AN ANALYSIS OF NON–OBLIQUE CORRECTIONS TO
THE $Zb\bar{b}$ VERTEX

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ABSTRACT

We present a model–independent analysis of the $Zb\bar{b}$ vertex, with the aim of constraining contributions of new physics to the left- and right–handed couplings of the $b$. We find that the left–handed coupling of the $b$ is quite narrowly constrained by present data, but that the right–handed coupling is still largely unconstrained.

1. Introduction

Recently there has been increasing interest in extensions of the Standard Model (SM) which predict sizable corrections to the $Zb\bar{b}$ vertex. This interest is motivated in part by the fact that a deviation from the SM prediction of $R_b = \Gamma_{\bar{b}b}/\Gamma_{\text{had}}$ has been observed at LEP. This quantity is particularly well suited for detecting non–SM vertex corrections since the leading QCD corrections cancel, to leading order, in the ratio. However, since a shift in the couplings of the $b$ will also affect observables such as $R_Z = \Gamma_{\text{had}}/\Gamma_{\ell^+\ell^-}$ and $\sigma^0_{\text{had}}$, it is important to analyze all the precision electroweak data in a systematic fashion for possible signatures of such corrections.

2. Sensitivity to oblique and non–oblique corrections

In the standard renormalization scheme where $\alpha$, $G_\mu$, and $m_Z$ are used as input to fix the theory, electroweak observables get their dependence on oblique corrections through the $\rho$ parameter and $\sin^2\theta_{\text{eff}}$. If we denote the contribution of new physics to these two quantities as $\delta\rho$ and $\delta\sin^2\theta_{\text{eff}}$, respectively, we have

$$\rho = [\rho]_{\text{SM}} + \delta\rho.$$
\[
\sin^2 \theta_{\text{eff}} = [\sin^2 \theta_{\text{eff}}]_{\text{SM}} + \delta s^2,
\]
where \([O]_{\text{SM}}\) denotes the Standard Model prediction of the observable \(O\).

The left and right handed couplings of the \(b\) quark to the \(Z\) are given by
\[
g^b_L = [g^b_L]_{\text{SM}} + \frac{1}{3} \delta s^2 + \delta g^b_L, \quad g^b_R = [g^b_R]_{\text{SM}} + \frac{1}{3} \delta s^2 + \delta g^b_R,
\]
where we have included possible non–oblique corrections from new physics, \(\delta g^b_L\) and \(\delta g^b_R\). Assuming that there are no other non–oblique corrections from new physics, we can calculate the dependence of various observables on \(\delta \rho, \delta s^2, \delta g^b_L, \text{ and } \delta g^b_R\). It is convenient to define the following linear combinations of \(\delta g^b_L\) and \(\delta g^b_R\):
\[
\xi_b \equiv (\cos \phi_b)\delta g^b_L - (\sin \phi_b)\delta g^b_R,
\]
\[
\zeta_b \equiv (\sin \phi_b)\delta g^b_L + (\cos \phi_b)\delta g^b_R,
\]
where \(\phi_b \equiv \tan^{-1} |g^b_R/g^b_L| \approx 0.181\). By expanding \(\Gamma_{bb}\) and \(A_b \equiv [(g^b_L)^2 - (g^b_R)^2]/[(g^b_L)^2 + (g^b_R)^2]\) about the point \(\delta s^2 = \xi_b = \zeta_b = 0\), we find
\[
\Gamma_{bb} = [\Gamma_{bb}]_{\text{SM}} \left(1 + \delta \rho - 1.25 \delta s^2 - 4.65 \xi_b \right),
\]
\[
A_b = [A_b]_{\text{SM}} \left(1 - 0.68 \delta s^2 - 1.76 \zeta_b \right).
\]

All the other observables get their dependence on \(\delta g^b_L\) and \(\delta g^b_R\) through either \(\Gamma_{bb}\) or \(A_b\) so they will depend on either \(\xi_b\) or \(\zeta_b\), but not both. The observables that depend on \(\Gamma_{bb}\) are:
\[
\Gamma_{Z} = [\Gamma_{Z}]_{\text{SM}} \left(1 + \delta \rho - 1.06 \delta s^2 - 0.71 \xi_b \right),
\]
\[
\sigma^0_{\text{had}} = [\sigma^0_{\text{had}}]_{\text{SM}} \left(1 + 0.11 \delta s^2 + 0.41 \xi_b \right),
\]
\[
R_Z \equiv \Gamma_{\text{had}}/\Gamma_{\ell^+\ell^-} = [R_Z]_{\text{SM}} \left(1 - 0.85 \delta s^2 - 1.02 \xi_b \right),
\]
\[
R_b \equiv \Gamma_{bb}/\Gamma_{\text{had}} = [R_b]_{\text{SM}} \left(1 + 0.18 \delta s^2 - 3.63 \xi_b \right),
\]
\[
R_c \equiv \Gamma_{cc}/\Gamma_{\text{had}} = [R_c]_{\text{SM}} \left(1 - 0.35 \delta s^2 + 1.02 \xi_b \right).
\]

Note that only \(\Gamma_{Z}\) depends on \(\delta \rho\). All of the other observables can be expressed as ratios of widths, so that the \(\rho\) dependence cancels between numerator and denominator. We will ignore \(\Gamma_{Z}\) in the following in order to keep the number of parameters at a manageable level. In an analogous way, we find
\[
A^b_{\text{FB}} = \frac{3}{4} A_e A_b = [A^b_{\text{FB}}]_{\text{SM}} \left(1 - 55.7 \delta s^2 - 1.76 \zeta_b \right).
\]

The relationship between our parameters and others that have appeared in the literature is as follows. The parameter \(\epsilon_b\) introduced in Ref. 1 is related to \(\delta g^b_L\) by
\[
\epsilon_b = [\epsilon_b]_{\text{SM}} - 2\delta g^b_L.
\]
The parameters \(\delta_{bV}\) and \(\eta_b\) introduced in Ref. 2 are related to \(\xi_b\) and \(\zeta_b\) by
\[
\delta_{bV} = [\delta_{bV}]_{\text{SM}} - 4.65 \xi_b,
\]
\[
\eta_b = [\eta_b]_{\text{SM}} - 1.76 \zeta_b.
\]
Table 1. Experimental measurements and Standard Model predictions for various observables

| Observable | Experiment       | SM prediction |
|------------|------------------|---------------|
| $\sin^2 \theta_{\text{eff}}$ | 0.2317 ± 0.0007 (LEP) 0.2294 ± 0.0010 (SLD) | 0.2320 |
| $\sigma_{\text{had}}^0$ | 41.49 ± 0.12 (nb) | 41.43 ± 0.03 |
| $R_Z$ | 20.795 ± 0.040 | 20.74 ± 0.04 |
| $R_b$ | 0.2202 ± 0.0020 | 0.2157 |
| $R_c$ | 0.1583 ± 0.0098 | 0.1711 |
| $A_{FB}^b$ | 0.0967 ± 0.0038 | 0.0957 |
| $A_b$ | 0.99 ± 0.14 | 0.934 |

3. Determination of $\xi_b$ and $\zeta_b$

In order to constrain $\xi_b$ and $\zeta_b$, we must first compute nominal Standard Model values for the various observables. This in turn requires that we specify nominal values for the top and Higgs masses. In the following, we use $m_t = 175$ GeV and $m_H = 300$ GeV. It is also necessary to specify the value of $\alpha_s$ used in computing the QCD corrections. Here we will use $\alpha_s = 0.120 \pm 0.006$, which is the value determined from hadronic event shapes, jet rates, and energy correlations. We use this value rather than the $0.123 \pm 0.006$ determined using lineshape data because it is independent of the $Z$ lineshape parameters we will be using in this analysis. For the top and Higgs masses given above, the Standard Model predictions for the relevant observables are summarized in Table 1, together with the most recent experimental determinations.

The errors on $[\sigma_{\text{had}}^0]_{\text{SM}}$ and $[R_Z]_{\text{SM}}$ are due to the uncertainty in $\alpha_s$.

The LEP value of $\sin^2 \theta_{\text{eff}}$ is the average over the leptonic asymmetries only; since the $b\bar{b}$ asymmetries are sensitive to vertex corrections as well as shifts in the value of $\sin^2 \theta_{\text{eff}}$, they should be handled separately.

![Fig. 1. The 1–σ limits placed on $\xi_b$ and $\delta s^2$.](image-url)
The constraints imposed by the various observables are illustrated in Figures 1 and 2. In Fig. 1, we show the experimentally preferred $1 - \sigma$ bands in the $\delta s^2 - \xi_b$ plane, and in Fig. 2 we show the corresponding figure for the $\delta s^2 - \zeta_b$ plane.

A fit to the data with $\delta s^2$, $\xi_b$, and $\zeta_b$ as parameters, including the correlation of -0.4 between $R_b$ and $R_c$, yields

$$
\begin{align*}
\delta s^2 &= -0.0009 \pm 0.0006, \\
\xi_b &= -0.003 \pm 0.002, \\
\zeta_b &= 0.018 \pm 0.027.
\end{align*}
$$

The 2-dimensional projections of the allowed regions onto the $\delta s^2 - \xi_b$ and $\delta s^2 - \zeta_b$ planes are shown in Figs. 3 and 4.

Fig. 2. The $1-\sigma$ limits placed on $\zeta_b$ and $\delta s^2$.

Fig. 3. The 68% and 90% confidence limits on $\xi_b$ and $\delta s^2$. Dashed contours show the positions of the 90% limit when only the LEP or SLD value of $\sin^2 \theta_{\text{eff}}$ is used. The SM points are plotted for $m_t = 150, 175, \text{ and } 200 \text{ GeV}$. Larger $m_t$ correspond to smaller $\delta s^2$. 
Fig. 4. The 68% and 90% confidence limits on $\zeta_b$ and $\delta s^2$. The meaning of the dashed contours and SM points are the same as in Fig. 3.

In terms of $\delta g_b^L$ and $\delta g_b^R$, Eq. 9 translates into

$$\delta g_b^L = -0.000 \pm 0.005, \quad \delta g_b^R = 0.018 \pm 0.027. \quad (10)$$

We see from this that the left–handed coupling of the $b$ is very tightly constrained by present data, while the right–handed coupling is more weakly constrained. This leaves considerable freedom for models containing extra right–handed gauge bosons or extended Higgs sectors, which would tend to modify the right–handed coupling of the $b$. It is also important to note that many observables, in addition to $R_b$, are sensitive to shifts in the couplings of the $b$.

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