Dynamic modeling and transient stability analysis of distributed generators in a microgrid system

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ABSTRACT

Increasing the penetration level of distributed generation units as well as power electronic devices adds more complexity and variability to the dynamic behaviour of the microgrids. For such systems, studying the transient modelling and stability is essential. One of the major disadvantages of most studies on microgrid modelling is their excessive attention to the steady state period and the lack of attention to microgrid performance during the transient period. In most of the research works, the behaviour of different microgrid loads has not been studied. One of the mechanisms of power systems stability studies is the application of state space modelling. This paper presents a mathematical model for connected inverters in microgrid systems with many variations of operating conditions. Nonlinear tools, phase-plane trajectory analysis, and Lyapunov method were employed to evaluate the limits of small signal models. Based on the results of the present study, applying the model allows for the analysis of the system when subjected to a severe transient disturbance such as loss of large load or generation. Studying the transient stability of microgrid systems in the stand-alone utility grid is useful and necessary for improving the design of the microgrid’s architecture.

Keywords: Method of Lyapunov, Microgrid model, Nonlinear model, Transient stability

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1. INTRODUCTION

Intelligent microgrids integrate different energy resources, especially renewable ones, to provide dependable and efficient operations while being connected to the grid or islanding mode. It can ensure an uninterrupted reliable flow of power and economic and environmental benefits while minimizing the energy loss through transmission over long distances. Local power generation and storage systems make the operation of grid and critical facilities possible independent of the public utility when necessary, thus eliminating blackouts. New technologies provide the option of automatic fixing in case of necessity and anticipate power disturbances [1]-[4]. Microgrids can also feed the public utility when power demand and cost are the highest by supplying electricity from renewable sources. Thus, the use of intelligent power interfaces between the renewable source and the grid is required. These interfaces have a final stage consisting of dc/ac inverters, which can be classified into current source inverters (CSIs) and voltage-source inverters (VSIs). Although CSIs are commonly used to inject current into the grid, for island or autonomous operation, VSIs are needed to maintain voltage stability [5]-[10]. Moreover, a system without master control of inverters is advantageous where every inverter is able to change voltage and frequency of the microgrid in
function of the PQ power into the microgrid. Besides, the fault of an inverter does not cause the collapse of the microgrid. In fact, each inverter could act as a plug-and-play entity to make expanding the microgrid system easier. Although the relationship between inverters is not necessary for the stability of the microgrid, it can be used to improve its performance [11]-[14].

Fast and flexible control of active and reactive power is an important requirement during the steady state and transient operation of the microgrid. A microgrid can experience low voltage instability when all distributed generation units are rotary machines with slow response speeds. Most of existing DG module technologies require power electronics converters as the intermediate interface in order to be connected to the network. A power electronics converter equipped with a quick control strategy can dynamically improve the microgrid stability by adjusting the instantaneous and reactive power, thereby increasing voltage quality and reducing the risk of angle instability. Due to different dynamics of distributed generator types compared to large power plants, the presence of distributed generation affects the dynamic characteristics of the grid. For this reason, modeling and controlling the behavior of these generators require careful consideration. This is more important in microgrids such that because of their small capacity, a distributed generation source can supply a significant percentage of load and its dynamic behavior greatly affects voltage and frequency control [15]-[17]. In [18] discusses the low voltage hybrid AC-DC microgrids power flow in islanding operation. The model simulation of hybrid AC-DC microgrid model have been carried out in two different cases, power flow from DC to AC sub microgrids and power flow from AC to DC sub microgrids. The result of the simulation shows that the model has a good response to power changes conditions.

In [19]-[21], appropriate simulation models and methods for investigating the dynamics of microgrids in transient stability have been developed. In the mentioned study, the aim is to investigate the interaction of microgrids in connected and islanded operation modes. All the components of the microgrids are modeled with sources, loads, lines, and power electronic interfaces.

This paper works with both linear and nonlinear tools. It uses phase-plane trajectory analysis and a method of Lyapunov to determine large variations in the systems. It is suggested that the new proposed model be utilized to analyze the system when it is subject to a severe transient disturbance, such as the loss of large loads or generation. In this paper, first, the mathematical model of the microgrid and basic equations are presented. Then, the system is described. The next step is the study of stability of Lyapunov. Finally, some simulations are presented.

2. MATHEMATICAL MODEL

Figure 1 presents a microgrid system in a stand-alone mode based on parallel connected inverters. It is assumed that the system is a balanced three-phase circuit. Each generator has a power DC renewable source, a DC/AC inverter, and a low pass filter and it is managed by two control loops. An inner loop is used to regulate the output voltage and current and the outer loop is used to share and trade-off the PQ power in the microgrid without communication between generators. This model does not consider the dynamics of the inner loop because it is of high frequency type [22]-[24].
For systems with two inverters connected, the signals can be represented as (1), (2):

\[ V_i = |V_i|e^{j\theta_i}, V_2 = |V_2|e^{j\theta_2}, \ldots, V_j = |V_j|e^{j\theta_j}, V_m = |V_m|e^{j\theta_m} \]  

\[ \theta_i = \text{Arc tan}\left(\frac{v_{ia}}{v_{al}}\right), \quad \theta_j = \text{Arc tan}\left(\frac{v_{ja}}{v_{al}}\right), \quad \ldots, \quad \theta_j = \text{Arc tan}\left(\frac{v_{ja}}{v_{al}}\right) \]  

and

\[ \theta_{j+1} = \theta_i - \theta_{i+1} \]  

Where \( V_i \) is nominal set points of d-axis output voltage, \( \theta_i \) is the power angle of the i’th generator, \( \theta_{i+1} \) is the power angle between i and i+1’th generators, \( v_{ai} \) and \( v_{al} \) are output voltages of the i-th islanded-inverter distributed in d-axis and q-axis respectively. The derived power angle between both generators can be defined by (4):

\[ \omega_i = \omega_0 - \omega_{j+1} \]  

\( \omega_0 \) is stator supply angular frequency.

The PQ controller uses an artificial droop control scheme with average signals P and Q, which are obtained using the low-pass filter, that can be expressed as (5), (6):

\[ \omega_i = \omega_0 - k_p P \]  

\[ V_i = V_0 - k_v Q \]  

where, \( \omega_0 \) and \( V_0 \) are system rated speed and voltage, P and Q are active and reactive average power respectively, also \( K_p \) and \( K_v \) are droop coefficients.

The cut-off frequency (\( \omega_j \)) used in (7) and (8) is a decade lower than frequency of the microgrid.

\[ P = \frac{\omega_i}{s + \omega_j} P_i \]  

\[ Q = \frac{\omega_i}{s + \omega_j} Q_i \]  

That, \( P_i \) and \( Q_i \) are instantaneous power. If (7) and (8) are replaced in (5) and (6),

\[ \omega_j = \omega_0 - k_v \frac{\omega_j}{s + \omega_j} P_i \]  

\[ V_i = V_0 - k_v \frac{\omega_j}{s + \omega_j} Q_i \]  

It is possible to rewrite the curve droop equation and the low pass filter in the time domain as in (11), and (12):

\[ \dot{\omega}_i = -\omega_i \omega_j + \omega_j \omega_0 - k_v \omega_j P_i \]  

\[ \dot{V}_i = -\omega_j V_i + \omega_j V_0 - k_v \omega_j Q_i \]  

For primary and end inverters connected (with index i):
\[ P = \frac{R_i}{R_i^2 + X_i^2} V_i^2 - \frac{V_i V_{0i}}{X_{i0i}} \sin(\theta_{i0i}) \]  \hspace{1cm} (13) \\
\[ Q = \frac{X_i}{R_i^2 + X_i^2} V_i^2 + \frac{V_i V_{0i}}{X_{i0i}} \cos(\theta_{i0i}) \]  \hspace{1cm} (14)

where, \( R_i \) and \( X_i \) are resistor and inductance of \( i \)th generator, also \( X_{i0i} \)is line inductance between \( i \) and \( i+1 \)th inverters.

Replacing (13) and (14) in (11) and (12), we will have:
\[ \dot{\omega}_i = -\omega_i \omega_a + \omega_j \omega_b - \frac{k \omega_j R_i}{R_i^2 + X_i^2} V_i^2 + \frac{k \omega_j}{X_{i0i}} V_i V_{0i} \sin(\theta_{i0i}) \]  \hspace{1cm} (15)
\[ \dot{V}_i = -\omega_j V_i + \omega_j V_0 - \left( \frac{k \omega_j X_i}{R_i^2 + X_i^2} - \frac{k \omega_j}{X_{i0i}} \right) V_i^2 + \frac{k \omega_j}{X_{i0i}} V_i V_{0i} \cos(\theta_{i0i}) \]  \hspace{1cm} (16)

For inner inverter connected (with index m):
\[ P_m = \frac{R_m}{R_m^2 + X_m^2} V_m^2 - \frac{V_m V_{0m}}{X_{m0m}} \sin(\theta_{m0m}) + \frac{V_m V_{0m}}{X_{m0m}} \sin(\theta_{m0m}) \]  \hspace{1cm} (17) \\
\[ Q_m = \frac{X_m}{R_m^2 + X_m^2} V_m^2 + \frac{V_m V_{0m}}{X_{m0m}} \cos(\theta_{m0m}) - \frac{V_m^2}{X_{m0m}} + \frac{V_m V_{0m}}{X_{m0m}} \cos(\theta_{m0m}) \]  \hspace{1cm} (18)

Replacing (17) and (18) in (11) and (12), we will have:
\[ \dot{\omega}_m = -\omega_m \omega_a + \omega_j \omega_b - \frac{k \omega_j R_m}{R_m^2 + X_m^2} V_m^2 + \frac{k \omega_j}{X_{m0m}} V_m V_{0m} \sin(\theta_{m0m}) \]  \hspace{1cm} (19) \\
\[ \dot{V}_m = -\omega_j V_m + \omega_j V_0 - \left( \frac{k \omega_j X_m}{R_m^2 + X_m^2} - \frac{k \omega_j}{X_{m0m}} \right) V_m^2 + \frac{k \omega_j}{X_{m0m}} V_m V_{0m} \cos(\theta_{m0m}) \]  \hspace{1cm} (20)

3. STUDY SYSTEM

Figure 2 show the circuit diagram of the microgrid systems in the stand-alone mode considered in this paper. This system has two inverters connected and the assumption is based on small signal modeling.

It is assumed that
\[ X_1 = \theta_3 , \ X_2 = \omega_3 , \ X_3 = \omega_2 , \ X_4 = |V| , \ X_5 = |V| \]  \hspace{1cm} (21)

According to (4), (15), and (16), we get:
\[ \dot{X}_1 = X_2 - X_3 \]  \hspace{1cm} (22) \\
\[ \dot{X}_2 = -\omega_3 X_2 + \omega_3 \omega_b - \frac{k \omega_j R_i}{R_i^2 + X_i^2} X_1^2 + \frac{k \omega_j}{X_{i0i}} X_1 X_3 \sin(X) \]  \hspace{1cm} (23)
\[
\dot{X}_1 = -\omega_i X_1 + \omega_i \omega_0 - \frac{k_\omega \omega_0 R_1}{R_1^2 + X_1^2} X_1^2 + \frac{k_\omega \omega_0}{X_{12}} X_1 X_2 \sin(X_2) \tag{24}
\]

\[
\dot{X}_2 = -\omega_i X_2 + \omega_i \omega_0 - \left( \frac{k_\omega \omega_0 X_2}{R_2^2 + X_2^2} - \frac{k_\omega \omega_0}{X_{12}} \right) X_2^2 + \frac{k_\omega \omega_0}{X_{12}} X_1 X_2 \cos(X_2) \tag{25}
\]

\[
\dot{X}_3 = -\omega_i X_3 + \omega_i \omega_0 - \left( \frac{k_\omega \omega_0 X_3}{R_3^2 + X_3^2} - \frac{k_\omega \omega_0}{X_{12}} \right) X_3^2 + \frac{k_\omega \omega_0}{X_{12}} X_1 X_3 \cos(X_3) \tag{26}
\]

The system shown in Figure 2 has the parameters presented in Table 1. Thus:

\[
\dot{X}_1 = X_2 - X_1 \tag{27}
\]

\[
\dot{X}_2 = -3.14 X_2 + 9.8 \times 10^{-3} - 2.8 \times 10^{-5} X_2^2 - 0.0027 X_3 X_2 \sin(X_2) \tag{28}
\]

\[
\dot{X}_3 = -3.14 X_3 + 9.8 \times 10^{-3} - 3.4 \times 10^{-5} X_3^2 + 0.0027 X_3 X_2 \sin(X_2) \tag{29}
\]

\[
\dot{X}_i = -3.14 X_i + 7.2 \times 10^{-5} - 0.0307 X_i^2 + 0.0307 X_4 X_i \cos(X_i) \tag{30}
\]

\[
\dot{X}_4 = -3.14 X_4 + 7.2 \times 10^{-5} - 0.0307 X_4^2 + 0.0307 X_4 X_3 \cos(X_3) \tag{31}
\]

The equilibrium points can be obtained if they make all the derivatives of the system equal to zero:

\[
f(X) = 0 = \begin{bmatrix} 0 = X_2 - X_1 \\ 0 = -3.14 X_2 + 9.8 \times 10^{-3} - 2.8 \times 10^{-5} X_2^2 - 0.0027 X_3 X_2 \sin(X_2) \\ 0 = -3.14 X_3 + 9.8 \times 10^{-3} - 3.4 \times 10^{-5} X_3^2 + 0.0027 X_3 X_2 \sin(X_2) \\ 0 = -3.14 X_i + 7.2 \times 10^{-5} - 0.0307 X_i^2 + 0.0307 X_4 X_i \cos(X_i) \\ 0 = -3.14 X_4 + 7.2 \times 10^{-5} - 0.0307 X_4^2 + 0.0307 X_4 X_3 \cos(X_3) \end{bmatrix} \tag{32}
\]

Mathematical software was used to solve (32) and find the equilibrium points. An equilibrium point in the range of the state variables for \( X_4 \) is shown by (33):

\[
X_0 = \begin{bmatrix} X_{01} & X_{02} & X_{03} & X_{04} & X_{05} \end{bmatrix} = \begin{bmatrix} 0.0019 & 320.18 & 320.18 & 239.7 & 239.6 \end{bmatrix} \tag{33}
\]

The stability of the equilibrium point, \( X_0 \), can be determined via linearization [25, 26] through the Jacobian matrix of \( f(X) \) at the equilibrium point. The values of matrix \( A \) are:

\[
A = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 \\ -153.9 & -32.04 & 0 & -0.145 & -0.0009 \\ 153.9 & 0 & -32.04 & -0.0009 & -0.0162 \\ -2.363 & 0 & 0 & -39.407 & 7.35 \\ -2.363 & 0 & 0 & 7.36 & -39.4 \end{bmatrix} \tag{34}
\]

The respective eigenvalues are:

\[
\lambda_1 = -16.02 + i7 \\
\lambda_2 = -16.02 - i7 \\
\lambda_3 = -32.05 \\
\lambda_4 = -46.8 \\
\lambda_5 = -32.05 \tag{35}
\]

It is shown that all the real part of eigenvalues are negative. Consequently, the equilibrium point is stable for small signals.
4. STABILITY OF LYAPUNOV

Variations in the angle, active power, and frequency of the first generator were analyzed and a negligible variation in the voltage was found. The second generator is an infinite bus with fixed frequency and voltage \((X_1=Y_1, X_2=Y_2, X_3=314 \text{ rad/s}, X_4=Y_3, X_5=230 \text{ v})\). The set of equations for the single generator is given by (36):

\[
\begin{align*}
\dot{Y}_1 &= Y_2 - 314 \\
\dot{Y}_2 &= -31.4V_2 + 9.8 \times 10^{-3} - 2.8 \times 10^{-3}Y_2^2 - 0.62Y_1 \sin(Y_1) \\
\dot{Y}_1 &= -31.4V_1 + 7.2 \times 10^{-3} - 0.0307Y_1^2 + 7.061Y_1 \cos(Y_1)
\end{align*}
\]  

Equation (36) reaches the equilibrium point by the solution to the nonlinear algebraic equation \(f(Y)=0\).

\[
Y_0 = \begin{bmatrix} Y_{01} & Y_{02} & Y_{03} \end{bmatrix} = \begin{bmatrix} -0.01 & 314.1 & 229.9 \end{bmatrix}
\]  

For Lyapunov stability analysis, it is convenient to transfer the stable equilibrium point to the origin by the transformation \(Z=Y-Y_0\). Thus, (36) becomes:

\[
\begin{align*}
\dot{Z}_1 &= Z_2 \\
\dot{Z}_2 &= -1.47 - 31.4Z_2 - 2.8 \times 10^{-3}Z_2^2 - 0.01Z_1 \\
&\quad + 0.006Z_1 \sin(Z_1) - 0.6Z_1 \cos(Z_1) - 140 \sin(Z_1) + 1.47 \cos(Z_1) \\
\dot{Z}_1 &= -1.6 \times 10^{-3} - 13.8Z_1 - 0.032Z_2 - 6.9Z_1 \cos(Z_1) \\
&\quad + 0.07Z_1 \cos(Z_1) - 140 \sin(Z_1) + 1.6 \times 10^{-3} \cos(Z_1) + 16.6 \sin(Z_1)
\end{align*}
\]  

This study assumes that the system is working in the stable equilibrium point and there is a large variation in the angle \(Z_i\). The voltage magnitude is constant. By working with the reduced model, we will have:

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\[
Z_1 = Z_1 \\
Z_2 = -31.4Z_1 - 1.47 - 140\sin(Z_1) + 1.47\cos(Z_1)
\]  

(39)

The phase portrait shown in Figure 3 can also be used to analyze whether all the trajectories between 3 and -3 move towards the equilibrium point at the origin. By assuming that

\[ f(Z_i) = 1.47 + 140\sin(Z_1) - 1.47\cos(Z_1) \]

(40)

Is a variable gradient method for constructing a Lyapunov function [27], [28] and if it is applied to (16), the Lyapunov function is:

\[
V(Z_i, Z_j) = \frac{1}{2} (31.4Z_i + Z_j)^2 + \int_{Z_j}^{Z_i} f(y)dy
\]

(41)

Equation (41) can be transformed into

\[
\dot{V}(Z_i, Z_j) = -31.4Z_i f(Z_i)
\]

(42)

The region of asymptotic stability is shown in Figure 4.

![Phase Portrait](image1)

![Derivative of Lyapunov Function](image2)

Figure 3. Phase portrait of \(Z_1-Z_2\). (\(x=Z_1\), \(y=Z_2\))

Figure 4. Derivate of Lyapunov function

5. SIMULATION RESULTS

The proposed microgrid is simulated in order to show the behavior of the systems, their equilibrium points, and the stability of the microgrid in general when there are possible changes in parameters. Figures 5 to 8 show the state variables of the systems. In this case, it is assumed that all the DG and impedance parameters given in Table 1 are involved. There are no faults during the operation. The simulation is repeated with a different line inductance values (0.15 \(\Omega\), 1 \(\Omega\), and 1.57 \(\Omega\)).

These figures have been carried out for verification of the nonlinear model of studied islanded-inverter-based microgrid system using the proposed generalized modelling approach. The time-domain simulation results have been obtained by solving a nonlinear differential-algebraic system of equations using numerical integration methods. According to Figure 5 when the system begins, the angle between \(V_1\) and \(V_2\) is zero. It has a transient response and it reaches steady point. As the line impedance increases, the overshoot of angle decreases due to the damping of the oscillations.

Figures 9 to 12 show the active and reactive power to each generator with different line inductor values (0.157 \(\Omega\), 1 \(\Omega\), and 1.57 \(\Omega\)). The process by which the stability is affected when the line inductance decreases can be witnessed. According to Figures 9 to 10, it can be said that the active capacities produced by the DG units vary for each unit, because the nominal power of them considered unequal. This confirms that the division of active power between units is proportional to their nominal power. According to Figures 11 to 12 the reactive power produced by each DG unit is proportional to its nominal power, because of existence the high resistance line in a low or medium voltage microgrid leads to a current.
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Power sharing can be modified by choosing different droop controller gains. The eigenvalues analysis, on the other hand, predicts poor damping for huge droop controller gains. Figures 13 to 16 show the variations in the P and Q power to each generator when the $K_p$ coefficient is increased.

![Figure 13. Active power of DG1 when the $K_p$ coefficient is increased](image1)

![Figure 14. Active power of DG2 when the $K_p$ coefficient is increased](image2)

![Figure 15. Reactive power of DG1 when the $K_p$ coefficient is increased](image3)

![Figure 16. Reactive power of DG2 when the $K_p$ coefficient is increased](image4)

Figure 17 show large disturbance in $V_2$. This disturbance occurs at time 2 (second) and it is 0.2 second long. The amplitude of disturbances is 20 v. The microgrid is recovered after that. Figures 18 to 23 show other parameters when disturbance occurs in $V_2$. According to Figure 18 voltage of DG1 unit is almost constant. Figure 19 illustrate that during the time of disturbance, the angle between $V_1$ and $V_2$ oscillates due to changes in the voltage of the second bus. But the microgrid is recovered after the disturbance. According to Figures 20 and 21, it is observed that in the presence of voltage sag, the active power of both DG units are changed. But according to Figures 22 and 23, reactive power of DG2 reduced more than DG1, which indicates a direct dependence between voltage and reactive power.

![Figure 17. Voltage of bus2 when the disturbance occurs at $V_2$](image5)

![Figure 18. Voltage of bus1 when the disturbance occurs at $V_2$](image6)
6. CONCLUSION

In this paper, a nonlinear state-space model of a microgrid was presented. The model included the most important dynamics. This modeling method could be extended to n-generators. The model was analyzed by means of both stability study of Lyapunov function and root locus plot. A general methodology to find a valid Lyapunov function for nonlinear stability analysis was presented. By using that Lyapunov function, the region of asymptotic stability can be determined. These tools will allow designing microgrid systems with loads, generators, and storage systems, hence global stability of the system.

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