A Note on Self-gravitating Radiation in AdS Spacetime

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Abstract

Recently Vaganov [arXiv:0707.0864] and Hammersley [arXiv:0707.0961] investigated independently the equilibrium self-gravitating radiation in higher \((d \geq 4)\) dimensional, spherically symmetric anti-de Sitter space. It was found that in \(4 \leq d \leq 10\), there exist locally stable radiation configurations all the way up to a maximum red-shifted temperature, above which there are no solutions; there is also a maximum mass and maximum entropy configuration occurring at a higher central density than the maximal temperature configuration. Beyond their peaks the temperature, mass and entropy undergo an infinite series of damped oscillations, which indicates the configurations in this range are unstable. In \(d \geq 11\), the temperature, mass and entropy of the self-gravitating configuration are monotonic functions of the central energy density, asymptoting to their maxima as the central density goes to infinity. In this note we investigate the equilibrium self-gravitating radiation in higher dimensional, plane symmetric anti-de Sitter space. We find that there exist essential differences from the spherically symmetric case: In each dimension \((d \geq 4)\), there are maximal mass (density), maximal entropy (density) and maximal temperature configurations; they do not appear at the same central energy density; the oscillation behavior appearing in the spherically symmetric case, does not happen in this case; and the mass (density), as a function of the central energy density, increases first and reaches its maximum at a certain central energy density and then decreases monotonically in \(4 \leq d \leq 7\), while in \(d \geq 8\), besides the maximum, the mass (density) of the equilibrium configuration has a minimum: the mass (density) first increases and reaches its maximum, then decreases to its minimum and then increases to its asymptotic value monotonically. The reason causing the difference is discussed.

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I. INTRODUCTION

Over the past decade, a lot of attention has been focused on anti-de Sitter (AdS) space and relevant physics due to the conjecture of AdS/CFT correspondence [1], which says that string theory/M theory on an AdS space (times a compact manifold) is dual to a strong coupling conformal field theory (CFT) residing on the boundary of the AdS space. According to the AdS/CFT correspondence, Witten [2] argued that thermodynamics of black holes in AdS space can be identified to that of dual strong coupling CFTs. Therefore one could study thermodynamics and phase structure of strong coupling CFTs by investigating thermodynamics and phase structure of AdS black holes. It is well-known that thermodynamics of AdS black holes is quite different from that of their counterparts in asymptotically flat space. For the AdS Schwarzschild black hole, there is a minimal temperature, below which there is no black hole solution, above which there are two black hole solutions with a same temperature. The large AdS Schwarzschild black hole is thermodynamically stable with positive heat capacity, while the small AdS Schwarzschild black hole is thermodynamically unstable with negative heat capacity like Schwarzschild black holes. Below the minimal temperature, only does the self-gravitating radiation (thermal AdS gas) solution exist. Between the large AdS black hole and thermal AdS gas, there is a one-order phase transition [3], named the Hawking-Page phase transition. According to the AdS/CFT correspondence, this phase transition can be identified as the confinement/deconfinement phase transition in the dual gauge theory [2]. Therefore it is of interest to study self-gravitating radiation configuration in AdS space.

Sorkin, Wald and Zhang [4] have studied the equilibrium configurations of self-gravitating radiation in a spherical box of radius $R$ in asymptotically flat space. It was found that for a locally stable configuration, the total gravitational mass of radiation obeys the inequality $M < \mu_{\text{max}} R$, where $\mu_{\text{max}} = 0.246$. In AdS space, one needs not any unphysical perfectly reflecting walls at finite radius. The rising gravitational potential in AdS space, plus natural boundary conditions at infinity, acts to confine whatever is inside [5]. In [5] Page and Philips examined the self-gravitating configuration of radiation in four dimensional AdS space. The configuration can be labeled as its mass, entropy and temperature versus the central density. They found that there exist locally stable radiation configurations all the way up to a maximum red-shifted temperature, above which there are no solutions; there is also a maximum mass and maximum entropy configuration occurring at a higher central density than the maximal temperature configuration. Beyond their peaks the temperature, mass and entropy undergo an infinite series of damped oscillations, which indicates the configurations in this
range are unstable. The self-gravitating radiation in five dimensional AdS space has been studied in [6] (see also [7]) with similar conclusions.

Recently, Hammersley [8] and Vaganov [9] independently discussed the self-gravitating radiation configurations in higher dimensional AdS spacetimes. They found that in the case of $4 \leq d \leq 10$, the situation is qualitatively similar to the case in four dimensions, while with $d \geq 11$, the oscillation behavior disappears. Namely, there is a critical dimension, $d = 11$ (very close, but not exact), beyond which, the temperature, mass and entropy of the self-gravitating configuration are monotonic functions of the central energy density, asymptoting to their maxima as the central density goes to infinity. For related discussions in asymptotically flat space see also [10].

On the other hand, let us note that in asymptotically AdS space, black hole horizon can be not only a positive constant curvature surface, but also a zero [11] or negative constant curvature surface [12]. Such black holes are called topological black holes in the literature. In particular, it was found that while the Hawking-Page phase transition exists for AdS black holes with spherically symmetric horizon in $d \geq 4$, it does not appear for a Ricci flat (plane symmetric) or hyperbolic horizon [13] (For Ricci flat black holes, if one of the spatial directions is compact, the Hawking-Page transition can happen due to the existence of AdS soliton, see [14] and references therein).

Note that Refs. [8] and [9] considered the spherically symmetric self-gravitating radiation configuration. It is therefore of great interest to see whether there is also a critical dimension in the plane symmetric ($k = 0$) self-gravitating radiation in AdS space, and to see whether there is any essential difference between the spherical symmetric case and the plane symmetric case. The organization of the paper is as follows. In the next section, we give a general formulism to describe the self-gravitating radiation in AdS space and study the plane symmetric case. The numerical results are given in Sec. III. Section IV is devoted to the conclusions.

II. SELF-GRAVITATING RADIATION IN ADS SPACE

Consider a $d$-dimensional asymptotically AdS space with metric

$$ds^2 = -e^{2\delta(r)}h(r)dt^2 + h^{-1}(r)dr^2 + r^2\gamma_{ij}dx^i dx^j,$$

(1)

where $\delta$ and $h$ are two functions of the radial coordinate $r$, and $\gamma_{ij}$ is the metric of a $(d - 2)$-dimensional Einstein manifold with constant scalar curvature $(d - 2)(d - 3)k$. Without loss of generality, one can take
\[ k = 1, 0 \text{ and } -1, \text{ respectively. We take the gauge } \lim_{r \to \infty} \delta(r) = 0, \text{ and rewrite the metric function } h(r) \text{ as} \]

\[ h(r) = k + \frac{r^2}{l^2} - \frac{16\pi G m(r)}{(d-2)\Sigma r^{d-3}}, \quad (2) \]

where \( l \) denotes the radius of the AdS space with cosmological constant \( \Lambda = -(d-1)(d-2)/2l^2 \), \( \Sigma \) is the volume of the Einstein manifold, and \( m(r) \) is the mass function of the solution. In our gauge, the total gravitational mass of the solution is just

\[ M = \lim_{r \to \infty} m(r). \quad (3) \]

The Einstein field equations with the cosmological constant and energy-momentum tensor \( T_{\mu\nu} \) are

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{(d-1)(d-2)}{2l^2} g_{\mu\nu} = 8\pi G T_{\mu\nu}. \quad (4) \]

Let us first briefly review the vacuum solution of (4). In this case, \( \delta(r) = 0 \) and \( m(r) \) in (2) is just an integration constant \( M \), which parameterizes the mass of the solution. When \( k = 1 \), the solution (1) with (2) describes a static, spherically symmetric black hole in AdS space. The black hole horizon \( r^+ \) is determined by \( h(r)|_{r=r^+} = 0 \). Thus the mass parameter \( M \) can also be expressed in terms of the horizon radius,

\[ M = \frac{(d-2)\Sigma r^{d-3} + \frac{16\pi G}{16\pi G}}{16\pi G} \left( k + \frac{r^2}{l^2} \right). \quad (5) \]

Of particular interest is that when \( k = 0 \) or \( -1 \), the metric is also of a black hole cause structure. When \( k = 0 \), namely, the plane symmetric case, the black hole horizon is \( r^{d-1}_+ = 16\pi G M l^2/(d-2)\Sigma \). Note that the volume of the Ricci flat surface \( \gamma_{ij}dx^idx^j \) could be divergent, in this case, but \( M/\Sigma \) is finite and can be viewed as the mass density of the solution. Of course, the volume can also be made finite by identification. When \( k = -1 \), some strange features appear. Note that in this case, the volume of the hyperbolic surface is also divergent, but one can make it finite by identification and the black hole horizon becomes a higher genus closed surface \[12\]. In the case of \( k = -1 \), one may notice that even when \( M = 0 \), the solution also describes a black hole with horizon \( r^+ = l \), although it is locally equivalent to a \( d \)-dimensional AdS space. Sometimes the black hole is called a “massless” black hole. Furthermore, note that when \( M > 0 \) in (2), the black hole has only a horizon, while \( M_{\text{crit}} < M < 0 \), the solution (2) can have two horizons, where

\[ M_{\text{crit}} = -\frac{(d-2)\Sigma r^{d-3}_{\text{min}}}{8\pi G(d-1)} \quad (6) \]

where \( r^{2}_{\text{min}} = (d-3)l^2/(d-1) \), which is the horizon radius for a minimal “negative” mass black hole. When \( M = M_{\text{crit}} \), the two horizons coincide with each other, and beyond which, the solution describes a naked singularity.
For the black hole solution with any $k$, the Hawking temperature is

$$T = \frac{(d-3)}{4\pi r_+} \left( k + \frac{d-1}{d-3} \frac{r_+^2}{l^2} \right),$$

and the entropy associated with black hole horizon still obeys the well-known area formula, namely $S = A/4G$, where $A = \Sigma r_+^{d-2}$ is the horizon area of the black hole. The free energy of the black hole is easy to calculate, which is given by

$$F = \frac{\Sigma r_+^{d-3}}{16\pi G} \left( k - \frac{r_+^2}{l^2} \right) - M_{\text{crit}} \delta_{k,-1}.$$  

(8)

Note that here in the case of $k = -1$, one has taken the extremal black hole with mass $6$ as the reference background, while in the cases of $k = 1$ and $k = 0$, the solutions $2$ with $m = 0$ have been considered as reference backgrounds. From $8$, we can see clearly that when $k = 0$ and $-1$, the free energy is always negative, which means that the black hole phase is always dominant, and no Hawking-Page phase transition can happen in this case.

However, when $k = 1$, the free energy is negative as $r_+ > l$ (large black hole), and positive as $r_+ < l$ (small black hole), which implies that a one-order phase transition occurs here when $r_+ = l$. This is just the Hawking-Page phase transition $2$. The phase transition temperature is $T_{\text{HP}} = (d-2)/2\pi l$. This phase transition means that when the temperature of thermal gas in AdS space reaches the critical temperature $T_{\text{HP}}$, the thermal gas will collapse to form a black hole in AdS space (in fact the phase transition temperature will be higher than $T_{\text{HP}}$ since in our calculation we neglect the contribution to the free energy of thermal gas in AdS $2$). Further, we can see from $7$ that when $k = 1$, there exists a minimal temperature of black hole, $T_{\text{min}} = \sqrt{\frac{d-1}{d-3}/2\pi l}$ with corresponding horizon radius $r_{\text{min}} = l\sqrt{(d-3)/(d-1)}$. Below this temperature, there is no black hole solution, only thermal gas configuration exists. When $r_+ < r_{\text{min}}$, the black hole has a negative heat capacity and is local thermodynamically unstable, while as $r_+ > r_{\text{min}}$, the AdS black hole has a positive heat capacity, it is thermodynamically stable, and it can be in thermal equilibrium with its surrounding Hawking radiation. Note that when the black hole has a horizon in the same order as the AdS radius $l$, the black hole mass $M \sim l^{d-3}$, that is, the mass is of order 1 in units of AdS radius.

When $k = 0$ and $-1$, one can see that such minimal temperature does not exist. The AdS black hole is always thermodynamically stable with positive heat capacity for any horizon radius. Therefore we see that thermodynamical properties of AdS black holes with $k = 1$ are quite different from those of black holes with $k = 0$ and $-1$. One of the aims of this work is to see whether there are any essential differences of the self-gravitating radiation between the cases $k = 1$ and $k = 0$. As for the case of $k = -1$, one will see shortly
that it is impossible to have regular self-gravitating radiation configuration in that case.

Now we turn to the self-gravitating radiation configuration in AdS space. In this case, the metric function $\delta(r)$ and the mass function $m(r)$ satisfy the following equations \[15\]

\[
\begin{align*}
\delta'(r) &= -\frac{8\pi G r}{(d-2) h(r)} (T^t_t - T^r_r), \\
m'(r) &= -\Sigma r^{d-2} T^t_t.
\end{align*}
\]

The energy-momentum tensor of the radiation is

\[ T^\mu_\nu = \text{diag}(-\rho, p, p, \cdots, p), \]

where $p = p_{\perp}$, and its equation of state obeys $\rho = (d-1)p$. Thus the equations in \[9\] reduces to

\[
\begin{align*}
\delta'(r) &= \frac{8\pi G d r}{(d-2)(d-1) h(r)} \rho, \\
m'(r) &= \Sigma r^{d-2} \rho,
\end{align*}
\]

From the conservation of the energy-momentum tensor, one can derive the energy density $\rho(r)$ satisfies the following equation

\[
\frac{d\rho}{dr} = -\frac{\rho d}{(d-2) r h(r)} \left( \frac{8\pi G \rho r^2}{d-1} + \frac{(d-2)r^2}{\ell^2} + \frac{8\pi G (d-3)m(r)}{\Sigma r^{d-3}} \right).
\]

The energy density of radiation can be expressed in terms of the local temperature in $d$-dimensional curved space as

\[
\rho(r) = a T^d_{\text{loc}}(r),
\]

where $a$ is a dimensional dependent constant describing the degrees of freedom of the radiation \[9\]: $a = (d-1)\pi^{-d/2}\Gamma(d/2)\zeta(d)g$ and $g = n_B + (1 - 2^{(d-1)})n_F$ with $n_B$ being the number of boson spin states and $n_F$ the number of fermion spin states. The local temperature has a relation to the proper temperature $T$ as

\[
T_{\text{loc}}(r) = e^{-\delta(r)} h^{-1/2}(r) T,
\]

according to the Tolman red-shift relation, here $T$ is a constant. For the equilibrium configuration, the entropy density of the radiation is $s = a d T_{\text{loc}}^{d-1}/(d-1)$. Thus the total entropy of the configuration can be obtained by integrating over the spatial volume

\[
S = \frac{d}{d-1} a^{1/d} \Sigma \int_0^\infty \rho^{(d-1)/d} h^{-1/2} r^{d-2} dr.
\]
Thus, for a fixed cosmological constant \( l^2 \), we can get a set of self-gravitating radiation configurations, labeled by the central energy density \( \rho_c \), by integrating (12) and (13), starting from \( r = 0 \) to \( r = \infty \) with the boundary conditions \( \rho(0) = \rho_c \) and \( m(0) = 0 \). The equilibrium temperature can be obtained by (14) as

\[
T = \lim_{r \to \infty} \left( a^{-1} \rho(r) h^{d/2} \right)^{1/d},
\]

where we have used the gauge with \( \delta(\infty) = 0 \). Once obtaining the proper temperature \( T \), to get another metric function \( \delta(r) \), one can use (14), rather than integrating the equation (11),

\[
\delta(r) = \ln \left( a^{1/d} \rho^{-1/d} h^{-1/2} T \right).
\]

In this note we are particularly interested in the relation between the total mass of configuration and the central energy density. To integrate (12) and (13), we make a scaling transformation as follows,

\[
r \to l r, \quad \rho \to l^{-2} \rho, \quad m(r) \to l^{d-3} m(r),
\]

so that \( r, \rho \) and \( m \) become dimensionless. In the numerical integration, to compare with [8] and [9], following [8], we will adopt the units \( 8\pi G = 1 \) and \( l = 1 \), and rescale the mass function as

\[
16\pi G m(r)/(d - 2)\Sigma \to m(r).
\]

In that case, the gravitational “mass” \( M \) in the plots in the next section in fact is the gravitational mass density, \( 16\pi GM/(d - 2)\Sigma \), of corresponding self-gravitating configurations. In this way the gravitational “mass” \( M \) becomes comparable for the two cases of \( k = 1 \) and \( k = 0 \).

Further, note that the equations (12) and (13) are singular at \( r = 0 \). To avoid this, in the numerical calculations, we will start the integration from \( r = \epsilon = 10^{-5} \) to \( r = L = 100 \) in the case of \( k = 1 \), and from \( r = \epsilon = 10^{-2} \) to \( r = L = 100 \) in the case of \( k = 0 \). Obviously, the accuracy of the numerical calculations depends on the values of \( \epsilon \) and \( L \).

III. NUMERICAL RESULTS

To be more clear to compare the two cases of \( k = 1 \) and \( k = 0 \), let us first revisit the spherically symmetric case. In this case, \( k = 1 \), and the volume of \( \gamma_{ij} dx^i dx^j \) is \( \Sigma = 2\pi(d-1)/\Gamma[(d-1)/2] \). In \( d = 4 \), Page and Philips [5] found that there is a maximal mass configuration. For larger central energy densities, the mass will undergo an infinite series of damped oscillations, corresponding to unstable configurations. The case of \( d = 5 \) is similar to the case of \( d = 4 \) [5]. Hammersley [8] and Vaganov [9] studied the case with higher dimensions.
FIG. 1: The spherically symmetric case: The mass of the self-gravitating radiation configuration versus the central energy density from spacetime dimension $d = 4$ to $d = 26$ (from the yellow to blue curves).

It was found that in $4 \leq d \leq 10$, the behavior is similar to the case of $d = 4$ and in $d \geq 11$, however, the mass, temperature and entropy of the configuration turn to be monotonic functions of the central energy density, asymptoting to their maxima as the central density goes to infinity \[9\]. However, the implication of the existence of the critical dimension $d = 11$ is not clear.

It can be seen from Fig. 1 that all of configuration mass is bounded from above by a relatively small value which is of order one in units of the AdS radius $l$ (note that in the numerical calculation, $l$ has been set to one). This means in particular that one of these configurations can at most have a total mass that equals that of a small AdS Schwarzschild black hole and never that of a large AdS Schwarzschild black hole. In addition, in $4 \leq d \leq 10$, the maximal mass configuration and maximal entropy configuration occur at the same central energy density, while the maximal temperature configuration appears at a smaller central energy density \[8, 9\].

In Fig. 2, we plot the mass, entropy and temperature of the self-gravitation radiation configuration in the case of $d = 4$, there the oscillation behavior can be clearly seen.

Now we turn to the plane symmetric case. In that case, we have $k = 0$. In Fig. 3 we plot the mass density versus the central energy density. Compared to the spherically symmetric case $k = 1$, some interesting new features appear here. First, the mass density $M$ is suppressed very much, compared to the case of $k = 1$, although the mass density is also bounded from above. For example, the mass density is of order one in the units of AdS radius in the case of $k = 1$, while the mass density is in the order $(10^{-6})$ in the case of $k = 0, d = 4$.

Note that in Fig. 3 we rescale the mass density $M$ with scale $10^{2d-2}$ in each dimension, which means that as the spacetime $d$ increases by one, the mass density reduces by two orders in magnitude. The suppression of the mass density is due to the fact that in the case of $k = 0$, the black hole is always thermodynamically
FIG. 2: The spherically symmetric case: The mass, entropy and temperature of the self-gravitating radiation configuration versus the central energy density in the case of dimension $d = 4$.

stable without any mass limit, while in the case of $k = 1$, the small mass AdS Schwarzschild black hole has a negative heat capacity and then is thermodynamically unstable. Second, for each dimension, there is a maximal mass configuration, beyond the corresponding central energy density, the mass density decreases. We notice that in $4 \leq d \leq 7$, the mass density increases and reaches its maximum at a certain central energy density and then decreases monotonically to its asymptotic value. In $d \geq 8$, the mass density increases and reaches its maximum at a certain central energy density, and then decreases to a minimum at a larger central energy density, and then increases to its asymptotic value monotonically. Third, differing from the spherically symmetric case, beyond the central energy density corresponding to the maximal mass configuration, there is no any oscillation behavior in the mass density as a function of the central energy density. In Figs. 4 and 5 the entropy density and temperature of the self-gravitating radiation configuration are plotted respectively, as functions of the central energy density. In the plots the entropy is scaled as $10^{2d-2}Sa^{-1/d}/(d-3)\Sigma$ and here $S$ is given by (16), the temperature is scaled as $a^{1/d}T$ and here $T$ is given by (17). Let us mention that although we have rescaled the mass, entropy and temperature with dimension dependent factors in Figs. 3, 4 and 5 the shapes of those curves will not change. In addition, note that the central energy density $\rho_0$ was not rescaled except that we have taken $l = 1$ as the case of $k = 1$.

In Figs. 6, 7 and 8 we plot the mass density, entropy density and temperature of the self-gravitating radiation configuration versus the central energy density in $d = 4$, $d = 7$ and $d = 11$, respectively. Note that in the figures, the mass density, entropy density and temperature are rescaled with different scales. We observe that different from the case of $k = 1$, the maximal mass and maximal entropy configurations do not appear at the same central energy density. In addition, the shapes of these curves are also different, in contrast to the case
FIG. 3: The plane symmetric case: The mass density of the self-gravitating radiation configuration versus the central energy density from spacetime dimension $d = 4$ to $d = 26$ (from the yellow to blue curves).

FIG. 4: The plane symmetric case: The entropy density of the self-gravitating radiation configuration versus the central energy density from spacetime dimension $d = 4$ to $d = 26$ (from the yellow to blue curves).

FIG. 5: The plane symmetric case: The temperature of the self-gravitating radiation configuration versus the central energy density from spacetime dimension $d = 4$ to $d = 26$ (from the yellow to blue curves).
FIG. 6: The plane symmetric case: The mass density, entropy density and the temperature of the self-gravitating radiation configuration versus the central energy density in the case of spacetime dimension $d = 4$.

FIG. 7: The plane symmetric case: The mass density, entropy density and the temperature of the self-gravitating radiation configuration versus the central energy density in the case of spacetime dimension $d = 7$.

of $k = 1$, where the curves of mass, entropy and temperature are similar, see, for example, Fig. 2 in the case of $d = 4$.

Finally, let us mention that in the case of $k = -1$, there does not exist regular equilibrium configuration of self-gravitating radiation. From (2) one can see that for any gravitation mass ($m > 0$), a black hole horizon always exists, there $h(r_+)=0$. And from (9) one can see that for radiation one has $T^t_t \neq T^r_r$, therefore it is impossible to have regular self-gravitating radiation configuration and regular black hole with thermal equilibrium radiation with equation of state $p = \rho/(d - 1)$. 

[15]
IV. CONCLUSION

In this note we studied thermal equilibrium self-gravitating radiation in higher dimensional, plane symmetric AdS space. It was found that there exist essential differences between the spherically symmetric configuration and plane symmetric configuration, like their black hole solution counterparts. In particular, in the spherically symmetric case, there is a critical dimension $d = 11$: in $4 \leq d \leq 10$, beyond the maximal mass, entropy and temperature configurations, there is an oscillation behavior, while in $d \geq 11$, the mass, entropy and temperature become monotonic functions of the central energy density, and approach their asymptotic values as the central energy density goes to infinity. In the plane symmetric case, the mass density is suppressed very much, compared to the case of $k = 1$. In each dimension, there are a maximal mass density configuration, a maximal entropy density configuration and a maximal temperature configuration. The maximal mass density configuration and maximal entropy density configuration do not appear at the same central energy density. Note that maximal mass configuration and maximal entropy configuration do appear at the same central energy density in the spherically symmetric case. Although we observe there is an essential difference in the behavior of the mass density beyond the maximal mass density configurations, between $4 \leq d \leq 7$ and $d \geq 8$, we guess that those configurations in the regime beyond the maximal mass density are dynamically unstable. If so, the difference between $4 \leq d \leq 7$ and $d > 8$ does not make any sense since it appears beyond the maximal mass configuration and those configurations are unstable and are of less interest physically. In addition, let us mention that although in the case of $k = 0$, there exist regular self-gravitating radiation configurations, the Hawking-Page phase transition still does not happen since as we have seen in the above, the contribution of
the self-gravitating radiation is negligible, compared to the case of $k = 1$. This result also can be understood from the thermodynamic properties of the Ricci flat AdS black holes as we discussed in the Sec. II.

Acknowledgments

This work was supported in part by a grant from the Chinese Academy of Sciences, grants from NSFC with No. 10325525 and No. 90403029. We thank K. Fang for her participation in the early stage of this work.

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