PION DISSOCIATION IN HOT QUARK MEDIUM

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Abstract

Pion dissociation in a medium of hot quark matter is studied. The decay width of pion is found to be large but finite at temperatures much higher than the so called critical temperature of chiral or deconfinement transition. Consequently, pions should coexist with quarks and gluons at such high temperatures. The result is in agreement with the lattice calculations. The implication of the above result in the study of Quark-Gluon plasma is discussed.

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A strong prediction of Quantum Chromodynamics (QCD), \emph{the underlying theory of strong interaction}, is that at very high temperature and/or density, the bulk properties of strongly interacting matter would be governed by the quarks and gluons, rather than the usual hadrons. Such a phase is called quark gluon plasma (QGP)\textsuperscript{4} in the

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literature and the search for such a novel phase of matter constitutes a major area of
current research in the field of high energy physics.

The properties and dynamics of QGP are obviously governed by QCD. This
conceptually straightforward task is, however, quite formidable in practice, particu-
larly because of the failure of perturbative QCD already in the temperature range in
the vicinity of $\Lambda_{QCD} (\sim$ few hundred MeV) [2]. Analytical non-perturbative methods
are not yet sufficiently developed to be of much use in this context and as such, the
lattice formulation of QCD has developed into the primary vehicle for the study of
QGP [3]. In addition to the intensive computation, both in terms of CPU time and
numerical complexities, one can only address static properties in the lattice. As a
result, the space - time evolution of the system formed in the ultrarelativistic heavy
ion collisions remains unapproachable in the framework of the lattice; thus the al-
ternate, classical picture of hydrodynamic evolution, which accounts for the overall
energy - momentum conservation in a collective manner and not much else, has been
used quite extensively to study the evolution of the QGP [4]. QCD inputs enter into
such a picture through the equation of state of the QGP, preferably evaluated on the
lattice ( but more often, through a phenomenological bag model [5] ).

An inescapable feature of the collision process is that the quarks and gluons
must, at some epoch, turn into hadrons which would ultimately be detected, never the
individual quarks and gluons. The actual process of hadronisation, however, continues
to elude us. It has been widely postulated that there could be an actual phase
transition (the order of which is an open issue), separating the QGP phase from the
hadronic phase [6]. The recent results, showing the lack of thermodynamic equilibrium
[4] in the quark-gluon phase in ultrarelativistic heavy ion collisions, indicate that such
an ideal situation is unlikely. It should also be noted at this juncture that although the
persistence of non-perturbative effects till very high temperatures was suggested in the
literature quite early on [8], it is only recently that the lattice results have confirmed that non-perturbative hadron like excitations could survive at temperatures far above the chiral phase transition temperature [9]. It is thus imperative to understand the behaviour of such hadronic resonances, their formation, stability and so on, in a quark gluon medium at high temperature. In this work we confine our attention to the case of pions alone, which is naturally a prototype of all hadrons. Being the lightest hadron, pions account for the bulk of the multiplicity.

Formation of pions, a bound state of light relativistic quarks, is an extremely difficult problem to handle in QCD. This is where all the troublesome features of non-perturbative QCD would make their presence felt. We therefore employ the usual practice of looking at the pion as a Goldstone boson arising from the spontaneous breaking of the chiral symmetry. The coupling of the pion to the quarks can then be obtained in a straightforward manner, by starting with the free lagrangian, imposing a chiral transformation and demanding invariance under such a transformation. Explicitly, the chiral angle is associated with the pion field and the quark field is rotated by the chiral transformation,

\[
q' = \exp \left[ i \frac{\pi \cdot \tau \gamma^5}{2f_\pi} \right] q
\]  

Expanding the exponential to first power in \( \frac{1}{f_\pi} \), we obtain the pion-quark coupling. The interaction term is given by [10],

\[
\mathcal{L}_{\text{int}} = \frac{m_q}{f_\pi} \bar{q} \gamma_5 \tau \cdot \pi q
\]

where \( m_q \) is the quark mass, \( f_\pi \) is the usual pion decay constant (=93 MeV), \( q \) is quark field and \( \tau \) is the usual Pauli matrix.
Even with such an interaction, the formation of pions from quarks and gluons would require an involved analysis through the Bethe-Salpeter equation. Such a study is very much on our agenda but we do not address this issue here. In the present work, we concern ourselves with the decay of pionic excitations, the properties of which we assume to be given by the lattice calculations. It should be reiterated that at temperatures above the critical temperature, these pionic excitations are more like resonances with large effective masses \[9, 11\]. The variation of the pion mass with
temperature, as calculated in the lattice [11], is shown in figure 1. In the following, we study the decay width of such pionic excitations in the hot quark medium as a function of temperature, starting with the interaction given above in equation (2).

The quark mass \( m_q \) appearing in eq. (2) is a very important ingredient in our calculation. In the absence of any medium and/or dynamic effect, \( m_q \) should assume the value of the current quark mass. On the other hand, we know that due to the spontaneous breakdown of the chiral symmetry, quarks attain the value of the constituent quark mass [10]. However, we are investigating the behaviour of pions in quark medium at very high temperatures (\( >> \) chiral symmetry restoration temperature) where quarks pick up a large thermal mass [12] due to medium effects. So, in our calculation, we have taken, for the sake of completeness, three different values for \( m_q \), namely the current quark mass (\( \sim 10 \text{MeV} \)), the constituent quark mass (\( \sim 300 \text{MeV} \)) as well as the thermal mass (see below).

The decay width of a pion in its rest frame is given by the usual expression,

\[
\Gamma = \int \frac{d^3p_1}{2p_1^0(2\pi)^3} \frac{d^3p_2}{2p_2^0(2\pi)^3} \frac{(2\pi)^4\delta^4(Q - p_1 - p_2)}{2Q_0} |M|^2 (1 - f(p_1))(1 - f(p_2))
\]  

where \( M \) is the matrix element, \( Q \) is the momentum of the pion and \( p_1 \) and \( p_2 \) are the momenta of \( q \) and \( \bar{q} \). \( f(p_1) \) and \( f(p_2) \) are the usual Fermi-Dirac distribution functions accounting for the Pauli blocking of final state quarks. The matrix element is given by

\[
M = \frac{m_q}{f_\pi} \bar{q} \gamma_5 q\tau
\]  

From (3) and (4), the final expression for the decay width of the pion to a quark anti-quark pair, in the rest frame of the pion, is given by,

\[
\Gamma = \frac{1}{4\pi m_\pi^2} \left[ \frac{m_q}{f_\pi} \right]^2 (m_\pi^2 - 4m_q^2)^{3/2}/(1 - f(E_1))(1 - f(E_2))
\]  

where \( E_1 \) and \( E_2 \) are the quark energies. The \( \delta \)-function in equation (3) yields \( E_1 = E_2 = m_\pi/2 \).
The thermal quark mass is defined as

\[ m_{th} = \sqrt{m^2_{curr} + \frac{g^2 T^2}{9}} \]  

where

\[ g_s = \sqrt{4\pi\alpha_s} \]
\[ \alpha_s = \frac{6\pi}{29} \ln \left( \frac{3T}{\Lambda} \right) \]

In the above equation, \( \Lambda \) is the QCD parameter and \( m_{curr} \) the current quark mass. We have considered three values of \( \Lambda \), 0.3, 0.2 and 0.1 GeV. The variation of the pion decay width with temperature for different quark masses and \( \Lambda \) are shown in figure 2. The decay width for the current quark mass is not included in the figure, as for this case, the decay width is very small; in fact it is practically zero in the scale of the present figure.

Figure 2 shows that the decay width is very high at high temperature (\( \sim 0.3 \) GeV) and decreases with decreasing temperature, going to zero at around \( T = 0.16 \) GeV. It is worth noticing that at around the same temperature, the effective pion mass attains the value of the free pion mass (figure 1). The dependence on \( \Lambda \) is very clear. The decay width increases with increase in \( \Lambda \). However, the temperature at which the decay width goes to zero does not depend sensitively on \( \Lambda \). The decay width is found to be maximum for the constituent quark mass. The fact that for the current quark mass the decay width is very small, whereas for the constituent quark mass it is fairly high, leads one to infer that the decay width decreases as the quark mass decreases. This explains the dependence on \( \Lambda \) too, as larger \( \Lambda \) corresponds to larger \( m_q \) at any given temperature. In figure 2 one should also note the 'shoulder' like structure in decay width around \( T = 200 MeV \). This 'shoulder' is a reflection of the temperature variation of pion mass (figure 1), which also has a 'shoulder' around that temperature.
Figure 2: Temperature dependence of pion decay width; (a) for the constituent quark mass, (b), (c) and (d) are for the thermal quark masses for $\Lambda = 0.3$, 0.2 and 0.1 GeV respectively.

The increases of the pion decay width with increasing $m_q$ is somewhat against the common notion that as the difference in the masses of pion and quark increases the decay width should increase. For the interaction used here, the decay width is proportional to $m_q^2$ (equation 2) and as a result the decay width increases with the increase in quark mass. The decay width is maximum when the quark mass is taken
to be its constituent mass \( i.e. \) Its value at \( T = 290 \) MeV is 0.8 GeV; as a result, the pions formed at that temperature will decay immediately to \( q \bar{q} \) pairs.

Our results will have a strong bearing on the study of hadronisation. As already mentioned, the lack of thermodynamic equilibrium in a QGP system implies that one may not get a clear-cut phase transition from QGP to hadrons. Thus, to understand the process of hadronisation, one should really start from a very high temperature (\( >> \) expected \( T_c \)) and then let the system evolve dynamically towards lower temperatures. Here what one would find, as indicated from our present calculation, is that initially a very small number of pions would be present in the system along with quarks and gluons. Then, even if additional pions are formed through \( q \bar{q} \) fusion and/or bound state formation, the total number of pions should not increase very fast, as most of them must decay immediately due to the large decay width at such high temperatures. Only in the vicinity of \( T \sim 170\)MeV, where the decay width is small, the number of pions would start increasing significantly and gradually become dominant compared to the number of quarks at some lower temperature. However, the exact value of the temperature, at which the decay width goes to zero, will depend on the value of the quark mass considered.

To summarise, we have calculated, for the first time, the decay of pions, which is a prototype of all hadrons, in a hot quark medium. The most interesting and noteworthy feature is that, even without any consideration of the detailed evolution and dynamics of the system, the pionic modes are found to dominate around a temperature of 160 MeV. Though the question whether this is a signature of a phase transition cannot be addressed within the framework of the present work, the fact that most of the pions decay into quarks, owing to a large decay width at temperatures higher than \( T_c \), is a remarkable finding.

It will be interesting to compare the pion decay width obtained here with the
decay widths of other mesons. Qualitatively, the same conclusion should hold but it remains to be seen if all hadronic modes start becoming important at about the same temperature. Work in this direction is in progress.

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