The process $D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$

**D. Delepine**  
Departamento de Física, DCI, Campus León, Universidad de Guanajuato, C.P. 37150, León, Guanajuato, México  
E-mail: delepine@fisica.ugto.mx

**Gaber Faisel**  
Department of Physics and Center for Mathematics and Theoretical Physics, National Central University, Chung-li, TAIWAN 32054.  
and Egyptian Center for Theoretical Physics, Modern University for Information and Technology, Cairo, Egypt  
E-mail: gfaisel@ncu.edu.tw

**Carlos A. Ramirez**  
Escuela de Física, Universidad Industrial de Santander, Bucaramanga, Colombia.  
E-mail: jpjdramirez@yahoo.com

**Abstract.** The main ingredients and the importance of the process $D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$ are described: its contribution to reveal the true nature of the scalar $K^*_0$ particle, what we know about the parametrization of its dynamics, its importance for CPV in the charm physics and possible new physics contributions.

1. Introduction  
The decay $D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$ is a Cabibbo Favored (CF [1]) process with large branching ratio (9.51(34) [2, 3]). It is dominated by the $K-\pi$ S-wave (7.3(2) % of the total), with two resonances: the $\kappa = K^*_0(800)$ and the $K^*_0(1430)$ (1.21(6) %). There is also a P-wave contribution by the $K^*(892)$ (1.1(1) %) and smaller components like the $\pi^+\pi^-$ ($I = 2, 1.4(3)$ %), etc. That makes this processes much simpler than for example $D^\pm \rightarrow K^0\pi^\pm\pi^0$ where one has the additional $\pi^\pm - \pi^0$ resonant interaction with scalar ($\sigma = f_0(600)$, etc.) and vector ($\rho$, etc.) contributions. Given that the dominant contribution is the $\kappa = K^*_0(800)$, this is a good place (maybe the best available) to study it. The same can be said about the strangeness scalar form factor [4]. Additionally one may be interested to see if the Watson’s theorem is violated [4, 5] and how, for the s-wave phase of the $K-\pi$ interaction [6]. Possible CP violations are interesting too, given that CPV has not been observed in any of the up quark sector.

The momenta involved in this process are defined as $p_D = p_K + p_1 + p_2 \equiv p_{K1} + p_2 \equiv p_{K2} + p_1 = p_K + p_{12}$. The Mandelstam variables and the partial decay rate are
\[ s = p_{K1}^2 = (p_K + p_1)^2, \quad t = p_{K2}^2 = (p_K + p_2)^2, \quad u = p_{12}^2 = (p_1 + p_2)^2 \]

\[ s + t + u = m_D^2 + m_K^2 + 2m_\pi^2 \]

\[ d\Gamma = \frac{|M|^2}{256\pi^3m_D^3}d|\mathbf{p}_K||\mathbf{p}_2'||d\sqrt{s}d\cos\theta' \] (1)

In order to obtain the amplitude \( \mathcal{M} \) one has to use the effective hamiltonian [1]

\[ \mathcal{H} = \frac{G_F}{\sqrt{2}} V_{ud}^* \left[ c_1 < K^- \pi^+ \pi^+ |\bar{s}c\bar{u}\bar{d}|D^+ > + c_2 < K^- \pi^+ \pi^+ |\bar{u}c\bar{s}|D^+ > \right] \]

\[ = \frac{G_F}{\sqrt{2}} V_{ud}^* \left[ a_1 < K^- \pi^+_1 |\bar{s}c|D^+ > < \pi^+_2 |\bar{u}d|0 > + a_2 < K^- \pi^+_1 |\bar{u}c|D^+ > \right] + (\pi_1 \leftrightarrow \pi_2) \]

\[ = \frac{G_F}{\sqrt{2}} V_{ud}^* \left[ a_1 A_1(s, t) + a_2 A_2(s, t) \right] \] (3)

with

\[ A_2 = \left[ t - u - \frac{\Delta^2_{D\pi}\Delta^2_{K\pi}}{m_K^2} \right] F_{\pi^+}^D(s) F_{K\pi}^K(s) + \frac{\Delta^2_{D\pi}\Delta^2_{K\pi}}{m_K^2} F_{0}^{D\pi}(s) F_{0}^{K\pi}(s) \]

\[ = -4F_{\pi^+}^D(s) F_{K\pi}^K(s) |\mathbf{p}_K'||\mathbf{p}_2'||\cos\theta' + \frac{\Delta^2_{D\pi}\Delta^2_{K\pi}}{s} F_{0}^{D\pi}(s) F_{0}^{K\pi}(s) \] (4)

The heavy quark FF can be taken as those of the dipole approximation:

\[ F_{\pi^+}^{D\pi}(s) = \frac{F_{\pi^+}^{D\pi}(0)}{1 - s/m_+^2}, \quad F_{\pi^+}^{D\pi}(0) = F_{0}^{D\pi}(0) = 0.624 \] (5)

with the masses \( m_+ = m_{D^0} = 2007 \text{ MeV}, \quad m_0 = m_{\pi^0} = 2352 \text{ MeV} \) [9, 2]. The \( A_1 \) is the dominant contribution and can be reduced to the form factors

\[ A_1 = < K^- \pi^+_1 |\bar{s}c|D^+ > < \pi^+_2 |\bar{u}d|0 > \]

\[ = -\frac{if_{\pi^+}^2 p_0^2}{m_D} \left[ f(p_K + p_1) + g(p_K - p_1) + r \rho_2 + \frac{ih}{m_D} \epsilon^{\mu\nu\rho\sigma} p^\nu(p_K + p_1)^\sigma(p_K - p_1)^\rho \right] \]

\[ = \frac{if_{\pi^+}^2}{2m_D} \left[ (s_K^2 - K^2_2) f + (u - t + K^2_2) g - 2m_\pi^2 r \right] \] (6)
2. Form Factors

In the previous section the amplitude was reduced to several Form factors (FF). There are two
types of them: those depending on the heavy physics (charm or $D$), like $F^{D\pi}_{+,0}(x)$ and those
depending on the light quarks physics. The former can be parametrized by a dipole dominance
(a single resonance) as in eq. (5 and so on). The remaining light quark form factors: $F_+$, $F_0$, $f$
and $g$ ($r$ is suppressed by the small pion mass) are more complicate. However one step further
can be obtained by reducing $f$ and $g$ to the $F_+$, $F_0$ form factors. In general $f$ and $g$ are functions
of $s$, $p_2^2$ and $\theta'$. The $p_2$ dependence is of course trivial given the on-shell condition $p_2^2 = m_\pi^2$.
The angular dependence is simple and can be taken into account by a partial wave expansion
(PWE), with only the $s$ and $p$ waves, given that tensor contributions are less than 1 % [9]. Thus
the only remaining dependence is on $s$.

Let us consider first the intermediate scalar contribution (see fig. 1), or $s$ wave:

$$J_\mu \simeq <p_K, p_1|J_{L, \mu}|p_D> = \sum_{K^0_\pi} <p_K, p_1|K^*_0 > BR^{K^*_0}(s) <K^*_0|J_{L, \mu}|p_D>$$

$$= c_d BR^{K^*_0}(-i) \left[ F^{DK^*_0}_+(p_2^2)(p_D + p_K1)_\mu + f^{DK^*_0}_-(p_2^2)p_{2\mu} \right]$$

$$= -ic_d BR^{K^*_0} \left( 2F^{DK^*_0}_+ p_{K1\mu} + [F^{DK^*_0}_+ + f^{DK^*_0}_-]p_{2\mu} \right)$$

$$= \frac{1}{m_D} \left[ fp_{K1\mu} + g(p_K - p_1)_\mu + rp_{2\mu} + \frac{ih}{m_D^2} \epsilon_{\mu\nu\alpha\beta} g^{\nu\alpha} p_{K1}^\beta (p_K - p_1)^\beta \right]$$

(7)

and

$$\frac{f}{m_D} = -2ic_d BW^{K^*_0}_0 F^{DK^*_0}_+ , \quad r \frac{m_D}{m_D} = -ic_d BW^{K^*_0}_0 [F^{DK^*_0}_+ + f^{DK^*_0}_-] , \quad g = h = 0$$

$$A_{1S} = f_{\pi} c_d BW^{K^*_0}_0 \left[ (s - m_D^2)F^{DK^*_0}_+(m_\pi^2) + m_\pi^2 f^{DK^*_0}_-(m_\pi^2) \right]$$

$$\simeq f_{\pi} \frac{\Delta_{K^*_0}^2}{f_{K^*_0}} (s - m_D^2) F^{DK^*_0}_+(0) F^{K\pi}_0(s)$$

(8)

with $p_2^2 = p_D - p_K - p_1 \equiv p_D - p_K1$, $s = p_{K1}^2$, $p_2^2 = m_\pi^2$, $BW_m(s) = [m^2 - s - i\sqrt{s} \Gamma(s)]^{-1}$
and given that $c_d BW^{K^*_0}_0(s) = \Delta_{K^*_0}^2 F^{K\pi}_0(s)/s f_{K^*_0}$.

This approximation (one resonance) seems too drastic but one has to remember that according
to the Watson theorem the $\pi - K$ scattering phase is the same in all cases, as long as one is in
the elastic range ($\sqrt{s} \leq 1.3$ GeV). Given analiticity and unitarity one can show that the form
factors are determinated up to multiplicative constant. This is what has been obtained in the
last eq. (8) and it is realized in the Omnes representation for the FF given below [4, 5]:

$$F_0(s) = F_0(0) \exp \left[ \frac{s}{\pi} \int_{s_{K\pi}}^{\infty} \frac{d\phi_0 d s'}{s'(s' - s - i\epsilon)} \right], \quad \sigma t_0 = \sin \phi_0 e^{i\phi_0}$$

(9)

where the elastic phase are measured in [6], and can be fitted as
Figure 1. Resonance (scalar, vector, etc.) approximation for the Form Factors

\[<\pi^+ K^-|T|\pi^+ K^-> = \frac{1}{3} \left( T^{3/2} + 2T^{1/2} \right) = \frac{2}{3} a_0 e^{i\phi_0} \]

\[a_0 e^{i\phi_0} = \frac{1}{2i} \left[ \left( \eta_0^{1/2} e^{2i\phi_0^{1/2}} - 1 \right) + \frac{1}{2} \left( \eta_0^{3/2} e^{2i\phi_0^{3/2}} - 1 \right) \right] \approx \frac{1}{2i} \left[ e^{2i\phi_0^{1/2}} + \frac{1}{2} e^{2i\phi_0^{3/2}} - \frac{3}{2} \right] \]

\[\delta_0^{1/2} = \tan^{-1} \left[ \frac{\Gamma_{K^0} m_{K^0}^2 p/\sqrt{s} p K_0^0}{m_{K^0}^2 - s} \right] + \tan^{-1} \left[ \frac{ap}{1 + bp^2 + cp^4} \right] \]

\[\delta_0^{3/2} = -\tan^{-1} \left[ \frac{a^{3/2} p}{1 + b^{3/2} p^2 + c^{3/2} p^4} \right] \] (10)

where \(p = p(s) = p(s = m_{K_0}^2)\). The parameters obtained in the fit are (all in the corresponding units of GeV): \(m_{K_0} = 1.44\) GeV, \(\Gamma_{K_0} = 0.32\) GeV, \(a = 1.91\), \(b = 1.5\), \(c = 0.82\), \(a^{3/2} = 1.01\), \(b^{3/2} = 0.5\), \(c^{3/2} = 0.9\). This expression is valid in the elastic range, \(4m_{K_0}^2 < s \leq (m_K + m_{\eta'})^2 \approx (1.3\) GeV\(^2\), before the \(\eta' - K\) channel is open. The inelasticity contribution from the \(\eta - K\) channel seems to be negligible and it is not going to be considered in this work [4].

The vector contribution or \(p\) wave, with \(p_2 = p_D - p_K - p_1 \equiv p_D - p_V\), \(s = p_V^2\)

\[J_\mu \equiv <p_K, p_1|J_{L,\mu}|p_D> \approx \sum_{K^*} <p_K, p_1|K^*> BK_{K^*}^{\dagger}(s) <K^*|J_{L,\mu}|p_D> \]

\[= \sum_{\lambda^*} g^{A_\mu} (p_K - p_1) \cdot \epsilon^{\lambda^*} BR_{K^*} \left[ F_{1A} \epsilon^{\lambda}_\mu + F_{2A} p_D \cdot \epsilon^{\lambda} p_D \mu + F_{3A} p_D \cdot \epsilon^{\lambda} p_D \mu + i F_V \epsilon_{\mu \lambda \alpha \beta} \epsilon^{\alpha \lambda} \mu \phi \right] \]

\[= -g^{A_\mu} BR_{K^*} \left[ F_{1A} (p_K - p_1) \mu + F_{2A} p_D \cdot (p_K - p_1) p_D \mu + F_{3A} p_D \cdot (p_K - p_1) q_\mu \right] \]

\[+ i F_V \epsilon_{\mu \lambda \alpha \beta} (p_K - p_1) p_D \lambda \mu \phi \frac{\Delta K_{\pi}}{m_{\pi}^2} \left( F_{1A} \epsilon^{\phi}_\mu + F_{2A} p_D \cdot \epsilon^{\phi} p_D \mu + F_{3A} p_D \cdot \epsilon^{\phi} p_D \mu \right) \]

\[= \frac{1}{m_D} \left[ f p_{V\mu} + g (p_K - p_1) \mu + r p_2 \mu + \frac{i}{m_D^2} \epsilon_{\mu \alpha \beta} \phi \right] (p_K - p_1)^{\beta} \] (11)

to obtain
\[ \frac{f}{m_D g^* BR_{K^*}(s)} = \frac{\Delta_{K^*}^2}{m^2} F_{1A} - \left[ p_D \cdot (p_K - p_1) - \frac{\Delta_{K^*}^2}{m^2} p_D \cdot p_V \right] F_{2A}, \quad \frac{g}{m_D g^* BR_{K^*}} = - F_{1A} \]

\[ \frac{r}{m_D g^* BR_{K^*}} = - \left[ p_D \cdot (p_K - p_1) - \frac{\Delta_{K^*}^2}{m^2} p_D \cdot p_V \right] (F_{2A} + F_{3A}), \quad \frac{h}{m_D g^* BR_{K^*}} = - F_{1(12)} \]

where the FF $F_{1A}$ and $F_V$ depend on the heavy quark physics and are evaluated at $p_2^2 = m_2^2$. Using the identities

\[ p_D \cdot (p_K - p_1) = \frac{1}{2} (t - u + \Delta_{K^*}^2), \quad p_D \cdot p_V = \frac{1}{2} (m_2^2 + \Delta_{D^*}^2) \]

\[ p_D \cdot (p_K - p_1) - \frac{\Delta_{K^*}^2}{m^2} p_D \cdot p_V = \frac{1}{2} \left( t - u - \frac{\Delta_{K^*}^2 \Delta_{D^*}^2}{m^2} \right) = - 2 |p_K| |p_2| |\cos \theta' \right) \]

one can obtain the vector contribution

\[ A_{1V}(s) = < K^- \pi^+_1 | \bar{s} c | D^+ > |\bar{u} d| 0 > = \frac{i f_\pi}{2 m_D} \left[ (s - \Delta_{D^*}^2) f + (u - t + \Delta_{K^*}^2) g - 2 m_2^2 r \right] \]

\[ = -i f_\pi g^* BW^*(s) \left[ 2 F_{1A} - (s - \Delta_{D^*}^2) F_{2A} + 2 m_2^2 (F_{2A} + F_{3A}) \right] |p_K| |p_2| |\cos \theta' \]

\[ = -4 f_\pi g^* BW^* A_0(s) \sqrt{s} |p_K| |p_2| |\cos \theta' = -4 f_\pi \frac{F_+(s)}{f_\pi m_2} A_0(s) \sqrt{s} |p_K| |p_2| |\cos \theta' \]

and the FF are parametrized again in the dipole approximation

\[ V(s) = \frac{V(0)}{1 - s/m_V^2}, \quad A_i(s) = \frac{A_i(0)}{1 - s/m_A^2} \]

with $m_A = 2.63(10)(13) \simeq m_{D^*} = 2.5$ GeV, $A_1(0) = 0.6226(56)(65)(74)$ and $m_V = m_{D^*} = 2.1$ GeV, $r_V = V(0)/A_1(0) = 1.463(17)(31)$, $r_2 = A_2(0)/A_1(0) = 0.801(20)(20)$ [11].

The Vector FF have been taken as $(q = P_P - P_V)$

\[ < V(p_V, \epsilon)|J_{\mu}^V|P(p_P) > = -i \left[ (m_V + m_P) \left( \epsilon^* - \frac{\epsilon^* \cdot q}{q^2} q \right) A_1(q^2) - (p_P + p_V - \frac{\Delta_{PV}^2}{q^2} q) \right] \]

\[ \frac{\epsilon^* \cdot q A_2(q^2)}{m_V + m_P} + 2 m_V q_{\mu} \frac{\epsilon^* \cdot q A_0(q^2)}{q^2} \]

\[ - \frac{2 V(q^2)}{m_P + m_V} \epsilon_{\mu \nu \alpha \beta} \epsilon^{* \nu} p_\nu p_\beta \]

\[ = F_{1A} \epsilon^*_\mu + F_{2A} p_{P_{\mu}} \epsilon^* + F_{3A} p_{P_{\mu}} \epsilon^* q^\mu + i F_V \epsilon_{\mu \nu \alpha \beta} \epsilon^{* \nu} p^\alpha p^\beta \]

and the two representation are related as

\[ F_{1A} = -i (m_V + m_P) A_1, \quad F_{2A} = \frac{2 i A_2}{m_V + m_P}, \quad F_V = \frac{2 i V}{m_V + m_P} \]

\[ F_{3A} = \frac{i m_V + m_P}{q^2} A_1 - \frac{2 i m_V}{q^2} A_0 - i \frac{1 + \Delta_{PV}^2 q^2}{m_V + m_P} A_2 \]

and to cancel the poles at $q^2 = 0$ one has that $2 m_V A_0(0) = (m_P + m_V) A_1(0) - (m_P - m_V) A_2(0), A_3(0) = A_0(0) = 0.71$ [12] (1991), in agreement with Babar measurement [11].
Figure 2. Form Factors with their phases. The black (red) line is the Vector (scalar)

Again one resonance has been taken into account (a much better approximation than in the scalar case), but the same considerations mentioned for the scalar case are valid here too.

The vector form factor $F_\pi$ is dominated by the $K^*(890)$ resonance, with $m_\pi \simeq g_\pi f_\pi = 891.7$ MeV and $\Gamma_\pi = 51$ MeV [2].

$$ F_\pi(s) = \frac{m_\pi^2}{m_\pi^2 - s - i\sqrt{s} \Gamma_\pi(s)}, \quad \Gamma_\pi(s) = \Gamma_\pi \frac{m_\pi^2}{s} \left( \frac{p}{p_\pi} \right)^3 $$

$$ p = \frac{1}{2\sqrt{s}} \lambda^{1/2}(s, m_K^2, m_\pi^2), \quad p_\pi = \frac{1}{2m_\pi} \lambda^{1/2}(m_\pi^2, m_K^2, m_\pi^2) $$  \hspace{1cm} (18)

Escribanos’s paper [9] obtains

$$ A_1 \simeq f_\pi \lambda_{S}^{\text{eff}}(m_\pi^2 - s) F_0^{K\pi}(s) - 4f_\pi \chi_{V}^{\text{eff}} N(s) F_0^{K\pi}(s) \left| P K^0 \right| P_\pi \left| \right|\cos \theta' \right) $$ \hspace{1cm} (19)

with [9]

$$ \chi_{S}^{\text{eff}} \simeq \frac{g_{K^*\pi} F_0^{DK^*}(m_\pi^2)}{\Gamma_{K^*}(m_{K^*}) F_0^{K\pi}(m_{K^*})} = 4.4(28) \text{ GeV}^{-1}, \quad \chi_{V}^{\text{eff}} \simeq f_{K^*}^{-1} = 4.9(2) \text{ GeV}^{-1} $$ \hspace{1cm} (20)

and $F_0^{DK^*}(m_\pi^2) = 1.24(7)$ and $F_{V}^{DK^*}(m_\pi^2) = 0.76(7)$ [9]

3. Experimental data and fitting

At the moment the best experimental results are those from Cleo [8], but earlier results and several analysis are available like those of the Focus collab. [10] and so on. The extraction of
Figure 3. The spectra: from Escribano, Total one (data are from Cleo), low and high energy. The black lines are the total, the red ones the dominant scalar contributions and the blue lines are the vector contributions. The BR=7.0 %, $\chi_S = 4.2$ and $\chi_P = 4.8$

| Mode           | BR[\%] | $A_{CP}$[\%] |
|----------------|--------|--------------|
| $D^0 \rightarrow K^- \pi^+$ | 3.95(5) | -            |
| $D^0 \rightarrow K^- K^+$      | 0.143(3) | 0.4(5)(1)    |
| $D^+ \rightarrow K^+ \pi^+$    | 0.394(7) | -0.04(3)(1)  |
| $D^{\pm} \rightarrow \pi^+ \pi^- \pi^\pm$ | 1.47(7) | -0.71(19)(20) |
| $D^{\pm} \rightarrow K^{\mp} \pi^\pm$ | 0.327(22) | 1.7(42)    |
| $D^{\pm} \rightarrow K^{\mp} \pi^\pm \pi^\pm$ | 9.51(34) | -0.5(4)(9) |
| $D^{\pm} \rightarrow K^{\mp} \pi^\pm \pi^0$ | 6.90(32) | 0.3(9)(3)   |
| $D^{\pm} \rightarrow K^{\mp} \pi^\pm$ | 0.98(4) | 0.39(61)    |

Table 1. Direct CP in $D$ nonleptonic decays, from table 156, p. 218 in HAFG [2]

the $s$ wave phase (to compare with Watson theorem) is not clear as the parametrization of the dominant $s$-wave is cumbersome, given that it is dominated by the broad $\kappa$ resonance with a large overlapping with the $K_0^*(1430)$ resonance. This is the case in the Isobar parametrization. For this reason another parametrization (the so called MIPWA) was proposed that allows to extract the $s$-wave phase and amplitude in a cleaner way. Unfortunately it seems that the Watson theorem is violated (there is no agreement with the phases obtained by the Lass coll. [6]), and additional contributions are present like $\pi^- \pi$ interactions [8], three body Final state interactions [7] and so on. From the theoretical side, the paper of Escribano, that we follow here seems to do a good job. The obtained fit is shown in Fig. 3. Unfortunately the $\pi^- \pi$ (this contribution is not included) spectra is not well fitted and there is no explanation for the violation of the Watson theorem (even if it is by an overall phase). In order to obtain a better fit it is necessary to include these contributions in a proper manner. New results from LHCB [13, 14, 15] are expected and certainly will contribute to clarify the situation.

4. CP and New Physics

CP has been predicted in the Standard Model (SM) to be of order $10^{-3}$ [16, 1] in several processes. This is of course interesting given that it may be at the reach of future experiments, as it is shown in table 1. However for this process no calculation has been done and we are exploring the SM and SUSY contributions in the present work. A more detailed calculation will be given in a future work [19]

In the case of CF processes, the contributions of the SM tree diagram have no weak CP violating phase required for non vanishing CP asymmetry. One has to look for loop diagrams
that can lead to the required weak CP violating phase. At one loop level, the only contribution can be obtained by considering the box diagram in Fig. 4 [18]

$$\Delta \mathcal{H} = \frac{G_F m_W^2}{2 \pi^2} \left[ V_{cd} V_{du} (V_{us} V_{ud} f_{ud} + V_{sc} V_{cd} f_{cd} + V_{st} V_{td} f_{td}) + V_{cs} V_{su} (V_{us} V_{ud} f_{us} + V_{sc} V_{cd} f_{cs} + V_{st} V_{td} f_{ts}) \right] \overline{u}_L \bar{s}_L \, (21)$$

using the unitarity relations for the CKM matrix one gets the box contribution for the Wilson coefficient

$$\Delta c_2 = \frac{G_F m_W^2}{\sqrt{2} \pi^2 V_{cs} V_{ud}} \left[ V_{cd} V_{du} [V_{sc} V_{cd} (f_{cd} - f_{ud}) + V_{st} V_{td} (f_{td} - f_{ud})] + V_{cs} V_{su} [V_{sc} V_{cd} (f_{cs} - f_{us}) + V_{st} V_{td} (f_{ts} - f_{us})] \right]$$

$$= \frac{G_F m_W^2}{\sqrt{2} \pi^2 V_{cs} V_{ud}} \left[ V_{cs} V_{su} [V_{sc} V_{cd} (f_{cs} - f_{cd} - f_{us} + f_{ud}) + V_{st} V_{td} (f_{ts} - f_{td} - f_{us} + f_{ud})] \right]$$

$$+ V_{cb} V_{bu} [V_{sc} V_{cd} (f_{cb} - f_{cd} - f_{ub} + f_{ud}) + V_{st} V_{td} (f_{tb} - f_{td} - f_{ub} + f_{ud})] \right] \right]$$

$$\simeq \frac{G_F m_W^2}{\sqrt{2} \pi^2 V_{cs} V_{ud}} \cdot 3.8 \cdot 10^{-7} e^{0.05 i} \simeq 10^{-8} e^{0.05 i} \, (22)$$

with

$$f_{xy} = \frac{7xy - 4}{4(1 - x)(1 - y)} + \frac{1}{x - y} \left[ \frac{y^2 \log y}{(1 - y)^2} (1 - 2x + \frac{xy}{4} - \frac{x^2 \log x}{(1 - x)^2} (1 - 2y + \frac{xy}{4} \right) \, (23)$$

It should be noted that penguin diagrams at one loop level do not contribute to the process under this study and can contribute only at two loop level (dipenguin diagrams). However, their weak phase is estimated to be one order of magnitude less than that generated by the box diagram with their absolute value of the same order as box diagram. Clearly from eq.22, the CP violating weak phase is so small and lead to a negligible CP asymmetry. Thus, it is interesting to look for new physics contributions that can enhance the weak CP phase. As an example of weak phases producing CPV, one can mention SUSY models. In the case of R-parity conservation SUSY models, the contributions to Wilson coefficients are of the same order of the SM and thus we do not except them to enhance the CP asymmetry. The charged Higgs contributions also are suppressed by the light quark masses. A larger contribution can be obtained in SUSY models when R-parity is violated [19].

**Acknowledgement**

Gaber Faisel’s work is supported by the National Science Council of R.O.C. under grants NSC 99-2112-M-008-003-MY3 and NSC 99-2811-M-008-085. D.D. has been supported by PROMEP and DINPO project from Guanajuato University. C.A.R. wish to thank to D.D. and to the Departamento de Física, DCI, Campus León, Universidad de Guanajuato, for its hospitality.
References
[1] M. Artuso, B. Meadows and A. A. Petrov, Ann. Rev. Nucl. Part. Sci. 58, 249 (2008) [arXiv:0802.2934 [hep-ph]].
[2] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).
[3] D. Asner et al. [Heavy Flavor Averaging Group HFAG], arXiv:1010.1589 [hep-ex].
[4] M. Jamin, J. A. Oller and A. Pich, Nucl. Phys. B 622, 279 (2002) [arXiv:hep-ph/0110193].
[5] J. F. Donoghue, J. Gasser and H. Leutwyler, Nucl. Phys. B 343, 341 (1990).
[6] D. Aston et al., Nucl. Phys. B 296, 493 (1988).
[7] B. Kubis, arXiv:1108.5866 [hep-ph].
[8] G. Bonvicini et al. [CLEO Coll.], Phys. Rev. D 78, 052001 (2008) [arXiv:0802.4214 [hep-ex]].
[9] D. R. Boito and R. Escribano, Phys. Rev. D 80, 054007 (2009) [arXiv:0907.0189 [hep-ph]].
[10] J. M. Link et al. [The FOCUS Collab.], Phys. Lett. B 681, 14 (2009) [arXiv:0905.4846 [hep-ex]].
[11] P. del Amo Sanchez et al. [The BABAR Coll.], Phys. Rev. D 83, 072001 (2011) [arXiv:1012.1810 [hep-ex]].
[12] J. G. Korner, K. Schilcher, M. Wirbel and Y. L. Wu, Z. Phys. C 48, 663 (1990).
[13] M. Gersabeck [LHCb Collaboration], PoS B E A U T Y 2 0 1 1, 0 2 0 (2011).
[14] B. O'Leary et al. [SuperB Collaboration ], [arXiv:1008.1541 [hep-ex]]. Super-B
[15] T. Aushev et al., arXiv:1002.5012 [hep-ex]. Super KEKB
[16] R. Aaij et al. [LHCb Collaboration], arXiv:1112.0938 [hep-ex].
[17] O. Gedalia and G. Perez, TASI 2009 Lectures - Flavor Physics, arXiv:1005.3106 [hep-ph].
[18] T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981) [Erratum-ibid. 65, 1772 (1981)].
[19] D. Delepine, G. Faisel and C. Ramirez, work in progress.