Abstract—Spectrum sharing between wireless networks improves the usage efficiency of radio spectrums. This paper addresses spectrum sharing between a cellular uplink and a mobile ad hoc networks. These networks use either all uplink frequency subchannels or their disjoint subsets, called spectrum overlay and spectrum underlay, respectively. Given these methods, the capacity tradeoff between the coexisting networks is analyzed in terms of transmission capacity. For a network with Poisson distributed transmitters, this metric is defined as the maximum density of transmitters subject to an outage constraint for a given signal-to-interference ratio (SIR). Using stochastic geometry, the transmission-capacity tradeoff between the coexisting networks is derived, where both spectrum overlay and underlay as well as successive interference cancellation (SIC) are considered. In particular, for small target outage probability, the transmission capacities of the coexisting networks are proved to satisfy a linear equation. Its coefficients depend on the spectrum sharing method and whether SIC is applied. This linear equation shows that spectrum overlay is more efficient than spectrum underlay.

I. INTRODUCTION

Despite spectrum scarcity, most licensed spectrums are underutilized according to Federal Communications Commission [1]. In particular, in existing cellular systems using frequency division duplex (FDD), equal bandwidths are allocated for uplink and downlink transmissions, even though uplink traffic is much lighter than downlink traffic [3]. This motivates the current study on sharing uplink spectrum between a cellular network and a mobile ad hoc network (MANET) to improve spectrum utilization. The performance of these networks (coexisting networks) is measured using the metric of transmission capacity [4]. The transmission capacities of the coexisting networks are defined as their maximum transmitter densities under an outage probability constraint for a target signal-to-interference ratio (SIR). We derive the transmission-capacity tradeoff between the networks for different spectrum-sharing methods. Such results are useful for controlling the network sizes to optimize uplink spectrum usage.

A licensed spectrum band is assigned to a network for exclusive use. Depending on whether holding a licence, a wireless network is referred to as the primary (e.g. cellular networks) or secondary network (e.g. MANETs). Accessing a licensed band, secondary transmitters must avoid significant interference to the primary receivers. To this end, secondary transmitters can spread signal energy over the whole band using spread spectrum techniques, suppressing the power spectrum density of resultant interference to the primary receivers. This method is called spectrum underlay [6].

Another method for sharing a licensed spectrum is called spectrum overlay, where secondary transmitters access frequency subchannels unused by nearby primary receivers. Recent research on spectrum overlay focuses on designing cognitive-radio algorithms for secondary transmitters to opportunistically access the spectrum by adapting to the traffic dynamics of the primary network [6]. These algorithms require complicated computation at the secondary transmitters. For this reason, we consider the case where base stations in the cellular (primary) network coordinates spectrum overlay. Thereby ad hoc (secondary) transmitters use a simple random access protocol.

There exist few theoretical results on the network capacity tradeoff between coexisting networks despite this being a basic question. In [8], the transmission capacities of a two-tier network are analyzed, which consists of a cellular network and a network of femtocell hot-spots. In [9], the transport capacities [10] of two coexisting multi-hop MANETs are shown to follow the optimum scaling laws for asymptotically large network sizes. In [8], [9], no analytical results exist on the network-capacity tradeoff between coexisting networks.

The transmission capacity provides the performance metric in this paper [4]. This metric has been applied to designing MANETs with Poisson distributed transmitters, addressing a wide range of issues such as spatial interference cancellation [13], opportunistic transmissions [4], bandwidth partitioning [11], and successive interference cancellation (SIC) [12]. Existent results target stand-alone networks and cannot be directly extended to coexisting networks. Analyzing their transmission capacities must address new challenges including heterogeneous network architectures, interference between networks and allocation of radio resource over networks.

This paper targets a cellular uplink network and a MANET sharing the uplink spectrum using either spectrum overlay or underlay, where uplink users, base stations, and ad hoc transmitters form independent Poisson point processes (PPPs) with different densities. Each transmitter modulates signals using frequency-hopping spread spectrum over the frequency subchannels assigned to the corresponding network. Our main contributions are summarized as follows. First, considering an interference-limited environment, bounds on the SIR outage probabilities are derived for spectrum overlay and underlay with and without using SIC at receivers. Second, for small target outage probability, the transmission-capacities of the coexisting networks are showed to satisfy a linear equation,
whose coefficients depend on the spectrum sharing method and whether SIC is used. Define the capacity region as the set of feasible combinations of transmitter densities under the SIR outage constraint. Third, for small target outage probability, the capacity region for spectrum underlay is shown to be no larger than that for spectrum overlay. Two regions can be equalized by choosing the derived transmission-power ratio, which depends on the interference-power threshold for qualifying canceled interferers. Moreover, SIC increases transmission capacities by a linear factor, which depends on the interference-power threshold for qualifying canceled interferers.

The remainder of this paper is organized as follows. The mathematical models are described in Section II. In Section IV, the bounds on outage probabilities are derived for different spectrum sharing methods. For small target outage probability, the transmission-capacity tradeoff is analyzed in Section V. Numerical results are presented in Section VI, followed by concluding remarks in Section VII.

II. MATHEMATICAL MODELS

A. Network Models

The coexisting networks are illustrated in Fig. 1. Their models are described as follows.

Following [4], [15], we make the following assumption.

Assumption 1. The transmitters in the MANET form a Poisson point process (PPP) on the two-dimensional plane, which is denoted as $\Pi$ with the density $\tilde{\lambda}$.

This Poisson assumption as well as the similar one in Assumption 2 are made for mathematical tractability. However, they are supported by experiments for networks with mobility. In particular, network models based on similar assumptions have been employed by cellular service providers to successfully predict network traffic load and blocking probabilities [16]. Next, each transmitter in the MANET is associated with a receiver located at a fixed distance $\tilde{d}$. The transmission power of transmitters is assumed fixed and denoted as $\tilde{P}$.

The cellular network is modeled based on the following assumption

Assumption 2. The base stations and uplink mobiles form two independent stationary PPPs denoted as $\Omega$ and $\Pi$, respectively.

Their corresponding densities are represented by $\lambda_b$ and $\lambda$. Let $B_n$, $U_m$, $D_{n,m}$ denote the two-dimensional coordinates of the $n$th base station, the $m$th uplink mobile, and their distance, respectively. Thus, $D_{n,m} = |B_n - U_m|^2$. To enhance the long-term link reliability, each mobile transmits to the nearest base station. Consequently, the cellular network forms a Poisson tessellation of the two-dimensional plane and each cell is known as a Voronoi cell [17]. The mobiles served by the $n$th base station form the set $V_n := \{U \in \Omega \mid |U - B_n| < |U - B| \forall B \in \Omega \{B_n\}\}$. A typical point of a PPP is defined as a point selected using the procedure where every point of the PPP has the same probability of being selected [17]. The typical points of the PPPs $\Omega$, $\Pi$ and $\Pi$ are referred to as the typical base station $B_0$, the typical mobile $U_0$, and the typical ad hoc transmitter $T_0$, respectively. Moreover, the intended receiver for $T_0$ is called the typical ad hoc receiver and denoted as $R_0$. Finally, represent the random distance between $B_0$ and $U_0$ as $D := |B_0 - U_0|$.

B. Channel and Modulation

The uplink spectrum is divided into $M$ frequency-flat subchannels by using orthogonal frequency division multiplexing (OFDM) [18]. Each of the coexisting networks uses a subset or the full set of subchannels, depending on the spectrum sharing methods discussed in Section II-C. In each network, a transmitter modulates signals using frequency-hopping spread spectrum, where signals hop randomly over all subchannels assigned to the affiliated network [5].

Consider the link between $U_0$ and $B_0$. The typical subchannel accessed by $U_0$ and $B_0$ comprises path loss and a fading factor denoted by $W$ such that the signal power received by $B_0$ is $P WD^{-\alpha}$, where $P$ is the transmission power. Similarly, the interference power from an interferer $X$ to $B_0$ is $P_X G_X R_X^{-\alpha}$, where $P_X \in \{P, \tilde{P}\}$, and $R_X = |X - B_0|$, and $G_X$ is the fading factor.

Similar channel models are used for the MANET. Specifically, the received signal power for $T_0$ is $\tilde{P} W \tilde{d}^{-\alpha}$ where $\tilde{W}$ is the fading factor; the interference power from an interferer $X$ to $T_0$ is $P_X G_X R_X^{-\alpha}$ where $\tilde{R}_X = |X - T_0|$.\footnote{Consideration of the randomness in $\tilde{d}$ provides little new insight. It is straightforward to extend the results in this paper to include the randomness in $\tilde{d}$. Specifically, if $\tilde{d}$ is random, Proposition 1-4 still apply except that the expectation operations in Proposition 1-2 must account for the randomness of $\tilde{d}$. Furthermore, $\tilde{d}$ in Theorem 1 should be replaced with $E[\tilde{d}]$.}

\footnote{The operator $|X|$ gives the Euclidean distance between $X$ and the origin if $X$ is two-dimensional coordinates, or the cardinality of $X$ if $X$ is a set.}
C. Spectrum Sharing Methods

1) Spectrum Overlay: For spectrum overlay, the \( M \) subchannels are divided into two disjoint subsets and assigned to two coexisting networks. Let \( \mathcal{A} \) and \( \mathcal{\tilde{A}} \) denote the index sets of the subchannels assigned to the cellular network and the MANET, respectively. Their cardinalities are represented by \( K := |\mathcal{A}| \) and \( \tilde{K} := |\mathcal{\tilde{A}}| \), where \( K + \tilde{K} = M \).

Spectrum overlay requires initialization where base stations broadcast to ad hoc nodes the indices of the available subchannels and the allowable transmitting node density. This density can be controlled by distributed adjustments of nodes’ transmission probability [4]. In practice, density can be controlled by distributed adjustments of nodes’ channels and the allowable transmitting node density. This allows for the dynamic allocation of bandwidth.

2) Spectrum Underlay: For spectrum underlay, both coexisting networks use all \( M \) subchannels. Therefore the transmitters accessing the same subchannel comprises both mobiles and ad hoc nodes. Consequently, spectrum overlaid networks are coupled rather than decoupled as for the case of spectrum overlay.

D. Successive Interference Cancelation

SIC decodes multiuser signals and subtracts them from the aggregate received signal sequentially [18]. This technique is effective for mitigating interference in the MANET given the disparity in received power of multiuser signals due to random node positions combined with path loss and fading [12]. The SIC decoding of a user’s signal treats the signals of undecoded nodes as noise. Therefore, for users with equal data rates, it is optimal to decode multiuser signals in the descending order of their power [19]. The above decoding order implies that canceled interferers’ signals are stronger than the desired one. This motivates the current simplified SIC model based on the following assumption.

**Assumption 3.** The interference power of each interferer targeted for cancelation must exceed a threshold equal to the desired signal power multiplied by a factor \( \kappa > 1 \).

This assumption is made for mathematical tractability. More accurate SIC models must account for cancelation errors and the decoding order [14], [18], [20]. Increasing \( \kappa \) decreases the average number of canceled interferers and vice versa. This model is very similar to that in [12] where only path-loss is considered and interferers within a fixed distance are canceled.

III. BACKGROUND

In this section, the shot noise process is introduced for modeling interference in the coexisting networks. Existing results on the shot-noise distribution are discussed, which are useful for analysis in the sequel. Finally, the transmission capacity and capacity region are defined.

A. Shot Noise Process

A general shot noise process refers to a functional resulting from feeding a memoryless and linear filter with impulses derived from a stationary PPP [21]. Such processes have been widely applied in modeling spatial interference in wireless networks (see e.g. [4], [22]). Consider a wireless network where transmitters form a stationary PPP \( \Phi \) with the density \( \lambda \) and channels comprise path-loss and fading. For a receiver \( A_0 \) located at the origin, the aggregate interference power \( I_\Phi \) is a shot noise process given as

\[
I_\Phi = \sum_{X \in \Phi \setminus \{F_0\}} P Q_X |X|^{-\alpha} \tag{1}
\]

where \( F_0 \) denotes the transmitter for \( A_0 \) and \( \{Q_X\} \) are i.i.d. fading factors. Note that the process \( \Phi \setminus \{F_0\} \) is identically distributed as \( \Phi \) accordingly to Slivnyak’s theorem [17]. Analyzing the outage probability for the transmission from \( F_0 \) to \( A_0 \) requires deriving the complementary cumulative density function (CCDF) of \( I_\Phi \). Unfortunately, this function has no closed-form expression [4], [21].

The CCDF of \( I_\Phi \) can be bounded using the approach in [4]. This approach separates the interferers in \( \Phi \setminus \{F_0\} \) into strong and weak interferers. Define the process of strong interferers as \( \Sigma_S = \{ X \in \Phi \setminus \{F_0\} \mid PQ_X|X|^{-\alpha} > t \} \), where each interferer alone guarantees the outage \( I_\Phi > t \). It follows that the process of weaker interferers is \( \Sigma_0 := (\Phi \setminus \{F_0\}) \setminus \Sigma_S \). Using these definitions

\[
\text{Pr}(I_\Phi > t) = \text{Pr}(\Sigma_S = \emptyset) \text{Pr}(I_0 > t) + \text{Pr}(\Sigma_S \neq \emptyset) \tag{2}
\]

where \( I_0 := \sum_{X \in \Sigma_S} PQ_X |X|^{-\alpha} \) represents the interference power from weak interferers. From (2), a lower bound on the outage probability is given as

\[
\text{Pr}(I_\Phi > t) > \text{Pr}(\Sigma_S \neq \emptyset) = 1 - e^{-\mu_S} \tag{3}
\]

where \( \mu_S := \mathbb{E}[\|\Sigma_S\|] \) is the average number of strong interferers. The upper bound on \( \text{Pr}(I_\Phi > t) \) is obtained by bounding the term \( \text{Pr}(I_0 > t) \) in (2) using Chebyshev’s inequality

\[
\text{Pr}(I_0 > t) \leq \frac{\text{var}(I_0)}{(t - \mathbb{E}[I_0])^2}, \quad t > \mathbb{E}[I_0] \tag{4}
\]

where the right-hand-side of the first inequality is Chebyshev’s upper bound. Close-form expressions for \( \mu_S, \text{var}(I_0) \) and \( \mathbb{E}[I_0] \) can be obtained using Campbell’s theorem [4], [17]. By substituting the resultant expressions into (3) and (4), the bounds on \( \text{Pr}(I_\Phi > t) \) are obtained in [4, Theorem 2] and shown in the following lemma.

**Lemma 1.** The CCDF of the shot noise process in (1) is bounded as

\[
P_{\text{out}}^\delta(t, \lambda) < \text{Pr}(I_\Phi > t) < P_{\text{out}}^u(t, \lambda)
\]

where

\[
P_{\text{out}}^\delta(a, b) = 1 - \exp(-\zeta a^{-\delta} b), \quad P_{\text{out}}^u(a, b) = 1 - \xi(a, b) \exp(-\zeta a^{-\delta} b),
\]

\[
\xi(a, b) = \begin{cases} 
1 - \frac{\delta}{\zeta} a^{-\delta} b \left( 1 - \frac{\delta}{\zeta} a^{-\delta} b \right)^{\frac{\delta}{1 - \delta}} & \text{if } 1 - \frac{\delta}{1 - \delta} a^{-\delta} b < 1 
0, & \text{otherwise}
\end{cases}
\]

with \( \zeta := \pi \theta^\delta \mathbb{E}[Q^\delta] \).
B. Transmission Capacity

We define two related performance metrics for the coexisting networks. As in [4], the networks are assumed to be interference limited and noise is neglected for simplicity. Hence the SIR measures the reliability of received data packets. Let $\text{SIR}$ and $\overline{\text{SIR}}$ represent the SIRs at $U_0$ and $R_0$, respectively. The correct decoding of received packets requires the SIRs to exceed a threshold $\theta$. To satisfy this constraint with high probability, the following outage constraints are applied for given $0 < \epsilon \ll 1$

\[
P_{\text{out}}(\lambda, \tilde{\lambda}) := \Pr(\text{SIR}(\lambda, \tilde{\lambda}) < \theta) \leq \epsilon
\]

\[
P_{\overline{\text{out}}}(\lambda, \tilde{\lambda}) := \Pr(\overline{\text{SIR}}(\lambda, \tilde{\lambda}) < \theta) \leq \epsilon
\]

(5)

where the functions $P_{\text{out}}$ and $\overline{P}_{\text{out}}$ map the SIRs to the outage probabilities. The transmission capacities $C$ of the cellular network and $\overline{C}$ of the MANET are defined as the maximum transmitter densities under the outage constraints in (5). This definition differs slightly from that in [4] by a linear factor $(1 - \epsilon)$, which has a negligible effect on the analysis.

Next, define the capacity region $\mathcal{R}$ as the set comprising all combinations $(\lambda, \tilde{\lambda})$ that satisfy the outage constraints in (5)

\[
\mathcal{R} = \left\{ (\lambda, \tilde{\lambda}) \in \mathbb{R}^2 \mid P_{\text{out}}(\lambda, \tilde{\lambda}) \leq \epsilon, \overline{P}_{\text{out}}(\lambda, \tilde{\lambda}) \leq \epsilon \right\}.
\]

(6)

As mentioned earlier, transmission capacities specify the boundary points of $\mathcal{R}$.

IV. NETWORK OUTAGE PROBABILITIES

In this section, the outage probabilities for the coexisting networks are derived for spectrum overlay and underlay with and without SIC.

For notation in this section, the superscripts $(a)$-$(d)$ identify four cases: $(a)$ spectrum overlay, $(b)$ spectrum underlay, $(c)$ spectrum overlay with SIC and $(d)$ spectrum underlay with SIC.

A. Outage Probabilities: Spectrum Overlay

For spectrum overlay, the SIRs for the coexisting networks are obtained as follows. Let $\Pi^{(a)}_\ell$ denote the process of mobile transmitters accessing the $\ell$th subchannel. Similarly, $\overline{\Pi}^{(a)}_m$ represents the process of ad hoc nodes transmitting over the $m$th subchannel. Since spectrum overlay decouples the networks, mobile receivers are interfered with only by unintended mobile transmitters, and ad hoc receivers by unintended ad hoc transmitters. Without loss of generality, consider the time slot where the receivers $B_0$ and $R_0$ decode signals from the $\ell$th and the $m$th subchannels, respectively, where $\ell \in \mathcal{A}$ and $m \in \mathcal{A}$. The interference power for $B_0$ can be written as $I^{(a)}_\ell := P \sum_{X \in \Pi^{(a)}_\ell} G_X \tilde{R}_X^{-\alpha}$ and that for $R_0$ as $\tilde{I}^{(a)}_m := \tilde{P} \sum_{X \in \overline{\Pi}^{(a)}_m} G_X \tilde{R}_X^{-\alpha}$. It follows that the SIRs are

\[
\text{Cellular} : \quad \text{SIR}^{(a)}_\ell = PW D^{-\alpha} / I^{(a)}_\ell
\]

\[
\text{MANET} : \quad \overline{\text{SIR}}^{(a)}_m = \tilde{P} \tilde{W} d^{-\alpha} / \tilde{I}^{(a)}_m.
\]

(7)

Given frequency-hopping spread spectrum, multiple subchannels assigned to each network contributes a processing gain for reducing the density of transmitters accessing the same subchannel. Specifically, the densities of $\Pi^{(a)}_\ell$ and $\overline{\Pi}^{(a)}_m$ are shown to be inversely proportional to $K$ and $\tilde{K}$, respectively. Define the mark $\mathbf{M}_X$ of a mobile transmitter $X$ as the index of the subchannel $X$ accesses in the current time slot. It follows from the marking theorem [23] that $\Pi^{(a)}_\ell$ is a stationary PPP with the density $\lambda \Pr(\mathbf{M}_X = \ell) = \lambda / K$, which is a thinned process generated by $\Pi^{(a)}$. Similarly, $\overline{\Pi}^{(a)}_m$ is a stationary PPP with the density $\tilde{\lambda} / \tilde{K}$.

Using the above results and (7), the bounds on the outage probabilities for spectrum overlay are obtained using Lemma 1. Note that the distribution of $\text{SIR}^{(a)}_\ell$ is independent of $\ell$. Thus the outage probability for the cellular network is

\[
P_{\text{out}}^{(a)} = \Pr \left( \text{SIR}^{(a)}_\ell < \theta \right)
\]

\[
= \mathbb{E} \left[ \Pr \left( I^{(a)}_\ell > WD^{-\alpha} \theta^{-1} \mid W, D \right) \right].
\]

(8)

Recognize that $I^{(a)}_\ell$ is a shot noise process (see Section III-A). Thus from Lemma 1 and (8), $P_{\text{out}}^{(a)}$ is bounded as shown in the following proposition. Similarly, we obtain the bounds on the outage probability $P_{\overline{\text{out}}}^{(a)}$ for the MANET as given in Proposition 1.

Proposition 1. For spectrum overlay, the outage probabilities are bounded as

\[
\text{Cellular} : \quad \mathbb{E} \left[ P_{\text{out}}^{(a)} \left( WD^{-\alpha} \lambda / K \right) \right] \leq P_{\text{out}}^{(a)}(K, \lambda)
\]

\[
\leq \mathbb{E} \left[ P_{\overline{\text{out}}}^{(a)} \left( WD^{-\alpha} \lambda / K \right) \right]
\]

\[
\text{MANET} : \quad \mathbb{E} \left[ P_{\text{out}}^{(a)} \left( \tilde{W} d^{-\alpha} \tilde{\lambda} / \tilde{K} \right) \right] \leq \tilde{P}_{\text{out}}^{(a)}(\tilde{K}, \tilde{\lambda})
\]

\[
\leq \mathbb{E} \left[ P_{\overline{\text{out}}}^{(a)} \left( \tilde{W} d^{-\alpha} \tilde{\lambda} / \tilde{K} \right) \right]
\]

where $P_{\text{out}}^{(a)}(\cdot, \cdot)$ and $P_{\overline{\text{out}}}^{(a)}(\cdot, \cdot)$ are given in Lemma 1.

Recall from Section II-B that $W$ and $\tilde{W}$ follow the same distributions as the fading factors of data links in the cellular network and the MANET, respectively; $G$ is identically distributed as the fading factors of the interference channels in the coexisting networks.

Finally, the CDF of $D$ is obtained. Note that the event $D \leq t$ is equivalent to that there exists at least one base station within a distance of $t$ from $U_0$. Lemma 2 follows.

Lemma 2. The CDF of $D$ is

\[
P(D \leq t) = 1 - e^{-\pi \lambda b t^2}.
\]

(9)

From (9), the CDF of $D$ depends on the density of base station $\lambda_b$. Intuitively, increasing $\lambda_b$ reduces the cell sizes and thus $D$ and vice versa.

B. Outage Probabilities: Spectrum Underlay

The SIRs for spectrum underlay are obtained as follows. Define the transmitter processes $\Pi^{(b)}$ and $\overline{\Pi}^{(b)}$ similarly as $\Pi^{(a)}$ and $\overline{\Pi}^{(a)}$ in the preceding section. Again, using the marking theorem, $\Pi^{(b)}$ and $\overline{\Pi}^{(b)}$ are shown to be stationary PPPs
with the densities \(\lambda/M\) and \(\tilde{\lambda}/M\), respectively. For spectrum underlay, \(B_0\) accessing the \(b\)th subchannel is interfered with by two processes of transmitters, namely \(\Pi_t(b) \setminus \{U_0\}\) and \(\bar{\Pi}_r(b)\). Thus the interference power for \(B_0\) is given as

\[
I_t^{(b)} := P \sum_{X \in \Pi_t(b) \setminus \{U_0\}} G_X R_X^\alpha + \tilde{P} \sum_{X \in \bar{\Pi}_r(b)} G_X R_X^{-\alpha}. \tag{10}
\]

Similarly, we can write the interference power for \(R_0\) accessing the \(m\)th subchannel as

\[
\tilde{I}_m^{(b)} := P \sum_{X \in \Pi_t(b)} G_X \tilde{R}_X^\alpha + \tilde{P} \sum_{X \in \bar{\Pi}_r(b) \setminus \{T_0\}} G_X \tilde{R}_X^{-\alpha}. \tag{11}
\]

Given the above definitions, the SIRs for spectrum overlay are readily written as

\[
\text{Cellular : } \text{SIR}_{t}^{(b)} = PW D^{-\alpha} / I_t^{(b)} \quad \text{MANET : } \text{SIR}_{t}^{(b)} = \tilde{P} \tilde{W} \tilde{d}^{-\alpha} / \tilde{I}_m^{(b)}
\]

where \(\ell, m \in \{1, 2, \ldots, M\}\).

We consider the combined process \(Y_\ell := \Pi_t(b) \cup \bar{\Pi}_r(b)\). A point in \(Y_\ell\) is either a mobile or ad hoc transmitter. As a result, its transmission power is a random variable with the support set \(\{P, \tilde{P}\}\). To analyze the shot noise process generated by \(Y_\ell\), we derive the distributions of \(Y_\ell\) and the transmission power of the points in \(Y_\ell\). These results are summarized below.

**Lemma 3.** \(Y_\ell\) is a stationary PPP with the density \((\lambda + \tilde{\lambda})/M\). Given \(X \in Y_\ell\), the transmission power \(P_X\) of \(X\) has the following distribution

\[
P_X = \begin{cases} 
P, & \text{w.p. } \frac{\lambda}{\lambda + \tilde{\lambda}} \\
\tilde{P}, & \text{w.p. } \frac{\tilde{\lambda}}{\lambda + \tilde{\lambda}}. \end{cases} \tag{13}
\]

**Proof:** See Appendix A.

In terms of \(Y_\ell\), (10) and (11) can be simplified as

\[
I_t^{(b)} := \sum_{X \in Y_t(b) \setminus \{U_0\}} P_X G_X R_X^\alpha \quad \tilde{I}_m^{(b)} := \sum_{X \in Y_m(b) \setminus \{T_0\}} P_X G_X \tilde{R}_X^\alpha.
\]

Using the above equation and Lemma 3 and following the approach in Section III-A, the bounds on the outage probabilities are obtained and shown below.

**Proposition 2.** For spectrum underlay, the outage probabilities are bounded as follows.

\[
\begin{align*}
\text{Cellular : } & \quad \mathbb{E} \left[ P_{\text{out}}^{\ell} \left( WD^{-\alpha}, \frac{\lambda + \eta^{-\delta} \tilde{\lambda}}{M} \right) \right] \leq P_{\text{out}}(\lambda, \tilde{\lambda}) \leq \mathbb{E} \left[ P_{\text{out}}^{\ell} \left( WD^{-\alpha}, \frac{\lambda + \eta^{-\delta} \tilde{\lambda}}{M} \right) \right] \\
\text{MANET : } & \quad \mathbb{E} \left[ P_{\text{out}}^{\ell} \left( \tilde{W} \tilde{d}^{-\alpha}, \frac{\eta^{-\delta} \lambda + \tilde{\lambda}}{M} \right) \right] \leq \tilde{P}_{\text{out}}(\lambda, \tilde{\lambda}) \leq \mathbb{E} \left[ P_{\text{out}}^{\ell} \left( \tilde{W} \tilde{d}^{-\alpha}, \frac{\eta^{-\delta} \lambda + \tilde{\lambda}}{M} \right) \right]
\end{align*}
\]

where the power ratio \(\eta := P/\tilde{P}\), and \(P_{\text{out}}^{\ell}(\cdot, \cdot)\) and \(P_{\text{out}}^{\ell}(\cdot, \cdot)\) are defined in Lemma 1.

**Proof:** See Appendix B.

Proposition 2 shows that the outage probability for each network depends on the transmitter densities of both networks. This coupling is due to spectrum underlay and the resultant mutual interference between the coexisting networks. As shown in Section V, such coupling may result in smaller transmission capacities for spectrum underlay than those for spectrum overlay. Moreover, Proposition 2 also shows that the outage probabilities for spectrum underlay depend on the transmission power ratio \(\eta\). The effect of \(\eta\) is characterized in Section V.

**C. Outage Probabilities: Spectrum Overlay with SIC**

As in the preceding sections, consider \(B_0\) and \(R_0\) accessing the \(b\)th and \(m\)th subchannels, respectively. SIC effectively removes the strongest interferers for \(B_0\) and \(R_0\). Based on the SIC model in Section II-D, the interferer process for \(B_0\) conditioned on the link power \(WD^{-\alpha}\) is

\[
\Pi_t^{(c)}(WD^{-\alpha}) := \left\{ X \in \Pi_t(a) \setminus \{U_0\} \middle| G_X R_X^\alpha \geq \kappa WD^{-\alpha} \right\} \tag{14}
\]

where \(\Pi_t^{(a)}\) is defined for spectrum overlay in Section IV-A. Similarly, the conditional interferer process for \(R_0\) is given by

\[
\tilde{\Pi}_m^{(c)}(\tilde{W} \tilde{d}^{-\alpha}) := \left\{ X \in \tilde{\Pi}_m(a) \setminus \{T_0\} \middle| G_X \tilde{R}_X^\alpha \geq \kappa \tilde{W} \tilde{d}^{-\alpha} \right\} \tag{15}
\]

where \(\tilde{\Pi}_m^{(c)}\) follows from Section IV-A. Thus the conditional interference power for \(B_0\) and \(R_0\) can be written as

\[
I_t^{(c)}(WD^{-\alpha}) := P \sum_{X \in \Pi_t^{(c)}(WD^{-\alpha})} G_X R_X^\alpha
\]

\[
\tilde{I}_m^{(c)}(\tilde{W} \tilde{d}^{-\alpha}) := \tilde{P} \sum_{X \in \tilde{\Pi}_m^{(c)}(\tilde{W} \tilde{d}^{-\alpha})} G_X \tilde{R}_X^\alpha.
\]

It follows that SIRs for spectrum overlay with SIC are

\[
\text{Cellular : } \text{SIR}_{t}^{(c)} = PW D^{-\alpha} / I_t^{(c)}(WD^{-\alpha})
\]

\[
\text{MANET : } \tilde{\text{SIR}}_{t}^{(c)} = \tilde{P} \tilde{W} \tilde{d}^{-\alpha} / \tilde{I}_m^{(c)}(\tilde{W} \tilde{d}^{-\alpha}).
\]

The above SIR derivation shows that spectrum overlay with and without SIC are closely related. This relation is specified in the following proposition.

**Proposition 3.** For spectrum overlay with SIC, the bounds on the outage probabilities \(P_{\text{out}}^{(c)}\) and \(\tilde{P}_{\text{out}}^{(c)}\) can be modified from those in Proposition I by replacing the functions \(P_{\text{out}}^{\ell}\) and \(\tilde{P}_{\text{out}}^{\ell}\) with \(P_{\text{out}}^{\ell}(\cdot, \cdot)\) and \(\tilde{P}_{\text{out}}^{\ell}(\cdot, \cdot)\) correspondingly, which are defined as

\[
P_{\text{out}}^{\ell}(a, b) := 1 - \exp \left( -\chi \alpha^{-\delta} b \right) \tag{17}
\]

\[
\tilde{P}_{\text{out}}^{\ell}(a, b) := 1 - \xi(a, b) \exp \left( -\chi \alpha^{-\delta} b \right) \tag{18}
\]

where \(\chi := 1 - \theta^{-\delta} \kappa^{-\delta}\).

**Proof:** See Appendix C.

Note that (17) and (18) differ from their counterparts in Lemma 1 only by the factor \(\chi\). The factor \(\chi < 1\) represents the SIC advantage of reducing outage probabilities with respect to the case of no SIC (\(\chi = 1\)). Moreover, decreasing the SIC factor \(\kappa\) reduces \(\chi\) and thus outage probabilities. Nevertheless, \(\kappa\) being too small may invalidate the assumption of perfect SIC. Specifically, small \(\kappa\) implies small SIR for the process of decoding interference prior to its cancelation and potentially results in significant residual interference after SIC [14].
D. Outage Probabilities: Spectrum Underlay with SIC

The outage probabilities for spectrum underlay with SIC can be obtained from those for spectrum overlay in Section IV-B. The procedure is similar to that in the preceding section. Specifically, the conditional interferer processes for \( B_0 \) and \( R_0 \) are defined similarly as in (14) and (15)
\[
\Pi^{(d)}_m(W \tilde{d}^{-\alpha}) := \{ X \in \Upsilon_m \mid \{ T_i \} | P_X G_X R_X^{-\alpha} \geq \kappa \tilde{P} W \tilde{d}^{-\alpha} \} 
\]
where \( \Upsilon_m \) is the combined PPP defined Section IV-B, and \( P_X \) is distributed as in Lemma 3. Thus the SIRs can be written as
\[
\text{Celluar : } \text{SIR}_{(d)} = \frac{P W \tilde{d}^{-\alpha}}{I^{(d)}_m(W \tilde{d}^{-\alpha})} \\
\text{MANET : } \text{SIR}_{(d)} = \frac{\tilde{P} \tilde{W} \tilde{d}^{-\alpha}}{I^{(d)}_m(\tilde{W} \tilde{d}^{-\alpha}).} (19)
\]
where the conditional interference power
\[
I^{(d)}_m(a) := P \sum_{X \in \Pi^{(d)}_{m}(a)} G_X R_X^{-\alpha}, \quad \tilde{I}^{(d)}_m(a) := \tilde{P} \sum_{X \in \tilde{\Pi}^{(d)}_{m}(a)} G_X \tilde{R}_X^{-\alpha}.
\]

The remarks on Proposition 3 are also valid for Proposition 4.

Proposition 4. For spectrum underlay with SIC, the bounds on the outage probabilities \( I^{(d)}_{\text{out}} \) and \( \tilde{I}^{(d)}_{\text{out}} \) can be modified from those in Proposition 2 by replacing the functions \( P^{(d)}_{\text{out}} \) and \( \tilde{P}^{(d)}_{\text{out}} \) defined in Proposition 3.

The remarks on Proposition 3 are also valid for Proposition 4.

V. NETWORK CAPACITY TRADEOFF: ASYMPTOTIC ANALYSIS

Using the results obtained in the preceding section, the tradeoff between the transmission capacities \( C \) and \( \tilde{C} \) of the coexisting networks is characterized in the following theorem for small target outage probability \( \epsilon \to 0 \).

Theorem 1. For \( \epsilon \to 0 \), transmission capacities of the coexisting networks satisfy
\[
\hat{\mu} \hat{C} + \mu \tilde{C} = \frac{M}{\varphi} + O(\epsilon^2) \tag{20}
\]
where the weights \( \hat{\mu} \) and \( \hat{\mu} \) are given as
\[
\hat{\mu}_o = \xi [\tilde{W}]^{-\delta} d^2, \quad \mu_o = 2\xi [W^{-\delta}](\pi \lambda_o)^{-1}, \quad \text{overlay} \quad \hat{\mu}_u = \hat{\mu}_o \lor (\eta^{-\delta} \mu_o), \quad \mu_u = (\tilde{\eta} \hat{\mu}_o) \lor \mu_o, \quad \text{underlay} \tag{21}
\]
and \( \varphi \) depends on if SIC is used
\[
\begin{align*}
\varphi &= 1, \quad \text{no SIC} \\
1 - \theta^{-\delta} \kappa^{-\delta} \leq \varphi &\leq \frac{2}{\beta^2} - \theta^{-\delta} \kappa^{-\delta}, \quad \text{SIC}. \quad \tag{22}
\end{align*}
\]
Proof: See Appendix D.

It can be observed that for spectrum overlay the numbers of subchannels assigned to the coexisting networks, namely \( K \) and \( \tilde{K} \), do not appear in Theorem 1. The reason is that \( \tilde{K} \) and \( K \) are merged into \( M \) based on the equality \( M = K + \tilde{K} \).

To facilitate discussion, define an outage limited network as one whose transmission capacity is achieved with the outage constraint being active. For instance, the cellular network is outage limited if \( P_{\text{out}}(C) = \epsilon \). For spectrum overlay, both the coexisting networks are outage limited. Nevertheless, for spectrum underlay, it is likely that only one of the two networks is outage limited as explained shortly. As implied by the proof for Theorem 1, for spectrum underlay, both networks are outage limited only if \( \mu_u = \hat{\mu}_u \), where \( \mu_u \) and \( \hat{\mu}_u \) are given in (21). Otherwise, \( \mu_u > \hat{\mu}_u \) correspond to only the cellular network being outage limited; \( \mu_u < \hat{\mu}_u \) indicates that only the MANET is outage limited.

Spectrum overlay is shown to be more efficient than spectrum underlay as follows. By definitions, the capacity region in (6) is the region enclosed by the capacity tradeoff curve in (20) and the positive axes of the \( C - \tilde{C} \) coordinates. This region contains all feasible combinations of the densities of the coexisting networks. Thus, the size of the capacity region measures the spectrum-sharing efficiency. The capacity regions for spectrum overlay and underlay are compared in the following corollary.

Corollary 1. For \( \epsilon \to 0 \), the capacity region for spectrum underlay is no larger than that for spectrum overlay. They are identical if and only if the transmission-power ratio is
\[
\eta = \left( \frac{\mu_o}{\tilde{\mu}_o} \right)^\frac{3}{\delta} \tag{23}
\]
where \( \mu_o \) and \( \tilde{\mu}_o \) are given in (21).

Proof: See Appendix E.

Corollary 1 shows that spectrum overlay is potentially more efficient than spectrum underlay due to network coupling for the latter. Specifically, the possibility that a network is not outage limited compromises the efficiency of spectrum underlay, which, however, can be compensated by setting \( \eta \) as given in (23). This optimal value of \( \eta \) ensures that both networks are outage limited for the case of spectrum underlay.

The next corollary specifies the effects of several parameters on transmission capacities of the coexisting networks.

Corollary 2. For \( \epsilon \to 0 \), transmission capacities vary with network parameters as follows.

1) Spectrum overlay: \( C \) increases linearly with the base station density \( \lambda_o \); \( \tilde{C} \) increases inversely with the ad hoc transmitter-receiver distance \( d \).
2) Spectrum underlay: If the cellular network is outage limited, both \( C \) and \( \tilde{C} \) increase linearly with \( \lambda_o \). Otherwise, both \( C \) and \( \tilde{C} \) increase inversely with \( d \).
3) For both spectrum sharing methods, \( C \) and \( \tilde{C} \) increase linearly with \( \epsilon \) and the number of subchannels \( M \), and inversely with \( \varphi \) related to SIC.
Finally, we analyze the transmission-capacity gains due to spatial diversity gains contributed by multi-antennas [24]. To obtain concrete results, the fading factors $W$ and $W$ are assumed to follow the chi-squared distributions with the complex degrees of freedom $L$ and $L$ respectively, which are called the diversity gains [18]. These fading distributions can result from using spatial diversity techniques such as beamforming over multi-antenna i.i.d. Rayleigh fading channels [25]. Thus

$$E|W|^{-2} = \frac{\Gamma(L-\delta)}{\Gamma(L)}, \quad E|W^{-2}| = \frac{\Gamma(L-\delta)}{\Gamma(L)}. \quad (24)$$

The following corollary is obtained by combining Theorem 1, (24) and the following Kershaw’s inequalities [26]

$$x + \frac{s}{2} \begin{array}{c} 1 \\ -s \end{array} < \frac{\Gamma(x+s)}{\Gamma(x+1)} \begin{array}{c} x \frac{1}{2} + \left(s + \frac{1}{4}\right) \end{array} \begin{array}{c} 1 \\ -s \end{array} \quad (25)$$

where $x \geq 1, \quad 0 < s < 1$.

**Corollary 3 (Spatial diversity gain).** Consider the diversity gains per link of $L$ and $L$ for the coexisting cellular and ad hoc networks, respectively.

1) **Spectrum overlay:** The spatial diversity gains multiply $C$ by a factor between $(L-1)^\delta$ and $L^\delta$, and $\tilde{C}$ by a factor between $(\tilde{L}-1)^\delta$ and $\tilde{L}^\delta$.

2) **Spectrum underlay:** The spatial diversity gains multiply both $C$ and $\tilde{C}$ by a factor between $(L-1)^\delta$ and $L^\delta$ if the cellular network is outage limited, or otherwise between $(L-1)^\delta$ and $\tilde{L}^\delta$.

**VI. SIMULATION AND NUMERICAL RESULTS**

In this section, the tightness of the bounds on outage probabilities derived in Section IV is evaluated using simulation. Moreover, the asymptotic transmission capacity tradeoff curves obtained in Theorem 1 are compared with the non-asymptotic ones generated by simulation. The simulation procedure summarized below is similar to that in [27]. The typical base station (or the ad hoc receiver) of the coexisting network lies at the centers of two overlapping disks, which contain interfering transmitters (either ad hoc nodes, users or both) and base stations respectively. Both the transmitters and the base stations follow the Poisson distribution with the mean equal to 200. The disk radiuses are adjusted to

---

Fig. 2. Comparison between the theoretical bounds on outage probabilities and the simulated values. For spectrum underlay, the densities of mobile and ad hoc transmitters are set equal. The sum density is referred to in the figures for spectrum underlay as the transmitter density.
provide the desired densities of transmitters or base stations. The simulation parameters are set as $d = 5$ m, $\theta = 3$ or 4.8 dB, $\alpha = 4$, $\lambda_0 = 10^{-3}$, $\kappa = 2$ dB, and $\eta = 5$ dB.

Fig. 2 compares the bounds on outage probabilities in Section IV and the simulated values. As observed from Fig. 2, for all cases, the outage probabilities converge to their lower bounds as the transmitter densities decrease; the upper and lower bounds differ by approximately constant multiplicative factors. Fig. 2 also shows that SIC reduces outage probabilities by a factor of about 0.54 approximately equal to $\chi$ in Proposition 3 and Proposition 4. Moreover, SIC loosens the bounds on outage probabilities for relatively large transmitter densities since SIC reduces the number of strong interferers to each receiver. Finally, outage probabilities become proportional to transmitter densities as they decrease.

Fig. 3 compares the asymptotic transmission-capacity tradeoff curves in Theorem 1 and those generated by simulations since SIC reduces the number of strong interferers to each receiver. Simulation results show that the derived bounds on outage probabilities are tight and the asymptotic bounds on outage probabilities are tight and the asymptotic tradeoff is valid in the non-asymptotic regime.

VII. Conclusion

In this paper, the transmission-capacity tradeoff between the coexisting cellular and ad hoc networks is analyzed for different spectrum sharing methods. To this end, bounds on outage probabilities for both networks are derived for spectrum overlay and underlay with and without SIC. For small target outage probability, the transmission capacities of the coexisting networks are shown to satisfy a linear equation, whose coefficients are derived for the cases considered above. These results provide a theoretical basis for adapting the node density of the ad hoc network to the dynamic traffic in cellular uplink under the outage constraints. The tradeoff relationship suggests that transmission capacities of the coexisting networks can be increased by adjusting various parameters such as decreasing the distances between intended ad hoc transmitters and receivers, increasing the base station density and link diversity gains, or by employing SIC. In particular, SIC increases the transmission capacities by a linear factor that depends on the interference power threshold for qualifying canceled interferers. Simulation results show that the derived bounds on outage probabilities are tight and the asymptotic linear capacity tradeoff is valid in the non-asymptotic regime.

APPENDIX

A. Proof for Lemma 3

Since $\Pi_\ell$ and $\tilde{\Pi}_\ell$ are both stationary PPPs, it follows from the superposition property of Poisson processes [23] that the combined process $\Upsilon_\ell$ is also a stationary PPP with the density $(\lambda + \tilde{\lambda})/M$, which is equal to the sum of those of $\Pi_\ell$ and $\tilde{\Pi}_\ell$. This proves the first part of Lemma 3.

Consider a point $X \in \Upsilon_\ell$. Let $B(X, r)$ denote a disk centered at $X$ and with the radius $r$, thus $B(X, r) = \{Y \in \mathbb{R}^2 | |Y - X| \leq r \}$. Moreover, the area of $B(X, r)$ is denoted as $A(B(X, r))$. Thus the probability for $P_X = P$ conditioned on $X$ is

$$\Pr(P_X = P \mid X) = \lim_{r \to 0} \frac{\Pr(\Pi_\ell^{(b)} \cap B(X, r) \neq \emptyset)}{\Pr(\Upsilon_\ell \cap B(X, r) \neq \emptyset)} = \lim_{r \to 0} \frac{1 - \exp(\frac{\lambda + \tilde{\lambda}}{M}A(B(X, r)))}{1 - \exp(\frac{\lambda}{M}A(B(X, r)))} = \frac{1 - e^{-\frac{\lambda}{M}}}{\lambda + \tilde{\lambda} - \lambda}.$$

Therefore $\Pr(P_X = \tilde{P} \mid X) = 1 - \Pr(P_X = P \mid X) = \frac{\lambda}{\lambda + \tilde{\lambda}}$. This completes the proof.

B. Proof for Proposition 2

The following proof focuses on the cellular network. The proof for the results concerning the MANET is similar and thus omitted. Based on $\Upsilon_\ell$, we define a marked PPP $\tilde{\Upsilon}_\ell$ [23], where the mark for a point $X \in \Upsilon_\ell$ is its transmission power $P_X$ and the fading factor $G_X$ (with respect to $B_0$)

$$\tilde{\Upsilon} := \{(X, P_X, G_X) \mid X \in \Upsilon_\ell \setminus \{U_0\}, P_X \in \{P, \tilde{P}\}, G_X \in \mathbb{R}^+\}.$$

Following the approach discussed in Section IV-A, $\tilde{\Upsilon}$ is divided into a strong-interferer sub-process conditioned on the link power $WD^{-\alpha}$, denoted as $\tilde{\Upsilon}_S(w, d)$ and given as

$$\tilde{\Upsilon}_S(WD^{-\alpha}) := \{(X, P_X, G_X) \mid (X, P_X, G_X) \in \tilde{\Upsilon}, P_X |X|^{-\alpha}G_X > PWD^{-\alpha} \theta^{-1}\}$$

and the weak-interferer process defined as $\tilde{\Upsilon}_W(WD^{-\alpha}) = \tilde{\Upsilon}/\tilde{\Upsilon}_S(WD^{-\alpha})$. Thus, the sum interference power from weak interferers to the typical base station $B_0$ can be written as

$$I_\ell(WD^{-\alpha}) = \sum_{(X, P_X, G_X) \in \tilde{\Upsilon}_W(WD^{-\alpha})} P_X |X|^{-\alpha}G_X,$$

which is a shot noise processes. To use the results in Lemma 1 for bounding the outage probability, it is sufficient to obtain $\mu_S(WD^{-\alpha}) := \mathbb{E}[\tilde{\Upsilon}_S(WD^{-\alpha})]$, $\mathbb{E}[I_\ell(WD^{-\alpha})]$ and var $[I_\ell(WD^{-\alpha})]$. Using the marking theorem [23] and Lemma 3,

$$\mu_S(a) = \frac{2\pi(\lambda + \tilde{\lambda})}{M} \left[ \Pr(P_X = P) \int_0^{\infty} (\sigma g)^{\frac{1}{2}} f_{G}(g) dg + \Pr(P_X = \tilde{P}) \int_0^{\infty} (\eta^{-1} - \sigma g)^{\frac{1}{2}} f_{G}(g) dg \right]$$

$$= \frac{\zeta a^{-\delta}(\lambda + \eta^{-\delta} \tilde{\lambda})}{M}$$

(26)

where $f_G(g)$ is the probability density function (PDF) of $G_X$ and $\zeta$ is defined in Lemma 1. Next, $\mathbb{E}[I_\ell(w, d)]$ and
(b) Spectrum underlay

Fig. 3. Comparison between the asymptotic and the simulated transmission-capacity tradeoff curves for the coexisting networks using (a) spectrum overlay or (b) spectrum underlay

var [$I^S_w(a, d)$] are derived using Campbell’s theorem [23] and Lemma 3

\[
\mathbb{E}[I^S_w(a)] = \frac{2\pi(\lambda + \lambda b)}{M} \left[ \int_0^{\infty} \int_0^{\infty} (Pr^{-\alpha} g) r f_G(g) dr dg \times \Pr(P_X = P) + \Pr(P_X = \tilde{P}) \right] \times \int_0^{\infty} \int_0^{\infty} \frac{(Pr^{-\alpha} g)^2 r f_G(g) dr dg}{(\eta \cdot a^{-1} \theta g)_{\pi^a}} \]

\[
= \frac{P}{1 - \delta} \left( \frac{\lambda + \lambda b}{M} \right) \zeta(a^{-1}) \delta^{-2} \theta^{-1} \quad (27)
\]

Substituting (26), (27), (28) into (5) and (4) gives the desired results.

C. Proof for Proposition 3

Only the bounds on $P_{\text{out}}$ are proved. The proof for those on $P_{\text{out}}$ is similar and thus omitted. Define the process of strong interferers after SIC as $\Sigma_S(WD^{-\alpha}) := \{ X \in \Pi^{(c)}_\theta \setminus \{U_0\} | \theta^{-1} WD^{-\alpha} \leq PG_X \leq \kappa WD^{-\alpha} \}$. Note that $\kappa WD^{-\alpha} > \theta^{-1} WD^{-\alpha}$ since $\kappa > 1$ and $\theta > 1$. Thus, the process of weak interferers can be defined as $\Sigma_w^c(WD^{-\alpha}) := \Pi^{(c)}_\theta \setminus \Sigma_S(WD^{-\alpha})$, which is observed to be identical to the counterpart for spectrum overlay without SIC considered in Section B. Since $\Sigma_S^c(WD^{-\alpha}) \cap \Sigma_S(WD^{-\alpha}) = \emptyset$, $\Sigma_S^c(WD^{-\alpha})$ and $\Sigma_S(WD^{-\alpha})$ are independent processes based on a basic property of Poisson processes [23]. From the discussion in Section III-A, the exponential terms in (3) and (4) depends only on $\Sigma_S(WD^{-\alpha})$, and the function $\xi(\cdot, \cdot)$ only on $\Sigma_S^c(WD^{-\alpha})$. Since $\Sigma^c_S(w, d)$ is invariant to SIC, and $\Sigma_S^c(w, d)$ and $\Sigma_S(w, d)$ are independent, the bounds on $P_{\text{out}}$ in Lemma 1 can be extended to the case of SIC by replacing the exponential terms in (3) and (4) with $\exp(-\mathbb{E}[[\Sigma_S(w, d)])$, where $\mathbb{E}[[\Sigma_S(w, d)])$ is obtained using Campbell’s theorem

\[
\mathbb{E}[[\Sigma_S(w, d)]) = 2\pi\lambda \int_0^{\infty} \int_0^{(\theta w^{-1} d^{-\alpha} g)^{\frac{1}{2}}} \exp((\xi(\cdot, \cdot) r f_G(g) dr dg)
\]

\[
= \chi \zeta w^{-\delta} d^{\frac{\lambda}{K}} \quad (29)
\]

and $\chi$ is defined in the statement of the proposition.

D. Proof for Theorem 1

1) Spectrum Overlay: The convergence $\epsilon \to 0$ implies $\lambda \to 0$ and $\lambda \to 0$. Using the series representation of the PDF of a power shot-noise process [21], the asymptotes of the outage probabilities follow from [4, Theorem 2]

\[
P_{\text{out}} = \frac{\lambda}{K} \mathbb{E}[W^{\delta}] \mathbb{E}[D^2] + O(\lambda^2)
\]

\[
P_{\text{out}} = \frac{\lambda}{K} \mathbb{E}[W^{\delta}] \mathbb{E}[D^2] + O(\lambda^2).
\]

By using Lemma 2, the term $\mathbb{E}[D^2]$ in (29) is obtained as follows

\[
\mathbb{E}[D^2] = \int_0^{\infty} 2\pi \lambda t e^{-\pi \lambda t^2} dt
\]

\[
= \int_0^{\infty} \pi \lambda t e^{-\pi \lambda t^2} dt = \frac{\Gamma(2)}{\pi \lambda b} = \frac{2}{\pi \lambda b}. \quad (29)
\]

Combining (5), (29), and (29) gives the desired asymptotic capacity tradeoff function for spectrum overlay.
2) Spectrum Underlay: By using the series expression of the PDF of the power shot noise [21] as well as Proposition 1, 
\[ P_{\text{out}}(\lambda, \tilde{\lambda}) = \frac{\lambda + \eta^{-\delta} \tilde{\lambda}}{M} \zeta \left( \mathbb{E}[W^{-\delta}] | \mathbb{E}[D^2] \right) + O(\max(\lambda^2, \tilde{\lambda}^2)) \] 
\[ P_{\text{out}}(\lambda, \tilde{\lambda}) = \eta^{\delta} \frac{\lambda + \tilde{\lambda}}{M} \zeta \left( \mathbb{E}[W^{-\delta}] | \mathbb{E}[D^2] \right) + O(\max(\lambda^2, \tilde{\lambda}^2)). \] (31) 
For \( \epsilon \to 0 \), the transmission capacities \( C \) and \( \tilde{C} \) satisfy the constraints \( P_{\text{out}}(C/M, \tilde{C}/M) \leq \epsilon \) and \( P_{\text{out}}(\tilde{C}/M) \leq \epsilon \). By combining these constraints, (30) and (31) 
\[ C + \eta^{-\delta} \tilde{C} \geq \frac{M}{\zeta} \max \left( \mathbb{E}[W^{-\delta}] | \mathbb{E}[D^2], \eta^\delta \mathbb{E}[W^{-\delta}] | \mathbb{E}[D^2] \right) = \epsilon + O(\epsilon^2). \] 
The desired result follows from the above equation.

3) Spectrum Sharing with SIC: Consider spectrum overlay with SIC. By canceling the strongest interferers using SIC, the PDF “upper-tail” of the power shot noise process is trimmed and its series expansion is difficult to find [21]. Nevertheless, the asymptotic transmission capacities can be characterized by expanding the bounds on \( P_{\text{out}}(\lambda, \tilde{\lambda}) \) in Proposition 3. Specifically 
\[ P_{\text{out}}(\lambda/K) = \frac{\lambda}{K} \zeta \mathbb{E}[W^{-\delta}] | \mathbb{E}[D^2] + O(\lambda^2) \] 
\[ P_{\text{out}}(\lambda/K) = 1 - \mathbb{E} \left[ \left( 1 - \frac{\delta}{2 - \delta} \mathbb{E}[W^{-\delta}] | \mathbb{E}[D^2] + O(\lambda^2) \right) \right] \] 
\[ = \left( \frac{2}{2 - \delta} - \theta^{-\delta} \kappa^{-\delta} \right) \zeta \mathbb{E}[W^{-\delta}] | \mathbb{E}[D^2] + O(\lambda^2). \] Thus 
\[ P_{\text{out}}(\lambda/K) = \chi \mathbb{E}[W^{-\delta}] | \mathbb{E}[D^2] \frac{\lambda}{K} \lambda \] (32) 
where \( 1 - \theta^{-\delta} \kappa^{-\delta} \leq \chi \leq \left( \frac{2^{1/\delta} - \theta^{-\delta} \kappa^{-\delta}}{2 - \delta} \right). \) Similarly 
\[ P_{\text{out}}(\lambda/K) = \chi \mathbb{E}[W^{-\delta}] | \mathbb{E}[D^2] \frac{\lambda}{K} \lambda \] (33) 
The desired results for spectrum overlay with SIC are obtained by combining (5), (32), and (33). The results for spectrum underlay with SIC are derived following a similar procedure.

E. Proof for Corollary 1

First, the capacity region for spectrum underlay is proved to be no larger than for spectrum overlay. It is sufficient to prove that \( \mu_o \geq \mu_o \) and \( \tilde{\mu}_o \geq \tilde{\mu}_o \), which follow from (21). Next, substituting (23) into (21) results in \( \mu_o = \mu_o \) and \( \tilde{\mu}_o = \tilde{\mu}_o \). This proves the second claim in the theorem statement.

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