Charged Higgs contribution to $\bar{B}_s \to \phi \pi^0$ and $\bar{B}_s \to \phi \rho^0$

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Abstract

We study the decay modes $\bar{B}_s \to \phi \pi^0$ and $\bar{B}_s \to \phi \rho^0$ within the frameworks of two-Higgs doublet models type-II and type-III. We adopt in our study Soft Collinear Effective Theory as a framework for the calculation of the amplitudes. We derive the contributions of the charged Higgs mediation to the weak effective Hamiltonian governing the decay processes in both models. Moreover we analyze the effect of the charged Higgs mediation on the Wilson coefficients of the electroweak penguins and on the branching ratios of $\bar{B}_s \to \phi \pi^0$ and $\bar{B}_s \to \phi \rho^0$ decays. We show that within two-Higgs doublet models type-II and type-III the Wilson coefficients corresponding to the electroweak penguins can be enhanced due to the contributions from the charged Higgs mediation leading into enhancement in the branching ratios of $\bar{B}_s \to \phi \pi^0$ and $\bar{B}_s \to \phi \rho^0$ decays. We find that, within two-Higgs doublet models type-II, the enhancement in the branching ratio of $\bar{B}_s \to \phi \pi^0$ can not exceed 18% with respect to the SM predictions. For the branching ratio of $\bar{B}_s \to \phi \rho^0$, we find that the charged Higgs contribution in this case is small where the branching ratio of $\bar{B}_s \to \phi \rho^0$ can be enhanced or reduced by about 4% with respect to the SM predictions. For the case of the two-Higgs doublet models type-III we show that the branching ratio of $\bar{B}_s \to \phi \pi^0$ can be enhanced by about a factor 2 of its value within two-Higgs doublet models type-II. However no sizable enhancement with respect to the SM predictions can be obtained for both $\bar{B}_s \to \phi \pi^0$ and $\bar{B}_s \to \phi \rho^0$ decays.

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I. INTRODUCTION

Within Standard Model (SM) flavour-changing neutral current (FCNC) decays are generated at the one loop level. As a result they are highly suppressed and can serve as a sensitive probe of possible New Physics (NP) beyond SM. Of particular interest are the purely isospin-violating decays $\bar{B}_s \rightarrow \phi\rho^0$ and $\bar{B}_s \rightarrow \phi\pi^0$ that are dominated by electroweak penguins [1]. They have been studied within SM in different frameworks such as QCD factorization as in Refs.[2, 3], in PQCD as in Ref.[4] and using Soft Collinear Effective Theory (SCET) as in Refs.[5, 6]. In Ref.[3] the study has been extended to include NP models namely, a modified $Z^0$ penguin, a model with an additional $U(1)'$ gauge symmetry and the MSSM using QCDF. Their results showed that the additional $Z'$ boson of the $U(1)'$ gauge symmetry with couplings to leptons switched off can enhance the electroweak penguin amplitude sizably leading to an enhancement in their branching ratios by up to an order of magnitude. This finding makes these decay modes are very interesting for LHCb and future $B$ factories searches [3]. Motivated by this possibility we extend the study to the two Higgs doublet models (2HDMs).

In 2HDMs, the Higgs sector of the SM can be extended to include extra $SU(2)_L$ scalar doublet. Accordingly, the simplest picture of the SM Higgs coupling to the quarks and leptons can be modified by the presence of the extra Higgs doublet. This results in several classes of 2HDMs such as 2HDMs type-I, type-II, type-III, type-X and type-Y [7–12]. For 2HDM type-I and type-II an investigation of the effect of the charged Higgs contributions to the electroweak penguins has been done in Ref.[13] where the interest was to explore their significance to $B \rightarrow K\pi$ decay modes. Their conclusion is that the significant contributions to the electroweak penguins are favored for small charged Higgs mass and $\cot\beta = 1$. However taking into account $B \rightarrow X_s\gamma$ constraints rule out this possibility.

In the present work we derive the new contributions to the electroweak penguins that are proportional to $m_b\tan^2\beta/m_t$ which were neglected in Ref.[13]. These new contributions become dominant when $\tan\beta$ becomes large as we will show in the following. Moreover the charged Higgs mediation at tree-level can lead to a set of new operators that can not be generated in the SM. We derive their contributions to the effective Hamiltonian governs the process under consideration and calculate their corresponding Wilson coefficients. Having all these new contributions we will give the predictions for the branching ratios of $\bar{B}_s \rightarrow \phi\pi^0$. 
and $B_s \rightarrow \phi \rho^0$ within 2HDMs type-II which has not been calculated in Ref. [13]. In addition, we extend our study to include 2HDMs type-III which has generic Yukawa structure that can allow for sizable effects in FCNC processes as shown in Ref. [8] and can also enhance CP violation in charm sector [14].

In this work we adopt SCET as a framework for the calculation of the amplitudes [15–18]. SCET provides a systematic and rigorous way to deal with the processes in which energetic quarks and gluons have different momenta modes such as hard, soft and collinear modes. The power counting in SCET reduces the complexity of the calculations. In addition, the factorization formula given by SCET is perturbative to all powers in $\alpha_s$ expansion.

This paper is organized as follows. In Sec. II, we briefly review the decay amplitude for $B \rightarrow M_1 M_2$ within SCET framework. Accordingly, we give a brief review of the SM contribution to the branching ratios of $\bar{B}_s \rightarrow \phi \pi^0$ and $\bar{B}_s \rightarrow \phi \rho^0$ decays within SCET framework. Then we derive the Wilson coefficients in the case of Two Higgs-doublets models type II and type III and analysis their contributions to the branching ratios of $\bar{B}_s \rightarrow \phi \pi^0$ and $\bar{B}_s \rightarrow \phi \rho^0$ in section III. Finally, we give our conclusion in Sec. V.

II. $B \rightarrow M_1 M_2$ IN SCET

At leading order in $\alpha_s$ expansion, the amplitude of $B \rightarrow M_1 M_2$ where $M_1$ and $M_2$ are light mesons can be written as

$$A_{B\rightarrow M_1 M_2}^{LO} = G_F m_B^2 \frac{1}{\sqrt{2}} \left( f_{M_1} \int_0^1 duT_{M_1 J}(u, z) \zeta_{M_1 J}^{BM2}(z) \phi_{M_1}(u) ight. $$

$$\left. + \zeta_{M_1 J}^{BM2} \int_0^1 duT_{M_1 J}(u) \phi_{M_1}(u) \right) + (M_1 \leftrightarrow M_2) \right) . \quad (1)$$

The hard kernels $T_{(M_1, M_2)\zeta}$ and $T_{(M_1, M_2)J}$ can be expressed in terms of the Wilson coefficients depending on the final states mesons $M_1$ and $M_2$. We refer to Refs. [19, 20] for explicit expressions of $T_{(M_1, M_2)\zeta}$ and $T_{(M_1, M_2)J}$ for different $M_1$ and $M_2$ final states mesons.

The hadronic parameters $\zeta_{BM}^M$ and $\zeta_{J}^{BM}$ that appear in Eq. (1) are related to the form factors for $B \rightarrow M$ transitions through the combination $\zeta_{BM}^M + \zeta_{J}^{BM}$ [21]. The power counting implies that $\zeta_{BM}^M \sim \zeta_{J}^{BM} \sim (\Lambda/m_b)^{3/2}$ [21]. Generally, we expect to have large number of $\zeta_{BM}^M$ and $\zeta_{J}^{BM}$ for the $87 B \rightarrow PP$ and $B \rightarrow VP$ decay channels. However, using symmetries
like SU(2) and SU(3) can reduce the number of these parameters [5, 19, 21]. On the other hand a model independent analysis requires to determine them from the experimental data as done for few decay modes of B mesons in Refs. [21, 22]. For a large number of B and B_s decays, the χ² fit method, using the experimental data of the branching fractions and CP asymmetries of the non leptonic B and B_s decays, have been used in Refs. [5, 19] to determine ζ_{BM} and ζ_{JBM}. We refer to refs. [21, 22] for details about the fit method to determine ζ_{BM} and ζ_{JBM}.

In our analysis, we follow ref. [19] and assume a 20% error in both ζ_{BM}^{B(M_1,M_2)} and ζ_{JBM}^{B(M_1,M_2)} due to the SU(3) symmetry breaking. In addition, we use the values of ζ_{BM}^{B(M_1,M_2)} and ζ_{JBM}^{B(M_1,M_2)} given in ref. [5] corresponding to the two solutions obtained from the χ² fit. For the light cone distribution amplitudes we use the same input values given in ref. [22]. Following our work in Ref. [6], the amplitudes of \( B_s \rightarrow φπ^0 \) and \( B_s \rightarrow φρ^0 \) decays corresponding to solution 1 of the SCET parameters are given as

\[
A(B_s^0 \rightarrow φπ^0) \times 10^6 \approx (-3.6C_{10} + 1.4\tilde{C}_{10} + 8.3C_7 - 8.3\tilde{C}_7 + 1.9C_8 - 1.9\tilde{C}_8 - 8.3C_9 + 6.6\tilde{C}_9)λ_i^s \\
+ (2.4C_1 - 0.9\tilde{C}_1 + 5.6C_2 - 4.4\tilde{C}_2)λ_u^s
\]

\[
A(B_s \rightarrow φρ^0) \times 10^6 \approx (-8.3C_{10} - 4.3\tilde{C}_{10} - 11.9C_7 + 11.9\tilde{C}_7 + 0.4C_8 - 0.4\tilde{C}_8 - 11.9C_9 + 0.05\tilde{C}_9)λ_i^s \\
+ (5.5C_1 + 2.9\tilde{C}_1 + 7.9C_2 - 0.03\tilde{C}_2)λ_u^s
\] (2)

while for solution 2 of the SCET parameters we have

\[
A(B_s^0 \rightarrow φπ^0) \times 10^6 \approx (-5.1C_{10} - 0.3\tilde{C}_{10} + 9.3C_7 - 9.3\tilde{C}_7 + 1.1C_8 - 1.1\tilde{C}_8 - 9.3C_9 + 5.2\tilde{C}_9)λ_i^s \\
+ (3.4C_1 + 0.2\tilde{C}_1 + 6.2C_2 - 3.4\tilde{C}_2)λ_u^s
\]

\[
A(B_s \rightarrow φρ^0) \times 10^6 \approx (-7.4C_{10} + 0.33\tilde{C}_{10} - 14.9C_7 + 14.9\tilde{C}_7 - 2.5C_8 + 2.5\tilde{C}_8 - 14.9C_9 + 8.3\tilde{C}_9)λ_i^s \\
+ (4.9C_1 - 0.22\tilde{C}_1 + 9.9C_2 - 5.5\tilde{C}_2)λ_u^s
\] (3)

here \( C_i \) and \( \tilde{C}_i \) are the Wilson coefficients that can be expressed as

\[
C_i = C_i^{SM} + C_i^{H^±}, \quad \tilde{C}_i = \tilde{C}_i^{H^±}
\] (4)

\( \tilde{C}_i \) are the Wilson coefficients corresponding to four-quark operators in the weak effective Hamiltonian that can be obtained by flipping the chirality from left to right and so in the SM \( \tilde{C}_i^{SM} = 0 \). It should be noted that the expressions of the amplitude of \( B_s \rightarrow φρ^0 \) considered
above is only for the decay of $\bar{B}_s$ to two longitudinally polarized $\phi$ and $\rho^0$ mesons. At leading order in the $1/m_b$ expansion expansions, one can match the weak effective Hamiltonian at the scale $\mu \sim m_b$ for $\Delta S = 1$ two body $B$ decays to a SCET Hamiltonian. The SCET Hamiltonian can be expressed in terms of two set of operators namely the leading order operators $Q_{ijf}^{(0)}$ and the relevant subleading operators $Q_{ijf}^{(1)}$ in the $\sqrt{\lambda}$ expansion. Here $f$ refer to $d$ and $s$ quarks and $i = 1, 2, \ldots$. These are the only relevant operators as higher order operators will be suppressed due to the smallness of the scaling parameter $\lambda$ that is defined as $\lambda = \Lambda_{QCD}/m_b$. The decay of $\bar{B}_s$ to two transversely polarized mesons, $\bar{B}_s \to V_\perp V_\perp$, do not receive contributions from $Q_{ijf}^{(0)}$ and $Q_{ijf}^{(1)}$ operators and thus the amplitude given in Eq.(11) is for $P P, PV$ and for two longitudinally polarized vector mesons, $B \to V_\parallel V_\parallel$, [19].

Here $P$ and $V$ stands for pseudoscalar and vector mesons respectively.

In Refs.[23, 24] it was pointed out that $\bar{B} \to V_\perp V_\perp$ decays can be enhanced by the presence of an enhanced $O(m_b)$ electromagnetic operator. This operator can lead to a contribution that are $m_b/\Lambda$ enhanced compared to the amplitudes for $B \to V_\parallel V_\parallel$, but which are, on the other hand, also $\alpha_{em}$ suppressed due to the exchanged photon [19]. Thus, numerically, the contribution from the electromagnetic operator can be expected to be smaller than the $O(m_0^0)$ terms in Eq.(11) [19]. Hence at leading order the only contributions to $B \to V_\perp V_\perp$ can arise from nonperturbative charming penguins $A_{cc}$ [20], which does not contribute to $\bar{B}_s \to \phi \rho^0$ decay, while the other terms are either $1/m_b$ or $\alpha_{em} m_b/\Lambda$ suppressed [19].

The predictions for the branching ratios of $\bar{B}_0^s \to \phi \pi^0$ and $\bar{B}_s \to \phi \rho^0$ within SM are presented in Table I. As can be seen from Table I the SCET predictions for the branching ratios are smaller than PQCD and QCDF predictions. This can be explained as the predicted form factors in SCET are smaller than those used in PQCD and QCDF [5].

As can be seen from Table I the branching ratios of $\bar{B}_s^0 \to \phi \rho^0$ are larger than the branching ratios of $\bar{B}_s^0 \to \phi \pi^0$. Both $\bar{B}_s^0 \to \phi \rho^0$ and $\bar{B}_s^0 \to \phi \pi^0$ decays are generated via the $\bar{B}_s \to \phi$ transition. Thus they have the same non perturbative form factors $\zeta^{B\phi}$ and $\zeta^{B\phi}$. However, using a non-polynomial model for the light cone distribution amplitude $\phi_r(u)$ in the case of $\bar{B}_s^0 \to \phi \rho^0$ decay can lead to a slightly different result from using the polynomial model for the light cone distribution amplitude $\phi_r(u)$ in the case of $\bar{B}_s^0 \to \phi \pi^0$ decay as pointed out in ref.[22]. Another reason for this difference is that the Wilson coefficients $C_7$ and $C_8$ enter the hard kernels, $T_{13}(u)$ and $T_{11}(u, z)$ of $\bar{B}_s^0 \to \phi \rho^0$ with opposite signs to the case in $\bar{B}_s^0 \to \phi \pi^0$ [6].
TABLE I. Branching ratios in units $10^{-8}$ of $\bar{B}_s \to \phi \pi^0$ and $\bar{B}_s \to \phi \rho^0$ decays. The last two columns give the predictions corresponding to the amplitudes in Eqs. (2,3) [6]. On the SCET predictions the errors are due to the CKM matrix elements and SU(3) breaking effects respectively. For a comparison with previous studies in the literature, we list the results evaluated in QCDF [3], PQCD [4].

III. MODELS WITH CHARGED HIGGS BOSONS

Charged Higgs can exist as one of the new Higgs particles in any possible extension of the Higgs sector of the SM such as two Higgs doublet models. In the literature, the 2HDM of type II has been investigated in many processes due to its simple Yukawa sector which respects flavor conservation by requiring that one Higgs doublet couple to down type-quarks and charged leptons while the other one couples to up-type quarks only such as the Higgs potential of the MSSM and so on. One way to achieve this is by imposing a symmetry on the Lagrangian such as $Z_2$ symmetry. Clearly, in the 2HDM of type II there are no FCNC at tree level can be induced by exchanging neutral Higgs particles and flavor violation can be induced only by the CKM matrix elements entering the charged Higgs vertex.

In the two Higgs doublet models type-III both Higgs can couple to up and down type quarks and upon taking some limits we restore back two Higgs doublet model type-II as we will show in the following. Thus the Yukawa sector of this model will allow for FCNC at tree level not only by the charged Higgs mediation but also with the exchanging of neutral Higgs particles. One can avoid the unwanted FCNC at tree level by imposing strong constraints on the new couplings from several observables in some processes as we show in the following. However some new couplings can still escape these constraints and thus can lead to interesting results as explaining the $B \to D^*\tau\nu$ anomaly which can not be explained in 2HDMs type-II [7]. In addition these new coupling can be in general complex and thus can lead to new sources of weak CP violating phases which can enhance direct CP asymmetries comparing to the SM.
The Yukawa Lagrangian of the 2HDMs type-III can be written as \[7, 25\]:

\[
L^\text{eff}_Y = \bar{Q}^a_i \left[ Y^{d}_{fi} \epsilon_{ab} H^b_d - \epsilon^d_{fj} H^a_u \right] d_i R
- \bar{Q}^a_i \left[ Y^{u}_{fi} \epsilon_{ab} H^b_u + \epsilon^u_{fj} H^a_d \right] u_i R + \text{h.c.},
\]

(5)

where \( \epsilon_{ab} \) is the totally antisymmetric tensor, and \( \epsilon^q_{ij} \) parameterizes the non-holomorphic corrections which couple up (down) quarks to the down (up) type Higgs doublet. After electroweak symmetry breaking the two Higgs doublets \( H_u \) and \( H_d \) result in the physical Higgs mass eigenstates \( A^0 \) (CP-odd Higgs), \( H^0 \) (heavy CP-even Higgs), \( h^0 \) (light CP-even Higgs) and \( H^\pm \). In our study we follow Refs.\[7, 25\] and assume a MSSM-like Higgs potential and thus the charged Higgs mass is given by

\[
m^2_{H^\pm} = m_{A^0}^2 + m_W^2
\]

(6)

where the \( W \) boson mass, \( m_W \), is related to the the vacuum expectation values of the neutral component of the Higgs doublets, \( v_u \) and \( v_d \), via

\[
m_W^2 = \frac{1}{2} g^2 (v_u^2 + v_d^2) = \frac{1}{2} g^2 v^2
\]

(7)

and the mass \( m_{A^0} \) is treated as a free parameter. It should be noted that in the limit \( v << m_{A^0} \) all heavy Higgs masses (\( m_{H^0} \), \( m_{A^0} \) and \( m_{H^\pm} \)) are approximately equal \[8\].

The effective Lagrangian \( L^\text{eff}_Y \) gives rise to the following charged Higgs-quarks interaction Lagrangian:

\[
L^\text{eff}_{H^\pm} = \bar{u}_f \Gamma^{H^\pm LR \text{eff}}_{u_f d_i} P_R d_i + \bar{u}_f \Gamma^{H^\pm RL \text{eff}}_{u_f d_i} P_L d_i,
\]

(8)

with \[7\]

\[
\Gamma^{H^\pm LR \text{eff}}_{u_f d_i} = \sum_{j=1}^{3} \sin \beta V_{fj} \left( \frac{m_{d_i}}{v_d} \delta_{ji} - \epsilon^d_{ji} \tan \beta \right),
\]

\[
\Gamma^{H^\pm RL \text{eff}}_{u_f d_i} = \sum_{j=1}^{3} \cos \beta \left( \frac{m_{u_f}}{v_u} \delta_{ji} - \epsilon^u_{ji} \tan \beta \right) V_{ji}
\]

(9)

Here \( V \) is the CKM matrix and \( \tan \beta = v_u/v_d \). Using the Feynman-rule given in Eq.(8) we can derive the contributions of the charged Higgs mediation to the weak effective Hamiltonian governs the \( b \to s \) transition. The weak effective Hamiltonian in this case is generated from diagrams similar to the case of the SM with the replacing of the charged \( W \) bosons.
with the charged Higgs bosons. Thus the weak effective Hamiltonian is the same as in the SM with only exception is that the presence of a new set of operators obtained from the SM ones by changing the chirality from left to right. For the left chirality operators we derived the corresponding Wilson coefficients due to the charged Higgs mediation and we find that they are given as:

\[ C_{1,2}^{(H^\pm)} = 0, \]
\[ C_{3}^{(H^\pm)} = -\frac{\sqrt{2} \alpha s \cos^2 \beta}{24 \pi G_F m_{H^\pm}^2} \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) I_1(x), \]
\[ C_{4}^{(H^\pm)} = \frac{\sqrt{2} \alpha s \cos^2 \beta}{8 \pi G_F m_{H^\pm}^2} \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) I_1(x), \]
\[ C_{5}^{(H^\pm)} = -\frac{\sqrt{2} \alpha s \cos^2 \beta}{24 \pi G_F m_{H^\pm}^2} \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) I_1(x), \]
\[ C_{6}^{(H^\pm)} = \frac{\sqrt{2} \alpha s \cos^2 \beta}{8 \pi G_F m_{H^\pm}^2} \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) I_1(x), \]
\[ C_{7}^{(H^\pm)} = \frac{\sqrt{2} \alpha s \cos^2 \beta}{6 \pi G_F m_{H^\pm}^2} \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) (I_2(x) + I_3(x)), \]
\[ C_{8}^{(H^\pm)} = 0, \]
\[ C_{9}^{(H^\pm)} = \frac{\sqrt{2} \alpha s \cos^2 \beta}{6 \pi G_F m_{H^\pm}^2} \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) \left( \frac{m_t}{v_u} - \epsilon_{33}^u \tan \beta \right) \left( I_2(x) + I_3(x) - \frac{1}{\sin^2 \theta_w} I_2(x) \right), \]
\[ C_{10}^{(H^\pm)} = 0. \]

Where the the loop functions \( I_{1,2,3}(x) \) are given by

\[
I_1(x) = \frac{x(7x^2 - 29x + 16)}{36(x-1)^3} + \frac{x(3x^2 - 2)}{6(x-1)^4} \log x
\]

and

\[
I_2(x) = \frac{x}{2(x-1)} - \frac{x}{2(x-1)^2} \log x \]
\[ I_3(x) = \frac{x(47x^2 - 79x + 38)}{108(x-1)^3} + \frac{x(-3x^2 + 6x - 4)}{18(x-1)^4} \log x
\]

with \( x = m_t^2/m_{H^\pm}^2 \). In Eq.\((\ref{10})\), we neglected the small contributions to the Wilson coefficients from the terms that are proportional to \( \epsilon_{13}^u \) and \( \epsilon_{23}^u \) due to the strong constraints on these parameters from \( b \rightarrow d \gamma \) and \( b \rightarrow s \gamma \) respectively arising at the one loop-level \(\text{[8]}\).

The charged Higgs mediation can give rise to new set of Wilson coefficients corresponding
to flipping the chirality in the effective Hamiltonian from left to right:

\[ \tilde{C}_{1,2}^{(H^\pm)} = 0, \]

\[ \tilde{C}_{3}^{(H^\pm)} = -\frac{\sqrt{2}}{24\pi G_F m_{H^\pm}^2} \left( \frac{m_b}{v_d} - \epsilon_{33}^d \tan \beta \right) \left( \frac{m_s}{v_d} - \epsilon_{22}^d \tan \beta \right) I_1(x), \]

\[ \tilde{C}_{4}^{(H^\pm)} = \frac{\sqrt{2}}{8\pi G_F m_{H^\pm}^2} \left( \frac{m_b}{v_d} - \epsilon_{33}^d \tan \beta \right) \left( \frac{m_s}{v_d} - \epsilon_{22}^d \tan \beta \right) I_1(x), \]

\[ \tilde{C}_{5}^{(H^\pm)} = -\frac{\sqrt{2}}{24\pi G_F m_{H^\pm}^2} \left( \frac{m_b}{v_d} - \epsilon_{33}^d \tan \beta \right) \left( \frac{m_s}{v_d} - \epsilon_{22}^d \tan \beta \right) I_1(x), \]

\[ \tilde{C}_{6}^{(H^\pm)} = \frac{\sqrt{2}}{8\pi G_F m_{H^\pm}^2} \left( \frac{m_b}{v_d} - \epsilon_{33}^d \tan \beta \right) \left( \frac{m_s}{v_d} - \epsilon_{22}^d \tan \beta \right) I_1(x), \]

\[ \tilde{C}_{7}^{(H^\pm)} = \frac{\sqrt{2}}{6\pi G_F m_{H^\pm}^2} \left( \frac{m_b}{v_d} - \epsilon_{33}^d \tan \beta \right) \left( \frac{m_s}{v_d} - \epsilon_{22}^d \tan \beta \right) \left( I_2(x) + I_3(x) \right), \]

\[ \tilde{C}_{8}^{(H^\pm)} = 0, \]

\[ \tilde{C}_{9}^{(H^\pm)} = \frac{\sqrt{2}}{6\pi G_F m_{H^\pm}^2} \left( \frac{m_b}{v_d} - \epsilon_{33}^d \tan \beta \right) \left( \frac{m_s}{v_d} - \epsilon_{22}^d \tan \beta \right) \left( I_2(x) + I_3(x) - \frac{1}{\sin^2 \theta_w} I_2(x) \right), \]

\[ \tilde{C}_{10}^{(H^\pm)} = 0, \]

(13)

As before, in the above equation, we neglected the small contributions to the Wilson coefficients from the terms that are proportional to \( \epsilon_{32}^d \) and \( \epsilon_{12}^d \) due to the strong constraints on these parameters from tree-level contributions to FCNC processes [8].

The charged Higgs mediation at tree level can lead to the following weak effective Hamiltonian

\[ \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ub} \sum_{i=11}^{14} C_i^H(\mu) Q_i^H(\mu), \]

(14)

where \( C_i^H \) are the Wilson coefficients obtained by perturbative QCD running from \( M_{H^\pm} \) scale to the scale \( \mu \) relevant for hadronic decay and \( Q_i^H \) are the relevant local operators at low energy scale \( \mu \approx m_b \). The operators can be written as

\[ Q_{11}^H = (\bar{u} P_L b)(\bar{s} P_R u), \]

\[ Q_{12}^H = (\bar{u} P_R b)(\bar{s} P_L u), \]

\[ Q_{13}^H = (\bar{u} P_L b)(\bar{s} P_L u), \]

\[ Q_{14}^H = (\bar{u} P_R b)(\bar{s} P_R u), \]

(15)
And the corresponding Wilson coefficients $C_i^H$ are given as

\[
C_{11}^H = \frac{\sqrt{2}}{G_F V_{us}^* V_{ub} m_W^2} \left( \sum_{j=1}^{3} \cos \beta V_{j2} \left( \frac{m_u}{v_u} \delta_{j1} - \epsilon_{j1}^u \tan \beta \right) \right) \left( \sum_{k=1}^{3} \cos \beta V_{k3} \left( \frac{m_u}{v_u} \delta_{k1} - \epsilon_{k1}^u \tan \beta \right) \right),
\]

\[
C_{12}^H = \frac{\sqrt{2}}{G_F V_{us}^* V_{ub} m_W^2} \left( \sum_{j=1}^{3} \sin \beta V_{1j} \left( \frac{m_d}{v_d} \delta_{j3} - \epsilon_{j3}^d \tan \beta \right) \right) \left( \sum_{k=1}^{3} \sin \beta V_{1k} \left( \frac{m_d}{v_d} \delta_{k2} - \epsilon_{k2}^d \tan \beta \right) \right),
\]

\[
C_{13}^H = \frac{\sqrt{2}}{G_F V_{us}^* V_{ub} m_W^2} \left( \sum_{j=1}^{3} \cos \beta V_{j3} \left( \frac{m_u}{v_u} \delta_{j1} - \epsilon_{j1}^u \tan \beta \right) \right) \left( \sum_{k=1}^{3} \sin \beta V_{1k} \left( \frac{m_d}{v_d} \delta_{k2} - \epsilon_{k2}^d \tan \beta \right) \right),
\]

\[
C_{14}^H = \frac{\sqrt{2}}{G_F V_{us}^* V_{ub} m_W^2} \left( \sum_{k=1}^{3} \cos \beta V_{k2} \left( \frac{m_u}{v_u} \delta_{k1} - \epsilon_{k1}^u \tan \beta \right) \right) \left( \sum_{j=1}^{3} \sin \beta V_{1j} \left( \frac{m_d}{v_d} \delta_{j3} - \epsilon_{j3}^d \tan \beta \right) \right),
\]

(16)

**IV. NUMERICAL RESULTS AND ANALYSIS**

In order to estimate the enhancements in the full Wilson coefficients $C_7$ and $C_9$ due to the charged Higgs contribution we define the ratios: $R_i^{H^\pm} = |C_i^{H^\pm}|/|C_i^{SM}|$ and $\tilde{R}_i^{H^\pm} = |\tilde{C}_i^{H^\pm}|/|C_i^{SM}|$ for $i = 7, 9$ where $C_i$ are the SM Wilson coefficients. These ratios will give us an indication about the magnitudes of the charged Higgs Wilson coefficients compared to the SM ones and thus can give a hint of the expected enhancement or reduction in the branching ratios of our decay channels. We also define the ratios $R_{bi}^M = (BR_{i}^{SM+H^\pm}(B_s \rightarrow \phi M) - BR_{i}^{SM}(B_s \rightarrow \phi M))/BR_{i}^{SM}(B_s \rightarrow \phi M)$ where $M = \pi, \rho$, $i = 1, 2$ refers to solutions 1, 2 for the SCET parameter space for which the corresponding amplitudes are given in Eqs. (213) and $BR_{i}^{SM+H^\pm}(B_s \rightarrow \phi M)$ and $BR_{i}^{SM}(B_s \rightarrow \phi M)$ are the branching ratios obtained when we consider the total contributions including charged Higgs and the SM contributions alone respectively. These ratios will give us the size of the enhancement or reduction to the branching ratios of our decay modes compared to the contribution from the SM.

**A. Two Higgs doublet model type-II**

We start by considering two Higgs doublets models type II. In this case the Wilson coefficients can be obtained from Eqs. (10, 13) by setting $\epsilon_{33}^u = \epsilon_{22}^d = \epsilon_{33}^d = 0$.

The requirement for the top and bottom Yukawa interaction to be perturbative results in a constraint on $\tan \beta$ namely, $0.4 \lesssim \tan \beta \lesssim 91$ [26]. LEP has performed a direct search for
a charged Higgs in 2HDM type-II and they have set a lower limit on the mass of the charged Higgs boson of 80 GeV at 95% C.L., with the process $e^+e^- \rightarrow H^+H^-$ upon the assumption $BR(H^+ \rightarrow \tau^+\nu) + BR(H^+ \rightarrow c\bar{s}) + BR(H^+ \rightarrow A^+\bar{W}^0) = 1$. If $BR(H^+ \rightarrow \tau^+\nu) = 1$ the bound on the mass of the charged Higgs is 94 GeV [27]. Recent results on $B \rightarrow \tau\nu$ obtained by BELLE [28] and BABAR [29] have strongly improved the indirect constraints on the charged Higgs mass in type II 2HDM [30]:

$$m_{H^+} > 240\text{GeV at 95\%CL}$$ \hspace{1cm} (17)

Other experimental bounds can be applied on the $(\tan\beta, m_{H^\pm})$ plane such as the bounds from $B \rightarrow X_s\gamma$ [8, 31], $B_s \rightarrow \mu^+\mu^-$, $B \rightarrow \tau\nu$, $K \rightarrow \mu\nu/\pi \rightarrow \mu\nu$ [8] and the bounds from ATLAS [32] and CMS [33] collaborations coming from $pp \rightarrow t\bar{t} \rightarrow b\bar{b}W^\pm H^\pm (\rightarrow \tau\nu)$.

We note from Eq.(10), after setting $\epsilon_{u3}^{33} = 0$, that the dependency of the Wilson coefficients $C_{u7,9}^{(H^\pm)}$ are on $\cos^2\beta/v_u^2 = 1/(v\tan\beta)^2$. Thus small values of $\tan\beta$ these Wilson coefficients $C_{u7,9}^{(H^\pm)}$ will blow up and can enhance sizeably the branching ratios of the decay channels under consideration. On the other hand we note from Eq.(13), after setting $\epsilon_{d22}^d = \epsilon_{d33}^d = 0$, the situation is reversed for $\tilde{C}_{d7,9}^{(H^\pm)}$ as the dependency in this case is on $\cot^2\beta$ and thus large values of $\tan\beta$ can enhance the branching ratios. In both cases small values of charged Higgs mass are required.

In Fig.(1) we plot $R_i^{H^\pm}$ and $\tilde{R}_i^{H^\pm}$ for $i = 7, 9$ verses $\tan\beta$ for a value of the charged Higgs mass $m_{H^\pm} = 380\text{GeV}$. This mass is the lower limit of the charged Higgs mass allowed by $B \rightarrow X_s\gamma$ constraints [31]. In the left diagram the blue (red) curve corresponds to $R_7^{H^\pm}$ ($\tilde{R}_7^{H^\pm}$) while in the right diagram it corresponds to $R_9^{H^\pm}$ ($\tilde{R}_9^{H^\pm}$). As expected from Eq.(10) the Wilson coefficients $C_{u7,9}^{(H^\pm)}$ vary inversely with $\tan^2\beta$ which can is clear in Fig.(1). Thus larger values of $C_{u7,9}^{(H^\pm)}$ can be obtained for smaller values of $\tan\beta$. For a value of $\tan\beta = 0.4$ allowed by the perturbativity of the top and bottom Yukawa interaction we find that $R_7^{H^\pm} \approx 400\%$. This indicates that $C_7^{(H^\pm)} \approx 4C_{7}^{SM}$ and represent the maximum value can be reached as $\tan\beta < 0.4$ is excluded by the perturbativity of the top and bottom Yukawa interaction constraints. For the case of the Wilson coefficients $C_9^{(H^\pm)}$ we find that $R_9^{H^\pm} \approx 3\%$. This indicates that $C_9^{(H^\pm)} \approx 0.03C_9^{SM}$. For larger values of $\tan\beta$ the ratios $R_i^{H^\pm}$ become so small and close to zero as shown in Fig.(1) indicating very small values of the Wilson coefficients $C_{d7,9}^{(H^\pm)}$ compared to their corresponding ones in the SM. Turning now to the Wilson coefficients $\tilde{C}_{d7,9}^{(H^\pm)}$ where the dependency in this case will be directly on
\( \tan^2 \beta \) as shown in Eqs. (13). Thus larger values of \( \tilde{C}_{7,9}^{(H \pm)} \) can be obtained for larger values of \( \tan \beta \). They are represented by the red curves in Fig. (1). For a value of \( \tan \beta = 91 \) allowed by the perturbativity of the top and bottom Yukawa interaction we find that \( \tilde{R}_7^{H \pm} \approx 8\% \). This indicates that \( \tilde{C}_{7}^{(H \pm)} \approx 0.08 C_{7}^{SM} \) and represent the maximum value can be reached as \( \tan \beta > 91 \) is excluded by the perturbativity of the top and bottom Yukawa interaction constraints. For the case of the Wilson coefficients \( \tilde{C}_{9}^{(H \pm)} \) we find that \( \tilde{R}_9^{H \pm} \approx 0.05\% \). For smaller values of \( \tan \beta \) the ratios \( \tilde{R}_7^{H \pm} \) become so small as shown in Fig. (1) indicating very small values of the Wilson coefficients \( \tilde{C}_{7}^{(H \pm)} \). We note also from Fig. (1) that \( \tilde{R}_7^{H \pm} \gg \tilde{R}_9^{H \pm} \) and similarly for \( \tilde{R}_7^{\tilde{H} \pm} \gg \tilde{R}_9^{\tilde{H} \pm} \) this is because in the denominators of these ratios \( C_{9}^{SM} \gg C_{7}^{SM} \).

Turning now to the Wilson coefficients \( C_{11}^{H} - C_{14}^{H} \) given in Eq. (16). By setting \( u_{ij}^{u,d} = 0 \) we find that \( C_{11}^{H}, C_{13}^{H} \) and \( C_{14}^{H} \) will be suppressed by the smallness of the product of quark masses \( m_u^2, m_u m_s \) and \( m_u m_b \) respectively. For \( C_{12}^{H} \) we find that it is proportional to \( m_s m_b \tan \beta \) which can be enhanced for large values of \( \tan \beta \) in a similar manner to \( C_{11}^{D} \) resulted from the charged Higgs mediation in the MSSM with large \( \tan \beta \) considered in Ref. [34]. Since all these Wilson coefficients have to be multiplied by the CKM factor \( \lambda_s \) they should be compared to the tree level Wilson coefficient of the SM. Clearly \( C_{11}^{H}, C_{13}^{H} \) and \( C_{14}^{H} \) can be

FIG. 1. Left diagram corresponds to \( R_7^{H \pm} (\tilde{R}_7^{H \pm}) \) in units of \( 10^{-2} \) blue (red) curve as a function of \( \tan \beta \). The right diagram corresponds to \( R_9^{H \pm} (\tilde{R}_9^{H \pm}) \) in units of \( 10^{-2} \) blue(red) curve as a function of \( \tan \beta \). In both plots we take \( m_{H \pm} = 380 \, \text{GeV} \).
safely drop and only $C_{12}^H$ can be comparable with the SM tree level Wilson coefficients only when $\tan\beta$ is large. However due to the constraints from $B^+ \to \tau^+\nu_\tau$, one find that $C_{12}^H$ is roughly one order of magnitude smaller than $C_2^{SM}$ as can be read from Eq.(24) in Ref.[34]. Thus we can also safely drop $C_{12}^H$ in our analysis.

In Fig.(2) we plot $R_{b_1}^\pi$ (R_{b_2}^\pi), blue(red) curve, as a function of $\tan\beta$ for $m_{H^\pm} = 380\,GeV$ left plot and the right plot is for $m_{H^\pm} = 1000\,GeV$.

For the lower bound on $\tan\beta = 0.4$ and for $m_{H^\pm} = 380\,GeV$ we find that $R_{b_1}^\pi \simeq 18\%$, $R_{b_2}^\pi \simeq 14\%$ which means charged Higgs contributions to the branching ratio of $\bar{B}_s \to \phi\pi$ can reach a maximum value 18% of the SM prediction. For $m_{H^\pm} = 1000\,GeV$ and $\tan\beta = 0.4$ the charged Higgs contributions to the branching ratio of $\bar{B}_s \to \phi\pi$ can reach 3% and 0.64% of the SM prediction corresponding to solutions 1 and 2 of the SCET parameter space respectively as shown in the plot. We note from Fig.(2) that $R_{b_1}^\pi > R_{b_2}^\pi$ for all values of $\tan\beta$. This can be explained by noticing that their denominators are $BR_1^{SM}(\bar{B}_s \to \phi\pi)$ and $BR_2^{SM}(\bar{B}_s \to \phi\pi)$ and form Table I we have $BR_2^{SM}(\bar{B}_s \to \phi\pi) > BR_1^{SM}(\bar{B}_s \to \phi\pi)$. Another remark is that $R_{b_2}^\pi$ varies with $\tan\beta$ and can have positive, zero and negative values. The reason is as follows: for $\tan\beta < 5$ we see from Fig.(1) that $C_7^{(H^\pm)} \gg \tilde{C}_7^{(H^\pm)}$. Note also $C_7^{(H^\pm)}$ has similar sign to $C_{SM}^{H^\pm}$ and thus it leads to instructive effect and enhance the amplitude. For values of $5 < \tan\beta < 20$ we find that the term in the amplitude proportional to $\tilde{C}_7^{(H^\pm)}$ starts to be non zero and have opposite sign to the total Wilson coefficient $C_7$ leading to a
destructive effect and almost Higgs contributions become negligible and thus we get $R_{b_2}^\pi = 0$. For $\tan \beta \geq 20$ we find that $\tilde{C}_7^{(H^\pm)} > C_7^{(H^\pm)}$ and thus it reduces the amplitude leading to $BR^{SM+H^\pm}(\bar{B}_s \to \phi M) < BR^{SM}(\bar{B}_s \to \phi M)$ and thus we obtain the negative values in the plot. Turning to $R_{b_1}^\pi$ we find the effect caused by the relative size of $\tilde{C}_7^{(H^\pm)}$ and $C_7^{(H^\pm)}$ is small as the coefficient of the $\tilde{C}_7^{(H^\pm)}$ term in the amplitude corresponding to solution 1 is smaller than its corresponding one in solution 2. This explains why we do not have zero and negative values for $R_{b_1}^\pi$ as we have for $R_{b_2}^\pi$ as shown in Fig.(2).

So far we have applied only the constraints from the requirement that the top and bottom Yukawa interaction to be perturbative to just give an estimation of the maximum enhancement can be obtained in 2HDMs type-II. We have selected two values of the charged Higgs mass and found that for the two values of the charged Higgs mass $m_{H^\pm} = 380\ GeV$ and $m_{H^\pm} = 1000\ GeV$ the maximum enhancement can be 18% of the SM prediction and correspond to solution 1 of the SCET parameter space. Thus for charged Higgs masses smaller than 380 GeV and values of $\tan \beta \lesssim 0.4$ the enhancement in the branching ratio of $\bar{B}_s \to \phi \pi$ can exceed 18%. This result motivates us to determine the regions in the $(\tan \beta, m_{H^\pm})$ plane which the enhancement in the branching ratio of $\bar{B}_s \to \phi \pi$ can be 18% or more of the SM prediction. In Fig.(3) we plot this region in the $(\tan \beta, m_{H^\pm})$ plane.

In Ref.[8], see Figure 1, an updated study of the possible constraints imposed on the $(\tan \beta, m_{H^\pm})$ plane of the two Higgs doublet model type-II from the experimental measurements in $B \to s\gamma$, $B \to D\tau\nu$, $B \to \tau\nu$, $K \to \mu\nu/\pi \to \mu\nu$, $B_s \to \mu^+\mu^-$ and $B \to D^*\tau\nu$ showed that no region in the $(\tan \beta, m_{H^\pm})$ plane is compatible with all these processes. Explaining $B \to D^*\tau\nu$ requires large values of $\tan \beta$ and very small Higgs mass and thus together with $B \to s\gamma$ constraints excludes the green region in Fig.(3). Thus we conclude that the enhancement in the branching ratio is always less than 18% for the allowed regions in the $(\tan \beta, m_{H^\pm})$ plane assuming no constraints from the anomaly in $B \to D^*\tau\nu$ observed by BABAR. However if this anomaly is confirmed in the near future by other experiments, such as Belle-II experiment, then taking into account $B \to D^*\tau\nu$ and $B \to s\gamma$ will rule out the whole parameter space of the charged Higgs in the two Higgs doublet model type-II.

As can be seen from Table I the errors of the SM predictions to the branching ratios are approximately 40% and thus it is clear that the enhancement in the branching ratio by 18% with respect to the SM predictions due to the charged Higgs mediation will be invisible within the theoretical uncertainties in the SM predictions.

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Turning now to the branching ratios of $\bar{B}_s \to \phi \rho$, we find that they can be enhanced or reduced by the charged Higgs contribution. However the enhancement or the reduction are always less than 4% of the SM prediction for the allowed regions in the $(\tan \beta, m_{H^\pm})$ plane.

![Graph showing allowed values of the parameter space which enhance $\text{Br} (\bar{B}_s \to \phi \pi^0)$ by more than or equal 18% for solution 1 of the SCET parameter space.]

**B. Two Higgs doublet model type-III**

We turn now to the case of two Higgs doublet models type III. In this case the Wilson coefficients are those given in Eqs. (10,13) and the parameter space contains extra parameters which are the couplings $\epsilon^q_{ij}$ where $q = u, d$ appears in the Yukawa Lagrangian.

We start our analysis by discussing the constraints on the parameters $\epsilon^u_{33}$, $\epsilon^d_{22}$ and $\epsilon^d_{33}$ relevant to our decay modes. Possible constraints on these parameters can be obtained by applying the naturalness criterion of ’t Hooft to the quark masses. According to this criterion the smallness of a quantity is only natural if a symmetry is gained in the limit in which this quantity is zero [8]. Hence applying this criterion to the quark masses in the 2HDM of type III we find that for $i \geq j$

$$|v_{u(d)} \epsilon^{d(u)}_{ij}| \leq \max [m_{d_i(u_i)}, m_{d_j(u_j)}] .$$

(18)
As can be seen from the above equation that $\epsilon_{22}^d$ will be severely constrained by the small mass of the strange quark. In addition the constraints are expected to become more stronger with increasing the value of $\tan \beta$ due to the inverse dependency on $v_u = v \sin \beta$ which increase with increasing $\tan \beta$. However we find that $v_u$ changes slightly with varying $\tan \beta$ and thus the constraints are insensitive to the values of $\tan \beta$. It is easy to check that the absolute values of the $\epsilon_{22}^d \tan \beta$ are always very small in comparison with the term $m_s/v_d$ for all values of $\tan \beta$ and thus we can safely drop $\epsilon_{22}^d \tan \beta$ terms in Eq.(13) comparing to $m_s/v_d$.

Turning now to $\epsilon_{33}^d$ we find also from Eq.(13) that it is less constrained compared to $\epsilon_{22}^d$ as the bottom quark mass is very large compared to the mass of the strange quark. Moreover, we find that the absolute values of $\epsilon_{22}^d \tan \beta$ are still comparable with the term $m_b/v_d$ and thus we can not drop these terms as we did for the case of $\epsilon_{22}^d \tan \beta$ terms. In Fig.(11) we show the allowed values of the real and imaginary parts of $\epsilon_{33}^d$ corresponding to two different values of $\tan \beta$. As can be seen from the figure that the constraints are insensitive to varying $\tan \beta$ as we discussed above.

The constraints imposed on $\epsilon_{33}^u$ by applying the naturalness criterion of 't Hooft to the top quark mass is expected to be even weaker than those obtained for $\epsilon_{33}^d$ due to the so large top quark mass compared to the bottom quark mass. Moreover we expect that the constrains becomes more loose with increasing the value of $\tan \beta$ due to the inverse dependency on $v_d = v \cos \beta$ which decrease significantly with increasing $\tan \beta$. Thus we can not rely on the naturalness criterion of 't Hooft to constrain $\epsilon_{33}^u$. In Ref.[8] an extensive study of the flavor physics in the context of two Higgs doublet model type-III has been performed to constrain the model both from tree-level processes and from loop observables. It is shown that possible constraints on $\epsilon_{33}^u$ can be obtained from $B_s - \bar{B}_s$ mixing and $B \rightarrow X_s \gamma$. Moreover the constraints on $\epsilon_{33}^u$ from $B \rightarrow X_s \gamma$ are the most important ones. For instance, applying $B \rightarrow X_s \gamma$ constraints, for $m_{H^\pm} = 500$ GeV and $\tan \beta = 50$ the coupling $\epsilon_{33}^u$ should satisfy $|\epsilon_{33}^u| \leq 0.55$ and the constrains become more strong for smaller values of $m_{H^\pm}$ and large values of $\tan \beta$. Thus in our analysis we take into account the constraints imposed on $\epsilon_{33}^d$ and $\epsilon_{33}^u$ discussed in Ref.[8].

In 2HDMs type-III the constraints on the charged Higgs mass from $B \rightarrow X_s \gamma$ become weaker comparing with their corresponding constraints in 2HDMs type-II. This because the off-diagonal parameter $\epsilon_{23}^u$ can lead to a destructive interference with the SM (depending on
FIG. 4. Constraints on $\epsilon_{33}^d$ obtained upon applying the naturalness criterion of 't Hooft to the quark masses. Left plot corresponding to $\tan \beta = 10$ while right plot corresponding to $\tan \beta = 50$. Its phase and thus reduces 2HDMs type-III contribution to the amplitude $[8]$. Thus the lower limit on the charged Higgs mass of 380 GeV in 2HDMs type-II can be pushed down in 2HDMs type-III.

We start by discussing the effects of the presence of the $\epsilon_{33}^d$ terms on the Wilson coefficients $\tilde{C}_{7,9}^{(H^\pm)}$. Since $\epsilon_{33}^d$ is generally complex, we expect that these terms can enhance or reduce $\tilde{C}_{7,9}^{(H^\pm)}$ comparing to their values in the two Higgs doublet model type-II. For $\tan \beta = 50$ and $m_{H^\pm} = 300$ GeV we find that $\tilde{R}_7^{H^\pm}$ varies in the range 3% − 7% for the allowed values of $\epsilon_{33}^d$ by the naturalness criterion of 't Hooft constraints. Setting the real and imaginary parts of $\epsilon_{33}^d$ to zeros leads to $\tilde{R}_7^{H^\pm} = 5%$ which we would obtain in two Higgs doublet model type-II. Thus the presence of $\epsilon_{33}^d$ terms would enhance or reduce $\tilde{C}_{7,9}^{(H^\pm)}$ by 2% only. For $\tan \beta = 30$ and $m_{H^\pm} = 300$ GeV we find that the enhancement or reduction is almost 1% while for $\tan \beta = 80$ the enhancement or reduction is almost 4%. For $\tilde{R}_9^{H^\pm}$ we find that the enhancements or the reductions are much smaller than the case of $\tilde{R}_7^{H^\pm}$ since $C_{9}^{SM} \gg C_{7}^{SM}$. As a result we conclude that the enhancements or the reductions of the Wilson coefficients $\tilde{C}_{7,9}^{(H^\pm)}$ due to the presence of the $\epsilon_{33}^d$ terms are not significant compared to the case of two Higgs doublet model type-II and they almost negligible for values of $\tan \beta \leq 30$.

We turn now to discuss the effects of the presence of the $\epsilon_{33}^u$ terms on the Wilson coefficients $C_{7,9}^{(H^\pm)}$ in a similar way as we did for $\epsilon_{33}^d$. Again as $\epsilon_{33}^u$ is generally complex we expect that these terms can enhance or reduce $C_{7,9}^{(H^\pm)}$ comparing with their values in the two Higgs
doublet model type-II. However since the allowed values for $\epsilon_{33}^u$ by $B \to X_s \gamma$ constraints exclude negative values of the real part of $\epsilon_{33}^u$, see figures 17 and 18 in Ref. [8], we find that the $\epsilon_{33}^u$ terms always enhance $C_7^{(H^\pm)}$ comparing with their values within two Higgs doublet model type-II. As before we expect the enhancements to be larger for the Wilson coefficient $C_7^{(H^\pm)}$ and thus we only focus on $R_7^{H\pm}$ in the following discussion. For $\tan \beta = 50$ and $m_{H^\pm} = 300 \text{GeV}$ we find that $R_7^{H\pm}$ can reach 13% which means that $C_7^{(H^\pm)}$ can reach 13% of $C_7^{SM}$. Setting $\epsilon_{33}^u = 0$ we obtain the value $R_7^{H\pm} < 1\%$ which is the limit within two Higgs doublet model type-II. This indicates that the presence of $\epsilon_{33}^u$ terms can enhance the value of $R_7^{H\pm}$ within two Higgs doublet model type-II by 13%. For $\tan \beta = 30$ the constraints become weaker than the case of $\tan \beta = 50$ and thus we expect to have larger enhancement. In this case we find that $R_7^{H\pm}$ can reach 40% indicating that within two Higgs doublet model type-III, $C_7^{(H^\pm)}$ can reach 40% of $C_7^{SM}$. Setting $\epsilon_{33}^u = 0$ we obtain the value $R_7^{H\pm} \approx 0.2\%$ which is the limit within two Higgs doublet model type-II. Clearly, the presence of $\epsilon_{33}^u$ terms enhance the value of $R_7^{H\pm}$ from 0.2% in 2HDMs type-II to 40% in 2HDMs doublet type-III.

For the Wilson coefficients $C_{11}^H - C_{14}^H$ given in Eq. (16) and keeping the $\epsilon_{ij}^{u,d}$ parameters we still find that they are still suppressed either by the smallness of the quark masses or the constraints applied on the $\epsilon_{ij}^{u,d}$ parameters and thus we drop their contributions in our analysis.

Turning now to the branching ratios of $\bar{B}_s \to \phi \pi^0$ and $\bar{B}_s \to \phi \rho^0$ we note from Eqs. (213) that an enhancement in $C_7$ will enhance the branching ratio of $\bar{B}_s \to \phi \pi^0$ and reduce at the same time $\bar{B}_s \to \phi \rho^0$ due to the opposite sign of the terms proportional to $C_7$. Since the enhancement is large for the case of $\tan \beta = 30$ we find that $\mathcal{R}_{\bar{b}_1}^\pi$ can be enhanced by about 4% of the SM prediction for solution 1 while for solution 2 it is still very small about 1%. Comparing the branching ratio of $\bar{B}_s \to \phi \pi^0$ corresponding to solution 1 in 2HDMs type-III with its value in 2HDMs type-II we find that $\mathcal{R}_{\bar{b}_1}^\pi$ is enhanced by about a factor 2. For smaller values of $\tan \beta$ where the constraints on $\epsilon_{33}^u$ becomes more weaker we find that the predictions for the branching ratios are close to their values for $\tan \beta = 30$ as $\epsilon_{33}^u$ is multiplied by $\tan \beta$ and thus enhancement in $\epsilon_{33}^u$ will not be significant when it is multiplied by small value of $\tan \beta$. Thus the branching ratios in 2HDMs type-III are approximately equal their values in 2HDMs type-II. For the case of $\bar{B}_s \to \phi \rho^0$ we find that the reductions by the presence of $\epsilon_{33}^{u,d}$ terms are almost negligible. Thus we conclude that although the presence of $\epsilon_{33}^{u,d}$ terms enhance the branching ratio of $\bar{B}_s \to \phi \pi^0$ by about a factor 2 of their values in
2HDMs type–II still the enhancement is not sizable compared to the SM predictions and will be also invisible within the theoretical uncertainties in the SM predictions for the branching ratios as for the case of 2HDMs type-II.

V. CONCLUSION

In this work we have studied the decay modes $\bar{B}_s \rightarrow \phi\pi^0$ and $\bar{B}_s \rightarrow \phi\rho^0$ within the frameworks of two-Higgs doublet models type-II and typ-III. We adopt in our study SCET as a framework for the calculation of the amplitudes. Within the framework of two-Higgs doublet models type-II and typ-III the charged Higgs boson can mediate the $b \rightarrow s$ transition at quark level and thus generate the decay modes $B_s \rightarrow \phi\pi^0$ and $B_s \rightarrow \phi\rho^0$. We have derived the contributions of the charged Higgs mediation to the weak effective Hamiltonian governing the decay processes and calculated the corresponding Wilson coefficients in both models. In addition we have analyzed the effect of the charged Higgs mediation on the Wilson coefficients of the electroweak penguins and on the branching ratios of $B_s \rightarrow \phi\pi^0$ and $\bar{B}_s \rightarrow \phi\rho^0$ decays.

Within two-Higgs doublet models type-II and type-III we find that the Wilson coefficients $C_7$ and $C_9$ can be enhanced due to the contributions from the charged Higgs mediation. As a consequence the branching ratios of $B_s \rightarrow \phi\pi^0$ and $\bar{B}_s \rightarrow \phi\rho^0$ decays are enhanced in turn. Moreover we have shown that the charged Higgs mediation can lead also to new set of Wilson coefficients obtained from the weak effective Hamiltonian by changing the chirality from left to right. The presence of these new Wilson coefficients can also lead to enhancement of the branching ratios of $\bar{B}_s \rightarrow \phi\pi^0$ and $\bar{B}_s \rightarrow \phi\rho^0$ decays.

We have shown that, within two-Higgs doublet models type-II, the enhancement in the branching ratio of $\bar{B}_s \rightarrow \phi\pi^0$ can not exceed 18% with respect to the SM predictions for a charged Higgs mass $380\, GeV$. For the branching ratio of $B_s \rightarrow \phi\rho^0$, we find that the charged Higgs contribution in this case is small where the branching ratio of $B_s \rightarrow \phi\rho^0$ can be enhanced or reduced by about 4% with respect to the SM predictions.

Turning to two-Higgs doublet models type-III we have shown for a value of the charged Higgs mass $300\, GeV$ and $\tan\beta = 30$ although the enhancement in BR ($\bar{B}_s \rightarrow \phi\pi^0$) can be about a factor 2 of its value within 2HDMs type-II however it is only 4% enhancement with respect to the SM predictions. For smaller values of $\tan\beta$ the predictions for the
branching ratios are close to their predictions in 2HDMs type-II. We show also that, since the errors of the SM predictions to the branching ratios are approximately 40% for $\bar{B}_s \rightarrow \phi \pi^0$, the enhancement in the branching ratios due to the charged Higgs mediation will be invisible within the theoretical uncertainties in the SM predictions. Clearly, charged Higgs contributions can not lead to a significant enhancement of the branching ratios of $\bar{B}_s \rightarrow \phi \pi^0$ and $\bar{B}_s \rightarrow \phi \rho^0$ decays by one order of magnitude over their SM predictions making them possible for detection at LHC.

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