Maximum Entropy Reinforcement Learning with Mixture Policies

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Abstract

Mixture models are an expressive hypothesis class that can approximate a rich set of policies. However, using mixture policies in the Maximum Entropy (MaxEnt) framework is not straightforward. The entropy of a mixture model is not equal to the sum of its components, nor does it have a closed-form expression in most cases. Using such policies in MaxEnt algorithms, therefore, requires constructing a tractable approximation of the mixture entropy. In this paper, we derive a simple, low-variance mixture-entropy estimator. We show that it is closely related to the sum of marginal entropies. Equipped with our entropy estimator, we derive an algorithmic variant of Soft Actor-Critic (SAC) to the mixture policy case and evaluate it on a series of continuous control tasks.

1. Introduction

Maximum Entropy (MaxEnt) reinforcement learning algorithms (Ziebart et al., 2008) are among the state-of-the-art methods in off-policy, continuous control reinforcement learning (RL) (Haarnoja et al., 2018b; Yin, 2002; Haarnoja et al., 2018a). MaxEnt algorithms achieve superior performance by offering a structured approach to exploration. Particularly, they add the entropy over actions to the discounted sum of returns. This modified objective encourages policies to maximize the reward while being as erratic as possible, which highly assists with exploration and stability (Ziebart, 2010). Nevertheless, contemporary MaxEnt techniques use simple policy classes (e.g. Normal) whose entropy function has a closed-form expression.

For the MaxEnt framework to remain competent in complex control tasks, it should be extended to handle richer classes of policies capable of accommodating complex behaviors. In this gap, much attention has been recently given to approaches that do not require knowledge of the probability of the performed action $a$ in a state $s$, $\pi(a|s)$ (Song & Zhao, 2020). For example, Distributional Policy Optimization (DPO) (Tessler et al., 2019) builds upon the policy gradient framework (Kakade, 2001), to propose a Generative Actor whose optimization is based on Quantile Regression (Koenker & Hallock, 2001). However, methods of this type usually involve challenging optimization, long training times, and lack the exploration and stability properties of MaxEnt algorithms.

Instead, we propose to use mixture models (Agostini & Celaya, 2010) to construct an expressive policy. Mixture models are universal density estimators (Carreira-Perpinan, 2000). They can approximate any distribution using elementary components with a closed-form probability density function (see Figure 1 for an illustration). Nevertheless, combining mixture policies with MaxEnt algorithms poses a challenge of estimating the entropy of the mixture, as the latter does not have a closed-form expression in many scenarios (Kim et al., 2015), nor is it easy to approximate (Huber et al., 2008).

In this paper, we attempt to effectively integrate mixture policies with the MaxEnt framework. We derive a tractable, low-variance mixture-entropy estimator, reminiscent of the weighted sum of the marginal entropies of the mixing components. As such, it is intuitive to understand and straightforward to calculate. In a series of experiments we empirically demonstrate that soft actor-critic algorithms can optimize mixture models comparably well to single-component policies. Optimizing the mixing weights as well, which is outside the scope of this work, may result in an additional performance gain.

The rest of the paper is organized as follows. Section 2 outlines the basic mathematical background. Section 3 presents our mixture MaxEnt low-variance estimator, followed by Section 4 where we describe our Soft Actor-Critic variant with mixture

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Figure 1. Illustration of Entropy vs Mixture Entropy regularization. Complex control tasks are more likely to elicit non-unimodal value functions (black line). In such situations, unimodal parameterized policies (left) run the risk of converging to suboptimal local minima. To explore both extremes of $Q$, we need to allow high entropy (gray area under the curve) by forcing high entropy regularization. In contrast, a mixture model (right) readily fits both centers without suffering high entropy (gray area under the curve). In this case, each component in the mixture can latch to a different extremum of $Q$. The resultant mixture policy is more diverse, and therefore robust since it assigns proportional weights to the two local maxima of $Q$.

2. Preliminaries

We assume a Markov Decision Process (MDP) (Puterman, 2014) which specifies the environment as a tuple $\mathcal{M} = (S, A, r, P, \gamma, \rho_0)$, consisting of a state space $S$, an action space $A$, a reward function $r : S \times A \mapsto [0, 1]$, a transition probability function $P : S \times A \times S \mapsto [0, 1]$, a discount factor $\gamma \in (0, 1)$, and an initial state distribution $\rho_0 : S \mapsto [0, 1]$. A policy $\pi : S \times A \mapsto [0, 1]$ interacts with the environment sequentially, starting from an initial state $s_0 \sim \rho_0$. At time $t = 0, 1, \cdots$ the policy produces a probability distribution over the action set $A$ from which an action $a_t \in A$ is sampled and played. The environment then generates a scalar reward $r(s_t, a_t)$ and a next state $s_{t+1}$ is sampled from the transition probability function $P(\cdot|s_t, a_t)$.

We use $\rho_{\pi}(s, a) \triangleq (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t P^\pi(s_t = s, a_t = a|s_0 \sim \rho_0)$ and $\rho_{\pi}(s) \triangleq \sum_{a} \rho_{\pi}(s, a)$ to denote the discounted state-action and state visitation distributions of policy $\pi$, respectively.

In this work, we assume a policy $\pi$ that is a mixture of $N$ components $\pi_1, \ldots, \pi_N$ with corresponding mixing weights $w_1, \cdots, w_N$. We can treat the set of component weights as the probabilities that outcomes $1 \cdots N$ take place, given a random variable $W$, where $P_{\pi}(W = i) = w_i$.

3. Maximum Mixture Entropy

MaxEnt methods promote stochastic policies by augmenting the standard expected sum of returns $J(\pi) = \sum_{t=0}^{T} \mathbb{E}_{\pi} [r(s_t, a_t)]$, with the expected entropy of the policy over $\rho_{\pi}$, $\sum_{t=0}^{T} \mathbb{E}_{\pi, a_t \sim \rho_\pi} [r(s_t, a_t) + \alpha \mathcal{H}(\pi(\cdot|s_t))]$. Here, $\alpha > 0$ is the entropy coefficient and the entropy, $\mathcal{H}(\pi(\cdot|s))$, is given by $\mathcal{H}(\pi(\cdot|s)) = -\sum_a \pi(a|s) \log \pi(a|s)$. The parameter $\alpha$ controls the stochasticity of optimal policy by balancing the reward and entropy terms.

3.1. Mixture Policies

Various types of mixture models are discussed in the RL literature. In episodic-mixture methods, a component is selected at the start of each episode and plays for the entire episode. Such methods are sometimes referred to as Ensemble RL methods (Wiering & Van Hasselt, 2008). A second kind, which we refer to as step-mixture methods, handpicks a new component to play after each or several steps. Step-mixtures describe popular RL setups such as macro actions (Pickett & Barto, 2002) or options (Precup & Sutton, 1998). In a third setting, termed as action-mixture methods, actions are produced by combining the predictions of multiple models, e.g. as with combinatorial action spaces (Vinyals et al., 2019). In this work, we chiefly ...
focus on the second type of step-mixture methods. The maximum entropy criterion in this setup is given by

$$J(\pi) = \sum_{t=0}^{T} \mathbb{E}_{s_t \sim p_{\pi}, a_t \sim \pi} [r(s_t, a_t) + \alpha \mathcal{H}(\pi(\cdot|s_t))]$$.

(1)

Evaluating the objective in Equation 1 can be described by the following three steps sampling process: (1) sample a state from the mixture distribution $\rho_{\pi}$, (2) sample a mixture component $\pi_i$ with probability $w_i$, and (3) sample an action $a \sim \pi_i(\cdot|s_t)$ and evaluate.

Notice that in Equation 1, there exists a separate marginal state distribution $\rho_{\pi_i}$ for each mixture component $\pi_i$. As such, one can independently optimize the data term denoting the expected sum of returns as in the non-mixture case. Nevertheless, in neither the episodic nor the step-mixture settings can the second entropy term be separately optimized. The use of mixture entropy that depends on the overall mixture is therefore still a joint optimization problem.

3.2. Mixture Entropy Estimation

Mixture entropy regularization maximizes the expected entropy of a mixture policy $\mathcal{H}(\pi)$. This is different from maximizing a weighted sum of marginal entropies $\sum_i w_i \mathcal{H}(\pi_i)$. As we show below, the mixture entropy depends on the pairwise distances between the components. Therefore, maximizing the mixture entropy creates diversity between the components, which can provide substantial improvement in exploration and robustness (Ziebart, 2010).

In a sequential decision-making setup, it is customary to define the entropy of $\pi$ as the expected entropy over $\rho_{\pi}$, i.e.,

$$\mathcal{H}(\pi) = \sum_{t=0}^{T} \mathbb{E}_{s_t \sim \rho_{\pi}} \mathcal{H}(\cdot|s_t).$$

Using laws of probability we can outline the connection between the joint and conditional mixture entropy:

$$\mathcal{H}(\pi, W) = \mathcal{H}(\pi|W) + \mathcal{H}(W) = \mathcal{H}(W|\pi) + \mathcal{H}(\pi),$$

(2)

where $\mathcal{H}(\pi|W)$ is given by $\mathcal{H}(\pi|W) = \sum_i w_i \mathcal{H}(\pi_i)$, and $\mathcal{H}(W|\pi)$ is defined as

$$\mathcal{H}(W|\pi) = - \sum_{a \in A} \sum_{t=0}^{T} \mathbb{E}_{s_t \sim \rho_{\pi}} \pi(a|s_t) \mathcal{H}(W|a, s_t)$$

$$= - \sum_{t=0}^{T} \mathbb{E}_{s_t \sim \rho_{\pi}} \sum_{a \in A} \pi(a|s_t) \sum_{w \in W} p(w|a, s_t) \log p(w|a, s_t)$$

$$= - \sum_{t=0}^{T} \mathbb{E}_{s_t \sim \rho_{\pi}} \sum_{a \in A} \sum_{w \in W} p(w, a|s_t) \log \frac{p(w, a|s_t)}{\pi(a|s_t)}.$$

A closer look at Equation 2 reveals that the explicit form of the mixture entropy is non-trivial as it requires calculating $p(w, a|s)$, i.e., the joint action-component probability. Eliminating the need to calculate this quantity requires some form of approximation. One method to approximate the mixture entropy is using Monte Carlo (MC) sampling. While MC may yield an unbiased estimate it comes with a high computational cost. A different approach is to use analytic estimators to approximate the mixture entropy. Such estimators have estimation bias but are efficient to compute.

A straightforward lower-bound estimate of the mixture entropy is given by

$$\mathcal{H}(\pi) \geq \mathcal{H}(\pi|W) = \sum_i w_i \mathcal{H}(\pi_i),$$

(3)

as conditioning can only decrease entropy. Alternatively, an immediate upper bound is given by the joint entropy

$$\mathcal{H}(\pi) \leq \mathcal{H}(\pi, W) = \mathcal{H}(\pi|W) + \mathcal{H}(W)$$

(4)
The upper and lower bounds remain the same regardless of the “overlap” between components. As such, they are poor choices for optimization problems that involve varying component locations.

Other estimators that depend on the “overlap” between components are based on Jensen’s inequality (Cover, 1999) or kernel density estimation (Joe, 1989; Hall & Morton, 1993). However, these exhibit significant underestimation (Kolchinsky & Tracey, 2017). To address this problem, Kolchinsky & Tracey (2017) proposed to estimate the mixture entropy using pairwise distances between mixture components. Specifically, let \( D(p_i||p_j) \) denote a (generalized) distance function between probability measures \( p_i \) and \( p_j \). Whenever \( D \) satisfies \( D(p_i||p_j) = 0 \) for \( p_i = p_j \), then \( \mathcal{H}_D(\pi) \) can be used to approximate \( \mathcal{H}(\pi) \) by

\[
\mathcal{H}_D(\pi) = \mathcal{H}(\pi|W) - \sum_i w_i \log \sum_j w_j e^{-D(p_i||p_j)}. \tag{5}
\]

Kolchinsky & Tracey (2017) also showed that for any distance function \( D \), \( \mathcal{H}_D(\pi) \) is bounded by the joint and conditional entropy

\[
\mathcal{H}(\pi|W) \leq \mathcal{H}_D(\pi) \leq \mathcal{H}(\pi, W). \tag{6}
\]

Combining Equation 6 with Equation 3 and Equation 4 we can bound the bias of \( \mathcal{H}_D(\pi) \) by

\[
|\mathcal{H}_D(\pi) - \mathcal{H}(\pi)| \leq \mathcal{H}(\pi, W) - \mathcal{H}(\pi|W) = \mathcal{H}(W). \]

Tighter bounds can be achieved for specific distance functions. We refer the reader to Kolchinsky & Tracey (2017) for further information.

3.3. Low Variance Estimation of \( \mathcal{H}_D(\pi) \)

\( \mathcal{H}_D(\pi) \) is attractive to use whenever \( D \) is easy to approximate. The first term, corresponding to the marginal entropies, can be approximated at state \( s \) by

\[
\mathcal{H}(\pi|W) = \sum_i w_i \mathcal{H}(\pi_i) \approx -\sum_i w_i \log \pi_i(\tilde{a}_i|s), \tag{7}
\]

where \( \tilde{a}_i \sim \pi_i(\cdot|s) \). Setting \( D \) to be the Kullback-Leibler divergence, the second term in \( \mathcal{H}_D(\pi) \) can be approximated at each state \( s \) by

\[
\sum_i w_i \log \sum_j w_j \exp(-D(\pi_i||\pi_j))
= \sum_i w_i \log \sum_j w_j \exp \left( \mathbb{E}_{a \sim \pi_i} \log \frac{\pi_j(a|s)}{\pi_i(a|s)} \right)
\approx \sum_i w_i \log \sum_j w_j \frac{\pi_j(\tilde{a}_i|s)}{\pi_i(\tilde{a}_i|s)}
= \sum_i w_i \log \sum_j w_j \pi_j(\tilde{a}_i|s) - \sum_i w_i \log \pi_i(\tilde{a}_i|s),
\]

where \( \tilde{a}_i \sim \pi_i(\cdot|s) \) is a second sample from policy \( \pi_i \). Setting \( \tilde{a}_i \equiv \tilde{a}_i \), will result in a biased estimation of \( \mathcal{H}_D(\pi) \) since we use the same sample twice. However, the variance of the estimation will be significantly smaller as

\[
\sum_i w_i \log \pi_i(\tilde{a}_i|s)
\]

cancel out from both terms (We refer the reader to Appendix A for an unbiased lower bound of \( \hat{\mathcal{H}}_D(\pi) \)). Specifically, we have that

\[
\hat{\mathcal{H}}_D(\pi) \approx -\sum_i w_i \log \pi_i(\tilde{a}_i|s) - \left[ \sum_i w_i \log \sum_j w_j \pi_j(\tilde{a}_i|s) - \sum_i w_i \log \pi_i(\tilde{a}_i|s) \right].
\]

\(^1\)Note that we do not assume \( D \) to obey the triangle inequality, nor that it is symmetric.
We thus have the following approximation for $\hat{H}_D(\pi)$

$$\hat{H}_D(\pi) \approx \sum_i w_i \hat{H}_i(\pi). \tag{8}$$

where $\hat{H}_i(\pi) = -\log \sum_j w_j \pi_j(\hat{a}_i|s)$. To see that $\text{Var}(\hat{H}_D(\pi)) \leq \text{Var}(\hat{H}_D(\pi))$ we denote by $X_i(s)$ the random variable $\log \pi_i(\cdot|s)$, and $Y_i(s)$ the random variable $\log \sum_j w_j \pi_j(\cdot|s)$. We have that

$$\text{Var}(\hat{H}_D(\pi)) = 4 \text{Var}\left( \sum_i w_i X_i(s) \right) + \text{Cov}\left( \sum_i w_i X_i(s), \sum_i w_i Y_i(s) \right) + \text{Var}\left( \sum_i w_i Y_i(s) \right) \geq \text{Var}\left( \sum_i w_i Y_i(s) \right) = \text{Var}(\hat{H}_D(\pi)).$$

Carefully examining Equation 8 with relation to the expressions for marginal entropies in Equation 7, our result indicates that the mixture entropy is defined as a weighted sum of mixed marginal entropies. That is, while the marginal entropy of each component, $\hat{H}_i(\pi)$, is approximated by $-\log \pi_i(\cdot|s)$, the mixed marginal entropy, $\hat{H}_i(\pi)$, is approximated by $-\log \sum_j w_j \pi_j(\cdot|s)$. While the $i$-th marginal entropy, $\hat{H}(\pi_i)$, measures the log probability of an action $a$ sampled from the $i$-th component, the $i$-th mixed marginal entropy, $\hat{H}_i(\pi)$, measures the log of the cumulative probability the mixture assigns to the same action. From an exploration viewpoint, the intrinsic bonus of any action will be silenced if other mixture components are already exploring it.

### 3.4. A Bellman Operator for Mixture Policies

In the infinite-horizon discounted setting we define the value function $v^\pi$ to include the entropy bonuses from every timestep

$$v^\pi(s) = \mathbb{E}_{\tau \sim \pi} \left[ \sum_{t=0}^{\infty} \gamma^t \left( r(s_t, a_t) + \alpha \hat{H}_D(\pi(\cdot|s_t)) \right) \bigg| s_0 = s \right],$$

and similarly for the $Q$-function $Q^\pi(s, a) = \mathbb{E}_{\tau \sim \pi} \left( \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) + \alpha \sum_{t=1}^{\infty} \gamma^t \hat{H}_D(\pi(\cdot|s_t)) \bigg| s_0 = s, a_0 = a \right)$. With this change, the soft value function can be written as

$$v^\pi(s) = \mathbb{E}_{a \sim \pi} Q^\pi(s, a) + \alpha \hat{H}_D(\pi(\cdot|s)) \tag{9},$$

and the Bellman backup operator associated with the $Q$-function of the mixture policy is, therefore $T^\pi Q^\pi(s_t, a_t) = r(s_t, a_t) + \gamma \mathbb{E}_{s_{t+1} \sim p} v^\pi(s_{t+1})$.

### 4. Soft Actor Critic Mixture

A natural algorithmic choice to optimize MaxEnt mixtures is by Soft Actor-Critic (SAC) algorithms (Haarnoja et al., 2018b). In this section we will describe Soft Actor Critic Mixture (SACM), our proposed framework. We use neural network function approximation for both the value function and the mixture policy, and alternate between policy evaluation steps (optimizing Q), and policy improvement steps (optimizing $\pi$).

Consider a parameterized soft Q-function $Q_\theta(s, a)$ and a set of parameterized policies $\{\pi_{\phi_i}\}$, e.g., a set of multivariate Gaussian distributions. We denote by $\theta$ the soft Q-function neural network parameters, and by by $\phi = \{\phi_i\}_{i=1}^N$ the parameters of the policy networks\(^2\). In what follows, we will derive update rules for $\theta$ and $\phi$.

\(^2\)It is common practice to represent $\{\pi_{\phi_i}\}$ using distinct “heads” over a shared feature extractor.
Algorithm 1 Soft Actor Critic Mixture (SACM)

Initialization:
\[
\begin{align*}
\bar{\theta} & : \{\phi_i, \alpha_i\}_{i=0}^N \\
\theta & \leftarrow \bar{\theta} \\
D & \leftarrow \emptyset
\end{align*}
\]

\text{\textbf{Output:} } \{\phi_i\}_{i=1}^N

1: for each environment step do
2: \hspace{1em} \(i \sim [w_1, w_2, \ldots, w_N]\)
3: \hspace{1em} \(a_i \sim \pi_i(\cdot|s_t)\)
4: \hspace{1em} \(s_{t+1} \sim p(s_{t+1}|s_t, a_t)\)
5: \hspace{1em} \(D \leftarrow D \cup \{s_t, a_t, r_t, s_{t+1}\}\)
6: end for
7: for each gradient step do
8: \hspace{1em} \(\theta \leftarrow \theta - \lambda_Q \nabla_{\theta} J_Q(\theta)\)
9: \hspace{1em} \(\theta \leftarrow \gamma \theta + (1 - \gamma)\bar{\theta}\)
10: for each mixture component \(j\) do
11: \hspace{2em} \(\phi_j \leftarrow \phi_j - \lambda_\pi \nabla_{\phi_j} J_\pi(\phi_j)\)
12: \hspace{2em} \(\alpha_j \leftarrow \alpha_j - \lambda_\alpha \nabla_{\alpha_j} J_\alpha(\alpha_j)\)
13: \hspace{1em} end for
14: end for
15: end for

4.1. Soft Mixture Policy Evaluation

The soft Q-function parameters are trained to minimize the Bellman error
\[
J_Q(\theta) = \mathbb{E}_{(s_t, a_t) \sim D} \left[ \frac{1}{2} (Q_\theta(s_t, a_t) - (r_{s_t, a_t} + \gamma \mathbb{E}_{s_{t+1} \sim p} v_\theta(s_{t+1}))^2) \right],
\]

where \(\bar{\theta}\) is a Polyak averaging (target network) of \(\theta\) which has been shown to stabilize training (Mnih et al., 2015). Note that \(v_\theta\), given by Equation 9, is not parameterized separately. The stochastic gradient of \(J_Q(\theta)\) is thus
\[
\nabla_{\theta} J_Q(\theta) = \nabla_{\theta} Q_\theta(s_t, a_t)(Q_\theta(s_t, a_t) - y).
\]

The target value \(y\) is computed by sampling an action \(a_{t+1}\) from the mixture policy in the next state \(s_{t+1}\)
\[
y = r_{s_t, a_t} + \gamma Q_\theta(s_{t+1}, a_{t+1}) - \alpha \log \sum_j w_j \pi_j(a_{t+1}|s_{t+1}).
\]

4.2. Soft Mixture Policy Improvement

Following (Haarnoja et al., 2018b;a), the policy parameters are trained to minimize the expected Kullback-Liebler with respect to the soft Q mixture function
\[
\pi_i \leftarrow \arg \min_{\pi_i \in \Pi} D_{KL}\left(\pi_i(\cdot|s_t) \parallel \frac{\exp(\frac{1}{\alpha} Q_\theta(s_i))}{Z(s_t)}\right),
\]

where \(Z\) is an intractable partition function needed to normalize the right-hand side of the KL divergence to a valid probability distribution. Fortunately, it does not contribute to the gradient and can thus be ignored. Equation 11, however, shows that all mixture components follow the same Boltzmann distribution induced by the shared soft Q-function. Therefore, without optimizing the mixing weights, we run the risk of having the mixture components converge to the same policy. As an intermediate solution, one may maintain an array of \(N\) soft Q-functions, \(\{Q_\theta_i\}_{i=1}^N\), one for each component, and train them using non-overlapping bootstrapped data samples. Exposing components to distinct data samples can assist in breaking the symmetry. (Osband et al., 2016). The update rule in Equation 11 is achieved by minimizing the following objective
\[
J_\pi(\phi_i) = \mathbb{E}_{s_t \sim D, a_t \sim \pi_{\phi_i}} \left[ \mathbb{E}_{s_t \sim D} \left( \alpha \log \pi_{\phi_i}(a_t|s_t) - Q_\theta(s_t, a_t) \right) \right].
\]
Unfortunately, the above objective is non differentiable, as it involves sampling from a probability distribution defined by $\pi_{\phi_i}$. Approximating the gradient is possible by using the likelihood ratio gradient estimator (REINFORCE, (Williams, 1992)) or using the reparametrization trick (Kingma & Welling, 2013; Rezende et al., 2014). Using the latter approach, actions are sampled according to

$$a_t = \mu_{\phi_i}(s_t) + \epsilon_t \cdot \sigma_{\phi_i}(s_t),$$

where $\epsilon_t \sim \mathcal{N}(0, 1)$ is a sample from a spherical Gaussian. The modified objective now becomes

$$J_\pi(\phi_i) = \mathbb{E}_{s_t \sim D_{\epsilon_t \sim \mathcal{N}(0, 1)}} [((\alpha \log \pi_{\phi_i}(a_t|s_t) - Q_{\theta}(s_t, a_t))],$$

and the stochastic gradient is given by

$$\nabla_{\phi_i} J_\pi(\phi_i) = \alpha \nabla_{\phi_i} \log \pi_{\phi_i}(a_t|s_t) + (\alpha \nabla_{a_t} \log \pi_{\phi_i}(a_t|s_t) - \nabla_{a_t} Q(a_t,s_t)) \nabla_{\phi_i} a_t(\phi_i).$$

Finally, we optimize a distinct entropy coefficient $\alpha_i$ for each component in the mixture. The alpha parameter is adjusted such that a pre-defined target entropy rate is maintained. This is possible by alternating the minimization of the following criterion concurrently while optimizing the $Q$ function and the policy, i.e.,

$$J(\alpha_i) = \mathbb{E}_{a_t \sim \pi_i} [-\alpha_i \log \pi_i(a_t|s_t) - \alpha_i \bar{H}].$$

The final $\alpha$ value in Equation 9 is then given by $\alpha = \sum_i w_i \alpha_i$. Pseudo code for the Soft Actor Critic Mixture (SACM) is outlined in Algorithm 1.

5. Related Work

Mixture policies serve several policy optimization settings. In multi-objective reinforcement learning, where complementary goals are defined simultaneously, components of mixture policies are used to optimize different objectives (Vamplew et al., 2009). The authors in Abbeel & Ng (2004), on the other hand, describe an apprenticeship learning algorithm that optimizes the feature expectation of a mixture policy to match that of an anonymous expert policy. In the same spirit, the algorithm presented in Fu et al. (2017) relies on the mixture distribution of expert demonstrations and an imitation policy to solve an inverse RL problem.

Mixture policies are useful in multi-agent problems as well. Lanctot et al. (2017) describes a multi-agent RL algorithm where the strategy of each player is the best response to a mixture policy, defined by the rest of the players. On the other hand, the work of Czarnecki et al. (2018) describes Mix & Match as a curriculum learning framework that defines a mixture of policies of increasing-learning complexity, while Henderson et al. (2017) uses a mixture policy to extract options. They view the policy-over-options, i.e., the agent that gates between options, as a single-step mixing policy. There is also the work of Calinon et al. (2013) that defined a multi-objective criterion that is optimized using a Gaussian mixture model - one of the most popular mixture models in RL literature. Finally, Agarwal et al. (2019) that presented Random Ensemble Mixture (REM), a Q-learning algorithm that enforces Bellman consistency on random convex combinations of multiple Q-value estimates.

Our work is also related to ensemble learning. In this context, a notable example is Bootstrapped DQN (Osband et al., 2016), which carries out temporally-extended exploration using a multi-head Q network architecture where each head trains on a distinct subset of the data. The ideas presented in Osband et al. (2016) were later on formalized by Lee et al. (2020) as part of the SUNRISE algorithm (Simple UNified framework for reinforcement learning using enSeMbles). SUNRISE is a three-ingredient framework: a) Bootstrap with random initializations, b) Weighted Bellman backup, and c) UCB exploration.

Ensemble methods are also a powerful "go-to" strategy for mitigating optimization hazards. Averaged-DQN shows how to use ensembles for stabilizing the optimization of Q learning algorithms. They average an ensemble of target networks to reduce approximation error variance (Anschel et al., 2017). In Fauber & Schwenker (2011), the authors combine the predictions of multiple agents to learn a robust joint state-value function, while in Marivate & Littman (2013) the authors show that better performance is possible when learning a weighted linear combination of Q-values from an ensemble of independent algorithms.
Figure 2. Training curves for the continuous control benchmarks. We use a mixture of $n = 3$ components and report the best one. We evaluate both the SMAC and Semi-SMAC (S2MAC) variants for the mixture model approach against the Soft Actor-Critic (SAC) baseline. The target entropy is set to $-\log |A|$ where $|A|$ is the size of the action vector. The results show the performance of stochastic action sampling from $\pi$ and not the behavior of playing the deterministic actions.

6. Experiments

As the mixing weights are not optimized, we anticipate all components to collapse toward some mean policy $\bar{\pi}$. Consequently, the mixture entropy will equal the marginal entropy of the mean policy $\bar{\pi}$

$$\mathcal{H}_D(\pi) = -\sum_i w_i \log \sum_j w_j \pi_j(a_i|s) = -\sum_i w_i \log \bar{\pi}(a|s) = -\log \bar{\pi}(a|s),$$

where $\bar{a}$ is a sample from $\bar{\pi}$ and we use the fact that $\sum_j w_j = 1$. Since there is no reason to believe that mixture policies outperform single component MaxEnt policies under such conditions, we ask the following questions: (1) Do MaxEnt mixture policies perform as well as single-component MaxEnt policies? (2) Can performance improve by enforcing asymmetry in the entropy, even if the components’ weights stay the same?

To answer the latter, we suggest to use a distinct soft Q-function, $Q_i$, for each component and a modified mixture entropy term

$$\hat{\mathcal{H}}_D(\pi) = -\sum_{i \leq s} \hat{\mathcal{H}}_i(\pi).$$ (14)

According to Equation 14, the soft Q-function $Q_i$ is regulated by a partial sum of the marginal mixture entropies. By doing so, we aim to increase diversity between the Q functions as well as policy components. We call this variant Semi-Soft Actor-Critic Mixture (S2ACM).

Testing Environment. To evaluate our approach, we applied SACM on a variety of continuous control tasks in the MuJoCo physics simulator (Todorov et al., 2012). Agents are comprised of linked bodies with distinct topologies. Some topologies are simple (Swimmer that contains 3 bodies) while others are complex (Humanoid that contains 17 bodies). The agent is rewarded for moving forward as much as possible in a given time frame. The state includes the positions and velocities of all bodies. The action is a vector of $n$ dimensions that defines the amount of torque applied to each joint.
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Figure 3. Training Curves for Variable Mixture Sizes. We run the S2MAC algorithm for 1 million steps on the Hopper-v2 task from the Mujoco control suite. The graph illustrates the training curves of mixtures of different sizes: $n = 1, 2, 3$. The results of this experiment indicate that the algorithm's performance varies with mixture size. We attribute this finding to enhanced stability typically associated with ensemble methods.

Implementation Details. A neural network was used to implement the mixture policy with two fully-connected layers of width 300 and 400 and a ReLU activation after each layer. Each policy component, modeled as a multivariate Gaussian distribution, produced two outputs, representing the mean and the standard deviation. We experimented varying sizes of mixtures and report the number of components in our results. The critic accepted a state-action pair as input. The output layer of the critic, with $N$ neurons, represents the Q value of all components.

We ran each task for 3 million steps and report an average over 5 seeds. We used the Adam optimizer (Kingma & Ba, 2014) with a learning rate of 0.0003 and a batch size of 256 to train the SACM algorithm. The algorithm collects experience for 1000 steps, followed by 1000 consecutive stochastic gradient steps. All other hyper-parameters are set to their default values as implemented in the Soft Actor-Critic algorithm as part of the stable-baselines framework (Hill et al., 2018). The results are presented in Figure 2. We also include a comparison to relevant baselines, including (1) SAC (Haarnoja et al., 2018a), (2) DDPG (Lillicrap et al., 2015), and (3) TD3 (Fujimoto et al., 2018).

Mixture Size Sweep. We tested the effect of different mixture sizes on overall performance. We ran S2MAC on the Hopper-v2 control task with different values of $N$. Figure 3 shows an improvement when using higher-order mixtures.

7. Conclusion

Solving continuous control tasks with arbitrary policy classes presents the challenge of calculating the log probability of actions. Policy gradient methods use it "as-is" to optimize the policy while MaxEnt algorithms require it to administer entropy regularization. To avoid using complicated techniques recently proposed to meet this challenge, we advocate the use of mixture policies.

This paper extends the MaxEnt framework to handle mixture-policies, a class of universal density estimators. The novelty we present lies in the derivation of a tractable and intuitive mixture entropy estimator. The estimation we propose relies on a set of mixed marginal entropies. Each mixed-marginal, $\bar{H}_i$, measures the cumulative probability that all components assign to a sample from $\pi_i$. This stands in contrast to the case where the entropy of the $i$-th component is evaluated exclusively from $\pi_i$. From an exploration viewpoint, with this change, the intrinsic reward of an action sampled from mixture-component $\pi_i$ will be low if other mixture components are already exploring it. Our estimator allows MaxEnt algorithms to be applied to a broader class of policies, making them competitive with recent policy gradient approaches capable of optimizing non-unimodal policies.

Our empirical testing demonstrates that mixture policies match the state-of-the-art results of continuous control algorithms.
However, we did not find clear evidence that mixture policies outperform their single-component counterparts. We attribute this finding to two reasons. First, benchmarked tasks are unimodal in nature, so a unimodal policy should do. Second, that the mixture policy collapses to a mean policy in the presence of equal mixing weights. Confirming the first hypothesis requires testing our algorithm in more complex control tasks or alternatively designing tasks with a controlled number of modes in the optimal Q function. Regarding the second hypothesis, it is possible to optimize the weights as part of the Hierarchical reinforcement learning framework (Dietterich, 1998), or by using policy gradient for example. Future research will have to explore this.

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A. Mixture Entropy Lower Bound

In the following we derive an unbiased lower bound for the mixture entropy. Denoting \( z_i = \log \sum_j w_j \exp (-D(\pi_i || \pi_j)) \), we can write Equation 5 more compactly as:

\[
\hat{H}_D(\pi) = H(\pi | W) - \sum_i w_i z_i.
\]  (15)

In the following, we wish to upper bound \( z_i \) for all \( i \in [N] \). For each \( j \in [N] \) define \( b_j = \frac{1}{N} \) and \( d_{i,j} = w_j \exp (-D(\pi_i || \pi_j)) \). Since \( b_j \geq 0 \) and \( d_{i,j} \geq 0 \) for all \( j \), we can write the following inequality:

**Lemma A.1 (Log Sum Inequality).** For non-negative numbers \( a_1, a_2, \cdots a_N \) and \( b_1, \cdots b_N \):

\[
\sum_{j=1}^N a_j \log \frac{a_j}{b_j} \geq \left( \sum_{j=1}^N a_j \right) \log \frac{\sum_{j=1}^N a_j}{\sum_{j=1}^N b_j}.
\]  (16)

We have that:

\[
\log \sum_j a_j \leq \left( \sum_{j=1}^N \frac{1}{a_j} \right) \sum_j a_j \log \frac{a_j}{b_j}
\]  (17)

\[
= \frac{1}{\sum_j a_j} \left( \sum_j a_j \log a_j - a_j \log b_j \right).
\]  (18)

Since \( 0 < b_j < 1 \) and \( 0 < a_j < 1 \) then \(-C < a_j \log b_j < 0\). Therefore:

\[
\frac{1}{\sum_j a_j} \left( \sum_j a_j \log a_j - a_j \log b_j \right)
\]  (19)

\[
\leq \frac{1}{\sum_j a_j} \left( \sum_j a_j \log a_j + C \right).
\]  (20)

Next, we plug in the expression for \( a_j = w_j \exp (-D(\pi_i || \pi_j)) \). Since all mixing weights \( w_i = w \) are equal we can further simplify:

\[
\frac{1}{\sum_j a_j} \left( \sum_j a_j \log a_j + C \right)
\]  (21)

\[
= \frac{1}{w \sum_j d_j} \left( w \sum_j d_j \log wd_j + \frac{wC}{w} \right)
\]  (22)

\[
= \frac{1}{\sum_j d_j} \left( \sum_j d_j \log w + d_j \log d_j + \frac{C}{w} \right).
\]  (23)

Since \( w = \frac{1}{N} < 1 \) and \( 0 < d_j < 1 \), then again \( d_j \log w < 0 \). Therefore:

\[
\frac{1}{\sum_j d_j} \left( \sum_j d_j \log w + d_j \log d_j + \frac{C}{w} \right)
\]  (24)

\[
\leq \frac{1}{\sum_j d_j \sum_j d_j} \left( \sum_j d_j \log d_j + \frac{C}{w} \right).
\]  (25)
Since $d_{ii} = 1$ then $\sum_j d_j \geq 1$. Therefore:

$$\frac{1}{\sum_j d_j} \sum_j d_j \log d_j + \frac{C}{w} \leq \sum_j d_j \log d_j + \frac{C}{w} = \frac{C}{w} - \sum_j e^{-D(\pi_i||\pi_j)} D(\pi_i||\pi_j) \leq \frac{C}{w} - \sum_j e^{\mathcal{H}(\pi_i) - \mathcal{H}(\pi_i,\pi_j)}.$$  

Since $\mathcal{H}(\pi) \geq 0$, we continue to get that:

$$\frac{C}{w} - \sum_j e^{\mathcal{H}(\pi_i) - \mathcal{H}(\pi_i,\pi_j)} \leq \frac{C}{w} - \sum_j e^{-\mathcal{H}(\pi_i,\pi_j)}.$$  

We can write our lower bound as:

$$\hat{\mathcal{H}}_D(\pi) \geq \mathcal{H}(\pi|W) + \sum_i w_i \sum_j e^{-\mathcal{H}(\pi_i,\pi_j)}$$  \hspace{1cm} (26)$$

where we have omitted the constant $\frac{C}{w}$ since it does not depend on $\pi$, and thus does not alter the optimization problem.

We finally arrive at the joint optimization function:

$$J(\pi) = \sum_{t=0}^T \sum_{i=1}^N \mathbb{E}_{s_t,a_t \sim \rho_{\pi_i}} \left[ r(s_t, a_t) + \alpha_i \mathcal{H}(\pi_i(|s_t)) \right] + \sum_{j} \beta_{i,j} e^{-\mathcal{H}(\pi_i(|s_t),\pi_j(|s_t))},$$

**B. Complementary Mixture**

Tuning a parameter for each pair results in a quadratic number of optimization problems, which can be cumbersome for mixtures of many components. In order to reduce the need to optimize for $N^2$ different values, we propose measuring a single distance between each component and its *complementary mixture*. For each index $i$, define the complementary mixture $\pi_{-i}$ as the mixture obtained after omitting $\pi_i$. That is: $\pi_0, \ldots, \pi_{i-1}, \pi_{i+1}, \ldots, \pi_N$. The distance between each policy and its complementary mixture is then given by:

$$D(\pi_i||\pi_{-i}) = \sum_{t=0}^T \sum_{j \neq i} \mathbb{E}_{s_t,a_t \sim \rho_{\pi_i}} w_j D(d_{\pi_i,t}||d_{\pi_j,t}).$$  \hspace{1cm} (27)$$

With this reduction we calculate a single distance measure, $D(\pi_i||\pi_{-i})$, for each mixture member, and tune a single temperature value $\beta_i$. Plugging Equation 27 into the estimator in Equation 5 we can now write the maximum entropy objective function for mixture policies:

$$J(\pi) = \sum_{t=0}^T \sum_{i=1}^N w_i \mathbb{E}_{s_t,a_t \sim \rho_{\pi_i}} \left[ r(s_t, a_t) + \alpha_i \mathcal{H}(\pi_i(s_t)) \right] - \beta_i \log \left( w_i + w_{-i} \exp \left( -D(d_{\pi_i,t}||d_{\pi_{-i},t}) \right) \right),$$  \hspace{1cm} (28)$$
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where \( w_{-i} = \sum_{j \neq i} w_j \), and we used the fact that \( D(p_i \| p_i) = 0 \). Examining Equation 1 and Equation 28, we see that it is now possible to decompose the joint maximum mixture entropy objective function into \( N \) marginal optimization problems:

\[
J_i(\pi) = \sum_{t=0}^{T} \mathbb{E}_{s_t, a_t \sim \rho_{\pi_i}} \left[ r(s_t, a_t) + \alpha_i \mathcal{H}(\pi_i(\cdot | s_t)) \right] + \beta_i D(d_{\pi_i, t} \| d_{\pi_{-i}, t}).
\]  

(29)

Note that we have absorbed the mixing weights \( w_i \) into the temperature coefficients and used the fact that \( \arg\max_x \log(1 + x) = \arg\max_x \log(x) \).

Calculating the distance \( D(\pi_i \| \pi_{-i}) \) between component \( i \) and its complementary mixture \( \pi_{-i} \) requires broadcasting the experience that \( \pi_i \) collects to the rest of the mixture. However, we recall that increasing the exploration via experience sharing was our original motivation in using mixture policies in the first place.