Experimental Reconstruction of Wigner Distribution Currents in Quantum Phase Space

Yi-Ru Chen,1 Hsien-Yi Hsieh,1 Jingyu Ning,1 Hsun-Chung Wu,1 Hua Li Chen,2 You-Lin Chuang,3 Popo Yang,1 Ole Steuernagel,4 Chien-Ming Wu,1 and Ray-Kuang Lee1,2,3,5

1Institute of Photonics Technologies, National Tsing Hua University, Hsinchu 30013, Taiwan
2Department of Physics, National Tsing Hua University, Hsinchu 30013, Taiwan
3Physics Division, National Center for Theoretical Sciences, Taipei 10617, Taiwan
4Department of Physics, Astronomy and Mathematics, University of Hertfordshire, Hatfield, AL10 9AB, UK
5Center for Quantum Technology, Hsinchu 30013, Taiwan

(Dated: November 17, 2021)

We experimentally reconstruct Wigner’s quantum phase current for the first time. Using an optical parametric oscillator, coupled to the environment, at varying pump powers, allows us to directly observe signatures of quantum phase space dynamics for the creation of squeezed states. Our approach allows identifying pure squeezing dynamics as well as dissipation and decoherence currents. The central stagnation point of the current shows a topological charge of -1, this is generic for the dynamics of squeezer setups. This work establishes a novel experimental paradigm for measuring quantumness and non-classicality of the dynamics of a quantum system.

Traditionally, when studying quantum systems, physicists concentrate on features of the system state more than its dynamics. Many measures of the quantumness of states exist [1], but few measures of the quantumness of the dynamics have been considered [2–8].

Here we establish experimentally that quantum dynamics can be directly studied in great detail. We thus open the door to investigations of the quantumness of dynamics.

We use Wigner’s representation of quantum systems [9], based on Wigner’s distribution $W(x, p)$ in quantum phase space (with coordinates $x$ for position and $p$ for momentum). $W$ is experimentally more directly measurable than a system’s density matrix [10, 11]. Additionally, $W$ conceptually allows for a more direct comparison between classical and quantum states than the commonly used density matrix $\rho$ [12–15].

In Wigner’s representation the phase space current $J = (J_x, J_p)$ describes the dynamics of the distribution $W$ via the continuity equation

$$\frac{\partial W(x, p)}{\partial \tau_{\text{eff}}} + \nabla \cdot J(x, p) = 0,$$

where $\nabla = (\partial_x, \partial_p)$. Wigner’s continuity equation (1) is the equivalent of von Neumann’s equation, but unlike this conventional approach, using Eq. (1) has the great advantage of allowing us to visualize quantum dynamics in phase space since $W$ is real-valued and $J$ a vector field that exists everywhere.

Here we determine Wigner’s current $J$ experimentally. It allows us to study the dynamics of a quantum system and to compare it directly to its classical counterpart $\dot{j}$. In classical phase space dynamics, the current $\dot{j} = \rho \nu$ carries the density $\rho$ via the divergence-free Hamiltonian velocity field $\nabla \cdot \nu = 0$. No such decomposition of the quantum current $J \neq W \psi$ exists, since the supposed velocity field $\psi$ would in the quantum case develop areas with infinity divergence $|\nabla \cdot \psi| = \infty$; but $J$ is always well behaved [6]. In the case investigated here phase space volumes are not conserved.

Path integrals along $J$ yield its field lines, reminiscent of classical phase portraits [5, 16]. But in contrast to the classical case, in the quantum case $J$ can have stagnation points anywhere in phase space, even when the momentum $p \neq 0$. These stagnation points do typically move under the dynamics whilst carrying a conserved topological charge which governs their splittings and mergers with other stagnation points [5, 17, 18].

Theoretical studies have confirmed that such behaviors are expected to occur in a large variety of systems such as anharmonic [16] and tunneling systems [5, 19], in systems described by effectively non-Hermitian parity-time symmetric hamiltonians [20] and even in discrete (spin) systems [17, 18].

Experimental data generated from balanced homodyne detectors via quantum state tomography enables us to reconstruct the Wigner distribution [9] and density matrix of quantum states [10, 11]. Quantum state tomography has been implemented in a variety of quantum systems, including quantum optics [21], ultracold atoms [22, 23], ions [24, 25], and superconducting devices [26]. However, so far all investigations of Wigner’s current are theoretical due to the lack of experimental capability in capturing the quantum dynamics in real time.

In this Letter, we use the advantages of machine learning-based quantum state tomography to reconstruct not only the Wigner distribution $W$, but also the associated Wigner current $J$ experimentally. Our neural network-enhanced tomography has demonstrated good performance when extracting detailed information about the degradation in a system undergoing decoherence dynamics [27].

Here, we generate squeezed vacuum states in a bow-tie optical parametric oscillator (OPO) cavity enclosing a periodically poled nonlinear KTiOPO 4 (PPKTP) crystal with second order nonlinear susceptibility $\chi^{(2)}$, operated below the lasing threshold at the wavelength 1064 nm [27].

Such systems predominantly generate photon pairs in degenerate processes well described by effective Hamiltonians of the form [28–31]

$$\hat{H} = -i\hbar \chi^{(2)}(\alpha^* \hat{a}^2 - \alpha \hat{a}^{\dagger 2}),$$

where $\alpha$ denotes the complex amplitude describing the pump field’s strength and phase. The corresponding formal time
FIG. 1. Measured quantum noise levels (in dB) of squeezing (SQ) and anti-squeezing (ASQ) quadratures at different pump powers. In the ideal case, SQ and ASQ levels should be equal (blue dashed line). Taking loss and phase noise, due to coupling to the environment, into account we instead observe degraded squeezing performance described by the solid green (Exp-fitting) line [27].

The evolution operator consequently has the form

$$\hat{U}(t) = \exp[-i\hat{H}t/\hbar] = \exp\left[\frac{i\chi^{(2)}}{2} (\alpha^* \hat{a}^2 - \alpha \hat{a}^\dagger 2)\right]$$

defined in Fig. 1.

FIG. 2. Experimental raw data of quadrature noise for squeezed vacuum states at pump powers (a) 0.25 mW and (b) 5 mW, respectively. The corresponding machine-learning determined [27] (c, d) density matrix in the photon number basis and (e, f) Wigner distributions $W(x, p)$ are obtained by machine learning-based quantum state tomography.

In other words, varying the pump power amounts to a formal variation of the evolution time and allows us to utilize the Wigner current $J$ of Eq. (1).

By injecting the AC signal of our balanced homodyne detection, the spectrum analyzer records the squeezing and anti-squeezing levels when scanning the phase of the local oscillator. In Fig. 1, we show the measured noise level curves for squeezing (SQ) and anti-squeezing (ASQ) in decibel (dB) while the pump power increases from 0 to 5 mW. The magnitude of squeezing and anti-squeezing levels are almost the same at low pump power levels, indicating that the generated squeezed states are almost pure. However, degradation arises due to the coupling to the environment, giving roughly 2.5 dB in SQ but 3.0 dB in ASQ at 5 mW.

The experimental balanced homodyne detector data are discretized into 2,048 levels, for pump powers 0.25 mW and 5 mW we show a sample in Fig. 2 panels (a) and (b), respectively. These discretized data are fed into a machine learning-based quantum state tomography algorithm [27]. The reconstructed density matrices in the photon number basis and (e, f) Wigner distributions $W(x, p)$ are obtained by machine learning-based quantum state tomography.

Information about the degradation of the purity, $\text{tr}(\rho^2)$, of the quantum state in our squeezing setup is shown in Fig. 3 (a) as a function of the pump power. The purity of our squeezed vacuum remains as high as $\approx 0.98$, even working at 5 mW pump power. However, unavoidable decoherence from the interaction with the environment is in evidence.

We use machine learning to extract information about this decoherence, based on the singular value decomposition of the reconstructed density matrix into its three dominant terms

$$\rho = \sigma_1 \rho^{sq} + c_1 \rho^{sq \text{th}} + d_1 \rho_{th}.$$  

FIG. 3. (a) Purity, $\text{tr}(\rho^2)$, of squeezed states and (b) machine learning-determined magnitude of weights $\sigma_1$, $c_1$ and $d_1$ in constituent mixed states (3) as functions of pump power.

Here, $\rho^{sq}$ denotes (pure) squeezed vacuum, $\rho^{sq} = \hat{S}\rho_0\hat{S}^\dagger$,
FIG. 4. Snapshots of Wigner current distributions (arrows, displayed in arbitrary units) of squeezed vacuum states, with the colored contours of corresponding Wigner distributions shown in the background. The First column (a-d): Wigner current, $J_{\text{exp}}$, reconstructed from experimental data, at OPO pump powers 0.25, 1.75, 3.25, 4.75 mW, respectively. The Second column (e-h): the ideal Wigner current, $J_{\text{sys}}$, fitted to pure squeezed states. The Third column (i-l): thermal contributions in the Wigner current $J_{\text{env}} = J_{\text{exp}} - J_{\text{sys}}$, extracted from the experimental data (by subtraction of the Second column from the First column data). The Fourth column (m-p): dissipative part of Wigner current, $J_{\text{damp}}$ (6). The Fifth column (q-t): diffusive part of Wigner current, $J_{\text{diff}}$ (6). Note, adding $J_{\text{damp}}$ and $J_{\text{diff}}$ in the Fourth and Fifth columns, respectively, yields $J_{\text{env}}$ shown in the Third column with $\rho_0$ the vacuum state. Due to coupling to the environment, the two dominant admixtures are from thermal states $\rho_{\text{th}}$ and squeezed thermal states $\rho_{\text{sq th}}^n = \hat{S} \rho_{\text{th}} \hat{S}^\dagger$ [32–35]. Fig. 3 (b) shows, that the respective three weights in Eq. (3) change little as the pump power is increased to 5 mW. This confirms that our system is stable since losses due to the environment are fairly constant.

Having experimentally determined the Wigner distribution $W_{\text{exp}}(\tau_j)$ for various effective times $\tau_j$, the corresponding Wigner current $J_{\text{exp}}$ can be obtained by the continuity Eq. (1), as now $\partial W_{\text{exp}}(x, p)/\partial \tau_{\text{eff}}$ is known. Here, we discretize the pump power into 20 effective time steps, i.e., using 0.25 mW pump power increments. In the First column of Fig. 4, a series of experimentally determined snapshots of Wigner current $J_{\text{exp}}$ are depicted as the pump power is increased, with the colored contours of corresponding Wigner distributions shown in the background.

To our knowledge this shows for the first time a quantum system’s experimentally determined Wigner current.

In order to extract further information about this open system’s dynamics, we decompose the contributions to $J_{\text{exp}} = J_{\text{sys}} + J_{\text{env}}$ due to OPO and the environment, respectively. The Wigner current of an ideal OPO system (2) driven at optical frequency $\omega_0$, denoted as $J_{\text{sys}}$, has the form

$$J_{\text{sys}} = \chi^2 |\alpha| \left( \begin{array}{c} x W \cos \theta + \frac{p}{\omega_0} W \sin \theta \\ \omega_0 x W \sin \theta - p W \cos \theta \end{array} \right)$$

for $\theta = \pm \pi/2$, (4)

where we assumed that the phase, $\theta$, of the driving field $\alpha = r \exp(i\theta)$ is chosen such that the squeezing parameter is $\xi = \chi^2 |\alpha|$.

In Fig. 4 (a-h), we observe that the currents for squeezed
vacuum states follow hyperbolic curves aligned with the squeezed and anti-squeezed quadratures, as predicted by theory for $J_{sy}$ in Eq. (5).

The stagnation point of the Wigner current of our OPO system is at the phase space origin, its orientation winding number $\omega(\mathcal{L}) \equiv \frac{\pi}{\hbar} \int \gamma^2 d\phi$ along a simple closed-loop $\mathcal{L}$ around the origin, has the value $\omega = -1$; here, $\phi$ is the orientation angle of Wigner current vectors $J_{exp}$ [5].

Compared to the pure system current $J_{sys}$, shown in Fig. 4 (e-h), the experimentally observed current $J_{exp}$, displayed in Fig. 4 (a-d), shows modifications due to decoherence processes: expansions of the Wigner distributions and some distortions in the currents. We extract the thermal contributions in the Wigner current by subtracting the ideal system current from the experimentally obtained data. This difference $J_{env} = J_{exp} - J_{sys}$ is displayed in the Third column of Fig. 4 (i-l). It graphically displays, to our knowledge for the first time, how the squeezing current is locally counteracted by the currents due to the coupling to the environment.

The Wigner current, $J_{env}$, due to the interaction with the environment has a dissipative part, $J_{damp}$, and a diffusive part, $J_{diff}$, of the form [36]

$$J_{env} = \frac{\gamma}{2} \left( xW \right) - \frac{\gamma}{2} \frac{\hbar}{\omega_0} (\pi + \frac{1}{2}) \left( \frac{\partial_x W}{\partial_p W} \right)$$

$$\equiv J_{damp} + J_{diff},$$

where $\gamma$ denotes the system energy damping rate, and $\pi^{-1} = [\exp(h\omega_0/k_B T) - 1]$ accounts for the average photon occupation number in the environmental thermal reservoir at temperature $T$.

We note that both, equation (4) and (6), imply that our system is ‘non-Liouvilian’ [6], phase space volumes change while our system evolves.

To analyze and deepen our understanding of the roles played by the environment currents further still, we use Eq. (6) to decompose the Wigner distribution current $J_{env}$ into the dissipative $J_{damp}$ and diffusive part $J_{diff}$, see the Fourth and Fifth columns of Fig. 4. Here, the experimentally determined Wigner function $W_{exp}$ is used to generate the dissipative and diffusive currents, $J_{damp}$ and $J_{diff}$, employing the two fitting parameters, $\gamma = 0.01$ and $\tilde{\gamma} = 0.1$.

The push-and-pull in the environmental part, $J_{env}$, of the Wigner current can be understood as the interplay between the damping (due to spontaneous emission, the Fourth column of Fig. 4) and diffusive (due to induced emission, the Fifth column of Fig. 4) processes resulting from interactions between OPO and its thermal environment. Our approach makes a phase space visualization of the roles played by Einstein’s A and B coefficients in a quantum system possible.

We emphasize that damping and diffusive parts of the currents do not have rotational symmetry in phase space but are modified by the shape of $W$ breaking this symmetry. In other words, dissipative and diffusive Wigner currents together form the overall environmental Wigner current (the Third column of Fig. 4), which is counteracting the system’s squeezing action.

In conclusion, with the help of machine learning-enhanced quantum state tomography, we experimentally reconstructed the Wigner distribution and its phase space current from squeezed vacuum states through the one-to-one mapping between the pump power and an effective time parameter. We found a Wigner current stagnation point at the origin and confirmed its orientation winding number topological charge as $\omega = -1$. The analysis of the Wigner current due to interactions with the thermal environment reveals a push-and-pull between its damping and diffusive parts.

In addition to the squeezed states investigated here, our methodology can be readily applied to other families of continuous variable states, such as single-photon states and ‘cat’ states [37–39], in order to study their evolution and interactions with outside systems. Our experimental implementation promises to provide us with a powerful diagnostic toolbox allowing us to probe details of a quantum state’s dynamics at levels previously not accessible in experiments [40, 41].

ACKNOWLEDGEMENT

This work is partially supported by the Ministry of Science and Technology of Taiwan (No. 108-2923-M-007-001-MY3 and No. 110-2123-M-007-002), Office of Naval Research Global, US Army Research Office (ARO), and the collaborative research program of the Institute for Cosmic Ray Research (ICRR), the University of Tokyo.

* rklee@ee.nthu.edu.tw

[1] F. Fröwis, P. Sekatski, W. Dür, N. Gisin, and N. Sangouard, “Macroscopic quantum states: Measures, fragility, and implementations,” Rev. Mod. Phys. 90, 025004 (2018).
[2] M. V. Berry and N. L. Balazs, “Evolution of semiclassical quantum states in phase space,” J. Phys. A. 12, 625 (1979).
[3] H. J. Korsch and M. V. Berry, “Evolution of Wigner’s phase-space density under a nonintegrable quantum map,” Phys. D Nonl. Phen. 3, 627, (1981).
[4] R. T. Skodje and H. W. Rohrs, and J. VanBuskirk, “Flux analysis, the correspondence principle, and the structure of quantum phase space,” Phys. Rev. A 40, 2894 (1989).
[5] O. Steuernagel, D. Kakofengitis, and G. Ritter, “Wigner flow reveals topological order in quantum phase space dynamics,” Phys. Rev. Lett. 110, 030401 (2013).
[6] M. Oliva, D. Kakofengitis, and O. Steuernagel, “Anharmonic quantum mechanical systems do not feature phase space trajectories,” Physica A, 502, 201 (2017).
[7] M. Oliva and O. Steuernagel, “Dynamic shear suppression in quantum phase space,” Phys. Rev. Lett. 122, 020401, (2019).
[8] A. E. Bernardini, “Testing nonclassicality with exact Wigner currents for an anharmonic quantum system,” Phys. Rev. A 98, 052128 (2018).
[9] E. Wigner, “On the Quantum Correction For Thermodynamic Equilibrium,” Phys. Rev. A 40, 749 (1932).
[10] Z. Hradil, “Quantum-state estimation,” Phys. Rev. A 55, R1561(R) (1997).
[11] A. I. Lvovsky and M. G. Raymer, “Continuous-variable optical quantum-state tomography,” Rev. Mod. Phys. 81, 299 (2009).
[12] U. Leonhardt and H. Paul, “Measuring the quantum state of light,” Prog. Quant. Elec. 19, 89 (1995).
[13] C. K. Zachos and D. B. Fairlie, and T. L. Curtright, Quantum Mechanics in Phase Space: An Overview with Selected Papers, (World Scientific, 2005).
[14] W. P. Schleich, Quantum Optics in Phase Space, (Wiley-VCH, 2001).
[15] W. H. Zurek, “Sub-Planck structure in phase space and its relevance for quantum decoherence,” Nature 412, 712 (2001).
[16] D. Kakofengitis and O. Steuernagel, “Wigner’s quantum phase-space current in weakly-anharmonic weakly-excited two-state systems,” Euro. Phys. J. Plus 132, 381, (2017).
[17] P. Yang, I. F. Valtierra, A. B. Klimov, S.-T. Wu, R.-K. Lee, L. L. Sanchez-Soto, and G. Leuchs, “The Wigner flow on the sphere,” Phys. Scripta 94, 044001 (2019).
[18] I. F. Valtierra, A. B. Klimov, G. Leuchs, and L. L. Sanchez-Soto, “Quasiprobability currents on the sphere,” Phys. Rev. A 101, 033803 (2020).
[19] R. T. Skodje, H. W. Rohrs, and J. VanBuskirk, “Flux analysis, the correspondence principle, and the structure of quantum-state tomography,” Rev. Mod. Phys. 81, 299 (2009).
[20] L. Praxmeyer, P. Yang, and R.-K. Lee, “Phase-space representation of a non-Hermitian system with PT symmetry,” Phys. Rev. A 93, 042122 (2016).
[21] U. L. Andersen, J. S. Neergaard-Nielsen, P. van Loock, and A. Furusawa, “Hybrid discrete- and continuous-variable quantum information,” Nature Phys. 11, 713 (2015).
[22] D. Barredo, S. de Leseleuc, V. Lienhard, T. Lahaye, and A. Browaeys, “An atom-by-atom assembler of defect-free arbitrary two-dimensional atomic arrays,” Science 354, 1021 (2016).
[23] M. Endres, H. Bernien, A. Keesling, H. Levine, E. R. Anschuetz, A. Krajenbrink, C. Senko, V. Vuletic, M. Greiner, and M. D. Lukin, “Atom-by-atom assembly of defect-free one-dimensional cold atom arrays,” Science 354, 1024 (2016).
[24] J. Zhang, G. Pagano, P. W. Hess, A. Kyprianidis, P. Becker, H. Kaplan, A. V. Gorshkov, Z.-X. Gong, and C. Monroe, “Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator,” Nature 551, 601 (2017).
[25] N. Fiuris, O. Marty, C. Maier, C. Hempel, M. Holzäpfel, P. Jurcevic, M. B. Plenio, M. Huber, C. Roos, R. Blatt, and B. Lanyon, “Observation of entangled states of a fully controlled 20-qubit system,” Phys. Rev. X 8, 021012 (2018).
[26] B. Vlastakis, G. Kirchmair, Z. Leghtas, S. E. Nigg, L. Frunzio, S. M. Girvin, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, “Deterministically encoding quantum information using 100- photon Schrodinger cat states,” Science 342, 607 (2013).
[27] H.-Y. Hsieh, Y.-R. Chen, H.-C. Wu, H. L. Chen, J. Ning, Y.-C. Huang, C.-M. Wu, and R.-K. Lee, “Extract the Degradation Information in Squeezed States with Machine Learning,” arXiv: 2106.04058 (2021).
[28] H. P. Yuen, “Two-photon coherent states of the radiation field,” Phys. Rev. A 13, 2226 (1976).
[29] H.-Y. Hsieh, Y.-R. Chen, H.-C. Wu, H. L. Chen, J. Ning, Y.-C. Huang, C.-M. Wu, and R.-K. Lee, “Extract the Degradation Information in Squeezed States with Machine Learning,” arXiv: 2106.04058 (2021).
[30] D. F. Walls and G. J. Milburn, Quantum Optics, 2nd Ed. (Springer, 1984).
[31] L. Praxmeyer, P. Yang, and R.-K. Lee, “Phase-space representation of a non-Hermitian system with PT symmetry,” Phys. Rev. A 93, 042122 (2016).
[32] U. L. Andersen, J. S. Neergaard-Nielsen, P. van Loock, and A. Furusawa, “Hybrid discrete- and continuous-variable quantum information,” Nature Phys. 11, 713 (2015).
[33] D. Barredo, S. de Leseleuc, V. Lienhard, T. Lahaye, and A. Browaeys, “An atom-by-atom assembler of defect-free arbitrary two-dimensional atomic arrays,” Science 354, 1021 (2016).
[34] M. Endres, H. Bernien, A. Keesling, H. Levine, E. R. Anschuetz, A. Krajenbrink, C. Senko, V. Vuletic, M. Greiner, and M. D. Lukin, “Atom-by-atom assembly of defect-free one-dimensional cold atom arrays,” Science 354, 1024 (2016).
[35] J. Zhang, G. Pagano, P. W. Hess, A. Kyprianidis, P. Becker, H. Kaplan, A. V. Gorshkov, Z.-X. Gong, and C. Monroe, “Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator,” Nature 551, 601 (2017).
[36] N. Fiuris, O. Marty, C. Maier, C. Hempel, M. Holzäpfel, P. Jurcevic, M. B. Plenio, M. Huber, C. Roos, R. Blatt, and B. Lanyon, “Observation of entangled states of a fully controlled 20-qubit system,” Phys. Rev. X 8, 021012 (2018).