Non-Uniform Pitch Helical Resonators for Dual-Passband Filter Design

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Abstract

This paper presents a study of a non-uniform pitch helical resonator (NPHR) structure and the coupling mechanisms to design dual-passband filters. The previous research analyzes NPHR as a type of step impedance resonator (SIR), however, it does not give analytical equations or prediction for the dual-resonance characteristics of the NPHR structure discussed in this paper. Consequently, a circuit model is proposed to analyze the dual-resonance characteristics of NPHRs. Analytical equations are derived, showing that the frequency ratio of a dual-band NPHR can be determined by the ratio of turns. EM simulation and experimental results have shown good agreement with the circuit analysis. The derived analytical equations from circuit model can be used for fast design of NPHRs. A step-width aperture is proposed to independently control the coupling coefficients at the each band of NPHRs. A third order dual-passband filter has been designed and fabricated. The filter has 3.3% and 3% fractional bandwidths at 789 MHz and 2402 MHz, respectively. The designed prototype filter shows that NPHRs can be utilized to realize compact filters with dual-band characteristics. The filter design can be extended from engineering perspective for application in wireless communication systems.

Keywords

Helical Filter, Dual-Band, Circuit Model, Step-Width Aperture

1. Introduction

Modern wireless communication standards demand multi-mode multi-functional systems which may require several RF/microwave transceiver chains in a system. In most scenarios, the wireless communication systems are required to be compact, light-weight and low-cost. Consequently, the engineers are searching for a
method to reduce the transceiver chains in system. RF/microwave components with multi-band characteristics have been studied and proposed to reduce the number of components and transceiver chains. In recent years, many multi-band antennas have been studied and applied in practical applications. Dual-passband filters have also drawn research interests of RF engineers.

The research on the realization of dual-passband filters can be categorized into two approaches. The first approach realises a dual-passband frequency response by introducing transmission zeros (TZs) within the original single-passband [1] [2]. The coupling matrix is manipulated to realise required TZs and only one resonant mode of the resonators is used. The other approach utilises two nondegenerate modes of the resonators to form the dual passbands respectively. The resonators must have dual-band characteristics, i.e. the dual resonant frequencies must be adjustable. The fundamental and second resonances are usually employed to realise the dual-passband filter response [3] [4].

Dual-band resonators, such as SIRs and dielectric resonators have been reported for dual-passband filter design [5] [6] [7]. However, their sizes are relatively large, thus not suitable for compact filter design. Zhou developed a dual-band (900 and 1800 MHz) antenna for mobile phones based on a non-uniform helical structure [8]. This structure has then been proposed for resonator and filter applications because of the dual-band characteristics and the excellent size reduction. The previous research analyses NPHRs as SIRs in helical configuration [9] [10] [11]. Analytical equations that define the dual resonances are derived for NPHR structure in [10]. Nevertheless, they cannot be used to predict the dual-band characteristics of NPHR structure in [9].

This paper demonstrates a circuit model that can be used to analyze the NPHR structure in [9]. Equations for prediction of dual-band characteristics of NPHR are derived. Multiple NPHRs are simulated and measured to verify the theoretical analysis. An inter-resonator coupling structure that independently controls the coupling intensity of the first and the second band is introduced. A dual-passband filter is designed and implemented to verify theoretical analysis and simulation.

2. Research Method

2.1. Uniform Pitch Helical Resonator

A traditional helical resonator consists of a coil enclosed in a circular or square cavity. The coil has uniform pitch, with one end short-circuited and one end open [12]. The uniform pitch helical resonator is usually analyzed as a quarter-wavelength transmission line, which has a uniformly distributed inductance, capacitance and resistance. The inductance per unit length \( L_0 \) and the capacitance per unit length \( C_0 \) of an air-filled helix with a cylindrical cavity are given by [13]

\[
L_0 = 0.025n_i^2d^2 \left[ 1 - \left( \frac{d}{D} \right)^2 \right] \mu \text{H/inch},
\]

where \( n_i \) is the index of refraction of the insulator, \( d \) is the radius of the wire, \( D \) is the diameter of the coil, and \( \mu \) is the magnetic permeability.
\[ C_0 = 0.75 \left( \log_{10} \frac{D}{d} \right)^{-1} \text{pF/inch}, \quad (2) \]

where \( d \) and \( D \) represent the diameters of the coil and the cavity in inches respectively, and the number of turns in unit length is represented by \( n_t \). A square cavity with a cross section length \( S \) can be used for substitution according to \( D = 1.2 \cdot S \) [13].

### 2.2. Non-Uniform Pitch Helical Resonator

This section analyses a dual-band NPHR structure using a circuit based model. An EM simulator and measured results of fabricated resonators have been used to verify the circuit model. The derived analytical equations of circuit analysis can be used in the design flow of dual-passband filters.

#### 2.2.1. NPHR Structure

A typical structure of the dual-segment NPHR is illustrated in **Figure 1**. The resonator comprises of two helical segments with different pitches and a supporting structure that connects the helical structure to the cavity. The helical segments are of equal height for this type structure, thus the non-uniform pitches

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**Figure 1.** A typical structure of proposed helical resonator with non-uniform pitches.
are determined by the number of turns in upper and lower segment.

The notations \( S \), \( d \), \( d_0 \), \( l_s \), and \( H \) represent the cross-section length of the square cavity, the diameter of the helical resonator, the diameter of the coil wire, the length of supporting structure, and the overall height of resonator. The heights of helical segments are represented by \( h_{\text{top}} \) and \( h_{\text{btm}} \) which are equal for this type of resonator. The upper segment has \( N_{\text{top}} \) turns while the lower segment has \( N_{\text{btm}} \) turns.

### 2.2.2. Circuit Model of NPHR

Miller [14] has analysed a helical antenna with an inductive or a capacitive load. This method can also be used to model and analyse a NPHR. Figure 2 shows the equivalent circuit for a uniform pitch helical resonator with different types of load. The notations \( L_e \) and \( C_e \) represents the equivalent lumped inductance and capacitance of the resonant segment, \( L_{\text{load}} \) and \( C_{\text{load}} \) are the inductance and capacitance of the load.

The resonant frequency of the lumped circuit in Figure 2(a) is then given by [14]

\[
f = \frac{1}{2\pi} \left( (L_{\text{load}} + L_e)C_e \right)^{-1/2},
\]

where \( L_e \), \( C_e \), \( L \) and \( C \) are defined by

\[
L_e = \frac{L}{2} \left[ \frac{1}{\sin^2 \left( \omega \sqrt{LC} \right)} - \cot \left( \omega \sqrt{LC} \right) \right],
\]

\[
C_e = C \left( \frac{\omega \sqrt{LC} \cot \left( \omega \sqrt{LC} \right)}{2} - \frac{\omega^2 LC}{2 \sin^2 \left( \omega \sqrt{LC} \right)} \right)^{-1},
\]

**Figure 2.** Equivalent circuit of a helical resonant segment. (a) with an inductive load. (b) with a capacitive load.
\[
f = \frac{1}{2\pi \left( (L_{\text{load}} + L_x)C_x \right)^{\frac{1}{2}}},
\]

\[L \text{ and } C \text{ are the static inductance, and capacitance of the resonant segment; } \omega \text{ represents the angular resonant frequency; and } m \text{ represents the odd harmonic modes where } m = 1, 3, 5, \text{ etc.} \]

Thus, the first and the second resonant frequencies corresponding to \( m = 1 \) and \( m = 3 \) are related by

\[
\frac{f_2}{f_1} = 4.01 \left( \frac{L_{\text{load}} + L}{L_{\text{load}} + 1.52L} \right)^{\frac{1}{2}}.
\]

The resonant frequencies of the equivalent circuit in Figure 2(b) are derived similarly in [14], resulting in the frequency ratio as

\[
\frac{f_2}{f_1} = 4.01 \left( \frac{C_{\text{load}} + C}{C_{\text{load}} + 1.52C} \right)^{\frac{1}{2}}.
\]

The NPHR structure in Figure 1 can be approximately analysed as an upper resonant segment in series with a lower inductive load segment when \( N_{\text{top}} \gg N_{\text{btm}} \). From (1) and (7), we have

\[
\frac{f_2}{f_1} = 4.01 \left( 1 + 0.5 \left( \frac{N_{\text{res}}}{N_{\text{load}}} \right)^{\frac{2}{3}} \right)^{\frac{1}{2}}.
\]

where \( N_{\text{res}} \) and \( N_{\text{load}} \) represent the number of turns of the resonant segment and the inductive load, respectively.

When \( N_{\text{top}} \ll N_{\text{btm}} \), the NPHR structure can be approximately analysed as a lower resonant segment in parallel with an upper capacitive load segment. In this case, (2) and (8) derives a constant value 3.09.

In practice, \( N_{\text{top}} \) and \( N_{\text{btm}} \) are comparable but the above derivation still gives a good approximation. The developed circuit model of the NPHR structure in Figure 1 can then be summarised as

\[
\frac{f_2}{f_1} = \begin{cases} 
4.01 \left( 1 + 0.5 \left( \frac{R_N}{1 - 1.52(R_N)^2} \right)^{\frac{2}{3}} \right)^{\frac{1}{2}} & \text{for } R_N \geq 1, \\
3.09 & \text{for } 0 < R_N < 1.
\end{cases}
\]

where \( R_N \) denotes the ratio of number of turns in upper and lower segment, i.e. \( R_N = N_{\text{top}}/N_{\text{btm}} \).

2.2.3. Simulation and Verification of NPHR

Figure 3 shows the frequency ratios obtained by EM simulations of various
resonators to validate the above circuit model and the derived analytical equations. Multiple resonators have been simulated using CST Microwave Studio to obtain their frequency ratios. The cross-section length of square cavity is 17.5 mm and the coil diameter is 7.3 mm. The wire has a diameter of 1.0 mm and the support length is 5 mm. The upper and lower segments are equally high \( h_{\text{top}} = h_{\text{btm}} = h_0 \) and the height of cavity \( H = 2 \times l_s + h_{\text{top}} + h_{\text{btm}} \). The outcome based on the developed circuit model and the EM simulation results mutually confirm the validity of each other.

Three NPHRs were manufactured to verify one of the simulated curves in Figure 3. The fabricated resonators are shown in Figure 4 with \( N_{\text{btm}} = 2.5 \) and \( h_0 = 18 \) mm. The helices were formed by winding insulated copper wires on 3D printed dual-segment moulds. The cavities are made of 0.3 mm thick brass sheet. The number of turns at the upper segment \( (N_{\text{top}}) \) is 1.5, 2.5, and 3.5 respectively.

Table 1 shows the measured fundamental and the second resonant frequencies of the resonators. The frequency ratios from measurement match very well with the simulated frequency ratios. In addition, the copper wire winding process increased diameter of helix, resulting in lower measured resonant frequencies.

![Figure 3. Developed circuit model and EM simulation results.](image)

![Figure 4. Fabricated resonators.](image)
This could be eliminated by utilising finer wire winding process.

### 2.3. Coupling Structure for NPHR

Microwave filters are generally implemented as coupled identical resonators. Coaxial cavity filters usually use apertures to realise magnetic couplings between resonators. Uniform width apertures limit the ability to control the coupling at multiple resonances independently, which is very significant for dual-passband filter design.

**Figure 5** shows the distribution of electric and magnetic fields of a NPHR. At the first resonance, the E-field strength at upper segment is apparently higher than that at lower segment. The H-field strength of lower segment is much higher than that at upper segment. However, at the second resonance, the E-field and H-field strengths are comparable in both lower and upper segment. Therefore, we can use a step-width aperture, as shown in **Figure 6**, to independently

| Frequency ratio \((f_2/f_1)\) | Number of turns for the upper helical segment \(N_{\text{top}}\) |
|-----------------------------|--------------------------|
|                             | 1.5  | 2.5  | 3.5  |
| Simulation                  | 3.021| 2.969| 2.771|
| Measurement                 | 3.052| 2.931| 2.820|

**Figure 5.** Field distribution of a NPHR. (a) E-field at the 1st resonance. (b) H-field at the 1st resonance. (c) E-field at the 2nd resonance. (d) H-field at the 2nd resonance.
control the inter-resonator couplings of the first and the second resonances.

The step change of aperture width is set at the height of pitch change. The fabricated NPHR with $N_{\text{top}} = 1.5$ is selected for simulation. The coupling coefficients ($k_1$ and $k_2$) at the first and the second resonances are obtained against aperture widths ($W_1$ and $W_2$).

The simulated coupling coefficients are shown in Figure 7 and Figure 8. It
can be found that $W_1$ and $W_2$ play comparable roles to control $k_1$, whereas $W_1$ is the dominant parameter that controls $k_2$. Therefore, when obtaining the aperture widths, $W_1$ is determined firstly from the required coupling coefficient at the 2nd band. Then $W_2$ is selected from the required coupling coefficient at the 1st band.

The lower aperture height ($H_1$) can also be utilised to control the intensity of inter-resonator couplings, however, it does not expand the ranges of the coupling coefficients.

3. Research Results

3.1. Dual-Passband Filter Design

A three-pole dual-passband filter is designed. The specification requires two passbands with 3% FBW at 820 and 2500 MHz, respectively.

The NPHR with $N_{\text{top}} = 1.5$ is selected for the filter as its resonant frequencies align with specification. The external coupling for the designed filter is realised by direct tapping. Step-width apertures are used for inter-resonator couplings. The geometrical parameters of the filter were optimized in EM simulation. Figure 9 shows the filter structure. The filter dimensions were optimized by employing Trust Region Framework Algorithm.

3.2. Measurement Results and Analysis

The filter is fabricated for verification. The simulated and measured frequency responses are shown in Figure 10. The sharp rejections at stopband attribute to the TZs which are generated by the cross-coupling between the first and the third NPHRs, and the coupling between the input and output probes. Figure 11 shows the frequency response of passbands. Apart from a frequency shift and...
low return loss, the measured response has a good agreement with the simulated response. The return loss and frequency shift can be improved in future designs with precise manufacture process and fine tuning.

Table 2 compares the measured parameters of the dual-passband filter with

![Figure 9. Filter structure and dimensions.](image)

![Figure 10. Simulated and measured frequency responses of the dual-passband filter.](image)
Figure 11. Passbands of the filter. (a) The first passband. (b) The second passband.

Table 2. Data of the designed filter.

| Parameters                              | Specification | Simulation | Measurement |
|-----------------------------------------|---------------|------------|-------------|
| Centre frequency $f_1$ (MHz)           | 820           | 820        | 789         |
| 3-dB FBW of the 1st band (%)            | 3             | 3          | 3.3         |
| Min. insertion loss of the 1st band (dB)| /             | 1.9        | 2.0         |
| Centre frequency $f_2$ (MHz)           | 2500          | 2524       | 2402        |
| 3-dB FBW of the 2nd band (%)            | 3             | 3.7        | 3           |
| Min. insertion loss of the 2nd band (dB)| /             | 1.2        | 1.3         |
| Frequency ratio $f_2/f_1$               | 3.05          | 3.08       | 3.05        |

Table 3. Comparison with the state-of-art.

| Filter                  | This work | [6]         |
|-------------------------|-----------|-------------|
| Resonator type          | NPHR      | Coaxial SIR |
| No. of Poles            | 3         | 2           |
| Centre frequencies (MHz)| 789/2402  | 906/1772    |
| Extracted Q             | 660/1000  | 2500/2700   |
| Filter size (mm$^3$)    | 53.7 × 18.1 × 46.6 | 125 × 70 × 135 |

specification and simulation. The measured 3-dB FBW is 3.3% and 3% for the first and the second passband, respectively. The minor discrepancies are mainly caused by mechanical tolerances. Minimum insertion loss for the passband is 2.0 dB and 1.3 dB, respectively. The measured frequency ratio of passbands has very good agreement with specification.

Table 3 shows the comparison between the designed dual-passband filter and the state-of-art. The NPHRs have lower Q but they are much smaller than the coaxial SIRs. The filter based on NPHRs saves up to 95% volume, compared with
the filter in [6]. It can be applied for a dual-band wireless communication system that has size limitation.

4. Conclusion

In this paper, we have studied a NPHR structure for the design of dual-passband filters. The dual-band characteristics of NPHR have been analysed by a circuit model and verified by simulation and measurement. Additionally, the inter-resonator coupling structure for NPHRs has been discussed. A step-width aperture can be used to independently control the coupling at each resonance. Finally, the paper demonstrated a prototype dual-passband filter utilising the proposed NPHRs and coupling structure. The measured results agree well with the simulation. The proposed circuit model offers a method for fast design of NPHRs. The designed dual-passband helical filter provides a compromise solution for applications that require low insertion loss with very limited size and cost, such as 4G and 5G microcell base stations, vehicular and satellite transceivers. The NPHRs can be further studied for a higher-order filter design so that it becomes applicable for practical use from engineering perspective. The tuning structures that separately tune the resonant frequencies of NPHR need further investigation in future works.

Acknowledgements

The authors would like to thank Mr. Mark Hough for fabricating resonators and filter.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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