An improved algorithm based on LAOMP

Bin Wang\(^1,3\), Deyang Wu\(^1,2\), Li Wang\(^1,2\), Fengming Xin\(^1\) and Xin Song\(^1\)

\(^1\)School of Computer and Communication Engineering, Northeastern University at Qinhuangdao, Qinhuangdao, China
\(^2\)School of Computer Science and Engineering, Northeastern University, Shenyang, China
\(^3\)Email: wangbinneu@qq.com

Abstract. It is extremely important to choose the optimal support set atoms in the reconstruction algorithm based on compressed sensing. The Look Ahead Orthogonal Matching Pursuit (LAOMP) is a commonly used reconstruction algorithm. However, in most cases, with the sparsity of signal unknown, we search every time for the fixed L iteration times not to guarantee that the selected atoms be optimal. To solve this problem, the Sparsity Adaptable Look Ahead Orthogonal Matching Pursuit (SA LAOMP) is proposed. Simulation results show that its Average Support Cardinality Error (ASCE) performance and Exact Reconstruction Probability (ERP) performance are superior to the LAOMP algorithm.

1. Introduction

Compressed sensing (CS), proposed by Donoho, Candès, Romberg, and Tao in 2004, is a novel theory of information acquisition, which enables signal linear measurement and high probability reconstruction with a sample rate less than the normal Nyquist rate, as long as the signal is sparse in a certain domain, increasingly reducing the amount of data generated by signal processing [1]. The core of compressed sensing refers to the signal reconstruction algorithm. LAOMP, proposed by Saikat Chatterjee et al, has been widely applied in many fields [2-3]. In the LAOMP algorithm, for the choice of a new atom, L best choices of new atoms are found, through a matched filtering operation with the look ahead prediction method. We consider its effect on the final performance in the sense of minimizing the residual, add the atom of the minimum norm of fitting residual to the support set, then update the intermediate support set, estimation the signal and residual, and finally generate the evaluation signal till the number of iterations exceeds K or it reaches terminate condition. Ceng proposes an orthogonal matching tracking algorithm combining forward prediction and backtracking, which divides all iterations into the former and later stages by setting the threshold, and in the early stage of iteration, the optimal atom is chosen by predicting the performance of the atom in the subsequent iteration. At the end of iteration, backtracking strategy is added, and every two iterations eliminate a previous atom selected erroneously, achieving better performance [4]. Xu makes the LAOMP intersect with the LAOMP and KOMP by employing the fusion strategy, which combines the estimated support set with the KOMP algorithm of low computing complexity to obtain high precision atom set. It reduces the reconstruction time of the algorithm and improves the computing efficiency [5]. The improvements above based on LAOMP algorithm can adaptively select the number of atoms with forward prediction, but still need to be performed in the case of the known sparsity.
In this paper, inspired by the research work above, SALAOMP is proposed to recover signal more efficiently. In the algorithm, without both knowing the sparsity of signal in advance and selecting $L$ highest amplitude components in each iteration by forward prediction to check, the only thing to realize is to set the length of the staged and select the biggest atoms in magnitude of the matched filter output with the regularization method, whose number is random in each iteration and always less than the corresponding iteration step [6-7]. In simulations, we will compare ASCE performance and ERP performance.

2. Compressed sensing and LAOMP algorithm
Compressed sensing is composed of three core parts, the sparsity of the signal (sparse matrix), the random projection of the signal (measurement matrix) and the reconstruction of the signal (reconstruction algorithm). As a new signal sampling theory, CS strives for developing the sparse characteristic of the signal, at far less than the Nyquist sampling rate, to reconstruct nonlinearly the perfect evaluation signal with the discrete sample of the signal obtained by the random sampling.

2.1. Compressed sensing theory
Assume a K-sparse signal $X \in R^N$ of $N$ dimensions via a certain sparse matrix ($\Psi = \{\psi_1, \psi_2, \Lambda, \psi_N\}^T$) transform.

$$X = \sum_{i=1}^{N} \psi_i \theta_i = \Psi \Theta$$

(1)

where $\Psi$ represents the orthogonal sparse matrix with $N \times N$, and the projection $\theta_i = \{X, \psi_i\}$ of $X$ on $\Psi$ is denoted as $\theta_i \in R^N$. Assume $\Theta$ is a $N$ dimensions signal with the sparsity of $K$ ($K << N$), which is to say there are $K$ nonzero values in $\Theta$. Thus signal $X$ is sparse on the base of the sparse matrix $\Psi$ with the sparsity of $K$.

The nonzero coordinates of the sparse signal $X$ have to be reduced while the signal needs compression sampling. This is because the signal $X$ is often N-dimension ($N$ is usually large) and the complexity is still relatively high. Therefore, a measurement matrix is needed to compress the dimension to reduce the computational complexity effectively. Assuming that $\Phi$ is a matrix with $M \times N$ ($K < M < N$), let $Y$ be a one-dimension observation signal with the length of $M$, then it can be expressed as

$$Y = \Phi X = \Phi \Psi \Theta$$

(2)

Let the sensing matrix $A$ with $M \times N$ be $A = \Phi \Psi$, then the equation above is equivalent to

$$Y = A \Theta$$

(3)

As the dimension of $Y$ is much lower than the dimension of $X$, (3) will have infinite solutions. That is to say, the problem is underdetermined and it is difficult to reconstruct the original signal. However, if the signal $X$ has only $K$ nonzero values, and the observation matrix satisfies the Restricted Isometry Property (RIP) [8], and that is

$$(1 - \delta_K)\|\Theta\|^2_2 \leq \|\Phi \Psi \Theta\|^2_2 \leq (1 + \delta_K)\|\Theta\|^2_2$$

(4)

where $0 < \delta_K < 1$, with the number of $M \geq cK \log(N/K)$ ($c$ is a quiet small constant) the $K$ coefficients can be reconstructed from the high probability of $M$ measurements.

2.2. Reconstruction core of LAOMP
The signal reconstruction process is to accurately recover the original signal by utilizing nonlinear projection of the measurement vector. The precision of LAOMP algorithm is improved by Saikat Chatterjee et al adding forward prediction into OMP algorithm. The principle of the algorithm is to select $L$ highest amplitude components of the Corresponding inner products, and then these atoms employ the forward prediction strategy. In addition, the Look Ahead Residual (LAR) algorithm is the
complete expression of the forward prediction strategy, judging its effect on the final residual, and then selecting the least residual, whose corresponding atom is added to the support set, to update the atomic intermediate set, estimate the signal and residue, and finally output the estimated signal till the number of iterations exceeds \(K\) or it reaches the end condition. The core of the LAOMP algorithm is the LAR algorithm, which is described as follows:

Algorithm 1 LAR

Inputs: Sensing Matrix \(A\), measurement Matrix \(Y\), sparsity level \(K\), previous support set \(I\) and new chosen index \(t\).

Initialization: \(I_k = I_{pre} Y t\), \(t \not\in I_{pre}\), \(\Theta_{k_i} = (A_{k_i}^T A_{k_i})^{-1} A_{k_i}^T Y\), \(r_k = Y - A_{j_k} \Theta_{j_k}\).

repeat
\(t_k = \arg \max a_{j_k}^T r_{k-1}\);
\(I_k = I_{pre} Y t_k\);
\(\Theta_{k_i} = (A_{k_i}^T A_{k_i})^{-1} A_{k_i}^T Y\);
\(r_k = Y - A_{j_k} \Theta_{j_k}\);
until \(\|r_k\|_2 > \|r_{k-1}\|_2\) or \(k > K\);

Output: \(r_k = Y - A_{j_k} \Theta_{j_k}\).

where \(A = \{a_{j_i}\}, \ i \in M, j \in N, Y \in R^M, t \in \{1,2,\ldots,N\}\), based on the above description, the forward prediction residuals are defined as
\[ r = (Y, A, K, I_{pre}, t) \]  

(5)

3. SALAOMP algorithm

LAOMP algorithm has some shortcomings [9-10]. First of all, the LAOMP algorithm is performed with the sparsity known, and the maximum inner product may not be the optimal value, and the corresponding index is not necessarily the optimal index. Therefore, the minimum residue is not sure to be the smallest one through the LAR. Secondly, the residue tends to be stable, which is smaller than the \(r\) usually set as \(10^{-6}\) after multiple iterations. The fixed \(L\) best atoms are added to LAR in each iteration, only one residue selected, which increases the number of iterations, leading to the high time complexity.

3.1. Regularization algorithm

SALAOMP algorithm selects the biggest atom set of the energy by regularization, which generally divide the correlation coefficients of the atoms corresponding to the indices in the intermediate set \(J\), to find a subset \(J_0\) satisfying the \(|\mu(i)| \leq 2|\mu(j)|\) \((i, j \in J_0)\), so as to select the largest atom in magnitude in all subsets that satisfies the condition. The core of SALAOMP algorithm is composed of two parts, namely regularization and LAR sub-algorithm. Since the LAR algorithm has been introduced before, then we introduce the regularization algorithm.

Algorithm 2 Regularization

Inputs: The inner production \(P\) of \(A^T r_{t-1}\), iteration step \(L\);

initialization: \(E_{max} = 0, t = 1\);
repeat
\[ 1) \text{ Choose the Corresponding indexes of the L biggest nonzero coordinates of } P, \text{ which are added to the set } J. \]
\[ 2) \text{ Choose the L biggest nonzero coordinates of } P, \text{ which are added to the set } J_{\text{val}}. \]
3) Outer Loop $k = 1: L$
   \[ J_{\text{pos}}(t) = J(k); \]
   \[ E_n = J_{\text{val}}(k)^2; \]

Inner Loop $m = k + 1: L$
   Choose $J_{\text{val}}(k) < 2 \times J_{\text{val}}(m)$
   \[ J_{\text{pos}}(t) = J(m); \]
   \[ E_n = E_n + J_{\text{val}}(m)^2; \]

if $E_n > E_{\text{max}}$
   Update: $J_t = J_{\text{pos}}(1:t)$, $E_{\text{max}} = E_n$

Output: $\text{pos} = J_t$, $\text{val} = E_{\text{max}}$.

According to the algorithm 2, the regularization process is defined as:
\[
[\text{val}, \text{pos}] = \text{Re}(P, L) \quad (6)
\]

3.2. Details of the SALAOMP algorithm

In the SALAOMP algorithm, first of all, the indices corresponding to the $F$ biggest inner products are selected by regularization, and then the $F$ indices are processed by LAR. Due to the great difference of the residues of early iteration, we need to iteratively estimate the residual performance in LAR algorithm, in order to get the smallest residue. As the number of iterations increases, the difference between adjacent inner product values is gradually reduced and repeated iterations are not necessary. Therefore, the number of iterations in this paper is selected as $\text{floor}(M / L)$ times. With the number of iterations increasing, the $L$ will also increase adaptively, so the number of iterations will decrease, and the number of iterations will be reduced, which meets our requirements perfectly. When the $f$ residual values of the LAR output is less than $\tau$, the $F$ indexes of the coordinates corresponding to the $f$ are added to the support set, with $\tau$ termed as a threshold value determining whether the residual performance tends to be stable. Otherwise, it will add directly the index of the smallest residual of the corresponding $F$ to the support set, updating the approximate coefficient, residual, and the iterative step length, which iterate consistently until the termination condition is satisfied to output the approximate coefficients.

The details of the SALAOMP algorithm are stated as follows:

Algorithm 3 SALAOMP

Inputs: Sensing Matrix $A_{M \times N}$, measurement Matrix $Y_{M \times 1}$, initial step $S$.

Initialization: Approximate coefficient $\Theta = 0$, residue $r_0 = Y$, support set $I = \emptyset$, step $L = S$,
\[ \text{pos} \_ \theta = \emptyset, \text{Stage} = 1, \text{Iter}_{\text{max}} = M, K = \text{floor}(M / L). \]

repeat
   $P = A^T r_{k-1}$;
   $[\text{val}, \text{pos}] = \text{Re}(P, L)$;
   $l = 1: f$;
   $r = (Y, A, K, \text{pos} \_ \theta, J_l)$;
   $n_f = \|r\|_2$;
   $I_k = I_{k-1} Y_{i_k}$; The number of index $i_k$ is updated according to the residual value.
   $\Theta_{I_k} = (A_{I_k}^T A_{I_k})^{-1} A_{I_k}^T Y$;

end repeat
\[ r_k = Y - \Theta_{i_k}; \]

\[ \text{when} \quad \|r_k\|_2 < \|r_k\|_2 \quad \text{or} \quad \text{norm}(r_k, 2) < \tau \]

update: \[ \text{Stage} = \text{Stage} + 1, \quad L = \text{Stage} \times S, \]

\[ \text{until} \quad k \geq \text{Iter}_{\text{max}} \quad \text{or} \quad L > M \]

Output: \[ \Theta_{i_k} = \left(A_{i_k}^T A_{i_k}\right)^{-1} A_{i_k}^T Y. \]

4. Simulation and results

In this experiment, the reconstruction performance (ASCE and ERP) without considering the noise of the SALAOMP algorithm is compared to the OMP, SAMP [11], LAOMP and ALAOMP algorithms. The parameters of simulation are set as follows: let the original signal \( X \) be Gaussian sparse signal and binary signal respectively. We also choose \( N \) (ambient dimension of the signal) to be 500. \( X \) is a Gaussian sparse signal, the compression ratio interval set to a total of 10 points on the interval \( [0.1, 0.19] \), while \( X \) is a binary signal, the compression ratio interval is set to a total of 21 points on the interval \( [0.1, 0.3] \). The set number of measurements is 1000 times. The sparsity \( K \) is set to 20. Besides, the forward parameter, threshold, initial step length and threshold of the ALAOMP algorithm are set to \( L = 5, \quad \mu = 0.7, \quad S \leq 5, \) and \( \tau = 10^{-12} \) respectively.

![Figure 1. ASCE in a Gauss random signal.](image1)

![Figure 2. ASCE in a binary sparse signal.](image2)

Figure 1 and figure 2 can show clearly that with the increase of compression ratio, the ASCE of the five algorithms decreases, and the decreasing trend of SALAOMP algorithm is the most obvious. In figure 1, when the compression ratio of the Gaussian sparse signal is 0.12, the ASCE of the algorithm SALAOMP is 0.260, obviously 54%, 42.9%, 20.5% and 12.5% lower than the ASCE of OMP (0.565), SAMP (0.455), LAOMP (0.327), and ALAOMP (0.297) respectively. In figure 2, when the compression ratio of the binary sparse signal is 0.23, the ASCE of SALAOMP algorithm is close to 0, and other four algorithms do not achieve the effect, which fully embodies the advantage of the proposed algorithm.

In figure 3 and figure 4, with the increase of compression rate, the ERP of all five algorithms increases, however, the effect of the proposed algorithm is sharper. When the compression rate is the same in the Gaussian sparse signal, the ERP of the algorithm is much better than other four algorithms, and the algorithm can achieve satisfactory results at a lower compression rate. Meanwhile, with the compression rate increasing ERP faster, when the compression rate reaches 0.17, ERP is close to 1, which is better than other four algorithms. In the binary sparse signal, the compression rate is less than 0.15 because of the binary invariance of the binary signal, but the ERP is clearly rising when the
compression rate is higher than 0.15. The ERP of SALAOMP algorithm is approaching 1 at 0.22, and other four algorithms do not achieve that, which fully demonstrates the superiority of the algorithm.

![Figure 3. ERP in a Gauss random signal.](image1)

![Figure 4. ERP in a binary sparse signal.](image2)

5. Conclusions
In this paper, the SALAOMP algorithm is proposed on the basis of the LAOMP algorithm, which possesses the advantage of having no knowledge of the sparsity of the signal, and not making sure of the number of the atoms in each selection, but can recover the estimation signal with high performance, merely needing to set the initial step length, selecting the biggest inner product in magnitude by regularization. In the sequel, these atoms are added to the LAR algorithm to check the effect on the final performance, and then some of the atoms that satisfy the threshold condition are added to the support set by judging the residual performance, and finally the subsequent iteration is carried out to decide whether the iteration step is updated. Compared and analyzed with The ASCE and ERP performance of OMP, SAMP, LAOMP, SALAOMP algorithm in two cases of Gaussian sparse signal and binary sparse signal, the performance of the proposed SALAOMP algorithm in this paper is better than other four algorithms.

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