Future universe with $w < -1$ Without Big Smash

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Abstract

It is demonstrated that if cosmic dark energy behaves like a fluid with equation of state $p = w\rho$ ($p$ and $\rho$ being pressure and energy density respectively) as well as generalized Chaplygin gas simultaneously, Big Rip or Big Smash problem does not arise even for equation of state parameter $w < -1$. Unlike other phantom models, here, the scale factor for the future universe is found regular for all time. PACS Number: 98.80.Cq. Keywords: Dark energy, phantom model, big rip and accelerated universe.

Experimental probes, during last few years suggest that the present universe is spatially flat as well as it is dominated by yet unknown form of dark energy [1,2]. Moreover, studies of Ia Supernova [3,4] and WMAP [5,6] show accelerated expansion of the present universe such that $\ddot{a} > 0$ with $a(t)$ being the scale factor of the Friedmann Robertson Walker line-element.
\[ dS^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]. \] \hspace{0.5cm} (1)

Theoretically accelerated expansion of the universe is obtained when the cosmological model is supposed to be dominated by a fluid obeying the equation of state (EOS) \( p = w \rho \) with \( p \) as isotropic pressure, \( \rho \) as energy density and \(-1 \leq w < -1/3\).

In the recent past, it was pointed out that the current data also allowed \( w < -1 \) [7]. Rather, in refs. [8,9,10], it is discussed that these data favor \( w < -1 \) being EOS parameter for phantom dark energy. Analysis of recent Ia Supernova data also support \( w < -1 \) strongly [11,12,13].

Soon after, Caldwell [8] proposed the phantom dark energy model exhibiting cosmic doomsday of the future universe, cosmologists started making efforts to avoid this problem using \( w < -1 \) [14,15]. In the braneworld scenario, Sahni and Shtanov has obtained well-behaved expansion of the future universe without Big Rip problem with \( w < -1 \). They have shown that acceleration is a transient phenomenon in the current universe and the future universe will re-enter matter-dominated decelerated phase [16].

It is found that GR (general relativity)-based phantom model encounters “sudden future singularity” leading to divergent scale factor \( a(t) \), energy density and pressure at finite time \( t = t_s \). Thus the classical approach to phantom model yields big-smash problem. For models with “sudden future singularity” Elizalde, Nojiri and Odintsov [17] argued that, near \( t = t_s \), curvature invariants become very strong and energy density is very high. So, quantum effects should be dominant for \( |t_s - t| < \) one unit of time, like
early universe. This idea is pursued in refs.[18, 19,20] and it is shown that an escape from the big-smash is possible on making quantum corrections to energy density $\rho$ and pressure $p$ in Friedmann equations.

In the framework of Robertson-Walker cosmology, Chaplygin gas (CG) is also considered as a good source of dark energy for having negative pressure, given as

$$p = -\frac{A}{\rho}$$

with $A > 0$. Moreover, it is the only gas having supersymmetry generalization [21,22]. Bertolami et al [12] have found that generalized Chaplygin gas (GCG) is better fit for latest Supernova data. In the case of GCG, eq.(2) looks like

$$p = -\frac{A}{\rho^{1/\alpha}},$$

where $1 \leq \alpha < \infty$. $\alpha = 1$ corresponds to eq.(2).

In this letter, a different prescription for GR-based future universe, dominated by the dark energy with $w < -1$, is proposed which is not leading to the catastrophic situations mentioned above. The scale factor, obtained here, does not possess future singularity. In the present model, it is assumed that the dark energy behaves like GCG, obeying eq.(3) as well as fluid with equation of state

$$p = w\rho \quad \text{with} \quad w < -1$$

simultaneously.

Connecting eq.(3) with the hydrodynamic equation
\[
\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + p\right) \tag{5}
\]

and integrating, it is obtained that

\[
\rho^{(1+\alpha)/\alpha}(t) = A + \left(\rho_0^{(1+\alpha)/\alpha} - A\right)(a_0/a(t))^{3(1+\alpha)/\alpha} \tag{6}
\]

with \(\rho_0 = \rho(t_0)\) and \(a_0 = a(t_0)\), where \(t_0\) is the present time.

Eqs.(3) and (4) yield \(w\) as

\[
w(t) = -\frac{A}{\rho^{(1+\alpha)/\alpha}(t)} \tag{7a}
\]

So, evaluation of eq.(7a) at \(t = t_0\) leads to

\[
A = -w_0\rho^{(1+\alpha)/\alpha}. \tag{7b}
\]

with \(w_0 = w(t_0)\). From eqs.(6) and (7), it is obtained that

\[
\rho = \rho_0\left[-w_0 + (1 + w_0)(a_0/a(t))^{3(1+\alpha)/\alpha}\right]^{\alpha/(1+\alpha)} \tag{8}
\]

with \(w_0 < -1\).

In the homogeneous model of the universe, a scalar field \(\phi(t)\) with potential \(V(\phi)\) has energy density

\[
\rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{9a}
\]

and pressure

\[
p_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{9b}
\]

Using eqs.(3), (4), (7) and (8), it is obtained that
\[ \dot{\phi}^2 = \frac{\rho^{(1+\alpha)/\alpha} + \rho_0^{(1+\alpha)/\alpha} w_0}{\rho}. \]  

(10)

Connecting eqs. (8) and (10), it is obtained that

\[ \dot{\phi}^2 = \frac{(1 + w_0) \rho_0^{(1+\alpha)/\alpha} (a_0/a)^{3(1+\alpha)/\alpha}}{[-w_0 + (1 + w_0)(a_0/a)^{3(1+\alpha)/\alpha}]^{\alpha/(1+\alpha)}}. \]  

(11)

This equation shows that \( \dot{\phi}^2 > 0 \) (giving positive kinetic energy) for \( w_0 > -1 \), which is the case of quintessence and \( \dot{\phi}^2 < 0 \) (giving negative kinetic energy) for \( w_0 < -1 \), being the case of super-quintessence (phantom). As a reference, it is relevant to mention that, long back, Hoyle and Narlikar used C-field (a scalar called creation field) with negative kinetic energy for steady-state theory of the universe [23].

Thus, it is shown that dual behaviour of dark energy fluid, obeying eqs. (3) and (4) is possible for scalars, frequently used for cosmological dynamics. So, this assumption is not unrealistic.

Now the Friedmann equation, with dominance of dark energy having double fluid behaviour, is

\[ \left( \frac{\dot{a}}{a} \right)^2 = H_0^2 \Omega_0 \left[ |w_0| + (1 - |w_0|)(a_0/a(t))^{3(1+\alpha)/\alpha} \right]^{\alpha/(1+\alpha)}, \]  

(12a)

where \( |w_0| > 1 \). \( H_0 \) is the present value of Hubble’s constant and \( \Omega_0 = \rho_0/\rho_{cr,0} \) with \( \rho_{cr,0} = 3H_0^2/8\pi G \) (\( G \) being the Newtonian gravitational constant).

Neglecting higher powers of \( 1 - |w_0|/(a_0/a(t))^{3(1+\alpha)/\alpha} \), eq. (12a) is written as

\[ \frac{\dot{a}}{a} \simeq H_0 \sqrt{\Omega_0 |w_0|^{\alpha/2(1+\alpha)}} \left[ 1 + \frac{\alpha(1 - |w_0|)}{2(1 + \alpha)|w_0|} (a_0/a(t))^{3(1+\alpha)/\alpha} \right] \]  

(12b)
Eq. (12b) is integrated to

\[ a(t) = \frac{a_0}{[2(1 + \alpha)|w_0|]^{\alpha/3(1+\alpha)}} \left[ (\alpha+2(1+\alpha)|w_0|)e^{\delta H_0|w_0|^\alpha/2(1+\alpha)} \sqrt{\Omega_0(t-t_0)} - \alpha(1-|w_0|) \right]^{\alpha/3(1+\alpha)}. \] (13)

yielding accelerated expansion of the universe with \( a(t) \to \infty \) as \( t \to \infty \), supporting observational evidences of Ia Supernova [3,4] and WMAP [5,6]. It is interesting to see that expansion, obtained here, is free from “finite time future singularity” unlike other GR-based phantom models. It is due to GCG behaviour of phantom dark energy.

Moreover, eq. (8) and (13) that energy density grows with time for \( w_0 < -1 \) and decreases for \( w_0 > -1 \). Also \( \rho \rightarrow \rho_0|w_0|^{\alpha/3(1+\alpha)} \) (finite) and \( p \rightarrow -p_0/w_0^{\alpha/3(1+\alpha)} \) as \( t \to \infty \). Eqs. (7) and (8) imply time-dependence of EOS parameter

\[ w = -|w_0|[|w_0| - (|w_0| - 1)(a_0/a(t))^{3(1+\alpha)/\alpha}]^{-1} \] (14)

with \( a(t) \), given by eq. (13). This equation shows that \( w \to -1 \) asymptotically.

The horizon distance for this case \( (a(t) \) given by eq. (16)) is obtained as

\[ d_H(t) \simeq \frac{3(1 + \alpha)a(t)}{\alpha a_0} \left[ \frac{2(1 + \alpha)|w_0|}{\alpha + (\alpha + 2)|w_0|} \right]^{\alpha/3(1+\alpha)} e^{\delta H_0|w_0|^\alpha/2(1+\alpha)} \sqrt{\Omega_0(\alpha t/3(1+\alpha))} \] (15a)

showing that

\[ d_H(t) > a(t). \] (15b)
So, horizon grows more rapidly than the scale factor implying colder and darker universe. It is like flat or open universe without dominance of dark energy.

In this case, Hubble’s distance is

\[
H^{-1} = \frac{3(1 + \alpha)}{\alpha H_0 \sqrt{\Omega_0}} \left\{ 1 - \frac{\alpha(1 - |w_0|)}{\alpha + (\alpha + 2)|w_0|} \exp \left[-H_0|w_0|^{\alpha/2(1+\alpha)} \sqrt{\Omega_0}(t-t_0) \right] \right\}
\]

showing its growth with time such that \(H^{-1} \to \frac{3(1+\alpha)}{\alpha H_0 \sqrt{\Omega_0}} |w_0|^{-\alpha/2(1+\alpha)} \neq 0\) as \(t \to \infty\). Here, \(H^{-1}_\infty\) is found large and finite. It means that, in the present case, galaxies will not disappear when \(t \to \infty\). It is unlike phantom models with future singularity expanding as \(|t - t_s|^n\) for \(n < 0\), where galaxies are expected to vanish near future singularity time \(t_s\) \[8\] as \(H^{-1} \to 0\) for \(t \to t_s\).

In Barrow’s model \[24\]

\[
H^{-1} = B + Ct^q + D(t_s - t)^n \quad \frac{qCt^{q-1} - Dn(t_s - t)^{n-1}}{qCt^q - Dn(t_s - t)^n - 1},
\]

where \(B, C, D\) are positive constants and \(q > 0\). Eq.(17) shows that, for \(n < 1, H^{-1} \to 0\) as \(t \to t_s\) and at \(t = t_s, H^{-1}\) is finite for \(n > 1\). In the model, taken by Nojiri and Odintsov \[18\]

\[
H^{-1} = [\tilde{H}(t) + A'|t_s - t|^n]^{-1},
\]

where \(\tilde{H}(t)\) is a regular function of \(t\) and \(A' > 0\). This equation shows that, for \(n < 0, H^{-1} \to 0\) as \(t \to t_s\) and it is finite at \(t = t_s\) for \(n > 0\).

Thus, it is found that if phantom fluid behaves like GCG and fluid with \(p = w\rho\), it is possible to get accelerated growth of scale factor of the future
universe for time $t_0 < t < \infty$ with no future singularity contrary to other phantom models. Here also, it is obtained that energy density and pressure increase with time, asymptotically approaching finite values $\rho_0|w_0|^{\alpha/(3(1+\alpha))} > \rho_0$ and $-p_0/|w_0|^{1/(3(1+\alpha))} > -p_0$ respectively. It is unlike GR-based models, driven by EOS $p = w\rho$, with $w < -1$ having future singularity at $t = t_s$, where $\rho$ and $p$ are divergent [8,14] or $\rho$ is finite and $p$ is divergent [18,24]. Based on Ia Supernova data, Singh et al [13] have estimated $w_0$ for models in the range $-2.4 < w_0 < -1.74$ upt0 95% confidence level. Taking this estimate as an example with $\alpha = 3$, $\rho_\infty = \rho(t \to \infty)$ is found in the range $1.15\rho_0 < \rho_\infty < 1.24\rho_0$. This does not yield much increase in $\rho$ as $t \to \infty$. But if this model is realistic and future experiments support large $|w_0|$, $\rho_\infty$ will be very high. In both cases, small or large values of $|w_0|$, increase in $\rho$ indicates creation of phantom dark energy in future. It may be due to decay of some other components of energy in universe, which is not dominating, for example cold dark matter.

It is interesting to see that big-smash problem does not arise in the present model. In refs.[17,18,19,20], for models with future singularity, escape from cosmic doomsday is demonstrated using quantum corrections in field equations near $t = t_s$. Here, using classical approach, a model for phantom cosmology, with accelerated expansion, is explored which is free from catastrophic situations. This model is derived from Friedmann equations using the effective role of GCG behaviour in a natural way.

References

[1] A.D. Miller et al , Astrophys. J. Lett. 524 (1999) L1; P. de Bernadis
et al, Nature (London) 400 (2000) 955; A.E. Lange et al, Phys. Rev. D 63 (2001) 042001; A. Melchiorri et al, Astrophys. J. Lett. 536 (2000) L63.

[2] S. Hanay et al, Astrophys. J. Lett. 545 (2000) L5.

[3] S. Perlmutter et al, Astrophys. J. 517 (1999) 565.

[4] A.G. Riess et al, Astron. J. 116 (1998) 1009.

[5] D.N. Spergel et al, astro-ph/0302209.

[6] L. Page et al, astro-ph/0302220.

[7] V. Faraoni, Phys. Rev. D 68 (2003) 063508; R.A. Daly et al, astro-ph/0203113; R.A. Daly and E.J. Guerra, Astron. J. 124 (2002) 1831; R.A. Daly, astro-ph/0212107; S. Hannestad and E. Mortsell, Phys. Rev. D 66 (2002) 063508; A. Melchiorri et al, Phys. Rev. D 68 (2003) 043509; P. Schuecker et al, astro-ph/0211480.

[8] R.R. Caldwell, Phys. Lett. B 545 (2002) 23; R.R. Caldwell, M. Kamionkowski and N.N. Weinberg, Phys. Rev. Lett. 91 (2003) 071301.

[9] H. Ziaeepour, astro-ph/0002400; astro-ph/0301640; P.H. Frampton and T. Takahashi, Phys. Lett. B 557 (2003) 135; P.H. Frampton, hep-th/0302007; S.M. Carroll et al, Phys. Rev. D 68 (2003) 023509; P. Singh, gr-qc/0502086.

[10] J.M. Cline et al, hep-ph/0311312.

[11] U. Alam, et al, astro-ph/0311364; astro-ph/0403687.
[12] O. Bertolami, et al, MNRAS, 353, (2004) 329 [astro-ph/0402387].

[13] P. Singh, M. Sami & N. Dadhich, Phys. Rev.D68 (2003) 023522; M. Sami & A. Toporetsky, gr-qc/0312009.

[14] B. McInnes, JHEP, 08 (2002) 029; hep-th/01120066.

[15] Pedro F. González-Díaz, Phys. Rev.D68 (2003) 021303(R); V.K.Onemli et al, Class. Quan. Grav. 19 (2002) 4607 (gr-qc/0204065); Phys.Rev. D 70 (2004) 107301 (gr-qc/0406098);Class. Quan. Grav. 22 (2005) 59 (gr-qc/0408080).

[16] V. Sahni & Yu.V.Shtanov, JCAP 0311 (2003) 014; astro-ph/0202346 ; G.Calcagni, Phys. Rev.D69 (2004) 103508 ; V. Sahni, astro-ph/0502032.

[17] E. Elizalde, S. Noriji and S. D. Odintsov, Phys. Rev.D70 (2004) 043539 [hep-th/0405034].

[18] S. Nojiri and S. D. Odintsov, Phys. Lett. B 595 (2004) 1 [hep-th/0405078]; Phys. Rev.D70 (2004) 103522 [hep-th/0408170].

[19] S.K.Srivastava, hep-th/0411221.

[20] S. Nojiri, S. D. Odintsov and S. Tsujikawa, hep-th/0501025.

[21] R. Jackiw, “(A particle field theorist’s) Lecture on Supersymmetric Non-Abelian Fluid Mechanics and d-branes”, physics/0010042.

[22] M. C. Bento, O. Bertolami, A.A.Sen, Phys. Rev. D 66 (2002)043507 [gr-qc/0202064]; N. Bilic, G.B.Tupper and R. Viollier, Phys. Lett. B 535
(2002) 17; J.S. Fabris, S.V. Goncalves and P.E. de Soyza, astro-ph/0207430; V. Gorini, A. Kamenshchik and U. Moschella, Phys. Rev. D 67 (2003) 063509 [astro-ph/0210476]; C. Avelino, L.M.G. Beca, J.P.M. de Carvalho, C.J.A.P. Martins and P. Pinto Phys. Rev. D 67 (2003) 023511 [astro-ph/0208528].

[23] F. Hoyle and J.V. Narlikar, MNRAS 108 (1948) 372; 109 (1949) 365; Proc. Roy. Soc. A 282 (1964) 191; MNRAS 155 (1972) 305; J.V. Narlikar and T. Padmanabhan Phys. Rev. D 32 (1985) 1928; F. Hoyle, G. Burbidge and J.V. Narlikar, *A Different Approach to Cosmology* (Cambridge University Press, Cambridge, England, 2000).

[24] J. Barrow, Class. Quan. Grav. 21 (2004) L79 - L82 gr-qc/0403084].