Abstract

Fairness-aware learning aims at constructing classifiers that not only make accurate predictions, but also do not discriminate against specific groups. It is a fast-growing area of machine learning with far-reaching societal impact. However, existing fair learning methods are vulnerable to accidental or malicious artifacts in the training data, which can cause them to unknowingly produce unfair classifiers. In this work we address the problem of fair learning from unreliable training data in the robust multisource setting, where the available training data comes from multiple sources, a fraction of which might not be representative of the true data distribution. We introduce FLEA, a filtering-based algorithm that allows the learning system to identify and suppress those data sources that would have a negative impact on fairness or accuracy if they were used for training. We show the effectiveness of our approach by a diverse range of experiments on multiple datasets. Additionally, we prove formally that –given enough data– FLEA protects the learner against corruptions as long as the fraction of affected data sources is less than half.

1 Introduction

Machine learning systems have started to permeate many aspects of our everyday life, such as finance (e.g. credit scoring), employment (e.g. judging job applications) or even judiciary (e.g. recidivism prediction). In the wake of this trend, other aspects besides prediction accuracy become important to consider. One crucial aspect is fairness, which aims at preventing learned classifiers from acting in a discriminatory way. To achieve this goal, fairness-aware learning methods adjust the classifier parameters in order to fulfill an appropriate measure of fairness. This strategy is highly successful, but only under idealized conditions of clean i.i.d.-sampled data. Unfortunately, fairness-aware learning methods are not robust against unintentional errors or intentional manipulations of the training data.

In this work, we propose a new algorithm, FLEA (Fair LEarning against Adversaries) that overcomes this problem in the setting where the training data is not one monolithic block, but rather comes as a collection of multiple data sources. This is, in fact, a common scenario, since obtaining fairness-aware datasets requires collecting both labels and protected attributes information, and is therefore expensive. Organizations such as hospitals or universities may need to share data about patients or job applicants (adhering to privacy considerations) to have a sufficient sample size for training. While sharing data in this way is clearly beneficial, none of the institutions can fully trust the quality of the data provided by the others, and so robustness concerns arise.

Briefly, FLEA adds a filtering step that effectively identifies and suppresses data sources that could have a negative impact on the classifier fairness or accuracy, so long as they constitute less than half of all sources. To achieve this, we introduce a new dissimilarity measure, disparity, that measures the achievable difference in classifier fairness between two data sources. We combine this with the existing discrepancy measure, which plays an analogous role for the classifier accuracy, and the
disbalance, which measures changes to the group composition of the training data. Together, these three notions provide a sufficient criterion for detecting harmful data, as we show both empirically and theoretically.

While previous method for robust fairness were only able to protect against special cases, such as random label flips, FLEA ensures that even a worst-case adversary is unable to negatively affect the training process: either the changes to the data are minor and will not hurt learning, or they are large, in which case the affected data sources are identified and removed. Our theoretical results certify the ability of FLEA to learn classifiers with optimal fairness and accuracy in the infinite sample size limit, as well as provide finite sample guarantees. Our extensive experimental evaluation demonstrates FLEA’s practical usefulness in suppressing the effect of corrupted data when learning fair models, even in cases where previous robust learning methods fail.

2 Preliminaries and Related Work

2.1 Fair classification

Throughout this work, we adopt a standard classification setting in which the task is to predict a binary label \( y \in \{0, 1\} \) for any \( x \in \mathcal{X} \). For a fixed data distribution \( p(x, y) \in \mathcal{P} (\mathcal{X} \times \mathcal{Y}) \), the classic goal of learning is to find a prediction function \( f : \mathcal{X} \rightarrow \mathcal{Y} \) with high accuracy, i.e. small risk, \( \mathcal{R}_p (f) = E_{p(x,y)}[y \neq f(x)] \), where \( || P || = 1 \) if a predicate \( P \) is true and \( || P || = 0 \) otherwise.

With the recent trend to consider not only the accuracy but also the fairness of a classifier, a number of statistical measures have been proposed to formalize this notion. In this work, we focus on the most common and simplest one, demographic parity (DP) \cite{Calders2009}. It postulates that the probability of a positive classifier decision should be equal for all subgroups of the population. Formally, we assume that each example \((x, y)\) also possesses a protected attribute, \( a \in \mathcal{A} \), which indicates its membership in a specific subgroup of the population. For example, \( a \) could indicate race, gender or a disability. For simplicity of exposition, we treat the protected attribute as binary-valued, but extensions to multi-valued attributes are straightforward by summing over all pairwise terms. Note that \( a \) might be a component or a function of \( x \), in which case it is available at prediction time, or it might be contextual information, in which case it would only be available for the learning algorithm at training time, but not for the resulting classifier at prediction time. We cover both aspects by treating \( a \) as an additional random variable, and write the underlying joint data distribution as \( p(x, y, a) \).

For a classifier \( f : \mathcal{X} \rightarrow \{0, 1\} \), the demographic parity violation, \( \Gamma_p \), and the empirical counterpart, \( \Gamma_S \) \cite{Calders2009, Dwork2012}, for a dataset \( S \subset \mathcal{X} \times \mathcal{Y} \times \mathcal{A} \) are defined as

\[
\Gamma_p (f) = \left| E_{p(x|a=1)} f(x) - E_{p(x|a=0)} f(x) \right| ,
\]

\[
\Gamma_S (f) = \left| \frac{1}{n_{a=1}} \sum_{x \in S} f(x) - \frac{1}{n_{a=0}} \sum_{x \in S} f(x) \right| ,
\]

where \( S_{a=z} = \{(x, y, a) \in S : a = z\} \) and \( n_{a=z} = |S_{a=z}| \) for \( z \in \{0, 1\} \). Smaller values of the demographic parity violation indicate more fair classifiers. Analogous quantities can be defined for related fairness measures, such as equality of opportunity or equalized odds \cite{Hardt2016, Zafar2017}. A detailed description of these and many others choices can be found in \cite{Barocas2019}.

Fairness-aware learning. In the last years, a plethora of algorithms have been developed that are able to learn classifiers that are not only accurate but also fair, see, for example \cite{Mehrabi2021} for an overview. They mostly rely on three core mechanisms. Postprocessing methods \cite{Chzhen2020, Hardt2016, Woodworth2017} adjust the acceptance thresholds of a previously trained classifier for each protected group, so that the desired fairness criterion is met. This is a simple, reliable and often effective method, but it requires the protected attribute to be available at prediction time. Penalty-based methods \cite{Chuang2021, Kamishima2012, Mandal2020, Zafar2017, Zemel2013} add a regularizer or constraints to the learning objective that penalize or prevent parameter choices that lead to unfair decisions. Adversarial methods \cite{Beutel2017, Lahoti2020, Wadsworth2018, Zhang2018} train an adversary in
We adopt the adversary model because it places no restrictions on the corruptions, and thus subsumes
We assume the following data corruption model
Wang et al., 2020]. However, as shown in Konstantinov and Lampert [2021], full protection against
where $R$ operates on them. This results in new data sources, $S_1, \ldots, S_N$, which the learning algorithm receives as input. The adversary is an arbitrary (deterministic or randomized) function with the only restriction that for a fixed subset of indices, $G \subset \{1, \ldots, N\}$, the data source remains unchanged. That is, $S_i = \tilde{S}_i$ for all $i \in G$, and $S_i$ is arbitrary for $i \not\in G$.

We adopt the adversary model because it places no restrictions on the corruptions, and thus subsumes all scenarios: malicious manipulations; common attack models, such as noisy labels; data-quality issues, such as biased sampling or data entry errors. This is in contrast to most prior works, which work under specific restrictive assumptions that may not apply in relevant real-world settings.

Multisource learning with protection against potential manipulations is known as robust multisource learning [Erfani et al., 2017]. A central concept in this context is the (empirical) discrepancy [Kifer et al., 2004; Mohri and Medina, 2012]. For two datasets, $S_1, S_2$, and a hypothesis set $\mathcal{H} \subset \{h : X \to Y\}$, it measures the maximal amount by which their estimates of the classification accuracy can differ:

$$\text{disc}(S_1, S_2) = \sup_{h \in \mathcal{H}} |\mathcal{R}_{S_1}(h) - \mathcal{R}_{S_2}(h)|,$$

where $\mathcal{R}_S(h) = \frac{1}{|S|} \sum_{(x,y) \in S}[y \neq h(x)]$ is the empirical risk of $h$ on $S$. In Konstantinov et al. [2020] the discrepancy is used to identify and suppress potentially harmful data sources. The associated algorithm is mostly of theoretic interest, however, as it requires too large training sets to

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1Formally, the situation we describe is the homogeneous multi-source (also called multi-batch) setting Qiao and Valiant [2018]; Konstantinov et al. [2020]; Jain and Orlišč [2020b], in which otherwise i.i.d. data is potentially manipulated. Heterogeneous multi-source learning, as e.g. in Crammer et al. [2008], is a variant of the domain adaptation problem and outside the scope of our work.

2Adversary is the common computer science term for a process whose aim it is to prevent a system from operating as intended. Our adversaries manipulate the training data and should not be confused with adversaries in adversarial machine learning, such as adversarial examples Goodfellow et al. [2015], or generative adversarial networks Goodfellow et al. [2014].
be effective. Konstantinov and Lampert [2019] follow a similar route, but the proposed algorithm requires access to a reference set that is guaranteed to be free of data manipulations. In Chen et al. [2019], Jain and Orlitsky [2020a], Qiao and Valiant [2018] robust multisource learning is addressed using tools from robust statistics in the context of discrete density estimation. All of the above works are tailored to the task of ensuring high accuracy of the learned classifiers or estimators, but they are not sensitive to issues of fairness.

3 Fair Multisource Learning

The goal of this work is to develop a method that allows fairness-aware learning, even if some of the available data sources are unrepresentative of the true training distribution. For this, we introduce FLEA, a filtering-based algorithm that identifies and suppresses those data sources that would negatively impact the fairness of the trained classifier. Its main innovation is the disparity measure for comparing datasets in terms of their fairness estimates.

**Definition 1 (Empirical Disparity).** For two datasets $S_1, S_2 \subset X \times Y \times A$, their empirical disparity with respect to a hypothesis class $H$ is

$$\text{disp}(S_1, S_2) = \sup_{h \in H} |\Gamma_{S_1}(h) - \Gamma_{S_2}(h)|. \quad (4)$$

where $\Gamma : H \rightarrow \mathbb{R}$ is an empirical (un)fairness measure, such as the demographic parity violation $\text{(2)}$.

The disparity measures the maximal amount by which the estimated fairness of a classifier in $H$ can differ between using $S_1$ or $S_2$ as the basis of the estimate. A small disparity value implies that if we construct a classifier that is fair with respect to $S_1$, then it will also be fair with respect to $S_2$.

Definition 1 is inspired by the empirical discrepancy $\text{(3)}$. Low discrepancy implies that a classifier learned on one dataset will have comparable accuracy as one learned on the other; low disparity means that the two classifiers will have comparable fairness. FLEA relies on the discrepancy as well as the disparity, because ensuring fairness alone does not suffice (e.g. a constant classifier is perfectly fair). As a third relevant quantity we introduce the (empirical) disbalance.

$$\text{disb}(S_1, S_2) = \left| \frac{|S_{a=1}^1|}{|S_1|} - \frac{|S_{a=1}^2|}{|S_2|} \right|. \quad (5)$$

The disbalance compares the relative sizes of the protected groups of two datasets. Its inclusion is a technical requirement to be able to also formally prove that demographic parity fairness remains unaffected by corruption.

In combination, disparity, discrepancy, and disbalance form an effective criterion for detecting dataset manipulations. If two datasets of sufficient size are sampled i.i.d. from the true data distribution, then by the law of large numbers we can expect all three measures to be small (they would be zero if not for the effect of finite sample sizes). If one of the datasets is sampled like this (called clean from now on) but the other is manipulated, then there are two possibilities. It is still possible that all three values are small. In this case, equations (3)–(5) ensure that neither accuracy nor fairness would be negatively affected, and we call such manipulations benign. If at least one of the values is large, training on such a manipulated datasource could have undesirable consequences. Such manipulations we will call malignant. Finally, when comparing two manipulated datasets, disparity, discrepancy, and disbalance can each have arbitrary values.

3.1 FLEA: Fair Learning against Adversaries

We now introduce the FLEA algorithm, which is able to learn fair classifiers even if some of the datasets are noisy, biased or have been manipulated. Similar to classic outlier rejection techniques [Barnett and Lewis, 1984], the main algorithm (Algorithm 1) takes a filtering approach. Given the available data sources and additional parameters, it calls a subroutine that identifies a subset of clean or benign sources, merges the training data from these, and trains a (presumably fairness-aware) learning algorithm on the resulting dataset.
FLEA’s crucial component is the filtering subroutine. This estimates the pairwise disparity, discrepancy and disbalance between all pairs of data sources and forms a matrix of dissimilarity scores (short: $D$-scores), from which \[D\] scores will be at least as small as the result of comparing two clean sources. For benign sources, the same reasoning applies, since their $D$-scores are indistinguishable from clean ones. For a malignant $S_i$, at least $K$ of the $D$-scores will be large, namely the ones where $S_i$ is compared to a clean source. Hence, there can be at most $N - K$ small $D$-scores for $S_i$. Because $K > N - K$, the $\alpha$-quantile $q_i$ will be at least as large as comparing a clean dataset to a malignant one.

As discussed above, large values indicate that at least one of the sources must be malignant. It is not a priori clear, though, how to use this information. On the one hand, we do not know which of the two datasets is malignant or if both are. On the other hand, malignant sources can also occur in pairs with small $D$-score, when both datasets were manipulated in similar ways. Finally, even the $D$-scores between two clean or benign sources will have non-zero values, which depend on a number of factors, in particular the data distribution and the hypothesis class.

FLEA overcomes this problem by using tools from robust statistics. For any dataset $S_i$, it computes a value $q_i$ (called $q$-value) as the $\alpha$-quantile of the $D$-scores to all other datasets. It then computes the $\alpha$-quantile of all such values and selects those datasets with $q$-values up to this threshold. To see that this procedure has the desired effect of filtering out malignant datasets, assume a setting with $N$ data sources of which at least $K > \frac{N}{2}$ are clean and set $\alpha = \frac{K}{N}$.

By assumption, for any clean dataset $S_i$, there are at least $K - 1$ other clean sources with which it is compared. We can expect the $D$-scores of these pairs are small, and also that $D_{ii} = 0$, of course. Because $\alpha N = K$, the quantile, $q_i$, is simply the $K$th-smallest of $S_i$’s $D$-scores. Consequently, $q_i$ will be at least as small as the result of comparing two clean sources. For benign sources, the same reasoning applies, since their $D$-scores are indistinguishable from clean ones. For a malignant $S_i$, at least $K$ of the $D$-scores will be large, namely the ones where $S_i$ is compared to a clean source. Hence, there can be at most $N - K$ small $D$-scores for $S_i$. Because $K > N - K$, the $\alpha$-quantile $q_i$ will be at least as large as comparing a clean dataset to a malignant one.

Choosing those sources that fall into the $\alpha$-quantile of $q_i$ values means selecting the $K$ sources of smallest $q_i$ value. By the above argument, these will either be not manipulated at all, or only in a way that does not have a negative effect on either the fairness or the accuracy of the training process. In practice, the regimes of large and small $D$-scores can overlap due to noise in the sampling process, and the perfect filtering property will only hold approximately. We later discuss a generalization bound that makes this reasoning rigorous.

Note that in the discussion above we use $\alpha = \frac{K}{N}$, which causes the filtering step to reject exactly the optimal number of $N - K$ sources. However, it is easy to see that FLEA remains effective also if run with a lower bound, $\alpha < \frac{K}{N}$. As long as $\frac{1}{2} < \alpha$, the quantile construction will ensure that all malignant sources are filtered out.

### 3.2 Implementation

FLEA is straightforward to implement, with only the discrepancy and disparity estimates in the FILTERSOURCES routine requiring some consideration. Naively, these would require optimizing combinatorial functions (the differences of fraction of errors or positive decisions) over all functions in the hypothesis class. This task is at least as hard as the problem of separating two point sets by a hyperplane, which is known to be NP-hard [Marcotte and Savard, 1992] and even difficult to approximate under any real-world conditions. Instead, we exploit the structure of the optimization.

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\[FLEA\] uses a max-operation to aggregate the scores as it allows proving formal guarantees. Other ways of aggregating the scores in practice, e.g. averaging, can also be imagined.
problems to derive tractable approximations. For the discrepancy (3) such a method was originally proposed in the domain adaptation literature [Ben-David et al. 2010]: finding the hypothesis with maximal accuracy difference between two datasets is equivalent to training a binary classifier on their union with the labels of one of the datasets flipped. From the solution one recovers the discrepancy value as $1 - 2E$, where $E$ denotes the (training) error rate.

For the disparity (4), we propose an analogous route. Intuitively, the optimization step requires finding a hypothesis that is as unfair as possible on $S_1$ (i.e. maximizes $\Gamma_{S_1}$) while being as fair as possible on $S_2$ (i.e. minimizes $\Gamma_{S_2}$, or vice versa. From Equation (2) one sees that a hypothesis $f$ is maximally unfair if it outputs $f(x) = 1$ on $S_{\tilde{\alpha}}^{\text{flip}}$ and $f(x) = 0$ on $S_{\tilde{\alpha}}^{\text{same}}$, or vice versa. This can be achieved by training a classifier to predict $f(x) = a$ on $S_1$. To give both protected groups equal importance, as the definition requires, we use per-sample weights that are inversely proportional to the group sizes. To trade off the unfairness on $S_1$ with the fairness on $S_2$, we simply add $\Gamma_{S_2}$ as a regularizer to this optimization problem. Consequently, estimating $\text{disp}(S_1, S_2)$ becomes as efficient as training a classifier with fairness regularization. As the above construction breaks the symmetry between the roles of $S_1$ and $S_2$, we apply it a second time with the roles of $S_1$ and $S_2$ interchanged, and we keep the larger of both values as estimate of $\text{disp}$.

### 3.3 Theoretical Guarantees

The informal justification of FLEA can be made precise in the form of a generalization bound. In this section we give a high-level formulation of the statement, and sketch the main proof ideas. For the formal statement and full proof, please see the supplemental material.

**Theorem 1.** Let $\tilde{S}_1, \ldots, \tilde{S}_N$ be $N$ datasets, each consisting of $n$ i.i.d.-samples from the data distribution $p$. Let $S_1, \ldots, S_N$ be the result of an adversary operating on these which leaves at least $K > \frac{N}{2}$ of them unchanged. For $\frac{1}{2} \leq \alpha \leq \frac{K}{N}$, let $I = \text{FILTEROURCES}(S_1, \ldots, S_N; \alpha)$ and set $S = \bigcup_{i \in I} S_i$. Denote the VC-dimension of the hypothesis class $H$ by $d$. Then, for any $\delta > 0$ there exists a constant $C = C(N, p, d, \delta)$, such that for any $n \geq C$, the following inequalities hold with probability at least $1 - \delta$ uniformly over all $f \in H$:

$$R_S(f) \leq R_p(f) + \tilde{O}(\sqrt{\frac{1}{n}}), \quad \Gamma_S(f) \leq \Gamma_p(f) + \tilde{O}(\sqrt{\frac{1}{n}}).$$

where $\tilde{O}$ indicates Landau’s big-O notation for function growth up to logarithmic factors [Cormen et al. 2009].

Theorem 1 implies that for large enough training sets the filtered training data $S$ becomes an arbitrarily good representative of the true underlying data distribution with respect to the classification accuracy as well as the fairness. Consequently, it is safe to be used for fairness-aware learning.

Note that despite its intuitive conclusion, Theorem 1 makes a highly non-trivial statement. For example, in the case of learning from a single datasource in which a constant fraction of the data can be manipulated, an analogous theorem is provably impossible [Kearns and Li 1993].

**Proof sketch** The proof consists of three steps. First, we characterize a set of values into which the empirical risks and empirical deviation measures of the clean data sources fall with probability at least $1 - \delta$. Then we show that because the clean datasets cluster in such a way, any individual dataset that is accepted by the FILTEROURCES algorithm provides good empirical estimates of the true risk and the true unfairness measure. Finally, we show that the same holds for the union of these sets, $S$, which implies the inequalities (6). For the risk, the last step is a straightforward consequence of the second. For the fairness, however, a careful derivation is needed that crucially uses the disbalance measure as well.

### 4 Experiments

FLEA’s claim is that it learns classifiers that are fair even in the presence of perturbations in the training data. Due to its filtering approach it can be used in combination with any existing learning method. For our experiments, we run it in combination with three fairness-aware learning methods as well as one fairness-unaware one against a variety of adversaries on four established fair classification
datasets. We benchmark our method against the corresponding base learning algorithms done with no pre-filtering, as well as against three robust learning baselines from the literature.

4.1 Experimental Setup

Datasets We use four standard benchmark datasets from the fair classification literature: COMPAS [Aigwina et al., 2016] (6171 examples), adult (48841), germancredit (1000) and drugs (1885) [Dua and Graff, 2017]. Details about the preprocessing and feature extraction steps can be found in the supplemental material. In all four cases, we use gender as the protected attribute. We train linear classifiers by logistic regression without regularization, using 80% of the data for training and the remaining 20% for evaluation. To achieve the multisource environment, we randomly split each training set into $N$ equal-sized parts, out of which the adversary can manipulate $\lfloor \frac{N - 1}{2} \rfloor$. All experiments are repeated ten times with different train-test splits and we report the mean accuracy and unfairness of the learned classifiers with error bars indicating standard deviation.

Fairness-Aware Learners We use FLEA on top of three fairness-aware learning methods that have found wide adoption in research and practice.
Score postprocessing [Hardt et al., 2016] first learns a fairness-unaware logistic regression classifier on the available data. Afterwards, it determines which decision thresholds for each protected group achieve (approximate) demographic parity on the training set, finally picking the fair thresholds with highest training accuracy.

Fairness regularization [Kamishima et al., 2012] directly learns a fair classifier by minimizing a linear combination of the classification loss and the empirical unfairness measure $\Gamma_S$, where for numeric stability, in the latter the binary-valued classifier decisions $f(x)$ are replaced by the real-valued confidences $p(f(x) = 1|\tilde{x})$.

Adversarial fairness [Wadsworth et al., 2018] learns by minimizing a weighted difference between two terms. One is the loss of the actual classifier; the other is the loss of a classifier that tries to predict the protected attribute from the real-valued outputs of the main classifier.

For completeness, we also include plain logistic regression as a fairness-unaware learner. The supplemental material contains details of the learners’ implementations and parameters.

Adversaries In a real-world setting, one does not know what kind of data quality issues will occur. Therefore, we test the baselines and FLEA for a range of adversaries that reflect potentially unintentional errors as well as intentional manipulations.

- **flip protected (FP), flip label (FL), flip both (FB)**: the adversary flips the value of protected attribute, of the label, or both, in all sources it can manipulate.
- **shuffle protected (SP)**: the adversary shuffles the protected attribute entry in each affected batch.
- **overwrite protected (OP), overwrite label (OL)**: the adversary overwrites the protected attribute of each sample in the affected batch by its label, or vice versa.
- **resample protected (RP)**: the adversary samples new batches of data in the following ways: all original samples of protected group $a = 0$ with labels $y = 1$ are replaced by data samples from other sources which also have $a = 0$ but $y = 0$. Analogously, all samples of group $a = 1$ with labels $y = 0$ are replaced by data samples from other sources with $a = 1$ and $y = 1$.
- **random (RND)**: the adversary randomly picks one of the strategies above for each source.
- **identity (ID)**: the adversary makes no changes to the data.

We include ID to certify that FLEA does not unnecessarily damage the learning process in the case when the training data is actually clean. The other adversaries either weaken the correlations between the protected attribute and the target data, thereby masking a potential existing bias in the data, or they strengthen the correlation between the protected attribute and the target label, thereby increasing the chance that the learned classifier will use the protected attribute as a basis for its decisions. In both cases, the dataset statistics at training time will differ from the situation at test time, and the efficacy of a potential mechanisms to ensure fairness at training time can be expected to suffer. For a more detailed discussion of the adversaries’ effects, please see the supplemental material.

As our result in the following section will show, even these relatively simple adversaries are highly effective in disrupting the fairness-aware learning process. More complex attack strategies have also been proposed in the literature. E.g., similar to Solans et al. [2020], for real-valued input data an adversary could run a gradient-based optimization in order to construct data with maximally adverse effect on a given unfairness measure. This would be difficult in the setting of our experiments, where the training data is mostly categorical. It is also not necessary, though: for the linear classifiers that we consider, the effect of the data manipulations can be assessed explicitly, as we do above. There is no need for an attacker to differentiate through the model.

Baselines To the best of our knowledge, FLEA is the only existing method to tackle fair learning under arbitrary data manipulations. To nevertheless put our results into context, we compare it to three baselines: 1) a robust ensemble (similar to Smith and Martinez [2018]), which learns separate classifiers on each datasource and then combines their decisions by a majority vote. 2) A distributionally robust optimization (DRO) approach as proposed in Wang et al. [2020] to address noisy protected attributes. 3) The filtering approach of Konstantinov et al. [2020] which identifies manipulated sources but does not specifically aim to preserve fairness. More details on these can be found in the supplemental material. Further candidates would be Roh et al. [2020], Konstantinov and
Lampert [2019], but these are not applicable in our setting, as they require access to guaranteed clean validation data.

4.2 Results

The results of our experiments show a very consistent picture across different datasets, base learners and adversaries. We display a representative subset of the outcomes in Fig. 1. The supplemental material contains further results.

For each of the datasets, we report the results (accuracy and unfairness) for five learning algorithms: an ordinary fairness-aware learner, a robust ensemble of fair learners, distributionally-robust learning (adapted from Wang et al. [2020]), fair learning after discrepancy-based filtering ([Konstantinov et al., 2020]) and the proposed FLEA. In each case, the fairness-aware learners are regularization-based, except DRO which uses fairness constraints.

Each panel contains 9 bars. The left-most one ("oracle") in each diagram shows the result of training only on the clean data sources, i.e. the ones which the adversary cannot modify. This is a hypothetical reference, since the necessary information is not available to practical algorithms. The remaining bars correspond to the outcome when different adversaries have perturbed the data. An ideal robust method should achieve results approximately as good as the oracle result, as this would indicate that the adversary was indeed not able to negatively affect the learning process.

Only FLEA comes close to this behavior. The top two rows of Fig. 1 show that for the two larger datasets, COMPAS and adult, FLEA reliably suppresses the effects of all the adversaries, producing classifiers whose accuracy and fairness match almost exactly those of a fair classifier trained only on the clean data sources ("oracle"). The robust ensemble is also able to improve fairness to some extent, but it does not reach the oracle results. The distributionally-robust optimization (DRO) approach shows highly volatile behavior. It sometimes improves fairness, but more often it does not. Compared to the other methods, it causes the largest changes to the accuracy of the classifier. The approach from [Konstantinov et al., 2020] has almost no effect, only for the largest dataset, adult, it is effective against the FL and FB adversaries. This can be explained by the fact that the method only removes sources that it can confidently identify as manipulated. The theory-derived thresholds for this are quite strict, so the method is ineffective unless a lot of data is available.

The observed characteristics of the different methods hold also when more or fewer data sources are available and for the other base learners, see the supplemental material.

The bottom two rows of Fig. 1 show the result for the two smaller datasets drugs and germancredit. The results are consistent with our previous observations. FLEA reliably restores fairness and accuracy to the level of the oracle classifier, except in a small number of cases where a certain loss of accuracy or suboptimal fairness can be observed. The reasons for this behavior is that for small dataset sizes, the estimates of the disc, disp and disb becomes less reliable. The robust ensemble achieves clearly worse results, often reducing rather than increasing the fairness. The DRO approach again shows inconsistent behavior, and the method from [Konstantinov et al., 2020] has no effect at all, since its thresholds never trigger.

Overall, our results confirm the findings of previous works: fair learners do not guarantee fair classifiers, if the quality of the training data cannot be guaranteed. But they also show that protection against perturbed data is possible, as FLEA almost perfectly recovers the accuracy and fairness of a classifier trained only on unperturbed data.

5 Conclusion

In this work, we studied the problem of fairness-aware classification in the setting when data from multiple sources is available, but some of them might by noisy, contain unintentional errors, or have even been maliciously manipulated. Ordinary fairness-aware learning methods are not robust against such manipulations, and – as our experiments confirm – they indeed often fail to produce fair classifiers.

4Note that these results should not negatively reflect on Wang et al., 2020, as our data manipulation model is different and substantially harder than what the DRO method was designed for.
We proposed a filtering-based algorithm, FLEA, that is able to identify and suppress those data sources that would negatively affect the training process, thereby restoring the property that fairness-aware learning methods actually produce fair classifiers. We showed the effectiveness of FLEA experimentally, and we also presented a theorem that provides formal guarantees of FLEA's efficacy.

Possible Improvements: Despite our promising results, we consider FLEA just a first step on the path toward making fairness-aware learning robust against unreliable training data. An obvious limitation is that we only discussed demographic parity as a fairness measure. We do not see fundamental problems to extending FLEA to other measures that are defined in terms of properties of the joint distribution of inputs, outputs and protected attributes, such as equality of opportunity or equalized odds. However, the theoretical analysis and the practical implementation would get more involved.

On the algorithmic side, FLEA currently requires computing all pairwise similarities between the sources. This could render it inefficient when the number of sources is very large (e.g. thousands). We expect that it will be possible to replace this step by working with a random subset of sources and still obtain probabilistic guarantees, but we leave this step to future work. Finally, the setting of federated learning is quite related to multisource learning, but it allows for certain differences in the true data distribution even between different clean data sources. Extending FLEA to this setting would be a major step forward to making robust fair learning applicable to even more real-world scenarios.

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Supplementary Material

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A Experimental Setup

A.1 Dataset Preparation

The datasets we use are publicly available and frequently used to evaluate fair classification methods. The COMPAS dataset was introduced by ProPublica. It contains data from the US criminal justice system and was obtained by a public records request. The dataset contains personal information. To mitigate negative side effects, we delete the name, first, last and dob (date of birth) entries from the dataset before processing it further. We then exclude entries that do not fit the problem setting of predicting two year recidivism, following the steps of the original analysis. Specifically, this means keeping only cases from Broward county, Florida, for which data has been entered within 30 days of the arrest. Traffic offenses and cases with insufficient information are also excluded. This steps leave 6171 examples out of the original 7214 cases. The categorical features and numerical features that we extract from the data are provided in Table 1b.

adult, germancredit, and drugs are available in the UCI data repository as well as multiple other online sources. We use them in unmodified form, except for binning some of the feature values; see Tables 1a–1c.

A.2 Training Objectives

All training objectives are derived from logistic regression classifiers. For data $S = \{(x_1, y_1), \ldots, (x_n, y_n)\} \subset \mathbb{R}^d \times \{-1, 1\}$ we learn a prediction function $g(x) = w^T x + b$ by solving

$$
\min_{w \in \mathbb{R}^d, b \in \mathbb{R}} \mathcal{L}_S(w, b) + \lambda \|w\|^2
$$

with

$$
\mathcal{L}_S(w, b) = \frac{1}{|S|} \sum_{(x, y) \in S} y \log(1 + e^{-g(x)}) + (1 - y) \log(1 + e^{g(x)})
$$

We use the LogisticRegression routine of the sklearn package for this, which runs a LBFGS optimizer for up to 500 iterations. By default, we do not use a regularizer, i.e. $\lambda = 0$. From $g(x)$ we obtain classification decisions as $f(x) = \text{sign} g(x)$ and probability estimates as $\sigma(x; w, b) = p(y = 1|x) = \frac{1}{1+e^{-g(x)}}$, where we clip the output of $g$ to the interval $[-20, 20]$ to avoid numeric issues.

To train with fairness regularization, we solve the optimization problem

$$
\min_{w \in \mathbb{R}^d, b \in \mathbb{R}} \mathcal{L}_S(w, b) + \eta \Gamma_S(w, b)
$$

with

$$
\Gamma_S(w, b) = \frac{1}{|S_{a=0}|} \sum_{x \in S_{a=0}} \sigma(x; w, b) - \frac{1}{|S_{a=1}|} \sum_{x \in S_{a=1}} \sigma(x; w, b)
$$

[1] https://github.com/propublica/compas-analysis
[2] adult: https://archive.ics.uci.edu/ml/datasets/adult
[3] germancredit: https://github.com/praisan/hello-world/blob/master/german_credit_data.csv
[4] drugs: https://raw.githubusercontent.com/deepak525/Drug-Consumption/master/drug_consumption.csv
where for reasons of numeric stability, we use $|t| = \sqrt{t^2 + \epsilon^2}$ with $\epsilon = 10^{-8}$. To do so, we use the `scipy.minimize` routine with "BFGS" optimizer for up to 500 iterations. The necessary gradients are computed automatically using `jax`\footnote{https://github.com/google/jax (version 0.2.12)}. To initialize $(w, b)$, we use the result of training a (fairness-unaware) logistic regression with $\lambda = 1$, where the regularization is meant to ensure that the parameters do not take too extreme values. When estimating the disparity, we use the same objective, but with different datasets, $S_1, S_2$ for the two terms in (9), with the protected attributes as target labels for $S_1$, and the inverse of the protected attributes as target labels for $S_2$.

To train with adversarial regularization, we parameterize an adversary $g' : \mathbb{R} \to \mathbb{R}$ as $g'(x') = w' x' + b'$ and solve the optimization problem

$$\min_{w \in \mathbb{R}, b \in \mathbb{R}} \max_{w' \in \mathbb{R}, b' \in \mathbb{R}} \mathcal{L}_S(w, b) - \eta \mathcal{L}'_S(w', b')$$  \hspace{1cm} (11)

with

$$\mathcal{L}'_S(w, b, w', b') = \frac{1}{|S|} \sum_{(x, a) \in S} a \log(1 + e^{-g'(g(x)))}) + (1 - a) \log(1 + e^{g'(g(x)))})$$ \hspace{1cm} (12)

To do so, we use the `optax` package with gradient updates by the Adam rule for up to 1000 steps. The learning rates for classifier and adversary are $0.001$. The gradients are again computed using `jax`. We initialize $(w, b)$ the same way as for (9). $(w', b')$ we simply initialize with zeros.

To perform score postprocessing, we evaluate the linear prediction function on the training set and determine the thresholds that result in a fraction of $r \in \{0, 0.01, \ldots, 0.99, 1\}$ positive decision separately for each protected group. For each $r$ we then compute the overall accuracy of the classifier that results from using these group-specific thresholds and select the value for $r$ that leads to the highest accuracy. We then modify the classifier to use the corresponding thresholds for each group by adjusting the classifier weights of the protected attributes.

### A.3 Baselines

In this section, we provide more details about the baselines.

**Robust ensemble** For this baseline, we train $N$ classifiers, one per data source, using the respective base learner. For prediction, we compute the median value of the predicted probabilities and threshold it at 0.5 to obtain a binary label. Since in our experiments the number of sources is always odd, this is also equivalent to classifying using the majority vote rule.

**Filtering method from [Konstantinov et al. 2020]** The method proposed in [Konstantinov et al. 2020] uses a filtering step to suppress unreliable sources, like we do, but that differs from FLEA’s in two main aspects: it uses only the discrepancy score for its decisions, and its decision criterion is threshold-based, not quantile-based.

For its implementation, one first computes the pairwise discrepancy scores, $\text{disc}(S_i, S_j)$, between all sources. Then, one determines a threshold, $t = \sqrt{\frac{8d \log(2en/d) + 8 \log(8N/\delta)}{n}}$, where $d$ is the VC dimension of the hypothesis class (for us: the dimensionality of the feature vectors plus 1). $\delta$ is a freely choosable confidence parameter. In the limited data regime of our experiments, its value has little influence on the threshold, so we leave it at a default of $\delta = 0.1$. Finally, for each source, $S_i$, we check for how many other sources, $S_j$, their pairwise discrepancy to $S_i$ is less than $t$ (i.e. $\sum_{j \neq i} \text{disc}(S_i, S_j) < t$). If the number of such sources is at least $K - 1$, the source $S_i$ is made part of the overall training set, otherwise it is discarded.

One can check that in the setting of our experiments, only for the adult dataset one obtains values for $t$ substantially below 1. Therefore, only for this dataset, the filtering step can have a non-trivial effect.

**DRO method from [Wang et al. 2020]** The DRO method was proposed originally for the *equal opportunity* or *equalized odds* fairness measures. We adapt it to *demographic parity* by imposing constraints on the fraction of positive decisions instead of the true and false positive rates.
Our implementation follows the publicly available github repository\(^8\) which implements an approximate version of the method described in the publication. The main step is learning a classifier with fairness constraints. This is implemented by deriving a Lagrangian objective and performing simultaneous gradient descent on the classifier parameters and gradient ascent on the Lagrange multipliers. This construction has one hyperparameter, \(\xi\), the permitted slack up to which the constraints have to be fulfilled. We set this adaptively, starting with a small value \(\xi = 0.01\), but then doubling \(\xi\) until the optimization results in a non-degenerate solution (i.e. not a constant classifier).

Additionally, the constraint term of the objective is optimized in a distributionally robust (DRO) way. For this, sample weights are introduced, and the Lagrangian term is maximized also with respect to these weights, subject to \(L^1\)-ball constraints around uniform weights, and \(L^1\)-simplex constraints to ensure that the weights encode a discrete probability distribution. Following the original code, we use a projected gradient algorithm for the ball constraint, while the simplex constraint is approximated by implicit renormalization. The DRO also has one hyperparameter, \(s\), the radius of the \(L^1\)-ball. Following the derivation in the original work, we set this to twice the maximal total variation distance between the data distribution of the protected attribute in the original data and in the manipulated data, which in our case is \(s = 2(1 - \alpha)\).

Additional hyperparameters are the learning rates of for the classifier parameters, for the Lagrangian multipliers, and for the sample weights. After some initial sanity checks we keep these at the values that worked best in the original publication, which is 0.01 in all three cases.

A.4 Computing resources

For the main manuscript, all experiments were run on a 3.1-GHz CPU workstation. For the supplemental material, we also used some addition CPU-only compute nodes. We did not use GPUs. For each train/test split of each dataset and each adversary, one experimental run across all learning methods takes between 3 and 18 minutes on two CPU cores, depending on the number of sources, the size of the data sets, and the CPU architecture. The combined time for all such reported experiments (4 datasets, 5 values for \(N\), 10 adversaries, 10 train-test splits, 4 base learners) is approximately 500 core hours.

For the baselines we were able to reuse many already computed parts. If implemented individually, we'd estimate that the training time for the robust ensemble would be slightly lower than for FLEA (but it is slower at prediction time), while the training time for [Konstantinov et al. 2020] and the DRO method from [Wang et al. 2020] would be comparable to FLEA's.

A.5 Hyperparameters

We avoid hyperparameter tuning as far as possible. We do not use \(L^2\)-regularization (hyperparameter \(\lambda\)) except to create initializers, where we found the value used to hardly matter. For the fairness-regularizer and fairness-adversary we use fixed values of \(\eta = \frac{1}{2}\). We found these to result in generally fair classifiers for unperturbed data without causing classifiers to degenerate (i.e. become constant). Hence we, did not tune these values on a case-by-case basis. When estimating the disparity, we use \(\eta = 1\) to be consistent with the theory.

As learning rate for the adversarial fairness training, \(lr_{\text{adv}} = 0.001\) was found by trial and error to ensure convergence at a reasonable speed. Once we identified a reliably working setting, we did not try to tune it further.

A.6 Adversaries

In this section, we describe the adversaries and their motivation in more detail.

- **flip protected (FP)**: the adversary flips the value of protected attribute.

  This is a straightforward attack on fairness. FP inverts the correlation between the protected attribute and the rest of the data. After the sources have been combined, the correlation is therefore weakened, which makes the training data look "less unfair". On the one hand, this can cause fairness-enforcing mechanisms as used, e.g., in postprocessing fairness,

\(^8\)https://github.com/wenshuoguo/robust-fairness-code
to erroneously believe that little or no compensation for dataset unfairness is required. Consequently, the resulting classifier is actually unfair when applied to future unmanipulated data. On the other hand, it is possible that the training process actually learns to ignore the protected attribute during training, because it is uncorrelated with the target labels. This could make the classifier more fair, e.g. when used with fairness-unaware training.

Our extended experimental results (Fig. 2-17) show that both of these effect do, in fact, occur. FP typically increases unfairness when regularization-based or postprocessing-based base learners are used, but it has the opposite effect for the fairness-unaware base learner.

- **flip label (FL):** the adversary flips the value of the label.
  
  This is a straightforward attack on accuracy. Following an analog reasoning as above, FL reduces the correlation between the target label and all other data, which makes it harder for the learner to identify a strong classifier.

  Indeed, the experiments shows that the FL adversary often succeeds in reducing the accuracy, while the fairness is relatively unaffected. The adverse effect is small for the large datasets (adult, COMPAS), and larger for the small ones (drugs, germancredit), presumably because having more data increases the robustness of the learners against mislabeled data.

- **flip both (FB):** the adversary flips the value of the protected attribute and the label.
  
  This attack influences fairness and accuracy at the same time. It preserves the correlation between the protected attribute and the labels, but reduces the correlation between these two and all the other features. Consequently, the learned classifier might rely heavily on the protected attribute to predict the label, which would make it maximally unfair, but potentially also less accurate.

  Our experiments show that this is, indeed, often the observed effect, though the exact amount depends strongly on the dataset and the base learner.

- **shuffle protected (SP):** the adversary shuffles the protected attribute entries of each batch it modifies, i.e. each example gets assigned the protected attribute of another example that has been chosen at random (without replacement).
  
  This adversary is similar to FP in that is reduces the overall correlation between the protected attribute and the other data. Its effect is weaker, since it does not explicitly introduce anti-correlation in the manipulated sources. However, its manipulations are less likely to be detected by automatic or manual inspection, since it does not change the marginal statistics of the data, i.e. even after the manipulation, the statistical distribution of each feature dimension, including the protected attribute, is the same as for clean sources.

  In experimental results, SP indeed performs similarly to FP for the fairness-aware base learners, and its effect are somewhat weaker for the fairness-unaware base learner.

- **overwrite protected (OP):** the adversary overwrites the protected attribute of each sample in the affected batch by its label.
  
  This manipulation creates a strong artificial correlation between the protected attribute and the target label. In fact, the maximally unfair classifier that predicts the label directly from the protected attribute will have perfect accuracy on the manipulated data, and still a much higher accuracy than what would be correct on the overall training data. Consequently, the learned classifier might make strong use of the protected attribute, which leads to unfair and potentially incorrect decisions on clean data.

  Our experiments show that OP indeed often leads to large increases in unfairness. However, there are also cases where the unfairness is actually reduced, but then typically this is accompanied by loss of accuracy.

- **overwrite label (OL):** the adversary overwrites the label of each sample in the affected batch by its protected attribute.
  
  Like the OP adversary, this manipulation leads to a perfect correlation between the target labels and the protected attributes. However, it achieves this without changing the marginal distribution of the protected attribute, instead influencing the statistics of the labels. Depending on the specific situation, it might be easier or harder to detect from automatic or manual inspection. OL is also more likely to negatively affect the accuracy, since the classifier will try to predict incorrect labels.

  The experiments show that OL indeed almost always reduces the accuracy, while at the same time often increasing unfairness.
• **resample protected (RP):** the adversary samples new batches of data in the following ways: all original samples of protected group \( a = 0 \) with labels \( y = 1 \) are replaced by data samples from other sources which also have \( a = 0 \), but \( y = 0 \). Analogously, all samples of group \( a = 1 \) with labels \( y = 0 \) are replaced by data samples from other sources with \( a = 1 \) and \( y = 1 \).

Like OL and OP, RP results in a perfect correlation between protected attributes and labels, thereby facilitating unfairness and reducing accuracy. It does so in a more subtle and harder-to-detect way, however, as it achieves the effect using original data samples. Indeed, in our experimental results RP influences fairness and accuracy in similar ways as the other two methods.

• **random (RND):** the adversary randomly picks one of the strategies above (except ID) for each source.

This adversary reflects the observation that different sources might be manipulated in different ways. One reason for this could be that in a real-world system, multiple adversaries exists who manipulate individual data sources without coordinating their actions. Alternatively, there might be just one adversary who manipulates all sources, but chooses to manipulate them in different ways, e.g. to avoid easy detection.

The experimental results show that this strategy does, indeed, work to some extent, with RND often having an effect where some of the other methods do not, but the effect is weaker.

• **identity (ID):** the adversary makes no changes to the data.

The ID adversary serves as a useful check that FLEA does not damage the learning process in the case that all data is actually clean. It also reflects the fact that even though the adversary has the power to manipulate the data it does not have to. Ideally, the learning method will notice this and achieve even better results in presence of the ID adversary than for the oracle.

In the experimental results, this is effect is only rarely visible for any method, though.

Note that even though we introduced the adversaries above as intentional manipulations, many of them could also occur accidentally when data from different sources is collected, e.g. as problems during data entering or numeric encoding.

### B Extended Experimental Results

In addition to the experiments with a regularization-based base learner that were reported in the main manuscript, we also run experiments with postprocessing-based fairness, adversarial fairness, and fairness-unaware learning. Additionally, besides the situation with \( N = 5 \) sources out of which \( K = 3 \) are clean, we also ran experiments with \( N = 3 \) (\( K = 2 \)), \( N = 7 \) (\( K = 4 \)), \( N = 9 \) (\( K = 5 \)), and \( N = 11 \) (\( K = 6 \)) in order to identify the breaking point of FLEA with respect to the size of the data sources. The results are depicted in Fig. 2–17 in the same format as Figure 1 in the main manuscript. Also included are results for two of the baselines, robust ensemble and Konstantinov et al. [2020]. The DRO method from Wang et al. [2020] that cannot be combined with arbitrary base learners is reported together with results for the regularization-based base learners, as these are methodologically the closest.

From the results, one can see that FLEA works almost perfectly in all cases with \( N = 3 \), and reliably also for \( N = 5 \). For larger values of \( N \), and therefore fewer examples in each data source, we see more filtering errors, and, therefore, worse classifier performance, in the smaller datasets. This is particular apparent for the germancredit, which is the smallest dataset (800 training examples in total), where for \( N = 11 \) FLEA is not able to reliably improve over the base learners anymore. However, this is also true for all other methods.

The results also show that different base learners achieve different accuracy/fairness trade-offs, but FLEA is effective with each of them.
### Table 1: Dataset information

(a) adult

| dataset size | 48842 |
|----------------------------------------|------|
| categorical features | workclass | federal-gov, local-gov, never-worked, private, self-emp-inc, self-emp-not-inc, state-gov, without-pay, unknown |
| | education | 1st-4th, 5th-6th, 7th-8th, 9th, 10th, 11th, 12th, Assoc-acdm, Assoc-voc, Bachelors, Doctorate, HS-grad, Masters, Preschool, Prof-school, Some-college |
| | hours-per-week | ≤ 19, 20–29, 30–39, ≥ 40 |
| | age | ≤ 24, 25–34, 35–44, 45–54, 55–64, ≥ 65 |
| | native-country | United States, other |
| | race | Amer-Indian-Eskimo, Asian-Pac-Islander, Black, White, other |
| numerical features | — |
| protected attribute | gender | values: female (33.2%), male (66.8%) |
| target variable | income | ≤ 50K (76.1%), > 50K (33.9%) |

(b) COMPAS

| dataset size | 6171 (7214 before filtering) |
|----------------------------------------|------|
| categorical features | c-charge-degree | values: F (felony), M (misconduct) |
| | age-cat | values: <25, 25–45, >45 |
| | race | values: African-American, Caucasian, Hispanic, Other |
| numerical features | priors-count |
| protected attribute | sex | Female (19.0%), Male (81.0%) |
| target variable | two-year-recid | 0 (54.9%), 1 (45.1%) |

(c) drugs

| dataset size | 1885 |
|----------------------------------------|------|
| categorical features | — |
| numerical features | Age, Gender, Education, Country, Ethnicity, Nscore, Escore, Oscore, Ascore, Cscore, Impulsive, SS |
| (precomputed numeric values in dataset) |
| protected attribute | Gender | female (31.0%), male (69.0%) |
| target variable | Coke | never used (55.1%), used (44.9%) |

(d) germancredit

| dataset size | 1000 |
|----------------------------------------|------|
| categorical features | Age | values: ≤ 24, 25–34, 35–44, 45–54, 55–64, ≥ 65 |
| | Saving accounts | little, moderate, quite rich, rich |
| | Checking account | little, moderate, rich |
| numerical features | Duration, Credit amount |
| protected attribute | Sex | female (31.0%), male (69.0%) |
| target variable | Risk | bad (30%), good (70%) |
Figure 2: adult dataset, regularization-based fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 3: adult dataset, postprocessing-based fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 4: adult dataset, adversarial fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 5: \texttt{adult} dataset, fairness-unaware

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 6: COMPAS dataset, regularization-based fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 7: COMPAS dataset, postprocessing-based fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 8: COMPAS dataset, adversarial fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 9: COMPAS dataset, fairness-unaware

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 10: drug dataset, regularization-based fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 11: Drugs dataset, postprocessing-based fairness

(a) $N = 3, K = 2$

![Graph showing ordinary fair training, robust ensemble, and FLEA (proposed) for $N = 3, K = 2$.]

(b) $N = 5, K = 3$

![Graph showing ordinary fair training, robust ensemble, and FLEA (proposed) for $N = 5, K = 3$.]

(c) $N = 7, K = 4$

![Graph showing ordinary fair training, robust ensemble, and FLEA (proposed) for $N = 7, K = 4$.]

(d) $N = 9, K = 5$

![Graph showing ordinary fair training, robust ensemble, and FLEA (proposed) for $N = 9, K = 5$.]

(e) $N = 11, K = 6$

![Graph showing ordinary fair training, robust ensemble, and FLEA (proposed) for $N = 11, K = 6$.]
Figure 12: drugs dataset, adversarial fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 13: drugs dataset, fairness-unaware

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 14: germancredit dataset, regularization-based fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 15: germancredit dataset, postprocessing-based fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 16: germancredit dataset, adversarial fairness

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
Figure 17: germancredit dataset, fairness-unaware

(a) $N = 3, K = 2$

(b) $N = 5, K = 3$

(c) $N = 7, K = 4$

(d) $N = 9, K = 5$

(e) $N = 11, K = 6$
C Complete Formulation and Proof of Theorem 1

C.1 Concentration tools and notation

We first present the two lemmas which demonstrate uniform convergence of the empirical risk and the empirical fairness deviation measure respectively, for any hypothesis set \( H \) with finite VC dimension.

The first is just the classic VC generalization bound, as given in Chapter 28.1 of Shalev-Shwartz and Ben-David [2014]. The proof of the second lemma closely follows the proofs of similar results from Woodworth et al. [2017], Agarwal et al. [2018], Konstantinov and Lampert [2021] and is presented in the last section for completeness.

**Lemma 1** (Uniform Convergence for Binary Loss). Let \( d \) be the VC-dimension of \( H \). Then for any dataset \( S \) of size \( n \) sampled i.i.d. from \( p \), \( \forall \delta \in (0,1) \),

\[
P \left( \sup_{h \in H} |R_S(h) - R_p(h)| > 2 \sqrt{8d \log \left( \frac{en}{d} \right) + 2 \log \left( \frac{4}{\delta} \right) n} \right) \leq \delta.
\]

**Lemma 2** (Uniform Convergence for demographic parity). Let \( d = VC(H) \geq 1 \) and let \( \tau = \min_{a \in \{0,1\}} \mathbb{P}_{(X,Y,A) \sim p}(A = a) \) for some constant \( \tau \in (0,0.5] \). Then for any dataset \( S \) of size \( n \geq \max \left\{ \frac{8 \log \left( \frac{\tau}{\delta} \right)}{d}, 1 \right\} \) sampled i.i.d. from \( p \), \( \forall \delta \in (0,1/2) \):

\[
P \left( \sup_{h \in H} |\Gamma_S(h) - \Gamma_p(h)| \geq \frac{16}{2} \sqrt{\frac{d \log \left( \frac{2en}{d} \right) + \log \left( \frac{24}{\delta} \right)}{n\tau}} \right) \leq \delta
\]  

(13)

For a dataset \( S_i \), denote by

\[
c_i := \sum_{(x,y,a) \in S_i} \mathbb{1}\{a = 0\} = |S_{1}^{a=0}|.
\]

(14)

Note that since \( \tau = \mathbb{P}(A = 0) \), for a clean data source we have that \( c_i \sim \text{Bin}(n, \tau) \). Therefore, by the Hoeffding bound, for any \( \delta > 0 \):

\[
P \left( |c_i - n\tau| \geq n \sqrt{\frac{\log \left( \frac{\tau}{\delta} \right)}{2n}} \right) \leq 2 \exp \left( - \frac{2 \left( \sqrt{n} \log \left( \frac{\tau}{\delta} \right) \right)^2}{n} \right) = \delta
\]

(15)

Denote by:

\[
\Delta(\delta) = \max \left\{ 2 \sqrt{\frac{8d \log \left( \frac{en}{d} \right) + 2 \log \left( \frac{4}{\delta} \right)}{n}}, 16 \sqrt{\frac{d \log \left( \frac{2en}{d} \right) + \log \left( \frac{24}{\delta} \right)}{n\tau}}, \frac{\log \left( \frac{\tau}{\delta} \right)}{2n} \right\}
\]

(16)

\[
= 16 \sqrt{\frac{d \log \left( \frac{2en}{d} \right) + \log \left( \frac{24}{\delta} \right)}{n\tau}}.
\]

(17)

The derivations above readily imply that:

\[
P \left( \sup_{h \in H} |R_S(h) - R_p(h)| \geq \Delta(\delta) \right) \leq \delta,
\]

(18)

\[
P_S \left( \sup_{h \in H} |\Gamma_S(h) - \Gamma_p(h)| \geq \Delta(\delta) \right) \leq \delta
\]

(19)

and

\[
P_S \left( |c_i - n\tau| \geq n\Delta(\delta) \right) \leq \delta.
\]

(20)
C.2 Proof

**Theorem 1.** Assume that $\mathcal{H}$ has a finite VC-dimension $d \geq 1$. Let $p$ be an arbitrary distribution and without loss of generality let $\tau = p(\alpha = 0) \leq 0.5$. Let $S_1, \ldots, S_N$ be $N$ datasets, each consisting of $n$ samples, out of which $K \geq \frac{N}{2}$ are sampled i.i.d. from the data distribution $p$. For $\frac{1}{2} < \alpha \leq \frac{K}{N}$ and $I = \text{FILTERSOURCES}(S_1, \ldots, S_N; \alpha)$ set $S = \bigcup_{i=1}^{I} S_i$. Let $\delta > 0$. Then there exists a constant $C = C(\delta, \tau, d, N)$, such that for any $n \geq C$, the following inequalities hold with probability at least $1 - \delta$ uniformly over all $f \in \mathcal{H}$ and against any adversary:

\[
\Gamma_S(f) \leq \Gamma_p(f) + \widetilde{O}\left(\sqrt{\frac{1}{n}}\right), \quad \mathcal{R}_S(f) \leq \mathcal{R}_p(f) + \widetilde{O}\left(\sqrt{\frac{1}{n}}\right).
\]  

**Proof.** The proof consists of three steps. First, we characterize a set of values into which the empirical risks and empirical deviation measures of the clean data sources falls with probability at least $1 - \delta$. Then we show that because the clean datasets cluster in such a way, any individual dataset that is modified by the adversary will not change the disbalance measure as well. For the fairness, however, a careful derivation is needed that crucially uses the disbalance measure as well.

**Step 1** Let $G \subset [N]$ be the set of indexes $i$, such that $S_i$ was not modified by the adversary. By definition, $|G| = K$. Now consider the following events that, as we will show, describe the likely values of the studied quantities on the clean datasets.

In particular, for all $i \in G$, let $c^R_i$ be the event that:

\[
\sup_{h \in \mathcal{H}} |\mathcal{R}_{S_i}(h) - \mathcal{R}_p(h)| \leq \Delta \left(\frac{\delta}{6N}\right),
\]

let $c^T_i$ be the event that:

\[
\sup_{h \in \mathcal{H}} |\Gamma_{S_i}(h) - \Gamma_p(h)| \leq \Delta \left(\frac{\delta}{6N}\right),
\]

let $c^{\text{bin}}_i$ be the event that:

\[
|c_i - n\tau| \leq n\Delta \left(\frac{\delta}{6N}\right)
\]

and finally, let $c^\text{count}_i$ be the event that:

\[
0 < c_i < n.
\]

Denote by $(c^R_i)^c, (c^T_i)^c$ and $(c^{\text{bin}}_i)^c, (c^\text{count}_i)^c$ the respective complements of these events. Then, by equations (18, 19, 20), we have:

\[
\mathbb{P}((c^R_i)^c) \leq \frac{\delta}{6N}, \quad \mathbb{P}((c^T_i)^c) \leq \frac{\delta}{6N}, \quad \mathbb{P}((c^{\text{bin}}_i)^c) \leq \frac{\delta}{6N}, \quad \forall i \in G.
\]

Moreover, whenever $n \geq C(\delta, \tau, d, N) = \max \left\{ \frac{1}{2\log(1 - \tau/\alpha)}, \frac{1}{2\log(1 - \tau/\alpha)} \right\}$, we have that:

\[
\mathbb{P}((c^\text{count}_i)^c) = (1 - \tau)^n + \tau^n \leq \exp \left(-n \log \left(\frac{1}{1 - \tau}\right)\right) + \exp \left(-n \log \left(\frac{1}{\tau}\right)\right) \leq \frac{\delta}{2N} + \frac{\delta}{2N} = \frac{\delta}{2N}.
\]

Therefore, setting $E := (\bigcap_{i \in G} c^R_i) \wedge (\bigcap_{i \in G} c^T_i) \wedge (\bigcap_{i \in G} c^{\text{bin}}_i) \wedge (\bigcap_{i \in G} c^\text{count}_i)$ then by the union bound the probability of $\mathbb{P}(E^c) \leq K \frac{\delta}{6N} + K \frac{\delta}{6N} + K \frac{\delta}{6N} + K \frac{\delta}{6N} \leq 3\frac{\delta}{6N} + 2\frac{\delta}{6N} = \delta$.

Hence the probability of the event $E$ that all of (22, 23, 24, 25) hold is at least $1 - \delta$. 

37
Step 2. Now we show that under the event $\mathcal{E}$, the inequalities in (6) are fulfilled. Indeed, assume that $\mathcal{E}$ holds. Fix any adversary $\mathcal{A}$ and any $h \in \mathcal{H}$.

For any pair of clean sources $i, j \in [N]$ the triangle law gives:

$$\text{disc}(S_i, S_j) = \sup_{h \in \mathcal{H}} |R_{S_i}(h) - R_{S_j}(h)| \leq 2\Delta \left( \frac{\delta}{3N} \right),$$

$$\text{disp}(S_i, S_j) = \sup_{h \in \mathcal{H}} |\Gamma_{S_i}(h) - \Gamma_{S_j}(h)| \leq 2\Delta \left( \frac{\delta}{3N} \right)$$

and

$$\text{disb}(S_i, S_j) = |c_i - c_j| \leq 2n\Delta \left( \frac{\delta}{3N} \right).$$

Therefore, under $\mathcal{E}$ we have that $q \leq 2\Delta \left( \frac{\delta}{3N} \right)$, where $q$ is the $\alpha$-th quantile of the $q_i$’s.

Denote by $I = \text{FILTERSOURCES}(S_1, \ldots, S_N; \alpha; \eta)$ the result of the filtering algorithm. Now for any $i \in I$, we have that $q_i \leq q \leq 2\Delta \left( \frac{\delta}{3N} \right)$. In addition, by the definition of $q_i$, $\text{disc}(S_i, S_j) \leq q_i$ for at least $|I| \geq \frac{N}{2}$ values of $j \in [N]$. Since $K > \frac{N}{2}$, this means that $\text{disc}(S_i, S_j) \leq q_i \leq 2\Delta \left( \frac{\delta}{3N} \right)$ for at least 1 value of $j \in G$. Therefore, we have:

$$\sup_{h \in \mathcal{H}} |R_{S_i}(h) - R_{\mathcal{A}}(h)| \leq \sup_{h \in \mathcal{H}} |R_{S_i}(h) - R_{S_j}(h)| + \sup_{h \in \mathcal{H}} |R_{S_j}(h) - R_{\mathcal{A}}(h)| \leq 3\Delta \left( \frac{\delta}{3N} \right)$$

(26)

because $\mathcal{E}$ holds. Similarly,

$$\sup_{h \in \mathcal{H}} |\Gamma_{S_i}(h) - \Gamma_{\mathcal{A}}(h)| \leq 3\Delta \left( \frac{\delta}{3N} \right)$$

(27)

and

$$|c_i - n\tau| \leq 3n\Delta \left( \frac{\delta}{3N} \right).$$

(28)

Step 3. Finally, we study the risk and disparity measures based on all filtered data $S = \cup_{i \in I} S_i$.

Denote by $R_S(h)$ the empirical risk across the entire trusted dataset $I$:

$$R_S(h) := \frac{1}{|I|} \sum_{i \in I} R_{S_i}(h).$$

(29)

Then the triangle law gives:

$$|R_S(h) - R_{\mathcal{A}}(h)| = \left| \frac{1}{|I|} \left( \sum_{i \in I} R_{S_i}(h) - R_{\mathcal{A}}(h) \right) \right| \leq \frac{1}{|I|} \sum_{i \in I} |R_{S_i}(h) - R_{\mathcal{A}}(h)| = 3\Delta \left( \frac{\delta}{3N} \right)$$

Since

$$3\Delta \left( \frac{\delta}{3N} \right) = 48 \sqrt{2 \frac{d \log \left( \frac{2en}{d} \right) + \log \left( \frac{72N}{3} \right)}{n\tau}} = \tilde{O} \left( \sqrt{\frac{d}{\tau n}} \right),$$

(30)

the bound on the risk follows.

Denote by $\Gamma_S(h)$ the empirical estimate of demographic parity across the entire trusted dataset $I$:

$$\Gamma_S(h) := \left| \frac{\sum_{j \in I} \sum_{i=1}^n 1\{h(x_i^{(j)}) = 1, a_i^{(j)} = 0\}}{\sum_{j \in I} \sum_{i=1}^n 1\{a_i^{(j)} = 0\}} - \frac{\sum_{j \in I} \sum_{i=1}^n 1\{h(x_i^{(j)}) = 1, a_i^{(j)} = 1\}}{\sum_{j \in I} \sum_{i=1}^n 1\{a_i^{(j)} = 1\}} \right|.$$ 

(31)

For convenience, denote $v_j = v_j(h) = \sum_{i=1}^n 1\{h(x_i^{(j)}) = 1, a_i^{(j)} = 0\}$ and $w_j = w_j(h) = \sum_{i=1}^n 1\{h(x_i^{(j)}) = 1, a_i^{(j)} = 1\}$, so that:

$$\Gamma_S(h) = \left| \frac{\sum_{j \in I} v_j}{\sum_{j \in I} c_j} - \frac{\sum_{j \in I} w_j}{\sum_{j \in I} (n - c_j)} \right|. \quad \text{Step 3}$$
Note that because of equation (28) we have that:

$$\frac{\sum_{j \in I} v_j}{\sum_{j \in I} c_j} - \frac{\sum_{j \in I} w_j}{\sum_{j \in I} (n - c_j)} \leq \frac{\sum_{j \in I} v_j}{|I|(n\tau - 3n\Delta(\frac{\delta}{3N}))} - \frac{\sum_{j \in I} w_j}{|I|(n(1 - \tau) + 3n\Delta(\frac{\delta}{3N}))}$$

whenever:

$$1 - \tau > 3\Delta \left(\frac{\delta}{3N}\right) = 48\sqrt{\frac{2d\log(\frac{2en}{d}) + 2\log(\frac{72N}{\delta})}{n\tau}}$$  

(33)

Since $\Delta(\delta) = \tilde{O}\left(\frac{1}{\sqrt{n}}\right)$, this holds whenever $n > C_2$ for some constant $C_2 = C_2(\delta, \tau, d, N)$.

Specifically, it suffices to take:

$$n \geq \frac{4(48^2) \log \left(\frac{72N}{\delta}\right)}{\tau^3} \Rightarrow \frac{\tau^2}{4(48^2)} \geq \frac{\log \left(\frac{72N}{\delta}\right)}{n\tau}$$

and

$$\log \left(\frac{2en}{d}\right) \leq \frac{\tau^3}{4(48^2)d_n} \Rightarrow \frac{d\log(\frac{2en}{d})}{n\tau} \leq \frac{\tau^2}{4(48^2)}.$$  

The second condition is fulfilled whenever

$$n \geq -W \left(-\frac{\frac{\tau^3}{4(48^2)d_n}}{\frac{3}{4(48^2)d_n}}\right) = -W \left(-\frac{\frac{\tau^3}{8(48^2)e}}{\frac{3}{4(48^2)e}}\right) \frac{4(48^2)d}{\tau^3},$$  

where $W(x)$ is the Lambert function. In summary, (33) and hence also (32) is fulfilled whenever:

$$n \geq C_2(\delta, \tau, d, N) := \max \left\{ \frac{9216 \log \left(\frac{72N}{\delta}\right)}{\tau^3}, -W \left(-\frac{\frac{\tau^3}{18432e}}{\frac{3}{18432e}}\right) \frac{9216d}{\tau^3} \right\}.$$  

(34)

Now, under this assumption, note that:

$$\frac{\sum_{j \in I} v_j}{\sum_{j \in I} c_j} - \frac{\sum_{j \in I} w_j}{\sum_{j \in I} (n - c_j)} \leq \frac{\sum_{j \in I} v_j}{|I|(n\tau - 3n\Delta(\frac{\delta}{3N}))} - \frac{\sum_{j \in I} w_j}{|I|(n(1 - \tau) + 3n\Delta(\frac{\delta}{3N}))} = \frac{1}{|I|} \left\{ \left(\frac{n(1 - \tau) + 3n\Delta(\frac{\delta}{3N})}{n\tau - 3n\Delta(\frac{\delta}{3N})} \right) \sum_{j \in I} v_j - \left(\frac{n(1 - \tau) + 3n\Delta(\frac{\delta}{3N})}{n(1 - \tau) + 3n\Delta(\frac{\delta}{3N})} \right) \sum_{j \in I} w_j \right\}$$

$$= \frac{1}{|I|} \sum_{j \in I} \left(\frac{v_j}{n\tau - 3n\Delta(\frac{\delta}{3N})} - \frac{w_j}{n(1 - \tau) + 3n\Delta(\frac{\delta}{3N})}\right)$$

$$= \frac{1}{|I|} \sum_{j \in I} \left(\frac{v_j}{c_j} - \frac{w_j}{n - c_j}\right)$$

$$\leq \frac{1}{|I|} \sum_{j \in I} \left(\frac{v_j}{c_j} - \frac{w_j}{n - c_j}\right)$$

$$+ \frac{1}{|I|} \sum_{j \in I} \left(\frac{v_j}{c_j} \left(\frac{c_j}{n\tau - 3n\Delta(\frac{\delta}{3N})} - 1\right) - \frac{w_j}{n - c_j} \left(\frac{n - c_j}{n(1 - \tau) + 3n\Delta(\frac{\delta}{3N})} - 1\right)\right)$$

$$\leq \frac{1}{|I|} \sum_{j \in I} \left(\frac{v_j}{c_j} - \frac{w_j}{n - c_j}\right)$$

$$+ \frac{1}{|I|} \sum_{j \in I} \left(\frac{v_j}{c_j} \left(\frac{n\tau + 3n\Delta(\frac{\delta}{3N})}{n\tau - 3n\Delta(\frac{\delta}{3N})} - 1\right) + \frac{w_j}{n - c_j} \left(1 - \frac{n(1 - \tau) - 3n\Delta(\frac{\delta}{3N})}{n(1 - \tau) + 3n\Delta(\frac{\delta}{3N})}\right)\right)$$

$$\leq \frac{1}{|I|} \sum_{j \in I} \left(\frac{v_j}{c_j} - \frac{w_j}{n - c_j}\right)$$

$$+ \frac{1}{|I|} \sum_{j \in I} \left(\frac{v_j}{c_j} \left(\frac{n\tau + 3n\Delta(\frac{\delta}{3N})}{n\tau - 3n\Delta(\frac{\delta}{3N})} - 1\right) + \frac{w_j}{n - c_j} \left(1 - \frac{n(1 - \tau) - 3n\Delta(\frac{\delta}{3N})}{n(1 - \tau) + 3n\Delta(\frac{\delta}{3N})}\right)\right)$$
Let $S = \{(x_i, y_i, a_i)\}_{i=1}^n$. For $a \in \{0, 1\}$, denote:

$$
\gamma^n_S(h) = \frac{\sum_{i=1}^n \mathbb{1}\{h(x_i) = 1, a_i = a\}}{\sum_{i=1}^n \mathbb{1}\{a_i = a\}}
$$

(36)
\[
\gamma_p^n(h) = \mathbb{P}(h(X) = 1|A = a),
\]
so that \( \Gamma_S(h) = |\gamma_0^S(h) - \gamma_1^S(h)| \) and \( \Gamma_p(h) = |\gamma_0^p(h) - \gamma_1^p(h)|. \)

First we use a technique of Woodworth et al. [2017], Agarwal et al. [2018] for proving concentration results about conditional probability estimates to bound the probability of a large deviation of \( \Gamma_S(h) \) from \( \Gamma_p(h) \), for a fixed hypothesis \( h \in \mathcal{H} \). Our result is similar to the one in Woodworth et al. [2017], but for demographic parity, instead of equal odds.

**Lemma 3.** Let \( h \in \mathcal{H} \) be a fixed hypothesis and \( p \in \mathcal{P}(X \times A \times Y) \) be a fixed distribution. Let \( \tau = \min_a \{\mathbb{P}(X,Y,A) = p(A = a) \in (0,0.5] \} \). Then for any dataset \( S \), drawn i.i.d. from \( p \), of size \( n \) and for any \( \delta \in (0,1) \) and any \( t > 0 \):

\[
\mathbb{P}(|\Gamma_S(h) - \Gamma_p(h)| > 2t) \leq 6 \exp\left(-\frac{t^2 \tau n}{8}\right).
\]

**Proof.** Denote by \( S_a = \{i \in [n]: a_i = a\} \) the set of indexes of the points in \( S \) for which the protected group is \( a \). Let \( c_a := |S_a| \) and \( P_a = \mathbb{P}_{(X,Y,A) \sim p}(A = a) \), so that \( \tau = \min_a P_a \). For both \( a \in \{0,1\} \), we have:

\[
P(|\gamma_0^S - \gamma_0^p| > t) = \sum_{S_a} \mathbb{P}(|\gamma_0^S - \gamma_a| > t|S_a) \mathbb{P}(S_a)
\]
\[
\leq \mathbb{P}(c_a \leq \frac{1}{2} P_a n) + \sum_{S_a: c_a > \frac{1}{2} P_a n} \mathbb{P}(|\gamma_0^S - \gamma_a| > t|S_a) \mathbb{P}(S_a)
\]
\[
\leq \exp\left(-\frac{P_a n}{8}\right) + \sum_{S_a: c_a > \frac{1}{2} P_a n} 2 \exp\left(-2t^2 c_a\right) \mathbb{P}(S_a)
\]
\[
\leq \exp\left(-\frac{P_a n}{8}\right) + 2 \exp\left(-t^2 P_a n\right)
\]
\[
\leq 3 \exp\left(-\frac{t^2 \tau n}{8}\right). 
\]

The triangle law gives:

\[
||\gamma_0^S - \gamma_1^S| - |\gamma_0^p - \gamma_1^p|| \leq |\gamma_0^S - \gamma_1^S| + |\gamma_0^p - \gamma_1^p| \leq |\gamma_0^S - \gamma_0^p| + |\gamma_1^S - \gamma_1^p|.
\]

Combining the previous two results:

\[
\mathbb{P}(||\gamma_0^S - \gamma_1^S| - |\gamma_0^p - \gamma_1^p|| > 2t) \leq \mathbb{P}(|\gamma_0^S - \gamma_0^p| + |\gamma_1^S - \gamma_1^p| > 2t)
\]
\[
\leq \mathbb{P}\left( (|\gamma_0^S - \gamma_0^p| > t) \lor (|\gamma_1^S - \gamma_1^p| > t) \right)
\]
\[
\leq \mathbb{P}(|\gamma_0^S - \gamma_0^p| > t) + \mathbb{P}(|\gamma_1^S - \gamma_1^p| > t)
\]
\[
\leq 6 \exp\left(-\frac{t^2 \tau n}{8}\right).
\]

Finally, we prove Lemma 2 by extending the previous result to hold uniformly over the whole hypothesis space, for any hypothesis space \( \mathcal{H} \) with a finite VC-dimension \( d := VC(\mathcal{H}) \). The extension is essentially identical to Konstantinov and Lampert [2021] and is included here for completeness.

**Lemma 2 (Uniform Convergence for demographic parity).** Let \( d = VC(\mathcal{H}) \geq 1 \) and let \( \tau = \min_{a \in \{0,1\}} \mathbb{P}_{(X,Y,A) \sim p}(A = a) \) for some constant \( \tau \in (0,0.5] \). Then for any dataset \( S \) of size \( n \geq \max\left\{ \frac{8 \log(\frac{2}{\delta})}{\tau}, \frac{d}{T} \right\} \) sampled i.i.d. from \( p \), for all \( \delta \in (0,1/2) \):

\[
\mathbb{P}_S\left( \sup_{h \in \mathcal{H}} |\Gamma_S(h) - \Gamma_p(h)| \geq 16 \sqrt{\frac{2 \log(\frac{2n}{\delta}) + \log(\frac{24}{\delta})}{n \tau}} \right) \leq \delta
\]
Proof. To extend Lemma 3 to hold uniformly over $\mathcal{H}$, we first prove a version of the classic symmetrization lemma [Vapnik 2013] for $\Gamma$ and then proceed via a standard growth function argument.

1) Consider a ghost sample $S' = \{(x'_i, a'_i, y'_i)\}_{i=1}^n$ also sampled i.i.d. from $p$. For any $h \in \mathcal{H}$, let $\Gamma_{S'}(h)$ be the empirical estimate of $\Gamma_p(h)$ based on $S'$.

We show the following symmetrization inequality for the $\Gamma$ measure:

$$\mathbb{P}_S \left( \sup_{h \in \mathcal{H}} |\Gamma_S(h) - \Gamma_p(h)| \geq t \right) \leq 2 \mathbb{P}_{S,S'} \left( \sup_{h \in \mathcal{H}} |\Gamma_{S'}(h) - \Gamma_S(h)| \geq t/2 \right), \tag{40}$$

for any constant $t \geq 8 \sqrt{\frac{2 \log(12)}{n\tau}}$.

Indeed, let $h^*$ be the hypothesis achieving the supremum on the left-hand side Then:

$$\mathbb{I}(|\Gamma_S(h^*) - \Gamma_p(h^*)| \geq t) \mathbb{I}(|\Gamma_{S'}(h^*) - \Gamma_p(h^*)| \leq t/2) \leq \mathbb{I}(|\Gamma_{S'}(h^*) - \Gamma_S(h^*)| \geq t/2).$$

Taking expectation with respect to $S'$:

$$\mathbb{I}(|\Gamma_S(h^*) - \Gamma_p(h^*)| \geq t) \mathbb{P}_{S'}(|\Gamma_{S'}(h^*), S') - \Gamma_p(h^*)| \leq t/2) \leq \mathbb{P}_{S'}(|\Gamma_{S'}(h^*) - \Gamma_S(h^*)| \geq t/2).$$

Now using Lemma 3

$$\mathbb{P}_{S'}(|\Gamma_{S'}(h^*) - \Gamma_p(h^*)| \leq t/2) \geq 1 - 6 \exp \left( - \frac{t^2 \tau n}{128} \right) \geq 1 - \frac{1}{2} = \frac{1}{2},$$

where the second inequality follows from the condition $t \geq 8 \sqrt{\frac{2 \log(12)}{n\tau}}$. Therefore,

$$\frac{1}{2} \mathbb{I}(|\Gamma_S(h^*) - \Gamma_p(h^*)| \geq t) \leq \mathbb{P}_{S'}(|\Gamma_{S'}(h^*) - \Gamma_S(h^*)| \geq t/2).$$

Taking expectation with respect to $S$:

$$\mathbb{P}_S(|\Gamma_S(h^*) - \Gamma_p(h^*)| \geq t) \leq 2 \mathbb{P}_{S,S'}(|\Gamma_{S'}(h^*) - \Gamma_S(h^*)| \geq t/2) \leq 2 \mathbb{P}_{S,S'}(\sup_{h \in \mathcal{H}} |\Gamma_{S'}(h) - \Gamma_S(h)| \geq t/2).$$

2) Next we use the symmetrization inequality to bound the large deviation of $\Gamma_S(h)$ uniformly over $\mathcal{H}$.

Specifically, given $n$ points $x_1, \ldots, x_n \in \mathcal{X}$, denote

$$\mathcal{H}_{x_1, \ldots, x_n} = \{ (h(x_1), \ldots, h(x_n)) : h \in \mathcal{H} \}.$$

Then define the growth function of $\mathcal{H}$ as:

$$G_{\mathcal{H}}(n) = \sup_{x_1, \ldots, x_n} |\mathcal{H}_{x_1, \ldots, x_n}|. \tag{41}$$

We will use that well-known Sauer’s lemma [Vapnik 2013], which states that whenever $n \geq d$,

$$G_{\mathcal{H}}(n) \leq \left( \frac{en}{d} \right)^d.$$

Notice that given the two datasets $S, S'$, the values of $\Gamma_S$ and $\Gamma_{S'}$ depend only on the values of $h$ on $S$ and $S'$ respectively. Therefore, for any $t \geq 8 \sqrt{\frac{2 \log(12)}{n\tau}}$,

$$\mathbb{P}_S \left( \sup_{h \in \mathcal{H}} |\Gamma_S - \Gamma_p(h)| \geq t \right) \leq 2 \mathbb{P}_{S,S'} \left( \sup_{h \in \mathcal{H}} |\Gamma_{S'}(h) - \Gamma_S(h)| \geq \frac{t}{2} \right) \leq 2G_{\mathcal{H}}(2n) \mathbb{P}_{S,S'} \left( |\Gamma_{S'}(h) - \Gamma_S(h)| \geq \frac{t}{2} \right). \tag{42}$$

If the supremum is not attained, the argument can be repeated for each element of a sequence of classifiers approaching the supremum.
\[ \leq 2G_{\mathcal{H}}(2n)\mathbb{P}_{S,S'} \left( \left| \Gamma_S(h) - \Gamma_p(h) \right| \geq \frac{t}{4} \right) \lor \left( \left| \Gamma_{S'}(h) - \Gamma_p(h) \right| \geq \frac{t}{4} \right) \]  

(44)

\[ \leq 4G_{\mathcal{H}}(2n)\mathbb{P}_S \left( \left| \Gamma_S(h) - \Gamma_p(h) \right| \geq \frac{t}{4} \right) \]  

(45)

\[ \leq 24G_{\mathcal{H}}(2n) \exp \left( -\frac{t^2}{516} \right) \]  

(46)

\[ \leq 24 \left( \frac{2en}{d} \right)^d \exp \left( -\frac{t^2}{516} \right). \]  

(47)

Here the second-to-last inequality is due to the same bound on the difference between \( \Gamma_S \) and \( \Gamma_p \) that was used in the previous lemma, and the last one follows from Sauer’s lemma. Now if we use the threshold \( t = 16\sqrt{2d \log \left( \frac{2d}{\tau n} \right) + \log \left( \frac{24}{\delta} \right)} \), we get:

\[ \mathbb{P}_S \left( \sup_{h \in \mathcal{H}} \left| \Gamma_S(h) - \Gamma_p(h) \right| \geq 16\sqrt{2d \log \left( \frac{2d}{\tau n} \right) + \log \left( \frac{24}{\delta} \right)} \right) < \delta \]  

(48)

\[ \square \]

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