Numerical simulation of turbulence arising in the free convection boundary layer after a cross row of rectangular obstacles

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Abstract. The contribution covers results of direct numerical simulation of transitional processes and turbulence developing in the vertical-plate free convection boundary layer downstream of a periodic row of 3D obstacles. The Prandtl number is set at 0.7. Periodicity conditions are prescribed in the spanwise direction. Streamwise position of the obstacles being spanwise-elongated rectangular parallelepipeds corresponds either to the local Rayleigh number, \( R_a \), equal to \( 10^9 \) or to \( R_a \approx 2.5 \times 10^8 \). In the first (basic) case, 3D vortex structures of a wide spectrum start to develop at a relatively short distance from the obstacles. The largest -in-size vortices, occupying the outer part of the layer, have a hairpin-like shape, and their heads are oriented opposite to the main flow direction. Streamwise evolution of the transitional and turbulent flow structure is analysed based on iso-surfaces of the Q-criterion covering the flow region up to \( R_a \approx 2.6 \times 10^{10} \). The predicted mean Nusselt number dependence on \( R_a \) is in a good agreement with the known measurement data, both for the transitional and the turbulent flow regions. For the second case, nonlinear spatial development of a nearly 2D instability wave occupying a large portion of the computational domain is predicted. Rapid destruction of this wave and occurrence of chaotic 3D eddies are observed in this case at \( R_a \approx 3 \times 10^9 \).

Introduction

Free convection flows developing along elongated vertical walls are common in practice. They are used in ventilation and heating, energy and fuel storage systems, nuclear reactors, passive solar heating applications, etc. In these applications, the flow can often reach a turbulent state characterized by an enhanced intensity of heat transfer.

Experimental data obtained for the canonical case of free convection boundary layer developing on a vertical plate under conditions of low external disturbances show that the laminar-to-turbulent transition zone has a considerable length: it is two to three times larger than the stable laminar flow region [1,2]. There are grounds to assume that reduction of the transitional region may produce a considerable positive effect in terms of overall heat transfer augmentation. For practical applications, the most attractive way for triggering the laminar-turbulent transition is to introduce roughness elements on the wall, covered by a free convection boundary layer. Consequently, a multi-parametric problem arises. It consists in searching optimal forms of the introduced macro-roughness elements, as well as their optimal position along the flow.

Based on current possibilities of computing system hardware and computational fluid dynamics (CFD) software it is reasonable to study the above indicated problem via direct numerical simulation
(DNS) of transitional processes and turbulence developing in the free convection boundary layer downstream of obstacles of varied geometrical forms.

In the literature, experience in application of DNS techniques for studying transitional and turbulent dynamics and heat transfer in free convection boundary layers adjacent to an isothermally heated vertical surface was accumulated, using the time-developing DNS technique that allows prescribing the periodic boundary condition in the streamwise direction and reducing computational efforts significantly [3,4]. Quite recently, an encouraging numerical study of ‘natural transition’ of free convection boundary layers has been reported. It was performed by means of space-developing DNS [5]. Triggering ‘natural transition’ was provided by introducing spatially and temporally random perturbations into the upstream boundary layer.

The present report covers first results obtained by means of DNS for the problem of turbulence development downstream of 3D obstacles introduced into the free convection boundary layer.

**Physical and mathematical models**

The flow configuration simulated here consists of a free convection boundary layer that develops along a heated isothermal vertical plate and runs on a row of disturbing obstacles in the form of elongated parallelepipeds. Plane ABCD of the rectangular computational domain used, as shown in Figure 1, is the plate surface. The vertical size of the plate is set at \( L = AB = DC = 1.92 \) m. The width of the computational domain is set at 0.24 m. Simulation is performed assuming periodicity boundary conditions (prescribed on planes ABFE and DCGH). Segment AD corresponds to the upstream edge of the plate \((x=0)\). The external boundary of the domain (plane EFGH) is placed at a distance of 0.24 m from the plate. Two adiabatic obstacles, 8x8x60 mm each, are positioned on the selected part of the heated plate, symmetrically with respect to the confining planes ABFE and DCGH. The spanwise distance between the obstacles is 60 mm; the upstream faces of the obstacles are placed at \( x = 0 = 0.646 \) m.

The simulation is performed on the basis of the three-dimensional incompressible-fluid Navier–Stokes and energy equations. The Boussinesq approximation is applied. Assuming the Prandtl number, \( \text{Pr} \), equal to 0.7, two computational cases are considered. In the first case (Case 1), thermal boundary conditions correspond to those adopted in the well known experimental study of air convection by Tsuji&Nagano [5]. Namely, the plate surface temperature, \( T_w, \) is set at 60°C. Ambient fluid temperature, \( T_0, \) equals 16°C. For the second case (Case 2), \( T_w = 23°C, T_0 = 16°C. \) Following [5], physical properties of air are evaluated at the mean temperature \( T_f = (T_w + T_0)/2, \) except that the thermal expansion coefficient \( \beta \) is evaluated at \( T = T_w. \)

Let us introduce the local Rayleigh number as \( \text{Ra}_x = \text{Pr} g \beta (T_w - T_0) x^3/\nu^2, \) where \( \nu \) is the fluid kinematic viscosity. For the first simulated case, \( \text{Ra}_x (x=x_0)=10^3 \) and \( \text{Ra}_x (x=L)=2.6 \times 10^{10}, \) whereas for the second case, \( \text{Ra}_x (x=x_0)=2.5 \times 10^9 \) and \( \text{Ra}_x (x=L)=6.5 \times 10^9. \)

Note also, that at \( x=x_0 \) the whole thickness, \( \delta_{x_0} \), of the undisturbed laminar boundary layer is about 25 mm in Case 1, and about 35 mm in Case 2. So, in both cases the obstacles, 8 mm height, are fully submerged into the disturbed layer.

**Computational aspects**

The computational grid used for simulating both cases consisted of about 34 million hexahedral cells with 1230 cells placed along the streamwise direction and 184 cells – along the spanwise direction. The grid was clustered near the plate. Figure 2 illustrates the grid nodes distribution near the obstacle.

Calculations were performed using ANSYS Fluent in version 17.0. This code was based on the finite volume method with the cell-centered variable arrangement. In addition to the boundary conditions described in Section 2, the following settings were applied. The slip adiabatic wall condition was imposed on the lower plane (plane ADHE in Figure 1). For the external boundary (EFGH) being in fact the inlet one, zero value of the reduced total pressure was prescribed. Zero value of the reduced static pressure was set on the upper plane (BCGF). The reference pressure was set as 101325 Pa.
The NITA solver with the Fractional-Step option was used when running the code. The second-order central scheme of spatial discretization was applied for evaluation of convective fluxes, both for the momentum and energy equations. The time advancing was carried out with a step of 0.002 seconds. Samples of duration of about 60 seconds were calculated for getting statistics after a transient period.

Results and discussion

Instantaneous flow pattern in the near wake and downstream of the obstacles computed for Case 1 is illustrated in Figure 3. This picture shows that strong spatial oscillations of the streamlines (colored by velocity magnitude) occur just after the end of the reverse flow zone adjacent to the obstacle back side. Figure 4 presents a plot of the Q-criterion iso-surfaces (colored with the streamwise velocity) for the same flow region. One can see that transitional vortex structures of a wide spectrum develop at a relatively short distance from the obstacles. Remarkably, the largest-in-size vortices, occupying the outer part of the layer, have a hairpin-like shape, and their heads are oriented opposite to the main flow direction. This peculiarity of the transitional free convection layer was established earlier from results of the time-developing direct numerical simulation [4].

Temporal variations of the y-component of the velocity vector at four monitoring points are illustrated in Figure 5. Measured in millimetres, coordinates $x,y,z$ of the points shown in Figure 3 are as follows: $P_1(750,15,60)$, $P_2(750,15,120)$, $P_3(950,15,60)$, $P_4(950,15,120)$. The velocity vector, $V$, is scaled by the buoyancy velocity, $V_b$, defined as $V_b = \sqrt{g \beta (T_w - T_a)}$, for the dimensionless velocity, $V'$, to be given by $V'=(V_x^*, V_y^*, V_z^*)=V/V_b$. Dimensionless time, $t'$, is calculated as $t' = t V_b^2/\nu$. Note that for Case 1 under consideration, $V_b=0.02927 \text{ m/s}$. As seen in Figure 5, strong oscillations of an easily selected frequency develop in the near wake of the obstacles, whereas downstream (points $P_3$ and $P_4$) the oscillations take a turbulent character.

Figures 6, 7 show instantaneous distributions of the skin friction coefficient over the plate, up to the plate upper edge. Note that here the depicted data are multiplied twice in the spanwise direction. The skin friction coefficient is defined as $c_f = \tau_w/\rho V_b^2$, where $\tau_w$ is the wall shear stress and $\rho$ is the fluid density. The plot given in Figure 6 for Case 1 highlights, in particular, a remarkable peculiarity of the transitional free convection boundary layer established previously in experiments [1,2]. Namely, the mean (time- and/or span-averaged) skin friction coefficient in the transitional region first decreases, reaching a minimum value at some coordinates, and then rapidly increases. Position of line A-A indicating a zone of minimal $c_f$ in Figure 6 corresponds to $Ra_x \approx 7 \times 10^7$.

The distribution of the skin friction coefficient given in Figure 7 for Case 2 shows that a large portion of the computational domain is occupied by a laminar flow zone with nonlinear spatial development of a nearly 2D instability wave. Rapid destruction of this wave and occurrence of chaotic motion components are observed in this case at $Ra_c \approx 3 \times 10^9$ (line B-B in Figure 7).
Figure 3. Instantaneous streamlines pattern in the wake of the obstacles for Case 1.

Figure 4. Visualization of transitional vortex structure in the obstacle wake for Case 1.

Figure 5. Temporal variations of the dimensionless normal velocity component at monitoring points shown in Figure 3: (a) P_1, (b) P_2, (c) P_3, (d) P_4.

Figure 6. Instantaneous distribution of skin friction coefficient over the plate in Case 1.

Figure 7. Instantaneous distribution of skin friction coefficient over the plate in Case 2.
Figure 8 presents a plot of the Q-criterion iso-surfaces that, unlike the plot in Figure 4, covers all the transitional region, as well as a section of turbulent flow simulated in Case 1. Note that the Q-criterion value taken to create this plot is four times higher than that used for the plot in Figure 4. Besides, the iso-surfaces are coloured by dimensionless temperature calculated as $T^* = (T - T_a)/(T_w - T_a)$. Due to this, one can clearly see that the mentioned-above hairpin-like vortices have a temperature close to the ambient fluid temperature, especially in the turbulent flow region. Another peculiarity of the flow highlighted by the plot is formation of large-scale “clouds” in the turbulent flow region. These “clouds” involve a number of hairpin-like vortices and are responsible for rapid thickening of the boundary layer. At that, one should recognize that the width of the computational domain used in the present simulation is not enough to get reliable quantitative data characterizing the size and spatial-temporal behavior of the “clouds”.

An analogous plot of the Q-criterion iso-surfaces but for Case 2 is given in Figure 9. Here the upper section part of the plate is covered by the transitional flow region. Comparing this plot with those given in Figures 4,8 for Case 1, one can conclude that excitation of turbulence in these two cases goes by considerably different scenarios.

Figure 8. Vortex structures in the transitional and turbulent regions in Case 1: iso-surfaces of the Q-criterion coloured by dimensionless temperature.

Figure 9. Vortex structures developing after destruction of non-linear instability waves in Case 2: iso-surfaces of the Q-criterion coloured by dimensionless temperature.

Figure 10 shows dependences of the Nusselt number on the local Rayleigh number obtained in the present simulations. Experimental data from [1] are given for comparison. The predicted Nusselt numbers were obtained by averaging time-dependent local Nusselt numbers over time and span. It is remarkable that the predicted Nusselt number values in Case 1 are in a good accordance with the measurement data despite that experiments [1] were carried out under “natural” transition with a low level of uncontrolled disturbances of a wide spectrum. As for Case 2, one can see that the beginning of transition is considerably delayed as compared with Case 1. However, the length of the transition region, in terms of the Rayleigh numbers, is shorter.
Conclusions
The DNS-based numerical analysis has been undertaken to study transitional processes and turbulence developing in the vertical-plate free convection boundary layer downstream of a periodic row of 3D obstacles that have a form of elongated rectangular bars and are introduced either at the streamwise coordinate corresponding to the local Rayleigh number of $10^9$ or at $Ra_x = 2.5 \cdot 10^8$.

It has been established that the most typical 3D structures developing in the most unstable, outer part of the boundary layer, characterised by a non-monotonic streamwise velocity profile, are hairpin-like vortices, which heads are oriented opposite to the main flow direction. At the end of the transition region, formation of large-scale “clouds” involving a number of such vortices is observed. Excitation of turbulence in the two cases differing in values of the obstacle-position Rayleigh number goes according to considerably different scenarios.

The predicted mean Nusselt number dependence on the local Rayleigh number obtained in the simulation does not contradict the known measurement data, both for the transitional and the turbulent flow regions.

Generally, the DNS data obtained can promote raising ideas for varying/choosing geometrical forms of the obstacles that would produce the most effective triggering transitional processes in the free convection boundary layer. Further research work in this area should also cover a more extended analysis of the effects related to variations of the obstacle coordinate, in terms of the local Rayleigh number, as well as a study of sensitivity of the prediction results to changing the width of the computational domain, if confined by planes with the prescribed periodicity boundary conditions.

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