Mathematical modeling of steel fiber reinforced concrete properties and selecting its effective composition

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Abstract. Design of optimal concrete compositions with specified quality indicators at minimal cost still remains one of the most actual problems. The paper deals with methodology for designing optimal fiber reinforced concrete composition using experimental-statistical models of concrete properties. The obtained mathematical models for compressive and flexural strength of steel fiber reinforced concrete and a model of superplasticizer consumption are used. The proposed method of fiber reinforced concrete composition design takes into account the specific characteristics of the studied materials. It allows relatively easy composition optimization by applying a specific minimum cost criterion. Experimental results obtained in the frame of the study confirm efficiency of the proposed concrete mix design methodology for producing steel fiber reinforced concrete with necessary properties.

1. Introduction
Steel fibers were for the first time proposed as dispersed reinforcement for concrete by Romualdi [1]. Since that time, using fibers in cement-based materials has developed considerably. Steel fiber reinforced concrete (SFRC) was applied in structures all over the world. Steel fiber high-strength concrete became a very popular material in structural engineering in the recent decades [2]. Many experimental studies were carried out to select effective fiber types and contents [3].

As for conventional concrete, the main common principles for SFRC mix design is based on selecting suitable components of concrete and determining their relative proportions that enable producing concrete with necessary properties in fresh and hardened states [4]. The most commonly specified features, considered in fiber reinforced concrete mix design, are workability, strength and durability.

In SFRC mix design fibers can be observed as aggregate with determined specific surface and very unfavorable shape, requiring more cement paste to provide desired workability [5]. As known, it is important to provide good adhesion between fibers and cement paste, so water/cement ratio should be as low as possible. To achieve higher workability and strength characteristics, superplasticizer and mineral micro fillers are usually used [6]. Kosmatka et al. [6] showed that the aggregate maximum size should be less than that of fiber.

Fibers play an important role in reaching a definite load bearing capacity after the matrix fracture, depending on allocation, orientation and embedded length [7]. Influence of various contents of steel fibers with different configurations on flexural tensile strength, fracture behavior and concrete mix
workability was studied experimentally. It was reported that adding steel fibers resulted in a more ductile behavior and higher load levels at post-cracking.

Basheerudeen and Anandan [8] proposed a design methodology for steel fiber self-compacting concrete. Aggregates optimization was based on packing density concept. Combinations of cement and ground granulated blast furnace slag were selected by paste consistency test. Superplasticizer dosage was chosen using Marsh cone tests and the steel fibers fraction was found from slump flow studies.

A method for designing concrete mixtures, aimed at achieving a densely compacted cementitious matrix, employing the modified Andreasen & Andersen particle packing model, was proposed [9]. The workability, air content, porosity, flexural and compressive strengths of the designed concrete were measured and analyzed. It was reported that the improved packing model allows design of ultra-high-performance fiber reinforced concrete (UHPFRC).

Koksal and Ilki [10] proposed an optimum mix design of steel concrete plates. According to the experimental results and multi-objective simultaneous optimization, optimum mix designs are proposed for minimizing the steel fiber content and cost while maximizing the SFRC plates toughness. Optimum solutions were obtained for mixes that contained fibers with aspect ratios between 65 and 80.

A method for SFRC composition optimization, considering the influence of concrete ingredients, properties of cement stone and aggregates, compacting quality and concrete mixture properties was presented [11]. The influence of water-cement ratio, fiber content, sand fineness and quantity on the concrete mixture stiffness, compressive and flexural strength of concrete was studied. A method for obtaining optimal steel fiber fine-grained concrete composition, taking into account concrete flexural strength, sand fineness and concrete mixture workability was proposed.

Mechanical properties of UHPFRC, considering various influential factors are required for its proper design and practical utilization [12]. A review of early-age strength and mechanical properties of hardened UHPFRC was presented. The effects of curing conditions, coarse aggregate, mineral admixtures, fiber properties, specimen size, and strain-rate on UHPFRC mechanical performance were discussed. It is noted that further research to reduce the production cost of UHPFRC should be addressed to make its widespread use more practical.

Experimental and statistical models proved high efficiency for design of optimal concrete compositions [13]. Such models allow proper solution of problems related to providing desired concrete properties, concrete mix design optimization for various compositions that were used in the frame of experimental studies as well as for those that are out of the study frame. Using mathematical models enables to perform effective technical and economical analysis of the technological solutions and optimization of economical parameters.

The present study is focused on using experimental-statistical models for design of SFRC compositions that enable producing steel fiber reinforced concrete with necessary properties.

2. Research aim and significance

Usually the main criteria for concrete mix design optimization are the minimisation of cement consumption or the minimum possible cost of concrete. For ordinary concrete, these criteria usually coincide. In both cases it is mandatory to provide a set of standardized properties of concrete mix and concrete.

Modern concrete is a multi-component system and the cost of individual components may be close to that of cement and even exceed it. This is actual for SFRC as well, in which the content of main components (cement, fibers and plasticizing admixtures) may vary in a wide range and have a major effect on the final cost. In practice, the task of minimizing the fiber reinforced concrete cost in the traditional approach is complicated. It is advisable to use the statistical method for this task [14].

In the present study we describe a method for solving such problem using experimentally-statistic models of SFRC compressive and flexural strength at different ages and a model of the required
superplasticiser content providing the specified parameters of fresh mix and hardened concrete. Efficiency of the method is demonstrated in numerical examples.

3. Problem statement
The problem of finding the optimal composition of SFRC with designed quality parameters can be formulated as follows: find the values of fiber reinforced concrete composition factors $x_1 \ldots x_n$, allowing to minimize the SFRC cost:

$$B_{FC} = B_C \cdot C + B_a \cdot A + B_f \cdot F \rightarrow \min$$

while the necessary quality parameters is provided

$$P_1 \geq f(x_1, x_2, \ldots, x_3)$$
$$P_2 \geq f(x_1, x_2, \ldots, x_3)$$

.........................
$$P_2 \geq f(x_1, x_2, \ldots, x_3)$$

while

$$x_1 \ldots x_n \in [a \ldots b]$$

where $B_c$, $B_a$, $B_f$ are the cost of cement, admixture (superplasticizer, active mineral additive, etc.) and fibers, respectively, USD / kg; $C, A, F$ are the consumption of cement, modifying admixture and fibers, correspondingly, kg / m$^3$ of SFRC; $P_1 \ldots P_m$ are the given quality parameters of SFRC; $x_1 \ldots x_n$ are the composition factors; $a, b$ are the limitations of factors’ possible values.

4. Experiments planning and materials properties
A series of experiments, based on algorithms according to the second order three-factor experiment plan (type B$_3$) [14], were implemented in order to determine the SFRC mix design parameters under the planning conditions given in Table 1. As components of the concrete mixture were used Portland cement CEM-I with a compressive strength of 52.5 MPa, aggregate in a form of a fractionated mixture of quartz sand (0.16 - 2 mm) and crushed granite (2 - 5 mm) mixed in a ratio of 0.45 / 0.55. A polycarboxylate type Melflux 2651F superplasticizer was added in order to ensure the mixture slump of 15 cm. Corrugated fibers Fibax F1 60/1 (length 60.0 ± 6.0 mm, diameter 1.0 ± 0.1 mm) made of low carbon steel were used.

| No. | Factors | Varying settings | Interval |
|-----|---------|------------------|----------|
| 1   | $x_1$   | Cement consumption, kg/m$^3$ (C) | 450 500 550 50 |
| 2   | $x_2$   | W/C              | 0.3 0.35 0.4 0.05 |
| 3   | $x_3$   | Fiber content, kg/m$^3$ (F)    | 80 100 120 20 |

The following mixing procedure was used for preparing the concrete specimens. It included dry mixing of cement and aggregate in mixer during 0.5 min, addition of water, mixing 3 min, addition of superplasticizer and fibers, mixing during 3 to 3.5 min. Fresh concrete was poured into standard forms and compacted on a vibration table. After 24 h of normal hardening, the samples were processed and stored for 27 days at a temperature of 18±2 °C and air humidity over 90% until the test was performed.
Compressive tests were carried out, at 28 days, on cubic specimens (100 × 100 × 100 mm). Tests were carried out using a hydraulic press. The ultimate compression load for each SFRC specimen was recorded. The number of tested samples was 3 for each concrete composition. Flexural strength tests were performed on SFRC prisms (70 × 70 × 280 mm). The loading was applied in an automatic manner with a constant speed of 0.05 MPa/s for flexural strength and 0.6 MPa/s for compressive strength according to requirements of Ukrainian National Standard for testing concrete [15].

5. Results and discussion

Experimental results are presented in Table 2. For this data corresponding statistical characteristics were obtained and coefficients of regression equation were calculated. After the significance of the coefficients were estimated, the adequacy equations was checked by calculating the adequacy dispersion, design value of Fisher's criterion (F-criterion) [16] and comparing the last with a given one.

Adequate experimental-statistical models of SFRC compressive and flexural strengths at 28 days ($f_{cm}$ and $f_{c,t}$, MPa, respectively), in the terms of the symbolic variables are as follows:

- compressive strength:

$$f_{cm} = 78.9 + 4.8x_1 - 13.8x_2 + 0.4x_3 - 1.53x_1^2 - 0.43x_2^2 - x_3 - 0.5x_1x_2$$  \hspace{1cm} (4)

- flexural tensile strength:

$$f_{c,t} = 17.2 + 0.59x_1 - 2.08x_2 + 2.5x_3 + 0.36x_1^2 - 0.38x_2^2 - 4.03x_3^2 - 0.98x_1x_2 - 0.14x_1x_3.$$  \hspace{1cm} (5)

| Batch No. | Coded factor value | Component consumption, kg/m$^3$ | W/C | AD-MIX, % | $f_{cm}$, MPa | $f_{c,t}$, MPa |
|-----------|-------------------|---------------------------------|-----|-----------|---------------|---------------|
| 1         | 1 1 1             | 550 1627 220 120                | 0.4 | 0.2       | 67.2          | 13.84         |
| 2         | 1 1 -1            | 550 1627 220 80                 | 0.4 | 0.1       | 66.6          | 9.08          |
| 3         | 1 -1 1            | 550 1776 165 120                | 0.3 | 1.1       | 96            | 19.92         |
| 4         | 1 -1 -1           | 550 1776 165 80                 | 0.3 | 0.8       | 95.2          | 15.23         |
| 5         | -1 1 1            | 450 1822 180 120                | 0.4 | 0.4       | 57.3          | 14.9          |
| 6         | -1 -1 1           | 450 1822 180 80                 | 0.4 | 0.2       | 56.9          | 9.59          |
| 7         | -1 -1 -1          | 450 1944 135 120                | 0.3 | 1.3       | 83.9          | 17.06         |
| 8         | -1 -1 -1          | 450 1944 135 80                 | 0.3 | 1         | 83.3          | 11.82         |
| 9         | 1 0 0             | 550 1702 193 100                | 0.35| 0.45      | 79.7          | 18.13         |
| 10        | -1 0 0            | 450 1883 158 100                | 0.35| 0.27      | 75.8          | 16.95         |
| 11        | 0 1 0             | 500 1724 200 100                | 0.4 | 0.3       | 65.3          | 15.49         |
| 12        | 0 -1 0            | 500 1860 150 100                | 0.3 | 0.5       | 92.4          | 19.64         |
| 13        | 0 0 1             | 500 1792 175 120                | 0.35| 0.3       | 79            | 15.67         |
| 14        | 0 0 -1            | 500 1792 175 80                 | 0.35| 0.1       | 77.5          | 10.67         |
| 15        | 0 0 0             | 500 1792 175 100                | 0.35| 0.2       | 78.3          | 17.05         |
| 16        | 0 0 0             | 500 1792 175 100                | 0.35| 0.2       | 78.1          | 17.17         |
| 17        | 0 0 0             | 500 1792 175 100                | 0.35| 0.2       | 77.9          | 17.25         |

To determine the optimal consumption of Melflux 2651F superplasticizer (ADMIX) the following mathematical model was developed, % of the cement mass:

$$ADMIX = 0.29 + 0.095x_1 - 0.35x_2 + 0.11x_3 - 0.05x_1^2 + 0.23x_2^2 + 0.03x_3^2 - 0.04x_2x_3.$$   \hspace{1cm} (6)

The conversion of the SFRC mixture composition parameters into the codified form is carried out as follows:
Founded polynomial models (Eqs. 4 - 6) enable to solve a number of practical problems. According to Voznesensky [17], there are 10 types of tasks that can be solved individually or jointly using such models. Among them are interpolation, extrapolation, tasks for achieving the minimum or maximum value of the output parameter, etc.

For graphic interpretation of the individual factors and assessing their effects, Eqs. 4 - 6 are converted to obtain one-factor models and corresponding graphs, which allow to assess the influence degree for each of the factors (so-called factors’ influence analysis task). The task of controlling the output parameter for two variables is solved, as a rule, by isoparametric analysis. For this purpose, isolines of the same output parameter values are obtained at different combinations of two factors. In this case, all other factors are fixed at a certain level (for example, at average one). However, it is quite difficult to determine the necessary factors’ values in order to achieve a certain output parameter by these methods, especially in the case of their large number (more than 5 - 6).

Tasks of minimizing resources and management with a fixed output parameter can be solved by a nomographic method [14]. The task of management is to determine such factors combinations that provide the given parameters of the output parameter. To do this, one of the factors is chosen from the obtained regression equation (for example, for tensile strength). As a result of the regression equation solution with respect to this factor, its necessary value is determined, which ensures a given value of the output parameter, when other factors are changed. Figure 1 presents a nomogram for determining the cement consumption at a given SFRC flexural tensile strength.

\[
 x_1 = \frac{C - 500}{50}, \quad x_2 = \frac{W / C - 0.35}{0.05}, \quad x_3 = \frac{F - 100}{20}
\]

Figure 1. Nomogram for SFRC flexural tensile strength at 28 days.

For SFRC, as already noted earlier, in addition to cement, the cost of fiber is significantly affected by the fibers content. For example, in Figure 1 is considered a case when the required tensile strength of 18 MPa can be achieved at the same \( W/C = 0.3 \), but with different fiber content (100 and 120 kg) and cement consumption (468 and 522 kg). If the superplasticizer cost, affecting both the properties of concrete and its cost, as well as the possibility of providing another quality index (for example,
compressive strength) should be considered, optimizing the SFRC composition by a nomographic method is practically impossible.

In order to obtain the optimum mix design for this case, it is necessary to solve the following mathematical programming problem: to find such a composition of SFRC mixture, which would allow to provide the necessary compressive and flexural tensile strength at 28 days at a minimum total cost in limits of admissible values of factors.

The calculation consequence is as follows.
- Substitute in the model (4) and (5) the values of strengths to be ensured, and in Eq. (1) - the SFRC components value.
- Set in Eq. (3) the limit values of factors (in coded values from -1 to 1).
- Picking up various combinations of factors, providing the given strength values by Eqs. (4) and (5) while minimizing function (1).

In order to determine the cost of SFRC during iterations, the required superplasticizer content in Eq. (6) is determined simultaneously with finding the intermediate values of factors \(x_1\) ... \(x_3\).

A result of such iterations is determining the optimal values of composition factors: cost of cement, fiber content, \(W/C\) and superplasticizer consumption. Water demand can be calculated from the obtained \(W/C\) and cement consumption:

\[
W = C \cdot (W / C)
\]  

The fine aggregate consumption is determined by the method of absolute volumes:

\[
\text{Aggr.} = \left\{ 1000 - \left( \frac{C}{\rho_c} + \frac{F}{\rho_f} + \frac{W}{\rho_w} \right) \right\} \cdot \rho_a, \tag{9}
\]

where \(\rho_c\), \(\rho_f\), \(\rho_w\), and \(\rho_a\) – respectively, the actual densities of cement, fiber, water and aggregates.

6. Numerical examples

6.1. Example 1

Determine the SFRC composition with the following properties at 28 days: compressive strength of 70 MPa; flexural tensile strength of 15 MPa as well as with a mixture slump of 15 cm. Use experimental-statistical models (4-6). Assume the cost of the main SFRC components as follows, c.u./kg: \(B_c = 0.1; B_f = 9; B_w = 1.8\).

Materials: Portland cement CEM 42.5, standard quality fine aggregate with a fineness modulus of 3.5 and real density \(\rho = 2.65\) kg/l, superplasticizer Melflux 2651f.

1. Substituting the compressive and flexural tensile strength values in Eqs. (4) and (5), obtain the restriction function (2) of the problem:

\[
f_{cm} : 78.9 + 4.8x_1 - 13.8x_2 + 0.4x_3 - 1.53x_1^2 - 0.43x_2^2 - x_3^2 - 0.5x_1x_2 \geq 70;
\]

\[
f_{ctf} : 17.2 + 0.59x_1 - 2.08x_2 + 2.5x_3 + 0.36x_1^2 - 0.38x_2^2 - 4.03x_3^2 - 0.98x_1x_2 - 0.14x_1x_3 \geq 15.
\]

2. Substitute the SFRC components values into Eq. (1), and specify the limitation of the factors values: from -1 to 1 (in coded form).

3. By varying different combination factors the values that satisfy the problem and minimize the total SFRC cost. The following parameters were obtained by using originally developed routines implemented using the MS Excel Solver software:

\(x_1 = -0.039; x_2 = 0.13; x_3 = -0.446\).

For values of the obtained factors, from Eqs. (4 - 5) it follows that \(f_{ctf} = 15\) MPa, which corresponds to the required flexural tensile strength value, and \(f_{cm} = 76.6\) MPa, which is higher than the required compressive strength value.
4. Determine the natural factors using Eq. (7):

\[ C = 50 \cdot x_1 + 500 = 50 \cdot (-0.026) + 500 = 498.1 \text{ kg/m}^3; \]
\[ W/C = 0.05 \cdot x_2 + 0.35 = 0.05 \cdot 0.13 + 0.35 = 0.356; \]
\[ F = 20 \cdot x_3 + 100 = 20 \cdot (-0.446) + 100 = 91.1 \text{ kg/m}^3. \]

5. The superplasticizer consumption according to Eq. (6) is:

- by %:

\[ ADMIX = 0.29 + 0.095x_1 - 0.35x_2 + 0.11x_3 - 0.05x_1^2 + 0.23x_2^2 + 0.03x_3^2 - 0.04x_2x_3 = 0.21\%. \]

- by mass:

\[ ADMIX_m = ADMIX \cdot C / 100 = 0.21 \cdot 498.1 / 100 = 1.046 \text{ kg/m}^3. \]

6. The minimum possible cost value per 1 m$^3$ of SFRC without taking into account the cost of aggregate and water is found during the iterations in the “Solver” application according to Eq. (1):

\[ BFC = 0.1 \cdot 498.1 + 9 \cdot 1.045 + 1.8 \cdot 91.1 = 222.3 \text{ USD}. \]

7. Water consumption following Eq. (8) is:

\[ W = 498.1 \cdot 0.356 = 177.5 \text{ l}. \]

8. Fine aggregate consumption according to Eq. (9) is:

\[ Aggr. = \left(1000 - \left(\frac{498.1}{3.1} + \frac{91.1}{7.85} + \frac{177.5}{1}\right)\right) \cdot 2.65 = 1723 \text{ kg}. \]

The final SFRC mix composition, kg/m$^3$ is:

\[ C = 498; \ W = 178; \ Aggr. = 1723; \ F = 91; \ ADMIX = 1.05. \]

At problem formulation stage for finding the SFRC composition the desired values of compressive strength and flexure tensile strength should be correctly set. Obviously, these values should be within the minimum and maximum possible value of the output parameter, since it is within these limits that the polynomial model adequately describes studied parameter. Such values can be easily found using the above-mentioned routines implemented in MS Excel Solver. For example 1, the limit values of strength within the factors variation range will be as follows:

\[ f_{cm}(\text{min}) = 58.2 \text{ MPa}; \ f_{cm}(\text{max}) = 96.1 \text{ MPa}; \ f_{c,t}(\text{min}) = 8.93 \text{ MPa}; \ f_{c,t}(\text{max}) = 21.93 \text{ MPa}. \]

Some deviation way beyond the output parameters limits is also possible. In this case, along with the optimization problem, an extrapolation problem is also solved, allowing to take the factors’ values outside the variation range (for example, \( x_1, x_2 \) = 1.1; 1.2; 1.3). However, it should be borne in mind that extrapolation may be due to certain errors, and these errors become more significant, the farther beyond the variation range limits. Extrapolation is possible, if according to the research results there is no doubt that outside the factors variation region the function nature remains unchanged.

Let’s consider another calculation example, which involves fixing one of the factors at a certain level.

6.2. Example 2

The conditions of the task fully correspond to the source data of Example 1, but additionally it is necessary to limit the fiber content to the minimum possible value, that is, \( F = 80 \text{ kg/m}^3 \).

1. Functions of task limitations are similar to pos. 1 (see example 1).
2. Set limitations of the factors’ values: \( x_1 = x_2 = -1 \; \ldots \; 1; \ x_3 = -1 \) (in code form). The value of the SFRC components is substituted into Eq. (1).
3. Using the routines implemented in MS Excel Solver find the factors values that satisfy the task limitations and minimize the total cost of SFRC:

\[ x_1 = 0.914; \ x_2 = -0.992; \ x_3 = -1. \]
For such factors values in Eqs. (4 - 5) \( f_{c.t} = 15 \text{ MPa} \), which corresponds to the required flexural tensile strength value, and \( f_{cm} = 94.3 \text{ MPa} \), which provides the required compressive strength one.

4. Determine the factors in kind in terms of Eqs. (7):

\[
\begin{align*}
C &= 50 \cdot x_1 + 500 = 50 \cdot 0.914 + 500 = 545.7 \text{ kg}; \\
W/C &= 0.05 \cdot x_2 + 0.35 = 0.05(-0.992) + 0.35 = 0.3; \\
F &= 20 \cdot x_3 + 100 = 20(-1) + 100 = 80 \text{ kg}.
\end{align*}
\]

5. Superplasticiser consumption following Eq. (6) is:

- by %: \( \text{ADMI}X = 0.854\% \);
- by mass: \( \text{ADMI}X_m = \text{ADMI}X \cdot C / 100 = 0854 \cdot 545.7 / 100 = 4.66 \text{ kg} \).

6. The minimum possible cost per 1 m\(^3\) of fiber-reinforced concrete without taking into account the aggregate and water cost:

\[
B_{FC} = 0.1 \cdot 545.7 + 9 \cdot 4.66 + 1.8 \cdot 80 = 6848.6 \text{ USD}.
\]

7. Water consumption is calculated according to Eq. (8):

\[
W = 545.7 \cdot 0.3 = 163.9 \text{ l}.
\]

8. Following Eq. (9), the fine aggregate consumption is:

\[
\text{Aggr} = \left( 1000 - \frac{545.7}{3.1} + \frac{80}{7.85} + \frac{163.9}{1} \right) \cdot 2.65 = 1722 \text{ kg}.
\]

Finally, the SFRC mix composition is, kg/m\(^3\):

\[
C = 546; \ W = 164; \ \text{Aggr} = 1722; \ F = 80; \ \text{ADMI}X = 4.66.
\]

Analyzing the obtained composition and comparing it with example 1, it is worth noting that when limiting the fibers’ content it becomes more complicated to achieve the specified flexural tensile strength and therefore it is necessary to go over the cement and superplasticizer overconsumption, which leads to excessive compressive strength of 94.3 MPa instead of the required value of 70 MPa.

7. Conclusions

Adequate mathematical models of SFRC properties (compressive and flexural tensile strengths, superplasticizer consumption required for achieving the desired concrete mix workability) were obtained using mathematical experiments planning methods. The models consider the influence of such main factors as cement consumption, W/C and fiber content.

Analysis of the obtained models shows that SFRC compressive strength depends mainly on W/C and on cement consumption. The fiber content is a main factor affecting the flexural tensile strength, however decreasing the W/C is also positive.

Based on the obtained mathematical models, a design method for SFRC composition was proposed. This method allows taking into account the special properties of the investigated materials and provides a very easy possibility for concrete composition optimization by a given minimum cost criterion.

An additional advantage of the proposed method is a possibility to add a certain number of limitations. It allows simultaneous satisfaction of many quality indexes according to the given value.

In the authors’ opinion, application of the proposed method in concrete industry will allow producing attractive elements and construction of effective structures at minimal cost.

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