Nonlinear mechanical properties estimated from images of a mechanical test

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Abstract. The problem of identifying the (non-linear) mechanical properties of a solid from images taken in a mechanical testing machine is considered. The two classical steps of i) measuring the displacement field with digital image (and volume) correlation and ii) identifying the mechanical properties from the displacement field are combined into a single “integrated” procedure to reduce the artificial bias introduced from the projection of the kinematic data onto an arbitrary displacement basis. It allows for the identification of nonlinear constitutive law parameters with arbitrarily complex kinematics. The example of the plastic behavior of thin Ti-foils is considered.

1. Introduction

1.1. Digital image and volume correlation
Due to the large amount of data it permits to extract, the digital image correlation (DIC) tools are now commonly used in the field of experimental mechanics. Given two images $f$ and $g$ representing e.g., the gray level or the density of the same part in two different mechanical loading conditions, DIC aims at finding how one of the two images is to be deformed to match the other one. In mechanical terms, the corresponding displacement field is an important piece of information for qualitative or quantitative observations.

Due to noise produced by usual acquisition devices, it is very common to obtain the displacement $u(x)$ from the minimization of the following quadratic difference

$$\int_\Omega \left[ g(x + u(x)) - f(x) \right]^2 \, dx$$ (1)

where $\Omega$ is the Region Of Interest (ROI).

Whether $f$ contains a lot of contrast or not, Eq. (1) does not contain enough constraints to find independent values of $u$ for each pixel or voxel of $\Omega$. To circumvent this difficulty, in its earlier developments DIC consisted in registering small zones of interest or ZOIs (i.e., small interrogation windows [1]) in the considered ROI. Each registration were performed over the ZOI area with various kinematic hypotheses, possibly accounting for their warping. Global approaches, which appeared more recently [2, 3, 4], consist in performing a registration over the whole ROI. The kinematic bases are then defined over the whole ROI (i.e., Rayleigh-Ritz formulations) or over a discretized ROI (i.e., Galerkin formulations). In this context, a finite element formulation can be considered since it gives a natural link with numerical simulations of mechanical problems for which the very same discretization may be considered.

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1.2. Finite Element Model Updating

DIC is a natural tool to provide the input data for Finite Element Model Updating (FEMU) procedures for identifying the mechanical properties of a solid which are consistent with the observed displacement. The principle of FEMU [5] is to tune the parameters of a constitutive law so as to minimize the distance between the “experimental” fields (obtained by DIC) and the numerical ones obtained from a direct simulation.

However, it is to be emphasized that the measured displacements are always corrupted by noise. Thus, to minimize the noise sensitivity, a “good” metric should account for this noise and weighting is definitely not uniform and uncorrelated over the whole ROI. When the same discretization can be used for DIC (providing a vector of measured degrees of freedom \( \{u_e\} \), and a correlation matrix \([M]\) and for the numerical simulations (providing a vector \( \{u_n\} \)), is has been proven [6] that

\[
\langle \{u_e - u_n\}, [M]\{u_e - u_n\} \rangle
\]

is the optimal choice for the distance.

Besides, as finite element simulations require boundary conditions (at least on non free borders), in most published work, they come from the “experimental” DIC values, which in turn may be highly sensitive to noise. However, it is possible to consider that the boundary conditions are additional unknowns of the FEMU procedure. In such cases, the boundary values given by DIC are simply ignored (or used as an initial value of the procedure).

1.3. Integrated DIC (IDIC)

The previous ingredients have been used in different cases where it has given full satisfaction even for situations in which the Signal to Noise Ratio (SNR) is small (i.e., in elasticity [7]).

However, in this two step DIC / FEMU procedure:

- the elements in the mesh have to remain big enough because of the aforementioned DIC limitations. In the case of complex kinematics (e.g., with singularities), such big elements may not be able to capture the mechanical reality requiring fine discretizations;
- in extreme cases, due to noise, the DIC solutions may be stuck in local minima, even if they do not correspond to the solution of a mechanical problem (typically the one that are used for the FEMU procedure).

Starting from the observation that the DIC fields are only intermediate results (they are generally not the final goal), an integrated DIC procedure has been developed. In this procedure, Eq. (1) is still to be minimized, but \( u \) has to be the solution to the finite element problem. Thus, Eq. (1) is minimized with respect to the unknown mechanical properties (and the boundary conditions). This procedure is presented in Figure 1. In practice, the finite element solutions are (automatically and symbolically) differentiated and then injected into Eq. (1) to obtain the updates to be applied to the sought parameters.

The basis of this procedure has been presented in [6] but for a linear test case, and without considering the boundary conditions in the unknown set. In the following, some results are presented with this integrated DIC procedure in the case of a nonlinear constitutive law.

2. Experimental setup

In the present example, a specimen made of pure titanium (T35 grade) has been photographed for different load levels in a standard hydraulic testing machine. The test is displacement-controlled at a constant speed, and images are taken at regular intervals of time. As can be seen in Figure 2, the specimen has a “dogbone” shape. Although it is unusual to use such irregular shapes for testing, DIC allows to make use of this type of geometry, and it gives the advantage
Figure 1. Integrated procedure for the identification of the parameters $p_i$. These parameters can be the constitutive law parameters, as in the present case, or boundary conditions. The initial correlation procedure, also with T3 (linear triangular) elements, can be used to initialize the identification procedure with known displacements at non-free boundaries (see Figure 2), but is not mandatory.

Figure 2. Specimen with the identified longitudinal $u_x$ displacement field for image 10. The displacement is expressed in pixels (1 pixel $\leftrightarrow 27 \, \mu m$). The green dots indicate the non free boundaries.

of providing several loading conditions on the same sample. Besides, it helps to ensure that the localization phenomena will occur in a given zone within the ROI.

Starting from macroscopic observations (i.e., from stress-strain curves), the nonlinear Ramberg-Osgood [8] expression has been chosen as a simple constitutive law for the simulation.
Figure 3. Residuals, obtained by FEMU (dashed lines) and by IDIC (solid ones). The gray level dynamics is the amplitude of observable gray level in the reference image.

Schematically, such a law describes the plastic strain as a power-law of the (deviatoric) stress. Only two parameters, namely the yield stress $\sigma_0$ for 0.2 % strain offset, and the hardening exponent $n$, are to be identified to define completely the constitutive law.

3. Exploitation

Figure 3 shows the gray level residuals (i.e., the rms difference between $f(x)$ and $g(x + u(x))$ at the end of the minimization procedure) for a given number of images (the contribution of each image pair is summed to obtain the same parameters across the interval). In the present case, the constitutive law has been implemented such as automatic symbolic differentiation was still possible for all unknown parameters. The accuracy problems that arise from numerical differentiation procedures has thus been avoided.

For the two cases (FEMU and IDIC), the values of $\sigma_0$ and $n$ are not completely stable. This means that the choice of the Ramberg-Osgood algebraic form is a too simple assumption. The important point to note however is that the residuals obtained by IDIC are significantly lower than the ones obtained by FEMU. The goal of developing IDIC procedures is to lower the gray level residuals, whereas that of FEMU is to reduce the distance to an intermediate result, namely, the measured displacement field. Hence, the final levels are different (the acquisition noise in terms of gray level was less than 1%). The automatic symbolic differentiation also allows to avoid the computation of as many finite element solutions as the number of parameters sought, and thus make this method less costly. Without this manipulation, the computation time is very similar to the one obtained with FEMU.
4. Conclusion
An integrated DIC procedure has been introduced to identify the parameters of a nonlinear constitutive law. In terms of fulfilment of the sought objective, the final results are clearly better than the ones obtained by the classical FEMU procedure. Furthermore, the proposed regularization allows the fields to be identified to better guide the numerical simulations in the bulk, notably for the cases where the boundary conditions do not give enough information.

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