Constraints on decay plus oscillation solutions of the solar neutrino problem

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We examine the constraints on non-radiative decay of neutrinos from the observations of solar neutrino experiments. The standard oscillation hypothesis among three neutrinos solves the solar and atmospheric neutrino problems. Decay of a massive neutrino mixed with the electron neutrino results in the depletion of the solar neutrino flux. We introduce neutrino decay in the oscillation hypothesis and demand that decay does not spoil the successful explanation of solar and atmospheric observations. We obtain a lower bound on the ratio of the lifetime over the mass of $\nu_2$, $(\tau_2/m_2) > 22.7 \text{ (s/MeV)}$ for the MSW solution of the solar neutrino problem and $(\tau_2/m_2) > 27.8 \text{ (s/MeV)}$ for the VO solution (at 90\% C.L.).

I. INTRODUCTION

Solar neutrino experiments with Chlorine\textsuperscript{1}, Gallium\textsuperscript{2}, and water cerenkov detectors\textsuperscript{3}, [4], show unequivocally that there is a deficit of $\nu_e$ at the Earth compared to the predictions of the standard solar model [5]. It is commonly accepted that vacuum oscillations (VO) or matter induced MSW conversions, with large mixing angle (LMA) or small mixing angle (SMA), can account for the deficit of solar neutrinos. The available experimental results allow not only a test of the neutrino oscillation hypothesis but also offer possibilities of constraining new physics, e.g. neutrino magnetic moment\textsuperscript{3}, neutrino decay, flavour changing neutral currents, etc. Here we will be concerned with neutrino decay in the context of neutrino oscillations. The possibility of solving the solar neutrino problem only through neutrino decay in vacuum was raised in [7] and ruled out mainly owing to the fact that the lower energy pp neutrinos observed in Gallium experiments are less suppressed compared to higher energy $^7$Be and $^8$B neutrinos observed in Chlorine experiments. The suppression in neutrino flux caused by the solar matter induced decay to majoron has correct energy dependence\textsuperscript{3} but the required fast rates cannot easily be obtained in the standard scenario without conflicting with other constraints [8]. The oscillation plus vacuum decay scenario has been studied more recently in a two generation model in [9], [10]. This analysis was prior to results from SNO [4]. These results in combination with earlier data from SuperKamioka\textsuperscript{3} can be used to separate the flux of the electron neutrino from that of other active flavours. This additional information provided by SNO can be quite useful in probing new physics. In the light of this, we study in this paper the three generation model of neutrino oscillation plus decay including the recent SNO\textsuperscript{4} result in our analysis.

Radiative decays of neutrinos are severely constrained by laboratory experiments and a variety of astrophysical and cosmological observations (see references in the PDG [1]). However, constraints on non-radiative decays are much less stringent. For a relatively heavy unstable neutrino, one can apply the argument that decay products would contribute to the energy density of the universe and thus obtain a limit [12]. However, for masses of the order of eV or less, there is no limit on the lifetime. Solar neutrino data constrains non-radiative neutrino decay in this mass regime.

We examine the scenario where the neutrinos from the Sun are depleted due to non-radiative decays like (a) Majoron emission decays $\nu_2 \to \bar{\nu}_1 + J$, where the $\bar{\nu}_1$ state is either sterile or the active antiparticle of $\nu_1$, or (b) $\nu_2 \to 3\nu$. We assume that the lowest mass neutrino $\nu_1$ is stable, or at least that it has lifetime $(\tau_1/m_1) \gg 20 \text{ (s/MeV)}$. (In the paper, we denote by $m_i$ and $\tau_i$ the mass and rest frame lifetime of the $\nu_i$ neutrino.) The neutrinos which arise as decay products in (a) or (b) can be either active or sterile. They are depleted in energy and carry on an average less than half of the solar neutrino energy. Since the flux of the electron neutrino sharply increases as we go down in energy, the relative contribution of the neutrinos produced in decay to the total signal in a given experiment will be quite small even if they are active. Because of this reason the net effect of the decay is depletion in the neutrino flux. This depletion is over and above the one caused by the neutrino oscillations or conversion which we assume to be the main cause of the solar deficit. The additional depletion caused by the decay is given by an exponential function

$$\exp\left(-\frac{R}{\tau_2} \frac{m_2}{E_\nu}\right)$$

where $E_\nu$ refers to the energy of the decaying neutrino and $R$ is the Sun-Earth distance. The solar neutrino survival probabilities are given in terms of the mixing $s_1$, mass difference $\delta m^2 = m_2^2 - m_1^2$ and the decay lifetime $\tau_2$. We
determine the regions in the $\tan^2 \theta_1 - \delta m^2$ plane allowed at 99% C.L. for different values of $\tau_2$. The allowed region is found to shrink as $\tau_2$ decreases and ultimately it disappears at some value which is taken to be the bound on the neutrino lifetime to invisible channel. The bound derived this way depends upon the specific solution of the solar neutrino problem and is stronger in case of the VO solution ($\tau_2/m_2 > 27.8$ (s/MeV) compared to the MSW-LMA and SMA solutions for which we get ($\tau_2/m_2 > 22.7$ (s/MeV). In an earlier pre-SNO analysis [14, 15], similar bounds were obtained for the LMA and vacuum solutions but there was no bound on $\tau_2$ for the SMA solution. We show in the final section that inclusion of SNO result is critical for obtaining a bound on $\tau_2$.

We assume the mixing among three neutrinos to be responsible for the deficit in the neutrino flux observed by the solar and atmospheric neutrino experiments. The mixing matrix $U$ needed in order to accomplish this has the following form:

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
c_1 & s_1 & 0 \\
-c_2 s_1 & c_2 c_1 & s_2 \\
-s_2 c_1 & -s_2 c_2 & c_2
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
$$

We have approximated $U_{e3} \sim 0$ in order to account for the negative results obtained in $\nu_e$ disappearance experiment at CHOOZ [13]. In addition to the above mixing one also needs $\delta m^2 = m_2^2 - m_1^2 \approx 10^{-4} - 10^{-11}$ eV$^2$ and $\delta A \equiv m_3^2 - m_2^2 \approx 10^{-3}$ eV$^2$ to account for the solar and atmospheric [14] neutrino scales. The required value of $\delta A$ is much larger than the value of the effective mass square $2\sqrt{2}G_F EN_c$ of the electron neutrino at the solar core. This suppresses mixing of the third neutrino in matter [13] which decouples from the rest in case of the MSW solution to the solar neutrino problem. The small value of $U_{e3}$ (taken here as zero) also prevents the mixing of the third neutrino in vacuum. As a result, the solar data cannot be used to constrain the lifetime of the heaviest mass eigenstate lying at the atmospheric scale. We therefore concentrate on limiting the $\nu_2$ lifetime.

II. VO PROBABILITIES

Neutrinos are produced and detected as flavor eigenstates $\nu_{\alpha}, \alpha = e, \mu, \tau$ but their time evolution operator is diagonal in the mass basis $\nu_i, i = 1, 2, 3$. The amplitude for flavor conversion during vacuum propagation is given by

$$
A_{\alpha\beta}(t) = \sum_i U_{\alpha i} U^*_{i\beta} A_i(t)
$$

where $U_{\alpha i}$ are elements of the mixing angle matrix and $A_i(t)$ is the time evolution operator for the $\nu_i$ mass eigenstate. For the decaying neutrino scenario the time evolution amplitude is given by

$$
A_i(t) = \exp(-E_i t) \exp\left(-\frac{t}{2\tau_i} \frac{m_i}{E_i}\right)
$$

where we allow for decay of the $\nu_i$ mass eigenstate with lifetime $\tau_i$ and we approximate $E_i \simeq p + m_i^2/2p$. The probability for flavor conversion during propagation in vacuum at time $t$ is then given by

$$
P_{\alpha\beta}(t) = \sum_{i>j} U_{\alpha i} U^*_{i\beta} U^*_{j\alpha} \cos \left(\frac{(m_i^2 - m_j^2)t}{2E_\nu}\right) \exp \left[-\left(\frac{m_i}{2\tau_i} + \frac{m_j}{2\tau_j}\right) \frac{t}{E_\nu}\right]
$$

with $E_\nu = p$.

Now we assume that the lightest mass state $\nu_1$ does not decay. Using the mixing matrix (2) the $\nu_e \rightarrow \nu_e, \nu_\mu, \nu_\tau$ conversion probability in vacuum from Sun to Earth is given by the expressions

$$
\begin{align*}
P_{ee} &= c_1^4 + s_1^4 \exp\left(-\frac{\alpha}{E_\nu}\right) + 2(c_1 s_1)^2 \cos \left(\frac{\delta m^2 R}{2E_\nu}\right) \exp\left(-\frac{\alpha}{2E_\nu}\right) \\
P_{e\mu} &= c_1^2 s_1^2 \left[1 + \exp\left(-\frac{\alpha}{E_\nu}\right) - 2 \cos \left(\frac{\delta m^2 R}{2E_\nu}\right) \exp\left(-\frac{\alpha}{2E_\nu}\right)\right] \\
P_{e\tau} &= c_1^2 s_1^2 \left[1 + \exp\left(-\frac{\alpha}{E_\nu}\right) - 2 \cos \left(\frac{\delta m^2 R}{2E_\nu}\right) \exp\left(-\frac{\alpha}{2E_\nu}\right)\right]
\end{align*}
$$

where
\[
\alpha = \frac{R m_2}{\tau_2}
\]  

\( R = 1.5 \times 10^{13} \text{ cm} = 500.3 \text{ s} \) being the Earth-Sun distance. We notice that the \( \nu_3 \) decay lifetime does not occur in the expressions \( \text{(10)} \) as \( U_{e3} \approx 0 \). The charged and the neutral current interactions in the detector respectively are related to \( P_{ee} \) and \( P_{\mu \mu} + P_{\tau \tau} \). Both of these are seen to be independent of the atmospheric mixing angle. The sum of all three probabilities is also independent of the solar \( \delta m^2 \),

\[
P \equiv P_{ee} + P_{\mu \mu} + P_{\tau \tau} = c_1^2 + s_1^2 \exp \left( -\frac{\alpha}{E_\nu} \right)
\]  

### III. MSW PROBABILITIES

If the flavor conversion of neutrinos in the Sun is by MSW mechanism then we have different expressions for probabilities for the decay plus conversion scenario. As discussed already, only two energy eigenstates \( \nu_1 \) and \( \nu_2 \) corresponding to the lighter neutrinos participate in the MSW conversion. In the core of the Sun \( E_1 > E_2 \), there is a level crossing at the resonance point after which \( E_2 > E_1 \) \( \text{i.e.} \ m_2 > m_1 \) in vacuum.

The probability of \( \nu_e \rightarrow \nu_1 \) just after level crossing is

\[
P_1 = P_J s_m^2 + (1 - P_J)c_m^2
\]  

Here, the first term stands for \( \nu_e \) going to \( \nu_2 \) by mixing with probability \( s_m^2 \) and then jumping to \( \nu_1 \) with probability \( P_J \) at the level crossing. The second term means that \( \nu_e \) goes to \( \nu_1 \) by mixing and then does not jump to \( \nu_2 \) with probability \( (1 - P_J) \) at the level crossing. The probability of \( \nu_e \rightarrow \nu_2 \) just after level crossing is

\[
P_2 = (1 - P_1) = (1 - P_J)s_m^2 + (P_J)c_m^2
\]  

In the formulas above, the Landau-Zener jump probability is given by

\[
P_J = \frac{\exp(-bs_1^2/E_\nu) - \exp(-b/E_\nu)}{1 - \exp(-b/E_\nu)}
\]  

\( b = \frac{\pi}{4} \left( \frac{\delta m^2}{|\Delta A/A|_{res}} \right) \approx 10^9 \frac{\delta m^2}{\text{eV}^2} \) \text{ MeV}  

and \( A = 2\sqrt{2} E_\nu G_F N_e \). The mixing angle in matter in the Sun is given in terms of the vacuum mixing angle by the expression

\[
\cos 2\theta_m = \frac{(-1 + \eta(1 - 2s_1^2))}{(1 - 2\eta(1 - 2s_1^2) + \eta^2)^{1/2}}
\]

\[
\eta = \frac{\delta m^2}{A} = 6.6 \times 10^{-5} \frac{b}{E_\nu}
\]

After level crossing the \( \nu_1 \) state stays as it is but the \( \nu_2 \) state can decay into antineutrinos or sterile neutrinos. Thus, at Earth the probability of detecting \( \nu_1 \) is \( P_1 \) and of detecting \( \nu_2 \) is \( P_2 \exp(-\alpha/E) \). We can now use the matrix \( (1) \) to find the \( \nu_e, \nu_\mu, \nu_\tau \) content of neutrinos at Earth,

\[
P_{ee} = c_1^2 P_1 + s_1^2 P_2 \exp(-\alpha/E_\nu)
\]

\[
P_{\mu \mu} = c_1^2 s_1^2 P_1 + c_1^2 c_2^2 P_2 \exp(-\alpha/E_\nu)
\]

\[
P_{\tau \tau} = s_1^2 c_2^2 P_1 + s_1^2 c_2^2 P_2 \exp(-\alpha/E_\nu)
\]

As in case of the VO, both \( P_{ee} \) and \( P_{\mu \mu} + P_{\tau \tau} \) are independent of the atmospheric angle. But unlike VO, the sum of the three probabilities in \( (13) \)

\[
P \equiv P_{ee} + P_{\mu \mu} + P_{\tau \tau} = P_1 + (1 - P_1) \exp(-\alpha/E_\nu)
\]

depends on the solar scale through the Landau-Zener formula \( (11) \).
IV. EXPERIMENTAL RATES AND BOUND ON LIFETIME

We use the probabilities derived above to obtain a bound on the neutrino lifetime. We consider only total rates for this purpose and include all experiments in our analysis. We do the analysis by two different methods. First we consider the total rates in Chlorine, Gallium, Super-K and SNO experiments and determine the allowed regions in the \( \tan^2 \theta_1 - \delta m^2 \) parameter space for different values of \( \tau_2 \). The Cl, Ga and the charged current rates of SNO by themselves cannot constrain \( \tau_2 \) in the SMA region of the MSW solution since these experiments measure \( P_{ee} \) which becomes independent of \( \tau_2 \) in the small mixing angle limit. This is not true however in case of \( P_{e\mu} + P_{e\tau} \) which is probed by neutral current events in SK. Since the neutral current rates can be inferred by combining SK and SNO data, such combination is expected to constrain the lifetime \( \tau_2 \). We show this explicitly by using the SK and SNO data alone.

The rates of neutrino capture in the Chlorine and Gallium experiments can be written as

\[
R_\alpha = \sum_{i=pp,Be,B} \int dE_\nu \Phi_i \sigma_\alpha P_{ee} \quad (17)
\]

where the subscript \( \alpha = Ga, Cl \) denotes the experiment and \( i = pp, Be, B \) denotes the type of neutrino flux from the Sun. The spectra of the \( pp \) and \( B \) neutrinos can be fitted with the analytical functions,

\[
\Phi_{pp} = (5.95 \times 10^{10})[193.9(0.931 - E_\nu)((0.931 - E_\nu)^2 - 0.261)^{1/2}E_\nu^2] \\
\Phi_B = (5.05 \times 10^6)[8.52 \times 10^{-6}(15.1 - E_\nu)^{2.75}E_\nu^2] \\
\Phi_{Be} = (4.77 \times 10^9)[\delta(E_\nu - 0.862)] \\
\]

where the neutrino fluxes are in units of \( cm^{-2}s^{-1} \) and \( E_\nu \) is in MeV. The first brackets in (18) give the total flux of neutrinos from the \( pp \), Be, and B reactions and are taken from BP2000, and the square brackets give the spectral shape.

The Ga experiments can detect all three types of neutrino fluxes and the neutrino absorption cross section of \( Ga \) is given in (17). The Chlorine experiment threshold is higher (0.8 MeV) and it detects only the Be and B neutrinos; we take the absorption cross section with the tables from ref. [18].

The electron scattering reaction in Super-K and the charge-current deuterium dissociation reaction at SNO can be written as

\[
R_{SK} = \frac{\int dE_\nu \sigma_{ee} \Phi_B P_{ee} + \int dE_\nu \sigma_{ee} \Phi_B (P - P_{ee})}{\int dE_\nu \sigma_{ee} \Phi_B} \\
R_{SNO}^{CC} = \frac{\int dE_\nu \sigma_{ee} \Phi_B P_{ee}}{\int dE_\nu \sigma_{ee} \Phi_B} \\
\]

(19) and (20)

The \( \nu_e e^- \) and \( \nu_{\mu,\tau} e^- \) elastic scattering cross section after folding with the detector response function are tabulated in ref. [19]. The deuterium dissociation cross section are taken from [20].

Using the flux spectrum in equation (18) and the cross sections (17–20) we can calculate the theoretical rates for MSW or VO conversion probabilities as a function of the three unknown parameters: the \( \tau_2 \) lifetime \( \tau_2 \), \( \delta m^2 \) and the vacuum mixing angle \( \theta_1 \). The experimental rates \( R^\text{expt}_\alpha \) with one-sigma combined (statistical and systematic) experimental errors \( \Delta_\alpha \) are as follows:

\[
R^\text{expt}_{Cl} = 0.335 \pm 0.029 \\
R^\text{expt}_{Ga} = 0.584 \pm 0.039 \\
R^\text{expt}_{SK} = 0.459 \pm 0.017 \\
R^\text{expt}_{SNO-CC} = 0.347 \pm 0.029 \\
\]

(21)

From the theoretical \( R_\alpha \), and the experimental \( R^\text{expt}_\alpha \), we compute the total \( \chi^2 \) for all experiments, defined as
\[
\chi^2 = \sum_{\alpha=\text{Cl, Ga, SK, SNO}} \frac{(R_{\alpha} - R_{\text{expt}})^2}{\Delta_{\alpha}^2}
\]  

Setting the parameter $\alpha = 0$ we reproduce the standard contours of LMA, SMA and VO solutions shown as dotted contours in Fig. 1 and Fig. 2. For $\alpha = 0$ the global minimum of $\chi^2$ occurs in the vacuum region with $\chi^2_{\text{min}} \approx 0.3$. The $\chi^2_{\text{min}} + 11$ contours which corresponds to 99\% C.L. bounds for $\alpha$ are plotted for various values of non-zero $\alpha$ as shown in the solid contours in Fig. 1 and Fig. 2. We find that the LMA and SMA allowed parameter space disappear for $\alpha = 18$ and $\alpha = 22$ (in MeV units) respectively. The VO allowed region disappears for $\alpha = 18$. These numbers translate into the bounds $\tau_2 > 22.7 \text{s} (m_2/\text{MeV})$ for the MSW solution (where we have taken the larger of the two bounds coming from SMA and LMA solutions) and $\tau_2 > 27.8 \text{s} (m_2/\text{MeV})$ for the VO solution.

As discussed before the bound on $\tau_2$ in the SMA region mainly comes from the SK data which probe $P_{e\mu} + P_{e\tau}$. It is thus interesting to consider only the SK and SNO results which together determine $P_{ee}$ and $P_{e\mu} + P_{e\tau}$ in the same energy range ($8 - 15 \text{ MeV}$). These two experiments are crucial in providing the bound on $\tau_2$. To see the impact of these results, we plot the 1.96$\sigma$ contours of the SNO ($R_{\text{SNO}} = 0.347 \pm 1.96 \cdot 0.029$ shown as continuous curves) and SK ($R_{\text{SK}} = 0.459 \pm 1.96 \cdot 0.017$ shown as dashed curves) rates. In Fig. 3 we show the allowed regions for $\alpha = 0$. In Fig. 4 we plot the same SNO and SK rates for $\alpha = 25$ and we can see that there is no overlap in the SMA region between the two experiments for this value of $\alpha$. The bound on $\alpha$ obtained from SK and SNO is marginally weaker than the one obtained in the combined analysis of all (SK, SNO, Ga and Cl) experiments.

In summary, we have used available results of the solar and atmospheric neutrino experiments to constrain neutrino lifetime within the three generation picture of neutrino oscillations. While the exact bound depends upon the specific solution, we typically find $\frac{\tau_2}{m_2} > 28 (\text{MeV}^{-1})$. The corresponding bound $\tau_2 > 2.8 \cdot 10^{-5} \text{sec}$ for eV mass neutrinos is not very strong but useful since it is the only one following from laboratory experiments in this mass range.

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FIG. 1. Allowed parameter space for VO plus decay solution, continuous curve is for decay parameter $\alpha = 0$ and dashed curve is for $\alpha = 10$.

FIG. 2. Allowed parameter space for the MSW plus decay solution, continuous curve is for decay parameter $\alpha = 0$ and dashed curve is for $\alpha = 10$. 
FIG. 3. The 1.96\(\sigma\) allowed parameter space for \(R_{SK}\) as region enclosed by dashed curves and \(R_{SNO}\) as region enclosed by continuous curves; for decay parameter \(\alpha = 0\).

FIG. 4. The 1.96\(\sigma\) allowed parameter space for \(R_{SK}\) as region enclosed by dashed curves and \(R_{SNO}\) as region enclosed by continuous curves; for decay parameter \(\alpha = 25\). There is no overlap in the SMA region for \(\alpha = 25\).