A First Look at Preheating after Axion Monodromy Inflation

Hossein Bazrafshan Moghaddam, Robert Brandenberger
Department of Physics, McGill University, Montreal, QC H3A 2T8 Canada

We take a first look at preheating after axion monodromy inflation, assuming a standard coupling between the inflaton field and a scalar matter field. We find that in spite of the fact that the oscillation of the inflaton about the field value which minimizes the potential is anharmonic, there is nevertheless a parametric resonance instability, and we determine the Floquet exponent which describes this instability as a function of the parameters of the axion monodromy potential.

INTRODUCTION

There has been a lot of recent interest in axion monodromy inflation models (see [1] for the initial paper and [2] for a recent review). Axion monodromy models are attractive since they may provide a natural realization of large field inflation in the context of superstring theory. Large field models of inflation [3, 4] are advantageous since in such a context the slow-roll trajectory is a local attractor in initial condition space [5], even including metric fluctuations [6]. In contrast, for small field models the initial velocity of the inflaton field needs to be fine tuned, thus creating a potential initial condition problem [7]. As is well known [8], from the point of view of observations, large field models are interesting since they may lead to a significant tensor to scalar ratio.

There are challenges to obtain large field inflation models. For field values $|\phi| > m_{pl}$ corresponding to large field inflation ($m_{pl}$ is the Planck mass), there is the danger that gravitational corrections will lift the potential and prevent slow rolling of the field, unless there is a symmetry such as shift symmetry [9] which protects the small mass required for large field inflation. In string theory, there is a further challenge of obtaining large field inflation: we expect the field range of the candidate inflatons, e.g. the moduli fields or the fields associated with brane separations, to be small, and hence incompatible with large field inflation. Monodromy inflation [10] provides a possible resolution of this problem, and axion monodromy is currently regarded as the most promising implementation of the idea of monodromy inflation in the context of string theory [2]. For this reason, there has been a lot of recent activity on this topic (see e.g. [11] for a selection of recent papers).

To our knowledge there has been very little work on reheating in axion monodromy inflation. Reheating was considered in the context of the initial brane monodromy inflation model of [10] in [12]. However, in terms of reheating the axion monodromy scenario is very different. The goal of this paper is to take a first look at reheating in the axion monodromy inflation scenario, making use of a generic coupling of the inflaton field to regular matter [30]. Our goal here is to study the changes in preheating which arise due to the fact that the oscillations of the inflaton about the field value which minimizes the potential are not harmonic. It a followup paper we will consider the actual coupling of string theoretic axions to matter fields and study the possibility of preheating in this more realistic context.

Reheating is an integral part of any inflationary model. Without reheating the universe after inflation would be empty and cold. Reheating was initially discussed using first order perturbation theory [10]. In [17] and [18] it was realized that an oscillating inflaton field will induce a parametric resonance instability in fields which couple to the inflaton (and also in the equations for the inflaton fluctuations themselves). This instability was then worked out in detail in [19,21] and given the name “preheating” [31].

Reheating has been analyzed mostly for inflaton potentials of the form $V(\phi) = \frac{1}{2}m^2\phi^2$ and $V(\phi) = \lambda\phi^4$, and for a matter field $\chi$ (treated for simplicity as a scalar field) coupled to $\phi$ via an interaction Lagrangian $L_{int} \sim \phi^2\chi^2$. For both types of inflaton potential, the linearized equation of motion for fluctuations of $\chi$ leads (neglecting the expansion of the universe) to a harmonic oscillator equation with a periodically varying mass. In the first case of a massive inflaton field, the equation for Fourier modes of $\chi$ is an equation of Mathieu type [22], in the second case, the case of a massless inflaton field with quartic potential, to a generalized equation. In both cases, the equation of motion for the Fourier modes $\chi_k$ is of Floquet type, and the general theory [23] tells us that the modes will evolve as

$$\chi_k \sim P_{1,k}(t)e^{\mu_k t} + P_{2,k}(t)e^{-\mu_k t},$$

(1)

where $P_{i,k}(t) (i = 1, 2)$ are periodic functions, and $\mu_k$ is a positive semi-definite exponent called the “Floquet exponent”. Values of $k$ for which $\mu_k > 0$ form resonance bands.

Past work on reheating has shown that the efficiency of the parametric resonance instability depends strongly on the details of the model. Axion monodromy inflation is given by an inflaton potential different from the ones studied

\[\text{arXiv:1502.06135v1 [hep-th] 21 Feb 2015} \]
in previous works on reheating. As in the canonical inflation models, the inflaton will be oscillating about its ground state. However, the oscillation is not harmonic. Given the recent interest in axion monodromy inflation it is hence of great interest to study the efficiency of reheating. This is the problem we address in this paper. We will study the structure of the resonance bands and estimate the value of the Floquet exponent within the resonance bands. Our results show that the parametric instability is of broad resonance type and hence efficient. It will lead to the transfer of the inflaton energy density from the homogeneous inflaton background to matter fluctuations within less than a Hubble expansion time.

In the next section we give a brief review of axion monodromy inflation. Section 3 is the main one in which we study the equation of motion for the fluctuations of a matter field coupled to the inflaton and establish that preheating is efficient. We work in the approximation that the expansion of space is negligible, but we argue that the expansion of space does not change the conclusions. In Section 4 we show that this approximation is self-consistent and we analyze various back-reaction effects.

We use standard notation in which $a(t)$ is the cosmological scale factor, $t$ is physical time, and $H(t)$ is the Hubble expansion rate.

**AXION MONODROMY INFLATION**

The low energy limit of string theory provides many scalar fields which can be candidates for an inflaton field (see e.g. [24] for a recent comprehensive review of inflation in the context of string theory). Examples are radii of extra dimensions or distances between brane-antibrane pairs. Typically, however, the range of field values is small and does not admit super-Planck values required for large-field inflation.

Monodromy [10] is a simple solution to this problem. A string connecting brane-antibrane pairs can be wrapped many times about an internal cycle, thus yielding large field values even if the basic field range is small. As was shown in [1], suitably wrapped branes induce trans-Planckian field ranges for the associated axion fields. In string compactifications, axions arise from integrating gauge potentials over cycles in the internal manifold. The potential for the axions induced by the presence of the branes of the axion field is not periodic but grows without bound, however with a power which is typically less than 2 [32]. One example studied in [1] has a linear potential, but potentials with fractional powers $p$, e.g. $p = 2/3$, also arise (see e.g. [10]). The potentials have oscillatory terms (caused by instanton effects) and thus can give interesting observational signals in the spectrum of cosmic microwave background anisotropies (see e.g. [25]). However, for powers $p < 2$ which we consider here the oscillatory terms are negligible for field values close to the minimum of the potential (which we take to be at $\phi = 0$). It is the dynamics near $\phi = 0$ which determines the efficiency of reheating, and hence the oscillating terms in the potential will have a negligible effects on the reheating efficiency. We will hence here consider simple power law potentials.

**EVOLUTION OF FLUCTUATIONS AFTER MONODROMY INFLATION**

As discussed in previous section, we will consider the effective potential for inflaton field to be

$$V_{\text{eff}}(\phi) = m^{4-p} |\phi|^p ,$$

where $m$ is a parameter with units of mass. We will also assume a simple coupling of the form

$$\mathcal{L}_{\text{int}} = \frac{1}{2} g^2 \phi^2 \chi^2$$

between the inflaton $\phi$ and a matter field $\chi$. Such a coupling arises at tree level (but at second order in the axion coupling constant) between the axion and gauge fields (which we can then represent in a simplifying way by a scalar matter field $\chi$). Therefore in the rest of the paper we will work with the potential

$$V(\phi, \chi) = m^{4-p} |\phi|^p + \frac{1}{2} g^2 \phi^2 \chi^2 .$$

If we take the matter field $\chi$ to be massless on the scale of inflation, and if we neglect self-interactions of $\chi$, then each Fourier mode $\chi_k$ of $\chi$ will evolve independently with an equation of motion

$$\ddot{\chi}_k + 3H \dot{\chi}_k + \left( \frac{k^2}{a(t)^2} + g^2 \phi(t)^2 \right) \chi_k = 0 .$$
This differential equation has a time-dependent frequency and a gravitational damping term. The time dependence is given by evolution of both the scale factor $a(t)$ and of the background inflaton field $\phi(t)$.

Following [17], we will neglect the expansion of the universe for the moment, and we will later show that this is a self-consistent approximation during the early stages of preheating) and thus take $a(t) = 1$. Then we can rewrite Eq. (5) as

$$\ddot{\chi} + f^2(t) \chi = 0,$$

where

$$f^2(t) \equiv k^2 + \frac{g^2}{2} \phi(t)^2.$$

If $f^2(t)$ is a periodic function of time, then the equation is of Floquet type [2, 22] and we hence there is the possibility of a parametric resonance instability for the $\chi$ fluctuations. The strength of the resonance depends sensitively on the dynamics of $\phi$ near the minimum of the potential. We will consider the case in which the homogeneous value of the $\chi$ field is zero. Since the effective potential for the inflaton in this case is given by (2), the equation of motion for the background inflaton field is

$$\ddot{\phi} + 3H \dot{\phi} + pm^{4-p} |\phi|^{p-2} \phi = 0.$$

As mentioned above, we will neglect the expansion of space and later check for the self consistency of this approximation. In this case, the equation becomes

$$\ddot{\phi} + pm^{4-p} |\phi|^{p-2} \phi = 0.$$

We consider values $0 < p \leq 2$ for the exponent in the potential. In spite of the fact that this equation is nonlinear and does not describe harmonic oscillation of inflaton field, the evolution of the inflaton field is periodic (periodic motion about the minimum of the potential).

As we know from previous studies of preheating [21], the efficiency of preheating is determined by the dynamics of $\phi$ near the minimum of its potential, i.e. by the value of $\dot{\phi}$ at the time $t_c$ when $\phi = 0$. This value can be determined by energy conservation. The energy density at the end of inflation ($t = t_{\text{end}}$) is given by

$$\rho_{\text{end}} = \frac{1}{2} \dot{\phi}^2 (t_{\text{end}}) + m^{4-p} |\phi|^p (t_{\text{end}})$$

from which we can find

$$\dot{\phi}(t_c) = \sqrt{2 \rho_{\text{end}}}.$$

Inflation ends when the slow-roll conditions break down. These are

$$\frac{1}{2} \dot{\phi}^2 < V(\phi) \quad \text{and} \quad 3H \dot{\phi} < V',$$

where the prime denotes the derivative with respect to $\phi$. The first condition breaks down once

$$|\phi| = \frac{p}{\sqrt{48\pi}}.$$  

For $1 \leq p \leq 2$ the second condition breaks down for a smaller value of $|\phi|$. For $0 < p < 1$ the number $p$ in the numerator of (15) is replaced by $\sqrt{p(2-p)}$. We will denote the end point of inflation by $\phi_e$ and will write

$$\phi_e \equiv \alpha m_{\text{pl}}$$

where the value of the constant $\alpha$ can be read off from (15), and we have used $G \equiv m_{\text{pl}}^{-2}$. It is convenient to parametrize the end point of inflation in terms of a dimensionless number $\sigma$ given by

$$\sigma m \equiv \alpha m_{\text{pl}}.$$

This will simplify some of the later equations.
Now we turn to the study of $\chi$ particle production, i.e. to the study of the structure of the resonance in the Eq. (6). While the adiabaticity condition
\[
\frac{df(t)}{dt} < f(t)^2
\] (17)
is satisfied then using the WKB approximation we conclude that the state tracks the instantaneous vacuum and we find the solution of the from
\[
\chi_k = \frac{\alpha_k}{\sqrt{2f(t)}} \exp(-i \int f(t) dt) + \frac{\beta_k}{\sqrt{2f(t)}} \exp(i \int f(t) dt),
\] (18)
where $\alpha_k$ and $\beta_k$ are coefficients determined by the initial conditions.

For small field values, the adiabaticity condition (17) is violated. To estimate the range of field values for which this is the case, we can replace $\dot{\phi}(t)$ by $\dot{\phi}(t_c)$, namely
\[
\dot{\phi} \approx 2\sqrt{\frac{m^4 - p(\alpha M_{pl})^p}{2g^2 \sigma_p^2 m^2}} = 2\sigma_p^p/2m^2,
\] (19)
since for small field values $\dot{\phi}$ is constant to a good approximation. Making use of the formula (7) for $f(t)$ we see that adiabaticity violation arises for field values obeying
\[
2g^2 \sigma_p^2 m^2 \phi \geq (k^2 + g^2 \phi^2)^{3/2}.
\] (20)
Considering modes in the far infrared (i.e. setting $k = 0$) we find
\[
|\phi| \leq \sqrt{2\sigma_p^2 m} \sqrt{\frac{1}{g}}.
\] (21)
We denote the resulting value of $|\phi|$ by $\phi_{max}$. Thus we conclude that for small values of $k$ the condition for violation of adiabaticity is
\[
-\sqrt{2\sigma_p^2 m} \sqrt{\frac{1}{g}} \leq \phi \leq \sqrt{2\sigma_p^2 m} \sqrt{\frac{1}{g}}.
\] (22)

In order to estimate the energy transfer via this parametric resonance instability it is important to estimate the width of the instability band. To a first order, this is given by the range of $k$ values for which the approximation made above of neglecting $k$ in the adiabaticity condition violation calculation is self-consistent, namely value of $k$ for which $k < g\phi_{max}$. This gives
\[
k_{max} = \sqrt{2g\sigma_p^2 m}.
\] (23)
To put this value in context, note that for $p < 4$ this value is parametrically larger than $H$, which means that the modes which dominate the phase space of modes which undergo parametric resonance have wavelength smaller than the Hubble radius $H^{-1}$ at the end of inflation, and that hence neglecting metric fluctuations have a negligible effect if the wavelength is smaller than the Hubble radius. Note that even modes with larger values of $k$ feel the resonance, but for a shorter time interval, namely a time interval corresponding to
\[
|\phi| < g^{-1}k.
\] (24)

Now we return to Eq. (6) and want to find amplification of $\chi_k$ modes after passing once through the range of $\phi$ which allows resonance. Using Eq. (19) for the range (22) of field values where resonance can take place we can write (by rescaling time)
\[
\phi \approx 2\sqrt{\frac{m^4 - p(\alpha M_{pl})^p}{2g^2 \sigma_p^2 m^2}} t.
\] (25)
Rewriting Eq. (6) in terms of new time variable
\[
\tau = 2g^{1/2} \sigma_p^2 m t
\] (26)
for a wave function $\psi$ the standard methods from quantum mechanics to solve the problem. When the field is in the range (22) corresponds to values of $x$ of reheating in [21], and in the case of reheating in the presence of noise in [27]. The time range in the reheating problem when the field is in the range (22) corresponds to the usual analogy where the classical variable $R$ becomes the spatial variable in the quantum problem, the equation (27) corresponds to the time-independent Schrödinger equation

$$\nabla^2 \psi + (E - V)\psi = 0,$$

for a wave function $\psi$ in a parabolic potential $V = \tau^2$ and with energy $E$. This analogy has been used in the context of reheating in [21], and in the case of reheating in the presence of noise in [27]. The time range in the reheating problem when the field is in the range (22) corresponds to values of $x$ where quantum tunneling occurs. We will use the standard methods from quantum mechanics to solve the problem.

Well before $\phi$ enters the range (22) the solution is of the form (18), namely

$$\chi_k^0 = \frac{\alpha_k^0}{\sqrt{2f(\tau)}} \exp(-i \int f(\tau)d\tau) + \frac{\beta_k^0}{\sqrt{2f(\tau)}} \exp(i \int f(\tau)d\tau).$$

Long after it passes through that region, the solution again is of the same form, namely

$$\chi_k^1 = \frac{\alpha_k^1}{\sqrt{2f(\tau)}} \exp(-i \int f(\tau)d\tau) + \frac{\beta_k^1}{\sqrt{2f(\tau)}} \exp(i \int f(\tau)d\tau).$$

The coefficients in Eq. (30) come from initial conditions. We can find the coefficients in Eq. (31) using amplitudes of reflection $R_k$ and transmission $D_k$ for the scattering. These coefficients are given by [21]

$$R_k = -\frac{ie^{i\varphi_k}}{\sqrt{1 + e^{-\pi q^2}}},$$

and

$$D_k = \frac{e^{-i\varphi_k}}{\sqrt{1 + e^{-\pi q^2}}},$$

where

$$|R_k|^2 + |D_k|^2 = 1$$

and $\varphi_k$ is given by

$$\varphi_k = \arg \Gamma(\frac{1 + iq^2}{2}) + \frac{q^2}{2} (1 + \frac{\ln \frac{2}{q^2}}{q^2}).$$

We also note that the dependence on the phase of incoming wave during resonance the amplitude can increase or decrease. Making use of these results, and including the phase $\theta_k^0$ of the incoming wave in the calculation, we obtain the following matrix relation between the coefficients of the positive and negative frequency modes before and after the scattering:

$$\begin{pmatrix} \alpha_k^1 \\ \beta_k^1 \end{pmatrix} = \begin{pmatrix} \sqrt{1 + e^{-\pi q^2}}e^{i\varphi_k} & ie^{-(\frac{q^2}{2})}q^2 + 2i\theta_k^0 \\ -ie^{-(\frac{q^2}{2})}q^2 - 2i\theta_k^0 & \sqrt{1 + e^{-\pi q^2}}e^{-i\varphi_k} \end{pmatrix} \begin{pmatrix} \alpha_k^0 \\ \beta_k^0 \end{pmatrix}$$

Returning to our reheating problem, we see that $|\beta_k^1|^2$ gives the occupation number of particles with momentum $k$ after one passage of $\phi$ through the minimum of its potential, given that the initial occupation number is $|\beta_k^0|^2$. Considering $n_k^1 = |\beta_k^1|^2$ and the same relation for incoming particles we get

$$n_k^1 = \exp(-\pi q^2) + (1 + 2\exp(-\pi q^2))n_k^0 - 2\exp(-\frac{\pi q^2}{2})\sqrt{1 + \exp(-\pi q^2)}\sqrt{n_k^0(1 + n_k^0)} \sin(\theta_{tot}).$$
where
\[ \theta^0_{\text{tot}} = 2\theta^0_k - \varphi_k + \arg \beta^0_k - \arg \alpha^0_k. \]  
(38)
The above relation between initial number density and final number density can be applied in sequence to all time intervals during which the adiabaticity condition is violated. Therefore, for the \( j \)'th scattering event we obtain (in the case of large occupation number)
\[ n^{j+1}_k \sim (1 + 2 \exp(-\pi q^2) - 2\sin(\theta^j_{\text{tot}}) \exp(-\frac{\pi}{2} q^2) \sqrt{1 + \exp(-\pi q^2)}) n^j_k. \]  
(39)
Defining the “Floquet exponent” \( \mu^j_k \) for the \( j \)'th scattering through
\[ n^{j+1}_k = \exp(2\pi \mu^j_k) n^j_k \]  
(40)
we obtain
\[ \mu^j_k = \frac{1}{2\pi} \ln(1 + 2 \exp(-\pi q^2) - 2\sin(\theta^j_{\text{tot}}) \exp(-\frac{\pi}{2} q^2) \sqrt{1 + \exp(-\pi q^2)}). \]  
(41)
Summing over all scattering events, considering the initial conditions \( n^0_k = 0, \alpha^0_k = 1, \beta^0_k = 0 \), and taking the phases \( \theta^j_k \) to be randomly distributed we obtain
\[ n_k(t) = \frac{1}{2} \exp(2\pi \sum_j \mu^j_k) \]  
(42)
We can rewrite the exponent as
\[ \sum_j \mu^j_k = \mu_{\text{eff}} N \]  
(43)
where \( N \) is the number of oscillations of inflaton field. At a time \( t \) after the end of inflation, the number \( N \) is given by
\[ N = \frac{t}{T/2} \]  
(44)
where \( T \) is the period of inflaton field oscillations is \( T \) and we divided by two since in each period of oscillation, the inflaton field passes through the non-adiabatic region two times.

The period \( T \) can be estimated taking \( \dot{\phi} \) to be the constant value given in (19) and solving
\[ T \dot{\phi} = 4\sigma m, \]  
(45)
since one quarter of the integrated field range over one period is \( \sigma m \). Hence
\[ T \sim \frac{2\sigma^{1-\frac{2}{p}} m}{\dot{\phi}}. \]  
(46)
Therefore
\[ \sum_j \mu^j_k \sim \mu_{\text{eff}} \frac{mt}{\sigma^{1-\frac{2}{p}}}. \]  
(47)
The next step is to find the value for \( \mu_{\text{eff}} \). Considering small values for \( q \) and averaging over \( \sin \theta_{\text{tot}} \), we find from (41)
\[ \mu_{\text{eff}} \simeq \frac{1}{2\pi} \ln(3). \]  
(48)
To find the energy density \( \rho_{\chi} \) of the particles produced during preheating we need to integrate \( n_k k \) over all momenta which experience parametric resonance, i.e. with \( |k| < k_{\text{max}} \)
\[ \rho_{\chi}(t) = \int_0^{k_{\text{max}}} d^3 k' \frac{1}{2} \exp(2\pi \mu_{\text{eff}} \frac{mt}{\sigma^{1-\frac{2}{p}}}) \]  
\[ \simeq \frac{4\pi}{9} k_{\text{max}}^4 \exp(\ln(3) \frac{mt}{\sigma^{1-\frac{2}{p}}}). \]  
(49)
where \( k_{\text{max}} \) is given by (23). Inserting the value for \( k_{\text{max}} \) we find

\begin{equation}
\rho_\chi(t) \simeq \frac{16\pi}{9} g^2 m^4 \sigma^p \exp(\ln(3) \frac{2nt}{\sigma^1 - \sigma^2}) ,
\end{equation}

which is to be compared with the inflaton density at the end of inflation which is

\begin{equation}
\rho(t_e) \simeq 2m^2 \sigma^p.
\end{equation}

We conclude that provided that the resonance lasts so long that the exponential growth factor can overcome the factor \( \frac{16\pi}{9} g^2 \) by which (51) is suppressed compared to (52), then the parametric resonance instability will be sufficiently strong to drain all of the energy stored in the inflaton field at the end of inflation. Whether this is the case or not depends on how soon back-reaction effects take over and how soon the approximations we have made cease to be self-consistent.

**BACK-REACTION EFFECTS**

In the analysis of the previous section we have neglected the expansion of space. In toy models such as those studied in [20, 21] this is a self-consistent approximation. To see whether this is true in our case we need to compare the exponential increase time in \( \rho_\chi \) with the Hubble expansion time at the end of inflation. From (51) we find that the condition is only marginally satisfied. However, our analysis shows that the non-adiabatic evolution of the solution of the equation of motion for \( \chi \) takes place in narrow time intervals \( \Delta t \) when \( \phi \) is close to \( \phi = 0 \). Combining (19), (22) and the value for \( H \) given by the potential energy density at the end of inflation, i.e. at \( \phi = \phi_e \) we obtain

\begin{equation}
\frac{H^{-1}}{\Delta t} \sim \frac{m_{pl}}{m} ,
\end{equation}

and hence we conclude that the cosmological expansion does not affect our analysis.

We will now consider some back-reaction effects which could terminate or even prevent preheating. First of all, the fluctuations of \( \chi \) which are generated in the preheating instability will lead to correction terms both in the equation of motion for \( \phi \) and in that of \( \chi \). We need to determine the length of time these effects can be neglected. Secondly, fluctuations in the inflaton field themselves could be amplified and then back-react both in the equation of motion for \( \phi \), shutting off the oscillations which drive preheating, and also in the equation of motion for \( \chi \), providing corrections to the mass term which will prevent the violation of adiabaticity which is required to obtain the resonance.

We first discuss the possible amplification of fluctuations of \( \phi \). The equation of motion for the \( \delta \phi_k \) mode is

\begin{equation}
\ddot{\delta \phi_k} + 3H \dot{\delta \phi_k} + \left[ \frac{k^2}{a^2} + p(p-1)m^{4-p} \phi^{p-2} \right] \delta \phi_k = 0 .
\end{equation}

To solve this equation and study possible resonance effects, we can for the moment neglect the expansion of the universe. Since we are interested in large scale modes we can also drop the \( k^2 \) term. Rewriting the equation and multiplying both sides by \( \phi^{2-p} \) gives

\begin{equation}
\phi^{2-p} \ddot{\phi} + p(p-1)m^{4-p} \delta \phi = 0 ,
\end{equation}

which (using 15 and 16) can be re-written as

\begin{equation}
A_1 t^{2-p} \ddot{\phi} + A_2 \delta \phi = 0 ,
\end{equation}

where

\begin{align}
A_1 & \equiv \left[ 2(\alpha m_{pl})^{\frac{3}{2}} m^{4-p} \right]^{2-p} \\
A_2 & \equiv p(p-1)m^{4-p} .
\end{align}

The solutions of (56) are Bessel functions of the first and second kind and take the following form

\begin{equation}
\delta \phi = c_1 \sqrt{t} J\left( \frac{1}{p} \frac{2 \sqrt{A_2}}{A_1} m^{p/2} \right) + c_2 \sqrt{t} Y\left( \frac{1}{p} \frac{2 \sqrt{A_2}}{A_1} m^{p/2} \right) .
\end{equation}
The solutions for $\delta \phi$ have a mild time dependence (much weaker than exponential). Hence, there is no parametric resonance of $\phi$ fluctuations, and thus no important back-reaction effects of these modes.

Thus, we now turn to the discussion of the back-reaction effects of the $\chi$ fluctuations. These fluctuations will lead to an induced potential for the inflaton in Eq. (8), the magnitude of which is given by $g^2 \chi^2 \phi^2$. This term will have a major effect on inflaton dynamics when the interaction term becomes as important as the bare potential term $m^{4-p}\phi^p$. Therefore the criterion for this back-reaction effect to be negligible is

$$\chi^2_{\text{eff}} \ll \frac{m^{4-p}}{g^2} \phi^{p-2},$$

(59)

where the value of the inflaton field to be inserted is the one at the end of inflation.

Due to nonlinearities in the $\chi$ sector (which will inevitably be present due to renormalization considerations) there is a term in the potential of the form $\frac{1}{4} \lambda \chi^4$ which will add a term to the mode equation (5). In the Hartree approximation, this term will be an extra mass of magnitude $\lambda < \chi^2 >$. Computing this effective mass by averaging over the modes which undergo parametric amplification, and comparing with the amplitude of the mass term which is driving the parametric resonance, we find that the induced mass term is negligible if

$$\chi^2_{\text{eff}} \ll \frac{g^2}{\lambda} \phi^2,$$

(60)

where once again the value of $\phi$ at the end of inflation is to be used.

The observed amplitude of density fluctuations constrains the value of the coupling constant $g^2$: at one loop level the interaction between $\chi$ and $\phi$ induces a quartic self coupling term of $\phi$. The coefficient of this term is proportional to $g^4$ but it is constrained to be smaller than $10^{-12}$. Hence the constraint on $g^2$ is $g^2 < 10^{-6}$. In turn, the same loop effects induce the self coupling of $\chi$, and we expect $\lambda \sim g^4$. Since the value of $m$ is also constrained to be small from the observed amplitude of the power spectrum of density fluctuations, we expect the first criterion (59) to be more stringent than the second one (60).

To see what the criterion (59) implies, note that when $\chi^2_{\text{eff}}$ reaches the limit given by (59), then the energy density in the $g^2 \phi^2 \chi^2$ term in the Lagrangian is of comparable magnitude to the initial energy density in the inflaton field, and (for values of $p < 2$) the energy density from the nonlinear terms in the potential for $\chi$ is larger. Thus, we conclude that the back-reaction effects studied here do not terminate the preheating instability before a fraction of order 1 of the initial inflaton energy density has been drained from the coherent oscillation of $\phi$. Hence, we conclude that the parametric resonance instability is very effective.

CONCLUSION

In this paper we have studied the effects on preheating which result when considering the “non-standard” inflaton potentials arising in axion monodromy inflation. We have shown that the parametric resonance instability is expected to be very effective [34].

However, in this work we have used a standard $\phi^2 \chi^2$ coupling between the inflaton and the preheat fields. Whereas such a coupling will arise at higher orders in the coupling constant in actual axion monodromy inflation models, in such models there are other couplings which are more important. In work in progress we are studying preheating for these more realistic couplings. In these more realistic models we can then also study the possible preheating of entropy modes of the metric fluctuations. Such metric instabilities could lead to important constraints on axion monodromy inflation models [28].

ACKNOWLEDGEMENTS

We thank A. Abolhasani, Y. Cai, E. Ferreira, H. Firouzjahi, E. McDonough, G. Moore, O. Ozsoy, Y. Wang and S. Watson for useful discussions. This work was supported in part by a NSERC Discovery grant and by funds from the Canada Research Chair program (RB).

[1] L. McAllister, E. Silverstein and A. Westphal, “Gravity Waves and Linear Inflation from Axion Monodromy,” Phys. Rev. D 82, 046003 (2010) [arXiv:0808.0706 [hep-th]].
R. Easther and R. Flauger, “Planck Constraints on Monodromy Inflation,” JCAP 1402, 037 (2014) [arXiv:1308.3736 [astro-ph.CO]].

T. Kobayashi, O. Seto and Y. Yamaguchi, “Axion monodromy inflation with sinusoidal corrections,” PTEP 2014, no. 10, 103C01 (2014) [arXiv:1404.5518 [hep-ph]].

T. Higaki, T. Kobayashi, O. Seto and Y. Yamaguchi, “Axion monodromy inflation with multi-natural modulations,” JCAP10 (2014) 025 [arXiv:1405.0775 [hep-ph]].

R. Flauger, L. McAllister, E. Silverstein and A. Westphal, “Drifting Oscillations in Axion Monodromy,” [arXiv:1412.1814 [hep-th]].

Y. F. Cai, F. Chen, E. G. M. Ferreira and J. Quintin, “A new model of axion monodromy inflation and its cosmological implications,” [arXiv:1412.3298 [hep-th]].

R. H. Brandenberger, “Lectures on the theory of cosmological perturbations,” Lect. Notes Phys. 646, 127 (2004) [hep-th/0306071].

V. Zanchin, A. Maia, Jr., W. Craig and R. H. Brandenberger, “Reheating in the presence of noise,” Phys. Rev. D 57, 4651 (1998) [hep-ph/9709273].

V. Zanchin, A. Maia, Jr., W. Craig and R. H. Brandenberger, “Reheating in the presence of inhomogeneous noise,” Phys. Rev. D 60, 023505 (1999) [hep-ph/9901207].

A. D. Dolgov, A. V. Popov and A. S. Rudenko, “Shape of the inflaton potential and the efficiency of the universe heating,” [arXiv:1412.0112 [astro-ph.CO]].

B. A. Bassett, C. Gordon, R. Maartens and D. I. Kaiser, “Restoring the sting to metric preheating,” Phys. Rev. D 61, 061302 (2000) [hep-ph/9909482].

B. A. Bassett and F. Viniegra, “Massless metric preheating,” Phys. Rev. D 62, 043507 (2000) [hep-ph/9909353].

F. Finelli and R. H. Brandenberger, “Parametric amplification of metric fluctuations during reheating in two field models,” Phys. Rev. D 62, 083502 (2000) [hep-ph/0003172].

H. B. Moghaddam, R. H. Brandenberger, Y. F. Cai and E. G. M. Ferreira, “Parametric Resonance of Entropy Perturbations in Massless Preheating,” [arXiv:1409.1784] [astro-ph.CO].

For a perturbative study see [12].

Very close to the origin the inflaton potential can be quadratic [10], but that field range is negligible in terms of setting up the overall dynamics of $\dot{\phi}$. As we will see, what matters for the efficiency of preheating is the amplitude of $\dot{\phi}$ near the minimum of the potential, and this is set by the asymptotics of the potential for field values where inflation ends. We thank E. McDonough for alerting us to this issue.

The expansion of space can be taken into account using the methods of [21, 22].

While our work was being written up, a paper [28] appeared which also studied the dependence of the efficiency of preheating on the shape of the inflaton potential and concluded that changes from the canonical shape do not decrease the efficiency of the loss of energy from the inflaton condensate.