Baryogenesis in false vacuum

Yuta Hamada\textsuperscript{1,a}, Masatoshi Yamada\textsuperscript{2,b}

\textsuperscript{1} KEK Theory Center, IPNS, KEK, Tsukuba, Ibaraki 305-0801, Japan
\textsuperscript{2} Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan

Received: 14 June 2017 / Accepted: 11 September 2017 / Published online: 23 September 2017
© The Author(s) 2017. This article is an open access publication

Abstract The null result in the LHC may indicate that the standard model is not drastically modified up to very high scales, such as the GUT/string scale. Having this in the mind, we suggest a novel leptogenesis scenario realized in the false vacuum of the Higgs field. If the Higgs field develops a large vacuum expectation value in the early universe, a lepton number violating process is enhanced, which we use for baryogenesis. To demonstrate the scenario, several models are discussed. For example, we show that the observed baryon asymmetry is successfully generated in the standard model with higher-dimensional operators.

1 Introduction

Although the standard model (SM) is complete after the discovery of the Higgs boson at the Large Hadron Collider [1,2], there are still mysteries in elementary particle physics, such as the finite neutrino mass and dark matter. Besides these the baryon asymmetry in the universe (BAU) is also one of the unsolved problems. That is, how has baryogenesis been realized in the evolution of the universe? The latest cosmological result from the Planck observations [3] tells us that the BAU is

\[ n_B = (8.67 \pm 0.05) \times 10^{-11}, \]

where \( n_B \) is the baryon number density and \( s \) is the entropy density.

In order to theoretically explain the BAU within elementary particle physics, the Sakharov conditions [4] have to be satisfied: there exists a process violating the baryon number conservation; \( C \) and \( CP \) invariances are violated; the system leaves its equilibrium state. The SM does not accommodate the departure from equilibrium. Although the baryon number is violated through the sphaleron process and \( CP \) symmetry is violated in the weak interaction, it is not enough to reproduce the BAU. Therefore the SM cannot satisfy these conditions and must be extended.

Some baryogenesis mechanisms satisfying the Sakharov conditions have been suggested, e.g. the grand unified theory [5] and the Affleck–Dine mechanism [6]. Leptogenesis is also one of the well-known mechanisms for baryogenesis [7] (see also the reviews [8,9]) where we use the fact that through the sphaleron process [10–13], the difference \( B - L \) between the baryon number \( B \) and the lepton number \( L \) is conserved whereas their sum \( B + L \) is not. The baryon number density in thermal equilibrium is provided by the \( B - L \) number density via the sphaleron process:

\[ n_B = \frac{8N_F + 4N_S}{22N_F + 13N_S} n_{B-L}, \]

where \( N_F \) is generation of quarks and leptons and \( N_S \) is that of scalar doublets. For instance, in the case of the SM where \( N_F = 3 \) and \( N_S = 1 \), the factor in the right-hand side is 28/79. Through the decay of the heavy particle, the lepton number is generated, and then its number density changes to the \( B - L \) number density \( n_{B-L} \), whose process is described by the coupled Boltzmann equations for these number densities.

In this paper, we study leptogenesis realized in the false vacuum of the Higgs field, in which the Higgs gains a vacuum expectation value far above the electroweak scale. The mass of the particles coupled to the Higgs field becomes supermassive, and the left-handed neutrinos can become heavier than the charged leptons and \( W \) boson in the presence of higher-dimensional lepton number violating operators. The decay of the left-handed neutrinos creates an \( L \) asymmetry. Shortly after, the phase transition of the Higgs takes place and the Higgs moves from the false vacuum to the true electroweak one (where thermal effects restore the electroweak symmetry allowing the sphalerons to reprocess the \( L \) into a \( B \) asymmetry). We consider the situation where the right-
handed neutrino masses are large compared with the reheating temperature. Therefore our scenario gives an alternative scenario for baryogenesis.

To demonstrate this scenario, two models are investigated. We first consider a minimal model depending on the SM with a high-dimensional operator,

$$\Delta L_5 = \frac{\lambda_{ij}}{\Lambda} H H \bar{L}_j L_i,$$

(3)

where $L_i$ is the lepton doublet, $\Lambda$ is a cutoff scale, and the Higgs doublet is defined as

$$H = \frac{1}{\sqrt{2}} \left( \chi_1 + i \chi_2 \right).$$

(4)

Such an operator is typically generated in the type I seesaw model by integrating out the right-hand neutrino. This effective interaction breaks the lepton number conservation saw model by integrating out the right-hand neutrino. This produced by this process actually is not adequate for the lepton asymmetry. In this subsection, to calculate the asymmetry of the universe, we evaluate the asymmetry by numerically solving them. We evaluate the asymmetry produced by the left-handed neutrino decay; however, we see that not enough baryon asymmetry is produced. To ameliorate the situation, next we add the new higher-dimensional operators. We demonstrate that, in this case, the decay of the neutrino can reproduce the observed amount of asymmetry.

2 Mechanism and Boltzmann equations

First, we consider a situation where the decay of the left-handed neutrino produces the baryon asymmetry. In this section, we present the Boltzmann equations and quantitatively evaluate the baryon asymmetry by numerically solving them. We evaluate the baryon asymmetry produced by the left-handed neutrino decay; however, we see that not enough baryon asymmetry is produced. To ameliorate the situation, next we add the new higher-dimensional operators. We demonstrate that, in this case, the decay of the neutrino can reproduce the observed amount of asymmetry.

2.1 The derivation of Boltzmann equations

In this subsection, to calculate the asymmetry of the universe, we follow Refs. [9,16,17] and derive the Boltzmann equations for the general case of leptogenesis. The change of the number density of a heavy particle is governed by

$$\dot{n}_X + 3Hn_X = \int d\Pi_X d\Pi_1 d\Pi_2 (2\pi)^4 \delta(4)(p_X - p_1 - p_2)$$

$$\times (-f(p_X)|\mathcal{M}(X \rightarrow 12)|^2$$

$$+ f(p_1)f(p_2)|\mathcal{M}(12 \rightarrow X)|^2)$$

$$+ \int d\Pi_X d\Pi_Y d\Pi_1 d\Pi_2 \cdots d\Pi_N (2\pi)^4 \delta(4)$$

$$\times (p_X + p_Y - p_1 - p_2 - \cdots - p_N)$$

$$\times (-f(p_X)f(p_Y)|\mathcal{M}(XY \rightarrow 12 \cdots N)|^2$$

$$+ f(p_1) \cdots f(p_N)|\mathcal{M}(12 \cdots N \rightarrow XY)|^2),$$

(7)

where $X$ and $Y$ represent the heavy particles; the numbers $1 \cdots N$ denote lighter particles; the dot on $n_X$ in the left-hand side denotes the time derivative; we have neglected the effects of the Pauli blocking and stimulated emission; $d\Pi_i = d^3p_i/(2\pi)^3E_i$ is the phase space integral; $H = \dot{R}/R$ is the Hubble parameter given by the scale factor $R$.

1 In Refs. [14,15], the operator (3) is used to realize leptogenesis as well as the $CP$ violating operator $\bar{L}_i \gamma^\mu L_j \bar{L}_i \gamma_\mu L_j$. These operators are naturally generated in the low energy effective theories of various seesaw models.
which is governed by the Friedmann equation. Here $f$ is the distribution function, approximately given by the Maxwell–Boltzmann distribution.

The first and second terms of the right-hand side in Eq. (7) correspond to the decay and annihilation of heavy particles, respectively. Let us rewrite the first term by using the definition of the decay rate,

$$
\Gamma_X = \frac{1}{2E_X} \int d\Pi_1 d\Pi_2 \, (2\pi)^4 \delta(4)(p_X - p_1 - p_2) |\mathcal{M}(X \to 12)|^2.
$$

(8)

We use the fact that the kinetic equilibrium allows us to make the replacement, \(^2\)

$$
f(p_1) f(p_2) = f^{\text{EQ}}(p_1) f^{\text{EQ}}(p_2) = f^{\text{EQ}}(p_X).
$$

(9)

Furthermore, at leading order, $|\mathcal{M}(X \to 12)|^2 = |\mathcal{M}(12 \to X)|^2$. Hence, we find that the first term in the right-hand side becomes

$$
\left(-n_X + n_X^{\text{EQ}}\right) \Gamma_X.
$$

(10)

The second term in Eq. (7) can be written in terms of the thermal average cross section of the pair annihilation $\langle \sigma_{\text{ann}} v \rangle$:

$$
\langle \sigma_{\text{ann}} v \rangle = \frac{\int d\Pi_X d\Pi_Y d\Pi_N \, (2\pi)^4 \delta(4)(p_X + p_Y - p_1 - \cdots - p_N) |\mathcal{M}(XY \to 1 \cdots N)|^2}{\int d\Pi_X d\Pi_Y (2E_X)(2E_Y) f(p_X) f(p_Y)}.
$$

(11)

We assume that $f(p_1) \propto f^{\text{EQ}}(p_1)$ thanks to the kinetic equilibrium, so that the second term in Eq. (7) becomes

$$
\langle \sigma_{\text{ann}} v \rangle \left(-n_X^2 + (n_X^{\text{EQ}})^2\right).
$$

(12)

To summarize, the Boltzmann equation of $n_X$ is given by

$$
n_X + 3H n_X = (-n_X + n_X^{\text{EQ}}) \Gamma_X + \langle \sigma_{\text{ann}} v \rangle \left(-n_X^2 + (n_X^{\text{EQ}})^2\right).
$$

(13)

In a similar manner, we can write the Boltzmann equation governing the lepton number density:

$$
\dot{n}_l + 3H n_l = \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 \, \delta(4)(p_X - p_l - p_1 - p_2 - p_3 - p_4)
\times \epsilon(-f(p_X) |\mathcal{M}(X \to lW)|^2 + f(p_1) f(p_2) |\mathcal{M}(lW \to X)|^2)
+ 2 \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 \, \delta(4)(p_1 + p_2 - p_3 - p_4)
\times (-f(p_1) f(p_2) |\mathcal{M}(l_1l_2 \to l_3l_4)|^2 + f(p_3) f(p_4) |\mathcal{M}(l_3l_4 \to l_1l_2)|^2).
$$

(14)

where, in the second equality, we have used

$$
\frac{dT}{dr} = -3H \frac{n_y}{dn_y/dT} = -HT.
$$

(17)

The right-hand side is

$$
- \Gamma_X (n_X - n_X^{\text{EQ}}) - \langle \sigma_{\text{ann}} v \rangle (n_X^2 - (n_X^{\text{EQ}})^2)
= -n_y H \gamma \left(\frac{\Gamma_X(z)}{H(z)z}\right) (N_X - N_X^{\text{EQ}})
- n_y H \gamma \left(\frac{\langle \sigma_{\text{ann}} v \rangle n_y}{H(z)z}\right) (N_X^2 - (N_X^{\text{EQ}})^2).
$$

(18)

In terms of $N_l$ and $z$, we can write the set of the Boltzmann equations as follows:

$$
\frac{d}{dz} N_X = - \left(\frac{\Gamma_X(z)}{H(z)z}\right) (N_X - N_X^{\text{EQ}})
- \left(\frac{\langle \sigma_{\text{ann}} v \rangle n_y}{H(z)z}\right) (N_X^2 - (N_X^{\text{EQ}})^2),
$$

$$
\frac{d}{dz} N_{B-L} = - e \left(\frac{\Gamma_X(z) Br}{H(z)z}\right) (N_X - N_X^{\text{EQ}})
- N_{B-L} \left(\frac{\Gamma_X(z) Br}{H(z)z}\right) N_X^{\text{EQ}}.
$$

(19)
Here we assume that $\lambda_{ij}$ is the order of 1 quantity, as in Eq. (5).

---

$\Gamma_X(z) = \left(\frac{1}{y}\right) \Gamma_X|_{z=\infty} \simeq \left(\frac{1}{y}\right) \frac{m_\nu}{8\pi} \\
\times \left(\frac{\lambda_{ii} \langle h \rangle^2}{\sqrt{2} \Lambda} + g_2^2 + y_\tau^2 \right),$

\begin{equation}
\text{Br} \simeq \frac{\langle h \rangle^2}{\left(\frac{\langle h \rangle}{\sqrt{2} \Lambda}\right)^2 + g_2^2 + y_\tau^2},
\end{equation}

where $\langle 1/y \rangle = K_1(z)/K_2(z)$ in the thermal bath, $K_1$ is for the modified Bessel functions of the first kind, $\lambda_{ii}$ is diagonalized by rotating the lepton field, and $y_\tau$ is the tau Yukawa coupling. We note that the branching ratio to the transverse gauge boson is important. This is because, in order to pick up the imaginary part of the amplitude, one needs to use the lepton Yukawa coupling rather than the SU(2) gauge coupling.\(^4\)

The CP asymmetry $\epsilon$ comes from the interference between the tree and the loop diagrams corresponding to the last three diagrams in Fig. 1, whose order is given by

\begin{equation}
\epsilon_i \simeq \frac{1}{8\pi} \sum_j \frac{\text{Im}[(YY^\dagger)]_{ij} y_{\tau j}}{y_\tau^2},
\end{equation}

where $Y$ is the charged lepton Yukawa matrix. Note that the imaginary part appears only if $m_\nu > M_W + M_\tau$,\(^5\) which yields

\(^4\) The coupling $\lambda_{ij}$ is not helpful in obtaining the imaginary part because this coupling becomes real by rotating the lepton field.

\(^5\) Even if $m_\nu < M_W + M_\tau$, the imaginary part appears in higher order. However, it is too small to obtain sufficient baryon asymmetry.
\[ \langle h \rangle > \frac{g_2^2}{2} \Lambda \simeq 1.5 \times 10^{14} \text{GeV} \left( \frac{\Lambda}{6 \times 10^{14} \text{GeV}} \right). \tag{28} \]

Here \( M_\tau \) is the mass of the tau lepton.

Let us estimate the imaginary parts of the Yukawa couplings in Eq. (27). The numerator of Eq. (27) is related to the Jarlskog invariant in the lepton sector \cite{18}, and the order is estimated as \cite{18}

\[ \epsilon_i \sim \frac{1}{8\pi^2} \sum_i \sin \delta \simeq 1.2 \times 10^{-7} \left( \frac{y_\tau}{10^{-2}} \right)^2 \left( \frac{\sin \delta}{\sin 1} \right). \tag{29} \]

where \( \delta \) is the Dirac CP phase of the neutrino sector.

We note that, by using the renormalization group equations, we obtain the values of the coupling constants at the high scale:\cite{19}

\[ g_2 \simeq 0.5, \quad y_\tau \simeq 1 \times 10^{-2}, \quad \alpha_2 \simeq \frac{g_2^2}{4\pi} = \frac{1}{50}, \tag{30} \]

### Numerical result in minimal model

The Planck observation \cite{3} tells us

\[ N_{B,\text{obs}} \simeq 6.1 \times 10^{-10} \times \frac{2387}{86} = 1.7 \times 10^{-8}, \tag{31} \]

where the factor \( 2387/86 \) is the photon production factor.\footnote{This factor comes from the ratio \( g_{88}(T_B)/g_{88}(\text{today}) \), where \( g_{88} \) is the effective degrees of freedom for the entropy, and \( T_B \) is the temperature of the baryogenesis. Here \( g_{88} \) is given by \( g_{88}(\text{today}) = \frac{43}{11}, \quad g_{88}(T_B) = \frac{217}{2} \). See e.g. Ref. \cite{19}.}

\[ N_{B-L,\text{obs}} \simeq 6.1 \times 10^{-10} \times \frac{2387}{86} \times \frac{79}{28} = 4.8 \times 10^{-8}. \tag{33} \]

Therefore, we numerically solve the Boltzmann equations given in Eqs. (19)-(23) and investigate whether or not the appropriate parameter space which satisfies the value (33) exists.

Unfortunately, we can easily see that the baryon asymmetry cannot be reproduced in this framework. We obtain

\[ \epsilon_i \text{Br} \sim 1.2 \times 10^{-7} \left( \frac{y_\tau^2}{\frac{y_{\tau}}{10^{-2}}^2} \right) \leq 2 \times 10^{-10}, \tag{34} \]

by combining Eqs. (26) and (29), and hence the resultant baryon asymmetry is too small to explain the current data. This indicates the necessity of an extension of the model. In the next subsection, we present the possible extension to realize the observed baryon asymmetry.

### 2.3 Extended model

A way to improve the situation is to add new operator. The smallness of the charged lepton Yukawa coupling results in the small baryon asymmetry. Therefore, if this coupling is modified in the false vacuum, the situation changes. Let us assume the existence of the higher-dimensional operator which contributes as the Yukawa coupling in the vacuum where Higgs takes the large VEV:

\[ y_{2,ij} \frac{H^+ H}{\Lambda_2^2} \bar{e}_i H L_j + \text{h.c.} \tag{35} \]

Similar, we consider the operator which gives the correction to Majorana neutrino masses:

\[ y_{3,ij} \frac{H^+ H}{\Lambda_3^2} \bar{H} \tilde{L}_j L_i + \text{h.c.} \tag{36} \]

In general, \( \Lambda_3 \) can be different from \( \Lambda_1 \) in Eq. (3). This structure may occur when we consider the right-handed neutrino model as a UV completion for example,\footnote{This is just a possibility of the UV completion, and we do not insist on this model in the following discussion.} where the action is

\[ M_{N_{ij}} \tilde{N}_j + \left( y_{N_{ij}} + y_{N_{ij}} \frac{H^+ H}{\Lambda_2^2} \right) \bar{N}_i H L_j + \text{h.c.} \tag{37} \]

We evaluate the order of the resultant asymmetry obtained by the decay of the SM neutrino again. The set of the thermal initial conditions of the Boltzmann equations is

\[ N_X(z_{\text{ini}}) = \frac{3}{4}, \quad N_{B-L}(z_{\text{ini}}) = 0. \tag{38} \]

In this case, the decay rate is\footnote{The last factor appears because of the phase space integral.}

\[ \frac{d}{dt} \langle h \rangle = \frac{g_2^2}{2} \Lambda \simeq 1.5 \times 10^{14} \text{GeV} \left( \frac{\Lambda}{6 \times 10^{14} \text{GeV}} \right) \]
and the functions which appear in Boltzmann equation are roughly given by

\[
\Gamma_{\chi}(z) = \left( \frac{1}{\gamma} \right) \frac{M_\nu}{8\pi} \left( g_2^2 + |Y_2|^2 + \frac{(\langle h \rangle)^2}{2\Lambda^2} + \left( \frac{(\langle h \rangle)^3}{2\sqrt{2}\Lambda^3} \right)^2 \right) \times \left[ 1 - \left( \frac{M_W^2}{M_\nu^2} \right) \right],
\]

and the functions which appear in Boltzmann equation are roughly given by

\[
\epsilon_i \approx \sum_j \frac{\text{Im}(Y_2 Y_2^*)_{ij}}{8\pi (Y_2 Y_2^*)_{ij}},
\]

\[
\langle \sigma_{\text{ann}} v \rangle \approx \alpha_2^2 \frac{1}{\text{Max}(M_W^2, T^2)},
\]

\[
\langle \sigma_L v \rangle \approx \left( \frac{|Y_2|^2}{4\pi} \right)^2 \frac{1}{\text{Max}(M_W^2, T^2)},
\]

\[
\text{Br} \approx \frac{g_2^2 |Y_2|^2}{|Y_2|^2} + 1 + \frac{(\langle h \rangle)^2}{2|Y_2|^2\Lambda^2} + \left( \frac{(\langle h \rangle)^3}{2\sqrt{2}|Y_2|\Lambda^3} \right)^2,
\]

where we define the effective charged lepton Yukawa coupling \( Y_{2ij} := Y_{ij} + y_{2,ij} (\langle h \rangle^2 / (2\Lambda^2)) \), and the neutrino mass \( M_{vi} := \text{diag}(\lambda_{ij} (\langle h \rangle^2 / \Lambda + y_{3,ij} (\langle h \rangle^4 / (2\Lambda^3))_{ij}) \). We further assume that the components of \( M_{vi} \) and \( Y_{2ij} \) are the same order of magnitude, respectively, and we denote \( M_{vi} = M_\nu, Y_{2ij} = Y_2 \) for simplicity. We focus on the asymmetry generated by the lightest neutrino in the false vacuum. In Fig. 2, we show the result assuming that \( CP \) phase is of the order of one, i.e. \( e^{i\delta} \sim 1 \). We use the following parameter set to draw the plot:

\[
\Lambda = 6 \times 10^{14} \text{ GeV}, \quad \Lambda_2 = 6 \times 10^{13} \text{ GeV},
\]

\[
\Lambda_3 = 3 \times 10^{13} \text{ GeV}, \quad \langle h \rangle = 2 \times 10^{13} \text{ GeV},
\]

\[
y_{2ij} = 1, \quad y_{3,ij} = 1.
\]

We can see that the BAU is reproduced in this extension. Notice that, unlike the minimal model, we obtain

\[
\epsilon_i \cdot \text{Br} \sim 3 \times 10^{-6}.
\]

In the extended model with the parameters in Eq. (41). This value is much larger than that in Eq. (34). This is one reason why we can obtain the realistic baryon asymmetry in the extended model. Numerically, the resultant asymmetry becomes smaller than Eq. (42) due to the wash-out effect of inverse decay process, as in the standard baryogenesis scenario by the decay of heavy particle.

### 3 Thermal history

In this section, we discuss the thermal history of the universe. We introduce a new scalar \( S \) to make the Higgs field stay at
false vacuum in the early universe, where $S$ is singlet under the SM gauge group. \footnote{In the mechanism we have proposed, it is important that the Higgs field obtains a large expectation value at higher temperature. To realize this situation, we introduce this singlet-scalar field $S$. If one can realize this situation by other ways, we do not have to introduce it. But, naively, introducing the singlet-scalar field is a simplest way.}

First, we explain the zero temperature scalar potential of the extended model with $S$ and the thermal correction to it. Then we discuss how the Higgs field is in false vacuum in the early universe.

3.1 Zero temperature Higgs potential

The tree level scalar potential is given by

$$V_{\text{tree}}(h, S) = -\kappa \frac{m_S^2}{4\lambda_S^2} h^2 + \frac{1}{4} h^4 + \kappa h^2 S^2 - \frac{1}{2} m_S^2 S^2 + \lambda_S S^4, \quad (43)$$

where $S$ is the new singlet-scalar field. We consider the region where all couplings take $O(0.1-1)$ value. Although $\lambda$ becomes small or negative at high scale in the SM (see e.g. Ref. [20]), now the running of $\lambda$ is modified, $\lambda$ can take values $O(0.1-1)$ since some scalar fields are added.

We note that the one-loop Coleman–Weinberg potential can be safely neglected because of $O(0.1-1)$ couplings, and therefore we do not include it for simplicity.

The potential (43) has an absolute minimum at $\langle h \rangle = 0, \quad \langle S \rangle = \frac{1}{2} \sqrt{\frac{m_S^2}{\lambda_S}} \equiv v_S. \quad (44)$

The quadratic term of the SM Higgs is added in order to make the Higgs massless in this vacuum.

3.2 Thermal potential

We follow Ref. [21] and show the thermal potentials. The thermal potentials are evaluated at the one-loop level where the loop effects of the massive Higgs boson, $W$, $Z$ boson, the top quark and the scalar $S$ are included. For the gauge fields, we employ the Landau gauge where the ghost fields are massless and do not have the $h$ field dependence. The NG bosons $\gamma_i$ in the Higgs doublet field (4) are neglected since their effects are small.

As the thermal effects, there are two components, namely $V_{\text{FT}}(h, T)$ and $V_{\text{ring}}(h, T)$. \footnote{The potential (43) has a minimum at $\langle h \rangle = \sqrt{\frac{m_S^2}{2\lambda_S}} = \sqrt{\frac{2\kappa}{\lambda_S}} v_S, \quad \langle S \rangle = 0$. This minimum does not becomes the absolute minimum but the local one for the parameter space we consider here.} The main contribution of thermal effects comes from $V_{\text{FT}}(h, T)$, which is

$$V_{\text{FT}}(h, T) = \frac{T^4}{2\pi^2} [J_{B}(\bar{m}_S^2/T^2) + J_B(m_S^2/T^2) + 6J_{B}(m_W^2/T^2) + 3J_B(m_Z^2/T^2) - 12J_F(m_t^2/T^2)], \quad (45)$$

where the mass for each particle is given by

$$m_W^2 = \frac{g_2^2}{4} h^2, \quad m_Z^2 = \frac{g_2^2 + g_Y^2}{4} h^2, \quad m_t^2 = \frac{y_t^2}{2} h^2, \quad \bar{m}_S^2 = 3\lambda_S h^2 - \kappa \frac{m_S^2}{2\lambda_S} + 2\kappa S^2, \quad m_S^2 = 12\lambda_S S^2 + 2\lambda h^2 - m^2; \quad (46)$$

the thermal functions are defined as

$$J_B(r^2) = \int_0^\infty dx x^2 \ln (1 - e^{-\sqrt{x^2 + r^2}}), \quad J_F(r^2) = \int_0^\infty dx x^2 \ln (1 + e^{-\sqrt{x^2 + r^2}}). \quad (47)$$

Remember here that the coupling constants $g_2, g_Y$ and $y_t$ are SU(2)$_L$, U(1)$_Y$ and top-Yukawa coupling constants, respectively. Since one cannot analytically and exactly evaluate these functions, the approximated expressions are made. \footnote{The high temperature expansion is often used. However, they are not useful for the case where we see the large field value of $h$. Therefore, the fitting functions (A23) are also employed [22]. See Appendix A for details.}

There are contributions to the ring diagrams (or the daisy diagrams) from the Higgs boson and the gauge boson:

$$V_{\text{ring}}(h, T) = -\frac{T}{12\pi} [(\bar{m}_h^2 + \Pi_h(T))^{3/2} - \bar{m}_h^2] - \frac{T}{12\pi} [(\bar{m}_S^2 + \Pi_h(T))^{3/2} - \bar{m}_S^2] - \frac{T}{12\pi} \left[2a^3/2 + \frac{1}{2\sqrt{2}} (a_\phi + c_\phi) - (a_\phi - c_\phi)^2 + 4b_\phi^2/3 \right]^{3/2} + \frac{1}{2\sqrt{2}} (a_\phi + c_\phi) + [(a_\phi - c_\phi)^2 + 4b_\phi^2/3]^{3/2} - \frac{1}{4} [(g_2^2/2 + g_Y^2)h^2]^{3/2}, \quad (48)$$

where the first and second terms correspond to the contribution from the Higgs and the scalar $S$; \footnote{Combining the ring contribution of the Higgs boson and the first term of Eq. (45), we can write}

$$\frac{T^4}{2\pi^2} J_B(\bar{m}_h^2/T^2) - \frac{T}{12\pi} \left[(\bar{m}_h^2 + \Pi_h(T))^{3/2} - \bar{m}_h^2\right] = \frac{T^4}{2\pi^2} J_B(\bar{m}_h^2(T)/T^2). \quad (49)$$
In the early universe, due to the finite temperature effect, the Higgs potential in the SM, we analyze the effective potential, $V_{\text{eff}}(h, S, T) = V_{\text{tree}}(h, S) + V_{\text{FT}}(h, S, T) + V_{\text{ring}}(h, S, T)$, where $V_{\text{tree}}(h, S)$ is given in Eq. (43). In next subsection, we investigate the phase transition of Higgs field by using this potential.

3.3 Thermal history

In the early universe, due to the finite temperature effect, $S$ and $H$ do not have the vacuum expectation value (VEV).\textsuperscript{15} They develop their respective VEVs at the temperature when the thermal mass term becomes comparable with their negative mass term. By utilizing the high temperature expansion (A21) and (A22), we estimate the critical temperatures which are given as the vanishing curvature of $V_{\text{eff}}(h, S, T)$ at the origin $(h, S) = (0, 0)$, namely

$$\left. \frac{\partial^2 V_{\text{eff}}(h, S, T_S)}{\partial S^2} \right|_{h=0, S=0} = 0,$$

$$\left. \frac{\partial^2 V_{\text{eff}}(h, S, T_h)}{\partial h^2} \right|_{h=0, S=0} = 0.$$

Solving these equations for $T$, we find\textsuperscript{16}

$$T_S = \frac{2 \sqrt{3} \sqrt{2 v^2 \lambda_S}}{\sqrt{\kappa} + 6 \lambda_S},$$

$$T_h = \frac{4 \sqrt{6} \sqrt{v_T \kappa}}{\sqrt{9 g_2^2 + 3 g_Y^2} + 12 \gamma^2 + 8 \kappa + 12 \lambda}.\tag{55}$$

Footnote 14 continued

where $\tilde{m}_S^2(T) = m_S^2 + \Pi_h(T)$ is the Debye mass of the Higgs boson. In the same manner, the thermal effects for the scalar $S$ also can be written as the same form.

\textsuperscript{15} Our thermal scenario is similar to Ref. [23], where the gravitational wave from electroweak phase transition at the high scale is discussed.

\textsuperscript{16} Here the $V_{\text{ring}}$ contribution is neglected. This should still provide an approximate estimation of the phase transition temperature.

Here $T_S$ and $T_h$ denote the critical temperatures of the phase transition of $S$ and $h$, respectively. Our scenario is as follows. The phase transition of Higgs field happens at $T = T_h$. At this time, $S$ and $h$ are in the false vacuum, $\langle S \rangle = 0$, $\langle h \rangle = \sqrt{\frac{2 \kappa}{\lambda_S}} v_S$, and the lepton number is created by the decay of heavy neutrinos. After that, at $T = T_S$, $S$ develops VEV $\langle S \rangle = v_S$, and then $\langle h \rangle$ comes back to the true vacuum Eq. (44).

In order to work with our scenario, we require

$$T_S < T_h.$$

Moreover, $S$ must have a negative mass at $\langle S \rangle = 0$, $\langle h \rangle = \sqrt{\frac{2 \kappa}{\lambda_S}} v_S$, namely $m_S < 0$, which yields

$$\lambda_S > \frac{\kappa^2}{\lambda}.\tag{57}$$

As an example of successful parameters, we take $\kappa = 0.7$, $\lambda_S \simeq 1.5$, $\lambda = 0.4$ and $\langle h \rangle = 2 \times 10^{13}$ GeV. $T_h$ and $T_S$ become

$$T_S \simeq 1.9 v_S, \quad T_h \simeq 2.0 v_S,$$

and Eq. (57) is satisfied. Here $g_Y = g_2 = y_\nu = 0.5$ is used.

Therefore, by solving the Boltzmann equations with

$$z_{\text{ini}} = \frac{M_v}{T_h}, \quad z_{\text{final}} = \frac{M_v}{T_S},$$

we can calculate the asymmetry. For example, we obtain\textsuperscript{17}

$$N_{B-L} \simeq 7.0 \times 10^{-7},$$

with the parameter set Eq. (41). Here we have taken into account the wash-out factor [15] in the symmetric phase,

$$\exp[-T_S/2 \times 10^{13} \text{ GeV}].$$

This implies that we can realize the observed value, $N_{B-L, \text{obs}} = 4.8 \times 10^{-8}$, by slightly changing the value of $CP$ phase. We notice that a numerical study is necessary to establish which values of the couplings return an acceptable pattern of symmetry breaking, as currently approximate estimates are provided in the paper.

Finally, let us briefly discuss the validity of the effective Lagrangian Eq. (3). The temperature of the phase transition is

$$T_h \simeq \sqrt{\frac{2 \lambda}{\kappa}} \langle h \rangle \simeq 2 \times 10^{13} \text{ GeV},\tag{62}$$

\textsuperscript{17} Here we take the thermal initial condition, $N_X = 3/4, N_{B-L} = 0$. 
while the cutoff scale in Eq. (3) is Eq. (6). It can be seen
that, as long as \( m_v \lesssim 0.1 \text{ eV} \), \( T_h \) is much smaller than \( \Lambda \) in
Eq. (41). Although \( T_h \) is close to \( \Lambda_2 \) and \( \Lambda_3 \), it is still below
these cutoffs. Hence, the effective Lagrangian would be valid
in this region.

4 Summary and discussion

We have considered the possibility of baryogenesis in a false
vacuum where the Higgs field develops a large field value
compared with the electroweak scale. Since all the SM par-
ticles receive mass from the coupling with the Higgs boson,
the large field value of the Higgs field means that they are
super-heavy. We have estimated the asymmetry produced by
the decay of the heavy left-handed neutrino. It has turned
out that the decay of the neutrino cannot realize the observed
baryon asymmetry. If the new higher-dimensional operators
are introduced, the decay of the neutrino can provide suffi-
cient asymmetry.

We have also presented the thermal history where the
Higgs field develops a large value in the early universe. It
has been found that, by adding the singlet scalar \( S \), our sce-
nario safely works.

Finally, we briefly mention the possibility of the high scale
electroweak baryogenesis. So far, we pursued the possibility
that the baryon asymmetry is created by the heavy particle,
while the cutoff scale in Eq. (3) is \( \Lambda \) in
Eq. (41). Although \( T_h \) is close to \( \Lambda_2 \) and \( \Lambda_3 \), it is still below
these cutoffs. Hence, the effective Lagrangian would be valid
in this region.

Appendix: The effective potential at finite temperature

In this appendix, following Ref. [21], we show the deriva-
tion of the thermal effects on the Higgs potential in the SM.
We consider the one-loop contribution of a particle with the
mass \( m(h) \) to the potential, which typically has the following form:

\[ V_{1\text{loop}}(h, T) = \pm \frac{1}{2} \text{Tr} \ln(k^2 + m^2(h)). \]  

(A1)

where “\( \text{Tr} \)” denotes the functional trace; \( k \) is the Euclidean
momentum; the boson (fermion) loop case has overall posi-
tive (negative) sign. For a particle with one degree of freedom, the potential is

\[ V_{1\text{loop}}(h, T) = \pm \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \ln(k^2 + m^2(h)). \]  

(A2)

At finite temperature, the time direction of momentum is
discretized and its loop integral changes to the Matsubara
summation:

\[ \int \frac{dk_0}{2\pi} \int \frac{d^3k}{(2\pi)^3} f(k_0, \vec{k}) = T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} f(\omega_n, \vec{k}), \]  

(A3)

with the Matsubara frequency,

\[ \omega_n = \begin{cases} 2n\pi T & \text{for bosons}, \\ (2n + 1)\pi T & \text{for fermions}. \end{cases} \]  

(A4)

Therefore, Eq. (A2) can be calculated as

\[ V_{1\text{loop}}(h, T) = \pm \frac{T}{2} \int \frac{d^3k}{(2\pi)^3} \beta\omega + 2 \ln(1 \mp e^{-\beta\omega}) \]

\[ = \pm \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \omega \pm T \int \frac{d^3k}{(2\pi)^3} \ln(1 \mp e^{-\beta\omega}). \]  

(A5)

where \( \omega = \sqrt{k^2 + m^2} \); the sign \((+\)) and \((-\)) in the logarithm
apply to fermions and bosons, respectively. The first term
does not depend on temperature and is rewritten as

\[ V_{\text{CW}}(h) = \pm \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \omega \]

\[ = \pm \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \ln(k^2 + m^2). \]  

(A6)
of particles, the thermal potential is given by

\[ V_{\text{TT}}(h, T) = \pm \frac{T}{2\pi^2} \int dk \, k^2 \ln(1 + e^{-\beta \omega}) = \pm \frac{T^4}{2\pi^2} J_B(F)(r^2), \]

(A7)

where the thermal functions for boson and fermion are defined as

\[ J_B(r^2) = \int_0^\infty dx \, x^2 \ln(1 - e^{-\sqrt{x^2 + r^2}}), \]
\[ J_F(r^2) = \int_0^\infty dx \, x^2 \ln(1 + e^{-\sqrt{x^2 + r^2}}), \]

(A8)

with \( x \equiv \bar{k}/T \) and \( r \equiv m(h)/T \). Note that in general case, the operator \( k^2 + m^2(h) \) is not diagonal, i.e. \( k^2 \delta_{ij} + m^2(h) \).

Therefore, the mass matrix \( m^2(h) \) has to be diagonalized.

In the SM case, taking account of the degrees of freedom of particles, the thermal potential is given by

\[ V_{\text{TT}}(h, T) = \sum_{i=W, Z, h} n_i \frac{T^4}{2\pi^2} J_B((m_i^B(h)/T)^2) - \sum_n n_i \frac{T^4}{2\pi^2} J_F((m_i^F(h)/T)^2), \]

(A9)

where \( n_W = 6, n_Z = 3, n_t = 12 \) and \( n_h = 1 \).

Next, we consider the ring (or daisy) contributions shown in Fig. 3, which are the next-higher-order corrections and are related to the infrared divergence; see e.g. [24] for a detailed discussion. The ring contribution for the Higgs field is given by

\[ V_{\text{ring}}(h, T) = -\frac{T}{12\pi} \sum_{\omega < \pi T} \int \frac{d^3k}{(2\pi)^3} \sum_i \frac{1}{i} \left( -\frac{1}{\omega^2 + k^2 + m^2(h)} \Pi_h(T) \right)^i \]
\[ = -\frac{T}{12\pi} \text{Tr} \left[ \left\{ m^2(h) + \Pi_h(T) \right\}^{1/2} - m^2(h) \right], \]

(A10)

where the thermal mass comes from the diagrams in the limit \( m(h)/T \ll 1 \) shown in Fig. 4 and becomes

\[ \Pi_h(T) = \Pi^{(A_\mu)}_\phi(T) + \Pi^{(B_\mu)}_\phi(T) + \Pi^{(\psi)}_\phi(T) + \Pi^{(\psi)}_B(T) \]
\[ = \frac{T^2}{12} \left( \frac{9}{4} g_2^2 + \frac{3}{4} g_Y^2 + 3 \lambda^2 + 6 \lambda \right), \]

(A11)

with

\[ \Pi^{(A_\mu)}_\phi(T) = \frac{3}{16} g_2^2 T^2, \quad \Pi^{(B_\mu)}_\phi(T) = \frac{1}{16} g_2^2 T^2, \]
\[ \Pi^{(\psi)}_\phi(T) = \frac{1}{4} g_Y^2 T^2, \quad \Pi^{(\psi)}_B(T) = \frac{1}{2} g_Y^2 T^2. \]

(A12)

Note that these contributions are evaluated by setting the external momentum to zero since we are interested in the infrared limit.

In a similar manner, one can obtain the ring contributions from the gauge bosons, which becomes

\[ V_{\text{ring}}^{gb}(h, T) = -\frac{T}{12\pi} \text{Tr} \left[ (M^2 + \Pi_{00}(T))^{1/2} - M^2(h) \right]. \]

(A13)

Here the mass matrices in the original gauge field basis \( (A_\mu', B_\mu') \) are

\[ M^2(h) = \begin{pmatrix} g_2^2 h^2/4 & 0 & 0 & 0 \\ 0 & g_2^2 h^2/4 & 0 & 0 \\ 0 & 0 & g_Y^2 h^2/4 & -g_Y g_2 h^2/4 \\ 0 & 0 & -g_Y g_2 h^2/4 & g_Y^2 h^2/4 \end{pmatrix}, \]

(A14)

\[ \Pi_{00}(T) = \begin{pmatrix} \Pi^{(2)}_{00}(T) & 0 & 0 & 0 \\ 0 & \Pi^{(2)}_{00}(T) & 0 & 0 \\ 0 & 0 & \Pi^{(2)}_{00}(T) & 0 \\ 0 & 0 & 0 & \Pi^{(2)}_{00}(T) \end{pmatrix}. \]

(A15)

where \( \Pi_{00}(T) \) is the \( 00 \) component of the polarization tensor in the infrared limit, namely \( \Pi_{\mu\nu}(\omega = 0, T) \) and

\[ \Pi^{(1)}_{00}(T) = \Pi^{(2)}_{00}(T) + \Pi^{(3)}_{00}(T) = \frac{11}{6} g_2^2 T^2, \]

(A16)

\[ \Pi^{(2)}_{00}(T) = \Pi^{(3)}_{00}(T) + \Pi^{(4)}_{00}(T) + \Pi^{(5)}_{00}(T) = \frac{11}{6} g_2^2 T^2, \]

(A17)

with

\[ \Pi^{(1)}_{\psi}(T) = \frac{1}{6} g_2^2 T^2, \quad \Pi^{(2)}_{\psi}(T) = \frac{5}{3} g_Y^2 T^2, \]
\[ \Pi^{(3)}_{\psi}(T) = \frac{2}{3} g_2^2 T^2, \quad \Pi^{(4)}_{\psi}(T) = \frac{1}{6} g_2^2 T^2, \quad \Pi^{(5)}_{\psi}(T) = g_Y^2 T^2. \]

(A18)

These thermal masses are obtained by calculating the two-point functions of SU(2) and U(1) gauge fields shown in Fig. 5. Evaluating the eigenvalues of \( M^2(h) + \Pi_{00}(T) \) and
U and dot lines denote the Higgs, single wave, double wave, solid thermal masses. The dashed, single wave, double wave and solid lines denote the Higgs, U(1) gauge, SU(2)L and top quark, respectively.

![Diagram](image)

*Fig. 4* The two-point functions of Higgs field at one-loop level which yield the thermal masses. The dashed, single wave, double wave and solid lines denote the Higgs, U(1) gauge, SU(2)L and top quark, respectively.

![Diagram](image)

*Fig. 5* The two-point functions of SU(2) and U(1) gauge fields at one-loop level which yield the thermal masses. The dashed, single wave, double wave, solid and dot lines denote the Higgs, U(1) gauge, SU(2)L, top quark and ghost, respectively.

![Diagram](image)

*Fig. 6* The diagrams which contribute to the thermal masses of the Higgs field and the new scalar one. The double dashed line denotes the new scalar field S.

\[ M^2(h) \] to the three-half power, and then taking trace of them, we have

\[ \nu_{\text{ring}}^{gb}(h, T) = -\frac{T}{12\pi} \left[ 2a_g^{3/2} + \frac{1}{2\sqrt{2}} (a_g + c_g) - \frac{1}{2\sqrt{2}} (a_g - c_g)^2 + \frac{1}{2\sqrt{2}} \right] \]

\[ \times \left[ \frac{1}{2\sqrt{2}} (a_g + c_g) + [(a_g - c_g)^2 + 4b_g^2]^{1/2} \right]^{3/2} - \frac{1}{2\sqrt{2}} \left[ \frac{1}{2\sqrt{2}} (a_g + c_g) + [(a_g - c_g)^2 + 4b_g^2]^{1/2} \right]^{3/2} \]

\[ - \frac{1}{4} (a_g + c_g)^{3/2} - \frac{1}{4} (a_g + c_g)^{3/2} \]

(20)

where \( a_g, b_g \) and \( c_g \) are given in Eq. (52).

Note that we have worked in the Landau gauge to evaluate the contributions from the gauge bosons. Although the thermal mass matrix (A14) can be diagonal only in the limit \( m_W(h)/T \ll 1 \) and \( m_Z(h)/T \ll 1 \), the ring contribution is still valid for the larger mass, thus the larger field value than temperature. This is because the ring contribution vanishes for the larger mass.

In case where the scalar \( S \) is introduced, the contribution from the diagram shown in Fig. 6 is added, and then the thermal masses of the Higgs field and \( S \) are given as Eqs. (50) and (51), respectively.

4.1 The thermal functions and their approximation

The high temperature expansion is often applied to the thermal functions (A8). However, it is not useful for investigat-
\[ J_{B(F)}(r^2) = e^{-r} \sum_{n=0}^{N_{B(F)}} c_n^{B(F)} r^n, \]  
(A23)

where \( N_{B(F)} \) and \( c_n^{B(F)} \) are the truncation order of the series and the fitting coefficients, respectively. For the fixed truncation order \( N_{F(B)} \), we find the coefficients \( c_n^{F(B)} \) by fitting the exact results numerically evaluated.

We show the comparisons between results of exact form Eq. (A8) and the approximated forms Eqs. (A21)–(A23) in Fig. 7. We see that the high temperature expansions are actually valid for \( r \leq 2 \) and the fitting functions with \( N_{B(F)} = 40 \) break down for \( r > 22 \). The fitting functions with \( N_{B(F)} = 100 \) are valid for large value of \( r \). Therefore, it is useful for evaluating a potential of large field values since \( r = m(\phi)/T \) and the mass \( m(\phi) \) is proportional to the value of the field \( \phi \).

References

1. ATLAS, G. Aad et al., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. Phys. Lett. B 716, 1–29 (2012). arXiv:1207.7214
2. CMS, S. Chatrchyan et al., Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. Phys. Lett. B 716, 30–61 (2012). arXiv:1207.7235
3. P.A.R. Ade et al., Planck 2015 results. XIII. Cosmological parameters. Astron. Astrophys. 594, A13 (2015). arXiv:1502.01589
4. A.D. Sakharov, Violation of CP invariance, c asymmetry, and Baryon asymmetry of the universe. Pisma Zh. Eksp. Teor. Fiz. 5, 32–35 (1967) [Usp. Fiz. Nauk 161, 61 (1991)]
5. M. Yoshimura, Unified gauge theories and the Baryon number of the universe. Phys. Rev. Lett. 41, 281–284 (1978) [Erratum: Phys. Rev. Lett. 42, 746 (1979)]
6. I. Affleck, M. Dine, A new mechanism for baryogenesis. Nucl. Phys. B 249, 361–380 (1985)
7. M. Fukugita, T. Yanagida, Baryogenesis without grand unification. Phys. Lett. B 174, 45–47 (1986)
8. W. Buchmuller, R.D. Peccei, T. Yanagida, Leptogenesis as the origin of matter. Ann. Rev. Nucl. Part. Sci. 55, 311–355 (2005). arXiv:hep-ph/0502169
9. C.S. Fong, E. Nardi, A. Riotto, Leptogenesis in the universe. Adv. High Energy Phys. 2012, 158303 (2012). arXiv:1301.3062
10. R.F. Dashen, B. Hasslacher, A. Neveu, Nonperturbative methods and extended hadron models in field theory. 3. Four-dimensional nonabelian models. Phys. Rev. D 10, 4138 (1974)
11. N.S. Manton, Topology in the Weinberg–Salam theory. Phys. Rev. D 28, 2019 (1983)
12. F.R. Klinkhamer, N.S. Manton, A saddle point solution in the Weinberg-Salam theory. Phys. Rev. D 30, 2212 (1984)
13. V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, On the anomalous electroweak baryon number nonconservation in the early universe. Phys. Lett. B 155, 36 (1985)
14. H. Aoki, H. Kawai, String scale baryogenesis. Prog. Theor. Phys. 98, 449–456 (1997). arXiv:hep-ph/9703421
15. Y. Hamada, K. Kawana, Reheating-era leptogenesis. Phys. Lett. B 763, 388–392 (2016). arXiv:1510.05186
16. E.W. Kolb, S. Wolfram, Baryon number generation in the early universe. Nucl. Phys. B 172, 224 (1980) [Erratum: Nucl. Phys. B 195, 542 (1982)]
17. E.W. Kolb, M.S. Turner, The early universe. Front. Phys. 69, 1–547 (1990)
18. Particle Data Group, C. Patrignani et al., Review of particle physics. Chin. Phys. C 40(10), 100001 (2016)
19. Y. Hamada, H. Kawai, K.-Y. Oda, Minimal Higgs inflation. PTEP 2014, 023B02 (2014). arXiv:1308.6651
20. Y. Hamada, H. Kawai, K.-Y. Oda, Bare Higgs mass at Planck scale. Phys. Rev. D 87(5), 053009 (2013). arXiv:1210.2538
21. M.E. Carrington, The effective potential at finite temperature in the Standard Model. Phys. Rev. D 45, 2933–2944 (1992)
22. K. Funakubo, E. Senaha, Electroweak phase transition, critical bubbles and sphaleron decoupling condition in the MSSM. Phys. Rev. D 79, 115024 (2009). arXiv:0905.2022
23. R. Jinno, K. Nakayama, M. Takimoto, Gravitational waves from the first order phase transition of the Higgs field at high energy scales. Phys. Rev. D 93(4), 045024 (2016). arXiv:1510.02697
24. J.I. Kapusta, C. Gale, Finite-Temperature Field Theory: Principles and Applications (Cambridge University Press, Cambridge, 2011)