Probing the sensitivity to leptonic $\delta_{CP}$ in presence of invisible decay of $\nu_3$ using atmospheric neutrinos

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Abstract

One of the main neutrino oscillation parameters whose value has not been determined very precisely is the leptonic $\delta_{CP}$ phase. Since neutrinos have a tiny but finite mass they can undergo decay both visibly and invisibly. The effect of invisible decay of the third mass eigen state $\nu_3$ on the sensitivity to $\delta_{CP}$ is analysed here using atmospheric neutrino and anti-neutrino events. Effects of detector resolutions and systematic uncertainties are studied to identify the optimum resolutions and efficiencies required by a detector to obtain a significant sensitivity even in presence of decay.

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I. INTRODUCTION

The value of the leptonic $\delta_{CP}$ phase is one of the most sought out unknowns in neutrino oscillation physics. Several accelerator based long baseline (LBL) experiments are taking data \cite{1, 2} and are being planned \cite{3, 4} to measure this quantity precisely. Since neutrino oscillations have proven that neutrinos have a tiny but finite mass, there is a possibility that they can decay. For Majorana neutrinos a possible decay mode allowed by Majoron model \cite{5, 6} is $\nu_i \rightarrow \nu_j + J$ or $\nu_i \rightarrow \bar{\nu}_j + J$, where $\nu_j$ and $\bar{\nu}_j$ are lighter neutrino and anti–neutrino states and $J$ is mostly a singlet Majoron \cite{6, 7}. This decay can be visible or invisible depending on whether the final state contains an active neutrino or a sterile neutrino respectively. In this paper the effect of invisible decay will be studied. The lifetimes of $\nu_1$ and $\nu_2$ are tightly constrained by solar neutrino data \cite{8, 18}. Detailed discussions on constraining neutrino lifetimes via cosmology and astrophysical experimental scales are performed in \cite{19–34}. Many of these papers have considered the invisible neutrino decay scenario, especially the Majoron model. Analyses for invisible neutrino decay for accelerator long and medium baseline and atmospheric neutrino experiments have been carried out in \cite{35–43} to obtain the limits for $\nu_3$ decay. The limits from visible decay in accelerator and reactor neutrino experiments are discussed in \cite{44–47}.

In this study the decay of $\nu_3$ into a light sterile neutrino state with which it does not mix \cite{41, 48} is considered. So there will be the dominant oscillations plus subdominant invisible decay. The study presented in this paper is mainly concerned with neutrino energies and distances available for terrestrial experiments - the energies and baselines corresponding to atmospheric neutrino experiments. The invisible decay will cause a depletion of observable events in the detector. This decay is characterised by a parameter $\alpha_3 = m_3/\tau_3$, where $m_3$ and $\tau_3$ are the mass and rest frame life time of $\nu_3$ respectively. It has been shown in \cite{35-38, 42, 43} that invisible decay will affect the measurement of other oscillation parameters, especially $\theta_{23}$. While the effect of decay on the measurement of $\delta_{CP}$ with LBL experiments has been studied in \cite{37}, this has not been studied in detail with atmospheric neutrinos. Atmospheric neutrinos offer a wide variety of baselines (from $\sim 15 \text{ km} – \sim 13000 \text{ km}$) and energies ($\sim 0.1–30.0 \text{ GeV}$) of neutrinos. Studies on sensitivity to $\delta_{CP}$ using sub-GeV atmospheric neutrinos have been conducted in \cite{49–51}. Low energy (sub-GeV) atmospheric neutrinos are a good probe for the effect of this invisible decay of $\nu_3$ due to several reasons.

1. Measurement of $\delta_{CP}$ unambiguous of neutrino mass hierarchy - The measurement of neutrino oscillation parameters is affected by degeneracies. Presence of more parameters mean more degeneracies and ambiguities and neutrino mass hierarchy has not been determined yet. At sub-GeV energies $\delta_{CP}$ can be measured unambiguous of hierarchy \cite{49, 51}. This opens up the possibility of determining the effect of other parameters like $\alpha_3$ on $\delta_{CP}$ measurement.

2. Effect of the invisible decay parameter $\alpha_3$ is more at lower (sub-GeV) energies \cite{42}. Hence, in the absence of other degeneracies, its effects on $\delta_{CP}$ measurement will me more evident at these energies.

3. Statistics - The flux of atmospheric neutrinos are large at sub-GeV energies \cite{52, 54}. Hence there will be more number of events available for the study.

In this paper a study of how the presence of invisible decay of $\nu_3$ affects the measurement of $\delta_{CP}$ with atmospheric neutrinos is conducted. The optimum detector configurations required
to achieve a good sensitivity to $\delta_{\text{CP}}$ even in the presence of $\alpha_3$ is also studied. The effect of invisible decay on the oscillation probabilities and event spectra relevant for this study are discussed in Sections III. The process of event generation for different types of analyses are discussed in Section IV. Sensitivities to $\delta_{\text{CP}}$ in presence of decay for ideal and realistic cases in the absence and presence of systematic uncertainties are discussed in Sections IV and V respectively. Summary and conclusions are given in Section VI.

II. EFFECT IN VISIBLE DECAY ON OSCILLATION PROBABILITIES IN MATTER

A full 3-flavour oscillations + decay in matter scenario is considered here. The mass eigen state $\nu_3$ decays invisibly via $\nu_3 \to \nu_s + J$, where $J$ is a pseudo-scalar Majaron and $\nu_s$ is a sterile neutrino which does not mix with the three active neutrinos. Hence the mixing matrix $U$ in vacuum will be:

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\ -c_{23} s_{12} - s_{23} s_{13} c_{12} e^{i \delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i \delta} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i \delta} & -s_{23} c_{12} - c_{23} s_{13} s_{12} e^{i \delta} & c_{23} c_{13} \end{pmatrix},$$

(1)

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$; $\theta_{ij}$ are the mixing angles and $\delta$ is the CP violating phase.

For true normal hierarchy, $m_s < m_1 < m_2 < m_3$, where $m_s$ is the mass of $\nu_s$ and $m_i$ are the mass of $\nu_i$, i=1,2,3. In the presence of Earth matter, the three-flavour evolution equation will be:

$$\frac{d \tilde{\nu}_i}{dt} = \frac{1}{2E} [U \tilde{M}^2 U^\dagger + A_{\text{CC}}] \tilde{\nu},$$

(2)

$$\tilde{M}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 - i \alpha_3 \end{pmatrix}, \quad \text{and} \quad A_{\text{CC}} = \begin{pmatrix} A_{\alpha \beta} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

(3)

where $E$ is the neutrino energy, $\alpha_3 = m_3/\tau_3$, is the decay constant in units of eV$^2$, $m_3$ is the mass of $\nu_3$ and $\tau_3$ its rest frame life time and $A_{\alpha \beta}$ is the matter potential.

$$A_{\alpha \beta} = 2\sqrt{2} G_F n_e E = 7.63 \times 10^{-5} \text{eV}^2 \rho (\text{gm/cc}) E (\text{GeV})$$

(4)

where, $G_F$ is the Fermi constant and $n_e$ is the electron number density in matter and $\rho$ is the matter density. For anti-neutrinos, both the sign of $A_{\alpha \beta}$ and the phase $\delta$ in Eq. (2) are reversed. Since the term $\alpha_3$ appears in the propagation equation along with $\Delta m_{31}^2$ they should have the same unit. The conversion factor to make $\alpha_3$ and $\Delta m_{31}^2$ have the same units (i.e eV$^2$) is $1 \text{eV/s} = 6.58 \times 10^{-16} \text{eV}^2$.

Transition probabilities, especially are mainly responsible for the sensitivity to $\delta_{\text{CP}}$ [55–57]. Since $\Phi_{\nu_\mu}/\Phi_{\nu_e} (\Phi_{\nu_\mu}/\Phi_{\nu_e}) \approx 2 : 1$, the contribution to $\delta_{\text{CP}}$ sensitivity from $\nu_\mu \to \nu_e$ ($\bar{\nu}_\mu \to \bar{\nu}_e$) events will be more. The sensitivity to $\alpha_3$ is more for $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ events, though their sensitivity to $\delta_{\text{CP}}$ is very low compared to the $\nu_\nu$ like events. Hence not to leave out any contribution from any channel all 8 channels - $\nu_{e3}, \nu_{e3}, \nu_{\mu3}, \nu_{\mu3}$, where $\beta = e, \mu$ are studied for this analysis. The difference between the 3-flavour oscillation probabilities in matter with $\delta_{\text{CP}} = -90^\circ$ and $\delta_{\text{CP}} = 0^\circ$ is shown as oscillograms in Figs. 1 and 2. The oscillograms for the lower energy range 0.1–2.0 GeV are shown. The central values of the oscillation parameters used for the analysis are given in Table 1.
It can be seen from the Fig. 1 that $\alpha_3$ has no significant effect on $P_{\mu e}$ and $\bar{P}_{\mu e}$. However, the minor changes in the several bins with the increase of $\alpha_3$ value can add up to a small contribution to the $\delta_{CP}$ sensitivity. As seen in Fig. 2 $\alpha_3$ does affect $P_{\mu \mu}$ and $\bar{P}_{\mu \mu}$ at very low energies, especially below $E_{\nu} = 0.4$ GeV. But the overall contribution of $P_{\mu \mu}$ and $\bar{P}_{\mu \mu}$ to $\delta_{CP}$ sensitivity is smaller compared to the electron like events. Atmospheric neutrino flux is lesser compared to accelerator based long base line experiments and neutrino physics experiments are low counting experiments. So even the smallest contribution to the sensitivity to a parameter cannot be neglected. Because of this, the study is done for events in the neutrino energy range 0.1–30 GeV. Normal hierarchy is assumed to be the true hierarchy. At low energies $\delta_{CP}$ measurement will be independent of hierarchy [49], but at higher energies (2.0–30 GeV) the effect of hierarchy will be present, hence the hierarchy is assumed to be known, for this study.

For low energies 0.1–1.0 GeV, both $P_{\mu e}$ and $\bar{P}_{\mu e}$ are affected by invisible decay. In the higher energy region, decay affects $P_{\mu e}$ in the resonance region. The effect of $\alpha_3$ on $\bar{P}_{\mu e}$ is not as much as for the neutrino case. From this figure we can see that the measurement of $\delta_{CP}$ will be affected by the presence of $\alpha_3$. The variation in sensitivity will depend on the value of $\alpha_3$, a lesser sensitivity is expected for larger $\alpha_3$ from electron like events.

### A. Effect of invisible decay of $\nu_3$ on the oscillated event spectra

The effect of the decay parameter $\alpha_3$ on the oscillated event spectra for different values of $\alpha_3$ and $\delta_{CP} = -90^\circ$ is shown in Fig. 3. The effect of $\alpha_3$ in the lower (0.1–2.0 GeV) and higher (2.0–30.0 GeV) energy regions can be separated. For $\nu_e$ and $\bar{\nu}_e$ events, $\alpha_3$ does not have any significant effect, both at lower and higher energies. But the oscillated event spectra get suppressed with increasing $\alpha_3$ values for both $\nu_\mu$ and $\bar{\nu}_\mu$ events. Thus, the sensitivity to $\delta_{CP}$ from $\nu_e$ and $\bar{\nu}_e$ events will not get affected much by $\alpha_3$, but the minor sensitivity from $\nu_\mu$ and $\bar{\nu}_\mu$ events will be. Electron like events are more sensitive to $\delta_{CP}$ than muon-like events.
FIG. 1: Top panels - $\Delta P_{\mu\mu}$ (left) for $\alpha_3 = 0$ eV$^2$ (no decay) and (right) $\alpha_3 = 1 \times 10^{-5}$ eV$^2$ in the neutrino energy range 0.1–2.0 GeV for true normal hierarchy. Bottom panels are for $\Delta \bar{P}_{\mu\mu}$.

like events. It can also be seen that the muon like events are more sensitive to $\alpha_3$ especially at low energies. This shows that at low energies (0.1–2.0 GeV), $\nu_e$ and $\bar{\nu}_e$ events are well suited to probe $\delta_{CP}$ while low energy $\nu_\mu$ and $\bar{\nu}_\mu$ events probe $\alpha_3$ better. At higher energies the effects are not much, but the small contributions from all bins can add up together.
FIG. 2: Top panels - $\Delta P_{\mu\mu}$ (left) for $\alpha_3 = 0$ eV$^2$ (no decay) and (right) $\alpha_3 = 1 \times 10^{-5}$ eV$^2$ in the neutrino energy range 0.1–2.0 GeV for true normal hierarchy. Bottom panels are for $\Delta \bar{P}_{\mu\mu}$. 

$\Delta P_{\mu\mu} = P_{\mu\mu}^{\delta \nu \rightarrow \delta \nu} - P_{\mu\mu}^{\delta \nu \rightarrow \delta \nu}$, $\alpha_3 = 0$ eV$^2$, NH

$\Delta P_{\mu\mu} = P_{\mu\mu}^{\delta \nu \rightarrow \delta \nu} - P_{\mu\mu}^{\delta \nu \rightarrow \delta \nu}$, $\alpha_3 = 1 \times 10^{-5}$ eV$^2$, NH

$\Delta \bar{P}_{\mu\mu} = \bar{P}_{\mu\mu}^{\delta \nu \rightarrow \delta \nu} - \bar{P}_{\mu\mu}^{\delta \nu \rightarrow \delta \nu}$, $\alpha_3 = 0$ eV$^2$, NH

$\Delta \bar{P}_{\mu\mu} = \bar{P}_{\mu\mu}^{\delta \nu \rightarrow \delta \nu} - \bar{P}_{\mu\mu}^{\delta \nu \rightarrow \delta \nu}$, $\alpha_3 = 1 \times 10^{-5}$ eV$^2$, NH
FIG. 3: Comparison of oscillated event spectra for different values of $\alpha_3$ for $\delta_{CP} = -90^\circ$. Top panels are for $\nu_e$ and bottom panels are for $\bar{\nu}_e$ events. $\alpha_3$ has no effect on the oscillated spectra in both the energy ranges.
FIG. 4: Comparison of oscillated event spectra for different values of $\alpha_3$ for $\delta_{CP} = -90^\circ$. Top panels are for $\nu_\mu$ and bottom panels are for $\bar{\nu}_\mu$ events. It is clear that $\alpha_3$ affects the $\nu_\mu$ and $\bar{\nu}_\mu$ spectra at low energies, which means that the sensitivity to $\delta_{CP}$ from these events, if any at all will be affected by $\alpha_3$. 
III. EVENT GENERATION AND $\chi^2$ ANALYSIS

Simulated charged current (CC) $\nu_\mu, \bar{\nu}_\mu, \nu_e$ and $\bar{\nu}_e$ events on an isoscalar target are used for this study. For atmospheric neutrinos, $\nu_e \rightarrow \nu_e$ survived events along with those from $\nu_\mu \rightarrow \nu_e$ transitions contribute to the (CC) $\nu_e$ event spectrum in the detector:

$$N^e = t \times n_d \times \int d\sigma_{\nu_e} \times \left[ P_{ee}^m \frac{d^2\Phi_e}{dE_\nu \, d \cos \theta_\nu} + P_{\mu e}^m \frac{d^2\Phi_\mu}{dE_\nu \, d \cos \theta_\nu} \right].$$

(5)

Here $P_{ee}^m$ and $P_{\mu e}^m$ are the oscillation probabilities in matter in presence of decay. Here $t$ is the exposure/run time, $n_d$ is the number of targets available for interaction in the detector, $d\sigma_{\nu_e}$ is the neutrino interaction cross section which is differential in final state charged lepton energy ($E_e$) and/or direction $\cos \theta_e$, and $d\Phi_{\nu_\mu}$ ($d\Phi_{\nu_e}$) is the $\nu_\mu$ ($\nu_e$) flux. Similarly for $\bar{\nu}_e$ and $\nu_\mu$ and $\bar{\nu}_\mu$ events also. Hereafter the charged current electron (muon) like events will be referred to as CCE (CCMU).

Sensitivity to $\delta_{CP}$ is studied for idealistic and realistic scenarios. The difference between these scenarios is given in Table. II.

| Idealistic | Realistic |
|------------|-----------|
| Perfect energy and direction resolution for the final state particles (nores) | Realistic energy for final state particles (wres) |
| Complete separation of $\nu_e$ ($\bar{\nu}_e$) like events from $\nu_\mu$ ($\bar{\nu}_\mu$) like events | $\nu_e$ and $\nu_\mu$ like events can be separated from each other |
| $\nu$ and $\bar{\nu}$ can be separated from each other (wcid) | No separation between $\nu$ and $\bar{\nu}$ (nocid) |
| Events binned in ($E_{obs}^\mu, \cos \theta_{obs}^\mu, E_{had}^\prime$) (3D) | Events binned in ($E_{obs}^\mu, \cos \theta_{obs}^\mu$) (2D) |
| No fluctuations | With fluctuations |

TABLE II: Criteria for idealistic and realistic cases for sensitivity studies

Unoscillated charged current (CC) events for an exposure of 100 years in a 50 kton detector (500 kton-years) are simulated using the NUANCE [59] neutrino generator. Honda 3D fluxes [52–54] for atmospheric neutrinos are used and the target is assumed to be a generic isoscalar one. For the perfect case analyses presented in Section IV the following procedure is used to generate “data” and theory events. For “data” events, each event in the 100 year sample is oscillated individually applying the central values of the oscillation parameters given in Table I. This is then scaled down to the required number of years (10 years). For theory events, the 100 year sample is oscillated event by event by varying the parameters in their respective $3\sigma$ ranges given in the same table. This method has no fluctuations. For the realistic case, the 10 years of events are selected randomly from the unoscillated 100 year sample and oscillated individually with the central values in Table I to generate “data”. The remaining 90 years of events are oscillated with parameters in their $3\sigma$ ranges and scaled to 10 years to generate theory. This method thus takes into account the fluctuations.

A poissonian $\chi^2$ analysis as described in [60] is performed with three final state observables $E_{\mu}^{obs}, \cos \theta_{\mu}^{obs}, E_{had}^{obs}$, which are the energy and direction of the observed muon and the energy of the observed hadron shower. The binning scheme is shown in Table III.

Systematic uncertainties are taken into account using pull method [60–67]. For the idealistic case where neutrino and anti-neutrino events can be separated from each other, the
| Observable | Range       | Bin width | No. of bins |
|------------|-------------|-----------|-------------|
| $E^{\text{obs}}_\mu$ (GeV) | [0.1, 0.2] | 0.1       | 1           |
|           | [0.2, 0.4] | 0.1       | 1           |
|           | [0.4, 0.5] | 0.1       | 1           |
|           | [0.5, 1.0] | 0.3       | 2           |
|           | [1.0, 4.0] | 0.5       | 6           |
|           | [4, 7]     | 1         | 3           |
|           | [7, 11]    | 4         | 1           |
|           | [11, 12.5] | 1.5       | 1           |
|           | [12.5, 15] | 2.5       | 1           |
|           | [15, 30]   | 15        | 1           |
| $E^{\text{obs}}_{\mu_{\text{had}}}$ (GeV) | [0, 2]     | 1         | 2           |
|           | [2, 4]     | 2         | 1           |
|           | [4, 15]    | 11        | 1           |
| $\cos \theta^{\text{obs}}_{\mu}$ (20 bins) | [-1.0, 1.0] | 0.1       | 20          |
| $E^{\text{obs}}_{\mu_{\text{had}}}$ (GeV) | [0.1, 0.2] | 0.1       | 1           |
|           | [0.2, 0.4] | 0.1       | 1           |
|           | [0.4, 0.5] | 0.1       | 1           |
|           | [0.5, 1.0] | 0.3       | 2           |
|           | [1.0, 4.0] | 0.5       | 6           |
|           | [4, 7]     | 1         | 3           |
|           | [7, 11]    | 4         | 1           |
|           | [11, 12.5] | 1.5       | 1           |
|           | [12.5, 15] | 2.5       | 1           |
|           | [15, 30]   | 15        | 1           |

TABLE III: Bins of the three observables used for the analysis.

11 pull $\chi^2$ analysis described in [60] is performed. For the realistic case when neutrino and anti-neutrino events cannot be separated from each other, the $\chi^2$ described by Eqn. 10 of [49] is used. The parameters $\theta_{23}, |\Delta m^2_{32}|$ and $\alpha_3$ are marginalised in their $3\sigma$ ranges. The other parameters $\theta_{12}, \theta_{13}$ and $\Delta m^2_{21}$ are measured precisely, so they are kept fixed in the analysis.

In the realistic case, the effect of final state lepton energy resolution on the sensitivity to $\delta_{CP}$ also is studied. For this, resolutions of the form [68]:

\[
\frac{\sigma}{E} = \frac{a\%}{\sqrt{E}} \oplus b\%
\]  

were taken, where $a = 2.5$ and $b = 0.5$ for electrons and $a = 3$ for muons.

IV. RESULTS-IDEALISTIC CASE

Sensitivity studies with and without final state lepton energy resolutions The results of sensitivity studies with and without pulls and energy resolutions are obtained.

A. No pulls

Fig. [5] shows the sensitivity to $\chi^2$ to the most ideal (and currently impractical) scenario. Several observations can be made from this figure.

- The sensitivity to $\delta_{CP}$ with CCE events is much higher than that with CCMU events as expected.
• While the sensitivity from CCE decreases with increase in the decay parameter, the sensitivity to $\delta_{CP}$ from CCMU events is slightly enhanced in the presence of invisible decay. The effect of $\alpha_3$ is opposite on CCE and CCMU events.

• In the absence of systematic uncertainties, for a given $\alpha_3$ value, energy resolution worsens the sensitivity only slightly.

• $\delta_{CP}^{test}$ values in the range $[-50^\circ, 110^\circ]$ can be excluded at 3$\sigma$ with CCE events alone, even in presence of invisible decay (with $\alpha_3 = 1 \times 10^{-5}$ eV$^2$). All values of $\delta_{CP}$ are allowed at 2$\sigma$ from CCMU events, $\delta_{CP} = \sim [-40^\circ, 90^\circ]$ can be excluded at 1$\sigma$ for all three values of $\alpha_3$.

The major contribution to the $\delta_{CP}$ sensitivity comes from the lower energy region where the effect of $\alpha_3$ is also high.

**B. Effect of pulls**

When systematic uncertainties are present the sensitivity decreases significantly for both CCE and CCMU events (for the idealistic CCE case this is very drasitc). For both CCE and CCMU, the sensitivities with finite detector resolutions are lesser than those with perfect resolutions even in the presence of all pulls. These are shown in Fig. 6.

• In the absence of any pull, effect of $\alpha_3$ on the sensitivity was clearly visible, especially for CCE events. With all 11 pull, not only does the $\delta_{CP}$ sensitivity for each value of $\alpha_3$ reduce, but the distinction between the sensitivities for different $\alpha_3$ values disappear.
in the region $\delta_{CP} \sim [-90^\circ, -20^\circ]$. For a detector with perfect resolutions, the no-decay case will have the most sensitivity even in the presence of all pulls in the region $\delta_{CP} \sim [-20^\circ, 180^\circ]$. There is a mildly significant separation between the no-decay and with decay cases in this $\delta_{CP}$ region. The trends are similar with finite detector resolutions.

- For CCMU events with 11 pulls, the effects for all 3 $\alpha_3$ values are similar until $\delta_{CP} \sim -40^\circ$. Unlike the CCE events the separation between the sensitivities with different $\alpha_3$ values can be seen well in $\delta_{CP} \sim [-40^\circ, 180^\circ]$ in the zoomed in version. The trend is similar for a finite resolution case.

When the sum is taken, the region between $[-14^\circ - 44^\circ]$ is excluded at 2$\sigma$. Also adding the $\chi^2$ contribution from CCMU events also restricts the region where $\alpha_3$ affects $\delta_{CP}$ sensitivity to $[44^\circ - 180^\circ]$. Here there is a reduction in sensitivity when $\alpha_3$ increases from 0 eV$^2$ to larger values, but there is no change in sensitivity while increasing $\alpha_3$ from $4.36 \times 10^{-6}$ to $1 \times 10^{-5}$ eV$^2$.

But these differences are very small and will be very difficult to separately identify in a very realistic case. To understand which systematic uncertainty is driving the loss of sensitivity to $\delta_{CP}$ let us look at $\delta_{CP} \chi^2$ for perfect resolution cases. The uncertainties - those in tilt, flux ratio and cross section are switched on one each at a time. The results are shown in Fig[8]. From the figure it can be seen that the flux and cross section uncertainties alone can result in the reduction of $\chi^2$ to about half of the no pull values for all three $\alpha_3$ values. Out of flux and cross section, the cross section has more effect than the flux uncertainty on CCE events. When both these uncertainties are combined we lose a significant amount of sensitivity as seen from the 11 pulls case in Fig[6]. Hence it is important that, we measure the neutrino fluxes and cross sections precisely.
V. RESULTS-REALISTIC CASE

The results for the realistic cases are discussed in this section. Here the effect of fluctuations are taken into account and the detector cannot separate between neutrinos and anti-neutrinos. Since there is no $\nu - \bar{\nu}$ separation, only 5 pulls are there - those on flux (20%), cross section (10%), tilt (5%), overall (5%) and zenith angle (5%) uncertainties. For the 0.1–2.0 GeV energy range, all values of $\delta_{CP}$ are allowed at 2$\sigma$. The left panel of Fig. 9 shows this result. When all five uncertainties are present and their values are large, all values of $\delta_{CP}$ are allowed at 2$\sigma$ for all values of $\alpha_3$. Also all the large uncertainties wash away the effect of decay and there is no way to distinguish if decay has any effect on the $\delta_{CP}$ sensitivity (except in the range [−180°,−90°] where the sensitivity to $\delta_{CP}$ is higher for the no decay case compared to the other two $\alpha_3$ values; but all of these are below 1$\sigma$ and hence are not very significant). The right panel of Fig. 9 shows the sensitivity when there are lesser and smaller uncertainties. Here, only 3 uncertainties are considered - 5% in cross section, 5% overall uncertainty and 5% tilt. Not only do the the sensitivities increase with smaller and fewer uncertainties, but the effect of $\alpha_3$ also becomes clearer between the no decay and the with decay cases. While the sensitivities of the with-decay cases are similar, the reduction in sensitivity with increase of $\alpha_3$ from a no decay to with decay is visible here, although it is small. For $\alpha_3 = 0$ eV$^2$ ($\alpha_3 = 4.36 \times 10^{-6}$ eV$^2$) the $\delta_{CP}$ region $\sim[−8°,73°]$ ($\sim[14.5°,51°]$) is ruled out at 2$\sigma$. All values of $\delta_{CP}$ are allowed at 2$\sigma$ for $\alpha_3 = 1 \times 10^{-5}$ eV$^2$.

Thus, in presence of uncertainties, finite resolutions and fluctuations, the effect of invisible decay of $\nu_3$ on $\delta_{CP}$ measurement is washed out. If we have to identify this effect, there should be a precise measurement of fluxes and cross sections as mentioned in Section IV B. It can also be seen that though the contribution of CCMU events itself is very small, adding it to the CCE $\chi^2$ can improve the sensitivities slightly. Since every event is valuable in low counting experiments, it is worthwhile keeping these events in the analysis.
FIG. 8: Comparison of sensitivity $\chi^2$ with $\delta_{CP}^{sel} = -90^\circ$ and true NH for CCE and CCMU events with pulls switched on one by one. Y-axes are not the same.

VI. SUMMARY AND CONCLUSIONS

Low energy (sub GeV) atmospheric neutrino oscillations are very interesting and can help us understand the neutrino oscillation parameters \cite{19,51,69} and new physics scenarios like invisible neutrino decay. The effect of invisible decay of $\nu_3$, which is a new physics scenario, on the measurement of $\delta_{CP}$, which is a standard neutrino oscillation parameter, using atmospheric neutrinos in the energy ranges 0.1–2.0 GeV and 0.1–30.0 GeV are studied for idealistic and realistic cases. In the absence of systematic uncertainties and with a detector having perfect resolutions the effect of $\alpha_3$ is identifiable. The major contribution to the sensitivity $\chi^2$ is from the energy range 0.1–2.0 GeV for both CCE and CCMU events. CCE events contribute more to $\delta_{CP}$ sensitivity. Sensitivity decreases (increases) with increase (de-
crease) in $\alpha_3$ for CCE (CCMU). Presence of systematic uncertainties reduce the sensitivities drastically - flux and cross section uncertainties are mainly responsible for this reduction. In the realistic case, any effect of invisible decay is washed out and the sensitivity is practically the same for all values $\alpha_3$ if there are large and more uncertainties. For smaller and fewer uncertainties the sensitivity improves and the effect of invisible decay is also discernible to a certain extend. The main uncertainties which affect the sensitivity are again those in flux and cross sections. Finite detector resolutions and fluctuations also contribute to the worsening of the sensitivity. Hence the limitations in the detector resolution and systematic uncertainties can result in the non-identification of the effect of invisible decay if $\nu_3$ indeed decays in nature. i.e, even if decay can affect the sensitivity to $\delta_{CP}$, with a detector without a high energy resolution and uncertainties in fluxes and cross sections we will not be able to identify that effect at all. This means that we need detectors with better energy resolutions and especially for atmospheric neutrinos where the fluxes cannot be controlled, a precise measurement of the neutrino–anti-neutrino fluxes [? ]. CCMU events get more affected by $\alpha_3$ than CCE events, especially in the very low energy bins. From the oscillograms in Fig.2 this is clearly visible at energies between $\sim 0.1–0.2$ GeV. To probe these energies, a detector with a very fine energy resolution is required. Also the separation of other events which can act as a background to the CCMU events in this extremely low energy range should also be possible. This study is beyond the scope of this paper and has to be done in a detailed manner elsewhere. In conclusion, invisible decay of $\nu_3$, if it exists in nature will have an effect on $\delta_{CP}$ measurement using atmospheric neutrinos. But this can be measured perfectly only in a very idealistic scenario or atleas in a case where we have good resolutions and lesser and fewer systematic uncertainties.
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