Combustion Process in a Spark Ignition Engine: Dynamics and Noise Level Estimation

T. Kamiński and M. Wendeker

Department of Combustion Engines, Technical University of Lublin,
Nadbystrzycka 36, PL-20-618 Lublin, Poland

K. Urbanowicz

Faculty of Physics, Warsaw University of Technology,
Koszykowa 75, PL-00-662, Warsaw, Poland.

G. Litak

Department of Mechanics, Technical University of Lublin,
Nadbystrzycka 36, 20-618 Lublin, Poland

(Dated: September 30, 2003)

Abstract

We analyse the experimental time series of internal pressure in a four cylinder spark ignition engine. In our experiment, performed for different spark advance angles, apart from usual cyclic changes of engine pressure we observed oscillations. These oscillations are with longer time scales ranging from one to several hundred engine cycles depending on engine working conditions. Basing on the pressure time dependence we have calculated the heat released per cycle. Using the time series of heat release to calculate the correlation coarse-grained entropy we estimated the noise level for internal combustion process. Our results show that for a smaller spark advance angle the system is more deterministic.
A combustion process in spark ignition engines is known as nonlinear and noisy one. Combustion instabilities are occurring as a cycle-to-cycle variations of internal cylinder pressure effecting directly on the power output. Examination of these variations can lead to better understanding of their sources and help in their eliminations in future. Improving engine efficiency requires achieving better combustion conditions without introducing additional disturbances. In the present paper we analyse the dynamics and estimate the noise level in combustion process basing on experimental time series of internal pressure and calculated from them heat release. In the following analysis we apply the nonlinear multidimensional methods which can distinguish random variations from a deterministic behaviour.

I. INTRODUCTION

Combustion in four stroke spark ignition (SI) engines is a complex cyclic process consisted of air intake, fuel injection, compression, combustion, expansion and finally gas exhaust phases (Fig. 1) where burned fuel power is transmitted through the piston to the crankshaft. In early beginning of SI engine development there were observed instabilities of combustion\(^1\). These instabilities are causing fluctuations of the power output making it difficult to control\(^2,3\). The problems of their sources identification and their elimination have became the main issues in SI engines technologies engineering and they have not been solved up to present time\(^4\). Among the the main factors of instabilities classified by Heywood\(^5\) are aerodynamics in the cylinder during combustion, amounts of fuel, air and recycled exhaust gas supplied to the cylinder and a local mixture composition near the spark plug.

Recently, Daw \textit{et al.}\(^6,7\) and Wendeker \textit{et al.}\(^8\) have done the nonlinear analysis of such process. Changing an advance spark angle they observed the considerable increase of pressure fluctuations level\(^8\) claiming that it is due to nonlinear dynamics of the process. In the other work\(^8\) Wendeker and coworkers have proposed intermittency mechanism to explain the route to eventually chaotic combustion.

Prompted by these findings we decided to analyse the the correlation entropy of the combustion process in different working conditions of the engine. With a help of entropy produced by the dynamical system we can quantify the level of measurement or dynami-
FIG. 1: Schematic picture of a combustion cycle in a four-stroke spark ignition engine.

In the present paper we shall start our analysis from examining experimental pressure time series. It should be noted that pressure is the best known quantity to analyse engine dynamics. Cylinder pressure together with volume data can be used to obtain indicated mean effective pressure (IMEP), calculate the engine torque, indicated efficiency and also burn rate, bulk temperature and heat release. Moreover, statistical analysis of the pressure data can also provide information about combustion process stability.

However, in practice, it is not easy to perform direct measurement of pressure\cite{11}, as one needs a good sensor persistent to high temperatures to be placed inside the engine cylinder. Therefore to obtain information about pressure some researchers developed alternative non-direct measurement procedures\cite{12}.

In our case we have been dealing with novel pressure fibre optical sensors\cite{13}. Due to applying them noise from measurement is very low, comparing to traditional piezo-electric ones\cite{14}. This enabled us to examine the dynamics more effectively than it was possible in earlier investigations.

The present paper is divided into 6 sections. After present introduction (Sec. 1), we will provide the description of our experimental standing and measurement procedure in Sec. 2. There we present some examples of cycle-to-cycle variations in pressure inside one of cylinder. In Sec 3. we examine the pressure with more detail. We also perform spatio-temporal analysis comparing the fluctuations of pressure in succeeding cycles for different advance angles. In Sec. 4 we calculate the heat release per cycle. Finally in Sec. 5 we analyse its time dependence and show our main result i.e. level of noise. We end up with
II. EXPERIMENTAL FACILITIES AND MEASUREMENTS OF INTERNAL PRESSURE

In our experimental stand (Fig. 2) pressure was measured directly inside cylinder by the use of the optical fibre sensor. Such equipment provides one of the most direct measures of combustion quality in an internal combustion engine. Internal pressure data were obtained from Engine Laboratory of Technical University of Lublin, where we conducted series of tests.

The pressure traces were generated on a 1998 cm$^3$ Holden 2.0 MPFI engine at 1000 RPM. The data was captured by use of NuDAC–TK v.2.0 data acquisition and data processing program. The original files contained cylinder pressure at crank angles 0-720 degrees. Each of three large file (about 990 MB per each) contained above 10000 combustion cycles. Data was taken at different spark timings (spark advance angles): 5, 15, 30 degrees before top dead center (BTDC). The engine speed, air/fuel ratio, and throttle setting were all held constant throughout the data collection period. Intake air pressure, in inlet pipe, was value 40 kPa. Torque for each of three spark timings were adequately: 21, 28 and 30 Nm.

To perform signal analysis we needed large enough data. In this aim we measured 10000 cycles for each of three spark advance angles $\Delta \alpha_z$. The results for first 1000 cycles are shown in Figs. 3a-c. Note that depending on an advance angle we have more or less broadened region of pressure fluctuations. The full line shows the pressure averaged over the first 1000 cycles. It is increasing with growing $\Delta \alpha_z$ and reaches its highest value for $\Delta \alpha_z = 30^\circ$.

Note also, in our for four stroke engine the combustion period (Figs. 1, 3) corresponds exactly to the double period of the crankshaft revolution synchronised with a single spark ignition. Every combustion cycle starts with initial conditions given by a mixture of air and
| variable                           | symbol |
|-----------------------------------|--------|
| position of the piston            | $h$    |
| internal cylinder pressure        | $P$    |
| actual cylinder volume            | $V(\alpha)$ |
| Heaviside step function           | $\Theta(z)$ |
| heat released                     | $Q$    |
| heat released in particular cycle $i$ | $Q_i$ |
| heat released vector in embedding space | $\mathbf{Q}$ |
| spark advance angle               | $\Delta \alpha_z$ |
| embedding time delay in cycles    | $m$    |
| cycle number                      | $i, j$ |
| embedding dimension               | $n$    |
| number of considered points in time series | $N$ |
| loading torque                    | $F$    |
| crank angle                       | $\alpha \in [0, 720^\circ]$ |
| threshold                         | $\varepsilon$ |
| correlation integral              | $C^n$  |
| coarse-grained correlation integral | $C^n(\varepsilon)$ |
| correlation entropy               | $K_2$  |
| coarse-grained correlation entropy | $K_2(\varepsilon)$ |
| calculated from time series       | $\hat{\chi}$ |
| coarse-grained entropy            | $D_2$  |
| correlation dimension             | $\text{NTS}$ |
| Noise-to-Signal ratio             | $\sigma_{\text{DATA}}$ |
| standard deviation of data        | $\text{Erf}(z)$ |
| error function                    | $\chi, a, b$ |
| fitting parameters                | $\sigma$ |
FIG. 3: Internal pressure of 1000 combustion cycles against a crank angle for a spark advance angle $\Delta \alpha_z = 5$, 15 and 30 degrees for Figs. 3a,b and c, respectively. Full lines correspond to the average angular pressure.

Fuel. All of succeeding cycles are separated by gas exhaust and intake stroke phases. That process is in general nonlinear and can be also mediated by stochastic disturbances coming from e.g. non homogenous spatial distribution of fuel/air ratio. After combustion exhaust gases are mixed with fresh portions of fuel and air.

Therefore the residual cylinder gases after each combustion cycle influence the process in a succeeding cycle leading to different initial conditions of air, fuel and residual gas mixture contents.
TABLE I: continuation

|                              |       |
|------------------------------|-------|
| cylinder diameter            | $D = 86\ mm$ |
| crank radius                 | $r = 43\ mm$ |
| connecting-rod length        | $l = 143\ mm$ |
| heating value of the fuel    | $W_u = 43000\ kJ/kg$ |
| compression ratio            | $\varepsilon = 8.8$ |
| Poisson constant             | $\kappa = \frac{c_p}{c_v} \approx 1.4$ |
| mass burned in cycle $i$     | $M_i$ |
| autocorrelation function     | $AC(j)$ |
| output torque                | $S$ |

III. ANALYSIS OF PRESSURE

During the combustion process the internal volume of engine cylinder is driven kinematically by the piston. In a consequence of above the internal it changes also periodically as a function of crank angle $\alpha$ and satisfying the relation

$$V(\alpha) = \pi \frac{D^2}{4} h + \pi \frac{D^2}{4} 2r \frac{1}{\varepsilon - 1},$$  \hspace{1cm} (1)

where the piston position $h$

$$h = r(1 - \cos \alpha) + l \left(1 - \frac{r}{l} \sqrt{\frac{l^2}{r^2} - \sin^2 \alpha}\right),$$ \hspace{1cm} (2)

and constants $r$, $l$, $D$ as well as $\varepsilon$ are defined in Tab. I.

In some sense the combustion initiated by ignition in each engine cycle is an independent combustion event. Such events are separated by the processes of exhaust and intake dependent on the amount of combusting fuel mass and quality of newly prepared fuel-air mixture. To illustrate this effect we are showing in Figs 4a-c spatio-temporal plots corresponding to first 1000 cycles of our pressure time series for different advance angle $\Delta \alpha_z = 5, 15, 30$ degrees, respectively. Each colour in Figs. 4a-c correspond to one of four interval of $[P_{\min}, P_{\max}]$: white $[P_{\min}, P_1]$, green $[P_1, P_2]$, red $[P_2, P_3]$ and blue $[P_3, P_{\max}]$. One can easily see that the pressure signal, especially in Fig. 4c seems to change in some regular manner of a time scale of about 100 cycles. Similar feature is also visible in Fig. 4a while it is difficult
FIG. 4: Four colour Spatio-temporal corresponding combustion process parameters as in Fig. 3a-c, respectively. Each colour correspond to one of four interval of $[P_{\text{min}}, P_{\text{max}}]$: white $[P_{\text{min}}, P_1]$, green $[P_1, P_2]$, red $[P_2, P_3]$ and blue $[P_3, P_{\text{max}}]$ ($P_{\text{min}} = -0.2\,\text{MPa}$, $P_1 = 0.1\,\text{MPa}$, $P_2 = 0.2\,\text{MPa}$, $P_3 = 0.6\,\text{MPa}$, $P_{\text{max}} = 2.0\,\text{MPa}$).

to find such regularity in Fig. 4b. However after more careful examination one can identify such a time scale consisting of about 450 cycles.

The broad angular interval of fluctuations visible in Fig. 3b. has its consequences in irregularly border between red and blue colours (Fig. 4b).

IV. VARIATIONS OF HEAT RELEASE

To capture the cycle-to-cycle changes in combustion process we decided to calculate of heat release in a sequence of combustion cycles. This quantity is more convenient to examine
stability of combustion process because enables us to concentrate on it. In contrast to that internal pressure was is effected by combustion and cyclic compression. Heat release has also practical meaning as it is proportional to burned fuel mass.

For an adiabatic process the heat released from chemical reactions during combustion in the engine is given by an differential equation coming from the first law of thermodynamics with respect to a crank angle $\alpha$:

$$\frac{dQ}{d\alpha} = \frac{\kappa}{\kappa - 1} p \frac{dV}{d\alpha} + \frac{1}{\kappa - 1} V \frac{dp}{d\alpha}. \quad (3)$$

Using it together with the pressure time series and parametric change of cylinder volume $V$ Eqs. 1-2 we have done calculation of heat released in succeeding cycles $Q_i$. 

FIG. 5: Heat release per cycle versus sequential cycles.
It is closely related to burned fuel mass in one cycle

\[ M_i = Q_i / W_u, \]  \hspace{1cm} (4)

where \( W_u \) is heating value of the fuel, listed in Tab. 1. It should be underlined that the last equation (Eq. 4) neglects the effects of heat exchange between the cylinder chamber and its walls. This is in spirit of an adiabatic process assumption (Eq. 3). Of course the consumed mass will be larger because it also depends on the quality of mixture and combustion process.

The calculated heat release \( Q_i \), for first 5000 cycles is plotted against cycles \( i \) in Fig. 5a-c for \( \Delta \alpha_z = 5, 15, 30 \) degrees, respectively.

Note that in all cases there is some modulation ranges from one to few hundred cycles. Interestingly, for a small advance angles \( \delta \alpha_z = 5^\circ \) or \( 15^\circ \) this modulation evolute indicating that the system can have quasiperiodic or chaotic nature. Note also for the first 1000 variations for \( \Delta \alpha_z = 5^\circ \) (Fig. 5a) resembles those for \( \Delta \alpha_z = 30^\circ \) (Fig. 5c) while for \( \Delta \alpha_z = 15^\circ \) the long time scale modulation is different. This is consistent with Fig. 4a-c. Generally, for \( \Delta \alpha_z = 30^\circ \) the oscillations of higher frequency and more regular. The high values of \( Q_i \) in the middle part of Fig. 5c (\( i \in [1300, 2400] \)) are connected with measurement instabilities. In that case however the average value of heat release \( \langle Q_i \rangle \) is \( 270 \) J (Fig. 5a) \( 237 \) J (Fig. 5b) \( 181 \) J (Fig. 5c) is the smallest indicating the lowest burning rate of fuel. In spite of that the output torque, for the same speed of a crankshaft, was relatively larger \( S = 30 \) Nm (in the case \( \Delta \alpha_z = 30^\circ \)) comparing to other levels 21 and 28 Nm for \( \Delta \alpha_z = 5^\circ \) and \( 15^\circ \), respectively. Obviously, there are better combustion conditions in the last case.

We have also calculated autocorrelation function from the whole 10000 cycles signal via

\[ AC(j) = \sum_i Q(i)Q(i + j) \]  \hspace{1cm} (5)

with appropriate normalisation to one. The results for all three advance angles \( \Delta \alpha_z \) are depicted in Fig. 6. One can see that the decay of \( AC(j) \) amplitude with growing \( j \) is comparable but frequency of modulation is different for all these cases. Clearly, this is higher for larger \( \Delta \alpha_z \).
FIG. 6: Correlation function calculated from heat release time series for different advance angle $\Delta \alpha_z$.

V. ESTIMATION OF NOISE LEVEL FROM HEAT RELEASE SERIES

In this section we shall examine the level of noise in heat release time series. In this aim we use nonlinear embedding space approach \cite{16}.

In the $n$ dimensional embedding space the state is represented by a vector

$$Q = \{Q_i, Q_{i+m}, Q_{i+2m}, \ldots, Q_{i+(n-1)m}\},$$

where $m$ denotes the embedding delay in terms of cycles. The correlation integral calculated in the embedding space can be defined as \cite{17, 18}

$$C^n(\varepsilon) = \frac{1}{N^2} \sum_i^N \sum_{j \neq i}^N \Theta(\varepsilon - ||Q_i - Q_j||),$$

where $N$ is the number of considered points corresponding to pressure peaks in cycles and $\Theta$ is the Heaviside step function. For simplicity we use maximum norm. The correlation integral $C^n(\varepsilon)$ is related to the correlation entropy $K_2(\varepsilon)$ and the system correlation dimension $D_2$ by the following formula \cite{17, 18}

$$\lim_{n \to \infty} C^n(\varepsilon) = D_2 \ln \varepsilon - nmK_2(\varepsilon).$$

The coarse-grained correlation entropy can be now be calculated as
FIG. 7: Noise to signal ratio NTS versus a spark advance angle $\Delta \alpha_z$.

$$K_2(\varepsilon) = \lim_{n \to \infty} \ln \frac{C^n(\varepsilon)}{C^{n+1}(\varepsilon)} \approx -\frac{d \ln C^n(\varepsilon)}{dn}.$$  \hspace{1cm} (9)

In such a case the correlation entropy is defined in the limit of a small threshold $\varepsilon$.

In presence of noise described by the standard deviation $\sigma$ of $Q_i$ time series, the observed coarse-grained entropy $K_{noisy}^{10,11}$ can be written as

$$K_{noisy}(\varepsilon) = -\frac{1}{m^2} \left( \frac{\varepsilon}{2\sigma} \right) \ln \varepsilon + \left[ \chi + b \ln(1-a\varepsilon) \right]$$

$$\times \left( 1 + \sqrt{\frac{\varepsilon^2/3 + 2a^2 - \varepsilon/\sqrt{3}}{\varepsilon}} \right).$$  \hspace{1cm} (10)

Function $g(z)$, present in the above formula, reads

$$g(z) = \frac{2}{\pi} \frac{ze^{-z^2}}{\text{Erf}(z)},$$  \hspace{1cm} (11)

where Erf(\cdot) is the Error Function. The parameters $\chi, a, b$ as well as $\sigma$ are unconstrained. They should be fitted in Eq. (10) to mimic the observed noisy entropy calculated from available data.

After application the above method to the heat release times series we estimated noise calculating Noise to Signal ratio (NTS):

$$\text{NTS} = \frac{\sigma}{\sigma_{DATA}}.$$  \hspace{1cm} (12)
where $\sigma_{DATA}$ is the standard deviation of data.

Figure 7 shows NTS versus a spark advance angle $\Delta \alpha_z$. It appeared that for any of examined cases the noise level is not high. Starting from a small spark advance angle $\Delta \alpha_z = 5^\circ$ the level of noise is the smallest $NTS \approx 13\%$. It grows to about 17\% for $\Delta \alpha_z = 15^\circ$ and 25.5\% for $\Delta \alpha_z = 30^\circ$, respectively. In fact easy to note that all points (in Fig. 7) lay on a line. and ratio may be described as increasing linearly with $\Delta \alpha_z$.

In all considered cases we have to do with some large scale signal modulation ranging to a few hundred engine cycles and fluctuations of a few tens or more as well as very fast ones (Figs. 4 and 5). Using the same method we have examined the nature of very fast fluctuations and found that they are purely stochastic. Fluctuation of large time scales may involve in the estimation of noise level to make them underestimated by a few percents. This is because the correlated noise can occur and such a noise is irrelevant to our method. We think that in all three cases the calculated noise level are biased similarly, because of the same experimental conditions, so the linear increasing behaviour of NTS versus advance angle $\Delta \alpha_z$ is preserved.

VI. REMARKS AND CONCLUSIONS

In this paper we analysed instabilities of combustion process. We started from analysing pressure time series. Using spatio-temporal methods we established that there is a long time scale in fluctuation of our experimental data. To examine this phenomenon in detail we calculated heat release and we performed the noise level estimation using nonlinear multidimensional methods. Our results clearly indicate that the noise in the time series is the highest for the largest advance angle. In case of $\Delta \alpha_z = 30^\circ$ we have got the signal with characteristic 100 cycles periodicity.

Heat released in cycle as a practical parameter closely related to burned fuel mass and enables to follow the stability of combustion process better. Our noise estimation basing on heat release time series is more credible than the analysis pressure histories itself, as pressure is effected by volume cyclic compression and expansion phases.

The method using correlation entropy which we applied here differs from the symbolic treatments\textsuperscript{19,20} used also for exploration of the engine dynamics by Daw et al.\textsuperscript{7} In their paper the signal was digitised and basing on the probabilities of 0 1 sequences probabilities the
information Shannon entropy was estimated. In our paper we estimate the NTS ratio by fitting coarse-grained entropy obtained from experimental time series to a general formula of correlation entropy evaluated in presence of noise. In our case the entropy has its dynamical meaning as a measure uncertainty of system state.

One should also note that our present examination was limited to only one crank rotational speed 1000 RPM. Using the above procedure we now preparing systematic analysis with the other speeds for a future report.

Acknowledgements

Two of authors (KU and GL) would like to thank Max Planck Institute for Physics of Complex Systems in Dresden for hospitality. During their stay in Dresden an important part of data analysis was performed. KU has been partially supported by KBN Grant 2P03B03224.

1 D. Clerk, The Gas Engine, (Longmans, London 1886).
2 M. Wendeker, A. Niewczas, B. Hawryluk, SAE Paper 00P-172 (1999).
3 J.B. Roberts, J.C. Peyton-Jones, K.J. Landsborough, SAE Paper 970059 (1997).
4 Z. Hu, SAE Paper 961197 (1996).
5 J.B. Heywood, Internal combustion engine fundamentals (McGraw-Hill, New York 1988).
6 C.S. Daw, C.E.A. Finney, J.B. Green Jr., M.B. Kennel, J.F. Thomas and F.T. Connolly, SAE Paper 962086 (1996).
7 C.S. Daw, M.B. Kennel, C.E.A. Finney, F.T. Connolly, Phys. Rev. E 57, 2811 (1998).
8 M. Wendeker, J. Czarnigowski, G. Litak and K. Szabelski, Chaos, Solitons & Fractals 18, 803 (2003).
9 M. Wendeker, G. Litak, J. Czarnigowski and K. Szabelski, Int. J. Bifurcation and Chaos 14, (2004).
10 K. Urbanowicz, J.A. Holyst, Phys. Rev. E 67 046218 (3003).
11 S. Leonhardt, N. Mller, R. Isermann, IEEE/ASME Transactions on Mechatronics 4 235 (1999).
12 Antoni I, Daniere J, Guillet F. Journal Sound and Vib. 257 839 (2002).
13 M. Wendeker, T. Kamiński, M. Krupa, Journal of KONES 10 373 (2003).

14 G. Litak, R. Taccani, R. Radu, K. Urbanowicz, J.A. Holyst, M. Wendeker, A. Giadrossi Chaos,
   Solitons & Fractals submitted (2003).

15 T. Kamiński, unpublished.

16 H. Kantz, T. Scheiber, (Cambridge University Press, Cambridge 1997).

17 K. Pawelzik and H.G. Schuster, Phys. Rev. A 35, 481 (1987).

18 P. Grassberger and I. Procaccia, Phys. Rev. Lett. 50, 346 (1983).

19 C.S. Daw, C.E.A. Finney and M.B. Kennel Phys. Rev. E 62, 1912 (2000).

20 C.S. Daw, C.E.A. Finney and E.R. Tracy, Rev. of Scien. Instruments 74, 915 (2003).