Conformal Dilatonic Cosmology

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Gravitation and the standard model of particle physics are incorporated within a single conformal scalar-tensor theory, where the scalar field is complex. The Higgs field has a dynamical expectation value, as has the Planck mass, but the relative strengths of the fundamental interactions are unchanged. Initial cosmic singularity and the horizon problem are avoided, and spatial flatness is natural. There were no primordial phase transitions; consequently, no topological defects were produced. Quantum excitations of the dilaton phase induced a slightly red-tilted spectrum of gaussian and adiabatic scalar perturbations, but no analogous primordial gravitational waves were generated. Subsequent cosmological epochs through nucleosynthesis are as in standard cosmology. A generalized Schwarzschild-de Sitter metric, augmented with a linear potential term, describes the exterior of stars and galaxies, such that there is no need for dark matter on galactic scales.

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Introduction.- The standard cosmological model has been very successful in parametrically fitting a wide range of observational data over a vast dynamical range extending from the big bang nucleosynthesis (BBN) to the Hubble scale, e.g. [1, 2]. Our understanding of the early universe down to (at least) the BBN era, is based on well-understood and experimentally established physics.

However, current understanding of the very early universe (energies TeV and higher) lacks direct experimental confirmation. A major underpinning of standard cosmology is the primordial inflationary phase thought to have ended at energies as high as $O(10^{16})$ GeV, e.g. [3-6]. A few fine-tuning (naturalness) and conceptual problems generically afflict inflation, e.g. [7-10], but its exceptional role in explaining and predicting a variety of cosmological phenomena is indeed remarkable.

On the largest scales, and rather recently, a non-clustering mysterious vacuum-like species, which is many orders of magnitude smaller than expected on theoretical grounds from a vacuum energy, e.g. [11], has come to dominate the background cosmological dynamics, e.g. [12, 13]. Additionally, on cosmological down to sub-galactic scales cold dark matter (CDM) is required for an observationally viable cosmological model.

In the absence of this species the global spatial flatness, non-Keplerian galactic rotation curves, strong gravitational lensing, and the observed abundance of nonlinear structure, could not be explained by a (largely) baryon-dominated cosmological model.

Moreover, in spite of the success of the standard cosmological model the microphysics of most of the cosmic energy budget is either unknown or fine-tuned to explain observations. Judging a physical theory based only on its (indeed remarkably small) number of free parameters is arguably unsatisfactory.

In this letter we explore a few cosmological implications of promoting fundamental physical constants, e.g. particle masses, Newton gravitational constant $G$, etc., to scalar fields. We adopt a conformal dilatonic framework for describing the fundamental interactions and formulate cosmology in a (field) frame-independent fashion. Important conceptual differences between our approach and standard cosmology, and departures in the physics of the early universe and galactic dynamics, are briefly highlighted. For a more complete and comprehensive presentation of the new approach summarized here, see [14]. Throughout, we adopt a mostly positive metric signature.

Conformal Gravity and the Standard Model.- Focusing on the gravitational sector of the fundamental interactions first, we consider a (complex) scalar-tensor theory of gravity that is formulated in terms of the action

$$I_{\text{gr}} = \int \left[ \frac{1}{6} |\phi|^2 R + \phi_{\mu} \phi^{* \mu} + L_M(|\phi|) \right] \times \sqrt{-g} \, d^4 x, \quad (1)$$

merely a generalization of the action discussed in [15] to the complex field case, where $g_{\mu\nu}$, $R$ and $L_M$ are the metric, curvature scalar, and matter lagrangian density, respectively. The dilaton field $\phi$ has a non-positive kinetic term, a feature which is usually considered a problem, but here we show that only its phase perturbation is a genuine degree of freedom subject to quantization in conformal dilatonic gravity. The matter lagrangian $L_M(|\phi|)$ may also contain terms $|\phi|^4$ which function as vacuum-like energy contributions (thereby effectively promoting the cosmological constant to a dynamical quantity). Eq. (1) is invariant under the generalized conformal (Weyl) transformation $\phi \rightarrow \phi/\Omega$, $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$, and $L_M \rightarrow L_M/\Omega^4$ where $\Omega(x)$ is an arbitrary function of spacetime, e.g. [16]. With $L_M$ independent of $\phi$ this theory is a version of a particular Brans-Dicke (BD) theory, with the dimensionless BD parameter $\omega_{BD} = -3/2$ [15]. In the presence of $L_M(|\phi|)$ it falls in the category of Bergmann-Wagoner scalar-tensor theories [17, 18].

Variation of Eq. (1) with respect to $g_{\mu\nu}$ and $\phi$ results in generalized Einstein equations, as well as equations for the scalar fields [19]. Combining these equations results in a generalized local energy momentum (non-) conser-

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\[ T_{M,\mu\nu} = \mathcal{L}_M, \phi_{,\mu}, \]

where \( T_{M,\mu\nu} \equiv \frac{1}{2} \delta ^{\lambda}_{\nu} \frac{\partial}{\partial g^{\lambda\mu}} \mathcal{L}_M \) is the matter energy-momentum tensor. Indeed, energy-momentum is clearly not conserved when Newton constant \( G \), the cosmological constant \( \Lambda \), or particle masses, are promoted to spacetime-dependent fields. Massless particles still travel along geodesics in this theory but massive particles do not. This fact is responsible for, e.g., cosmological redshift in a comoving frame where the metric field is static, and non-Keplerian behavior of galactic rotation curves – phenomena that are usually attributed to space expansion and galactic CDM, respectively [14].

Embedding the standard model (SM) of particle physics in this conformal theory of gravity is straightforward, and is summarized by the following action that accounts for both the gravitational and the SM interactions

\[
I_{tot} = \int d^4x \sqrt{-g} \left[ \frac{1}{6} (\phi^2 - H^4) R + (\phi^* \phi - D^\mu D_\mu H) - \lambda_{SM} (H^2 - v^2)^2 + \mathcal{L}_{SM}(v, \psi, A_\mu, g^{\mu\nu}) \right].
\]

(3)

Here, \( H \) is the Higgs isospin doublet, \( \lambda_{SM} \) is its dimensionless self-interaction constant, and \( v \equiv a(\phi) \) is its vacuum expectation value (VEV) where \( \alpha = O(10^{-16}) \) is the dimensionless ratio of the Higgs VEV and the dynamical (reduced) Planck mass. The lagrangian \( \mathcal{L}_{SM} \) is a function of \( v \), fundamental fermions \( \psi \), gauge bosons \( A_\mu \), and the metric \( g_{\mu\nu} \). Eq. (3) differs from a similar action discussed in [22] by one crucial aspect: here, unlike in [22], \( \phi \) is complex. In addition, unlike in [22] and similar works, we never fix \( \phi \) to a constant value; masses (essentially \( v \)), \( G \), and \( \Lambda \) are dynamical.

**Cosmological Model.**- Defining \( \phi \equiv \rho e^{i\eta} \), \( a \equiv a \rho \) and \( \mathcal{H} \equiv \dot{a}/a \) with \( f \equiv \frac{df}{d\eta} \) for any \( f \), where \( \eta \) is the conformal time related to cosmic time \( t \) via \( dt \equiv a(\eta) d\eta \), and \( a(\eta) \) is the scale factor describing the time-dependence of the Friedmann-Robertson-Walker (FRW) metric \( g_{\mu\nu} = a^2 \cdot \text{diag}(-1, 1, r^2, r^2 \sin^2 \theta) \) in spherical coordinates and conformal time units, the field equations derived from Eq. (1) can be cast in a manifestly frame-invariant fashion [14]

\[
\mathcal{H}^2 + K = \dot{a}^2 \rho_M + \lambda \dot{a}^2 - \theta^2
\]

(4)

\[
\mathcal{H}^2 + K = \frac{1}{2} (1 - 3w_M) \dot{a}^2 \rho_M + 2\lambda \dot{a}^2 + \theta^2.
\]

(5)

Here, \( K \) is the spatial curvature, the matter energy-momentum tensor of a perfect fluid is \( T_{M,\mu\nu} = \rho_M \cdot \text{diag}(-1, w_M, w_M, w_M) \), where \( w_M \equiv \rho_M / P_M \) is the equation of state (EOS), and \( \rho_M \) and \( P_M \) are the energy density and pressure, respectively. The analog of the energy density scales as \( \rho_M \equiv \sum_i \rho_{MB_i} \dot{a}^{-3(1 + w_{M,i})} \), where the index \( i \) runs over the species. The effective analog energy densities associated with \( \theta \) and \( K \) are \( \rho_{\theta} \equiv -\theta^2 / \dot{a}^2 \) and \( \rho_K \equiv -K / \dot{a}^2 \), respectively. In addition to matter we include a cosmic term with an effective \( w_\Lambda = -1 \) which is characterized by \( T_{\Lambda,\mu\nu} = \lambda \cdot \text{diag}(-1, 1, 1, 1) \), where \( \lambda \) is a fixed dimensionless constant. The \( \lambda \dot{a}^2 \) term appearing in Eq. (4) drives a very early conformal phase of evolution and is also responsible for the recent vacuum-like dominated era.

The generalized Friedmann equations, Eqs. (4) & (5), are invariant under any simultaneous change of \( \rho \) and \( a \) (i.e. particle masses and the scale factor, respectively) that leaves their product, \( \dot{a} \), unchanged. Note that the EOS of perfect fluids does not change under conformal transformations. Since the global U(1) symmetry of Eq. (1) implies that \( \theta \propto a^{-2} \), the dilatonic phase term \( \propto \theta^2 \) in Eqs. (4) & (5) can be described as an effective stiff matter \((w_\theta = 1) \) carrying negative energy density. In [14] it is shown that the linear order perturbation equations governing the metric and matter perturbations are frame-invariant as well. Moreover, consistency of the perturbation equations associated with the dilaton with the perturbed trace of the Einstein equations, implies that (cosmological) metric perturbations in the theory described by Eq. (1) must be adiabatic, i.e. \( \delta P_M = w_M \delta \rho_M \mid [14] \). In addition, perturbations of the modulus \( \delta \rho_M \) can be systematically absorbed in ‘renormalized’ metric and matter perturbations. In particular, this implies that real fields, such as the inflaton, cannot seed metric perturbations if conformally coupled to gravity. For this reason, quantum and thermal fluctuations of \( \rho \) are absent, implying no catastrophic particle production is associated with the negative kinetic term of Eq. (1) and no primordial phase transitions occurred, respectively. The observed non-clustering of the DE contribution to the cosmic energy budget is consistent with \( \rho_\Lambda = \lambda a^{-1} \) being a dimensionless constant, not a field. In any case, the DE terms appearing in Eqs. (4) & (5) are \( \propto a^2 \dot{a}^2 \) and \( \dot{a} \) is unperturbed then the effective DE contribution to the perturbed Friedmann equation identically vanishes. As shown below, quantum fluctuations of the phase \( \theta \) that seed scalar perturbations are governed by a Mukhanov-Sasaki-like wave equation.

In contrast to the standard cosmological model where cosmic time, \( t \), is effectively replaced by \( \eta \) for massless particles, in our proposed cosmological model the ‘cosmic clock’ ticks universally for both massless and massive particles. Time is parametrized by \( \eta \), with cosmological redshift generally explained by a combination of space expansion and dynamical masses, i.e. varying Rydberg ‘constant’. Redshift is explained solely by the latter effect in the comoving frame.

The very early universe scenario described in this letter begins with a very large deflating \( \dot{a} \) in a ‘conformal era’ when the right hand sides of Eqs. (4) & (5) are dominated by the \( ax \dot{a}^2 \) terms. Therefore, in the absence of dimensional constants \( \dot{a} \propto (\eta - \eta')^{-1} \), where \( \eta' \) is an arbitrarily negative integration constant (since our cosmological model is non-singular as shown below), scales according to its canonical dimension, \( [\dot{a}] = [\rho] = \text{length}^{-1} \), and the
spontaneously broken conformal symmetry is ‘restored’.

An important aspect of the conformal era \((w_M = -1)\) in the cosmological context, and within a theory featuring a conformally-coupled complex scalar field, is the generation of gaussian and adiabatic scalar metric perturbations which are characterized by a flat spectrum [23]. The mechanism proposed here is different in a few crucial aspects, e.g. unlike in [23] we consider the effect of \(\theta' \neq 0\) as well as coupling of phase- to metric-perturbations. These differences result in fundamentally different explanations for the tilt of the power spectrum and the adiabaticity of scalar perturbations [14], and avoids a few difficulties with [23] stemming from coupling of \(\delta \rho\) to \(\theta\).

Working in the shear-free gauge, and neglecting stress anisotropy, the linear perturbation equations in the very early conformal epoch satisfy [14]

\[
\varphi'' + 6 \ddot{H} \varphi' + (q^2 + 6 \dot{a}^2) \varphi = 0, \tag{6}
\]

\[
\delta \theta = (\varphi' + H \varphi)/(3 \dot{\theta}), \tag{7}
\]

where \(\varphi\) is the ‘renormalized’ Newtonian potential, \(q^2 \equiv k^2 - 8K\). The phase dynamics \((\theta' \neq 0)\) plays a crucial role in mediating phase perturbations \(\delta \theta\) to scalar metric perturbations \((\text{Eq. 7})\). Neglecting spatial curvature, and using \(\dot{a} = (\sqrt{\dot{a}^2})^{-1}\) during the conformal era (where we set an integration constant to 0 for simplicity), the general solution of Eq. (6) is \(\varphi = c_1 \eta^3 J_{5/2}(k \eta) + c_2 \eta^2 Y_{5/2}(k \eta)\), where \(J_{5/2}(k \eta)\) and \(Y_{5/2}(k \eta)\) are the Bessel functions of the first and second kinds, respectively. Requiring the appropriate Bunch-Davis vacuum for \(\delta \theta\) at \(k \eta \gg 1\) determines the coefficients \(c_1\) and \(c_2\) and results in \(\varphi \propto k^{-1} \eta^{5/2} H^{(1)}_{5/2}(k \eta)\) where \(H^{(1)}_{5/2}(k \eta)\) is the Hankel function of the first kind and of order 5/2, which in the \(k \eta \ll 1\) limit corresponds to \(k^3 P_\eta(k) = k^{-4}\). This translates to \(k^3 P_\eta(k) = \text{constant}\) by virtue of the Poisson equation. Here \(P_\phi\) & \(P_\rho_\phi\) are the power spectra of metric and density perturbations, respectively. A pseudo-conformal epoch with \(w_M \gtrsim -1\) similarly results in regular perturbation modes at the bounce which are characterized by a red-tilted power spectrum. In contrast, \(w_M \lesssim -1\) corresponds to singular perturbation modes at the bounce, thereby undermining the underlying homogeneity of the cosmological model (and is therefore not a viable solution), and are described by a blue-tilted spectrum [14].

This mechanism of phase-perturbation-induced metric perturbations is exclusive to perturbations of the scalar type. A positive detection of primordial gravitational waves (PGW) would definitely challenge the proposed mechanism. Since Eq. (6) is linear in \(\varphi\) there is no mode-mode coupling and because we assume (as is common in inflationary models) that the underlying fluctuating \(\theta\) is a quantum vacuum, and is therefore gaussian, then one expects the induced metric perturbations to be likewise gaussian.

Curvature and the effective stiff matter contributions may still be dynamically important at sufficiently small \(\dot{a}\). Assuming all matter was relativistic (near the minimal \(\dot{a}\)) with energy density \(\rho_r\), and integrating Eq. (4) (while ignoring the potential term) then results in

\[
\dot{a}^2 \approx \left( \frac{\rho_{\rho_\phi}}{K} + \left( \frac{\rho_{\phi \phi}}{2K} \right)^2 \right)^{1/2} \cosh(2\sqrt{-K}(\eta - \eta_*)) - \frac{\rho_{\phi \phi}}{2K}\tag{8}
\]

and its minimum is attained at \(\eta = \eta_*\), where (recall that) \(\rho_\theta \leq 0\) and \(K < 0\). When the \(\rho_\theta\) and the effective \(\rho_k\) terms become subdominant to \(\rho_r\), BBN begins, followed by all standard cosmological epochs, including radiation-matter equality, recombination, and the recent acceleration.

In case that \(K = 0\) Eq. (4) integrates to

\[
\dot{a}^2 = \rho_{\rho_\phi}(\eta - \eta_*)^2 - \rho_{\phi \phi}/\rho_r.\tag{9}
\]

In could be easily shown that in this case not only \(\eta\) is extended to \(-\infty\), but also \(t = \int a(\eta)d\eta\). In other words, both timelike and null geodesics are freely extended through the non-singular bounce. Since the initial cosmic singularity is avoided no matter generation mechanism is required – spacetime and matter always existed. In addition, it is straightforward to show that \(\rho_{\rho_\phi}\) can be naturally chosen such that scalar metric perturbations are finite at the bounce and its vicinity [14], thereby not undermining the underlying homogeneity on cosmological scales. By selecting \(\rho_{\rho_\phi}\) sufficiently small the post-bounce era starts at sufficiently large number densities to guarantee very effective double-Compton and bremsstrahlung processes at its minimum \(\dot{a}\) value and thereby the effective thermalization of the CMB is guaranteed. Structure formation history is exactly as in standard cosmology since both the background equations (Eqs. 4 & 5) and the perturbation equations [14] are unchanged in the relevant post radiation-dominated era.

The flatness problem, which is addressed in standard cosmology by inflation, does not exist in our early universe scenario. The problem essentially arises in the standard model since space monotonically expands, but in a non-singular bouncing scenario such as the one advocated here no fine-tuning of the curvature is required. In the deflationary phase the matter density grows in proportion to the energy density associated with curvature. Since the model is symmetric in \(\dot{a}\) this implies that for curvature to dominate over matter at present the matter density should have been extremely fine-tuned to zero at \(\eta = -\infty\). From that perspective, matter domination at any finite time is actually an attractor rather than an unstable point.

On galactic and sub-galactic scales (and possibly on extra solar system scales) we consider a spherically symmetric static line element (with conformal time)

\[
ds^2 = -B(r)dt^2 + B^{-1}(r)dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \tag{9}
\]

The metric \(B(r)\) is determined (up to conformal transformations of \(g_{\mu\nu}\) by the field equations, in vacuum and with no cosmological constant, to be [14]

\[
\rho = \frac{\rho_0}{1 + \gamma r/(2 - 3\beta\gamma)} \tag{10}
\]

\[
B = (1 - 3\beta\gamma) - \frac{\beta(2 - 3\beta\gamma)}{r} + \gamma r - Kr^2. \tag{11}
\]
where $\rho_0$, $\beta$ and $\gamma$ are integration constants. We note that $B(r)$ coincides with the corresponding quantity obtained in fourth order Weyl gravity [20, 21]. In accord with what has been done in [20, 21], we considered the limit $\beta \gamma \ll 1$. Note that $\kappa$ plays the role of an effective cosmological constant ($\Lambda = 3\kappa$) even though Eqs. (10) & (11) are obtained as a vacuum solution of the field equations; this can be understood if the cosmological constant is viewed (in static spacetimes) as an arbitrary integration constant.

As noted in, e.g., [20, 21] the linear term appearing in Eq. (11) may account for the shapes of galactic rotation curves and strong lensing data with no recourse to CDM on these scales. This would set a lower bound on the CDM particle mass of $\gtrsim 10^{-22}eV/c^2$ [14] if it is assumed that CDM does cluster on galaxy cluster scales, as may be implied by bullet-like clusters. We note that the metric and scalar field in Eqs. (10) & (11) are determined only up to a conformal rescaling which we determine to be $g_{\mu\nu} \rightarrow \tilde{\alpha}^2 g_{\mu\nu}$ and $\rho \rightarrow \rho/\tilde{\alpha}$, commensurate with observations of emission by sources residing within gravitationally bound objects [14].

Summary.- This work advocates abandoning the standard units convention that underlies general relativity (GR) and the SM of particle physics – local energy-momentum conservation – in favor of local scale invariance, i.e. conformal (Weyl) symmetry. One might argue that forcing energy-momentum conservation on galactic scales required cosmologists to introduce CDM in order to explain the observed anomalous rotation curves and strong lensing data. While invoking the CDM hypothesis has been rather successful in parametrically fitting observations to GR predictions, the essence of CDM remains elusive.

Whereas conformal dilatonic gravity naturally accommodates quartic potential inflation, perturbations of scalar fields moduli are systematically absorbed in renormalized metric and matter perturbations. Therefore, they cannot be used to explain the seed density perturbations if the inflaton field is conformally coupled to gravity. We consider an alternative bouncing cosmological scenario which is symmetric in $\tilde{\alpha}$ around the bounce. The deflating pre-bounce era involves a conformal cosmic epoch followed (possibly) by a (negative) curvature-dominated (CD) era, nonrelativistic matter, radiation, and an effective ‘stiff’ matter component; the latter reflects the dynamics of the transversal mode of the complex scalar field, and is characterized by an effective negative energy density, thereby providing a ‘centrifugal barrier’ that is responsible for the bounce at a finite $\tilde{\alpha}$.

The deflating pseudo-conformal evolutionary phase ($w_M \gtrsim -1$), $\tilde{\alpha} \propto t^{-1}$, where $\tilde{\alpha}$ rolls down its (nearly) quartic potential dominates the evolution for sufficiently large $\tilde{\alpha}$. Perturbations of the dilaton phase induce a nearly flat, red-tilted spectrum of gaussian and adiabatic scalar metric perturbations. No analogous production of either PGW or vector perturbations is expected, rendering this mechanism (and possibly the entire framework) falsifiable. The flatness problem is naturally addressed by the non-singular bouncing scenario; matter domination over curvature is an attractor point at any finite time. In addition, the horizon problem is avoided by the fact that the model is non-singular. No primordial phase transitions occur in conformal dilatonic gravity, and consequently no primordial relic problem arises in the first place. These cosmic epochs are subsequently followed by the conventional radiation- and matter-domination, recombination, etc. Structure formation history is unchanged.

Conformal dilatonic gravity admits spherically symmetric vacuum solutions for a modified Schwarzschild-de Sitter spacetime augmented by a linear potential term. When applied to galactic scales, this approach results in significant departures from standard interpretations of observations. This pertains, in particular, to our understanding of the nature of cosmological redshift, CDM, and DE. The implication is that CDM may not be required on galactic and sub-galactic scales, but may be required on galaxy cluster scales and larger for a proper phenomenological description of ‘bullet’-like systems. This fact alone already sets a lower bound $m_{CDM} \gtrsim 10^{-22}eV/c^2$ on the mass of CDM particles.

We have shown that the dynamics of conformally-coupled complex scalar fields can account for cosmological redshift in a (field) frame-independent fashion, explain away the horizon and flatness problems, and avoid initial cosmic singularity and primordial relics. Additionally, our theoretical formulation naturally explains the spectrum of primordial density perturbations, their gaussianity and adiabacity, removes the need for invoking CDM on galactic scales, provides the dynamic VEV for the Higgs field (up to a large hierarchy constant), and thereby all fundamental length (and mass) scales, all in a single unified framework that underscores the unique role played by conformal symmetry, possibly an overarching symmetry of the four fundamental interactions.

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