Comparison of PERT and M-PERT scheduling for a construction project in Malang, Indonesia

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Abstract. Construction projects planned using deterministic method suffers from delay caused by uncertainties that are not considered in advance when determining activity durations. Probabilistic method offers an alternative to consider such uncertainties using PERT. Several studies on PERT however, proved that PERT results underestimated the project duration mean and overestimated the project duration variance. In 2017, PERT was developed into M-PERT. This study aims to compare project duration obtained from both the PERT and M-PERT with project duration obtained from Monte Carlo simulation software. Results of PERT and M-PERT on the same network model showed that M-PERT produced smaller value of project duration variance compared to PERT at the same equal value of project duration mean. M-PERT also produced project duration with RMSE value towards the simulation results of 0.0001818. This value is smaller compared to PERT’s RMSE value towards the simulation results of 0.0053961. This shows that M-PERT is able to produce results that are closer to computer simulation results compared to PERT. This study suggests that a probabilistic scheduling software which operates according to M-PERT principles should be developed in order for M-PERT to be completely utilized in actual construction projects.

1. Introduction

Project scheduling in construction projects commonly use deterministic method, where a certain value is determined as the duration of the activity. This is done based on historical data from previous projects or the scheduler’s judgement and experience. However, every construction project is unique from one to another [1], meaning a successful plan from one project does not ensure success when applied in another project. Another scheduling method that is less popular in project management is the probabilistic method, where a project duration may be calculated when the activity duration is uncertain. One of the probabilistic methods used in project management is the Program Evaluation and Review Technique (PERT). It is used to calculate project duration from the optimistic, pessimistic and most likely duration of the activities. However, PERT has its shortcoming in the form of result accuracy [2]. The cause of this problem is the network diagram modelling and no definitive solution for network diagram without a dominant critical path [2], [3]. Ballesteros-Pérez [4] then reformulated PERT and renamed it as M-PERT which allows more ways to model activity relationships to visualize their real-life conditions in the network diagram. M-PERT calculates project duration by merging activities in the network diagram repeatedly, providing a closer result to computer simulation compared to PERT [4], [5]. This research is conducted by collecting time related data from an actual project and implementing both PERT and M-PERT using that data to calculate the project duration. The results from both methods
are compared one another, and then compared with the results from Monte Carlo Simulation using a computer software.

2. Method and materials

Research method used on this research are as follows:

2.1. Collection of Project Data

The required project data for this research was the activities in the project and their optimistic, most likely and pessimistic duration. The type of project suitable for this research is a simple project such as regular building with or less than 5 stories constructed in 2017 to 2018. List of activities were prepared using the project time schedule, which contained information about all the activities that must be completed and their planned completion time. This list of activities was then brought to the expert who had experience and qualification in the project, to confirm how the activities were actually implemented on site. The activities from the schedule may be divided into one or more detailed activities, or combined into one major activity. The expert then suggested the optimistic, most likely and pessimistic duration for each activity according to his or her experience on site.

2.2. Analysis Method

The three-point-estimate (optimistic, most likely and pessimistic duration) were used to calculate the activity duration mean and variance. To calculate the project duration using PERT, the activity mean previously calculated may be used as a single duration in the Arrow Diagram Network (ADN). After completing the ADN, the critical path was identified. The project duration mean and variance for PERT were calculated by directly adding up the mean and variance of the critical activities. As for M-PERT, the activity variance must first be multiplied by correction factor K as explained in [6]. The network diagram for M-PERT was modelled as Precedence Diagram Network (PDN) finish-to-start. Activity relationships other than finish-to-start must be converted [7], [8]. Project duration mean and variance for M-PERT were calculated through merging procedure where two or more activities in the network are merged into a single activity with specific value of mean and variance. Merging procedure provided in [4] was unique for every scenario in the network diagram [9], [10], [11]. Project duration mean and variance from PERT and M-PERT were then compared with project duration mean and variance obtained from Monte Carlo simulation to check their accuracy [12], [13], [14].

3. Results and discussion

Results obtained from this research were divided into the following sections:

3.1. Project Information

The selected project for this research was a 3 story supermarket building in Malang, East Java, Indonesia. It was done from January 2017 to September 2017 for 38 weeks (266 days). The activities in this project were divided into seven major categories: (1) preliminary works, (2) earthmoving works, (3) concrete structure works, (4) MEP / utilities structure works, (5) steel structure works, (6) architectural works and (7) external works.

3.2. Activity Duration Mean and Variance

The three-point-estimate obtained from the expert’s suggestions showed that in most activities, the optimistic duration was the most likely duration. This is because the expert was confident that the construction method implemented on site was sufficient to ensure the activity would be finished in the allotted duration. Table 1 is a sample consisting of some activities in the project and their three-point-estimate. The mean and variance for each activity were first calculated using PERT’s formula for mean and variance. In accordance to the statement from Herrerías-Velasco et al. [6], the activity mean for M-PERT is equal to PERT-calculated mean; as for the activity variance, M-PERT required that the PERT-
calculated variance multiplied by a correction factor (K) if the scheduler decided to use three-point-estimate. The K factor was also calculated using the optimistic, most likely and pessimistic duration.

Table 1. List of activities with their three point estimate, mean and variance

| LIST OF ACTIVITIES       | Act Code | Optimistic Dur O (weeks) | Realistic Dur M (weeks) | Pessimistic Dur P (weeks) | Estimated Mean Te (weeks) | PERT Variance vTe | Correction Factor K | M-PERT Variance vTe |
|--------------------------|----------|--------------------------|-------------------------|--------------------------|--------------------------|-------------------|---------------------|---------------------|
| PRELIMINARY WORKS        | A1       | 2                        | 2                       | 3                        | 2.17                     | 0.0278            | 0.7143              | 0.0198              |
| EARTHMOVING WORKS        | B        | 6                        | 6                       | 8                        | 6.33                     | 0.1111            | 0.7143              | 0.0704              |
| MAIN STRUCTURE           |          |                          |                         |                          |                          |                   |                     |                     |
| 3.1 STRIP FOUNDATION + GROUND TIE BEAM | C1      | 4                        | 4                       | 6                        | 4.33                     | 0.1111            | 0.7143              | 0.0704              |
| 3.2 GROUND FLOOR COLUMN  | C2       | 4                        | 4                       | 5                        | 4.17                     | 0.0278            | 0.7143              | 0.0198              |
| 3.3 GROUND FLOOR SLAB (ELEV + 0.00) | C3      | 2                        | 3                       | 5                        | 3.17                     | 0.2500            | 1.2222              | 0.3056              |
| 3.4 1ST FLOOR BEAM AND SLAB (ELEV + 4.95) | C4     | 2                        | 4                       | 4                        | 3.67                     | 0.1111            | 0.7143              | 0.0704              |
| 3.5 1ST FLOOR COLUMN     | C5       | 2                        | 3                       | 4                        | 3.00                     | 0.1111            | 1.2222              | 0.3056              |
| 3.6 ROOF FLOOR BEAM AND SLAB (ELEV + 9.95) | C6     | 2                        | 3                       | 5                        | 3.17                     | 0.2500            | 1.2222              | 0.3056              |
| 3.7 MEZANINE FLOOR COLUMN | C7      | 2                        | 2                       | 4                        | 2.33                     | 0.1111            | 0.7143              | 0.0704              |
| 3.8 MEZANINE FLOOR BEAM AND SLAB (ELEV + 14.85) | C8   | 2                        | 2                       | 4                        | 2.33                     | 0.1111            | 0.7143              | 0.0704              |
| 3.9 CONCRETE CONSOLE / LISTPLANK | C9     | 1                        | 1                       | 2                        | 1.17                     | 0.0278            | 0.7143              | 0.0198              |
| 3.10 PARAPET             | C10      | 1                        | 1                       | 2                        | 1.17                     | 0.0278            | 0.7143              | 0.0198              |
| 3.11 STAIRS              | C11      | 2                        | 4                       | 5                        | 3.83                     | 0.2500            | 1.2222              | 0.3056              |

3.3. Project Duration Mean and Variance of PERT and M-PERT

The Arrow Diagram Network (ADN) for PERT is given in Figure 1 and Figure 2. Figure 1 is the first half of the project consisting of preliminary works, earthmoving works and concrete structure works. Figure 2 is the second half of the project consisting the MEP structure, steel structure, architectural works, and external works. The blue, green and yellow node in Figure 1 will continue in the node with the same colour in Figure 2. Notice that in these two figures, there are single activities that are divided into two activities, for example activity C1. This is done to indicate that the next activity would start a certain number of weeks after the current activity started. The activity mean (Te) as shown in Table 1 is entered as the single duration of each activity. Using the CPM, the project duration was calculated and the critical activities were determined. Critical activities were marked in Figure 1 and Figure 2 with the yellow colour and red arrow. The project duration obtained from PERT was 38.34 weeks and the variance was 2.7222 weeks, where both are the sum of critical activity mean and variance respectively.

Figure 1. PERT Arrow Diagram Network of case study project (part 1)
PERT’s PDN model has been made when modelling the Precedence Diagram Network (PDN) for M-PERT is given in Figure 3 and Figure 4. As the finish-to-start model has been made when modelling PERT’s ADN, the layout for the M-PERT’s PDN model is no different. Activities are written in the nodes instead of on the arrows. However, it is slightly different than the usual PDN as in M-PERT, the variance of activity duration must also be considered when calculating through merging procedure. Both mean and variance must be written in the activity boxes as seen in Figure 3 and Figure 4. The node with the red coloured topside in Figure 3 and Figure 4 has the same function as the coloured node in Figure 1 and 2; to connect the two figures. Other things that can be noticed from both figures are that division of single activities just like those in PERT’s ADN must also be carried out and there is no critical path because the merging procedure do not require it.

Figure 2. PERT Arrow Diagram Network of case study project (part 2)

Figure 3. M-PERT Precedence Diagram Network of case study project (part 1)
Figure 4. M-PERT Precedence Diagram Network of case study project (part 2)

Merging procedure must be applied in order to calculate M-PERT project duration mean and variance. In this case however, the project duration mean and variance could not be calculated manually. This was due to the fact that at one point where the paths must be untangled into parallel paths (see illustration in Figure 5), too many parallel paths were formed. As mentioned by Ballesteros-Pérez [4], correlation factor must be calculated when there are tangled paths in the network that must be untangled. This means even if somehow the scheduler was able to identify all the possible paths, he/she must still calculate the correlation factor for all of that possible paths with a formula for manual calculation that was limited. Ballesteros-Pérez [4] formula for correlation factor could only calculate up to four parallel paths, while there were 420 possible parallel paths in this project from start to finish that must be merged. The original formula used by Sculli and Shum [11] in their research were also applied to merge up to 14 paths only and there were no detailed calculations provided in the research. In order to obtain results from both PERT and M-PERT that can be compared, the next section of this research analyzed one of the subnetworks in the project, which was the external works category.

Figure 5. Illustration of untangling paths into parallel paths
3.4. External Works Subnetwork Duration Mean and Variance of PERT and M-PERT

The External Works Subnetwork consisted of activities that were listed on Table 2. The activity mean and variance for PERT and M-PERT were calculated the same way as Table 1.

Table 2. External works list of activities three point estimate, mean and variance

| NO | LIST OF ACTIVITIES       | Act Code | Optimistic O (week) | Most Likely M (week) | Pessimistic P (week) | Act Mean Te (week) | PERT Var vTe (week) | Correction Factor K | M-PERT Var vTe (minggu) |
|----|--------------------------|----------|---------------------|----------------------|----------------------|--------------------|---------------------|---------------------|------------------------|
| 4  | STEEL STRUCTURE          | E3       | 1                   | 1                    | 3                    | 1.3333             | 0.1111              | 0.71429             | 0.07937                |
| 4.5| MOTOR CANOPY             | E3       | 1                   | 1                    | 3                    | 1.3333             | 0.1111              | 0.71429             | 0.07937                |
| 4.9| BILLBOARD                | E6       | 1                   | 1                    | 2                    | 1.1667             | 0.0278              | 0.71429             | 0.01984                |
| 6  | EXTERNAL WORKS           |          |                     |                      |                      |                    |                     |                     |                        |
| 6.1| RIGID PAVEMENT           | H1       | 2                   | 2                    | 4                    | 2.3333             | 0.1111              | 0.71429             | 0.07937                |
| 6.2| PAVING BLOCK PAVEMENT    | H2       | 2                   | 2                    | 4                    | 2.3333             | 0.1111              | 0.71429             | 0.07937                |
| 6.3| ASPHALT PAVEMENT         | H3       | 1                   | 1                    | 2                    | 1.1667             | 0.0278              | 0.71429             | 0.01984                |
| 6.4| CANSTEEN                 | H4       | 1                   | 1                    | 2                    | 1.1667             | 0.0278              | 0.71429             | 0.01984                |
| 6.5| PARKING LINE PAINTING    | H5       | 1                   | 1                    | 2                    | 1.1667             | 0.0278              | 0.71429             | 0.01984                |
| 6.6| DRAINAGE SYSTEM          | H6       | 2                   | 2                    | 3                    | 2.1667             | 0.0278              | 0.71429             | 0.01984                |
| 6.7| LANDSCAPING              | H7       | 1                   | 1                    | 2                    | 1.1667             | 0.0278              | 0.71429             | 0.01984                |
| 6.8| SITE ENCLOSURE AND DIVISION | H8   | 2                   | 3                    | 3                    | 2.8333             | 0.0278              | 0.71429             | 0.01984                |
| 6.9| MISCELLANEOUS            | H9       | 1                   | 2                    | 3                    | 2.0000             | 0.1111              | 1.28571             | 0.14286                |

The ADN for the external works subnetwork are given in Figure 6. After completely modelling the network using forward pass and backward pass calculations, it was clear that the critical activities were H2-H8-H6-H7-H9. The critical path produced a project duration mean and variance of 10.50 weeks and 0.3056 weeks respectively.

Figure 6. PERT Arrow Diagram Network of external works category

The PDN and merging procedure for M-PERT are shown through Figure 7. To sum it up, all of the activities in the same path were first merged. After all the paths has been formed, the three paths were sorted from top to bottom according to their total mean value. Path A and Path B were merged, and its results (Path AB) would then be merged with Path C. In the end, the merging produced M-PERT mean and variance value of 10.50 weeks and 0.2817 weeks respectively. It produced the same mean value as PERT and lesser variance than PERT. The different variance value indicated that M-PERT would provide a project duration with a more homogenous distribution to its mean than PERT. If these two results were to be drawn as normal distribution curve, M-PERT’s normal curve would be leaner than PERT’s curve as it has greater number of data with value closer to its mean.
Figure 7. Merging Procedure of External Works PDN for M-PERT

3.5. External Works Subnetwork Duration Mean and Variance: Monte Carlo Simulation
Monte Carlo simulation was performed using @RISK (trial version), an add-in software for Microsoft Excel. The desired activity distribution is selected, then the three-point-estimate of activities were entered into the software as input. Simulation results after 1,000,000 times of iteration are displayed in Figure 8. The parameters in the right side of Figure 8 necessary to compare PERT and M-PERT results with Monte Carlo simulation were the mean of 10.5002, standard deviation of 0.5301, minimum value of 8.5364, and maximum value of 13.1692.
Comparison of the Monte Carlo Simulation, PERT and M-PERT results were created using Microsoft Excel. The comparison was presented in the form of cumulative distribution from the three methods (PERT, M-PERT and simulation) for X value (external works duration) of 8.50 to 13.00 weeks. As seen in Figure 9, the results from PERT, M-PERT and Monte Carlo simulation were so close that the cumulative distribution curve of the three methods were too difficult to be observed visually. Hence, a new indicator was introduced to measure the difference between Monte Carlo simulation and PERT or M-PERT; the Root Mean Square Error (RMSE) value.

![Figure 8. Distribution of external works duration produced by @RISK (trial version)](image)

**Figure 8.** Distribution of external works duration produced by @RISK (trial version)

Comparison of external works duration’s cumulative distribution from simulation, PERT, and M-PERT.

![Figure 9. Comparison of external works duration’s cumulative distribution from simulation, PERT, and M-PERT](image)

**Figure 9.** Comparison of external works duration’s cumulative distribution from simulation, PERT, and M-PERT
The RMSE calculation was done twice, as there were two comparisons observed; the simulation results to PERT results; and the simulation results to M-PERT results. First, difference of probability obtained from two methods were calculated for each X-value from 8.50 to 13.00. To calculate the RMSE value of those two methods, the difference value must be squared and then summed. The total was divided by number of observation and the square root value of that is the RMSE value. In Microsoft Excel, simply list the cumulative probability for each observation (X-value) from the two methods as shown in Table 3. The cumulative probability value of PERT was computed by Excel formula of normal distribution (“=NORM.DIST(…”) using the mean, standard deviation and the desired X-value. The same also goes for M-PERT. Difference is calculated by subtracting the “PERT” cell data from the “Simulation” cell data. After difference for all observation has been calculated, enter the following formula at the bottom cell as the RMSE formula: “=SQRT (SUMSQ (all DIFFERENCE cells)/COUNTA (all DIFFERENCE cells))”. From Table 3, the RMSE value between simulation results and PERT results was 0.0053961.

Table 3. Sample for RMSE calculation between simulation and PERT

| X      | SIMULATION (μ = 10,5002; σ = 0,5301) | PERT (μ = 10,50; σ = 0,55277) | DIFFERENCE (SIMULATION - PERT) |
|--------|-------------------------------------|--------------------------------|--------------------------------|
| 8,5    | 0.0001                              | 0.0001                         | -0.0001                        |
| 8,6    | 0.0002                              | 0.0003                         | -0.0001                        |
| 8,7    | 0.0003                              | 0.0006                         | -0.0002                        |
| 8,8    | 0.0007                              | 0.0011                         | -0.0004                        |
| 8,9    | 0.0013                              | 0.0019                         | -0.0006                        |
| 9      | 0.0023                              | 0.0033                         | -0.0010                        |
| 13     | 1.0000                              | 1.0000                         | 0.0000                         |

ROOT MEAN SQUARE ERROR (SIMULATION VS PERT) 0.0053961

The same steps were also performed for RMSE value between simulation and M-PERT. Results were shown in Table 4. The RMSE value between simulation and M-PERT was 0.0001818. From here the conclusion was clear that M-PERT was more accurate than PERT as it produced closer results to Monte Carlo simulation than PERT.

Table 4. Sample for RMSE calculation between simulation and PERT

| X      | SIMULATION (μ = 10,5002; σ = 0,5301) | M-PERT (μ = 10,50; σ = 0,5308) | SELISIH (SIMULATION - M-PERT) |
|--------|-------------------------------------|--------------------------------|--------------------------------|
| 8,5    | 0.0001                              | 0.0001                         | 0.0000                         |
| 8,6    | 0.0002                              | 0.0002                         | 0.0000                         |
| 8,7    | 0.0003                              | 0.0003                         | 0.0000                         |
| 8,8    | 0.0007                              | 0.0007                         | 0.0000                         |
| 8,9    | 0.0013                              | 0.0013                         | 0.0000                         |
| 9      | 0.0023                              | 0.0024                         | 0.0000                         |
| 13     | 1.0000                              | 1.0000                         | 0.0000                         |

ROOT MEAN SQUARE ERROR (SIMULATION VS M-PERT) 0.0001818

4. Conclusion
A research has been conducted to compare two probabilistic scheduling methods; PERT and M-PERT. Using the external works as the analyzed project, M-PERT provided better estimation results than PERT.
as it produced the same value of mean duration with smaller value of variance. M-PERT also surpassed PERT in terms of accuracy as it produced smaller RMSE value to the Monte Carlo simulation than PERT. Although implementing M-PERT in a complex project was a challenge that has yet to be answered, its principles can be used to develop a probabilistic scheduling software to assist scheduler’s work in a construction site.

5. References

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