Birth of the GUP and its effect on the entropy of the Universe in Lie-N-algebra

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In this paper, the origin of the generalized uncertainty principle (GUP) in an M-dimensional theory with Lie-N-algebra is considered. This theory which we name GLNA-Generalized Lie-N-Algebra-theory can be reduced to M-theory with M = 11 and N = 3. In this theory, at the beginning, two energies with positive and negative signs are created from nothing and produce two types of branes with opposite quantum numbers and different numbers of timing dimensions. Coincidence with the birth of these branes, various derivatives of bosonic fields emerge in the action of the system which produce the r GUP for bosons. These branes interact with each other, compact and various derivatives of spinor fields appear in the action of the system which leads to the creation of the GUP for fermions. The previous predicted entropy of branes in the GUP is corrected as due to the emergence of higher orders of derivatives and different number of timing dimensions.

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I. INTRODUCTION

An interesting result of respective hypotheses of quantum gravity, M-theory and field theory is the creation of a minimum measurable length. This leads to a modification of the Heisenberg uncertainty principle (HUP) or equivalently, modified commutation relations between position coordinates and momenta, which is well known as the generalized uncertainty principle (GUP). There are different versions of the GUP. One version and the comparable modified commutators between position and momentum coordinates contain the quadratic terms, and are given by [8, 9]:

\[ \Delta x_i \Delta p_i \geq \frac{\hbar}{2} \left[ 1 + \beta ( (\Delta p)^2 + \langle p \rangle^2 ) \right] + \]

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We begin the action for the Dp-brane \[22–25\], a new theory which includes all the properties of string theory and N three dimensional brackets \[26–29\]. Now, by using the Lie-N-algebra \[15–25\]. Also, all \(M_p\)-branes in string theories are built from \(N\)-branes which depend on the dimension of the branes and the algebra. We observe that the number of terms and their orders change from zero to the dimensions of the branes. After that, by replacing these derivatives with momenta, we obtain exact forms of the GUP for each brane in the universe. We observe that the number of terms and their orders change from zero to higher numbers differing numbers of timing dimensions and quantum numbers. During the formation of branes, various bosonic fields emerge such that the order of derivatives in their time-spatial wave equations changes from zero to higher numbers differing numbers of timing dimensions and quantum numbers. During the formation of branes, various bosonic fields emerge and their wave equations contain various orders of derivatives. Thus, different versions of branes, fermions emerge and their wave equations contain various orders of derivatives. Therefore, different versions of the GUP are produced in which their shapes depend on the number of dimensions of branes and the algebra. Each of these versions produces a special entropy for which the order of terms in it grows with the increase in the number of dimensions of the brane.

In section II, we consider the process of the emergence of GUP during the formation of branes in the Lie-N-algebra. In section III, we have obtained the corrected entropy by using the exact form of the GUP for a \(p\)-dimensional brane. The summary and conclusion are given in the last section. The units used throughout the paper are: \(\hbar = c = 8\pi G = 1\).

### II. EMERGENCE OF GUP IN LIE-N-ALGEBRA

In this section, first, we obtain the action of \(p\) dimensional branes in a universe with \(M\)-dimensions and Lie-N-algebra. Then, by extracting the wave equation of motion, we show that various derivatives appear for which their orders change from zero to the dimensions of the branes. After that, by replacing these derivatives with momenta, we obtain exact forms of the GUP for each brane in the universe. We observe that the number of terms and their orders depend on the dimension of the branes and the algebra.

Firstly, we should introduce the general form of actions in Lie-N-algebra. Previously, it has been shown that all \(Dp\)-branes in string theories are built from \(D0\)-branes which follow from Lie-two-algebras with two dimensional brackets \[15\ [23\]. Also, all \(M_p\)-branes in \(M\)-theory are constructed from \(M0\)-branes which arise from a Lie-three-algebra with three dimensional brackets \[20\ [24\]. Now, by using the Lie-N-algebra and generalizing dimensions to \(M\), we construct a new theory which includes all the properties of string theory and \(M\)-theory and resolve the puzzles in them. To show this, we begin of the action for the Dp-brane \[22\ [23\]:

\[
S = -\frac{T_{Dp}}{2} \int d^{p+1}x \sum_{n=1}^{p} \beta_{\mu} \chi^{\mu} \chi^{\nu} \chi^{\nu} \chi^{\nu} \chi^{\nu} \] (3)

\[
\chi^{\nu} = \delta^{\nu}_{\alpha} STr \left( -det(P_{ab}[E_{mn}E_{mi}(Q^{-1} + \delta)^{ij}E_{jn}] + \lambda F_{ab})det(Q^j) \right)^{1/2} \] (4)

where

\[
E_{mn} = G_{mn} + B_{mn}, \quad Q^j = \delta^j + i\lambda[X^j, X^k]E_{kj} \] (5)
\[ \lambda = 2 \pi l_s^2 \text{ and } G_{ab} = \eta_{ab} + \partial_a X^i \partial_b X^i \text{ and } X^i \text{ are scalar strings that link to branes. In this equation, } a, b = 0, 1, \ldots, p \text{ are the world-volume indices of the } D_p \text{-branes, } i, j, k = p + 1, \ldots, 9 \text{ are the indices of the transverse space, and } m, n \text{ are related to ten-dimensional spacetime indices. Also, } T_{D_p} = \frac{1}{g_s(2\pi l_s^2)^{p+1}} \text{ refers the tension of the } D_p \text{-brane, } l_s \text{ is the string length and } g_s \text{ refers to the string coupling. Now, we can show that the action of the } D_p \text{-brane can be constructed by summing over actions of } D0 \text{-branes. To this end, we make use of the rules below [15, 17, 22–25]:} \]

\[ \sum_{a=0}^{9} \sum_{m=0}^{9} \frac{1}{(2\pi l_s)^p} \int d^{p+1} \sigma \Sigma_{m=0}^{9} \Sigma_{a=0}^{9} \lambda = 2 \pi l_s^2 \]

\[ i, j = p + 1, \ldots, 9 \quad a, b = 0, 1, \ldots, p \quad m, n = 0, 1, \ldots, 9 \]

\[ \frac{1}{Q} \sum_{n=1}^{P} \frac{1}{Q} (\partial_a X^i \partial_b X^i + \frac{\lambda^2}{4}(F^{ab})^2) \]

\[ \text{det}(Q^i_j) \to \text{det}(Q^i_j) \prod_{n=1}^{P} \text{det}(\partial_a X^i \partial_b X^i + \frac{\lambda^2}{4}(F^{ab})^2) \]

Using the above laws, we can show that the action of } D_p \text{-branes can be written in terms of two dimensional brackets [15, 17, 22–25]:}

\[ S_{D_p} = -(T_{D0})^p \int dt \sum_{n=1}^{P} \beta_n (\delta_{a_1 a_2 \ldots a_n} L_{b_1}^{b_2 \ldots b_n} L_{a_1} \ldots L_{a_n})^{1/2} \]

\[ (L_a)^b = Tr \left( \sum_{a=0}^{9} \sum_{b=1}^{9} ([X^a, X^b] [X_b, X_j]) + [X^a, X^b] [X_b, X^j] [X_i, X_j] \right) \]

where we have made use of the antisymmetric properties for \( \delta \). At this stage, we can show that the above action is built by summing over the actions of } D0 \text{-branes by using the definition below for the } D0 \text{-brane [17, 25]:}

\[ S_{D0} = -T_{D0} \int dt Tr(\sum_{m=0}^{9} [X^m, X^n]^2) \]

Replacing the two dimensional brackets in the Lie-two-algebra by three dimensional brackets in the Lie-three-algebra, we obtain the action of the } M0 \text{-branes [15, 22–26, 29]:}

\[ S_{M0} = T_{M0} \int dt Tr(\sum_{M,N,L=0}^{10} ([X^M, X^N, X^L], [X^M, X^N, X^L])) \]

where } X^M = X^M_\alpha T^\alpha \text{ and }

\[ [T^\alpha, T^\beta, T^\gamma] = f^{\alpha\beta\gamma}_{\eta} T^\eta \]

\[ \langle T^\alpha, T^\beta \rangle = h^{\alpha\beta} \]

\[ [X^M, X^N, X^L] = [X^M_a T^a, X^N_\alpha T^\alpha, X^L_\beta T^\beta] \]

\[ \langle X^M, X^M \rangle = X^M_\alpha X^M_\beta \langle T^\alpha, T^\beta \rangle \]

where } X^M(i=1,3,10) \text{ are scalar strings which are linked to the } M0 \text{-brane. By substituting } N \text{-dimensional brackets instead of three dimensional brackets in the action of } [3], \text{ we obtain the action of the } G0 \text{-brane in } GLNA-theory \text{ as:}

\[ S_{G0} = T_{G0} \int dt Tr(\sum_{L_1=L_2=\ldots=L_N=0}^{M} ([X^{L_1}, X^{L_2}, \ldots, X^{L_N}], [X^{L_1}, X^{L_2}, \ldots, X^{L_N}]))) \]

where } X^M = X^M_\alpha T^\alpha \text{ and }
\[ [T^{\alpha_1}, T^{\alpha_2}, ..., T^{\alpha_N}] = f^{\alpha_1 \alpha_2 \alpha_N} T^L \]
\[ \langle T^\alpha, T^\beta \rangle = h^{\alpha \beta} \]
\[ [X^{L_1}, X^{L_2}, ..., X^{L_N}] = \{X^{L_1}_{\alpha_1} T^{\alpha_1}, X^{L_2}_{\alpha_2} T^{\alpha_2}, ..., X^{L_N}_{\alpha_N} T^{\alpha_N} \} \]
\[ \{X^M, X^M \} = X^M_\alpha X^M_\beta \langle T^\alpha, T^\beta \rangle \]

The above action reduces to the action of M0-branes by putting \( N = 3 \) and \( M = 10 \) and also to the action of D0-branes for \( N = 2 \) and \( M = 9 \). Now, the question that arises is what is the origin of this brane in GLNA-theory? To answer this question, we suppose that first, two energies with opposite signs are produced such that the sum over them is zero. Then, these energies create 2M degrees of freedom which each two of them cause to creation of new dimension. After that, \( M - N \) of degrees of freedom are hidden by compacting half of M-N dimensions on a circle to create Lie-N-algebra. During this process, the behaviour of some dimensions changes and they form times. Also, for second energy, the properties of more dimensions change which lead to the emergence of more time coordinates. Thus, the physics of branes is completely different from anti-branes.

Let us to begin with two oscillating energies which are created from nothing and expands in the \( M^{th} \) dimension. We obtain:

\[ E \equiv 0 \equiv E_1 + E_2 \equiv \begin{cases} 0 \equiv N_1 + N_2 \equiv k((X^M)^2 - (X^M)^2) = k \int d^2\varepsilon (\frac{\partial}{\partial x})^2 ((X^M)^2 - (X^M)^2) \end{cases} \]

where \( N_{1/2} \) are the number of degrees of freedom for first and second energies. These energies are oscillating, excited and produce \( M \) dimensions with 2M degrees of freedom. This can be shown clearly by rewriting equation (13) as follows:

\[ E \equiv 0 \equiv k \int d^2x \varepsilon^{i_1 i_2 \cdots i_M} \varepsilon^{i_1 i_2 \cdots i_M} \left( \frac{\partial}{\partial x_{i_1}} \frac{\partial}{\partial x_{i_2}} \cdots \frac{\partial}{\partial x_{i_{M-1}}} \right)^2 (X^M)^2 - k \int d^2x \varepsilon^{i_1 i_2 \cdots i_M} \varepsilon^{i_1 i_2 \cdots i_M} \left( \frac{\partial}{\partial x_{i_1}} \frac{\partial}{\partial x_{i_2}} \cdots \frac{\partial}{\partial x_{i_{M-1}}} \right)^2 (X^M)^2 \]

where, we apply the definition \( \varepsilon^{i_1 i_2 \cdots i_M} \varepsilon^{i_1 i_2 \cdots i_M} = -1 \). We can substitute some brackets instead of derivatives and write \[ \varepsilon^{i_1 i_2 \cdots i_M} \varepsilon^{i_1 i_2 \cdots i_M} = -1 \]. We can substitute some brackets instead of derivatives and write \[ \varepsilon^{i_1 i_2 \cdots i_M} \varepsilon^{i_1 i_2 \cdots i_M} = -1 \]. We can substitute some brackets instead of derivatives and write

Applying the relations of Eq. (15) in Eq. (14), we get:

\[ E \equiv 0 \equiv E_1 + E_2 \equiv \]
\[ E_1 = k \int d^2x ([X_{i_1}, X_{i_2}, ..., X_{i_M}], [X_{i_1}, X_{i_2}, ..., X_{i_M}]) \]
\[ E_2 = -k \int d^2x ([X_{i_1}, X_{i_2}, ..., X_{i_M}], [X_{i_1}, X_{i_2}, ..., X_{i_M}]) \]

(16)
The shape of the above energies is similar to the shape of the action of $G_0$-branes, however their algebra has an $M$ dimensional bracket, while the $G_0$-brane has an $N$ dimensional bracket and for producing this algebra, we should remove $M - N$ degrees of freedom by compactification. To achieve this aim, we apply the mechanism in [20–26] and replace $X_{i\in1,3,5,\ldots,M-N}$ by $i\frac{R}{l_p}$, where $l_p$ is the Planck length. We derive the action below for the first energy:

$$E_1 \equiv k \int d^2M x([X_{i1}, X_{i2}, \ldots, X_{iM}],[X_{i1}, X_{i2}, \ldots, X_{iM}]) =$$

$$k \int d^2M x\xi^{i1\ldots iM} \xi^{j1\ldots jM} X_{i1}X_{i2}\ldots X_{iM}X_{j1}X_{j2}\ldots X_{jM} =$$

$$(i)^{2(M-N)}k \int d^N x\left(\frac{R^{M-N}}{l_p^2}\right)^{\frac{1}{2}}\xi^{i1\ldots jN} X_{i1}X_{j2}\ldots X_{jN} =$$

$$(i)^{2(M-N)}k \int d^N x\left(\frac{R^{M-N}}{l_p^2}\right)^{\frac{1}{2}}([X_{i1}, X_{i2}, \ldots, X_{iN}],[X_{j1}, X_{j2}, \ldots, X_{jN}]) =$$

$$k \int d^N x\left(\frac{R^{M-N}}{l_p^2}\right)^{\frac{1}{2}}([iX_{i1}, iX_{i2}, \ldots, iX_{iM-N+1}, X_{jN}],[iX_{j1}, iX_{j2}, \ldots, iX_{jM-N+1}, X_{jN}])$$

where we have defined $\xi^{i1\ldots iM} \xi^{j1\ldots jM} = (-i)^{N-M}\xi^{i1\ldots jN} \xi^{j1\ldots jN}$. Some scalar strings take one extra (i) and expand in the time directions. Obviously, in GLNA-theory, there exists $M - N$ time coordinates where $M$ is the dimension of the universe and $N$ is the dimension of the algebra. The reason that we only observe one dimension is our living in a four dimensional universe and observing the three dimensional brackets of M-theory. For this reason, for us, $M = 4$ and $N = 3$ and thus we have only one time dimension. For the second energy, for which its sign is opposite to that of first energy, we have some extra time dimensions:

$$E_2 = -E_1 \equiv (i)^2E_1 =$$

$$k \int d^N x\frac{R^{M-N}}{l_p^2}\left(\frac{1}{2}\right)^{\frac{1}{2}}([iX_{i1}, iX_{i2}, \ldots, iX_{iM-N+1}, X_{jN}],[iX_{j1}, iX_{j2}, \ldots, iX_{jM-N+1}, X_{jN}])$$

Thus, properties of anti-branes which are created by this energy, are different and we have more time dimensions. For example, in one four dimensional anti-universe, there exists two time coordinates and all things are changed. In our universe, the length of an object can be defined by $l^2 = -t^2 + x_1^2 + x_2^2 + x_3^2$ where $t$ is time and $x_i$ are coordinates of space, while, in an anti-universe, length is obtained by $l^2 = -t^2 - x_1^2 + x_2^2 + x_3^2$. Also, energy and momentum, which is related to the mass with the equation for our universe $(m^2 = -E^2 + P^2_1 + P^2_2 + P^2_3)$, has this relation $(m^2 = -E^2 - P^2_1 + P^2_2 + P^2_3)$ for an anti-universe.

Until now, we have considered symmetrical compactification. However, maybe during this process, the symmetry of the system is broken and only the upper or the lower part of one dimension is compactified, and fermions emerge $(X \rightarrow \psi^\dagger \psi^L \psi^R)$. To include non-symmetrical compactification, we use the mechanism in [17], and compactly the $M^{th}$ dimension of the branes on a circle with radius $R$ by choosing $< X^M > = i\frac{R}{l_p}P^M$ for bosons and $< \psi^L^*, M > = i\frac{R^{3/2}}{l_p^2}T^{L,M}$ for fermions in the action of [17]. We get:

$$E_1 \equiv k \int d^N x\left(\frac{R^{M-N}}{l_p^2}\right)^{\frac{1}{2}}([iX_{i1}, iX_{i2}, \ldots, iX_{iM-N}, X_{jN}],[iX_{j1}, X_{j2}, \ldots, iX_{jM-N}, X_{jN}])$$

$$-i\left(\frac{R^{M-N}}{l_p^2}\right)^{\frac{1}{2}}([iX_{i1}, iX_{i2}, \ldots, iX_{iM-N}, \gamma_{jM}, \psi_{R,jN}],[iX_{j1}, iX_{j2}, \ldots, iX_{jM-N}, \gamma_{jM}, \psi_{R,jN}])$$

By choosing $\gamma_{jm} = T_{jm} \frac{R^2}{l_p}$, where the $\gamma_{jm}$’s are the Pauli matrices in $M$ dimensions, we obtain the action of the initial energy as follows:

$$E_1 \equiv k \int d^N x\left(\frac{R^{M-N}}{l_p^2}\right)^{\frac{1}{2}}([iX_{i1}, iX_{i2}, \ldots, iX_{iM-N}, X_{jN}],[iX_{j1}, X_{j2}, \ldots, iX_{jM-N}, X_{jN}])$$

$$-i\left(\frac{R^{M-N}}{l_p^2}\right)^{\frac{1}{2}}([iX_{i1}, iX_{i2}, \ldots, iX_{iM-N}, \gamma_{jM}, \psi_{R,jN}],[iX_{j1}, iX_{j2}, \ldots, iX_{jM-N}, \gamma_{jM}, \psi_{R,jN}])$$

(20)
This action includes both fermionic and bosonic degrees of freedom and supersymmetry emerges. If we assume that all scalars depend only on one time coordinate and \((R = i^{1/2})\), we achieve the action of \(G0\)-branes:

\[
S_{G0} \equiv kV_{N-1} \int dt \left( \langle \left[ iX_{j1}, iX_{j2}, \ldots, iX_{jM-N} \ldots, X_{jN} \right], \left[ iX_{j1}, X_{j2}, \ldots, iX_{jM-N} \ldots, X_{jN} \right] \rangle - i\langle \left[ iX_{j1}, iX_{j2}, \ldots, iX_{jM-N} \ldots, \gamma_{jM'}, \psi_{R_{jN}} \right], \left[ iX_{j1}, iX_{j2}, \ldots, iX_{jM-N} \ldots, \gamma_{jM'}, \psi_{R_{jN}} \right] \rangle \right) \tag{21}
\]

where \(V_{N-1}\) is the volume of space which is formed by the remaining coordinates. Also, by adding one negative sign and using equation \((13)\), we obtain the action of the anti-\(G0\)-brane:

\[
S_{\text{Anti-}G0} \equiv kV_{N-1} \int dt \left( \langle \left[ iX_{j1}, iX_{j2}, \ldots, iX_{jM-N+1} \ldots, X_{jN} \right], \left[ iX_{j1}, X_{j2}, \ldots, iX_{jM-N+1} \ldots, X_{jN} \right] \rangle - i\langle \left[ iX_{j1}, iX_{j2}, \ldots, iX_{jM-N+1} \ldots, \gamma_{jM'}, \psi_{R_{jN}} \right], \left[ iX_{j1}, iX_{j2}, \ldots, iX_{jM-N+1} \ldots, \gamma_{jM'}, \psi_{R_{jN}} \right] \rangle \right) \tag{22}
\]

These actions for the \(G0\)-brane and the \(G0\)-anti-brane contain both bosonic and fermionic fields which are created due to symmetrical or non-symmetrical compactification of dimensions. Also, time which is a puzzle in cosmology, is produced by compactification. Thus, we conclude that all fields have the same origin and begin from nothing. Then, by different compactification, different shapes of matter emerge.

By substituting \(N\)-dimensional brackets in equations \((21)\) and \((22)\) instead of two dimensional brackets, and increasing dimensions from 10 in action \((7)\), we can obtain the action of the \(Gp\)-brane and anti-\(Gp\)-brane:

\[
S_{Gp} = - (T_{G0})^p \int dt \sum_{n=1}^{p} \beta_n \left( \delta_{b_1 b_2 \ldots b_n} L_{a_1} L_{a_n} \right)^{1/2}
\]

\[
(L)^{\delta a b}_n = \delta^a b_n \text{Tr} \left( \sum_{L=0}^{N-L^p} \sum_{H=0}^{N-\sum_p} \sum_{a_1 \ldots a_L=0}^{\Sigma^M} \sum_{j_1 \ldots j_H=p+1}^{(N-N-1)} \right)
\]

\[
i^2(p-N) \left( \langle X_{j1}, \ldots, X_{jH-1}, X_{a1}, \ldots, X_{aL}, X_{jH} \rangle, \langle X_{j1}, \ldots, X_{jH-1}, X_{a1}, \ldots, X_{aL}, X_{jH} \rangle \right) +
\]

\[
i^2(p-N) \left( \sum_{L=0}^{N-L^p} \sum_{H=0}^{N-\sum_p} \sum_{a_1 \ldots a_L=0}^{\Sigma^M} \sum_{j_1 \ldots j_H=p+1}^{(N-N-1)} \right)
\]

\[
S_{\text{Anti-}Gp} = - (T_{\text{Anti-G0}})^p \int dt \sum_{n=1}^{p} \beta_n \left( \delta_{b_1 b_2 \ldots b_n} L_{a_1} L_{a_n} \right)^{1/2}
\]

\[
(L)^{\delta a b}_n = \delta^a b_n \text{Tr} \left( \sum_{L=0}^{N-L^p} \sum_{H=0}^{N-\sum_p} \sum_{a_1 \ldots a_L=0}^{\Sigma^M} \sum_{j_1 \ldots j_H=p+1}^{(N-N-1)} \right)
\]

\[
i^2(p-N+1) \left( \langle X_{j1}, \ldots, X_{jH-1}, X_{a1}, \ldots, X_{aL}, X_{jH} \rangle, \langle X_{j1}, \ldots, X_{jH-1}, X_{a1}, \ldots, X_{aL}, X_{jH} \rangle \right) +
\]

\[
i^2(p-N+1) \left( \sum_{L=0}^{N-L^p} \sum_{H=0}^{N-\sum_p} \sum_{a_1 \ldots a_L=0}^{\Sigma^M} \sum_{j_1 \ldots j_H=p+1}^{(N-N-1)} \right)
\]

To write the actions in terms of gauge fields and derivatives with respect to fields, we have to use some laws. Extending the rules in equation \((6)\) for \(M\)-theory to \(N\)-dimensional brackets in GLNA-theory, we can obtain the following laws \((13)-(18)\), \((23)-(29)\):

\[
\sum_{L=0}^{N-L^p} \sum_{a_1 \ldots a_L=0}^{\Sigma^M} \sum_{j_1 \ldots j_H=p+1} \langle X_{j1}, \ldots, X_{jH-1}, X_{a1}, \ldots, X_{aL}, X_{jH} \rangle =
\]

\[
\frac{1}{2} \sum_{L=0}^{N-L^p} \sum_{a_1 \ldots a_L=0}^{\Sigma^M} \sum_{j_1 \ldots j_H=p+1} (X_{j1} X_{jH-1})^2 \langle \partial_{a_1} \ldots \partial_{a_L} X^i, \partial_{a_1} \ldots \partial_{a_L} X^i \rangle
\]
\[S_{Gp} = -(T_{Gp}) \int dt \sum_{\alpha=1}^{p} \beta_{\alpha} \left( \delta_{\alpha 1} a_{a_1 a_2 \ldots a_n} L_{a_1} \right)^{1/2}
\]

\[\left( L \right)^a_{b_0} = \delta^a_{b_0} T \left( \frac{1}{2} \right) (2^{p-N}) \sum_{H=0}^{N} \sum_{a_1 \ldots a_p} \lambda^2 \frac{1}{2 \ldots N} (X_{j_1} \ldots X_{j_p})^2 \langle \delta_{a_1 \ldots a_p} \rangle + \]

\[\left( L \right)^a_{b_0} = \delta^a_{b_0} T \left( \frac{1}{2} \right) (2^{p-N+1}) \sum_{H=0}^{N} \sum_{a_1 \ldots a_p} \lambda^2 \frac{1}{2 \ldots N} (X_{j_1} \ldots X_{j_p})^2 \langle \delta_{a_1 \ldots a_p} \rangle - \]
$$\frac{1}{2}i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i) -$$

$$i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}\frac{\lambda^2}{1.2...N}g_{a_L-1}(X^{j_1}...X^{j_H})^2\langle \tilde{F}^{a_1...a_{L-1}}, \tilde{F}^{a_1...a_L}\rangle (27)$$

These actions reduce to the action of $Dp$-branes (7) for $N = 2$ and $M = 9$ and the action of $Mp$-branes for $N = 3$ and $M = 10$ [13,29]. Also, it is clear that for each scalar, there is a fermion, and for each bosonic gauge field with each rank, there exists a fermionic gauge field with the same rank. This means that these actions are super-symmetric and the number of degrees of freedom for both fermions and bosons are the same. For $\beta_2 \neq 0$ and $\beta_n = 0, n \neq 2$, the above actions can be simplified to:

$$S_{GP} = -(T_{GP}) \int dt \beta_2 (\delta_{a_1 a_2} (\delta_{a_1} \delta_{a_2} - \delta_{a_2} \delta_{a_1})) \times$$

$$Tr\left(\frac{1}{2}i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2(\partial_{a_1}...\partial_{a_L}X^i,\partial_{a_1}...\partial_{a_L}X^i) +
\right.$$}

$$\left.i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i)
\right) (28)$$

$$S_{Anti-GP} = -(T_{Anti-GP}) \int dt \beta_2 (\delta_{a_1 a_2} (\delta_{a_1} \delta_{a_2} - \delta_{a_2} \delta_{a_1})) \times$$

$$Tr\left(\frac{1}{2}i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2(\partial_{a_1}...\partial_{a_L}X^i,\partial_{a_1}...\partial_{a_L}X^i) +
\right.$$}

$$\left.i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i)
\right) (29)$$

Now, we can extract the wave functions from the actions of (28) for branes:

$$i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i) +$$

$$i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i) +$$

$$i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i) +$$

$$i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i) = 0 (30)$$

Also, the wave equations for anti-branes can be obtained from the action of (29):

$$i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i) +$$

$$i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i) +$$

$$i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i) +$$

$$i^{2(p-N)+1}\sum_{L=0}^{N-p}\sum_{a_1...a_L=0}^{M-p}\sum_{j_1...j_H=p+1}(X^{j_1}...X^{j_{H-1}})^2\gamma_{a_L-1}(\partial_{a_1}...\partial_{a_{L-1}}\psi^i,\partial_{a_1}...\partial_{a_L}\psi^i) = 0 (30)$$
\[ i^{2(p-N)+1} \Sigma_{L=0}^{N} \Sigma_{H=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p+1}(X^{j})^{jH-1} 2 \gamma^{a_{L}-1} \partial_{\alpha_{i}}..\partial_{\alpha_{L}}..\partial_{\alpha_{L}-1} \partial_{\alpha_{L}} \psi^{i} + \]
\[ i^{2(p-N)+1} \Sigma_{L=0}^{N} \Sigma_{H=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p+1}(\partial_{\alpha_{i}}..\partial_{\alpha_{L}}..\partial_{\alpha_{L}-1}(X^{j})^{jH-1} 2 \gamma^{a_{L}-1} \partial_{\alpha_{i}}..\partial_{\alpha_{L}} \psi^{i} - \]
\[ i^{2(p-N)+1} \Sigma_{L=0}^{N} \Sigma_{H=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p+1}(X^{j})^{jH-1} 2 \gamma^{a_{L}-1} \partial_{\alpha_{i}}..\partial_{\alpha_{L}} \psi^{i} - \]

By assuming that all gauge fields are zero \((F = \tilde{F} = 0)\), and substituting \(\partial \equiv -iP\) and \(X^{i} = e^{ipx}\), we obtain the following relations from equations \([30] \) and \([31] \):

\[
\left( \sum_{N=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p}(1)^{a_{1}+..a_{L}} \right)^{2} \left( \sum_{N=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p}(1)^{a_{1}+..a_{L}} \right) \psi^{i} = 0
\]

\[
\left( \sum_{N=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p}(1)^{a_{1}+..a_{L}} \right)^{2} \left( \sum_{N=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p}(1)^{a_{1}+..a_{L}} \right) \psi^{i} = 0
\]

These equations show that momenta for bosons and spinors, and also for branes and anti-branes, are different. For example, if the energy-momentum tensor on the four dimensional brane contains only one energy with extra \((i)\)factor, this tensor on a four dimensional anti-brane includes two components of energy with the extra \((i)\)factor. Now, we compare these wave equations with wave equations in four dimensions:

\[
\square^{2} X^{i} = 0 \Rightarrow P_{0}^{2} X^{i} = 0
\]
\[
i \gamma^{a_{L}-1} P_{0, \mu} \psi^{i} = 0 \Rightarrow i \gamma^{a_{L}-1} P_{0, \mu} \psi^{i} = 0
\]

Comparing the equation \([33] \) with equations \([32] \) and \([33] \), we can obtain the explicit form of the momenta in a four dimensional universe in terms of the momenta in Lie-N-algebra:

\[
P_{0, \text{Uni, boson}}^{2} = \sum_{L=0}^{N} \sum_{H=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p}(1)^{a_{1}+..a_{L}} \left( \sum_{N=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p}(1)^{a_{1}+..a_{L}} \right)
\]

\[
2 \sum_{L=0}^{N} \sum_{H=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p}(1)^{a_{1}+..a_{L}} \left( \sum_{N=0}^{N-L} \eta_{\alpha_{i}..\alpha_{L}..L_{j}..H_{j}-p}(1)^{a_{1}+..a_{L}} \right) \psi^{i} = 0
\]
modified commutation relations between position coordinates and momenta are:

For this reason, the relations for momenta on the branes and anti-branes are different. Using the above relations, the

on, and also on the dimensions of the algebra of the universe. Also, bosonic momenta are related to lower orders of

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\[ x_0, \phi \] = Σ(N-H=0) a_{i,j-H}=p+1(-1)^{a_1+..+a_L} \times

P_{0,anti-Uni,fermion} = \sum_{L=0}^{N} N-L \sum_{a_{i,j-H}=p+1(-1)^{a_1+..+a_L} \times

P_{a_1,anti-brane,fermion} \cdot P_{a_L,anti-brane,fermion} +

2\sum_{L=0}^{N} N-L \sum_{a_{i,j-H}=p+1(-1)^{a_1+..+a_L} \times

((i)^{a_1+..+a_l-1} P_{a_1,anti-brane,fermion} \cdot P_{a_L-1,anti-brane,fermion})^{jH-1}) \times

P_{a_1,anti-brane,fermion} \cdot P_{a_L,anti-brane,fermion}

These equations show that the momenta in a four dimensional universe can be given in terms of momenta in Lie-N-algebra. The order of terms depends on the number of dimensions of the brane which our universe is placed on, and also on the dimensions of the algebra of the universe. Also, bosonic momenta are related to lower orders of momenta on the brane with respect to fermionic momenta. On the other hand, energy-momentum tensors on the brane contain less number of timing components (which have extra (i)-factor) with respect to one on the anti-brane. For this reason, the relations for momenta on the branes and anti-branes are different. Using the above relations, the modified commutation relations between position coordinates and momenta are:

\[
[x_0, P_{0,Uni,fermion}] = i\hbar \sum_{L=0}^{N} N-L \sum_{a_{i,j-H}=p+1(-1)^{a_1+..+a_L} \times

P_{a_1,brane,fermion} \cdot P_{a_L,brane,fermion} +

2\sum_{L=0}^{N} N-L \sum_{a_{i,j-H}=p+1(-1)^{a_1+..+a_L} \times

((i)^{a_1+..+a_l} P_{a_1,brane,fermion} \cdot P_{a_L,brane,fermion})^{jH} \times

P_{a_1,brane,fermion} \cdot P_{a_L,brane,fermion}

(37)
These equations indicate that commutation relations between coordinates and momenta or GUP in a four dimensional universe can be written in terms of different orders of momenta in a Lie-$N$-algebra. The order of momenta in this version of the GUP is related to the number of dimensions of the brane which our universe is located on. Also, this GUP depends on the dimensions of the algebra and universe, and has a more exact form with respect to other versions. In this model, bosonic commutation relations depend on lower orders of momenta on the brane with respect to the fermionic one. On the other hand, because of the difference between the number of timing components, the GUP on the brane is different from the one on the anti-brane. Thus, the shape of the GUP depends on the behaviour of particles and changes by changing the properties of the system.

III. CALCULATING THE ENTROPY OF BRANES IN LIE-$N$-ALGEBRA

In this section, we calculate the entropy of branes in the GUP and show that it includes different orders of area of branes from zero to $N^2pM$ where $N$ is the dimension of the algebra, $p$ is the dimension of the brane and $M$ is the dimension of the universe. This entropy can be reduced to previous predictions for $N = 3$ and eleven dimensions of $M$-theory. To show this, we replace $\delta p \sim (\frac{\hbar}{\delta x})$ in Eqs. (35 and 36) and obtain:

\[
E_{0,Uni,boson} \sim 2\delta p_{0,Uni,boson} = \left( \sum_{L=0}^{N} \sum_{H=0}^{p-1} \delta x_{a_{1}} \delta x_{a_{2}} \delta x_{d_{1}} \delta x_{d_{2}} \right) \sum_{j_{1} \cdot j_{H}} = p+1 (-1)^{a_{1}+..+a_{L}} \left( \frac{\hbar}{\delta x} \right)_{a_{1},brane,boson} \left( \frac{\hbar}{\delta x} \right)_{a_{L},brane,boson} + \frac{\hbar}{\delta x}_{a_{1},brane,fermion} \left( \frac{\hbar}{\delta x} \right)_{a_{L},brane,fermion}
\]

\[
E_{0,Uni,fermion} \sim 2\delta p_{0,Uni,fermion} = \left( \sum_{L=0}^{N} \sum_{H=0}^{p-1} \delta x_{a_{1}} \delta x_{a_{2}} \delta x_{d_{1}} \delta x_{d_{2}} \right) \sum_{j_{1} \cdot j_{H}} = p+1 (-1)^{a_{1}+..+a_{L}} \left( \frac{\hbar}{\delta x} \right)_{a_{1},brane,fermion} \left( \frac{\hbar}{\delta x} \right)_{a_{L},brane,fermion} + \frac{\hbar}{\delta x}_{a_{1},brane,boson} \left( \frac{\hbar}{\delta x} \right)_{a_{L},brane,boson}
\]

\[
E_{0,anti-Uni,boson} \sim 2\delta p_{0,anti-Uni,boson} = \left( \sum_{L=0}^{N} \sum_{H=0}^{p-1} \delta x_{a_{1}} \delta x_{a_{2}} \delta x_{d_{1}} \delta x_{d_{2}} \right) \sum_{j_{1} \cdot j_{H}} = p+1 (-1)^{a_{1}+..+a_{L}} \left( \frac{\hbar}{\delta x} \right)_{a_{1},anti-brane,boson} \left( \frac{\hbar}{\delta x} \right)_{a_{L},anti-brane,boson} + \frac{\hbar}{\delta x}_{a_{1},anti-brane,fermion} \left( \frac{\hbar}{\delta x} \right)_{a_{L},anti-brane,fermion}
\]

\[
E_{0,anti-Uni,fermion} \sim 2\delta p_{0,anti-Uni,fermion} = \left( \sum_{L=0}^{N} \sum_{H=0}^{p-1} \delta x_{a_{1}} \delta x_{a_{2}} \delta x_{d_{1}} \delta x_{d_{2}} \right) \sum_{j_{1} \cdot j_{H}} = p+1 (-1)^{a_{1}+..+a_{L}} \left( \frac{\hbar}{\delta x} \right)_{a_{1},anti-brane,fermion} \left( \frac{\hbar}{\delta x} \right)_{a_{L},anti-brane,fermion} + \frac{\hbar}{\delta x}_{a_{1},anti-brane,boson} \left( \frac{\hbar}{\delta x} \right)_{a_{L},anti-brane,boson}
\]
We can rewrite equations (39 and 40) in terms of the area of the branes as:

\[
(i)^{a_1+\ldots+a_{L-1}} \left( \frac{\hbar}{\delta x} \right)_{a_1,\text{anti-brane,fermion}} \left( \frac{\hbar}{\delta x} \right)_{a_{L-1},\text{anti-brane,fermion}}^{jH-1} \times \left( \frac{\hbar}{\delta x} \right)_{a_1,\text{anti-brane,fermion}} \left( \frac{\hbar}{\delta x} \right)_{a_L,\text{anti-brane,fermion}}^{jH-1} \times
\]

(40)

On the other hand, the area of each brane can be obtained as: \((\frac{\hbar}{\delta x})_{\text{brane}} = \delta x \times (\frac{\hbar}{\delta x})_{\text{brane,fermionic}} + (\frac{\hbar}{\delta x})_{\text{brane,bosonic}}\) in which \((\frac{\hbar}{\delta x})_{\text{brane,fermionic}}\) denotes the area of the brane which can be observed by fermions and \((\frac{\hbar}{\delta x})_{\text{brane,bosonic}}\) is the area of the brane which is observed by bosons. Also, \((\frac{\hbar}{\delta x})_{\text{brane,fermionic/bosonic}} \approx \delta x \cdot \delta x_p\) and thus \(\delta x \approx (\frac{\hbar}{\delta x})^{1/p}.

We can rewrite equations (39) and (40) in terms of the area of the branes as:

\[
E_{U_{\text{uni, boson}}} \sim \delta P_{0, U_{\text{uni, boson}}} = \left( \sum_{L=0}^{N} \sum_{j=0}^{L} \frac{1}{p} \right)^{2} a_{1,\text{brane,fermion}} \cdot \left( \frac{\hbar}{\delta x} \right)_{\text{brane,fermion}}^{jH-1} \times \left( \frac{\hbar}{\delta x} \right)_{\text{brane,fermion}}^{jH-1} \times
\]

(41)

\[
E_{U_{\text{uni, fermion}}} \sim \delta P_{0, U_{\text{uni, fermion}}} = \sum_{L=0}^{N} \sum_{j=0}^{L} \frac{1}{p} \right)^{2} a_{1,\text{brane,fermion}} \cdot \left( \frac{\hbar}{\delta x} \right)_{\text{brane,fermion}}^{jH-1} \times \left( \frac{\hbar}{\delta x} \right)_{\text{brane,fermion}}^{jH-1} \times
\]

(41)
\[
2\sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{p=0}^{\pi L \cdot a_{L-1,anti-brane,fermion}} (-1)^{a_1+.+a_L} \left( \frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
((i)^{a_1+.+a_L-1} \left( \frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
((i)^{a_1+.+a_L-1} \left( \frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion )^{j_H-1} \times \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \\
(42)
\]

These equations show that the energy of bosons and fermions have a direct relation with different orders of \((\frac{A}{L_p})^{-1}\), where \((\frac{A}{L_p})\) is the area of the universe. This is because for small area, particles are very close to each other, interact more and the energy of the system increases, while for large area, particles become distant, interact less and the energy of the system decreases. Previously it has been shown that the entropy of the system can be obtained from the following relation [30]:

\[
\tilde{S} \approx \int d(A) \left( \frac{\Delta S}{(\frac{A}{L_p})^{1/p}} \right)_{min}
\]

(43)

In agreement with the results of information theory [30, 31], we can predict that the minimal increase of entropy should be independent of the value of the area, simply one bit of information; let us assume this fundamental unit of entropy as \((\Delta S)_{min} = b = \ln 2\). On the other hand, for a system which includes particles with energy \(E\) and length \(\delta x\), the minimum increase in area can be given by [33]:

\[
(\Delta (\frac{A}{L_p}))_{min} = 8\pi L_p^2 E \delta x = 8\pi L_p^2 \left( E_{Uni,boson} + E_{Uni,fermion} \right) \delta x = \\
8\pi L_p^2 \left( E_{Uni,boson} + E_{Uni,fermion} \right) \left( \frac{A}{L_p} \right)^{-1/p}
\]

(44)

Thus, entropy for branes and anti-branes can be obtained as:

\[
\tilde{S}_{Universe} \approx \int d(A) \left( \frac{\Delta S}{(\frac{A}{L_p})^{1/p}} \right) = \\
b \left( \frac{\sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{p=0}^{\pi L \cdot a_{L-1,anti-brane,fermion}} (-1)^{a_1+.+a_L} \left( \frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
((i)^{a_1+.+a_L-1} \left( \frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
((i)^{a_1+.+a_L-1} \left( \frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion )^{j_H-1} \times \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \right)
\]

\[
\tilde{S}_{anti-Universe} \approx \int d(A) \left( \frac{\Delta S}{(\frac{A}{L_p})^{1/p}} \right) = \\
b \left( \frac{\sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{p=0}^{\pi L \cdot a_{L-1,anti-brane,fermion}} (-1)^{a_1+.+a_L} \left( \frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
((i)^{a_1+.+a_L-1} \left( \frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
((i)^{a_1+.+a_L-1} \left( \frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion )^{j_H-1} \times \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \times \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \\
\left(\frac{\hbar}{(\frac{A}{L_p})^{1/p}} \right) a_1,anti-brane,fermion \right)
\]

(45)
Higher numbers whose values depend on the number of dimensions of branes (\(M\)) order changes from zero to higher values depending on the number of dimensions of the Lie-algebra. Different from the surface that is observed by bosons, these entropies depend on fermionic and bosonic areas, whose number of dimensions of the branes, higher orders of their areas appear, and the entropy increases.

Various derivatives of fermions are produced which lead to the creation of different orders of momenta for spinors, momenta in momentum space. This is known as the generalized uncertainty principle or GUP. By compacting branes, these branes contain various derivatives of bosonic fields which cause the production of terms with different orders and consequently the emergence of a fermionic GUP. We have calculated the entropy of the branes, and find that the number of terms depend upon the dimension of the branes and the applied algebra. In fact, with increasing the number of dimensions of the branes, higher orders of their areas appear, and the entropy increases.

These equations show that the predicted entropy on the universe is different from the one on the anti-universe. This is because areas of the brane on which our universe is located have less timing coordinates with respect to areas of the anti-brane on which the anti-universe is placed. On the other hand, the surface area, which is seen by fermions is different from the surface that is observed by bosons. These entropies depend on fermionic and bosonic areas, whose order changes from zero to higher values depending on the number of dimensions of branes and the Lie-algebra.

### IV. SUMMARY AND CONCLUSION

In this research, we have shown that the GUP can arise during the formation of branes and anti-branes in an \(M\)-dimensional universe whose fields obey a Lie-\(N\)-algebra. The order of terms in the GUP changes from zero to higher numbers whose values depend on the number of dimensions of branes (\(p\)), number of dimensions of the universe (\(M\)) and dimensions of the Lie-algebra (\(N\)). For \(N = 3\) and an 11-dimensional universe, the results are consistent with \(M\)-theory. In this model, firstly, two energies with opposite signs are created from nothing, excited and produce various branes and anti-branes with different timing dimensions and quantum numbers. The related actions for these branes contain various derivatives of bosonic fields which cause the production of terms with different orders of momenta in momentum space. This is known as the generalized uncertainty principle or GUP. By compacting branes, various derivatives of fermions are produced which lead to the creation of different orders of momenta for spinors, and consequently the emergence of a fermionic GUP. We have calculated the entropy of the branes, and find that the number of terms depend upon the the dimension of the branes and the applied algebra. In fact, with increasing the number of dimensions of the branes, higher orders of their areas appear, and the entropy increases.

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