Self-organization in cold atomic gases: a synchronization perspective

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We study non-equilibrium spatial self-organization in cold atomic gases, where long-range spatial order spontaneously emerges from fluctuations in the plane transverse to the propagation axis of a single optical beam. The self-organization process can be interpreted as a synchronization transition in a fully connected network of fictitious oscillators, and described in terms of the Kuramoto model.

1. Introduction

In recent years, cold and ultracold matter have proved to be a formidable tool for the investigation of phase transitions and collective behaviour in non-equilibrium systems. When coupling the dynamics of light and the centre-of-mass degrees of freedom of laser-cooled atoms, the dynamics becomes nonlinear and, above a critical value for the energy injected into the system, a transition is observed from a spatially homogeneous state to a state displaying some form of long-range order. This can be obtained in various configurations: transversally pumped cavities [1] where collective dynamics and self-organization in cold [2,3] and ultracold [4] gases have been investigated; collective atomic recoil lasing (CARL) where the spontaneous generation of a back-scattered beam within a monodirectional cavity is self-sustained by atomic bunching in the resulting optical potential [5–7]; in a counter-propagating geometry super-radiance and high-order nonlinearities stemming from atomic bunching have been studied [8–10]. The spontaneous breaking of...
a continuous translational symmetry in the presence of a strong viscous damping was investigated for both cavity [11,12] and counter-propagating [10,13,14] geometries, whereas a single-mirror geometry in the absence of damping was the focus of recent theoretical [15] and experimental [16] research. The distinguishing feature of these studies is that the spatial scale of the emerging spatial structure is self-selected, so that the spontaneous breaking of two continuous symmetries is observed (rotations and translations in the plane). A fully connected network is implemented through effective long-range interactions mediated by the optical fields, so that a mean-field model effectively captures the dynamics of the system [11,15]. The same general concept lies at the heart of much recent research, ranging from the atom-optical simulation of condensed matter phenomena [1] to the study of spin-glass transitions in cold atoms [17,18].

Together with spatial self-ordering, another prominent feature of nonlinear systems is that of temporal spontaneous ordering, i.e. synchronization. The emergence of synchronization is a pervasive feature of nonlinear science, ranging from biology and chemistry to neuroscience and social networks [19]. Broadly speaking, the spatial ordering into a periodic structure and the synchronization of oscillators on a limit cycle can be thought of as the same phenomenon, if the extended nature of the spatial coordinate is ignored and only the spatial phase is considered. Implementing light-mediated atom–atom interactions opens the possibility for tunable and controllable realizations of long-range interacting and mean-field models for synchronization [20], and indeed this was exploited in [7,21] to connect the viscous CARL dynamics to the Kuramoto model for synchronization of coupled oscillators [22]. We show in the following that this connection is not limited to CARL, but applies also to the symmetry-breaking transverse instabilities studied in [15,16]. Moreover, the connection made in [7,21] referred to the case where strong damping is present in the system (hence the denomination ‘viscous’ CARL), which in the Kuramoto analogy translates into the case where the oscillators have zero natural frequencies (their distribution is a Dirac delta function). We extend here the Kuramoto analogy to the situation analysed in [15,16], where no damping is present and a finite spread exists in the natural frequency spread of the fictitious oscillators.

2. Single-mirror optomechanical instabilities

(a) Basic scheme

We consider the single-mirror set-up [23,24] depicted in figure 1, where a cold gas of two-level atoms is illuminated by a pump beam of amplitude $F$, frequency $\omega_0$ and wavenumber $k_0$. The pump beam is detuned by $\delta = \omega_0 - \omega_{at}$ from the atomic transition (frequency $\omega_{at}$, linewidth $\Gamma$), and is retroreflected by a mirror of reflectivity $R$ placed at distance $d$ from the cloud to form the backward beam (amplitude $B$). We can thus envisage a situation where fluctuations in the atomic spatial properties can modify the refractive index distribution in the atomic ensemble and in turn the phase profile of the field. These phase fluctuations are converted into amplitude fluctuations by the free-space propagation to the mirror and back (related to the Talbot effect [25]) [26,27]. If the medium is nonlinear, then it will react to these amplitude perturbations resulting in a runaway, self-organization process where initial fluctuations are exponentially amplified and a macroscopic structure, emerging from noise, is encoded in the spatial properties of the atomic cloud. These spatial perturbations can be in the internal states of the atoms, for example a change of population of the excited state (electronic or two-level nonlinearity) or Zeeman sublevel (optical pumping nonlinearity) or in the external degrees of freedom of the atoms. In particular, we are interested here in the centre-of-mass, motional degrees of freedom of the atoms, which can be excited by dipole forces. As was first established for dielectric beads [28–30], linear Rayleigh scatterers can exhibit a significant optomechanical nonlinearity: for a negative polarizability (corresponding to $\delta > 0$ for atoms), the dielectric scatterers are low-field seekers and are expelled from high-intensity regions. Because for $\delta > 0$ the refractive index of atoms is smaller than one, a decrease in atomic density implies an increase of refractive index. Hence, the refractive index increases where
Figure 1. Sketch of the single-mirror set-up. A pump beam with an intensity corresponding to a saturation parameter $p_0 = |F|^2$, frequency $\omega_0$ and wavenumber $k_0$ illuminates a dense cloud of cold $^{87}$Rb (temperature $T$, optical density in line centre $b_0$). The beam is phase-shifted and transmitted from the cloud, and then retroreflected as the backward ($B$) beam after propagation to the mirror (distance $d$, reflectivity $R$) and back. (Online version in colour.)

the intensity is high, leading to an effective self-focusing nonlinearity. The result is a transverse optomechanical instability.

(b) Basic experimental observations

In the experiments motivating our study [16], the $D_2$ line of $^{87}$Rb is exploited (transition wavelength $\lambda_0 = 780.27$ nm) with an excited state lifetime of $\Gamma^{-1} = 26$ ns. An atomic ensemble is laser-cooled in a magneto-optical trap (MOT) to a temperature of about $290 \mu$K. At this temperature, Doppler broadening is negligible compared with the linewidth of the atomic transition ($\Gamma/(2\pi) = 6.06$ MHz). The cold sample obtained has a roughly Gaussian density profile with dimensions (full width at half maximum, FWHM) of $10 \times 10 \times 5$ mm ($10$ mm along the propagation direction) and contains about $5 \times 10^{10}$ atoms. The optical density in line centre is $b_0 = 150$. Then, the MOT (trapping lasers and magnetic field) is shut down, and the pump beam is turned on for a duration $t_{\text{pump}}$. This pump beam is spatially filtered by a single-mode fibre and collimated to a spot size of $1.9$ mm (FWHM). The experiment is performed in the vicinity of the $F = 2 \rightarrow F' = 3$ hyperfine transition, which is closed. A repumper tuned to the $F = 1 \rightarrow F' = 2$ transition counteracts hyperfine pumping owing to the residual excitation of other states. The polarization of the pump beam is linear. Details of the set-up can be found in [16].

Spontaneous breaking of transverse symmetries and pattern formation is observed for a wide range of positive detunings to the $F = 2 \rightarrow F' = 3$ transition, most experiments being done with a pump detuning in the range $\delta = 7$–$10 \Gamma$. The observed patterns have hexagonal symmetry and consist of well-developed peaks. As we are going to compare the results with a one-dimensional theory, we are presenting here only cross sections of the two-dimensional structures. Figure 2a shows a section through the light pattern in the pump beam. A high modulation is evident with peaks reaching $3.5$ times the background of the input beam. The pattern wavelength is about $110 \mu$m and is found to depend on the feedback distance [16].

Ten microseconds after the pump beam is switched off, a weak probe beam, which is orthogonally polarized to the pump and does not experience feedback, is injected into the medium. It is detuned a few linewidths to the low-frequency side of the resonance and shows a honeycomb pattern. For the dispersive imaging situation used here, one expects that high-intensity levels are obtained where the refractive index in the sample is high. For negative detunings, these are the regions with a high atomic density. The observation of honeycombs is hence consistent with the expectation for complementary patterns in the light field and the atomic density: atoms are expelled from the pump filament and gather along the ridges of the honeycomb pattern.
Figure 2. Cross sections of typical hexagonal patterns observed in the transmitted pump beam (a) and probe beam (b). Both stem from the re-imaged intensity distribution 10 mm after the cloud (i.e. corresponding to the re-entrant backwards beam) and are normalized to the input intensity without atoms. Parameters for the pump beam: $I = 129 \text{ mW cm}^{-2}$, $\delta = +7\Gamma$, and $d = 5 \text{ mm}$; for the probe beam $\delta = -7\Gamma$. (Online version in colour.)

Figure 3. Decay of the density pattern (as measured by the probe) as the delay between pump (duration 215 µs) and probe (duration 10 µs) pulses is increased. Other parameters are pump detuning $+7\Gamma$, probe detuning $+5\Gamma$, pump power 6.6 mW, probe power 20 µW. Pump and probe are circularly polarized. The contrast is normalized to the contrast just after the pump switch-off. (Online version in colour.)

Within 10 µs, all excited state populations created by the pump will have decayed. Hence, the presence of a structure a few microseconds after the pump beam is switched off excludes electronic excitation as the source of the atomic grating evidenced by the probe. As other
measurements exclude hyperfine and Zeeman gratings [16], we conclude on the presence of a significant density grating in the atomic cloud. This can be further substantiated by investigating the dependence of the contrast of the probe pattern on the delay time between pump and probe pulse. The result is shown in figure 3, which gives a decay time of around 80 µs (corresponding to a decrease of the contrast by a factor of two). As an atom with a one-dimensional thermal speed of 0.17 m s$^{-1}$ would traverse 55 µm (half the transverse period) in about 300 µs, this is a reasonable order of magnitude for a washout of a density pattern owing to the velocity spread of the ensemble. This is explored further in §3.

3. Theoretical model

The analysis of the experimental results is somewhat complicated by the fact that in general electronic and optomechanical nonlinearities are simultaneously present, so that an internal state and a density pattern can be simultaneously present in the atomic ensemble. Internal-state nonlinear effects can be accounted for in our theoretical model [16], but we neglect them here by assuming low saturation values, because, at low temperatures, a spatial instability is expected to occur even at low saturation levels owing to density redistribution effects only [15]. The results presented in [16] provide experimental support for this claim in the appropriate parameter regimes.

For a theoretical treatment, we consider in the following the case of a cigar-shaped cloud elongated along the coordinate $x$ transverse to the propagation direction $z$, so that transverse self-organization will lead to one-dimensional structures. Our theoretical analysis [15] indicates in fact that no qualitative differences are expected for the dynamics of one- and two-dimensional systems, and restricting to one dimension allows for a connection with temporal synchronization phenomena. We neglect propagation inside the cloud and account for diffraction of the transmitted field only in the free-space propagation to the mirror and back. For the parameters considered here, this simplifying assumption provides good qualitative agreement with more detailed models in which diffraction within the medium is also taken into consideration, as discussed in [16]. We also assume the pump to be substantially detuned from the atomic resonance, so that the atoms act as linear scatterers, and the transmitted field $F_{tr}$ at the exit of the cloud is

$$F_{tr}(x,t) = F \exp(i\chi_0 n(x,t)),$$

(3.1)

where $\chi_0 = b_0 \Delta / (1 + 4 \Delta^2)$ parametrizes the cloud susceptibility and $\Delta = \delta / \Gamma$. Scaling of the field is chosen, so that $p_0 \equiv |F|^2 = I_{pump} / I_{sat}(1 + 4 \Delta^2)$ denotes the off-resonance saturation parameter of the pump, where $I_{pump}$ is the pump intensity and $I_{sat}$ is the on-resonance saturation intensity ($I_{sat} = 1.6 - 3.6 \text{ mW cm}^{-2}$ for the $D_2$ line of $^{87}\text{Rb}$, depending on the exact excitation conditions). In the limit of large number of atoms $N$ ($N \sim 10^{10}$ in a typical experiment), we describe the cloud dynamics in terms of a phase-space distribution $f(x,v,t)$, where $v$ is the transverse velocity coordinate, so that the atomic density is obtained as

$$n(x,t) = \int_{-\infty}^{+\infty} dv f(x,v,t).$$

As the gas is cooled at relatively low temperatures, $T \sim 100 \mu \text{K}$, optical forces become relevant for the centre-of-mass dynamics of the gas. In particular, at large detuning radiation pressure is reduced, and the atoms are subjected to the conservative dipole potential $U_{\text{dip}} = 1/2\hbar \delta \log(1 + s(x,t)) \simeq 1/2\hbar \delta s(x,t)$, where the last approximation was taken by assuming the saturation
Negligible and the dynamics of the gas is captured by the Vlasov equation

\[ q \left( 1 + \frac{\delta}{h} \right) = \frac{1}{\sqrt{2\pi}} \int \exp \left( -\frac{\delta^2}{2\sigma^2} \right) \sqrt{2\pi} \sigma \, \mathrm{d}\sigma. \]

The above-described coupled light-atom dynamics (without internal-state dynamics) is predicted to lead to a self-organizing transition when the injected power exceeds a critical value, \( p_0 \geq p_0^\text{th} = (2R\sigma_0 \chi_0)^{-1} \), where \( \sigma = \hbar\delta/2k_B T \) measures the strength of dipole forces relative to thermal effects [15]. The spatial scale \( \Lambda_c = 2\pi/q_c \) is set by the critical (most unstable) wavenumber \( q_c = \sqrt{\pi k_0/2d} \), and can be continuously tuned through the mirror distance \( d \). As \( q_c \) is selected
among a continuum of wavenumbers, the self-structuring mechanism discussed here leads to the spontaneous breaking of a continuous translational symmetry (figure 4c,d). Moreover, in two dimensions, the rotational symmetry is also broken [16]. This aspect fundamentally differentiates the optomechanical self-structuring investigated here from other self-organizing systems involving optomechanics and cold atoms, such as CARL [6,7] and transversely pumped cavities [1–4], as the spatial scale of the emerging structures is not predetermined by the geometrical set-up or interference conditions.

4. Synchronization dynamics and connection with the Kuramoto model

We now discuss the connection between the optomechanical spatial instabilities discussed above and the Kuramoto model for synchronization. As a starting point, we remark that the spontaneous emergence of a periodic pattern (identified by a single spatial frequency \( q_c \)) can always be interpreted in terms of a synchronization transition. Focusing on the phase \( \theta = q_c x \) instead of the spatial coordinate \( x \) itself, the bunching of the atoms in the minima of the self-organizing optical potential corresponds to the transition from a homogeneous state where the atoms have a uniformly distributed phase to a state where the distribution of the phases is peaked around a certain value \( \psi \). It is convenient in the following to exploit the periodicity of the pattern and confine the phase in the range \((−\pi, \pi)\) as \( \theta_j = \text{mod}(q_c x_j, 2\pi) − \pi \). This connection between spatial and temporal self-organization was exploited, for instance, in showing that the spatial instability of viscous CARL can be interpreted in terms of the Kuramoto model [7,21]. Similar results also hold for ‘viscous’ single-mirror instabilities (see [11,12] for a cavity analogue), but we wish to focus our attention here on the inviscid regime in connection with the experimental results presented in [16].

In order to describe the dynamics of the \( N \) atoms composing the gas, we consider the 2\( N \) coupled equations (\( j = 1, \ldots, N \))

\[
\dot{x}_j = v_j, \quad \dot{v}_j = \frac{f_{\text{dip}}(x_j)}{M} = -\frac{\hbar \delta}{2M} \frac{\partial |B|^2}{\partial x} \bigg|_{x_j},
\]

where the force is given by the dipole force \( f_{\text{dip}} = -\partial U_{\text{dip}}/\partial x \) as above. Because the pump intensity is assumed to be spatially homogeneous, the optical gradients are due only to the backward field modulations. We now obtain an approximate expression for the backward field valid when the system is driven just above the threshold for self-organization. Close to the critical point, a spatial modulation for the atomic density is obtained at the critical wavenumber, \( n(x) = 1 + r \cos(\psi - q_c x) \). As the system is translationally invariant, the phase \( \psi \) of the pattern is self-selected by the system (see also [21]). The amplitude \( r \) and the phase \( \psi \) of the pattern define the Fourier mode of the density at the critical wavenumber,

\[
re^{i\psi} = \frac{1}{L} \int_0^L n(x) \exp(i q_c x) \, dx,
\]

so that \( r \) acts as an order parameter for the instability. In line with the CARL literature, we refer to \( r \) as the bunching factor. We stress that, while the density spectral properties are directly accessible in numerical simulations, this is not the case in the experimental realizations. A definition analogous to the bunching factor, but based on the Fourier properties of the optical fields, was termed contrast in [16] and used to experimentally monitor the self-organizing dynamics. The same procedure was also used in figure 3. In the following, we typically assume that the system is close to the critical point, so that \( 0 < r \ll 1 \). The critical wavenumber is selected by the system as the one that most efficiently converts phase modulations in the pump beam into amplitude
modulations for the backward beam [23, 24]. This implies that the far-field sideband at the critical wavenumber is shifted by a factor $e^{i\pi/2} = i$ in the propagation to the mirror and back, so that $B$ is obtained from the transmitted field as (see equation (3.3))

$$F_{tr} \simeq \sqrt{p_0} e^{i\chi_0} [1 + i\chi_0 r \cos(\psi - q_c x)] \implies B = \sqrt{p_0} e^{i\chi_0} [1 - \chi_0 r \cos(\psi - q_c x)].$$

To first order in $r$, the dipole force is thus found as

$$\frac{f_{\text{dip}}}{M} = -\frac{\hbar \delta}{2M} \frac{\partial |B|^2}{\partial x} \simeq \frac{\hbar \delta}{M} R_0 \chi_0 q_c r \sin(\psi - q_c x) \equiv f q_c r \sin(q_c x - \psi),$$

where in the last step we defined $f = (\hbar \delta / M) R_0 \chi_0$.

In order to obtain a connection with models for temporal synchronization, we now wish to use equation (4.2) to obtain a single set of $N$ equations for the phases $\theta_j = q_c x_j$. Close to the critical point, the modulation depth of the optical potential is small ($r \ll 1$), and the dynamical behaviour of the atoms is force-free in first approximation. This is correct up to a certain time $t^*$, which we term as the *dephasing time*, and has the following interpretation: close to the critical point, the atoms can move freely in the emerging optical potential without affecting the dynamics. This is strictly correct only in the limit of vanishing modulations, $r \to 0$ (i.e. below threshold), but we exploit critical slowing down at the onset of the instability to assume that no feedback on the optical field is exerted by the atomic motion for a characteristic time defined by $t^* = (q_c v_{th})^{-1}$. This definition is chosen as the time an atom at the thermal speed needs to travel the characteristic distance of the pattern, $\Lambda_c \sim q_c^{-1}$. The equation for the frequencies $\Omega_j = q_c v_j$ is then solved up to $t^*$ as

$$\Omega_j(t^*) \simeq \Omega_j(0) + f q_c^2 r \sin(\theta_j - \psi) t^*, $$

where $\Omega_j(0)$ denotes the ‘natural’ frequencies $\Omega_j(0) = q_c v_j(0)$ determined by the initial atomic velocities. The phases then evolve as

$$\dot{\theta}_j = \Omega_j(0) + f q_c r \sin(\theta_j - \psi).$$

Equation (4.3) is in the form of a Kuramoto equation [22], with a coupling strength $K = f q_c / v_{th}$. We remark that in equation (4.3) an effective long-range interaction between the ‘oscillators’ (i.e. the atoms) is mediated by the light field, which thus offers the possibility of implementing a mean-field model with all-to-all coupling in a simple and powerful way.

With a Lorentzian initial condition for the natural frequencies $\Omega_j(0)$, the Kuramoto model (4.3) is known to lead to a synchronization transition when the driving exceeds the critical value $K \geq K_{th} = 2\Omega_{th} = 2 q_c v_{th}$ [22]. Hence, we recover the power threshold obtained in [15] and reported above: $p_{th}^0 = (2R \sigma \chi_0)^{-1}$. As discussed in [15], with a Gaussian initial condition, a slightly different growth rate is found for the instability, but the threshold condition is unchanged. Because the synchronization transition leads to a bunching of the fictitious oscillators around the (self-selected) phase $\psi$, in the $x$-space this corresponds to the formation of complementary periodic structures $\sim \cos(q_c x - \psi)$ for the atomic density and the optical intensity. We remark that the sign of the optical potential is positive for blue detuning ($\delta > 0$) and negative for red detuning ($\delta < 0$), so that when $\delta > 0$ the density and optical profiles are shifted by half a wavelength (figures 2 and 4). The onset of the optomechanical instability can thus be reinterpreted as a Kuramoto transition to a synchronized state with a self-selected phase $\psi$ where the threshold for this process is determined by the initial spread of the oscillator frequencies. Physically, this spread is determined by the initial temperature of the gas and is therefore tunable experimentally, whereas the coupling strength $K$ is tunable through the injected pump $p_0$.

We numerically solved the coupled dynamics (3.1)–(3.2) of the phase-space distribution $f(x, v, t)$ and the optical field in one transverse dimension using a semi-Lagrangian method with spline interpolation [15]. The initial condition of the gas is set to be a Lorentzian distribution with FWHM $v_{th}$: $f_0(v) = v_{th} / [\pi (v^2 + v_{th}^2)]$. Our numerical results confirm the theoretical predictions reported in the previous sections, because, above threshold, a spatially homogeneous cloud is spontaneously converted into a periodic pattern with periodicity $\Lambda_c = 2\pi / q_c$. Driving
the system 5% above threshold, we monitor the bunching parameter $r$ and observe a transition from an $r = 0$ state to a state characterized by $r > 0$, followed by oscillations with a period of about $T_{\text{slosh}} \approx 800 \mu s$ (figure 4a). This is analogous to what is observed in inviscid CARL [31], and is due to the atoms sloshing and periodically amplifying in the spontaneously formed optical potential. Figure 4b–d shows the formation of complementary periodic spatial structures for the optical intensity $s(x)$ and the atomic density $n(x)$. The spatial scale of the pattern is $\Lambda_c \approx 120 \mu m$ for our choice of parameters, and is in good agreement with the experimental observations [16].

The depth of the modulated potential shown in figure 4d in units of temperature is $T_{\text{pot}} \approx \hbar (s_{\text{max}} - s_{\text{min}})/k_B \approx 440 \mu K$, where $s_{\text{max}}$ and $s_{\text{min}}$ are the maximum and minimum values of $s$, respectively, and $k_B$ is the Boltzmann constant. For each of these states, we calculate the probability distribution of the phase $\theta$ by sampling the density distribution $n(\theta)$ with $N = 10^5$ particles, which shows a peak around the value $\psi \approx \pi/2$ in correspondence with the synchronization transition.

Figure 5 shows the dynamics of the pattern obtained for the same parameters as in figure 4 when the injected pump is switched off after 5 ms. As the system is left with no driving ($K = 0$), the ballistic motion of the atoms leads to a dephasing dynamics in which the bunching factor $r$ decreases monotonically (figure 5d). The phase-space distribution $f(x, v)$ is monitored just before switching off the pump (figure 5a), and then after 100 $\mu$s (figure 5b) and 500 $\mu$s (figure 5c). The characteristic dephasing time can be inferred from figure 5d, and is of the order of 100 $\mu$s. This is consistent with the time scale of atomic motion at the considered temperature, and agrees with our experimental results (figure 3).

5. Conclusion

Self-structuring of the atomic density in a cloud of a cold atomic gas that is optically pumped can be interpreted as a transition to a synchronized state of Kuramoto oscillators. Although this connection was made in cases where strong damping is present [7,21], we have shown that the
analogy can be extended to cases where no damping is present such as those of the experimental realization in [16]. In this case, a finite spread exists in the natural frequency distribution of the fictitious oscillators, which sets the threshold for the synchronization transition.

We note that a similar coupling between the optical field and the motion of atoms can also arise in the context of cooperative scattering of light by an optically thick cloud of atoms [32]. It is possible to derive the optical forces acting on each atom in an ensemble of $N$ atoms including both the quasi-resonant radiation pressure (dominant close to the atomic resonance) as well as the optical dipole forces (more relevant for larger detuning) [33]. In contrast to the situation considered in this paper, in [32,33] these forces arise from direct dipole–dipole coupling between atoms and are not mediated via a propagation and feedback through a mirror. Furthermore, we expect that synchronization between different atoms not only manifests itself in the motional degrees of freedom, but also can affect the internal degrees of freedom of the atoms, leading to synchronized cooperative scattering of a large cloud. Such cooperative scattering was investigated in pioneering work by Dicke [34] for samples smaller than the wavelength but also for ‘radiation of gas of large extent’.

Finally, it is important to mention that our theoretical and numerical predictions of the Kuramoto analogy for inviscid systems can be applied outside the interaction of light with cold atoms. For example, it is possible to connect cold atoms and plasmas via the correspondence between dipole and ponderomotive forces. Attractive (shadow) and repulsive (radiation pressure) forces exist inside magneto-optically trapped samples, which introduce an effective charge between the atoms and thus simulate electrostatic interactions. Self-structuring and its Kuramoto interpretation are then to be expected in the investigation of various plasma systems without friction, possibly including quantum plasmas.

In the context of the special issue, it is important to mention that optomechanical solitons were predicted in single-pass propagation [13] and cavity schemes [12] with velocity damping. Corresponding structures are expected for the single-mirror geometry.

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