Nonadiabatic multichannel dynamics of a spin-orbit coupled condensate

Bo Xiong\textsuperscript{1,2}, Jun-lui Zheng\textsuperscript{1}, and Daw-wei Wang\textsuperscript{1,3}

\textsuperscript{1}Department of Physics, National Tsing-Hua University, Hsinchu, Taiwan 300, Republic of China
\textsuperscript{2}Institute for Advanced Study, Tsinghua University, Beijing 100084, China
\textsuperscript{3}Institute of Physics, National Center for Theoretical Science, Hsinchu, Taiwan 300, Republic of China

(Dated: October 31, 2014)

We systematically investigate the nonadiabatic dynamics of a driven spin-orbit coupled Bose-Einstein condensate, and find that the standard Landau-Zener (LZ) tunneling fails in the regime of weak driven force and/or strong spin-orbital coupling. We show both analytically and numerically that the full nonadiabatic dynamics requires a new mechanism through multichannel quantum interference, which is beyond semi-classical approach and provides an oscillating power-law decay in the long time limit. Furthermore, the condensate density profile is found to be dynamically fragmented by the multichannel effects and enhanced by interaction effects. Experimental indication of these nonadiabatic dynamics is discussed.

PACS numbers: 03.75.Lm, 33.80.Be

I. INTRODUCTION

The experimental realization of synthetic gauge field and spin-orbit (SO) coupling in neutral quantum gases paves the way for studying some exotic many-body physics, including spin-Hall effect, Majorana fermions, and topological insulator. The ground state of a SO coupled condensate has been extensively studied and found to have many interesting properties not observed in conventional condensates due to the presence of external coupling field. Recently the condensate dynamics are investigated experimentally in the presence of SO coupling, which can be controlled in highly flexible parameter range by the external field. This provides an opportunity to investigate the nonadiabatic dynamics, which may not easily be controlled or investigated in the conventional solid state system.

From the theoretical point of view, nonadiabatic dynamics of a many-body system are important but in general highly challenging, because very few analytic or numerical tools are available and the associated results are conventionally non-universal. Nevertheless, there are still some theoretical works along this direction, e.g., universal adiabatic dynamics near the regime of quantum critical point, quench-induced phase transition in the quantum Ising model, and nonadiabaticity induced by fluctuations in a driven Landau-Zener system. By far, the most important and controllable system in experiments for studying nonadiabatic dynamics is probably the system of a Bose-Einstein condensate, which has a macroscopic occupation in a single particle state and hence can be understood by the celebrated Landau-Zener (LZ) mechanism (for example, the inter-band tunneling in a tilted optical lattice). However, the dynamics can be much more complicated when a SO coupling accesses the condensate, where the tunneling rates for particles in different momentum channels are distinct. How these quantum interference between different momentum channels competes with the single channel LZ dynamics as well as the interaction effect thus becomes an intriguing and important question.

In this paper, we investigate the nonadiabatic dynamics of a SO coupled condensate driven through the avoid crossing point opened by spin-orbital coupling, including the multichannel effects in momentum space. We find that the full dynamics is controlled by the relative amplitude of the initial driven kinetic energy compared to strength of spin-orbital coupling: When the kinetic energy dominates, the condensate dynamics can be approximated by a semi-classical “particle” in the momentum space, leading to a single-channel Landau-Zener tunneling. However, in the regime of strong spin-orbital coupling, the nonadiabatic dynamics is dominated by the quantum interference effects through multichannel coherent tunneling in momentum space, leading to a universal oscillating power-law decay. We further find a dynamical fragmentation of the condensate when the nonadiabatic dynamics evolves for a longer time, which is also resulted from the SO coupling and enhanced by inter-particle interaction. All of these novel features of nonadiabatic dynamics can be observed in the current experimental condition.

In Sec. II, we will first introduce the underlying many-body system Hamiltonian for a SO coupled condensate. Analytic studies and numerical results for the nonadiabatic tunneling dynamics in noninteracting regime by multichannel interference are shown in Sec. III while the results in the semi-classical limit is in Sec. IV. In Sec. V we show the interaction effects toward to dynamics and we summarize our results in Sec. VI.

II. MODEL AND HAMILTONIAN

We consider the quantum tunneling problem of an interacting condensate with SO coupling of equal Rashba and Dresselhaus types, described by the Hamiltonian: $\hat{H} = \hat{H}_\text{SO} + \hat{H}_\text{trap} + \hat{H}_\text{int}$. The SO coupling term is...
\[ \hat{H}_{SO} = \sum_{k} \left[ \hat{\Psi}^{+}_{+}(k) \hat{\Psi}^{+}_{-}(k) \right] H_0 \left[ \hat{\Psi}^{-}_{+}(k) \hat{\Psi}^{-}_{-}(k) \right] \]

\[ \hat{H}_{\text{trap}} = -\frac{m\omega^2}{2} \sum_{k,\sigma} \hat{\Psi}^{\dagger}\sigma(k) \nabla^2 \hat{\Psi}_{\sigma}(k), \]

and the interacting term

\[ \hat{H}_{\text{int}} = \frac{g_{||}}{V} \sum_{k,k',q} \hat{\Psi}^{\dagger}_{+}(k) \hat{\Psi}^{\dagger}_{-}(k') \hat{\Psi}_{-}(k' - q) \hat{\Psi}_{+}(k + q) + \frac{g_{\perp}}{2V} \sum_{k,k',q,\sigma} \hat{\Psi}^{\dagger}_{\sigma}(k) \hat{\Psi}^{\dagger}_{\sigma}(k') \hat{\Psi}_{\sigma}(k' - q) \hat{\Psi}_{\sigma}(k + q). \]

Here \( \sigma_{x,y,z} \) are the spin-1/2 representation of Pauli matrix and \( \sigma = \pm \) denotes the two spin states, which are coupled by two lasers in an effective field. The trapping potential \( \hat{H}_{\text{trap}} \) is the single-spin interaction strength \( \Omega \) and the trapping frequency \( \omega \) that we will use to avoid crossing.

The system Hamiltonian described above has been experimentally realized recently (e.g., see [4, 13]), and the associated level diagram is shown in Fig. 1 (a).

Without trapping potential and the interaction, the single particle Hamiltonian, \( \hat{H}_{\text{SO}} \), can be easily diagonalized and displays a two-band structure, where the Raman coupling opens a finite band gap as shown in Fig. 1 (b). The typical LZ dynamical problem we will consider is illustrated in Fig. 1 (b). The initial wavefunction is prepared in the \(|+)\rangle \) state. The harmonic trap potential \( \hat{H}_{\text{trap}} \) can drive the condensate to move toward the anti-crossing point and lead to a change of internal spin state due to SO coupling. If the dynamics is adiabatic, all particles shall stay in the upper band and are changed into the \(|-)\rangle \) state. However, due to the nonadiabatic tunneling between the two energy eigenstates, a certain amount of particles are linking to the lower energy level. Furthermore, the condensate wavefunction may also affected during the process, as we will show further below.

### III. Quantum Limit: Multichannel Interference

To investigate the full quantum dynamical problem in such SO system, we first consider the noninteracting system, i.e., \( g_{||} = g_{\perp} = 0 \). Even in such simple case, the nonadiabatic decay of the condensate can be still caused by two distinct mechanisms: the multichannel quantum interference due to the spin-orbital coupling and the temporal variation of the quantum number (here it refers to the momentum). We will show that the former leads to a special universal power-law decay in the long time limit, while the later can be classified into the celebrated Landau-Zener mechanism in the semi-classical limit. For simplicity, we consider 1D case with zero detuning \( (\delta = 0) \), while our result should be qualitatively applicable in higher dimensional systems or with a finite detuning.

#### A. Wavefunction in momentum space

In the noninteracting limit and when the external harmonic trapping potential is negligible compared to the SO coupling, the energies of the SO coupling system subject to the Hamiltonian \( \hat{H} \) reads

\[ \Lambda_{\pm}(k) = E_k + E_r \pm \frac{1}{2} \Omega_k, \]

where \( \Omega_k \equiv \sqrt{16E_kE_r + \Omega^2} \) with the kinetic energy \( E_k \equiv k^2/2m \) and the recoil energy \( E_r \equiv k_t^2/2m \). Correspondingly, at zero temperature, the dynamics of the system wavefunction, \( |\Psi(t)\rangle \), can be calculated by transforming to the eigenstate basis for evolution. In principle, similar calculation can be performed in finite temperature case with a canonical ensemble, but here we will concentrate on zero temperature case. The exact expression of final wavefunction, \( |\Psi(t)\rangle = \sum_{k,\sigma} \Psi_{\sigma}(k, t)|\sigma, k\rangle \), can be obtained by calculating \( \Psi_{\sigma}(k, t) \).

After some straightforward algebra by assuming all particles are initially prepared in \(|+)\rangle \) state (i.e.,
i.e., $\Psi$ function is uniformly distributed in the momentum space, first consider a generic example, where the initial wave-momentum distribution, respectively for $\Omega = 0$.

$|\Psi(k,t)|^2$ has a sharp valley at $k$ wavefunction at different momentum components respectively with an initial condition of a uniform distribution ($k_F = 0.2 k_r$). (a) and (b) are for noninteracting system, while (c) and (d) are from the same calculation with $g_1 = g_\perp = 0.0002 E_r / k_r$. Here $\Omega/E_r = 0.1$ and $N = 1000$ for both calculation.

\[ |\Psi(k,t)|^2 = \left[ \frac{e^{-i\Lambda_+(k)t}}{1 + e^{-2\Lambda_0 k}} + \frac{e^{-i\Lambda_-(k)t}}{1 + e^{2\Lambda_0 k}} \right] |\Psi(k,0)|^2 \quad (4) \]

\[ |\Psi(k,t)|^2 = \frac{\text{sech}\theta_k}{2} \left[ e^{-i\Lambda_+(k)t} - e^{-i\Lambda_-(k)t} \right] |\Psi(k,0)|^2 \quad (5) \]

where we define $\cosh \theta_k \equiv \sqrt{1 + 16 E_k E_r / \Omega^2} = \Omega k / \Omega$. Thus, the corresponding probability to find a particle remaining in the same spin state ($|+\rangle$) and in the other state ($|-\rangle$) is given by:

\[ P_+(t) = \frac{1}{N} \sum_k \left[ 1 - \frac{\Omega^2}{\Omega_k^2} \sin^2(\Omega k t / 2) \right] |\Psi_+(k,0)|^2 \quad (6) \]

\[ P_-(t) = \frac{1}{N} \sum_k \frac{\Omega^2}{\Omega_k^2} \sin^2(\Omega k t / 2) |\Psi_+(k,0)|^2 \quad (7) \]

with $N = \sum_k |\Psi_+(k,0)|^2$ is the total number of particles.

Above results show that the noninteracting condensate wavefunction at different momentum $k$ has different oscillation frequency as well as the oscillation amplitude. In Fig.2 (a) and (b), we show the time evolution of the momentum distribution, $|\Psi_+(k,t)|^2$ and $|\Psi_-(k,t)|^2$ respectively for $\Omega = 0.1 E_r$. Without loss of generality, we first consider a generic example, where the initial wavefunction is uniformly distributed in the momentum space, i.e., $|\Psi_+(k,0)|^2 = \sqrt{N / 2k_F}$ for $|k| \leq k_F$ and 0 otherwise. Fig.2 (a) shows that in the short time regime, $|\Psi_+(k,t)|^2$ has a sharp valley at $k = 0$ due to coherently transferring the atoms in $|+\rangle$ state to the $|-\rangle$ state. For a sufficiently long time, a Fresnel interference pattern appears in both bands owing to the momentum-dependent Rabi oscillation term $\sin(\Omega k t)$, giving the width of the central peak to be $\Delta k_k / k_r \sim \sqrt{\pi \Omega / 8 r^2 E_r}$ [by setting $(\Omega k = \Delta k_k - \Omega k = 0)t \sim \pi$ ]. The momentum dependent oscillation frequency, $\Omega_k$, is proportional to $|k|$ in large momentum regime. Therefore, we can expect the temporal spatial distribution of the condensate wavefunction may have significant changes due to the fast oscillating phase of the large momentum regime. This will be shown more clearly below.

**B. Dynamical fragmentation of condensate wavefunction**

Besides the distribution in momentum space, the temporal distribution in real space also manifest interesting properties of multichannel interference. Precisely, the wavefunction in $|-\rangle$ state by means of the Fourier transform of Eq.4 yields,

\[ \Psi_-(x,t) = \int \frac{dk}{2\pi} e^{ikx} \frac{\text{sech}\theta_k}{2} \left[ e^{-i\Lambda_+(k)t} - e^{-i\Lambda_-(k)t} \right] \Psi_+(k,0) \quad (8) \]

It is stressed that $\text{sech}\theta_k = \Omega / \Omega_k$ is peaked at $k = 0$ with a width $\Delta k_k$ when $16 E_\Delta E_r / \Omega^2 \sim 1$ or $\Delta k_k \sim m \Omega / 2 k_r$.

Consequently, the integral is dominantly contributed by small $k$ regime, i.e., $|k| < \Delta k_k$. On the other hand, in the long time limit, $t \rightarrow \infty$, the time-dependent phase term $e^{-i\Lambda_\pm(k)t}$ performs fast oscillation if $\Lambda_\pm(k) \neq 0$. Hence the major contribution of the integral is arisen from $\Lambda_-(k) \sim 0$; in other words, $k = \pm k_1, \pm k_2$ where $k_{1,2} \equiv k_r \sqrt{1 \mp \Omega / 2 E_r}$ according to the zeros of $\Lambda_-(k)$ (Note $\Lambda_+(k) \sim 1 / 2$ is the upper band, see Fig.2 (a) and (d)). On basis of two analysis above, we find that in the weak SO coupling regime, $\Omega < 2 E_r$, $\Psi_-(x,t)$ is mostly contributed from $k \sim \pm k_1 = \pm k_r \sqrt{1 - \Omega / 2 E_r}$, and the condensate prefers to be separated by moving in two opposite directions with velocities, $\pm v_1$, which is the dispersion velocity near $k_1$ (see Fig.2 (c)). For strong SO coupling, $\Omega > 2 E_r$, $\pm k_1$ disappear and $\pm k_2$ are in general much larger than the initial momentum distribution of the condensate (see Fig.2 (d)). While all the phase in each momentum channels oscillates fast, the distinction of oscillation frequencies in different momentum channels is greatly suppressed. Therefore, the real-space condensate wave stops moving away and, and its amplitude varies fast but the configuration remains well in contrast to weak SO coupling (see Fig.2 (f)).

According to weak SO coupling, the situation for $|+\rangle$ state is distinct from $|-\rangle$ state, as shown in Fig.2 (b): for $\Omega = 0.1 E_r$ the condensate of $|+\rangle$ state moves in the positive $x$ direction and gradually becomes fragmented as time goes on. This can be well interpreted from the
The condensate wavefunction in the long time limit is from the selective, for strong SO coupling so that the dynamic behavior for momentum-dependent frequencies are suppressed greatly with different velocities in different space. However, the channels, which makes the condensate starts fragmented, there are still finite contribution from other momentum frequencies become more manifest for weak SO coupling, proportional to $(1 + e^{2i\theta})^{-1}$, which is finite only in the negative $k$ regime. As a result, most contribution of the condensate wavefunction in the long time limit is from $k \sim -k_1$, which gives a positive velocity of the condensate, $v_1$. Since the momentum-dependent oscillation frequencies become more manifest for weak SO coupling, there are still finite contribution from other momentum channels, which makes the condensate starts fragmented with different velocities in different space. However, the momentum-dependent frequencies are suppressed greatly for strong SO coupling so that the dynamic behavior for $|\Psi_+(x,t)|^2$ is highly similar to $|\Psi_-(x,t)|^2$ for large $\Omega$ (see Fig.3 (c)).

C. Universal survival probability, $P_r(t)$

It is preferable and relevantly easy for experimentalists to measure the nonadiabatic dynamics of SO coupling BEC by means of counting the number of particles in the different spin states. Since $P_{-}(t) = 1 - P_{+}(t)$, we merely investigate the analytic expression of the survival probability $P_{+}(t)$ in both short-time and long-time limit.

As $t \to 0$, the asymptotic expression of Eq.(6) shows $P_{+}(t) = 1 - \Omega^2 t^2 / 4 + O(t^4)$, which matches a typical Rabi oscillation. In the long time limit, for simplicity, we just consider the generic case, where the initial density distribution in momentum space is a constant up to $|k| < k_F$ as above. We will show below that more complicated initial wavefunction will only quantitatively affect the asymptotic behavior and can be easily adjusted by a single fitting parameter. Within such simplification of uniform initial wavefunction, we can evaluate the summation in the momentum channel by integration and derive the leading order expression in the long time limit by considering the major contribution only. After some calculation (see Appendix A for details), we find that:

$$P_{+}(t) = 1 - \frac{\alpha}{8} \tan^{-1} \left( \frac{4}{\alpha} \right) + \frac{\sqrt{2\pi\alpha \cos(\Omega t + \pi/4)}}{16\Omega} + \frac{\alpha^2 \sin(\Omega k_F t)}{32 \Omega^2} + O(t^{-3/2})$$

where $\alpha \equiv \Omega / \sqrt{E_F E_r}$ is a dimensionless parameter and $E_F \equiv E_{k_F}$ is the “effective Fermi energy”.

Again, the dominant contribution comes from the momentum near $\pm k_{1,2}$ according to the oscillation of the phase factor. In contrast to Eq.(5), the weighting function behaves differently: the second term above is proportional to $(1 + e^{2i\theta})^{-1}$, which is finite only in the negative $k$ regime. As a result, most contribution of the condensate wavefunction in the long time limit is from $k \sim -k_1$, which gives a positive velocity of the condensate, $v_1$. Since the momentum-dependent oscillation frequencies become more manifest for weak SO coupling, there are still finite contribution from other momentum channels, which makes the condensate starts fragmented with different velocities in different space. However, the momentum-dependent frequencies are suppressed greatly for strong SO coupling so that the dynamic behavior for $|\Psi_+(x,t)|^2$ is highly similar to $|\Psi_-(x,t)|^2$, which is highly similar to $|\Psi_-(x,t)|^2$ for large $\Omega$ (see Fig.3 (c)).
(i) We can find that the multichannel quantum interference effects can also lead to a "decay" with an oscillating amplitude, which is a special feature of SO coupled condensate due to the momentum dependence of the Rabi oscillation frequency, $\Omega_k$. (ii) The saturated value, $P_+(\infty) = 1 - \frac{4}{\pi} \tan^{-1} \left( \frac{1}{\alpha} \right)$, approaches 1/2 as expected for a single mode Rabi oscillation if $E_F < \Omega (\alpha \to 0)$. On the other hand, it reaches $P_+(0) = 1$ if $E_F \gg \Omega (\alpha \to 0)$, showing that the energy band width (uncertainty in energy) can reduce the many-body quantum tunneling through interference effects. (iii) These results also apply to noninteracting fermions, since Pauli exclusion principle requires $\Psi_+(k,0) = 1$ for $|k| \leq k_F$ and the sign of wavefunction exchange should not affect the probability to find a particle in any spin channel. (iv) For a general initial wavefunction, $\Psi_+(k,t)$, the long-time behavior of $P_+(t)$ can be still applied except one can use $\alpha$ as a fitting parameter, characterizing the energy uncertainty of the initial wavefunction in the momentum space. In Fig. 1 we show the time dependence of the survival probability, $P_+(t)$, as a function of time for both uniform distribution and a Gaussian distribution in the initial wavefunction. Results from the analytic expression (Eq. (11)) are also shown together (the value of $\alpha$ for the Gaussian distribution is given by single parameter fitting). They both agree with the numerical results very well as $t > \Omega^{-1}$.

IV. CLASSICAL LIMIT: LANDAU-ZENER TUNNELING

Now we consider another region in which the "kinetic energy" arisen from the external trapping potential dominates the nonadiabatic dynamics. Here we will employ a semi-classical approach in this region and show its connection with the well-known Landau-Zener effect. In the semi-classical limit, one can treat the center-of-mass position, $x(t)$, and momentum, $k(t)$, as a classical particle at time $t$, and neglect the spatial or momentum distribution induced by the condensate wavefunction. As a result, the system dynamics can be described by a two-component state, $[\psi_+(t), \psi_-(t)]^T$, which is controlled by the Hamiltonian as Eq. (1) in the moving frame of a momentum $k(t)$:

$$i \hbar \frac{d}{dt} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} = \frac{\hbar^2}{2m} \begin{bmatrix} \frac{\Omega}{2} & \Omega/2 \\ \Omega/2 & \frac{\hbar^2}{2m} \end{bmatrix} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix},$$

where the "quasi-momentum" $k(t)$ becomes a time-dependent external parameter, controlling the time-dependence of the single-particle energies as a typical LZ-type problem. If the condensate is initially prepared in momentum state $k(0) = -k_0 < 0$ of the $|+\rangle$ state (see Fig. 1(b)), the time-dependence of $k(t)$ in the semi-classical approximation fulfills $k(t) = -(k_0 + k_r) \cos(\omega t) + k_r$ in a harmonic potential well.

Following the standard treatment of LZ tunneling, we perform the time variation of $k(t)$ approximately as a linear function of $t$ when the condensate is at $k = 0 (t = t_c)$ point. From $k(t_c) = 0$ we can have $\cos(\omega t_c) = k_r/(k_r + k_0)$. The probability for such a particle to be transferred into the $|-\rangle$ state after a long time has the form, $P_{LZ}(\infty) = \exp \left[ -\frac{\pi \Omega^2}{8 \omega E_r \left( (k_0/k_r)^2 + 2(k_0/k_r) \right)} \right]$. (11)

It is explicitly shown that the results of LZ in the semi-classical limit has totally different dependence of Raman coupling strength from the one derived in Eq. (11) in the quantum limit.

Fig. 5 shows typical results of $P_+(\infty)$ from numerically solving Eq. (12) and compare with the semiclassical analysis derived above. Our results show that the LZ formula matches the full numerical results very well in the small coupling regime, while it becomes deviated in the large $\Omega$ limit. This might be because, when $\Omega$ is comparable to the recoil energy, $E_r$, the assumption of a linear time dependence of $k(t)$ at the avoid crossing point $k = 0(t = t_c)$ fails seriously by touching the bottom of the energy band (see Fig. 1(b)).

V. INTERACTING SPIN-ORBIT COUPLED BEC

Now we explore the interaction effects on the nonadiabatic dynamics. Following the standard approach by assuming all particles condensate in a single particle wave-
function, i.e. $\hat{\Psi}_\sigma = \Psi_\sigma$, the mean-field Gross-Pitaevskii equation (GPE) can be written as following:

$$i\partial_t \Psi_\pm(k) = \left[\frac{(k \mp k_r)^2}{2m} - \frac{m\omega^2}{2} \frac{\partial^2}{\partial k^2}\right] \Psi_\pm(k) + \frac{\Omega}{2} \Psi_\mp(k)$$

$$+ \frac{g_L}{L} \sum_{k',q} \Psi_+^*(k') \Psi_+(k' - q) \Psi_\pm(k + q)$$

$$+ \frac{g_R}{L} \sum_{k',q} \Psi_-^*(k') \Psi_-^*(k' - q) \Psi_\pm(k + q),$$

where $\sum_{k,\sigma} |\Psi_\sigma(k)|^2 = N$.

There are two temporal effects raised by the contact interaction: the first is the nonlinear dynamics induced by the variation of the mean-field energy, and the second is the re-arrangement of the particle density distribution due to the scattering between different momentum channels. As we will see below, the combination of the SO coupling and interatomic interaction can lead to interesting condensate behavior during the nonadiabatic dynamical process.

### A. Nonlinear dynamics in single channel

**Rabi-oscillation**

While our paper concentrates on the multichannel effects in the momentum space, it is still instructive to compare results of single channel (single mode) dynamics, especially when the interaction is included. The most relevant example is by assuming all particles condensate at $k = 0$ state in the momentum space without external trapping potential. The corresponding meanfield dynamical equation can be written as following:

$$i\partial_t \Psi_\pm = \left[g_\| |\Psi_\pm|^2 + g_\perp |\Psi_\mp|^2\right] \Psi_\pm + \frac{\Omega}{2} \Psi_\mp,$$

which is nothing but a coupled nonlinear equations and we have removed the constant term $E_r \Psi_\parallel$. Here $\Psi_\parallel(t) \equiv \Psi_\parallel(k = 0, t)$ is the wavefunction at zero momentum. Since the kinetic energy completely disappear, the SO effects turns off and the Raman coupling term makes the symmetry is not preserved as in most cases with pseudo-spins, the tunneling dynamics changes in various way, depending on the relative strength of the two interactions. In contrast to multichannel tunneling, however, all of these dynamics is coherent and periodic because no decoherence or dissipation is involved in such two component systems. Such simple two-component model can be extended to the case of few components. Interaction merely plays a role of nonlinearity, while it is not the case of multichannel systems as shown in a SO coupled condensate.

### B. Multichannel scattering in quantum interference limit

In addition to the meanfield type energy shift described above, inter-particle interaction can also scatter particles in different momentum channels, which makes the multichannel dynamics much more complicated than the noninteracting case. To get first insight, we begin by exploring the interaction effect on the condensate dynamics without trapping potential (i.e. no driven force).

Fig.4(c) and (d) display the temporal momentum distribution with a finite interaction strength. In the short-time limit, the repulsive interaction certainly broadens the initial uniform distribution to momentum larger than $k_F$ so that the initial distribution of jump type around $k_F$ becomes smooth (see Fig.4(c)). At late times, the competition between more pronounced tunneling occurring around $k = 0$ (c.f., see the amplitude term of Eq.13) and the atoms scattered preferably to low-density region lead to the disappearance of the Fresnel oscillation pattern shown in Fig.4(a) and (b).

In Fig.7 we show the survival probability, $P_+(t)$, with respect to various interaction strengths. Compared to the oscillating power-law decay in noninteracting case (see Fig.4), an additional time scale for the decay of $P_+(t)$ as well as arriving at dynamic equilibrium state is required. Specifically, in the initial “collapse” of $P_+(t)$, i.e., $t < \pi/\Omega$, the repulsive interaction scatters the atoms into high-momentum channels where the tunneling amplitude is approximately reduced with $1/k^2$ (c.f., see Eq.13) and thereby it suppresses the whole tunneling. This is manifested in Fig.7 where for $t < \pi/\Omega$, $P_+(t)$ for interacting case is conventionally larger than for noninteracting case. Afterwards, due to SO coupling, the more number of atoms tunneling around low-momentum regime
than high-momentum regime results in a low-density region around \( k = 0 \) (see the valley of \( |\Psi_+(k, t)|^2 \) shown in Fig. 3 (c)), while the interaction prefers to scatter atoms into low-density region. Consequently, the repulsive interaction enhances the whole tunneling of the system, and \( P_+(t) \) for interacting case is always smaller than for noninteracting case for \( t > 2\pi/\Omega \), as shown in Fig. 4.

C. Interaction effect in classical limit

In this part, we explore the interaction effects on a general driven SO coupled BEC. As shown above, there are two fundamentally distinct mechanism for the nonadiabatic dynamics: the multichannel interference in the quantum limit and the LZ effects in the semi-classical (single channel) regime. In the later, one treats the whole condensate as a classical particle with definite position and momentum at the same time, so that the quantum oscillation effects are neglected. However, in any realistic situation, the condensate always has a finite distribution in momentum space, and hence the inter-particle interaction can still scatter particles between momentum channels and hence mix these two mechanisms. Since such complicated dynamics cannot be readily studied by analytic approach, here we show the numerically simulating results by solving the full GP equations within the meanfield approximation.

In Fig. 5 we show a typical tunneling dynamics of an interacting driven condensate with SO coupling. As we can see, when the condensate wavefunction approach the anti-cross point (i.e., \( k = 0 \)), particles start tunneling to the \( |−> \) state. For the noninteracting case, the original Gaussian shape in both \(|+>\) and \(|−>\) channels is basically kept, because the noninteracting Hamiltonian (see Eq. 1) is just like a particle moving a simple harmonic potential in momentum space as \((k \pm k_r)/2m \gg \Omega\) in the long time limit, making each component of a condensate oscillating with the same frequency. This is true even when part of the condensate are split into the other spin state through SO coupling.

However, in comparison with the noninteracting condensate, the density profile of the condensate with a finite interaction strength displays in different way as it passes the critical point. When the interaction is increased from zero, the condensate density distribution becomes highly distorted after passing through the critical point: the head of the condensate is compressed to be much narrower peak, while the tail is destroyed without any smooth profile. Such results can be understand from the scattering between multichannel momenta: the condensate broadened from its initial profile by the repulsive interaction during the motion (note that the condensate profile is not broaden in momentum space if no interaction), so that the “velocity” of the wave head is faster than the “velocity” of the wave tail, making much more particles tunneling into the \(|+>\) state in the tail part. The small density oscillation shows the interference effect of such two “velocity” in momentum space. We propose that this special feature of the condensate distortion can be a feature of many-body effects and can be also observed in current experiment setup [4, 13].

FIG. 7. Illustration of the influence of contact interaction on multichannel tunneling by means of measuring the survival probability, \( P_+(t) \), for the uniform distribution of initial wave function in momentum distribution (\( k_F = 0.2k_r \)), \( g_\parallel = g_\perp = 0 \) for the black solid line, \( g_\parallel = g_\perp = 0.0002/k_r \) for the red dots, \( g_\parallel = g_\perp = 0.0008/k_r \) for the blue cross, and \( g_\parallel = g_\perp = 0.001/k_r \) for the green dashed line. The other parameters are identical to Fig. 3. The points at time \( t = \pi/\Omega \) and \( 2\pi/\Omega \) are labeled by the lower left and upper right arrows, respectively.

FIG. 8. Temporal momentum density distribution of a condensate driven by external potential with an initial momentum, \( k_0 = −k_r \). Red dashed and black solid lines display the density distribution for \(|+>\) and \(|−>\) states, respectively, at time \( tE_r = 0 \) (a), 2.4 (b), 12 (c), and 19.6 (d). The associated parameters are \( \Omega/E_r = 1.6 \), \( g_\parallel = g_\perp = 0.001E_r/k_r \), and \( N = 1000 \). For comparison, the inserts in (a), (b), (c), and (d) are the momentum distribution at the same time under identical parameters but without interaction.
VI. EXPERIMENTAL PARAMETERS AND SUMMARY

In principle, any geometry and dimension of the condensates with SO coupling are applicable for exploring experimentally multichannel tunneling dynamics and interaction-induced collapse. We propose a one-dimension (1D) or quasi-1D configuration, which is probably optimal for experimentists to observe the phenomena by controlling and adjusting physical parameters fewer than higher dimension, in accordance with our calculation. We consider a $^{87}$Rb BEC with the $s$-wave scattering length $a = 5.4 \text{mm}$, confined in the harmonic potential with the transverse frequency, $\omega_y = 2\pi \times 1000 \text{Hz}$ and the longitudinal frequency $\omega_z = 2\pi \times 160 \text{Hz}$. Supposing that Raman coupling laser has a wavelength $\lambda$, in our calculation, as the wavefunction fragmentation due to multichannel tunneling, so that the complete nonadiabatic dynamics is much more than higher dimension, in accordance with our calculation. We consider a $^{87}$Rb BEC with the $s$-wave scattering length $a = 5.4 \text{mm}$, confined in the harmonic potential with the transverse frequency, $\omega_y = 2\pi \times 1000 \text{Hz}$ and the longitudinal frequency $\omega_z = 2\pi \times 160 \text{Hz}$. Supposing that Raman coupling laser has a wavelength $\lambda = 804 \text{nm}$, the quasi-1D effective scattering amplitude is approximately $g_{1D} = 2\hbar a \sqrt{\omega_y \omega_z} \approx 0.02 E_r/k_r$.

In our calculation, $g_{\parallel} = g_{1D}/2\pi$ so that $g_{\perp} = g_{\perp} \approx 0.003 E_r/k_r$. This type of interaction couplings can match our calculation: note that the mean-field dynamics are essentially determined by $g_{\parallel} N$ and $g_{\perp} N$ and thus total particle number $N$ can be also adjusted to control the dynamics.

In summary, we systematically investigate the full dynamics of a driven SO coupled condensate. We show that SO coupling is a special technique to address and to detune the tunneling rate in each momentum channel, so that the complete nonadiabatic dynamics is much richer than the traditional LZ mechanism. The quantum interference between different momentum channels and scattering between them gives a qualitatively new phenomena not reported before. We show how these different mechanism can be addressed in different experimental regime within the same frame work. Our prediction on the power-law decay of the transition rate as well as the wavefunction fragmentation due to multichannel quantum interference can be observed within current experimental set-up.

ACKNOWLEDGMENTS

We thank Hui Zhai, and Yu-Ju Lin for the inspiring discussions. The work was supported by NCTS and MoST in Taiwan. B. X. also acknowledges support of NSFC Grants No. 11347025.

Appendix A: Power-law decay of the survival rate for the condensate with uniform distribution in momentum space

The survival probability of noninteracting particles in the initial spin state yields,

$$P(t) = \frac{1}{N} \sum_k \left[ 1 - \frac{\Omega^2}{\Omega_k^2} \sin^2(\Omega_k t/2) \right] |\Psi_+(k, 0)|^2,$$  \hspace{1cm} (A1)

where $\Omega_k = \sqrt{16E_k E_r + \Omega^2}$ with $E_k \equiv k^2/2m$ and $E_r \equiv k_r^2/2m$.

Considering $\Psi_+(k, 0) = \sqrt{\frac{N}{2\pi}}$ for $|k| \leq k_F$ and otherwise 0, Eq. (A1) can be written into

$$P(t) = \frac{1}{k_F} \int_0^{k_F} dk \left[ 1 - \frac{\Omega^2}{2\Omega_k^2} (1 - \cos(\Omega_k t)) \right]$$

$$= 1 - \frac{1}{8} \int \frac{\Omega^2}{E_F E_r} \tan^{-1} \left( \frac{4k_F t}{\Omega} \right) + A,$$ \hspace{1cm} (A2)

where $\tilde{k} \equiv k/k_F$ and $\tilde{\Omega} = \Omega/E_r$. The critical oscillation part has the form,

$$A = \frac{1}{k_F} \int_0^{k_F} dk \left( \frac{\Omega^2}{2\Omega_k^2} \right) \cos(\tilde{\Omega} \tilde{k} t),$$ \hspace{1cm} (A3)

where $\tilde{t} = tE_r$ and $\tilde{\Omega}_k = \sqrt{16k^2 + \Omega^2}$.

The following procedure is to obtain the analytic expression for $A$ in some proper treatment. We arrange $A$ into

$$A = \frac{\tilde{\Omega}}{8k_F} \left[ F(\tilde{t}, 1) - F(\tilde{t}, a) \right],$$ \hspace{1cm} (A6)

where $a = \sqrt{1 + \left(4k_F/\tilde{\Omega} \right)^2}$. For $F(\tilde{t}, a)$ with $a > 1$, we can expand the square root term and obtain

$$F(\tilde{t}, a) = \int_a^\infty dy \frac{\cos(\tilde{t}y)}{y^2} \left[ 1 + \frac{1}{2} \frac{1}{y^2} + \frac{3}{8} \frac{1}{y^4} + \cdots \right]$$

$$= F_2(\tilde{t}) + \frac{1}{2} F_4(\tilde{t}) + \frac{3}{8} F_6(\tilde{t}) + \cdots,$$ \hspace{1cm} (A7)

where $F_{2n}(\tilde{t}) = \int_{\tilde{t}}^\infty \frac{\cos(\tilde{t}z)}{z^{2n}} dz$. Let $z = \tilde{t}y$, so

$$F_{2n}(\tilde{t}) = \frac{\tilde{t}^{2n-1}}{2} \int_{\tilde{t}}^\infty \frac{\cos^2z}{z^{2n}} dz$$

$$= \frac{\tilde{t}^{2n-1}}{2} \int_{\tilde{t}}^\infty \left( e^{iz - \eta z} + e^{-iz - \eta z} \right) z^{-2n} dz,$$

where $\eta \to 0^+$ is to make the integral convergent.

Since $\int_{\tilde{t}}^\infty z^{-2n} e^{-(\eta + i\tilde{t})z} dz = (\pm i)^{2n} e^{-(\eta \mp i\tilde{t})z} - 2n e^{\pm i\tilde{t}z}$ for $\tilde{t} \to \infty$,

$$F_{2n}(\tilde{t}) = -\frac{1}{\tilde{t}^{2n}} \frac{\sin(\tilde{t}a)}{\tilde{t}a} + O(t^{-2}).$$ \hspace{1cm} (A8)
Correspondingly,

\[ F(\tilde{t}, a) = -\frac{\sin(\tilde{t}a)}{\tilde{t}} \frac{1}{\sqrt{a^2 - 1}} + \mathcal{O}(t^{-2}) \, . \tag{A9} \]

For \( F(\tilde{t}, 1) \) and when \( \tilde{t} \to \infty \), the “high-frequency” wave according to large \( y \) can be approximately neglected and the part of small \( y \), i.e., \( y \to 1 \), dominates the integral. So we can treat approximately

\[ F(\tilde{t}, 1) \approx \frac{1}{\sqrt{2}} \int_1^\infty dy \cos(\tilde{t}y) \sqrt{y - 1} \tag{A10} \]

As \( t \to \infty \), Eq.\( \text{(A10)} \) has a solution,

\[ F(\tilde{t}, 1) = \sqrt{\frac{\pi}{\tilde{t}}} \cos \left( \tilde{t} + \frac{\pi}{4} \right) + \mathcal{O}(t^{-3/2}) \tag{A11} \]

Through Eq.\( \text{(A2)} \), Eq.\( \text{(A6)} \), Eq.\( \text{(A9)} \), and Eq.\( \text{(A11)} \), one can obtain, at \( t \to \infty \),

\[ P(t) = 1 - \frac{\alpha}{8} \tan^{-1} \left( \frac{4}{\alpha} \right) + \frac{\sqrt{2\pi\alpha} \cos(\Omega t + \pi/4)}{16 \sqrt{\Omega t}} + \frac{\alpha^2 \sin(\Omega k_F t)}{32 \Omega t} + \mathcal{O}(t^{-3/2}) \tag{A12} \]

where \( \alpha \equiv \Omega / \sqrt{E_F E_r} \) is a dimensionless parameter to control the decaying amplitude and \( E_F \equiv E_{k_F} \) is the “effective Fermi energy”.

[1] Y.-J. Lin, R. L. Compton, K. Jiménez-García, W. D. Phillips, J. V. Porto, and I. B. Spielman, Nat. Phys. 7, 531 (2011).
[2] Y.-J. Lin, R. L. Compton, K. Jiménez-García, J. V. Porto, and I. B. Spielman, Nature, 462, 628 (2009).
[3] Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman, Phys. Rev. Lett. 102, 130401 (2009).
[4] Y.-J. Lin, K. Jiménez-García, and I. B. Spielman, Nature, 471, 83 (2011).
[5] J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. 92, 126603 (2004).
[6] C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 146802 (2005).
[7] J. D. Sau, R. M. Lutchyn, S. Tewari, and S. Das Sarma, Phys. Rev. Lett. 104, 040520 (2010).
[8] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. 82, 3045 (2010).
[9] V. Galitski and I. B. Spielman, Nature, 494, 49 (2013).
[10] C. J. Wang, C. Gao, C. M. Jian, and H. Zhai, Phys. Rev. Lett. 105, 160403 (2010).
[11] T.-L. Ho and S. Z. Zhang, Phys. Rev. Lett. 107, 150403 (2011).
[12] Q. Zhou and X. L. Cui, Phys. Rev. Lett. 110, 140407 (2013).
[13] A. J. Olson, S.-J. Wang, R. J. Niffenegger, C.-H. Li, C. H. Greene, and Y. P. Chen, Phys. Rev. A 90, 013616 (2014).
[14] A. Polkovnikov, Phys. Rev. B 72, 161201(R) (2005).
[15] W. H. Zurek, U. Dorner, and P. Zoller, Phys. Rev. Lett. 95, 105701 (2005).
[16] A. Altland, V. Gurarie, T. Kriecherbauer, and A. Polkovnikov, Phys. Rev. A 79, 042703 (2009).
[17] L. D. Landau, Phys. Z. Sowjetunion 2, 46 (1932).
[18] C. Zener, Proc. R. Soc. London, Ser. A 137, 696 (1932).
[19] E. C. G. Stueckelberg, Helv. Phys. Acta 5, 369 (1932).
[20] E. Majorana, Nuovo Cimento 9, 45 (1932).
[21] Biao Wu and Qian Niu, Phys. Rev. B 61, 023402 (2000).
[22] D. Wittmann, E. M. Graefe, and H. J. Korsch, Phys. Rev. A 73, 063609 (2006).
[23] A. Zenesini et al., Phys. Rev. Lett. 103, 090403 (2009).
[24] A. Smerzi, S. Fantoni, S. Giovanazzi, and S. R. Shenoy, Phys. Rev. Lett. 79, 4950 (1997).