A Process Algebra for Games

Yong Wang
College of Computer Science and Technology,
Beijing University of Technology, Beijing, China

Abstract. Using formal tools in computer science to describe games is an interesting problem. We give games, exactly two person games, an axiomatic foundation based on the process algebra ACP (Algebra of Communicating Process). A fresh operator called opponent’s alternative composition operator (OA) is introduced into ACP to describe game trees and game strategies, called GameACP. And its sound and complete axiomatization system is naturally established. To model the outcomes of games (the co-action of the player and the opponent), correspondingly in GameACP, the execution of GameACP processes, another operator called playing operator (PO) is extended into GameACP. We also establish a sound and complete axiomatization system for PO. Finally, we give the correctness theorem between the outcomes of games and the deductions of GameACP processes.

Keywords: Games; Process Algebra; Algebra of Communicating Processes; Axiomatization

1. Introduction

Game theory [1] is a great theoretical outcome in 20th century and is used widely in interpreting social and economic phenomena. Because of universality of games, games are also widely used in science and engineering. On one side, wide using of game theory into various application domain has gained great successes. On the other side, using different tools to interpret game theory is also an interesting direction.

In computer science, there are various kind of tools to capture the computation concept which concerns the nature of computability. There is no any doubt that process algebra [2] is one of the most influential tools. Milner’s CCS (Calculus of Communicating Systems) [3], Hoare’s CSP (Communicating sequential processes) [4] and ACP (Algebra of Communicating Process) [5] are three dominant forms of process algebra, and there are also several kinds of process calculi. Such process algebras often have a formal deductive system based equational logic and a formal semantics model based on labeled transition system, can be suitable to reason about the behaviors of parallel and distributed systems.

The combination of games and computer science is a fascinating direction, and it gains great successes, such as the so-called game semantics [9]. Since there exist lots of game phenomena in parallel and distributed systems, especially interactions between a system and its environment, interactions among system
components, and interactions among system components and outside autonomous software agents, the introduction of games into traditional computation tools, such as the above mentioned process algebra, is attractive and valuable. The extended computation tools with supporting of games can be used to reason about the behaviors of systems in a new viewpoint.

Using these computation tools to give game theory an interpretation is an interesting problem [26]. This direction has subtle difference with introducing games and ideas of games into the computation tools. It not only can make these tools having an additional ability to using games in computation, but also gives game theory a new interpretation which will help the human to capture the nature of games and also the development of game theory.

In this paper, we give games an axiomatic foundation called GameACP based on process algebra ACP [5]. Because of ACP’s clear semantic model based on bisimulation or rooted branching bisimulation [6] and well designed axiomatic system, GameACP inherits ACP’s advantages in an elegant and convenient way. This is the first step to use computation tools to interpret games in an axiomatic fashion as we known.

This paper is organized as follows. In Section 2, we analyze the related works. Application scenarios called SubmittingOrder, Transacting and Purchasing are illustrated in Section 3. In Section 4, we briefly introduce some preliminaries, including equational logic, structural operational semantics, process algebra ACP and also games. In Section 5, the extension of BPA for games is done, which is called GameBPA, including opponent’s alternative composition operator and another new operator called playing operator of GameACP processes, and their transition rules and the properties of the extension, and we design the axioms of opponent’s alternative composition and playing operator, including proving the soundness and completeness of the axiomatical system. In Section 6, we do another extension based on ACP, which is called GameACP. We give the correctness theorem in Section 7. In Section 8, we show the support for multi-person games. Finally, conclusions are drawn in Section 9.

2. Related Works

As mentioned above, the combination of computation tools and game semantics includes two aspects: one is introducing games or idea of games into these computation languages or tools to give them a new viewpoint, and the other is using these computation tools to interpret games. The first one has plenty of works and gained great successes, but the second one has a few works [25] [26] as we known. We introduce the main works on the two ones in the following.

It is no doubt that the so-called game semantics gained the most great successes in introducing games into computer science. Game semantics models computations as playing of some kind of games, especially two person games. In the two person game, the Player (P) represents the system under consideration and the Opponent (O) represents the environment in which the system is located. In game semantics, the behaviors of the system (acts as P) and the environment (acts as O) are explicitly distinguished. So the interactions between the system and the environment can be captured by game plays between the opponent and the player, and successful interactions can be captured by the game strategy.

For example, the function \( f(x) = 3x + 10 \) where \( x \in \mathbb{N} \) can be deemed as the games played in Fig. 1. Firstly, the opponent (the environment) moves to ask the value of \( f(x) \), then the player (the function) moves to ask the value of \( x \), and then the opponent moves to answer that the value of \( x \) is 5, the player moves to answer that the value of \( f(x) \) is 25 finally.

Game semantics has gained great successes in modeling computations, such as an initial success of modeling the functional programming language PCF (Programming Computable Functions) [7] [8] [9], multiplicative linear logic [10], idealized Algol [11], general reference [12], etc. To model concurrency in computer science with game semantics, a new kind of game semantics called asynchronous game [13] [14] [15] [16]

\[
\begin{align*}
\text{N} &\rightarrow \text{N} \\
q &\rightarrow \text{O} \\
5 &\rightarrow \text{O} \\
25 &\rightarrow \text{P}
\end{align*}
\]

Fig. 1. Game Semantics for the Function \( f(x) = 3x + 10 \) where \( x \in \mathbb{N} \).
[17] is established and a bridge between the asynchronous game and traditional game semantics is founded. Moreover, asynchronous games perfectly model propositional linear logic and get a full completeness result. Another kind of game semantics to describe concurrency is concurrent game [19] [20], and a work to bridge asynchronous game and concurrent game is introduced in [18].

Algorithmic game semantics [21] is the premise of implementation of game semantics for further automatic reasoning machine based on some specific game semantics model. And game semantics can be used to establish the so-called interaction semantics [22] among autonomous agents, and can be used to model and verify compositional software [23] [24].

Game semantics utilizes such dialogue games to model interactions between the system under consideration and the environment, and pays more attention to the playing process of the two players. And it develops some key concepts which have correspondents to traditional computation concepts, such as innocence to context independence and bracketing to well-structured property. Different to game semantics, there are also several works to use computation tools to model games of two agents.

Game-CTR [25] introduces games into CTR (Concurrent Transaction Logic) to model and reason about runtime properties of workflows that are composed of non-cooperative services – such as Web Services. Game-CTR includes a model and proof theory which can be used to specify executions under some temporal and causality constraints, and also a game solver algorithm to convert such constraints into other equivalent Game-CTR formulas to be executed more efficiently. Chatzikokolakis et al [26] develop a game semantics for a certain kind of process calculus with two interacting agents. Games and strategies on this process calculus are defined, and strategies of the two agents determine the execution of the process. And also, a certain class of strategies correspond to the so-called syntactic schedulers of Chatzikokolakis and Palamidessi. In these works, the games used are not dialogue games, and there are no interactions such as questions and answers and also no winning concept.

More like Game-CTR [25] and Chatzikokolakis’s work [26], we introduce games into ACP, or we use ACP to give games an interpretation. Unlike [25] and [26], our work GameACP is an attempt to do axiomatization with an extension of process algebra ACP for games. It has the following characteristics:

1. As a result of axiomatization, GameACP has not only an equational logic, but also a bisimulation semantics.
2. The conclusions of GameACP are without any assumption or restriction, such as epistemic restrictions on strategies in [26].
3. Though the discussions of GameACP are aimed at two person games, GameACP can be naturally used in multi-person games.
4. GameACP provides a new viewpoint to model interactions between one autonomous agent and other autonomous agents, and can be used to reason about the behaviors of parallel and distributed systems with game theory supported.

3. Application Scenarios

In this section, we will illustrate the universality of game phenomena that exist in computer systems through three different examples. Using these examples throughout this paper, we illustrate our core concepts and ideas.

3.1. Graphical User Interface – SubmittingOrder

Graphical user interface is the most popular human-machine interface now. Fig. 2-a illustrates the flow of submitting an order for a user through a graphical interface. The flow is as follows.

1. The interface program starts.
2. The user writes an order via the interface.
3. When the order is completed, the user can decide to submit the order or cancel the order.
4. If the order is submitted, then the order is stored and the program terminates.
5. If the order is canceled, then the program terminates.
In this SubmittingOrder example, the selection of submitting or canceling the order is done by the user, but not the program according to its inner states. This situation is suitable to be captured by use of a game between the user and the interface program.

3.2. Transaction Processing – Transacting

Transaction processing is the core mechanism of database and business processing. Traditional transaction has ACID properties and is illustrated in Fig. 3-a. The flow of traditional database transaction is following.

1. The transaction is started.
2. Operations on the data are done by a user.
3. The user can decide to submit the transaction or abort the transaction.
4. If the transaction is submitted, the data are permanently stored and the transaction terminates.
5. If the transaction is aborted, the data are rollbacked and the transaction also terminates.

In this Transaction example, the selection of submitting or aborting the transaction is also done by the user, but not the database or business processing system according to its inner states. This situation is also suitable to be modeled by use of a game between the user and the database or business processing system.
3.3. Web Service Composition – Purchasing

Web Service is a quite new distributed object and Web Service composition created new bigger Web Services from the set of smaller existing Web Services. A composite Web Service is defined by use of a kind of Web Service composition language and is executed by interpreting the definition of the composite Web Service. WS-BPEL[27] is a kind of such language. In WS-BPEL, the atomic function units are called atomic activities and the corresponding structural activities define the control flow among these atomic activities. Pick activity is a kind of choice structural activity in which the decision is made by outside autonomous Web Services, and is different from the If activities, in which the decision is made by the composite Web Service according to its inner states.

In Fig. 4, a composite Web Service implements the following flow of purchasing goods and can be used by a user through a user agent Web Service.

1. The composite Web Service is started by a user through a user agent Web Service.
2. The user shops for goods.
3. After the shopping is finished, the user can select the shipping way: by truck, by train or by plane.
4. If the truck way is selected, then the user should order a truck and pay online for the fees.
5. If the train way is selected, then user should order a train and pay online for the fees.
6. If the plane way is selected, then the user should order a plane, if the money amount is greater than 1000 dollars, he/she should pay offline for the fees; and if not, he/she should pay online.

The WS-BPEL skeleton of the Purchasing composite Web Service is shown in Fig. 5. Note that the first choice is modeled by use of a Pick activity and the second choice is modeled by use of an If activity.

In this Purchasing composite Web Service, the selection of shipping ways is also done by the user through a user agent Web Service, and not the composite Web Service according to its inner states. This situation is also suitable to be modeled by use of a game between the user (or the user agent Web Service) and the composite Web Service.

4. Preliminaries

In this section, we introduce some preliminaries, including equational logic, structural operational semantics, process algebra ACP and games.
Fig. 5. WS-BPEL Skeleton of Purchasing Goods Example.

In the following, the variables \( x, x', y, y', z, z' \) range over the collection of process terms, the variables \( \nu, \omega \) range over the set \( A \) of atomic actions, \( a, b, c \in A \), \( s, s', t, t' \) are closed items, \( \tau \) is the special constant silent step, \( \delta \) is the special constant deadlock, \( \xi \) is the special constant non-determinacy, and the predicate \( \frac{a}{a} \rightarrow \sqrt{\cdot} \) represents successful termination after execution of the action \( a \).

4.1. Equational Logic

We introduce some basic concepts about equational logic briefly, including signature, term, substitution, axiomatization, equality relation, model, term rewriting system, rewrite relation, normal form, termination, weak confluence and several conclusions. These concepts are coming from [5], and are introduced briefly as follows. About the details, please see [5].

Definition 4.1.1 (Signature). A signature \( \Sigma \) consists of a finite set of function symbols (or operators) \( f, g, \cdots \), where each function symbol \( f \) has an arity \( ar(f) \), being its number of arguments. A function symbol \( a, b, c, \cdots \) of arity zero is called a constant, a function symbol of arity one is called unary, and a function symbol of arity two is called binary.

Definition 4.1.2 (Term). Let \( \Sigma \) be a signature. The set \( T(\Sigma) \) of (open) terms \( s, t, u, \cdots \) over \( \Sigma \) is defined as the least set satisfying: (1) each variable is in \( T(\Sigma) \); (2) if \( f \in \Sigma \) and \( t_1, \cdots, t_{ar(f)} \in T(\Sigma) \), then \( f(t_1, \cdots, t_{ar(f)} \in T(\Sigma)) \). A term is closed if it does not contain variables. The set of closed terms is denoted by \( T(\Sigma) \).

Definition 4.1.3 (Substitution). Let \( \Sigma \) be a signature. A substitution is a mapping \( \sigma \) from variables to the set \( T(\Sigma) \) of open terms. A substitution extends to a mapping from open terms to open terms: the term \( \sigma(s) \) is obtained by replacing occurrences of variables \( x \) in \( t \) by \( \sigma(x) \). A substitution \( \sigma \) is closed if \( \sigma(x) \in T(\Sigma) \) for all variables \( x \).

Definition 4.1.4 (Axiomatization). An axiomatization over a signature \( \Sigma \) is a finite set of equations, called axioms, of the form \( s = t \) with \( s, t \in T(\Sigma) \).

Definition 4.1.5 (Equality relation). An axiomatization over a signature \( \Sigma \) induces a binary equality relation \( = \) on \( T(\Sigma) \) as follows. (1) (Substitution) If \( s = t \) is an axiom and \( \sigma \) a substitution, then \( \sigma(s) = \sigma(t) \). (2) (Equivalence) The relation \( = \) is closed under reflexivity, symmetry, and transitivity. (3) (Con-
text) The relation $=$ is closed under contexts: if $t = u$ and $f$ is a function symbol with $\text{ar}(f) > 0$, then $f(s_1, \ldots, s_i-1, t, s_{i+1}, \ldots, s_{\text{ar}(f)}) = f(s_1, \ldots, s_{i-1}, u, s_{i+1}, \ldots, s_{\text{ar}(f)})$.

**Definition 4.1.6 (Model).** Assume an axiomatization $\mathcal{E}$ over a signature $\Sigma$, which induces an equality relation $\equiv$. A model for $\mathcal{E}$ consists of a set $\mathcal{M}$ together with a mapping $\phi : \mathcal{T}(\Sigma) \to \mathcal{M}$. (1) $(\mathcal{M}, \phi)$ is sound for $\mathcal{E}$ if $s = t$ implies $\phi(s) \equiv \phi(t)$ for $s, t \in \mathcal{T}(\Sigma)$; (2) $(\mathcal{M}, \phi)$ is complete for $\mathcal{E}$ if $\phi(s) \equiv \phi(t)$ implies $s = t$ for $s, t \in \mathcal{T}(\Sigma)$.

**Definition 4.1.7 (Term rewriting system).** Assume a signature $\Sigma$. A rewrite rule is an expression $s \to t$ with $s, t \in \mathcal{T}(\Sigma)$, where: (1) the left-hand side $s$ is not a single variable; (2) all variables that occur at the right-hand side $t$ also occur in the left-hand side $s$. A term rewriting system (TRS) is a finite set of rewrite rules.

**Definition 4.1.8 (Rewrite relation).** A TRS over a signature $\Sigma$ induces a one-step rewrite relation $\to$ on $\mathcal{T}(\Sigma)$ as follows. (1) (Substitution) If $s \to t$ is a rewrite rule and $\sigma$ a substitution, then $\sigma(s) \to \sigma(t)$. (2) (Context) The relation $\to$ is closed under contexts: if $t \to u$ and $f$ is a function symbol with $\text{ar}(f) > 0$, then $f(s_1, \ldots, s_{i-1}, t, s_{i+1}, \ldots, s_{\text{ar}(f)}) \to f(s_1, \ldots, s_{i-1}, u, s_{i+1}, \ldots, s_{\text{ar}(f)})$. The rewrite relation $\to^*$ is the reflexive transitive closure of the one-step rewrite relation $\to$: (1) if $s \to t$, then $s \to^* t$; (2) $t \to^* t$; (3) if $s \to^* t$ and $t \to^* u$, then $s \to^* u$.

**Definition 4.1.9 (Normal form).** A term is called a normal form for a TRS if it cannot be reduced by any of the rewrite rules.

**Definition 4.1.10 (Termination).** A TRS is terminating if it does not induce infinite reductions $t_0 \to t_1 \to t_2 \to \ldots$

**Definition 4.1.11 (Weak confluence).** A TRS is weakly confluent if for each pair of one-step reductions $s \to t_1$ and $s \to t_2$, there is a term $u$ such that $t_1 \to^* u$ and $t_2 \to^* u$.

**Theorem 4.1.1 (Newman’s lemma).** If a TRS is terminating and weakly confluent, then it reduces each term to a unique normal form.

**Definition 4.1.12 (Commutativity and associativity).** Assume an axiomatization $\mathcal{E}$. A binary function symbol $f$ is commutative if $\mathcal{E}$ contains an axiom $f(x, y) = f(y, x)$ and associative if $\mathcal{E}$ contains an axiom $f(f(x, y), z) = f(x, f(y, z))$.

**Definition 4.1.13 (Convergence).** A pair of terms $s$ and $t$ is said to be convergent if there exists a term $u$ such that $s \to^* u$ and $t \to^* u$.

Axiomatizations can give rise to TRSs that are not weakly confluent, which can be remedied by Knuth-Bendix completion [29]. It determines overlaps in left hand sides of rewrite rules, and introduces extra rewrite rules to join the resulting right hand sides, which are called critical pairs.

**Theorem 4.1.2.** A TRS is weakly confluent if and only if all its critical pairs are convergent.

### 4.2. Structural Operational Semantics

The concepts about structural operational semantics include labelled transition system (LTS), transition system specification (TSS), transition rule and its source, source-dependent, conservative extension, fresh operator, panth format, congruence, bisimulation, etc. These concepts are coming from [5], and are introduced briefly as follows. About the details, please see [6].

We assume a non-empty set $S$ of states, a finite, non-empty set of transition labels $A$ and a finite set of predicate symbols.

**Definition 4.2.1 (Labeled transition system).** A transition is a triple $(s, a, s')$ with $a \in A$, or a pair $(s, P)$ with $P$ a predicate, where $s, s' \in S$. A labeled transition system (LTS) is possibly infinite set of transitions. An LTS is finitely branching if each of its states has only finitely many outgoing transitions.

**Definition 4.2.2 (Transition system specification).** A transition rule $\rho$ is an expression of the form $\frac{H}{\pi}$, with $H$ a set of expressions $t \overset{a}{\to} t'$ and $tP$ with $t, t' \in \mathcal{T}(\Sigma)$, called the (positive) premises of $\rho$, and $\pi$ an expression $t \overset{a}{\not\to} t'$ or $tP$ with $t, t' \in \mathcal{T}(\Sigma)$, called the conclusion of $\rho$. The left-hand side of $\pi$ is called the source of $\rho$. A transition rule is closed if it does not contain any variables. A transition system specification (TSS) is a (possible infinite) set of transition rules.

**Definition 4.2.3 (Proof).** A proof from a TSS $T$ of a closed transition rule $\frac{H}{\pi}$ consists of an upwardly branching tree in which all upward paths are finite, where the nodes of the tree are labelled by transitions such that: (1) the root has label $\pi$; (2) if some node has label $l$, and $K$ is the set of labels of nodes directly
above this node, then (a) either \( K \) is the empty set and \( l \in H \), (b) or \( \frac{K}{T} \) is a closed substitution instance of a transition rule in \( T \).

**Definition 4.2.4 (Generated LTS).** We define that the LTS generated by a TSS \( T \) consists of the transitions \( \pi \) such that \( \frac{\pi}{\pi} \) can be proved from \( T \).

**Definition 4.2.5.** A set \( N \) of expressions \( t \rightarrow^a \) and \( t \rightarrow P \) (where \( t \) ranges over closed terms, \( a \) over \( A \) and \( P \) over predicates) hold for a set \( \mathcal{S} \) of transitions, denoted by \( \mathcal{S} \models N \), if: (1) for each \( t \rightarrow^a \in N \) we have that \( t \rightarrow^a t' \notin \mathcal{S} \) for all \( t' \in T(\Sigma) \); (2) for each \( t \rightarrow P \in N \) we have that \( tP \notin \mathcal{S} \).

**Definition 4.2.6 (Three-valued stable model).** A pair \( (\mathcal{C}, \mathcal{U}) \) of disjoint sets of transitions is a three-valued stable model for a TSS \( T \) if it satisfies the following two requirements: (1) a transition \( \pi \) is in \( \mathcal{C} \) if and only if \( T \) proves a closed transition rule \( \frac{\pi}{\pi} \) where \( N \) contains only negative premises and \( \mathcal{C} \cup U \models N \); (2) a transition \( \pi \) is in \( \mathcal{C} \cup \mathcal{U} \) if and only if \( T \) proves a closed transition rule \( \frac{\pi}{\pi} \) where \( N \) contains only negative premises and \( \mathcal{C} \lor N \).

**Definition 4.2.7 (Ordinal number).** The ordinal numbers are defined inductively by: (1) 0 is the smallest ordinal number; (2) each ordinal number \( \alpha \) has a successor \( \alpha + 1 \); (3) each sequence of ordinal number \( \alpha < \alpha + 1 < \alpha + 2 < \cdots \) is capped by a limit ordinal \( \lambda \).

**Definition 4.2.8 (Positive after reduction).** A TSS is positive after reduction if its least three-valued stable model does not contain unknown transitions.

**Definition 4.2.9 (Stratification).** A stratification for a TSS is a weight function \( \phi \) which maps transitions to ordinal numbers, such that for each transition rule \( \rho \) with conclusion \( \pi \) and for each closed substitution \( \sigma ; (1) \) for positive premises \( t \rightarrow^a t' \) and \( tP \) of \( \rho \), \( \phi(\sigma(t) \rightarrow^a \sigma(t')) \leq \phi(\sigma(\pi)) \) and \( \phi(\sigma(t)P \leq \phi(\sigma(\pi)) \), respectively; (2) for negative premise \( t \rightarrow^a \) and \( tP \) of \( \rho \), \( \phi(\sigma(t) \rightarrow^a t') \geq \phi(\sigma(\pi)) \) for all closed terms \( t' \) and \( \phi(\sigma(t)P \geq \phi(\sigma(\pi)) \), respectively.

**Theorem 4.2.1.** If a TSS allows a stratification, then it is positive after reduction.

**Definition 4.2.10 (Process graph).** A process (graph) \( p \) is an LTS in which one state \( s \) is elected to be the root. If the LTS contains a transition \( s \xrightarrow{t} s' \), then \( p \rightarrow_1 p' \) where \( p' \) has root state \( s' \). Moreover, if the LTS contains a transition \( sP \), then \( pP \). (1) A process \( p_0 \) is finite if there are only finitely many sequences \( p_0 \xrightarrow{a_1} p_1 \xrightarrow{a_2} \cdots \xrightarrow{a_k} p_k \). (2) A process \( p_0 \) is regular if there are only finitely many processes \( p_k \) such that \( p_0 \xrightarrow{a_1} p_1 \xrightarrow{a_2} \cdots \xrightarrow{a_k} p_k \).

**Definition 4.2.11 (Bisimulation).** A bisimulation relation \( \mathcal{B} \) is a binary relation on processes such that: (1) if \( pBq \) and \( p \xrightarrow{a} p' \) then \( q \xrightarrow{a} q' \) with \( p'Bq' \); (2) if \( pBq \) and \( q \xrightarrow{a} q' \) then \( p \xrightarrow{a} p' \) with \( p'Bq' \); (3) if \( pBq \) and \( pP \), then \( qP \); (4) if \( pBq \) and \( qP \), then \( pP \). Two processes \( p \) and \( q \) are bisimilar, denoted by \( p \leftrightarrow q \) if there is a bisimulation relation \( \mathcal{B} \) such that \( pBq \).

**Definition 4.2.12 (Congruence).** Let \( \Sigma \) be a signature. An equivalence relation \( \mathcal{B} \) on \( T(\Sigma) \) is a congruence if for each \( f \in \Sigma \), if \( sBf_i \) for \( i \in \{1, \ldots, \text{ar}(f)\} \), then \( f(s_1, \ldots, s_{\text{ar}(f)})Bf(t_1, \ldots, t_{\text{ar}(f)}) \).

**Definition 4.2.13 (Pan format).** A transition rule \( \rho \) is in pan format if it satisfies the following three restrictions: (1) for each positive premise \( t \rightarrow^a t' \) of \( \rho \), the right-hand side \( t' \) is single variable; (2) the source of \( \rho \) contains no more than one function symbol; (3) there are no multiple occurrences of the same variable at the right-hand side of positive premises and in the source of \( \rho \). A TSS is said to be in pan format if it consists of pan rules only.

**Theorem 4.2.2.** If a TSS is positive after reduction and in pan format, then the bisimulation equivalence that it induces is a congruence.

**Definition 4.2.14 (Branching bisimulation).** A branching bisimulation relation \( \mathcal{B} \) is a binary relation on the collection of processes such that: (1) if \( pBq \) and \( p \xrightarrow{a} p' \) then either \( a \equiv \tau \) and \( p'Bq' \) or there is a sequence of (zero or more) \( \tau \)-transitions \( q \rightarrow \cdots \rightarrow q_0 \) such that \( pBq_0 \) and \( q_0 \xrightarrow{a} q' \) with \( p'Bq' \); (2) if \( pBq \) and \( q \xrightarrow{a} q' \) then either \( a \equiv \tau \) and \( pBq' \) or there is a sequence of (zero or more) \( \tau \)-transitions \( p \rightarrow \cdots \rightarrow p_0 \) such that \( p_0Bq_0 \) and \( q_0 \xrightarrow{a} q' \) with \( p'Bq' \); (3) if \( pBq \) and \( p \rightarrow P \), then there is a sequence of (zero or more) \( \tau \)-transitions \( q \rightarrow \cdots \rightarrow q_0 \) such that \( pBq_0 \) and \( q_0P \); (4) if \( pBq \) and \( q \rightarrow P \), then there is a sequence of (zero or more) \( \tau \)-transitions \( p \rightarrow \cdots \rightarrow p_0 \) such that \( p_0Bq_0 \) and \( p_0P \). Two processes \( p \) and \( q \) are branching bisimilar, denoted by \( p \leftrightarrow_B q \), if there is a branching bisimulation relation \( \mathcal{B} \) such that \( pBq \).

**Definition 4.2.15 (Rooted branching bisimulation).** A rooted branching bisimulation relation \( \mathcal{B} \) is a binary relation on processes such that: (1) if \( pBq \) and \( p \xrightarrow{a} p' \) then \( q \xrightarrow{a} q' \) with \( p'Bq' \); (2) if \( pBq \) and \( q \xrightarrow{a} q' \) then \( p \xrightarrow{a} p' \) with \( p'Bq' \); (3) if \( pBq \) and \( p \rightarrow P \), then \( qP \); (4) if \( pBq \) and \( q \rightarrow P \), then \( pP \). Two processes \( p \)}
and $q$ are rooted branching bisimilar, denoted by $p \underset{\tau \rightarrow}{\sim} q$, if there is a rooted branching bisimulation relation $B$ such that $p B q$.

**Definition 4.2.16 (Lookahead).** A transition rule contains lookahead if a variable occurs at the left-hand side of a premise and at the right-hand side of a premise of this rule.

**Definition 4.2.17 (Patience rule).** A patience rule for the $i$-th argument of a function symbol $f$ is a panth rule of the form

$$x_{i} \xrightarrow{\tau} y$$

$$f(x_{1}, \cdots, x_{ar(f)}) \xrightarrow{\tau} f(x_{1}, \cdots, x_{i-1}, y, x_{i+1}, \cdots, x_{ar(f)})$$

**Definition 4.2.18 (RBB cool format).** A TSS $T$ is in RBB cool format if the following requirements are fulfilled. (1) $T$ consists of panth rules that do not contain lookahead. (2) Suppose a function symbol $f$ occurs at the right-hand side the conclusion of some transition rule in $T$. Let $\rho \in T$ be a non-patience rule with source $f(x_{1}, \cdots, x_{ar(f)})$. Then for $i \in \{1, \cdots, ar(f)\}$, $x_{i}$ occurs in no more than one premise of $\rho$, where this premise is of the form $x_{i}P$ or $x_{i} \xrightarrow{\tau} y$ with $\alpha \neq \tau$. Moreover, if there is such a premise in $\rho$, then there is a patience rule for the $i$-th argument of $f$ in $T$.

**Theorem 4.2.3.** If a TSS is positive after reduction and in RBB cool format, then the rooted branching bisimulation equivalence that it induces is a congruence.

**Definition 4.2.19 (Conservative extension).** Let $T_{0}$ and $T_{1}$ be TSSs over signatures $\Sigma_{0}$ and $\Sigma_{1}$, respectively. The TSS $T_{0} \oplus T_{1}$ is a conservative extension of $T_{0}$ if the LTSs generated by $T_{0}$ and $T_{0} \oplus T_{1}$ contain exactly the same transitions $t \xrightarrow{\tau} t'$ and $tP$ with $t \in T(\Sigma_{0})$.

**Definition 4.2.20 (Source-dependency).** The source-dependent variables in a transition rule of $T$ are defined inductively as follows: (1) all variables in the source of $\rho$ are source-dependent; (2) if $t \xrightarrow{\tau} t'$ is a premise of $\rho$ and all variables in $t$ are source-dependent, then all variables in $t'$ are source-dependent. A transition rule is source-dependent if all its variables are. A TSS is source-dependent if all its rules are.

**Definition 4.2.21 (Freshness).** Let $T_{0}$ and $T_{1}$ be TSSs over signatures $\Sigma_{0}$ and $\Sigma_{1}$, respectively. A term in $T(T_{0} \oplus T_{1})$ is said to be fresh if it contains a function symbol from $\Sigma_{1} \setminus \Sigma_{0}$. Similarly, a transition label or predicate symbol in $T_{1}$ is fresh if it does not occur in $T_{0}$.

**Theorem 4.2.4.** Let $T_{0}$ and $T_{1}$ be TSSs over signatures $\Sigma_{0}$ and $\Sigma_{1}$, respectively, where $T_{0}$ and $T_{0} \oplus T_{1}$ are positive after reduction. Under the following conditions, $T_{0} \oplus T_{1}$ is a conservative extension of $T_{0}$. (1) $T_{0}$ is source-dependent. (2) For each $\rho \in T_{1}$, either the source of $\rho$ is fresh, or $\rho$ has a premise of the form $t \xrightarrow{\alpha} t'$ or $tP$, where $t \in T(\Sigma_{0})$, all variables in $t$ occur in the source of $\rho$ and $t'$, $\alpha$ or $P$ is fresh.

4.3. Process Algebra – ACP

ACP[5] is a kind of process algebra which focuses on the specification and manipulation of process terms by use of a collection of operator symbols. In ACP, there are several kind of operator symbols, such as basic operators to build finite processes (called BPA), communication operators to express concurrency (called E), deadlock constants and encapsulation enable us to force actions into communications (called ACP), linear recursion to capture infinite behaviors (called ACP with linear recursion), the special constant silent step and abstraction operator (called ACP with guarded linear recursion) allows us to abstract away from internal computations.

Bisimulation or rooted branching bisimulation based structural operational semantics is used to formally provide each process term used the above operators and constants with a process graph. The axiomatization of ACP (according the above classification of ACP, the axiomatizations are $E_{BPA}$, $E_{PAP}$, $E_{ACP}$, $E_{ACP} + RDP$ (Recursive Definition Principle) + RSP (Recursive Specification Principle), $E_{ACP} + RDP + RSP + CFAR$ (Cluster Fair Abstraction Rule) respectively) imposes an equation logic on process terms, so two process terms can be equated if and only if their process graphs are equivalent under the semantic model.

ACP can be used to formally reason about the behaviors, such as processes executed sequentially and concurrently by use of its basic operator, communication mechanism, and recursion, desired external behaviors by its abstraction mechanism, and so on.

ACP can be extended with fresh operators to express more properties of the specification for system behaviors. These extensions are required both the equational logic and the structural operational semantics.
to be extended. Then the extension can use the whole outcomes of ACP, such as its concurrency, recursion, abstraction, etc.

4.3.1. SubmittingOrder Described by ACP

The process graph of the SubmittingOrder example is illustrated in Fig. 2-b. Since ACP does not distinguish the choice decision made by outside agent or inner states, the process of the SubmittingOrder example can be expressed by the following process term in ACP.

\[
\text{start} \cdot \text{write} \cdot (\text{submit} \cdot \text{store} + \text{cancel}).
\]

4.3.2. Transaction Described by ACP

The process graph of the Transaction example is illustrated in Fig. 3-b. The process of the Transaction example can be expressed by the following process term in ACP.

\[
\text{start} \cdot \text{operate} \cdot (\text{submit} \cdot \text{store} + \text{abort} \cdot \text{rollback}).
\]

4.3.3. Purchasing Described by ACP

The process graph of the Purchasing composite Web Service is illustrated in Fig. 6. The process of the Purchasing composite Web Service can be expressed by the following process term in ACP.

\[
\text{start} \cdot \text{shopping} \cdot (\text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} + \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} + \text{sPlane} \cdot \text{oPlane} \cdot (\text{pOnLine} + \text{pOffLine})).
\]

4.4. Games

In the above application scenarios, one agent interacts with other autonomous agents or human beings. In the agent’s viewpoint, some branch decisions are made by outside agents or human beings, but not the inner states. In this situation, a two person game is suitable to model the interaction. In the game, the agent is modeled as the Player (denoted as P) and the other agent or the human being is modeled as the opponent (denoted as O).

Corresponding to a process graph, there exists a game tree, for example, the game tree corresponding to process graph in Fig. 6 is illustrated in Fig. 7.

We define move and strategy as follows.

**Definition 4.4.1 (Move)** Every execution of an action \( a \) in the process graph causes a move \( a \) in the corresponding game tree.

And we do not distinguish the action \( a \) and the move \( a \).

**Definition 4.4.2 (P strategy)** A strategy \( \lambda_P \) of P in a game tree is a subtree defined as follows:
1. the empty move $\epsilon \in \lambda_P$;
2. if the move $m \in \lambda_P$ is a P move, then exactly one child move $m'$ of $m$ and $m' \in \lambda_P$;
3. if the move $m \in \lambda_P$ is an O move, then all children $M'$ of $m$ are in $\lambda_P$, that is $M' \subseteq \lambda_P$.

Since P and O are relative, the strategy $\lambda_O$ of O can be defined similarly.

Definition 4.4.3 (O strategy) A strategy $\lambda_O$ of O in a game tree is a subtree define as follows:

1. the empty move $\epsilon \in \lambda_O$;
2. if the move $m \in \lambda_O$ is a O move, then exactly one child move $m'$ of $m$ and $m' \in \lambda_O$;
3. if the move $m \in \lambda_O$ is an P move, then all children $M'$ of $m$ are in $\lambda_O$, that is $M' \subseteq \lambda_O$.

In the game tree illustrated in Fig. 7 of Purchasing example, there are two choice decisions. One is made by the user agent (or the user), and the other is made by the composite service. In this game, we model the composite service as P and the user agent (or the user) as O.

A strategy of P is illustrated as Fig. 8 shows. And a strategy of O is as Fig. 9 illustrates.

We can see that the actual execution a game tree are acted together by the P and the O. For a P strategy $\lambda_P$ and an O strategy $\lambda_O$ of a game tree, $\lambda_P \cap \lambda_O$ has the form $\lambda_P \cap \lambda_O = \{\epsilon, m_1 \cdot m_2, \ldots, m_1 \cdot \ldots \cdot m_n\}$ according to the definition of strategy. We can get that the maximal element $m_1 \cdot \ldots \cdot m_n$ of $\lambda_P \cap \lambda_O$ exactly defines an execution of the game tree.

For the P strategy $\lambda_P$ illustrated in Fig. 8 and O strategy $\lambda_O$ illustrated in Fig. 9, the maximal element $start \cdot shopping \cdot sPlane \cdot oPlane \cdot pOffLine$ of $\lambda_P \cap \lambda_O$ defines an execution of the process as illustrated in process graph Fig. 6. This is shown in Fig. 10.

5. Extension of BPA for Games – GameBPA

GameBPA is based on BPA. In BPA, there are two basic operators called alternative composition $+$ and sequential composition $\cdot$. We give the transition rules for BPA as follows.
The axioms of BPA are in Table 1.

The main results on BPA are the following ones.

**Theorem 5.1** Bisimulation equivalence is a congruence with respect to BPA.
A Process Algebra for Games

Table 1. Axioms of BPA

| No. | Axiom                                      |
|-----|--------------------------------------------|
| A1  | \(x + y = y + x\)                         |
| A2  | \((x + y) + z = x + (y + z)\)             |
| A3  | \(x + x = x\)                             |
| A4  | \((x + y) \cdot z = x \cdot z + y \cdot z\) |
| A5  | \((x \cdot y) \cdot z = x \cdot (y \cdot z)\) |

Theorem 5.2 \(\mathcal{E}_{\text{BPA}}\) is sound for BPA modulo bisimulation equivalence.
Theorem 5.3 \(\mathcal{E}_{\text{BPA}}\) is complete for BPA modulo bisimulation equivalence.

5.1. Scenarios Described by GameBPA

5.1.1. SubmittingOrder Described by GameBPA

In SubmittingOrder example, we model the interface program as P and the user as O. Then in the P’s view, the process can be expressed by the following process term in GameBPA.

\[
\text{start} \cdot \text{write} \cdot (\text{submit} \cdot \text{store} \downarrow \text{cancel}).
\]

And the subtree corresponding to the above term is the only strategy of P.

In the O’s view, the process can be expressed by the following process term in GameBPA.

\[
\text{start} \cdot \text{write} \cdot (\text{submit} \cdot \text{store} + \text{cancel}).
\]

So the subtrees corresponding to term \(\text{start} \cdot \text{write} \cdot \text{submit} \cdot \text{store}\) and \(\text{start} \cdot \text{write} \cdot \text{cancel}\) are all strategies of O.

5.1.2. Transaction Described by GameBPA

In Transaction example, we model the database as P and the user as O. Then in the P’s view, the process can be expressed by the following process term in GameBPA.

\[
\text{start} \cdot \text{operate} \cdot (\text{submit} \cdot \text{store} \downarrow \text{abort} \cdot \text{rollback}).
\]

And the subtree corresponding to the above term is the only strategy of P.

In the O’s view, the process can be expressed by the following process term in GameBPA.

\[
\text{start} \cdot \text{operate} \cdot (\text{submit} \cdot \text{store} + \text{abort} \cdot \text{rollback}).
\]

So the subtrees corresponding to term \(\text{start} \cdot \text{operate} \cdot \text{submit} \cdot \text{store}\) and \(\text{start} \cdot \text{operate} \cdot \text{abort} \cdot \text{rollback}\) are all strategies of O.

5.1.3. Purchasing Described by GameBPA

In Purchasing example, we model the composite service as P and the user agent (or the user) as O. Then in the P’s view, the process can be expressed by the following process term in GameBPA.

\[
\text{start} \cdot \text{shopping} \cdot (\text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} \downarrow \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} \downarrow \text{sPlane} \cdot \text{oPlane} \cdot (\text{pOnLine} + \text{pOffLine})).
\]

So the subtrees corresponding to term \(\text{start} \cdot \text{shopping} \cdot (\text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} \downarrow \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} \downarrow \text{sPlane} \cdot \text{oPlane} \cdot \text{pOnLine} \downarrow \text{sPlane} \cdot \text{oPlane} \cdot \text{pOffLine})\) and \(\text{start} \cdot \text{shopping} \cdot (\text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} \downarrow \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} \downarrow \text{sPlane} \cdot \text{oPlane} \cdot \text{pOffLine})\) are all strategies of P.

In the O’s view, the process can be expressed by the following process term in GameBPA.

\[
\text{start} \cdot \text{shopping} \cdot (\text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} + \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} + \text{sPlane} \cdot \text{oPlane} \cdot (\text{pOnLine} \downarrow \text{pOffLine})).
\]

So the subtrees corresponding to term \(\text{start} \cdot \text{shopping} \cdot \text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine}, \text{start} \cdot \text{shopping} \cdot \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} \text{ and start} \cdot \text{shopping} \cdot \text{sPlane} \cdot \text{oPlane} \cdot (\text{pOnLine} \downarrow \text{pOffLine})\) are all strategies of O.
5.2. Transition Rules of Opponent’s Alternative Composition Operator

Based on the above discussions, the transition rules of opponent’s alternative composition operator \( \dagger \) are given as follows. \( x, x', y, y' \) range over \( A \), the variables \( \upsilon \) range over the set \( A \) of atomic actions. We define two set of \( A_{\lambda_P} \) and \( A_{\lambda_O} \) which denote the set of atomic actions in \( \lambda_P \) and the set of atomic actions in \( \lambda_O \).

\[
\begin{align*}
\frac{x \xrightarrow{\upsilon} \sqrt{} }{x \dagger y \xrightarrow{\upsilon} \sqrt{}} \\
\frac{x \xrightarrow{\upsilon} x'}{x \dagger y \xrightarrow{\upsilon} x'} \\
\frac{y \xrightarrow{\upsilon} \sqrt{} }{x \dagger y \xrightarrow{\upsilon} \sqrt{}} \\
\frac{y \xrightarrow{\upsilon} y'}{x \dagger y \xrightarrow{\upsilon} y'}
\end{align*}
\]

where \( \upsilon \in A_{\lambda_P} \cap A_{\lambda_O} \). That is, in the view of O, the opponent’s alternative composition is the same as the traditional alternative composition.

In the O’s view, the first transition rule of opponent’s alternative composition operator \( \dagger \) says if \( t \) terminates successfully by executing an action \( a \) then the process term \( t \dagger s \) will terminate successfully by executing the action \( a \). The second one says if \( t \) evolves into \( t' \) by executing an action \( a \) then the process term \( t \dagger s \) will evolve into \( s' \) by executing the same action \( a \). The third transition rule of opponent’s alternative composition operator \( \dagger \) says if \( s \) terminates successfully by executing an action \( a \) then the process term \( t \dagger s \) will terminate successfully by executing the action \( a \). The fourth one says if \( s \) evolves into \( s' \) by executing an action \( a \) then the process term \( t \dagger s \) will evolve into \( s' \) by executing the action \( a \).

But, in P’s view, the choice of the opponent’s alternative composition can not be done according to its own inner states or its own knowledge. That is, the choice of the opponent’s alternative composition is depending on the O in the game. In the P’s view, \( x \dagger y \) is non-deterministic in nature and we call it as a GameBPA Process. A GameBPA process can not execute according to its own knowledge without the co-action with the O. That is, a GameBPA process is not a real process in process algebra and the opponent’s alternative composition \( \dagger \) is only a symbol, but not a real operator, just because all processes in process algebra are executable.

To make a GameBPA process executable, or to make the opponent’s alternative composition \( \dagger \) to be a real operator, we introduce a special constant called non-determinacy constant which is denoted as \( \xi \). Let a GameBPA process term \( t \dagger s = \xi \) which means that the choice of \( t \) or \( s \) is non-deterministic.

5.3. Properties of GameBPA

We can get the following two properties of GameBPA.

**Theorem 1** GameBPA is a conservative extension of BPA (see Section 4.3).

**Proof.** This theorem follows from the following two facts.

1. The transition rules of BPA (see Section 4.3) are all source-dependent (see Section 4.2).
2. The sources (see Section 4.2) of the four transition rules for the opponent’s alternative operator all contain an occurrence of \( \dagger \).

Since the transition rules of BPA is source-dependent, and the transition rules for the opponent’s alternative operator contain a fresh operator (see Section 4.2) in their sources, Theorem 4.2.4 says that GameBPA is a conservative extension of BPA.

**Theorem 2** Bisimulation equivalence is a congruence with respect to GameBPA.

**Proof.** The transition rules for the opponent’s alternative operator, as well as of BPA, are all in panth format.
Table 2. Axioms of the opponent’s alternative composition operator and the non-determinacy constant

| No. | Axiom |
|-----|-------|
| OA1 | In the view of P, $x \vdash y = \xi$ |
| OA2 | In the view of O, $x \vdash y = x + y$ |
| ND1 | $\xi \cdot x = \xi$ |
| ND2 | $x + \xi = \xi$ |

(see Section 4.2). So the bisimulation equivalence (see Section 4.2) that they induce is a congruence (see Section 4.2 Theorem 4.2.2).

5.4. Axioms of Opponent’s Alternative Composition Operator and Non-determinacy Constant

By extending the opponent’s alternative composition operator $\vdash$ to model ACP for games, it is clear to construct a sound and complete axiomatical system. We design axioms of opponent’s alternative composition operator $\vdash$ shown in Table 2.

The axioms OA1 and OA2 are presented for the opponent’s alternative composition $\vdash$, and the axioms ND1 and ND2 are for the non-determinacy $\xi$.

5.5. Properties of the Axiomatic System

The following are two properties of the axiomatical system $E_{\text{GameBPA}}$.

**Theorem 3** $E_{\text{GameBPA}}$ is sound for GameBPA modulo bisimulation equivalence.

**Proof.** Since bisimulation is both an equivalence and a congruence, we only need to check that the first clause in the definition of the relation $=_{\mathcal{E}}$ is sound. That is, if $s = t$ is an axiom in GameBPA and $\sigma$ is a closed substitution that maps the variables in $s$ and $t$ to process terms, then we need to check that $\sigma(s) \leftrightarrow \sigma(t)$.

We only provide some intuition for soundness of the axioms in Table 2.

1. The axiom OA1 says a GameBPA process term $t \vdash s$ is non-deterministic in the P’s view.
2. The axiom OA2 says a GameBPA process term $t \vdash s$ is the same as the process term $t + s$ in the view of O.
3. The axioms ND1 and ND2 say the non-determinacy $\xi$ is non-deterministic, so that the GameBPA process term $\xi \cdot t$ and the GameBPA process term $t + \xi$ are also non-deterministic.

**Theorem 4** $E_{\text{GameBPA}}$ is complete for GameBPA modulo bisimulation equivalence.

**Proof.** The proof is based on the proof of the completeness theorem of BPA. (See Section 4.3 and [5]).

The proof consists of three main step: (1) we will show that the axioms OA1, OA2 and ND1, ND2 can be turned in to rewrite rules (see Section 4.1), and the resulting TRS is terminating (see Section 4.1); (2) we will show that norm forms (see Section 4.1) do not contain occurrences of the fresh opponent’s alternative composition operator $\vdash$; (3) we will prove that $E_{\text{GameBPA}}$ is complete for GameBPA modulo bisimulation equivalence.

(1) The axioms OA1, OA2 and ND1, ND2 are turned into rewriting rules directly from left to right, and added to the three rewriting rules in the proof the completeness of $E_{\text{BPA}}$ (see [5]). The resulting TRS is terminating modulo AC (Associativity and Commutativity) of $+$ operator through defining new weight functions on process terms.

$$weight(\xi) \triangleq 2$$

$$weight(s \vdash t) \triangleq weight(s) + weight(t)$$
We can get that each application of a rewriting rule strictly decreases the weight of a process term, and that moreover process terms that are equivalent modulo AC of + have the same weight. Hence, the TRS is terminating modulo AC of +.

(2) We will show that the normal form \( n \) are not of the form \( s \uparrow t \). The proof is based on induction with respect to the size of the normal form \( n \).

- If \( n \) is an atomic action, then it does not contain \( \uparrow \).
- Suppose \( n = _{AC} s + t \) or \( n = _{AC} s \cdot t \). Then by induction, the normal forms \( s \) and \( t \) do not contain \( \uparrow \), so \( n \) does not contain \( \uparrow \).
- \( n \) cannot be of the form \( s \uparrow t \), because in that case, the directed version of OA1 or OA2 would apply to it, contradicting the fact that \( n \) is a normal form.

We proved that normal forms are all basic process terms.

(3) We proceed to prove that the axiomatization \( E_{GameBPA} \) is complete for GameBPA modulo bisimulation equivalence. Let the process terms \( s \) and \( t \) be bisimilar. The TRS is terminating modulo AC of the +, so it reduces \( s \) and \( t \) to normal forms \( n \) and \( n' \), respectively. Since the rewrite rules and equivalence modulo AC of the + can be derived from \( E_{GameBPA} \), \( s = n \) and \( t = n' \). Soundness of \( E_{GameBPA} \) then yields \( s \leftrightarrow n \) and \( t \leftrightarrow n' \). We shown that the normal forms \( n \) and \( n' \) are basic process terms. Then it follows that \( n \leftrightarrow n' \) implies \( n = _{AC} n' \). Hence, \( s = n = _{AC} n' = t \).

5.6. Execution of GameBPA Processes

Execution of GameBPA processes needs the co-action of the P and the O, that is, the playing of the game between the P and the O. We introduce a new binary playing operator \( \sqcap \) over the P’s GameBPA process and the O’s GameBPA process. A single GameBPA process is non-deterministic, but the playing operator \( \sqcap \) of GameBPA processes for the P and the O will eliminate this determinacy and will result in a real execution of two GameBPA processes. To eliminate the mismatched branches in the alternation composition in the co-action of GameBPA processes, a special constant called deadlock \( \delta \) is also introduced.

5.6.1. Transition Rules of Playing Operator

The transition rules of playing operator \( \sqcap \) are following.

\[
\begin{align*}
  x \overset{u}{\rightarrow} & \sqrt{y} \overset{v}{\rightarrow} \sqrt{y} \\
  x \sqcap y \overset{u}{\rightarrow} & \sqrt{y} \\
  x \overset{u}{\rightarrow} & \sqrt{y} \overset{v}{\rightarrow} y' \\
  x \sqcap y \overset{u}{\rightarrow} & y' \\
  x \overset{u}{\rightarrow} x' & \overset{v}{\rightarrow} \sqrt{y} \\
  x \sqcap y \overset{u}{\rightarrow} & x' \\
  x \overset{u}{\rightarrow} x' & \overset{v}{\rightarrow} y' \\
  x \sqcap y \overset{u}{\rightarrow} & x' \sqcap y' 
\end{align*}
\]

The first transition rule of playing operator \( \sqcap \) says if \( t \) terminates successfully by executing an action \( a \) and \( s \) terminates successfully by executing the same action \( a \), then the process term \( t \sqcap s \) will terminate successfully by executing the action \( a \). The second one says if \( t \) terminates successfully by executing an action \( a \) and \( s \) evolves into \( s' \) by executing the same action \( a \), then the process term \( t \sqcap s \) will evolve into \( s' \) by executing the same action \( a \). The third one says if \( t \) evolves into \( t' \) by executing an action \( a \) and \( s \) terminates successfully by executing the same action \( a \), then the process term \( t \sqcap s \) will evolve into \( t' \) by executing the same action \( a \). The fourth one says if \( t \) evolves into \( t' \) by executing an action \( a \) and \( s \) evolves into \( s' \) by executing the same action \( a \), then the process term \( t \sqcap s \) will evolve into \( t' \sqcap s' \) by executing the same action \( a \).
The above four transition rules intuitively capture the co-action of the P’s GameBPA process and the O’s GameBPA process.

To eliminate the mismatched branches in the alternation composition in the co-action of GameBPA processes, we introduce a special constant called deadlock $\delta$. The deadlock $\delta$ means do nothing. That is, when the execution sequence of the P’s process is not matched that of the O’s process, a deadlock will be caused.

### 5.6.2. Properties of Playing Operator

**Theorem 5** GameBPA with playing operator and deadlock constant is a conservative extension of GameBPA.

**Proof.** This theorem follows from the following two facts.

1. The transition rules of GameBPA are all source-dependent (see Section 4.2).
2. The sources (see Section 4.2) of the four transition rules for the playing operator all contain an occurrence of $\sqcap$.

Since the transition rules of GameBPA is source-dependent, and the transition rules for the playing operator contain a fresh operator (see Section 4.2) in their sources, Theorem 4.2.4 says that GameBPA with playing operator is a conservative extension of GameBPA.

**Theorem 6** Bisimulation equivalence is a congruence with respect to GameBPA with playing operator and deadlock constant.

**Proof.** The transition rules for the playing operator, as well as of GameBPA, are all in panth format (see Section 4.2). So the bisimulation equivalence that they induce is a congruence(see Section 4.2 Theorem 4.2.2).

### 5.6.3. Axioms of Playing Operator and Deadlock Constant

We design the axioms of the playing operator and the deadlock constant as Table 3 shows.

The axioms DL1-DL2 are presented for the deadlock constant $\delta$, and the axioms PO1-PO14 are for the playing operator $\sqcap$. There are not axioms for the association of the deadlock constant $\delta$ and the playing operator $\sqcap$, just because the function of the playing operator $\sqcap$ is eliminating all non-deterministic factors.

### 5.6.4. Properties of the Axiomatic System

**Theorem 7** $\mathcal{E}_{\text{GameBPA}} + \text{DL1-DL2} + \text{PO1-PO14}$ is sound for GameBPA with playing operator and deadlock constant modulo bisimulation equivalence.
Proof. Since bisimulation is both an equivalence and a congruence, we only need to check that the first clause in the definition of the relation $\equiv$ is sound. That is, if $s = t$ is an axiom in GameBPA and $\sigma$ is a closed substitution that maps the variables in $s$ and $t$ to process terms, then we need to check that $\sigma(s) \leftrightarrow \sigma(t)$.

We only provide some intuition for soundness of the axioms in Table 3.

1. The axiom DL1 says that $\delta$ displays no behavior, so the process term $t + \delta$ is equal to the process term $t$.
2. The axioms PO1 and PO2 say that the co-action of two same actions will lead to the only action, otherwise, it will cause a deadlock.
3. The axioms PO5-PO10 say that $s \sqcap t$ makes as initial transition a playing of initial transitions from $s$ and $t$. If the execution sequence of $s$ is not matched with that of $t$, a deadlock will be caused.
4. The axioms PO11-PO12 say that the function of playing operator makes two non-deterministic GameBPA processes deterministic.
5. The axioms PO13-PO14 say that the playing operator satisfies right and left distributivity to the operator $\sqcap$.

Theorem 8. $E_{\text{GameBPA}} + \text{DL1-DL2} + \text{PO1-PO14}$ is complete for GameBPA with playing operator and deadlock constant modulo bisimulation equivalence.

Proof. The proof is based on the proof of the Theorem 4.

The proof consists of three main step: (1) we will show that the axioms DL1, DL2 and PO1-PO14 can be turned in to rewrite rules (see Section 4.1), and the resulting TRS is terminating (see Section 4.1); (2) we will show that norm forms (see Section 4.1) do not contain occurrences of the fresh opponent’s alternative composition operator $\sqcap$; (3) we will prove that $E_{\text{GameBPA}} + \text{DL1-DL2} + \text{PO1-PO14}$ is complete for GameBPA with playing operator and deadlock constant modulo bisimulation equivalence.

(1) The axioms OA1, OA2 and PO1-PO14 are turned into rewriting rules directly from left to right, and added to the seven rewriting rules in the proof the completeness of $E_{\text{GameBPA}}$ (see proof of Theorem 4). The resulting TRS is terminating modulo AC (Associativity and Commutativity) of $+$ operator through defining new weight functions on process terms.

\[
\begin{align*}
\text{weight}(\delta) & \triangleq 2 \\
\text{weight}(\nu) & \triangleq 2 \\
\text{weight}(\omega) & \triangleq 2 \\
\text{weight}(s \sqcap t) & \triangleq (\text{weight}(s) \cdot \text{weight}(t))^2
\end{align*}
\]

We can get that each application of a rewriting rule strictly decreases the weight of a process term, and that moreover process terms that are equivalent modulo AC of $+$ have the same weight. Hence, the TRS is terminating modulo AC of $+$.

(2) We will show that the normal form $n$ are not of the form $s \sqcap t$. The proof is based on induction with respect to the size of the normal form $n$.

- If $n$ is an atomic action, then it does not contain $\sqcap$.
- Suppose $n = s + t$ or $n = s \cdot t$. Then by induction, the normal forms $s$ and $t$ do not contain $\sqcap$, so $n$ does not contain $\sqcap$.
- Suppose $n = s \sqcap t$. By induction, the normal form $s$ does not contain $\sqcap$. We distinguish the possible forms of the normal form $s$:
  - if $s \equiv a$, then the directed version of PO1, PO2, PO5 or PO6 apply to $s \sqcap t$;
  - if $s = s \sqcap \nu$, then the directed version of PO7-PO10 apply to $s \sqcap t$;
  - if $s = s \sqcap \omega + \omega'$, then the directed version of PO13 applies to $s \sqcap t$;
6. GameACP – A Full Extension of ACP for Games

GameBPA extends to process algebra BPA and does not use the full outcomes of ACP, such as concurrency, recursion, abstraction, etc. Now, we make GameBPA be based on the full ACP (exactly ACP, with guarded linear recursion) and this extension is called GameACP. GameACP remains the opponent’s alternative composition operator \( \parallel \), the playing operator \( \sqsubseteq \) and the non-determinacy constant \( \xi \). Because the deadlock constant is already existing in ACP, we remove the duplicate definition of deadlock constant in GameACP.

The transition rules of the opponent’s alternative composition operator \( \parallel \) and the playing operator \( \sqsubseteq \) are the same as those in GameBPA. Through defining \( x \parallel \tau = x \) and \( x \sqsubseteq \tau = x \), we extend \( A \) to \( A \cup \{ \tau \} \).

We can get the following two conclusions.

**Theorem 9** GameACP (exactly ACP, with guarded linear recursion, opponent’s alternative composition operator \( \parallel \), playing operator \( \sqsubseteq \) and non-determinacy \( \xi \)) is a conservative extension of ACP (exactly ACP, with guarded linear recursion) (see Section 4.3).

**Proof.** The sources of transition rules of opponent’s alternative composition operator \( \parallel \) and playing operator \( \sqsubseteq \) contain one fresh function symbol \( \parallel \) and \( \sqsubseteq \). And it is known that the transition rules of ACP, with guarded linear recursion are source-dependent. According to the definition of conservative extension (see Section 4.2), GameACP is a conservative extension of ACP, with guarded linear recursion. \( \Box \)

**Theorem 10** Rooted branching bisimulation equivalence is a congruence with respect to GameACP (exactly ACP, with guarded linear recursion, opponent’s alternative composition operator \( \parallel \), playing operator \( \sqsubseteq \) and non-determinacy \( \xi \)).

**Proof.** We introduce successful termination predicate \( \downarrow \). A transition rule \( \overrightarrow{\downarrow \parallel} \) is added into transition rules of GameACP. Replacing transition rules occurring \( \overrightarrow{a} \parallel \sqrt{A} \) by \( \overrightarrow{a} \downarrow \), the result transition rules of GameACP are in RBB cool format according to the definition of RBB cool format. So rooted branching bisimulation equivalence is a congruence with respect to GameACP according to the definition of congruence (see Section 4.2). \( \Box \)

Because of the removal of the deadlock constant in GameACP, the axiomatization \( \mathcal{E}_{\text{GameACP}} \) of GameACP (exactly ACP, with guarded linear recursion, opponent’s alternative composition operator \( \parallel \), playing operator \( \sqsubseteq \) and non-determinacy \( \xi \)) only contains \( \mathcal{E}_{\text{ACP}} + \text{RDP}, \text{RSP}, \text{CFAR} \) and \( \text{OA1-OA2}, \text{ND1-ND2}, \text{PO1-PO14} \).

Now, we get the following two conclusions.

**Theorem 11** \( \mathcal{E}_{\text{GameACP}} \) (exactly ACP, with guarded linear recursion, opponent’s alternative composition operator \( \parallel \), playing operator \( \sqsubseteq \) and non-determinacy \( \xi \)) modulo rooted branching bisimulation equivalence is sound for GameBPA (exactly ACP, with guarded linear recursion, opponent’s alternative composition operator \( \parallel \), playing operator \( \sqsubseteq \) and non-determinacy \( \xi \)) modulo rooted branching bisimulation equivalence.

**Proof.** Because rooted branching bisimulation is both an equivalence and a congruence, we only need to check that if \( t = u \) is an axiom and a closed substitution \( \sigma \) replacing the variables in \( t \) and \( u \) to get \( \sigma(t) \) and \( \sigma(u) \), then \( \sigma(t) \parallel \sigma(u) \).

We only provide some intuition for soundness of the axioms AO1-AO2, ND1-ND2, PO1-PO14.
1. The axiom OA1 says a GameBPA process term \( t \not\equiv s \) is non-deterministic in the P’s view.
2. The axiom OA2 says a GameBPA process term \( t \not\equiv s \) is the same as the process term \( t + s \) in the view of O.
3. The axiom ND1 and ND2 say the non-determinacy \( \xi \) is non-deterministic, so that the GameBPA process term \( \xi \cdot t \) and the GameBPA process term \( t + \xi \) are also non-deterministic.
4. The axioms PO3 and PO4 say that \( \delta \) blocks the behavior of the process term \( \delta \cdot t \), \( \delta \cap t \) and \( t \cap \delta \).
5. The axioms PO1 and PO2 say that the co-action of two same actions will lead to the only action, otherwise, it will cause a deadlock.
6. The axioms PO5-PO10 say that \( s \cap t \) makes as initial transition a playing of initial transitions from \( s \) and \( t \). If the execution sequence of \( s \) is not matched with that of \( t \), a deadlock will be caused.
7. The axioms PO11-PO12 say that the function of playing operator makes two non-deterministic GameBPA processes deterministic.
8. The axioms PO13-PO14 say that the playing operator satisfies right and left distributivity to the operator +.

\[\square\]

**Theorem 12** \( \mathcal{E}_{\text{GameACP}} \) (\( \mathcal{E}_{\text{ACP}} + \text{RDP, RSP, CFAR} + \text{OA1-OA2 + ND1-ND2 + PO1-PO14} \)) is complete for GameBPA (exactly \( \text{ACP}_\mathcal{R} \) with guarded linear recursion, opponent’s alternative composition operator \( | \), playing operator \( \cap \) and non-determinacy \( \xi \)) modulo rooted branching simulation equivalence.

**Proof.** We need to prove that each process term \( t \) in GameACP is equal to a process term \( \langle X|E \rangle \) with a guarded linear recursive specification \( E \). That is, if \( \langle X|E_1 \rangle \mapsto \_ \varepsilon \langle Y_1|E_2 \rangle \) for guarded linear recursive specifications \( E_1 \) and \( E_2 \), then \( \langle X|E_1 \rangle = \langle Y_1|E_2 \rangle \) can be gotten from \( \mathcal{E}_{\text{GameACP}} \).

This proof is based on the completeness proof\(^5\) of \( \mathcal{E}_{\text{ACP}} + \text{RDP, RSP, CFAR} \). We apply structural induction the size of process term \( t \). The new case is \( t = s \cap r \). First assuming \( s = \langle X|E \rangle \) with a guarded linear recursive specification \( E \) and \( r = \langle Y|F \rangle \) with a guarded linear recursive specification \( F \), we prove the case of \( t = \langle X|E \rangle \cap \langle Y|F \rangle \). Let \( E \) consists of guarded linear recursive equations

\[X_i = a_{i1}X_{i1} + \ldots + a_{ik}X_{ik} + b_{i1} + \ldots + b_{il_i}\]

for \( i \in 1, \ldots, N \). Let \( F \) consists of guarded linear recursive equations

\[Y_j = c_{j1}Y_{j1} + \ldots + c_{jm_j}Y_{jm_j} + d_{j1} + \ldots + d_{jn_j}\]

for \( j \in 1, \ldots, M \).

\[\langle X|E \rangle \cap \langle Y|F \rangle \]

\[
\begin{align*}
\text{RDP} & \quad (a_{i1}X_{i1} + \ldots + a_{ik}X_{ik}) + (b_{i1} + \ldots + b_{il_i}) \cap \langle Y|F \rangle \\
\text{PO13} & \quad a_{i1}X_{i1} \cap \langle Y|F \rangle + \ldots + a_{ik}X_{ik} \cap \langle Y|F \rangle + b_{i1} \cap \langle Y|F \rangle + \ldots + b_{il_i} \cap \langle Y|F \rangle \\
\text{RDP} & \quad a_{i1}X_{i1} \cap (c_{j1}Y_{j1} + \ldots + c_{jm_j}Y_{jm_j}) + (d_{j1} + \ldots + d_{jn_j}) \\
& \quad + \ldots + (a_{ik}X_{ik}) \cap (c_{j1}Y_{j1} + \ldots + c_{jm_j}Y_{jm_j}) + (d_{j1} + \ldots + d_{jn_j}) \\
& \quad + (b_{i1}) \cap (c_{j1}Y_{j1} + \ldots + c_{jm_j}Y_{jm_j}) + (b_{il_i}) \cap (d_{j1} + \ldots + d_{jn_j}) \\
\text{PO14} & \quad (a_{i1}X_{i1}) \cap (c_{j1}Y_{j1}) + \ldots + (a_{i1}X_{i1}) \cap (c_{jm_j}Y_{jm_j}) + (a_{i1}X_{i1}) \cap (d_{j1}) + \ldots + (a_{i1}X_{i1}) \cap (d_{jn_j}) \\
& \quad + \ldots + (a_{ik}X_{ik}) \cap (c_{j1}Y_{j1}) + \ldots + (a_{ik}X_{ik}) \cap (c_{jm_j}Y_{jm_j}) + (a_{ik}X_{ik}) \cap (d_{j1}) + \ldots + (a_{ik}X_{ik}) \cap (d_{jn_j}) \\
& \quad + (b_{il_i}) \cap (c_{j1}Y_{j1}) + \ldots + (b_{il_i}) \cap (c_{jm_j}Y_{jm_j}) + (b_{il_i}) \cap (d_{j1}) + \ldots + (b_{il_i}) \cap (d_{jn_j})
\end{align*}
\]

Then we can use the axioms PO1-PO10 into the above equation. This will lead to several cases and we
Illustrated in Fig. 10.

OnLine in Fig. 3, the maximal element rollback as illustrated in process graph Fig. 3-b.

Illustration of the GameACP processes.

Illustration of the correctness theorem through three examples in Section 3.

For the P strategy \( \lambda_P \) corresponding to the GameACP process term \( \text{start} \cdot \text{write} \cdot (\text{submit} \cdot \text{store} \downarrow \text{cancel}) \) and the O strategy corresponding to the GameACP process term \( \text{start} \cdot \text{write} \cdot \text{submit} \cdot \text{store} \) in Fig. 2, the maximal element \( \text{start} \cdot \text{write} \cdot \text{submit} \cdot \text{store} \) of \( \lambda_P \cap \lambda_O \) defines an execution of process graph Fig. 2-b.

\[
\begin{align*}
\text{PO9} & \quad (\text{start} \cdot \text{write} \cdot (\text{submit} \cdot \text{store} \downarrow \text{cancel})) \cap (\text{start} \cdot \text{write} \cdot \text{submit} \cdot \text{store}) \\
\text{PO11} & \quad \text{start} \cdot \text{write} \cdot ((\text{submit} \cdot \text{store} \downarrow \text{cancel}) \cap (\text{submit} \cdot \text{store})) \\
\text{DL1} & \quad \text{start} \cdot \text{write} \cdot (\text{submit} \cdot \text{store} + \delta) \\
\end{align*}
\]

For the P strategy \( \lambda_P \) corresponding to the GameACP process term \( \text{start} \cdot \text{operate} \cdot (\text{submit} \cdot \text{store} \downarrow \text{abort} \cdot \text{rollback}) \) and the O strategy corresponding to the GameACP process term \( \text{start} \cdot \text{operate} \cdot \text{abort} \cdot \text{rollback} \) in Fig. 3, the maximal element \( \text{start} \cdot \text{operate} \cdot \text{abort} \cdot \text{rollback} \) of \( \lambda_P \cap \lambda_O \) defines an execution of the process as illustrated in process graph Fig. 3-b.

\[
\begin{align*}
\text{PO9} & \quad (\text{start} \cdot \text{operate} \cdot (\text{submit} \cdot \text{store} \downarrow \text{abort} \cdot \text{rollback})) \cap (\text{start} \cdot \text{operate} \cdot \text{abort} \cdot \text{rollback}) \\
\text{PO11} & \quad \text{start} \cdot \text{operate} \cdot ((\text{submit} \cdot \text{store} \downarrow \text{abort} \cdot \text{rollback}) \cap (\text{abort} \cdot \text{rollback})) \\
\text{PO13, PO6} & \quad \text{start} \cdot \text{operate} \cdot (\delta + \text{abort} \cdot \text{rollback}) \\
\text{DL1} & \quad \text{start} \cdot \text{operate} \cdot \text{abort} \cdot \text{rollback} \\
\end{align*}
\]

For the P strategy \( \lambda_P \) corresponding to the GameACP process term \( \text{start} \cdot \text{shopping} \cdot (sTruck \cdot oTruck \cdot pOnLine \downarrow sTrain \cdot oTrain \cdot pOnLine \downarrow sPlane \cdot oPlane \cdot pOffLine) \) in Fig. 8 and the O strategy corresponding to the GameACP process term \( \text{start} \cdot \text{shopping} \cdot \text{sPlane} \cdot \text{oPlane} \cdot \text{pOffLine} \) in Fig. 9, the maximal element \( \text{start} \cdot \text{shopping} \cdot \text{sPlane} \cdot \text{oPlane} \cdot \text{pOffLine} \) of \( \lambda_P \cap \lambda_O \) defines an execution of the process as illustrated in Fig. 10.
8. Support for Multi-person Games – Extended Purchasing Example

In fact, the axioms in Table 2 and Table 3 can be naturally used in multi-person games without any alternation. For a three-person game, let $t$ be a GameACP process term corresponding to a strategy of the first player, $s$ a GameACP process term corresponding to a strategy of the second player and $u$ a GameACP process term corresponding to a strategy of the third player. The process term $t \sqcap s \sqcap u$ can be deduced to an execution of these strategies by use of the above axioms. We show this situation in the section.

The process graph of the extended Purchasing composite Web Service is illustrated in Fig. 11. The process of the Purchasing composite Web Service can be expressed by the following process term in ACP:

$$
\text{start} \cdot \text{shopping} \cdot (s\text{Truck} \cdot o\text{Truck} \cdot p\text{OnLine} \sqcap s\text{Train} \cdot o\text{Train} \cdot p\text{OnLine} \\
\sqcap s\text{Plane} \cdot o\text{Plane} \cdot p\text{OffLine})
$$

The game tree to process graph in Fig. 11 is illustrated in Fig. 12.

In the game tree illustrated in Fig. 12 of the extended Purchasing example, there are three choice decisions.
The first is made by the user agent (or the user), and the second is made by the composite service, and the third is made by the air corporation. In this game, we model the user agent as Player 1, the composite service as Player 2 and the air corporation as Player 3.

A strategy of Player 1 is illustrated as Fig. 13 shows. And a strategy of Player 2 is as Fig. 14 illustrates. And also Fig. 15 shows a strategy of Player 3.

We can see that the actual execution a game tree are acted together by all players. For a strategy
λ₁, a strategy λ₂ and a strategy λ₃ of a game tree, λ₁ ∩ λ₂ ∩ λ₃ has the form λ₁ ∩ λ₂ ∩ λ₃ = {ε, m₁̈, m₂, ..., mₙ̈, m₁, ..., mₙ} according to the definition of strategy. We can get that the maximal element

\[ m₁ \cdot m₂ \cdot ... \cdot mₙ \]

of λ₁ ∩ λ₂ ∩ λ₃ exactly defines an execution of the game tree.

For the strategy λ₁ illustrated in Fig. 13, the strategy λ₂ illustrated in Fig. 14, and the strategy λ₃ illustrated in Fig. 15 the maximal element

\[ \text{start} \cdot \text{shopping} \cdot \text{sPlane} \cdot \text{oPlane} \cdot \text{pOffLine} \cdot \text{ByBank} \]

of λ₁ ∩ λ₂ ∩ λ₃ defines an execution of the process as illustrated in process graph Fig. 5. This is shown in Fig. 16.

In extended Purchasing example, in the view of the Player 1, the process can be expressed by the following process term in GameACP.

\[ \text{start} \cdot \text{shopping} \cdot (\text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} + \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} + \text{sPlane} \cdot \text{oPlane} \cdot (\text{pOnLine} \downarrow \text{pOffLine} \cdot (\text{ByCheck} \uparrow \text{ByBank}))) \]

So the subtrees corresponding to term \( \text{start} \cdot \text{shopping} \cdot \text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} \), \( \text{start} \cdot \text{shopping} \cdot \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} \) and \( \text{start} \cdot \text{shopping} \cdot \text{sPlane} \cdot \text{oPlane} \cdot (\text{pOnLine} \downarrow \text{pOffLine} \cdot (\text{ByCheck} \uparrow \text{ByBank})) \) are all strategies of the Player 1.

In the view of the Player 2, the process can be expressed by the following process term in GameACP.

\[ \text{start} \cdot \text{shopping} \cdot (\text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} \downarrow \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} \downarrow \text{sPlane} \cdot \text{oPlane} \cdot (\text{pOnLine} \downarrow \text{pOffLine} \cdot (\text{ByCheck} \uparrow \text{ByBank}))) \]

So the subtrees corresponding to term \( \text{start} \cdot \text{shopping} \cdot (\text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} \downarrow \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} \downarrow \text{sPlane} \cdot \text{oPlane} \cdot \text{pOnLine} \cdot (\text{ByCheck} \uparrow \text{ByBank})) \) and \( \text{start} \cdot \text{shopping} \cdot (\text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} \downarrow \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} \downarrow \text{sPlane} \cdot \text{oPlane} \cdot \text{pOffLine} \cdot (\text{ByCheck} \uparrow \text{ByBank})) \) are all strategies of the Player 2.

In the view of the Player 3, the process can be expressed by the following process term in GameACP.

\[ \text{start} \cdot \text{shopping} \cdot (\text{sTruck} \cdot \text{oTruck} \cdot \text{pOnLine} \downarrow \text{sTrain} \cdot \text{oTrain} \cdot \text{pOnLine} \downarrow \text{sPlane} \cdot \text{oPlane} \cdot (\text{pOnLine} \downarrow \text{pOffLine} \cdot (\text{ByCheck} \uparrow \text{ByBank}))) \]
So the subtrees corresponding to term \(\text{start} \cdot \text{shopping} \cdot (s\text{Truck} \cdot o\text{Truck} \cdot p\text{OnLine} \cdot s\text{Plane} \cdot o\text{Plane} \cdot (p\text{OnLine} \cdot p\text{OffLine} \cdot By\text{Bank})))\) and \(\text{start} \cdot \text{shopping} \cdot (s\text{Truck} \cdot o\text{Truck} \cdot p\text{OnLine} \cdot s\text{Train} \cdot o\text{Train} \cdot p\text{OnLine} \cdot s\text{Plane} \cdot o\text{Plane} \cdot (p\text{OnLine} \cdot p\text{OffLine} \cdot By\text{Bank})))\) are all strategies of the Player 3.

If \(\lambda_{P1}\) is a strategy of the Player 1, \(\lambda_{P2}\) is a strategy of Player 2 and \(\lambda_{P3}\) is a strategy of Player 3 as illustrated in Section 4.4, and if the GameACP process term \(s\) corresponds to \(\lambda_{P1}\), the GameACP process term \(t\) corresponds to \(\lambda_{P2}\) and the GameACP process term \(r\) corresponds to \(\lambda_{P3}\), then the process term \(s \sqcap t \sqcap r\) exactly defines an execution of \(\lambda_{P1}, \lambda_{P2}\) and \(\lambda_{P3}\).

For the strategy \(\lambda_{P1}\) corresponding to the GameACP process term \(\text{start} \cdot \text{shopping} \cdot s\text{Plane} \cdot o\text{Plane} \cdot (p\text{OnLine} \cdot p\text{OffLine} \cdot (By\text{Check} \cdot By\text{Bank})))\) in Fig.13, the strategy \(\lambda_{P2}\) corresponding to the GameACP process term \(\text{start} \cdot \text{shopping} \cdot (s\text{Truck} \cdot o\text{Truck} \cdot p\text{OnLine} \cdot s\text{Train} \cdot o\text{Train} \cdot p\text{OnLine} \cdot s\text{Plane} \cdot o\text{Plane} \cdot p\text{OffLine} \cdot (By\text{Check} \cdot By\text{Bank})))\) in Fig.14, and the strategy \(\lambda_{P3}\) corresponding to the GameACP process term \(\text{start} \cdot \text{shopping} \cdot (s\text{Truck} \cdot o\text{Truck} \cdot p\text{OnLine} \cdot s\text{Train} \cdot o\text{Train} \cdot p\text{OnLine} \cdot s\text{Plane} \cdot o\text{Plane} \cdot (p\text{OnLine} \cdot p\text{OffLine} \cdot By\text{Bank})))\) in Fig.15, the maximal element \(\text{start} \cdot \text{shopping} \cdot s\text{Plane} \cdot o\text{Plane} \cdot p\text{OffLine} \cdot By\text{Bank}\) of \(\lambda_{P} \sqcap \lambda_{O}\) defines an execution of the process as illustrated in Fig.16.

\[
\begin{align*}
\text{(start} \cdot \text{shopping} \cdot s\text{Plane} \cdot o\text{Plane} \cdot (p\text{OnLine} \cdot p\text{OffLine} & \cdot (By\text{Check} \cdot By\text{Bank}))) \\
\sqcap & (\text{start} \cdot \text{shopping} \cdot (s\text{Truck} \cdot o\text{Truck} \cdot p\text{OnLine} \cdot s\text{Train} \cdot o\text{Train} \cdot p\text{OnLine} \cdot s\text{Plane} \cdot o\text{Plane} \cdot p\text{OffLine} \cdot (By\text{Check} \cdot By\text{Bank})))) \\
\sqcap & (\text{start} \cdot \text{shopping} \cdot (s\text{Truck} \cdot o\text{Truck} \cdot p\text{OnLine} \cdot s\text{Train} \cdot o\text{Train} \cdot p\text{OnLine} \cdot s\text{Plane} \cdot o\text{Plane} \cdot (p\text{OnLine} \cdot p\text{OffLine} \cdot By\text{Bank}))) \\
\end{align*}
\]

\[PO^{1} \equiv PO^{14}\]

\[\text{start} \cdot \text{shopping} \cdot s\text{Plane} \cdot o\text{Plane} \cdot p\text{OffLine} \cdot By\text{Bank}\]

9. Conclusions

In order to describe game theory in ACP, we do extension of ACP with an opponent’s alternative composition operator \(\triangleleft\) and a non-determinacy constant \(\xi\) which is called GameACP. To model the playing process of games, an extension of GameACP with a playing operator \(\sqcap\) and a deadlock constant \(\delta\) is also made. And two sound and complete axiomatical system are designed. As a result of axiomatization, GameACP has several advantages, for example, it has both a proof theory and also a semantics model, it is without any assumptions and restrictions and it can be used in multi-person games naturally. GameACP can be used to reason about the behaviors of parallel and distributed systems with game theory supported. And also, GameACP gives games an axiomatization interpretation naturally, this will help people to capture the nature of games.

It must be explained that any computable process can be represented by a process term in ACP (exactly ACP, with guarded linear recursion) [28]. That is, ACP may have the same expressive power as Turing machine. Although GameACP cannot improve the expressive power of ACP, it still provides an elegant and convenient way to model game theory in ACP.

As [26] pointed out, the combination of computation tools and game theory, not only includes using computation tools to interpret games or introducing games into the computation tools, but also includes using game theory to give computation concepts an interpretation. The second one remains an open problem and is a future work we would do.

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