Learning Quantization in LDPC Decoders

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Abstract—Finding optimal message quantization is a key requirement for low complexity belief propagation (BP) decoding. To this end, we propose a floating-point surrogate model that imitates quantization effects as additions of uniform noise, whose amplitudes are trainable variables. We verify that the surrogate model closely matches the behavior of a fixed-point implementation and propose a hand-crafted loss function to realize a trade-off between complexity and error-rate performance. A deep learning-based method is then applied to optimize the message bitwidths. Moreover, we show that parameter sharing can both ensure implementation-friendly solutions and results in faster training convergence than independent parameters. We provide simulation results for 5G low-density parity-check (LDPC) codes and report an error-rate performance within 0.2 dB of floating-point decoding at an average message quantization bitwidth of 3.1 bits. In addition, we show that the learned bitwidths also generalize to other code rates and channels.

I. INTRODUCTION

Channel coding is an essential must in any communication system. The most widely used channel codes nowadays are low-density parity-check (LDPC) codes (e.g., in WiMAX, WLAN 802.11n, DVB-S2/T2/C2 and 5G data channel) motivated by the presence of a well-understood belief propagation (BP) decoder. Unfortunately, most of the power dissipation in the receiver side is attributed to the channel decoder operations (i.e., error correction consumes most energy and chip area) [1]. Thus, possible improvements might bring large gains in terms of energy efficiency.

BP decoding of LDPC codes involves passing real-valued infinite precision log-likelihood ratio (LLR) messages between the variable nodes (VNs) and the check nodes (CNs). However, in order to reduce the hardware implementation complexity those messages are typically discretized to a practical finite-precision representation. On one hand, reducing the message bitwidths degrades the error-rate performance especially at high signal-to-noise ratios (SNRs) (i.e., error floor region) [2]. On the other hand, reducing the bitwidths of the processed quantities inside the decoder leads to a reduced area/memory requirement, possibly increased maximum operating frequency, reduced routing congestion and, thus, high throughput decoder implementations.

In this paper, we focus on optimizing the quantization of the messages involved in the hardware-popular min-sum LDPC decoder (i.e., channel LLRs, VN-to-CN and CN-to-VN messages) using machine learning approaches (i.e., stochastic gradient descent (SGD) optimizers). However, as the quantization is a discontinuous operation whose gradient is zero almost everywhere, we propose the usage of a surrogate model for training instead. It is worth mentioning that this surrogate model approach is also applicable to other applications where quantization optimization is needed.

We investigate the usage of the general concept of the surrogate model in the context of message quantization of an LDPC decoder (i.e., learn quantization bitwidths) in order to reduce the overall decoding complexity without sacrificing the error-rate performance. This model is used only for training (i.e., finding optimal message bitwidths) and is replaced by a regular decoder with “real” quantization afterwards. Note that, here, we focus on decoding LDPC codes from the 5G standard, however, it is straightforward to apply the proposed techniques to other (de)coding approaches.

We propose a hand-crafted loss function based on a fixed performance-complexity trade-off parameter \( \lambda_c \) (i.e., trade-off between error-rate performance and decoding complexity). The obtained results use different quantization bitwidths for different messages in the decoding graph which can be interpreted as message importance. Moreover, we show that weight sharing is possible across different BP iterations.

II. PRELIMINARIES

A. LDPC Codes

The key aspect of an LDPC code is that the corresponding \((M \times N)\) parity-check matrix \( H = [h_{m,l}]_{M \times N} \) contains a small number of \( 1 \)'s and is therefore sparse. This enables efficient storage of the \( H \)-matrix and effective usage of BP decoding. The dimensions \( M = N - K \) and \( N \) define the code rate \( R = k/N \), where \( K \) is the number of information bits and \( N \) is the length of a codeword.

B. Min-Sum Decoding

The min-sum decoder [3] is an iterative message passing decoder that descends from the log-domain version of the BP algorithm introduced by Gallager [4]. The messages, in form of LLRs, are exchanged over the Tanner graph [5] of the code, which is a bipartite graph that divides nodes into \( N \) VNs and \( M \) CNs. We denote messages from VNs to CNs by \( q_{m,l} \) and messages from CNs to VNs by \( r_{m,l} \), where the subscript \( l = 1, \ldots, N \) indices a certain VN and \( m = 1, \ldots, M \) a
specific CN. The iterative process of exchanging messages is performed until a stopping condition is met (e.g., in this work decoding stops after $N_{\text{iters}}$ iterations). The first $q$ messages are initialized by the channel LLRs as

$$q_{m,l} = \ell_{\text{ch},l}, \forall m.$$  

Then, the CNs are updated and extrinsic $r$ messages are computed as

$$r_{m,l} = \left( \prod_{l' \in V(m) \setminus l} \text{sign}(q_{m',l}) \right) \cdot \min_{l' \in V(m) \setminus l} |q_{m',l}|,$$  

where $V(m)$ is the set of VNs that is connected to CN $m$. Correspondingly, the update equation of the VNs is

$$q_{m,l} = \ell_{\text{ch},l} + \sum_{m' \in C(l) \setminus m} r_{m',l},$$

where $C(l)$ is the set of CNs that are connected to VN $l$. Note that the incoming message $r_{m,l}$ is omitted to calculate the extrinsic update of $q_{m,l}$. After the final iteration, marginal LLRs $\ell_{\text{total}}$ are obtained by adding up all incoming $r$ messages at each VN as

$$\ell_{\text{total},l} = \ell_{\text{ch},l} + \sum_{m' \in C(l)} r_{m',l}.$$  

A hard decision on $\ell_{\text{total}}$ retrieves the codeword bit estimates of the decoder. Eq. (1) contains the name-giving approximation by $\min(\cdot)$ function. In contrast, the exact formulation results in a product of costly $\tanh(\cdot)$ functions. As the approximated CN update results in significantly lower complexity at the price of slightly degraded error-rate performance, the min-sum decoder lays the foundation of state-of-the-art implementations for LDPC decoders. Note that the degradation in error-rate performance can be reduced or even nullified by enhancements, such as an attenuation factor or an added offset [6]. However, as both are straightforward extensions in our quantization framework, we stick to the basic min-sum decoder.

C. Quantized Decoding

In a hardware-friendly implementation of a message passing LDPC decoder (e.g., min-sum decoder) the messages are usually quantized using a uniform quantization with $b$-bits per message representation. In [6], [7], 5-bit and 7-bit quantizers were used, while in [8] a 4-bit quantizer was used to implement an unrolled LDPC decoder. Note that typically the message alphabet does not change over the decoding iterations in order to constrain the hardware implementation complexity. However, in [2], a quasi-uniform quantization with an adaptive message bitwidth is used, where the messages involved in later iterations are quantized with a larger bitwidth when compared to earlier iterations. In other words, different message representations are used for different iterations.

A quantized min-sum decoding algorithm for LDPC codes is proposed in [9], where the VN update rule is replaced with a look-up table (LUT) that is designed using an information-theoretic criterion (e.g., using density evolution). Optimizing the LUT-based VN update can accommodate lower bitwidths for all messages involved in the decoding process without sacrificing the error-rate performance (e.g., 3-bit quantizers for internal messages and 4-bit quantizers for the channel LLRs).

Another method of designing finite-precision decoders is the information bottleneck algorithm. Again, the aim is to design lower complexity decoding algorithms with error-rate performance close to high-precision decoders. Information bottleneck can be used to design LUT-based node update computations. For more details we refer the interested reader to [10] and the references therein.

III. LEARNING QUANTIZATION

A. Quantizer Implementation

For maximum efficiency, we allow each value in the decoding algorithm to take a different bitwidth. This way, the memory, wiring and circuit complexity maybe minimized. However, arithmetic operations may require their operands to be the same bitwidth. Therefore, we desire that conversion from a lower bitwidth to a higher bitwidth can be done without
any overhead. For this, we first formulate the constraint that the set of quantization levels of lower bitwidths is always a subset of the set of quantization levels of higher bitwidths. If $\mathcal{D}_b$ denotes the set of quantization levels for a given bitwidth $b$, this constraint can be expressed as

$$\mathcal{D}_b \subseteq \mathcal{D}_{b+1}.$$  

Moreover, 0 should be a valid quantization level (for all bitwidths) and the quantizer should be symmetric. Thus, there are $2^b - 1$ valid quantization levels and the quantization step size is defined as

$$\alpha(b) = \frac{L_{\text{limit}}}{2^b - 1},$$

where $L_{\text{limit}}$ is an overall scaling parameter that defines the dynamic range of the quantizer

$$L_{\text{limit}} = \lim_{b \to \infty} \max \{\mathcal{D}_b\}.$$  

Note that the maximum representable value is dependent on $b$ and given by

$$L_{\text{clip},b} = \max \{\mathcal{D}_b\} = L_{\text{limit}} - \alpha(b).$$

Fig. 2 shows the quantization curves for the proposed scheme for $L_{\text{limit}} = 8$ and $b \in \{2, 3, 4\}$ bits. Note that a two’s complement labeling works naturally with this quantizers, as going to a larger bitwidth corresponds to appending zeros.

**B. Surrogate Model**

Fig. 1 shows the block diagram of the considered communications model, consisting of LDPC encoding, binary phase shift keying (BPSK) mapping, additive white Gaussian noise (AWGN) channel, demapping and min-sum decoding. The goal is finding suitable bitwidths for a fixed-point implementation of the decoder. However, in our proposed approach, we use a floating-point version with a quantizer after each operation, as indicated by blocks labeled $Q(\cdot)$. In particular, we quantize the channel LLR $\ell_{\text{ch}}$, the VN-to-CN messages $q$ and the CN-to-VN messages $r$. The corresponding quantization steps are the independent trainable parameters $\alpha_{q,i,e}$, $\alpha_{r,i,e}$ and $\alpha_{\ell,l}$, where $i$ denotes the iteration and $e$ denotes the edge of the Tanner graph $(m,l)$ with $h_{m,l} = 1$.

![Fig. 2: Quantization curves for the proposed “compatible” quantization scheme, $L_{\text{limit}} = 8$.](image)

To optimize the quantization steps (and, thus, the bitwidths) of the quantizers, we propose to use a surrogate model, where each quantizer is replaced by the addition of a random variable $n$. This random variable models the quantization error (or quantization noise). We use the common assumption that the quantization noise for a uniform quantizer is uniformly distributed and independent of the input signal [11]. For a quantization with quantization step $\alpha$ that rounds to the nearest quantization level, the quantization error is therefore distributed according to $U(-0.5\alpha, 0.5\alpha)$. Equivalently, we can draw $n$ from $U(-0.5, 0.5)$ and scale it by $\alpha$. Lastly, we account for the limited number of quantization steps by clipping the result of the addition at $\pm L_{\text{clip}}$. A block diagram of a single surrogate quantizer is shown in Fig. 3. Each quantizer in the surrogate model has its own trainable parameter $\alpha$ that can be optimized using gradient-based methods. For this, we need a suitable differentiable loss function with respect to the trainable parameters $\alpha$.

**C. Loss Function**

A suitable loss function to optimize the error-rate performance using bit-metric decoding is the binary cross entropy (BCE) loss [12]. The reason is that the minimization of the loss is equivalent to maximizing the bit-wise mutual information. As $\ell_{\text{total},l}$ are the logits of the message estimate, we apply the sigmoid function in the loss computation. We assume that the code is systematic and, thus, the first $K$ indices of $\ell_{\text{total}}$ correspond to the information vector $u$. As a result, the total loss function is given by

$$\mathcal{L} = \sum_{l=1}^{K} \text{BCE}(\text{sigmoid}(\ell_{\text{total},l}), u_l) + \lambda_c \cdot C,$$

(2)

where $C$ models the (bitwidth-based) complexity of the decoding and $\lambda_c$ is a fixed hyper parameter specifying the trade-off between error-rate performance and complexity. In practice, Eq. (2) is estimated using batches of $B$ transmissions of random codewords over the simulated channel. $C$ is defined as the mean of the corresponding quantizer bitwidths

$$C = \frac{1}{2N_{\text{iters}} \cdot N_{\text{msg}} + N \cdot N_{\text{iters}}} \left[ \sum_{i=1}^{N_{\text{iters}}} \sum_{l=1}^{N} \left( \log_2 \left( \frac{L_{\text{limit}}}{\alpha_{q,i,e}} \right) + 1 \right) \right] + \sum_{i=1}^{N_{\text{iters}}} \sum_{e=1}^{N_{\text{msg}}} \left( \log_2 \left( \frac{L_{\text{limit}}}{\alpha_{r,i,e}} \right) + 1 + \log_2 \left( \frac{L_{\text{limit}}}{\alpha_{\ell,l}} \right) + 1 \right).$$
D. Model Conversion

To convert the trained surrogate model to a fixed-point implementation (i.e., real quantizers), the real-valued parameters $\alpha_i, e_i$, and $\alpha_j$ are counted $N_{\text{iters}}$ times in the complexity formula.

The surrogate model is then trained (i.e., the parameters $\alpha_i, e_i$, and $\alpha_j$) using a SGD-based optimizer, e.g., Adam [13], with batches of $B$ transmissions per epoch.

### IV. RESULTS

We evaluate the surrogate model-based quantization bitwidth training using the 5G LDPC code [14], [15] with parameters $N = 264$ and $K = 132$, which corresponds to the second base graph (BG) using lifting size $Z = 22$. We fix the number of iterations to $N_{\text{iters}} = 10$.

#### A. Surrogate Model Performance

First, we assess how close the surrogate model matches a real fixed-point quantization. In Fig. 4, we compare the error-rate performance of a 3-bit fixed-point model with the (untrained) surrogate model with an equivalent bitwidth of 3-bit. As a baseline, we also plot the performance of a floating-point implementation. We see that there is a gap of approximately 0.3 dB of the fixed-point implementation to the floating-point baseline. Moreover, the surrogate model performance is slightly worse than that of the fixed-point implementation. We can explain that by the mismatch of the distribution of the quantization error, which is not quite uniform in the actual fixed-point scenario. Hence, the surrogate model’s quantization error has a larger variance than the real model.

Fig. 4: Error-rate performance comparison of unquantized, quantized and surrogate min-sum decoding for the (264,132) 5G LDPC code.

As the channel LLRs $\ell_{ch}$ are required in every iteration, they are counted $N_{\text{iters}}$ times in the complexity formula.

#### B. Trained Model

For training, we set the hyper parameter $\lambda_c = 0.15$, fix $L_{\text{lim}} = 8$ and initialize all trainable parameters to $\alpha = 1$, corresponding to 4-bit quantization. The system is trained in an AWGN scenario at an SNR of $E_b/N_0 = 2.5$ dB using Adam [13] with learning rate $\gamma = 2.5 \cdot 10^{-3}$ and a batch size $B = 2048$ for 3000 epochs. The resulting bitwidths (not rounded) for $q$ and $r$ are depicted in Fig. 5 for iterations $i = 1, 6, 10$, respectively. The values are arranged according to the parity-check matrix $H$ of the code. We observe that in the first iteration, only very few checks are important; the other checks do not require any accuracy (i.e., the values are just 0). This can be explained by the punctured message bits of the 5G LDPC code. The first 2$Z = 44$ VN are initialized with LLRs $= 0$, therefore, all checks involving two of these punctured VNs only output zero-valued messages. Only the $q$ and $r$ values needed to reconstruct the punctured message bits are active. Similarly, the $r$ messages of the extension parity-bits are quantized with a low resolution. This makes sense, as the extension VNs are degree-1 and therefore, the values are not used by the VN update. The $q$ messages outgoing of the extension parity-bits are generally more important than other messages and are quantized with a higher resolution. Finally, in the last iteration, only the $r$ messages corresponding to the $K$ message bits are quantized with a reasonable bitwidth. This is easily explained by the fact that those are the only bits used in the BCE loss calculation. The learned bitwidths for the channel LLRs remained approximately at 4, resulting in an overall average complexity of 3.1 bits per message.

#### C. Weight Sharing

One can observe from Fig. 5 that messages corresponding to the same BG edge should be quantized with a similar bitwidth.
Fig. 6: Loss vs. training epoch for different weight sharing methods, using the (264,132) 5G LDPC code.

| Method                              | No WS | BG-WS  | CN-WS  |
|-------------------------------------|-------|--------|--------|
| # of parameters                     | 19228 | 874    | 161    |
| Avg. bitwidth, surrogate            | 3.289 | 3.304  | 3.486  |
| Avg. bitwidth, converted            | 3.103 | 3.126  | 3.452  |

Table I: Resulting complexity of different weight sharing methods for the (264,132) 5G LDPC code.

Therefore it is reasonable to share a single trainable parameter for $Z$ edges, reducing the training complexity. Another possible weight sharing (WS) approach is to share the same bitwidth across each BG-CN (i.e., groups of $Z$ rows in the parity-check matrix). The number of parameters and their resulting complexity (after training and conversion) is listed in Tab. I. Fig. 6 shows the development of the loss value $\mathcal{L}$ over the training epochs. We can see that WS does not only reduce the number of trainable parameters, it also speeds up the model convergence. While CN-based WS converges similarly fast, it cannot reach loss values as low as the other methods. This is expected by the previously observed diversity in the message bitwidths of the inbound and outbound messages to one CN. Consequently, training cannot achieve complexity values as low as in the other WS approaches. The error-rate performance of the surrogate and converted decoders are depicted in Fig. 7 and Fig. 8, respectively. We do not observe any difference in error-rate performance in case of the surrogate models for different WS methods. However, the converted model with CN-WS seems to achieve a lower error floor than the other methods.

D. Generalization

Finally, we evaluate how the proposed method can generalize to other channels or rates. For this, we train a model for one scenario and evaluate the learned parameters in a different scenario. Fig. 9 shows the performance of the AWGN trained model in an ergodic Rayleigh fading channel scenario with perfect channel state information. As we can see, the generalized model has almost the same performance as a model that was trained in this environment (same complexity of 3.1 bits, using $\lambda_c = 0.15$ at $E_b/N_0 = 5$ dB).

Next, we evaluate how the model can generalize to other code rates. For this we train the surrogate model at a low code rate of $R = 1/4$, corresponding to $K = 132$ and $N = 528$, using $\lambda_c = 0.003$. This performance is shown in Fig. 10; the corresponding complexity is 3.6 bits. Then, we take the bitwidths from this model and evaluate it for a rate of $R = 2/3$, i.e., $N' = 198$. This is possible, as in the 5G standard, all codes with the same code dimension use the same parity-check matrix, and merely differ in the number of transmitted/punctured extension parity-bits. Fig. 11 shows the results where again it can be seen that the model performs very similarly to the one trained at this rate (same complexity of 3.95 bits, using $\lambda_c = 0.011$ at $E_b/N_0 = 3$ dB). Note that the higher rate code has fewer of the coarsely quantized extension checks, increasing its average complexity.

V. Conclusion and Outlook

We propose a novel method to optimize the bitwidths of a fixed-point min-sum LDPC decoder. The method is based on a surrogate model where quantization has been replaced by the
addition of noise, such that the model remains differentiable and gradient-based optimization can be applied.

Weight sharing across base graph edges reduces the training complexity and improves the convergence speed. Furthermore, as all parameters are on base graph level, it allows for easy generalization to other blocklengths and rates via the standard rate-matching schemes.

The proposed methodology can be straightforwardly extended to attenuated or offset min-sum decoding [6]. Moreover, many other parts of a communications system, where fixed-point implementations are used, can be optimized using the surrogate approach.

REFERENCES

[1] M. Kuschnerov, T. Bex, and P. Kainzmaier, “Energy efficient digital signal processing,” in Optical Fiber Commun. Conf. and Exhibition (OFC), Mar. 2014, pp. 1–3.

[2] X. Zhang and P. H. Siegel, “Quantized iterative message passing decoders with low error floor for LDPC codes,” IEEE Trans. Commun., vol. 62, no. 1, pp. 1–14, Jan. 2014.

[3] M. Fossorier, M. Mihaljevic, and H. Imai, “Reduced complexity iterative decoding of low-density parity check codes based on belief propagation,” IEEE Trans. Commun., vol. 47, no. 5, pp. 673–680, May 1999.

[4] R. Gallager, “Low-density parity-check codes,” IRE Trans. Inf. Theory, vol. 8, no. 1, pp. 21–28, Jan. 1962.

[5] R. Tanner, “A recursive approach to low complexity codes,” IEEE Trans. Inf. Theory, vol. 27, no. 5, pp. 533–547, Sep. 1981.

[6] J. Chen, A. Dholakia, E. Eleftheriou, M. Fossorier, and X.-Y. Hu, “Reduced-complexity decoding of LDPC codes,” IEEE Trans. Commun., vol. 53, no. 8, pp. 1288–1299, Aug. 2005.

[7] C. Studer, N. Preys, C. Roth, and A. Burg, “Configurable high-throughput decoder architecture for quasi-cyclic LDPC codes,” in 42nd Asilomar Conference on Signals, Systems and Computers, Oct. 2008, pp. 1137–1142.

[8] P. Schläfer, N. Wehn, M. Alles, and T. Lehmkugel-Emden, “A new dimension of parallelism in ultra high throughput LDPC decoding,” in IEEE Workshop on Signal Processing Systems (SiPS), Oct. 2013, pp. 153–158.

[9] A. Balatsoukas-Stimming, M. Meidlinger, R. Ghanaatian, G. Matz, and A. Burg, “A fully-unrolled LDPC decoder based on quantized message passing,” in IEEE Workshop on Signal Processing Systems (SiPS), Oct. 2015, pp. 1–6.

[10] M. Stark, L. Wang, G. Bauch, and R. D. Wesel, “Decoding rate-compatible 5G-LDPC codes with coarse quantization using the information bottleneck method,” IEEE Open Journal of the Communications Society, pp. 646–660, Mar. 2020.

[11] R. Gray and D. Neuhoff, “Quantization,” IEEE Trans. Inf. Theory, vol. 44, no. 6, pp. 2325–2383, Oct. 1998.

[12] S. Cammerer, F. A. Aoudia, S. Dörner, M. Stark, J. Hoydis, and S. ten Brink, “Trainable communication systems: Concepts and prototype,” IEEE Trans. Commun., vol. 68, no. 9, pp. 5489–5503, Sep. 2020.

[13] D. P. Kingma and J. L. Ba, “Adam: A method for stochastic optimization,” ArXiv e-prints, Jan. 2017.

[14] 3GPP TSG RAN Meeting no. 71, RP-160671, “New SID proposal: Study on new radio access technology,” NTT DOCOMO Inc., Göteborg, Sweden, Mar. 2016.

[15] T. Richardson and S. Kudekar, “Design of low-density parity check codes for 5G new radio,” IEEE Commun. Mag., vol. 56, no. 3, pp. 28–34, Mar. 2018.