Yang-Mills theories using only extended fields (vectorial and scalar) as gauge fields

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**Abstract**

A few years ago H. Morales and the author introduced a type of generalized derivative that contained both vector and scalar boson fields. Here it is shown how to construct a full-fledged generalized Yang-Mills theory through the introduction of extended field multiplets. These are mixed fields that include both a vector and a scalar part. It is shown how the standard model of high energy physics appears naturally in a Yang-Mills theory that uses extended field multiplets through two spontaneous symmetry breakings, one due to the VEV of a scalar field and another to the VEV of a vector field.

About 4 years ago in a series of papers H. Morales and I showed how to write a generalized covariant derivative that used both vector and scalar bosons. Some of the Lie group generators $T^a$ were associated with scalar and some with vector fields. I am presenting here related but more general results I have recently obtained, on how to build a full nonabelian gauge theory in terms of extended fields, which are boson fields that have both vector and scalar components. In the old way of doing it some fields were chosen to be vectorial and other scalar; now they are all mixed. I will then show that the high energy standard model is the unitary gauge of one of these theories after spontaneous symmetry breaking (SSB).

I will first give a representative example of a nonabelian gauge theory, and then show how to generalize it to a theory with extended fields. Take the Lie Group to be $SU(N)$. Let $A^a_\mu$, $a = 1,\ldots,N^2-1$, be the fields to be associated with the generators $T^a$. The fermions $\psi(x)$ are in a spinorial representation of the Lorentz group, say the fundamental $N$ of $SU(N)$. In this context we would have $A^a_\mu T^a \rightarrow UA^a_\mu T^a U^{-1}$, where $U$ is an element of the fundamental representation, that is, the $A^a_\mu$ transforms in the adjoint.

Treating the $A^a_\mu$ as dynamical fields obeying a gauge principle, we postulate the gauge transformation

$$A_\mu \rightarrow UA_\mu U^{-1} + U(\partial_\mu U^{-1})$$
and the covariant derivative:

\[ D_\mu = \partial_\mu + A_\mu, \quad A_\mu = igA_\mu^a(x)T^a, \]

where \( g \) is a coupling constant. It follows then that under the gauge transformation the covariant derivative transforms as

\[
D_\mu f = (\partial_\mu + A_\mu)f \\
\rightarrow \partial_\mu f + U(\partial_\mu U^{-1})f + U A_\mu U^{-1} \\
= U(\partial_\mu + A_\mu)(U^{-1}f),
\]

or simply

\[ D_\mu \rightarrow UD_\mu U^{-1}. \]

The Lagrangian of the YMT is then the obviously invariant

\[
\mathcal{L}_{YMT} = \bar{\psi} i[D_{\mu}, \psi] + \frac{1}{2g^2} \widetilde{\text{Tr}} ([D_\mu, D_\nu][D_\mu, D_\nu]),
\]

which can be written in the more common form

\[
\mathcal{L}_{YMT} = \bar{\psi} i(\bar{\psi} + A)\psi + \frac{1}{2g^2} \widetilde{\text{Tr}} \left( (\partial_{[\mu} A_{\nu]} + [A_{\mu}, A_{\nu}])^2 \right).
\]

The \( g \) is a coupling constant. Notice that after doing all the algebra there are no partials left acting to the right. The trace \( \widetilde{\text{Tr}} \) is over the \( SU(N) \) group generators, where the tilde is used to differentiate it from the trace over Dirac matrices.

The covariant derivative has to be generalized in the spinorial representation; this cannot be done in the vectorial representation. We need the following theorem, whose proof can be found in Refs. 2 and 4.

**Theorem.** Let \( D_\mu = \partial_\mu + B_\mu \), where \( B_\mu \) is a vector field (either abelian or nonabelian). Then:

\[
((\partial_{[\mu} B_{\nu]} + [B_\mu, B_\nu])^2 = \frac{1}{8} \text{Tr}^2 \varphi^2 - \frac{1}{2} \text{Tr} \varphi^4.
\]

Using the Theorem we can write the kinetic energy of a YMT in the spinorial representation of the Lorentz group:

\[
\frac{1}{2g^2} \widetilde{\text{Tr}} \left( (\partial_{[\mu} A_{\nu]} + [A_{\mu}, A_{\nu}])^2 = \frac{1}{2g^2} \widetilde{\text{Tr}} \left( \frac{1}{8} \text{Tr}^2 \varphi^2 - \frac{1}{2} \text{Tr} \varphi^4 \right) .
\]

We proceed to define the extended fields

\[ \Upsilon \equiv \bar{\Upsilon} + \Phi = \gamma^\mu A_\mu + \gamma^5 \varphi \quad (1) \]

where

\[ A_\mu = igA_\mu^a(x)T^a, \quad \varphi = -g \varphi^a T^a; \]
the $\varphi^a$ are boson fields. (Alternatively we could define $\tilde{\Upsilon} = \Upsilon^a T^a$, $\Upsilon^a \equiv \gamma^\mu i g A^a_\mu - \gamma^5 g \gamma^a$.) We require these dynamical fields to obey the gauge transformation law

$$\tilde{\Upsilon} \rightarrow U \tilde{\Upsilon} U^{-1} - (\partial U) U^{-1}.$$ 

The generalized covariant derivative is defined to be

$$D = \partial + \tilde{\Upsilon}, \quad (2)$$

so that the covariant derivative has to transform as

$$D \rightarrow U D U^{-1}.$$ 

Finally, we define the gauge invariant Lagrangian of a Yang-Mills theory with extended fields to be

$$\mathcal{L} = \bar{\psi}iD\psi + \frac{1}{2g^2} \tilde{\text{Tr}} \left( \frac{1}{8} \text{Tr}^2 D^2 - \frac{1}{2} \text{Tr} D^4 \right),$$

where the $\psi$ are the fermion fields.

Expanding the covariant derivative and taking the Dirac traces of this Lagrangian one obtains:

$$\mathcal{L} = \bar{\psi}(i\partial + A)\psi + \frac{1}{2g^2} \tilde{\text{Tr}} \left( \left( \partial_\mu A_\nu \right) [A_\mu, A_\nu] \right)^2 - g \bar{\psi} i\gamma^5 \varphi^a T^a \psi + \frac{1}{g^2} \tilde{\text{Tr}} \left( \left( \partial_\mu \varphi^a \right) [A_\mu, \varphi^a] \right)^2. \quad (3)$$

The first line is a Yang-Mills theory; the second line is Yukawa terms; the third is the usual kinetic energy of the scalar bosons in a Yang-Mills theory. Notice that there are no interactions between the scalar fields higher than quadratic.

As an example we work out the theory resulting from the gauge group $SU(6)$; it is similar to $SU(5)$ unification but with some advantages. We put the extended gauge fields in the adjoint $35$ so that the covariant derivative is

$$D = \partial + (35) = \partial + \tilde{\Upsilon}.$$ 

A maximal subgroup of $SU(6)$ is $SU(5) \times U(1)$, and the obvious course is to try to obtain the $SU(5)$ grand unified theory (GUT) through SSB. This is the way Morales and I proceeded a few years ago. The branching rule for this maximal subgroup is

$$35 = 1 + 5_6 + 5_{-6} + 24_0,$$

or, in a simplified symbolism,

$$(35) = \left( \begin{array}{cc} 24 & 5 \\ 5^t & 1 \end{array} \right).$$
We took the 5 to be scalar Higgs field that gives masses to the $W^\pm$ and the $Z^0$ and the 24 to contain the gauge vectors of the $SU(5)$ Yang-Mills.

Besides involving rather arbitrary choices of which fields to make scalar or vector, this way of doing things has a problem that was not noticed at first. In the usual $SU(5)$ GUT, the Higgs field that eventually generate the very high masses for the leptoquark vector bosons are contained in another 24. After SSB the 12 scalar fields on the block diagonal vanish in the unitary gauge and reappear as the third degrees of freedom of the 12 off block diagonal vector bosons that have become massive. However, in the scheme Morales and I used the symmetry breakings are done by Higgs fields associated with diagonal generators of the 35, singlets of the $24 \subset 35$, so there are no dynamical degrees of freedom available to cover for the longitudinal degree of freedom the vector bosons develop after they have become massive.

What I am proposing now is to always assume all gauge fields are extended. That is, the gauge field associated to each generator of the covariant derivative must be extended in the way specified by (1). It turns out that the standard model appears if one then assumes two SSBs. In particular, the vector part of the 5 obtains a large mass, so that the massless Higgs structure appears in a natural fashion.

The Lie group $SU(6)$ has 5 diagonal generators, which we, with the benefit of hindsight, will take to be (writing the diagonal components only):

\[
\begin{align*}
T^1 &= (1 -1 0 0 0 0), \\
T^2 &= (1 1 -2 0 0 0), \\
T^3 &= (0 0 0 1 -1 0), \\
T^4 &= (2 2 2 -3 -3 0), \\
T^5 &= (1 1 1 -1 -1 -1).
\end{align*}
\]

This is not quite the same original choice Morales and I decided upon. We used a different fifth generator,

\[
T^5 = (1 1 1 1 1 -5), \quad \text{(not used)}
\]

in accord with our idea that what was involved was the maximal subalgebra $SU(6) \supset SU(5) \times U(1)$. The choice I am giving above corresponds to the maximal subalgebra $SU(6) \supset SU(3) \times SU(2) \times U(1) \times U(1)$. This is a full bona fide maximal subalgebra, although it has the peculiarity of having two $U(1)$ subgroups. It is generated by having VEVs associated with the $T^4$ and $T^5$ generators. The crucial point here, in order to obtain precisely the standard model out of pure extended fields, is that one of the two fields with a VEV must be a vector field. It would not be acceptable to have the vector field developing a nonzero VEV being one of the vector carriers of the known forces, but, as we shall see, the vector field with the VEV is the one associated with the generator $T^5$ and is not a carrier of one of the known forces. Making the assumption, usual in GUTs, that the mass of a Higgs boson is of the order of its VEV, it would be an extremely heavy vector boson, and thus would not be observable. This situation raises the possibility of a small Lorentz breaking. Different such scenarios,
particularly due to high-level corrections from string theory, has been studied in the
last few years.\(^5\)

In expansion (3) of the Lagrangian of the generalized Yang-Mills theory there are
two terms that allow mass generation:

\[
\frac{1}{g^2} \tilde{\text{Tr}} \left(\left( [A_\mu, \varphi] \right)^2 \right), \quad \frac{1}{2g^2} \tilde{\text{Tr}} \left(\left( [A_\mu, A_\nu] \right)^2 \right). \tag{4}
\]

We conclude from their study that a scalar boson with a VEV can generate masses in
vector fields but not in other scalar fields, and that a vector with a VEV can generate
masses in both vector fields and scalar fields.

There is a coupling between two fields when their commutator is not zero. Let us
assume that the vector part of the extended field associated with \(T^5\) has a VEV \(v\).
Then the couplings of the extended fields in \(\Upsilon\) with the VEV are given by

\[
[vT^5, \bar{\Upsilon}] = v \begin{pmatrix}
0 & 0 & 0 & 2\Upsilon_- & 2\Upsilon_- & 2\Upsilon_- \\
0 & 0 & 0 & 2\Upsilon_- & 2\Upsilon_- & 2\Upsilon_- \\
-2\Upsilon_+ & -2\Upsilon_+ & -2\Upsilon_+ & 0 & 0 & 0 \\
-2\Upsilon_+ & -2\Upsilon_+ & -2\Upsilon_+ & 0 & 0 & 0 \\
-2\Upsilon_+ & -2\Upsilon_+ & -2\Upsilon_+ & 0 & 0 & 0
\end{pmatrix}.
\]

Thus all fields (vector or scalar) on those two corners acquire large masses. Let us
now assume that the scalar part of the extended field associated with \(T^4\) has a VEV \(w\). Then the coupling of this scalar field to the others in \(\Upsilon\), as given by (4), is only
to their vector part. The fields that get a mass are:

\[
[wT^4, \bar{\Upsilon}] = w \begin{pmatrix}
0 & 0 & 0 & 5\Upsilon^V_- & 5\Upsilon^V_- & 2\Upsilon^V_- \\
0 & 0 & 0 & 5\Upsilon^V_- & 5\Upsilon^V_- & 2\Upsilon^V_- \\
-5\Upsilon^V_+ & -5\Upsilon^V_+ & -5\Upsilon^V_+ & 0 & 0 & -3\Upsilon^V_- \\
-5\Upsilon^V_+ & -5\Upsilon^V_+ & -5\Upsilon^V_+ & 0 & 0 & -3\Upsilon^V_- \\
-2\Upsilon^V_+ & -2\Upsilon^V_+ & -2\Upsilon^V_+ & 3\Upsilon^V_- & 3\Upsilon^V_- & \Phi
\end{pmatrix},
\]

where the superindex \(V\) emphasizes that only the vector part of the extended field is
left in the matrix component.

The gauge fields that remain massless throughout both SSB are:

\[
\bar{\Upsilon}_{\text{massless}} = \begin{pmatrix}
\Upsilon^- & \Upsilon^- & \Upsilon^- & \times & \times & \times \\
\Upsilon_+ & \Upsilon^- & \Upsilon^- & \times & \times & \times \\
\Upsilon_+ & \Upsilon_+ & \Upsilon^- & \times & \times & \times \\
\times & \times & \times & \Upsilon^- & \Upsilon^- & \Upsilon^S \\
\times & \times & \times & \Upsilon_+ & \Upsilon^- & \Upsilon^S \\
\times & \times & \times & \Upsilon_+ & \Upsilon^S & \Upsilon
\end{pmatrix}.
\tag{5}
\]

The Higgs fields \(\Upsilon^S_\pm\) are an isospin doublet of the maximal subalgebra \(SU(3) \times SU(2) \times U(1) \times U(1) \subset SU(6)\). The 3 \times 3 block is the \(SU(3)\) adjoint and the 2 \times 2 is the
SU(2). In the description above of the massless fields I have not written explicitly the different diagonal fields. Let us do their bookkeeping. The two large blocks take up three of the five extended fields along the diagonal. The vector part of the $T^4$ is the usual vector boson associated in SU(5) GUTs with hypercharge (the scalar part has a nonzero VEV). The scalar part of the $T^5$ decouples completely from all fields since it commutes with them (the vector part has the other nonzero VEV), so the only visible interacting fields are in $SU(3) \times SU(2) \times U(1)$.

The vector fields off the block diagonal have become massive and acquired an extra degree of freedom. As it is also the situation in customary GUTs these degrees of freedom “eat” the scalar bosons of the irrep that have remained massless after the SSB. One must go to the unitary gauge in order to get rid of these spurious fields, and these means using some of the gauge freedom to transform to them away. In customary SU(5) GUTs this is done using the gauge freedom of SU(5) that does not include $SU(3) \times SU(2) \times U(1)$. The same thing happens here. One could think that, due to the fact that the gauge group in our case is larger there would be additional gauge freedom, yet, this is not true. The reason is that the Higgs fields that have appeared, I mean, the $\Upsilon_S^-$, cannot be mixed with the other fields in the diagonal block matrix, since the $\Upsilon_S^-$ are not fixed, but transform as a 2 of $SU(2)$. The forecloses the use of the gauge degrees along the sixth row and column of the transformation matrices.

Another way of understanding this same point is recalling that there are two SSBs, and the remnant symmetry cannot mix massive particles with the massless, for either case; again, we cannot used the gauge freedom of the sixth row and column.

In SU(6) there is a $15 = 6 \times 6|_{\text{antisymmetric}}$ that contains the fermions. The term $\bar{\psi}iD\psi$ gives the correct couplings and quantum numbers for the fermions, as been pointed out in previous papers.

The whole Standard Model can be written using only two terms and two irreps, provided we have at our disposal the two required SSBs. No light is shed in this model on the origin of the VEVs. The triplet-doublet problem of GUTs and supersymmetric GUTs does not appear at all, as the Higgs 5 are naturally cut in two due to the way the SSBs occurred.

It is important to notice the following general point. Nowadays the standard model is pictured as an effective low-mass quantum field theory of some ultimate theory with a large energy scale. The masslessness of the vector bosons of the standard model is protected by gauge invariance, and the masslessness of the fermions is protected by chiral symmetry. But nothing protects the scalar bosons from the large energy scale. An important function of supersymmetry is doing precisely that, at least in the limit of small supersymmetry breaking. In the present theory we have an alternative mechanism to protect the masslessness of the scalar bosons. In terms of the extended fields the theory has its own current conservation and Ward identities that can be used to show the good behavior of scalar bosons self-energies.

References.
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Additional related papers can be found in references 2, 3 and 4.

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