A New Improved Algorithm for Aeromagnetic Compensation

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Abstract. Magnetic compensation is a necessary step in aeromagnetic data processing. The aeromagnetic compensation model is a linear regression model, but the model has multiple collinearity problems, which will reduce the performance of the compensation model. In view of this problem, we propose a deep autoencoder (DAE) aeromagnetic compensation algorithm. The DAE network extracts the features of the data by learning the compressed representation of the coefficient matrix, thereby weakening the correlation between the coefficient matrix variables. The feature obtained after dimension reduction is used for compensation calculation. The DAE algorithm is verified by Unmanned Aerial Vehicles, and the results show that the compensation quality of the DAE is better than the least squares algorithm.

1. Introduction

Aeromagnetic exploration has been widely used in geological structure and mineral resources exploration [2]. During the aeromagnetic measurement, the data collected by the magnetic probe on the aircraft includes the magnetic interference generated during the maneuver of the aircraft [12]. Eliminating the magnetic interference from aircraft can improve the quality of aeromagnetic data, which is an important problem in aeromagnetic data processing.

The interference compensation model was first proposed by Tolles and Lawson (1950) [12], who established the mathematical equation of the magnetic interference (Tolles & Lawson (TL) equation). Leliak (1961) established a coefficient equation to compensate for magnetic interference based on TL equation [4]. Leach (1979) treated the magnetic compensation problem from the perspective of linear regression. Due to the correlation problem between model variables, he proposed a ridge regression algorithm to solve multicollinearity in the compensation model [5]. Wu (2018) proposed to use the principal component analysis to mitigate the multicollinearity of the model [16]. At present, most of the coefficient equation solving algorithms are proposed based on the linear regression theory of traditional mathematics [1].

Deep learning has developed rapidly, but has fewer applications in the aeromagnetic field. To date, Williams (1993) has employed a neural network for aeromagnetic compensation purposes [15]. We propose a deep autoencoder (DAE) algorithm to deal with the compensation model. The DAE is a special form of neural network. It compresses data through the specific structure of deep neural network to achieve data dimensionality reduction [9]. The DAE eliminates the correlations of the coefficient matrix while obtaining useful features, thus making the least squares (LS) algorithm more accurate. Compared with the LS algorithm, this improved method has better performance.
2. Algorithm

2.1. TL equation

Here, we briefly describe the TL equation. The aircraft magnetic interference model includes the following three components: permanent ($H_p$) magnetic fields generated by various parts of the aircraft, induction ($H_i$) effect created by the earth’s magnetic field and eddy-current ($H_{ec}$) magnetic fields produced by the aircraft’s maneuvers [6]. The total ($H_t$) magnetic interference field can be expressed as:

$$H_t = H_p + H_i + H_{ec}$$

$$= T[(c_1u_1/T + c_2u_2/T + c_3u_3/T) + (c_4u_1^2 + c_5u_1u_2 + c_6u_1u_3 + c_7u_2^2 + c_8u_2u_3 + c_9u_3^2) + (c_{10}u_1u_1' + c_{11}u_1u_2' + c_{12}u_1u_3' + c_{13}u_2u_1' + c_{14}u_2u_2' + c_{15}u_2u_3' + c_{16}u_3u_1' + c_{17}u_3u_2' + c_{18}u_3u_3')]$$

$$= c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5 + c_6x_6 + c_7x_7 + c_8x_8 + c_9x_9 + c_{10}x_{10} + c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{15}x_{15} + c_{16}x_{16} + c_{17}x_{17} + c_{18}x_{18}. \quad (1)$$

where $c_1, c_2, \cdots, c_{18}$ are the 18 compensation coefficients, $u_i$ is the derivative of the $u_i$, $T$ is the earth’s magnetic field. Leach (1979) proposed using the three-axis fluxgate magnetometer data ($T_x, T_y$, and $T_z$) to calculate the $u_i$ [7]:

$$u_1 = \frac{T_x}{T_t}, u_2 = \frac{T_y}{T_t}, u_3 = \frac{T_z}{T_t}. \quad (2)$$

$$T_t = \sqrt{T_x^2 + T_y^2 + T_z^2}. \quad (3)$$

The least squares solution of the TL equation is as follows:

$$XC = H_t, \quad (4)$$

$$C = (X^TX)^{-1}X^TH_t. \quad (5)$$

where $H_t$ and $C$ are column vectors consisting of $H_t$ and $c_i$ ($i = 1, 2, \cdots, 18$) and $X$ is a matrix that can be written as follows:

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,18} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,18} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,18} \end{bmatrix}$$

$$= \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,18} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,18} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,18} \end{bmatrix}. \quad (6)$$

When we get $C$, bring it back to Equation (4), and get the calculated value of interference $\tilde{H_t}$. The compensated magnetic interference ($H_{t,c}$) is calculated as follows:

$$H_{t,c} = H_t - \tilde{H_t} = H_t - XC. \quad (7)$$

There are multicollinearities between the vectors $x_{n,1}$, $x_{n,2}$, $\cdots$, $x_{n,18}$, which are the column vectors of the matrix $X$. If there is exact collinearity between the independent variables. The rank of the matrix $X$ is less than $k + 1$ ($k$ is the number of independent variables), which means that $|X^TX| = 0$ (i.e., $(X^TX)^{-1}$ does not exist). This will result the inability of the LS method; If there is a large degree of approximate collinearity between the independent variables. Although the LS method parameter estimator can be obtained, the expression of the parameter estimator variance is $\text{Cov}(\beta) = \sigma^2 (X^TX)^{-1}$. Since $|X^TX| \approx 0$, the elements on the main diagonal of $X^TX$ are larger, so that the variance of the parameter estimates increases, and the least squares parameter estimator is not effective.

Therefore, we propose a feature extraction algorithm based on deep autoencoder to solve the problem of multicollinearity.

2.2. Deep autoencoder

A deep autoencoder (DAE) is a stack of multiple autoencoders. The structure of an autoencoder includes input layer, hidden layer, and output layer. The deep autoencoder includes multiple hidden layers [12, 16]. The DAE contains two parts: encoding and decoding (Fig. 1). There are certain characteristics of the input data that can represent the entire data set. Coding means that the number of nodes in the hidden layer is less than the number of nodes in the input layer, which forces the network to learn the compressed representation of the input samples (i.e., get the characteristics of the input data). Decoding
means reconstructing the input data set of through the acquired features and the network (i.e., obtaining a structure similar to the input data) [17]. What we need is the features we get from the encoding.

![Deep Autoencoder Diagram](image)

**Figure 1.** The structure of the deep autoencoder.

The activation function $f(\cdot)$ for encoding and decoding in the network uses the sigmoid function in Logistic Regression:

$$f(z) = \frac{1}{1 + e^{-z}},$$

The loss function uses mean square error (MSE):

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x}_i)^2,$$

For a given data set $X_{n \times 18}$, its overall cost function ($J(W, b)$) is:

$$J(W, b) = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{1}{2} \|x_i - \bar{x}_i\|^2 + \frac{\lambda}{2} \sum_{l=1}^{L} \sum_{i=1}^{s_l} \sum_{j=1}^{s_{l+1}} (W_{ij}^{(l)})^2 \right).$$

The first term in $J(W, b)$ is the mean squared term. The second term is a regularization term, which aims to reduce the magnitude of the weights and prevent overfitting. Where $L$ is the number of layers of the neural network. This model has five layers, and each layer is 18, 16, 14, 16, 18 respectively. $s_l$ is the number of nodes in the first layer (excluding $b$). $W_{ij}^{(l)}$ is the weight between the $j$-th unit of the $l$-th layer and the $i$-th unit of the $l+1$-th layer. The initial assignment of $W_{ij}^{(l)}$ is a random value close to zero (i.e., the random value is generated using the normal distribution ($Normal(0, \epsilon^2)$, $\epsilon$ is 0.01), and calculate the value of $W_{ij}^{(l)}$ by the conjugate gradient descent. First, perform feedforward conduct-tion calculation to get the activation value of each layer. Then, the trainscg function is used in the training process to perform gradient descent error backpropagation. By repeating the iterative steps of the conjugate gradient method to reduce the value of the cost function $J(W, b)$. Finally, the weights of each part of the neural network is calculated, and the output terms is obtained.

Actually, the DAE can get a low-dimensional feature of input data similar to the principal component analysis results [9,10,17]. We use the feature matrix ($F_{DAE} = [a_i^{(3)}] = [f \left( \Sigma_{l} W_{ij}^{(l)} x_i + b^2 \right)]$) obtained by encoding calculation between the second and third layers of the coefficient matrix.

Hardwick (1984) suggested using the standard deviation (STD) and improvement ratio (IR) of the signal to evaluate data quality improvement. The STD and IR defined as follows [3]:

$$\text{STD} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2},$$

$$\text{IR} = \frac{\text{STD}_{\text{old}} - \text{STD}_{\text{new}}}{\text{STD}_{\text{old}}},$$

where $\mu$ is the mean of the data set.
\[ IR = \frac{STD_u}{STD_c} \]  \hspace{1cm} (12)

where, \( \mu \) is the arithmetic mean of the variable, \( STD_u \) is the standard deviation of the uncompensated data, \( STD_c \) is the standard deviation of the compensated data.

A more accurate compensation scheme yields smaller the \( STD_c \) values and larger IR values. The smoother the curve of the data after compensation.

2.3. Result and discussion

We use an Unmanned Aerial Vehicles (UAV) to perform a ‘Figure-of-Merit (FOM) flight’ (Noriega 2011) at an experimental site in southern China. During the FOM flight, the UAV is equipped with a potassium pump magnetometer and a three-axis fluxgate magnetometer in order to measure the total geomagnetic field and the three components of the geomagnetic field, respectively. The fluxgate magnetometer data are shown in Figure 2. The potassium pump data are shown in Figure 3.

![Figure 2. Fluxgate magnetometer data recorded during the compensation flight](image)

![Figure 3. Potassium pump magnetometer data recorded during the compensation flight](image)

We examined the differences between the DAE and LS methods for the data collected during the FOM flight (Fig. 4). The DAE method reduced the STD value from 15.434 to 0.679, resulting in an IR value of 22.730. The LS technique reduced the STD value from 15.434 to 1.219, resulting in an IR value of 12.661. We have demonstrated that the IR value for the DAE algorithm is higher than that of the LS algorithm.

3. Conclusions

Most aeromagnetic compensation research is mostly based on linear regression algorithms. We introduce deep learning into the field of aeromagnetic compensation. For the problem of multicollinearity in the compensation model of aeromagnetic compensation, we propose a deep autoencoder to solve
the aeromagnetic compensation problem. The DAE weakens the collinearity in the model by obtaining the characteristics of the input data. The DAE-based least squares method can remove the magnetic interference from UAV platforms. The UAV flight experiments prove that the improved algorithm is effective.

![Figure 4. Compensation results for the DAE and LS methods applied to actual data recorded during a compensation flight.](image)

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