Analysis of Energy Conversion during Co- and Counter-Helicity Spheromak Merging by Particle-In-Cell Simulation

Kento NISHIDA, Ritoku HORIUCHI and Yasushi ONO
Graduate School of Engineering, The University of Tokyo, Tokyo 133-0032, Japan
1)National Institute for Fusion Science, Toki 509-5292, Japan
2)Graduate School of Frontier Science, The University of Tokyo, Kashiwa 277-8561, Japan
(Received 10 January 2019 / Accepted 20 June 2019)

Plasma merging has been proposed as an attractive startup method for obtaining compact and high-beta configurations, utilizing the fast energy conversion of magnetic reconnection. In this paper, we investigate energy conversion during spheromak mergings using particle-in-cell computations in cylindrical coordinates. The simulations reveal differences in the heating characteristics that depend upon the polarity of the toroidal field. We find symmetry breaking of the energy conversion downstream from the reconnection point, which leads to symmetry breaking of the energy gain \( E \cdot J \). The energy gained by the electrostatic field in the perpendicular direction is the dominant term for the ions, while the electrons gain energy inductively, mainly parallel to magnetic field.

Keywords: particle-in-cell, plasma merging, magnetic reconnection, energy conversion, case-I/O, polarity effect

DOI: 10.1585/pfr.14.3401145

1. Introduction

Plasma merging is a widely used technique for obtaining high-beta configurations without requiring center-solenoid coils. It has become a promising candidate for non-inductive startup, utilizing magnetic reconnection as the heating source, because it can convert magnetic energy into plasma thermal energy much faster than can Joule heating [1]. There are several types of merging methods depending on the seed configurations to be collided as depicted in Fig. 1. A Spheromak is one prominent candidate for such a seed configuration because of its ready availability, well-understood formation process, and good scalability [2, 3]. There are two types of spheromak merging methods which are classified according to the sign of the toroidal field. The merging of two spheromaks with toroidal fields in the same direction is called “co-helicity merging”. Only the poloidal magnetic fields reconnect during such a merging, and the combined plasma relaxes toward a spheromak. The other, “counter-helicity” spheromak merging is a collision of two spheromaks with oppositely directed toroidal fields. The toroidal magnetic fields of the two spheromaks are completely dissipated during the merging, and combined plasma relaxes to a field-reversed configuration (FRC) [4, 5]. A large increase in the ion temperature occurs in the short duration of recon-nection, but the underlying cause of the heating remains unknown. Much work has been devoted to explaining the energization mechanisms, especially for Harris-sheet-type reconnection simulations. It is well-known that the electrostatic potential plays an important role in accelerating particles during reconnection [6]. On the other hand, it has also been suggested that ion-pickup processes play a key role in gaining energy from the reconnected field in the presence of a guiding field [7]. However, it remains unknown how kinetic processes around the x-line connect to large-scale MHD processes in the downstream recon-nection.
tion region. Thus, it is necessary to investigate the energization process not only in the vicinity of the x-line but also throughout the entire merging configuration.

Several numerical simulations of plasma merging have previously been reported. First, MHD simulations of spheromak merging were performed in cylindrical coordinates. They showed that the bouncing motion caused by pressure balance affects the reconnection rate and makes the reconnection impulsive. Hall-MHD simulations of the mergings of spherical tori (ST-ST mergings) were also studied, and they revealed that the ion and electron temperatures relax into figure-eight shapes along the magnetic field in the presence of heat conduction [8,9]. In addition to the reconnection rate and temperature relaxation, it has been pointed out that the polarity of the toroidal field can alter the structure of the current sheet in counter-helicity mergings both in experiments [10] and in Hall-MHD simulations [11]. These investigations demonstrated that decoupling of the electrons and ions causes a radial shift of the reconnection point. Deformation of the current sheet caused by the Hall-effect leads to modifications of the ion and electron flow structures. In regard to energy conversion, it is crucial to include the particle kinetic effects in magnetic reconnection. References [12,13] studied the merging of two plasmoid-like flux tubes using particle-in-cell simulation in Cartesian coordinates. The ion temperature was found to peak in counter-helicity merging due to the meandering motion of the ions, while the ion temperature profile tends to be flat or hollow in co-helicty merging [12].

In this paper, we focused on global structure of converted energy in spheromak merging. In order to simulate whole spheromak merging process with kinetic effects, we developed a new 2D cylindrical particle-in-cell code as described in the following section.

2. Numerical Setup

Most magnetic-reconnection simulations start from a Harris-type current sheet in Cartesian (XYZ) coordinates, even though plasma merging experiments are performed in cylindrical (RTZ) coordinates. We have therefore developed a new particle-in-cell code in the 2D RZ plane, assuming axisymmetry. We solve the time integrations explicitly in both the particle and the field solver by utilizing Buneman-Boris method and finite-difference time-domain (FDTD) calculations [14]. The electric field E and magnetic field B are defined on staggered grids for easier implementation of the boundary conditions and better conservation of \( \mathbf{v} \cdot \mathbf{B} \) and of the charge density \( \rho_e \). The current density \( \mathbf{J} \) is calculated using density-decomposition method (DDM) [15]. This method rigorously conserves the charge density on the grid, so the code does not have to solve the Poisson’s equation for the charge density correction, which is numerically expensive for a highly parallelized code. In order to reduce the particle noise, we use a 2nd-order shape function to represent the particles, which leads to better total energy conservation.

It is important to maintain a sufficient number of super-particles (≥ 100/ cell) to obtain accurate treatment of the velocity distribution function. However, when the density varies by an order of magnitude, it is necessary to use an excessive number of super-particles to express the large densities. In addition, the volume element \( V \) of super-particles varies depending upon its radial position \( R \) in cylindrical geometry (\( V = 2\pi R dZ \)). Such density non-uniformities make it difficult to use the particle-in-cell method for plasma merging simulations. To overcome this problem, we have introduced adaptive particle-refinement (APR) scheme originally based on Assous’s method [16], which we have improved to be applicable to higher-order shape functions. We employ weighted particles to control the number of the particles per cell, and we perform particle refinement in the cell with excessive numbers of particles. It is confirmed that charge, momentum and energy for overall particles and the charge and current density in each cell are rigorously conserved during particle refinements [17]. In addition to the APR, we have adopted dynamic domain decomposition (DDD) as the parallelization method to provide better load balancing among processing elements.

As the initial condition for the plasma merging simulation, we prepared two spheromaks in the simulation region, as described in [8]. Here, neither spheromak contains an external toroidal field (\( B_{T0} \)), so, \( B_{T0} \) in [8] is set to zero. We assume the density and pressure to be constant with \( \beta \sim 15\% \) over the entire computational region. We consider the plasma currents \( I_p \) in the spheromaks to be carried solely by the electrons, and we assume the initial velocity distributions of electrons \( f_e \) to be shifted Maxwellians with \( v_{th e}/c = 0.12 \), and \( u_e/I_p/(-e\Delta_e \Delta_t \nu_{const}) \). Note that the plasma beta \( \beta \) or initial density \( \nu_{const} \) must be high enough to avoid both instability caused by centrifugal force and the Kelvin-Helmholtz instability. The distribution functions of the ions are determined by their thermal speed \( v_{th i} \).

We conducted particle-in-cell simulations using the parameters, \( T_i/T_e = 1 \), \( \nu_{th e}/c = 0.12 \), \( \omega_{pe}/\omega_{ce} = 5 \), \( \Delta_e/\Delta_{De} = \Delta_i/\Delta_{Di} = 1.3 \), where \( T_i/T_e \) is the ratio of the ion and electron temperatures, \( \nu_{th e}/c \) is the electron thermal speed normalized by the speed of light, \( \omega_{pe}/\omega_{ce} \) is the ratio of the electron plasma frequency to the electron gyro frequency, and \( \Delta_e/\Delta_{De} = \Delta_i/\Delta_{Di} = 1.3 \). This is the ratio of the grid size to the electron Debye length. These parameters correspond to following typical dimensional parameters; magnetic field \( B_0 = 0.027 \) [T], temperature \( T_0 = 7.4 \) [keV], density \( n_0 = 2 \times 10^{17} \) [m\(^{-3}\)] and length \( L = 1.0 \) [m]. The dimensional parameters for the initial spheromak configurations are: \( B_{T0} = 0.13 \) [T], \( \nu_{const} = 1.7 \times 10^{17} \) [m\(^{-3}\)], \( I_p = 100 \) [kA]. Perfectly conducting wall is located at the limit of the Z-direction (\( Z = \pm Z_0 \)), and at the outer edge of the R-direction (\( R = R_0 \)). The boundary condition is given by
\[ E \times n = 0, \text{ where } n \text{ is a unit vector normal to the wall. The particles are reflected elastically at the perfect conducting wall.} \]

3. Simulation Results

3.1 Overview of merging process

First, we show an overview of the spheromak merging from our particle-in-cell code. Two-dimensional profiles of the ion temperature (color) and magnetic flux (lines) are shown in Fig. 2. In Fig. 2 (a), the upper spheromak \((Z > 0)\) has a negative toroidal field at \(t = 0\), while the lower spheromak \((Z < 0)\) has a positive one. In the process of merging, oppositely directed toroidal fields of two spheromaks cancel each other and the plasma finally relaxes into a FRC, which has no toroidal field. From \(\omega_{pe} t \sim 60\), the magnetic field begins to change its topology, forming a current sheet between the two spheromaks. The current sheet can be decomposed into toroidal and radial components, and the sign of each current is determined by the magnetic field to be reconnected. With our given initial conditions, both co-helicity and counter-helicity mergings have negative toroidal currents. In addition to a toroidal component, the current sheet in a counter-helicity merging has a radial component, and its sign is determined by the reconnecting toroidal field. In the case shown in Fig. 2 (a), the radial current has a negative sign, which is mainly carried by electrons flowing in the \(+R\) direction. Because the electrons tend to be strongly magnetized, the structure of the current sheet is modified by the electron outflow in the presence of the Hall effect. As shown in the panel with \(\omega_{pe} t = 137.5\) in Fig. 2(a), the radial position at which reconnection takes place shifts outward. Thus, this type of counter-helicity merging is named as “case-O”. Conversely, the upper \((Z > 0)\) spheromak in Fig. 2 (b) has a positive toroidal field, while the lower \((Z < 0)\) one has a negative field. This combination of toroidal fields leads to a radially inward movement of the reconnection point, so this type of merging is called “case-I”. This deformation of the structure of the current sheet has been identified previously both in experiments [10] and in Hall-MHD simulation [11].

In Fig. 2, it is obvious that the ion temperature, which is quite low in the initial stages of merging, increases substantially in the downstream region as a result of magnetic reconnection. This demonstrates that most of the ion heating takes place inside the last closed flux surface (LCFS) of the newly formed FRC. However, Fig. 2 displays totally different 2D profiles for the ion temperatures in the two modes. In Fig. 2 (a) (case-O), ion heating occurs mainly at the inboard side of the reconnection point, while the peak ion temperature can be seen at the outer side of the reconnection point in case-I. This is attributed to biased ion outflow structures that result from the current sheet deformation. In case-O, the outward movement of the x-line causes a strong reconnection-driven ion flow in the radially inward direction, as shown in Fig. 2 (a). In contrast, the ion flow in the \(+R\) direction is stronger than that in the \(−R\) direction in Fig. 2 (b) (Case-I). We conclude that this symmetry breaking of the kinetic-energy flux of the ions is the cause of the one-sided ion temperature profile. The bifurcation of the temperature profile—depending on whether we are considering the case-I or case-O—is consistent with the experimental data reported in [18]. Although the panel with \(\omega_{pe} t = 183.3\) in Fig. 2 (b) shows that significant thermal energy exists outside the LCFS, a large portion of this energy bounces back into the core region after a short time. This is because the ion Larmor radius in counter-helicity case is comparable to the size of the FRC. Because its guiding center is inside the LCFS, energized ions come back into the core region in the later stages and are efficiently confined within the LCFS.

The time evolution of the merging rate \(\eta_{merg}\) and the radial position of the reconnection point \(r_X\) are plotted in Figs. 3 (a) and (b). We define the merging rate, which represents the level of completion of the merging, as \(\eta_{merg} = (\Psi_X − \Psi_X(t = 0))/\max(\Psi_{O1}, \Psi_{O2})\), where \(\Psi_X, \Psi_{O1}, \Psi_{O2}\) are the poloidal magnetic flux function at the x-line, and the magnetic axis (O-point) of each spheromak, respectively. Figure 3 (a) shows that the merging time is \(\omega_{pe} t = 36 - 149\) for co-helicity merging, and is \(\omega_{pe} t = 36 - 184\) for counter-helicity merging, where the start time and the end time of the merging are defined as the times required to reach \(\eta_{merg} = 0.02\) and \(\eta_{merg} = 0.98\), respectively. The radial shift of the x-line caused by the polarity effect is
clearly shown in Fig. 3 (b). While a large displacement of the x-line position can be seen for case-O, the movement of the x-line is more subtle for case-I. However, this is no surprise because the radially outward shift of the x-line is also observed in the co-helicity case which is not affected by the polarity effect.

Figure 3 (c) shows the time evolution of the volume-integrated magnetic (green), electric (blue), kinetic (magenta) and total (gray) energies for three types of mergings, counter-helicity (O), counter-helicity (I) and co-helicity. At the beginning of the simulation, the magnetic energy comprises 85% of the total energy, while remaining 15% is owned by the particles. During the merging, magnetic energy is converted into kinetic or thermal energy of the plasmas conserving the total energy. Because magnetic reconnection in co-helicity merging does not release the toroidal magnetic field energy, the amount of exchanged energy is approximately 20% of that of counter-helicity merging. It is noteworthy that the volume integral of the converted energy is almost the same for both case-I and case-O, although there are significant differences in the ion temperature profile.

3.2 Electrostatic potential and $E \cdot J$ profile

Figure 4 shows the two dimensional profiles of $(E \cdot J)$total (left column), $(E \cdot J)_{tor}$ (right column). It is clearly shown that the ions mainly gain their kinetic energy near the separatrix. The ions gain kinetic energy at
the outboard side of the reconnection separatrix in case-
I (Fig. 4(a)), while most of the energy conversion takes
place at the inboard side in case-O (Fig. 4(c)). The re-
region where the ions gain their energy is consistent with
the ion temperature profile shown in Fig. 2. Although the
apparent symmetry breaking of \((E \cdot J)_{\text{pol}}\) that occurs in
counter-helicity merging is not seen in the co-helicity case
(Fig. 4(e)), the \((E \cdot J)_{\text{pol}}\) profile tilts slightly, as can be seen
in the guide-field reconnection. It is noteworthy that more
than 95% of the energy conversion takes place inside the
LCFS. This leads to efficient confinement of the converted
thermal energy after the merging phase.

Meanwhile, the toroidal energy gain of the electrons
is concentrated in the current sheet in all three types of
mergings (Figs. 4(b) (d) (f)). The electron energy gain in
the toroidal direction \((E \cdot J)_{\text{tor}}\) is larger than that of the
poloidal direction \((E \cdot J)_{\text{pol}}\) by a factor of three. Considering
that there are no significant differences in the \((E \cdot J)_{\text{tor}}\)
profile, we conclude that energization of the electrons does
not depend on the type of merging.

For the further investigation of electric field respon-
sibility for accelerating the particles, Poisson’s equation,
\[ \Delta \phi_{st} = -\nabla \cdot E = \rho/e, \] is solved for the electrostatic potential
\(\phi_{st}\). Bird’s-eye views of the electrostatic potentials near the
x-line are shown in Figs. 5(a) (b). Large potential drops
can be seen in the downstream region \((20 < R/d_e < 45,
-12 < Z/d_e < 12\) for case-O (Fig. 5(a)), and \(55 < R/d_e <
70, -12 < Z/d_e < 12\) for case-I (Fig. 5(b)). The potential
well is formed mainly at one side of the exhaust region;
that is, the profile symmetry is broken again as is also the
case for the ion temperature profiles. Considering that the
symmetry breaking of \(T_i\) and \((E \cdot I)_{\text{pol}}\) agree with that of
\(\phi_{st}\), the electrostatic field seems likely to be the cause of
the symmetry breaking of the ion energy gain.

Two types of typical ion trajectories are shown in
Fig. 5(c). One trajectory (white line: trajectory #.1) pro-
cceeds toward the inboard side of the reconnection point
through the potential well, while the other trajectory (green
line trajectory #.2) proceeds in the +R direction. Kinetic
energy \(m_i v_i^2/2\), the work done by electric field \(v_i \cdot E\),
the electrostatic potential \(\phi_{st}\), and the adiabatic invariant \(\mu_i\)
along the trajectory are shown in Figs. 5(d) - (g). The ki-
etic energy and the work done by electric field are de-
composed into components parallel and perpendicular to
the magnetic field. (Here, we use the two normal vectors
\(\vec{n}_1, \vec{n}_2\) to distinguish the two components, \(\vec{n}_2 = \vec{B}_p \times \vec{\phi}\)
and \(\vec{n}_1 = \vec{n}_2 \times \vec{B}\), where \(\vec{B}, \vec{B}_p\) are unit vectors parallel
to the magnetic field and to its poloidal component, and \(\vec{\phi}\)
is a unit vector in the toroidal direction. Thus, \(\vec{n}_1\) and \(\vec{n}_2\)
are unit vectors in the direction \(\perp 1\) and \(\perp 2\). The \(\perp 2\) component
is perpendicular to the magnetic field, and it lies in the poloidal plane. Conversely, the \(\perp 1\) component is
perpendicular to both the magnetic field and the \(\perp 2\)
component.) This clearly shows that the kinetic energy in-
creases sharply after \(\omega_{pe} t \sim 130\). At the same time, the
electrostatic potential \(\phi_{st}\) at the position of ions (shown in
Fig. 5(g)) decreases as much as the particle’s kinetic energy
gain \(m_i v_i^2/2 \sim 0.4\). Figure 5(e) also shows that the main component of the work done by electric field is elec-
sstatic \(v_i \cdot E_{st}\). Therefore, it is confirmed again — from
the particle point of view—that the ions tend to gain their kinetic energy electrostatically.

From Fig. 5(d), it is shown that the ions tend to gain energy in the \( \perp 2 \) direction. Considering that there is no toroidal electrostatic field, most of the acceleration by electrostatic field works in the \( \perp 2 \) direction. This explains why the ions first gain their energy in the \( \perp 2 \) direction and then convert it into the \( \perp 1 \) or \( \parallel \) directions.

### 3.3 Decomposition of energy conversion

In order to investigate what component of the electric field dominates the energy conversion, we decomposed \( \mathbf{E} \cdot \mathbf{J} \) into several terms. First, we decomposed the electric field into electrostatic and inductive components \( \mathbf{E} = \mathbf{E}_{\text{st}} + \mathbf{E}_{\text{id}} \). In addition, these two electric field components accelerate particles in directions parallel and perpendicular to the magnetic field. Thus, the energy conversion for particle \( s \) is decomposed into four terms as follows:

\[
\mathbf{E} \cdot \mathbf{J}_s = \mathbf{E}_{\text{st},\parallel} \cdot \mathbf{J}_{s,\parallel} + \mathbf{E}_{\text{st},\perp} \cdot \mathbf{J}_{s,\perp} + \mathbf{E}_{\text{id},\parallel} \cdot \mathbf{J}_{s,\parallel} + \mathbf{E}_{\text{id},\perp} \cdot \mathbf{J}_{s,\perp}.
\]

(1)

Here, \( \mathbf{E}_{\text{st},\parallel} \) and \( \mathbf{E}_{\text{st},\perp} \) are the parallel and perpendicular components of the electrostatic field, \( \mathbf{E}_{\text{id},\parallel} \) and \( \mathbf{E}_{\text{id},\perp} \) are the electromagnetic fields in the parallel and perpendicular direction, and \( \mathbf{J}_{s,\parallel}, \mathbf{J}_{s,\perp} \) are the parallel and perpendicular current densities associated with particle \( s \). We calculate the cumulative energy conversion \( \int \int \mathbf{E} \cdot \mathbf{J} \, dx \, dt \) from time and space integrations of each term of \( \mathbf{E} \cdot \mathbf{J}_s \). Since more than 95% of the energy conversion takes place inside the LCFS, the volume integrated energy conversion over the entire simulation area shows good agreement with that inside the LCFS. Figure 6 summarizes the contributions of energy exchanged between particles and fields.

In Fig. 6, the contribution from the electrostatic field in the parallel direction is negative, and the other three terms are positive, regardless of the type of the merging or the particle species. Negative energy conversion means the electrons lose their kinetic energy and transfer it to the electrostatic field. Thus, we consider the electrostatic potential to be formed by the electrons. The major cause of electron acceleration is the inductive field along the magnetic field, which is greater than 50% in counter-helicity (I) mergings. Although the spatial profile of the energy conversion differs for case-I and case-O, we found no significant difference in the cumulative energy conversion. It is difficult for electrons to gain energy from the electrostatic field in the perpendicular direction to the magnetic field due to the magnetization. On the other hand, more than 40% of the ion energy gain consists of contributions from the electrostatic field. This tends to reduce its fraction in co-helicity merging. It is concluded that symmetry breaking of the ion temperature is caused by the one-sided acceleration of the ions by electrostatic field. On the other hand, \( \mathbf{E} \cdot \mathbf{J}_e \) exhibits a symmetry profile, because the main cause of the electron energy gain is inductive.

### 4. Conclusion

We performed 2D particle-in-cell simulations in cylindrical coordinates in order to investigate the energy conversion in co- and counter-helicity spheromak mergings. Our particle-in-cell simulations confirmed the radial shift of the x-line, depending on the polarity of the toroidal field, which is consistent with previous reports from experiments and Hall-MHD simulations. We found that not only the current sheet but also the spatial structure of the energy conversion is modified by the Hall-effect. It was shown that the two-dimensional profile of the energy conversion \( \mathbf{E} \cdot \mathbf{J}_s \) and the electrostatic potential \( \phi_{\text{st}} \) to be in good agreement with the ion temperature profile. It was indicated that more than 60% of the ion energy gain in counter-helicity merging consists of perpendicular acceleration by the electrostatic field. Taking this into account, the symmetry breaking of the ion temperature resulted from modifications of the electrostatic potential associated with the current sheet deformation. Although we observed significant differences in the ion temperature profiles, the total amount of converted kinetic energy was almost the same, regardless of the polarity of the toroidal magnetic field.

**Acknowledgement**

This work was supported by Grant-in-Aid for JSPS Research Fellows No.16J07924. This work is performed on “Plasma Simulator” (FUJITSU FX100) of NIFS with the support and under the auspices of the NIFS Collaboration Research program (NIFS17KNS091). This research was conducted using the SGI Rackable C2112-4GP3/C1102-GP8 (Redbush-U/H/L) in the Information Technology Center, The University of Tokyo.
[1] M. Yamada, R. Kulsrud et al., Rev. Mod. Phys. 82, 603 (2010).
[2] C.G.R. Geddes, T.W. Kornack et al., Phys. Plasmas 5, 1027 (1998).
[3] M. Yamada, H. Ji et al., Plasma Fusion Res. 2, 004 (2007).
[4] E. Kawamori, Y. Ono et al., Phys. Rev. Lett. 13, 3401060 (2018).
[5] L.C. Steinhauer, Phys. Plasmas 18, 070501 (2011).
[6] R. Horiuchi and T. Sato, Phys. Plasmas 4, 277 (1997).
[7] J.F. Drake, M. Swisdak et al., J. Geophys. Res. 114, A05111 (2009).
[8] A. Stanier, P.K. Browning et al., Phys. Plasmas 20, 122302 (2013).
[9] P.K. Browning, S. Cardnell et al., Plasma Phys. Control. Fusion 58, 014041 (2016).
[10] M. Inomoto, S.P. Gerherdt et al., Phys. Rev. Lett. 97, 135002 (2006).
[11] Y. Kaminou, X. Guo et al., Phys. Plasmas 24, 032508 (2017).
[12] R. Horiuchi, T. Moritaka et al., Plasma Fusion Res. 13, 3403035 (2018).
[13] K.D. Makwana, R. Keppens et al., Phys. Plasmas 25, 082904 (2018).
[14] C.K. Birdsall and A.B. Langdon, Plasma Physics Via Computer Simulation (McGraw-Hill, 1985).
[15] T.Zh. Esirkepov, Comput. Phys. Commun. 135, 144 (2001).
[16] F. Assous, T.P. Dulimbert et al., J. Comput. Phys. 187, 550 (2003).
[17] K. Nishida, X. Guo et al., Plasma Fusion Res. 13, 3401060 (2018).
[18] A. Kuwahata, Y. Kaminou et al., IEEJ Trans. Fundamentals and Materials 136, No.4, 212 (2015).