Does Newton’s gravitational constant vary sinusoidally with time? Orbital motions say no

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Received 30 April 2015, revised 8 December 2015
Accepted for publication 15 December 2015
Published 21 January 2016

Abstract
A sinusoidally time-varying pattern of the values of Newton’s constant of gravitation $G$ measured in Earth-based laboratories over the last few decades has been recently reported in the literature. We put to the test the hypothesis that the aforementioned harmonic variation may pertain to $G$ itself in a direct and independent way. We numerically integrated the ad hoc modified equations of motion of the major bodies of the Solar System, finding that the orbits of the planets would be altered by an unacceptably larger amount in view of the present-day high accuracy astrometric measurements. In the case of Saturn, its geocentric right ascension $\alpha$, declination $\delta$ and range $\rho$ would be affected by up to $10^4 - 10^5$ milliarcseconds and $10^5$ km, respectively; the present-day residuals of such observables are as little as about 4 milliarcseconds and $10^{-1}$ km, respectively. We analytically calculated the long-term orbital effects induced by the putative harmonic variation of $G$ at hand, finding non-zero rates of change for the semimajor axis $a$, the eccentricity $e$ and the argument of pericenter $\omega$ of a test particle. For the LAGEOS satellite, an orbital increase as large as 3.9 m yr$^{-1}$ is predicted, in contrast with the observed decay of $-0.203 \pm 0.035$ m yr$^{-1}$. An anomalous perihelion precession as large as 14 arcseconds per century is implied for Saturn, while latest observations constrain it to the $10^{-4}$ arcseconds per century level. The rejection level provided by the Mercury’s perihelion rate is of the same order of magnitude.

Keywords: experimental studies of gravity, experimental tests of gravitational theories, ephemerides, almanacs, calendars

(Some figures may appear in colour only in the online journal)
1. Introduction

Newton’s gravitational constant $G$ (Gillies 1997, Mohr et al 2012), measured for the first time\(^1\) by Cavendish (1798) at the end of the 18th century\(^2\), is one of the fundamental parameters of Nature, setting the magnitude of the gravitational interaction (Uzan 2003, 2009, 2011, Chiba 2011). In both the Newtonian and the Einsteinian theories it is assumed that it does not depend on either spatial or temporal coordinates, being a truly universal constant.

In the 20th century, prominent scientists (Milne 1935, 1937, Dirac 1937, Jordan 1937, 1939), mainly on the basis of cosmological arguments, argued that, actually, $G$ may experience slow time variations over the eons. Current research took over such a fascinating idea, so that nowadays there are several theoretical scenarios encompassing it; see, e.g., Brans and Dicke (1961), Brans (1962), Wu and Wang (1986), Ivaschuk and Mel’Nikov (1988), and Melnikov (2002, 2009).

Recently, Anderson et al (2015a) showed that a set of measurements of $G$ obtained over the years with different techniques (see, e.g., Gundlach and Merkowitz 2000, Quinn et al 2001, Schlamminger et al 2006, Fixler et al 2007, Lamporesi et al 2008, Parks and Faller 2010, Luo et al 2009, Tu et al 2010, Quinn et al 2013, Rosi et al 2014, Schlamminger 2014) in terrestrial laboratories over the latest decades (Speake and Quinn 2014) can be satisfactorily modeled with the following harmonic time-dependent signature:

$$G(t) = G_0 + \Delta G(t) = G_0 + A_G \sin (\omega_G t + \phi),$$

with (Anderson et al 2015a)

$$G_0 = 6.673899 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2},$$

$$A_G = 1.619 \times 10^{-14} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2},$$

$$\phi = 80.9^\circ,$$

$$\omega_G = \frac{2\pi}{P_G},$$

$$P_G = 5.899 \text{ yr}.$$  

Anderson et al (2015a) did not suggest that the reported pattern may be due to some modifications of the currently accepted laws of gravity. They remarked that a correlation with recently reported harmonic variations in measurements of the length of the day (LOD) (Holme and de Viron 2013) is present. Later, a compilation of all published measurements of $G$ made since 1980, including also some additions and corrections with respect to the values analyzed by Anderson et al (2015a), was offered by Schlamminger et al (2015). Their least-square fit to such an expanded dataset confirmed the existence of the sinusoidal component with a 5.9 yr periodicity by Anderson et al (2015a), although the correlation with the LOD turned out to be weakened; in the minimization procedure, a second periodicity of about 1 yr was found by Schlamminger et al (2015). Cautiously, Schlamminger et al (2015) did not

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\(^1\) For a critical discussion of earlier attempts (Bouguer 1749, Maskelyne 1775), see, e.g., Poynting (1894) and Smallwood (2010).

\(^2\) In view of the unit conventions in use at the time of Cavendish’s work, the gravitational constant did not appear explicitly in it. Indeed, one of the first explicit formulations of the Newtonian gravity in terms of a gravitational constant $G$ appeared not before 1875 (Cornu and Baille 1875). Thus, the English scientist actually measured the mean density of the Earth (Clotfelter 1987). His results were later reformulated in modern terms as a determination of $G$ (Poynting 1894, Mackenzie 1900, Boys 1894).
explicitly mention any possible variation of the gravitational laws, warning that the harmonic pattern they found may be due to some unspecified systematic errors, with underestimated systematic uncertainty. Pitkin (2015), by using the compiled measurements of $G$ by Schlamminger et al (2015) and a Bayesian model comparison, claimed that a constant $G$ measurement model with an additional Gaussian noise term would fit the experimental data better than a model containing periodic terms. In the reply they offered, Anderson et al (2015b) were not able to independently confirm the claim by Pitkin (2015), and stood by their conclusions of potential periodic terms in the reported $G$ measurements. In particular, Pitkin (2015) found that a model with two harmonic components would be slightly favored with respect to their early proposal (Anderson et al 2015a), allowing us also to cope with the issue of the weakened correlation with the LOD pointed out by Schlamminger et al (2015).

At the time of writing, none of the putative systematic errors allegedly affecting the Earth-based measurements of $G$, reasonably conjectured by Anderson et al (2015a) and Schlamminger et al (2015), have been yet disclosed. Things standing thus unknown, it is not unreasonable to follow a complementary approach, which could help in effectively selecting the directions of further experimental analyses, and look at the potentially intriguing—although admittedly unlikely—possibility that some modifications of the currently accepted laws of gravity may be at work in the present case. According to Klein (2015), the observed discrepancies between the $G$ values determined by different experiments may be connected with a differential interpretation of MOND theory applied to the galaxy rotation curves.

In this spirit, we would like to quantitatively put to the test the daring conjecture that (some of) the observed sinusoidal pattern(s) in the $G$ data records may reflect an unexpected physical phenomenon in an independent way by looking at the consequences that such an effect, if real, would have on systems other than those used to collect the measured values of $G$ on the Earth. With this aim, for the sake of simplicity, we will consider the simpler model of equations (1)–(6) and the changes which would occur in the motions of the major bodies of the Solar System to check if they are compatible with the current stringent limits posed on their standard dynamics by accurate astrometric measurements.

Here, we will use recently released Cassini data analyses spanning ten years (2004–2014) of the orbit of Saturn in terms of its geocentric range $\rho$, right ascension $\alpha$ and declination $\delta$ (Hees et al 2014, Jones et al 2015) to independently test the $G(t)$ scenario of equations (1)–(6). In particular, we will suitably compare numerically simulated perturbations $\Delta \alpha(t)$, $\Delta \delta(t)$, $\Delta \rho(t)$ induced by equations (1)–(6) on the Kronian celestial coordinates with the currently existing residuals for them (Hees et al 2014, Jones et al 2015). Full details of the methodology adopted are given in section 2. Section 3 summarizes our findings.

2. Methods and results

A striking feature of the alleged time variation of $G$ investigated here is its relatively short characteristic timescale, which, according to equation (6), amounts to only about 6 yr. This is in neat contrast with virtually all the theoretical models predicting a $G(t)$ varying over typically cosmological timescales. This distinctive feature has also direct phenomenological consequences. Indeed, the validity of the numerous bounds on the percent variation of $G$ existing in the literature, of the order of (Williams et al 2004, Müller and Biskupek 2007, Pitjev and Pitjeva 2013, Pitjeva and Pitjev 2013)

3 A search for them is outside the scope of the present work.
may not be straightforwardly extended to the present case, since they were inferred from least-square reductions of planetary and lunar positional data by modeling $\Delta G(t)$ as a secular trend. Such a choice, reasonable in view of the extremely slow changes assumed in the literature for $G$ with respect to the typical orbital frequencies of the major bodies of the Solar System, does not apply to equation (1). Thus, a dedicated analysis should be performed in the present case: it will be the subject of the present section. In particular, in section 2.1 a numerical approach will be followed, while an analytical calculation will be offered in section 2.2.

2.1. A numerical approach

As a first step, we simultaneously integrate the barycentric equations of motion of all of the currently known major bodies of the Solar system in rectangular Cartesian coordinates over a centennial time-span (1914–2014) with the standard package MATHEMATICA\(^4\). The dynamical accelerations modeled include the general theory of relativity to the first post-Newtonian level, and all the major known Newtonian effects such as the Sun’s oblateness, pointlike mutual perturbations by the eight planets and the three largest asteroids, two massive rings accounting for the minor asteroids (Kuchynka et al 2010) and the Kuiper Belt’s objects (Pitjeva and Pitjev 2013). The initial conditions are taken from tables 1 and 2: they come from an adjustment of the suite of measurement and dynamical models of the EPM2013 ephemerides (Pitjeva and Pitjev 2014) to an extended data record of more than 800 000 observations ranging covering the last century, and are referred to the epoch $t_0 = \text{JD} 2446000.50$ (27 October 1984, h: 00.00.0 0).

Thus, keeping the other parameters of the numerical integration unchanged, we repeat the same step by including also the putative variation of $G$ according to equations (1)–(6). Both numerical integrations, with and without $\Delta G(t)$, share the initial conditions for the known bodies of the Solar system retrieved from tables 1 and 2. From the resulting time series of the Earth and Saturn, we numerically compute the time series of $\rho$, $\alpha$, $\delta$ of Saturn, with and without $\Delta G(t)$; then, for all of the three Kronian celestial coordinates, we compute differential time series $\Delta \rho(t)$, $\Delta \alpha(t)$, $\Delta \delta(t)$, which show up the impact of $\Delta G(t)$ over the 2004–2014 interval of time covered by the most recent Cassini data analyses (Hees et al 2014, Jones et al 2015); Gaussian white noise is also added to properly simulate the impact of the measurement errors. The results are displayed in figure 1. Finally, we compare our simulated residuals to the corresponding existing Kronian residuals in figure 5 of Hees et al (2014) and figure 2 of Jones et al (2015), which were produced without explicitly modeling the perturbing action of $\Delta G(t)$.

Some technical considerations about the general validity of the approach followed are in order. Possible objections of lacking of meaningfulness concerning such kinds of direct comparison among theoretically calculated signatures of a certain dynamical effect and actual data processed without modeling the effect itself have recently proved to be ineffective, at least in some specific cases in which its parameterization is relatively simple. Indeed, apart from the fact that such an approach had been proven successful since the time of the Pioneer

\[^4\] The MATHEMATICA method adopted is ExplicitRangeKutta, with a working precision used in internal computations of $53 \log_{10} 2 \approx 16$, and eight digits of precision and of absolute accuracy. The computer used has a 64-bit operating system.
anomaly (Iorio and Giudice 2006, Standish et al 2008, Standish 2010, Fienga et al 2010), the latest constraints on a certain form of the MOND theory, equivalent to the action of a remote trans-Plutonian body located in the direction of the Galactic Center, which were obtained by explicitly modeling it in a dedicated planetary data reduction (Hees et al 2014), turned out to be equivalent to those previously established by comparing theoretically computed effects to their observationally inferred counterparts determined without modeling it (Iorio 2010).

Finally, one might raise consistency issues about our analysis since our initial conditions come from the EPM2013 ephemerides (Pitjeva and Pitjev 2014), while the post-fit residuals (Hees et al 2014, Jones et al 2015) with which our simulated signatures are contrasted were obtained with the DE430 ephemerides. Actually, it is not so. First, the EPM2013 initial planetary state vectors differ from the DE431 coordinates at $t_0$, retrieved from the HORIZONS Web interface at http://ssd.jpl.nasa.gov/? horizons, by just $\epsilon = 2(x_{EPM} - x_{DE})/(x_{EPM} + x_{DE}) \approx 10^{-9}$. Then, from figure 7 of Pitjeva (2013) it can be noticed that the differences between the Kronian celestial coordinates calculated with the EPM2013 and DE424 ephemerides over a time interval as little as 10 years (2004–2014) are

\begin{tabular}{ccc}
\textbf{Table 1.} Solar System barycentric (SSB) initial positions $x_0$, $y_0$, $z_0$ of the Sun, the eight planets and the dwarf planet Pluto estimated with the EPM2013 ephemerides (Pitjeva and Pitjev 2014). They are referred to in the International Celestial Reference Frame (ICRF2) at the epoch JD 2446000.5 (27 October 1984, h: 00.00.00) (E.V. Pitjeva, private communication).

|       | $x_0$ (au) | $y_0$ (au) | $z_0$ (au) |
|-------|-----------|-----------|-----------|
| Sun   | 0.0005203216237770 | 0.0084630932909853 | 0.0035304787263118 |
| Mercury | -0.1804160787007675 | -0.3766844130175976 | -0.183416156676606 |
| Venus | 0.3542041782003010 | -0.5632690218672094 | -0.2760410432968001 |
| Earth | 0.8246646037849634 | 0.5178999919107320 | 0.2244221978752543 |
| Mars | 1.1810287466202694 | -0.6327726058206141 | -0.325338798256297 |
| Jupiter | 1.6184148186514691 | -4.4994366329965958 | -1.9681878191442090 |
| Saturn | -6.5110893457860746 | -6.9749292526618341 | -2.6006192842623910 |
| Uranus | -5.3939091822416660 | -16.7458210494500021 | -7.2578236561504630 |
| Neptune | 0.5400549266512163 | -27.9860579257785673 | -11.4683596423229197 |
| Pluto | -24.1120716069166541 | -17.4030370013573190 | 1.8337998706332588 |
\end{tabular}

\begin{tabular}{ccc}
\textbf{Table 2.} SSB initial velocities $\dot{x}_0$, $\dot{y}_0$, $z_0$ of the major bodies of the Solar system. The other details are as in table 1.

|       | $\dot{x}_0$ (au d$^{-1}$) | $\dot{y}_0$ (au d$^{-1}$) | $\dot{z}_0$ (au d$^{-1}$) |
|-------|--------------------------|--------------------------|--------------------------|
| Sun   | -0.0000082057731502      | -0.0000014976919650      | -0.0000004352388239      |
| Mercury | 0.0202454878200986      | -0.0077363989584885      | -0.006233734254165       |
| Venus | 0.0175313054842932      | 0.0093063903885975       | 0.0030763136627655       |
| Earth | -0.0099025409715828     | 0.0130300000107108       | 0.005649920171062        |
| Mars | 0.0078002404125410      | 0.0120303517832832       | 0.005306780995448        |
| Jupiter | 0.0070681002624717      | 0.0025516512292944       | 0.000921536833663        |
| Saturn | 0.0038982964656769      | -0.0033491220519229     | -0.0015508256594232      |
| Uranus | 0.0037423336736737      | -0.0011699461011685     | -0.000565407338528       |
| Neptune | 0.0031184870780294      | 0.0000964171123061      | -0.000381527005270       |
| Pluto | 0.0019439712118417      | -0.0025828344901677     | -0.0013916896197374       |
\end{tabular}
Figure 1. Numerically produced time series of $\rho$, $\alpha$, $\delta$ of Saturn perturbed by $\Delta G(t)$ as in equations (1)–(6). For each of the three Kronian observables considered, they were calculated as differences between two numerical integrations of the SSB barycentric equations of motion of the Sun, its eight planets and the dwarf planet Pluto from 1914 to 2014 with and without $\Delta G(t)$. Both integrations shared the same initial conditions in rectangular Cartesian coordinates, retrieved from tables 1 and 2, and the same standard dynamical models, apart from $\Delta G(t)$ itself. Thus, such curves represent the expected $\Delta G(t)$-induced signatures $\Delta \rho$, $\Delta \alpha$, $\Delta \delta$. The patterns and the size of the present signals can be compared with the range residuals by Hees et al (2014) ($\Delta \rho_{\text{exp}} \lesssim 0.1 \, \text{km}$) and those for RA and DEC by Jones et al (2015) ($\Delta \alpha_{\text{exp}}, \Delta \delta_{\text{exp}} \lesssim 4 \, \text{milliarcseconds}$).
smaller than the residuals in Hees et al (2014), Jones et al (2015) obtained with the DE430 ephemerides.

2.2. An analytical calculation

The sinusoidal part of $G$ induces a time-dependent component of the gravitational acceleration, which, to the Newtonian level, can be written as

$$A_G = - \frac{\Delta G(t) M}{r^2}. \tag{8}$$

In view of equation (3), equation (8) can be thought of as a small radial correction to the usual inverse-square law. As such, it can be treated with the standard methods of the perturbation theory (Bertotti et al 2003, Xu 2008) by assuming a Keplerian ellipse as unperturbed, reference trajectory.

For a test particle orbiting a primary of mass $M$ along a Keplerian ellipse with semimajor axis $a_0$, orbital period $P_0 = 2\pi n_0^{-1} = 2\pi \sqrt{a_0^3 G M^{-1}}$ and eccentricity $e_0$, the time $t$ is connected with the true anomaly $f$, which gives the instantaneous position along the orbit, through (Capderou 2005)

$$n_0 (t - t_p) = 2 \arctan \left[ \frac{1 - e_0}{1 + e_0} \tan \left( \frac{f}{2} \right) \right] - \frac{e_0}{1 + e_0} \tan f. \tag{9}$$

In equation (9), $n_0$ is the unperturbed Keplerian mean motion, while $t_p$ is the time of passage at the pericenter. The true anomaly $f$ usually starts to be reckoned just at the crossing of the pericenter, so that $f(t_p) = 0$. For the sake of simplicity, in the following we will assume $t_p = 0$.

By inserting equation (8), with equation (1) and $t$ given explicitly by equation (9), into the right-hand sides of the Gauss equations (Bertotti et al 2003, Xu 2008) for the variation of the Keplerian orbital elements$^5$

$$\frac{da}{dt} = \frac{2}{n_0 \sqrt{1 - e_0^2}} \left[ e_0 A_R \sin f + A_T \left( \frac{r_0}{r_0} \right) \right], \tag{10}$$

$$\frac{de}{dt} = \sqrt{1 - e_0^2} \left\{ A_R \sin f + A_T \left[ \cos f + \frac{1}{e_0} \left( 1 - \frac{r_0}{a_0} \right) \right] \right\}, \tag{11}$$

$$\frac{d\Omega}{dr} = \frac{1}{n_0 a_0 \sqrt{1 - e_0^2} \sin I_0} A_N \left( \frac{r_0}{a_0} \right) \sin (\Omega_0 + f), \tag{12}$$

$$\frac{d\omega}{dr} = \frac{\sqrt{1 - e_0^2}}{n_0 a_0 e_0} \left[ -A_R \cos f + \left( 1 + \frac{r_0}{r_0} \right) \sin f \right] - \cos I_0 \frac{d\Omega}{dr}. \tag{13}$$

$^5$ In equations (10)–(13), $A_R$, $A_T$, $A_N$ are the components of the disturbing acceleration $A$ onto the radial, transverse and out-of-plane directions, $\Omega$ is the longitude of the ascending node, $\omega$ is the argument of the pericenter, and $I$ is the orbital inclination to the reference $\{x, y\}$ plane.
evaluated on the unperturbed Keplerian ellipse

\[ r_0 = \frac{p_0}{1 + e_0 \cos f}, \quad p_0 = a_0(1 - e_0^2), \]  

(14)

and averaging them over one orbital period \( P_0 \) by means of (Capderou 2005)

\[ \frac{dt}{df} = \frac{(1 - e_0^2)^3/2}{n_0 (1 + e_0 \cos f)^2}, \]  

(15)

non-vanishing long-term rates of change of the semimajor axis, the eccentricity and the pericenter are obtained. They are

\[ \frac{\langle da \rangle}{dt} = \frac{e_0 A_G M P_0 P_G^2}{\pi^2 a_0^2 (P_0^2 - P_G^2)} \cos \left( \frac{\pi P_0}{P_G} + \phi \right) \sin \left( \frac{\pi P_0}{P_G} \right) + \mathcal{O}(e_0^2), \]  

(16)

\[ \frac{\langle de \rangle}{dt} = \frac{A_G M P_0 P_G^2}{4\pi^2 a_0^3 (P_0^2 - P_G^2)} \left[ \sin \phi - \sin \left( \frac{2\pi P_0}{P_G} + \phi \right) \right] + \mathcal{O}(e_0), \]  

(17)

\[ \frac{\langle d\omega \rangle}{dt} = \frac{A_G M P_0 P_G^2}{\pi^3 a_0^4 (P_0^2 - 4P_G^2)} \sin \left( \frac{\pi P_0}{P_G} \right) \sin \left( \frac{\pi P_0}{P_G} + \phi \right) + \mathcal{O}(e_0). \]  

(18)

While the leading component of the rate of change of the semimajor axis is of order \( \mathcal{O}(e_0) \), the next-to-leading order terms of the rates of the eccentricity and the pericenter are of order \( \mathcal{O}(e_0) \).

To the approximation level in \( e \) indicated, which is quite adequate in the Solar System, equations (16)–(18) are exact in the sense that no a priori assumptions on the relative magnitudes of \( P_0 \) and \( P_G \) were made. Thus, they can be applied to a variety of systems ranging, e.g., from Earth’s fast artificial satellites to the slowest planet of the Sun. As such, equations (16)–(18) also hold for any putative modified model of gravity yielding possibly a harmonic variation of \( G \) over arbitrary timescales (Morikawa 1990, Barrow 1993b, 1993a, Barrow and Mimoso 1994, Barrow 1995, Barrow and Carr 1996, Barrow and Parsons 1997, Stadnik and Flambaum 2015).

Let us start with the LAGEOS satellite orbiting the Earth along a nearly circular orbit with \( a_0 = 12274 \) km, \( e_0 = 0.0039, \quad P_0 = 3.7 \) hr \( = 4.2 \times 10^{-4} \) yr. It has been known for a long time (Rubincam 1982) that its semimajor axis experiences a secular decrease due to a variety of physical mechanisms. Its latest measurement amounts to (Sośnica et al. 2014, Sośnica 2014)

\[ \dot{a}_{\text{LAGEOS}} = -0.203 \pm 0.035 \text{ m yr}^{-1}. \]  

(19)

Instead, equation (16) predicts a secular increase as large as

\[ \dot{a}_{\text{LAGEOS}} = 3.9 \text{ m yr}^{-1}, \]  

(20)

which is in disagreement with equation (19).

Moving to Saturn, which orbits the Sun in about 29 yr at 9.5 au, it turns out that equations (16) and (18) predict

\[ \dot{a}_{\text{♃}} = -668 \text{ m yr}^{-1}, \]  

(21)

\[ \dot{\omega}_{\text{♃}} = 14'' \text{ cty}^{-1}. \]  

(22)

Actually, there is no trace of such an enormous secular decrease of the Kronian semimajor axis in the observational data records. As far as its perihelion is concerned, the current
admitted range for a putative anomalous precession is as little as\(^6\) (Pitjeva and Pitjev 2013, Fienga et al 2011)

\[
\Delta \dot{\omega}_{\text{P}}^\text{EPM} = (-3.2 \pm 4.7) \times 10^{-4} \text{ cty}^{-1},
\]

\[
\Delta \dot{\omega}_{\text{P}}^\text{INPOP} = (1.5 \pm 6.5) \times 10^{-4} \text{ cty}^{-1}.
\]

Similar results also hold for other planets, e.g. Mercury. Its predicted perihelion rate is as large as

\[
\dot{\omega}_\text{I}^{\text{pred}} = -108 \text{ cty}^{-1},
\]

while any anomalous precession is constrained within (Pitjeva and Pitjev 2013, Fienga et al 2011)

\[
\Delta \dot{\omega}_{\text{I}}^\text{EPM} = (-2 \pm 3) \times 10^{-3} \text{ cty}^{-1},
\]

\[
\Delta \dot{\omega}_{\text{I}}^\text{INPOP} = (4 \pm 6) \times 10^{-4} \text{ cty}^{-1}.
\]

3. Summary and conclusions

In this paper, we have independently tested the hypothesis that the harmonic temporal variation in the time series of laboratory measurements of the Newtonian constant of gravitation made over the years, which has been recently reported in the literature, may be due to some unknown physical mechanism affecting \(G\) itself. Cautiously, its discoverers did not mention such a possibility, claiming instead that it may be due to some systematic errors. Nonetheless, at present, none of them has been either identified or even explicitly suggested.

With this aim, we looked at the effects that such a putative time-dependent behavior of the fundamental parameter characterizing the strength of the gravitational interaction, if real, would have on the orbital motions of the major bodies of the Solar System. We numerically integrated their equations of motion with and without the proposed modification, and calculated the differences of the resulting time series for the observables used in real astrometric data reductions (right ascension \(\alpha\), declination \(\delta\), range \(\rho\)) to produce simulated residual signals \(\Delta \alpha(t), \Delta \delta(t), \Delta \rho(t)\). We remark that, given the specific functional dependence of the effect considered and its relatively short characteristic timescale compared to the Solar System’s typical orbital periods, it would be incorrect to straightforwardly extend the existing bounds on \(G/G\) to the present case since they were obtained by modeling a secular variation of \(G\). It turned out that the resulting anomalous signatures in right ascension, declination and range are far too large to have escaped detection in the residuals produced so far with the existing standard ephemerides, even if the putative variation of \(G\) was not explicitly modeled in all of them. Suffice it to say that, in the case of Saturn, \(\alpha, \delta\) and \(\rho\) would be affected up to \(10^4 - 10^5\) milliarcseconds and \(10^5\) km, while the current residuals are as little as about 4 milliarcseconds and \(10^{-1}\) km, respectively.

Complementarily, we also performed an analytical calculation of the long-term perturbations which a harmonic variation of \(G\) would induce on the orbital elements of a test particle orbiting a central mass. In the limit of small eccentricities, we obtained non-vanishing secular variations of the semimajor axis \(a\), the eccentricity \(e\) and the pericenter \(\omega\). They have a general validity since no a priori assumptions concerning the relative magnitudes of the orbital and \(G\) frequencies were made; as such, they can be applied, in principle, to any

\(^6\) Here, \(\varpi = \dot{\Omega} + \omega\) is the longitude of the pericenter.
physical mechanism predicting a sinusoidally time-dependent variation of \( G \). A comparison with the latest observational determinations for the LAGEOS satellite and Saturn confirmed the outcome of the numerical analysis. Indeed, the recently measured orbital decay of the geodetic satellite, of the order of \(-0.2 \pm 0.03 \text{ m yr}^{-1}\), rules out the predicted increase of approximately \(+4 \text{ m yr}^{-1}\). Furthermore, the expected anomalous apsidal rate of 14 arcseconds per century for Saturn falls neatly outside of the allowed range for any possible anomalous Kronian perihelion precession, which is set by the observations to approximately the \( 10^{-4} \) \( \text{cty}^{-1} \) level. Also Mercury yields an analogous outcome since the magnitude of its predicted anomalous perihelion precession amounts to \( 10^{-8} \) \( \text{cty}^{-1} \), while the observations constrain any possible deviations from standard physics down to approximately \( 10^{-10} \).

In conclusion, our analysis quantitatively rules out the possibility that some long-range modification of the currently accepted laws of the gravitational interaction may be at work in the present case, accounting for the observed harmonic pattern of the laboratory-measured values of \( G \). As such, it may contribute to further direct future investigations towards the discovery of the even more likely systematic uncertainties allegedly plaguing the set of measurements of \( G \) analyzed so far.

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