We investigate the Casimir pressure between two parallel plates made of magnetic materials at nonzero temperature. It is shown that for real magnetodielectric materials only the magnetic properties of ferromagnets can influence the Casimir pressure. This influence is accomplished through the contribution of the zero-frequency term of the Lifshitz formula. The possibility of the Casimir repulsion through the vacuum gap is analyzed depending on the model used for the description of the dielectric properties of the metal plates.

Keywords: magnetic materials; Lifshitz formula; plasma model; Drude model.

1. Introduction

The Casimir effect which results in a force acting between two parallel electrically neutral material plates separated with a gap of width \( a \) finds many prospective applications ranging from fundamental physics to nanotechnology. For real dissimilar material plates described by the dielectric permittivities \( \varepsilon^{(1,2)}(\omega) \) and magnetic permeabilities \( \mu^{(1,2)}(\omega) \) at temperature \( T \) in thermal equilibrium the generalized Lifshitz formula for the Casimir (van der Waals) pressure takes the form \[ P(a, T) = -\frac{k_B T}{\pi} \sum_{l=0}^{\infty} \int_{0}^{\infty} q k_{\perp} d k_{\perp} \left\{ \frac{\varepsilon^{2aq_l}}{r_{TM}^{(1)}(i\xi_l, k_{\perp}) r_{TM}^{(2)}(i\xi_l, k_{\perp})} - 1 \right\}^{-1} \]

(\xi_l = 2\pi k_B T l / \hbar \text{ with } l = 0, 1, 2, \ldots \text{ are the Matsubara frequencies.})

Here, \( k_B \) is the Boltzmann constant, prime adds a multiple one half to the term with \( l = 0 \), \( k_{\perp} \) is the projection of the wave vector on the plane of the plates, and \( \xi_l = 2\pi k_B T l / \hbar \) with \( l = 0, 1, 2, \ldots \) are the Matsubara frequencies. The reflection coefficients for the transverse magnetic and transverse electric polarizatons of the electromagnetic field are given by

\[
\begin{align*}
    r_{TM}^{(n)}(i\xi_l, k_{\perp}) &= \frac{\varepsilon_l^{(n)} q_l - k_l^{(n)}}{\varepsilon_l^{(n)} q_l + k_l^{(n)}}, \\
    r_{TE}^{(n)}(i\xi_l, k_{\perp}) &= \frac{\mu_l^{(n)} q_l - k_l^{(n)}}{\mu_l^{(n)} q_l + k_l^{(n)}},
\end{align*}
\]

(2)
where $\varepsilon^{(n)}_l \equiv \varepsilon^{(n)}(i\xi_l)$, $\mu^{(n)}_l \equiv \mu^{(n)}(i\xi_l)$, the index $n = 1$, 2 numerates the plates and

$$q_l^2 = k^2_{\perp} + \frac{\xi^2}{c^2}, \quad k^{(n)}_l = k^2_{\perp} + \varepsilon^{(n)}_l \mu^{(n)}_l \frac{\xi^2}{c^2}. \quad (3)$$

As was noticed long ago,[6] “In the majority of cases, the contribution to the van der Waals interaction due to the magnetic properties of real materials is extremely small.” A large contribution (including the Casimir repulsion through a vacuum gap for some range of parameters) was found[7] using the approximation of frequency-independent $\varepsilon$ and $\mu$. Later, however, it was shown[8] that for real materials $\mu$ is nearly equal to unity in the range of frequencies which gives a major contribution to the Casimir pressure. This problem was reconsidered[4] at both zero and nonzero temperature for one metallic and one magnetodielectric plate using the description of a metal by means of the Drude model and of magnetodielectric by a simplified model of the Drude-Lorentz type. At $T \neq 0$ the Casimir force was found to be always attractive.

In this paper we investigate the thermal Casimir pressure between plates made of ferromagnetic metal, ferromagnetic dielectric and nonmagnetic metal taking into account realistic dependences of $\varepsilon$ and $\mu$ on the frequency and using different approaches to the theory of the thermal Casimir force suggested in the literature. We demonstrate how the use of different approaches influences the Casimir pressure and find when the Casimir repulsion through a vacuum gap is feasible. In Sec. 2 we provide a brief review of magnetic properties. Sec. 3 deals with ferromagnetic metals and Sec. 4 with ferromagnetic dielectrics. In Sec. 5 we consider the behavior of the Casimir pressure in the vicinity of Curie temperature. Sec. 6 contains our conclusions.

2. Review of magnetic properties

The magnetic permeability along the imaginary frequency axis is represented in the form

$$\mu(i\xi) = 1 + 4\pi\chi(i\xi), \quad (4)$$

where $\chi(i\xi)$ is the magnetic susceptibility. The magnitude of $\chi(i\xi)$ decreases monotonously when $\xi$ increases. All materials possess diamagnetic polarization for which[6][7] $\chi(0) < 0$, $\mu(0) < 1$ and $|\mu(0) - 1| \sim 10^{-5}$. Diamagnets (such materials as, for instance, Au, Si, Cu and Ag) do not possess any other type of magnetic polarization. For them one can put $\mu_l = 1$, $l = 0, 1, 2, \ldots$ in computations using[11] so that magnetic properties of diamagnets do not influence the Casimir force.

Some materials also possess paramagnetic polarization (in a broad sense) which is larger in magnitude than the diamagnetic one and leads to[11] $\chi(0) > 0$, $\mu(0) > 1$. Paramagnets (in a narrow sense) are materials with $\mu(0) > 1$ if the interaction of magnetic moments of their constituent particles is negligibly small. Paramagnets may consist of microparticles which are paramagnetic magnetizable but
have no intrinsic magnetic moment (the Van Vleck polarization paramagnetism)\(^{12}\) and of microparticles possessing a permanent magnetic moment (the orientational paramagnetism)\(^{9–12}\). For all paramagnets in a narrow sense it is true that \(\chi(0) < 10^{-4}\) and one can put \(\mu_l = 1\) for all \(l\). This conclusion is unchanged for all paramagnets in a broad sense (with the single exception of ferromagnets) because \(\chi(0)\) remains as small as mentioned above and takes only a slightly larger values in the vicinity of \(T = 0\) even at temperatures below the critical temperature \(T_{cr}\) of the magnetic phase transitions\(^{9–11,13–15}\) (for different materials \(T_{cr}\) varies from a few K to more than thousand K).

For the subdivision of paramagnetic materials called ferromagnets it is true that \(\mu(0) \gg 1\) at \(T < T_{cr}\) (in this case \(T_{cr}\) is referred to as the Curie temperature, \(T_{cr} \equiv T_C\)). There is a lot of ferromagnetic materials, both metals and dielectrics\(^{16}\). The rate of decrease of \(\mu(i\xi)\) for ferromagnets depends on their electric resistance. Thus, for ferromagnetic metals and dielectrics \(\mu(i\xi)\) becomes approximately equal to unity for \(\xi\) above \(10^4\) and \(10^9\) Hz, respectively. Keeping in mind that the first Matsubara frequency \(\xi_1 \sim 10^{14}\) Hz at \(T = 300\) K we arrive at the conclusion that ferromagnets can affect the Casimir force between macroscopic bodies only through the contribution of the zero-frequency term of the Lifshitz formula (1). In all terms of this formula with \(l \geq 1\) one can put \(\mu_l = 1\). Note that below we do not consider so-called hard ferromagnetic materials possessing a spontaneous magnetization because the magnetic interaction between the plates made of such materials far exceeds any conceivable Casimir force. The subject of our interest is the soft ferromagnetic materials which do not possess a spontaneous magnetization. It is well known that the magnetic permeability of ferromagnets depends on the applied magnetic field.\(^{9–11}\) Since in the Casimir interaction the mean applied field is equal to zero, below we consider the so-called initial (zero field) permeability, i.e., \(\mu = \mu(H = 0)\).

3. Ferromagnetic metals

First we consider the case when both Casimir plates are made of common ferromagnetic metal Co, Cd, Fe or Ni. The dielectric properties of a metal are described by the Drude\(^{17,18}\) or the plasma\(^{19,20}\) model approaches, i.e., using the dielectric functions of the form

\[
\varepsilon_D(i\xi) = 1 + \frac{\omega_p^2}{\xi(\xi + \gamma)}, \quad \varepsilon_D(i\xi) = 1 + \frac{\omega_p^2}{\xi^2}, \quad (5)
\]

where \(\omega_p\) is the plasma frequency, \(\gamma\) is the relaxation parameter. By considering different models proposed in the literature we aim to determine whether or not the magnetic properties influence the magnitude of the Casimir pressure and is it possible to experimentally distinguish between alternative theoretical predictions. For all \(l \geq 1\) we put \(\mu_l = 1\).

For two similar plates Eq. (2) leads to the following reflection coefficients at
\( \xi = 0 \) if the Drude and plasma models are used

\[
\begin{align*}
\tau_{TM,D}(0, k_{\perp}) &= \tau_{TM,p}(0, k_{\perp}) = 1, \\
\tau_{TE,D}(0, k_{\perp}) &= \frac{\mu(0) - 1}{\mu(0) + 1} = r_\mu, \\
\tau_{TE,p}(0, k_{\perp}) &= \frac{\mu(0)ck_{\perp} - [c^2k_{\perp}^2 + \mu(0)\omega_p^2]^{1/2}}{\mu(0)ck_{\perp} + [c^2k_{\perp}^2 + \mu(0)\omega_p^2]^{1/2}}.
\end{align*}
\]

In the limiting case of large separations (high \( T \)) only the zero-frequency term in Eq. (1) contributes to the Casimir pressure and all calculations can be performed analytically. When the Drude model is used, the result is

\[
P_D(a, T) = -\frac{k_B T}{8\pi a^3} \left[ \zeta(3) + \text{Li}_3(r_\mu^2) \right],
\]

where \( \zeta(z) \) is the Riemann zeta function and \( \text{Li}_n(z) \) is polylogarithm function. Under the conditions \( \mu(0) \gg 1 \) (valid for ferromagnetic metals) and \( \mu(0) = 1 \) (valid for nonmagnetic metals) Eq. (7) leads to

\[
P_{D, fm}(a, T) = -\frac{k_B T}{4\pi a^3} \zeta(3), \quad P_{D, nm}(a, T) = -\frac{k_B T}{8\pi a^3} \zeta(3),
\]

respectively. As can be seen from Eq. (8), if the Drude model is used, the account of magnetic properties of ferromagnetic metals doubles the magnitude of the Casimir pressure at large separations. If, however, the plasma model is used at large separations under the condition \( \sqrt{\mu(0)}\delta_0/a \ll 1 \), Eq. (1) results in

\[
P_{p, fm}(a, T) = -\frac{k_B T}{4\pi a^3} \zeta(3) \left[ 1 - 3\sqrt{\mu(0)}\frac{\delta_0}{a} \right],
\]

where \( \delta_0 = c/\omega_p \) is the skin depth. The same expression, but with \( \mu(0) = 1 \), is obtained for nonmagnetic metals described by the plasma model. At \( T = 300 \text{ K} \) Eqs. (7)–(9) are applicable for \( a > 6 \mu m \).

From an experimental point of view the most interesting region is from \( a = 0.5 \mu m \) to \( a = 1 \mu m \) (for \( a < 0.5 \mu m \) the contribution of the zero-frequency term and, thus, of magnetic properties is not large enough). We have performed numerical

Fig. 1. The relative Casimir pressure as a function of separation in the configuration of two parallel Co plates with inclusion of magnetic properties (the solid lines) and with magnetic properties neglected (the dashed lines). Computations are performed with the dielectric permittivity (a) of the Drude model and (b) of the plasma model.
computations of the Casimir pressure, Eq. (1), in the region from 0.5 to 6 µm for Co with parameters $\omega_{p,Co} = 3.97$ eV, $\gamma_{Co} = 0.036$ eV and $\mu_{Co}(0) = 70$. In Fig. 1 we plot the ratio of the Casimir pressure $P$ between two Co plates at $T = 300$ K to $P_0 = -\pi^2\hbar c/(240a^4)$ computed using (a) the Drude model and (b) the plasma model. The solid lines take into account the magnetic properties and the dashed lines are computed with magnetic properties neglected. Note that the solid line in Fig. 1(a) is almost coincident with the dashed line in Fig. 1(b). At small separations $a < 1$ µm, the difference between the dashed line in Fig. 1(a) and the solid line in Fig. 1(b) is also not observable in the limits of the experimental precision. Thus the experiments on an indirect measurement of the Casimir pressure by means of a micromechanical oscillator at separations of about 500–600 nm can allow us to choose one of the following situations.

1. The experimental data are in favour of the solid line in Fig. 1(a) and the dashed line in Fig. 1(b). This means that either the magnetic properties affect the Casimir pressure and metals should be described by the Drude model or the magnetic properties do not affect the Casimir pressure and metals should be described by the plasma model.

2. The experimental data are in favour of the dashed line in Fig. 1(a) and the solid line in Fig. 1(b). In this case either the magnetic properties affect the Casimir pressure and metals should be described by the plasma model or the magnetic properties do not affect the Casimir pressure and metals should be described by the Drude model.

Now let one plate be made of a ferromagnetic metal ($n = 1$) and the other of a nonmagnetic metal ($n = 2$). Here, in the limit of large separations one obtains

$$P_D(a, T) = -\frac{k_B T}{8\pi a^3} \zeta(3),$$

if the Drude model is used. For the plasma model under conditions $\sqrt{\mu(0)}\delta_{01}/a \ll 1$ and $\delta_{02}/a \ll 1$ it follows

$$P_p(a, T) = -\frac{k_B T}{4\pi a^3} \zeta(3) \left[ 1 - \frac{3(\sqrt{\mu(0)}\delta_{01} + \delta_{02})}{2a} \right].$$

Note than when the Drude model is used $r^{(2)}_{TE}(0, k_{\perp}) = 0$ and, thus, the magnetic properties of a ferromagnetic plate entering only through $r^{(1)}_{TE}(0, k_{\perp})$ do not influence the result. The results of the numerical computations for the Co plate interacting with the Au plate ($\omega_{p,Au} = 9.0$ eV, $\gamma_{Au} = 0.035$ eV) in the case when the plasma model is used are shown in Fig. 2(a). It is seen that here the inclusion of the magnetic properties (the solid line) decreases the magnitude of the Casimir pressure. The influence of the magnetic properties is, however, very moderate and can be observed only in the experiment on measuring the difference Casimir pressure above a patterned plate, one section of which is made of Co and the other of Au. Such an experiment allows one to choose between the two alternatives in each of the situations described above. This will provide a complete experimental answer to
Fig. 2. The relative Casimir pressure as a function of separation in the configuration of two parallel plates, one made of Au and the other of (a) Co and (b) ferromagnetic dielectric with inclusion of magnetic properties (the solid lines) and with magnetic properties neglected (the dashed lines). Computations are performed using the plasma model for the dielectric permittivity of metals.

questions whether the magnetic properties influence the Casimir force and what dielectric model should be used in the Lifshitz theory to describe real metals.

4. Ferromagnetic dielectrics

Ferromagnetic dielectrics are materials that, while displaying physical properties characteristic of dielectrics, show ferromagnetic behavior under the influence of an external magnetic field. Such materials are widely used in different magneto-optical devices. As an example, we consider a composite material of polystyrene with a volume fraction \( f \) of ferromagnetic metal nanoparticles in the mixture. The permittivity of such a material can be presented in the form:

\[
\varepsilon_{id}(i\xi) = \varepsilon_{d}(i\xi) \left(1 + \frac{3f}{1-f}\right),
\]

where \( \varepsilon_{d} \) is the permittivity of polystyrene.

We have performed computations of the Casimir pressure for two parallel plates one of which is made of ferromagnetic dielectric \([f = 0.25, \varepsilon_{id}(0) = 5.12, \mu(0) = 25]\) and the other of Au described by the plasma model. Recall that if Au is described by the Drude model the magnetic properties do not influence the Casimir pressure as explained in Sec. 3. The computational results for \( P/P_0 \) as a function of \( a \) are presented in Fig. 2(b) where the solid line takes the magnetic properties into account and the dashed line neglects them. As can be seen in Fig. 2(b), magnetic properties have an important influence on the Casimir pressure and even lead to the change of sign of the force (from attraction to repulsion).

This important conclusion can be confirmed analytically in the limiting case of large \( a \). If the metallic properties of Au plate are described by the Drude model, one obtains

\[
P_D(a, T) = -\frac{k_B T}{8\pi a^3} \text{Li}_3(r_\varepsilon), \quad r_\varepsilon = \frac{\varepsilon_{id} - 1}{\varepsilon_{id} + 1}.
\]
Fig. 3. (a) The static magnetic permeability of Gd at the magnetic phase transition as a function of temperature. (b) The relative Casimir pressure as a function of temperature in the configuration of two parallel Gd plates at the separation $a = 0.5 \, \mu \text{m}$. The solid and dashed lines include and neglect the magnetic properties, respectively. The pairs of lines marked 1 and 2 indicate the respective computational results obtained using the Drude and the plasma models.

This does not depend on the magnetic properties. If, however, the plasma model is used, then, under the condition $\delta_{02}/a \ll 1$, one arrives at

$$P_{\mu}(a, T) = -\frac{k_B T}{8\pi a^3} \left[ \text{Li}_3(r_\varepsilon) + \text{Li}_3(-r_\mu) \left( 1 - 3\frac{\delta_{02}}{a} \right) \right].$$  \hspace{1cm} (14)

The expression on the right-hand side of Eq. (14) is positive and the respective Casimir force is repulsive if the following condition is satisfied:

$$\text{Li}_3(r_\varepsilon) < \left| \text{Li}_3(-r_\mu) \left( 1 - 3\frac{\delta_{02}}{a} \right) \right|. \hspace{1cm} (15)$$

This condition is easily satisfied for real materials.

5. Vicinity of the Curie temperature

At the Curie temperature $T_C$, specific for each material, ferromagnets undergo a magnetic phase transition. At higher temperature they lose ferromagnetic properties and become paramagnets in the narrow sense. Thus, for Fe, Co, Ni and Gd the Curie temperature is equal to 1043 K, 1388 K, 627 K and 293 K, respectively. Here, we consider the behavior of the Casimir pressure under the magnetic phase transition which occurs with the increase of $T$ in the configuration of two similar plates made of Gd. The Drude parameters of Gd are equal to $\omega_p, \text{Gd} = 9.1 \, \text{eV}$, $\gamma_{\text{Gd}} = 0.58 \, \text{eV}$.

Computations of the Casimir pressure between two parallel plates made of Gd in the vicinity of the Curie temperature require respective values of $\mu(0)$ for Gd at $T < T_C$ [at $T > T_C$, $\mu_{\text{Gd}}(0) = 1$ to high accuracy]. In Fig. 3(a) the magnetic permeability of Gd is shown as a function of temperature in the region from 280 K to 300 K on the basis of the experimental data. The Casimir pressure as a function of temperature was computed at the separation $a = 500 \, \text{nm}$ between the plates using Eq. (1). The computational results obtained using the Drude and the plasma models are shown in Fig. 3(b) by the pairs of lines 1 and 2, respectively. In each pair the
solid line takes into account the magnetic properties and the dashed line is computed with these properties disregarded. As can be seen from Fig. 3b, experiments on the magnetic phase transition can also be used to determine the influence of magnetic properties on the Casimir force and as a test for different models of the dielectric properties of metals.

6. Conclusions

The investigation of the influence of magnetic properties on the Casimir force performed above leads to the following conclusions.

1. Of all the real materials, only ferromagnets might affect the Casimir force.
2. At all feasible temperatures the possible influence of ferromagnets on the Casimir force occurs solely through the contribution of the zero-frequency term in the Lifshitz formula.
3. In the framework of the Lifshitz theory the Casimir repulsion of two macroscopic bodies separated by a vacuum gap arises for only the case when one body is made of ferromagnetic dielectric and the other is metallic. In doing so the metal is described by the plasma model.
4. Modern experimental techniques present good opportunities to check whether the magnetic properties of the plate material influence the Casimir force. Experiments with magnetic bodies allow independent test of the plasma and Drude model approaches to the description of the dielectric properties of metals.

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