A worldsheet perspective on string inflation

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Abstract: We investigate the constraints of slow-roll inflation on the string worldsheet. A general gravity-matter set-up is used in which the worldsheet consists of an abstract CFT coupled to a 3+1 dimensional non-linear sigma model. The empirical slow-roll parameters are expressed in terms of the beta functions of operators in the matter/internal CFT and the beta function of the dilaton. The result confirms that inflation is only sensitive to macroscopic properties of the matter sector, and that in string theory inflation is a non-perturbative (in $g_s$) phenomenon and one must go beyond tree-level string theory.

Keywords: Time-dependent string theory, inflation
1. Introduction

The last ten years many attempts have been made to understand inflation from a more fundamental level within string theory [1–6]. Cosmological observations strongly suggest an era of inflation in the early Universe, and string theory, being a quantum theory of gravity with a unique UV-completion, should be able to describe this. In addition, inflation generically probes energy scales that are unobtainable in accelerator experiments, and there is a chance that string scale effects may be detectable in future cosmological observations [7–12].

One of the essential characteristics of inflation is that it solves the flatness and horizon problem within classical general relativity [13–15]. In string theory the equations of motion of classical general relativity are the conditions of conformal invariance of the worldsheet string theory. Moreover, inflation is a very coarse phenomenon that only depends on the energy density and pressure in the Universe without a need to specify any details of the matter content. As such, a string theoretic description of inflation should only depend on very generic scaling properties of the conformal field theory on the worldsheet.

Extending worldsheet descriptions of tachyon condensation scenarios [16–18], we will attempt to describe inflation with a worldsheet theory that is a combination of a spacetime
and matter-part, which mix via spacetime dependent couplings $u^a(X)$ for operators $O_a$ of an abstract internal CFT. From the viewpoint of the internal CFT alone such a deformation induces an internal renormalization group flow. Total conformal invariance of the combined theory can only be kept if the background fields adjust themselves in such a way that the running induced by the scaling behavior of the operators $O_a$ of the internal CFT is cancelled. The renormalization group flow can therefore be seen to define the possible dependence of $u^a(X)$ on the spacetime coordinates $X^\mu$, or in other words the beta functions of the full theory determine the equations of motion for the background fields $u^a(X)$. These equations can be compared to slow-roll inflation to find conditions on the internal CFT. We shall indeed find that, from the worldsheet perspective, the inflationary slow-roll parameters are completely characterized by the central charge and the scaling behavior of the couplings of the CFT, in line with our expectation that inflation is a phenomenon that only depends on generic properties of the matter sector.

This is not to say that we have solved inflation in string theory. Describing strings in a time-dependent background is notoriously difficult. In a large part this is due to our lack of a background independent description of the theory. At low energies we can resort to a supergravity description, but inflation fits awkwardly in the low energy supergravity framework (eta-problem, Lyth-bound). The worldsheet approach is conceptually different from supergravity calculations, but it has its own drawbacks when trying to describe a string in a de Sitter-like background. At tree-level (in $g_s$), we are only able to describe small deviations from Minkowski spacetime rather than de Sitter spacetime. Conversely de Sitter spacetimes cannot be described at string tree-level, as is well known [19–23]. For this reason we may already anticipate problems to describe inflationary solutions. Our result is indeed only formal. Substituting the solutions to the beta functions into the formal expressions, we find a divergence due to the fact that the dilaton cannot be stabilized in tree-level string theory (and for a dynamical dilaton inflation does not occur). This is of course the Fischer-Susskind phenomenon [19, 20]. This, however, is not the main point. We wish to show that, inflation being a coarse phenomenon, it only depends on coarse details of the internal CFT. That we do, while at the same time we recover the known Fischler-Susskind result that any tree-level string theory model is ruled out as a theory for inflation.

Our paper is structured as follows: first we describe the worldsheet set-up suitable for inflation and derive the equations of motion. We review multi-field slow-roll inflation in section 3, so that in section 4 we can state our main result. We shortly discuss the possibility to generalize the results to higher loop order. We conclude discussing the relation between our results with results known from the literature [22, 23].

2. Background dynamics for a generic worldsheet theory

2.1 Conformal perturbation of a coupled gravity and matter system

We wish to describe a realistic model of inflation in string theory, i.e. there is a $3+1$-dimensional homogeneous and isotropic cosmological spacetime which inflates. Similar to phenomenological model building, we are naturally led to consider a worldsheet CFT.
consisting of two parts: a non-linear sigma model accounting for four-dimensional gravity in combination with a matter/internal theory [16, 17]. The non-linear sigma model is a curved bosonic string in 4 dimensions, $\mu, \nu \in \{0, 1, 2, 3\}$,

$$S_{NL\sigma M} = S_{G(X)} + S_{\Phi(X)}, \quad (2.1a)$$

$$S_{G(X)} = \frac{1}{2\pi\alpha'} \int d^2 z G^{(S)}_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu, \quad (2.1b)$$

$$S_{\Phi(X)} = \frac{1}{4\pi} \int d^2 z \sqrt{g} \Phi(X) R^{(2)}, \quad (2.1c)$$

with $G^{(S)}_{\mu\nu}$ the four-dimensional string frame metric and $g_{\alpha\beta}$ the worldsheet metric. Insisting on homogeneity and isotropy as in standard cosmology, we set the Neveu-Schwarz form to zero, $B_{\mu\nu} = 0$, but we do consider the effect of the dilaton. The dilaton is a (light) scalar and is naturally a part of cosmological dynamics or any time-dependent scenario, e.g. tachyon condensation [17]. More importantly, the dilaton is closely related to the scale factor of the Einstein frame metric and as such could be driving part of the cosmological expansion.

The internal theory will be some two-dimensional CFT $S_{CFT_O}$ with central charge $c_{internal}$ and (primary and descendant) operators $O_a$ with scaling dimensions $\Delta_a$. We purposely leave the theory unspecified. The goal of this paper is to deduce what type of internal CFT, i.e. which constraints on the central charge and operator dimensions and couplings, could give rise to a realistic model for inflation. Since FRW cosmological dynamics only cares about coarse characteristics of the matter, viz. pressure and energy, we expect that only coarse information about the internal CFT should be needed to deduce cosmological dynamics. Because time-dependent backgrounds must break supersymmetry, we can incorporate all the fermionic partners to $X^\mu$, and the worldsheet diff$\times$Weyl and supersymmetry ghosts into the internal CFT.\footnote{One could keep supersymmetry manifest in principle but it is technically far more involved: with the worldsheet supersymmetric string one needs to track the GSO projection carefully whereas the superspace Green-Schwarz string does not lend itself easily to non-supersymmetric backgrounds. Essentially all these technicalities reside in the internal sector and it is not clear what one would gain by tracking them closely.} The internal CFT will exhibit characteristic scaling behavior under a deformation by nonzero couplings $u^a$ to the primary operators,

$$S = S_{CFT_O} + S_{\Phi} + S_u, \quad (2.2a)$$

$$S_{\Phi} = \frac{1}{4\pi} \int d^2 z \sqrt{g} \Phi R^{(2)}, \quad (2.2b)$$

$$S_u = \int d^2 z u^a O_a. \quad (2.2c)$$

This behavior is intrinsic to the internal theory and fully captured by the beta functions $\tilde{\beta}^a(u)$ of the couplings $u^a$, whose lowest order (classical) contribution is given by $(\Delta_a - 2)u^a$. We have included the dilaton $\Phi$ here as a (non $X$-dependent) coupling to the worldsheet curvature $R^{(2)}$ in order to easily incorporate the Weyl anomaly of the theory. At a renormalization group fixed point of this perturbed CFT, $\tilde{\beta}^\Phi(u)$ will just be proportional to the
central charge of the internal CFT,
\[ \bar{\beta}^\Phi(u) = \frac{2c_{\text{internal}}}{3} + \mathcal{O}(u). \] (2.3)

Due to the conformal perturbations of the internal theory, higher order effects in \( u \) will result in a “running” of \( \bar{\beta}^\Phi \) [24, 25].

To obtain spacetime dynamics driven by the matter sector we couple the internal theory plus dilaton to the Polyakov non-linear sigma model into a full worldsheet theory with a cross-coupling \( u^a(X)\mathcal{O}_a \) between the two sectors,
\[
S_{\text{tot}} = S_{G(X)} + S_{\text{CFT}} + S_{\Phi(X)} + S_u(X),
\]
(2.4a)
\[
S_u(X) = \int d^2 z u^a(X)\mathcal{O}_a.
\]
(2.4b)
The couplings \( u^a(X) \) to the internal CFT operators \( \mathcal{O}_a \) depend on the spacetime coordinates \( X^\mu \). Since a consistent string theory is described by a conformal worldsheet theory, the full operators \( u^a(X)\mathcal{O}_a \) are assumed to be exactly marginal deformations of the theory. That is the total theory must remain conformally invariant and the spacetime equations of motion are given by the requirement that the beta functions of the full theory vanish [26–28].

The beta functions are readily computed using worldsheet techniques and conformal perturbation theory [17]. We give a brief summary in appendix A. Here we simply state the result,
\[
0 = \frac{1}{\alpha'} \beta^G_{\mu\nu} = R_{\mu\nu} - M_{ab}(u)\nabla_\mu u^a\nabla_\nu u^b + 2\nabla_\mu \nabla_\nu \Phi,
\]
(2.5a)
\[
0 = \frac{1}{\alpha'} \beta^a = \frac{1}{\alpha'} \bar{\beta}^a(u) - \frac{1}{2} D\nabla u^a + \nabla^\rho \Phi \nabla_\rho u^a,
\]
(2.5b)
\[
0 = \frac{1}{\alpha'} \beta^\Phi = U(u) - \frac{1}{2} \nabla^2 \Phi + (\nabla \Phi)^2,
\]
(2.5c)
where \( M_{ab}(u) \) is the positive definite Zamolodchikov metric on the space of coupling constants \(^2 [17, 25],
\[
M_{ab}(u) = 4\pi^2 \langle \mathcal{O}_a(\epsilon)\mathcal{O}_b(0) \rangle_u;
\]
(2.6)
we denote its connection by \( K^a_{bc} \) and we have defined a covariant derivative [29] \( D\nabla u^a \) and scalar function \( U(u) \) respectively by
\[
D\nabla u^a = \nabla^\rho \nabla_\rho u^a + K^a_{bc} \nabla^\rho u^b \nabla_\rho u^c,
\]
(2.7a)
\[
U(u) = \frac{2c_{\text{st}}}{3\alpha'} + \frac{1}{\alpha'} \bar{\beta}^\Phi(u) = \frac{2c_{\text{st}}}{3\alpha'} + \frac{1}{\alpha'} \bar{\beta}^\Phi(u).
\]
(2.7b)
The scalar function \( U(u) \) accounts for the different quantum Weyl anomalous effects. There are contributions from the central charges of the two components of the theory, \( c_{\text{st}} = 4 \) and \( c_{\text{internal}} \equiv \frac{3}{2} \bar{\beta}^\Phi(0) \), and in addition there are higher order effects in \( u \), which are collected in the non-constant parts of \( \bar{\beta}^\Phi(u) \).

\(^2\)For later convenience we have rescaled the metric by a factor of \( 4\pi^2 \) compared to more conventional definitions.
The actual computation of the beta functions combines two methods with distinct perturbative expansions: conformal perturbation theory where $u^a$ and $\delta_a = \Delta_a - 2$ are small and $\beta^a(u) = \delta_a u^a + \ldots$ is known exactly, and separately the background field method where $u^a$ can be large but $\beta^a(u)$ and $\nabla u^a$ are required to be small. By allowing for arbitrary $\bar{\beta}^a(u)$ and $\bar{\beta}^\Phi(u)$ these methods can be combined in a mixed $\alpha'$-expansion: it can be made “exact” to all orders in $u^a$, but only to second order in $\nabla u^a$ by capturing all $u$-dependence in the arbitrary unknown functions $M_{ab}(u)$, $\bar{\beta}^a(u)$ and $\bar{\beta}^\Phi(u)$. Note that $\beta^G_{\mu\nu}(u)$ only depends on $\nabla u^a$ as the two sectors of the total theory decouple when $u^a$ is $X$-independent. Limiting ourselves to two derivatives is not an impediment, since inflation should be captured by a two derivative description, especially slow-roll inflation.

2.2 String dynamics from an action

The condition for Weyl invariance $\beta^G_{\mu\nu} = \beta^a = \beta^\Phi = 0$ determines the equations of motion for the background fields $\Phi(X)$, $G_{\mu\nu}(X)$ and $u^a(X)$. A crucial ingredient for the consistency of this interpretation is the coupling between the dilaton field $\Phi(X)$ and the other matter fields $u^a(X)$. The potential terms, $\bar{\beta}^a(u)$ and $\bar{\beta}^\Phi(u)$ in (2.3), are not independent but related via

$$M_{ab}(u)\bar{\beta}^b(u) = \partial_a \bar{\beta}^\Phi(u),$$

(2.8)

to all orders in $u$. This result may be derived from the fact that the conformal anomaly $\bar{\beta}^\Phi$ is a $c$-number rather than an operator by the Wess-Zumino consistency condition [17, 26, 28, 30, 31]. In particular $\bar{\beta}^\Phi$ is $X$-independent and hence $\nabla_\mu \bar{\beta}^\Phi$ vanishes. Since $\beta^G_{\mu\nu} = \beta^a = 0$, we can verify

$$0 = -\frac{4}{\alpha'} \nabla_\nu \bar{\beta}^\Phi = -\frac{4}{\alpha'} \nabla_\nu \left( \bar{\beta}^\Phi - \frac{1}{4} \beta^G_{\mu\nu} \right) = \frac{4}{\alpha'} \left( M_{ab}\bar{\beta}^b - \partial_a \bar{\beta}^\Phi \right) \nabla_\nu u^a. \quad (2.9)$$

The last step follows from the explicit formulae for the beta functions (2.5). Recall that the beta functions are derived up to second order in $\nabla u^a$ but are exact in powers of zeroth derivatives of $u$ due to the incorporation of all zeroth derivatives of $u$ in the potential functions $\bar{\beta}^a(u)$ and $\bar{\beta}^\Phi(u)$. Whereas our result is only an effective description for the connection between spacetime and matter sector, the matter sector itself is described exactly.

As a result of the relation (2.8) between $\bar{\beta}^\Phi(u)$ and $\bar{\beta}^a(u)$ the equations of motion can be integrated to an action

$$S_{SF} = \frac{1}{2\kappa^2} \int d^4 x \sqrt{-G} e^{-2\Phi} \left[ R + 4(\nabla \Phi)^2 - M_{ab} \nabla^a u^a \nabla^b u^b - 4U(u) \right]. \quad (2.10)$$

Transforming to the Einstein frame $\tilde{G}_{\mu\nu}^{(E)} = e^{\Phi_0 - \Phi} G_{\mu\nu}^{(S)} = e^{-\Phi} G_{\mu\nu}^{(S)}$, we obtain an action that can be directly compared to standard cosmological models,

$$S_{EF} = \frac{1}{2\kappa^2} \int d^4 x \sqrt{-G} \left[ \tilde{R} - 2\tilde{\nabla}_\mu \tilde{\Phi} \tilde{\nabla}^\mu \tilde{\Phi} - M_{ab} \tilde{\nabla}_a \tilde{u}^a \tilde{\nabla}^b \tilde{u}^b - 4e^{2\Phi} U(u) \right]. \quad (2.11)$$

Here $\kappa = \kappa_0 e^{\Phi_0} = \sqrt{8\pi G_N}$ is the gravitational coupling. The action (2.11) is simply that of a multi-scalar field model coupled to gravity,

$$S_{inflation} = \frac{1}{\kappa^2} \int d^4 x \sqrt{-G} \left[ \frac{1}{2} \tilde{R} - \frac{1}{2} g_{ij} \partial^i \phi^j \partial_j \phi - V(\phi) \right], \quad (2.12)$$
with the potential

\[ V(\phi) = 2e^{-2\phi_0} e^{2\Phi} U(u), \tag{2.13} \]

where we have defined a multi-scalar field \( \phi^i = (\Phi, u^a)^t \) and a metric on the space of fields \( g_{ij} = \left( \begin{array}{cc} 2 & 0 \\ 0 & M^2 \end{array} \right) \). Since we will be working in the Einstein frame from here on, we have dropped the tilde on the spacetime metric \( G_{\mu\nu}(X) \). The question we wish to investigate is whether the potential (2.13) is flat enough to provide realistic slow-roll inflation. Since \( V(\phi) \) is proportional to the beta function \( \bar{\beta}(u) \) of the internal sector and the central charge \( c \) of the total theory, demanding slow-roll inflation is equivalent to a set of phenomenological constraints on the internal conformal field theory. Before we turn to this question, we quickly review slow-roll inflation in multi-field models.

3. Multi-field slow-roll inflation

The rapid acceleration of the universe that characterizes inflation arises when the system is potential energy dominated. Current observations favor an adiabatic slow-roll inflationary model of early universe cosmology, whose phenomenology can be described by gravity coupled to a single scalar field. The single field inflationary case was formalized in [32]. Fundamentally there is no reason to have only one scalar field. Indeed in string theory or supergravities one generically has multiple scalar fields, although its characteristic singularity, isocurvature fluctuations, is at most 10% of the primordial powerspectrum and is at this time not a better fit to the data [33]. The connection to the powerspectrum for multi-field slow-roll inflation [14, 15, 34, 35] was formalized in [29, 36, 37]. We shall follow [29].

Minimally coupled multi-field inflation is described by the action (2.12), where \( V(\phi) \) is the scalar potential and \( g_{ij} \) is the positive definite metric on the space of scalar fields. For a homogeneous and isotropic FRW universe, the independent equations of motion for the generic multi-field action (2.12) are\(^3\)

\[ H^2 = \frac{1}{3} \left( \frac{1}{2} g_{ij} \ddot{\phi}^i \ddot{\phi}^j + V \right), \tag{3.1a} \]
\[ 0 = \nabla \ddot{\phi}^i + 3H \ddot{\phi}^i + g^{ij} \partial_j V, \tag{3.1b} \]

where \( H \) is the Hubble parameter, \( \Gamma^i_{jk} \) are the connection coefficients for the metric \( g_{ij} \) and where we define

\[ \nabla \ddot{\phi}^i = \ddot{\phi}^i + \Gamma^i_{jk} \ddot{\phi}^j \ddot{\phi}^k, \tag{3.2} \]

similar to (2.7a). The field equations (3.1) completely determine the dynamics of the model. Unfortunately equations (3.1) are difficult to solve exactly and one generally studies them

\(^3\)There is another equation of motion, \( \dot{H} = -|\dot{\phi}|^2/2 \), from the spatial part of the variation with respect to the metric, but this also follows from (3.1).
in a slow-roll approximation. To this end slow-roll parameters are defined \[29\],\(^4\)

\[
\epsilon = -\frac{\ddot{H}}{H^2}, \quad \eta^i = \frac{\dot{D}\phi^i - \frac{\dot{\phi}^i\dot{\phi}}{|\phi|^2}}{H|\phi|}, \quad (3.3)
\]

The vector \(\eta\) can be decomposed in components parallel \(\eta^\parallel\) and perpendicular \(\eta^\perp\) to the field velocity \(\dot{\phi}\). Define

\[
e_1^i = \frac{\dot{\phi}^i}{|\phi|}, \quad e_2^i = \frac{\dot{D}\phi^i - \frac{\dot{\phi}^i\dot{\phi}}{|\phi|^2}}{|\dot{D}\phi - \frac{\dot{\phi}^i\dot{\phi}}{|\phi|^2}|}, \quad (3.4)
\]

then

\[
\eta^\parallel = e_1 \cdot \eta = \frac{\dot{D}\phi \cdot \dot{\phi}}{H|\phi|^2}, \quad \eta^\perp = e_2 \cdot \eta = \frac{|\dot{D}\phi - \frac{\dot{\phi}^i\dot{\phi}}{|\phi|^2}|}{H|\phi|}, \quad (3.5)
\]

and

\[
\eta^i = \eta^\parallel e_1^i + \eta^\perp e_2^i. \quad (3.6)
\]

The parameter \(\epsilon\) is a direct measure for inflation, since \[32\]

\[
\ddot{a} > 0 \Leftrightarrow \epsilon < 1. \quad (3.7)
\]

Furthermore \(\epsilon\) and \(\eta\) together quantify the relative energy contributions of kinetic and potential energy. One can reexpress \(\text{(3.1)}\) in terms of the slow-roll parameters,

\[
H^2 = \frac{V}{3} (1 - \frac{1}{3\epsilon})^{-1}, \quad (3.8a)
\]

\[
\dot{\phi}^j + \frac{1}{\sqrt{3V}} g^{ij} \partial_j V = -\frac{1}{3} \sqrt{\frac{2\sqrt{\epsilon V}}{3}} \left( \eta^i + \frac{\epsilon \dot{\phi}^i}{|\phi|} \right), \quad (3.8b)
\]

As it is given here equation \(\text{(3.8)}\) is exact. It shows precisely which approximation is made by assuming that “potential energy strictly dominates over kinetic energy”, which is often the explanation behind slow-roll inflation. Using \(\text{(3.8)}\) one could also obtain results which are any order in slow-roll \[29, 32\]. Limiting ourselves to first order in the approximation, in which \(\epsilon, \sqrt{\epsilon \eta}^\parallel, \sqrt{\epsilon \eta}^\perp \ll 1\), equation \(\text{(3.8)}\) reduces to

\[
H^2 = \frac{1}{3} V, \quad (3.9a)
\]

\[
\dot{\phi}^j = -\frac{1}{\sqrt{3V}} g^{ij} \partial_j V. \quad (3.9b)
\]

The second equation tells us that slow-roll approximation implies \textit{gradient flow}. Using these equations we see that in the slow-roll approximation

\[
\ddot{H} = \frac{1}{2H} \frac{1}{3} \dot{V} = \frac{1}{6\sqrt{V}} \partial_i V \dot{\phi}^i = -\frac{\sqrt{3}}{6\sqrt{V}} \frac{1}{\sqrt{3V}} g^{ij} \partial_i V \partial_j V = -\frac{1}{6V} |\nabla V|^2, \quad (3.10a)
\]

\[
\dot{D}\phi^i = \partial_i \left( -\frac{1}{\sqrt{3V}} g^{ij} \partial_j V \right) + \Gamma^i_{jk} \frac{1}{3V} g^{kl} \partial_j V \partial_m V = \frac{1}{6} \nabla_i |\nabla V|^2 / V, \quad (3.10b)
\]

\(^4\)The definition for \(\eta\) differs from the definition \(\tilde{\eta} = V''/V\) normally used in single field slow-roll inflation. Specified to the single field case, \(\tilde{\eta}\) and \(\eta\) are related by \(\eta^\parallel = \epsilon - \tilde{\eta}\) to lowest order in the slow-roll approximation, cf. \(\text{(3.11)}\).
and hence in the slow-roll regime,
\[ \epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{2} \frac{\left| \nabla V \right|^2}{V^2}, \tag{3.11a} \]
\[ \eta^i = \frac{\mathbf{D} \phi^i}{H|\phi|} = \frac{1}{2 \left| \nabla V \right|} \nabla^i \frac{\left| \nabla V \right|^2}{V}, \tag{3.11b} \]
\[ \eta^\parallel = \frac{\mathbf{D} \phi \cdot \phi}{H|\phi|^2} = -\frac{1}{2 \left| \nabla V \right|^2} \nabla V \cdot \nabla \frac{\left| \nabla V \right|^2}{V} = \epsilon - \frac{\nabla^i V \nabla^j V \nabla_i \nabla_j V}{V \left| \nabla V \right|^2}, \tag{3.11c} \]
\[ \eta^\perp = \frac{\mathbf{D} \phi - \frac{\mathbf{D} \phi \cdot \phi}{|\phi|^2} \phi}{H|\phi|} = \frac{1}{2 \left| \nabla V \right|} \sqrt{\frac{\nabla^2 \left| \nabla V \right|^2}{V^2} - \frac{\left( \nabla V \cdot \nabla \frac{\left| \nabla V \right|^2}{V} \right)^2}{\left| \nabla V \right|^2}} - (\eta^\parallel)^2. \tag{3.11d} \]

4. Inflation from the worldsheet

4.1 Slow-roll parameters for tree-level worldsheet string theory

We are now in a position to address our question: how do we describe slow-roll inflation in terms of worldsheet dynamics? That is, we need to verify that the potential \( V(\Phi, u) = 2 \left( \frac{a_0}{\alpha'} \right)^2 e^{2\Phi} U(u) \) is capable of driving a slowly rolling inflaton field. We shall assume the spacetime part of the worldsheet theory to describe an accelerating (i.e. inflationary) flat, homogeneous and isotropic FRW universe, \( G^{(E)} = \text{diag}(-1, a^2(t), a^2(t), u^2(t)) \), which is driven by a homogeneous dilaton \( \Phi(X) = \Phi(t) \) and homogeneous internal fields \( u(X) = u(t) \). The demand that the slow-roll parameters are small then provides restrictions on \( V(\Phi, u) \) and hence, as conjectured, on the coarse characteristics of the internal CFT, \( c, \frac{\beta^b}{\alpha'}(u) \) and \( \beta^a(u) \). Direct calculation of (3.11) for \( V(\phi) = 2 \left( \frac{a_0}{\alpha'} \right)^2 e^{2\Phi} U(u) \) reveals

\[ \epsilon = 1 + \frac{1}{2} \gamma^2, \tag{4.1a} \]
\[ \eta^\parallel = -\epsilon - \frac{D}{2 + \gamma^2}, \tag{4.1b} \]
\[ \eta^\perp = \sqrt{\frac{1}{4} (2 + \gamma^2)^2 + D + \frac{\gamma^6 \alpha_{ab} \nabla_a \nabla_b \beta^b}{\alpha'^2 U^2} - \frac{2 \gamma^2 D - 2 \gamma^6 + \gamma^4}{2 + \gamma^2} - (\eta^\parallel)^2}, \tag{4.1c} \]

where we have defined the combinations,

\[ \gamma_a(u) = \frac{M_{ab} \beta^b}{\alpha' U} = \partial_a \ln U = \partial_a \ln \left[ \frac{2 c_{st}}{3 \alpha'} + \frac{1}{\alpha'} \beta^b \right], \tag{4.2a} \]
\[ D = \frac{\gamma^a \gamma^b \nabla_a \nabla_b U}{U} - \gamma^4. \tag{4.2b} \]

From (4.1) we immediately see that \( V(\Phi, u) = 2 \left( \frac{a_0}{\alpha'} \right)^2 e^{2\Phi} U(u) \) is incapable of driving inflation: \( \epsilon \) is always larger than unity. Regardless of the specific form of \( \gamma_a = \partial_a \ln \left[ \frac{2 c_{st}}{3 \alpha'} + \frac{1}{\alpha'} \beta^b \right] \), the positive definiteness of the Zamolodchikov metric \( M_{ab} \) ensures that \( \gamma^2 > 0 \).
Tracing back we see that the coefficient 1 in $\epsilon$, characteristic of an exponential potential, is due to the dynamics of the dilaton. One could wonder whether taking $\Phi$ constant, i.e. excluding it from the cosmological dynamics, would modify the model into one which does allow for inflation. Because the field space metric $g_{ij}$ is block diagonal, equation (3.1) implies that for a constant $\Phi$, $\Phi$ must be stabilized at $\partial V = 4 \left( \frac{\kappa_0}{\kappa} \right)^2 e^{2\Phi} U = 0$. However, excluding $\Phi = -\infty$, the relation (2.8) precludes a constant dilaton, as $U$ is not allowed to vanish. In our set-up fields $u^a(X)$ that undergo a time evolution in four-dimensional spacetime are described by a renormalization group flow of the couplings, i.e. $\tilde{\beta}^a \neq 0$. Equation (2.8) then implies that $U$ cannot vanish, which forces the dilaton to be non-constant by the requirement (2.5c) of a vanishing $\beta^\Phi$. Turning the argument around, suppose one magically stabilizes the dilaton at tree-level. Then $\epsilon = U^{-2} \beta^a \beta_a$ but $U \sim \partial V$ which must vanish by the assumption that the dilaton is stabilized.

Within tree-level worldsheet string theory, the dilaton is therefore always part of the cosmological dynamics and its tree-level exponential potential rules out an inflationary universe.

4.2 Inflation from the Ramond sector, string loop corrections and inflation from open strings

Clearly to describe inflation in string theory we must have a more complicated potential for the dilaton. One guess could be to supersymmetrize the worldsheet and include RR fields. Technically this is a far from trivial task, as it is not yet known how to compute beta functions for RR vertex operators. However at the end of the day, even including fermionic dynamics, the resulting worldsheet theory must be of the form (2.4). On the worldsheet the dilaton/vertex-operator interactions are such that they always lead to an action $S = \int e^{-2\Phi} L$ in the string frame [38]. Thus one always deduces equation (2.10) and the remainder of the analysis is the same.

Thus one is naturally led to consider string loop corrections or non-perturbative effects, i.e. open strings. From the worldsheet point of view these two additions roughly boil down to the same thing. Both are obtained by including more general worldsheet topologies than just the spherical worldsheet of tree-level string theory. The corrections from including closed string loops could convert $\epsilon$ into a more sensible expression. We can expect this based on the well-known dilaton tadpoles of Fischler-Susskind [19, 20]. Our results are an extension of the Fischler-Susskind result that to obtain a worldsheet description of strings in a de Sitter space, there must be a one loop (in $g_s$) contribution to the dilaton to have vanishing beta functions, i.e. to satisfy the equations of motion. Slow-roll inflation is in essence an adiabatic continuation of de Sitter space to a slowly varying vacuum energy.

It is interesting to see what happens if we suppose that the higher loop contributions allow us to consistently stabilize the dilaton at weak coupling independent of the value of
Then one finds the slow-roll parameters

\[ \epsilon = \frac{\beta^a \beta_a}{2(\beta^\Phi + \frac{2c_s^2}{3})}, \]

\[ \eta^\parallel = \epsilon - \frac{\beta^a \beta^b \nabla_d \beta_b}{(\beta^\Phi + \frac{2c_s^2}{3}) \beta^c \beta_c}, \]

\[ \eta^\perp = \sqrt{\frac{1}{4 \beta^c \beta_c} \left| \nabla^a \beta^b \beta_b \right|^2 - (\eta^\parallel)^2}. \]

The dilaton stabilization needs to be such that \( \alpha' U = \beta^\Phi + \frac{2c_s^2}{3} \) is no longer proportional to \( \partial \Phi V \) and hence the above expressions make sense. Of course dilaton stabilization at weak coupling has its own problems [22, 23].

The inclusion of open strings, in addition to the closed strings considered here, may yield more promising results for describing worldsheet theories on inflationary backgrounds. In the supergravity literature the usefulness/necessity of open string corrections has already been recognized [1–6, 39, 40]. Open strings have been extensively investigated from a low energy effective field theory point of view, e.g. DBI inflation, and all known viable supergravity inflationary models have an open string component.

5. Conclusion

Inflation does not care about anything but very coarse features of the matter sector, only its pressure and energy. This suggests that in string theory inflation is determined by coarse features of the internal conformal field theory on the worldsheet. Qualitatively this is what we find. At the same time our result shows that it is not possible to have an inflationary cosmology described by a tree-level string worldsheet. The exponential potential for the dilaton ensures that \( \epsilon \) is strictly larger than unity, completely independent of the internal CFT. At first sight this conclusion may be puzzling, as inflation is a classical phenomenon and one therefore may expect tree-level string theory to be sufficient for a consistent description. Nevertheless the result simply recovers that de Sitter backgrounds are known to only arise at one-loop level in worldsheet string theory through the Fischler-Susskind mechanism [19–21]. For inflation to occur the dilaton must be stabilized through such higher loop effects. If this stabilization happens at weak coupling, then inflation is possible with slow-roll parameters that only depend on the beta functions of the internal CFT.

In a way Fischler-Susskind and the result here are special cases of Dine-Seiberg runaway [22, 23]: within string theory one cannot probe a nearby vacuum from the original vacuum because in string perturbation theory as currently understood all higher order corrections are larger than the first order — string theory is either free or strongly coupled. Whereas the result in [22, 23] is obtained by general reasoning, Fischler-Susskind specifically attempt

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5In supergravity language open strings add D-terms in addition to F-term potentials. The closed string worldsheet only captures a dilaton type F-term inflation.
to describe a de Sitter cosmology from a Minkowski worldsheet, and we attempt to obtain inflation. We can be even more explicit: in our tree-level analysis the time-dependent process of inflation requires on the one hand a non-constant dilaton to satisfy the equations of motion, while on the other hand only a constant dilaton makes sense observationally. In the tree-level limit we therefore have found a clear inconsistency of the approach. A strong coupling analysis is necessary to realize inflation within string theory. The reader should be aware that we have not ruled out a non-constant dilaton scenario at all, we simply have found out that a zeroth order weak coupling approach is insufficient to describe inflation. In the strong coupling regime the dilaton may turn out to be non-constant after all.

It is interesting to note that our result confirms a conjecture in [23], that a cosmological solution in which the world is slowly sliding to its free Minkowski vacuum cannot be studied from this final state. From the reasoning in [22, 23] this appears to be a perfectly fine solution, if unlikely. Our result confirms their expectation that such a slow-roll inflationary scenario is not possible within tree-level worldsheet string theory.

To conclude: we have provided a proof of principle that the coarse characteristics of the internal CFT determine whether and how inflation occurs by expressing the slow-roll parameters in terms of the beta functions of the internal CFT. As de Sitter-like solutions only arise at one-loop in a Minkowski string worldsheet, a necessary requirement for real and realistic worldsheet models of string inflation is to include higher order string loop corrections to the analysis. This remains subject to further investigation.

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A. Calculating the beta functions

In this appendix we will review the calculation of the beta functions (2.5) of the total theory (2.4). For more details concerning this calculation we refer to [17].

A.1 Conformal perturbation theory

For a general CFT $\mathcal{C}$ that is perturbed by adding operators to the action,

$$ S = S_\mathcal{C} + \int d^2 z \; u^I \mathcal{O}_I, $$

the beta functions $\beta^I$ for the couplings $u^I$ can be defined as the coefficients of the trace of the stress-energy tensor

$$ \Theta = -\pi \beta^I \mathcal{O}_I. $$

In the Zamolodchikov renormalization group scheme these can be computed in an expansion in $u^I$ with $\delta_I = |\Delta_I - 2|$ small [17, 38, 41],

$$ \beta^I = (\Delta_I - 2) u^I + 2\pi C^I_{JK} u^J u^K + \mathcal{O}(u^3), $$

(A.3)
where \( C_{JK}^I \) are the OPE coefficients defined via

\[
O_J(z)O_K(y) = \sum I C_{JK}^I |z - y|^{\Delta_I - \Delta_J - \Delta_K} O_I \left( \frac{z + y}{2} \right),
\]

(A.4)

In the coupled system \( CFT_X \otimes CFT_O \) that is deformed by the term \( S_{\alpha}(X) = \int u^a(X)O_a \) as described in the main text (2.4), the operators in (A.2) are the three (types of) operators,

\[ O_G^{\mu\nu} = \frac{1}{2\pi\alpha'} \partial X^\mu \bar{\partial} X^\nu, \quad O_a, \quad O_\Phi = \frac{1}{8\pi} R^{(2)}, \]

(A.5)

which couple to the coupling functionals \( G_{\mu\nu}(X), u^a(X) \) and \( \Phi(X) \) respectively. By a Fourier transform these coupling functionals may be seen as an infinite set of coupling constants \( G_{\mu\nu}(p), u^a(k) \) and \( \Phi(q) \) that couple to the dressed operators \( O_p^{\mu\nu} = \frac{1}{2\pi\alpha'} \partial X^\mu \bar{\partial} X^\nu \epsilon^{\mu
u\lambda} X^\lambda 1, O_{(k,a)} = \mathcal{C}_a e^{ik \cdot X} \) and \( O_q^\Phi = \frac{1}{8\pi} R^{(2)} e^{iq \cdot X} \) with dimensions

\[
\Delta_p^{\mu\nu} = 2 + \frac{\alpha'}{2} p^2, \quad \Delta_{(k,a)} = \Delta_a + \frac{\alpha'}{2} k^2, \quad \Delta_q^\Phi = 2 + \frac{\alpha'}{2} q^2.
\]

(A.6)

We are not constraining the graviton momentum or dilaton momentum to be lightlike. \( p^2 = 0 \) and \( q^2 = 0 \) would be the on-shell condition for a free graviton and free dilaton, whereas we wish to consider the coupled gravity-matter system. The OPE coefficients can be readily computed to be

\[
C_{(k_1,a)(k_2,b)}^{(p,1)} = -\frac{\alpha'}{8\pi} (k_1 - k_2)_\mu (k_1 - k_2)_\nu \delta^4 (p - k_1 - k_2) M_{ab},
\]

(A.7a)

\[
C_{(k_2,b)(k_3,c)}^{(k_1,a)} = \delta^4 (k_1 - k_2 - k_3) C_b^a,
\]

(A.7b)

where \( C_b^a \) are the OPE coefficients of the internal CFT and we have denoted the Zamolodchikov metric by \( M_{ab} = 4\pi^2 C_{ab}^1 \). Applying (A.6) and (A.7) to (A.3) and Fourier-transforming back to position-space, yields

\[
\frac{1}{\alpha'} \beta_{\mu\nu}^G = -\frac{1}{2} \partial^\rho G_{\mu\nu} + \frac{1}{2} M_{ab} \left( u^a \partial_\mu u^b - \partial_\mu u^a \partial_\nu u^b \right),
\]

(A.8a)

\[
\frac{1}{\alpha'} \beta^a = \frac{1}{\alpha'} \left( (\Delta_a - 2) u^a + 2\pi C_b^a u^b u^c \right) - \frac{1}{2} \partial^\rho \partial_\nu u^a = \frac{1}{\alpha'} \bar{\beta}^a(u) - \frac{1}{2} \partial^\rho \partial_\nu u^a,
\]

(A.8b)

\[
\frac{1}{\alpha'} \beta^\Phi = -\frac{1}{2} \partial^\rho \partial_\nu \Phi,
\]

(A.8c)

where we use (A.3) in reverse to express \( \beta^a \) in terms of \( \bar{\beta}^a \).

A.2 Weyl anomaly and classical dilatonic contribution

In addition to the operator effects from (A.8c), the beta function for \( \Phi \) receives a further contribution from the well-known Weyl anomaly, a worldsheet contribution proportional to the worldsheet curvature. Its contribution is determined by the central charge of the
spacetime non-linear sigma model as well as by that from the (perturbed) internal theory as explained in the main text,

$$\Theta_{1\text{-loop}} = -\left(\frac{cX}{12} + \frac{1}{8}\beta^\Phi\right)R^{(2)} = -\pi \left(\frac{2cX}{3} + \beta^\Phi(u)\right)\frac{1}{8\pi}R^{(2)}$$

$$= -\pi \left(\frac{2cX}{3} + \beta^\Phi(u)\right)\mathcal{O}_\Phi. \tag{A.9}$$

Comparing this expression with the definition of the beta functions as coefficients in the stress-energy tensor (A.2), we find a contribution

$$\beta^\Phi_{1\text{-loop}} = 2\alpha' X^3 + \beta^\Phi(u)\text{ to the beta function of the dilaton.}$$

The final contribution to all of the beta functions comes from the dilaton term (2.1c) in the worldsheet action, which breaks Weyl invariance already at the classical level. Due to an additional overall $\alpha'$-factor compared to the other terms in the worldsheet, this classical contribution to the beta functions is of the same order as loop effects from the classically Weyl invariant terms. On a curved worldsheet the easiest way to determine deviation from Weyl invariance is by calculating the trace of the stress-energy tensor via

$$\Theta = -\pi \sqrt{g} \frac{\delta S}{\delta g^{\alpha\beta}} g^{\alpha\beta}. \tag{A.10}$$

This definition for $\Theta$ in terms of a variation of the worldsheet metric differs by a factor from more common definitions, which is necessary to relate the result properly with our earlier definition (A.2). One can check that this leads to the right result by looking at the metric and dilaton field, whose contributions are well-known [26, 38]. Making use of the equations of motion for $X^\mu$,

$$\partial\partial X^\rho = -\Gamma^\rho_{\mu\nu}\partial X^\mu \partial X^\nu + \pi \alpha' \partial^\rho u^a \mathcal{O}_a + \frac{\alpha'}{8} \partial^\rho \Phi R^{(2)}, \tag{A.11}$$

the classical violation of Weyl invariance by the dilaton term (2.1c) is,

$$\Theta_{\text{classical}} = -\pi \frac{\delta S_{(X)}}{\sqrt{g} \delta g^{\alpha\beta} g^{\alpha\beta}} \bigg|_{g_{z\bar{z}}=1/2} = -\partial \partial \Phi(X) = -\left(\partial_\mu \partial_\nu \Phi \partial X^\mu \partial X^\nu \partial_\rho \Phi \partial \partial X^\rho\right)$$

$$= -\pi \left(2\alpha' \nabla_\mu \nabla_\nu \Phi \mathcal{O}_G^{\mu\nu} + \alpha' \nabla^\rho \Phi \nabla_\rho u^a \mathcal{O}_a + \alpha' (\nabla \Phi)^2 \mathcal{O}_\Phi\right). \tag{A.12}$$

Again comparing with (A.12), we find contributions

$$\beta^G_{\text{classical}} = 2\alpha' \nabla_\mu \nabla_\nu \Phi, \tag{A.13a}$$

$$\beta^a_{\text{classical}} = \alpha' \partial^\rho \Phi \partial_\rho u^a, \tag{A.13b}$$

$$\beta^\Phi_{\text{classical}} = \alpha' \partial^\rho \Phi \partial_\rho \Phi. \tag{A.13c}$$

### A.3 Covariantization

The beta functions $\beta^G_{\mu\nu}$, $\beta^a$, $\beta^\Phi$ found so far are (partially) non-covariant. The terms obtained using conformal perturbation methods are not spacetime covariant at first. This is inherent to the conformal perturbation method, which uses correlation functions defined
with respect to flat spacetime. Conformal perturbation is an expansion in \( \delta G_{\mu\nu} = G_{\mu\nu} - \eta_{\mu\nu} \). If one corrects for this by evaluating the Weyl transformation of all terms of the coherent state of gravitons \( G_{\mu\nu}(X) \), the expressions will become covariant. Covariantization is necessary because the true beta functions are gravitationally only consistent when all orders in \( \delta G_{\mu\nu} \) are taking into account. Using background field methods one obtains the following spacetime covariant expressions \[17, 38\],

\[
\frac{1}{\alpha'} \beta^G_{\mu\nu} = R_{\mu\nu} + \frac{1}{2} M_{ab} \left( u^a \nabla_\mu u^b - \nabla_\mu u^a u^b \right) + 2 \nabla_\mu \nabla_\nu \Phi, \tag{A.14a}
\]

\[
\frac{1}{\alpha'} \beta^a = \frac{1}{\alpha'} \bar{\beta}^a(u) - \frac{1}{2} \nabla^2 u^a + \nabla^\rho \Phi \nabla_\rho u^a, \tag{A.14b}
\]

\[
\frac{1}{\alpha'} \beta^\Phi = U(u) - \frac{1}{2} \nabla^2 \Phi + (\nabla \Phi)^2. \tag{A.14c}
\]

A further adaptation needs to be made to \( \beta^a \), which is not covariant on the space of couplings \( u^a(X) \). The right expression for \( \beta^a \) should be

\[
\frac{1}{\alpha'} \beta^a = \frac{1}{\alpha'} \bar{\beta}^a(u) - \frac{1}{2} \nabla^2 u^a - \frac{1}{2} K^a_{bc} \nabla^\rho u^b \nabla_\rho u^c + \nabla^\rho \Phi \nabla_\rho u^a, \tag{A.15}
\]

where \( K^a_{bc} \) is the connection coefficient associated to the Zamolodchikov metric \( M_{ab} \). In a general renormalization scheme it arises from contact terms in the OPE. It has not appeared explicitly in the Zamolodchikov scheme because in that scheme \( K^a_{bc} \) is already of first order in \( u \) \[17\], as a result of which \( K^a_{bc} \nabla^\rho u^b(X) \nabla_\rho u^c(X) \) is beyond leading order in the calculation of the beta functions. In the Zamolodchikov scheme (A.15) is correct to leading order and by general covariance it holds in any renormalization scheme.

The last step to obtain beta functions which are convenient to work with, is to remove the double derivatives \( \nabla \nabla u \) in \( \beta^G_{\mu\nu} \). This can be achieved by a shift in the dilaton

\[
\Phi \rightarrow \Phi - \frac{1}{8} M_{ab} u^a u^b. \tag{A.16}
\]

The term in \( \beta^a \) and \( \beta^\Phi \) that result from this shift are all of higher order, whereas \( \beta^G_{\mu\nu} \) precisely becomes

\[
\frac{1}{\alpha'} \beta^G_{\mu\nu} = R_{\mu\nu} - M_{ab} \nabla_\mu u^a \nabla_\nu u^b + 2 \nabla_\mu \nabla_\nu \Phi + \frac{1}{2} e^{2\Phi} \left( e^{-2\Phi} u^a \nabla_\nu u^b \right) M_{ab} . \tag{A.17}
\]

Here all covariantization on the scalar manifold of couplings with respect to \( M_{ab} \) is implicit.

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\(^6\)In [17] this is done by way of a diffeomorphism that is not entirely clear to the authors.
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