Spin alignment of vector mesons in unpolarized hadron-hadron collisions at high energies

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Abstract

We argue that spin alignment of the vector mesons observed in unpolarized hadron-hadron collisions is closely related to the single spin left-right asymmetry observed in transversely polarized hadron-hadron collisions. We present the numerical results obtained from the type of spin-correlation imposed by the existence of the single-spin left-right asymmetries. We compare the results with the available data and make predictions for future experiments.

13.88.+e, 13.85.Ni, 13.85.-t, 13.65.+i
1. Introduction

It has been observed\(^1\) for a long time that hyperons with moderately large transverse momenta in unpolarized hadron-hadron collisions are polarized transversely to the production plane. Data for different hyperons at different energies in reactions using different projectiles and/or targets have been available\(^2\). Different efforts\(^3-5\) have been made to understand them. Progress has been achieved in last years, but the role of spin in the production dynamics is still not well understood. It should be interesting and helpful to see whether similar effects exist for the production of other kinds of hadrons. In this connection, we note the measurements on the polarization of vector mesons along the normal of the production plane in unpolarized hadron-hadron or hadron-nucleus collisions\(^6-8\), especially those of \(K^*+\) in \(K^+p\) and neutron-carbon collisions\(^7,8\). The obtained results show an obvious “spin alignment” for \(K^*+\). Clearly, the study of them may shed new light on the searching of the origin of the hyperon polarization in unpolarized hadron-hadron collisions.

In this note, we study the spin alignment of vector mesons in unpolarized hadron-hadron collisions by relating it to the single spin left-right asymmetries \((A_N)\) observed in transversely polarized hadron-hadron collisions\(^9\). We argue that these two effects are closely related to each other and make calculations for the spin alignment of vector mesons using the same method as that used in \(e^+e^-\) annihilation at \(Z^0\) pole\(^10\). We compare the results obtained with the available data\(^6-8\) and make predictions for future experiments.

2. Qualitative arguments

A. Spin alignment of a vector meson in the fragmentation of a polarized quark

The polarization of a vector meson is described by the spin density matrix \(\rho\) or its element \(\rho_{mm'}\), where \(m\) and \(m'\) label the spin component along the quantization axis. The diagonal elements \(\rho_{11}, \rho_{00}\) and \(\rho_{-1-1}\) for the unit-trace matrix \(\rho\) are the relative intensities for \(m\) to take the values 1, 0, and \(-1\) respectively. In experiment, \(\rho_{00}\) can be measured from the
angular distribution of the decay products of the vector mesons. If the meson is unpolarized, we have \( \rho_{00} = \rho_{11} = \rho_{-1-1} = 1/3 \). Hence, a deviation of \( \rho_{00} \) from 1/3 indicates that the spin of the vector meson has a larger (if \( \rho_{00} < 1/3 \), or smaller if \( \rho_{00} > 1/3 \)) probability to be parallel or anti-parallel to the quantization direction. Such a phenomenon is referred as spin alignment of the vector meson along the quantization axis.

Recently, high accuracy data on \( \rho_{00} \) have been published for different vector mesons in \( e^+e^- \) annihilation at \( Z^0 \) pole\(^{11} \). The results show a clear spin alignment for the vector mesons along the moving direction of the produced quark in particular in the large momentum fraction region. The \( \rho_{00} \) measured in the helicity frame is, e.g., larger than 0.5 for \( z > 0.5 \), where \( z \equiv 2p_V/\sqrt{s} \), \( p_V \) is the momentum of the vector meson, \( \sqrt{s} \) is the total \( e^+e^- \) center of mass (c.m.) energy. We note that the vector mesons with large \( z \) are predominately fragmentation results of the initial quarks, and according to the Standard Model for electroweak interactions the initial quarks produced at \( e^+e^- \) annihilation vertex at \( Z^0 \) pole are longitudinally polarized.\(^12 \) These data\(^{11} \) lead us to the following conclusion: In the longitudinally polarized case, the vector mesons produced in the fragmentation of a polarized quark have a significant spin alignment with the fragmenting quark. The resulting \( \rho_{00} \) for the vector mesons is significantly larger than 1/3 in the frame where the polarization direction of the fragmenting quark is chosen as the quantization axis.

The above-mentioned conclusion is derived from the \( e^+e^- \) annihilation data\(^{11} \), and applies to the fragmentation of a longitudinally polarized quark. If we now extend it to the fragmentation of a quark polarized in any direction, we obtain the following general conclusion: In the fragmentation of a polarized quark, the vector mesons produced have a significant spin alignment with the fragmenting quark. The resulting \( \rho_{00} \) for the vector meson is significantly larger than 1/3 in the frame where the polarization direction of the fragmenting quark is chosen as the quantization axis.

We should emphasize that the extension of the above-mentioned conclusion from the longitudinally polarized case to the transversely polarized case is not obvious. This can be seen from the difference between the helicity distribution of the quarks in longitudinally...
polarized nucleon and the transversity distribution\textsuperscript{13}. Because of the relativistic effects, the magnitudes and/or shapes (i.e. the \(x\)-dependences, where \(x\) is the fractional momentum carried by the quark in the nucleon) of them are in general different from each other. But it seems that the qualitative features in particular the signs of them are the same, especially in the large \(x\) region\textsuperscript{13,14}. The relativistic effects do not change the signs of them. Similarly, we expect that the magnitude and the \(z\)-dependence of \(\rho_{00}\) for the vector meson obtained in the fragmentation of a longitudinally polarized quark can be different from those in the fragmentation of a transversely polarized quark, even if the quantization axis is chosen as the polarization direction in each case. (Here, \(z\) is the fractional momentum carried by the vector meson.) But, we also expect that the qualitative features in particular whether \(\rho_{00}\) is larger or smaller than \(1/3\) should be the same. The differences in the two cases should be reflected in the calculations of \(\rho_{00}\) as a function of \(z\) and we will come back to this point in next section when formulating the calculation method. Since we are, at the present stage, more interested in the qualitative features, we will assume the conclusion is in general true and use it as a starting point for the following discussions.

\section*{B. Spin alignment of the vector meson in unpolarized and left-right asymmetry in singly polarized hadron-hadron collisions}

Having the conclusion obtained above in mind, we now ask what we can obtain from the data\textsuperscript{7,8} mentioned in the introduction for the spin alignment of \(K^{*+}\) in unpolarized \(K^+p\) and neutron-carbon collisions. These experiments show in particular that\textsuperscript{8}, \(\rho_{00} = 0.424 \pm 0.011 \pm 0.018\) for \(K^{*+}\) with \(p_V > 12\text{GeV}\) produced in unpolarized neutron-carbon collisions with \(p_{inc} = 60\text{GeV}\). We see that the \(\rho_{00}\) obtained for the vector meson \(K^{*+}\) is significantly larger than \(1/3\) if the normal of the production plane is chosen as the quantization axis. Together with the conclusion obtained above, this data leads immediately to the following statement: There should be a significant polarization of the quark which contributes to the vector meson production and the polarization is transverse to the production plane. In other
words, there exists a spin-correlation of the type $\vec{s}_q \cdot \vec{n}$ in the reaction. [Here, $\vec{s}_q$ is the spin of the fragmenting quark; $\vec{n} \equiv (\vec{p}_{\text{inc}} \times \vec{p}_V)/|\vec{p}_{\text{inc}} \times \vec{p}_V|$ is the unit vector in the normal direction of the production plane, $\vec{p}_{\text{inc}}$ and $\vec{p}_V$ are respectively the momentum of the incident hadron and that of the produced hadron.] The spin alignment of the vector meson in unpolarized hadron-hadron or hadron-nucleus collisions is one of the manifestations of the $\vec{s}_q \cdot \vec{n}$ type of spin correlation in the reaction.

Now, we recall that, in another class of hadron-hadron collision experiments where a transversely polarized beam is used, a significant single-spin left-right asymmetry $A_N$ has been observed\textsuperscript{9}. The existence of the non-zero $A_N$ implies a significant spin correlation of the form $\vec{s}_P \cdot \vec{n}$ in such processes, where $\vec{s}_P$ is the spin of the projectile. We recall that this effect exists mainly in the fragmentation region and hadrons in this region are predominately the fragmentation results of the valence quarks of the incident hadron. Having in mind that the valence quarks are in general also transversely polarized if the beam hadron is transversely polarized, we reach the conclusion that the kind of spin-correlation indicated by the existence of $A_N$ is very similar to that inferred from the existence of spin alignment of vector mesons in unpolarized hadron-hadron collisions. Hence, we suggest that they are two different manifestations of the same spin-correlation of the form $\vec{s}_q \cdot \vec{n}$ in hadron-hadron or hadron-nucleus collisions. This is similar to [4] where it is suggested that hyperon polarization in unpolarized hadron-hadron collisions has the same origin as the single-spin left-right asymmetry $A_N$. Both of them are also manifestations of the $\vec{s}_q \cdot \vec{n}$ type of spin correlation in hadron-hadron or hadron-nucleus collisions.

3. A quantitative calculation

Since the purpose of this paper is to demonstrate the close relation between the existence of $A_N$ and that of spin alignment in unpolarized hadron-hadron collisions in an explicit and possibly quantitative manner, we follow the same way as that adopted in [4] and do the following calculations. We use the results derived from the $A_N$ data\textsuperscript{9} for the strength of the
above-mentioned \( \vec{z}_q \cdot \vec{n} \) type of spin-correlation as input to calculate the polarization of the valence-quark before the hadronization. Then, to calculate \( \rho_{00}^V \) in unpolarized hadron-hadron collisions, we need only to calculate \( \rho_{00}^V \) in the hadronization of a transversely polarized quark. We do the calculation using the same method as that used in calculating \( \rho_{00}^V \) in \( e^+e^- \) annihilation at \( Z^0 \) pole where \( \rho_{00}^V \) is calculated in the hadronization of a longitudinally polarized quark. The details of each step is given in the following.

A. Calculating \( \rho_{00} \) in \( e^+e^- \to Z^0 \to VX \)

The method for calculating \( \rho_{00}^V \) in \( e^+e^- \) annihilations at \( Z^0 \) pole is given in Ref.[10] and can be summarized as follows: To do the calculations, we need to divide the final state vector mesons into the following two groups and consider them separately: (A) those which are directly produced and contain the polarized fragmenting quark; (B) the rest. Then, \( \rho_{00}^V \) is given by\(^{10} \)

\[
\rho_{00}^V(z) = \frac{\rho_{00}^V(A)N_V(z, A) + \rho_{00}^V(B)N_V(z, B)}{N_V(z, A) + N_V(z, B)},
\]

(1)

where \( N_V(z, A) \) and \( N_V(z, B) \) are the number densities of vector mesons of group (A) and (B) respectively, \( \rho_{00}^V(A) \) and \( \rho_{00}^V(B) \) are their corresponding \( \rho_{00}^V \)'s, and \( z \) is the momentum fraction carried by the vector meson.

The vector mesons in group (B) do not contain the polarized fragmenting quarks and there are in general many different possibilities to produce them in a collision process. Hence, they are simply taken as unpolarized\(^{10} \), i.e. \( \rho_{00}^V(B) = 1/3 \). For those in group (A), the spin density matrix \( \rho^V(A) \) is obtained\(^{10} \) from the direct product of the spin density matrix of the fragmenting quark, \( \rho^q \), and that of the anti-quark, \( \rho^\bar{q} \), which combines with the fragmenting quark to form the vector meson. Taking the most general form for \( \rho^\bar{q} \), we obtained that\(^{10} \), if the polarization direction of the fragmenting quark is chosen as the quantization axis, \( \rho_{00}^V(A) \) is given by,

\[
\rho_{00}^V(A) = (1 - P_qP_z)/(3 + P_qP_z),
\]

(2)
where $P_q$ is the polarization of the fragmenting quark, and the polarization direction is taken as $z$-direction; $P_z$ is the $z$-component of the polarization of the anti-quark. By comparing the results with the $e^+e^-$ annihilation data\(^{11}\), we see that\(^{10}\), to fit the data, we have to take the anti-quark as polarized in the opposite direction as the fragmenting quark, and the polarization $P_z$ is proportional to $P_q$, i.e.,

$$P_z = -\alpha P_q,$$

where $\alpha \approx 0.5$ is a constant. Inserting Eq.(3) into Eq.(2), we obtain that, $\rho_{00}^V(A) = 1/3 + \Delta \rho_{00}^V(A)$, and

$$\Delta \rho_{00}^V(A) = 4\alpha P_q^2/[3(3 - \alpha P_q^2)].$$

Using this, we can fit\(^{10}\) the $e^+e^-$ annihilation data reasonably. We note that two assumptions are used in these calculations: First, the spin of the vector meson is taken as the sum of the spin of the fragmenting quark and that of the anti-quark which combines with the fragmenting quark to form the meson and $\alpha$ is a constant. Second, the influence from the polarization of the fragmenting quark on the polarization of the vector mesons which are produced in the fragmentation but do not contain the fragmenting quark are neglected, thus $\rho_{00}^V(B)$ is taken as $1/3$. It should be emphasized both of them should be tested and the situations can be different for longitudinally or transversely polarized case. In other words, these are the places where the differences between $\rho_{00}^V$ in the longitudinally polarized case and that in the transversely polarized case can come in. Because of the influences of the non-perturbative effects, we cannot derive them from QCD at the moment. We therefore applied the calculation method to calculate $\rho_{00}^V$ not only in $e^+e^-$ annihilations but also in deeply inelastic lepton-nucleon scattering and high $p_T$ jets in polarized $pp$ collisions\(^{15}\) for both longitudinally and transversely polarized cases. The results can be used to check whether these assumptions are true. Since we are now mostly interested in the qualitative behavior of the $\rho_{00}^V$ in unpolarized hadron-hadron collisions, the two assumptions seem to be quite safe and we will just use the method in the following.
B. Calculating $\rho^V_{00}$ in unpolarized hadron-hadron collisions

According to the method described above, to calculate $\rho^V_{00}$ in unpolarized hadron-hadron collisions at moderately large transverse momentum, we need to find out the number densities for those from group (A) and (B) respectively. For this purpose, we follow the same way as that in Ref.[16] where single-spin left-right asymmetries $A_N$ are studied. In Ref.[16], the hadrons produced in the moderately large transverse momentum region are divided into the direct-formation part and the non-direct-formation part. The number density of the former is denoted by $D(x_F, V)$ and that of the latter by $N_0(x_F, V)$. (Here, $x_F = 2p_{V||}/\sqrt{s}$, $p_{V||}$ is the component of the momentum of $V$ parallel to the beam direction, and $\sqrt{s}$ is the c.m. energy of the incoming hadron system.) The direct-formation part denotes those directly produced and contain the fragmenting valence quarks. They are described by the “direct-formation” or “direct fusion” process, $q_v + \bar{q} \rightarrow M$, in which the valence quark $q_v$ picks up an anti-quark $\bar{q}$ to form the meson $M$ observed in experiments. Obviously, this part just corresponds to the vector mesons of group (A) mentioned above. The non-direct-formation part denotes the rest which is just the mesons of group (B). More precisely, we have, $N_V(z, A) \leftrightarrow D(x_F, V)$, $N_V(z, B) \leftrightarrow N_0(x_F, V)$. Hence, the $\rho^V_{00}(x_F)$ in the unpolarized hadron-hadron collisions is given by,

$$\rho^V_{00}(x_F) = \frac{1}{3} + \Delta \rho^V_{00}(A) \frac{D(x_F, V)}{D(x_F, V) + N_0(x_F, V)},$$

(5)

The quantization axis is now taken as the normal direction of the production plane of the vector meson, and $P_q$ is the polarization of the valence quark of the projectile with respect to this axis before the hadronization or the direct-formation takes place. We see from Eq.(5) that, $\rho^V_{00}(x_F)$ can be calculated if we know $P_q$ and the ratio $D(x_F, V)/N_0(x_F, V)$.

1. Determination of $P_q$ from the $\vec{s}_q \cdot \vec{n}$ spin correlation

The polarization $P_q$ of the quark before the direct formation of the mesons is determined by the strength of the $\vec{s}_q \cdot \vec{n}$ type of spin-correlation in the process. Assuming the same origin
for the single spin left-right asymmetry in transversely polarized hadron-hadron collisions and the spin alignment of vector mesons in unpolarized hadron-hadron collisions, we can extract $P_q$ from the experimental results$^9$ for $A_N$. This is exactly the same as what we did in Ref.[4] and now described in detail in the following.

We recall that$^9, 16$, in the language commonly used in describing $A_N$, the polarization direction of the incident proton is called upward, and the incident direction is forward. The single-spin asymmetry $A_N$ is just the difference between the cross section where $\vec{p}_V$ points to the left and that to the right, which corresponds to $\vec{s}_q \cdot \vec{n} = 1/2$ and $\vec{s}_q \cdot \vec{n} = -1/2$ respectively. The data$^9$ on $A_N$ show that if a hadron is produced by an upward polarized valence quark of the projectile, it has a large probability to have a transverse momentum pointing to the left. $A_N$ measures the excess of hadrons produced to the left over those produced to the right. The difference of the probability for the hadron to go left and that to go right is denoted$^{16}$ by $C$. $C$ is a constant in the region of $0 < C < 1$. It has been shown that$^{16}$, to fit the $A_N$ data$^9$, $C$ should be taken as, $C = 0.6$.

Now, in terms of the spin-correlation discussed above, the cross section should be expressed as,

$$\sigma = \sigma_0 + (\vec{s}_q \cdot \vec{n})\sigma_1$$

(6)

where $\sigma_0$ and $\sigma_1$ are independent of $\vec{s}_q$. The second term just denotes the existence of the $\vec{s}_q \cdot \vec{n}$ type of spin-correlation. $C$ is just the difference between the cross section where $\vec{s}_q \cdot \vec{n} = 1/2$ and that where $\vec{s}_q \cdot \vec{n} = -1/2$ divided by the sum of them, i.e., $C = \sigma_1/(2\sigma_0)$.

Now, we assume the same strength for the spin-correlation in vector meson production in the same collisions and use Eq.(6) to determine $P_q$. For a vector meson produced with momentum $\vec{p}_V$, $\vec{n}$ is given. The cross-section that this vector meson is produced in the fragmentation of a valence quark with spin satisfying $\vec{s}_q \cdot \vec{n} = 1/2$ is $(\sigma_0 + \sigma_1/2)$, and that with spin satisfying $\vec{s}_q \cdot \vec{n} = -1/2$ is $(\sigma_0 - \sigma_1/2)$. Hence, the polarization of the valence quarks which lead to the production of the vector mesons with that $\vec{n}$ is given by,

$$P_q = \frac{(\sigma_0 + \sigma_1/2) - (\sigma_0 - \sigma_1/2)}{(\sigma_0 + \sigma_1/2) + (\sigma_0 - \sigma_1/2)} = \frac{\sigma_1}{2\sigma_0} = C.$$ 

(7)
It should be emphasized that, similar to hyperon polarization in unpolarized hadron-hadron collisions, \( P_q \neq 0 \) just means that the strength of the spin-correlation of the form \( \vec{s}_q \cdot \vec{n} \) is non-zero in the reaction. It means that, due to some spin-dependent interactions, the quarks which have spins along the same direction as the normal of the production plan have a larger probability to combine with suitable anti-quarks to form the mesons than those which have spins in the opposite direction. It does not imply that the quarks in the unpolarized incident hadrons were polarized in a given direction, which would contradict the general requirement of space rotation invariance. In fact, in an unpolarized reaction, the normal of the production plane of the mesons is uniformly distributed in the transverse directions. Hence, averaging over all the normal directions, the quarks are unpolarized.

2. Qualitative behavior of \( \rho^V_{00} \) as a function of \( x_F \)

Using the the numerical values \( P_q = C = 0.6 \) and \( \alpha \approx 0.5 \), we obtain \( \Delta \rho^V_{00}(A) \approx 0.085 \) from Eq.(4). We see clearly from Eq. (5) that, \( \rho^V_{00}(x_F) \) is larger than \( 1/3 \) as long as \( D(x_F, V) \) is nonzero. Since \( 0 \leq D(x_F, V)/[D(x_F, V) + N_0(x_F, V)] \leq 1 \), \( \rho^V_{00}(x_F) \) should be in the range of \( 1/3 \leq \rho^V_{00}(x_F) \leq 0.42 \). The detailed form of the \( x_F \)-dependence of \( \rho^V_{00}(x_F) \) is determined by that of the ratio \( D(x_F, V)/N_0(x_F, V) \).

To see the qualitative behavior of \( D(x_F, V)/N_0(x_F, V) \) as a function of \( x_F \), we recall that the valence quarks usually take the large momentum fractions of the incident hadrons. Hence, at moderately large transverse momenta, hadrons which contain the valence quarks of the incident hadrons dominate the large \( x_F \) region. It is therefore clear that, to discuss the behavior of \( \rho^V_{00}(x_F) \) as a function of \( x_F \), we should divide the vector mesons into the following two classes according to their flavor compositions: (1) those which have a valence quark of the same flavor as one of the valence quarks of the incident hadron; (2) those which have no valence quark in common with the incident hadron. The behavior of \( \rho^V_{00}(x_F) \) as a function of \( x_F \) for these two classes of vector mesons should be quite different from each other.
Example of class (1) are:

\[ p + A \rightarrow (\rho^+, \rho^- \text{ or } \rho^0) + X, \]  
(8)
\[ p + A \rightarrow (K^{*+} \text{ or } K^{*0}) + X, \]  
(9)
\[ K^+ + A \rightarrow (\rho^+ \text{ or } \rho^0) + X, \]  
(10)
\[ K^+ + A \rightarrow (K^{*+} \text{ or } K^{*0}) + X. \]  
(11)

For such vector mesons, \( D(x_F, V) \) dominate the large \( x_F \) region and \( D(x_F, V)/N_0(x_F, V) \) should increase with increasing \( x_F \). We therefore expect that, for such vector mesons, \( \rho^V_{00}(x_F) \) increases with increasing \( x_F \). It should start from 1/3 at \( x_F = 0 \), increase monotonically with increasing \( x_F \), and reach about 0.42 as \( x_F \rightarrow 1 \). For different vector mesons in this class, there will be only some small differences in the detailed forms of the \( x_F \)-dependences, but the qualitative features remain the same.

Examples of class (2) are:

\[ p + A \rightarrow (K^{*-} \text{ or } \bar{K}^{*0}) + X, \]  
(12)
\[ K^+ + A \rightarrow \rho^- + X, \]  
(13)
\[ K^+ + A \rightarrow (K^{*-} \text{ or } \bar{K}^{*0}) + X. \]  
(14)

Here, there is no kinematic region where the vector mesons of origin (A) play an important role. More precisely, for the vector mesons in this class, the mesons of origin (B) dominate for all \( x_F \) in \( 0 < x_F < 1 \) at moderately large transverse momenta. Hence, we should see no significant spin alignment for such vector mesons, i.e. \( \rho^V_{00}(x_F) \approx 1/3 \) for all \( x_F \).

We see that although the difference between the \( \rho^V_{00} \)’s of different vector mesons in the same class is negligible, there is a distinct difference between the \( \rho^V_{00} \) of the vector mesons in class (1) and that of the vector mesons in class (2). This can be checked easily in experiment. Presently, there are not enough data available to check whether these features are true. But they seem to be in agreement with the existing data. Here, we note that \( \rho^0_{00} = 0.424 \pm 0.011 \pm 0.018 \) were obtained for \( K^{*+} \) with \( p_V > 12 \text{GeV} \) produced in neutron-carbon
collisions with $p_{\text{inc}} = 60\text{GeV}$. This is an example in the first class and the corresponding $x_F$ of the produced $K^{*+}$'s in this experiment is quite large so that the $K^{*+}$'s are mainly from group (A). The resulting $\rho_{00}^V$ should be very close to 0.42 which is in agreement with the data. Another data is $8$, $\rho_{00} = 0.393 \pm 0.025 \pm 0.018$ for $K^{*-}$ in neutron-carbon interaction. This is an example of the second class and it is also consistent with the theoretical result within the experimental errors. Further measurements are needed to check whether the theoretical predictions are true.

3. A rough estimation of $\rho_{00}^V$ as a function of $x_F$

The detailed form of the $x_F$-dependence of the ratio $D(x_F, V)/N_0(x_F, V)$ is independent of the spin properties and can be obtained from the model calculations and/or unpolarized experimental data. A detailed procedure was given in [16]. It has been shown that, from the general constraints imposed by the conservation laws such as energy-momentum and flavor conservation on the direct-formation process, $D(x_F, M)$ should be proportional to the product of the distribution function of the valence quark and that of the sea anti-quark which combines the valence quark to form the vector meson. The proportional constant is determined by fitting the data for unpolarized cross section at large $x_F$. The non-direct formation part $N_0(x_F, M)$ can be obtained by parameterizing the difference between the unpolarized data for the number density in the corresponding transverse momentum region and the direct formation part $D(x_F, M)$. We can now follow this procedure to get $D(x_F, V)/N_0(x_F, V)$. But unfortunately, there is no appropriate unpolarized data available for the differential inclusive cross section of vector meson production in the corresponding reactions. On the other hand, we expect that the qualitative behavior of $D(x_F, V)/N_0(x_F, V)$ for the vector mesons should be quite similar to those for the pseudo-scalar mesons with the same flavor. Hence, we simply use the results for the corresponding pseudo-scalar mesons to replace them to make a rough estimation of $\rho_{00}^V(x_F)$ as a function of $x_F$. E.g., for $pp \rightarrow K^{*+}X$, we take $D(x_F, K^{*+})/N_0(x_F, K^{*+}) \approx D(x_F, K^+)/N_0(x_F, K^+)$, where the latter has been discussed in
detail in the second paper of Ref.[16]. Using this, we obtain $\rho_{00}^{V}(x_F)$ for $K^{*+}$ as a function of $x_F$ from Eq.(5) as shown in Fig.1. We see that the obtained $\rho_{00}^{K^{*+}}(x_F)$ increases from $1/3$ to 0.42 with increasing $x_F$. In the fragmentation region, e.g., $x_F > 0.5$, it is almost equal to 0.42. This is consistent with the available data and can also be checked by future experiments.

We note that it is in principle also possible to calculate the transverse momentum dependence of the $\rho_{00}^{V}$. For that calculation, we need to know transverse momentum dependence of $A_N$ thus that of the corresponding $C$. Presently, there are not enough data available for such a calculation. Recent data from BNL E0925 Collaboration show a significant energy dependence of $A_N$. It seems that this energy dependence can be obtained from the energy dependence of $D(x_F, s|M)$ in the above-mentioned formulation. Presently, we are working on this and if it is true, we can also apply it here to obtain the energy dependence of $\rho_{00}^{V}$.

4. Conclusion and discussion

In summary, we argue that the spin alignment of the vector mesons in unpolarized hadron-hadron collisions and the single-spin left-right asymmetry are closely related to each other. Both of them are different manifestations of the $\vec{s}_{q} \cdot \vec{n}$ type of spin correlation in high energy hadron-hadron or hadron-nucleus collisions. We calculate the spin alignment of vector mesons in unpolarized hadron-hadron collisions from the spin-correlation derived from the single-spin left-right asymmetries in hadron-hadron collisions. The obtained results are consistent with the available data and predictions for future experiments are made.

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Fig. 1. Spin alignment of $K^{*+}$ along the normal of the production plane in unpolarized $pp \rightarrow K^{*+}X$ at $p_{\text{inc}} = 200 \text{ GeV}$. 
Figure Captions

Fig.1: Spin alignment of $K^{*+}$ along the normal of the production plane in unpolarized $pp \rightarrow K^{*+}X$ at $p_{inc} = 200$ GeV.