Quantum Hall Stripe States in a Tilted Magnetic Field

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Abstract. A strong anisotropy in the longitudinal resistivity of a 2D electron system has been observed at half-filled high Landau levels. We report on detailed Hartree-Fock calculations of the unidirectional charge density wave (UCDW) orientation energy induced by a tilted magnetic field. We find that stripes can orient both parallel or perpendicular to the in-plane field depending on the sample parameters and field strength. The close agreement between complex experimental data on different sample geometries and our theoretical results strongly support the UCDW picture as the origin of the observed anisotropies in high Landau levels.

1 Introduction

Recently, a strongly anisotropic transport has been observed in high quality GaAs/AlxGa1−xAs single heterostructures at filling factors ν = 9/2, 11/2, etc., and in 2D hole systems starting at ν = 5/2. The origin of the magnetotransport anisotropy has not been firmly established yet. The most appealing interpretation suggests that the 2D electron gas spontaneously breaks the translational symmetry by forming a unidirectional charge density wave, as predicted by Hartree-Fock theory. Because of uncertainty about the reliability of this Hartree-Fock prediction, a special emphasis is placed on tests of its ability to explain experimental results on stripe orientation in tilted magnetic fields.

Many-body RPA/Hartree-Fock calculations using a model, parabolic quantum well system showed that the stripe orientation relative to the in-plane field is not universal. In narrow quantum wells, stripes are expected to orient perpendicular to the in-plane field. For wider quantum wells, with two occupied subbands at zero tilt angle, a more complex behavior is predicted. Isotropic 2D electron systems with half-filled lowest Landau level (LL) of the second subband can undergo a transition into a UCDW state due to level (anti)crossings induced by the in-plane magnetic field. The orientation of these stripe states can change from parallel to perpendicular as the strength of the in-plane field increases.

In this paper we present calculations of the field-tilt UCDW anisotropy energy that include a self-consistent local-spin-density-approximation (LSDA) description of one-particle states in experimental sample geometries. Our results are compared to magnetotransport experiments on GaAs/AlxGa1−xAs single heterostructures with different 2D electron gas densities and on a square quantum well sample with two occupied electric subbands.

2 Theoretical model and results

The energy per electron of the UCDW state at fractional filling ν∗ of the valence LL is given by

\[ E = \frac{1}{2\nu^*} \sum_{n=-\infty}^{\infty} \Delta_n^2 U \left( \frac{2\pi n \hat{e}}{a} \right) \], \[ \Delta_n = \nu^* \sin\left( (n\nu^* \pi) / n\nu^* \pi \right) \],

where \( a \) is the period of the UCDW state, \( \hat{e} \) is the direction of charge variation, and \( \Delta_n \) is the Fourier transform of the guiding center occupation function at wave vector \( n2\pi/a \). In Hartree-Fock theory, the UCDW state energy depends only on \( a \) and \( \hat{e} \) and the optimal UCDW is obtained by minimizing Eq.(1) with respect to these parameters. In Eq.(1), \( U(q) \) can be separated into direct, \( H(q) \), and exchange, \( X(q) \), contributions with

\[ H(q) = \frac{1}{2\pi^2} V(q) \]

\[ X(q) = - \int \frac{d^2p}{(2\pi)^2} e^{i(p_x q_x - p_y q_y)\ell^2} V(p) \],

where \( V(q) \) is the RPA-screened effective Coulomb interaction.

To evaluate \( V(q) \) we choose the in-plane component \( B_{ip} \) of the magnetic field to be in the \( \hat{z} \)-direction and use the following Landau gauge for the vector potential, \( A = (0, B_{ip} z, 0) \). The one-particle orbitals are calculated using the self-consistent LSDA method and for any sample geometry, with the \( z \)-dependent confining potential, can be written as \( e^{iky}/\sqrt{\ell_y} \varphi_i(x - \ell^2 k, z) \), where \( k \) is the wave vector which labels states within LL \( i \), \( \sigma \) is the spin index, and \( \ell^2 = \hbar c/eB_{ip} \). The translational symmetry responsible for LL degeneracy leads to a 2D wavefunction \( \varphi_i(x, z) \) which is independent of the state label \( k \), except for the rigid shift by \( \ell^2 k \) along \( x \)-axis. This in turn leads to two-particle matrix elements of the Coulomb interactions with a dependence on state labels which is identical to that for the lowest LL of a zero-thickness 2D electron system provided the 2D Coulomb interaction is replaced by the effective interaction \( V(q) \).

For an infinitely narrow electron layer the orbital effects of \( B_{ip} \) vanish and the 2D Coulomb interaction, \( V(q) \), reduces to \( e^{-q^2\ell^2/2}/q \left( L_N(q^2\ell^2/2) \right)^2 2\pi e^2/\epsilon \) where \( L_N(x) \) is the Laguerre polynomial and \( N \) is the orbit radius quantum number. Starting from \( N = 1 \), zeros of \( L_N(q^2\ell^2/2) \) occur at finite \( q = q^* \), producing a zero in the repulsive Hartree interaction at wave vectors where
the attractive exchange interaction is strong. For the half-filled valence LL the corresponding UCDW state consists of alternating occupied and empty stripes of electron guiding center states with a modulation period \( a \approx 2\pi/q^* \).

In finite-thickness 2D systems the dependence of the effective interaction on wavevector magnitude \( q \) and orientation \( \phi \) relative to the in-plane field direction can be accurately approximated \( V(q) \) by \( V(q) = V_0(q) + V_2(q) \cos(2\phi) \). At weak \( B_{ip} \), the isotropic term \( V_0(q) \) has a wavevector-dependence similar to that of the effective interaction in the infinitely narrow 2D layer. The corresponding curve for the valence LL at \( \nu = 9/2 \) for a single heterojunction with 2D density \( n_e = 2.67 \times 10^{11} \text{ cm}^{-2} \) is shown in the lower inset of Figure 1.

The non-zero anisotropy coefficient \( V_2(q) \) of the effective interaction at \( B_{ip} > 0 \), also shown in the inset, is responsible for the non-zero UCDW anisotropy energy \( E_A \), defined \( E_A \) as the total Hartree-Fock energy of stripes oriented parallel with \( B_{ip} \) minus the total energy of stripes perpendicular to \( B_{ip} \). The main plot of Figure 1 shows \( E_A \) calculated for single heterojunction samples. The two curves correspond to high and low density 2D electron systems with one occupied electric subband in perpendicular magnetic field. The subband wavefunctions are plotted in the upper inset. We found that stripes are oriented perpendicular to the in-plane field for the considered interval of densities \( n_e = 1.5 - 2.67 \times 10^{11} \text{ cm}^{-2} \), consistent with available experimental data on single heterojunctions \( \text{[4,5,10]} \). Similar agreement between theory and experiment is also obtained for other half-filled high LLs.

In Figure 2 we present results for a 350 Å wide GaAs quantum well with \( n_e = 4.6 \times 10^{11} \text{ cm}^{-2} \) \( \text{[1]} \). At \( \nu = 9/2 \) and 11/2, and \( B_{ip} = 0 \), the valence LL is the \( N = 0 \) LL of the second subband. The effective interaction \( V_0(q) \) has no zeros at finite wavevector, as seen in the inset of Figure 2, and the UCDW state is not expected to form, consistent with experiment \( \text{[1]} \). An in-plane field \( B_{ip} \sim 3 \text{ T} \) induces a level anticrossing accompanied by a development of a minimum in \( V_0(q) \) at finite \( q \). As discussed above, it is the minimum of the interaction energy at finite wavevector that opens the possibility for the formation of the UCDW state. The theoretical and experimental \( \text{[1]} \) in-plane fields corresponding to the onset of the UCDW and transport anisotropy, respectively, are remarkably close.

The main graph in Figure 2 indicates that, in this sample, the sign of \( E_A \) changes at \( B_{ip} \sim 10 \text{ T} \). The stripes orient parallel with \( B_{ip} \) at low fields while perpendicular orientation is obtained in the high in-plane field region, consistent with the experimentally observed interchange of easy and hard current axes at similar \( B_{ip} \).

We conclude that the close agreement between complex experimental data and theoretical results for all studied samples leave little doubt as to the UCDW origin of the observed transport anisotropies.

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