Asymptotically stationary and static spacetimes and shear free null geodesic congruences

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Abstract

In classical electromagnetic theory, one formally defines the complex dipole moment (the electric plus $i$ magnetic dipole) and then computes (and defines) the complex center of charge by transforming to a complex frame where the complex dipole moment vanishes. Analogously in asymptotically flat spacetimes, it has been shown that one can determine the complex center of mass by transforming the complex gravitational dipole (mass dipole plus $i$ angular momentum) (via an asymptotic tetrad transformation) to a frame where the complex dipole vanishes.

We apply this procedure to such spacetimes which are asymptotically stationary or static, and observe that the calculations can be performed exactly, without any use of the approximation schemes which must be employed in general. In particular, we are able to exactly calculate complex center of mass and charge world-lines for such spacetimes, and—as a special case—when these two complex world-lines coincide, we recover the Dirac value of the gyromagnetic ratio.

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1. Introduction

It is the purpose of this work to examine the special case of asymptotically stationary or static spacetimes in the context of the recently developed method for the identification of physical quantities from the geometric quantities in general asymptotically flat spacetimes [1–3]. In the general case, due to the complexities and nonlinearities, approximations and restrictions on the spherical harmonic expansions must be employed. In this special case, however, the analysis can be done simply and exactly and is in full agreement with the more general approximate work. In addition, it gives a clearer picture of what the physical identification procedure is.

The principle stage on which this identification method is applied is the future conformal boundary of the spacetime, future null infinity (i.e. Penrose’s $\mathcal{I}^+$). $\mathcal{I}^+$, which is a null (3D)
surface, is coordinatized by the so-called Bondi coordinates \((u, \zeta, \bar{\zeta})\) with \(u = \text{constant}\) cross-sections and \((\zeta, \bar{\zeta})\) labeling its null generators (null geodesics). The asymptotically flat Einstein equations in the spin-coefficient (SC) formalism are integrated in the neighborhood of \(I^+\) leading to the asymptotic behavior of the SC version of the Weyl tensor. These asymptotic Weyl tensor components are objects that live on \(I^+\) (i.e. are functions only of \((u, \zeta, \bar{\zeta})\)). From a standard procedure \([2, 3]\), a spherical harmonic term \((l = 1)\) of a specific Weyl tensor component is identified with the complex dipole moment, 
\[
D^C_{\l} = \left( D^\text{mass}_l + i c^{1} J^l \right),
\]
(mass dipole plus ‘\(i\) × angular momentum).

The next step is to consider how the Weyl tensor, and hence \(D^C_{\l}\), transforms under a change in the Bondi coordinates and tetrad (or a generalization of this) at \(I^+\) and thereby find the transformation that produces the new \(D^C_{\l}\) \([2, 3]\). Setting the new \(D^C_{\l} = 0\) defines the complex center of mass which turns out to be a complex world-line in complex Minkowski space. Ordinarily, the calculations required to transform this Weyl component to zero are quite complicated for a general asymptotically flat spacetime; indeed, exact computations are usually impossible, and approximation schemes must be employed. In practice, this is done in two ways: perturbation terms are expanded only to second order, and harmonic expansions are truncated at the \(l = 2\) contributions (e.g. \([2–4]\)). In the present instance of asymptotically stationary (or static) spacetimes, the calculations simplify and may be performed exactly.

A most important technical tool for these calculations comes from an analysis of null geodesic congruences (NGCs) and specifically from the regular shear-free or asymptotically shear-free NGCs \([5, 6]\), all of which can be constructed from the solutions of the so-called good cut equation \([7]\). The important point is the observation \([2, 3]\) that regular asymptotically shear-free NGCs in asymptotically flat spacetimes (or regular shear-free congruences in Minkowski spacetime) are determined by the free choice of a complex world-line in an auxiliary complex Minkowski spacetime: the freedom in the choice of the solutions to the good cut equation. It turns out that setting the new \(D^C_{\l}\) to zero uniquely determines a specific world-line, referred to as the complex center of mass. This procedure is analogous (with a complex generalization) to the situation in classical electromagnetic theory, where the center of charge world-line is found by transforming to a system where the electric dipole vanishes. The asymptotic Bianchi identities \([8]\) determine the evolution equations for this line, which in turn allows for the physical identifications. Furthermore, these results agree exactly with those from already well-known stationary (or static) spacetimes such as the Kerr metric.

In this paper, we show how the asymptotic Weyl tensor can be found and the asymptotic Bianchi identities easily integrated. From these results, it is simple to show from supertranslation in the asymptotic symmetry group (the Bondi–Metzner–Sachs (BMS) group \([10]\)) that a Bondi coordinate/tetrad system can be constructed so that the asymptotic (Bondi) shear vanishes. This allows us to work with the homogeneous good cut equation, meaning that we have no need to force the termination of spherical harmonic expansions. In turn, we are then able to compute exactly the asymptotic tetrad transformation necessary to find the complex center of mass world-line and that allows us to identify and give a kinematic descriptions of the mass, linear 3-momentum and intrinsic spin of the system.

For completeness, we include the Maxwell field in the discussion, i.e. we are considering the asymptotically flat stationary Einstein–Maxwell equations.

### 2. Complex center of mass in an asymptotically stationary or static spacetime

A spacetime is stationary if it has a time-like Killing vector field (e.g. the Kerr metric) and static if the Killing field is surface forming (e.g. the Schwarzschild metric) \([9]\). In both cases, the metric can be written in coordinates that make it manifestly time independent. For our
purposes, we define asymptotically stationary (or static) spacetimes to mean that only all asymptotic variables of the spacetime under consideration are time independent. Since we will utilize the well-known Bondi coordinate system \((u, r, \zeta, \bar{\zeta})\), this simply means that all asymptotic variables are \(u\) independent. For instance, when considering the Peeling property of the Weyl tensor in the SC formalism \[8\]

\[
\begin{align*}
\psi_0 &= \psi_0^0 r^{-5} + O(r^{-6}), \\
\psi_1 &= \psi_1^0 r^{-4} + O(r^{-5}), \\
\psi_2 &= \psi_2^0 r^{-3} + O(r^{-4}), \\
\psi_3 &= \psi_3^0 r^{-2} + O(r^{-3}), \\
\psi_4 &= \psi_4^0 r^{-1} + O(r^{-2}),
\end{align*}
\]

it follows that

\[
\partial_u \psi_k^0 \equiv \dot{\psi}_k^0 = 0, \quad k = 0, 1, 2, 3, 4,
\]

or that

\[
\psi_k^0 = \psi_k^0 (\zeta, \bar{\zeta}).
\]

The same applies to the asymptotic Maxwell field:

\[
\begin{align*}
\phi_0 &= \phi_0^0 \frac{r^3}{r^3} + O(r^{-4}), \\
\phi_1 &= \phi_1^0 \frac{r^2}{r^2} + O(r^{-3}), \\
\phi_2 &= \phi_2^0 \frac{r}{r} + O(r^{-2}), \\
\phi_3^0 &= \phi_3^0 (\zeta, \bar{\zeta}).
\end{align*}
\]

Recall the asymptotic Bianchi identities for the Weyl and Maxwell tensors in the SC formalism \[8\]:

\[
\begin{align*}
\dot{\psi}_0 &= -\bar{\sigma} \psi_3^0 + \sigma^0 \psi_2^0 + k \phi_2^0 \bar{\sigma}_0^0, \\
\dot{\psi}_1 &= -\bar{\sigma} \psi_2^0 + 2\sigma^0 \psi_3^0 + 2k \phi_1^0 \bar{\sigma}_0^0, \\
\dot{\psi}_2 &= -\bar{\sigma} \psi_1^0 + 3\sigma^0 \psi_1^0 + 3k \phi_0^0 \bar{\sigma}_0^0, \\
\dot{\psi}_3 &= -\bar{\sigma} \sigma^0, \\
\dot{\psi}_4 &= -\bar{\sigma} \sigma^0, \\
\phi_1 &= -\bar{\sigma} \phi_2^0, \\
\phi_0 &= -\bar{\sigma} \phi_1^0 + \sigma^0 \phi_2^0, \\
k &= 2Gc^{-4},
\end{align*}
\]

where \(\sigma^0\) is the complex Bondi shear (the free characteristic data) for the spacetime, coming from the expansion of the full shear

\[
\sigma = \frac{\sigma^0}{r^2} + O(r^{-4}),
\]

and \(\bar{\sigma}\) is the well-known spin-weight operator on \(S^2\). Additionally, we have the Bondi mass aspect as

\[
\Psi \equiv \psi_2^0 + \bar{\sigma}^2 \sigma^0 + \sigma^0 \bar{\sigma}^0,
\]

satisfying the familiar reality condition (derived from the SC equations)

\[
\Psi = \bar{\Psi}.
\]
For the stationary (or static) case, we have
\[ \sigma^0 = \sigma^0(\zeta, \bar{\zeta}). \]

In an asymptotically stationary (or static) spacetime, equations (6)–(12) reduce to
\[ 0 = k\phi_0^0\phi_2^0, \quad (17) \]
\[ 0 = -\bar{\psi}_0 + 2k\phi_1^0\phi_2^0, \quad (18) \]
\[ 0 = -\bar{\psi}_2 + 3\sigma^0\phi_2^0 + 3k\phi_0^0\phi_2^0, \quad (19) \]
\[ \psi_4^0 = 0, \quad (20) \]
\[ \psi_3^0 = 0, \quad (21) \]
\[ 0 = -\bar{\psi}_1 + 2\sigma^0\phi_2^0, \quad (22) \]
\[ \phi_0 = \phi_0^0 Y_{11}(\zeta, \bar{\zeta}) + \cdots, \quad (24) \]
\[ \phi_0^0 = \text{const}, \quad (25) \]

which immediately simplify to
\[ \phi_2^0 = 0, \quad (26) \]
\[ \bar{\psi}_2 = 0, \quad (27) \]
\[ \bar{\psi}_1 = 3\sigma^0\psi_2^0, \quad (28) \]
\[ \bar{\phi}_1^0 = 0, \quad (29) \]

while the mass aspect becomes
\[ \Psi = \psi_2^0 + \bar{\psi}^0. \quad (30) \]

Since \( \psi_2 \) is a spin-weight zero quantity, equation (27) tells us immediately that \( \psi_2^0 \) contains only the \( l = 0 \) harmonic in a spherical harmonic expansion. Keeping this in mind, if we apply the reality condition of (16)–(30), we obtain
\[ \psi_2^0 + \bar{\psi}^0 = \bar{\psi}_2 + \bar{\psi}_2^0, \quad (31) \]

or
\[ \text{Im}(\psi_2^0) = -\frac{i}{2}(\bar{\psi}^0 - \bar{\psi}^0_2) = \text{Im}(\bar{\psi}_2^0). \quad (32) \]

The imaginary part of \( \psi_2^0 \) is thus determined by the ‘magnetic’ part of the asymptotic shear. Since \( \sigma^0 \) has spin-weight 2 and contains only \( l \geq 2 \) harmonics in its harmonic expansion, it follows that the magnetic portion of \( \sigma^0 \) must vanish since we already established that \( \psi_2^0 \) contains only \( l = 0 \) harmonic contributions.

Now, under the operation of a supertranslation subgroup of the BMS group [10], the Bondi time coordinate transforms as
\[ u \rightarrow u' = u + \alpha(\zeta, \bar{\zeta}), \quad (33) \]

for an arbitrary analytic function on the two-sphere. By the Sachs theorem, the asymptotic shear transforms under such a supertranslation according to [8]:
\[ \sigma^0 \rightarrow \sigma'^0 = \sigma^0 + \bar{\alpha}^2(\zeta, \bar{\zeta}), \quad (34) \]
so by the proper choice of the function $\alpha(\zeta, \bar{\zeta})$, the real (‘electric’) portion of the asymptotic shear may be set equal to 0. Hence, we can make both the ‘electric’ and the ‘magnetic’ parts of the asymptotic shear to vanish, and in an appropriate Bondi frame, we have that

$$\sigma^0 = 0.$$  \hspace{1cm} (35)

Using the vanishing of $\sigma^0$, we have from equations (27)–(29) that $\psi^0_2$ and $\phi^0_1$ have only an $I = 0$ part, while $\psi^0_1$ only has an $I = 1$ part. They turn out to be proportional to the mass, the charge and the complex gravitational dipole moment, respectively.

Before turning to the recently developed procedure for the identification of the physical variables hidden in the geometry, we summarize our situation. In a preferred Bondi system, we have the geometric results (using standard canonical Bondi identifications) for the major terms [3]:

\begin{align*}
\psi &= \psi^0_2 = \psi^0_0 + \psi^1_0 Y^0_1, \\
\psi^0_0 &= \psi^0_2 = -\frac{2\sqrt{2}G}{c^2} M_B, \\
\psi^1_0 &= -\frac{6G}{c^3} p^i = 0, \hspace{1cm} (37) \\
\psi^0_1 &= -\frac{6\sqrt{2}G}{c^2} (D^i_{(mass)} + i\zeta^{-1} J^i) Y^i_{11}, \hspace{1cm} (39) \\
\psi^0_0 &= Q_{ij}^C Y^2_{ij} + \cdots, \hspace{1cm} (40) \\
\phi^1_0 &= q, \hspace{1cm} (41) \\
\phi^0_0 &= 2(D^i_E + iD^i_M) Y^i_{11}, \hspace{1cm} \sigma^0 = 0, \hspace{1cm} (42)
\end{align*}

where $(M_B, q, D^i_E, D^i_M, D^i_{(mass)}, J^i, Q_{ij}^C)$ are, respectively, the Bondi mass, the Coulomb charge, the (stationary) electric, magnetic and mass dipoles, angular momentum and complex quadrupole (mass and spin) moments.

The recently developed identification procedure [2, 3] begins with two related ideas. (1) In flat space Maxwell theory, one can transform the origin of coordinates to the center of charge so that the electric dipole moment associated with this origin vanishes. This suggests that the same can be done in general relativity (GR) to the center of mass so that the mass dipole vanishes. (2) It raises the question that: can one generalize this transformation in GR and relate it to a remarkable property of regular shear-free or asymptotically shear-free NGCs: each such congruence is determined uniquely by a complex world-line in an auxiliary complex Minkowski space [3, 6]?

The answer is yes. For general asymptotically flat spacetimes [11], one can choose, in the neighborhood of $\mathcal{I}^+$, a family of (in general twisting) null geodesics, i.e. a NGC, that ‘appears’ to come from a complex world-line in complex Minkowski space [2]. By an appropriate choice of the congruence followed by rotating the Bondi tetrad to a tetrad based on these new null geodesics, one can set the mass dipole to zero. As an added bonus, since the world-line is complex, it can be chosen so that the angular momentum can be made to vanish. It is this general procedure that we apply to the special case of asymptotically stationary spacetimes.

In general, shear-free or asymptotically shear-free NGCs are generated by the solutions

$$u = G(\tau, \zeta, \bar{\zeta})$$

of the so-called good cut equation

$$\partial^2 G = \sigma^0(G, \zeta, \bar{\zeta}),$$

where $\sigma^0 = 0$. (43)
which in our case of vanishing Bondi shear becomes the homogeneous good equation
\[ \Box G = 0. \] (43)

Its general regular solution is given by
\[ u = G(\tau, \zeta, \bar{\zeta}) = \xi^a(\tau) \overline{l}_a(\zeta, \bar{\zeta}) = \sqrt{2} \xi_0(\tau) - \frac{1}{2} \xi^i(\tau) Y^0_1(\zeta, \bar{\zeta}), \] (44)

with
\[ \overline{l}_a(\zeta, \bar{\zeta}) = \left( \frac{\sqrt{2}}{2},\frac{1}{2} Y^0_1(\zeta, \bar{\zeta}) \right), \] (45)

where \( z^a = \xi^a(\tau) \) is the complex Minkowski space world-line parameterized by the complex \( \tau \). The relation given by equation (44) describes a one-parameter family of complex cuts on the complexified \( I^+ \). Though we are interested in the real values of \( u \), the local complexification is essential. The reason lies in the following: the asymptotically shear-free NGC is determined from equation (44) by taking its derivative and defining the stereographic angle field on \( I^+ \) by
\[ L(u, \zeta, \bar{\zeta}) = (\tau) G(\tau, \zeta, \bar{\zeta}) \big|_{\tau = T(u, \zeta, \bar{\zeta})} = \xi^a(\tau) |_{\tau = T(u, \zeta, \bar{\zeta})} \nu_a(\zeta, \bar{\zeta}), \] (46)

where \( \tau = T(u, \zeta, \bar{\zeta}) \) is the inverse function to equation (44), and \( \partial_{(\tau)} \) indicates the application of the \( \partial \) operator, while the variable \( \tau \) is held constant. The real values of \( u \) must be used only after the action of the derivative.

The \( L(u, \zeta, \bar{\zeta}) \) constructed in this manner determines a null direction field at \( I^+ \) that in turn determines an asymptotically shear-free NGC. In addition, it plays a key role in the physical identifications.

We now single out a particular world-line \( \xi^a \) which is to represent a ‘complex center of mass’ for the system, thus choosing a particular good cut function, equation (44). To do this, we recall (39) that the \( l = 1 \) contribution from \( \psi^0_1 \) is identified as the complex gravitational dipole [3]:
\[ \psi^0_1 = -\frac{6\sqrt{2}G}{c^2} D_\zeta = -\frac{6\sqrt{2}G}{c^2} (D_{(\text{mass})} + ic^{-1} J^i), \] (47)

where \( D_{(\text{mass})} \) is the mass dipole and \( J^i \) is the angular momentum (differences with other angular momentum identification theories should vanish in this case, due to the shear freeness and dynamical simplicity). Hence, in a center of mass frame, we expect \( \psi^0_1 = 0 \). The transformation of the Bondi tetrad to the tetrad associated with the shear-free congruence is given asymptotically by the null rotation [8]:
\[ l^a \rightarrow l^{*a} = l^a - \frac{L}{r} m^a - \frac{L}{r} m^a + O(r^{-2}), \] (48)
\[ m^a \rightarrow m^{*a} = m^a - \frac{L}{r} n^a + O(r^{-2}), \] (48)
\[ n^a \rightarrow n^{*a} = n^a. \]
where $L$ is a holomorphic, spin-weight one function given by equation (45) and $\{l^a, n^a, m^a, \bar{m}^a\}$ is the well-known Bondi null tetrad, chosen in this case so that $l^a$ is tangent to geodesics of the constant $u$ null hypersurfaces in the spacetime. Under such a rotation, $\psi_{1}^{0}$ transforms, for the asymptotically stationary (or static) case, as

$$\psi_{1}^{0} \rightarrow \psi_{1}^{0*} = \psi_{1}^{0} - 3L\psi_{2}^{0|1}.$$  \hfill (49)

Setting $\psi_{1}^{0*} = 0$, then using $\psi_{1}^{0} = \frac{-6\sqrt{2}G}{c^2} D_{C}^{l}$, $\psi_{2}^{0} = -\frac{2\sqrt{2}G}{c^2} M_{B}$ and $L(\tau, \zeta, \bar{\zeta}) = \xi^{i}(\tau)Y_{1}^{i}(\zeta, \bar{\zeta})$, equation (49) immediately leads to

$$\psi_{1}^{0} = -\frac{6\sqrt{2}G}{c^2} M_{B}\xi^{i},$$  \hfill (50)

or, the kinematic expression for the complex dipole,

$$D_{C}^{l} = M_{B}\xi^{i},$$  \hfill (51)

$$D_{(mass)}^{l} = M_{B}\xi^{i},$$  \hfill (52)

$$c^{-1} J^{l} = M_{B}\xi^{i}.$$  \hfill (53)

Now, by the time independence of the system (i.e. $P^{l} = 0$), it follows that the ‘spatial’ part of our world-line, $\xi^{i}$, must be a constant 3-vector. Although as it is written now, $\xi^{i}$ is a function of $\tau$, we can invert (44) to obtain the world-line as a function of $u$; thus $u$ independence is equivalent to $\tau$ independence. For such a constant vector, a Poincaré translation can be chosen to set the real part, $\xi^{i}_{R}$, equal to 0, and an ordinary rotation can be made to set the imaginary part to

$$\xi^{i} = (0, 0, \xi^{3}).$$  \hfill (54)

Thus, if we also demand that the tangent vector $v^{a} = \partial_{\tau}\xi^{a}$ be normalized to length one, we obtain the center of mass world-line as

$$\xi^{a} = (\tau, 0, 0, i\xi^{3}).$$  \hfill (55)

From equations (44) and (46), we thus have that

$$u = G(\tau, \zeta, \bar{\zeta}) = \frac{\tau}{\sqrt{2}} - \frac{i}{2}\xi^{3}Y_{1}^{0},$$  \hfill (56)

$$L(\tau, \zeta, \bar{\zeta}) = i\xi^{3}Y_{1}^{1}. $$

Finally, we flesh out our physical identification via equation (47) to get

$$D_{(mass)}^{l} = 0,$$  \hfill (57)

$$J^{l} = S^{l} = cM_{B}\xi^{3}\delta^{l}_{3} = cM_{B}\xi^{i}_{l},$$  \hfill (58)

where we have made the conventional identification of the intrinsic spin as $S^{l} = cM_{B}\xi^{i}_{l}$.

We note that there was no discussion of the higher stationary moments that would appear in the higher powers of $r^{-1}$ in the expansion for the Weyl component $\psi_{0} = \psi_{0}^{0}r^{-5} + \psi_{1}^{1}r^{-6} + \cdots$. They are indeed there, but play no role in the transformation to the complex center of mass. If they happen to vanish, we are left with the asymptotic Kerr or Kerr–Newman metrics.

Note also that the Maxwell field also played no role in the discussion, due to (26). However, there is a parallel discussion, given in the following section, for the Maxwell field when one can transform to the complex center of charge where both the electric and magnetic dipoles vanish.
3. Center of charge world-line

From the asymptotic Maxwell equations, we obtained equations (26), (41) and (42)

\[ \phi_2^0 = 0 \]  
\[ \phi_1^0 = q \]  
\[ \phi_0^0 = 2(D_E^i + iD_M^i)Y_{11}^i. \]  

(59)  
(60)  
(61)

The higher multipole moments, hidden in the higher \( r^{-n} \) terms in the expansion of \( \phi_0 \) (see equation (4)), are not needed or used in this discussion.

We now transform via the asymptotic null rotation, equation (48), to the complex center of charge world-line where both the electric and magnetic dipoles vanish. The \( L \) and its associated complex world-line used in this section, i.e. \( L = \eta i Y_{11}^i \), are in general independent of the \( L \) used in the previous section, \( L = \xi i Y_{11}^i \).

Under this rotation, equation (48), the Maxwell tensor component \( \phi_0^0 \) transforms as

\[ \phi_0^0 \rightarrow \phi_0^{0*} = \phi_0^0 - 2L\phi_1^0 + L^2\phi_2^0. \]  

(62)

Using equations (46) and (59)–(61), with the assumption that \( \phi_0^{0*} = 0 \) at the center of charge, we obtain from (62) that

\[ q\eta^i = D_C^i = D_E^i + iD_M^i. \]  

(63)

In analogy to the complex center of mass line, this determines the complex center of charge world-line,

\[ z'' = \eta'' = (\tau, q^{-1}D_C^i), \]

a straight time-like world-line displaced into the complex.

Note that this is just a complex generalization of finding the real center of charge in electrostatics.

Since we have already fixed the Bondi system via the complex center of mass argument, we cannot further simplify the Maxwell field. We however can consider the very special case where the complex center of charge coincides with the complex center of mass, \( \eta'' = \xi'' \). This leads to the real relation

\[ q^{-1}D_M^i = c^{-1}M_B^{-1}S', \]

or

\[ \frac{D_M^i}{S'} = \frac{q}{cM_B}, \]  

(64)

which leads to the Dirac value of the gyromagnetic ratio, \( g = 2 \).

4. Conclusion

As we can see from equations (55), (57) and (58), the case of an asymptotically stationary (or static) spacetime provides us with a very nice example for the application of the recently developed physical identification theory based on the null rotation (48) to an asymptotically shear-free NGC. In particular, the complex center of mass world-line, Bondi mass and linear momentum, gravitational dipole and spin-angular momentum were all calculated quite easily, and without the need to impose constraints on the harmonic expansions used or the order of the expressions in terms of the world-line. Additionally, we found that these results concurred exactly with what was to be expected from a stationary or static spacetime \( a \ priori. \)
In particular, we found that such spacetimes have vanishing Bondi 3-momentum (as expected) and that all of their angular momenta are intrinsic, or in other words, all angular momenta take the form of intrinsic spin. Furthermore, the spin expression obtained in (58) is exactly that of the Kerr solution, probably the most important example of a stationary spacetime [12]. Our results represent the most general asymptotic results for the stationary or static cases and are unaffected by higher multipole considerations. Of course, for a static spacetime, there should be no spin at all, and this would correspond to a world-line whose spatial part is real (i.e. $\xi^I = 0$). When the two world-lines, complex centers of mass and charge, coincide, we have a `Dirac-like’ particle with $g = 2$ from equation (64).

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