Coexistence of two dissipative mechanisms in two-dimensional turbulent flows

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Abstract. Two distinct dissipative mechanisms occurring in two-dimensional fully developed turbulent flows in the limit of vanishing viscosity have been highlighted by means of direct numerical simulation. First, molecular energy dissipation is triggered by the production of localized vortices at the walls. Second, instabilities intrinsic to the flow itself generate a noisy component which can be quantified by wavelet analysis. The possibilities of competition and coexistence of the two mechanisms are discussed.

1. Introduction

The topic of dissipation by macroscopic flows is approached by considering one of its most representative occurrences, namely dissipation by fully developed turbulent flows. We explore the applicability of a conditional statistical modelling approach: can we sort out what part of the information should be kept, and what part should be modelled statistically, or, in other words, “dissipated”? It is argued that dissipation can arise either due to the residual effect of a microscopic coupling parameter, or due to purely macroscopic nonlinear mixing effects. In this work, we emphasize the difference between these two phenomena, by presenting two situations where they occur independently of each other.

The first situation is 2D homogeneous isotropic turbulence, for which the macroscopic concept of dissipation we introduce relies on a split of the flow between coherent and incoherent contributions (Farge et al., 1999): the coherent flow is constructed from the wavelet coefficients of the vorticity field whose modulus is above a certain threshold, and the incoherent flow from the remaining wavelet coefficients. In previous work, a unique threshold was applied to all wavelet coefficients, while here we also consider the effect of a scale by scale thresholding algorithm, called scale-wise coherent vorticity extraction (Nguyen van yen et al., 2011).

The second idealized situation that we consider is a dipole-wall collision (Orlandi, 1990), which is a generic event in two-dimensional incompressible flows with solid boundaries. A first
specific study of energy dissipation in this context was conducted by Clercx & van Heijst (2002), where the Navier-Stokes equations (NSE) with no-slip boundary conditions were approximated using Galerkin discretization over a basis of Chebychev polynomials, both in the wall parallel and wall normal directions. The numerical solutions satisfied the no-slip boundary conditions to machine accuracy but it was later found out (Clercx & Bruneau, 2006) that they were not converged beyond the collision time. In contrast, we have focused on using a numerical scheme which resolves scales at least as fine as $Re^{-1}$ (Nguyen van yen et al., 2011), to cope with a theorem of Kato (1984). Such a stringent requirement on resolution has not been enforced, to our knowledge, in any previous numerical experiments at similar Reynolds numbers. The best way that we have found to comply with it was to work with a numerical model known as volume penalization (Angot et al., 1999). The counterpart is that the no-slip boundary conditions are replaced by Navier boundary conditions with a slip length tending to zero when $Re \to \infty$.

2. Numerical method

The two series of datasets that we analyze were obtained by solving numerically the initial value problem for the Navier-Stokes equations in a periodic domain with the effect of solid boundaries modeled by a volume penalization term:

$$
\begin{aligned}
\partial_t u + (u \cdot \nabla) u &= -\nabla p + \frac{1}{Re} \nabla^2 u - \frac{1}{\eta} \chi u \\
\nabla \cdot u &= 0, \quad u(\cdot, t = 0) = u_0,
\end{aligned}
$$

where $u$ is the velocity field, $p$ is pressure, $\nu$ is the kinematic viscosity, $\chi$ is a mask function which equals zero in the fluid domain and one in the solid domain, and $\eta$ is the penalization parameter. The equations are discretized in space using a fully dealiased pseudo-spectral Fourier method, and in time using a third order Runge-Kutta scheme.

| $Re$   | $1.7 \cdot 10^4$ | $6.6 \cdot 10^4$ | $2.66 \cdot 10^5$ | $1.062 \cdot 10^6$ | $4.248 \cdot 10^6$ | N/A |
|--------|------------------|------------------|------------------|------------------|------------------|-----|
| $\alpha$ | 1                | 1                | 1                | 1                | 1                | 2   |
| $N$      | 512              | 1024             | 2048             | 4096             | 8192             | 8192 |

Table 1. Parameters of numerical experiments concerning homogeneous isotropic turbulence.

To study homogeneous isotropic turbulence we formally set $\eta = +\infty$. We use a resolution up to $N^2 = 8192^2$, and perform splittings of the field thanks to a wavelet decomposition of the vorticity field, while varying the Reynolds number according to $Re \sim N^{-2}$, everything else being fixed (Table 1). In addition, we consider a case where the Laplacian operator in (1) is replaced by a squared Laplacian, the constant in front being adjusted accordingly to preserve proper resolution of the flow, see (Nguyen van yen et al., 2011) for details. Classical diagnostics concerning the obtained solutions, reported in Fig. 1, are consistent with the available literature on 2D turbulence.

To study the dipole-wall collision, we use a resolution up to $N^2 = 16384^2$, and we vary the Reynolds number as $Re \sim N^{-1}$, and the penalization parameter as $\eta \propto N^{-1}$ (Table 2).

| $Re$   | 985              | 1970             | 3940             | 7880             |
|--------|------------------|------------------|------------------|------------------|
| $N$      | 2048             | 4096             | 8192             | 16384            |
| $\eta$ | $4 \cdot 10^{-5}$ | $2 \cdot 10^{-5}$ | $8192$           | $10^{-5}$        | $0.5 \cdot 10^{-5}$ |

Table 2. Parameters of numerical experiments concerning the dipole-wall collision.
3. Results

For 2D homogeneous turbulence, we split the vorticity field using the scale-wise coherent vorticity extraction method (Nguyen Van Yen et al., 2011). The vorticity is thus split into a coherent component, which contains organized structures corresponding to the large, infrequent, wavelet coefficients within each scale, and an incoherent component, which behaves statistically as a correlated noise (Fig. 2). By measuring the enstrophy and energy content of both components as a function of Reynolds number (Fig. 4), we find that 2D homogeneous turbulence

![Energy spectra (left) and vorticity pdfs (middle) at t = 50, and time evolution of enstrophy (right) for 2D homogeneous isotropic turbulence at various Reynolds numbers.](image)

**Figure 1.** Energy spectra (left) and vorticity pdfs (middle) at $t = 50$, and time evolution of enstrophy (right) for 2D homogeneous isotropic turbulence at various Reynolds numbers.

![Left to right: total, coherent, and incoherent components of the vorticity field as a result of scale-wise coherent vorticity extraction in fully developed 2D homogeneous isotropic turbulence.](image)

**Figure 2.** Left to right: total, coherent, and incoherent components of the vorticity field as a result of scale-wise coherent vorticity extraction in fully developed 2D homogeneous isotropic turbulence.

![Snapshots of the vorticity field at several instants during the collision of a dipole into a wall at Re = 7880. The whole vorticity field at t = 0 is shown in the inset on the left, and the successive frames correspond to zooms at t = 0.36, 0.4, 0.45 and 0.495 near the impact region of the positive (red) vortex.](image)

**Figure 3.** Snapshots of the vorticity field at several instants during the collision of a dipole into a wall at $Re = 7880$. The whole vorticity field at $t = 0$ is shown in the inset on the left, and the successive frames correspond to zooms at $t = 0.36, 0.4, 0.45$ and $0.495$ near the impact region of the positive (red) vortex.
produces incoherent enstrophy and incoherent energy. We interpret this process as macroscopic dissipation.

For the dipole-wall collision, Fig. 3, we directly measure the molecular energy dissipation rate due to the localized structure produced at the wall as a result of the collision. We find (Fig. 4) that, as the Reynolds number increases, this rate seems to converge towards a value which is independent on the Reynolds number.

4. Discussion

The coexistence of the two dissipative mechanisms we have studied could be readily triggered by integrating a flow in the presence of walls for times longer than have been considered there. Indeed, due to vortex sheet instabilities, the randomization effect that we have observed in homogeneous turbulence is expected to occur sooner or later during the evolution of any flow whose Reynolds number is sufficiently large. In opposition with the most commonly accepted view, our results suggest that molecular dissipation in such flows will be essentially due to very localized coherent structures produced by the wall, whereas the incoherent background flow resulting from randomization undergoes very weak molecular dissipation because it is produced mostly at intermediate scales in the inertial range, and not in the dissipative range.

The behavior of drag as a function of Reynolds number for three-dimensional wall-bounded flows has puzzled engineers and mathematicians alike for centuries. Experimentally, it is observed that the drag does not tend to zero when the Reynolds number is increased up to the highest attainable values. This is directly related to the ability of the flow to efficiently dissipate the power injected by the drag force at the walls, and thus to the type of phenomena we have been considering here. Although the essential phenomenon of vortex stretching which occurs in three-dimensional flows is completely absent in our model, we argue that the phenomenology which it reproduces makes its study of great practical relevance. Surprisingly, very few numerical studies of two-dimensional turbulence in the presence of walls have been reported in the literature (Li & Montgomery, 1996; Clercx & van Heijst, 2000; Schneider & Farge, 2005; Clercx & van Heijst, 2009). This may stem from the fact that the most important system featuring quasi 2D flows, namely the Earth’s atmosphere, does not have boundaries. However, if two-dimensional wall-bounded flows are considered as a toy model for flows which become both noisy and singular in the limit of infinite Reynolds number, their study may bring in new ideas to tackle the old riddle of turbulent dissipation, and this is the essential message of this paper.

Figure 4. Left: decomposition of enstrophy as a result of scale-wise coherent vorticity extraction at a fixed time as a function of Reynolds number for homogeneous isotropic 2D turbulence. Middle: same with energy. Right: energy dissipation as a function of Reynolds number in the region corresponding to the dashed square in Fig. 3 for the dipole-wall collision.
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