Dall’Ava, Luca

Approximations of the balanced triple product $p$-adic $L$-function. (English) Zbl 07650798
J. Number Theory 246, 189-226 (2023)

Let $O$ be a valuation ring finite flat over $\mathbb{Z}_p$. Let $I$ be a normal domain finite flat over the Iwasawa algebra $\Lambda = O[[\Gamma]]$ of the topological group $\Gamma = 1 + p\mathbb{Z}_p$, and let $X^+_{\Gamma}$ be the set of arithmetic points of $I$. Let $f_i$ $(i = 1, 2, 3)$ be a primitive cuspidal family of the tame conductors. Let $\rho_f$ Gal$(\overline{\mathbb{Q}}/\mathbb{Q}) \to \text{GL}_2(\text{Frac} I)$ be the big Galois representation associated to $\rho_f$, and let $V_f$ denote the natural realization of $\rho_f$ inside the étale cohomology groups of modular curves. Let $R = \mathbb{I}(\mathbb{O}(\mathbb{O} I$, and

$$X^+_R = \left\{ Q = (Q_1, Q_2, Q_3) \in (X^+_{\Gamma})^3 \mid k_{Q_1} + k_{Q_2} + k_{Q_3} \equiv 0 \pmod{2} \right\}.$$

Put

$$X^f_r = \left\{ Q = (Q_1, Q_2, Q_3) \in X^+_R \mid k_{Q_1} + k_{Q_2} + k_{Q_3} \leq 2k_{Q_i} \right\},$$

and

$$X^{bal}_r = \left\{ Q = (Q_1, Q_2, Q_3) \in X^+_R \mid k_{Q_1} + k_{Q_2} + k_{Q_3} > 2k_{Q_i} \right\}.$$

Let $\mathbb{V} = V_1 \otimes V_2 \otimes V_3$, and for $Q = (Q_1, Q_2, Q_3) \in X^+_R$ put $V_{Q_i} = V_{1, Q_1} \otimes V_{2, Q_2} \otimes V_{3, Q_3}$. Let $X$ be a $\mathbb{R}$-adic $p$-ramified Galois character such that $X(\epsilon) = (-1)^n$ with $\epsilon$ the complex conjugate and let $V_\epsilon = V \otimes X^{-1}$. Let $V_{Q_1}$ be its specialization for $Q \in X^+_R$, and let $L(V_{Q_1}, s)$ be the complete $L$-function attached to $V_{Q_1}$. The $p$-adic $L$-function $L_{p}^{bal}[f]$ interpolates $L(V_{Q_1}, s)$ for $Q \in X^{bal}_R$. In the paper under review, the author provides an algorithm for approximating the value of $L_{p}^{bal}[f]$ at $(2, 1, 1)$. Actually, the $p$-adic $L$-function is constructed as the limit of certain theta-elements described explicitly in Proposition 4.9 and the author provides an algorithm for computing their values when evaluated at a triple of arithmetic points of the form $(2, (2, \epsilon), (2, \epsilon))$, for $\epsilon$ a primitive $p$-adic character of conductor $p^n$. This allows to approximate the value $L_{p}^{bal}[f](2, 1, 1)$ as the limit over the increasing conductor $p^n$ of such theta-elements evaluated at $(2, (2, \epsilon), (2, \epsilon))$.

Reviewer: Sami Omar (Sukhair)

MSC:
11F67 Special values of automorphic $L$-series, periods of automorphic forms, cohomology, modular symbols
11R52 Quaternion and other division algebras: arithmetic, zeta functions
11Y16 Number-theoretic algorithms; complexity

Keywords:
triple product $p$-adic $L$-function; quaternionic modular forms; finite-length geodesics; Bruhat-Tits tree

Full Text: DOI arXiv

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Wiebe, Jordan, Constructing non-maximal orders in quaternion algebras (2018)

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