Feedback control–based active cooling with pre-estimated reliability for stabilizing the thermal error of a precision mechanical spindle

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Abstract
The thermal error stability (STE) of the spindle determines the machining accuracy of a precision machine tool. The “active cooling-spindle” system is regarded as a feedback control system, and the data-driven thermal error model is utilized to output feedback. In this way, the spindle thermal error can be stabilized by the homeostasis ability of the feedback control system under disturbance. Structural temperature measurements are considerably interfered by the active cooling, so the regression models trained with experimental data might output inaccurate feedback in unseen work conditions. Such inaccurate feedbacks are the primary cause for excessive fluctuations and failures of the thermal error control processes. The independence of the thermal data is analyzed, and a V-C (Vapnik–Chervonenkis) dimension–based approach is presented to estimate the generalization error bound of the regression models. Then, the model which is most likely to give acceptable performance can be selected, the reliability of the feedback can be pre-estimated, and the risk of unsatisfactory control effect will be largely reduced. Experiments under different work conditions are conducted to verify the proposed strategy. The thermal error is stabilized to be within a range smaller than 1.637 μm, and thermal equilibrium time is advanced by more than 78.3%.

Keywords Precision mechanical spindle · Active cooling · Thermal error feedback control · Thermal error stabilization · Thermal equilibrium

1 Introduction

Thermal error contributes to about 70% of the total machining error in precision machine tools [1–4]. The spindle is one core component of a precision machine tool; the thermal error of the spindle is the primary factor that leads to the deterioration of machining accuracy for machine tools [5–7]. Precision machine tools usually preheat for a long period to achieve thermal equilibrium before machining. Then, the relative position between the workpiece coordinate system and the machine coordinate system will be adjusted by tool setting, and only the thermal error variation will affect the machining accuracy. We can conclude that the variation range or stability of the spindle thermal error is the dominant factor that affects the machining accuracy.

Active cooling is an effective way to suppress the thermal deformation of a precision spindle. Liu et al. developed a closed-loop bath recirculation system for temperature control of a motorized spindle; the coolant supply power is adjusted to control the temperature rise between the outlet and inlet of the cooling channel, that is, the spindle heat dissipation from cooling [8]. A power matching–based heat dissipation strategy is then proposed; the coolant temperature varies with the estimated heat generation of the component under cooling [9]. Grama et al. proposed the Cooler Trigger Model strategy for temperature control of a spindle, which dynamically controls the switching frequency of the cooler compressor so that the heat extraction is in accordance with the estimated heat generation rate [10]. Ge and Deng presented external cooling equipment for a spindle, including a cooling unit, CFRP (carbon fiber reinforced polymer) bars,
and thermoelectric modules [11]. The external cooling essentially reduced the thermal deformation of the spindle. According to the heat generation estimation of the spindle components, a constant coolant temperature cooling experiment for a precision spindle is carried out by our research team [12]; the spindle thermal error is significantly reduced but still not stabilized.

The aforementioned active cooling strategies aim to minimize the spindle structure’s overall deformation to reduce the thermal error to the minimum. But such strategies are generally performed based on constant coolant temperature or offline empirical and theoretical heat generation models; hence the heat dissipation rate cannot accurately adapt to the rapidly changing dynamic thermal characteristics of the spindle, and the thermal error can hardly be kept stable in the long term. Moreover, strategies aimed at dissipating all generated heat and minimizing overall structural deformation usually consume large amounts of energy and bring extra costs such as complex cooling channels structures and a multiple loops cooling system.

The “active cooling-spindle” system is regarded as a feedback control (closed-loop) system, which takes the coolant temperature as the control variable, the spindle thermal error as the controlled variable, the cooler as the actuator, and the spindle as the controlled object; a thermal error regression model with temperature monitoring is employed to output the feedback real-timely. In this way, the spindle thermal error can be stabilized by the homeostasis ability of the feedback control system under disturbances of the heat generation of bearings. Then, most of the spindle thermal error can be eliminated by setting the tool. Compared to the aforementioned active cooling strategies, the proposed thermal error feedback control strategy can effectively stabilize the spindle thermal error in the long term without reducing much of the magnitude; thus, the thermal-induced error in machining can be essentially reduced by tool setting and the cooling energy consumption is low.

Much effort has been made to study high-quality data-driven regression modeling methods for the spindle thermal error, and the modeling methods are widely used in the thermal error compensation for various spindles [6, 13–15]. It seems such regression modeling methods can also be used for the thermal error feedback control, the online-temperature measurements can be used as the inputs for the spindle thermal error regression model, and the model can output feedback in real time for the thermal error feedback control. Then, the coolant temperature will be adjusted according to the difference between the estimated thermal error and the RITE (reference input of thermal error).

However, as studies have reported, the temperature measurements of the spindle can be interfered by the cooling, so the temperature curves of some measuring points are inconsistent with the curve of the thermal error [2, 14]. As for the spindle under cooling of varying coolant temperature, such inconsistency is more severe. Thermal data analysis (Sect. 3.1) also indicates that the correlation coefficients between the temperature variables and the thermal error variable are considerably low. As a result, the estimations of the regression model can be inaccurate, and the actual thermal error will deviate from the input reference. Usually, the effect of systematic error of the feedback (model outputted thermal error) is equivalent to that of a different RITE; it usually affects the thermal error magnitude but not the STE. Few inaccurate estimations with large residuals mainly cause significant coolant temperature deviation. Hence, the thermal error continuously deviates from the input reference and leads to the accumulated deviations of thermal error. This is the primary reason for occasional excessive thermal error fluctuations, even failure of the feedback control processes.

The long-term output accuracy of the thermal error model, namely the feedback accuracy, is vital for the thermal error control effect. As an experimental fact, the thermal error can be stabilized for any normal speed conditions if the real-time thermal error measurement is used as feedback. However, the performance of a thermal error model cannot be ensured or reasonably estimated when facing unseen data from various work conditions; hence, the thermal error stabilization level and its duration time in the feedback control processes are uncertain and unstable.

We seek to address the issue of pre-estimating the reliability of feedbacks for the not-yet-in-progress thermal error control process so that we can prejudge whether an acceptable model (regressor) can be trained with existing data and regression algorithms, or further data diversifying and cleansing, and algorithm modification is still needed. The independence of the spindle thermal data is verified, so that the independence hypothesis-based algorithms, such as the V-C (Vapnik–Chervonenkis) dimension analysis, can be effectively applied for the training data. Then, a V-C dimension–based [16] generalization error bound [17] for the regression model is presented, by performing which the generalization error bound of feedbacks can be estimated for the not-yet-in-progress thermal error control processes with high confidence.

The spindle thermal error feedback control method is first presented in this study. Then, the independence of the experimental thermal error training data is analyzed, the V-C dimension for several thermal error models is calculated, and the generalization error bounds are estimated for the model with the confidence of 0.95. In this way, the model that is most likely to give acceptable performance for unseen work conditions can be employed. The unqualified models that might output inaccurate feedback can be excluded. Thus the risks of excessive fluctuations and failures for the thermal error feedback control processes will be essentially reduced. Finally, experiments are conducted under different
work conditions to verify the thermal error feedback control strategy for the mechanical spindle.

2 Thermal error feedback control–based active cooling strategy for the precision spindle

2.1 The feedback control system for the mechanical spindle

The mechanical spindle of a precision boring machine is taken as the study object. The mechanical spindle possesses no internal cooling channels, so an extra cooler of the mechanical spindle is needed for the experimental investigation. The thermal error feedback control system is developed, including the computation module (the host computer) to run the algorithms, the real-time temperature monitoring module of the spindle structure and coolant, the cooling module, and the inter-communication of modules through the PLC and SQL server (runs on the host computer).

2.1.1 The schematic diagram of the spindle active cooling system

The schematic diagram of the spindle active cooling system is presented in this paper as shown in Fig. 1. A Siemens S7-200 PLC is used as the controller of the system. Configuration software (Force Control) is adopted to control and monitor the cooling devices run on a host computer. The coolant temperature is adjusted according to the control instruction; then, the coolant is pressurized with a recycle pump and stabilized with a turbine flowmeter and flow control valve during the circulation. The coolant flows in the helical cooler and finally circulates to the coolant temperature controller.

Temperature sensors and pressure sensors are installed in the coolant inlet and outlet of the helical cooler to monitor the coolant flow conditions and installed on the cooling area of the spindle to measure its temperature. Data acquisition module (co coolant flow velocity, pressure, temperature of the cooler inlet and outlet, structural temperature) is developed by the secondary development of Force Control configuration software. The data exchange between the active cooling system and the host computer is through the SQL Server database. This is the same for the data exchange between the acquisition software in a host computer and the synchronous measurement system. The control module uses SQL query operation to write the latest data in a database table and execute the feedback control algorithm. The coolant temperature \( u_k \) instructions are reported to another table in the database. Data in the table is then transferred to a real-time database using an ODBC router. Finally, the \( u_k \) is sent to the PLC of the coolant temperature controller by an I/O driver.

2.1.2 Cooler design for the mechanical spindle

Due to that the mechanical spindle of the precision boring machine possesses no inner cooling channel, an external cooling scheme is proposed. Thermal simulation for the spindle under cooling is conducted to pre-validate the external cooling scheme before developing it. The boundary condition calculation and thermal simulation method for the spindle is referring to [12, 18]. The calculated boundary conditions and the material properties of the thermal simulation are listed in Tables 1 and 2.

Thermal simulation results (Fig. 2) show that the external cooling largely reduces the thermal error; the thermal
error of the spindle with external cooling can quickly reach a stable state and has significantly superior stability than that without cooling, thus proving the effectiveness of the external cooling scheme. Then, the external helical cooler, which can take effect, is installed on the spindle to dissipate heat from the internal components and restrict the thermal housing deformation [11], thus suppressing the thermal error of the spindle without substantially changing the dynamic characteristics of the spindle. The helical cooler is installed and adjusted to fit the actual size and shape of the spindle, as shown in Fig. 3a, and an insulation cover can prevent unnecessary convection heat transfer between the helical cooler and the ambient air (Fig. 3b, c). Silicone-based thermal grease is fully applied on the joint surfaces to fill gaps and improve the cooling efficiency. Figure 3d shows the coolant temperature controller and the recirculation loop.

### 2.1.3 The online synchronous thermal data testing system

The thermal behavior of the mechanical spindle can be investigated experimentally based on our previous works [19]. Synchronous temperature and displacement data are gathered by online measurement with an acquisition card (NI SCXI-1600 series). The sampling interval in the measurement is 1 s, the precision of the temperature sensor is $\leq \pm 0.1 ^\circ C$, and the nonlinearity in the displacement sensor is $\leq \pm 1\%$.

Photographs of the sensors setup and synchronous acquisition system are shown in Fig. 4. The eddy current displacement sensor directly faced the end of the spindle detection rod, so the thermal error of the spindle can be measured. Regarding the PT100 temperature sensors, $T_{S1}$ is embedded beneath the housing and direct contact with the spindle sleeve, and $T_{S2}$ and $T_{S3}$ are installed on the rear of the housing. The temperature of the measuring locations distant from the cooler is considered potential input feature variables for modeling. The influences from the temperature-varying coolant can be alleviated.

The structural thermal sensitivity points for the spindle are analyzed using fuzzy clustering [19]. The selected positions for temperature measuring are relatively distant from the cooler, so they can be less affected. Finally, the temperature variables for thermal error modeling are optimized to be $\{T_{S1}, T_{S2}, T_{S3}, T_{C}\}$ (Fig. 4) plus coolant temperature $T_{C}$; they are considerably strongly associated with the thermal error and relatively independent from each other. The original sampling frequency is 1 Hz. In the thermal error feedback control processes, the feedback renews every 3 min, and the temperature signals for every 3 min are processed to mean moving average for model input.

The thermal equilibrium of the spindle refers to the stable state of its temperature field and thermal deformation field. The stability of the most concerned variable, thermal error $E$, can determine whether the spindle is in thermal equilibrium.

### 2.2 Thermal error feedback control strategy

The thermal error feedback control system adopts the proportional-integral-derivative (PID) algorithm. The

| Table 1 Boundary conditions | Components | Conditions | Value                  |
|-----------------------------|------------|------------|------------------------|
| Thermal contact resistance at junction | Bearing-shaft | | $1.603 \times 10^{-4}$ (W/m·°C) |
| | Bearing-sleeve | | $1.778 \times 10^{-4}$ (W/m·°C) |
| The thermal resistance of grease-coated joint | Cooler-housing | Chambered-flat contact | $3.424 \times 10^{-4}$ (m·°C/W) |
| Heating generation rate of bearings | Rear (7011C) | 3000 rpm, 500 N | 54.54 W |
| | Front (7013C) | 3000 rpm, 500 N | 86.58 W |
| Convective heat transfer coefficient hair | Housing-air | | $9.5$ (W/m·°C) |
| Ambient temperature | | | Constant | 16, 18 °C |

| Table 2 Material properties | Specific heat c (J/kg·°C) | Thermal conductivity k (W/m·°C) | Mass density $\rho$ (kg/m3) | Thermal expansion $\alpha$ (1×10⁶/°C) |
|-----------------------------|--------------------------|-------------------------------|----------------------------|-----------------------------------|
| Gr15 (bearing)             | 460                      | 44                            | 7830                       | 12                                |
| HT250 (housing)            | 510                      | 45                            | 7280                       | 8.2                               |
| CuZn (cooler)              | 381                      | 401                           | 8300                       | 18                                |
| 45Cr (shaft)               | 460                      | 44                            | 7870                       | 12                                |
| 45# (sleeve)               | 450                      | 48                            | 7280                       | 11.7                              |
PID-based thermal error feedback control scheme for the spindle thermal error is shown in Fig. 5. The PID controller is given as follows:

\[ u_k = u_{k0} + k_p \cdot e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt} \]  

Fig. 2 Thermal deformation simulation of the spindle with helical cooler: a thermal deformation field; b cooling effect on the spindle thermal error

Fig. 3 The precision mechanical spindle with helical cooler: a machine tool; b installation of the helical cooler; c cooler with insulation; d the coolant temperature controller and recirculation loop
where $u_k$ is the instruction for coolant temperature ($T_C$ is the actual value of the coolant temperature), $u_{k0}$ is the bias and also the initial $u_k$ value when the controlling starts, and $e_1(t)$ is the control error which equals that of RITE (reference input of thermal error) minus the estimated thermal error $E$. The controller parameters are the proportional gain $k_p$, integral gain $k_i$, and derivative gain $k_d$.

To tune the PID parameters, thermal error feedback control–based active cooling experiments are conducted with the real-time thermal error measurements as feedbacks. The PID parameters are tuned using the common trial and error method based on experimental observation: (i) adjust the response sensitivity by increasing or decreasing the gain parameter $k_p$. (ii) Derivative parameter $k_d$ is set to reduce...
overadjustment. The PID tuning experiments use real-time measurement as feedback so that the influences from the inaccurate thermal error model can be avoided.

Finally, the PID parameters are empirically and experimentally set to: \( k_p = 1.5 \), \( k_i = 0 \) and \( k_d = 50 \).

3 Thermal data preprocessing and independence testing

Based on thermal testing experiments for the mechanical spindle, experimental thermal data, including structural and coolant temperature, are presented in this section. The thermal data are cleansed by detecting and removing the outliers, and the independence of the observations (data points) is tested.

3.1 Descriptive analysis of the experimental thermal data

The experimental training data (Fig. 6) for thermal error modeling includes five sub-datasets obtained from early tryouts of the thermal error feedback control. The thermal behavior of the spindle under varying coolant temperature cooling can be well reflected. Work conditions are similar (constant 3000 rpm). The training sub-datasets are adopted because they are different and typical among the many tryout datasets in the datasets, and the variety of the training data is fairly good. Sub-datasets 1 and 2 are obtained from the PID tuning experiments with real-time thermal error measuring as feedback. Sub-datasets 3, 4, and 5 are measured in early experimental attempts of thermal error feedback controlling, which adopt the unsuccessful thermal error models trained with insufficient and limited data. The experimental data of sub-datasets 1–5 can well reflect the effect of rapidly varying temperature’s coolant acts on the spindle’s structural temperature and thermal error. The above experiments are all conducted in a constant temperature environment. With a similar cool down time, so the influences from the ambient temperature and initial structural temperature/deformation field are trivial.

The structural temperature \( T_S \) and coolant temperature \( T_C \) are denoted as \( T_M = [T_{S1}, T_{S2}, T_{S3}, T_{S4}, T_C] \), which are the explanatory variables. Thermal error \( E \) is the response variable. The structural temperature measuring points are distant from the helical cooler, so the variables \( T_{S1}, T_{S2}, T_{S3}, T_{S4} \) and \( T_C \) are less affected by the coolant temperature variation and are relatively independent of the coolant temperature variable \( T_C \). The variation trends of \( T_{S1}, T_{S2}, T_{S3}, \) and \( T_{S4} \) agree with the variation trend of \( E; T_C \) affects the drastic change of \( E \) in Fig. 6b and relates to the \( E \) fluctuations in Fig. 6d, e.

Correlation analysis is conducted on the 5 sub-datasets of thermal data and the correlation coefficients are shown in Table 3. The correlation coefficients between the structural temperature variable \( (T_{S1}, T_{S2}, T_{S3}, T_{S4}) \) and the thermal error \( E \) are within [0.7481, 0.9208], and the correlation coefficients between the coolant temperature \( T_C \) and \( E \) are within [0.6051, 0.7181]. The correlation coefficients are considerably low for the thermal error regression modeling, compared to that of the spindle thermal data without cooling [19] which are usually larger than 0.96.

The spindle structural temperature and deformation field (thermal state) relate strongly to the previous thermal state; the observations in Fig. 5 are dependent and associated. But it is unclear whether the data of the thermal state variations are independent. For all the above thermal data, we denote each of their variation data points (observations) at time \( t \) as \( D_t = [T_M, E] \), \( t = 2, 3, \ldots, T \). The thermal variation data are denoted as \( D \). The mean and variance of each variable vector \( (\Delta T_{S1}, \Delta T_{S2}, \Delta T_{S3}, \Delta T_{S4}, \Delta T_C, \Delta E) \) are calculated and compared among time periods and sub-datasets. They are close in value, so each column of \( D \) can be regarded as stationary data. Finally, the thermal error \( E_t \) at any time point \( t \) can be estimated by summing \( \Delta E \):

\[
E_t = \sum_{i=1}^{t} \Delta E_i
\]  

3.2 Boosting-based outliers detection for the thermal data

The boosting-based outliers detection approach is employed for cleansing the outliers for dataset \( D \) [19]. The training dataset \( D \) and a robust SVR weak learner are given first, and the algorithm requires a maximum number of iterations \( M \) to be chosen a prior. In each iteration step \( I \in \{1, \ldots, M\} \), training sample at this iteration is randomly drawn from \( p^I = [p^I_1, p^I_2, \ldots, p^I_n] \), with replacement according to the normalized probability weights distribution \( D \), which is a sample \( D^I \) of size \( n \). And the SVR regression function \( f^I \) is established with the sample \( D^I \). The initial iteration step \( I \) is set to one and the initial probability weights. And the SVR regression function \( f^I \) is established with the sample \( D^I \). The initial iteration step \( I \) is set to one and the initial probability weights \( p^I \) are \( \left\{ \frac{1}{n}, \ldots, \frac{1}{n} \right\} \). Initial \( f^I \) is estimated using the initial probability weights \( p^I \). Then, at every iteration \( I, I > 1 \), the updated \( p^I \) would be normalized, and the regression model \( f^I \) is established according to the normalized \( p^I \). The model \( f^I \) would be evaluated by using an overall error measure, which is calculated on the basis of regression error rates \( e^I_1, e^I_2, \ldots, e^I_n \) at each iteration \( I \). Error rate \( e^I_1, [1, 2, \ldots, n], I \in \{1, 2, \ldots, M\} \) is defined as \( \frac{\sum_{i=1}^{n} e^I_i}{n} \).

The regression error rates are given by dividing the absolute residuals by the data point value, such that the error
The error rate of every data point is obtained and lies in the interval $[0, 1]$. Then the error rate of every data point is compared with the preset error rate threshold $\tau$, and the data points whose error rates exceed the threshold $\tau$ are considered as the poorly estimated. Finally, the overall error measure $\varepsilon^I$ at iteration $I$ is calculated as:

$$\varepsilon^I = \sum_{i: e_i^I > \tau} p_i^I$$

where $i \in \{1, 2, \ldots, n\}$ and $e_i^I$ is the absolute residual of the $i$th data point. The calculated overall error $\varepsilon^I$ is the weighted sum of the normalized error rate which also lies in the interval $[0, 1]$. The probability weights are updated according to a rule depending on threshold $\tau$. The weights associated with data points with residuals smaller than $\tau$ are decreased, while the weights of poorly estimated data points with residuals larger than $\tau$ would remain constant. The weights of

*Fig. 6* Experimental data (3000 rpm, initial $T_c = 16$ °C) for training the thermal error model: a sub-dataset 1; b sub-dataset 2; c sub-dataset 3; d sub-dataset 4; e sub-dataset 5
well-estimated data points are decreased as \( \beta \leftarrow (\epsilon^I)^r \), and \( p'_t \leftarrow p'_t \cdot \beta^I \), where \( r \) is the power coefficient which controls the decreasing rate of the weights for well-estimated data points.

After the probability weights of data points for iteration \( I + 1 \) are updated, the updated probability weight distribution of iteration \( I + 1 \) would be normalized so the sum is 1, and the probability weights of the poorly learned data points be in fact increased. Iterations would stop when overall error rate \( \epsilon^I \) is low enough or after the algorithm has iterated \( M \) times.

Finally, three data points are suspected as outliers because they are associated with prominent large normalized probability weights, 0.1722, 0.1638, and 0.2414, while the most significant normalized weight for the other training data points is 0.0511. The dataset \( D \) is cleansed by removing these data points of suspected outliers.

### 3.3 Independence testing for the spindle thermal data

The lagged autocorrelation and portmanteau statistic are adopted to characterize the independence of the multidimensional spindle thermal dataset to determine whether the independence hypothesis-based algorithms are appropriate to be applied.

#### 3.3.1 The multivariate portmanteau statistic

The measured and preprocessed spindle thermal data matrix \( D \) can be denoted as a multivariate vector \((T \times S)\):

\[
D = \begin{bmatrix}
  x_{11} & x_{12} & \cdots & x_{1S} \\
  x_{21} & x_{22} & \cdots & x_{2S} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{T1} & x_{T2} & \cdots & x_{TS}
\end{bmatrix}
\]  

(4)

where \( S \) is the total number of variates, \( T \) is the total number of total observations, and \( s \) and \( t \) are respectively the coordination for variate and observation \( (x_{ts}) \). \( D_{t:T-l} \) denotes the \( t \)th to \((T-l)\)th observations, and \( D_{t+l:T} \) contains the \((t+l)\)th to \(T\)th observations.

Denoting the diagonal matrix of standard deviations by \( S_a = \text{Diag}(\sigma_1, \sigma_2, \sigma_3) \), where \( \sigma_s \) is the standard deviation of the \( s \)th variate \( x_s \), \( \sigma_s = \sqrt{T^-1 \sum_{l=1}^{T} x_{ts}} \). \( R_l \) is the cross-correlation for \( D_{t:T-l} \) and \( D_{t+l:T} \), and \( C_l \) is the auto-covariance matrix at lag \( l \):

\[
C_l = S_a R_l S_a = \begin{bmatrix}
  \sigma_1 & 0 & \cdots & 0 \\
  0 & \sigma_2 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & \sigma_S
\end{bmatrix}
\begin{bmatrix}
  E_{x_1x_1} & E_{x_1x_2} & \cdots & E_{x_1x_S} \\
  E_{x_2x_1} & E_{x_2x_2} & \cdots & E_{x_2x_S} \\
  \vdots & \vdots & \ddots & \vdots \\
  E_{x_Sx_1} & E_{x_Sx_2} & \cdots & E_{x_Sx_S}
\end{bmatrix}
\begin{bmatrix}
  \sigma_1 & 0 & \cdots & 0 \\
  0 & \sigma_2 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & \sigma_S
\end{bmatrix}
\]  

(5)

Then, the portmanteau statistic \( Q \) is defined [20, 21]:

\[
Q = T^2 \sum_{l=1}^{L} (T - l)^{-1} \left( C_l^T C_l^{-1} C_l C_l^{-1} \right)^T
\]  

(6)

#### 3.3.2 Independence testing for the spindle thermal data

The \( S \) variates of \( D \) are the variations of structural temperature \( \Delta T_s \) and coolant temperature \( \Delta T_c \) and the thermal error \( \Delta E \). The independence testing is repeated with \( l = 1:150 \). In each test, \( D_{t:T-l} \) and \( D_{t+l:T} \) are normalized to a \((0, \sigma^2)\) distribution.

The null distribution (independent) is approximated using the inverse cumulative distribution function of standard \( \gamma \) distribution:

\[
g(l) = \frac{1}{\gamma} \int_{0}^{l} \frac{1}{\Gamma(\gamma)} e^{-t} dt, \quad l \geq 0, \quad \gamma > 0
\]  

(7)

For each lag \( l \), the null hypothesis of independence is supported if \( Q < g(l) \). \( Q > g(l) \) means that the null hypothesis is rejected; the larger \( Q \) means the stronger violation for the independence hypothesis of the tested data. \( Q \) against different lag \( l (= 1:150) \) is calculated for the thermal data of the spindle under active cooling (Dataset I). Thermal data of the precision spindle without cooling (Dataset II), which is repeatedly adopted in thermal error modeling and proved high-quality [13, 22, 23], their \( Q \) against same lags (\( l = 1:150 \)) are also calculated for comparison. Finally, in order to test the data independence, \( Q \) of the above datasets are compared with that of the inverse cumulative distribution function which is standard \( \gamma \) distribution (Fig. 7).

For the thermal Dataset I (spindle under active cooling), the IID hypothesis is supported in 18 of 150 values, and the \( Q \) curve is close to that of the inverse cumulative distribution function (standard \( \gamma \) distribution), so there is only slight violation to the null hypothesis (Fig. 7). The violation for
independence assumption of the Dataset I is much weaker than that of the Dataset II (spindle without cooling). In view of the fine modeling effect with Dataset II using independence-based algorithms, we conclude that the independent violation is weak enough to be ignored for Dataset I.

4 Thermal error modeling and reliability pre-estimation of the feedback

As is analyzed in Sect. 3.3, the independence assumption–based algorithms are appropriate to be applied on the dataset $D$ (Dataset I). The regression modeling algorithms are performed to establish thermal error models, and then the generalization bounds of the models are estimated. Finally, the model which is most likely to give acceptable performance for unseen work conditions is employed, and the maximum $T_C$ deviation (from the ideal $T_C$) for the upcoming thermal error feedback control processes is pre-estimated.

4.1 Regression modeling algorithms for the thermal error modeling

4.1.1 Support vector machine for regression theorem

In the regression context of SVM, the training dataset is $(X_i, Y_i), i = 1, 2, ..., n$, where $X$ is a $S$-dimensional input and $X \in R^S$. The $Y$ is a one-dimensional output and $Y \in R$. In SVR, the input $X$ is mapped onto a high dimension feature space using a kernel function at first, and then a model is established in this high dimension feature space [24–26]. The multivariates linear model $f(X_i, \Omega)$ is given by:

$$f(X_i, \Omega) = \sum_{k=1}^{m} \omega_k g_k(X)$$  \hfill (8)

where $g_k(X)$ denotes a set of nonlinear transformations, $\omega$ is the set of variable coefficients in the multi-linear model, and $\Omega = \{\omega_1, \omega_2, ..., \omega_n\}$. The quality of estimation of the established model is measured by the loss function $L(Y, f(X_i, \Omega))$. SVR uses the $\varepsilon$-insensitive loss function:

$$L(Y, f(X_i, \Omega)) = \begin{cases} 0 & \text{if } |Y - f(X, \Omega)| \leq \varepsilon \\ |Y - f(X, \Omega)| - \varepsilon & \text{otherwise} \end{cases}$$ \hfill (9)

where the parameter $\varepsilon$ defines the width of insensitive margin. The empirical risk is:

$$R_{emp} = \frac{1}{n} \sum_{i=1}^{n} L(Y_i, f(X_i, \Omega))$$ \hfill (10)

SVR model is established by minimizing:

$$\min \frac{1}{2} ||\Omega||^2 + C \sum_{i=1}^{n} (\xi_i + \xi_i^*)$$ \hfill (11)

s.t.:

$$y_i - f(X_i, \Omega) \leq \varepsilon + \xi_i$$

$$f(X_i, \Omega) - y_i \leq \varepsilon + \xi_i^*$$

$$\xi_i, \xi_i^* \geq 0, i = 1, ..., n$$

In the minimization problem (1), $\varepsilon$ defines the width of the insensitive margin (tolerance margin), and $\xi$ is called “slack variable” which determines the deviation distances from the insensitive margin. $C$ is the penalty that characterizes the amount of tolerance for the data points lying outside the insensitive margin [27]. The function $f$ is as follows:
where $K(X_i, X_j)$ is the kernel function which is the inner product of the high-dimensionally mapped input feature vectors, $K(X_i, X_j) = \Phi(X_i)^T \Phi(X_j)$. Kernel function makes the dot product calculation practical, and it can be considered as a measure of similarity among the training data. Many kernel forms are commonly used in SVR, and the radial basis function (RBF) kernel function is adopted for SVR:

$$K(X_i, X_j) = \exp(-\frac{||X_i - X_j||^2}{2\gamma^2}), \text{ if } i \neq j$$

where $\gamma$ is the (Gaussian) kernel width parameter.

Moreover, the $\varepsilon$-insensitive zone width parameter $\varepsilon$ is already empirically set to 0.01, and the SVR model is established with two parameters, the penalty parameter $C$ and the RBF kernel width parameter $\gamma$. The GA method is adopted to optimize the parameters of penalty parameter $C$ and RBF kernel width parameter $\gamma$ [19]. The GA is based on the mechanics of natural selection and genetics. Procedures of GA in this paper are carried out using references [28], which involves three stages, (1) population initialization, (2) operators, and (3) chromosome evaluation.

### 4.1.2 Random forest regression theorem

A random forest regression (RFR) model [29, 30] consists of a collection of classification and regression trees (CARTs) [31]. Two random sampling procedures are adopted in RFR to alleviate over-fitting:

1. Bootstrap aggregating. For each CART, a different sub-dataset is used for training, and one-third of the training data (out-of-bag data, OOB) estimates the general error. By sampling with replacement, some observations may be repeated in each subset. Finally, the CARTs are assembled by averaging the output.

2. Feature projection. At first, a certain number of input variables are randomly selected for the tree training process. The data subset of the randomly selected input variables is then used to split each node, resulting in less correlation among trees and a lower error rate.

The above procedures can vastly reduce the variance error of RFR, but the bias can still be significant. Application of the bias-correction approach can alleviate this problem [30, 32] by estimating the systematic residuals and subtracting them from the estimation:

1. Fit the training dataset using RFR. Compute the estimated values and residuals $r = y - y^*$, where $r$ is the residual, $y$ is the test value, and $y^*$ is the RFR models’ estimation with input $X$.

2. Fit an RFR model regarding $r$ as the response variable (explanatory variables are also $X$), the output of the model is $r^*$, and the final estimation output $y_o$ is:

$$y_o = y^* + r^*$$

### 4.2 Generalization bounds estimation via V-C dimension

#### 4.2.1 Estimating the V-C dimension

Let $X$ be the explanatory variable leading to the response variable $Y$, and the assuming $(X, Y)$ values are on the $x \times y$. $\alpha_k \in \Lambda, k = 1, \ldots, K, \Lambda$ is an index set for the regression functions, and the hypothesis space is finite, so the regression function is denoted as $Y = f(X, \alpha_k)$. Let $D$ be the randomly sampled dataset with the length of $2nl$, $(Z_1 = (X_1, Y_1), \ldots, Z_{2nl} = (X_{2nl}, Y_{2nl}))$ be a data set of size $2nl$ of independent and identically distributed copies of $(X, Y)$. Write $D_1=((X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n))$ for the first half and $D_2=(Z_{nl+1}, \ldots, Z_{2nl}, (X_{n+1}, Y_{n+1}), \ldots, Z_{2nl+1}, (X_{2n+1}, Y_{2n+1}), \ldots, Z_{2nl+1}, (X_{2n+1}, Y_{2n+1})))$ for the second half of $D$. Data length of $D_1$ and $D_2$ are all $n_l$.

Let $L(\alpha_k)$ be the unknown actual loss of the function $f$ at $\alpha_k$, $Q((X, Y), \alpha) = (Y - f(X, \alpha))^2$ be the bounded real-value loss function. For a fixed $\alpha$, the value of $Q((X, Y), \alpha)$ is within $[0, B]$, where $B$ is the upper bound of the losses. Discretize $Q((X, Y), \alpha)$ using $m$ disjoint intervals:

$$Q^m((X, Y), \alpha) = \sum_{j=0}^{m-1} \frac{(2j+1)B}{2m} I(Q((X, Y), \alpha)$$

$$I(Q((X, Y), \alpha)) = \begin{cases} 1, & \text{if } Q((X, Y), \alpha) \in [\frac{jB}{m}, \frac{(j+1)B}{m}], \\ 0, & \text{otherwise} \end{cases}$$

where $I$ is an indicator function taking value 1 when its argument is true and value 0 when it is false; the discretization is based on the uniform left-closed, right-open partition of $[0, B]$ into $m$ subintervals, here denoted $[\frac{jB}{m}, \frac{(j+1)B}{m})$, which are the midpoints.

Let $\alpha_1, \alpha_2 \in \Lambda$, and $v$ be the empirical loss function, so the empirical functions on the first and the second half of the training data $(D_1$ and $D_2$) are:

$$v(D_1, \alpha_2) = \frac{1}{n} \sum_{i=1}^{n} Q((X_{n+1}, Y_{n+1}), \alpha_2)$$

$$v(D_2, \alpha_2) = \frac{1}{n} \sum_{i=1}^{n} Q((X_{n+1}, Y_{n+1}), \alpha_2)$$
Based on formula (15), we introduce the empirical counts of the data points whose losses belong to \( \left\{ \frac{B}{m}, \frac{(j+1)B}{m} \right\} \), which can be written as:

\[
C_j^m(D_1, \alpha_1) = \frac{1}{n} \sum_{i=1}^{n} I(Q((X_{n+1}, Y_{n+1}), \alpha_1))
\]

(19)

\[
C_j^m(D_2, \alpha_1) = \frac{1}{n} \sum_{i=1}^{n} I(Q((X_{n+1}, Y_{n+1}), \alpha_1))
\]

(20)

This means we are counting the errors of the \( \alpha_1 \) model on the second half of the data \( D_2 \) and the errors of the \( \alpha_2 \) model on the first half of the data \( D_1 \). This begins the setup of the cross-validation form of the error that we use and leads to the following expressions for the empirical losses of the discretized loss functions:

\[
v_j^m(D_2, \alpha_1) = \frac{1}{n} \sum_{i=1}^{n} C_j^m(D_2, \alpha_1)(2j+1)B - \frac{2m}{2m}
\]

(21)

\[
v_j^m(D_1, \alpha_1) = \frac{1}{n} \sum_{i=1}^{n} C_j^m(D_1, \alpha_1)(2j+1)B - \frac{2m}{2m}
\]

and the empirical loss in the \( j \)th discretized subinterval is:

\[
v_j^m(D_2, \alpha_1) = \frac{1}{n} \sum_{i=1}^{n} C_j^m(D_2, \alpha_1)(2j+1)B - \frac{2m}{2m}
\]

(23)

\[
v_j^m(D_1, \alpha_1) = \frac{1}{n} \sum_{i=1}^{n} C_j^m(D_1, \alpha_1)(2j+1)B - \frac{2m}{2m}
\]

(24)

Referring to, with the \( \varepsilon \geq 0, m \in \mathbb{N} \) and \( j = 0, 1, \ldots, m-1 \) and the V-C dimension \( d_{vc}(L; \alpha; \alpha \in \Lambda) \) is finite, the expected maximum difference between the empirical losses \( \Delta \) is:

\[
\Delta = E(\sup_{\alpha_1, \alpha_2 \in \Lambda} [v_j^m((X_{n+1}; Y_{n+1}; 2n), \alpha_1) - v_j^m((X_{1:n}; Y_{1:n}); \alpha_2)])
\]

(25)

where \( \sup() \) means the support functions of \( v_j^m \).

\( \delta_{d_{vc}}(n_1) \) is the upper bound for \( \Delta \) of each sub-dataset \( D_1 \) and \( D_2 \) for each \( n_1 \):

\[
\delta_{d_{vc}}(n_1) = \sqrt{\frac{d_{vc}}{n_1} \log\left(\frac{2n_1 e}{d_{vc}}\right)}
\]

(26)

where \( c \) is a constant empirically chosen to 8.

We design a series of data length \( n_t \) \((t = 1, 2, \ldots, T)\) for randomly sampling sub-datasets. For each \( t = 1, 2, \ldots, T \), a bootstrap sample \( D \) of size \( 2n_t \) is picked and is randomly divided into 2 sub-datasets \( D_1 \) and \( D_2 \). Two regression functions are respectively trained on \( D_1 \) and \( D_2 \), which are denoted as \( f(\alpha_1) \) and \( f(\alpha_2) \). The loss function is estimated by computing the squared error \( SE_1 = (f(x_1, \alpha_2) - y_1)^2 \), \( SE_2 = (f(x_2, \alpha_2) - y_2)^2 \).

For the \( m \) discretized subintervals, empirical counting \( C_j^m(D_1, \alpha_1) \) and \( C_j^m(D_1, \alpha_2) \) for each subinterval are obtained using the squared error \( SE_1 \) and \( SE_2 \) (Eqs. (19) and (20)), and \( v_j^m(D_1, \alpha_1) \) and \( v_j^m(D_1, \alpha_2) \) can be estimated.

The bootstrapping method is used to iteratively resample a dataset with replacement and estimate the statistics. In the bootstrapping, an observation for \( \delta_{d_{vc}} \) is introduced, which is denoted as \( \delta_{d_{vc}}^* \). The \( \delta_{d_{vc}}^* \) for each bootstrap sample \( D \) can be obtained by taking the mean and sum across all subintervals:

\[
\delta_{d_{vc}}^*(n_t) = \frac{1}{m} \sum_{j=1}^{m-1} \text{mean}[v_j^m(D_1, \alpha_2) - v_j^m(D_2, \alpha_1)]
\]

(27)

Finally, the V-C dimension \( d_{vc} \) can be obtained by minimizing the squared distance \( (f_\alpha^* \) between the observation \( \delta_{d_{vc}}^*(n_t) \) and the true upper bound \( \delta_{d_{vc}}(n_t) \):

\[
f_\alpha(d_{vc}) = \frac{1}{m} \left( \delta_{d_{vc}}^*(n_t) - \delta_{d_{vc}}(n_t) \right)^2 = \sum_{j=1}^{m} \left( \delta_{d_{vc}}^*(n_t) - \delta_{d_{vc}}(n_t) \right)^2
\]

(28)

4.2.2 Loss estimation for the regression function

Let \( Q_{emp}(\alpha_k) \) be the empirical risk at \( \alpha_k \). For any \( p \in (0, 1) \), with probability at least \( 1-p \), the inequality [17, 33] is:

\[
Q(\alpha_k) \leq Q_{emp}(\alpha_k) + m \sqrt{\frac{1}{n} \log\left(\frac{2m}{p} \left(\frac{2n_1 e}{d_{vc}}\right)^{d_{vc}}\right)}
\]

(29)

where the empirical risk \( Q_{emp}(\alpha_k) \) is defined as the sum squared error of the fitting \( MSE \) of the function \( f(\alpha_k) \) on the training dataset:

\[
Q_{emp}(\alpha_k) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i^* - y_i)^2}
\]

(30)

where \( y^* \) is the prediction and \( y \) is the measured data for the response variable \( Y \). Besides, the Vapnik–Chervonenkis (V-C) dimension \( d_{vc} \) needs to be calculated for estimating the upper bound of \( Q(\alpha_k) \). The best model is the one which minimizes \( Q(\alpha_k) \).

The GA-SVR and RFR regression algorithms are employed to establish the thermal error models with the experimental data presented in Sect. 3.1. Then the generalization error bound estimating approach is applied to the established models. With \( p = 0.05 \) (probability \( 1-p \) equals 0.95), the bounded loss function is discretized into ten
disjoint intervals, and the $d_{VC}$, $Q_{emp}$, and $Q$ are calculated and listed in Table 4.

With the probability of 0.95, the upper bounds (absolute value) of generalization error for the MLR, GA-SVR, and RFR models are, respectively, 0.7342 μm, 0.6882 μm, and 0.6211 μm (Table 4).

4.2.3 Reliability pre-estimation for the thermal error feedback control

The generalization error bound of the RFR model is the smallest among the three models. Thus, the feedback accuracy with the RFR model is expected to be the highest for unseen conditions.

Normal systematic residual of the feedback ($E$) is equivalent to a different RITE, so it usually affects the thermal error magnitude but not the STE. Few inaccurate variation (Δ$E$) estimations with large residuals at specific time points are the primary cause for the $T_C$ deviations (from the ideal $T_C$) and excessive thermal error fluctuations. But, if the estimation residual of Δ$E$ is bounded to a small range, only small fluctuations and slight sustained increment/decrement tendencies will occur, even if the estimation residual accumulates for a period. The experimentally tuned proportional gain $k_p$ is 1.5, so the maximum $T_C$-deviation is expected to be 0.93 °C with the RFR model. We can assume that the thermal error will be stabilized to within an acceptable range in the thermal error feedback control processes.

5 Experiments of the thermal error feedback control–based spindle active cooling

5.1 Pre-determination of RITE

Adjusting and maintaining coolant temperature consumes major energy in the active cooling for the spindle. Since the “spindle-active cooling” system is regarded as a feedback control system and the constant input of the system is not one of the main factors that affect the response stability of the system, the magnitude of RITE mainly affects the cooling energy consumption of the thermal error controlling process and can hardly affect STE. Maintaining STE at a relatively high thermal error value can reduce the amount of heat that needs to be dissipated by coolant circulations, thus decreasing the energy consumption of adjusting $T_C$ in the coolant temperature controller.

In the constant temperature cooling, the ambient temperature and the initial temperature of the spindle structure are 18 °C. Minimum energy would be consumed in heat convection with ambient air if $T_C$ keeps close to 18 °C, and an instruction($u_k$) for coolant temperature ($T_C$) that is distant from 18 °C leads to energy consumption in the coolant temperature controller. Thus, the coolant temperature should be close to 18 °C in the control process, so the energy consumption is low. Regarding the cooling with a constant 16 °C, the preheating time is about 120 min until thermal error stabilizes. Then, for the thermal error feedback controlling with the rotation speed of 3000 rpm, RITE is empirically determined to be 17 μm where the coolant temperature changes around 16 °C. As for the thermal error feedback controlling with the rotation speed of 2000 rpm, RITE is set to 12 μm where the coolant temperature also varies around 16 °C.

5.2 Evaluation method of STE

To quantitatively evaluate the STE using the discrete derivative, the fluctuation in the measured thermal error curves should be avoided, and only the major variation trend of the thermal error should be reflected [12]. The spindle thermal error, which varies nonlinearly with time, is fitted to a smooth curve:

$$f(t) = p_5 \cdot t^5 + p_4 \cdot t^4 + p_3 \cdot t^3 + p_2 \cdot t + p_1$$  \hspace{1cm} (31)

The least-square method is used to fit the 4-order polynomial regression model. STE is the discrete derivative of fitted thermal error $f(t)$ to time $t$:

$$\text{STE} = df(t)/dt$$  \hspace{1cm} (32)

In practical applications, a precision machine tool is usually preheated before machining. The spindle thermal error can hardly affect the machining accuracy when it is stable because most of the spindle thermal error will be compensated by tool setting. The thermal error can be considered stable when STE becomes less than a threshold value.

5.3 Experimental results of the thermal error feedback control

In the active cooling experiments, the thermal error feedback control strategy is performed on the mechanical spindle under rotation speed conditions similar to that in common boring machining. Each experiment is conducted after the spindle is cooled down in the natural environment.

| Table 4 Models evaluation results |
|---|---|---|---|
| | $m$ | $d_{VC}$ | $Q_{emp}$ | $Q$ |
| MLR | 10 | 6 | 0.2379 | 0.7342 |
| GA-SVR | 10 | 5 | 0.1402 | 0.6882 |
| RFR | 10 | 5 | 0.1270 | 0.6211 |
Fig. 8 Experimental results of the thermal error feedback control: a Case 1 (work condition #1, $T_{S0} = 18.3 ^\circ C$); b Case 2 (work condition #1, $T_{S0} = 17.7 ^\circ C$); c Case 3 (work condition #2, $T_{S0} = 18.1 ^\circ C$); d Case 4 (work condition #2, $T_{S0} = 18.6 ^\circ C$); e Case 5 (work condition #3, $T_{S0} = 18.5 ^\circ C$)
The spindle can hardly reach a thermal equilibrium state with constant temperature cooling. If we relax the judging criteria for the thermal equilibrium state to an STE value of 0.25 \( \times 10^{-3} \) \( \mu \text{m/s} \), compared with that of constant temperature cooling, the time for reaching the equilibrium state is respectively advanced by 78.3\%, 86.7\%, 86.9\%, 83.8\%, and 87.8\% in the five cases of thermal error feedback control.

### 5.4 Model performance analysis

Five experiments are conducted with the RFR model, for which the maximum \( T_C \) deviation at any time point is pre-estimated to be 0.93 \( ^\circ \text{C} \). For the five cases of feedback (uncontrolled environmental temperature), so the season and weather would affect the spindle structure’s initial temperature/thermal deformation distribution, hence the different spindle thermal behavior in the experiments under the same work condition. Five experiments are conducted on successive winter days, and the ambient temperature is within 18 \( \pm 1 \) \( ^\circ \text{C} \) for all the experiments. In the active cooling experiments, the spindle structure’s initial temperature \( T_{SO} \) distribution is assumed to be uniform, but \( T_{SO} \) varies in different experiments due to different weather, season, and cooldown time. As a result, the thermal experiment results of the spindle might be different due to different \( T_{SO} \) under the same work condition.

The rotation speed of constant 3000 rpm and rotation speed conditions of varying 2000 rpm, 3000 rpm, and 4000 rpm are adopted, which are common operating conditions of the boring machine. The constant and varying rotation speed conditions are adopted to validate the proposed thermal error feedback control strategy comprehensively. Cases 1 and 2 are conducted under work condition #1 of the constant rotation speed of 3000 rpm. Cases 3 and 4 are conducted under work condition #2: the mechanical spindle operates at 3000 rpm before 80 min, 2000 rpm before 140 min, and 4000 rpm until the end. Case 5 is performed under work condition #3: the mechanical spindle runs at 3000 rpm before 80 min, 4000 rpm before 140 min, and 2000 rpm until the end. The experimental results of the thermal error feedback control based on active cooling are presented in Fig. 8: they are compared with the experimental cooling results with a constant 16 \( ^\circ \text{C} \). In this way, the robustness of the thermal error feedback control strategy can be tested.

The \( E \) curves with constant temperature cooling \( (T_C=16 \ ^\circ \text{C}) \) rose steadily during the experiment process (Fig. 8). In contrast, the \( E \) curve with thermal error feedback control stabilized around the pre-set RITE (17 \( \mu \text{m} \)) after preheating of 30–50 min. For the experiments in Fig. 8, the temperature data (sampling frequency is 1 Hz) of every 3 min are characterized to a single mean moving average value. Hence, the model outputted feedback value renews every 3 min. The thermal error measurements are fitted to smooth curves using the five-order polynomial regression method, so the STE which reflects the major variation trend can be obtained (Fig. 9). The fitted \( E \) polynomials are presented in Table 5.

The thermal error is considered stable when the absolute value of STE becomes less than \( 0.25 \times 10^{-3} \) \( \mu \text{m/s} \), which is the thermal equilibrium state. For the presented five thermal error feedback control cases, the STE curves become less than \( 0.25 \times 10^{-3} \) \( \mu \text{m/s} \) after 51, 31, 45, 49, and 43 min. The thermal equilibrium state is maintained for 249, 169, 205, 151, and 157 min. The EVR (thermal error variation range) are respectively 1.513 \( \mu \text{m} \), 1.487 \( \mu \text{m} \), 1.637 \( \mu \text{m} \), 1.482 \( \mu \text{m} \), and 1.508 \( \mu \text{m} \).

The spindle can hardly reach a thermal equilibrium state with constant temperature cooling. If we relax the judging criteria for the thermal error equilibrium state to an STE value of \( 0.5 \times 10^{-3} \) \( \mu \text{m/s} \), compared with that of constant temperature cooling, the time for reaching the equilibrium state is respectively advanced by 78.3\%, 86.7\%, 86.9\%, 83.8\%, and 87.8\% in the five cases of thermal error feedback control.

### Table 5 Coefficients of the fitted polynomials

| \( p_1 \) | \( p_2 \) | \( p_3 \) | \( p_4 \) | \( p_5 \) | \( p_6 \) | \( T_C=16 \ ^\circ \text{C} \) |
|-------|-------|-------|-------|-------|-------|------------------|
| 7.331 \( \times 10^{-21} \) | 1.034 \( \times 10^{-19} \) | 1.387 \( \times 10^{-17} \) | 2.231 \( \times 10^{-19} \) | 1.154 \( \times 10^{-18} \) | 4.305 \( \times 10^{-21} \) |
| \( -4.724 \times 10^{-16} \) | \( -2.78 \times 10^{-14} \) | \( 2.881 \times 10^{-13} \) | \( -8.01 \times 10^{-15} \) | \( 4.234 \times 10^{-14} \) | \( -3.937 \times 10^{-17} \) |
| 1.193 \( \times 10^{-16} \) | \( 2.821 \times 10^{-10} \) | \( 2.335 \times 10^{-9} \) | \( 1.074 \times 10^{-10} \) | \( 5.927 \times 10^{-10} \) | \( 1.322 \times 10^{-11} \) |
| \( -1.516 \times 10^{-7} \) | \( -1.378 \times 10^{-6} \) | \( 9.051 \times 10^{-6} \) | \( 6.426 \times 10^{-7} \) | \( 3.917 \times 10^{-6} \) | \( -3.951 \times 10^{-7} \) |
| 0.001039 | 0.003657 | 0.01651 | 0.001775 | 0.01234 | 0.005098 |
control–based active cooling experiments, the online thermal error estimations are compared with the actual thermal error measurements (Fig. 10). The time interval for the model to output an estimation is 180 s, which equals the time interval that the online temperature measurements are processed to mean moving average.

Cases 1 and 2 are conducted under the rotation speed condition of 3000 rpm, similar to the experiments for collecting
training data. Cases 3, 4, and 5 are conducted under the unseen work conditions of varying rotation speed. In the thermal equilibrium state, the maximum differences (estimation residual) between measured and estimated \( E \) (by summing \( \Delta E \)) are respectively 1.0178 (127.3 min), 0.2079 (166.7 min), and 0.8366 (137.3 min). Above large \( E \) residuals are formed by the accumulation of \( \Delta E \) estimation residuals, which is the systematic error of feedback. As a result, spindle thermal error kept increasing slowly in Cases 1–5 in the thermal equilibrium state but is within the expected variation range all the time.

Suppose we exclude the observations of the preheating stage and consider only those in the equilibrium state. In that case, the model’s goodness of fit will reflect whether reliable feedback is outputted for the thermal error feedback control so that the thermal error will be stabilized in the long term. Three evaluation criteria, the \( MSE \), the determination coefficient \( R \), and the accuracy \( \eta \), are introduced to assess the model performance in the thermal error feedback control process:

\[
MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y}_i)^2
\]

\[
R = 1 - \frac{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}
\]

\[
\eta = 1 - \frac{\sum_{i=1}^{n} |y_i - \bar{y}_i|}{\sum_{i=1}^{n} |y_i|}
\]

and the model performance evaluation results are presented in Table 6. The \( MSE \) values are smaller than 0.5159; the predicting accuracies (\( \eta \)) are more than 9.494%; the determination parameters (\( R \)) are more than 0.9640.

| Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
|--------|--------|--------|--------|--------|
| MSE    | 0.2897 | 0.4105 | 0.3499 | 0.4451 | 0.5159 |
| \( R \) | 0.9640 | 0.9656 | 0.9748 | 0.9663 | 0.9742 |
| \( \eta \) | 0.9736 | 0.9650 | 0.9703 | 0.9791 | 0.9494 |

Table 6 Model performance evaluation for the equilibrium stage

### 6 Conclusion

A thermal error feedback control–based active cooling strategy is proposed to stabilize the thermal error of a mechanical spindle. The spindle thermal variation data is tested to be independent, and a V-C dimension–based approach is presented to estimate the error bound of the model that outputs feedback; thus, the reliability of the thermal error control process can be forecasted. The core conclusions of this study are as follows:

1. The multivariate portmanteau statistic testing results showed that the thermal variation data could be regarded as independent, so it is reasonable to perform the V-C dimension estimation algorithm on the thermal variation data.

2. The V-C dimension–based analysis indicates that the RFR model is most likely to give acceptable performance, where the estimated maximum deviation of \( T_C \) is 0.93 °C.

3. Compared to the active cooling of constant coolant temperature, the thermal error feedback control strategy can primarily advance the time for reaching the equilibrium state and effectively reduce the thermal error variation range.

For further research, this method should be extended for various kinds of spindles in precision machine tools under diverse work conditions, and the preheating time should be shortened.

### Author contribution

Mohan Lei: Conceptualization, methodology, writing—original draft preparation. Feng Gao: Investigation, validation. Yan Li: Software. Ping Xia: Software, writing—reviewing and editing. Mengchao Wang: Validation. Jun Yang: Supervision.

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### Availability of data and materials

All experimental data of this study are available.

### Code availability

Not applicable.

### Declarations

#### Ethics approval

Not applicable.

#### Conflict of interest

The authors declare no competing interests.

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