A study on human intelligence using Fuzzy Grey Relational Maps (FGRMs)

S. Arokiamary¹, M. Mary Mejullo Merlin²*

¹Department of Mathematics, Mother Gnanamma Women’s College of Arts and Science, Varadarajanpet, Ariyalur, Tamilnadu, India.
²PG & Research Department of Mathematics, Holy Cross College (Autonomous), Trichirapalli, India.

E-mail: merlinprashanth@yahoo.com

Abstract- Fuzzy-grey system is an ambiguous system where unclear, incomplete and only small data is available. Fuzzy Grey relational maps model the causal relation between complex systems. In this fuzzy model, the nodes are fuzzy while the edges are grey and they represent the grey causal relationship between two disjoint sets of concepts. Intelligence is one of the most admired and coveted features of human mind for, human intelligence has played a greater role in placing human beings at the top of the evolutionary pyramid of creation. Intelligence is widely accepted as hereditary, but at the same time there are researches going on to prove that it is also influenced by the beliefs the individual holds. In this paper, the causal influence between intelligence and self-beliefs is studied using Fuzzy Grey relational maps.

Key words: Fuzzy relational maps, FGRM, beliefs, intelligence, grey system

1. Introduction

The cognitive processes in human beings such as reasoning and decision-making take place in a highly uncertain environment. Modelling such a complex system using classical methods is a challenging task. The uncertainty and hesitancy in human thinking make the information highly approximate and not absolutely reliable. The Fuzzy Grey Relational Maps reduce the ambiguity in the available data with the help of Grey System Theory. The grey numbers capture the uncertainty and hesitancy in the data provided by an individual. Identifying and assessing human intelligence is certainly a very indecisive process. In this paper, the causal relationship between different types of intelligence and the underlying beliefs is studied using the Fuzzy Grey Relational Maps.

2. Description of the Problem

Intelligence is a much-appreciated trait in every creature for, it enables them to find ways and means to a meaningful life and creative solutions in times of challenges. In particular, human intelligence has taken human life and evolution to what it is today. The world moves by the power of human intelligence. Human Intelligence has played a greater role in making this planet earth an advanced place to live. Human beings have worked so hard, since time immemorial, to reach this far. From the researches that have been carried out on intelligence, it can be inferred that beliefs are
strappingly associated with human intelligence and their performance [1,2]. The strong association between beliefs and intelligence of an individual prompts to study the causal association between them.

2.1. Beliefs and Intelligence

Human intelligence is the capacity of an individual to understand, comprehend, learn, create, plan, solve problems, make decisions and so on. These abilities define a person’s intelligence and gives them the power to carry on their life successfully. The capacities of individuals vary person to person to a greater extend. One individual who is good in one aspect may not do well in another. Assessing an individual’s capacity is also another indecisive process where one may not be able to give an exact measure of the level or type of intelligence [3-6]. Because everyone has different opinions and ideas about intelligence [7, 8], it is difficult to come to conclusion as to what is the ability of an individual to become the fittest to survive. Human intelligence is not a fixed idea, but an evolving concept that varies with time and experience. What brings in the change in one’s capacity to cope with life to survive and succeed in life attracts our attention [1, 2,9]. In this paper, an attempt is made to study the relationship between self- beliefs and different types of human intelligence.

2.2. Simulating the causal relationship between Beliefs and Intelligence

Simulating psychological constructs to study their properties and behaviours is definitely a difficult process as it involves a great deal of uncertainty and vagueness. The ambiguity in human thinking and understanding makes it even harder to analyse the mental constructs. The hesitancy and indecisiveness associated with human reasoning and decision-making processes have to be incorporated intelligently into the model. Producing new models that is enabled with capacity to measure and process the imprecise information is the need of the hour.

There are different theories on human intelligence which suggest the components, types or features of intelligence. Here, the model proposed by Harvard University neuropsychologist and educator Howard Gardener in his book Frames of Mind: The theory of Multiple Intelligences which describes eight kinds of intelligence is adopted [9]. Human belief system is not a well-defined structure, yet there are several primary beliefs which are universal and indispensable in every individual. The description of domains of beliefs and types of intelligence is given in table-1. In this study, seven areas of common beliefs and eight types of intelligences are chosen. The causal relationship between personal beliefs and types of intelligence using a fuzzy-grey mathematical model called Fuzzy Grey Relational Maps (FGRM) as given in figure-1.

| Table-1: FGRM nodes and their description |
| Self-Beliefs (Domain Space) | Human Intelligence (Range Space) |
|---------------------------|-------------------------------|
| \( B_1 \) Survival       | \( I_1 \) Linguistic/Verbal   |
| \( B_2 \) Security        | \( I_2 \) Logical/Mathematical|
| \( B_3 \) Connection      | \( I_3 \) Visual/Spatial      |
| \( B_4 \) Vulnerability   | \( I_4 \) Bodily/Kinaesthetic |
| \( B_5 \) Judgement       | \( I_5 \) Musical/Rhythmical |
| \( B_6 \) Responsibility   | \( I_6 \) Interpersonal       |
| \( B_7 \) Recognition     | \( I_7 \) Intrapersonal       |
|                           | \( I_8 \) Naturalist          |
The relationship between human beliefs and intelligence is completely grey. No one can absolutely estimate the level or identify the type of one’s intelligence or one’s own for that matter. The FGRM is obtained in the form of a matrix from parents and teachers who deal with children of age 3 to 6 years. The relation between the nodes of domain space and range space is given by a grey number which is an interval taking values from [-1, +1]. The relational matrix that captures the grey relationship between these two disjoint concepts is obtained from experts and the augmented matrix is calculated. The augmented relational matrix that describes the grey relation between the domain space and the range space is given in table-2.

**Table-2: The augmented relational matrix $R^{aug} (\otimes)$ based on Expert’s Opinion**

| $I_1$  | $I_2$  | $I_3$  | $I_4$  | $I_5$  | $I_6$  | $I_7$  | $I_8$  |
|--------|--------|--------|--------|--------|--------|--------|--------|
| $B_{1+}$ | [0.6, 0.8] | [0.4, 0.7] | [0.8, 0.9] | [0.4, 0.7] | [0.6, 0.8] | [0.7, 0.9] | [0.3, 0.5] | [0.4, 0.6] |
| $B_{1-}$ | [-0.6, -0.2] | [-0.4, -0.3] | [-0.7, -0.5] | [-0.4, -0.3] | [-0.5, -0.2] | [-0.5, -0.3] | [-0.8, -0.6] | [-0.5, -0.3] |
| $B_{2+}$ | [0.4, 0.6] | [0.3, 0.7] | [0.6, 0.7] | [0.7, 0.9] | [0.5, 0.8] | [0.7, 0.9] | [0.2, 0.5] | [0.4, 0.6] |
| $B_{2-}$ | [-0.5, -0.3] | [-0.5, -0.4] | [-0.5, -0.2] | [-0.6, -0.4] | [-0.6, -0.3] | [-0.4, -0.2] | [-0.8, -0.6] | [-0.7, -0.4] |
| $B_{3+}$ | [0.6, 0.8] | [0.5, 0.8] | [0.5, 0.7] | [0.4, 0.7] | [0.4, 0.6] | [0.6, 0.8] | [0.3, 0.5] | [0.5, 0.8] |
| $B_{3-}$ | [-0.7, -0.4] | [-0.6, -0.3] | [-0.5, -0.2] | [-0.6, -0.3] | [-0.7, -0.3] | [-0.5, -0.3] | [-0.8, -0.6] | [-0.7, -0.5] |
| $B_{4+}$ | [0.6, 0.9] | [0.5, 0.8] | [0.4, 0.7] | [0.6, 0.8] | [0.5, 0.8] | [0.6, 0.9] | [0.3, 0.5] | [0.4, 0.7] |
| $B_{4-}$ | [-0.7, -0.3] | [-0.7, -0.4] | [-0.6, -0.4] | [-0.7, -0.4] | [-0.8, -0.5] | [-0.5, -0.2] | [-0.8, -0.6] | [-0.6, -0.4] |
| $B_{5+}$ | [0.8, 0.9] | [0.6, 0.8] | [0.5, 0.7] | [0.4, 0.7] | [0.4, 0.6] | [0.6, 0.9] | [0.3, 0.5] | [0.3, 0.6] |
| $B_{5-}$ | [-0.4, -0.2] | [-0.5, -0.3] | [-0.5, -0.4] | [-0.5, -0.2] | [-0.6, -0.3] | [-0.4, -0.3] | [-0.7, -0.5] | [-0.6, -0.3] |
| $B_{6+}$ | [0.7, 0.9] | [0.6, 0.8] | [0.6, 0.9] | [0.4, 0.7] | [0.5, 0.7] | [0.7, 0.9] | [0.3, 0.6] | [0.4, 0.7] |
| $B_{6-}$ | [-0.6, -0.3] | [-0.8, -0.4] | [-0.7, -0.3] | [-0.6, -0.3] | [-0.5, -0.3] | [-0.5, -0.2] | [-0.8, -0.6] | [-0.6, -0.3] |
| $B_{7+}$ | [0.7, 0.9] | [0.6, 0.8] | [0.5, 0.8] | [0.6, 0.8] | [0.4, 0.7] | [0.7, 0.9] | [0.5, 0.7] | [0.4, 0.6] |
| $B_{7-}$ | [-0.6, -0.3] | [-0.6, -0.4] | [-0.5, -0.3] | [-0.6, -0.3] | [-0.6, -0.3] | [-0.3, -0.2] | [-0.8, -0.6] | [-0.7, -0.4] |

Figure-1: FGRM between beliefs and intelligence
3. Fuzzy Grey Relational Maps

Fuzzy Grey Cognitive Maps (FGCMs) was proposed by Salmeron J.L in his research article Modelling grey uncertainty with Fuzzy Grey Cognitive Maps in 2010 [10]. The Fuzzy Grey Relational Maps (FGRMs) are constructed analogous to FGCM. Fuzzy Grey Cognitive Map (FGRM) is an extension of Fuzzy Relational Map (FRM) [11] where two uncertain systems, namely fuzzy system and grey system are combined. Like the Fuzzy system, grey system is also an ambiguous system where unclear, incomplete and only small data is available [12-21]. In FGRM, the nodes are fuzzy while the edges are grey and they represent the grey causal relationship between two disjoint concepts. FGRM are useful in studying the causal relationship between two disjoint concepts [11] while the relationship could be either, black or white or grey [10].

Vasantha Kandasamy et al. introduced Fuzzy Relational Maps model to study the real-world problems wherein the cause and effect relationship between two disjoint concepts is analysed in 2000. Fuzzy Relational Maps (FRM) are structurally similar to the state-of-the-art model to study the complex systems namely, Fuzzy Cognitive Maps (FCM). In FRMs the concepts (nodes) are divided into two disjoint fuzzy sets as domain space and range space. Like FCMs, the FRMs can also be extended in a similar manner to adapt to different scenarios. The greatest challenge that is encountered in formulating and functioning FRMs is assigning weight to the edges which describe the influence of one concept on another. The uncertainty and hesitancy of the experts in assigning the proper weight results in subjective valuations which may not be precise in bringing out the relationship between the two sets of nodes. Consequently, lots of efforts have gone into finding methods and techniques to suggest proper weight that would take into consideration the uncertainty and hesitancy involved in the processes of determining the weights and their inference in the complex system.

The Fuzzy Grey Relational Maps is yet another approach to study the relationship between concepts in a complex system by choosing the proper edge strength. The Fuzzy Grey Cognitive Maps (FGCM) that were first introduced by Jose L. Salmeron in 2010 [10] wherein the author describes several new ideas such as grey environment, grey values and arithmetic operation on grey numbers and whitenisation of grey numbers. Jose L. Salmeron and Elpiniki I. Papageorgiou studied Decision Support System for radiotherapy treatment planning using FGCM tool in 2012[16]. Jose L. Salmeron and Ester Gutierrez used FGCM in reliability engineering 2012[20]. Wojciech Froelich and Jose L. Salmeron analysed the evolutionary learning of FGCM in 2015 [14]. Subsequently Jose L. Salmeron and Pedro R. Palos-Sanchez studied the uncertainty propagation in FGCM With Hebbian-Like Learning Algorithms in 2017 [22]. FGCMs are used in various grey environments to study the highly uncertain systems efficiently. The FGRM tool is a combination of Grey system theory and Fuzzy Relational maps which is very much useful in highly uncertain multiple meaning environments. This tool is advantageous over regular FRMs as it includes the uncertainty and hesitancy that may arise due to shortcomings in human perception and comprehension. This fuzzy model best suited to obtain the cause and effect relationship of the problem considered for the analysis.

3.1. Dynamics of Fuzzy Grey Relational Maps

Fuzzy Grey Relational Maps (FGRM) is a fuzzy tool that models the causal relations between two disjoint set of concepts that may involve discrete, small and incomplete data [12-21, 23]. The nodes of FGRM are fuzzy and their edges are grey. The causal influence between the nodes of the domain space and the range space are given by grey weights which are interval valued numbers. The weight of each edge is given by the grey weight:

\[ \otimes w_{ij} = \left[ \frac{w_{ij}}{\overline{w_{ij}}}, \overline{w_{ij}} \right], \forall i, j w_{ij} < \frac{1}{\overline{w_{ij}}}, \left\{ w_{ij}, \overline{w_{ij}} \right\} \in [-1, +1] \]

where i and j denote the concepts in Domain Space and Range Space respectively.
The resultant inference of the one node influencing the other is given by [21]
\( \otimes C_{j}^{t+1} = f \left( (\otimes C_j^t) + \sum_{i=1}^{N} \otimes w_{ij} \cdot \otimes C_i^t \right) \)

\(= f(\otimes C^t) = f \left( \left[ C^t, \overline{C^t} \right] \right) \)

\(= \left[ f(C^t), f(\overline{C^t}) \right] = \left[ C^{t+1}, \overline{C^{t+1}} \right] \)

where \( f(\cdot) \) is the bipolar sigmoid function.

Since the concepts’ states map in the range \([-1, +1]\) the hyperbolic tangent function which is a bipolar sigmoid function is used. The component \( i \) of the state vector \( \otimes \hat{C}_{i}^{t+1} \) after the inference is denoted as follows.
\( \otimes \hat{C}_{i}^{t+1} \in \left[ e^{\lambda C^t} - e^{-\lambda C^t}, e^{\lambda C^t} + e^{-\lambda C^t} \right] \)

The output of the function is restricted to the interval \([-1,1]\). Additionally, FGRM includes greyness as an uncertainty measurement and it is computed as follows. [21]
\( \varnothing(\otimes C_i) = \frac{l(\otimes C_i)}{l(\otimes \psi)} \)

Where \( l(\otimes C_i) \) is the absolute value of the grey influence \( \otimes C_i \) state value and \( l(\otimes \psi) \) is the absolute value of the information space range denoted by \( \otimes \psi \). Higher values of greyness imply higher degree of uncertainty. FGRM maps nodes’ states within the interval \([-1, +1]\) since negative values are allowed. In this case,
\( l(\otimes \psi) = f(x) = \begin{cases} 1, & \text{if } \{ \otimes C_i, \otimes w_i \} \subseteq [0,1] \ 
\forall \otimes C_i, \otimes w_i \ 
2, & \text{if } \{ \otimes C_i, \otimes w_i \} \subseteq [-1,1] \ 
\forall \otimes C_i, \otimes w_i \end{cases} \)

The grey relational matrices obtained from different experts is converted into augmented relational matrix \( R^{Aug}(\otimes) \) which gives the improved approximation of all the experts who have studied the problem. The dynamics of FGRM is similar to that of FGCM which in turn is similar to FCM [12-21]. The initial vector that represents the initial uncertainty is also with grey values, i.e.,
\( \otimes \hat{C}_0^i = [C_0^i, \overline{C_0^i}] \). The different initial vectors interact with the augmented relational matrix \( R^{Aug}(\otimes) \).

### 3.2. Analysis of the Problem Using FGRM model.

The choice of initial vector the output that is expected from the system. The interaction of the initial Vector with the dynamic system of causal relationship results in a stable vector. The initial vector: \([0.8, 1], [0, 0], [0,0], [0,0], [0,0], [0,0], [0,0] \) is chosen to analyse the relationship between the different beliefs in the domain space and verbal intelligence \( (I_1) \) of the range space.

The FGRM method takes in the uncertainty and hesitancy present in the experts’ opinion for causal influences between concepts as well as within the initial vector. The resultant stable vector is also a grey number with reduced greyness compared to the original value [23]. The stable vector is converted into white number by taking appropriate values for \( \alpha \) in the expression \( C_i = C_n^i \alpha + (1 - \alpha) C_0^i \). Here, in this problem under study, equal mean weight is calculated by taking \( \alpha = 0.5 \). The process of finding the white numbers corresponding to the grey numbers is displayed on table-3. This table captures the causal influence of the beliefs and the verbal intelligence \( (I_1) \) of the range space. The
greyness value is the measure of uncertainty. If the white numbers for two causal associations are the same, then the number with less greyness is chosen.

Table-3: Calculation of stable vector and whitenisation process

|    | $I_1$ | Stable Vector | Length $|\mathcal{L}(\otimes C_i)|$ | Greyness $\varphi(\otimes C_i)$ | Whitenisation |
|----|-------|---------------|-------------------------------|-------------------------------|---------------|
| $B_{1+}$ | $[0.6, 0.8]$ | [0.48, 0.80] | 0.32 | 0.16 | 0.64 |
| $B_{1-}$ | $[-0.6, -0.2]$ | [-0.60, -0.16] | 0.44 | 0.22 | -0.38 |
| $B_{2+}$ | $[0.4, 0.6]$ | [0.32, 0.60] | 0.28 | 0.14 | 0.46 |
| $B_{2-}$ | $[-0.5, -0.3]$ | [-0.50, -0.24] | 0.26 | 0.13 | -0.37 |
| $B_{3+}$ | $[0.6, 0.8]$ | [0.48, 0.80] | 0.32 | 0.16 | 0.64 |
| $B_{3-}$ | $[-0.7, -0.4]$ | [-0.70, -0.32] | 0.38 | 0.19 | -0.51 |
| $B_{4+}$ | $[0.6, 0.9]$ | [0.48, 0.90] | 0.42 | 0.21 | 0.69 |
| $B_{4-}$ | $[-0.7, -0.3]$ | [-0.70, -0.24] | 0.46 | 0.23 | -0.47 |
| $B_{5+}$ | $[0.8, 0.9]$ | [0.64, 0.90] | 0.26 | 0.13 | 0.77 |
| $B_{5-}$ | $[-0.4, -0.2]$ | [-0.40, -0.16] | 0.24 | 0.12 | -0.28 |
| $B_{6+}$ | $[0.7, 0.9]$ | [0.56, 0.90] | 0.34 | 0.17 | 0.73 |
| $B_{6-}$ | $[-0.6, -0.3]$ | [-0.60, -0.24] | 0.36 | 0.18 | -0.42 |
| $B_{7+}$ | $[0.7, 0.9]$ | [0.56, 0.90] | 0.34 | 0.17 | 0.73 |
| $B_{7-}$ | $[-0.6, -0.3]$ | [-0.60, -0.24] | 0.36 | 0.18 | -0.42 |

4. Discussion

The equal mean white values of causal relationship between the personal beliefs and types of intelligence is given in table-4. From this table it is easy to compare the influence of different beliefs on different kinds of intelligence.

Table-4: Causal influences between Beliefs and Intelligence

| $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | $I_8$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| $B_{1+}$ | 0.64 | 0.51 | 0.77 | 0.51 | 0.64 | 0.73 | 0.37 | 0.46 |
| $B_{1-}$ | -0.38 | -0.32 | -0.55 | -0.28 | -0.33 | -0.37 | -0.64 | -0.37 |
| $B_{2+}$ | 0.46 | 0.47 | 0.59 | 0.73 | 0.60 | 0.73 | 0.33 | 0.46 |
| $B_{2-}$ | -0.37 | -0.40 | -0.33 | -0.46 | -0.43 | -0.28 | -0.64 | -0.51 |
| $B_{3+}$ | 0.64 | 0.60 | 0.55 | 0.51 | 0.46 | 0.64 | 0.37 | 0.60 |
| $B_{3-}$ | -0.51 | -0.43 | -0.33 | -0.42 | -0.47 | -0.37 | -0.64 | -0.55 |
| $B_{4+}$ | 0.69 | 0.60 | 0.55 | 0.64 | 0.60 | 0.69 | 0.37 | 0.51 |
| $B_{4-}$ | -0.47 | -0.51 | -0.46 | -0.51 | -0.60 | -0.33 | -0.64 | -0.46 |
| $B_{5+}$ | 0.77 | 0.64 | 0.55 | 0.51 | 0.46 | 0.69 | 0.37 | 0.42 |
| $B_{5-}$ | -0.28 | -0.33 | -0.41 | -0.33 | -0.42 | -0.32 | -0.55 | -0.42 |
| $B_{6+}$ | 0.73 | 0.64 | 0.69 | 0.51 | 0.55 | 0.73 | 0.42 | 0.51 |
| $B_{6-}$ | -0.42 | -0.56 | -0.47 | -0.42 | -0.37 | -0.33 | -0.64 | -0.42 |
| $B_{7+}$ | 0.73 | 0.64 | 0.60 | 0.64 | 0.51 | 0.73 | 0.55 | 0.46 |
| $B_{7-}$ | -0.42 | -0.46 | -0.37 | -0.42 | -0.42 | -0.23 | -0.64 | -0.51 |
5. Conclusion

The table-4 gives the estimated causal relationship between human beliefs and types of intelligence in white numbers. From this table it is inferred that the beliefs which arise from the belief-domain $B_5$ (Judgement) influence the linguistic/verbal intelligence $I_1$ very much. The primitive beliefs from the belief-domains $B_1$ (survival) and $B_2$ (security) play a significant role in promoting logical/mathematical intelligence $I_2$. Positive beliefs about one’s own survival gives them the power to visualize and foresee things in life which in turn increases their skills related to visual/spatial intelligence $I_3$. People who have positive beliefs about security of their life and existence develop bodily/kinaesthetic intelligence $I_4$. Those who feel confident about the survival of their being approach life with balanced attitude and thus develop musical/rhythmical intelligence $I_5$. All the positive beliefs have equal influence in promoting musical/rhythmical intelligence. All the positive beliefs are equally active in promoting interpersonal intelligence $I_6$. People who have reasonable level of positive beliefs seem to feel equanimous and this improves their intrapersonal intelligence $I_7$. Those who have strong positive beliefs emerging from the $B_3$ (connection) domain influence a person to have naturalistic intelligence $I_8$. The FGRM tool is very efficient in analysing the causal relationship between the disjoint sets of concepts. In this model the strength of the edges would be characterized with white, black or grey values. In order to make this system more competent we can include sophisticated training methods that would train the network to produce better and optimum results in a grey environment.

References

[1] Armstrong, T. 2014. You're smarter than you think: A kid's guide to multiple intelligences. Minneapolis, MN: Free Spirit Publishing.
[2] Armstrong, Thomas. 2009. Multiple Intelligences in the Classroom, 3rd ed. Alexandria, VA: Association for Supervision and Curriculum Development, 2009. ISBN 978-1-4166-0789-2
[3] Concepción L., Nápoles G., Bello R., Vanhoof K. 2020 On the Behavior of Fuzzy Grey Cognitive Maps. In: Bello R., Miao D., Falcon R., Nakata M., Rosete A., Ciucci D. (eds) Rough Sets. IJCRS 2020. Lecture Notes in Computer Science, vol 12179. Springer, Cham. https://doi.org/10.1007/978-3-030-52705-1_34
[4] Furnham, A., & Akanda, A. 2004. African parents' estimation of their own and their children's multiple intelligences. Current Psychology, 22(4), 281–294.
[5] Furnham, A., & Fukumoto, S. 2008. Japanese parents' estimates of their own and their children's multiple intelligences: Cultural modesty and moderate differentiation. Japanese Psychological Research, 50(2), 63–76.
[6] Furnham, A., & Wu, J. 2008. Gender differences in estimates of one's own and parental intelligence in China. Individual Differences Research, 6(1), 1–12.
[7] Sternberg, R. J. 2013. Intelligence. In D. K. Freedheim & I. B. Weiner (Eds.), Handbook of psychology: History of psychology (p. 155–176). John Wiley & Sons, Inc.
[8] Sternberg, Robert J. 1999. “The Theory of Successful Intelligence.” Review of General Psychology, 3 (4), pp. 292–316, doi:10.1037/1089-2680.3.4.292.
[9] Gardner, H. 1983. Frames of mind: The theory of multiple intelligences. New York: Basic Books.
[10] Salmeron,J.L. 2010, Modelling grey uncertainty with Fuzzy Grey Cognitive Maps, Expert Systems with Applications., 37 (12), 7581-7588. https://doi.org/10.1016/j.eswa.2010.04.085
[11] Vasantha Kandasamy, W.B., and Smarandache, F. 2004 Fuzzy Relational Maps and Neutrosophic Relational Maps, HEXIS, Church Rock.
[12] Harmati I.Á., Köczy L.T. 2020. Improvements on the Convergence and Stability of Fuzzy Grey Cognitive Maps. In: Lesot MJ. et al. (eds) Information Processing and Management of Uncertainty in Knowledge-Based Systems. IPMU 2020. Communications in Computer and
Information Science, vol 1239. Springer, Cham. https://doi.org/10.1007/978-3-030-50153-2_38

[13] Harmati, I. Á., & Kóczy, L. T. 2019. On the Convergence of Sigmoidal Fuzzy Grey Cognitive Maps, International Journal of Applied Mathematics and Computer Science, 29(3), 453-466. doi: https://doi.org/10.2478/amcs-2019-0033

[14] Papageorgiou, E. I. and J. L. Salmeron. 2012. “Learning fuzzy grey cognitive maps using nonlinear Hebbian-based approach,” Int. J. Approx. Reason., 53 (1), pp. 54–65.

[15] Salmeron J.L., Papageorgiou E.I. 2014. Using Fuzzy Grey Cognitive Maps for Industrial Processes Control. In: Papageorgiou E. (eds) Fuzzy Cognitive Maps for Applied Sciences and Engineering. Intelligent Systems Reference Library, vol 54. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-39739-4_14

[16] Salmeron, J. L. and E. I. Papageorgiou. 2012. “A fuzzy grey cognitive maps-based decision support system for radiotherapy treatment planning,” Knowl. Based Syst., vol. 30, no. 1, pp. 151–160, https://doi.org/10.1016/j.knosys.2012.01.008

[17] Salmeron, J. L. and E. I. Papageorgiou.2014. “Fuzzy grey cognitive maps and nonlinear Hebbian learning in process control,” Appl. Intell., vol. 41, no. 1, pp. 223–234.

[18] Salmeron, J. L. and P. R. Palos-Sanchez. 2019. "Uncertainty Propagation in Fuzzy Grey Cognitive Maps With Hebbian-Like Learning Algorithms," in IEEE Transactions on Cybernetics, 49 (1), pp. 211-220, doi: 10.1109/TCYB.2017.2771387.

[19] Salmeron, J.L. 2016. "An autonomous FGCM-based system for surveillance assets coordination." The Journal of Grey System, 28(1), p. 27-35.

[20] Salmeron, J.L., Gutierrez, E. 2012. Fuzzy Grey Cognitive Maps in Reliability Engineering. Applied Soft Computing 12 (12), 3818-3824. https://doi.org/10.1016/j.asoc.2012.02.003

[21] Salmeron,J.L. 2010, Modelling grey uncertainty with Fuzzy Grey Cognitive Maps, Expert Systems with Applications., 37 (12), 7581-7588. https://doi.org/10.1016/j.eswa.2010.04.085

[22] Seyed Amin Seyed Haeri., JafarRezaei. 2019. A grey-based green supplier selection model for uncertain environments, Journal of Cleaner Production, 221, 768-784, https://doi.org/10.1016/j.jclepro.2019.02.193

[23] Zanon, L. G. and L. Cesar Ribeiro Carpinetti. 2018. "Fuzzy Cognitive Maps and Grey Systems Theory in the Supply Chain Management Context: a literature review and a research proposal," 2018 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Rio de Janeiro, pp. 1-8, doi: 10.1109/ FUZZ-IEEE.2018.8491473