Monte Carlo Simulation for Exploring Mechanical Properties of Porous Materials Based on Scaled Boundary Finite Element Method

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Abstract: The existence of pores is a very common feature of nature and of human life, but the existence of pores will alter the mechanical properties of the material. Therefore, it is very important to study the impact of different influencing factors on the mechanical properties of porous materials and to use the law of change in mechanical properties of porous materials for our daily lives. The SBFEM (scaled boundary finite element method) method is used in this paper to calculate a large number of random models of porous materials derived from Matlab code. Multiple influencing factors can be present in these random models. Based on the Monte Carlo simulation, after a large number of model calculations were carried out, the results of the calculations were analyzed statistically in order to determine the variation law of the mechanical properties of porous materials. Moreover, this paper gives fitting formulas for the mechanical properties of different materials. This is very useful for researchers estimating the mechanical properties of porous materials in advance.

Keywords: scaled boundary finite element method; Monte Carlo simulation; equivalent modulus; porous material

1. Introduction

With the development and progress of modern science, people can produce a variety of porous materials beneficial to social development, such as metals, ceramics, and glass. Due to their good sound insulation, heat insulation, and damping effect, these materials are widely used in aerospace, automobiles, machinery, construction, packaging, and other engineering fields [1–8]. However, the existence of pores will affect the mechanical properties of materials. When people understand how mechanical characteristics of porous materials vary as a result of various influencing variables. Porous materials can make use of all of their benefits and be used in all aspects of human life. The SBFEM (scaled boundary finite element method) employed in this study combines the benefits of both finite element and boundary element methods while avoiding the drawbacks of both. In complicated issues, it offers greater accuracy and efficiency, and it is ideal for investigating stochastic models with multiple uncertainties. The computation results of a large number of specimens may be studied and analyzed using Monte Carlo simulation. Obtain the probability distribution and change the law of the specimen intuitively and precisely. It may also use the digital properties of probability distribution to assess the quality of simulation results. Scholars have spent a lot of time studying the modeling of porous materials in a variety of domains [9–16]. Passive noise control performances of nanofibrous textiles manufactured through electrospinning were investigated in [17,18] from a numerical standpoint, via the boundary element method, and from an experimental standpoint in a semi-anechoic chamber. The adoption of nanofibrous textiles turned out to produce high advantages in terms of in-cabin noise abatement, also in a critical low-frequency range (nearly 1 KHz).
for modern aircraft. Chen Yang [19] applied some research methods of particle reinforced composites to the study of porous materials. The equivalent shear modulus of porous materials under equal volume deformation was studied by the numerical method. It was found that the macroscopic mechanical behavior of porous materials under equal volume biaxial deformation can be predicted by the equivalent shear modulus under pure shear deformation, and the strain energy function of porous materials under equal volume deformation was also obtained. Li Li and Xue Yong [20] derived the loading function, non-correlation potential, and flow law of isotropic porous materials based on powder metallurgy materials, which played a certain role in promoting the numerical analysis of plastic processing of porous materials. Based on the characteristics that concrete and rock mass will form crack grids under external loads and other erosion environments, Li Le [21] established a micromechanical model of the permeability of porous materials with crack connectivity. The applicability of the model to crack networks with different geometric characteristics was verified. Cao et al. [22] developed a numerical homogenization analysis method to describe the elasticity and plasticity of a class of porous materials. Gharehghani Ayat [23] comprehensively reviewed the application of porous materials in a combustion system. Siegelman RL et al. [24] used porous materials for carbon dioxide separation and achieved good results. Tepina MS, Gorlenko NV et al. [25] prepared porous composite adsorbents based on the characteristics of porous composite materials of artificial industrial wastes. Wang YY, Zhu YM et al. [26] developed a group of similar materials that can simulate the physical and mechanical properties of shale and have five different porosities. Through the uniaxial compression test, splitting test, and triaxial compression test, it is concluded that the Poisson’s ratio of the materials increases with the increase in porosity, and the compressive strength, tensile strength, elastic modulus, cohesion, and internal friction angle decrease with the increase in porosity.

Many academics have provided analytical solutions to compute the equivalent modulus of porous materials and other mechanical indicators during the creation and research process of porous materials. The calculating approach is thus only suited for evaluating a small number of random models with substantial uncertainty. As a result, the goal of this study is to discover the variation law of porous materials under various influencing circumstances and to derive an appropriate formula for predicting the mechanical characteristics of various porous materials. It allows researchers to grasp the mechanical characteristics of porous materials in a more efficient manner. The SBFEM utilized in this study offers a built-in advantage when it comes to solving random complicated models. With the advent of contemporary computers capable of simulating a huge number of specimens in recent years, the Monte Carlo approach now avoids not only the challenge of a high number of specimens but also the dimensionality of the study problem. The higher the number, the more precise the result. The more advantageous scaled boundary finite element approach is utilized in this research to compute a material with a large number of pores. This research employs a simpler and more practical Monte Carlo approach to investigate the variation law of mechanical characteristics of porous materials. In summary, this study combines the benefits of the two techniques, calculates the random model of a large number of pore materials, examines the change law of their mechanical properties with connected influencing variables, and also provides the fitting formula for related mechanical characteristics.

2. Theoretical Method
2.1. Scaled Boundary Finite Element Method

The scaled boundary element method is a semi-analytical calculation method based on finite element and boundary element theory, which was first proposed by Wolf and Chongming [27]. Because only two-dimensional meshes are needed in the calculation of scaled boundary elements, which greatly reduces the amount of calculation, its calculation efficiency is higher than that of finite element and boundary element. The scaled boundary element method can accurately deal with stress singularity and boundaryless domain problems, and these two problems are not solved by the finite element method. When
solving the element, a new coordinate system, scaled boundary coordinate system, is adopted. Each sub domain of the scaled boundary finite element has a scaling center. The selection of the scaling center requires that the boundary of the whole sub domain is visible on the scaling center, which is generally selected on the geometric center of the sub domain. Use the radial coordinates \( \xi(\xi = 1 \text{ at the boundary and } \xi = 0 \text{ at the scaling center}) \) as the scaling factor to scale the boundary of the S domain, as shown in Figure 1a.

![Figure 1](image)

**Figure 1.** (a) S domain (b) S domain of unit circle.

For the convenience of calculation, the S domain in the system is converted into the unit circle domain with radial coordinates and angular coordinates, as shown in Figure 1b. According to references [27,28], song proposed that the scaled boundary finite element control equation is derived from the virtual work principle and its solution process. References [29,30] for the derivation and solution of the formula.

2.2. Introduction of Basic Model

Shale rock containing a large number of pores is used as the research model in this paper. Generally, the porosity of shale rock is between 0.4% and 10%. A square sheet with a side length of 2 \( \mu m \) was taken with minimal thickness, as illustrated in Figure 2. It has an elastic modulus of \( E = 22.5 \text{ GPa} \) and a Poisson’s ratio of \( \nu = 0.2 \). The model’s boundary conditions were as follows: the model’s left side was fixed, while the right side was given a 0.002 \( \mu m \) displacement. The unit of equivalent modulus and stress output is MPa. The goal of this work is to investigate the change law of mechanical characteristics of porous materials by varying parameters such as porosity, pore concentration degree, and pore number. This model is used in all of the investigations in this article, with the exception that the influence factors are adjusted according to the study topic.

![Figure 2](image)

**Figure 2.** Pore model with 15 pores and 10% porosity.
Generate a Random Model by Matlab

(a) Generate a circle of control points at any place within the 4 \( \mu m^2 \) rectangle, as indicated in the red points in Figure 3a. Many points are formed equally over the rectangle, forming set \( A \), as shown in the blue points in Figure 3b. It should be mentioned that the pores generated in the model are round or elliptical to prevent the influence of pore shape.

(b) Delete all of the points in set \( A \) that are surrounded by control points, but just a portion of the points that are not. Set \( B \) is formed by the remaining points in the rectangle area, as represented by the black points in Figure 3c. The distribution of points around the control point can be seen to be dense. The distribution of points at a distance from the control point is sparse. In the area surrounding the control point, there are no points.

(c) According to the two-dimensional Delaunay triangulation method, all of the points in set \( B \) form an equilateral triangle grid as much as possible, as shown in Figure 3d.

(d) A Voronoi grid is formed, as shown in Figure 3e. The model’s main grid is generated after the preceding steps are completed. Adding boundary conditions, applying load, and eliminating rigid body displacement on the corresponding nodes can be used to perform subsequent calculations.

![Figure 3](image-url)

**Figure 3.** (a) The control point for the pore (b) Control point and set \( A \) (c) Set \( B \) (d) Delaunay grid (e) Voronoi grid.

2.3. Solution Process

The homogenization method is to represent the macroscopic scale in a region by averaging each meso-scale. In this paper, the average stress and strain fields of a single model in the macrostructure are obtained by homogenization of the actual stress and strain fields of a single model. The equivalent elastic modulus and Poisson’s ratio of each model are obtained by substituting the above-average value into the generalized Hooke’s law.
The homogeneous equivalent stress and homogeneous equivalent strain are obtained by applying uniform boundary conditions in the above model.

\[
\sigma = \frac{1}{V} \int_{\Omega} \sigma_i d\Omega, \quad \varepsilon_x = \frac{1}{V} \int_{\Omega} \varepsilon_{ix} d\Omega, \quad \varepsilon_y = \frac{1}{V} \int_{\Omega} \varepsilon_{iy} d\Omega
\]

where \( \varepsilon_{ix} \) and \( \varepsilon_{iy} \) are the average strains of each point in X direction and Y direction respectively. \( \varepsilon_x \) and \( \varepsilon_y \) are the average strain. \( \sigma \) is the average stress. The volume unit’s volume is represented by the letter \( V \).

\[
M^* = \frac{\sigma}{\varepsilon}, \quad \nu^* = \frac{\varepsilon_y}{\varepsilon_x}
\]

\( M^* \) is the equivalent modulus and \( \nu^* \) is the Poisson’s ratio. The generalized Hooke’s law is used to calculate the equivalent elastic modulus and Poisson’s ratio.

2.4. Monte Carlo Simulation and Research Procedures

The Monte Carlo simulation method works on the principle that when a problem or object has probability characteristics, computer simulation can generate sampling results. Based on the sampling results, calculate the values of statistics or associated parameters. A steady conclusion can be established by averaging the estimated values of statistics or parameters as the simulation time increases, the authors of [31,32] used Monte Carlo simulations to explore the propagation of cracks during the operation of complex mechanical components from a probabilistic perspective. Sources of uncertainty included material variability, loading circumstances, and modeling approximations, and their impact on component life prediction was assessed and ranked accordingly. This is the basis for the Monte Carlo simulation used in this paper.

2.4.1. Construct Stochastic Variables

In the Monte Carlo approach, each model created with Matlab code is a random variable. Each model’s boundary conditions, external stresses, and material attributes are predetermined. Each model created as a result is self-contained. The control model’s other influencing factors remain unaltered, and a huge number of random models are formed. The model’s equivalent modulus, major principal stress, and Poisson’s ratio are computed, and the results of all models’ calculations are recorded.

2.4.2. Obtain the Probability Distribution

A great number of random models are produced, and the results are analyzed to produce the Monte Carlo probability model. The equivalent modulus is used to describe how to solve the probability distribution of the model’s mechanical properties. If there is enough statistical data, the arithmetic mean of the data approaches the mathematical expectation of the data, according to the law of large numbers. The probability of an event converges to the frequency of data occurrence. The frequency can be used to replace the probability that the equivalent modulus appears in each interval as the number of models grows (Figure 4a). The result calculated by this group of models under given influencing factors is thought to be the modulus value represented by the fraction interval with relatively large probability.
With the follow-up study of a single influencing factor, this problem caused by insufficient numbers of models can be avoided. As a result, the probability of a given modulus value calculated by the random model can also be considered normally distributed. As a result, the normal distribution function is applied to the Monte Carlo simulation method to improve simulation accuracy.

3. Determination of the Number of Stochastic Models

The model in this paper, in particular, is generated by Matlab code, and it is controlled by a number of variables. However, the model with the most uncertain factors is chosen for calculation and analysis in each group of random models generated in this section. It is to obtain more representative results when there are more influencing factors in the model. With the follow-up study of a single influencing factor, this problem caused by insufficient numbers of models can be avoided.

A random model number should be determined first before the Monte Carlo simulation. The law of large numbers and the Monte Carlo principle dictate that as the number of models increases, the mean value of the whole group of models will become more stable, and the mean value will become more and more near the expectation. The mean values of equivalent modulus, Poisson’s ratio, and maximum principal stress were calculated for 400, 600, 800, 1000, and 1500 random models, respectively. In Figure 5, the analysis results for each group of models are presented.

![Figure 4. (a) Histogram of probability distribution (b) Normal distribution simulation diagram.](image-url)
Figure 5. Relationship between the number of models and mechanical properties.

Figure 5 shows that when the number of models reaches 1000, the mean fluctuation of the three calculations is decreasing, indicating that the numerical value of the mechanical properties is generally stable. Overall consideration, the number of random models to be calculated for each study is 1000.

4. Factors Affecting Mechanical Properties of Porous Materials

4.1. Circular Pores

4.1.1. Concentration and Dispersion of Pores

The porosity of each model is regulated to remain unchanged at 10% on the basis of the specified model, and the number of pores is controlled to remain unchanged at 10. Pore position dispersion, relative dispersion, relative concentration, and concentration can all be achieved by modifying the pore generating area. The pore-forming area accounts for 56% of the total area, indicating that the pores are dispersed. Moreover, 44% of the total area indicates that the pore position is relatively dispersed, 30% is relatively concentrated, and 18% is very concentrated. The four situations of pore formation areas are shown in Figure 6.

Figure 6. Region of pores formation.

The results of Poisson’s ratio and equivalent modulus are independent of the pore location distribution, as shown in Figure 7 and Table 1, and the calculation results are essentially stable. The variance is influenced by the distribution of pore positions. As pores become increasingly concentrated, the variance also increases. This shows that the greater the deviation of the calculation result from the expected value. This is also evident in the results of equivalent modulus and Poisson’s ratio. The first principal stress has
obvious regularity compared with the former two. The more concentrated the pore location distribution is, the greater the first principal stress is. However, from the perspective of variance, the simulation results have great volatility. In practice, the more concentrated the pores are, the easier it is to produce stress concentration. In fact, when the area where pores can be generated exceeds 44% of the whole area, the location of pores in the whole area will be more and more random. The dispersion degree of pores has little effect on the mechanical properties of materials and basically does not change. In this case, it is meaningless to discuss the dispersion degree of pores. When the number of pores is large, the influence of pore dispersion on the mechanical properties of porous materials can be ignored. As a result, the pore dispersion affecting factor is no longer taken into account in the following research.

![Figure 7](image-url)

**Figure 7.** (a) Influence of pore location distribution on equivalent modulus of porous materials. (b) Influence of pore location distribution on Poisson’s ratio of porous materials. (c) Influence of pore location distribution on major principal stress of porous materials.
Table 1. Values of mechanical properties of porous materials with different degrees of dispersion.

| Dispersion Degree of Pores | Equivalent Modulus (MPa) | Variance of Equivalent Modulus | Poisson's Ratio | Variance of Poisson's Ratio | First Principal Stress (MPa) | Variance of the First Principal Stress |
|----------------------------|-------------------------|-------------------------------|----------------|-----------------------------|----------------------------|---------------------------------------|
| 18%                        | 19320.06                | 166.51                        | 0.221          | 0.006                       | 10.87                      | 1.37                                  |
| 30%                        | 19529.37                | 97.81                         | 0.218          | 0.004                       | 7.88                       | 0.48                                  |
| 44%                        | 19547.45                | 61.10                         | 0.217          | 0.003                       | 7.18                       | 0.26                                  |
| 56%                        | 19543.74                | 49.15                         | 0.217          | 0.002                       | 6.98                       | 0.20                                  |

4.1.2. Number of Pores

On the basis of a given material and boundary conditions, the porosity of each group is controlled at 10%. Models containing 4, 6, 8, 10, 15, and 20 pores were generated and calculated in each group. As shown in Figure 8, the following results are obtained:

Figure 8. Models of different pore numbers.

From Figure 9 and Table 2, the results show that the values of Poisson’s ratio and equivalent modulus basically do not change with the number of pores. The change in the number of pores has no effect on the calculation results of Poisson’s ratio and equivalent modulus. As the number of pores increases, however, the variance of the two results decreases, meaning the simulation results become increasingly focused on the expected value. It can be seen from Figure 9c and Table 2 that the value of the first principal stress does not change significantly with the increase in the number of pores. The variance decreases gradually and then tends to be stable. It shows that the simulation results become increasingly stable when the number of pores increases. The above shows that the increase or decrease in the number of pores within a certain range has little impact on the mechanical properties of the pore material and can be ignored.

Table 2. Values of mechanical properties of porous materials with different pore numbers.

| Number of Pores | Equivalent Modulus (MPa) | Variance of Equivalent Modulus | Poisson’s Ratio | Variance of Poisson’s Ratio | First Principal Stress (MPa) | Variance of the First Principal Stress |
|-----------------|--------------------------|-------------------------------|----------------|-----------------------------|----------------------------|---------------------------------------|
| 4               | 17129.54                 | 231.00                        | 0.231          | 0.009                       | 6.88                       | 0.70                                  |
| 6               | 17063.64                 | 184.34                        | 0.233          | 0.007                       | 7.13                       | 0.61                                  |
| 8               | 17066.14                 | 145.11                        | 0.232          | 0.006                       | 7.14                       | 0.54                                  |
| 10              | 17048.23                 | 132.40                        | 0.233          | 0.005                       | 7.24                       | 0.57                                  |
| 15              | 17015.27                 | 121.75                        | 0.233          | 0.005                       | 7.36                       | 0.43                                  |
| 20              | 17023.49                 | 88.81                         | 0.233          | 0.003                       | 7.27                       | 0.42                                  |
4.1.3. Porosity

Based on the given model, the number of pores in each group is controlled to remain unchanged at 10. The models of 5%, 10%, 15%, and 20% porosity are shown in Figure 10.

![Figure 10. Change in model porosity.](image)

According to Table 3 and Figure 11, Poisson’s ratio and equivalent modulus change strongly and have obvious rules. The equivalent modulus decreases and its variance...
increases with the increase in porosity. It shows that the results of equivalent modulus fluctuate increasingly. With the increase in porosity, Poisson’s ratio gradually increases, and its variance also increases. Reference [26] also concluded that porosity changes. In particular, Poisson’s ratio also has a corresponding quantitative relationship: for every 5% increase in porosity, the variance increases by about 5‰. Therefore, it is verified that when the porosity increases by 2%, whether the value of variance increases by 2‰, it can be found that the results also have a corresponding quantitative relationship. From Figure 11c, it can be found that the major principal stress does not change significantly with the increase in porosity, and its variance does not change much. The Poisson’s ratio and equivalent modulus of porous materials are clearly affected by changes in porosity, but the major principal stress of porous materials is unaffected.

Table 3. Values of mechanical properties of porous materials with different porosity.

| Porosity (\%) | Equivalent Modulus (MPa) | Variance of Equivalent Modulus | Poisson’s Ratio | Variance of Poisson’s Ratio | First Principal Stress (MPa) | Variance of The First Principal Stress |
|---------------|---------------------------|-------------------------------|----------------|---------------------------|----------------------------|---------------------------------------|
| 5%            | 19473.01                  | 134.67                        | 0.220          | 0.005                     | 7.67                       | 1.12                                  |
| 10%           | 16864.78                  | 303.17                        | 0.238          | 0.010                     | 7.72                       | 1.09                                  |
| 15%           | 14605.79                  | 393.03                        | 0.253          | 0.015                     | 7.70                       | 1.01                                  |
| 20%           | 12648.98                  | 489.04                        | 0.267          | 0.020                     | 7.64                       | 1.06                                  |

Figure 11. (a) Influence of porosity on equivalent modulus of porous materials. (b) Influence of porosity on Poisson’s ratio of porous materials. (c) Influence of porosity on major principal stress of porous materials.
In order to adapt to a variety of materials, three materials with an initial modulus of 1.67 GPa, 3.4 GPa, and 4.2 GPa, and Poisson’s ratio of 0.2 are selected to generate a large number of models for calculation. Studying the law of calculation results yielded the following fitting formulas for mechanical properties:

\[ M_p^* = M_0 \times 10^{-4} \left( 28702.9P^2 - 27185.0K + 9945.42 \right) \]  
\[ S_p^* = 0.6594(M_0 \times 10^{-4})^2 - 0.0335(M_0 \times 10^{-4}) + 4.4922 \]

where \( M_0 \) represents the initial modulus of the material, \( M_p^* \) represents the calculated modulus with the change in porosity and its unit is MPa, \( P \) represents porosity and \( S_p^* \) represents the calculated major principal stress and its unit is MPa. The first principal stress is independent of the porosity but will change with the initial elastic modulus of the material.

The equivalent modulus of 8.5% porosity silica gel material is estimated. According to the data, the elastic modulus of the silica gel matrix is 1.2 Gpa, Poisson’s ratio is 0.48. Table 4 shows the numerical comparison between the fitting formula and the Monte Carlo simulation. It is evident that the fitting formula has a small error. Researchers can use the fitting formula to estimate the modulus and maximum stress of materials accurately.

Table 4. Porosity = 8.5%, comparison of calculation results between Monte Carlo simulation and fitting formula.

| Equivalent Modulus (MPa) | First Principal Stress (MPa) | Error of Equivalent Modulus | Error of First Principal Stress |
|-------------------------|----------------------------|-----------------------------|-------------------------------|
| Monte Carlo simulation value | 9410.08 | 5.21 | 0.004% | 2.31% |
| Calculation value of fitting formula | 9410.49 | 5.33 | | |

Note that when an initial Poisson’s ratio is given, the initial elastic modulus is changed. The change in Poisson’s ratio caused by the change in initial elastic modulus is very small and can be ignored (see Appendix Table A1). However, the value of Poisson’s ratio is always related to the initial Poisson’s ratio. Similarly, when an initial elastic modulus is specified, the material’s initial Poisson’s ratio is changed. The equivalent modulus and the first principal stress do not vary dramatically when the initial Poisson’s ratio changes. As a result, the Poisson’s ratio fitting formula is not provided in this paper (see Appendix Table A2).

4.2. Elliptical Pore

Axis \( a \), axis \( b \), and angle \( \theta \) make up the shape of an ellipse. The ellipse shape parameter \( K (K = b/a) \) is now defined. The effects of \( K \) and \( \theta \) on the model’s mechanical properties are investigated, and the laws are obtained. Figure 12 is as follows, the shape parameter \( K = 0.4 \), angle from 0° to 180°. The a-axis and b-axis of the ellipse are defined, and the \( \theta \) is the angle between the loading direction and the a-axis. The porosity used in each group of experiments in this section was controlled at 10%.

Figure 12. \( K = 0.4 \), ellipses changing from 0° to 180°.
4.2.1. Angle of Elliptical Pore

When the given model and the control conditions of this part are constant (porosity = 10%), the elliptic parameter \( K = 0.5 \) is controlled to remain unchanged. The angle changes regularly from 0° to 90° and the results are as follows:

Table 5 shows that the equivalent modulus decreases with increasing angle. The variance of equivalent modulus also increases gradually with angle. The larger the modulus calculation result deviates from the expected value, the flatter the curve is shown in Figure 9a. The Poisson’s ratio decreases with the increase in the angle from 0° to 90°. The variance of the Poisson’s ratio increases first and then decreases (shown in Figure 13b that the curve becomes flat first and then becomes thin and high). The decreased amplitude is larger than the increased amplitude. The major principal stress increases with the increase in angle, and its variance shows an increasing trend. However, when the angle is larger, the increasing trend is not obvious.

Table 5. Values of mechanical properties of porous materials at different angles.

| \( \theta \) | Equivalent Modulus (Mpa) | Variance of Equivalent Modulus | Poisson’s Ratio | Variance of Poisson’s Ratio | First Principal Stress (Mpa) | Variance of the First Principal Stress |
|---|---|---|---|---|---|---|
| 0° | 18621.92 | 119.82 | 0.258 | 0.009 | 5.14 | 0.28 |
| 15° | 18286.02 | 142.53 | 0.253 | 0.010 | 5.63 | 0.40 |
| 30° | 17395.98 | 194.33 | 0.241 | 0.011 | 6.99 | 0.65 |
| 45° | 16216.89 | 250.88 | 0.226 | 0.011 | 8.72 | 1.03 |
| 60° | 15146.41 | 327.97 | 0.212 | 0.009 | 10.08 | 1.08 |
| 75° | 14372.00 | 361.64 | 0.203 | 0.008 | 11.22 | 1.33 |
| 90° | 14114.00 | 394.93 | 0.200 | 0.007 | 12.10 | 1.31 |

The elliptical angle has a significant effect on the mechanical properties of porous materials. The fitting formula for mechanical properties of porous materials is also given, which can be applied to a variety of materials.

\[
M_\theta^\ast = M_0 \times 10^{-4}(7484.3 + 512.4\cos(0.034426\theta) - 26.654\sin(0.034426\theta))
\]

\[
S_\theta^\ast = M_0 \times 10^{-4}(3.6397 - 0.852\cos(0.033773\theta) + 0.005638\sin(0.033773\theta))
\]

where \( M_\theta^\ast \) represents the calculated modulus with the change in the ellipse angle and its unit is MPa, \( \theta \) represents the angle and \( S_\theta^\ast \) represents the calculated major principal stress with the change in the ellipse angle and its unit is MPa. Based on the fitting formula and Monte Carlo simulation, the values of equivalent modulus and first principal stress were obtained for the silica gel material with an elliptical angle of 37.4°. Table 6 compares the values calculated by the Monte Carlo method and fitting formula.

Table 6. \( \theta = 37.2^\circ \), Comparison of calculation results between Monte Carlo simulation and fitting formula.

| Method | Equivalent Modulus (Mpa) | First Principal Stress (Mpa) | Error of Equivalent Modulus | Error of First Principal Stress |
|---|---|---|---|---|
| Monte Carlo simulation value | 9099.16 | 4.13 | 0.30% | 1.77% |
| Calculation value of fitting formula | 9126.43 | 4.058 | | |
Figure 13. (a) Influence of elliptical angle on equivalent modulus of porous materials. (b) Influence of elliptical angle on Poisson’s ratio of porous materials. (c) Influence of elliptical angle on major principal stress of porous materials.

4.2.2. Elliptical Shape Parameters

On the basis of the given model and the constant control conditions in this part (porosity = 10%), the elliptical angle ($\theta = 90^{\circ}$) is controlled to remain unchanged, and $K$ changes from 0.4 to 0.5 to 0.6 to 2.5. Similarly, this section also provides the fitting function of parameter $K$.

It can be seen from Figure 14a and Table 7 that with the increase in $K$, the equivalent modulus increases, and the value of each increase gradually decreases. The increasing trend is increasingly gentle. With the increase in $K$, the equivalent modulus of the model should gradually increase and then tend to stabilize. The variation gradually reduces, and the magnitude of the decline increases, indicating that the fluctuation is becoming increasingly smaller. It demonstrates that the equivalent modulus calculation findings are becoming increasingly stable. Poisson’s ratio also gradually increases with the increase in $K$, and the increasing trend gradually decreases. Its variance increases first and then decreases and then gradually tends to be stable, and there is a maximum value of variance. The model’s major principal stress gradually decreases as $K$ increases, as shown in Figure 14c. The variance is gradually reduced as well.
Figure 14. (a) Influence of $K$ on equivalent modulus of porous materials. (b) Influence of $K$ on Poisson’s ratio of porous materials. (c) Influence of $K$ on major principal stress of porous materials.

Table 7. The values of mechanical properties of porous materials with different elliptical shape parameters.

| $K$ | Equivalent Modulus (Mpa) | Variance of Equivalent Modulus | Poisson’s Ratio | Variance of Poisson’s Ratio | First Principal Stress (Mpa) | Variance of the First Principal Stress |
|-----|--------------------------|--------------------------------|-----------------|-----------------------------|-------------------------------|----------------------------------------|
| 0.4 | 12939.89                 | 381.79                         | 0.184           | 0.006                       | 12.57                         | 0.18                                   |
| 0.5 | 14139.62                 | 359.53                         | 0.200           | 0.006                       | 12.07                         | 0.20                                   |
| 0.6 | 14976.78                 | 357.76                         | 0.210           | 0.007                       | 11.087                        | 0.21                                   |
| 0.7 | 15632.15                 | 360.72                         | 0.219           | 0.008                       | 10.22                         | 0.22                                   |
| 1.0 | 16867.24                 | 295.42                         | 0.237           | 0.010                       | 8.58                          | 0.24                                   |
| 1.2 | 17467.59                 | 228.15                         | 0.244           | 0.010                       | 7.14                          | 0.24                                   |
| 1.6 | 18173.00                 | 153.97                         | 0.253           | 0.010                       | 5.85                          | 0.25                                   |
| 2.0 | 18610.51                 | 122.44                         | 0.258           | 0.009                       | 5.16                          | 0.26                                   |
| 2.5 | 18962.58                 | 99.48                          | 0.264           | 0.009                       | 4.62                          | 0.26                                   |
Various material properties are given by the fitting formula of mechanical properties of porous materials, which is applicable to the change in elliptical shape parameters:

\[
M^*_K = M_0 \times 10^{-4}(\varepsilon_{-863.77} K^2 + 3679.23 K + 4584.13)
\]  
(7)

\[
S^*_K = M_0 \times 10^{-4}(1.03K^2 - 4.78K + 7.3)
\]  
(8)

where \(M^*_K\) represents the calculated modulus with the change in the elliptical shape parameters. \(K\) represents the elliptical shape parameters and \(S^*_K\) represents the calculated major principal stress with the change in the elliptical shape parameters. Similarly, silica gel materials with the same parameters were selected for comparison. At this point, the shape parameter is set to 0.84. The values calculated by the Monte Carlo method and fitting formula were compared. As it can be seen in Table 8, the fitting formula calculation results are basically consistent with the Monte Carlo simulation results.

Table 8. \(K = 0.84\), Comparison of calculation results between Monte Carlo simulation and fitting formula.

|                      | Equivalent Modulus (MPa) | First Principal Stress (MPa) | Error of Equivalent Modulus | Error of First Principal Stress |
|----------------------|--------------------------|----------------------------|-----------------------------|-------------------------------|
| Monte Carlo simulation value | 8679.05                 | 4.96                      |                             |                               |
| Calculation value of fitting formula | 8478.25                 | 5.00                      | 0.001%                      | 0.0001%                       |

According to the conclusion of the elliptical part, this study analyzed the model and finds that the flatter the pore shape is in the loading direction, the higher the equivalent modulus and Poisson’s ratio. The red line in Figure 15 represents the value of the initial elliptical shape parameter \(K\), and the ellipse’s shape change is clearly visible.

Figure 15. Changes in elliptical parameters.

5. Conclusions

In this paper, the scaled boundary finite element method, which has greater adaptability, efficiency, and accuracy, was used to calculate a large number of models. The variation laws of mechanical properties of porous materials under various influencing factors were obtained using a Monte Carlo simulation. Fitting formulas for different materials were provided. According to the fitting formula, researchers can calculate the equivalent modulus and the first principal stress of different materials and different influencing factors. Moreover, in this paper, a method for calculating the number of Monte Carlo simulation models of porous materials was first presented. On the assumption that the porosity does not change, the pore location distribution and the number of pores showed no effect on the mechanical characteristics of the porous material, indicating that the porosity is the primary influencing factor of the pore material’s mechanical qualities. The mechanical properties of materials change as porosity changes; this is self-evident. The elliptical pore was investigated under the assumption that porosity does not change. The shape parameter \(K\) of the elliptical pore and the angle of the ellipse were found to have a significant impact on the material’s mechanical properties, with the maximum modulus appearing when the elliptical pore is flat along the loading direction.
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Appendix A

Table A1. The effect of initial elastic modulus change on Poisson’s ratio.

| Porosity | Initial Poisson’s Ratio = 0.2 | Initial Poisson’s Ratio = 0.25 | Initial Poisson’s Ratio = 0.3 | Initial Poisson’s Ratio = 0.35 |
|----------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 5%       | 14453.03                      | 19473.01                      | 29476.48                      | 36358.21                      |
| 10%      | 12527.89                      | 16864.77                      | 25599.14                      | 31467.72                      |
| 15%      | 10837.87                      | 14605.78                      | 22353.12                      | 27220.74                      |
| 20%      | 9383.30                       | 12648.98                      | 19461.00                      | 23693.14                      |

Table A2. The effect of varying the initial Poisson’s ratio on the equivalent modulus and first principal stress.

| Porosity | Initial Modulus (MPa) | Initial Poisson’s Ratio = 0.2 | Initial Poisson’s Ratio = 0.25 | Initial Poisson’s Ratio = 0.3 | Initial Poisson’s Ratio = 0.35 |
|----------|-----------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 5%       | 19473.01              | 19491.32                      | 19511.46                      | 19507.36                      |
| 10%      | 16864.78              | 16918.23                      | 17032.49                      | 16927.33                      |
| 15%      | 14605.78              | 14737.63                      | 14889.24                      | 14736.98                      |
| 20%      | 12648.98              | 12888.49                      | 12920.97                      | 12821.30                      |
| 5%       | 0.22                  | 0.26                          | 0.31                          | 0.35                          |
| 10%      | 0.24                  | 0.27                          | 0.31                          | 0.35                          |
| 15%      | 0.26                  | 0.28                          | 0.31                          | 0.35                          |
| 20%      | 0.27                  | 0.29                          | 0.32                          | 0.35                          |
| 5%       | 7.67                  | 7.25                          | 6.96                          | 7.22                          |
| 10%      | 7.71                  | 7.30                          | 6.83                          | 7.32                          |
| 15%      | 7.70                  | 7.12                          | 6.68                          | 7.17                          |
| 20%      | 7.64                  | 6.94                          | 6.84                          | 6.91                          |
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