A Softmax-free Loss Function Based on Predefined Optimal-distribution of Latent Features for CNN Classifier

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Abstract—In the field of pattern classification, the training of convolutional neural network classifiers is mostly end-to-end learning, and the loss function is the constraint on the final output (posterior probability) of the network, so the existence of Softmax is essential. In the case of end-to-end learning, there is usually no effective loss function that completely relies on the features of the middle layer to restrict learning, resulting in the distribution of sample latent features is not optimal, so there is still room for improvement in classification accuracy. Based on the concept of Predefined Evenly-Distributed Class Centroids (PEDCC), this article proposes a Softmax-free loss function based on predefined optimal-distribution of latent features—POD Loss. The loss function only restricts the latent features of the samples, including the cosine distance between the latent feature vector of the sample and the center of the predefined evenly-distributed class, and the correlation between the latent features of the samples. Finally, cosine distance is used for classification. Compared with the commonly used Softmax Loss and the typical Softmax-free loss function AM-Softmax Loss, COT-Loss and PEDCC-Loss, experiments on several commonly used datasets on a typical convolutional neural network show that the classification performance of POD Loss is always better and easier to converge. Code is available in https://github.com/TianYuZu/POD-Loss.

Index Terms—Softmax-free, POD Loss, latent features, PEDCC, image classification

I. INTRODUCTION

In recent years, convolutional neural networks have achieved great success in image classification [1], semantic segmentation [2], face recognition [3], target tracking [4] and target detection [5]. The algorithm based on convolutional neural networks has become one of the leading technologies in these fields. In order to further improve the performance of convolutional neural networks, researchers start with the network structure, from AlexNet [1] to VGGNet [6], and then to the deeper ResNet [7], DenseNet [8], etc. Many effective solutions have also been put forward in other aspects such as data enhancement, batch normalization [9] and various activation functions.

Loss function is an indispensable part of CNN model. It assists in updating the parameters of CNN model during the training phase. The traditional Softmax loss function is composed of softmax plus cross-entropy loss function. The output of the neural network passes through the softmax layer to get the posterior probability. Because of its advantages of fast learning speed and good performance, it is widely used in image classification. However, Softmax loss function adopts an inter-class competition mechanism, only cares about the accuracy of the prediction probability of the correct label, ignores the difference of the incorrect label, and can not ensure the compactness of intra-class and the discreteness of inter-class. L-Softmax [10] adds an angle constraint on the basis of Softmax to ensure that the boundaries of samples of different classes are more obvious. A-Softmax [11] also improves Softmax, and proposes weights normalized and angular spacing to realize the recognition standard that the maximum intra-class distance is smaller than the minimum intra-class distance. In addition, AM-Softmax [12] further improves A-Softmax. In order to improve the convergence speed, Euclidean feature space is converted to cosine feature space, and is changed to . Center Loss [13] adds the constraints of sample features before the classification layer for the first time on the basis of Softmax (the mean square error between the sample features and the calculated class centers is used to restrict the intra-class distance and the inter-class distance), but the distance between similar classes cannot be well separated.

The above improvement to Softmax Loss improves the classification accuracy in the field of face recognition (the inter-class distance is relatively close), but in general image classification (the inter-class distance is farther), the effect of Softmax Loss is still the best. For the cross entropy commonly used in the loss function, it mainly uses the information from the correct label to maximize the posterior probability of the sample, which largely ignores the information of the remaining incorrect labels. Therefore, Complement Objective Training [14] proposed a method of alternate training using correct label and incorrect label information, which not only improves the performance of the model, but also is more robust to single-step adversarial attacks, but does not consider the non-entropy-based complement objectives. Liang et al. [15] proposed a new loss function Near Classifier Hyper-Plane (N-CHP) Loss under the new CNN training framework, so that the learned sample features have the minimum intra-class distance and are close to the classifier hyperplane. The learned knowledge is transferred to Softmax Loss by using Loss Transferring, which greatly improves the classification performance. However, when the classification task has a large number of classes, N-CHP Loss is easy to fuse the sample features, resulting in poor performance.
From the perspective of maximize inter-class distance and minimize intra-class distance, PEDCC-Loss \([16] [17]\) predefines evenly-distributed class centroids to replace the continuously updated class centers in Center Loss, so as to maximize the inter-class distance. Meanwhile, using AM-Softmax and mean square error (MSE) loss to restrict the compactness of feature distribution of intra-class and the discreteness of feature distribution of inter-class. PEDCC-Loss improves the classification accuracy in both face recognition and general image classification, but still contains the constraint of the loss function of the posterior probability, which makes the sample feature distribution still not in the optimal state.

In the application of pattern recognition, the structure of convolutional neural network classifier generally includes convolutional layers and fully connected layers, in which convolutional layers are used to extract the features of input information and fully connected layers are used for classification. In the field of classification, convolutional neural networks mostly use end-to-end learning, that is, the output of the classification linear layer in the convolutional neural network output. On the one hand, the predefined evenly-distributed class centroids (PEDCC) is used, and the weight of the classification linear layer in the convolutional neural network (the weight is fixed in the training phase to maximize the inter-distance) is replaced, so as to optimize the distribution of latent features. On the other hand, by introducing the decorrelation mechanism, the improved Cosine Loss restricts the cosine distance between the sample feature vector and the PEDCC. At the same time, the correlations between the sample features are limited, so that the extracted latent features are the most effective. POD Loss discards the constraints on the posterior probability in the traditional loss function, and only restricts the extracted sample features to achieve the optimal distribution of latent features. Finally, the cosine distance is used for classification to obtain high classification accuracy.

The location of POD Loss and the whole network structure are shown in Fig. 1. Section III gives the details of the method.

Our main contributions are as follows:

- The PEDCC we proposed before is adopted in our classification model, and only the output of the feature extraction layer in the convolutional neural network is restricted to achieve the optimal distribution of latent features. The Softmax loss function is discarded, the weight of the classification layer is fixed, and the cosine distance is used for classification.
- The output of the feature extraction layer is constrained. On the one hand, the improved mean square error (MSE) loss function is used to constrain the cosine distance between the sample features and the PEDCC class centroid. On the other hand, the correlation between the latent feature dimensions is minimized to improve the classification accuracy.
- For the classification tasks, experiments on multiple datasets were conducted. Compared with the traditional cross-entropy loss plus softmax, and the typical Softmax related loss functions AM-Softmax Loss, COT-Loss and PEDCC-Loss, the classification accuracy of POD Loss is obviously better, and the training of network is easier to converge.

II. RELATED WORKS

A. Loss function of classification

In the field of classification, there are many different loss functions in convolutional neural networks for end-to-end learning. For multi-classification problems, Softmax loss function is generally selected, which has good performance and is easy to converge. The Softmax loss function is as follows:

$$L_{\text{Softmax}} = -\frac{1}{N} \sum_{j=1}^{M} \log \frac{e^{y_j}}{\sum_{j=1}^{M} e^{y_j}}$$

where \(N\) represents the number of samples, \(y_i\) represents the output value of the last fully connected layer of the correct class \(y_i\), and \(z_j\) is the output value of the last fully connected layer of the \(j\)-th class. Therefore, \(z_{y_i} = W^T y_i x_i\), that is, \(z_{y_i} = ||W^T y_i|| \cdot ||x_i|| \cdot \cos(\theta_{y_i})\), \(W^T y_i\) is the corresponding weight in the fully connected layer and \(x_i\) represents the input feature of the \(i\)-th sample. Therefore, the Softmax loss function becomes the following formula:

$$L_{\text{Softmax}} = \sum_{i=1}^{N} -\log \frac{e^{y_i}}{\sum_{j=1}^{M} e^{y_j}}$$

In general image classification tasks, there is no doubt about the performance of Softmax Loss, but for face recognition, due to the small inter-class distance, its classification accuracy is not satisfactory.

L-Softmax increases the coefficient \(m\) before the angle \(\theta\), \(m\) is a positive integer, and the cosine function is monotonically decreasing in the range of \(0 \to \pi\). Therefore, \(\cos(m\theta) \leq \cos(\theta)\). In this way, the inter-class distance learned by the model is larger, and the intra-class distance is smaller. The greater the value of \(m\), the greater the difficulty of learning.

A-Softmax is similar to L-Softmax in that it increases the angular spacing and normalizes the weight \(||W||\). Based on this, AM-Softmax proposes to change \(\cos(m\theta)\) to \((\cos\theta - m)\), changes the multiplication in the formula to addition, and also normalizes the input feature \(||x_i||\). Compared with A-Softmax, the formula is simpler in form and calculation. The final AM-Softmax is written as:

$$L_{\text{AM-Softmax}} = -\frac{1}{N} \sum_{i=1}^{N} \log \frac{e^{\cos(\theta_{y_i} - m)}}{e^{\cos(\theta_{y_i} - m)}} + \sum_{j=1}^{M} \sum_{i \neq j} e^{\cos(\theta_{y_i} - \theta_y_j)}$$

Center Loss is no longer limited to the constraints of neural network output. On the basis of Softmax, an additional mean square error (MSE) loss function is added to calculate the sample feature and the class center feature of the sample to reduce the intra-class distance and increase the inter-class distance. The class feature centers are continuously updated during the process of network training.

$$L_{\text{Center}} = L_{\text{Softmax}} + \frac{\lambda}{2} \sum_{i=1}^{M} ||x_i - c_{y_i}||^2$$

$M$ represents the number of samples, $x_i$ is the input feature of the $i$-th sample, $c_{yi}$ represents the central feature of the class to which the $i$-th sample belongs, which is continuously updated during the learning process after initialization. $\lambda$ represents the weighting factor.

Cross entropy mainly uses the information from the correct label to maximize the possibility of data, but largely ignores the information from the remaining incorrect labels. COT believes that in addition to correct labels, incorrect labels should also be used in training, which can effectively improve the performance of the model. The training strategy is to alternate training between the correct label and the incorrect labels. The constraint of the correct label is the commonly used cross entropy, and the constraint of the incorrect labels is the complement entropy expression as follows:

$$C(\hat{y}_i) = -\frac{1}{N} \sum_{i=1}^{N} \sum_{j=1,j\neq g}^{K} \frac{\hat{y}_{ij}}{1-\hat{y}_{ig}} \log(\frac{\hat{y}_{ij}}{1-\hat{y}_{ig}})$$  \hspace{1cm} (5)$$

where $N$ is the number of samples, $K$ is the number of classes, $\hat{y}_{ij}$ represents the predicted probability of the incorrect label $j$ of the $i$-th sample, and $\hat{y}_{ig}$ represents the predicted probability of the correct label $g$ of the $i$-th sample. When the predicted probabilities of all labels are equal, the entropy will be maximized, so the entropy makes $\hat{y}_{ij}$ approximate to $\frac{1}{K}$, essentially offsetting the predicted probability of incorrect labels as $K$ increases.

PEDCC-Loss [15, 17] proposes predefined evenly-distributed class centroids instead of the continuously updated class centers in Center Loss, and uses the fixed PEDCC weight instead of the weight of the classification linear layer in the convolutional neural network to maximize the inter-class distance. At the same time, the constraint of latent features is added (a constraint similar to Center Loss is added to calculate the mean square error (MSE) loss of sample features and PEDCC center), and AM-Softmax is also applied. This method makes the distribution of intra-class features more compact and the distribution of inter-class features more distant. The overall system diagram is shown in Fig. 2. The PEDCC-Loss expression is as follows:

$$L_{PEDCC-AM} = -\frac{1}{N} \sum_{i=1}^{N} \log \frac{e^{s(\cos \theta_{y_i}, m)}}{\sum_{j=1,j\neq y_i}^{K} e^{s \cos \theta_{j}}}$$

$$L_{PEDCC-MSE} = \frac{1}{2N} \sum_{i=1}^{N} \|x_i - pedcc_{y_i}\|^2$$ \hspace{1cm} (7)$$

$$L_{PEDCC-Loss} = L_{PEDCC-AM} + \lambda \sqrt{L_{PEDCC-MSE}}$$ \hspace{1cm} (8)$$

where $N$ is the number of samples, $\lambda$ is a weighting coefficient, and $n \geq 1$ is a constraint factor of $L_{PEDCC-MSE}$, $x_i$ represents the input feature of the $i$-th sample (normalized), and $pedcc_{y_i}$ represents the PEDCC central feature of the class to which the sample belongs (normalized).

B. Loss function for latent features in self-supervised learning

In self-supervised learning, Barlow Twins [13] proposes an innovative loss function, adding a decorrelation mechanism to the loss function to maximize the variability of representation learning. Barlow Twins Loss is written as:

$$L_{BT} = \sum_{i} (1 - C_{ii})^2 + \lambda \sum_{i} \sum_{j \neq i} C_{ij}^2$$ \hspace{1cm} (9)$$

Where $\lambda$ is a weighting coefficient, which is used to weigh the two terms in the loss function, and $C$ is the cross-correlation matrix of the output features of the sample and its enhanced sample under the same batch of two same network. The addition of the latter redundancy reduction term in the loss function reduces the redundancy between the network output features, so that the output features contains the non-redundant information of the sample, and achieves a better feature representation effect.

On this basis, VICReg [19] combines the variance term with a decorrelation mechanism based on redundancy reduction and covariance regular term. The covariance criterion removes the correlation between the different dimensions of the learned...
representation, and the purpose is to spread information between dimensions to avoid dimension collapse. The criterion mainly punishes the off-diagonal terms of the covariance matrix, which further improves the classification performance of features.

In this article, PEDCC is also used to generate the predefined evenly-distributed class centroids, instead of the continuously updated class centers in Center Loss, and the solidified PEDCC weights replace the weights of the classification linear layer to maximize the inter-class distance. On the one hand, Cosine Loss calculates and constrains the distance between the sample feature and the central features of the PEDCC. On the other hand, similar to self-supervised learning, a decorrelation mechanism is introduced. The self-correlation matrix of the difference between the sample features and the central features of PEDCC in each batch is calculated, and the correlation between any pair of features is restricted to improve the classification accuracy.

In the task of image classification, the method in this article no longer restricts the posterior probability of neural network output, but restricts the latent features extracted from sample information, and realizes the optimal distribution of latent features for classification.

III. METHOD

A. Cosine Loss

The mean square error (MSE) loss function can be used to constrain the distance between the sample feature and its class PEDCC feature center. In this article, the PEDCC center and the sample feature vector are normalized before calculation, so the MSE Loss expression and its derivation are as follows:

$$L_{PEDCC-MSE} = \frac{1}{2N} \sum_{i=1}^{N} ||x_i - pedcc_{y_i}||^2$$ (10)

$$= \frac{1}{2N} \sum_{i=1}^{N} (||x_i||^2 + ||pedcc_{y_i}||^2 - 2x_i \cdot pedcc_{y_i})^2$$ (11)

$$= \frac{1}{N} \sum_{i=1}^{N} (1 - cos\theta_{y_i})$$ (12)

where $N$ represents the number of samples. It can be seen that the loss function is essentially a constraint on the cosine distance between the sample feature and the center of the PEDCC. Taking $\theta_{y_i}$ as the independent variable to derive the derivative of $(1 - cos\theta_{y_i})$, the derivative is $sin\theta_{y_i}$, as shown in Fig. 3. It can be seen from Fig. 3 that in the range of $0^\circ$ to $90^\circ$, the derivative value becomes larger and larger, and in the range of $90^\circ$ to $180^\circ$, the derivative value becomes smaller and smaller, and the derivative value is always less than or equal to 1 in the whole range of $0^\circ$ to $180^\circ$.

$\theta_{y_i}$ represents the angle between the sample feature and the predefined center feature of the $i$-th class. The angle between the sample feature point and the predefined center of the class to the origin must also be in the range of $0^\circ$ to $180^\circ$. In the process of network learning, what we want is let $\theta_{y_i}$ approach $0^\circ$ as quickly as possible. Therefore, the faster the falling speed of $\theta_{y_i}$, the better, that is, the greater the derivative value, the better, and $(1 - cos\theta_{y_i})$ does not completely conform to the mathematical formula we want.

Based on above, Cosine Loss is proposed, whose expression is as follows:

$$L_{Cosine} = \frac{1}{N} \sum_{i=1}^{N} (1 - cos\theta_{y_i})^2$$ (13)

where $N$ represents the number of samples, and $cos\theta_{y_i}$ is the
cosine value of the angle between the sample feature and its own predefined central feature. Using \( \theta_{yi} \) as the independent variable to derive the derivative of \((1 - \cos \theta_{yi})^2\), the derivative is \(2 \cdot (1 - \cos \theta_{yi}) \cdot \sin \theta_{yi}\), as shown in Fig. 4. It can be seen from the figure that the derivative value becomes larger and larger in the range of \(0^\circ\) to \(120^\circ\), and the derivative value reaches 1 around \(65^\circ\) and the maximum value reaches about 2.5. In network learning, Cosine Loss is easy to converge and converges faster. Softmax improved loss AM-Softmax gives the selection details between the number of classes and the dimension of features. The element in the correlation matrix of the difference matrix formed by the latent features of samples. Therefore, Self-correlation Constraint Loss is proposed:

\[
L_{SC} = \sum_{i=1}^{n} \sum_{j \neq i} n R_{ij}^2
\]

where \(n\) represents feature dimension, \(R\) represents the self-correlation matrix of the difference matrix formed by the difference between the sample features and the predefined central features. The element in the \(i\)-th row and \(j\)-th column of the self-correlation matrix is the correlation coefficient between the \(i\)-th column and the \(j\)-th column of the difference matrix. \(B\) is the number of samples in each batch. \(X\) is the normalized feature matrix of \(B\) samples in Fig. 1, where each column vector represents a sample. \(X_{pedcc}\) is the PEDCC matrix corresponding to the sample feature.

**B. Selection of latent feature dimension**

For a given type of dataset, \(k\) gives the theorem: For arbitrarily generated \(k\) point \(a_i (i = 1, 2, ..., k)\) evenly-distributed on the unit hypersphere of \(n\) dimensional Euclidean space, if \(k \leq n+1\), such that \((a_i, a_j) = -\frac{1}{\sqrt{k}}, i \neq j\). Corresponding to the convolutional neural network, \(k\) evenly-distributed PEDCC class centroids can be obtained when \(k(\text{number of classes}) \leq n(\text{feature dimension}) + 1\) is satisfied.

Different feature representations can be regarded as different knowledge of the images. The more comprehensive the knowledge, the better the classification performance, and the multi-feature dimension is indeed easier to find a hyperplane to separate images of different classes. However, too many feature dimensions will cause the classifier to emphasize the accuracy of the training set too much, and even learn some wrong or abnormal data, resulting in over-fitting problems. Therefore, under the premise of \(k \leq n+1\), selecting the appropriate feature dimension is also crucial to the classification performance. In the experiments of this article, Section IV gives the selection details between the number of classes and the dimension of features.

**C. Decorrelation between latent features**

The features of predefined evenly-distributed class centroids are irrelevant, but the sample features during the training process only approximate the features of the class centroids to which they belong, which can not be completely equal. Therefore, there is always a certain correlation between the features. Meanwhile, under the premise of ensuring that the number of center points \(k(\text{the number of classes})\) and the space dimension \(n\) satisfy \(k \leq n + 1\), \(n\) is generally taken greater than \(k\). For example, when the number of classes \(k\) is 10, \(n\) is 256. When the network training is over, due to the effect of loss function, the features of the training samples are basically distributed in the \(k - 1\) dimensions subspace. From this point of view, it seems that the remaining \(n - k + 1\) dimensions are useless, but the fact is that if \(n\) is set to \(k - 1\) at the beginning, the classification accuracy drops significantly, which indicates that these extra dimensions play a role in optimizing classification features during the training process.

In this case, in order to further improve the utilization of all latent features, constraining the correlation between the dimensions is proposed. For example, for the image classification of cats and mice, if the correlation between the dimensions is not restricted, one of the learned features may represent the body size and the other dimension represents the facial contour size. Obviously, these two features have a strong correlation, which means a large body must have a large facial contour. For classification, the classification performance based on the two features is very similar or even the same as that based on one of the two features. Therefore, the resources occupied by one of the dimensions are wasted. When the constraint of the correlation between the dimensions is added, the two features learned in the above example may be hair color and body size, which are almost irrelevant. Compared with the classification based on a single feature, it is obvious that the combination of the two is better for classification and achieves the purpose of making full use of features.

Barlow Twins Loss adds a decorrelation mechanism to reduce the redundancy between network output features, so that the output features contain non-redundant information of samples. Barlow Twins uses a cross-correlation matrix and punishes the non-diagonal terms of the calculated cross-correlation matrix. Our method draws on this decorrelation mechanism, but instead of using the cross-correlation matrix, the self-correlation matrix of the difference between the latent features of the samples and the predefined central features is used. The non-diagonal terms are also punished to constrain the correlation between the different dimensions of the latent features of samples. Therefore, Self-correlation Constraint Loss is proposed:

\[
R = \frac{1}{B-1} (X - X_{pedcc})(X - X_{pedcc})^T
\]
D. POD Loss

On the one hand, Cosine Loss is used to constrain the distance between the sample feature and its class PEDCC feature center, and on the other hand, it accelerates the convergence speed of network training. At the same time, Self-correlation Constraint (SC) Loss is defined as the constraint on the correlation between the different dimensions of sample latent features, that is, the decorrelation term, which includes punishing the off-diagonal terms of the self-correlation matrix of the difference between the sample features and the predefined central features. The final expression of POD Loss is:

\[ L_{POD} = L_{Cosine} + \lambda L_{SC} \]  

(16)

where \( L_{Cosine} \) is the cosine loss function, \( L_{SC} \) is the loss function that restricts the correlation between feature dimensions, and \( \lambda \) is the weighting coefficient. For some experiments, the adjustment of \( \lambda \) may improve the classification performance.

IV. EXPERIMENTS AND RESULTS

Experiment is implemented using Pytorch1.0 [21]. The network structure used in the whole experiment is ResNet50 [21]. The datasets used include CIFAR10 [22], CIFAR100 [22], Tiny ImageNet [23], FaceScrub [24] and ImageNet [23] dataset. In order to make the network structure more suitable for the image sizes of different datasets, some modifications are made to the original ResNet50 structure. For CIFAR10 (images size is 32 × 32), CIFAR100 (images size is 32 × 32) and Tiny ImageNet (images size is 64 × 64), the convolution kernel of the first convolution layer is changed from the original 7 × 7 to 3 × 3, and the step size is changed to 1. The maximum pooling layer in the second convolutional layer is also eliminated (except Tiny ImageNet). The above changes are not made for ImageNet. In addition, the number of feature dimensions input by the PEDCC layer in the network structure varies with the number of classes of these datasets. All experimental results are the average of three experiments.

A. Experimental datasets

CIFAR10 dataset contains 10 classes of RGB color images, and the images size is 32 × 32. There are 50,000 training images and 10,000 test images in the dataset. CIFAR100 dataset contains 100 classes of images, the images size is 32 × 32, a total of 50,000 training images and 10,000 test images. FaceScrub dataset contains 100 classes of images, the images size is 64 × 64, there are 15896 training images and 3896 test images in the dataset. Tiny ImageNet dataset contains 200 classes of images, the images size are 64 × 64, there are a total of 100,000 training images and 10,000 test images. For these datasets, standard data enhancement is performed, that is, the training images are filled with 4 pixels, randomly cropped to the original size, and horizontally flipped with a probability of 0.5, and the test images are not processed.

ImageNet dataset contains 1000 classes of images. The images size is not unique, and the height and width are both greater than 224. There are 1282166 training images and 51000 test images in the dataset. For this dataset, the training images are randomly cropped to different sizes and aspect ratios, scaled to 224 × 224, and flipped horizontally with a probability of 0.5. The test images are scaled to 256 proportionally with a small side length, and then the center is cropped to 224 × 224.

For the above datasets, in the training phase, using the SGD optimizer, the weight decay is 0.0005 (ImageNet is 0.0001), and the momentum is 0.9. The initial learning rate is 0.1, and a total of 100 epochs are trained. At the 30th, 60th, and 90th epoch, the learning rate drops to one-tenth of the original, and the batchsize is set to 128 (ImageNet is 96).

B. Experimental results

1) Selection of feature dimensions: According to the theoretical analysis of feature dimensions in Section III, comparison experiments are carried out on the selection of multiple different feature dimensions for datasets of different classes. Table I shows the classification performance under different feature dimensions on CIFAR10 dataset. Within a certain range, increasing the feature dimension will get better classification performance, but when the number of features reaches a certain scale, the performance of the classifier is declining. The feature dimensions with the best performance on different datasets under multiple experiments are shown in Table II. For CIFAR10 dataset, 10 evenly-distributed class centroids are predefined in a 256 dimensions hyperspace. For CIFAR100, FaceScrub and Tiny ImagNet dataset, several evenly-distributed class centroids of corresponding classes are respectively predefined in a 512 dimensions hyperspace. For ImageNet dataset, the top 30 classes, the top 100 classes, the top 200 classes, the top 500 classes, and all the 1000 classes are selected for experiments. The optimal feature dimensions are 256, 512, 512, 1024 and 2048.

| Feature Dimension | Accuracy(%) |
|-------------------|-------------|
| 9                 | 92.71       |
| 32                | 93.51       |
| 64                | 93.77       |
| 128               | 93.98       |
| 256               | **94.31**   |
| 512               | 94.25       |
| 1024              | 93.82       |

TABLE I: Classification accuracy (%) under different feature dimensions on CIFAR10 dataset

| Dataset          | Number of Classes | Feature Dimension |
|------------------|-------------------|-------------------|
| CIFAR10          | 10                | 256               |
| CIFAR100         | 100               | 512               |
| FaceScrub        | 100               | 512               |
| Tiny ImageNet    | 200               | 512               |
| ImageNet(30)     | 30                | 256               |
| ImageNet(100)    | 100               | 512               |
| ImageNet(200)    | 200               | 512               |
| ImageNet(500)    | 500               | 1024              |
| ImageNet(1000)   | 1000              | 2048              |

TABLE II: The number of classes and the optimal feature dimensions of different datasets
2) Role of Cosine Loss: Comparative experiments on Softmax, AM-Softmax Loss ($s = 5, m = 0.25$), COT, PEDCC-Loss ($s = 10, m = 0.5$) and Cosine Loss are conducted on CIFAR100 dataset and FaceScrub dataset. The experimental results are shown in Table III. Among the five loss functions, Cosine Loss has the highest classification performance. In the network training process, the convergence speed of AM-Softmax Loss, PEDCC-Loss and Cosine Loss is shown in Fig. 5. As can be seen from Fig. 5, only Cosine Loss training has a faster and more stable convergence speed.

| Loss Function | CIFAR100 | FaceScrub |
|---------------|----------|-----------|
| Softmax Loss  | 73.80    | 89.10     |
| AM-Softmax Loss ($s = 5, m = 0.25$) | 73.03 | 91.67 |
| COT           | 74.03    | 90.40     |
| PEDCC-Loss ($s = 10, m = 0.5$) | 75.58 | 90.98 |
| Cosine Loss   | 76.23    | 92.11     |

3) Role of SC Loss: The variance of the 2048 dimensions feature eigen-vectors (normalized) before the fully connected layer of the network is respectively calculated after POD Loss and Cosine Loss training, as shown in Table IV. The variance of eigen-vectors after POD Loss training is smaller than that of Cosine Loss, indicating that the existence of SC Loss can balance the energy of the features and gain some benefits for subsequent classification.

| Loss Function | Variance of Feature Eigen-vectors |
|---------------|-----------------------------------|
| Cosine Loss   | 1.82e-8                           |
| POD Loss      | 1.53e-8                           |

Comparative experiments of Cosine Loss and POD Loss have also been carried out on some datasets, and the experimental results are shown in Table V. The experimental results show that POD Loss with SC Loss is better than a single Cosine Loss in classification accuracy on multiple datasets.

| Dataset       | Cosine Loss | POD Loss |
|---------------|-------------|----------|
| CIFAR10       | 93.83       | 94.31    |
| CIFAR100      | 76.23       | 77.70    |
| Tiny ImageNet | 62.01       | 62.16    |
| FaceScrub     | 92.11       | 93.01    |

4) POD Loss: Comparative experiments of Softmax Loss and POD Loss are conducted on the above datasets. The experimental results are shown in Table VI. Experimental results show that the classification performance of POD Loss is higher than that of Softmax Loss on multiple datasets, and the classification accuracy is much higher than Softmax Loss on some datasets. Experiments on the ImageNet dataset show that with the increase of the number of classes in the dataset, the classification accuracy of POD Loss is gradually approaching Softmax Loss. The reason lies in the structure of the network itself (the maximum output feature dimension of ResNet50 is 2048, the ratio of the feature dimension to the number of large classes is much smaller than the ratio of the feature dimension to the number of small classes, and there are fewer or no extra dimensions to benefit classification). So, on large-class datasets, the advantages of POD Loss over Softmax Loss are limited.

| Dataset       | Loss Function | Softmax Loss | POD Loss |
|---------------|---------------|--------------|----------|
| CIFAR10       | 93.71         | 94.31        |
| CIFAR100      | 75.56         | 77.70        |
| Tiny ImageNet | 60.29         | 62.16        |
| FaceScrub     | 89.10         | 93.01        |
| ImageNet(30)  | 79.86         | 85.40        |
| ImageNet(100) | 78.25         | 82.06        |
| ImageNet(200) | 82.06         | 83.08        |
| ImageNet(500) | 82.14         | 82.24        |
| ImageNet(1000)| 75.65         | 75.71        |

On the other hand, in the process of network training, the convergence speed of POD Loss and Softmax Loss is shown in Fig. 6. As can be seen from Figure 6, POD Loss training converges faster.

5) Discussion on the last classification method: The constraint on latent features only solves the optimization problem of latent feature distribution, and the pattern classification method needs to be matched later. On the one hand, the POD Loss proposed in this article includes Cosine Loss that constrains the latent features and the cosine distance between the PEDCC centroids. When POD Loss converges, the cosine distance between the latent feature vector and the PEDCC centroid of the correct label reaches the minimum, and the cosine distances between the latent feature vector and the PEDCC centroids of the incorrect labels reach the maximum in a uniform state. The classification method is:

$$I = \arg\max_{i=1,2,...,k} (x \cdot \text{pedcc}_i)$$

$$= \arg\max_{i=1,2,...,k} \left(\|x\| \cdot \|\text{pedcc}_i\| \cdot \cos\theta_{x,\text{pedcc}_i}\right)$$

$$= \arg\max_{i=1,2,...,k} \cos\theta_{x,\text{pedcc}_i}$$

where $k$ represents the number of categories, $x$ is the sample feature, $\text{pedcc}_i$ is the center of the PEDCC of the $i$-th class, and $\cos\theta_{x,\text{pedcc}_i}$ represents the cosine value of and .

On the other hand, under the premise that the features of samples satisfy the Gaussian distribution, the mean and covariance of each PEDCC are different, and each class has different class conditional probability densities, so Gaussian...
Discriminant Analysis (GDA) method can also be used for classification.

Therefore, this article conducts a comparative experiment on the two classification methods of cosine distance and GDA based on POD Loss. The experimental results on multiple datasets are shown in Table VII. It can be seen from Table VII that the classification method of cosine distance is better than GDA on multiple datasets. Therefore, cosine distance is adopted as the final classification method in this article.

| Dataset       | Method                      | Cosine distance | Gaussian discrimination analysis |
|---------------|-----------------------------|-----------------|----------------------------------|
| CIFAR10       | 94.31                       | 94.23           |                                  |
| CIFAR100      | 77.70                       | 73.37           |                                  |
| Tiny ImageNet | 62.16                       | 50.09           |                                  |

![Fig. 5: Variation of classification accuracy of AM-Softmax Loss, PEDCC-Loss and Cosine Loss with epoch during training.](image1.png)

![Fig. 6: Variation of classification accuracy of Softmax Loss and POD Loss with epoch during training.](image2.png)

V. CONCLUSION

For convolutional neural network classifier, this article proposes a Softmax-free loss function (POD Loss) based on predefined optimal-distribution of latent features. The loss function discards the constraints on the posterior probability in the traditional loss function, and only constrains the output of feature extraction to realize the optimal distribution of latent features, including the cosine distance between sample feature vector and predefined evenly-distributed class centroids, the decorrelation mechanism between sample features, and finally the classification through the solidified PEDCC layer. The experimental results show that, compared to the commonly used Softmax Loss and its improved Loss, POD Loss achieves better performance on image classification tasks, and is easier to train and converge. In the future, new loss function based on latent feature distribution optimization will be further studied from the perspective of distinguishing classification features and representation features, so as to further improve the recognition performance of the network.

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