On the (Non-)Applicability of a Small Model Theorem to Model Checking STMs*

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Abstract. Software Transactional Memory (STM) algorithms provide programmers with a synchronisation mechanism for concurrent access to shared variables. Basically, programmers can specify transactions (reading from and writing to shared state) which execute “seemingly” atomic. This property is captured in a correctness criterion called opacity. For model checking opacity of an STM algorithm, we – in principle – need to check opacity for all possible combinations of transactions writing to and reading from potentially unboundedly many variables.

To still apply automatic model checking techniques to opacity checking, a so called small model theorem has been proven which states that model checking on two variables and two transactions is sufficient for correctness verification of STMs. In this paper, we take a fresh look at this small model theorem and investigate its applicability to opacity checking of STM algorithms.

1 Introduction

Today, multi-core processors are widely utilized since their usage yields a large increase in computing power. This additional computing power can best be employed in concurrent programs. When writing programs with concurrent threads accessing shared state, programmers – however – have to provide appropriate synchronisation among threads to avoid access to inconsistent memory values.

Software Transactional Memory (STM) (as proposed by Shavit and Touitou [20]) aims at providing programmers with an easily usable synchronisation technique for such an access to shared state.

STMs allow programmers to define software transactions, much alike database transactions [18]. A transaction consists of a number of read and write operations to the shared state, and the STM algorithm should guarantee these operations to take place “seemingly atomically”, while ideally also allowing transactions to run concurrently. This seeming atomicity is formalized in a correctness criterion called opacity [10]. STMs typically try to avoid strict locking schemes in order to allow for good performance. This often comes at the price of complexity in verification as the high degree of concurrency leads to intricate interleavings.

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Moreover, opacity verification is a parameterized verification problem: STMs have to be proven correct for any number of transactions operating on any number of variables with moreover an unbounded number of possible values written to variables, i.e., for infinitely many possible instantiations.

A number of approaches have so far studied verification of STMs. The proposed techniques range from model checking approaches of fixed instantiations over techniques employing data independence arguments to reduce the number of instantiation to look at to interactive proofs. Interactive approaches typically show refinement between the STM algorithm and an abstraction called TMS2 which is known to be opaque. Lesani furthermore developed a specific (also non-automatic) proof method for opacity by splitting opacity into a number of other conditions (called markability). In addition, Lesani and Palsberg have proposed conditions for disproving opacity. A survey of verification approaches for STMs can be found in.

Interactive proofs provide results for the parameterized verification problem of a specific STM but are often very laborious, requiring several weeks of work in particular for defining invariants. Hence, it seems to be attractive to employ automatic model checking for all instantiations at once as developed by Guerraoui, Henzinger and Singh (in the following referred to as the GHS approach). This approach is based on a small model theorem allowing to reduce the parameterized verification problem to a model checking problem over 2 transactions and 2 variables. GHS applied their technique to a number of STMs, including DSTM and TL2. Abdulla et al. have furthermore employed the same technique for verification of a hybrid TM. Our interest was thus in the applicability of this small model theorem to STMs like the pessimistic STM in or like FastLane, both of which have been interactively verified. This paper reports about the outcome of this investigation: We re-investigate the applicability of the small model theorem of GHS to opacity checking of software transactional memory algorithms.

2 Background

We start by explaining software transactional memory algorithms and the property of interest, opacity. In this, we follow GHS, not the standard definition, because their small model theorem is given for their own, specific version of opacity. Later, we will comment on differences to the standard definition.

2.1 Basics

Software transactional memory algorithms allow for concurrent access to shared state. The locations to be accessed by the STM are a set \( \text{Var} \) of variables. Programmers can use commands \( C = \{\text{cmt}\} \cup \{\text{rd}, \text{wr}\} \times \text{Var} \) (commit, read, write) to interact with STMs (plus typically an operation \( \text{begin} \) which is however not formalized by GHS). The STM algorithm might respond to these commands by aborting a transaction, thus we let \( \hat{C} = C \cup \{\text{abrt}\} \). We use \( T = \{1, \ldots, n\} \).
as the set of transaction or thread identifiers and let $S = C \times T$, $\hat{S} = \hat{C} \times T$.

We write such statements $((c, v), t)$ as $c_t(v)$ as e.g. in $\text{rd}_t(x)$ stating that thread $t$ reads from variable $x$. Note that Guerraoui et al. do not consider the values (of variables) as passed as parameters to writes or returned as outputs of reads.

Opacity is defined by looking at the histories an STM algorithm produces. A history is a word $h \in \hat{S}^*$, i.e., a sequence of statements. The projection of a history $h$ on a transaction $t$, $h\rvert_t$, consists of the statements in $h$ for transaction $t$ only. We assume transaction identifiers to be unique. In a projection $h\rvert_t = s_0 \ldots s_m$, either $s_m$ is an abort or commit statement or these statements do not occur in $h\rvert_t$ at all. In the latter case, the transaction is live in $h$, in the former it is finished. If the last statement is $\text{cmt}$, the transaction is committing; if it is $\text{abrt}$, the transaction is aborting.

For two transactions $t_1, t_2$, we say that $t_1$ precedes $t_2$ in $h$, $t_1 <_h t_2$, if the last statement of $t_1$ occurs before the first statement of $t_2$. If neither $t_1 <_h t_2$ nor $t_2 <_h t_1$, then transactions $t_1$ and $t_2$ are concurrent in the history $h$. A history is sequential if no transactions are concurrent.

A transaction $t$ writes to a variable $x$ in a history $h$ if $h$ contains a statement $\text{wr}_t(x)$. A statement $s = \text{rd}_t(x)$ in $h$ is a global read of variable $x$ if there is no $\text{wr}_t(x)$ before $s$ in $h$, i.e., $t$ does not read from its own write. As an example, consider the sequential history

$$h_1 = \text{wr}_1(x)\text{cmt}_1\text{rd}_2(x)\text{cmt}_2$$

consisting of transactions 1 and 2. Both are committing transactions, 1 precedes 2; transaction 2 has a global read of variable $x$, transaction 1 writes to $x$.

Note that operations in histories are often split into invocations and responses in other formalizations of opacity, but in this we again stick to the formalization of GHS.

### 2.2 Opacity

STM algorithms can broadly be categorized as using direct or deferred updates. In a direct update algorithm, the variables in shared state are directly updated during write operations; in deferred update algorithms the actual update takes place during commit operations. For the standard definition of opacity as given in [10] this differentiation does not matter: the same definition of opacity is applicable to direct and deferred update algorithms. The definition of GHS is however based on conflicts between statements which allows to ignore the actual values of variables – at the price of needing to introduce a definition of opacity differing from the original one.

A statement $s_1$ of transaction $t_1$ is in conflict with a statement $s_2$ in $t_2$, $t_1 \neq t_2$, in a history $h$ if (i) $s_1$ is a global read of some variable $x$, $s_2$ is a $\text{cmt}$ statement and $t_2$ writes to $x$, or (ii) both $s_1$ and $s_2$ are $\text{cmt}$ statements and $t_1$…

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1 Guerraoui et al. distinguish between transactions and threads, but for reasons of simplicity we have refrained from doing so here.
and \(t_2\) both write to some variable \(x\). Opacity requires “seeming atomicity” of transactions which is formalized by the existence of a sequential history reordering the concurrent one, however by keeping the real-time order and the order of conflicting statements. The following gives a definition of opacity for STMs with deferred update.

**Definition 1.** A history \(h\) is opaque if there is a sequential history \(h'\) such that \(h\) and \(h'\) are strictly equivalent, i.e.,

- for all transactions \(t\): \(h|_t = h'|_t\),
- for all transactions \(t_1, t_2\), if \(t_1 <_h t_2\) and the last statement of \(t_1\) is a commit or abort, then \(t_1 <_{h'} t_2\) and
- for every pair of statements \(s_i, s_j\) in \(h\), if \(s_i\) is in conflict with \(s_j\) and \(i < j\), then \(s_i\) occurs before \(s_j\) in \(h'\).

We refer to such a definition as a conflict-based definition of opacity. As GHS say, a corresponding definition can be given for STMs with direct update, but their paper does not contain it and also does not provide a small model theorem for STMs with direct update.

As an example, consider again history \(h_1 = \text{wr}_1(x)\text{cmt}_1\text{rd}_2(x)\text{cmt}_2\). In \(h_1\), statement \(\text{rd}_2(x)\) and \(\text{cmt}_1\) conflict. The sequential history witnessing opacity of \(h_1\) is \(h_1\) itself. As an example of a non-opaque history consider \(h_2\):

\[
h_2 = \text{wr}_1(x)\text{wr}_1(y)\text{rd}_2(x)\text{cmt}_1\text{wr}_2(y)\text{cmt}_2
\]

Here, \(\text{rd}_2(x)\) and \(\text{cmt}_1\) conflict as well as \(\text{cmt}_1\) and \(\text{cmt}_2\). Thus a sequential history strictly equivalent to \(h_2\) would need to order transaction 1 before 2 and 2 before 1 which is impossible.

### 3 An Example STM: DSTM

As an example STM, we use the DSTM algorithm (Dynamic STM) of Herlihy et al. [11] in the version given by Lesani and Palsberg [16]. This version is called CoreDSTM.

Though not formalized in the histories of conflict-based opacity definitions, STM algorithms write values into variables and read operations need to return such values. We let \(\text{Val}\) be the set of values for variables. CoreDSTM employs the following so called meta data to ensure opacity:

- \(\text{status} : T \rightarrow \{C, A, R\}\) (the status of every transaction),
- \(\text{rdSet} : T \rightarrow 2^{\text{Var} \times \text{Val}}\) (the variables read by a transaction), and
- \(\text{state} : \text{Var} \rightarrow T \times \text{Val} \times \text{Val}\) (last writer with old and new value).

Initially, \(\text{rdSet} = \lambda t.\emptyset\), \(\text{status} = \lambda t.C\) and \(\text{state} = \lambda x.(t_0, 0, 0)\) where \(t_0\) is some dedicated transaction initially setting all variable values to 0. The component \(\text{state}\) stores the last transaction having written to a variable as well as the old and new value. We access the three components of a state \(st\) by \(st.\text{writer} \in T\),
$st. newVal \in Val$ and $st. oldVal \in Val$. The current status of transactions ($C = \text{committed}$, $A = \text{aborted}$ and $R = \text{running}$) is recorded in $status$. The STM furthermore tracks the set of variables read by transactions together with the value read.

Algorithm 1 gives the code for read, write and commit operations. The operation CAS used therein has the following meaning: in a statement $\text{var.CAS}(o,n)$ the value of $\text{var}$ is compared to $o$ (old) and – if equal – is set to $n$ (new). This compare-and-set is done in one atomic step. The CAS operation returns the result of the comparison, i.e., a boolean operation. We see that the write operation first of all stores the value to be written in $newVal$ (line 8 within write). Procedures $\text{validate}$ and $\text{stableValue}$ only retrieve the new value if the writing transaction has committed. Hence, this is a deferred update algorithm.

As observed by Lesani and Palsberg, CoreDSTM is not opaque. The actual implementation however seems to differ from this version and is opaque. For demonstration purposes here it makes sense to look at the non-opaque version.

The non-opacity of CoreDSTM can be seen in the following execution: Transactions 1 and 2 first both read from variables $x$ and $y$ (the initial value 0).

Afterwards transaction 1 writes to $x$ (say, value 7) and transaction 2 to $y$ (say, value 8). Then they commit concurrently: first, transaction 1 executes lines 15 and 16 of commit, then transaction 2 does so (both having their local variable $valid$ being true afterwards), and after that they successfully end their commit operation. As a history, this gives

$$h = \text{rd}_1(x)\text{rd}_2(x)\text{rd}_1(y)\text{rd}_2(y)\text{wr}_1(x)\text{wr}_2(y)\text{cmt}_1\text{cmt}_2 \quad (1)$$

This history is not opaque as $\text{rd}_2(x)$ and $\text{cmt}_1$ as well as $\text{rd}_1(y)$ and $\text{cmt}_2$ are in conflict, so the required ordering for the sequential history is that 2 has to precede 1 and 1 has to precede 2 which cannot be fulfilled at the same time.

4 The Small Model Theorem

GHS aim at an automatic way of checking opacity (as well as strict serializability). To this end, they develop four properties of STMs which are sufficient for reducing the general verification problem to a model checking problem over 2 variables and 2 transactions. We (informally) introduce these four properties here, and study one of them in more detail later.

The properties refer to the executions of a particular STM algorithm $M$ as seen in its histories.

**P1** Transaction projection: Let $h$ be a history of an STM $M$ and $T' \subseteq T$ the set of all committed plus some of the live transactions of $h$. Then $h|_{T'}$ is a history of $M$ as well.
Algorithm 1 CoreDSTM

1: procedure READ_t(x)
2:   s := status(t);
3:   if (s = A) then
4:       return A;
5:   st := state(x);
6:   v := stableValue_t(st);
7:   wr := st.writer;
8:   if (wr ≠ t) then
9:       rdSet(t).add((x,v));
10:  valid := validate_t();
11:  if (¬valid) then
12:      return A;
13:  return v;

14: procedure COMMIT_t
15:   valid := validate_t();
16:   if (¬valid) then
17:      return A;
18:   b := status(t).CAS(R,C);
19:   if (b) then
20:      return C;
21:   else
22:      return A;

23: procedure stableValue_t(st)
24:   t’ := st.writer;
25:   s’ := status(t’);
26:   if (t’ ≠ t ∧ s’ = R) then
27:      status(t’).CAS(R,A);
28:   s’ := status(t’);
29:   if (s’ = A) then
30:      v := st.oldVal;
31:   else
32:      v := st.newVal;
33:   return v;

1: procedure WRITE_t(x,v)
2:   s := status(t);
3:   if (s = A) then
4:      return A;
5:   st := state(x);
6:   wr := st.writer;
7:   if (wr = t) then
8:      st.newVal := v;
9:   return ok;
10:  v’ := stableValue_t(st);
11:  st’ := (t,v’,v);
12:  b := state(x).CAS(st, st’);
13:  if (b) then
14:     return ok;
15:  else
16:     return A;

17: procedure validate_t
18:   for all ((x,v) ∈ rdSet(t)) do
19:     st := state(x);
20:     t’ := st.writer;
21:     s’ := status(t’);
22:     if (s’ = C) then
23:        v’ := st.newVal;
24:     else
25:        v’ := st.oldVal;
26:     if (v ≠ v’) then
27:        return false;
28:     s := status(t);
29:     return (s = R);
Thread symmetry: Plays no role in our formalization as we do not distinguish between threads and transactions.

Variable projection: Let $h$ be a history of an STM $M$ without aborting transactions, and let $V \subseteq \text{Var}$. Then $h|_{V}$ is a history of $M$ as well (where the projection of $h$ onto some set of variables removes all reads and writes to other variables).

Monotonicity\(^4\): Let $h \cdot s$ be a history of an STM such that $h$ (a history) is opaque, $s$ (a single statement) is not an abort statement, $h$ has exactly one live transaction and $s$ is a statement of this transaction. Then there is some $h'$ which is strictly equivalent to $h$ and sequential, and $h' \cdot s$ is a history of $M$.

Here, $\cdot$ is concatenation. We will see below that when evaluated for concrete STMs property P4 is subject to interpretation. Intuitively, P4 states that whenever a history is allowed by an STM, then more sequential versions of the history are allowed as well.

These four properties allow to reduce opacity checking of STMs to 2 transactions and 2 variables. An STM is said to be $(n,k)$-opaque if all histories with $n$ transactions (i.e., at most $n$ concurrent transactions) and $k$ variables are opaque.

**Theorem 1.** If a TM $M$ ensures $(2,2)$-opacity and satisfies the properties P1, P2, P3 and P4 for opacity, then $M$ ensures opacity.

For the proof, see [21]. The proof proceeds by constructing for every non-opaque history of $M$ another non-opaque history with just 2 variables and 2 transactions. Properties P1 to P4 ensure that this new history is still possible for $M$. Thus every violation of opacity can be seen in histories with 2 transactions and 2 variables. Hence automatic model checking of STMs is possible by inspecting instantiations with 2 transactions and 2 variables only.

## 5 Applicability

There are a number of issues making this small model theorem and its associated automatic model checking procedure difficult to apply to concrete STM algorithms.

**Issue 1** It is unclear how to automatically show properties P1, P2, P3 and P4 for some concrete STM algorithm.

Given that the technique is supposed to make opacity model checkable, this is a realistic difficulty. The PhD thesis of Singh [21] employs three other conditions (abort isolation, pending isolation and conflict commutativity) to guarantee P1 and P4. We exemplify Issue 1 on one of them, namely abort isolation.

\(^4\) This is the version of property P4 for monotonicity taken from the PhD thesis of one of the authors [21] to align it with the discussion on further conditions in [21].
Definition 2. A TM algorithm is abort isolated if for every history $h$ and every aborted transaction $t$ in $h$, the following holds: if an instruction of $t$ changes the value of a global variable $g$ and a transaction $t'$ observes the value of $g$ before $t$ aborts, then $t'$ aborts in the step of observing $g$.

Abort isolation together with a similar condition for live transactions guarantees property P1 [21]. Singh writes that DSTM is abort isolated because “an aborted transaction does not change the state in DSTM”. An automatic way of showing abort isolation is not proposed. However, CoreDSTM (as well as DSTM) can produce histories of the following form: First, a transaction $t$ writes to a variable (thus setting the writer of this variable to $t$ thereby changing the value of a global variable). Afterwards a further transaction $t'$ writing to the same variable would see $t$ as writer and abort $t$. Transaction $t'$ might successfully commit later. So DSTM is not abort isolated.

Issue 2 The opacity definition (and hence the theorem) refers to STMs with deferred update only.

It is not clear whether such a reduction theorem also holds for STMs with direct update and what the exact formulation of properties P1 to P4 would be in that case. Moreover, not all STMs strictly fall in one or the other category. An example for this is the STM FastLane [22] which provides two different modes for transactions: one master transaction uses direct update while all helper transactions employ deferred update. FastLane has been shown to be opaque using interactive theorem proving [19].

The next issue refers to the definition of opacity. The standard reference for the definition of opacity (as also given by GHS) is that of Guerraoui and Kapalka [10]. There are two key differences between the definition given there and the one employed for the small model theorem:

1. Transactions operate on shared variables and these possess a state, i.e., there are values associated with variables and these values appear in histories as arguments or return values of write and read operations,
2. operations are divided into invocations and responses (i.e., instead of a $\text{cmt}$ operation there are operations $\text{inv}(\text{cmt}, \ldots)$ and $\text{res}(\text{cmt}, \ldots)$).

As a consequence of the first difference, opacity can and is then defined by looking at the values returned by reads (instead of by looking at conflicts) and by defining when these values are legal (namely when the last committing writer before a read has written this value). As a consequence of the second difference, histories can then directly describe interleavings of transactional operations (e.g., a commit of one transaction occurring concurrently with a commit of another transaction). Both of these differences have consequence for the applicability of the small model theorem to model checking opacity. The following observation has already been made by us before [12].

Issue 3 The value-based definition of opacity is not the same as the conflict-based definition.
These two notions are in fact incomparable. The following two histories show that a value-based and a conflict-based definition of opacity do not coincide. For this, we extend the operations write and read with arguments and return values, respectively. That is, an operation \( \text{wr}_1(x, 7) \) is a write of transaction 1 on shared variable \( x \) with value 7.

\[
\begin{align*}
  h_3 &= \text{wr}_1(x, 7) \text{cmt}_1 \text{rd}_2(x, 3) \text{cmt}_2 \\
  h_4 &= \text{wr}_1(x, 5) \text{wr}_2(x, 5) \text{wr}_1(y, 42) \text{wr}_2(y, 43) \text{cmt}_1 \text{rd}_3(x, 5) \text{cmt}_2 \text{rd}_3(y, 43) \text{cmt}_3
\end{align*}
\]

History \( h_3 \) is opaque under the conflict-based definition: statements \( \text{rd}_2(x, 3) \) and \( \text{cmt}_1 \) are in conflict, thus transactions 1 and 2 need to be ordered as \( 1 < 2 \) in the sequential history which is possible without violating other constraints on orderings. In a value-based definition of opacity \( h_3 \) is clearly not opaque since transaction 2 is reading an incorrect value from variable \( x \).

On the other hand, under a value-based version history \( h_4 \) is opaque (as justified by the sequential order \( 1 < 2 < 3 \)). For the conflicts, we however get constraint \( 2 < 3 \) (since \( \text{cmt}_2 \) and \( \text{rd}_3(y, 43) \) are in conflict) as well \( 3 < 2 \) (since \( \text{rd}_3(x, 5) \) and \( \text{cmt}_2 \) are in conflict) which cannot both be satisfied by a sequential history.

As a follow-up of moving to a conflict-based definition, GHS had to change the STM algorithms they employ as examples (as these store values of shared variables, modify them by writes and return their values during reads). For instance, they also give the DSTM algorithm (in the repaired version), but instead of recording old and new values of variables and using the stability check, they introduce an ownership set. The stated proof of opacity of DSTM thus refers to an abstraction of DSTM only.

The next issue is the level of atomicity considered in the formalization of the GHS approach and thus concerns the second difference in the opacity definition. First, because operations are not split into invocations and responses, all operations seem to be considered to be atomic. On a more detailed look\footnote{Guerraoui et al. give the TM algorithms in a very unusual form. This makes it difficult to determine what the actual runs of a TM are, and what the histories derived from these runs are.}, TM algorithms considered by GHS still allow for runs interleaving statements of operations (via extended commands and specific \( \bot \) responses). As this is invisible in the histories, the notion of “a history being sequential” gets unclear and as a consequence the interpretation of property \( P_4 \) is unclear.

**Issue 4** It is unclear on which level of granularity property \( P_4 \) is to be interpreted.

For this, consider again history \( h \) of CoreDSTM given in \( \text{[1]} \). History \( h \) is not opaque, but

\[
h' = \text{rd}_1(x) \text{rd}_2(x) \text{rd}_1(y) \text{rd}_2(y) \text{wr}_1(x) \text{wr}_2(y) \text{cmt}_1
\]
is. With \( h \) and \( h' \) we have the situation required for property P4: \( h' \cdot s \) where \( s = \text{cmt}_2 \) is not opaque, \( s \) is not an abort, \( h' \) is opaque. Property P4 states that the sequentialisation of \( h' \) witnessing opacity of \( h' \) which is

\[
h'' = \text{rd}_2(x)\text{rd}_2(y)\text{wr}_2(y)\text{rd}_1(x)\text{rd}_1(y)\text{wr}_1(x)\text{cmt}_1
\]

and its extension by \( s \) needs to be a history of the STM. The question is now what the execution of the STM is which we need to look at when trying to get this history. The natural expectation is that \( h'' \) corresponds to an execution of DSTM in which no statements of \( \text{cmt}_2 \) are executed. However, then \( h'' \cdot s \) is not a history of DSTM (transaction 2 will abort when it starts its commit after 1’s commit) and consequently P4 is not satisfied. If we allow \( h'' \cdot s \) to belong to an execution interleaving statements of the two commit operations, \( h'' \cdot s \) is a history of DSTM (and P4 would be fulfilled). From the way property P4 is stated, such an \( h'' \) would however intuitively not be allowed as these histories are supposed to be “sequential” (which one can expect to be interpreted as “no interleaving of operations at all”).

The final observation also refers to the level of granularity in operations and hence possible interleavings of concurrent transactional operations.

**Issue 5** The level of granularity of STM operations given by GHS differs to the published algorithms.

For example, for the (correct version of) DSTM as given by GHS, the read operation is an atomic step, write consists of two atomic steps (write and own) and commit of two steps (validate and commit, the validate for instance aborting several other transactions in a single atomic step). This simplifies in particular the proof of property P4 for DSTM, e.g., Singh [21] argues that DSTM is conflict commutative (which guarantees P4) by saying “As the read consists of a single instruction, it cannot be concurrent with a commit instruction”. This clearly does not hold for CoreDSTM.

STM implementations typically use specific instructions to achieve atomicity (like the CAS in DSTM) and use them only on a very fine-grained level as to allow a high degree of concurrency. By employing formal models of STMs extending atomicity to blocks of statements (as GHS do), critical interleavings leading to non-opaque behaviour can easily be missed.

**6 Conclusion**

In this paper, we have re-investigated the applicability of a small model theorem to the model checking of STMs with respect to opacity. While neither the small model theorem is incorrect nor the model checking results for the algorithms as given in the paper are, the general applicability of this theorem remains unclear. The usage of this theorem presupposes a number of abstractions carried out on (a) the opacity definition itself and (b) the STM algorithms being checked. The results employed by this technique are thus not directly transferable to the original STM algorithms.
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