On the Gaussian surface area of spectrahedra

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Abstract

We show that for sufficiently large \( n \geq 1 \) and \( d = Cn^{3/4} \) for some universal constant \( C > 0 \), a random spectrahedron with matrices drawn from Gaussian orthogonal ensemble has Gaussian surface area \( \Theta(n^{1/8}) \) with high probability.

1 Introduction

A spectrahedron \( S \subseteq \mathbb{R}^n \) is a set of the form

\[
S = \left\{ x \in \mathbb{R}^n : \sum_i x_i A^{(i)} \preceq B \right\},
\]

for some \( d \times d \) symmetric matrices \( A^{(1)}, \ldots, A^{(n)}, B \in \text{Sym}_d \). Here we will be concerned with the Gaussian surface area of \( S \), defined as

\[
\text{GSA}(S) = \liminf_{\delta \to 0} \frac{G^n(S^\text{out}_\delta)}{\delta},
\]

where \( S^\text{out}_\delta = \{ x \in S : \text{dist}(x, S) \leq \delta \} \) denotes the outer \( \delta \)-neighborhood of \( S \) under Euclidean distance and \( G^n(\cdot) \) denotes the standard Gaussian measure on \( \mathbb{R}^n \) whose density is \( (2\pi)^{-n/2} \exp(-\|x\|^2/2) \). Ball showed that the GSA of any convex body in \( \mathbb{R}^n \) is \( O(n^{1/4}) \) [Bal93], which was later shown to be tight by Nazarov [Naz03]. Moreover, Nazarov [KOS08] showed that the GSA of a \( d \)-facet polytope\(^1\) in \( \mathbb{R}^n \) is \( O(\sqrt{\log d}) \) and this fact has found application in learning theory and constructing pseudorandom generators for polytopes [KOS08, HKM13, ST17, CDS19]. We refer the interested reader to [KOS08, HKM13] for more details. Motivated by recent work [AY21], this raises the question of whether the GSA of spectrahedra is also small. In this note we answer this question in the negative. Recall that a matrix \( A \) drawn from the Gaussian orthogonal ensemble is a symmetric matrix whose entries \( \{A_{i,j}\}_{i \leq j} \) are all independent normal random variables of mean 0 having variance 1 if \( i < j \) and variance 2 if \( i = j \).

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\(^1\)A \( d \)-facet polytope is the special case of a spectrahedron when the matrices, \( A^{(1)}, \ldots, A^{(n)}, B \) are diagonal.
Theorem 1. For a universal constant $C > 0$ and any integers $n, d \geq 1$ satisfying $d \leq n/C$ the following hold. If $A^{(1)}, \ldots, A^{(n)}$ are i.i.d. drawn from the $d \times d$ Gaussian orthogonal ensemble, then the spectrahedron
\[
\mathcal{T} = \left\{ x \in \mathbb{R}^n : \sum_{i} x_i A^{(i)} \preceq 2\sqrt{nd} \cdot \mathbb{I} \right\}
\]
satisfies $\text{GSA}(\mathcal{T}) \geq c \cdot \sqrt{n/d}$ for some absolute constant $c > 0$ with probability at least $1 - C \exp(-dn^{-3/4}/C)$. Moreover, for any integer $d$ satisfying $d \leq n/C$, $\text{GSA}(\mathcal{T}) \leq 2\sqrt{n}/(\sqrt{\pi d})$ holds with probability at least $1 - \exp(-n/50)$.

The theorem shows the existence of spectrahedra with $\text{GSA}$ of $\Omega(n^{1/8})$. (In fact, a random spectrahedron as above satisfies this with constant probability). This lower bound can be contrasted with the $\text{GSA}$ upper bound of Ball [Bal93] of $O(n^{1/4})$ for arbitrary convex bodies. Moreover, the lower bound shows that in contrast to the case of polytopes, the $\text{GSA}$ of spectrahedra can depend polynomially on $d$. A natural open question is how large the $\text{GSA}$ of arbitrary spectrahedra can be; can spectrahedra with small $d$ (say, polynomial in $n$) achieve a $\text{GSA}$ of $\Theta(n^{1/4})$?

2 Preliminaries

For a matrix $A$, $\lambda_{\text{max}}(A)$ is the maximum eigenvalue of $A$. We use $g, x, A$ to denote random variables. We let $\mathcal{G}(0, \sigma^2)$ be the normal distribution with mean 0 and variance $\sigma^2$. We denote by $\mathcal{H}_d$ the $d \times d$ Gaussian orthogonal ensemble (GOE). Namely, $A \sim \mathcal{H}_d$ if it is a symmetric matrix with entries $\{A_{i,j}\}_{i \leq j}$ independently distributed satisfying $A_{i,j} \sim \mathcal{G}(0, 1)$ for $i < j$ and $A_{i,i} \sim \mathcal{G}(0, 2)$. To keep notations short, for $b \geq 0$ we use $[a \pm b]$ to represent the interval $[a - b, a + b]$. For every $c \geq 0$, we use $c \cdot [a \pm b]$ to represent the interval $[ac \pm bc]$. We denote the set of $n$-dimensional unit vectors by $S^{n-1}$. Finally, we let $\chi_n$ be the $\chi$ distribution with $n$ degrees of freedom, which is the square root of the sum of the squares of $n$ independent standard normal variables. The following are some simple facts about the $\chi$ distribution.

Fact 2. Let $n \in \mathbb{Z}_{\geq 0}$ and $h(\cdot)$ be the pdf of $\chi_n$. Then the following hold.

1. $h(x) \geq c$ for $x \in [\sqrt{n} \pm c]$, where $c > 0$ is an absolute constant.

2. $h(x) \leq \sqrt{n}/(\sqrt{\pi} \cdot |x|)$ for $x \in \mathbb{R}$.

Proof. Recall that by definition
\[
h(x) = \frac{1}{2^{n-1} \Gamma(n/2)} x^{n-1} e^{-x^2/2}
\]
for $x \geq 0$, where $\Gamma(\cdot)$ denotes the gamma function, and $h(x) = 0$ otherwise. By elementary calculus, $x^{n-1} e^{-x^2/2}$ monotonically increases for $0 \leq x < \sqrt{n-1}$ and monotonically decreases for $x > \sqrt{n-1}$. We therefore have
\[
x^{n-1} e^{-x^2/2} \geq \min \left\{ (\sqrt{n} + c)^{n-1} e^{-(\sqrt{n}+c)^2/2}, (\sqrt{n} - c)^{n-1} e^{-(\sqrt{n}-c)^2/2} \right\}
\]
for $0 < c \leq 1$ and $x \in [\sqrt{n} \pm c]$. Item 1 now follows from Eq. (3) and the fact that $\Gamma(z) \leq \sqrt{2\pi} z^{z-1/2} e^{-z+1/(12z)}$ for all $z > 0$ [AAR99, Jam15].

Item 2 is trivial for $x \leq 0$. For $x > 0$, it follows from the inequalities $\Gamma(z) \geq \sqrt{2\pi} z^{z-1/2} e^{-z}$ for all $z > 0$ [AAR99, Jam15] and $x^n e^{-x^2/2} \leq n^{n/2} e^{-n/2}$, which follows from the same argument as above. \qed
Lemma 3 ([LM00, comment below Lemma 1]). For \( n \geq 1 \), let \( r \) be a random variable distributed according to \( \chi_n \). Then for every \( x > 0 \), we have

\[
\Pr \left[ n - 2\sqrt{nx} \leq r^2 \leq n + 2\sqrt{nx} + 2x \right] \geq 1 - 2e^{-x}.
\]

For our purposes, it will be convenient to use an alternative definition of Gaussian surface area in terms of the inner surface area. Namely, for \( S_\delta^\text{in} = \{ x \in S : \text{dist}(x, S^c) \leq \delta \} \) where \( S^c \) is the complement of the body \( S \), we define,

\[
\text{GSA}(S) = \lim_{\delta \to 0} \frac{G^n(S_\delta^\text{in})}{\delta}.
\] (4)

It follows from Huang et al. [HXZ21, Theorem 3.3] that this definition is equivalent to the one in Eq. (1) when \( S \) is a convex body that contains the origin, which is sufficient for our purposes.

To prove our main theorem, we use the following facts, starting with a well-known bound on the size of an \( \varepsilon \)-net of the \( n \)-dimensional sphere.

Fact 4 ([Tao12, Lemma 2.3.4]). For every \( d \geq 1 \) and any \( 0 < \varepsilon < 1/2 \) there exists an \( \varepsilon \)-net of the sphere \( S_{d-1} \) of cardinality at most \( (3/\varepsilon)^d \).

The following claim gives a formula for the pdf of the product of two real-valued random variables.

Claim 5 ([RS15, Page 134, Theorem 3]). Let \( x, y \) be two real-valued random variables and \( f \) be the pdf of \( (x, y) \). Then the pdf of \( z = x \cdot y \) is given by

\[
g(z) = \int_{-\infty}^\infty f \left( \frac{z}{x}, \frac{z}{y} \right) \cdot \frac{1}{|x|} dx.
\]

Theorem 6 ([LR10, Theorem 1]). Let \( A \sim \mathcal{H}_d \). For every \( 0 < \eta < 1 \), it holds that

\[
\Pr \left[ \lambda_{\max}(A) \in 2\sqrt{d}[1 \pm \eta] \right] \geq 1 - C \cdot e^{-dn^{3/2}/C},
\]

for some absolute constant \( C > 0 \).

3 Proof of main theorem

The core of the argument is in the following lemma, bounding \( q(2\sqrt{nd}) \) where \( q \) is the pdf of the largest eigenvalue of the matrix showing up in Eq. (2). We will later show that this value is essentially the same as \( \text{GSA}(T) \), where \( T \) is the spectrahedron in the statement of the theorem.

Lemma 7. For \( n, d \geq 1 \) and \( A^{(1)}, \ldots, A^{(n)} \in \text{Sym}_d \), let \( q(\cdot) \) be the probability density function of

\[
\lambda_{\max} \left( \sum_i x_i A^{(i)} \right),
\]

where \( x = (x_1, \ldots, x_n) \) is a random vector and each entry is i.i.d. drawn from \( \mathcal{G}(0, 1) \). If \( A^{(1)}, \ldots, A^{(n)} \) are i.i.d. drawn from the \( d \times d \) Gaussian orthogonal ensemble, then \( q(2\sqrt{nd}) \geq c \cdot \sqrt{1/d} \) with probability at least \( 1 - C \exp(-dn^{-3/4}/C) \) (over the choice of \( A^{(1)}, \ldots, A^{(n)} \)) where \( c, C > 0 \) are universal constants. Moreover, for any integer \( d \) and any \( d \times d \) matrices \( A^{(1)}, \ldots, A^{(n)} \), \( q(2\sqrt{nd}) \leq 1/(2\sqrt{\pi d}) \).
Proof. Let $y \sim S^{n-1}$ be chosen uniformly from the unit sphere and for matrices $A^{(1)}, \ldots, A^{(n)}$, denote by $p$ the pdf of $\lambda_{\max}(\sum_i y_i A^{(i)})$. Let $r \sim \chi_n$ and notice that $r y$ is distributed like $x$ (since both are spherically symmetric and by definition, have equally distributed norms). Denote by $h$ the pdf of $r$. By Claim 5, we have

$$q(2\sqrt{nd}) = \int_{-\infty}^{\infty} h\left(2\sqrt{nd}/z\right) p(z) \frac{1}{|z|} dz. \quad (5)$$

Using Item 2 of Fact 2, $h(2\sqrt{nd}/z)/|z| \leq 1/(2\sqrt{\pi d})$ for all $z$. Hence Eq. (5) can be bounded as $(1/(2\sqrt{\pi d})) \cdot \int_{-\infty}^{\infty} p(z)dz = 1/(2\sqrt{\pi d})$, establishing the claimed upper bound on $q$.

To prove the lower bound on $q$, let $A^{(1)}, \ldots, A^{(n)} \sim \mathcal{H}_d$ be $n$ matrices chosen i.i.d. from the Gaussian orthogonal ensemble. Observe that by Theorem 6, we have

$$\Pr\left[\lambda_{\max}\left(\sum_{i=1}^{n} y_i A^{(i)}\right) \in I\right] \geq 1 - C \exp(-dn^{-3/4}/C), \quad (6)$$

where

$$I = 2\sqrt{d} \cdot |1 \pm c/\sqrt{n}|,$$

for some universal constants $C, c > 0$. Define the set of matrices

$$G = \left\{\left(A^{(1)}, \ldots, A^{(n)}\right) : \Pr\left[\lambda_{\max}\left(\sum_{i=1}^{n} y_i A^{(i)}\right) \in I\right] \geq \frac{1}{2}\right\}.$$

Then, using the definition of $G$ and Eq. (6), we have

$$\Pr\left[\left(A^{(1)}, \ldots, A^{(n)}\right) \in G\right] \geq 1 - 2C \exp(-dn^{-3/4}/C).$$

Now fix any $(A^{(1)}, \ldots, A^{(n)}) \in G$. By definition of $G$, $\int_I p(z)dz \geq 1/2$, and therefore the right-hand side of Eq. (5) is at least

$$\int_I h\left(2\sqrt{nd}/z\right) p(z) \frac{1}{|z|} dz \geq c \cdot \int_I p(z) \frac{1}{|z|} dz \geq \frac{c}{2\sqrt{d}(1 + c/\sqrt{n})} \cdot \int_I p(z)dz \geq \frac{c}{5\sqrt{d}}, \quad (7)$$

for some absolute constant $c > 0$, where we used Item 1 of Fact 2 to conclude that $h(2\sqrt{nd}/z) \geq c$ for all $z \in I$. \hfill \Box

We next relate $q(2\sqrt{nd})$ to $\text{GSA}(T)$. For a vector $v \in S^{d-1}$, and $d \times d$ symmetric matrices $A^{(1)}, \ldots, A^{(n)}$, define the vector

$$W_v = \left(v^T A^{(1)} v, v^T A^{(2)} v, \ldots, v^T A^{(n)} v\right) \in \mathbb{R}^n. \quad (8)$$

Notice that $T$ can be written as

$$T = \left\{x \in \mathbb{R}^n : \sum_i x_i A^{(i)} \leq 2\sqrt{nd} \cdot \|v\|\right\} = \left\{x \in \mathbb{R}^n : \forall v \in S^{d-1}, \left\langle x, W_v \right\rangle \leq 2\sqrt{nd}\right\}.$$

We say that $A^{(1)}, \ldots, A^{(n)}$ are good if

$$\forall v \in S^{d-1}, \frac{1}{2}\sqrt{n} \leq \|W_v\| \leq 2\sqrt{n}. \quad (9)$$
Lemma 8. There exists a constant $C \geq 1$ such that for all integers $n$ and $d \leq n/C$, random matrices $A^{(1)}, \ldots, A^{(n)}$ drawn i.i.d. from $\mathcal{H}_d$ are good with probability at least $1 - \exp\left(-n/50\right)$.

Proof. For a fixed $v \in S^{d-1}$, we claim that

$$\Pr[n \leq \|W_v\|^2 \leq 3n] \geq 1 - 2\exp\left(-n/40\right).$$

To see this, observe that by definition of the Gaussian orthogonal ensemble, for $A \sim \mathcal{H}_d$ and unit vector $v \in \mathbb{R}^d$, $v^T Av = \sum_{i,j} v_i v_j A_{i,j}$ is distributed according to

$$\left(4 \sum_{i<j} v_i^2 v_j^2 + 2 \sum_i v_i^4\right)^{1/2} \cdot \mathcal{G}(0, 1) = \sqrt{2} \cdot \mathcal{G}(0, 1).$$

Therefore, each entry in $W_v$ is distributed according to $\mathcal{G}(0, 2)$, and Lemma 3 implies Eq. (9). We next prove that with high probability (over the $A^{(i)}$’s), for every unit vector $z$, $\|W_v\|$ is large. First, by Fact 4, there exists a set $\mathcal{V} = \{v_1, \ldots, v_{10^d}\} \subseteq \mathbb{R}^d$ of unit vectors that form a $10^{-4}$-net of the unit Euclidean sphere. Applying a union bound on $\mathcal{V}$, we have

$$\Pr[\forall v \in \mathcal{V} : n \leq \|W_v\|^2 \leq 3n] \geq 1 - 2\exp\left(-n/40\right) \cdot 10^{5d} \geq 1 - \exp\left(-n/50\right),$$

here we used that $d \leq n/C$ for a sufficiently large $C$.

To conclude the proof, it suffices to show that if $A^{(1)}, \ldots, A^{(n)}$ are such that

$$\forall v \in \mathcal{V}, \ n \leq \|W_v\|^2 \leq 3n,$$

then also

$$\forall z \in S^{d-1}, \ \|W_z\| \geq \frac{1}{2} \sqrt{n}.$$

Let $b_{\max} = \max_{z \in S^{d-1}} \|W_z\| = b_{\min} = \min_{z \in S^{d-1}} \|W_z\|$. Let $z_{\max}$ and $z_{\min}$ be the vectors achieving the maximum and the minimum respectively. Let $v_{\max}$ and $v_{\min}$ be the vectors in $\mathcal{V}$ that are closest to $z_{\max}$ and $z_{\min}$, respectively. For any vectors $z, v \in S^{d-1}$ with $\|z - v\| \leq 10^{-4}$, applying the spectral decomposition of $zz^T - vv^T$, there exist unit vectors $u_1, u_2$ and $0 \leq \lambda = \frac{1}{100}$ such that

$$zz^T - vv^T = \lambda \cdot (u_1 u_1^T - u_2 u_2^T).$$

Hence

$$\|W_z - W_v\|^2 = \sum_{i=1}^n \left(z_i^T A^{(i)} z - v_i^T A^{(i)} v\right)^2 = \sum_{i=1}^n \left(\text{Tr}(A^{(i)}(zz^T - vv^T))\right)^2 \leq \frac{1}{10^4} \sum_{i=1}^n \left(u_i^T A^{(i)} u_1 - u_2^T A^{(i)} u_2\right)^2 \leq \frac{1}{5000} \sum_{i=1}^n \left(u_i^T A^{(i)} u_1\right)^2 + \left(u_2^T A^{(i)} u_2\right)^2 \leq \frac{b_{\max}^2}{2500}.$$

Choosing $z = z_{\max}$ and $v = v_{\max}$, we have

$$\|W_{z_{\max}}\| \leq \|W_{v_{\max}}\| + \frac{b_{\max}}{50}.$$
Now, since \( \|W_{z_{\text{max}}}\| = b_{\text{max}} \), we have
\[
b_{\text{max}} \leq \frac{50}{49} \|W_{v_{\text{max}}}\| \leq \frac{50}{49} \sqrt{3n} \leq 2\sqrt{n}.
\]
Similarly, we set \( z = z_{\text{min}} \) and \( v = v_{\text{min}} \) and obtain
\[
b_{\text{min}} \geq \|W_{v_{\text{min}}}\| - \frac{b_{\text{max}}}{50} \geq \sqrt{n} - \frac{1}{25} \sqrt{n} > \frac{1}{2} \sqrt{n}.
\]
This concludes the result. \( \square \)

For the following claim, we define the inner and outer shells of \( \mathcal{T} \) as
\[
\mathcal{D}_\delta^{\text{in}} = \left\{ x : \lambda_{\text{max}} \left( \sum_i x_i A^{(i)} \right) \in \sqrt{n} \cdot [2\sqrt{d} - \delta, 2\sqrt{d}] \right\},
\]
\[
\mathcal{D}_\delta^{\text{out}} = \left\{ x : \lambda_{\text{max}} \left( \sum_i x_i A^{(i)} \right) \in \sqrt{n} \cdot [2\sqrt{d}, 2\sqrt{d} + \delta] \right\}.
\]
Also recall the inner and outer neighborhoods of \( \mathcal{T} \), defined as
\[
\mathcal{T}_\delta^{\text{in}} = \{ x \in \mathcal{T} : \exists y \notin \mathcal{T} : \| x - y \| \leq \delta \},
\]
\[
\mathcal{T}_\delta^{\text{out}} = \{ x \notin \mathcal{T} : \exists y \in \mathcal{T} : \| x - y \| \leq \delta \}.
\]

**Claim 9.** For sufficiently small \( \delta > 0 \) and any good \( A^{(1)}, \ldots, A^{(n)} \), we have \( \mathcal{D}_\delta^{\text{in}} \subseteq \mathcal{T}_\delta^{\text{in}} \) and \( \mathcal{T}_\delta^{\text{out}} \subseteq \mathcal{D}_\delta^{\text{out}} \).

**Proof.** For every \( x \in \mathcal{D}_\delta^{\text{in}} \), let \( v \) be a unit eigenvector of \( \sum_i x_i A^{(i)} \) with the eigenvalue \( \lambda_{\text{max}}(\sum_i x_i A^{(i)}) \). Therefore,
\[
\langle x, W_v \rangle = v^T \left( \sum_i x_i A^{(i)} \right) v \geq (2\sqrt{d} - \delta) \sqrt{n}.
\]
Setting \( y = 2\delta \sqrt{n} W_v / \|W_v\|^2 \), we have
\[
\langle x + y, W_v \rangle = \langle x, W_v \rangle + 2\delta \sqrt{n} \geq (2\sqrt{d} - \delta) \sqrt{n} + 2\delta \sqrt{n} = (2\sqrt{d} + \delta) \sqrt{n},
\]
and so \( x + y \notin \mathcal{T} \). Moreover, since \( A^{(1)}, \ldots, A^{(n)} \) are good, \( \|y\| = 2\delta \sqrt{n} / \|W_v\| \leq 4\delta \) and therefore \( x \in \mathcal{T}_\delta^{\text{in}} \), as desired. For the other containment, let \( x \in \mathcal{T}_\delta^{\text{out}} \). Then for any unit vector \( v \), by Cauchy-Schwarz and using \( \|W_v\| \leq 2\sqrt{n} \),
\[
\langle x, W_v \rangle \leq 2\sqrt{nd} + 2\delta \sqrt{n},
\]
implying that \( x \in \mathcal{D}_\delta^{\text{out}} \), as desired. \( \square \)

We now prove our main theorem.

**Proof of Theorem 1.** By Lemmas 7 and 8, if \( A^{(1)}, \ldots, A^{(n)} \) are i.i.d. drawn from the \( d \times d \) Gaussian orthogonal ensemble, then with probability at least \( 1 - C \exp(-dn^{-3/4}/C) \), we have that \( q(2\sqrt{nd}) \geq c \cdot \sqrt{1/d} \) (where \( q(\cdot) \) is as defined in Lemma 7) and that \( A^{(1)}, \ldots, A^{(n)} \) are good, where \( c, C > 0 \) are some constants. Since \( q(\cdot) \) is continuous, the former implies that \( \mathcal{G}^n(\mathcal{D}_\delta^{\text{in}}) \geq c\delta \sqrt{n/(2d)} \) for sufficiently small \( \delta > 0 \). Thus, \( \mathcal{G}^n(\mathcal{T}_\delta^{\text{in}}) \geq c\delta \sqrt{n/(2d)} \) by Claim 9. By definition of \( \text{GSA}(S) = \lim_{\delta \to 0} \mathcal{G}^n(S^\delta) / \delta \), we obtain the desired lower bound on \( \text{GSA}(\mathcal{T}) \). Similarly, by Lemmas 7 and 8, if \( A^{(1)}, \ldots, A^{(n)} \) are i.i.d. drawn from the \( d \times d \) Gaussian orthogonal ensemble, then with probability at least \( 1 - \exp(-n/50) \), \( \mathcal{G}^n(\mathcal{D}_\delta^{\text{out}}) \leq \delta \sqrt{n}/(\sqrt{\pi d}) \) for sufficiently small \( \delta > 0 \). Thus, \( \mathcal{G}^n(\mathcal{T}_\delta^{\text{out}}) \leq \delta \sqrt{n}/(\sqrt{\pi d}) \) by Claim 9. We complete the proof using \( \text{GSA}(S) = \lim_{\delta \to 0} \mathcal{G}^n(S^\delta) / \delta \). \( \square \)
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