Modelling observable properties of rapidly rotating stars

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Abstract. To fully understand the Be star phenomenon, one must have a reasonable degree of knowledge about the star beneath the disk, which is often found to be rapidly rotating. Rapid rotation complicates modelling because fundamental properties like the stellar luminosity and effective temperature require knowledge of the angle of inclination at which the star is observed. Furthermore, our knowledge of the structure of rapidly rotating stars is on a less sure foundation than for non-rotating stars. The uncertainties in the inclination and the surface properties of a few rapidly rotating stars have been substantially reduced by interferometric observations over the last decade, and these stars can be used as tests of rotating stellar models, even if those stars themselves may not be Be stars.

Vega, as an MK standard, is historically a very important star because it is used for calibration purposes. However, several studies have suggested that Vega is a rapidly rotating star viewed at a very low inclination angle, raising questions as to how well we really know its properties. Appropriate modelling has been challenging and there is still room for debate over the actual properties of Vega, as opposed to its observed properties. We have previously shown that under certain conditions both the stellar surface properties and the deduced surface properties scale from one model to another with the same surface shape. We used this scaling algorithm with realistic 2D models to compute high-resolution spectral energy distributions and interferometric visibilities to determine the best rotating model fit to Vega. Detailed comparisons between the computed and observed data will be presented.

1. Introduction

Be stars tend to be the most rapidly rotating near main sequence stars with an average $V \sin i$ as much as 150 km s$^{-1}$ faster than similar B stars (Slettebak 1949). Be stars are also more complex systems composed of a rapidly rotating star surrounded by a disk, which makes modelling them much more complicated. Cases like Achernar (Domiciano de Souza et al. 2003; Carciofi et al. 2008) have shown that even distinguishing the stellar surface from the disk is challenging.

Realistic modelling of rotating stars may be an important factor in understanding the Be phenomenon. Advances in the general study of rotating stars, however, have been limited by both theoretical and observational difficulties. A key example is that fundamental properties such as the effective temperature ($T_{\text{eff}}$) and luminosity ($L$) that one would deduce from observations now depend significantly on the angle of inclination ($i$) between the line of sight and the star’s rotation axis for sufficiently rapidly
rotating stars (e.g., Collins & Harrington 1966; Hardorp & Strittmatter 1968; Maeder & Peytremann 1970; Lovekin et al. 2006; Gillich et al. 2008; Dall & Sbordone 2011). This greatly complicates the determination of the star's position on the H-R diagram and hence nearly all other useful information unless its inclination can be determined.

As for any kind of star, the usual information available for a rotating star consists of a spectral energy distribution (SED), broadband photometry and a deduced luminosity (deduced \( L \)). The complication for rotating stars is that the SED is strongly dependent on the inclination. Traditionally, analysis of the line profile broadening provides a way to measure the projected rotational velocity (\( V \sin i \)) of the rotating star, but determination of the inclination and surface shape, however, are not easily measurable with standard observing methods. Fortunately, important advances in interferometric instrumentation over approximately the last decade have permitted resolved observations of some nearby rapid rotators (e.g., van Belle et al. 2001; Domiciano de Souza et al. 2003; Aufdenberg et al. 2006; Monnier et al. 2007; Zhao et al. 2009; Che et al. 2011). This type of observations is able to measure \( i \) as well as the surface shape of the star.

The strong dependence of the observed properties of a rotating star and its inclination requires a distinction between its measured properties (i.e. deduced \( T_{\text{eff}} \) and deduced \( L \)) and its intrinsic properties, like the actual \( L \), which is defined as the total energy leaving the star per unit time, and the actual \( T_{\text{eff}} \) which we define as \( (L/(4\pi A\sigma))^{1/4} \) where \( A \) is the surface area of the star and \( \sigma \) is the Stefan-Boltzmann constant. The deduced \( T_{\text{eff}} \) is obtained applying non-rotating color - \( T_{\text{eff}} \) relations to the observed colors of the star.

The direct process for finding appropriate models that fit the observations requires performing the calculations with different models until the observed SED properties are matched to the extent possible. This can be laborious and it would be far preferable to be able to start with the observed SED properties and work backward to what the luminosity and the latitudinal variation of the effective temperature must be. It is of great interest to find ways that can provide a bridge between the deduced properties of a rotating star and its intrinsic properties. In the next section we will discuss a methodology that may help with this problem.

2. Scaling of observed properties in rotating stars

Deupree (2011) showed that a number of properties of rotating models, particularly the surface effective temperature as a function of latitude, are proportional between models as long as the surface shape (which we shall indicate by \( R_p/R_{\text{eq}} \), although we mean having the same surface radius ratio applies at all latitudes) remains the same. Furthermore, for the surface shapes to be exactly the same, the two models must have the same rotation law to within a multiplicative constant. The independence of latitude for the actual effective temperature and radius ratios suggested that observable properties such as the deduced luminosity and deduced effective temperature as functions of inclination could scale as well. This scaling might be able to at least place constraints on models and parameters that could produce the observed properties. It would also allow the deduction of the actual luminosity and effective temperature from observations in a straightforward way for cases in which both the inclination and surface shape are known.

Recently, we showed (Castañeda & Deupree 2014) that observable properties of rotating stars that have the same shape are proportional within each other. This effec-
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tively means that
\[
\frac{\text{deduced } T_{\text{eff,2}}(\theta_j)}{\text{deduced } T_{\text{eff,1}}(\theta_j)} = c_{T_{\text{eff}}} \quad \text{and} \quad \frac{\text{deduced } L_2(\theta_j)}{\text{deduced } L_1(\theta_j)} = c_L \quad \forall \ j
\]
(1)

where \( \theta_j \) is the inclination and \( c \) denotes a constant that is independent of inclination. Here the designations '1' and '2' refer to the two models. Figure 1 shows the deviation from constancy of the scale factors \( c_{T_{\text{eff}}} \) and \( c_L \) for stellar models with masses between \( 1.875M_\odot \) and \( 3M_\odot \). The figure was created by calculating the percentage difference between the constants at different inclinations and their values at \( i = 50^\circ \) for both plots. These scaling relations become useful when trying to know what the actual \( T \) and actual \( L \) are, as well as to provide surface properties such as and \( T_{\text{eff}}, g_{\text{eff}}, \) the surface radius \( R \) and the rotational velocity \( V \), as functions of latitude.

Figure 1. Constancy of scale factors \( c_{T_{\text{eff}}} \) and \( c_L \) Left: Deduced \( T_{\text{eff}} \) Right: Deduced \( L \)

There are some limitations, however, because some assumptions are required. We have assumed that the surface is an equipotential, which only exists if the rotation law is conservative. Even for conservative rotation laws, it remains an assumption that the surface is an equipotential. This likely matters for the part of the solution that makes an estimate of the mass, but it need not affect the scaling of the observable properties as long as whatever mechanism determines the surface shape determines it in the same way for both the unknown and comparison objects. Our very limited knowledge of the surfaces of rotating stars does not allow an answer to this question.

In order to find an application for these scaling relations, we found in the literature rapidly rotating stars that have been observed with interferometry. The next section will present the work done on one of those rotating stars.

3. Test Case: Vega

Vega has been a photometric standard for a good part of the last century, but suggestions of anomalies in its luminosity, radius and shapes of the weak lines when compared to other A0 V type stars indicate that Vega is not the perfect standard it was once thought to be. First hints of Vega’s abnormal luminosity came from calibration studies of the Hy
equivalent width to absolute magnitude relationship (Petrie 1964; Millward & Walker 1985). The radius was found to be larger than expected (Hanbury Brown et al. 1967; Ciardi et al. 2001). Finally, another important element of Vega’s peculiarities that hinted the rotating nature of Vega was the flat-bottomed shapes of its weak lines (Gulliver et al. 1991). Work done by Elste (1992) and Gulliver et al. (1994) showed that the shape of the lines could be explained if Vega was a rapidly rotating star ($V_{eq} = 240 \text{ km s}^{-1}$) viewed nearly pole-on ($i = 5.5^\circ$). Subsequent interferometric studies would confirm the rapidly rotating nature of Vega (Peterson et al. 2006; Aufdenberg et al. 2006), and also establish that the star was rotating at nearly $V_{eq} = 270 \text{ km s}^{-1}$ ($\sim 90\%$ of its breakup velocity). Recent spectroscopic analysis, however, has lead to different results when trying to determine the rotating properties of Vega. In particular Takeda et al. (2008) found that Vega had to be rotating at a much lower rotational rate ($V_{eq} \approx 175 \text{ km s}^{-1}$) and at a slightly higher inclination angle of $i \approx 7^\circ$ in order to match the observed spectral features found on VEGA’s SED. Recently, findings of a periodic weak magnetic field in Vega (Lignières et al. 2009) allowed an independent measurement of a rotating period of $P = 0.71 \pm 0.03$ days (Petit et al. 2010; Alina et al. 2012), which is compatible with the period range predicted by the line profile studies ($P \sim 0.7 - 0.9$ days). In light of these latest results Monnier et al. (2012) used the latest CHARA array data to obtain new interferometric data for Vega, and after applying better calibration methods for the observations together with new rotating models it was found that it is possible to get more slowly rotating models that match the interferometric observations. It is important to note that part of the analysis in this work shows the strong dependence of the gravity darkening parameter $\beta$ to the fit of the observed interferometric data. Depending on the value of $\beta$ it is possible to find models that agree with both previous interferometric analysis as well as with the spectroscopic work that suggest a slower rotating case.

We believe we can contribute to the analysis of Vega by having realistic 2D rotating models from which we can apply the scaling relationships described before to find the appropriate surface properties of Vega that match the observed interferometric data, SED and corresponding weak lines used in previous studies. A virtue of these models and this method is that the relationship between $T_{eff}$ and $g_{eff}$ is based on a grey atmosphere relation between the surface temperature and $T_{eff}$, effectively removing the need to arbitrarily impose a gravity darkening parameter $\beta$. Interestingly, calculated values of $\beta$ for our models were compared with recent gravity darkening studies (Espinosa Lara & Rieutord 2011), finding similar results.

A first step to obtaining an appropriate model is the selection of target parameters of Vega. Table 1 shows the set of parameters chosen to use the scaling relations. The method uses this information as input as well as the surface properties of pre-computed rotating stellar models with the same $R_p/R_{eq}$, to return the surface properties ($T_{eff}$, $g_{eff}$, the radius $R$ and the rotational velocity $V$) as functions of latitude. A more detailed description of the procedure is given in Castañeda & Deupree (2014).

With the surface parameters obtained, it is possible to calculate a synthetic image of the star in the sky, which at the same time allows the calculation of synthetic interferometric data to compare with observed information. The procedure is the same as the one described by Aufdenberg et al. (2006). We used this to compare how well the scaled model fit the data used by Monnier et al. (2012) and the result is shown in Figure 2. The result is comparable to the recent studies, but it goes against the recent findings that suggest that Vega is rotating at a much slower rate.
Table 1. Selected properties of Vega

| Parameter | Value   | Source       |
|-----------|---------|--------------|
| Deduced $T_{\text{eff}}$ | 9550K   | SED          |
| $i$       | 5°      | Interferometry|
| $V \sin i$ | 21.9km s$^{-1}$ | Line profiles |
| $R_{\text{eq}}$ | 2.728R$_\odot$ | Interferometry and parallax |
| $R_p$     | 2.20R$_\odot$ | Assumed     |

Figure 2. Comparison between the observed visibility curve of Vega and a synthetic visibility curve obtained by scaling the properties of a reference rotating model

After the scaled surface parameters are obtained, one can compute an SED by performing a weighted integral over all the contributions of the intensities from every point on the stellar surface visible to the observer to obtain the flux the observer would see. This approach for calculating the SED has been frequently used (e.g., Slettebak et al.)
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1980; Linnell & Hubeny 1994; Frémat et al. 2005; Aufdenberg et al. 2006; Gillich et al. 2008; Yoon et al. 2008, Dall & Sbordone 2011); the specific details of our calculations are outlined by Lovekin et al. (2006). We then compared some of the lines considered in the analysis made by Takeda et al. (2007). Figure 3 shows comparison of four selected weak lines. Like other authors (Takeda et al. 2008; Yoon et al. 2008, 2010) have done in previous studies, we renormalized the line depth by multiplying by an arbitrary factor to make the theoretical line strength consistent with the observed one. This can be done because the weak lines involved (their maximum depth ~ 1%) are on the linear part of the curve of growth so their line depth should scale closely with $\epsilon g f$ (where $\epsilon$ is the abundance, $g$ is the statistical weight, and $f$ the oscillator strength). These quantities depend on the atomic physics modelling and do not carry information about the rotating properties of the star, which is the information we want to compare.

4. Conclusion

Interferometric observations combined with traditional observations provide a great opportunity to test realistic rotating stellar models. With this in mind, the preliminary analysis of Vega using the scaling relationships of observable properties in rotating

Figure 3. Selected weak line comparison: Top Left: CI-4775.8Å, Top Right: CrI-4616.6Å, Bottom Left: FeI-4528.6Å, Bottom Right: MgI-4703Å

Takeda et al. (2008) considered 196 weak lines. We compared our results with about 20 of them, finding similar agreement in the shape of the line profiles. Although there are small discrepancies in the shapes of some lines, this is an interesting result because the fit is achieved using the model that matches the higher rotational velocity parameters.

So far we have only performed this test by using one set of accepted observed properties of Vega. A next step would be to consider a broader range of parameter space and document how different results we get in those cases.
stars is used to translate what we observe from a rotating star to physically useful information under the appropriate conditions. The study of the Be phenomenon, although more complex because of the presence of the disk, could benefit from this technique if one can know both the inclination and the surface shape of the star.

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References

Alina, D., Petit, P., Lignières, F., Wade, G. A., Fares, R., Aurière, M., Böhm, T., & Carfantan, H. 2012, in American Institute of Physics Conference Series, edited by J. L. Hoffman, J. Bjorkman, & B. Whitney, vol. 1429 of American Institute of Physics Conference Series, 821107.5639

Aufdenberg, J. P., Merand, A., du Foresto, V. C., Absil, O., Di Folco, E., Kervella, P., Ridgway, S. T., Berger, D. H., ten Brummelaar, T. A., McAlister, H. A., Sturmann, J., Sturmann, L., & Turner, N. H. 2006, ApJ, 645, 664

Carciofi, A. C., Domiciano de Souza, A., Magalhães, A. M., Bjorkman, J. E., & Vakili, F. 2008, ApJ, 676, L41. 0801.4901

Castañeda, D., & Deupree, R. G. 2014, ApJ, 794, 13

Che, X., Monnier, J. D., Zhao, M., Pedretti, E., Thureau, N., Mérand, A., ten Brummelaar, T., McAlister, H., Ridgway, S. T., Turner, N., Sturmann, J., & Sturmann, L. 2011, ApJ, 732, 68

Ciardi, D. R., van Belle, G. T., Akeson, R. L., Thompson, R. R., Lada, E. A., & Howell, S. B. 2001, ApJ, 559, 1147

Collins, I., George W., & Harrington, J. P. 1966, ApJ, 146, 152

Dall, T. H., & Sbordone, L. 2011, Journal of Physics Conference Series, 328, 012016

Deupree, R. G. 2011, ApJ, 735, 69

Domiciano de Souza, A., Kervella, P., Jankov, S., Abe, L., Vakili, F., di Folco, E., & Paresce, F. 2003, A&A, 407, L47. astro-ph/0306277

Elste, G. H. 1992, ApJ, 384, 284

Espinosa Lara, F., & Rieutord, M. 2011, A&A, 533, A43. 1109.3038

Frémat, Y., Zorec, J., Hubert, A.-M., & Floquet, M. 2005, A&A, 440, 305. astro-ph/0503381

Gillich, A., Deupree, R. G., Lovekin, C. C., Short, C. I., & Toqué, N. 2008, ApJ, 683, 441

Gulliver, A. F., Adelman, S. J., Cowley, C. R., & Fletcher, J. M. 1991, Astrophys. J., 380, 223

Gulliver, A. F., Hill, G., & Adelman, S. J. 1994, ApJ, 429, L81

Hanbury Brown, R., Davis, J., Allen, L. R., & Rome, J. M. 1967, MNRAS, 137, 393

Hardorp, J., & Strittmatter, P. A. 1968, ApJ, 151, 1057

Lignières, F., Petit, P., Böhm, T., & Aurière, M. 2009, A&A, 500, L41. 0903.1247

Linnell, A. P., & Hubeny, I. 1994, ApJ, 434, 738

Lovekin, C. C., Deupree, R. G., & Short, C. I. 2006, ApJ, 643, 460

Maeder, A., & Peytremann, E. 1970, A&A, 7, 120

Millward, C. G., & Walker, G. A. H. 1985, ApJS, 57, 63

Monnier, J. D., Che, X., Zhao, M., Ekström, S., Maestro, V., Aufdenberg, J., Baron, F., Georgy, C., Kraus, S., McAlister, H., Pedretti, E., Ridgway, S., Sturmann, J., Sturmann, L., ten Brummelaar, T., Thureau, N., Turner, N., & Tuthill, P. G. 2012, ApJ, 761, L3. 1211.6055

Monnier, J. D., Zhao, M., Pedretti, E., Thureau, N., Ireland, M., Muirhead, P., Berger, J.-P., Millan-Gabet, R., Van Belle, G., Ten Brummelaar, T., McAlister, H., Ridgway, S., Turner, N., Sturmann, L., Sturmann, J., & Berger, D. 2007, Science, 317, 342

Peterson, D. M., Hummel, C. A., Pauls, T. A., Armstrong, J. T., Benson, J. A., Gilbreath, G. C., Hindsley, R. B., Hutter, D. J., Johnston, K. J., Mozurkewich, D., & Schmitt, H. R. 2006, Nature, 440, 896
Questions asked after the talk

Q: "Maybe you’re aware that Vega is the prototype of a possibly large class of stars in which very weak surface magnetic fields are present. How do your derived period and radius compare with those inferred from Zeeman Doppler Imaging by Petit et al.?"

A: Please refer to Page 4, line 14 in this document.

Q: "You mentioned that your models allow for latitudinal variation of gravity darkening that can differ from von Zeipel’s law. Could explain more what more general conditions are used?"

A: A comment about this aspect of our models was made in page 4: End of first paragraph and second paragraph.