Method for solving problems of the theory restrictions of infocommunication systems using linear equations with many unknowns

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Abstract. The article presents a method for solving problems of theory of constraint in the form of linear equations with many unknowns encountered in practice when managing the life cycle of engineering products. The development of any type of infocommunication systems may include mathematical modeling of current processes based on a system of linear equations. The use of the method is meant for automating the process of substantiating the decisions taken at the stage of preparation for production and managing the product life cycle when managing dimensional, temporary and economic relations. The example of the first problem describes the key aspects of pricing and remuneration taking into account the productivity of workers. The second problem is to determine the economically achievable tolerance of the closing link of a dimension chain. The third problem describes the aspects of the enterprise load, taking into account mandatory contributions. These problems allow us to find a rational solution with a limited number of input data. The presented problems reveal a method for solving the problems of the theory of constraint and the prospects of using equations for practice in managing the life cycle and building automated systems.

1. Introduction

An analysis of studies in the field of solving linear equations with many unknowns revealed that the authors use various search methods and create systems for finding unknowns [1-11]. To solve the problems of forestry management, the use of infocommunication systems based on mathematical models and theory of constraints represents an interesting scientific task. The creation of infocommunication systems requires new approaches in the field of mathematical modeling and solving problems based on linear equations with many unknowns. It is established that such equations have many solutions. For example, one equation with four unknowns has infinitely many solutions, and it is possible to give arbitrary values to three unknowns, and two equations with four unknowns have infinitely many solutions, and arbitrary values can be given to two unknowns. For three equations with four unknowns, there are infinitely many solutions, and arbitrary values can be given to one unknown, and four equations with four unknowns have only one solution.
2. Methods and Materials

In this paper, we present a technique with which it is possible to limit the options for finding arbitrary unknowns by an algorithm using probability theory for systems of the theory of constraints.

Types of linear equations with many unknowns can be classified as follows:

The first type with many positive coefficients \( K \) for unknown \( X \) and a free term \( D \):

\[
K_1X_1 + K_2X_2 + \ldots + KnXn = D
\]

(1)

The second type with positive and negative coefficients \( K \):

\[
-K_1X_1 - K_2X_2 - \ldots - KnXn = D
\]

(2)

The third type with positive and negative coefficients \( K \) and a free term \( D = 0 \):

\[
K_1X_1 - K_2X_2 - \ldots - KnXn = D
\]

(3)

The equations are attractive in that a lot of practical problems are presented with their help. We formulate a methodology for solving using equations on the following applied problems.

2.1. The solution to the first problem

The team consists of three workers. Workers have a \( Ki \) ratio of labor productivity [units / shift]. The task is to distribute a cash reward of 600 rubles to the brigade.

Decision:

Draw up an equation (see formula 1) with data, where the coefficients denote productivity (rubles):

\[
3X_1 + 2X_2 + 1X_3 = 600
\]

(4)

We transform the equation by dividing the coefficients by their sum \( \sum_{i=1}^{n} K_i = 6 \), as a result we get (rubles):

\[
\frac{3}{6}X_1 + \frac{2}{6}X_2 + \frac{1}{6}X_3 = 100 \text{ or } 0.5X_1 + 0.34X_2 + 0.16X_3 = 100
\]

(5)

Equation 4 is the mathematical expectation of a random discrete quantity.

\[
M(x) = m_x
\]

(6)

Representation of the equation in the form of mathematical expectation is the main innovation of the technique.

Assuming the distribution of the random variable to be uniform, we construct a curve (figure 1). A graphical representation of the distribution of a random variable is the second innovation of the method.

From the graph we make up the ratio and find random variables (rubles):

\[
X_1 = 100; X_2 = \frac{0.34}{0.5} \cdot 100 = 68; X_3 = \frac{0.16}{0.5} \cdot 100 = 32
\]

(7)

Substituting \( Xi \) in equation (5) we obtain (rub.):

\[
0.5 \cdot 100 + 0.34 \cdot 68 + 0.16 \cdot 32 = 78.24
\]

(8)

Equality of equations (4) and equation (6) is not satisfied. In order for equality to be fulfilled, we apply two methods.

2.1.1. The first variant. Correction or scaling is found [1]:

\[
P = \frac{100}{78.28} = 1.28
\]

(9)

Multiplying \( Xi \) by the correction, we get 1 solution (rub.): \( X_1 = 128; X_2 = 87; X_3 = 49 \);
2.1.2. The second variant. Using two arbitrary values of $X_i$ from the relations, and the third unknown term of the equation we obtain by calculation (rubles):

$$0.5 \cdot X_1 + 0.34 \cdot 68 + 0.16 \cdot 32 = 100, X_1 = 143.5,$$

$$0.5 \cdot 100 + 0.34 \cdot X_2 + 0.16 \cdot 32 = 100, X_2 = 132,$$

$$0.5 \cdot 100 + 0.34 \cdot 68 + 0.16 \cdot X_3 = 100, X_3 = 168,$$

The calculation results of all options (rubles):

$$X_1 = 128; X_2 = 87; X_3 = 49,$$

$$X_1 = 143; X_2 = 68; X_3 = 32,$$

$$X_1 = 100; X_2 = 132; X_3 = 32,$$

$$X_1 = 100; X_2 = 68; X_3 = 168,$$

For a uniform distribution of money between workers, the first option is chosen.

Analysis of the solutions showed that the best option is option 16, since in this option the minimum number of grams. The (13) and (14) options with negative decisions are not accepted.

2.2. The solution to the second problem

Distribute the economically achievable tolerance of the closing link $T\Delta$ into the component links depending on the size of the nominal size (figure 2).

The general tolerance formula for the closing link, calculated by the method of equal tolerances, has the form (17):

$$TA\Delta = \sum_{i}^{n-1} TA_i = \sum_{i}^{n-1} K_i \cdot X_i \text{ or } TA\Delta = \sum_{i}^{n-1} 0.45 \sqrt{A_i} + 0.001A_i \cdot a_i,$$

(17)
where, \( a_i \) is the number of tolerance units of the component link an unknown quantity, in the equation \( X_i = a_i \), and the value \( K_i = \sum_{l}^{n-1} 0.455^l A_l + 0.001 A_l \) - tolerance unit, characterized by the value of the nominal links of the dimensional chain. Numerical equations (18, 19):
\[
4X_1 + 3X_2 + 2X_3 + 1X_4 = 100, \quad (18)
\]
\[
0.4X_1 + 0.3X_2 + 0.2X_3 + 0.1X_4 = 10, \quad (19)
\]
To find the unknowns, we construct a graph, see figure 3.

![Graph of the distribution function of a discrete random variable with 4 unknowns.](image)

On the graph there are two abscissa axes of different mathematical expectations 10 and 0. From the similarity of the triangles of the graph we find the relations:
\[
X_1 = 0.4/0.5 \cdot 10 = 8; X_2 = 0.2/0.5 \cdot 10 = 4, \quad (20)
\]
\[
X_3 = 0.3/0.5 \cdot 10 = 6, X_4 = 0.1/0.5 \cdot 10 = 2, \quad (21)
\]
Substituting in equation (19) we get:
\[
0.4 \cdot 8 + 0.3 \cdot 6 + 0.2 \cdot 4 + 0.1 \cdot 2 = 6, \quad (22)
\]
To fulfill the equality, we find solutions in 2 variants:
\[
0.4 \cdot X_1 + 0.3 \cdot 6 + 0.2 \cdot 4 + 0.1 \cdot 2 = 10; X_1 = 18, \quad (23)
\]
\[
0.4 \cdot 8 + 0.3 \cdot X_2 + 0.2 \cdot 4 + 0.1 \cdot 2 = 10; X_2 = 19.3, \quad (24)
\]
\[
0.4 \cdot 8 + 0.3 \cdot 6 + 0.2 \cdot 4 + 0.1 \cdot 2 = 10; X_3 = 24, \quad (25)
\]
\[
0.4 \cdot 8 + 0.3 \cdot 6 + 0.2 \cdot 4 + 0.1 \cdot X_4 = 10; X_4 = 42, \quad (26)
\]
Results of decisions of all options:
\[
X_1 = 18; X_2 = 6; X_3 = 4; X_4 = 2, \quad (27)
\]
\[
X_1 = 8; X_2 = 19.3; X_3 = 4; X_4 = 2, \quad (28)
\]
\[
X_1 = 8; X_2 = 6; X_3 = 24; X_4 = 2, \quad (29)
\]
\[
X_1 = 8; X_2 = 6; X_3 = 4; X_4 = 42, \quad (30)
\]
Of the four solutions for problem 2, we accept (27) option. Larger nominal size - greater tolerance.

The third type of tasks is divided into 2 types: 1-with positive coefficients; 2-with positive and negative. The complexity of solving this problem lies in the fact that \( D = 0 \) and the first correction method is not applicable.
2.3. The solution to the third problem
Consider an example of the task of ensuring the production cycle, taking into account mandatory contributions. It is necessary to determine how many kilograms of products need to be sold and purchased by the company so that 100 units remain after the trade.

We compose the equation and make the transformations (31,32):

\[
4X_1 + 3X_2 - 2X_3 - 1X_4 = 100, \tag{31}
\]
\[
0.4X_1 + 0.3X_2 - 0.2X_3 - 0.1X_4 = 10, \tag{32}
\]

In equation (32), the coefficients show the cost

From the likeness of the triangles of the graph in Figure 3, we find the relations:

\[
X_1 = \frac{0.4}{0.5} \cdot 10 = 8; \quad X_2 = \frac{0.2}{0.5} \cdot 10 = 4, \tag{33}
\]
\[
X_3 = \frac{0.3}{0.5} \cdot 10 = 6; \quad X_4 = \frac{0.1}{0.5} \cdot 10 = 2, \tag{34}
\]

To determine the solutions, we use the 2 method:

\[
0.4X_1 + 0.3 \cdot 6 - 0.2 \cdot 4 - 0.1 \cdot 2 = 10; \quad X_1 = 23 \tag{35}
\]
\[
0.4 \cdot 8 + 0.3 \cdot X_2 - 0.2 \cdot 4 - 0.1 \cdot 2 = 10; \quad X_2 = 26 \tag{36}
\]
\[
0.4 \cdot 8 + 0.3 \cdot 6 - 0.2 \cdot X_3 - 0.1 \cdot 2 = 10; \quad X_3 = -26 \tag{37}
\]
\[
0.4 \cdot 8 + 0.3 \cdot 6 - 0.2 \cdot 4 - 0.1 \cdot X_4 = 10; \quad X_4 = -58 \tag{38}
\]

Results of decisions of all options:

\[
X_1 = 21; \quad X_2 = 6; \quad X_3 = 4; \quad X_4 = 2, \tag{39}
\]
\[
X_1 = 8; \quad X_2 = 26; \quad X_3 = 4; \quad X_4 = 2, \tag{40}
\]
\[
X_1 = 8; \quad X_2 = 6; \quad X_3 = -26; \quad X_4 = 2, \tag{41}
\]
\[
X_1 = 8; \quad X_2 = 6; \quad X_3 = 4; \quad X_4 = -58, \tag{42}
\]

The best option for problem (3) is 39 solution that reflects minimal costs.

3. Results and Discussion
Many solutions to the equation of the theory of restriction of systems with many unknowns can be limited by presenting the equation in the form of a mathematical expectation of a discrete quantity and a strict solution algorithm. This part presents three tasks revealing the proposed method for solving and the prospects for using the equations for practice in managing the life cycle and building information and communication systems. Building high-quality infocommunication systems, based on mathematical methods for solving problems in the forest sector, can improve the quality of information support for forestry.

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