Projectile $\Delta$ Excitations in $p(p,n)NN\pi$ Reactions

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Abstract

It has recently been proven from measurements of the spin-transfer coefficients $D_{xx}$ and $D_{zz}$ that there is a small but non-vanishing $\Delta S = 0$ component $\sigma_0$, in the inclusive $p(p,n)NN\pi$ reaction cross section $\sigma$. It is shown that the dominant part of the measured $\sigma_0$ can be explained in terms of the projectile $\Delta$ excitation mechanism. An estimate is further made of contributions to $\sigma_0$ from s-wave rescattering process. It is found that s-wave rescattering contribution is much smaller than the contribution coming from projectile $\Delta$ excitation mechanism. The addition of s-wave rescattering contribution to the dominant part, however, improves the fit to the data.

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The \( p(p, n)N\pi \) reaction at intermediate energies has been a subject of a number of studies from both experimental \([1, 3]\) and theoretical \([6, 9]\) point of views. The understanding of the reaction is important in its own sake; it is one of the basic processes in the intermediate nuclear physics. One of the dynamical process involved in the reaction is the projectile \( \Delta \) excitation process (PDP). The PDP is usually ignored in the inclusive \( (p, n) \) cross section \( \sigma \) calculations, since \( \sigma \) is dominated by the contribution coming from the target \( \Delta \) excitation process (TDP). The contribution from PDP gives only a small correction to the dominant TDP cross section. Therefore, it has been difficult to test the predicted PDP cross section by the inclusive cross section data.

Recently, however, several measurements of the spin-transfer coefficients \( D_{xx} \) and \( D_{zz} \) have been made \([1, 2, 5]\). Using these coefficients, it is possible to extract the no spin-transfer(\( \Delta S = 0 \)) component \( \sigma_0 \) from the inclusive cross section \( \sigma \). In fact, we show that the measured \( \sigma_0 \) can be explained well in terms of PDP. In the present study, we restrict our interests to the zero-degree case, i.e., the case where the neutron is emitted at zero-degree \( (\theta_n = 0^\circ) \). Under this restriction, \( \sigma_0 \) can be expressed, in terms of the observed inclusive cross section \( \sigma \) and the spin-transfer coefficients \( D_{xx} \) and \( D_{zz} \), as

\[
\sigma_0 = \frac{1}{4} \sigma (1 + 2D_{xx} + D_{zz}). \tag{1}
\]

In order to present the theoretical cross sections \( \sigma_0 \) and \( \sigma \), let us denote the \( p(p, n)N\pi \) reaction as \( a + A \rightarrow b + B + \pi^\alpha \), where \( a \) (\( b \)) and \( A \) (\( B \)) denote the projectile (ejectile) and target (residual nucleus) respectively, and \( \pi^\alpha \) is the emitted pion that carries the charge \( \alpha \). In the center of mass system, total inclusive cross section \( \sigma \) may be written as

\[
\sigma = \frac{d^2\sigma}{dE_Bd\Omega_b} \bigg|_{\theta_b=0} = \frac{m_am_cm_AM_cm_B}{(2\pi)^5} \frac{p_B}{\sqrt{s}} \int \frac{d\Omega_{\pi}}{s_d} \frac{p_{\pi^d}^d}{[T]^2}, \tag{2}
\]

where \( m_i \) and \( p_i \) (\( i = a, b, A, B \)) are the mass and the 4-momentum of the particle \( i \), \( s_d \) is the invariant mass of the final \( N + \pi \) system and \( s = (p_a + p_A)^2 \). Further, \( \Omega_{\pi}^d \) and \( p_{\pi^d}^d \) are the solid angle and the momentum of the emitted pion in the \( N + \pi \) rest frame, while \( T \) is
the Lorentz invariant transition amplitude. \([T]^2\) means to take the sum and average of the final and initial spin states respectively.

In calculating \(\sigma\), we take into account both PDP and TDP as well as contributions from the s-wave rescattering processes. These processes are schematically shown in Fig. 1(a) and 1(b) (target and projectile \(\Delta\) excitations) and Fig. 1(c) and (d) (s-wave rescattering processes).

First, the \(\Delta\) excitation processes (for both PDP and TDP in Figs. 1(a) and 1(b) respectively) are treated by means of the transition amplitude \(\hat{t}_{NN\Delta}\) used in Ref. [11] and \(\Delta\) decay Hamiltonian. The explicit form of \(\hat{t}_{NN\Delta}\) is

\[
\hat{t}_{NN\Delta} = V_L(\hat{q} \cdot \sigma)(\hat{q} \cdot \vec{S}^\dagger) + V_T(\hat{q} \times \sigma) \cdot (\hat{q} \times \vec{S}^\dagger),
\]

(3)

where \(\hat{q}\) is the unit vector whose direction is that of the momentum transfer involved in the excitation process, \(\sigma\) is the Pauli spin operator and \(\vec{S}^\dagger\) is the spin operator for the \(N \rightarrow \Delta\) transition. \(V_L\) and \(V_T\) are strength parameters of the spin-longitudinal(LO) and spin-transverse(TR) which are used in Ref. [11]. The Hamiltonian for the \(\Delta\) decay is

\[
H_{\pi N\Delta} = \frac{f^*}{\mu} (\vec{p}_\pi \cdot \vec{S}^\dagger) T^\alpha + \text{h.c.},
\]

(4)

where \(\mu\) denotes pion mass and \(T^\alpha\) is isospin transition operator with charge \(\alpha\). For the coupling constant we take \(f^*^2/4\pi = 0.36\).

Second, the s-wave rescattering processes are calculated as in Ref. [10]. The basic couplings in this process are \(NN\pi\) coupling and \(N\pi \rightarrow N\pi\) s-wave amplitude. The \(NN\pi\) coupling is given by

\[
H_{\pi NN} = \frac{f}{\mu} (\vec{p}_\pi \cdot \vec{S}^\dagger) T^\alpha,
\]

(5)

where \(\vec{p}_\pi\) is the momentum of the pion and the coupling is given as \(f^2/4\pi = 0.08\). The Hamiltonian for the s-wave \(N\pi \rightarrow N\pi\) is given as

\[
H_{\pi\pi NN} = 4\pi\delta_{m_s m'_s} \left\{ \frac{2\lambda_1}{\mu} \delta_{m_t m'_t} \delta_{\lambda\lambda'} + i\epsilon_{\alpha\lambda\lambda'} \frac{2\lambda_2}{\mu} < m'_t | T^\alpha | m_t > \right\},
\]

(6)
where indices \( m_s, m'_s, m_t, m'_t \) in Eq. 6 are the spin and isospin variables of the incoming and outgoing nucleons. For the couplings, we take

\[
\lambda_1 = 0.0075 \quad \text{and} \quad \lambda_2 = 0.0528.
\]

The total T-amplitude can then be given as

\[
-i T = \sum_{s_1 \mu_1 s_2 \mu_2} (-1)^{\frac{1}{2} - m_a} < \frac{1}{2}, m_b; \frac{1}{2}, -m_a | s_1, \mu_1 >
\times (-1)^{\frac{1}{2} - m_A} < \frac{1}{2}, m_B; \frac{1}{2}, -m_A | s_2, \mu_2 > C_{s_1 \mu_1 s_2 \mu_2},
\]

where \((s_1, \mu_1)\) and \((s_2, \mu_2)\) represent the spin transfers involved in the \(a \rightarrow b\) and \(A \rightarrow B\) transition processes respectively. The partial amplitude \(C_{s_1 \mu_1 s_2 \mu_2}\) may be decomposed into the two contributions \(A_{s_1 \mu_1 s_2 \mu_2}\) and \(B_{s_1 \mu_1 s_2 \mu_2}\), coming from the \(\Delta\) excitation and s-wave rescattering processes respectively:

\[
C_{s_1 \mu_1 s_2 \mu_2} = A_{s_1 \mu_1 s_2 \mu_2} + B_{s_1 \mu_1 s_2 \mu_2},
\]

where

\[
A_{0000} = 0.0,
\]

\[
A_{1\mu_00} = -\frac{4}{3} f \left[ (\hat{q} \cdot \bar{p}_\pi) \hat{q}_\mu^* V_L + (\bar{p}_\pi^* - (\hat{q} \cdot \bar{p}_\pi)) \hat{q}_\mu^* V_T \right] G_t C_t,
\]

\[
A_{001\mu} = \frac{4}{3} f \left[ (\hat{q}' \cdot \bar{p}_\pi') \hat{q}_\mu^* V_L + (\bar{p}_\pi'^* - (\hat{q}' \cdot \bar{p}_\pi')) \hat{q}_\mu^* V_T \right] G_p C_p,
\]

\[
A_{1\mu_1\mu_2} = -\frac{2\sqrt{2}}{3} \frac{f}{\mu} \left[ \hat{q}_{\mu_1} (\hat{q} \cdot \bar{p}_\pi)^*_{\mu_2} (V_L - V_T) G_t C_t 
\quad + (-1)^{\mu_1} < 1, -\mu_1; 1, \nu_2 | 1, \mu_2 > \bar{p}_\pi^* V_T G_t C_t 
\quad - \hat{q}_{\mu_2}^* (\hat{q}' \cdot \bar{p}_\pi')^*_{\mu_1} (V_L - V_T) G_p C_p 
\quad + (-1)^{\mu_2} < 1, -\mu_2; 1, \nu_2 | 1, \mu_1 > \bar{p}_\pi'^* V_T G_p C_p \right]
\]

and

\[
B_{s_1 \mu_1 s_2 \mu_2} = 8\pi f \left[ -\delta_{s_1 1} \delta_{s_2 0} \hat{q}_{\mu_1}^* \sqrt{2} \lambda_+ + \delta_{s_1 0} \delta_{s_2 1} \hat{q}_{\mu_2}^* \lambda_0 \right] D_\pi F_\pi.
\]

In the above expressions, \(\lambda_0\) and \(\lambda_+\) are given as
\[ \lambda_0 = -\frac{2\sqrt{2}}{\mu} \lambda_2 , \quad \lambda_+ = \frac{2}{\mu} (\lambda_1 + \lambda_2) , \]  

(16)

\[ C_i \] is the isospin factor for \( \pi N \Delta \) vertex and index \( i \) refers both target(\( t \)) and projectile(\( p \)) \( \Delta \) excitations. The propagators and the pion form factor are defined as follows.

\[ G_i = \frac{1}{\sqrt{s_i - M_\Delta + i\Gamma(s_i)/2}} ; \quad \Delta \text{ propagator}, \]

(17)

\[ D_\pi = \frac{1}{\omega^2 - q^2 - \mu^2} ; \quad \pi \text{ propagator}, \]

(18)

\[ F_\pi = \frac{\Lambda^2 - m^2_\pi}{\Lambda^2 - t} ; \quad \pi NN \text{ form factor with } \Lambda = 1200 \text{ MeV}. \]

(19)

It is then easy to see that

\[ |T|^2 = \frac{1}{4} \sum_{s_1\mu_1 s_2\mu_2} |C_{s_1\mu_1 s_2\mu_2}|^2. \]

(20)

We note further that \( \sigma_0 \) can be evaluated by simply picking up the component with \((s_1, \mu_1) = (0, 0)\), which comes from both PDP and s-wave rescattering from the projectile. Thus, defining \( |T_0|^2 \) as

\[ |T_0|^2 = \frac{1}{4} \sum_{s_2\mu_2} |C_{0 s_2\mu_2}|^2, \]

(21)

\( \sigma_0 \) can be given as

\[ \sigma_0 = \left. \frac{d^2\sigma_0}{dE_b d\Omega_b} \right|_{\theta_b=0} = \frac{m_a m_b m_A m_B}{p_b} \int d\Omega_\pi \int_{s_d}^{2\pi} \frac{p'_d}{s_d} |T_0|^2. \]

(22)

Fig. 2(a) and 2(b) show the final results of \( \sigma \) and \( R \equiv \sigma_0/\sigma \). They are compared with the experimental data. The solid lines are our final results including both PDP and s-wave scattering, while the dotted line shown in Fig. 2(b) represents the result obtained when only the contribution from PDP is taken into account. Comparing the two theoretical cross sections in Fig. 2(b), it can be seen that the PDP dominates \( \sigma_0 \). The contribution from the s-wave rescattering process to \( \sigma_0 \) is thus small, though it helps to improve the fit of the calculated final \( \sigma_0 \) to the experimental data, particularly at the off-resonance region. The inclusive cross section data \( \sigma^{exp} \) are taken from Ref. 4, while the experimental \( R \) (\( R^{exp} \)) are
obtained by $D_{xx}$ and $D_{zz}$ of Refs. [4,5] and $\sigma^{\exp}$ of Ref. [3]. As seen in the Fig. 2(a), $\sigma^{\exp}$ is reproduced very well by the calculation. In the resonance region, the $R^{\exp}$-values are rather small; $R^{\exp} \approx 0.025$, implying that $\sigma^{\exp}$ contributes only about 2.5% to the total exclusive cross section. However, $R^{\exp}$ becomes larger at both tail regions of the resonance.

The good fit of the calculated $R$ to the data seems to support strongly that the observed $\sigma_0$ indeed comes from PDP. This conclusion is further supported by the data of $R$ for nuclear targets available for the $d$, $^{12}$C, $^{40}$Ca and $^{208}$Pb targets. For the deuteron target, the $R^{\exp}$-values are larger by about a factor of $2 \sim 4$ as compared with those of the proton target. The $R$-values for other nuclear target are about the same as those of the deuteron target. The observed increase of the $R$-values may easily be understood if one assumes that $\sigma_0$ comes from PDP. Since the dominant part of $\sigma$ comes from TDP, $\sigma$ for the deuteron target is expected to be about $4/3$ of $\sigma$ for the proton target due to isospin, while $\sigma_0$ of the deuteron should be about 4 times of $\sigma_0$ for the proton target. Thus, it is expected that the $R$-values may become larger by about a factor of 3 for the deuteron target case, as compared with the proton target case. This agrees very well with the experimental factor $2 \sim 4$. Since the ratio of the number of protons to that of neutrons contributing to the reaction may roughly stay to be unity, the $R$-value for the heavy nuclei should roughly be equal to that of the deuteron target case, which also agrees with the observation.

Finally, we remark that $\sigma_0$ may come from the TDP via the $\Delta S = 0$ interaction term involved in the $\hat{t}_{NN,\Delta N}$. Such a term has recently been determined from the analysis of the $p(p,n)\Delta^{++}$ reaction data [9]. Using the $\hat{t}_{NN,\Delta N}$ operator determined in Ref. [9], one can estimate $\sigma_0$. It has been found that both magnitude and energy dependence of $\sigma_0$ thus estimated do not fit the data very well; the magnitude is larger by about a factor of 2 than $R^{\exp}$, and also the $\omega$-dependence is quite different from what is observed. This might have been caused by the fact that the analysis made in Ref. [9] is done without taking into account PDP.

In summary, we have shown that $\sigma_0$, deduced from the data of the spin transfer data
$D_{xx}$ and $D_{zz}$ together with the inclusive cross section $\sigma$, can be well explained by the calculations that take into account PDP and the s-wave rescattering effects.

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REFERENCES

[1] G. Glass, M. Jain, M. L. Evans, J. C. Hiebert, L. C. Northcliffe, B. E. Bonner, J. E. Simmons, C. Bjork, P. Riley, and C. Cassapakis, Phys. Rev. D 15, 36 (1977)

[2] G. Glass, M. Jain, M. L. Evans, J. C. Hiebert, L. C. Northcliffe, B. E. Bonner, J. E. Simmons, C. Bjork, P. Riley, and C. Cassapakis, Phys. Lett. B 129, 27 (1983).

[3] F. Shimizu, H. Koiso, Y. Kubota, F. Sai, S. Sakamoto, and S. Yamamoto, Nucl. Phys. A389, 445 (1982).

[4] A. B. Wicklund, M. Arenton, D. Ayres, R. Diebold, E. May, L. Nodulman, J. Sauer, E. Swallow, M. Calkin, M. Corcoran, J. Hoftiezer, H. Miettinen and G. Mutchler, Phys. Rev. D 35, 2670 (1987).

[5] D. Prout, J. McClelland, E. Sugarbaker, T. Taddeucci, B. Luther, L.J. Rybarcyk, D.G. Marchlenski, S. Delucia, D. Cooper, B. Park, D.J. Mercer, X.Y. Chen, G. Gaarde, T. Sams, W.C. Sailor, T.A. Carey, R.C. Byrd, J. Rapaport, C.D. Goodman, W. Huang, Y. Yang, E. Gülmez, C.A. Whitten and W.A. Alford, RIKEN International Workshop on Delta Excitation in Nuclei, ed. H. Toki, M. Ichimura and M. Ishihara, RIKEN, May 1993 (World Scientific, Singapore)

[6] B. K. Jain and A. B. Santra, Phys. Rep. 230, 1 (1993).

B. K. Jain and A. B. Santra, Nucl. Phys. A519, 697 (1990).

[7] R. R. Silbar, R. J. Lombard, and W. M. Kloet, Nucl. Phys. A381,381 (1982).

[8] T. Mizutani, C. Fayard, G. H. Lamot, and B. Saghai, Phys. Rev. C 47, 56 (1993).

[9] L. Ray, Phys. Rev. C 49, 2109 (1994).

[10] E. Oset, E. Shiino, and H. Toki, Phys. Lett. B 224, 249 (1989).

[11] T. Udagawa, P. Oltmanns, F. Osterfeld, and S. W. Hong, Phys. Rev. C 49, 3162 (1994).
FIGURES

FIG. 1. Feynman diagrams for $p(p, n)N\pi$ reaction. Figures (a) and (b) show the p-wave interaction($\Delta$ excitations) in the target and projectile respectively, while figures (c) and (d) show s-wave rescatterings in the target and projectile.

FIG. 2. Zero degree neutron spectra for the reaction $p(p, n)N\pi$ at $E_p = 795$ MeV. The inclusive cross section (a) and the ratio $R = \sigma_0/\sigma$ (b) are shown. See text for details.
This figure "fig1-1.png" is available in "png" format from:

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