The Lorentz force and superconductivity

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To change the velocity of an electron requires that a Lorentz force acts on it, through an electric or a magnetic field. We point out that within the conventional understanding of superconductivity electrons appear to change their velocity in the absence of Lorentz forces. This indicates a fundamental problem with the conventional theory of superconductivity. A hypothesis is proposed to resolve this difficulty. This hypothesis is consistent with the theory of hole superconductivity.

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In the microscopic realm, electrons do not change their state of motion in the absence of an electromagnetic force (we omit consideration of gravitational forces throughout this paper). For example, in the Stark effect the electron changes its wavefunction when an electric field is applied because of the electric Lorentz force acting on the electron. In paramagnetic atoms, orientation of atomic magnetic moments under application of a magnetic field can be understood as arising from the magnetic Lorentz force on orbital or intrinsic (due to spin) electric currents. In a diamagnetic atom, the wavefunction of the electron does not change upon application of a magnetic field (to lowest order) but its velocity does. From the relation between velocity and canonical momentum \( \vec{p} \) in the presence of a magnetic vector potential \( \vec{A} \),

\[
\vec{v} = \frac{\vec{p}}{m_e} - \frac{e}{m_e c} \vec{A} \quad (1)
\]

one finds that the change in velocity when the magnetic field is increased from zero to its finite value is

\[
\Delta \vec{v} = -\frac{e}{m_e c} \vec{A} \quad (2)
\]

since the wavefunction and consequently the canonical momentum do not change to first order \( \hat{1} \). Here, \( m_e \) is the free electron mass. This change of velocity can be understood as arising from the Lorentz electric force generated by the changing magnetic field through Faraday’s law \( \hat{1} \). I argue that we know of no example in the microscopic quantum-mechanical world where electrons would change their velocity in the absence of an applied Lorentz force.

\[
\vec{F} = e(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}) \quad (3)
\]

which requires either an electric field, or a magnetic field together with a non-zero velocity.

In the macroscopic world, new phenomena may occur when many degrees of freedom are at play \( \hat{2} \). For example, a ferromagnet will 'spontaneously' develop a magnetic moment when cooled below its critical temperature. Still, even in that case no net magnetic moment will be observed in the absence of an applied magnetic field because of domain formation. If the metal is cooled in the presence of a magnetic field the magnetic moments of the domains will orient in the direction of the external field because of the torque that a magnetic moment experiences in the presence of a magnetic field, which we can also attribute to the Lorentz force Eq. (3).

In contrast, in superconductors new phenomena are observed that appear to be more 'mysterious' than those seen in both the microscopic world as well as in other macroscopic phase transitions: electrons change their state of motion in certain specific ways that appear to be independent of forces acting on the electrons. I argue that these situations present a puzzle within the conventional theory of superconductivity. An explanation of these phenomena is proposed that requires a revision of the conventional understanding of superconductivity.

A. The Meissner effect

The expulsion of magnetic flux from the interior of a type I superconductor when a magnetic field is turned on can be understood, just as diamagnetism of atomic electrons, as arising from the current generated through the Lorentz electric force generated by the changing magnetic field acting on the superfluid electrons \( \hat{3} \). However when a simply connected type I superconductor is cooled below \( T_c \) in the presence of a constant magnetic field, the same final state needs to be achieved, in the absence of electric forces generated by changing magnetic fields. Furthermore since the superfluid is supposed to be at rest, no magnetic Lorentz force can act. How is it possible then that a state with finite screening currents is reached?

London postulates the London equation \( \hat{4} \)

\[
\vec{v}_s = -\frac{e}{m_e c} \vec{A} \quad (4)
\]

for the superfluid velocity that should exist in the presence of the vector potential \( \vec{A} \), regardless of how that state was reached. No justification for Eq. (4) exists within the standard theory of electromagnetism. As a plausibility argument it is argued that the wavefunction of the superconductor is 'rigid' and that Eq. (1) with
\( \vec{p} = 0 \) applies for a simply connected superconductor independent of history, so that Eq. (4) is the only possible state due to this rigidity. However this begs the question: rigidity or not, in the atom the diamagnetic current is generated through a real force generated by a real electric field acting on the atomic electron. Why can’t the final state of the superconductor be understood in the same way? How do the electrons in the superconductor 'know' to start moving when the metal is cooled below \( T_c \) in the presence of a magnetic field?

B. The rotating superconductor

In a simply connected superconductor rotating with angular velocity \( \vec{\omega} \), a magnetic field exists throughout its interior given by [9]

\[ \vec{B} = -\frac{2m_e c}{e} \vec{\omega} \]  

(conventionally called 'London field') [5]. This has been verified experimentally for both conventional [6, 7] and high \( T_c \) superconductors. Theoretically it was predicted first based upon the theory of perfect conductors [5], for the case where the metal in the superconducting state is put into rotation. In that framework it can be understood as arising because the electrons near the surface 'lag behind' when the body is put in rotation, and a surface current is generated. When the ions start moving the resulting electric current due to ionic motion generates a changing magnetic field which in turn generates an electric field that makes the electrons follow suit.

The existence of the field Eq. (5) also follows from London’s equation [3], and hence is predicted to exist also when a rotating normal metal is cooled below its superconducting transition temperature, and indeed is so found experimentally [5]. However we face then a similar problem as for the Meissner effect discussed above. If the electrons are rotating with the lattice in the normal state, what makes the electrons near the surface 'lag behind' when the metal becomes superconducting to generate the interior magnetic field Eq. (5)? No magnetic field nor electric field should initially exist, so no Lorentz force acts on the electron.

Furthermore there is another mysterious consequence of Eq. (5). In the interior of the superconductor the electrons are rotating at the same angular velocity as the lattice. Assuming no force is exerted by the ionic lattice on the superfluid, the centripetal force for the electron to rotate needs to be provided by the magnetic field. However the magnetic field required for a charge \( e \) and mass \( m_e \) to rotate with angular velocity \( \vec{\omega} \) is half the value of the magnetic field Eq. (5)! In other words, an electron rotating in a magnetic field rotates at the cyclotron frequency \( \omega = eB/m_e c \) rather than the Larmor frequency \( \omega = eB/2m_e c \). Consequently, for mechanical equilibrium, an electric field in the interior of the superconductor needs to exist:

\[ \vec{E} = \frac{\vec{B} \times (\vec{\omega} \times \vec{r})}{2c} \]  

This electric field points towards the interior of the superconductor. Hence it requires the negative superfluid to move slightly in towards the interior of the superconductor to generate this field, as shown schematically in Figure 1. However, one would expect exactly the opposite: if in a rotating metal the electrons become 'free' as the metal enters the superconducting state, the centrifugal force would push the electrons out rather than in. However if such was the case the resulting electric field would point out, which would be incompatible with Eq. (5) and mechanical equilibrium.

We should point out that previous discussions of the electric field inside rotating superconductors within the conventional framework erroneously concluded that an electric field pointing out exists [6, 7], compatible with the expectation [5] that electrons should move out due to the centrifugal force. This is because in analyzing the situation in the rotating frame, the contribution to the electric field arising from Lorentz-transforming the magnetic field Eq. (5) was omitted.

C. The quantized flux

Consider a metal ring with magnetic flux through its center in a well-localized region that does not overlap the ring, as shown in Figure 2. The flux quantization condition is

\[ \oint \vec{p} \cdot d\vec{l} = nh \]  

FIG. 1: Charge configuration in a rotating superconductor implied by the conventional theory (qualitative). For mechanical equilibrium, an electric field in the interior of the superconductor needs to exist; generated by negative electrons moving in slightly giving a non-uniform charge distribution.
FIG. 2: Superconducting ring threading magnetic flux. The magnetic field lines are confined to a small central region far away from the inner surface of the ring. No magnetic field exists anywhere in the ring. A current \( j \) will exist near the ring surfaces if the applied flux is not an integer multiple of the flux quantum.

\[
(n=\text{integer, } h=\text{Planck’s constant}) \text{ requires that if the ring is in the superconducting state the magnetic flux enclosed is an integer multiple of the flux quantum}[12]:
\]

\[
\Phi = \int \vec{B} \cdot d\vec{S} = n\Phi_0 \quad (8a)
\]

\[
\Phi_0 = \frac{hc}{2e} \quad (8b)
\]

If the applied magnetic field does not satisfy the condition Eq. (8), surface currents develop in the ring so that Eq. (8) is satisfied. If the external magnetic field is changed from a value that satisfies Eq. (8) to one that does not, while the ring is in the superconducting state, the development of these ring currents can be understood: as the magnetic flux is changed, magnetic field lines will move across the superconducting ring and exert a Lorentz force on the superfluid electrons that will drive the ring surface currents necessary to satisfy Eq. (8).

However, if the ring is cooled from the normal to the superconducting state while enclosing a flux that does not satisfy Eq. (8), how do the ring currents develop? In that case no magnetic field ever exists in the ring itself, as well as no electric field, so the Lorentz force is zero. How do the electrons know to start moving?

We argue that the three examples discussed above represent unsolved puzzles in the conventional understanding of superconductivity. In the following we propose a hypothesis that explains these puzzles.

A hint to explain these puzzles arises from consideration of the electron in a diamagnetic atom. In the atom, it is the change in the electron velocity that obeys the London-like equation (2). This can be simply understood classically. Assume the electron is rotating in an orbit of radius \( r \). The centripetal force is provided by the ionic electric field \( E_{\text{ion}} \):

\[
\frac{m_e v^2}{r} = eE_{\text{ion}}
\]

(9)

On applying a magnetic field, the change in the centripetal force is provided by the magnetic Lorentz force. In absolute value,

\[
\frac{2m_e v \Delta v}{r} = \frac{ev}{c}B
\]

(10)

leading to

\[
\Delta v = \frac{e}{m_e c} \frac{Br}{2}
\]

(11)

which is equivalent to Eq. (2) for \( \vec{A} = \vec{B} \times \vec{r}/2 \). The left-hand side of Eq. (10) follows from a variation of Eq. (9) only if \( \Delta v \ll v \), which is consistent with the fact that quantum-mechanically Eq. (2) only holds to first order in the magnetic field.

From an identical consideration it is clear that we will have mechanical equilibrium for the superfluid electrons in the rotating superconductor with the correct factor of 2 in the London field Eq. (5) if the superfluid electron is already rotating at a high angular velocity before the body is set into rotation, so that Eq. (10) applies just as for the electron in the atom (from Eq. (10), Eq. (5) follows for rigid rotation with \( \Delta v = \omega r \)). If this is so, an electric field has to exist in the interior of the superconductor which provides the centripetal force to sustain the superfluid electron rotation. This implies that a non-zero positive charge density exists in the interior of the superconductor, which in turn leads us to conclude that negative charge is expelled from the interior of the metal towards the surface when the metal enters the superconducting state.
FIG. 4: Explanation of the flux quantization puzzle. When the superconductor expels electrons from its interior, the electronic wave function 'leaks out' of the body of the superconductor. The tail of the electronic wave function has to extend into the region where the magnetic field is non-zero to feel the Lorentz force and start moving.

Remarkably, this hypothesis then provides us with an explanation of the Meissner effect. When the system goes superconducting electrons in the interior are expelled towards the surface. In the presence of a magnetic field, the Lorentz force on the radially outgoing electron will give rise to a tangential force in the direction needed to generate the surface currents that will screen the magnetic field, as shown schematically in Figure 3.

Furthermore this assumption provides a natural explanation for why the superfluid electrons near the surface 'lag behind' when a rotating metal becomes superconducting: electrons flow out, and if the body is rotating there is a Coriolis force on the outward flowing electrons that makes them lag behind when they approach the surface. Simply put, the electron from the interior has a smaller tangential velocity so that when it flows out it will lag the faster motion occurring at larger r.

Finally, how do the electrons in the superconducting ring threaded by magnetic flux get set into motion when neither a magnetic field nor an electric field nor a preexistent superfluid velocity exist in the ring? When the superconductor expels electrons from its interior we need to assume that the wavefunction of these electrons near the surface 'leaks out' and reaches the region where the magnetic field is non-zero, as shown in Figure 4. This occurs through a radially inward velocity, which when the wavefunction reaches the region of non-zero magnetic field gives rise to a Lorentz force in the tangential direction that can set the surface current in the superconducting ring into motion. Note that this implies that the current generated will always be such that the magnetic field generated by the ring currents opposes the pre-existing flux, hence will 'round down' the non-integral flux quantum number generated by the applied field.

We point out that the theory of hole superconductivity predicts that electrons are expelled from the interior of superconductors when the transition to superconductivity occurs, that a radially outward electric field exists in the interior of superconductors, and that the wavefunctions of electrons will leak out from the body of the superconductor, as required by the explanations discussed above. Of course other explanations may also be possible.

The reader may note that the proposed solution to these puzzles raises another puzzle. If superfluid electrons are rotating in the absence of body rotation and magnetic fields, why is no current observed? The reason is that when electrons are expelled from the interior of the superconductor, interaction of the electron spin of the radially outgoing electron with the ionic lattice will deflect electrons of opposite spin tangentially in opposite directions. As a consequence, macroscopic spin currents are predicted to exist in superconductors if this scenario is correct. This phenomenon and some experimental consequences are also discussed in ref. [15].

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