Updated Analysis of $a_2/a_1$ in Exclusive B Decays $^a$

Hai-Yang Cheng and Kwei-Chou Yang

Institute of Physics, Academia Sinica, Taipei, Taiwan 115, R.O.C.

Using recent experimental data and various theoretical calculations on form factors, we reanalyze the effective parameters $a_1$ and $a_2$ and their ratio.

Nonleptonic two-body decays of mesons were conventionally studied in the generalized factorization approach, in which the factorized decay amplitude is associated with the effective coefficients

$$a_{1,2}^{\text{eff}} = c_{1,2}^{\text{eff}} + c_{2,1}^{\text{eff}}[(1/N_c) + \chi_{1,2}] ,$$  \hspace{1cm} (1)

where $c_{1,2}^{\text{eff}}$ are the effective Wilson coefficients and the nonfactorized terms are expressed by $\chi_{1,2}$. For simplicity, in what follows we will drop the superscript “eff” of $a_{1,2}^{\text{eff}}$. Based on the generalized factorization assumption, one can catalog the decay processes into three different classes. For class-I decays, the decay amplitudes, dominated by color-allowed external W-emission, are proportional to the $a_1$ parameter. For class-II decays, the decay amplitudes, governed by color-suppressed internal W-emission, are described by $a_2$. The decay amplitudes of the class-III involve a linear combination of $a_1$ and $a_2$.

From the data analysis, contrary to the case of charmed meson decays in which $a_2$ is negative and $\chi \sim -1/N_c$, $a_2$ is positive in two-body decays of the $B$ meson. $\chi_{2(1)}$ coming from the contribution of the matrix element of the color octet-octet and singlet-singlet currents, since it is made of nonfactorizable soft gluon effects in itself, will become more important if the final particles move slower $^1$. Therefore, we naively expect that $|\chi_{2}(D \to V P)| \gtrsim |\chi_{1}(D \to P P)| > |\chi_{2}(B \to V P)|$, where $P$ is the pseudoscalar meson and $V$ the vector meson. The data of $B \to J/\Psi K$ which are class-II decay modes can be used to study the absolute value of $a_2$. However, the experimental results are quite difficult to account for the observed longitudinal polarization fraction $\Gamma_L/\Gamma$ and the ratio $R \equiv \Gamma(B \to J/\Psi K^*)/\Gamma(B \to J/\Psi K)$ simultaneously.

$^a$presented by K.-C. Yang at “The Fourth International Workshop on Particle Physics Phenomenology”, Kaohsiung, Taiwan, 18-21 January 1998.

Email address: kcyang@phys.sinica.edu.tw
using the existing form factor calculations. To accommodate the data, it has been advocated a modification to the generalized factorization hypothesis so that \( a_2 \) is process dependent. In this talk, we focus on the extraction of the value \( a_1, |a_2|, \) and \( a_2/a_1 \) from the current experimental data and various form factor calculations, the BSWI model, the BSWII model, the LF model, the NS model, the QCD sum rule results (Yang), and the light-cone sum rule calculations (LC). Since the accuracy of these values still suffers from reliability of the form factor calculations, we desire that an objective estimation can be obtained.

Here will consider the decay amplitudes of processes which have been measured experimentally:

- Class I: \( B^- \rightarrow D^{(*)0} D^{(*)-}_s, \bar{B}_d \rightarrow D^{(*)+} D^{(*)-}_s, \) and \( \bar{B}_d \rightarrow D^{(*)+} \pi^- (\rho^-), \)
- Class II: \( B^- \rightarrow J/\Psi K^{(*)-} \) and \( B^0 \rightarrow J/\Psi K^{(*)0} . \)
- Class III: \( B^- \rightarrow D^{(*)0} \pi^- (\rho^-), \)

The value of \( a_{1,2} \) can be extracted from the above data of the class-(I,II) type transitions, while their ratio can be obtained by the following ratios:

\[
R_1 = \frac{\mathcal{B}(B^- \rightarrow D^{(*)0}_s \pi^-)}{\mathcal{B}(B^0 \rightarrow D^{(*)0}_s \pi^-)}, \quad R_2 = \frac{\mathcal{B}(B^- \rightarrow D^{(*)+}_s \pi^-)}{\mathcal{B}(B^0 \rightarrow D^{(*)+}_s \pi^-)}, \quad R_3 = \frac{\mathcal{B}(B^- \rightarrow D^{(*)+}_s \pi^-)}{\mathcal{B}(B^0 \rightarrow D^{(*)+}_s \pi^-)}, \quad R_4 = \frac{\mathcal{B}(B^- \rightarrow D^{(*)0}_s \rho^-)}{\mathcal{B}(B^0 \rightarrow D^{(*)0}_s \rho^-)}. \tag{2}
\]

The hadronic matrix element of effective operators \( O_{1,2} \) can be redefined in the scheme- and scale-\((\mu)\)-dependent ways \( \langle O(\mu) \rangle = g(\mu) \langle O \rangle_{\text{tree}} \), where \( \langle O \rangle_{\text{tree}} \)
The results are listed in Tables 1, 2, 3, and 5. In Tables 1 and 2, the updated theoretical results of form factors and the experimental data from the Particle Form-factor models.

Table 2: The effective parameter $a_1$ extracted from $B_d^0 \to D^{(*)} \pi^- (\rho^-)$ using different form-factor models.

| Model       | BSWI | BWSH | LF   | SR+HQET | NS  |
|-------------|------|------|------|---------|-----|
| $B^0 \to D^+ \pi^-$ | 0.89±0.06 | 0.89±0.06 | 0.88±0.06 | 1.15±0.08 | 1.31±0.09 |
| $B^0 \to D^+ \rho^-$ | 0.91±0.08 | 0.90±0.08 | 0.89±0.08 | 1.13±0.10 | 1.29±0.11 |
| $B^0 \to D^{++} \pi^-$ | 0.98±0.04 | 0.98±0.04 | 0.84±0.03 | 0.93±0.04 | 0.89±0.03 |
| $B^0 \to D^{++} \rho^-$ | 0.86±0.21 | 0.87±0.21 | 0.75±0.18 | 0.83±0.20 | 0.80±0.20 |
| Average     | 0.95±0.03 | 0.94±0.03 | 0.83±0.03 | 0.98±0.03 | 0.96±0.03 |

Table 3: The effective parameter $|a_2|$ extracted from $B \to J/\Psi K^{(*)}$ using different form-factor models.

| Model       | BSWI | BSWII | LF   | NS  | Yang | LC  |
|-------------|------|-------|------|-----|------|-----|
| $B^+ \to J/\Psi K^+$ | 0.33±0.02 | 0.22±0.01 | 0.28±0.01 | 0.36±0.02 | 0.37±0.02 | 0.30±0.02 |
| $B^0 \to J/\Psi K^0$ | 0.32±0.02 | 0.21±0.01 | 0.27±0.02 | 0.35±0.02 | 0.36±0.02 | 0.29±0.02 |
| Average     | 0.32±0.01 | 0.22±0.01 | 0.28±0.01 | 0.36±0.01 | 0.36±0.01 | 0.30±0.01 |
| $B^+ \to J/\Psi K^{*+}$ | 0.20±0.02 | 0.21±0.02 | 0.25±0.02 | 0.24±0.02 | 0.39±0.04 | 0.20±0.02 |
| $B^0 \to J/\Psi K^{*0}$ | 0.19±0.01 | 0.21±0.01 | 0.25±0.02 | 0.25±0.02 | 0.39±0.03 | 0.19±0.01 |
| Average     | 0.20±0.01 | 0.22±0.01 | 0.26±0.01 | 0.25±0.01 | 0.40±0.02 | 0.20±0.01 |

is $\mu$-independent. We thus can rewrite the matrix element of the effective Hamiltonian ($H_{\text{eff}}$) as the product of $c_{\text{eff}}$ and $\langle O \rangle_{\text{tree}}$, both of which are scheme- and $\mu$-independent. The only relevant scale, which is implicit and of order $m_b$, separating $c_{\text{eff}}$ and $\langle O \rangle_{\text{tree}}$ is the energy release of B decays. Here we use $c_{\text{eff}} = 1.149$ and $c_{\text{eff}} = -0.325$, to the next-to-leading order \textsuperscript{5}. Using the theoretical results of form factors and the experimental data from the Particle Data Group (PDG)\textsuperscript{6}, one can easily extract the values of $a_1$, $|a_2|$, and $a_2/a_1$. The results are listed in Tables 1 and 2, while the updated data are used for the parameters of NS, while the results of SR+HQET are gotten by the NS formula but the parameters are obtained from the sum rule results and the predictions of the heavy quark effective theory \textsuperscript{7}. All the results in Tables 1 and 2 indicate that the value of $a_1$, which is very close to 1, seems to follow the pattern $a_1(B \to D^{(*)} D_{s}^{(*)}) \gtrsim a_1(B \to D^{(*)} \pi (\rho))$.

For $B \to J/\Psi K^{(*)}$, since they are color suppressed and sensitive to the nonfactorizable contributions, therefore, they are very good examples to examine.

\textsuperscript{5}We will use the $B \to D^{(*)}$ form-factor results of SR+HQET as substitutes of the Yang’s and LC calculations, since Yang’s sum rules and LC only show the form factors of the B meson into a light meson.

\textsuperscript{6}The class-I transitions determine the absolute value of $a_1$. However, since this kind of processes is color allowed, we expect the sign of $a_1$ is the same as that of $c_{\text{eff}}$ and is therefore positive.
Table 4: $R$, $\Gamma_L/\Gamma$, and $|P|^2$ from various form factor calculations based on the generalized factorization hypothesis together with the CLEO new data.

|       | BSWI | BSWII | LF | NS   | Yang | LC            | CLEO          |
|-------|------|-------|----|------|------|---------------|---------------|
| $R$   | 4.15 | 1.58  | 1.79| 3.17 | 3.48 | 1.45 ± 0.26   |               |
| $\Gamma_L/\Gamma$ | 0.57 | 0.36  | 0.53| 0.49 | 0.47 | 0.52 ± 0.08   | 0.42 ± 0.07   |
| $|P|^2$ | 0.09 | 0.24  | 0.09| 0.12 | 0.19 | 0.23 ± 0.09   | 0.16 ± 0.09   |

Table 5: $a_2/a_1$ extracted from the PDG data.

|       | BSWI | BSWII | LF | NS   | Yang | LC            |
|-------|------|-------|----|------|------|---------------|
| $R_1$ | 0.29±0.10 | 0.29±0.10 | 0.38±0.13 | 0.28±0.10 | 0.29±0.10 | 0.27±0.10 |
| $R_2$ | 0.59±0.30 | 0.52±0.27 | 0.56±0.28 | 0.43±0.22 | 1.21±0.62 | 0.34±0.17 |
| $R_3$ | 0.23±0.06 | 0.20±0.06 | 0.31±0.09 | 0.30±0.09 | 0.28±0.08 | 0.25±0.07 |
| $R_4$ | 0.55±0.40 | 0.67±0.49 | 0.85±0.62 | 0.79±0.58 | 1.50±1.09 | 0.62±0.45 |
| Average | 0.26±0.05 | 0.23±0.05 | 0.35±0.07 | 0.31±0.06 | 0.30±0.06 | 0.27±0.05 |

explore the generalized factorization hypothesis to see if $a_2$ is universal. On the other hand, since the energy release in $B \to J/\Psi K$ is close to that in $B \to J/\Psi K^*$, we expect that the extracted values of $|a_2|$ from these two kinds of processes can be slightly different (the so-called minimal modified factorization hypothesis). However, from Table 4, we know that only BSWII, LF, and Yang’s sum rules can satisfy our expectation.

The other ways to examine the generalized factorization hypothesis and/or the quality of various form factor results are to study the production ratio $R = \mathcal{B}(B \to J/\Psi K^*)/\mathcal{B}(B \to J/\Psi K)$, the fraction of longitudinal polarization $\Gamma_L/\Gamma$, and the $P$–wave transverse polarization $|P|^2$ measured in the transversity basis in $B \to J/\Psi K^*$ decays. The predictions from various form factor calculations based on the generalized factorization hypothesis together with the CLEO new data are shown in Table 4. We find again that only BSWII, LF, and Yang’s sum rules can accommodate the data, expect that the value of $\Gamma_L/\Gamma$ in BSWII is a little smaller than the CLEO data. Furthermore, if allowing the nonfactorizable contribution $\chi_2$ to be slightly different in $B \to J/\Psi K^{(*)}$ and in $B \to D^{(*)} \pi(\rho)$, we find that the values of $a_2(B \to J/\Psi K^{(*)})$ of BSWII/LF are consistent with that obtained from the analyses of $R_1$–$R_4$ (see Tables 4 and 5) if assuming $a_1 \approx 1$. However, note that the Yang’s sum rule results do not behave like the BSWII or LF model. The value of $a_2$ in the sum rule analysis of Yang is larger. There are two possibilities to explain it. One is $\chi_2 \approx 0.3$, a larger value compared to that in BSWII or LF ($\chi_2 \approx 0.14$ in BSWII, 0.18 in LF if we have adopted $a_2 > 0$). The other possibility is both the nonfactorizable values $\chi_2$ and $a_2$ become negative in $B \to J/\Psi K^{(*)}$, as the D meson decays. The QCD sum rule calculation on the
nonfactorizable contribution in $B \to J/\Psi K$ was reported to be negative, contrary to the generalized nonfactorization hypothesis. However, it is difficult to understand why $a_2$ is negative in $B \to J/\Psi K(\ast)$ while it becomes positive in $B^- \to D^{(\ast)} \pi^-(\rho^-)$. One way to solve directly this problem is to evaluate the class-III or class-II decay channels of $B \to D \pi(\rho)$, within framework of the QCD sum rules to see if $\chi_2$ becomes positive.

From the analyses of $R_{1-4}$ defined in Eq. (2) one can obtain $a_2/a_1$. Since we have adopted that $a_1$ is positive, we can thus determine the sign of $a_2$. The results are listed in Table 5 if using PDG data. All of the results show that $a_2$ is positive. Two remarks are in order. First, $a_2/a_1$ of $R_{1,3}$ for all of form factor results are quantitatively stable, while that of $R_{2,4}$ have large central values and errors. Second, the prediction of $a_2/a_1$ in LF is slightly large is because $a_1$ in LF is a smaller value (see Tables 1 and 2).

CLEO II has also reported the data on various class-III and class-I branching ratios of $B \to D^{(\ast)} \pi(\rho)$. We show that the analysis results of $a_2/a_1$ in Table 4. But now the values of $a_2/a_1$ from $R_{2,4}$ is much smaller the corresponding values in Table 4. Eventually, if we account of the fact that $a_1$ is little small in LF, then $a_2$ is lying in $0.2 - 0.3$, which is a reasonable result for all of form factor calculations.

To conclude, we have used the current experimental data and various theoretical results of form factors to analyze the effective coefficients $a_1$ and $a_2$. Our results have shown that if allowing a minimal modification to the generalized factorization hypothesis, i.e., the nonfactorizable contribution $\chi_{1,2}$ can be slightly different in different decay processes, then we have the following conclusions: (1) $a_1 \approx 1$ and $a_1(B \to D^{(\ast)} D_s^{(\ast)}) \gtrsim a_1(B \to D^{(\ast)} \pi(\rho))$. (2) Only the BSW II model, the light-front model, and Yang’s sum rules can satisfy this factorization assumption in $B \to J/\Psi K(\ast)$ analyses. The results are shown in Tables 3 and 4. (3) From the results of $a_2/a_1$ listed in Tables 3 and 4 we obtain that $a_2$ is lying in $0.2 - 0.3$, which is a reasonable result for all of form factor calculations in the $B \to D^{(\ast)} \pi(\rho)$ decay processes.

This work was supported in part by the National Science Council of R.O.C.
under Grant No. NSC88-2112-M-001-006.

1. H.Y. Cheng and B. Tseng, *Phys. Rev.* D **51**, 6529 (1995); H.Y. Cheng, *Z. Phys.* C **69**, 647 (1996).
2. H.Y. Cheng, *Phys. Lett.* B **395**, 345 (1997).
3. C. Cao *et al.* (Particle Data Group), Eur. Phys. J. C **3**, 1 (1998).
4. For a recent similar work, see F.M. Al-Shamali and A.N. Kamal, [hep-ph/9806270](https://arxiv.org/abs/hep-ph/9806270).
5. M. Bauer, B. Stech, and M. Wirbel, *Z. Phys.* C **34**, 103 (1987).
6. H.Y. Cheng, C.Y. Cheung, and C.W. Hwang, *Phys. Rev.* D **55**, 1559 (1997).
7. M. Neubert, B. Stech, CERN-TH-97-099, in second edition of Heavy Flavours, ed. by A.J. Buras and M. Lindner, World Scientific, Singapore.
8. The detailed results will be published in a separate work. The basic formulas can be found in K.C. Yang, *Phys. Rev.* D **57**, 2983 (1998).
9. P. Ball and V.M. Braun, CERN-TH/98-162 [hep-ph/9805422](https://arxiv.org/abs/hep-ph/9805422).
10. H.Y. Cheng and B. Tseng, *Phys. Rev.* D **58**, 094005 (1998).
11. M. Neubert, CERN-TH-97-024 [hep-ph/9702375](https://arxiv.org/abs/hep-ph/9702375); Phys. Rep. **245**, 259 (1994).
12. CLEO Collab, C.P. Jessop *et al.*, *Phys. Rev. Lett.* **79**, 4533 (1997).
13. A. Khodjamirian and R. Rückl, WUE-ITP-97-049 [hep-ph/9801443](https://arxiv.org/abs/hep-ph/9801443).
14. J.L. Rodriguez, [hep-ex/9801028](https://arxiv.org/abs/hep-ex/9801028).