Omnidirectional bending of light in $\mu$-near-zero metamaterials

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We theoretically study the transmission of light in $\mu$-near-zero metamaterials. We find the effect of omnidirectional bending of light, which means that the direction of energy flow in a wave transmitted to a $\mu$-near-zero metamaterial does not depend on the incident angle if an incident wave is $s$-polarized. This effect is similar to the omnidirectional bending of light recently found in $\varepsilon$-near-zero metamaterials for a p-polarized incident wave [S. Feng, Phys. Rev. Lett. 108, 193904 (2012)]. To provide a specific example, we consider the transmission of light in a negative-index metamaterial in the spectral region with a permeability resonance, and show that omnidirectional bending of light indeed takes place at the wavelength for which the real part of permeability is vanishingly small.

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I. INTRODUCTION

Refraction of light is the fundamental optical phenomenon. Significant progress in fabrication of nanoscale structures led to creation of optical metamaterials which allow us to manipulate the way light refracts. For example, at the interface of negative-index metamaterials (NIMs), the angle of refraction turns out to be negative. In Ref. [8] an array of optically thin resonators with subwavelength separation was used to modulate the phase of incident light along the interface. It was demonstrated that, depending on the designed phase gradient, the refraction angle can be arbitrary for any incident angle. Omnidirectional bending of light was predicted at the interface of metamaterials with a vanishingly small real part of permittivity, so-called $\varepsilon$-near-zero metamaterials. In Ref. [9] Feng showed that the direction of energy flow bends towards the interface normal for any incident angle when p-polarized (transverse magnetic) light enters an $\varepsilon$-near-zero metamaterial. He clarified that such omnidirectional bending is a result of material losses.

In our work we show that similar omnidirectional bending can be realized with s-polarized (transverse electric) incident light at the interface of metamaterials with a vanishingly small real part of permeability, so-called $\mu$-near-zero metamaterials. A vanishingly small real part of permeability can be found in NIMs near their permeability resonance which is used to achieve a negative index of refraction. To confirm our idea we calculate the transmission angle of the Poynting vector at the interface with the NIM recently reported by García-Meca et al.

II. INHOMOGENEOUS WAVES IN A LOSSY METAMATERIAL

Since omnidirectional bending of light is a consequence of losses in a medium and the propagation of light in lossy media differs from that in lossless media, we first summarize the basic features of light waves in lossy media. Unlike the case of light waves in lossless media, the equi-amplitude and equi-phase planes of light waves in lossy media are not parallel, and such waves are called inhomogeneous waves. The summarized results in this section will be used in the following sections to determine the direction of the Poynting vector of the wave transmitted through a metamaterial.

We consider an interface between two isotropic media (see Fig. 1). The first medium is a lossless dielectric with a real refractive index $n_0$ and the second medium is a lossy metamaterial with complex permittivity $\varepsilon = \varepsilon' - i\varepsilon''$ and permeability $\mu = \mu' - i\mu''$. An incident plane wave with a real wave vector $k_0$ comes from the first medium. The incident angle $\theta_0$ is an angle between $k_0$ and the unit vector normal to the interface $\hat{q}$, which is pointing to the second medium. The complex electric and magnetic fields, $E$ and $H$, respectively, of the transmitted wave are written as

$$E = e e^{i (\omega t - k \cdot r)},$$
$$H = h e^{i (\omega t - k \cdot r)},$$

where $e$ and $h$ are complex amplitude vectors, and $k$ and $r$ are wave vectors and a position vector, respectively, with $\omega$ and $\tau$ being the wave frequency and time. Since the second medium is lossy, the wave vector of the transmitted wave is complex: $k = k' - i k''$, where $k'$ and $k''$ are the real phase and attenuation vectors, respectively, and they are written as

$$k' = p + q' \hat{q},$$
$$k'' = q'' \hat{q}.$$
where $e_s = s^{-2}(e \cdot s)s$ and $e_p = s^{-2}[s \times (e \times s)] = s^{-2}(e \cdot \hat{q})[s \times k]$, or
\[ e = A_s s + A_p [s \times k], \]
where $A_s = s^{-2}(e \cdot s)$ and $A_p = s^{-2}(e \cdot \hat{q})$ are the complex amplitudes of the $s$- and $p$-polarized components, respectively. Amplitudes $A_s$ and $A_p$ of the transmitted wave are connected with the corresponding amplitudes of the incident wave by Fresnel coefficients, i.e., $A_p = 0$ if the incident wave is $s$-polarized, and $A_s = 0$ if it is $p$-polarized. To find the decomposition of the complex magnetic vector amplitude $\mathbf{h}$, we substitute Eq. (5) into the identity $\mathbf{h} = (\mu_0 \mu \omega)^{-1} [k \times e]$, and obtain
\[ \mathbf{h} = \varepsilon_0 \varepsilon_0 \omega A_p s - \frac{A_s}{\mu_0 \mu \omega} [s \times k]. \]

Now, with the help of Eq. (1), we write a complex time-averaged Poynting vector as $S = \frac{1}{2}[E \times H^*] = \frac{1}{2}[e \times h^*] \exp[-2(k'' \cdot r)]$, where "*" means complex conjugate. Substituting Eqs. (6) and (5) into the last expression, we find that vector $S$ can be written as the sum of three components: $S = S_s + S_p + S_{sp}$, where
\[ S_s = e^{-2(k'' \cdot r)} \frac{|A_s|^2}{\mu \mu_0 \omega^2} [s \times (k^* \times s)], \]
\[ S_p = e^{-2(k'' \cdot r)} \frac{\varepsilon_0 \varepsilon_0 \omega |A_p|^2}{\mu \mu_0 \omega} [s \times [k \times s]], \]
\[ S_{sp} = -e^{-2(k'' \cdot r)} \frac{A_s^* A_p}{\mu \mu_0 \omega^2} [k \times s] \times [k^* \times s]. \]
The $s$-polarized component $S_s$ depends only on $A_s$, while the $p$-polarized component $S_p$ depends only on $A_p$. The cross-polarized component $S_{sp}$ is determined by both $A_s$ and $A_p$. According to Eq. (7c), the cross-polarized component $S_{sp}$ exists only in lossy media where the wave vector $k$ is complex.

A real time-averaged Poynting vector $P$ corresponds to the real part of $S$, i.e., $P = \Re(S)$. Expanding the vector products and taking the real parts of Eq. (7), we find that $P = P_s + P_p + P_{sp}$, where
\[ P_s = e^{-2(k'' \cdot r)} \frac{\varepsilon_0 \varepsilon_0 s^2 |A_s|^2}{\mu \mu_0 \omega^2} (\mu' k' + \mu'' k''), \]
\[ P_p = e^{-2(k'' \cdot r)} \frac{\varepsilon_0 \varepsilon_0 s^2 |A_p|^2}{\mu \mu_0 \omega^2} (\varepsilon' k' + \varepsilon'' k''), \]
\[ P_{sp} = \frac{e^{-2(k'' \cdot r)} \varepsilon_0 \varepsilon_0 s^2}{2 \mu \mu_0 \omega^2} \times (\mu' \Re(A_s^* A_p) - \mu'' \Im(A_s^* A_p))^2. \]
with \( \Re \{ \} \) and \( \Im \{ \} \) being the real and imaginary parts of the corresponding expressions. Equation (8) says that s- and p-polarized components of the Poynting vector are proportional to the sum of vectors \( k' \) and \( k'' \), while the cross-polarized component \( P_{sp} \) is proportional to vector \( s \) and thus normal to the incidence plane. The latter component is responsible for the transversal shift of the transmitted light beam. A similar shift named Imbert-Fedorov shift takes place for the reflected light beam for the case of total internal reflection. Despite the fact that these two shifts look similar, there are several different viewpoints on the component \( P_{sp} \). The authors of Ref. 13 argue that "there is no mechanism for energy transport in the direction perpendicular to the plane of incidence" and set \( P_{sp} \) equal to zero, based on the fact that "the Poynting vector is defined only up to an arbitrary, additive, solenoidal vector". Fedorov in his book believes that this component is real and responsible for the light pressure in the direction perpendicular to the incidence plane. The authors of Ref. 14 came to the conclusion that the appearance of \( P_{sp} \) is caused by excitation of surface electric polariton mode or surface magnetic mode by the resonant or non-resonant manner. In this work, however, we only consider s- \((A_s = 0)\) or p-polarized \((A_p = 0)\) incident light for which \( P_{sp} = 0 \).

Looking at Eq. (8), we find that both \( P_s \) and \( P_p \) are parallel to \( k'' \) if \( \mu' \) or \( \varepsilon' \) is equal to zero, respectively. Meanwhile, for any incident angle, the attenuation vector \( k'' \) is always normal to the interface [see Eq. (2b)]. Therefore, we conclude that, at the interface of a material with a vanishingly small real part of permeability \((\mu' = 0)\), the s-polarized incident wave gives rise to the transmitted wave whose energy flow is directed normally to the interface, irrespective of the incident angle. A similar argument holds for the p-polarized incident wave at the interface of a material with a vanishingly small real part of permittivity \((\varepsilon' = 0)\), as was shown by Feng.

### IV. TRANSMISSION ANGLES

Now we consider the transmission angles \( \psi_s \) and \( \psi_p \) for s- and p-polarized components of the Poynting vector. Angles \( \psi_s \) and \( \psi_p \) are defined as the angles between vectors \( P_s \) and \( P_p \), respectively, and the unit normal \( \hat{q} \). Consider, for example, the angle \( \psi_s \). We can find this angle from the equation, \( \tan \psi_s = |(P_s \times \hat{q})|/(P_s 
 \cdot \hat{q}) \). Using Eq. (8) and taking into account that \( |k' \times \hat{q}| = s \) and \( k'' \parallel \hat{q} \), we find that \( \tan \psi_s = \mu'|s|/(\mu'q' + \mu''q'') \). Similarly we obtain \( \tan \psi_p = \varepsilon'|s|/(\varepsilon'q' + \varepsilon''q'') \). This form of equations for \( \psi_s \) and \( \psi_p \) was previously obtained in Ref. 13. Using the equations of \( |s| = m' \sin \theta_c \), \( q' = m' \sin \theta_c \), and \( q'' = m'' \sin \theta_c \), we finally obtain

\[
\tan \psi_s = \frac{\mu' m' \sin \theta}{\mu' m' \cos \theta + \mu'' m''}, \tag{9a}
\]

\[
\tan \psi_p = \frac{\varepsilon' m' \sin \theta}{\varepsilon' m' \cos \theta + \varepsilon'' m''}. \tag{9b}
\]

Here we see another manifestation of omnidirectional bending. Namely, in case of \( \mu' = 0 \) or \( \varepsilon' = 0 \) the corresponding transmission angle, \( \psi_s \) or \( \psi_p \), is equal to zero, irrespective of the incident angle. Moreover, Eq. (9) says that the transmission angles \( \psi_s \) and \( \psi_p \) are not equal, which means that the direction of the energy flow in a lossy material is different for s- and p-polarized incident light. However, for any natural material this difference is negligibly small, since usually \( \mu''/\mu' \ll 1 \) and \( \varepsilon''/\varepsilon' \ll 1 \), which, according to Eq. (9), means that

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**FIG. 2.** (color online). (a) Variation of the real \( \mu' \) and imaginary \( \mu'' \) parts of the permeability \( \mu \) as a function of the wavelength \( \lambda \). (b) and (c) The transmission angles \( \psi_s \) and \( \psi_p \) for the s- and p-polarized components of the Poynting vector as functions of the wavelength \( \lambda \) and incident angle \( \theta_0 \). All above functions are calculated for the interface between vacuum and the metamaterial reported in Ref. 6. The spectral region of negative refraction is located between the two vertical dotted lines.
ψ_s ≃ ψ_p ≃ θ. Nevertheless, the difference between ψ_s and ψ_p can be significant in metamaterials where the above inequalities may not hold.

To be more quantitative we calculate the values of ψ_s and ψ_p for the case in which the first medium is vacuum (n_0 = 1) and the second medium is the NIM reported in Ref. 6. We retrieve the relevant parameters for the permittivity ε and permeability μ of this NIM, performing the parameter fitting for the Drude model. Figure 2(a) shows the dependence of the real and imaginary parts of the retrieved permeability μ on the wavelength λ. We see that this dependence has a resonance feature and the real part μ’ of the permeability is equal to zero at the wavelengths 732 and 768 nm. At these wavelengths we expect omnidirectional bending for incident s-polarized light.

Using the retrieved functions for ε and μ, we calculate ψ_s and ψ_p by Eq. (9) where values of m’, m’’, and θ have been obtained using the equations in section II. Figures 2(b) and 2(c) show the dependencies of the transmission angles ψ_s and ψ_p on the wavelength λ and the incident angle θ_0. In spite of negative refraction, we consider ψ_s and ψ_p as angles between two vectors and set them positive.

As expected, we see in Fig. 2(b) that at wavelengths where μ’ = 0, the transmission angle ψ_s is zero for any incident angle. Therefore, the direction of energy flow in the second medium will be normal to the interface for any incident angle.

By comparing Figs. 2(b) and 2(c), we clearly see the difference between ψ_s and ψ_p. This difference is more significant at the wavelength λ = 732 nm where we have omnidirectional bending for the s-polarized component of the Poynting vector. We hope that this observation will help to experimentally verify the difference between the transmission angles for s- and p-polarized incident light.

V. CONCLUSIONS

In conclusion we have theoretically studied the transmission of light in μ-near-zero metamaterials. Similar to the case of ε-near-zero metamaterials, we have found the effect of omnidirectional bending of light in μ-near-zero metamaterials for the s-polarized incident wave. We have presented specific results for the negative-index metamaterial with a permeability resonance where the real part of the permeability becomes zero. Additionally, we have shown that the transmission angle of the Poynting vector depends on the polarization of the incident wave, and this difference is very significant in the spectral region where omnidirectional bending of light takes place.

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