Pseudoscalar decay constant in heavy light systems

Joachim Hein

\textsuperscript{a}Dept. Physics & Astronomy, University of Glasgow, G12 8QQ, Scotland, UK; UKQCD Collaboration.

We discuss the size of the higher order terms in the NRQCD expansion of the pseudoscalar decay constant. Power law divergences in the matrix elements contributing to \( f_{PS} \) are also investigated.

1. Introduction

When investigating the physics of the heavy light mesonic system within the framework of non-relativistic QCD (NRQCD), the action and observables are expanded in powers of \( \Lambda_{QCD}/M_Q \), \( M_Q \) denoting the heavy quark mass. If the regularisation, in our case the lattice cutoff, would be removed, divergences will arise in the matrix elements and coefficients of the expansion. To circumvent this, we keep the cutoff finite and use an improved action to minimise residual effects on physical results. Large residual divergences would become obvious as scaling violations of the contributing matrix elements.

We use 278 quenched \( 12^3 \times 24 \) configurations with \( \beta = 5.7 \), generously provided by UKQCD. For the light quarks the tadpole improved clover action with \( \kappa = 0.1400 \) has been used. This is slightly lighter than the strange quark mass as determined from the \( K \) meson, \( \kappa_s |_K = 0.1398 \). At \( \beta = 6.0 \) such a mismatch leads to a shift of 1\% in \( f_{PS} \) which is negligible compared to other errors in our calculation. From \( m_\rho \) one gets \( a^{-1} = 1.103(11)(50) \) GeV \cite{Hodges:1997ym}. For the heavy quarks we used a tadpole improved action corrected up to order \( \mathcal{O}(1/M^2) \) in the bare heavy quark mass, details on which can be found in \cite{Hodges:1997ym}. We use heavy quark masses in the range \( 20 \leq aM \leq 0.6 \), which ranges from heavy quarks much heavier than the \( b \) and slightly lighter than the \( c \) quark.

All results presented are preliminary.

2. Size of \( f_{PS} \) contributions

The pseudoscalar decay constant \( f_{PS} \) is defined by the matrix element

\[
p_{\mu} f_{PS} := \langle 0 | A_\mu | PS \rangle,
\]

of a pseudoscalar meson state \( | PS \rangle \) and the axial vector current \( A_\mu \). In lattice NRQCD \( A_\mu \) has to be matched by a set of local operators \( J_{A,\text{lat}}^{(i)} \)

\[
A_0 = \sum_{i=0}^{2} c_i (\alpha_s, aM) J_{A,\text{lat}}^{(i)} + \sum_{i=3}^{5} J_{A,\text{lat}}^{(i)} + \mathcal{O}(\alpha_s^2, a^2, \frac{\alpha_s}{\Lambda}, \frac{a}{\Lambda}) ,
\]

\[
J_{A,\text{lat}}^{(0)} = \bar{q} \gamma_5 \gamma_0 Q ,
\]

\[
J_{A,\text{lat}}^{(1)} = -\frac{1}{2M} \bar{q} \gamma_5 \gamma_0 (\bar{c} B) ,
\]

\[
J_{A,\text{lat}}^{(2)} = \frac{1}{2M} (\bar{c} \gamma_5 \gamma_0) Q ,
\]

\[
J_{A,\text{lat}}^{(3)} = \frac{1}{8M^2} \bar{q} \gamma_5 \gamma_0 D^2 Q ,
\]

\[
J_{A,\text{lat}}^{(4)} = \frac{g}{8M^2} \bar{q} \gamma_5 \gamma_0 \bar{c} B Q ,
\]

\[
J_{A,\text{lat}}^{(5)} = -\frac{i g}{4M^2} \bar{q} \gamma_5 \gamma_0 \alpha_5 \gamma_i \bar{c} D^2 Q .
\]

\( M \) denotes the bare heavy quark mass. The \( c_i \) are determined in one loop PT \cite{Hodges:1997ym}. Since the scale \( q^* \) of the coupling constant \( \alpha_s \) has not been calculated, we will average the final results over \( a q^* = 1 \) and \( \pi \). The difference is treated as a systematic error of the pert. expansion.

In figure \ref{fig:matrix_elements} we display the size of the matrix element contributing to \( f_{PS} \) in different orders of the bare heavy quark mass. At the \( b \)-quark the dif-

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different orders in $1/M$ are nicely suppressed by one order of magnitude with respect to the previous. However at the $c$-quark the $\mathcal{O}(\frac{1}{M})$ contribution has the same size as the $\mathcal{O}(\frac{1}{M^2})$ part.

If you look it in detail, this is not that surprising. For the term in $\mathcal{O}(\frac{1}{M})$ we observe a suppression of 0.36 GeV/$M$ at the $B_s$ meson and 0.29 GeV/$M$ at the $B_s$ meson with respect to the leading term. For the $\mathcal{O}(\frac{1}{M^2})$ term the suppression is 0.26 GeV$/M^2$ resp. 0.22 GeV$^2$/M$^2$. These numbers are compatible with the expectation of a factor $\approx \Lambda_{\text{QCD}}/M$ for each order in $\frac{1}{M}$, but for the $\mathcal{O}(\frac{1}{M^2})$ term there is apparently a prefactor of $\approx 2$ to $3$. At the $c$ quark $aM \leq 1$, which could have some affect on the outcome, requiring further investigation.

3. Power law contributions

Due to operator mixing, the coefficients $c_i$ of eqn. (3) would diverge for $a \to 0$ as $\mathcal{O}(\frac{1}{aM})$, $\mathcal{O}(\frac{1}{aM^2})$, \ldots , which is unphysical. Therefore the divergences have to cancel against similar terms arising from the matrix elements of the $J^{(i)}_{A,\text{lat}}$. To investigate the size of the unphysical part of the matrix elements in comparison to their physical, we compare to the results of [2], which were obtained at $a^{-1}(m_\rho) = 1.92$ GeV. For the comparison we study

$$O^{(1)}_{A,\text{lat}} = \langle 0|MJ^{(1)}_{A,\text{lat}}|PS \rangle,$$

$$O^{(M^2)}_{A,\text{lat}} = \langle 0|M^2(J^{(3)}_{A,\text{lat}} + J^{(4)}_{A,\text{lat}} + J^{(5)}_{A,\text{lat}})|PS \rangle,$$

which have a reduced $M$ dependence and a non-zero static limit. The result for $O^{(1)}_{A,\text{lat}}$ is shown in figure 2. No scaling violation is found within the quite small error bars. For the ratio of these values we obtain 1.06(5) at the $B_s$, which seems incompatible with a pure $\alpha_s/aM$ power law behaviour. Using $\alpha_s(aq^* = 2)$ this would lead to a ratio of 1.39. Our ratio of the $O^{(1)}_{A,\text{lat}}$ is independent of the quark mass and its error stays below 0.1 even for the heaviest quark mass.

For $O^{(M^2)}_{A,\text{lat}}$ we observe some scaling violation as displayed in figure 3. At the $B_s$ the result on the fine lattice is by a factor of 1.35(7) larger than the one from the coarse one. However a power law behaviour $\alpha_s/(aM)^2$ would lead to a factor of 2.4. Again the scaling violations are independent of the heavy quark mass.
4. Pseudoscalar decay constant

In fig. 3 we show the scaling behaviour of $f_{PS}$ itself. The error bars encompass the statistical errors and the uncertainty of $q^*$ for both graphs, since the latter need not to be equal when changing the $a$ value. Apart from the heaviest meson masses, the error bars on the $\beta = 5.7$ graph are dominated by the uncertainty of $q^*$. The central values differ by 15% ± 10%, which is compatible with the expected $O((a\Lambda_{QCD})^2)$ discretisation correction. Furthermore, a plot of the ratio of these results is flat within error bars.

For the decay constant of the $B_s$ we obtain from the data set at $\beta = 5.7$

$$f_{B_s} = 201(6)(15)(7) \text{ MeV}. \tag{5}$$

Here we only give the statistical error, the uncertainty arising from $q^*$ and the determination of the bare $b$ quark mass.

5. Conclusion

In this talk we discuss the size of the contributions to $f_{PS}$ in a large range of heavy quark masses, encompassing the $b$ and the $c$ quark. At the $b$ we find higher orders to be nicely suppressed. Scaling studies find no evidence for large power divergent contributions to higher order matrix elements, which appear to be dominated by their physical $O(\Lambda_{QCD}/M)$ resp. $O(\Lambda_{QCD}^2/M^2)$ term. For the decay constant itself we observe reasonable scaling behaviour.

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