Can particle-creation phenomena replace dark energy?

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Abstract

Particle creation at the expense of the gravitational field might be sufficient to explain the cosmic evolution history, without the need of dark energy at all. This phenomena has been investigated in a recent work by Lima et al (Class. Quantum Grav. 2008 25 205006) assuming particle creation at the cost of gravitational energy in the late Universe. However, the model does not satisfy the WMAP constraint on the matter-radiation equality (Steigman et al 2009 J. Cosmol. Astropart. Phys. JCAP06(2009)033). Here, we have suggested a model, in the same framework, which fits perfectly with SNIa data at low redshift as well as an early integrated Sachs–Wolfe effect on the matter-radiation equality determined by WMAP at high redshift. Such a model requires the presence of nearly 26% primeval matter in the form of baryons and cold dark matter.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Recently released 7 year WMAP data [1, 2] have found no trace of deviation from the standard \( \Lambda \)-cold dark matter (\( \Lambda \)CDM) model. But the problem in connection with the cosmological constant remains unresolved. The vacuum energy density, as calculated by the field theorists, is some \( 10^{120} \) order of magnitude greater than the cosmological constant \( \Lambda \) required by the cosmologists to explain late time cosmic acceleration, which is of the order of \( H_0^2 \), \( H_0 \) being the present Hubble parameter. So far, many alternatives to the standard \( \Lambda \)CDM model have been proposed and explored and as a matter of fact, all of these models have been found suitable to explain late time cosmic acceleration. The problems associated with these models are that they cannot be distinguished from the standard \( \Lambda \)CDM model on one hand, and most of them are not suitable to explain the early Universe on the other. However, the \( \Lambda \)CDM model
requires 26% of matter in the form of pressureless dust, out of which only 4% are baryons and the rest, about 22%, are CDM. Since dark matter interacts only with the gravitational field, it plays a key role in the structure formation. The gravitational Jeans instability allows compact structures to form and is not opposed by any force such as radiation pressure in the case of dark matter. As a result, dark matter begins to collapse into a complex network of dark matter Halos, well before ordinary baryonic matter, which is impeded by pressure force. Without dark matter the epoch of Galaxy formation would have occurred at a substantially later stage, than observed. Thus, the amount of CDM (22%) must have been created in the very early Universe, prior to the radiation-dominated era together with the baryons, by some sort of mechanism, namely supersymmetry breaking, cosmic string decay or particle creation at the expense of the gravitational field. These particles are usually supposed to be weakly interacting massive particles (WIMP). For example, as a heavy stable particle, the lightest neutralino is an excellent candidate to comprise the Universe’s CDM. In many models (see [3] for a nice review) the lightest neutralino can be produced in the hot early Universe and leave approximately the right relic abundance to account for the observed dark matter, i.e. 22% as required by the $\Lambda$CDM model. Now, if phenomenologically one considers that CDM may also be produced by gravitational particle-creation mechanism, even at a very slow rate, during the late time evolution of the Universe, namely during the matter dominated era, as considered by Lima et al [5], then it may be possible to explain the presently observable acceleration of the Universe, without taking dark energy into account. The very advantage of the creation of CDM over dark energy is that it avoids coincidence problem and also may be detectable in future experiments. In this work, our focus is on the cosmological consequences of particle production on the evolution of late stage of the Universe which was initiated recently by Alcaniz and Lima [4] and Lima, Silva and Santos (lss) [5].

We remember that in the 1980s, the motivation that initiated to go after the inflationary scenario was the fundamental three problems associated with the Friedmann model, namely the horizon, the flatness and the observed isotropy and homogeneity in the cosmic microwave background radiation (CMBR). There was an additional problem in the form of huge entropy per baryon in the observable Universe ($\sim 10^{87}$). Einstein’s equations are purely adiabatic and reversible. Consequently, these equations can hardly provide, by themselves, an explanation relating to the origin of cosmological entropy. This problem was resolved [6] by taking into account the cosmological consequence of irreversible particle-creation phenomena in the framework of Einstein’s equation, classically.

Particle-creation phenomena were explored largely during the last century to explain the early Universe. Matter constituents may be produced quantum mechanically [7–9] in the framework of Einstein’s equations. Cosmological consequence of the particle-creation mechanism is studied taking into account an explicit phenomenological balance law (see the appendix) for the particle number [6, 10, 11] in addition to the familiar Einstein’s equations. In view of such a balance law, Prigogine et al [6] successfully explained the cosmological evolution of the early Universe.

Recently, Lima et al [5] have developed a late time model Universe taking into account the creation phenomena in the matter dominated era. The model admits early deceleration followed by a recent acceleration of the Universe as suggested by present observations and fits SNIa data to some extent. However, they [5] have shown that in their model the creation phenomena are never ending; as a result, cosmic evolution does not ever include the standard radiation-dominated or the matter-dominated Friedmann era. This definitely creates problem in explaining the structure formation of the Universe and the CMBR. Later, Steigman et al [12] analysed the model in the two limits of high and low redshifts. They have observed a clear conflict between the WMAP constraint on the matter-radiation equality $\tilde{z}_{eq}$ at high redshift
and SNIa data at low redshift. The main criticisms of the $\beta-\gamma$ model proposed by Lima et al [5] are that they have not taken into account the amount of CDM created in the very early Universe at one end, and that their creation rate $\Gamma = 3\beta H + 3\gamma H_0$ depends on the present Hubble parameter on the other. If these problems are alleviated, then the phenomenologically particle-creation process obviously unifies early inflation with late stage of cosmic acceleration in an elegant fashion.

In this work, we propose a model where, instead of choosing the creation parameter $\Gamma$ arbitrarily, we have considered the experimentally verified fact that the Universe has recently entered an accelerated phase of expansion. As a result, we have chosen the scale factor judiciously, such that particle creation could start again in the matter-dominated era. This naturally alleviates the said problems and unifies the early and the late stages of cosmic evolution in an elegant fashion. Additionally, the model fits perfectly with the WMAP constraint on the matter-radiation equality $z_{eq}$ only if one considers the presence of nearly 26% of primeval matter in the form of baryons and CDM. In view of such a model, particle-creation phenomena are now able to explain the history of cosmic evolution from the very early Universe till date, without requiring dark energy at any stage and thus avoiding the coincidence problem.

In the following section we write down the field equations incorporating the phenomena of particle creation, which is apparent through the balance equation. In section 3, we show how the conflict between the high redshift and the low redshift data may be reduced in the model presented by Lima et al [5] just by accounting for some amount of CDM produced in the very early Universe. However, we also mention some more problems which are still associated with their model [5]. In section 4, instead of choosing a form of the creation rate arbitrarily, we rather choose a form of the scale factor, suitable for a transition from the early deceleration to the late time cosmic acceleration, to understand the associated problems. This gives us insight to find a form of the creation rate $\Gamma$ associated with the so-called intermediate inflation [13]. Such a form of $\Gamma$ appears to be much too elegant to study the cosmological evolution alleviating all the problems discussed. This has been done in section 5. Finally, we end up with the conclusion in section 6 and an appendix in section 7 to calculate the balance law.

2. Balance law and the field equations

As mentioned in the introduction, cosmological consequence taking into account particle-creation phenomena is studied by using an explicit phenomenological balance law for the particle number [6, 10]. Such a balance law in the process of particle production is modelled by $(nu^a)_{;a} = \Psi$, with a source term $\Psi$. For a vanishing $\Psi$ the particle number is conserved. In the isotropic and homogeneous background metric,

$$ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right],$$  

the above balance law reduces to $\frac{\dot{N}}{N} = \frac{\dot{u}}{u} + \Theta = \frac{\Psi}{N} = \Gamma$, where, $u^a, N, n$ and $H$ are the fluid four velocity vector, the total number of particles, particle number density and the Hubble parameter respectively, while $\Theta = u^a_{;a} = 3H$ is the expansion scalar. Thus, the field equations in the spatially flat ($k = 0$) Robertson–Walker metric (1) associated with particle-creation phenomena are

$$2\dot{H} + 3H^2 = -8\pi G(p_m + p_{cm}),$$

$$3H^2 = 8\pi G(\rho_m + \rho_{cm}) = 8\pi G\rho,$$
\[
\frac{3H + \dot{n}}{n} = \Gamma = \frac{\Psi}{n},
\]

\[
p_{cm} = -\frac{\rho + \rho_m}{3H} \Gamma.
\]

For the last equation (5) please see the appendix. In the above set of equations, \(H = \frac{\dot{a}}{a}\) is the Hubble parameter, \(p_m\) and \(\rho_m\) are the pressure and the energy density, respectively, of the matter existing in the Universe in the form of a barotropic fluid containing baryons and cold dark matter, created in the very early Universe. \(p_{cm}\) and \(\rho_{cm}\), respectively, are the pressure and the energy density of the CDM in the form of WIMP created at the late stage of cosmic evolution, i.e. during the matter-dominated era. \(\Gamma\) is the creation parameter, and \(n\) is the particle number density. It has been shown in the appendix that the second law of thermodynamics allows the creation of particle from the gravitational field and the process is irreversible. Thus, the creation parameter \(\Gamma > 0\), and so it is clear from equation (5) that the particle-creation phenomena is always associated with a negative pressure \(p_{cm}\), which may be responsible for acceleration at the late stage of cosmic evolution. We would like to mention at this stage that, other than CDM and baryons, different types of particles may be created in view of quantum field theory in curved spacetime [7–9] and all are associated with a negative pressure, as mentioned, and also, as mentioned in the introduction, that the structure formation requires 22\% that are constituted by dark matter, which must have been created in the early Universe. The present estimated amount of baryons is 4\% and the rest amount required for present cosmic acceleration is usually treated as dark energy. The other form of matter (like hot dark matter, say) has negligible contribution. Here, we proceed to show that creation of the same amount of CDM also solves the puzzle. Further note that the creation of baryons and CDM in the early Universe was also associated with a large negative pressure. However, the creation phenomena weakened and finally stopped, when the Universe had expanded sufficiently, thereby giving way to the hot big bang followed by the radiation-dominated era of Friedmann type (\(a \propto t^{1/2}\)) [6]. Thereafter, the baryons and the CDM created in the very early Universe obviously start acting as pressureless dust, so that we can take \(p_m = 0\) and hence \(\rho_m = \rho_0 a^{-3}\), \(\rho_0\) being a constant. Thus, we can simplify the above equations to obtain

\[
\Gamma = 3H + \frac{2}{H} = 3H \left( \frac{2H + 3H^2}{3H^2} \right) = -3H w_e,
\]

\[
\rho_{cm} = \frac{3H^2}{8\pi G} - \rho_0 a^{-3},
\]

\[
p_{cm} = -\frac{1}{8\pi G} HT\Gamma,
\]

\(w_e\) being the effective state parameter. In view of the above set of three equations (6)–(8), we need to find the scale factor \((a)\) (and consequently \((H)\), the Hubble parameter), the creation rate \(\Gamma\), the creation pressure \(p_{cm}\) and the creation matter density \(\rho_{cm}\). Obviously, we need yet another suitable condition to solve the system of equations. Lima et al [5] studied these equations under the assumption of a form of the creation rate \(\Gamma\). Afterwards, while further studying their model in connection with data fitting, they found a clear conflict between the low (SNIa) and high (WMAP constraint on \(z_{eq}\)) [12] redshift limits. In the following section, we review the problem and show how the production of CDM in the very early Universe alleviates the problem.
3. A brief review of the lss model

The Friedmann equation, taking into account the created matter, baryonic matter and radiation, reads

\[
\left( \frac{H}{H_0} \right)^2 = \Omega_r (1 + z)^4 + \Omega_b (1 + z)^3 + \frac{\rho_{cm}}{\rho_c},
\]

where \( \rho_c \) is the present value of critical density. To calculate the last term let us take the total number of created particles at an instant to be \( N = nV \), where \( V = V_0(1 + z)^{-3} \) is the comoving volume at that instant. Thus, the creation rate is given as

\[
\frac{1}{N} \frac{dN}{dt} = \frac{d[\ln(\rho_{cm}V)]}{dt} = \Gamma,
\]

which yields

\[
\rho_{cm} = \rho_{cm0}(1 + z)^3 \exp \left( - \int_{t_0}^t \Gamma \, dt' \right),
\]

where \( \rho_{cm0} \) is the present value of the created matter density. So, the Friedmann equation (9) finally reads

\[
\left( \frac{H}{H_0} \right)^2 = \Omega_r (1 + z)^4 + \Omega_b (1 + z)^3 + \Omega_{cm}(1 + z)^3 \exp \left( - \int_{t_0}^t \Gamma \, dt' \right).
\]

Now, under the assumption \( \Gamma = 3\beta H + 3\gamma H_0 \), where \( \beta \) and \( \gamma \) are constants and \( H_0 \) is the present Hubble parameter, lss [5] obtained a solution of the scale factor in the form

\[
a(t) = a_0 \left[ 1 - \gamma - \beta (e^{\frac{\gamma(t)}{\beta}} - 1) \right] \frac{1}{\sqrt{1 - \gamma}}.
\]

which admits the observed transition from early deceleration to late time acceleration. In a later investigation [12], this model was found to produce a conflict between SNIa data at low redshift and the WMAP 5 year data constraint [14] on the matter-radiation equality \( z_{eq} = 3141 \pm 157 \), occurred at the high redshift limit of the observed ISW effect. Let us review the situation to find the real problem associated with the conflict. The Friedmann equation (13) in the model under consideration reads

\[
\left( \frac{H}{H_0} \right)^2 = \Omega_r (1 + z)^4 + \Omega_b (1 + z)^3 + \Omega_{cm}(1 + z)^{3(1-\beta)} \exp 3\gamma(\tau - \tau_0),
\]

where \( \tau = H_0t \) and \( \tau_0 = H_0t_0 \) are the age at any instant and the present age of the Universe, respectively, in the units of Hubble’s age (\( H_0^{-1} \)). Setting \( \Omega_{cm} = 1 - \Omega_B \), the \( \gamma - \beta \) relation is obtained (see equation (34) in [12]) as

\[
\gamma = (1 - \beta)[(1 - \Omega_b)^2 - \left( \Omega_r (1 + z_{eq}) - \Omega_b \right)^{\frac{3}{2}}(1 + z_{eq})^{\frac{3}{2}}].
\]

This model fits SNIa data for \( \beta = 0 \) and \( \gamma = 0.66 \pm 0.04 \), while \( 1 + z_{eq} = 1798_{-556}^{+536} \), taking \( \Omega_B = 0.042 \). Clearly, the model does not fit with the WMAP 5 year data constraint [14] on the mater-radiation equality \( z_{eq} = 3141 \pm 157 \), occurred at the high redshift limit of the observed ISW effect. This contradiction may be alleviated easily, if we consider the existence of CDM that was created in the early Universe and which was responsible for inflation. As already mentioned, this amount of CDM created in the very early Universe behaves now as pressureless
Figure 1. Distance modulus $(M - m)$ versus the redshift $z$ plot of the present model (blue) shows a perfect fit with the $\Lambda\text{CDM}$ model (red) for $\alpha = 4$. In fact, for $\alpha > 1.5$ the fit is perfect and it remains so up to $\alpha = 200$.

Dust and has been redshifted like baryons. If we now add the corresponding density parameter $\Omega_{\text{CDM}}$, associated with the CDM created in the very early Universe in equation (13), it reads

$$\left(\frac{H}{H_0}\right)^2 = \Omega_r (1 + z)^4 + \Omega_m (1 + z)^3 + \Omega_{\text{cm}} (1 + z)^3 \exp\left(-\int_{0}^{t} \Gamma \, dt'\right), \quad (16)$$

where $\Omega_m = \Omega_B + \Omega_{\text{CDM}}$ and $\Omega_{\text{cm}} = 1 - \Omega_m$. In the absence of matter creation phenomena in the late Universe, $\beta, \gamma$ vanish, and hence $\Gamma = 0$. Thus, there is no creation pressure $p_m$ as well as creation matter density $\rho_{\text{cm}}$. Hence, $\Omega_{\text{cm}} = 0$. Thus, at the matter-radiation equality ($z_{eq} = \frac{\Omega_r}{\Omega_m} - 1$), taking $\Omega_m = \Omega_B + \Omega_{\text{CDM}} = 0.26$ and $\Omega_r = 8 \times 10^{-5}$, one recovers $z_{eq} = 3249$, which is at par with WMAP data. Equation (15) now takes the form

$$\gamma = (1 - \beta)[(1 - \Omega_m)^{1/2} - \{\Omega_r (1 + z_{eq}) - \Omega_m\}^{1/2}(1 + z_{eq})^{3/2}] \quad (17)$$

If we now consider that 16% of CDM (say) were produced in the very early Universe, then $\Omega_m = \Omega_B + \Omega_{\text{CDM}} = 0.2$ and thus for $\beta = 0$ and $\gamma = 0.66 \pm 0.04$, $z_{eq} = 3186^{+254}_{-214}$, which is very much at par with WMAP data [1, 2, 14]. This clearly indicates that one should include the contribution of CDM created at the very early Universe. The creation of CDM in the very early Universe, as mentioned in the introduction, was halted and the Universe entered the usual Friedmann radiation-dominated era. Thereafter, this amount of CDM is being redshifted like baryons.

In the above analysis, while we have showed how the conflict encountered between low and high redshift data [12] may be reduced, nevertheless it does not support the model [5]. First, in their $\beta-\gamma$ model, $\beta = 0$, somehow fits SNIa data, which is not a very good fit at all (see [12], figure 1). Furthermore, $\beta = 0$ turns out to give a constant creation rate throughout the evolution of the Universe, which is highly objectionable. Also, the choice of the creation parameter ($\Gamma$) as a function of the present Hubble parameter ($H_0$) implies that the model is plagued by the coincidence problem. Finally, we could accommodate only 16% of CDM out of 22% to alleviate the conflict [12]. Addition of another 6% of CDM shifts $z_{eq}$ to a much higher value. In view of the above criticism we pose to present a more realistic model.
4. Case 1

To get an explicit solution of the field equations (6) through (8), we need yet another physically reasonable assumption. However, we really have no idea of the rate of matter creation $\Gamma$ either from the quantum field theoretical (QFT in CST) or from the classical kinetic approach. Nevertheless, it is clear from equation (5) that such phenomena are associated with a negative pressure $p_{cm}$. If the creation pressure is sufficiently negative, it might lead to an accelerating phase of cosmological evolution. Additionally, in order to get an idea about the form of the creation rate we can also depend on the presently available cosmological data and the best fit models. Since the $\Lambda$CDM model has excellent fit with the SNIa and WMAP data, we can infer certain important aspects of cosmological evolution. First of these is definitely that the Universe has encountered a transition from early deceleration to late time acceleration. Next is that the early growth of perturbation should track the $\Lambda$CDM model closely. These facts allow us to choose a suitable form of the scale factor at par with the present experimental results and in the process we expect to get an idea on the form of $\Gamma$. In view of the two aspects of the late time cosmological evolution just discussed, we choose the scale factor in the matter-dominated era as

$$a = a_0 t^\frac{4}{3} + b_0 t^\alpha = b_0(qt^\frac{4}{3} + t^\alpha),$$  \hspace{1cm} (18)

where $a_0$, $b_0$ and $\alpha > 1$ are constants with $q = \frac{a_0}{b_0} > 0$. With such a form of the scale factor, the first term of equation (18) plays the leading role in the early stage of cosmological evolution in the matter-dominated era and the Universe tracks the Friedmann model $a \propto t^\frac{2}{3}$ closely. Thus, the growth of perturbation in connection with the structure formation tracks the $\Lambda$CDM model closely. The second term appears in connection with the creation of matter, which is associated with a negative creation pressure. As $t$ increases, the second term starts playing a significant role and it starts dominating as $t^{\alpha - 2/3} > q$, which implies that the creation phenomena starts rather late. Finally, since $\ddot{a} = b_0[-\frac{2q}{9t^2} + \alpha(\alpha - 1)t^{\alpha - 2}],$ acceleration ($\ddot{a} > 0$) starts, only when $t > \left[\frac{2q}{9t^2(\alpha - 1)}\right]^\frac{1}{\alpha - 2}$. Thus, equation (18) clearly depicts early deceleration and late time acceleration of the Universe. Now, in the matter-dominated era, $p_m = 0$, the solutions in view of the chosen form of the scale factor (18) are

$$H = \frac{\frac{2}{3}qt^{-\frac{1}{3}} + \alpha t^{\alpha - 1}}{qt^\frac{4}{3} + t^\alpha}. \hspace{1cm} (19)$$

$$\Gamma = -3Hw_e = 2\frac{H}{H} + 3H = \frac{(3\alpha - 2)(2(3\alpha + 1)qt^\frac{4}{3} + 9\alpha t^\alpha)t^{(\alpha - 1)}}{3(qt^\frac{4}{3} + t^\alpha)(2qt^\frac{4}{3} + 3\alpha t^\alpha)}. \hspace{1cm} (20)$$

$$p_{cm} = -\frac{1}{8\pi G}\Gamma H = -\frac{(3\alpha - 2)(2(3\alpha + 1)qt^\frac{4}{3} + 9\alpha t^\alpha)t^{(\alpha - 2)}}{72\pi G(qt^\frac{4}{3} + t^\alpha)^2}. \hspace{1cm} (21)$$

$$\rho = \rho_m + \rho_{cm} = \frac{3H^2}{8\pi G} = \frac{(2qt^\frac{4}{3} + 3\alpha t^\alpha)^2}{24\pi Gt^\frac{4}{3}(qt^\frac{4}{3} + t^\alpha)^2}. \hspace{1cm} (22)$$

$$w_e = -\frac{2\dot{H} + 3H^2}{3H^2} = -\frac{(3\alpha - 2)(2(3\alpha + 1)qt^\frac{4}{3} + 9\alpha t^\alpha)t^{\alpha}}{3(2qt^\frac{4}{3} + 3\alpha t^\alpha)^2}. \hspace{1cm} (23)$$

$$w_{cm} = -\frac{2\dot{H} + 3H^2}{3H^2 - 8\pi G\rho_m} = -\frac{2\dot{H} + 3H^2}{3H^2 - 3H_0^2\Omega_m(1+z)^3}.$$
1. State parameters $w(z)$ (continuous) and $w_{cm}(z)$ (dashed) have been plotted against the redshift parameter $z$ for $\alpha = 4$. Both remain nearly zero till $z = 2.5$. The transition redshift is $z_a \approx 0.56$. The present value of $w_e$ is $-1$. A smooth transient crossing for $w_{cm}$ is observed. The first crossing occurs in the past while the other will occur in the future. For $\alpha < 3.2$, no such crossing is observed.

$$z = \frac{q t^\alpha}{q t^\alpha + t^\alpha} = 1. \quad (24)$$

In the above set of solutions $w_e$ and $w_{cm}$ are the effective state parameter and the state parameter corresponding to the created matter respectively. Since $\alpha > 1$, the expression (23) for $w_e$ is clearly negative which, as mentioned earlier, implies that the created matter is such that it is always associated with a negative creation pressure.

**Fitting the observational data.** The present model is parametrized by the two parameters $\alpha$ and $q$. With $\alpha = 4$, $h = 0.78 = 0.7 \, \text{Gyr}^{-1}$ and fixing $H_0 = 1$, $q$ is automatically fixed and the distance modulus versus the redshift curve (blue) is found to fit perfectly with the $\Lambda$CDM model (red). In fact, the two are practically indistinguishable (figure 1). We obtain the present value of the effective state parameter $w_e(0) = -1$ and the transition redshift $z_a = 0.56$, which are in excellent agreement with the $\Lambda$CDM model. It is observed that the effective state parameter encounters a transient double crossing of the phantom divide line in the future (figure 2) and so the Big-Rip singularity is bypassed. For $\alpha \leq 3.2$, the phantom divide line is never crossed, while for $\alpha > 4$, the first crossing occurs in the past but the second one always occurs in the future. The model fits perfectly with SNIa data for a wide range of values of $\alpha$. One can also observe that the state parameter $w_e$ remains nearly zero till $z = 2.5$ (figure 2), which confirms that the growth of perturbation in the present model tracks the concordance model closely.

Thus, the present model fits perfectly with the $\Lambda$CDM model without any problem whatsoever. However, the problem arises while one tries to fit the recently released 7 year WMAP [1, 2] constraint on the redshift of the matter-radiation equality at the early ISW effect. The value of the integral in equation (16) is $X = \exp(\int_0^H \Gamma dr') = 0.3236$, taking $\alpha = 4$. As a result the redshift at the matter-radiation equality is pushed far away to $z_{eq} = 4380$, taking only baryons into account, i.e. $\Omega_m = \Omega_B = 0.042$. It goes even further if some amount of $\Omega_{CDM}$ is incorporated. The situation is even worse for $\alpha > 4$. The whole situation taking
The creation rate $\Gamma$ versus the redshift parameter $z$ for $\alpha = 4$ shows that it is nearly vanishing in the past and started developing only recently at $z \approx 30$. For higher values of $\alpha$, $\Gamma$ starts developing from even smaller redshift value, while for $\alpha < 4$, it starts earlier. Thus, the WMAP constraint on $z_{\text{eq}}$ can be fitted for $\alpha < 3.5$, accommodating some amount of CDM.

### Table 1. Cosmological parameters for different values of $\alpha$.

| $\alpha$ | $X$    | Nature of $\Gamma$ | $z_{\alpha}$ | $w_{r0}$ | $\Omega_{m}$ | $\Omega_{\text{CDM}}$ | $w_{r0\alpha}$ | $z_{eq}$ | Fit with SN1a |
|----------|--------|---------------------|--------------|----------|---------------|------------------------|----------------|----------|---------------|
| 4        | 0.3236 | Rises from $z = 30$ | 0.56         | -1       | 0.04          | Nil                    | -1.20          | 4380     | Indistinguishable from $\Lambda$CDM. |
| 2        | 0.1876 | Rises from $z = 2500$ | 1.08         | -0.56    | 0.09          | 0.05                   | -0.62          | 3257     | Indistinguishable from $\Lambda$CDM. |
| 1.8      | 0.1563 | Rises from $z = 7000$ | 1.2          | -0.51    | 0.12          | 0.08                   | -0.58          | 3217     | Indistinguishable from $\Lambda$CDM. |
| 1.5      | 0.0962 | Very large initially | 1.46         | -0.44    | 0.18          | 0.14                   | -0.545         | 3234     | Fit is not the very best. |

some lower values of $\alpha$ and with the same values of $h = 0.7$, and $H_{0}d_{0} = 1$ is depicted in table 1.

We are in search of a model where additional creation starts some time after the matter-radiation equality $z_{\text{eq}}$. The $\Gamma$–$z$ plot (figure 3) depicts that the creation rate is almost vanishing ($\sim 10^{-8}$) till $z = 30$ for $\alpha = 4$ and only thereafter it rises sharply. However, the problem is that the creation rate is not sufficiently large to fit the WMAP constraint on $z_{\text{eq}}$. For $\alpha > 4$, the situation is even worse. The behaviour is the same also for lower values up to $\alpha = 2$. The only difference is that the creation starts earlier in this case at $z = 2500$, and so the created matter is a little large. Thus, the WMAP constraint on $z_{\text{eq}}$ is satisfied taking into account only a small amount of CDM, namely $\Omega_{\text{CDM}} = 0.05$. Thus, the problem with the lss model is encountered here too. For $\alpha \leq 2$, creation starts very early, much before $z_{\text{eq}}$ and so these cases are discarded.
5. Case 2

The main problem, we repeat, associated with the above model and the lss model [5] is that the creation of matter is not sufficient to fit the WMAP constraint on the redshift of the matter-radiation equality at the early ISW effect, accommodating 22% that are constituted by dark matter created in the very early Universe. Thus, the choice of $\Gamma$ should be such that the creation of matter starts near $z \approx 3000$ and in the later epoch it should increase considerably, to make $X$ sufficiently small. So, to find a suitable form of $\Gamma$, we try with a scale factor associated with the so-called intermediate inflationary solution [13], namely

$$a = a_0 \exp \left[ A t f \right],$$

$a_0$ being a constant. Such a solution for $A > 0$ and $0 \leq f < 1$ was presented by Barrow [13], and was shown to lead to late time acceleration [15] in different models. To appreciate the underlying beauty of the ansatz (25), let us expand it as

$$a = a_0 \left[ 1 + A t f + \frac{1}{2!} A^2 t^2 f^2 + \cdots \right],$$

and observe that for $f = \frac{2}{3}$, the standard matter-dominated era of the Friedmann model is recovered in the early Universe when the second term dominates, and the third term becomes responsible for accelerated expansion in the late stage of cosmological evolution. For $f = \frac{1}{3}$, the third term leads to the standard Friedmann model and acceleration starts a little late. For even smaller values of $f$, the model tracks decelerated expansion for a longer time recovering the standard Friedmann model at some intermediate stage of evolution and leads to accelerated expansion at much later stage of cosmic evolution.

The redshift parameter $1 + z = \frac{a(t_0)}{a(t)}$, where $t_0$ is the present time, is found as

$$1 + z = \exp \left[ A (t_0^f - t^f) \right].$$

Hence, the Hubble parameter takes the following form:

$$H = \frac{A f}{t^{(1-f)}} = \frac{A f}{\left[ t_0^f - \ln(1+z) \right]^{1/f}},$$

where we have used equation (26) to obtain the second equality in the expression of $H$. In view of equation (6), we can now find a form of $\Gamma$ as

$$\Gamma = 3H - 2(1 - f) \left( \frac{H}{A f} \right)^{1/f}. $$

This form of $\Gamma$ is clearly different from the $\beta$–$\gamma$ model [5]. The most important difference is that the creation rate $\Gamma$ here starts developing only when the Hubble parameter

$$H \gtrless \left[ \frac{3}{2(1-f)} A f \right]^{(1-f) \over 1/f},$$

since, as already mentioned in the introduction, $\Gamma < 0$ is not allowed by the second law of thermodynamics (see the appendix). The creation pressure and the creation matter density are now found as

$$8\pi G P_{cm} = -\Gamma H = H \left[ 3H - 2(1 - f) \left( \frac{H}{A f} \right)^{1/f} \right].$$

$$8\pi G \rho_{cm} = 3H^2 - 8\pi G \rho_m. $
Table 2. Cosmological best fit parameters keeping $0.67 \leq h \leq 0.7$.

| $A$ | $f$ | $z_{\Gamma=0}$ | $z_{a}$ | $w_{0}$ | $\Omega_{m}$ | $\Omega_{c}$ | $\Omega_{CDM}$ |
|-----|-----|---------------|--------|--------|-------------|------------|-------------|
| 15  | 0.056 | 468          | 0.70   | -0.35  | 0.246       | 0.754      | 0.204       |
| 16  | 0.053 | 658          | 0.71   | -0.35  | 0.249       | 0.751      | 0.207       |
| 17  | 0.051 | 1145         | 1.32   | -0.36  | 0.255       | 0.745      | 0.213       |
| 18  | 0.048 | 1350         | 0.82   | -0.35  | 0.255       | 0.745      | 0.213       |
| 19  | 0.046 | 2050         | 1.04   | -0.36  | 0.257       | 0.743      | 0.215       |
| 20  | 0.044 | 2914         | 1.09   | -0.36  | 0.258       | 0.742      | 0.216       |
| 21  | 0.0415| 3080         | 0.40   | -0.33  | 0.256       | 0.744      | 0.214       |
| 22  | 0.0392| 3157         | -0.11  | -0.33  | 0.256       | 0.744      | 0.214       |

where

$$8\pi G \rho_m = 8\pi G \rho_{m0}(1+z)^3 = \frac{\rho_{m0}}{\rho_c} 3H_0^2(1+z)^3 = 3H_0^2\Omega_m(1+z)^3,$$

(32)

in which $\rho_c$, $\rho_{m0}$ and $\Omega_m$ are the present values of critical density, matter density and the matter density parameter respectively. We can find the effective state parameter and also the state parameter of the created matter as

$$w_e = -\frac{2H + 3H^2}{3H^2} = -1 + \frac{2}{3} \left(1 - f\right).$$

(33)

$$w_{cm} = -\frac{2H + 3H^2}{3H^2 - 8\pi G \rho_m} = -\frac{3A^2 f^2 - 2Af(1-f)}{3A^2 f^2 - 3\Omega_m H_0^2(1+z)^3 \left[t_0^0 - \ln(1+z) \right]^{21/7}}.$$

(34)

Now let us see how far this model parametrized by the two parameters $A$ and $f$ fits with the observed data. We have kept $0.96 \leq H_0/t_0 \leq 1$ and $0.67 \leq h(= 9.78 H_0^{-1} \text{ Gyr}^{-1}) \leq 0.7$ as par with the HST project [16]. As already mentioned, the restriction on the parameters are $A > 0$ and $0 < f < 1$. We have tested the model by choosing $A$ and $f$ which fit SNIa data from a wide range of values between $0.08 \leq A \leq 25$ and $0.03 \leq f \leq 0.99$. The fit requires large $A$ for small $f$ and vice versa. We have presented our results briefly in table 2, taking only some integral values of $A$ starting from $A = 15$, since for lower values this model does not probe to the large redshift $z$. We have taken $z_{eq} = 3300$, which is very much at par with recently released WMAP data [1, 2] and $\Omega_B = 0.042$, to find the amount of matter produced in the late stage of cosmic evolution restricting the amount CDM produced in the very early Universe.

**Fitting the observational data.** In table 2, $z_{\Gamma=0}$ and $z_{a}$ symbolize the redshift values at which the creation of matter and the acceleration start respectively, while $w_{0}$ is the present value of the effective state parameter. Let us list our observation point by point.

1. The distance modulus versus the redshift curve fits between the present and the $\Lambda$CDM model (taking $\Omega_{\Lambda} = 0.74$ and $\Omega_m = 0.26$) almost perfectly for a wide range of values of the parameters $A$ and $f$.

2. It is observed that for the combinations of $A$ and $f$, which can probe to a distant redshift, the present value of the state parameter is nowhere near $-1$, yet the model fits both the experimental data, namely SNIa and WMAP. Particularly, for $A \geq 22$, the acceleration is yet to start.

3. The most important point is to note that for $A > 10$, $z_{eq}$ is at par with the recent 7 year WMAP data [1, 2], only if 24–26% of matter (baryons and CDM) is assumed...
Creation starts in the matter-dominated era around \( z = 2050 \) and its rate has a maxima around \( z_{\Gamma_{\text{max}}} = 1100(= z_{\text{recombination}}) \). Presently, the creation rate is insignificantly small. The behaviour is the same for all other combinations of \( A \) and \( f \), only \( z_{\Gamma=0} \) and \( z_{\Gamma_{\text{max}}} \) are different.

Figure 5. The figure shows the combined plot of the effective state parameter \( w_{\Delta} \) and the state parameter \( w_{cm} \) of the created matter versus the redshift parameter \( z \), since creation started, taking \( A = 19 \) and \( f = 0.046 \). While figure 4 depicts that creation started at \( z = 2050 \) and reaches its maxima at \( z = 1100 \), figure 5 shows that most of the time Universe undergoes decelerated expansion while acceleration started recently at \( z = 1.04 \).

(4) The behaviour of the creation parameter \( \Gamma' \) given in equation (28) has been plotted in figure (4) for a particular pair of the parameters \( A = 19 \) and \( f = 0.046 \). It shows that the creation started at \( z = 2050 \) reaches a maxima during the reionization era and presently it is insignificantly small. The behaviour is the same for all other pairs of \( A \) and \( f \), only the redshift values at which creation starts \( (z_{\Gamma=0}) \) and its maxima change.

The table shows that the density parameter \( 0.74 \leq \Omega_{cm} \leq 0.76 \), which corresponds to 74–76% of matter created in the matter-dominated era. Thus, instead of taking into account 74% of dark energy, creation of dark matter by the same amount in the matter-dominated era solves the cosmic puzzle.
Figure 6. The contour plots of $H_0 t_0 = 1$ (dotted line) and $\Omega_m$ for different parametric values of $A$ and $f$, which fit SNIa data, have been combined together. The plot shows that the $H_0 t_0 = 1$ line lies within the two lines $\Omega_m = 0.24$ and $\Omega_m = 0.26$, which are calculated taking $z_{eq} = 3300$. Thus, the present model fits SNIa data, satisfies the WMAP constraint on $z_{eq}$, demanding the correct amount of $\Omega_m$ required for structure formation, and agrees with the experimental constraint on $H_0 t_0 = 1$.

(5) Taking the same values of $A$ and $f$, figure 5 has been plotted. It represents the combined plot of the effective state parameter $w_e$ and the state parameter $w_{cm}$ corresponding to the created matter versus the redshift parameter $z$, since creation started. Figures 4 and 5 depict that though creation started rather early at $z = 2050$ and reaches its maxima around $z = 1100$, acceleration started only recently at $z = 1.04$. Using relation (26) it is found that it requires nearly 13.99 Gyr to create 74% of matter. On the other hand, inflation is supposed to start at $10^{-42}$s and ends at around $10^{-32}$ s. Thus, 22% that are constituted by dark matter have been created in $10^{-32}$ s only, in the very early Universe. This gives a comparison of creation phenomena in curvature-dominated and low-curvature regions.

(6) We have also presented a suitable contour plot in figure (6) to explore the data presented in table 2 at a glance. The plot presents all the successful combinations of the parameters $A$ and $f$, which fit SNIa data and satisfy the WMAP constraint on $z_{eq} = 3300$, keeping $H_0 t_0 \approx 1$, and $24\% \leq \Omega_m \leq 26\%$. Calculation shows that the WMAP constraint on $z_{eq}$ is not satisfied for lower or higher values of $\Omega_m$, with the same parametric combination of $A$ and $f$. Particularly, for $\Omega_m = 0.2$, $2400 \leq z_{eq} \leq 2500$, while for $\Omega_m = 0.3$, $3800 \leq z_{eq} \leq 3900$. Thus, nearly 26% of primordial matter in the form of baryons and CDM is required to fit presently observable data, in view of particle-creation phenomena.

Finally, it is no less important to understand if the adiabatic process is simulated due to the long-drawn particle-creation phenomena in the low-curvature region. It is known [8] that if the expansion rate is very weak, the production of high mass particles is exponentially small. This is due to the fact that large amount of energy must emerge from changing gravitational field to supply particle’s rest mass. Thus, the particle number remains adiabatic invariant and a comoving particle detector remains unexcited, which means that the probability of detecting particles falls sharply to zero. However, in discussing quantum particle production in curved spacetime [8], the balance law (5) has never been accounted for, which is crucial in the present
analysis. The balance law introduces back-reaction phenomena. As soon as some particles are produced, they impart the negative pressure $p_{\text{cm}}$, which enhances the expansion rate causing more particles of higher mass to produce. The process continues as long as the Universe expands sufficiently so that the curvature fluctuation is further reduced and the creation rate falls. This feature is present in the $\Gamma-z$ plot of figure (4). Thus, it appears that the adiabatic process will not occur.

6. Concluding remarks

We have presented a phenomenological cosmological model based on particle creation in the matter-dominated era, which fits SNIa data and the redshift of the early integrated Sachs–Wolfe effect at the matter-radiation equality, $z_{\text{eq}} = 3145^{+140}_{-139}$ [1] and $z_{\text{eq}} = 3196^{+134}_{-133}$ [2], determined by WMAP. The value of the $h$-parameter ($h \approx 0.70$) and $H_0 t_0 \approx 1$ also are very much within the observational limit obtained from the HST project [16]. Such a model constrains the amount of primeval matter to $0.24 \leq \Omega_m \leq 0.26$, out of which CDM amounts to $0.20 \leq \Omega_{\text{CDM}} \leq 0.22$, which is again the same amount required for structure formation.

Quantum particle production phenomena (QFT in CST) have been discussed in the literature in detail [8, 9]. The energy of these particles may then be extracted from the gravitational field [17]. To study the classical consequence of particle-creation phenomena, kinetic collision theory may be adopted to find a balance law in addition to standard Einstein’s equations, if weakly interacting massive particles (WIMP) are taken into account. In view of such a balance law a cosmological model of the early Universe has been explored [6]. In that model, as the particle production rate becomes comparable to the expansion scalar ($\Theta = 3H$), it builds up a large negative creation pressure that pushes the Hubble parameter $H$ to an approximately constant value. As a result, inflationary behaviour due to a large particle production rate is realized. Consequently, the universe starts with a de Sitter phase avoiding cosmological singularity. As inflation continues, the expansion rate becomes too large comparable to the particle production rate. In such a dilute cosmic fluid, the particle production rate decreases and halts at some time. At this stage, in the absence of sufficient negative pressure, inflation ends giving way to reheating and the resulting Universe smoothly approaches the familiar Friedmann–Lemaître–Robertson–Walker behaviour. Since the radiation-dominated era, in such a model, is of standard Friedmann type (i.e. $a \propto t^{1/2}$, $a$ being the scale factor), the standard Big-Bang nucleosynthesis (BBN) remains unaltered. Thus, cosmological evolution of the early Universe may be explained successfully in view of particle-creation phenomena.

Inflation makes the Universe almost spatially flat, which has also been confirmed by recent observations. However, at the end of inflation, if the Universe remains slightly away from spatial flatness, it may cause particle production in the matter-dominated era again. If this happens, then even a slow particle production rate may cause sufficient negative pressure in billions of years to cause recently observed cosmic acceleration. These phenomena have been studied earlier [4, 5], but the model suffered from a clear conflict [12] between the low (SNIa) and high redshift (WMAP) data. This problem has been resolved in the present model, which constrains primeval matter to $\Omega_m \approx 26\%$. Since, particle production starts long after matter-radiation equality, so the early growth of perturbation in connection with the structure formation also remains unaltered following the $\Lambda$CDM concordance model closely. Thus, the particle production process may successfully explain the late stage of cosmic evolution also. As a result, the cosmological evolution of the Universe may be explained successfully in view of particle-creation phenomena from early time till date.
Presently, we have numerous dark energy models, explaining the late time cosmic phenomena. Most of these models do not explain the early Universe on one hand, and it is not possible to identify these models from one another in any of the future experiments on the other. Cosmological consequence of particle-creation phenomena does not require dark energy at all at any stage, and some programmes have been taken in the recent years to detect lightest neutralino of roughly 10–10000 GeV—one of the weakly interacting massive particles (WIMP). Thus, creation phenomena of CDM can solve the presently observed cosmic puzzle single handedly, without taking into account dark energy at all, which may be resolved in future experiments.

7. Appendix. Thermodynamics of adiabatic particle creation

In this appendix we formulate the balance equation in connection with particle-creation phenomena. This has been done in [5, 6]. However, the approaches are slightly different; hence, we produce a straightforward calculation. Adiabatic cosmological evolution in the presence of particle creation can be treated in the open system, and so the first law of thermodynamics is modified as

\[ d(\rho V) + p_m dV - \frac{h}{n} d(nV) = 0, \]  

(A.1)

where, \( \rho, p_m, V, n \) and \( h \) are the total energy density, the true thermodynamical pressure, any arbitrary co-moving volume, the number of particles per unit volume and the enthalpy per unit volume respectively. In the case under consideration, the system receives heat only due to the transfer of energy from gravitation to matter. So, creation of particles acts as a source of internal energy. Thus, for adiabatic transformation the second law of thermodynamics reads

\[ T dS = \frac{h}{n} d(nV) - \mu d(nV) = T \sigma dN, \]  

(A.2)

Combination of the two laws (A.1) and (A.2) gives

\[ T dS = \frac{h}{n} d(nV) - \mu d(nV) = T \sigma dN, \]  

(A.3)

where we have used the usual expression for the chemical potential as \( \mu n = h - TS \) and define \( s = \frac{T}{\rho} \) to be the entropy per unit volume and \( \sigma = \frac{s}{n} \) as the specific entropy. Thus, we observe that the second law of thermodynamics, namely \( dS \geq 0 \), implies \( dN \geq 0 \), and the reverse process is thermodynamically impossible, i.e. particle can only be created and cannot be destroyed. Further, expressing \( S \) in terms of \( \sigma \), the above equation can also be expressed as

\[ T N d\sigma = 0 \Rightarrow \dot{\sigma} = 0. \]  

(A.4)

Hence, in the adiabatic particle-creation phenomena, entropy increases, while the specific entropy remains constant. The first law given by equation (A.1) can also be expressed as

\[ V d\rho + \rho dV + p_m dV - h dV - \frac{hV}{n} dn = 0 \Rightarrow V d\rho - \frac{hV}{n} dn = 0 \Rightarrow \rho = \frac{n}{h}. \]  

(A.5)

Now, the energy–momentum tensor \( T^{\mu\nu} \) along with the conservation law when creation phenomena is incorporated is

\[ T^{\mu\nu} = (\rho + p_m + p_{cm}) u^\mu u^\nu - (p_m + p_{cm}) g^{\mu\nu}, \quad T_{\nu,\mu}^{\mu} = 0, \]  

(A.6)

where \( \rho = \rho_m + \rho_{cm} \) is the total energy density and \( p_m \) is the thermodynamic pressure, as already stated, while \( p_{cm} \) is the creation pressure and \( u^\mu \) is the component of the four-velocity vector. The energy conservation law (A.6) in homogeneous cosmological models reads

\[ \dot{\rho} + \Theta (\rho + p_m + p_{cm}) = 0, \]  

(A.7)
where $\Theta = 3H$ is the expansion scalar, $H$ being the Hubble parameter. If we now plug in $\dot{\rho}$ from equation (A.5) in the above equation (A.6), we obtain

$$p_{cm} = -\dot{\rho} + \rho_m \theta \left(\frac{\dot{\theta}}{\theta} + \frac{\dot{n}}{n}\right) = -\frac{\dot{\rho} + \rho_m}{\Theta} \Gamma,$$

(A.8)

where $\Gamma = \theta + \frac{\dot{n}}{n}$ is the creation rate.

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References

[1] Komatsu E et al 2010 arXiv:1001.4538v2 [astro-ph.CO]
[2] Larson D et al 2010 arXiv:1001.4635v1[astro-ph.CO]
[3] Bertone G, Hooper D and Silk J 2005 Phys. Rep. 405 279 (arXiv:hep-ph/0404175) (FERMILAB-Pub-04-047-A)
[4] Alcaniz J S and Lima J A S 1999 Astron. Astrophys. 349 729 (arXiv:astro-ph/9906410v1)
[5] Lima J A S, Silva F E and Santos R C 2008 Class. Quantum Grav. 25 205006
[6] Prigogine I, Geheniau J, Gunzig E and Nardone P 1989 Gen. Rel. Grav. 21 767
[7] Parker L 1969 Phys. Rev. 183 1057
[8] Birrell N D and Davies P C W 1982 Quantum Fields in Curved Space (Cambridge: Cambridge University Press)
[9] Mukhanov V F and Winitzki S 2007 Introduction to Quantum Fields in Gravity (Cambridge: Cambridge University Press)
[10] Calvlo M O, Lima J A S and Waga I 1992 Phys. Lett. A 162 223
[11] Lima J A S, Germano A S M and Abramo L R W 1996 Phys. Rev. D 53 4287
[12] Steigman G, Santos R C and Lima J A S 2009 J. Cosmol. Astropart. Phys. JCAP06(2009)033 (arXiv:0812.3912[astro-ph])
[13] Barrow J D 1990 Phys. Lett. B 235 40
[14] Komatsu E et al 2009 Astrophys. J. (Suppl.) 180 330 (arXiv:0803.0547[astro-ph])
[15] Sanyal A K 2007 Phys. Lett. B 645 1 (arXiv:astro-ph/0608104)
Sanyal A K 2008 Adv. High Energy Phys. 2008 630414 (arXiv:astro-ph/0704.3602)
Sanyal A K 2009 Adv. High Energy Phys. 2009 612063 (arXiv:astro-ph/0710.3486)
Sanyal A K 2009 Gen. Rel. Grav. 41 1511 (arXiv:astro-ph/0710.2440)
[16] Freedman W L et al 2001 Astrophys. J. 553 47
[17] Brout R, Englert F and Gunzig E 1978 Ann. Phys. 115 78
Brout R, Englert F and Gunzig E 1979 Gen. Rel. Grav. 11
Brout R et al 1980 Nucl. Phys. B 170 228
Brout R, Englert F and Spindel P 1979 Phys. Rev. Lett. 43 417