Comments on branon dressing and the standard model

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Abstract. This paper shows how electrodynamics and a Yukawa model are dressed after integrating out perturbative branon fluctuations. We discovered that first-order corrections in the inverse of the branon tension occur for the fermion and scalar wave functions, the couplings and the masses. Nevertheless, field redefinitions actually lead to effective actions, where only masses are dressed to this first order. We compare our results with the literature and find discrepancies at the next order, which, however, might not be measurable in the valid regime of low-energy branon fluctuations.

Contents

1. Introduction
2. Branon–electrodynamics interactions
3. Effective action for electrodynamics
   3.1. Derivation
   3.2. Comments
   3.3. Phenomenological implication
4. Coupling to a Yukawa model
   4.1. Corrections to the bare Lagrangian
   4.2. Phenomenological implications
5. Conclusion
Acknowledgments
References

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1. Introduction

In the context of braneworld scenarios and higher-dimensional theories, branons are modes that correspond to perturbative quantum fluctuations of the brane, about its equilibrium position in extra dimensions. Using an effective description of a four-dimensional brane embedded in a five-dimensional universe, branons can be thought of as a scalar field living on the brane, representing fluctuations in the position of the brane, as measured in the fifth dimension \( f^4 \). Branons can then be thought of as particles that couple to the standard model (SM), with a dimensionful coupling constant proportional to the inverse of the brane tension \( f^{-4} \).

Branons are interesting to study in the context of cosmology and collider physics, since there could be phenomenological effects arising from branons, which may be detectable in colliders such as the Large Hadron Collider (LHC). In this paper, we study the coupling of branons to electrodynamics and a Yukawa model, and the corresponding possible phenomenological effects. We will see that, although the interaction of branons with SM particles is suppressed by \( f^{-4} \), the derivative interactions between branons and the SM can compensate for this suppression. The idea is then to integrate out branon degrees of freedom, in order to obtain the effective theory for the SM degrees of freedom. In the low-energy approximation for branons, the coupling to the SM is quadratic in branons, such that the integration of the latter can be performed exactly. As a consequence, the one-loop result is exact and will be expanded in powers of the dimensionful coupling constant \( f^{-4} \). The resulting effective theory contains corrections to SM interactions, but also new interactions, such as four-fermion interactions, all with dimensions larger than 4. We will concentrate here on the corrections to the SM, since the other interactions are suppressed by higher orders of the inverse brane tension, \( f^{-8} \).

It has been shown in previous studies [1] that warped extra dimensions lead to massive branons, but we will consider a flat extra dimension, leading to massless branons: the dressing effects that we study here are mainly due to UV dynamics, where the branon mass does not play an important role.

Similar studies were carried out in [2], where the authors investigate the coupling of branons to the SM, and integrate the former to obtain the corresponding effective action for SM particles. We compare this to our approach, and find the same conclusion to the order \( f^{-4} \), but a discrepancy to the next order \( f^{-8} \), for the Yukawa model. The reason for the discrepancy is that the authors of [2] assume that the equations of motion hold, without taking into account the dressing from branons. We do not assume that SM degrees of freedom satisfy the equations of motion, and find that the Yukawa coupling gets a correction. Nevertheless, we argue that this \( f^{-8} \) correction is certainly negligible in the low-energy approximation, where the branon model is valid.

The paper is organized as follows. Section 2 describes the construction of the low-energy theory describing the branons–electrodynamics interaction. Starting from the general expression for the electrodynamics action in curved spacetime, we expand the metric in terms of the branon degree of freedom, which leads us to the branon–electrodynamics interactions. We then derive, in section 3, the effective action for electrodynamics, obtained after the exact integration of branons. We give technical comments on this derivation by comparing our work to the one given in [2]. The agreement between our approaches, in this specific case, is a consequence of gauge symmetry. Indeed, the gauge coupling and the fermion wave function renormalization need to obtain the same corrections for gauge symmetry to hold, such that, after a redefinition of...
the fields, the correction to the gauge coupling exactly vanishes and only the fermion mass gets a correction. Another important fact is that the Maxwell free theory is conformally invariant, independent of the equations of motion, such that the gauge field does not get any correction. Section 4 shows how a Yukawa model is dressed by branons. In this case, because no gauge invariance is required, we find that the Yukawa coupling does get a correction, leading to the discrepancy with [2]. Our conclusion questions the concept of branon, which, in order to be well defined, needs a low-energy approximation, where the branon dressing might not be measurable in experiments.

2. Branon–electrodynamics interactions

We consider a five-dimensional flat universe with generic coordinates $X^M = (x^\mu, y)$, where $x$ are the coordinates on the brane, which is defined by the equation $y = Y(x)$. The brane coordinates are denoted with the indices $\mu$ and $\nu$, and the induced metric $h_{\mu\nu}$ on the brane is

$$h_{\mu\nu}(x) = \eta_{\mu\nu} - \partial_\mu Y \partial_\nu Y. \quad (1)$$

If $f^4$ is the brane tension, the brane action is then

$$S_{\text{brane}} = -f^4 \int d^4x \sqrt{-h}$$

$$= -f^4 \int d^4x \left( 1 - \frac{1}{2}\eta^{\mu\nu} \partial_\mu Y \partial_\nu Y - \frac{1}{8}(\eta^{\mu\nu} \partial_\mu Y \partial_\nu Y)^2 + \cdots \right), \quad (2)$$

where dots represent higher orders in derivatives of $Y$. Our dynamical variable is the canonically normalized branon field $\phi = f^2 Y$, with mass dimension 1, and the brane ground state is $Y = 0$. Ignoring field-independent terms, since we are investigating an effective theory in flat spacetime, and taking into account the low-energy branon approximation, the resulting effective action for branons is then

$$S_{\text{branon}} = \int d^4x \left( \frac{\eta^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi \right), \quad (3)$$

and describes a free theory. Interactions will occur, though, with particles propagating in the brane over cosmological distances, and we now derive the corresponding action for electrodynamics.

In what follows, Latin indices refer to the local inertial frame and are contracted with $\eta_{ab}$, whereas Greek indices are contracted with the induced metric $h_{\mu\nu}$. The action describing electrodynamics in the curved background is [3]

$$S_{\text{em}} = \int d^4x \sqrt{-h} \left\{ -\frac{1}{4} F_{\mu\nu} F^{\rho\sigma} h^{\mu\rho} h^{\nu\sigma} + \frac{i}{2} \tilde{\psi} \gamma^a e^\mu_a (\partial_\mu + \Gamma_\mu) \psi - \frac{i}{2} e^\mu_a (\partial_\mu \tilde{\psi} + \tilde{\psi} \Gamma_\mu) \gamma^a \psi - g_0 \tilde{\psi} e^\mu_a \gamma^a A_\mu \psi - m_0 \tilde{\psi} \psi \right\}, \quad (4)$$

where the spin connection is

$$\Gamma_\mu = \frac{1}{8} \{ \gamma^a, \gamma^b \} e^\nu_a \nabla_\mu e^\nu_b, \quad (5)$$

and the gamma matrices $\gamma^a$ are defined in the local inertial frame, therefore satisfying

$$\{ \gamma^a, \gamma^b \} = 2\eta^{ab}. \quad (6)$$
Neglecting higher orders in the branon derivatives $\partial Y$, we find the following approximate vierbeins on the brane:

\[
e^\mu_a = \delta^\mu_a - \frac{1}{2} \partial_\mu Y \partial^a Y + \mathcal{O}(\partial Y)^4,
\]

\[
e^\mu_a = \delta^\mu_a + \frac{1}{2} \partial_a Y \partial^\mu Y + \mathcal{O}(\partial Y)^4,
\]

which lead to the expected definition $e^\mu_a e^\nu_b \eta_{ab} = h_{\mu\nu}$, up to higher orders in $\partial Y$. The Christoffel symbols for the induced metric are then

\[
\Gamma^\rho_{\mu\nu} = -(\partial^\rho Y)(\partial_\mu \partial_\nu Y) + \mathcal{O}(\partial(\partial Y)^4),
\]

and the spin connection is

\[
\Gamma_\mu = \frac{1}{8} \left[ \gamma^a, \gamma^b \right] (\partial_\mu \partial_a Y)(\partial_b Y) + \mathcal{O}(\partial(\partial Y)^4),
\]

such that the total action for the branons–electrodynamics system is, after integrations by parts,

\[
S_{\text{total}}[\phi, \bar{\psi}, \psi, A] = \int d^4x \left\{ -\frac{1}{4} F^2 + \bar{\psi}(i \gamma^\mu - g_0 A_\mu) \psi - m_0 \bar{\psi} \psi 
+ \frac{1}{2} \partial^\mu \phi \partial_\mu \phi
+ \frac{1}{8} f^4 F_{\mu\nu} F_{\rho\sigma} \eta^{\mu\nu} \left[ \eta^{\rho\sigma} (\partial \phi)^2 - 4 \partial^\rho \phi \partial^\sigma \phi \right]
+ \frac{1}{2} f^4 \bar{\psi} \left( \partial^2 \phi \partial^\mu \phi - \partial^\mu \phi \partial_\mu \phi \right) \psi
+ \frac{1}{2} f^4 (\partial^\mu \phi \partial_\mu \phi) m_0 \bar{\psi} \psi \right\},
\]

where the indices are raised and lowered with the Minkowski metric. We note that, although the coupling between branons and electrodynamics is proportional to $f^{-4}$, it is not actually negligible, because of derivative interactions. Finally, the Greek indices appearing in the effective action (10) all denote flat four-dimensional spacetime coordinates.

3. Effective action for electrodynamics

3.1. Derivation

We assume from now on that branons have energies up to some value $\epsilon f$, where $\epsilon < 1$ for the low-energy branon approximation (3) to be valid. We integrate out their degrees of freedom from the theory described by action (10), which can be performed exactly, since this action is quadratic in the branon field. The resulting effective action for electrodynamics is then

\[
S_\epsilon[\bar{\psi}, \psi, A] = \int d^4x \left\{ -\frac{1}{4} F^2 + \bar{\psi}(i \gamma^\mu - g_0 A_\mu) \psi - m_0 \bar{\psi} \psi 
+ \frac{1}{2} \text{Tr}_\epsilon \left\{ \ln \left( \delta^2 S_{\text{total}} / \delta \phi \delta \phi \right) \right\} \right\},
\]

where the trace is taken over the branon momentum $0 \leq |p| \leq \epsilon f$. This regularization does not contradict gauge invariance, since the momenta of photons and fermions are not restricted, but only the branon momentum is, as can be seen from the Fourier transform (13), when the trace
is taken \((p + q = 0)\). The second functional derivative of \(S_{\text{total}}\) is
\[
\frac{\delta^2 S_{\text{total}}}{\delta \phi_x \delta \phi_y} = -\partial^2 \delta^{(4)}(x - y) - \frac{m_0}{f^4} \partial^\mu \left( \tilde{\psi} \psi \partial_\mu \delta^{(4)}(x - y) \right) \\
- \frac{1}{4f^4} \partial_\mu \left( F^2 \partial^\mu \delta^{(4)}(x - y) \right) + \frac{1}{f^4} \partial_\mu \left( F^{\mu \nu} F_{\rho \psi} \partial^\rho \delta^{(4)}(x - y) \right) \\
+ \frac{i}{2f^4} \delta_\rho \left[ \tilde{\psi} \left( -\gamma^\rho \partial^\mu \delta^{(4)}(x - y) \partial_\mu - \partial^\rho \delta^{(4)}(x - y) \Phi \right) \right] \psi \\
+ \frac{i}{2f^4} \delta_\rho \left[ \partial^\rho \left( \tilde{\psi} \partial_\rho \delta^{(4)}(x - y) \right) - \tilde{\psi} \delta^2 \delta^{(4)}(x - y) \gamma^\rho \psi \right] \\
+ \tilde{\psi} \partial^\rho \delta^{(4)}(x - y) \psi - \partial_\rho \left( \tilde{\psi} \gamma^\nu \partial^\rho \delta^{(4)}(x - y) \right) \psi,
\]
(12)
where \(\partial_\rho = \partial_\mu + ig_\mu A_\mu\), and has the following Fourier transform:
\[
\int d^4x d^4y \frac{\delta^2 S_{\text{total}}}{\delta \phi_x \delta \phi_y} e^{ipx+ipy} = p^2 (2\pi)^4 \delta^{(4)}(p + q) - \frac{pq}{f^4} \int_k m_0 \tilde{\psi}(k) \psi(p + q - k) \\
- \frac{pq}{f^4} \int_k F^{\mu \nu}(k) F_{\mu \nu}(p + q - k) + \frac{p_\rho q^\rho}{f^4} \int_k F^{\mu \nu}(k) F_{\rho \psi}(p + q - k) \\
- \frac{i}{2f^4} \int_k \tilde{\psi}(k) \left[ (\Phi q^\mu + q p^\mu) \partial_\mu(l) - 2pq \Phi(l) \right] \psi(p + q - k - l) \\
+ \frac{1}{2f^4} \int_k \tilde{\psi}(k) \left[ p^2 q + q^2 \Phi - pq(q + \Phi) \right] \psi(p + q - k),
\]
(13)
where
\[
\int_k (\cdots) = \int \frac{d^4k}{(2\pi)^4} (\cdots).
\]
(14)
We then expand the logarithm in the effective action (11) around the diagonal part (proportional to \(\delta^{(4)}(p + q)\)) and keep the first order in the inverse brane tension, to obtain
\[
\frac{1}{2} \text{Tr} \left\{ \ln \left( \frac{\delta^2 S_{\text{total}}}{\delta \phi \delta \phi} \right) \right\} = \frac{\epsilon^4}{64\pi^2} \int_k m_0 \tilde{\psi}(k) \psi(-k) - \frac{3\epsilon^4}{256\pi^2} \int_k \tilde{\psi}(k) i \Phi(l) \psi(-k - l) \\
= \frac{\epsilon^4}{64\pi^2} \int d^4x \left\{ m_0 \tilde{\psi}(x) \psi(x) - \frac{3}{4} \tilde{\psi}(x) i \Phi(x) \psi(x) \right\},
\]
(15)
where field-independent terms were ignored. Note that the contributions for the correction to \(F^2\) cancel each other after taking the trace, because
\[
\frac{1}{4} \int_{p, q} \delta^{(4)}(p + q) p^\rho q_\rho F^{\mu \nu} F_{\mu \nu} = \int_{p, q} \delta^{(4)}(p + q) p_\mu q^\rho F^{\mu \nu} F_{\rho \nu},
\]
(16)
and the terms arising from the spin connection (the last line in equation (13)) also cancel in the trace. We will discuss these two points in the next subsection. We can also note that, although
branons are massless, no IR divergences appear in the branon loop integrals because of the derivative interactions compensating the possible divergences at $p = 0$. The effective action for electrodynamics is finally

$$S_\epsilon[\bar{\psi}, \psi, A] = \int d^4x \left\{ -\frac{1}{4} F^2 + \left(1 - \frac{3\epsilon^4}{256\pi^2}\right) \bar{\psi} (i\partial^0 - g_0 A) \psi - \left(1 - \frac{\epsilon^4}{64\pi^2}\right) m_0 \bar{\psi} \psi \right\}, \quad (17)$$

and we find a correction to the fermion wave function, the coupling and the mass. Also, as expected, the corrections to the fermion wave function and to the coupling are the same, which is a consequence of gauge invariance.

3.2. Comments

We now explain how the effective action (17) could be obtained differently, as was done in [2].

In this work, the authors start from the action

$$\tilde{S} = \int d^4x \left\{ L_{SM} + \frac{1}{2} \partial^\mu \phi \partial^\nu \phi + \frac{1}{2} f^4 \partial^\mu \phi \partial^\nu \phi (T_{SM}^{\mu\nu}) \right\}, \quad (18)$$

where $L_{SM}$ is the flat spacetime SM Lagrangian, and $T_{SM}^{\mu\nu}$ is its energy–momentum tensor. After integration over branons, they find that the first-order correction to the SM action is proportional to

$$\frac{\eta_{\mu\nu}}{f^4} \int d^4x \ T_{SM}^{\mu\nu}, \quad (19)$$

which would vanish if the SM was scale invariant and if one assumes the equations of motion to be satisfied.

To put our results in a similar context, we bear in mind that not only the metric but also its derivatives depend on the branon field

$$\partial_\rho h^{\mu\nu} = \frac{1}{f^4} \partial_\rho (\partial^\mu \phi \partial^\nu \phi) + O(\partial \phi)^4, \quad (20)$$

such that $h^{\mu\nu}$ and $\partial_\rho h^{\mu\nu}$ cannot be considered independent variables. As a consequence, expanding the following action up to the first order in $(\partial \phi)^2$:

$$S_{em} = \int d^4x \sqrt{-h} \left\{ - f^4 + L_{em}(h^{\mu\nu}, \partial_\rho h^{\mu\nu}) \right\}, \quad (21)$$

we obtain (neglecting constant terms)

$$S_{em} = \int d^4x \left\{ (L_{em})_{flat} + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \frac{1}{2} f^4 \partial^\mu \phi \partial^\nu \phi (T_{em}^{\mu\nu})_{flat} \right\}$$

$$+ \frac{1}{f^4} \int d^4x \left( \frac{\partial L_{em}}{\partial (\partial_\rho h^{\mu\nu})} \right)_{flat} \partial_\rho (\partial^\mu \phi \partial^\nu \phi) + O(\partial \phi)^4, \quad (22)$$

where ‘flat’ denotes the corresponding quantity for vanishing branon field, and

$$T_{em}^{\mu\nu} = \frac{\delta S_{em}}{\delta h^{\mu\nu}} = -L_{em} h_{\mu\nu} + 2 \frac{\partial L_{em}}{\partial h^{\mu\nu}}. \quad (23)$$

As will be seen in equation (24), the term $F^2$ is conformally invariant, independent of the equations of motion, and we repeat here the general argument given in [2].
Our results can be understood as follows:

- The trace of the energy–momentum tensor is

\[
\text{tr} \left\{ T^\text{cm}_{\mu\nu} \right\}_\text{flat} = \eta^{\mu\nu} \left\{ - \frac{1}{4} F_{\rho\sigma} F_{\mu\nu} \eta^{\rho\sigma} \eta^{\tau\tau} + 4 \delta^\mu_\rho \delta^\nu_\tau \right\} - \frac{i}{2} \eta^{\mu\nu} ( \bar{\psi} \gamma^\rho \psi - \partial_\rho \bar{\psi} \gamma^\rho \psi ) \\
+ \frac{i}{2} ( \bar{\psi} \gamma^\rho \partial_\rho \psi - \partial_\rho \bar{\psi} \gamma^\rho \psi ) - g_0 ( - A \eta^{\mu\nu} + \gamma_\mu A_\nu ) \psi + \eta^{\mu\nu} m_0 \bar{\psi} \psi \\
= - \frac{3}{2} i ( \bar{\psi} \gamma^\rho \partial_\rho \psi - \partial_\rho \bar{\psi} \gamma^\rho \psi ) + 3 g_0 \bar{\psi} A \psi + 4 m_0 \bar{\psi} \psi, 
\]

where the following derivative has been used:

\[
\left( \frac{\partial e^\rho_a}{\partial h^{\mu\nu}} \right)_\text{flat} = \frac{1}{2} \delta^\rho_a \eta_{\mu\nu},
\]

and leads to the corrections we obtain. Note that the ratio between the coefficients of the correction to the coupling and the correction to the mass is 3/4, which was found in equation (17). The Maxwell term \( F^2 \) does not get any correction since its energy–momentum tensor is traceless anyway.

- The only term in the Lagrangian that contains derivatives of the metric is the spin connection. We have seen that the corresponding correction vanishes, which is necessary to maintain gauge invariance, since this correction would dress the fermion kinetic term and not the coupling. Hence, the second line of expansion (22) should not play a role, and this can be understood since the corresponding correction is

\[
\text{Tr} \left\{ \frac{\delta^2}{\delta \phi \delta \phi} \int d^4x \left( \frac{\partial L^\text{em}}{\partial (\partial^\rho h^{\mu\nu})} \right)_\text{flat} \partial_\rho ( \partial^\mu \phi \partial^\nu \phi ) \right\} \\
= 2 \int d^4x \int d^4y \delta^4(x - y) \partial_\mu \partial_\rho \left( \frac{\partial L^\text{em}}{\partial (\partial^\rho h^{\mu\nu})} \right)_\text{flat} \partial^\nu \delta^4(x - y) \\
= \delta^4(0) \int d^4x \partial_\rho \partial^\mu \partial^\nu \left( \frac{\partial L^\text{em}}{\partial (\partial^\rho h^{\mu\nu})} \right)_\text{flat}, 
\]

which is a surface term.

### 3.3. Phenomenological implication

The effective action (17) implies a change in the parameters describing electrodynamics. To identify the change, we consider the following rescaled fermion field:

\[
\Psi = \psi \sqrt{1 - \frac{3 \epsilon^4}{256 \pi^2}},
\]

which is our new dynamical variable and has a canonical kinetic term. The resulting effective action for electrodynamics becomes

\[
S_\epsilon[\bar{\Psi}, \psi, A] = \int d^4x \left\{ - \frac{1}{4} F^2 + \bar{\Psi} (i \gamma^\rho \partial_\rho - g_0 A_\rho ) \psi - m_0 \bar{\psi} \psi \right\},
\]

\[
(28)
\]
where the effective mass is
\[ m_\alpha = \frac{1 - \epsilon^4/64\pi^2}{1 - 3\epsilon^4/256\pi^2} m_0 \approx (1 - \alpha) m_0, \quad \text{with} \quad \alpha = \frac{\epsilon^4}{256\pi^2} \ll 1. \quad (29) \]

As a consequence, after field redefinition, we find that only the fermion mass gets a correction from the branon dressing, with a value agreeing with [2], since \( \alpha \ll 1 \), which leads to hardly any measurable effect.

It is worth remembering that in experiments we only measure the dressed parameters. Therefore, in order to distinguish the branon-dressed parameters from the bare ones, we need to compare an experiment with branons with a similar experiment without branons, which is difficult to set up.

4. Coupling to a Yukawa model

In the previous example of electrodynamics, we found no dressing for the coupling after field redefinition, since corrections to the coupling, before field redefinition, have to be the same as corrections to the fermion kinetic term, because of gauge invariance. We consider here a Yukawa model, not ‘protected’ by a gauge symmetry, and we will see that the coupling does get a correction, of the order \( f^{-8} \) though.

4.1. Corrections to the bare Lagrangian

We briefly present here similar arguments for branons coupled to a Yukawa model.

We start with the action
\[ S_Y = \int \! d^4x \sqrt{-h} \left\{ L_Y + \frac{1}{2} \partial^\mu \bar{\psi} \partial_\mu \psi - \frac{i}{2} e^a_\mu (\bar{\psi} \gamma^a \partial_\mu \psi - \partial_\mu \bar{\psi} \gamma^a \psi + \bar{\psi} \gamma^a \Gamma_\mu \psi) - f^4 - \lambda_0 \tilde{\psi} \bar{\psi} - \frac{M_0^2}{2} \bar{\psi}^2 - m_0 \bar{\psi} \psi \right\}, \quad (30) \]

where the induced metric \( h_{\mu\nu} \) is given by equation (1). We use here directly the approach given in [2] and write the action (30) in the form (neglecting higher orders in branon derivatives)
\[ S_Y = \int \! d^4x \left\{ L_Y + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \left( \eta_{\mu\nu} + \frac{1}{f^4} T^Y_{\mu\nu} \right) \right\}, \quad (31) \]

where \( L_Y \) is the Yukawa Lagrangian in flat space time, and \( T^Y_{\mu\nu} \) the corresponding energy momentum tensor. The second functional derivative of \( S_Y \) with respect to the branon field has Fourier transform
\[ \frac{\delta^2 S_Y}{\delta \phi \delta \bar{\phi}} = p^2 (2\pi)^4 \delta^{(4)}(p + q) - \frac{p^\mu q^\nu}{f^4} \tilde{T}^Y_{\mu\nu}(p + q), \quad (32) \]

where \( \tilde{T}^Y_{\mu\nu} \) is the Fourier transform of \( T^Y_{\mu\nu} \). The integration over branons, up to the energy \( \epsilon f \), leads to the following effective action (ignoring field-independent terms):
\[ S'_{\epsilon} = S_Y + \frac{1}{2} \text{Tr} \left\{ \ln \left( (2\pi)^4 \delta^{(4)}(p + q) - \frac{p^\mu q^\nu}{p^2 f^4} \tilde{T}^Y_{\mu\nu}(p + q) \right) \right\}. \quad (33) \]
When expanding the logarithm, we obtain (again, ignoring field-independent terms)

\[ S'_Y = S_Y - \frac{1}{2f^4} \text{Tr} \left\{ \frac{\mu^i \nu^j \tilde{T}^Y_{\mu\nu}(p+q)}{p^2} \right\} + \mathcal{O}(T_Y)^2 \]

\[ = S_Y + \frac{1}{2f^4} \int d^4x \eta^{\mu\nu} T^Y_{\mu\nu}(x) + \mathcal{O}(T_Y)^2 \]  

(34)

and the trace over Lorentz indices of \( T_{\mu\nu}^Y \) is

\[ \eta^{\mu\nu} T^Y_{\mu\nu} = - (\partial \xi)^2 - \frac{3i}{2} (\bar{\psi} \gamma^\mu \partial^\mu \psi - \partial^\mu \bar{\psi} \gamma^\mu \psi) + 4\lambda_0 \bar{\xi} \bar{\psi} \psi + 2M_0^2 \xi^2 + 4m_0 \bar{\psi} \psi. \]

(35)

In the situation where \( m_0 = M_0 = 0 \) (scale-invariant theory), trace (35) can also be written as

\[ \eta^{\mu\nu} T^Y_{\mu\nu} = \partial^\mu \left( -\xi \partial^\nu \xi + \frac{3i}{2} \bar{\psi} \gamma^\mu \psi \right) + \xi \left( \partial^2 \xi + \lambda_0 \bar{\psi} \psi \right) - 3 \bar{\psi} \left( i \gamma^\mu \partial^\mu \psi - \lambda_0 \bar{\psi} \psi \right). \]

(36)

and is therefore the sum of a divergence plus the terms that vanish if one assumes that the equations of motions hold, before dressing. But because of branon dressing, these equations of motion hold only up to terms of order \( f^{-4} \), such that, because of the overall additional factor \( f^{-4} \) in equation (34), we expect a difference of order \( f^{-8} \) from \([\text{2}])\. Therefore, we do not assume any cancellation of the trace (36) and, from equation (34) and by analogy with the calculations performed for the electrodynamics case, we are led to the following effective Lagrangian:

\[ \mathcal{L}_{\text{eff}} = \frac{1}{2} \left( 1 - 2\alpha \right) (\partial \xi)^2 + i(1 - 3\alpha) \bar{\psi} \gamma^\mu \partial^\mu \psi - (1 - 4\alpha) \left( \lambda_0 \bar{\xi} \bar{\psi} \psi + \frac{M_0^2}{2} \xi^2 + m_0 \bar{\psi} \psi \right), \]

(37)

where \( \alpha \) is given in equation (29).

4.2. Phenomenological implications

In terms of the rescaled fields

\[ \Xi = \xi \sqrt{1 - 2\alpha} \quad \text{and} \quad \Psi = \psi \sqrt{1 - 3\alpha}, \]

(38)

we obtain from the effective Lagrangian (37)

\[ \mathcal{L}_{\text{eff}} = \frac{1}{2} (\partial \Xi)^2 + i \bar{\Psi} \gamma^\mu \partial^\mu \Psi - \lambda_\alpha \Xi \bar{\Psi} \Psi - \frac{M_\alpha^2}{2} \Xi^2 - m_\alpha \bar{\Psi} \Psi, \]

(39)

where the effective masses are

\[ M_\alpha = M_0 \sqrt{\frac{1 - 4\alpha}{1 - 2\alpha}} = \left( 1 - \alpha - \frac{5}{2} \alpha^2 + \mathcal{O}(\alpha^3) \right) M_0, \]

\[ m_\alpha = m_0 \sqrt{\frac{1 - 4\alpha}{1 - 3\alpha}} = \left( 1 - \alpha - 3\alpha^2 + \mathcal{O}(\alpha^3) \right) m_0, \]

(40)

and the effective coupling is

\[ \lambda_\alpha = \frac{\lambda_0 (1 - 4\alpha)}{\sqrt{1 - 2\alpha(1 - 3\alpha)}} = \lambda_0 \left( 1 - \frac{5}{2} \alpha^2 + \mathcal{O}(\alpha^3) \right). \]

(41)

Hence, although correction (41) occurs in the order \( f^{-8} \) only, it does not vanish exactly, as one could conclude from [2]. Note that this \( f^{-8} \) correction has nothing to do with higher orders
in the expansion of the logarithm in equation (33), since these higher orders lead to four-fermion interactions and other operators with dimensions larger than 4. The Yukawa coupling receives corrections from the first order only, in the expansion of the logarithm appearing in equation (33).

Nevertheless, if one considers a low-energy branon regime, consistent with the initial approximation (3), one should assume that $\epsilon < 1$, such that the correction (41) satisfies

$$\frac{5}{2}\alpha^2 < 4 \times 10^{-7},$$

and the discrepancy with Cembranos et al [2] actually does not produce measurable consequences, since the Yukawa coupling of the SM is measured via fermion masses: correction (42) is smaller than the error bars on the Cabibbo–Kobayashi–Maskawa matrix.

Again, as with the electrodynamics case, it is worth remembering that in experiments we only measure the dressed parameters and the comments at the end of section 3.3 apply here as well.

5. Conclusion

In this paper we have discussed, in the framework of brane models, how two sectors of the SM are dressed by branon fluctuations. This was achieved by integrating out a massless scalar degree of freedom, associated with perturbative brane fluctuations, the branon. In order to check the results published previously, we performed this integration explicitly, in the case of electrodynamics, without assuming any symmetry of the energy–momentum tensor, and found the expected result. But in the situation of the Yukawa interaction, we find that the coupling does get a correction, which differs from previous studies, but with a difference hardly measurable experimentally.

More generally, the concept of branon gives an elegant, effective description of low-energy branon fluctuations, but the feedback on the coupling constants of the SM is of order $f^{-8}$ only, making this effective description hard to test experimentally, especially if the brane tension is expected to be larger than $(100 \text{ GeV})^4$ [4]. Also, it is not really clear how to interpret the cut-off $\epsilon f$ in branon energy, making the experimental tests even more difficult. One possibility is to assume that $\epsilon f$ corresponds to a center of mass energy in a collision, in which case the branon dressing can be compared with quantum corrections. But the corresponding dressing of coupling constants, of order $f^{-8}$, can be in the error bars of the experimental measurements if the brane tension is large. Bounds have been discussed in [2] though. Another possibility is to assume a branon bath of typical temperature $\epsilon f$, in a finite temperature collision, but it is not clear how efficiently branons thermalize to SM particles. One can also assume that branons are produced by an astrophysical catastrophic event, but in this case, the parameter $\epsilon$ is then certainly so small that no effect can be detected.

Furthermore, we note that, in order to distinguish the branon-dressed parameters from the bare ones, we need to compare an experiment with branons with a similar experiment without branons, which is difficult to set up. It seems more realistic to check the effects of branons at the next order in the inverse brane tension, since new interactions appear when the logarithm in the effective action (11) is developed further. These effects are of order $f^{-16}$ though, and have been calculated in [2]. The discrepancy we find here with this reference would not change their results substantially, since the corresponding correction occurs of the order $f^{-16}$.
We would also like to remark that the initial approximation (3) takes into account the first order only in \((\partial\phi)^2\), to get the action describing the branon–SM interaction. This approximation is based on the occurrence of low-energy branons, and contains two derivatives of the branon field. But the resulting coupling \(F^2(\partial\phi)^2\) contains four derivatives, such that, should we follow a strict gradient expansion scheme, we would also have to take into account terms of order \((\partial\phi)^4\) in the expansion of the induced metric \(h_{\mu\nu}\). This was not done and would spoil the exact integration over branon degrees of freedom. These additional terms, though, might be relevant in the limit of the parameter \(\epsilon\) going to 1, where the low-energy branon approximation is not valid anymore. These ambiguities show that a proper study of brane fluctuations feedback on the SM might need to be non-perturbative. The concept of branon degrees of freedom would then be difficult to define, but phenomenological effects might be more realistically predictable.

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