Elastoplastic buckling of a cylindrical shell with initial geometric imperfections and an elastic filler at external pressure

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Abstract. A technique has been developed for the numerical calculation of deformation and elastoplastic stability loss nonlinear problems rotating shells based on the Tymoshenko hypotheses for nonshallow shells taking into account geometric nonlinearities. Kinematic relations are formulated in velocities and constructed in the current status metric taking into account large deformations, displacements and rotation angles of shell elements. Plastic flow theory with nonlinear isotropic hardening describes physical relations. The motion equations result from the virtual working power balance. Winkler foundation models the revolution shell and the elastic filler contact interaction. The numerical calculation is based on an explicit "cross" type mesh scheme. The dependence of the elastoplastic revolution shell stability loss form and the load critical value on the filler stiffness under various values of the initial imperfections amplitude was studied.

1. Introduction

Modern computer engineering, mathematical models, and computer complexes capabilities allow us to solve the stability problems of shells under plastic deformations, taking into account geometric and physical nonlinearities, edge effects, and initial imperfections for loading types arbitrary combinations. In the course of the evaluation, subcritical structural changes and limit, rather than bifurcational, loads are calculated. Numerical calculation of geometrically nonlinear elastoplastic problems of cylindrical metal shells deformation and stability are obtained under quasistatic and dynamic loading with small deformations and considerable displacements in axisymmetric [1–3] and non-axisymmetric formulations [4–7]. Note that only the dynamic problem statement allows modeling the elastoplastic structure postbuckling behaviour – a jump to a new balanced condition.

For stability margin improvement of shell structures and blocking non-axisymmetric buckling forms, an internal filler is used [1, 2, 8–11]. A review of studies on the stress-strain determination and stability thin shells with a filler and analysis of their interaction is given in [12]. The interaction between elastic filler and the shell is often modeled by the Winkler or Pasternak foundation [9, 13–
17]. The Winkler (proportionality) coefficient dependences on the wave number, physical and geometric parameters of the considered elastic structure are revealed [9, 12]. The applicability of the Winkler model for elastoplastic shells with a filler has not been sufficiently studied.

2. The numerical calculation method

The deformation and elastoplastic buckling problem of rotation shells with a filler under combined static and dynamic loads are formulated in a two-dimensional (plane or axisymmetric) formulation based on the technique [3]. The defining equation system is written in a Cartesian (or cylindrical) system of Eulerian coordinates $O\mathbf{r}z$ ($Oz$ – rotation axis). A local Lagrangian coordinate system $O\xi \zeta$ is introduced for each shell element, associated with the general system by the following relations $ds = \psi_z dr - \psi_r dz$, $d\zeta = \psi_z dr + \psi_r dz$, where $s$ is the arc length of the shell element meridian; $\zeta$ is coordinate line normal to the median surface; $\psi_z = r_s, \psi_r = -z_s$ are the direction cosines of the normal to the median surface.

The shell elements are considered thin, the metric in thickness is invariable. The kinematic relations are formulated in velocities and are constructed in the current status metric:

$$\mathbf{u}_s(s, \zeta, t) = \mathbf{u}_s(t) + \mathbf{u}_\phi(t), \quad \mathbf{u}_\zeta(s, \zeta, t) = \mathbf{u}_\zeta(t)$$

$$\dot{\varepsilon}_{ii} = \dot{\varepsilon}_{ii} + \varepsilon_{\phi ii}, \quad i = s, \beta, \quad \dot{\varepsilon}_{ss}^0 = \dot{u}_{r,s}\psi_z - \dot{u}_{z,s}\psi_r, \quad \dot{\chi}_{\beta \beta} = \dot{\mathbf{u}}_\phi$$

$$\dot{\varepsilon}_{s\zeta} = \dot{\varepsilon}_{s\zeta}^0 \left(1 - \frac{(2\zeta/h)^2}{2}\right), \quad \dot{\varepsilon}_{s\beta} = \dot{\varepsilon}_{s\zeta}^0 = 0$$

Where $\mathbf{u}_s(t)$ and $\mathbf{u}_\zeta(t)$ are the displacement velocities of the median surface in the tangent and normal direction; $\mathbf{u}_\phi(t)$ is the cross sections turning angular velocity in the meridional section plane; $\dot{\varepsilon}_{ii}^0$ and $\varepsilon_{\phi ii}$ are the rates tensors components of the median surface strain and the bend, $\nu$ is the symmetry parameter (if $\nu = 0$ then we have a plane deformation and if $\nu = 1$ – then deformation is axisymmetric).

The elastoplastic behavior of the shell material is taken into account within the framework of the flow theory with nonlinear isotropic hardening. It is assumed that elastic deformations are small and plastic deformations can be large. Strain rate tensor components $\dot{\varepsilon}_{ij}$ can be represented as the sum of the elastic $\dot{\varepsilon}_{ij}^e$ and plastic $\dot{\varepsilon}_{ij}^p$ components:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^e + \dot{\varepsilon}_{ij}^p, \quad \dot{\varepsilon}_{ij}^p = 0, \quad i, j = s, \beta$$

The Cauchy stress tensor components which is normal to the shell median surface are taken to be equal to zero. From this condition and the generalized Hooke’s law the components $\dot{\varepsilon}_{s\zeta}^e$, the tensors components of the stress rates $\dot{\sigma}_{ij}$ and stress $\sigma_{ij}$ are determined:

$$\dot{\varepsilon}_{s\zeta}^e = \frac{\mu}{\mu - 1} \left(\dot{\varepsilon}_{ss}^e + \dot{\varepsilon}_{\beta\beta}^e\right), \quad \dot{\varepsilon}_{ss}^e = \frac{E}{1 - \mu} \left(\dot{\varepsilon}_{ss}^e + \mu \dot{\varepsilon}_{\beta\beta}^e\right), \quad \dot{\sigma}_{\beta\beta} = \frac{E}{1 - \mu} \left(\varepsilon_{\beta\beta}^e + \mu \varepsilon_{ss}^e\right)$$

(3)
\[ \sigma_{s\xi} = \frac{E}{1 + \mu} \dot{\varepsilon}_{s\xi}^p, \quad \sigma_{\xi\xi} = \sigma_{s\beta} = \sigma_{s\alpha} = 0, \quad \sigma_{ij} = \int_{0}^{t} \dot{\sigma}_{ij} dt, \quad i, j = s, \xi, \beta \]

where \( E \) is Young's modulus, \( \mu \) is Poisson's ratio. The velocities of the plastic strain components are determined by the associated flow law:

\[ \dot{\varepsilon}_{ij}^p = \dot{\lambda} n_{ij}, \quad \sigma_{ij}^p \sigma_{ij} = \frac{2}{3} \sigma_{ij}^2 (\kappa), \quad \sigma_{ij} = \sigma_{ij} - \sigma \delta_{ij}, \quad \sigma = \left( \sigma_{ss} + \sigma \beta \beta \right), \quad \kappa = \frac{2}{3} \int_{0}^{t} \dot{\varepsilon}_{ij}^p \dot{\varepsilon}_{ij}^p dt \]

where \( \sigma_{ij}^p(\kappa) \) is the yield surface radius; \( \kappa \) is the Odquist parameter, \( \delta_{ij} \) is the Kronecker delta; the parameter \( \dot{\lambda} \) is determined from the condition of passage of the instant yield surface through the end of the additional load vector.

The variational equation of shell motion is derived from the general dynamic equation taking into account the accepted shell hypotheses:

\[ \int_{S} \left[ Q \psi_{r} + N_{s} \psi_{z} \right] \ddot{u}_{r,s} + \left[ Q \psi_{z} - N_{s} \psi_{r} \right] \ddot{u}_{z,s} + \left[ vr^{-1} N_{\beta} + M \rho \ddot{u}_{r} - p_{r} - q_{r} \right] \ddot{u}_{r} + \\
+ M_{s} \ddot{u}_{\rho,s} + \left[ M \rho \ddot{u}_{z} - p_{z} - q_{z} \right] \ddot{u}_{z} + \left[ Q + vr^{-1} M \beta \psi_{r} + J \rho \ddot{\psi}_{\rho} \right] \ddot{u}_{\rho} \right] r \nu ds - \\
- r^{\nu} \left[ P_{r} \ddot{u}_{r} + P_{z} \ddot{u}_{z} + M_{0}^{0} \ddot{u}_{\rho} \right]_{s = 0, L} = 0 \]

Where \( P_{r} \), \( P_{z} \) and \( M_{0}^{0} \) are external forces and bending moment applied to the shell contour; \( p_{r} \) and \( p_{z} \) are the surface loads; \( q_{r} \) are \( q_{z} \) are the contact pressures, \( N_{s} \), \( N_{\beta} \), \( Q \), \( M_{s} \) and \( M_{\beta} \) are the internal forces and the moments; \( M_{0}^{0} \) and \( J \rho \) are the mass and the moment of inertia.

The contact between the shell and the deformable filler is modelled based on the normal no-penetration condition and the free slip condition along the tangent. The no-penetration condition determines the normal force:

\[ \begin{cases} \ddot{u}_{\xi} = \ddot{\xi}_{\xi}, & q_{\xi} - q_{\xi}^* = 0, \quad q_{\xi} \geq 0 \\ q_{\xi} = -\ddot{\xi}_{\xi}, & q_{\xi} \leq 0 \end{cases} \]

and the tangential forces are taken to be equal to zero \( q_{\xi}' = q_{\xi}^* = 0 \).

Functions with a stroke refer to the shell contact surface and functions with two strokes — to the filler. The no-penetration conditions (5) are fulfilled only in the contact interaction active phase. If the contact is interruption, they are replaced by conditions on the free surface. We assume that the contact pressure in the interaction area between the shell and the elastic filler is proportional to the shell deflection (Winkler foundation):

\[ q_{\xi}^* = k \ddot{\xi}_{\xi} \]

Where \( k \) is the proportionality factor (Winkler coefficient), \( \ddot{\xi}_{\xi}(s,t) \) is the shell contact surface deflection.

The loading velocity is assumed to be such that the inertia forces are negligible in quasi-static problems. The numerical solution of the determining system of equations is carried out according to an explicit finite-difference scheme of time integration of the “cross” type of the second order of
accuracy. This research method describes nonlinear sub-critical deformation of elastoplastic rotating shell with a filler and defines limit (critical) loadings in a wide speeds range of loading taking into account form geometrical imperfections.

3. The results of numerical simulation

The stability problem of the cylindrical shell with a filler under quasi-static loading by linearly increasing external pressure \( p = p_0 f \), \( p_0 = 50 M / I a \) in a flat formulation was considered to test the technique. Shell material is a steel X18H10T, the radius to thickness initial ratio is \( R_0 / h_0 = 14.5 \), the initial thickness is \( h_0 = 10^{-3} M \) [1]. In the numerical simulation, it was assumed that the stress-strain behaviour along the forming shell is homogeneous. Edge effects caused by boundary conditions set at the ends of the shell were not considered. A shell cross-section was the computational region. The initial deflection was given in the form \( \Delta R_0 = A h_0 \cos n\beta \leq \beta \leq 2\pi \). To analyse the calculations results the following symbols are introduced:

\[
\begin{align*}
\bar{w} &= \frac{1}{L} \sum_{i=1}^{N} u^L_i, \\
\hat{w} &= -\frac{w}{h_0}, \\
N_s &= \int_{0}^{h} \sigma_{ss} d\xi, \\
\hat{N} &= -\frac{N_s}{\sigma_T h_0}.
\end{align*}
\]

Where \( w \) is the average deflection, \( L \) is the difference mesh nodes number in the circumferential direction, \( u^L_i \) is the deflection in the node number \( i \), \( N_s \) is circumferential stress, \( \sigma_T \) is yield stress, \( \hat{w} \) and \( \hat{N} \) are dimensionless average deflection and circumferential load.

Calculations have shown that there is a nonaxisymmetric stability loss in the second mode in the circumferential direction when the hollow cylindrical shell is loaded with external pressure. Before both elastic and elastoplastic shell stability loss, a linear increase in the circumferential load is observed until the pressure reaches a critical value \( p^* \) and then there is a steep decline. At the moment of buckling, the relative average deflection value \( \hat{w} \leq 0.02 \), that is the stability loss occurs at small elastoplastic deformations. It is established that for an elastic shell the critical load value \( p^* \) does not depend on the initial geometric imperfections amplitude \( A \). In elastoplastic deformation, the critical load \( p^* \) significantly depends (up to 20 %) on the initial deflection amplitude \( A \) in the range from \( 10^{-5} \) to \( 10^{-2} \) of the thickness \( h_0 \).

We studied the load critical value dependence (external pressure) and the buckling mode of the elastoplastic shell with an elastic filler on the rigidity of the filler at different initial imperfection amplitude \( A \). For each parameters pair \( (A, k) \), a series of calculations were performed when specifying the initial imperfections \( n = 2, 3 \) or 4. In each calculation series, a variant was chosen in which the critical load \( p^* \) is minimal, that is, the stability loss occurs earlier than in others \( n \). The research results are presented in figures 1, 2. Figure 1 shows the dependences of the dimensionless average deflection \( \hat{w} \) (a, b) and circumferential force \( \hat{N} \) (c, d) on the load (external pressure) \( p \) for the initial imperfection \( A = 10^{-5} \) (a, c) and \( A = 10^{-2} \) (b, d). Figure 2 shows the dependence of the critical (maximum) load value on the Winkler coefficient \( k \) at different initial imperfection amplitude values \( A \). Points indicate the computation variants in which the stability loss occurs in the second mode, squares – in the third, and triangles – with a local bulge formation. The critical load \( p^* \) increases with the Winkler coefficient \( k \) and decreases with the initial imperfection amplitude \( A \).

In the calculations, the contact pressure value acting from the filler side to the shell is significantly (by two orders of magnitude) lower than the specified external pressure value. Until the shell loses stability, the contact pressure distribution is close to homogeneous. In the process of stability loss, a local separation of the shell from the filler occurs in the bulging zones, and the distribution of contact pressure becomes inhomogeneous.
Figure 1. Dependence of the average deflection $\hat{w}(a, b)$ and circumferential force $\hat{N}(c, d)$ on the load (external pressure) $p$ at the magnitude of the initial imperfection amplitude: $A = 10^{-5}$ (a), $A = 10^{-2}$ (b) and Winkler coefficient values $k = 0$ (curves 1), $k = 10 \text{ MPa/m}$ (2), $k = 1000 \text{ MPa/m}$ (3), $k = 2000 \text{ MPa/m}$ (4), $k = 5000 \text{ MPa/m}$ (5), $k = 10000 \text{ MPa/m}$ (6).

Figure 2. Load critical (peak) value dependence $p^*$ on the Winkler coefficient $k$ under the initial imperfection amplitude of the elastic-plastic shell $A = 10^{-2}$ (curve 1), $A = 5 \cdot 10^{-3}$ (2), $A = 10^{-3}$ (3), $A = 10^{-4}$ (4), $A = 10^{-5}$ (5).
Conclusions
At quasistatic loading by external pressure buckling of elastoplastic hollow and filled shells with \( R_0/h_0 = 14.5 \) occurs at small elastoplastic deformations (conditional circumferential deformation does not exceed 2%). The contact pressure arising from the elastic filler interaction and the shell is approximately by two orders of magnitude less than the external pressure value specified in the calculations. The initial deflection amplitude and the rigidity of the elastic filler affect the stability loss mode of the elastoplastic shell. With the Winkler coefficient increase, the number mode of the stability loss changes from the second to the third, at an elastic filler high rigidity, a local bulge is formed. At the same time, the critical load value increases. When the initial imperfection amplitude increases, the critical load value decreases.

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References
[1] Bazhenov V G, Baranova M S, Kibets A I, Lomunov V K and Pavlenkova E V 2010 Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki 152 (4) 86–105 (in Russian)
[2] Abakumov A I, Kvaskov G A, Novikov S A, Sinitsin V A and Uchayev A A 1988 J. of Appl. Mechanics and Technical Phys. 3 150–153 (in Russian)
[3] Bazhenov V G and Lomunov V K 1975 Prikladnye probemny prochnosti i plasticnosti: Vsesoyuznyi mezhvuzovskii sbornik. Gor'kovskii universitet 2 44–50 (in Russian)
[4] Nagornykh E V, Bazhenov V G, Kibets A I and Zhegalov D V 2019 XXX International Innovation-Oriented Conference of Young Scientists and Students (MIKUMS - 2018): Proc. conf. (Moscow, November 20-21, 2018) (Moscow: Publishing House IMASH RAS) pp 149-152
[5] Artem'eva A A, Bazhenov V G, Nagornykh E V, Kazakov D A and Kuzmicheva T V 2017 PMM J. of Appl. Mathematics And Mechanics 81 (5) 420–428
[6] Farhat C, Wang K G, Main A, Kyriakides S, Lee L H, Ravi-Chandar K, Belytschko T 2013 Int. J. of Solids and Structures 50 2943–61
[7] Shamass R, Alfano G and Guarracino F 2015 Thin-Walled Structures 95 347–362
[8] Volf'mir A S 1967 Stability of Deformable Systems (Nauka, Moskow) (in Russian)
[9] Ivanov V A 2011 Vestnik Kazanskogo tekhnologicheskogo universiteta 8 224–228 (in Russian)
[10] Gonik E G, Kibets A I, Petrov M V and Fedorova T G 2013 Problemy prochnosti i plasticnosti 75 (3) 215–220 (in Russian)
[11] Eksi S, Kapti A O and Genel K 2017 Acta Phys. Pol. A No. 3-II 132 875-878
[12] Il'gamov M A, Ivanov V A and Gulin B V 1977 Strength, Stability and Dynamics of Shells with Elastic Filler (Moskow: Nauka) (in Russian)
[13] Gao K, Gao W, Wu D and Song C 2018 J. of Sound and Vibration 415 147–168
[14] Lugovoy P Z and Prokopenko N Ya 2015 Int. Appl. Mechanics 51 (5) 116–124 (in Russian)
[15] Shaterzadeh A R and Foroutan K 2017 J. of Solid Mechanics 9 (4) 849-864
[16] Nobili A, Radi E and Lanzoni L 2014 J. Eur. Ceram. Soc. 34 (11) 2737–44
[17] Sato M, Wadee M A, Iiboshi K, Sekizawa T and Shima H 2012 Int. J. of Mechanical Sciences 59(1) 22-30