Andreev reflection in unitary and non-unitary triplet states

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The quasiparticle reflection and transmission properties at normal conductor-superconductor interfaces are discussed for unitary and non-unitary spin triplet pairing states recently proposed for Sr$_2$RuO$_4$. We find resonance peaks in the Andreev reflection amplitude, which are related to surface bound states in the superconductor. They lead to conductance peak features below the quasiparticle gap in the superconductor. The position of the peaks depends on the angle of the incident quasiparticles in a different way for unitary and non-unitary states. Based on this observation we propose a possible experiment which allows to distinguish between the two kinds of superconducting states.

I. INTRODUCTION

The recent observation of superconductivity in Sr$_2$RuO$_4$ has attracted much attention because of the close structural similarity with some of the high-temperature superconductors [1]. The critical temperature, $T_c \sim 1\,\text{K}$, is, however, rather low. In fact there is little similarity between Sr$_2$RuO$_4$ and high-temperature superconductors beyond the crystal structure. In strong contrast to the Cu$_2$O$_2$-systems Sr$_2$RuO$_4$ is a good metal as a stoichiometric compound, displaying clear Fermi liquid properties. All three $t_{2g}$-d-orbitals of Ru$^{4+}$ ($4d^4$) yield bands which cross the Fermi energy and lead to two electron-like and one hole-like Fermi surfaces observed in the beautiful de Haas-van Alphen experiments of Mackenzie et al. [2]. The layered structure of the system leads to a comparatively small dispersion of the bands along the c-axis that is reflected in the highly anisotropic resistivity. Therefore we find an essentially two-dimensional electron system and the nearly cylindrical Fermi surfaces are open along the c-axis. There are various indications for strong correlation effects. Among others the indication of a considerable mass enhancement, $m^* \sim 4m_0$, has been found [3], which causes an enhanced linear specific heat coefficient $\gamma$. Also the Pauli spin susceptibility is enhanced.

Sr$_2$RuO$_4$ is a member of a homologous series of compounds, Sr$_{n+1}$Ru$_n$O$_{3n+1}$, where $n$ is the number of RuO$_2$-layers per unit cell. The single layer compound Sr$_2$RuO$_4$ seems to have unique properties among the members investigated so far. Both the compound with $n = 2$ (double layer) and $n = \infty$ (infinite layer) are itinerant ferromagnets. This suggests that also in the single layer compound ferromagnetic spin fluctuations should be very important, again in contrast to the CuO$_2$-compounds where antiferromagnetic spin fluctuations are dominant.

The presence of strong correlation effects implies that it is difficult for electrons to form Cooper pairs in the usual s-wave channel. Thus a higher relative angular momentum would be more favorable. There are also interesting similarities with $^3\text{He}$ which led to the suggestion that Cooper pairing in the spin triplet (odd-parity) configuration might be favored [3]. Clearly also the presence of strong ferromagnetic spin fluctuations and the Hund’s rule coupling of the d-orbitals on the Ru-anion point into the same direction.

At present, there is no unambiguous experimental indication for triplet pairing. Nevertheless, there is strong evidence for unconventional superconductivity. The superconducting state is extremely sensitive to disorder. Only very clean samples show superconductivity and a small concentration of non-magnetic impurities (Al) suppresses the transition temperature to zero. The nuclear quadrupolar relaxation (NQR) deviates from that of conventional superconductors [4]. It does not have any sign of a Hebel-Slichter peak and shows a low temperature Korringa-like behavior. Furthermore, the specific heat data indicate a large residual density of states in the superconducting phase whith a linear-$T$ coefficient $\gamma$ with about half of the normal state value even in the purest samples [5] consistent with NQR.

The number of possible spin triplet Cooper pairing states is large and the question arises whether any of those states could explain the presence of the large residual density of states. One proposal was a non-unitary state which can only occur for spin triplet pairing [6]. For such a state the quasiparticle excitation spectrum consists of two branches, one of which could be gapless leading to finite density of states at zero energy which is half the normal state value. The nature of this state is very similar to the $A_1$-phase of superfluid $^3\text{He}$ which is stabilized in a magnetic field. It is still an open problem whether there is a mechanism that could give rise to such a state in zero external magnetic field. An alternative explanation which avoids this difficulty was recently proposed by Agterberg et al. [6] based on the fact that there are three distinct Fermi surface sheets. For symmetry reasons two sheets are nearly decoupled from...
the third one for Cooper pair scattering. Therefore the superconducting gap would have different values on different sheets. This leaves space for rather low-lying quasiparticle excitations on certain Fermi surfaces which would appear as residual density of states in the specific heat measurement. There is no clear experimental distinction between the two proposals so far.

There is good experimental evidence that also the heavy fermion superconductor UPt$_3$ has spin-triplet pairing ($T_c \approx 0.5$K) \[10\]. In this case two distinct superconducting phases, a low- and a high-temperature phase, occur. The low-temperature phase has presumably broken time reversal symmetry and may also have a non-unitary state \[12,13\]. The discussion we do here for Sr$_2$RuO$_4$ may also apply to this compound to some extent.

In this paper we investigate the quasiparticle tunneling from a normal metal to the spin triplet superconductor (NS) for both the unitary and non-unitary pairing state. This type of problem was first analyzed by Bruder many years ago \[9\]. We would like to focus here on the interesting aspect of broken time reversal symmetry in the non-unitary state. This property influences the presence of so-called Andreev bound states at the NS interface. We will show that the energy of the bound states depends on the angle of the incident quasiparticle (hole) which is a direct consequence of the polarized internal angular momentum of the non-unitary state. These bound states appear as anomalies in the current-voltage (IV) characteristics of the interface. In general, measurements of the IV-characteristics include an average over a wide range of incidence angles. We will show, however, that it is possible to introduce a certain selection of angles by the method of magnetic focusing previously used for the study of the Andreev effect in conventional superconductors \[20\].

II. THE SPIN-TRIPLET PAIRING STATES

For spin-triplet superconducting states the Cooper pairs possess a spin-1 degree of freedom. This state is described by a gap function which is, in general, a 2 × 2-matrix in spin space and can be parameterized by a vector function $\mathbf{d}(\mathbf{k})$

$$\hat{\Delta}(\mathbf{k}) = i\sigma_2(\mathbf{d}(\mathbf{k}) \cdot \hat{\mathbf{\sigma}})$$

(1)

which is odd in $\mathbf{k}$ \[6,13\]. The energy spectrum of the superconducting quasiparticles is given by

$$E^\pm_k = \sqrt{\epsilon^2(\mathbf{k}) + |\mathbf{d}(\mathbf{k})|^2 \pm |\mathbf{q}(\mathbf{k})|}$$

(2)

where the vector $\mathbf{q}(\mathbf{k}) = i\mathbf{d} \times \mathbf{d}^*$. $\epsilon(\mathbf{k})$ is the band energy measured with respect to the chemical potential. It is $\mathbf{q}$ which distinguishes between two classes of spin-triplet pairing states, the unitary and non-unitary state. If $\mathbf{q} = 0$ the state is unitary and the two branches $E^\pm_k$ are identical. On the other hand, for $\mathbf{q} \neq 0$ the state is called non-unitary. It has two distinct quasiparticle energy spectra and breaks time-reversal symmetry.

For simplicity we will restrict ourselves to the case of a single cylindrical symmetrical Fermi surface and analyze an example of both types of spin-triplet pairing states. These states result from a classification of pairing states including spin-orbit coupling and crystal field effects \[3,7\].

The example for the non-unitary state has the form

$$\mathbf{d}(\mathbf{k}) \propto (\hat{\mathbf{x}} + i\hat{\mathbf{y}}) (k_x - ik_y),$$

(3)

or

$$\hat{\Delta}(\mathbf{k}) = \Delta_0 \begin{pmatrix} k_x - ik_y & 0 \\ 0 & 0 \end{pmatrix}.$$  

(4)

In this state, the Cooper pairs have $S_z = +1$ and $L_z = -1$ \[6\]. It is degenerate to a similar state with opposite orientation of spin and angular momenta. The essential feature of the two states is that the energy gap for one spin projection of quasiparticles (spin-up for (3), $E^{+}_{k}$-branch) has a constant magnitude, while it vanishes for the branch $E^{-}$ on the whole Fermi surface as can be seen immediately by inserting (3) into (2). As mentioned above these gapless quasiparticles could account for the large residual density of states.

We consider the analogue of the Balian-Werthamer (BW) state \[14\] as an example for a unitary state,

$$\hat{\Delta}(\mathbf{k}) = \Delta_0 \begin{pmatrix} k_x - ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix}.$$  

(5)

Note that in two dimensions this is an equal spin pairing state and has a gap with constant modulus for both spins on the cylindrical Fermi surface. This state is stable in the weak coupling approach and could be the basis for the scenario of orbital dependent superconductivity \[8\].
III. THE MODEL FOR THE NS-INTERFACE

In this section we analyze the properties of the normal-metal-superconductor (NS) interface for the two types of pairing states (3) and (5). We apply the formulation by Blonder, Tinkham and Klapwijk (BTK) \cite{15} for NS interfaces. This allows us to derive expressions for the reflection probabilities of electrons. The geometry of our problem contains the following restrictions. The particles move in the $xy$-plane and the boundary between normal metal ($x < 0$) and superconductor ($x > 0$) is the $yz$-plane located at $x = 0$. We will use a step function for the spatial dependence of the gap magnitude, $\hat{\Delta}(k, x) = \Theta(x) \hat{\Delta}(k)$. The surface potential will be taken as a delta potential $U(x) = H \delta(x)$. Then following BTK \cite{15} the amplitudes for quasiparticle (Andreev-) reflection and transmission can be calculated using the appropriate boundary conditions for the wave functions $\psi_N$ on the normal and $\psi_S$ on the superconducting side of the NS interface. Since for triplet superconductors we have to take care of the spin structure of the gap function, the quasiparticle wave functions are four-spinors in Nambu (particle-hole $\otimes$ spin) space. Their particle and hole components are determined by the solutions of the Bogoliubov-deGennes equations \cite{13}, i.e. the diagonalization of the corresponding BCS mean field Hamiltonian

$$\hat{u}_k (\hat{E}_k - \epsilon(k)) = \hat{\Delta}(k) \hat{v}^*_{-k},$$
$$\hat{v}^*_{-k} (\hat{E}_k + \epsilon(k)) = \hat{\Delta}^\dagger(k) \hat{u}_k.$$  \hspace{1cm} (6)

Here $\hat{u}_k$ and $\hat{v}^*_{-k}$ are matrices in spin-$\frac{1}{2}$ space, the Bogoliubov quasiparticle operators in the superconductor are then given by

$$\gamma_{k\uparrow} = v_{-k,11}^* \hat{c}_{-k \uparrow} + v_{-k,21}^* \hat{c}_{-k \downarrow} + u_{k,11}^\dagger \hat{c}_{k \uparrow} + u_{k,21} \hat{c}_{k \downarrow},$$
$$\gamma_{k\downarrow} = v_{-k,12}^* \hat{c}_{-k \uparrow} + v_{-k,22}^* \hat{c}_{-k \downarrow} + u_{k,12}^\dagger \hat{c}_{k \uparrow} + u_{k,22} \hat{c}_{k \downarrow}.$$  \hspace{1cm} (7)

For equal-spin pairing (ESP) states ((3) or (5)), the spin-up and spin-down components decouple, therefore we obtain two-spinors with one electron and one hole component. For triplet states, the Andreev hole is a missing electron of the same spin projection as the incoming particle. Following BTK \cite{15}, we denote the amplitude for Andreev reflection by $a$ and for normal reflection by $b$. Moreover we introduce the amplitudes $c$ for transmission into the superconductor as a Bogoliubov quasiparticle on the same branch of the spectrum and $d$ for transmission with backscattering through the Fermi surface, i.e. reverting the momentum $x$ component. The momenta involved into the reflection processes are shown in Fig.1.

![FIG. 1. Momenta (solid lines) and velocities (dashed lines) of the quasiparticles involved in the Andreev reflection amplitude $a(E, \theta)$.](image)

Now assume that the incoming particles are electron-like quasiparticles with excitation energy $E$ with respect to the Fermi energy, let their spins be polarized along the “gapped” direction (in our convention spin-up). Then the wave function on the normal conducting side is the sum of the wave functions of the incoming particle and the reflected particles multiplied with their amplitudes,

$$\psi_N = \psi_{\text{inc.}} + a \psi_a + b \psi_b,$$

and on the superconducting side we have

$$\psi_S = c \psi_c + d \psi_d.$$

Limiting ourselves to sufficiently small angles of incidence $\theta$ in the $xy$-plane, we can approximate all magnitudes of $x$ components of the involved momenta by $k_x = k_F \cos \theta$. If we assume conservation of the quasiparticle momenta...
parallel to the boundary the transverse wave function of all excitations on both sides is \(e^{ik_F \sin \theta y}\). Writing two-spinors with upper component for spin-up electrons and lower component for missing spin-up electrons, the \(x\)-depending parts are

\[
\psi_{\text{inc.}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_s x},
\psi_a = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_s x},
\psi_b = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_s x},
\psi_c = \begin{pmatrix} u(\theta) \\ \eta^* (\theta) v(\theta) \end{pmatrix} e^{ik_s x},
\psi_d = \begin{pmatrix} \eta (\pi - \theta) v(\pi - \theta) \\ u(\pi - \theta) \end{pmatrix} e^{-ik_s x}.
\]

where

\[
u(\theta) = \sqrt{\frac{1}{2} \left(1 + \frac{(E^2 - |\Delta(\theta)|^2)^{1/2}}{E}\right)},
\]
and

\[
u(\theta) = \sqrt{\frac{1}{2} \left(1 - \frac{(E^2 - |\Delta(\theta)|^2)^{1/2}}{E}\right)},
\]

and \(\eta(\theta) = \Delta(\theta)/|\Delta(\theta)|\) is the gap phase. For the non-unitary state \((\eta)\) and for the spin-up channel of the unitary state \((\eta)\) we have \(\eta(\theta) = \exp(-i\theta)\) while for the spin-down channel of the unitary state \(\eta(\theta) = \exp(i\theta)\). For time-reversal conserving singlet states \(\eta(\theta)\) can be taken real and, in particular, for \(s\)-wave pairing with \(\eta(\theta) = 1\) we recover the BTK results.

Using the boundary conditions we obtain the following expressions for \(a\) and \(b\):

\[
a_c(E, \theta) = \frac{\eta^* (\theta) v(\theta) u(\pi - \theta)}{u(\theta) u(\pi - \theta) + Z^2 \delta (E, \theta)},
\]

\[
b_c(E, \theta) = \frac{(Z^2 + i Z) \delta (E, \theta)}{u(\theta) u(\pi - \theta) + Z^2 \delta (E, \theta)}
\]

with

\[
\delta (E, \theta) = u(\theta) u(\pi - \theta) - \eta^* (\theta) \eta (\pi - \theta) v(\theta) v(\pi - \theta).
\]

\(Z(\theta) = H/(v_F \cos(\theta))\) measures the barrier strength. The subscript \(e\) denotes the coefficient for the incident electrons. Incoming holes (subscript \(h\)) with the same excitation energy \(E\) and group velocity in the same direction \(\theta\) have momenta opposite to those of incoming electrons. Performing the appropriate substitutions in \((\eta)\) we obtain

\[
|a_h (E, \theta)|^2 = |a_c (E, -\theta)|^2. \tag{9}
\]

For the states \((\eta)\) and \((\eta)\) and the given geometry the gap function seen by the holes is the complex conjugate of that seen by the electrons under the same angle, i.e. Cooper pairs with reversed angular momentum along the \(z\) axis.

IV. RESULTS FOR THE NON-UNITARY STATE

In Fig.3 we show results of Andreev and normal reflection probabilities for various angles of incidence and two barrier strengths, \(Z = 0.35\) and \(Z = 1.5\). These curves correspond to the gapped channel (in our convention the incoming spin-up quasiparticles) of the non-unitary state \((\eta)\). For incoming spin-down particles, the superconducting gap is zero, therefore we obtain N-N junction results, which are not shown here.

For the unitary state \((\eta)\) both spin orientations have a gapped spectrum and there is a simple relation between spin-up and -down component. For the spin-down electrons the behavior for given angle of incidence \(\theta\) are those of the spin-up electrons with \(-\theta\), because of the reversed angular momentum of the spin-down Cooper pairs \((\eta(\theta) \rightarrow \eta^* (\theta)\) or \(k_y \rightarrow -k_y)\). The angle-independent gap magnitude sets the energy scale for the reflection amplitudes, but their behavior is also determined by the phase factors \(\eta(\theta)\) occurring in the denominator of \((\eta)\). The latter give rise to interesting properties.
A. Asymmetry with respect to $\theta \rightarrow -\theta$

If we consider the triplet states (3), (5) and incoming particles of one spin projection (say spin-up)

$$\eta^*(\theta)\eta(\pi - \theta) \neq \eta^*(-\theta)\eta(\pi + \theta).$$

As a consequence the Andreev and normal reflection amplitudes (8) are asymmetric with respect to the boundary normal. This can be seen in Fig. 2. Similar asymmetries have been discussed for $s + id$ superconducting gaps by Matsumoto and Shiba [16]. But note that for the triplet states (4) and (6) this asymmetry occurs for arbitrary boundary orientation, because changing the boundary orientation is equivalent to multiplying the gap function with a constant phase. In the unitary BW state (5) the asymmetry cancels if we sum over the spin projections because as mentioned above for incident spin-down quasiparticles the $\eta$’s change into their complex conjugates, which is equivalent to $\theta \rightarrow -\theta$.

![Graph showing Andreev and normal reflection probability currents](image)

FIG. 2. Andreev ($A = |a_c(E, \theta)|^2$, solid line) and normal reflection ($B = |b_c(E, \theta)|^2$, dashed line) probability currents for an N-S boundary with non-unitary superconducting state (6) and incoming spin-up electron with energy $E$. The plots show the results for various angles of incidence $\theta$ and two different barrier strengths $Z$. The $\ast$ denotes the resonance energy $E_A = -\Delta_0 \sin \theta$. 


B. Sub-gap resonances

For certain sub-gap energies and angles of incidence, \( \delta(E, \theta) \) in the denominator of (8) vanishes. This occurs under the condition

\[
\eta^*(\theta) \eta(\pi - \theta) = u(\theta) u(\pi - \theta) \frac{v(\pi - \theta)}{v(\theta)}. \tag{10}
\]

Then the effect of the boundary barrier is turned off and we obtain \( |a_e(E, \theta)|^2 = 1 \) which corresponds to a resonance. The corresponding peaks in the Andreev reflection probability become sharper with increasing \( Z \), their width decreases \( \propto Z^{-2} \). At the resonance condition (10), the transmitted wave in the superconductor is a superposition of a surface bound state (note that this expression is only sensible for \( Z \to \infty \)) formed by repeated Andreev and normal reflection (see Fig.3, consider the limit \( d \to 0 \)) and a \( c \)-type \( \gamma \dagger \) quasiparticle, which carries the charge \( 2e \) into the superconductor.

Both parts of the transmitted wave decay on a length scale \( v_F \cos \theta/\sqrt{\Delta^2_0 - E^2} \). The condition \( \delta(E, \theta) = 0 \) is equivalent to a vanishing bound state wave function at the barrier at \( x = 0 \). In the quasi-classical picture of Fig.3 the bound state is formed if the phases accumulated by the electron and hole components of the two-spinors during one round trip through the normal layer sum up to an integer multiple of \( 2\pi \). Since the phase shifts for the Andreev reflections are angle-dependent, the resonance energies also vary with \( \theta \).

In the non-unitary state (3) \( \delta(E, \theta) \) vanishes for \( E_A = -\Delta_0 \sin \theta \), i.e. negative angles of incidence. This leads to resonances in the Andreev amplitude, \( |a_e(E_A, \theta)|^2 = 1 \), at negative angles of incidence. Note that the bound state has a non-vanishing angular momentum along the \( z \) axis, therefore its asymmetric position at negative \( \theta \) can be understood as a consequence of the finite relative angular momentum of the Cooper pairs in non-unitary states. This is in agreement with the fact that according to (9) for injected holes the resonance peaks in the Andreev amplitude change side, i.e. they occur at positive angles of incidence. Since its angular momentum \( L_z \) is fixed by the gap function, the bound state is only accessible for electrons with angle of incidence \( \theta \) and holes with angle of incidence \(-\theta\). In the spin down channel of the unitary state (5) the analogous bound state with opposite \( L_z \) does exist, therefore the resonances occur symmetrically if we assume unpolarized incoming particles.

The bound state condition \( \delta(E, \theta) = 0 \) is essentially equivalent to the equations recently given by Hu [17] and Tanaka et al. [18] for \( d \)-wave and other singlet superconductors. They report zero energy states for certain orientations of the boundary with respect to the pair wave function. This is in contrast to the triplet states (3) and (5) where the sub-gap resonances are not sensitive to the boundary orientation in the plane.

C. Zero-bias conductance peaks

The charge current through the NS boundary by an incoming electron is increased by an Andreev hole with probability \( |a_e(E, \theta)|^2 \) and diminished by normal reflection with probability \( |b_e(E, \theta)|^2 \). Therefore at \( T = 0 \)K for the non-unitary state (3) and applied voltage \( V \) such that we have incoming electrons from the N side, the conductance of the gapped spin-up channel is given by [15]...
Here, the reflection amplitudes have to be evaluated at energy \( E = eV \) and at the angle of incidence in the plane \( \theta \), \( \rho^\uparrow(eV,k_\perp) \) and \( v_g(k_\perp) \) denote the spin-up density of states and group velocity on the N side at transverse momentum \( k_\perp \) parallel to the boundary, over which we average with probability distribution \( P(k_\perp) \). The latter depends on the type of experiment one is interested in. For the non-unitary state \([3]\) the spin-down conductance is equal to the normal state conductance.

For s-wave superconducting states (all \( \eta \)'s = 1), the Andreev amplitude is largest at \( E = \Delta \). Therefore the conductance \( G_{NS} \) is also peaked at gap energy and resembles the bulk density of states for low transparency boundaries. This so called gap-like feature is entirely absent for the triplet states \([3]\) and \([5]\). Due to the bound states at \( E = \pm \Delta_0 \sin \theta \), the Andreev amplitude for small angles of incidence is largest around \( E \approx 0 \). This leads to zero-bias conductance peaks (ZBCP) which become increasingly pronounced when the junction favors smaller angles of incidence. As explained above for the two-dimensional triplet states \([3]\) and \([5]\) these ZBCPs do not depend on the boundary orientation. Therefore they should provide reliable evidence whether a nodeless triplet state with \( \Delta(\theta) \propto \exp(\pm i \theta) \) is realized in \( \text{Sr}_2\text{RuO}_4 \).

Yamashiro and Tanaka [19] have also reported that ZBCPs should occur for \((100)\ NS \) boundaries with the non-unitary superconducting triplet states \([3]\) and \([5]\) and \( \Delta_{\uparrow\uparrow}(\theta) \propto \cos \theta + \sin \theta \), which has nodes at \( \theta = -\pi/4 \) and \( \theta = 3\pi/4 \) and two lobes with different signs of \( \Delta(\theta) \) in between. By looking at the phase structure of the latter gap function, we expect the ZBCP to be strongest for a \((110)\) boundary (one of the two lobes is directed towards the interface which implies zero-energy resonances for all angles of incidence) and a smeared-out gap-like feature and no ZBCP for \((110)\) boundaries (lobes parallel to the interface, no sub-gap resonances). Thus such states should be experimentally distinguishable from the nodeless states \([3]\) and \([5]\).

V. EXPERIMENTAL DISTINCTION BETWEEN UNITARY AND NON-UNITARY STATES

The interplay between asymmetries and bound states can be utilized in an experiment in order to distinguish between the unitary and non-unitary states proposed for \( \text{Sr}_2\text{RuO}_4 \). A feasible way for such an experiment is two-point spectroscopy described by Benistant et al. [20]. Here, one injects unpolarized electron-like quasiparticles through a point contact into a normal conductor of thickness \( d \) attached to the superconductor with a certain distribution \( P(\phi) \) of the injection angle \( \phi \). Both the electrons and the Andreev reflected holes are deflected by a magnetic field applied parallel to the \( z \)-axis. The holes are now collected by a second point contact. Obviously, experiments of this kind require a normal conducting crystal of high quality where quasiparticles move ballistically. Suppose we have many hole-collectors along the transverse direction of the field, i.e. we are able to measure the intensity of Andreev holes arriving at the surface of the normal conductor as a function of the transverse coordinate \( y \). If the probability for Andreev reflection is angle-independent, e.g. for a conventional s-wave superconductor, one obtains a hole distribution which is peaked at a minimal distance \( y = y_0 \) from the injection point at \( y = 0 \). This feature is known under the name ”electron focusing” (EF) [21]. The intensity peak at \( y = y_0 \) can be also seen in Fig.4 in the density of the hole trajectories arriving at the surface. It should be noted that for the focusing to work any voltage drop should take place at the point contacts only, otherwise the motion of the particles would be influenced by inhomogeneous electric fields inside the normal metal. For voltages \( eV \) comparable to \( \Delta_0 \) the quasiparticle trajectories are independent of the voltage drop, because in this case the velocity deviation from \( v_F \) is negligible.

For a non-s-wave superconducting state, we have to take care of the angular dependent Andreev reflection probability. This additional factor results for the triplet states \([3]\) and \([5]\) in a modification of the collected intensities due to the resonance peaks, particularly for higher barrier strengths \( Z \), where the Andreev reflectivity is low for energies away from the resonances. In the presence of a magnetic field screening currents lead in general to a change in the resonance energy \( \Delta_k \) due to a shift in the quasiparticle spectrum by \( \Delta_k = v_s \cdot k = v_s k_F \sin \theta \), where \( v_s \) is the superfluid velocity. Since both transmitted quasiparticles responsible for the bound state formation have the same transverse momentum (see Fig.4) and the bound state energies have the same angular dependence \( \propto \sin \theta \) as the energy shift, this only leads to a renormalization of the energy scale and the qualitative picture drawn here remains valid. If the weight of the quasiparticles which satisfy the resonance condition \([1]\) is sufficiently high, they give rise to a broader second peak (besides the focusing peak) in the ”count rate” at a different collector position (see Fig.4 at \( y \approx -0.5d \)). The magnetic field leads to an asymmetric distribution of angles of incidence at the NS boundary. For the non-unitary state \([3]\) the resonances occur only for negative angles of incidence (e.g. at \( -\theta \) in Fig.4), for which the corresponding electron paths have small injection angles \( \phi \).

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Inversion of the voltage difference between emitter and collector is equivalent to injecting holes instead of electrons. The injected holes and Andreev reflected electrons follow the same paths as the electrons and reflected holes did before the voltage reversal. But according to (9) the holes face a gap potential with reversed angle of incidence $-\theta$, which makes the resonance peaks change sides for the non-unitary state (3) (of course the sign of the detected voltage is also reversed). Now the paths corresponding to the resonance peaks at $\theta$ have large angle of incidence $\phi$ (see Fig. 4). If the angular probability distribution $P(\phi)$ of the injected particles is peaked at small injection angles this will lead to observable changes in the "count rate" of Andreev particles. In Fig. 5 we show the count rates versus collector position for a non-unitary superconducting state (3) and two different energies of the injected particles. Note that the assignment of the two curves to the electron and hole injection depends on which one of the degenerate non-unitary states is realized in the superconductor. In the calculations, we assumed that the angular distribution $P(\phi)$ is Gaussian with width $\sigma = 20^\circ$. For increasing $\sigma$ the asymmetry between the electron and hole injection becomes less pronounced.

FIG. 4. Scheme of a two-point-contact measurement. The three curves in the upper half of the plot show the distributions due to focusing (dashed-dotted line), and including the angular dependent Andreev reflection amplitude in the non-unitary state (3) for resonances at negative angle of incidence ($-\theta$, solid line) and resonances at positive $\theta$ (dotted line). All curves assume a Gaussian distribution for the injection angle $\phi$ with $\sigma = 20^\circ$.

FIG. 5. Non-unitary state: Intensities of collected Andreev particles versus collector position for barrier strength $Z = 2.5$, fixed cyclotron radius $r = 4d$ and two energies. The 'electron' curves denote injection of electrons into the normal metal layer and are obtained from the 'hole' curves by inverting the voltage between emitter and collector.
FIG. 6. Non-unitary state: Intensities of collected Andreev particles versus applied magnetic field for $E = 0.3\Delta_0$ (left plot) and $E = 0.6\Delta_0$ (right plot), $Z = 2.5$. Fixing the sign of the voltage drop at the emitter (i.e. electron or hole injection) corresponds to selecting one of the two solid lines. The magnetic field is measured in $1/r$, where $r$ denotes cyclotron radius measured in units of the thickness of the normal conducting layer.

In a real experiment one will vary the applied field instead of the collector position. In Fig. 6 we plot the count rates for a fixed collector position at energies $E = 0.3\Delta_0$ and $E = 0.6\Delta_0$, $Z = 2.5$, $\sigma = 20^\circ$. For collector positions away from the focusing peak the asymmetry between electron and hole injection is very pronounced because the probability for Andreev reflection is small outside the resonance peaks. For Sr$_2$RuO$_4$ the lower critical field along the $c$ direction is $H_{c1}(0) = 11$ mT [24]. In a focusing field of $H = 9$ mT and using the numbers given by Benistant et. al. [20] for a silver normal conducting layer, the radius of the electron paths is $r \approx 0.88$ mm. For example the right plot in Fig. 6, i.e. $d = r/4 \approx 0.22$ mm, corresponds to a distance of $y = 0.5d \approx 0.11$ mm between collector and injection point. All these values are in the same range as those of the experiments performed by Benistant et. al. for conventional superconductors.

FIG. 7. Non-unitary state: Intensities of collected Andreev particles versus energy $E$ of the injected particles (applied voltage) for fixed collector position $y$ and two magnetic field strengths, $Z = 2.5$.

For the two-dimensional BW state [3] we do not expect any differences in the count rates if we reverse the voltage between the two contacts, because for unpolarized injected particles there is no asymmetry with respect to $\theta \leftrightarrow -\theta$ and consequently no difference between electron and hole injection. For fixed voltage, we have to add the two curves...
denoted by "electrons" and "holes" in the plots above in order to obtain the spin-averaged count rate for one type of quasiparticles injected.

VI. SUMMARY

Our analysis of the properties of two typical spin-triplet pairing states at NS interfaces has shown the existence of angular dependent sub-gap resonances in the Andreev reflection. These resonances correspond to bound states and are responsible for zero-bias conductance peaks independent of the boundary orientation in the xy-plane within our model. The angular dependence of the resonance has important implications for the non-unitary state. The reflectivity of electrons and holes (averaged over both spin components) is not equal for the same angle of incidence. This property is connected with the broken time reversal symmetry of the non-unitary state. The electron and the hole probe the time reversed pairing states. This effect could be used to identify the non-unitary state experimentally. The EF technique explained above allows to probe a certain range of incident angles and would reveal the broken time reversal symmetry as an asymmetry between positive and negative bias voltages.

Recent investigations on high-temperature superconductors suggest that a state with locally broken time reversal symmetry could appear for certain orientations of the interface. This state, the so-called $d + is$-wave state, would lead to a splitting of the zero-bias anomaly seen in the pure $d$-wave state for such an interface [22]. This more simple probe does not apply in our case where for both triplet states (3) and (5) there is no significant qualitative difference in the I-V characteristics.

An additional interesting aspect of the non-unitary states is that one spin orientation generates an excess current in the NS-interface while the other one does not. Therefore, the quasiparticle current of such an NS-interface transports not only charge but also spin. In this context, however, the spin-orbit coupling of Sr$_2$RuO$_4$ plays an important role in determining the spin polarization direction. Hence, a more careful consideration of the transfer of the pseudospin states at the NS-interface is necessary.

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