HEATING OF GAS INSIDE RADIO SOURCES TO MILDLY RELATIVISTIC TEMPERATURES VIA INDUCED COMPTON SCATTERING

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Measured values of the brightness temperature of low-frequency synchrotron radiation emitted by powerful extragalactic sources reach $10^{11}–10^{12}$ K. If some amount of nonrelativistic ionized gas is present within such sources, it should be heated as a result of induced Compton scattering of the radiation. If this heating is counteracted by cooling due to inverse Compton scattering of the same radio radiation, then the plasma can be heated up to mildly relativistic temperatures $kT \sim 10$–100 keV. The stationary electron velocity distribution can be either relativistic Maxwellian or quasi-Maxwellian (with the high-velocity tail suppressed), depending on the efficiency of Coulomb collisions and other relaxation processes. We derive several easy-to-use approximate expressions for the induced Compton heating rate of mildly relativistic electrons in an isotropic radiation field, as well as for the stationary distribution function and temperature of electrons.

We give analytic expressions for the kernel of the integral kinetic equation (one as a function of the scattering angle and another for the case of an isotropic radiation field), which describes the redistribution of photons in frequency caused by induced Compton scattering in thermal plasma. These expressions can be used in the parameter range $h\nu \ll kT \ll 0.1mc^2$ (the formulae earlier published in Sazonov, Sunyaev, 2000 are less accurate).

1 Introduction

Milliarcsecond-resolution interferometric radio observations of the central parsec-scale regions of active galactic nuclei often reveal sub-structure emitting low-frequency radiation with inferred rest-frame brightness temperatures $T_b \sim 10^{11}–10^{12}$ K. If there were thermal plasma present in these regions, it could be heated efficiently as a result of induced Compton scattering of the radio radiation on the electrons (Levich, Sunyaev, 1971).

Such a plasma has not been observed so far in extragalactic sources. On the other hand, there is evidence that relatively cold matter constitutes a significant fraction of the total mass contained in the jets, including their inner ($\lesssim 10^{12}$ cm) regions, of the famous galactic source SS433 (see Vermeulen, 1992 for a review). Of course, these jets are only mildly relativistic ($v \sim 0.26c$) and therefore quite different from the relativistic jets in active galactic nuclei (which have bulk Lorentz factors $\gamma \sim 5$), but it is reasonable to expect that some amount of quasi-thermal plasma may be present in extragalactic jets as well (see, e.g., Celotti et al., 1998).

One can derive directly from the Kompaneets kinetic equation the induced Compton heating rate of thermal electrons located in an isotropic radiation field (Levich, Sunyaev, 1971). The
resulting expression is, however, only applicable when the electron temperature is relatively low, \( kT \ll \text{a few keV} \). As the electrons become more relativistic, their heating rate monotonically decreases. Theoretical efforts in the 70-s (Vinogradov, Pustovalov, 1972; Blandford, 1973; Blandford, Scharlemann, 1975) resulted in a number of useful formulae that allow one to determine the heating rate of relativistic electrons and their distribution function in various limits, e.g., for ultrarelativistic electrons, for narrow radiation beams, or for some specific radiation spectra. Illarionov and Kompaneets (1976) have derived a general expression that gives the heating rate for an electron moving with an arbitrary velocity in a given isotropic radiation field. This formula is, however, rather complex: one needs to compute a 3-dimensional integral in order to find the heating rate of an ensemble of electrons with a given velocity distribution (e.g., Maxwellian).

In the present paper we demonstrate that electrons can be heated by Compton scattering up to mildly relativistic temperatures \( kT \sim \text{a few tens of keV} \), but not more, in an isotropic radiation field with \( T_b < 10^{12} \text{ K} \). For these plasma temperatures, the nonrelativistic estimates for the heating rate and some relevant quantities become inaccurate. However, these expressions can be modified by adding to them a few correction terms, thereby retaining the original simple structure, as shown below.

It should be noted that although we mentioned above only active galactic nuclei, the induced Compton effect may play a major role also in other astrophysical situations. We also note that the problem of plasma heating near, but outside, a source of low-frequency radiation requires a special study.

2 Heating and cooling of thermal electrons during Compton scattering

Let us derive the rates of heating and cooling of mildly relativistic thermal electrons \( (kT \lesssim 0.1mc^2 \text{, where } T \text{ is the electron temperature}) \) in an isotropic radiation field, as a result of both the spontaneous and induced Compton scattering processes.

The radiation field may be defined by its spectral energy density, \( \epsilon_\nu \) (measured in units of \( \text{erg cm}^{-3} \text{ Hz}^{-1} \)), or, equivalently, by the occupation number in photon phase space, \( n = \epsilon_\nu c^3/(8\pi h\nu^3) \). We require that the spectrum be sufficiently broad, which means that its characteristic width \( \Delta \nu/\nu \) should be much larger than the typical fractional frequency shift acquired by a photon during a scattering event either due to the Doppler effect, \( \sim \pm (kT/mc^2)^{1/2} \), or due to Compton recoil, \( \sim -h\nu/mc^2 \). In this case the integral kinetic equation describing the time evolution of \( \epsilon_\nu \) (caused by the Compton interaction of the radiation with the electrons) can be transformed into a Fokker-Planck-type differential equation. In the limit of \( kT, h\nu \ll mc^2 \), the resulting equation is the Kompaneets (1957) equation.

From the Kompaneets equation one can derive the expressions for the electron heating and cooling rates (Levich, Sunyaev, 1971), which are fairly accurate for electron and photon energies \( \lessapprox 0.01mc^2 \). The Kompaneets equation can be extended into the mildly-relativistic domain \( (kT, h\nu \lesssim 0.1mc^2) \) by adding to it relativistic correction terms \( \sim (kT/mc^2)^m(h\nu/mc^2)^n \) (Itoh et al., 1998; Challinor, Lasenby, 1998; Sazonov, Sunyaev, 2000). For example, the first-order generalization of the Kompaneets equation (which obtains by Fokker-Planck-type
expansion of the integral kinetic equation to the fourth order in \( \Delta \nu \) is

\[
\frac{\partial n}{\partial t} = \frac{\sigma_T N_e h}{mc} \frac{1}{\nu^2} \frac{\partial}{\partial \nu} \nu^4 \left\{ n(1 + n) + \frac{kT}{h} \frac{\partial n}{\partial \nu} + \frac{7}{10} \frac{h}{mc^2} \frac{\partial}{\partial \nu} \right\} + \frac{kT}{mc^2} \left[ \frac{5}{2} \left( n(1 + n) + \frac{kT}{h} \frac{\partial n}{\partial \nu} \right) + \frac{21}{5} \nu \frac{\partial}{\partial \nu} \left( n(1 + n) + \frac{kT}{h} \frac{\partial n}{\partial \nu} \right) \right] + \frac{7}{10} \nu^2 \left( -2 \left( \frac{\partial n}{\partial \nu} \right)^2 + 2(1 + 2n) \frac{\partial^2 n}{\partial \nu^2} + \frac{kT}{h} \frac{\partial^3 n}{\partial \nu^3} \right) \right\},
\]

(1)

where \( \sigma_T \) is the Thomson cross section and \( N_e \) is the number density of electrons.

Multiplying Eq. (1) by \( h\nu \) and integrating over the photon phase space leads to an expression for the variation of the radiation total energy density, \( \epsilon = \int_0^\infty \epsilon_\nu d\nu \), and, at the same time, for the change of the mean electron energy, \( E_e \approx 1.5kT(1 + 1.25kT/mc^2) \):

\[
- \frac{1}{N_e} \frac{dE_e}{dt} = \frac{8\pi \sigma_T h}{c^2} \left[ -\frac{4}{mc^2} \int_0^\infty \nu^3 n d\nu + \frac{h}{mc^2} \int_0^\infty \nu^4 (n + n^2) d\nu -10 \left( \frac{kT}{mc^2} \right)^2 \int_0^\infty \nu^3 n d\nu - \frac{21}{5} \left( \frac{h}{mc^2} \right)^2 \int_0^\infty \nu^5 n d\nu \right] + \frac{kT}{mc^2} \frac{h}{mc^2} \left( \frac{47}{2} \int_0^\infty \nu^4 (n + n^2) d\nu - \frac{21}{5} \int_0^\infty \nu^6 \left( \frac{\partial n}{\partial \nu} \right)^2 d\nu \right). 
\]

(2)

The first two terms in the square brackets in Eq. (2) describe the electron heating and cooling rates in the nonrelativistic limit (Peyraud, 1968; Zel’dovich, Levich, 1970; Levich, Sunyaev, 1971). The additional terms represent relativistic corrections of the order of \( kT/mc^2 \) or \( h\nu/mc^2 \). Note that the correction to the heating rate associated with the induced Compton effect (the corresponding terms are nonlinear in \( n \)) includes a term which is expressed through the square of the derivative, \( (\partial n/\partial \nu)^2 \). Higher-order \([kT/mc^2]^2, \text{etc.}\] corrections to the induced-Compton energy transfer rate, which are not given in Eq. (2), depend on higher-order derivatives of \( n \), as is explicitly shown below (in §3).

Using Eq. (2), we can, for example, find the energy transfer rate between electrons and black-body radiation of temperature \( T_r \),

\[
\frac{d\epsilon}{dt} = 4\epsilon \sigma_T N_e c \left( \frac{kT}{mc^2} - \frac{kT_r}{mc^2} \right) \left( 1 + \frac{5}{2} \frac{kT}{mc^2} - \frac{2\pi^2 kT_r}{mc^2} \right). 
\]

(3)

This expression was originally derived (in a different way) by Woodward (1970).

We note that the problem of the energy transfer between radiation and a thermal plasma via spontaneous Compton scattering is well studied, and the corresponding part of Eq. (2) follows directly from the first moment of the Compton scattering kernel (Shestakov et al., 1988; Sazonov, Sunyaev, 2000; also see the Appendix). We are concerned here with the terms describing the contribution of induced scattering.
3 Induced Compton heating of thermal electrons

Powerful extragalactic radio sources emit low-frequency continuum radiation that in some cases has a very high brightness temperature, $T_b = nh\nu/k \sim 10^{11} - 10^{12}$ K, so that $kT_b > mc^2$. Induced Compton scattering can be a major mechanism of heating of free electrons situated in such a radiation field. Moreover, nonrelativistic estimates lead to the conclusion that electrons can be heated up to relativistic temperatures $kT \sim a$ fraction of $kT_b$ (Levich, Sunyaev, 1971). Clearly, nonrelativistic treatment is inappropriate here. Below we show that in reality, electron temperatures achievable in this situation are only mildly relativistic (i.e., $kT$ is significantly less than $mc^2$). As a result, all relevant physical quantities can be described by simple analytic expressions.

We consider throughout an isotropic radiation field. In this paragraph we also assume a relativistic Maxwellian distribution of electrons (which can be maintained, e.g., by Coulomb collisions). This last assumption is dropped in §4.

The rate at which energy is transferred by induced Compton scattering from an isotropic radiation field to a single electron moving at a speed $v = \beta c$ is given by (Illarionov, Kompaneets, 1976)

$$W^+(\beta) = \frac{12\pi\sigma_T h^2}{mc^4} \int_0^\beta \Phi(y') G(\beta, \beta') d\beta',$$

$$\Phi(y) = \int_0^\infty n(\nu)n(y\nu)\nu^4 d\nu,$$

$$G(\beta, \beta') = \frac{\beta'^2}{\gamma^6 \beta^8 (1 + \beta')^5} \left[ (30 - 24\beta^2 + 2\beta^4) \ln \frac{y'}{y} \
+ 28\beta^3 - 60\beta + 5(3 - \beta^2)^2\beta' + (5 - \beta^2)(3 + \beta^2)\beta' \left( \frac{\gamma'}{\gamma} \right)^2 \right],$$

$$y = \frac{1 - \beta}{1 + \beta}, \quad y' = \frac{1 - \beta'}{1 + \beta'},$$

$$\gamma = (1 - \beta^2)^{-1/2}, \quad \gamma' = (1 - \beta'^2)^{-1/2}. \quad (4)$$

Eq. (4) is valid for arbitrary electron velocities, provided that $h\nu \ll mc^2$. This formula can be simplified in some limits, in particular, for spectrally narrow radiation lines ($\Delta\nu \ll \nu$), such as those produced in cosmic masers and laboratory lasers (Vinogradov, Pustovalov, 1972; Blandford, Scharlemann, 1975). We are instead interested in the $\Delta\nu \gg \nu$ case.

In order to calculate the instantaneous heating rate of an actual plasma, it is necessary to weight the result of Eq. (4) with a given distribution of electron velocities, $f_\beta$. The computation procedure to determine the heating rate will therefore be three-dimensional integration: 1) over the radiation spectrum [when calculating $\Phi(y')$], 2) over $d\beta'$ and 3) over $f_\beta$. However, if the electrons obey a relativistic Maxwellian distribution and are only mildly relativistic ($kT \ll 0.1mc^2$), it is possible to considerably simplify the computation procedure, by reducing it to integration over the radiation spectrum, as demonstrated below.

When $kT \ll 0.1mc^2$, the majority of electrons have $\beta \ll 0.5$, and therefore the typical random Doppler frequency shift caused by a scattering is relatively small: $\delta\nu \ll \nu$. The following
Taylor expansion of the $\Phi(y')$ function entering Eq. (4) is then justified:

$$\Phi(y') = \int_0^\infty n^2 \nu^4 d\nu + (y' - 1) \int_0^\infty n \frac{\partial n}{\partial \nu} \nu^5 d\nu + \frac{1}{2} (y' - 1)^2 \int_0^\infty n \frac{\partial^2 n}{\partial \nu^2} \nu^6 d\nu + ..., \quad (5)$$

where, in turn,

$$y' - 1 = -2 \beta' + 2 \beta^2 + ..., \quad \frac{1}{2} (y' - 1)^2 = 2 \beta^2 + ..., \text{etc.} \quad (6)$$

A similar expansion is possible for the kernel $G(\beta, \beta')$ of Eq. (4):

$$G = \frac{4}{\beta} \left[ \left( \frac{\beta'}{\beta} \right)^3 - 2 \left( \frac{\beta'}{\beta} \right)^5 + 2 \left( \frac{\beta'}{\beta} \right)^7 \right] + 20 \left[ - \left( \frac{\beta'}{\beta} \right)^4 + 2 \left( \frac{\beta'}{\beta} \right)^6 - 2 \left( \frac{\beta'}{\beta} \right)^8 \right] + ... \quad (7)$$

The subsequent straightforward calculation leads to

$$W^+(\beta) = \frac{\sigma_T c^2}{8 \pi m} \left[ J_0 + \left( -\frac{17}{30} J_0 - \frac{7}{5} J_1 \right) \beta^2 + \left( -\frac{47}{600} J_0 + \frac{17}{150} J_1 + \frac{11}{30} J_2 \right) \beta^4 \right. \right.$$  

$$+ \left. \left( -\frac{899}{19600} J_0 + \frac{2833}{29400} J_1 + \frac{4261}{14700} J_2 - \frac{64}{1575} J_3 \right) \beta^6 + \right.$$  

$$\left. J_0 = \int_0^\infty \epsilon^2 \nu^{-2} d\nu, \quad J_1 = \int_0^\infty \left( \frac{\partial \epsilon}{\partial \nu} \right)^2 d\nu, \quad J_2 = \int_0^\infty \left( \frac{\partial^2 \epsilon}{\partial \nu^2} \right)^2 \nu^2 d\nu, \quad J_3 = \int_0^\infty \left( \frac{\partial \epsilon}{\partial \nu^3} \right)^2 \nu^4 d\nu. \quad (8)$$

Note that the Planck constant, $\hbar$, is absent from Eq. (8), which reflects the well-known fact that the process of induced Compton scattering is classical (see, e.g., Zel’dovich, 1975).

Our next step is to replace $\beta^2$, $\beta^4$, $\beta^6$ in Eq. (4) by their average values $\langle \beta^2 \rangle$, $\langle \beta^4 \rangle$, $\langle \beta^6 \rangle$, calculated for a mildly-relativistic Maxwellian distribution at temperature $T$, which is given by (e.g., Sazonov, Sunyaev, 2000)

$$f_\beta = \left( \frac{2 \pi kT}{mc^2} \right)^{-3/2} \left[ 1 + \frac{15}{8} \frac{kT}{mc^2} + \frac{105}{128} \left( \frac{kT}{mc^2} \right)^2 - \frac{315}{1024} \left( \frac{kT}{mc^2} \right)^3 + \right. \right.$$  

$$\left. \left( 1 + \frac{5}{2} \beta^2 - \frac{3 \beta^4 mc^2}{8 kT} + \frac{35}{8} \beta^4 - \frac{5 \beta^6 mc^2}{4 kT} + \frac{9 \beta^8 mc^2}{128 k^2 T^2} + \frac{105 \beta^10 mc^2}{256 k^2 T^2} \right) \exp \left( -\frac{\beta^2 mc^2}{2kT} \right). \quad (9)$$

The resulting heating rate as a function of the plasma temperature is

$$W^+(T) = \frac{\sigma_T c^2}{8 \pi m} \left[ J_0 - \left( \frac{17}{10} J_0 + \frac{21}{5} J_1 \right) \frac{kT}{mc^2} + \left( \frac{123}{40} J_0 + \frac{61}{5} J_1 + \frac{11}{2} J_2 \right) \left( \frac{kT}{mc^2} \right)^2 \right.$$  

$$- \left( \frac{1989}{280} J_0 + \frac{453}{14} J_1 + \frac{1899}{140} J_2 + \frac{64}{15} J_3 \right) \left( \frac{kT}{mc^2} \right)^3 + \right.$$  

$$\left. \right]. \quad (10)$$
where $J_0$, $J_1$, $J_2$ and $J_3$ were introduced in Eq. (8).

In the case of cold electrons ($kT \ll mc^2$), only the leading term of the series in powers of $kT/mc^2$ in Eq. (10) is important, and the heating rate due to induced Compton scattering is described by the well-known (Zel’dovich, Levich, 1970; Levich, Sunyaev, 1971) formula

$$W_0^+ = \frac{\sigma_T c^2}{8\pi m} \int_0^\infty \epsilon_0^\nu \nu^{-2} d\nu.$$

(11)

Both the nonrelativistic expression (11) and the first-order relativistic correction to it (see Eq. [10]) were already obtained in §2 following a different approach; see Eq. (2). Thus, two independent methods give the same result.

### 3.1 Heating in a field of self-absorbed synchrotron radiation

Let us now consider one particular spectral distribution, namely the spectrum of synchrotron radio emission with self-absorption at low frequencies. The radiation spectrum generated by a spherically-symmetric homogeneous source is (Gould, 1979)

$$\epsilon_\nu^0(\nu) = A \left(\frac{\nu}{\nu_0}\right)^{5/2} \left[\frac{1}{2} + \frac{\exp(-t)}{t} - \frac{1 - \exp(-t)}{t^2}\right],$$

$$t = \left(\frac{\nu_0}{\nu}\right)^{0.5p+2}. \quad (12)$$

The shape of the distribution (12) is defined by a single parameter, $p$, which is the index of the power-law energy distribution of the electrons producing the synchrotron radiation, $dN_e \sim \gamma^{-p} d\gamma$. The other two parameters that appear in Eq. (12), $\nu_0$ and $A$, determine the position of the spectrum along the frequency axis and its amplitude, respectively. Far enough from the peak of intensity ($\nu_{\text{peak}} \approx 1.15\nu_0$), the spectrum (12) assumes a power-law shape: $\epsilon_\nu^0 \sim \nu^{5/2}$ in the region of self-absorption ($\nu \ll \nu_0$) and $\epsilon_\nu^0 \sim \nu^{(1-p)/2}$ in the optically thin part ($\nu \gg \nu_0$).

Since the integral $\epsilon = \int_0^\infty \epsilon_\nu^0 d\nu$ diverges at $\infty$ for $p \leq 3$, and it is this quantity that determines the Compton cooling rate, which will be taken into account below, we modify Eq. (12) as follows:

$$\epsilon_\nu(\nu) = \begin{cases} 
\epsilon_\nu^0(\nu), & \nu \leq \nu_b \\
(\nu/\nu_b)^{-0.5} \epsilon_\nu^0(\nu), & \nu > \nu_b.
\end{cases} \quad (13)$$

Here we have assumed that a steepening of the spectrum (increase in the slope by 0.5) takes place above some "break" frequency, $\nu_b \gg \nu_0$, due to the fast synchrotron cooling of more energetic electrons.

To be concrete, we take the slope of the optically thin part of the spectrum (prior to the slope break) to be $\alpha = -0.7$, which corresponds to $p = 2.4$. We additionally adopt $\nu_b = 10^3\nu_0$ as a fiducial value for our treatment of electron cooling, although this parameter just enters as a multiplicative factor in the relevant formulae as long as $\nu_b \gg \nu_0$. The resulting spectrum is plotted in Fig. 1. Note that the low-frequency breaks observed in the spectra of some radio sources are sometimes interpreted as being caused by mechanisms other then synchrotron self-absorption, which include free-free absorption in the ambient medium (e.g., Bicknell et
al., 1997) and induced Compton scattering either outside or inside the radio source (e.g., Sunyaev, 1971; Sincell, Krolik, 1994; Kuncic et al., 1998). However, the radiation field inside the source, which we are concerned with, may well be the superposition of self-absorbed synchrotron spectra generated by individual plasma components.

Fig. 2a shows the heating rate of thermal electrons exposed to self-absorbed synchrotron radiation as a function of the electron temperature. The exact result was obtained by weighting the heating rate for monoenergetic electrons, given by Eq. (4), with the relativistic Maxwellian distribution,

\[
\frac{dN_e}{d\gamma} = \text{const} \frac{\gamma}{(\gamma^2 - 1)^{1/2}} \exp \left( -\frac{\gamma mc^2}{kT} \right) d\gamma.
\]

We have additionally verified this dependence with Monte-Carlo simulations (using the Comptonization code described in Pozdnyakov et al., 1983 with a slight modification to allow for induced Compton scattering). The computation proves to be faster with the semi-analytic formula of Illarionov and Kompaneets. Also presented in Fig. 2a are various approximations for the heating rate that result from retaining a different number of temperature terms in Eq. (10).

One can see that the heating rate begins to decrease appreciably (> 5%) at \( kT \sim 5 \text{ keV} \). The temperature domain where this decrement is described very well by the approximate relation extends to \( kT \sim 30 \text{ keV} \), when the heating rate is already smaller by > 30% than in the case of cold electrons. The further decrease in the heating rate that takes place at yet higher temperatures cannot be described properly in the framework of the Fokker-Planck approach, which led to Eq. (10). We found it convenient to describe the exact dependence presented in Fig. 2a by an approximate formula, which is accurate to within 3% for \( kT < 5mc^2 \):

\[
W^+(T) = \left\{ 0.8 \exp \left[ -3.7 \left( \frac{kT}{mc^2} \right)^{0.8} \right] + 0.2 \exp \left[ -1.8 \left( \frac{kT}{mc^2} \right)^{0.6} \right] \right\} W_0^+.
\]

(14)

where, as follows from Eq. (11), (12):

\[
W_0^+ = 1.1 \cdot 10^{24} A^2 \nu_0^{-1} \text{ eV s}^{-1}
\]

(15)

\((A \text{ is measured in } \text{erg Hz}^{-1} \text{ cm}^{-3} \text{ and } \nu_0 \text{ in GHz}).

It is worth mentioning that the heating rate approaches an asymptote \( W^+(T) \sim (mc^2/kT)^3 \) when \( kT \gg mc^2 \). This results from the fact that only the lower-energy part of the relativistic Maxwellian distribution, i.e., electrons with \( \gamma \ll 1 \), significantly contributes to the net heating rate (because \( W^+ \sim \gamma^{-5} \) for \( \gamma \gg 1 \) — see Illarionov, Kompaneets, 1976), and the relative number of such electrons is proportional to \((mc^2/kT)^3\).

Although we have assumed a particular slope \((-0.7)\) for the optically-thin part of the radiation spectrum, it turns out that the dependence of the heating rate on the electron temperature changes very slowly as the slope varies. Quantitatively, \( W^+(T) \) remains the same to within 10% for \( \alpha \) in the range \([-0.9; -0.5]\). Thus, formula (14) is quite useful in that it allows obtaining reasonably good estimates for the heating rate of substantially relativistic Maxwellian electrons in an isotropic field of self-absorbed synchrotron radiation.

Of course, the mildly-relativistic Eq. (10), which is applicable when \( kT \lesssim 30 \text{ keV} \), explicitly depends on the spectral distribution, and so can be used to calculate the heating rate for an arbitrary (broad) radiation spectrum.
3.2 Stationary temperature of electrons

Zel’dovich and Levich (1970) studied, in the nonrelativistic approximation, the problem about the establishment of a stationary distribution of electrons during their interaction with an isotropic field of low-frequency radiation of high brightness temperature. These authors showed that if the induced Compton heating is counteracted by cooling due to inverse (spontaneous) Compton scattering, then the stationary distribution will be Maxwellian with a temperature

\[ kT_{eq}^0 = \frac{c^3}{32\pi e} \int_0^\infty e^2 \nu^{-2} d\nu, \]  

(16)

where \( \epsilon = \int_0^\infty \epsilon \nu d\nu \).

It is reasonable to suggest that in the mildly-relativistic regime there will be only small deviations from a Maxwellian distribution. Furthermore, if other mechanisms, e.g., Coulomb collisions should play a significant role in the redistribution of electrons in momentum space, then a Maxwellian distribution can be maintained easily. We postpone a detailed discussion of questions related to the electron distribution until §4. Here we will continue to assume as before that the electrons obey a relativistic Maxwellian distribution, and will find the stationary electron temperature, considering inverse Compton scattering (of the same low-frequency synchrotron radiation that is heating the electrons) the only cooling agent. Other possible cooling mechanisms, which may prove more important under certain conditions, will be mentioned in §3.3.

The inverse Compton cooling rate is given by (e.g., Pozdnyakov et al., 1983)

\[ W^{-}(T) = \left( \langle \gamma \rangle + \frac{kT}{mc^2} \right) W_0^{-}(T), \]  

(17)

where the mean electron energy (in units of \( mc^2 \)) is

\[ \langle \gamma \rangle = \frac{3kT K_2(mc^2/kT) + mc^2 K_1(mc^2/kT)}{2kT K_1(mc^2/kT) + mc^2 K_0(mc^2/kT)}. \]  

(18)

\( K_p(x) \) are modified Bessel functions, and the cooling rate in the nonrelativistic limit (\( kT \ll mc^2 \)) is

\[ W_0^{-}(T) = \frac{4\sigma_T kT}{mc}. \]  

(19)

We can expand Eq. (17) in powers of \( kT/mc^2 \) to obtain a formula applicable in the mildly-relativistic limit, which is similar in structure to the corresponding relation for the heating rate (Eq. [10]):

\[ W^{-}(T) = \frac{4\sigma_T kT}{mc^2} \left[ 1 + \frac{5kT}{2mc^2} + \frac{15}{8} \left( \frac{kT}{mc^2} \right)^2 - \frac{15}{8} \left( \frac{kT}{mc^2} \right)^3 + ... \right]. \]  

(20)

An excellent fit to the exact formula (17) in the range \( kT \ll 5mc^2 \) is provided by

\[ W^{-}(T) = \left[ 1 + 3.4 \left( \frac{kT}{mc^2} \right)^{1.1} \right] W_0^{-}(T), \]  

(21)
with $W_0^-$ given by Eq. (19).

The dependences $W^-(T)$ described by Eqs. (17) and (20) are plotted in Fig. 2b and can be compared to the corresponding dependences for the heating rate (Fig. 2a). We note that the relationship $W^-(T)$, of course, does not depend on the shape of the radiation spectral distribution (depending only on the total energy density), as opposed to $W^+(T)$.

We can now find the equilibrium electron temperature by solving the equation $W^+(T_{eq}) = W^-(T_{eq})$. In order to proceed, we need to specify the constant $A$ appearing in Eq. (12), which defines the amplitude of the spectral distribution. It is natural to express this coefficient through the maximal brightness temperature,

$$T_{b}^{\max} = \max\left[\epsilon_{\nu}(\nu)c^3/(8\pi\nu^2)\right].$$

$T_b$ peaks at $\nu_{\max} = 0.61\nu_0$ for $\alpha = -0.7$ (although the position of the peak is only marginally dependent on $\alpha$ for typically observed radiation spectra of extragalactic radio sources). We find:

$$A \approx 22\pi k T_{b}^{\max} \nu_0^2/c^3.$$  (22)

Fig. 3 shows the equilibrium electron temperature, $T_{eq}$, as a function of $T_{b}^{\max}$. The exact result is compared with the nonrelativistic result (Eq. (16)) and different-order mildly-relativistic estimates that were obtained by equating Eq. (10) and Eq. (20). One can see that the nonrelativistic Eq. (16) is still valid for $T_{eq} \lesssim 5$ keV, which corresponds in our case to brightness temperatures $T_{b}^{\max} \lesssim 3 \cdot 10^{10}$ K. In this regime

$$k T_{eq} = 1.9 T_{b}^{\max} 10^{10} K \left(\frac{\nu_b}{1000 \nu_0}\right)^{-0.33} \text{ keV}.$$  (23)

The next decade of values of the equilibrium temperature, up to $kT_{eq} \sim 40$ keV, is well described by the approximate formulae (10) and (20). Note that $kT_{eq} = 40$ keV corresponds to $T_{b}^{\max} \sim 4 \cdot 10^{11}(\nu_b/1000\nu_0)^{0.33}$ K. An important conclusion can be made: electrons can be heated up to mildly relativistic temperatures, $kT \sim$ a few tens of keV, as a result of induced Compton scattering of synchrotron radiation with $T_b \sim 10^{11}$ – $10^{12}$ K, but not above these temperatures.

### 3.3 Evolution of the electron temperature during Compton interaction

Let us now address a related timing problem, namely, examine how rapidly electrons can be heated to mildly relativistic temperatures through induced Compton scattering.

We will first estimate the basic characteristic quantities in the nonrelativistic limit. Substituting $A$ given by Eq. (22) into Eq. (13), we find

$$W_0^+ = 1.4 \cdot 10^{-9} \left(\frac{T_{b}^{\max}}{10^{11} K}\right)^2 \left(\frac{\nu_0}{1 \text{ GHz}}\right)^3 \text{ eV s}^{-1}.$$  (24)

If there were no cooling, then initially cold electrons would acquire a kinetic energy of $kT_{eq}^0$, given by Eq. (23), during

$$t_{\text{heat}} = \frac{kT_{eq}^0}{W_0^+} = 1.4 \cdot 10^{13} \left(\frac{T_{b}^{\max}}{10^{11} K}\right)^{-1} \left(\frac{\nu_0}{1 \text{ GHz}}\right)^{-3} \left(\frac{\nu_b}{1000 \nu_0}\right)^{-0.33} \text{ s}.$$  (25)
We notice that the heating time depends strongly on the characteristic frequency of the synchrotron self-absorption: $t_{\text{heat}} \sim \nu_0^{-3}$.

Using Eq. (19), we can find the corresponding cooling rate:

$$W_0^{-}(T) = 3.8 \times 10^{-8} \frac{kT}{mc^2} \frac{T_b^{\text{max}}}{10^{11}\text{K}} \left( \frac{\nu_0}{1\text{GHz}} \right)^3 \left( \frac{\nu_b}{1000\nu_0} \right)^{0.33} \text{eV s}^{-1}. \quad (26)$$

In the mildly-relativistic regime, the heating and cooling rates will be respectively smaller and larger than those given by Eqs. (24) and (26). We have computed the evolution of the electron temperature for a set of $T_b^{\text{max}}$ values by integrating the following differential equation:

$$\frac{dT}{dt} = \left( \frac{d\langle \gamma \rangle(T)}{dT} \right)^{-1} \frac{W^+(T) - W^-(T)}{mc^2}, \quad (27)$$

using Eq. (18) to represent the dependence $\langle \gamma \rangle(T)$ and the fitting formulae (14) and (21) for $W^+(T)$ and $W^-(T)$, respectively. The resulting time histories are presented in Fig. 4, for which the value $\nu_0 = 1$ GHz was used. For a given value of $\nu_0$, one should simply rescale the time axis in Fig. 4 as $(\nu_0/1\text{GHz})^{-3}$.

Each of the time histories presented in Fig. 4 can be divided into two intervals. During the earlier period, the temperature grows linearly because $W^+ \gg W^-$. As $T$ becomes $\gtrsim 0.5T_{\text{eq}}$, the second (longer) period starts, during which the cooling plays a major role and the temperature slowly approaches the equilibrium value. This transition period is additionally lengthened by relativistic effects (compare the different solutions in Fig. 4). We can define a characteristic time of induced heating as the time needed to heat the plasma to $\sim 0.5T_{\text{eq}}$. To give an example, for $T_b^{\text{max}} = 10^{12}$ K, a time of $\sim 3 \times 10^4 (\nu_0/1\text{GHz})^{-3}$ years is required to heat the electrons to $kT = 0.5kT_{\text{eq}} \sim 34$ keV. If further, $\nu_0 \sim 10$ GHz, the heating time becomes $\sim 30$ years. Interestingly enough, the simple nonrelativistic Eq. (25) provides a good estimate (within a factor of 2) for the heating time even for values of $T_b^{\text{max}}$ as high as $\sim 10^{13} (\nu_b/1000\nu_0)^{0.33}$ K (of course, the equilibrium temperature in this case is much less than the nonrelativistic estimate).

In real situations, there may be mechanisms operating by which the plasma cools more efficiently than by inverse Compton scattering of the synchrotron radiation. One should then modify accordingly the cooling rate $W^-(T)$ in our treatment above. If the energy density of a possible high-frequency radiation component is larger than that of the low-frequency synchrotron emission, then the contribution of this component to the inverse Compton cooling rate will be accordingly larger. Also, cooling due to free-free transitions will become more important than inverse Compton cooling if the plasma is dense enough, namely when $N_e T^{-1/2} \epsilon^{-1} > 10^4 \text{K}^{-1/2} \text{erg}^{-1}$. For the synchrotron spectrum described by Eqs. (12) and (13), this condition translates into $N_e > 6(T_b^{\text{max}}/10^{11}\text{K})^{3/2}(\nu_0/1\text{GHz})^3(\nu_b/1000\nu_0)^{0.166} \text{cm}^{-3}$. Another possible cooling mechanism is adiabatic expansion of a plasma cloud. The characteristic time scale for this process is $t_{\text{ad}} = 3 \times 10^{10} (R/1\text{pc})(U/10^3\text{km s}^{-1})^{-1} \text{s}$, where $R$ is the size of the cloud and $U$ is the expansion velocity (assuming spherical expansion).
4 Effect of induced Compton scattering on the electron distribution

So far we have assumed that the distribution of the electrons in momentum space remains relativistic Maxwellian during their interaction with the low-frequency radiation. This should be the case if the plasma is dense enough that electron-electron collisions can quickly smooth out any arising deviations from a Maxwellian distribution. In order to find out whether this thermalization really takes place, one should compare the induced Compton heating time, \( t_{\text{heat}} \), given by Eq. (25), with the time scale on which electrons can relax to a Maxwellian distribution (Spitzer, 1978),

\[
t_{e-e} = 2.5 \cdot 10^{12} \left( \frac{\ln \Lambda}{40} \right)^{-1} \left( \frac{T_{b}^{\text{max}}}{10^{11} \text{K}} \right)^{5/2} \left( \frac{\nu_{0}}{1 \text{GHz}} \right)^{3} \left( \frac{\nu_{b}}{1000 \nu_{0}} \right)^{-0.166} N_{e}^{-1} < 1.
\]  

Let us consider a few examples. At \( T_{b}^{\text{max}} = 10^{11} \text{ K} \) and \( \nu_{0} = 1 \text{ GHz} \), Maxwellization of the electron spectrum occurs when \( N_{e} \gtrsim 10^{-3} \text{ cm}^{-3} \). For \( T_{b}^{\text{max}} = 5 \cdot 10^{11} \text{ K} \) and \( \nu_{0} = 10 \text{ GHz} \), the corresponding range is \( N_{e} \gtrsim 10^{2} \text{ cm}^{-3} \). It should be noted here that the establishment of the high-velocity tail of the Maxwellian distribution occurs on a larger time scale than \( t_{e-e} \) given above. In the nonrelativistic limit, the appropriate characteristic time is proportional to \( (v/\langle v \rangle)^{3} \) when \( v \gg \langle v \rangle \), where \( v \) is the velocity of an electron and \( \langle v \rangle \) is the typical thermal velocity of electrons (e.g., Krall, Trivelpiece, 1973).

We now consider an extreme situation when \( t_{e-e} \gg t_{\text{heat}} \), i.e., no Maxwellization of electrons takes place due to collisions. To find out what momentum distribution, \( f(p) \), results in this case, we need to consider the diffusion of electrons in momentum space caused by induced and spontaneous (inverse) Compton scattering. The corresponding Fokker-Planck equation for the electron momentum distribution is (Illarionov, Kompaneets, 1976)

\[
\frac{\partial f}{\partial t} = \frac{1}{p^{2}} \frac{\partial}{\partial p} p^{2} \left[ D \frac{\partial f}{\partial p} - F_{\text{sp}} f \right],
\]  

where the diffusion coefficient

\[
D(\beta) = \frac{12\pi \sigma_{T} h^{2}}{c^{4}} \int_{0}^{\beta} \Phi(y') G_{D}(\beta, \beta') d\beta'
\]

\[
G_{D}(\beta, \beta') = \frac{2\beta^{2}}{\gamma^{4} \beta^{6}(1 + \beta')^{5}} \left[ (3 - \beta^{2}) \ln \frac{y}{y'} + 2\beta - 2\beta' \left( \frac{\gamma'}{\gamma} \right)^{2} + 2(\gamma^{2} + \gamma^{-2})(\beta - \beta') \right],
\]  

and the breaking force due to spontaneous scattering (Landau & Lifshits 1975)

\[
F_{\text{sp}}(\beta) = -\frac{4}{3} \sigma_{T} \epsilon \beta \gamma^{2}.
\]  

The electron velocity is related to the electron momentum via

\[
\beta = \frac{p}{\gamma mc} = \frac{p}{mc} \left[ 1 + \left( \frac{p}{mc} \right)^{2} \right]^{-1/2}.
\]
The remaining quantities appearing in Eqs. (30) and (31) are the same as in Eq. (4).

The cumbersome expression (30) can be simplified if we assume that \( \beta \ll 1 \). Series expansion can then be carried out, which is completely analogous to those written down in §3 for the induced Compton heating rate:

\[
D(\beta) = \frac{\sigma T c^2}{24\pi} \int_0^\infty \epsilon^2 \nu^{-2} d\nu \left[ 1 + \beta^2 \left( \frac{4}{25} - \frac{21}{50} \int_0^\infty \frac{\partial \epsilon}{\partial \nu} \frac{d\nu}{\epsilon^2 \nu^{-2}} \right) + \ldots \right].
\]  

(33)

The equilibrium momentum distribution is given by

\[
f_{\text{eq}}(p) = \text{const} \exp \left[ \int_0^p \frac{F_{sp}(p')}{D(p')} dp' \right],
\]

(34)

In the nonrelativistic limit, \( D(p) = \text{const} \) and \( F_{sp}(p) \sim p \); hence \( F_{\text{eq}} \sim \exp(-p^2/2m k T_{\text{eq}}^0) \) with \( T_{\text{eq}}^0 \) given by Eq. (16). Therefore, the equilibrium distribution is Maxwellian in this limit, which was first shown by Zel’dovich and Levich (1970).

We can find the first-order relativistic correction to the nonrelativistic equilibrium distribution from Eq. (34), by making use of the approximate expression (33) for the diffusion coefficient and transforming Eq. (31) to

\[
F_{sp} = -\frac{4}{3} \sigma T c (p/mc) \left[ 1 + 0.5 \left( \frac{p}{mc} \right)^2 + \ldots \right].
\]

The result is

\[
f_{\text{eq}}(p) = \text{const} \exp \left\{ -\frac{p^2}{2m k T_{\text{eq}}^0} \left[ 1 + \left( \frac{p}{mc} \right)^2 \left( \frac{17}{100} + \frac{21}{50} \int_0^\infty \frac{\partial \epsilon}{\partial \nu} \frac{d\nu}{\epsilon^2 \nu^{-2}} \right) \right] \right\},
\]

(35)

where \( T_{\text{eq}}^0 \) is given by Eq. (16).

In the case of synchrotron radiation with self-absorption at low frequencies (Eqs. (12), (13)), the equilibrium distribution is

\[
f_{\text{eq}}(p) = \text{const} \exp \left\{ -\frac{p^2}{2m k T_{\text{eq}}^0} \left[ 1 + 0.69 \left( \frac{p}{mc} \right)^2 \right] \right\}.
\]

(36)

In Fig. 5, we have plotted the equilibrium distributions for two values of \( T_{b}^{\text{max}} \): \( 5 \cdot 10^{11} \) K and \( 5 \cdot 10^{12} \) K. The corresponding stationary temperatures, as estimated using nonrelativistic Eq. (16), are \( k T_{\text{eq}} = 95 \) keV and 950 keV. The exact result was obtained by numerical evaluation of Eq. (34) using Eqs. (30) and (31). Also plotted are the nonrelativistic Maxwellian distribution at temperature \( T_{\text{eq}}^0 \) and the mildly-relativistic approximation given by Eq. (36). One can see that the right wing of the distribution is substantially suppressed compared to the nonrelativistic Maxwellian distribution. This takes place because of the decreasing diffusion coefficient and increasing breaking force with increasing \( p \). Surprisingly enough, the approximate formula (36) provides an excellent fit even when \( k T_{\text{eq}}^0 \gg mc^2 \).

Since the distribution (34) becomes a Maxwellian one in the limit \( k T_{\text{eq}}^0 \ll mc^2 \) and assumes a quasi-Maxwellian shape in the mildly-relativistic regime, it is natural to characterize this type of distribution by some effective temperature, \( T_{\text{eff}} \). We define \( T_{\text{eff}} \) to be the temperature of the relativistic Maxwellian distribution for which the mean electron energy, \( \langle \gamma \rangle mc^2 = \langle (p^2 + 1)^{1/2} \rangle \), is equal to that for a given quasi-Maxwellian distribution, i.e.,

\[
\frac{\int (p^2 + 1)^{1/2} p f_{\text{eq}}(p) dp}{\int p^2 f_{\text{eq}}(p) dp} = \langle \gamma \rangle (T_{\text{eff}}) mc^2,
\]

(37)
where the dependence of $\langle \gamma \rangle$ on temperature is given by Eq. (18).

For the two example distributions presented in Fig. 5, $kT_{\text{eff}}$ takes values of 45 and 140 keV — these should be compared with $kT_{\text{eq}} = 95$ and 950 keV, respectively. The relativistic Maxwellian distributions that correspond to these $T_{\text{eff}}$ values are shown in Fig. 5.

In Fig. 6, we have plotted $T_{\text{eff}}$, calculated using both the exact formula (34) and its mildly-relativistic approximation (35), as a function of $T_{\text{max}}$. For comparison, the dependence $T_{\text{eq}}(T_{\text{max}})$ for Maxwellian electrons is reproduced from Fig. 3. One can see that the two exact dependencies are nearly coincident (the difference is less than 10%) in an extremely broad range of parameter values: $kT_{\text{eff}}, kT_{\text{eq}} \lesssim mc^2$. Even more surprising is the nearly perfect agreement between the exact solution for $T_{\text{eq}}(T_{\text{max}})$ and the mildly-relativistic approximation (Eq. [35]) for $T_{\text{eff}}(T_{\text{max}})$. We should emphasize here that the differences between the different dependences shown in Fig. 6 are real (as is confirmed by the fact that they diverge significantly when $T_{\text{eff}} \gg mc^2$), although very small. An important conclusion follows: the energy that can be accumulated by an ensemble of electrons as a result of induced Compton heating (with inverse Compton scattering serving to cool the electrons) is almost independent in the sub-relativistic regime on whether the electrons are maintained Maxwellian while being heated or not.

We have also checked that the contribution of the heated electrons to the gas pressure, which is proportional to $\langle p\beta \rangle \sim \int (p^2 + 1)^{1/2} [1 - (1 + p^2)^{-1}] p^2 f(p) dp$ (compare with Eq. [37] for the mean energy), proves to be nearly the same (to within 2% for arbitrary values of $T_{\text{eff}}$) for a quasi-Maxwellian plasma with the distribution function (34) and for the thermal plasma with $T = T_{\text{eff}}$ (note that in the limit $kT_{\text{eff}} \gg mc^2$ both pressures must be equal, because $\beta \to 1$). Therefore, the effective temperature $T_{\text{eff}}$ perfectly characterizes the thermodynamic properties of a mildly relativistic quasi-Maxwellian plasma that has been heated by means of the induced Compton process.

The above discussion also suggests that one can make use of the simple expression (35) to estimate the mean stationary electron energy with high accuracy in a very broad parameter range, at least up to $T_{\text{eff}} \sim mc^2$, which for our model spectrum corresponds to as high as $T_{\text{max}} \sim 10^{15}$ K, independent of whether thermalization takes place or not.

Suppose that plasmas whose electrons are in the distribution given by Eq. (35) do exist. Would they be different from thermal plasmas observationally? In particular, will the energy spectrum of hard X-ray bremsstrahlung emission from such plasmas be peculiar? We have computed such a spectrum for the momentum distribution shown in Fig. 5a ($kT_{\text{eff}} = 45$ keV). The result is presented in Fig. 7. The computation consisted of weighting the Bethe-Heitler formula (Jauch, Rohrlich, 1976) for the cross-section of electron-ion bremsstrahlung with the given momentum distribution (Eq. [36]). One can see that the deviation from the spectrum produced by an ensemble of relativistic Maxwellian electrons with temperature $T_{\text{eff}}$ is negligible. Only when $kT_{\text{eff}} \gg 100$ keV, does the bremsstrahlung spectrum formed in the quasi-Maxwellian plasma become noticeably different from that corresponding to a thermal plasma.

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Appendix

In a recent paper (Sazonov, Sunyaev, 2000) we have given an analytic expression for the kernel of the integral kinetic equation describing the redistribution of photons in frequency as a result of induced Compton scattering in a mildly relativistic thermal plasma. This kernel allows one, in principle, to derive the terms associated with the induced Compton process in Eq. (31) — the generalized Kompaneets equation. Unfortunately, the published expression contains a minor error (in the leading coefficient), and as a consequence, the formula does not correspond to the stated accuracy (it is valid when \( kT \lesssim 0.01mc^2 \), instead of \( kT \lesssim 0.1mc^2 \)). We use the opportunity to give here the correct expression:

\[
K_{\text{ind}}(\nu, \Omega, \nu', \Omega') = \left( \frac{\nu'}{\nu} \right)^2 K(\nu', \Omega' \rightarrow \nu, \Omega) - K(\nu, \Omega \rightarrow \nu', \Omega')
\]

\[
= \frac{3}{32\pi} \sqrt{\frac{2}{\pi}} \left( \frac{kT}{mc^2} \right)^{-3/2} \frac{h\nu(\nu' - \nu)}{mc^2 \nu \nu'} \left[ 1 + \mu^2 + \left( \frac{1}{8} - \mu - \frac{63}{8} \mu^2 + 5\mu^3 \right) \frac{kT}{mc^2} \right.
\]

\[
- \mu(1 - \mu^2) \left( \frac{\nu' - \nu}{g} \right)^3
- \frac{3(1 + \mu^2)mc^2}{8kT} \left( \frac{\nu' - \nu}{g} \right)^4 \exp \left[ -\frac{(\nu' - \nu)^2mc^2}{2g^2kT} \right],
\]

\[
g = |\nu\Omega - \nu'\Omega'| = (\nu^2 - 2\nu\nu'\mu + \nu'^2)^{1/2}.
\]

Here, \( \nu \) and \( \Omega \) are the frequency and direction of propagation of a photon before scatter, \( \nu' \) and \( \Omega' \) are the corresponding values after scatter, and \( \mu = \Omega\Omega' \) is the scattering angle.

An analogous correction is due for the kernel averaged over the scattering angle (see Sazonov, Sunyaev, 2000):

\[
P_{\text{ind}}(\nu, \nu') = \left( \frac{\nu'}{\nu} \right)^2 P(\nu' \rightarrow \nu) - P(\nu \rightarrow \nu') = 2\sqrt{\frac{2}{\pi}} \left( \frac{kT}{mc^2} \right)^{-3/2} \frac{h\nu(\nu' - \nu)}{mc^2 \nu \nu'} (p_0 + p_t),
\]

\[
p_0 = \left( \frac{11}{20} + \frac{4}{5} \delta^2 + \frac{2}{5} \delta^4 \right) F + |\delta| \left( -\frac{3}{2} - 2\delta^2 - \frac{4}{5} \delta^4 \right) G,
\]

\[
p_t = \left[ \left( \frac{-1091}{1120} - \frac{507}{560} \delta^2 + \frac{57}{35} \delta^4 + \frac{68}{35} \delta^6 \right) F + |\delta| \left( \frac{9}{4} + 2\delta^2 - \frac{26}{5} \delta^4 + \frac{136}{35} \delta^6 \right) \right] \frac{kT}{mc^2},
\]

\[
F = \exp(-\delta^2), \quad G = \int_{|\delta|}^\infty \exp(-t^2) \, dt = 0.5\pi^{1/2} \text{Erfc}(|\delta|),
\]

\[
\delta = \left( \frac{2kT}{mc^2} \right)^{-1/2} \frac{\nu' - \nu}{\nu' + \nu}.
\]

The formulae (38) and (39) are applicable in the range \( h\nu \ll kT \lesssim 0.1mc^2 \), \( nh\nu = kT_b \gg kT \) (the latter condition means that the inverse Compton effect is small with respect to the induced one).

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Figure 1: Radiation spectrum produced by a self-absorbed synchrotron source, described by Eqs. (12) and (13). The frequency is measured in units of the characteristic frequency $\nu_0$. 
Figure 2: (a) Deviation of the induced Compton heating rate of Maxwellian electrons located in an isotropic field of self-absorbed synchrotron radiation from the nonrelativistic estimate, $W_0^+$ (Eq. [11]), as a function of temperature. The solid line — the exact result, obtained by weighting Eq. (4) with a relativistic Maxwellian distribution. The dashed and dash-dotted lines represent the results of the calculation by the mildly-relativistic formula (10), in which retained are, respectively, only the correction term $O(kT/mc^2)$ and all quoted terms up to $O((kT/mc^2)^3)$. (b) Deviation of the inverse Compton cooling rate from the nonrelativistic estimate, $W_0^-(T)$ (Eq. [19]). The solid line — the exact result (Eq. [17]). The dashed and dash-dotted lines represent the results of the calculation by the mildly-relativistic formula (20), similarly as in (a).
Figure 3: Stationary electron temperature vs. peak radiation brightness temperature. This dependence results from the balance $W^+(T_{eq}) = W^-(T_{eq})$, with $W^+(T)$ and $W^-(T)$ as plotted in Fig. 2. The types of the lines have the same meaning as in Fig. 2. Also shown (the dotted line) is the nonrelativistic result (Eq. [16] or Eq. [23]).
Figure 4: Evolution of the temperature of Maxwellian electrons during their Compton interaction with self-absorbed synchrotron radiation. The curves are labelled with the corresponding values (in K) of the peak brightness temperature $T_b^{\text{max}}$. At moment $t = 0$ the plasma is cold.
Figure 5: (a) Equilibrium distribution of electrons located in an isotropic field of self-absorbed synchrotron radiation with $T_{\text{max}} = 5 \cdot 10^{11}$ K. The solid line — the exact result, obtained using Eqs. (34), (30) and (31). The dotted line — the nonrelativistic Maxwellian distribution with $kT_{\text{eq}} = 95$ keV, a value found from Eq. (16). The dashed line (almost coincident with the solid line) — the mildly-relativistic approximation (Eq. (36)). The dash-dotted line — the relativistic Maxwellian distribution with $kT = kT_{\text{eff}} = 45$ keV, a value found from Eq. (37). The mean electron energy for this distribution is equal to that for the equilibrium distribution. (b) The same as (a), but $T_{\text{max}} = 5 \cdot 10^{12}$ K, in which case $kT_{\text{eq}} = 950$ keV and $kT_{\text{eff}} = 140$ keV.
Figure 6: Effective temperature of the stationary electron distribution, defined by Eq. (37), vs. the peak radiation brightness temperature. The solid line — the exact result, obtained using Eqs. (34), (30) and (31). The dashed line — the result for the distribution given by the approximate formula (35). The dependence of the stationary temperature of Maxwellian electrons on $T_b^{\text{max}}$ is reproduced from Fig. 3 for comparison (the dash-dotted line).
Figure 7: Spectrum of hard X-ray bremsstrahlung emission from a plasma whose electrons are in the distribution plotted in Fig. 5a (the solid line), compared with the spectrum (the dashed line) that corresponds to the relativistic Maxwellian distribution with $kT = kT_{\text{eff}} = 45$ keV.