Phonons and Phonon Confinement in Transport through Double Quantum Dots

S. Debald\textsuperscript{1}, T. Vorrath\textsuperscript{1}, T. Brandes\textsuperscript{1,2}, B. Kramer\textsuperscript{1}

\textsuperscript{1} I. Institut f"{u}r Theoretische Physik, Universit"{a}t Hamburg, Jungiusstr. 9, 20355 Hamburg, Germany, e-mail: debald@physnet.uni-hamburg.de
\textsuperscript{2} Department of Physics, University of Manchester Institute of Science and Technology (UMIST), P.O. Box 88, Manchester M60 1QD, United Kingdom

Abstract

We calculate the electron–phonon interaction coefficients for surface acoustic waves and for phonons in free standing quantum wells. These are used to derive the inelastic current through a double quantum dot caused by spontaneous emission of phonons. For the case of the free standing structure (phonon cavity), we predict a staircase–like inelastic current superimposed by van Hove singularities. Therefore, the phonon confinement can be detected by electron transport measurements.

1 Introduction

The fabrication of free–standing or suspended nanoscale structures\cite{1,2} has opened new perspectives for the investigation of mesoscopic phonon effects such as phonon confinement, thermal conductance quantization, or single phonon transport. Recent experiments are designed to show and emphasize mesoscopic phonon effects of few or even a single phonon\cite{3}. At the same time, sensitive detectors for phonons become important\cite{4}. Electron transport measurements have already been used successfully for this task.

In double quantum dots, the emission of phonons has been observed to dominate the non–linear electron transport at mK–temperatures\cite{5}. This effect could be explained by an X–ray singularity–like boson shake–up effect for tunneling of single electrons due to coupling to phonons\cite{6}. Different phonon modes (surface, bulk, or confined) and features in phonon densities of states can therefore (at least in principle) be identified in electronic transport measurements. The energy difference between the two dot ground states should be used to scan the relevant phonon energy window.

2 Surface Acoustic Phonons

The knowledge of the electron phonon interaction coefficient is necessary in order to quantitatively understand the influence of different kinds of phonons in electron transport measurements. Here, we consider surface acoustic waves (SAW) and their interaction with electrons. In most experiments, the relevant electrons are part of a 2DEG which is located a small distance \(d\) beneath the surface of the sample (typically \(d \approx 100\)nm). If the SAW wavelength becomes comparably small, it is important to take into account the non–monotonic decrease of its amplitude into the sample. For an anisotropic crystal like GaAs, we calculate the piezoelectric electron–phonon interaction coefficient as a function of the depth

\[
\gamma_q(z) = \frac{1}{L} \sum_q \gamma_q(z) e^{iqz} (b_q + b_q^\dagger)
\]

and compare it with results for an isotropic crystal. Finally, we apply the result to estimate the inelastic tunnel current through a double quantum dot, caused by emission of SAW.

We consider a SAW propagating in [110] direction on the [001] surface of an anisotropic cubic crystal. This wave causes an electric potential \(\phi\) by the piezo effect. Hence, electrons in a 2DEG or a quantum dot close to the surface interact with the SAW as they are exposed to the potential \(V = e\phi\). The interaction potential can be written in the common form

\[
V_{\text{int}}(x, z) = \frac{1}{L} \sum_q \gamma_q(z) e^{iqx} (b_q + b_q^\dagger)
\]

with wave number \(q\), normalization length \(L\) and boson annihilation and creation operators \(b_q, b_q^\dagger\). \(\gamma_q\) is the interaction coefficient, that is \(z\)–dependent for the case of surface waves. Calculating the piezoelectric potential\cite{5}, we find for \(\gamma_q\) the expression

\[
\gamma_q(z) = \tilde{C} \{ A_1 e^{i q z} + A_2 e^{i b q z} + A_3 e^{-q z} \}
\]

where all prefactors are combined in \(\tilde{C}\) which does not depend on \(q\). \(A_1, A_2, A_3, b_1, b_2\) are complex functions of
the material constants. For GaAs we find for \( \gamma_q \) on the surface a value of \( 8.6 \times 10^{-13} \text{eVm} \). This is slightly smaller as the value found in the isotropic model of the crystal \( \text{[4]} \). The dependence on the depth is shown in the inset of Fig. \( \text{[4]} \). It can be seen that the isotropic model slightly exaggerates the penetration depth of the interaction coefficient.

We use the interaction coefficient \( \text{[4]} \) to estimate the inelastic tunnel current through a double quantum dot caused by spontaneous emission of surface acoustic phonons. Such a current was measured in recent experiments \( \text{[4]} \). Following the theoretical approach in \( \text{[4]} \), we calculate an effective density of states \( \rho(\varepsilon) \) which serves as an approximation for the inelastic tunnel current. The result is shown in Fig. \( \text{[4]} \) as well as the result for interaction with bulk phonons from \( \text{[4]} \). Both results possess the oscillations in the inelastic current that were found in the experiment, although the inelastic current in the case of emission of bulk phonons is stronger.

3 Confinement in Phonon Cavities

In (partly) freestanding structures \( \text{[4]} \), electron transport through lateral double dots is expected to show signatures of phonon confinement. We investigate a free standing quantum well of finite thickness as a phonon cavity model. Due to the confinement, a quantization of the phonon modes similar to the electronic case occurs. Families of shear, flexural and dilatational modes can be classified as in classical acoustics.

We calculate the dispersion relation of the cavity phonons using a numerical approach and find a staircase-like phonon density of states \( \nu(\omega) \) superimposed by van Hove singularities as shown in Fig. \( \text{[4]} \). The characteristic energy scale is given by \( \omega_b = \epsilon_i / b \) with the longitudinal velocity of sound and the cavity width \( 2b \).

For small enough cavity confinement widths one can show that the deformation potential is the dominant interaction mechanism between electrons and confined phonons. For a cavity confined in \( x \) direction the deformation potential is \( \text{[4]} \)

\[
V_{\text{def}} = \sum_{\mathbf{q}_{||},\mathbf{q}_l} \gamma(\mathbf{q}_l) e^{i \mathbf{q}_l \cdot \mathbf{r}_l} \left\{ \cos q_l n x \sin q_l n x \right\} \left[ b_n(\mathbf{q}_l) + b_n^\dagger(-\mathbf{q}_l) \right],
\]

where \( \mathbf{q}_l \) and \( \mathbf{q}_l \) are the in–plane and transversal components of the wavevector, resp. The upper row is for dilatational and the lower is for flexural modes. Dilatational phonons induce a symmetrical potential with respect to the cavity midplane, flexural phonons an antisymmetrical potential. Shear waves do not induce a deformation potential. For a double quantum dot placed in the middle of the cavity we calculate the inelastic current in the same model as in Sec. 2 using the deformation potential interaction between electrons and confined phonons. We find that due to the symmetry of the different phonon families, one can suppress the emission of such phonons. This can be done by changing the double dot axis relative to the confinement direction of the phonon cavity. Therefore, one can modify the emission characteristics of the double dot. Furthermore, we calculate the inelastic current through the double dot caused by flexural phonons corresponding to the emission characteristic of the dots oriented in direction of the confinement. We find that the traces of phonon confinement seen in the thermodynamical density of states remain in the inelastic current as shown in Fig. \( \text{[4]} \). For a phonon cavity made of GaAs and a width of \( 2b = 1 \mu m \), the characteristic energy is \( \hbar \omega_b = 7.5 \mu \text{eV} \). Therefore, it should be possible to detect the phonon confinement in electron transport experiments through coupled quantum dots.

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**Fig. 2** Thermodynamical density of states of the cavity phonons \( \nu(\omega) \) (dashed) and inelastic current through the double dot in the cavity \( \text{[4]} \). The \( 2^{\text{nd}} \) and \( 13^{\text{th}} \) flexural modes lead to van Hove singularities in the density of states and the inelastic current. For GaAs and a cavity width of \( 2b = 1 \mu m \) the characteristic energy is \( \hbar \omega_b = \hbar c_i / b = 7.5 \mu \text{eV} \).

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