Orbital resonances in exoplanetary systems

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Abstract. At present, more than 700 exoplanetary systems are known to have been discovered. They incorporate more than 130 multiplanet systems, i.e. those hosting two or more planets. The orbital resonance and near-resonance phenomena are ubiquitous in them. We present a statistical and dynamical analysis of the resonance structure of the multiplanet systems and planetary systems of binary stars. We have built distributions of the orbital period ratios, separately considering the cases of inner and outer location of the massive perturber. The histograms reveal apparent peaks close to the first order orbital resonances $2/1$ and $3/2$ in both cases; this confirms previous findings. We have performed analytical modelling of the histograms, and obtained exact positions of the peaks. Moreover, we have built the “period ratio – eccentricity” diagrams, with collision curves superimposed, so that to find anomalous systems.

1. Introduction
We analyze statistics of resonances in multiplanet systems, as well as in binary stellar systems with planets, using exoplanet database [1]. Orbital period ratios are calculated for each pair of objects in all discovered systems suitable for analysis. Differential distributions of the orbital period ratios are constructed separately for the following two cases: (a) the orbit of the dominating (in mass) body in the pair is inside the orbit of the smaller body in the pair (in the Solar system, this configuration is similar to the “Neptune – TNO (trans-Neptunian object)” configuration); (b) the opposite case (in the Solar system, this is similar to the “(main belt) asteroid – Jupiter” configuration). The dominating body in the pair is considered to be the “perturber” (analogously to the situation in the Solar system, where Neptune and Jupiter are the main perturbers for the TNOs and main-belt asteroids, respectively). In the both cases (a) and (b), the outer body is indexed with number 1 (e.g., its orbital period is $T_1$), and the inner body with number 2 (the orbital period $T_2$).

2. The period ratio distributions
We build the histograms of period ratios, using exoplanet database [1]. The obtained histograms are presented in figures 1 and 2 in the defined above cases (a) and (b), respectively. As one can see, peaks are prominent near the resonances $3/2$, $2/1$, $5/2$, $3/1$, and $4/1$. However, some peaks seem to be shifted (mostly to the right) relative to their nominal positions both in (a) and (b) cases. This is a known phenomenon in exoplanetary systems, though discovered quite recently (see [2, 3] and references therein). Possible dynamical and cosmogonical mechanisms for such shifts were discussed in [3, 4, 5]. E.g., the shift can appear, if the planetary masses in the pair increase with time [3]. In [4, 5], the effect is explained by tidal dissipation.
Figure 1. The period ratio histogram in case (a) (the inner perturber). The best-fit function (1) is shown by the solid curve.

Figure 2. The same as in figure 1, but for case (b) (the outer perturber).

We fit the histograms in figures 1 and 2 with the curves described by the multi-peaked function

$$f \left( \frac{T_1}{T_2} \right) = \sum_{i=1}^{5} f_i \left( \frac{T_1}{T_2} \right) + \phi \left( \frac{T_1}{T_2} \right),$$

(1)

where

$$f_i \left( \frac{T_1}{T_2} \right) = \frac{A_i}{\sigma_i \sqrt{2\pi}} e^{-\left(\frac{T_1/T_2 - \mu_i}{2\sigma_i^2}\right)^2},$$

(2)

$$\phi \left( \frac{T_1}{T_2} \right) = B \cdot \left( \frac{T_1}{T_2} - C \right)^5 e^{-\left(\frac{T_1/T_2 - C}{D}\right)}.$$

(3)

Every function $f_i$ is nothing but the normal distribution of the period ratio values near resonance. The function $\phi$ describes the wide bulge and decay in the distribution tail. The values of parameters $A_i$, $\mu_i$, $\sigma_i$, $B$, $C$, and $D$ are obtained by the Levenberg–Marquardt method, using “Mathematica” software. The fits are shown in figures 1 and 2 by solid curves.

The best-fit positions of the peaks are given in Tables 1 and 2. It turns out that the peak shifts are much smaller in case (a) than in case (b). Moreover, almost no shift is present for resonances of order higher than 1 in case (a).

3. Collision curves and anomalous systems

In order to find anomalous (unstable or close to instability) systems, we have constructed the “period ratio – eccentricity” diagrams, using the same sample of objects. These diagrams are shown in figures 3 and 4 for cases (a) and (b), respectively.

The dots in both diagrams correspond to the planetary pairs in the sample. In figure 3, two squares correspond to the circumbinary systems Kepler-16 and Kepler-35, and the triangle
Table 1. Peak locations in case (a)

| Resonance | Peak position |
|-----------|---------------|
| 2/1       | 2.122 ± 0.007 |
| 3/2       | 1.547 ± 0.024 |
| 3/1       | 3.000 ± 0.039 |
| 4/1       | 4.033 ± 0.046 |
| 5/2       | 2.500 ± 0.018 |

Table 2. Peak locations in case (b)

| Resonance | Peak position |
|-----------|---------------|
| 2/1       | 2.131 ± 0.018 |
| 3/2       | 1.599 ± 0.025 |
| 3/1       | 3.168 ± 0.024 |
| 4/1       | 3.991 ± 0.039 |
| 5/2       | 2.517 ± 0.056 |

Figure 3. The “period ratio — eccentricity” diagram in case (a). The collision curve is superimposed (solid line).

Figure 4. The same as in figure 3, but for case (b).

corresponds to the circumbinary system Kepler-47 (the b planet is taken into account solely). In figure 4, the triangle corresponds to the planetary pair b–c in the Kepler-47 system.

In order to identify anomalous systems, we have superimposed collision curves on the plots. These curves are given by the functions

\[ e_1 \left( \frac{T_1}{T_2} \right) = 1 - \left( \frac{T_1}{T_2} \right)^{-2/3} \]  
(4)

and

\[ e_2 \left( \frac{T_1}{T_2} \right) = \left( \frac{T_1}{T_2} \right)^{2/3} - 1 \]  
(5)

in cases (a) and (b), respectively. These formulas can be obtained easily from basic relations of the two-body problem theory [6].
Using these diagrams, we have identified anomalous systems, i.e. those with planets located outside the zones of conditional stability (zones limited by the collisional curves). The dots located higher than the collision curves betray anomalous systems that host planets with intersecting orbits. It turns out that two such systems are unambiguously identified: HD102272 (in case (a)) and HD215152 (in case (b)).

![Figure 5](image-url)  
**Figure 5.** Configuration of the planetary orbits in the HD102272 system at zero time (when integration starts).

**Table 3.** Data on planets in the HD102272 system [9]

| Parameters              | b      | c      |
|-------------------------|--------|--------|
| Mass $m \sin i$ [$M_J$] | 5.9 ± 0.2 | 2.6 ± 0.4 |
| Semimajor axis $a$ [AU] | 0.614 ± 0.001 | 1.57 ± 0.05 |
| Period $P$ [d]          | 127.58 ± 0.3 | 520 ± 26 |
| Eccentricity $e$        | 0.05 ± 0.04 | 0.68 ± 0.06 |
| Argument of pericenter $\omega$ [°] | 118 ± 58 | 320 ± 10 |

Unfortunately, the observational data available are not enough for modelling the second system, HD215152. In what concerns the HD102272 system, it is close to the mean motion resonance $4/1$. We have integrated the system motion, using the software [7, 8], allowing one to compute the Lyapunov exponents of the motion. Figure 5 shows the initial orbital configuration. The planetary parameters and initial conditions, as given in [9], are collected in Table 3; the mass of the star is equal to $1.9 \pm 0.3 M_\odot$. Our numerical modelling shows that the system is stable on time intervals up to $10^6$ yr, notwithstanding the fact of the intersection of orbits.

**4. Conclusions**

We have built distributions of the orbital period ratios, considering separately the cases of inner and outer location of the massive perturber. The histograms have prominent peaks close to the first order orbital resonances $2/1$ and $3/2$, as well as less significant peaks at resonances $5/2$, $3/1$
and $4/1$. We have performed analytical modelling of the histograms, and obtained the best-fit positions of the peaks. It turns out that the peak shifts are much smaller in case (a) than in case (b). Moreover, almost no shift is present for resonances of order higher than 1 in case (a).

Moreover, we have constructed the “period ratio – eccentricity” diagrams, with collision curves superimposed. Using these diagrams, we have identified anomalous systems, i.e. those with planets located outside the zones of conditional stability (zones limited by the collisional curves).

References

[1] Schneider J 2013 http://exoplanet.eu
[2] Fabrycky D C et al 2012 Preprint arXiv:1202.6328
[3] Petrovich C, Malhotra R, and Tremaine S 2013 Astrophys. J. 770 16
[4] Batygin K and Morbidelli A 2013 Astron. J. 145 10
[5] Lithwick Y and Wu Y 2012 Astrophys. J. 756 L11
[6] Kholshevnikov K and Titov V 2002 The Two-Body Problem (St. Petersburg: St. Petersburg Univ.), in Russian
[7] Shevchenko I and Kourpianov V 2002 Astron. Astrophys. 394 663
[8] Kourpianov V and Shevchenko I 2005 Icarus 176 224
[9] Niedzielski A et al 2009 Astrophys. J. 693 276