Article

Aggregate Simulation with Statistical Approach Considering Substituting

Byeong Hun Woo 1, Jeong Bae Lee 2, Hyunseok Lee 3,* and Hong Gi Kim 1,*

1 Civil and Environmental Engineering Department, Hanyang University, Seoul 04763, Korea; dimon123@hanyang.ac.kr
2 Department of Civil Engineering, Daejin University, Pocheon-si 11159, Korea; dlwjdqo@nate.com
3 Division of Horticulture and Medicinal Plant, College of Life Science and Biotechnology, Andong National University, Andong 36729, Korea
* Correspondence: hyunseoklee@anu.ac.kr (H.L.); dmkg1404@hanyang.ac.kr (H.G.K.); Tel.: +82-2-2220-4323 (H.G.K.)

Abstract: This work focused on reflecting the substituting ratio of fine aggregate in an aggregate simulation. The existing simulation studies showed superior performance on generating the particles; however, the studies did not and could not reflect the substituting ratio of fine aggregate. Therefore, a statistical approach with the Monte Carlo simulation method was tried to improve the lacking part. According to the fitting of the distributions, the Cauchy distribution was best for the natural sand and the log-normal distribution was best for the substituting materials. The chosen two distributions were mixed and applied, using the Monte Carlo method with the mixed model, rather than the existing particle generation formula of the simulation. The substitution ratio was considered to be 0, 30, 50, 70, 100%. The fraction of small particles was gradually increased by the substituting ratio. As a result, the simulated particle distribution reflected well the statistical model. In addition, the simulation was almost the same as that of real particle distribution, according to the CT scanning.

Keywords: simulation; aggregate; substituting; Monte Carlo simulation; statistical approach

1. Introduction

The materials used for cement composites are now changing rapidly. In particular, the fine and coarse aggregates are now using other materials instead of natural materials, because these natural resources are running out, according to the rapid development of cities and buildings [1]. The demand for concrete is increasing; therefore, the demand for aggregates is also increasing. Considering the environment on Earth, many studies are focused on alternative aggregate, such as recycled aggregate [2,3], artificially developed aggregate [4,5], bottom ash from various fields [6–8], and new materials, including silicon carbide [9–12]. Due to the active use of alternative aggregate, the PSD may be disturbed by the mixing of the aggregate materials, meaning that the effect of the various PSD type needs to be studied.

Woo et al. [8] studied the use of municipal waste incinerated bottom ash as fine aggregate and they found that the volume fraction of the fine particles was increased. Jeon et al. [12] used silicon carbide as a replacement material for natural sand. They demonstrated that silicon carbide has sufficient strength to be used as a fine aggregate. Furthermore, silicon carbide has high thermal properties. Woo et al. [11] aimed to enhance the thermal properties, using silicon carbide as fine aggregate for ice-melting technology. In addition, Woo et al. [11] used silicon carbide as substituting material to fine aggregate, with a replacing ratio of 0, 30, 50, 70, and 100%. In another study, Kim et al. [9] used phase change material in the slag-based lightweight coarse aggregate, to improve the insulating
performance of cement composites. Kim et al. [9] also used their developed aggregate to a maximum of 100% of the natural aggregate fraction. Likewise, many studies were focused on using alternative aggregates that were substituted to a maximum of 100% of the natural aggregate fraction. Therefore, a study for using alternative aggregates should be considered because the substituting ratio up to 100% can now be reached.

In recent days, the most important work has been simulation. The research on the construction field was dramatically changed after adopting AI. Following this trend, the simulating studies were highlighted in the construction research field. Prior to the AI era, simulation studies were focused on the theoretical aspect [13–15]. However, the importance of the statistical approach was highlighted after AI was adopted. As a result, simulation studies were changed by applying the statistical approach. Achenbach et al. [16] studied heat transfer simulation using an MC simulation method. They estimated the related parameters and established the process of how to simulate the heat transfer of cement composites [16]. Liu et al. [17] focused on the prediction of concrete deterioration with GPR. They made a process for predicting the deterioration of concrete and simulated it [17]. The GPR method gave high accuracy based on the data [18,19]; therefore, the prediction accuracy of concrete was high [17]. Wang et al. [20] simulated the mesoscale fracture of concrete with randomly generated coarse aggregate using the MC method. Their purpose was to confirm the size effect, so they used the MC simulation for making program specimens with different sizes [20]. Peng et al. [21] studied something similar to Wang et al. [20], but more advanced. Peng et al. [21] simulated their samples and analyzed them through the statistical approach. In addition to these studies, various simulation-based studies are being actively carried out [22].

The value of simulation is in predictions, before practical application or without experiment. The accuracy of the simulation was gradually increased and the simulation works showed high accuracy. In particular, aggregate simulations were remarkably improved [23–26]. However, there was a problem in reflecting the substituting ratio of aggregate. Following the studies of simulating the aggregate [23–26], various prediction simulations, such as cracking estimation and making voids [20,21] have become possible. However, the trend of experimental studies and practical application on aggregate is rapidly changing; therefore, a simulation study on reflecting the substitution of aggregate should be carried out. It was expected that the combination of discrete distributions would be derived. The MC method showed a simple operation and superior performance from the existing studies [16,20,21] and, therefore, we believe that the MC method was a suitable method to treat the discrete distributions.

In this study, a simulation of fine aggregate generation was performed. This study focused on the PSD of aggregate and the PSD was established using the statistical approach with the MC method. The PSD data was based on the experimental data, including the direct experiments for this study and existing studies. The PSD data include the PSD of natural sand and the PSD of other substituting materials.

2. Data Collecting and Simulation Method

2.1. Particle Size Distribution Data of Fine Aggregates

The PSD of the fine aggregate data was collected by experiments and from other studies. Most studies indicated the PSD trend in the cumulated percentile [2,10–12]. However, the PSDs of this study should be presented in a non-cumulative percentile way, in order to approach the data in the statistical method. The PSDs are indicated in Figure 1.
According to Figure 1b, various styles of PSDs of the natural sand were used [10,12,27–29]. On the other hand, all the PSDs of Figure 1c were the results from direct measurement. Silicon carbide was referenced by Woo et al. [10,11] and Jeon et al. [12]. Slag aggregate was referenced by Kim et al. [9], but the slag aggregate was crushed in order to make the data. Bottom ash was referenced by Woo et al. [8]. Based on these data, the statistical approach on the particle sizes was carried out. In this state, the particle size range of natural sand was 0.075 mm to 4.75 mm and the particle size of small particle material was 0 mm to 0.25 mm.

2.2. Statistical Fitting of the PSD Data

First, PSD data-based PDF fitting work was carried out. Considering the characteristics of PSD data, a total of 5 functions were confirmed to fit. The first function was the ND of Equation (1) [30].

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

In this condition, $\mu = \sum_{i=1}^{n} \frac{x_i}{n}$ and $n$ is the number of data, $\sigma^2 = \sum_{i=1}^{n} (x_i-\mu)^2 / n-1$. ND is a basic function in statistical analysis, therefore it needed to be confirmed to fit well.

The second function was the BD of Equation (2) [31]. The main parameters of BD are $\alpha$ and $\beta$.

$$f(x|\alpha, \beta) = x^{\alpha-1}(1-x)^{\beta-1}$$

In the case of the BD, the mean of data is $E(X) = \frac{\alpha \beta}{\alpha + \beta}$ and the variance of data is $Var(X) = \frac{\alpha \beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$.

The third function was the GD of Equation (3) [32]. GD is different from gamma function. The main parameters of GD are also $\alpha$ and $\beta$, and the parameter condition is the same as the BD.

$$f(x|\alpha, \beta)_{gamma} = x^{\alpha-1}e^{-x/\beta}$$

In the case of the GD, the mean of data is $E(X) = \frac{\alpha \beta}{\alpha + \beta}$ and the variance of data is $Var(X) = \frac{\alpha \beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$. The difference from BD is that the GD can allow the data range of data higher than 1.

The fourth function was the SCD [33]. The SCD is a very simple distribution, needing only the data. The structure of SCD is shown in Equation (4).
\[ f(x) = \frac{1}{\pi(1 + x)^2} \]  

(4)

The original Cauchy distribution considers the location parameter and scale parameter, however, the simple test of data fitting showed SCD behavior. Therefore, SCD was considered in this study.

The last function was the LND. It has a similar form of ND but considers the data with log form. The structure of the LND is expressed as Equation (5) [34].

\[
f(x|\mu, \sigma) = \frac{1}{x \sqrt{2\pi\sigma_{\text{LND}}^2}} e^{-\frac{\text{log}(x) - \mu_{\text{LND}}}{2\sigma_{\text{LND}}^2}}
\]  

(5)

In the case of the LND, the parameters are \( \mu_{\text{LND}} = \log \left( \frac{m^2}{\sqrt{v + m^2}} \right) \) and \( \sigma_{\text{LND}} = \sqrt{\log \left( \frac{v}{m^2} + 1 \right)} \), where \( m \) is the same as \( \mu \) of ND and \( v \) is the same as \( \sigma \) of ND. Total of 5 cases of distributions were fitted and checked. This is detailed in Section 3.

2.3. Aggregate Simulation

Unger et al. [24] established a good aggregate simulation. Their aggregate simulation results showed that the shape of aggregates was ellipsoid. In order to make the particles in the shape of an ellipsoid, a factor needs to be followed and the factor is as in Equation (6).

\[
\sum_{i=1}^{3} \left( \frac{X_i}{r_i} \right)^2 = 1
\]  

(6)

For simplifying the simulation, the radii were assumed to be \( r_1 \geq r_2 \geq r_3 \) [24]. Here, \( r_2 \) is the core factor to make the particles. Because \( r_1 \) and \( r_3 \) were determined after setting the \( r_2 \), the \( r_2 \) naturally settles to the median value. After setting the \( r_2 \) value, \( r_1 \) and \( r_3 \) values were determined from Equations (7) and (8).

\[
r_1 = \left( 1 + u_1 \frac{f - 1}{f + 1} \right) r_2
\]  

(7)

\[
r_3 = \left( 1 + u_3 \frac{f - 1}{f + 1} \right) r_2
\]  

(8)

If \( f = 1 \), this condition indicates the shape of particles to be an ideal sphere. On the other hand, when \( f \geq 1 \), then the shape of the particles alter to be ellipsoid. In this condition, \( r_1 \) should be in the range of \( [r_2, r_2 \left( \frac{2f}{f+1} \right)] \), and \( r_3 \) should be in the range of \( [r_2 \frac{2f}{f+1}, r_2] \) [24]. Particle size is very important in simulation, however, the volume of a single particle is also important too. The volume of simulated single particles can be expressed by Equation (9) [24].

\[
V_{\text{particle}} = \frac{4}{3} \pi r_2^2 \left( 1 - \frac{\left( f - 1 \right)}{2 f + 1} \right)
\]  

(9)

Unger et al. [24] mentioned the \( d_{\text{eq}} \) of the simulated single particle as this is important, because it is assumed to be \( d_{\text{eq}} = 2r_2 \) [24]. In this condition, it can be inferred that the \( r_2 \) value can be calculated from \( d_{\text{eq}} \), \( r_2 = d_{\text{eq}}/2 \). The formula for calculating \( d_{\text{eq}} \) is expressed as Equation (10).

\[
d_{\text{eq}} = \frac{d_{\text{min}} d_{\text{max}}}{\sqrt{R d_{\text{min}}^3 + (1 - R) d_{\text{max}}^3}} \quad (0 \leq R \leq 1)
\]  

(10)
According to the assumed condition and Equation (10), $d_{eq}$ is the main factor for determining the particle size. From Unger et al. [24], the deriving process of $d_{eq}$ was very complex, however, the key point was Equation (10). In Equation (10), $R$ indicates the random variable with the range of 0 to 1. Here, this study focused on the behavior of Equation (10), in short, Equation (10) is the key material in this study.

Figure 2 indicates the advanced point of this study. The main point was determining the $d_{eq}$ from Equation (10), because the other factors like $r_1$, $r_2$, $r_3$, $V_{\text{particle}}$ could be calculated after determining $d_{eq}$. The state of Equation (10) could not reflect the substituting ratio of fine aggregate, therefore, it was necessary to replace the mechanism in order to reflect the substituting ratio. The mechanism was applying the MC method with the fitted mixed distributions. In this state, the simulation considering the substituting ratio was ready to simulate.

Figure 2. The main difference between the original simulation and this study.

The next step was the placing of the simulated particles. Since placing was a big problem, generated particles should not overlap each other, therefore, it must be controlled in equations. According to Unger et al. [24], the least distance of particles was controlled by a form of a parameter. The method was given a parameter in $r_2$ and the shape is like that in Equation (11).

$$\Delta p = \varepsilon p r_2$$  \hspace{1cm} (11)

The generated particles are discretized forcibly by Equation (11). When the gaps between the particles were determined, the position of the particles could be calculated. However, it was not enough to be sure Equation (11) may be perfect. It needed a further check, to be sure the particles did not overlap. All conditions should be satisfied with the condition of Equation (12), as follows:

$$x^T Ex = 0$$  \hspace{1cm} (12)

where, $x = [x, y, z, 1]^T$ are homogeneous coordinates and $E$ is a $4 \times 4$ matrix that integrates the radii of the ellipsoid with its position and direction in space [24].

The location of the particles was based on their center point in 3D coordinates which are $c_x$, $c_y$, $c_z$ [24]. In addition, the rotation angles of the particles are also considered in order to give a realistic with $\theta_x$, $\theta_y$, $\theta_z$. Following Equations (11) and (12), the possible estimate of $E$ is given in Equation (13).
With Equation (13), Equations (14) and (15) should be considered because of the positioning and rotating of the particles [24].

\[
\mathbf{D}_r = \begin{bmatrix}
1 & 0 & 0 & -c_x \\
0 & 1 & 0 & -c_y \\
0 & 0 & 1 & -c_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

In this condition, the final form of \( \mathbf{E} \) for an ellipsoid can be expressed by Equation (16).

\[
\mathbf{E} = \mathbf{D}_r^T \mathbf{E} \mathbf{D}_p \mathbf{D}_r
\]

Following these processes with the code program, the core contents of particle generation can be simulated. The shape of space and other minor considerations are considered by Unger et al. [24].

3. Results

3.1. Probability Distribution Fitting Results

The PSD data of Figure 1 were all applied to the statistical analysis. In short, the natural sand range of 0.075 mm to 4.75 mm was fitted as in Figure 3a and the small particles’ range of 0 mm to 0.25 mm was fitted like in Figure 3b. In this state, the particle size range of 0.075 mm to 0.25 mm overlapped. However, the overlapped range could be controlled by the MC simulation. The results are indicated in Figures 3–6.

![Figure 3. Fitting check of each distribution case through histogram: (a) Natural sand; (b) Small particle aggregate.](image)

According to Figure 3, the fitted functions showed their expected characteristics. In the case of natural sand, BD could not be fitted because the range of data was not suitable for BD. Except for BD, SCD and LND showed good fitting in the case of natural sand. On the other hand, small particle aggregate showed that all functions were fitted. LND and GD showed excessive camber, near zero, but the other range was reflected well. In the case of small particle aggregate, BD, GD, SCD, and LND seemed well fitted. However, it...
was not able to judge what distribution was best. Therefore, the sub-data analysis methods were performed.

The Q–Q plot of natural sand showed that ND was not suitable for the natural sand case. However, LND and SCD showed a good trend in natural sand that followed the reference line of Figure 4a well. Therefore, LND and SCD were considered good functions in natural sand. In the case of small particle aggregate, it was able to narrow down to two functions, which were GD and LND. The other distribution functions in small particle aggregate showed great deviation from the reference line of Figure 4b.

Figure 4. Confirming the suitability of distributions by Quantile–Quantile plot: (a) Natural sand; (b) Small particle aggregate.

In the case of natural sand, SCD exhibited a fairly poor performance. However, LND and GD showed a good fitting that followed the reference line of Figure 5a well. On the other hand, in the case of small particle aggregate, the fitting trend was almost the same as the Q–Q plot of Figure 4b.

Figure 5. Confirming the suitability of distributions by Probability–Probability plot: (a) Natural sand; (b) Small particle aggregate.

Comprehensively, the SCD was chosen as the best fitting in the case of natural sand. Although SCD demonstrated a relatively bad performance compared to LND in the P–P plot, it showed good performance in the fitting shape and Q–Q plot. Most of all, the simple method that SCD only required input data was the critical considering factor. In the case of small particle aggregate, LND was chosen as the best fitting. LND showed good performance in the sub-checking and shape fitting. Although SCD was better in the shape fitting, in the data of small particles, it was able to decrease the quality of statistical analysis by double, considering SCD. The important factor is the harmonizing of the various
functions. Therefore, the distribution was fitted with SCD for natural sand and LND for small particle aggregates. The distributions were fixed; therefore, the next step involved making an MC process and discretizing the mixed distribution, as in Figure 6.

![Mixed distribution of NLD and SCD](image)

**Figure 6.** The mixed distribution of SCD and LND.

Figure 6 and the MC method were able to overcome the limit of fine aggregate simulation. All existing studies showed the particle distribution with Fuller’s curve [24–26], which cannot reflect the substituting rate. However, reflecting the substituting rate of fine aggregate with the mixed distribution and MC method is possible.

### 3.2. Fine Aggregate Simulation Results with Simulated PSD

The simulation results are indicated in Figure 7. The substituting ratio was applied to 0, 30, 50, 70 and 100% of the fine aggregate. To evaluate the performance of the advanced simulation, the extreme substituting ratio was needed. Through the extreme substitution, it could be demonstrated that the statistical approach with the MC method could simulate the substitution. The dimensions of the simulated specimens were 5 mm in diameter and 5 mm in height.

![3D rendering and section views](image)
Figure 7. Aggregate simulation with substituting ratio: (a) 0%; (b) 30%; (c) 50%; (d) 70%; (e) 100%.

The green particles indicate a size of below 0.075 mm and the light pink particles indicate a size larger than 0.075 mm. According to the 0% substitution simulation, the
distribution of simulated particles reflected the data well [10,12,27–29]. In addition, the simulation, including substituting ratio, also reflected the data well. To provide evidence this claim, CT images were provided in Figure 8.

**Remark**
Size of CT specimen: Φ 100 mm
Simulated specimen: Φ 5 mm

(a) Real aggregate distribution of 0% substitution
(b) Real aggregate distribution of 30% substitution
(c) Real aggregate distribution of 50% substitution
(d) Real aggregate distribution of 70% substitution
The CT specimens were 100 mm in diameter and 50 mm in height and were cylindrical in shape. The CT data were measured by a CT machine, Vtomex M 240D, from General Electric corporation of Boston, US. Considering the relative scale between simulated specimens and CT measured specimens, we believe that the simulation of aggregate with the proposed process performed well. The ratio of natural sand and small particle aggregate was clearly simulated when considering the CT results. In particular, the green particles spread randomly, and this phenomenon could also be confirmed from the CT images; in short, almost the same trend appeared between the simulation and reality. However, the pore simulation could not be reflected in this study; therefore, it should be studied on reflecting the pore simulation, in terms of fractal theory [35]. Huang et al. [35] showed a simple distribution method in terms of fractal theory and it had the possibility to combine with this work. Comprehensively, the proposed process, with the statistical approach and MC method, can reflect real phenomena in the simulation, which was proved by this work. In addition, there will be further points of advancement when combining the pore simulation with fractal theory.

4. Conclusions

This study was focused on reflecting the substitution ratio of fine aggregate in the aggregate simulation, using a statistical approach with the MC method, to find the best distribution in the data and fitting the best distribution. In addition, the substituting ratio should be considered to reflect the aggregate substitution in the simulation and we used mixed distribution with the MC method. The comprehensive results are as follow:

1. The best distributions should be confirmed in order to apply the MC method. Through the fitting of the distribution, the validation works of Q–Q plot, P–P plot, and histogram were performed. From the validation, the SCD was best for natural sand and the LND was best for substituting materials. However, this is the result from an experiment; therefore, it should be focused on the process when someone follows this method, because the aggregate conditions are different case by case.

2. From the simulation results, the statistical approach was reflected in the simulation well. However, a condition should be strictly kept by discretizing the mixed distribution with the MC method.

3. It was demonstrated that reflecting the established mixed distribution brought realistic simulation results in the particle simulation. This was confirmed by comparing the CT results and the simulated section.

4. In a further study, pore simulation should be considered. The possibility was confirmed that this study can be combined with fractal theory. The existing fractal theory showed a simple simulation method; therefore, it can be combined with this study.
Author Contributions: Conceptualization, B.H.W. and H.G.K.; methodology, B.H.W.; validation, B.H.W., H.L. and J.B.L.; data curation, B.H.W. and H.L.; writing—original draft preparation, B.H.W.; visualization, B.H.W. and J.B.L.; supervision, H.L. and H.G.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by R&D Program for Forest Science Technology (grant number 2021357B10-2123-AC03) provided by Korea Forest Service (Korea Forestry Promotion Institute).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgment: This research was funded by R&D Program for Forest Science Technology (grant number 2021357B10-2123-AC03) provided by Korea Forest Service (Korea Forestry Promotion Institute).

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

| Acronym | Description |
|---------|-------------|
| AI      | Artificial intelligence |
| BD      | Beta distribution |
| GD      | Gamma distribution |
| GPR     | Gaussian process regression |
| LND     | Log normal distribution |
| MC      | Monte Carlo |
| ND      | Normal distribution |
| PDF     | Probability density function |
| P–P     | Probability–probability |
| PSD     | Particle size distribution |
| Q–Q     | Quantile–quantile |
| SCD     | Standard Cauchy distribution |

Symbols

| Symbol | Description |
|--------|-------------|
| α, β   | Parameter of BD and GD |
| d_\text{eq} | Equivalent diameter |
| d_\text{max} | Largest diameter |
| d_\text{min} | Smallest diameter |
| ε_p    | Enlargement factor |
| f      | Flatness factor |
| μ      | Mean of data in ND |
| σ^2    | Variance of data in ND |
| ν_{i1}, ν_{i2} | Random variable for realization with the range of 0 to 1 |
| r_1, r_3 | Principal radius of the ellipsoid |
| x      | Data |

References

1. Hu, Z.; Kong, Z.; Cai, G.; Li, Q.; Guo, Y.; Su, D.; Liu, J.; Zheng, S. Study of the properties of full component recycled dry-mixed masonry mortar and concrete prepared from construction solid waste. *Sustainability* 2021, 13, 8385.

2. Suchithra, S.; Jayashree, S. A review on recent developments in the recycled aggregate concrete. In Proceedings of AIP Conference Proceedings; 2021, p. 100001.

3. Yehia, S.; Helal, K.; Abusharkh, A.; Zaher, A.; Istaitiyeh, H. Strength and durability evaluation of recycled aggregate concrete. *Int. J. Concr. Struct. Mater.* 2015, 9, 219–239.

4. Manju, K.; Kumar, D.; Kumar, N.; Kumar, S.N. Behaviour of concrete by using artificial aggregates a review. *Eng. Technol.* 2018, 7, 255–258.

5. George, G.K.; Revathi, P. Production and Utilisation of Artificial Coarse Aggregate in Concrete—a Review. In Proceedings of IOP Conference Series: Materials Science and Engineering; Kerala, India, 14–15 July 2020, p. 012035.

6. Onprom, P.; Chaimoon, K.; Cheerarot, R. Influence of bottom ash replacements as fine aggregate on the property of cellular concrete with various foam contents. *Adv. Mater. Sci. Eng.* 2015, 1-11, https://doi.org/10.1155/2015/381704.

7. Singh, N.; Mithulraj, M.; Arya, S. Influence of coal bottom ash as fine aggregates replacement on various properties of concretes: A review. *Resour. Conserv. Recycl.* 2018, 138, 257–271.
15. Achenbach, M.; Lahmer, T.; Morgenthal, G. Identification of the thermal properties of concrete for the temperature calculation of concrete slabs and columns subjected to a standard fire—Methodology and proposal for simplified formulations.

16. Liu, Y.; Song, Y.; Keller, J.; Bond, P.; Jiang, G. Prediction of concrete corrosion in sewers with hybrid Gaussian processes regression model. *RSC Adv.* **2017**, *7*, 30894–30903.

17. Vasudevan, S.; Ramos, F.; Nettleton, E.; Durrant-Whyte, H. Gaussian process modeling of large-scale terrain. *J. Field Robot.* **2009**, *26*, 812–840.

18. Unger, J.F.; Eckardt, S. Multiscale modeling of concrete. *Arch. Comput. Methods Eng.* **2011**, *18*, 341–393.

19. Lee, E.; Ko, J.; Yoo, J.; Park, S.; Nam, J. Analysis of the aggregate effect on the compressive strength of concrete using dune sand. *Appl. Sci.* **2021**, *11*, 1952.

20. Mahawish, A.; Bouazza, A.; Gates, W.P. Effect of particle size distribution on the bio-cementation of coarse aggregates. *Acta Geotech.* **2018**, *13*, 1019–1025.

21. Jakhani, S.H.; Kim, H.G.; Jeon, I.K.; Ryoo, J.S. Effect of saturated tea waste and perlite particles on early age hydration of high-strength cement mortars. *Materials* **2019**, *12*, 2269.

22. Guan, J.; Yuan, P.; Hu, X.; Qing, L.; Yao, X. Statistical analysis of concrete fracture using normal distribution pertinent to maximum aggregate size. *Theor. Appl. Fract. Mech.* **2019**, *101*, 236–253.

23. Campos, J.A.G. Distributional assumptions in educational assessments analysis: Normal distributions versus generalized beta distribution in modeling the phenomenon of learning. *Proc. Soc. Behav. Sci.* **2013**, *106*, 886–895.

24. Wang, J.; Zheng, W.; Zhao, Y.; Zhang, X. Prediction of concrete failure time based on statistical properties of compressive strength. *Appl. Sci.* **2020**, *10*, 815.

25. Balakrishnan, N.; Nevzorov, V.B. *A Primer on Statistical Distributions*; John Wiley & Sons: Hoboken, NJ, USA, 2004.

26. Diamond, S.; Dolch, W.L. Generalized log-normal distribution of pore sizes in hydrated cement paste. *J. Coll. Interface Sci.* **1972**, *38*, 234–244.

27. Huang, J.; Li, W.; Huang, D.; Wang, L.; Chen, E.; Wu, C.; Wang, B.; Deng, H.; Tang, S.; Shi, Y. Fractal analysis on pore structure and hydration of magnesium oxysulfate cements by first principle, thermodynamic and microstructure-based methods. *Fractal Fract.* **2021**, *5*, 164.