Analytical Investigation into Effects of Capillary Force on Condensate Film Flowing over Horizontal Semicircular Tube in Porous Medium

Tong-Bou Chang, Bai-Heng Shiue, Yi-Bin Ciou, and Wai-IO Lo

Department of Mechanical and Energy Engineering, National Chiayi University, Chiayi, Taiwan

Correspondence should be addressed to Tong-Bou Chang; tbchang@mail.nchu.edu.tw

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A theoretical investigation is performed into the problem of laminar filmwise condensation flow over a horizontal semicircular tube embedded in a porous medium and subject to capillary forces. The effects of the capillary force and gravity force on the condensation heat transfer performance are analyzed using an energy balance approach method. For analytical convenience, several dimensionless parameters are introduced, including the Jakob number $Ja$, Rayleigh number $Ra$, and capillary force parameter $Bo_c$. The resulting dimensionless governing equation is solved using the numerical shooting method to determine the effect of capillary forces on the condensate thickness. A capillary suction velocity can be obtained mathematically in the calculation process and indicates whether the gravity force is greater than the capillary force. It is shown that if the capillary force is greater than the condensate gravity force, the liquid condensate will be sucked into the two-phase zone. Under this condition, the condensate film thickness reduces and the heat transfer performance is correspondingly improved. Without considering the capillary force effects, the mean Nusselt number is also obtained in the present study as $Nu_{V=0} = (2Ra Da/Ja)^{1/2} \int_0^\pi (1 + \cos \theta)^{1/2} d\theta$.

1. Introduction

The problem of laminar film condensation in porous media has received considerable attention in the literature due to its importance in many engineering applications, including packed-bed heat exchangers, chemical reactors, petroleum recovery, heat pipes, casting solidification, geothermal reservoirs, and oil recovery in reservoir engineering. In such applications, the capillary force has a significant effect on the heat transfer performance and must therefore be taken into account when designing and evaluating the system. The problem of laminar film condensation was first investigated by Nusselt [1], who considered a condensate film flowing down a vertical plate. In developing a theoretical solution for the heat transfer rate, Nusselt imposed four assumptions, namely, (1) the vertical wall had a constant temperature; (2) the laminar film had a linear temperature gradient; (3) the shear force between the liquid and the steam was sufficiently small to be ignored; and (4) the inertial force in the liquid film could also be ignored. However, these assumptions do not necessarily hold in practical situations. For example, the liquid film thickness is often nonconstant in experimental tests, and hence a linear temperature gradient cannot be guaranteed. Thus, the theoretical heat transfer coefficient predicted by Nusselt is frequently slightly lower than that observed experimentally. Consequently, many scholars have attempted to modify and improve the original assumptions in [1] in order to achieve a better fit between the theoretical and experimental results. Cheng [2] investigated the heat transfer coefficient of a condensate film on an inclined plate in a porous medium. The results showed that, for an inclination angle close to zero, the flow condition at the edge of the plate could not be found and the value can only be found when the inclined
angle is larger than zero due to the ambiguity of the pressure gradient. In a later study, this problem was resolved by Yang and Chen [3] using an open channel hydraulics concept. Chang [4] and Wang et al. [5] investigated the heat transfer performance of a condensate layer on a horizontal plate in a porous medium in the absence of capillary forces. However, in many of these problems, the effective pore radii are small, and hence the influence of capillary forces on the heat transfer performance is potentially significant and should be considered. Accordingly, Majumdar and Tien [6] studied the effect of capillary forces on film condensate flow along a vertical wall. The results showed that capillary forces actually play a significantly effect on condensate flow. Udell [7, 8] used a sand-water-steam system to perform a series of theoretical and experimental investigations into the effect of capillary forces on the heat transfer process in porous media saturated with both vapor and liquid phases.

Heat transfer and fluid flow in porous media had been studied by many researchers. EL-Hakiem and Rashad [9] investigated the effects of both radiation and the nonlinear Forchheimer terms on free convection from a vertical cylinder embedded in a fluid-saturated porous medium numerically by using the second-level local nonsimilarity method. EL-Kabeir et al. [10] applied the group theoretic method to solve problem of combined magnetohydrodynamic heat and mass transfer of non-Darcy natural convection about an impermeable horizontal cylinder in a non-Newtonian power law fluid embedded in porous medium under coupled thermal and mass diffusion, inertia resistance, magnetic field, and thermal radiation effects. Bakier et al. [11] developed a linear transformation group approach method to simulate the problem of hydromagnetic heat transfer by mixed convection along vertical plate in a liquid saturated porous medium. The steady two-dimensional laminar forced flow and heat transfer of a viscous incompressible electrically conducting and heat-generating fluid past a permeable wedge embedded in non-Darcy high-porosity ambient medium with uniform surface heat flux have been studied by Bakier and Rashad [12]. Rashad and EL-Kabeir [13] studied the coupled heat and mass transfer in transient flow by a mixed convection boundary layer past an impermeable vertical stretching sheet embedded in a fluid-saturated porous medium in the presence of a chemical reaction effect. Mansour et al. [14] developed a thermal nonequilibrium modeling of steady natural convection inside wavy enclosures with the effect of thermal radiation. Mansour et al. [15] numerically investigated the problem of double-diffusive convection in inclined triangular porous enclosures with sinusoidal variation of boundary conditions in the presence of heat source or sink. Sameh and Rashad [16] investigated the effects of an electromagnetic force on the heat transfer by natural convection flow of nanofluids in wavy enclosure filled by an isotropic porous medium. They found that the increase in the Hartmann number and the decrease in Darcy number obstruct the fluid movement but they increase the rate of heat transfer.

Liu et al. [17] found that the injection of copper bubbles into a porous medium increased the heat transfer rate under joint boiling and condensation conditions. Lee et al. [18] used a nanoscale surface to increase the heat transfer rate in dropwise condensation. Xie et al. [19] investigated the condensation heat transfer performance of R245fa foam insulation in tubes with and without lyophilic porous-membrane-tube inserts, respectively. Wen et al. [20] investigated the condensation heat transfer performance by using hydrophobic copper nanowires to induce a capillary force. The results showed that a smaller hole diameter of the hydrophobic copper nanowires induces a larger capillary force and then enhances the condensation heat transfer performance. Buleiko et al. [21] examined the effects of the capillary force on the phase behavior of liquid and gaseous propane and the dynamics of hydrate formation and dissociation in porous media. It was shown that the reactant with water and propane required more pressure to extract than a simple material in a porous medium.

Fluid flow around a semicircular tube had been carried out by many researchers [22–27]. This geometry is of particular interest for chemical and food processing industries, heat pipe designs, solar energy applications, and the design of heat exchangers. Moreover, since the capillary forces have a significant effect on the heat transfer performance in porous media and must therefore be taken into consideration, accordingly, the present study investigates the influence of capillary forces on the laminar film condensation flow on a finite-size horizontal semicircular tube embedded in a porous medium and maintained at a constant temperature. In order to determine the effects of the capillary forces, the present study employs the concepts of capillary pressure and a two-phase zone to analyze the liquid flow. The dimensionless heat transfer coefficient is then solved for various values of the Darcy number Da, Jakob number Ja, Rayleigh number Ra, Prandtl number Pr, and capillary parameter \( Bo \).

2. Analysis

Figure 1 shows a schematic illustration of the physical model considered in the present study with curvilinear coordinates \((x, y)\) aligned along the tube surface and surface normal, respectively. The tube is assumed to be horizontal, clean, and semicircular (with a radius \( R \)) and to be embedded in a porous medium. Pure quiescent vapor in a saturated state and with a uniform temperature \( T_{\text{sat}} \) condenses on the tube surface. If the wall temperature is lower than the vapor temperature, the resulting condensation forms a thin condensate layer and wets the tube surface ideally. Under the effects of gravity, the liquid film boundary layer thickness, \( \delta \), has a minimum value at the top of the tube and increases gradually as the liquid flows downward over the surface of the tube. Notably, the gravity force acting on the liquid film in the downward direction is opposed by the capillary force induced by the porous medium, which pulls the film in the upward direction with a velocity \( v_2 \).

The following assumptions are made for analyzing the heat transfer characteristics of the condensate film:

1. The flow is steady and laminar. Hence, the effects of inertia and convection in momentum equation are
negligible. Moreover, the condensate film travels very slowly and thus has negligible kinetic energy.

(2) The vapor temperature, wall temperature, and physical properties of the dry vapor, condensate, and porous medium, respectively, remain constant.

(3) The liquid film flow in the porous medium is governed by Darcy’s law.

Given these assumptions, from the concept of convection in porous media [28], the governing equations for the liquid film can be formulated as follows:

Continuity equation is as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$  
(1)

Momentum equation in x-direction is as follows:

$$(\rho - \rho_v) g \sin \theta - \frac{\mu_e}{K} u = 0.$$  
(2)

Energy equation is as follows:

$$\alpha_e \frac{\partial^2 T}{\partial y^2} = 0,$$  
(3)

where $\mu_e$ and $\alpha_e$ are the dynamic viscosity of the liquid condensate and effective thermal diffusivity, respectively; $\rho$ and $\rho_v$ are the liquid density and vapor density, respectively; and $u$ and $v$ are the velocity components in the $x$- and $y$-directions, respectively.

Assuming that the vapor density $\rho_v$ is much less than the liquid density $\rho$, (2) can be rewritten as

$$u = \frac{\rho g K}{\mu_e} \sin \theta.$$  
(4)

According to Fourier’s conduction law, the first law of thermodynamics and Nusselt’s classical analysis method, the following energy balance exists for each element of the liquid film with height $\delta$ and width $dx$:

$$\frac{d}{dx} \left[ \int_0^\delta \rho u \left( h_{f,g} + C_P (T_{sat} - T) \right) dy \right] dx + \rho h_{f,g} v_2 dx = K_e \frac{\partial T}{\partial y} dx,$$  
(5)

where $k_e$ is the effective thermal conductivity of the porous medium with saturated liquid and subscript 2 indicates that the referred properties belong to the two-phase zone.

The right-hand side of (5) represents the energy transferred from the liquid film to the tube surface. Meanwhile, the first term on the left-hand side of the equation expresses the net energy flux across the liquid film (from $x$ to $x + dx$), while the second term expresses the net energy sucked into the two-phase zone by the capillary force.

The continuity equation at the interface between the two-phase zone and the liquid film has the form

$$\frac{\partial u_2}{\partial x} \bigg|_{y_2=0} + \frac{\partial u_3}{\partial x} \bigg|_{y_2=0} = 0.$$  
(6)

Moreover, the no slip condition between the two-phase zone and the liquid film is given by

$$u_2 \big|_{y_2=0} = u_l \big|_{y_2} = \frac{\rho g K}{\mu_e} \sin \theta.$$  
(7)

Applying the relation $dx = R d\theta$, (7) can be rewritten as

$$\frac{\partial u_2}{\partial x} \bigg|_{y_2=0} = \frac{\partial u_3}{\partial x} \bigg|_{y_2=0} = \frac{\rho g K \cos \theta}{\mu_e R},$$  
(8)

According to Udell [7], the suction velocity produced by the capillary force can be written as

$$v_2 = \frac{K_{re}}{\mu_e} \left( \frac{\partial P_c}{\partial y_2} - \rho g \cos \theta \right),$$  
(9)

where

$$K_{re} = s^3,$$  
(10a)

$$P_c = \frac{\sigma}{\sqrt{K/e}} f(s),$$  
(10b)

$$f(s) = 1.417 (1 - s) - 2.120 (1 - s)^3 + 1.263 (1 - s)^3,$$  
(10c)

in which $s$ is the dimensionless saturation and has a value of $s = 1$ at $y_2 = 0$.

Substituting (10a)–(10c) into (9), the suction velocity is obtained as

$$v_2 = \frac{K}{\mu_e} s^3 \left( \frac{\sigma}{\sqrt{K/e}} f(s) \frac{\partial s}{\partial y_2} - \rho g \cos \theta \right).$$  
(11)
Substituting (11) and (8) into (6), the continuity equation at the interface between the two-phase zone and the liquid film can be rewritten as

$$\frac{\rho g K}{\mu_e R} \cos \theta + \frac{K}{\mu_e} \left\{ \frac{\sigma}{\sqrt{K/\varepsilon}} \left[ s^3 f'' \left( \frac{\partial s}{\partial y_2} \right)^2 + 3s^2 f^2 \left( \frac{\partial s}{\partial y_2} \right)^2 + 3s^3 f_1 \frac{\partial^2 s}{\partial y_2^2} - 3s^2 \rho g \cos \theta \frac{\partial s}{\partial y_2} \right] \right\} = 0.$$  \hspace{1cm} (12)

According to Majumdar and Tien [6], the saturation profiles near the interface of the two-phase zone and the liquid film is almost linear when \( y_2 = 0 \). In other words, \((\partial^2 s/\partial y_2^2) = 0 \) at \( y_2 = 0 \). Equation (12) can then be rewritten as

$$\frac{\partial s}{\partial y_2} \bigg|_{y_2=0} = \frac{3s^2 \rho g \cos \theta \pm \sqrt{3 (3 \rho gs^2 \cos \theta)^2 - 4 (\sigma / \sqrt{(K/\varepsilon)}) \left[ s^3 f'' + 3s^2 f' \right] (\rho g / R) \cos \theta}}{2 (\sigma / \sqrt{(K/\varepsilon)}) \left[ s^3 f'' + 3s^2 f' \right]}.$$  \hspace{1cm} (13)

The suction velocity \( v_2 \) can be calculated by substituting (13) into (11) and then rearranging to give

$$v_2 = \frac{K}{\mu_e} s^3 \left\{ \frac{\sigma}{\sqrt{K/\varepsilon}} f(s) \left[ 3s^2 \rho g \cos \theta \pm \sqrt{3 (3 \rho gs^2 \cos \theta)^2 - 4 (\sigma / \sqrt{(K/\varepsilon)}) \left[ s^3 f'' + 3s^2 f' \right] (\rho g / R) \cos \theta} \right]}{2 (\sigma / \sqrt{(K/\varepsilon)}) \left[ s^3 f'' + 3s^2 f' \right]} - \rho g \cos \theta \right\}.$$  \hspace{1cm} (14)

For analytical convenience, let the following dimensionless parameters be introduced:

$$B_0 = \frac{\delta \sqrt{\varepsilon}}{\rho g K},$$  \hspace{1cm} (15)

$$Da = \frac{K}{R^2},$$

$$v_2 \bigg|_{y_2=0} = \frac{\rho g K}{\mu_e} \left\{ 0.2503 \sqrt{\cos^2 \theta + 3.774 B_0 \sqrt{Da} \cos \theta - 0.7497 \cos \theta} \right\}.$$  \hspace{1cm} (16a)

As described above, \( v_2 \) is the suction velocity produced by the capillary force. As a result, it cannot have a negative value physically. However, a value of \( v_2 \big|_{y_2=0} < 0 \) can be obtained mathematically in the calculation process and implies that the gravity force is greater than the capillary force. In other words, the condensate film flows entirely in the downward direction and no liquid condensate is sucked into the two-phase zone (i.e., \( v_2 \big|_{y_2=0} = 0 \) physically).

For analytical convenience, let the suction velocity produced by the capillary force be expressed in the following dimensionless form:

$$V_2^* = \frac{v_2 \big|_{y_2=0}}{(\rho g K / \mu_e)} = 0.2503 \sqrt{\cos^2 \theta + 3.774 B_0 \sqrt{Da} \cos \theta - 0.7497 \cos \theta},$$  \hspace{1cm} (16b)
where \((\rho gK/\mu_e)\) is the characteristic velocity (i.e., the Darcy velocity, \(u_d = (\rho gK/\mu_e)\)).

Equation 16(a) can then be rewritten as follows:

\[
\nu_2|_{y=0} = \frac{\rho gK}{\mu_e} \cdot V_2^*
\]

or \(\nu_2|_{y=0} = u_d \cdot V_2^*\).

Substituting (4) and (17) into (5) yields

\[
\frac{\delta}{dx} (\delta \sin \theta) + \frac{h_f g}{h_f g + (1/2)C p \Delta T} \delta \cdot V_2^* = \frac{\Delta T \mu_e k_e}{\rho^2 g K (h_f g + (1/2)C p \Delta T)}
\]

Fortunately, for the particular case where the capillary force is neglected (i.e., \(V_2^* = 0\)), the analytical solution of the dimensionless local liquid film thickness can be derived using the separation of variables method as follows:

\[
\delta^*|_{V_2^*=0} = \left(\left(\frac{2Ja}{Ra Da}\right) \left(\frac{1}{1 + \cos \theta}\right)^{1/2}\right).
\]

In order to solve \(\delta^*\) from (20) (i.e., for the case where the capillary force cannot be neglected), the following effective capillary suction function, \(F\), is introduced to represent the effect of the capillary force on the thickness of the condensate layer:

\[
\frac{\delta^*}{\delta^*|_{V_2^*=0}} = 1 - F,
\]

or

\[
\delta^* = (1 - F) \left(\left(\frac{2Ja}{Ra Da}\right) \left(\frac{1}{1 + \cos \theta}\right)^{1/2}\right).
\]

Substituting (23) into (20) yields

\[
- (1 - F) \left\{ \left(\frac{2Ja}{Ra Da}\right) \left(\frac{1}{1 + \cos \theta}\right) \right\} \sin \theta \frac{dF}{d\theta} + \left\{ -2 \frac{Ja}{Ra} - \left(1 - \frac{1}{2}Ja\right) \left(\frac{2Ja}{Ra Da} \left(\frac{1}{1 + \cos \theta}\right)^{1/2} \cdot V_2^*\right) \right\} F
\]

\[
+ \frac{Ja}{Ra Da} F^2 + \left(1 - \frac{1}{2}Ja\right)V_2^* \left(\frac{2Ja}{Ra Da} \left(\frac{1}{1 + \cos \theta}\right)^{1/2}\right) = 0.
\]

Equation (24) is a first-order differential equation of \(F\) with respect to \(\theta\). By setting \(\theta = 0\), the following polynomial equation with respect to the initial boundary condition, \(F|_{\theta=0}\) (i.e., \(F(0)\)), can be derived:

\[
AF_0^2 + BF|_{\theta=0} + C = 0,
\]

where

\[
A = \frac{Ja}{Ra Da},
\]

\[
B = \left\{ -2 \frac{Ja}{Ra} - \left(1 - \frac{1}{2}Ja\right) \left(\frac{Ja}{Ra Da}\right)^{1/2} \cdot V_2^* \right\},
\]

\[
C = \left(1 - \frac{1}{2}Ja\right)V_2^* \left(\frac{Ja}{Ra Da}\right)^{1/2}.
\]
Substituting the derived value of $F(0)$ into (17) and using the forward difference shooting method [29], the variation of $F$ in the $\theta$ direction can be calculated.

Let the local Nusselt number be defined as

$$ Nu_\theta = \frac{h_\theta D}{k_e}, $$

(27)

where

$$ h_\theta = \frac{k_e}{\delta}. $$

(28)

Substituting $h_\theta$ into (27), the local Nusselt number can be rewritten as

$$ Nu_\theta = \frac{D}{\delta} = \frac{2}{(1 - F)} \left(\frac{2RaDa}{Ja} \right)^{1/2} (1 + \cos \theta)^{1/2}. $$

(29)

The mean Nusselt number can then be obtained as

$$ \overline{Nu} = \frac{1}{\pi} \int_0^\pi \left(\frac{2RaDa}{Ja} \right)^{1/2} \frac{(1 + \cos \theta)^{1/2}}{(1 - F)} d\theta. $$

(30)

By substituting $F = 0$ into (29) and (30), respectively, the local Nusselt number and mean Nusselt number for the special case of $V_2^* = 0$ can be derived as

$$ Nu_{\theta}|_{V_2^*=0} = \left(\frac{2RaDa}{Ja} \right)^{1/2} (1 + \cos \theta)^{1/2}, $$

(31)

$$ \overline{Nu}|_{V_2^*=0} = \left(\frac{2RaDa}{Ja} \right)^{1/2} \int_0^\pi (1 + \cos \theta)^{1/2} d\theta. $$

### 3. Results and Discussions

The physical parameters used in (20) (i.e., $R, Ja, Pr_e, Ra, Da,$ and $Bo_c$) must all be assigned reasonable values for simulating practical engineering problems. The following analyses take water-vapor as the working liquid and use the dimensional and dimensionless parameter values cited in Udell [7,8] and Bridge et al. [30], as summarized in Table 1.

Figures 2 and 3 show the distributions of the dimensionless film thickness and local Nusselt number, respectively, along the surface of the tube as a function of the Jacob number, $Ja$. (note that the remaining parameters are assigned the characteristic values shown in Table 1, i.e., $Da = 6.4 \times 10^{-11}, Ra = 2 \times 10^3$, and $Bo_c = 6.1 \times 10^3$). In general, the results show that as $Ja$ increases, the dimensionless liquid film thickness increases, while the local Nusselt number decreases. Furthermore, the dimensionless film thickness has a minimum value at the top of the tube $(\theta = 0^\circ)$ and increases with increasing $\theta$. This result is intuitive since the analyses consider the case of falling film condensation, and thus the effects of gravity minimize the film thickness on the upper surface of the tube but cause the film thickness to increase toward an infinite value as the liquid flows over the tube and collects at the lower surface. The minimum film thickness on the upper surface of the tube results in the maximum Nusselt number. However, as the film thickness increases, the heat transfer performance reduces due to the lower temperature gradient. Consequently, the minimum Nusselt number occurs at the lower surface of the tube.

Figure 4 shows the variation of the effective capillary suction function $F$ with $\theta$ as a function of the Jacob number, $Ja$. As shown, $F$ has a positive value for all values of $Ja$ and $\theta$ but decreases as $Ja$ and $\theta$ increase. Figure 5 shows the distribution of the dimensionless suction velocity, $V_2^*$, along the surface of the tube as a function of $Bo_c$. It is seen that $V_2^*$ increases significantly with increasing $Bo_c$, particularly at smaller values of $\theta$. This finding is reasonable since a higher value of $Bo_c$ implies a stronger capillary force, which implies in turn a greater dimensionless suction velocity. It is additionally seen that the dimensionless suction velocity has a maximum value at the top of the tube and decreases with increasing $\theta$. This finding is again intuitive since the current analyses consider the case of falling film condensation, and hence the suction effect is the greatest at the upper surface of the tube and causes a spreading of the liquid in the horizontal direction. However, as the sucked liquid gradually falls in the downward direction under the effects of gravity, the capillary ability of the interface reduces and hence the suction velocity also reduces.

Figure 6 shows the variation of the mean Nusselt number, $\overline{Nu}$, with the capillary parameter, $Bo_c$, as a function of $Ja, Ra,$ and $Da$. As shown, $\overline{Nu}$ increases with increasing $Bo_c$ once $Bo_c$ increases beyond a certain critical value (where this value depends on the values assigned to $Ja, Ra,$ and $Da$). The turning points in the figure indicate at which the capillary force effect should be taken into consideration. To the left of the turning point, the capillary force has only a small effect and can be neglected. In other words, the condensate flow is dominated by the gravitational force, and hence no liquid condensate is sucked into the two-phase zone. As a result, the value of $\overline{Nu}$ is close to that calculated for $\overline{Nu}$ using $V_2^* = 0$ in (31). However, to the right of the turning point, the value of $\overline{Nu}$ increases significantly with increasing $Bo_c$. This result is to be expected since a higher value of $Bo_c$ implies a stronger capillary force, and hence a greater amount of liquid is sucked into the two-phase zone.
Table 1: Physical parameters used in present analyses.

| Symbol | Interpretation                                                                 | Typical value                   |
|--------|-------------------------------------------------------------------------------|---------------------------------|
| $K$    | Permeability                                                                  | $6.4 \times 10^{-13} \text{m}^2$ |
| $\rho$ | Liquid density                                                                | $957.9 \text{kg/m}^3$           |
| $R$    | Radius of tube                                                                | $0.1 \text{m}$                  |
| $C_p$  | Specific heat at constant pressure                                           | $4217 \text{J/kg}^\circ \text{C}$ |
| $\mu_e$| Liquid viscosity                                                              | $2.79 \times 10^{-4} \text{kg/m-s}$ |
| $\sigma$| Surface tension                                                               | $5.94 \times 10^{-2} \text{N/m}$  |
| $h_{fg}$| Heat of vaporization                                                          | $2257 \text{kJ/kg}$             |
| $\Delta T$ | Saturation temperature minus wall temperature $(T_{sat}-T_w)$                   | $5^\circ \text{C}$, $10^\circ \text{C}$, $25^\circ \text{C}$ |
| $\epsilon$| Porosity                                                                    | $0.38$                          |
| $Ja$   | $(C_p\Delta T/h_{fg} + (1/2)C_p\Delta T)$                                    | $0.01$, $0.02$, $0.05$          |
| $Pr_e$ | $(\mu_e C_p/k_e)$                                                             | $1.76$                          |
| $Da$   | $(K/R^2)$                                                                     | $6.4 \times 10^{-11}$           |
| $Ra$   | $Ra = (\rho^2 g Pr_e R^4 / \mu_e^2)$                                         | $2 \times 10^{11}$              |
| $Bo_e$ | $(\sigma \sqrt{\epsilon} / \rho g K)$                                       | $6.1 \times 10^5$               |

Figure 2: Variation of dimensionless film thickness $\delta^*$ with radial position $\theta$ as function of $Ja$.  
Figure 3: Variation of local Nusselt number $Nu_\theta$ with radial position $\theta$ as function of $Ja$.  

Ra = $2 \times 10^{11}$, Da = $6.4 \times 10^{-11}$, Bo_e = $6.1 \times 10^5$
Consequently, the film thickness reduces and the heat transfer rate increases due to the steeper temperature gradient.

4. Conclusion

This study has analyzed the problem of laminar film condensation on a semicircular tube embedded in porous medium and subjected to a capillary force. An effective capillary suction function, \( F \), has been introduced to model the effect of the capillary force on the thickness of the condensation film, thereby allowing both the local condensate film thickness and the local Nusselt number to be derived using a simple numerical method. The main conclusions of this study can be summarized as follows:

1. If the capillary force effect is simply ignored, the local liquid film thickness can be expressed analytically as 
   \[ \delta |_{V^2=0} = \left(\frac{2Ja}{Ra Da} \right) \left(1 + \cos \theta \right)^{1/2}, \]
   while the corresponding mean Nusselt number can be derived as 
   \[ Nu |_{V^2=0} = \left(\frac{2Ra Da}{Ja} \right)^{1/2} \int_0^\theta (1 + \cos \theta)^{1/2} d\theta. \]
2. When the capillary effect is considered, but the capillary force is much less than the gravitational
force, the capillary effect can be ignored. However, if the capillary force is greater than the gravitational force, the liquid condensate is sucked into the two-phase zone. Consequently, the condensate film thickness reduces and the Nusselt number, $Nu$, increases.

(3) A dimensionless parameter, $Boc$, has been introduced to characterize the liquid flow induced by the capillary force. The results have shown that when $Boc$ has a higher value (i.e., the capillary force is greater than the gravitational force), the liquid film thickness reduces, and the heat transfer rate is enhanced as a result of the higher temperature gradient.

Abbreviations

$Boc$: Ratio of surface tension and gravity forces defined in (15)
$Cp$: Specific heat at constant pressure
$Da$: Darcy number defined in (15)
$F$: Effective capillary suction function defined in (22)
$f$: Saturation parameter defined in (10c)
$g$: Acceleration of gravity
$h$: Heat transfer coefficient
$hfg$: Heat of vaporization
$Ja$: Jakob number defined in (19)
$k$: Thermal conductivity
$K$: Permeability of the porous medium
$Ke$: Saturation parameter defined in (10a)
$m$: Condensate mass flux
$Nu$: Nusselt number defined in (27)
$Pc$: Capillary pressure
$Pr$: Capillary number
$Re$: Reynolds number
$s$: Dimensionless saturation
$T$: Temperature
$\Delta T$: Saturation temperature minus wall temperature
$u, v$: Velocity component in $x$ and $y$ direction
$u_d$: Darcy velocity defined in (17)
$V$: Dimensionless capillary suction velocity defined in (16b).

Greek symbols

$\delta$: Condensate film thickness
$\delta_0$: Condensate film thickness at top of tube
$\theta$: Angle measured from top of tube
$\mu$: Liquid viscosity
$\rho$: Liquid density
$\alpha$: Thermal diffusivity
$\sigma$: Surface tension
$\varepsilon$: Porosity.

Subscripts

2: Properties in the two-phase zone
$o$: Quantity at top of tube
c: Capillary
sat: Saturation property
w: Quantity at wall
e: Effective property.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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