Development of a general correlation for free convection vapor condensation over a horizontal tube in the presence of a noncondensable gas

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ABSTRACT

In applications involving external condensation in the presence of noncondensable gases (NCG), there is a growing interest in horizontal tubes because they minimize condensate resistance, and hence reduce the adverse effects of NCGs. In the literature, experimental heat transfer rates are summarized in empirical correlations that fit the underlying data, but do not cover the spectrum of possible steam-NCG mixtures and thermal conditions. In this work, we aim at developing a general correlation that is applicable to a wide range of physical and geometrical conditions.

We first present a CFD model for laminar free convection regimes, taking into account the resistance of the gas phase and the liquid condensate. The model is validated against recent condensation data on steam-N\textsubscript{2} and steam-CO\textsubscript{2} mixtures. The predictions agree well with the data, being mostly within the experimental uncertainty.

Subsequently, close to 200 parametric CFD simulations are conducted for steam-air mixtures across a wide range of steam mass fraction (0.05–0.99), wall temperature subcooling (5–50 K), pressure (1–5 bar) and tube diameter (0.01–0.04 m). The resulting data are cast in a compact correlation that is compatible with the heat and mass transfer analogy. The proposed correlation is in good agreement with empirical correlations in the literature.

1. Introduction

Condensation in the presence of a noncondensable gas (NCG) has a wide range of practical applications, ranging from nuclear power to chemical industries to the environment. It has long been established that NCGs cause a drastic decrease in condensation rates even in minute concentrations. For example, in free convection along a vertical isothermal plate, Al Diwani and Rose [1] reported a 50% reduction in the condensation heat flux with just 1.5% of air by mass in the mixture.

In environmental applications, the CO\textsubscript{2} exiting thermal plants needs to be first separated from the steam before dispersal. The purification process involves the use of shell-and-tube condensers [2]. In desalination, many studies have proposed special condenser designs to collect fresh water from mixtures consisting of very high air content [3].

The mechanisms leading to heat transfer deterioration due to NCGs are also well established. In the seminal work of Sparrow et al. [4], it was shown that vapor which is drawn to the cold wall entrains with it the NCG. By definition, the latter is unable to penetrate the gas-liquid interface, and hence accumulates there. Given that the total pressure is constant in the gas boundary layer, the high concentration of the NCG at the interface mechanically results in a lower vapor concentration there, with a lower saturation temperature and thus lower condensation rates.

Ideally, one would like to extract the NCG from the mixture to prevent such loss of condensation performance. However, in many applications, the presence of the NCG is unavoidable and condensers need to perform in environments with medium to high NCG content. For instance, advanced nuclear power plants rely on so-called Passive Containment Cooling Systems (PCCS) to condense steam in the containment space where large amounts of NCGs are present [5]. These PCCS have typically been designed as vertical tube-and-shell heat exchangers with vapor condensing on the shell side. Recently, however, horizontal or nearly horizontal tube-and-shell PCCS have been proposed [6]. The rationale for this is to minimize condensate resistance to heat transfer, since the liquid film thickness is obviously much smaller on a horizontal tube compared to a vertical one.

Some experimental and analytical works have aimed at quantifying...
2. A CFD model for condensation over a horizontal tube in laminar flows

2.1. Problem description

An infinitely long horizontal tube is assumed to be immersed in a quiescent medium comprised of a vapor and a noncondensable gas. The tube wall is held at constant temperature $T_w$, which is lower than the bulk vapor saturation temperature $T_{sat}$, resulting in partial vapor condensation over the surface of the tube. We assume moreover that condensation is of the filmwise type, and that the gravity force acts in the vertically downward direction.

The problem set up is shown schematically in Fig. 1. A 2D symmetric
treatment of the problem is adequate. The gas mixture is introduced in
the channel from the top at very small velocity (e.g. 0.001 m/s), so that
free convection around the cold tube is dominant. Zero gauge pressure is
prescribed along the bottom section. Two co-existing boundary layers
develop for the film condensate and the gas mixture. The interaction
between the boundary layers results in the emergence of an interface
with an unknown temperature $T_i$. The latter is computed by requiring
continuity of the heat flux at the interface, in addition to some simpli-
fying assumptions that will be described later on. The NCG concentra-
tion increases close to the gas-film interface, leading to a corresponding
reduction in the vapor partial pressure and hence temperature, and a
subsequent deterioration in heat transfer rates compared to pure vapors.
(liquid film thickness is exaggerated) and Right: boundary
conditions.

2.2. CFD model of the gas boundary layer

In this section, we succinctly describe the CFD model used to predict
condensation over a horizontal cylinder in laminar flows. The model
follows closely our previous work [12], but includes liquid condensate
modeling as well as a more suitable free shear boundary condition at the
liquid-gas interface. We define the species mass fractions for the vapor
and NCG as follows:

$$W_v \equiv \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_i + \rho_{nc}}$$
(1)

$$W_{nc} \equiv \frac{\rho_{nc}}{\rho} = \frac{\rho_{nc}}{\rho_i + \rho_{nc}}$$
(2)

The mixture is assumed an ideal gas, and as a result, we can write:

$$\rho = \frac{PM}{RT}$$
(3)

Above $M$ represents the effective mixture molecular weight. Using
mixture rules, the latter can be computed from the individual molecular
weights $M_i$ and species mass fractions:

$$M = \frac{M_i M_{nc}}{M_v + (M_{nc} - M_i) W_v}$$
(4)

Condensation will occur because of interactions between vapor
diffusion in the gas mixture and heat conduction in the liquid film.
Vapor and NCG mass fluxes at the interface are composed of convective
and diffusive contributions. Following the approach of Bird et al. [13],
one has:

$$\dot{m}_{vi} = \rho W_v v_i \rho D \frac{\partial W_i}{\partial n}$$
(5)

$$\dot{m}_{ni} = \rho W_{nc} v_i \rho D \frac{\partial W_i}{\partial n}$$
(6)

Above $v_i$ is the wall-normal mixture velocity, $D$ is the mass diffusion
coefficient and $n$ refers to the normal to the liquid film. Requiring the
interface to be impermeable to the NCG, one can show [12] that the
vapor mass flux is reduced to:

$$\langle \rho v_i \rangle_i = \dot{m}^\nu_i = \frac{1}{(W_i - 1)} \left( \rho D \frac{\partial W_i}{\partial n} \right)_i$$
(7)

Knowing the interface temperature and total pressure, one can
deduce the vapor mass fraction at the interface and hence the conden-
sation mass flux. In the ANSYS Fluent [14] code, the latter is translated
into a volumetric mass sink term for the mixture continuity equation in
the cell adjacent to the wall:

$$\dot{m}_i^\nu = \dot{m}^\nu_i \frac{A_{cell \ wall}}{V_{cell}}$$
(8)

Above $A_{cell \ wall}$ represents the wall area and $V_{cell}$ the cell volume
of the wall-adjacent cell. Combining Eqs. (7) and (8), the volumetric mass
sink term reduces to the following expression:

$$\dot{m}_i^\nu = \frac{1}{(W_i - 1)} \left( \rho D \frac{\partial W_i}{\partial n} \right) \frac{A_{cell \ wall}}{V_{cell}}$$
(9)

Correspondingly, a vapor species sink term is also prescribed as
follows:

$$\dot{m}_{i,vi}^\nu = \dot{m}_i^\nu W_{vi} = \frac{W_{vi}}{(W_i - 1)} \left( \rho D \frac{\partial W_i}{\partial n} \right) \frac{A_{cell \ wall}}{V_{cell}}$$
(10)

Vapor condensation involves as well volumetric sink terms in the
momentum and energy equations:

\[ S_{\text{energy}} = \frac{m_i n^w h_{\text{fg}}}{W_{i,j}} \]  
\[ S_{\text{momentum}} = \frac{m_{i,j} v_j}{W_{i,j}} \]  

In the equation above, the mixture velocity in the jth direction \( v_j \) is defined in the center of the wall-adjacent cell. If the no-slip velocity condition at the interface is prescribed, the equations above will result in lower heat transfer rates than in experiments. Indeed, condensation involves suction effects that enhance heat and mass transfer, but no-slip conditions do not allow for a proper representation of such suction effects.

CFD approaches to take into account suction effects fall in three categories. In the first one [12], an enhanced effective diffusion coefficient is specified in the cells near the wall such that greater heat transfer rates are obtained, replicating correction factors such as the one proposed by Bird et al. [13]. While this approach has produced good predictions compared to experiments, it is a case-dependent, ad-hoc procedure, and hence a calibration is required for each specific set of geometric and physical conditions.

The second approach uses the Volume of Fluid (VOF) framework whereby the gas-film interface is dynamically tracked. While more rigorous, this method requires very large numbers of cells to accurately resolve the interface, and hence is prohibitive if one desires to perform wide-ranging parametric studies as planned in the current investigation.

The third approach due to Yoon et al. [16] involves replacing the velocity no-slip assumption at the interface by a free surface boundary condition, in which case a zero shear stress is specified. Doing so automatically results in non-zero wall mixture velocity components at the interface, hence naturally accounting for suction without further corrections. Yoon et al. [16] showed that the zero shear assumption produces close agreement with the data in slow momentum flows. The free surface boundary condition will be adopted throughout this study since weak convective flows are prevalent.

2.3. Heat transfer resistance in the condensate film

CFD investigations of condensation in the presence of NCG usually neglect film resistance, which is a reasonable assumption for high NCG fractions [12,17–19]. In this study, the goal is to address condensation up to very high vapor fractions, in which case neglecting film resistance is not justified and would result in significant over-estimation of heat transfer rates.

To account for film resistance, we adopt an iterative procedure as follows: we first solve the heat transfer problem disregarding film resistance. This yields an approximate estimate of the total heat flux transferred to the condensate film. The heat transfer rate from the gas side is comprised of latent and sensible parts, although the latter is quite small. Thus the total heat flux is therefore:

\[ q_i = q_i^\text{cond} + q_i^\text{conv} = n_i^w h_{\text{fg}} + k_i \left( \frac{dT}{dn} \right)_i = h_{\text{cond}}(T_w - T_i) + h_{\text{conv}}(T_w - T_i) \]  

In a second step, we use the analytical solution for the filmwise condensation flux along the circumference of the tube, and prescribe continuity of the heat flux across the gas-liquid interface. Thus:

\[ q_i^\text{cond} + q_i^\text{conv} = n_i^w h_{\text{fg}} + k_i \left( \frac{dT}{dn} \right)_i \]  

We need now to provide an expression for \( q_i^\text{cond} \), the local heat flux from the liquid side of the interface. The classical Nusselt analysis is adopted for the liquid flow around the cylinder, with no-slip velocity at the solid wall, and zero shear at the liquid-film interface. The film Nusselt number around a tube has been derived by many authors, e.g. [20] and can be written as a function of the angle from the vertical \( \theta \):

\[ N_{\text{Nu}} \equiv \frac{h_{\text{film}}d}{k_l} = \left( \frac{2\rho_l(\rho_l - \rho_v)g h_{\text{fg}} l}{\mu_l k_l(T_s - T_l)} \right)^{1/4} F(\theta) \]  

In the above, the function \( F(\theta) \) is defined as:

\[ F(\theta) \equiv (\sin \theta)^{1/3} \left( \frac{4}{\theta} \int_0^\theta (\sin \theta)^{1/3} d\theta \right)^{-1/4} \]  

The function \( F(\theta) \) does not have a closed analytical expression. It has therefore been computed numerically between 0 (topmost point of tube) and \( \pi \) (bottommost point of tube). Results were curve-fitted as follows:

![Fig. 2. The function F(\theta).](image-url)
average Nusselt number over the surface of the tube is thus given by:

$$\overline{Nu} \equiv \frac{h_{\text{film}}d}{k_l} = \frac{1}{\pi} \int_0^\pi Nu d\theta = \frac{1}{\pi} \left( \frac{2\rho_i (\rho_l - \rho_i) g h_{\text{film}}}{\mu_l k_l (T_i - T_\infty)} \right)^{1/4} \int_0^\pi F(\theta) d\theta$$  \hspace{1cm} (18)

Performing the numerical integration, one obtains for the average Nusselt number:

$$\overline{Nu} \equiv \frac{h_{\text{film}}d}{k_l} = 0.728 \left( \frac{\rho_l (\rho_l - \rho_i) g h_{\text{film}}}{\mu_l k_l (T_i - T_\infty)} \right)^{1/4}$$ \hspace{1cm} (19)

The expression above is identical to the one provided in the literature [20]. The local heat flux and film heat transfer coefficient (HTC) at any angle $\theta$ are given by the following expressions, respectively:

$$q_i^{\text{film}} = h_{\text{film}}(T_i - T_\infty) = \left( \frac{2\rho_i (\rho_l - \rho_i) k_l^2 g h_{\text{film}}}{\mu_l d} \right)^{1/4} (T_i - T_\infty)^{1/4} F(\theta)$$  \hspace{1cm} (20)

$$h_{\text{film}} = \left( \frac{2\rho_i (\rho_l - \rho_i) k_l^2 g h_{\text{film}}}{\mu_l (T_i - T_\infty)d} \right)^{1/4} F(\theta)$$ \hspace{1cm} (21)

Combining Eqs. (14) and (20), one can compute an improved value for the interface temperature profile $T_i(\theta)$ and repeat the CFD flow field computation with this new thermal boundary condition. The iteration proceeds until the interface temperature profile converges. The convergence rate is fast for low to medium vapor fractions, as the interface temperature is close to the wall temperature. The process slows down considerably as the pure vapor conditions are approached, requiring e.g. order of 100 profile iterations for $W_e = 0.99$ and above. Beyond $W_e = 0.001$, the problem became numerically very stiff, and it was not possible to get converged solutions. With the converged solution, one finally obtains the local heat flux and heat transfer coefficient:

$$q_i \equiv h(T_w - T_\infty) = \frac{(T_w - T_\infty)}{h_{\text{cond}} + h_{\text{film}}}$$ \hspace{1cm} (22)

It is assumed that the film remains in the laminar regime, i.e. has a Reynolds number less than 30. This assumption is confirmed a-posteriori after convergence of each calculation. In fact, because of the cylindrical geometry, the film thickness is very thin, and the condensate Reynolds number is always much smaller than the threshold value.

\[ \text{Fig. 3. The numerical grid with a close-up near the tube wall (right).} \]

2.4. Numerical settings, boundary conditions and grid convergence studies

In the simulations conducted in this work, CFD best practice guidelines [21] are followed to minimize numerical errors and the commercial code ANSYS Fluent is used [14]. The numerical grid features in particular high spatial resolution near the tube wall. The wall nearest cell is constructed such that, under no-slip conditions, $y +$ is of the order of 0.3 or less. In addition, 50 to 100 cells are present in the gas boundary layer region. The quadratic meshes consist respectively of a total of 64 K, 125 K and 190 K cells. Mild inflation ratios between 1.03 and 1.05 are used for the boundary layer region where cells are structured, as shown in Fig. 3. Second order spatial discretization accuracy is adopted for all conservation equations.

Three grid resolutions are built for the most stringent physical conditions, that is, for the largest condensation rates which correspond to a vapor fraction of 0.99 and wall subcooling of 50 K. We assume the isothermal tube has an OD of 0.01 m, and impose an inlet velocity of 0.001 m/s from the top to initiate the free convective flow around the cylinder. In addition, zero gauge pressure is set for the bottom outlet of
the channel.

The plot of the condensation heat flux for the different grid resolutions is shown in Fig. 4. The grids provide almost identical results, with differences in the mean heat flux value of less than 0.2% between the medium and fine meshes. The medium grid is subsequently used throughout this study, although it is over-resolved for cases with low to medium condensation rates.

2.5. Inspection of a typical gas flow field around the cylinder

In Fig. 5 we show a typical gas velocity flow field around the cylinder for medium condensation rates, i.e. for a steam mass fraction of 0.70 and wall subcooling of 50 K at 3 bars total pressure. Due to wall condensation, the gas mixture is sucked towards the cold tube wall. The imposed free shear at the gas-liquid interface results in non-zero velocity components at the wall, in at least qualitative agreement with the expected physical behavior. A no-slip boundary condition at the wall will show exactly zero velocity components at the gas-liquid interface, which is physically incorrect.

We show in Fig. 6 the corresponding heat flux and interface temperature profiles. As expected, the wall heat flux profile is similar in shape to the pure vapor cases (Fig. 2), although somewhat flatter in most of the domain up to $\theta = 3$. This is due to the less rapid relative thickening of the liquid film in the case where a NCG is present. Keeping in mind that $T_w = 399$ K and $T_w = 349$ K, the interface temperature profile is close to the wall temperature. In the high flat heat flux region, i.e. for $\theta$ up to 3, the interface temperature increases mildly to compensate for the falling $F(\theta)$. Afterwards, the heat flux decreases sharply and approaches zero, and as a result, the interface temperature tends towards the wall value.

3. Model verification and validation

3.1. Model verification for noncondensing free convection

We begin by first verifying the CFD model for pure laminar free convection of air around an isothermal cylinder at atmospheric pressure. The bulk and wall temperatures are set respectively to 360 K and 340 K. Based on a comprehensive study of data in the literature, Morgan et al. [22] have given the following correlation for the convective Nusselt
The range of $Ra$ numbers covers the region of interest to most engineering applications. The CFD predictions of the Nusselt number for various tube diameters are compared to the correlation above in Fig. 7. As can be seen, excellent agreement is reached, with a discrepancy less than 5%.

### 3.2. Model validation against the experiments by Lu et al. [10,11]

Recently, Lu et al. [11] have conducted a series of atmospheric tests on condensation over an isothermal cylinder with steam-$N_2$ and steam-$CO_2$ mixtures. The condenser consisted of a 0.48 m long horizontal copper tube with an outer diameter of 0.024 m. Conditions approaching free convection were prevalent inside the small mixing chamber where the cylinder tube stood.

#### 3.2.1. Validation against the Lu et al. tests [11] at atmospheric pressure

3.2.1.1. Lu et al. experiments: steam-$N_2$ mixtures. We performed simulations of these tests using the CFD model described earlier. For steam-$N_2$ mixtures, the full range of steam mass fractions (0.39–0.92) was simulated. Comparison of the CFD results with the data is shown in Fig. 8.

Globally, the predictions are good agreement with the data and within the reported experimental uncertainty of 14%, except for the lowest heat transfer rates corresponding to the smallest subcooling level of 5 K, where the under-prediction reaches up to of 35%. This may be due to greater experimental uncertainties at these lower temperature differences. In fact, as shown in the similar data of Ge [9], heat transfer rate scatter is in a band of 30–50% for low subcoolings of 5 K. Another cause for the under-prediction may be the small forcing flows inherent to the modest volume in which the tube was placed. Such conditions invariably deviate from the ideal free convection assumed in the simulations. As will be shown later on in the sensitivity study section, forcing velocity of just a few cm/s result in appreciable enhancements in the heat transfer rates compared to ideal free convection.

3.2.1.2. Lu et al. experiments: steam-$CO_2$ mixtures. Similar simulations were conducted for steam-$CO_2$ mixtures and comparison of the CFD results with the data is displayed in Fig. 9. Here as well, the CFD predictions are mostly within the experimental uncertainty, except for the very high NCG fractions, where the heat transfer is small and weak forcing flows influence the heat removal rate.
3.2.2. Validation against the Lu et al. tests [10] at sub-atmospheric pressure

Earlier, Lu et al. [10] have conducted a series of sub-atmospheric tests (0.05–1 bar) with steam-CO$_2$ mixtures using the same set-up. The experiments showed that, under fixed NCG fraction and wall subcooling, the heat transfer diminishes significantly as the total pressure decreases. We provide in Fig. 10 a sample validation of our CFD model for a representative sub-atmospheric pressure of 0.3 bar. As can be seen, the agreement of the model with the data is excellent both qualitatively and quantitatively.

4. Sensitivity of the predictions to interface boundary condition and inlet velocity

4.1. Sensitivity to wall boundary condition

To gain more insight into the CFD model developed, we present in the following section a sensitivity study on the imposed momentum boundary condition at the liquid-gas interface. There are four boundary condition possibilities to model the gas-liquid interface:

1) Free shear + film resistance (FS-FILM)
2) Free shear + no film resistance (FS-NOFILM)
3) No-slip + film resistance (NS-FILM)
4) No-Slip + no film resistance (NS-NOFILM)

Going back to the validation data of the Lu et al. steam-N$_2$ database [11], we repeat the computations for boundary condition items (2) through (4) (The full model (1) is already computed). Results are summarized by Fig. 11. Overall, the model adopted (1) matches the data best, as it includes the most complete set of sub-models, i.e. a more realistic heat transfer enhancement due to suction as well as the additional resistance due to the presence of the condensate film. We note that suction and film resistance work in opposite directions as regards their influence on the total heat transfer rate, and thus one should caution against the possibility of incomplete models achieving a good agreement with the data because of canceling errors.

Enforcing the free shear as opposed to the no-slip boundary condition at the interface improves substantially the predictions across the range of steam fraction considered (0.3 to 0.92). Not including film resistance leads to appreciable errors (as high as 20%) for high steam fractions, but the error is small for low to medium steam fraction below 0.5. One can therefore conclude that the most accurate predictions are obtained with the free slip boundary condition as well as the inclusion of the condensate film resistance.

4.2. Sensitivity to slow forced flows

It was conjectured earlier that the under-prediction of the Lu et al. [11] data at low heat transfer rates might be traced to slight forcing flows present in the experiments due to the small volumes of the
chamber in which the tube was installed. In contrast, simulations were conducted with an inlet velocity of 0.001 m/s, which is representative of ideal free convection conditions. In this section, we perform some simulations with weak inlet velocities and compare the outcome with earlier results at virtually no forcing flow (HTC denoted as $h_b$).

As seen in Fig. 12, forcing velocities of 0.05 m/s and 0.1 m/s engender enhancements in heat transfer of up to 25% and 50% percent, respectively. For low heat transfer rates, i.e. low steam fractions ($W_s = 0.40$), the lower the subcooling, the higher the sensitivity of heat transfer to forcing velocities. At high steam fractions ($W_s = 0.85$), the heat transfer enhancement is independent of subcooling for the ranges considered.

5. Development of a general correlation from the validated CFD model

5.1. Proposed general correlation

To make the best use of the validated model described earlier, it is necessary to summarize its predictions in a more compact way. We therefore performed a wide range of parametric CFD simulations for steam-air mixtures at thermal and geometric conditions outlined in Table 1. These conditions span the range of values expected in most engineering applications. In total, 189 simulations were performed to cover almost the entire range of steam fractions (0.05 to 0.99). Although the model computes the local HTC around the circumference of the tube, the focus is on the average HTC value.

For practical purposes, we seek to cast the CFD results of the parametric study in a compact correlation with the aid of the heat and mass transfer analogy (HMTA). We start with the correlation of Morgan et al. [22] for laminar free convection heat transfer for medium to high Ra numbers:

$$Nu = 0.48 \cdot Ra^{1/4} = 0.48 \cdot (Gr \cdot Pr)^{1/4}$$

Using the HMTA, the mass transfer rate over a horizontal cylinder can be deduced from the following correlation of the Sherwood number $Sh_b$ under low mass transfer rates:

$$Sh_b = \frac{h_b \cdot L}{\rho \cdot D} = 0.48 \cdot (Gr \cdot Sc)^{1/4}$$

The condensation mass flux can thus be expressed as a function of the mass transfer coefficient $h_b$ as follows:

$$\dot{m}'' = h_b \cdot \frac{W_s - W_w}{1 - W_s} = \frac{\rho \cdot D \cdot Sh_b \cdot W_s - W_w}{d} \cdot \frac{1}{1 - W_s}$$

The Grashof number is appropriately formulated in terms of differences in densities rather than temperatures in order to allow for a wide variation in the physical parameters such as mass fraction, pressure and temperature:

$$Gr = \frac{\rho_r \cdot (\rho_s - \rho_w)}{\mu^2} \cdot d^4$$

The Schmidt number is expressed as:

$$Sc = \frac{\mu}{\rho D}$$

The properties are evaluated at the mean temperature between the wall and the bulk and prescribed according to the methodology detailed in the Appendix.

For high suction rates due to condensation, the mass transfer is enhanced, and the correction factor is applied based on the formulation of Bird et al. [13]:

$$\Theta = \frac{ln(1 + B)}{B}$$

In the above, the suction parameter $B$ is given by:

$$B \equiv \frac{W_s - W_w}{1 - W_w}$$

As a result, the corrected mass transfer rate can be deduced from the following relationship:

$$Sh = \Theta \cdot Sh_b = \Theta \cdot 0.48 \cdot (Gr \cdot Sc)^{1/4}$$

Hence, the condensation flux is:

$$\dot{m}'' = \Theta \cdot \frac{\rho \cdot D \cdot Sh_b \cdot W_s - W_w}{d} \cdot \frac{h_b (T_w - T_s)}{h_{bHMTA}}$$

After some algebra, the heat transfer coefficient predicted by the HMTA reduces to:

$$h_{HMTA} = \frac{0.48 \cdot \rho \cdot D^{1/4} \cdot (\rho_r - \rho_w)}{d^{1/4} \cdot \mu} \cdot \frac{h_b (T_w - T_s)}{(T_w - T_{\infty}) \cdot ln(1 - W_w)}$$

It is to be noted that the heat transfer coefficients computed in this section by the CFD model based on the test matrix (Table 1) vary

\[ \text{Table 1} \]

| Parameter                  | Value                      |
|----------------------------|----------------------------|
| Tube diameter, m           | 0.01, 0.02, 0.04           |
| Pressure, bars             | 1, 2, 5                    |
| Wall subcooling, K         | 5, 20, 50                  |
| Steam mass fraction, –     | 0.05, 0.2, 0.5, 0.7, 0.9, 0.95, 0.99 |

Fig. 12. Sensitivity of the predictions to the prescribed inlet velocity

Fig. 13. CFD results for low heat transfer region.
between 25 W/m²K to 16,600 W/m²K, i.e. over three orders of magnitude. A single functional fit of the data would result in large discrepancies at the extremes of the range. We elected instead to break down the data into a linear region up to a value of 800 W/m²K, and a non-linear region beyond that threshold.

In the low heat transfer region (Fig. 13), the CFD data collapse around a straight line with very little scatter (standard deviation 2.1%). In those conditions, the film resistance is negligible, and the HTC coefficient predicted by the CFD analysis is about 20% higher than that predicted by the HMTA expression (33).

Incidentally, this means that the Bird corrected factor for suction (29) needs to be augmented by 20% in order for the HMTA estimates of heat transfer to align with the CFD predictions. One should remember that the Bird correction given by Eq. (29) was developed based on the stagnant film theory whereby the convective velocity towards the interface is zero, and only diffusive transport is important. These conditions are somewhat different in condensing flows, and thus the Bird formula is only a first approximation of the underlying physics.

The form of the HMTA expression is valid if one does not consider the film resistance. As seen earlier, at higher condensation rates, the film resistance becomes increasingly important, and the total HTC is smaller than that implied by the HMTA. A reasonable assumption is that the correction for film resistance is a function of the driving potential \( W_\infty - W_w \). Thus, for the film correction, we seek a relationship for the CFD HTC of the form:

\[
h_{\text{CFD}} = f(h_{\text{HMTA}}, W_\infty - W_w) \tag{34}
\]

In the non-linear region where film resistance is important, it was found that the HTC data correlate reasonably well with the following grouping:

\[
h_{\text{CFD}} = f(h_{\text{HMTA}} (1 + 0.3(W_\infty - W_w))) \tag{35}
\]

The data are shown in Fig. 14. The polynomial of degree 2 fits the data quite well with a standard deviation of 7.6%. To summarize the findings, the HTC data computed with the CFD model can be cast in the following correlation:

\[
h = \begin{cases} 
1.197 h_{\text{HMTA}} & h \leq 800 \text{ W/m}^2\text{K} \\
-117 + 1.26 h_{\text{HMTA}} (1 + 0.3(W_\infty - W_w)) & 800 < h < 16600 \text{ W/m}^2\text{K} \\
-2.33 \times 10^{-1} (h_{\text{HMTA}} (1 + 0.3(W_\infty - W_w)))^2 & h \geq 16600 \text{ W/m}^2\text{K}
\end{cases}
\tag{36}
\]

This expression was deduced from simulations with steam fractions from 0.05 to 0.99, pressures between 1 bar and 5 bar, wall temperature subcooling between 5 K and 50 K and tube diameters between 0.01 and 0.04 m. The fact that the correction for film resistance in the high heat transfer region depends only on the driving potential \( W_\infty - W_w \) but not on any physical property of the NCG points to the suitability of the correlation for NCGs other than air. In addition, the use of the HMTA as the basis for the correlation indicates that the latter should be applicable to thermal conditions beyond the range in Table 1. Both of these
observations will be quantitatively examined in the next section.

5.2. Out of sample predictions of the correlation for steam-CO₂ mixtures

The correlation given by Eq. (36) was obtained from simulations of steam-air mixtures. Since its form is cast to comply with the HMTA with corrections for film resistance, it should provide reasonable estimates of heat transfer rates with other steam-NCG mixtures and thermal conditions. In the following section, we verify this expectation by comparing the correlation with condensation heat transfer data collected by Lu et al. [10] for steam-CO₂ mixtures at 0.3 bar and 0.7 bar, respectively.

Results are shown in Fig. 15. The correlation is in good agreement with the data, and the mean deviation is 11%. Predictions are slightly conservative for reasons invoked earlier. The proposed correlation can thus generally be used to estimate condensation heat transfer over horizontal tubes for conditions beyond the ranges of Table 1, as long as laminar free convection is the prevailing regime.

5.3. Comparison with other correlations in the literature

In this section, we compare the correlation developed in this work with three available correlations: the ones recently proposed by Lu et al. [11] and Ge et al. [9], as well as the older correlation of Henderson et al. [8].

5.3.1. Correlation by Lu et al. [11]

Lu et al. [11] have recently presented the following correlation for condensation on a horizontal cylinder with a NCG present:

$$ h = 0.64 D^{0.14} \left( \frac{\rho_v + \rho_l}{2} \right)^{0.5} \left( \frac{\mu_l}{\mu_l} \right)^{0.25} \frac{h_{fg}}{(T_w - T_{\infty})^{0.4} \ln \left( \frac{1 - W_s}{1 - W_w} \right)} $$

(37)

Our correlation (Eq. (36)) and that of Lu et al. [11] were applied for steam-N₂ mixtures and wall subcooling levels as shown in Fig. 16. It appears that our correlation matches reasonably well the data and is mostly within the experimental uncertainty. On the other hand, the correlation by Lu et al. significantly under-predicts the data. The main reason is that the authors define the mixture Grashof number in terms of temperature differences (hence the presence of the thermal expansion coefficient β) rather than density differences. This choice is clearly inadequate because of complex dependencies of the mixture density in the boundary layer (see Eq. (38)), and for this reason the Boussinesq approximation fails to represent the density differences which drive the free convection.

5.3.2. Correlation of Henderson et al. [8]

Henderson et al. [8] have conducted atmospheric pressure condensation tests of steam-air mixtures over an isothermal copper tube of length 1.22 m and OD 0.0286 m. The tests were performed in free convection laminar regime with medium to very high steam mass fractions (0.650–0.997). The authors correlated their data as follows:

$$ \frac{h}{h_{lu}} = \frac{1}{1 + 51(1 - X_s)} $$

(38)

Above $h_{lu}$ refers to the pure vapor condensation HTC given by expression (19), and $X_s$ is the bulk steam mole fraction. Unfortunately, the authors did not report the temperature subcooling levels they used, but state that the correlation they were comparing their results against (Othmer’s data [23]) covers the same range as their correlation. Othmer’s range of subcooling varied between 2.2 K and 33.3 K. We take here the mean of this range, which is 17.8 K. In any case, the ratio of the heat transfer coefficient normalized with the pure condensation value (Eq. (19)) is not very sensitive to the temperature subcooling, so there are grounds for a meaningful comparison between Henderson’s correlation and the one developed in this work (Eq. (36)). Results of the comparison are displayed in Fig. 17.

The two correlations provide similar predictions of heat transfer (within 25%), except at the very high steam fraction of 0.99. A look at the data collected by Henderson et al. shows large scatter at the highest steam fractions considered (0.99 and above), with values ranging from 0.58 to 0.67. Keeping this in mind, we conclude that the correlation developed in this work is fully consistent with the database of

Fig. 16. Predictions of our correlation (Eq. (36)) and that of Lu et al. [11] (Eq. (37)) compared to experimental data [11].

Fig. 17. Comparison of our correlation (Eq. (36)) with that of Henderson et al. (Eq. (38)) for atmospheric steam-air mixtures.

Fig. 18. Comparison of our correlation (Eq. (36)) and that of Ge et al. (Eq. (39)) for atmospheric steam-CO₂ mixtures.
5.3.3. Correlation of Ge et al. [9]

Ge et al. [9] have performed atmospheric condensation tests from steam-CO$_2$ mixtures on a horizontal tube condenser of OD 0.016 m for CO$_2$ mole fractions in the range of 0.112 to 0.897 (steam mass fractions in the range of 0.10–0.89). The mixture was directed vertically downward at a velocity of 0.21 m/s, and the wall temperature subcooling varied between 5 K and 35 K. The authors correlated their database as follows:

$$h = \frac{1}{h_n} - \frac{1}{h_n} + \exp\left(\frac{4.3(0.96 - X_b)^{0.58}}{1 + \exp\left(0.96 - X_b\right)}\right)$$  (39)

The velocity over the tube was deduced from the boiler heating power, which is not a very accurate method. Nonetheless, the conditions are not too far from free convection, in which case a comparison with our correlation is meaningful. We chose a subcooling of 20 K, which is in the mean of the range used in the tests. The comparison is summarized in the plot of Fig. 18. The latter shows that the two correlations provide similar predictions of heat transfer. As with the comparison in the preceding section, using pressures other than 1 bar results in larger discrepancies up to 100%, stressing the fact that the correlation of Ge et al. [9], like the one by Henderson et al. [8], can only be used at atmospheric pressure.

6. Conclusion

We present a CFD model to predict condensation rates of vapor over a horizontal tube in the presence of a NCG. The prevalent regime is laminar free convection. The model differs from traditional CFD analyses of condensation with NCGs in two important ways: firstly, the enhancement in mass transfer due to suction effects is naturally included by prescribing a zero shear boundary condition at the gas-liquid interface rather than a velocity no-slip. This eliminates the need for suction correction factors and removes empiricism. Secondly, the resistance due to the liquid condensate is included by introducing an iterative procedure to compute the gas-liquid interface temperature.

In a first series of simulations, the model is validated against recent condensation data of steam-N$_2$ and steam-CO$_2$ mixtures. It is shown that the predictions are in general good agreement with the data and mostly within the experimental uncertainty, except for high NCG concentrations where under-prediction is noted, probably because of small experimental forcing currents that cannot be avoided, departing somewhat from the ideal free convection assumed in the simulations.

Subsequently, close to 200 parametric CFD simulations are conducted for steam-air mixtures across a wide range of steam mass fraction (0.05–0.99), wall temperature subcooling (5-50 K), pressure (1–5 bar) and tube diameter (0.01–0.04 m). For the sake of practicality, results are cast in a compact correlation that is compatible with the heat and mass transfer analogy and includes a correction for liquid film resistance. The proposed general correlation is found to be in good agreement with empirical correlations in the literature that were developed over narrow ranges of conditions. In addition, out of sample use of the correlation for steam-CO$_2$ mixtures and sub-atmospheric pressures shows good agreement with the data, demonstrating the adequacy of the correlation for conditions beyond the limits of the simulation range.

The correlation developed in this work will typically provide the lowest heat transfer rates that can be expected from a horizontal tube condenser immersed in a quiescent vapor-NCG mixture. It will be of interest to extend the methodology and develop a similar correlation for forced flows over horizontal tubes. Investigations are underway to accomplish this goal.

Credit author statement

I am the sole author of this paper.

Declaration of competing interest

None.

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Appendix A

The specification of the NCG-steam diffusion coefficient is accomplished following the approach outlined by Wilke and Lee [24]:

$$D = \frac{10^{-3}T^{1.5}}{M'ab(M')^{0.5}}$$

$$M_{ab} \text{ is given by:}$$

$$M_{ab} = 2\left(1 + \frac{1}{M'}\right)^{-1}$$

$M'$s are the species molecular weights in g/mol, $T$ is the mean temperature between bulk and the wall, $P$ the total pressure in bar, $\sigma_{ab}$ the Lennard-Jones collision diameter, and $\Omega$ the collision integral. $\Omega$ is a function of temperature and is provided by Poling et al. [25]:

$\Omega = \frac{\alpha_1}{T^n} + \frac{\alpha_2}{e^{\alpha_3T}} + \frac{\alpha_4}{e^{\alpha_5T}} + \frac{\alpha_6}{e^{\alpha_7T}}$

$\alpha_i$ are defined as:

$\alpha_1 = 1.06036; \alpha_2 = 0.1561; \alpha_3 = 0.1930; \alpha_4 = 0.47635; \alpha_5 = 1.03587; \alpha_6 = 1.52996; \alpha_7 = 1.76474; \alpha_8 = 3.89411$

The dimensionless temperature $T^*$ is expressed as follows:

$T^* = \frac{T}{(\epsilon_{ab}/k)}$

Above $k$ is the Boltzmann constant. The Lennard-Jones parameters are given as:
\[
\sigma_{AB} = 0.5 (\sigma_A + \sigma_B)
\]
\[
\frac{\mu_i}{\kappa} = \left( \frac{\mu_i \kappa}{\kappa} \right)^{1/2}
\]

For the different gases in this work, we have respectively:
\[
\sigma_{\text{steam}} = 2.655 \text{ A} \} \sigma_{\text{steam}}/\kappa = 363.0 \text{ K};
\]
\[
\sigma_{\text{air}} = 3.711 \text{ A} \} \sigma_{\text{air}}/\kappa = 78.6 \text{ K};
\]
\[
\sigma_{\text{N}_2} = 3.7798 \text{ A} \} \sigma_{\text{N}_2}/\kappa = 71.4 \text{ K};
\]
\[
\sigma_{\text{CO}_2} = 3.941 \text{ A} \} \sigma_{\text{CO}_2}/\kappa = 195.2 \text{ K};
\]

The NGC-vapor molecular viscosity is specified according to the prescription of Wilke [26]:
\[
\mu_i = \left[ 1 + \left( \mu_i/\mu_j \right)^{0.5} \left( M_j/M_i \right)^{0.25} \right]^{2/3} \mu_j
\]
\[
\phi_i = \frac{X_i}{\sum_j X_j} \left[ \frac{1}{2} \sqrt{1 + M_j/M_i} \right]^{15}
\]

Above \( \mu_i \) is the viscosity of the \( i \)th species evaluated at the average temperature, and \( X_i \) is the mole fraction of the \( i \)th species.

The mixture thermal conductivity is computed using an identical approach:
\[
k_s = \sum_i \frac{X_i k_i}{\sum_j X_j \phi_i}
\]

\( k_i \) is the thermal conductivity of the \( i \)th species computed at the average temperature.

The liquid film properties are evaluated at the arithmetic mean temperature between the interface and the wall using appropriate fits.

References

[1] H.K. Al-Diwany, J.W. Rose, Free convection film condensation of steam in the presence of non-condensing gases, Int. J. Heat Mass Transf. 16 (7) (1973) 1359-1369.
[2] M.H. Ge, J. Zhao, S.X. Wang, Experimental investigation of steam condensation with high concentration CO2 on a horizontal tube, Appl. Therm. Eng. 61 (2013) 334-343.
[3] A. Kasearian, S. Babaei, M. Jahanpanah, H.J. Sarrafzadeh, A.S. Alasgari, S. Ghaffarian, W.-M. Yan, Solar humidification-dehumidification desalination systems: a critical review, Energy Convers. Manag. 201 (2019) 1-26.
[4] E.M. Sparrow, S.H. Lin, Condensation heat transfer in the presence of a noncondensable gas, J. Heat Transf. 86 (1964) 430-436.
[5] T. Leonardi, M. Ishii, Condensation heat transfer with noncondensable gas for passive cooling of nuclear reactors, Nucl. Eng. Des. 236 (2006) 1789-1799.
[6] M. Haag, P.K. Selvam, S. Leyer, Effect of condenser tube inclination on the flow dynamics and instabilities in a passive containment cooling system (PCCS) for nuclear safety, Nucl. Eng. Des. 367 (2020) 110780.
[7] E.O. Olukanni, J. Su, Z. Sun, M. Ding, G. Fan, Analysis of experiments for the effect of noncondensable gases on steam condensation over a vertical tube external surface under low wall subcooling, Nucl. Eng. Des. 278 (2014) 644-650.
[8] E. Ozdemir, M. Kondo, N. Erkan, K. Oomoto, CFD based wall condensation model for evaluating PCV conditions in Fukushima Daiichi Unit-1, Nucl. Eng. Des. 352 (2019) 110170.
[9] J.H. Lienhard IV, J.H. Lienhard V, A Heat Transfer Textbook, Fourth ed., Dover Publications, 2011.
[10] ERCOFTAC, European Research Community on Flow Turbulence and Combustion (ERCOFTAC) Best Practice Guidelines, Version 1, 2000.
[11] V.T. Morgan, Heat transfer by natural convection from a horizontal isothermal circular cylinder in air, Heat Transf. Eng. 18 (1997) 25-33.
[12] D.F. Othmer, The condensation of steam, Ind. Eng. Chem. 21 (6) (1929) 576-583.
[13] A. Dehbi, F. Janaziz, B. Bell, Prediction of steam condensation in the presence of noncondensable gases using a CFD-based approach, Nucl. Eng. Des. 258 (2013) 199-210.
[14] R.B. Bird, W.E. Stewart, E.N. Lightfoot, Transport Phenomena. 2002, John Wiley Sons N. Y. (2004) 582-611.
[15] A.N. Apte, A. Shah, J. Luo, J. Li, Prediction of steam condensation heat and mass transfer in the presence of noncondensable gas CO2 on a horizontal tube at sub-atmospheric pressure, Exp. Thermal Fluid Sci. 105 (2019) 278-288.