Time Travel Paradoxes and Multiple Histories

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Abstract

If time travel is possible, it seems to inevitably lead to paradoxes. These include consistency paradoxes, such as the grandfather paradox, and bootstrap paradoxes, where something is created out of nothing. One proposed class of resolutions to these paradoxes allows for multiple histories (or timelines), such that any changes to the past occur in a new history, independent from the one where the time traveler originated. In this paper we introduce a simple model for a spacetime with a time machine and multiple histories, and suggest three possible physical manifestations of multiple histories: one branching universe, many parallel universes, or one universe with many parallel histories. We use our model to study questions such as how many histories are needed to resolve time travel paradoxes and whether one can ever come back to a previously visited history. Interestingly, we find that the histories may be cyclic under certain assumptions, in a way which combines the Novikov self-consistency conjecture with the multiple-histories approach. We give an example of this novel hybrid scenario in the context of the grandfather paradox, and discuss its consequences. Finally, we discuss how observers may experimentally distinguish between the different manifestations of multiple histories, and between multiple histories and the Hawking and Novikov conjectures.

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1 Introduction

The theory of general relativity, which describes the curvature of spacetime and how it interacts with
matter, has been verified to very high precision over the last 100 years. As far as we can tell, general
relativity seems to be the correct theory of gravity, at least in the regimes we can test. However,
within this theory, there exist certain spacetime geometries which feature closed timelike curves (CTCs),
or more generally, closed causal curves (CCCs), thus allowing the violation of causality [1, 2, 3, 4]. The
fact that these geometries are valid solutions to Einstein’s equations of general relativity indicates
crucial gaps in our understanding of gravity, spacetime, and causality.

Wormhole spacetimes and cosmological models admitting CTCs were first explored in the decades
following the discovery of general relativity [5, 6, 7]. Although these spacetimes were clearly unphys-
ical – the wormholes were non-traversable and the cosmologies unrealistic – they were followed, sev-
eral decades later, by traversable wormholes, warp drives, and other spacetimes supporting time travel
[8, 9, 10, 11, 12, 13].

These exotic geometries which allow violations of causality almost always violate the energy conditions
[14], a set of assumptions imposed by hand and thought to ensure that matter sources in general
relativity are “physically reasonable”. However, it is unclear whether these conditions themselves
are justified, as many realistic physical models – notably, quantum fields – also violate some or all of
the energy conditions.

In this paper, we consider two types of causality violations: consistency paradoxes and bootstrap para-
doaxes. A familiar example of a consistency paradox is the grandfather paradox, where a time traveler
prevents their own birth by going to the past and killing their grandfather before he met their grand-
mother; this then means that the time traveler, having never been born, could not have gone back in
time to prevent their own birth in the first place.

More precisely, we define a consistency paradox as the absence of a consistent evolution for appro-
priate initial conditions under appropriate laws of physics. Following Krasnikov [15], “appropriate
initial conditions” are those defined on a spacelike hypersurface in a causal region of spacetime – that
is, a region containing no CTCs – and “appropriate laws of physics” are those which respect locality
and which allow consistent evolution for all initial conditions in entirely causal spacetimes.

1Here, by “causal” we mean either timelike or null.
Bootstrap paradoxes arise when certain information or objects exist only along CTCs, and thus appear to be created from nothing. These paradoxes are more accurately classified as pseudo-paradoxes, as unlike consistency paradoxes, they do not indicate any physical contradictions arising from reasonable assumptions [3]. Nevertheless, they might make one feel slightly uncomfortable. Information in a bootstrap paradox has no clear origin and does not appear to be conserved, and events can occur which are impossible to predict from data in a causal region of spacetime [2]. Therefore, we explore these pseudo-paradoxes as well, identifying the situations in which they do or do not occur in our model.

There exist three primary paths to resolving time travel paradoxes [2]. First, the Hawking Chronology Protection Conjecture suggests that “the laws of physics do not allow the appearance of [CTCs]” [17]. Under this conjecture, quantum effects or other laws of physics ensure that the geometry of spacetime cannot be manipulated to allow CTCs.

Second, the Novikov Self-Consistency Conjecture holds that “the only solutions to the laws of physics that can occur locally in the real universe are those which are globally self-consistent” [18]. Thus, whether or not CTCs are physically allowed, they can never cause valid initial conditions to evolve in a causality-violating fashion. Finally, the multiple-histories (or multiple-timelines) approach encompasses several models which resolve time travel paradoxes by allowing events to occur along different distinct histories.

Of these, the multiple-histories approach supports the broadest range of exotic physics: CTCs are allowed, and a time traveler may freely change history without causing any inconsistencies, as the modified history will be independent of the history from which the time traveler originated. This approach has traditionally been presented as a branching spacetime model, utilizing non-Hausdorff (or perhaps non-locally-Euclidean [19]) manifolds to allow distinct futures with shared pasts [20]. However, the actual mechanics of resolving paradoxes using a branching spacetime has been underdeveloped in the literature, and such constructions present considerable mathematical challenges.

Another popular model for resolving time travel paradoxes is Deutsch’s quantum time travel model, also known as D-CTCs [22]. This model takes advantages of the properties of quantum mechanics in order to resolve paradoxes, and qualifies as a multiple-histories approach if one employs the Everett (“many-worlds”) interpretation. On the other hand, the model involves a significant modification of quantum mechanics, such that the equation of motion is no longer unitary nor linear in the presence of CTCs. Furthermore, as Deutsch himself noted, this model does not actually allow one to avoid the mathematical issues discussed above.

With this in mind, we consider a slightly simpler multiple-histories model, where we merely allow time machines to associate regions of spacetime between different manifolds, which we interpret as different “histories”. As we discuss below, this model can be constrained to mimic a branching spacetime model. Moreover, it enables us to explore a variety of multiple-histories formulations, differing in the number of histories used and whether or not CTCs or bootstrap paradoxes arise, as well as a variety of possible physical explanations.

Of course, before resolving any paradoxes, it is necessary to demonstrate that time travel paradoxes actually exist. This task is surprisingly non-trivial: many scenarios which on first glance might contain paradoxes actually support consistent solutions for all initial conditions [23]. However, clear paradoxes have been formulated; in particular, Krasnikov [15] developed a simple set of physical rules and examined them in a causality-violating spacetime. As one of few examples of true time travel paradoxes in the literature, Krasnikov’s model is a natural environment for us to explore the multiple-histories approach.

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2 Even in the absence of consistency paradoxes, CTCs occur in causality-violating regions and thus behind a Cauchy horizon [16]. Consider a wormhole whose mouths are surrounded by vacuum and separated more in time than in space. Then an object may, at any time, emerge from the earlier mouth and travel to the later mouth along a CTC, in a way unpredictable from outside the causality-violating region.

3 A topology satisfies the Hausdorff condition (or “is Hausdorff”) if and only if for any two distinct points \(x_1 \neq x_2\) there exist two open neighborhoods \(U_1 \ni x_1\) and \(U_2 \ni x_2\) such that \(U_1 \cap U_2 = \emptyset\).
This paper is organized as follows. First, in Chapter 2, we describe the twisted Deutsch-Politzer time machine and Krasnikov’s paradox model. Then, in Chapter 3, we generalize this model to include additional colors and particles, and most importantly, multiple histories. We discuss several different physical manifestations of multiple histories, along with some of their advantages and disadvantages. In Chapter 4 we show how allowing unlimited histories – such that every instance of time travel creates a new history and a time traveler may never return to a previously visited history – solves both consistency and bootstrap paradoxes for any number of particles and colors. In this approach, the number of histories is effectively infinite, as there may be an arbitrarily large number of instances of time travel.

Next, in Chapter 5, we discuss whether a finite number of histories could be sufficient to resolve time travel paradoxes, and if so, how many histories are needed and what it means to be able to go back to a previous history. We suggest that if the histories are cyclic, then the Novikov conjecture and the multiple-histories approach both apply simultaneously. We also show that, although consistency paradoxes are resolved, bootstrap paradoxes still exist if the histories are cyclic – but can be avoided by reinterpreting the particle interactions in our model.

Chapter 6 makes experimental predictions that could distinguish the different manifestations of multiple histories from one another, as well as the multiple-histories approach from the Hawking and Novikov conjectures. Finally, in Chapter 7 we discuss our results and suggest avenues for future exploration.

2 Krasnikov’s Paradox Model

2.1 The Deutsch-Politzer Time Machine

An early attempt at formalizing a consistency paradox was proposed by Polchinski: perhaps a billiard ball traversing a wormhole time machine might emerge in the past and collide with its past self, ensuring that it cannot enter the wormhole in the first place. However, Echeverria, Klinkhammer, and Thorne failed to find a paradox in this system, instead finding an infinite set of solutions for many reasonable initial conditions [23]. Furthermore, no paradox was found even after considering more general physical possibilities [25].

A similar construction was attempted using the Deutsch-Politzer (DP) space [22, 26] in [27], and although this construction was shown to be flawed in [28], a modification of this construction known as the twisted Deutsch-Politzer (TDP) space was used in [15] to construct a more compelling paradox [3].

In 1+1 dimensions with coordinates \((t, x)\), the DP space is constructed by associating the line \((1, x)\) with \((-1, x)\) for \(-1 < x < 1\) in Minkowski space. The TDP space is constructed in an analogous way, by instead associating the line \((1, x)\) with \((-1, -x)\) for \(-1 < x < 1\). This means that particles entering the line at \(t = 1\) will emerge at \(t = -1\) “twisted”, that is, with their spatial orientation inverted. The DP and TDP spaces are illustrated in Figures 1 and 2.

In both spacetimes, there must be singularities at \((t, x) = (\pm 1, \pm 1)\), as these points cannot be included without violating the Hausdorff condition [3]. At all other points, the spacetime is flat, and we can use the same coordinates we used in the original Minkowski space [26]. Although this 1+1-dimensional space is non-orientable, the 3+1-dimensional analogue is orientable. In this paper, we will ignore the presence of the singularities for the sake of simplicity, motivated by the fact that traversable wormholes in 3+1 dimensions, for which the DP and TDP spaces are a toy model, do not in general possess singularities.

The causality-violating region, denoted \(J^0(M)\) where \(M\) is the spacetime manifold, is the set of all points \(p\) which are connected to themselves by a closed causal curve. Each such point is in its own future and past. This is depicted for the TDP space in Figure 3.
Figure 1: In the DP space, the line $(1, x)$ is associated with $(-1, x)$ for $-1 < x < 1$ in Minkowski space. This is a simplified model for a wormhole time machine [2]. After traversing the wormhole, the particle emerges at an earlier value of $t$ and travels in the same direction in $x$.

Figure 2: In the TDP space, $(1, x)$ is instead associated with $(-1, -x)$ for $-1 < x < 1$. After emerging from the wormhole, the particle will travel in the opposite direction in $x$. 
2.2 The Model

Krasnikov \cite{15, 3} constructs a paradox in the TDP space by introducing point particles accompanied by a set of physical laws:

1. The particles are massless, and thus follow null geodesics\(^4\).
2. Whenever two particle worldlines intersect, the two particles interact. This interaction can be interpreted as an elastic collision, with each particle flipping its direction of movement. Later we will see that this can lead to bootstrap paradoxes, and suggest a different interpretation, where particles instead go through each other, continuing in the same direction.
3. Each particle has one of two colors\(^5\). In every interaction, each particle flips its color (independently of the color of the other particle), as illustrated in Figure 4.

The first law considerably simplifies the discussion by allowing us to ignore timelike paths, and the second follows the spirit of Polchinski’s paradox. However, these two laws alone still permit consistent solutions analogous to those that have been found for the Polchinski paradox, so the third law is introduced to prevent this\(^6\). These physical laws respect locality and allow consistent evolution.

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\(^4\)Note that this means we should discuss CCCs and not CTCs, although both types exist in the TDP space.

\(^5\)This property is named “flavor” in \cite{15} and “charge” \cite{3}. Here we adopt the name “color” in order to make the visualization clearer, and also to avoid the impression that this quantity is conserved.

\(^6\)Krasnikov also considers that particles appearing from the singular points \((t, x) = (-1, \pm 1)\) may allow for consistent solutions, and introduces a fourth law to prevent this. This law adds another property – named “color” in both \cite{15} and \cite{3}, but not to be confused with the property we call “color” here – such that particles only interact with other particles of the same “color”, and the “color” itself never changes. Having three such “colors” is sufficient for the purpose of preventing consistent solutions, since there are two singularities, so they can produce consistent solutions for at most two of the “colors”. In this paper, we will ignore the singularities for the sake of simplicity, and thus the only property we will need is the one defined in law number 3.
Figure 4: The four possible distinct vertices for particle interactions in Krasnikov’s model. Note how each blue particle changes into a green particle, and vice versa, in every collision.

for all initial conditions in entirely causal spacetimes. Importantly, one must also assume that the particles are all test particles, and do not influence the geometry of spacetime via Einstein’s equation. We can unite all four possible vertices into a more readily generalizable form by enumerating the two colors as $0, 1 \in \mathbb{Z}_2$, so that a particle’s color increases by 1 (mod 2) after each collision. Using these physical laws, both types of paradoxes – consistency and bootstrap – are illustrated in Figure 5.

Figure 5: An illustration of the consistency and bootstrap paradoxes in Krasnikov’s model. The blue and green lines represent the two possible particle colors, as above. The gray lines indicate a particle which cannot be assigned a consistent color.

First, the particle emerging from the time machine (in gray) ends up falling into the time machine again, so it appears out of nowhere and only exists within the CCC – causing a bootstrap paradox. Second, when it collides with the particle coming from the causal region (in blue), both must flip color. For the blue particle, this is not a problem – it simply changes into a green particle. However, if the gray particle was initially blue, then it would have to change into green, but this means it would enter the time machine as a green particle and exit as a blue particle – an inconsistency. Of course, the same inconsistency also applies if the gray particle was initially green. Therefore, there is no choice of color which is consistent along the particle’s entire path – a consistency paradox.

Not every set of initial conditions in this model necessarily causes a paradox. However, even ini-
tial conditions which have consistent solutions still exhibit unusual properties. In Figures 6 and 7, one particle approaching from each side of the causality-violating region leads to a system with two consistent color configurations; thus, the evolution inside the causality-violating region cannot be predicted from the initial conditions. We will argue below that this is a hint that multiple histories may appear in the presence of a causality-violating region even in non-paradoxical scenarios.

3 Generalizing the Model

Let us now generalize Krasnikov’s model in several ways.

3.1 Additional Colors

In the previous chapter we expressed the two possible particle colors as elements of \( Z_2 \) and phrased Krasnikov’s rule for color evolution as an increase by 1 (mod 2) to the color of each particle after a collision. We can use the same rule for an arbitrary number of colors.

Let \( C \in \mathbb{N} \); then particle colors are elements of the cyclic group \( Z_C = \{0, \ldots, C - 1\} \) and follow the same color evolution rule, increasing by 1 (mod \( C \)) after a collision. When \( C = 2 \), we capture the physical system described in the previous chapter. For \( C \geq 2 \), we describe a more general system that will be useful for exploring causality violations in a variety of cases, such as multiple histories. As long as \( C \neq 1 \), a single incoming particle leads to the same paradoxes as in Figure 5 above. When \( C > 2 \), particle interactions are no longer time-reversible. However, our system has not completely lost its symmetry. In particular, if we define color conjugation as mapping a color \( c \) to \(-c\) (mod \( C \)) in \( Z_C \), then CT symmetry is satisfied (with \( C \) representing color, not charge). In fact, our interactions are symmetric under parity transformations, so the system also satisfies CPT symmetry, as depicted in Figure 8. Although the colors in each leg of the resulting vertices will in general be different, the system as a whole is invariant under these symmetries.

3.2 Additional Particles

We also expand the physical system under consideration by considering an arbitrary number of particles. We will explore the general case of \( m \) particles approaching from the \(-x\) direction and \( n \) particles approaching from the \(+x\) direction for \( m, n \in \mathbb{N} \), assuming \( m \leq n \) without loss of generality since our spacetime is symmetric under parity transformations \( x \mapsto -x \). Considering this general case will allow us to not only demonstrate paradoxes, but also prove the absence of paradoxes in certain systems, motivating the utility of the multiple-histories approach.

3.3 Additional Histories

The final and most important generalization is to introduce a parameter, \( h \in H \), where \( H \) is a set of labels (finite, countably infinite, or even uncountably infinite), which indexes independent histories. The exact nature or physical manifestation of these histories is left open; however, we will propose three different approaches here:

1. **Branching universe**: There is one universe, and it branches into two whenever time travel occurs. This is the traditional model which has been discussed in the literature [20, 21], but a concrete and well-defined mathematical model has not yet been found, as the branching violates either the Hausdorff condition or the condition that the manifold should be locally Euclidean [19], which presents difficult challenges. In this case, our parameter \( h \) labels each unique choice of path among the branches.
Figure 6: One of the two consistent solutions obtained by sending particles towards the causality-violating region from both sides.

Figure 7: The second of the two consistent solutions obtained by sending particles towards the causality-violating region from both sides. Note that the initial conditions and final outcomes are the same as in Figure 6 – two blue particles coming in and two green particles coming out – but the evolution inside the causality-violating region is different. Thus, evolution in this region cannot be predicted.
Figure 8: (a) Given the identification between colors and elements of $\mathbb{Z}_C$, this general vertex captures all four vertices of Figure 4 for $C = 2$, as well as those for any other values of $C$. For illustration, the four colors in the figure – blue, green, orange, and magenta – represent any of the $C$ possible colors, for the case $C \geq 4$.

(b) This vertex is the result of reversing time and parity and conjugating color with respect to the vertex in (a). Since each particle still leaves with a color one greater than it starts with, the result is a valid vertex. In fact, performing CT or P transformations independently also yields a valid vertex. In this example we took blue = 0, orange = 1, green = 2, magenta = 3, $c = 0$, $c' = 2$, and $C = 4$ in both (a) and (b).
2. **Parallel universes**: There are many copies – perhaps even an infinite number of copies – of the universe. This differs from the branching model in that the universes need not be the same up to the point where time travel has occurred. Of course, if they are completely different, then “time travel” may not be the right word to describe travel between different universes. Therefore, we should assume that “adjacent” universes (e.g. such that their values of $h$ are within some $\delta > 0$ of each other) are only different “up to $\epsilon$” in some well-defined way. In this case, our parameter $h$ labels each parallel universe.

3. **Parallel histories**: There is only one universe, and the different histories are simply independent possible evolutions of the particles in this universe from the same initial conditions, given the occurrence of different events in the causality-violating region. The different histories evolve in parallel, and particles may only interact with other particles in the same history, allowing different outcomes in each. In this case, our parameter $h$ labels the different histories.

![Figure 9: Three different approaches to multiple histories.](image)

While all three options are very similar, there are also important differences. For example, in the parallel histories approach there is only one universe, and thus only one spacetime geometry. This geometry interacts with matter by virtue of the Einstein equation, and so one must decide how to deal with multiple different configurations of matter at the same place in each history.

This will not be relevant in our discussion, since we are explicitly only dealing with test particles. However, it is interesting to note that the parallel histories approach does not allow one to “turn off” a time machine. The branching universe approach might allow that, since each branch can have a different geometry, while the parallel universes approach definitely allows that, since the universes are independent.

Importantly, these approaches exist independently of time travel. For example, the branching universe approach is intimately related to the Everett (“many-worlds”) interpretation of quantum mechanics, as each measurement would result in a branching. Similarly, the parallel universes approach is compatible with some speculative multiverse theories.

However, these examples are still very different from multiple-histories resolutions to time travel paradoxes, in that it is generally accepted that one cannot travel between the “worlds” of the Everett interpretation or between different universes in a multiverse, while the multiple-histories resolutions to the paradoxes explicitly require that one can travel between histories. Nevertheless, it is entirely possible that the multiple histories have more than one use; indeed, Deutsch’s model, mentioned

\[^7\text{Perhaps we could consider some kind of average, similar to how the semi-classical Einstein equation } G_{\mu\nu} = \langle T_{\mu\nu} \rangle \text{ uses the expectation value of the quantum stress tensor } T_{\mu\nu}.\]
in the introduction, invokes the Everett interpretation for the purpose of resolving time travel paradoxes.

We claim that the existence of a causality-violating region seems to be naturally associated with multiple histories, in two different ways. One, of course, is that multiple histories resolve time travel paradoxes. However, even non-paradoxical scenarios seem to lead to multiple histories. Consider, for examples, the two solutions presented in Figures 6 and 7.

This situation, where the same initial conditions – two blue particles coming in, one from the left and one from the right – lead to two different consistent evolutions, usually does not appear in the absence of CCCs, as classical physics is in general deterministic. How will the universe “decide” which evolution to use? Choosing a specific one would require additional assumptions that would explain what is special about that particular evolution.

We claim that a more elegant resolution to this problem is to posit that both evolutions must exist in parallel. These multiple histories are more akin to the “worlds” of the Everett interpretation than to the histories solving time travel paradoxes, as one cannot travel between different histories. Still, it is remarkable that causality-violating spacetimes seem to give rise to multiple histories in two distinct, but related, ways.

4 The Case of Unlimited Histories

In the previous chapters, we introduced a spacetime and physical laws that admit initial conditions for \( C \geq 2 \) for which there is no consistent evolution, generating a paradox. However, we also introduced the possibility for multiple histories \( h \in \mathcal{H} \). This allows the same association of points in spacetime to be made as in the TDP space described above, but at different values of \( h \). Let us denote a point in spacetime together with a history as \((t, x, h)\). Then for each history \( h \in \mathcal{H} \), we associate the points \((+1, x, h)\) with \((-1, -x, h')\) for all \(-1 < x < 1\) for some corresponding choice of \( h' \in \mathcal{H} \).

Let us also assume that a particle in a given history may never return to the same history after leaving it. In the case of countably infinite histories, with \( \mathcal{H} = \mathbb{N} \), this may be implemented by associating \((+1, x, h)\) with \((-1, -x, h+1)\) for all \( h \). In this case, there is a distinguished first history, \( h = 1 \), which is the only one where nothing comes out of the time machine. In that history, the line at \( t = -1 \) is not associated to anything.

As above, we denote our spacetime manifold as \( \mathcal{M} \) and the causality-violating region as \( J^0(\mathcal{M}) \). In the parallel histories approaches, all possible histories \( h \) – at least in the case under consideration, where there is a single time machine – must be identical anywhere outside the causal future of the causality-violating region, denoted \( J^+ (J^0(\mathcal{M})) \). However, they can certainly differ inside \( J^+ (J^0(\mathcal{M})) \) and on \( J^0(\mathcal{M}) \) itself. This ensures that the effects of the causality-violating region only manifest themselves within its future Cauchy horizon.

In the branching universe approach, not only are the two histories identical up to the point of branching, they are in fact the same history. In the parallel universes approach, the histories of “adjacent” universes are instead constrained to be “almost identical” up to a certain point (in some well-defined way), but they don’t have to be completely identical.

This assumption seems reasonable because it provides observers going into the time machine the appearance of time travel, without any associated paradoxes. If one wanted to go back in time to see the dinosaurs, they can certainly do that, and the world they will arrive at will indeed be the same world from which they left, with the same dinosaurs, up to the moment of arrival. However, as soon as they arrive, they inadvertently change history – by their mere presence, whether they want to or not. Thus, by definition, they must have arrived at a different history \( h \).

This framework allows consistent solutions to our previously paradoxical initial conditions. Particles which would have followed CCCs in one history now traverse multiple histories, and since they may

\[\text{\[29\]}\]
never return to the same history, they can never complete a closed loop. Consistency paradoxes arise
from conditions enforced along closed causal curves, and bootstrap paradoxes arise from particles
existing only inside these closed curves; neither situation is possible in the unlimited histories case,
and thus both paradoxes are avoided. In other words, we have avoided paradoxes created due to
CCCs by simply avoiding any actual CCCs. This is shown in Figure 10.

Figure 10: In the first history, nothing comes out of the time machine, since no particles
have entered the time machine “yet”. When the blue particle enters the time machine at
\( h = 1 \), it comes out twisted (since we are in a TDP space) at \( h = 2 \). The new history has
an identical copy of the initial blue particle, but this time it encounters itself (or more
precisely, its copy from \( h = 1 \)) and the two particles change their colors. A green particle
then enters the time machine, and continues to \( h = 3 \), and so on. Thus we have avoided
both consistency and bootstrap paradoxes.

5 The Case of Cyclic Histories

Above we assumed that \( h \) increases monotonically, so that the time traveler may never return to their
original history. This means that the total number of histories must be effectively (countably) infinite,
since the time traveler may in principle travel as many times as they want. However, we will now
show that, at least within this particular model, it is in fact possible for a time traveler to return to
a previous history. Specifically, we will focus on the case where the histories are cyclic – one can go
from the last history back to the first one.

Unlike the case of unlimited histories, this model does admit CCCs. On the other hand, although
there are only a finite number of histories, and not an infinity of them, there nevertheless exist con-
sistent solutions to otherwise paradoxical scenarios, as illustrated in Figure 11. We will now explore
when exactly this model resolves time travel paradoxes.

5.1 Analyzing Particle Collisions

In this section, we will examine the TDP space in more detail in order to determine the constraints
implied by arbitrary initial conditions. Although initial conditions defined outside \( J^0(M) \) cannot
uniquely determine the physics inside this region – consider the two consistent color configurations
Figure 11: When $C = 2$, the consistency paradox can be solved with two cyclic histories. The blue particle entering the time machine in $h = 1$ comes out of the time machine in $h = 2$, and the green particle entering the time machine in $h = 2$ comes out of the time machine back in $h = 1$. Since we interpret the vertices as elastic collisions, we now have a bootstrap paradox; the particle traveling along the CCC only exists within the CCC itself. We will discuss how to resolve this in Section 5.3.

in Figures 6 and 7 — they do determine the trajectories of all the particles in this region. Thus, the paths of all particles in the spacetime are a set of null lines, where a “path” can include the trajectory of more than one particle if the vertices are elastic collisions. For example, in Figure 11, the blue path in $h = 1$ is considered to be one path whether the original blue particle continues in the same direction or not after the interaction at the vertex; this path then continues to $h = 2$ and exits to infinity.

Null lines in TDP space must be extendible to $t = \pm \infty$. This is certainly true for paths which do not enter $J^0(M)$. As for other paths, they may traverse the wormhole only once. Indeed, suppose a path enters the wormhole at $(t, x) = (1, x_0)$ and exits at $(t, x) = (-1, -x_0)$. Without loss of generality, we assume that the path then moves along the $+x$ direction. Then the path, parametrized by $\lambda \in \mathbb{R}$, will be such that

$$ (t, x) = (\lambda - 1, \lambda - x_0). $$

The path will reach $t = 1$, where the time machine is located, at $\lambda = 2$. However, at this point it will be at $x = 2 - x_0$. The wormhole is located at $x \in (-1, 1)$, so $x_0$ must be in that range, and in particular $x_0 < 1$. Hence, we see that we must have $x > 1$, and the point of intersection with $t = 1$ is outside of the wormhole. Therefore, a path can never intersect the wormhole twice.

We conclude that all null lines entering the time machine must originate at $t = -\infty$ and, upon traversing the wormhole once, must terminate at $t = +\infty$. Consequently, the paths of all particles inside $J^0(M)$ other than those originating in singularities are determined by initial data prior to the region. Since this data is the same for each history by assumption, the trajectories followed in each history must be identical; the only differences between histories will be in other properties of the particles.

Additionally, the color evolution of particles is determined entirely by the choice of $m$ and $n$, the number of particles entering from the left ($-x$) and the right ($+x$) respectively. The initial positions

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9Other than those originating at singular points, as considered by Krasnikov, but as noted above, we will ignore this subtlety here.
of the particles impact the positions of particles in $J^0(\mathcal{M})$, but not the ordering of vertices, which entirely determine how colors change since colors are constant outside of the vertices.

Moreover, the initial colors of the incoming particles do not impact the colors of particles in $J^0(\mathcal{M})$. Since each null line approaching $J^0(\mathcal{M})$ traverses the wormhole and then leaves in the same direction it came from (due to the twist at the wormhole), it is apparent that $m$ particles must leave $J^0(\mathcal{M})$ going in the $-x$ direction and $n$ particles must leave going in the $+x$ direction.

Since we are considering a spacetime with one spatial dimension, the spatial ordering of a set of particles or paths cannot change over time, except when it is flipped passing through the wormhole. Thus, the particles which leave $J^0(\mathcal{M})$ must be the same particles as those which enter it in the first place. Consequently, the remaining particles are confined to $J^0(\mathcal{M})$, and their colors are impacted only by the number of collisions they have with the incoming particles, not by the incoming particles' colors.

Given this structure, we can identify three zones in $J^0(\mathcal{M})$ where there are group collisions, sets of individual particle collisions arising from multiple particles approaching from either direction and scattering off one another. These zones are illustrated in Figure 12.

![Figure 12](image)

Figure 12: $J^0(\mathcal{M})$ can be partitioned into three zones, each of which contains a group collision of particles.

Since all the incoming particles collide such that they scatter away from $J^0(\mathcal{M})$ without ever traversing the wormhole, particles approaching from $-x$ may only participate in collisions in zone I and particles approaching from $+x$ may only participate in collisions in zone II. Thus, $m$ particles leave zone III going in the $-x$ direction, participate in a group collision in zone I, and are scattered back into zone III; similarly, $n$ particles leave zone III going in the $+x$ direction, participate in a group collision in zone II, and are scattered back into zone III. These $m + n$ particles follow CCCs, and thus they impose consistency constraints that must be satisfied to produce a legitimate solution to a given set of initial conditions.

The particles following CCCs collide in a group collision in zone III. However, it’s not initially clear that the structure of this collision is the same as that of those in zones I and II: the group collision is interrupted by a wormhole that flips the spatial ordering of the particles. Nevertheless, we can treat this collision in the same way as the others. This can be visualized by stacking a copy of $J^0(\mathcal{M})$, flipped in $x$, on top of itself, as shown in Figure 13. This construction illustrates that the zone III.
collision takes the same form as the others, and we don’t have to account for the wormhole’s spatial flip until after the collision, when we hope to compare to the particles leaving zone III for group collisions in zones I and II.

For a group collision of \( m \) and \( n \) particles coming from \(-x\) and \(+x\) respectively, there are three such collisions in each history. Without loss of generality, suppose that \( m < n \). Since \( m \) lines cross with \( n \) lines in one of these collisions, \( m \times n \) vertices arise, as illustrated in Figure 14. If we assign each line a number, as in the figure, then it is straightforward to label the vertices with tuples of these numbers. Since the spatial ordering of the particles is constant we can determine the first and last collisions each particle will participate in.

In order to determine how particle colors change in one group collision, let \( x_k \) be the color of the particle number \( k \) counting from the left. We can assign each particle non-unique initial and final vertices corresponding to the first and last collisions they participate in. As particles traverse the group collision between their initial and final vertices, they always travel from some vertex \((a, b)\) to one of two adjacent vertices: \((a + 1, b)\) or \((a, b + 1)\). Thus, if a particle enters the group collision at \((a_i, b_i)\) and leaves at \((a_f, b_f)\), the total number of collisions it participates in is \((a_f - a_i) + (b_f - b_i) + 1\).
Figure 14: A collision of $m$ particles from the left and $n$ particles from the right.
where the +1 accounts for the initial vertex.

Particle number $k$ first collides at $(m - k + 1, 1)$ if $k \leq m$ or at $(1, k - m)$ if $k > m$, and last collides at $(m, k)$ if $k \leq n$ or at $(m + n - k + 1, n)$ if $k > n$. Thus, over the course of a group collision,

$$x_k' = x_k + \begin{cases} 2(k - 1) + 1 & k \leq m, \\ 2m & m < k \leq n, \\ 2(m + n - k) + 1 & k > n. \end{cases}$$

(5.2)

Note the special case where $m = n$, and

$$x_k' = x_k + \begin{cases} 2(k - 1) + 1 & k \leq m, \\ 2(2m - k) + 1 & k > m. \end{cases}$$

(5.3)

### 5.2 Consistency Constraints

Now, let us consider the set of particles following CCCs: the $m + n$ particles which never leave the causality-violating region of the TDP space. We are interested in whether there exists an assignment of colors to these particles that remains consistent after the particles have traversed $f^0(M)$ through $H$ histories. Let $y_k$ be the color of particle number $k$, counting from the left among those following CCCs, after the zone III collision has taken place but before entering zones I or II. We denote by $(p, q)$ a particle collision with $p$ particles coming from one side and $q$ from the other.

We first determine how these values of $k$ relate to those used in Eq. (5.2) and (5.3). For $k \leq m$, particle $y_k$ corresponds to particle $x_{m+k}$ in a $(m, m)$ particle collision in zone I, and then to particle $x_k$ in an $(m, n)$ particle collision in zone III. For $k > m$, particle $y_k$ corresponds to particle $x_{k-m}$ in a $(n, n)$ particle collision in zone II, and then to particle $x_k$ in an $(m, n)$ particle collision in zone III. Thus, evolving through $f^0(M)$ in one history,

$$y_k' = y_k + \begin{cases} 2(2m - (m + k)) + 1 & k \leq m, \\ 2((k - m) - 1) + 1 & m < k \leq n, \\ 2((k - m) - 1) + 1 & k > n, \end{cases} + \begin{cases} 2(k - 1) + 1 & k \leq m, \\ 2m & m < k \leq n, \\ 2(m + n - k) + 1 & k > n. \end{cases}$$

Zone I and II collisions

Zone III collision

$$y_k' = y_k + \begin{cases} 2m & k \leq m, \\ 2k - 1 & m < k \leq n, \\ 2n & k > n. \end{cases}$$

After passing through the wormhole, the spatial ordering of the particles is reversed. Thus, after traversing an odd number of histories, $y_k'$ must equal $y_{m+n-k}$ for consistency; after $H$ traversals, where $H$ is odd, the consistency constraint is:

$$y_{m+n-k} \equiv \begin{pmatrix} 2(k - 1) + 1 & k \leq m, \\ 2m & m < k \leq n \\ (H + 1)m + (H - 1)n & k > n. \end{pmatrix} \mod C,$$

(5.4)

where $C$ is the number of colors. For even $H$, the consistency constraint is:

$$y_k \equiv y_k + H(m + n) \mod C.$$

(5.5)
Note that both of these consistency requirements are in the group $Z_C$. We first consider the former constraint, for odd $H$. When $k \neq \frac{m+n+1}{2}$ (that is, for all particles except the middle particle when $m+n$ is odd) it implies that $2H(m+n) \equiv 0 \mod C$, and is satisfied for all $m+n$ if and only if $C \mid 2H$ (i.e. $C$ divides $2H$, or $2H$ is a multiple of $C$). However, when $m+n$ is odd, the $k = \frac{m+n+1}{2}$ condition implies that $H(m+n) \equiv 0$, and is satisfied for all $m+n$ if and only if $C \mid H$. The requirement for even $H$ is satisfied if and only if $H(m+n) \equiv 0 \mod C$; since all values of $m+n$ are possible, this requires that $C \mid H$.

Thus, in general, $C \mid H$ is equivalent to the non-existence of paradoxes for that system, since we can find a consistent solution for the particle colors. However, if $m+n$ is even, only $C \mid 2H$ is required. In either case, we can determine the number of free color variables, and consequently the number of distinct consistent solutions. In particular, for even $H$ each particle color is independent and there are $C^{m+n}$ possible solutions, whereas for odd $H$ most equations are coupled, giving $C^{\lceil \frac{m+n}{2} \rceil}$ solutions.

5.3 Avoiding Bootstrap Paradoxes

Although we have found conditions in which consistency paradoxes can be avoided using a finite number of histories, these solutions still have bootstrap paradoxes: the particles entering the causality-violating region are the same ones leaving it, and the remaining particles travel on CCCs through this region. However, the presence of bootstrap paradoxes depends on our interpretation of the vertices as elastic collisions.

In [15] it is suggested that the two-color system can instead be interpreted as intersections of penetrable particles which flip colors when they cross each other’s paths. We can extend this interpretation to our system, where it involves a more complicated interaction: particles pass each other, adopting the other particle’s color plus 1. This re-interpretation is sufficient to remove the possibility for bootstrap paradoxes, as depicted in Figure 15, where we represented one particle with a solid path and the other with a dashed path.

Figure 15: In this illustration, with $C = 2$ and $H = 2$, we see a scenario in which the particles do not collide; instead, they pass through each other, thus avoiding a bootstrap paradox. However, the same vertices in Figure 4 still apply. One particle is solid, while the other is dashed.
Note how the solid particle starts blue in \( h = 1 \) and passes through the dashed particle (which came out of the time machine), interacting using vertex (c) in Figure 4 – which now means, instead of the particles changing both their directions and colors, they change neither! It then goes through the time machine and exits in \( h = 2 \), where it again passes through the dashed particle, now interacting using vertex (a) of Figure 4. It changes its color to green, and goes out to infinity. The dashed particle follows a similar path.

Neither of the particles actually follows a CCC, and both of them have a clear start and end outside of the causality-violating region: the solid particle enters from the right in \( h = 1 \) and exits to the right in \( h = 2 \), while the dashed particle enters from the right in \( h = 2 \) and exits to the right in \( h = 1 \). Thus, we avoid a bootstrap paradox.

### 5.4 Revisiting Previous Histories and the Novikov Conjecture

In this chapter we have discovered that, assuming the number of colors \( C \) is finite, it is sufficient to have \( C \) different cyclic histories in order to resolve every possible paradox, both consistency and bootstrap. In other words, contrary to popular opinion, one does not need to prevent going back to previously visited histories in order to avoid paradoxes.

Note that, since we allowed going back to the very first history, the time machine will always emit particles from the future as soon as it is created, which is what one would expect if the Novikov conjecture is true, but not from a traditional multiple-histories scenario, where the first history should, by definition, be the one where no one has “yet” traveled back in time.

The scenario where travel to the first history is possible will thus, in fact, be a combination of multiple histories and the Novikov self-consistency conjecture. Indeed, in the Novikov conjecture, since there is only one history, when we open the time machine at \( t = -1 \), particles must come out since they went (will go) into the time machine at \( t = +1 \). This is similar to how, in the traditional Novikov conjecture scenario, a time traveler who goes back in time to kill themselves has, in fact, already gone back and already failed. There is no history where the time traveler did not go back in time “yet”, since there is only one history.

To illustrate this more precisely, consider a scenario where Alice wants to travel back in time from 2020 to 1950 and kill her grandfather, Bob, before he met her grandmother. Let us first assume that the Novikov conjecture is correct, but there is only one history. Then in this one history, in chronological order, Bob is born in 1930, Alice emerges from a time machine in 1950 and tries to kill Bob – but fails, Alice is born in 1990, and Alice goes into a time machine in 2020. This is a completely consistent chain of events, and there is no “other” universe or history where Alice did not travel back to 1950.

Next, let us assume that there are multiple histories, and they are cyclic all the way back to the first history. Then, again, there is a completely self-consistent chain of events – however, now it encompasses more than one history. We will denote the year 2020 in history A as 2020A, and so on, and similarly denote Alice from history A as Alice A, and so on.

In history A, Bob A is born in 1930A, Alice B emerges from a time machine in 1950A and tries to kill Bob A by releasing a crocodile – but fails, Alice A is born in 1990A, Bob A tells Alice A in 2010A a story about a woman who looked remarkably like an older version of her who tried to kill him back in 1950A by releasing a crocodile, and Alice A goes into a time machine in 2020A determined to kill her grandfather by another, more efficient method.

In history B, Bob B is born in 1930B, Alice A emerges from a time machine in 1950B and tries to kill Bob B by dropping a piano on him – but fails, Alice B is born in 1990B, Bob B tells Alice B in 2010B a story about a woman who looked remarkably like an older version of her who tried to kill him back in 1950B by dropping a piano on him, and Alice B goes into a time machine in 2020B determined to kill her grandfather by another, more efficient method.

This is a Novikov-like scenario, but with two distinct histories which are not self-consistent individually, since the murder attempts in each history are different; when Alice B tries to kill Bob A by
releasing a crocodile, she is deliberately doing something that she knows not to be consistent with her own history (B), as she is trying to change history. Although she does manage to change history (into history A), Novikov’s conjecture still conspires to prevent her from changing it in an inconsistent way; the combination of histories A and B together represents a completely self-consistent chain of events, spanning two distinct histories.\footnote{Furthermore, the traditional Novikov scenario, with only one history, does not leave any room for “free will”, since Alice cannot make any choice that will change the past; if Alice already knows how her future self attempted to kill Bob in the past, then she will simply not be able to choose to try killing him in another way. However, in the hybrid Novikov-multiple-histories scenario, Alice does, in fact, have the capacity to change history – as Alice A and B did in the example above. If she ever succeeds, then the chain of histories is simply terminated; however, it is also possible that she fails every single time, in which case the histories can be cyclic. Thus this scenario provides at least the illusion of “free will”.
}

6 Experimental Predictions

Let us now consider a scenario where there is an unlimited number of non-cyclic histories, as discussed in Chapter 4, and Novikov’s conjecture does not apply. Imagine that Alice – let’s call her Alice 1 – builds the time machine described by the TDP space. Then when she turns it on at $t = -1$, she does not see herself come out of the time machine – because she has not “yet” entered it.

When she enters the time machine at $t = +1$, she finds herself at $t = -1$, staring at a younger copy of herself. Indeed, this must be a copy of Alice, let’s call it Alice 2, since Alice 1 did not witness herself coming out of the time machine at $t = -1$. If Alice 1 now prevents Alice 2 from entering the time machine, no paradoxes occur, and the chain of histories is terminated.

As for Alice 2, she builds the time machine, but against her expectation, even though she has not “yet” entered the time machine, she suddenly finds a copy of herself – Alice 1 – exiting the machine at $t = -1$. Alice 2 then concludes that she, in fact, belongs to an alternate history, and not the original history, even though both histories share the same past before the time machine was turned on.

Both Alices recount their individual experiences, and they deduce that the unlimited-multiple(histories) approach must, in fact, be the correct one, since otherwise, Alice 1 would have seen herself coming out of the time machine as soon as it was turned on.

Now consider Bob, who was watching Alice as she was testing her time machine. Bob must also split into two copies. Bob 1, in the original history, sees Alice 1 enter the time machine, and then never sees her again. Bob 2, on the other hand, sees Alice 1 exit the time machine as soon as Alice 2 turns it on.\footnote{If, furthermore, Alice 1 prevents Alice 2 from entering the time machine, then this history contains two Alices after $t = 1$, while the original history contains zero Alices after $t = 1$. Thus there is a “conservation of Alices” if all of the histories together are taken into account.}

Bob 1 and Bob 2 independently deduce that the unlimited-multiple-histories approach must be the correct one.

The unlimited-multiple-histories approach therefore has observable consequences which differ from the Hawking and Novikov conjectures. Indeed, in Hawking’s case it would be impossible to build the time machine in the first place, so this is trivially a very different outcome. In Novikov’s case, since there is only one history, there is just one copy of Alice, so it is impossible for Alice 1 and Alice 2 to see different things, as we described above. Hence, as soon as the two Alices compare their observations, they can easily distinguish between the multiple-histories and Novikov models.

Furthermore, it should in principle be possible to distinguish between the three different manifestations of multiple histories listed in Section 3.3. Both Alices can compare their histories up to $t = -1$ and check if they are exactly the same. If there are small differences between their pasts, they conclude that they are probably in a parallel universes scenario. If their histories are exactly the same, then the scenario can either be a branching universe or parallel histories. The Alices can then try

\footnote{Perhaps in a way that conspires into “creating” Alice 1 at the time machine at $h = 2$ from some earlier set of initial conditions which did not exist at $h = 1$.}
to turn off the time machine; if they fail to do so, then they are likely in the parallel histories approach, where the geometry of spacetime is fixed in each history. Otherwise, they are most likely in a branching universe.13

7 Discussion and Future Plans

In this paper, we introduced a 1+1-dimensional model for a spacetime with a time machine and multiple histories, and showed how time travel paradoxes within this model are inevitable unless one allows for sufficiently many histories. An infinite number of histories is certainly sufficient; however, we also showed that a finite number of cyclic histories is sufficient within our particular model, producing a variation of Novikov’s conjecture which spans multiple histories. The latter is a more compact resolution, which requires less extreme assumptions than just Novikov or multiple histories alone, in line with the principle of Occam’s razor. In addition, we suggested three different possible manifestations of multiple histories, and discussed how to experimentally differentiate between them, as well as between multiple histories and the Hawking and Novikov conjectures.

There are several important issues we did not discuss here, including:

- We did not provide an actual physical mechanism for creating new histories; we merely assumed them, as is usually done in the literature.
- We ignored subtle mathematical issues which proved too complicated for the scope of this paper. Chief among them is the question of how different histories connect to one another in terms of the topology and geometry of spacetime. We expect to have a different answer to this question for each of the suggested physical manifestation of multiple histories.
- We asserted that, in the TDP space, a time traveler moves from one history to another while traversing the wormhole. However, we did not develop a prescription for determining at which point along a closed timelike or causal curve this transition between histories happens. This question is of particular concern in the case of more “realistic” time travel models, such as those using warp drives and less trivial wormholes. As time travel in this case involves traversing a nonzero distance, it is unclear where exactly along this journey the new history should be created. This problem becomes even more complicated when one considers that closed curves, by definition, do not have a beginning or end.

We hope to address these issues in future work. Other intriguing avenues of future research include generalizing our model in different ways, such as:

- Formulating the model in 2+1 and 3+1 spacetime dimensions,
- Employing realistic physical laws, ideally given by a well-defined Lagrangian,
- Allowing particles to travel along timelike paths in addition to null paths,
- Allowing additional time machines,
- Allowing time machines to be turned on and off,
- Incorporating quantum mechanics or quantum field theory.

If we also consider the cyclic multiple histories scenario, things get more complicated. Since now there is not necessarily a first history, two Alices alone cannot tell if they are in the middle of a finite cyclic chain of histories, or merely somewhere along an infinite chain of histories. We leave the deliberation of this problem for future work.

13If we also consider the cyclic multiple histories scenario, things get more complicated. Since now there is not necessarily a first history, two Alices alone cannot tell if they are in the middle of a finite cyclic chain of histories, or merely somewhere along an infinite chain of histories. We leave the deliberation of this problem for future work.
Multiple histories are, in our opinion, the most compelling of the existing approaches for resolving time travel paradox. Hawking’s conjecture simply prevents time travel from happening in the first place, while Novikov’s conjecture allows time travel, but in an extremely limited way, where the past cannot be changed and the time traveler cannot exercise their “free will”. If either conjecture is true, it would make life much less interesting.

In contrast, the multiple histories approach allows changing the past and at least the illusion of “free will”, thus making the universe considerably more exciting. In addition, it challenges many fundamental notions in mathematics, physics, and philosophy, and opens up stimulating new avenues of research. Yet, there is surprisingly little literature about it.

We hope that this paper will inspire mathematicians, physicists, and philosophers to work on the formulation of a consistent and well-defined framework for physics with multiple histories, both in relation to time travel paradoxes and in other contexts, such as the Everett interpretation of quantum mechanics.

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