MX/G/1 retrial queue with different modes of failure

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**M/G/1 retrial queue with different modes of failure**

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**Abstract.** M/G/1 retrial queue with general service is considered. The server is prone to random breakdown of different modes. There are two phases of repair – essential and optional. The essential repair of the failed server starts after a random amount of time and varies according to the mode of failure. Optional repair is instantaneous and common to all modes of failure. After repair completion the server provides the remaining service to the interrupted customer before accepting the new customer. The stationary distributions of the server status and the number of customers in the orbit are obtained. Analytical expressions for the expected number of customers in the orbit and reliability measures are derived. Numerical illustrations are provided.

1. **Introduction**

The bulk arrival retrial queueing systems were analysed by many authors, like, Artalejo and Atencia [1], Dudin et al. [3], Falin [4] and Deepak et al. [2]. Li and et al. [11] considered repairable retrial queue. State dependent bulk arrival retrial queue with server breakdown was examined by Jain and Mishra [8] and obtained reliability indices. Jain and Bhargava [7] investigated priority repairable retrial queue.

In most of the articles, the repair of the breakdown server starts instantaneously at the epoch of failure. But in several real life situations the system has to wait for the repair to start. This time is known as setup time or delay time. Prakash Rani et al. [13] analysed retrial queueing models with server breakdown and delayed repair. Ke and Huang [9] derived system size distributions for bulk arrival queueing system with vacation, breakdown and delayed repair at random epoch, at departure epoch and at the initiation epoch of a busy period. Khalaf et al. [10] analysed bulk arrival queue with server vacation, random breakdown and delay times. Singh et al. [14] investigated repairable retrial queue with vacation. Li and Zhang [12] discussed repairable retrial queue with negative customers. Failure of a service mechanism may require several repair steps, since the failure of a component can cause a series of failures due to the difficult communications. Hsieh et al. [6] studied a queueing model with repair process occurs in two steps. Gray et al. [5] considered a different kinds of breakdown in a single repairable queueing model. In this paper bulk arrival retrial queue with different modes of failure, setup time and two stages of repair is considered.

2. **Model description**

Consider a single server retry queue where clients arrive collectively according to the combined Poisson process of rate $\lambda$. The batch size $U$ is a random variable with distribution function $P(U=k) = C_k$, $k = 1, 2, ...$ and first two moments $m_1$ and $m_2$. In an incoming batch, if the server is identified as idle, one of the batch members immediately starts the service and the remaining subscribes to the retry group and requests the service after each random time. The retrial time is generally distributed with distribution function...
A(u), Laplace Stieltjes transform \( A^*(s) \) and hazard rate function \( \eta(u) = dB(u) / (1 - B(u)) \). The service time is generally distributed with distribution function \( B(u) \). Laplace Stieltjes transform \( B^*(s) \), hazard rate function \( \mu(u) = dB(u) / (1 - B(u)) \) and \( n \)th moments \( \mu^{(n)} \), \( n \geq 1 \).

While the server is active, it is subject to one of the \( L \) types of breakdowns. It is assumed that inter-breakdown time of \( i \)th type is exponentially distributed with rate \( \omega_i \). The failed server requires first essential repair corresponding to the type of breakdown and second optional common repair. As soon as the server fails it stops serving the customers and waits for a period of time known as setup time to begin essential repair.

As soon as the essential repair is completed, the server opts for common repair with probability \( p \) or returns to serve the customers with its complementary probability \( q = 1 - p \). The setup time during \( i \)th type breakdown is generally distributed with distribution function \( F_{i3}(u) \). Laplace Stieltjes transform \( F_{i3}^*(s) \), hazard rate function \( \gamma_i(u) = dF_{i3}(u) / (1 - F_{i3}(u)) \) and \( n \)th moments \( \gamma_i^{(n)} \), \( n \geq 1 \), \( 1 \leq i \leq L \). The essential repair time of \( i \)th type (optional repair time) is generally distributed with distribution function \( F_{32}(u) (F_{32}(u)) \), Laplace Stieltjes transform \( F_{32}^*(s) \), hazard rate function \( \beta(u) = dF_{32}(u) / (1 - F_{32}(u)) \) and finite moments \( \beta^{(n)} \).

Immediately before a service failure occurs, the customer remains at the service location with probability \( r \) or is likely to be out of the service area with probability \( T \) and returning at exponentially distributed times with a rate \( \tau \). When the repair is complete, the server will either complete the rest of the service to the customer who has stopped or wait for the same customer. This waiting time is called the reserved time. Until the suspended customer leaves the system, the server will not be able to approve the new customer. If the server is busy or down, the server is blocked.

3. Analysis of steady state probabilities

Let \( U(t) \) be the number of customers in the retrial queue at time \( t \).

Define \( \{ S(t), e_1(t), e_2(t) \} \) as follows:

If \( S(t) = 0 \) then server is idle and \( e_1(t) = \) lapsed retrial time, if \( S(t) = 1 \) then server is busy and \( e_1(t) = \) lapsed service time, if \( 2 \leq S(t) \leq L+1 \) then the server with \( L \) type of different breakdown and is in setup time, \( e_1(t) = \) lapsed service time and \( e_2(t) = \) lapsed setup time, if \( L+2 \leq S(t) \leq 2L+1 \), then server is under essential repair, \( e_1(t) = \) lapsed service time and \( e_2(t) = \) lapsed repair time, if \( S(t) = 2L+3 \), \( e_1(t) = \) lapsed service time and \( e_2(t) = \) lapsed reserved time. Let \( S^*(t) = 0 \) denote interrupted customer remains in service position, \( S^*(t) = 1 \) otherwise.

The stochastic behaviour of this retrial queueing system can be described as the Markov process

\[
\{ N(t), t \geq 0 \} = \{ S(t), S^*(t), U(t), e_1(t), e_2(t) \}
\]

Define the probabilities

\[
\begin{align*}
J_0(t) &= P[S(t) = 0, U(t) = 0] \\
J_n(t, u) \, du &= P[S(t) = 0, U(t) = n, u \leq e_1(t) < u + du], n \geq 1 \\
M_n(t, u) \, du &= P[S(t) = 1, U(t) = n, u \leq e_1(t) < u + du]
\end{align*}
\]

and for \( n \geq 0 ; i = 1, 2, \ldots, L ; j = 0, 1 \)

\[
\begin{align*}
P_{1,i,i}(t, u, v) \, dv &= P[S(t) = 1 + i, S^*(t) = j, U(t) = n, u \leq e_1(t) < u + du, v \leq e_2(t) < v + dv] \\
P_{2,i,i}(t, u, v) \, dv &= P[S(t) = L+1+i, S^*(t) = j, U(t) = n, u \leq e_1(t) < u + du, v \leq e_2(t) < v + dv] \\
P_{3,i,j}(t, u, v) \, dv &= P[S(t) = 2L+2, S^*(t) = j, U(t) = n, u \leq e_1(t) < u + du, v \leq e_2(t) < v + dv] \\
R_n(t, u, v) \, dv &= P[S(t) = 2L+3, U(t) = n, u \leq e_1(t) < u + du, v \leq e_2(t) < v + dv]
\end{align*}
\]
4. Governing equations
Let $J_0$, $M_0(u)$, $P_{1,i,j,n}(u, v)$, $P_{2,i,j,n}(u, v)$, $P_{3,j,n}(u, v)$, and $R_n(u, v)$ be respectively the steady state probabilities of $I_0(t)$, $I_a(t, u)$, $M_a(t, u)$, $P_{1,i,j,n}(t, u, v)$, $P_{2,i,j,n}(t, u, v)$, $P_{3,j,n}(t, u, v)$ and $R_n(t, u, v)$ under the stability condition $\lambda \mu_1^{\mu(1)} G_1 + m_1 (1 - A^\mu(\lambda)) < 1$, where $G_1 = 1 + \sum_{i=1}^L \omega_{i} (\gamma_i^{(1)} + \beta_i^{(1)} + \rho^{(1)} + \tau / \tau)$. The steady state equations corresponding to this model are given below.

\[
\lambda J_0 = \int_0^\infty M_0(u) \mu(u) \, du \quad (1)
\]

\[
\frac{dJ_i(u)}{du} = - (\lambda + \eta(u)) \quad J_i(u), \quad n \geq 1 \quad (2)
\]

\[
\frac{dM_n(u)}{du} = - (\lambda + \mu(u) + \sum_{i=1}^L \omega_{i}) M_n(u) + \int_0^\infty P_{3,0,n}(u, v) \beta(v) \, dv
\]

\[
+ (n \sum_{i=1}^L \int_0^\infty P_{2,i,0,n}(u, v, \omega) \beta(v) \, dv + \tau \int_0^\infty R_n(u, v) \, dv
\]

\[
+ \lambda (1 - \delta_{0n}) \sum_{k=1}^n \omega_{k} M_{n-k}(u), \quad n \geq 0 ; i = 1, 2, ..., L \quad (3)
\]

\[
\frac{\partial}{\partial v} P_{1,i,j,n}(u, v) = - (\lambda + \gamma_j(v)) P_{1,i,j,n}(u, v) + \lambda (1 - \delta_{0n}) \sum_{k=1}^n \omega_{k} P_{1,i,j,n-k}(u, v),
\]

\[
n \geq 0 ; i = 1, 2, ..., L ; j = 0, 1 \quad (4)
\]

\[
\frac{\partial}{\partial v} P_{2,i,j,n}(u, v) = - (\lambda + \beta_j(v)) P_{2,i,j,n}(u, v) + \lambda (1 - \delta_{0n}) \sum_{k=1}^n \omega_{k} P_{2,i,j,n-k}(u, v),
\]

\[
n \geq 0 ; i = 1, 2, ..., L ; j = 0, 1 \quad (5)
\]

\[
\frac{\partial}{\partial v} P_{3,j,n}(u, v) = - (\lambda + \beta_j(v)) P_{3,j,n}(u, v) + \lambda (1 - \delta_{0n}) \sum_{k=1}^n \omega_{k} P_{3,j,n-k}(u, v), \quad n \geq 0; j=0,1(6)
\]

\[
\frac{\partial}{\partial v} R_n(u, v) = - (\lambda + \tau) R_n(u, v) + \lambda (1 - \delta_{0n}) \sum_{k=1}^n \omega_{k} R_{n-k}(u, v), \quad n \geq 0 \quad (7)
\]

\[
J_0(0) = \int_0^\infty M_0(u) \mu(u) \, du, \quad n \geq 1 \quad (8)
\]

\[
M_0(0) = \lambda C_1 J_0 + \int_0^\infty J_1(u) \eta(u) \, du \quad (9)
\]

\[
M_n(0) = \lambda C_{n+1} J_0 + \int_0^\infty J_{n+1}(u) \eta(u) \, du + \lambda \sum_{k=1}^n \omega_{k} \int_0^\infty J_{n-k+1}(u) \, du, \quad n \geq 1 \quad (10)
\]

\[
P_{1,i,0,n}(u, 0) = \tau \omega_i M_0(u), \quad n \geq 0 ; i = 1, 2, ..., L \quad (11)
\]

\[
P_{1,i,1,n}(u, 0) = \tau \omega_i M_0(u), \quad n \geq 0 ; i = 1, 2, ..., L \quad (12)
\]

\[
P_{2,i,j,n}(u, 0) = \int_0^\infty P_{1,i,j,n}(u, v) \gamma(v) \, dv, \quad n \geq 0 ; i = 1, 2, ..., L ; j = 0, 1 \quad (13)
\]

\[
P_{3,j,n}(u, 0) = \rho \sum_{i=1}^L \int_0^\infty P_{2,i,j,n}(u, v) \beta(v) \, dv, \quad n \geq 0 ; j = 0, 1 \quad (14)
\]

\[
R_n(u, 0) = q \sum_{i=1}^L \int_0^\infty P_{3,i,n}(u, v) \beta(v) \, dv + \int_0^\infty P_{3,1,n}(u, v) \beta(v) \, dv, \quad n \geq 0 \quad (15)
\]
Define the joint probability generating functions

\[ J(w, u) = \sum_{n=0}^{\infty} J_n(u) w^n, \quad M(w, u) = \sum_{n=0}^{\infty} M_n(u) w^n \]

\[ P_{k,i,j}(w, u, v) = \sum_{n=0}^{\infty} P_{k,i,j,n}(u, v) w^n, \quad P_{3,j}(w, u, v) = \sum_{n=0}^{\infty} P_{3,j,n}(u, v) w^n, \text{ and} \]

\[ R(w, u, v) = \sum_{n=0}^{\infty} R_n(u, v) w^n \quad \text{where} \quad i=1, 2, ..., L; \quad j = 0, 1; \quad k=1, 2 \]

Under the stability condition, the partial probability generating function of the steady state distributions of \{N(t), t \geq 0\} are obtained as

\[ J(w) = J_0[1 - A^*(\lambda)][w - C_b(w) B^*(G(h(w)))] / D_r(w) \quad (16) \]

\[ M(w) = \lambda J_0 A^*(\lambda) [1 - C_b(w)][1 - B^*(G(h(w))] / [D_r(w)G(h(w))] \quad (17) \]

\[ P_{1,i,0}(w) = r \omega J_0 A^*(\lambda) [1 - F_{1,i}^w(h(w)][1 - B^*(G(h(w))] / [D_r(w)G(h(w))], i = 1, 2, ..., L \quad (18) \]

\[ P_{1,i,1}(w) = \tilde{r} \omega J_0 A^*(\lambda) [1 - F_{1,i}^w(h(w)][1 - B^*(G(h(w))] / [D_r(w)G(h(w))], i = 1, 2, ..., L \quad (19) \]

\[ P_{3,0}(w) = \rho J_0 A^*(\lambda) [1 - F_{3}^w(h(w)][1 - B^*(G(h(w))] \sum_{i=1}^{M} \omega_i F_{3,i}^w(h(w)) F_{2,i}^w(h(w)) / [D_r(w)G(h(w))] \quad (22) \]

\[ P_{3,1}(w) = \rho \tilde{r} J_0 A^*(\lambda) [1 - F_{3}^w(h(w)][1 - B^*(G(h(w))] \sum_{i=1}^{M} \omega_i F_{3,i}^w(h(w)) F_{2,i}^w(h(w)) / [D_r(w)G(h(w))] \quad (23) \]

\[ R(w) = \tilde{r} \lambda J_0 A^*(\lambda) [1 - C_b(w)][1 - B^*(G(h(w))] [q + p F_{3}^w(h(w))] \sum_{i=1}^{M} \omega_i F_{3,i}^w(h(w)) F_{2,i}^w(h(w)) \]

\[ / [D_r(w)G(h(w))(h(w)+\tau)] \quad (24) \]

where

\[ h(w) = \lambda (1 - C_b(w)) \]

\[ J_0 = N_1 / A^*(\lambda) \quad (25) \]

\[ D_r(w) = \lambda [A^*(\lambda) + C_b(w) (1 - A^*(\lambda))] B^*(G(h(w))) - w \quad (26) \]

\[ G(u) = u + \sum_{i=1}^{M} \omega_i [1 + F_{1,i}^w(u) F_{2,i}^w(u)] [q + p F_{3}^w(u)] [ru + \tau] / [u + \tau] \quad (27) \]

\[ N_1 = \lambda m_1 \mu^{(1)} G_1 - m_2 (1 - A^*(\lambda)) \quad (28) \]

The PGF of the number of customers in the orbit is

\[ P_0(w) = J_0 + J(w) + M(w) + \sum_{j=0}^{L} \sum_{i=1}^{M} \left[ P_{1,i,j}(w) + P_{2,i,j}(w) \right] + P_{3,j}(w) + R(w) = N_1 (1 - w) / D_r(w) \quad (29) \]

The expected number of customers in the orbit is

\[ L_q = \lim_{w \to 1} P'_0(w) = N_2 / N_1 \quad (30) \]

where

\[ N_2 = \lambda m_1^2 \mu^{(1)} G_1 + m_2 / 2 \quad [1 - A^*(\lambda)] + G_2 / 2 \quad (31) \]

and

\[ G_2 = \lambda^2 m_1^2 \mu^{(2)} G_1^2 + 2 \mu^{(1)} \sum_{i=1}^{M} \omega_i \left[ \bar{\tau} / \tau + \gamma_i^{(1)} + \beta_i^{(1)} + p\beta_i^{(1)} + \gamma_i^{(1)} + \beta_i^{(1)} + p\beta_i^{(1)} / 2 \right] \]

\[ + \lambda m_2 \mu^{(1)} [1 + \sum_{i=1}^{M} \omega_i \left[ \bar{\tau} / \tau + \gamma_i^{(1)} + \beta_i^{(1)} + p\beta_i^{(1)} + \gamma_i^{(1)} p\beta_i^{(1)} + p\beta_i^{(1)} \beta_i^{(1)} \right] \quad (32) \]

The PGF of the number of customers in the system is
\[ P_s(w) = N_1 (1 - w) \frac{B'(G(h(w)))}{D_r(w)} \] (33)

The expected number of customers in the system is
\[ L_s = L_q + \lambda m_1 \mu^{(1)} G_1 \] (34)

The availability of the server in steady state is
\[
A = J_0 + \sum \int J_n(u) \, du + \sum \int M_n(u) \, du + \sum \int R_n(u, v) \, du \, dv \\
= 1 - \lambda m_1 \mu^{(1)} \sum \omega_i [Y_i^{(1)} + \beta_i^{(1)} + p \beta_i^{(1)}] 
\] (35)

Steady state Failure frequency of the server is
\[
F = \sum \omega_i \int M_n(u) \, du = \lambda m_1 \mu^{(1)} \sum \omega_i 
\] (36)

5. Numerical study

Extensive numerical work has been derived for the model under discussion by assuming retrial time, service time, setup time and repair time follow exponential distribution with parameters \( \eta, \mu, \gamma, \beta, (i = 1, 2, \ldots, L) \) and batch size follows geometric distribution with mean \( l/\sigma, \sigma \in (0, 1] \).

| Table 1 Performance Measures for various values of \( \lambda \) and \( \mu \) | Table 2 Performance Measures for different values of \( \omega, \beta \) and \( \gamma \) |
|---|---|
| \( \lambda \) | \( \mu \) | \( J_0 \) | \( L_s \) | \( \omega \) | \( \beta \) | \( \gamma \) | \( J_0 \) | \( L_q \) | \( A \) | \( F \) |
| 0.1000 | 25.0000 | 0.8086 | 0.2766 | 2 | 0.6711 | 1.2917 | 0.7486 | 0.2286 |
| 30.0000 | 0.8994 | 0.2299 | 2 | 0.7488 | 0.8011 | 0.8428 | 0.2286 |
| 35.0000 | 0.9128 | 0.1976 | 2 | 0.7643 | 0.725 | 0.84 | 0.2286 |
| 0.2000 | 25.0000 | 0.7597 | 0.6614 | 2 | 0.7488 | 0.8011 | 0.8428 | 0.2286 |
| 30.0000 | 0.7975 | 0.5355 | 2 | 0.8265 | 0.4484 | 0.901 | 0.2286 |
| 35.0000 | 0.8245 | 0.4522 | 2 | 0.8421 | 0.3948 | 0.9162 | 0.2286 |
| 0.3000 | 25.0000 | 0.6373 | 1.2190 | 2 | 0.7643 | 0.725 | 0.84 | 0.2286 |
| 30.0000 | 0.6944 | 0.9523 | 2 | 0.8421 | 0.3948 | 0.9162 | 0.2286 |
| 35.0000 | 0.7352 | 0.7859 | 2 | 0.8576 | 0.3448 | 0.9314 | 0.2286 |
| 0.4000 | 25.0000 | 0.5134 | 2.0782 | 2 | 0.1115 | 25.547 | 0.2457 | 0.6857 |
| 30.0000 | 0.5900 | 1.5413 | 2 | 0.3447 | 5.3045 | 0.4743 | 0.6857 |
| 35.0000 | 0.6448 | 1.2333 | 2 | 0.3913 | 4.2308 | 0.52 | 0.6857 |
| 0.5000 | 25.0000 | 0.3880 | 3.5384 | 6 | 0.3447 | 5.3045 | 0.4743 | 0.6857 |
| 30.0000 | 0.4844 | 2.4188 | 6 | 0.5778 | 1.7159 | 0.7029 | 0.6857 |
| 35.0000 | 0.5533 | 1.8532 | 6 | 0.6245 | 1.37 | 0.7486 | 0.6857 |
| 0.6000 | 25.0000 | 0.2611 | 6.4914 | 10 | 0.3913 | 4.2308 | 0.52 | 0.6857 |
| 30.0000 | 0.3776 | 3.8361 | 10 | 0.6245 | 1.37 | 0.7486 | 0.6857 |
| 35.0000 | 0.4608 | 2.7528 | 10 | 0.6711 | 1.0833 | 0.7943 | 0.6857 |
| 0.7000 | 25.0000 | 0.1327 | 15.3188 | | | | | |
| 30.0000 | 0.2695 | 6.4555 | | | | | |
| 35.0000 | 0.3672 | 4.1522 | | | | | |
| 0.8000 | 25.0000 | 0.0029 | 829.360 | | | | | |
| 30.0000 | 0.1602 | 12.7727 | | | | | |
| 35.0000 | 0.2725 | 6.5830 | | | | | |
Table 1 depicts the effect of $\lambda$ (arrival rate) and $\mu$ (service rate) on performance measures with input parameters $\eta = 15$, $L = 4$, $\omega_1 = 5$, $\omega_2 = 6$, $\omega_3 = 7$, $\omega_4 = 8$, $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 5$, $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 10$, $\beta = 5$, $p = r = \sigma = 0.5$ and $\tau = 5$. From the table it is clear that

- Increase in $\lambda$ decreases $I_0$ and increases other measures
- Increase in $\mu$ increases $I_0$ and decreases other measures.

For the selected parameters $\lambda = 0.5$, $\mu = 35$, $\eta = 25$, $L = 4$, $\beta = 5$, $p = r = \sigma = 0.5$ and $\tau = 5$, the comparative study of performance measures on varying failure rate $\omega (= \omega_0)$, repair rate $\beta (= \beta_0)$ and set up rate $\gamma (= \gamma_i)$ is presented in Table 2. From the table it is inferred that

- $I_0$ and $A$ increase with $\beta$ and $\gamma$ and decrease with $\omega$
- $L_0$ decreases with $\beta$ and $\gamma$ and increases with $\omega$

6. Conclusion

M$^S$/G/1 retrial queue with general service is analysed. The server is subject to random breakdown of different modes. Mandatory recovery of a failed server starts after a certain amount of time and depends on the failure mode. Optional repair is immediate and common to all modes of failure. Analytical expressions discussed in this article are analysed numerically.

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