STATISTICAL ANISOTROPY IN INFLATIONARY MODELS WITH MANY VECTOR FIELDS AND/OR PROLONGED ANISOTROPIC EXPANSION

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Abstract. We study the most general contributions due to scalar field perturbations, vector field perturbations, and anisotropic expansion to the generation of statistical anisotropy in the primordial curvature perturbation \( \zeta \). Such a study is done using the \( \delta N \) formalism where only linear terms are considered. Here, we consider two specific cases that lead to determine the power spectrum \( P_\zeta(k) \) of the primordial curvature perturbation. In the first one, we consider the possibility that the \( n \)-point correlators of the field perturbations in real space are invariant under rotations in space (statistical isotropy); as a result, we obtain as many levels of statistical anisotropy as vector fields present and, therefore, several preferred directions. The second possibility arises when we consider anisotropic expansion, which leads us to obtain \( I + a \) additional contributions to the generation of statistical anisotropy of \( \zeta \) compared with the former case, being \( I \) and \( a \) the number of scalar and vector fields involved respectively.

Keywords: Statistical anisotropy, Prolonged anisotropic expansion, Vector fields.
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INTRODUCTION

The most important quantity in modern cosmology is the primordial curvature perturbation \( \zeta \) since it is responsible for the formation of structures such as galaxies and galactic clusters; besides, \( \zeta \) is directly related to the temperature fluctuations in the cosmic microwave background (CMB) through the Sachs-Wolfe effect [1]. Hence, the statistical properties of the CMB temperature anisotropies can be described in terms of the spectral functions, like the spectrum, bispectrum, trispectrum, etc., of \( \zeta \); in this way, we may compare theory with observation. On the other hand, we may see, from the distribution of temperatures in the CMB [2] and from the distribution of matter that form the large-scale structure that our Universe exhibits, departures from the exact homogeneity and isotropy.

More precisely, recent observations of the CMB anisotropies in the Wilkinson Microwave Anisotropy Probe (WMAP) experiment show that there are certain features of the full sky maps which seem to be anomalous [3, 4, 5, 6, 7]. These anomalies indicate an alignment of the lowest multipoles in the quadrupole moment of the power spectrum also known as the “axis of evil” [8], and an asymmetry in power between the northern and southern ecliptic hemispheres (the hemispherical anisotropy) [4, 7]. These anomalies also indicate the presence of statistical anisotropy [3] which indicates one preferred direction in the Universe; in other words, when we analyze the statistical properties of the distribution of temperatures, the power spectrum shows some dependence on the direction of the wavevector, signaling in turn violation of the rotational invariance in the two-point correlator of \( \zeta \), a feature which is called statistical anisotropy and is parameterized through the level of statistical anisotropy \( g_\zeta \). However, it has been suggested that systematic and statistical errors in the CMB signal may lead to the observed anomalies since the preferred direction lies near the plane of the solar system [3]. Moreover, the forthcoming observations of the Planck satellite may as well detect statistical anisotropy in the near future which would lead us to compare theory and observation more accurately [10, 11].

The probability distribution function for \( \zeta \) has well defined statistical descriptors which depend directly upon the particular inflationary model (once the action has been defined) and that are suitable for comparison with present observational data. The right framework to propagate the statistical properties of the field perturbations to the
statistical properties of $\zeta$ is the cosmological perturbation theory (CPT) \cite{12}; however, this normally involves lengthy calculations, even more when the nature of the fields is not scalar. Despite of this, the CPT is valid throughout all scales, leaving no room for discrepancies attributed to not considered subhorizon phenomena. A different approach is the $\delta N$ formalism, where $\zeta$ is identified with the perturbation in the amount of expansion $N$ from an initial time in a flat slicing to a final time in a uniform energy density slicing (the threading must be comoving) \cite{13, 14, 15, 16}. The $\delta N$ formalism gives an expression for $\zeta$ which is valid to all orders in CPT; however, it is only valid for superhorizon scales (in absolute contrast with CPT). Of course, extracting the statistical properties of $\zeta$ in the $\delta N$ formalism requires also to do some “perturbation theory”: to expand $N$ in a Taylor series and to cut it out at the desired order.

This paper is structured as follows. Initially we provide the theoretical framework for building the $n$-point correlators of $\zeta$. After that, we obtain the levels of statistical anisotropy for $\zeta$ when isotropic (anisotropic) expansion and multi-vector field perturbations are considered. When we consider isotropic expansion only, one generic expression for $g_\zeta$ is obtained; moreover there exist a preferred directions in contrast with the parameterization of a single vector field. Another possibility arises when anisotropic expansion is present in the background metric which leads to obtain $I + 2a$ contributions to the generation of statistical anisotropy in the primordial curvature perturbation. Finally, we discuss our results and present our conclusions.

**THEORETICAL FRAMEWORK**

We are interested in studying the statistical properties of a perturbation map through the $n$-point correlators of the perturbations \cite{17}. Let’s define a scalar perturbation $\beta(x)$ in real space, which can be expressed in terms of a Fourier integral expansion as

$$\beta(x) = \int \frac{d^3k}{(2\pi)^3} \beta(k) e^{i k \cdot x}. \quad (1)$$

The $n$-point correlators of the perturbations $\beta(x)$ are defined as averages over the ensemble of universes of the products $\langle \beta(x_1) \beta(x_2) \ldots \beta(x_n) \rangle$, where $x_1, x_2, \ldots, x_n$ represent different points in space \cite{17}.

$$\langle \beta(x_1) \beta(x_2) \ldots \beta(x_n) \rangle = \int \frac{d^3k_1 d^3k_2}{(2\pi)^3(2\pi)^3} \ldots \frac{d^3k_n}{(2\pi)^3} \langle \beta(k_1) \beta(k_2) \ldots \beta(k_n) \rangle e^{i(k_1 \cdot x_1 + k_2 \cdot x_2 + \ldots + k_n \cdot x_n)}. \quad (2)$$

Hence, the correlation functions in real space may be studied via the correlation functions in momentum space. Let’s see now the meaning of statistical homogeneity and statistical isotropy in the $n$-point correlators of perturbations.

**Statistical homogeneity**

According to the observation, the perturbation map is not homogeneous, but it may be that the probability distribution function governing $\beta(x)$ is. This feature can be expressed in the following way: if the $n$-point correlators in real space are invariant under translations in space, it is said that there exists statistical homogeneity \cite{18, 19}.

$$\langle \beta(x_1 + d) \beta(x_2 + d) \ldots \beta(x_n + d) \rangle = \langle \beta(x_1) \beta(x_2) \ldots \beta(x_n) \rangle, \quad (3)$$

where $d$ is some vector in real space establishing the amount of spatial translation. The above condition may be achieved if the $n$-point correlators in momentum space are proportional to a Dirac delta function (which is not the only possibility).

$$\langle \beta(k_1) \beta(k_2) \ldots \beta(k_n) \rangle \equiv (2\pi)^3 \delta^3(k_1 + k_2 + \ldots + k_n) M_\beta(k_1, k_2, \ldots, k_n), \quad (4)$$

where the function $M_\beta(k_1, k_2, \ldots, k_n)$ is called the $(n-1)$-spectrum. Statistical homogeneity is absolutely necessary as an hypothesis of the ergodic theorem \cite{12}. 

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1 The ensemble average inside the integral is over the Fourier mode functions only since they are the stochastic variables.
Statistical isotropy

Once statistical homogeneity has been secured, we ask about the invariance under spatial rotations of the $n$-point correlators in real space (i.e. statistical isotropy). Of course again, the perturbation map is not isotropic, but it may be that the probability distribution function governing $\hat{\beta}(x)$ is, which is called statistical isotropy [18, 19]. This means that the $n$-point correlators in real space are invariant under rotations in space.

$$\langle \hat{\beta}(\mathbf{x}_1)\hat{\beta}(\mathbf{x}_2)\ldots\hat{\beta}(\mathbf{x}_n) \rangle = \langle \beta(\mathbf{x}_1)\beta(\mathbf{x}_2)\ldots\beta(\mathbf{x}_n) \rangle,$$

(5)

where $\mathbf{x}_i = R \mathbf{x}_i$, $R$ being a rotation operator. To satisfy the above requirement, the $(n - 1)$-spectrum must satisfy the condition

$$M_\beta(\mathbf{k}_1, \mathbf{k}_2, \ldots, \mathbf{k}_n) = M_\beta(\mathbf{k}_1, \mathbf{k}_2, \ldots, \mathbf{k}_n),$$

(6)

where the tildes over the momenta represent as well a spatial rotation, parameterized by $R$, in momentum space. We assume that the $n$-point correlators are invariant under translations in space (statistical homogeneity):

$$\langle \hat{\beta}(\mathbf{k}_1)\beta(\mathbf{k}_2) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2)P_\beta(\mathbf{k}_1).$$

(7)

In the above expressions, $P_\beta$, is called the spectrum. When we consider statistical isotropy, the spectrum depends only on the wavenumber. Hence, the argument in the spectrum may be considered as $k = |k_1| = |k_2|$.

Considering violation of rotational invariance due to the presence of a vector field which points out in the preferred direction $\hat{\mathbf{d}}$, the form of the power spectrum changes according to [9]

$$P_\zeta(k) = P_\zeta^{iso}(k)\left[1 + g_\zeta(\mathbf{k} \cdot \hat{\mathbf{d}})^2\right].$$

(8)

In the above expression $P_\zeta^{iso}(k)$ is the average over all directions, $\hat{\mathbf{k}}$ is a unit vector and $g_\zeta$ is the level of statistical anisotropy whose value is in the range $g_\zeta = 0.290 \pm 0.031$ and rules out statistical isotropy at more than $9\sigma$ [3].

THE $\delta N$ FORMALISM WITH MANY VECTOR AND SCALAR FIELDS

Starting with an initial flat slicing such that the locally-defined scale factor is homogeneous, and ending with a slicing of uniform energy density, we can then express the primordial curvature perturbation $\zeta$ through the $\delta N$ formula [20]

$$\zeta(x, t) = N(x, t) - N_0(t) = \delta N(x, t),$$

(9)

where $N$ is the associated amount of expansion. Thus, the evolution of $\zeta$ depends directly on the evolution of $N$ and this in turn depends on the nature of the fields considered. In this work, we will consider $n$ scalar fields and $m$ vector fields. To deal with the different contributions from the fields involved, we introduce the notation:

$$\delta \Phi_A = \{ \delta \phi_I, \delta A^i_a \}.$$  

(10)

The index $A$ is separated into two sets: a set of indices $I$ labelling the scalar fields which run from 1 to $n$ and another set of indices $a$ labelling vector fields which run from 1 to $m$. The index $i$ specifies the component of any vector field and runs from 1 to 3. Accordingly, the derivatives of $N$ with respect to the fields are separated as follows [17]

$$N_A = \{ N_I, N_i^e \},$$

(11)

$$N_{AB} = \{ N_{IJ}, N^b_{ij}, N^b_{ij} \}.$$  

(12)

In the above notation, we represent the mixed second derivative with respect to $\phi$ and $A^b_j$ as $N^b_{ij} = \partial^2 N / \partial \phi_i A^b_j$. In terms of the above introduced notation, the curvature perturbation from the multi-scalar and multi-vector field case is written in terms of the mode functions by means of the following truncated expansion up to first order:

$$\zeta(k, t) = N_A \delta \Phi(k, t) + \frac{1}{2} N_{AB} \int \frac{d^3k_1}{(2\pi)^3} \delta \Phi_A(\mathbf{k} - \mathbf{k}_1) \delta \Phi_B(\mathbf{k}_1).$$

(13)
We only consider linear terms of $\zeta$ in Eq. (13) (corresponding to what we call the tree level contributions) because we are interested in studying the power spectrum of $\zeta$ and not in other spectral functions. Hence, once the product of the respective perturbations has been made, we perform the average over the ensemble in order to obtain the two-point correlator associated to $\zeta$

$$\langle \zeta(k_1)\zeta(k_2) \rangle = N_A N_B \langle \delta\Phi_A(k_1)\delta\Phi_B(k_2) \rangle$$

$$= N_0^2 \langle \delta\phi(k_1)\delta\phi(k_2) \rangle + N_A N_0 \langle \delta\Lambda_A(k_1)\delta\Lambda_A(k_2) \rangle$$

$$+ N_0 N_f \langle \delta\phi(k_1)\delta\Lambda_f(k_2) \rangle + N_f N_0 \langle \delta\Lambda_A(k_1)\delta\phi(k_2) \rangle.$$  \hspace{1cm} (14)

We can see that the two-point correlator of $\zeta$ depends on the two-point correlators of the involved fields, (either scalar, vector, or in the most general case both fields). In view of this fact, it is necessary to assume some properties of these correlators; hence, we will assume the following proposition as a conjecture to obtain the power spectrum of the curvature perturbation and therefore the levels of statistical anisotropy \cite{21}: if we consider anisotropic expansion (the field perturbations live in the anisotropic background metric), at least one of the correlators of the involved field perturbations is not invariant under rotations in space; it is reasonable to think this since there is no a conceivable way to get statistical isotropy if the perturbations are defined in an anisotropic background. Let’s consider anisotropic expansion so that the two-point correlator of $\zeta$ is not invariant under rotations in space; however, it will be absolutely necessary to assume statistical homogeneity as an hypothesis of the ergodic theorem. These features lead to express the two-point correlator of the field perturbations as

$$\langle \delta\Phi_A(k_1)\delta\Phi_B(k_2) \rangle = (2\pi)^3 \delta(k_1 + k_2) \Pi_{AB}(k_1),$$

where the power spectra of the scalar, vector and mixed fields perturbation are $\Pi_{AB} = \{\Pi_{JJ}(k), \Pi_{J\bar{J}}(k), \Pi_{IJ}(k), \Pi_{I\bar{J}}(k)\}$ respectively. Thus, the power spectrum of $\zeta$ is determined by the power spectra of the field perturbations and the derivatives of the amount of expansion $N_A$ which, in turn, depend on the background metric:

$$P_\zeta(k_1) = N_A N_B \Pi_{AB}(k_1).$$

(17)

In addition, the power spectrum of each field could be different since each one of the associated spectra exhibits some dependence on the wavevectors due to the lack of symmetry in the background and therefore in the $n$-point correlators. These reasons lead us to have an anisotropic spectrum for $\zeta$ sourced by the statistical anisotropy in the two-point correlator of the field perturbations.

On the other hand, the scalar field perturbation power spectra $\Pi_{JJ}(k)$, related to the two-point correlators of the scalar fields $\phi_i$, is defined as

$$\langle \delta\phi_i(k_1)\delta\phi_j(k_2) \rangle = (2\pi)^3 \delta(k_1 + k_2) \Pi_{IJ}(k_1),$$

whereas for the vector field perturbation power spectra $\Pi^{ab}_{ij}$, whose origin lies again in the anisotropic expansion, we have

$$\langle \delta A_i^a(k_1)\delta A_j^b(k_2) \rangle = (2\pi)^3 \delta^2(k_1 + k_2) \Pi_{ij}^{ab}(k_1).$$

(19)

According to Ref. \cite{18}, $\Pi^{ab}_{ij}$ is given by

$$\Pi^{ab}_{ij}(k) = \Pi^\text{even}_{ij}(k) P^a_{i+}(k) + i \Pi^\text{odd}_{ij}(k) P^a_{i-}(k) + \Pi^\text{long}_{ij}(k) P^a_{i\text{long}}(k),$$

which is written in terms of the longitudinal component of the power spectra $P^a_{i\text{long}}$ and the parity-conserving and parity-violating power spectra $P^a_{i+}$ and $P^a_{i-}$ respectively. These spectra are defined as

$$P^a_{i\pm} = \frac{1}{2} (P^a_R + P^a_L).$$

(21)

The scalar-vector field perturbation power spectra do not contribute to the power spectrum of $\zeta$ because there is no correlation between fields of different nature.
where \( P_{\text{ab}}^R \) and \( P_{\text{ab}}^L \) denote the power spectra for the transverse components with right-handed and left-handed circular polarizations, and again all of them depend on the wavevector because we are considering anisotropic expansion. The formal definitions of the polarization spectra are given by
\[
\langle \delta A_a^b(k_1) \delta A_{a'}^{b'}(k_2) \rangle = (2\pi)^3 \delta^3(k_1 - k_2) P_{\text{ab}}^R(k_1),
\]
where we have used the reality condition \( \beta(-k) = \beta^*(k) \) and \( \lambda \) denotes the different polarizations \( L, R, \text{or long} \). The basis \( \Pi_{ij}^{\text{even}}(k) \), \( \Pi_{ij}^{\text{odd}}(k) \) and \( \Pi_{ij}^{\text{long}}(k) \) in this case is given by
\[
\Pi_{ij}^{\text{even}}(k) \equiv \delta_{ij} - \hat{k}_i \hat{k}_j, \quad \Pi_{ij}^{\text{odd}}(k) \equiv \epsilon_{ijk} \hat{k}_k, \quad \Pi_{ij}^{\text{long}}(k) \equiv \hat{k}_i \hat{k}_j,
\]
where \( \epsilon_{ijk} \) is the totally antisymmetric tensor. Now, it is possible to calculate the power spectrum of \( \zeta \) in terms of the components associated to the scalar and vector field perturbation power spectra defined respectively above. Starting from Eq. \((21)\), we can obtain the remarkable result \((22)\)
\[
P_{\zeta}(k_1) = (N_0^a)^2 P_{\delta \phi}^L(k_1) + N_1^a N_2^b ((\delta_{ij} - \hat{k}_i \hat{k}_j) P_{ab}^R(k_1) \delta_{ab} + i \epsilon_{ijk} \hat{k}_k P_{ab}^L(k_1) \delta_{ab})
+ \hat{k}_1 \cdot \hat{k}_1 P_{\text{long}}(k_1) \delta_{ab},
\]
In this expression, we have not considered correlation between different scalar field perturbations and different vector field perturbations; however, it is the most general expression for \( P_{\zeta} \) since parity violation in the Lagrangian has been initially allowed (although, this term really does not contribute to \( P_{\zeta} \) due to symmetry arguments) and the anisotropic expansion has been considered. Besides, we have considered the possibility of having several scalar and vector fields in the inflationary dynamics.

**Case I. Isotropic expansion**

As a particular case, when isotropic expansion is considered (which leads to have invariance under rotations in space for the two-point correlators of the fields involved) we obtain a similar expression for the power spectrum of \( \zeta \):
\[
P_{\zeta}(k_1) = (N_0^a)^2 P_{\delta \phi}^L(k_1) + N_1^a N_2^b ((\delta_{ij} - \hat{k}_i \hat{k}_j) P_{ab}^R(k_1) \delta_{ab} + i \epsilon_{ijk} \hat{k}_k P_{ab}^L(k_1) \delta_{ab}),
\]
and hence, we obtain \( a \) levels of statistical anisotropy:
\[
\xi_{\zeta}^a = \frac{(r_{\text{long}} - 1)(N_1^a)^2 P_{\phi}^L(k_1)}{(N_0^a)^2 P_{\phi}^L(k_1) + (N_1^a)^2 P_{\phi}^R(k_1)},
\]
The latest result can be compared with that obtained in Ref. \([18]\). Moreover, in this case there are as many preferred directions as the number of vector fields present; namely, there now exist \( a \) preferred directions or levels of statistical anisotropy in contrast with the parameterization given by Eq. \((8)\). In the above expression, \( P_{\phi}^{\text{iso}} \) has been identified as
\[
P_{\zeta}^{\text{iso}} = (N_0^a)^2 P_{\delta \phi}^L(k_1) + (N_1^a)^2 P_{\phi}^L(k_1),
\]
and we have defined \( N^a \equiv N_0^a \hat{N} \) and \( r_{\text{long}} = P_{\text{long}} / P_+ \).

**Case II. Anisotropic expansion**

The most interesting case arises when anisotropic expansion is assumed which is equivalent to allow violation of the statistical isotropy in the two-point correlators of the field perturbations whose associated spectra are \( P_{\delta \phi} \), \( P_+ \), \( P_- \), and \( P_{\text{long}} \) respectively. Considering the parameterization given by Eq. \((8)\) and linear contributions only, the power spectrum of \( \zeta \) may now be written as \((21)\)
\[
P_{\zeta}(k_1) = P_{\zeta}^{\phi}(k_1)[1 + \xi_{\phi}^L (\hat{k}_1 \cdot \hat{d}_L^i)^2 + \xi_{\phi}^R (\hat{k}_1 \cdot \hat{d}_R^i)^2 + \xi_{\phi}^{\text{iso}} (N_1^a)^2 (\hat{k}_1 \cdot \hat{N}_1^i)^2].
\]
This result clearly exhibits the nature of the origin of $2\alpha + I$ preferred directions determined by $\hat{d}_\delta^I$, $\hat{d}_+^a$ and $\hat{N}_1^a$. The levels of statistical anisotropy $\tilde{g}^{I\delta\phi}$ in Eq. (28) were precisely obtained in the isotropic expansion case; however, in the present case, $N$ depends on the anisotropic background. Furthermore, although there is no explicit dependence for the scalar field on the wavevector, the anisotropic contribution of these fields is due to the fact that we are considering anisotropic expansion. The levels of statistical anisotropy associated with each field are given by

$$
\tilde{g}^{I\delta\phi} = \frac{g^{I\delta\phi}(N_\phi^a)^2 P_{\delta\phi}^{iso}(k_1)}{(N_\phi^a)^2 P_{\delta\phi}(k_1) + (N_\phi^a)^2 P_{\delta\phi}^{iso}(k_1)},
$$

(29)

$$
\tilde{g}^{a+} = \frac{g^{a+}(N_\phi^a)^2 P_{a+}^{iso}(k_1)}{(N_\phi^a)^2 P_{a+}(k_1) + (N_\phi^a)^2 P_{a+}^{iso}(k_1)},
$$

(30)

$$
\tilde{g}^{N} = \frac{(r_{\text{long}}^I - 1)(N_\phi^a)^2 P_{a+}^{iso}(k_1)}{(N_\phi^a)^2 P_{a+}(k_1) + (N_\phi^a)^2 P_{a+}^{iso}(k_1)}.
$$

(31)

In these expressions, the isotropic spectra do not correspond to the isotropic case because the nature of the background metric is now anisotropic. Rather than calculating these spectra in an isotropic background, which is incoherent, we will refer to them as the pieces of the spectra that do not depend on the wavevector.

As a remarkable result, we have considered the most general case to the generation of statistical anisotropy in the primordial curvature perturbation $\zeta$ (and therefore to the power spectrum in Eq. (28)) due to the contributions of multiple field perturbations and anisotropic expansion. This latter effect, is clearly indicated by the $2\alpha + I$ contributions. On the other hand, in order to obtain the magnitude of the levels of statistical anisotropy when one particular setup is established (in this new scenario), it will be necessary to study the anisotropic particle production for the scalar and vector fields involved and to calculate the amount of expansion in an anisotropic background; in this way, we may directly compare our theoretical predictions with observations.

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REFERENCES

1. R. K. Sach and A. M. Wolfe, Astrophys. J. 147, 73 (1967).
2. G. Hinshaw, et. al., arXiv:1212.5226 [astro-ph.CO].
3. N. E. Groeneboom, L. Ackerman, I. K. Wehus, and H. K. Eriksen, Astrophys. J. 722, 452 (2010).
4. J. Hoftuft et. al., Astrophys. J. 699, 985 (2009).
5. C. Armendariz-Picon and L. Pekowsky, Phys. Rev. Lett. 102, 031301 (2009).
6. N. E. Groeneboom and H. K Eriksen, Astrophys. J. 690, 1807 (2009).
7. I. K. Wehus, L. Ackerman, H. K. Eriksen, and N. E. Groeneboom, Astrophys. J. 707, 343 (2009).
8. K. Land and J. Magueijo, Phys. Rev. Lett. 95, 071301 (2005).
9. L. Ackerman, S. M. Carroll, and M. B. Wise, Phys. Rev. D 75, 083502 (2007).
10. The PLANCK Collaboration, arXiv:astro-ph/0603489.
11. A. R. Pullen and M. Kamionkowski, Phys. Rev. D 76, 103529 (2007).
12. S. Weinberg, Cosmology, Oxford University Press, Oxford - UK (2008).
13. A. A. Starobinsky, Pis’ma Zh. Eksp. Teor. Fiz. 42, 124 (1985). [JETP Lett. 42, 152 (1985)].
14. M. Sasaki and E. D. Stewart, Prog. Theor. Phys. 95, 71 (1996).
15. M. Sasaki and T. Tanaka, Prog. Theor. Phys. 99, 763 (1998).
16. D. H. Lyth, K. A. Malik, and M. Sasaki, JCAP 0505, 004 (2005).
17. C. A. Valenzuela-Toledo, Y. Rodríguez, and J. P. Beltrán Almeida, JCAP 1110, 020 (2011).
18. K. Dimopoulos, M. Karciuskas, D. H. Lyth, and Y. Rodríguez, JCAP 0905, 013 (2009).
19. L. R. Abramo and T. S. Pereira, Adv. Astron. **2010**, 378203 (2010).
20. D. H. Lyth and A. R. Liddle, *The Primordial Curvature Perturbation*, Cambridge University Press, Cambridge - UK (2009).
21. L. G. Gómez and Y. Rodríguez, work in preparation.