Penguins and Mixing Dependent CP Violation

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Abstract

Constraints on angles of Unitarity triangle are reviewed, and in particular constraint on $\gamma$ from limit on $\Delta m_s$ is emphasized. Effects of penguin diagram on measurement of $\beta$ and $\alpha$ are then reviewed. New measurements on $B \rightarrow \pi^+\pi^-$ in QCD improved factorization approximation suggest large penguin effects. It is possible to estimate the error in measurement of $\alpha$ as a function of $\gamma$ for different $|V_{ub}/V_{cb}|$ values.

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1 Constraints on Angles of Unitarity Triangle

Constraints on unitarity triangle expressed in terms of Wolfenstein parameterization are given below:

[a ] From charmless semileptonic B decays [1]:

\[ \left| \frac{V_{ub}}{V_{cb}} \right| = 0.08 \pm 0.02 \] (1)

which yields

\[ \left( \rho^2 + \eta^2 \right)^{1/2} = 0.36 \pm 0.09 \] (2)

[b ] From \( B_d - \bar{B}_d \) mixing. Error is dominated by \( f_B \) and the bag factor \( B_B \):

\[ |V_{td}| = 0.009 \pm 0.003 \] (3)

which yields

\[ |1 - \rho - i\eta| = 1.0 \pm 0.3 \] (4)

[c ] Value of \( \epsilon \) in \( K \) system. Error is dominated by hadronic matrix elements:

\[ \eta (1 - \rho + 0.35) = 0.48 \pm 0.20 \] (5)

[d ] Constraints from \( B_s - \bar{B}_s \) [2]

\[ (\Delta m_s) > 14.3ps^{-1} (90\%CL) \] (6)

Using the relation from Box diagrams

\[ \left| \frac{V_{td}}{V_{ts}} \right| = \xi \left[ \frac{m_{B_s} \Delta m_d}{m_{B_d} \Delta m_s} \right]^{1/2} \] (7)
where
\[ \xi = \frac{f_{B_s}}{f_{B_d}} \sqrt{\frac{B_{B_s}}{B_{B_d}}} \]  
(8)

Lattice calculations yield for \( \xi \) the value \[ \xi = 1.15 \pm 0.05 \]  
(9)

This translates into the bound
\[ |V_{td}/V_{ts}| < 0.214 \]  
(10)

or
\[ |1 - \rho - i\eta| < 0.96 \]  
(11)

The last constraint in particular implies \( \gamma < 90^\circ \) and \( 75^\circ < \alpha < 120^\circ \). This allowed range of \( \gamma \) leads to unique estimate of errors in \( \alpha \) as we shall see.

2 CP Violation Through Mixing

Strategy to measure \( \beta \) and \( \alpha \) involve measuring time dependent asymmetry in \( B \) decays to CP eigenstate. Defining the long and short lived eigenstates of \( B \) as
\[ |B\rangle_{L,S} = p |B^0\rangle \pm q |\bar{B}^0\rangle, \]  
(12)

the amplitudes for decays into CP eigenstates are defined as
\[ A = \langle f_{CP}|H_w|B^0\rangle \]  
(13)
\[ \bar{A} = \langle f_{CP}|H_w|\bar{B}^0\rangle. \]  
(14)
The asymmetry is then defined by

\[ \text{Asy}(t) = \left( (1 - |\lambda|^2) \cos(\Delta M t) - 2 \text{Im}(\lambda) \sin(\Delta M t) \right) / \sqrt{1 + |\lambda|^2} \]  

(15)

where \( \lambda = (q/p) \frac{\bar{A}}{A} \). In the standard model \((q/p) = e^{-2i\beta}\). If \( A \) is dependent on a single weak phase,

\[ \left( \frac{\bar{A}}{A} \right) = e^{-2i\phi_{\text{weak}}} \]  

(16)

then we have the expression

\[ \text{Asy}(t) = -\text{Im}(\lambda) \sin(\Delta M t) \]  

(17)

### 2.1 Measurement of \( \beta \)

The mode that has the least theoretical uncertainty is \( B \to \psi K_s \). The amplitude for this mode can be written in terms of Tree and Penguin contribution as

\[ A = V_{cb} V_{cs}^* T + V_{tb} V_{ts}^* P = V_{cb} V_{cs}^* (T - P) + V_{ub} V_{us}^* P \]  

(18)

since \( |V_{ub} V_{us}^*/V_{cb} V_{cs}^*| \approx 1/50 \), and the Penguin contribution has predominantly \( \bar{c}c \) in a color octet state, the contribution due to penguin diagram is less than 1%.

If \( B \to D^+ D^- \) mode is used instead, the penguin contribution is much larger, and there is no color suppression either.

\[ A = V_{cb} V_{cs}^* T + V_{tb} V_{ts}^* P = V_{cb} V_{cs}^* (T - P) - V_{ub} V_{us}^* P \]  

(19)

The value of \( |V_{ub} V_{us}^*/V_{cb} V_{cs}^*| \approx 0.3 \), and although \( P \) is suppressed compared to \( T \) due to small Wilson coefficients, one can expect a contamination due to penguin of a few percent.
2.2 Measurement of $\alpha$

The mode $B^o \to \pi^+\pi^-$ lends itself to the earliest measurement of $\alpha$. For this mode the amplitude is

$$ A = V_{ub} V_{ud}^* T + V_{tb} V_{td}^* P = V_{ub} V_{ud}^* (T - P) + V_{cb} V_{cd}^* P $$  \hspace{1cm} (20)

The value of $|V_{cb} V_{cd}^* / V_{ub} V_{ud}^*| \approx 3$ giving a crude estimate of around 15% for the penguin contamination. Gronau and London [4] have presented a method of extracting $\alpha$ from measurements of $B^o \to \pi^+\pi^-$, $B^o \to \pi^0\pi^\pm$, $B^+ \to \pi^0\pi^\pm$, and $B^+ \to \pi^\pm\pi^0$. However, the most recent theoretical estimates of $B^o \to \pi^0\pi^0$ branching ratio are around $5 \times 10^{-7}$, making this method academic at present. However, we now discuss theoretical developments that may allow us to extract the correct $\alpha$ from measurements of asymmetry in $B^o \to \pi^+\pi^-$ alone.

3 Determination of $\alpha$ from $B^o \to \pi^+\pi^-$

This is based on recent work of Agashe and Deshpande [5]. Recently, the CLEO collaboration has reported the first observation of the decay $B \to \pi^+\pi^-$ [6]. The effective Hamiltonian for $B$ decays is:

$$ H_{eff} = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{ud}^* (C_1 O_1^{u} + C_2 O_2^u) ight. $$

$$ + V_{cb} V_{cd}^* (C_1 O_1^{c} + C_2 O_2^c) - V_{tb} V_{td}^* \sum_{i=3}^{6} C_i O_i \right]. $$  \hspace{1cm} (21)

The $C_i$’s are the Wilson coefficients (WC’s). In a recent paper, Beneke et al. found that the matrix elements for the decays $B \to \pi\pi$, in the large $m_b$ limit, can be written as [7]

$$ \langle \pi\pi | O_i | B \rangle = \langle \pi | j_1 | B \rangle \langle \pi | j_2 | 0 \rangle $$

$$ \times \left[ 1 + \sum r_n \alpha_s^n(m_b) + O(\Lambda_{QCD}/m_b) \right], $$  \hspace{1cm} (22)
where \( j_1 \) and \( j_2 \) are bilinear quark currents. If the radiative corrections in \( \alpha_s \) and \( O(\Lambda_{QCD}/m_b) \) corrections are neglected, then the matrix element on the left-hand side factorizes into a product of a form factor and a meson decay constant so that we recover the “conventional” factorization formula. These authors computed the \( O(\alpha_s) \) corrections. In this approach, the strong interaction (final-state rescattering) phases are included in the radiative corrections in \( \alpha_s \) and thus the \( O(\alpha_s) \) strong interaction phases are determined \([7]\). The matrix element for \( B \to \pi^+\pi^- \) is \([7]\):

\[
i \bar{A} \left( B_d \to \pi^+\pi^- \right) = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{ud}^* \left( a_1 + a_4^u + a_6^u r_\chi \right) + V_{cb} V_{cd}^* \left( a_4^c + a_6^c r_\chi \right) \right] \times X. \tag{23}
\]

Here

\[
X = f_\pi \left( m_B^2 - m_\pi^2 \right) F_{0}^{B \to \pi^-} \left( m_\pi^2 \right), \tag{24}
\]

where \( f_\pi = 131 \text{ MeV} \) is the pion decay constant and \( F_{0}^{B \to \pi^-} \) is a form factor. In the above equations, the \( a_i \)'s are (combinations of) WC’s with the \( O(\alpha_s) \) corrections added. The values of the \( a_i \)'s are given in Table \([7]\). The imaginary parts of \( a_i \)'s are due to final-state rescattering. For the \( CP \) conjugate processes, the CKM elements have to be complex-conjugated. We discuss two values of the form factors: \( F_{B \to \pi^-} = 0.27 \) and 0.33. Model calculations indicate that the \( SU(3) \) breaking in the form factors is given by \( F_{B \to K^-} \approx 1.13 F_{B \to \pi^-} \) \([8, 9]\). The large measured \( BR(B \to K\eta') \) requires \( F_{B \to K^-} \approx 0.36 \) \([10]\) which, in turn, implies a larger value of \( F_{B \to \pi^-} \) (\( \approx 0.33 \)). If \( F_{B \to K^-} \approx 0.36 \), then we require a “new” mechanism to account for \( BR(B \to K\eta') \): high charm content of \( \eta' \) \([11]\), QCD anomaly \([12]\) or new physics. Also, if \( F_{B \to \pi^-} < 0.27 \), then the value of \( F_{B \to K} \) is too small to explain the measured BR’s for \( B \to K\pi \) \([13]\). We use \( |V_{cb}| = 0.0395 \), \( |V_{ud}| = 0.974 \), \( |V_{cd}| = 0.224 \), \( m_B = 5.28 \text{ GeV} \) and \( \tau_B = 1.6 \text{ ps} \) \([1]\). In Fig. \([4]\) we show the \( CP \)-averaged BR for \( B \to \pi^+\pi^- \) as a functions of \( \gamma \) for \( F_{B \to \pi^-} = 0.33 \) and 0.27 and for \( |V_{ub}/V_{cb}| = 0.1, 0.08 \) and 0.06.
Figure 1: $CP$-averaged $BR(B \rightarrow \pi^+\pi^-)$ as a function of $\gamma$ for $F^{B \rightarrow \pi} = 0.27$ (left) and 0.33 (right) and for $|V_{ub}/V_{cb}| = 0.1$ (solid curves), 0.08 (dashed curves) and 0.06 (dotted curves). The BR measured by the CLEO collaboration lies (at the 1 $\sigma$ level) between the two horizontal (thicker) solid lines. The errors on the CLEO measurement have been added in quadrature to compute the 1 $\sigma$ limits.
\[
\begin{array}{|c|c|}
\hline
a_1 & 1.047 + 0.033 i \\
\hline
a_2 & 0.061 - 0.106 i \\
\hline
a_4^u & -0.030 - 0.019 i \\
\hline
a_4^c & -0.038 - 0.009 i \\
\hline
a_6^{u,c} r_X & -0.036 \\
\hline
\end{array}
\]

Table 1: The factorization coefficients for the renormalization scale \( \mu = m_b/2 \).

The CLEO measurement is \( B \to \pi^+\pi^- = (4.7^{+1.8}_{-1.5} \pm 0.6) \times 10^{-6} \). If \( F_{B \to \pi^-} = 0.33 \) and for \( \gamma \approx 90^\circ \), we see from the figures that smaller values of \( |V_{ub}/V_{cb}| \approx 0.06 \) are preferred: \( |V_{ub}/V_{cb}| = 0.08 \) is still allowed at the 2\( \sigma \) level for \( \gamma \sim 100^\circ \). The smaller value of \( |V_{ub}/V_{cb}| \) leads to greater penguin contamination. However, if the smaller value of the form factor (0.27) is used, then the CLEO measurement is consistent with \( |V_{ub}/V_{cb}| \approx 0.08 \). We obtain similar results using “effective” WC’s (\( C^{eff} \))’s and \( N = 3 \) in the earlier factorization framework.

Since the \( B_d - \bar{B}_d \) mixing phase is \( 2\beta \), if we neglect the (QCD) penguin operators, i.e., set \( a_{4,6} = 0 \) in Eq. (23), we get
\[
\frac{\bar{A}}{A} = e^{-i2\gamma}
\]
and
\[
\text{Im}\lambda = \sin (-2(\beta + \gamma)) = \sin 2\alpha.
\]
In the presence of the penguin contribution, however, \( \bar{A}/A \neq e^{-i2\gamma} \) so that \( \text{Im}\lambda \neq \sin 2\alpha \). We define
\[
\text{Im}\lambda = \text{Im} \left( e^{-i2\beta} \frac{\bar{A}}{A} \right) \equiv \sin 2\alpha_{\text{meas}}.
\]
as the “measured” value of \( \sin 2\alpha \), i.e., \( \sin 2\alpha_{\text{meas}} = \sin 2\alpha \) if the penguin operators can be neglected. In Fig. 3 we plot the error in the measurement of
Figure 2: The error in the measurement of CKM phase $\alpha$ using (only) time-dependent $B \to \pi^+\pi^-$ decays as a function of $\gamma$ for $|V_{ub}/V_{cb}| = 0.1$ (solid curve), 0.08 (dashed curve) and 0.06 (dotted curve).

$\alpha$, $\Delta \alpha \equiv \alpha_{\text{meas}} - \alpha$, where $\alpha_{\text{meas}}$ is obtained from Eq. (27) and $\alpha$ is obtained from $\gamma$ and $|V_{ub}/V_{cb}|$. Note that $\Delta \alpha$ is independent of $F_{B \to \pi^-}$ since the form factor cancels in the ratio $\bar{A}/A$. We see that for the values of $|V_{ub}/V_{cb}| \approx 0.06$ preferred by the $B \to \pi^+\pi^-$ measurement (if $F_{B \to \pi^-} \approx 0.33$), the error in the determination of $\alpha$ is large $\sim 15^\circ$ (for $\gamma \sim 90^\circ$). If $F_{B \to \pi^-} \approx 0.27$, then $|V_{ub}/V_{cb}| \approx 0.08$ is consistent with the $B \to \pi^+\pi^-$ measurement which gives $\Delta \alpha \sim 10^\circ$ (for $\gamma \sim 90^\circ$).

The computation of Beneke et al. [7] includes final state rescattering phases, i.e., it is exact up to $O(\Lambda_{QCD}/m_b)$ and $O(\alpha_s^2)$ corrections. Thus, the value of $\sin 2\alpha$ “measured” in $B \to \pi^+\pi^-$ decays (Eq. (27)) is a known function of $\gamma$ and $|V_{ub}/V_{cb}|$ only (in particular, there is no dependence on
Figure 3: The “true” value of $\sin 2\alpha$ as a function of the value of $\sin 2\alpha$ “measured” in $B \to \pi^+\pi^-$ decays for $|V_{ub}/V_{cb}| = 0.1$ (solid curve), 0.08 (dashed curve) and 0.06 (dotted curve).
the phenomenological parameter $\xi \sim 1/N$ and strong phases are included unlike in the earlier factorization framework [11]). Since, the “true” value of $\alpha$ can also be expressed in terms of $\gamma$ and $|V_{ub}/V_{cb}|$, we can estimate the “true” value of $\sin 2\alpha$ from the “measured” value of $\sin 2\alpha$ for a given value of $|V_{ub}/V_{cb}|$ (of course, up to $O(\Lambda_{QCD}/m_b)$ and $O(\alpha_s^2)$ corrections); this is shown in Fig. 3 where we have restricted $\gamma$ to be in the range $(40^\circ, 120^\circ)$ as indicated by constraints on the unitarity triangle from present data. If $0^\circ \leq \gamma \leq 180^\circ$ is allowed, then there will be a discrete ambiguity in the determination of $\sin 2\alpha$ from $\sin 2\alpha_{\text{meas.}}$.

4 Conclusions

We have shown how $\alpha$ can be obtained from the measured value of $\sin 2\alpha$ inspite of large penguin effects. The theoretical work can be extended to $K\pi$ modes to obtain values of $\gamma$ from the measured branching ratios. Naive factorization suggest $\gamma \approx 100^\circ$ [14, 13].

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