Prediction of spark ignition engine fuel consumption for different functional conditions using a theoretical model

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Abstract. The theoretical model estimates the characteristic parameters of a car spark ignition engine by using complex functions. Analyzing the variation of the efficiencies and the fuel consumption at various operating regimes it is possible to optimize them according to the operating regime of the car. It is also studied a method for estimation of the specific fuel consumption of the spark ignition engine by considering engine operation in variable mode. The theoretical study presented by authors contributes actively to the optimization of the interaction between vehicle and propulsion system by identifying low fuel consumption modes.

1. Introduction
The study of the combustion process in internal combustion engines shows that the actual quantity of air, \( L_G \), in kg/kg of fuel, differs from the minimum quantity of air, \( L_{\text{min}G} \), in many operating regimes. The minimum quantity of air is also known as the theoretical (stoichiometric) quantity of air, required for the complete combustion. The ratio of these two parameters defines the air excess ratio [4, 8]

\[
\lambda = \frac{L_G}{L_{\text{min}G}} = \frac{L_M}{L_{\text{min}M}}.
\]  

(1)

In formula (1), \( L_M \) and \( L_{\text{min}M} \) are the molar quantities of air to fuel, in kmol/kg of fuel.

2. Theoretical model
The theoretical model developed by authors is a complex one, proposing an analysis of the energy parameters of the engine. Preliminary variants of this model have been presented by authors in other articles [9-12]. Starting from the variations of the volumetric efficiency, \( \eta_v \), and the indicated mean pressure, \( p_i \), obtained in the model, the indicated specific fuel consumption and the brake specific fuel consumption functions were developed. This was possible by assuming a variation of the mechanical efficiency, \( \eta_m \). Complexity of the problem also comes from the fact that, as well known, engine operation involves variation of \( \lambda \) in a very narrow range around the stoichiometric value. Hence, the quantity of fresh air drawn into the cylinders of a spark ignition engine, in kmol/kg of fuel, is given by

\[
M_a = \lambda \cdot L_{\text{min}M} + \frac{1}{M_C},
\]  

(2)

where the term \( M_C \), in kg/kmol, represents the combustion molecular mass. In the case of spark ignition engines operating with gasoline, the recommended values of \( M_C \) are in the range 110...120
kg/kmol. Since gasoline is a complex mixture of hydrocarbons, it is recommended to consider that it consists entirely of a single and representative pure hydrocarbon, which is n-octane \((C_8H_{18})\). The molecular mass of this chemical compound is \(M_c = 114\) kg/kmol \([1, 2]\).

By considering that \(M_r\), in kmol, is the quantity of residual exhaust gas retained inside the cylinders from the last combustion, it results that cylinders charge, comprising of the fresh air and the residual gas, is

\[ M_i = M_a + M_r. \quad (3) \]

The parameter \(M_i\) is expressed function by the coefficient of residual gases, which is defined as

\[ \gamma_r = \frac{M_r}{M_a}. \quad (4) \]

From equations (4) and (2) it results

\[ M_i = \gamma_r \cdot M_a = \gamma_r \cdot \left( \lambda \cdot \frac{L_{\text{init}}}{M_c} + \frac{1}{M_c} \right). \quad (5) \]

In these conditions, equation (3) becomes

\[ M_i = M_a \cdot (1 + \gamma_r) = \left( \lambda \cdot \frac{L_{\text{init}}}{M_c} + \frac{1}{M_c} \right) \cdot (1 + \gamma_r). \quad (6) \]

Variation of \(\gamma_r\) with the engine load, \(\chi\), is given by

\[ \gamma_r(\chi) = a_{\gamma_r} \cdot \chi + b_{\gamma_r}, \quad (7) \]

where the coefficients \(a_{\gamma_r} = -0.3\) and \(b_{\gamma_r} = -0.35\) were determined by authors on the basis of data from the literature \([2, 4]\).

By considering \(L_{\text{init}} \approx \frac{1}{28.96} \cdot L_{\text{init}}\) for liquid petroleum fuels, from equations (6) and (7) it results

\[ M_i(\chi) = \left( \lambda \cdot \frac{L_{\text{init}}}{28.96} + \frac{1}{M_c} \right) \cdot \left[ 1 + \left( a_{\gamma_r} \cdot \chi + b_{\gamma_r} \right) \right]. \quad (8) \]

![Figure 1. Variation of the air excess coefficient.](image-url)
If there are also taken into account the mass fractions of the fuel components \((C = 0.854; \ H_2 = 0.142; \ O_2 = 0.004)\) as well as \(L_{\text{min}}G \cong 14.82\ \text{kg of air/kg of fuel}\) and \(L_{\text{min}}M \cong 0.5118\ \text{kmol}\), then the cylinder charge in the initial state is expressed as a function of engine load and speed, as follows:

\[
M_i(\chi) = (0.5118 \cdot \lambda_{n,\chi} + 0.00877) \cdot \left[1 + (a_p \cdot \chi + b_p)\right].
\] (9)

In equation (9), the values of the air excess coefficient, \(\lambda_{n,\chi}\), changes with the operating regime according to the matrix \(L = \left[\lambda_{n,\chi}\right]\), which was determined by processing experimental data corresponding to medium-capacity spark ignition engines and collected from literature [6, 7]. The values in this matrix correspond to the diagram \(\lambda(n, \chi)\), shown in Figure 1.

On the other hand, the indicated efficiency, \(\eta_i\), as known, is given by the ratio of indicated work, \(L_i\) (mechanical work generated by gas expansion in cylinders), to fuel’s input energy, \(Q_c\):

\[
\eta_i = \frac{L_i}{Q_c}.
\] (10)

It should be mentioned that cycle losses can be estimated on the basis of the indicated efficiency, so the capacity of the actual cycle to convert heat into mechanical work can be predicted. This can characterize the economy of the cycle.

According to the literature [3-5], the indicated efficiency can be expressed as

\[
\eta_i = 8.315 \cdot \frac{M_i}{H_i} \cdot \frac{p_i \cdot T_0}{\eta_i \cdot p_0},
\] (11)

where \(H_i\) is the lower calorific value of the fuel, while \(p_0\) and \(T_0\) are the initial pressure and the initial temperature of the environment. In the developed model, the initial environmental conditions are taken into consideration by introducing the parameter

\[
\phi_0(p_0, T_0) = \frac{p_0}{T_0}.
\] (12)

Variation of the mean indicated pressure, described by matrix \(\Pi_i = \left[p_{n,\chi}\right]\),

\[
\Pi_i = \begin{bmatrix}
2.52 & 3.08 & 3.10 & 2.76 \\
3.84 & 4.64 & 5.02 & 6.79 \\
4.89 & 5.89 & 6.01 & 6.88 \\
5.96 & 6.85 & 7.24 & 7.88 \\
6.96 & 7.72 & 8.18 & 8.44 \\
7.30 & 8.41 & 8.90 & 9.00 \\
7.84 & 8.89 & 9.41 & 9.81 \\
8.37 & 9.52 & 9.92 & 10.33 \\
9.85 & 10.53 & 10.87 & 10.90 \\
10.54 & 11.02 & 11.11 & 11.13
\end{bmatrix}.
\] (13)

was expressed as
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\[
p_i(x) = \left[ \frac{\varphi_i \cdot \phi_i \cdot (1-\varepsilon^{-1})}{\varepsilon-1} \left(1-\varphi_{pu} + \gamma_i\right) \right] \frac{1}{n \cdot \varepsilon} 
\]

\[
\begin{align*}
&\{0.69 \cdot \chi + 0.13 \} &\{0.3775 \} &\{n=5 \cdot 10^3 \text{ rpm}\} \\
&\{0.656 \cdot \chi + 0.212 \} &\{0.3883 \} &\{n=4 \cdot 10^3 \text{ rpm}\} \\
&\{0.594 \cdot \chi + 0.277 \} &\{0.3937 \} &\{n=3 \cdot 10^3 \text{ rpm}\} \\
&\{0.475 \cdot \chi + 0.4 \} &\{0.397 \} &\{n=2 \cdot 10^3 \text{ rpm}\}
\end{align*}
\]

\[
\mu_i \cdot \tau_i \left(1 - \frac{1}{\varepsilon^i - 1}\right)
\]

\[
\begin{align*}
&\{278 \cdot \chi^2 + 473 \cdot \chi + 225 \} &\{0.3775 \} &\{n=5 \cdot 10^3 \text{ rpm}\} \\
&\{167 \cdot \chi^2 + 284 \cdot \chi + 263 \} &\{0.3883 \} &\{n=4 \cdot 10^3 \text{ rpm}\} \\
&\{28 \cdot \chi^2 + 48 \cdot \chi + 311 \} &\{0.3937 \} &\{n=3 \cdot 10^3 \text{ rpm}\} \\
&\{56 \cdot \chi^2 + 95 \cdot \chi + 299 \} &\{0.397 \} &\{n=2 \cdot 10^3 \text{ rpm}\}
\end{align*}
\]

\[
\eta_v = a_v \cdot \chi + b_v
\]

\[
\begin{align*}
&\{0.69 \cdot \chi + 0.13 \} &\{n=5 \cdot 10^3 \text{ rpm}\} \\
&\{0.656 \cdot \chi + 0.212 \} &\{n=4 \cdot 10^3 \text{ rpm}\} \\
&\{0.594 \cdot \chi + 0.277 \} &\{n=3 \cdot 10^3 \text{ rpm}\} \\
&\{0.475 \cdot \chi + 0.4 \} &\{n=2 \cdot 10^3 \text{ rpm}\}
\end{align*}
\]

Notations used in equation (14) are:
- \(\varepsilon\) - compression ratio;
- \(\varepsilon_c, \varepsilon_e\) - polytropic compression exponent, polytropic expansion exponent;
- \(T_z\) - combustion temperature;
- \(\varphi_{pu}\) - overfilling degree;
- \(\varphi_t\) - rounding factor of the main indicated diagram.

Taking into account the mean indicated pressure, the volumetric efficiency and the parameter describing initial environmental conditions as defined by formulas (14), (15) and (12), equation (11) can be written as

\[
\eta_i(x) = \frac{8.315}{H_i} \frac{1}{\phi_0 a_v \cdot \chi + b_v} \{0.5118 \cdot \|\lambda_{n,z}\| + 0.00877\} \left[1 + \left(a_y \cdot \chi + b_y\right)\right]
\]

Hence, the dependence of the indicated efficiency on the engine operating mode can be analyzed.

By considering formula (16), the indicated specific fuel consumption can be computed with formula

\[
c_i(x) = \frac{3600}{8.315 \frac{1}{\phi_0 a_v \cdot \chi + b_v} \{0.5118 \cdot \|\lambda_{n,z}\| + 0.00877\} \left[1 + \left(a_y \cdot \chi + b_y\right)\right]}
\]

Similar to the case of the air excess coefficient, the mechanical efficiency has associated the matrix.
The brake specific fuel consumption can be estimated using the formula
\[
c_e = \frac{c_i}{\eta_m}.
\]

From formulas (17) and (19) it results
\[
c_e (\chi) = \frac{3600}{8.315 \cdot \frac{1}{\phi_b} \cdot \frac{P_{n,x}}{a_n \cdot \chi + b_n} \cdot (0.5118 \cdot \frac{\rho_{n,x}}{\eta_{n,x}} + 0.00877) \cdot \left[1 + (a_{ry} \cdot \chi + b_{ry})\right]}.
\]

3. Results
The model developed by authors contributes to the study of the indicated power of a spark ignition engine. The results presented below are obtained by simulation with this model and correspond to an existing engine with the following characteristic parameters:

- \( \varepsilon = 9.5 \) (compression ratio);
- \( V_i = 0.3995 \text{ dm}^3 \) (displacement);
- \( i = 4 \) (number of cylinders);
- \( n = 5500 \text{ rpm} \) (nominal engine speed);
- \( S = 80.5 \text{ mm} \) (stroke);
- \( \tau = 4 \) (engine’s strokes).
- \( D = 79.5 \text{ mm} \) (bore);

The variation of the indicated specific fuel consumption with the engine speed at different loads is graphically represented in Figure 2.

![Figure 2. Variation of the indicated specific fuel consumption with the engine speed.](image-url)
It is also possible to use the developed model in order to study the variation of $c_i$ with the engine load for various preset speeds, as shown in figure 3.

![Figure 3. Variation of the indicated specific fuel consumption with the engine load.](image)

Besides, by using the values of the mechanical efficiency, $\eta_m$, it is possible to predict the evolution of the actual brake specific fuel consumption with the engine speed and with engine load, as shown in figure 4 and figure 5, respectively.

![Figure 4. Variation of the brake specific fuel consumption with the engine speed.](image)

![Figure 5. Variation of the brake specific fuel consumption with the engine load.](image)
4. Conclusions

By using the proposed theoretical model, which represents a further development of a model previously developed by authors, the analytical expressions of some characteristic parameters of the engine, i.e. $p_i$, $c_i$, $\eta_i$ and $c_e$, are obtained. These parameters, which affect the operation of the engine, are defined within the model as functions of the engine load, $\chi$.

The model also offers the possibility to create the brake specific fuel consumption map, as shown in figure 6, which highlights the economical operating modes of the engine. Besides, it is possible to study the variation of $c_i$ with the engine load for various preset speeds.

It can be considered that the theoretical study performed by authors on the basis of the developed model makes an important contribution to the optimization of the interaction between car and engine, by identifying low fuel consumption regimes. The theoretical model is about to be validated with laboratory experimental data.

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