Quantum simulation is at the heart of the ongoing “second” quantum revolution, with various synthetic quantum matter platforms realizing evermore exotic condensed matter and particle physics phenomena at high levels of precision and control. The implementation of gauge theories on modern quantum simulators is especially appealing due to three main reasons: (i) it offers a new probe of high-energy physics on low-energy tabletop devices, (ii) it allows exploring condensed matter phenomena that are prominent in gauge theories even without a direct connection to high-energy physics, and (iii) it serves as a banner of experimental benchmarking given the plethora of local constraints arising from the gauge symmetry that need to be programmed and controlled. In order to faithfully model gauge-theory phenomena on a quantum simulator, stabilizing the underlying gauge symmetry is essential. In this brief review, we outline recently developed experimentally feasible methods introduced by us that have shown, in numerical and experimental benchmarks, reliable stabilization of quantum-simulator implementations of gauge theories. We explain the mechanism behind these linear gauge protection schemes, and illustrate their power in protecting salient features such as gauge invariance, disorder-free localization, quantum many-body scars, and other phenomena of topical interest. We then discuss their application in experiments based on Rydberg atoms, superconducting qubits, and in particular ultracold neutral atoms in optical superlattices. We hope this review will illustrate some facets of the exciting progress in stabilization of gauge symmetry and in gauge-theory quantum simulation in general.

CONTENTS

I. Introduction 1
II. Setting 2
III. Linear gauge protection 4
   A. In terms of the local generator 4
   B. In terms of a local pseudogenerator 5
IV. Stabilizing exotic far-from-equilibrium phenomena 6
   A. Quantum many-body scars 6
   B. Disorder-free localization 6
V. Experimental applications 7
VI. Conclusion 8
Acknowledgments 9
References 9

* jad.halimeh@physik.lmu.de
† philipp.hauke@unitn.it
uses Gauss’s law to integrate out redundant degrees of freedom, i.e., it expresses the state of the electric fields through charge configurations or vice versa [16–20]. This approach led to the first successful quantum simulations of lattice gauge theories in trapped ions [21] and Rydberg atoms [22, 23], which were followed briefly thereafter by various other experiments and refined proposals [24–30]. This approach has advantages in making efficient use of resources and it ensures the conservation of Gauss’s law by hardcoding it into the simulator. However, the resultant interactions can sometimes be of complicated local or nonlocal form, and errors in the quantum devices can lead to unphysical interpretations, such as a particle with infinite attached electric-field string appearing out of the vacuum [21, 31]. The second approach, which historically was at the basis of the first series of proposals for gauge-theory quantum simulations [32–39], keeps both matter and gauge fields as dynamical degrees of freedom in the quantum simulator. This gives some flexibility in realizations, e.g., through degenerate perturbation theory, and has led to successful experiments in various platforms [40–46]. Maybe even more interestingly, as Gauss’s law is not imposed a priori, this second approach permits one to test fundamental questions about the possible emergence of gauge invariance in nature [47–52]. However, Gauss’s law then needs to be stabilized by some other means.

In this article, we review recent theoretical and experimental progress in stabilizing gauge symmetry in quantum simulators, embarking from our personal and thus necessarily limited point of view. We focus mostly on low-dimensional systems, where the bulk of current experimental implementations have been realized and where numerical benchmarks are possible thanks to powerful tensor-network methods, as well as on Abelian symmetries. In particular, we review the framework of linear gauge protection, where simple single-particle terms proportional to Gauss’s law are added to a faulty Hamiltonian, which dynamically stabilizes gauge symmetry. These are akin to, but experimentally significantly simpler for nonequilibrium quantum simulation, than energy penalties that are quadratic in Gauss’s law, which force the low-energy manifold to approximately obey Gauss’s law [32–34, 36, 37, 39, 53–61]. We discuss what has been shown for linear gauge protection rigorously analytically as well as in numerical studies through exact diagonalization (addressing long times) and infinite matrix product states (addressing systems in the thermodynamic limit). We further illustrate the power of linear gauge protection for stabilizing salient phenomena such as disorder-free localization (DFL) and quantum many-body scars (QMBS), and we give a glimpse on recent experimental implementations and proposals.

In this proceedings article, we focus mostly on works we have been personally involved in, but it is worth stressing that the program of ensuring gauge invariance in quantum simulators is a community effort that is seeing a strong current momentum. Other closely related approaches for protecting gauge symmetry employ, e.g., engineered dissipation [53], dynamical decoupling [62], or pseudo-random gauge transformations [63]. Symmetry protection in a general setting, i.e., also of global symmetry, is also a prospect of large current interest [64, 65], and has also been employed beneficially in a trapped-ion simulation of a lattice gauge theory [30]. Finally, the complexity of models inspired from the interacting degrees of freedom of gauge theories, but without respecting the underlying gauge symmetry, can give rise to intriguing many-body phenomena and is worthwhile of investigations in its own right [66, 67].

This review is structured as follows: In Sec. II, we introduce the setting of our problem, which involves faulty gauge theories with dynamical matter and gauge fields, and we review previous methods utilized to stabilize them on quantum simulators. In Sec. III, we discuss the concept of linear gauge protection based on local generators and also on local “pseudogenerators”. We showcase how linear gauge protection stabilizes quantum many-body scars and disorder-free localization in Sec. IV. In Sec. V, we discuss experimental applications and proposals involving linear gauge protection. Finally, in Sec. VI, we conclude and provide an outlook.

II. SETTING

In this work, we focus on Abelian lattice gauge theories (LGTs), described ideally, i.e., in an error-free setting, by Hamiltonian $\hat{H}_0$ [explicit examples are given further below, see Eqs. (3) and (9)]. The system has $L$ lattice sites, where matter fields reside, and $L$ links, where gauge and electric fields exist. Unless otherwise stated, we will assume periodic boundary conditions throughout this work. The gauge symmetry of $\hat{H}_0$ is generated by the operators $\hat{G}_j$ (Gauss’s law generators), and gauge invariance is encoded through the commutation relations $[\hat{H}_0, \hat{G}_j] = 0, \forall j$. Gauge-invariant states $|\phi\rangle$ are simultaneous eigen-
states of all local generators: \( \hat{G}_j |\phi\rangle = g_j |\phi\rangle, \forall j \). The eigenvalues \( g_j \) can be seen as background charges, a set of which over the volume of the system defines a gauge superselection sector \( g = (g_1, g_2, \ldots, g_L) \). A given superselection sector imposes a specific relation between matter occupation at site \( j \) and the allowed electric-field configurations at its two neighboring links; see Fig. 1. In the basis of the generators \( G_j \), the Hamiltonian \( \hat{H}_0 \) can be block-diagonalized, with each block representing a unique superselection sector; see yellow blocks in Fig. 2. Due to its gauge symmetry, \( \hat{H}_0 \) can only drive dynamics within each superselection sector (intrasector dynamics) and cannot couple different superselection sectors (intersector dynamics).

In realistic SQM implementations of LGTs with dynamical matter and gauge fields [40–46] (the second approach mentioned in Sec. I), gauge-breaking errors \( \lambda \hat{H}_1 \) of some strength \( \lambda \), even if small, will unavoidably arise. If left uncontrolled, such errors, even when merely perturbative, can drastically modify the system dynamics at sufficiently long times or in its ground state, such that it can no longer be associated with a gauge theory [68, 69]. It is thus essential to devise experimentally feasible approaches to stabilize the gauge symmetry in such SQM implementations. In recent years, various proposals have been presented that mitigate such unavoidable experimental errors through energetic constraints [32–34, 36, 37, 39, 53–61, 63].

Most of the original proposals involved turning the target superselection sector \( g^{\text{tar}} = (g_1^{\text{tar}}, g_2^{\text{tar}}, \ldots, g_L^{\text{tar}}) \) into a ground-state manifold by introducing a quadratic protection term of the general form [61]

\[
V \hat{H}_{\text{quad}} = V \sum_{j=1}^{L} \left( \hat{G}_j - g_j^{\text{tar}} \right)^2, \tag{1}
\]

with strength \( V \). For a U(1) lattice gauge theory similar to QED in equilibrium [Eq. (3) below], this has been shown to lead to a gauge-symmetry violation quantum phase transition in the system of Hamiltonian \( \hat{H}_0 + \lambda \hat{H}_1 + V \hat{H}_{\text{quad}} \) at a critical value of \( V/\lambda \), above which the system is in a phase characterized by a renormalized gauge theory that retains the critical features of the original model \( \hat{H}_0 \) [68], in the spirit of a Higgs transition [48, 55, 71, 72] and similar to what has been discussed for topological states of matter [73, 74].

Such quadratic protection schemes have also been demonstrated through classical simulations and analytic arguments to work reliably in out-of-equilibrium scenarios. A useful figure of merit to quantify the performance of the protection is the gauge violation, defined as the average variance of the Gauss’s law generators with respect to the target sector,

\[
\varepsilon(t) = \frac{1}{L t} \int_0^t ds \sum_{j=1}^{L} \langle \psi(s) | (\hat{G}_j - g_j^{\text{tar}})^2 |\psi(s)\rangle, \tag{2}
\]

where \( |\psi(t)\rangle = e^{-i\hat{H}t} |\psi_0\rangle \) and \( \hat{H} = \hat{H}_0 + \lambda \hat{H}_1 + V \hat{H}_{\text{quad}} \) is the faulty theory. At sufficiently large \( V \), the gauge violation can be shown to settle into a plateau of value \( \propto \lambda^2/V^2 \) at a timescale \( \propto 1/V \), which lasts up to all numerically accessible times for finite systems [61] and also in the thermodynamic limit [75]. This method has even been shown to work well for non-Abelian gauge theories [76].

However, a major drawback of the quadratic protection scheme is that Eq. (1) is very challenging to realize experimentally. Indeed, \( G_j^2 \) usually includes terms that
are at least quadratic (in case \( \hat{G}_j \) is composed of at most single-body terms). Furthermore, quadratic protection necessarily energetically isolates a given target superselection sector from all its counterparts, which is not useful in applications where the dynamics is across several sectors simultaneously. In the following, we will review an approach we have developed that solves both these issues \cite{77}.

### III. LINEAR GAUGE PROTECTION

Let us first consider a setting where we are interested in the dynamics occurring only within a given target superselection sector \( \mathbf{g}^\text{tar} \). In an SQM implementation, this would entail quantum-simulating the quench dynamics of a given LGT, starting in a gauge-invariant initial state \( |\psi_0\rangle \) residing in the target gauge sector: \( \hat{G}_j |\psi_0\rangle = g_j^\text{tar} |\psi_0\rangle \), \( \forall j \). In an ideal situation, the time-evolved wave function \( |\psi(t)\rangle \) should also reside in the superselection sector \( \mathbf{g}^\text{tar} \), which would indeed be the case if the dynamics was propagated by only \( \hat{H}_0 \). However, as mentioned, in a realistic SQM implementation there will be unavoidable gauge-breaking errors \( \Delta \hat{H}_1 \) that will cause \( |\psi(t)\rangle \) to “leak” out of the target sector. An experimentally feasible way to stabilize the gauge symmetry of the model in presence of these errors is to introduce protection terms linear in the gauge-symmetry local generator \cite{77}. In case the local generator is itself complicated to implement, a further experimental simplification can be obtained by enforcing the protection through a local pseudogenerator (LPG) \cite{78}.

#### A. In terms of the local generator

If the LGT possesses a local generator that is composed of at most single-body terms, its experimental addition to the Hamiltonian is straightforward and there is no need for an LPG. This is, e.g., the case of the spin-\( S \) U(1) quantum link model (QLM) \cite{79–81}

\[
\hat{H}_0 = \sum_{j=1}^{L} \left[ \frac{J}{2\sqrt{S(S+1)}} (\hat{\sigma}_j^- \hat{s}_{j,j+1}^z + \text{H.c.}) + \frac{\mu}{2} \hat{\sigma}_j^x + \frac{\eta}{2} (\hat{s}_{j,j+1}^z)^2 \right].
\]

Equation (3) converges to the lattice Schwinger model in the Kogut–Susskind limit \( S \to \infty \). Often, even surprisingly small spin representations already allow for reliable extrapolations to the limit of the lattice Schwinger model \cite{82, 83}, a feature that has been observed also in other truncation schemes of the gauge field \cite{24, 56, 84–86}. The QLM formulation has the advantage over some of these that it preserves the canonical commutation relations between electric field and parallel transporter (exponential of vector potential) \cite{79–81}.

The generator of the U(1) gauge symmetry of Eq. (3) is

\[
\hat{G}_j = (-1)^j \left[ \hat{s}_{j-1,j}^z + \hat{s}_{j,j+1}^z + \frac{\hat{\sigma}_j^z + 1}{2} \right],
\]

which can be interpreted as a discretized version of Gauss’s law from QED. Importantly, this generator is composed of only single-body terms, which add little experimental overhead in typical SQM setups \cite{43, 46}. We now introduce the linear gauge protection term

\[
V \hat{H}_G = V \sum_{j=1}^{L} c_j \hat{G}_j.
\]

The protection sequence \( c_j \) is pivotal to the efficacy of the linear gauge protection (5). Ideally, it must be tailored in such a way that, at sufficiently large \( V \), the target sector is energetically completely separated from other superselection sectors. This can be achieved when \( c_j \) is composed of rational numbers obeying the compliance condition

\[
\sum_{j=1}^{L} c_j (g_j - g_j^\text{tar}) = 0 \iff g_j = g_j^\text{tar}, \forall j.
\]

Indeed, as proven by the Gauge-Protection Theorem detailed in Ref. \cite{77}, such a sequence guarantees a controlled gauge violation (2) up to a timescale exponential in a volume-independent \( V \) \cite{87}. However, an experimentally nontrivial challenge arises in the case of such a compliant sequence \( c_j \), as it would have to grow exponentially with system size for a fixed value of \( V \). Thus, although \( V \) itself is volume-independent \cite{77}, \( c_j \) is not. While this issue may be secondary for small-size realizations, it poses a problem for large-scale SQM implementations.

Nevertheless, dependent on the error model, one can employ simpler noncompliant periodic sequences, even as simple as \( c_j = (-1)^j \). These can be shown analytically through the concept of quantum Zeno dynamics \cite{88–91} to stabilize gauge invariance up to timescales at least linear in \( V \), during which the dynamics under the faulty LGT \( \hat{H} = \hat{H}_0 + \lambda \hat{H}_1 + V \hat{H}_G \) is reproduced by the effective
Zeno Hamiltonian

\[ \hat{H}_{\text{QZE}}^{\text{tar}} = \hat{H}_0 + \lambda \hat{P}_{g^{\text{tar}}} \hat{H}_1 \hat{P}_{g^{\text{tar}}}, \]  

(7)

Here, \( \hat{P}_{g^{\text{tar}}} \) is the projector onto the target sector \( g^{\text{tar}} \), i.e., the dynamics of the effective Zeno Hamiltonian falls into the ideal sector of conserved Gauss’s law. The linear gauge protection (5) with \( c_j = (-1)^j \) has been demonstrated to suppress the gauge violation (2) in the target sector up to all numerically accessible times in both finite systems through exact diagonalization (ED) [77] and in the thermodynamic limit using infinite matrix product state (iMPS) techniques [75], for experimentally relevant gauge-breaking errors.

It is important to note that Eq. (7) is the effective Hamiltonian when restricting to the target sector \( g^{\text{tar}} \). However, as we will discuss in Sec. IV B, situations can be of interest where the initial state is a superposition of an extensive number of superselection sectors [92, 93], instead of a single one as above. In such cases, and with the appropriate sequence \( c_j \), the Zeno Hamiltonian takes the more general form [94],

\[ \hat{H}_{\text{QZE}} = \hat{H}_0 + \lambda \sum_g \hat{P}_g \hat{H}_1 \hat{P}_g, \]  

(8)

where \( \hat{P}_g \) is the projector onto the superselection sector \( g \). Though no longer restricted to a single superselection sector, the effective dynamics—again up to timescales at least linear in \( V \)—nevertheless does not couple different superselection sectors.

B. In terms of a local pseudogenerator

The generator (4) of the spin-\( S \) U(1) QLM (3) is ideal for linear gauge protection, given that it is composed of only single-body terms. However, not all gauge theories possess such a simply gauge-symmetry local generator. For example, a paradigmatic model that has recently been at the center of several experiments [40, 41, 45, 46] is the \( Z_2 \) LGT, described by the Hamiltonian [69, 95–99]

\[ \hat{H}_0 = \sum_{j=1}^{L} \left[ J (\hat{b}_{j+1}^{\dagger} \hat{b}_{j+1} \hat{b}_{j} \hat{b}_{j+1} + \text{H.c.}) - h \hat{\tau}_x^{j,j+1} \right]. \]  

(9)

The associated \( Z_2 \) gauge-symmetry local generator is the three-body term

\[ \hat{G}_j = (-1)^{n_j} \hat{\tau}_x^{j-1,j} \hat{\tau}_x^{j,j+1}, \]  

(10)

where \( [\hat{H}_0, \hat{G}_j] = 0, \forall j \). Here, the hard-core bosonic ladder operators \( \hat{b}_j, \hat{b}_j^{\dagger} \) represent the matter field on site \( j \), with \( n_j = \hat{b}_j^{\dagger} \hat{b}_j \) the corresponding boson number operator, and the Pauli operator \( \hat{\tau}_x^{j,j+1} \) represents the gauge (electric) field at the link between the sites \( j \) and \( j+1 \).

Implementing the linear gauge protection scheme (5) with the generator in Eq. (10) in modern SQM devices is challenging as it requires the realization of three-body terms. It is therefore necessary to search for alternative, experimentally more feasible approaches. For this purpose, the local pseudogenerator (PG) has been proposed, [78]

\[ \hat{W}_j = \hat{\tau}_x^{j-1,j} \hat{\tau}_x^{j,j+1} + 2g_j^{\text{tar}} \hat{n}_j, \]  

(11)

which obeys the relation

\[ \hat{W}_j | \phi \rangle = g_j^{\text{tar}} | \phi \rangle \iff \hat{G}_j | \phi \rangle = g_j^{\text{tar}} | \phi \rangle, \]  

(12)

where \( \{| \phi \rangle \} \) is the set of all gauge-invariant states. An implication of relation (12) is that \( \hat{W}_j \) and \( \hat{G}_j \) are identical to each other in the target gauge superselection sector \( g^{\text{tar}} \). One can therefore employ the linear gauge protection term

\[ V \hat{H}_W = V \sum_{j=1}^{L} c_j \hat{W}_j, \]  

(13)

for which the formalism of Ref. [77] can be extended to obtain the Zeno Hamiltonian

\[ \hat{H}_{\text{QZE}}^{\text{tar}} = \hat{H}_0 + \lambda \hat{P}_{g^{\text{tar}}} \hat{H}_1 \hat{P}_{g^{\text{tar}}}, \]  

(14)

for quenches starting in the target sector [78].

However, and quite intriguingly, for applications when the dynamics involves an extensive number of superselection sectors, an enriched Zeno Hamiltonian emerges,

\[ \hat{H}_{\text{QZE}} = \sum_w \hat{P}_w (\hat{H}_0 + \lambda \hat{H}_1) \hat{P}_w, \]  

(15)

where \( \hat{P}_w \) is the projector onto the superselection sector \( w \) of the local symmetry associated with the LPG. Unlike Eq. (14), the Hamiltonian in Eq. (15) encodes the full structure of the local symmetry associated with the LPG \( \hat{W}_j \). This is nontrivial because this symmetry is richer than the \( Z_2 \) gauge symmetry generated by \( \hat{G}_j \) of Eq. (10), and we will discuss later how this can lead to rich physics in the case of DFL. In particular, the local symmetry associated with \( \hat{W}_j \) contains the \( Z_2 \) gauge symmetry:

\[ [\hat{H}', \hat{W}_j] = 0, \forall j \Rightarrow [\hat{H}', \hat{G}_j] = 0, \forall j, \]  

(16)

for a given Hamiltonian \( \hat{H}' \), but the converse is generally not true. Noting that \( [\hat{H}_0, \hat{W}_j] \neq 0 \), this means that \( \hat{H}_{\text{QZE}} \), which commutes with both \( \hat{W}_j \) and \( \hat{G}_j \), hosts a richer local symmetry than the \( Z_2 \) LGT.

IV. STABILIZING EXOTIC FAR-FROM-EQUILIBRIUM PHENOMENA

The linear gauge protection scheme has been employed to protect dynamics within a target superselection sector for both the U(1) QLM and \( Z_2 \) LGT, where it has demonstrated reliable stability and suppression of gauge
in the spin-1

FIG. 3. (Color online). Stabilizing quantum many-body scars state is quenched with the faulty theory $\hat{c} \propto Krylov$-subspace methods. Figure adapted from Ref. [100].

all investigated evolution times. Results were calculated using strong, the ensuing dynamics in (a) fidelity and (b) electric flux. Figure 3 for the case of the former for

A paradigm of strong ergodicity breaking, DFL emerges in gauge theories in the wake of quenches starting in special initial states. In the case of the ideal model ($\lambda = V = 0$), as seen in the inset of Fig. 3(a). Taking errors into account, we find that the revivals vanish even when the error strength is perturbative ($\lambda = 0.5J$). The errors used for these results are

$$\lambda \hat{H}_1 = \lambda \sum_{j=1}^{L-1} \left( \hat{\sigma}_j^- \hat{\sigma}_{j+1}^- + \hat{\sigma}_j^+ \hat{\sigma}_{j+1}^+ + \hat{\sigma}_{j,j+1}^z \right),$$

but our conclusions remain the same for other experimentally relevant gauge-breaking errors.

Upon employing linear gauge protection (5) with $c_j = (-1)^j$, we find that the revivals are almost perfectly restored over all investigated evolution times at experimentally accessible protection strengths $V \gtrsim 8J$. This is also reflected in the electric flux, shown in Fig. 3(b). Whereas unprotected errors destroy persistent oscillations typical of scarred dynamics of the ideal case, linear gauge protection restores such oscillations qualitatively and quantitatively for moderate values of $V$.

These results not only indicate the efficacy and experimental feasibility of linear gauge protection in stabilizing a very fine-tuned phenomenon such as QMBS, but it also shows the intimate connection between QMBS and gauge invariance, as elucidated in further detail in Ref. [100].
the dynamical emergence of an effective disorder over the background charges associated with the superselection sectors $g$ involved in the superposition. In an ideal situation, this leads to localized dynamics for all evolution times, where for example the imbalance

$$I(t) = \frac{1}{L} \int_0^t ds \sum_{j=1}^L p_j \langle \psi(s) | \hat{n}_j | \psi(s) \rangle,$$

(19)

with $p_j = 2 \langle \psi_0 | \hat{n}_j | \psi_0 \rangle - 1$, does not decay to zero (as would be expected in the case of thermalization) but rather settles into a finite plateau persisting indefinitely. However, also in this case it turns out that a great deal of fine-tuning is required in order to witness DFL, while even perturbative gauge-breaking errors can quickly destroy it [117]; see red curves in Fig. 4(a,b), illustrating this for both the U(1) QLM and $Z_2$ LGT, respectively. Nevertheless, it has been demonstrated recently that linear gauge protection can stabilize DFL in U(1) QLMs [94], and even enhance it in the case of $Z_2$ LGTs [70]; see Fig. 4. For both models, the initial state is prepared as a domain wall in the matter fields (left half fully occupied while the right half is empty), and the electric fields are prepared locally as a superposition of both local eigenstates, such that the initial states form a superposition over an extensive number of superselection sectors. Krylov-subspace methods are employed for the time evolution, with $L = 8$ lattice sites and periodic boundary conditions in place.

In the case of the U(1) QLM, the linear gauge protection (5) with $c_j = J(-1)^j$ was utilized to protect against the error term (18), but with periodic boundary conditions, in Ref. [116]. This scheme of Stark gauge protection (named so due to $c_j$ involving a linear staggered potential) goes beyond the concept of quantum Zeno dynamics, and stabilizes DFL up to all accessible evolution times. The emergent gauge theory still hosts the same U(1) gauge symmetry as the U(1) QLM, and DFL is restored to the same plateau of the ideal case, as indicated by the imbalance dynamics in Fig. 4(b). Note how even a moderate value of $V = 10J$ yields almost perfect quantitative agreement with the ideal case. Note that in the case of the U(1) QLM, $\tilde{n}_j = (\hat{\sigma}_j^z + 1)/2$ in Eq. (19).

For the case of the $Z_2$ LGT, errors inspired from the implementation of Ref. [40] are considered. The linear gauge protection (13) based on the LPG (11) are employed for all evolution times. The emergent gauge theory still hosts the same $U(1)$ gauge symmetry as the U(1) QLM, and DFL is restored to the same plateau of the ideal case, as indicated by the imbalance dynamics in Fig. 4(b). Note how even a moderate value of $V = 10J$ yields almost perfect quantitative agreement with the ideal case. Note that in the case of the U(1) QLM, $\tilde{n}_j = (\hat{\sigma}_j^z + 1)/2$ in Eq. (19).

V. EXPERIMENTAL APPLICATIONS

Our work on linear gauge protection has been used experimentally in large-scale implementations of the spin-1/2 U(1) QLM [43, 44]. In these cold-atom experiments, the U(1) QLM has been mapped onto the Bose–Hubbard model

$$\hat{H}_{\text{BH}} = -J_{\text{BH}} \sum_{j=1}^{L_{\text{BH}}-1} (\hat{b}_j^\dagger \hat{b}_{j+1} + \text{H.c.}) + U \sum_{j=1}^{L_{\text{BH}}} \tilde{n}_j (\tilde{n}_j - 1)$$

$$+ \sum_{j=1}^{L_{\text{BH}}} \left[ (-1)^j \frac{\delta}{2} + j \Delta \right] \tilde{n}_j,$$

(20)

where $J_{\text{BH}}$ is the tunneling strength, $U$ is the on-site interaction strength, $\Delta$ is the strength of a linear potential generated by the gravitational force, and $\delta$ is the strength of the staggered chemical potential. Further, $L_{\text{BH}}$ is the
total number of sites of the bosonic superlattice, corresponding to \( L = L_{\text{BH}}/2 \) matter sites and \( L_{\text{BH}}/2 - 1 \) gauge sites of the QLM. The bosonic ladder operators \( \hat{b}_j, \hat{b}^\dagger_j \) on site \( j \) satisfy the canonical commutation relations \([\hat{b}_j, \hat{b}_i] = 0 \) and \([\hat{b}_j, \hat{b}^\dagger_i] = \delta_{j,i}\), and \( \hat{n}_j = \hat{b}^\dagger_j \hat{b}_j \) is the bosonic number operator at site \( j \). Figure 1 illustrates the optical superlattice of the model in Eq. (20), and the mapping to the spin-1/2 U(1) QLM within the target sector \( \hat{G}_j |\phi\rangle = 0, \forall j \).

The U(1) QLM can be obtained from second-order perturbation theory in the limit of \( U, \delta \gg J_{\text{BH}} \) and \( U \approx 2\delta \) [43]. The parameters of Eq. (3) for \( S = 1/2 \) can further be related to those of Eq. (20) from degenerate perturbation theory as

\[
\kappa = 2\sqrt{2} J_{\text{BH}}^2 \left( \frac{\delta}{\delta^2 - \Delta^2} + \frac{U - \delta}{(U - \delta)^2 - \Delta^2} \right),
\]

\[
\mu = \delta - \frac{U}{2},
\]

where \( \kappa = J/\sqrt{S(S+1)} \). Note that for \( S = 1/2 \), the term \( \propto \eta^2 \) in Eq. (3) can be neglected as it is an inconsequential constant energy term, because \((\hat{s}_j^\dagger s_{j+1})^2 = 1\) in this case.

One can then show that gauge-breaking processes in this mapping are suppressed by the linear gauge protection term \( \sum \kappa \hat{G}_\ell \), where [77, 118]

\[
c_\ell = (-1)^\ell \left[ \Delta \ell + \left( U - \delta + \frac{\Delta}{2} \right) \right],
\]

\[
\hat{G}_\ell = (-1)^\ell \left[ \frac{1}{2} (\hat{n}_{\ell-1,\ell} + \hat{n}_{\ell,\ell+1}) + \hat{n}_\ell - 1 \right],
\]

with \( \ell \) the index of a matter site and \( \hat{n}_{\ell,\ell+1} \) the bosonic number operator at the gauge site between matter sites \( \ell \) and \( \ell + 1 \). Additional energy penalties due to \( U \) and \( \delta \) enforce a maximum of single- and double-occupations on matter and gauge sites, respectively. Within that occupation subspace, the “proto-Gauss’s law” operators \( \hat{G}_\ell \) stabilize the gauge theory in the quantum simulator, based on the concept of linear gauge protection.

The above mapping has allowed the large-scale quantum simulation of the spin-1/2 U(1) QLM in up to 71 superlattice sites, where in Ref. [43] an adiabatic mass ramp was employed to probe Coleman’s phase transition and the adherence to Gauss’s law was certified for the first time in a quantum-simulation experiment. In Ref. [44], global quenches were performed to probe the thermalization dynamics of gauge theories. Recently, we have also proposed a modification of this setup that allows introducing a tunable topological \( \theta \)-angle to study confinement in gauge theories [118] (see also the related work [119]). We emphasize that all these experiments are possible due to the control and suppression of gauge-breaking errors by linear gauge protection.

The linear gauge protection can also be employed fruitfully in other experimental platforms. For example, we have proposed an experiment in Rydberg atoms with optical tweezer for probing DFL, exploiting linear gauge protection in terms of the LPG [70]. Recently, linear gauge protection has further been used in a superconducting qubit processor within the Early Access Program of Google Quantum AI to tune from a \( \mathbb{Z}_2 \) gauge symmetry to the larger continuous U(1) gauge symmetry [46]. This was achieved by adding a term proportional to the U(1) Gauss’s law generators given in Eq. (4), which required only extremely simple single-qubit gates without any relevant experimental overhead. In that experiment, a stabilization of the U(1) gauge symmetry over 15 Trotter steps has been demonstrated using a very moderate strength of \( V = 6J \). Further, it was shown how the resulting change from a \( \mathbb{Z}_2 \) to an effective U(1) gauge symmetry leads to a drastic modification of the system dynamics, in particular a freezing of the dynamics due to the severe constraints given by the U(1) Gauss’s law generator.

VI. CONCLUSION

To conclude, the control and stabilization of gauge symmetry in quantum simulators has made great strides in the recent years, theoretically and experimentally. Though important questions for the future will be how the stabilization strategies perform for higher-dimensional systems [11] and non-Abelian symmetries [27, 29, 37, 38, 62, 76, 120, 121], the achieved advances are already laying the foundations for future quantum simulations of effects of importance to high-energy and nuclear physics, including in the near term confinement [23, 32, 33, 35, 38, 45, 46, 122], string breaking [34, 56, 123], topological terms [23, 118, 119, 124–130], transport coefficients [131], the question how gauge theories reach thermal equilibrium [44, 132, 133], and the Schwinger effect of particle production [81, 134], to just give some salient examples.

The topics treated in this review also open the horizon to a fresh view on gauge-symmetry violation: while in fundamental models of nature, such as the Standard Model of Particle Physics, gauge symmetry is a postulate, quantum simulators that do not use the approach of integrating out are physically described by an enlarged Hilbert space, where also configurations live that violate Gauss’s law. Apart from the important question how gauge-symmetry can emerge in such a setting [47–52, 68, 69, 73, 74], it then becomes natural to investigate violations of gauge symmetry not as an error but as an effect that generates new physical phenomena. An example has been mentioned in this work, where superpositions of different gauge superselection sectors in the initial state induce slow dynamics in the form of disorder-free localization [92, 93]. Another example is a phenomenon that we have dubbed *staircase prethermalization*, where a small term that breaks the gauge symmetry drives the system into a series of long-
lived plateaus of successively stronger gauge-symmetry violations [135, 136].

As all these examples show, the quantum simulation of lattice gauge theories opens up to an extremely fertile field of rich physical phenomena.

ACKNOWLEDGMENTS

We are grateful to the groups of Fabian Grusdt, Johannes Knolle, Jian-Wei Pan, Zlatko Papić, Bing Yang, Zhen-Sheng Yuan, Zhang Jiang and the Google Quantum AI team, as well as to Debasish Banerjee, Luca Barbiero, Annabelle Bohrdt, Markus Heyl, Haifeng Lang, Julius Mildenberger, Ian P. McCulloch, Arnab Sen, Maarten Van Damme, and Hongzheng Zhao for collaborations on related work. We are grateful to Lukas Homeier for assistance with Fig. 2. J.C.H. acknowledges funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programm (Grant Agreement no 948141) — ERC Starting Grant SimUcQuam, and by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy – EXC-2111 – 390814868. P.H. acknowledges support by the ERC Starting Grant StrEnQTh (project ID 804305), the Google Research Scholar Award ProGauge, Provincia Autonoma di Trento, and Q@TN — Quantum Science and Technology in Trento.

[1] Proceedings of the 2021 Quantum Simulation for Strong Interactions (QuaSi) Workshops (2022), https://iquis.uw.edu/.
[2] Immanuel Bloch, Jean Dalibard, and Wilhelm Zwerger, “Many-body physics with ultracold gases,” Rev. Mod. Phys. 80, 885–964 (2008).
[3] Waseem S. Bakr, Jonathon I. Gillen, Amy Peng, Simon Fölling, and Markus Greiner, “A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice,” Nature 462, 74–77 (2009).
[4] Philipp Hauke, Fernando M Cucchietti, Luca Tagliacozzo, Ivan Deutsch, and Maciej Lewenstein, “Can one trust quantum simulations?” Reports on Progress in Physics 75, 082401 (2012).
[5] Yuri Alexeev, Dave Bacon, Kenneth R. Brown, Robert Calderbank, Lincoln D. Carr, Frederic T. Chong, Brian DeMarco, Dirk Englund, Edward Farhi, Bill Fefferman, Alexey V. Gorshkov, Andrew Houck, Jungsang Kim, Shelby Kimmel, Michael Lange, Seth Lloyd, Mikhail D. Lukin, Dmitri Maslov, Peter Mauz, Christopher Monroe, John Preskill, Martin Roetteler, Martin J. Savage, and Jeff Thompson, “Quantum computer systems for scientific discovery,” PRX Quantum 2, 017001 (2021).
[6] Natalie Kico, Alessandro Roggero, and Martin J. Savage, “Standard model physics and the digital quantum revolution: Thoughts about the interface,” (2021), arXiv:2107.04769 [quant-ph].
[7] Mari Carmen Bañuls, Rainer Blatt, Jacopo Catani, Alessio Celi, Juan Ignacio Cirac, Marcello Dalmon, Leonardo Fallani, Karl Jansen, Maciej Lewenstein, Simone Montangero, Christine A. Muschik, Benni Reznik, Enrique Rico, Luca Tagliacozzo, Karel Van Acoleyen, Frank Verstraete, Uwe-Jens Wiese, Matthew Wingate, Jakub Zakrzewski, and Peter Zoller, “Simulating lattice gauge theories within quantum technologies,” The European Physical Journal D 74, 165 (2020).
[8] M. Dalmon and S. Montangero, “Lattice gauge theory simulations in the quantum information era,” Contemporary Physics 57, 388–412 (2016), https://doi.org/10.1080/00107514.2016.1151199.
[9] Erez Zohar, J Ignacio Cirac, and Benni Reznik, “Quantum simulations of lattice gauge theories using ultracold atoms in optical lattices,” Reports on Progress in Physics 79, 014401 (2015).
[10] Monika Aidelsburger, Luca Barbiero, Alejandro Bermudez, Titas Chanda, Alexandre Dauphin, Daniel González-Cuadra, Przemysław R. Grzybowski, Simon Hands, Fred Jendrzejewski, Johannes Jinnemann, Gedinnias Juzeliūnas, Valentin Kasper, Angelo Piga, Shi-Ju Ran, Matteo Rizzi, Germán Sierra, Luca Tagliacozzo, Emmanuele Tirrito, Torsten V. Zache, Jakub Zakrzewski, Erez Zohar, and Maciej Lewenstein, “Cold atoms meet lattice gauge theory,” Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 380, 20210064 (2022).
[11] Erez Zohar, “Quantum simulation of lattice gauge theories in more than one space dimension—requirements, challenges and methods,” Philosophical Transactions of the Royal Society of London Series A 380, 20210069 (2022), arXiv:2106.04609 [quant-ph].
[12] Christian W. Bauer, Zohreh Davoudi, A. Baha Balantekin, Tammy Bhattacharya, Marcela Carena, Wibe A. de Jong, Patrick Draper, Aida El-Khadra, Nate Gemelke, Masanori Hanada, Dmitri Kharzeev, Henry Lamn, Ying-Ying Li, Junyu Liu, Mikhail Lukin, Yanick Meurice, Christopher Monroe, Benjamin Nachman, Guido Pagano, John Preskill, Enrico Rinaldi, Alessandro Roggero, David I. Santiago, Martin J. Savage, Irfan Siddiqi, George Siopsis, David Van Zanten, Nathan Wiebe, Yukari Yamauchi, Kübra Yeter-Aydiniz, and Silvia Zorzetti, “Quantum simulation for high energy physics,” (2022), 10.48550/ARXIV.2204.03381.
[13] S. Weinberg, The Quantum Theory of Fields, Vol. 2: Modern Applications (Cambridge University Press, 1995).
[14] C. Gattringer and C. Lang, Quantum Chromodynamics on the Lattice: An Introductory Presentation, Lecture Notes in Physics (Springer Berlin Heidelberg, 2009).
[15] A. Zee, Quantum Field Theory in a Nutshell (Princeton University Press, 2003).
[16] C. J. Hamer, Zheng Weihong, and J. Oitmaa, “Series expansions for the massive schwinger model in hamiltonian lattice theory,” Phys. Rev. D 56, 55–67 (1997).
[17] M. C. Bañuls, K. Cichy, J. I. Cirac, and K. Jansen, “The mass spectrum of the schwinger model with matrix
product states,” Journal of High Energy Physics 2013, 158 (2013).
[18] Mari Carmen Bañuls, Krzysztof Cichy, J. Ignacio Cirac, Karl Jansen, and Hana Saito, “Matrix product states for lattice field theories,” (2013), 10.48550/ARXIV.1310.4118.
[19] H. Saito, M. C. Bañuls, K. Cichy, J. I. Cirac, and K. Jansen, “Thermal evolution of the one-flavour schwinger model using matrix product states,” (2015), 10.48550/ARXIV.1511.00794.
[20] Mari Carmen Bañuls, Krzysztof Cichy, Karl Jansen, and Hana Saito, “Chiral condensate in the schwinger model with matrix product operators,” Phys. Rev. D 93, 094512 (2016).
[21] Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller, and Rainer Blatt, “Real-time dynamics of lattice gauge theories with a few-qubit quantum computer,” Nature 534, 516–519 (2016).
[22] Hannes Bernien, Sylvain Schwartz, Alexander Keesling, Harry Levine, Ahmed Omran, Hannes Pichler, Soon-won Choi, Alexander S. Zibrov, Manuel Endres, Markus Greiner, Vladan Vuletić, and Mikhail D. Lukin, “Probing many-body dynamics on a 51-atom quantum simulator,” Nature 551, 579–584 (2017).
[23] Federica M. Surace, Paolo P. Mazza, Giuliano Giudici, Alessio Lerose, Andrea Gambassi, and Marcello Dalmonte, “Lattice gauge theories and string dynamics in rydberg atom quantum simulators,” Phys. Rev. X 10, 021041 (2020).
[24] N. Klco, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage, “Quantum-classical computation of schwinger model dynamics using quantum computers,” Phys. Rev. A 98, 032331 (2018).
[25] C. Kokail, C. Maier, R. van Bijnen, T. Brydges, M. K. Joshi, P. Jurcevic, C. A. Muschik, P. Silvi, R. Blatt, C. F. Roos, and P. Zoller, “Self-verifying variational quantum simulation of lattice models,” Nature 569, 355–360 (2019).
[26] Erez Zohar and J. Ignacio Cirac, “Removing staggered fermionic matter in u(n) and su(n) lattice gauge theories,” Phys. Rev. D 99, 114511 (2019).
[27] Natalie Klco, Martin J. Savage, and Jesse R. Stryker, “Su(2) non-abelian gauge field theory in one dimension on digital quantum computers,” Phys. Rev. D 101, 074512 (2020).
[28] Anthony Ciavarella, Natalie Klco, and Martin J. Savage, “Trailhead for quantum simulation of su(3) yang-mills lattice gauge theory in the local multiplet basis,” Phys. Rev. D 103, 094501 (2021).
[29] Yasar Y. Atas, Jinglei Zhang, Randy Lewis, Amin Jahanpour, Jan F. Haase, and Christine A. Muschik, “Su(2) hadrons on a quantum computer via a variational approach,” Nature Communications 12, 6499 (2021).
[30] Nhung H. Nguyen, Minh C. Tran, Yingyue Zhu, Alaina M. Green, C. Huerta Alderete, Zohreh Davoudi, and Norbert M. Linke, “Digital quantum simulation of the schwinger model and symmetry protection with trapped ions,” (2021), 10.48550/ARXIV.2112.14262.
[31] Christine Muschik, Markus Heyl, Esteban Martinez, Thomas Monz, Philipp Schindler, Berit Vogell, Marcello Dalmonte, Philipp Hauke, Rainer Blatt, and Peter Zoller, “U(1) wilson lattice gauge theories in digital quantum simulators,” New Journal of Physics 19, 103020 (2017).
[32] Erez Zohar and Benni Reznik, “Confinement and lattice quantum-electrodynamic electric flux tubes simulated with ultracold atoms,” Phys. Rev. Lett. 107, 275301 (2011).
[33] Erez Zohar, J. Ignacio Cirac, and Benni Reznik, “Simulating compact quantum electrodynamics with ultracold atoms: Probing confinement and nonperturbative effects,” Phys. Rev. Lett. 109, 125302 (2012).
[34] D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, U.-J. Wiese, and P. Zoller, “Atomic quantum simulation of dynamical gauge fields coupled to fermionic matter: From string breaking to evolution after a quench,” Physical Review Letters 109 (2012), 10.1103/physrevlett.109.175302.
[35] L. Tagliacozzo, A. Celi, A. Zamora, and M. Lewenstein, “Optical abelian lattice gauge theories,” Annals of Physics 330, 160–191 (2013).
[36] Erez Zohar, J. Ignacio Cirac, and Benni Reznik, “Simulating (2 + 1)-dimensional lattice qed with dynamical matter using ultracold atoms,” Phys. Rev. Lett. 110, 055302 (2013).
[37] D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, U.-J. Wiese, and P. Zoller, “Atomic quantum simulation of U(n) and SU(n) non-abelian lattice gauge theories,” Phys. Rev. Lett. 110, 125303 (2013).
[38] L. Tagliacozzo, A. Celi, P. Orland, M. W. Mitchell, and M. Lewenstein, “Simulation of non-abelian gauge theories with optical lattices,” Nature Communications 4, 2615 (2013).
[39] P. Hauke, D. Marcos, M. Dalmonte, and P. Zoller, “Quantum simulation of a lattice schwinger model in a chain of trapped ions,” Phys. Rev. X 3, 041018 (2013).
[40] Christian Schweizer, Fabian Grusdt, Moritz Berngruber, Luca Barbiero, Eugene Demler, Nathan Goldman, Immanuel Bloch, and Monika Aidselsburger, “Floquet approach to Z2 lattice gauge theories with ultracold atoms in optical lattices,” Nature Physics 15, 1168–1173 (2019).
[41] Frederik Görg, Kilian Sandholzer, Joaquín Minguzzi, Rémi Desbuquois, Michael Messer, and Tilman Esslinger, “Realization of density-dependent peierls phases to engineer quantized gauge fields coupled to ultracold matter,” Nature Physics 15, 1161–1167 (2019).
[42] Alexander Mil, Torsten V. Zache, Apoorva Hegde, Andy Xia, Rohit P. Bhatt, Markus K. Oberthaler, Philipp Hauke, Jürgen Berges, and Fred Jendrzejewski, “A scalable realization of local u(1) gauge invariance in cold atomic mixtures,” Science 367, 1128–1130 (2020).
[43] Bing Yang, Hui Sun, Robert Ott, Han-Yi Wang, Torsten V. Zache, Jad C. Halimeh, Robert Ott, Hui Sun, Philipp Hauke, Bing Yang, Zhen-Sheng Yuan, and Norbert M. Linke, “Observation of gauge invariance in a 71-site bose–hubbard quantum simulator,” Nature 587, 392–396 (2020).
[44] Zhao-Yu Zhou, Guo-Xian Su, Jad C. Halimeh, Robert Ott, Hui Sun, Philipp Hauke, Bing Yang, Zhen-Sheng Yuan, Jürgen Berges, and Jian-Wei Pan, “Thermalization dynamics of a gauge theory on a quantum simulator,” (2021), arXiv:2107.13563 [cond-mat.quant-gas].
Yi Guo, Luhong Su, Kai Xu, Dongning Zheng, and Heng Fan, “Observation of emergent $Z_2$ gauge invariance in a superconducting circuit,” (2021), 10.48550/ARXIV.2111.05048.

[46] Julius Mildenberger, Wojciech Mruczkiewicz, Jad C. Halimeh, Zhang Jiang, and Philipp Hauke, “Probing confinement in a $Z_2$ lattice gauge theory on a quantum computer,” (2022), 10.48550/ARXIV.2203.08905.

[47] D. Fuerster, H.B. Nielsen, and M. Ninomiya, “Dynamical stability of local gauge symmetry creation of light from chaos,” Physics Letters B 94, 135 – 140 (1980).

[48] E. Poppitz and Y. Shang, “Light from chaos” in two dimensions,” International Journal of Modern Physics A 23, 4545–4556 (2008), https://doi.org/10.1142/S0217751X08041281.

[49] C. Wetterich, “Gauge symmetry from decoupling,” Nuclear Physics B 915, 135 – 167 (2017).

[50] Edward Witten, “Symmetry and emergence,” Nature Physics 14, 116–119 (2018).

[51] Carlos Barceló, Raúl Carbálo-Rubio, Luis J. Garay, and Gerardo García-Moreno, “Emergent gauge symmetries: Yang-mills theory,” Phys. Rev. D 104, 025017 (2021).

[52] Steven D. Bass, “Emergent gauge symmetries: making symmetry as well as breaking it,” Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 380, 20210559 (2022), https://royalsocietypublishing.org/doi/pdf/10.1098/rsta.2021.0055.

[53] K. Stannigel, P. Hauke, D. Marcos, M. Hafezi, S. Diehl, D. González-Cuadra, A. Dauphin, P. R. Grzybowski, and Alexandre Dauphin, “Intertwined topological phases induced by emergent symmetry protection,” Nature Communications 10, 2094 (2019).

[54] D. González-Cuadra, A. Dauphin, P. R. Grzybowski, M. Lewenstein, and A. Bermudez, “Dynamical solitons and boson fractionalization in cold-atom topological insulators,” Phys. Rev. Lett. 125, 265301 (2020).

[55] Maarten Van Damme, Jad C. Halimeh, and Philipp Hauke, “Gauge-symmetry violation quantum phase transition in lattice gauge theories,” (2020), arXiv:2010.07338 [cond-mat.quant-gas].

[56] Umberto Borla, Ruben Verresen, Jeet Shah, and Sergej Moroz, “Gauging the Kitaev chain,” SciPost Phys. 10, 148 (2021).

[57] Jad C. Halimeh, Lukas Homeier, Hongzheng Zhao, Annabelle Bohrdt, Fabian Grusdt, Philipp Hauke, and Johannes Knolle, “Enhancing disorder-free localization through dynamically emergent local symmetries,” (2021), arXiv:2111.08715 [cond-mat.quant-gas].

[58] A. Bazavov, Y. Meurice, S.-W. Tsai, J. Unmuth-Yockey, and Jin Zhang, “Gauge-invariant implementation of the abelian-higgs model on optical lattices,” Phys. Rev. D 92, 076003 (2015).

[59] Jochen Heitger, “Numerical simulations of gauge-higgs models on the lattice,” PhD Thesis (1997).

[60] M. B. Hastings and Xiao-Gang Wen, “Quasidiabatic continuation of quantum states: The stability of topological ground-state degeneracy and emergent gauge invariance,” Phys. Rev. B 72, 045141 (2005).

[61] Subir Sachdev, “Topological order, emergent gauge fields, and fermi surface reconstruction,” Reports on Progress in Physics 82, 014001 (2018).

[62] Maarten Van Damme, Haifeng Lang, Philipp Hauke, and Jad C. Halimeh, “Reliability of lattice gauge theories in the thermodynamic limit,” (2021), arXiv:2104.07040 [cond-mat.quant-gas].

[63] Jad C. Halimeh, Haifeng Lang, and Philipp Hauke, “Gauge protection in non-abelian lattice gauge theories,” New Journal of Physics 22, 113040 (2020).

[64] Jad C. Halimeh, Haifeng Lang, Julius Mildenberger, Zhang Jiang, and Philipp Hauke, “Gauge-symmetry protection using single-body terms,” PRX Quantum 2, 040311 (2021).
[78] Jad C. Halimeh, Lukas Homeier, Christian Schweizer, Monika Aidelsburger, Philipp Hauke, and Fabian Grusdt, “Stabilizing lattice gauge theories through simplified local pseudo generators,” (2021), arXiv:2108.02203 [cond-mat.quant-gas].

[79] S Chandrasekharan and U.-J. Wiese, “Quantum link models: A discrete approach to gauge theories,” Nuclear Physics B 492, 455 – 471 (1997).

[80] U.-J. Wiese, “Ultracold quantum gases and lattice systems: quantum simulation of lattice gauge theories,” Annalen der Physik 525, 777–796 (2013).

[81] V Kasper, F Hebenstreit, F Jendrzejewski, M K Oberthaler, and J Berges, “Implementing quantum electrodynamics with ultracold atomic systems,” New Journal of Physics 19, 023030 (2017).

[82] Torsten V. Zache, Maarten Van Damme, Jad C. Halimeh, Philipp Hauke, and Debasish Banerjee, “Achieving the continuum limit of quantum link lattice gauge theories on quantum devices,” (2021), arXiv:2104.00025 [hep-lat].

[83] Jad C. Halimeh, Maarten Van Damme, Torsten V. Zache, Debasish Banerjee, and Philipp Hauke, “Achieving the quantum field theory limit in far-from-equilibrium quantum link models,” (2021), arXiv:2112.04501 [cond-mat.quant-gas].

[84] Boye Buyens, Simone Montangero, Jutho Haegeman, Daniel Burgarth, Paolo Facchi, Hiromichi Nakazato, Mari Carmen Bañuls and Krzysztof Cichy, “Review on many-body localization dynamical limit using local pseudogenerators,” (2021), arXiv:2110.08041 [quant-ph].

[85] Jean-Yves Desaules, Debasish Banerjee, Ana Hudomal, Zlatko Papic, Arnab Sen, and Jad C. Halimeh, “Weak Ergodicity Breaking in the Schwinger Model,” arXiv e-prints , arXiv:2203.08830 (2022), arXiv:2203.08830 [cond-mat.quant-gas].

[86] Sanjay Moudgalya, Stephan Rachel, B. Andrei Bernevig, and Nicolas Regnault, “Exact excited states of nonperturbative gauge errors in the thermodynamic limit using local pseudogenerators,” (2021), arXiv:2204.01745.

[87] Thomas Iadecola and Michael Schecter, “Quantum many-body scar states with emergent kinetic constraints and finite-entanglement revivals,” Phys. Rev. B 101, 024306 (2020).

[88] Sanjay Moudgalya, Stephan Rachel, B. Andrei Bernevig, and Nicolas Regnault, “Exact excited states of Affleck-Kennedy-Lieb-Tasaki models: Exact results, many-body scars, and violation of the strong eigenstate thermalization hypothesis,” Phys. Rev. Lett. 119, 030601 (2017).

[89] Sanjay Moudgalya, Stephan Rachel, B. Andrei Bernevig, and Nicolas Regnault, “Exact excited states of Affleck-Kennedy-Lieb-Tasaki models: Exact results, many-body scars, and violation of the strong eigenstate thermalization hypothesis,” (2022), arXiv:2203.08830 [cond-mat.str-el].

[90] Adith Sai Aramthottil, Utso Bhattacharyya, Daniel González-Cuadra, Maciej Lewenstein, Luca Barbiero, and Jakub Zakrzewski, “Scar states in deconfined $Z_2$ lattice gauge theories,” (2022), 10.48550/ARXIV.2201.01745.
thermalization hypothesis,” Phys. Rev. B 98, 235156 (2018).

[109] Cheng-Ju Lin and Oleksi I. Motrunich, “Exact quantum many-body scar states in the Rydberg-blockaded atom chain,” Phys. Rev. Lett. 122, 173401 (2019).

[110] Michael Schecter and Thomas Iadecola, “Weak ergodicity breaking and quantum many-body scars in spin-1 XY magnets,” Phys. Rev. Lett. 123, 147201 (2019).

[111] Daniel K. Mark, Cheng-Ju Lin, and Oleksi I. Motrunich, “Unified structure for exact towers of scar states in the Affleck-Kennedy-Lieb-Tasaki and other models,” Phys. Rev. B 101, 195131 (2020).

[112] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papic, “Weak ergodicity breaking from quantum many-body scars,” Nature Physics 14, 745–749 (2018).

[113] Federica Maria Surace, Matteo Votto, Eduardo Gonzalez Lazo, Alessandro Silva, Marcello Dalmonte, and Giuliano Giudici, “Exact many-body scars and their stability in constrained quantum chains,” Phys. Rev. B 103, 104302 (2021).

[114] Cleve Moler and Charles Van Loan, “Nineteen dubious ways to compute the exponential of a matrix,” SIAM Review 45, 3–49 (2003), https://doi.org/10.1137/S00361445024180.

[115] R. B. Sidje, “EXPKIT. A software package for computing matrix exponentials,” ACM Trans. Math. Softw. 24, 130–156 (1998).

[116] Haifeng Lang, Philipp Hauke, Johannes Knolle, Fabian Grusdt, and Jad C Halimeh, “Disorder-free localization with stark gauge protection,” arXiv preprint arXiv:2203.01338 (2022).

[117] Adam Smith, Johannes Knolle, Roderich Moessner, and Dmitry L. Kovrizhin, “Dynamical localization in $z_2$ lattice gauge theories,” Phys. Rev. B 97, 245137 (2018).

[118] Jad C. Halimeh, Ian P. McCulloch, Bing Yang, and Philipp Hauke, “Tuning the topological $\theta$-angle in cold-atom quantum simulators of gauge theories,” (2022), 10.48550/ARXIV.2204.0570.

[119] Yunting Cheng, Shang Liu, Wei Zheng, Pengfei Zhang, and Hui Zhai, “Tunable confinement-deconfinement transition in an ultracold atom quantum simulator,” (2022), 10.48550/ARXIV.2204.06586.

[120] Erez Zohar, J. Ignacio Cirac, and Benni Reznik, “Cold-atom quantum simulator for su(2) Yang-Mills lattice gauge theory,” Phys. Rev. Lett. 110, 125304 (2013).

[121] A. Mezzacapo, E. Rico, C. Sabín, I. L. Egusquiza, L. Lamata, and E. Solano, “Non-abelian su(2) lattice gauge theories in superconducting circuits,” Phys. Rev. Lett. 115, 240502 (2015).

[122] D. Banerjee, S. Caspar, F. J. Jiang, J. H. Peng, and U. J. Wiese, “Nematic confined phases in the $u(1)$ quantum link model on a triangular lattice: An opportunity for near-term quantum computations of string dynamics on a chip,” (2021), 10.48550/ARXIV.2107.01283.

[123] F. Hebenstreit, J. Berges, and D. Gelfand, “Real-time dynamics of string breaking,” Phys. Rev. Lett. 111, 201601 (2013).

[124] T. V. Zache, N. Mueller, J. T. Schneider, F. Jendrzejewski, J. Berges, and P. Hauke, “Dynamical topological transitions in the massive schwinger model with a $\theta$ term,” Phys. Rev. Lett. 122, 050403 (2019).

[125] Dmitri E. Kharzeev, Larry D. McLerran, and Harmen J. Warringa, “The effects of topological charge change in heavy ion collisions: “event by event p and cp violation”,” Nuclear Physics A 803, 227–253 (2008).

[126] Kenji Fukushima, Dmitri E. Kharzeev, and Harmen J. Warringa, “Chiral magnetic effect,” Phys. Rev. D 78, 074033 (2008).

[127] D.E. Kharzeev, J. Liao, S.A. Voloshin, and G. Wang, “Chiral magnetic and vortical effects in high-energy nuclear collisions—a status report,” Progress in Particle and Nuclear Physics 88, 1–28 (2016).

[128] Volker Koch, Soeren Schlichting, Vladimir Skokov, Paul Sorensen, Jim Thomas, Sergej Voloshin, Gang Wang, and Ho-Ung Yee, “Status of the chiral magnetic effect and collisions of isobars,” Chinese Physics C 41, 072001 (2017).

[129] Dmitri E. Kharzeev and Yuta Kikuchi, “Real-time chiral dynamics from a digital quantum simulation,” Phys. Rev. Research 2, 023342 (2020).

[130] Angus Kan, Lena Funcke, Stefan Kühn, Luca Dellantonio, Jinglei Zhang, Jan F. Haase, Christine A. Muschik, and Karl Jansen, “$3+1d$ $\theta$-term on the lattice from the Hamiltonian perspective,” (2021), 10.48550/ARXIV.2111.02238.

[131] Thomas D. Cohen, Henry Lamm, Scott Lawrence, and Yukari Yamauchi (NuQS Collaboration), “Quantum algorithms for transport coefficients in gauge theories,” Phys. Rev. D 104, 094514 (2021).

[132] J. Berges, “Scaling up quantum simulations,” Nature 569, 339–340 (2019).

[133] Niklas Mueller, Torsten V. Zache, and Robert Ott, “Quantum thermalization of gauge theories: chaos, turbulence and universality,” (2021), 10.48550/ARXIV.2111.01155.

[134] T V Zache, F Hebenstreit, F Jendrzejewski, M K Oberthaler, J Berges, and P Hauke, “Quantum simulation of lattice gauge theories using wilson fermions,” Quantum Science and Technology 3, 034010 (2018).

[135] Jad C. Halimeh and Philipp Hauke, “Staircase prethermalization and constrained dynamics in lattice gauge theories,” (2020), arXiv:2004.07248 [cond-mat.quant-gas].

[136] Jad C. Halimeh and Philipp Hauke, “Origin of staircase prethermalization in lattice gauge theories,” (2020), arXiv:2004.07254 [cond-mat.str-el].