Auxiliary Field Meson Model at Finite Temperature and Density

H. Kouno\(^1\), T. Sakaguchi\(^2\), K. Kashiwa\(^2\), M. Hamada\(^2\),
H. Tokudome\(^1\), M. Matsuzaki\(^2\) and M. Yahiro\(^2\)

\(^1\) Department of Physics, Saga University, Saga 840-8502, Japan
\(^2\) Department of Physics, Kyushu University, Fukuoka 812-8581, Japan
\(^3\) Department of Physics, Fukuoka University of Education, Munakata, Fukuoka 811-4192, Japan

Abstract

Starting from many quark interactions, we construct a nonlinear \(\sigma\)-\(\omega\) model at finite temperature and density. The mesons are introduced as auxiliary fields. Effective quark-meson couplings are strongly related to effective meson masses, since they are derived simultaneously from the original many quark interactions. In this model, even if the effective \(\omega\)-meson mass decreases due to the partial chiral restoration, the equation of state (EOS) of nuclear matter can become soft.

1 Introduction

The \(\omega\)-meson is important for the nuclear structure. It is reported that the reduction of the effective \(\omega\)-meson mass makes the nuclear matter EOS stiffer. [1] However, if we require that the \(\omega\)-meson mean field be proportional to the baryon density, the effective \(\omega\)-nucleon coupling also becomes smaller as the effective \(\omega\)-meson mass becomes smaller and the EOS of nuclear matter becomes softer. [2]

In this paper, we show that, at finite temperature and density, effective meson-quark couplings are strongly related to effective meson masses, if the meson fields are introduced as auxiliary fields which consist of quarks and anti-quarks. Consequently, if the effective \(\omega\)-meson mass decreases, the effective \(\omega\)-quark (or nucleon) coupling decreases and the EOS of nuclear matter becomes softer. Therefore, even if the effective \(\omega\)-meson mass decreases due to the partial chiral restoration, the EOS of nuclear matter can become soft in this model.

2 Auxiliary field method for nonlinear \(\sigma\)-\(\omega\) model

In this section, using the auxiliary field method, [3,4] we construct a nonlinear \(\sigma\)-\(\omega\) model. (For details, see the reference [5].) We start from the many quark interactions [4,5]

\[
\int dt \; V = \sum_{m+n \geq 2} \frac{1}{m!n!} \int d^4x_1 \cdots d^4x_m d^4y_1 \cdots d^4y_m d^4u_1 \cdots d^4u_n d^4v_1 \cdots d^4v_n
\]

\[
\times V^{(m,n)}_{\mu_1, \cdots, \mu_n}(x_1, \cdots, x_m, y_1, \cdots, y_m, u_1 \cdots u_n, v_1 \cdots v_n)
\]

\[
\times :\bar{\psi}(x_1)\psi(y_1) \cdots \bar{\psi}(x_m)\psi(y_m)\bar{\psi}(u_1)\gamma^{\mu_1}\psi(v_1) \cdots \bar{\psi}\gamma^{\mu_n}\psi(v_n) : ,
\]

where \(\psi\) is the quark field. The quantum transition amplitude is given by

\[
Z_{\bar{A}} = \int D\bar{\psi} D\psi \exp \left(i \int d^4x L \right) ,
\]

where \(L\) is the Lagrangian of the model.
where $L$ is the Lagrangian density of the system. Inserting the identity

$$1 = \int \prod_{x,y} D\Sigma_\sigma(x,y) D\Sigma_\omega(x,y) D\sigma(x,y) D\omega^\mu(x,y)$$

$$\exp \left( i \int dx dy \Sigma_\sigma \{ \sigma(x,y) - \bar{\psi}(x) \psi(y) \} \right)$$

$$\times \exp \left( i \int dx dy \Sigma_\omega \{ \bar{\psi}(x) \gamma^\mu \psi(y) - \omega^\mu(x,y) \} \right) ,$$

we introduce the auxiliary meson fields $\sigma(= \bar{\psi}\psi)$ and $\omega^\mu(= \bar{\psi}\gamma^\mu \psi)$ as well as the quark self-energies $\Sigma_\sigma$ and $\Sigma_\omega$. In this model, the expectation values of $\sigma$ and $\omega_0$ fields are proportional to the quark scalar density and the baryon density, respectively.

Integrating the quark field, we obtain by means of the mean field approximation

$$Z_\mu = \int D\sigma D\omega_\mu \exp (i \Gamma[\sigma, \omega_\mu]) ,$$

where $\Gamma$ is the effective action and is given by

$$\Gamma[\sigma, \omega_\mu] = W_0[\Sigma_\sigma[\sigma, \omega_\mu], \Sigma_\omega[\sigma, \omega_\mu]] - \sum_{m+n \geq 2} \int V^{(m,n)}_{\mu_1\ldots\mu_n} \sigma^m \omega^{\mu_1} \cdots \omega^{\mu_n}$$

$$+ \text{Tr} (\sigma \Sigma_\sigma[\sigma, \omega_\mu] - \omega_\mu \Sigma_\omega[\sigma, \omega_\mu]) .$$

The $W_0$ represents the quark energy and the remaining parts represent the meson potential. The quark self-energies $\Sigma_\sigma$ and $\Sigma_\omega$ are determined by the following conditions.

$$\frac{\delta \Gamma}{\delta \sigma} = - \frac{\partial}{\partial \sigma} \left( \sum_{m+n \geq 2} \int V^{(m,n)}_{\mu_1\ldots\mu_n} \sigma^m \omega^{\mu_1} \cdots \omega^{\mu_n} \right) + \Sigma_\sigma = 0 .$$

(6)

$$\frac{\delta \Gamma}{\delta \omega_\mu} = - \frac{\partial}{\partial \omega_\mu} \left( \sum_{m+n \geq 2} \int V^{(m,n)}_{\mu_1\ldots\mu_n} \sigma^m \omega^{\mu_1} \cdots \omega^{\mu_n} \right) - \Sigma_\omega = 0 .$$

(7)

### 3 Effective meson masses, effective couplings and EOS

Because of the conditions (6) and (7), the quark self-energies are strongly related to the meson potential. Therefore, at finite temperature and density, the effective meson-quark couplings are strongly related to the effective meson masses. In the uniform and rotationally invariant matter, we obtain

$$\frac{m^*_\sigma^2}{m_\sigma^2} = \frac{\partial^2 \epsilon}{\partial \sigma^2} = \frac{g_{\sigma\sigma} \Pi g_{\omega\omega}}{m_\sigma^2} + \frac{g_{\sigma\omega}^*}{g_\sigma} \text{ and } \frac{m^*_\omega^2}{m_\omega^2} = - \frac{\partial^2 \epsilon}{\partial \omega^2} = - \frac{g_{\omega\omega} \Pi g_{\omega\omega}^*}{m_\omega^2} + \frac{g_{\omega\omega}^*}{g_\omega} ,$$

(8)

where $g_{\sigma\sigma}^* \equiv - \frac{\partial \sigma}{\partial \sigma}$, $g_{\omega\omega}^* \equiv - \frac{\partial \omega}{\partial \omega}$, $g_{\sigma\omega}^* \equiv - \frac{\partial \sigma}{\partial \omega}$, and $\Pi$ is the polarization function. The $m_\sigma$, $m_\omega$, $g_\sigma$, and the $g_\omega$ are the $\sigma$-meson mass, the $\omega$-meson mass, the $\sigma$-quark coupling and the $\omega$-quark coupling at zero temperature and zero density, respectively, and $\epsilon$ is the energy density of the system. If the effects of the mixing interaction, the term including $g_{\omega\omega}^*$, can be neglected, the square of the effective $\omega$-meson mass is proportional to the effective $\omega$-quark coupling. Therefore, the effective $\omega$-quark coupling decreases as the effective $\omega$-meson mass decreases.
In Fig. 1, we show the baryon density ($\rho_B$) dependence of the binding energy of nuclear matter at zero temperature. In the calculation, we assume that $g_{N\sigma}^\ast = 3g_{\sigma}^\ast$ and $g_{N\omega}^\ast = 3g_{\omega}^\ast$, where $g_{N\sigma}^\ast$ and $g_{N\omega}^\ast$ are the effective $\sigma$-nucleon and $\omega$-nucleon couplings, respectively. In the nonlinear model (NLM) $g_{N\omega}^\ast/g_{N\omega} = m_\omega^2/m_\omega^2 \sim 0.94$ at the normal density $\rho_{B0}$, whereas $g_{N\omega}^\ast/g_{N\omega} = m_\omega^2/m_\omega^2 = 1$ in the linear model (LM). (See Fig. 2.) Although the effective $\omega$-meson mass decreases in the NLM, the EOS in the NLM becomes much softer than that in the LM.

4 Summary

In summary, starting from the many quark interaction, we have constructed the nonlinear $\sigma$-$\omega$ model. The mesons are introduced as auxiliary fields. Effective quark-meson couplings are strongly related to effective meson masses, since they are derived simultaneously from the original many quark interactions. In this model, even if the effective $\omega$-meson mass decreases due to the partial chiral restoration, the effective $\omega$-quark (or nucleon) coupling decreases and the EOS of nuclear matter can become soft.

References

[1] F. Weber, Gy. Wolf, T. Maruyama and S. Chiba, preprint nucl-th/0202071; C.H. Hyun, M.H. Kim and S.W. Hong, Nucl.Phys. A718 (2003) 709.
[2] H. Kouno, Y. Horinouchi and K. Tuchitani, Prog. Theor. Phys. 112 (2004) 831; K. Tuchitani et al., preprint, nucl-th/0407004.
[3] See, e.g. T. Kashiwa and T. Sakaguchi, Phys. Rev. D68 (2003) 065002.
[4] H. Reinhardt, Phys. Lett. B208 (1988) 15.
[5] H. Kouno et al., in preparation.