The recently reported coexistence of a magnetic order, with the critical temperature $T_M = 35 \mu K$, and superconductivity, with the critical temperature $T_S = 207 mK$, in AuIn$_2$ is studied theoretically. It is shown that superconducting (S) electrons and localized nuclear magnetic moments (LM’s) interact dominantly via the contact hyperfine (EX) interaction, giving rise to a spiral or domain-like magnetic order in superconducting phase. The electromagnetic interaction between LM’s and S electrons is small compared to the EX one giving minor contribution to the formation of the oscillatory magnetic order. In clean samples ($l > \xi_0$) of AuIn$_2$ the oscillatory magnetic order should produce a line of nodes in the quasiparticle spectrum of S electrons giving rise to the power law behavior. The critical field $H_c(T = 0)$ in the coexistence phase is reduced by factor two with respect to its bare value.

The problem of the coexistence of magnetic (M) order and superconductivity (S) is a long-standing one, which was first considered in 1956 theoretically by V. L. Ginzburg and then intensively discussed after the discovery of the ternary rare earth (RE) compounds (RE)Rh$_2$B$_4$ and (RE)Mo$_6$X$_8$ ($X=S,Se$). In many of these compounds both ferromagnetic (F) and antiferromagnetic (AF) orderings, which coexist with S, have been observed. It turns out that S and AF orderings coexist in several of these compounds usually down to $T = 0$ K, while S and modified F (spiral) orderings coexist only in limited temperature interval in ErRh$_4$B$_4$, HoMo$_6$S$_8$ and HoMo$_6$Se$_8$, due to their antagonistic character. A general theory of magnetic superconductors has been developed in Refs. 3, 4, 5, where possibilities for the coexistence of S and spiral or domain-like magnetic order (which is the modified F order in the presence of superconductivity) have been elaborated quantitatively by including the exchange and electromagnetic interaction of superconducting electrons and localized magnetic moments (LM’s). To the same conclusion came also Blount and Varma by taking into account the electromagnetic interaction only. Note, that some heavy fermions UPt$_3$, URu$_2$Si$_2$ etc. show the coexistence of the AF and S orderings, while S and modified M order coexist in quaternary intermetallic compounds (RE)Ni$_2$B$_2$C, see Ref. 6.

However, until recently it was impossible to investigate the interplay of S and nuclear magnetic order, because of lack of suitable materials. Thanks to the pioneering work of Pobel’s group in Germany, 7–8, and Lounasmaa’s one in Finland, 9–10 at least two materials were discovered where S and nuclear M order seem to coexist. The first one is metallic Rb, which is superconducting at $T_S = 325 \mu K$ and whose nuclear moments might be ordered antiferromagnetically at $T_N \sim 1$ mK, see Refs. 7, 8. There are also some hints on the AF order at negative nuclear temperature $T_N$. Rb is an interesting system because of its rather large Korringa constant $\kappa(= \frac{\tau}{T_n}) \leq 10$ s K, where $\tau$ is the spin-lattice relaxation time and $T_n$ is the electronic temperature. Large $\kappa$ (or $\tau$) in Rb allows to achieve very low nuclear temperatures $T_n \ll T_s$, as well as a realization of negative $T_n$. The problem of the competition of nuclear magnetism and S order in Rb will be studied elsewhere.

A remarkable achievement in this field was recently done by Pobel’s group by investigating AuIn$_2$, where the coexistence of the nuclear ferromagnetism and superconductivity ($T_S = 207 mK$) was found below $T_M = 35 \mu K$. Because of good thermal coupling of nuclear magnetic moments to the conduction electrons in AuIn$_2$ (Korringa constant $\kappa = 0.1$ s K) the experiments were performed in thermal equilibrium $T_n = T_s$ down to $T = 25 \mu K$. It was also found that AuIn$_2$ is a type-I superconductor with the bare critical field $H_{c0}(T = 0) = 14.5$ G, which would be in absence of the F ordering, while in its presence $H_{c0}(T)$ is decreased, i.e. $H_c = 8.7$ G at $T = 25 \mu K$. The latter result is a hint that S and F orderings might coexist in the bulk down to $T = 0$.

In the following the coexistence of S and M order in AuIn$_2$ is studied in the framework of the microscopic theory of magnetic superconductors. 11–12 It considers interactions between LM’s and conduction electrons: a) via the direct hyperfine interaction – because of simplicity it is called the exchange (EX) one; b) via the dipolar magnetic field $B_m(r) = 4\pi M(r)$, which is created by LM’s, it is called electromagnetic (EM) interaction. The general Hamiltonian which describes conducting electrons and nuclei moments in AuIn$_2$ is given by

$$\hat{H} = \int d^3r \{ \psi^\dagger(r) (\hat{\mathbf{p}} - \frac{e}{c} \mathbf{A}) \psi(r) + [\Delta(r) \psi^\dagger(r) i\sigma_y \psi(r)] + c.c. \} + \frac{|\Delta(r)|^2}{V} + \sum_{\mathbf{I}} J_{nu} \delta(r - r_I) \psi^\dagger(r) \mathbf{I}_I \psi(r)$$
Here, \( \epsilon(p) \), \( \Delta(r) \), \( A \), \( J_{en}(r) \) and \( V \) are the quasiparticle energy, the superconducting order parameter, the vector potential, the hyperfine contact coupling between electronic spins \( \sigma \) (Pauli matrices) and localized nuclear moments (LM’s) \( I_i \) and the electron-phonon coupling constant respectively. The first three terms in Eq. (1) describe the superconducting mean-field Hamiltonian in the magnetic field \( B(r) = \text{curl}A(r) \) due to LM’s and screening current, while the term \( \hat{H}_{\text{imp}} \) describes the electron scattering (including also the spin-orbit one) on nonmagnetic impurities. The term \( -B(r_i)g_n\mu_n I_i \) describes the dipole-dipole interaction of LM’s, as well as their interaction with the magnetic field due to screening superconducting current – see more below. \( \hat{H}_0(I_i) \) is (together with the dipole-dipole interaction) responsible for magnetic anisotropy of LM’s. In the case of AuIn2, which is simple cubic crystal, its form is unknown – see discussion below. Later we show that under experimental conditions reported in Refs. 1,10,11 the ferromagnetic structure, which would be in absence of \( S \) order, is transversal. The characteristic parameters of AuIn2 were obtained by measurements \( 6 \). The characteristic parameters of AuIn2 are very small compared to \( T_S \approx 0.2 \) K, and it is much larger than the characteristic dipole-dipole temperature \( \Theta_{em}(\approx 1 \mu K) \), see below. This fact allows us to estimate the hyperfine contact interaction between electrons and LM’s, which is characterized by the parameter \( h_{ex} = J_{en}(0)n_m | \langle I_i \rangle | \), where \( n_m \) is the concentration of LM’s. The indirect exchange energy (via conduction electrons) between the LM’s of nuclei is characterized by the RKKY temperature \( \Theta_{ex} = N(0)h_{ex}^2/n_m \), where \( N(0) \) is the electronic density of states at the Fermi level. The crystallographic structure gives \( n_m \approx 3 \times 10^{22} \) cm\(^{-3} \). \( N(0) \) is obtained by knowing \( H_{0}(T = 0) = 14.5 \) G, see Refs. 10, which gives \( N(0) \approx 0.64 \times 10^{24} \) erg\(^{-1} \) cm\(^{-3} \). Since \( T_M(=35 \mu K) \) is predominantly due to the indirect exchange interaction between \( I_i \) nuclei moment \( S \) one has \( T_M \approx \Theta_{ex} \), which gives \( h_{ex} \approx 1 \) K. Note that one has \( h_{ex} > \Delta_0 (\approx 0.36 \) K), which gives rise to a gapless quasiparticle spectrum in \( S \) state below \( T_M \) in clean samples \( (l > \xi_0) \) of AuIn2, see below.

The electromagnetic (EM) dipole-dipole interaction between LM’s is characterized by \( \Theta_{em} = 2\pi n_m \mu^2 \), where \( \mu = g_n\mu_n I \). In case of the In nuclei in the AuIn2 cubic crystal one has \( \mu \approx 5.5 \mu_n \), i.e. \( \Theta_{em} \approx 1.2 \mu K (\ll T_M) \), which means that the dipole-dipole interaction does not contribute to \( T_m \) in AuIn2. However, it makes the magnetic structure transverse in \( S \) state, see below.

From \( \Delta_0 (\approx 1.76 T_S) \) and \( v_F \approx 1.68 \times 10^6 \) cm/s, \( \xi_0 \approx 10^{5} \) A, while from the resistivity measurements \( 5 \), where \( RRR = 500 \), one gets \( \xi_0 \approx 3.6 \times 10^4 \) A. Accordingly, the spin-orbit scattering mean-free path is very large, i.e. \( l_{so} > 3.6 \times 10^4 \) A, because one always has \( l_{so} > l \). Note that \( l < \xi_0 \) and the system is in the dirty (but not very dirty) limit. The London penetration depth \( \lambda_L \approx 200 \) A is estimated from \( H_S \) and by knowing \( \xi_0 \) and \( l \), which means that AuIn2 is the type I superconductor at temperatures where \( S \) and \( M \) orderings coexist. \( \xi \) From the above analysis we estimate the parameter \( (h_{ex}/\hbar)^2 = 0.1 \). It is small and dirty limit may be used to treat effect of exchange field on superconductivity. This simplifies the theoretical analysis given below. Here \( \tau = t/\tau_F \) is the electron scattering time.

B. Theoretical analysis of AuIn2

It was shown in Refs. 1,10,11 that when the electron spin-orbit interaction is weak, i.e. when \( l_{so}/l > (k_F^{-1}\xi_0/l)^{2/3} \), there is a peak in the spin susceptibility in the superconducting state at nonzero wave vector \( Q \). This means that in the superconducting state an oscillatory magnetic order is more favorable than the F one. In AuIn2 one has \( k_F = 1.45 \) Å\(^{-1} \) and thus the condition for an oscillatory magnetic order is \( l_{so} \gg 10^{-2} l \). Since by definition \( l_{so} > l \) we see that the spin-orbit interaction does not play any role in the formation of the magnetic structure in the coexistence phase of AuIn2. The magnetic order can be a spiral or domain-like one, depending on the magnetic anisotropy, see below.

The problem is now reduced to the study of electrons moving in an oscillatory (with the wave vector \( Q \)) and magnetic field \( h_{ex}(R) = h_{ex}(S(R)) \), \( B(R) = \text{curl}A(R) \) respectively. By using the Eilenberger equations for the normal \( g_n(v, R) \) and anomalous \( f_{\omega}(v, R) \) electronic Green’s function, the superconducting order parameter \( \Delta(R) \) is a solution of the self-consistency equation \( \Delta(R) = 2\pi g \sum f_{\omega}(R, n)/4\pi \), (\( g = N(0)V \) the electron-phonon coupling constant) one obtains the free-energy functional of the system \( F_{SM}(\Delta, Q) \). It may be presented as a sum of magnetic \( (F_M) \), superconducting \( (F_S) \) and interacting \( (F_{int}) \) parts, i.e.

\[
F_{SM}(\Delta, Q) = F_S(\Delta) + F_M(\Delta, Q) + F_{int}(\Delta, Q). \]

By assuming that: a) \( \pi E_S/F_{int} > 1, E_S = N(0)\Delta^2/2n_m \) – this is indeed fulfilled in AuIn2, where \( \pi E_S/F_{int} \approx 100 \), and b) the Fermi surface is isotropic – it is also fulfilled in AuIn2, one gets the free-energy \( F_{SM} \)

\[
F_S(\Delta) = -\frac{1}{2} N(0)\Delta^2 \ln \frac{e\Delta_0^2}{\Delta^2},
\]

\[
F_{int} = F_{ext} + F_{em} = F_{ext} + \sum Q \frac{3\pi^2 \Theta_{em} \Delta |S_Q,\omega|}{v_F Q(\lambda_L Q)^2}. \tag{2}
\]

Here the terms \( F_{ext} \) and \( F_{em} \) in Eq. (2) describe the exchange EX and EM interaction of superconducting electrons with LM’s respectively. \( F_{ext} \) and \( F_M \) depend on \( h_{ex}, S_Q, l, \xi_0 \) etc. We consider only these cases which might be important for the physics of AuIn2.

1. Dirty case \((l < \xi_0)\) This case is realized in AuIn2 as reported in Refs. 4, where \( l \approx 3.6 \times 10^4 \) A and \( \xi_0 \approx 10^5 \) A. Immediately below the magnetic critical temperature
The present experiments were only higher order terms contribute to transverse (nuclei, although small (Θmagnetic structure is due to the effective EX interaction with the wave vector Q).

Fig. 1. (a) The spiral magnetic structure S(x), with the period L_S in the superconducting phase for small anisotropy D/Θ_{ex} < 10^{-3}. (b) The domain-like magnetic structure S(x), with the period L_D in the superconducting phase for appreciable anisotropy D/Θ_{ex} > 10^{-3}.

2. Clean limit (l > ξ_0). The present experiments were performed on dirty (but not very dirty) AuIn_{2} samples, where l < ξ_0. In that case the motion of Cooper pairs in the coexistence phase is diffusive and there is an isotropization of the quasiparticle spectrum. This means that the oscillatory magnetic structure acts like magnetic impurities – for similarity and differences of effects of magnetic impurities and the oscillatory magnetic structure see Ref. [3]. However, it would be interesting to perform experiments on clean AuIn_{2} samples with l > ξ_0 – for instance on samples with a residual resistivity ratio RRR > 1500. Namely, it was shown in Refs. [3,7] that the oscillatory magnetic order in clean superconductors can give rise to the gapless quasiparticle spectrum with nodes on a line at the Fermi surface if h_{ex} > Δ_0. This is just the case in AuIn_{2}, where h_{ex} ≈ 1 K and Δ_0 ≈ 0.36 K. In the clean limit the quasiparticle motion is anisotropic in the presence of an oscillatory magnetic structure with the wave vector Q.
quasiparticle energy vanishes on lines at the Fermi surface given by \( \mathbf{v}_F \cdot \mathbf{Q} = 0 \) if \( h(T) = h_{2D} S_Q(T) > \Delta_0 \), see Ref. 2. In this case the density of states for \( E < \Delta_0 \) is

\[
N_s(E) = N(0)(\pi E h/\Delta_0 v_p Q_d) \ln(4\Delta_0/\pi E)
\]

for the domain structure and \( N_s(E) = N(0)(\pi E h/\Delta_0 v_p Q_d) \) for the spiral one. \( N_s(E) \) can be experimentally obtained by measuring voltage dependence of the tunneling conductivity in the S–N junction with AuIn\(_2\) being in the coexistence phase.

C. Effect of magnetic field

Because of very small interaction energy one expects that the critical field \( H_c(T) \) does not vanish down to \( T = 0 \). Indeed, equating Gibbs energy for superconducting state and that of normal ferromagnetic state one gets

\[
F_S + F_M + F_{int} = F_M^0 - \frac{H^2}{8\pi} - M \cdot \mathbf{H}_c, \tag{4}
\]

and if one defines \( H_{SM} = [8\pi(F_M^0 - F_M - F_S - F_{int})]^{1/2} \)

\[
H_c = \sqrt{H_{SM}^2 + (4\pi M_0)^2 - 4\pi M}. \tag{5}
\]

At \( T \ll T_M \) the magnetization \( \mathbf{M} \) is saturated, i.e. \( M \approx M_0 = 5.5m_\mu \). Because \( F_{int} \ll F_S, F_M \) one has small difference in magnetic energy of oscillatory state, \( F_M \) and that of ferromagnetic state, \( F_M^0 \). As result, \( H_{SM} \approx H_{00}(0) = [8\pi(-F_M^0)]^{1/2} \). The experimental values \( H_{00}(0) \approx 14.5 \) G and \( 4\pi M_0 \approx 11 \) G which gives \( H_0(0) \approx 7 \) G. It was found experimentally that \( H_c \approx 8.7 \) G at \( T = 25 \) \( \mu \)K, and thus our estimate is reasonable. Nonzero value of \( H_c(0) \) in AuIn\(_2\) is in contrast to the case of magnetic type II superconductors ErRh\(_3\)B\(_4\) and HoMn\(_6\)S\(_8\), where \( H_c(T) \) tends to zero [as well as \( H_{2D}(T) \)] at \( T \to T_M \), because in these compounds one has \( F_{int} \approx |F_N - F_S| \) near some temperature \( T_S < T_M \). At \( T > T_M \) one obtains \( H_c = H_c^0/(1 + 4\pi \chi_M) \) in absence of demagnetization effects, where \( \chi_M(T) = \Theta_{em}(T - \Theta_{ex}) \) and \( \Theta_{em} = \Theta_{em}/6\pi \). The change of the slope of \( H_c(T) \) takes place at \( T \) very near \( T_M \). The experimental broadening of the transition in the magnetic field can be due to the polycrystallinity of the sample, where even small magnetic anisotropy of the crystallites can produce percolation like broadened resistive transition in magnetic field.

In conclusion, we found that superconductivity coexists with a domain-like magnetic structure if the anisotropy parameter \( D \) is not too small, i.e. \( D/\Theta_{ex} > 10^{-3} \). We estimate the period \( L_D \approx 300 \) \( \text{Å} \) for \( D \approx 1 \) \( \mu \)K. In the opposite case, \( D/\Theta_{ex} < 10^{-3} \), the magnetic structure is spiral with the period \( L_S \approx 120 \) \( \text{Å} \). The realization of the spiral structure in AuIn\(_2\) is more probable due to the simple cubic structure of this compound and accordingly due to small magnetic anisotropy.

It is also proposed that in the case of very clean AuIn\(_2\) samples with RRR\( > 1500 \) there is a line, given by \( \mathbf{v}_F \cdot \mathbf{Q} = 0 \), at the Fermi surface with nodes in the quasiparticle spectrum in the coexistence phase. It would be interesting to study this regime experimentally, because in that case thermodynamic and transport properties show power law behavior.

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