Statistical data analysis in the DANSS experiment

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Abstract. DANSS is a one cubic meter highly segmented solid scintillator detector. DANSS is placed under a 3.1 GW industrial reactor at the Kalininskaya NPP (Russia) on a movable platform. The distance from the reactor core center is varied from 10.9 m to 12.9 m on-line. The inverse beta decay (IBD) process is used to detect antineutrinos. DANSS detects about 5000 IBD events per day with the background from cosmic muons at the level of 2%. Sterile neutrinos are searched for assuming a 4 neutrino model (3 active and 1 sterile neutrino). The exclusion area in the sterile neutrino parameter plane is obtained using a ratio of positron energy spectra collected at different distances. New analysis that uses information about relative counting rates as well as changes in positron energy spectra shapes is described. We present recent results based on 3.5 million IBD events collected at 3 distances from detector to reactor core. The excluded area covers a wide range of the sterile neutrino parameters up to $\sin^2 2\theta_{ee} < 0.008$ in the most sensitive region. The significance of the best-fit point in the parameters space is $1.5\sigma$.

1. Introduction
The number of active neutrinos is limited to 3 by the measurements of the Z boson decay width [1]. However, existence of additional sterile neutrinos is not excluded. The deficit of $\nu_e$ in calibration runs with radioactive sources in the Ga solar neutrino experiments SAGE and GALEX [2, 3] (“Galium Anomaly”) and deficit in reactor antineutrino fluxes [4] (“Reactor Antineutrino Anomaly”) can be explained by active-sterile neutrino oscillations [5, 6]. MiniBooNE collaboration recently presented an evidence for electron (anti)neutrino appearance in the muon (anti)neutrino beams [7]. The effect significance reaches $6.0\sigma$ when the MiniBooNE and LSND results are combined. Collaboration Neutrino-4 claimed an observation of oscillations to sterile neutrino with significance of $3.2\sigma$ [8].

If a sterile neutrino exists, some electron antineutrinos would undergo a transition to the sterile ones, which would lead to oscillations in the number of electron antineutrinos depending on the distance. The survival probability of a reactor $\bar{\nu}_e$ at short distances in the 4$\nu$ mixing scenario (3 active and 1 sterile neutrino) is described by the formula:

$$1 - \sin^2 2\theta_{ee} \sin^2 \left( \frac{1.27\Delta m_{41}^2 [eV^2] L [m]}{E_{\nu}[MeV]} \right),$$

where $\Delta m_{41}^2 = m_4^2 - m_1^2$ is the difference in the squared masses of neutrino mass states, $\sin^2 2\theta_{ee}$ is the mixing parameter, $E_{\nu}$ is the antineutrino energy, and $L$ is the distance from the production...
point to the detection point. A fairly large value is expected for $\Delta m^2_{41}$ (∼1 eV$^2$, which is much larger than the difference of the squared masses of the known neutrinos). Therefore, the known neutrino oscillations can be neglected at short distances. Thus, the existence of sterile neutrinos would manifest itself in distortions of the $\tilde{\nu}_e$ energy spectrum at short distances. At longer distances these distortions are smeared out and only the rate is reduced by a factor of $1 - \sin^2 2\theta_{ee}/2$. Measurements at only one distance from a reactor core are not sufficient since the theoretical description of the $\tilde{\nu}_e$ energy distribution is considered not to be reliable enough. The most reliable way to observe such distortions is to measure the $\tilde{\nu}_e$ spectrum with the same detector at different distances. In this case, the shape and normalization of the $\tilde{\nu}_e$ spectrum as well as the detector efficiency are canceled out. The DANSS experiment uses this strategy and measures $\tilde{\nu}_e$ spectra at 3 distances from the reactor core centre: 10.9 m, 11.9 m, and 12.9 m to the detector centre. Antineutrinos are detected by means of the Inverse Beta Decay (IBD) reaction

$$\tilde{\nu}_e + p \rightarrow e^+ + n \text{ with } E_{\tilde{\nu}} \approx E_{e^+} + 1.8 \text{ MeV.} \quad (2)$$

2. The DANSS Detector
DANSS is highly segmented plastic scintillator detector. It consists of 2500 polystyrene-based scintillator strips with a thin Gd-containing surface coating. The coating serves as a light reflector and a $(n, \gamma)$-converter simultaneously. The detector is placed inside a composite shielding of copper, borated polyethylene and lead to suppress the external radiation and neutron background. Detector is surrounded on 5 sides (excluding bottom) by double layers of scintillator plates to veto cosmic muons. DANSS is installed under the core of a 3.1 GW$_{th}$ industrial power reactor at the Kalininskaya Nuclear Power Plant (KNPP) on a moving platform. The reactor materials provide a good shielding equivalent to ∼ 50 m of water, which removes the hadronic component of the cosmic background and reduces the cosmic muon flux by a factor of 6. More detailed description of the detector design can be found elsewhere [9]. Results of the first year of data taking are already published [10].

3. Calculations of positron spectra
For each point in the $\Delta m^2_{41}$, $\sin^2 2\theta_{ee}$ plane predictions for the positron spectra at the different detector positions were calculated. Several factors were taken into account:

- $\tilde{\nu}_e$ spectrum. Here we used Huber [11] and Mueller [4] calculations, but this choice does not influence final results, since the ratio of spectra is compared with the experimental data.
- IBD cross-section [12].
- Reactor and detector size.
- Distance between reactor and detector.
- Reactor fuel burning profile (provided by KNPP). Distribution averaged over the campaigns was used in this analysis.
- Detector resolution including tails obtained from GEANT4 MC simulations. Observed energy resolution was slightly worse than MC predictions (33% instead of predicted 31% at 1 MeV). Therefore additional smearing had been added to MC predictions ($\sigma_{additional}/E = 12%/\sqrt{E} \oplus 4\%$). After that MC describes well all calibration sources. This additional smearing was taken into account in the positron spectra calculations.
- Antineutrino survival probability (1).

4. $\chi^2$ statistics
The calculated positron spectra ratios were compared with observed ones. The fit was performed for (1.5-6) MeV positron energy range using 36 energy bins. We divided the whole dataset into
two parts: phase I and phase II. During the first phase the experimental data were collected at three different distances from the detector to the reactor core: top, middle and bottom. During the second phase we used only top and bottom detector positions. The difference in average fuel composition for spectra collected at different positions is negligible (< 0.01% in IBD counting rate) within each phase. Without such splitting of the data the average fuel composition would significantly differ for the middle position.

The systematic uncertainties are treated as nuisance parameters. The stability of the detector operation allows us to include relative counting rates into the analysis. The corresponding uncertainties are included as penalty terms for nuisance parameters into the test statistics. This approach differs from the previously used one [13]. Another improvement in the analysis is the inclusion of the third detector position into $\chi^2$ by introducing the second ratio $R_2 = \text{Middle}/\sqrt{\text{Bottom} \cdot \text{Top}}$ (with such a choice of the second ratio off-diagonal terms in the covariance matrix are very small). The inclusion of the third position adds little to the sensitivity, but results are potentially more stable with respect to possible additional systematic uncertainties. Previously, the third position was used for cross-checks only.

As a result, the test statistics is defined as follows:

$$\chi^2 = \min_{\eta, k}(\chi^2_1 + \chi^2_II + \chi^2_{\text{penalty}})$$

(3)

$$\chi^2_1 = \sum_{i=1}^{N} (Z_{1i} - Z_{2i}) \cdot W^{-1} \cdot (Z_{2i})$$

(4)

$$\chi^2_{II} = \sum_{i=1}^{N} \frac{Z^2_{1i}}{\sigma^2_{1i}} \sum_{i=1}^{N} \frac{(R^\text{obs}_{ij} - k_1 \cdot R^\text{pre}_{ij}(\eta))^2}{\sigma^2_{ij}}$$

(5)

$$\chi^2_{\text{penalty}} = \sum_{j=1,2} (k_j - k^0_j)^2 \sum_{l} \frac{(\eta_l - \eta^0_l)^2}{\sigma^2_{l}}$$

(6)

where $i$ – energy bin, $Z_j = R^\text{obs}_{ij} - k_j \times R^\text{pre}_{ij}(\Delta m^2, \sin^2 2\theta, \eta)$ for each energy bin, $R_1 = \text{Bottom}/\text{Top}, R_2 = \text{Middle}/\sqrt{\text{Bottom} \cdot \text{Top}}$, where Top, Middle, Bottom – absolute counting rates per day for each detector position, $W$ – covariance matrix, $k$ – relative efficiency (nominal values $k^0_1 = k^0_2 = 1$), $\eta$ – other nuisance parameters. Thus, the $\chi^2$ does not depend on the absolute IBD counting rate, but includes information about relative counting rates at different positions. Systematic uncertainties related to the relative detector efficiency are discussed in [14]. Information about relative counting rates allows to increase sensitivity areas considerably. The comprehensive list of all systematic uncertainties which were taken into account is given in Table 1.

Predicted spectra were calculated separately for different types of systematic uncertainties (not related to relative efficiency) for small deviations from nominal values. During the fit each absolute spectrum $S(E, \eta)$ was approximated using the first-order Taylor expansion:

$$S(E, \eta) = S(E, \eta_0) + \sum_i \frac{\partial S}{\partial \eta_i} d\eta_i$$

(7)

where $\eta$ is the vector of nuisance parameters and $\eta_0$ describes their nominal values.

Figure 1 shows the distribution of $\Delta \chi^2 = \chi^2_{3\nu} - \chi^2_{\nu}$ for the parameter space $\Delta m^2_{3\nu}$, $\sin^2 2\theta_{ee}$. There are several areas where $\chi^2_{3\nu} < \chi^2_{\nu}$. The best point in the parameter space is $\Delta m^2_{3\nu} = 1.3eV^2$, $\sin^2 2\theta_{ee} = 0.02$ and $\Delta \chi^2 = -5.5$. Figure 2 shows the Bottom/Top spectra ratio for two phases combined.
Table 1. Systematic uncertainties considered in the analysis and corresponding 1σ values used in the penalty terms in the equation (6). These values correspond to changes from nominal values.

| type of systematic                          | σ       |
|--------------------------------------------|---------|
| relative detector efficiencies            | 0.2%    |
| distance to the fuel burning profile center | 5 cm    |
| cosmic background                          | 25%     |
| fast neutron background                    | 30%     |
| additional smearing in energy resolution added to MC | 25%     |
| energy scale                               | 2%      |
| energy shift                               | 50 keV  |

Figure 1. Distribution of \( \Delta \chi^2 = \chi^2_{4\nu} - \chi^2_{3\nu} \) for the parameter space \( \Delta m^2_{41}, \sin^2 2\theta_{ee} \) (color online). Blue color indicates areas with \( \chi^2_{4\nu} > \chi^2_{3\nu} \), red color indicates areas with \( \chi^2_{4\nu} < \chi^2_{3\nu} \). The color axis is limited, \( \Delta \chi^2_{\text{max}} = \chi^2_{\text{best 4}\nu} + 11.83 \). The value 11.83 corresponds to the exclusion at 3σ level in case of a \( \chi^2 \) distribution with 2 d.o.f. In case of sterile neutrino searches \( \Delta \chi^2 \) statistics doesn’t follow the \( \chi^2 \) distribution with 2 d.f., so here we use it just for illustration. Exclusion areas are calculated with a Gaussian CLs method.

Figure 2. Ratio of positron energy spectra measured at the bottom and top detector positions (statistical errors only) for the whole period of time (both phases). The solid curve is the prediction for 3ν case, the dotted curve corresponds to the best fit in the 4ν mixing scenario (\( \sin^2 2\theta_{ee} = 0.02, \Delta m^2_{41} = 1.3 \text{ eV}^2 \)), the dashed curve is the expectation for the optimum point from the RAA and GA fit [5] (\( \sin^2 2\theta_{ee} = 0.14, \Delta m^2_{41} = 2.3 \text{ eV}^2 \)).

5. Exclusion and sensitivity areas
Exclusion areas were calculated using a Gaussian CLs method [15]. This method doesn’t require computationally extensive calculations and allows to combine results from different experiments easily. The boundary of the 90% C.L. exclusion area is shown in figure 3 (solid line). In order to estimate sensitivity area many toy MC experiments were carried out. The dashed line in figure 3 is the boundary of the 90% C.L. sensitivity area. Figure 4 demonstrates the influence of systematic uncertainties on the sensitivity area. For small values of \( \Delta m^2_{41} \) systematic
uncertainties related to relative counting rates influence the sensitivity area considerably. Contribution from the rest of systematic uncertainties is small. The best point of the RAA and GA lies well inside $5\sigma$ level in case of the CL$_s$ method. It is computationally hard to estimate the significance of the exclusion with a very large $\Delta\chi^2 = \chi^2_{RAA+GA} - \chi^2_{\text{min}} = 68$ using the Feldman-Cousins method [16]. However, we have checked that at lower significance levels the two methods agree well even CL$_s$ method usually provides more conservative results.

**Figure 3.** Exclusion area at 90% C.L. obtained with Gaussian CL$_s$ method (filled area) and 90% C.L. sensitivity contour (dashed line).

**Figure 4.** 90% C.L. sensitivity contours without systematic uncertainties (dotted line), with systematic uncertainties related to relative counts (solid line), with all the systematic uncertainties (dashed line).

6. Best point significance

Although the Gaussian CL$_s$ method has many advantages, it doesn’t allow to estimate the significance of the best point in the parameter space $\Delta m^2_{41}$, $\sin^2 2\theta_{ee}$. In order to evaluate the significance of the best point we used a method proposed by Feldman and Cousins [16]. Here we define the test statistics for some point in the parameter space as follows:

$$\Delta\chi^2 = \chi^2_{\text{point}} - \chi^2_{\text{min}}$$

The best point in the parameter space has $\Delta\chi^2 = \chi^2_{3\nu} - \chi^2_{4\nu\text{min}} = 5.5$. In order to obtain $\Delta\chi^2$ distribution multiple toy-MC experiments were carried out. Toy datasets were generated by taking the model without oscillations at each position and varying the bin content according to the statistical uncertainty in the corresponding bin. Then the test statistics (8) was calculated for each toy-MC dataset as well as for the observed dataset. Calculated $\Delta\chi^2$ distribution is shown in figure 5. 14% of toy experiments result in $\Delta\chi^2$ values larger than observed. This means that $3\nu$ hypothesis lies inside a $1.5\sigma$ confidence interval, so the best point is not significant.

7. Conclusions

Larger statistics and new analysis allowed us to broaden the exclusion area in the $\Delta m^2_{41}$, $\sin^2 2\theta_{ee}$ parameter space up to $\sin^2 2\theta_{ee} < 0.008$ in the most sensitive region. Our results exclude a large
fraction of the RAA and GA predictions. The significance of the best-fit point in the parameter space was evaluated using the Feldman and Cousins approach. The obtained value is 1.5σ, only. It is important to stress that our analysis is based only on the comparison of the shapes of the positron energy distributions and relative counting rates at the three distances from the reactor core measured with the same detector. Obtained results don’t depend on absolute detector efficiency or theoretical predictions.

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References
[1] Zyla P et al. (Particle Data Group) 2020 PTEP 2020 083C01
[2] Abdurashitov J N et al 2006 Phys. Rev. C 73 045805
[3] Kaether F et al 2010 Phys. Rev. B 685 47
[4] Mueller T A et al 2011 Phys. Rev. C 83 054615
[5] Mention G et al 2011 Phys. Rev. D 83 073006
[6] Carlo Giunti and Marco Laveder 2011 Phys. Rev. C 83 065504
[7] Aguilar-Arevalo A A et al 2018 Phys. Rev. Lett. 121 221801
[8] Feldman G J and Cousins R D 1998 Phys. Rev. D 57(7) 3873–3889