Sherborne Missile Fire Frequency with Unconstraint Parameters

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Abstract. For the modeling problem of shipborne missile fire frequency, the fire frequency models with unconstant parameters were proposed, including maximum fire frequency models with unconstant parameters, and actual fire frequency models with unconstant parameters, which can be used to calculate the missile fire frequency with unconstant parameters.

1. Introduction
Ship borne missile fire frequency is an important impact indicator of target kill probability. Kenan Teng¹ and Fei Li etc² proposed ship borne missile fire frequency models by not considering parameters of both sides. In this paper, ship borne missile fire frequency models with unconstraint parameters are proposed, including maximum fire frequency models with unconstraint parameters, and actual fire frequency models with unconstraint parameters.

2. Maximum fire frequency models with unconstraint parameters
Assume that trh be the moment when the target is detected by warning radar of ship borne missile system, \( \zeta_{trh} \) be the reciprocal of the average time required for the target detected by the radar.

Assume that trh1 and trh2 be the moments when the target reached the far boundary and near boundary of the missile launch area respectively, \( t_{rh2} \geq t_{rh1} \). Assume that ttr be the required time for the missile system to execute the target detection, the target threat evaluation, fire control computing and so on, \( t_{ths} \) be the missile system reaction time between pressing the first missile launch button and the missile taking off, \( \Delta t_g \) be the missile launch interval, \( t_{thy} \) be the time between the missile taking off and flying at collision point on the far boundary. The time for the target being initially intercepted by the missile system is \( t_{th} + t_{hy} \).

2.1. Case one
When trh+tfy\( \leq \)trh1, the target can be detected by the radar before the target reached the far boundary of the missile launch area, the target can be intercepted by the missile system.

Assume that \( \Delta t_g \) be a constant, trh1 follows a normal distribution with a mean of trh10 and a standard deviation of \( \sigma_{r1} \), trh2 follows a normal distribution with a mean of trh20 and a standard deviation of \( \sigma_{r2} \), trhs follows a normal distribution with a mean of trhs0 and a standard deviation of \( \sigma_{r3} \), trhy follows a normal distribution with a mean of trhy0 and a standard deviation of \( \sigma_{r4} \). Based on Eq.(4), C_{rh2} follows a
normal distribution with a mean of $Crh_{20} = \left( t_{rh20} - t_{rh0} - t_{rh0} - t_{rh0} \right) / \Delta t_g + 1$ and a standard deviation of

$$\sigma_{c2} = \sqrt{\sigma_{t1}^2 + \sigma_{t2}^2 + \sigma_{u1}^2 + \sigma_{u2}^2}.$$ 

Assume that $P(C_{rh2})$ be the probability that the fire frequency is $Crh_2$, it can be calculated by

$$P(C_{rh2}) = \begin{cases} \Phi(C_{rh2} - C_{rh20}) & C_{rh2} \leq C_{rh20} \\ 1 - \Phi(C_{rh2} - C_{rh20}) & C_{rh2} > C_{rh20} \end{cases}$$

Where $\Phi(\ )$ is standard normal distribution function.

Assume that $P_{mc2}$ be the target kill probability when the fire frequency is $Crh_2$, $P_{mdh}$ be the target kill probability with single fire, and $P_{mc2}$ can be calculated by

$$P_{mc2} = 1 - (1 - P_{mdh})^{C_{rh2}}$$

The maximum fire frequency with unconstraint parameters in case two is the $C_{rh2}$ value corresponding to the maximum value of $P_{mc2} P(C_{rh2})$.

### 2.2. Case two

Assume that $t_{th}$ follows a normal distribution with a mean of $t_{th0}$ and a standard deviation of $\sigma_t$, $t_y$ follows a normal distribution with a mean of $t_{y0}$ and a standard deviation of $\sigma_{y}$. Based on Eq.(6), $C_{rh3}$ follows a normal distribution with a mean of $C_{rh30} = \left( t_{rh20} - t_{rh0} - t_{rh0} - t_{rh0} \right) / \Delta t_g + 1$ and a standard deviation of $\sigma_{c3} = \sqrt{\sigma_{t1}^2 + \sigma_{t2}^2 + \sigma_{y1}^2 + \sigma_{y2}^2}.$

Assume that $P(C_{rh3})$ be the probability that the fire frequency is $Crh_3$, it can be calculated by

$$P(C_{rh3}) = \begin{cases} \Phi(C_{rh3} - C_{rh30}) & C_{rh3} \leq C_{rh30} \\ 1 - \Phi(C_{rh3} - C_{rh30}) & C_{rh3} > C_{rh30} \end{cases}$$

Assume that $P_{mc3}$ be the target kill probability when the fire frequency is $Crh_3$, it can be calculated by

$$P_{mc3} = 1 - (1 - P_{mdh})^{C_{rh3}}$$

The maximum fire frequency with unconstraint parameters in case three is the $C_{rh3}$ value corresponding to the maximum value of $P_{mc3} P(C_{rh3})$.

### 3. Actual fire frequency models with unconstraint parameters

#### 3.1. Actual Fire Frequency Models with Unconstraint Parameters under Single Shot

Assume that $t_{pd}$ is a constant.

#### 3.1.1. Case one. $D_{rh1}$ follows a normal distribution with a mean of $D_{rh10} = \left( t_{rh20} - t_{rh0} - t_{rh0} - t_{rh0} \right)/ \left( \Delta t_g + t_{pd} \right) + 1$ and a standard deviation of $\sigma_{d1} = \sqrt{\sigma_{t1}^2 + \sigma_{t2}^2 + \sigma_{u1}^2 + \sigma_{u2}^2}.$

Assume that $P(D_{rh1})$ be the probability that the fire frequency is $D_{rh1}$, it can be calculated by
\[ P(D_{a1}) = \begin{cases} \Phi(D_{a1} - D_{a10}) & D_{a1} \leq D_{a10} \\ 1 - \Phi(D_{a1} - D_{a10}) & D_{a1} > D_{a10} \end{cases} \] (5)

Assume that \( P_{md1} \) be the target kill probability when the fire frequency is \( D_{rh1} \), it can be calculated by

\[ P_{md1} = 1 - (1 - P_{md0})^{D_{a1}} \] (6)

When \( P_{md1} \geq P_{ru} \), the actual fire frequency with unconstant parameters under single shot in case two is the minimum \( D_{rh1} \) value satisfied the equation \( P_{md1} \geq P_{ru} \). Otherwise, the actual fire frequency is the \( D_{rh1} \) value corresponding to the maximum value of \( P_{md1} \).

### 3.1.2. Case two.

\( D_{rh2} \) follows a normal distribution with a mean of \( D_{rh20} = \left( (t_{rh20} - t_{rh0} - t_{rh0} - t_{rh0})/\Delta t_g + t_{pq2} \right) + 1 \) and a standard deviation of \( \sigma_{D2} = \sqrt{\sigma_{i1}^2 + \sigma_{i2}^2 + \sigma_{u}^2 + \sigma_{y}^2} \).

Assume that \( P(D_{a2}) \) be the probability that the fire frequency is \( D_{rh2} \), it can be calculated by

\[ P(D_{a2}) = \begin{cases} \Phi(D_{a2} - D_{a20}) & D_{a2} \leq D_{a20} \\ 1 - \Phi(D_{a2} - D_{a20}) & D_{a2} > D_{a20} \end{cases} \] (7)

Assume that \( P_{md2} \) be the target kill probability when the fire frequency is \( D_{rh2} \), it can be calculated by

\[ P_{md2} = 1 - (1 - P_{md0})^{D_{a2}} \] (8)

When \( P_{md2} \geq P_{ru} \), the actual fire frequency with unconstraint parameters under single shot in case three is the minimum \( D_{rh2} \) value satisfied the equation \( P_{md2} \geq P_{ru} \). Otherwise, the actual fire frequency is the \( D_{rh2} \) value corresponding to the maximum value of \( P_{md2} \).

### 3.2. Actual Fire Frequency Models with Unconstraint Parameters under Salvo

#### 3.2.1. Case one.

\( D_{rh3} \) follows a normal distribution with a mean of \( D_{rh30} = \left( t_{rh20} - t_{rh0} - t_{rh0} - t_{rh0} \right) / \left[ (N_{c} - 1) \Delta t_g + t_{pq2} \right] \) and a standard deviation of \( \sigma_{D3} = \sqrt{\sigma_{i1}^2 + \sigma_{i2}^2 + \sigma_{u}^2 + \sigma_{y}^2} \).

Assume that \( P(D_{a3}) \) be the probability that the fire frequency is \( D_{rh3} \), it can be calculated by

\[ P(D_{a3}) = \begin{cases} \Phi(D_{a3} - D_{a30}) & D_{a3} \leq D_{a30} \\ 1 - \Phi(D_{a3} - D_{a30}) & D_{a3} > D_{a30} \end{cases} \] (9)

Assume that \( P_{md3} \) be the target kill probability when the fire frequency is \( D_{rh3} \), it can be calculated by

\[ P_{md3} = 1 - (1 - P_{md0})^{D_{a3}} \] (10)
When $P_{md3} P(D_{rh3}) \geq P_{ru}$, the actual fire frequency with unconstraint parameters under salvo in case two is the minimum $D_{rh3}$ value satisfied the equation $P_{md3} P(D_{rh3}) \geq P_{ru}$. Otherwise, the actual fire frequency is the $D_{rh3}$ value corresponding to the maximum value of $P_{md3} P(D_{rh3})$.

3.2.2. Case two. $D_{rh4}$ follows a normal distribution with a mean of $D_{rh40} = (t_{rh20} - t_{rh0} - t_{rh0} - t_{rh0} - t_{rh0})/\left( (N_c - 1) \Delta t_g + t_{pq2} \right)$ and a standard deviation of $\sigma_{D_{rh}} = \sqrt{\sigma_{r}^2 + \sigma_{z}^2 + \sigma_{v}^2 + \sigma_{v}^2}$.

Assume that $P(D_{rh4})$ be the probability that the fire frequency is $D_{rh4}$, it can be calculated by

$$P(D_{rh4}) = \begin{cases} \Phi(D_{rh4} - D_{rh40}) & D_{rh4} \leq D_{rh40} \\ 1 - \Phi(D_{rh4} - D_{rh40}) & D_{rh4} > D_{rh40} \end{cases}$$

(11)

Assume that $P_{md4}$ be the target kill probability when the fire frequency is $D_{rh4}$, it can be calculated by

$$P_{md4} = 1 - \left(1 - P_{mdh} \right)^{D_{rh4}}$$

(12)

When $P_{md4} P(D_{rh4}) \geq P_{ru}$, the actual fire frequency with unconstraint parameters under salvo in case three is the minimum $D_{rh4}$ value satisfied the equation $P_{md4} P(D_{rh4}) \geq P_{ru}$. Otherwise, the actual fire frequency is the $D_{rh4}$ value corresponding to the maximum value of $P_{md4} P(D_{rh4})$.

4. Conclusion
Ship borne missile fire frequency is an important impact indicator, target kill probability is affected by it. For the fire frequency models with unconstraint parameters, the maximum fire frequency values with unconstraint parameters can be obtained through the maximum fire frequency models with unconstraint parameters in case one and case two, the actual fire frequency values with unconstraint parameters under single shot can be obtained through the actual fire frequency models with unconstraint parameters under single shot in case one and case two, the actual fire frequency values with unconstraint parameters under salvo can be obtained through the actual fire frequency models with unconstraint parameters under salvo in case one and case two.

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