Frequency analysis of tangential force measurements on a vertical axis wind turbine

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Frequency analysis of tangential force measurements on a vertical axis wind turbine

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Abstract. This paper presents experimental results of the torque ripple obtained from a three bladed 12 kW experimental H-rotor prototype. The measurements are performed by means of load cells installed on the base of the struts and by electrical measurements on the generator. The resulting torques are analysed in terms of frequency spectrum and order spectrum (synchronized with rotation). The measurements are compared to aerodynamic simulations of the turbine. The expected large torque ripple at three times the rotational speed (3p) is only weakly represented at the hub and in the generator. This suggests that the system is filtering the ripple and/or that the simulations are overestimating the 3p component. The torque ripple loads on the drive train are therefore lower than anticipated. Even if highly attenuated, most of the low frequencies correlating to aerodynamics are still represented in the generator electrical torque. Given a certain baseline, this opens for possible online monitoring of unbalances in the turbine by electrical measurements.

1. Introduction
The majority of the wind turbines operating today are horizontal axis turbines (HAWT). However, there is a growing interest in vertical axis wind turbines (VAWT), since they have the potential to further reduce energy cost [1]. Some of the advantages of VAWTs are the omnidirectional nature, ground level placement of generator and constant gravity loads.

There are also concerns associated with VAWTs. One of the main challenges is the varying aerodynamic forces on the blades that lead to a force oscillation on the structure and a torque ripple in the drive train [1]. To better understand these forces, simulations and verifying experiments have been performed. Sandia National Laboratories conducted studies on force measurements of large-scale VAWTs with troposkein shaped blades in the 1980s [2, 3, 4].

The presented study is based on force measurements on the 12 kW straight bladed VAWT (H-rotor) prototype built by the Division of Electricity at Uppsala University in 2006. Design and construction of the turbine prototype and its permanent magnet generator is further described in [5, 6, 7, 8].

The experimental method for obtaining the measurements used in this work is described in [9]. The measured normal force of the blade using the method has been presented earlier [10]. The generator used in the experiments is passively rectified using diodes that introduce a high frequency torque ripple [11]. The measured tangential force of the blade using the method also relieved large low frequency oscillations likely caused by the blade and support arms dynamics, which are further investigated in this study.
Several signal processing techniques have been studied for wind turbines, since they can be useful for online monitoring and failure detection [12]. In this paper, the frequency and order spectrum are used for analysing the tangential force as torque. The electric torque output of the generator is also used in the analysis. The aim is to investigate how vibrations and aerodynamic torque variations propagate through the wind turbine system as well as to make initial assessments of whether measurements of electrical torque can reveal mechanical properties.

2. Method

The H-rotor VAWT prototype is directly driven with a permanent magnet generator with 16 pole-pairs, see Figure 1. The forces between one blade with support arms and the hub are measured using four single axis load cells. The generator output is passively rectified using diodes and the dc-link voltage is converted to three phase ac-voltage by a two level voltage source converter. The converter power output is set by a PI-controller running at 10 Hz with a low pass filtered generator speed feedback. The controller aims to keep the rotational speed fixed during the experiments. The sample rate of all measurements was 2 kHz, except for the wind speed which was sampled at 1 Hz. The details of the measurement system, the turbine prototype, the experimental method and accuracy discussions are presented in [9].

Figure 1. The experimental setup with a magnified view of the load cells setup.
2.1. Blade torque

The tangential force is calculated from measurements by four single axis load cells installed between the support arms and the hub on one of the three blades. The load cells are the only attachment of the support arm to the hub and all force from the blade and support arm must pass through the cells. The two additional blades of the turbine have spacers installed of the same dimensions as the load cell assembly to compensate for the offset in radius, see Figure 1. The tangential force is obtained from the load cells as

\[ F_T = \frac{L_1}{2L_B} (F_0 + F_2 - F_1 - F_3 - F_{T,\text{zero}}) \]  

where \( F_0, F_1, F_2 \) and \( F_3 \) are the measured forces from the load cells, \( L_1 \) is the horizontal distance between the sensors, \( L_B \) is the distance from load cells to the blade, \( R_t \) is the radius of the turbine and \( F_{T,\text{zero}} \) is the zero calibration to compensate for the weight and the tension of the structure.

The four load cells are positioned in a square and higher values of \( F_0 \) and \( F_2 \) compared to \( F_1 \) and \( F_3 \) indicate that a positive torque is transmitted through the load cell assembly. The zero calibration is determined by:

\[ F_{T,\text{zero}} = F_{0,\text{zero}} + F_{2,\text{zero}} - F_{1,\text{zero}} - F_{3,\text{zero}} \]  

where \( F_{0,\text{zero}}, F_{1,\text{zero}}, F_{2,\text{zero}} \) and \( F_{3,\text{zero}} \) are the no-load forces measured by the load cells. These forces change over time and with temperature and an average was used to calibrate all measurements. Finally, the tangential force of the blade acting at the turbine radius \( R_t \) provides a torque as

\[ \tau_b = F_T R_t \]  

2.2. Generator electrical torque

The electrical torque of a permanent magnet generator with non-salient poles can be determined as

\[ \tau_{el} = \frac{\mathcal{E}_a i_a + \mathcal{E}_b i_b + \mathcal{E}_c i_c}{\Omega_g} = \frac{1}{\Omega_g} \left( i_a v_a + i_b v_b + i_c v_c + L \left( \frac{di_a}{dt} + \frac{di_b}{dt} + \frac{di_c}{dt} \right) + R (i_a^2 + i_b^2 + i_c^2) \right) \]  

where \( \mathcal{E}_a, \mathcal{E}_b \) and \( \mathcal{E}_c \) are the internally generated EMFs, \( R \) is the winding resistance, \( L \) is the equivalent series inductance, \( v_a, v_b \) and \( v_c \) are the terminal voltages as line to neutral voltages, \( i_a, i_b \) and \( i_c \) are the currents and \( \Omega_g \) is the rotational speed of the generator [13].

2.3. Spectral analysis

The spectral analysis is performed by a Fast Fourier Transform (FFT) with a Minimum 4-term Blackman-Harris window. For signals where the interesting frequency components are likely to change linearly with the average value of the signal, the FFT can be scaled by its average as

\[ \frac{\text{fft}(x)}{\langle x \rangle} = \frac{\text{fft}(x)}{\text{fft}(x)|_{\omega=0}} \]  

In this paper we note this self scaled FFT as the relative spectral amplitude. A convenient side effect from the relative spectral amplitude is the automatic compensation for the amplitude change caused by the windowing.
2.4. Frequency spectrum versus order spectrum
For rotating mechanical systems it is common to express frequencies in the unit \( p \), defined as

\[
p = \frac{2\pi f}{\Omega} \tag{6}
\]

where \( f \) is the frequency in (Hz) and \( \Omega \) is the rotational speed in (rad/s). The benefit, compared to frequency units such as (rad/s), (rpm) or (Hz) is that \( p \) is independent of the actual speed of the system. The spectrum in \( p \) is the order spectrum. For a wind turbine, the order spectrum in low frequency should mainly correspond to aerodynamically induced oscillations. The frequency spectrum in (Hz) can be useful for study of the natural oscillations of the system. Some care is needed when looking at the order spectrum. For example, consider a three-bladed turbine where a 1 \( p \) frequency force is introduced by one blade. The frequency may appear as a 1 \( p \) frequency on the shaft. Additionally, if the two other blades also introduce a 1 \( p \) frequency, a 3 \( p \) component is also expected on the shaft.

2.5. Aerodynamic simulations
Aerodynamic simulations were performed according to the vortex model described in [14]. The obtained forces give a purely aerodynamic torque since the simulations are without any aero elasticity or mechanical modelling of the system. The simulations were performed for tip speed ratios (TSRs or \( \lambda \)) from 2.5–4.5 for three different wind speeds. During each simulation, the TSR and the wind speed were constant and a full 200 turbine revolutions was simulated. The first half of the simulation stabilized the wake and the last 100 turbine revolutions were used for determining the relative spectral amplitude of the torque as an order spectrum (see sections 2.3 and 2.4). The relative spectral amplitudes at 1–6 \( p \) were picked and plotted against the TSR. The process was done for the torque of one blade and for the total torque provided by all blades. The resulting two plots are given in Figure 2. The relative spectral amplitudes at 7–20 \( p \), were <0.05 for the blade torque and <0.03 for the total torque. At those frequencies, the relative spectral amplitudes are not as smooth functions of TSR as for the lower frequencies 1–6 \( p \).

These simulations ignore aero elasticity, mechanical dynamics and electrical properties of the system and can not be compared directly with the measurements of the blade torque and generator electrical torque. Still, from the simulations some conclusions can be drawn about the expectations of the measurements of the blade torque:

- The order spectrum with relative spectral amplitudes mainly depends on the TSR of the turbine for the frequency range 1–6 \( p \).
- The different Reynolds numbers (due to different wind speeds) cause some variation in the relative spectral amplitude of the order spectrum, mainly for 1 \( p \), 3 \( p \), 4 \( p \) and 5 \( p \) at low TSRs.
- Most of the torque of one blade is represented by the spectral amplitudes at 1–6 \( p \).
- Quadratic curve fits, one for each \( p \), seem sufficient to capture the correlation between the TSR and relative spectral amplitude.
- The order spectrum at frequencies >6 \( p \) has low amplitudes and lacks smooth correlation to TSR (more random behaviour).

Additionally, it is noted that most of the total torque is represented by the relative spectral amplitude at 3 \( p \) with a small contribution at 6 \( p \) and 9 \( p \). The other harmonics cancel out due to the three blade symmetry. The symmetry also causes the relative amplitudes at 3 \( p \), 6 \( p \) and 9 \( p \) to be the same for the total torque as for the one blade torque.
2.6. Data treatment and study

The full measurement campaign was from 13 September to 4 December 2014. Sets of data were extracted from the measurement campaign to get a spread in rotational speed, TSR and torque while the conditions (wind speed, wind direction, rotational speed and average power output) for the duration of 10 full turbine revolutions were stable. Data sets of 10 turbine revolutions where the relative standard deviation of the 1 s averages were within 1 % for the rotational speed and 5 % for the electrical torque were selected. Given that the rotational speed and electric torque were stable, the other conditions were also assumed stable. A total of 21 datasets were collected, with TSR ranging 2.0 – 4.2, average torque 90 – 460 Nm and rotational speed 40 – 73 rpm. The statistics of these sets are presented in Appendix A. The blade torque and electrical torque were determined as described in section 2.1 and 2.2. The relative spectral amplitudes in frequency spectrum were obtained as described in section 2.3. The relative spectral amplitudes in order spectrum were obtained in the same way, but from rotation synchronized data. The rotational synchronization was performed by re-sampling of the time domain signals based on the zero crossings of the voltage in one of the generator phases. Spline interpolation was used in the re-sampling process.

3. Experimental results and discussion

The results from the analysis are divided into frequency spectrum and order spectrum. The difference is illustrated in Figure 3. From the blade torque $\tau_b$ (Figure 3a and 3b) it seems that synchronizing with rotation truly simplifies comparison and the relative spectral amplitudes indeed are of similar amplitudes for a specific TSR. By comparing the blade torque to the generator electrical torque $\tau_{el}$ (Figure 3c and 3d) it is clear that the relative spectral amplitudes are considerably lower for the generator. We also note a small amplitude at 16 p that is unrepresented at the blade. This correlates with the 16 pole pairs of the generator and likely causes are reluctance cogging and/or uneven magnet placement. The passive rectification causes a large 96 p component in the electric power from the generator. However, since we here focus on lower frequencies it is omitted in figures. The rectification impact is further studied in [11]. The difference in amplitudes between the frequency spectrum and the order spectrum is due to the signal processing used to achieve the order spectrum. In the two following sections all of the 21 data series are further analysed in order spectrum and frequency spectrum respectively.
Figure 3. Example result at three different rotational speeds with similar TSR in frequency spectrum (Hz) Figure (a) and (c), and in order spectrum (p) Figure (b) and (d).

3.1. Order spectrum
The relative spectral amplitudes of the blade torque and the generator electrical torque in order spectrum, are compared in Figure 4. Quadratic curve fits are added since the correlations are expected to be quadratic, according to section 2.5. First, it should be considered that the measurements are not directly comparable to the simulations in Figure 4. For the blade, the measurements were made at the hub, while the simulated force is acting on the blade. There is added dynamics caused by aero elasticity and the inertia as well as the spring and damper properties of the blade and support arms. For the generator torque, while correlated to the total torque, it is affected by the additional dynamics of the shaft and the generator on top of that from the blade and the support arms. With these considerations in mind, an analysis of the results can be made.

For the blade, the spread in the measured relative spectral torque is quite low and the quadratic curve fits look appropriate, see Figure 4. Comparing the spread around the curve fits with the expected spread caused by variations in wind speed, the results show good coherence. Furthermore, the 2p is indeed the most dominating frequency, but the amplitude is increasing more at higher TSR than in the simulations. Perhaps a bit surprising, the amplitude at 3p is the lowest one in the measurements, but the second largest in the simulations. The amplitude at 1p is larger in measurements compared to simulations. One interpretation is that the dynamics of the system filter the aerodynamic torque. Another explanation could be that the simulations are overestimating the amplitude at 3p.

For the generator, the measured relative electrical torque is first of all much lower in amplitude compared to the blade. There are also only small differences in amplitude at
different $p$. According to the simulations, the total torque is expected to mainly consist of spectral amplitudes at $3p$ with small contributions at $6p$ and $9p$, and the relative spectral amplitude of one blade should be the same as the relative spectral amplitude of the total torque. Comparing amplitudes of the curve fits at $3p$ and $6p$ between blade and generator in Figure 4 gives the relative spectral amplitude at $3p$ for the generator to be at most 41% of that of the blade. The same comparison for $6p$ gives at most 11%. This suggests some attenuation of the hub torque ripple before seen on the generator electrical output, and that the attenuation may change with frequency (since $6p$ is more attenuated than $3p$). The presence of some $1p$, $2p$, $4p$ and $5p$ in generator electrical torque indicates a possible unbalance in the torque provided by the three blades. The relative amplitudes are in the same order of magnitude as at $3p$ and $6p$, which are known to be small based on the blade measurement, indicating that the possible unbalance is probably small.

The measured relative spectral amplitudes at $7–10p$ are plotted in Figure 5. As predicted by the aerodynamic simulations (see section 2.5), the correlations to TSR are more random for both the blade and the generator electrical torque. However, spectral amplitudes of the blade are at least 10 times larger than the simulations. One explanation could be that the natural frequencies of the system are synchronizing with some of the $p$-frequencies and are therefore mistakenly passing as one of them. The spectral amplitudes at the generator actually match the simulated total turbine amplitudes for $7–10p$. This indicates that the rather high amplitudes measured at the blade are not progressing to the generator.

The maximum error in the torque measurement is carefully estimated in [9]. Since only one blade is used here, the error is divided by three and presented in Appendix A. Estimating the error in terms of the relative spectral amplitudes used in this paper is not as straightforward. However, we can conclude that the deviation between simulation and experiment regarding especially the $3p$ component in Figure 2a and 4a is unlikely explained as a measurement error. The measurement errors in the rotational speed and the wind speed contributes to an error in the TSR. Without any estimations of its size, we still realize that an error in the TSR would only shift the values in $\lambda$-direction (sideways).

The accuracy and resolution of the voltage and current measurements of the generator are assumed fairly good, however, at these low amplitudes, accuracy may be an issue. The rotational speed of the generator has been considered constant when estimating the electrical torque, which is also a possible source of error.

3.2. Frequency spectrum

The relative spectral amplitude of the blade torque in frequency spectrum is illustrated in Figure 6. All 21 datasets are plotted on top of each other, with some transparency on each line. From Figure 4 it is expected to see large peaks at especially $2p$, $1p$ and $4p$. Most of the significant spikes in Figure 6a in the range $0–8$ Hz are from those $p$-peaks. To find natural frequencies it is instead reasonable to focus on the areas where most of the spectrum seems to coincide. These points indicate over-represented frequencies from all measurements. Two such regions can be spotted; at bit fuzzy around $6$ Hz and more distinctly slightly above $8$ Hz. From hammer tests in [9], tangential modes were found at $8.0$ Hz and $29$ Hz for the measurement blade and at $9.1$ Hz and $29$ Hz/$36$ Hz for the other blades, see Appendix B. These frequencies are also slightly visible in the spectrum, see Figure 6b.

The relative spectral amplitude of the generator electrical torque in frequency spectrum is illustrated in Figure 7. The relative spectral amplitude of the electrical torque for frequencies $1–20$ Hz is $<3\%$ of the relative spectral amplitude of the blade. The possible reasons have already been discussed in section 3.1. The $16p$ spikes turn up, as mentioned before. To spot any coinciding areas is even more difficult than in the blade torque plot and this approach to monitor natural frequencies in the system seems difficult. Apart from all the $p$-peaks cluttering
Figure 4. The measured relative spectral amplitude of the torque at 1–6 p plotted versus the TSR (dots). Quadratic curve fits have been added (solid lines). Figure (a) is the blade and Figure (b) the generator electric torque.

Figure 5. The measured relative spectral amplitude of the torque at 7–10 p plotted versus the TSR. The scale of the y-axis is the same as in Figure 4.

the spectrum, the overall amplitudes are low, making noise a larger part of the spectrum. In Figure 7b the frequency range is extended to 12–40 Hz in an attempt to locate the second tangential mode of the blades, that was visible in blade torque spectrum. No clear evidence of 29 Hz or 36 Hz is present in the generator electric torque spectrum.

Since the amplitude response of excited natural frequencies may change non-linearly with average force, the use of relative spectral amplitude to monitor the natural frequencies should be considered a source of error.

3.3. General discussion

Advanced methods can be used for spectral estimations, and the FFT used here should be considered a possible source of error. Also, any possible impact caused by the passive rectification of the generator or the load controller has not been considered in this work. Furthermore, even if data with low variations has been selected, the measurements are performed in an open site where wind speed, wind direction and rotational speed of the turbine are not constant.
Figure 6. The measured relative spectral amplitude of the blade torque as frequency spectrum at 0–12 Hz Figure (a) and 12–40 Hz Figure (b). For reference; 1 p = 0.67–1.21 Hz. All 21 datasets are plotted on top of each other. The blade first tangential natural frequency is visible at 8 Hz. The second tangential natural frequencies are visible at 29 Hz and 36 Hz respectively.

Figure 7. The measured relative spectral amplitude of the generator electrical torque as frequency spectrum at 0–12 Hz Figure (a) and 12–40 Hz Figure (b). For reference; 1 p = 0.67–1.21 Hz. All 21 datasets are plotted on top of each other.

4. Conclusions
The torque provided by one blade has been measured at the hub of a VAWT. The time domain signals of the blade torque were distorted by the dynamics of the blade and support arms. Aerodynamic simulations predict the order spectrum to mainly depend on TSR. The experimental results analysed in order spectrum indeed provided fairly consistent relative torque spectrum as a function of TSR for one blade. According to the aerodynamic simulations, the order spectrum of the total torque is limited to 3 p with a small contribution at 6 p and 9 p. The rest of the order spectrum in the range 1–10 p cancel out due to symmetry. However, the spectral behaviour in the measurements differ from the aerodynamic simulations. The measured electrical torque, based on generator voltage and current, has only 3% of the relative spectral amplitude of the blade torque in the range 1–12 Hz. The results suggest either that the system acts as a mechanical filter, highly attenuating even the 3 p component from the blade.
torque and/or that the aerodynamic simulations overestimate the 3p component. An interesting consequence is that the torque ripple entering the shaft of the turbine is surprisingly low. This may change during more turbulent conditions, when the blades provide uneven torque.

Even if filtered to lower values, the most dominating amplitudes in the order spectrum of the turbine torque are measurable in the generator electrical torque. The process is straightforward, given that the data can be re-sampled in synchronisation with the rotational speed. An unbalance in the turbine could possibly be identified by monitoring changes from a given baseline in the order spectrum.

The attempt to track natural frequencies in the system by simple FFT of the relative torque was promising for the blade, but unsuccessful for the generator electrical torque. Two challenges were discovered; the FFT of the generator electrical torque is cluttered by the p-frequencies and the spectral amplitudes are small. Tracking of natural frequencies in a VAWT using this method may still be possible with more advanced signal processing and/or improved measurement performance.

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### Appendix A. Statistics of the datasets

| $\Omega_g$ (rpm) | Wind speed (m/s) | TSR | $\langle \tau_{el} \rangle$ (Nm) | $\langle \tau_b \rangle \pm \Delta \tau_b$ (Nm) | Time             |
|------------------|------------------|-----|-------------------------------|---------------------------------|------------------|
| 41.0             | 4.6              | 3.03| 88.9                          | 40.4 ± 4                        | 2014-09-25 11:15 |
| 40.3             | 6.8              | 2.00| 106.0                         | 46.0 ± 4                        | 2014-10-07 13:49 |
| 40.3             | 6.1              | 2.26| 111.4                         | 47.1 ± 4                        | 2014-10-07 13:55 |
| 40.6             | 6.7              | 2.06| 110.2                         | 46.6 ± 4                        | 2014-10-07 13:59 |
| 40.3             | 5.2              | 2.63| 108.5                         | 47.5 ± 4                        | 2014-10-08 12:33 |
| 50.3             | 5.5              | 3.08| 177.3                         | 72.1 ± 4                        | 2014-09-23 07:22 |
| 50.0             | 4.7              | 3.63| 90.1                          | 38.9 ± 4                        | 2014-09-23 07:29 |
| 50.7             | 5.0              | 3.06| 170.3                         | 66.5 ± 4                        | 2014-09-23 07:31 |
| 50.4             | 4.9              | 3.51| 121.1                         | 51.7 ± 4                        | 2014-09-23 07:34 |
| 51.1             | 5.2              | 3.30| 126.9                         | 55.7 ± 4                        | 2014-10-06 11:50 |
| 60.7             | 8.8              | 2.34| 321.4                         | 120.1 ± 5                       | 2014-09-26 14:28 |
| 60.5             | 7.6              | 2.72| 345.9                         | 125.9 ± 5                       | 2014-10-08 05:39 |
| 60.2             | 8.9              | 2.30| 314.9                         | 121.3 ± 5                       | 2014-10-08 05:39 |
| 59.8             | 10.2             | 2.00| 253.6                         | 97.2 ± 5                        | 2014-10-08 05:40 |
| 60.4             | 7.0              | 2.91| 327.1                         | 120.5 ± 5                       | 2014-10-08 05:44 |
| 65.6             | 5.3              | 4.17| 137.1                         | 59.0 ± 4                        | 2014-10-07 19:22 |
| 66.4             | 6.8              | 3.34| 248.1                         | 95.4 ± 5                        | 2014-10-07 19:25 |
| 65.2             | 5.7              | 3.90| 130.8                         | 57.0 ± 4                        | 2014-10-07 19:25 |
| 66.6             | 6.2              | 3.69| 200.6                         | 80.3 ± 4                        | 2014-10-07 19:47 |
| 65.5             | 6.6              | 3.37| 303.2                         | 112.9 ± 5                       | 2014-10-07 20:04 |
| 72.6             | 8.9              | 2.77| 456.8                         | 164.8 ± 5                       | 2014-10-27 08:28 |

### Appendix B. Tangential hammer test spectrum

![Figure B1](attachment:image.png)

**Figure B1.** The frequency spectrum of the tangential force measured by the load cells after impacting the front of the blade at about half of the blade length. The plot illustrates two subsequent tests.