Recent Developments in Heavy Quarkonium Phenomenology

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• Summary
Polarization of Prompt $J/\psi$
at the Fermilab Tevatron
$p_T$ Distribution of Prompt $J/\psi$

at the Fermilab Tevatron

At Large $p_T$

- Color-singlet fusion does not explain the data.
- Color-singlet fragmentation corrects the shape only.
- Inclusion of color-octet gluon fragmentation resolves the problem in both shape and normalization.
**J/ψ Production Mechanism**

**Gluon Fusion**

\[
d\sigma/dp_T^2 \propto \alpha_s^3 M^4/p_T^8
\]

\[
c\bar{c}(^3S_1^{(1)}, ^1S_0^{(8)}, ^3P_J^{(8)})
\]

**Color-Singlet / Gluon Fragmentation**

\[
d\sigma/dp_T^2 \propto \alpha_s^5/p_T^4
\]

Braaten, Yuan (93)

\[g^* \rightarrow 2g + c\bar{c}(^3S_1^{(1)}) \rightarrow \psi'\]

**Color-Octet / Gluon Fragmentation**

\[
d\sigma/dp_T^2 \propto \alpha_s^3 v_c^4/p_T^4
\]

Braaten, Fleming (95)

\[g^* \rightarrow c\bar{c}(^3S_1^{(8)}) \rightarrow \psi' + \text{(soft)}\]

- The gluon-fragmentation diagrams are suppressed by \(\alpha_s^2\) (singlet) or \(v_c^4\) (octet) compared to the color-singlet fusion diagrams.
- At large \(p_T\), such suppression factors are overcome by the large kinematic enhancement because the virtuality of the fragmenting gluon is of order \(m_{J/\psi}\).
NRQCD Lagrangian

\[ \mathcal{L} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}} + \delta \mathcal{L} \]

\[ \mathcal{L}_{\text{heavy}} = \psi^\dagger \left( iD_t + \frac{D^2}{2M} \right) \psi \]
\[ + \chi^\dagger \left( iD_t - \frac{D^2}{2M} \right) \chi \]

\( \psi \rightarrow Q \) annihilation operator
\( \chi \rightarrow \bar{Q} \) creation operator

**Bilinear Interaction Terms**

\[ \delta \mathcal{L} = \frac{c_1}{8M^3} (\psi^\dagger (D^2)^2 \psi - \chi^\dagger (D^2)^2 \chi) + \cdots \]
\[ + \frac{c_4}{2M} (\psi^\dagger (gB \cdot \sigma) \psi - \chi^\dagger (gB \cdot \sigma) \chi) \]

**4-Fermion Interaction Terms**

\[ \delta \mathcal{L} = \sum_n \frac{f_n(\Lambda)}{M^{d_n-4}} \mathcal{O}_n(\Lambda) \]

\( f_n: \) short distance coefficient \hspace{1cm} \( \mathcal{O}_n: \) 4 quark operator
NRQCD Factorization Approach

Factorization Formula (incl. prod.)
Bodwin, Braaten, Lepage (95)

\[ \sigma(H) = \sum_n F_n(\text{short}) \langle 0 | \mathcal{O}^H_n | 0 \rangle(\text{long}) \]

\[ \mathcal{O}^H_n = \chi^\dagger \mathcal{K}_n \psi \sum_{X} |H + X\rangle \langle H + X| \psi^\dagger \mathcal{K}'_n \chi \]

\[ n = 3 \quad S_1^{(1,8)}, 1 \quad S_0^{(1,8)}, 3 \quad P_J^{(1,8)}, \ldots \]

\[ \mathcal{K}_n = (1, \sigma) \otimes (1, D) \otimes (1, T^a) \]

\begin{align*}
\text{Scale} & \quad p \geq M \quad p \leq M\nu \\
\text{in } F_n & \quad \text{in } \langle \mathcal{O}^H_n \rangle
\end{align*}

Important Channels for $J/\psi$

\begin{align*}
^3S_1^{(1)} & \quad 1 \quad \text{Color-Singlet} \\
^3S_1^{(8)} & \quad v_c^4 \quad \text{Octet, Non-Spin-Flip}^2, \text{Dominates in high } p_T \\
^1S_0^{(8)} & \quad v_c^3 \quad \text{Octet, Spin-Flip} \\
^3P_J^{(8)} & \quad v_c^4 \quad \text{Octet, Non-Spin-Flip, } P-\text{wave} \\
& \quad (\text{Chromo-electric/magnetic Dipole Approximation})
\end{align*}

- Separates long-distance and short-distance contributions in a factorized form.
- Expansion is made systematically in powers of $\alpha_s$ and $\nu$. 
How to Determine Long-distance NRQCD Matrix Elements

ME's are Universal

Production Matrix Elements

Only Phenomenological Determination

\[ \sigma(H) = \sum_n F_n \times \langle O_n^H \rangle \]
measure \( \sigma(H) \)
calculate \( F_n \) with pert. QCD
determine \( \langle O_n^H \rangle \)

Decay Matrix Elements

Singlet
Lattice NRQCD
Potential Model
Phenomenological

Octet
Lattice NRQCD
\( c\bar{c}, b\bar{b} \) (quenced): Bodwin, Sinclair, Kim, PRL(1996)
\( b\bar{b} \) (unquenced): Bodwin, Sinclair, Kim, PRD65 (02)

Phenomenological

• Phenomenological determination may have limitations if an observable depends on many independent matrix elements.
Polarization Predictions

HIGH $p_T$

gluon fragmentation

$\rightarrow$ transversely polarized

$\psi'$ Polarization

Cho, Wise (95)
Beneke, Rothstein (96)
Beneke, Krämer (97)
Leibovich (97)

Prompt $J/\psi$ Polarization

Braaten, Kniehl, Lee (99)

• Introduction of color-octet gluon fragmentation explained the production rate.
• Observation of transverse $J/\psi$ at large $p_T$ may provide a strong confirmation of the color-octet gluon-fragmentation dominance.
Polarization Parameter $\alpha$

$$\alpha = \frac{\sigma - 3\sigma_L}{\sigma + \sigma_L}$$

$$\frac{dT}{d\cos\theta}(J/\psi \rightarrow \mu^+\mu^-) \sim 1 + \alpha \cos^2 \theta \quad \theta = \angle(p_{J/\psi}, \mu^+) \text{ in } J/\psi \text{ rest frame}$$

- $\sigma(\sigma_L)$: unpolarized (longitudinal) contribution to $d\sigma/dp_T$ at a given $p_T$.
- $\alpha$ can be determined phenomenologically by measuring angular distribution of $\mu$ at $J/\psi$ rest frame.
- Quantization axis $\hat{z}$ is chosen so that $\hat{z} \propto p_{J/\psi}$ in the $p\bar{p}$ CM frame.

| Polarization | $\alpha$ | $\alpha$ |
|--------------|----------|----------|
| unpolarized  | $+1$     | 0        |
| 100% $T$     | 0        | -1       |

Color Evaporation Model: $\alpha = 0$

Prompt $J/\psi$ and Inclusive $\gamma(nS)$

direct + Feed-down

easy to measure

more samples than direct

theoretically complicated
Feeddowns to $J/\psi$ and $\Upsilon(1S)$
Polarization of Prompt $J/\psi$ at the Tevatron: RUN I

CDF, PRL (2000)
Braaten, Kniehl, Lee, PRD (2000); Kniehl, Lee, PRD (2000)

Agree in intermediate $p_T$
Disagree in the highest $p_T$ bin
Error bars are large
Polarization of Prompt $J/\psi$ at the Tevatron: RUN II

CDF, PRL (2007)

- Prompt polarization does not show the trend towards transverse polarization

CDF RUN II data disagree with both CDF RUN I and NRQCD

Problem still not resolved
E866 Measurement of $\Upsilon$ Polarization in p-Cu

$\Upsilon(1S)$ : UNPOLARIZED
$\Upsilon(2 + 3S)$ : TRANSVERSELY POL (~ 100%)

- No fragmentation dominance
- Feeddown dilutes polarization
- Color-evaporation model disagrees with data

a) $\alpha$ versus $p_T$ (E866)
Drell-Yan ($m_{\mu^+\mu^-} = 8.1 - 8.45, 11.1 - 15.0$ GeV)
$\Upsilon(1S)$ ($8.8 < m_{\mu^+\mu^-} < 10.0$ GeV)
$\Upsilon(2S+3S)$ ($10.0 < m_{\mu^+\mu^-} < 11.1$ GeV)
$0.28 < \alpha_{LO}^{\Upsilon(1S)} < 0.31$ : Tkabladze, PLB462 (99)

b) $\alpha$ versus $x_F$ (E866)
The errors shown are statistical, there is an additional systematic error not shown of 0.02 in $\alpha$ for Drell-Yan polarizations and 0.06 in $\alpha$ for onium polarizations.
Polarization of Prompt $J/\psi$

Vaia Papadimitriou, Physics in Collision 2003
Braaten, Lee, PRD (2001)

Run I: 
PRL 88 (2002)161802

- Prediction less dramatic as $J/\psi$
  - no fragmentation dominance
- Large errors from uncertainties in NRQCD matrix elements

similar to $c\bar{c}$ → as yet inconclusive

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Polarization of Upsilon at the Tevatron: RUN II (D0)

- Strong longitudinal polarization at low $p_T$.
- As $p_T$ increases, $\alpha$ increases, but not up to the NRQCD Predictions.

- $\alpha$ increases as $p_T$ increases.
- Low $p_T$ prediction not available.
- $\Upsilon(2S)$ is more transverse (less feeddown) : consistent with NRQCD
- D0 data disagree with CDF

D0, arXiv:0804.2799
Braaten, Lee, PRD (2001)
NLO Corrections to Color-Singlet Contribution
(Estimating NNLO Contributions)

F. Maltoni, QWG07
Atroisenet, Lansberg, Maltoni, PLB (2007)
Campbell, Maltoni, Tramontano, PRL (2007)

- NLO corrections: real + virtual
- Fragmentation approximation disregarded (direct calculation)
- Incomplete NNLO corrections
- Improved in size and shape, but smaller than data
  → Still needs the color-octet contributions
Transverse momentum distribution of differential cross section with $\mu_r = \mu_f = \sqrt{(2m_c)^2 + p_T^2}$ at Tevatron.
Center mass energy is $\sqrt{s_{\text{Tevatron}}} = 1.98 \text{ TeV}$
NLO+ denotes result including contribution from $gg \to J/\psi c\bar{c}$ at NLO.

- Includes virtual + real corrections
- Confirms Maltoni et al’s results (cross section smaller than data)
- Dramatic change in polarization, but cross section disagrees with data
Gong, Li, Wang, arXiv:0805.4751

- Includes only $^1S_0$, $^3S_1$ octet
- $^3P_J$ octet not included
- $\langle O_8 \rangle_{^1S_0}$, $\langle O_8 \rangle_{^3S_1}$ are fitted to the data
- Does not include feeddown from $\chi_{cJ}, \psi(2S)$ (direct only)
- Polarization is still transverse
- Discrepancy in polarization not resolved
Resolution of
\[ e^+e^- \rightarrow J/\psi + \eta_c \]
puzzle at the B factories
Resolution of $e^+e^- \rightarrow J/\psi + \eta_c$ puzzle

Recoil mass distribution in $e^+e^- \rightarrow J/\psi + X$ at the B factories

BELLE, PRD (2004)

- Bumps are for the resonances of 2-body final states.
- **BELLE**:
  \[ \sigma[e^+e^- \rightarrow J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4 \text{ fb}. \]
- **BABAR**:
  \[ \sigma[e^+e^- \rightarrow J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb}. \]
Resolution of $e^+e^- \rightarrow J/\psi + \eta_c$ puzzle

\[ e^+e^- \rightarrow J/\psi + \eta_c \]

Puzzle

- NRQCD at LO in $\alpha_s$ and $\nu$:
  - Braaten, Lee, PRD (2003) : (QCD+QED)
    \[ \sigma[e^+e^- \rightarrow J/\psi + \eta_c] = 3.78 \pm 1.26 \text{ fb} \]
  - Liu, He, Chao, PLB (2003) : (QCD)
    \[ \sigma[e^+e^- \rightarrow J/\psi + \eta_c] = 5.5 \text{ fb} \]
- Much smaller than the measured cross sections
Resolution of $e^+e^- \rightarrow J/\psi + \eta_c$ puzzle

**NLO Corrections**

NLO corrections in $\alpha_s$ give a large $K$ factor ($\sim 1.96$), but not large enough to resolve the problem.

Zhang, Gao, Chao, PRL (2006)
Necessity of the Resummation of Relativistic Corrections

• Braaten and Lee showed that the relativistic corrections is large with huge uncertainties.
• Order-$v^2$ NRQCD matrix element was known with large uncertainties.
• Bodwin, Kang, Lee introduced the first reliable determination of order-$v^2$ matrix element.
NRQCD Matrix Elements and Potential Model

- **LO NRQCD matrix elements** in terms of potential-model WF:
  \[
  \psi(0) = \int \frac{d^3 p}{(2\pi)^3} \tilde{\psi}(\mathbf{p}) = \frac{1}{\sqrt{2N_c}} \langle 0 | \chi^\dagger \psi(x = 0) | H(1S_0) \rangle, \\
  \epsilon \psi(0) = \frac{1}{\sqrt{2N_c}} \langle 0 | \chi^\dagger \sigma \psi(x = 0) | H(3S_1) \rangle.
  \]

- **Order-\(v^2\) NRQCD matrix element** in terms of potential-model WF:
  \[
  \psi^{(2)}(0) \equiv \int \frac{d^3 p}{(2\pi)^3} \mathbf{p}^2 \tilde{\psi}(\mathbf{p}) = \frac{1}{\sqrt{2N_c}} \langle 0 | \chi^\dagger (-\nabla^2) \psi(x = 0) | H \rangle.
  \]
  Appear in the relativistic corrections to many quarkonium decay and production processes.

- \(\langle \mathbf{p}^2 \rangle \equiv \psi^{(2)}(0)/\psi(0), \quad \langle v^2 \rangle = \langle \mathbf{p}^2 \rangle / m_c^2\).
Previous Attempts to Determine $\psi^{(2)}(0)$

- **Phenomenological attempts:**
  large uncertainties from uncalculated higher orders in $\alpha_s, m_c$.

- **Lattice attempts:**
  large uncertainties because of large cancellations in converting from lattice regularization to dimensional regularization.

- **Gremm-Kapustin relation:**
  \[
  \frac{\psi^{(2)}(0)}{\psi(0)} = \langle p^2 \rangle \approx (M_H - 2m_c)m_c = \epsilon_B m_c.
  \]
  $\epsilon_B$ is the binding energy. Large uncertainties in $m_c$ make the method unreliable: For $m_c = 1.4 \pm 0.2$ GeV
  \[-0.35 < \langle p^2 \rangle < 0.84, \quad -0.14 < \langle v^2 \rangle < 0.58.\]

- **Even the sign of $\psi^{(2)}(0)$ is not known with great confidence.**
Strategy to Calculate $\psi^{(2)}(0)$

- Use the Bethe-Salpeter equation to expose one loop.
- Large momentum contribution dominates.
- Coulomb-gluon dominates over the transverse gluon.
- UV divergence regularized with Dim Reg.

$$\psi^{(2)}_{DR}(0) = -m\tilde{\gamma}_B^2 \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3p}{(2\pi)^3} \frac{\tilde{V}(k-p)\tilde{\psi}(p)}{k^2 - \tilde{\gamma}_B^2 + i\epsilon}.$$ 

- Use Schrödinger equation

$$ (k^2 - \tilde{\gamma}_B^2)\tilde{\psi}(k) = -\int \frac{d^3p}{(2\pi)^3} m\tilde{V}(k-p)\tilde{\psi}(p).$$

- $V$: Use Cornell potential model to get

$$\psi^{(2)}_{DR}(0) = \tilde{\gamma}_B^2 \int \frac{d^3k}{(2\pi)^3} \tilde{\psi}(k) = \tilde{\gamma}_B^2 \psi(0).$$

- Obtain generalized Gremm-Kapustin relation

$$\langle \nu^2 \rangle = \frac{\psi^{(2)}(0)}{m_c^2 \psi(0)} = \frac{\epsilon_B}{m_c} + \mathcal{O}(\nu^4).$$

$$\epsilon_B = \tilde{\gamma}_B^2 / m : \text{Binding energy}$$

Bodwin, Kang, Lee, PRD (2006)
Resummation

- NRQCD matrix elements of higher order in $v$ are
  \[ \psi^{(2n)}(0) \equiv \int \frac{d^3p}{(2\pi)^3} p^{2n} \tilde{\psi}(p) = \frac{1}{\sqrt{2N_c}} \langle 0| \chi^\dagger (-\nabla^2)^n \psi(x = 0)|H\rangle. \]
  \[ \langle p^{2n} \rangle \equiv \psi^{(2n)}(0)/\psi^{(0)}(0). \]

- Using the equation of motion, we obtain the generalized Gremm-Kapustin relation:
  \[ \langle p^{2n} \rangle = \langle p^2 \rangle^n. \]

- This formula allows us to resum a class of the relativistic corrections to the $S$-wave quarkonium amplitude to all orders in $v$.

  Bodwin, Kang, Lee, PRD (2006)
Calculation of Matrix Elements

- Make use of $\Gamma[J/\psi \to e^+ e^-]$ and $\Gamma[\eta_c \to \gamma \gamma]$ to compute $\langle O_1 \rangle_{J/\psi}$ and $\langle O_1 \rangle_{\eta_c}$. A class of relativistic corrections is resummed.

- Make use of the potential model to compute $\langle q^2 \rangle_{J/\psi}$ and $\langle q^2 \rangle_{\eta_c}$.

- Results:
  
  $\langle O_1 \rangle_{J/\psi} = 0.440^{+0.067}_{-0.055}$ GeV$^3$,
  
  $\langle O_1 \rangle_{\eta_c} = 0.437^{+0.111}_{-0.105}$ GeV$^3$,
  
  $\langle q^2 \rangle_{J/\psi} = 0.441^{+0.140}_{-0.140}$ GeV$^2$,
  
  $\langle q^2 \rangle_{\eta_c} = 0.442^{+0.143}_{-0.143}$ GeV$^2$.

  Bodwin, Chung, Kang, Lee, Yu, PRD (2008)
Relativistic Corrections to $e^+e^- \rightarrow J/\psi + \eta_c$

- Indirect relativistic corrections account for a change of 72%
- Direct relativistic correction: 40%
- The effect of resummation is small, being -12% of the direct relativistic correction.

Result:

Theory: $\sigma[e^+e^- \rightarrow J/\psi + \eta_c] = 17.6^{+8.1}_{-6.7} \text{ fb}$

Belle: $\sigma[e^+e^- \rightarrow J/\psi + \eta_c] = 25.6 \pm 2.8 \pm 3.4 \text{ fb}$

BaBar: $\sigma[e^+e^- \rightarrow J/\psi + \eta_c] = 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb}$

- It seems fair to say that the discrepancy between theory and experiment has been resolved.

Bodwin, Lee, Yu, PRD (2008)
Resolution of $e^+e^- \rightarrow J/\psi + \eta_c$ puzzle

**NLO Corrections to**

$$e^+e^- \rightarrow J/\psi + \chi_{c0}$$

![Diagram](image)

- NLO Corrections give a large $K$ factor ($\sim 2.8$)
- Using $m_c = 1.5\text{GeV}$, $\langle O_1 \rangle \chi_{c0} = 0.0688\text{GeV}^5$ and $\alpha_s = 0.259$, 
  $$\sigma[e^+e^- \rightarrow J/\psi + \chi_{c0}] \approx 17.9\text{fb}$$
- Resummation of relativistic corrections not available

Zhang, Ma, Chao, arXiv:0802.3655
Inclusive Charm Production in Bottomonium Decays
Inclusive Charm Production in $\chi_b$ Decays

(a) $b\bar{b}_1(3P_J) \rightarrow c\bar{c}g \propto C_J^{(c)}, C_8^{(c)}$
(b) $b\bar{b}_8(3S_1) \rightarrow g^* \rightarrow c\bar{c} \propto C_8^{(c)}$

$$d\Gamma(c) = dC_J^{(c)}(\Lambda) \frac{\langle O_1(3P_J) \rangle}{m_b^4} + dC_8^{(c)} \frac{\langle O_8(3S_1) \rangle(\Lambda)}{m_b^2}$$

$$O_8(3S_1) = O_8(3S_1)^{(\Lambda)} + \frac{(4\pi e^{-\gamma})^{\epsilon}}{\epsilon_{UV}} \frac{2C_F\alpha_s}{3\pi N_c m_b^2} \sum_{j=0}^{2} O_1(3P_J)$$

- Color-singlet model for $\chi_Q \rightarrow LH$ has infrared divergences
- It has been resolved by the color-octet mechanism of NRQCD factorization approach [Bodwin, Braaten, Lepage, PRD (92)]
- $\chi_b \rightarrow c + X$ has the same feature, but the $c$ momentum distribution is calculable and accessible experimentally
- Direct investigation of color-octet mechanism
Matrix elements for $\chi_{bJ}$

Lattice simulation

$$\langle O_1 \rangle_{\chi_b(1P)} = 3.2 \pm 0.7 \text{ GeV}^5,$$

$$\frac{\langle O_8 \rangle_{\chi_b(1P)}^{(\Lambda)}}{\langle O_1 \rangle_{\chi_b(1P)}} = 0.0021 \pm 0.0007 \text{ GeV}^{-2}.$$ 

Bodwin, Sinclair, Kim, PRD65(’02)

$$\rho_8 \equiv \frac{m_b^2 \langle O_8 \rangle_{\chi_b}^{(m_b)}}{\langle O_1 \rangle_{\chi_b}} = 0.044 \pm 0.015.$$ 

Potential model (Buchmüller–Tye potential)

$$\langle O_1 \rangle_{\chi_b(1P)} \approx 2.03 \text{ GeV}^5,$$

$$\langle O_1 \rangle_{\chi_b(2P)} \approx 2.37 \text{ GeV}^5.$$ 

Bodwin, Braaten, Kang, Lee, PRD76(’07)

From the solution to the RG equation

$$\langle O_8 \rangle_{\chi_b}^{(m_b)} = \langle O_8 \rangle_{\chi_b}^{(\Lambda)} + \frac{4C_F}{3N_c\beta_0} \log \left( \frac{\alpha_s(\Lambda)}{\alpha_s(m_b)} \right) \frac{\langle O_1 \rangle_{\chi_b}}{m_b^2}.$$ 

$$\Lambda = m_b v.$$ 

$$\rho_8 \gtrsim 0.068.$$
Inclusive Charm Production in $\chi_b$ Decays

Color-octet contributions

$y_1$ : scaled momentum of the charm quark

$r = m_c^2/m_b^2 = 4m_D^2/m_{\chi_{b,j}}^2,$

$x_1 = E_1/m_b,$

$y_1 = \sqrt{\frac{x_1^2 - r}{1 - r}}.$

Singular at the end point.

Bodwin, Braaten, Kang, Lee, PRD76 (07)

• Color-singlet P-wave contributions diverge at the endpoint.
• The IR divergence cancels that of 1-loop correction to color-octet S-wave NRQCD matrix element.
• Remaining UV divergence is absorbed by renormalizing the color-octet matrix element.
• $\chi_{b1} \rightarrow g^* g$ survives:
  \[
  \text{Br}[\chi_{b1} \rightarrow c + X] \text{ is significant because } \chi_{b1} \rightarrow gg \text{ is forbidden.}
  \]
Inclusive Charm Production in \( \chi_b \) Decays

| \( \chi_b \) | \( \rho_b \) | Theory | \( \rho_b = 0.1 \) |
|--------------|-------------|--------|------------------|
| \( \chi_{b0}(1P) \) | 13 ± 7 ± 2 | 6      | 5                |
| \( \chi_{b1}(1P) \) | 31 ± 5 ± 5 | 25     | 23               |
| \( \chi_{b2}(1P) \) | 13 ± 4 ± 2 | 12     | 8                |
| \( \chi_{b0}(2P) \) | 8 ± 6 ± 1  | 6      | 5                |
| \( \chi_{b1}(2P) \) | 19 ± 3 ± 2 | 25     | 23               |
| \( \chi_{b2}(2P) \) | 1 ± 3 ± 1  | 12     | 8                |

B. Heltsley QWG@DESY, Oct 18, 2007

CLEO Preliminary

\( \rho_b \)

Bodwin, Braaten, Kang, Lee, PRD76 (07)

- The ratio \( \rho_b \equiv m_b^2 \langle \mathcal{O}_8 \rangle_{\chi_b} / \langle \mathcal{O}_1 \rangle_{\chi_b} \) can be determined phenomenologically.
- CLEO measurement will provide a strong constraint to the ratio.
Plot $E_\gamma$ for tagged $D_0$ near $M_D$
- D-sideband subtraction
- Smooth bgd subtraction
- Fit using lineshapes from inclusive $\gamma$'s

>7$\sigma$ signals for $\chi_{b1}(1P)$, $\chi_{b1}(2P)$

Correct for efficiency
- Assume $\rho_8 = 0.1$ (non-perturbative model parameter) for $p$>2.5 GeV/c cut

Subtract secondary sources of $\chi_{bJ}$

Correct for $\chi_{bJ} \rightarrow \gamma X$: quote $B^*$

| $B^*(\chi_{bJ}(nP) \rightarrow c\bar{c} X)$ (%) | Theory | $\rho_8 = 0.1$ |
|----------------------------------------------|--------|----------------|
| $\chi_{b0}(1P)$: 13 ± 7 ± 2 | 6      | 5              |
| $\chi_{b1}(1P)$: 31 ± 5 ± 5 | 25     | 23             |
| $\chi_{b2}(1P)$: 13 ± 4 ± 2 | 12     | 8              |
| $\chi_{b0}(2P)$: 8 ± 6 ± 1 | 6      | 5              |
| $\chi_{b1}(2P)$: 19 ± 3 ± 2 | 25     | 23             |
| $\chi_{b2}(2P)$: 1 ± 3 ± 1 | 12     | 8              |

CLEO Preliminary
Inclusive Charm-hadron Production in $\chi_b$ Decays

- Charm hadron distribution is obtained by convolving charm quark momentum distribution with a fragmentation function for $c \rightarrow$ charm hadron $D$.

\[
\frac{d\Gamma}{dy_D} = \frac{dz_D}{dy_D} \int_{z_D}^1 \frac{dz_1}{z_1} D(z_D/z_1) \frac{dy_1}{dz_1} \frac{d\Gamma}{dy_1}
\]

- We used the fragmentation function whose parameters were determined by the Belle Collaboration.

Belle, PRD73, 032002 (2006)

Unphysical negative rates near at the end point are the artifacts of fixed-order calculation. Resummation of logarithmic corrections to all orders may cure the problem.
Inclusive Charm Production in Bottomonium Decays

\[ b(p) \rightarrow c(p_1) \]

\[ b(p) \rightarrow \bar{c}(p_2) \]

\[ g(p_3) \]

\[ g(p_4) \]

\[ (a) \quad b\bar{b}_1(3\,S_1) \rightarrow c\bar{c}gg \propto C_1^{(c/g^a)} \]

\[ \Rightarrow d\Gamma^{(c)} = dC_1^{(c)} \left< \mathcal{O}_1(3\,S_1) \right> \frac{1}{m_b^2} \]

- CLEO can also access the charm production in the inclusive \( \Upsilon \) decays.
- There is a significant QED contribution as well as QCD one.
- There are no infrared singularities unlike \( Xb \) decays.
- At leading order in \( \nu \), the color-octet terms do not contribute to the decay rate.
Charm-quark Momentum Distribution in $\Upsilon$ Decays

- QCD contribution is broad.
- QED contribution is localized at the kinematic end point.

$y_1 :$ scaled momentum of the charm quark

\[ r_c = \frac{m_c^2}{E_b^2}, \]
\[ x_1 = \frac{E_1}{E_b}, \]
\[ y_1 = \sqrt{\frac{x_1^2 - r_c}{1 - r_c}}. \]
Charm-hadron Momentum Distribution in $\Upsilon$ Decays

• Convolution with the fragmentation function significantly soften the spectrum.
• QED contribution, which was localized at the end point, is smeared out to wider ranges of $y_D$.
• QED dominates over QCD contribution.
• Include feeddowns from $D^*$:
  $D^{*0} \rightarrow D^0$ (100%), $D^{*+} \rightarrow D^0$ (68%), $D^{*+} \rightarrow D^+$ (31%)
Summary

- **Polarization of the Prompt $J/\psi$ at the Tevatron**
  - CDF Run II data disagree with Run I as well as NRQCD
  - Problem still open

- **$e^+e^- \rightarrow J/\psi + \eta_c$ Puzzle at the B factories**
  - Large-relativistic corrections associated with one-loop QCD corrections have resolved the problem

- **Inclusive Charm Production in Bottomonium Decays**
  - Direct investigation of color-octet mechanism
  - CLEO may provide strong constraint to the ratio $\rho_8$ of octet/singlet matrix elements for $\chi_b$