On the Clock of the Combinatorial Clock Auction*

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Abstract

The Combinatorial Clock Auction (CCA) has frequently been used in recent spectrum auctions. It combines a dynamic clock phase and a one-off supplementary round. The winning allocation and the corresponding prices are determined by the VCG rules. These rules should encourage truthful bidding, whereas the clock phase is intended to reveal information. We inquire into the role of the clock when bidders have lexicographic preferences for raising rivals’ costs. We show that in an efficient equilibrium the clock cannot fully reveal bidders’ types. In the spirit of the ratchet effect, competitors will extract surplus from strong bidders whose type is revealed. We also show that if there is substantial room for information revelation, that is, if the uncertainty about the final allocation is large, all equilibria are inefficient. Qualitative features of our equilibria are in line with evidence concerning bidding behavior in some recent CCAs.

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1 Introduction

In recent years, many regulators around the world have chosen the Combinatorial Clock Auction (CCA) to allocate telecommunication spectrum. The CCA has partially replaced the older Simultaneous Ascending Auction (SAA) for two reasons. First, in the SAA bidders may have an incentive to strategically reduce demand to establish a “reasonable” allocation at low prices. The sophisticated design of the CCA should overcome this issue as it incorporates (i) a generalized second-price (Vickrey) rule providing bidders with an incentive to bid truthfully (Cramton, 2013), while (ii) a clock phase should facilitate “price and package discovery” (Ausubel, Cramton and Milgrom, 2006). Second, in contrast to the SAA, bidders can express bids for packages in the CCA. Package bidding is deemed to be important as current spectrum auctions allocate multiple units where bidders may value the units as complements. If that is the case the SAA, but not the CCA, suffers from the well-known exposure problem. The focus of this paper is the first issue: is it the case that the CCA provides bidders with an incentive to bid truthfully and that the clock phase facilitates “price and package discovery”? 

The CCA is a dynamic version of the Vickrey-Clarke-Groves (VCG) mechanism and consists of two integrated phases. In the first clock phase bidders express their demand on packages at given prices in every round. If for a certain good demand is larger than supply in a given round, then the price for that good is increased in the next round. The clock phase ends when demand is not larger than supply for all the auctioned goods. Importantly, no goods are allocated and no prices are determined at the end of the clock phase. Instead, the clock phase imposes constraints on the bids in the second supplementary phase. In that phase, bidders can bid on as many additional packages as they like and they may raise bids on packages they have bid on in the clock phase. At the end of the supplementary phase, goods are allocated, and prices are determined. The regulator uses all the bids from the clock phase and the supplementary phase to determine the value-maximizing combination of bids. The auctioneer uses the Vickrey-pricing rule to determine the prices winners have to pay (e.g., Milgrom, 2004).

Without the clock phase, the CCA reduces to the VCG mechanism. As the number of packages is an exponential function of the number of commodities, bidders in a VCG auction may need to consider bidding on a vast number of packages. In particular, if the uncertainty concerning competitors is large, bidders

\footnote{In practice, there is a third phase—the assignment phase. In this phase, generic packages are allocated. We abstract away from this phase since it does not affect our analysis.}
may have a fairly limited idea about the package they may eventually win and at which price. The clock phase is meant to reveal this kind of information. Bidders can then focus their bidding in the supplementary round on the packages that may still be winning.

Under standard preferences, truthful bidding in the clock and supplementary phase is indeed an equilibrium. If bidders bid truthfully the outcome is efficient. However, truthful bidding is not a strict equilibrium as bidders may be indifferent across all their permissible bids in the supplementary round (Levin and Skrzypacz, 2016). To eliminate the payment relevant indifferences we consider bidders that ceteris paribus prefer outcomes where competitors pay more. We model this objective as a secondary dimension in a lexicographic way.

Our first main result is that an efficient equilibrium, where the clock phase fully reveals bidders’ types, does not exist if bidders have a lexicographic preference for raising rivals’ costs. This result implies that the CCA exhibits a fundamental trade-off between efficiency and information revelation in the clock phase. The trade-off follows from the fact that if bidders bid truthfully in the clock phase, the clock reveals information about the bidders’ types. Bidders would like to use this information to maximally raise the rival’s cost by placing bids in the supplementary phase on large packages that they know cannot be winning. The stronger their competitor, the more they can raise their price. The rules of the CCA are such that bidders are only able to raise the rival’s cost if they expand demand in the clock phase as this relaxes the constraints on the supplementary phase bids. Predicting that the clock phase eventually will fully reveal information, bidders can expand demand in the early phase of the clock without the risk of affecting the final allocation. Knowing that the competitor is able and inclined to raise their cost if their types are fully revealed, stronger bidders have an incentive to pool with weaker types in the clock phase.

This result is best understood from the perspective of the ratchet effect from the dynamic principal-agent literature (Laffont and Tirole, 1988). In that literature, an agent may have an incentive not to reveal his type to a principal if the principal could use that information to extract more surplus from the agent in future interactions. In our case, knowing the competitor is strong, a bidder (by bidding more aggressively in the supplementary phase) may increase the price the competitor has to pay beyond what they would do if the competitor’s type were unknown. Rationally anticipating this exploitation, stronger bidders prefer to pool with weaker types. The intuition for our first main result differs in two dimensions from the traditional ratchet effect. First, unlike the principal-agent
model, the roles of bidders in an auction are symmetric to one another so that each bidder is both the object of and the initiator of surplus extraction. Second, the extent to which bidders can raise the rival’s cost in the supplementary round is not exogenously given, but endogenously determined by their behavior in the clock phase. Thus, bidders will only be able to raise the rival’s cost if they expand demand in the clock phase.

The result that fully revealing efficient equilibria do not exist does not rule out the existence of efficient equilibria. Even with lexicographic preferences for raising rival’s cost efficient equilibria can exist. We present examples of efficient equilibria. In any of these equilibria, to be able to raise rival’s cost, bidders need to relax the constraints their clock phase bidding imposes on the bids they are permitted to place in the supplementary phase. They will do so by demanding the full supply (even if prices are such that truthful bidding would tell them to demand less). The demand expansion ends with a sudden switch to truthful bidding. In one type of equilibrium, if the ex ante uncertainty concerning the rival’s type is relatively small, then there exists a range of prices such that the truthful demand of weak bidders is positive at these prices, while the truthful demand of strong bidders is smaller than their proportional share of the available supply. Thus, when all bidders drop demand the clock stops immediately. In such an equilibrium, there is no price or package discovery whatsoever. This clock phase development allows all bidders to bid their true marginal values in the supplementary round. As a result, the final allocation is efficient. We show that any efficient equilibrium of the CCA involves demand expansion in the clock phase.

Our second main result is that if the uncertainty concerning the competitor’s type is sufficiently large, all equilibria of the CCA are inefficient. Efficiency requires that at relatively high clock prices, a weak bidder drops demand in the clock phase such that if both bidders are relatively weak, the clock phase ends. However, if a bidder turns out to be relatively strong, the clock phase continues as this bidder needs to express a very high demand in an efficient equilibrium. A relatively weak bidder then infers from the continuation of the clock that the competitor is relatively strong. This learning effect creates the opportunity for a relatively weak bidder to make the supplementary round behavior conditional on the price at which the clock phase stops. Knowing the competitor is strong, the weak bidder can raise the rival’s cost more without running the risk of winning an inferior share. Consequently, a strong bidder has an incentive to obfuscate his

\[ \text{This is in line with, for example, the Austrian 2013 auction where (as we mention below) bidders were bidding very aggressively in the clock phase.} \]
type and does this by reducing demand towards the end of the clock phase. This
demand reduction rules out expressing true marginal values for all shares in the
supplementary round, which leads to inefficient final allocations. As we also show
that the static VCG mechanism always has efficient equilibria, we claim that it is
the clock phase that creates this inefficiency.

Ausubel (2004, p. 1452) has stated that “The auctions literature has provided
us with two fundamental prescriptions guiding effective auction design”: first,
“the winner’s price should depend solely on opposing participants’ bids—as in
the sealed-bid, second-price auction—so that each participant has full incentive to
reveal truthfully her value for the good. Second, an auction should be structured
in an open fashion that maximizes the information made available to each partic-
ipant at the time she places her bids”. Our results show that following these two
prescriptions can be at the expense of generating efficient outcomes in multi-unit
auctions where bidders have a weak incentive to raise the rival’s cost. If efficiency
is preserved, then the information that is revealed through the open format is
fairly limited.

The lexicographic modeling of bidders’ preference for raising the rivals’ costs
implies that if two bidding strategies yield the same expected surplus to a bidder,
the bidder chooses the strategy where the rival pays more.\(^3\) The raising rivals’
costs motive may stem from (i) principal-agent issues within a firm (bidder)\(^4\) or
from (ii) the fact that (in spectrum auctions) bidders face weaker competitors in
the market after an auction if competitors have paid more for their licenses. If
firm A makes B pay more for spectrum, B’s credit rating will fall, and its cost of
capital will go up, weakening its strategic position. Milgrom (2004) and Cramton
and Ockenfels (2014) mention fairness as a reason for why bidders may want to
raise rivals’ costs. Of course, if bidders repeatedly interact in the same market,
bidders may be able to collude as all may realize they are better off if they do not
raise each others’ cost. In that case, it is more likely bidders behave to reduce
(rather than to raise) rivals’ costs.

The raising rivals’ costs motive has become a concern in designing auctions.\(^5\)

\(^3\)The analysis with lexicographic preferences provides a robustness check on the equilibria
under standard preferences: equilibria under our preferences are also equilibria under standard
preferences, but the reverse does not necessarily hold true.

\(^4\)In spectrum auctions, given the considerable uncertainty concerning future technological
developments and uptake of data services, it is difficult for bidders to evaluate what the spec-
trum is worth. Valuations are highly subjective. Accordingly, if a bidder wants to have a more
objective evaluation measure of its bidding team’s performance, it might be better to evalu-
ate performance relative to other bidders, than relative to the uncertain and subjective own
valuation.

\(^5\)See, e.g., (i) BAKOM (2012) on the outcome of the Swiss auction and subsequent discussion
After the 2013 auction, the Austrian regulator RTR attributed the high revenue to overly aggressive behavior by bidders: during the clock phase, bidders were bidding very aggressively, and the majority of the supplementary bids were on very large packages that had a low probability of winning but played a crucial role in determining other bidders’ prices. The fact that payments in the Austrian auction were essentially the same as the final clock prices is a clear signal of aggressive bidding, as with Vickrey pricing and “downward sloping demand” one would not expect marginal and average prices to be identical. The observed behavior, however, is reminiscent of the equilibria we describe. Moreover, in a consultation document on the award of the 2.3 GHz and 3.4 GHz bands, the British regulator Ofcom (2014, p. 38, 6.73-6.77) explicitly mentions the possibility of price driving by placing “risk-free bids” in the supplementary phase as a problematic aspect of the CCA. Some of the potential bidders’ responses share this concern (e.g., BT, 2015). None of these arguments for raising rivals’ costs implies that bidders should have a lexicographic preference for doing so; lexicographic preferences are a useful modeling approach to inquire into the effects of this motive.

This is the first paper that provides a full equilibrium analysis of the CCA when bidders have a lexicographic preference for raising rivals’ costs. The most closely related paper is Levin and Skrzypacz (2016). Like us, they consider two bidders competing for a perfectly divisible good in the CCA. Common is also that in some parts of their analysis they consider bidders with a lexicographic preference for raising rival’s cost. A minor difference is that we study a broader class of utility functions. The main difference is that Levin and Skrzypacz (2016) do not perform a full equilibrium analysis. Instead, they restrict at least one bidder to continuous demand functions in the clock phase, which is inconsistent with the rules of the auction and the incentive to raise rivals’ costs. Our full equilibrium analysis provides results that are qualitatively and quantitatively different. In contrast to Levin and Skrzypacz (2016), we find that efficient equilibria can exist even when bidders relax the constraints of the activity rule. It turns out that bidders use discontinuous clock demand functions to relax these constraints. We also show how inefficiencies may result from the incentives of strong bidders to obfuscate their type. Janssen and Karamychev (2016) also analyze bidders with a lexicographic preference for raising rivals’ costs in the CCA. They focus on dominated strategies in the supplementary phase, whereas we consider the interaction between the clock and the supplementary phase in equilibrium.

on why Sunrise paid much more for comparable spectrum than other bidders, (ii) Ofcom (2012, page 122, point 7.9.) in response to an earlier consultation on the UK LTE auction in 2013.
A variant of the CCA has first been suggested by Ausubel et al. (2006) and further developed in Cramton (2013). Ausubel and Baranov (2014) discuss the evolution of the CCA. Gretschko et al. (2017) discuss why bidding can be complicated in a CCA. Bichler et al. (2013) report experimental evidence on the CCA and present a simple example in which one bidder submits a spiteful bid. This paper also contributes to the growing literature that explores real-world auction mechanisms. Ausubel et al. (2014) analyze the discriminatory and the uniform price auction in a similar framework. Goeree and Lien (2014) derive equilibria for the SAA and find that the exposure problem is indeed problematic for efficiency and revenue. However, as the number of items grows large, outcomes converge to VCG outcomes.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 proves our first main result that there do not exist efficient equilibria of the CCA where the clock phase fully reveals rivals’ types. Section 4 presents our second main result, namely that if the uncertainty concerning the competitor’s type is large, the CCA does not have efficient equilibria. Both sections illustrate the main results through examples of equilibria. Section 5 analyzes the VCG mechanism as a benchmark. We show that under standard preferences, iterated elimination of weakly dominated strategies always results in an efficient outcome, but it leaves the bids of weak types on large shares undetermined. The lexicographic preferences impose that bids on large shares are chosen to raise the rival’s cost. Section 6 concludes with a discussion where we also discuss the relevance of our paper for interpreting real-world auctions. All proofs are in the appendix.

2 Model and Auction Rules

We consider auctions where two bidders compete to get a share of a perfectly divisible good that is in unit supply. Throughout the paper, when a bidder has label \( i = 1, 2 \), the other bidder’s label is \( j = 3 - i \). Bidder \( i \)’s privately known type is denoted by \( a_i \). A bidder’s type is randomly drawn from an atom-less and commonly known distribution with support \([a, \overline{a}]\). The set of type profiles \( a = (a_1, a_2) \) is denoted by \( A = [a, \overline{a}]^2 \). The utility a bidder of type \( a_i \) derives from acquiring a share \( x \) is denoted by \( U(a_i, x) \). The utility function \( U(a_i, x) \) is strictly increasing in \( a_i \) and \( x \), twice continuously differentiable, and concave. The marginal utility is increasing in \( a_i \), i.e., \( \partial^2 U(a_i, x)/\partial a_i \partial x > 0 \) for \( x > 0 \) and non-negative for \( x = 0 \). When convenient we write \( U_i(x) \) instead of \( U(a_i, x) \). Throughout the paper, we denote utility and bidding functions with capital letters.
and the respective derivatives with small letters. For example, we write \( U_i \) for the utility function and \( u_i \) for marginal utility. Also, \( \bar{U} = U(a, \cdot) \) denotes the utility function of the weakest possible bidder.

Besides these standard preferences, the bidders have a spite motive. We model this spite motive in a lexicographic way. In the first dimension, bidders maximize their surplus from the auction, and in the second dimension, they maximize the payment of the other bidder. This spite motive is relatively weak since bidders do not want to harm the other bidder if this implies getting a lower surplus themselves. Introducing a spite motive in a lexicographic manner resolves indifferences concerning auction outcomes in favor of outcomes that harm the other bidder most.\(^6\) Studying auction outcomes under lexicographic preferences can be seen as a robustness check for equilibria under standard preferences. An equilibrium under standard preferences might not be an equilibrium under lexicographic preferences, while any equilibrium under lexicographic preferences is an equilibrium with standard preferences.

For every type profile \( a \in A \), we define the efficient allocation \( x^* = (x^*_1, x^*_2) \) as \( x^*(a) \in \text{arg max}_x U(a_1, x_1) + U(a_2, x_2) \) such that \( x_1 + x_2 \leq 1 \) and \( x_i \geq 0 \) for \( i = 1, 2 \). From the above assumptions, a few results are immediate. Since the utility functions are strictly increasing and concave, there exists a unique efficient allocation, which may involve one bidder not getting anything. As the objective function of the constrained maximization problem is supermodular in \( (a_i, x_i) \), Topkis’ Monotonicity Theorem implies that bidder \( i \)'s efficient share \( x^*_i(a) = x^*_i(a_i, a_j) \) is non-decreasing in \( a_i \), and hence, it is non-increasing in \( a_j \). It follows that for each type \( a_i \) there exists a lowest possible efficient share \( \min_j x^*_i(a_i, a_j) = x^*_i(a_i, \bar{a}) = \underline{x}_i \), and a largest possible efficient share \( \max_j x^*_i(a_i, a_j) = x^*_i(a_i, a) = \bar{x}_i \). Concavity of \( U \) implies that the allocation \((1/2, 1/2)\) is efficient for any symmetric type profile. As a consequence, for types \( \underline{a} < a_i < \bar{a} \) we have that \( \underline{x}_i < 1/2 < \bar{x}_i \). In any efficient allocation, the lowest type will never win more than 1/2, while the strongest type \( \bar{a} \) will not win less than 1/2. Berge’s Maximum Theorem implies that \( x^*(a) \) is continuous in \( a \). Hence, for any \( x \in [\underline{x}_i, \bar{x}_i] \) there exists a type profile \( a_j \) such that \( x = x^*(a) \). Finally, we note that \( u(a_i, \bar{x}(a_i)) \) is non-decreasing in \( a_i \).\(^7\) When there is no danger of causing confusion, we sometimes drop subscripts.

\(^6\)When we talk about efficiency we talk about efficiency in the first dimension of the preferences only.

\(^7\)This can be seen as follows. Let \( a'_i > a_i \), so \( \bar{x}'_i \geq \bar{x}_i \). Suppose \( \bar{x}'_i = \bar{x}_i = 1 \). Then clearly \( u(a'_i, 1) > u(a_i, 1) \). If \( \bar{x}'_i = 1 \), but \( \bar{x}_i < 1 \), then \( u(a'_i, 1) \geq u(a, 0) \geq u(a_i, 1 - \bar{x}_i) = u(a_i, \bar{x}_i) \), by decreasing marginal values and necessary conditions of efficiency. If \( 1 > \bar{x}'_i \), then efficiency requires \( u(a_i, \bar{x}_i) = u(a, 1 - \bar{x}_i) \) and \( u(a'_i, \bar{x}'_i) = u(a, 1 - \bar{x}'_i) \). Since \( 1 - \bar{x}_i \geq 1 - \bar{x}'_i \), decreasing marginal values imply \( u(a, 1 - \bar{x}_i) \leq u(a, 1 - \bar{x}'_i) \).
The value function of the maximization problem defining the efficient allocation is\[ V(a) = U(a_1, x_1^*) + U(a_2, x_2^*). \]It is non-decreasing in \( a_i \) for all \( i \), since by the envelope theorem,\[ \frac{\partial V(a)}{\partial a_i} = \frac{\partial U(a_i, x_i^*)}{\partial a_i} \geq 0. \]Let \( V_i(a) = V(a_i, a) \) denote the minimal value of the efficient allocation when bidder \( i \) has type \( a_i \).

Levin and Skrzypacz (2016) consider a particular case of the above description. In their model, bidders have a strictly increasing quadratic utility function of the form\[ U(a_i, x) = a_i x - \frac{b}{2} x^2, \]with \( a \geq b > 0 \) and \( x \in [0, 1] \). The condition \( a \geq b \) makes the utility function increasing in \( x \) for all types. Levin and Skrzypacz (2016) adopt the assumption \( a - a_j < b \), which guarantees that the efficient allocation is always in the interior of \([0, 1]\) as \( u_i(0) > u_j(1), j \neq i \). The efficient share of bidder \( i \) is then\[ x_i^*(a_i, a_j) = \frac{a_i - a_j + b}{2b}. \]

We use the quadratic utility functions in our examples. Note that in our general set-up we do not assume that the efficient allocation is always in the interior. We will now discuss the auction rules.

**VCG Rules**

In the VCG mechanism, every bidder submits a bidding function \( S_i : [0, 1] \rightarrow \mathbb{R}_+ \). As bidding usually depends on the type of the player, we also use \( S(a_i, x) \) to denote a bidder's bid on quantity \( x \) when he is of type \( a_i \). The auctioneer chooses the allocation \( x = (x_1, x_2) \) with \( x \in \arg \max_x S_1(x_1) + S_2(x_2) \) such that \( x_1 + x_2 \leq 1 \) and \( x_i \geq 0 \) for \( i = 1, 2 \). If two allocations solve the maximization problem, the auctioneer implements the allocation in which the distance to the allocation \((1/2, 1/2)\) is minimized. Bidder \( i \) receives share \( x_i \) and pays the VCG price \( \max_y S_j(y) - S_j(x_j) \), i.e., the opportunity cost he imposes on the other bidder. We sometimes abuse notation, as \( x \) can denote both the allocation \( x = (x_1, x_2) \) and the share bidder \( i \) gets. In the latter case and with strictly increasing bidding functions, the final allocation is \((x, 1-x)\) and bidder \( i \)'s surplus from the auction is \( U_i(x) - S_j(1) + S_j(1-x) \).

**CCA Rules**

The CCA is an auction with two stages. In the first stage, the clock phase, the auctioneer increases the price of the good and bidders report demands. The second
stage, the supplementary phase, is a VCG auction in which bidders submit bids subject to so-called activity rules that are described below. Put differently, the clock phase elicits a demand function, whereas in the supplementary phase bidders submit an inverse demand function. Activity rules ensure the consistency of the two functions.

Bidder $i$’s action in the clock phase is a weakly decreasing demand function $x_i: \mathbb{R}_+ \to [0, 1]$ that maps prices to demand. The clock phase begins at an initial price $p_0 = 0$. The clock price is increased continuously as long as there is excess demand. The clock phase stops at $\hat{p}$ if excess demand is smaller than or equal to zero, i.e., if $x_1(\hat{p}) + x_2(\hat{p}) \leq 1$. Bidders are not allowed to increase their demand during the clock. We analyze a CCA where bidders do not receive any information concerning total demand in the clock phase.\(^8\) Hence, bidders can only condition their demand on the price, but not on their rival’s previous demand. This makes this type of auction easier to analyze.

In the supplementary phase, bidders submit bidding functions $S_i: [0, 1] \to \mathbb{R}_+$. The choice of the function $S_i$ is constrained by three activity rules. First, clock bids remain valid, that is, if bidder $i$ demanded $x$ at clock price $p$, then it has to be the case that $S_i(x) \geq p \cdot x$. This is a minimal requirement to make clock bidding meaningful. Second, supplementary bids must satisfy the so-called final cap, i.e., $S_i(x) \leq S_i(\hat{x}_i) + \hat{p}(x - \hat{x}_i), x \neq \hat{x}_i$, where $\hat{p}$ is the final clock round price and $\hat{x}_i$ is bidder $i$’s demand in the final clock round. The final cap rule essentially requires that supplementary round bids satisfy the axiom of revealed preference with respect to the final clock round behavior. These constraints bound the supplementary bidding function from above. If the clock ends with market clearing, then the final cap implies that the final clock allocation is the final allocation (Levin and Skrzypacz, 2016). Finally, if in the clock phase bidder $i$ was demanding $x$ at a price $p$, then for any $x' > x$, bidder $i$ cannot express an incremental bid for $x'$ in the supplementary round that is larger than $p$, i.e.,

\[^8\]In practice, CCAs have different regimes concerning the information that is released to the bidders in the clock phase. In one regime, bidders are only informed about the fact that there is still excess demand and that the clock phase continues. In another regime, bidders are informed about total demand at every clock price. The first regime was used in the first part of the Austrian auction and seems to be favored in case there is some suspicion that collusion between bidders may be something to worry about.

In the consultation document on the award of the 2.3 GHz and 3.4 GHz bands, Ofcom (2014) proposes to use either the CCA or the SAA without demand disclosure. In a reaction for Hutchinson 3G, Power Auctions LLC (2015) claims that a dynamic auction with no demand disclosure is basically a sealed-bid auction. We show, however, that the equilibria that can be sustained in a CCA without demand disclosure during the clock phase differ from the equilibria of the VCG.
\( S_i(x') \leq S_i(x) + p(x' - x) \). For differentiable bidding functions, this implies that \( s_i(x') \leq p \). This so-called relative cap rule imposes constraints on the slope of the bidding function. A bid on \( x' \) cannot be larger than the area under the expressed clock demand curve.

Given the bids in the clock and the supplementary phase, the auctioneer uses the same rules as described above for the VCG mechanism to compute the final allocation and individual CCA prices.\(^9\)

Bidders use the information about the clock development to update their beliefs about the type of the rival bidder. Even though no information about demand is revealed, bidders can infer information about the competitor’s type from the equilibrium strategies and the duration of the clock phase. We denote by \( \mathcal{A} \) the support of the posterior of the other bidder’s type distribution. In a pooling equilibrium, a bidder does not learn anything about the other bidder’s type, so \( \mathcal{A} = [a, \bar{a}] \). On the contrary, the equilibrium might be separating so that the final clock price reveals the rival’s type. The posterior in such an equilibrium is then the singleton \( \mathcal{A} = \{a_j\} \). In any case, the set \( \mathcal{A}(p) \) denotes the support of the posterior if the clock ends at price \( p \).

### Preferences for raising rivals’ costs and equilibrium notion

For the VCG auction, we define the preference for raising rival’s cost as follows. Given the other bidder’s strategy \( S_j \), bidder \( i \) strictly prefers strategy \( \hat{S}_i \) over strategy \( S_i \), if and only if, \( \hat{S}_i \) yields a strictly higher primary expected utility than \( S_i \), or the primary expected utility is the same and \( \hat{S}_i \) leads to a weakly higher VCG price for bidder \( j \) for all \( a_j \in \mathcal{A} \) and to a strictly higher VCG price for a least one \( a_j \in \mathcal{A} \). More formally, let \( \hat{x}(a) \) be the allocation implemented by \( (\hat{S}_i, S_j) \) and let \( x(a) \) be the allocation implemented by strategy profile \( (S_i, S_j) \). The strategy \( \hat{S}_i \) is preferred to \( S_i \) in the spite dimension if

\[
\max_y \hat{S}_i(y) - \hat{S}_i(\hat{x}(a)) \geq \max_y S_i(y) - S_i(x(a)).
\]

For the CCA, the definition of raising rival’s cost has to be slightly adapted as follows. First, in the CCA a strategy consists of a clock demand function \( x_i \) and a supplementary bidding function \( S_i^p \) for every possible final clock price \( p \). Accordingly, the VCG strategy \( S_i \) has to be replaced by the CCA strategy \( (x_i, \{S_i^p\}_p) \).

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\(^9\)We do not consider the “core-selecting” elements in the pricing rule of real-world CCAs auctions (see, e.g., Day and Milgrom (2008), Day and Cramton (2012), and Erdil and Klemperer (2010), as well as Goeree and Lien (2016) and Ausubel and Baranov (2013)).
Second, the dynamic nature of the CCA needs to be taken care of. A strategy \((\hat{x}_i, \{\hat{S}_i^p\}_p)\) is then weakly preferred to another strategy \((x_i, \{S_i^p\}_p)\) if for any history of the clock phase the continuation strategy is weakly preferred. Conditional on the clock price \(\hat{p}\) being reached, the difference with the VCG mechanism is that we use the posterior \(\mathcal{A}(\hat{p})\) rather than the prior belief \(\mathcal{A}\) to evaluate a bidder’s preference. This means that if the clock ended at price \(\hat{p}\), the supplementary bidding function \(\hat{S}_i^\hat{p}\) is weakly preferred to \(S_i^\hat{p}\). When the clock has not ended at price \(\hat{p}\), then the evaluation of whether the continuation strategy \((\hat{x}_i|_{p \geq \hat{p}}, \{\hat{S}_i^p\}_{p \geq \hat{p}})\) is weakly preferred to \((x_i|_{p \geq \hat{p}}, \{S_i^p\}_{p \geq \hat{p}})\) again uses the posterior \(\mathcal{A}(\hat{p})\) and not the prior.

The equilibrium concept we use is a refinement of the standard ex post equilibrium applied to the first dimension of the preferences. That is we only consider ex post equilibria that are such that given the prior beliefs and the strategies of the others no bidder prefers to use a different strategy as defined above, including the preference for raising rival’s cost. We cannot use the notion of ex post equilibrium using the full preferences as in equilibrium we must allow for the fact that knowing the type of the competitor ex post, a bidder may want to change the rival’s cost raising bids.

### 3 Efficiency and Information Revelation

This section presents the fundamental trade-off between efficiency and information revelation in the clock phase. The clock phase can reveal information on the competitor’s type through the price at which the clock phase ends. In particular, along the equilibrium path, the clock may stop at different prices for different type profiles allowing bidders to learn the competitor’s type. For example, if bidders bid truthfully\(^{10}\) in the clock phase, then for a given clock price each type expresses a different demand so that given a bidder’s type, the final clock price depends on the competitor’s type.

Bidders face two constraints in raising the rival’s cost. The rival’s payment is increased by raising the supplementary bid on the full supply or any other high share. Bids on large shares need to be low enough so that they do not become winning. Besides, they also need to satisfy the constraints imposed by the activity rule. We will subsequently see that information revelation might relax the first constraint. The second constraint can only be relaxed by appropriate clock

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\(^{10}\) A bidder bids *truthfully* in the clock phase if at price \(p\) he demands an amount \(x_i\), such that \(u_i(x_i) = p\).
behavior. Throughout the paper, we will use the bid on the full supply to raise rival’s cost.

Learning the competitor’s type allows increasing the other bidder’s CCA price by allowing the bidder to target more precisely by how much he can raise the bid on the full supply without winning. In Section 2 we have seen that the total value of the efficient allocation is increasing in the other bidder’s type. Hence, the constraint that the bid used to raise the CCA price does not become winning becomes relaxed if one learns that the other bidder has a strong type.

We will now show how bidders can bid in the clock phase to relax the constraints imposed by the activity rule to raise the rival’s cost in the supplementary round. In particular, we first show that demand expansion is part of any efficient equilibrium when bidders have a preference for raising rival’s cost. We will say that a bidder expands demand in the clock phase if there are clock prices such that the bidder demands an amount $x_i$ with $u_i(x_i) < p$. As marginal utilities are decreasing in $x_i$, it is clear that this inequality can only hold if bidders demand more than their truthful demand.

The next proposition states that in any efficient equilibrium weak bidders expand demand in the early stages of the clock phase. The intuition is simple. First, in an efficient equilibrium, the clock phase cannot end at a price smaller than $u(a,1/2)$ as otherwise, the relative cap prevents bidders from expressing true marginal values on all possibly efficient shares in the supplementary phase. Second, expanding demand at prices smaller than $u(a,1/2)$, while keeping fixed the rest of the clock phase bidding allows the bidder to raise the rival’s cost in the supplementary round in a way that does not risk winning these bids. Such a deviation is preferred by bidders if they have a preference for raising rival’s cost.

Proposition 1. In any efficient equilibrium of the CCA, there is a price $\tilde{p} > u(a,1/2)$ and an open set of (low) types that expand demand for prices $p < \tilde{p}$.

Knowing how bidders can relax the constraints imposed by the activity rule to be able to raise rival’s cost in the supplementary round, we now consider how this ability eliminates the possibility of fully revealing efficient equilibria. To this end, the following function helps to analyze equilibrium information revelation. Let $\tau : A \to \mathbb{R}_+$ be the function that assigns, for a given equilibrium, to every type profile the final equilibrium clock price, i.e., $\tau(a) = \inf\{p : \sum_i x_i(a_i,p) \leq 1\}$. There are two extreme cases regarding information revelation. First, no information is revealed if the clock ends at the same price for all type profiles. In terms of the function $\tau$, there exists a price $p$ such that $\tau(A) = \{p\}$. We call an equilibrium
in which this is the case clock-pooling since no bidder learns anything about the other bidder’s type. Another extreme case is a clock-separating equilibrium which is defined as an equilibrium where the function $\tau(a)$ is non-decreasing in $a_i$ for all $i$ and strictly increasing in $a_j$ for all bidders $j$ who win a positive share $x_j < 1$.\footnote{The restriction $x_j < 1$ is needed for the following reason. Suppose $a \in A$ is such that bidder 1 wins 1 in the efficient allocation. If bidder $j$ bids truthfully, for example, then the clock would end by all bidder $j$ dropping out truthfully. In this case the clock provides a lower bound for $a_1$, but the final clock price is not increasing in $a_1$.} In a clock-separating equilibrium, the final clock price conveys detailed information about the competitor’s type. A prominent example of clock-separation would be truthful bidding.

The following proposition shows that there is no efficient clock-separating equilibrium in the CCA. Truthful bidding is therefore not an equilibrium. In addition, if one interprets a clock-separating equilibrium as a formal definition of the notion of price and package discovery, mentioned in the introduction (see, e.g., Ausubel et al., 2006), then it follows that under a weak preference for raising rivals’ costs the CCA cannot deliver its two main objectives: efficiency and price and package discovery.

**Proposition 2.** There does not exist an efficient clock-separating equilibrium in the CCA.

In the proof of the proposition, we show that in a clock-separating equilibrium all bidders must bid truthfully in the clock phase for all prices $p \in [u_i(x_i), u_i(x_i)]$. However, weak types have an incentive to expand demand at least until the clock price is in the interior of this interval. Then they drop demand discontinuously and demand truthfully from then on. They can “correct” their deviation in the supplementary round by generating “missing” truthful bids in the interval $[x_i, x_i]$. The competitor will not observe the deviation and simply (mistakingly) infer from the final clock price that the deviating bidder is a stronger type than his true type. As the competitor’s clock demand does not change, they will continue expressing the same supplementary bids so that the deviating player can continue acquiring the efficient share at the same price as without the deviation. The deviation weakens, however, the constraints imposed by the activity rules and allows the deviating bidder to raise rival’s costs beyond what he could do if he had not deviated.

The intuition for the result is similar to the ratchet effect in the dynamic contracting literature (Laffont and Tirole, 1988). In this literature, high types do not want to reveal their type in the first period, because they would get a worse
contract in the second period. In the CCA, the driving forces are similar. Suppose a low type observes that the clock has not ended yet, indicating that the other bidder has a high type. A low type can use this information to raise the high type’s CCA price if the constraints of the activity rule are not binding, which is the case if the low type has expanded demand for a sufficiently long period. The high type will then best respond by pooling with low types in order to obfuscate his type.

Example of an efficient clock-semi-separating equilibrium

We illustrate these results through an example where some, but not full, information revelation may arise in an efficient equilibrium. In the example, bidders have quadratic utility functions. In the equilibrium we analyze here, bidders demand the entire supply at prices lower than $\tilde{p} \leq a - \frac{b}{4}$. At $\tilde{p}$ bidders start demanding truthfully. The clock ends at $\tilde{p}$ if both bidders are sufficiently weak, i.e., if $a_i + a_j \leq 2\tilde{p} + b$. Define $\tilde{a}(\tilde{p}) = 2\tilde{p} + b - \pi$ to be the highest type for which the clock always ends at $\tilde{p}$. Types lower than $\tilde{a}$ never reveal any private information in the clock. For other types, the clock will not end if the competitor is of a sufficiently strong type. In this case, bidders continue to bid truthfully at prices larger than $\tilde{p}$, i.e., they use the clock demand function

$$ x_i(p) = \begin{cases} 
1 & \text{if } p < \tilde{p} \\
\max\left\{\frac{a_i - p}{b}, 0\right\} & \text{if } p \geq \tilde{p},
\end{cases} $$

and update their prior about the other bidder. As the clock proceeds after $\tilde{p}$, bidders gradually learn their competitor’s type as the lower bound of the belief concerning the competitor’s type is increasing in $p$. Eventually, the clock will end with market clearing at $p$ and bidder $i$ knows the type of the other bidder to be $2p + b - a_i$. We call this equilibrium an efficient clock-semi-separating equilibrium as some, but not all, types reveal their private information during the clock phase.

Consider the supplementary phase if the clock ends at $\tilde{p}$. Whatever bidders bid on their last clock round package $\tilde{x}_i$ in the supplementary round, the final and the relative cap imply they can maximally bid $\tilde{p}(1 - \tilde{x}_i)$ more on the full supply. However, if their bid on the full supply is more than $S_i(\pi_i) - S_i(\tilde{x}_i) + S(1 - \pi_i)$ higher than their bid on $\tilde{x}_i$ they run the risk of winning the full supply if the rival bidder’s type is low. In equilibrium, bidders bid value $U_i(x_i)$ for $x_i \in [0, 1)$ and maximally raise the bid on the full supply without the risk of winning it. Hence,
bidders bid $S_i(1) = \min\{U_i(\bar{x}_i) + U(1 - \bar{x}_i), U_i(\hat{x}_i) + \hat{p}(1 - \hat{x}_i)\}$. It turns out that there is a cutoff type $\hat{a}(\hat{p}) = \hat{p}(2 + \sqrt{2}) - a(1 + \sqrt{2}) + b$ such that the inequality $U_i(\hat{x}_i) + \hat{p}(1 - \hat{x}_i) < U_i(\bar{x}_i) + U(1 - \bar{x}_i)$ is true if and only if $\hat{a}(\hat{p}) < a_i$. As a result, only bidders with a low type $a_i < \hat{a}(\hat{p})$ bid $S_i(1) = U_i(\bar{x}_i) + U(1 - \bar{x}_i)$.

If the clock ends with market clearing at $p > \hat{p}$, all types submit

$$S^p_i(x) = \begin{cases} U_i(x) & \text{if } x \in [0, 1) \\ U_i(\hat{x}_i) + \hat{p}(1 - \hat{x}_i) & \text{if } x = 1. \end{cases}$$

All types fully raise the bid on the full supply, since the final allocation is determined by the clock phase bidding (given market clearing at the final clock price). The relative cap requires that $s_i(x_i) \leq u_i(x_i)$ for $x_i \in [x_i(p), x_i(\hat{p})]$. Bidders maximally raise their (marginal) bids on $[x_i(p), x_i(\hat{p})]$ as this allows them to raise the bid on the full supply.

The proposed strategy profile is part of an equilibrium if $\hat{a}(\hat{p}) \geq \hat{a}(\hat{p})$. In this case, all types $a_i > \hat{a}(\hat{p})$ for which the clock phase possibly continues at prices $p > \hat{p}$ cannot raise their supplementary bid on the full supply if the clock stops at $\hat{p}$ without running the risk of sub-optimally winning the full supply. If, however, $\hat{a}(\hat{p}) > \hat{a}(\hat{p})$, then there exist types $a_i \in [\hat{a}(\hat{p}), \hat{a}(\hat{p})]$ for which the clock phase does not necessarily stop at $\hat{p}$ and their bid on the full supply depends on whether the clock stopped at $\hat{p}$ or not. If the clock ended at $\hat{p}$ they will bid $U_i(\bar{x}_i) + U(1 - \bar{x}_i)$ as they do not want to risk winning the full supply, which happens if the competitor’s type is close to $\hat{a}$. If the clock ends at a higher price, due to market clearing, they can safely bid $S_i(1) = U_i(\hat{x}_i) + \hat{p}(1 - \hat{x}_i)$. Thus, the fact that the clock did not stop at $\hat{p}$ makes that for any fixed type in the interval $(\hat{a}, \hat{a})$ their bid on the full supply jumps discretely by $U_i(\hat{x}_i) + \hat{p}(1 - \hat{x}_i) - (U_i(\bar{x}_i) + U(1 - \bar{x}_i))$. Knowing this, it is profitable for some types higher than $\hat{a}(\hat{p})$ to reduce demand at $\hat{p}$ to end the clock for sure. To guarantee that an efficient clock-semi-separating equilibrium exists it suffices that $\hat{a}(\hat{p}) \geq \hat{a}(\hat{p})$, which together with $\hat{p} \leq a - \frac{b}{4}$ holds if $\bar{a} - a < \frac{b\sqrt{2}}{4}$.\footnote{This condition also implies that one can find prices $\frac{\bar{a} + a - b}{4} < \hat{p} < a - \frac{b}{4}$. Along the equilibrium path it should be the case that even for the highest possible type $\bar{a}$ the clock phase may possibly stop at $\hat{p}$. This requires that $\hat{p} > \frac{\bar{a} + a - b}{4}$. The reason is that if a bidder with type $\bar{a}$ knows that the clock will not stop even if he bids truthfully, he does not have an incentive to reduce demand and instead prefers to continue bidding on the full supply. As $\hat{a}(\hat{p}) < \bar{a}$ the candidate equilibrium strategy implies he is restrained raising the rival’s cost if the clock stops at $\hat{p}$ and continuing bidding on the full supply would allow him to further raise the rival’s cost without affecting the final allocation (and the price he pays).}
solution arises.

The example shows that there are efficient equilibria with some information revelation. In this equilibrium, low types pool at a certain price, and high types need to be constrained by the activity rule so that they cannot exploit new information for raising rival’s cost.

The example is noteworthy as it shows that even if bidders anticipate that the competitor engages in raising their cost in the supplementary phase, they do not have to reduce demand in the clock phase. This is in contrast to Levin and Skrzypacz (2016) who restrict themselves to linear proxy strategies and show that bidders will engage in demand reduction in the clock phase, assuming (against the auction rules) that a demand reduction strategy in the clock phase does not affect the ability to raise rival’s cost. The example also shows that, in contrast to what some observers of the CCA have argued, the clock phase may well end up with excess supply, while bidders are still able to raise rival’s cost.13

A variant of this equilibrium arises if the clock stops for all types at \( \hat{p} \). In an efficient clock-pooling equilibrium, the clock does not reveal any information to bidders. There are three constraints on the clock-pooling price \( \hat{p} \). First, all types need to demand less than half of the full supply at \( \hat{p} \) so that the clock ends. In an efficient equilibrium, this happens if \( \hat{p} \geq \overline{a} - b/2 \) (the marginal utility of the highest type at his smallest efficient share, which is equal to 1/2). Second, bidders must not have an incentive to expand demand until \( p > \hat{p} \). They do not have such an incentive if they can already screen the lowest possible type at \( \hat{p} \), that is, when \( \overline{a} \leq \hat{a}(\hat{p}) \). This is the case when \( \hat{p} \) is larger than \( \frac{\pi + a(1 + \sqrt{2}) - b}{2(1 + \sqrt{2})} \). Third, the final clock price \( \hat{p} \) must be low enough so that the weakest possible type still derives non-negative utility if he acquires his highest possible efficient share, i.e., \( U(1/2) \geq \hat{p}/2 \), which is equivalent to \( \overline{a} - \frac{b}{4} \geq \hat{p} \). The constraints on \( \hat{p} \) specify how large the range \( \overline{a} - \underline{a} \) can be for such a clock-pooling equilibrium to exist.

13See, e.g., Levin and Skrzypacz (2016, remark 2 on page 2542) where they observe that “If we allowed bidder 2 to create excess supply at the end of the clock phase, she could increase bidder 1 payment even more. ... Such extreme predatory behavior is even more difficult to execute and even more risky for player 2 than what we describe. Moreover, analyzing equilibria in this case is difficult, so we maintain the assumption that player 2 is not allowed to create excess supply in the clock phase.” Similarly, Kroemer et al. (2015, p. 38) observe that “In recent spectrum auction implementations, the regulator decided not to reveal excess supply in the last round, in order to make spiteful bidding risky. It depends on the market specifics, if this risk is high enough to eliminate spiteful bidding”. In a recent consultation document on annual license fees, the UK regulator Ofcom (2015, A8.48 on page 16) also writes in a similar vein when they consider the Austrian 2013 CCA outcome: “We also noted that at the end of the clock rounds there was an excess supply of 2x10 MHz in each of the 900 MHz and 1800 MHz bands. ... This further suggests a possible reason why bidders may have considered price driving in the supplementary bids to be a risky strategy, ... .”
4 No efficient equilibrium with large uncertainty

So far we have shown that efficient clock-revealing equilibria do not exist. There may, however, exist efficient equilibria in the CCA when bidders have a preference for raising the rival’s cost. The example in the previous section uses parameter values where the uncertainty concerning the final allocations, measured by $\bar{\sigma} - \underline{\sigma}$, is relatively small. In this section, we will consider auctions where the ex ante uncertainty concerning the final allocations is relatively large and, consequently, information revelation might also be more important. Our second main result of the paper shows, however, that the CCA does not have efficient equilibria when the uncertainty about the other bidder’s type is sufficiently large.

**Proposition 3.** There is no symmetric and efficient equilibrium when the ex ante uncertainty about the final allocation is high, i.e., $u(\bar{\sigma}, 1) > u(\underline{\sigma}, 0)$.

To prove this result, we introduce the following notation. Denote by $\rho(a_i)$ the lowest clock price at which, along the equilibrium path, the clock phase may end for type $a_i$. Formally, if the demand in the clock phase is non-decreasing in a bidder’s type, then $\rho(a_i) = \tau(a_i, a)$, where the function $\tau(a_i, a_{-i})$ is introduced in Section 3. In the efficient clock-pooling and clock-semi-separating equilibria presented in the previous section, we have that $\rho(a_i) = \tau(a_i, a) = \tilde{p}$ for all types $a_i \in [\underline{\sigma}, \bar{\sigma}]$, and thus, the image of $\rho$ is a singleton. In Janssen and Kasberger (2016) we present an example of an efficient equilibrium where the image of $\rho$ is a pair of two prices.

The proof of the proposition first shows that if the uncertainty is large, there does not exist an efficient equilibrium where the image of the function $\rho$ is a singleton or a pair. The reason is simple. If the uncertainty is large, ex post efficiency requires that strong types continue bidding on the full supply for relatively large clock prices, while at these high clock prices weak types have to express bids above their utility to remain active in the clock phase. Stronger types will then have an incentive to raise the prices of the weak types so much that they make losses. Next, the proof establishes that independent of how large the uncertainty is, there is no efficient equilibrium where the image of $\rho$ has more than two elements. The argument here is that if $\rho$ would have more than two elements, then the lowest possible types can condition their supplementary bidding function on the final clock price in such a way that the strongest types pay more than moderate types. From an efficiency point of view, the strongest would need to continue the clock phase at prices where the moderate types would like to stop. Individually, however, a stronger bidder increases primary expected utility by pooling with weaker
bidders through demand reduction to prevent weak types from extracting more surplus. This demand reduction leads to ex post inefficient outcomes.

**Example of an inefficient equilibrium with large uncertainty**

Knowing that efficient equilibria do not exist if the uncertainty concerning a rival’s type is large, we now provide an example of an inefficient equilibrium. The inefficiency is due to demand reduction in the clock phase at relatively high prices as the strongest bidders know that if they would bid truthfully (or engage in demand expansion) at these prices, the clock phase may continue and rival bidders will be able to raise their prices discontinuously. Demand reduction takes on a different form, however, from what is discussed in Levin and Skrzypacz (2016). Demand reduction occurs at relatively high prices and bidders first expand demand to be able to raise the rival’s cost as much as possible. Thus, bidders do not use continuous demand functions in the clock phase as assumed by Levin and Skrzypacz (2016).

The example also provides further detail how an equilibrium can be constructed where the image of \( \rho \) is a pair. In particular, there are two prices \( \tilde{p}_1 \) and \( \tilde{p}_2 \) so that \( \rho(a_i) = \tilde{p}_1 \) for all types \( a_i \in [\underline{a}, \underline{a}^1) \) and \( \rho(a_i) = \tilde{p}_2 \) for all types \( a_i \in [\overline{a}^1, \overline{a}] \).

We define weak types as types in \([\underline{a}, \underline{a}^1)\), where \( \underline{a}^1 \) is the highest type for which the clock can possibly end if both bidders demand truthfully at \( \tilde{p}_1 \): \( \underline{a}^1 = 2\tilde{p}_1 + b - \underline{a} \). Weak bidders demand the full supply for all prices \( p < \tilde{p}_1 \) and demand truthfully at \( \tilde{p}_1 \). The price \( \tilde{p}_1 \) is chosen such that no weak type can further raise the rival’s CCA price if he learns that the opponent is a strong bidder. This is the case if \( \hat{a}(\tilde{p}) \) as defined in Section 3, is not larger than \( \underline{a} \) and we choose \( \tilde{p}_1 \) to be the largest price \( \tilde{p} \) such that this is the case: \( \tilde{p}_1 = \underline{a} - b/(2 + \sqrt{2}) \).\(^{14}\) It follows that \( \overline{a}^1 = \underline{a} + b/\sqrt{2} = \underline{a} + b (\sqrt{2} - 1) \), which is smaller than \( \overline{a} \) if the uncertainty is large enough.

Strong types demand the full supply for all prices \( p < \tilde{p}_2 \), where \( \tilde{p}_2 > \tilde{p}_1 \). For simplicity, and to easily characterize the equilibrium structure, we choose \( \tilde{p}_2 = \underline{a} \). If \( (\pi - \underline{a})/b > 1/2 \), there is a positive mass of types whose truthful demand at \( \tilde{p}_2 = \underline{a} \), \( (a_i - \underline{a})/b \), is more than 1/2. We stipulate in the equilibrium we construct that these types reduce demand to 1/2 in order to end the clock for sure at \( \tilde{p}_2 \). To distinguish these types larger than \( \underline{a} + b/2 \) from the strong bidders that drop demand truthfully to less than 1/2 at \( \tilde{p}_2 = \underline{a} \), we call these types “super-strong”.

\(^{14}\)Note that there is a multiplicity of inefficient equilibria, as \( \tilde{p}_1 \) and \( \tilde{p}_2 \) can be chosen smaller than the values we chose here. We focus on precise values not to complicate the example too much. We refer to the working paper (Janssen and Kasberger, 2016) for further details.
Thus, we have the clock demand function

\[
x_i(p) = \begin{cases} 
1 & \text{if } p < \hat{p}^2 \\
\max\{\min\left\{\frac{a_i - p}{b}, \frac{1}{2}\right\}, 0\} & \text{if } p \geq \hat{p}^2
\end{cases}
\]

of strong and super-strong types. Let \(\tilde{x}_j^i\), \(j = 1, 2\), denote the truthful demand at price \(\tilde{p}^j\).

To finish the description of the clock phase, we still have to describe how weak types bid for prices \(p\) such that \(\tilde{p}^1 < p < \tilde{p}^2\). We specify that they bid according to true marginal values until they learn that the other bidder is strong. Given this strategy, a weak type \(a_i\) learns that the rival bidder is a strong bidder if the clock phase is not over at a price \((\bar{a}^1 + a_i - b)/2\). Once they learn their rival is strong, they keep demand at the level \(\tilde{x}_1^i = x_i((\bar{a}^1 + a_i - b)/2)\) to be maximally able to raise their rival’s cost as long as \(p \leq \frac{U_i(x)}{x}\). At this price, they demand their truthful demand.

Figure 1 summarizes the description of clock phase behavior. Type profiles in the gray area are sufficiently weak so that the clock phase stops at price \(\tilde{p}^1\) and that it ends with excess supply with probability 1. If the bidders’ types are weak, but their profile is not in the gray area, then the clock phase stops at a price \(p\) with \(\tilde{p}^1 < p < \tilde{p}^2\). The interaction of these weak bidders is similar to what is described by the semi-separating equilibrium in Section 3. If the types \((a_i, a_j)\) are such that at least one bidder’s type \(a_i > \bar{a}^1\), then the clock phase stops at price \(\tilde{p}^2\). If both types are strong, then the interaction between the bidders is similar to what is described as a clock-pooling equilibrium.

We will now specify the behavior in the supplementary phase. If the clock ends at \(\tilde{p}^1\), then weak bidders submit the supplementary bidding function

\[
S_{\tilde{p}^1}^i(x) = \begin{cases} 
U_i(x) & \text{if } 0 \leq x < 1 \\
U_i(\tilde{x}_1^i) + \tilde{p}^1(1 - \tilde{x}_1^i) & \text{if } x = 1.
\end{cases}
\]

Weak bidders bid true utility on all shares, but the full supply and maximally raise the rival’s cost by maximally raising their bid on 1. This is what is achieved by the bidding function (4).

If the clock phase ends at price \(\tilde{p}^1 < p^* < \tilde{p}^2\), it ends with market clearing and bidder \(i\) believes that the efficient share \(x_i^*\) has been implemented. Since they have submitted positive, truthful, bids for \(x \in [x_i^*, \tilde{x}_1^i]\), they submit the bidding

\[As \ (\bar{a}^1 + a_i - b)/2 < a \text{ all weak types learn this before the clock reaches price } \tilde{p}^2.\]
Figure 1: Illustration of the inefficient equilibrium

function

\[ S^p_i(x) = \begin{cases} 
U_i(x) & \text{if } x \in [0, \bar{x}_1^i] \\
U_i(\bar{x}_1^i) & \text{if } x \in (\bar{x}_1^i, 1) \\
U_i(\bar{x}_1^i) + \bar{p}^i(1 - \bar{x}_1^i) & \text{if } x = 1.
\end{cases} \] (5)

Bidders cannot further raise the competitor’s cost compared to the situation when the clock ends at \( \bar{p}^1 \) since the relative cap was already binding at \( \bar{p}^1 \). If the bidder is a weak type and the clock phase ends at \( \bar{p}^2 \) he will also bid according to (5).

Finally, if the clock ends at \( \bar{p}^2 \) strong bidders submit the following supplementary bidding function

\[ S^{p2}_i(x) = \begin{cases} 
U_i(x) & \text{if } 0 \leq x < 1 \\
U_i(\bar{x}_i) + U(1 - \bar{x}_i) & \text{if } x = 1,
\end{cases} \] (6)

while super-strong types bid according to the following strategy:

\[ S^{p2}_i(x) = \begin{cases} 
0 & \text{if } x \in [0, 1/2) \\
U_i(\bar{x}_i^2) + \bar{p}^2(x - \bar{x}_i^2) & \text{if } x \in [1/2, \bar{x}_i^2) \\
U_i(x) & \text{if } x \in [\bar{x}_i^2, 1) \\
U_i(\bar{x}_i) + U(1 - \bar{x}_i) & \text{if } x = 1.
\end{cases} \] (7)

If the clock ends at \( \bar{p}^2 \) along the equilibrium path, strong bidders have received no information where their efficient share may lie and any share in the interval \( [\underline{x}_i, \bar{x}_i] \) is possible. As it is well possible that their competitor is the weakest type possible,
they will not use the ability to fully raise the rival’s cost as they risk winning the full supply at too high a price. In addition, super-strong bidders cannot bid true marginal values in the supplementary round that are higher than \( \tilde{p}^2 \) as their clock phase behavior in combination with the local revealed preference rule does not allow them to do so. Thus, on shares smaller than \( \tilde{x}_i^2 \), but larger than 1/2 they bid their maximum bids.

To complete the description of the equilibrium strategies, we specify some aspects of bidder behavior if the clock continues out-of-equilibrium at prices \( p > \tilde{p}^2 \). What is important here is that bidders believe that the reason why the clock did not stop at \( \tilde{p}^2 \) is that the super-strong bidders have deviated and demanded more than 1/2. Given this belief, the strong and super-strong bidders that did not deviate will respond to this deviation by adapting their bid on the full supply in the supplementary phase from what is specified in (7) to \( U_i(\tilde{x}_i^2) + \tilde{p}^2(x - \tilde{x}_i^2) \).

If these strategies are chosen, the allocation is \((1/2, 1/2)\) if both bidders are super-strong, as both bidders demanded 1/2 in the final clock round and the clock ended with market clearing. The final cap rule implies that this is the final allocation independent of the bidders’ true types. As this allocation is independent of bidders’ types, it is clear that this allocation is inefficient. If a super-strong bidder \( i \) meets another bidder \( j \) and the sum of their truthful demands \( (a_i - \tilde{p}^2)/b + (a_j - \tilde{p}^2)/b \) at the final clock price \( \tilde{p}^2 \) is larger than 1, then the final allocation is \((1 - \tilde{x}_j^2, \tilde{x}_j^2)\), with \( 1 - \tilde{x}_j^2 < \tilde{x}_i^2 \). It is clear that this allocation is also inefficient as it would be more efficient to give bidder \( i \) more than \( 1 - \tilde{x}_j^2 \) and bidder \( j \) less than \( \tilde{x}_j^2 \). Clearly, the inefficiencies follow from the fact that super-strong bidders cannot express their true marginal utilities on all possibly efficient shares. If the bidders’ types are such that the above two cases do not arise, then the allocation in this equilibrium is efficient.

Intuitively, understanding why these strategies form an equilibrium requires a discussion of three crucial aspects:\(^{16}\) (i) why do superstrong bidders engage in demand reduction to end the clock phase, (ii) why do strong bidders not want to reduce demand to stop the clock phase earlier, and (iii) why do weak bidders not want to raise the cost of the strong bidders further? To understand the first aspect it is easiest to consider the case where \( x(\bar{p}, \tilde{p}^2) \) is slightly larger than 1/2. By reducing demand to 1/2 at \( \tilde{p}^2 \), superstrong bidders do not lose much primary utility they get from their final allocation compared to the situation where they would deviate and bid truthfully. Deviating gives rise to the clock phase continuing if the competitor is superstrong (which happens with positive probability) and in this

\(^{16}\) A formal proof is given in the working paper of this paper (Janssen and Kasberger, 2016).
case, rivals will be able to raise the payment of a deviating bidder discontinuously. Thus, it is better to engage in demand reduction.

The second aspect follows from the fact that we have constructed \( \hat{p} \) in such a way that \( \hat{a}(\hat{p}) = a \), i.e., in equilibrium all weak types already raise the rival’s cost to the maximal extent possible and they cannot increase this cost further even if they learn the rival is a strong bidder. Strong bidders, therefore, have nothing to gain by reducing demand.

The third aspect requires an argument that weak bidders do not want to raise the cost of the strong bidders further by extending their demand on the full supply at prices \( p > \hat{p} \) is more involved. Consider a type \( a_i = a + b(\sqrt{2} - 1) - 2b\epsilon < \bar{a}_i \) for some \( \epsilon > 0 \). We will argue that there are some types of the rival bidder such that the only time type \( a_i \) can get the efficient share is by bidding truthfully at \( \hat{p} \). It is clear that the bidder does not want to reduce demand as this will prevent him from always getting the efficient share at a price he wants to pay for it. Compare then the situation where he bids truthfully at \( \hat{p} \) and one where he expands demand. If he bids truthfully and the rival is of type \( a_j = a + b\epsilon \), then their truthful demands at \( \hat{p} \) are

\[
\hat{x}_1^i = \frac{a + b(\sqrt{2} - 1) - 2b\epsilon - a - b\left(\frac{1}{\sqrt{2}} - 1\right)}{b} = \frac{1}{\sqrt{2}} - 2\epsilon
\]

\[
\hat{x}_1^j = \frac{a + b\epsilon - a - b\left(\frac{1}{\sqrt{2}} - 1\right)}{b} = 1 - \frac{1}{\sqrt{2}} + \epsilon
\]

and under truthful demand the clock ends at \( \hat{p} \) with excess supply, i.e.,

\[
\hat{x}_1^i + \hat{x}_1^j = \frac{1}{\sqrt{2}} - 2\epsilon + 1 - \frac{1}{\sqrt{2}} + \epsilon = 1 - \epsilon < 1.
\]

Importantly, note that the efficient share \( x_j^* = \frac{a + b\epsilon - a - b(\sqrt{2} - 1) + 2b\epsilon + b}{2b} \) for bidder \( j \) is larger than his demand \( \hat{x}_j^1 \). Given that the supplementary bidding function (4) applies in this case, the efficient allocation will be implemented. If, however, bidder \( i \) expands demand so that the clock phase does not end at \( \hat{p} \), bidder \( j \) believes that \( a_i > a + b(\sqrt{2} - 1) - b\epsilon \) and that his efficient share is smaller than \( \hat{x}_j^1 \). Given the specification of the supplementary bidding function (5), he will only bid true marginal values on shares that are smaller than \( \hat{x}_j^1 \), while the true efficient share is larger. Thus, the only time in the auction when \( a_j \) submits true marginal values on efficient shares \( x_j^* \) is when the clock ends at \( \hat{p} \). Consequently, if bidder \( a_i \) does not drop demand to \( \hat{x}_1^i \), there is a positive probability he misses
the chance of acquiring the efficient share and this reduces his expected surplus since \( U_i(x_i^*) + U_j(x_j^*) > U_i(1 - \tilde{x}_j^1) + U_j(\tilde{x}_j^1) \). Thus, as the other weak bidder bids truthfully at \( \tilde{p}^1 \), all weak bidders want to bid truthfully too, at least until they learn that the other bidder is not weak. This argument does not hold true for types \( a_i \geq \overline{a}^1 \) as they have zero probability that the clock ends at \( \tilde{p}^1 \) under truthful bidding.

Finally, it is interesting to note that in this equilibrium types that are close to each other may pay very different amounts of money for very similar shares of the full supply. To see this, consider the difference in the CCA price if a type \( \overline{a}^1 - \varepsilon \) faces a type \( \overline{a}^1 + \varepsilon \) when \( \varepsilon \) is arbitrarily small. The CCA price for the strong bidder \( \overline{a}^1 + \varepsilon \) (facing a weak bidder \( \overline{a}^1 - \varepsilon \)) is approximately equal to \( U_i(\overline{x}_i) + U_i(1 - \overline{x}_i) - U_i(\frac{1}{2}) \), where \( a_i = \overline{a}^1 \), while the same price for the weak bidder \( \overline{a}^1 - \varepsilon \) (facing a strong bidder \( \overline{a}^1 + \varepsilon \)) is approximately equal to \( U_i(\tilde{x}_i^1) + \tilde{p}^1(1 - \tilde{x}_i^1) - U_i(\frac{1}{2}) \). Thus, the relative price of the strong bidder \( \overline{a}^1 + \varepsilon \) is approximately

\[
\frac{U_i(\overline{x}_i) + U_i(1 - \overline{x}_i) - U_i(\frac{1}{2})}{U_i(\tilde{x}_i^1) + \tilde{p}^1(1 - \tilde{x}_i^1) - U_i(\frac{1}{2})} - 1 = 2 - \frac{4\sqrt{2}}{3} \approx 0.1143
\]

lower than what type \( \overline{a}^1 - \varepsilon \) pays. In this equilibrium, the marginally weaker bidder is restricted to raise the rival’s CCA price due to his behavior in the clock phase, where the marginally stronger bidder does not face such restrictions.

5 The VCG mechanism

To better understand the implications of having a clock phase for raising the rival’s cost, we now briefly analyze the VCG mechanism. The purpose of this section is twofold. First, we want to establish how we see the weak preference for raising rival’s cost as an alternative way to select among the many equilibria of the VCG. Second, we want to show that independent of the size of the uncertainty the VCG always has efficient equilibria that are consistent with the preference for raising rival’s costs. The contrast with the result on the CCA of the previous section reinforces the point that it is the dynamic element of the CCA, i.e., the clock phase, that is responsible for the inefficiency result.

We first show that under standard preferences the outcome of applying iterative elimination of weakly dominated strategies (IEDS) to the VCG mechanism is always efficient, but that the payments are undetermined and depend on the way IEDS is implemented. Truthful bidding is one of the strategies that survives IEDS,
but, depending on the order of elimination, other strategies may survive IEDS as well. Bidders have to bid true marginal values on possible efficient shares in the interval \([x_i, \pi_i]\) in order to get the efficient share. Outside the interval \([x_i, \pi_i]\), bidders may bid differently as depending on the order of elimination bids on these shares may not be pivotal. As for weaker bidders, it is always the case that \(\pi_i < 1\), these bidders have a range of shares for which the bid is undetermined by IEDS and the choice of these bids determines how much competitors have to pay. Accordingly, the payments in the VCG mechanism may well differ from the payments under truthful bidding.

**Proposition 4.** In the VCG mechanism with standard preferences, any strategy profile that survives any process of IEDS implements the efficient allocation. The VCG payments depend, however, on the order in which weakly dominated strategies are eliminated and on the choice of strategy profile that survives IEDS.

The elimination of weakly dominated strategies does lead to efficient outcomes, but the auction revenue is undetermined. The raising the rival’s cost motive may be viewed as a way to resolve the indeterminacy related to payments in an alternative way to imposing that bidders play their weakly dominant strategy. Under spiteful preferences truthful bidding is not an equilibrium in the VCG mechanism. To see this, suppose other bidders bid truthfully and consider a weak enough bidder with type \(a_i\) for whom the maximal efficient share \(\pi_i < 1\). Without lexicographic preferences, bidder \(i\) is indifferent between some bids on \((\pi_i, 1]\). A lexicographic bidder knows, however, that he can increase the price other bidders have to pay. The easiest way to do so is to increase the bid \(S_i(1)\) on the full supply as much as possible under the constraint that it is not winning.\(^{17}\) He never wins the full supply in an efficient equilibrium if for all \(a_j \in A\)

\[
S_i(1) \leq S_i(x_i^*(a_i, a_j)) + S_j(x_j^*(a_j, a_i)) .
\] (8)

The right-hand side of (8) depends on the type of the other bidder and is minimized if the other bidder has the lowest possible type \(a\). Hence, given our formulation of the spite motive bidder \(i\) wants to set the bid on 1 equal to the minimal value of the efficient allocation given bidder \(i\)’s type. If all bid true utility on \([x_i, \pi_i]\), then the optimal bid is \(S_i(1) = V_i(a)\). Thus, bidders can use their private information and their knowledge about the lowest possible type of bidders to raise the bid on the full supply. Types that can win everything in an efficient equilibrium maximize

\(^{17}\)He could also increase his bid on other \(x \in (\pi_i, 1]\), but this does not create any benefit.
the rival’s payment by bidding truthfully, in which case \( S_i(1) = U_i(1) = V_i(a) \).

The next proposition determines an efficient equilibrium under lexicographic preferences where bidders bid truthfully on all possible shares, apart from 1 if the type is low.\(^{18}\) The equilibrium strategies are increasing in \( x \), but not necessarily continuous at 1.

**Proposition 5.** The strategy profile in which bidder \( i = 1, 2 \) with type \( a_i \) plays

\[
S_i(x) = \begin{cases} 
U_i(x) & \text{for } 0 \leq x < 1 \\
V_i(a) & \text{for } x = 1,
\end{cases}
\]

forms an equilibrium of the VCG auction. There is a process of IEDS such that this strategy profile is iteratively undominated. This equilibrium is consistent with lexicographic preferences for raising the rival’s cost.

In strategy profile (9), all bidders bid true utility on all shares smaller than 1. No bidder cannot further raise the VCG price without running the risk of winning as the other bidder’s type may be such that the value of the efficient allocation is minimal. Hence, the strategy profile is an equilibrium under lexicographic preferences for raising the rival’s cost. Note that the strategy profile in (9) implements the efficient allocation and survives the IEDS of the proof of Proposition 4.

It is also important to note that all types \( a_i > a \) make positive surplus. This is because bidders do not want to risk winning the full supply and therefore are restricted in raising rival’s cost by the lowest possible efficient value \( V_i(a) \). If bidders would know their rival’s type, they would fully expropriate them in any equilibrium where bidders bid valuation on the possibly efficient shares \([x_i, \bar{x}_i] \). Thus, in the VCG mechanism, bidders benefit from rivals being uncertain about their type.

### 6 Discussion and Conclusion

This paper provides a full equilibrium analysis of the CCA where the strategic interaction between the clock phase and the supplementary round is studied in an environment where bidders not only care about their own payoff but also (lexicographically) about how much rivals pay. We have argued that it is quite likely that

\(^{18}\)This is not the only equilibrium when bidders have lexicographic preferences. It is clear that bidders never want to bid above value on possible efficient shares. To protect themselves against others raising their price, bidders may however reduce their own bids on the domain of possibly efficient allocations without affecting their marginal bids.
bidders in real life auctions have an incentive to raise rivals’ costs and the lexicographic modeling of this preference is a very mild way of introducing this motive, while at the same time it provides a robustness check of the claims regarding the CCA.

We have two main results. First, there does not exist an efficient equilibrium of the CCA that fully reveals the type of the competitor in the clock phase. If the rival’s type is fully revealed, bidders can raise their cost through bids in supplementary phase, providing an incentive to reduce demand and pretend to be a weaker type. This argument is related to the *ratchet effect* in the dynamic principal-agent literature where the agent does not want to reveal her type to a principal as the latter will use that information to extract more surplus from the agent in future interactions. In a CCA, the future interaction is represented by the supplementary phase. Under preferences for raising rivals’ costs, a bidder both wants to extract surplus from the competitor (as the principal) and knows his surplus can be extracted by the other bidder (as the agent).

Our second main result is that the CCA is inefficient if the uncertainty concerning final allocations is relatively large. In an efficient equilibrium, bidders want to expand demand in the clock phase to be able to extract more surplus from their competitor in the supplementary phase. When the uncertainty is large, one cannot find final clock prices that conceal enough information about types and are such that bidders can express their true marginal utilities on all possibly efficient shares. This creates the inefficiency. If the uncertainty concerning rivals’ types is small enough, efficient equilibria exist, but they do not fully reveal information concerning rivals’ types. In this sense, the clock phase does not fulfill its role as a price and allocation discovery process.

It is difficult to assess whether or not real-world CCAs have been efficient as this would require knowing bidders’ utility functions. However, many of the equilibrium features of the CCAs we have highlighted show similarities to observed features of CCAs. Without pretending that there are no alternative explanations for these phenomena, we will give the following observations. First, after the 2013 auction the Austrian regulator RTR observed that during a large part of the clock phase, bidders’ demanded close to their full spectrum caps. This is in line with our examples on clock-pooling and clock-semi-separating equilibria and explained by our result on demand expansion in the clock phase. Second, the Austrian mobile network operator Telekom Austria (2013) indicates in a press release after the auction that the clock phase ended with excess supply in key spectrum bands. According to the Austrian regulator RTR this did not withhold
the bidders to bid aggressively in the supplementary round.\footnote{See, \url{https://www.rtr.at/en/pr/P28102013TK}} This is also in line with our examples on clock-pooling and clock-semi-separating equilibria, where we argued that bidders create excess supply purposefully to obfuscate their type to prevent rivals from raising their costs.\footnote{The clock phase of the Canadian 700 MHz auction also ended with excess demand even though these units were allocated in the supplementary round.} Finally, the 2012 Swiss auction became known for bidders having to pay very different amounts for very similar spectrum shares. Again, this is in line with our example of an inefficient equilibrium where very similar bidders pay very different prices for almost identical spectrum shares.

Ausubel and Baranov (2015) have worked on alternative activity rules with the purpose of providing bidders with stricter incentives to bid according to their intrinsic preferences. They propose to replace the relative cap we used in this paper by GARP (the generalized axiom of revealed preference). We observe that in none of the equilibria we constructed bidders violate GARP, and we conclude therefore that most of our results continue to hold if we would adopt the GARP activity rule.

We end our paper by briefly discussing the robustness of our results to changes in the model. First, many of our findings hold when there are more than two bidders. Equilibria with more than two bidders feature coordination on how the costs of the rivals’ are raised. In equilibrium, this coordination is not an issue, but characterizing the equilibrium is more difficult. Restricting the analysis to two bidders also has a certain virtue beyond simplicity, as all relevant information may be revealed in a fully revealing equilibrium. When two bidders bid truthfully in the clock, the final clock price reveals the other bidder’s type. With more than two bidders, however, one only learns at most the “sum of types”. The two bidder case, therefore, allows the clearest test of the CCA. Second, it might be that values are not private but interdependent. In this case, information revelation in the clock may improve efficiency. While this may be true, the underlying economic forces described in this paper still hold. Low types know that they cannot win the full supply, so they expand demand in order to place high bids on the full supply to raise rivals’ costs. Bidders are therefore still reluctant to reveal their private information because they know that by doing so they will be exploited. Thus, the trade-off between information revelation and efficiency we have uncovered in this paper is likely to remain important in more complicated settings.
Proof. For the proof we use terminology introduced later in Section 3 and ideas introduced in the proof of Proposition 2. Please note that the proof of Proposition 2 does not depend on Proposition 1. Let \( \tilde{p} = \tau(a) \).

It is clear that in an efficient equilibrium, the clock phase cannot end at a price smaller than \( u(a, 1/2) \), because otherwise bidders would be restricted by the relative cap and could not bid true marginal utility on all possibly efficient shares. The final outcome could not be efficient. The proof of Proposition 2 shows that there is no efficient equilibrium in which \( \tau(a) = u(a, 1/2) \). Hence, it must be the case that \( \tau(a) > u(a, 1/2) \). Low types, that is, types in an open neighborhood of \( a \in A \) always have an incentive to expand demand until the lowest possible final clock price, as they would not be able to fully raise the rival’s cost without demand expansion. \( \blacksquare \)

**Proposition 2.** There does not exist an efficient clock-separating equilibrium in the CCA.

Proof. Suppose an efficient clock-separating equilibrium exists. Clock separation requires that demand is monotone in type, i.e., \( a'_i \geq a_i \) implies \( x_i(a'_i, p) \geq x_i(a_i, p) \). Let \( A' \) be an open neighborhood of the type profile \( a \in A \) so that for any \( a \in A' \) all bidders are winners in the efficient allocation. The equilibrium strategy profile must have the following properties.

First, the clock must end with market clearing almost surely. Suppose there is a positive probability, i.e., an open set of type profiles \( A'' \subseteq A' \), that the clock ends with excess supply. The clock ends with excess supply only if a bidder uses a demand function with discrete downward jumps. Without loss of generality, let bidder 1 make a jump that ends the clock for type profile \( a \in A'' \) at \( \tau(a) = p \). Consider type \( a'_2 \) being slightly smaller than \( a_2 \). Then the clock must end with excess supply at \( \tau(a_1, a'_2) \) by bidder 2 making downward jumps, because \( \tau \) is increasing in \( a_2 \). Fix \( a'_2 \) slightly smaller than \( a_2 \) such that \( \tau(a_1, a'_2) = p' < p \) and bidder 1’s demand with type \( a_1 \) has no jumps on \( [p', p) \). Note that for a given \( a_j \), the function \( \tau_i(a_i) = \tau(a_i, a_j) \) is strictly increasing in \( a_i \), and therefore almost everywhere continuous. Hence, there exists a type \( a'_1 \) slightly larger than \( a_1 \) such that \( \tau(a'_1, a'_2) = p'' \) and \( p' < p'' < p \). At \( p'' \) it is bidder 1’s discrete decrease that ends the clock. Since demand functions are monotone in type, it must be that
type $a_1$ drops demand at $p''$, a contradiction. It cannot be that the clock ends with excess supply with positive probability.

Second, the relative cap is binding for relevant shares in $(x_i^*, 1]$ in any supplementary phase on the equilibrium path. Since the clock ends with market clearing almost surely and the equilibrium is efficient, demand in the final clock round must be the respective efficient shares. This follows from the definition of the final cap rule. If the relative cap was not binding, then as the clock continues one can increase supplementary bids on shares that determine other bidders’ CCA prices relative to the efficient share. As a consequence, the price paid by a rival bidder would be dependent on the final clock price. Demand reduction would then be a best response for some types.

Third, bidders need to demand truthfully for $p \in [u_i(\bar{x}_i), u_i(x_i)]$. The clock ends with market clearing and the relative cap is binding. Suppose $u_i(x_i^*) < \tau_i(a)$, that is, the bidders did not bid truthfully in the final clock round, but every bidder demanded the efficient share. Then the marginal bid $s_i(x) > u_i(x)$ for $x > x_i^*$ and therefore the marginal CCA price is not such that bidders want to win the efficient share. This cannot be the case in an efficient equilibrium. As a result, every bidder bids $s_i(x) = u_i(x)$ for $x \in [x_i(p^*), \bar{x}_i]$ for any final clock price $p \in [u_i(\bar{x}_i), u_i(x_i)]$.

We now show that given these properties there is a profitable deviation from the clock-separating equilibrium strategy. This deviation leads to the same expected utility in the first dimension of the preferences, but to strictly higher CCA prices for some possible final clock prices. Bidder $i$ deviates by first expanding demand for some prices strictly above $u_i(\bar{x}_i)$ and bids truthfully at some price $p > u_i(\bar{x}_i)$ and from then on. If the clock ends at $p$, it almost surely ends with excess supply. Other bidders do not see the deviation and believe that the clock ended with market clearing. They fully raise the supplementary bids to $s_j(x) = u_j(x)$ for $x \in [x_j(p), \bar{x}_j]$. Hence, a suitable level of $S_j(x_i(p))$ and true marginal values $s_i(x) = u_i(x)$ on $[x_i(p), \bar{x}_i]$ implement the efficient allocation. The CCA price for the deviating bidder is the same as under the initial strategy, since the CCA price is independent of the final clock price. The CCA price for the other bidder is not less than the ‘equilibrium’ price if the clock ends at $p$. If the clock does not end at $p$, it will end at a higher clock prices with market clearing. The deviation weakened the constraints of the activity rule, hence the bids on $(\bar{x}_i, 1]$ are strictly larger than of the initial strategy and lead to a higher CCA price for the other bidder.

**Proposition 3.** There is no symmetric and efficient equilibrium when the ex ante uncertainty about the final allocation is high, i.e., $u(\bar{\pi}, 1) > u(\underline{\pi}, 0)$.  

30
Proof. We first prove two properties any efficient and symmetric equilibrium needs to satisfy. Then we use these properties to show that if the uncertainty is large, i.e., \( u(\bar{\pi}, 1) > u(\bar{a}, 0) \), no symmetric and efficient equilibrium exists where the image of the function \( \rho \) is a singleton or a pair. Finally, we show that independent of the uncertainty there cannot be an efficient and symmetric equilibrium where the image of the function \( \rho \) is neither a singleton nor a pair. Throughout the proof, we will say that the relative cap is binding if the bid on 1 relative to the bids on efficient shares is the same in every supplementary phase, i.e., if, from bidder \( j \)'s perspective, \( \bar{S}_i^p(1) - \bar{S}_i^p(x_i^*) \) only depends on \( a_i \) but not on final clock price \( p \).

The first property is that the relative cap must be binding in every supplementary phase along the equilibrium path if it is not the case that \( \tau(a_i, a_j) \) is independent of \( a_i \) and \( a_j \), i.e., there are at least two distinct final clock prices possible. If this was not true, then a bidder could increase the price paid by the other bidder once it is known that the clock continues for prices above the smallest \( \tau(a_i, a_j) \).

The second property is that (as the relative cap must be binding along the equilibrium path) bidders will expand demand in the clock phase to relax the relative cap if they can do so without affecting the equilibrium allocation. There are two instances in the clock where they can do so. The first instance is at the beginning of the clock, implying bidder \( i \) with type \( a_i \) demands 1 until the price is \( \rho(a_i) \). The second instance is the following. Suppose bidder \( i \) with type \( a_i \) knows that for \( a_j < \bar{\pi}' \) the clock must end no later than \( p' = \sup_{a_j < \bar{\pi}'} \tau(a_i, a_j) \). Suppose that \( \tau(a_i, \bar{\pi}') > p' \) and the clock has not ended at \( p' \). Then bidder \( i \) can expand demand in the clock phase by keeping demand constant between \( p' \) and \( \tau(a_i, \bar{\pi}') \).

We now prove that if \( u(\bar{\pi}, 1) > u(\bar{a}, 0) \), it cannot be that the image of \( \rho \) is a singleton or a pair. We will prove this as follows. Denote the two clock prices in the image of \( \rho \) by \( \hat{p}^1 \) and \( \hat{p}^2 \) with \( \hat{p}^1 \leq \hat{p}^2 \). If we can prove that the price \( \hat{p}^2 \) has to be strictly smaller than \( u(\bar{a}, 0) \), then it follows from \( u(\bar{\pi}, 1) > u(\bar{a}, 0) \) that \( \hat{p}^2 < \min\{u(\bar{\pi}, 1), u(\bar{a}, 0)\} \) and then in an efficient equilibrium it must be that the clock will necessarily continue beyond \( \hat{p}^2 \) for at least some types close to \( \bar{\pi} \), which is in contradiction to the fact that the image of \( \rho \) is a pair. So, the only thing we need to establish is that \( \hat{p}^2 \) has to be smaller than \( u(\bar{a}, 0) \). Suppose to the contrary that \( \hat{p}^2 \geq u(\bar{a}, 0) \). Then the lowest possible type has to expand demand between \( \hat{p}^1 \) and \( \hat{p}^2 \) (or has to engage in demand expansion until \( \hat{p}^1 = \hat{p}^2 \) if the image is a singleton), because otherwise the clock ends at a price below \( \hat{p}^2 \). As a consequence

\[21\text{In Section 3 in the clock-semi-separating equilibrium, the binding relative cap requirement manifests itself in } \hat{a} \leq \bar{a}.\]
of the demand expansion, the lowest type needs to bid above utility for possible efficient shares (and also on 0). We will show that bidding above utility leads to negative primary surplus for some type combinations. Define $a'$ to be the lowest type for which $\pi_i = 1$, i.e., $u(a, 0) = u(a', 1)$. As $u(\bar{\pi}, 1) > u(a, 0)$ such a type $a'$ exists. Let $a_i$ be slightly smaller than $a'$. Type $a_i$ needs to demand truthfully at $\hat{p}_i$ in order to implement the efficient allocation. Suppose the relative cap binds for the bid $S_i(1) = S_i(\hat{x}_i^2) + \hat{p}_i^2(1 - \hat{x}_i^2)$, and bidding $S_i(x) = U_i(x) - U_i(\hat{x}_i^2) + S_i(\hat{x}_i^2)$ for $x \in [\hat{x}_i^2, 1)$ implements the efficient allocation. When the lowest type meets $a_i$, the efficient share is implemented and the lowest type’s surplus is

$$U(1 - \pi_i) - S_i(\hat{x}_i^2) - \hat{p}_i^2(1 - \hat{x}_i^2) + U_i(\pi_i) - U_i(\hat{x}_i^2) + S_i(\hat{x}_i^2) < 0 \iff \frac{U_i(\hat{x}_i^2) + \hat{p}_i^2(1 - \hat{x}_i^2)}{U(1 - \pi_i) + U_i(\pi_i)}.$$ 

The last inequality is true, because the right-hand side is just slightly larger than $U_i(1)$, but bidder $i$ expanded demand, so the left-hand side is discretely larger than $U_i(1)$. As a result, if the relative cap is binding and the efficient allocation is implemented, then the lowest type makes a loss in the efficient allocation. When a binding relative cap does not implement the efficient allocation, then the proposed strategy profile cannot be an efficient equilibrium, as the argument above shows. Thus, $\hat{p}_i^2 < u(a, 0)$ and the image of $\rho$ cannot be a pair.

The last substantial part of the proof is to show that independent of the uncertainty there cannot be an efficient and symmetric equilibrium where the image of the function $\rho$ is neither a singleton nor a pair. First, it is important to note that due to the arguments given in the proof of Proposition 2 it cannot be the case that the image of $\rho$ contains a range of prices $[\hat{p}_1, \hat{p}_1 + \epsilon)$ for some $\epsilon > 0$, where $\hat{p}_1$ is the smallest of the prices in the image of $\rho$. It remains to be shown that it cannot be the case that the image of $\rho$ has two final clock prices $\hat{p}_1$ and $\hat{p}_2$ as its lowest two prices and that the clock continues after $\hat{p}_2$ for some set of types $[a, a']$. If this were the case then it should be that there exists $\bar{\pi}'$ and $\bar{\pi}''$ with $a < a' < \bar{\pi}' < \bar{\pi}'' \leq \bar{\pi}$ and $\bar{\pi}''$ being at most the lowest type for which the highest possible efficient share is 1, i.e., $x^*(a, a_j) > 0$ for all $a_j < \bar{\pi}'$, such that low types drop demand at the same price, i.e., $\rho([a, \bar{\pi}']) = \hat{p}_1$ and higher types drop demand at a higher clock price and this clock price is such that the clock ends when meeting the lowest type, so $\rho([\bar{\pi}', \bar{\pi}'']) = \hat{p}_2$.

As $\rho([a, \bar{\pi}']) = \hat{p}_1$ it is clear that $x(a, \hat{p}_1) < 1/2$ so that there must exists some $\delta > 0$ such that if both types are in $[a, a + \delta)$ the clock phase will stop at $\hat{p}_1$. We will now show that all types in $[a, a + \delta)$ must demand truthfully. It is clear that
a type \( a_i \in [a, a + \delta) \) cannot reduce demand at \( \tilde{p}^1 \), i.e., \( x_i^1 < \tilde{x}_i^1 \) since it prevents bidding true marginal values on efficient shares, and it limits the possibility to raise rival’s costs as well. Let us then consider the case where their demand \( x_i^1 \) is in \([\tilde{x}_i^1, 1/2]\). In any supplementary phase on the equilibrium path, bidder \( i \)'s bid on efficient shares must be the same relative to the bid on 1 and the relative cap must be binding, so bidder \( i \) bids \( S_i(x) = U_i(x) - U_i(x_i^1) + S_i(x_i^1) \) for \( x \in [x_i, \bar{x}_i] \) and on 1 raises the bid maximally to \( S_i(1) = S_i(x_i^1) + \tilde{p}^1(1 - x_i^1) \). The CCA price for an efficient share \( S_i(1) - S_i(x_i^1) = S_i(x_i^1) + \tilde{p}^1(1 - x_i^1) - U_i(x_i^1) + U_i(x_i^1) - S_i(x_i^1) \) is then maximized at \( x_i^1 = \tilde{x}_i^1 \), since \( \tilde{x}_i^1 \) satisfies the first order condition with respect to \( x_i^1 \), \( u_i(\tilde{x}_i^1) = \tilde{p}^1 \). Hence, all types in \([a, a + \delta) \) must demand truthfully at \( \tilde{p}^1 \).

Given that weak bidders in \([a, a + \delta) \) must demand truthfully at \( \tilde{p}^1 \), it is clear that in an efficient equilibrium these weak bidders cannot reduce demand at prices \( p \) with \( \tilde{p}^1 < p < \tilde{p}^2 \) to levels smaller than their truthful demand \( \tilde{x}_i(p) \) as this will imply they are restricted to bid true marginal utility in the supplementary phase on possibly efficient shares.

We will now show that there cannot be an equilibrium where these weak types bid truthfully at all prices \( p \) with \( \tilde{p}^1 < p < \tilde{p}^2 \) as they will earlier learn that their competitor is of a type \( a_j > \bar{a} \) and use this information to raise the rival’s cost. Truthful bidding leads to the supplementary bidding constraint \( s_i(x) \leq u_i(x) \). This constraint must be binding in equilibrium, since it maximally raises the CCA price of the competitor. Hence, types above \( \bar{a}' \) have no incentive to reduce demand at prices higher than \( \tilde{p}^1 \), because they can get the efficient share at later final clock prices, too. These types expand demand and drop demand at \( \tilde{p}^2 \). Thus, there must exists some \( \delta_1 > 0 \), such that types in \([a, a + \delta_1) \) learn that the other bidder’s type is larger than \( \bar{a} \). They learn this at \( p = u(a, \bar{a}') \geq \tilde{p}^1 \) and for small enough \( \delta_1 \) we have \( u(a, \bar{a}') < \tilde{p}^2 \). From this price bidders with type in \([a, a + \delta_1) \) keep demand constant until \( \tilde{p}^2 \). As \( \tilde{p}^2 \leq u(a, 0) \) these weak types demand a positive amount and the clock will continue with positive probability. If it continues, they can raise the rival’s CCA price. In the supplementary round, this bidder will use different supplementary round bid functions, depending on whether the clock phase stopped at a price \( p \leq \tilde{p}^2 \) or whether the clock phase stopped at a price \( p > \tilde{p}^2 \). In particular, if the clock phase stopped at a price \( p \leq \tilde{p}^2 \) these types will bid true marginal values on shares \( \tilde{x}_i^2 \leq x \leq \tilde{x}_i^1 \) and raise the price the competitor pays to \( \tilde{p}^1(1 - \tilde{x}_i^1) + U_i(x_i^1) - U_i(\tilde{x}_i^2) \). On the other hand, if the clock phase stops at a price \( p > \tilde{p}^2 \) these types in \([a, a + \delta_1) \) will express true marginal values only on shares \( x < \tilde{x}_i^2 \). Hence, they can use the demand expansion before \( \tilde{p}^2 \) to raise the CCA price to \( U_i(x_i^1) - U_i(\tilde{x}_i^2) + \tilde{p}^2(\tilde{x}_i^1 - \tilde{x}_i^2) + \tilde{p}^1(1 - \tilde{x}_i^1) \). This will give types
just above $\bar{\pi}''$ an incentive to reduce their demand in the clock phase to imitate types just below $\bar{\pi}''$.

Finally, suppose some types in $[a, a+\delta_2)$ keep demand constant at $\hat{x}_i^1 = x(a_i, \hat{p}^1)$ for prices $p > \hat{p}^1$, i.e., they expand demand until some $p' > \hat{p}^1$ and demand truthfully at $p'$. A similar argument as in the preceding paragraph can be used to argue that these bidders will be able to raise the rival’s cost further if they learn the clock does not stop at price $\hat{p}^2$ and that types just above $\bar{\pi}''$ then have an incentive to reduce demand and imitate types just below $\bar{\pi}''$.

**Proposition 4.** In the VCG mechanism with standard preferences, any strategy profile that survives any process of IEDS implements the efficient allocation. The VCG payments depend, however, on the order in which weakly dominated strategies are eliminated and on the choice of strategy profile that survives IEDS.

**Proof.** First we show that with standard preferences, any strategy profile that survives any process of IEDS implements the efficient allocation. This proof has three parts. First, we show that bidding truthfully is an always optimal strategy. Therefore, it survives any process of iteratively eliminating weakly dominated strategies. Second, we argue that any bidder must be indifferent between any strategy that survived the IEDS and truthful bidding. In the third and final step we show that only the efficient allocation can be implemented by strategies that survive IEDS. We will use the following notation. The set $S_i$ is the set of strategies that survived IEDS for bidder with type $a_i$. The set of iteratively undominated strategy profiles is denoted as $S = S_1 \times S_2$.

First, bidding truthfully is an always optimal strategy, i.e., it is a best response against any strategy profile of the other bidder $S_j$. To see this, let the other bidder use $S_j$ and let $\hat{x}$ denote the allocation implemented by the profile $(U_i, S_j)$, that is, $U_i(\hat{x}_i) + S_j(\hat{x}_j) \geq U_i(x_i) + S_j(x_j)$ for all other feasible allocations $x$. This inequality also says that the surplus of bidder $i$ is at least as large under the allocation $\hat{x}$ than under any other allocation, because one can simply subtract the constant $\max_{y \in S_j(y)}$ on both sides. Truthful bidding is always optimal and therefore $U_i \in S_i$.

Second, bidder $i$ must be indifferent between all $S_i \in S_i$ and $U_i$. For all $S_i \in S_i$ it holds that for all other bidding functions $T_i$ of bidder $i$ and all bidding functions $S_j \in S_j$ the surplus of $S_i$ is at least as large as for $T_i$ or strictly higher than for $T_i$ for at least one $S_j$. Recall that the surplus from $U_i$ is at least as large as from $S_i$. As a result, the strategy $S_i$ is iteratively not dominated if and only if for all $S_j \in S_j$ the surplus of $S_i$ and $U_i$ is the same for all $S_j \in S_j$. 

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We will now prove that any profile $S \in \mathcal{S}$ strictly implements the efficient allocation, i.e., $\sum S_i(x_i^*) > \sum S_i(x_i)$ for all feasible allocations $x \neq x^*$. First, note that the only share implemented by $(U_i, S_j)$ is the efficient share, that is,

$$U_i(x_i^*) + S_j(x_j^*) > U_i(x_i) + S_j(x_j) \text{ for all } x \neq x^*. \tag{10}$$

To see this, suppose there is an allocation $y \neq x^*$ with $U_i(y_i) + S_j(y_j) \geq U_i(x_i^*) + S_j(x_j^*)$. As bidder $j$ is indifferent between $U_j$ and $S_j$, we have that $U_j(y_j) + U_i(y_i) = U_j(x_j^*) + U_i(x_i^*)$. Strict concavity of $U$ implies that there is a unique efficient allocation, implying that $y = x^*$, a contradiction. Hence, $(U_i, S_j)$ only implements the efficient share. The next step uses this property to show that also $(S_i, S_j)$ implements the efficient allocation. Again by contradiction, let $\hat{x} \neq x^*$ be the allocation implemented by $(S_i, S_j)$. Bidder $i$ is indifferent between $S_i$ and $U_i$, so $U_i(\hat{x}_i) + S_j(\hat{x}_j) = U_i(x_i^*) + S_j(x_j^*)$, contradicting inequality (10). As a result, $(S_i, S_j)$ must implement the efficient allocation.

The proof that the VCG prices depend on the process of IEDS is constructive. We construct a sequence of eliminations that ends with a set of undominated strategies. Strategies in the set will have the desired properties. In order to show that a strategy is dominated, one needs to find an alternative strategy that yields weakly higher utility against all admissible strategy profiles of the other bidders and a strictly higher utility against some admissible strategy profiles. Above we have seen that bidding $U_i$ is always optimal. In the subsequent three steps of iterative elimination, we only have to find a strategy $S_j$ to show that the alternative of truthful bidding is strictly preferred.

Let $\mathcal{B}$ be the set of all bidding functions, i.e., the set of all $S : [a, \bar{a}] \times [0, 1] \rightarrow \mathbb{R}_+$. Note that the optimality of a function depends on the type $a_i$.

**Step 1:** Strategies $S_i$ for which there exists $\hat{x} < 1$ such that $S_i(\hat{x}) > U_i(\hat{x})$ are dominated. Bidder $j$ uses the bidding function

$$S_j(x) = \begin{cases} \max_y S_i(y) + S_i(\hat{x}) & \text{for } x = 1 \\ \max_y S_i(y) & \text{for } x = 1 - \hat{x} \\ 0 & \text{else.} \end{cases}$$

The bidding profile $S$ implements the allocation in which bidder $i$ wins $\hat{x}$ and bidder $j$ wins $1 - \hat{x}$; since ties are broken in favor of interior allocations. Bidder $i$’s surplus is $U_i(\hat{x}) - S_i(\hat{x}) < 0$, whereas the surplus from bidding truthfully is non-negative. Remove these dominated strategies to obtain $\mathcal{S}^1 \subset \mathcal{B}$. 

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Step 2: Strategies are dominated that satisfy \( S_i(1) > V_i(a) \). Note that for low types \( V_i(a) > U_i(1) \). For high types it can be the case that \( U_i(1) = V_i(a) \). Bidder \( j \) bids \( S_j(x) = 0 \) for \( x < 1 \) and \( S_j(1) = S_i(1) - \varepsilon \), with \( \varepsilon \in (0, S_i(1) - V_i(a)) \). Bidder \( i \) wins the full supply at a price higher than utility. Truthful bidding is therefore strictly better against this strategy profile of other bidders. Remove these dominated strategies to get \( S_2 \subset S_1 \).

Step 3: Strategies are dominated for which there exists \( \tilde{x} \in [x_i, \bar{x}_i] \) with 
\[ U_i(\tilde{x}) > S_i(\tilde{x}). \]
Let \( x' \in \arg\max_y S_i(y) \). Let \( \varepsilon \in (0, U_i(\tilde{x}) - S_i(\tilde{x})) \). In the case of \( \tilde{x} < 1 \), suppose bidder \( j \) uses the bidding function \( S_j(1) = S_i(x') + \varepsilon, S_j(1 - \tilde{x}) = S_i(x') - S_i(\tilde{x}) \) and \( S_i(x) = 0 \) for all other \( x \). Under this bidding function, bidder \( i \) wins nothing and gets zero surplus. If the bid on \( \tilde{x} \) is raised to \( U_i(\tilde{x}) \), then bidder \( i \) wins \( \tilde{x} \) and gets strictly positive surplus. For \( \tilde{x} = 1 \), let \( S_j(1) = S_i(1) + \varepsilon \) and 0 otherwise. Bidder \( i \) wins nothing if the bid is below true utility level and the full supply if the bid equals utility. The set \( S \subset S^2 \) is obtained by eliminating these dominated strategies.

After the three steps of elimination, all remaining strategies implement the efficient allocation. To see this, let bidders use the admissible strategy profile \( S \in S \). The value jointly expressed for the efficient allocation is higher than the value jointly expressed for any other feasible allocation \( x \). Let \( x_i < 1 \) for all \( i \).

Then

\[ S_i(x_i) + S_j(x_j) \leq U_i(x_i) + U_j(x_j) \leq U_i(x^*_i) + U_j(x^*_j) = S_i(x^*_i) + S_j(x^*_j), \]

where the first inequality follows from step 1, the second inequality from the definition of efficiency, and the last equality from steps 1, 2 and 3. For an allocation such that there is an \( i \) with \( x_i = 1 \) we have

\[ S_i(x_i) + S_j(x) = S_i(1) \leq V_i(a) \leq U_i(x^*_i) + U_j(x^*_j) = S_i(x^*_i) + S_j(x^*_j), \]

where the first equality follows from step 1, the first inequality from step 2, the second inequality from the definition of efficiency, and the last equality from steps 1, 2 and 3. All strategy profiles in \( S \) implement the efficient allocation. There are no further dominated strategies since any strategy that survives IEDS yields the same expected utility as bidding truthfully.

To see that the VCG prices depend on the chosen strategy profile, consider a bidder with \( a_i \) sufficiently small so that \( V_i(a) < U_i(1) \) and all the other player having the lowest possible type \( a \). The value of the efficient allocation is \( V(a_i, a) \).
Suppose bidder $i$ chooses $S_i \in \mathcal{S}_i$ with $S_i(x) = U_i(x)$ for $x < 1$ and $S_i(1) = V_i(a)$ and the other bidder plays $S_j = U_j$. Hence, the VCG price for bidder $j$ is $V_i(a) - U_i(x^*_i)$. If the strategy profile was such that $S_i = U_i$, then the VCG price would be strictly less than that and equal to $U_i(1) - U_i(x^*_i)$. Note that the strict inequality and continuity imply that the difference in VCG prices holds for an open set of types. Similarly, if step 1 was such that we eliminate also $S_i(1) > U_i(1)$, then the first VCG price would not be possible.

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