The New Charged Massless States of Quantum Electrodynamics

E.C.MARINO

Joseph Henry Laboratories
Princeton University
Princeton, NJ 08544

Abstract

Quantum Electrodynamics can be formulated as the theory of an antisymmetric tensor gauge field. In this formulation the topological current of this field appears as an additional source for the electromagnetic field. The topological charge therefore acts physically as an electric charge. The topologically nontrivial, electrically charged, sector contains massless states orthogonal to the vacuum which are created by a gauge invariant local operator and can be interpreted as coherent states of photons.

\[1\] On sabbatical leave from Departamento de Física, Pontifícia Universidade Católica, Rio de Janeiro, Brazil. E-mail: marino@puhep1.princeton.edu
The use of nonperturbative methods in quantum field theory has always revealed surprising and unsuspected features. Despite the tremendous success of perturbation theory in describing the physical properties of a variety of systems, including the gauge theories of the fundamental interactions, it is well known that a wealth of new phenomena can be uncovered by nonperturbative investigations. A very interesting class of nonperturbative problems in quantum field theory is the one concerning the existence of topological sectors in the Hilbert space. These sectors are characterized by the fact that the states they contain bear a charge whose associated current is identically conserved. In the most frequent situations there are classical finite energy solutions of the field equations (solitons) carrying the topological charge and manifesting themselves as massive quantum states in the topological sector. In less common cases, despite the absence of classical finite energy solitons the theory still has quantum states (usually massless) carrying the topological charge.

In the present work, we study Quantum Electrodynamics in 3+1 D. We show that the vector field (photon) part of the theory possesses a hidden topological sector, whose topological charge behaves as the electric charge itself. In spite of the fact that the field equations do not admit classical finite energy solutions, we construct an operator which creates the states in this hidden sector and evaluate its correlation functions. From the large distance behavior of these correlation functions we can deduce that the topological charge carrying states are massless and orthogonal to the vacuum.

There is actually an extremely simple system which displays essentially the same features we discovered in QED, namely the massless scalar field in two-dimensional spacetime. The field equation $\Box \phi = 0$ implies the conservation of the current $j^\mu = \partial^\mu \phi$. It also implies the identity $j^\mu = \tilde{j}^\mu$, where $\tilde{j}^\mu = \epsilon^{\mu\nu} \partial_\nu \tilde{\phi}$ is the topological current of the scalar field $\tilde{\phi}$ which is itself defined by the above identity between currents. It is clear that no polynomial in $\phi$ could create states carrying the charge associated with the current $j^\mu$. It is not difficult to see, however, that the operator $\exp\{ib\phi\} = \exp\{ib \int^x \tilde{\phi} dx\}$ carries $b$ units of this charge and creates (massless) states orthogonal to the vacuum even though the theory possesses no classical solitons. We
will see that the photon sector of QED presents a structure completely analog to the one just described.

Let us start showing that QED can be formulated in terms of an antisymmetric tensor gauge field. Indeed, starting from

\[ \mathcal{L}_W + \mathcal{L}_I = -(1/12) W_{\mu\nu\alpha} (-\Box)^{-1} W^{\mu\nu\alpha} - (1/2) \varepsilon^{\mu\nu\alpha\beta} A_\mu \partial_\nu W_{\alpha\beta} \]

where \( W_{\mu\nu\alpha} = \partial_\mu W_{\nu\alpha} + \partial_\nu W_{\alpha\mu} + \partial_\alpha W_{\mu\nu} \) is the field intensity tensor of the antisymmetric field, we can readily obtain the pure Maxwell lagrangian \( \mathcal{L}_M = -(1/4) F_{\mu\nu} F^{\mu\nu} \) upon functional integration over \( W_{\mu\nu} \). We will therefore write the photon part of the QED lagrangian as

\[ \mathcal{L}_W = (1/2) [ \mathcal{L}_W + \mathcal{L}_I + \mathcal{L}_M ] \]

Integration over \( W_{\mu\nu}(A_\mu) \) in \( \mathcal{L} \) will produce the lagrangian \( \mathcal{L}_M(\mathcal{L}_W) \) (for the gauge fixing terms, see below). The nonlocality of \( \mathcal{L}_W \) is only apparent since \((-\Box)^{-1}\) has support on the light-cone surface (we choose the Feynman prescription). The field equations associated with this lagrangian are

\[ \partial_\nu F^{\nu\mu} = (1/2) \varepsilon^{\mu\nu\alpha\beta} \partial_\nu W_{\alpha\beta} \]

\[ \partial_\alpha W^{\alpha\mu\nu} = (-\Box) \varepsilon^{\mu\nu\alpha\beta} \partial_\alpha A_\beta \]

We immediately see that the topological current \( J^\mu \equiv (1/2) \varepsilon^{\mu\nu\alpha\beta} \partial_\nu W_{\alpha\beta} \) becomes a source for the electromagnetic field. We could as well consider the coupling of \( A_\mu \) to a matter current \( j^\mu_m \). In this case we would have the sum \( J^\mu + j^\mu_m \) as the total source of the electromagnetic field in the Maxwell equation in (1). The analogy with the 1+1 D massless scalar field is established by:

\[ A_\mu \leftrightarrow \phi, \ W_{\mu\nu} \leftrightarrow \tilde{\phi}, \ \partial_\nu F^{\nu\mu} \leftrightarrow \partial_\nu \tilde{\phi}, \ J^\mu \leftrightarrow \epsilon^{\mu\nu} \partial_\nu \tilde{\phi} . \]

It is not difficult to find a classical configuration of the antisymmetric field having nonzero topological charge. Consider, for instance the configuration [1]:

\[ W_{ij} = -\frac{1}{4\pi} \varepsilon_{ijk} \frac{x^k}{|x|^4}, \quad \bar{W}_{0i} = 0. \]

For this we have \( J^0 = \delta^3(\bar{x}) \) and \( J^i = 0 \) and therefore unit topological charge. The energy of this configuration however is infinite. This would be exactly the situation if we tried to probe a Sine-Gordon soliton in our 1+1 D theory of the massless scalar field \( \tilde{\phi} \): it would have nonzero topological charge but infinite energy.

The field equations (1) are solved by

\[ F^{\nu\mu} = (1/2) \varepsilon^{\mu\nu\alpha\beta} W_{\alpha\beta} + \Lambda^{\mu\nu} \]

\[ W^{\alpha\mu\nu} = (-\Box) \varepsilon^{\mu\nu\alpha\beta} A_\beta + \Lambda^{\alpha\mu\nu} \]

3
where $\Lambda^{\mu\nu} = -(1/2)\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha \Lambda_\beta - \partial_\beta \Lambda_\alpha)$ and $\Lambda_{\mu\nu} = -(\Box)\epsilon^{\mu\nu\alpha\beta}\partial_\beta \Lambda$ for arbitrary $\Lambda_\beta$ and $\Lambda$. We see that we can always choose these last two quantities so as to cancel the variations of $W_{\alpha\beta}$ and $A_\beta$, respectively, under gauge transformations in such a way that the gauge invariance of the l.h.s. of (2) is guaranteed. Inverting the first equation in (2) and choosing the Lorentz gauge for both the electromagnetic and antisymmetric fields, namely, $\partial_\mu A^\mu = 0$ and $\partial_\mu W^{\mu\nu} = 0$ we have

$$W^{\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}, \quad W_{\alpha\mu} = -(\Box)\epsilon^{\mu\nu\alpha\beta}A_\beta$$

(Note that since in the Lorentz gauge $\Box \Lambda = 0$, $W_{\alpha\mu}$ remains invariant under $A_\mu \to A_\mu + \partial_\mu \Lambda$). We see that in this particular gauge the antisymmetric field can be identified with the dual field intensity tensor $\tilde{F}^{\mu\nu}$ and the Lorentz gauge condition (on $W^{\mu\nu}$) becomes the Bianchi identity (observe that the configuration $\tilde{W}^{\mu\nu}$ corresponds to the field of a static point electric charge).

We can now obtain the basic commutation rules that will be needed later. Using a covariant gauge (Lorentz) quantization (see [2], for instance) we have the equal-time commutation relations $[A^i(x), E^j(y)] = i\delta^i_j \delta^3(x - y)$ where $E^i = F^{i0}$ is the electric field. Using this equation and the Lorentz gauge solution for $W$ in terms of $A_\mu$ we find

$$[W^{0ij}(x), W_{kl}(y)] = i\Delta^{ij}_{kl}(-\Box)\delta^3(x - y); \quad [W^{0ij}(x), E^k(y)] = i\delta^{ijk}(-\Box)\delta^3(x - y)$$

where $\Delta^{ij}_{kl} \equiv \delta^i_k \delta^j_l - \delta^i_l \delta^j_k$.

Let us introduce now the operator $-\mu$ — which will create the states carrying topological charge. This belongs to a class of operators which was introduced in two, three, and four dimensional spacetime as the creation operators of the respective topological excitations, namely—kinks, vortices and magnetic monopoles [3, 4]. As in these related operators, a basic ingredient here will be the external antisymmetric tensor field

$$\tilde{A}_{\mu\nu}(z, x) = (b/4\pi) \int_{T_3(S)} d^3\xi \Phi_\nu(\xi - x)\delta^4(z - \xi) - (\mu \leftrightarrow \nu)$$

(3)

where $b$ is an arbitrary dimensionless real parameter and $\Phi_\nu = (0, 0, 0, \frac{1 - \cos \theta}{r \sin \theta})$ (in the coordinate system $(t, r, \theta, \varphi)$). The integral in (3) is performed over the three-
dimensional hypersurface $T_x(S)$ whose surface element $d^3\xi_\mu$ has only the 0-component nonvanishing. $T_x(S)$ is the region of the $\mathbb{R}^3$ space external to the surface S at $z^0 = x^0$. This surface consists of a piece of sphere of radius $\rho$ centered at $\vec{x}$ ($0 \leq \theta \leq \pi - \delta$) superimposed to an infinite trunk of cone with vertex at $\vec{x}$ and angle $\delta$ (cut a distance $\rho \cos \delta$ from the tip) with axis along $\theta = \pi$.

The $\mu-$operator is then constructed in the following way

$$\mu(x) = \lim_{\rho,\delta \to 0} \exp\{-i/6\int d^4z W_{\mu\nu\alpha}(z,x)\}$$

where $\tilde{A}_{\mu\nu\alpha}$ is the field intensity tensor of $\tilde{A}_{\mu\nu}$. The operator $\mu$, as it stands, depends on the hypersurface $T_x(S)$. Nevertheless, as it happens in the case of the above mentioned related operators \[3, 4\] all the hypersurface dependence of the $\mu$ correlation functions can be eliminated by the introduction of a renormalization counterterm whose form is uniquely determined solely by the requirement of hypersurface invariance. The parameters $\rho$ and $\delta$ will be used as regulators which will be eliminated at the end of the calculations.

Inserting (3) in (4) and observing that the surface element in (3) only possesses the 0-component nonvanishing we get

$$\mu(x) = \lim_{\rho,\delta \to 0} \exp\{(ib/4\pi)\int_{T_x(S)} d^3\xi W_{\mu\nu\alpha}(\vec{\xi})(-\Box)^{-1}\partial_\nu \Phi_j(\vec{\xi} - \vec{x})\}$$

Using the above solution for W in terms of $\tilde{A}_\mu$ we could also express $\mu$ in terms of the electromagnetic field. Observe that inserting the identity (which is valid in $T_x(S)$) $\tilde{\nabla} \times \Phi \equiv \tilde{\nabla}[-\frac{1}{|\vec{x}|}]$ in (4) and thereby expressing $\mu$ in terms of the field configuration of a point charge, the singularity along the $\theta = \pi$ axis would disappear and we could already take the limit $\delta \to 0$ safely, thereby eliminating the piece of cone and just keeping the sphere of radius $\rho$ in the definition of the surface S.

Let us show now that $\mu$ is indeed a charge (or topological charge) bearing operator. The charge and topological charge densities, which are identified by the first equation in (2) are given, respectively by $j^0 = \partial_t E^i$ and $J^0 = \varepsilon^{ijk} \partial_i W_{jk}$. Using the expansion for $e^A B e^{-A}$ and the above commutation rules, we find the equal times
\[ \left[ \rho(y), \mu(x) \right] = \left( b/4\pi \right) \mu(x) \lim_{\rho, \delta \to 0} \int_{S(x)} d^3 \xi \, \epsilon^{ijk} \partial_k^{(y)} \delta^3(\xi - y) \partial_j^{(z)} \Phi_j(\xi - z) \]

where \( \rho(x) \) stands either for the charge or topological charge density. Using the Gauss theorem and the identity involving \( \nabla \times \Phi \) we can straightforwardly evaluate the above integral (we can see that the cone piece of \( S \) does not contribute) obtaining

\[ \left[ \rho(y), \mu(x) \right] = b \mu(x) \]

This relation shows that \( \mu \) does indeed create states bearing \( b \) units of charge. A very important relation involving \( \mu(x) \) that can be derived along the same lines is the commutator with the electric field

\[ \left[ E^k(y), \mu(x) \right]_{ET} = b \left( \frac{\vec{y} - \vec{x}}{4\pi |\vec{y} - \vec{x}|^3} \right) \mu(x) \]

This implies that the vacuum expectation value of the electric field in the states created by \( \mu \) is the field configuration of a point electric charge of magnitude \( b \):

\[ \langle \mu(x) | E^k(y) | \mu(x) \rangle_{ET} = b \left( \frac{\vec{y} - \vec{x}}{4\pi |\vec{y} - \vec{x}|^3} \right) \mu(x) \]

This expression characterizes \( |\mu(x)\rangle \) as a coherent state of photons.

Let us study now the \( \mu \) correlation function. Taking the expression of \( \mu \), Eq. (4), the lagrangian \( \mathcal{L} \) and going to euclidean space (in which we will work henceforth) we may write

\[ \langle \mu(x) | \mu^\dagger(y) \rangle = Z^{-1} \int D\mu D\mu' DA \exp\left\{ - \int d^4z \left[ \mathcal{L} + \mathcal{L}_{GF} + \mathcal{L}_{CT} \right. \right. \]

\[ \left. \left. - \left( 1/6 \right) W^{\mu\nu\alpha} (-\Box)^{-1} \tilde{A}_{\mu\nu\alpha}(z; x, y) \right] \right\} \]

where \( \mathcal{L}_{GF} = \mathcal{L}_{GFW} + \mathcal{L}_{GFA} \) is the gauge fixing term, with

\[ \mathcal{L}_{GFW} = - (\xi_1/8) W_{\mu\nu} K^{\mu\nu\alpha} (-\Box)^{-1} W_{\alpha\beta} \]

where \( K^{\mu\nu\alpha\beta} = \partial_\mu \partial_\nu \delta^{\alpha\beta} + \partial_\nu \partial_\alpha \delta^{\mu\beta} - (\alpha \leftrightarrow \beta) \). The \( A_\mu \) gauge fixing term is \( \mathcal{L}_{GFA} = - (\xi_2/4) A_\mu \partial^\mu \partial^\nu A_\nu \). In (3) \( \tilde{A}_{\mu\nu\alpha}(x, y) = \tilde{A}_{\nu\mu\alpha}(x) - \tilde{A}_{\mu\nu\alpha}(y) \) and \( \mathcal{L}_{CT} \) is the above mentioned hypersurface renormalization counterterm to be determined below. We
see that \(< \mu \mu^\dagger >= e^{F[\hat{A}_{\mu\nu}]}\) is the vacuum functional in the presence of the external field \(\hat{A}_{\mu\nu}\). This property of the correlation functions of \(\mu\) is common to all the above mentioned topological charge bearing related operators \([3]\) and follows from the general fact that topological charge carrying operators are closely related to the disorder variables of Statistical Mechanics \([5]\). Indeed, treating these operators as disorder variables \([3, 4]\) immediately leads in general \([6]\) to a form of \(\mu\) which is expressed in terms of the coupling of the lagrangian field to an external field like \(\hat{A}_{\mu\nu}\) and also to a renormalization counterterm consisting in the self-coupling of this external field both with the same form as the kinetic term. Also here, we will see explicitly that the renormalization counterterm \(\mathcal{L}_{CT} = (1/12)\hat{A}^{\mu\alpha}(\xi)^{-1}\hat{A}_{\mu\alpha}\) will absorb the external field infinite self-energy and render the \(\mu\) correlation function hypersurface independent.

Integrating over \(A_{\mu}\) and then over \(W_{\mu\nu}\) in (6) with the help of the euclidean propagator of this field, namely

\[
D^{\mu\nu\alpha\beta}(x) = (1/4) \lim_{m \rightarrow 0} [(-\Box)^{\mu\nu\alpha\beta} + (1 - \xi_{1}^{-1})K^{\mu\nu\alpha\beta}] - (1/8\pi^2) \ln m\gamma|x|\]

where \(\gamma\) is the Euler constant and \(m\) is an infrared regulator—used to define the inverse Fourier transform of \(1/k^4\)—we obtain the following result

\[
< \mu(x)\mu^\dagger(y) > = \lim_{\rho,\delta,m,\epsilon \rightarrow 0} \exp\left\{ \left( b^2/2 \right) \sum_{i,j=1}^{2} \lambda_i \lambda_j \int_{T_{x_i}} d^3\xi \partial^\alpha_{\xi} \Phi_{\nu}(\xi - \vec{x}_i) \times \int_{T_{x_j}} d^3\eta \partial^\beta_{\eta} \Phi_{\rho}(\eta - \vec{x}_j) \epsilon^{\mu\nu\sigma\epsilon} \epsilon^{\gamma\rho\beta\lambda} \times [-\Box_{\sigma\lambda} + \partial_{\nu}^{(\xi)}\partial_{\lambda}^{(\xi)}] - (1/8\pi^2) \ln m\gamma[|\xi - \eta| + |\epsilon|] - \mathcal{S}_{CT} \right\} \tag{7}
\]

In this expression, \(x_1 \equiv x\), \(x_2 \equiv y\), \(\lambda_1 \equiv +1\) and \(\lambda_2 \equiv -1\). We also introduced the ultraviolet cutoff \(|\epsilon|\) in the euclidean \(W_{\mu\nu}\) propagator. Only the first term of the \(W\)-propagator contributed to (7). In particular, all the gauge dependent terms were cancelled. This happens because of the gauge invariant way in which the external field is coupled in (3). The first term in (4) is nothing but \(S_{CT}\) and therefore is exactly
canceled. Using Gauss theorem and the identity for $\vec{\nabla} \times \vec{\Phi}$ it is not difficult to see that the crossed terms (with $i \neq j$) vanish in the limit $\rho \to 0$. The self-interaction terms (with $i = j$) on the other hand, diverge in this limit. We conclude therefore that the renormalization counterterm $S_{CT}$ contains only the unphysical self interaction terms.

Each of the integrals in (7) can be evaluated straightforwardly by the use of the Gauss theorem and of the identity for $\vec{\nabla} \times \vec{\Phi}$. The result is $\exp[-F(x-y)+F(\epsilon)]$ where $F(x) = (b^2/8\pi^2) \ln m\gamma |x|$ and we still must make $m, \epsilon \to 0$. Note that the $m\gamma$ factors cancel out. In a charge selection rule violating correlation function (like $<\mu\mu>$, for instance) we would have the sign of the $F(x-y)$ term reversed and the $m\gamma$ factors would no longer cancel, actually forcing the correlation function to vanish in the limit $m \to 0$ and thereby enforcing the charge selection rule. The ultraviolet divergence at $|\epsilon| \to 0$ can be eliminated by a multiplicative renormalization of the field $\mu$, namely $\mu_R(x) = \mu(x)|\epsilon|^{-b^2/16\pi^2}$. Using this we finally get $<\mu_R(x)\mu_R^\dagger(y)> = |x-y|^{-b^2/8\pi^2}$.

This is our final expression for the $\mu$ field euclidean correlation function. Observe that $|x-y|$ is a distance in 4-dimensional euclidean space. An arbitrary 2n-point correlation function could be obtained in a straightforward manner by just inserting additional external fields $\tilde{A}_{\mu\nu}$ in an expression like (8). It would be given by a product of monomials of the type found in $<\mu\mu^\dagger>$. The form of the $\mu$ correlation functions characterizes it inequivocally as a local operator.

From the long distance behavior $\lim_{|x-y| \to \infty} <\mu_R(x)\mu_R^\dagger(y)> = 0$, we can infer that $<\mu(x)> = 0$ and therefore that the states $|\mu(x)>$, created by $\mu$ are orthogonal to the vacuum. The power law decay of the correlation function on the other hand implies that these states are massless.

As far as we can see, the charge of the states created by $\mu$ is not quantized, in the same way as in the two-dimensional analogous system. It would be interesting to investigate whether some additional coupling — as it happens with the Sine-Gordon coupling in the case of the scalar field in 1+1 D — would produce a charge quantization for these states.

Including the coupling of the electromagnetic field $A_\mu$ to a matter current $j^\mu_m$ (in (8), for instance) we would no longer be able to obtain an exact expression for
the $\mu$ correlation functions because the integration over the matter fields could not be done exactly. Nevertheless, due to the well known infrared asymptotic freedom of QED we still may conclude that the long distance behavior of $< \mu \mu^\dagger >$ will be the same. The result that the operator $\mu(x)$ creates massless states orthogonal to the vacuum, therefore, also holds in the presence of matter fields.

Let us mention a well known operator which shares some of the properties of $\mu$ (as the commutator with the charge operator, for instance), namely, $\exp\{-i(b/4\pi) \int x A_\mu dx^\mu\}$. This operator is unacceptable because for it we would have nonvanishing string dependent crossed ($i \neq j$) interaction terms (see the remarks after Eq. (7)) which would inevitably lead to a nonlocal correlation function.

It would be extremely interesting to investigate the conditions under which the states we found here could be experimentally observed. These should certainly include the presence of high intensity electromagnetic fields which would be needed to populate the coherent photon state. An interesting feature of the massless charged states is that they will not radiate since a massless charge cannot be accelerated.

The results we found in this work show that a physical quantity which is usually associated with matter, namely, charge, can be generated as an attribute of some coherent states of the electromagnetic field itself. It is not inconceivable that other quantities like spin, mass, flavor, color and so on could be generated as well as properties of some peculiar states of the gauge fields in general. This would lead to the outstanding possibility of describing both matter and the fields which mediate its interactions within the same unified framework. We hope this work could provide a small contribution towards this end.

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