An extension for direct gauge mediation of metastable supersymmetry breaking

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Abstract – We study the direct mediation of metastable supersymmetry breaking by a \( \Phi^2 \)-deformation to the ISS model and extend it by splitting both \( \text{Tr}\Phi \) and \( \text{Tr}\Phi^2 \) terms in the superpotential and gauging the flavor symmetry. We find that with such an extension enough-long-lived metastable vacua can be obtained and the proper gaugino masses can be generated. Also, this allows for constructing a kind of models which can avoid the Landau pole problem. Especially, in our metastable vacua there exist a large region for the parameter \( m_3 \) which can satisfy the phenomenology requirements and allow for a low SUSY-breaking scale (\( h_{\mu 2} \sim 100 \text{TeV} \)).

Introduction. – Dynamical supersymmetry (SUSY) breaking is a convincing scenario to solve the gauge hierarchy problem, but it seemed an exceptional phenomenon because its realistic models in general have to satisfy many theoretical requirements. On the other hand, the phenomenological considerations are complex when these dynamical SUSY-breaking effects are mediated to the visible sector.

Recently, Intriligator, Seiberg and Shih (ISS) [1] discovered the metastable supersymmetry breaking in the surprising context of a vector-like theory, which offers a natural framework for dynamical SUSY breaking and its mediation. Their model (called ISS model) has received a great deal of attention. However, this model also has some problems. One is the Landau pole problem which implies that the unification cannot be simply realized in this model. Another problem is that the presence of an accidental R-symmetry (a generic property of SUSY breaking models) forbids the gaugino masses. To tackle these problems, many approaches have been proposed [2-6].

In the ISS model the mediation of SUSY breaking can proceed through gauge interaction. Actually, the gauge mediation of dynamical SUSY breaking was once proposed in [7,8], which tries to use a QCD-like strong interaction to break supersymmetry dynamically and identify the standard model gauge group as a subgroup of the flavor symmetry. Nevertheless, these early models suffer from some phenomenological problems, such as the Landau pole problem and the gaugino mass problem, and thus the idea of gauge mediation of dynamical SUSY breaking was discarded for a long time. With the advent of the ISS model, this idea was revived [3,4].

Note that with some deformations the ISS model can give the required phenomenology. In [2] a \( \text{Tr}\Phi^2 \) term is introduced into the superpotential (called \( \Phi^2 \)-deformation) and further in [3] the \( \text{Tr}\Phi \) term in the superpotential is splitted. In this work we consider a more general deformation to the ISS model by splitting both \( \text{Tr}\Phi \) and \( \text{Tr}\Phi^2 \) terms in the superpotential. We find that such a general deformation can satisfy the phenomenological constraints like generating the appropriate gaugino masses and avoiding the Landau pole.

This work is organized as follows. In the second section we give a brief description for the ISS model and its \( \Phi^2 \)-deformation. In the third section we present our general deformation and discuss its phenomenology. Finally, in the last section we give our conclusion.

The ISS model and its \( \Phi^2 \)-deformation. – In the ISS model, the hidden sector is just the \( N=1 \) supersymmetric QCD and has massive quarks with \( N_c < N_f < \frac{3}{2}N_c \), with \( N_f \) being the flavor number and \( N_c \) the color number. Its superpotential in the dual magnetic theory is

\[ W = h_q \Phi_{ij} \bar{q}_j - h_{\mu 2} \text{Tr}\Phi, \]

where \( q \) and \( \bar{q} \) are, respectively, the quark and anti-quark, \( \Phi \) is the meson field, and \( i \) and \( j \) running from 1 to \( N_f \) are the flavor indices. In the low-energy region, the \( F \)-terms of
Φ cannot be simultaneously set to zero because the rank of \( q\bar{q} \) is \( N_f - N_c \) which is smaller than the rank of \( \Phi = N_f \), and then we get the SUSY-breaking vacuum energy as

\[
V = N_c |h\mu|^2. \tag{2}
\]

However, in the high-energy range below the scale \( \langle h\Phi \rangle \) the quarks are integrated out and the effective theory is then \( SU(N_f - N_c) \) pure Yang-Mills where the non-perturbative correction to the superpotential restore the SUSY vacua low energy is only metastable.

The \( \Phi^2 \)-deformation to the ISS model is proposed by Giveon and Kutasov (GK) \[2\], whose superpotential can be derived from the brane configuration \[9-11\] and takes the form

\[
W = h q \Phi_{ij} \bar{q}_j - h \mu^2 \text{Tr} \Phi + \frac{1}{2} h^2 \mu \Phi \text{Tr} \Phi^2 \tag{3}
\]

with \( \mu \) being a new energy scale. Note that there is no non-perturbative superpotential from gaugino condensation effects. We assume

\[
\Phi \leq \frac{\mu^2}{h \mu} \tag{4}
\]

to ensure the \( \Phi^2 \)-deformation to be just a perturbation to the ISS model in the low-energy region. This deformed supersymmetric QCD has a rich landscape of supersymmetric and non-supersymmetric vacua. The expectation values of the fields are given by

\[
\langle h\Phi \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \mu^2 \end{pmatrix}, \tag{5}
\]

\[
\langle q\bar{q} \rangle = \begin{pmatrix} \mu^2 I_{N_f} & 0 \\ 0 & 0 \end{pmatrix},
\]

in supersymmetric vacua (\( I_n \) denotes a \( n \times n \) unit matrix), and

\[
\langle h\Phi \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{\mu^2}{\mu} I_{N_f - k-n} \\ 0 & 0 & \frac{\mu^2}{\mu} I_{N_f - k-n} \end{pmatrix},
\]

\[
\langle q\bar{q} \rangle = \begin{pmatrix} \mu^2 I_k & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\]

in metastable non-supersymmetric vacua. In the latter case it is necessary to consider the one-loop contribution to the potential, which to leading order is given by \[1\]

\[
V_{1-loop} = b|\mu|^2 |\text{Tr} \Phi_n|^2, \tag{7}
\]

with \( b \) being a constant given by

\[
b = \frac{\ln 4 - 1}{8\pi^2} (N_f - N_c). \tag{8}
\]

In our following analysis we fix \( b = 0.01 \) for convenience. The full potential for \( \Phi_n, q, \bar{q} \) then takes the form

\[
V = \frac{V}{|\mu|^2} = |\Phi_n q|^2 + |\Phi_n \bar{q}|^2
\]

\[
+ |q \bar{q} - \mu^2 I_n + h \mu \Phi_n|^2 + b|\mu|^2 |\text{Tr} \Phi_n|^2 \Phi_n. \tag{9}
\]

To be free of tachyons in the dual quark direction and to ensure supersymmetric vacua to be far enough from metastable vacua, we have (neglecting the phase factor in the energy scale and coupling constants)

\[
\frac{2 \mu}{V} \lesssim \mu \lesssim \frac{2 \mu}{V}. \tag{10}
\]

Here the first constraint comes from the requirement of enough-long-lived metastable vacua (this will be discussed in the following), and the second constraint comes from the special property of this deformation (that can also be applied to our more general deformation discussed in the proceeding section) and can be obtained from the analysis of the potential in eq. (9) through calculating \( \partial V/\partial \Phi \). Considering the above constraints, we see that a small \( h \) is favored.

To check if the GK metastable vacua is long-lived enough, we estimate its decay rate by evaluating the Euclidean action \( S_1 \) from the GK metastable vacua to their corresponding true vacua and the action \( S_2 \) from the GK metastable vacua to the ISS metastable vacua. Using the triangle approximation \[12,13\], we estimate the bounce action as

\[
S_1 \sim \frac{\Delta \Phi^4}{V} \sim \frac{1}{nh^2} \left( \frac{\mu}{\mu h} \right)^4, \tag{11}
\]

\[
S_2 \sim \frac{\Delta \Phi^4}{V} \sim \frac{1}{nh^6} \left( \frac{\mu}{\mu h} \right)^4, \tag{12}
\]

where \( \Delta \Phi \) is the corresponding interval between the two vacua. Therefore, if we have the conditions in eq. (10), we can obtain a sufficiently long lifetime for the universe even if \( h \) is not too small.

After building a dynamical supersymmetric breaking model, the next task is to mediate the breaking effects to the visible sector. Although the constraints from the long lifetime of the metastable vacua are weak, the gaugino masses may be much smaller than the sfermion masses when we use the direct gauge mediation model by gauging the flavor subgroup in the ISS sector. In the following section we propose a more general deformation by splitting both the Tr\( \Phi \) and Tr\( \Phi^2 \) terms in the superpotential. With such a deformation, we can avoid the hierarchy between gaugino and sfermion masses and, furthermore, can avoid the Landau pole problem.

**A more general deformation to the ISS model.**

- In \[3\] the term Tr\( \Phi \) in the superpotential is splitted. Here we propose a more general deformation and take the
As discussed in the preceding section, we have the condition $m_2/\sqrt{b} \ll \mu_2 \leq m_2/(hh)$ for the long-lived metastable vacua and no tachyonic quark. Note that in the above we took $k = N_1$. In our following analysis we also discuss the case of $k = 0$ without presenting the explicit structure. There are, of course, other metastable vacua in our model, but the above vacua are enough for our purpose of getting an appropriate phenomenological model.

In our meta-stable vacua in eq. (16), the flavor symmetry $SU(N_2)$ would be broken to $SU(n) \times SU(N_2-n)$. We can gauge the flavor symmetry $SU(n)$ or $SU(N_2-n)$ and embed the standard model gauge group into the gauged flavor symmetry to realize gauge mediation of SUSY breaking.

Now we examine the gaugino masses in our deformation. The gaugino masses in gauge mediation [14,15] (with a superpotential explicitly breaking $R$-symmetry) are given by [16]

$$m_\lambda = \frac{g^2 N}{(4\pi)^2} F_{X_i} \frac{\partial}{\partial X_i} \log (\det \mathcal{M}),$$

where $N$ is a constant, $\mathcal{M}$ is the mass matrix of messenger fields, $X_i$ denotes a superfield in the hidden sector and $-F_{X_i} = \partial W/\partial X_i$. In our deformation the form of $\mathcal{M}$ is determined by which flavor symmetry, $SU(n)$ or $SU(N_2-n)$, is gauged.

1) If we choose to gauge $SU(N_2-n)$ flavor symmetry and embed the standard model group $SU(3) \times SU(2) \times U(1)$ into it, the messenger fields would be $\rho_2$, $R$, $Z_2$. In our analysis we use the notation

$$\Phi = \begin{pmatrix} Y & \Phi_1 & \Phi_2 \\ \bar{Z}_1 & \Phi_1 & \Phi_2 \\ \bar{Z}_2 & \Phi_1 & \Phi_2 \end{pmatrix}, \quad q = \begin{pmatrix} \chi \\ \rho_1 \\ \rho_2 \end{pmatrix},$$

where $\Phi_1$ is the $n \times n$ matrix and $\Phi_2$ is the $(N_2-n) \times (N_2-n)$ matrix. The mass matrix $\mathcal{M}$ is given by

$$\mathcal{M}/h = \begin{pmatrix} \Phi_2 & \mu_1 & 0 & 0 \\ \mu_1 & \mu_{m_3} & 0 & 0 \\ 0 & 0 & \Phi_2 & \mu_2 \\ 0 & 0 & \mu_2 & \mu_{m_2} \end{pmatrix},$$

in the basis

$$\begin{pmatrix} \rho'_2 \\ Z_2 \\ \rho''_2 \\ R \end{pmatrix},$$

where $\rho_2$ includes $\rho'_2$ and $\rho''_2$ which couple with different components of $\chi$. Therefore, under the assumption $m_2 m_3 \gg b h^2$, we have

$$m_\lambda = \frac{g^2 N}{(4\pi)^2} F_{X_i} \frac{\partial}{\partial X_i} \log (\det \mathcal{M}),$$

$$\simeq \frac{\alpha}{4\pi} F_{\Phi_2} \left( \frac{h m_3}{m_2 \Phi_2} - \mu_1 + \frac{h m_2}{\Phi_2} - \mu_2 \right),$$

$$\simeq \frac{\alpha}{4\pi} F_{\Phi_2},$$

where $F_{\Phi_2} = h \mu_2^2$ and $\langle \Phi_2 \rangle = m_2/(hh)$ denotes the expectation value in the meta-stable vacuum shown in eq. (16). On the other hand, we have the squark masses as

$$m_{\tilde{s}} \simeq \frac{\alpha}{4\pi} \left( \frac{F_{\Phi_2}}{\mu_1} + \frac{F_{\Phi_2}}{\Phi_2} \right) \simeq \frac{\alpha}{4\pi} F_{\Phi_2},$$
where we assumed $\mu_1 \gg \langle \Phi_2 \rangle$ and considered $\langle \Phi_2 \rangle = m_2/\langle hh \rangle \gg \mu_2$ as required by the non-existence of tachyonic messenger fields. In this way, we obtain the same-order masses for gauginos and squarks. Note that the assumptions $m_2m_3 \gg b\mu_1^2$ and $\mu_1 \gg \langle \Phi_2 \rangle$ are easy to be satisfied if we let $m_3$ large enough. We checked that these conditions do not affect our vacua structure in eqs. (15), (16) and also do not affect our calculations about the lifetime of the vacua (in our calculations we use eq. (11) and replace $\mu_\phi$ and $\mu$ with $m_2$ and $\mu_2$, respectively).

Compared with [3], where the two independent scales are required to be nearly equal, i.e., $m_3 \sim \mu_1$, in our study we have a larger appropriate region for the parameter $m_3$. Actually, as shown from our following analysis of Landau pole, a large $m_3$ is favored, which enables us to obtain a SUSY-breaking scale ($\hmu_2 \sim 10^3$ TeV) lower than the value obtained in [3].

2) If we choose to gauge the $SU(n)$ flavor symmetry and embed the standard model group into it, the messenger fields would be $\rho_2, \chi, R$ and $Z_1$. Then the mass matrix $\mathcal{M}$ is given by

$$\mathcal{M}/h = \begin{pmatrix}
\Phi_2 & \mu_2 & 0 & 0 \\
\mu_2 & hm_2 & 0 & 0 \\
0 & 0 & Y & \mu_2 \\
0 & 0 & \mu_2 & hm_3
\end{pmatrix}$$

in the basis

$$\begin{pmatrix}
\rho_2 \\
R \\
\chi \\
\bar{Z}_1
\end{pmatrix}.$$  

Therefore, if we assume the $F$-term of the $Y$ field is not zero (for example, we can take $k = 0$ and a non-zero $N_1$, and then in eq. (16) the first diagonal element is $(m_1/b)I_{N_1}$ for $\langle h\Phi \rangle$ and $0I_{N_1}$ for $\langle q\bar{q} \rangle$) and further assume $m_1m_3 \gg b\mu_2^2$, we have

$$m_\lambda = \frac{g^2 N}{(4\pi)^2} F_X \frac{\partial}{\partial X_1} \log(\det \mathcal{M})$$

$$\sim \frac{\alpha}{4\pi} \left( h^2m_2 + \frac{F_Yhm_3}{hm_3(Y) - \mu_2^2} \right)$$

$$\sim \frac{\alpha}{4\pi} \frac{F_Y}{Y},$$

where $F_Y = h\mu_2^2$ and $\langle Y \rangle = m_1/\langle hh \rangle$ denotes the expectation value in the metastable vacuum. On the other hand, we have the squark masses as

$$m_s \sim \frac{\alpha}{4\pi} \left( \frac{F_{\Phi_2}/\langle \Phi_2 \rangle + F_Y/\langle Y \rangle}{} \right).$$

Therefore, in this case if $\mu_1 \sim \mu_2$ the squark masses can also be of the same order as the gaugino masses.

Finally, we check if our deformation is free of the Landau pole problem. The mass spectrum can be read out from the metastable vacua and is dependent on which flavor symmetry, $SU(n)$ or $SU(N_2 - n)$, is gauged. We found that our model has the Landau pole problem if we choose to gauge the $SU(n)$ symmetry, but is free of the Landau pole problem if we choose to gauge the $SU(N_2 - n)$ symmetry. In the following we demonstrate how to avoid the Landau pole problem in case of gauging the $SU(N_2 - n)$ symmetry.

When we gauge $SU(N_2 - n)$ and embed the standard model group into it, the $\rho_2$ and $Z_2$ have a mass of $O(m_2/b)$ and $O(hm_3)$, respectively. The $R$ and $\rho_2'$ have a mass near the scale $h\mu_2$, and the pseudo-modulus $\Phi_2$ has a mass of similar size to the gauginos. In our following calculation we take $m_2/b \sim h\mu_2$ for simplicity. The beta-function coefficients of the $SU(3)$ gauge coupling $b_3$ are given by

$$b_3(\mu_R < m_\lambda) = b_3^{SM} = -7,$$

$$b_3(\mu_R < h\mu_2) = -3 + N_2 - n,$$

$$b_3(h\mu_2 < \mu_R < hm_3) = -3 + N_2 + N_f - N_c,$$

$$b_3(hm_3 < \mu_R < \Lambda) = -3 + 2N_f - N_c,$$

$$b_3(\mu_R > \Lambda) = -3 + N_c,$$

where $\mu_R$ is the renormalization scale. In our analysis we use the definition

$$\frac{d\alpha_3}{d\mu_R} = \frac{b_3}{16\pi^2} \frac{g_3^2(M_3)}{\alpha_3} \equiv \beta_3,$$

and take the input parameters as

$$M_{GUT} = 10^{16} \text{ GeV}, \quad M_2 \approx 90 \text{ GeV},$$

$$m_\lambda \approx 10^3 \text{ GeV}, \quad \frac{g_3^2(M_3)}{4\pi} \approx 0.18.$$

The $SU(3)$ coupling is obtained as

$$\alpha_3^{-1}(M_{GUT}) \approx 5.6 - \frac{7}{2\pi} \log M_3$$

$$+ \frac{4 + N_2 - n}{2\pi} \log m_\lambda + \frac{N_f - N_c + n}{2\pi} \log(h\mu_2)$$

$$+ \frac{N_f - N_c}{2\pi} \log(hm_3) + \frac{2N_c - 2N_f}{2\pi} \log M_{GUT},$$

where we take $\Lambda = M_{GUT}$. For example, taking $N_2 - n = 5$, $N_f = 11$, $N_c = 9$, $n = 1$ and $N_1 = N_f - N_2 = 5$, we find that for the SUSY-breaking scale $h\mu_2 \sim 10^4$ GeV and $hm_3 \geq 10^7$ GeV, the Landau pole can be avoided below the unification scale, i.e., $\alpha_3^{-1}(M_{GUT}) > 1$. Under this condition, proper gaugino mass can be obtained from eq. (26), and we checked it is consistent with our other assumptions.

**Conclusion.** – In this work we considered a more general deformation to the ISS model by splitting both $Tr\Phi$ and $Tr\Phi^2$. Then we found that the corresponding metastable vacua can be long-lived enough and the proper
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gaugino masses can be generated. In particular, there can exist a kind of models which can avoid the Landau pole problem if we gauge the $SU(N_2-n)$ flavor group and embed the standard model group into it.

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