STRING COSMOLOGY: AN UPDATE

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Abstract

A string cosmology scenario (“pre-big-bang”) postulates that the evolution of the Universe starts from a state of very small curvature and coupling, undergoes a long phase of dilaton-driven kinetic inflation and at some later time joins smoothly standard radiation dominated cosmological evolution, thus giving rise to a singularity free inflationary cosmology. I report on recent progress in understanding some outstanding issues such as initial conditions, graceful exit transition and generation of inhomogeneity perturbations.

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1 Introduction

String theory predicts gravitation, but the gravitation it predicts is not that of standard general relativity. In addition to the metric, string gravity also contains a scalar dilaton, that controls the strength of coupling parameters. An inflationary scenario \( [1,2] \), is based on the fact that cosmological solutions to string dilaton-gravity come in duality-related pairs, an inflationary branch in which the Hubble parameter increases with time and a decelerated branch that can be connected smoothly to a standard Friedmann–Robertson–Walker expansion of the Universe with constant dilaton. The scenario (“pre-big-bang”) is that the evolution of the Universe starts from a state of very small curvature and coupling and then undergoes a long phase of dilaton-driven kinetic inflation. Then, after spending some time in a high-curvature “string phase” \([3]\), a graceful exit transition occurs and the evolution joins smoothly standard radiation dominated cosmology, thus giving rise to a singularity free inflationary cosmology.

The two duality-related branches of lowest order solutions are singular. The inflationary branch has a singularity in the future and the decelerated branch has a singularity in the past, therefore, they cannot be connected smoothly to form a single cosmology. However, because the Universe evolves towards higher curvatures and stronger coupling, there will be some time when the lowest order effective action can no longer reliably describe the dynamics and it must be corrected. Corrections come from two sources. The first are classical corrections, due to the finite size of strings, arising when the fields are varying over the string length scale \( \lambda_s = \sqrt{\alpha'} \). These terms are important in the regime of large curvature. The second are quantum loop corrections. The loop expansion is parameterized by powers of the string coupling parameter \( e^\phi = g_{\text{string}}^2 \), which in the models that we consider is time dependent. So quantum corrections will become important when the dilaton \( \phi \) becomes large. As we will see, the two types of corrections can induce a graceful exit transition. Even in the absence of a detailed description of the high curvature string phase, assuming that a graceful exit transition did occur, it is possible to find experimentally observable consequences of string cosmology models.

Most of the experimentally observable consequences of string cosmology models based on the pre-big-bang scenario arise from the inhomogeneity perturbations generated around the homogeneous and
isotropic background. Inhomogeneities get produced during the dilaton-driven phase by the standard mechanism of amplification of vacuum fluctuations [4, 2, 5]. Deviations from homogeneity and isotropy are generated by quantum fluctuations around the homogeneous and isotropic background and then amplified by the accelerated expansion of the Universe. In addition to the dilaton and metric, string theory contains many other fields that have trivial expectation values and do not affect the classical solutions, but they do fluctuate.

2 Initial Conditions

Initial conditions are an essential part of any cosmology, as stressed by Hawking [6]. Since the simplest dilaton-driven inflationary solution has a future singularity, and is completely regular in the past, it might have given the wrong impression that the evolution could be extended backwards in time indefinitely. We now know that this is only correct for some strictly homogeneous and isotropic solutions. The question “how did the Universe find itself in the dilaton-driven inflationary phase” is a valid question, although it may well be that its resolution lies outside of physics.

In all inflationary models, to solve the problems of standard cosmology, the initial conditions have to be rather special. For the simplest string cosmology models the Hubble parameter $H$, of the inflationary branch satisfies $H = \frac{1}{\sqrt{3}}(t_0 - t)^{-1}$ and $\dot{\phi} = (1 + \sqrt{3})(t_0 - t)^{-1}$ for $t < t_0$. The number of e-folds from $t_i$ until $t_f$ is $Z = a_{(t_f)}H_{(t_f)} - a_{(t_i)}H_{(t_i)} = e^{1/\sqrt{3}((\phi(t_f) - \phi(t_i)))}$. To ensure at least 60 e-folds of inflation during the dilaton-driven phase, the dilaton has to start such that the coupling $e^{\phi/2}$ is very small and remains perturbative throughout the evolution.

The basic postulate of the scenario is that the initial curvature and the initial coupling are very small, however this does not necessarily mean that the Universe starts in a completely homogeneous and isotropic state. The issue of initial conditions for the dilaton-driven phase has been investigated by several authors [7]-[10] and we review the results. We introduce the problem by looking at the equation for $H$ in the presence of spatial curvature to model the evolution of under- and over-dense regions, $H = H\sqrt{3H^2 - 6k/a^2 - 2k/a^2}$, where $k = 0, \pm 1$ determines the spatial curvature. Even without solving the equation we see that the flat ($k = 0$) inflationary solution is an at-
tractor, since the contribution of the curvature term decreases rapidly as the Universe expands. Running the equation backwards in time we see that the effects of curvature increase and the solution deviates substantially from the dilaton-driven solution.

Effects of inhomogeneities were first discussed in [7]. It was found that the inflationary branch is an attractor for a class of inhomogeneous initial conditions and that a range of initial conditions will lead to the inflationary branch. The conclusion of [8] is that the results of [7] are correct, but the onset of inflation is delayed by initial inhomogeneities and curvature and therefore the requirement on the initial value of the dilaton and curvature are more severe depending on the initial curvature and inhomogeneities. The conclusion of [10] is that the results of [8] are correct, but there are two classical moduli of string theory, the initial scale factor and initial coupling that are free and there is always a choice that will guarantee a long duration of inflation.

\[
Z \sim \min \left[ e^{1/\sqrt{3}(\phi(t_f)-\phi(t_i))}, \left( \lambda_2^2 R(t_i) \right)^{-\frac{\sqrt{3}+1}{2\sqrt{3}}} \right],
\]

where \( R(t_i) \) is the initial curvature. That the initial coupling has to be small enough, we have already seen, in addition, the initial curvature has to be small enough so that it does not become too big too early and disrupt the dilaton-driven inflation. The conclusion of [7] is that the results of [8] are correct but that in estimating the total number of e-folds, the expansion during the string phase should also be taken into account and therefore the constraints on the initial conditions should be relaxed.

The results show, in my opinion, that as in all models of inflation, initial conditions do need to be specified, and that they are rather special. To decide quantitatively how likely or unlikely they are, a theory of initial conditions which does not exist at the moment has to be developed.

### 3 Graceful Exit

The two duality related branches of solutions of the lowest order string effective equations are separated by a singularity. Additional fields or correction terms need to be added to make a smooth “graceful exit”
transition between branches possible. In \cite{11, 12} it was shown that the transition is forbidden for a large class of fields and potentials, a result which was reinforced by many subsequent investigations \cite{13}. In \cite{14} we proposed to use an effective description in terms of sources that represent arbitrary corrections to the lowest order equations and were able to formulate a set of necessary conditions for graceful exit and to relate them to energy conditions appearing in singularity theorems of general relativity \cite{15}. We showed that a successful exit requires violations of the null energy condition (NEC) and that this violation is associated with the change from a contracting to an expanding universe (bounce) in the “lowest order Einstein frame” \(\mathcal{E}\), defined by a conformal change of variables. Since most classical sources obey NEC this conclusion hints that quantum effects, known to violate NEC in some cases, may be the correct sources to look at.

The \(00\) equation of motion is quadratic and may be conveniently written in the string frame,

\[
\dot{\phi} = 3H \pm \sqrt{3H^2 + e^\phi \rho},
\]

where we have fixed our units such that \(16\pi\alpha' = 1\). The choice of sign here corresponds to our designation of (+) and (−) branches.

In \cite{16}, relying on a demonstration that classical corrections can limit curvature \cite{17}, we were able to find an explicit model that satisfies all the necessary conditions and to produce the first example of a complete exit transition.

The explicit correction Lagrangian we use to produce the concrete example presented in Fig. 1 is

\[
\frac{1}{2} \mathcal{L}_c = e^{-\phi} \left( \frac{R_{\mu\nu}^2}{4} - \nabla^2 \phi + 1000(\nabla \phi)^4 \right) + 1000e^\phi (\nabla \phi)^4.
\]

The first term is in the form of \(\alpha'\) corrections examined in \cite{17}, the second and third are plausible forms for the one and two loops corrections respectively. The large coefficients account for the expected large number of degrees of freedom contributing to the loop. The signs of these terms are deliberately chosen to force the exit. We have checked that qualitatively similar evolution is obtained for a range of coefficients, of which the evolution in Fig. 1 is a representative.

We set up initial conditions in weak coupling near the (+) branch vacuum and a numerical integration yields the evolution shown in Fig. 1 in the \(\dot{\phi}, H_S\) phase space. We have also plotted lines marking important landmarks in the evolution, the (+) and (−) vacuum (\(\rho = 0\)
in (2), the line of branch change (+) \(\to\) (−) (square root vanishing in (2)) and the position of the \(E\) bounce \((H_E = 0, \dot{\phi} = 2H_S)\).

The evolution falls into distinct phases. **Phase (i)** This is the inflationary phase. As curvature becomes large we see deviation induced by the \(\alpha'\) corrections, and without influence from other corrections the solution would settle into the fixed point noted in [17], marked with a ‘+’. The solution does cross the line of branch change, but does not execute the \(E\) bounce required by a complete exit, corresponding to the fact it does not violate NEC in the \(E\). **Phase (ii)** Eventually the loop corrections become important, while the coupling is still perturbative because of the large correction coefficient. The first to do so is the one loop correction. Since we require further NEC violation to complete the \(E\) bounce, we have chosen the sign of the one loop correction to provide this and in this phase corrections are dominated by this term. As a result a bounce occurs and the evolution proceeds into the \(\rho > 0\) region. We checked that other forms of loop correction will have the same effect if they are introduced with a coefficient allowing NEC violation. Without further corrections this solution would continue to grow into regions of larger curvature and stronger coupling. We refer to this era as “correction dominated” and we also find there are obstacles to stabilizing the dilaton with standard mechanisms like capture in a potential or radiation production. **Phase (iii)** To offset the destabilizing NEC violation we have introduced the two loop correction with the opposite sign, allowing it to overturn the NEC.
violation when it becomes dominant as \( \phi \) continues to grow. Indeed during this phase we see the expansion decelerating, dilaton growth stabilizing, and the corrections vanishing. We have also checked that in this phase the dilaton can be captured into a potential minimum or halted by radiation production. This phase can be smoothly joined to standard cosmologies.

4 Inhomogeneity perturbations and particle production

Until recently, most of the produced particle spectra were found to rise sharply with frequency, a property with interesting consequences [3, 18, 19, 20], but also some disadvantages. In particular, at very large wavelengths there is almost no power, making these inhomogeneity perturbations an unlikely source for large scale anisotropy. The standard explanation given for the generic spectral frequency dependence is that since the curvature increases sharply, particle production also increases and hence the resulting spectrum. However, we now know that this standard explanation is not always correct. The first hint came from axionic vacuum perturbations [21] which exhibited a decreasing spectrum. Although sharply rising spectra are indeed common, flatter spectra or even decreasing spectra are just as likely [22, 23]. In addition to the obvious dependence on the background solution, the spectrum depends also on the spin of the particle, the type of dilaton prefactor, the coupling of the particle to internal moduli, and whether it is massless or massive, revealing a rich range of spectral shapes, of which many more deserve further individual attention.

The action for each field’s perturbation is obtained by expanding the 4 dimensional effective action of strings, which generically, for a tensor field of rank \( N \), has the form

\[
\frac{1}{2} \int d^4x \sqrt{-g} e^{l\phi} L^{(2)}(T_{\mu_1,\ldots,\mu_N}),
\]

where the parameter \( l \) in the dilaton prefactor is determined by the type of field. Setting the dilaton and metric at their background values, \( g_{\mu\nu}(\eta) = a^2(\eta) \eta_{\mu\nu}, \phi(\eta) \) (where \( \eta \) is conformal time) results in a quadratic action for each physical component of the perturbation \( \psi \),

\[
A = \frac{1}{2} \int d\eta \, d^3x \, S^2(\eta) \left( \psi'^2 - (\nabla \psi)^2 - M^2 a^2 \psi^2 \right)
\]

where \( ' \) denotes \( \partial/\partial \eta \) and \( M \) is the mass of the perturbed field. The
function $S(\eta)$ (equal to $a^m e^{\phi/2}$ for the simplest case but can be quite complicated in general) is the “pumping-field”.

The linearized equation of motion, satisfied by the Fourier modes of the field perturbation $\psi$, derived from the action (3) is the following

$$\psi''_k + \frac{2S'}{S} \psi'_k + \left( k^2 + M^2 a^2 \right) \psi_k = 0. \quad (4)$$

The general solution of the perturbation equation is a linear combination of two modes, one which is approximately constant outside the horizon and one which is generically time dependent outside the horizon. We understand the appearance of a constant mode as the freezing of the perturbation amplitude, since local physics is no longer active on such scales. The existence of the time dependent mode can be most easily understood in terms of a constant mode of the conjugate momentum of $\psi$, $\Pi = S^2(\eta) \psi'$. The amplitude of the conjugate momentum also freezes outside the horizon, since local physics is no longer active on such scales. This forces $S^2(\eta) \psi'$ to be approximately constant leading to a “kinematical” time dependence of $\psi$ \[21\]. The number density and energy density of the produced particles can be easily read off from the solutions of eq. (4) in a standard way \[4\]. The function $S(\eta)$ determines the resulting spectrum of produced inhomogeneities (or equivalently, particles) and may depend in a non-trivial way on $a(\eta)$, $\phi(\eta)$, and additional variables. If the background solution changes, so does $S$, even if its functional dependence on the dilaton and scale factor remains the same. Therefore, a rich array of spectral indices appears, as the example of Table 1. shows. For a comprehensive list of possibilities, and detailed explanation see \[22, 23\].

In \[22\] we solved the perturbation equation by first solving the early time equation with boundary conditions of normalized vacuum fluctuations. We assumed that both the constant mode of $\psi$ and the constant mode of $\Pi$ remain constant while outside the horizon during the string phase, thus bridging the gap of unknown background evolution during the string phase, at the expense of introducing only two string phase parameters. We then matched the early time solutions to the late time solutions. In \[23\], and in previous calculations, an explicit background solution during the string phase was assumed. In all cases where a comparison is possible the results using the two methods agree.
Table 1: A variety of spectral indices for different fields, assuming the simplest background evolution. For internal axions a range of possible internal backgrounds was considered. The spectral index $n$ determines the energy density spectrum ($n = 1$ corresponds to a flat spectrum).

| field                        | spectral index $n$ |
|------------------------------|--------------------|
| gravitons, dilatons          | 4                  |
| model-independent axions     | 0.54               |
| internal axions              | from 0 to 4        |
| RR axions                    | 2.26               |
| heterotic gauge bosons       | 3.27               |

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