Numerical simulation in diagrams for concrete deformation in computations of strength of reinforced concrete elements in deformation model

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Abstract. The paper establishes correlation between the integral parameters of nonlinear diagrams for concrete deformation in the versions of Russian and foreign regulatory documents applied to computation of strength of the reinforced concrete elements in the deformation model. It is recommended to use the fragments of the areas boarded by the branches of diagrams, as well as the coefficient of the diagram completeness as integral parameters of the diagrams for concrete deformations. The diagram area with the predetermined values of the element curvature is equivalent to the value of the ultimate strain in the compressed zone of concrete in the bendable elements under off-center compression, and the geometry of the stress diagram corresponds to this diagram. The study offers analytical expressions establishing the connection of strength and deformation parameters for the classes of concrete, which have standard values of deformation and stress in the reference points of curvilinear diagrams. The article presents the results of the computation and establishes the interconnection between the parameters of diagrams obtained by analytic correlation between stresses and deformations in the considered regulatory documents. There are recommendations for defining standard integral parameters of diagrams in calculating the strength of reinforced concrete elements using the deformation model.

1. Introduction

For the design of the structures using the deformation model in regulating documents [1-4], it is recommended to use various kinds of diagrams for concrete deformation and analytical correspondence demonstrating the interrelations of relative deformations with the strains $\varepsilon_b - \sigma_b$ in case of uniaxial compression and stretching. Curvilinear diagram with the ascending and descending branches for deformation fully corresponds to the physical properties of concrete and experimental data of testing standard concrete samples. In the description of diagrams for concrete deformation during compression in the versions [5-9], the following reference points are used: at the top of the diagram on the ascending branch; at the end of the descending branch, in which the deformations reach maximum values. The dependencies of diagram descriptions, methods of defining standard deformations in the reference points in the versions of Russian [1,4] and foreign [2,3] regulatory documents bring to disagreements in the values of ultimate strain in calculating the strength of reinforced concrete components. Table presentation of standard deformation values for the class of concrete compressive strength complicates the operation of the automatic non-linear calculation.
process for defining the strength of the reinforced concrete element in the deformation model. In the regulatory documents, the value of nonlinear deformations decreases with the increase of the concrete class; thus, the descending branch is excluded from the calculation for high-strength concrete types. Nevertheless, the limits of the descending branch depend not only on the physical properties of the material, but also on the stress state of the element. In case of small deformation gradients, stress redistribution is impossible, and the limiting deformations should be restricted by their values at the top of the diagram. In such a case, it is necessary to disclose the correlation of parameters in the limit state according to the truncated and complete diagrams for all classes of concrete. There are important objectives: to disclose the correlation between the concrete classes in terms of compressive strength in the versions of Russian and foreign standards; to determine unambiguous values of deformations in the reference points of the diagrams by approximating their standard values. The present research is important for perfecting the computational models of reinforced concrete strength resistance [10-12] and developing the theory of durability of structural systems of buildings and premises [13].

2. Setting the aim and the tasks of the research
To develop an efficiency suggestion to complement the automated methods for the computation of strength of the construction elements by the analytical form of determining the standard values of deformations in the reference points of concrete diagrams, taking into account the acknowledged functional connections and the rules of their applications to the tables of the regulatory documents. To establish the correlation of the integral parameters of the diagrams, recommended by the regulatory documents for the classes of concrete compressive strength, taking into account the physical properties of concrete and the stress state of the component.

3. Methods of study
In the regulatory documents [2,3], the class of concrete in terms of axial compression strength is marked by the letter C and numbers, for example, C12/15 (before the slash there are values of standard resistance \( f_{ck} \) - compressive strength of cylinders, 150 mm in diameter and 300 mm in height, tested at the age of 28 days; after the slash there are values \( f_{c,cube}^{G} \) - guaranteed strength of concrete cube, size 150×150×150 mm, with a parameter 0,95 measured in MPa. Russian standards are based on the strength of the cube, that is why further in the text (Table 1.2), the correspondence between classes C and B is established according to this principle (for example, class C12/15 corresponds to the class of concrete B15, etc.). When assessing the deformation properties of concrete in [2, 3], the average values of compressive strength are introduced: \( f_{cu} = f_{ck} + \Delta f \) , where \( \Delta f = 8 \) MPa. Diagrams of concrete in compression are constructed with the coordinates “\( \varepsilon_{bc} \) – \( \varepsilon_{bc} \)”, \( \sigma_{bc} \) – \( \sigma_{bc} \)” (in the brackets there are the markings of deformations and stresses used in [2, 3]). In curvilinear diagrams, the following points are used as reference points in strength calculations: the top of the diagram on the ascending branch with coordinates \( \varepsilon_{c,b} \), \( \sigma_{c,b} \); at the end of the descending branch with maximum values of deformations and coordinates \( \varepsilon_{cu} \), \( \sigma_{cu} \) . The analytical dependence, which establishes interconnection of deformations and stresses in [2,3], is presented in the following way in calculating the strength of the elements:

\[
\frac{\sigma_{cu}}{f_{cu}} = \frac{k\eta - \eta^2}{1 + (k-2)\eta}
\]

where: \( \eta = \varepsilon_{c,b}/\varepsilon_{c,b} \); \( |\varepsilon_{c,b}| \leq |\varepsilon_{cu}| \); \( k = 1,1E_{cm} \varepsilon_{c,b}/f_{cu} \), here \( E_{cm} \) is a module of elasticity for the concrete at the age of 28 days, \( f_{cu} \) is a design resistance of concrete to the axial compression.

Standard values of deformation in the reference points \( \varepsilon_{c,b} \) and \( \varepsilon_{cu} \) in the design of stresses using the formula (1) for different classes of concrete are presented in the table format [2] (Table. 6.1). Table values of deformations in the reference points during the computation of the first and second limit
states of the reinforced concrete components are recommended to be approximated by analytical dependencies:

\[ \varepsilon_{c1} = -1.6\left(\frac{f_{cm}}{10\text{MPa}}\right)^{0.25} / 1000 \quad \varepsilon_{cu1} = \varepsilon_{c1}\left(1 - \frac{f_{cm} - f_{cm}^*}{81\text{MPa}}\frac{10\text{MPa}}{f_{cm}}\right)^{0.2} \]  

(2)

where \( f_{cm}^* \) is a fixed value of the average concrete strength for the class of concrete, which is accompanied by excluding the descending branch from the calculations and applying the equations: \( |\varepsilon_{c1}| = |\varepsilon_{cu1}| \) and \( f_{cd} = f_{cm} \).

Analytical correlation of deformation and strain in case of the axial compression of concrete in versions [1, 5, 6, 7, 8] is presented as

\[ \varepsilon_i = \frac{\sigma_i}{E_i\nu_i} \]  

(3)

where \( \nu_i \) is a coefficient of the secant module of deformation.

Deformations \( \hat{\varepsilon}_b \) at the top of the diagram, unlike in [2,3], are defined not according to the tables of standards, but to the equation, which takes into account class B and the type of concrete. Nevertheless, the design values of deformations \( \hat{\varepsilon}_b \) and \( \varepsilon_{bu} \), in case the concrete belongs to the class for compression, can be different from the standard values, which, finally, influence the accuracy of determining the applied strength in the limiting state of the element. For the heavy-weight kinds of concrete, it is necessary to introduce analytical dependencies of defining the deformation in the reference points \( \hat{\varepsilon}_b \) and \( \varepsilon_{bu} \), which approximate their standard values.

\[ \hat{\varepsilon}_b = 1.75\left(\frac{B}{10\text{MPa}}\right)^{0.2} / 1000 \quad \varepsilon_{bu} = \hat{\varepsilon}_b\left(1 - \frac{B - B^*}{98\text{MPa}}\frac{10\text{MPa}}{B}\right)^{0.2} \]  

(4)

where \( B^* \) is a fixed class of concrete, causing the exclusion of the descending branch from design and the application of the following equations: \( |\hat{\varepsilon}_b| = |\varepsilon_{cu}| \) and \( \sigma_{bu} = R_b \).

The values of deformations in the reference points are determined by average strains \( f_{cm} \) in formula (2), and for concrete class B, in formula (4) correspondingly. It means that the values of deformation in the reference points do not depend on the values of design resistance of concrete, and the formulae for the computation of deformations in the reference points can be used in computations for the limit states of the first group, as well as the second one. There follows a common rule for working with the diagrams. If in the construction of the diagram, the deformations are predetermined, and the strains are calculated, the maximum values of deformation are limited by the values \( \varepsilon_{cu} \) (2) and \( \varepsilon_{bu} \) (4). If the strains are predetermined, and the deformations are calculated \([1,4]\), then the minimum values of strains on the descending branch are limited by the value of relative strains \( \eta_{bu} \), calculated according to the formula:

\[ \eta_{bu} = 1 + \hat{\lambda}_b \frac{B - B^*}{B + B^*} \]  

(5)

where \( \eta_{bu} = \sigma_{bu} / R_b \), here \( B^* \) is a fixed class of concrete, and the descending branch of concrete is, accordingly, excluded from calculations.

If to consider foreign experience, then formula (1) leads to the conclusion that the minimum value of relative strains on the descending branch \( \eta_{bu} = f_{cm} / f_{cm}^* \) (if \( \sigma_{e} = f_{cm} \)), changes in linear dependence in the range from 0.9 to 1 with the increase of the class of concrete compressive strength. According to
the standards [1, 4], for the types of concrete with low strength, it is recommended to take the value 0.85; in that case, in the formula (5) \( \lambda_b = 0.2 \), and there is a linear dependence \( \eta_{bu} \) within the range from 0.85 to 1.

The descending branch is constructed on the basis of the expression:
\[
\eta_b = 1 + (\eta_{bu} - 1) \left( \frac{\eta_d - 1}{\eta_{du} - 1} \right)^2
\]

where \( \eta_{du} = \hat{\varepsilon}_{bu} / \hat{\varepsilon}_b \), \( \eta_d = \varepsilon_{bu} / \varepsilon_b \), these are the current values of the relative deformations.

4. Results of calculations
After bringing the classes of concrete B and C (on the basis of the concrete cube strength) in accordance using the tables of the regulatory documents, correspondingly, [1] and [2] for classes of concrete compressive strength with the concrete classes of compressive strength \( B_{15} \pm 105 \) and \( C_{12} \pm C_{90} \), the standard resistance of concrete to compression \( R_{bn} \) and \( f_{ck} \) are defined. Design resistances \( R_b \) and \( f_{cd} \) are calculated by dividing the standard compressive resistances of concrete to compression, accordingly, \( R_{bn} \) is divided by the reliability coefficient of concrete \( \gamma_b = 1.3 \), while \( f_{ak} \) is divided by the safety coefficient of concrete \( \gamma_c = 1.5 \) when calculating the limit states of the first group (a special coefficient \( \gamma_{HSC} \) [2] is taken into account for high-strength concrete types). For a truncated diagram of concrete (without taking into account the descending branch), formulae (2) and (4) are used to calculate the deformations in the tops of the diagrams \( \varepsilon_{c1} \) and \( \hat{\varepsilon}_b \), the values of which for these diagrams are maximum (Table 1).

| SNB [3] | Class of concrete | \( \varepsilon_{c1} \) [%] | C12 | C25 | C35 | C50 | C60 | C70 | C80 | C90 |
|---------|-----------------|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| S, MPa* [%] | 19.9 | 40.29 | 57.4 | 80.4 | 95.0 | 104.6 | 117 | 125 |
| \( \omega \) | 0.82 | 0.785 | 0.764 | 0.74 | 0.73 | 0.729 | 0.722 | 0.724 |

| SP [1] | Class of concrete | \( \hat{\varepsilon}_b \) [%] | 1.9 | 2.18 | 2.36 | 2.5 | 2.62 | 2.68 | 2.75 | 2.8 |
|---------|-----------------|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| S, MPa* [%] | 19.9 | 41.0 | 60.8 | 80.0 | 94.9 | 103.5 | 112.5 | 120 |
| \( \omega \) | 0.81 | 0.781 | 0.76 | 0.74 | 0.729 | 0.723 | 0.717 | 0.711 |

For the complete diagrams of concrete constructed with the formulae (2) and (4), the maximum values of deformations are calculated on the descending branch of the diagrams \( \varepsilon_{cu} \) and \( \hat{\varepsilon}_{bu} \), while formulae (1) and (5) are used to calculate the values of stress \( f_{cu} \) and \( \sigma_{bu} \), corresponding to these deformations (Table 2). Design dependencies (2) \( \eta \) and (4) are accurate enough in approximating the standard table values of deformations in the reference points of the diagrams. The general tendency for changing the stresses in the reference end points of the diagrams is preserved.
Table 2. Design parameters of complete diagrams of concrete deformation.

| Class of concrete | of | C12 | C25 | C35 | C50 | C60 | C70 | C80 | C90 |
|-------------------|----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\varepsilon_{u1}$ [%o] |    | 3.5 | 3.5 | 3.47| 3.35| 3.24| 3.11| 2.98| 2.83|
| $f_{cu}$, MPa    |    | 7.4 | 15.5| 21.9| 31.8| 38.1| 42.1| 47.5| 50.2|
| $S$, MPa *[%o]   |    | 26.7| 53.3| 72.3| 97.8| 110.0|116.5|123.8|125  |
| $\omega$         |    | 0.89| 0.859|0.836|0.803| 0.78 | 0.767|0.742|0.724|
|                |    |     |     |     |     |     |     |     |     |
| Class of concrete | of | B15 | B30 | B45 | B60 | B75 | B85 | B95 | B105 |
| $\varepsilon_{u2}$ [%o] |    | 3.5 | 3.5 | 3.44| 3.31| 3.20| 3.04| 2.92| 2.8  |
| $\sigma_{iu}$, MPa |    | 7.3 | 15.2| 23.2| 31.3| 37.7| 41.8| 45.6| 49.0 |
| $S$, MPa *[%o]   |    | 27.2| 53.8| 76.3| 96.0| 108.0|113.4|117.6|119.4|
| $\omega$         |    | 0.874|0.851|0.827| 0.8  |0.773 |0.754|0.734|0.711|

The correlation between the diagrams is set by integral parameters, which are: $S$ as the area of the fragment bounded by the branches of the diagrams; $\omega$ as the coefficient of diagram completeness, equal to the ratio of area $S$ to the area of the rectangle, the sides of which pass through the reference points of the diagrams (Tables 1, 2). The parameters $S$ and $\omega$ may differ in the diagrams under consideration since the strength and deformation characteristics of concrete in the corresponding classes of concrete compressive strength $B$ and $C$ in the regulatory documents [1] and [2] have different values. However, the unambiguous values of deformations in the reference points of the diagrams obtained with the help of expressions (2, 4, 5, 6), allow us to find corresponding values of integral diagrams parameters, and, in case of equal curvature of the element, to find the value of forces applied in the limit state. In case of insignificant centering errors for deformations when redistribution of pressure is impossible, it is necessary to use the truncated diagrams in the design of element durability (without taking into consideration the descending branch). Taking into account the stressed state of the element is important for the classes of concrete with low strength. As for the classes of concrete with high strength, the value of non-linear deformations in the regulatory documents decreases, and the physical properties of concrete take deformation gradient into account.

5. Conclusions
Numerical modeling of parameters of nonlinear diagrams demonstrating concrete deformation recommended by Russian and foreign regulatory documents for the design of reinforced concrete elements in the first and second ultimate strain states allows to establish the bilateral correlation of the computation results for ultimate strain applied to classes of concrete compressive strength.

6. Recommendations
Aiming to simplify the method of computation for the strength of the concrete elements in the deformation model, it is recommended to turn from standardizing the deformation values in the reference points of the diagram to standardizing their integral parameters. The uniformed approach in computations of ultimate strain allows avoiding the procedure of numerical integration of stresses in the bars taking into account the height of the element while using automated methods.

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