THE SCALAR-ISOSCALAR SPECTRAL FUNCTION OF STRONG MATTER IN A LARGE N APPROXIMATION

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The enhancement of the scalar-isoscalar spectral function near the two-pion threshold is studied in the framework of an effective linear $\sigma$ model, using a large $N$ approximation in the number of the Goldstone bosons. The effect is rather insensitive to the detailed $T = 0$ characteristics of the $\sigma$ pole, it is accounted by a pole moving with increasing $T$ along the real axis of the second Riemann sheet towards the threshold location from below.

1. Introduction

With the advent of the new heavy-ion experiments, the significance of the theoretical study of models at high temperature and density is growing. The study started by Hatsuda et al. of the $\sigma$ particle has recently received particular interest. At nonzero temperature $T$ and baryon density $n_B$ they conjectured the narrowing of the broad $\sigma$ resonance: during the restoration of chiral symmetry, the mass of the $\sigma$ meson decreases which entails the squeezing of the phase space available for the $\sigma \to 2\pi$ decay. The gradual enhancement of the spectral function near the two-pion threshold means that in spite of the elusive nature of the $\sigma$ particle in the vacuum, at $T = 0$, $n_B = 0$, there is a chance to see it more clearly in medium.

Our aim is to study in the leading order large N limit of the linear $\sigma$
model the trajectory of the sigma pole and its imprint on the shape of the scalar-isoscalar spectral function. Here, we point out only the main features of the investigation, referring for a more comprehensive treatment to Ref. 2 for the chiral limit and Ref. 3 for the physical value of the pion mass.

2. The model and the quantities of interest

The Lagrangian of the linear $\sigma$ model including an explicit symmetry breaking external field $h$ is parameterised as:

$$L = \frac{1}{2}[(\partial \vec{\phi})^2 - m^2 \vec{\phi}^2] - \frac{\lambda}{24N}[\vec{\phi}]^2 + \sqrt{N}h\phi_1.$$

In the broken symmetry phase one separates the vacuum expectation value of the $\sigma$ field by the replacement: $\phi_a \rightarrow (\sqrt{N}\Phi(T) + \phi^1, \phi^i)$. For the $\sigma - \pi$ system, $N = 4$ and one has $\Phi(0) = f_\pi / 2$ where $f_\pi = 93$ MeV. The role of the external field $h$ is to give a mass to the pions: $m^2_\pi(T) = h/\sqrt{N}$. There are several reasons that favour the use of a large $N$ approximation. First of all since the expansion goes in powers of $1/N$, a strongly self-coupled theory does not pose any limitation to this expansion. This approach is insensitive to the choice of the renormalisation point. It leads to a 2nd order chiral transition and provides correct critical description near $T_c$. It bears also some nice features required in phenomenology: satisfies at LO the scalar-isoscalar channel unitarity and the Adler-zero condition.

The quantities of interest in our investigation are the equation of state $\langle \phi^1 \rangle = 0$, the $\sigma$ propagator $G_{\sigma}(p)$, whose pole trajectory we are interested in, and its spectral function: $\rho_{\sigma}(p_0, T) = -\frac{1}{\pi} \lim_{\epsilon \rightarrow +0} \text{Im} G_{\sigma}(p_0 + i\epsilon, T)$.

Using the equation of state $m^2 + \frac{1}{3}\Phi^2(T) + \frac{1}{8\sqrt{N}}(\phi^a\phi^a) = \frac{h}{\sqrt{N}}$ in the pion propagator, we check the general Goldstone theorem for pions, i.e. their propagator turns out to be $G_{\pi}(p) = 1/(p^2 - m^2_\pi(T))$. This propagator is used when calculating the pion bubbles both in the equation of state and in the $\sigma$ propagator.

At leading order in the $1/N$ expansion $G_{\sigma}^{-1}(p)$ is given by

$$G_{\sigma}^{-1}(p) = p^2 - m^2 - \left[\pi^{\pi} + \phi^{\phi^1} + \phi^{\phi^i} + \cdots + \phi^{\phi^1} \circ \cdots \circ \phi^{\phi^i} \cdots \circ \phi^{\phi^a}\right].$$

With the help of the equation of state, after performing a coupling constant renormalisation $\frac{1}{1} + \frac{1}{16\pi^2} \ln \frac{N^2}{\lambda_R^2} = \frac{1}{\lambda_R}$, one can express $G_{\sigma}^{-1}(p)$ as

$$G_{\sigma}^{-1}(p) = p^2 - \frac{h}{\Phi(T)} - \frac{\lambda_R \Phi^2(T)/3}{1 - \lambda_R b_R(p)/3},$$

where $b(p) = \phi^{\phi^a}$. Notice, that the imaginary part of $G_{\sigma}$ is due exclusively to the imaginary pieces in $b_R(p)$ ($b_R(p)$ is the finite part of $b(p)$). In what follows we restrict ourselves to the propagator with zero spatial momentum ($p = 0$).
3. Analytical continuation

Since we are looking for poles of $G_\sigma(p_0)$ on the unphysical, second Riemann sheet, where usually resonances lie, we have to perform the analytical continuation of the expression of $G_\sigma(p_0)$. This actually means the continuation of the function $b_R(p_0)$, which initially is defined on the physical sheet. We have chosen to perform the continuation of $b(p_0)$ across the positive real axis where we have to deal with the discontinuity along the real axis of the original expression for the temperature dependent part of the bubble, $b_T(p_0)$.

The mass $M_\sigma$ and the width $\Gamma$ of the $\sigma$ particle are the real and the imaginary parts of the solution of $G_\sigma^{-1}(p_0) = 0$, respectively. To determine the value of the coupling constant $\lambda_R$, we use the phenomenological value of $M_\sigma/\Gamma$ ratio at $T = 0$ and the restriction imposed by the existence of a tachyonic pole in the theory. The tachyon, which is a pole of $G_\sigma(p_0)$ on the imaginary axis of the physical sheet, is intimately related to the effective nature of the $\lambda \Phi^4$ theory. By looking at the coupling renormalisation formula we see that, if one wants to evade triviality ($\lambda_R = 0$) maintaining the stability of the theory (i.e. $\lambda > 0$) one has to keep the value of the cutoff finite. In order to have a stable effective theory the cutoff has to be below the scale of the tachyon.

The value of the ratio $M_\sigma/\Gamma$ approaches the phenomenologically preferred value with increasing $\lambda_R$, but in the same time the tachyon mass comes closer to $M_\sigma$. Remaining below the tachyonic scale by a factor of 3 to 5, the closest value is achieved for $\lambda_R = 310$ ($M_\sigma = 3.5 f_\pi, M_\sigma/\Gamma \sim 1$) in the limiting chiral case and $\lambda_R = 400$ ($M_\sigma = 3.95 f_\pi, M_\sigma/\Gamma = 1.4$) for $m_\pi(0) = 140$ MeV.

4. Results

Fig. 1 shows the basic features of the temperature driven $\sigma$ pole trajectory for different values of the $T = 0$ pion mass. The imaginary part of the pole will eventually decrease with increasing $T$ and the pole approaches the two-pion threshold. But in a first stage the imaginary part actually increases, a feature observed also in the context of chiral perturbation theory.

One can also see that with decreasing $m_\pi(0)$ the pole approaches more and more the imaginary axis. Below a certain mass value actually two oppositely moving poles emerge from the collision on the imaginary axis of the pole in the 4th quadrant with its 3rd quadrant mirror. The one that moves towards the origin comes down from the imaginary axis switching
over to the real axis only when colliding with one pole belonging to the infinite chain of poles which are on the imaginary axis due to the Bose-Einstein distribution function. In the chiral limit no second collision occurs, the pole goes to the origin as \( T \to T_c \) along the negative imaginary axis accounting for the observed critical behaviour \(^2\).

The l.h.s of Fig. 2 presents in more details the trajectory of the \( \sigma \) pole for the physical pion mass \( m_\pi(0) = 140 \) MeV. At \( T^{**} \approx 0.69m_\pi(0) \) the real part of the pole goes below the threshold while the imaginary part is still finite. Then at \( T_{\text{reg}} \) the pole collides with its 1st quadrant mirror on the real axis of the 2nd Riemann sheet, below the threshold and splits up in two poles moving along the real axis. At \( T^* \approx 1.07m_\pi(0) \) one of the poles reaches the threshold, climbs up to the 1st Riemann sheet and becomes physical, the other remains on the real axis of the unphysical sheet. A pole moving on the real axis of the unphysical sheet towards the threshold was recently reported in Ref. \(^5\) using an alternative approach to the \( \sigma \) model.

We observe by looking at the spectral function presented on the r.h.s of Fig. 2 that only for \( T < T^{**} \) there is trace of the \( \sigma \) pole in the spectral function. For higher temperature the approximate Lorentzian shape is distorted and in this way no resonance can be identified. A gradual threshold enhancement occurs in the interval \( T \in (T^{**}, T^*) \), where the real part of the pole is smaller than \( 2m_\pi(T) \). The maximum of the enhancement develops for \( T = T^* \), around which \( \rho_\sigma(p_0, T^*) \approx (1 - 4m_\pi^2(T^*)/p_0^2)^{-1/2} \). The details of how this asymptotic behaviour sets in depend crucially on the fact that the pole approaches the threshold along the real axis.

To test the generic nature of pole evolution presented above, we have investigated at \( T = 0 \) the dependence of the pole trajectory on the baryon
charge density $n_B$. Following Ref. 6 the decrease of the chiral order parameter with increasing values of $n_B$ in matter was taken into account by linearly rescaling the $T = 0$ vacuum expectation value. Qualitatively the same pattern of the pole trajectory was obtained as $n_B$ varies, like the one presented in Fig. 2. This shows that the scenario is independent of the specific type of thermodynamical driving force applied to the system.

5. Conclusions

In the LO large N approximation of the $\pi - \sigma$ system we presented a generic scenario for the $\sigma$ pole evolution. For a physical value of $m_\sigma(0)$ the pole hits the real axis of the unphysical sheet below the threshold and produces the threshold enhancement in the spectral function when moving on the real axis towards the threshold. This represents a qualitative prediction for heavy ion collision experiments since the shape of the spectral function in a wide temperature range would lead to a high intensity narrow peak in the $2\gamma$ spectra coming from the $\sigma \rightarrow 2\pi \rightarrow 4\gamma$ decay chain.

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