EVOLUTION OF DARK MATTER PHASE-SPACE DENSITY DISTRIBUTIONS IN EQUAL-MASS HALO MERGERS

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ABSTRACT

We use dissipationless $N$-body simulations to investigate the evolution of the true coarse-grained phase-space density distribution $f(x,v)$ in equal-mass mergers between dark matter (DM) halos. The halo models are constructed with various asymptotic power-law indices $\rho \propto r^{-\gamma}$ ranging from steep cusps to core-like profiles and we employ the phase-space density estimator "Enbid" developed by Sharma & Steinmetz to compute $f(x,v)$. The adopted force resolution allows robust phase-space density profile estimates in the inner $\sim 1\%$ of the virial radii of the simulated systems. We confirm that merger events result in a decrease of the coarse-grained phase-space density in accordance with expectations from Mixing Theorems for collisionless systems. We demonstrate that binary mergers between identical DM halos produce remnants that retain excellent memories of the inner slopes and overall shapes of the phase-space density distribution of their progenitors. The robustness of the phase-space density profiles holds for a range of orbital energies, and a variety of encounter configurations including sequences of several consecutive merger events, designed to mimic hierarchical merging, and collisions occurring at different cosmological epochs. If the progenitor halos are constructed with appreciably different asymptotic power-law indices, we find that the inner slope and overall shape of the phase-space density distribution of the remnant are substantially closer to that of the initial system with the steepest central density cusp. These results explicitly demonstrate that mixing is incomplete in equal-mass mergers between DM halos, as it does not erase memory of the progenitor properties. Our results also confirm the recent analytical predictions of Dehnen (2005) regarding the preservation of merging self-gravitating central density cusps.

Subject headings: cosmology: theory — dark matter — halos — halos: structure — halos: phase-space density profiles — halos: evolution — methods: numerical

1. INTRODUCTION

In the standard hierarchical cold dark matter (CDM) structure formation paradigm (e.g., Blumenthal et al. 1984), galaxy assembly commences with the gravitational collapse of overdense regions into halos of dark matter (DM). Bound in the potential wells established by these halos, baryons cool, condense, and form galaxies with a variety of properties (e.g., White & Rees 1978). An essential step toward gaining insight into the physical processes of galaxy formation involves understanding the origin and evolution of halo structural properties.

During the past two decades, numerical simulations of structure formation set within the CDM paradigm have revealed two intriguing facts which highlight the simplicity of the equilibrium structure of DM halos. First, their spherically-averaged density profiles, $\rho(r)$, are well described by simple two- or three-parameter profiles with the slope steadily becoming shallower with decreasing radius (e.g., Dubinski & Carlberg 1991; Navarro et al. 1994, 1995, 1996; Moore et al. 1999; Klypin et al. 2001; Power et al. 2003; Diemand et al. 2005; Merritt et al. 2006; Stadel et al. 2008; Navarro et al. 2008). Second, the spherically-averaged quantity $Q(r) = \rho(r)/\sigma^3(r)$, where $\sigma(r)$ denotes the velocity dispersion of DM particles, and which has dimensions of phase-space density, displays an approximately power-law behavior, $Q \propto r^{-\beta}$ with $\beta \simeq 1.9$, over a large range of radii (Taylor & Navarro 2001; Ascasibar et al. 2003; Hoffman et al. 2007; Stadel et al. 2008; Navarro et al. 2008).

Establishing the degree to which the hierarchical assembly of structure prescribed by the prevailing CDM paradigm affects the evolution of halo properties can have profound implications for our current understanding of galaxy formation and evolution. Cosmological $N$-body simulations constitute a powerful tool in gaining insight into the origin of the phase-space density distributions of DM halos (e.g., Taylor & Navarro 2001; Ascasibar et al. 2003; Hoffman et al. 2007; Wang & White 2008; Knollmann et al. 2008; Vass et al. 2008). However, the complexity of halo formation in a cosmological context, with continuous mergers, smooth accretion, and rapidly changing potential wells, poses difficulties in isolating the mechanisms that drive the evolution of halo phase-space density profiles.

In addition, while numerous theoretical studies have considered the effect of mergers on the evolution of configuration-space density profiles of DM halos using controlled numerical experiments (e.g., White 1978; Sver & White 1998; Barnes 1999; Boylan-Kolchin & Ma 2004; Moore et al. 2004; Kazantzidis et al. 2006), comparatively much less attention has been devoted to investigating the evolution of the corresponding phase-space density distributions during merger events. Given these facts, we are motivated to examine for the first time the effect of equal-mass mergers on the evolution of phase-space density distributions of DM halos using a large ensemble of dissipationless, controlled $N$-body simulations. Our simulation set is carefully...
designed to permit an investigation of a large parameter space and our primary goal is to establish conclusively the degree to which the phase-space density distributions of merger remnants are related to those of their progenitors over a wide range of initial conditions for the internal structure of the progenitor halos and encounter configurations.

Our choice to focus on equal-mass mergers is motivated by the fact that they constitute the most violent events in the formation history of a cosmological structure. Thus, they are likely to result in the greatest amount of mixing in phase-space and, consequently, to be associated with substantial evolution of the phase-space density distribution. For our numerical experiments, we construct self-consistent realizations of DM halo models at different cosmic epochs with various asymptotic power-law indices ranging from steep cusps to core-like profiles. We also consider a wide variety of orbital energies and encounter configurations, including binary mergers and sequences of consecutive merger events in an attempt to mimic the hierarchical mass assembly that characterizes CDM-like cosmological models.

As we illustrate below, the phase-space density profiles of DM halos are remarkably robust during dissipationless merging and the overall shapes of remnant phase-space density profiles retain a faithful memory of those of their progenitors, despite the relaxation that accompanies merger activity. Specifically, in mergers between identical systems, the inner slopes and overall shapes of the remnant phase-space density distributions are virtually identical to those of the initial systems. On the other hand, if the progenitor DM halos are constructed with appreciably different asymptotic power-law indices, the remnant phase-space density profiles are indistinguishable from those of the initial system with the steepest cusp. These conclusions hold regardless of the number of mergers and initial conditions associated with the encounters and confirm the predictions of analytical studies pertaining to the evolution of phase-space density and the robustness of central density cusps (Dehnen 2005).

The outline of this paper is as follows. In Section 2 we describe the halo models and numerical simulations used in this study. In Section 3 we provide a brief introduction to the theoretical understanding of mixing in phase-space and describe various methods used to compute phase-space densities. Section 4 contains our results regarding the evolution of the phase-space density distribution of DM in equal-mass halo mergers. Finally, Section 5 summarizes our main results and concludes.

2. NUMERICAL METHODS

The primary goal of the present study is to examine the evolution of the coarse-grained phase-space density distribution during dissipationless, equal-mass mergers between DM halos. For this purpose, we analyze a subset of the $N$-body merger simulations of Kazantzidis et al. (2006). In what follows, we provide a brief description of their methods and experiments and refer the reader to Kazantzidis et al. (2006) for more details.

2.1. Halo Models

The initial halo models followed density profiles that are described by the general $(\alpha, \beta, \gamma)$ spherical density law (Eq. 1) e.g., Zhao (1996), where $\gamma$ denotes the asymptotic inner slope of the profile, $\beta$ corresponds to the outer slope, and $\alpha$ determines the sharpness of the transition between the inner and outer profile,

$$\rho(r) = \frac{\rho_{\star}}{(r/r_{\star})^{2} + (r/r_{\star})^{\beta - \gamma}} \quad (r \leq r_{\text{vir}}).$$

The normalization of the density profile is set by $\rho_{\star}$, while $r_{\star}$ corresponds to the scale radius. The virial radius, $r_{\text{vir}}$, is defined as the radius enclosing an average density equal to the virial overdensity, $\Delta_{\text{vir}}$, times the critical density for a flat universe, $\rho_{\text{crit}}$. For a flat CDM cosmological model ($\Omega_{m} = 0.3, \Omega_{\Lambda} = 0.7, h = 0.7$) and a redshift $z = 0$, $\Delta_{\text{vir}} \approx 103.5$. Kazantzidis et al. (2006) considered three initial halo models specified by particular choices of the parameters $\alpha, \beta,$ and $\gamma$ in Eq. 1. The first model followed the Navarro et al. (1996, hereafter NFW) profile with $[\alpha, \beta, \gamma] = [1, 3, 1]$, while the second and third models correspond to profiles with steeper $([\alpha, \beta, \gamma] = [2, 3, 1.7])$ and shallower $([\alpha, \beta, \gamma] = [2, 3, 0.2])$ inner slopes. This choice of parameters ensures that the initial halo phase-space density distributions have significantly different shapes, a fact which will be crucial for the interpretation of the results.

Due to the fact that density profiles with outer slopes of $\beta = -3$ correspond to divergent cumulative mass distributions, halo profiles were truncated beyond $r_{\text{vir}}$ with an exponential law (Kazantzidis et al. 2004a). Lastly, the $N$-body halo models used in the simulations of Kazantzidis et al. (2006) were constructed from the exact phase-space distribution function (DF) under the assumptions of spherical symmetry and an isotropic velocity dispersion tensor (Kazantzidis et al. 2004a).

2.2. Description of Merger Simulations

In this paper we analyze several of the $N$-body merger simulations described in Kazantzidis et al. (2006). These simulations were conducted with the multi-stepping, parallel, tree $N$-body code PKDGRAV (Stadel 2001). The first set of experiments comprised binary, parabolic encounters both between identical progenitors and between systems with different values of the asymptotic density power-law index $\gamma$ (“hybrid” mergers). Specifically, we considered the parabolic collisions between: (a) two NFW halos, (b) two halos with inner density slopes of $\gamma = 0.2$, and (c) one halo with an inner slope of $\gamma = 1.7$ and one with $\gamma = 0.2$. The analysis of these experiments will aid in investigating the effect of the asymptotic inner slope of the density profile of the merging halos, $\gamma$, on the evolution of the phase-space distribution.

In a second set of experiments, we considered the merger of two NFW halos on a (a) circular orbit and (b) radial orbit with zero orbital angular momenta (Moore et al. 2004). These experiments will provide insight into the effect of the orbital energy of the encounter on the evolution of the phase-space density distribution.

The initial DM halos were constructed at $z = 0$ with a virial mass of $M_{\text{vir}} = 10^{13}M_{\odot}$ and a concentration of $c \equiv r_{\text{vir}}/r_{\star} = 12$ (Bullock et al. 2001). This choice results in a virial radius of $r_{\text{vir}} \approx 256.7$ kpc and a scale radius of $r_{\star} \approx 21.4$ kpc.

The binary mergers were initialized by generating pairs of halo models and placing them at a distance equal to twice their virial radii. The initial models were set on orbits with pericentric distances of $20\%$ of the halo virial radii. We note that this

$^6$ Note that the value of $M_{\text{vir}}$ serves merely practical purposes and does not imply anything special about the particular choice of mass scale. Because we do not consider non-gravitational processes such as gaseous dissipation, the scale-free nature of gravity allows the rescaling of these models to any system of units and hence the extension of our conclusions to mergers between equal-mass systems of any mass scale.
particular orbital configuration is the most typical of merging halos in cosmological simulations (e.g., [Khochof & Burkert 2006]. The initial center of mass velocity of each pair was determined from the corresponding Keplerian orbit of two point masses. In the case of the radial mergers, the initial relative velocities were set to be equal in magnitude to the velocities of point particles in a circular orbit about the common center of mass. In this way, both the radial and circular mergers had precisely the same total orbital energy.

Besides focusing on binary encounters, one of our additional goals is to elucidate the net effect of a hierarchy of mergers on the evolution of the phase-space distribution of DM. Such a set-up is a more appropriate characterization of the mass assembly in CDM-like cosmological models. To this end, we analyzed a sequence of binary, parabolic mergers between NFW halos. This sequence of mergers consisted of two simulations. The first stage was simply the binary merger between the initial halo models described above. In the second stage, identical copies of the remnants from the first stage were merged after randomly changing the initial relative orientation of their principal axes.

To assess the generality of our conclusions for earlier cosmological epochs when densities are higher and consequently orbital times are shorter, we also analyzed a sequence of parabolic encounters occurring at redshift \( z = 3 \), rather than at \( z = 0 \). The initial systems followed the steep density profile with \( \gamma = 1.7 \) and had the same \( M_{\text{vir}} \) as the standard halo models. Their concentration parameter was scaled according to \( c \propto (1+z)^{-1} \) which is valid for fixed mass objects (e.g., [Bullock et al. 2001]). This modeling results in substantially denser initial systems compared to those at \( z = 0 \) with \( r_{\text{vir}} \approx 79.5 \) kpc. This sequence of mergers consisted of three simulations. As before, the first stage of the merging hierarchy was the binary encounter between the initial halo models. In the second and third stage, identical copies of the remnants from the previous stage were merged according to the scheme described above.

Lastly, all initial halo models were sampled with \( N = 2 \times 10^5 \) particles. For mergers occurring at \( z = 0 \), forces were softened with a spline gravitational softening length equal to \( \epsilon = 1.5 \) kpc. For encounters occurring at \( z = 3 \), the softening lengths were scaled correspondingly to allow for the same fraction of the virial radius of the simulated systems (\( \sim 1\% \)) to be resolved. Extensive convergence tests carried out by Kazantzidis et al. (2006) indicate that numerical convergence has been achieved with this choice of numerical parameters. Table I contains a summary of all merger simulations analyzed in this study.

3. MIXING IN PHASE-SPACE AND PHASE-SPACE DENSITY ESTIMATORS

The fundamental quantity that characterizes the state of a collisionless dynamical system is the six-dimensional (6D) phase-space density \( f(x,v,t) \), which is also known as the “fine-grained” phase-space density DF. Indeed, at any given time \( t \), a collisionless system can be completely described by specifying the mass \( f(x,v,t)dv \) contained in an infinitesimal phase-space volume element \( dv \) centered on \( (x,v) \). The evolution of \( f(x,v,t) \) is governed by the collisionless Boltzmann equation (CBE) [sometimes also called the Vlasov equation] which constitutes a continuity equation in phase-space

\[
\frac{\partial f}{\partial t} + v \cdot \nabla f - \nabla \Phi \cdot \frac{\partial f}{\partial v} = 0 , \tag{2}
\]

where \( \Phi \) is the gravitational potential. An immediate consequence of the CBE is that the fine-grained DF \( f(x,v,t) \) is conserved during the evolution of collisionless systems. In addition, the volume of phase-space occupied by phase-space elements whose density lies in the range \( (f, f+df) \), \( V(f)df \), is also conserved.

However, mixing in phase-space during the dynamical evolution of collisionless systems in non-equilibrium situations (e.g., galaxy mergers, collapse) renders the measurement of \( f(x,v,t) \) extremely intricate (see, however, [Vogelsberger et al. 2008]). This is because the mixing of high and low phase-space density material in the course of dynamical evolution causes the phase-space elements to become increasingly thinner. To overcome this difficulty, one typically computes the average of \( f \) over some finite volume of phase-space, known as the “coarse-grained” DF, \( \bar{f}(x,v) \), and the associated volume DF denoted by \( V(f) \). The present study also focuses on the evolution of the coarse-grained phase-space density \( \bar{f} \) and does not consider the behavior of its fine-grained counterpart, \( f \). For this reason, we simplify our notation and use \( f \) in the remainder of the paper to represent the former quantity.

Although the coarse-grained DF does not obey the CBE, a theoretical framework does exist that aids in the understanding of its evolution. Indeed, the Mixing Theorems (Tremaine et al. 1986, Binney & Tremaine 1987, Mathur 1988) postulate that processes operating during the relaxation of collisionless systems (e.g., phase mixing, chaotic mixing, and the mixing of energy and angular momentum that accompanies violent relaxation) result in a decrease of the coarse-grained phase-space density. This is because in the course of dynamical evolution towards equilibrium high and low phase-space density material gets mixed in any region of phase-space. This mixing reduces \( f \) such that, at any given time, \( f < f_{\text{max}} \), where \( f_{\text{max}} \) denotes the initial, maximum phase-space density.
space density.

It is worth noting that Dehnen (2005) has recently proved another form of the Mixing Theorem in terms of a new concept, the excess-mass function, \( D(f) \). This quantity is defined as the mass of material with coarse-grained phase-space density greater than \( f \) and it is additive so that the excess mass of a combination of non-overlapping systems is the sum of their individual \( D(f) \). Dehnen (2005) showed that the mixing of phase-space elements changes \( f \) in such a way that the excess-mass function decreases for all values of \( f \) and, among other things, applied this new form of the Mixing Theorem to the merging of self-gravitating density cusps. We emphasize that a comparison between the predictions of Dehnen (2005) for the evolution of power-law phase-space density profiles during mergers of collisionless systems and the results of our numerical simulations will be presented in Section 5.

In recent years, three independent numerical codes have been developed for computing the 6D coarse-grained phase-space density of a discrete set of \( N \) sample points (Arad et al. 2004; Ascasibar & Binney 2005; Sharma & Steinmetz 2006). While all of these methods rely on tessellation schemes to define phase-space volumes within which the phase-space density is computed, they differ in the scheme used to tessellate the 6D phase-space as well as in the adopted phase-space density estimators. Arad et al. (2004) uses a “Delaunay tessellation field estimator” which computes the phase-space densities of a set of points from the volume of the Delaunay cells to which they belong. In a space of \( d \) dimensions, a “Delaunay cell” is defined as a \( d \)-dimensional polyhedron made by connecting every set of \( d+1 \) points, such that a \((d-1)\)-dimensional sphere passes through all of them but does not encompass any other point from the sample. The main criticism regarding the Delaunay tessellation scheme is that the phase-space volumes, and thus, the phase-space density estimates derived, are not metric-free.

Ascasibar & Binney (2005) and Sharma & Steinmetz (2006) developed the algorithms “FiEstAS” and “EnBiD”, respectively, which are both metric-free and very fast. FiEstAS is based on a repeated division of each dimension of phase-space into two regions that contain roughly equal numbers of particles. The process of subdivision continues until each volume element contains only one particle. The dimension to be divided is chosen either randomly or by providing an equal number of divisions in each dimension. The highest resolution is obtained for the dimension with the largest number of tessellations (or subdivisions). EnBiD closely follows the FiEstAS algorithm, but instead the phase-space is tessellated along the dimension having minimum entropy. This scheme optimizes the number of divisions to be made in a particular dimension and extracts maximum information from the data. Due to the fact that it employs a minimum entropy criterion, EnBiD is more accurate at measuring high values of phase-space density compared to FiEstAS.

In what follows, we compare the accuracy of the EnBiD and FiEstAS codes in estimating coarse-grained phase-space densities. The comparison was performed by applying both algorithms on a particle realization with a given DF. Specifically, we employed the spherical and isotropic NFW DM halo constructed at \( z = 0 \) with a virial mass of \( M_{\text{vir}} = 10^{12} \odot \) and sampled with \( N = 2 \times 10^5 \) particles for the comparison. Results are presented as a function of radius in units of the virial radius, \( r_{\text{vir}} \). Top: Median values, \( F \), of coarse-grained phase-space densities. The solid line corresponds to the theoretical phase-space density, while the remaining curves show results using the FiEstAS code (dotted), the EnBiD code with no smoothing (dot-dashed), the EnBiD code with FiEstAS smoothing (long-dashed), and the EnBiD code with a \( n = 10 \) kernel smoothing (dashed-triple-dotted). Bottom: Ratio of numerically estimated to the theoretical median phase-space density \( F \). EnBiD resolves the high phase-space density regions better than does FiEstAS. Employing kernel smoothing in EnBiD results in higher accuracy in the estimation of the coarse-grained phase-space density.

will show later (Figure 4) there is a large range of \( f \) values at any given radius \( r \) in the halo. We chose to show the median values of \( f \), hereafter represented by \( F(r) \), at any radius \( r \) rather than the mean of \( f \), since it is less sensitive to the extremes of the distribution of \( f \) in each bin. It has been demonstrated that this choice is particularly important in cosmological DM halos where the presence of subhalos and streams tends to significantly affect the mean of the distribution of \( f \) (Vass et al. 2008). We note that the numerically estimated values of \( F \) differ not only in the algorithms employed (EnBiD versus FiEstAS), but also in the number of smoothing neighbors around each point used to reduce the noise in the phase-space density estimate.

The results presented in Figure 1 demonstrate that EnBiD overall resolves the high phase-space density regions more accurately compared to FiEstAS. Specifically, EnBiD with a smoothing kernel which includes 10 nearest neighbors about each point (\( n = 10 \)) recovers the analytic phase-space density profile in the density cusp more than a factor of \( \sim 4 \) bet-
ter compared to FiEstAS. These findings confirm the results of Sharma & Steinmetz (2006) who conducted similar analysis for a $N = 10^6$ particle spherical and isotropic Hernquist particle realization (Hernquist 1990) with an analytic DF. In the rest of this paper we present all results pertaining to the coarse-grained phase-space density using EnBiD with a $n = 10$ kernel smoothing.

Lastly, the simplest method for estimating the phase-space density of a cosmological structure is undoubtedly through the proxy $Q(r) = \rho(r)/\sigma^2(r)$ (Taylor & Navarro 2001). It is important to emphasize that although $Q(r)$ has dimensions of phase-space density, it is not a true coarse-grained phase-space density that obeys any of the Mixing Theorems (Dehnen & McLaughlin 2005). In fact it has been shown that $Q(r)$ is closely related to the inverse of the global entropy distribution of DM particles and that its behavior is similar to that of the entropy distributions of hot gas in clusters of galaxies (Hoffman et al. 2007; Vass et al. 2008).

The main criticism for using $Q(r)$ as a measure of phase-space density is related to the fact that it is obtained through the computation of spherically-averaged density and velocity dispersion profiles in large arbitrary volumes in configuration-space, rather than in phase-space. Nevertheless, the universality of $Q(r)$ is intriguing since there is no a priori reason to expect that $\rho(r)$ and $\sigma(r)$ should change in such a way as to preserve their ratio over more than 2.5 orders of magnitude in radius, regardless of mass and background cosmology (Taylor & Navarro 2001; Ascasibar et al. 2004; Rasia et al. 2004; Hoffman et al. 2007). Given this fact and also because $Q(r)$ is widely used in the literature, we will present results for its evolution as well.

Figure 2 shows results for the previously described NFW test halo. It shows the spherically-averaged quantity $Q(r)$ and the median phase-space density $F(r)$ obtained by EnBiD in spherical bins along with power-law fits to both profiles and the associated residuals. Results are presented after the system was evolved in isolation for 15 Gyr. We have explicitly checked that there is no discernible evolution in the $Q(r)$ and $F(r)$ profiles throughout the evolution in isolation confirming further the self-consistency of the initial model. This simulation serves as an important control experiment with which to compare the evolution in the merger experiments. Figure 2 illustrates that both $Q(r)$ and $F(r)$ are well described by power-laws over a significant radial range. Specifically, the power-law fits were performed over the radial range $0.01 - 0.7 r_{vir}$, and correspond to $Q \propto r^{-1.88 \pm 0.010}$ and $F \propto r^{-1.92 \pm 0.009}$. A bootstrapping technique was used to determine the errors in the fitting parameters. The results of the power-law fits suggest that $Q(r)$ and $F(r)$ have very similar slopes. We also note that $Q(r)$ and $F(r)$ differ quite significantly in amplitude at any radius. This is only because the standard definition of $Q(r)$ is only dimensionally motivated and does not specify the shape of volume elements in phase-space.

4. RESULTS

4.1. Binary Halo Mergers

We begin with the binary merger simulations. In all parabolic and radial mergers, the systems merge in three to five pericenter passages (between $\sim 5.5$ to $\sim 7$ Gyr), with the specific value depending on the inner power-law indices of the systems and the initial orbital configurations of the mergers. These timescales are larger than those reported in earlier studies of binary equal-mass mergers (e.g., Barnes & Hernquist 1996) due to the larger and more realistic pericentric distances adopted here. The merger timescale is set by the combination of dynamical friction, which dissipates the orbital energy of the dense halo cores, and mass loss processes (e.g., Taffoni et al. 2003). Models with steeper inner density distributions merge faster due to the stronger gravitational drag exerted by the galaxy cores. The differences in merging times can be of the order of $\sim 1$ Gyr or even larger between shallow and steep density profiles. Finally, for circular orbits the merger process takes substantially longer time to complete $\sim 25$ Gyr owing to the much slower rate at which orbital energy is dissipated.

The first results that we present are those of the binary parabolic merger involving identical NFW halo models (run Bp1). Figure 3 compares the evolution of $F(r)$ and $Q(r)$ dur-
Fig. 3.— Evolution of $F(r)$ and $Q(r)$ profiles in the binary parabolic merger involving identical NFW halo models (run Bp1). Results are presented for one of the progenitor halos as a function of distance from its most bound particle. In the first three panels, radii are scaled to the virial radius of the progenitors, while in the last three panels radii are normalized to the virial radius of the remnant. The initial power-law halo phase-space density profiles $Q(r)$ and $F(r)$ are well-preserved during the merger for $r \lesssim 0.7r_{\text{vir}}$. Both $F(r)$ and $Q(r)$ flatten out and deviate from these simple power-laws at larger radii.

Due to the fact that the initial and final virial radii differ by less than 15% (Table I), we do not expect any of our conclusions to be affected by this choice. The comparison between the initial and final distributions of $F(r)$ shows that the coarse-grained phase-space density decreases during the merger. This behavior is entirely consistent with expectations.
from the Mixing Theorems.

In addition, as the merger progresses, both $Q(r)$ and $F(r)$ extend to substantially larger radii. This is because material is ejected during the collision (Kazantzidis et al. 2006).
Indeed, the merger remnant contains significant fraction of its bound mass, ∼ 40%, outside of what we would formally identify as the virial radius. Interestingly, at times between ∼ 2 and ∼ 3 Gyr, we report a temporary shrinkage with radius and a large associated fluctuation in both quantities. These times are important as they correspond to the first pericentric and the first apocentric passage, respectively.

For $r \lesssim 0.7r_{\text{vir}}$, both $Q(r)$ and $F(r)$ remain roughly power-laws throughout the evolution ($Q(r) \propto r^{-1.88}$ and $F(r) \propto r^{-1.92}$). This finding indicates that the power-law halo phase-space density profiles of DM halos are well-preserved during the hierarchical assembly of larger systems, at the very least during periods of equal-mass mergers, despite the relaxation that accompanies merger activity. This is intriguing as there is no a priori reason to expect that $\beta(r)$ and $\sigma(r)$ should change during the merger in such a way as to preserve $Q(r)$ over ∼ 2.5 orders of magnitude in radius. In fact, Kazantzidis et al. (2006) have demonstrated that both configuration and velocity space distributions are altered during the same merger events analyzed in the present study.

Note that while it is customary to present density and phase-space density profiles only for material within one virial radius from the center of the halo, it has been shown recently that a better outer radius for DM halos is the so-called "static radius", defined as the radius at which the mean radial velocity of particles is zero (Cuesta et al. 2008). This radius is typically about $2r_{\text{vir}}$ for a galaxy-sized DM halo. For the previously described merger experiment, we find that ∼ 40% of the initial total mass lies beyond the virial radius, $r_{\text{vir}}$, of the remnant and ∼ 20% lies beyond $2r_{\text{vir}}$. Motivated by these findings, in the rest of this paper and unless otherwise explicitly stated we present results for particles at all radii.

It is also worth pointing out the presence of wave-like features in both $F(r)$ and $Q(r)$ profiles seen at distances $r \gtrsim 0.2r_{\text{vir}}$ (over and above the deviation from power-law seen in the initial profiles). These features are clearly noticeable both in the late stages of the merger and after the collision is deemed complete ($t \gtrsim 5.5$ Gyr). They are associated with "shells" of particles left over after the merger and persist for several billion years of evolution subsequent to the completion of the collision. Therefore, even though the progenitor halos were smooth and structureless in phase-space, the phase-space density profiles of the remnants are not completely smooth. Similar conclusions were reached by Kazantzidis et al. (2006) who reported the presence of wave-like features in the configuration-space density profiles of the remnants at the same distances.

Besides knowing the median value of $f$ at a given radius, it is useful to examine how the values of $f$ are distributed at each radius. This analysis is presented in Figure 3. This figure shows color-coded isodensity contours representing the number density of particles in the log($f$) versus log($r$) plane. Results are presented for all particles in one of the progenitor NFW halos. The yellow/orange contours correspond to the highest number density of particles, while the blue/black contours correspond to the lowest number density of particles. Contours are spaced at logarithmic density intervals relative to the maximum density contour. The white solid lines in each panel show the median $F(r)$ presented in Figure 3.

At any radius from the center there exist particles with phase-space densities spanning between 2–4 decades in $f$. The presence of islands of high particle number density (yellow/orange peaks) at different radii is clearly noticeable at late times (8 and 10 Gyr), suggesting that the system is not yet completely mixed. These together with the wiggles seen in the outer contours at large radii illustrate again the formation of phase-space shells due to the ejection of particles.

Comparing this figure with the corresponding one for cosmological DM halos presented in Vass et al. (2008) (see their Figure 5) illustrates the main difference between these merger remnants and cosmological halos. This difference lies in the presence of rich amounts of substructure in phase-space that is not present in the former. Halo substructure results in a much wider range of $f$ at a given radius $r$ (the scatter around the median in merger remnants is relatively small compared to cosmological halos) and in stronger deviation from power-law of $F(r)$ at large radii.

Following up on the findings reported in Figs. 2 and 4, Figure 5 compares the initial and final distributions of $F(r)$ in three characteristic parabolic mergers between: (a) two NFW halos (run Bp1) (b) two halos with inner density slopes of $\gamma = 0.2$ (run Bp2), and (c) one halo with an inner slope of $\gamma = 1.7$ and one with $\gamma = 0.2$ (run hBp). The initial distributions correspond to one of the progenitor halos in mergers between halos with identical density cusps $\gamma$ except in mergers between systems with different values of $\gamma$. In the latter case, we show results for both progenitors. The final distributions correspond to the merger remnants and are presented at a time when the merger would conventionally be deemed complete ($t \approx 5.5$ Gyr).

In agreement with the Mixing Theorems, this figure shows that in all cases the coarse-grained phase-space density decreases during the merger process. Moreover, in binary mergers between identical DM halos both the inner power-law index and the overall shape of the remnant phase-space density distribution at radii $r \lesssim r_{\text{vir}}$ are strikingly similar to those of the progenitor systems. On the other hand, if the progenitor halos are constructed with appreciably different asymptotic power-law indices, the inner slope and overall shape of the phase-space density distribution of the remnant is substantially closer to that of the steepest of the initial systems. These results extend previous findings regarding the evolution of configuration-space density profiles of merging DM halos (e.g., Boylan-Kolchin & Ma 2004; Kazantzidis et al. 2006).

Although it is instructive to plot the median values of $f$ at each radius, much more information is contained in the full coarse-grained phase-space density. The six-dimensional function $f(x, v)$ can be most easily visualized using the volume DF $V(f)$ which also obeys a Mixing Theorem (Mathur 1988). $V(f)$ is formally defined as

$$V(f = f_0) = \int \delta(f(x, v, t) - f_0) \, dx \, dv,$$

(3)

the volume of phase-space occupied by phase-space elements whose density lies in the range $(f, f + df)$. Arad et al. (2004) showed that at $z = 0$, $V(f)$ for cosmological DM halos follows a power-law profile $V(f) \propto f^{-\alpha}$ with power-law index $\alpha \approx 2.3$ over 4 orders of magnitude in $f$. It is instructive to investigate the evolution of the coarse-grained volume element of phase-space $V(f)$ using our merger simulations.

Numerically, $V(f)$ is evaluated by binning the particles in logarithmically-spaced bins of $f$. If $n_{\text{bin}}$ is the total number of particles in the $i^{th}$ bin (the model assumes equal mass particles), the density of the bin is given by

$$V(f_{\text{bin}}) = n_{\text{bin}} / f_{\text{bin}} (f_{i+1} - f_i),$$

$$f_{\text{bin}} = (f_{i+1} + f_i)/2$$

(4)
Evolution of Dark Matter Phase-Space Density Distributions in Equal-Mass Halo Mergers

Figure 5.— Phase-space density distributions $F(r)$ for initial systems and remnants in binary mergers between DM halos (runs Bp1, Bp2, and hBp). The profiles are plotted as a function of radius in units of the virial radius of the progenitors, $r_{\text{vir}}$, and the asymptotic density power-law indices, $\gamma$, of the initial systems are indicated in each panel. In mergers between identical halos, the central slope and overall shape of the remnant phase-space density distribution are remarkably similar to those of the progenitors. If the progenitor halos are constructed with appreciably different asymptotic power-law indices, the remnant phase-space density distribution constitutes a reflection of that of the steepest of the initial systems.

Figure 6.— Evolution of the volume DF $V(f)$ (see Equation 3), for the binary parabolic merger involving identical NFW halo models (run Bp1). Results are presented for all particles in both the progenitor halos. A power-law fit $V(f) \propto f^{-2.50 \pm 0.016}$ is included for both the initial system ($t = 0$ Gyr) and the remnant after the collision is deemed complete ($t \gtrsim 5.5$ Gyr). Results are presented for all particles in both the progenitor halos. A power-law fit $V(f) \propto f^{-2.50 \pm 0.016}$ is included at $t = 0, 6, 8, 10$ Gyr for $f$ values in the range of $[10^{-3.4}, 10^{-0.2}]$. $V(f)$ profiles are notably robust to equal-mass mergers.

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not demonstrated explicitly in Figure 6 $V(f)$ for the isolated halo does not evolve with time because there is negligible mixing in this case.

The volume of phase-space density $V(f)$ associated with low phase-space density $f$ increases after the merger is completed. However, we note that the $V(f)$ associated with higher phase-space density values exhibits very little change during the collision. The increase in volume of phase-space at low phase-space densities is predicted to be a consequence of the Mixing Theorems. Nonetheless, the preservation of the power-law distribution of $V(f)$ over three orders of magnitude in $f$ during the evolution is intriguing. The conservation of the $V(f)$ profile shape over the range $10^{-4} < f < 1$ appears to be a consequence of the robustness of the phase-space density profile.

The phase-space DF’s of collisionless stellar systems are expected to be functions of three isolating integrals of motion. For non-spherical systems, these are the energy $E$ and two quantities resembling the angular momentum $J$. In general it is intricate to find three global integrals of motion, and thus it is common to define DF’s in the form $f(E,J)$. Recently, Wojtak et al. (2008) studied the DF’s of cosmological DM halos in the mass range $10^{14} - 10^{15} M_\odot$. They also derived a analytic DF $f(E,J)$, which is dependent on three parameters that describe the anisotropy profile. They showed that their derived DF was a good match to the DF of cosm-
affected by the presence of the second halo. This panel reveals that particles with the highest phase-space density have the most negative energies. It also demonstrates that the larger the energy $E$ of the DM particles, the greater their spread in $J$ and the lower their phase-space density. In addition, at each energy particles are uniformly distributed in angular momenta since the initial DF is designed to have an isotropic velocity dispersion profile. In fact, it is the isotropic velocity distribution that gives rise to the uniformly colored vertical bands. This initial distribution arises because the initial NFW halos were constructed to have DF’s dependent solely upon the energy, $f = f(E)$.

The second panel of Figure 8 shows again the $f(E, J)$ contours that correspond to the remnant of the parabolic merger between NFW halos (run Bp1). This distribution, while not strictly $f(E)$, is qualitatively similar to the initial $f(E)$ distribution for the isolated halo. The principal differences seen are that the former extends over a much larger range in both energy and angular momentum. The most tightly bound particles with the highest phase-space density have more negative energies compared to the most tightly bound particles in the isolated NFW halo. This simply reflects the increase of the depth of the potential well in the remnant. Moreover, the absolute range of $J$ for the least bound particles is significantly larger than in the initial halo.

However, we note that while the final remnant has a much larger volume of phase-space occupied by low phase-space density material, most of this material lies beyond the virial radius ($E > -1$). For $E < -1$, the bands of constant phase-space density are no longer vertical as in the initial distribution. Instead, at a given energy $E$ they curve to the left toward higher $J$ values, indicating that particle maximum angular momentum at a given energy $E$ is lower than in the isolated case. Such curvature in the bands of constant $J$ towards lower $E$ therefore signify increase in radial anisotropy. Indeed, although the initial progenitor models are fully isotropic (i.e., $\beta = 1 - \sigma_t^2/2\sigma_r^2 = 0$ at all radii, where $\sigma_t$ and $\sigma_r$ are the tangential and radial velocity dispersion, respectively), merger remnants exhibit anisotropy that changes from mildly radial with $\beta \approx 0.2$ at small radii ($r/r_{\text{vir}} < 0.1$) to strongly radial with $\beta \approx 0.3 - 0.4$ on the outskirts (Kazantzidis et al. 2006).

The last two panels of Figure 8 show similar $f(E, J)$ contours corresponding to remnants of mergers with different orbital configurations, including radial (run Br) and circular (run Bc) encounters. These experiments provide insight into the effect of the orbital energy of the collision on the final $f(E, J)$ distribution. In all cases, analysis is performed after each system is globally relaxed. As discussed before, this timescale is markedly different depending on the orbital parameters. These panels indicate that the circular encounter results in the largest spread in angular momenta. Similarly to the parabolic merger, in the radial encounter the bands of uniform phase-space density $f$ at a given energy $E$ curve toward higher values of $J$. This is again indicative of a velocity distribution profile with a fewer number of orbits with the highest angular momenta (i.e., a more radially anisotropic velocity distribution). In contrast, the curvature of the colored bands is toward larger $E$ values and higher $J$ values in the case of a circular merger indicating that the remnant is significantly more tangentially anisotropic.

Lastly, Figure 9 compares the initial and final distributions of $F(r)$ in the parabolic, radial, and circular mergers between identical NFW halos (runs Bp1, Br, and Bc). This figure demonstrates that the remnants retain faithful memories of the

![Image of color-coded contours of equal phase-space density $f$ of particles as a function of their total energy ($E$) and angular momentum ($J$). Results are presented for all particles which end up within the virial radius of the merger remnant. Contours are spaced at logarithmic density intervals relative to the maximum phase-space density contour. Purple contours correspond to the highest phase-space density regions.](image-url)
Evolution of Dark Matter Phase-Space Density Distributions in Equal-Mass Halo Mergers

4.1. Sequential Mergers

Next, we consider the merger sequences. In the sequential merger simulations, identical copies of the remnants from the NFW halo mergers discussed in § 4.1 were collided with each other after removing the small number (\(\sim 1-3\%\) in all cases) of unbound particles. We initialized the mergers on parabolic or radial orbits, placed the systems at relative distances equal to twice their virial radii and oriented the principal axes of the merger remnants randomly with respect to each other. We shall refer to the two levels of mergers as “level one,” and “level two,” respectively. The level one mergers refer simply to the parabolic mergers discussed in § 4.1. Note that the pericentric distances for all merger levels are kept equal to 20% of the halo virial radii. This choice is motivated by the results of the binary merger simulations which demonstrated that the phase-space density structure of the remnants is largely insensitive to the details of the encounter orbital energy (Figure 9).

In all sequences of mergers, we identified a time after which the central density profile of the remnant did not evolve significantly (changes of the order of few percent were considered acceptable). We further allowed the remnants to settle into equilibrium for a timescale equal to one crossing time at the virial radius of the system, \(t_{\text{cross}}(r_{\text{vir}})\). Then their equilibrium state was analyzed. The phase-space density distributions \(F(r)\) for this set of experiments (runs Bp1 and Sp) are shown in Figure 10. Not surprisingly, the remnant of the second level of the hierarchy exhibits faithful memories of the inner power-law indices of the phase-space density distribution of its progenitors. This indicates that, even in a hierar-

Fig. 8.— Color-coded contours of equal phase-space density \(f\) of particles as a function of their total energy \((E)\) and angular momentum \((J)\). Results are presented for one of the progenitor NFW halos (top panel) and various remnants of mergers with different orbital configurations (runs Bp1, Br, and Bc). Only particles with \(r < r_{\text{vir}}\) in each case are included. Color-coding is as in Figure 7.

Fig. 9.— Phase-space density distributions \(F(r)\) for initial systems (solid line) and remnants in binary mergers between identical NFW DM halos. The profiles are plotted as a function of radius in units of the virial radius and each is normalized to its own \(r_{\text{vir}}\). Results are presented for the initial halo and for remnants obtained from mergers with a variety of orbital configurations including parabolic (dotted line), radial (dot-dashed), and circular (dashed) encounters (runs Bp1, Br, and Bc). The central slopes and overall shapes of the remnant phase-space density distributions exhibit faithful memories of those of their progenitors independently of the orbital energy of the encounter.

inner power-law indices and overall shapes of the phase-space density distribution of their progenitors, independently of the orbital energy of the encounter. \(F(r)\) deviates significantly from power-law at \(r > r_{\text{vir}}\) where it flattens out considerably at large radii, even increasing slightly in the outer regions. There are some differences in the shape of \(F(r)\) that manifest at large radii where the phase-space density is fairly low, but the agreement is excellent at \(r < r_{\text{vir}}\).

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In this paper we have examined the evolution of the coarse-grained phase-space density distribution during dissipationless, equal-mass mergers between DM halos. For this purpose, we have analyzed a subset of the N-body merger simulations presented in Kazantzidis et al. (2006). The initial halo models were constructed with various asymptotic power-law indices $\gamma$. We compared the abilities of different codes (EnBiD and Fiestas) and various smoothing parameters for EnBiD to reproduce the phase-space density $DF$ for a NFW halo which was constructed to be of the form $f = f(E)$. Based on this comparison, we selected the EnBiD algorithm developed by Sharma & Steinmetz (2006) with $n = 10$ kernel smoothing to compute phase-space densities in the present study. We have analyzed an ensemble of simulations comprising not only binary encounters, but also sequences of mergers as well as collisions occurring at different cosmological epochs in order to study the range of behaviors that are realized in the context of cosmological structure formation. Lastly, we have also probed a wide range of parameter space by considering various initial orbital configurations and orbital energies in an effort to establish the generality of our main results.

Our main conclusions can be summarized as follows.

**5. CONCLUSIONS AND DISCUSSION**

...
1. Equal-mass mergers between DM halos result in a slight decrease of the median $F(r)$ of the coarse-grained phase-space density as expected from the Mixing Theorems (Tremaine et al. 1986; Binney & Tremaine 1987; Mathur 1988). This decrease occurs in a scale-free manner so that its magnitude is nearly independent of radius.

2. Binary mergers between identical DM halos with various inner density distributions ranging from steep cusps to core-like profiles produce remnants that retain faithful memories of the inner slopes and overall shapes of the phase-space density distribution of their progenitors. This conclusion remains valid for a range of orbital energies, and a variety of encounter configurations including sequences of several consecutive merger events, designed to mimic hierarchical merging, and collisions occurring at different cosmological epochs.

3. If the progenitor halos are constructed with appreciably different asymptotic power-law indices, the inner slope and overall shape of the phase-space density distribution of the remnant are substantially closer to that of the initial system with the steepest central density cusp.

4. Our results explicitly demonstrate that mixing is incomplete in equal-mass mergers between DM halos as it does not erase memory of the properties of the merging progenitors.

5. During the merger of two NFW halos, the volume distribution function, $V(f)$, increases at low phase-space densities $f$ due to the ejection of matter at large radii. However, at intermediate to high values of $f$ and over two orders of magnitude, the $V(f)$ profile is preserved. The robustness of $V(f)$ is not a consequence of similar levels of mixing at different radii. In fact, $V(f)$ at small radii is found to be essentially constant, demonstrating, once again, the robustness of the cusp. Much of the increase in $V(f)$ at low values of $f$ is the result of matter ejected from the outer 30% of the halos during the merger.

6. During the merger of two NFW halos, the true coarse-grained phase-space density in spherical radial bins, $F(r)$, and its spherically-averaged proxy, $Q(r) = ρ(r)/a^3(r)$, remain power-laws for $r ≤ 0.7r_{vir}$ ($Q(r) ∝ r^{-0.88}$ and $F(r) ∝ r^{-1.92}$). Both $F(r)$ and $Q(r)$ flatten out and deviate from these simple power-laws beyond $r_{vir}$.

By design, the distribution functions used to construct the initial DM halo models depended solely on the energy, $f(E)$. In an equal-mass merger of two NFW halos on a parabolic or a radial orbit, we find that the remnant is characterized by a distribution function that is more radially anisotropic, which means that particles at a given $E$ have a smaller range of $J$ values (Figure 3). In contrast, in the case of merger remnants that arise from circular orbits, particles at the lowest energies have significantly larger range of angular momenta compared to both radial and parabolic cases indicating that the remnant is significantly more tangentially anisotropic. These results are expected from previous studies (Moore et al. 2004) and the representation of $f(E,J)$ in this paper serves as a basis for understanding distribution functions that arise from cosmological simulations of halo formation.

An analytical framework exists that aids in the understanding of our numerical results. Indeed, Dehnen (2005) recently showed that for phase-space density $f$, the excess mass function $D(f)$, defined as the difference between the mass with phase-space density $> f$ and the product of $f$ and the volume of phase-space with density $> f$, can only decrease upon mixing. This mixing theorem and the simple properties of $D(f)$ as $f → ∞$ for self-gravitating density cusps can then be used to prove that a merger remnant cannot have a density cusp steeper than that of the steepest progenitor. Dehnen (2005) also argued that reducing the slope of the density cusp would require arbitrarily large dilution of the phase-space density so that it seems implausible that the remnant could have an inner cusp shallower than the steepest progenitor cusp.

Our findings are in agreement with the conclusions of Dehnen (2005). Indeed, Figure 5 illustrates that in a merger of two identical power-law phase-space density profiles, the remnant exhibits a phase-space density distribution with an inner slope and overall shape that are remarkably similar to those of their progenitors. In addition, Figure 5 demonstrates that in mergers between halos of very different inner density power-laws, the phase-space density distribution of the remnant is substantially closer to that of the steepest progenitor.

However, it is worth emphasizing that the results of Dehnen (2005) apply asymptotically as $r → 0$ and our simulations simply may not resolve sufficiently small radii for the asymptotic result to hold. In this regard, we see no evidence that the second conclusion of Dehnen (2005) is violated but higher-resolution numerical simulations of merging density cusps are required to test this conjecture more extensively.

Our findings also shed light on the resilience of density cusps to gravitational interactions. Kazantzidis et al. (2004b) elucidated the dynamical effect of tides on the central density profiles of cuspy satellite halos. These authors employed collisionless, high-resolution cosmological N-body simulations with simulations of the tidal stripping of individual satellites orbiting within a static host potential and showed that the density distribution retains its initial inner power-law index as the satellite experiences mass loss. Their analysis revealed that the central density cusp is notably stable to tidal shocks and gravitational stripping. The results reported in this paper combined with those of Kazantzidis et al. (2004b) as well as those of Kazantzidis et al. (2006) who highlighted the robustness of configuration-space density profiles during dissipationless mergers, suggest that density cusps, once formed, are extremely difficult to disrupt or even to modify in any significant way.

In our modeling, we have also implicitly assumed that cuspy halos have a higher phase-space density in the central regions compared to their cored counterparts. However, it is not difficult to imagine merging scenarios where this situation can be reversed as, for example, in cases of unequal-mass mergers or encounters between systems with considerably different concentration parameters. It will be important to perform a detailed numerical study of these considerations.

While equal-mass mergers, such as those analyzed in the present study, are instructive since they represent the most extreme form of mixing in phase-space, they are not the major mode of halo growth in the Universe. Moreover, while the current experiments are ideal to address the evolution of the phase-space density distributions of DM halos they were not designed to illuminate the formation of these profiles. In order to gain a better understanding of the origin of the power-law phase-space density distributions of cosmological DM halos.
one needs to resort to hierarchical structure formation simulations. In a recent paper Vass et al. (2008) have investigated the evolution of the phase-space distribution function in 4 Milky Way-sized systems obtained from such structure formation simulations. Their analysis and simulations complement these of our study and show that the phase-space density of cosmological DM halos also evolves in a manner that is consistent with the Mixing Theorems. These authors found that while $Q(r)$ within the halo virial radius is well described by a power-law at all redshifts, the median phase-space density $F(r)$ is not a simple power-law at any redshift.

In addition, Vass et al. (2008) demonstrated that $F(r)$ has a slope that decreases with radius to $0.6r_{vir}$ but increases thereafter. The increase in $F(r)$ in the outer halo regions reflects the fact that DM subhalos begin to play a major role in the evolution of phase-space density of DM. Vass et al. (2008) also showed that these subhalos have high central phase-space densities which are representative of DM phase-space densities at much higher redshifts ($z \sim 9$). Lastly, the aforementioned work showed that while mixing and relaxation inside virialized DM halos is the major cause of the decrease of the median phase-space density of DM, a significant decrease in DM phase-space density arises from cosmological expansion of the Universe. These authors showed that while the overall phase-space density of DM decreases as approximately $f(a) \propto a^{-1.5}$, the contribution from cosmic expansion of the Universe is described by $f(a) \propto a^{-4.5}$.

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