Family Non-universal $Z'$ effects on $\bar{B}_q - B_q$ mixing, $B \to X_s \mu^+ \mu^-$ and $B_s \to \mu^+ \mu^-$ Decays

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Abstract

Motivated by the large discrepancy of CP-violating phase in $\bar{B}_s - B_s$ mixing between the experimental data and the Standard Model prediction, we pursue possible solutions within a family non-universal $Z'$ model. Within such a specific model, we find that both the $\bar{B}_s - B_s$ mixing anomaly and the well-known “$\pi K$ puzzle” could be moderated simultaneously with a nontrivial new weak phase, $\phi_s^L \sim -72^\circ$ (S1) or $-82^\circ$ (S2). With the stringently constrained $Z'$ coupling $B_{sb}^L$, we then study the $Z'$ effects on the rare $B \to X_s \mu^+ \mu^-$ and $B_s \to \mu^+ \mu^-$ decays, which are also induced by the same $b \to s$ transition. The observables of $B \to X_s \mu^+ \mu^-$, at both high and low $q^2$ regions, are found to be able to put strong constraints on the $\mu - \mu - Z'$ coupling, $B_{\mu \mu}^{L,R} \sim 10^{-2}$. It is also shown that the combined constraints from $\bar{B}_s - B_s$ mixing, $B \to \pi K$ and $B \to X_s \mu^+ \mu^-$ do not allow a large $Z'$ contribution to the pure leptonic $B_s \to \mu^+ \mu^-$ decay.

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1 Introduction

As particle physics is entering the era of LHC, one may expect direct evidences to be available to establish whether new particles and interactions are present. Meanwhile, high sensitivity studies of low energy phenomena would complement the direct discovery physics at LHC. The flavor changing neutral current (FCNC) processes, such as $b \rightarrow s$ transitions, arise only from loop effects within the Standard Model (SM), and are therefore very suitable for testing the SM and probing its various extensions. Recently, both CDF and D0 collaborations have announced the measurements of CP violation parameters in $B_s$ system, with the obtained CP-violating phase

$$\phi_s = -0.57^{+0.24}_{-0.30} \, \text{(stat)} \pm 0.07 \, \text{(syst)} \quad \text{D0 collaboration} \ [1],$$

$$\phi_s \in [-2.82, -0.32] \, \text{(68\%C.L.)} \quad \text{CDF collaboration} \ [2],$$

while within the SM this phase is expected to be

$$\phi_s^{SM} = -2 \beta_s^{SM} = -2 \ \text{arg} \left[ -V_{ts}V_{tb}^*/(V_{cs}V_{cb}^*) \right] = -2 \times (0.018 \pm 0.001),$$

which deviates from the D0 measurement Eq. (1) by more than $2\sigma$. Combining all the available experimental information on $\bar{B}_s - B_s$ mixing, the UTfit collaboration claims that the divergence of $\phi_s$ between the experiment measurements and the SM prediction is more than $3\sigma$ [3]. Taking into account the deviation of $\phi_s$ in a generic scenario of NP, the CKM-fitter group has found that the SM is disfavored at $2.5\sigma$ [4]. Interestingly, comparing an updated theoretical predication of $B_s - \bar{B}_s$ mixing with D0 [5] and CDF [6] early results based on $1fb^{-1}$ data, the authors of Ref. [7] have found the mixing phase $2\sigma$ deviated from the SM expectation. Such a large observed phase, if still persisting in the upcoming experimental measurements, would indicate a signal of new physics (NP) manifested in $b \rightarrow s$ transitions. In the following numerical analyses, we would use the UTfit results of $\phi_s$ [3] as benchmarks.

Motivated by the above observed anomaly, in this paper we shall pursue possible solutions within a family non-universal $Z'$ model [8], which could be naturally derived in certain string constructions [9], $E_6$ models [10] and so on. Searching for such an extra $Z'$ boson is an important mission in the experimental programs of Tevatron [11] and LHC [12]. Performing constraints on the new $Z'$ couplings through low-energy physics is, on the other hand, very important and
complementary for direct experimental searches. It is interesting to note that, within such a specific scenario, both the CP-violating phase problem and the well-known “πK puzzle” in hadronic $B \to \pi K$ decays could be resolved [13, 14, 15]. Since both the $\bar{B}_s - B_s$ mixing and the $B \to \pi K$ decays involve the same $b - s - Z'$ couplings, it is worthwhile to perform a constraint on these couplings with all the available experimental data taken into account simultaneously. At the same time, we could also get the allowed ranges for flavor-conserving $u - u - Z'$ and $d - d - Z'$ couplings.

The FCNC $b \to s l^+ l^-$ ($l = e, \mu, \tau$) transition, which gives rise to the rare inclusive $B \to X_s \mu^+ \mu^-$ and the purely leptonic $B_s \to \mu^+ \mu^-$ decays, is another important process to probe NP. Averaging the recent experimental data from BABAR [16], Belle [17] and CLEO [18], the Heavy Flavor Averaging Group (HFAG) presents the following total branching ratio [19]

$$B(B \to X_s \mu^+ \mu^-) = (4.3^{+1.3}_{-1.2}) \times 10^{-6}.$$  (4)

As for the ones in the low $(1.0 \text{ GeV}^2 < q^2 < 6.0 \text{ GeV}^2)$ and high $(14.4 \text{ GeV}^2 < q^2 < 25 \text{ GeV}^2)$ $q^2$ regions, after naively averaging the BABAR [16] and Belle [17] measurements, we get respectively

$$B^L(B \to X_s \mu^+ \mu^-) = (1.6 \pm 0.5) \times 10^{-6},$$  (5)
$$B^H(B \to X_s \mu^+ \mu^-) = (0.44 \pm 0.12) \times 10^{-6}.$$  (6)

Theoretically, with the up-to-date input parameters, the SM predictions [20, 21] for the above three observables are about $5.0 \times 10^{-6}$, $1.8 \times 10^{-6}$ and $0.45 \times 10^{-6}$ respectively, which agree with the experimental data well. It implies that such observables in $B \to X_s \mu^+ \mu^-$, together with the measurements of $\bar{B}_s - B_s$ mixing and hadronic $B \to \pi K$ decays, may provide strict constraints on the new $Z'$ couplings involving the lepton sector.

As for the $B_s \to \mu^+ \mu^-$ decay, in addition to the electro-weak loop suppression, the decay rate is helicity suppressed in the SM and predicted to be about $3 \times 10^{-9}$ [20, 22, 23], which is still one order of magnitude lower than the CDF upper bound [24]

$$B(B_s \to \mu^+ \mu^-) < 4.7 \times 10^{-8} \text{ (90\% C.L.)}.$$  (7)

It is expected that precise measurements would be available at the upcoming experiments at LHC and super B factories. As a consequence, we shall also investigate the $Z'$ contribution to
this decay mode within the parameter spaces constrained by \( \bar{B}_s - B_s \) mixing, \( B \to \pi K \) and \( B \to X_s \mu^+ \mu^- \) decays.

Our paper is organized as follows. In Section 2, after a brief review of \( B_q - \bar{B}_q \) mixing within the SM, we pursue possible solutions to the \( \bar{B}_s - B_s \) mixing anomaly within a family non-universal \( Z' \) model, taking into account the constrains from \( B \to \pi K \) decays [15]. In Section 3, the effects of such a NP scenario on \( B \to X_s \mu^+ \mu^- \) and \( B_s \to \mu^+ \mu^- \) decays are investigated in detail. Our conclusions are summarized in Section 4. Appendix. A includes all of the theoretical input parameters.

2 Constraints on \( Z' \) couplings from \( \bar{B}_q - B_q \) mixing and \( B \to \pi K \) decays

2.1 Theoretical framework

Within the SM, the effective Hamiltonian \( \mathcal{H}_{eff}^{SM}(\Delta B = 2) \) for \( \bar{B}_q - B_q \) mixing, relevant for scales \( \mu_b = \mathcal{O}(m_b) \) is given by [20]

\[
\mathcal{H}_{eff}^{SM}(\Delta B = 2) = \frac{G_F^2}{16\pi^2} M_W^2 (V_{tb} V_{tq}^*)^2 C_Q(\mu_b) Q(\Delta B = 2) + \text{h.c.},
\]

where \( Q(\Delta B = 2) = (\bar{q}b)_{V-A} (\bar{q}b)_{V-A} \). Accurate to next-to-leading order (NLO) in QCD, the off-diagonal term \( M_{12}^{SM}(q) \) in the neutral B-meson mass matrix is given by

\[
2m_{B_q} M_{12}^{SM}(q) = \langle B_q^0 | \mathcal{H}_{eff}^{SM}(\Delta B = 2) | \bar{B}_q^0 \rangle = \frac{G_F^2}{6\pi^2} M_W^2 (V_{tb} V_{tq}^*)^2 (\bar{B}_{B_q} f_{B_q}^2) m_{B_q}^2 \eta_B S_0(x_t),
\]

where \( M_W \) is the mass of W boson, \( \bar{B}_{B_q} \) the “bag” parameter, and \( f_{B_q} \) the B-meson decay constant. Explicit expressions for the short-distance QCD correction function \( \eta_B \) and the “Inami-Lim” function \( S_0(x_t) \), with \( x_t = \frac{m_t^2}{m_W^2} \), could be found in Ref. [20].

Recently UTfit collaboration has performed a model-independent analysis of NP effects to \( \bar{B}_q - B_q \) mixing in terms of two parameters \( C_{B_q} \) and \( \phi_{B_q} \), with the following parametrization [3]

\[
C_{B_q} e^{2i\phi_{B_q}} \equiv \frac{\langle B_q | \mathcal{H}_{eff}^{full} | \bar{B}_q \rangle}{\langle B_q | \mathcal{H}_{eff}^{SM} | B_q \rangle} = \frac{A_q^{SM} e^{i\phi_q^{SM}} + A_q^{NP} e^{i(2\phi_q^{NP} + \phi_q^{SM})}}{A_q^{SM} e^{i\phi_q^{SM}}}. \tag{10}
\]
Table 1: Fit results for $\bar{B}_q - B_q$ mixing parameters $C_{B_q}$ and $\phi_{B_q}$ by UTfit collaboration [3]. The two solutions for $\phi_{B_s}$, S1 and S2, result from measurement ambiguities, see Ref. [3] for details.

| NP parameters | $C_{B_d}$ | $\phi_{B_d}$ [$^\circ$] | $C_{B_s}$ | $\phi_{B_s}$ [$^\circ$] (S2 ∪ S1) |
|---------------|-----------|----------------|-----------|-----------------------------------|
| 68% prob.     | 0.96 ± 0.23 | $-2.9 \pm 1.9$ | 1.00 ± 0.20 | $(-68.0 \pm 4.8) \cup (-20.3 \pm 5.3)$ |
| 95% prob.     | [0.57, 1.50] | $[-6.7, 1.0]$ | [0.68, 1.51] | $[-77.8, -58.2] \cup [-30.5, -9.9]$ |

Within the SM, the modulus $C_{B_q}$ and the phase $\phi_{B_q}$ are predicted to be one and zero, respectively. Combining all the available experimental information on $\bar{B}_q - B_q$ mixing, the fitting results at 68% and 95% probabilities from Ref. [3] are listed in Table 1. For each probability, UTfit has found two solutions for $\phi_{B_s}$ due to measurement ambiguities [3]: one is close to, but still $3\sigma$ deviated from, the SM expectations (denoted as S1 hereafter); another one is much more distinct from the SM and even require dominant NP contributions (S2). Such large deviations may suggest the first evidence of NP exhibited in $b \rightarrow s$ induced processes [3]. So, in the following we pursue possible solutions within a family non-universal $Z'$ model [8].

While the general framework for $Z'$-induced FCNC transitions has been formulated by Langacker and Plümacher [8], our discussion throughout this paper for the $Z'$ contributions goes under the following simplifications: (1) neglecting kinetic mixing since it only amounts to a redefinition of the unknown $Z'$ couplings; (2) neglecting the $Z - Z'$ mixing, which has been constrained to be tiny by the $Z$-pole measurements at LEP [25, 26], but can be easily incorporated [8, 27]; (3) no significant renormalization group (RG) evolution effects between $M_{Z'}$ and $M_W$ scales; (4) although there are no severe constraints on right-handed $q - \bar{q} - Z'$ couplings, we follow the simplification in the literature [13, 27, 28] and assume that right-handed couplings are flavor-diagonal and hence real due to the hermiticity of the effective Hamiltonian, while flavor-off-diagonal left-handed coupling terms will result in sizable FCNC $b_L - s_L - Z'$ couplings.

Then, the effective Hamiltonian $\mathcal{H}^{Z'}_{eff}(\Delta B = 2)$ induced by $Z'$ contribution at $M_W$ scale could be written as

$$\mathcal{H}^{Z'}_{eff}(\Delta B = 2) = \frac{G_F}{\sqrt{2}}(B_{q_b})^2 Q(\Delta B = 2) + \text{h.c.}, \quad (11)$$
where $B^L_{q b}$ is the $Z' - b - q$ coupling, whose definition is different from the one used in our previous paper [15] by a factor $g_2 M_Z/g_1 M_{Z'}$, with $g_1$ and $g_2$ being the gauge couplings of $Z$ and $Z'$ bosons, respectively. Due to our assumed simplifications, the RG running of the Wilson coefficient induced by $Z'$ boson is the same as that of the SM, with the corresponding evolution matrix $U^{LL}(\mu_b, M_W)$ given to the NLO level by [20]

$$U^{LL}(\mu_b, \mu_W) = \left[ 1 + \frac{\alpha_s(\mu_b)}{4\pi} J_5 \right] U^0(\mu_b, \mu_W) \left[ 1 - \frac{\alpha_s(\mu_W)}{4\pi} J_5 \right],$$

with $U^0(\mu_b, \mu_W) = (\alpha_s(\mu_W)/\alpha_s(\mu_b))^{\frac{\gamma_0}{2\pi}}$, $\gamma_0 = 4$, $\beta_0 = 23/3$, and $J_5 = 1.627$ in naive dimensional regularization (NDR) scheme with 5 effective quark flavors.

After some simple derivations, one can get the final $Z'$ contribution to $M_{12}(q)$

$$2m_{B_q} M_{12}(q) = \frac{G_F}{\sqrt{2}} U'_{LL} |B^L_{q b}|^2 e^{2\phi_q^L} \frac{8}{3} m_{B_q}^2 (\hat{B}_{B_q} f_{B_q}^2),$$

with $U'_{LL} \equiv (\alpha_s(\mu_W)/\alpha_s(\mu_b))^{\frac{\gamma_0}{2\pi}} \left[ 1 - \frac{\alpha_s(\mu_W)}{4\pi} J_5 \right].$

Finally, we get the total contribution to the off-diagonal mass matrix term

$$M_{12}(q) = M_{12}^{SM}(q) + M_{12}^{Z'}(q).$$

The mass difference, which describes the strength of the $\bar{B}_q - B_q$ mixing, is then given by $\Delta M_q = 2|M_{12}(q)|$. An early general investigation of $Z'$ effects in $\bar{B}_q - B_q$ mixing could be found in Ref.[27].

2.2 Numerical results and discussions

With the UTfit results at 68% and 95% probabilities listed in the Table, as constraints, respectively, we get the allowed ranges for the $Z'$ parameters as shown in Fig. with the corresponding numerical results given in Table. We find that the new $b - s - Z'$ coupling, with a new weak phase $\phi_s^L \sim -58^\circ$ ($\phi_q^L \sim -80^\circ$) and strength $|B^L_{s b}| \sim 1.2 \times 10^{-3}$ ($2.2 \times 10^{-3}$) corresponding to the UTfit result S1 (S2), is crucial to resolve the observed $\bar{B}_s - B_s$ mixing phase anomaly. On the other hand, the $\Delta M_d$ is well measured and in good agreement with the SM predictions, the strength of $b - d - Z'$ coupling involved in $B^0_d - \bar{B}^0_d$ mixing should be much weaker than the one of the SM box diagrams. Numerically, $|B^L_{d b}|$ is found to to be about

6
the experimental data available so far, which may imply that the flavor-changing interactions from the flavor-changing model of Minimal Flavor Violation (MFV) type, the low energy effective Hamiltonian resulting

\[ V_\text{tree} = \frac{g_\phi}{\sqrt{2}} \left( \phi^{\dagger} \partial_i \phi \right) \]


\[ \phi = \sum_i n_i \phi_i \]

From Eqs. (10), (13), and (14), we find that the parameters \( C_{B_q} \) and \( \phi_{B_q} \) are independent of the theoretical uncertainties associated with the non-perturbative factor \( \hat{B}_{B_q} f_{B_q}^2 \) within such a

1.2 \times 10^{-4}. Combining with the constraints by \( B^0_s - \bar{B}^0_s \) mixing and \( B \to \pi K \) decays, we find that the relative strength, \( |B_{sB}^L|/|B_{sB}^L| \sim O(10^{-1}) \), is quite similar to the hierarchy of CKM matrix elements within the SM, \( |V_{td}^* V_{tb}/V_{ts}^* V_{tb}| \sim 0.2 \). On the theoretical side, it is noted that such a hierarchy is not required by the \( Z' \) model itself. Although the \( Z' \) model considered here is not a model of Minimal Flavor Violation (MFV) type, the low energy effective Hamiltonian resulting from the flavor-changing \( Z' \) couplings also obey the so-called MFV hypothesis [29, 30] driven by the experimental data available so far, which may imply that the flavor-changing interactions in the \( Z' \) model are also linked to the known structure of the SM Yukawa couplings.

From Eqs. (10), (13), and (14), we find that the parameters \( C_{B_q} \) and \( \phi_{B_q} \) are independent of the theoretical uncertainties associated with the non-perturbative factor \( \hat{B}_{B_q} f_{B_q}^2 \) within such a
family non-universal $Z'$ model under our assumed simplifications. However, to get the mass difference $\Delta M_q$, such uncertainties are unavoidable. With the relevant input parameters listed in the Appendix A, the final numerical results for $\Delta M_q$ are listed in Table 3. It can be seen that, after including the $Z'$ contributions, our predictions for $\Delta M_q$ also agree with the experiment data, taking into account the respective theoretical uncertainties.

In our pervious paper [15], we found that a nontrivial new weak phase $\phi_s^L \sim -86^\circ$ associated with the $b - s - Z'$ coupling is helpful to resolve the so-called “$\pi K$ puzzle”, which is similar to our present fitting result $\phi_s^L \sim -58^\circ$ in S1 ($\phi_s^L \sim -80^\circ$ in S2) from $\bar{B}_s - B_s$ mixing. However, as found in Ref. [15], the range $\phi_s^L > -50^\circ$ is almost excluded by the CP-averaged branching ratios and direct CP asymmetries of $B \to \pi K$ decays. So, it is very necessary and interesting to re-evaluate the ranges of $Z'$ couplings under the constraints from $\bar{B}_s - B_s$ mixing and $B \to \pi K$ decays simultaneously.

Like the Case IV in Ref. [15], we give up any simplifications on the flavor-diagonal $u - u - Z'$ and $d - d - Z'$ couplings, and use the QCD factorization (QCDF) [31] approach to calculate the amplitudes of $B \to \pi K, \pi K^*$ and $\rho K$ decays. As for the end-point divergence appearing in twist-3 spectator and annihilation amplitudes, instead of the parametrization scheme, we quote an infrared finite dynamical gluon propagator derived by Cornwall [32] to regulate it. Explicitly we quote $m_g = 0.50 \pm 0.05 \text{GeV}$, which is a reasonable choice so that most of the observables for $B \to \pi K, \pi K^*$ and $\rho K$ decays are in good agreement with the experimental data [33]. In this way, we find that the time-like annihilation amplitude could contribute a large strong-interaction phase, while the space-like spectator-scattering amplitude is real [33]. The explicit comparison for the two schemes have been systemically discussed in our previous papers [15, 33]. Although numerically these two schemes have some differences, both of their predictions are consistent with most of the experimental data within errors.

Table 3: Numerical results for the mass difference $\Delta M_q$ (ps$^{-1}$).

| Solutions | Exp. [19] | SM  | S1  | S2  |
|-----------|----------|-----|-----|-----|
| $\Delta M_d$ | $0.508 \pm 0.005$ | $0.525 \pm 0.057$ | $0.522 \pm 0.077$ | — |
| $\Delta M_s$ | $17.77 \pm 0.12$ | $18.18 \pm 1.47$ | $17.14 \pm 2.30$ | $17.42 \pm 2.40$ |
Figure 2: The allowed regions for the parameters $B_{uu,dd}^{L,R}$ under the constraints from $C_{B_s}$, $\phi_{B_s}$ (95% prob. only) and $B \rightarrow \pi K$ decays.

Table 4: Numerical results for the parameters $|B_{sb}^{L}|$, $B_{uu,dd}^{L,R}$ and $\phi_{s}^{L}$ under the constraints from $\bar{B}_s - B_s$ mixing and $B \rightarrow \pi K$ decays. The other captions are the same as the ones in Table 2.

| Solutions | $|B_{sb}^{L}|(\times 10^{-3})$ | $\phi_{s}^{L[\circ]}$ | $B_{uu}^{L}$ | $B_{uu}^{R}$ | $B_{dd}^{L}$ | $B_{dd}^{R}$ |
|-----------|-----------------|-----------------|-------------|-------------|-------------|-------------|
| S1        | 1.18 ± 0.16     | -62 ± 5         | 0.66 ± 0.38 | -0.13 ± 0.12| 0.88 ± 0.36 | -0.18 ± 0.14|
|           | 1.09 ± 0.22     | -72 ± 7         | 0.34 ± 0.55 | -0.04 ± 0.18| 0.70 ± 0.48 | -0.07 ± 0.20 |
| S2        | 2.19 ± 0.06     | -81 ± 2         | -0.02 ± 0.34| 0.01 ± 0.12 | 0.22 ± 0.32 | 0.01 ± 0.12  |
|           | 2.20 ± 0.15     | -82 ± 4         | 0.02 ± 0.34 | -0.01 ± 0.12| 0.27 ± 0.32 | -0.04 ± 0.24 |

Including the constraints from $\bar{B}_s - B_s$ mixing and $B \rightarrow \pi K$ decays, the final allowed ranges for the $Z'$ couplings are presented in Figs. 1 and 2. As shown in Fig. 1, the range of $\phi_{s}^{L}$ in S1 is now further restricted with the constraints from $B \rightarrow \pi K$ decays included (i.e., the range $\phi_{s}^{L} > -50^\circ$ is now excluded), while their effect for S2 case is tiny. Our numerical results for $Z'$ couplings are summarized in Table 4.

For evaluating $B \rightarrow \pi K$ decays, since the $Z'$ mediated effects can occur not only in the coefficients of the electro-weak penguin operators but also in the strong penguin ones, we do not assume $B_{dd}^{R}/B_{uu}^{R} = e_d/e_u = -1/2$ (if assumed, the $Z'$ effects in terms of the Wilson coefficients of the SM QCD penguin operators will then vanish, as usually adopted in the literature, see for example Refs. [13, 14, 15]). So, compared with $|B_{dd}^{R}| = | - B_{uu}^{R}/2| < 0.1$ [14], our fitting result in Fig. 2 shows that a larger ranges for $B_{uu,dd}^{L,R}$ are still allowed. Furthermore, as can be seen
3 Constraints on \( Z' \) couplings from \( B_s \rightarrow \mu^+\mu^- \) and \( B \rightarrow X_s\mu^+\mu^- \)

3.1 Theoretical Framework

Within the SM, after dropping the negligible charm contributions, the effective Hamiltonian for purely leptonic \( B_s \rightarrow l^+l^- \) decay is given as [20, 34, 35]

\[
\mathcal{H}_{eff}^{SM}(B_s \rightarrow l^+l^-) = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} V_{tb} V_{ts}^* Y(x_t)(\bar{s}b)_{V-A}(\bar{u}l)_{V-A} + \text{h.c.}, \tag{16}
\]

where \( \alpha = \frac{e^2}{4\pi} = 1/137, \sin^2 \theta_W = 0.23119 \) [36], and the function \( Y(x_t) \) is defined as [20, 34]

\[
Y(x_t) = Y_0(x_t) + \frac{\alpha_s}{4\pi} Y_1(x_t), \\
Y_0(x_t) = \frac{x_t}{8} \left[ x_t^2 - 4 \right] \ln x_t, \\
Y_1(x_t) = \frac{4x_t + 16x_t^2 + 4x_t^3}{3(1-x_t)^2} \ln(1-x_t) + 8x_t \frac{\partial Y_0(x_t)}{\partial x_t} \ln \frac{\mu_s^2}{M_W^2}.
\]

Within our approximations for the non-universal \( Z' \) couplings, the effective Hamiltonian for \( b \rightarrow sl^+l^- \) transition induced by the new \( Z' \) boson could be written as

\[
\mathcal{H}_{eff}^{Z'}(b \rightarrow sl^+l^-) = -\frac{2G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ -\frac{B_{sb}^L B_{ll}^R}{V_{tb} V_{ts}^*} (\bar{s}b)_{V-A}(\bar{u}l)_{V-A} - \frac{B_{sb}^L B_{ll}^R}{V_{tb} V_{ts}^*} (\bar{s}b)_{V-A}(\bar{u}l)_{V-A} \right] + \text{h.c.}. \tag{18}
\]

Then, the full expression for the branching ratio of \( B_s \rightarrow l^+l^- \) is

\[
B(B_s \rightarrow l^+l^-) = \tau_{B_s} \frac{G_F^2}{4\pi} f_{B_s}^2 m_{l_s}^2 m_B \left[ 1 - \frac{4m_{l_s}^2}{m_B^2} |V_{tb} V_{ts}^*|^2 \right] \\
\times \left| \frac{\alpha}{2\pi \sin^2 \theta_W} Y(x_t) - 2 \frac{B_{sb}^L (B_{ll}^R - B_{ll}^R)}{V_{tb} V_{ts}^*} \right|^2. \tag{19}
\]
To clarify the differences between Eq. (19) and the ones in the literature, a detailed derivation of Eq. (19) is presented in Appendix B.

The SM effective Hamiltonian for rare $b \to s l^+ l^-$ decay at scale $\mu$ is given by

$$
\mathcal{H}^\text{eff}_{s l^+ l^-}(b \to s l^+ l^-) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=1}^{8} C_i(\mu) Q_i + C_{9W}(\mu) Q_{9W} + C_{10A}(\mu) Q_{10A} \right] + \text{h.c.} \quad (20)
$$

Here we choose the operator basis given by Refs. [20, 21], in which the 1-loop QCD correction factor $\kappa$ is presented in Appendix B. To clarify the differences between Eq. (19) and the ones in the literature, a detailed derivation of the Wilson coefficient has been detailed in Refs. [20, 21, 35].

Introducing the normalized dilepton invariant mass $\hat{s} = (p_{l^+} + p_{l^-})^2 / m_b^2$, the differential decay rate with respect to $\hat{s}$ for $b \to s l^+ l^-$ reads

$$
R(\hat{s}) \equiv \frac{d \mathcal{B}(b \to s l^+ l^-)}{d \hat{s}} \mathcal{B}(b \to c l^- \bar{\nu}) = \frac{\alpha^2}{4\pi^2} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} (1 - \hat{s})^2 f(\chi) |\kappa(\chi)| \sqrt{1 - \frac{4t^2}{s}} D(\hat{s}), \quad (25)
$$

with

$$
D(\hat{s}) = (1 + 2\hat{s})(1 + \frac{2t^2}{s}) |\tilde{C}_9^{eff}|^2 + 4(1 + \frac{2t^2}{s})(1 + \frac{2t^2}{s}) |\tilde{C}_7^{eff}|^2
$$

$$
+ \left[ (1 + 2\hat{s}) + \frac{2t^2}{s} (1 - 4\hat{s}) \right] |\tilde{C}_{10}^{eff}|^2 + 12(1 + \frac{2t^2}{s}) C_7^{eff} \text{Re}(\tilde{C}_9^{eff} \ast), \quad (26)
$$

where $t = m_t / m_b$, $\chi = m_c / m_b$ and $\mathcal{B}(B \to X_c l^- \bar{\nu}) = (10.1 \pm 0.4)\% \ [30]$. The phase-space factor $f(\chi)$ and the 1-loop QCD correction factor $\kappa(\chi)$ for $B \to X_c l^- \bar{\nu}$ decay are given respectively

\[ A sign in Eq. (B1) in Ref. [13] and Eq. (15) in Ref. [28] mistyped, and the interference terms missed in Eq. (B1) of the first version Ref. [14]. We thank T. Liu and C.W. Chiang for confirmation. \]
by [37]

\[ f(\chi) = 1 - 8\chi^2 + 8\chi^6 - \chi^8 - 24\chi^4 \ln \chi, \quad (27) \]

\[ \kappa(\chi) = 1 - \frac{2\alpha_s(\mu)}{3\pi} \left[ \left( \pi^2 - \frac{31}{4} \right) (1 - \chi)^2 + \frac{3}{2} \right]. \quad (28) \]

The effective coefficient \( \tilde{C}_9^{\text{eff}} \) is defined as [21]

\[ \tilde{C}_9^{\text{eff}} = \tilde{C}_9 \eta(\hat{s}) + h(\chi, \hat{s})(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \frac{1}{2} h(1, \hat{s})(4C_4 + 4C_4 + 3C_5 + C_6) \]

\[ - \frac{1}{2} h(0, \hat{s})(C_3 + 3C_4) + \frac{2}{9}(3C_3 + 4C_4 + 3C_5 + C_6) \quad (29) \]

where the function \( \eta(\hat{s}) \) in the first term represents one gluon corrections to the matrix element of \( Q_{9V} \), the other terms arise from the insertions of four-quark operators (indicated by the \( C_i \)) to the one-loop matrix element of \( Q_{9V} \) [21, 38, 39].

\[ \eta(\hat{s}) = 1 + \frac{\alpha_s(\mu)}{\pi} \left[ - \frac{2}{9}\pi^2 - \frac{4}{3}\text{Li}_2(\hat{s}) - \frac{2}{3}\ln \hat{s} \ln(1 - \hat{s}) - \frac{5 + 4\hat{s}}{3(1 + 2\hat{s})} \ln(1 - \hat{s}) \right. \]

\[ - \left. \frac{2\hat{s}(1 + \hat{s})(1 - 2\hat{s})}{3(1 - \hat{s})^2(1 + 2\hat{s})} \ln \hat{s} + \frac{5 + 9\hat{s} - 6\hat{s}^2}{6(1 - \hat{s})(1 + 2\hat{s})} \right], \quad (30) \]

\[ h(\chi, \hat{s}) = -\frac{8}{9} \ln \frac{m_b}{\mu} - \frac{8}{9} \ln \chi + \frac{8}{27} + \frac{4}{9} x \]

\[ - \frac{2}{9}(2 + x)|1 - x|^2 \begin{cases} \ln \left| \frac{\sqrt{x + 1}}{\sqrt{x - 1}} \right| - i\pi & \text{for } x \equiv \frac{4y^2}{s} < 1, \\ 2\arctan \frac{1}{\sqrt{x - 1}} & \text{for } x \equiv \frac{4y^2}{s} > 1 \end{cases} \quad (31) \]

\[ h(0, \hat{s}) = \frac{8}{27} - \frac{8}{9} \ln \frac{m_b}{\mu} - \frac{4}{9} \ln \hat{s} + \frac{4}{9} i\pi. \quad (32) \]

Besides these well defined short distance contributions, there are long distance corrections related to the \( c\bar{c} \) intermediate states. Phenomenologically they are estimated with Breit-Wigner approximation [40, 41, 42] which results in a modification to \( \tilde{C}_9^{\text{eff}} \) by

\[ Y_{\text{res}}(\hat{s}) = \frac{3\pi}{\alpha^2} \kappa(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \sum_{V_n = \Psi(nS)} \frac{\Gamma(V_n \rightarrow l^+l^-) m_{V_n}}{m_{V_n}^2 - s m_b^2 - i m_{V_n} \Gamma_{V_n}}. \quad (33) \]

Usually the factor \( \kappa \simeq 2.3 \) (more precisely 2.3 times an arbitrary strong phase [41]) is introduced phenomenologically to include the known factorizable and the unknown nonfactorizable contributions to account for the present experimental data on \( B \rightarrow \Psi(nS)X_s \) decays. This

\[^2\text{we thank the referee for bringing this point to us}\]
approximation, however, will cause a double counting of partonic and hadronic degrees of freedom\textsuperscript{13}. To avoid the double counting, it has been suggested that the long-distance effects could be estimated by means of experimental data on $R_c(s) = \sigma(e^-e^+ \rightarrow c\bar{c})/\sigma(e^-e^+ \rightarrow \mu^+\mu^-)$ using a dispersion relation \textsuperscript{13} (KS approach). In KS approach, only factorizable effect \textit{i.e.} the $c\bar{c}$ in color singlet, could be estimated with $R_c(s)$. It still needs the phenomenological enhancement factor $\kappa$ to model possible nonfactorizable effects to match the aforementioned large rate of $B \rightarrow \Psi(\Psi')X_s$. However, as discussed in detail in Ref. \textsuperscript{44}, unlike the $c\bar{c}$ contribution to the $e^-e^+ \rightarrow \text{hadrons}$ cross section where the imaginary part of a current-current correlator is integrated over phase space, the huge charm-resonance contributions to $B \rightarrow X_s\ell^+\ell^-$ are related to a drastic failure of quark-hadron duality in the narrow-resonance region for integrating the absolute \textit{square} of the correlator over the phase space. Further detailed discussions on these non-perturbative effects could be found in Ref. \textsuperscript{44}. In this paper, we concentrate on the short-distance effects and use the low- and the high-$s$ data, i.e., away from the $\Psi$ and $\Psi'$ peaks, to constrain NP effects, and ignore the resonance effects. It should be noted that, unlike $\Psi$ and $\Psi'$ (large data samples, known structures, etc.), the backgrounds due to higher $J^{PC} = 1^{--}$ charmonium resonances in the high-$s$ region may be very hard to be vetoed experimentally. Although their effects are expected to be much smaller than the former ones, they still cause sizable uncertainties which are hard to be estimated \textsuperscript{44}. In a recent study of exclusive $B \rightarrow K\ell^+\ell^-$ decay \textsuperscript{46}, it is argued that duality violations from the higher resonances in high-$s$ region are at a moderate level which may spoil the precision of theoretical predictions for (partially) integrated branching ratios of $B \rightarrow K\ell^+\ell^-$ at the level of several percentage. In view of the large uncertainties included in our numerical analyses, we may expect the effects of higher $c\bar{c}$ resonances would not alter our conclusion much.

Finally, the normalized forward-backward (FB) asymmetry distribution is defined as

\begin{equation}
A_{FB}(\hat{s}) = \frac{\int_0^1 dz \frac{d\Gamma}{dsdz} - \int_{-1}^0 dz \frac{d\Gamma}{dsdz}}{\int_0^1 dz \frac{d\Gamma}{dsdz} + \int_{-1}^0 dz \frac{d\Gamma}{dsdz}} = -3 \sqrt{1 - \frac{4t^2}{\hat{s}}} E(\hat{s}),
\end{equation}

with

\begin{equation}
E(\hat{s}) = \text{Re}(\tilde{C}_9^{eff} \tilde{C}_9^{*} \hat{s} + 2 \tilde{C}_7^{eff} \tilde{C}_7^{*} \hat{s}).
\end{equation}
3.2 Numerical analyses and discussions

With the relevant theoretical formulas collected in section 3.1 and the input parameters summarized in the Appendix A, we now proceed to present our numerical analyses and discussions. The rare $B \to X_s \mu^+ \mu^-$ and $B_s \to \mu^+ \mu^-$ decays involve not only the coupling $B_{\mu \mu}^L$, which has been severely constrained by $\bar{B}_s - B_s$ mixing and $B \to \pi K$ decays discussed in section 2, but also the unrestricted $\mu - \mu - Z'$ couplings, $B_{\mu \mu}^L$ and $B_{\mu \mu}^R$. So, our analyses are further divided into the following three cases with different simplifications for our attention, namely

- **Case I**: with $B_{\mu \mu}^L$ arbitrary, while taking $B_{\mu \mu}^R = 0$;
- **Case II**: with $B_{\mu \mu}^R$ arbitrary, while taking $B_{\mu \mu}^L = 0$;
- **Case III**: with both $B_{\mu \mu}^L$ and $B_{\mu \mu}^R$ arbitrary.

In the following discussions, we quote the fitting results for $|B_{\mu \mu}^L|$ and $\phi_{\mu \mu}^L$ under the constraints from $C_{B_s}$, $\phi_{B_s}$ (95% prob.) and $B \to \pi K$ decays as inputs. In our numerical evaluations, we don’t consider the LD contribution to $\tilde{C}_9^{\mu \mu}$. In each case, our fitting is performed with the experimental data on $\mathcal{B}(B \to X_s \mu^+ \mu^-)$, $\mathcal{B}^H(B \to X_s \mu^+ \mu^-)$ and $\mathcal{B}^L(B \to X_s \mu^+ \mu^-)$ varying randomly within their respective 1σ error bars, while the theoretical uncertainties are obtained by varying the input parameters within the regions specified in Appendix A. Moreover, we leave $\mathcal{B}(B_s \to \mu^+ \mu^-)$, $A_{FB}(B \to X_s \mu^+ \mu^-)$ and $A_{FB}^{L,H}(B \to X_s \mu^+ \mu^-)$ as our theoretical prediction, which could be tested by more precise measurements in the coming years.
Table 5: Numerical results for the parameters $B_{\mu\mu}^L$ and $B_{\mu\mu}^R$ (in unit of $\times 10^{-2}$).

| Cases     | Case I | Case II | Case III |
|-----------|--------|---------|----------|
|           | S1     | S2      | S1       | S2      | S1   | S2   |
| $B_{\mu\mu}^L$ | -2.5 $\pm$ 2.7 | -0.55 $\pm$ 1.0 | -2.7 $\pm$ 2.5 | -0.59 $\pm$ 0.93 |
| $B_{\mu\mu}^R$ | -       | -       | 0.78 $\pm$ 2.0 | 0.23 $\pm$ 0.97 | 0.61 $\pm$ 2.4 | 0.19 $\pm$ 0.88 |

![Figure 4: The dependence of $d\mathcal{B}(B \to X_{s}\mu^+\mu^-)/d\hat{s}$ on $B_{\mu\mu}^{L(R)}$](image)

Case I: with $B_{\mu\mu}^L$ arbitrary, while taking $B_{\mu\mu}^R = 0$.

In order to investigate the effects of $B_{\mu\mu}^L$, we neglect the $Z'$ contributions involving $B_{\mu\mu}^R$ in this case. Corresponding to the two solutions S1 and S2 for $B_{sb}^L$, we obtain two allowed regions for $B_{\mu\mu}^L$ as shown in Fig. 3 and the corresponding numerical results are listed in Table 5. Our predictions for $\mathcal{B}(B \to X_{s}\mu^+\mu^-)$, $A_{FB}(B \to X_{s}\mu^+\mu^-)$, including the results at both low and high $q^2$ regions, and $\mathcal{B}(B_{s} \to \mu^+\mu^-)$ are given in Table 6. Due to the fact that the SM predictions, $\mathcal{B}(B \to X_{s}\mu^+\mu^-) = (5.0 \pm 0.3) \times 10^{-6}$, $\mathcal{B}^L(B \to X_{s}\mu^+\mu^-) = (1.8 \pm 0.1) \times 10^{-6}$, $\mathcal{B}^R(B \to X_{s}\mu^+\mu^-) = (0.45 \pm 0.06) \times 10^{-6}$, and $\mathcal{B}(B_{s} \to \mu^+\mu^-) = (3.1 \pm 0.2) \times 10^{-9}$, agree quite well with the experimental measurements as given by Eqs. (4), (5), (6), and (7), the parameter space with $B_{\mu\mu}^L \sim 0$ is still allowed. However, as shown latter, nonzero $B_{\mu\mu}^L$ may have significant impacts on the other observables.

With the central values of theoretical input parameters, Fig. 4 (a) shows the dependence of $d\mathcal{B}(B \to X_{s}\mu^+\mu^-)/d\hat{s}$ on $B_{\mu\mu}^L$ at different $\hat{s}$, from which we can see that the minimal value
Figure 5: The dependence of $d\mathcal{B}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ and $dA_{FB}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ on $\hat{s}$ with $|B_{s\mu}^L| = 1.09 \times 10^{-3}(2.20 \times 10^{-3})$, $\phi_s^L = -72^\circ(-82^\circ)$, and $B_{\mu\mu}^R = 0$.

of the differential decay rate appears at $B_{\mu\mu}^L \sim -0.03 (-0.005)$ in S1 (S2). Furthermore, the dependence of $d\mathcal{B}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ and $dA_{FB}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ on $\hat{s}$, with different values of $B_{\mu\mu}^L$, is shown in Fig. 5. From Fig. 5(a), one can find that $d\mathcal{B}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ is reduced at $B_{\mu\mu}^L \sim -0.03$ but enhanced at $B_{\mu\mu}^L \sim -0.06$ and 0.02 in S1. In S2, with $B_{\mu\mu}^L = -0.02$, the $Z'$ contributions induced by $B_{\mu\mu}^L$ could enhance $d\mathcal{B}(B \to X_s \mu^+ \mu^-)/d\hat{s}$. However, as shown in Fig. 5(c), it nearly can’t reduce $B(B \to X_s \mu^+ \mu^-)$. So, if the future refined experimental measurement on $B(B \to X_s \mu^+ \mu^-)$ is significantly smaller than the SM prediction, S2 will be excluded first. For both S1 and S2 cases, the $Z'$-induced effects on $dA_{FB}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ are tiny as shown in Figs. 5(b) and (d).

In our fitting, we find that the constraints on $B_{\mu\mu}^{L,R}$ are dominated by $\mathcal{B}^{(L,H)}(B \to X_s \mu^+ \mu^-)$, while the constraint from $\mathcal{B}(B_s \to \mu^+ \mu^-)$ is very weak due to the fact that there exits only upper bound at the moment. At the quark level, since both $B \to X_s \mu^+ \mu^-$ and $B_s \to \mu^+ \mu^-$ involve the same $b \to s\mu^+ \mu^-$ transition, it is interesting to see the $Z'$ contributions to $B_s \to \mu^+ \mu^-$ within the parameter spaces constrained by $B \to X_s \mu^+ \mu^-$. 

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Table 6: Numerical results for $B(B_s \to \mu^+\mu^-)$ on $B_{\mu\mu}^L - B_{\mu\mu}^R$ at $\phi_s^L = -80^\circ (-85^\circ)$, $-70^\circ (-80^\circ)$ and $-60^\circ (-75^\circ)$ with $|B_{s\mu}| = 1.09 \times 10^{-3} (2.20 \times 10^{-3})$ in S1 (S2).

| Cases                        | Exp. | SM   | Case I  | Case II | Case III |
|------------------------------|------|------|---------|---------|----------|
| $B(B_s \to \mu^+\mu^-)$     | < 47 | 3.1 ± 0.5 | 3.0 ± 1.1 | 3.3 ± 0.8 | 3.1 ± 1.4 | 3.5 ± 1.1 | 3.2 ± 1.5 | 3.5 ± 1.1 |
| $B(B \to X_s\mu^+\mu^-)$    | $43_{-12}^{+13}$ | 50 ± 7 | 46 ± 10 | 49 ± 7 | 50 ± 7 | 49 ± 7 | 46 ± 11 | 49 ± 7 |
| $B^L(B \to X_s\mu^+\mu^-)$  | $16_{-4.8}^{+5.2}$ | 18 ± 3.2 | 17 ± 4.3 | 18 ± 3.2 | 18 ± 3.1 | 18 ± 3.1 | 16 ± 4.5 | 18 ± 3.2 |
| $B^H(B \to X_s\mu^+\mu^-)$  | 4.4 ± 1.2 | 4.5 ± 0.6 | 4.4 ± 1.2 | 4.4 ± 1.2 | 4.5 ± 1.1 | 4.4 ± 1.1 | 4.4 ± 1.2 | 4.4 ± 1.1 |
| $A_{FB}(B \to X_s\mu^+\mu^-)$ | — | 28 ± 0.1 | 29 ± 2 | 29 ± 1.2 | 22 ± 6.6 | 22 ± 5.6 | 18 ± 13 | 23 ± 6.7 |
| $A_{FB}^L(B \to X_s\mu^+\mu^-)$ | — | 0.5 ± 0.3 | 1.1 ± 1.2 | 1.0 ± 0.8 | 0.2 ± 0.7 | 0.2 ± 0.5 | 0.2 ± 2.0 | 0.4 ± 0.9 |
| $A_{FB}^H(B \to X_s\mu^+\mu^-)$ | — | 17 ± 1.3 | 17 ± 1.4 | 17 ± 1.4 | 14 ± 4.4 | 15 ± 3.9 | 11 ± 7.3 | 14 ± 4.5 |

Fig. 6 shows the dependence of $\mathcal{B}(B_s \to \mu^+\mu^-)$ on $B_{\mu\mu}^L - B_{\mu\mu}^R$ at different $\phi_s^L$. In S1, with $B_{\mu\mu}^R = 0$, we find that $\mathcal{B}(B_s \to \mu^+\mu^-)$ is easier to be reduced with a smaller $|\phi_s^L|$. Numerically, with $\phi_s^L \sim -65^\circ$ and $B_{\mu\mu}^L \sim -0.05$, we get $\mathcal{B}(B_s \to \mu^+\mu^-) = 2.5 \times 10^{-9}$, which is 19% smaller than the SM prediction $3.1 \times 10^{-9}$. However, since $|\phi_s^L(S2)| > |\phi_s^L(S1)|$, $\mathcal{B}(B_s \to \mu^+\mu^-)$ is sensitive to seizable $|B_{\mu\mu}^L - B_{\mu\mu}^R|$ and could be enhanced for most parameter space of $\phi_s^L - |B_{\mu\mu}^L - B_{\mu\mu}^R|$. For S1 (S2), with $\phi_s^L = -80^\circ (-86^\circ)$, $B_{\mu\mu}^L = -0.06 (-0.02)$, we find that the $Z'$ contributions in case I could enhance $\mathcal{B}(B_s \to \mu^+\mu^-)$ by about 18% (12%) compared with the SM prediction.
Case II: with $B^R_{\mu\mu}$ arbitrary, while taking $B^L_{\mu\mu} = 0$.

Taking $B^L_{\mu\mu} = 0$, we are going to evaluate the $Z'$ effects induced by $B^R_{\mu\mu}$. The allowed regions of the $Z'$ parameters are shown in Fig. 7. From Fig. 4 (b), which shows the dependence of...

Figure 7: The allowed regions for the parameters $B^L_{\mu\mu}$ and $B^R_{\mu\mu}$.

Figure 8: The dependence of $dB(B \to X_s \mu^+ \mu^-)/d\hat{s}$ and $dA_{FB}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ on $\hat{s}$ with $|B^L_{s\mu}| = 1.09 \times 10^{-3} (2.20 \times 10^{-3})$, $\phi^{L}_{s} = -72^\circ (-82^\circ)$, and $B^R_{\mu\mu} = 0$. The other captions are the same as in Fig. 5.

Figure 8: The dependence of $dB(B \to X_s \mu^+ \mu^-)/d\hat{s}$ and $dA_{FB}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ on $\hat{s}$ with $|B^L_{s\mu}| = 1.09 \times 10^{-3} (2.20 \times 10^{-3})$, $\phi^{L}_{s} = -72^\circ (-82^\circ)$, and $B^R_{\mu\mu} = 0$. The other captions are the same as in Fig. 5.

Case II: with $B^R_{\mu\mu}$ arbitrary, while taking $B^L_{\mu\mu} = 0$.

Taking $B^L_{\mu\mu} = 0$, we are going to evaluate the $Z'$ effects induced by $B^R_{\mu\mu}$. The allowed regions of the $Z'$ parameters are shown in Fig. 7. From Fig. 4 (b), which shows the dependence of...
$B(B \to X_s \mu^+ \mu^-)$ on $B_{\mu \mu}^R$, one would observe that the minimal $B(B \to X_s \mu^+ \mu^-)$ corresponds to the point $B_{\mu \mu}^R = 0$. So, the $Z'$ contributions induced by $B_{\mu \mu}^R$ are nearly can’t reduce $B(B \to X_s \mu^+ \mu^-)$, which is confirmed by Figs. 8 (a) and (c).

In Case I, from Figs. 5 (b) and (d), we find $dA_{FB}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ is not very sensitive to $B_{\mu \mu}^L$. However, comparing Figs. 5 (b) and (d) with Figs. 5 (b) and (d), we find $dA_{FB}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ is very sensitive to the $Z'$ contributions induced by $B_{\mu \mu}^R$. Since the $Z'$ contributions to $E(\hat{s})$ are smaller than that to $D(\hat{s})$, $A_{FB}(B \to X_s \mu^+ \mu^-)$ can be reduced easily rather than enhanced. With $B_{\mu \mu}^R = 0.03 (0.015)$ and the central values of the other theoretical parameters, $A_{FB}(B \to X_s \mu^+ \mu^-)$ could be reduced by a factor 12% (9%) in S1 (S2) compared to the SM prediction. At high/low $q^2$ region, it could be reduced by 12% (13%)/17% (55%).

Taking $B_{\mu \mu}^L = 0$, Fig. 6 shows the dependance of $B(B_s \to \mu^+ \mu^-)$ on $B_{\mu \mu}^R$. At $B_{\mu \mu}^R = 0.03$ with $\phi_{L}^{B_s} = 65^\circ$, we get the small $B(B_s \to \mu^+ \mu^-) \sim 2.5 \times 10^{-9}$, which is 18% smaller than the SM prediction. With $B_{\mu \mu}^R = 0.03$, $\phi_{L}^{B_s} = -80^\circ$, $B(B_s \to \mu^+ \mu^-)$ is enhanced by about 10%.

Case III: with both $B_{\mu \mu}^L$ and $B_{\mu \mu}^R$ arbitrary.

More generally, we give up any assumptions for the $Z'$ couplings $B_{\mu \mu}^L$ and $B_{\mu \mu}^R$. Because of the interference effect between $B_{\mu \mu}^L$ and $B_{\mu \mu}^R$, the allowed regions for these two parameters are now larger than the ones in Case I and Case II, which are shown in Fig. 9. From the figure, one can find the correlotions between the coupling parameters. Since $B_{s \bar{s}}^L$ and its phase have been constrained by $B_s - \bar{B}_s$ mixing, $B_{\mu \mu}^L$ and $B_{\mu \mu}^R$ are found to be small. It is interesting to note that a model independent constraint on $C_{10}^{NP}$ and its phase has been performed in Ref. [17] with $A_{FB}(B \to X_s \ell \ell)$. Combining our constraints on $B_{s \bar{s}}^L$, $B_{\mu \mu}^L$ and $B_{\mu \mu}^R$, and re-scaling the combination by $V_{ts}$, one can find our constraints are in good agreement with the magnitude of $C_{10}^{NP}$ in Ref. [17], but with a much stronger constraint on its phase.

The dependence of $dB(B \to X_s \mu^+ \mu^-)/d\hat{s}$ and $dA_{FB}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ on $B_{\mu \mu}^L$ and $B_{\mu \mu}^R$ has been discussed separately in the last two cases. So, including all of the theoretical uncertainties, we just present the dilepton invariant mass spectrum and the differential normalized forward-backward asymmetry in Fig. 10. We find that in S1, as shown in Fig. 10 (a), $B(B \to X_s \mu^+ \mu^-)$ could be either enhanced or reduced by $Z'$ contributions. Since the $Z'$ contributions induced by $B_{\mu \mu}^R$ can hardly reduce $B(B \to X_s \mu^+ \mu^-)$ as discussed in Case II, the $Z'$ effects for reducing
Figure 9: The allowed regions for the parameters $B_{\mu\mu}^L$ and $B_{\mu\mu}^R$.

$\mathcal{B}(B \to X_s \mu^+ \mu^-)$ are dominated by $B_{\mu\mu}^L$ as discussed in Case I, while the enhancement is due to both $B_{\mu\mu}^L$ and $B_{\mu\mu}^R$.

However, in S2, as shown in Fig. (c), $\mathcal{B}(B \to X_s \mu^+ \mu^-)$ can hardly be reduced, which confirms our analysis in Case. I and II. In both S1 and S2, as shown in Figs. (b) and (d), $A_{FB}(B \to X_s \mu^+ \mu^-)$ can be easily reduced but hardly be enhanced by $B_{\mu\mu}^R$.

With both $B_{\mu\mu}^L$ and $B_{\mu\mu}^R$ included, the branching ratio for $B_s \to \mu^+ \mu^-$ is affected by $B_{\mu\mu}^L - B_{\mu\mu}^R$. Numerically, with $B_{\mu\mu}^L - B_{\mu\mu}^R = -0.05$, $\phi_s^L = -65^\circ$ and the central value of the other
Figure 10: The effects of the $Z'$ contributions induced by $B_{\mu \mu}^{L,R}$ on $d\mathcal{B}(B \to X_s \mu^+ \mu^-)/d\hat{s}$ and $dA_{FB}(B \to X_s \mu^+ \mu^-)/d\hat{s}$. The other captions are the same as in Fig. 5.

theoretical inputs, $\mathcal{B}(B_s \to \mu^+ \mu^-)$ is reduced by about 19%, which is the same as in Case I. With $B_{\mu \mu}^L = -0.05$, $B_{\mu \mu}^R = 0.03$, $\phi_s^L = -80^\circ$ and the central value of the other theoretical inputs, we find that $\mathcal{B}(B_s \to \mu^+ \mu^-) = 4.5 \times 10^{-9}$, which is about 46% larger than the SM prediction. So, if the coming measurements at LHCb and super B factories present $\mathcal{B}(B_s \to \mu^+ \mu^-) \sim 10^{-8}$, the family non-universal $Z'$ model will suffer a serious challenge.

4 Conclusion

In conclusion, motivated by the observed $B_s - B_s$ mixing phase anomaly and the so-called “$\pi K$ puzzle”, we have studied a family non-universal $Z'$ model to pursue possible solutions. With the constrained $b - s - Z'$ coupling by $B_s - B_s$ mixing and $B \to \pi K$ decays, we focus on the $Z'$ effects on the rare $B \to X_s \mu^+ \mu^-$ (including both the high and the low $q^2$ regions) and the purely leptonic $B_s \to \mu^+ \mu^-$ decays, both of which are also induced by FCNC $b \to s$ transitions. Our main conclusions are summarized as:
• $B_s - B_s$ mixing anomaly and “$\pi K$ puzzle” could be moderated simultaneously within such a family non-universal $Z'$ model. Corresponding to the two fitting results $S1$ and $S2$ by UTfit collaboration, a new weak phase $\phi_s \sim -72^\circ$ and $-82^\circ$ are crucial to resolve these two problems.

• Similar to the hierarchy of the CKM elements $|V_{td}^\ast V_{tb}|/|V_{ts}^\ast V_{tb}| \sim 0.2$, we find $|B_{db}/B_{sb}| \sim O(10^{-1}) (\lesssim 0.2)$. So, such a hierarchy should be hold within the model. Our results also imply the relations $B_{uu}^L < B_{dd}^L$ and $B_{uu}^R > B_{dd}^R$.

• Combing $B_{sb}^L$ restricted by $\bar{B}_s - B_s$ mixing and $B \to \pi K$ decays, and $B_{\mu\mu}^{L,R}$ by $B \to X_s \mu^+ \mu^-$, we find $B_{\mu\mu}^{L,R} \sim O(10^{-2})$. For observable $B(B \to X_s \mu^+ \mu^-)$, the reduction effects is dominated by the $Z'$ contributions induced by $B_{\mu\mu}^L$ in $S1$. And, both the $Z'$ contributions induced by $B_{\mu\mu}^L$ and $B_{\mu\mu}^R$ are helpful to enhance it. The forward-backward symmetry $A_{FB}(B \to X_s \mu^+ \mu^-)$ is sensitive to the $Z'$ contributions induced by $B_{\mu\mu}^R$ but dull to the one induced by $B_{\mu\mu}^L$.

• With the strictly constrained $Z'$ couplings by $\bar{B}_s - B_s$ mixing, $B \to \pi K$ and $B \to X_s \mu^+ \mu^-$, comparing with the SM prediction, we find $B(B_s \to \mu^+ \mu^-)$ could be reduced/enhanced about 19%/46% by $Z'$ contributions at most. The minimal value of $B(B_s \to \mu^+ \mu^-)$ appears at the point $B_{\mu\mu}^L - B_{\mu\mu}^R \sim -0.05$ with the minimal new weak phase $\phi_s \sim -65^\circ$.

The refined measurements for the (semi-)leptonic $B(s)$ decay in the upcoming LHCb and super B factory will provide a fertile testing ground for the SM and possible NP. Our analysis about the $Z'$ effects on the observables $B^{(H,L)}(B \to X_s \mu^+ \mu^-)$, $A_{FB}^{(H,L)}(B \to X_s \mu^+ \mu^-)$ and $B(B_s \to \mu^+ \mu^-)$ are helpful to confirm or refute the family non-universal $Z'$ model.

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Appendix A: Theoretical input parameters

For the CKM matrix elements, we adopt the fitting results from UTfit collaboration \[3, 48\]

\[
\begin{align*}
\bar{\rho} &= 0.154 \pm 0.022 (0.177 \pm 0.044), & \bar{\eta} &= 0.342 \pm 0.014 (0.360 \pm 0.031), \\
|V_{td}/V_{ts}| &= 0.209 \pm 0.0075 (0.206 \pm 0.012), \\
|V_{cb}| &= (4.13 \pm 0.05) \times 10^{-2} ((4.12 \pm 0.05) \times 10^{-2}),
\end{align*}
\]

(36)

with \(\bar{\rho} = \rho (1 - \frac{\lambda^2}{2})\) and \(\bar{\eta} = \eta (1 - \frac{\lambda^2}{2})\). The values given in the brackets are the CKM parameters in presence of generic NP, and used in our calculation when the \(Z'\) contributions are included.

As for the quark masses, there are two different classes appearing in our calculation. One type is the current quark mass which is scale dependent. Here we take

\[
\begin{align*}
\overline{m}_s(\mu) &= 27.4 \pm 0.4 [49], & m_s(2 \text{ GeV}) &= 87 \pm 6 \text{ MeV} [49], & \overline{m}_c(m_c) &= 1.27^{+0.07}_{-0.11} \text{ GeV} [36], \\
\overline{m}_b(m_b) &= 4.20^{+0.17}_{-0.07} \text{ GeV} [36], & \overline{m}_t(m_t) &= 164.8 \pm 1.2 \text{ GeV} [36],
\end{align*}
\]

(37)

where \(\overline{m}_q(\mu) = (m_u + m_d)(\mu)/2\), and the difference between \(u\) and \(d\) quark is not distinguished. The other one is the pole quark mass. In this paper, we take \[36, 50\]

\[
\begin{align*}
m_u &= m_d = m_s = 0, & m_c &= 1.61^{+0.08}_{-0.12} \text{ GeV}, \\
m_b &= 4.79^{+0.19}_{-0.08} \text{ GeV}, & m_t &= 172.4 \pm 1.22 \text{ GeV}.
\end{align*}
\]

(38)

As for the B-meson lifetimes and decay constants, we take \[36, 51\]

\[
\begin{align*}
\tau_{B_u} &= 1.638 \text{ ps}, & \tau_{B_d} &= 1.530 \text{ ps}, \\
f_{B_{u,d}} &= (190 \pm 13) \text{ MeV}, & \sqrt{\hat{B}_{B_u} f_{B_d}} &= (216 \pm 15) \text{ MeV}, \\
f_{B_s} &= (231 \pm 15) \text{ MeV}, & \sqrt{\hat{B}_{B_s} f_{B_s}} &= (266 \pm 18) \text{ MeV}.
\end{align*}
\]

(39)

(40)
Appendix B: Derivation for the Eq. (19)

From the effective Hamiltonian for $B_s \to l^+l^-$ decay given by Eqs. (16) and (18), the amplitude for the $B_s \to l^+l^-$ decay can be written as

$$ A = A_{SM} + A_{Z'}, $$  (41)

$$ A_{SM} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{2\pi \sin^2 \theta_W} Y(x_t) \langle \mu^+ \mu^- | \bar{s} \gamma^\mu (1 - \gamma_5) b \otimes \bar{\mu} \gamma_\mu (1 - \gamma_5) \mu | \bar{B}_s \rangle, $$  (42)

$$ A_{Z'} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \frac{-2B_{sb}^L B_{mu}^L}{V_{tb} V_{ts}} (\mu^+ \mu^- | \bar{s} \gamma^\mu (1 - \gamma_5) b \otimes \bar{\mu} \gamma_\mu (1 - \gamma_5) \mu | \bar{B}_s \rangle + \frac{-2B_{sb}^L B_{mu}^R}{V_{tb} V_{ts}} (\mu^+ \mu^- | \bar{s} \gamma^\mu (1 - \gamma_5) b \otimes \bar{\mu} \gamma_\mu (1 + \gamma_5) \mu | \bar{B}_s \rangle \right], $$  (43)

After parameterizing the hadron parts, the above equations could be rewritten as

$$ A_{SM} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* i f_{B_s} P_{B_s} \frac{\alpha}{2\pi \sin^2 \theta_W} Y(x_t) \bar{\mu} \gamma_\mu (1 - \gamma_5) \mu, $$  (44)

$$ A_{Z'} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* i f_{B_s} P_{B_s} \left[ \frac{-2B_{sb}^L B_{mu}^L}{V_{tb} V_{ts}} \bar{\mu} \gamma_\mu (1 - \gamma_5) \mu + \frac{-2B_{sb}^L B_{mu}^R}{V_{tb} V_{ts}} \bar{\mu} \gamma_\mu (1 + \gamma_5) \mu \right], $$  (45)

where we have defined $\langle 0 | \bar{s} \gamma^\mu (1 - \gamma_5) b | \bar{B}_s \rangle = -if_{B_s} P_{B_s}^\mu$, with $f_{B_s}$ being the $B_s$-meson decay constant. For simplicity, we introduce

$$ A_1 \equiv \bar{\mu} P_{B_s} (1 - \gamma_5) \mu, \quad A_2 \equiv \bar{\mu} P_{B_s} (1 + \gamma_5) \mu, $$

$$ B \equiv \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* i f_{B_s}, \quad C \equiv \frac{\alpha}{2\pi \sin^2 \theta_W} Y(x_t), $$

$$ D_1 \equiv -\frac{2B_{sb}^L B_{mu}^L}{V_{tb} V_{ts}}, \quad D_2 \equiv -\frac{2B_{sb}^L B_{mu}^R}{V_{tb} V_{ts}}. $$  (46)

Then the total decay amplitude can be written as

$$ A = B[(C + D_1)A_1 + D_2 A_2], $$  (47)

$$ |A|^2 = |B|^2 |(C + D_1)A_1|^2 + |D_2|^2 |A_2|^2 + (C + D_1)^* D_2 A_1^* A_2 + (C + D_1) D_2^* A_1 A_2^* . $$  (48)

It is easy to get

$$ |A_1|^2 = |A_2|^2 = 8m_\mu^2 m_{B_s}^2, $$  (49)

$$ A_1^* A_2 = A_1 A_2^* = -8m_\mu^2 m_{B_s}^2. $$  (50)
So,

\[ |A|^2 = |B|^2 8m^2 \mu m^2 B_s (C + D_1) - D_2 |^2, \]

\[ = \frac{G_F^2}{2} |V_{tb} V_{ts}^*|^2 8m^2 \mu m^2 B_s \left| \frac{\alpha}{2\pi \sin^2 \theta_W} Y(x_t) - \frac{2B_{sL}(B_{sL} - B_{sR})}{V_{tb} V_{ts}^*} \right|^2. \] (51)

Finally, with \( |P_c| = \frac{1}{2} \sqrt{m^2_{B_s} - 4m^2_\mu}, \) we get

\[ B(B_s \to \mu^+ \mu^-) = \tau_{B_s} \frac{|P_c|}{8\pi m^2_{B_s}} |A|^2 \]

\[ = \tau_{B_s} \frac{G_F^2}{4\pi} f_{B_s}^2 m^2_\mu m_{B_s} \sqrt{1 - \frac{4m^2_\mu}{m^2_{B_s}}} |V_{tb} V_{ts}^*|^2 \]

\[ \times \left| \frac{\alpha}{2\pi \sin^2 \theta_W} Y(x_t) - \frac{2B_{sL}(B_{sL} - B_{sR})}{V_{tb} V_{ts}^*} \right|^2. \] (52)

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