D-brane configuration and black hole thermodynamics

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Abstract

We consider a configuration of strings and solitons in the type IIB superstring theory on $M^5 \times T^5$, which is composed of a set of arbitrarily-wound D-fivebranes on $T^5$ and a set of arbitrarily-wound D-strings on $S^1$ of the torus. For the configuration, it is shown that number of microscopic states is bounded from above by the exponential of the Hawking-Bekenstein entropy of the corresponding black hole and the temperature of closed string radiation from the D-branes is bounded from below by the Hawking temperature of the black hole. After discussing the necessary and sufficient condition to saturate these bounds, we give some speculations about black hole thermodynamics.
I. INTRODUCTION

Black hole thermodynamics [1] [2] is one of the most interesting topics in the theory of black holes. Recently Strominger and Vafa [3] gave a microscopic origin of the Hawking-Bekenstein entropy of a black hole in the framework of superstring theory by using the D-brane technology [4]. Hawking radiation was also explained in the D-brane picture [5].

In this letter we consider a configuration of strings and solitons in the type IIB superstring theory on $M^5 \times T^5$. The corresponding classical black-brane solution was obtained in [6] and is characterized by four charges $(Q_1, Q_5, n_L, n_R)$. In the case of the extreme limit $(n_R = 0)$, Callan and Maldacena have thought of it as a configuration composed of $Q_5$ D-fivebranes wrapped on the torus and $Q_1$ D-strings wrapped on $S^1$ of the torus, based on the fact that a D-string has charge $(Q_1, Q_5) = (1, 0)$ and a D-fivebrane has charge $(Q_1, Q_5) = (0, 1)$. On the contrary Maldacena and Susskind [7] regarded it as a configuration of a single long D-string and a single long D-fivebrane. In both interpretations $P_L \equiv n_L/R$ is the momentum along the circle which is a sum over left moving massless modes of open strings on the D-branes, where $R$ is the radius of the circle. In this letter we consider a more general configuration of D-branes and investigate open strings on it.

In Sec. II we give the number of microscopic states, which is related to the black hole entropy. In Sec. III temperature of a canonical ensemble of open strings on D-branes is obtained. It is related to the temperature of a decay of D-brane excitations to a closed-string mode, which seems to be the Hawking radiation. Finally in Sec. IV we summarize this letter and give some speculations.

II. NUMBER OF MICROSCOPIC STATES

First let us consider a set of $Q_1$ D-strings wrapped on $S^1$ whose length is $2\pi R$. They may connect up [7] to form a set of multiply-wound D-strings, which is composed of $N_{q_1}^{(1)}$ D-strings of length $2\pi R q_1$ ($q_1 = 1, 2, \cdots$). Next consider a set of $Q_5$ D-fivebranes wrapped
on $T^5$ and a set of $Q_1$ D-strings wrapped on $S^1$ of the torus. The corresponding non-extremal supergravity solution is obtained by setting $N_1 = N_5 = 0$ in the solution found in [6] and is characterized by four parameters $(Q_1, Q_5, n_L, n_R)$. Here $Q_1$ and $Q_5$ are charges of a $U(1)$ gauge field; $n_L$ ($n_R$) is left (right) moving momentum along $S_1$ multiplied by $R$. By suppressing reference to the other four directions we may think of the D-fivebranes as D-strings wrapped on the circle $S^1$. The D-fivebranes may also connect up to form a set of multiply-wound D-fivebranes, which is composed of $N_{q_5}^{(5)}$ D-fivebranes of length $2\pi R q_5$ ($q_5 = 1, 2, \cdots$) along the circle. Note that $N_{q_1}^{(1)}$ and $N_{q_5}^{(5)}$ must satisfy the following constraints:

\[ Q_1 = \sum_{q_1} q_1 N_{q_1}^{(1)}, \]
\[ Q_5 = \sum_{q_5} q_5 N_{q_5}^{(5)}, \]

since a D-string which winds $q_1$ times has charge $(Q_1, Q_5) = (q_1, 0)$ and a D-fivebrane which winds $q_5$ times has charge $(Q_1, Q_5) = (0, q_5)$.

There are several types of open strings on the D-brane configuration: both boundaries are on a common D-string; one boundary is on a D-string and another is on a different D-string; one is on a D-string and another is on a D-fivebrane; etc. Among them, as an approximation, we consider only strings which connect a $q_1$-wound D-string and a $q_5$-wound D-fivebrane and neglect contribution from other strings. The spectrum of the strings is

\[ p_n(q_1, q_5) = \pm \frac{n}{R(q_1, q_5)_{LCM}}, \quad (n = 1, 2, \cdots) \]  
\[ (\, \prime+\prime \text{ for left moving } ; \, \prime-\prime \text{ for right moving}), \]

where $(q_1, q_5)_{LCM}$ is the least common multiple of $q_1$ and $q_5$, since the boundary condition of the strings is

\[ X^5 \sim X^5 + 2\pi R(q_1, q_5)_{LCM}. \]  

We also regard the momentum $P_L \equiv n_L/R$ ($P_R \equiv -n_R/R$) along the circle as a sum over the left (right) moving massless modes of the strings. Hence the number of microscopic states $d(n_L, n_R)$ is given by
d(n_L, n_R) = d(n_L)d(n_R),

\[
\sum_N d(N)w^N = \prod_{q_1} \prod_{q_5} \prod_{n=1}^{\infty} \left[ \frac{1}{1 - w^{n/(q_1,q_5)_{\text{LCM}}}} \right]^{4N_{q_1}^{(1)}N_{q_5}^{(5)}}.
\]

By using the asymptotic form of the θ-function and the saddle point method, we can obtain \(d(n_{L,R})\) in the limit

\[
\sqrt{\frac{Q_1Q_5}{n_{L,R}}} \ll \min_{(q_1,q_5) \in A} (q_1,q_5)_{\text{LCM}},
\]

where \(A \equiv \{(q_1,q_5)|N_{q_1}^{(1)}N_{q_5}^{(5)} \neq 0\}\). The result is

\[
d(n_{L,R}) \approx \exp \left[ 2\pi \sqrt{\frac{n_{L,R}}{Q_1Q_5} \sum_{q_1} \sum_{q_5} N_{q_1}^{(1)}N_{q_5}^{(5)} (q_1,q_5)_{\text{LCM}}} \right] \leq \exp(2\pi \sqrt{n_{L,R}Q_1Q_5}).
\]

Therefore, for a configuration satisfying (3), \(d(n_L, n_R)\) is bounded from above:

\[
d(n_L, n_R) \leq \exp \left[ 2\pi \left( \sqrt{n_L} + \sqrt{n_R} \right) \sqrt{Q_1Q_5} \right],
\]

where the bound is saturated if and only if all \((q_1,q_5) (\in A)\) are relatively prime.

### III. CANONICAL ENSEMBLE OF OPEN STRINGS

In the D-brane approach to Hawking radiation [5] [8] [9], the thermal spectrum of the decay of non-BPS D-brane excitations to closed string modes was obtained by summing up decay rates over all consistent initial states of open strings on the D-branes. The summation was approximately performed by replacing the microcanonical ensemble by a canonical ensemble. The resulting temperature of the decay spectrum is as follows [9]:

\[
T_{\text{decay}} = T,
\]

where \(T\) is the temperature of the canonical ensemble.

Therefore, in this section we consider a canonical ensemble of open strings on the D-brane configurations introduced in Sec. [11]. The partition function is
\[ Z(\beta, \alpha) = Z_L(\beta, \alpha)Z_R(\beta, \alpha), \]

\[ Z_{L,R}(\beta, \alpha) = \prod_{q_1} \prod_{q_5} \prod_{n=1}^{\infty} \left[ \frac{1 + e^{-\beta e_n(q_1,q_5)-\alpha p_n(q_1,q_5)}}{1 - e^{-\beta e_n(q_1,q_5)-\alpha p_n(q_1,q_5)}} \right]^{4N_{q_1}^{(1)}N_{q_5}^{(5)}}. \]

where \( p_n \) is given by (1) and \( e_n = |p_n| \). By using the asymptotic form of the \( \theta \)-function, we can obtain \( Z_{L,R}(\beta, \alpha) \) in the limit (3). The result is

\[ \ln Z_{L,R}(\beta, \alpha) = \frac{\pi^2 R}{\beta \pm \alpha} \sum_{q_1} \sum_{q_5} N_{q_1}^{(1)}N_{q_5}^{(5)}(q_1,q_5)_{LCM}, \]

\[ (\prime + \text{ for } 'L'; \prime - \text{ for } 'R'). \]

\( \beta \) and \( \alpha \) are fixed by

\[ E_L = P_L = n_L/R, \]
\[ E_R = -P_R = n_R/R, \]

where \( E_L \) (\( E_R \)) is the left (right) movers’ contribution to the expectation value of the energy and \( P_L \) (\( P_R \)) is the left (right) movers’ contribution to the expectation value of the momentum:

\[ E_L + E_R = -\frac{\partial \ln Z(\beta, \alpha)}{\partial \beta}, \]
\[ P_L + P_R = -\frac{\partial \ln Z(\beta, \alpha)}{\partial \alpha}. \]

Thus, for a configuration satisfying (3), the entropy \( S \) and the inverse-temperature \( 1/T = \beta \) are

\[ S = 2\pi \left( \sqrt{n_L} + \sqrt{n_R} \right) \sqrt{\sum_{q_1} \sum_{q_5} N_{q_1}^{(1)}N_{q_5}^{(5)}(q_1,q_5)_{LCM}}, \]

\[ \frac{1}{T} = \frac{\pi R}{2} \left( \frac{1}{\sqrt{n_L}} + \frac{1}{\sqrt{n_R}} \right) \sqrt{\sum_{q_1} \sum_{q_5} N_{q_1}^{(1)}N_{q_5}^{(5)}(q_1,q_5)_{LCM}}. \]

They are bounded from above as follows:

\[ S \leq 2\pi \left( \sqrt{n_L} + \sqrt{n_R} \right) \sqrt{Q_1Q_5}, \]  
\[ \frac{1}{T} \leq \frac{\pi R}{2} \left( \frac{1}{\sqrt{n_L}} + \frac{1}{\sqrt{n_R}} \right) \sqrt{Q_1Q_5}. \]
where the bounds are saturated if and only if all \((q_1, q_5) \in A\) are relatively prime. Finally note that the following relation is trivially satisfied:

\[
S \approx \ln d(n_L, n_R). \tag{8}
\]

IV. SUMMARY AND SPECULATIONS

In summary we have investigated open strings on the general D-brane configurations. For configurations satisfying (3), the number of microscopic states \(d(n_L, n_R)\) is bounded from above and the temperature of a decay of D-brane excitations to closed strings is bounded from below. Moreover, for any configurations not satisfying (3), the same bounds seem to exist without saturation [7]. Therefore it is expected that the quantities calculated from the microscopic point of view are related as follows to the macroscopic quantities of the black hole.

\[
\ln d(n_L, n_R) \leq S_{BH}, \tag{9}
\]

\[
T_{\text{decay}} \geq T_{BH}, \tag{10}
\]

where \(S_{BH}\) and \(T_{BH}\) are the Hawking-Bekenstein entropy and the Hawking temperature of the black hole [6, 9]:

\[
S_{BH} = 2\pi (\sqrt{n_L} + \sqrt{n_R}) \sqrt{Q_1 Q_5},
\]

\[
\frac{1}{T_{BH}} = \frac{\pi R^2}{2} \left( \frac{1}{\sqrt{n_L}} + \frac{1}{\sqrt{n_R}} \right) \sqrt{Q_1 Q_5}.
\]

Note that the Hawking-Bekenstein entropy and the Hawking temperature are defined by macroscopic quantities (area and surface gravity of the event horizon). The bounds (9) and (10) are saturated if and only if all \((q_1, q_5) \in A\) are relatively prime and (3) is satisfied.

Thus several speculations may be possible.

- Inside a black hole characterized by the four parameters \((Q_1, Q_5, n_L, n_R)\), some dynamical processes may occur. The processes may be described in the D-brane picture:
D-branes repeat fission and fusion to settle down to one of the states for which all 
\((q_1, q_5) (\in A)\) are relatively prime and \((3)\) is satisfied.

- During the process the microscopic entropy increases to reach the Hawking-Bekenstein entropy of the corresponding black hole. Moreover the temperature of closed string radiation from the D-branes decreases to reach the Hawking temperature of the black hole.

These speculations may be significant to investigate the microstates of dynamical black holes. For example let us consider a merger of two black holes \(B_1\) and \(B_2\) and suppose that \(B_1\) corresponds to a D-brane configuration \(\{N^{(11)}_{q_1}, N^{(15)}_{q_5}\}\) and \(B_2\) corresponds to a configuration \(\{N^{(21)}_{q_1}, N^{(25)}_{q_5}\}\). Just after merging, the large black hole \(B\) formed by the merger corresponds to a configuration \(\{N^{(1)}_{q_1} = N^{(11)}_{q_1} + N^{(21)}_{q_1}, N^{(5)}_{q_5} = N^{(15)}_{q_5} + N^{(25)}_{q_5}\}\), provided that directions of the D-strings are the same for \(B_1\) and \(B_2\). In general the last configuration does not saturate the bounds \((9)\) and \((10)\) even when the configurations for \(B_1\) and \(B_2\) saturate the bounds. Thus, in general just after the merger the microscopic entropy of \(B\) does not agree with the corresponding Hawking-Bekenstein entropy and the temperature of the closed string radiation does not agree with the corresponding Hawking temperature. However, after a sufficiently long time the D-branes’ fission and fusion settle the entropy and the temperature to the Hawking-Bekenstein entropy and the Hawking temperature. To say something quantitative about the above process, we have to find a supergravity solution which represents the merger of branes and have to investigate the D-branes’ fission and fusion in terms of the elementary interactions. However, without doing these, we could reach the above conclusion about the dynamical process inside the black hole by analyzing the microscopic entropy of open strings on D-branes.
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