Single $\pi^+$ Electrophotoproduction on the Proton in the First and Second Resonance Regions at 0.25 GeV$^2 < Q^2 < 0.65$ GeV$^2$ Using CLAS

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The structure of the nucleon and its excited states has been one of the most extensively studied subjects in nuclear and particle physics for many years. It allows us to understand important aspects of the underlying theory of the strong interaction, QCD, in the confinement regime where solutions are very difficult to obtain. Elastic electron scattering experiments provide information on the ground state of the nucleon, while studying the Q^2 evolution of the transition amplitudes from the nucleon ground state into the excited states provides insight into the internal structure of the excited nucleon. Single-pion electroproduction is one of the most suitable processes for studying the transitions to states with masses below 1.7 GeV because of the large πN coupling for these states [1]. The detection of two out of three outgoing particles is sufficient to achieve a complete measurement of the differential cross sections in order to attempt the extraction of the amplitudes for the individual resonances. The kinematic quantities of the ep → e'π^+n reaction is shown in Fig. 1. The virtual photon is described by the four-momentum transfer Q^2, energy transfer ν and the polarization parameter ϵ:

\[ Q^2 = 4E_iE_f \sin^2 \frac{\theta_e}{2}, \]
\[ ϵ = \left[ 1 + 2 \left( 1 + \frac{\nu^2}{Q^2} \right) \tan^2 \frac{\theta_e}{2} \right]^{-1}, \]
\[ ν = E_i - E_f, \]

where E_i and E_f are the initial and final energies of the electron and θ_e is the electron scattering angle. The mass of the hadronic system is given by:

\[ W = \sqrt{M^2 + 2Mν - Q^2}, \]

where M is the proton mass. The two hadron production angles θ and φ are defined in the center-of-mass (c.m.) reference frame, with θ being the angle between the outgoing pion and the direction of the three-momentum transfer, and φ being the angle between the electron scattering plane and the hadron production plane. The unpolarized cross section for single-pion electroproduction can be written as [2]:

\[ \frac{\partial^2σ}{\partial E_f \partial Ω_π} = \frac{\partial σ}{\partial Ω_π}, \]
\[ \Gamma = \frac{α}{2\pi^2Q^2E_i} \frac{W^2 - M^2}{1 - \epsilon}, \]
\[ \frac{dσ}{dΩ_π} = σ_T + ϵσ_L + ϵσ_{TT} \cos 2φ + \sqrt{2ϵ(1 + ϵ)}σ_{TL} \cos φ, \]
where $\Gamma$ is the virtual photon flux, and $\frac{d\sigma}{d\Omega}$ is the virtual photoproduction cross section. The $\sigma_T$, $\sigma_L$, $\sigma_{LL}$ and $\sigma_{TT}$ structure functions are bilinear combinations of the helicity amplitudes, depending only on the variables $Q^2$, $W$ and $\theta$. The analysis of the angular distributions provides information for extracting the electroproduction amplitudes for different resonances.

The main tree-level Feynman diagrams contributing to the $ep \rightarrow e'\pi^+ n$ process are shown in Fig. 2. The $s$-channel resonance excitation process is represented by the diagram in Fig. 2a. The hadronic vertex of this process is known from $\pi N$ elastic scattering experiments [3]. Therefore studies of pion electroproduction can yield the $Q^2$ evolution of the photocoupling amplitudes describing the $\gamma^* NN^*$ vertex. For the purpose of studying the excitation of nucleon resonances, the other diagrams are considered as physical background. The largest non-resonant contribution to the cross section comes from the $t$-channel pion exchange diagram, shown in Fig. 2b. Although this process mainly contributes in the forward region due to the pion propagator pole, it still accounts for a significant part of the cross section even at large angles. The diagrams in Fig. 2a and Fig. 2b correspond to the $s$-channel nucleon pole and $t$-channel $\rho$-meson exchange amplitudes. Sophisticated analysis procedures are necessary to separate the resonant contributions from the non-resonant background, and to extract the resonant amplitudes for different overlapping excited states. The extraction of resonance multipoles is beyond the scope of this paper. In this contribution we describe the experiment and data analysis, and the extraction of fully exclusive and differential cross sections, and determination of response functions.

Electroexcitation of a nucleon resonance can be described in terms of three photocoupling amplitudes $A_{1/2}, A_{3/2}$ and $S_{1/2}$. The first two are due to the coupling of transverse photons with the proton resulting in a combined helicity $h = \frac{1}{2}$ or $h = \frac{3}{2}$ respectively. The $S_{1/2}$ amplitude is present due to the possibility of a longitudinal polarization for virtual photons. Alternatively, pion electroproduction can be described using multipole amplitudes $E_{1\pm}, M_{1\pm}$ and $S_{1\pm}$. The $l$-index represents the orbital angular momentum of the $\pi N$ system, and the $\pm$ sign indicates how the nucleon spin is coupled to the orbital momentum. For each excited state the helicity amplitudes can be expressed in terms of multipole amplitudes and vice versa [2].

Quark models predict that the $\frac{E_{1+}}{M_{1+}}$ and $\frac{S_{1+}}{M_{1+}}$ ratios for the $P_{33}(1232)$ are small at low $Q^2$ [4, 5], while perturbative QCD predicts $\frac{E_{1+}}{M_{1+}} = 1$ and $\frac{S_{1+}}{M_{1+}}$ is independent of $Q^2$ as $Q^2 \rightarrow \infty$ [6]. A transition between these two regimes is expected at some finite $Q^2$. At low $Q^2$ the deviations of these ratios from zero can be interpreted as non-spherical deformation of the nucleon or the $\Delta(1232)$ [7]. Usually these ratios for $\Delta(1232)$ are obtained through measurements in the $\pi^0 p$ decay channel with an assumption that the uncertainty due to the isospin $I = \frac{1}{2}$ background is negligible. High quality data in the $\pi^+ n$ channel will enable us to separate the isospin $I = \frac{1}{2}$ and $I = \frac{3}{2}$ components of the transition form-factors for the $P_{33}(1232)$ and to determine these ratios with smaller uncertainties coming from non-resonant contributions.

The second resonance region is dominated by the three known isospin $I = \frac{1}{2}$ states, $P_{11}(1440)$, $D_{13}(1520)$ and $S_{11}(1535)$. These resonances, produced in electron-proton scattering, are twice as likely to decay through the $\pi^+ n$ channel than through $\pi^0 p$. Therefore, cross section measurements of the $ep \rightarrow e' \pi^+ n$ process are crucial for understanding the properties of these states. The nature of the $P_{11}(1440)$ resonance is not understood in the framework of the constituent quark model (CQM) [8].

![FIG. 1](image1.png) Kinematic diagram of single-pion electroproduction.

![FIG. 2](image2.png) (Color online) Some of the main diagrams contributing to single $\pi^+$ electroproduction.
and there are suggestions that the Roper resonance may be a hybrid state or a small quark core with a large vector meson cloud. The $Q^2$ evolution of the $A_{1/2}$ photocoupling amplitude for the Roper is predicted to be different for 3-quark and hybrid states. Previous analyses indicate a rapid fall-off of $A_{1/2}$ between $Q^2 = 0$ and $Q^2 = 0.5$ GeV$^2$, therefore high quality data in this region will be very valuable in understanding the nature of the $P_{11}(1440)$.

The experimental data for the $A_{1/2}$ transition amplitude for $S_{11}(1535)$ show a significantly slower $Q^2$ fall-off than predicted by constituent quark models. Most of these results are obtained through analysis of $\eta$-meson electroproduction data, where there can be no $I = \frac{1}{2}$ background. The proximity of the $S_{11}(1535)$ mass to the $\eta$-production threshold complicates the analysis of the data. High quality single $\pi^+$ data currently exist only at the photoproduction point, and there is very little data for non-zero $Q^2$. The results from analyses of pion and $\eta$ photoproduction data are significantly different, and the source of these discrepancies is still not understood. New electroproduction data will allow for a similar comparison between the results from the two channels from CLAS to check the consistency of the analysis frameworks. These data will also allow for a future combined analysis of pion and $\eta$ production data, which will provide more stringent constraints on the fit.

Until now there have only been three experiments measuring single $\pi^+$ electroproduction cross section in the resonance regions in this range of $Q^2$. In all of these experiments the lack of angular coverage in the center-of-mass reference frame significantly reduced the sensitivity to the resonant amplitudes. The aim of the present experiment is to provide differential cross sections for the $\pi^+n$ channel over a large kinematic region and with high statistical accuracy, that can be used together with other channels to obtain more reliable results on the resonance photocoupling amplitudes.

**EXPERIMENT**

The measurement was carried out using the CEBAF Large Acceptance Spectrometer (CLAS) at the Thomas Jefferson National Accelerator Facility (Jefferson Lab), located in Newport News, Virginia. CLAS is a nearly $4\pi$ detector, providing almost complete angular coverage for the $ep \rightarrow e'\pi^+n$ reaction in the center-of-mass frame. It is well suited for conducting experiments which require detection of two or more particles in the final state. Such a detector and the continuous beam produced by CEBAF provide excellent conditions for measuring the $ep \rightarrow e'\pi^+n$ electroproduction cross section by detecting the outgoing electron and pion in coincidence.

The main magnetic field of CLAS is provided by six superconducting coils, which produce an approximately toroidal field in the azimuthal direction around the beam axis. The gaps between the cryostats are instrumented with six identical detector packages, also referred to here as “sectors”, as shown in Fig. 3. Each sector consists of three regions (R1, R2, R3) of Drift Chambers (DC) to determine the trajectories of the charged particles, Čerenkov Counters (CC) for electron identification, Scintillator Counters (SC) for charged particle identification using the Time-Of-Flight (TOF) method, and Electromagnetic Calorimeters (EC) used for electron identification and detection of neutral particles. The liquid-hydrogen target was located in the center of the detector on the beam axis. To reduce the electromagnetic background resulting from Møller scattering off atomic electrons, a second smaller normal-conducting toroidal magnet (mini-torus) was placed symmetrically around the target. This additional magnetic field prevented the Møller electrons from reaching the detector volume. A totally absorbing Faraday cup, located at the very end of the beam line, was used to determine the integrated beam charge passing through the target. The CLAS detector can provide $\delta_p < 0.5\%$ momentum resolution, and $\approx 80\%$ of $4\pi$ solid-angle coverage. The efficiency of detection and reconstruction for stable charged particles in fiducial regions of CLAS is $\epsilon > 95\%$. The combined information from the tracking in the DC and the TOF systems allows us to reliably separate protons from positive pions for momenta up to 3 GeV.

**Apparatus**

FIG. 3: Three dimensional view of CLAS.
Data taking and data reduction

The data were taken in the spring of 1999 as part of the experimental program of the CLAS collaboration. The CEBAF 1.5 GeV electron beam was incident on a 5-cm long liquid hydrogen target at 20.5 K temperature. The data were taken at 3 nA nominal beam current, with ±0.04 nA current fluctuations, at luminosities of ∼4 × 10^{33} cm^{-2}s^{-1}. The size of the beam spot at the target was ∼0.2 mm, with position fluctuations of ±0.04 mm. The main torus current was set at 1500 A, which created a magnetic field of about 0.8 Tesla in the forward direction. The magnetic field of the spectrometer is significantly lower at large angles. The CLAS event readout was triggered by a coincidence of signals from the electromagnetic calorimeter and the Čerenkov counters in a single sector, generating an event rate of ∼2 kHz. The total number of accumulated triggers at these detector settings was about 4.5 × 10^8. The raw data were written onto a tape silo of the Jefferson Lab Computer Center. During the off-line processing each file was retrieved from the tape silo and analyzed to produce files for general use containing 4-vectors of the reconstructed particles. ROOT files, containing the specific information relevant for single π^{±} electroproduction, were created and stored on a disk. These files were further analyzed to extract the differential cross sections for the $ep \rightarrow e'\pi^{±}n$ reaction.

Particle identification

One of the key issues in electron scattering experiments is the ability of the detector to reliably identify electrons. Electron identification at the trigger level was accomplished by requiring a minimum amount of deposited energy in the electromagnetic calorimeter in coincidence with a signal in the Čerenkov counter in the same sector. Additional requirements were applied in the off-line analysis to select events containing an electron. First, a geometrical matching was required between the EC and CC hits and the associated negatively charged tracks in the drift chambers. The values of the geometrical cuts in the software are given in Table I. A sampling fraction cut was imposed on the dependence of the EC visible energy on the momentum to reject the background coming from negative pions (see Fig. 4). The electron identification in the off-line analysis can be summarized by:

$$EID = TRK \otimes CC \otimes EC \otimes SF,$$

where $TRK$ stands for track reconstruction in the drift chambers, $CC$ and $EC$ are the Čerenkov counter and the calorimeter geometrically matched hits and $SF$ is the sampling fraction cut described above. In order to avoid inefficiencies due to the trigger threshold in the electromagnetic calorimeter, only events containing an electron with at least 500 MeV momentum were used in the analysis. In addition, fiducial cuts, discussed later, were applied to select only electrons in the regions where the Čerenkov counter efficiency was greater than 92%. The final cross sections were corrected for the remaining inefficiency of the Čerenkov counters.

Charged hadron identification in the CLAS detector is accomplished using the momentum determined from the tracking and the timing information from the scintillation counters. Fig. 5 shows the distribution of positively charged particles at 1.515 GeV electron beam energy plotted versus velocity $\beta$ and momentum $P$. Bands due to positrons, pions, protons and deuterons can be easily identified. At low momentum the muon band is visible as well. The deuterons are produced from electron scattering on the aluminum windows of the target cell. All positive particles in the region outlined by the dashed lines were considered as π^{±}. Positrons can be separated from pions at low momenta, but at higher momenta the pion and positron bands merge. Background due to muons and positrons is significantly reduced by the missing mass and vertex cuts described below. The remaining contamination is evaluated as a systematic uncertainty.

| Matching | Tolerance |
|----------|-----------|
| $TRK \otimes EC$ | 30 cm |
| $TRK \otimes CC$ | 5° |
| $EC \otimes CC$ | 5° |

TABLE I: Cuts for the geometrical matching in the offline analysis software.
Momentum corrections

When extracting the resonant parameters for excited states, it is important to have the correct value for the invariant mass of the hadronic state. Therefore, it is necessary to measure the electron momentum with high accuracy. For this reason additional corrections were applied to the electron momentum reconstructed by the standard CLAS software package. These corrections were determined using elastic scattering events from the same runs that were used in the single pion analysis. It was found that the missing mass determined from the elastically scattered electrons is typically ∼ 5 MeV below the proton mass \( M_p = 0.938 \text{ GeV} \). Assuming that the scattering angle of the electron is measured correctly, and using:

\[
E_f = \frac{2M E_i - (W^2 - M^2)}{2M + 4E_i \sin^2 \frac{\theta_e}{2}}
\]

(9)

the momentum correction factor can be found as:

\[
C_p = \frac{\delta E_f}{E_f} = -\frac{W^2 - M^2}{2ME_i},
\]

(10)

where \( E_i \) is the electron beam energy and \( W \) is the measured recoil mass. This quantity was calculated for different bins in \( \theta_e \in [15^\circ, 55^\circ] \) and \( \phi_e^* \in [-30^\circ, +30^\circ] \) in the laboratory frame for each sector and stored in a look-up table. The azimuthal angle \( \phi_e^* \) is defined within a sector, with \( \phi_e^* = 0 \) corresponding to the mid-plane of the sector.

The momenta of the electrons from single \( \pi^+ \) production data were corrected using this table on an event-by-event basis. This procedure relies on the fact that the relative momentum offset is independent of \( W \) for a fixed value of \( \theta_e \). It was found that after these corrections were applied, the neutron peak in the missing mass of the \( ep \rightarrow e'\pi^+X \) reaction was within ±2.0 MeV of the neutron mass for 1.1 GeV < \( W < 1.6 \text{ GeV} \) range. Fig. 6 shows the difference between the missing mass in \( ep \rightarrow e'\pi^+X \) reaction and the neutron mass with and without momentum corrections. The six gaps between the points are due to the six coils of the magnet. The dependence of the peak position on \( \phi_e \) is practically eliminated by this procedure, and the peak is located much closer to the neutron mass.

Fiducial cuts

Although CLAS is a nearly 4π detector, it still contains significant inactive volumes without particle detectors. In addition, some of the detection inefficiencies in the ac-

FIG. 5: Distribution of the number of positively charged particles versus \( \beta \) and \( p \). The visible bands are due to positrons, muons, pions, protons and deuterons. All positive particles within the area outlined by the dashed lines are considered as \( \pi^+ \)'s in this analysis.

FIG. 6: Position of the missing mass peak in the \( ep \rightarrow e'\pi^+X \) reaction versus \( \phi_e \) in the lab frame: a) - before, b) - after momentum corrections are applied. The error bars represent the width of the distribution around the neutron mass.

FIG. 7: Čerenkov counter efficiency versus the \( \theta_e \) and \( \phi_e \) electron angles in the laboratory frame for Sector 4. The black curves indicate the outer edges of the electron fiducial regions. Each momentum bin is 200 MeV wide.
tive volumes are not adequately reproduced in the detector simulation software. These areas are near the edges of the electromagnetic calorimeter, Čerenkov counter mirrors, the main torus and mini-torus coils, regions with broken wires in the drift chambers, and malfunctioning phototubes in the TOF system. To eliminate events with particles traveling through these regions, a set of fiducial cuts was developed. For electrons the main boundary of the fiducial region was defined by the efficiency of the Čerenkov counters and the edges of the electromagnetic calorimeter. Fig. 4 shows the dependence of the Čerenkov counter efficiency versus the $\theta_e$ and $\phi_e$ electron angles in the laboratory frame for six 200 MeV wide momentum bins in Sector 4. Due to the optics design of the CLAS Čerenkov counters [15] there are areas with relatively lower efficiency shown with the lighter shade. These features are difficult to implement in the detector simulation. Only events in the regions within the black curves and with the Čerenkov counter efficiency above 92% were used in the analysis. An additional set of geometrical cuts was applied to reject electrons hitting malfunctioning scintillator counters or traversing regions with missing or inefficient wires in the drift chambers.

Two sets of fiducial cuts were used to define the outer boundary of the fiducial regions for the positive pions. The first set, similar to the electron cuts, was defined in such a way that the $\phi_e^{lab}$ distributions of the number of events be uniform within the fiducial region. The second set of cuts was applied to ensure equivalent solid angle coverage for pions in the Monte-Carlo simulation and the real data. This mismatch was due to distortions of the minitorus coils which were not implemented in the detector simulation package. As in the case with the electrons, tracks in the regions with malfunctioning scintillator counters or traversing regions with missing or inefficient wires were rejected by another set of fiducial cuts.

**Kinematic cuts**

The exclusive final state was selected by detecting the outgoing electron and the $\pi^+$, and by requiring that the missing particle be a neutron. The missing-mass spectrum in Fig. 3b shows a prominent neutron peak as well as events from the radiative tail and from multi-pion production channels. The arrows indicate the cuts used in the analysis. The number of rejected events in the tails is recovered by imposing the same cuts on the simulated events in the acceptance calculations.

GEANT-based Monte-Carlo studies showed that about 18% of the positive pions decay in-flight into $\mu^+\nu_\mu$. Most of the momentum of the original pion is carried by the $\mu^+$, which is, therefore, often detected and reconstructed as a $\pi^+$ with a significantly different momentum vector. In order to reduce the number of the events with decaying pions, a vertex cut $|Z_\pi - Z_e| < 2$ cm was applied on the difference of the $Z$-coordinates along the beam-line for the electron and the pion tracks in the same event (see Fig. 5b). This led to a reduction in the number of events with decaying $\pi^+$ to 4% with less than 1% losses in the number of events when the pion did not decay.

The kinematic coverage of this experiment is shown in Fig. 8. The grating on the figures shows the binning of the data. In the first resonance region this experiment covers a $Q^2$ range from 0.25 GeV$^2$ to 0.65 GeV$^2$, while in the second resonance region the upper boundary of the $Q^2$ coverage is reduced to 0.45 GeV$^2$. The angular coverage in the hadronic center-of-mass frame is nearly complete, with the exception of the region $\theta > 140^\circ$. This limitation at larger angles is related to the fact that the CLAS coverage for charged particles is limited to $140^\circ$ in laboratory frame. The number and the sizes of the cross section bins are given in Table 1.
TABLE II: The number and the sizes of the data bins. Values for the limits indicate the upper and lower edges of the bins, rather than the bin centers.

| Variable | # of bins | Lower limit | Upper limit | Width |
|----------|-----------|-------------|-------------|-------|
| $Q^2$    | 4         | 0.25 GeV$^2$ | 0.65 GeV$^2$ | 0.10 GeV$^2$ |
| $W$      | 25        | 1.1 GeV     | 1.6 GeV     | 20 MeV |
| $\theta$ | 12        | $0^\circ$   | $180^\circ$ | $15^\circ$ |
| $\phi$   | 12        | $0^\circ$   | $360^\circ$ | $30^\circ$ |

Acceptance corrections

In order to relate the experimental yields to cross sections, acceptance correction factors were calculated using the Monte-Carlo method. The GEANT-based detector simulation package GSIM incorporated the survey geometry of CLAS, realistic drift chamber and timing resolutions along with missing wires and malfunctioning photomultiplier tubes. Because CLAS is a complicated detector covering almost 4π of solid angle, it is virtually impossible to separate the efficiency calculations from the geometrical acceptance calculations. In this work the term acceptance correction refers to a combined correction factor due to the geometry of the detector and the inefficiencies of the detection and reconstruction. It is defined as the ratio of the number of reconstructed Monte-Carlo events to the number of simulated events in a given bin:

$$A = \frac{N_{\text{rec}}}{N_{\text{sim}}}.$$  \hspace{1cm} (11)

With this definition of the acceptance it is desirable to have a realistic physics model in the event generator because of the finite bin sizes and bin migration effects, which are described later. In this work the MAID2000 model [23], which reasonably reproduces both $p\pi^0$ [24] and the current $n\pi^+$ CLAS data, was used as an input to the Monte-Carlo event generator. The simulated 200 million events were processed using the same software package and analyzed with the same cuts that were applied to the real data. An acceptance table with $8 \times 25 \times 24 \times 48$ bins in $Q^2$, $W$, $\theta$ and $\phi$, respectively, was calculated using the definition in Eq. (11). The fine binning of the acceptance look-up table reduces the model dependence of the acceptance calculations. In this work geometrical acceptance calculations. In this work acceptance corrections were estimated using the binomial distribution:

$$\delta A_{\text{bin}} = \sqrt{\frac{A_{\text{bin}}(1 - A_{\text{bin}})}{N_{\text{gen}} - 1}},$$  \hspace{1cm} (12)

where $N_{\text{gen}}$ is the number of the Monte-Carlo events generated in the bin. These errors are included in the statistical error of the final cross sections.

The acceptance of CLAS for single-pion electroproduction at $Q^2 = 0.3$ GeV$^2$ and $W = 1.23$ GeV is shown in Fig. 10. The $\theta$-dependence of the acceptance (see Fig. 10a) exhibits a dip at $\sim 45^\circ$, which is due to the forward beam pipe. Six sectors of CLAS can be clearly identified in the plot showing acceptance versus $\phi$-angle (see Fig. 10b). The width of the curves in these graphs represent the statistical error bands. Since a single cross section bin contains sixteen acceptance bins, the contribution of the acceptance statistical error to the total uncertainty of cross sections is on average approximately four times smaller than the errors seen in these plots.

Radiative corrections

In addition to processes which result in the exclusive $e^+\pi^+\pi^-$ final state, there are radiative processes represented by Feynman diagrams similar to the original single photon exchange diagrams, but with an additional photon leg, that also contribute to the cross sections. The experimentally measured cross sections must be corrected for such processes, also known as internal radiation. The radiative cross section for an exclusive process can be written as [25]:

$$d\sigma_r = \frac{(4\pi\alpha)^3 dQ^2 dW^2 d\Omega^*_\nu}{2(4\pi)^2 S^2 W^2} \times \int d\Omega_L d\Omega_\lambda \frac{\lambda W}{J_W^2 Q^4} L^{(\nu)}_{\mu
u} W_{\mu\nu},$$  \hspace{1cm} (13)

where $S \equiv 2E_\pi M_{\nu}$, $d\Omega^*_\nu$ is the differential center-of-mass solid angle of the $\pi^+$, $v \equiv M_\pi^2 - M_{\nu}^2$, $L^{(\nu)}_{\mu
u}$ and $W_{\mu\nu}$ are the leptonic and the hadronic tensors respectively, and

$$\lambda W = (W^2 - m_{\pi^+}^2 - M_{\nu}^2)^2 - 4m_{\pi^+}^2 W^2,$$

$$J_W = W - E_\pi + p_\pi (\cos \theta_\pi \cos \theta_k + \sin \theta_\pi \sin \theta_k \cos (\phi_\pi - \phi_k)).$$  \hspace{1cm} (14)
Corrections for binning effects

Because of the finite detector resolution and finite bin size, the measured values of the cross section in the center of the data bin can be distorted by up to 10%. The experimentally measured quantity is the cross section averaged over a full data bin, while usually it is more desirable to determine the value of the cross section at the center of the bin. To account for such distortions, multiplicative corrections were introduced as the ratio of the cross section in the center of a bin to the average cross section in that bin:

$$B = \frac{\frac{d\sigma}{d\Omega} |_{\text{ctr}}}{\frac{d\sigma}{d\Omega} |_{\text{avg}}}. \quad (16)$$

The averaged cross sections were evaluated using two models: the $Q^2$ dependence of the cross sections was taken from the MAID2000 model [23], while the cross sections at fixed values of $Q^2$ were obtained using a unitary isobar [24] fit to these data in the first iteration.

Normalization

The integrated charge of the electron beam passing through the target was measured using the Faraday cup located at the end of the Hall B beam line. It generated pulses with a frequency proportional to the beam current with 10 Hz per 1 nA linear slope parameter. The calibration parameters of this device are known with less than 0.5% uncertainty. The measured charge was corrected for the data acquisition live-time, calculated as the ratio of the counts from two scalers. These scalers were connected to a single 100 kHz pulse generator. One of them was ungated, while the other one was gated by the data acquisition “live” signal. To ensure the quality of the analyzed data sample, software cuts were imposed on the live-time, elastic scattering and single $\pi^+$ electroproduction rates. The portions of the runs for which these quantities were outside of the imposed limits were excluded from the analysis, with the corresponding beam charge being subtracted from the total charge. As was mentioned above, the Čerenkov counter efficiencies were parameterized during the calibration procedure [19], and the appropriate corrections were applied to the cross sections. The comparison of the elastic scattering cross sections versus $\theta_e$ from CLAS and the model calculation using a parameterization [20] for the elastic form factors is shown in Fig. 12. The model cross section includes radiative effects, according to the Mo and Tsai formalism [20]. The error bars on the data points represent statistical uncertainties only. The solid line at $R = 1.015$ shows the result of fitting a constant to the ratio of the measured cross sections to the parameterization [20]. The fluctuations around this line can be used to estimate the

Here, $\theta_{\pi}$, $\phi_{\pi}$, $\theta_k$ and $\phi_k$ are the pion and radiated photon’s angles in the hadronic center-of-mass reference frame. The integral in Eq. (15) is taken over the photon angles and variable $v$.

In addition, there is also a nonzero probability that in the presence of the electromagnetic field of the atoms of the target the electron will emit one or more photons before or after interacting with the nucleus of the target (external radiation). The probability of emitting a real photon of a particular energy is proportional to the path length of the electron in the target material. The size of the external radiative corrections for these measurements was significantly smaller than for internal because of the small amount of the target material ($t = 0.5\%$ of radiation length).

The internal radiative corrections for the cross sections were calculated using the ExcluRad program [22] as multiplicative correction factors for each data bin. The external radiative corrections were done using the Mo and Tsai formalism [20]. The unradiated structure functions, needed as an input for the correction procedure, were calculated using a parameterization of the multipole amplitudes using a fit of these CLAS data based on the unitary isobar model [24]. The size of the required corrections varied up to 55%, depending on the kinematics. Fig. 11 shows dependences of the radiative corrections $R \equiv \frac{\sigma_{\text{meas}}}{\sigma_{\text{para}}}$ on the variables $W$, $\theta$ and $\phi$ at $Q^2 = 0.3$ GeV$^2$. The dotted line shows the correction due to external radiation, and the dashed line is the correction factor obtained using the ExcluRad program. The solid line is the combined correction factor calculated as the product of the two. Because of the short length of the target, the external radiative corrections are much smaller than the internal corrections.

FIG. 11: Sample plots of the radiative correction factor $R$ at $Q^2 = 0.3$ GeV$^2$. The dotted lines are the external corrections, the dashed lines are internal corrections, and the solid lines show the combined radiative correction.
systematic uncertainty of electron detection and reconstruction.

The contributions from scattering off the target cell walls was estimated to be 1.5% using empty target runs. This correction factor was applied to the cross section in every data bin.

**SYSTEMATIC ERRORS STUDIES**

A number of studies were carried out to estimate the systematic uncertainties on the measured cross sections. The primary method used in these studies was to vary different independent parameters of the analysis to determine the corresponding change in the resulting cross sections and the structure functions.

Because of the finite bin size, the result of averaging the acceptance over an acceptance bin depends on the distribution of events in that bin. If the physics model used in the Monte-Carlo simulation differs from the real data, then the averaging over a bin may result in an incorrect cross section. The introduced error depends on the shape of the acceptance function and the cross sections as well as on the acceptance bin size. In addition, because of the finite detector resolution, some of the events produced in one acceptance bin will be reconstructed in a different bin. This may cause significant distortions in the final cross section distributions. In order to correctly account for these effects, a realistic physics generator and detector simulation are required. To estimate the errors of the final results due to the model used in the acceptance calculations, we calculated the acceptance table with two different models. The comparison of the results with the two acceptance corrections allowed us to estimate the systematic errors due to the physics model in the acceptance calculations.

As was mentioned above, we use missing mass and vertex cuts to select the single-pion production events and to reduce the number of events with decaying pions. These cuts cause losses of some single-pion events as well. The true number of events is expected to be recovered by applying the acceptance corrections by using exactly the same cuts on the Monte-Carlo data. The remaining systematic errors associated with these cuts were estimated by varying the sizes of the windows. The absolute value of the cross section variations calculated with different cut windows, averaged over $\phi$ at fixed $Q^2$, $W$ and $\theta$, was considered as the systematic uncertainty for all $\phi$ for that fixed $Q^2$, $W$ and $\theta$.

One of the possible sources of systematic errors in this experiment is the uncertainty in the normalization. This can arise from miscalibrations of the Faraday cup, target density variations, errors in determining the target length and its temperature along with data acquisition live-time and other factors. However, the presence of the elastic events in the data set allows us to account for the normalization uncertainties of the cross sections by comparing the elastic cross sections to the parameterization of the world data [28]. This way we were able to combine the normalization, electron detection, electron tracking and electron identification errors into one global uncertainty factor. A comparison of the measured elastic cross sections for different $\theta_e$ and $\phi_e$ electron angles allowed us to assign a 5.2% global uncertainty due to the normalization and electron efficiency uncertainties.

The systematic uncertainty due to the model used in the radiative corrections was estimated by performing a second iteration. The radiatively corrected experimental cross sections from CLAS were fitted once more, and using the fit the new correction factors were calculated and compared with the previous iteration. The comparison indicated an uncertainty on the order of 2% due to the model dependence of the radiative corrections.

Using the kinematically over-determined reaction $ep \rightarrow e'\pi^+\pi^-p$ allows us to determine the $\pi^+$ efficiency by detecting the outgoing electron, $\pi^-$ and proton. The efficiency of the $\pi^+$ detection can be found as the ratio of the number of events where the $\pi^+$ was detected to the number of events where the $\pi^+$ was expected to be detected. A comparison of the pion efficiency calculated from the real data with the efficiency from GEANT-based Monte-Carlo simulation lead to a systematic error estimate of 2.5%.

In order to estimate the background coming from two-pion production, a sample of two-pion Monte-Carlo events was processed as if it were the actual data sample. The analysis of these events showed that this background would contribute less than 1% uncertainty to the differential cross sections. The systematic error due to the $\pi^+$ misidentification was estimated to be about $\sim 0.5\%$ by varying the cut in the proton-pion separation in the

![FIG. 12: The ratio of the measured elastic cross section to the parameterization of the world data [28]. The error bars represent the statistical uncertainty only. The solid line is from the fit of the data points to a constant.](attachment:image.png)
analysis.

The total systematic error in each bin was calculated as the square root of the sum of the squares of these different contributions. The size of the systematic errors is typically slightly larger than the size of the statistical uncertainties and is shown in Fig. 13 as the shaded bands.

RESULTS

Cross sections

The experimental differential cross section for each data bin was determined using the following formula:

$$\frac{\partial^2 \sigma}{\partial E_f \partial Q^2} = R \cdot B \sum_{\text{events}} \frac{1}{A L_{\text{ccc}}} \cdot \frac{1}{\Delta Q^2 \Delta W \sin \theta \Delta \theta \Delta \phi} \cdot \frac{\partial(W, Q^2)}{\partial(E_f, \cos \theta_e)} \cdot \frac{\partial(W, Q^2)}{\partial(E_f, \cos \theta_e)} \cdot \frac{T}{W},$$

where the sum in Eq. 7 runs over the $e\pi^+(n)$ events reconstructed in the fiducial region of CLAS. Here $A$ is the acceptance correction factor for an event, $L$ is the integrated luminosity, $N_A$ is Avogadro’s number, $\rho$ is the target density, $L_T$ is the target length, $Q_{\text{tot}}$ is the integrated charge corrected for the data acquisition live-time, $e$ is the electron charge, $e_{\text{ccc}}$ is the Čerenkov efficiency correction factor, $\Delta Q^2$, $\Delta W$, $\Delta \theta$, $\Delta \phi$ are the bin sizes, $\frac{\partial (W, Q^2)}{\partial (E_f, \cos \theta_e)}$ is the Jacobian between the $(W, Q^2)$ and $(E_f, \cos \theta_e)$ sets of variables, and $R$ and $B$ are the radiative and binning correction factors, respectively. The values of all kinematic variables are calculated for each particular event, as opposed to being taken at the center of the bin. The virtual photoproduction cross section can be obtained, according to Eq. 8, by dividing the left-hand side of Eq. 17 by the virtual photon flux $\Gamma$ factor defined in Eq. (6).

Sample plots of the differential cross sections compared with models are shown in Fig. 14 and Appendix A. The solid curves em the Sato-Lee model [30] and the dashed curves are from the Sato-Lee model [30]. The table of the structure functions are presented in Appendix B. The error column in the table shows the statistical and systematic uncertainties added in quadrature.

Structure functions

The structure functions $\sigma_{TT}$, $\sigma_{TL}$ and the linear combination $\sigma_T + e \sigma_L$ were obtained by fitting the $\phi$-dependence of the cross section to a function of the form:

$$F(\phi) = A + B \cos \phi + C \cos 2\phi.$$
A typical $W$-distribution of the $\sigma_T + \epsilon\sigma_L$ and $\sigma_{TT}$ terms features a distinct $\Delta(1232)$ peak, followed by the less prominent second resonance region. The $\theta$-dependence of $\sigma_T + \epsilon\sigma_L$ is mostly flat at low values of $W$ near the single-pion production threshold. This is consistent with $E_{0+}$ dominance at low energies. With increasing invariant mass, structures are developing at $\theta \approx 90^\circ$, which is characteristic of resonance production. Above the first resonance region the $\theta$-dependence of $\sigma_T + \epsilon\sigma_L$ becomes monotonically falling, consistent with the $t$-channel pion exchange mechanism dominance.

From the plots in Figs. 19,21,23,25 in Appendix A, one can see that in the first resonance region the measurements of $\sigma_{TT}$ agree very well with the Sato-Lee model. For these values of hadronic center-of-mass energy this structure function is dominated by the $\Delta(1232)$ resonance contributions. But the model predictions for the $\sigma_T + \epsilon\sigma_L$ linear combination disagree with the measured values, although the discrepancy is within the systematic errors. The present experiment does not separate the longitudinal and transverse structure functions. Given the successful description of the $\sigma_{TT}$ term, which can be expressed in terms of helicity amplitudes as $\sigma_{TT} \sim \text{Re}(H_2^2 - H_3 H_4^\dagger)$, one may assume that the structure function $\sigma_T \sim |H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2$, which depends on the same helicity amplitudes, may also be described reasonably well. Then the discrepancies in the sum $\sigma_T + \epsilon\sigma_L$ could be due to incomplete knowledge of the non-resonant physical background contributing directly to $\sigma_L$. Therefore, these data can be used to improve our understanding of the non-resonant background in the first resonance region. The measured $\sigma_{TL}$ structure function is the smallest, and the relative systematic uncertainties are large. The predictions for $\sigma_{TL}$ from Sato-Lee are in agreement with the measured values within the error bars.

The MAID2003 model, which is a fit to predominantly $\pi^0 p$ channel, describes our data surprisingly well, with the curves in the plots following most of the features of the experimental data. But the absolute values for the $\sigma_{TT}$ and $\sigma_T + \epsilon\sigma_L$ structure functions are typically overestimated, especially in the second resonance region. This may be indicative of our relatively poor knowledge of the $D_{13}(1520)$ and $S_{11}(1535)$ strength in the $e p \to e'\pi^+n$ channel. We also observe a distinct structure in the $\theta$ dependence of the $\sigma_{TL}$ amplitude for $W > 1.32$ GeV, which is not reproduced by MAID2003, where the sign is in fact opposite (see Fig. 14). Inclusion of these data in the MAID fit can improve our knowledge of the resonance parameters, background terms and the branching ratios for the states in the second resonance region.

**SUMMARY**

In conclusion, for the first time we have measured the unpolarized electroproduction cross sections for the $e p \to e'\pi^+n$ process covering a large angular range in the center-of-mass frame, and we have extracted the $\sigma_{TT}$, $\sigma_{TL}$ and $\sigma_T + \epsilon\sigma_L$ linear combinations of the structure functions. The combined statistical and systematic errors are of the order of 10% in most of the measured kinematic region. In the first resonance region the measured cross sections and the structure functions are in qualitative agreement with the MAID2003 and the Sato-Lee models, with a quantitative discrepancy with the MAID2003 model. In the second resonance region MAID2003 overestimates the height of the resonance peak. Together with $p\pi^0$ channel these data will provide the basis for the analysis of resonance transition form-factors in a coupled-channel analysis.

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**APPENDIX A: PLOTS OF THE STRUCTURE FUNCTIONS**
FIG. 15: (Color online) Structure functions versus \( W \) at \( Q^2 = 0.3 \text{ GeV}^2 \). Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.

FIG. 16: (Color online) Structure functions versus \( W \) at \( Q^2 = 0.4 \text{ GeV}^2 \). Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.
FIG. 17: (Color online) Structure functions versus \( W \) at \( Q^2 = 0.5 \text{ GeV}^2 \). Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.

FIG. 18: (Color online) Structure functions versus \( W \) at \( Q^2 = 0.6 \text{ GeV}^2 \). Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.
FIG. 19: (Color online) Structure functions versus c.m. \( \theta \) at \( Q^2 = 0.3 \text{ GeV}^2 \). Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.

FIG. 20: (Color online) Structure functions versus c.m. \( \theta \) at \( Q^2 = 0.3 \text{ GeV}^2 \). Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.
FIG. 21: (Color online) Structure functions versus c.m. $\theta$ at $Q^2 = 0.4$ GeV$^2$. Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.

FIG. 22: (Color online) Structure functions versus c.m. $\theta$ at $Q^2 = 0.4$ GeV$^2$. Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.
FIG. 23: (Color online) Structure functions versus c.m. $\theta$ at $Q^2 = 0.5$ GeV$^2$. Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.

FIG. 24: (Color online) Structure functions versus c.m. $\theta$ at $Q^2 = 0.5$ GeV$^2$. Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.
FIG. 25: (Color online) Structure functions versus c.m. $\theta$ at $Q^2 = 0.6$ GeV$^2$. Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.

FIG. 26: (Color online) Structure functions versus c.m. $\theta$ at $Q^2 = 0.6$ GeV$^2$. Solid curves represent MAID2003 calculations, while the dashed curves show the predictions of the Sato-Lee model. Shaded areas represent the systematic uncertainties.
### APPENDIX B: TABLE OF THE STRUCTURE FUNCTIONS

| $Q^2$ | $W$ | $\theta$ | $\theta_T + \theta_L$ | $\theta_T$ | $\theta_L$ | $\theta_T + \theta_L$ |
|-------|-----|---------|----------------------|---------|---------|----------------------|
| 0.31  | 7.98 | 0.17 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 1.79 | 0.15 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 7.98 | 0.17 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 1.79 | 0.15 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 7.98 | 0.17 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 1.79 | 0.15 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 7.98 | 0.17 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 1.79 | 0.15 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 7.98 | 0.17 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 1.79 | 0.15 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 7.98 | 0.17 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 1.79 | 0.15 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 7.98 | 0.17 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 1.79 | 0.15 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 7.98 | 0.17 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 1.79 | 0.15 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 7.98 | 0.17 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |
| 0.31  | 1.79 | 0.15 | 2.00 | 0.19 | -0.073 | 0.30 | 1.100 | 0.06 |

- $W$: The invariant mass squared of the system.
- $\theta$: The angle between the momentum transfer direction and the axis of the production.
- $\theta_T + \theta_L$: The sum of the target and laboratory angles.
- $\theta_T$ and $\theta_L$: The individual target and laboratory angles.

The table lists values for various $Q^2$ and $W$ combinations, along with the calculated angles and their combinations. The data is presented in a tabular format with columns for each variable, allowing for the analysis of the structure functions across different conditions.
| $Q^2$ | $W$ | $\theta$ | $\sigma_T$ | $\sigma_{EL}$ | $\sigma_{TL}$ | $\sigma_{TT}$ |
|-------|-----|--------|--------|--------|--------|--------|
| 0.4 | 0.15 | 0.529 | 67.5 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | 65.3 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | 62.5 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | 60.2 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | 45.3 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | 42.5 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | 12.5 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | 10.2 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | 7.8 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | 4.7 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | 2.5 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |
| 0.4 | 0.15 | 0.529 | ±0.4 | 3.641 ± 0.25 | 0.385 ± 0.10 | $3.983 ± 0.03$ |

**Table 2:**

| $Q^2$ | $W$ | $\theta$ | $\sigma_T$ | $\sigma_{EL}$ | $\sigma_{TL}$ | $\sigma_{TT}$ |
|-------|-----|--------|--------|--------|--------|--------|
| 0.5 | 0.129 | 0.791 | 97.5 | 4.211 ± 0.27 | $-0.605 ± 0.10$ | $3.983 ± 0.03$ |
| 0.5 | 0.129 | 0.791 | 112.5 | 3.307 ± 0.21 | $-0.282 ± 0.11$ | $3.105 ± 0.03$ |
| 0.5 | 0.129 | 0.791 | 127.5 | 2.436 ± 0.18 | $-0.092 ± 0.10$ | $3.788 ± 0.03$ |
| 0.5 | 0.129 | 0.791 | 142.5 | 1.525 ± 0.15 | $-0.529 ± 0.17$ | $3.073 ± 0.03$ |
| 0.5 | 0.129 | 0.791 | 157.5 | 0.675 ± 0.12 | $-0.379 ± 0.17$ | $2.939 ± 0.03$ |
| 0.5 | 0.131 | 0.848 | 7.5 | 13.846 ± 1.19 | $-0.518 ± 0.79$ | $1.473 ± 0.07$ |
| 0.5 | 0.131 | 0.848 | 22.5 | 4.049 ± 0.24 | $-0.221 ± 0.15$ | $0.953 ± 0.05$ |
| 0.5 | 0.131 | 0.848 | 37.5 | 6.613 ± 0.36 | $-0.506 ± 0.38$ | $1.627 ± 0.07$ |
| 0.5 | 0.131 | 0.848 | 52.5 | 8.609 ± 0.49 | $-0.366 ± 0.58$ | $2.102 ± 0.08$ |
| 0.5 | 0.131 | 0.848 | 67.5 | 11.203 ± 0.64 | $-0.434 ± 0.76$ | $2.612 ± 0.10$ |
| 0.5 | 0.131 | 0.848 | 82.5 | 14.570 ± 0.82 | $-0.635 ± 0.96$ | $3.279 ± 0.13$ |
| 0.5 | 0.131 | 0.848 | 97.5 | 18.029 ± 1.02 | $-0.909 ± 1.16$ | $3.945 ± 0.15$ |
| 0.5 | 0.131 | 0.848 | 112.5 | 21.780 ± 1.22 | $-1.215 ± 1.33$ | $4.506 ± 0.20$ |
| 0.5 | 0.131 | 0.848 | 127.5 | 25.671 ± 1.46 | $-1.614 ± 1.62$ | $5.103 ± 0.26$ |
| 0.5 | 0.131 | 0.848 | 142.5 | 29.507 ± 1.87 | $-2.160 ± 2.12$ | $5.590 ± 0.32$ |
| 0.5 | 0.131 | 0.848 | 157.5 | 38.982 ± 2.53 | $-3.008 ± 3.21$ | $6.903 ± 0.41$ |
| 0.5 | 0.131 | 0.848 | 172.5 | 47.937 ± 3.21 | $-3.908 ± 4.36$ | $8.134 ± 0.52$ |
| 0.5 | 0.131 | 0.848 | 187.5 | 56.032 ± 4.03 | $-4.898 ± 5.63$ | $9.273 ± 0.65$ |
| 0.5 | 0.131 | 0.848 | 202.5 | 63.101 ± 4.97 | $-5.898 ± 7.04$ | $10.352 ± 0.81$ |
| 0.5 | 0.131 | 0.848 | 217.5 | 69.003 ± 6.02 | $-6.898 ± 8.60$ | $11.333 ± 0.97$ |
| 0.5 | 0.131 | 0.848 | 232.5 | 72.698 ± 7.12 | $-7.898 ± 10.21$ | $12.242 ± 1.15$ |
| 0.5 | 0.131 | 0.848 | 247.5 | 74.796 ± 8.26 | $-8.898 ± 11.85$ | $13.061 ± 1.32$ |
| 0.5 | 0.131 | 0.848 | 262.5 | 75.394 ± 9.44 | $-9.898 ± 13.58$ | $13.800 ± 1.50$ |
| 0.5 | 0.131 | 0.848 | 277.5 | 75.267 ± 10.67 | $-10.898 ± 15.32$ | $14.488 ± 1.69$ |
| \(Q^2\) | W | \(\theta\) | \(\theta_T + \epsilon L\) | \(\theta_{PL}\) | \(\theta_{PT}\) |
|-----|-----|-----|-----|-----|-----|
| 0.1 | 0.16 | 0.405 | 97.5 | 2.417 | -0.755 | 0.25 | -3.945 |
| 0.5 | 0.19 | 0.455 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.19 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.5 | 0.19 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.19 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.21 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.21 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.23 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.23 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.25 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.25 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.27 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.27 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.29 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.29 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.31 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.31 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.33 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.33 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.35 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.35 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.37 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.37 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.39 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.39 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.41 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.41 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.43 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.43 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |
| 0.6 | 0.45 | 0.465 | 112.5 | 2.167 | -0.377 | 0.59 | -1.028 |
| 0.6 | 0.45 | 0.465 | 97.5 | 2.035 | -0.520 | 0.59 | -1.028 |

**Q^2** | W | \(\theta\) | \(\theta_T + \epsilon L\) | \(\theta_{PL}\) | \(\theta_{PT}\) |
|-----|-----|-----|-----|-----|-----|
| 0.6 | 0.13 | 0.065 | 97.5 | 2.411 | -0.607 | 0.25 | -3.785 |
| 0.6 | 0.13 | 0.065 | 112.5 | 1.976 | -0.724 | 0.31 | -1.807 |
| 0.6 | 0.13 | 0.065 | 97.5 | 2.411 | -0.607 | 0.25 | -3.785 |
| 0.6 | 0.13 | 0.065 | 112.5 | 1.976 | -0.724 | 0.31 | -1.807 |
| 0.6 | 0.13 | 0.065 | 97.5 | 2.411 | -0.607 | 0.25 | -3.785 |
| 0.6 | 0.13 | 0.065 | 112.5 | 1.976 | -0.724 | 0.31 | -1.807 |
| 0.6 | 0.13 | 0.065 | 97.5 | 2.411 | -0.607 | 0.25 | -3.785 |
| 0.6 | 0.13 | 0.065 | 112.5 | 1.976 | -0.724 | 0.31 | -1.807 |