Abstract
In this proceeding, we present our recent investigations on hydrodynamic collectivity in high-multiplicity proton–proton collisions at $\sqrt{s} = 13$ TeV using the EBE–VISHNU hybrid model with different initial condition models, called HIJING, super–MC and TRENTo. We find that with carefully tuned parameters, hydrodynamic simulations can give reasonable descriptions of the measured two-particle correlations. However, multi-particle single and mixed harmonics cumulants can not be described by hydrodynamics with these three initial conditions, even for the signs in a few cases. Further studies show that the non-linear response plays an important role in the hydrodynamic expansion of the p–p systems. Such an effect can change $c_2$ from a negative value in the initial state to a positive value in the final state. The failure of the hydrodynamic description of multi-particle cumulant triggers the questions on whether the hydrodynamics can rule all collision systems, including p–p collisions at the LHC.

Keywords: hydrodynamics, flow, proton–proton collisions

1. Introduction
Ultra-relativistic heavy-ion collisions at the LHC provide a unique opportunity to study the Quark–Gluon Plasma (QGP). With the new developments on both flow measurements and model calculations, the understanding of the properties of the QGP and its fluctuating initial conditions have been improved to an unprecedented level. In addition to the detailed study of flow in Pb–Pb and Xe–Xe collisions, the flow phenomena in small collision systems like p–Pb and p–p collisions, which initially expected to serve as a reference data, have been studied in great detail [1,2,3]. Surprising observation of flow phenomena in these smaller collision systems has attracted a lot of attention. It challenges both the hydrodynamic model as the “standard model” in heavy-ion physics and the PYTHIA model as the “standard tool” for the minimum-bias p–p physics at the same time. While it has been commonly accepted that the observed large anisotropic flow is attributed to the creation of the QGP in heavy-ion collisions, the origin of the similar size of flow
measured in small systems is less clear and currently under intense debate. In this proceeding, we will show the latest developments of hydrodynamic flow in p–p collisions, and discuss if hydrodynamics could be the right mechanism to describe the observed flow feature in experiments.

2. Hydrodynamic model

In this study, iEVE-VISHNU has been implemented to simulate the high multiplicity p–p collisions at √s = 13 TeV. iEVE-VISHNU [4] is an event-by-event version of VISHNU hybrid model that combines 2+1D viscous hydrodynamics and UrQMD hadron cascades to simulate the evolution of the QGP and hadronic matter. For simplicity, the bulk viscosity, net baryon density, and heat conductivity are neglected, and the specific shear viscosity η/s is assumed to be constant. In order to investigate the dependence on the initial condition models, we implement three different initial condition models, namely, modified HIJING [5], super–MC [6], and TRENTo [7]. For more details please refer to [8]. It is found that with careful tuning of the input parameters, the iEVE-VISHNU with each of the three different initial conditions is able to describe the measured integrated flow coefficients v2, v3, and v4 with two-particle correlations as shown in Fig. 1.

![Figure 1](image_url)

Fig. 1. (Color online) Multiplicity dependence of v2, v3, and v4 in p–p collisions at √s = 13 TeV, calculated by iEVE-VISHNU with HIJING (left), super–MC (middle) and TRENTo (right) initial conditions, respectively. Para-I–IV represent different parameter sets tuned for each initial condition, which are taken from Ref. [8].

3. Results and discussions

Figure 2 shows the four-particle cumulants of the second harmonics, c2[4], in high multiplicity p–p collisions at √s = 13 TeV. It shows that the hydrodynamic calculations of c2[4] are always positive, no matter which of the above-mentioned initial conditions has been applied. We also noticed that the hydrodynamic model MUSIC with IP-Glasma initial conditions also do not generate a negative c2[4] in p–p collisions [11]. Thus, we emphasize that hydrodynamic expansion does not necessarily produce final state correlations with negative c2[4], and the observed negative c2[4] in experiments does not necessarily associate with the hydrodynamic flow in small collision systems [5].

In addition, we also find that the positive c2[4] from hydrodynamics is not due to the contaminations of non-flow or due to multiplicity fluctuations [5], but due to the large non-linear response of hydrodynamic evolution, as shown in the correlation between initial eccentricity ε2 and final state v2 in Fig. 3 (left). A clear deviation of v2 from the linear relationship with ε2 is seen in the region of ε2 > 0.45, where a cubic response becomes significant [12] [13]. The non-negligible non-linear response indicates that the scaled distribution of v2, P(v2/ε2), doesn’t follow the scaled distribution of ε2, P(ε2/ε2), which can be seen in Fig. 3 (right). Such non-linear response leads to additional fluctuations of v2 in the final states, which could even change the sign of c2[4] and eventually fail to reproduce the negative c2[4] as observed in experiment. Because two- and multi-particle cumulants have different sensitivities to flow fluctuations, the successful
Fig. 2. (Color online) $c_2(4)$ as a function of $N_{ch}$ in p–p collisions at $\sqrt{s} = 13$ TeV, calculated by iEBE–VISHNU with HIJING (left), super–MC (middle) and TRENTo (right) initial conditions, using standard cumulant method. The CMS data with standard cumulant method and the ATLAS data with three-subevent method are taken from [9] and [10], respectively.

Fig. 3. (Color online) Left panel: the scatter points between the $v_2$ and $c_2$, together with a linear fitting and a non-linear fitting with cubic-term. Right panel: the comparison between the scaled event-by-event $c_2$ distribution and scaled $v_2$ distributions of Para-III of HIJING initial conditions at 0–0.1% centrality bin in p–p collisions at $\sqrt{s} = 13$ TeV.

descriptions of two-particle correlations but failure descriptions of four-particle cumulants suggest that the presented hydrodynamic simulations in this proceeding could not describe the flow and the flow fluctuations simultaneously.

Besides four-particle cumulants $c_2(4)$, we also study the multi-particle mixed harmonic cumulants in the high multiplicity p–p collisions at $\sqrt{s} = 13$ TeV, using iEBE–VISHNU with different initial conditions. The three-particle asymmetric cumulant $ac_{2m,n,n} = \langle v_2^m v_2^n \cos 2(m(\Psi_n - \Psi_m)) \rangle$ is sensitive to the correlations between flow magnitudes and flow angles. The four-particle symmetric cumulants $SC(m,n) = \langle v_2^m v_2^n - \langle v_2^m \rangle \langle v_2^n \rangle \rangle$ quantifies the correlation between $v_2^m$ and $v_2^n$ [13]. It was observed (but not shown in these proceedings) that the hydrodynamic calculations could qualitatively or even quantitatively describe $ac_{2}[3]$ and $nSC_{2,4}[4]$ after proper tuning of the parameters. Nevertheless, the results in Fig. 5 show that the hydrodynamic calculations, independently on the initial conditions, could not describe the data. It seems that the currently applied hydrodynamic framework, which works nicely in the description of two-particle correlations, have some difficulties to describe the multi-particle single and mixed harmonic cumulants unless the implementation of the initial conditions are done with the assumption of the simple elliptic shape of the proton (when the proton is traveling in the near speed of light). In order to answer whether or not hydrodynamic flow has been produced in high multiplicity p–p collisions, further developments on initial conditions and the hydrodynamic modelling are both necessary.
4. Summary

In this proceeding, we reported our recent investigations on hydrodynamic flow in high-multiplicity proton–proton collisions at $\sqrt{s} = 13$ TeV within the framework of iEBE–VISHNU hybrid model with HIJING, super–MC and TRENTo initial conditions. We have shown successful descriptions of two-particle correlations and the current challenge to reproduce the multi-particle single and mixed harmonics cumulants. We leave the implementation of 3+1D hydrodynamics with dynamical initial conditions and longitudinal fluctuations to the future work to see if the hydrodynamic calculation has a chance to describe the existing fruitful data and thus confirm the creations of one tiny droplet of hydrodynamic fluid in p–p collisions at the LHC.

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