Alpha clustering and condensation in nuclei

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Abstract. Low density states near the 3α and 4α breakup thresholds in ¹²C and ¹⁶O, respectively, are discussed in terms of the α-particle condensation. Calculations are performed in OCM (Orthogonality Condition Model) and THSR (Tohsaki-Horiuchi-Schuck-Röpke) approaches. The 0⁺2 state in ¹²C and the 0⁺6 state in ¹⁶O are shown to have dilute density structures and give strong enhancement of the occupation of the S-state c.o.m. orbital of the α-particles. The possibility of the existence of α-particle condensed states in heavier nα nuclei is also discussed.

1. Introduction

It is well established that α-clustering plays a very important role for the structure of lighter nuclei [1, 2]. The importance of α-cluster formation has also been discussed in infinite nuclear matter, where α-particle type condensation is expected at low density [3]. In nuclei the bosonic constituents always are only very few in number, nevertheless possibly giving rise to clear condensation characteristics, as is well known from nuclear pairing [4]. Concerning α-particle condensation, the Hoyle state, i.e. the 0⁺2 state in ¹²C has clearly been established. Several papers of the past [5, 6, 7, 8] and also more recently [9, 10, 11, 12, 13] have by now established beyond doubt that the Hoyle state, only having about one third of saturation density, can be described, to good approximation, as a product state of three α-particles, condensed, with their c.o.m. motion, into the lowest mean field 0S-orbit [14, 15]. This shall be the definition of an α-particle condensed state in finite nuclei, clearly reflecting the situation found in infinite matter in Ref. [3]. The establishment of this novel aspect of the Hoyle state naturally leads us to the speculation about 4α-particle condensation in ¹⁶O.

In the present contribution, a brief review of the status of work on the Hoyle state is given, and the recent progress on the investigation of 4α-particle condensate state in ¹⁶O is shown.

2. Description of the Hoyle state as the α condensate

Based on the investigations of the possibility of α-particle condensation in low-density nuclear matter [3], the present authors proposed a conjecture that near the nα threshold in self-conjugate 4n nuclei there exist excited states of dilute density which are composed of a weekly interacting
gas of self-bound α particles and can be considered as an nα condensed state [9]. This conjecture was backed by examining the structure of $^{12}$C and $^{16}$O using a new α-cluster wave function of the α-cluster condensate type, which has been denoted as THSR (Tohsaki-Horiuchi-Schuck-Röpke) wave function, as follows: $\Phi_{n\alpha} \propto A \prod_{i=1}^{n} \exp(-2(\mathbf{X}_i - \mathbf{X}_G)^2/B^2)\phi(\alpha_i)$. Here $\phi(\alpha_i)$, $\mathbf{X}_i$, $\mathbf{X}_G$, and $B$ are the internal wave functions, c.o.m. coordinates, of the $i$-th alpha cluster, the total c.o.m. coordinate, and a variational parameter, respectively. It should be noted that this wave function contains two limits exactly: the one of a shell-model relevant at higher densities and the one of an ideal α-particle condensate in the dilute limit [9]. An interesting analysis of the applicability of the THSR wave function can be performed by comparing with stochastic variational calculations [14] and OCM calculations [15]. The α density matrix $\rho(\mathbf{r}, \mathbf{r}')$ is diagonalized to study the single-α orbits and occupation probabilities in $^{12}$C states. FIG. 1 shows the occupation probabilities of the L-orbits with $S$, $D$ and $G$ waves belonging to the $k$-th largest occupation number (denoted by $L_k$), for the ground and Hoyle state of $^{12}$C obtained by diagonalizing the density matrix $\rho(\mathbf{r}, \mathbf{r}')$. We found that in the Hoyle state the α-particle $S$ orbit with zero node is occupied to more than 70 % by the three α-particles (see also Ref. [14] and Fig. 1). Taking into account the finite size of the nucleus, a reduction of the condensate fraction from 100 % to about 70 % is not surprising, and the remaining fraction (about 30 %) is due to higher orbits originating from antisymmetrization among nucleons. This huge percentage means that an almost ideal α-particle condensate is realized in the Hoyle state.

### Figure 1. Occupation of the single-α orbitals of the ground state of $^{12}$C compared with the Hoyle state [15]. For explanation see the text.

#### 3. $4\alpha$ condensation in $^{16}$O

The situation in $^{16}$O with respect to cluster states is already much more complicated than in $^{12}$C. In $^{12}$C, exciting one α-particle out of the ground state necessarily leads to a dilute α-gas state, since the remaining nucleus, $^{8}$Be, is itself a loosely bound two α-object. On the other hand exciting an α-particle out of the $^{16}$O ground state can lead to multiple α+$^{12}$C configurations. It is well documented [17] that the $0^+_2$ state at 6.06 MeV is an α-particle orbiting in a 0S-wave around the ground state of $^{12}$C. However, there are many more possibilities. The α particle can be in higher nodal $S$-wave, the α can orbit in a 0D-wave around the $2^+_1$ of $^{12}$C and couple to a 0$^+$ state. It also can orbit in an odd parity wave around the 1$^-$ and 3$^-$ states in $^{12}$C, etc. The $^{12}$C can also be in the Hoyle state which then leads us to the four α gas state in $^{16}$O, which is the state of our interest. Experimentally, there are six $0^+$ states in $^{16}$O up to around the four α
disintegration threshold at 14.4 MeV: the ground state $0^+_1$, $0^+_2$ at 6.06 MeV, $0^+_3$ at 12.05 MeV, $0^+_4$ at 13.6 MeV [18], $0^+_5$ at 14.01 MeV, and $0^+_6$ at 15.1 MeV.

Figure 2. (a): Comparison of energy spectra between experiments [23] and 4α OCM calculation [19]. The $0^+_4$ state in experiment is given in Ref. [18]. (b): Single-α orbits with $L = 0$ belonging to the largest occupation number, for the ground and $0^+_6$ states. The radial part of the orbits are shown.

In analogy to the above mentioned OCM calculation for $^{12}$C [15], we recently performed a quite complete OCM calculation also for $^{16}$O, including many of the cluster configurations [19]. We were able to reproduce the full spectrum of the observed $0^+$ states. As shown in FIG. 2(a), we tentatively make a one to one correspondence of our states with the six lowest $0^+$ states of the experimental spectrum. In view of the complexity of the situation, the agreement can be considered as very satisfactory. Considering the properties of the various states in more detail, a certain number of rather big surprises came up. The analysis of the diagonalisation of the α-particle density matrix $\rho(r, r')$ (as was done in [14, 15, 20, 21]) showed that the newly discovered $0^+_5$ state at 13.6 MeV [18], as well as the well known $0^+_6$ state at 14.01 MeV, corresponding to our states at 12.6 MeV and 14.1 MeV, respectively, have, contrary to what we assumed previously [22], very little condensate occupancy of the $0^S$-orbit (about 20 %). On the other hand, the sixth $0^+$ state at 16.5 MeV calculated energy, to be identified with the experimental state at 15.1 MeV, has 61 % of the α-particles being in the $0^S$-orbit. The corresponding single-α $0^S$ orbit is shown in FIG. 2(b). It has a strong spatially extended behaviour, and in the interior no node ($0^S$), showing almost no effect of the Pauli principle acting on nucleons belonging to different alpha particles. This indicates that α particles are condensed into the very dilute $0^S$ single-α orbit, see also Ref. [24]. Thus, the $0^+_6$ state clearly has a 4α condensate character. We should note that the orbit is very similar to the single-α orbit of the Hoyle state [14, 15]. We also show in FIG. 2(b) the single-α orbit for the ground state. It has maximum amplitude at around 3 fm and oscillations in the interior with two nodal (2S) behaviour, due to the Pauli principle and reflecting the shell-model configuration. The $0^+_6$ state also has a very large radius of 5.6 fm, though this value may be somewhat over estimated because of the 10 % too high energy of the $0^+_6$ state.

4. Conclusion and remarks
Multiple successful theoretical investigations, concerning the Hoyle state in $^{12}$C, have established, beyond doubt, that it is a dilute gas-like state of three α-particles, held together only by the Coulomb barrier, and describable very well by a wave function of the THSR type, where the
three bosons are condensed into the 0S-orbital. There is no objective reason, why in \( ^{16}\text{O}, ^{20}\text{Ne}, \cdots \) there should not exist similar “Hoyle”-like states. At least the calculations with THSR and OCM approaches show this to be the case, systematically. In this contribution, we give results of a complete OCM calculation which reproduces the six first 0\(^+\) states of \( ^{16}\text{O} \) to rather good accuracy. In that calculation the 0\(^+_0\) state at 16.5 MeV, to be identified with the experimental 0\(^+_0\) state at 15.1 MeV, shows the characteristics typical for the Hoyle state.

An interesting question is how many \( \alpha \)’s can maximally be in a self bound \( \alpha \)-gas state. In this respect, a schematic investigation in an \( \alpha \)-gas mean field calculation of the Gross-Pitaevskii type was performed \([25]\). Because of the increasing Coulomb repulsion, the Coulomb barrier fades away and our estimate yields a maximum of about eight \( \alpha \)-particles that can be held together in a condensate. However, a few extra neutrons can have a strong additional binding effect (see \( ^9\text{Be} \) and \( ^{10}\text{Be} \) \([26, 27]\)) and may stabilize larger condensates.

We should remark that an interesting idea concerning \( \alpha \)-particle condensates was put forward by W. von Oertzen and collaborators \([28, 29]\). \( \alpha \)-particles outside a strongly bound core (e.g. \( ^{40}\text{Ca} \)) can form a condensate at the multi-\( \alpha \)-particle threshold \([28]\). For the condensate with a fixed particle number, the emission of two \( \alpha \)’s and three \( \alpha \)’s must be enhanced. In fact the observation of the emission of \( ^{12}\text{C} \) in the state from the compound nucleus \( ^{52}\text{Fe} \) has been observed \([30]\) and a very strong deviation from statistical model predictions is observed. Also interesting recent experimental works on loosely bound \( \alpha \)-structures in light nuclei have been performed \([31, 32, 33]\).

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