A method on path optimization modeling of UAV based on probabilistic model checking

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Abstract. To optimize the design of flight path on unmanned aerial vehicle (UAV for short) and improve its efficiency and safety, a method on path optimization modeling of UAV based on probabilistic model checking is provided. In the proposed method, the problem for the uncertainty of the flight path of UAV is studied, it is caused by arbitrary angle selection and repeated round-trip flight of UAV. When UAV interacts with the operator on duty, the influence of operator’s characteristics are considered. In this paper, the scene of road network monitoring by UAV is taken as a case study, accepting the image from UAV, the operator on duty judges whether the image is qualified according to monitoring points and non-monitoring points. Besides, the dangerous area of UAV is avoided and repetitive flight of the initial node in the flight strategy of UAV is limited. Finally, Markov decision process model is extended to specify the formal model for UAV by the target monitoring scene, and multi-objective attributes for UAV is verified by the model checking tool for PRISM. Formal verification results indicate the workload of the operator is reduced, and the efficiency for the operator and the safety of UAV is improved in the proposed method, which provides an excellent way for establishing a safety flight model for UAV.

1. Introduction
In recent years, UAV has been widely used in some fields such as farmland dust removal, weather monitoring and land and sea target monitoring [1, 2]. Furthermore, with the rapid development of UAV related technologies, the flexibility of UAV has been enhanced obviously, and relevant applications for UAV have been extended in civil and military fields. How to optimize the design of UAV flight path and improve the efficiency and safety of UAV has become a hot topic.

There are some researches on UAV path planning and operator interacts with UAV in recent years. Adopting simulation tool MATLAB study trajectory planning of UAV is a classic method [3, 4], whose results show that UAV can move from the initial point to the target point under the premise of avoiding obstacles smoothly. The fact that the UAV is out of track indicates that reasonable coordination between the operator and the UAV is very important [5, 6]. The data in [7] show that the

¹ This work has been supported by the national natural science foundation of China (No. 61462034, No. 61561024), and Scientific Research Fund of Jiangxi Provincial Education Department, China (No. GJJ160632, No. GJJ170517).
rate for human error is raised when pressure level of human is increased, and decreases as the operator's proficiency is improved. The above papers have carried out simulation verification on the path planning of UAV or analyzed the impact on human characteristics in the system. However, in safety critical systems, simulation verification cannot fully verify system’s safety. The formal analysis method is a rigorous mathematical method, which can verify the correctness and errors of system. Probabilistic model checking [8, 9] is used in distributed system, it is a good way to verify and analyze the properties with probability in the model checking tool PRISM [10].

This paper takes the road condition monitoring scene in [11] as a case study, and proposes a method on path optimization modeling of UAV based on probabilistic model checking (MPOMU algorithm for short). MPOMU algorithm takes into account the effects of the operator's proficiency, workload, fatigue and other performance characteristics on UAV path planning, and verifies the multi-objective attributes to find out the best mission planning, furthermore re-plan the path to avoid obstacle. In the optimization of UAV path planning, the method of classifying nodes is provided and the angle selection is classified into target node and non-target node. A specific angle is selected at the target node, and arbitrarily angle is selected at the non-target node. When the operator processes images which are sent by UAV, whether the image is taken at the target node needs to be judged, if the image is taken at the non-target node, then the next path selection is performed directly, and visiting initial node repeatedly is limited. Finally, a safety model is established by avoiding dangerous paths, the safety of the model is verified.

![Figure 1. Road condition monitoring scene graph.](image_url)

2. MPOMU algorithm
The proposed MPOMU algorithm extend Markov decision process (MDP for short) model, and the extended MDP model 1 and the extended MDP model 2 are acquired, which are used to represent the road condition monitoring scenario shown in Figure 1. In addition, Probabilistic Computing Tree Logic (PCTL for short) attribute formula [12] is adopted to verify multi-objective attributes, and the minimum cumulative return value of the system. The MDP model and its extended models are defined as follows.

**Definition 1** The MDP model is a tuple \( M = (Q, q_1, Act, \delta) \), where \( Q \) is a finite set of states, \( q_1 \in Q \) is the initial state, \( Act \) is a set of actions, \( \delta: S \rightarrow 2^{Act \times Dist(q)} \) a transition probability function, and for any \( (a, \mu) \in \delta(q) \), there is \( \sum_{q' \in Q} \mu(q') = 1 \).

**Definition 2** The extended MDP model 1 is a tuple \( EM_1 = (Q, q_1, As, Rs, Act, \delta) \), where \( Q, q_1, Act, \delta \) are the same as Definition 1, and \( As \) is a set of angles, according to Figure 1, \( As=\{a1, a2, a3, a4, a5, a6, a7, a8\} \), \( Rs=\{r1, r2, r3, ..., rm\} \) is a path set, \( m \) is the total number of path.
**Definition 3** The extended MDP model 2 is a tuple $EM_2=(Q,q_1,L(q),As,Rs,Act,\delta)$ where $Q,q_1,As,Act,\delta:S \rightarrow 2^{Act\times Dist(q)}$ are the same as Definition 2, $L(q):q \rightarrow 2^{stro\times tro\times k}$, $L(q)$ is not empty, where stro represents the set of picture quality, tro represents the set of workload levels, and $k$ represents the fatigue parameter.

MPOMU algorithm is mainly divided into sub-algorithm for UAV path optimization and Ope operator, which are respectively described as follows.

**2.1. Sub-algorithm for UAV path optimization**

Sub-algorithm for path optimization assumes that the UAV road condition monitoring scene is shown in Figure 1, where $q_2, q_5, q_6$ are target detection points, $q_1, q_3$ and $q_4$ are non-target detection points, and $q_1$ is the initial node, when $q_2$, $q_5$, $q_6$ are visited, the task of UAV flight is finished. The input and the output of sub-algorithm for path optimization are represented as follows.

**Input:** $EM_2=(Q,q_1,As,Rs,Act,\delta)$ where $Q = \{q_1, q_2, q_3, q_4, q_5, q_6\}$, $q_1$ is the initial node, and $qi \in Q$, $a \in As$, $r \in Rs$, $Act=\{$fly, wait, image, wait, camera, go$\}$.

**Output:** If the state sequence meets system’s requirements, then Boolean variable stop=true, which indicates visiting the next node is stop, otherwise stop=false, continues to access the next node.

Sub-algorithm for path optimization starts from the initial node $q_0$ and sends an image to the operator. When the operator sends a qualified picture, the next state is selected by action and angle, path selection are performed until stop=true. The specific description of sub-algorithm for path optimization is shown in figure 2.

**sub-algorithm for path optimization**

**Variable:** $a$: angle, $g$: position node, $r$: path, send: whether to take a picture and send a picture, in: whether to enter the location node point

**Initialization:** $a \leftarrow 0, g \leftarrow 1, r \leftarrow 0, send \leftarrow true, in \leftarrow true$

while stop=true do
    If($q_1=0$&$in=true$&$send=false$)
        send \leftarrow true;
        The operator processes the image
        send \leftarrow false; Continue;
    End
    If($in=false$&$send=false$&$r=0$&$a=0$)
        If($q_1=2$&$q_1=5$&$q_1=6$)
            $a \leftarrow \{1,2,3,8\}$; Continue;
        End
    End
    If($q_2=2$) \hspace{1em} //Angle selection at q2
        $a \leftarrow \{2,3,5\}$; Continue;
    End
    If($q_5=5$) \hspace{1em} //Angle selection at q5
        $a \leftarrow \{1,5,6,7,8\}$; Continue;
    End
    If($q_6=6$) \hspace{1em} //Angle selection at q6
        $a \leftarrow \{5,7\}$; Continue;
    End
    If($in=false$&$send=false$&$r=0$&$a=0$)
        Road-Chose(); Continue;
    End
End

**Figure 2.** Sub-algorithm for path optimization.

**2.2. Sub-algorithm for Ope operator**

**Input:** $EM_2=(Q,q_1,L(q),As,Rs,Act,\delta)$, where $L(q):q \rightarrow 2^{stro\times tro\times k}$, stro $\in \{0,1,2\}$ is the quality of the
image, 0 is the initial value, 1 is qualified, 2 is unqualified; \( t \in \{1,2,3\} \) is the workload level, 1 is the initial value, 2 is the low load level, 3 is the high workload level; \( k \) is the fatigue degree, \( \text{Act}=\{\text{process, image, wait, go}\} \).

Output: If the state sequence meets requirements of the system, then Boolean variable \( \text{stop}=\text{true} \), which indicates visiting the next node is stopped, otherwise \( \text{stop}=\text{false} \).

**Variable:**\( k \): fatigue parameter, \( t \): workload level, \( \text{stro} \): image quality, \( fd \): fatigue discount, \( pl \): accuracy under low workload, \( ph \): accuracy under high workload, \( Pro \): the probability that other factor caused additional work.

**Initialization:**\( k \leftarrow 0; \ t \leftarrow 1; \ \text{stro} \leftarrow 0; \)

while \( \text{stop}=\text{true} \) do

If \( (p=2|p=5|p=6) \) then

If \( t = 1 \) then

with a probability \( Pro:\text{stro} \leftarrow 2; \) // low workload

with a probability \( (1-Pro)\text{stro} \leftarrow 3; \) // high workload

 Continue;

End

If \( t = 2 \& k \leq \text{COUNTER} \) then // under low workload and less than fatigue

with a probability \( pl\text{stro} \leftarrow 1; \) // image qualified under low workload

\( k \leftarrow k+1; \)

with a probability \( 1-pl\text{stro} \leftarrow 2; \)

\( k \leftarrow k+1; \) Continue;

End

// Under low workload and greater than the fatigue threshold

If \( t = 2 \& k > \text{COUNTER} \) then

with a probability \( pl\times fd\text{stro} \leftarrow 1; \)

\( k \leftarrow k+1; \)

with a probability \( 1-pl\times fd\text{stro} \leftarrow 2; \)

\( k \leftarrow k+1; \) Continue;

End

If \( t = 3 \& k \leq \text{COUNTER} \) then // under high workload and less than fatigue

with a probability \( ph\text{stro} \leftarrow 1; \)

\( k \leftarrow k+1; \)

with a probability \( 1-ph\text{stro} \leftarrow 2; \)

\( k \leftarrow k+1; \) Continue;

End

If \( t = 3 \& k > \text{COUNTER} \) then // Under high workload and greater than the fatigue threshold

with a probability \( ph\times fd\text{stro} \leftarrow 1; \)

\( k \leftarrow k+1; \)

with a probability \( 1-ph\times fd\text{stro} \leftarrow 2; \)

\( k \leftarrow k+1; \) Continue;

End

End

End

If \( (p=2|p=5|p=6) \) then

\( \text{stro} \leftarrow 1; \) Continue;

End

If \( (\text{stro}=2) \) then

\( \text{stro} \leftarrow 0; \ t \leftarrow 1; \) Continue;

End

If \( (\text{stro}=1) \) then

\( \text{go}=\text{true}; \) Continue;

End

End

**Figure 3.** Sub-algorithm for Ope operator
Sub-algorithm for Ope operator starts from the initial node q1, and selects the next state by the probability transfer matrix δ and Act. If st=1, the following operation for selecting path is continued. If st=2, the picture is unqualified, so it is necessary to wait for the image to be taken again. Specific algorithm description is shown in figure 3.

3. Experiment analysis

According to the MPOMU algorithm, the formal model of UAV road condition monitoring scene is established, and it serves as a functional model in UVA control system. The model checking tool PRISM is used to verify and analyze the system attributes, and compared with the results in [11]. Considering different conditions for human characteristics such as fatigue degree, proficiency, workload level, the relationship between the cost of completing the task and the access to the dangerous area is analyzed by Pareto curve under.

The safety model for obstacle avoidance is established and verified. The experimental results are shown in Figure 4 to Figure 7. The dotted line represents the experimental results in [11], and the solid line represents the experimental results of MPOMU algorithm. The corresponding experiment representation is as follows.

1) This paper analyzes the time cost of the operator on different performance characteristics as the work task is completed, and verifies the attribute $R\{\text{time}\}\min=\omega[Fw2\&w5\&w6]$, the results are shown in Figure 4 and Figure 5. Figure 4 shows the impact of the operator's fatigue threshold COUNTER and the discount factor $fd$ on UAV to complete the task of visiting target nodes. The experimental results show that the higher value of the discount factor $fd$ and the greater value of the fatigue threshold COUNTER will lead to the shorter time. Figure 5 shows the operator's probabilistic $pl(k)$ at low workload and the probabilistic $ph(k)$ at high workload for the minimum expected time to complete the task. Experimental results show that operators with higher $pl(k)$ and $ph(k)$ values are more proficient and can complete tasks faster. Through a lot of experimental exploration, the accuracy decreases more rapidly, the $ph(k)$ value is smaller, the time to complete the task is longer.

![Figure 4. Impact of COUNTER and fd on UAV completion tasks.](image)

2) The relationship between the minimum times spent on completing the task and the probability of access to dangerous area are analyzed. The attribute to be verified is multi($R\{\text{time}\}\min=\omega[C], R\{\text{ROZ}\}\min=\omega[C]$), the verification results are shown in Figure 6 and
Figure 7. Figure 6 shows the effect of different initial values for pl(k) and ph(k) of the operator under different workloads on the two mission objectives, and corresponding Pareto curves are generated. Experimental results show that the values of pl(k) and ph(k) decrease, Pareto curves migrate to the right, and the time to complete the task is increased gradually. Figure 7 shows the offset of the Pareto curve when changing the values of the COUNTER and the fd. Compared with Figure 6 and Figure 7, it is indicated that the effects of the parameters for COUNTER and fd are not as great as the parameters for pl(k) and ph(k).

(3) Compared with the experimental results in Figure 4 to Figure 7, it is shown that the proposed MPOMU algorithm are obviously better than the paper [11], and the time taken to complete the task is shorter, the efficiency is higher, and the system performance is better.

Figure 6. Pareto curves of pl(k) and ph(k) on UAV completion tasks.

Figure 7. Pareto curves of COUNTER and fd on UAV completion tasks.

(4) A safety model is built and analyzed by circumventing the dangerous area. First of all, whether there is a non-hazardous path is analyzed in the functional model which is established at the beginning of this experiment. By verifying the attribute of R("ROZ") min = ? [F w2 & w5 & w6], it can be concluded that the cumulative value of the return of a path to visit dangerous areas is 0. Therefore, by re-planning path and angle of the functional model, a model circumvented the dangerous area can be obtained.

The relevant position of the dangerous area can be obtained by analyzing Figure 1. It should be avoided in the process of modeling. The dangerous area (roz) can be described by the formula roz = (r=8) | (w=3 & a=1) | (w=3 & a=2) | (w = 5 & a = 2), and the evasive strategy for dangerous areas is as follows:

(a) circumventing the path: r!7; r!=8;
(b) evading the angle: w=5 & a=2; (w=3 & a = 1) ; (w=3 & a=2);

The detected road condition is re-modeled by the evasive strategy mentioned above, the model is verified by the attribute P<=0 [ (F roz & w2 & w5 & w6)]. The verification results indicate that there is no reachable path for the dangerous area in the system.

4. Conclusions
In this paper, the PRISM tool is used to analyze the impact of human characteristics on the mission objectives during the human interacting with UAV process. By improving the operator’s processing and path selection for UAV, the efficiency of the UAV control system is improved, and the trade-off between multiple mission objectives is analyzed by the Pareto curve. Finally, the route of the UAV...
flight is re-planned to establish a safety model which can avoid dangerous area and will benefit future research for UAV control system.

Acknowledgments
This work has been supported by the national natural science foundation of China (No. 61462034, No. 61561024, No. 61562037, No. 11461031), Jiangxi Provincial Natural Science Foundation of China (No.20151BAB207035, No.20181BBE58018), and Scientific Research Fund of Jiangxi Provincial Education Department, China (No. GJJ160632, No. GJJ170517).

References
[1] Peng Wang, Fan Zhang, Lei Dong, et al. Modeling and safety analysis of civil aircraft head-up display system based on probability model test, Electronics Optics & Control. 11 (2017) 64-69.
[2] Mingshan Yang, Yuhua Fu, Shanghui Li, et al. Blasting and safety control technology of 67m thick wall chimney in complex environment, Journal of Jiangxi University of Science and Technology. 38 (2017) 1 55-60.
[3] Pan Yong, Ding Dongdong, Xu Xiangrong, Design and simulation of trajectory planning for airborne robots (UAV), Journal of Jiamusi University (Natural Science Edition). 36 (2018) 04 563-566+577.
[4] L. Humphrey, E. Wolff, and U. Topcu, Formal specification and synthesis of mission plans for unmanned aerial vehicles, In Proc. of the AAAI Spring Symposium, 2014.
[5] T. Chen, V. Forejt, M. Kwiatkowska, D. Parker, and A. Simaitis, PRISM-games: A model checker for stochastic multi-player games, In TACAS, 2013.
[6] D. Donath, A. Rauschert, and A. Schulte, Cognitive assistant system concept for multi-UAV guidance using human operator behaviour models, In HUMOUS, 2010.
[7] K. R. Boff and J. E. Lincoln, Engineering data compendium: human perception and performance, AAMRL, Wright-Patterson AFB, OH, 1988.
[8] Chunrui Xia, Rui Wang, Xiaojuan Li, et al. Path planning method based on probabilistic model detection in dynamic environment, Computer Engineering and Applications. 52 (2016) 12 5-11.
[9] Feng C, Zhang H, Yan S, et al. Reliability evaluation for distribution system based on probabilistic model checking, International Conference on Reliability Systems Engineering, IEEE, 2017, pp. 1-6.
[10] Feng L, Wiltsche C, Humphrey L, et al. Controller synthesis for autonomous systems interacting with human operators, 2015, pp. 70-79.
[11] Feng L, Wiltsche C, Humphrey L, et al. Synthesis of Human-in-the-Loop Control Protocols for Autonomous Systems, IEEE Transactions on Automation Science & Engineering. 13 (2016) 2 450-462.
[12] Vojtĕch Forejt, Kwiatkowska M, Norman G, et al. Automated verification techniques for probabilistic systems, 2011.