Quantum state engineering with single atom laser

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Abstract. On the basis of quantum stochastic trajectories approach it is shown that a single
atom laser with coherent pumping can generate not only coherent states, but squeezed and
Fock states, when different schemes of detection are followed by coherent feedback pulses or
feedforward actions.

1. Introduction
A single atom laser (SAL) is the limiting case of lasers. This miniaturized model naturally
appears considering the resonant interaction of an atom with the quantum field of high-Q cavity
both an optical [1], and microwave [2, 3] ranges. To describe it, over fifty years ago Jaynes and
Cummings introduced the model [4], and a numerous theoretical and experimental researches
of their model led to the discovery of many quantum effects, including Rabi oscillation [5],
collapse-revival effect [6], phase bistability [7], effect of compression (sub-Poisson distribution of
the number of photons) [8] and others.

Of particular interest is the consideration of the Jaynes-Cummings model supplemented
taking into account the relaxation processes of atomic and field dissipation and pumping. In
this case, all radiated photons in the resonator mode are indistinguishable. Thus the conclusions
based on the difference between spontaneous and stimulated photons emission, become invalid.
In particular, the proposition that the two level system cannot provide the generating radiation.
As shown in [9, 10] steady-state distribution of photons emitted by the single atom laser can
produce Poisson light.

In this paper by using the method of quantum stochastic trajectories [11, 12] we show that
the single atom laser also can be used for engineering another quantum states, for example the
Fock states or squeezed states.

2. Model
The model of SAL is a two-level atom (with ground \( |g⟩ \) and excited \( |e⟩ \) states) in a high-
Q cavity interacting with a single field mode via the Jaynes-Cummings Hamiltonian
\( H_{JC} = \hbar g (a^† \sigma_- + a \sigma_+) \), where \( g \) is the interaction constant, \( \sigma_+ = |e⟩⟨g| \), \( \sigma_- = |g⟩⟨e| \) are atomic
transition operators and \( a^†, a \) are field creation and annihilation operators. The field and atomic
decay rates \( 2k \) and \( \Gamma \) are taken into account through the corresponding Lindblad superoperators
\[ 2L(x)ρ = 2xρx^† - x^†xρ - ρx^†x \]
in master equation for the density matrix \( ρ \). Also we consider
the possibility of switching on and off of the coherent pumping in specific times. For simplicity
the amplitude of external field is supposed constant. The coherent pump is described by the
interaction operator
\[ V = \frac{E_0}{2} (D_{ge}\sigma_- + D_{eg}\sigma_+), \]  
(1)
where \( D_{ge} = D_{eg} = D \) is the dipole transition moment of atom and \( E_0 \) is the amplitude of external field. Resulting master equation is the following
\[ \dot{\rho} = -\frac{i}{\hbar} [H_{JC} + V, \rho] + 2kL(a)\rho + \Gamma L(\sigma_-)\rho. \]
(2)

3. Coherent feedback excitation with atomic and field decay detection
It can be shown for SAL with incoherent pumping that possibility of atomic decay and pumping and field decay detection allows to determine the state into cavity in time between clicks of detector as
\[ |\Psi\rangle = c(t)|g\rangle \otimes |n\rangle + f(t)|e\rangle \otimes |n-1\rangle. \]
(3)
It is also performed for SAL without pumping (click of any detector means decreasing \( n \) by one). Because at some time coefficient \( f(t) \) in (3) will be zero, we can act on system by short coherent \( \pi/2 \)-pulse [13]. This action increases the amount of excitation by one. Thus, we can represent evolution of the system as quantum stochastic trajectory, which consists of two interchangeable stages. The first is the stage of preparing (we wait time when atom will be in ground state). The second is the stage of pumping (we transfer atom from ground to excited state). After the registration of atomic decay in any time of evolution we pump system by \( \pi/2 \)-pulse at once. After the field decay registration we must wait a specific time for \( \pi/2 \)-pulse again.

For modeling of quantum stochastic trajectory rewrite (2):
\[ \dot{\rho} = -iH_{eff}^p\rho + i\rho H_{eff}^d + J\rho, \]
(4)
where \( H_{eff}^p \) and \( H_{eff}^d \) are effective non-hermit Hamiltonians for stages of preparing and pumping respectively:
\[ H_{eff}^d = (H_{JC} + V)/\hbar - iR, \]
(5)
\[ H_{eff}^p = H_{JC}/\hbar - iR, \]
(6)
\[ 2R = \Gamma \sigma_+ \sigma_- + 2ka^\dagger a, \]
(7)
and superoperator of reduction \( J \) is consisted of two parts:
\[ J = \sum_{\zeta=B,F} J_{\zeta}, \]
(8)
that act as:
\[ J_Bx = \Gamma \sigma^- x \sigma^+ \equiv \Gamma \langle b|x|b\rangle |a\rangle \langle a|, \]
(9)
\[ J_Fx = 2kaxa^\dagger \equiv 2k \sum_{m,n} \sqrt{mn} \langle m|x|n\rangle |m-1\rangle \langle n-1|. \]
(10)
It means that \( J_B \) operator by acting on \( x \) creates new operator with \( R_{ba} \langle b|x|b\rangle |a\rangle \langle a|. \) Physical meaning of \( J_B\rho(t) \) is committing atomic decay in time \( t \) (commits \( B \)-type jump). Analogically, \( J_F \) operator transforms field components of \( x \) \( \langle m|x|n\rangle |m\rangle \langle n| \) into \( 2k \sqrt{mn} \langle m|x|n\rangle |m-1\rangle \langle n-1| \), and \( J_F\rho(t) \) is committing field decay in time \( t \) (commits \( F \)-type jump).
After each stage of evolution we redefine density matrix: \( \rho_{t_i-1} = \rho_i^{(0)} \). We also need probabilities of committing jumps:

\[
P_B^{(i)} = \int_0^{t_i} J_B S^{(\tau,0)}_\kappa \rho_i^{(0)} d\tau, \tag{11}
\]

\[
P_F^{(i)} = \int_0^{t_i} J_F S^{(\tau,0)}_\kappa \rho_i^{(0)} d\tau, \tag{12}
\]

\( S^{(t_i, t_{i-1})}_\kappa \) is non-unitary evolution operator:

\[
S^{(t_i, t_{i-1})}_\kappa \rho(0) = e^{-iH_{\text{eff}}(t_i-t_{i-1})} \rho(0)e^{iH_{\text{eff}}^*(t_i-t_{i-1})}. \tag{13}
\]

Because of the additivity of the probabilities, each stage contributes to the general probabilities of committing jump:

\[
P_k = \sum_{i=1}^{k} (P_B^{(i)} + P_F^{(i)}), \tag{14}
\]

so, if general probability \( P_{k-1} \) on \( k-1 \) stages is more than generating random number \( \chi \) \((0 \leq \chi \leq 1)\), jump happens on stage \( k \) (if \( P_{k-1} < \chi < P_{k-1} + P_B^{(k)} \) then \( B \)-type take place, if \( P_{k-1} + P_B^{(k)} < \chi < P_{k-1} + P_B^{(k)} + P_F^{(k)} \) then \( F \)-type follows).

**Figure 1.** Quantum trajectory with the detection of decay clicks. Black line is the evolution of excitation number \( m \) (photon number plus unity if atom occupies excited state); red lines indicate \( \pi/2 \)-pulses; green lines represent the times of field decay detecting; orange lines represent the times of atom decay detecting and immediately after their detection runs \( \pi/2 \)-pulse.

**Figure 2.** The quantity of relaxation acts for obtaining the \( m \) excitation level with probability 1-90%, 2-95%, 3-99.5%.

The second random number \( \beta \) allows to find the time of committing jump \( t \):
\[ \beta = \frac{\int_0^{t_k} J_\xi S_\kappa^{(\tau,0)} \rho_\kappa^{(0)} d\tau}{\int_0^{t_k} J_\xi S_\kappa^{(\tau,0)} \rho_\kappa^{(0)} d\tau}, \] (15)

where \( \xi = B \) or \( F \) respectively and \( t_k \) is the waiting time counted from time of atomic transition. We will use parameters for numerical calculation from paper [1]: \( 2g = 2\pi \ast 68MHz \), \( \Gamma = 2\pi \ast 2.6MHz \), \( k = 2\pi \ast 4.1MHz \). Normalize parameters are \( \eta = g^2/k^2 = 68.8 \), \( \nu = (\Gamma - 2k)/4k = -0.34 \). The value \( d = E_0D/(2h) = 20000 \) is taken from consideration that the duration of the pumping stage \((\pi/(2d) = 19ps)\) was several orders of magnitude less than the duration of the preparing.

On Figure 1 a quantum trajectory is modeled by the approach described above. The proposed pump method can be used to generate Fock states, but in this case, the obtaining of any state is a statistical problem, and it is necessary to estimate the time at which the probability of generation of a state will be given (for example, at which time will state \( |10\rangle \) appear with a probability of \( P = 99.5\% \)?)

It is necessary to solve recurrent equation:

\[ W_k^i = \sum_{l=0}^{m} \left( P_F^l (1 - P_F^{l-1}) - P_B^l (1 - P_B^{l-1}) \right) W_{l-1}^{i-1} + P_B^l W_k^{i-1}, \] (16)

\[ W_0^0 = W_1^1 = \prod_{l=0}^{m} (1 - P_B^l - P_F^l), \] (17)

\[ P = \sum_{j=0}^{s} W_j^j. \] (18)

Then we will know amount of click of detectors \( s \) that needed for generating Fock state \( m \) with probability \( P \) (see Figure 2).

AS example, for time 51\mu s the ten-photons Fock state will be generated with probability 99.5\% (see Figure 2, line 3, amount of jumps is \( s = 1083 \)).

4. Photon clicks detection: feedforward strategy

In the absence of detection of acts of atomic decay, the state inside the resonator will no longer be pure. Therefore, the main reason for using the pumping by the short pulses is irrelevant. For this case we consider continuous coherent pumping.

Similar to the previous section we can modulate the quantum stochastic trajectory of single atom laser. Rewrite (2):

\[ \dot{\rho} = -i\hat{H}_{eff}\rho + i\rho \hat{H}_{eff}^\dagger + J_F \rho, \] (19)

here \( J_F \) is jump operator (10) and \( \hat{H}_{eff} \) is effective Hamiltonian from which the non-unitary operator is built:

\[ -i\hat{H}_{eff}\rho + i\rho \hat{H}_{eff}^\dagger = \frac{i}{\hbar} [H_{JC} + V, \rho] + \Gamma L_{\sigma - \rho} - k \sigma^1 a \rho - k \rho a^1, \] (20)

\[ \tilde{S}^{(t_1-t_{l-1})} \rho(0) = e^{-i\hat{H}_{eff}^{(t_1-t_{l-1})} \rho(0)} e^{i\hat{H}_{eff}^{(t_1-t_{l-1})}}. \] (21)

We need only one random number to find the time of committing \( F \)-type jump:

\[ \chi = \int_0^{t_i} \tilde{J}_i S^{(\tau,0)} \rho_i^{(0)} d\tau, \] (22)
since probability of committing jump always is equal to one.

Figure 3 shows a simulated quantum trajectory. The effect of the decay of the field on the state of the system leads to the fact that short intervals between jumps transform the state inside the cavity into a state with almost exact Poisson statistics (inset a). And even a strong change in the state of the field due to the long absence of decay (inset b) is leveled by a further evolution (inset c).

![Figure 3. Quantum trajectory with the detection of field decay clicks only. The blue vertical lines indicate the times at which the detector that detects the decay of the photon from the cavity has triggered. Insets a), b), c) show the distribution of the system from photon numbers at the time before (red histogram) and after (blue histogram) of the jump ($k\tau$ is time waiting of jump); black points marked coherent state with $|\alpha|^2 = 17.04$.](image)

Thus, it is possible to generate states whose statistics coincide with the statistics of coherent states with great accuracy.

5. No clicks strategy

The last considered case is a single atom laser without detection of atomic and field decays. It is of interest to analyze the state of the system under the action of alternating stages of switching on / off coherent pumping. We represent the solution of the master equation (2) in the form:

$$\rho(t) = S_{\kappa}^{(t,0)} \rho,$$

where $S_{\kappa}^{(t,0)}$ with $\kappa = d$ is evolution operator with coherent pumping, and with $\kappa = g$ without pumping.

Then the state of the system through $n$ alternating stages can be written as:

$$\rho(t_n, t_{n-1}, ..., t_1) = S_g^{(t_n, t_{n-1})} S_d^{(t_{n-1}, t_{n-2})} ... S_g^{(t_2, t_1)} S_d^{(t_1, 0)} \rho(0).$$

The obtained state can be investigated from different points of view, for example, find the values $t_1, ... t_n$ for which the state inside the system will be the most squeezed in the coordinate
quadrature:

\[(\Delta X_1)^2 = \langle \left( \frac{a + a^\dagger}{2} \right)^2 \rho(t_n, t_{n-1}, ..., t_1) \rangle - \langle \left( \frac{a + a^\dagger}{2} \right) \rho(t_n, t_{n-1}, ..., t_1) \rangle^2. \tag{25} \]

Numerical calculations establish the values \( t_j, j = 1, 6 \), at which the minimal value of the coordinate quadrature is equal to \( \langle \Delta X_1^2 \rangle = 0.12559 \) (see Figure 4). It is achieved by three pumping pulses, and further alternating pumping / absence of pumping will not reduce it. The distribution of excitations of the system at time of maximum of squeezing is shown on the Figure 5.

![Figure 4.](image)

**Figure 4.** The value of the coordinate quadrature of the state of the system as a function of time; blue segments are pumping stages, red are stages of absence of pumping. The minimum value of \( \langle \Delta X_1^2 \rangle = 0.12559 \) is reached.

![Figure 5.](image)

**Figure 5.** The distribution of excitations of the system at time of maximum of squeezing.

6. Conclusion

In this paper it is shown that a single atom laser can be of interest as a practical device generating a wide class of quantum states. The class type of states depends on the observation of the occurring relaxation processes in the system, which was demonstrated using the parameters realized in a particular experiment [1].

By registering all decays of the system into both the cavity and non-cavity modes, the state of the system becomes localized. The use of pulsed coherent pumping allows us to change the values of the number of excitations inside the resonator, and if we can provide sufficient time observations of the system, we obtain the desired Fock state.

By registering only decays of the system into the cavity mode, with continuous coherent pumping, we can generate coherent states.

Without detecting of the relaxation of the system, the generated states do not belong to any nominal class of states, but we can influence their statistical properties by switching on and off coherent pumping, and as a result obtain states with the necessary properties. For example, it has been shown that a squeezed state can be obtained.

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