Stellar structures and the enigma of pulsars rotation frequency decay

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Abstract. Pulsars are astrophysical objects normally modelled as compact neutron stars that originated from the collapse of another star. This model, that we name canonical, assumes that pulsars are described by spherical magnetized dipoles that rotate, usually with the magnetic axis misaligned to the rotation axis. This misalignment would be responsible for the observation of radiation emitted in well-defined time intervals in a certain direction (lighthouse effect), the typical observational characteristic of this kind of star. It has been noticed that the rotation frequency of pulsars is slowly decaying with time (spin down), implying a gradual decrease of the rotational angular velocity (Ω). Such decay can be quantified by a dimensionless parameter called “braking index” (\(n\)), given by \(n = \Omega \ddot{\Omega} / (\dot{\Omega})^2\), where a dot indicates a time derivative. The canonical model predicts that this index has one only value for all pulsars, equal to three. However, observational data indicate that actual braking indices are less than three, representing an enigma. The main goal of this research is the exploration of a more precise model for pulsars' rotation frequency decay.

1. Introduction

In July 1967, scientists at the University of Cambridge detected signals using a radio telescope. The observed signals were not only isolated peaks, but periodic pulses and, according the authors, “clearly displays the regular periodicity and also the characteristic irregular variation of pulse amplitude.” [1]

Pulsar became the name of the new phenomenon, an abbreviation of pulsating stars, “The constancy of frequency in the recently discovered pulsed radio sources can be accounted for by the rotation of a neutron star.” [2]

According to Gold [3], pulsars have a strong magnetic field and high rotational speeds, and the radiation is released as a pattern of a rotating beacon.

Neutron stars are the remnants of the supernova process. The compression of the remains of the star after the supernova effect also implies a compression of magnetic field leaving it stronger where the field strength is greatest at its poles due to a larger magnetic density. The gravitational attraction of the remains of the star that went through a supernova process results in an extremely compact object with radius of about 10 km, which has high-speed rotation due to conservation of angular momentum.

The discovery of the first pulsar and curiosity by this object led the analysis and discovery of more pulsars. The pulsar B0531+21, in the Crab Nebula, had an increase in its period [4] and...
brought more support to the theory that the period of pulsars increases.

2. The Lighthouse Model

![Image of a Pulsar](http://www.astron.nl/about-astron/press-public/news/neutron-stars-laboratory/neutron-stars-laboratory)

**Figure 1.** Artistic illustration of a Pulsar, available at: “http://www.astron.nl/about-astron/press-public/news/neutron-stars-laboratory/neutron-stars-laboratory”

This model is accepted as a way to explain the behavior of neutron stars. These neutron stars have a high magnetic field, where the intensity is greater at its poles, since the field lines are closer than at its equator, and with its high rotation speed particles are accelerated along these field lines forming a beam through which these particles emit radiation at radio frequencies.

2.1. The Model for Pulsars’ Spin-down

We treat the pulsar as a rotating magnetic dipole, we can see that its rotational kinetic energy equals [5]

\[ E_{\text{rot}} = \frac{1}{2} I \Omega^2 \]  \hspace{1cm} (1)

and the time derivative of (1)

\[ \frac{dE_{\text{rot}}}{dt} = \frac{1}{2} \cdot 2 \cdot I \cdot \Omega \cdot \dot{\Omega} + \frac{1}{2} \cdot \Omega^2 \cdot \dot{I} \]  \hspace{1cm} (2)

where the second term of the above expression is very small in magnitude. Soon, we will have,

\[ \dot{E}_{\text{rot}} = I \cdot \Omega \cdot \dot{\Omega} \]  \hspace{1cm} (3)

and the magnetic energy radiated by this pulsar is the radiated energy of a magnetic dipole rotating [6],

\[ \dot{E}_{\text{mag}} = -\frac{2}{3c^3}|\vec{m}|^2 \]  \hspace{1cm} (4)
being \( m \) the magnetic dipole moment, and \( |\ddot{m}|^2 \) equal to [5],

\[
|\ddot{m}|^2 = m^2 \cdot \sin^2 \alpha \cdot (\Omega^4 + \dot{\Omega}^2)
\]

where \( \dot{\Omega}^2 \ll \Omega^4 \),

\[
|\ddot{m}|^2 = m^2 \cdot \sin^2 \alpha \cdot \Omega^4
\]

then, the derivative of the magnetic energy has the form,

\[
\dot{E}_{mag} = -\frac{2}{3c^3} m^2 \sin^2 \alpha \cdot \Omega^4.
\] (5)

Since the magnetic dipole \( m = \frac{1}{2} B_P \cdot R^3 \) [5], we get the expression for the magnetic energy,

\[
\dot{E}_{mag} = -\frac{1}{6c^3} B_P^2 R^6 (\sin \alpha)^2 \Omega^4
\] (6)

where \( R \) is the radius of the sphere and \( B_P \) the magnetic field at the poles. And being \( k \) positive constant equal to,

\[
k = \frac{1}{6c^3} \frac{R^6}{I} (B_P \sin \alpha)^2
\]

and with that, we come to a simplified expression for the rate of change of radiated energy by the dipole in time,

\[
\dot{E}_{mag} = kI \Omega^4.
\] (7)

In the canonical model, one of the hypotheses is the conversion of rotational energy (\( E_{rot} \)) into magnetic energy (\( E_{mag} \)). From this we found that the expected value of \( n \) by the pulsars’ spin-down model is equal to 3,

\[
E_{rot} + E_{mag} = 0.
\] (8)

Substituting the values of the time derivatives of the kinetic energy of rotation (3) and magnetic energy (7) found,

\[
I \Omega \dot{\Omega} + kI \Omega^4 = 0
\]

\[
\dot{\Omega} = -k \cdot \Omega^3.
\] (9)

(10)

The expression in equation (10) relates the increasing of the rotation period of pulsars, i.e., the orbital velocity decay. The energy conservation leading into account the model of rotating magnetic dipole radiating its rotational energy shows that the spin down occurs with a power 3 of its angular velocity. If we derive the equation (10), we obtain

\[
\ddot{\Omega} = -3 \cdot k \cdot \Omega^2 \cdot \dot{\Omega}
\] (11)

where we can organize it and replace \(-k \cdot \Omega^3\) by \( \ddot{\Omega} \),

\[
\ddot{\Omega} = 3 \cdot \frac{(\ddot{\Omega})}{\dot{\Omega}} \cdot \ddot{\Omega}
\] (12)

\[
\ddot{\Omega} = 3 \cdot \frac{\ddot{\Omega}}{\Omega^2} \cdot \Omega
\] (13)

and we find that the product of \( \ddot{\Omega} \) with \( \Omega \) divided by \( \ddot{\Omega} \) should have exactly the value of 3.

\[
\frac{\ddot{\Omega} \cdot \Omega}{\ddot{\Omega}^2} = 3
\] (14)
being, the value 3 in the equation (10) and (14) represents an intensity factor of braking. It is the called braking index of pulsars. The observable $\Omega$, $\dot{\Omega}$ and $\ddot{\Omega}$ are used to calculate the braking index and obtained experimentally, and should result in the value of 3. However, the values found for braking index of pulsars does not corresponds the expected value. We can observe that the generalization of the equation (10),

$$\dot{\Omega} = -k \cdot \Omega^n$$

and of (14) we have,

$$\frac{\ddot{\Omega} \cdot \Omega}{\Omega^2} = n$$

even if, we generalize the equations we can see that the equation (16) would only make sense to a value of $n = 3$, but the first equation become a definition [7] where the angular deceleration is associated with decreased angular velocity raised to the exponent $n$.

3. Divergence in the observed values with the predicted

In the following table, we present the pulsars’ with the value of braking index well observed [8] [9] [10] [11] [12].

| Data    | PSR B0531+21 | PSR B0833-45 | PSR B0540-69 | PSR B1509-58 | PSR J1119-6127 | PSR J1846-0258 |
|---------|--------------|--------------|--------------|--------------|----------------|----------------|
| $f$ (Hz) | 30.2         | 11.2         | 19.2         | 6.63         | 2.45           | 3.06           |
| $\frac{df}{dt} \times 10^{-10} s^{-2}$ | -3.86        | -0.15        | -1.88        | -0.67        | -0.24          | -0.66          |
| $\frac{d^2f}{dt^2} \times 10^{-21} s^{-3}$ | 12.4         | 0.031        | 3.81         | 1.95         | 0.63           | 3.13           |
| $n$     | 2.51         | 1.54         | 2.07         | 2.88         | 2.67           | 2.20           |

4. Conclusions

The current model does not corresponds the reality observations, and a possible fix is necessary to obtain correctly the values of braking index. However, the mathematical model must be able to relate any implication that may cause changes in the values of braking index.

From this, arise questions that, we lead to think, which conditions the pulsars’ are subject to modify it value expected for the braking index, and hence, we realize that, we can propose an alternative way to calculate the braking index with the development a new model with related changes of variables (mass, radius, momentum of inertia, magnetic field, ...) that can to be related directly to the distortion of the predicted and observed values.

This work aims to present to the scientific community the problem which we believe to be important for nuclear astrophysics, thus to develop a model to solve correctly the problem of braking index.
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