Multi-frequency Electromagnetic Tomography for Acute Stroke Detection Using Frequency-Constrained Sparse Bayesian Learning

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Abstract—Imaging the bio-impedance distribution of the brain can provide initial diagnosis of acute stroke. This paper presents a compact and non-radiative tomographic modality, i.e. multi-frequency electromagnetic tomography (mfEMT), for the initial diagnosis of acute stroke. The mfEMT system consists of 12 channels of gradiometer coils with adjustable sensitivity and excitation frequency. To solve the image reconstruction problem of mfEMT, we propose an enhanced Frequency-Constrained Sparse Bayesian Learning (FC-SBL) to simultaneously reconstruct the conductivity distribution at all frequencies. Based on the Multiple Measurement Vector (MMV) model in the Sparse Bayesian Learning (SBL) framework, FC-SBL can recover the underlying distribution pattern of conductivity among multiple images by exploiting the frequency constraint information. Both simulations and experiments were conducted to evaluate the performance of FC-SBL. Results showed that the proposed FC-SBL method is robust to noisy data for image reconstruction problems of mfEMT compared to the single measurement vector model, which is promising to detect acute stroke in the brain region with enhanced spatial resolution and in a calibration-free manner.

Index Terms—Acute stroke, electromagnetic tomography, multi-frequency, multiple measurement model, sparse Bayesian learning.

I. INTRODUCTION

ROKE is the second most common cause of death worldwide, and the third most common cause of disability [1]. There is a significant increase in stroke burden across the world, especially in developing countries. There are two types of strokes: ischaemic, and hemorrhagic. Among them, around 80 out of 100 are ischaemic strokes [2]. Luckily, it is now possible to treat acute stroke with thrombolytic drugs but it must be executed within 3-6 hours of stroke onset. Brain imaging must be conducted before treatment, in order to differentiate these two strokes as the thrombolytic drugs would worsen the case of hemorrhagic stroke [3]. Existing imaging techniques for stroke diagnosis include Positron Emission Tomography (PET), diffusion/perfusion-weighted Magnetic Resonance Imaging (MRI), and Computed Tomography (CT) [4]. But on this occasion, their applications are restrained due to long diagnosis time or limited accessibility. A compact, fast, and cost-effective solution for early acute stroke detection is highly desirable.

In the recent decade, Electrical Impedance Tomography (EIT) has been investigated for acute stroke detection through indirectly imaging the bio-impedance change induced by acute stroke [3], [5]–[7]. However, the presence of highly resistive skull blocks the excitation current flowing through the head. In addition, skin-to-electrode contact impedance varies with surface conditions which is unpredictable [8]. These issues make the practical application of EIT in acute stroke detection very challenging.

Multi-frequency Electromagnetic Tomography (mfEMT) is a non-contact and non-invasive imaging technique [9]. In contrast to EIT that applies excitation current through contact electrodes, mfEMT employs inductive coils to generate magnetic fields based on the eddy current effect. Thus, it can penetrate the highly resistive skull easily. With mfEMT, bio-impedance can be measured by multi-frequency excitation spreading across the bandwidth of interest. There are two imaging modes of mfEMT, i.e. Time-Difference (TD) imaging and Frequency-Difference (FD) imaging. TD imaging requires a before-lesion dataset and a measurement dataset. For acute stroke imaging, the before-lesion dataset cannot be obtained as patients present after the event. FD imaging measures multi-frequency data in a short time interval without a reference dataset, and uses the differences between selected frequencies for imaging, which makes it more promising for calibration-free acute stroke imaging.

Thus far, the potential application of mfEMT for intracranial hemorrhage detection has been preliminarily investigated in [8], [10], [11]. But these work is mainly based on analytical models and simulation data. There are still needs to (i) perform proof-of-concept validation on a feasible experimental platform, and (ii) develop high-resolution image reconstruction algorithms to effectively visualize small abnormality.

Motivated by this, in this paper, we first report a 12-channel mfEMT system and then propose a FD image reconstruction approach named Frequency-Constrained Sparse Bayesian Learning (FC-SBL) for calibration-free, small abnormality detection under noisy scenarios. Sparse Bayesian Learning (SBL) has attracted attention in recent years for solving the inverse problem [12]–[17]. The fundamental idea of FC-SBL is to exploit the correlation among images under a set
of excitation frequencies by extending the SBL framework. Mathematically, such correlations lead to a constrained optimization problem that promotes the signal’s group-sparsity and its rank-deficiency. We design a frequency-constrained block sparsity prior that incorporates both the frequency and spatial correlation of conductivity distribution. The signal and noise statistics are learned directly from the data by sparse Bayesian learning. We then demonstrate the image reconstruction improvement of the proposed method through both simulations and experiments.

The paper is organized as follows. In Section II we present the sensing principle and measurement system of mfEMT. In Section III we formulate two fundamental problems of mfEMT, i.e. the forward and inverse problem. Then in Section IV we present the FC-SBL algorithm to solve the inverse problem. Section V gives experimental results and discussions. Finally, Section VI concludes the paper.

II. mfEMT SYSTEM

A. Sensing Principle

mfEMT images the conductivity of an object by measuring the mutual inductance between coils placed around its periphery. The object is excited in a magnetic field (primary field) produced by a current flowing in a coil. A resulting electrical field in the object is generated to induce eddy current. Consequently, a secondary magnetic field can be measured externally. Our previous work reported a gradiometer coil with significantly improved sensitivity [9] to measure the secondary field. The sensitivity of the gradiometer coil is governed by:

\[ s_g = \frac{\Delta \varphi_g}{\Delta \sigma} = -\frac{V_0 (P_1 - P_2) \omega \mu_0}{V_{res}} \]  

where \( \Delta \varphi_g \) denotes the phase response of the secondary field caused by conductivity change \( \Delta \sigma \). \( V_0 \) is the voltage caused by the primary magnetic field. \( P_1 \) and \( P_2 \) are the geometrical factors concerning the size and shape of the object and its position relative to the coil. \( \omega \) is the excitation frequency and \( \mu_0 \) is the permeability in free space. \( V_{res} \) is a key parameter of sensitivity which is tuned by the residual voltage of two differential receiver coils.

B. System Structure

The developed mfEMT system comprises 4 modules (see Fig.1): (1) sensor array consisting of 12 gradiometer coils; (2) excitation module to drive gradiometer coils; (3) front-end circuit and data acquisition modules based on Red Pitaya, an open-source hardware platform with dual ADCs and DACs [18]; (4) phase measurement by Fast Fourier Transform (FFT) and image reconstruction.

The operation principle of the mfEMT system is as follows. First, one of the 12 excitation channels is enabled and the rest of the other 11 excitation channels are disabled as open circuits. Second, multi-frequency sine waves are generated by Red Pitaya. Then, the 12 differential sensing coils are sequentially selected and the sensing signals are multiplexed to be acquired. One frame of data covering all the excitation/sensing coil combinations consists of 144 (12 × 12) measurements.

III. FORWARD AND INVERSE PROBLEM OF mfEMT

A. Forward Problem

Two problems of mfEMT need to be solved, i.e. the forward problem and inverse problem. The forward problem is to determine measurements (phase values) given the conductivity distribution [19], as expressed by:

\[ \varphi = F(\sigma) + \nu \]  

where \( \sigma \in \mathbb{R}^M \) is the conductivity distribution; \( \varphi \in \mathbb{R}^N \) represents the noisy measurements, and \( N \ll M \); \( \nu \) is the noise vector; \( F \) is a nonlinear function mapping the conductivity distribution to measurements. A simplified linear model is commonly used:

\[ \Delta \varphi \approx \frac{\partial F(\sigma)}{\partial \sigma} \Delta \sigma + \nu = J \cdot \Delta \sigma + \nu \]  

The sensitivity matrix \( J \in \mathbb{R}^{N \times M} \) (see Fig.2(a)) maps the conductivity distribution to measurements, which is solved by [20]:

\[ J(\Omega) = \frac{k}{I_1 I_2} (B_1 \cdot B_2) \]  

where \( \Omega \) is the spatial coordinates; \( B_1 \) is the magnetic field produced by a current \( I_1 \) injected into the excitation coil; \( B_2 \) is the magnetic field produced by a current \( I_2 \) injected into the differential sensing coil; \( k \) is a coefficient.

B. Inverse Problem

Image reconstruction of mfEMT is a typical inverse problem. The objective is to estimate \( \sigma \) from \( \varphi \). A general optimization framework can be formulated as:

\[ \hat{\sigma} = \arg \min_{\sigma \in \mathbb{R}^M} \{ d(\sigma) + r(\sigma) \} \]  

where \( d(\sigma) \) is the data-fidelity term that penalizes the mismatch to the measurements and \( r(\sigma) \) is the regularizer that imposes a prior information of the conductivity. Two common regularizers include the spatial sparsity-promoting penalty \( r(\sigma) \equiv \| \sigma \|_{\ell_1} \) and Total Variation penalty \( r(\sigma) \equiv \| D \sigma \|_{\ell_1} \), where \( D \) is a discrete gradient operator [21].

In this work, we turn to a statistical perspective on sparsity-promoting regularization. Optimization (5) is interpreted in a Bayes perspective within the SBL framework to maximize the posterior \( p(\sigma|\varphi) \):

\[ \arg \max_{\sigma} p(\sigma|\varphi) \triangleq \arg \min_{\sigma} \left[ -\log p(\varphi|\sigma) - \lambda \log p(\sigma) \right] \]  

where, the data-fidelity term \( d(\sigma) = -\log p(\varphi|\sigma) \) models the data likelihood. This term encapsulates the physics model for generation of measurement \( \varphi \). \( r(\sigma) = -\lambda \log p(\sigma) \) is the regularization term by using prior \( p(\sigma) \) that is elaborately handcrafted or learned in accordance with the prior knowledge of \( \sigma \). The SBL framework seeks to recover a probabilistic distribution instead of deterministic values. Compared to the conventional approaches, it is capable of quantifying and learning the uncertainty and the statistical properties of data and noise [15], [22], making it more robust to noise.
IV. IMAGE RECONSTRUCTION WITH FC-SBL

A. MULTI-FREQUENCY MEASUREMENT PREPROCESSING

Given that we acquire a sequence of multi-frequency measurements \{φ_{f_0}, φ_{f_1}, \ldots, φ_{f_L}\} of mfEMT. The measurement at the lowest frequency \(φ_{f_0}\) is selected as a reference. Generally, both the target object (acute stroke) and the background (other matters in the brain) are frequency-dependent. Thus, simple frequency difference \(Δφ_{f_i} = φ_{f_i} - φ_{f_0}\) cannot cancel out the background signal. Assume that there exists a small stroke inside a large background. Due to the size difference, the frequency-dependent local change should be different from the frequency-dependent global change. Decompose \(φ_{f_i}\) by

\[
φ_{f_i} = φ_{f_0} + \Delta φ_{bg} + \Delta φ_{obj}
\]

where \(Δφ_{bg}\) is the phase response induced by the background, whereas \(Δφ_{obj}\) by the target objects. Since the background is assumed to be homogeneously conductive, it could be represented by \(Δφ_{bg} = α_iJ1\). Therefore, the variance induced by the target is

\[
Δφ_{obj} = (φ_{f_i} - φ_{f_0}) - α_iJ1
\]

where \(α_i = \frac{(φ_{f_i} - φ_{f_0})}{J1}\) is a projection coefficient (see Fig. 2b); \(\langle \cdot \rangle\) denotes the inner product. In consequence, the weighted differences of multiple measurements are:

\[
\begin{align*}
Δφ_{f_1} &= (φ_{f_1} - φ_{f_0}) - α_1J1 \\
Δφ_{f_2} &= (φ_{f_2} - φ_{f_0}) - α_2J1 \\
\vdots \\
Δφ_{f_L} &= (φ_{f_L} - φ_{f_0}) - α_LJ1
\end{align*}
\]

As revealed in (1), the sensitivity of the gradiometer coil is linearly proportional to the excitation frequency. To use one identical sensitivity matrix \(J\) for algorithm development, the measurements are further normalized in accordance with frequency:

\[
Δφ_{norm} = [Δφ_{f_1}, \ldots, Δφ_{f_L}] \left[ \begin{array}{c} f_1 \\
0 \\
\vdots \\
0 \\
f_L \end{array} \right]^{-1}
\]

Thereafter, we use \(\{y_i\}, i = 1 \ldots, L\) to denote \(Δφ_{norm}\).

B. FC-SBL

In mfEMT, a sequence of measurements are acquired and then preprocessed as described in Section IV-A. Each measurement is governed by the same equation,

\[
\begin{align*}
y_1 &= Jσ_1 + v_1 \\
y_2 &= Jσ_2 + v_2 \\
\vdots \\
y_L &= Jσ_L + v_L
\end{align*}
\]

where \(\{y_1, y_2, \ldots, y_L\}, y_i \in \mathbb{R}^{N \times 1} (i = 1, 2, \ldots, L)\) denotes a sequence of preprocessed measurements; \(J \in \mathbb{R}^{N \times M} (N \ll M)\) represents the sensitivity matrix; \(\{σ_1, σ_2, \ldots, σ_L\}, σ_i \in \mathbb{R}^{M \times 1} (i = 1, 2, \ldots, L)\) are the conductivity to be solved and \(\{v_1, v_2, \ldots, v_L\}, v_i \in \mathbb{R}^{N \times 1} (i = 1, 2, \ldots, L)\) are noise.

To exploit the frequency constraints of mfEMT measurements, we extend (1) to the MMV model (8):

\[
Y = Jσ + V
\]

where \(Y = [y_1, y_2, \ldots, y_L] \in \mathbb{R}^{N \times L}, \sigma = [σ_1, σ_2, \ldots, σ_L] \in \mathbb{R}^{M \times L}, V = [v_1, v_2, \ldots, v_L] \in \mathbb{R}^{N \times L}\). Each column of \(σ\) corresponds to a measurement at a given frequency and each row represents a pixel (conductivity) in the image.

Since the block partition of spatial conductivity is unknown, without requiring any prior knowledge of the block distribution pattern, we consider a general case that 2D square blocks with equal size of \(h \times h\) overlap with each other (see Fig. 2c). This block structure is different from the 1D block in previous literatures [13], [15]. Note that though the resulting algorithms are not very sensitive to the block structure (1D or 2D) since real partition can be learned during the SBL process, empirical evidence showed that algorithmic performance can be further improved with 2D blocks. A 2D block partition structure is embedded with a matrix \(A\):

\[
\sigma_j \triangleq Ax_j \triangleq \begin{bmatrix} A_1 & \cdots & A_g \end{bmatrix} \begin{bmatrix} x_{i1}^T, \ldots, x_{ij}^T, \ldots, x_{ik}^T \end{bmatrix}^T = x_{ij}^T
\]

where \(j = 1, \cdots, L; g \approx M\) is the total number of possible blocks in \(σ_j, \forall i = 1, 2, \ldots, g, A_i \triangleq [e_{b_1}, \cdots, e_{b_h}] \in \mathbb{R}^{M \times h}\) is the \(i\)-th block structure; \(e_{b_k} = [0, \cdots, 0, 1, 0, \cdots, 0]^T\) is a base vector where the 1 appears at the \(b_k\)-th position, \(k = 1, 2, \cdots, h\). \(\{b_k\}\) is a set recording the pixel index of each element in the \(i\)-th block. \(x_{ij} = [x_{(ih+1)j}, \ldots, x_{(ih+j)}]^T \in \mathbb{R}^{1 \times h}\) is a slice recording the 1D block signal of the \(i\)-th block.
\( \mathbb{R}^{h \times 1} \) denotes the weights of each element in the block.

Then, the spatial block-sparse underlying model in [12] is further expressed as
\[
Y = J\sigma + V = \Phi X + V \quad (14)
\]
where \( \Phi = JA; X = [x_{1}, \ldots, x_{j}, \ldots, x_{L}] \in \mathbb{R}^{gh \times L} \). Note from Fig. [2] (d) that \( X \) has both inter-row (spatial block) and inter-column (frequency constraints) correlation. We can formulate this structure into a block matrix form:
\[
X = \begin{bmatrix}
X_{[1]}^T, X_{[2]}^T, \ldots, X_{[g]}^T
\end{bmatrix}^T \quad (15)
\]
\( \forall i = 1, \ldots, g, X_{[i]} \in \mathbb{R}^{h \times L} \) denotes the \( i \)-th block of all the column. It incorporates the spatial and frequency information of multiple measurements into each block matrix. This is the core idea of the FC-SBL method.

To solve \( X \) in (14), we reformulate the MMV model to frequency-constrained block SMV model [24]:
\[
y_{F} = D x_{F} + v_{F} \quad (16)
\]
by letting \( y_{F} = \text{vec} (Y^{T}) \in \mathbb{R}^{N_{L} \times 1}, D = \Phi \otimes I_{L} \in \mathbb{R}^{N_{L} \times ghL}, x_{F} = \text{vec} (X^{T}) \in \mathbb{R}^{ghL \times 1}, v_{F} = \text{vec} (V^{T}) \in \mathbb{R}^{ghL \times 1} \). \( \otimes \) represents the Kronecker product of the two matrices. \( \text{vec}(\cdot) \) denotes the vectorization of the matrix by stacking its columns into a single vector column.

To exploit the spatial block correlation and frequency-constrained information simultaneously, we design a covariance matrix \( \Sigma_{0} \) as a prior of the weights \( x_{F} \) using a zero-mean Gaussian distribution
\[
p (x_{F}; \{\gamma_{i}, C_{i}\}_{i=1}^{g}) \sim \mathcal{N}_{x_{F}} (0, \Sigma_{0}) \quad (17)
\]
where \( \Sigma_{0} \) is
\[
\Sigma_{0} = \begin{bmatrix}
\gamma_{1}C_{1} & 0 & \cdots & 0 \\
0 & \ddots & \cdots & 0 \\
0 & \cdots & \gamma_{g}C_{g}
\end{bmatrix} \in \mathbb{R}^{ghL \times ghL} \quad (18)
\]
\( \Sigma_{0} \) is a block diagonal matrix. \( \gamma_{i} \geq 0 (i = 1, \ldots, g) \), most of which are zeros, determines the sparsity pattern of \( X \). \( C_{i} \in \mathbb{R}^{hL \times hL} (i = 1, \ldots, g) \) is a covariance matrix.

It is suggested that for many applications, the intra-block elements can be sufficiently represented by a first-order Auto-Regressive (AR) process to model the correlation [25]. [26] and thus, the corresponding covariance matrix is a Toeplitz matrix. Since the covariance matrix \( \{C_{i}\}_{i=1}^{g} \) incorporate the spatial and frequency information together, we model it with two different AR processes, i.e., \( \{A_{i}\}_{i=1}^{g} = \text{Toeplitz} (1, r_{s}, \ldots, r_{s}^{h-1}) \) for spatial correlation; \( B = \text{Toeplitz} (1, r_{f}, \ldots, r_{f}^{L-1}) \) for frequency constraints. Thus, \( C_{i} \) is regularized as the Kronecker product of two Toeplitz matrices (not commutative):
\[
C_{i} = A_{i} \otimes B \quad (19)
\]
In this way, the resulting prior \( \Sigma_{0} \) is written as:
\[
\Sigma_{0} = \begin{bmatrix}
\gamma_{1}A_{1} & 0 & \cdots & 0 \\
0 & \ddots & \cdots & 0 \\
0 & \cdots & \gamma_{g}A_{g}
\end{bmatrix} \otimes B = \Pi \otimes B \quad (20)
\]
Moreover, we assume that the elements in the noise vector \( \{v_{i}\} \) are independent and each has a Gaussian distribution i.e., \( p (v_{i}) \sim \mathcal{N}_{v_{i}} (0, \lambda I) \). Considering the frequency constraints \( B \) in \( v_{F} \), we get \( p (v_{F}) \sim \mathcal{N}_{v_{F}} (0, \lambda I \otimes B) \).

By imposing the prior in (20), we solve [16] obeying the Bayesian rule to optimize the posterior of \( p (x_{F} | y_{F}) \), i.e.,
\[
p (x_{F} | y_{F}; \lambda, \{\gamma_{i}, A_{i}\}_{i=1}^{g}, B) \sim \mathcal{N}_{x_{F} | y_{F}} (\mu_{x}, \Sigma_{x}) \quad (21)
\]
where \( \mu_{x}, \Sigma_{x} \) are given by:
\[
\mu_{x} = \Sigma D^{T} (\lambda I \otimes B)^{-1} y_{F} = \text{vec} \left( Y^{T} \left( \lambda I + \Phi \Pi \Phi^{T} \right)^{-1} \Phi \Pi \right) \quad (22)
\]
\[
\Sigma_{x} = \left( (\Pi \otimes B)^{-1} + D^{T} (\lambda I \otimes B^{-1} D)^{-1} \right)^{-1} = \left[ \Pi - \Pi \Phi^{T} (\lambda I + \Phi \Pi \Phi^{T})^{-1} \Phi \Pi \right] \otimes B \quad (23)
\]
The final estimation of conductivity is:
\[
\hat{\sigma} = A \mu_{x} = A \Pi \Phi^{T} (\lambda I + \Phi \Pi \Phi^{T})^{-1} Y \quad (24)
\]
where \( \mu_{x} \) is obtained by reshaping \( \mu_{x} \) to \( \mathbb{R}^{ghL \times 1} \) dimension. The estimation of hyperparameter \( \Theta = \{\lambda, \{\gamma_{i}, A_{i}\}_{i=1}^{g}, B\} \) is the main body of FC-SBL.

C. Hyper Parameter Estimation

The hyper parameters \( \Theta = \{\lambda, \{\gamma_{i}, A_{i}\}_{i=1}^{g}, B\} \) can be estimated by a Type II maximum likelihood procedure [27].
Algorithm 1: Frequency-Constrained Sparse Bayesian Learning (FC-SBL) for mfEMT Image Reconstruction

Input: \{\(\mathbf{Y}, \mathbf{J}, h, \epsilon_{\text{min}}, \vartheta_{\text{max}}\)\} \(\triangleright\) multi-frequency measurement vectors \(\mathbf{Y} \in \mathbb{R}^{N \times L}\), sensitivity matrix of mfEMT \(\mathbf{J} \in \mathbb{R}^{N \times M}\), spatial block size \(h\), the minimum error bound \(\epsilon_{\text{min}}\), and the maximum iteration steps \(\vartheta_{\text{max}}\).

Output: \(\hat{\sigma} = \{\sigma_1, \ldots, \sigma_L\}\) \(\triangleright\) reconstructed conductivity distribution of \(L\) frequencies

1: Initialize:
   \(\epsilon = 1, \vartheta = 0, \mu_x = 0_{ghL \times L}, \Sigma_x = 0_{ghL \times ghL}\)
   \(\gamma_i = 1, i = 1, \ldots, g\)
   \(\lambda = 0.01 \times \frac{1}{L-1} \sum_{i=1}^{L} \left( \frac{1}{M-1} \sum_{j=1}^{M} |y_{ji} - \mathbf{Y}_i|^2 \right)\)
   \(\mathbf{A}_i = \text{Toeplitz} \left( \begin{bmatrix} 0.9^{0}, \ldots, 0.9^{h-1} \end{bmatrix} \right), i = 1, \ldots, g\)
   \(\mathbf{B} = \text{Toeplitz} \left( \begin{bmatrix} 0.9^{0}, \ldots, 0.9^{L-1} \end{bmatrix} \right)\)

2: while \(\epsilon > \epsilon_{\text{min}}\) and \(\vartheta < \vartheta_{\text{max}}\) do
   3: Update \(\mu_x\) using (22);
   4: Update \(\Sigma_x\) using (23);
   5: Update \(\{\gamma_i\}_{i=1}^g\) using (26);
   6: Update \(\mathbf{B}\) using (27) and (28);
   7: Update \(r_f\) with (32), regularize \(\mathbf{B}\) in Toeplitz matrix;
   8: Update \(\mathbf{A}_i\) using (27) and (28);
   9: Update \(\mathbf{r}_s\) with (33), regularize \(\mathbf{A}_i\) in Toeplitz matrix;
   10: Update \(\lambda\) using (30);
   11: Estimate \(\epsilon = \left\| \mu_x^{(0)} - \mu_x^{(\vartheta - 1)} \right\|^2 / \left\| \mu_x^{(0)} \right\|^2\)
   12: Iteration update \(\vartheta \leftarrow \vartheta + 1\).
   13: end while

14: return \(\hat{\sigma} = \mathbf{A} \hat{\mu}_x\) \(\triangleright\) \(\mathbf{A} \in \mathbb{R}^{M \times ghL}\) is a predefined matrix of embedding block structure.

yielding the effective cost function:
\[
\mathcal{L}(\theta) = \mathbf{y}^T \Sigma_x^{-1} \mathbf{y} + \log |\Sigma_y| + \text{TR} \sum_{i=1}^{g} \frac{\mu_x^{(0)} - \mu_x^{(\vartheta - 1)}}{\mu_x^{(0)}}
\]
(25)
where \(\Sigma_y = \lambda \mathbf{I} \otimes \mathbf{B} + \mathbf{D}(\mathbf{I} \otimes \mathbf{B}) \mathbf{D}^T\).

By optimizing the cost function with respective to each parameter in \(\theta\), we derive the following learning rules [24], [28]. Let, \(\bar{\mathbf{Y}} \triangleq \mathbf{Y}B^{1/2}\), then
\[
\gamma_i \leftarrow \frac{\text{Tr} \left( \mathbf{X}_i^T \mathbf{B}^{-1} \mathbf{X}_i \mathbf{A}_i^{-1} \right) / L}{\text{Tr} \left( \left( \lambda \mathbf{I} + \Phi \Pi \Phi^T \right)^{-1} \Phi \Pi \mathbf{A}_i \Phi^T \right)}
\]
(26)
\[
\bar{\mathbf{B}} \leftarrow \frac{\sum_{i=1}^{g} \mathbf{X}_i^T \mathbf{A}_i^{-1} \mathbf{X}_i}{\gamma_i} + \eta \mathbf{I}
\]
(27)
\[
\mathbf{B} \leftarrow \left\| \mathbf{B} \right\|_{F}^{-1} \bar{\mathbf{B}}
\]
(28)
\[
\mathbf{A}_i \leftarrow \frac{1}{L} \sum_{l=1}^{L} \left( \mathbf{\Sigma}_i + \hat{\mu}_x \mathbf{F}_i \mathbf{F}_i^T \right) / \gamma_i
\]
(29)
\[
\lambda \leftarrow \frac{1}{NL} \left\| \bar{\mathbf{Y}} - \Phi \hat{\mu} \right\|_F^2 + \frac{1}{N} \sum_{i=1}^{g} \frac{1}{\gamma_i} \text{Tr} \left( \mathbf{\Sigma}_i \mathbf{\Phi}_i^T \mathbf{\Phi}_i \right)
\]
(30)

where \(\Phi_i\) denotes the consecutive columns in \(\Phi\) which correspond to the \(i\)-th block in \(\mathbf{X}\), i.e., \(\mathbf{X}_{[i]}\), \(\mathbf{\Sigma}_i \in \mathbb{R}^{h \times h}\) is the \(i\)-th diagonal block of \(\Sigma\), and
\[
\mathbf{\Sigma} = \Pi - \Pi \Phi^T \left( \lambda \mathbf{I} + \Phi \Pi \Phi^T \right)^{-1} \Phi \Pi
\]
(31)

To mitigate the overfitting problem, estimation of \(\mathbf{B}\) and \(\mathbf{A}_i\) in (28), (29) is further regularized in Toeplitz matrix as [19]. Instead of deriving it from the cost function, we make empirical estimations of \(r_f\) and \(r_s\):
\[
\begin{align*}
\hat{r}_f &= \frac{\text{diag} \left( \mathbf{B} \right)}{\text{diag} \left( \mathbf{B} \right)} \\
\hat{r}_s &= \frac{\text{diag} \left( \mathbf{A}_i \right)}{\text{diag} \left( \mathbf{A}_i \right)}
\end{align*}
\]
(32)
(33)
where \(\text{diag} \left( \cdot \right)\) is the average of the elements along the main diagonal; \(\text{diag} \left( \cdot, 1 \right)\) is the average along the main subdiagonal.

Algorithm 1 summarizes the FC-SBL in pseudo-code.

V. RESULTS AND DISCUSSIONS

In this section, we conducted numerical simulation and phantom experiments to evaluate the performances of the mfEMT system and algorithm. Comparisons with state-of-the-art reconstruction methods, such as Total Variation regularization [29], [30], Adaptive Group Sparsity (AGS) [31], and Structure-Aware Sparse Bayesian Learning (SA-SBL) [15] were presented.

A. Numerical Simulation

We established a phantom model with three abnormalities representing hemorrhagic or ischaemic strokes, as shown in Fig. [3]. The diameters of the sensing region and abnormalities are 120 mm and 10 mm, respectively. Five excitation frequencies are \(\{f_0, f_1, f_2, f_3, f_4\} = [1/16, 1/8, 1/4, 1/2, 1] \times 6.25 MHz\). These frequencies are selected to meet full-cycle sampling in order to avoid spectral leakage in FFT, considering that the sampling rate of the system is 62.5 MHz. The frequency dependent conductivities of the background and target objects are \([\sigma_{f_0}, \sigma_{f_1}, \sigma_{f_2}, \sigma_{f_3}, \sigma_{f_4}] = [0.14, 0.15, 0.17, 0.21, 0.25] S/m\) and \([\sigma_{f_0}, \sigma_{f_1}, \sigma_{f_2}, \sigma_{f_3}, \sigma_{f_4}] = [0.1, 0.15, 0.25, 0.45, 0.7] S/m\).

These parameters are selected in accordance with the conductivity spectral in [32], [33]. The reference frequency is \(f_0\). In order to simulate the background noise of the real system, a same quantity of white noise is added to each measurement. Since the conductivity changes with respect to \(f_0\) increase with frequency, the resulting SNR also increase correspondingly. We added a strong noise to a level that makes SNR \(f_4 = 30 dB\).

Table [4] illustrates the image reconstruction results of the simulated phantom. The first row shows results using Total Variation. Calculations were carried out multiple times to solve each individual measurement. There are obvious artifacts in the images and the shapes of objects are hardly visible. As
for SA-SBL in the second row, the reconstruction process is performed multiple times for each frequency as well. The reconstructed images of SA-SBL are much better. At $f_3$, $f_4$, boundary and shape of objects are restored with less artifacts. Whilst at $f_1$, $f_2$ with smaller conductivity changes, SA-SBL failed to find a satisfactory solution. As indicated from [15], although SA-SBL has stronger regularization compared with other approaches, the performance degrades at low SNR levels (35 dB or less). We can see that the proposed FC-SBL outperforms the others in terms of better edge preservation and shape estimation. This is because FC-SBL exploits the frequency constraints information of multiple measurements and thus, the underlying distribution pattern could be better restored even under strong noise levels.

To quantitatively evaluate the performance of algorithms, we adopt two widely used criteria, i.e. Image Correlation Coefficient (ICC) and Relative Image Error (RIE):

$$ICC = \frac{\sum_{i=1}^{M} (\sigma_i - \bar{\sigma})(s_i - \bar{s})}{\sqrt{\sum_{i=1}^{M} (\sigma_i - \bar{\sigma})^2 \sum_{i=1}^{M} (s_i - \bar{s})^2}}$$  \hspace{0.5cm} (34)

$$RIE = \frac{\|\sigma - \bar{s}\|_2}{\|s\|_2}$$  \hspace{0.5cm} (35)

where $s$ is the ground-truth conductivity; $\sigma$ is the reconstructed conductivity.

We did 1000 trials of each algorithm by varying the SNR from 20 dB to 50 dB. Fig. 4 and 5 show the comparison results in terms of ICC and RIE. We selected the lowest and the highest frequency of $f_1$ and $f_4$ as representatives. It can be observed that FC-SBL outperforms SA-SBL for both ICC and RIE, especially at lower SNR cases. When the SNR approaches 45 dB, SA-SBL and FC-SBL show comparative performance. The TV method gives the largest RIE and the smallest ICC, indicating the worst performance.

**B. System Evaluation**

Fig. 6 shows the experiment setup of the mfEMT system which consists of a 12-channel coil array with a diameter of 120 mm, a signal generation and data acquisition module based on Red Pitaya, a computer for image reconstruction, and a circuit board that incorporates multiplexer, excitation and sensing electronics.

The sensitivity of 12 channels is calibrated using a sequence of NaCl solution with conductivity ranges from 0.01 S/m to 5.13 S/m. The NaCl solution in plastic bottles was placed closely to the coil which is driven by a 6.25 MHz sinusoidal signal. As shown in Fig. 7, the sensitivity of the gradiometer coils ranges from $0.77 \, \text{S/m}^{-1}$ to $0.98 \, \text{S/m}^{-1}$. The measured phase response is highly linear with conductivity. Due to the manufacturing inconsistency of coils, the sensitivity of 12 channels are calibrated before experiment.

**C. Phantom Experiments**

We acquired experimental data using the 12-channel mfEMT system. Two phantoms were designed using an acrylic cylindrical tank. The tank was filled with background objects, which is a mixture of 0.9% sodium chloride solution and small pieces of carrot. Three banana cylinders with a diameter of approximately 20 mm and two cucumber cylinders with a diameter of about 20 mm were placed in the background to simulate the abnormalities in the human brain. The excitation frequencies in the experiment are $[f_0, f_1, f_2, f_3, f_4] = [1/16, 1/8, 1/4, 1/2, 1] \times 6.25 \, \text{MHz}$.

Table II shows the image reconstruction results of two phantoms using experimental data. Comparison with the most recent SA-SBL method was presented as its superiority over...
other methods has been demonstrated in [15]. The termination conditions of SA-SBL and FC-SBL are set as $\epsilon_{\text{min}} = 1 \times 10^{-5}$ and $\vartheta_{\text{max}} = 120$. The 2D block size of FC-SBL is $h = 9$ (length of 2D block is 3). Others parameter are determined as default in the paper.

Overall, for both methods, the image quality increases with frequency, which is reasonable as the conductivity of biological tissues is monotonically increasing which results in a larger signal response. For phantom 1, SA-SBL restored the two target objects and their location, whilst there are some visible artifacts between two objects. By comparing, FC-SBL reconstructed more accurate images with clear boundaries and less artifacts. With respect to phantom 2, the superiority of FC-SBL over SA-SBL is more obvious. Reconstruction of multiple objects are more challenging than one or two objects, because the nonlinear effect is more severe as the complexity increases and other problems, e.g. the inter-tissue inductive coupling [34], might cause image error as well. Therefore, the performance of SA-SBL for phantom 2 degraded considerably, especially at lower frequencies, i.e. $f_1$ and $f_2$. Differently, FC-SBL recovered the three objects well throughout the whole frequency range. Although there are a little shape deformation, each individual object and its location are clearly resolved.

| Table I: Image reconstruction results of simulated model (SNR $f_4 = 30$ dB) |
|---------------------------------------------------------------|
| $f_1 = 0.7813\, MHz$ | $f_2 = 1.5625\, MHz$ | $f_3 = 3.125\, MHz$ | $f_4 = 6.25\, MHz$ |
| TV Reg. [30] | SA-SBL [15] (SMV) | FC-SBL (MMV) | Fig. 6: Experimental setup. | Fig. 7: Sensitivity calibration 12 channel sensor. |
### TABLE II: Image reconstruction results using experimental data.

| Frequency (MHz) | SA-SBL Phantom 1 | FC-SBL Phantom 1 | SA-SBL Phantom 2 | FC-SBL Phantom 2 |
|----------------|------------------|------------------|------------------|------------------|
| $f_1 = 0.7813$ | ![Image](image1) | ![Image](image2) | ![Image](image3) | ![Image](image4) |
| $f_2 = 1.5625$ | ![Image](image5) | ![Image](image6) | ![Image](image7) | ![Image](image8) |
| $f_3 = 3.125$  | ![Image](image9) | ![Image](image10) | ![Image](image11) | ![Image](image12) |
| $f_4 = 6.25$   | ![Image](image13) | ![Image](image14) | ![Image](image15) | ![Image](image16) |

**VI. CONCLUSION**

To tackle the challenge of early diagnosis of acute stroke, this paper presented a mfEMT system to perform bioimpedance imaging in a non-radiative, noncontact and calibration-free manner. To enhance image quality while dealing with small abnormalities, a frequency constrained sparse Bayesian learning (FC-SBL) approach is proposed for mfEMT image reconstruction. The performance of the proposed method is evaluated through simulations and experiments and the results indicate the superiority of FC-SBL over other state-of-the-art methods in stroke detection application. This work not only presents a novel imaging modality for biomedical applications but also provides a framework of inverse problems with Bayesian inversion that has plug-and-play specifications of the forward model and strong regularizing and parameter-free properties. Future work will extend the experimental setup to assess the imaging performance of the dynamic evolution of acute stroke.

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