs bosons in doublet interact with \( Z' \) and three point couplings can be obtained such that

\[
(D_{\mu}H_1) \supset \frac{g_X}{3} Z^\mu(w_+^{\nu} \partial_\mu w_1^- - w_1^+ \partial_\mu w_1^-) + \frac{2g_X}{3} Z^\mu(h_1 \partial_\mu z_1 - z_1 \partial_\mu h_1)
\]

\[
\supset \frac{g_X}{3} \sin^2 \beta Z^\mu(H^+ \partial_\mu H^- - H^- \partial_\mu H^+) + \frac{2g_X}{3} \sin \beta \sin \alpha Z^\mu(h \partial_\mu A - A \partial_\mu h)
\]

\[
+ \frac{2g_X}{3} \sin \beta \cos \alpha Z^\mu(A \partial_\mu H - H \partial_\mu A). \tag{II.25}
\]

Thus \( Z' \) can decay into \( HA, hA \) and \( H^+H^- \) pair.

**III. NEUTRINO MASS AND FLAVOR CONSTRAINTS**

In this section we formulate neutrino mass matrices (both Dirac and Majorana mass matrices), and explore constraints from flavor physics such as \( \mu \to e\gamma, \mu \to e \) conversion and \( B_s - \bar{B}_s \) mixing.

**A. Neutrino mass matrices**

The Yukawa interactions for leptons are given by

\[
-\mathcal{L} \supset y_{\alpha a}^L \bar{L}_{aL} e_{aR}\Phi_2 + y_{\alpha a}^\nu \bar{L}_{aL}\nu_{aR}\tilde{\Phi}_2 + \tilde{y}_{12}^\nu \bar{L}_{1L}\mu_{R}\Phi_1 + \tilde{y}_{21}^\nu \bar{L}_{2L}\nu_{1R}\tilde{\Phi}_1
\]

\[+ M\tilde{\nu}_{1R}\nu_{1R} + Y_{12}\tilde{\nu}_{1R}\nu_{2R}\Phi_1^* + Y_{23}\tilde{\nu}_{2R}\nu_{3R}\Phi_2^* + h.c., \tag{III.1}
\]
where $a = 1, 2, 3$ and $Y_{ab} = Y_{ba}$. After the symmetry breaking, Dirac and Majorana mass matrices for neutrinos have the structure of

$$
M_D = \begin{pmatrix}
(M_D)_{11} & 0 & 0 \\
(M_D)_{21} & (M_D)_{22} & 0 \\
0 & 0 & (M_D)_{33}
\end{pmatrix},
\quad
M_\nu = \begin{pmatrix}
(M_\nu_R)_{11} & (M_\nu_R)_{12} & 0 \\
(M_\nu_R)_{21} & 0 & (M_\nu_R)_{23} \\
0 & (M_\nu_R)_{32} & 0
\end{pmatrix},
$$

(III.2)

where the elements of the mass matrices are given by

$$(M_D)_{aa} = \frac{1}{\sqrt{2}} y_{aa}^\nu v^2, \quad (M_D)_{21} = \frac{1}{\sqrt{2}} \tilde{y}_{21} v_1,$$

$$(M_\nu_R)_{11} = M, \quad (M_\nu_R)_{12(21)} = \frac{1}{\sqrt{2}} Y_{12} v_{\phi_1}, \quad (M_\nu_R)_{23(32)} = \frac{1}{\sqrt{2}} Y_{23} v_{\phi_2}.$$

(III.3)

The active neutrino mass matrix is given by type-I seesaw mechanism:

$$m_\nu \simeq -M_D M^{-1}_\nu M^T_D$$

$$= \begin{pmatrix}
\frac{(M_D)_{11}^2}{(M_\nu_R)_{11}} & \frac{(M_D)_{11}(M_D)_{21}}{(M_\nu_R)_{11}} & \frac{(M_D)_{11}(M_D)_{21}}{(M_\nu_R)_{11}} \\
\frac{(M_D)_{11}(M_D)_{21}}{(M_\nu_R)_{11}} & \frac{(M_D)_{21}^2}{(M_\nu_R)_{11}} & \frac{(M_D)_{21}(M_D)_{22}}{(M_\nu_R)_{11}} \\
\frac{(M_D)_{11}(M_D)_{32}}{(M_\nu_R)_{11}} & \frac{(M_D)_{11}(M_D)_{32}}{(M_\nu_R)_{11}} & \frac{(M_D)_{32}^2}{(M_\nu_R)_{11}}
\end{pmatrix} \cdot
\begin{pmatrix}
\frac{(M_\nu_R)_{11} (M_\nu_R)_{12}}{(M_\nu_R)_{11} (M_\nu_R)_{32}} & \frac{(M_\nu_R)_{11} (M_\nu_R)_{21}}{(M_\nu_R)_{11} (M_\nu_R)_{32}} & \frac{(M_\nu_R)_{11} (M_\nu_R)_{21}}{(M_\nu_R)_{11} (M_\nu_R)_{32}} \\
\frac{(M_\nu_R)_{21} (M_\nu_R)_{12}}{(M_\nu_R)_{11} (M_\nu_R)_{32}} & \frac{(M_\nu_R)_{21} (M_\nu_R)_{22}}{(M_\nu_R)_{11} (M_\nu_R)_{32}} & \frac{(M_\nu_R)_{21} (M_\nu_R)_{22}}{(M_\nu_R)_{11} (M_\nu_R)_{32}} \\
\frac{(M_\nu_R)_{32} (M_\nu_R)_{12}}{(M_\nu_R)_{11} (M_\nu_R)_{32}} & \frac{(M_\nu_R)_{32} (M_\nu_R)_{22}}{(M_\nu_R)_{11} (M_\nu_R)_{32}} & \frac{(M_\nu_R)_{32} (M_\nu_R)_{22}}{(M_\nu_R)_{11} (M_\nu_R)_{32}}
\end{pmatrix}.
$$

(III.4)

Note that our neutrino mass matrix does not have zero structure and neutrino oscillation data can be easily fit. Here we do not carry out further analysis of the neutrino phenomenology in this paper.

**B. Charged lepton mass matrices**

The charged lepton mass matrix is given by

$$M^e = \frac{1}{\sqrt{2}} \begin{pmatrix}
y_{11}^e v_2 & \tilde{y}_{12}^e v_1 & 0 \\
0 & y_{22}^e v_2 & 0 \\
0 & 0 & y_{33}^e v_2
\end{pmatrix} \equiv \begin{pmatrix}
m_{11}^e & \delta m_{12}^e & 0 \\
0 & m_{22}^e & 0 \\
0 & 0 & m_{33}^e
\end{pmatrix}.$$

(III.5)
For $\delta m_{12} \ll m_{22}$, the mass matrix can be diagonalized in good approximation as

$$
\begin{pmatrix}
    m_e & 0 & 0 \\
    0 & m_\mu & 0 \\
    0 & 0 & m_\tau
\end{pmatrix}
≈
V^e_L M^e (V^e_R)^\dagger,
$$

(III.6)

where $\epsilon = \delta m_{12} / m_{22}$. We also find $m_e \simeq m_{11}$, $m_\mu \simeq m_{22}$ and $m_{33} = m_\tau$.

C. Charged lepton flavor violation

Here we consider charged lepton flavor violation (cLFV) in the model associated with $Z'$. The $Z'$ gauge interactions for mass eigenstates of charged leptons are given by

$$
\mathcal{L} \supset -\frac{g_X}{3} \bar{\ell}_i \gamma^\mu V^e_L \begin{pmatrix}
    0 & 0 & 0 \\
    0 & 0 & 4 \\
    0 & 0 & 4
\end{pmatrix} V^e_L^\dagger \begin{pmatrix}
    P_L \ell_j Z'_\mu - \frac{g_X}{3} \bar{\ell}_i \gamma^\mu & 0 & -1 & 0 \\
    0 & 0 & 0 & 0
\end{pmatrix},
$$

(III.8)

where the flavor violating structure for left-handed charged lepton currents is given by

$$
V^e_L \begin{pmatrix}
    0 & 0 & 0 \\
    0 & -1 & 0 \\
    0 & 0 & 4
\end{pmatrix} V^e_L^\dagger \simeq \begin{pmatrix}
    -\epsilon^2 & \epsilon & 0 \\
    \epsilon & -1 & 0 \\
    0 & 0 & 4
\end{pmatrix}.
$$

(III.9)

Thus we have LFV interaction for $e$ and $\mu$. Then we first consider $\mu \rightarrow e\gamma$ process induced by $Z'$ loop in Fig. 2 where the left diagram gives dominant contribution due to suppression.
by $\epsilon$. Estimating the loop diagram we obtain dominant contribution to the decay width for the $\mu \to e\gamma$ process such that

$$
\Gamma_{\mu \to e\gamma} \simeq \frac{e^2 m_{\mu}^3}{16\pi} |a_R|^2, \quad (III.10)
$$

and

$$
a_R \simeq \frac{e\epsilon g_{X}^2 m_{\mu}}{144\pi^2} \int_0^1 dx dy dz \delta(1 - x - y - z) \frac{2x(1 + y)}{[(x^2 - x) + xz + y + z]m_{\mu}^2 + x m_{Z'}^2}, \quad (III.11)
$$

Branching ratio for the LFV process is given by

$$
BR(\mu \to e\gamma) = \frac{\Gamma_{\mu \to e\gamma}}{\Gamma_{\mu \to e\bar{\nu}_e\nu_e}} \simeq \frac{12\alpha}{G_F^2 m_{\mu}^2} |a_R|^2, \quad (III.12)
$$

where $G_F \simeq 1.17 \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant and $\alpha \simeq 1/137$ is the fine structure constant. In Fig. 3 we show $BR(\mu \to e\gamma)$ on $\{g_X, \log |\epsilon|\}$ plane fixing $m_{Z'} = 1.5(2.0)$ TeV where the shaded regions are excluded by the current constraint $BR(\mu \to e\gamma) \lesssim 4.2 \times 10^{-13}$ by the MEG experiment [44]. Further parameter region will be explored in future with improved sensitivity [45].

Here we also discuss $\mu \to e$ conversion via $Z'$ exchange. In our case, the relevant effective Lagrangian for the process is derived as follows [46-48]

$$
\mathcal{L}_{eff} = -\frac{4G_F}{\sqrt{2}} \sum_{N=p,n} \left[ C_{VN}^{NN} \bar{e}_\gamma^\alpha P_L \bar{\mu} N \gamma_\alpha N + C_{AL}^{NN} \bar{\epsilon}_\gamma^\alpha P_L \bar{\mu} \bar{N} \gamma_\alpha \gamma_5 N \right], \quad (III.13)
$$
TABLE III: A summary of parameters for the $\mu - e$ conversion formula for $^{27}_{13}$Al and $^{197}_{79}$Au nuclei [47, 49].

| Nucleus $^{A}_{Z}$N | $V^{p}$ | $V^{n}$ | $\Gamma_{\text{capt}}$ [10$^{6}$ sec$^{-1}$] |
|---------------------|--------|--------|------------------|
| $^{27}_{13}$Al      | 0.0161 | 0.0173 | 0.7054           |
| $^{197}_{79}$Au     | 0.0974 | 0.146  | 13.07            |

where the corresponding coefficients are given by

$$C_{\nu L}^{\nu n(n)} = -C_{\nu L}^{\nu n(n)} = (2) \frac{\sqrt{2} g_{X}^{2} |V_{d}|^{2}}{216 G_{F} m_{Z'}^{2}}.$$  \hspace{1cm} (III.14)$$

Then we obtain the spin-independent contribution to the BR for $\mu \to e$ conversion on a nucleus such that

$$BR(\mu \to e) = \frac{32 G_{F}^{2} m_{\mu}^{5}}{\Gamma_{\text{cap}}} |C_{\nu L}^{\nu n} V^{(p)} + C_{\nu L}^{\nu n} V^{(n)}|^{2},$$  \hspace{1cm} (III.15)$$

where $\Gamma_{\text{cap}}$ is the rate for the muon to transform to a neutrino by capture on the nucleus, and $V^{(p,n)}$ is the integral over the nucleus for lepton wavefunctions with corresponding nucleon density. The values of $\Gamma_{\text{cap}}$ and $V^{(n,p)}$ depend on target nucleus and those for $^{197}_{79}$Au and $^{27}_{13}$Al are given in Table. III [47, 49]. In Fig. 4, we show $BR(\mu \to e)$ for $^{27}_{13}$Al on \{$g_{X}$, log $|\epsilon|$\} plane fixing $m_{Z'} = 1.5(2.0)$ TeV in left(right)-panel where gray(light-gray) shaded region is excluded by current $\mu \to e\gamma$ BR ($\mu \to e$ BR on $^{197}_{79}$Au [50]) constraints. We find that large parameter region can be explored by $\mu \to e$ conversion measurement since its sensitivity will reach $\sim 10^{-16}$ on $^{27}_{13}$Al nucleus in future experiments [51, 52].

We next consider the LFV B decay $B_{s} \to \mu^{+}e^{-}$ which is related to $C_{9}^{\mu}$ above. It is because that the process is induced from $C_{10}^{\mu e}$ which is obtained as $C_{10}^{\mu e} = -\epsilon \Delta C_{9}^{\mu}$ in the model. The branching ratio can be given by

$$BR(B_{s} \to \mu e) = \left| \frac{C_{10}^{\mu e}}{C_{10}^{SM}} \right|^{2} BR(B_{s} \to \mu^{+}\mu^{-})_{SM} \simeq |0.24 \times \epsilon \Delta C_{9}^{\mu}|^{2} BR(B_{s} \to \mu^{+}\mu^{-})_{SM},$$  \hspace{1cm} (III.16)$$

where we used $C_{10}^{SM}(\mu_{b}) \simeq -4.2$ and $BR(B_{s} \to \mu^{+}\mu^{-})_{SM} = (3.65 \pm 0.23) \times 10^{-9}$ is the SM prediction for the BR of $B_{s} \to \mu^{+}\mu^{-}$. We find that $BR(B_{s} \to \mu e) < 10^{-11}$ in the parameter region satisfying the constraint from $BR(\mu \to e\gamma)$ which is well below the current constraint.
FIG. 4: $BR(\mu \rightarrow e)$ on $^{27}_{13}\text{Al}$ as a function of $\{g_X, \log |\epsilon|\}$ fixing $m_{Z'} = 1.5(2.0)$ TeV for left(right) plot where gray(light-gray) shaded region is excluded by current $\mu \rightarrow e\gamma$ $BR$ ($\mu \rightarrow e$ $BR$ on $^{197}_{79}\text{Au}$ [50]) constraints.

D. Constraint from neutrino trident process and $Z'$ contribution to muon $g - 2$

$U(1)_X$ gauge coupling and $Z'$ mass are constrained by the neutrino trident process $\nu N \rightarrow \nu N \mu^+ \mu^-$ where $N$ is a nucleon [53]. The bound is approximately given by $m_{Z'}/g_X \gtrsim 550$ GeV for $m_{Z'} > 1$ GeV. We then consider parameter region of $\{m_{Z'}, g_X\}$ satisfying this bound.

The observed muon magnetic dipole moment is deviated from the SM prediction as $\Delta a_\mu = (26.1 \pm 8.0) \times 10^{-10}$ [54] (muon $g - 2$). The $Z'$ boson can contribute to muon $g - 2$ at one loop level as

$$\Delta a_\mu^{Z'} \approx \frac{g_X^2 x_\mu^2}{4\pi^2} \int_0^1 da \frac{ra(1 - a)^2}{r(1 - a)^2 + a}, \quad (III.17)$$

where $r \equiv (m_\mu/M_{Z'})^2$. We find that the $Z'$ contribution is small for the parameter region providing $\Delta C_9 \sim -1$; for example $\Delta a_\mu^{Z'} \sim 1.7 \times 10^{-12}$ with $m_{Z'} = 1500$ GeV and $g_X = 0.6$. 
E. Constraint from $B_s$–$\bar{B}_s$ mixing

In our model, $Z'$ and neutral scalar bosons induce flavor changing neutral current (FCNC) interactions. Here we consider constraints from $B_s$–$\bar{B}_s$ mixing where other $\Delta F = 2$ processes are more suppressed by CKM factors.

The effective Hamiltonian for the $B_s$–$\bar{B}_s$ mixing is given by

$$H_{\text{eff}} = C_1 (\bar{s} \gamma_\mu P_L b)(\bar{s} \gamma_\mu P_L b) + C_2' (\bar{s} P_R b)(\bar{s} P_R b).$$  \hspace{1cm} (III.18)

The relevant Wilson coefficients are

$$C_1 = \frac{1}{2} \frac{g_X^2}{9 m_{Z'}^2} \left( \Gamma_{sb}^d \right)^2, \quad C_2' = \sum_{\eta=h,H,A} \frac{-1}{2 m_\eta^2} \left( \Gamma_{sb}^\eta \right)^2,$$  \hspace{1cm} (III.19)

where $\Gamma_{\eta q \bar{q}'}$ is couplings for $\eta q \bar{q}'$ interactions ($\eta = h, H, A$), the explicit expressions of which are given in the Appendix. Using these Wilson coefficients we obtain ratio between $\Delta m_{B_s}$ in our model and the SM prediction $\Delta m_{B_s}^{SM}$, under large $\tan \beta$ and small $\alpha$, such that

$$R_{B_s} = \frac{\Delta m_{B_s}}{\Delta m_{B_s}^{SM}} \approx \frac{g_X^2 (V_{tb} V_{ts}^*)^2}{9 m_{Z'}^2} \left( 8.2 \times 10^{-5} \text{ TeV}^{-2} \right)^{-1} \left[ 0.12 \cos^2 (\alpha - \beta) \tan^2 \beta + 0.19 \tan^2 \beta \left( \frac{(200 \text{ GeV})^2}{m_H^2} - \frac{(200 \text{ GeV})^2}{m_A^2} \right) \right],$$  \hspace{1cm} (III.20)

where the first and second terms in the right-hand side corresponds to contributions from $Z'$ and scalars, respectively \cite{14, 55, 56}. The allowed range of $R_{B_s}$ is estimated by \cite{55, 56}

$$0.83 < R_{B_s} < 0.99.$$  \hspace{1cm} (III.21)

We find that $R_{B_s}$ will be deviated from the allowed range by $Z'$ contribution when $\Delta C_9^\mu \approx -1$ is required. Thus cancellation between $Z'$ and scalar contribution is necessary to satisfy the experimental constraint\footnote{Similar phenomena were also observed in the flavor gauge model where $U(1)'$ gauge interaction couples only to the right-handed top quark in the interaction basis in the context of the top forward-backward asymmetry and the same sign top pair productions at hadron colliders \cite{57, 58}. In that model, cancellation between the amplitudes with $t$-channel exchanges of vector and (pseudo)scalar bosons occur in the same sign top pair production through $u_R u_R \rightarrow t_R t_R$, which saves the $U(1)$ flavor model from the stringent constraints from the same sign top pair production at the Tevatron and the LHC.}.

Here we derive allowed parameter region on $\{m_H, m_A - m_H\}$
\[ \tan \beta = 10 \quad \cos(\alpha - \beta) \sim 0 \]

FIG. 5: The allowed region on \( \{m_H, m_A - m_H\} \) plane satisfying \( B_s - \bar{B}_s \) constraints with fitting \( C_9 \) to explain \( b \to s\ell^+\ell^- \) anomalies where the yellow/light yellow region corresponds to that in Fig. 1. Here we take \( \tan \beta = 10 \) and \( \cos(\alpha - \beta) \sim 0 \) as reference values.

plane satisfying \( B_s - \bar{B}_s \) constraints when we fit \( C_9^\mu \) to explain \( b \to s\ell^+\ell^- \) anomalies choosing \( \tan \beta = 10 \) and \( \cos(\alpha - \beta) \sim 0 \) as reference values. In Fig. 5 we show the allowed parameter region where the yellow/light yellow region corresponds to that in Fig. 1.

F. Prediction on \( B \to K^{(*)}\tau^+\tau^- \)

Here we discuss \( B \to K^{(*)}\tau^+\tau^- \) process in our model. The branching ratios are given by Wilson coefficient \( C_9 \) associated with \( \tau \) such that \[ 10^7 \times BR(B \to K\tau^+\tau^-)^{[15,22]} = (1.20 + 0.15\Delta C_9^\tau + 0.02(\Delta C_9^\tau)^2) \]
\[ \pm (0.12 + 0.02\Delta C_9^\tau), \] \quad (III.22)

\[ 10^7 \times BR(B \to K^*\tau^+\tau^-)^{[15,19]} = (0.98 + 0.38\Delta C_9^\tau + 0.05(\Delta C_9^\tau)^2) \]
\[ \pm (0.09 + 0.03\Delta C_9^\tau + 0.01(\Delta C_9^\tau)^2), \] \quad (III.23)

where the superscript indicates the \( q^2 \) range for the dilepton invariant mass in unit of [GeV^2].

For the \( b \to s\tau^+\tau^- \) channel, we obtain \( \Delta C_9^\tau = -4C_9^\mu \) from our charge assignments, and the BRs are slightly enhanced from the SM prediction by factor \( \sim 1.5 \). However current upper bounds of the BRs are much larger than the prediction as \( BR(B \to K\tau^+\tau^-) < \)
2.25 \times 10^{-3}$. Therefore it is difficult to test the enhancement effect.

IV. COLLIDER PHYSICS AND DARK MATTER

In this section we explore collider physics focusing on $Z'$ production at the LHC and estimate relic density of our DM candidate searching for parameter region providing observed value.

A. $Z'$ production at the LHC

Here we discuss $Z'$ production at the LHC 13 TeV where $Z'$ can be produced via interaction in Eq. (1.6), followed by decay modes of $Z' \rightarrow \mu^+\mu^-$ and $Z' \rightarrow \tau^+\tau^-$ [Drell-Yan (DY) productions]. In this model $Z'$ mainly decays into $\tau^+\tau^-$ model with $BR(Z' \rightarrow \tau^+\tau^-) \sim 0.5$ and BR of $\mu^+\mu^-$ mode is suppressed by factor of $1/16$. The production cross section is estimated by CalcHEP [63] using the CTEQ6 parton distribution functions (PDFs) [64]. In Fig. 6 we show $\sigma(pp \rightarrow Z') BR(Z' \rightarrow \ell^+\ell^-/\tau^+\tau^-)$ ($\ell = e, \mu$) as a function of the $Z'$ mass for several values of $g_X$. The cross sections are compared with constraints from LHC data; from Refs. [65] and [66] for $ \ell^+\ell^-$ and $\tau^+\tau^-$ modes. We thus find that $\ell^+\ell^-$ mode (mostly $\mu^+\mu^-$) provides more strict bound although $BR(Z' \rightarrow \mu^+\mu^-) : BR(Z' \rightarrow \tau^+\tau^-) = 1 : 16$. Here we set masses of $H$, $A$ and $H^\pm$ as 400 GeV and apply $\tan \beta = 10$ and $\cos(\alpha - \beta) = 0$ where the effects of the $Z'$ decays into scalar bosons are small. Also right-handed neutrinos and DM $\chi$ are taken to be heavier than $m_{Z'}/2$ so that $Z'$ does not decay into on-shell right-handed neutrinos and DM.

Our $Z'$ boson also decays into neutrinos with BR value of $BR(Z' \rightarrow \nu_\tau \bar{\nu}_\tau) = 16 BR(Z' \rightarrow \nu_\mu \bar{\nu}_\mu) \simeq 0.25$. Thus we can also test our model by $pp \rightarrow Z' g \rightarrow \nu \bar{\nu} g$ process at the LHC experiments searching for signal with mono-jet plus missing transverse momentum. The cross section of $pp \rightarrow Z' g \rightarrow \nu \bar{\nu} g$ process is, for example, $\sim 1$ fb with $g_X = 0.6$ and $m_{Z'} = 1500$ GeV estimated by CalcHEP with $p_T > 25$ GeV cut. We thus need large integrated luminosity to analyze the signal [67] and it will be tested in future LHC experiments.
FIG. 6: Left(right) plot: $\sigma(pp \rightarrow Z')BR(Z' \rightarrow \ell^+\ell^- (\tau^+\tau^-))$ with $\ell = e, \mu$ for several values of $g_X$ compared with LHC limit; from Refs. [65] and [66] for $\ell^+\ell^-$ and $\tau^+\tau^-$ modes.

B. Dark matter

We consider a Dirac fermion $\chi$ as our DM candidate, and the relic density is determined by the DM annihilation process $\chi\bar{\chi} \rightarrow Z' \rightarrow f_{SM}f_{SM}/HA/H^+H^-$ where $f_{SM}$ is a SM fermion and/or $\chi\bar{\chi} \rightarrow Z'Z'$ depending on kinematic condition. Then we estimate relic density of our DM using micrOMEGAs 4.3.5 [68] implementing relevant interactions. Fig. 7 shows the relic density $\Omega h^2$ as a function of DM mass $m_X$ where we apply several values of $g_X$ and $m_{Z'} = 1.5$ TeV as reference values, and indicate observed $\Omega h^2$ value by horizontal dashed line [69]. We see that the relic density drops at around $m_{Z'} \sim 2m_X$ due to resonant enhancement of the annihilation cross section.

In addition, we scan parameters in the range of

$$m_X \in [200, 3100] \text{ GeV}, \quad m_{Z'} \in [500, 7000] \text{ GeV}, \quad g_X \in [0.01, 1.5], \quad (IV.1)$$

with assuming that $\tan \beta = 10$ and $\cos(\alpha - \beta) = 0$ as reference values. We note that the effects of scalar bosons are subdominant. The left panel of Fig. 8 shows the parameter region which accommodates the observed relic density of DM, $\Omega h^2 = 0.1206 \pm 0.0063$, taking 3σ range of observed value by the Planck collaboration [69]. Moreover the right panel of the figure indicates the region in which both observed relic density and $b \rightarrow s\ell^+\ell^-$ anomalies are explained within 2σ. Notice that the allowed region with $m_{Z'} < m_X$ is partly excluded by or
FIG. 7: Relic density of DM as function of DM masses for different values of $U(1)_X$ gauge couplings, $g_X = 0.1, 0.2$ and 0.3. We have fixed $m_{Z'} = 1.5$ TeV.

close to LHC constraint shown in Fig. 6 and will be explored in future LHC experiments. In addition DM-nucleon scattering cross section by $Z'$ exchange is suppressed by CKM factor and the allowed region is not constrained by the DM direct detection experiments.

Before closing this section we discuss possibility of indirect detection of our DM. In this model DM pair annihilates mainly through $\chi \bar{\chi} \to Z' \to \tau^+\tau^-$ and/or $\chi \bar{\chi} \to Z'Z' \to 2\tau^+\tau^-$ and gamma-ray search gives the strongest constraint on the annihilation cross section by Fermi-LAT observation [70, 71]. In our parameter region of $m_{Z'} > 500$ GeV, DM annihilation cross section explaining the relic density is well below the constraint for the $\tau^+\tau^-$ dominant case [70] unless there is large enhancement factor; constraint on cross section for four $\tau$ mode would be similar. Thus our model is safe from indirect detection cross section and will be tested with larger amount of data in future.

V. SUMMARY AND DISCUSSIONS

We have discussed a flavor model based on $U(1)_{B_3-x_\mu L_\mu-x_\tau L_\tau}(\equiv U(1)_X)$ gauge symmetry in which two Higgs doublet fields are introduced to obtain the observed CKM matrix. Favor changing $Z'$ interactions with the SM quarks are obtained after diagonalizing quark mass matrix, and $b \to s\ell^+\ell^-$ anomalies can be explained due to lepton flavor non-universal charge assignment when $x_\mu$ is taken to be negative value. Then we have considered minimal set up
explaining $b \to s\ell^+\ell^-$ anomalies and generated neutrino mass matrix where two SM singlet scalar fields and Dirac fermionic DM candidate are introduced.

We have computed the $Z'$ contribution to the Wilson coefficient $C_9^\mu$ relevant for $b \to s\mu^+\mu^-$, as well as neutrino mass matrices, charged lepton flavor violations and the $B_s-\bar{B}_s$ mixing, including the relevant experimental constraints. We have found that cancellation between $Z'$ and scalar bosons contributions to $B_s-\bar{B}_s$ is required to satisfy experimental constraint, while explaining $b \to s\ell^+\ell^-$ anomalies. In addition, we have shown constraints from lepton flavor violation process $\mu \to e\gamma$ and future prospects for $\mu \to e$ conversion measurements.

Then collider physics regarding $Z'$ production at the LHC and relic density of DM are explored. We have shown cross sections for the DY processes, $pp \to Z' \to \mu^+\mu^-(\tau^+\tau^-)$, where constraints on the $\{m_{Z'}, g_X\}$ parameter space dominantly come from the data of dimuon resonance search at the LHC. The relic density of DM further constrains $\{m_{Z'}, g_X\}$ parameter space since the relic density is determined by DM pair annihilation process via $Z'$ interactions. The preferred parameter region can be further tested in future LHC experiments and observations for flavor physics such as LFVs.
Acknowledgments

The work of CY was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT), NRF-2017R1A2B4011946 and NRF-2017R1E1A1A01074699.

Appendix

Here we summarize Yukawa interactions in two Higgs doublet sector which are taken from ref. [14].

\[
\mathcal{L}_Y = -\bar{u}_L \left( \frac{\cos \alpha}{v \sin \beta} m_u^D - \frac{\cos(\alpha - \beta)}{\sqrt{2} \sin \beta} \tilde{\xi}^u \right) u_R h - \bar{d}_L \left( \frac{\cos \alpha}{v \sin \beta} m_d^D - \frac{\cos(\alpha - \beta)}{\sqrt{2} \sin \beta} \tilde{\xi}^d \right) d_R h \\
- \bar{u}_L \left( \frac{\sin \alpha}{v \sin \beta} m_u^D - \frac{\sin(\alpha - \beta)}{\sqrt{2} \sin \beta} \tilde{\xi}^u \right) u_R H - \bar{d}_L \left( \frac{\sin \alpha}{v \sin \beta} m_d^D - \frac{\sin(\alpha - \beta)}{\sqrt{2} \sin \beta} \tilde{\xi}^d \right) d_R H \\
- i\bar{u}_L \left( \frac{m_u^D}{v \tan \beta} - \frac{1}{\sqrt{2} \sin \beta} \tilde{\xi}^u \right) u_R A + i\bar{d}_L \left( \frac{m_d^D}{v \tan \beta} - \frac{1}{\sqrt{2} \sin \beta} \tilde{\xi}^d \right) d_R A \\
- \left[ \bar{u}_R \left( \frac{\sqrt{2}}{v \tan \beta} m_u^D V - \frac{1}{\sin \beta} (\tilde{\xi}^u)^\dagger \right) d_L + \bar{u}_L \left( \frac{\sqrt{2}}{v \tan \beta} V m_d^D - \frac{1}{\sin \beta} V \tilde{\xi}^d \right) d_R \right] H^+ \\
+ h.c. ,
\]

(V.1)

where flavor indices are omitted and the non-diagonal coupling matrices are defined as

\[
\tilde{\xi}^u = U_L^\dagger \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \tilde{y}_{31}^u & \tilde{y}_{32}^u & 0 \end{pmatrix} U_R, \quad \tilde{\xi}^d = D_L^\dagger \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & \tilde{y}_{13}^d \\ 0 & 0 & \tilde{y}_{23}^d \\ 0 & 0 & 0 \end{pmatrix} D_R.
\]

(V.2)

Under the approximation \( V \simeq D_L \) and \( D_R \simeq 1 \), we obtain

\[
\tilde{\xi}^d \simeq V^\dagger \xi^d \simeq \frac{\sqrt{2}}{\cos \beta} \frac{m_b}{v} \begin{pmatrix} 0 & 0 & -V^*_{tb} V_{tb} \\ 0 & 0 & -V^*_{tb} V_{tb} \\ 0 & 0 & 1 - |V_{tb}|^2 \end{pmatrix}.
\]

(V.3)

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