Spectral hardening as a viable alternative to disc truncation in black hole state transitions

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ABSTRACT
Constraining the accretion flow geometry of black hole binaries in outburst is complicated by the inability of simplified multicolour disc models to distinguish between changes in the inner disc radius and alterations to the emergent spectrum, parametrized by the phenomenological colour-correction factor, $f_{\text{col}}$. We analyse Rossi X-ray Timing Explorer observations of the low-mass Galactic black hole X-ray binary, GX 339–4, taken over seven epochs when the source was experiencing a state transition. The accretion disc component is isolated using a pipeline resulting in robust detections for disc luminosities, $10^{-3} \lesssim L_{\text{disc}}/L_{\text{Edd}} \lesssim 0.5$. Assuming that the inner disc remains situated at the innermost stable circular orbit over the course of a state transition, we measure the relative degree of change in $f_{\text{col}}$ required to explain the spectral evolution of the disc component. A variable $f_{\text{col}}$ that increases by a factor of $\sim 2.0–3.5$ as the source transitions from the high/soft state to the low/hard state can adequately explain the observed disc spectral evolution. For the observations dominated by a disc component, the familiar scaling between the disc luminosity and effective temperature, $L_{\text{disc}} \propto T_{\text{eff}}^4$, is observed; however, significant deviations from this relation appear when GX 339–4 is in the hard intermediate and low/hard states. Allowing for an evolving $f_{\text{col}}$ between spectral states, the $L_{\text{disc}}-T_{\text{eff}}^4$ law is recovered over the full range of disc luminosities, although this depends heavily on the physically conceivable range of $f_{\text{col}}$. We demonstrate that physically reasonable changes in $f_{\text{col}}$ provide a viable description for multiple state transitions of a black hole binary without invoking radial motion of the inner accretion disc.

Key words: accretion, accretion discs – black hole physics – X-rays: binaries – X-rays: individual: GX 339–4.

1 INTRODUCTION
The nature of the accretion flow for black hole binaries (BHBs) undergoing state transitions remains unclear and is an active topic of observational and theoretical pursuit. The phenomenology describing the predominant spectral states experienced by BHBs include the high/soft state (HSS), characterized by high luminosity and a standard thin accretion disc as the dominant spectral component; the low/hard state (LHS), characterized by low luminosity with a dominant hard power-law spectral component that is likely produced by inverse Compton scattering of soft photons by a relativistic electron corona (Shapiro, Lightman & Eardley 1976); and the intermediate state (IS), which is itself further divided into the hard intermediate state (HIS) and soft intermediate state (SIS) (see, e.g. Belloni 2010, for an extensive description of these phenomenological states).

Currently, the picture of a black hole accreting at a significant fraction of the Eddington mass accretion rate, $\dot{M}_{\text{Edd}}$, from a geometrically thin, optically thick disc extending to the innermost stable circular orbit (ISCO) is the accepted description of the HSS. However, there is a lack of consensus regarding a universal geometry of the disc and innermost accretion flow in the LHS when the black hole feeds at a low fraction of $\dot{M}_{\text{Edd}}$ and/or the flow has a low radiative efficiency. According to the paradigm put forth by Esin, McClintock & Narayan (1997), the accretion disc in the LHS is radially recessed (i.e. truncated) with the interior region being composed of a radiatively inefficient accretion flow (RIAF) (Narayan & Yi 1994). Alternatively, models where the inner disc remains at the ISCO in the LHS have been proposed by invoking magnetically...
Motivated by the lack of consensus on accretion disc evolution over the course of a BHB state transition, we aim to address the question: Can physically reasonable changes in the vertical disc structure alone explain the observed evolution of disc properties during a state transition without invoking a truncated disc geometry? In light of this question, we sought a source that met the following criteria. (1) The source was observed over multiple state transitions, permitting a comparison of disc variations between different transitions. (2) Given that our methodology hinges on the presence of a disc, a disc component must have been robustly detected in the LHS. (3) Conflicting claims for both a severely recessed disc and a disc extending to the ISCO in the LHS are published for the source. (4) Variable low-frequency quasi-periodic oscillations (LFQPOs), which may be linked to the disc evolution, have been observed coincident with state transitions. (5) Reasonable constraints are known on mass, distance and inclination; however, this is of ancillary importance. Based on these criteria, we selected GX 339–4 as the focus of our analysis.

2.1 GX 339–4

GX 339–4, discovered by Markert et al. (1973), is a low-mass X-ray binary and recurring transient hosting a dynamically confirmed black hole with a mass \( M \geq 5.8 \, M_\odot \) (Hynes et al. 2003; see also Muñoz-Darias, Casares & Martinez-Pais 2008) and distance \( D = 8 \pm 2 \, \text{kpc} \) (Zdziarski et al. 2004; see also Hynes et al. 2004). Supposing that the inner disc inclination, \( i \) (0° for face-on, 90° for edge-on), can be assumed to align with the binary inclination, then the lack of observed eclipses constrain \( i \leq 60° \) (Cowley et al. 2002), while radio observations imply \( i \leq 30° \) (Gallo et al. 2004). Results from modelling the X-ray reflection features of GX 339–4 in the SIS also suggest a lower disc inclination of \( i \sim 10°–30° \) (Miller et al. 2004, 2009a) and in the LHS (Reis et al. 2008). Owing to the uncertainties on the intrinsic parameters (namely \( M, D, \) and) of GX 339–4 and the limitations of existing X-ray data, seeking absolute measurements that depend on these parameters presents a serious challenge.

GX 339–4 experienced multiple outbursts since its discovery, making it an appealing source for temporal evolution studies. The seven state transitions we select to study are listed in Table 1. The notation convention we adopt for referring to a transition is the letter \( R \) or \( D \) specifying a rise or decay type, respectively, followed by the year associated with the transition. A rise (decay) type indicates a...
transition from the LHS (HSS) to the HSS (LHS). The intervals of each transition were selected based on previously published dates and the emergence/disappearance of LFQPOs (Shaposhnikov & Titarchuk 2009; Motta et al. 2011; Stiele et al. 2011).\footnote{The end date listed for transition ID GX339-D03 in table 1 of Shaposhnikov & Titarchuk (2009) is mistyped and should read 2003/06/05 (2003 June 05).} We chose to extend the transition start/end times by \pm 5 d beyond the intervals quoted in the literature to ensure that we capture the full transition behaviour in our subsequent analysis.

Alarming discrepancies in the location of the inner accretion disc edge, particularly in the LHS, appear in the literature based on spectral fits to X-ray observations of GX 339–4. The strongest case for disc truncation is presented in Tomsick et al. (2009), which used fluorescent iron emission lines to find \( R_\text{in} > 175 R_g \) (90 per cent confidence, assuming \( g = 30 \)) when GX 339–4 was in the LHS with luminosity \( L_X \approx 1.4 \times 10^{33} L_{\odot} \) (assuming \( D = 8 \) kpc). Claims of a truncated disc in the LHS also appear based on modelling of direct disc emission accounting for irradiation of the inner disc (Cabana et al. 2009). Even in the SIS, Yamada et al. (2009) suggest a recessed disc with \( R_\text{in} = 5–32 R_g \) (68 per cent confidence, assuming \( g = 30 \)). These measurements suggest that the inner portion of the disc is absent in low-luminosity states, perhaps replaced by a RIAF. In stark contrast, there are numerous claims of the inner disc radius in GX 339–4 remaining consistent with the ISCO in the LHS. Using physically motivated, sophisticated spectral fitting techniques, the disc inner radius was determined to lie within \( R_\text{in} = 2.0–3.0 R_g \) in the SIS (Miller et al. 2004) and \( R_\text{in} = 3.0–5.0 R_g \) in the LHS with luminosity \( L_X \approx 0.05 L_{\odot} \) (assuming \( D = 8 \) kpc) (Miller et al. 2006). Both of these results were confirmed by a systematic reanalysis that employed detailed disc reflection modelling (Reis et al. 2008). Using the same data set as in Miller et al. (2006), Wilkinson & Uttley (2009) also measured a disc extending down to \( 4 R_g \) from their best-fitting iron line model. Tomsick et al. (2008) observed GX 339–4 in the LHS at luminosities \( L_X \approx 0.023 L_{\odot} \) and \( L_X \approx 8 \times 10^{32} L_{\odot} \), finding the disc to extend within \( 10 R_g \) and being consistent with the disc remaining fixed at \( \sim 4 R_g \). Reis et al. (2010) performed a systematic study of eight BHBs (including GX 339–4) in the LHS, robustly detecting a thermal component in all sources and placing a stringent upper limit of \( 10 R_g \) on the truncation radius for six sources confirmed by both broad iron line and thermal disc modelling independently.

In order to measure black hole spin, one must associate \( R_\text{in} \) with the ISCO, located at \( R_{\text{ISCO}} \). A measurement of the inner disc radius is the crucial diagnostic for measuring black hole spin, as \( R_{\text{ISCO}} \) is a monotonically decreasing function of the dimensionless spin parameter, \( a_* = Jc/GM^2 \), where \( J \) is the black hole angular momentum (Bardeen, Press & Teukolsky 1972). Perhaps unsurprisingly, conflicting claims regarding the behaviour of \( R_\text{in} \) have led to discrepancies in spin measurements of GX 339–4. Spin measurements employing the relativistically broadened iron line method (Fabian et al. 1989; Laor 1991) suggest that GX339–4 harbours a near-maximally spinning black hole with \( a_* > 0.94 \pm 0.02 \) (Reis et al. 2008; Miller et al. 2009a), while the thermal disc continuum fitting technique (Zhang, Cui & Chen 1997) supports a low-to-moderately spinning black hole with \( a_* < 0.4 \) (Yamada et al. 2009; see also Kolehmainen & Done 2010; Kolehmainen, Done & Díaz Trigo 2011).

LFQPOs exhibit variability simultaneous with transitions between the LHS and HSS (Rutledge et al. 1999a). Variable LFQPOs were observed in each GX 339–4 transition studied here, with the measured frequency ranges listed in Table 1 and typically spanning 0.2–8 Hz (Belloni et al. 2005; Shaposhnikov & Titarchuk 2009; Motta et al. 2011; Stiele et al. 2011). The idea that LFQPOs may originate from oscillating modes of a quasi-spherical RIAF that is interior to, or partially overlapping with, a truncated thin disc was proposed by Giannios & Spruit (2004). Long time-scale variability of the disc blackbody component of GX 339–4 was associated with disc instabilities and interpreted as arising from intrinsic variability of the disc (Wilkinson & Uttley 2009). This suggests that the variable LFQPOs may be intricately linked to disc truncation; however, the truncation radius cannot exceed \( 20 R_g \) if the observed variations are limited by the viscous time-scale.

\section*{2.2 Rossi X-ray Timing Explorer}

The monitoring campaign of the RXTE (Bradt, Rothschild & Swank 1993; Swank 1999) observed GX 339–4 transitioning between states in 2002, 2003, 2004, 2007, 2010 and 2011, obtaining well-sampled (in time) observations spanning each transition before being decommissioned in early 2012. We examined 283 RXTE observations of GX 339–4 covering seven time intervals, during which the source experienced state transitions (see Table 1). The RXTE standard products, comprised of the source and background spectra along with instrument response matrices, were obtained from the High Energy Astrophysics Science Archive Research Center data archive.\footnote{The proportional counter array (PCA); Jahoda et al. 1996 and High Energy X-ray Timing Experiment (HEXTE; Rothschild et al. 1996).} The RXTE counting rate measurements were normalized to the RXTE standard products, comprised of the source and background spectra along with instrument response matrices, were obtained from the High Energy Astrophysics Science Archive Research Center data archive.\footnote{http://heasarc.gsfc.nasa.gov/cgi-bin/W3Browse/w3browse.pl}
et al. 1998) standard source spectra were analysed jointly. The PCA energy spectrum (Standard 2 mode) is built from summing all layers from various proportional counter units (PCUs). The PCU-1, PCU-2 and PCU-3 are used in all observations prior to 2006 December 25 (i.e. R02, D03, R04), while only the PCU-2 and PCU-3 are used in observations thereafter (i.e. R07, D07, R10, D11) due to a propane loss in the top layer of PCU-1.4 For observations corresponding to transitions R02, D03, R04, R07 and D07, we used the HXTE Cluster B spectrum, but could not use these data for R10 and D11 because the HEXTE Cluster B was permanently left in an off-source pointing position from 2010 March 29 onwards.5 The HEXTE Cluster A data for transitions R10 and D11 are affected by strong residuals, resulting from difficulty in determining the appropriate contribution of the background spectra. Consequently, we excluded all HEXTE data while analyzing transitions R10 and D11, using PCA data only for these observations. This resulted in many observations with a strong soft component, which is indicative of a disc, being excised from our sample due to poor spectral modelling of the high-energy continuum (see Section 4.1).

While the time sampling capability of the RXTE satellite makes it well suited for a study of BHB state transitions, which are a highly time-dependent phenomenon, RXTE has important limitations. Determining the spectral properties of the disc component for a given observation is made difficult by the limited low-energy coverage of RXTE, which only extends down to ~3 keV, while the disc spectrum peaks at ~1 keV. Especially in the HIS and LHS, the diminished disc component, if present, is difficult to characterize due to a combination of its weakness and the dominance of a Comptonized component. Notably, Fig. 1 presents a hardness–intensity diagram (HID) of all 283 RXTE observations of GX 339–4 considered in this work. Observations that do not statistically require a disc component (see Section 4.1) are indicated by grey crosses, while the various glyphs indicate the transition associated with the observation. The plethora of IS and HSS observations (i.e. low-to-moderate hardnesses) deemed to not require a disc comes from the R10 and D11 data sets, where the absence of HEXTE data prevented acceptable fits to disc-dominated spectra. Notably, Fig. 1 shows that the set of observations considered in the subsequent analysis span all spectral states, which indicates that we are studying the complete evolution of state transitions in GX 339–4.

For a given observation, hardness and intensity are calculated from the PCA data as follows. The distribution of counts across all channels is obtained from both the standard PCA source spectral file and background spectral file. The background counts are then subtracted from the source counts channel-by-channel, yielding the counts in each channel associated with GX 339–4. The hardness is then defined as the ratio of GX 339–4 counts in channels 15–24 (6.3–10.5 keV) to those in channels 8–14 (3.8–6.3 keV) (Belloni et al. 2005). A count rate is obtained by summing the counts across all channels and dividing by the exposure time. Good time intervals (GTIs) are identified from the GTI file included in the standard products. The average number of PCUs that are turned on during the GTIs is determined from the filter file included in the standard products, which indicates that we are studying the complete evolution of state transitions in GX 339–4.

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To justify using the $\chi^2$ fitting statistic for deriving parameter constraints, both the PCA and HXTE spectra were binned to require a minimum of 20 counts per energy bin using the {	t FTOOL} {	t grppha}. A systematic error of 0.6 per cent was added to the PCA spectrum

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4 http://heasarc.nasa.gov/docs/xte/recipes/plotprod_guide.html
5 http://heasarc.nasa.gov/docs/xte/xhp_new.html

Figure 1. HID for all observations of GX 339–4 considered in this work generated from the results of the POW/BKNPOW model (see Section 4.2). The disc fraction for each observation is given by the glyph colour and is an indicator of the spectral state. Data points marked by a grey cross correspond to observations that were not deemed to statistically require a disc component. The key in the bottom left identifies the glyph associated with each transition ID. See Section 2.2 for a description of how the HID was generated. A wide range of hardness is considered in this work.

**3 MCD BLACKBODY MODELS**

The accretion disc in BHBs manifests itself as a soft, quasi-blackbody component with a typical peak inner disc temperature of ~1 keV (e.g. Fender & Belloni 2004). While the accepted description for the origin of the soft thermal component in the HSS is a geometrically thin, optically thick accretion disc (Shakura & Sunyaev 1973) extending down to the ISCO, no such consensus has been reached for the nature of the innermost flow in the HIS or...
3.1 The standard MCD model

The most commonly adopted disc model is the MCD blackbody model (DISKBB in XSPEC) (Mitsuda et al. 1984; Makishima et al. 1986). The attractiveness of the MCD model is owed to its legacy, applicability and simplicity. However, the familiarity and broad acceptance of DISKBB often mask subtle features of the model, possibly leading to unintended misuse. Here, we dissect the DISKBB model in order to elucidate its underlying assumptions and expose its limitations. Assuming a geometrically thin, optically thick accretion disc that locally emits a blackbody spectrum in concentric annuli, the colour temperature for an MCD model at a given radius is described as

$$T_{\text{col}}(r) = T_c \left( \frac{r}{R_{\text{in}}} \right)^{-3/4} \left[ 1 - \beta \left( \frac{r}{R_{\text{in}}} \right)^{-1/2} \right]^{1/4}, \quad (1)$$

where $R_{\text{in}}$ is the radius of the inner disc edge and $\beta$ describes the boundary condition at this location. For discs in the HSS, $R_{\text{in}}$ is commonly associated with $R_{\text{ISCO}}$. A zero-torque boundary condition at $R_{\text{ISCO}}$ corresponds to $\beta = 1$, while $\beta = 0$ allows for non-zero stresses at the inner disc and is the choice adopted in DISKBB (Zhang et al. 1997; Gierliński et al. 1999; Zimmerman et al. 2005). The leading factor, which roughly describes the colour temperature at the inner disc, is given by

$$T_\ast = f_{\text{col}} \left( \frac{3 GM \dot{M}}{8 \pi \sigma R_{\text{in}}^4} \right)^{1/4}, \quad (2)$$

where $G$ is the gravitational constant, $\sigma$ is the Stefan–Boltzmann constant, $M$ is the compact object mass, $\dot{M}$ is the mass accretion rate and $f_{\text{col}}$ is the colour-correction factor (Shimura & Takahara 1995), also referred to as the spectral hardening factor. For a zero-torque condition at $R_{\text{in}}$ (i.e. $\beta = 1$ in equation 1), $T_\ast$ is related to the maximum temperature attained in the disc by $T_{\text{col, max}} = 0.488T_\ast$, which occurs at $r = (49/36)R_{\text{in}}$. Since the disc temperature does not reach $T_\ast$ at any radius, $T_{\text{col, max}}$ is a more physically meaningful characterization of the blackbody temperature associated with the spectrum. For a thin disc that is torqued at the inner boundary, as is the case for DISKBB, $T_{\text{col, max}}$ coincides with $T_\ast$ because the temperature profile grows asymptotically with decreasing $r$ (see fig. 1 of Zimmerman et al. 2005). One of the two free parameters of the DISKBB model is $T_\ast$, implicitly assuming $f_{\text{col}} = 1$. The ‘normalization’ is the other free parameter of DISKBB and is given by

$$K_{\text{bb}} = 100 \frac{1}{f_{\text{col}}} \left( \frac{R_{\text{in, km}}}{D_{\text{pc}}} \right)^2 \cos i, \quad (3)$$

where, again, $f_{\text{col}} = 1$, $D_{\text{pc}}$ is the distance to the source in kiloparsecs, $R_{\text{in, km}}$ is the inner disc radius in kilometres and $i$ is the inner disc inclination. Inner disc radii are commonly derived from $K_{\text{bb}}$ by measuring intrinsic system properties and imposing a constant $f_{\text{col}}$ (not necessarily unity as assumed by DISKBB), regardless of the source luminosity state (e.g. Zdziarski et al. 2004). Remarkably, the simple two-parameter DISKBB model fully describes the accretion disc spectrum under the appropriate assumptions of a non-zero torque inner disc boundary condition and a fixed colour-correction factor for characterizing the overall deviation from a blackbody spectrum.

3.2 Zero-torque boundary condition and disc vertical structure

If the inner disc radius can be identified with the ISCO, a zero-torque boundary condition becomes attractive from the physical consideration that the gas viscous inspiral time-scale is long relative to the free-fall time-scale for gas at the ISCO; therefore, the gas interior to $R_{\text{in}}$ does not have time to radiate appreciably before being accreted (Novikov & Thorne 1973; Shakura & Sunyaev 1973; Page & Thorne 1974; Abramowicz & Kato 1989; Frank, King & Raine 1992; Afshordi & Paczyński 2003). In principle, for a geometrically thin disc in the presence of a magnetic field, non-negligible torque resulting from magnetic stresses may be present at the ISCO (Gammie 1999; Krolik 1999; Agol & Krolik 2000; Afshordi & Paczyński 2003). However, magnetohydrodynamic three-dimensional simulations of global thin accretion discs around black holes demonstrate that a weak magnetic field is unable to couple the gas within the plunging region to the disc, despite large fluctuating magnetic stresses inside the ISCO, in accordance with the zero-torque boundary condition (Armitage, Reynolds & Chiang 2001; Reynolds & Armitage 2001). Similar simulations exploring initially purely poloidal and toroidal field topologies instead observe significant stress at the ISCO and continuing deep within the plunging region, refuting the zero-torque assumption (Hawley & Krolik 2001, 2002). A difficulty with allowing for a non-zero torque in disc models is that there is no obvious choice for the magnitude of the torque, necessitating the introduction of a free parameter. The zero-torque choice is both physically motivated and is not arbitrary; however, the presence of significant torque on the inner disc may be important in real systems depending on highly unknown factors such as the disc thickness, dissipation properties and magnetization.

The underlying physics responsible for altering the locally emitted spectrum of an optically thick disc from a pure blackbody are parametrized into the colour-correction factor. The result of this approximation is the colour-corrected, or ‘diluted’, blackbody,

$$I_\nu = \frac{1}{f_{\text{col}}} B_\nu (f_{\text{col}}T_{\text{col}}), \quad (4)$$

where $I_\nu$ is the specific intensity, $B_\nu$ is the Planck function, $T_{\text{col}} = T_{\text{col}}/f_{\text{col}}$ is the effective temperature and $f_{\text{col}} = 1$ refers to a pure blackbody spectrum. In reality, deviations in the emergent disc spectrum from a canonical blackbody arise from frequency-dependent opacities determined from the disc vertical structure, which is neglected in the standard thin disc treatment. The degree of spectral modification will be governed by the combined effects of electron scattering (i.e. Comptonization in the disc), free–free emission/absorption and bound-free absorption, resulting in a departure from local thermodynamic equilibrium and a hardening of the spectrum (Felten & Rees 1972; Shimura & Takahara 1995; Zavlin, Pavlov & Shibanov 1996; Rutledge et al. 1999b; McClintock, Narayan & Rybicki 2004). A surrounding corona acting as a deposition for some fraction of the disc accretion power provides yet another means for altering the emergent disc spectrum (Svensson & Zdziarski 1994; Merloni, Fabian & Ross 2000). Furthermore, magnetic pressure support likely contributes significantly to hydrostatic balance in the disc photosphere and may act to vertically extend the disc atmosphere, producing a harder spectrum (Blaes et al. 2006; Begelman & Pringle 2007). The phenomenological colour-correction prescription avoids all of these complications by neglecting any frequency dependence, only permitting transnational (i.e. frequency-independent) hardening and attenuation of the Planck function. Provided that this model yields a satisfactory fit
to the observed disc spectrum, one presumes that $f_{\text{col}}$, which is assumed to be constant for all radial locations in the disc, provides an adequate description of the disc vertical structure in an average sense.

The two disc models we appeal to in this work differ from DISKBB in that they both adopt a zero-torque inner boundary condition and allow for the possibility of a variable colour-correction factor. The EZDISKBB model (Zimmerman et al. 2005) is the analogue of DISKBB, but with the zero-torque boundary condition enforced. The free parameters are the maximum disc colour temperature, $T_{\text{col, max}}$ defined above, and the normalization, given by

$$K_{\text{col}} = 100 \frac{1}{f_{\text{col}}} \left( \frac{R_{\text{in,km}}}{D_{\text{eqc}}} \right)^2 \cos i,$$

where $f_{\text{col}}$ is not specified. We emphasize that changes in $K_{\text{col}}$ do not necessarily imply changes in the disc inner radius, due to the parameter degeneracy between $R_{\text{in}}$ and $f_{\text{col}}$. The DISKPN model (Gierliński et al. 1999), along with the zero-torque boundary condition, incorporates a pseudo-Newtonian potential (Paczyński & Wiita 1980) to more accurately calculate the radial temperature distribution in the disc at the expense of introducing the inner disc radius as a free parameter. The two remaining free parameters are $T_{\text{col, max}}$ and the normalization,

$$K_{\text{PN}} = 1 \frac{M_{\text{sun},0}}{f_{\text{col}}} \left( \frac{M_{\text{sun},0}}{D_{\text{eqc}}} \right)^2 \cos i,$$

where $M_{\text{sun},0}$ is the compact object mass in solar mass units. In all of the MCD model normalizations, $f_{\text{col}}$ enters to the fourth power. Therefore, asking how much $f_{\text{col}}$ could conceivably change during a state transition is not unreasonable, as only small changes in $f_{\text{col}}$ are required to explain the observed spectral evolution, which may not necessitate disc truncation.

The disc models described up to this point neglect many important physical considerations. We wish to emphasize that sophisticated, physically motivated models are available in XSPEC that account for irradiation of the disc (DISKIR; Gierliński, Done & Page 2008), general relativistic effects around a Schwarzschild (GRAD; Ebisawa, Mitsuda & Hanawa 1991) and Kerr black hole (KERRBB; Li et al. 2005), and detailed modelling of the disc vertical structure around a Kerr black hole (BHSPEC; Davis et al. 2005). Realistic disc models come with the expense of introducing additional parameters. The limited quality of X-ray observations and the desire to limit the number of model parameters forces the use of oversimplified disc models, with the hope that they can adequately capture the most important aspects of the accretion physics. The purpose of the subsequent analysis is to demonstrate that simplistic disc models produce good spectral fits but they cannot be used to make reliable claims regarding the behaviour of the inner disc radius.

### 4 ANALYSIS AND RESULTS

In what follows, we exploit the timing and monitoring capabilities of RXTE, which provide multiple spectral samples of moderate resolution tracking the transition between spectral states, to study the evolution of the accretion disc in GX 339–4. The ultimate goal of our spectral analysis is to isolate the disc component in order to measure relative changes in disc properties during transitions over the rise (LHS → HSS) and decay (HSS → LHS) stages of an outburst. We show that allowing for physically reasonable changes in $f_{\text{col}}$ provides an alternative explanation to disc truncation for the observed disc spectral evolution during state transitions.

### Table 2. Best-fitting parameters and χ² standard uncertainty in parentheses for all observations, where a disc was deemed required. The spectral models employed for the disc and continuum were DISKBB and POWER-LAW (or BKNPOWER, respectively). The Fe line at 6.4 keV is fitted with the GAUS model. The entire model is absorbed using PHABS with $N_{\text{H}} = 5.7 \times 10^{20}$ cm$^{-2}$ (Miller et al. 2009b), and an energy-independent constant is applied to the HEXTE data to account for offset relative to the PCA. A 0.6 per cent systematic error is added to all PCA energy spectrum points. Parameters are defined above, and the normalization is given by $K_{\text{col}}$ or all fits used in this work is available as an electronic supplement.

| Observation ID | Trans. ID | MJD | Best-fitting model | $\Gamma_{\text{in}}$ | $\Gamma_{\text{out}}$ | $\Gamma_{\text{warm}}$ | $\Gamma_{\text{hot}}$ | $E_0$ (keV) | $\sigma$ | $\sigma$ | $\chi^2$/DF |
|---------------|----------|-----|-------------------|----------------|----------------|----------------|----------------|-----------|-------|--------|--------|
| R02 | 40031-03-02-00 | 523 | 83.103 | bkn | 1.52(1) | 32(1) | 2.34(6) | 1.38(4) | 2.37(3) | 1.43(3) | 1.31(3) | 1.24(4) | 1.04(1) | 0.024(4) | 1.37(8) | 27(9) | 0.72 | 70 | 0.84 | 0.86 (2) | 1.62 (4) |
| R02 | 40031-03-02-01 | 523 | 84.147 | bkn | 1.52(6) | 30(2) | 2.44(8) | 1.42(6) | 2.40(6) | 1.43(6) | 1.31(3) | 1.29(5) | 1.04(1) | 0.024(4) | 1.31(3) | 24(10) | 0.72 | 70 | 0.84 | 0.86 (2) | 1.62 (4) |
| R02 | 40031-03-02-02 | 523 | 84.979 | bkn | 1.52(8) | 32.0(9) | 2.37(3) | 1.43(3) | 2.44(8) | 1.43(6) | 1.31(3) | 1.29(5) | 1.04(1) | 0.024(4) | 1.31(3) | 24(10) | 0.72 | 70 | 0.84 | 0.86 (2) | 1.62 (4) |
| R02 | 70110-01-05-00 | 523 | 85.469 | bkn | 1.58(2) | 33(1) | 2.40(6) | 1.43(6) | 2.40(6) | 1.43(6) | 1.31(3) | 1.29(5) | 1.04(1) | 0.024(4) | 1.31(3) | 24(10) | 0.72 | 70 | 0.84 | 0.86 (2) | 1.62 (4) |
| R02 | 70110-01-06-00 | 523 | 87.490 | bkn | 1.59(2) | 34.0(9) | 2.42(4) | 1.43(6) | 2.40(6) | 1.43(6) | 1.31(3) | 1.29(5) | 1.04(1) | 0.024(4) | 1.31(3) | 24(10) | 0.72 | 70 | 0.84 | 0.86 (2) | 1.62 (4) |
| R02 | 70109-01-04-00 | 523 | 87.646 | bkn | 1.54(8) | 32.0(9) | 2.37(3) | 1.43(3) | 2.44(8) | 1.43(6) | 1.31(3) | 1.29(5) | 1.04(1) | 0.024(4) | 1.31(3) | 24(10) | 0.72 | 70 | 0.84 | 0.86 (2) | 1.62 (4) |
| R02 | 40031-03-02-05 | 523 | 88.065 | bkn | 1.52(1) | 32.1(9) | 2.40(6) | 1.43(6) | 2.40(6) | 1.43(6) | 1.31(3) | 1.29(5) | 1.04(1) | 0.024(4) | 1.31(3) | 24(10) | 0.72 | 70 | 0.84 | 0.86 (2) | 1.62 (4) |
| R02 | 40031-03-02-06 | 523 | 89.017 | bkn | 1.52(8) | 34.0(9) | 2.41(2) | 1.43(6) | 2.40(6) | 1.43(6) | 1.31(3) | 1.29(5) | 1.04(1) | 0.024(4) | 1.31(3) | 24(10) | 0.72 | 70 | 0.84 | 0.86 (2) | 1.62 (4) |

Applies for fits using the BKNPOWER model; + applies for using the HSS model.
provides a sample of the best-fitting parameters for the fits used in this work. Table 2 in its entirety is available as an electronic supplement.

4.1 Requirement of a disc component

We wish to verify the presence of a thermal disc component with adequate confidence using phenomenological models that have proven successful in characterizing the general features of BHB X-ray spectra. To determine whether or not a disc component is required in a given RXTE spectrum, we adopt a methodology based on $\chi^2$ statistics that uses phenomenological models to fit numerous observations in various spectral states, as motivated by previous works (Dunn et al. 2008, 2010, 2011; Reynolds & Miller 2013). The PCA and HEXTE data are fit jointly, allowing an energy-independent constant factor to vary for the HEXTE data to account for any normalization offset relative to the PCA and linking all parameters between the PCA and HEXTE spectra. The spectral models are modified by photoelectric absorption using the PHABS model with a fixed equivalent neutral hydrogen column density of $N_H = 5.7 \times 10^{21}$ cm$^{-2}$ (Miller et al. 2009a). Both Galactic and intrinsic absorption are folded into the PHABS model, leading to a potential ambiguity as to whether low-energy spectral evolution is due to changes in the intrinsic absorption or to evolution of the source. Miller, Cackett & Reis (2009b) fitted high-resolution BHB spectra (including GX 339–4) over a wide range in luminosity and spectral states, demonstrating that the low-energy spectral evolution is a consequence of the evolution of the source itself, rather than a variable neutral photoelectric absorption. Notably, Cabanac et al. (2009) present the case for possible variations in $N_H$ during outbursts of GX 339–4; however, the restricted low-energy coverage of RXTE does not permit $N_H$ to be a free parameter in our spectral fits.

As a first attempt, the spectra are fitted with a simple model consisting of an absorbed power-law and MCD blackbody (i.e. PHABS x (POWERLAW+DISKBB) in XSPEC) to model the continuum and accretion disc, respectively. In many observations, this simple prescription does not yield an adequate fit due to the presence of a broad Fe Kα emission line. To check for the requirement of an iron line component, a Gaussian line profile (GAUSS in XSPEC) is added to the model at fixed centroid energy of 6.4 keV with the width, $\sigma$, and normalization, $K_{\text{line}}$, as free parameters. An $F$-test is inappropriate for determining whether the inclusion of a spectral line is a statistically significant improvement (Protassov et al. 2002). The alternative diagnostics outlined by Protassov et al. (2002) for testing the presence of a line are not easily incorporated into the spectral fitting framework used here. We are concerned with obtaining an acceptable fit in order to determine the presence of a disc component and do not wish to make any strong statements regarding Fe Kα emission; therefore, we adopt the line detection procedure of Dunn et al. (2008). In order for a line to be considered significant, the following two criteria must be met. (1) An $F$-test must determine that the line is significant at $\geq 3\sigma$ ($F$-statistic probability $P \leq 0.0027$). (2) Given the $\sigma$ uncertainty in the line normalization, $\sigma_{\text{line}}$, obtained with the XSPEC error command, we require $K_{\text{line}} \geq 3\sigma_{\text{line}}$. This procedure for line determination is more conservative than appealing to an $F$-test alone, though we note that some detected line features may be erroneous.

We next test for the presence of a high-energy break in the spectrum by replacing the power-law component with the broken power-law model, BKNPOWER, and repeating the aforementioned fitting procedure. If an $F$-test determines that the broken power-law model is a significant improvement over the power-law model at $\geq 3\sigma$ confidence, then we proceed with the broken power-law model imposing the parameter ranges for the spectral indices and break energy of $1 \leq \Gamma \leq 4$ and $E_b \geq 10$ keV, respectively. In instances where a line component was deemed necessary in the power-law model but not the broken power-law model, or vice versa, we select the best-fitting model determined by the lowest reduced $\chi^2$-statistic, $\chi^2_\nu \equiv \chi^2 / \nu$, where $\nu$ is the number of degrees of freedom in the model. For cases when the best fit converges to the parameter limits of the BKNPOWER model, the pipeline reverts to the POWERLAW model for that observation. If the best-fitting POWERLAW model also runs into a parameter bound, the observation is discarded.

At this point in the phenomenological fitting pipeline, for a fit to be considered acceptable for use in subsequent analysis, we require $\chi^2_\nu \leq 2$. Often, a poor fit to the high-energy spectrum is the culprit for failure to meet this criterion. For instance, a poor fit may result from the inability of simplistic, unphysical models to properly fit a reflection component or a complicated spectrum arising from a dynamic corona. Since we are using phenomenological models to characterize the soft disc component and are not interested in a detailed characterization of the high-energy spectrum, if the best-fitting model to the combined PCA/HEXTE data gives $\chi^2_\nu > 2$, then we repeat the above fitting procedure on the PCA data alone in search of an acceptable fit. In these cases where only the PCA data are being fitted, we use the POWERLAW model only because any spectral energy break is expected to lie beyond the high-energy cutoff of the PCA, rendering the use of the BKNPOWER model inappropriate.

The disc component (DISKBB) is then removed from the best-fitting model and the spectrum is re-fitted. An $F$-test is performed to determine whether or not the inclusion of the disc improved the fit by $\geq 3\sigma$. Observations that meet these criteria are kept for further analysis, while those that do not statistically require a disc are discarded. Running this model fitting procedure on 283 total observations of GX 339–4 in different stages of transition between the low/hard and high/soft states resulted in 177 observations where a disc component was deemed statistically required from spectral fitting techniques (see Table 1).

We caution that the absence of a quasi-blackbody component in a given RXTE spectrum does not imply the absence or truncation of the physical accretion disc. The low-energy cutoff of the RXTE PCA is restricted to $\sim 3$ keV; thus, only the Wien tail of the MCD blackbody spectrum will be in the observable band. Although we may not detect a disc, especially in the HIS and LHS, this is not enough to claim the absence of a disc component, which may be dominant at energies below the sensitivity of RXTE (e.g. Dunn et al. 2008) and has been observed with XMM–Newton in the LHS of GX 339–4 (e.g. Miller et al. 2006; Reis et al. 2008).

Fig. 2 shows the data-to-model ratios with the disc component removed from the best-fitting model for representative observations of each state transition. Disc fractions are defined by

$$DF \equiv \frac{F_{\text{disc}}}{F_{\text{total}}} \frac{0.1-10 \text{ keV}}{1-100 \text{ keV}}. \quad (7)$$

where $F_{\text{disc}}$ and $F_{\text{total}}$ are the unabsorbed fluxes of the disc component and overall model, respectively, taken over the energy ranges indicated by the superscripts. Fluxes were obtained with the cflux command in XSPEC. Disc components are robustly detected by the pipeline described above, even for moderately low disc fractions, which is essential for tracking the evolution of the disc throughout each transition in the subsequent analysis.
produces a power-law spectrum to first order. The phenomenological power-law model extends to arbitrarily low energies, which does not mimic a physical Comptonization process where a sharp cutoff is expected at an energy comparable to the characteristic energy of the input seed photon distribution, namely the quasi-blackbody disc spectrum. Therefore, the extension of the power-law component towards arbitrarily low energies results in ‘flux stealing’ from the disc component, making a disc more difficult to detect than had we adopted a more physical Comptonization model.

To investigate the sensitivity of fitting the disc component on the choice of Comptonization model, we explore two different models to characterize the continuum. The Comptonization model COMPTT (Titarchuk 1994; Hua & Titarchuk 1995) provides an analytic description of the spectrum produced from a soft photon distribution propagating through a hot plasma cloud. We re-ran the disc detection pipeline with COMPTT in place of the power-law component, allowing only the plasma temperature, $T_p$, plasma optical depth, $\tau_p$, and normalization, $K_{\text{comptt}}$, to be free parameters. The remaining COMPTT parameters that we chose were a redshift $z = 0$, a seed photon temperature, $T_0$, and normalization, $K_0$, to be free parameters. A total of 150/177 observations required a disc component when COMPTT was used, compared to the 177/283 using the standard phenomenological POWERLAW and BKNPOWER models.

An alternative empirical substitute for a power-law model is SIMPL (Steiner et al. 2009), which truncates the power-law spectrum at an energy comparable to the characteristic input seed photon energy; thus, avoiding the unphysical low-energy extension of the POWERLAW and BKNPOWER models. Since SIMPL is a convolution model, we cannot apply the disc detection pipeline of Section 4.1 to determine whether or not a disc component is required. Instead, the previously determined 177 best-fitting models are re-fitted with SIMPL as the component responsible for modelling Comptonization, allowing for both up- and down-scattering of the input seed spectrum. Again, as was done for the disc detection pipelines using POWERLAW/BKNPOWER and COMPTT as the continuum models, the observation is discarded if the best fit converges on a parameter limit. This returned 150/177 observations where the best fit does not encounter a parameter limit. 

Figure 2. Ratios of the RXTE data to the best-fitting models with the DISKBB component removed to highlight the need for a quasi-blackbody feature to model the soft excess. For each transition, three data/model ratios are shown that representatively span the range in measured disc fractions. Glyph colours denote the disc fraction for each fit. For comparison, the best-fitting models where Comptonization is modelled by the POWERLAW or BKNPOWER (left-hand column) and COMPTT (right-hand column) XSPEC models are shown. For each row, the observation IDs are given in the upper-right corner of the plots and the transition ID on the rightmost ordinate.

4.2 Sensitivity of spectral fits to the continuum model

The motivation for employing a simple power-law model with limited free parameters is to fit the portion of the spectrum that is thought to arise from Comptonization of thermal disc photons by a low-density, energetic electron corona of unknown geometry, which
4.3 Disc evolution results from spectral fitting

A parameter degeneracy exists between the inner disc radius and the colour-correction factor within the normalization of the standard DISKBB model. Therefore, decomposing the contributions to the disc evolution due to changes in the vertical disc structure, parametrized into $f_{\text{eq}}$, and a migrating inner disc region is non-trivial. Combining the models DISKPN and EZDISKBB, while appealing to relative measurements of the disc model normalization evolution, we examine the degree of change in $f_{\text{eq}}$ and $R_{\text{in}}$ required to explain the disc spectral evolution in state transitions of GX 339–4.

4.3.1 Relative measurements

Obtaining reliable absolute measurements of quantities locked up in the disc normalization is limited by constraints on the observables, namely the mass, distance and disc inclination, which all appear in the normalization. However, relative changes in $f_{\text{eq}}$ and $R_{\text{in}}$ are accessible from the evolution of the disc normalization because the intrinsic parameters do not change between successive observations.

Imposing a small systematic error on RXTE PCA data is common practice, resulting in a substantial improvement in the quality of spectral fits. Applying model fits to data where no systematic error is added will result in poor fits but with the best-fitting parameter errors dramatically reduced. While the best-fitting parameter values may be unreliable, they should be affected in a systematic sense and systematic errors associated with instrument calibration will be removed from the parameter errors. Provided that relative, rather than absolute, measurements are of interest, the evolution of a parameter may be tracked from fits to spectra without systematic error incorporated into the PCA data.

To test the validity of this proposition, we re-fitted all spectra deemed to require a disc component with the best-fitting models determined from the aforementioned fitting pipeline, but with no systematic error added to the PCA data. Fig. 5 compares the distribution of the DISKBB normalization relative error, defined as
Spectral hardening in state transitions

Figure 5. Histogram of relative errors on the best-fitting DISKBB normalization, $\sigma_{K_{bb}}/K_{bb}$, for all RXTE observations requiring a disc component. Models fitted with a systematic error of 0.6 per cent added to all PCA energy channels and no systematic error added to the PCA data are shown as red forward slashes and grey bars, respectively. The relative error on the disc normalization is reduced when systematic error is not added to the data.

For each transition, Fig. 6 shows $K_{bb,\text{rel}}=K_{bb,\text{nosys}}/K_{bb,\text{w}}$, which is defined as the best-fitting DISKBB normalization for fits with no systematic error added, $K_{bb,\text{nosys}}$, relative to the normalization for fits with 0.6 per cent systematic error added, $K_{bb,\text{w}}$. The constancy of this ratio throughout the spectral evolution of all transitions suggests that excluding systematic error for the PCA data to study the evolution of the disc normalization in a relative sense is justified in practice.

Occasionally, an observation that was adequately fitted when a 0.6 per cent systematic error was added to the PCA data will converge on parameter limits or grossly unphysical values when the data without systematic error are fitted. In these situations, the data without systematic error added are fitted by hand in order to achieve a fit with more realistic best-fitting parameters. All errors presented in this work are $1\sigma$ and are appropriately propagated when making relative measurements.

$\sigma_{K_{bb}}/K_{bb}$ for fits to all GX 339-4 spectra when no systematic error and the standard 0.6 per cent systematic error were added to the PCA data (Jahoda et al. 2006). As expected, not including systematic errors results in improved relative errors on the DISKBB normalization. Adopting an analysis technique based on omitting systematic errors from the PCA data requires that the disc normalization values are only affected in a systematic way as a result. For each transition, Fig. 6 shows $K_{bb,\text{rel}}=K_{bb,\text{nosys}}/K_{bb,\text{w}}$, which is defined as the best-fitting DISKBB normalization for fits with no systematic error added, $K_{bb,\text{nosys}}$, relative to the normalization for fits with 0.6 per cent systematic error added, $K_{bb,\text{w}}$. The constancy of this ratio throughout the spectral evolution of all transitions suggests that excluding systematic error for the PCA data to study the evolution of the disc normalization in a relative sense is justified in practice.

Occasionally, an observation that was adequately fitted when a 0.6 per cent systematic error was added to the PCA data will converge on parameter limits or grossly unphysical values when the data without systematic error are fitted. In these situations, the data without systematic error added are fitted by hand in order to achieve a fit with more realistic best-fitting parameters. All errors presented in this work are $1\sigma$ and are appropriately propagated when making relative measurements.

Two identical data sets, one with a small systematic error added ($X$) and one without ($Y$), are nearly perfectly correlated (i.e. the correlation coefficient, $\rho_{X,Y}=\text{cov}(X, Y)/\sigma_X\sigma_Y \approx 1$). Since both $X$ and $Y$ only differ in the uncertainties associated with the data, their covariance is given by, $\text{cov}(X, Y)=\text{cov}(X, X)=\sigma_X^2$. The resulting correlation coefficient becomes $\rho_{X,Y}=\sigma_X/\sigma_Y \approx 1$, assuming that the systematic errors added to $Y$ are small, which is true for our purposes.

4.3.2 Disc spectral evolution: variable $f_{\text{col}}$

A variable colour-correction factor has been considered from both a numerical (e.g. Shimura & Takahara 1995; Merloni et al. 2000;...
Davis et al. (2005) and observational (e.g. Dunn et al. 2011; Reynolds & Miller 2013) approach, with $1.4 < f_{col} < 3$ generally believed to be a conservatively broad allowable range, implying at most a factor of $\sim 2$ change in $f_{col}$. However, this range in $f_{col}$ does not consider regimes extending to the low disc luminosities associated with the HIS and LHS nor the effects of disc inhomogeneities and strong magnetization. Spectra that are less disc dominated are harder; thus, requiring an increased $f_{col}$. The possibility of extending the upper $f_{col}$ bound for discs in the HIS and LHS (i.e. non-disc-dominated regimes where $L_{disc} \lesssim 0.01 L_{Edd}$) and whether a colour-corrected blackbody is an appropriate description of the disc spectrum in this regime remain open questions.

Using the DISKPN model with the inner disc radius fixed ensures that the evolution of the disc normalization is due entirely to a variable colour-correction factor, which can be computed directly from equation (6), as demonstrated by Reynolds & Miller (2013). We elect to explore the degree of change in $f_{col}$ alone that is required to explain the disc spectral evolution for a fixed inner disc radius of $6R_g$. This choice for $R_{in}$ is motivated by considering a disc extending down to the ISCO for a Schwarzschild black hole and is the minimum value allowed by DISKPN.

To measure the evolution of absolute $f_{col}$ values required to explain the disc evolution in each transition for a fixed inner disc radius, we re-fit all of the spectra with 0.6 per cent systematic error added to the PCA data using the best-fitting models, but replacing the DISKBB component with DISKPN and freezing $R_{in}$ at $6R_g$. Fig. 7 shows the evolution in $f_{col}$ that is necessary to account for each transition, with the error bars incorporating the best-fitting $K_{pn}$ error and the range in intrinsic parameters of GX 339–4. The values adopted for the GX 339–4 observables, along with their (lower, upper) limits, are $M = 7 (5.8, 12) M_\odot$, $D = 8 (6, 10)$ kpc, and $i = 29^\circ (10^\circ, 60^\circ)$ (see Section 2.1 and references therein). Fig. 7 shows that, within the uncertainties, the disc spectral evolution can potentially be explained by a variable $1.4 < f_{col} < 3$ for an inner disc radius that remains at $6R_g$ throughout each state transition.

Studying the evolution of absolute $f_{col}$ values for a fixed $R_{in}$ is limited by the large uncertainties in the mass, distance and inclination of GX 339–4. Given that an inner disc that remains at $6R_g$ combined with a modestly variable $f_{col}$ may be a viable model for accretion disc evolution in the state transitions studied here, we use the relative measurement technique introduced in Section 4.3.1 to better understand the degree of change in $f_{col}$ that is required to explain the observations without appealing to disc truncation.

The PCA data are replaced with a duplicate copy where no systematic errors have been added. Next, the DISKPN fits where $R_{in} = 6R_g$ are re-fitted and a best fit is found for each observation. This is done for each of the three Comptonization models POW/BKNPOW, SIMPL and COMPTT. For each observation in a given transition, $f_{col}$ is computed from $K_{pn}$ using equation (6), without specifying the intrinsic parameters (i.e. $M$, $D$, $i$). The resulting $f_{col}$ values are normalized to the $f_{col}$ from a representative HSS observation, $f_{col,HSS}$, and further normalized to the adopted $f_{col} = 1.4$ value for the HSS (Davis et al. 2005), yielding the relative colour-correction factor

$$f_{col,rel} = 1.4 \left( \frac{f_{col}}{f_{col,HSS}} \right).$$

Supposing that the inner disc remains fixed at $6R_g$ throughout each state transition, Figs 8–10 show the relative change in $f_{col}$ alone required to explain the observed disc spectral evolution. Notably, the POW/BKNPOW model requires more drastic change in $f_{col}$ compared to COMPTT and SIMPL. Under the presumption that $f_{col,rel} = 3$ is a conservative upper bound, the notion of a variable $f_{col}$ alone is perhaps possible for transitions R02, R07, R10 and D11; however, an evolving $f_{col}$ is an inadequate description of the evolving disc spectrum for transitions D03, R04 and D07, regardless of the Comptonization model employed. For the later set of

Figure 7. Evolution of the colour-correction factor over all state transitions derived from the best-fitting DISKPN normalizations where the inner disc radius was held at $6R_g$. The left-hand, middle and right-hand panels correspond to fits using the POW/BKNPOW, COMPTT and SIMPL continuum models, respectively. The abscissa shows time elapsed since the start time, $t_0$, of the transition (see Table 1). Error bars account for the maximum conceivable ranges of the mass, distance and inclination of GX 339–4 (see the text). The dashed lines enclose $1.4 < f_{col} < 3$, which is the adopted allowable range in $f_{col}$. The transition ID corresponding to each row is indicated on the rightmost ordinate. Within the large uncertainties, a non-truncated disc with a variable $f_{col}$ can adequately explain the observed disc spectral evolution.
Spectral hardening in state transitions

Figure 8. Relative measurements of the evolution of $f_{col}$ for each transition derived from fits using POW/BKNPOW to model Comptonization. The two lower dashed lines enclose $1.4 < f_{col} < 3$, while the upper dashed line marks $f_{col} = 5$. The $f_{col, rel}$ measurements for each transition were made relative to the filled glyphs and normalized to $f_{col} = 1.4$, which is appropriate for the HSS. The disc evolution can be explained for transitions R02, R07, R10 and D11 by a non-truncated disc with $1.4 \leq f_{col, rel} \leq 5$.

transitions, one must invoke disc truncation or an alternative disc model to satisfactorily explain the observed disc evolution. Reynolds & Miller (2013) speculate that if the colour-corrected blackbody is a viable model of the LHS, then $f_{col}$ may be as large as $\sim 5$ based on spectral fits to a comprehensive set of BHB spectra. Inspecting Figs 9–10, where the COMPTT and SIMPL models were adopted, all seven transitions essentially conform to this extended $f_{col}$ range; however, making definitive claims regarding the physically plausible $f_{col}$ range and applicability of the MCD model in the LHS is beyond our scope.

4.4 A hot inner disc can masquerade as an increased $f_{col}$

Observational evidence suggests that GX 339–4 harbours a near-maximally spinning black hole (see Section 2.1). Consequently, the inner disc extends deep into the black hole potential and relativistic effects become non-negligible, rendering Newtonian disc models inappropriate. The disc models used in this work to derive $f_{col}$ measurements impose an effective lower limit on the inner disc radius of $6R_g$, whereas the ISCO of a Kerr black hole with $a_\ast = 0.998$ lies at
Using the XSPEC command fakeit none, we generate a suite of simulated disc spectra with the KERRBB model of a spinning black hole with an RXTE response matrix and integration time of 2000 s. The cases considered are a Schwarzschild black hole ($a_*=0, \eta=0.057$) and maximal Kerr black hole ($a_*=0.998, \eta=0.30$) (Thorne 1974), each having a mass of $7M_\odot$ located at a distance of $8$ kpc with a disc inclination of $30^\circ$. The inner disc torque is set to zero and the effects of both self-irradiation and limb darkening on the disc spectrum are switched on. We pair mass accretion rates with colour-correction factors that are reasonably representative of the spectral state corresponding to each $M$ that we select. For the $a_* = 0$ black hole, we simulate disc spectra for $M = 0.05M_{\text{Edd}}$ with $f_{\text{col}} = 1.4$, attempting to mimic a HSS/SIS accretion disc; and $M = 0.005M_{\text{Edd}}$ with $f_{\text{col}} = 3$, which is more representative of a HIS/LHS observation. Similarly, for the $a_* = 0.998$ black hole, we choose $M = 0.01M_{\text{Edd}}$ with $f_{\text{col}} = 1.4$ and $M = 0.001M_{\text{Edd}}$ with $f_{\text{col}} = 3$. Extending $M$ beyond these ranges for a fixed $f_{\text{col}}$ resulted in either simulated data with unconstrained uncertainties or unacceptable fits (i.e. $\chi^2 > 1$) using the procedure described below.

Each spectrum is fitted over the energy range 2.8–10 keV using the DISKPN model with $R_a$ fixed at $6R_g$. The best-fitting colour-correction factor is computed from the normalization using equation (6). Table 3 presents the results of this exercise. In general, the pseudo-Newtonian model, DISKPN, reliably recovers the true $f_{\text{col}}$ value for a non-spinning black hole, but requires a factor of $\sim 1.9$ increase in $f_{\text{col}}$ for a maximally spinning black hole. This demonstrates that the inability of Newtonian disc models to properly model $R_a$ and $T_{\text{eff}}$ for Kerr black holes can result in unphysically large $f_{\text{col}}$ measurements to account for the hotter inner disc.

One may wonder if relative measurements of $f_{\text{col}}$ derived from Newtonian disc models, as done in this work, are affected by the inability of these models to properly represent the hotter discs around spinning black holes. A colour-correction factor evolving in an HSS → LHS transition from $f_{\text{col}} = 1.4 \rightarrow 3$ corresponds to a factor of $2.1$ increase in $f_{\text{col}}$. Inspection of Table 3 indicates that the DISKPN model requires a factor of $\sim 2.1$ increase in $f_{\text{col}}$ for an order of magnitude decrease in $M$ for both the Schwarzschild and maximal Kerr black holes. This result is fully consistent with the ‘true’ $f_{\text{col}}$ evolution and shows that relative measurements of $f_{\text{col}}$ made with Newtonian disc models are reliable, even when the black hole under consideration is maximally spinning.

| $a_*$ | $M$ ($M_{\text{Edd}}$) | $T_{\text{col, max}}$ (keV) | $K_{\text{fit}}$ | $f_{\text{col, true}}$ | $f_{\text{col}}$ | $\chi^2$ |
|-------|------------------------|-----------------------------|-----------------|------------------------|-----------------|---------|
| 0     | 0.05               | 0.416(2)                   | 0.177(8)        | 1.4                    | 1.39(2)         | 1.35    |
| 0     | 0.005              | 0.493(5)                   | 0.0098(8)       | 3.0                    | 2.87(6)         | 1.26    |
| 0.998 | 0.01               | 0.455(5)                   | 0.015(2)        | 1.4                    | 2.58(9)         | 0.87    |
| 0.998 | 0.001              | 0.56(1)                    | 0.0006(1)       | 3.0                    | 5.8(2)          | 0.71    |

Figure 10. Same as Fig. 8, but with the Comptonization model SIMPL replacing POW/BKNPOW. The required evolution in $f_{\text{col, rel}}$ is systematically reduced compared to results from the POW/BKNPOW continuum model.
4.5 Disc temperature and luminosity

The luminosity of a geometrically thin, optically thick disc is theoretically expected to obey a relation of the form \( L_{\text{disc}} = \sigma A T^4 \), where \( A \) is the radiating surface area. The disc luminosity depends on both \( A \), which scales as \( R_g^2 \), and effective temperature, which depends on \( f_{\text{col}} \). The purpose of this section is to explore the \( L_{\text{disc}} - R_g T_{\text{eff}} \) relation, particularly extended to low luminosities (i.e. the HIS and LHS), and the effect of introducing the possibility of a variable colour-correction factor.

For the EZDISKBB disc model of a Newtonian disc with a zero-torque inner boundary condition, Zimmerman et al. (2005) finds the disc luminosity is given by

\[
L_{\text{disc}} = 73.9 \sigma \left( \frac{T_{\text{col}, \text{max}}}{f_{\text{col}}} \right)^4 R_{\text{ht}}^2.
\]

(9)

Observationally, we measure an unabsorbed disc flux

\[
F_{\text{disc}} = \frac{L_{\text{disc}}}{4 \pi D^2}.
\]

(10)

In practice, \( F_{\text{disc}} \) is obtained by operating the XSPEC cx1ux command on the disc component with the normalization frozen to its best-fitting value. Unabsorbed disc fluxes and unabsorbed total model fluxes, \( F_{\text{unabs}} \), are calculated over 0.1–10 and 0.1–100 keV, respectively. Fig. 11 shows the relation between unabsorbed disc flux and maximum effective temperature for each of the three Comptonization models POW/BKNPOW, COMPTT and SIMPL. \( T_{\text{col}, \text{max}} \) is measured from fits to all observations with the disc component modelled by EZDISKBB and \( T_{\text{eff, max}} = T_{\text{col}, \text{max}} / f_{\text{col}} \), assuming a constant \( f_{\text{col}} = 1.4 \). The disc fractions, defined by equation (7), are given by the glyph colours. We note that this work includes observations with very low disc fractions compared to previous investigations of the luminosity–temperature relation for BHBs, which tend to be restricted to the HSS where \( DF \gtrsim 0.8 \) (e.g. Gierliński & Done 2004).

If one assumes a constant disc geometry, the disc emitting area is unchanging and we expect to recover the scaling, \( F_{\text{disc}} \propto T_{\text{eff}}^4 \), denoted by the diagonal lines in Fig. 11 for various choices of \( R_g \) and \( f_{\text{col}} \). While the theoretical relation is achieved for disc luminosities, \( L_{\text{disc}} \gtrsim 0.05L_{\text{Edd}} \), clear departure is evident below this threshold where the BHB is categorized in the HIS or LHS.

The DISKBB and EZDISKBB models are purely Newtonian and correction factors accounting for various general relativistic and viewing angle effects (Cunningham 1975; Zhang et al. 1997) have been applied in previous observational studies of the luminosity–temperature relation for BHBs (e.g. Gierliński & Done 2004; Dunn et al. 2011). Selecting the proper correction factors requires knowledge of the disc inclination and spin, for which there is considerable debate for GX 339–4 (see Section 2.1). Fortunately, the correction factors are independent of flux and temperature; therefore, we can safely neglect incorporating these modifications without altering the shape of the \( F_{\text{disc}} - T_{\text{eff}} \) diagram, irrespective of the accuracy of the \( F_{\text{disc}} \) and \( T_{\text{eff}} \) values.

The effective temperature plotted in Fig. 11 is calculated under the assumption of a constant colour-correction factor, which will subsequently be relaxed. Deriving relative measurements for the colour-correction factor according to equation (8) and as described in Section 4.3.2, Fig. 12 shows the relationship between \( f_{\text{col}} \) and the maximum colour temperature, which is a parameter in the disc models. Supposing that the inner disc remains fixed at \( R_g = 6R_g \), which is the implicit assumption behind all of our \( f_{\text{col}, \text{rel}} \) measurements, the colour-correction factor increases with colour temperature approximately linearly. The majority of observations yield colour corrections that fall within the range \( 1.4 \lesssim f_{\text{col}, \text{rel}} \lesssim 5 \), in agreement with Reynolds & Miller (2013). Notably, the models SIMPL and COMPTT, which are rooted in physical considerations of Comptonization, constrict the range of \( f_{\text{col}, \text{rel}} \) as compared to the phenomenological and widely used POWERLAW and BKNPOWER models. Particularly for best-fitting colour temperatures, \( T_{\text{col}, \text{max}} \gtrsim 1 \) keV, the POWERLAW and BKNPOWER continuum models may return erroneously low values for the disc normalization.
Figure 12. Relative measurements of the colour-correction factor shown against the maximum colour temperature of the disc from best fits using the XSPEC Comptonization models POW/BKNPOW (top panel), SIMPL (centre panel) and COMPTT (bottom panel). Results from all transitions are shown together, with the disc fraction given by the glyph colour. The key in the upper left portion of the top panel identifies the glyph associated with each transition ID. $T_{\text{col, max}}$ is obtained from the best-fitting EZDISKBB parameter and $f_{\text{col, rel}}$ is computed using the same procedure as in Fig. 8 (see the text). Horizontal dashed lines mark $f_{\text{col, rel}} = 1.4, 3, 5$. For a disc inner radius fixed at $6R_g$, the majority of observations yield $1.4 \lesssim f_{\text{col, rel}} \lesssim 5$ and show a roughly linear relation between $f_{\text{col}}$ and $T_{\text{col}}$.

We attempt to explore whether the luminosity–temperature relation can be extended to lower luminosities characteristic of the HIS and LHS by allowing for a variable $f_{\text{col}}$. Fig. 13 shows the $F_{\text{disc}}$–$T_{\text{eff}}$ diagram with the only difference to Fig. 11 being that the effective temperature is now calculated by $T_{\text{eff, max}} = T_{\text{col, max}}/f_{\text{col, rel}}$. In other words, Fig. 11 is recomputed using the $f_{\text{col, rel}}$ values of Fig. 12 in place of the constant $f_{\text{col}} = 1.4$. Accepting the dynamic range in $f_{\text{col}}$ suggested by Fig. 12, $F_{\text{disc}} \propto T_{\text{eff}}^{4}$ is recovered for disc luminosities, $L_{\text{disc}} \gtrsim 10^{-3} L_{\text{Edd}}$, when $f_{\text{col}}$ is considered to be variable.

Caution must be exercised when combining different disc models. The relative measurements $f_{\text{col, rel}}$ used to recompute $T_{\text{eff, max}}$ in Fig. 13 called upon DISKPN and EZDISKBB separately to model the disc component in order to take advantage of the strengths of each model for our purposes. One may reasonably ask if these two disc models, which both adopt the zero-torque inner bound-
LFQPOs observed in GX 339–4 and their frequency migration over the course of the state transitions studied here has received considerable attention in the literature. The frequency of LFQPOs is observed to decrease (increase) over a decay (rise) transition with a typical range $0.1 \lesssim v_{\text{QPO}} \lesssim 10$ Hz, implying that accretion rate and QPO frequency are intricately linked. In an attempt to test the truncated disc LFQPO production model, which predicts that $R_{\text{Kep}} \approx R_\text{m} \approx 100R_g$ in the LHS, we collect measured $v_{\text{QPO}}$ values for all of the GX 339–4 state transitions considered (Belloni et al. 2005; ShapiroNov & Tittarchuk 2009; Motta et al. 2011; Stiele et al. 2011; Nandi et al. 2012). Given the uncertainty in black hole mass for GX 339–4, we seek relative measurements of $R_{\text{Kep}}$ normalized to the ISCO for a non-spinning black hole in the HSS,

$$R_{\text{Kep,rel}} \equiv 6R_g \left( \frac{R_{\text{Kep}}}{R_{\text{Kep,HSS}}} \right), \quad (13)$$

where $R_{\text{Kep,HSS}}$ is the Keplerian radius computed for a given transition from the $v_{\text{QPO}}$ of the observation nearest to the HSS in time using equation (12), without specifying $M$. Supposing that $R_{\text{Kep}}$ is associated with $R_m$, Fig. 15 shows the evolution in disc inner radius implied by the observed migrating LFQPOs for each transition. With the exception of transition D11, the Keplerian frequencies implied by the LFQPOs reach out to $\sim 50-100R_g$ in the LHS, which is broadly consistent with the degree of recession required by the RIAF/truncated disc LFQPO excitation model. If LFQPOs are associated with the inner disc in a similar manner to that described by Giannios & Spruit (2004), then a truncated disc geometry may be a plausible description for state transitions. However, the spectral fits in this work suggest that $R_m$ does not truncate to such extreme disc locations, which would demand reconciling.

5 DISCUSSION

We have shown that a moderately increasing $f_{\text{col}}$ as the disc fraction decreases can adequately and consistently describe the spectral evolution of the accretion disc in GX 339–4 over multiple state transitions. The purpose of this work is not to debunk a truncated disc geometry as a viable model for the LHS of BHBs. We recognize that the observations in this work interpreted with simplistic disc models are unable to distinguish between changes in $R_m$ and/or $f_{\text{col}}$. Rather, we demonstrate that reasonable variations in $f_{\text{col}}$ provide an alternative explanation to disc truncation for accretion disc evolution. Motivated by this result and supposing that $f_{\text{col}}$ is variable, we consider the impacts that this would have on the interpretation of BHB observations, ultraluminous X-ray sources (ULXs) and models of the accretion flow across spectral states. We then discuss the physically expected range of attainable values for $f_{\text{col}}$ and conclude the discussion with a cautionary commentary on the widely used disc models.

5.1 Implications of a variable colour correction

Imposing an unchanging inner accretion disc radius located at the ISCO of a Schwarzschild black hole, we measured the relative changes in $f_{\text{col}}$ necessary to adequately fit RXTE spectra of GX 339–4 over seven state transitions. Arbitrarily selecting a normalization of $f_{\text{col}} = 1.4$ for the HSS and denoting measurements of $f_{\text{col}}$ relative to this value by $f_{\text{col,rel}}$ (see equation 8), we tracked the evolution of $f_{\text{col,rel}}$ for each transition (see Figs 8–10). Transitions R02, R07, R10 and D11 require colour corrections conforming to the...
5.1.1 Recovering $L-T_{\text{col}}$ at low luminosities

A blackbody source of constant emitting area is expected to obey the $L-T_{\text{col}}$ scaling, which relates the source luminosity to its effective temperature. Accretion flows corresponding to the HSS of BHBs, where the thermal accretion disc dominates the total flux and the inner disc location does not deviate from the ISCO, nicely follow this expected relation (e.g. Gierlinski & Done 2004); however, deviations arise in the HIS and LHS (e.g. Dunn et al. 2011). This work encompasses the disc luminosity range, $10^{-3} \lesssim L_{\text{disc}}/L_{\text{Edd}} \lesssim 0.5$, finding that an $f_{\text{col}}$ that evolves with spectral state can reproduce the theoretical $L_{\text{disc}}-T_{\text{col}}$ relation across all spectral states. Studying the disc-dominated states (i.e. $\text{DF} \gtrless 0.8$) of many BHBs over a typical luminosity range, $0.01 \lesssim L_{\text{disc}}/L_{\text{Edd}} \lesssim 0.5$, Gierlinski & Done (2004) found that the $L_{\text{disc}}-T_{\text{col}}$ relation holds for a constant $f_{\text{col}}$. We observe similar and consistent behaviour for GX 339–4 in the HSS. As the disc fraction drops and the source enters the HIS or LHS, significant departures from $L_{\text{disc}}-T_{\text{col}}^{4}$ are apparent in Fig. 11, in agreement with Dunn et al. (2011). These ‘spurs’ on the luminosity–temperature diagram can be interpreted as either a decrease in the inner disc radius, and hence the emitting area, or the result of neglecting to account for a decrease in $T_{\text{eff}} = T_{\text{col}}/f_{\text{col}}$ caused by an increased $f_{\text{col}}$ in the LHS. Of course, some combination of these two effects is possible as well. Dunn et al. (2011) measured the incremental degree of change in $f_{\text{col}}$ from a canonical HSS value that was required to return the spurs to the expected $L_{\text{disc}} \propto T_{\text{eff}}^{4}$ law for a constant emitting area (see their figure A.1). For GX 339–4, they found that fairly modest changes in $f_{\text{col}}$, consistent with the relative changes we determined, could restore the theoretical relation; however, they argue that the most severe departures from $L_{\text{disc}}-T_{\text{col}}^{4}$, which occur for the lowest disc fractions, are not likely the result of a variable $f_{\text{col}}$. Here, we suggest that the physically realizable range of $f_{\text{col}}$ is perhaps not as narrow as commonly assumed (see Section 5.2) and that a thin disc geometry with a fixed inner radius and evolving $f_{\text{col}}$ is a viable phenomenological model for BHB state transitions.

We expand on the work of Dunn et al. (2011) by breaking the observations of a BHB into individual transitions, exploiting relative measurements, and exploring empirical and physical Comptonization models. Comparing Figs 8–10, we find that employing physically motivated Comptonization models that avoid the ‘flux stealing’ phenomenon from the disc component require less severe $f_{\text{col}}$ evolution for any given transition. Inspection of the panels of Fig. 13 shows that the $L_{\text{disc}}-T_{\text{col}}$ relation is recovered when a changing $f_{\text{col}}$ is incorporated, regardless of the choice of Comptonization model. Relative measurements of $f_{\text{col}}$ broken down by transition in Figs 8–10 hint at an underlying difference in the details of the accretion flow depending on whether the source is in the rise or decay stage of outburst. Qualitatively, the decay transitions demand more extreme and perhaps physically unattainable $f_{\text{col}}$ evolution, while the rise transitions fit more comfortably into the idea of a moderately variable $f_{\text{col}}$. Given that decoupling the true behaviour of the inner disc from that of the colour-correction factor is impossible with simplistic disc models and RXTE data, we leave the possibility of different accretion scenarios between rise and decay transitions as pure speculation.

5.1.2 The ultraluminous state

Adopting a variable colour-correction factor parametrizes the various physical processes in BHB accretion into a single factor. This work suggests that moderate changes in $f_{\text{col}}$ may conceivably...

Figure 15. Relative measurements of the evolution of $R_{\text{Kep, rel}}$ for each transition (see the text for definition) derived from LFQPO frequency evolution. The dashed lines mark $R_{\text{in}} = 6, 20, 100 R_g$. The $R_{\text{Kep, rel}}$ measurements for each transition were made relative to the filled glyphs and normalized to $R_{\text{in}} = 6 R_g$, which is appropriate for a Schwarzschild black hole in the HSS. range $1.4 \lesssim f_{\text{col, rel}} \lesssim 3$, while transition R04 demands the range be extended to $1.4 \lesssim f_{\text{col, rel}} \lesssim 5$. These ranges in $f_{\text{col}}$ evolution, which can account for the majority of the transitions, correspond to a factor of $\sim 2.0$–3.5 increase in $f_{\text{col}}$ for a HSS $\rightarrow$ LHS transition. Transitions D03 and D07 require more extreme evolution in $f_{\text{col, rel}}$, with values as high as $f_{\text{col, rel}} \sim 10$. Our main finding is to demonstrate that an evolving $f_{\text{col}}$ provides an alternative and adequate description of the changing disc spectrum during a state transition, which is perhaps a more attractive scenario to imagine than the sudden evacuation or replenishment of the inner accretion flow.
account for the wide range of spectral states observed when a BHB undergoes a state transition. Recently, a new spectral class, dubbed the ultraluminous state (ULS), has been postulated for the class of ULXs (Gladstone, Roberts & Done 2009). Here, we explore whether the ULS, which has never been observed in a confirmed BHB, is consistent with the interpretation of spectral states arising from physical processes that are observationally manifested as an evolving colour-correction factor.

ULXs are X-ray sources that exceed the Eddington luminosity of stellar mass black holes ($L_X \gtrsim 10^{39}$ erg s$^{-1}$), but are not spatially coincident with galactic nuclei; therefore, ULXs are distinct from AGNs. Two possible explanations for ULXs are (1) intermediate mass black holes (IMBHs; $M \sim 10^2$–$10^4 M_\odot$) accreting at sub-Eddington rates, but the production rate of binaries composed of an IMBH and donor star is likely too low to account for the number of observed ULXs (Madhusudhan et al. 2006) and/or (2) stellar mass black holes accreting at super-Eddington rates, which requires a mechanism for achieving super-Eddington accretion, such as beaming/viewing effects of outflows (e.g. King et al. 2001; Begelman, King & Pringle 2006) or disc inhomogeneities (Begelman 2001, 2002, 2006).

The spectral characteristics of ULXs were studied by Gladstone et al. (2009), leading them to propose that ULXs are super-Eddington accreting stellar mass black holes with ubiquitous spectral features. These commonalities suggest that ULXs are indeed a spectral class of BHBs, dubbed the ULS; however, this accretion state does not appear to arise in the well-studied Galactic BHBs. The ULS is characterized by both a broadened, hardened thermal disc component and a break in the power-law component at lower energies ($\gtrsim 3$ keV) than typically observed in less luminous BHB states. The Gladstone et al. (2009) ULX sample was adequately fitted with a physically motivated disc plus Comptonizing corona model, where the best-fitting models called for optically thick coronae ($\tau_p \sim 5$–30) and cool disc colour temperatures ($T_{\text{col, max}} \sim 0.2$ keV). Physically, the broadened/hardened disc and low-energy spectral break may be attributed to a complicated inner geometry where an optically thick corona obscures and alters the energetics of the inner disc. Normally, disc temperatures this low would imply a black hole of mass $\sim 10^3 M_\odot$ (e.g. Kaaret et al. 2003); however, the measured disc temperature may be lowered if a significant fraction of the accretion energy is dissipated in the corona (e.g. Svensson & Zdziarski 1994). Based on the requirement of an optically thick corona, as opposed to the $\tau_p \sim 1$ coronae typically observed in BHBs, sub-Eddington accretion on to an IMBH was ruled out.

Fig. 16 shows the coronal optical depth versus the maximum disc colour temperature for all transitions, where $\tau_p$ and $T_{\text{col, max}}$ are measured using COMPTT and DISKPN with $R_{\text{in}} = 6R_\odot$, respectively. The disc fraction is given by the glyph colour. The results from a sample of 12 ULXs (table 8 of Gladstone et al. 2009) are overlaid in black crosshairs. The HIS/LHS GX 339–4 RXTE data analysed in this work permit moderately optically thick coronae in the same range as found for ULXs.

![Figure 16. Coronal optical depth versus maximum disc colour temperature for all transitions, where $\tau_p$ and $T_{\text{col, max}}$ are measured using COMPTT and DISKPN with $R_{\text{in}} = 6R_\odot$, respectively. The disc fraction is given by the glyph colour. The results from a sample of 12 ULXs (table 8 of Gladstone et al. 2009) are overlaid in black crosshairs. The HIS/LHS GX 339–4 RXTE data analysed in this work permit moderately optically thick coronae in the same range as found for ULXs.](https://example.com/figure16.png)

for discs with temperatures $T_{\text{col, max}} \lesssim 0.5$ keV ($T_{\text{col, max}} \lesssim 0.5$ keV). Fig. 16 challenges the claim that spectral fits to sub-Eddington BHBs do not permit moderately optically thick coronae.

Although the data are consistent with an optically thick corona model, here we point out a strong parameter degeneracy between the colour-correction factor and the coronal optical depth. Of the three models we considered to represent Comptonization, all of which provided equally good fits to the data, only COMPTT incorporates the coronal temperature and optical depth as physical parameters. Inspection of Fig. 12 reveals that the POW/BKNPOW and SIMPL models require higher colour-correction factors than COMPTT. Particularly for the HIS/LHS observations of GX 339–4, introducing a physical corona drives $f_{\text{col}}$ to lower values with preference for an optically thick corona (see Fig. 12). Therefore, the data cannot distinguish between an increased colour-correction factor and an optically thick corona.

In the IMBH model, ULXs are massive scaled-up versions of BHBs in the LHS (i.e. IMBHs accreting at sub-Eddington rates). The IMBH model for ULXs was discarded by Gladstone et al. (2009) because the measured coronal optical depths were larger than commonly observed in BHBs. In the case of GX 339–4 in the HIS/LHS, using the same disc plus Comptonization model, we see that the best-fitting coronal optical depths lie precisely in the same moderately optically thick range as the ULX sample of Gladstone et al. (2009). Therefore, the data do permit optically thick coronae in the HIS/LHS of GX 339–4, but we stress that the measurement of coronal optical depth from basic disc+corona modelling cannot be made definitive due to the degeneracy between $\tau_p$ and $f_{\text{col}}$. This possibility alleviates the hesitation for supposing that ULXs are analogous to scaled up BHBs on the grounds of coronal optical depth. Allowing for a variable colour-correction factor, the
A changing mass accretion rate plays a key role in state transitions, but it is likely not the whole picture, nor is it clear what mechanism drives changes in $M$. For an inner disc remaining at the ISCO during a state transition, a large change in $M$ is implied. In the HSS, the inner portions of the disc are expected to be radiation pressure dominated, where instabilities may lead to an inhomogeneous disc structure and a low $f_{\text{col}} \sim 1$ (Begelman 2006). As $M$ decreases, the ratio of radiation-to-gas pressures drops and below some $L_{\text{disc}}/L_{\text{Edd}}$, the density inhomogeneities would disappear and $f_{\text{col}}$ would increase. If the disc is strongly magnetized, values of $f_{\text{col}} \lesssim 5$ are conceivable (Begelman & Pringle 2007). For a disc that instead transitions from a standard thin disc to a truncated disc with an interior RIAF, a relatively constant $M$ is implied at this stage with a decreased radiative efficiency. Both scenarios of an inner disc remaining extended to the ISCO and a truncated disc+RIAF predict different $M$ behaviour, which is difficult to distinguish observationally.

The phenomenological picture of a variable colour-correction factor being capable of describing the disc spectral evolution observed in BHB outbursts advocates for the development and consideration of models that appeal to an evolving vertical disc structure and/or intricate disc–corona–jet connection. Recent observational work supports the idea of a coupled disc+corona playing a critical role in the LHS. Adopting sophisticated, self-consistent modelling techniques, Reis et al. (2013) studied the BHB XTE J1650−500 during its 2001/2002 outburst and found definite correlations between the reflected X-ray flux and power-law flux associated with the corona. These results fit into the interpretation of a light bending+collapsing corona model for state transitions (e.g. Minutti & Fabian 2004) and do not lend support to the truncated disc picture. An organized campaign of contemporaneous and frequent X-ray/radio monitoring of Cygnus X-1 in the LHS reveal that the reflected X-ray flux and the radio flux are positively correlated (Miller et al. 2012), which is consistent with the coronal ejection model of Beloborodov (1999). As the promising prospect of disc+corona coupling in BHB outbursts receives more attention, we hope that the relative measurements of $f_{\text{col}}$ presented here serve as a useful reference for physical models that seek to generate predictive diagnostics of spectral hardening in accretion systems.

5.2 Physical range of the colour-correction factor

The colour-correction factor phenomenologically parametrizes our ignorance of spectral alterations arising from the highly unknown and complicated combinations of disc vertical structure, accretion power dissipation properties, magnetization and inhomogeneities. Lumping the effects of these physical processes into the frequency-independent $f_{\text{col}}$ parameter is attractive from the standpoint of limiting the parameters in MCD blackbody models; however, whether this treatment is an appropriate description of BHB discs in all spectral stages remains disputed. Here, we discuss the results from theoretical endeavours to constrain the physically realizable range of the colour-correction factor.

5.2.1 Radiative transfer in one-dimensional disc atmospheres

Investigations into the physically reasonable $f_{\text{col}}$ range and the regimes for which the colour-corrected blackbody model is justified...
have focused on one-dimensional atmospheric disc models. These atmospheres are assumed to have no azimuthal structure and are constructed at many disc radii from self-similar standard thin disc equations. In brief, frequency-dependent opacities are computed at each radial position, allowing the modification of the locally emitted blackbody spectrum to be determined for multiple concentric annuli. The emergent spectrum, which is modified by the vertical structure, is computed for each annulus. The integrated disc spectrum is obtained by summing over all annuli and is fitted with the colour-corrected blackbody model, yielding a best-fitting $f_{\text{col}}$.

Following the technique of self-consistently computing the radiative transfer and vertical disc structure (Ross, Fabian & Mineshige 1992), Shimura & Takahara (1995) considered the effects of electron scattering and free–free emission/absorption on the emergent disc spectrum. Regardless of the black hole mass and radial position in the disc, the local spectra (i.e. the emergent spectrum from an annulus) were adequately fitted with $f_{\text{col}} \approx 1.8–2.0$ and $f_{\text{col}} \approx 1.7$ for $M \approx M_{\text{Edd}}$ and $M \approx 0.1M_{\text{Edd}}$, respectively, suggesting a relatively constant $f_{\text{col}}$ over a large range of mass accretion rates expected in the HSS. For $M \lesssim 0.1M_{\text{Edd}}$, which is characteristic of the LHS, the local spectrum was not well described by the colour-corrected blackbody model due to ineffective (i.e. unsaturated) Comptonization within the disc.

Merloni et al. (2000) extended the study of Shimura & Takahara (1995) by considering the effects of Doppler blurring, gravitational redshift and allowing the dissipated accretion energy to be partitioned between the corona and disc (Svensson & Zdziarski 1994). Assuming that all of the accretion energy is dissipated within the disc, Merloni et al. (2000) found $f_{\text{col}} \approx 1.8$ for $M = 0.1–0.3M_{\text{Edd}}$, fully consistent with Shimura & Takahara (1995). Exploring the lower accretion rate, $M = 0.05M_{\text{Edd}}$ and allowing a significant fraction of the energy to be dissipated in the corona, Merloni et al. (2000) instead found a variable $f_{\text{col}}$ ranging from $1.9$ to $2.7$. Therefore, if coronal activity is important, $f_{\text{col}}$ cannot be considered constant, with the suggested physical range being $1.7 \lesssim f_{\text{col}} \lesssim 3$.

Davis et al. (2005) presents the most sophisticated study to date of the effects of disc vertical structure on the emergent spectrum, incorporating bound-free opacities and a fully relativistic radiative transfer treatment. For disc luminosities, $L_{\text{disc}} = 0.01–0.3L_{\text{Edd}}$, which spans the luminosity regimes of the LHS and HSS, Davis et al. (2005) found a weakly evolving $f_{\text{col}} \approx 1.4–1.6$, concluding that $f_{\text{col}}$ remains fairly constant over broad ranges in mass accretion rate. The results of Davis et al. (2005) are at odds with those of Merloni et al. (2000); however, a direct comparison is complicated by the different methodologies used in each work.

5.2.2 Magnetization

Although the details of magnetic fields (e.g. topology, strength, variability) in accreting black hole systems are poorly understood, magnetization is central to many aspects of accretion physics. Perhaps most notable is the role of the MRI (Balbus & Hawley 1991), which is widely accepted as the mechanism for angular momentum transport, hence, allowing accretion to persist. The turbulence generated within the disc by the MRI amplifies the toroidal component of magnetic field, perhaps reaching a state where magnetic pressure dominates over the combined gas and radiation pressures. Begelman & Pringle (2007) present analytic arguments for the maximal toroidal magnetic field strength achievable by the MRI and compute the corresponding disc structure, finding that magnetically dominated discs are thicker and have harder spectra compared to the standard Shakura–Sunyaev thin disc. Under the conjecture that the toroidal field attains the limiting value set by the MRI, Begelman & Pringle (2007) derive a lower limit for the colour-correction factor

$$f_{\text{col}} \geq 6.9 \left( \frac{\rho_*}{\rho} \right)^{-2/9} \alpha^{5/27} \left( \frac{M}{M_{\text{Edd}}} \right)^{-1/108} \left( \frac{M}{M_{\text{Edd}}} \right)^{5/108} \left( \frac{r}{R_{\text{g}}} \right)^{25/108},$$

where $\rho$ is the mid-plane density, $\rho_*$ is the density corresponding to the thermalization layer (i.e. effective photosphere) of the disc and $\rho_* / \rho < 1$. Shearing box simulations place the disc photosphere a few scaleheights above the mid-plane, corresponding to density contrasts, $\rho_* / \rho \approx 10^{-2}–10^{-4}$ (Hirose et al. 2006, 2009). Therefore, if the disc around a stellar mass black hole is magnetically dominated, the colour-correction factor can far surpass the values predicted by thin disc treatments that neglect the effects of a magnetic field on the disc structure.

Conceivably, the details of vertical dissipation and coronal properties could alter the emergent disc spectrum considerably. Beloborodov (1999) envisaged a picture where buoyant magnetic loops subject to Parker instability rise into the corona and the magnetic energy available within the loops is subsequently dissipated by reconnection. Recently, Uzdensky (2012) analytically computed the vertical structure of a gas pressure-dominated disc threaded by a poloidal magnetic field where turbulent (i.e. dissipative) heating arising from the MRI is balanced by radiative cooling. Based on this disc model, Uzdensky (2012) argue that the minimum fraction of available accretion energy that ends up being released in the corona is $f_{\text{min}} \approx \rho_{*0,\max}^{1/3} \approx 0.4$, where $\rho_{*0,\max} \approx 100$ is a reasonable estimate of the gas-to-magnetic pressure ratio at the disc mid-plane. Under conditions where a substantial fraction of accretion power is dissipated in the corona, Merloni et al. (2000) demonstrate that substantial spectral hardening is expected.

Numerical simulations show that magnetically supported, gas pressure-dominated discs have harder emergent spectra than their unmagnetized counterparts (Blaes et al. 2006). Blaes et al. (2006) calculated local accretion disc spectra from stratified shearing box simulations and found that the disc spectrum hardened due to magnetic pressure support dominating in the upper atmosphere. This magnetic pressure acts to vertically extend the disc atmosphere and decrease the density at the effective photosphere, leading to saturated Comptonization and spectral hardening. In deriving the local disc spectrum, Blaes et al. (2006) considered two vertical dissipation profiles: (1) a constant dissipation per unit mass and (2) a broken power-law fit to the numerical dissipation profile. Both treatments produced identical emergent spectra, implying that the local emergent disc spectrum is insensitive to the vertical dissipation profile and that magnetic pressure support is solely responsible for the observed spectral hardening. Motivated by the result that magnetic pressure support drives the spectral hardening and assuming the dissipation profile has no radial dependence, Blaes et al. (2006) constructed global disc spectra and found only modest hardening, with $f_{\text{col}}$ increasing from 1.48 to 1.74 upon introducing magnetic pressure support. Blaes et al. (2006) caution that the numerical dissipation occurs at the grid scale and may be sensitive to grid resolution. Recent work focusing on decaying magnetohydrodynamic turbulence suggests that the main effect of incorporating physical dissipation terms in grid-based simulations is to move the dissipation scale to larger spatial scales, without affecting the physics of energy transfer within the turbulence at scales larger than the dissipation scale (Salvesen et al. 2013). Adopting dissipation profiles motivated by numerical simulations is an important step towards merging accretion disc theory with simulations.
5.2.3 Inhomogeneous discs

Strong disc inhomogeneities are expected to arise in moderate-to-strongly magnetized, radiation pressure-dominated regions of accretion discs (Begelman 2002, 2006) due to photon bubble instability (Arons 1992; Gammie 1998; Begelman 2001). In this picture, strong density contrasts arise on scales smaller than the disc scale-height, resulting in a porous disc where radiation preferentially ‘leaks’ through the underdense regions. Due to the typical densities in photon bubble-dominated discs being higher than in standard thin discs of the same luminosity, thermalization of radiation is more effective in inhomogeneous discs. Consequently, radiation that is thermalized near the disc mid-plane and escapes through optically thin chutes can result in a colour-correction factor near unity for inhomogeneous discs (Begelman 2006). Dexter & Quataert (2012) computed inhomogeneous disc spectra based on a toy model (Dexter & Agol 2011) and found that the disc spectrum broadens and hardens as the amplitude of the local temperature fluctuations increases. Inhomogeneous discs can substantially alter the standard disc spectrum and further investigation is required to characterize their properties.

5.3 Limitations of simplistic accretion disc models

While significant strides have been made in exploring the imprints of the disc structure on the emergent spectrum in the HSS, the applicability of the colour-corrected blackbody model to discs in the LHS remains elusive. From a theoretical perspective, thin discs with low-mass accretion rates will be fully gas pressure dominated with absorption overtaking electron scattering as the dominant opacity source, nullifying the approximation of the emergent spectrum as a colour-corrected blackbody (e.g. Ebisuzaki, Sugimoto & Hanawa 1984). Nevertheless, standard MCD models consistently produce exceptional fits to LHS accretion disc spectra, giving credence to the notion that the MCD model successfully describes real astrophysical discs in the LHS. Analogously, standard thin discs are expected to suffer from thermal (Shakura & Sunyaev 1976) and viscous (Lightman & Eardley 1974) instabilities, yet the MCD model provides an acceptable description of the observed disc spectrum, which should not be the case if the disc were plagued by catastrophic instabilities. While it may turn out that $f_{\text{col}}$ has a very narrow allowable range and/or it is an inappropriate parametrization to use for low disc luminosities, there is currently no compelling evidence that this should be so based on fits to observed disc spectra alone.

Direct measurements of $f_{\text{col}}$ should not be made with Newtonian disc models when the black hole is known to possess a non-zero spin and an inner disc extending to the ISCO. Newtonian disc models have the strength of being able to describe a standard black hole accretion disc spectrum with very few parameters; however, the important physical effects of light bending, Doppler blurring and gravitational redshift inherent to Kerr black holes are neglected. The pseudo-Newtonian disc model, DISKPN, provides accurate fits to fully relativistic disc spectra from a non-spinning black hole, but requires uncomfortably large values of $f_{\text{col}}$ when fitting a disc spectrum from an extreme Kerr black hole (see Section 4.4). When attempting to fit the hotter disc spectra intrinsic to spinning black holes, Newtonian disc models are forced to artificially increase the colour-correction factor to achieve the colour temperature demanded by the data. Fortunately, relative measurements of $f_{\text{col}}$ do not appear to be appreciably affected in the way that absolute measurements are at least for the low disc inclination considered in this work. This implies that the relative measurements of $f_{\text{col}}$ evolution during state transitions of GX 339–4 are sound, regardless of the intrinsic spin possessed by GX 339–4. The possibility that the inner disc recedes radially outwards or progresses inwards between successive observations cannot be ruled out and would manifest itself in the relative measurements of $f_{\text{col}}$, rendering them moot.

The main outcome of this work is that an evolving colour-correction factor in a standard thin disc with inner radius fixed at the ISCO is a viable, albeit not the only, explanation for the changing disc spectrum of the BHB GX 339–4 over many state transitions. Given the uncertainty in accretion flow geometry as BHs transition between spectral states, we emphasize that inner disc radii cannot be reliably computed from spectral fits made with simplified disc models. Instead, self-consistent, physically motivated spectral modelling applied to high-quality CCD X-ray spectra is necessary to distinguish between opposing accretion scenarios. Future insights into the complicated disc–corona–jet connections in BHBs will likely progress by concentrated observing campaigns and the development of physical models that can provide testable diagnostics.

6 SUMMARY AND CONCLUSIONS

The purpose of this work is to show that physically reasonable changes in the colour-correction factor provide a valid, alternative explanation to disc truncation for explaining the accretion disc spectral evolution observed in BHB state transitions. To this end, we systematically performed spectral fits to RXTE observations of the low-mass X-ray binary GX 339–4 over seven state transitions and collected the observations that statistically required a disc component. The disc detection pipeline measured discs with luminosities as low as $L_{\text{disc}} \lesssim 10^{-3} L_{\text{Edd}}$ and disc fractions down to DF $\approx 0.5$. In a suite of spectral fits to the observations deemed to require a disc, we imposed an unchanging inner disc radius located at the ISCO for a Schwarzschild black hole in order to ensure that a changing colour-correction factor, rather than a radially receding/advancing inner disc, was responsible for any spectral evolution of the disc component. We give our conclusions as follows.

(i) For nearly all state transitions of GX 339–4 investigated here, relative changes in $f_{\text{col}}$ in the range $1.4 \lesssim f_{\text{col,rel}} \lesssim 5$, which corresponds to a factor of $\sim 3.5$ change, can fully account for disc evolution in a state transition without invoking a truncated disc geometry.

(ii) Accounting for an $f_{\text{col}}$ evolution in the range quoted above over the course of state transitions in GX 339–4, the $F_{\text{disc}} \propto T_{\text{eff}}^4$ relation is recovered for a constant emitting area (i.e. non-truncated disc) down to the low luminosities characteristic of the HIS and LHS.

(iii) The validity of the results mentioned above are insensitive to the Comptonization model adopted for fitting the high-energy continuum. Adopting a continuum model that avoids the low-energy flux stealing character of a simple power law reduces the degree of $f_{\text{col}}$ variability required to explain the observed disc evolution in state transitions.

(iv) The practice of measuring the inner disc radius from the best-fitting disc normalization is unreliable when both $R_{\text{in}}$ and $f_{\text{col}}$ are worked into the disc normalization. Instead, we suggest that disc models where these two parameters are decoupled (e.g. DISKPN) be used to break the degeneracy.

(v) We stress that this work does not demonstrate that the inner accretion disc remains at the radius of marginal stability during a state transition. Instead, we argue that variations of $f_{\text{col}}$ within a
theoretically reasonable range can account for most, if not all, of the disc spectral evolution observed over the course of a state transition.

(vi) Relative measurements, unlike absolute measurements, of $f_{\text{col}}$ are not appreciably affected by our ignorance of black hole spin.

(vii) When a physically motivated Comptonization model is used in conjunction with a disc component to fit the RXTE data of GX 339–4 in the HIS and LHS, we observe a parameter degeneracy between the coronal optical depth and the colour-correction factor. This calls into question the association of ULXs with super-Eddington stellar mass BHBs based on measurements of coronal optical depth without allowing for a variable $f_{\text{col}}$.

Simple disc models of the MCD family are attractive and useful from the standpoint that they consistently achieve superb fits with limited free parameters to accretion disc spectra, where the intrinsic system parameters are often poorly constrained. This work illustrates that allowing for the possibility of reasonable variations in the colour-correction factor provides an adequate description of disc spectra over a wide range in luminosity states. We advocate that $f_{\text{col}}$ may be dynamic and should not be dismissed as a mere constant in studies of accretion disc spectra spanning a broad range in luminosity, as is often done in practice. Numerous factors may contribute to altering the emergent disc spectrum including the vertical disc structure, details of accretion energy dissipation, coronal dynamics, magnetization and disc inhomogeneities. Given that the available X-ray data cannot isolate these effects, common practice is to adopt a blanket term, the colour-correction factor, which is supposed to account for the overall deviations from a multicolour blackbody disc spectrum. We encourage an increased awareness for the implications of presuming a constant $f_{\text{col}}$. Important physics is wrapped up into $f_{\text{col}}$ that should be considered before making definitive claims based on the assumption of an invariant $f_{\text{col}}$.

The standard theoretical emergent disc spectrum hinges on the assumptions that the accretion energy is dissipated within the disc (i.e. where the majority of the mass resides) and that the density structure is smooth and homogeneous. The effects of magnetization, which surely play a crucial role in accretion physics and disc structure, are also ignored in the disc models applied to observations. Radiation pressure-dominated, magnetized discs, as expected in the HSS, are also ignored in the disc models applied to observations. Radiation surely play a crucial role in accretion physics and disc structure, are also ignored in the disc models applied to observations. Numerous factors may contribute to altering the emergent disc spectrum including the vertical disc structure, details of accretion energy dissipation, coronal dynamics, magnetization and disc inhomogeneities. Given that the available X-ray data cannot isolate these effects, common practice is to adopt a blanket term, the colour-correction factor, which is supposed to account for the overall deviations from a multicolour blackbody disc spectrum. We encourage an increased awareness for the implications of presuming a constant $f_{\text{col}}$. Important physics is wrapped up into $f_{\text{col}}$ that should be considered before making definitive claims based on the assumption of an invariant $f_{\text{col}}$.

The standard theoretical emergent disc spectrum hinges on the assumptions that the accretion energy is dissipated within the disc (i.e. where the majority of the mass resides) and that the density structure is smooth and homogeneous. The effects of magnetization, which surely play a crucial role in accretion physics and disc structure, are also ignored in the disc models applied to observations. Radiation pressure-dominated, magnetized discs, as expected in the HSS, are subject to clumping instabilities, resulting in a porous disc structure and lowered colour-correction factor (Begelman 2006). Considerations of the corona as an energy dissipation reservoir (Svenson & Zdziarski 1994; Merloni et al. 2000; Merloni & Fabian 2002) and of strongly magnetized discs (Beloborodov & Pringle 2007) indicate that $f_{\text{col}}$ may extend well beyond the canonical value of $\sim 1.7$ and into a realm consistent with the values required here and in similar observational studies (Dunn et al. 2011; Reynolds & Miller 2013). With the advent of sophisticated codes capable of treating magnetohydrodynamics with radiative transport and general relativity, state-of-the-art numerical simulations provide a promising avenue for studying accretion disc structure with the aforementioned assumptions relaxed. Striving towards the development of sophisticated models that capture the relevant physics of realistic accreting black hole systems, provide predictive diagnostics and are accessible to observers is a crucial step towards disentangling the details of the accretion flow and distinguishing between different accretion scenarios.

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SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article:

Table 2. Best-fitting parameters and 1σ standard uncertainty in parentheses for all observations where a disc was deemed required (http://mnras.oxfordjournals.org/lookup/suppl/doi:10.1093/mnras/stt436/-/DC1).

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