Fig. 1(a)
Fig. 1(b)

- $w = 8.0$
- $h_0 = 0.5$
Fig. 1(c)
Fig. 1(d)

\[ w = 8.0 \]
\[ h_0 = 2.0 \]
Fig. 2(a)
$\frac{Q}{w} = 1.65 \quad h_0 = 1.40$

Fig. 2(b)
Zero-temperature dynamic transition in the random field Ising model: A Monte Carlo study

Muktish Acharyya*

Institute for Theoretical Physics
University of Cologne, 50923 Cologne, Germany

Abstract: The dynamics of a random (quenched) field Ising model (in two dimension) at zero temperature in the presence of an additional sinusoidally oscillating homogeneous (in space) magnetic field has been studied by Monte Carlo simulation using the Metropolis single spin flip dynamics. The instantaneous magnetisation is found to be periodic with the same periodicity of the oscillating magnetic field. For very low values of amplitude of oscillating field and the width of randomly quenched magnetic field, the magnetisation oscillates asymmetrically about a nonzero value and the oscillation becomes symmetric about a zero value for higher values of amplitude of oscillating field and the width of the quenched disorder. The time averaged magnetisation over a full cycle of the oscillating magnetic field defines the dynamic order parameter. This dynamic order parameter is nonzero for very low values of amplitude of oscillating magnetic field and the width of randomly quenched field. A phase boundary line is drawn in the plane formed by the amplitude of the oscillating magnetic field and the width of the randomly quenched magnetic field. A tricritical point has been located, on the phase boundary line, which separates the nature (discontinuous/continuous) of the dynamic transition.

Keywords: Random field Ising model, Monte Carlo simulation
PACS Numbers: 05.50 +q

*E-mail: muktish@thp.uni-koeln.de
I. Introduction

The Ising model in a quenched random field or the so called random field Ising model (RFIM) is an active field of modern research over a few decades. The behaviour of this model at zero temperature and the zero temperature ferro-para phase transition has been studied widely [1]. Recently, a simple model was proposed [2] for hysteresis in magnetic model systems incorporating the return point memory and Barkhausen noise [3]. For the smaller values of the quenched disorder, the steady-state magnetisation has a jump discontinuity at a critical value of the uniform external magnetic field and it vanishes continuously for larger values of disorder strength. The transition from ‘jump’ to ‘no jump’ in magnetisation was observed [4] at a critical value of the width of the Gaussian disorder and the scaling behaviour was observed in the vicinity of this critical value of the disorder. The exact solution [5] of this model in one dimension shows no jump discontinuity for Gaussian disorder.

Recently, Dhar et al [6] studied the zero-temperature hysteresis in the RFIM on a Bethe lattice. Surprisingly, they observed for coordination number $z = 3$ that there is no jump discontinuity for any nonzero strength of Gaussian disorder. However, for larger values of coordination number and for very weak disorder, the magnetisation shows a jump discontinuity as a function of external uniform field, which disappears for strong disorder.

All these transitions discussed above are static transitions characterised by the steady state magnetisation. No attempt has been made to investigate the dynamic transition in this model. The dynamic transition in the absence of quenched random disorder and at finite (nonzero) temperatures are studied widely [7-9]. For an introduction, it would be convenient to review briefly the previous studies on dynamic transition in the following paragraph.

The dynamic transition, in the kinetic Ising model in presence of a sinusoidally oscillating magnetic field, was first studied by Tome & Oliveira [7]. They solved the mean-field dynamic equation for the averaged magnetisation and defined the time averaged magnetisation over a full cycle of the oscillating field as the dynamic order parameter. In the plane formed by the temperature and the amplitude of the oscillating field, they have drawn a dynamic phase boundary below which the dynamic phase is ordered (nonzero value of the dynamic order parameter) and above which dynamic order parameter vanishes characterising a disordered phase. They have also located a tricritical point (TCP) on the phase boundary line which separates the nature (discontinuous/continuous) of this transition. However, this transition is not truly dynamic in nature since it exists even in the static (zero frequency) limit due to the absence of any fluctuation. The true dynamic transition (in presence of thermal fluctuation) was studied extensively [8,9]. There are some indications in the recent studies [10] that this transition possesses some features analogous to equilibrium thermodynamic phase transitions. It may be mentioned here that the evidence of
dynamic transition, associated to the dynamic symmetry breaking, is found in the kinetic Ising model in presence of a randomly varying (in time but uniform in space) magnetic field [10].

In this paper, the dynamic transition has been studied, in the random field Ising model driven by an (additional) sinusoidally varying (in time) homogeneous (in space) magnetic field, by Monte Carlo simulation using the Metropolis single spin-flip dynamics at zero temperature. This has been organised as follows: in section II the model and the simulation scheme are discussed, the simulation results are given in section III and the paper ends with a summary of the work in section IV.

II. The model and simulation

A square lattice of linear size $L$ is taken. Each site is labelled by an integer $i$ and carries an Ising spin $S_i$ ($S_i = \pm 1$) which interacts with all its nearest neighbours (spins) with a ferromagnetic interaction strength $J$. At each site $i$, there is a local quenched random field $h_i$. The random fields $h_i$ are assumed to be independent and identically distributed random variables with a rectangular probability distribution $P(h_i)$. The random field $h_i$ can take any value from $-w/2$ to $+w/2$ with the same probability. The width of the distribution is $w$. In addition, there is a uniform (in space) magnetic field $h(t)$ which is varying sinusoidally ($h(t) = h_0 \cos(\omega t)$) in time. The amplitude and the frequency are denoted by $h_0$ and $\omega$ respectively. This kind of model is described by the Hamiltonian

$$H = -J \sum_{<ij>} S_i S_j - \sum_i h_i S_i - h(t) \sum_i S_i,$$  \hspace{1cm} (1)

under the periodic boundary condition. For simplicity, the interaction strength $J$ has been set equal to unity throughout the study.

The zero-temperature single spin-flip dynamics is specified by the transition rates (W) [11]

$$W(S_i \rightarrow -S_i) = \Gamma, \text{ if } \Delta E \leq 0 \text{ and } W(S_i \rightarrow -S_i) = 0, \text{ otherwise} \hspace{1cm} (2)$$

where $\Delta E$ is the change in energy due to spin flip. In words the algorithm is: never flip the chosen spin if this process would increase the energy and flip otherwise. We have started with all spins are up ($S_i = +1$) as an initial condition and updated the lattice sequentially using the above flipping algorithm. One such full scan over the entire lattice consists a Monte Carlo step per spin (or MCS). The instantaneous magnetisation (per site) $m(t)$ is easily calculated,

$$m(t) = \frac{1}{L^2} \sum_i S_i.$$ \hspace{1cm} (3)
After an initial transient period the instantaneous magnetisation \( m(t) \) has been found to be stabilised and periodic with the same periodicity of the applied oscillating field. The dynamic order parameter \( Q \), is defined as

\[
Q = \frac{\omega}{2\pi} \oint m(t) dt,
\]

which can be viewed as the time averaged magnetisation over a full cycle of the oscillating field. For a particular values of \( h_0, \omega \) and \( w \) the dynamic order parameter \( Q \) is calculated by averaging over 20 different random disorder (quenched) realisations.

### III. Results

The simulations are performed on a square lattice of linear size \( L = 100 \) and a particular value of the frequency \( (\omega = 0.01 \times 2\pi) \) of the oscillating magnetic field. The time is measured in units of Monte Carlo steps per spin or MCS and the values of random field and the oscillating field are measured in units of interaction strength \( J \).

It has been observed numerically that, for fixed values of \( h_0 \) and \( w \), the magnetisation becomes periodic (in time) with the same periodicity as the applied sinusoidal magnetic field. For the smaller values of the quenched disorder \( (w = 8.0) \) and the field amplitude \( (h_0 = 0.5) \), the magnetisation oscillates asymmetrically about the zero line (Fig. 1a) i.e., the system remains in a dynamically symmetry broken phase. Consequently, the hysteresis \( (m - h) \) loop resides on the upper half plane formed by \( h(t) \) and \( m(t) \) (Fig. 1b). So, the time averaged magnetisation over a full cycle of the oscillating field, the dynamic order parameter, is nonzero in the symmetry broken phase. By increasing the field amplitude \( (h_0 = 2.0) \) keeping \( w \) fixed \( (w = 8.0) \), it was observed that the system acquires a dynamically symmetric phase, i.e., the magnetisation oscillates symmetrically about the zero line (Fig. 1c). The hysteresis loop is also symmetric (Fig. 1d). Consequently, the value of the dynamic order parameter \( Q \) is zero in this dynamically disordered (symmetric) phase.

In the dynamically disordered phase, the dynamic order parameter \( Q \) can be kept at zero in two ways, either by increasing the random field width \( w \) for a fixed field amplitude \( h_0 \) or vice versa. So, in the plane formed by the field amplitude \( (h_0) \) and the width \( (w) \) of the quenched disorder (random field), one can think of a boundary line, below which \( Q \) is nonzero and above which it vanishes. Figure 2a displays such a phase boundary in the \( h_0 - w \) plane obtained by Monte Carlo simulation. The nature (discontinuous/continuous) of the transition depends on the value of \( w \) and \( h_0 \) on the phase boundary line. The transition across the upper part of phase boundary line is discontinuous and it is continuous for the rest part of the boundary. A tricritical point on the phase boundary line separates these natures. Figure 2b demonstrates two typical transitions for two sets of values of \( w \) and \( h_0 \) lying just in the left and
right sides of the tricritical point (TCP). In the case of discontinuous transition, the dynamic order parameter $Q$ jumps to a small nonzero value and then vanishes continuously. The uncertainty in the location of the TCP on the phase boundary are shown by the circle enclosing it.

IV. Summary

The zero-temperature dynamics of random field Ising model in presence of an additional sinusoidally varying homogeneous field has been studied by Monte Carlo simulation using the Metropolis single spin-flip algorithm. The instantaneous magnetisation has been calculated and found to be periodic with the same periodicity of the applied oscillating field. For quite small values of amplitude of the oscillating field and the width of randomly quenched field, the magnetisation oscillates asymmetrically about a nonzero value giving an asymmetric hysteresis loop, and the systems remains in a dynamically ordered (nonzero dynamic order parameter) symmetry broken phase.

On the other hand, for quite large values of disorder-strength and field-amplitude, the magnetisation oscillates symmetrically about zero. As a consequence, the hysteresis loop is symmetric and the system remains in a symmetric dynamically disordered (dynamic order parameter vanishes) phase. The phase boundary line is sketched in the plane formed by the disorder-strength and field-amplitude. The nature (discontinuous/continuous) of the transition across the phase boundary depends on the value of disorder strength. For low values of the disorder strength the transition is discontinuous and becomes continuous for higher values. A tricritical point (located on the phase boundary) separates the nature of the transition.

Experimental evidence of a dynamic transition via a dynamical symmetry (of the hysteresis loop) breaking has been found [12] in the ultrathin ferromagnetic Co/Cu(001) sample by magneto-optic Kerr effect. However, the phase boundary and its details (TCP) are not yet investigated experimentally.

Recently, a theory of the hysteresis loop in ferromagnets controlled by the domain wall motion has been developed [13], where the domain walls are considered as plane or linear interfaces moving in a random medium under the action of the external $ac$ magnetic field.

Acknowledgments: This work is supported by SFB 341.

References
[1] Y. Imry, J. Stat. Phys, 34 (1984) 849; For a recent review of the random-field Ising model, see T. Nattermann in Spin glasses and random field Ising models (1997) Ed. A. P. Young (World-Scientific) (in press; cond-mat/9705295)

[2] J. P. Sethna, K. Damen, S. Kartha, J. A. Krumhansl, B. W. Roberts and J. D. Shore, Phys. Rev. Lett., 70 (1993) 3347; O. Percovic, K. Dahmen and J. P. Sethna, Phys. Rev. Lett., 75 (1995) 4528; K. Dahmen and S. Kartha, Phys. Rev. B, 53 (1996) 14872

[3] K. P. O. Brien and M. B. Weissman, Phys. Rev E, 53 (1994) 3446.

[4] O. Perkovic, K. Dahmen and J. P. Sethna, Unpublished (1996) cond-mat/960972

[5] P. Shukla, Physica A, 233 (1996) 235; Physica A, 233 (1996) 242

[6] D. Dhar, P. Shukla and J. P. Sethna, J. Phys. A:Math & Gen. 30 (1997) 5259

[7] T. Tome and M. J. de Oliveira, Phys. Rev. A, 41 (1990) 4251

[8] W. S. Lo and R. A. Pelcovits, Phys. Rev. A, 42 (1990) 7471; S. W. Sides, R. A. Ramos, P. A. Rikvold and M. A. Novotny, J. Appl. Phys, 81 (1997) 5597; J. Appl. Phys., 79 (1996) 6482

[9] M. Acharyya and B. K. Chakrabarti, in Annual Reviews of Computational Physics, Vol 1, pp. 107, Ed. D. Stauffer, (World-Scientific, Singapore), 1994; Phys. Rev. B 52 (1995) 6550; M. Acharyya, Physica A, 235 (1997) 469; Phys. Rev. E, 56 (1997) 2407; Phys. Rev. E, 56 (1997) 1234

[10] M. Acharyya, (1997) Unpublished

[11] See, for example, K. Kawasaki, Phase transition and critical phenomena, Vol 2, Ed. C. Domb and M. S. Green (London, Academic) 1972.

[12] Q. Jiang, H. N. Yang and G. C. Wang, Phys. Rev. B 52, (1995) 14911

[13] I. F. Lyuksyutov, T. Nattermann and V. Pokrovsky, Preprint (1997) cond-mat/9709194
Figure Captions

Fig. 1. (a) The time variation of magnetisation $m(t)$ and the field $h(t)$ for $h_0 = 0.5$ and $w = 8.0$, (b) the corresponding hysteresis $(m(t) - h(t))$ loop (see the scale of y-axis), (c) the time variations of magnetisation $m(t)$ and $h(t)$ for $h_0 = 2.0$ and $w = 8.0$, (d) the corresponding hysteresis $(m(t) - h(t))$ loop.

Fig. 2. (a) The phase boundary (of the dynamic transition) in $w - h_0$ plane. The tricritical point (TCP) lies within the encircled region. The boundary of the circle is the uncertainty associated in locating the TCP, (b) two typical transitions just below and above the tricritical point which show the different natures (discontinuous/continuous) of the transitions.