Double Beta Decay, Nuclear Structure and Physics beyond the Standard Model.

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Abstract. The Neutrinoless Double Beta Decay ($0\nu\beta\beta$) allows to determine the absolute scale of the neutrino masses. For this one must assume, that the light Majorana neutrino exchange is the leading mechanism for $0\nu\beta\beta$ and that the matrix element of this transition can ba calculated reliably. The different methods: Quasi-particle Random Phase Approximation (QRPA), Shell Model (SM), Projected Hartree-Fock-Bogoliubov (PHFB) and Interacting Boson Model (IBM2) to calculate these matrix elements are reviewed. The question, how one can determine the leading mechanism or mechanisms from the data of the $0\nu\beta\beta$ decay in different nuclei is studied in the later part of this article. Possible interference terms allow to test CP (Charge and Parity conjugation) violation.

1. Introduction
The neutrinoless double beta decay ($0\nu\beta\beta$) provides a method to determine the absolute scale of the neutrino masses. The determination of the masses is possible, if one assumes, that the light left handed Majorana neutrino exchange is the leading mechanism for the neutrinoless double beta decay and one is also able to calculate reliably the transition matrix element. To determine the absolute masses the matrix element $M^{0\nu}$ is as important as the data for the $0\nu\beta\beta$ transition. The different methods used to calculate these matrix elements are presented and compared with their advantages and their drawbacks [1, 2, 3, 4, 5, 6], [7], [8, 9, 10], [11].

If one gives up the assumption, that the light left handed Majorana neutrino exchange is the leading mechanism [12, 13] one can have cases, where two or even more equally strong mechanisms interfere. There the relative phases can test the CP (combined Charge conjugation and Parity) conservation or violation.

In addition to the light left handed Majorana neutrino exchange, one has other possible mechanisms as cause for the neutrinoless double beta decay: Grand Unification (GUT), Supersymmetry (SUSY) and extensions to extra dimensions. We shall discuss here extension to GUT’s and SUSY.

\[ T^{0\nu} = M^{0\nu}_\nu \cdot < m_\nu > + M_\theta < \tan \vartheta > + M_{WR} < \left( \frac{M_1}{M_2} \right)^2 > + M_{SUSY} \cdot \lambda_{111}^2 + M_{NR} < \frac{m_p}{M_{MR}} > + ... \] (1)
\[ < m_{\nu} > = \sum_{k=1,2,3} (U_{ek})^2 \cdot m_{k\nu} = \sum_{k=1,2,3} e^{2i\alpha_k} \cdot |U_{ek}|^2 \cdot m_{k\nu} \]  

(2)

The first term on the right hand side of eqn. (1) with the matrix element \( M_{0\nu} \) and the effective Majorana mass \( < m_{\nu} > \) of eqn. (2) is often called the gold plated term.

2. The different Many Body Approaches for the \( 0\nu\beta\beta \) Matrix Elements.

The groups in Tübingen, Bratislava and Jyväskylä [1, 2, 3, 4, 5, 6] use the Quasi-particle Random Phase Approach (QRPA), the group in Strasbourg-Madrid the Shell Model (SM) [7], Tuebingen, Rath et al. and Martínez-Pinedo et al. HFB and methods built on it [8, 9, 10] and Iachello [11] the Interacting Boson Model.

The expressions for the matrix elements \( M_{\nu} \) and the corresponding \( 0\nu\beta\beta \) transition operators are given, e.g., in Ref. [2]:

\[ M_{\nu}^{(0\nu)} = M_{\nu \nu}^{GT} - \left( \frac{g_V}{g_A} \right)^2 M_{\nu \nu}^{0\nu} - M_{\nu \nu}^{T} \]  

(3)

The SM approach has been applied by the Strasbourg-Madrid group [7]. It is compared with QRPA in fig.1 and in fig. 2.

The projected HFB approach for a real transformation, with axial symmetry and no parity mixing is restricted [16] to contributions of neutron pairs with angular momenta \( 0^+, 2^+, 4^+,... \). In addition the contributions of transition of higher angular momentum neutron to proton pairs \( 2^+, 4^+,... \) are drastically reduced compared to the QRPA and the SM. The reason for this is obvious: in a spherical nucleus the HFB solution contains only seniority zero and no stronger higher angular momentum pairs.

The IBM (Interacting Boson Model) [11] can only change \( 0^+ \) (S) and \( 2^+ \) (D) fermionic pairs from two neutrons into two protons.

3. Including the Nuclear Deformation in QRPA.

| initial (final) nucleus | \( \beta_2 \) | \( g_{pp} \) | \( \langle BCS_i|BCS_f \rangle \) |
|-------------------------|-----------------|-----------------|-----------------|
| \(^{76}\text{Ge} \) \(^{76}\text{Se} \) | 0.10 (0.16) | 0.71 | 0.74 |
| | 0.0 (0.0) | 0.68 | 0.81 |
| \(^{150}\text{Nd} \) \(^{150}\text{Sm} \) | 0.240 (0.153) | 1.05 | 0.52 |
| | 0.0 (0.0) | 1.01 | 0.85 |
| \(^{160}\text{Gd} \) \(^{160}\text{Dy} \) | 0.303 (0.292) | 1.00 | 0.74 |
Figure 1. (Color online) Contributions of the transforming neutron pairs with different angular momenta $J^z$ to the total $M^{0\nu}$ calculated within the QRPA and different basis sizes for the $0\nu\beta\beta$ decay $^{82}$Se$\rightarrow^{82}$Kr. The left bar is calculated with the same basis of four levels, $1p_{3/2}, 0f_{5/2}, 1p_{1/2}$ and $0g_{9/2}$, used in the shell model calculations [7]. The Ikeda Sum Rule (ISR) [14] is exhausted only by 50%. The second bar from the left includes in addition the $1f_{7/2}$ level, one of the two missing spin-orbit partners given for the $^{82}$Se nucleus in ref. [7] for the shell model. The ISR is exhausted by 66%. The third bar from the left includes both missing spin-orbit partners $0f_{7/2}$ and $0g_{7/2}$ amounting in total to 6 single-particle levels. The ISR is fulfilled by 100%. This leads to the increase in the neutrinoless matrix element from 1.12 to 4.07. The right bar represents the QRPA result with 9 single-particle levels ($1f_{7/2}, 2p_{3/2}, 1f_{5/2}, 2p_{1/2}, 1g_{9/2}, 2d_{5/2}, 3s_{1/2}, 2d_{3/2}, 1g_{7/2}$). The matrix element gets only slightly increased from 4.07 to 4.27. The spin-orbit partners are essential to fulfill the Ikeda Sum Rule (ISR). In all four QRPA calculations the QRPA “renormalization” factor $g_{pp}$ (given in the figure) of the particle-particle strength of the Bonn CD nucleon-nucleon interaction is adjusted to reproduce the experimental $2\nu\beta\beta$ decay rates.

We have also calculated [5, 6] the transition matrix elements in deformed nuclei for the light left handed Majorana neutrino exchange $M^{0\nu}$. Different deformations for the initial and the final nuclei are allowed. The BCS overlaps are listed in table 1. The quadrupole deformations
**Table 2.** The total calculated nuclear matrix elements (NME) \( M_0^\nu \) for \( 0\nu\beta\beta \) decays \( ^{76}\text{Ge} \rightarrow ^{76}\text{Se}, ^{150}\text{Nd} \rightarrow ^{150}\text{Sm}, ^{160}\text{Gd} \rightarrow ^{160}\text{Dy} \) including deformations. The BCS overlaps from table 1 are taken into account. In the last two columns the \( 0\nu \) matrix element \( M_0^\nu \) and the half-lives for assumed \( < m_\nu > = 50 \text{ meV} \) (see eqn. 2) are shown.

| \( A \) | Def. | \( g_A \) | \( M_0^\nu \) | \( T_{1/2}^0 \cdot [10^{26}\text{y}] \) |
|-------|------|-------|-------------|------------------|
| 76    | "1"  | 1.25  | 4.69        | 7.15             |
|       | "0"  | 1.25  | 5.30        | 5.60             |
| 150   | "1"  | 1.25  | 3.34        | 0.41             |
|       | "0"  | 1.25  | 6.12        | 0.12             |
| 160   | "1"  | 1.25  | 3.76        | 2.26             |

are taken from the reorientation Coulomb excitation of the \( 2^+ \) states.

### 4. How to find the Leading Mechanisms for the Neutrinoless Double Beta Decay?

Normally one assumes, that the first term of eq. (1) is the leading one and with the experimental data and the matrix element for the light left handed Majorana neutrino exchange \( M_0^\nu \) one can determine the effective Majorana neutrino mass \( < m_\nu > \) (2). But in Grand Unification (GUT) and Supersymmetry (SUSY) additional mechanisms for the neutrinoless Double Beta Decay \( (0\nu\beta\beta) \) are possible [12, 13].

The inverse half life is given by:

\[
\frac{1}{T_{1/2}^{0\nu}} = \frac{\omega_{0\nu}^L}{\ln 2} \approx G^{0\nu}(E_0, Z) \cdot \left| [\eta_\nu M_0^{0\nu} + \eta_{NL} M_{NL}^{0\nu} + \eta_X M_N^{0\nu}]^2 + |\eta_{NR}|^2 |M_{NR}^{0\nu}|^2 \right| \quad (4)
\]

with \( \eta \) given in ref. [13].

To test, if the left handed Majorana neutrino exchange is the leading mechanism, one needs at least experimental data of the neutrinoless double beta decay and reliable transition matrix elements in two systems. To determine the absolute values of the two strength parameters for two non-interfering mechanisms e.g. \( \eta_\nu \) and \( \eta_{NR} \), one needs at least two decay systems. To verify, that these are indeed the leading mechanisms one needs at least a third measurement. But if one forms ratios of half lives using the Tuebingen matrix elements for the light Majorana neutrino exchange and the heavy right handed neutrino exchange one obtains a very restricted allowed interval for these ratios. These ratios are quite insensitive to the different parameters of the model.

\[
0.15 \leq \frac{T_{1/2}^{0\nu}(^{100}\text{Mo})}{T_{1/2}^{0\nu}(^{76}\text{Ge})} \leq 0.18; \quad 0.17 \leq \frac{T_{1/2}^{0\nu}(^{130}\text{Te})}{T_{1/2}^{0\nu}(^{76}\text{Ge})} \leq 0.22; \quad 1.14 \leq \frac{T_{1/2}^{0\nu}(^{130}\text{Te})}{T_{1/2}^{0\nu}(^{100}\text{Mo})} \leq 1.24; \quad (5)
\]

If the two leading mechanisms like the light Majorana neutrino exchange and the SUSY mechanism with gluino or neutralino exchange can interfere, the situation is a bit more complicated: One needs three decay systems to determine the absolute values of the three parameters \( \eta_\nu, \eta_N \) and the relative phase angle \( \theta_{\nu, N} \). At least one additional system is needed to verify, that indeed these two mechanisms are the leading ones. Again the ratios of the half lives are allowed to lie only in narrow regions [13]. If this is not the case, the chosen mechanisms are
Figure 2. (Color online) Neutrinoless double beta decay transition matrix elements for the different approaches: QRPA [2, 3], the SM [7], the projected HFB method [9], the projected HFB with the Generator Coordinate Method (GCM) over deformations [10] (GCM-PNAMP) and the IBM2 [11]. The error bars of the filled circles for the QRPA are calculated as the highest and the lowest values for three different single-particle basis sets, two forces (Bonn CD and Argonne V18), two different axial charges $g_A = 1.25$ and the quenched value $g_A = 1.00$ and two different treatments of short range correlations (Jastrow-like [18] and the Unitary Correlator Operator Method (UCOM) [19]). The radius parameter is as in this whole work $r_0 = 1.2$ fm. The triangle with the tip up are the SM results [7]. The triangle with the tip down represent the transition matrix element of the Interacting Boson Model 2 (IBM2) [11]. The squares have been calculated by Pradfulla Rath and coworkers [9]. The star (GCM-PNAMP) is a projected HFB calculation with the Gogny force by Rodriguez and Martinez-Pinedo [10] allowing for superposition of different deformations with the Generator Coordinate Method (GCM).

not the leading ones [13]. With CP conservation the strength parameters $\eta$ must be real and thus the relative phase angle is zero or 180 degrees. So the determination of $\theta_{\mu,\nu}$ allows to test CP conservation or violation.

5. The effective Majorana Neutrino Mass.
Before we summarize the results let us assume Klapdor-Kleingrothaus et al. [17] have indeed measured the neutrinoless double beta decay in $^{76}\text{Ge}$, although the general belief is, that this still needs confirmation. From the half life given by Klapdor et al. [17] one can derive with
our matrix elements the effective Majorana neutrino mass under the assumption, that the light Majorana exchange is the leading mechanism.

\[ < m_{\nu} > = 0.24 [eV] \text{(exp \pm 0.02; theor. \pm 0.01)} [eV] \]  

(6)

The uncertainty (error) from experiment is 0.02 [eV], while the theoretical error originates from the uncertainties of the QRPA matrix elements as indicated in figure 2. The theoretical error is 0.01 [eV].

6. Conclusions

The Shell Model (SM) [7] has a severe handicap due to the restricted single-particle basis. The matrix elements in the \(^{76}\text{Ge}\) region are by a factor 2 smaller than the results of the Quasiparticle Random Phase Approximation (QRPA) [1, 2, 3, 4], the projected Hartree Fock Bogoliubov approach [9, 10] and the Interacting Boson Model (IBM2) [11]. With the same restricted basis as used by the SM the QRPA obtains roughly the same results as the SM, but the Ikeda sum rule [14] is strongly violated due to the missing spin-orbit partners in the SM single-particle basis.

The angular momentum projected Hartee-Fock-Bogoliubov (HFB) method [9] is restricted in its scope. With a real Bogoliubov transformation without parity mixing and with axial symmetry, one can only describe neutron pairs with angular momenta and parity \(0^+, 2^+, 4^+, 6^+, \ldots\) [16] changing into two protons for ground state-to-ground state transitions. The restriction for the Interacting Boson Model (IBM) [11] is even more severe: one is restricted to \(0^+\) and \(2^+\) neutron pairs changing into two protons.

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