Increasing the mobility of tracked vehicles during curvilinear motion with partial skidding

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Abstract. A method for determining the forces and moment of resistance for curvilinear motion of a tracked vehicle on deformable soil is presented. It is assumed that in the patch of contact with the ground, the track performs both rotational and translational motion. Under these conditions, the real law of friction has a fundamental difference from the classical Coulomb’s law. To describe this process, a friction model based on the first-order Padé expansion is proposed, generalized to a flat rectangular contact pattern of the tracked mover with the ground. The determining factor in this process is sliding of the tracked mover relative to the ground. It limits the realizable traction force and the boundary conditions for the adhesion of the tracks to the ground in the contact pattern. The interpretation of these physical processes occurring in the contact pattern, based on the theory of combined friction, makes it possible to use this mathematical model in the structure of the algorithm of an automatic motion control system to solve the problem of controllability in case of partial side skid, which will further increase mobility and maneuverability of tracked vehicles.

1. Introduction

At present, the possibilities of increasing average speeds of high-speed tracked vehicles when turning by improving the design of the turning mechanism are practically exhausted. The main limitation of the speed of movement is partial lateral skid, which is characterized by the loss of traction of the tracks with the ground and the beginning of uncontrolled side slip under certain parameters of the vehicle turning in specific road and ground conditions. At the same time, the results of studies of the turning dynamics accumulated to date show dependence of the stability of movement on the displacement of the center of inflexions and the lateral skid of the lower track wheels, which is presented in the works of A. Blagonravov, V.B. Derzhanskii, V.I. Krasnenkov, F.A. Opeyko [1, 2, 3]. The work of foreign experts is also devoted to the determination of dependences of kinematic and power parameters of the turning of tracked vehicles [4, 5, 6, 7, 8]. When making a turn at speeds above 35-40 km/h, the driver cannot take proactive actions to prevent skidding due to a delay in the driver’s own reaction to a change in the movement trajectory, as well as due to a delay in the reaction of the vehicle to a change in the position of the control (steering wheel) [9, 10]. Currently, high-speed tracked vehicles use proportional motion control systems, which have a low control quality due to a large phase lag in the response of the vehicle to control, and also do not provide the function of automatic control of the parameters of curvilinear motion and prevention of stability loss and side
skid. To improve the quality of control, it is proposed to increase the control system sensitivity by introducing a booster circuit [11, 12]. To implement the algorithm for the operation of such a control system, in this work, a mathematical model is proposed that describes the skidding processes of a tracked vehicle based on considering the displacement of the center of inflexions and the lateral skid of the lower track wheels, when the movement stability of the vehicle is lost. The basis is formalization of the processes of interaction of the support surface of the tracked mover with the ground in the contact pattern [15]. This model can be used to solve the problem of determining the boundaries of the turn stability [13] and optimum control [14].

2. Main part

When constructing mathematical models of interaction of the tracked mover’s support surface with the soil in the contact pattern, the determining factor is the type of soil - non-deformable or deformable. For non-deformable soil, the processes of formation of resistance forces in the contact pattern are currently well studied and described by empirical dependences [1, 2]. For deformable soil, the interaction processes are more complex and, in the general case, substantially nonlinear. The currently known dependences of the determining forces (moments) of resistance in the contact pattern do not take into account the effect of the angular velocity of rotation of the tracked vehicle, which becomes an essential factor when simulating motion in unsteady modes, including side skidding.

To obtain the necessary dependencies when constructing a mathematical model, nonholonomic kinematic connections between the theoretical and actual speeds of the tracks are taken into account, which are expressed through the slip (skid) coefficients [15].

The forces of resistance to the movement of vehicles \( R_1 \) and \( R_2 \) are the sum of the forces of rolling resistance \( R_{ij} \) and the forces of friction \( R_{tj} \). The resultant forces are applied at points \( O_1 \) and \( O_2 \) - the geometric centers of the tracks.

To simulate the interaction process of tracks with the ground and determine the resistance forces, the vertical load is represented in the form of concentrated forces \( G_{ij}(i = 1, \ldots, n; j = 1, 2) \) acting on the ground under separate lower track wheels [15]. The axes of the lower track wheels have a longitudinal coordinate \( l_i \) relative to the middle of the bearing surface.

Resistance forces are considered as the totality of the forces of the tracks’ rolling resistance and friction of the tracks against the ground:

\[
R_j = R_{ij} + R_{tj}, \quad (j = 1, 2).
\] (1)

The rolling resistance of the tracks is proportional to the normal load acting on each \((i, j)\) lower track wheel

\[
R_{ij} = f \cdot \sum_{i=1}^{n} G_{ij}, \quad (j = 1, 2)
\] (2)

and is directed parallel to the longitudinal axis of the tracked vehicle.

When the mover interacts with the soil, the latter is subjected to crushing, shear in different directions, as a result of which fields of normal and tangential stresses arise in the soil. Simply, this interaction model can be represented by a generalized friction model. The friction forces of the tracks relative to the ground in the classical formulation of the problem are also proportional to the normal load

\[
R_{tj} = \mu_j \cdot \sum_{i=1}^{n} G_{ij}, \quad (j = 1, 2),
\] (3)

where \( \mu_j \) - coefficient of resistance to friction of the tracks against the ground.

The friction forces are directed in the direction opposite to the direction of vectors of instantaneous velocities of motion of the geometric centers of the tracks \( V_1 \) and \( V_2 \).
We assume that friction between the surface and the ground obeys the Coulomb’s law and is anisotropic when moving along deformable soil. In this case, coefficient of friction \( \mu \) depends on the coefficient of tracks’ slipping \( \delta \) and is determined by the expression

\[
\mu_{i,2} = \mu_{\text{max}} \left( 1 - e^{-k\delta_{i,2}} \right),
\]

where \( \mu_{\text{max}} \) - maximum coefficient of friction-adhesion,

\( k \) - experimental constant.

Thus, the resultants of all elementary friction forces of the tracks with the ground \( R_{ij} \) are directed at an angle \( \gamma_{ij} \) to the longitudinal axis of the vehicle (Fig. 1). In this regard, dependence of the longitudinal and lateral components of the friction force on the direction of the resultant one is of interest. First of all, as shown by Opeyko F.A. [1], the principle of superposition is not preserved in kinematic connections with friction. The presence of skidding indicates a limitation of the resulting adhesion force, therefore, an increase in the lateral component leads to a decrease in the longitudinal component and a change in the direction of the resulting force. That is, there is an inverse relationship. In this case, the magnitude of the friction forces is a function of the angle relative to the longitudinal axis \( R_{ij} = f(\gamma_{ij}) \) and is determined by the friction hodograph for a given type of soil and the profile of the working surface of the tracks with lugs. The closed curve representing friction hodograph determines the adhesion limit value of the friction force. The presence of skidding when turning the tracked vehicle indicates that the resultant of friction forces has reached the adhesion limit. When the adhesion limit is reached, the sliding force is less than the adhesion limit.

![Figure 1](image_url). Direction of the friction force in the contact pattern of the track with the ground.

\( R_{ij} \) - friction force of tracks with the ground under the i-th lower track wheel; \( O_{ij} \) - center of vertical load application on the i-th lower track wheel; \( \gamma_{ij} \) - angle between the friction force direction and the longitudinal axis of the vehicle; \( O_{1j} \) - center of inflexions of j-th track; \( \chi \) - longitudinal displacement of the center of inflexions; \( y_j \) - lateral displacement displacement of the center of inflexions.

The elementary forces arising during sliding in the contact pattern are limiting in adhesion. With instant rotation of the track support surface, the speed of each point is directed perpendicular to its
distance to the instantaneous center of rotation. When skidding, all the elementary forces reach their maximum adhesion value. The resulting frictional force $R_{Tj}$ can reach its limiting value even in the absence of a moment (i.e., with a rectilinear translational motion). The resulting moment appears when a shoulder of the resultant force component occurs relative to the center of inflexions.

The frictional forces of the tracks relative to the ground and the resistance to turning are formed by the corresponding projections of the elementary frictional forces in the contact pattern. This makes it possible to determine the moment of resistance to turning $M$ in the system of equations of motion [15] as a set of moments created by the projections of the resultant friction forces under each lower track wheel relative to the center of inflexions, based on geometric representations. In this case, the moment of resistance to rotation is

$$
M = \sum_{j=1}^{n} \sum_{i=1}^{n} \mu_j \cdot G_{ij} \cdot \sqrt{y_j^2 + (l_j - \chi)^2} \cdot \sin \left( \gamma_j - \arctg \frac{y_j}{l_j - \chi} \right),
$$

where $y_j$ - lateral displacement of the center of inflexions;
$
\chi$ - longitudinal displacement of the center of inflexions;
$
\gamma_j$ - angle between the direction of the friction forces under each lower track wheel $R_{Tij} = \mu_j \cdot G_{ij}$ and the longitudinal axis of the vehicle.

A feature of expression (5) is the nature of friction coefficients, which are limiting in adhesion. Thus, when turning the tracked vehicle, with an increase in the angular speed of turning, the longitudinal component of the friction force decreases, which is a geometric factor and is determined by the friction hodograph. However, there is another effect, which is practically not described explicitly in the literature on the theory of tracked vehicles movement. This effect consists in the fact that with an increase in the angular rate of turning, the coefficient of friction in the contact pattern decreases. This largely determines the peculiarity of transient processes in case of partial side skid of the tracked vehicle. To describe this process, it is advisable to use modern models of friction.

This case is analogous to the so-called combined friction, which occurs when the surface of a body performs both rotational and translational motion. Under these conditions, the real law of friction has a fundamental difference from the classical Coulomb’s law.

Using the friction model based on the first-order Pade expansion, and generalizing it to a flat rectangular contact pattern of the tracked mover with the ground, the following expressions were obtained for the friction force and the moment of resistance to turning [15] - [18]:

$$
R'_{Tj} = R_{Tj} \cdot \frac{V_j}{V_j + b \cdot \psi \cdot \sqrt{y_j^2 + (l_j - \chi)^2}},
$$

$$
M' = M \cdot \frac{\psi \cdot \sqrt{y_j^2 + (l_j - \chi)^2}}{\psi \cdot \sqrt{y_j^2 + (l_j - \chi)^2} + a \cdot V_j},
$$

$$
R'_{Tj} = R_{Tj} \bigg|_{\psi = 0},
$$

$$
M' = M \bigg|_{V_j = 0},
$$

$$
1 = \psi \frac{\partial R'_{Tj}}{\partial V_j} \bigg|_{V_j = 0},
$$

$$
\frac{1}{a} = \frac{\partial R_{Tj}}{\partial V_j} \bigg|_{V_j = 0}.
$$
Here $M$ - moment of friction resistance forces (5), limiting in adhesion; $R_{Tj}$ - friction resistance force (4); $V_j$ - linear speed of translational motion (skidding) of the geometric center of the contact pattern in a fixed coordinate system; $\psi$ - angular speed of rotation relative to the instantaneous center of inflexion; the area

$$\sqrt{y_j^2 + (l_{ij} - \chi)^2}$$

-radius of rotation of the geometric center of the contact pattern relative to the instantaneous center of inflexion;

$a$ and $b$ - empirical coefficients.

A distinctive feature of the friction models (4), (5), (7)…. (11) was that they were all built on the assumption of the validity of the Coulomb’s law in differential form for a small element of the area inside the contact pattern. The known results of experimental studies of real dry friction processes [7], which differ from the classical Coulomb processes, make it possible to represent the characteristic of the friction force at a non-zero sliding velocity by a nonlinear dependence

$$R_{Tj} = \mu_j \cdot \sum_{i=1}^{n} G_i \cdot \left( \text{sign} V_j - \gamma_1 \cdot V_j + \gamma_2 \cdot V_j^3 \right),$$

where $\gamma_1$ and $\gamma_2$ - empirical coefficients.

In the case of combined kinematics, dependence (142) can be used in differential form [8]. In this case, additional polynomial terms appear in the representations for the force and moment of friction. Then the two-dimensional model of first-order friction (7) - (11) has the following form

$$\begin{pmatrix}
V_j \\
V_j + b \cdot \psi \cdot \sqrt{y_j^2 + (l_{ij} - \chi)^2} \\
+ 2 \cdot \pi \cdot \left( \gamma_1 \cdot V_j^3 - \gamma_2 \cdot V_j \cdot I_3 + 2 \cdot \gamma_1 \cdot V_j \cdot \psi \cdot I_3 ight)
\end{pmatrix},$$

$$\begin{pmatrix}
\frac{1}{a} = \frac{\psi}{R_{Tj}} \cdot \frac{\partial R_{Tj}'}{\partial V_j} \bigg|_{V_j=0}
\end{pmatrix},$$

$$\begin{pmatrix}
\frac{1}{b} = \frac{V_j}{M'} \cdot \frac{\partial M'}{\partial \psi} \bigg|_{\psi=0} 
\end{pmatrix}. $$
The coefficients of the polynomial terms of this model are the first moments of distribution of normal contact stresses in contact [9]. The moments of the first, third and fifth order are determined by the following expressions:

\[ I_1 = \int_0^1 r \sigma(r) \, dr, \]  
(17)

\[ I_3 = \int_0^1 r^3 \sigma(r) \, dr, \]  
(18)

\[ I_5 = \int_0^1 r^5 \sigma(r) \, dr, \]  
(19)

where \( \sigma(\kappa) \) - distribution of contact stresses inside the contact pattern; \( r \) - radius-vector of an elementary area inside the contact pattern.

The values of moments (17) - (19) are determined by the distribution law of normal contact stresses in the contact pattern, which can be presented in analytical form [10] or by approximation by a grid set.

The quantitative differences in the behavior of the forces and moments of frictional resistance for the presented model, in comparison with the classical case, are of interest for determining the boundaries of the stability region of the tracked vehicle. In particular, the results of numerical simulation of the movement of high-speed tracked vehicles weighing 22 tons and 16.1 tons along a trajectory of the ‘snake’ type in the range of speeds of 32 ... 38 km/h showed displacement of the stability boundary to the region of lower speeds by 7-10%, compared with the traditional representation of forces and moments of resistance. It also affects the phase frequency characteristics of tracked vehicles, determining the delay in the response of the vehicle to the control action in the angular speed and heading angle, and limiting the speed by the condition of fitting into the dynamic corridor of movement, which is confirmed by the results of experimental studies [9].

To study the stability of the curvilinear movement of tracked vehicles, it is of interest to interpret real friction processes different from the classical Coulomb processes. In particular, an analogue of the movement of a machine in the full skid mode is the process of friction-sliding, in which it can be assumed that the instantaneous center of velocities lies inside the contact pattern. In this case, for any nonzero angular velocity of rotation of the object, the friction force in the vicinity of low sliding velocities behaves like a viscous friction force [6]. With respect to sliding, the friction at rest practically disappears, and any arbitrarily small perturbing force acting parallel to the surface of the contact pattern leads to the occurrence of skidding. In conditions of combined kinematics, the use of the classical Coulomb’s law is not correct. The closest approximation to the real situation is provided by the use of the first-order two-dimensional model, which was considered above. This also implies the long-known fact of slipping (skidding) of the tracks when turning. This is explained within the framework of the combined theory of friction by the fact that in the presence of arbitrarily small spinning, the condition for the absence of skidding in the contact pattern cannot be realized.

3. Conclusion

The new method considered above for determining the forces and moment of resistance during curvilinear movement of a tracked vehicle based on a combined model of first-order friction makes it possible to clarify the physical meaning of the processes of violation of the movement stability of a tracked vehicle. The combined friction model makes it possible to substantially supplement the method for evaluating the conditions for a tracked vehicle to enter and exit a skid, obtained from the condition of the presence of a limit set for a local integral manifold. Of course, the presented model of combined friction simplifies the real processes of interaction of the tracked mover with the ground when turning, which is also subjected to a complex process of crushing and shear in different directions. Nevertheless, the determining factor of this process is precisely the skidding of the tracked
mover relative to the ground. It limits the realizable traction force and the boundary conditions for the adhesion of the tracks to the ground in the contact patch. The interpretation of these physical processes occurring in the contact pattern, based on the theory of combined friction, makes it possible to use this mathematical model in the structure of the algorithm of an automatic motion control system to solve the problem of controllability in case of partial side skid, which will further increase the mobility and maneuverability of tracked vehicles.

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