Seismicity and earthquake risk in western Sicily

P. Consentino (*)

Received on July 13th, 1978

ABSTRACT

The seismicity and the earthquake risk in Western Sicily are here evaluated on the basis of the experimental data referring to the historical and instrumentally recorded earthquakes in this area (from 1248 up to 1968), which have been thoroughly collected, analyzed, tested and normalized in order to assure the quasi-stationarity of the series of events.

The approximated magnitude values — obtained by means of a compared analysis of the magnitude and epicentral intensity values of the latest events — have allowed to study the parameters of the frequency-magnitude relation with both the classical exponential model and the truncated exponential one previously proposed by the author.

So, the basic parameters, including the maximum possible regional magnitude, have been estimated by means of different procedures, and their behaviours have been studied as functions of the threshold magnitude.

(*) Istituto di Geofisica Mineraria, University of Palermo. C.so Calatafimi, 260. 90129 Palermo (Italy).
RIASSUNTO.

Vengono calcolati, attraverso un metodo precedentemente elaborato dall’autore, i parametri utili per la valutazione della sismicità della Sicilia occidentale.

I dati presi in esame si riferiscono ai terremoti avvenuti in quest’area dal 1248 al 1968, che sono stati minuziosamente raccolti e catalogati.

Per mezzo dei valori approssimati di magnitudo (ricavati attraverso un’analisi comparata della magnitudo e dell’intensità epicentrale dei terremoti più recenti) la serie dei dati viene studiata e normalizzata per tener conto delle possibili perdite d’informazione.

Vengono quindi ricavati i parametri della relazione frequenza-magnitudo applicando sia il classico modello esponenziale, sia quello esponenziale troncato, precedentemente proposto dall’autore.

Tali parametri di base, tra i quali la magnitudo regionale massima possibile, vengono ottenuti attraverso differenti metodi di stima, e ne viene studiato l’andamento in funzione del valore di magnitudo scelto come soglia per l’utilizzazione dei dati.

Infine vengono calcolati e discussi la magnitudo regionale media, il periodo di ritorno medio ed il rischio di terremoto.

INTRODUCTION

The seismicity of Western Sicily has never been thoroughly studied mostly because of the lack of a suitable network of seismological stations in this area.

In order to remedy this situation, some contributions have been already carried out for the location of both a first framework of seismological network in the area (Cimino et al., 1974) and other geophysical equipments useful for the study of some earthquake premonitory phenomena (Cosentino et al., 1976).

Meanwhile the seismicity of the Sicilian area has been investigated in the frame of the Italian region (for instance, Caputo et al., 1973; Kárník, 1971; Shenkareva, 1971), and, in particular, some papers on the seismicity of the Sicilian region have been carried out on the basis of a statistical approach.
As a matter of fact, a number of historical and instrumentally recorded events have been thoroughly collected and analysed (Cosentino and Ficarra, 1975), the reliability of which is however limited by the unavoidable defects of homogeneity and completeness.

Nevertheless, using the collected series of data, a first statistical approach has been executed (Cosentino and Ficarra, 1974) considering all the events with magnitude $M < 4.6$. These data have been later on used (Cosentino and Enescu, 1976) for the earthquake risk evaluation as well as for an attempt of statistical forecast of the seismic activity in Sicily. Furthermore, a study about the maximum possible magnitude in Sicily has been carried out (Cosentino and Luzio, 1977) in which an attempt of regionalization has been accomplished.

In order to have a more careful evaluation of the statistical parameters and a more suitable zoning (Cosentino, 1976; idem, 1977), the available smaller events ($M > 3.0$) have been included in the temporal series of the earthquakes of Western Sicily, which is studied and statistically processed in this paper.

**EXPERIMENTAL DATA**

By means of an analytical study of the historical earthquakes reported by a number of papers (Cosentino and Ficarra, 1975), it has been possible to collect 142 events referring to Western Sicily from 1248 up to 1968.

The values of epicentral intensity (M.M. scale) have been assigned to all the events, and, by means of the following empirical relation (Cosentino and Ficarra, 1974):

\[ M = -0.45 + 2.47 \exp(0.106 I_{cp}) , \]  

[1]

the corresponding magnitude values have been obtained.
Fig. 1 - Epicenter map of the earthquakes of Western Sicily (1248-1968). The size of the small rings indicates different values of the epicentral intensity (M.M.). The cross indicates instrumental location, while the dashed outlined areas included probable locations. All the events having the same location and the same intensity are indicated with a single ring labelled with their number.
Of course, as simplifier assumption, the quasi-uniformity of the ipocentral depth has been supposed, just so as generally verified in the Western Sicily normal earthquakes.

Furthermore, the values of the energy released have been calculated using the relation (Bath, 1973):

$$\log E_s = 12.24 + 1.44 M . \quad [2]$$

All the events have been located and are presented in the epicenter map shown in Fig. 1; the location of the earliest events obviously is rather uncertain, so that a clustering of the epicenters in correspondence with the principal towns can be noticed. In the south-eastern part of the Fig. 1 the extremity of the large graben including the plastic series (Agrigento, Caltanissetta and Enna districts) can be observed, which is characterized by a very low seismicity level.

Even if in the considered series many events, especially in the low magnitude range, are missing, nevertheless the space distribution shown in the map is representative enough of the space behaviour of the seismicity in Western Sicily.

**FREQUENCY-MAGNITUDE RELATION**

The events contained in the catalogue have been processed in order to evaluate the completeness as function of time (Caputo, 1974).

The data have been grouped in 21 magnitude classes, 0.2 in width, and in 15 time periods, as presented in the matrix of Table 1.

For each class of magnitude the time stability of the frequency of occurrence has been controlled going backwards from the latest events by considering the behaviour of the slope of the frequency-time linear regression.
Matrix of the events of the catalogue grouped in classes of magnitude (1, ..., 21) and in periods of time. The first class of magnitude starts from 2.85 and all classes are 0.2 in width. The lower part of the matrix, separated by the full line, is considered complete (or quasi-complete).

\[
\begin{array}{cccccccccccccccccccc}
\Delta T & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 \\
1221-1270 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1271-1320 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1321-1370 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1371-1420 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1421-1470 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1471-1520 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1521-1570 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1571-1620 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1621-1670 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1671-1720 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1721-1770 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1771-1820 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1821-1870 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1871-1920 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1921-1970 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
Table 2

| N | \(a_e\) | \(a_{te}\) | \(b_L\) | \(b_{L.M.}\) | \(b_{M.M.}\) | \(b_{M.L.}\) | \(M.L.\) | \(M.M.\) | \(M.P.\) |
|---|---|---|---|---|---|---|---|---|---|
| \(L.S.\) | \(L.S.\) | \(L.S.\) | \(L.S.\) | \(M.M.\) | \(M.L.\) | \(M.L.\) | \(M.L.\) | \(M.L.\) | \(M.L.\) |
| 2.85 | 31.186 | 6.880 | 0.610 | 0.046±0.084 | 0.476 | 0.237 | 0.410±0.010 | 7.55 | 5.70 | 7.07 | 7.02±0.08 |
| 3.05 | 38.631 | 11.834 | 0.626 | 0.471±0.093 | 0.530 | 0.384 | 0.485±0.011 | 8.15 | 6.19 | 7.19 | 7.05±0.13 |
| 3.25 | 45.978 | 15.020 | 0.639 | 0.491±0.102 | 0.554 | 0.402 | 0.513±0.014 | 8.75 | 6.27 | 7.25 | 7.05±0.18 |
| 3.45 | 54.867 | 19.888 | 0.652 | 0.530±0.112 | 0.578 | 0.463 | 0.561±0.016 | 9.35 | 6.58 | 7.41 | 7.00±0.29 |
| 3.65 | 64.237 | 19.287 | 0.664 | 0.505±0.122 | 0.578 | 0.354 | 0.545±0.020 | 9.95 | 6.19 | 7.36 | 7.04±0.26 |
| 3.85 | 79.619 | 26.142 | 0.680 | 0.543±0.133 | 0.601 | 0.401 | 0.595±0.024 | 10.55 | 6.31 | 7.55 | 6.88±0.45 |
| 4.05 | 96.393 | 56.356 | 0.694 | 0.699±0.153 | 0.658 | imp | 0.764±0.028 | 11.15 | imp | imp | imp |
| 4.25 | 96.350 | 18.586 | 0.694 | 0.492±0.153 | 0.582 | 0.068 | 0.655±0.035 | 12.25 | 6.11 | 10.37 | imp |
| 4.45 | 139.824 | 40.749 | 0.720 | 0.636±0.174 | 0.632 | 0.463 | 0.733±0.040 | 8.05 | 6.53 | imp | imp |
| 4.65 | 159.321 | 173.406 | 0.730 | 0.899±0.207 | 0.740 | imp | imp | 11.15 | imp | imp | imp |
| 4.85 | 111.495 | 54.408 | 0.704 | 0.738±0.231 | 0.664 | 0.709 | 0.845±0.064 | 10.15 | 7.80 | imp | imp |

Table of the results obtained by the processing of the data. \(b = \beta \log\epsilon; a = \) exponential model, \(te =\) = truncated exponential model, L.S. = least squares, M.L. = maximum likelihood, M.M. = moment method. The limits indicated in the M.L. columns indicate 95% confidence intervals. imp = impossible estimation, tmv = total mean value, \(\Delta tmv = 95\%\) confidence interval of tmv, pmv = partial mean value obtained using the estimates referring to \(M_n>4, \Delta pmv = 95\%\) confidence intervals of pmv.
So, for each class a break point has been chosen where the slope of the regression line presents the minimum before the final obvious slow increasing. In Table 1 the broken line, which has been obtained by connecting the break points, separates the upper part from the lower one which can be retained complete (or quasi-complete).

For homogeneity reasons the annual frequencies of the events pertaining to the different classes have been calculated taking into account a more conservative separation marked by a full line in Table 1.

In order to study the cumulated frequency-magnitude relation, both the exponential statistical model (Gutenberg and Richter, 1944):

\[ N(M) = \alpha_{M_0} \exp \left\{ -\beta (M - M_0) \right\} \text{ for } M \leq M_0, \quad [3] \]

and the truncated exponential one (Cosentino and Luzio, 1976):

\[ N(M) = \alpha_{M_0} \frac{1 - \exp \left\{ \beta (M_0 - M_0) \right\}}{1 - \exp \left\{ \beta (M_0 - M_0) \right\}} \text{ for } M_0 \leq M < M_p, \quad [4] \]

have been used. In the relations [3] and [4] \( M_c \) is the threshold magnitude value, \( \alpha_{M_0} \) is the frequency of the earthquakes with magnitude larger than or equal to \( M_0 \), \( \beta \) is the scale parameter and \( M_p \) is the maximum possible magnitude in the considered area. Obviously, the relations [3] and [4] coincide for \( M_p = \infty \).

The parameters of the relations [3] and [4] have been estimated by means of different procedures: the maximum likelihood method (only for \( \beta \)) and the least squares method for the exponential model, and the maximum likelihood method (Cosentino et al., 1978), the moment method and the least squares method for the truncated exponential one. All the estimates, carried out for various \( M_0 \) values with the help of computer HP 9830A, are presented in Table 2, where \( b = \beta \log_{10} e \) has been reported instead of \( \beta \).
It can be noticed the high variability of the estimated values of some parameters as function of $M_a$. In particular, the values of $\alpha_0$ (Fig. 2) generally increase for larger $M_a$ values, and the largest confidence intervals (Table 2) have been obtained by means of the truncated exponential model. A trend towards the stability can be noticed for $M_a$ larger than 4.

In Fig. 3 the punctual estimated values of the parameter $b$ as function of $M_a$ are represented showing a general trend as the estimates of $\alpha_0$ do.
On the contrary the punctual estimates of $M_p$ (Fig. 4), which for some cases are impossible to be obtained, show a notable stability, except the least squares estimates the trend of which is similar to that of $\alpha_o$ and $b$.

Fig. 3 - Estimates of $b = \beta \log_{10}e$ as function of $M_a$. Squares = exponential model, Least Squares. Triangles = exponential model, Moment Method. Small dots = truncated exponential model, Maximum Likelyhood.

It is difficult to carry out a reliable estimate of the minimum magnitude value $M_p$ from which the frequency-magnitude relation shows a Poissonian behaviour (Cosentino, 1977), nevertheless from $M_o=4$ the punctual estimates of both $\alpha_o$ and $b$ show a trend towards the stability. So, in order to choice the values of the parameters, the partial mean values (pmv) of the punctual estimates ($M_o>4$) have been calculated for $\alpha_o$ and $b$. 
The selected value of $M_p$ is the mean value of all the unbiased M.L. punctual estimates which have been obtained (tmv).

Fig. 4 - Estimates of $M_p$ as function of $M_0$. Dots = Least Squares estimates, Square = Moment Method estimates, Circles = Maximum likelyhood estimates, Small dots = Unbiased Maximum Likelyhood estimates.

**Mean regional magnitude, mean return period and earthquake risk**

These statistical quantity have been calculated both with the exponential model (Epstein and Lomnitz, 1966) and the truncated exponential on (Cosentino and Luzio, 1977).

Obviously, in order to compute the values of the various quantity, for each model the basic parameters obtained by means of the same model should be used.

The basic difference is represented by the main statement
of the truncated exponential model, i.e. no one earthquake with magnitude larger than $M_p$ is possible in the region as long as the seismotectonic conditions will not change. In the exponential model there is no limit of magnitude to the possible earthquakes.

The mean regional magnitude in the truncated exponential model (te) becomes:

$$M_{te} (M_o) = M_o + 0.58 + \frac{701 - M_o}{1 - \exp (12.087 - 1.725 M_o)}$$

for $4 < M_o \leq 7.01$, \[5\]

while, in the exponential model (e), it is:

$$M_e (M_o) = M_o + 0.627 \quad \text{for} \quad M < 4 \text{ .} \quad \[6\]$$

The mean return period of the earthquakes with magnitude larger than or equal to $M$ can be evaluated, in years, by means of (te-model):

$$T_{te} (I_{ep}) = \frac{2582}{\exp (12.087 - 1.725 M) - 1} \quad \text{for} \quad 4 < M \leq 7.01 \text{ ,} \quad \[7\]$$

while in the e-model it is:

$$T_e (M) = 0.0083 \exp (1.596 M) \quad \text{for} \quad M > 4 \text{ .} \quad \[8\]$$

It can be observed that using the e-model large underestimates of the mean return periods can be achieved, especially in the large magnitude range.

An attempt to calculate the mean return period as a function of the epicentral intensity $I_{ep}$ can be performed by using the relations [1] and [7], so that it is possible to obtain:

$$T_{te} (I_{ep}) = \frac{2582}{\exp \{12.863 - 4.261 \exp (0.106 I_{ep})\} - 1} \quad \text{for} \quad 5.55 < I_{ep} \leq 10.7 \text{ .} \quad \[9\]$$
It is evident that the use of the te-model avoids the discrepancies noticed by Rothé (1976), so that a more real description of the seismicity is obtained.

Finally the earthquake risk $R_d(M)$, i.e. the probability of occurrence of at least one earthquake with magnitude larger than or equal to $M$ in a $D$ year period in Western Sicily, can be expressed in the following two different forms depending on the used model:

$$R_{wd}(M) = 1 - \exp \left\{ -68.737 D \left( \exp \left( -1.725 M \right) - 0.0000056 \right) \right\}$$

for $4 < M \leq 7.01$ , \[10\]

and

$$R_{wd}(M) = 1 - \exp \left\{ -120.677 D \exp \left( -1.596 M \right) \right\}$$

for $M > 4$ . \[11\]

The relations [10] and [11] are plotted in Fig. 5 for $D = 30$ years. In the same figure the conservative evaluations of the

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig5.png}
\caption{Earthquake risk for a 30 years period in Western Sicily. Full and broken lines refer to the truncated exponential model, dotted lines refer to exponential model. \(n\) = normal estimates, \(c\) = conservative estimates, \(vc\) = very conservative estimate.}
\end{figure}
earthquake risk (Cosentino and Luzio, 1977) are presented, obtained using as basic parameters $\alpha_0 + \Delta \alpha_0$, $\beta - \Delta \beta$, and $M_p + \Delta M_p$. Furthermore, a very conservative evaluation is obtained using, for each model, the largest obtained $\alpha_0$ and $M_p$ values and the lowest $\beta$ value.

It can be observed that the earthquake risk is overestimated if calculated by the e-model, especially in the large magnitude range, which is very important in problems of earthquake engineering.

In particular, large differences can be noticed in Fig. 5 between the values of the design magnitude $M_R$—i.e. the magnitude which is exceeded with a specified probability $R$—obtained using the two models. For instance, the earthquake risk of 5% corresponds to a design magnitude $M = 6.0$ if calculated by the te-model, and to $M = 7.0$ if calculated by the e-model.

**Conclusions**

The seismic area on which the basic statistical parameters and the connected ones have been calculated is characterized by a fairly high seismicity level, but a very poor network of geophysical equipments, so that many difficulties arise in carrying out a detailed study.

In particular, from a statistical point of view, the available series of seismic events, even if collected with a lot of efforts and researches, seems to be not complete. Nevertheless some appreciable results can be obtained on the basis of a careful statistical procedure applied to restricted and normalized data.

The obtained results would be considered rather as a kind of mean values on the whole area. In fact a more detailed zoning in this area is very difficult to be carry out on statistical basis, due to the scarcity of the events pertaining to more restricted areas.
On the other hand, as the considered area is characterized by a fairly uniform distribution of the seismicity, the values obtained by means of the relations [5], [7], [9] and [10] can be considered as a good approaching to the real values in every area of the Western Sicily, provided that the relations [7], [9] and [10] are suitably normalized for smaller areas.
REFERENCES

BATH M., 1973 - Introduction to seismology. Birkhauser Verlag, Basel.

CAPUTO M., 1974 - Analysis of seismic risk. In J. Solnes « Engineering Seismology and Earthquake Engineering », Noordnof Leiden.

CAPUTO M., KEILIS-BOROK V., KRONROD T., MOLCHAN G., PANZA G.F., PIVA A., PODAEZKJA V., POSTPISCHL D., 1973, - Models of earthquake occurrence and isoseismals in Italy. « Ann. Geof. », 26, 23, pp. 421-444.

CIMINO A., COSENTINO P., GIUNTA G., LIGUORI V., 1974 - Studio preliminare per il controllo sistematico dell'attività sismica nella Sicilia occidentale. « Riv. Min. Sic. », 25, 145-147, pp. 62-70.

COSENTINO P., 1976 - Difficulties and related criticism in applying the Gutenberg and Richter relation to the seismic regions in statistical seismology. « Boll. Geof. Teor. Appl. », 70, pp. 79-91.

COSENTINO P., 1977 - Frequency-magnitude statistical parameters with reference to some zoning problems. Internat. School of Appl. Geophys., Erice, Trapani (Italy).

COSENTINO P., FICARRA V., 1974 - I terremoti in Sicilia dall'anno 1000 al 1968: uno studio analitico e statistico. « Ann. Geof. », 27, 3-4, pp. 517-537.

COSENTINO P., FICARRA V., 1975 - Catalogo dei terremoti più intensi avvenuti nell'area Siciliana dall'anno 1000 al 1968. « Lav. Ist. Geof. Min. », Palermo Univ.

COSENTINO P., LUZIO D., 1976 - A generalization of the frequency-magnitude relation in the hypothesis of a maximum regional magnitude. « Ann. Geof. », 29, 1-2, pp. 3-8.

COSENTINO P., LUZIO D., 1977 - Truncated exponential frequency-magnitude relation applied to the Sicilian earthquakes. Publ. « Inst. Geoph. Pol. Acad. Sc. », A-5 (116), pp. 211-220.

COSENTINO P., LUZIO D., 1977 - Earthquake risk and connected statistical parameters in the truncated exponential model. « Riv. It. Geof. SS. AA. », 34, pp. 119-121.
COSENTINO P., LUZIO D., RIGGIO A.M., 1976 - Geomagnetic survey for siting a magnetic observatory in Western Sicily. «Boll. Geof. Teor. Appl.», 71, pp. 143-150.

COSENTINO P., LUZIO D., RIGGIO A.M., 1978 - Seismicity and earthquake risk in the Balkan region. Proceedings of the Symposium on the Analysis of Seismicity and on Seismic Risk (October 1977, Liblice), Academia, Praga.

EPSTEIN B., LOMNITZ C., 1966 - A model for the occurrence of large earthquakes. «Nature», 211, 5052, pp. 954-956.

GUTENBERG B., RICHTER C.F., 1944 - Frequency of earthquakes in California. «Bull. Seism. Soc. Am.», 34, pp. 185-188.

Kárník V., 1971 - Seismicity of the European area., Vol. I (1969) and II. Reidel Pub. Co., Dordrecht, Holland.

ROTHIE J.P. 1976 - Le recherches de séismicité historique et la magnitude maximale à envisager en un site donné. Proc. of Symp. on earthquake risk for nuclear power plants, edited by A.R. Ritsema, De Bilt, Nederland.

SHENKAREVA G.A., 1971 - Seismicity of Italy. «Boll. Geof. Teor. Appl.», 51-52, pp. 271-297.