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In-process identification of milling parameters based on digital twin driven intelligent algorithm

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Abstract: The potential benefits of Industry 4.0 have led to an increased interest in smart manufacturing. To facilitate the self-diagnosis and adaptive ability in smart milling system, a digital twin driven intelligent algorithm for monitoring in-process milling parameters is proposed here. The algorithm can extract the radial width of cut, axial depth of cut, cutter runout parameters and cutting constants in the end milling process at the same time only by using force sensor. It is an important breakthrough in this paper to converge two different force models to realize cyber-physical fusion for identifying milling parameters in the milling process. By using the convolution force model, digital twin technology can extract the approximate solution of milling parameters in the machining process in advance, so as to narrow the range of solution. Furthermore, the subsequent artificial intelligence algorithm can find the accurate solution of the current milling parameters in a short calculation time by cyber-physical fusion with the numerical force model considering cutter runout effect. Milling experiments are carried out to validate the proposed algorithm. It is shown that due to the complementary advantages of the convolution force model and numerical force model, the algorithm proposed in this paper can give consideration to the identification accuracy and calculation efficiency.

Keywords: cyber-physical fusion; digital twin; smart manufacturing; milling
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**Nomenclature**

| Symbol | Description |
|--------|-------------|
| $\mathbf{A}$ | vectors of the Fourier series coefficients for the milling force |
| $\alpha, N, D$ | helix angle, number of cutter flutes and cutter diameter |
| $\beta_a$ | axial angular range of cut |
| $CWD$ | Fourier transform of the chip width density function |
| $d_a, d_r$ | axial depth of cut, radial width of cut |
| $\Delta \phi$ | phase shift between the starting angle position of force measurement and the origin of force model coordinate |
| $\phi$ | cutter angular displacement |
| $k_t, k_r$ | cutting constants of lumped shearing force model in the tangential and radial directions |
| $k_{ts}, k_{rs}$ | shearing force constants in the tangential and radial directions |
| $k_{tp}, k_{rp}$ | ploughing force constants in the tangential and radial directions |
| $\lambda$ | angular location of the cutter runout |
| $\theta$ | angular position of cutting edge at the workpiece |
| $\theta_1, \theta_2$ | cutting angles of entry and exit |
| $\rho$ | magnitude of cutter offset |
| $N_z$ | number of axial disk element |
| $P_1(Nk), P_2(Nk)$ | Fourier transforms of local tangential forces per unit width at the tooth passing frequency $Nk$ |
| $t_c(i, j, k)$ | the chip thickness of the $i$-th disk element and the $k$-th flute in the cut at the cutter angular position $\phi_j$ |
| $t_s$ | feed per tooth |

**1. Introduction**

Obviously, if the cutting tool is worn to a certain extent while milling the workpiece, the quality of the parts may be degraded due to large metal removing rate. On the other hand, for cutting tools in the initial wear stage, productivity may be reduced under conservative cutting conditions. Accordingly, an adaptive control strategy is usually employed to determine the cutting conditions in a smart milling system. In such a system, a given cutting power or motor load is usually used as a threshold to decide whether to replace the cutting tool or adjust feed speed or stop the machine tool [1-2]. In fact, it is difficult to perform this adaptive strategy effectively without knowing the current cutting depths (including axial depth of cut and radial width of cut) in milling, because even if the cutting tool is not worn enough to be replaced, the cutting power or motor load may exceed the threshold due to the increase of the cutting depths. Similarly, when the cutting tool is worn to the extent that it must be replaced, the cutting power or motor load may be below the threshold due to the
decrease of the cutting depths. In addition, if the information of the current cutting depths can be obtained in the milling process, the adaptive strategy can also be used to adjust the feed rate to improve the machining efficiency or surface quality of workpiece. In real-life applications, due to the different stages of rough milling and finish milling, the cutting depths of each machining pass may be different. Even in the same machining pass, different radial width of cut and axial depth of cut may be produced due to the change of workpiece shape. Unfortunately, it should be noted that the machining data of these cutting depths are not included in the NC machining program. Tarng et al. [3] suggested that the software of the CAD / CAM system can be used to identify the sweep area of the tool path across the workpiece, then the radial width can be calculated through the identified sweep area. However, the adaptive strategy presented in [3] must limit the axial depth of cut to a constant. A method of using ultrasonic sensor to monitor the axial depth of cut in end milling was proposed in [4]. The measurement system uses two ultrasonic sensors installed on both sides of the spindle. One sensor measures the workpiece surface in front of the tool path during machining, and the other sensor measures the machined surface behind the cut. Accordingly, the axial depth of cut is equal to the offset distance between the two heights measured by the ultrasonic sensors. However, the radial width can’t be measured simultaneously by this measurement system. The other approach to detect the axial depth of cut automatically was presented by Gaja and Liou [5]. In their work, an acoustic emission (AE) sensor, SAS statistical software and artificial neural network-based model are used to identify the axial depth of cut. In their study, it is found that the signal measured from the AE sensor is complex with both nonlinearity and nonstationarity. Although the rules established by the artificial neural network model can well determine the relationship between the signal obtained from the AE sensor and the axial depth of cut, they need to be re-learned under different radial depth of cut. Similarly, if this method is used to identify the radial depth of cut, one must spend a lot of time to establish the relationships between AE signal and radial depth of cut for a constant axial depth of cut and rules applicable to various relationships through the artificial neural network. When above mentioned methods are implemented in a smart manufacturing, only a single information of radial width of cut or axial depth of cut can be extracted, so the benefits of the smart manufacturing system are limited. Although it is possible to integrate multiple sensors according to above mentioned methods to identify the current axial depth of cut and radial width in milling at the same time, it will increase the equipment cost and the difficulty of information integration.

On the other hand, in the context of Industry 4.0, smart manufacturing makes people more and more interested in digital twin technology which is considered as a core technology to support smart manufacturing by realizing cyber-physical fusion.
Some digital twin technologies for identifying the current cutting depths in milling have been established by the previous works [10-14]. Through the digital twin technology, it is found that both radial width and axial depths of cut can be simultaneously extracted only using force sensor based on the milling force models as presented in [15-18]. There are two key strategies for extracting cutting depths from the measured milling force signals in the previous studies [10-14]. The first is to locate the angular positions of entry cut and exit cut according to the measured cutting force signals while one of the cutting flutes swept the cutting area, and the second is to determine the cutting depths based on the measured average cutting forces in the feed direction and the cross-feed direction. When using the first strategy, one can only need to pay attention to the pulsation shape of the cutting force, and do not need to study the magnitude of cutting force. However, under the cutting condition of multi-tooth engaging simultaneously in the cutting area, which is the most cases in milling, the first strategy to determine the angular positions of entry cut and exit cut of a cutting tooth is difficult to carry out. Compared with the first strategy, the second strategy using the method of measuring average cutting force will be easier. In addition, the average cutting force model can be applied to various cutting conditions without the need to consider whether the cutting condition in the cutting area is single-tooth engaging or multi-tooth engaging. However, the application of the second strategy needs a prior knowledge regarding the specific cutting coefficients, which requires a series of cutting experiments in advance. Even if the specific cutting coefficients are given, the increase of tool wear will lead to some deviations of the pre-established data. In addition, when the dynamometer is used for a long time, an average force drift may occur due to the charge leakage in the force sensor [19-20].

To solve those problems existing in the second strategy, this paper proposes an algorithm to extract the cutting parameters using force sensor without requiring average force signals. The algorithm proposed in this paper can simultaneously extract the axial depth of cut, radial width and specific cutting coefficients only by using a cutting force sensor, which can improve the self-diagnosis and self-adaptation ability of milling system in a more effective and cheap way.

Digital twin framework used to realize cyber-physical fusion for in-process identification of milling parameters are presented in section 2. Section 3 illustrates the identification process for extracting the cutting depths and specific cutting coefficients. Experiment verification and discussion are afforded in section 4 followed by conclusions.

2. Digital twin framework

Figure 1 shows the digital twin framework used in this paper to realize cyber-physical fusion.
The framework includes physical milling space and virtual milling space, in which physical milling space and virtual milling space are connected through the measured cutting force data and milling parameters to form a closed loop. The physical milling space includes machine tool, cutting tool, workpiece, dynamometer and related infrastructure. Physical milling space really completes the specific cutting task and generates the cutting force, and then transmits the measured cutting force data to the virtual milling space.

The virtual milling space consists of a set of models to describe the physical counterpart. It is the mapping image of physical milling space in the cutting process, and has the characteristics of real-time synchronization. The in-process information of milling parameters (including radial width of cut, axial depth of cut, cutter runout parameters and specific cutting coefficients) generated by virtual simulation is fed back to the physical milling space to direct the corresponding actual milling process. The in-process specific cutting coefficient can be used to diagnose tool wear condition, the identified cutting depths in milling can be used to adjust feed rate or spindle speed, so as to optimize the manufacturing benefit. It should be pointed out that cutting force model is the most important model in virtual milling space for the identification of milling parameters. In order to give consideration to the identification efficiency and accuracy of milling parameters, two cutting force modes are integrated in this paper. The two cutting force modes used here have been introduced in the previous literatures [17,21], so only a brief overview is given as follow.

2.1 Numerical cutting force model

The numerical cutting force model used here was developed by Kline et al. [17], which was widely used in previous research work [22-24]. In the numerical force model, within the axial depth of cut, the milling cutter with \( N \) flute is divided into \( N_z \) axial disc elements along the cutter axis as shown in Fig. 2.
Therefore, by summing local forces $DFX(i, j)$ and $DFY(i, j)$ over all the axial disk elements, the total cutting forces in the X and Y directions at the cutter angular position $\phi_j$ can be expressed as:

\[
\begin{align*}
F_x(\phi_j) &= \sum_{i=1}^{N} DFX(i, j) \\
F_y(\phi_j) &= \sum_{i=1}^{N} DFY(i, j)
\end{align*}
\]  

(1)

where $DFX(i, j)$ and $DFY(i, j)$ are the local forces in the X and Y directions on the $i$th disk element at the cutter angular position $\phi_j$ and are given by:

\[
\begin{align*}
DFX(i, j) &= \sum_{k=1}^{N} f_t(i, j, k) \cos[\theta(i, j, k)] + f_r(i, j, k) \sin[\theta(i, j, k)] \\
DFY(i, j) &= \sum_{k=1}^{N} f_t(i, j, k) \sin[\theta(i, j, k)] - f_r(i, j, k) \cos[\theta(i, j, k)]
\end{align*}
\]  

(2)

where $f_t(i, j, k)$ and $f_r(i, j, k)$ are tangential and radial cutting forces on the $i$th disk element and $k$th flute at the cutter angular position $\phi_j$, and $\theta(i, j, k)$ is the angular position of the $i$th disk element and the $k$th flute in the cut at the cutter angular position $\phi_j$. In the milling process as shown in Fig. 2, when $\theta(i, j, k)$ is limited by $\theta_1 \leq \theta(i, j, k) \leq \theta_2$, $f_t(i, j, k)$ and $f_r(i, j, k)$ are written as:
\[
\begin{align*}
\begin{cases}
  f_t(i, j, k) = k_t d_z t_c(i, j, k) \\
  f_r(i, j, k) = k_r f_t(i, j, k)
\end{cases}
\tag{3}
\end{align*}
\]

where \( t_c(i, j, k) \) is the chip thickness of the \( i \)th disk element and the \( k \)th flute in the cut at the cutter angular position \( \phi_j \), and \( d_z \) is the chip width of the disk element (i.e. \( d_z = d_a/N_z \)). It should be noted that if \( \theta(i, j, k) \) is not between cutter entry angle \( \theta_1 \) and cutter exit angle \( \theta_2 \), both \( f_t(i, j, k) \) and \( f_r(i, j, k) \) are equal to zero.

Without considering the effect of cutter runout, the expression of \( t_c(i, j, k) \) is approximated by
\[
t_c(i, j, k) = t_x \sin \theta(i, j, k) \tag{4}
\]

### 2.2 Convolution cutting force model

Wang et al. [21] proposed a model of total cutting forces based on convolution integral approach. In this model, the total milling forces in X and Y directions can be expressed as Fourier series in complex form:
\[
F(\phi) = \begin{pmatrix}
  F_x(\phi) \\
  F_y(\phi)
\end{pmatrix} = \sum_{k=0}^{\infty} A_x(Nk) e^{iNk\phi} + \sum_{k=0}^{\infty} A_y(Nk) e^{iNk\phi}, k = 0, \pm 1, \pm 2...
\tag{5}
\]

The Fourier series coefficients \( A_x(Nk) \) and \( A_y(Nk) \) can be written as an algebraic representation of helix angle \( \alpha \), number of cutter flutes \( N \), cutter diameter \( D \), axial depth of cut \( d_a \), cutter entry angle \( \theta_1 \), cutter exit angle \( \theta_2 \), feed per tooth \( t_x \) and cutting constants \( (k_t \) and \( k_r) : \)
\[
\begin{pmatrix}
  A_x(Nk) \\
  A_y(Nk)
\end{pmatrix} = \frac{N}{2\pi} \text{CWD}(Nk) \begin{pmatrix}
  P_x(Nk) \\
  P_y(Nk)
\end{pmatrix}
\tag{6}
\]

where
\[
\text{CWD}(Nk) = \frac{D \sin \frac{Nk\beta_x}{2} e^{-\frac{Nk\beta_y}{2}}}{Nk \tan \alpha}, \quad \beta_x = \frac{2d_a \tan \alpha}{D}
\tag{7}
\]
\[
\begin{pmatrix}
  P_x(Nk) \\
  P_y(Nk)
\end{pmatrix} = k_r \begin{pmatrix} 1 & k_r \\ -k_r & 1 \end{pmatrix} \begin{pmatrix}
  P_x(Nk) \\
  P_y(Nk)
\end{pmatrix}
\tag{8}
\]
The first feature of representing total milling force as Fourier series is to generate the volume of massive data, because the Fourier series has infinite terms, and each term is valuable. The second feature is data variety. In light of the nature of Fourier series, the coefficients of each series exist independently of each other, so infinite groups of independent simultaneous equations can be used to extract the various cutting information of interest to the users. The third feature is that it can increase the calculation velocity of the solution, since each Fourier coefficient in the Fourier series can be clearly written as an analytical expression of the cutting parameters, which greatly improves the calculation speed of the solution. With the 4V (Volume, Velocity, Variety and Value) characteristics of big data, the convolution force model can provide an efficient strategy for extracting the online milling parameters, which will be presented in next section.

3. Milling parameters identification from cutting force model

Equation (5) shows that the total cutting force can be decomposed into the frequency components at the tooth passing frequency \( \omega = Nk \). The normalized frequency of \( "Nk" \) stands for an absolute frequency of \( \omega = Nk \omega_s \), where \( \omega_s \) is spindle rotating frequency. Through the frequency components of each tooth passing frequency in X and Y directions, the current milling parameters, including axial depth of cut, radial width of cut and cutting constants, can be identified directly by using digital twin technology.

3.1 Extracting radial width of cut and radial cutting constant \( k_r \)

By letting \( k=1 \) in Eq. (6), a ratio of frequency components may be given by

\[
\frac{A_x(N)}{A_y(N)} = \frac{A_x'(N)}{A_y'(N)} = b + jc
\]

where \( A_x'(N) \) and \( A_y'(N) \) are the force data measured in frequency domain from the physical milling space, so the values of \( b \) and \( c \) can be obtained directly from the force measurement. Substituting Eqs. (6)-(8) into Eq. (10), the left-hand side of Eq. (10) can be written as

\[
\frac{A_x(N)}{A_y(N)} = \frac{P_1(N) + k_r P_2(N)}{-k_r P_1(N) + P_2(N)} \quad (11)
\]
of digital twin can be found as:

\[
b + jc = \frac{P_1(N) + k P_2(N)}{-k_r P_1(N) + P_2(N)} = \frac{[P_{1r}(N) + jP_{1i}(N)] + k [P_{2r}(N) + jP_{2i}(N)]}{-k_r [P_{1r}(N) + jP_{1i}(N)] + [P_{2r}(N) + jP_{2i}(N)]}
\]  

(12a)

or

\[
(b + jc) [k [P_{1r}(N) + jP_{1i}(N)] + [P_{2r}(N) + jP_{2i}(N)]] = [P_{1r}(N) + jP_{1i}(N)] + k [P_{2r}(N) + jP_{2i}(N)]
\]

(12b)

where \( P_{1r}(N), P_{2r}(N), P_{1i}(N), P_{2i}(N) \) represent the real and imaginary parts of \( P_1(N) \) and \( P_2(N) \), and can be expressed as:

\[
\begin{align*}
\left( \frac{P_{1r}(N)}{P_{1i}(N)} \right) &= \frac{-1}{8 - 2N^2} \left[ \begin{array}{c} 2 \cos 2\theta \cos N\theta + N \sin 2\theta \sin N\theta \\ N \sin 2\theta \cos N\theta - 2 \cos 2\theta \sin N\theta \end{array} \right] \\
\left( \frac{P_{2r}(N)}{P_{2i}(N)} \right) &= \frac{1}{8 - 2N^2} \left[ \begin{array}{c} N \cos 2\theta \sin N\theta - 2 \sin 2\theta \cos N\theta \\ 2 \sin 2\theta \sin N\theta + N \cos 2\theta \cos N\theta \end{array} \right] + \frac{1}{2N} \left[ \begin{array}{c} \sin N\theta \\ \cos N\theta \end{array} \right]
\end{align*}
\]

(13a) \hspace{1cm} (13b)

By splitting both sides of Eq. (12b) into their real and imaginary parts, the radial cutting constant \( k_r \) can be solved from the real part as follows:

\[
k_r = \frac{b P_{2r}(N) - c P_{2i}(N) - P_{1r}(N)}{b P_{1r}(N) - c P_{1i}(N) + P_{2r}(N)}
\]

(14)

On the other hand, solving for the radial cutting constant \( k_r \) from the imaginary part gives

\[
k_r = \frac{b P_{2r}(N) + c P_{2i}(N) - P_{1r}(N)}{b P_{1r}(N) + c P_{1i}(N) + P_{2r}(N)}
\]

(15)

Equating Eq. (14) and Eq. (15) yields the following equation

\[
\frac{b P_{2r}(N) - c P_{2i}(N) - P_{1r}(N)}{b P_{1r}(N) - c P_{1i}(N) + P_{2r}(N)} = \frac{b P_{2r}(N) + c P_{2i}(N) - P_{1r}(N)}{b P_{1r}(N) + c P_{1i}(N) + P_{2r}(N)}
\]

(16)

for extracting the radial width of cut. When the number of cutter flutes \( N \) is given, the two ratios in both sides of Eq. (16) are the function of cutting angles of entry and exit. For the case in up milling, the cutter entry angle \( \theta_1 \) is defined as zero, so the only unknown, cutter exit angle \( \theta_2 \), can be solved by Eq. (16). Referring to the cutting geometry as shown in Fig. 2, once the cutter exit angle is identified, the radial width of cut \( d_r \) can be easily determined by the following formula:

\[
d_r = \frac{D (1 - \cos \theta_2)}{2}
\]

(17)

For the case in down milling, the cutter exit angle \( \theta_2 \) is defined as \( \pi \), so to evaluate the radial width of cut is from the cutter entry angle \( \theta_1 \) which is solved by Eq. (16). This is done as follows:
Finally, substituting \( \theta_1 \) (or \( \theta_2 \)) which is solved by Eq. (16) into Eq. (14) (or Eq. (15)) gives the value of radial cutting constant \( k_r \).

3.2 Extracting axial depth of cut and cutting constant \( k_t \)

In this section, the first and second harmonic force components in X direction are used to identify the axial depth of cut and cutting constant \( k_t \). By setting \( k=1 \) and \( k=2 \) in Eq. (6), the magnitude ratio of \( A_1(N) \) to \( A_2(2N) \) may be expressed as

\[
\frac{|A_1(N)|}{|A_2(2N)|} = \frac{|A_1(N)|}{|A_2(2N)|} = |d + je| = r_a
\]

where \( A_1'(N) \) and \( A_2'(2N) \) are the first and second harmonic force components measured in X direction, respectively. Therefore, the values of \( d, e \) and \( r_a \) can be directly determined from the force measurement. In general milling conditions, \( r_a \) is a real number greater than 1. An analytical expression of magnitude ratio in left-hand side of Eq. (19) can be obtained by substituting Eqs. (6)-(8) in Eq. (19). After rearrangement, the expression of magnitude ratio can be derived as:

\[
\frac{A_1(N)}{A_2(2N)} = \frac{CWD(N)[P_1(N) + k_1P_2(N)]}{CWD(2N)[P_1(2N) + k_1P_2(2N)]} \Rightarrow \frac{|A_1(N)|}{|A_2(2N)|} = \frac{CWD(N)}{CWD(2N)} \cdot \frac{|P_1(N) + k_1P_2(N)|}{|P_1(2N) + k_1P_2(2N)|} = \frac{|P_1(N) + k_1P_2(N)|}{\cos \frac{N\beta_a}{2} |P_1(2N) + k_1P_2(2N)|}
\]

By equating the absolute value ratio \( r_a \) which is from physical milling space to the absolute value ratio of the virtual milling space in Eq. (20), the expression of digital twin can be given as:

\[
\frac{|P_1(N) + k_1P_2(N)|}{\cos \frac{N\beta_a}{2} |P_1(2N) + k_1P_2(2N)|} = r_a
\]

Since the radial width of cut \( d_r \) and cutting constant \( k_r \) are given in section 3.1, Eq. (21) gives the closed form expression for solving the angular parameter \( \beta_a \) which is related to axial depth of cut. From the Eq. (21), there exist two possible values of \( \beta_a \):

\[
\beta_{a1} = \frac{2}{N} \cos^{-1} \left( \frac{|P_1(N) + k_1P_2(N)|}{r_a |P_1(2N) + k_1P_2(2N)|} \right), \quad \beta_{a2} = \frac{2}{N} \cos^{-1} \left( -\frac{|P_1(N) + k_1P_2(N)|}{r_a |P_1(2N) + k_1P_2(2N)|} \right)
\]

The correct value of \( \beta_a \) in Eq. (22) is the one that results in a phase angle of \( A_1(N)/A_2(2N) \) closer to the phase angle \( \gamma_0 = \tan^{-1}(e/d) \). Once the angular parameter \( \beta_a \) is obtained, the axial depth of cut can be determined from Eq. (7):
Formula for the tangential cutting constant $k_t$ can be derived by equating the first harmonic force component of physical milling space $A_{nx}^\prime(N)$ to the first harmonic force component of virtual milling space $A_x(N)$. It is noted that starting angle position of force measurement and the coordinate origin of the force model may exist a phase shift $\Delta \phi$. Therefore, the relation between $A_{nx}^\prime(N)$ and $A_x(N)$ could be expressed as:

$$A_{nx}^\prime(N) = A_x(N)e^{jN\Delta \phi} = \frac{Nk_t}{2\pi} CWD(N)[P_1(N) + k_x P_2(N)]e^{jN\Delta \phi}$$

(24)

With the axial depth of cut $d_a$ known, the value of $CWD(N)$ in Eq. (24) can be given from Eq. (7). Equating the magnitude of $A_{nx}^\prime(N)$ to that of $A_x(N)e^{jN\Delta \phi}$, then the tangential cutting constant $k_t$ can be obtained with

$$k_t = \frac{2\pi |A_{nx}^\prime(N)|}{Nt_x |CWD(N)[P_1(N) + k_x P_2(N)]|}$$

(25)
Start

Read the cutter geometry and feed per tooth: \( N, D, \alpha, t \)

Read the measured force components: \( A_1(Nk), A_1(Nk) \)

Set digital twin expression:
\[
\frac{A_1(N)}{A_1(N)} = A_1(N) = b + je
\]

Find radial width of cut \( d_r \) from Eqs. (15)-(17)

Find radial cutting constant \( k_r \) from Eq. (13) or (14)

Set digital twin expression:
\[
\left| \frac{A_1(N)}{A_1(2N)} \right| = \left| d + je \right| = r_e
\]

Calculate \( \beta_{a1} \) and \( \beta_{a2} \) from Eq. (21)

Calculate \( \gamma_1 \) and \( \gamma_2 \): (two phase angles of \( A_1(N)/A_1(2N) \) corresponding to \( \beta_{a1} \) and \( \beta_{a2} \)) from Eq. (19)

Calculate \( \gamma_0 \) by \( \gamma_0 = \tan^{-1}(e/d) \)

Yes

\( \gamma_1 - \gamma_0 < |\gamma_2 - \gamma_0| \)

\( \beta_a = \beta_1 \)

\( d_a = \frac{D\beta_a}{2 \tan \alpha} \)

No

\( \beta_a = \beta_2 \)

Find tangential cutting constant \( k_t \) from Eq. (24)

End

**Fig. 3** Identification procedure for extracting milling parameters based on convolution force model

It should be pointed out that although the use of Eq. (25) to determine the specific cutting constant is independent of the phase angle \( \Delta \phi \), but the phase angle will play an important role in the identification of parameters using digital twin technology in the angular domain. Further application of the phase angle \( \Delta \phi \) will be described in section 3.3. A flowchart for extracting the milling parameters through the digital twin
technology proposed in section 3.1 and section 3.2 is provided in Fig. 3.

With the given cutter geometry (including helix angle, number of cutter flutes and cutter diameter) and feed per tooth, it is shown that the current information of milling parameters can be identified by equating the measured force data of physical milling space and convolution force model of virtual milling space. The procedure starts with extracting the radial width of cut \( d_r \) and radial cutting constant \( k_r \), then the axial depth of cut \( d_a \) and tangential cutting constant \( k_t \) can be identified in turn.

### 3.3 Calibrating the milling parameters

Although the online information of radial width of cut and axial depth of cut can be directly obtained through the identification procedure shown in the Fig. 3, in the convolution force model proposed in section 3.2, ignoring the effects of cutter runout and ploughing force may result in errors that can’t be ignored in estimating milling parameters under some cutting conditions. In this section, the numerical cutting force model presented in section 2.1 will be modified to consider the effects of cutter runout and ploughing mechanism, and then the milling parameters identified from the convolution force model will be more accurately updated by digital twin technology.

The effect of cutter runout in milling on cutting force is related to chip thickness. A chip thickness model considering with cutter runout was proposed in [25]:

\[
t_c(\theta, \beta) = \min_{m=1}^{N} \left\{ t_m(\theta, \beta) = [mt, \sin \theta - 2\rho \sin \frac{m\pi}{N} \sin(\beta - \frac{m\pi}{N})]w(\theta) \right\}
\]

\[
w(\theta) = \begin{cases} 1, & \theta \leq \theta_s \\ 0, & \text{otherwise} \end{cases}
\]

if \( t_m(\theta, \beta) < 0 \), then \( t_m(\theta, \beta) = 0 \)

In Eq. (26), the chip thickness affected by cutter runout is regarded as the minimum positive possible chip thickness that can be formed between the current cutting flute and all other previous \( m \) flutes with \( m = 1 \) to \( N \). The cutter runout related parameters in Eq. (26) are magnitude of cutter runout \( \rho \) and angular location of cutter runout \( \lambda \). These two parameters will be determined by an intelligent algorithm in this study. For a ideal milling case, \( \rho = 0 \), it can be shown that \( m = 1 \) and \( t_c(\theta, \beta) = t_s \sin \theta \), which is identical to Eq. (4). Therefore, Eq. (4) is the special case of Eq. (26), and the chip thickness expression in Eq. (4) will be replaced by Eq. (26) in this section. In addition, in order to incorporate the ploughing mechanism into the cutting force model, the Eq. (3) shown in section 3.1 will be modified to:

\[
\begin{align*}
f_r(i, j, k) &= k_{ts}dz c(\theta, \beta) + k_{tp}dz \\
f_r(i, j, k) &= k_{rs}k_{ts}dz c(\theta, \beta) + k_{rp}k_{tp}dz
\end{align*}
\]

where \( k_{ts} \) and \( k_{rs} \) are cutting constants related to shearing mechanism in milling. The \( k_{tp} \) and \( k_{rp} \) are defined as ploughing cutting constants. Setting ploughing cutting constants
in Eq. (27) as zeros reduces the Eq. (27) to Eq. (3). Through the milling tests shown in [26], it can be found that the local cutting force model in Eq. (27) has better predictability than the model in Eq. (3).

Based on the modified numerical cutting force model, PSO (particle swarm optimization) [27] can be applied to update the radial width of cut and axial depth of cut determined from Fig. 3. By equating the measure cutting force which is from physical milling space to the modified numerical cutting force model of the virtual milling space, the objective function can be expressed as

$$\min E(\phi_j) = \sum_{j=1}^{m_j} [(F'_x(\phi_j)-F_x(\phi_j))^2 + (F'_y(\phi_j)-F_y(\phi_j))^2$$

(28)

where $F'_x(\phi_j)$ and $F'_y(\phi_j)$ represent the measured cutting force components in X and Y directions, respectively. In practice, there may exist a phase shift angle $\Delta \phi$ between the starting angle position of force measurement and the coordinate origin of the force model. Therefore, the $F'_x(\phi_j)$ and $F'_y(\phi_j)$ in Eq. (28) should be calibrated by $F'_x(\phi_j-\Delta \phi)$ and $F'_y(\phi_j-\Delta \phi)$. In addition, the average force drift, $\Delta F_x$ and $\Delta F_y$, may also exist due to the problem of charge leakage in the dynamometer, which is a common problem of the dynamometer with piezoelectric sensor [19-20]. By taking into account the effects of the phase shift angle and average force drift, Eq. (28) can be modified as follows:

$$\min E(\phi_j) = \sum_{j=1}^{m_j} [(F'_x(\phi_j-\Delta \phi)-\Delta F_x-F_x(\phi_j))^2 + (F'_y(\phi_j-\Delta \phi)-\Delta F_y-F_y(\phi_j))^2$$

(29)

By using the PSO standard operating procedures reported in [27], Eq. (29) is able to serve as a generalized objective function for extracting the milling parameters. However, in this case, up to 11 undetermined parameters as listed in Table 1 need to be identified by the PSO standard operating procedures, which may consume a lot of calculation time and may not be able to identify the current milling parameters in time.

| Table 1 Undetermined milling parameters in the PSO |
|------------------------------------------------|
| axial depth of cut and radial width of cut | specific cutting constants | cutter runout parameters | phase shift angle and average force drifts |
| $d_a, d_r$ | $k_{rs}, k_{rp}, k_{sp}$ | $\rho, \lambda$ | $\Delta \phi, \Delta F_x, \Delta F_y$ |

Reducing the number of iterations in the PSO program may be a feasible method to
reduce the calculation time, but the effective use of this method is to obtain the approximate values of these 11 undetermined parameters before executing the PSO program. Otherwise, too few iterations may not guarantee that the milling parameters can be obtained correctly through the PSO program. Although the cutter runout effect is not considered in the identification process of Fig. 3, the axial depth of cut and radial width of cut identified from the Fig. 3 may be used as approximate values for executing the PSO program. Furthermore, through a few initial cutting tests, the approximate values of specific cutting constants and cutter runout parameters can be obtained from [28], which are calculated under the assumption that chip thickness is always formed by the two succeed flutes. In this case, the chip thickness model in Eq. (26) can be approximated as:

\[ t_s(\theta, \beta) = t_s \sin \theta - 2 \rho \sin \frac{\pi}{N} (\beta - \lambda - \frac{\pi}{N}) \]  

(30)

If the cutter is not worn too seriously, the experimental results show that the identified values of specific cutting constants and cutter runout parameters under other milling conditions are not significantly different from those determined by the initial cutting test [28]. However, different from specific cutting constants and cutter runout parameters, the phase shift angle and average force drifts in each force measurement are random variables, and can’t be obtained from the initial cutting tests. Ignoring the cutter runout effect, the phase shift angle \( \Delta \phi \) may be approximated from Eq. (24) by

\[ \Delta \phi \approx \frac{\phi_x' - \phi_x}{N} \]  

(31)

where \( \phi_x' \) and \( \phi_x \) are the phase angles of complex Fourier coefficients \( A_x'(N) \) and \( A_x(N) \), respectively. When the phase shift angle \( \Delta \phi \) is found in Eq. (31), \( \phi_x' \) can be obtained directly from the force measurement, and \( \phi_x \) can be calculated by using Eq. (6) with the milling parameters given in Section 3.2. In addition, the difference between the measured average force and the predicted average force from the cutting force model in Eq. (5) may provide an approximate value of the average force drift. Therefore, \( \Delta F_x \) and \( \Delta F_y \) may be approximated by

\[
\begin{bmatrix}
\Delta F_x \\
\Delta F_y
\end{bmatrix}
\approx
\begin{bmatrix}
A_x'(0) - A_x(0) \\
A_y'(0) - A_y(0)
\end{bmatrix}
\]  

(32)

where \( A_x'(0) \) and \( A_y'(0) \) are the measured average forces in X and Y directions, respectively. Similar to the phase angle \( \phi_x' \) in Eq. (30), the predicted average forces, \( A_x(0) \) and \( A_y(0) \), in Eq. (32) are also obtained with the assumption that there is no
cutter runout in milling, which can be found from Eq. (5) by using $k=0$.

4 Experimental verification and discussion

To verify the proposed digital twin driven intelligent algorithm, milling experiments were carried out on a three-axis CNC vertical milling machine. The work material Al7075-T6 was machined by using a three-fluted HSS, 10-mm diameter end mill with a 30° helix angle. Cutting forces in X and Y directions were measured with a Kistler-9255B dynamometer mounted on the machining table. In addition, Kistler-5007 charge amplifier and the acquisition system were used to perform signal amplification and data acquisition, respectively. In this way, the measured force signal can be analyzed on the PC. The force measurement system is given in Fig. 4.

Firstly, initial cutting tests were performed to estimate the specific cutting constants, $k_{ts}$, $k_{rs}$, $k_{tp}$, $k_{rp}$, and cutter runout parameters by using the identification method proposed in [28]. The identified values as shown in Table 2 were used as approximate values for executing the PSO program which was used to calibrate the milling parameters.

|                | specific cutting constants | cutter runout parameters |
|----------------|---------------------------|--------------------------|
| $k_{ts}$      | 890 MPa                   | 0.283                    |
| $k_{rs}$      | 10 $N/mm$                 | 10 $N/mm$                |
| $k_{tp}$      | 0.52                      | 14.9 $\mu m$            |
| $k_{rp}$      |                           |                          |
| $\rho$        |                           | 30°                      |
| $\lambda$     |                           |                          |

It is noted that the values determined by the initial cutting test are only used for the first calibration, and the specific cutting constants and cutter runout parameters
obtained from the first calibration will be used as the latest approximate values for the second calibration, and so on. Table 3 lists the cutting conditions for the experimental verification.

Table 3. The cutting conditions for the identification of axial depth of cut and radial depth of cut. $D = 10$ mm, $N=3$, $\alpha=30^\circ$, down milling.

| No. | Spindle speed (rpm) | Radial depth of cut (mm) | Axial depth of cut (mm) | Feed rate (mm/min) |
|-----|---------------------|--------------------------|------------------------|-------------------|
| 1   | 1000                | 1                        | 3                      | 150               |
| 2   | 1000                | 1                        | 3                      | 130               |
| 3   | 1000                | 2                        | 3                      | 110               |
| 4   | 1000                | 2                        | 3                      | 150               |
| 5   | 1000                | 2                        | 3                      | 130               |
| 6   | 1000                | 2                        | 3                      | 110               |
| 7   | 1000                | 2                        | 3.5                    | 150               |
| 8   | 1000                | 2                        | 3.5                    | 150               |

Table 4. The identified milling parameters for the cutting conditions in Table 3.

| No. | $d_r$ (mm) | $d_a$ (mm) | $k_r$ (MPa) | $k_t$ | $\Delta F_x$ (N) | $\Delta F_y$ (N) | $\Delta \phi$ (Deg) |
|-----|------------|------------|-------------|-------|-----------------|-----------------|---------------------|
| 1   | 1.02       | 3.14       | 0.35        | 1257  | 8.7             | 7.5             | 15.2                |
| 2   | 1.05       | 3.16       | 0.37        | 1266  | 6.8             | 6.2             | 26.3                |
| 3   | 1.13       | 3.17       | 0.41        | 1258  | 9.4             | 4.8             | 10.2                |
| 4   | 1.94       | 3.65       | 0.43        | 1141  | 5.3             | 9.1             | 36.1                |
| 5   | 1.93       | 3.72       | 0.5         | 1189  | 4.8             | 7.6             | 51.4                |
| 6   | 1.94       | 3.8        | 0.55        | 1269  | 8.4             | 8.1             | 8.7                 |
| 7   | 1.88       | 4.32       | 0.51        | 1145  | 6.2             | 5.1             | 29.4                |
| 8   | 1.87       | 4.35       | 0.56        | 1241  | 7.3             | 7.8             | 44.7                |

According to the flowchart in Fig. 3, $d_r$, $k_r$, $d_a$, and $k_t$ are estimated in turn. The identified results are shown in Table 4. Although the effect of cutter runout is not considered in the flowchart, the identified values of axial depth of cut and radial width of cut from the first three experiments are shown to be quite consistent with the specified values as shown in Table 4. However, the identified values of axial depth of cut for cases 4-8 show greater discrepancy from the specified values as in Table 3, which could be explained by the larger ploughing force in the five experiments. It should be noted that the flowchart in Fig. 3 is derived from a lumped shearing force model which do not explicitly consider the individual effects of ploughing mechanism and shearing mechanism on cutting force. With the increase of tool wear, the influence
of ploughing mechanism on cutting force will be more important, which result in
greater percentage error for the identification of axial depth of cut in cases 4-8. On the
other hand, the lumped shearing force model is shown to have accurate identification
for the radial width of cut, the increase of ploughing force does not seem to have a
significant impact on the identification of radial width of cut. It is also shown in Table
4 that the identified values of radial width of cut have better accuracy than those of
axial depth of cut. It can be attributed to the fact that the identification of radial width
of cut has nothing to do with axial depth of cut, but the identification of axial depth of
cut needs to rely on the identification result of radial width of cut. This means that the
identification error of radial width of cut will accumulate into the identification error of
axial depth of cut, resulting in the identification error of axial depth of cut being
greater than that of radial width of cut.

The execution time of the identification process in Figure 3 takes only 0.08
seconds by using a common notebook computer with processor speed of 2.3 GHz,
which is suitable for online monitoring of radial width of cut and axial depth of cut. However, the accuracy of the identification results may not be satisfactory, especially
when the tool is worn greatly after being used for a period of time. In order to reduce
the identification errors, the PSO program described in Section 3.3 was further
performed to improve the identification accuracy of the radial width of cut and axial
depth of cut. Since the identified values of the milling parameters as shown in Table 2
and Table 4 are close enough to the exact values, the search range of milling parameter
solutions in the PSO program is set to 0.8 to 1.1 times of each determined approximate
value. The number of samples $m_j$ which is used to calculate the PSO objective function
in Eq. (29) is given by 10. Other parameter values for executing the PSO program are
set as shown in Table 5.

| Table 5. Parameters for executing the PSO program |
|-----------------------------------------------|
| Iterations | Population size | Inertia weight | Learning factors |
|------------|-----------------|----------------|-----------------|
| 10         | 10              | 1              | 2.05, 2.05      |

The milling parameter values calibrated by the PSO program are listed in Table 6. Comparing the identification values of radial width of cut and axial depth of cut in
Table 4 and Table 6, the results show that the proposed PSO algorithm can find a more
accurate identification values of milling parameters with lower average percentage
error. Table 6 also indicates that the shear constants ($k_{ts}$ and $k_{rs}$) remain almost constant
during the cutting tests, but the ploughing constants ($k_{tp}$ and $k_{rp}$) increase significantly
with the increase of the number of cutting tests, which can be explained as the
ploughing effect increases with the increase of tool wear. In addition, using the
identified milling parameters in Table 6, the numerical cutting force model considering
the cutter runout effect is employed to predict the cutting forces. Figs. 5 and 6 show
that the force model with the identified milling parameters in Table 6 has fairly good
prediction. From the predicted results, the identification values of milling parameters in Table 6 can be further justified.

Table 6. Calibrated values of the milling parameters through the PSO program

| No. | $d$, $d_a$ (mm) | $k_{rs}$ | $k_{tp}$ (MPa) | $k_{tp}$ (N/mm) | $\lambda$ (Deg) | $\Delta F_x$ (N) | $\Delta F_y$ (N) | $\Delta \phi$ (Deg) |
|-----|----------------|----------|---------------|-----------------|-----------------|-----------------|-----------------|------------------|
| 1   | 1.03, 3.08     | 0.31     | 908           | 10.2            | 29.5            | 9.4             | 7.8             | 14.9             |
| 2   | 1.06, 3.04     | 0.29     | 920           | 10.5            | 28.7            | 7.5             | 6.5             | 25.8             |
| 3   | 1.07, 2.93     | 0.3      | 931           | 11.6            | 31.4            | 8.7             | 4.5             | 9.9              |
| 4   | 1.95, 3.19     | 0.32     | 915           | 12.3            | 29.1            | 4.9             | 8.7             | 35.8             |
| 5   | 2.06, 3.21     | 0.31     | 926           | 13.1            | 29.7            | 4.6             | 7.9             | 50.9             |
| 6   | 2.07, 3.25     | 0.33     | 922           | 13.8            | 30.3            | 7.8             | 7.6             | 8.3              |
| 7   | 1.95, 3.71     | 0.32     | 933           | 14.8            | 31.4            | 6.4             | 4.9             | 28.7             |
| 8   | 1.92, 3.79     | 0.34     | 935           | 16              | 28.5            | 7.1             | 7.4             | 43.6             |

It can be found that the additional execution time of the procedure for calibrating the milling parameters is 1.27 seconds. The reward of spending the 1.27 seconds is to lower the identification error of the milling parameters, and the maximum percentage error of identified axial depth of cut can be reduced from 24.3% to 8.3%. Therefore, the total execution time of the program to identify the accurate value of milling parameters by the proposed algorithm is about 1.35 seconds, which is still acceptable for the application of online monitoring of axial depth of cut and radial width of cut in milling. It should be noted that even without identifying the approximate values of the milling parameters from the flow chart in Figure 3 in advance, the PSO program proposed in Section 3.3 can still be used to identify the accurate values of milling parameters. However, this will greatly enlarge the search range of solutions, population size and iterations in the PSO program, and the calculation time will be more than 50 times of the original time, which is unacceptable for the online monitoring of milling parameters and adaptive control in milling.
5. Conclusions
A digital twin driven intelligent algorithm with two cutting force models has been proposed for monitoring in-process milling parameters by realizing cyber-physical fusion. It is shown that the accurate identification of milling parameters can be achieved in a short calculation time by using PSO program and numerical cutting force model with complexed cutter runout effect under the guidance of convolution cutting force model. Without considering the influence of cutter runout effect, the simplified
convolution cutting force model can provide the approximate value of the milling parameters in 0.08 seconds by virtue of the 4V (Volume, Velocity, Variety and Value) characteristics of big data, which greatly reduces the search range of solutions, population size and numbers of iteration in the PSO program. The other important conclusions of this paper are listed as follows:

1) In this paper, a fast identification method of milling parameters based on simplified convolution force model is proposed. Although the identification accuracy of the axial depth of cut is not accurate enough, it can quickly extract the axial depth of cut and radial width of cut through one-time force measurement in the milling process.

2) By using the proposed algorithm, radial width of cut, axial depth of cut, cutter runout parameters and specific cutting constants can be extracted simultaneously only by the dynamometer, so as to improve the intelligent manufacturing ability of milling system in a more effective and cheaper way.

3) In the proposed algorithm, the milling parameters can be identified without requiring average force signals. Therefore, even if the piezoelectric dynamometer has the problem of zero-drift, the identification results of milling parameters will not be affected.

4) It is shown that the phase angle between starting angular position of measured cutting forces and coordinate origin of the force model can be estimated by the simplified convolution cutting force model in frequency domain, which is helpful to use the numerical cutting force model for cyber-physical fusion in angular domain.

5) Through the complementary advantages of the two cutting force modes, the algorithm proposed in this paper can take into account the accuracy of identification results and computational efficiency. It is shown that the total execution time of the program is only about 1.35 seconds, and the maximum percentage error of milling parameters is less than 8.3%.
Declarations

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**Consent to Participate:** Not applicable

**Conflicts of interest/Competing interests:** Not applicable

**Consent to Publish:** Applicable

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**Code availability:** Software application

**Authors' contributions:**
The name and the contribution of each author are listed below.

Dr. Charles Ming Zheng: first author, interpreting data, completing the equation for the identification of milling parameters based on convolution force model.

Mr. Lu Zhang: completing the equation for the identification of milling parameters based on numerical force model.

Dr. Yaw-Hong Kang: corresponding author, checking the proposed method, editing the manuscript.

Dr. Youji Zhan: completing the PSO program for the identification of milling parameters.

Dr. Yongchao Xu: drafting, performing the milling experiments.
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