GLOBAL ANALYSIS OF SIRI KNOWLEDGE DISSEMINATION MODEL WITH RECALLING RATE

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ABSTRACT. In order to study the dissemination mechanism of knowledge, a
SIRI dynamics model with the learning rate, the forgetting rate and the recall-
ing rate is constructed in this paper. Stability of equilibria and global dynamics
of the SIRI model are analyzed. Two thresholds that determine whether knowl-
edge is disseminated are given. We describe the stability of the equilibria for
the SIRI model in which there are an equilibrium and a line of equilibria. In
particular, we find the dividing curve function which is used to partition in-
variant set in order to discuss the local stability, and obtain the equation of
the wave peak value or wave trough value in the process of knowledge dissemi-
nation. Numerical simulations are provided to support the theoretical results.
The complicated dynamics properties exhibit that the model is very sensitive
to variation of parameters, which play an important role on controlling and
administering the knowledge dissemination.

1. Introduction. Knowledge refers to data, information, ideas, rules, procedures,
intuition, experiences, and models. Commonly, we consider that the method of
knowledge dissemination is school education [3], which is the traditional learning
style with organization and purpose in a fixed location, including various training
and learning. The popularity of the Internet is quietly changing the traditional
learning methods. By means of multitudinous social network learning [4, 6] and
various kinds of education mode [1], for instance, open course, there exists the
ubiquitous communication of knowledge [2, 5]. The open course is a formal open

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course teaching activity for specific people with organization, opportunity and purpose, which plays an important role in the dissemination of knowledge. For similar purposeful learning style, with knowledge forgetting, there may be secondary learning at the end of the learning process, or reviewing and recalling during the learning process. We are interested in what the mechanism for this type of knowledge dissemination is, how to effectively administrate the dissemination of knowledge in the multiple sources, how to control the speed and the efficiency in the process of knowledge dissemination. In this case, it is very necessary and valuable to analyze the dynamics properties of knowledge dissemination models that incorporate the information spreading and experience transferring.

In the present paper, the primary objective is to reveal a simple and biologically reasonable mechanism that can explain the learning effect in knowledge dissemination. Considering the spread and dissemination of varied knowledge in a population of size $N$, we can partition the population into compartments, which are exclusive groups according to the natural process of the learning. For the dissemination of a class of knowledge, possible compartments may be:

- $S$: students, who are uneducated about relevant knowledge and want to study the knowledge.
- $I$: intellectuals, who are well-educated and have mastered the knowledge.
- $R$: recalling people, who have forgotten part of the knowledge and will recall to mind the knowledge by learning process again.

Obviously, it is a decomposition of $N$ into three sets of mutually disjoint.

From the standpoint of biology and the simplified model, we do not think about the migration. In other words, we only consider no in-out migration which implies the population movement between three compartments. The knowledge dissemination with no migration is not biologically unrealistic, for example, the school education. For physiology reason, considering that the existence of forgetting can be regarded as a biological setting, the study of the rate of forgetting makes sense in knowledge dissemination process. Besides, the rate of learning and the rate of recalling are considered as the important factors of knowledge dissemination. The schematic diagram of transmission process with recalling rate is sketched in FIG.1. In the schematic diagram, the arrows indicate movements of individuals among compartments.

![Schematic diagram for the knowledge dissemination model.](image)

The rest of this paper is organized as follows. In the following section, the SIRI model with the recalling rate is formulated. In Section 3, the global dynamics analysis of the system is investigated. In Section 4, the numerical simulations are presented to illustrate the theoretical results. At last, the results obtained in this article are summarized and discussed.

2. **Model formulation.** In the process of knowledge dissemination with recalling rate, we consider that the goal of modeling is to track the number of members in each
of the three compartments at any given time $t$. To formulate the compartmental model, we will denote these numbers by $S(t)$, $I(t)$, $R(t)$.

Let $S(t)$ denote the number of members who want to learn a class of knowledge, and maybe learn the knowledge when they are in contact with $I$, then those who become intellectual person will move into $I$ at time $t$. Let $I(t)$ denote the number of members who are intellectual and have learned knowledge at time $t$. Let $R(t)$ denote the number of members who forget a portion of the knowledge and move into $R$ from $I$ at time $t$. Besides, if $R$ is in contact with $I$, then recalling individual might become intellectual individual.

Consider the type of knowledge dissemination by means of direct contact with people, we make the following assumptions about the dissemination process of knowledge.

1. The number of contacts between members from different compartments depends only on the number of member of each compartment. In particular, the learning rate - number of new $I$ from $S$ member per unit time - can be expressed as $\beta SI$, where $\beta$ is called the learning coefficient. And the recalling rate - number of new $I$ from $R$ member per unit time - can be expressed as $\lambda IR$, where $\lambda$ is called the recalling coefficient.

2. The rate of transfer from a compartment is proportional to the member size of the compartment. For instance, the rate of transfer from $I$ to $R$, the forgetting rate, can be written as $\alpha I$, where $\alpha$ is called the forgetting coefficient.

3. Migration is not considered in the population, which implies no input of new members and no removal from any compartments. We can regard as zero migration.

4. The total population remains a constant. This is a direct result of the third assumption, but we clearly emphasize this fact here.

5. The parameters in the model are positive constant for biological feasibility.

Based on the above assumptions, we can exhibit that the dissemination process is demonstrated in the following transfer diagram (see FIG.2).

![Transfer diagram](image)

**FIGURE 2.** Transfer diagram for knowledge dissemination SIRI model.

By mathematical descriptions, we establish the following system of differential equations for knowledge dissemination SIRI model with recalling rate as follows:

\[
\begin{align*}
\dot{S} &= -\beta SI, \\
\dot{I} &= \beta SI - \alpha I + \lambda IR, \\
\dot{R} &= \alpha I - \lambda IR,
\end{align*}
\]

with initial conditions $S(0) \geq 0$, $I(0) \geq 0$, $R(0) \geq 0$.

Denote $\Omega_1 = \{(S, I, R) : S + I + R = K, S \geq 0, I \geq 0, R \geq 0\}$, where $K = S(0) + I(0) + R(0)$. For system (1), we have the following property.

**Theorem 2.1.** If $S(0) \geq 0$, $I(0) \geq 0$, $R(0) \geq 0$, $\Omega_1$ is positive invariant with respect to system (1).
Proof. By equation \( \dot{S} = -\beta SI \), we get \( S(t) = S(0)e^{-\beta \int_0^t I(\tau) d\tau} \geq 0 \) for \( S(0) \geq 0 \). By equation \( I = \beta SI - \alpha I + \lambda IR \), we obtain \( I(t) = I(0)e^{\int_0^t (\beta S(\tau) - \alpha + \lambda R(\tau)) d\tau} \geq 0 \) for \( I(0) \geq 0 \). From above, it is easy to see \( \dot{S} = -\beta SI \leq 0 \) for all \( t > 0 \). Since \( S(t) \geq 0 \) and \( \dot{S} \leq 0 \), we conclude that \( \lim_{t \to \infty} S(t) \) exists and \( 0 \leq S(t) \leq S(0) \).

By equation \( \dot{R} = I(\alpha - \lambda R) \), we note that \( R(t) = \frac{\alpha}{\lambda} \) is a constant solution. Using the existence-uniqueness theorem, when \( I(0) \geq 0 \), we obtain that in the case of \( R(0) \geq \frac{\alpha}{\lambda} \), it is easy to see \( R(t) \geq \frac{\alpha}{\lambda} > 0 \), and in the case of \( 0 \leq R(0) < \frac{\alpha}{\lambda} \), we get \( \dot{R} \geq 0 \), then \( 0 \leq R(t) < \frac{\alpha}{\lambda} \). Hence, \( R(t) \geq 0 \).

From the above analysis, we have shown that all solutions starting in \( R^3_+ \) remain in \( R^3_+ \) for all \( t > 0 \).

Let \( N(t) = S(t) + I(t) + R(t) \) denote the total population. Adding all three equation of system (1), we obtain \( \dot{N} = 0 \), which implies \( \dot{N} = K(\text{constant}) \) for all \( t \geq 0 \), where \( K = S(0) + I(0) + R(0) \). Hence, the feasible region \( \Omega_1 \) is positive invariant.

In system (1), it is easy to see that if \( I(0) = 0 \), then \( I(t) \equiv 0 \), \( S(t) \equiv S(0) \) and \( R(t) \equiv R(0) \) for \( t \geq 0 \); and if \( S(0) = 0 \), then system (1) is epidemic IR model. In order to generally discuss the dynamical behavior of system (1) in the feasible region, we give the initial conditions \( S(0) > 0, I(0) > 0, R(0) \geq 0 \) in this paper.

3. Stability of equilibria. In this section, we will focus on the existence and global stability of equilibria. Considering that the total number of population is constant \( K \), the system is reduced to a two-dimensional system, then system (1) can be rewritten as the following equivalent system:

\[
\begin{align*}
\dot{S} &= -\beta SI, \\
\dot{I} &= \beta SI - \alpha I + \lambda I(K - S - I),
\end{align*}
\]

in a two-dimensional feasible region \( \Omega_2 = \{(S, I) : 0 \leq S + I \leq K, S \geq 0, I \geq 0 \} \).

Theorem 2.1 indicates that region \( \Omega_2 \) is positive invariant with respect to system (2). In the rest of the paper, we will study the dynamics of system (2) with the initial conditions \( S(0) > 0, I(0) > 0 \) in region \( \Omega_2 \).

Dividing the two equations of system (2), one can obtain

\[
\frac{dI}{dS} = \frac{\lambda - \beta}{\beta} + \frac{\lambda}{\beta S} + \frac{\alpha - \lambda K}{\beta S}.
\]

Then, the solutions of equation (3) can be written as

\[
I(t) = -S(t) + \left(K - \frac{\alpha}{\lambda}\right) + \frac{S(0) + I(0) - \left(K - \frac{\alpha}{\lambda}\right)}{S(0)} S(t)^\frac{1}{2}.
\]

Let \( R_0 = \frac{\lambda\alpha}{\beta} \) denote the threshold of SIRI model, which is used to determine whether knowledge is disseminated. Based on the threshold definition, equation (4) can be rewritten as follows:

\[
I(t) = -S(t) + \frac{\alpha}{\lambda} (R_0 - 1) + \frac{S(0) + I(0) - \frac{\alpha}{\lambda} (R_0 - 1)}{S(0)} S(t)^\frac{1}{2}.
\]

We further give the following theorem.

**Theorem 3.1.** For all \( S(0) > 0, I(0) > 0 \), \( S(t) \) and \( I(t) \) are convergent as \( t \to \infty \), and they satisfy \( 0 \leq S^* < S(0) \) and \( I^* \geq 0 \), where \( \lim_{t \to \infty} S(t) = S^* \), \( \lim_{t \to \infty} I(t) = I^* \). If \( S^* > 0 \), then \( I^* = 0 \). If \( S^* = 0 \), then \( I^* = \frac{\alpha}{\lambda} (R_0 - 1) \) for \( R_0 \geq 1 \).
Proof. By Theorem 2.1, it is easy to see that $S(t)$ is convergent as $t \to \infty$, and it satisfies $0 \leq S^* < S(0)$ for $S(0) > 0$. By equation (5), we obtain that there exists $\lim_{t \to \infty} I(t)$ for $I(t) \geq 0$ in region $\Omega_2$, then $I^* \geq 0$.

Since $\lim_{t \to \infty} S(t) = S^*$ and $\lim_{t \to \infty} I(t) = I^*$, it is easy to get $\lim_{t \to \infty} \dot{S}(t) = -\beta S^* I^*$.

We further arrive at the conclusion that $\lim_{t \to \infty} S(t) = 0$. Hence, if $S^* > 0$ then $I^* = 0$. Besides, when $I^* = 0$, $S^*$ can be calculated by the following implicit function:

$$-S^* + \frac{\alpha}{\lambda}(R_0 - 1) + \frac{S(0) + I(0) - \frac{\lambda}{\alpha}(R_0 - 1)}{S(0)^\frac{\alpha}{\lambda}}(S^*)^{\frac{\alpha}{\lambda}} = 0.$$  \tag{6}

By equation (5), if $S^* = 0$, then $I^* = \frac{\lambda}{\alpha}(R_0 - 1)$ for $R_0 \geq 1$.

In order to obtain all equilibria of system (2), let the right sides of (2) be equal to zero. Obviously, there exists a line of equilibria which is described as the half-line $\{(S, I) : S \geq 0, I = 0\}$. Define $E_1(0, \frac{\lambda}{\alpha}(R_0 - 1))$, called equilibrium of knowledge dissemination. Define $E_2(S^*, 0)$, called equilibria of no knowledge dissemination. All equilibria can be given by $E_1(0, \frac{\lambda}{\alpha}(R_0 - 1))$ and $E_2(S^*, 0)$.

In the feasible region, the half-line is denoted by a set $D = \{(S, I) : S \geq 0, I = 0\} \cap \Omega_2 = \{(S, I) : 0 \leq S \leq K, I = 0\}$. Obviously, we get the following results. If $R_0 \geq 1$, the equilibria of system (2) are $E_1(0, \frac{\lambda}{\alpha}(R_0 - 1))$ and $E_2(S^*, 0)$ for $S^* \in [0, K]$ in region $\Omega_2$. If $R_0 < 1$, the equilibria of the system are $E_2(S^*, 0)$ for $S^* \in [0, K]$ in region $\Omega_2$.

We further consider the isocline of system (2) in region $\Omega_2$. In the coordinate systems, the vertical isocline are $S = 0$ and $I = 0$, the horizontal isocline are $I = 0$ and $I = (\frac{\lambda}{\alpha} - 1)S + \frac{\alpha}{\lambda}(R_0 - 1)$.

Define the isocline line $l : I = (\frac{\beta}{\lambda} - 1)S + \frac{\alpha}{\lambda}(R_0 - 1)$.

Region $\Omega_2$ is divided into following three subregions:

$$A = \{(S, I) : I > \left(\frac{\beta}{\lambda} - 1\right)S + \frac{\alpha}{\lambda}(R_0 - 1)\} \cap \Omega_2,$$

$$B = \{(S, I) : I < \left(\frac{\beta}{\lambda} - 1\right)S + \frac{\alpha}{\lambda}(R_0 - 1)\} \cap \Omega_2,$$

$$C = \{(S, I) : I = \left(\frac{\beta}{\lambda} - 1\right)S + \frac{\alpha}{\lambda}(R_0 - 1)\} \cap \Omega_2.$$

Denote $A = A_1 \cup A_2$, where $A_2 = A - A_1$, $A_1$ is defined as follows:

$$A_1 = \{(S, I) : I > \frac{\beta}{\lambda}(\frac{\lambda - \beta}{\alpha(R_0 - 1)})^{\frac{1}{\beta - 1}}S^{\frac{\beta}{\beta - 1}} - S + \frac{\alpha}{\lambda}(R_0 - 1)\} \cap A.$$

Obviously, $\Omega_2 = A \cup B \cup C$. It is easy to get $\dot{S} < 0$ in region $A$. $\dot{I} > 0$ in region $B$ and $\dot{I} = 0$ in region $C$. In particular, the point of intersection of the isocline line $l$ and the S-axis (resp. I-axis) is $\left(\frac{\alpha(R_0 - 1)}{\alpha - \beta}, 0\right)$ (resp. $\left(0, \frac{\lambda}{\alpha}(R_0 - 1)\right)$).

Denoting a new threshold by $R_1 = \frac{K\beta}{S}$, we will discuss local stability of equilibria of system (2). The conclusion can be summarized as follows:

**Theorem 3.2.** For system (2) in region $\Omega_2$,

(1) $R_0 = 1$. (a) If $R_1 \geq R_0$, $E_1(0, 0)$ is locally asymptotically stable and $E_2(S^*, 0)$ for $S^* \in (0, K]$ is unstable. (b) If $R_1 < R_0$, $E_2(S^*, 0)$ for $S^* \in (0, K)$ is locally
asymptotically stable and $E_1(0,0)$ is unstable, besides, $E_2(S^*,0)$ for $S^* \in \{K\}$ is unstable.

(2) $R_0 < 1$. (a) If $R_1 > R_0$, $E_2(S^*,0)$ for $S^* \in (0, \min\{\frac{\alpha(R_0-1)}{\lambda-\beta}, K\})$ is locally asymptotically stable, and $E_2(S^*,0)$ for $S^* \in [\min\{\frac{\alpha(R_0-1)}{\lambda-\beta}, K\}, K] \cup \{0\}$ is unstable. (b) If $R_1 \leq R_0$, $E_2(S^*,0)$ for $S^* \in (0,K]$ is locally asymptotically stable, and $E_2(S^*,0)$ for $S^* \in \{0,K\}$ is unstable.

(3) $R_0 > 1$. (a) If $R_1 \geq R_0$, $E_1(0, \frac{\alpha}{\beta}(R_0 - 1))$ is locally asymptotically stable and $E_2(S^*,0)$ for $S^* \in [0,K]$ is unstable. (b) If $R_1 < R_0$, in the case of $R_1 \geq 1$, $E_1(0, \frac{\alpha}{\beta}(R_0 - 1))$ is locally asymptotically stable and $E_2(S^*,0)$ for $S^* \in [0,K]$ is unstable; in the case of $R_1 < 1$, $E_1(0, \frac{\alpha}{\beta}(R_0 - 1))$ is locally asymptotically stable in region $\Omega_2 - A_2$, $E_2(S^*,0)$ for $S^* \in (\frac{\alpha(R_0 - 1)}{\lambda - \beta}, K)$ is locally asymptotically stable in region $A_2$, and $E_2(S^*,0)$ for $S^* \in [0, \frac{\alpha(R_0 - 1)}{\lambda - \beta}) \cup \{K\}$ is unstable in region $\Omega_2$.

Proof. Before that, recall equation (5), some important information about the trajectory, depending on initial conditions, is given as follows:

In the case of $\lambda = \beta$, by taking the derivative of $S$, we get that $\frac{dS}{d\tau} = \frac{I(0) - \frac{\alpha}{\beta}(R_0 - 1)}{S(0)}$ and $\frac{d^2S}{d\tau^2} \equiv 0$. The intersection of the trajectory curve and S-axis (resp. I-axis) is $(S^*,0)$ (resp. $(0, \frac{\alpha}{\beta}(R_0 - 1))$), where $S^*$ satisfies equation (6). Besides, we further get $S^* = \frac{-\frac{\alpha}{\beta}(R_0 - 1)S(0)}{I(0) - \frac{\alpha}{\beta}(R_0 - 1)}$ for $I(0) \neq \frac{\alpha}{\beta}(R_0 - 1)$.

In the case of $\lambda \neq \beta$, by taking the derivative of $S$, we get that $\frac{dS}{d\tau} = -1 + \frac{S(0) + I(0) - \frac{\alpha}{\beta}(R_0 - 1)}{S(0)} \cdot \frac{\lambda}{\beta} S^{\lambda - 1}$ and $\frac{d^2S}{d\tau^2} = \frac{S(0) + I(0) - \frac{\alpha}{\beta}(R_0 - 1)}{S(0)} \cdot \frac{\lambda}{\beta} S^{\lambda - 2}$. Furthermore, if $S(0) + I(0) - \frac{\alpha}{\beta}(R_0 - 1) \leq 0$, then $\frac{dS}{d\tau} < 0$; if $S(0) + I(0) - \frac{\alpha}{\beta}(R_0 - 1) > 0$, then $\frac{dI}{d\tau} < 0$ for $R_1 > R_0$ (resp. $R_1 < R_0$). According to convexity-concavity identification, we can obtain that there exists a maximum (resp. minimum) at a point $(\hat{S}, \hat{I})$, where $\hat{S}$ and $\hat{I}$ satisfy the following equations

$$
\begin{aligned}
\hat{S} &= \left[\frac{S(0) + I(0) - \frac{\alpha}{\beta}(R_0 - 1)}{S(0)} \cdot \frac{\lambda}{\beta} S^{\lambda - 1}\right]^{\frac{\beta}{\lambda - \beta}}, \\
\hat{I} &= \left(\frac{\beta}{\lambda - 1}\right)\hat{S} + \frac{\alpha}{\beta}(R_0 - 1).
\end{aligned}
$$

If $(\hat{S}, \hat{I}) \in \Omega_2$ and $0 < \hat{S} < S(0)$, we think there exists an extreme value $\hat{I}$, if not, $I$ with respect to $S$ is a monotone function in region $\Omega_2$. From above all, if $S(0) + I(0) - \frac{\alpha}{\beta}(R_0 - 1) > 0$, we can derive $I_{\max} = \hat{I}$ (resp. $I_{\min} = \hat{I}$) for $R_1 > R_0$ (resp. $R_1 < R_0$), that is, the wave peak value (resp. trough value) in the process of knowledge dissemination can be forecasted by equation (7).

We further get that the intersection of the trajectory curve and S-axis (resp. I-axis) is $(S^*,0)$ (resp. $(0, \frac{\alpha}{\beta}(R_0 - 1))$) in region $\Omega_2$, where $S^*$ satisfies equation (6).

In the positive invariant region, according to the subregions divided by isochronal lines, we obtain that if $\frac{d^2\hat{S}}{d\tau^2} < 0$, then the number of intersection of the trajectory curve and S-axis is no more than one; if $\frac{d^2\hat{I}}{d\tau^2} > 0$, then maybe the intersections of the trajectory curve and S-axis are $(S_1^*,0)$ and $(S_2^*,0)$, where $0 < S_1^* < \frac{\alpha(R_0 - 1)}{\lambda - \beta} < S_2^* < S(0)$, which means $\hat{I} < 0$. However, by using existence-uniqueness theorem and continuous dependence of solution on initial value, with $S(t)$ decreasing, the trajectory could not pass through the half-line $\{(S,I) : S \geq 0, I = 0\}$, but only tend to the bigger one $(S_2^*,0)$. Hence, we denote $S^* = S_2^*$. 
Next, we will discuss dynamics behaviors of equilibria in three cases as follows.

Case 1. $R_0 = 1$. Obviously, $E_1(0, \frac{1}{\lambda}(R_0 - 1)) = (0, 0)$, which is denoted as $E_1(0, 0)$. By equation (5), the trajectory curve passes through $(0, 0)$. In the case, we consider the following three subcases.

Case 1.1. $\lambda = \beta$, it is equivalent to $R_1 = R_0$. Obviously, $\Omega_2 = A$ and $\frac{dS}{dt} > 0$. By equation (5), we can get that the trajectory curve will only pass through $(0, 0)$ on the axes. It is easy to see that the trajectory depending on initial conditions will tend to $E_1(0, 0)$, hence, $E_2(S^*, 0)$ for $S^* \in (0, K]$ is unstable.

We sketch the phase portraits in FIG. 3(a), which shows the result that the trajectories tend to $E_1(0, 0)$ in the interior of $\Omega_2$.

Case 1.2. $\lambda < \beta$, it is equivalent to $R_1 > R_0$. Obviously, $\Omega_2 = A \cup B \cup C$ and $\frac{dI}{dS} < 0$. The trajectory curve passes through $(0, 0)$ and $(S^*, 0)$ on the axes, where $S^* > S(0)$. It is easy to see that the intersection of the trajectory curve and coordinate axes is only $(0, 0)$ in region $\Omega_2$. The trajectory depending on initial conditions will tend to $E_1(0, 0)$, hence, $E_2(S^*, 0)$ for $S^* \in (0, K]$ is unstable.

The phase portraits are sketched (see FIG. 3(b)) which shows the result that the trajectories tend to $E_1(0, 0)$ in the interior of $\Omega_2$.

Case 1.3. $\lambda > \beta$, it is equivalent to $R_1 < R_0$. Obviously, $\Omega_2 = A$ and $\frac{dI}{dS} > 0$. The intersections of the trajectory curve and coordinate axes are $(0, 0)$ and $(S^*, 0)$ in region $\Omega_2$, where $0 < S^* < S(0) < K$. From the previous conclusion, it means that the trajectory depending on initial conditions will only tend to $E_2(S^*, 0)$ for $S^* \in (0, K)$, hence, $E_1(0, 0)$ is unstable and $E_2(S^*, 0)$ for $S^* \in \{K\}$ is also unstable.

FIG. 3(c) shows the result that the trajectories tend to $E_2(S^*, 0)$ in the interior of $\Omega_2$.

Case 2. $R_0 < 1$. Obviously, $E_1 \in \Omega_2$. In the case, we consider the following three subcases.

Case 2.1. $\lambda = \beta$, it is equivalent to $R_1 = R_0$. Obviously, $\Omega_2 = A$ and $\frac{dI}{dS} > 0$. By equation (5), in the feasible region, the intersection of the trajectory curve and coordinate axes is only $(S^*, 0)$, where $0 < S^* < S(0) < K$. Hence, the trajectory depending on the initial conditions will tend to $E_2(S^*, 0)$ for $S^* \in (0, K)$, which implies $E_2(S^*, 0)$ for $S^* \in \{0\} \cup \{K\}$ is unstable.

FIG. 4(a) shows the result that the trajectories tend to $E_2(S^*, 0)$ in the interior of $\Omega_2$.

Case 2.2. $\lambda < \beta$, it is equivalent to $R_1 > R_0$. Obviously, $\Omega_2 = A \cup B \cup C$ and $\frac{dI}{dS} < 0$. The trajectory curve is convex, then the intersection of the curve and coordinate axes is $(S^*, 0)$, where $0 < S^* < \min\{\frac{\alpha(R_0 - 1)}{\lambda - \beta}, S(0)\} < K$. Hence, the trajectory depending on the initial conditions will tend to $E_2(S^*, 0)$ for $S^* \in (0, \min\{\frac{\alpha(R_0 - 1)}{\lambda - \beta}, K\})$, which implies $E_2(S^*, 0)$ for $S^* \in \min\{\frac{\alpha(R_0 - 1)}{\lambda - \beta}, K\} \cup \{0\}$ is unstable.

The phase portraits are sketched in FIG. 4(b1) (resp. FIG. 4(b2)) for $\frac{\alpha(R_0 - 1)}{\lambda - \beta} < K$ (resp. $\frac{\alpha(R_0 - 1)}{\lambda - \beta} > K$), which shows the result that the trajectories tend to $E_2(S^*, 0)$ in the interior of $\Omega_2$.

Case 2.3. $\lambda > \beta$, it is equivalent to $R_1 < R_0$. Obviously, $\Omega_2 = A \cup B \cup C$ and $\frac{dI}{dS} > 0$. It is easy to get that the trajectory curve is concave. By equation (5), in the feasible region, the intersection of the curve and coordinate axes is $(S^*, 0)$,
FIGURE 3. When $R_0 = 1$ and keeping the fixed parameter $\alpha = 0.72$, the phase portraits with different other parameters are given.
(a) Parameters are $\lambda = \beta = 0.0036$, $K = 200$. (b) Parameters are $\lambda = 0.0072$, $\beta = 0.0172$, $K = 100$. (c) Parameters are $\lambda = 0.0072$, $\beta = 0.0033$, $K = 100$.

where $0 < S^* < S(0) < K$. The trajectory depending on the initial conditions will tend to $E_2(S^*, 0)$ for $S^* \in (0, K)$, which implies $E_2(S^*, 0)$ for $S^* \in \{0\} \cup \{K\}$ is unstable.

FIG. 4(c) shows the result that the trajectories tend to $E_2(S^*, 0)$ in the interior of $\Omega_2$.

Case 3. $R_0 > 1$. In the case, we consider the following three subcases.

Case 3.1. $\lambda = \beta$, it is equivalent to $R_1 = R_0$. Obviously, $\Omega_2 = A \cup B \cup C$. We further get that if $I(0) \geq \frac{\alpha}{\lambda}(R_0 - 1)$ (resp. $I(0) < \frac{\alpha}{\lambda}(R_0 - 1)$), then $\frac{dI}{dS} \geq 0$ (resp. $\frac{dI}{dS} < 0$). By equation (5), in the feasible region, the intersection of the trajectory curve and coordinate axes is only $(0, \frac{\alpha}{\lambda}(R_0 - 1))$. Hence, the trajectory depending on the initial conditions will tend to $E_1(0, \frac{\alpha}{\lambda}(R_0 - 1))$, and $E_2(S^*, 0)$ for $S^* \in [0, K]$ is unstable.

FIG. 5(a) shows the result that the trajectories tend to $E_1(0, \frac{\alpha}{\lambda}(R_0 - 1))$ in the interior of $\Omega_2$.

Case 3.2. $\lambda < \beta$, it is equivalent to $R_1 > R_0$. Obviously, $\Omega_2 = A \cup B \cup C$ and $\frac{dI}{dS} < 0$, which means that the trajectory curve is convex. Furthermore, the intersection of the curve and coordinate axes is only $(0, \frac{\alpha}{\lambda}(R_0 - 1))$ in region $\Omega_2$. Hence, the trajectory depending on the initial conditions will tend to $E_1(0, \frac{\alpha}{\lambda}(R_0 - 1))$, and $E_2(S^*, 0)$ for $S^* \in [0, K]$ is unstable.

FIG. 5(b) shows the result that the trajectories will tend to $E_1(0, \frac{\alpha}{\lambda}(R_0 - 1))$ in the interior of $\Omega_2$. 
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Figure 4. When $R_0 < 1$ and keeping the fixed parameter $\alpha = 0.72$, the phase portraits with different others parameters are given. (a) Parameters are $\lambda = \beta = 0.0033$, $K = 200$. (b1) Parameters are $\lambda = 0.0033$, $\beta = 0.005$, $K = 200$. (b2) Parameters are $\lambda = 0.0033$, $\beta = 0.00357$, $K = 200$. (c) Parameters are $\lambda = 0.005$, $\beta = 0.0033$, $K = 130$.

Case 3.3. $\lambda > \beta$, it is equivalent to $R_1 < R_0$. Obviously, $\Omega_2 = A \cup B \cup C$. It is easy to see the fact that $\frac{dI}{dS} > 0$ in region $A$ and $\frac{dI}{dS} < 0$ in region $B$. Thus, with $S(t)$ decreasing, $I(t)$ decreasing in region $A$ and $I(t)$ increasing in region $B$. We can get that the trajectory curve only pass through $(0, \alpha \lambda (R_0 - 1))$ on the coordinate axes in region $B$. By equation (5), we further obtain $\frac{d^2I}{dS^2} > 0$ in region $A$, and the curve will pass through $(S^*, 0)$ as the parameters satisfy $0 < \frac{\alpha(R_0 - 1)}{\lambda - \beta} < K$, where $\frac{\alpha(R_0 - 1)}{\lambda - \beta} \leq S^* < S(0) < K$. Then some detailed issues about the following two subcases are discussed.

(1) $\frac{\alpha(R_0 - 1)}{\lambda - \beta} \geq K$, it is equivalent to $R_1 \geq 1$. The intersection of the trajectory curve and coordinate axes is only $(0, \frac{\alpha}{\lambda}(R_0 - 1))$ in region $\Omega_2$. Since $\Omega_2 = A \cup B \cup C$, the trajectory depending on the initial conditions will tend to $E_1(0, \frac{\alpha}{\lambda}(R_0 - 1))$, and $E_2(S^*, 0)$ for $S^* \in [0, K]$ is unstable.

In FIG. 5(c1), the phase portraits are sketched, which shows the result that the trajectories tend to $E_1(0, \frac{\alpha}{\lambda}(R_0 - 1))$ in the interior of $\Omega_2$.

(2) $0 < \frac{\alpha(R_0 - 1)}{\lambda - \beta} < K$, it is equivalent to $R_1 < 1$. The intersections of the trajectory curve and coordinate axes are $(0, \frac{\alpha}{\lambda}(R_0 - 1))$ and $(S^*, 0)$ in region $\Omega_2$. By the existence-uniqueness theorem, using the curve which passes through $(\frac{\alpha(R_0 - 1)}{\lambda - \beta}, 0)$, the dividing curve is given as follows:

$$h : I = \frac{\beta}{\lambda} (\frac{\lambda - \beta}{\alpha(R_0 - 1)})^{\frac{1}{\lambda - \beta}} S^{\frac{1}{\lambda - \beta}} - S + \frac{\alpha}{\lambda}(R_0 - 1).$$
The trajectory depending on the initial conditions under curve $h$ will tend to $E_2(S^*, 0)$ for $\frac{\alpha(R_0 - 1)}{\lambda - \beta} < S^* < S(0) < K$, otherwise, the trajectory will tend to $E_1(0, \frac{\alpha}{\lambda}(R_0 - 1))$. In other words, the trajectory will tend to $E_2(S^*, 0)$ for $S^* \in [\frac{\alpha(R_0 - 1)}{\lambda - \beta}, K]$ in region $A_2$, the trajectory will tend to $E_1(0, \frac{\alpha}{\lambda}(R_0 - 1))$ in region $\Omega_2 - A_2$, which implies that $E_2(S^*, 0)$ for $S^* \in [0, \frac{\alpha(R_0 - 1)}{\lambda - \beta}] \bigcup \{K\}$ is unstable in region $\Omega_2$.

The phase portraits are sketched, which shows the complication of the trend of trajectories in region $\Omega_2$. FIG. 5(c2) (resp. FIG. 5(c3)) places emphasis on the trajectories in region $B$ (resp. $A$).

![Figure 5](image)

**Figure 5.** When $R_0 > 1$ and keeping the fixed parameter $\alpha = 0.72$, the phase portraits with different others parameters are given. (a) Parameters are $\lambda = \beta = 0.0032$, $K = 300$. (b) Parameters are $\lambda = 0.0033$, $\beta = 0.005$, $K = 250$. (c1) Parameters are $\lambda = 0.004$, $\beta = 0.0033$, $K = 230$. (c2) Parameters are $\lambda = 0.004$, $\beta = 0.0033$, $K = 200$. (c3) Parameters are $\lambda = 0.005$, $\beta = 0.0034$, $K = 150$.

To sum up, all the possibilities are discussed. This completes the proof.
From Theorem 3.1 and Theorem 3.2, the dynamic behaviors of SIRI system are summarized as the following theorem.

**Theorem 3.3.** For system (2) in region $\Omega_2$,

1. $R_0 = 1$. (a) If $R_1 \geq R_0$, $E_1(0,0)$ is globally asymptotically stable and $E_2(S^*, 0)$ for $S^* \in (0, K]$ is unstable. (b) If $R_1 < R_0$, $E_2(S^*, 0)$ for $S^* \in (0, K)$ is globally asymptotically stable and $E_1(0,0)$ is unstable, besides, $E_2(S^*, 0)$ for $S^* \in \{K\}$ is unstable.

2. $R_0 < 1$. (a) If $R_1 > R_0$, $E_2(S^*, 0)$ for $S^* \in \{0, \min\{\frac{\alpha(R_0-1)}{\lambda - \beta}, K\}\}$ is unstable. (b) If $R_1 \leq R_0$, $E_2(S^*, 0)$ for $S^* \in \{0, K\}$ is globally asymptotically stable, and $E_2(S^*, 0)$ for $S^* \in \{0, K\}$ is unstable.

3. $R_0 > 1$. (a) If $R_1 \geq R_0$, $E_1(0, \frac{\alpha}{\lambda - \beta}(R_0 - 1))$ is globally asymptotically stable and $E_2(S^*, 0)$ for $S^* \in [0, K]$ is globally asymptotically stable and $E_2(S^*, 0)$ for $S^* \in \{0, K\}$ is unstable. (b) If $R_1 < R_0$, in the case of $R_1 \geq 1$, $E_1(0, \frac{\alpha}{\lambda - \beta}(R_0 - 1))$ is globally asymptotically stable and $E_2(S^*, 0)$ for $S^* \in [0, K]$ is unstable; in the case of $R_1 < 1$, $E_1(0, \frac{\alpha}{\lambda - \beta}(R_0 - 1))$ is locally asymptotically stable in region $\Omega_2 - A_2$, $E_2(S^*, 0)$ for $S^* \in \{\frac{\alpha(R_0-1)}{\lambda - \beta}, K\}$ is locally asymptotically stable in region $A_2$, and $E_2(S^*, 0)$ for $S^* \in \{0, \frac{\alpha(R_0-1)}{\lambda - \beta}\} \cup \{K\}$ is unstable in region $\Omega_2$.

**Remark 1.** According to Theorem 3.3, there are two cases with respect to the trajectory of system (2). In the case of $R_0 \leq 1$ for $(S(0), I(0)) \in \Omega_2$ and $R_1 < 1 < R_0$ for $(S(0), I(0)) \in A_2$, $\lim_{t \to \infty} I(t) = 0$, otherwise, $\lim_{t \to \infty} I(t) = \frac{\alpha}{\lambda - \beta}(R_0 - 1)$.

In a biological view, we obtained some useful conclusions as follows. If $R_0 \leq 1$, we know there must be no knowledge dissemination. If $1 < R_0 \leq R_1$, or if $1 < R_1 < R_0$, we know there must exist knowledge dissemination. In the above two cases, we can clearly know the ultimate scale of $I$ compartment, which depends on parameters. In particular, if $R_1 < 1 < R_0$, different initial conditions lead to different final states, which is that when $(S(0), I(0)) \in A_2$, there is no knowledge dissemination, or else, there exists knowledge dissemination. In this case, we can administrate and control the knowledge dissemination by changing the initial conditions.

4. **Numerical simulations.** In this section, numerical simulations are given to illustrate the theoretical results.

Recall $E_1(0, \frac{\alpha}{\lambda - \beta}(R_0 - 1))$ is the equilibrium of knowledge dissemination; $E_2(S^*, 0)$ is the equilibrium of no knowledge dissemination. By equation (7), under some conditions, the wave peak value or wave trough value can be calculated.

Choose $\alpha = 0.4$, $\beta = 0.01$, $\lambda = 0.004$, and $K = 61$, it is easy to get $R_0 = 0.61$, $R_1 = 1.525$, then $R_0 < 1 < R_1$. By Theorem 3.3, we can know that there is no knowledge dissemination and $\lim_{t \to \infty} (S(t), I(t), R(t)) = (S^*, 0, R^*)$, where $R^* = K - S^*$, then there exists $S^*_{num} = 10.1739$ by equation (6). Furthermore, by equation (7), with initial conditions $S(0) = 60$, $I(0) = 1$, $R(0) = 0$, we obtain the numerical solution of the wave peak value point $(\hat{S}_{num}, \hat{I}_{num}) = (30.5257, 6.7886)$.

FIG. 6(a) shows that when we extract data from this graph, there are $S^*_{data} \approx 10.17$, $I^*_{data} \approx 0$, $R^*_{data} \approx 50.83$, and $I_{\max} \approx 6.79$, which are consistent with our theoretical conclusion. Biologically speaking, there is no knowledge dissemination for sufficiently large time, however, $I$ can arrive at a wave peak in the progress of knowledge dissemination.

Choose $\alpha = 0.4$, $\beta = 0.01$, $\lambda = 0.04$, and $K = 61$, it is easy to get $R_0 = 6.1$, $R_1 = 1.525$, then $1 < R_1 < R_0$. By Theorem 3.3, we can know that there exists
the knowledge dissemination and \( \lim_{t \to \infty} (S(t), I(t), R(t)) = (0, \frac{\alpha}{\beta} (R_0 - 1), K - \frac{\alpha}{\beta} (R_0 - 1)) = (0, 51, 10) \). Furthermore, by equation (7), with initial conditions \( S(0) = 60, I(0) = 1, R(0) = 0 \), we obtain the numerical solution of the wave trough value point \( (\hat{S}_{\text{num}}, \hat{I}_{\text{num}}) = (3.1880, 48.6090). \)

FIG. 6(b) shows that when we extract data from this graph, there are \( S^*_{\text{data}} \approx 0, I^*_{\text{data}} \approx 51, R^*_{\text{data}} \approx 10 \), and \( I_{\text{min}} \approx 48.61 \), which are consistent with our theoretical conclusion. Biologically speaking, there exists knowledge dissemination, however, \( I \) initially arrives at a wave trough in a short time, then goes up, until holds the steady state.

We fix initial conditions \( S(0) = 60, I(0) = 1, R(0) = 0 \), and choose parameters \( \alpha = 0.4, \beta = 0.01, \lambda = 0.004 \). In this case, there is no knowledge dissemination. We will consider deeply whether changing one of parameters value can achieve good results, which is that there exists knowledge dissemination. Next we will vary one of parameters in order to observe the dynamics behaviors of system (2) and perform sensitivity analysis of parameters.

Let \( \alpha_1 = 0.2, \alpha_2 = 0.4, \) and \( \alpha_3 = 0.8 \), obviously, the smaller \( \alpha \), the bigger \( R_0 \). By simple calculation, we can get \( R_{01} = 1.22, R_{02} = 0.61, \) and \( R_{03} = 0.305 \), furthermore, \( R_{11} = 3.05, R_{12} = 1.525 \) and \( R_{13} = 0.7625 \). By Theorem 3.3, when \( \alpha_1 = 0.2 \), then \( \lim_{t \to \infty} (S(t), I(t), R(t)) = (0, 11, 50) \), that is, we consider there exists knowledge dissemination; when \( \alpha_2 = 0.4 \) or \( \alpha_3 = 0.8 \), there is no knowledge dissemination, that is, \( I_{\text{num}} = I_{\text{num}} = 0 \). By equation (7), we further obtain the numerical solution of extreme value point \( (\hat{S}_{\text{num}}, \hat{I}_{\text{num}}) = (9.6150, 25.4225), (\hat{S}_{\text{num}}, \hat{I}_{\text{num}}) = (96.9131, 6.3696) \). Since \( (\hat{S}_{\text{num}}, \hat{I}_{\text{num}}) \in \Omega_2 \) and \( (\hat{S}_{\text{num}}, \hat{I}_{\text{num}}) \in \Omega_2 \), we can get that when \( \alpha_1 = 0.2 \), there is peak value in the process of knowledge dissemination; when \( \alpha_3 = 0.8 \), \( I \) is monotonous and there is no trough value in the process of knowledge dissemination.

In FIG. 7, extracting data from the graph, they are shown as \( S^*_\text{data} \approx 0, I^*_\text{data} \approx 11, R^*_\text{data} \approx 50, \) and \( \bar{I}_{\text{data}} \approx 0, \bar{I}_{\text{data}} \approx 0 \). Besides, we can obtain that the maximum value of \( I(t) \) in region \( \Omega_2 \) is \( I_{\text{max}} \approx 25.42 \) which is almost equal to \( \bar{I}_{\text{num}} \), and when \( \alpha_3 = 0.8, I(t) \) is monotone decreasing. From above, Fig.7 shows that data are consistent with our theoretical conclusion. By biological significance, in the case of \( R_{11} > R_{01} > 1 \), a kind of knowledge dissemination phenomenon
will ultimately hold the steady state, that is, as time goes on, the trajectory of knowledge dissemination goes up to the peak at first, then falls, until it maintains stable for a long time; in the case of $R_{02} < 1$ (resp. $R_{03} < 1$), there is no knowledge dissemination. From the point of view about data $S^{*}_{1\text{data}} \approx 0$, $S^{*}_{2\text{data}} \approx 10.17$, and $S^{*}_{\text{data}} \approx 57.16$, it means that $S^{*}$ increases obviously with increasing of $\alpha$.

Let $\beta_{1} = 0.005$, $\beta_{2} = 0.01$, and $\beta_{3} = 0.02$, it is easy to obtain that $R_{01} = R_{02} = R_{03} = 0.61$, and the numerical solution of extreme value point are $(\hat{S}_{1\text{num}}, \hat{I}_{1\text{num}}) = (252.8395, 24.2099)$, $(\hat{S}_{3\text{num}}, \hat{I}_{3\text{num}}) = (15.1957, 21.7869)$. Obviously, there is no knowledge dissemination for every case. It is easy to obtain $(\hat{S}_{1\text{num}}, \hat{I}_{1\text{num}}) \in \Omega_{2}$ and $(\hat{S}_{3\text{num}}, \hat{I}_{3\text{num}}) \in \Omega_{2}$. Hence, when $\beta_{1} = 0.005$, there is no wave trough value; when $\beta_{3} = 0.02$, there exists a wave peak value.

In FIG. 8, the phase diagram shows there exists no knowledge dissemination, which implies that changing the parameter does not influence the stability of the equilibrium as $R_{0} < 1$. Graphically, when $\beta_{1} = 0.005$, $I(t)$ is monotonically decreasing, hence, there is no wave trough. By taking the data in this picture, it is shown as $\hat{I}_{3\text{max}} \approx 21.79$, which is consistent with the numerical conclusion. We further get $S^{*}_{1\text{data}} \approx 57.06$, $S^{*}_{2\text{data}} \approx 10.17$, and $S^{*}_{\text{data}} \approx 0.58$, which means that with the increase of $\beta$, $S$ decreases obviously.

Let $\lambda_{1} = 0.002$, $\lambda_{2} = 0.004$, and $\lambda_{3} = 0.008$. Obviously, the bigger $\alpha$, the bigger $R_{0}$. We get $R_{01} = 0.305$, $R_{02} = 0.61$, $R_{03} = 1.22$, and $R_{11} = R_{12} = R_{13} = 1.5250$. By Theorem 3.3, in the case of $\lambda_{1} = 0.002$ (resp. $\lambda_{2} = 0.004$), there is no knowledge dissemination; in the case of $\lambda_{3} = 0.008$, since $R_{13} > R_{03} > 1$, there exists the phenomenon of knowledge dissemination, and it is easy to get $\lim_{t \to \infty} (S(t), I(t), R(t)) = (0, 11, 50)$. Furthermore, the wave peak can be given by $(\hat{S}_{1\text{num}}, \hat{I}_{1\text{num}}) = (36.1441, 5.5763)$ and $(\hat{S}_{3\text{num}}, \hat{I}_{3\text{num}}) = (7.9012, 12.9753)$.

In FIG. 9, we can get $I_{1\text{max}} \approx 5.577$, $I_{3\text{max}} \approx 12.98$ and $I_{3\text{data}} \approx 11$, which are consistent with our theoretical conclusion. By taking the data, they are shown.

**Figure 7.** The Numerical simulation for system (2) with $\beta = 0.01$, $\lambda = 0.004$, $\alpha_{1} = 0.2$, $\alpha_{2} = 0.4$, $\alpha_{3} = 0.8$. 
Figure 8. The Numerical simulation for system (2) with $\alpha = 0.4$, $\lambda = 0.004$, $\beta_1 = 0.005$, $\beta_2 = 0.01$, $\beta_3 = 0.02$.

as $S_{1\text{data}}^* \approx 17.73$, $S_{2\text{data}}^* \approx 10.17$, and $S_{3\text{data}}^* \approx 0$, which means that with the increasing of $\lambda$, $S^*$ decreases obviously.

Figure 9. The Numerical simulation for system (2) with $\alpha = 0.4$, $\beta = 0.01$, $\lambda_1 = 0.002$, $\lambda_2 = 0.004$, $\lambda_3 = 0.008$. 
Comparing the phase with $\alpha_1 = 0.2$ in FIG.7 and the phase with $\lambda_3 = 0.008$ in FIG.9, they are all stable, however, there exists the different changing process. FIGS. 7, 8, 9 show that the model is very sensitive to variation of parameters, which demonstrates that it has important influence on controlling the knowledge dissemination by two thresholds. We conclude that $R_0$ is the more important threshold of knowledge dissemination and $R_1$ plays an important role in knowledge dissemination as the secondary important threshold.

5. Conclusions and discussion. For the knowledge dissemination model without recalling rate, which is the classic epidemic SIR model, there exists one threshold $R_0 = \frac{K\beta}{\alpha}$. However, with introducing the recalling rate, there are two thresholds in SIRI model, that is, $R_0 = \frac{K\lambda}{\alpha}$ and $R_1 = \frac{K\beta}{\alpha}$, then the dynamics is changed dramatically.

In this paper we completely study the existence of equilibria and dynamics behaviors of the knowledge dissemination model with recalling rate. Numerical simulations are provided to support the qualitative conclusions and theoretical results. We highlight the importance of two thresholds of knowledge dissemination which are $R_0 = \frac{K\lambda}{\alpha}$ and $R_1 = \frac{K\beta}{\alpha}$.

If $R_0 \leq 1$, it implies that $I(t)$ will die out ultimately as $t \to \infty$. If $R_1 < 1 < R_0$, $I(t)$ will die out ultimately for $(S(0), I(0)) \in A_2$, otherwise, $\lim_{t\to\infty} I(t) = \frac{\alpha}{\lambda}(R_0 - 1)$.

Furthermore, by discussing the progress of the $I$ compartment, we know the peak value of $I$ can be calculated by equation (7), which is of great significance to the study of knowledge dissemination.

Biologically speaking, our results have important consequences for researching knowledge dissemination mechanism. A key of administrating knowledge dissemination is provided in the SIRI model, that is, we can vary initial conditions and three parameters in order to control the speed and effectiveness for knowledge dissemination. The SIRI model either can be applied to the knowledge dissemination or the spreading of public opinion, rumors and so on.

Although it is necessary to introduce recalling rate in order to estimate accurately the effectiveness of knowledge dissemination, we emphasize that there is no migrant. In terms of two thresholds $R_0$ and $R_1$, the feasible region is divided into two partitions in which different part has different character. Based on above analysis, we will consider what the knowledge dissemination model with in-out migrant is, which is our further studies.

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