On the SINR Distribution of SWIPT MU-MIMO with Antenna Selection

Hadi Saki, M. Shikh Bahae
Institute of Telecommunications, King’s College London, London, United Kingdom
Email: {hadi.saki, m.sbahaei}@kcl.ac.uk

Abstract—In this paper, we provide a closed-form approximation of the Full-duplex Simultaneous wireless information and power transfer (SWIPT) multi-user MIMO (MU-MIMO) system signal-to-interference-and-noise ratio (SINR) distribution for the received signal at the sensor users with perfect and imperfect channel state information (CSI) and transmit antenna selection scheme at the transmitter. We also studied the SINR distribution of the uplink with Zero-Forcing beamforming (ZFBF) and antenna selection at the aggregator (AGG) receiver with transmit antenna selection. The downlink SINR with perfect and imperfect CSI is modeled by a multivariate Beta type II distribution and the uplink with perfect and imperfect CSI is modeled with multivariate Wishart distributions. We compared the proposed distributions analytical results to the Monte-Carlo simulations and we obtained a perfect match.

Index Terms—SWIPT, MU-MIMO, beamforming, power splitting, energy harvesting system, full-duplex radios, antenna selection.

I. INTRODUCTION

Real-time and reliable data transmission is an essential requirement for next-generation autonomous platforms where power consumption, latency, and delay are important parameters. In terms of communication systems, research has addressed these requirements in several ways including cross layer design [1, 2], energy efficient communication technologies [3, 4], and improved reliable and fast communication systems. Recently, Full Duplex (FD) [5, 6] Simultaneous Wireless Information and Power Transfer (SWIPT) multi-user MIMO (MU-MIMO) has become a promising technology in wireless communication systems due to its application in the short-range in-body and out-body wireless sensor networks. However, a particular challenge in implementing these methods is how to define and evaluate the outage performance and the ergodic capacity of the system. To this end deriving a simple closed-form SINR distribution is essential. Since the introduction MU-MIMO, many works have been pursued to derive a closed-form approximation of the outage capacity where the emphasis is on how to avoid the interference of the MU-MIMO network.

The closed-form expressions for pdf of the maximum eigenvalue of a product of multivariate independent complex Gaussian in single user MIMO is driven in single user MIMO Channels is derived in [7]. Authors in [8] proposed a closed-form model for the distributions of constrained SINR in MIMO under imperfect CSI at the transmitter. The work in [9] studied the MU-MIMO fading broadcast channel and the achievable ergodic capacity is determined when CSI is obtained at the receivers and it is delivered to the transmitter. Furthermore, ZFBF is also has become a promising beamforming technique due to the performance in the imperfect channel knowledge availability compared to the matched filter (MF), block diagonalization (BD) and singular value decomposition (SVD) beamforming techniques. SNR distribution of the uplink MIMO systems with ZFBM with perfect and imperfect CSI is calculated in [10]. Generalized Selection Criterion performance evaluation of ZF-Precoded MU-MIMO also studied in [11]. In this work, the SNR distribution is calculated to evaluate the ergodic sum-rate of MU-MIMO. Even though the MU-MIMO SINR distribution is derived in the previous work, they have neglected the antenna selection and energy harvesting performance and little attention have been paid to performance analysis of SWIPT MU-MIMO. Thus, we propose a closed-form approximation of the downlink SWIPT MU-MIMO and uplink MU-MIMO with ZF-precoded with received antenna selection system SINR using random matrices.

A. Contributions of our Work

In this work, we investigate the problem in an IBFD SWIPT MU-MIMO system. Our objective is to derive a closed-form distribution of the SINR in the present of the energy harvesting and antenna selection. The primary contributions of this paper can be summarized as follows,

- We derived a new closed-form multivariate Beta distribution of type II formulation for the received SINR of the SWIPT MU-MIMO at the SUs under perfect and imperfect CSI.
- A closed-form low complexity multivariate Wishart distribution formulation of the SWIPT MU-MIMO SINR at the AGG with ZFBM under perfect and imperfect CSI is calculated.

B. Organization and Notation

The rest of this work is as follows. The system model is given in Section II. SINR distribution of the downlink SWIPT MU-MIMO for perfect and imperfect CSI is presented in section III and in Section IV the SINR distribution of uplink MU-MIMO with antenna selection and ZF beamforming is presented. Finally, simulation results and the conclusion are discussed in Section V and Section VI.

Notation: We use the following notations throughout this paper; bold upper case and lower case letters are used for
matrices and vectors respectively, while small normal letters are kept for scalars. \( \mathbf{X}^*, \mathbf{X}^{-1}, \mathbf{X}^T, Tr(\mathbf{X}), \mathbf{X}^H, \) and \( \mathbf{X}^{1/2} \) are used to denote the complex conjugate, transpose, inverse, pseudo-inverse, trace, Hermitian and the square-root of \( \mathbf{X} \) respectively. \( \mathbf{I} \) and \( \mathbf{O} \) denote an identity matrix and an all-zero matrix, respectively, with appropriate dimensions; \( \mathbf{X} \succeq \mathbf{O} \) and \( \mathbf{X} \succ \mathbf{O} \) mean that \( \mathbf{X} \) is positive semi-definite and positive definite, respectively; \( \mathbb{E}[\cdot] \) denotes the statistical expectation; The distribution of a CSCG random variable with zero mean and variance 2 is denoted as \( \mathcal{CN}(0, \sigma^2) \), and \( \sim \) means distributed as; \( \mathbb{C}^{x \times y} \) denotes the space of \( x \times y \) matrices with complex entries; \( \|x\| \) denotes the Euclidean norm of a vector \( x \); the unit-norm vector of a vector \( x \) is denoted as \( \mathbf{x} = x/\|x\| \); the quantity \( \min(x,y) \) and \( \max(x,y) \) represents the minimum and maximum between two real numbers.

II. SYSTEM MODEL

We consider a bidirectional FD multi-user MIMO system as shown in Fig. 1, where \( K \) sensor user (SU) indexed with \( k \in \mathcal{K} \triangleq \{1, 2, \ldots, K\} \) and each sensor is equipped with \( N_u \) antennas and communicates with an AGG equipped with \( N_T \) and \( N_R \) transmit and receive antennas respectively, where similarly indexed with \( n_t \in \mathcal{N}_T \triangleq \{1, 2, \ldots, N_T\} \) and \( n_r \in \mathcal{R} \triangleq \{1, 2, \ldots, N_R\} \). Without loss of generality, we assume equal number of transmit and receive antennas at the AGG and the SUs are equipped with small number of antennas compared to the AGG, i.e. \( N_T = N_R \gg N_U \). The AGG is connected to a constant power supply. Uncorrelated antennas are assumed at the AGG. The SUs are energy limited devices and harvest their energy from transmitted signal by the AGG. SUs can split the received signal by using power splitter into two different energy harvesting (EH) and information detection (ID) elements. The power splitting (PS) ratio of the \( k_{th} \) SU for the EH and ID elements are denoted by \( \rho \) and \( 1 - \rho \) respectively. SUs are also equipped with a limited capacity rechargeable battery that store the harvested energy.

III. DOWNLINK SWIPT MU-MIMO

A. Perfect channel knowledge at the AGG

We consider an SWIPT system as shown in Fig. 1. The transmitted signal by \( k_{th} \) SU is given by

\[
x_{k}^u = w_{k}^u s_{k}^u.
\]

where \( x_{k}^u \) is the transmitted signal, \( w_{k}^u \in \mathbb{C}^{N_u \times 1} \) is the beamforming vector and \( s_{k}^u \in \mathbb{C} \) and \( \mathbb{E}[|s_{k}^u|^2] = 1 \) is the transmitted information symbol. In this section we consider a perfect channel knowledge flat fading channel at the AGG. Therefore the received signal at the AGG can be modeled as,

\[
y_{k}^u = \sqrt{p^u} H_{k} w_{k}^u s_{k}^u + \sqrt{p^u} \sum_{i \neq k, i \in K} H_{i} w_{i}^u s_{i}^u + n_{k}^u,
\]

where \( p^u \) is the uplink maximum average transmit power, \( H_{k} \in \mathbb{C}^{N_r \times N_u} \) denotes the channel matrix between the \( k_{th} \) SU and AGG. Downlink transmit power is denoted as \( \sqrt{p^d} \) and \( w^d \) is the downlink precoding matrix. \( G^d \in \mathbb{C}^{N_r \times N_k} \) is the self-interference (IS) noise at the AGG. \( N_r, N_t, N_u \) are the number of receive and transmit antenna at the AGG and the number of FD antennas at the user respectively. \( s^d \) is the downlink symbol vector, and \( n^u \in \mathbb{C}^{N_u \times 1} \) is the additive Whit Gaussian noise (AWGN) at the AGG. To be able to reduce the computational complexity and also to reduce the energy consumption at the AGG with large number of receive antenna arrays, we use the antenna selection in the AGG. We select \( N_t \) antennas out of total \( N_t \) receive antennas in a way that \( N_t \ll N_r \). This is mathematically applied by the use of the diagonal antenna selection matrix \( \mathbf{V}^r_k \in \mathbb{C}^{N_r \times N_r} \), where \( [\mathbf{V}]_{i,i} = 1 \) if the antenna \( i \) is selected and \( [\mathbf{V}]_{i,i} = 0 \) if the \( i_{th} \) antenna is not selected. \( \sum_{i=1}^{N_r} [\mathbf{V}]_{i,i} = N_R \). Therefore the received signal will be denoted as

\[
y_{k,s}^u \triangleq \sqrt{p^d} \mathbf{V}^r_k H_{k} w_{k}^u s_{k}^u + \sqrt{p^u} \sum_{i \neq k, i \in K} \mathbf{V}^u_i H_{i} w_{i}^u s_{i}^u + n_{k,s}^u.
\]

The eq. 3 can be rearranged as,

\[
y_{k,s}^u = \tilde{H}_{k} \left[ \sum_{i \neq k, i \in K} \mathbf{V}^u_i w_{i}^u s_{i}^u \mathbf{C}_i^{-1} + \bar{G}^d s^d \right] + n_{k,s}^u,
\]

where \( \tilde{H}_{k} \triangleq [H_k \quad A_k] \), \( A_k C_i = \mathbf{V}^u_i H_{i} w_{i}^u \), \( \forall i \in \mathcal{K} \& i \neq k \), and \( A_k \bar{G}^d = \mathbf{V}^u_i G^d s^d \) are the uplink and downlink interference coefficient respectively.

To derive the received signal distribution at the AGG, we first assume that the transmit matrix is fixed, and then the zero-forcing (ZF) processing technique is used for the data reception at AGG. The ZF beamforming equalizer at the AGG receiver can be implemented by precoder \( \mathbf{C}_i \). The ZF is a standard method widely implemented in the massive MIMO processing systems [13] [14]. Furthermore, since \( H_k \) is not correlated with \( A_k \) and \( \mathbf{w}^d_k \), therefore the \( N_r \times N_r \) matrix of \( [H_k \quad A_k] \) is full rank. Consequently, the ZF beamforming equalizer at the AGG receiver can be expressed as,

\[
U_{k}^r = [\mathbf{I}_{d_k} \quad 0] \tilde{H}_{k}^{-1},
\]
\[
\gamma_{k,s}^{u} = \frac{p^{u}\bar{H}_{k}^{H}V_{u}\bar{w}_{k}^{H}w_{k}^{u}V_{u}\bar{H}_{k} + p^{d}G^{d}V_{d}^{H}w_{d}^{H}w_{d}^{u}V_{d}^{d} + \sigma_{k}^{u}}{p^{u}\sum_{i \neq k,i \in \mathcal{K}} H_{i}^{H}V_{i}\bar{w}_{k}^{H}w_{k}^{u}V_{u}H_{i} + p^{d}G^{d}V_{d}^{H}w_{d}^{H}w_{d}^{u}V_{d}^{d} + \sigma_{k}^{u}},
\]
\[
\gamma_{k,s}^{u,zf} = \frac{(1 - \alpha_{k}^{2})P_{s}V_{s}^{H}w_{k}^{H}w_{k}^{u}V_{u}/\bar{d}_{k}}{Z_{k}^{H}(\text{tr}(\alpha^{2}V_{d}^{H}w_{d}^{H}(P_{w}u^{H}V_{u}))(\bar{H}_{k}\bar{H}_{k})^{-1} + \sigma_{k}^{u}(\bar{H}_{k}\bar{H}_{k})^{-1})Z_{k}^{(6)}},
\]

where we define \(Z_{k} = [I_{d_{k}} \ 0]\).

**Proposition 1.** For multi user uplink FD MIMO system with perfect CSI and adopting ZF beamforming \(U_{k}^{zf}\), the processed SINR follows an \(N_{r} \times N_{r}\) complex central Wishart distribution with identity covariance matrix of the rank \((U_{k}^{zf}\bar{H}_{k}) = d_{k}\), and \(2N_{y}d_{k}\) degree of freedom.

**Proof:** By applying the ZF beamforming to eq[2] the \(k_{th}\) user received signal at \(\hat{s}_{k}\) changes to,
\[
y_{k}^{zf} = Z_{k}\bar{H}_{k}^{-1}\left[\sum_{i \neq k,i \in \mathcal{K}} V_{i}\bar{w}_{k}^{H}w_{k}^{i}V_{i} + \bar{G}_{d}\bar{s}_{d}\right] + Z_{k}\bar{H}_{k}^{-1}n_{k,s}^{u},
\]
and the \(k_{th}\) user received SINR at eq[4] will be,
\[
\gamma_{k,s}^{u,zf} = \frac{p^{u}V_{k}^{H}w_{k}^{H}w_{k}^{u}V_{u}/\bar{d}_{k}}{Z_{k}(\bar{H}_{k}\bar{H}_{k})^{-1}Z_{k}^{H}d_{k}\sigma_{k}^{u}}
\]
we define \(\eta_{k}^{u} = \sqrt{p^{u}V_{u}^{H}w_{u}^{H}}/\sqrt{d_{k}\sigma_{k}^{u}}\). From [15] we have,
\[
Z_{k}(\bar{H}_{k}\bar{H}_{k})^{-1}Z_{k}^{H} = Z_{k}\left[H_{k}\bar{H}_{k} \bar{H}_{k}^{H}A_{1}^{H}A_{1}\right]^{-1}Z_{k}^{H}. \quad (10)
\]

Since multiplication of \((\bar{H}_{k}\bar{H}_{k})^{-1}\) by \(Z_{k}\) and \(Z_{k}^{H}\) only keeps the \(d_{k}\) row and columns respectively, the \(Z_{k}(\bar{H}_{k}\bar{H}_{k})^{-1}Z_{k}^{H}\) can be simplified to \((H_{k}^{H}H_{k} - H_{k}^{H}A_{1}A_{1}^{H}H_{k})^{-1} = (H_{k}^{H}(I_{N_{y}} - A_{1}A_{1}^{H})H_{k})^{-1}\), where \((I_{N_{y}} - A_{1}A_{1}^{H})\) is the projection matrix of the rank \(d_{k}\). Since \(H_{k} \in \mathbb{C}^{N_{y} \times N_{u}}\) is a complex Gaussian random vector with zero mean and unit variance, \(H_{k}^{H}H_{k}\) follows a central Wishart distribution with \(I_{N_{y}}\), covariance matrix and \(2N_{y}d_{k}\), degree of freedom. Therefore, considering \((I_{N_{y}} - A_{1}A_{1}^{H})\) as a projection matrix of \(d_{k}\), then \((H_{k}^{H}(I_{N_{y}} - A_{1}A_{1}^{H})H_{k})\) distributed as Wishart distribution with \(I_{d_{k}}\) covariance matrix and \(2N_{y}d_{k}\) degrees of freedom. Therefore, according to [16] (theorem 3.3.11), SINR has a Wishart distribution with \(H_{k}^{H}H_{k}\) \(d_{k}\) \(d_{k}\) covariance matrix and \(2N_{y}d_{k}\) degrees of freedoms, 
\[
f(\gamma_{k,s}^{u,zf}) = \frac{\gamma_{k,s}^{u,zf}(2N_{y} - N_{s} - 1)/2}{\sqrt{(2\pi)^{d_{k}/2}/\Gamma_{d_{k}}(N_{y})}} \exp\left[-\frac{1}{2}tr\left(\eta_{k}^{u}H_{k}\eta_{k}^{u}\right)\right].
\]
\[
\eta_{k}^{u} = \sqrt{\frac{1 - \alpha_{k}^{2}P_{k}V_{k}^{H}w_{k}^{H}w_{k}^{u}V_{u}^{d}}{\sqrt{d_{k}(\bar{d}_{k}/\sigma_{k}^{u})}}}
\]
\[
\Delta = \left|\Delta_{1}, \Delta_{2}, \ldots, \Delta_{K}\right|, \quad w_{u} = [w_{u}^{1}, w_{u}^{2}, \ldots, w_{u}^{K}] \quad \text{and} \quad s^{u} = [s_{u}^{1}, s_{u}^{2}, \ldots, s_{u}^{K}].
\]

**Proposition 2.** For multi user uplink FD MIMO system with imperfect CSI at AGG and adopting ZF beamforming \(\hat{U}_{k}^{zf}\), the processed SINR distribution is as follows:
\[
f(\gamma_{k,s}^{u,zf}) = \frac{\gamma_{k,s}^{u,zf}(2N_{y} - N_{s} - 1)/2}{\sqrt{(2\pi)^{d_{k}/2}/\Gamma_{d_{k}}(N_{y})}} \exp\left[-\frac{1}{2}tr\left(\eta_{k}^{u}H_{k}\eta_{k}^{u}\right)\right],
\]
\[
\eta_{k}^{u} = \sqrt{\frac{1 - \alpha_{k}^{2}P_{k}V_{k}^{H}w_{k}^{H}w_{k}^{u}V_{u}^{d}}{\sqrt{d_{k}(\bar{d}_{k}/\sigma_{k}^{u})}}}
\]
\[
\Delta = \left|\Delta_{1}, \Delta_{2}, \ldots, \Delta_{K}\right|, \quad w_{u} = [w_{u}^{1}, w_{u}^{2}, \ldots, w_{u}^{K}] \quad \text{and} \quad s^{u} = [s_{u}^{1}, s_{u}^{2}, \ldots, s_{u}^{K}].
\]

**Proof:** The post-processing received signal at the AGG after applying the [14] ZF beamforming is given by,
\[
y_{k,s}^{u,zf} = \sqrt{1 - \alpha_{k}^{2}}Z_{k}\bar{H}_{k}^{-1}\bar{H}_{k}^{H}V_{u}^{H}w_{u}^{H}V_{u}^{d} + \bar{G}_{d}\bar{s}_{d}
\]
\[
\eta_{k}^{u} = \sqrt{\frac{1 - \alpha_{k}^{2}P_{k}V_{k}^{H}w_{k}^{H}w_{k}^{u}V_{u}^{d}}{\sqrt{d_{k}(\bar{d}_{k}/\sigma_{k}^{u})}}}
\]
\[
\Delta = \left|\Delta_{1}, \Delta_{2}, \ldots, \Delta_{K}\right|, \quad w_{u} = [w_{u}^{1}, w_{u}^{2}, \ldots, w_{u}^{K}] \quad \text{and} \quad s^{u} = [s_{u}^{1}, s_{u}^{2}, \ldots, s_{u}^{K}].
\]

The post-processing received SINR at the AGG for the \(k_{th}\) user is given in eq.[6].

**B. Imperfect channel knowledge at the AGG**

In the realistic wireless communication system, due to the feedback delays and/or estimation error, actual channel is different from estimated channel and can be modeled as,
\[
\bar{H}_{k} = \sqrt{1 - \alpha_{k}^{2}}\bar{H}_{k} + \alpha\Delta_{k}.
\]
\[
\bar{H}_{k} \sim \mathcal{C}\mathcal{N}(0, I)
\]

where $\Theta$ is the $\Delta$ variance matrix and $P = \mathbb{E}((s^H s)^{2/\nu})$. Since $\Delta$ is i.i.d. with zero-mean and unit-variance elements, eq. ?? can be simplified as eq [28]. It can be more simplified to,

$$\gamma_{k,s}^{wz} = \frac{(1 - \omega_s^2) P_k V_k^H w^H_k V_k/d_k}{Z_k^2 ((\omega_s^2 P_k + \sigma_n^2)(H_k^H H_k)^{-1}) Z_k = (1 - \omega_s^2) P_k V_k^H w^H_k V_k/d_k \ (1 - \omega_s^2 P_k + \sigma_n^2)(H_k^H (I_N - \bar{A}^H A) H_k)^{-1}}.$$  

(18)

Therefore, similar to the perfect CSI case ($I_{N_k} - \bar{A}^H A$) is a projection matrix of rank $d_k$, then $(H_k^H (I_N - \bar{A}^H A) H_k)$ distributed as Wishart distribution with projection matrix of rank $d_k$.

Assuming that full CSI is available at the AGG, the received $\eta_k$ harvested power by $k_{th}$ SU is given and also from [16] (theorem 3.3.11) the received SINR distribution have also with a Wishart distribution with $\eta_k^T I_d \eta_k$ covariance and $2N_r$ degrees of freedom in eq. [13].

IV. DOWNLINK SWIPT MU-MIMO

A. Downlink with perfect CSI

Similar to the uplink scenario, here also we first consider a fixed beamforming at the transmitter and to derive the SINR at the AGG, the transmitted signal is expressed as,

$$x^d = \sum_{k=1}^K w^d_k e_k,$$  

(19)

where $x^d$ is the transmitted signal at the AGG, $w^d_k \in \mathbb{C}^{N_t \times N_u}$ is the SU's uplink beamforming matrix, and $G^u \in \mathbb{C}^{K \times N_u}$ is the self-interference (IS) noise at the SU. $s^u$ is the uplink symbol vector, and $n^d_k \in \mathbb{C}^{N_t \times 1}$ is the additive White Gaussian noise (AWGN) at the $k_{th}$ SU. We assume that each SU antenna is equipped with a PS device that coordinates energy harvesting and the information decoding from the received signal. Therefore the received ID and EH signal elements are respectively denoted as,

$$y_{k}^{d} = \sqrt{\rho_k} [\sqrt{p^d} F_k V_k^d w^d_k e_k] + \sum_{i \neq k, i \in K} \sqrt{\rho_i} P^d F_i V^d_i w^d_i e_i + \sqrt{\rho^d} G^u w^d u + n^d_k,$$  

(20)

where $n^d_k$ is the power split processing noise at the $k_{th}$ SU. By assuming that full CSI is available at the AGG, the received signal to interference ratio (SINR) at $k_{th}$ SU is given and the harvested power by $k_{th}$ SU is given in equations [23] and (??) on top of Page 5 respectively. Here $\eta_k \in (0, 1]$ is the energy conversion efficiency of the $k_{th}$ user energy harvesting. The received SINR at the $k_{th}$ user therefore can be noted as eq [23].

Corollary 1. Assuming $X$ is a multivariate Gaussian distribution matrix as $N_d(\mu_k, \sigma)$ then $X^H Q X$ has a Wishart distribution denoted as $X^H Q X \approx W_d(\mathbf{r}, \sigma, \delta)$, where $Q$ is a symmetric $d \times d$ matrix and $i = r$ is the number of the independent column $s$ of $X^H$, and

$$\delta = \sigma^{-1/2} M^H Q M \sigma^{-1/2},$$  

(25)

$$M = (\mu_1, \mu_2, ..., \mu_r).$$  

(26)

Proposition 3. For multi user downlink FD MIMO system with perfect CSI and the processed SINR follows a matrix variate Beta type II distribution with parameters $(N_1, N_2)$ and defined as,

$$\gamma_k^d \sim B_{N_1}^H(N_1, N_2) \sim \frac{\det(\gamma_k^{d})^{(2N_1-N_2-1)}}{\det(I_{N_k}^{H} + \gamma_k^{d})^{N_1-N_2}} \beta(N_1, N_2),$$  

(27)

where

$$\beta(N_1, N_2) = \frac{\Gamma_{N_k}(N_1) \Gamma_{N_k}(N_2)}{\Gamma_{N_k}(N_1 + N_2)},$$  

(28)

and $\Gamma_{N_k}(x)$ is the multivariate Gamma function given as,

$$\Gamma_{N_k}(x) = \pi^{N_k(N_k-1)/4} \prod_{i=1}^{N_k} \Gamma(x - (i - 1)/2).$$  

(29)

Proof: For simplicity we denote the received SINR of MU-MIMO with energy harvesting for $k_{th}$ user at eq [23] as $\gamma_k^d = \frac{\gamma_k^d}{\gamma_k^d}$ and $\sigma_k^d = \frac{\sigma_k^d}{\sigma_k^d}$. Since $F_k \in \mathbb{C}^{N_u \times N_t}$ is a complex Gaussian random vector with zero mean and unit variance, $F_k F_k^H$ follows a central Wishart distribution with $I_{N_k}$ covariance matrix and $2N_2$ degrees of freedom as,

Therefore, considering $w^d_k \in \mathbb{C}^{N_u \times N_t}$, $V_k^H w^d_k V_k^H$ as a projection matrix of $\eta_k$, then $F_k^H(V_k^H w^d_k V_k^H) F_k^H$ distributed as Wishart distribution with $I_{N_k}$ covariance matrix and $2N_2$ degrees of freedom. Therefore, $\Phi$ has a central multivariate matrix Wishart distribution with $I_{N_k}$ covariance matrix and $2N_2$ degrees of freedom as follows,

$$\Phi = W^d \approx W_{N_k}(2N_1, I_{N_k}) = \gamma_k^{d(2N_1-N_2-1)/2} \exp \left[ -\frac{1}{2} tr((I_{N_k}) - 1 - \frac{d}{2}) \right] \frac{2N_2 N_k \Gamma(N_k)}{\det(I_{N_k}) \Gamma(N_k)},$$  

(30)

where $W_{N_k}(y, z)$ is a central multivariate Wishart distribution. Furthermore, the the first term of $\Psi$ in eq [27] is a sum of central Wishart distribution with similar identity covariance matrices can be simplified as follow,

$$\Phi = \sum_{k=1}^{K} \sum_{i=1, i \neq k}^{K} F_k^H V_k^H w^d_k V_k^H F_k = \sum_{k=1}^{K} \sum_{i=1, i \neq k}^{K} W_i^1 \approx W_{N_k}(2N_1 \times (K - 1), I_{N_k}),$$  

(31)

where $W_i^1 \sim W_{N_k}(2N_1, I_{N_k})$ has a multivariate Wishart distribution with $I_{N_k}$ covariance matrix and $2N_2$ degrees of freedom. In general, if the covariance matrices are linear proportional to identity matrix, then the sum of Wishart random
variable follows a semi-correlated Wishart distribution \[18\]. However, if the covariance matrices are not proportional to the identity matrix, then deriving the sum of Wishart distribution is nontrivial \[19\]. The second term also is a Wishart distribution as follow,

\[ G^u^Hw^u^Hw^uG^u = W^P \sim \mathcal{W}_{N_u}(N_i, I_q) \]  

(32)

Therefore, the sum of two weighted central Wishart random variable where approximation for the distribution can be obtained as follows,

\[
\sum_{i=1,i\neq k}^K F_k^H V_k^H w_i^H V_k w_i^H F_k + \frac{p^u}{p^d} G^u^H w^u^H w^u G^u
\]

\[
\sum_{i=1,i\neq k}^K W_i^H + \frac{p^u}{p^d} W^P = \eta^u W^u,
\]

where, \( \eta^u = \frac{K(1+\frac{(N-1)^2}{N+2})}{K(1+\frac{2}{N-1})} \) and \( W^u \sim \mathcal{W}_{N_u}(N_s, I_q) \) with the following degree of freedom,

\[
N_s = \frac{N_t(p^d/K + K - 1)^2}{(p^d)^2(K+1)},
\]

(34)

therefore \( \Psi \) is given by,

\[
\Psi = \eta^u W^u + \sigma^d_0 = \eta^u W^u_k,
\]

(35)

where, \( \eta^u = \frac{N_s}{N_v + \sigma^d_0} \) and \( W^u_k \sim \mathcal{W}_{N_u}(2N_v, I_q) \) with the following degree of freedom,

\[
N_v = N_s/2 + \sigma^d_0(2N_s + \sigma^d_0)/2N_s.
\]

(36)

According to the definition of \( \Phi \) and \( \Psi \), Olkin in \[18\] show that if the \( \Psi^{1/2} \) is selected as a symmetric square root of \( \Psi \) and also if the covariance matrices of \( \Phi \) and \( \Psi \) are proportional to identity matrix then we have the relationship among them as

\[
\gamma^d_k = \frac{\Phi^{1/2} \Psi^{-1/2} \Phi^{1/2}}{\eta^u} W^v^{-1/2} W^d W^v^{-1/2}.
\]

(37)

From \[16\], \( W^v^{-1/2} W^d W^v^{-1/2} \sim \mathcal{B}^u_{N_u}(N_t, N_v) \) distributed as a multivariate beta type II distribution. By invoking the first and second moment, the distribution of \( \gamma^d_k \sim \mathcal{B}^u_{N_v}(N_1, N_2) \), where \( N_1 \) and \( N_2 \) are the degrees of freedom given as follows,

\[
N_1 = N_t(N_v + (N_v - 2)\eta^v + 1),
\]

(38)

\[
N_2 = N_v(N_t - 3\eta^v + 2 + N_v^2\eta^v + 2(\eta^v - 1)),
\]

(39)

and the pdf of the distribution is given in eq. \[27\]

\[ B^u_{N_v}(N_1, N_2) \]

B. Downlink with imperfect CSI

Similar to the uplink case, we consider imperfect CSI for CSI. The actual and the estimated channel are defined as,

\[
F_k = \sqrt{1 - \alpha^2} \hat{F}_k + \alpha \Delta^d_k,
\]

(40)

where similarly, \( \hat{F}_k \sim \mathcal{C}(0, I) \) is the downlink imperfect estimated channel with zero mean and unit variance and \( \Delta^d_k \sim \mathcal{C}(0, I) \) is the estimated downlink channel Gaussian noise. The received signal at the \( k_{th} \) SU will be as follows,

\[
y^d_k \triangleq \sqrt{p^d}(\hat{F}_k + \alpha \Delta^d_k) V^d w^d s^d_k + \sum_{i \neq k} \sqrt{p^d}(\hat{F}_i + \alpha \Delta^d_i) V^d w^d s^d_i
\]

\[
+ \sqrt{p^u} G^u w^u s^u + n^k_d.
\]

(41)

Similar to the perfect CSI case the received ID and EH signal elements are respectively denoted as,

\[
y^d_k = \sqrt{p^d} \sqrt{1 - \alpha^2} \hat{F}_k + \alpha \Delta^d_k V^d w^d s^d_k + \sum_{i \neq k} \sqrt{p^d}(\hat{F}_i + \alpha \Delta^d_i) V^d w^d s^d_i
\]

\[
+ \sqrt{p^u} G^u w^u s^u + n^k_d.
\]

(42)

\[
y^d_{ek} = \sqrt{1 - \rho_k} \sqrt{p^d}(\hat{F}_k + \alpha \Delta^d_k) V^d w^d s^d_k + \sum_{i \neq k} \sqrt{p^d}(\hat{F}_i + \alpha \Delta^d_i) V^d w^d s^d_i
\]

\[
+ \sqrt{p^u} G^u w^u s^u + n^k_d.
\]

(43)

\[ \text{Proposition 4.} \] For multi user downlink FD MIMO system with imperfect CSI and the processed SINR follows a matrix variate Beta type II distribution with parameters \( (a, b) \) and
defined as,
\[ \gamma_d^k = B_{N_u}^k \left( N_1, N_2 \right) \sim \frac{\det(\gamma_d^k) \left( 2N_1 - N_u - 1 \right)}{\beta(N1, N2)} \det(I_{k_k} + \gamma_d^k - N1 - N2), \]  
(44)
where

\[ \beta(N1, N2) = \frac{\Gamma_N(N1) \Gamma_N(N2)}{\Gamma_N(N1 + N2)}, \]  
(45)
and \( \Gamma_N(x) \) is the multivariate Gamma function given as,

\[ \Gamma_N(x) = \pi^{N_u(N_u - 1)/4} \prod_{i=1}^{N_u} \Gamma(x - (i - 1)/2). \]  
(46)

Proof: Similar to the perfect case, for simplicity we denote the received SINR of MU-MIMO with energy harvesting for \( k_{th} \) user at eq.24 as \( \gamma_d^k = \frac{\hat{\Phi}_k}{\hat{\Psi}} \) and \( \hat{\sigma}_d^k = \frac{\sigma_s^2}{(1 - \alpha^2)} \rho_d + \frac{\sigma_d^2}{1 - \rho_d^2}. \) Therefore, \( \hat{\Phi}_k \) has a central multivariate matrix Wishart distribution with \( I_{k_k} \) covariance matrix and \( 2N_t \) degrees of freedom as follows,

\[ \hat{\Phi}_k = \hat{W}_d^k \sim W_{\Gamma}(2N_t, I_{k_k}) = \frac{\gamma_d^k \left( 2N_1 - N_u - 1 \right)^2 e^{\frac{1}{2}tr(I_{k_k}^{-1} \gamma_d^k)}}{\beta(N1, N2) \det(I_{k_k})_{N1, N2}}, \]  
(47)
where \( W_{\Gamma}(y, z) \) is a central multivariate Wishart distribution. Furthermore, the first term of \( \hat{\Psi} \) in eq.24 is a sum of central Wishart distribution with similar identity covariance matrices can be simplified as follow,

\[ \sum_{i=1, i \neq k}^{K} \hat{F}_i^H V_i^d w_i^d H \hat{V}_d^i \hat{V}_d^i = \hat{W}_s^k, \]  
(48)
where \( \hat{W}_s^k \sim W_{\Gamma}(2N_t(K - 1), I_{k_k}) \) has a multivariate Wishart distribution. Similarly, in the second term of \( \hat{\Psi} \) in eq.24 is a sum of central Wishart distribution with similar identity covariance matrices can be simplified as follow,

\[ \sum_{i=1}^{K} \Delta_i^H V_i^d w_i^d H \hat{V}_d^i \Delta_i^d = W^A \]  
(49)
where $\hat{W}^\Delta \sim \mathcal{W}_N(K, \text{I}_{\text{I}_k})$ has a multivariate Wishart distribution with $\text{I}_{\text{I}_k}$ covariance matrix and $2N_i$ degrees of freedom. The third term also is a Wishart distribution as follows,

$$G^uHwu^Hw^uG^u = \hat{W}^p \sim \mathcal{W}_N(K, \text{I}_{\text{I}_k}).$$

Therefore, the sum of three weighted central Wishart random variables where approximation for the distribution can be obtained as follows,

$$\sum_{i=1,i \neq k}^K \hat{W}_i^\alpha + \frac{\hat{W}_k^\alpha}{1-\alpha} = \frac{\hat{W}_k^\alpha}{1-\alpha} \hat{W}^g,$$

where, $\hat{W}^g = \frac{1}{K(1+\frac{\eta}{K})\eta - 1}$ and $\hat{W}^g \sim \mathcal{W}_N(N_{\text{g}}, \text{I}_{\text{I}_k})$ with the following degree of freedom,

$$N_{\text{g}} = \frac{2N_i(\frac{\eta}{K} - 1)K - 1}{(\frac{\eta}{K} - 1)K - 1},$$

and

$$\hat{W}^q = \frac{\eta^2}{N_{\text{q}} - 1} \hat{W}^q + \frac{\eta^2}{N_{\text{q}} - 1} \hat{W}^q = \frac{\eta^2}{N_{\text{q}} - 1} \hat{W}^q,$$

where, $\hat{W}^q = \frac{\eta^2}{N_{\text{q}} - 1} \hat{W}^q$ and $\hat{W}^q \sim \mathcal{W}_N(2N_i, \text{I}_{\text{I}_k})$ with the following degree of freedom,

$$N_{\text{q}} = \frac{(\frac{2N_iK\alpha^2}{(\alpha - 1)^2} + N_{\text{q}}\eta^2)}{2N_i(K + 1)},$$

therefore $\psi$ is given by,

$$\psi = \frac{\eta^2}{N_{\text{q}} - 1} \hat{W}^q + \frac{\eta^2}{N_{\text{q}} - 1} \hat{W}^q,$$

where,

$$\hat{W}^q = \frac{\eta^2}{N_{\text{q}} - 1} \hat{W}^q$$

and

$$\hat{W}^q \sim \mathcal{W}_N(2N_i, \text{I}_{\text{I}_k})$$

with the following degree of freedom,

$$N_{\text{q}} = \frac{(\frac{2N_iK\alpha^2}{(\alpha - 1)^2} + N_{\text{q}}\eta^2)}{2N_i(K + 1)}.$$

Similar to the perfect CSI case, we have,

$$N_1 = \frac{N_k(N_k + (N_v - 2)\hat{q}^k + 1)}{\hat{q}^k(N_k + N_v - 1)},$$

$$N_2 = \frac{N_k(N_k - 3\hat{q}^k + 2) + N^2\hat{q}^k + 2(\hat{q}^k - 1)}{N_k + N_v - 1},$$

and the pdf of the distribution is given in eq. 44.

V. SIMULATION RESULTS

In this section, we study the performance of the proposed closed-form approximation of the SINR of the downlink SWIPT-MIMO with antenna selection and the uplink MU-MIMO with ZFBF. We assume that the number of receive and transmit antennas at the AGG is equal and larger than the number of SUs. The total bandwidth is 10 MHz. We also consider 3 SUs facilitated with 2 transceiver antennas. The channel uncertainty is considered as $\alpha = 0.2$. The power split factor is assumed to be $\rho = 0.3$ and the power efficiency at the EH unit is $\eta_{\text{I}} = 40\%$ for all SUs and the harvested energy is stored in a 3.2 v 20Ah battery. Fig. 4 and Fig. 5 show the empirical data and chi-square distribution as a simplified form of the proposed Wishart distribution for a different number of receive antennas for perfect an imperfect CSI. Note that the number of received antennas increases, the distributions pdf moves to the higher SINR region and the estimation becomes more accurate. Fig. 4 and Fig. 5 show the empirical data and Beta distribution of type II as a simplified form of the proposed multivariate Beta distribution of type II for a different number of receive antennas of the downlink SWIPT MU-MIMO with antenna selection for perfect an imperfect CSI. Numerical results show that the proposed closed-form approximation offers a perfect match with the empirical data in a wide range of the number of antennas.

VI. CONCLUSION

In this paper a new low complexity closed-form approximation of the Full-duplex SWIPT MU-MIMO system SINR distribution for the received signal at the sensor users with perfect and imperfect CSI and transmit antenna selection scheme at the transmitter is proposed. SINR distribution of the uplink with ZFBF and antenna selection at the AGG receiver with transmit antenna selection also studied. The uplink SINR with perfect and imperfect CSI is modeled with multivariate Wishart distributions and the downlink SWIPT with perfect and imperfect CSI is modeled by a multivariate Beta type II distribution. The proposed SINR distributions analytical results are compared to the Monte-Carlo simulations and we obtained a perfect match.

REFERENCES

[1] A. Shadmand, K. Nehra, and M. Shikh-Bahaei, “Cross-Layer Design in Dynamic Spectrum Sharing Systems,” EURASIP Journal on Wireless Communications and Networking, vol. 2010, no. 1, p. 458472, 2010.

[2] K. Nehra, A. Shadmand, and M. Shikh-Bahaei, “Cross-Layer Design for Interference-Limited Spectrum Sharing Systems,” in 2010 IEEE Global Telecommunications Conference GLOBECOM 2010. IEEE, dec 2010, pp. 1–5.

[3] A. Olaf and M. Shikh-Bahaei, “Optimum power and rate adaptation with imperfect channel estimation for MQAM in rayleigh flat fading channel,” in VTC-2005-Fall. 2005 IEEE 62nd Vehicular Technology Conference, 2005., vol. 4. IEEE, pp. 1793–1797, 2005.

[4] M. M. Mahyari, A. Shojaiifar, and M. Shikh-Bahaei, “Probabilistic Radio Resource Allocation Over CDMA-Based Cognitive Radio Networks,” pp. 3560–3565, 2015.

[5] V. Towsifudin and M. Shikh-Bahaei, “Cooperative ARQ in full duplex cognitive radio networks,” in 2016 IEEE 27th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC). IEEE, sep 2016, pp. 1–5.

[6] M. Nasirzadeh, S. A. Ghorbani, and M. Shikh-Bahaei, “Full-duplex Device-to-Device collaboration for low-latency wireless video distribution,” in 2017 24th International Conference on Telecommunications (ICT). IEEE, may 2017, pp. 1–5.

[7] Shi Jin, M. McKay, Kai-Kit Wong, and Xiqi Gao, “Transmit Beamforming in Rayleigh Product MIMO Channels: Capacity and Performance Analysis,” IEEE Transactions on Signal Processing, vol. 56, no. 10, pp. 5204–5211, oct 2008.

[8] S. Malla and G. T. F. de Abreu, “SINR Distributions for Robust MIMO TX-Power Minimization With Controlled Outage,” IEEE Systems Journal, pp. 1–12, 2017.

[9] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, “Multiuser MIMO Achievable Rates With Downlink Training and Channel State Feedback,” IEEE Transactions on Information Theory, vol. 56, no. 6, pp. 2845–2866, jun 2010.

[10] B. Nosrat-Makouei, J. G. Andrews, and R. W. Heath, “MIMO Interference Alignment Over Correlated Channels With Imperfect CSI,” IEEE Transactions on Signal Processing, vol. 59, no. 6, pp. 2783–2794, jun 2011.

[11] D. Lee, “Performance Analysis of ZF-Preceded Scheduling System for MU-MIMO with Generalized Selection Criterion,” IEEE Transactions on Wireless Communications, vol. 12, no. 4, pp. 1812–1818, apr 2013.

[12] C. Yuk-Fan Ho, B. Wing-Kuen Ling, Zhi-Wei Chi, M. Shikh-Bahaei, Yan-Qun Liu, and Koki-Lay Tay, “Design of Near-Allpass Structurally Stable Minimal-Phase Real-Valued Rational IIR Filters,” IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 55, no. 8, pp. 781–785, aug 2008. [Online]. Available: http://ieeexplore.ieee.org/document/4642070.

[13] T. Bergel, V. Veres, and S. Shamma, “Uplink Downlink Rate Balancing and Throughput Scaling in FDD Massive MIMO Systems,” IEEE Transactions on
[14] D. Hwang, D. I. Kim, and T.-J. Lee, “Throughput Maximization for Multiuser MIMO Wireless Powered Communication Networks,” IEEE Transactions on Vehicular Technology, vol. 65, no. 7, pp. 5743–5748, jul 2016. [Online]. Available: http://ieeexplore.ieee.org/document/7151842/

[15] D. S. Watkins, Fundamentals of matrix computations. Wiley, 2010.

[16] A. K. Gupta, Matrix Variate Distributions. Chapman & Hall/CRC, 1999. [Online]. Available: [http://www.amazon.com/exec/obidos/redirect?tag=citeulike07-20&path=ASIN/1584880465]

[17] G. Arthur Frederick Seber, A matrix handbook for statisticians, 2008.

[18] M. Ivrlac, W. Utschick, and J. Nossek, “Fading correlations in wireless MIMO communication systems,” IEEE Journal on Selected Areas in Communications, vol. 21, no. 5, pp. 819–828, jun 2003. [Online]. Available: http://ieeexplore.ieee.org/document/1203167/

[19] S. Kumar, “Eigenvalue statistics for the sum of two complex Wishart matrices,” EPL (Europhysics Letters), vol. 107, no. 6, p. 60002, sep 2014. [Online]. Available: http://stacks.iop.org/0295-5075/107/i=6/a=60002?key=crossref.c939d6ca2966b7c9588be85c95851800