Unambiguous discrimination between mixed quantum states based on programmable quantum state discriminators

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We discuss the problem of designing an unambiguous programmable discriminator for mixed quantum states. We prove that there does not exist such a universal unambiguous programmable discriminator for mixed quantum states that has two program registers and one data register. However, we find that we can use the idea of programmable discriminator to unambiguously discriminate mixed quantum states. The research shows that by using such an idea, when the success probability for discrimination reaches the upper bound, the success probability is better than what we do not use the idea to do, except for some special cases.

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I. INTRODUCTION

The discrimination of quantum states is a basic task in quantum information and quantum communication [1]. A great deal of attention has been attracted into this field these years, especially the unambiguous discrimination (UD) of quantum states. UD is a sort of discrimination that never gives an erroneous result, but sometimes it may fail. In the case of pure states, UD has been widely considered. In the case of two pure states, the optimum measurement for the UD of two pure states was found decade ago [2-5]. A sufficient and necessary condition for unambiguously distinguishing arbitrary pure states and upper bound on the success probability for UD of arbitrary pure states have also been given (see, for example, [6] and some related references therein). Indeed, a complete overview of UD of pure states can be found in two recent review articles [7]. In the case of mixed quantum states, lots of work also has been done this years [8-14], which focuses on the upper bound and how to get the upper bound of the success probability for discrimination. For the case of two mixed quantum states, a necessary and sufficient condition for discriminating two mixed states to reach upper bound has been derived in [12].

As we know, if we want to unambiguously discriminate quantum states, we need construct some positive operator valued measurements (POVMs) according to the states. However, if the states are unknown, we can not construct such POVMs, which means that we can not discriminate unknown states directly. Recently, a programmable quantum state discriminator for unambiguous discrimination was first proposed by Bergous and Hillery [15] to resolve this problem. Bergous and Hillery’s discriminator is a fixed measurement that has two program registers and one data register. The quantum states in the data register is what we want to identify, which is confirmed to be one of the two states in program registers. That is to say, if we want to discriminate two states \(|\psi_1\rangle\) and \(|\psi_2\rangle\), we assign the two states into the two program registers, and the data register is assigned with the state which we want to identify. Here we have no idea of these two states. Now we have two input states

\[
|\psi_1^{in}\rangle = |\psi_1\rangle|\psi_1\rangle, |\psi_2^{in}\rangle = |\psi_1\rangle|\psi_2\rangle|\psi_2\rangle. \tag{1}
\]

It is easy to see that if we can discriminate \(|\psi_1^{in}\rangle\) and \(|\psi_2^{in}\rangle\), then we can discriminate states \(|\psi_1\rangle\) and \(|\psi_2\rangle\). Bergous and Hillery’s discriminator makes this target successful with a fixed measurement.

Based on Bergous and Hillery’s discriminator, Zhang et al [16] recently presented an unambiguous programmable discriminator for \(n\) arbitrary quantum states in an \(m\)-dimensional Hilbert space, where \(m \geq n\). If \(m = n\), an optimal unambiguous programmable discriminator for \(n\) arbitrary states was given in [16]. Notably, the unambiguous programmable discriminator for two states with a certain number of copies has been discussed in [17,18].

However, all the discriminators mentioned above concentrate on pure states. As we are aware, the unambiguous programmable discriminators for mixed quantum states still have not been discussed. In this paper, we try to deal with the problem of designing an unambiguous programmable discriminator for mixed quantum states. Our purpose is to see whether or not an programmable unambiguous discriminator for mixed quantum states can be realized.

This paper is organized as follows: In Sec. II we prove that there does not exist an unambiguous programmable discriminator for mixed quantum states that has two program registers and one data register. Then, however, in Sec. III we find that we can still use the idea of programmable quantum state discriminator to unambiguously discriminate mixed quantum states. The research shows that by using this idea, when the success probability for discrimination reaches the upper bound, the success probability is better than what we do not use the idea to do, except for some special cases. At last, we conclude the paper with a short summary.

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II. NONEXISTENCE OF PROGRAMMABLE DISCRIMINATOR FOR MIXED STATES BASED ON BERGOUSS AND HILLERY’S MODEL

First we try to design an unambiguous programmable discriminator for mixed quantum states based on Bergouss and Hillery’s model [15]. Our purpose is to see whether or not such an unambiguous programmable discriminator for mixed quantum states can be realized. To begin with, we prove a theorem here.

**Theorem 1.** Two mixed quantum states \( \rho_1, \rho_2 \) can be unambiguously discriminated if and only if \( \rho_1^{in}, \rho_2^{in} \) can be unambiguously discriminated, where

\[
\rho_1^{in} = \rho_1 \otimes \rho_2 \otimes \rho_1, \quad \rho_2^{in} = \rho_1 \otimes \rho_2 \otimes \rho_2.
\]

**Proof.** First let

\[
\rho_1 = \sum_{i=1}^{n_1} \alpha_i |\varphi_i\rangle \langle \varphi_i|, \quad \rho_2 = \sum_{j=1}^{n_2} \beta_j |\psi_j\rangle \langle \psi_j|, \quad \rho_3 = \sum_{k} \gamma_k |\psi_k\rangle \langle \psi_k|.
\]

be the spectral decompositions of \( \rho_1, \rho_2, \rho_3 \). Then

\[
\rho_1^{in} = \sum_{i,j,k} \alpha_i \beta_j \gamma_k \langle \varphi_i \psi_j \psi_k | \langle \varphi_i \psi_j \psi_k |,
\]

\[
\rho_2^{in} = \sum_{i,j,k} \alpha_i \beta_j \gamma_k \langle \varphi_i \psi_j \psi_k | \langle \psi_i \psi_j \psi_k |.
\]

where \( i, j, k = 1, \ldots, n_1, n_2 \). Clearly, formula (4) is also the spectral decompositions of \( \rho_1^{in} \) and \( \rho_2^{in} \).

Suppose that \( \rho_1, \rho_2, \rho_3 \) can be unambiguously discriminated. Then there exist POVM elements \( \Pi_0, \Pi_1, \Pi_2 \) such that \( \Pi_0 + \Pi_1 + \Pi_2 = I \) and \( \text{Tr}(\Pi_j \rho_j) = p_i \delta_{ij} \) for some \( p_i > 0 \), where \( i, j = 1, 2 \). Now we construct a new set of POVM elements \( \Pi_0^{in} = I \otimes \Pi_0, \Pi_1^{in} = I \otimes \Pi_1, \Pi_2^{in} = I \otimes \Pi_2 \), where \( I \) denotes the identity operator on \( \rho_1 \otimes \rho_2 \). We can easily prove that \( \Pi_0^{in} + \Pi_1^{in} + \Pi_2^{in} = I \) and \( \text{Tr}(\Pi_j^{in} \rho_j^{in}) = p_i \delta_{ij} \) for the above \( p_i > 0 \), where \( i, j = 1, 2 \).

It means that there exists a set of POVM elements which can unambiguously discriminate \( \rho_1^{in}, \rho_2^{in} \), i.e., \( \rho_1^{in}, \rho_2^{in} \) can be unambiguously discriminated.

On the other side, suppose that \( \rho_1^{in}, \rho_2^{in} \) can be unambiguously discriminated. Then \( \text{supp}(\rho_1^{in}) \neq \text{supp}(\rho_1^{in}, \rho_2^{in}) \) and \( \text{supp}(\rho_2^{in}) \neq \text{supp}(\rho_1^{in}, \rho_2^{in}) \) [10]. Here \( \text{supp}(\rho_1, \ldots, \rho_n) \) is defined by the Hilbert space spanned by the eigenvectors of the mixed states \( \rho_1, \ldots, \rho_n \) with corresponding nonzero eigenvalues. For \( \text{supp}(\rho_2^{in}) \neq \text{supp}(\rho_1^{in}, \rho_2^{in}) \), it means that there exist some \( i, j, k \), where \( 1 \leq i, j, k \leq n_1 \) and \( 1 \leq j \leq n_2 \), satisfying

\[
|\varphi_i \psi_j \psi_k \rangle \neq \sum_{i', j', k'} a_{i', j', k'} |\varphi_{i'} \psi_{j'} \psi_{k'} \rangle
\]

where \( i' = 1, \ldots, n_1, j', k' = 1, \ldots, n_2 \). Specifically, if we choose \( i' = i, j' = j \), then

\[
|\varphi_i \psi_j \psi_k \rangle \neq |\varphi_i \psi_j \rangle \sum_{i'} a_{i'} |\psi_{i'} \rangle.
\]

and, as a result,

\[
|\varphi_k \rangle \neq \sum_{i'} a_{i'} |\psi_{i'} \rangle.
\]

It implies \( \text{supp}(\rho_2) \neq \text{supp}(\rho_1, \rho_2) \). With similar discussion, we can also have \( \text{supp}(\rho_1) \neq \text{supp}(\rho_1, \rho_2) \). Therefore, \( \rho_1, \rho_2 \) can be unambiguously discriminated. This completes the proof.

In terms of **Theorem 1**, we discuss whether or not there exists an unambiguous programmable discriminator for mixed quantum states based on Bergouss and Hillery’s model [15]. Indeed, we have the following result.

**Theorem 2.** There does not exist an unambiguous programmable discriminator for mixed quantum states that has two program registers and one data register.

**Proof.** Suppose that there exists such an unambiguous programmable discriminator for mixed quantum states. Then there also exists a fixed measurement that can unambiguously discriminate \( \rho_1^{in}, \rho_2^{in} \), where \( \rho_1^{in} = \rho_1 \otimes \rho_2 \otimes \rho_1, \rho_2^{in} = \rho_1 \otimes \rho_2 \otimes \rho_2 \), and \( \rho_1, \rho_2 \) are guaranteed to be unambiguously discriminated. We here assume that the fixed POVM elements are \( \Pi_0, \Pi_1, \Pi_2 \), which satisfy

\[
\Pi_1 \rho_2^{in} = 0, \Pi_2 \rho_1^{in} = 0, \quad \text{Tr}(\Pi_1 \rho_1^{in}) > 0, \quad \text{Tr}(\Pi_2 \rho_2^{in}) > 0,
\]

\[
\Pi_0 + \Pi_1 + \Pi_2 = I,
\]

for any \( \rho_1, \rho_2 \) when they can be unambiguously discriminated.

Now, we have three special mixed quantum states as follows

\[
\rho_1 = a_1 |\gamma_1 \rangle \langle \gamma_1| + a_2 |\gamma_2 \rangle \langle \gamma_2|,
\]

\[
\rho_2 = b_1 |\gamma_2 \rangle \langle \gamma_2| + b_2 |\gamma_3 \rangle \langle \gamma_3|,
\]

\[
\rho_3 = c_1 |\gamma_1 \rangle \langle \gamma_1| + c_2 |\gamma_3 \rangle \langle \gamma_3|,
\]

where \( \gamma_1, \gamma_2, \gamma_3 \) are mixed quantum states in \( m \)-dimension Hilbert space (\( m \geq 3 \)), and \( \{ |\gamma_1 \rangle, |\gamma_2 \rangle, |\gamma_3 \rangle \} \) consists of an orthonormal basis in this space. It is no doubt that any two of these three stats can be unambiguously discriminated. Now we use the discriminator to discriminate any two of these states.

(1) Let \( \rho_1 = \rho_1', \rho_2 = \rho_2' \). Then \( \rho_1^{in} = \rho_1 \otimes \rho_2 \otimes \rho_1', \rho_2^{in} = \rho_1 \otimes \rho_2 \otimes \rho_2' \). According to (8), \( \Pi_1 \rho_2^{in} = 0, \text{Tr}(\Pi_1 \rho_1^{in}) > 0 \), and we have

\[
\Pi_1 |\gamma_1 \gamma_2 \gamma_2 \rangle = 0, \Pi_1 |\gamma_1 \gamma_2 \gamma_3 \rangle = 0, \Pi_1 |\gamma_1 \gamma_3 \gamma_2 \rangle = 0,
\]

\[
\Pi_1 |\gamma_1 \gamma_3 \gamma_3 \rangle = 0, \Pi_1 |\gamma_2 \gamma_2 \gamma_2 \rangle = 0, \Pi_1 |\gamma_2 \gamma_2 \gamma_3 \rangle = 0,
\]

\[
\Pi_1 |\gamma_2 \gamma_3 \gamma_2 \rangle = 0, \Pi_1 |\gamma_2 \gamma_3 \gamma_3 \rangle = 0,
\]

and

\[
\text{Tr}(\Pi_1 \rho_1^{in}) = \sum_{i,j,k=1}^{i,j,k=2} a_{i} b_{j} a_{k} \langle \gamma_i \gamma_{j+1} \gamma_k \rangle |\Pi_1 |\gamma_i \gamma_{j+1} \gamma_k \rangle > 0.
\]
(2) Let \( \rho_1 = \rho_2 \), then \( \rho_1^{in} = \rho_2^{in} = \rho_1 \). According to (8), \( \Pi \rho_2^{in} = 0 \), and we have
\[
\Pi_1 |\gamma_1^2 \gamma_2^1 \rangle = 0, \Pi_1 |\gamma_2^2 \gamma_1^1 \rangle = 0, \\
\Pi_1 |\gamma_1^2 \gamma_2^1 \rangle = 0, \Pi_1 |\gamma_2^2 \gamma_1^1 \rangle = 0. \tag{12}
\]

(3) Let \( \rho_1 = \rho_2 = \rho_3 \). Then \( \rho_1^{in} = \rho_2^{in} = \rho_3^{in} = \rho_1 \). According to (8), \( \Pi \rho_2^{in} = 0 \), and we have
\[
\Pi_1 |\gamma_1^1 \gamma_2^1 \rangle = 0, \Pi_1 |\gamma_1^1 \gamma_3^2 \rangle = 0, \Pi_1 |\gamma_1^2 \gamma_3^1 \rangle = 0, \\
\Pi_1 |\gamma_2^2 \gamma_3^3 \rangle = 0, \Pi_1 |\gamma_2^2 \gamma_3^2 \rangle = 0. \tag{13}
\]

(4) Let \( \rho_1 = \rho_2 = \rho_3 \), then \( \rho_1^{in} = \rho_2^{in} = \rho_3^{in} = \rho_1 \). According to (8), \( \Pi \rho_2^{in} = 0 \), and we have
\[
\Pi_1 |\gamma_1^1 \gamma_2^1 \rangle = 0, \Pi_1 |\gamma_1^1 \gamma_2^2 \rangle = 0, \Pi_1 |\gamma_1^2 \gamma_2^1 \rangle = 0, \\
\Pi_1 |\gamma_2^2 \gamma_3^3 \rangle = 0, \Pi_1 |\gamma_2^2 \gamma_3^2 \rangle = 0. \tag{14}
\]

Now using (10) and (12)-(14), we find that \( Tr(\Pi_1 \rho_1^{in}) \) in (11) is equal to zero, which contradicts (11) that is \( Tr(\Pi_1 \rho_1^{in}) > 0 \). It means that there does not exist such a fixed measurement. In other words, such an unambiguous programmable discriminator for mixed quantum states does not exist. The proof is completed.

Why does not there exist such an unambiguous programmable discriminator for mixed quantum states? The reason is not hard to find from the above proof. It is because the mixed states \( \rho_1^{in}, \rho_2^{in} \) loose the symmetry which \( |\psi_1^{in} \rangle, |\psi_2^{in} \rangle \) have. Or we can say that the difference between mixed states and pure states results in Theorem 2. Also, from Theorem 2 we have seen some special features that mixed states have but pure states do not.

### III. Unambiguous Discrimination Between Mixed Quantum States Based on Programmable Discriminator

It is disappointed that we do not have such an unambiguous programmable discriminator for mixed quantum states that was indicated above. We do not know whether there exists other type of discriminators for mixed quantum states, either. However, if we think about it from a different angle, we can find that the unambiguous programmable discriminator is a very good idea for discriminating states. We can still use the idea of unambiguous programmable discriminators here to discriminate mixed states. That is to say, if we want to discriminate two known mixed states \( \rho_1, \rho_2 \), then we can try to discriminate two mixed states \( \rho_1^{in}, \rho_2^{in} \). We use the idea of unambiguous programmable discriminators which have two program registers and \( n \) data registers. Specifically, if we want to discriminate two known mixed states \( \rho_1, \rho_2 \), then we try to discriminate the following states
\[
\rho_1^{in} = \rho_1 \otimes \rho_2 \otimes \rho_1^{sn}, \quad \rho_2^{in} = \rho_1 \otimes \rho_2 \otimes \rho_2^{sn}. \tag{15}
\]

It is clear that if we can discriminate \( \rho_1^{in}, \rho_2^{in} \), then we can also discriminate \( \rho_1, \rho_2 \).

First we consider whether \( \rho_1^{in}, \rho_2^{in} \) can be unambiguously discriminated when \( \rho_1, \rho_2 \) can be unambiguously discriminated. The answer is yes. We can use the similar method in Theorem 1 to prove it. Now based on the two known states \( \rho_1^{in}, \rho_2^{in} \), we can construct POVMs to distinguish them. Before dealing with the success probability for unambiguous discrimination between \( \rho_1^{in} \) and \( \rho_2^{in} \), we have a simple lemma as follows.

**Lemma 1.** Let \( \rho_1, \rho_2 \) be two arbitrary mixed states, and let \( \rho_1^{in} = \rho_1 \otimes \rho_2 \otimes \rho_1^{sn}, \rho_2^{in} = \rho_1 \otimes \rho_2 \otimes \rho_2^{sn} \). We have \( F(\rho_1^{in}, \rho_2^{in}) = F(\rho_1, \rho_2)^n \), where \( n \geq 1 \) and \( F(\cdot, \cdot) \) is the definition of fidelity in [1], i.e., \( F(\rho_1, \rho_2) = Tr(\sqrt{\rho_1 \rho_2}) \).

The proof of lemma 1 follows from the simple fact as follows.

\[
F(\rho_1 \otimes \rho_2, \rho_1 \otimes \rho_3 \otimes \rho_4) = F(\rho_1, \rho_3) \times F(\rho_2, \rho_4). \tag{16}
\]

Now we discuss the failure probability of the unambiguous discrimination between \( \rho_1^{in}, \rho_2^{in} \). According to Raynal and Lütkenhaus’ work [12], if \( supp(\rho_1^{in}) \cap supp(\rho_2^{in}) = \{0\} \) and some conditions are satisfied, the failure probability of the unambiguous discrimination between \( \rho_1^{in}, \rho_2^{in} \) can reach its low bound. Let \( F_1^{in} \) and \( F_2^{in} \) denote \( \sqrt{\rho_1^{in} \rho_2^{in} \rho_1^{in}}, \sqrt{\rho_1^{in} \rho_2^{in} \rho_2^{in}} \), respectively. Let \( F(\rho_1^{in}, \rho_2^{in}) \) be the fidelity of the two states \( \rho_1^{in}, \rho_2^{in} \). Then \( F(\rho_1^{in}, \rho_2^{in}) = F(\rho_1, \rho_2)^n \). We denote by \( P_1^{in} \) and \( P_2^{in} \), the projectors onto the supports of \( \rho_1^{in} \) and \( \rho_2^{in} \), respectively. Let \( P_1 \) and \( P_2 \) be the projectors onto the supports of \( \rho_1 \) and \( \rho_2 \), respectively. Then \( P_1^{in} = P_1 \otimes P_2 \otimes P_1^{in} \) and \( P_2^{in} = P_1 \otimes P_2 \otimes P_2^{in} \). We can prove \( Tr(P_1^{in} \rho_2^{in}) = Tr(P_1 \rho_2)^n \) and \( Tr(P_2^{in} \rho_1^{in}) = Tr(P_2 \rho_1)^n \) using the similar method as lemma 1. Let \( \eta_1 \) and \( \eta_2 \) be the prior probabilities of \( \rho_1 \) and \( \rho_2 \), respectively. Now according to [12], we have
where $Q_{in}^{opt}$ denotes the optimal failure probability of the unambiguous discrimination between $\rho_1^{in}, \rho_2^{in}$. Here $Tr(P_2\rho_1) \leq 1$, $Tr(P_1\rho_2) \leq 1$, $F(\rho_1, \rho_2)^2 \leq Tr(P_2\rho_1)$ and $F(\rho_1, \rho_2)^2 \leq Tr(P_1\rho_2)$ (the more details are referred to [12]).

The first question is whether or not $\text{supp}(\rho_1^{in}) \cap \text{supp}(\rho_2^{in}) = \{0\}$ can be satisfied? Actually, we can easily prove that if $\text{supp}(\rho_1) \cap \text{supp}(\rho_2) = \{0\}$, then $\text{supp}(\rho_1^{in}) \cap \text{supp}(\rho_2^{in}) = \{0\}$. It means that $\text{supp}(\rho_1^{in}) \cap \text{supp}(\rho_2^{in}) = \{0\}$ is not a stricter constraint.

Let $Q_{in}$ denote the failure probability of the unambiguous discrimination between $\rho_1^{in}, \rho_2^{in}$. From [12], we know that $Q_{in}$ here can reach $Q_{in}^{opt}$ sometimes. When $Q_{in}$ reaches $Q_{in}^{opt}$, that is, $Q_{in} = Q_{in}^{opt}$, we find that $Q_{in}$ is smaller than $Q$ (here $Q$ denotes the failure probability of the unambiguous discrimination between $\rho_1, \rho_2$), except for some special cases. We discuss this in what follows.

If $F(\rho_1, \rho_2) = 0$, i.e., it means that the two states can be perfectly discriminated, then $Q_{in} = Q = 0$. When $n = 1$, we find that if $Q_{in}$ reaches $Q_{in}^{opt}$, then $Q$ can also reach its optimal value, and thus $Q_{in} = Q = Q_{in}^{opt}$. Now we consider the situation where $0 < F(\rho_1, \rho_2) < 1$ and $n > 1$:

1. If $\frac{Tr(P_2\rho_1)}{F(\rho_1, \rho_2)} \leq 1$ and $\frac{Tr(P_1\rho_2)}{F(\rho_1, \rho_2)} \geq 1$, then no matter which regime $\sqrt{\frac{2n}{\eta_1}}$ is, we will find that if $Q_{in}$ reaches $Q_{in}^{opt}$, then $Q_{in} = Q_{in}^{opt} < Q$.

2. If $\frac{Tr(P_2\rho_1)}{F(\rho_1, \rho_2)} \leq \frac{F(\rho_1, \rho_2)}{Tr(P_1\rho_2)} < 1$, then, except for the regime $\frac{F(\rho_1, \rho_2)}{Tr(P_1\rho_2)} \leq \sqrt{\frac{2n}{\eta_1}}$, we cannot compare, we will find that if $Q_{in}$ reaches $Q_{in}^{opt}$, then $Q_{in} = Q_{in}^{opt} < Q$.\[Q_{in} \geq Q_{in}^{opt} < Q.\]

3. If $\frac{Tr(P_2\rho_1)}{F(\rho_1, \rho_2)} > 1$, then, except for the regime $\frac{Tr(P_2\rho_1)}{F(\rho_1, \rho_2)} \leq \sqrt{\frac{2n}{\eta_1}} \leq \frac{Tr(P_2\rho_1)}{Tr(P_1\rho_2)}$, that we cannot compare, we will find that if $Q_{in}$ reaches $Q_{in}^{opt}$, then $Q_{in} = Q_{in}^{opt} < Q$.

From the above discussion we can see that if the failure probability of the unambiguous discrimination between $\rho_1^{in}, \rho_2^{in}$ reaches its optimization, then the failure probability of the unambiguous discrimination between $\rho_1^{in}, \rho_2^{in}$ is better than that between $\rho_1, \rho_2$ mostly. It is easy to find that the bigger $n$ is, the smaller $Q_{in}^{opt}$ will be. That means that if $Q_{in}$ can reach $Q_{in}^{opt}$ with the bigger $n$, then the smaller $Q_{in}$ will be. Considering the conditions of $Q_{in}$ being able to reach $Q_{in}^{opt}$ in (17), we find that such conditions are not stricter when $n$ is bigger. Especially, the conditions in the first and the third regime of (17) can be derived from $n = 1$. On the other hand, even if $n$ is small, such as $n = 2$, and $F(\rho_1, \rho_2)$ is much smaller than 1, then we can also have a very small $Q_{in}^{opt}$ here.

A rest question is what about the situation when $Q_{in}$ does not reach its optimization? We have no answer yet. The solution of such a question depends on the solution of how to discriminate two arbitrary mixed states optimally. However, how to discriminate optimally two arbitrary mixed quantum states still is an open question now.

\[4\]

IV. CONCLUSIONS

In this paper, we try to design an unambiguous programmable discriminator for mixed quantum states based on Bergous and Hillery’s model [15]. We have proved that there does not exist a universal unambiguous programmable discriminator for mixed quantum states that has two program registers and one data register. However, we found that we can use the idea of programmable discriminators to unambiguously discriminate mixed quantum states. The research shows that by using such an idea, when the success probability for discrimination reaches the upper bound, the success probability is better than what we do not use the idea to do, except for some special cases. We have discussed this result in detail and presented some prospects for it.

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