Experiments in Dilute Atomic Bose-Einstein Condensation

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1. – Introduction

1’1. Why BEC? – In the month we began writing this paper, fourteen papers on the explicit topic of Bose-Einstein condensation (BEC) in a dilute gas appeared in the pages of Physical Review. Both theoretical and experimental activity in BEC has expanded dramatically in the three years since the first observation of BEC in a dilute atomic gas. One is tempted to ask, why? Why is there so much interest in the field? Is this burst of activity something that could have been foreseen, in, say, 1990?

We feel the answer to this last question is yes: while “interesting” is a quality which is impossible to define, there was good reason to anticipate, even in 1990, that dilute-gas BEC was going to be something worth investigating. To see why, one need only perform the following simple test. Ask any physicist of your acquaintance to compile a list of what he or she considers the six most intriguing physical phenomena that occur at length scales greater than a picometer and smaller than a kilometer. In our experience, such lists almost always include at least two of the following three effects: superfluidity, lasing, and superconductivity. These three topics all share two common features: counter-intuitive behavior and macroscopic occupation of a single quantum state. This then would have been our clue — one might have anticipated that dilute-gas BEC was to be interesting because it shares the same underlying mechanism with the widely appreciated topics of lasing, superfluidity, and superconductivity, and yet is quite different from any of them: Dilute-gas BEC is more amenable to microscopic analysis than superfluidity,
more strongly self-interacting than laser beams, and it possesses a range of experimental observables which are complementary to those of superconductivity.

This paper will not pretend to provide a thorough survey of the status of dilute-gas BEC experiment. Any attempt to do so at this time would surely be obsolete by the time it appears in print — such is the pace of developments these days. Rather, we will try to cover the same pedagogical ground as the set of lectures delivered by one of us (EAC) at the 1998 Enrico Fermi summer school on Bose-Einstein condensation. The lectures were coordinated with other experimenter-lectures at Varenna, and thus were not meant to be comprehensive. This paper as well will be selective in its focus.

An outline of this paper is as follows: We begin with an introductory section including a review of “ideal-gas” BEC theory [1]. The second section presents a history of the subject, and the third a forward-looking summary of BEC technology. The fourth section introduces the important topic of interactions in the condensate and surveys experimental work on interactions. The fifth section discusses, mainly by reference to published articles, experiments on condensate excitations. The sixth section is a brief essay on the meaning of the “phase” of a condensate. The seventh section surveys what is known about heating processes in trapped atom samples, and examines the implications for trapping very large samples. The eighth section contains some general observations on atomic collisions relevant to evaporation.

1.2. BEC in an Ideal Gas. – It is conventional to begin a lecture on ideal Bose statistics with a ritual chant, “Imagine a system of $N$ indistinguishable particles...” On occasion, however, it is worth pausing to reflect on what an exotic thing it is to imagine, a system of indistinguishable particles. One is willing enough to admit that yes, two atoms can be so similar one to the other as to allow no possibility of telling them apart. Why not? That is not so much to concede. The problem arises only when one is forced to confront the physical implications of the concept of indistinguishable bosons. For example, if there are ten particles to be arranged in two microstates of a system, the statistical weight of the configuration \{ten particles in one state and zero in the other\} is exactly the same as the weight as the configuration \{five particles in one state, five in the other.\} We find this ratio of 1 : 1 disquieting and counter-intuitive. The corresponding ratio for distinguishable particles, “Boltzmannons,” would be 1 : 252.

In the second [2] of Einstein’s two papers [2, 3, 4] on Bose-Einstein statistics, Einstein acknowledges that in his statistics, “The ... molecules are not treated as statistically independent ...,” and comments that the differences between distinguishable and indistinguishable state counting “express indirectly a certain hypothesis on a mutual influence of the molecules which for the time being is of a quite mysterious nature.” The mysterious statistics of indistinguishable bosons in turn mandates a variety of exotic behavior, e.g. the famous enhanced probability for scattering into occupied states and of course Bose-Einstein condensation.

Why should the presence or absence of distinguishing tags on the atoms so profoundly change their statistical behavior? For the particular case of photons, the problem may be more palatably expressed in terms of fields, rather than particles. When many photons
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occupy a particular microstate, we talk about the electric field amplitude in a particular mode. It is relatively easy, at least for the authors of this paper, to believe that having equal field amplitudes in two adjacent modes is no more entropic than having a large field in one mode and a small field in the other. In some ways, this hasn’t helped much. For as soon as we try to apply the notion of fields back to atoms, we are left with a new set of troubling notions – can the probability amplitude for a large collection of atoms ever really be like an electric field – coherent, continuous, macroscopically occupied, macroscopically observable? The answer is “absolutely yes!” Troubling or no, this is exactly what a Bose condensate is about.

Proceeding conventionally, then, let us imagine a system of \(N\) indistinguishable particles, distributed among the microstates of a confining potential, such that any occupation number is allowable. The mean distribution, the Bose-Einstein distribution (BED), may be derived in several different ways. (See for instance [5].) In the end one may always understand the BED as the most random way to distribute a certain amount of energy among a certain number of particles in a certain potential. The BED gives the mean number of particles in the \(i\)th state as

\[
\begin{aligned}
\langle n_i \rangle &= \frac{1}{e^{(\epsilon_i - \mu)/kT} - 1} \\
\end{aligned}
\]

where \(\epsilon_i\) is the energy of a particle in the \(i\)th state, \(k\) is Boltzmann’s constant, and \(T\) and \(\mu\) are identified as the temperature and chemical potential, respectively. In the microcanonical understanding of the BED, \(\mu\) and \(T\) are determined from the constraints on total number \(N\) and total energy \(E\):

\[
\begin{aligned}
N &= \sum_i \frac{1}{e^{(\epsilon_i - \mu)/kT} - 1} \\
E &= \sum_i \frac{\epsilon_i}{e^{(\epsilon_i - \mu)/kT} - 1} \\
\end{aligned}
\]

For large systems, the constraints may be written as integrals

\[
\begin{aligned}
N &= \int d\epsilon \frac{g(\epsilon)}{e^{(\epsilon - \mu)/kT} - 1} \\
E &= \int d\epsilon \frac{\epsilon g(\epsilon)}{e^{(\epsilon - \mu)/kT} - 1} \\
\end{aligned}
\]

where \(g(\epsilon)\) is the density of states in the confining potential.
Equations (1)-(3) contain nearly all the ideal gas physics of BEC. The number of spatial dimensions, and the effects of a confining potential, are all taken care of in the power law of the density of states, \(g(\epsilon)\). The effects of finite number, and of very asymmetric potentials \[6\], can be determined by using the sums rather than the integrals to constrain \(\mu\) and \(T\). The critical temperature, the ground-state occupation fraction and the specific heat can all be calculated without difficulty. Only number fluctuations, which require a more careful consideration of the underlying ensemble statistics, are left out of this picture. The overall picture is sufficiently easy to understand that, if the system truly were an ideal gas, there would be little left to study at this point.

As it actually has turned out, interactions between particles add immeasurably to the richness of the system, and work in dilute-gas BEC is in its early days.

1.3. Some Real Systems. – Einstein’s original conception of BEC was in an ideal gas, but the first experiments in BEC were in superfluid helium, a liquid, (which is to say a strongly correlated fluid, the opposite limit from an ideal gas). The beautiful and startling experiments on viscosity, vortices, and heat-flow in liquid helium, and the ground-breaking theory those experiments inspired, more or less defined the field of Bose-Einstein condensation for four decades and more \[7\]. The BEC concept has been put to use in broader contexts over the years, however, in such diverse topics as Kaons in neutron stars \[8\], cosmogenesis, and exotic superconductivity \[9\]. Using the term in its broadest meaning (“macroscopic number of bosons in the same state”) one needn’t have a fluid of any sort – lasers and masers produce macroscopically occupied states of optical and microwave photons, respectively. For that matter, a portable telephone, or even a penny-whistle produce macroscopic occupation numbers of identical bosons (radio-frequency photons in one case, acoustic-frequency phonons, in the other.)

In superfluid He-4, the bosons exist independently of the condensate process. The fermionic neutrons, protons, and electrons that make up a He-4 atom bind to form a composite boson at energies much higher than the superfluid transition temperature. There exists a broad family of physical systems, however, in which the binding of the fermions to form composite bosons, and the condensation of those bosons into a macroscopically occupied state, occurs simultaneously. This is of course the famous BCS mechanism. Best-known for providing the microscopic physical mechanism of superconductivity, the Bose-condensed “Cooper pairs” of BCS theory occur as well in superfluid He-3 and may also be relevant to the dynamics of large nuclei \[10\] and of neutron stars.

Prior to the observation of BEC in a dilute atomic gas, the laboratory system which most closely realized the original conception of Einstein was excitons in cuprous oxide \[11\]. Excitons are formed by pulsed laser excitation in cuprous oxides. There exist metastable levels for the excitons which delay recombination long enough to allow the study of a thermally equilibrated Bose gas. The effective mass of the exciton is sufficiently low that the BEC transition at cryogenic temperatures occurs at densities which are dilute in the sense of the mean particle spacing being large compared to the exciton radius. Recombination events, which can be detected either electrically \[12\] or by collecting their fluorescence \[11\], are the main experimental observable. The most convincing evidence
for BEC in this system is an excess of fluorescence from “zero-energy” excitons. In addition, anomalous transport behaviour evocative of superfluidity has been observed [13].

As an experimental and theoretical system, BEC in a dilute gas is nicely complementary to the variety of BEC-like phenomena described above. In terms of strength of interparticle interaction, atomic gas BEC is intermediate between liquid helium, for which interactions are so strong that they can not be treated by perturbation theory, and photon lasers, for which “interactions” (or nonlinearity) is small except in effectively two-dimensional configurations. In terms of the underlying statistical mechanics, atomic-gas BEC is most like He-4. Unlike in superconductivity, the bosons are formed before the transition occurs, and unlike in lasers, the particle number is conserved. The properties of the phase transition, then, should most closely resemble liquid helium. Finally, in terms of experimental observables, atomic-gas BEC is in a class entirely by itself. The available laboratory tools for characterizing and manipulating atomic-gas BEC are essentially orthogonal to those available for excitonic systems or for liquid helium.

Although dilute-gas atomic BEC provides a nice foil to more traditional BEC systems, it comes at a price: one is forced to work deep in the thermodynamically forbidden regime.

1.4. Why BEC in Dilute BEC is hard. – Figure 1 is a qualitative phase-diagram which shows the general features common to any atomic system. At low density and relatively high temperature, there is a vapor phase. At high density there are various condensed matter phases. But the intermediate densities are thermodynamically forbidden, except at very high temperatures. The Bose-condensed region of the $n - T$ plane is entirely forbidden, except at such high densities that the equilibrium configuration of nearly all known atoms or molecules is crystalline. (The existence of a crystal lattice rules out a Bose condensate.) The sole exception is helium, which remains a liquid below the BEC transition. Reaching BEC under dilute conditions, say at densities ten or 100 times lower than conventional liquid helium, is as forbidden to helium as it is to any other atom.

All the same, it is possible to do forbidden things, as long as one doesn’t attempt to do them for very long! While making a dilute-gas BEC in equilibrium is impossible, one can exploit metastability to stray temporarily into the forbidden region. In the absence of artificial nucleation sites, the process by which a supersaturated atomic gas finds its way from a gas to a solid begins with molecular recombination. Radiative recombination, by which two atoms collide and emit a photon to stabilize a dimer molecule, has a vanishingly small rate. At any reasonable density the dominant channel is three-body recombination. This then is what makes BEC possible. A gas of atoms can come into kinetic equilibrium via two-body collisions, whereas it requires three-body collisions to achieve chemical equilibrium (i.e. to form molecules and thence solids.) At sufficiently low densities, the two-body rate will dominate the three-body rate, and a gas will reach kinetic equilibrium, perhaps in a metastable Bose-Einstein condensate, long before the gas finds its way to the ultimately stable solid-state condition.

The need to maintain metastability usually dictates a more stringent upper limit on density than does the desire to create a dilute system. Densities around $10^{20}$ cm$^{-3}$,
for instance, would be a hundred times more dilute than a condensed-matter helium superfluid. But creating such a gas is quite impractical – even at an additional factor of a thousand lower density, say $10^{17}$ cm$^{-3}$, metastability times would be on the order of a few ms; more realistic are densities on the order of $10^{14}$ cm$^{-3}$. The low densities mandated by the need to maintain long-lived metastability in turn challenge one’s ability to achieve the necessary low temperatures for BEC. The critical temperature for ideal-gas BEC in a gas with mass $m$ is related to density $n$ by

$$T_{c,\text{ideal}} = \frac{\hbar^2}{2\pi mk} \left( \frac{n}{\zeta(3/2)} \right)^{2/3}$$

where $\zeta(z)$ is the Riemann zeta function and $\zeta(3/2) \approx 2.612$. Realistically, one needs to be able to achieve temperatures in the microkelvin scale or still lower, which is a challenge indeed.

The desire to create a dilute BEC forces one to work in a forbidden, metastable region of the $n-T$ plane. The need to maintain metastability pushes one to still lower densities, and low densities in turn suppress the phase transition temperature to temperatures that were all but unimaginable two decades ago. Thus, making a dilute-gas BEC was an
experimentally difficult thing to do, and the history of the effort is worth reviewing.

2. – History

2.1. Conceptual beginnings. – The notion of Bose statistics dates back to a 1924 paper in which Satyendranath Bose used a statistical argument to derive the black-body radiation spectrum [14]. Albert Einstein extended the statistical model to include systems with conserved particle number [2, 3]. The result was Bose-Einstein statistics. Particles that obey Bose-Einstein statistics are called bosons, and today it is known that all particles with integer spin (and only those particles) are bosons. Einstein immediately noticed a peculiar feature of the distribution: at low temperature, it saturates. “I maintain that, in this case, a number of molecules steadily growing with increasing density goes over in the first quantum state (which has zero kinetic energy) while the remaining molecule separate themselves according to the parameter value \( A = 1 \) [in modern notation, \( \mu = 0 \)] … A separation is effected; one part condenses, the rest remains a ‘saturated ideal gas.’” [2, 4] Thus began the concept of Bose-Einstein condensation.

BEC has not always been a particularly reputable character. In the decade following Einstein’s papers, doubts were cast on the reality of the model [15]. Fritz London and L. Tisza [16, 17] resurrected the idea in the mid 1930s as a possible mechanism underlying superfluidity in liquid helium 4. London’s view was either disbelieved or else felt to be not particularly illuminating. Certainly the influential helium theory papers of the 50s and 60s make little or no mention of BEC. [18, 19, 20]. The authors of the present review are not well-versed in the history of that era, but it is evident that sometime in the intervening years the bulk of expert opinion has shifted. Experiment and theory (neutron scattering [21] and path-integral Monte Carlo simulations [22], respectively) now support the idea that the microscopic physics underlying superfluidity is a zero-momentum BEC. Due to interactions, only about 10% of the helium atoms participate in the condensate, even at temperatures so low that empirically 100% of the liquid appears to be in the super-fluid state.

2.2. Spin-polarized hydrogen. – Efforts to make a dilute BEC in an atomic gas were spurred by provocative papers by Hecht [23] and Stwalley and Nosonov [24]. They argued on the basis of the quantum theory of corresponding states that spin-polarized hydrogen would remain a gas down to zero temperature, and thus would be a great candidate for making an weakly interacting BEC. A number of experimental groups [25, 26, 27, 28] in the late 70s and early 80s began work in the field. Spin-polarized hydrogen was first stabilized by Silvera and Walraven in 1980 [25], and by the the mid-80s spin-polarized hydrogen had been brought within a factor of 50 of condensing [27]. These experiments were performed in a dilution refrigerator, in a cell whose walls were coated with superfluid liquid helium. A radiofrequency (rf) discharge dissociated hydrogen molecules, and a strong magnetic field preserved the polarization of the resulting atoms. Individual hydrogen atoms can thermalize with a superfluid helium surface without becoming depolarized. The atoms were compressed using a (conceptually) simple piston-in-cylinder
arrangement [29], or inside a helium bubble [30]. Eventually the helium surface became problematic, however. If the cell is made relatively cold, the surface density of hydrogen atoms becomes so large that they undergo recombination there. If the cell is too hot, the volume density of hydrogen necessary for BEC becomes so high that, before that density can be reached, the rate of three-body recombination becomes too high [31].

2.3. Laser cooling and the ascendancy of alkalis. – Contemporaneous with (but quite independent from) the hydrogen work, an entirely different kind of cold-atom physics was evolving. The remarkable story of laser cooling has been reviewed elsewhere [32, 33, 34, 35] but we mention some of the highlights in compressed form below. The idea that laser light could be used to cool atoms was suggested in early papers from Wineland and Dehmelt [36], from Hänsch and Schawlow [37], and from Letokhov’s group [38]. Early optical force experiments were performed by Ashkin [39]. Trapped ions were laser-cooled at the University of Washington [40] and at the National Bureau of Standards (now NIST) in Boulder [41]. Atomic beams were deflected and slowed in the early 80s [42, 43, 44]. Optical molasses was first studied at Bell Labs [45]. Measured temperatures in the early molasses experiments were consistent with the so-called Doppler limit, which amounts to about 300 microkelvin in most alkalis. Coherent stimulated forces of light were studied [46, 47]. The dipole force of light was used to confine atoms as well [48]. In 1987 and 1988 there were two major advances that became central features of the method of creating BEC. First, a practical spontaneous-force trap, the Magneto-Optical trap (MOT) was demonstrated [49], and second, it was observed that under certain conditions, the temperatures in optical molasses are in fact much colder than the Doppler limit [50, 51, 52]. These were heady times in the laser-cooling business. With experiment yielding temperatures mysteriously far below what theory would predict, it was clear that we all lived under the authority of a munificent God. The MOT had generated high densities in trapped gases. Perhaps still colder temperatures and still higher densities were due to arrive shortly. Indeed, it seemed reasonable enough, at the time, to speculate that the methods of laser cooling and trapping might soon lead directly to BEC!

2.4. Cold reality: limits on laser cooling. – It didn’t happen that way, of course. By 1990 it was clear that there were fairly strict limits to both the temperature and density obtainable with laser cooling. Theory caught up with experiment and showed that the sub-Doppler temperatures were due to a combination of light-shifts and optical pumping that became known as Sysiphus cooling. Random momentum fluctuations from the rescattered photons limit the ultimate temperature to about a factor of ten above the recoil limit [53]. The rescattered photons are also responsible for a density limit – the light pressure from the reradiated photons gives rise to an effective inter-atom repulsive force [54]. Momentum fluctuations and trap loss arising from light-assisted collisions limited temperature and density as well [55]. The product of the coldness limit and the density limit works out to a phase-space density of about $10^{-5}$, which is five orders of magnitude too low for BEC.

Since 1990, advances in laser cooling and trapping have allowed both the temperature
The density limit to be circumvented. But it seems that in most cases higher densities have been won at the cost of high temperatures, and lower temperatures have been achievable only in relatively low density experiments. The peak phase-space density in laser cooling experiments has increased hardly at all in the decade since 1989 [59].

The alkali-atom work of the 1980s included another development, however, which was to have a large impact on BEC work. Sodium atoms were trapped in purely magnetic traps [60, 61, 62].

2.5. Evaporative cooling and the return of hydrogen. – Harold Hess from the MIT Hydrogen group realized the significance that magnetic trapping had for their BEC effort. Atoms in a magnetic trap have no contact with a physical surface and thus the surface-recombination problems could be circumvented. Moreover, thermally isolated atoms in a magnetic trap were the perfect candidate for evaporative cooling. In a remarkable paper Hess laid out in 1986 most of the important concepts of evaporative cooling of trapped atoms [31]. Let the highest energy atoms escape from the trap, and the mean energy, and thus the temperature, of the remaining atoms will decrease. In a dilute gas in an inhomogeneous potential, decreasing temperature in turn means decreasing occupied volume. One can actually increase the density of the remaining atoms even though the total number of confined atoms decreases. The Cornell University Hydrogen group also considered evaporative cooling [63]. By 1988 the MIT group had implemented these ideas and had demonstrated that the method was as powerful as anticipated. In their best evaporative run, they obtained, at a temperature of 100 $\mu$K, a density only a factor of five too low for BEC [64]. Further progress was limited by dipolar relaxation, but perhaps more fundamentally by loss of signal-to-noise, and the need for a more accurate means of characterizing temperature and density in the coldest clouds [65]. Evaporative work was also performed by the Amsterdam group [66].

Both the MIT and the Amsterdam group began constructing laser sources capable of accessing optical transitions in atomic hydrogen, reasoning that with the powerful tool of laser spectroscopy they would be better positioned to understand and surpass the low temperature limits [66, 67, 68]. The Amsterdam group performed some laser cooling on atomic hydrogen [69, 70].

The hydrogen evaporation results made a big impression on the JILA alkali group. It seemed to us that a hybrid approach combining laser cooling and evaporation had an excellent chance of working. Evaporation from a magnetic trap seemed like a very appealing way to circumvent the limits of laser cooling. Laser cooling could serve as a pre-cooling technology, replacing the dilution refrigerator of the hydrogen work. With convenient lasers in the near-IR, and with the good optical access of a room-temperature glass cell, detection sensitivity could approach single-atom capability. The elastic cross-section, and thus the rate of evaporation, should almost certainly be larger in an alkali atom than it is in hydrogen. Encouraged by these thoughts (and by other collisional considerations, see sections 2.7 and 8.1 (below) the JILA group set out (See Ref. [71] and a presentation at the 1991 Varenna summer school [72]) to combine the best ideas
of alkali and of hydrogen experiments, in an attempt to see BEC in an alkali gas.

2.6. Hybridizing MOT and evaporative cooling techniques. – In a sense efforts to hybridize optical cooling with magnetic trapping are as old as atomic magnetic trapping itself. The original NIST sodium magnetic trap [61] and the first Ioffe-Pritchard (IP) trap [62] were loaded from Zeeman-tuned optical beam-slowers. Modern alkali BEC apparatuses, however, can trace their conceptual roots through a series of devices built at JILA during an era beginning in late 1989 and continuing into the early 90s [71, 72, 73, 74, 75, 76, 77]. As things stood at the end of the 1980s, optical cooling on the one hand and magnetic trapping on the other were both somewhat heroic experiments, to be undertaken only by advanced and well-equipped AMO laboratories. The prospect of trying to get both working, and working well, on the same bench and on the same day was daunting.

The JILA vapor-cell MOT, with its superimposed IP trap (figure 2), represented a much-needed technological simplification and introduced a number of ideas that are now
in common use in the hybrid trapping business: [71, 72]: i) Vapor-cell (rather than beam) loading; ii) fused-glass rather than welded-steel architecture; iii) extensive use of diode lasers; iv) magnetic coils located outside the chamber; v) over-all chamber volume measured in cubic centimeters rather than liters; vi) temperatures measured by imaging an expanded cloud; vii) magnetic-field curvatures calibrated in situ by observing the frequency of dipole and quadrupole (sloshing and pulsing) cloud motion; viii) the basic approach of a MOT and a magnetic trap which are spatially superimposed (indeed, which often share some magnetic coils) but temporally sequential; and ix) optional use of additional molasses and optical pumping sequences inserted in time between the MOT and magnetic trapping stages. In the early experiments [71, 73, 72, 74] a number of experimental issues came up that continue to confront all BEC experiments: the importance of aligning the centers of the MOT and the magnetic trap; the density-reducing effects of mode-mismatch; the need to account carefully for the (previously ignored) force of gravity; heating (and not merely loss) from background gas collisions; the usefulness of being able to turn off the magnetic fields rapidly; the need to synchronize many changes in laser status and magnetic fields together with image acquisition. At the time the design was quite novel, but by now it is almost standard. It is instructive to note how much a modern, IP-based BEC device (figure 3) resembles its ancestor (figure 2).

2.7. Collisional concerns. – From the very beginning, dilute-gas BEC experiments had to confront the topic of cold collisions. Even before evaporation was considered, when cooling was simply a matter of conduction to the chamber walls, the rate of three-body decay determined the lifetime of samples of spin-polarized hydrogen samples. With the advent of evaporation, there was a demand for understanding several different collisional processes – elastic collisions, dipolar relaxation, three-body recombination, and (to a lesser extent) spin-exchange.

Atomic collisions at very cold temperatures is now a major branch of the discipline of AMO physics, but in the 1980s there was almost no experimental data, and what there was came in fact from the spin-polarized hydrogen experiments [80]. There was theoretical work on hydrogen from Shlyapnikov and Kagan [81, 82], and from Silvera and Verhaar [83]. An early paper by Pritchard [84] includes estimates on low-temperature collisional properties for alkalis. His estimates were extrapolations from room-temperature results, but in retrospect several were surprisingly accurate.

Experiments on cold collisions between alkali atoms were initially all performed on light-induced collisions in molasses and MOTs [85, 86, 87]. The earliest ultra-cold ground-state on ground-state measurements were based on pressure-shifts in the cesium clock transition [88] and thermalization rates in magnetically trapped Cs [74], Rb [77] and Na [89]. S-wave collisions were observed directly in cesium [90]. Eventually experiment and theory of excited state collisions, in particular photoassociative events, yielded so much information on atom-atom potentials that ground-state cross-sections could be extracted [91, 92, 93, 94, 95, 96]. By the time BEC was observed in rubidium, sodium, and lithium, there existed, at least at the 20% level, reasonable estimates for the respective elastic scattering length of each species. Evaporation efforts on all three species were initially
begun, however, under conditions of relatively large uncertainty concerning elastic rates, and near-ignorance on inelastic rates.

In 1992 the JILA group came to realize that dipolar relaxation in alkalis should in principle not be a limiting factor. As explained in section 8.1 below, collisional scaling with temperature and magnetic field is such that, except in pathological situations, the problem of good and bad collisions in the evaporative cooling of alkalis is reduced to the ratio of the elastic collision rate to the rate of loss due to imperfect vacuum; dipolar relaxation and three-body recombination can be finessed. It was reassuring to move ahead
on efforts to evaporate with the knowledge that, while we were essentially proceeding in
the dark, there were not as many monsters in the dark as we had originally imagined. Of
course, even given successful evaporative cooling, there was still the question of the sign
of scattering length, which must be positive to ensure the stability of a large condensate.
The JILA group had sufficient laser equipment, however, to trap either $^{85}\text{Rb}$, $^{87}\text{Rb}$, or
$^{133}\text{Cs}$. Given the “modulo” arithmetic that goes into determining a scattering length, it
seemed fair to treat the scattering lengths of the three species as statistically independent
events, and the chances then of Nature conspiring to make all three negative were too
small to worry about [97].

2.8. Increasing elastic collisions per background loss. – A simple MOT, with operating
parameters adjusted to maximize collection rate, will confine atoms at a density limited
by reradiated photon pressure to a relatively low value [54], with correspondingly low
collision rate after transfer to the magnetic trap.

2.8.1. Multiple Loading. Early efforts to surmount the density limit included a multiple-
loading scheme pursued at JILA [73]. Multiple MOT-loads of atoms were launched in
moving molasses, optically pumped into an untrapped Zeeman level, focused into a mag-
netic trap, then optically repumped into a trapped level. The repumping represented the
necessary dissipation, so that multiple loads of atoms could be inserted in a continuously
operating magnetic trap. In practice, each step of the process involved some losses and
the final result hardly justified all the additional complexity. It is interesting to note,
however, that multiple transfers from MOT to MOT [90, 76], rather than from MOT to
magnetic trap, is a technique currently in widespread practice.

2.8.2. Adiabatic compression. Also in wide, current use is the practice of magnetic
compression. After loading into a magnetic trap, the atoms can be compressed by further
increasing the curvature of the confining magnetic fields. This is a technique which is
available to experiments which use MOTs to load the magnetic trap, but not to experi-
ments which use cryogenic loading (in the latter, the magnetic fields are usually already
at their maximum values during loading, in order to maximize capture.) This method is
discussed in [72] and was implemented first in early ground-state collisional work [74].

2.8.3. Enhancing MOT density. An important advance came from the MIT Sodium
group in 1992, when they developed the Dark-spot MOT [58]. A shadow is arranged in
the repumping light, such that atoms that have already been captured and concentrated
in the middle of the trap are pumped to an internal state that is relatively unperturbed
by close neighbors. In effect, the MOT is divided into two spatial regions, one for
efficient collection, and one for efficient compression and storage. A related technique is
to modulate the MOT parameters temporally, rather than spatially. This approach has
been dubbed CMOT (Compressed MOT) [98]. A very thorough study of related MOT
behavior came out of a British-French collaboration [99].

2.8.4. Reducing background loss. Since the most important figure of merit in the
evaporation business in the collision rate compared to the trap lifetime, one can do
almost as much good improving vacuum as one can improving MOT density. Early machines for cooling atomic beams were relatively dirty, by the standards of modern UHV practice. Except for the Pritchard group’s cryogenic trap [62], confinement times in optical trapping experiments were usually on the order of two or three seconds. Improved atomic beam design [100, 101] and the adoption of modern UHV practice has made 100 s lifetimes standard. An experiment at JILA saw a MOT lifetime of 1000 s [75]. Cryogenic MOTs have had lifetimes of greater than 3600 s [102].

2.9. Forced evaporation. – Cooling by evaporation is a process found throughout nature. Whether the material being cooled is an atomic nuclei or the Atlantic ocean, the rate of natural evaporation, and the minimum temperature achievable, are limited by the particular fixed value of the work-function of the evaporating substance. In magnetically confined atoms, no such limit exists, because the “work function” is simply the height of the lowest point in the rim of the confining potential. Hess pointed out [31] that by perturbing the confining magnetic fields, the work-function of a trap can be made arbitrarily low; as long as favorable collisional conditions persist, there is no lower limit to the temperatures attainable in this forced evaporation. Pritchard [103] pointed out that evaporation could be performed more conveniently if the rim of the trap were defined by an rf-resonance condition, rather than simply by the topography of the magnetic field; experimentally, his group made first use of position-dependent rf transitions to selectively transfer magnetically trapped sodium atoms between Zeeman levels and thus characterized their temperature [104]. From 1993-1995, a number of other experimental groups launched their own efforts to create BEC in alkali species using the hybrid laser-cooling/evaporation approach. Progress accelerated and by summer of 1994 three groups had announced the successful use of evaporation to increase the phase-space density of trapped alkalis [105, 106, 89]. By the DAMOP meeting in Toronto, in May of 1995, the number of groups with clear evidence of evaporative cooling had increased to four [107]. The Rice, JILA, and MIT alkali groups were all seeing significant increases in phase-space density. The JILA alkali group returned from Toronto with a shared impression that “there is no time left to fiddle around” [108].

2.10. Magnetic trap improvements. – Each of the groups reporting significant evaporation at Toronto, had recently implemented a major upgrade in magnetic trap technology. In a harmonic trap, the collision rate after adiabatic compression scales as the final confining frequency squared [72]. Clearly, there was much to be gained by building a more tightly confining magnetic trap, but the requirement of adequate optical access for the MOT, along with engineering constraints on power dissipation, make the design problem complicated. When constructing a trap for weak-field seeking atoms, with the aim of confining the atoms to a spatial size much smaller than the size of the magnets, one would like to use linear gradients. In that case however one is confronted with the problem of the minimum in the magnitude of the magnetic fields (and thus of the confining potential) occurring at a local zero in the magnetic field. This zero represents a “hole” in the trap, a site at which atoms can undergo Majorana transitions [109] and thus escape
from the trap. If one uses the second-order gradients from the magnets to provide the confinement, there is a marked loss of confinement strength. This scaling is discussed in reference [105]. The three experimental groups each solved the problem differently. The Rice group built a trap with permanent magnets [101] which can be stronger than electromagnets. The MIT and JILA groups each built traps with linear gradients, and then “plugged the hole,” in the MIT case with a beam of blue-detuned (repulsive) light [89], and in the JILA case with a time-dependent dither field [105]. As it turned out, all three methods worked well enough. In Spring of 1995 (less than three weeks after the DAMOP meeting), the JILA group saw BEC in rubidium 87, and by Autumn of 1995 all three groups had either observed BEC or else had preliminary evidence for it [110, 111, 112].

If one is willing to accept tight confinement in two directions only, it turns out that the original IP design, operated with a very low bias field, is more than adequate to produce BECs. In 1996, BEC was observed in second-generation traps at both MIT [113] and JILA [78]. The fields were in the IP configuration; additional coils were used to nearly null the central magnetic field.

2.11. Imaging techniques. – The fact that a wealth of quantitative information can be extracted from simple spatial images of the atom cloud is one of the features that most distinguishes BEC in dilute atomic gas from other BEC experiments, such as excitons and liquid helium. It is worth mentioning then some issues in imaging trapped atoms. Early imaging of magnetically trapped atoms was done by collecting fluorescence [71]. While acceptable for large, dilute clouds, fluorescence imaging does not work well when the clouds get small and dense because of optical thickness and motional blurring. Absorption imaging minimizes motional blurring because the mean spontaneous force from the absorbed light is directly along the line of sight (assuming good optical alignment.) The presence of magnetic fields in the trap can make absorption images more difficult to interpret, although the MIT Sodium group has shown that with careful modeling even images taken of atoms in full-strength quadrupole magnetic fields can be correctly understood [89]. The more fundamental problem is that as the clouds get colder, they get both smaller and denser. Two issues arise. First, the cloud can become smaller than the resolution limit of the optics, which makes it easy to misinterpret images [112, 114]. Second, very dense clouds can give rise to “lensing,” in which the real part of the index of refraction becomes as important as the absorption. In the early experiments, the solution was to turn off the fields suddenly and let the cloud expand before imaging. The expansion decreases the optical thickness and increases the effective resolution. For the original JILA observation, it was necessary to magnetically support the atoms against gravity in order to allow the atoms to expand sufficiently without dropping out of the imaging field-of-view. More recently in situ imaging techniques have been demonstrated to be accurate (see Section 3.6 below).
3. – Survey of BEC technology, present and future

The list of groups currently observing BEC, and the basic technology each uses (Table I) will doubtless be outdated by the time this article appears in print, but it is interesting to note the variations on a theme. There are many obvious generalizations to make, but most of them have exceptions. In this section we survey the range of existing technology and speculate on what the future may bring.

3.1. Magnetic (and other) Traps. – 1. As a rule, these are of the IP configuration. But TOP traps continue to be built, and the IP traps have yet to zero-in on a standard configuration. As of this writing, the eleven groups producing BEC with IP fields are using nine different coil geometries. Still more exotic trap designs have been proposed [115]. It seems likely that over the next few years there will be something of a shake-out in coil design, with a small subset of today’s designs being found to be adequate to cover the needs of most experiments.

2. As a rule, the coils are water-cooled. But the MIT hydrogen trappers use superconducting coils, and the Orsay group uses so little power that they can air-cool.

3. It seems likely that evaporative cooling to BEC can eventually be accomplished by purely optical means, either in FORTs [106], or their near-dc cousins, the QUESTs [116, 117]. Already, the MIT group has confined condensates in purely optical traps [118], although the clouds are still precooled in magnetic traps. Still more exotic traps have been proposed, including boxes fashioned from blue-detuned evanescent optical waves or from planes of tiny magnetic domains [119].

3.2. Atomic species. – A majority of groups are currently working in rubidium 87, with most of the remainder using sodium 23. But the Rice group has had success in lithium 7 and the hydrogen trappers recently have seen condensation [127]. There is considerable experimental effort around the world on cesium [132, 133, 134, 135, 136], rubidium 85 [79], and potassium 39 and 41 [137, 138] and on the fermionic isotopes lithium 6 [139] and potassium 40 [140, 141]. To the best of our knowledge, all sodium condensates to date have been formed in the $F = 1$ state. There seems to be some suggestion that the collisional properties of the $F = 2, m_F = 2$ state are unfavorable for condensation. With the recent successes of buffer-gas loading of molecules and non-alkali atoms [142, 143, 144] it seems likely that almost any atomic species with a ground-state magnetic moment, and a great many molecular species, can be magnetically trapped. The success of sympathetic cooling [78] lifts most of the requirements on collisional properties. A species with recalcitrant collisional properties can be trapped together with, and cooled sympathetically by, a “good evaporating” species such as $^{87}$Rb; sympathetic cooling requires a far smaller good/bad collision ratio to succeed. Thus the number of different atomic and molecular species that could eventually be cooled to the BEC transition may be in the hundreds.

3.3. Precooling and compression. – Evaporative cooling can work only if the sample is initially cold and dense enough to permit tight confinement and large collision rates. By
far the most popular approach is to collect, cool, and compress atoms in a MOT [49], then rapidly turn off the MOT and turn on the fields of the magnetic trap immediately around it [71]. The density of the MOT is often enhanced by a darkspot [58], or by modulating the detuning and field gradients of the MOT just before transfer to the magnetic trap [98]. Sometimes a stage of optical cooling is included after the MOT magnetic fields have been turned off and before the magnetic trap has been turned on [71], but for traps with large number, this often provides only marginal benefit. The MOT is a useful but, as it has turned out, hardly an essential tool. The Rice group, which cannot rapidly reconfigure their magnetic fields from MOT to IP trap, uses a beam-loaded molasses and dispenses with a MOT altogether. The hydrogen trapping groups have never needed molasses cooling of any kind to prepare atoms for magnetic trapping and evaporative cooling – they rely on thermalization with a dilution-refrigerated He-II surface. In the future, buffer-gas loading techniques [142] may rival MOTs as the primary precooling method. Optical cooling may also continue even after the atoms are in their final evaporation

| Place                        | Atom | Collector/Cooler       | Magnetic Trap                  |
|------------------------------|------|------------------------|--------------------------------|
| JILA [110]                   | $^{87}$Rb | MOT/MOT                | i) TOP                         |
|                              |      |                        | ii) IP (baseball)              |
| MIT [111]                    | $^{23}$Na | Beam/MOT              | IP (cloverleaf)                |
|                              |      |                        | (hybrid optical)               |
| Rice [112]                   | $^7$Li | Beam/Molasses          | IP (permanent magnet)          |
| Rowland Institute [120]      | $^{23}$Na | Beam/MOT              | IP (four-dee)                  |
| Yale (was Stanford) [121]    | $^{87}$Rb | Vapor/MOT             | TOP                            |
| UTexas [122]                 | $^{87}$Rb | Beam/MOT            | TOP                            |
| Konstanz [123]               | $^{87}$Rb | MOT/MOT             | IP (bars)                      |
| München [124]                | $^{87}$Rb | MOT/MOT             | IP (3 coils)                   |
| NIST, Gaithersburg [125]     | $^{23}$Na | Beam/MOT             | TOP (transverse)               |
| Paris [126]                  | $^{87}$Rb | MOT/MOT             | IP (3 coils)                   |
| MIT [127]                    | H    | Dilution Refrigerator/He-II Surface | IP (Superconducting) |
| Orsay [128]                  | $^{87}$Rb | Beam/MOT             | IP (Pole-Piece)                |
| Hannover [129]               | $^{87}$Rb | Beam/MOT             | IP (Cloverleaf)                |
| Otago [130]                  | $^{87}$Rb | MOT/MOT             | TOP                            |
| Sussex [131]                 | $^{87}$Rb | MOT/MOT             | IP (Baseball)                  |

Table I. – Current BEC technology at a glance — Where, What and How
trap. Optical cooling has been shown to work in magnetic traps \([145, 77]\) and in optical dipole traps \([146, 147]\).

3.4. MOT loading. – Among the experiments that do use MOTs, there are variants in the methods for loading them. The simplest approach is to collect atoms in the MOT directly from background thermal vapor \([71, 110]\), but one encounters a dilemma – Should one operate at high vapor pressure, so as to fill the MOT quickly with large numbers of atoms, but then suffer rapid loss during evaporation, or should one operate at low vapor pressure, so as to have longer time available for evaporation, but then live with slow, meager TOP trap fills? The original JILA apparatus found a workable range of compromise pressures, with the help of a darkspot used to suppress collisional loss from the MOT during the 200-second fill-time \([75]\). The desire for larger condensates and faster repetition rates has prompted most groups to abandon the simplicity of a single vapor cell in favor of more elaborate loading schemes. Roughly half of the groups fill their MOTs with Zeeman-slowed beams \([43]\); most of the remainder use a double-MOT scheme – a “dirty” MOT in a relatively high pressure region cools atoms which are transferred to a “science MOT” at UHV \([90, 76, 148]\). In the future, LVIS \([149]\) or funnel-type beam generators \([150]\) may replace the Zeeman slowed beams. A range of other possibilities exist: one could devise a way to rapidly modulate the vapor pressure of the desired species, initially high for MOT collection, then low for evaporation. Alternatively, it has been shown also to be possible to spatially separate the MOT from the magnetic trap, and accumulate many MOT loads in a single magnetic trap \([73]\); perhaps such a technique could eventually find use. In yet another direction, the group at JILA is investigating mechanical means for moving atoms from high-density collection regions to low-density evaporation regions.

3.5. Chamber Design. – As a rule, the chambers tend to be small glass cells with magnetic coils wound outside of the vacuum, but the more traditional conflat-based stainless-steel cans with vacuum-compatible coils inside are in use as well, for instance at the Rowland Institute and at the University of Texas. Most likely stainless-steel chambers will continue to find use in certain applications – in cryogenic systems, for instance, or in experiments for which the condensate is to be deposited on or scattered off of samples. Such experiments may benefit from the greater flexibility of reconfigurable steel chambers.

3.6. Imaging technique. – Imaging the cloud after ballistic expansion has the advantage of being relatively easy to do, but the technique has its drawbacks. Some traps are difficult to turn off rapidly enough to ensure that no unintentional (and uncharacterized) work is done on the expanding gas. Also, although in most cases the expansion process is relatively easy to model \([151, 152]\), it is an additional and sometimes unwelcome link in the chain of inference that connects an image with the underlying physical behavior. Finally, post-expansion imaging is not consistent with acquiring multiple images of the same cloud. The alternative to post-expansion imaging, imaging \textit{in situ}, has the previously mentioned drawbacks that the cloud can be small compared to the resolution limit, and
that the cloud can be so optically thick as to give rise to lensing systematics. If sufficient care is taken, quantitative information can be extracted from an absorption image even when the object is smaller than the resolution limit [153, 154], or when the object is very optically thick [120]. The problems of optical thickness can be circumvented altogether with imaging methods based on far-detuned probe beams sensitive solely to the real part of index of refraction: Polarization rotation [153], and dark-field and phase-contrast imaging [155].

For work requiring very high spatial resolution, or high signal-to-noise, expansion followed by absorption imaging will continue to be a useful method. For work in which the (relatively thin) thermal component is of primary interest, in situ absorption imaging will probably be preferable. For studies of time-dependent behavior (as long as they do not require very high spatial resolution) in situ phase-contrast methods are probably best.

4. – Effects of interactions

As discussed in Section 1.3 above, the topic of Bose-Einstein condensation develops its full richness only after interactions are included. Most recent discussions of interactions in a $T=0$ condensate begin with the Gross-Pitaevski (GP) equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = \left( -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{ext}} + \frac{4\pi \hbar^2 a}{m} |\Psi|^2 \right) \Psi$$

where $a$ is the 2-body scattering length, $V_{\text{ext}}$ is the external potential of the trap and $\Psi$ is the order parameter. In a dilute gas at zero temperature, the density at any point is space and time is just $|\Psi|^2$. For an instructive comparison of several quite distinct routes to arrive at this “starting point” see [156]. For recent surveys of the status of the theory in this area, see [157, 158, 159, 160, 161]. We will review here the experimental studies to date on interactions. All published experiments on BEC to date have shown signs of these interactions, and these experiments have tested the theoretical predictions to a greater or lesser degree. For convenience we will discuss excitations in a separate section.

4'1. Energy and size. – In an ideal gas, the statistical mechanics of Bose condensation ensures that the majority of the atoms will accumulate in the lowest energy state of the confining potential. In particular, in a harmonic trap, the atoms accumulate in the $(0,0,0)$ state of the 3-D harmonic oscillator, with a zero-point energy given by $(\hbar/2)(\omega_x + \omega_y + \omega_z)$ per atom. The onset of inter-particle interactions does not change the basic behavior of BEC: atoms still accumulate in the lowest energy state. The difference is only that the spatial extent, and total energy, of the interacting ground-state is larger than its ideal gas predecessor.

The energy of the condensate is proportional to the square of the size of its time-of-flight image. Interactions perturbed even the earliest images of 2000-atom condensates
Fig. 4. – Comparison of the measured condensate energy (○) to the prediction from mean-field theory (solid line) as a function of interaction strength. By non-adiabatically releasing the condensate from the magnetic trap, the kinetic and interaction energies of the trapped atoms become the kinetic energy of the released and expanding atom cloud. The kinetic energy of expansion is obtained from a sequence of cloud widths measured at different expansion times. The inset shows experimental widths in the horizontal (○) and vertical (×), compared to the mean-field predictions (dashed and solid lines), for the data point at $10^{-4} N \nu^{1/2} = 0.53 \text{Hz}^{1/2}$. Used with permission of M. Holland from ref. [164].
Quantitative checks on the in situ size of the interacting condensate have been performed at MIT [165] and the Rowland institute [120], and are in good agreement with theory.

In addition to total energy, and in situ size of the condensate, eq. (5) also yields predictions on the aspect ratio of the expanded cloud and on the detailed shape of the condensate density profile, in the trap or expanded, (in the TF limit, the density profile is an inverted paraboloid). See [157] for a review of this theory. In one high-precision measurement, the aspect ratio of an expanded cloud was measured to be in agreement with theory at the 2% level [166]. In the presence of noise and various image distortions, it can be difficult to quantify how well a “shape” agrees with theory, but no zero-$T$ measurements we are aware of [152] show any major discrepancies with the basic theory.

4.2. Two-species condensates. – In thinking about experiments in condensate mixtures, it is easiest to imagine mixtures of two different elements, rubidium and sodium, for instance [167]. As it turns out, experiments with two-species condensates have to date mainly been performed in mixtures of two hyperfine states of $^{87}$Rb [78, 168, 169, 170, 171]. The two-component rubidium work has been recently reviewed [170], so we will not discuss it extensively here. A range of two-fluid behavior, including mutual repulsion, component separation, relative-center-of-mass oscillation and damping, and residual steady-state overlap have been observed. Qualitatively, the mutual interaction behavior can be accounted for in the framework of two, coupled, GP equations [172, 173, 167, 174]. Quantitatively, this model has not been tested to any degree of accuracy.

It is worth noting that the rf output-coupler experiment of MIT [175] was also sensitive to interactions between different spin-states of the same atomic species. The crescent shape of the out-coupled pulse of atoms arose from the repulsion between the out-coupled $F = 1, m_F = 0$ atoms, and the still-trapped $F = 1, m_F = -1$ atoms [176].

Experiments have also been recently performed in a multiple-component system in which spontaneous interconversion of components is an important process in the system’s dynamics. These are the $F = 1$ spinor condensates of the MIT sodium group. This work is reviewed in another article in this volume [177].

4.3. Negative scattering length. – Bose condensates composed of atoms with negative scattering lengths pose a particular challenge to theory. In a homogenous system, such a condensate can not exist, because it is unstable to small density perturbations — for wavelengths larger than a critical value, excitation frequencies become negative. However, if a condensate is confined in a potential such that its size is smaller than the critical wavelength, it is stable (or at least metastable) to density fluctuations. Given a particular atomic species and a particular confining potential, there is a critical number of condensate atoms. If the condensate grows beyond that number, it will implode. Currently the only published experimental data on this phenomenon are from the Rice Lithium group [153]. At present they do not have the stability or sensitivity to watch a single condensate approach the critical value and then collapse. Rather, they collect many images and show that they never observe condensates larger than a certain size, a
size consistent with the predicted “implosion limit.”

In future work, improved stability and statistical analysis of observed cloud sizes may shed more light on the issue of metastability [178]. Alternatively, experiments utilizing atomic Feshbach resonances may be able to study implosion dynamics more directly.

4.4. Finite temperature. – The theory of Bose condensation at nonzero temperature [159, 160, 161] is a more subtle topic than $T = 0$ BEC. After all, at $T = 0$ the phenomenon of “Bose condensation,” in the sense of a large fraction of the particles occupying the lowest available orbital, would occur even in Maxwell-Boltzmann statistics [179]. In the decades following the original Einstein papers, it was even believed the phase transition predicted in 1925 [2] would not survive a proper treatment of interactions at finite temperature [15, 179]. Experimentally, we now know that indeed the phase transition does occur [180] at close to the (non-zero) temperature predicted in [2], and that the associated coherent behavior [181, 148, 182] survives the effects of interactions and finite temperature. Beyond that, there has been relatively few critical experimental tests of finite-$T$ theory. There are several published experimental studies on the effect of temperature on excitations, but these data, as discussed in section 5.4 below, are taken in a regime for which theoretical modeling is particularly difficult. There are however solid theoretical predictions for the effects of finite-$T$ interactions on several static and thermodynamic properties:

4.4.1. Critical temperature. Interactions should affect the temperature at which the condensate first appears in two different ways. First, the mean-field repulsion arising from the atoms concentrated at the lowest point in the confining potential will reduce the density there; the net effect is to reduce the critical temperature relative to what it would be for the same number of non-interacting atoms in the same external potential. See for instance [183]. Second, many-body effects among the confined particles, effects which can be described as a break-down of the dilute limit, are predicted to modestly increase the critical temperature [184, 185]. So far there has been no decisive experimental observation of either effect. The most precise measurement of $T_c$ to date [180] quoted an error of about 5%, which was comparable to the expected effect of interactions. Improvements in the accuracy of measurements of $N$, $N_0$, and $T$ will probably allow an unambiguous confirmation that interactions affect $T_c$, but a quantitative sorting out of the relative effects of mean-field and many body interactions will be an enormous experimental challenge [186, 187].

4.4.2. Energy content. Interactions will also affect the specific heat of a confined Bose gas, or equivalently, the total energy content, which is the temperature integral of the specific heat [190, 191, 192]. There has been an experimental study of the energy content as a function of temperature, based on analysis of images expanded clouds [180]. The data (figure 5) clearly show a deviation from ideal gas behavior, which can be accounted for by finite-$T$ theory (figure 6). The experimental procedure is probably not sufficiently accurate to provide a definitive and quantitative test of the theory.
Fig. 5. – The measured scaled energy per particle of a trapped Bose gas is plotted vs. the temperature scaled by the critical temperature, $T_0$, for a non-interacting (ideal) gas of bosons in a 3-D harmonic oscillator. The straight, solid line is the energy for a classical, ideal gas, and the dashed line is the predicted energy for a finite number of ideal bosons [188, 189]. The solid, curved lines are separate polynomial fits to the data above and below the empirical transition temperature of $0.94T_0$. The measured energy is actually the kinetic and interaction energies of the trapped atoms and is measured by suddenly releasing the atoms from the magnetic trap and calculating the second-moment of the velocity distribution of the released atoms. (inset) The difference $\Delta$ between the data and the classical energy emphasizes the change in slope of the measured energy-temperature curve near $0.94T_0$ (vertical dashed line). Figure taken from ref. [180].

4.4.3. Density distributions. Interactions of the non-condensed fraction with the condensed fraction, and interactions of the non-condensed fraction with itself, should modify the spatial distribution of the non-condensed fraction, particularly just at the edge of the condensate cloud. There exist some carefully acquired images of this region [193] but a theoretical study of the sensitivity of these images to alternate models of finite-$T$. 
interactions has not been undertaken.

4'5. Future directions. – There are a number of exciting experimental paths to explore.

4'5.1. Finite $T$. As discussed just above, our understanding of finite temperature interactions would benefit from quantitative measurements taken in a regime that facilitates comparison with theory.

4'5.2. Feshbach resonances. Some years after their significance for BEC was first pointed out [194, 195], Feshbach resonances have been observed now in sodium [196] and in rubidium [197, 79]. The interactions between atoms is now in principle an experimentally tunable quantity. Whether the Feshbach resonances will in the end be useful remains an open question. In sodium at least, there appears to be an enormous increase
in the inelastic scattering rate associated with the Feshbach resonance [196]. If this turns out to be a general feature of Feshbach resonances, it will make tuned-interaction experiments much less convenient.

4.5.3. Vortices. Vortices are the heart and soul of superfluidity. Experimental studies of vortices will be critical to our understanding of superfluidity in dilute-gas BEC.

4.5.4. Many-body effects. Producing experimental evidence for interactions beyond the mean-field in dilute-gas BEC will be difficult, but quite worthwhile.

4.6. A perspective. – Equation (5) above looks superficially similar to the Landau-Ginzburg equation one finds throughout condensed matter theory. It is worthwhile to reflect, however, on the important differences. The coefficient ‘a’ in the nonlinear term is not a temperature-dependent quantity, nor is it a factor empirically determined from observed fluid behavior. Rather, ‘a’ is the scattering length for a two-body collision in free space. For the species which have to date been made to condense, the value of a has been determined from spectroscopic studies of pair-wise collisions, entirely independently from the measurements of collective behavior discussed in this paper. Equation (5), above, and for that matters Eqns. (1)-(4), make quantitative predictions for Bose condensation behavior with no adjustable parameters.

The ability to predict macroscopic quantities such as critical temperatures, healing lengths, specific heats, etc., from calculations based on independently measured microscopic interactions, with no adjustable parameters, is a rare occurrence in condensed matter physics. Power-law behavior can be extracted from renormalization group calculations, but the actual values are almost always determined empirically. It is common for modern condensed matter physicists to dismiss the actual value of a critical temperature, or the actual length of a particular correlation function, as “uninteresting,” when perhaps what they really mean is “too difficult.” In any case, we find very appealing the idea that someone will soon be able to begin with the elementary two-body collision potential and proceed systematically through to a quantitative prediction for the temperature-dependent time constant of the decay of the spiral motion of a vortex core. The scenario is all the more pleasing because within a year before or after the prediction is published, an experimenter will very likely measure that same quantity.

5. – Excitations

In this section, by “excitations” we mean “coherent fluctuations in the density distribution.” Excitation experiments in dilute-gas BEC have been motivated by two main considerations. First, BEC is expected to be a superfluid, and a superfluid is defined by its dynamical behavior. Studying excitations is an obvious first step towards understanding dynamical behavior. Second, in experimental physics a precision measurement is almost always a frequency measurement, and the easiest way to study an effect with precision is to find an observable frequency which is sensitive to that effect. In the case of dilute-gas BEC, the observed frequency of standing-wave excitations in a condensate
is a precise test of our understanding of the effect of interactions. In this section we will review work to date on excitations, concentrating almost exclusively on the experimental side.

5.1. Probing excitations. – BEC excitations were first observed in expanded clouds [198]. The clouds were coherently excited (see below), then allowed to evolve in the trap for some particular dwell time, and then rapidly expanded and imaged via absorption imaging. By repeating the procedure many times with varying dwell times, the time-evolution of the condensate density profile can be mapped out. From that data, frequencies and damping rates can be extracted. In axially symmetric traps, excitations can be characterized by their projection of angular momentum on the axis. The perturbation on the density distribution caused by the excitation of lowest-lying $m = 0$ and $m = 2$ modes can be characterized as simple oscillations in the condensate’s linear dimensions. Figure 7 shows the widths of an oscillating condensate as a function of dwell-time.

The use of in situ imaging is particularly useful in studying condensate excitations. In a single measurement, many observations of the width can be made. Simultaneous calibration of the trap frequency can be extracted from observing any residual dipolar “slosh” of the condensate (this mode is insensitive to interactions) superimposed on the driven oscillation. Ketterle’s group has developed this technique to the point where that a frequency can be determined to much better than 1% in a single shot [199].

Higher-order modes do not result in oscillations of the overall condensate width but can be observed directly as density waves propagating through the condensate [165].

5.2. Driving the excitations. – A frequency-selective method for driving the excitations is to modulate the trapping potential at the frequency of the excitation to be excited [198]. Experimentally this is accomplished by summing a small ac component onto the current in the trapping magnets. In a TOP trap, it is convenient enough to independently modulate the three second-order terms in the transverse potential. By controlling the relative phase of these modulations, one can impose $m = 0, m = 2$ or $m = −2$ symmetry on the excitation drive. The frequency selectivity of this method can in instances be a disadvantage — one can miss what one is not looking for. A still simpler technique for driving the oscillations is to impose a single “step-function” on the confining potential [200]. This approach provides an inherently very broadband excitation, which has its own advantages and disadvantages. Condensates can be manipulated in lossless ways by a focused beam of far-detuned blue light [111]. This technology has been exploited to excite high-order modes in condensate [165].

The techniques mentioned above all involve exciting the condensate by manipulating the external potential. Much the same ends can be achieved by manipulating the internal interactions of the condensate. One would ideally like to modulate the condensate interaction strength arbitrarily and at will, for instance by the use of Feshbach resonances. The JILA group was able to drive excitations in the condensate by discontinuously transferring the atoms from one internal state to another [166]. The different internal states
Fig. 7. – Zero temperature excitation data from [198]. A weak $m = 0$ modulation of the magnetic trapping potential is applied to a $N \approx 4500$ condensate in a 132 Hz (radial) trap. Afterward, the freely evolving response of the condensate shows radial oscillations. Also observed is a sympathetic response of the axial width, approximately 180° out of phase. The frequency of the excitation is determined from a sine wave fit to the freely oscillating cloud widths.

had different interaction strengths and the discontinuous change in mean-field energy resulted in the simultaneous excitation of two $m = 0$ modes. See figure 8.

5.3. *Connection to theory.* – There have been a very large number of theory papers published on excitations; much of this work is reviewed in [157]. All the zero-temperature, small amplitude excitation experiments published to date have been very successfully modeled theoretically. Quantitative agreement has been by and large very good; small discrepancies can be accounted for by assuming reasonable experimental imperfections with respect to the $T = 0$ and small-amplitude requirements of theory.
5.4. finite T. – The excitation measurements discussed above have also been performed at non-zero temperature [201, 199]. The frequency of the condensate excitations are observed to depend on the temperature, and the damping rates show a strong temperature dependence. This work is important because it represents the only observations so far performed that bear on the finite-temperature physics of interacting condensates. Connection with theory remains somewhat tentative. The damping rates, which are observed to be roughly linear in temperature, have been explained in the context of Landau damping [202, 203]. The frequency shifts are difficult to understand, in large part because the data so far have been collected in an awkward, intermediate regime: the cloud of noncondensate atoms is neither so thin as to have completely negligible effect on the condensate, nor so thick as to be deeply in the “hydrodynamic (HD) regime.” In this context, “hydrodynamic regime” means that the classical mean free path in the thermal cloud is much shorter than any of its physical dimensions. In the opposite limit, the “collisionless regime,” there are conceptual difficulties with describing the observed density fluctuations as “collective modes.” The MIT Sodium group has published data [199] on mixed-cloud excitations which are tending to but perhaps not safely within the hydrodynamic limit; the JILA group’s published data [201] describe experiments in which the noncondensate cloud was thin enough to be in the collisionless limit but which still had distinct effects on the condensate excitations. In order to make better contact with finite-T theory, future experiments could be performed on much bigger clouds, so as to

Fig. 8. – Oscillation in the width of a pure BEC cloud in both axial and radial direction due to the instantaneous change in scattering length. The widths are for condensates as a function of free evolution time in units of \( \omega_r^{-1} = 9.4 \) ms, followed by 22 ms of ballistic expansion. Each point is the average of approximately 10 measurements. The solid line is a fit of a Gross-Pitaevskii model to the data, with only the amplitude of oscillation and initial size as free parameters. Figure taken from ref. [166].
make the HD limit more accurate. Alternatively work could be performed in traps with spherical symmetry \[204\], which would make more tractable the theory of excitations in the collisionless or the intermediate regime.

5.5. **nonlinear excitations.** – The GP equation is manifestly nonlinear; the observations discussed so far in this section are performed in a small amplitude, linear limit. Some data exist characterizing the lowest-order anharmonic behavior \[201, 200\], but in general there has been little work on nonlinear dynamics. Theorists have predicted a range of really interesting behavior. Varying the aspect ratio of the confining potential causes the small-amplitude frequencies of various standing-wave modes to move around. One can arrange for accidental degeneracies – for instance at a certain aspect ratio the frequency of the lowest-order \( m = 0 \) mode is exactly half that of the next higher-order \( m = 0 \) mode. As one approaches the magic trap parameter, the anharmonic coefficient diverges and one should be able to observe frequency doubling effects \[205\]. In the presence of gravity, the aspect ratio of an axially symmetric TOP trap can be varied from \( \sqrt{8} : 1 \) (oblate) down to slightly under \( 1 : 2 \) (prolate) \[204\]. An IP trap can be varied from extremely prolate (greater than \( 30 : 1 \)) to slightly oblate, although in the near-spherical regime, a IP trap’s axial symmetry will be compromised by the effects of gravity \[71\].

When the condensate is driven very hard, the motion will become chaotic, and energy will couple into many different modes \[206\]. Dilute-gas BEC may be an instructive model system for experiments to probe the boundary between chaotic and truly thermal behavior.

A variety of soliton and soliton-like behavior is also predicted, and is likely to be observable experimentally. See for example ref. \[207\].

5.6. **Many-body effects.** – As a condensate becomes more dense, the mean-field GP equation will begin to break down. The first-order corrections to excitation frequencies have been calculated \[208\]. Precision measurements of condensate excitation frequencies may provide sensitive tests of this nontrivial theory.

5.7. **Perspective.** – The two most recent published experiments have in one case measured an excitation frequency that agrees with theory to well under 1% \[199\], and in the other determined amplitude data from which a ratio of interaction strengths was extracted \[166\]. The ratio agreed with the ratio determined from two-body spectroscopic data, also to better than 1%.

It is worth pausing to think about what the accuracy of these two measurements, and the associated theory, tells us about the field of dilute-gas BEC. As discussed in Section 4.6 above, this sort of connection between macroscopic interacting behavior and calculations beginning *ab initio* with two-body potentials is something a little out of the ordinary.
6. – Condensates and Quantum Phase

Bose condensates are a wonderful environment in which to investigate the subtle concept of quantum phase. Experimental work on phase is really only just now getting started. There are several papers already in print, notably the Ketterle group’s observation of interference between two separate condensates [181] and a collection of papers on mixed-state Rb-87 condensates from the JILA group [169]. We recently reviewed the mixed-condensate work in a separate publication [170], and so will not discuss it here. For the separate-condensate interference experiment, see the original paper [181] or ref. [177]. We know of interesting phase experiments going on at Yale [209] and at Gaithersberg [210]. Presumably papers from these groups will have appeared in print by the time this article is published.

We postpone to a future paper a thorough review of phase experiments. For the remainder of this section, we would like instead to discuss some of the terminology of the topic of quantum phase in condensates, terminology which has led, in our opinion, to some unfortunate misconceptions. Some may find the content of this section trivial — it is aimed at the confused nonspecialist.

6.1. A common misconception. – In informal conversations, we have heard it asserted that a precise measurement of the number of atoms in a condensate must “destroy” a condensate, so that it is “not a condensate anymore.” The argument goes like this: (i) the defining characteristic of a condensate is its coherence; (ii) if you measure the number of atoms in a condensate very precisely, it will be in a number state; (iii) if a collection of particles is in a number state, it can not be in a coherent state; (iv) since it is not in a coherent state, it is no longer a condensate! The careful reader will see the flaw in this syllogism, but in oral discussions, the argument can be compelling. The problem of course is that the phrase “coherent state” is a technical term from the field of quantum optics [211]. It refers to only a small portion of a broader concept which we could call “having coherence.” It is certainly true that if the atoms have no coherence, then the condensate has been destroyed. And it is equally true that a precision measurement of the number destroys a coherent state. The fallacy is to equate the destruction of the coherent state with the destruction of coherence. The fallacy is perpetuated by a tendency to treat the concept of spontaneous symmetry breaking as a fundamentalist Truth, rather than as a useful metaphor.

6.2. The basic phase thought experiment. – We turn now from the topic of what is wrong to the topic of what is correct. We will couch our discussion in terms of particular thought experiments, rather than in the usual language of expectation values of field operators. In our experience the formal notation of the field operators and brackets has sufficient ambiguity that it as likely to confuse as to enlighten. Here, then, is the basic experimental module: take a not-too-well determined number of atoms from the vicinity of a particular point in space ($x_1$), and a similar bunch from from the vicinity of another point ($x_2$), and cause cause both bunches to impinge on an atom counter in such a way as to ensure there is no way the counter can tell from which point any
given atom originated. The number of atoms detected at the counter will be sensitive to an interference term between the amplitude of atoms arriving from point one and the amplitude of atoms arriving from point two, and this interference term will be a function of \( \cos\delta\phi(x_1, x_2) \), where \( \delta\phi(x_1, x_2) \) is the difference in local quantum phase between the two points. The interference term will also depend on the additional differential phase which may accumulate while the atoms are being transported to the detector, of course.

6.3. How is a condensate coherent? – In descriptions of condensates, “coherence” means “predictability of \( \delta\phi \).” A condensate is “coherent” because one can predict, for any given time \( t_1 = t_2 \), the \( \delta\phi \) between any two points \( x_1 \) and \( x_2 \) within the condensate. For a ground-state condensate at rest in the measurement frame, predicting \( \delta\phi \) is particularly easy because \( \delta\phi(x_1, x_2) = 0 \) for all \( x_1 \) and \( x_2 \) in the condensate. For a condensate moving with a velocity \( v \), one can with confidence predict that \( \delta\phi(x_1, x_2) = mv(x_1 - x_2)/\hbar \). For a condensate with a vortex, the dependence of \( \delta\phi \) on \( x_1 \) and \( x_2 \) is more complicated [212], but again completely predictable. Note that a measure of the total number of atoms in a condensate, even an arbitrarily precise measurement, need have no effect on the state of the condensate’s coherence, as defined in this subsection.

6.4. Relative phase of two condensates. – Although completely irrelevant to the question of whether a condensate is a condensate, the “coherent state” of quantum optics is a useful idea when discussing the difference in phase between two different condensates. In this context, \( \delta\phi \) is indeed the conjugate variable to \( \delta N \), the difference in number between two condensates, and yes, a precise measurement of \( \delta N \) must perforce change unpredictably the \( \delta\phi \) between the two condensates, such that there remains no coherence between the two condensates. (Although the two condensates individually can remain “coherent” objects!)

There is a useful insight due to J. Javanainen [213] and to Castin and Dalibard [214] concerning the relative phase of two condensates. They have pointed out that, starting with two condensates in a relative number state, if one proceeds to measure the relative phase, the experiment will yield a perfectly well-defined precise value. We will not review the formalism of their arguments here, but the gist of it is that the act of sequentially detecting particles builds up correlations in the remaining particles and uncertainties in the remaining relative number. After a relatively small number of particles are detected, the remaining two condensates find themselves in a relative coherent state. The phase of that state is unknowable before the measurement begins: in the view of Castin and Dalibard, the act of measurement has projected out, in a quantum mechanical sense, a particular relative phase.

If we return to the “building-block” thought experiments on phase described above, we can see right away what we mean when we say “a measurement of the relative phase between two condensates always yields a particular well-defined value.” We take two condensates, called A and B, for simplicity we assume both are ground-state condensates at rest in the measurement frame, and we take a small number of atoms from near a point \( x_A \) in condensate A and a small number of atoms from near a point \( x_B \) in
condensate B, and beat them on a detector, and from the interference term, we determine 
\(\cos(\delta \phi(x_A, x_B))\). Immediately thereafter, we pick two new points, \(x'_A\) and \(x'_B\), again in condensates A and B, respectively, and find that we get exactly the same value for \(\cos(\delta \phi)\). And in general we find that no matter what point we pick in A and what point we pick in B, we always find the same relative phase as we found in the first measurement, although we may not have been able to predict ahead of time what phase we would measure in the very first measurement. This then is what we mean by saying that a “measurement of the relative phase of two condensates always yields a particular well-defined relative value.” If we assume a sort of “transitive property of relative phase,” (i.e., that \(\delta \phi(x'_A, x'_B) = \delta \phi(x'_A, x_A) + \delta \phi(x_A, x_B) + \delta \phi(x_B, x'_B)\) ) then the fact that a measurement of the relative phase of two condensates always yields a particular, well-defined relative value, is equivalent to the fact that each condensate is independently “coherent” in the sense defined in section 6.3 above.

6.5. Noncondensate clouds. – One can not talk about a single relative phase between two conventional, noncondensed, thermal atomic clouds (unless one has filtered out all but a volume within a single thermal coherence length for each cloud – which in fact is what is done in non-condensate atom-interferometry experiments.) The significance of the MIT two-condensate interference experiment [181] and of other condensate coherence studies [148, 182] is that they demonstrated once and for all that condensates are not simply very cold, relatively dense clouds of atoms.

6.6. Coherence and decoherence. – The relative phase between two condensates can be determined either by measuring it, as described above, or by splitting a single condensate into two with some “gentle” technology [166, 169] which does not too greatly perturb the condensate(s). Immediately after the relative phase has been determined, one can of course predict what will be the result of a new phase measurement. Because the relative phase is known, we say that there is a relative coherence between the condensates. If at some later time we are no longer able to predict to within an uncertainty of \(\pi\) radians the relative phase, we say the two condensates have undergone decoherence. The decoherence can arise in many ways. For instance, if between our initial determination of relative phase and our final measurement of it, someone should perform a precise measurement of the relative number, we know the measurement would cause decoherence. On the other hand suppose the two condensates are at slightly different heights, so that they feel slightly different gravitational potentials. Their relative phase will then evolve at a rate proportional to the difference in the gravitational potential. If the difference in their heights remains very constant over time, we can perform several measurements, determine accurately the rate at which their relative phase evolves, and thereafter be able to predict at some future time what their relative phase will be. If the difference in their heights varies in time in some uncharacterized way, we can lose track of the relative phase and be unable to make an accurate prediction of it. From an experimenter’s point-of-view, then, decoherence needn’t (and usually doesn’t) arise from anything particularly fundamental.
6.7. **Mixtures and superpositions.** – Experimentally, the distinction between “a mixture of two condensates” and a “single condensate of atoms in a superposition state,” is exactly the distinction between decoherence and coherence. A mixture is a superposition in which we have forgotten the relative phase. A superposition is a mixture for which we still remember the relative phase. The distinction between a mixture and a superposition has little to do with the intrinsic nature of the sample in question, and much to do with the state of the experimenter’s knowledge.

6.8. **A final warning on vocabulary.** – Much has been written about a particular mechanism for decoherence, one which arises from the nonlinearity of condensate self-interactions: (i) In a finite-sized condensate, the energy per atom depends on the number of atoms. (ii) When the phase between two condensates is measured, an uncertainty in the relative number of atoms is always introduced. Therefore, there will be an uncertainty in the difference in the energy per particle for two systems, and over time an uncertainty in the relative phase must develop [215]. The mechanism has been given the deceptive title “quantum phase diffusion.” While the mechanism does involve the quantum phase, and while that phase does change unpredictably (which is to say, it diffuses), “quantum phase diffusion” is only one of many ways in which the phase can become unknowable. For reasonably sized condensates, finite temperature effects will almost surely be more important [216].

7. – **Stray Heating: a Pessimistic note**

7.1. **Introduction to the problem.** – Ultracold atoms confined in a magnetic trap have a tendency, in the absence of ongoing evaporative cooling, to warm up. Given that the ambient temperature of the trapping environment is typically nine orders of magnitude higher than the temperature of the trapped atoms, the surprise perhaps is not that the atoms heat, but that they heat so slowly. To an optimist, a heating rate of 100 nanokelvin/sec means that the time constant for the atoms to thermally equilibrate with the 300 K environment is about one hundred years. Hardly objectionable! To a pessimist, (or to a realist) that same heating rate means that the temperature of a sample initially at 200 nK will double in only two seconds, which can be a major experimental inconvenience.

The phenomenon of trapped-atom heating is observed, as far as we know, in all magnetic trapping laboratories. Relatively little has been published on it, however. In this, the final section of our paper, we will survey what is known about the various mechanisms for heating. While the topic has for the most part not attracted much scientific study, it is worth investigating for two principle reasons. First, this ubiquitous effect represents a systematic which often complicates the interpretation of condensate studies, such as those on condensate formation or on collisional loss. Second, heating seems to scale unfavorably with increasing atom number, and may give rise to an upper limit on the size of condensates one can produce.

It is difficult to give a satisfactory summary of the experimental observations. Some
quantitative studies are described in [217, 218], but most of what is known in the community has been passed around anecdotally, by word of mouth. Here are some general observations: A typical heating rate for a cloud of $10^7$ Rb-87 atoms in a Ioffe-Pritchard trap, in the $F = 1, m_F = -1$ state, at 1 $\mu$K, is perhaps 150 nK/s. There is anecdotal evidence, although no careful comparisons, that indicate the problem is worse in Rb-87 than it is in Na. The problem seems to be worse in the Rb-87 $F = 2, m_F = 2$ state than it is in the $F = 1, m_F = -1$ state. Heating is observed to get worse with increasing trapped-atom number, with increasing density, with increasing column density, with increasing elastic collision rate, and with decreasing bias field. Unfortunately, it is difficult to vary each of the above-mentioned quantities independently, therefore it is not clear what really matters. Both the MIT Sodium group and the JILA BEC group have both observed that the presence of an “rf shield” (described in Subsection 7.5 below) profoundly affects the heating rate, in most situations. Heating rates as low as 10 nK/s have been observed in well-shielded traps [199].

Efforts on the part of the JILA group to come up with an experimentally validated, comprehensive model for the heating mechanism(s) have not been very successful. Our experimental results have been somewhat inconclusive, and our models tend to become too complicated to give simple numerical predictions. The discussion in this section is meant to be taken as a collection of general observations, as suggestions for directions of research, rather than as definitive results.

In the remainder of this section we discuss in turn various candidate mechanisms for heating. We believe most of these mechanisms are inadequate by themselves to account for the observed heating. In the final subsection we explore what we feel is the best explanation for the heating — the ultra-cold trapped sample is often surrounded by a very dilute haze of much hotter atoms, also trapped, which gradually thermalizes with the ultra-cold sample. Glancing collisions with background atoms, and inelastic decay products, while inadequate to directly cause heating, help populate the high-energy haze and thus indirectly contribute significantly to the heating.

7.2. Miscellaneous small effects.

7.2.1. Anti-evaporation. The earliest paper on trapped-atom evaporation [31] discusses one of the most ubiquitous mechanisms for heating: inelastic collisions. These can be thought of as giving rise to “anti-evaporation” for the following reason: at the typical temperatures of trapped atoms, the rates for three-body recombination and dipolar relaxation are independent of the collision velocities, and the per-volume rates are simple functions of the local density. Since in thermal equilibrium the density is highest where the potential is deepest, inelastic collisions selectively remove atoms with low potential energy, thus increasing the mean energy of the remaining atoms. Heating from anti-evaporation is very common; most experimenters have probably encountered it.

In samples below the critical temperature, most of the inelastic collisions occur in the condensate, because its density is much higher than the thermal cloud. If the sample is not too large, the decay products can pass cleanly out of the cloud. In this case, the
inelastic collisions consume the condensate but do not strictly speaking cause heating, because the density and temperature of the thermal cloud is unchanged. But because the overall number of atoms is decreasing, the value of the critical temperature goes down. Antievaporation therefore increases the value of $T/T_c$.

Antievaporation usually arises from inelastic collisions, but more generally anything which preferentially removes atoms from the center of the trap gives rise to antievaporation. For instance, if there is trap loss due to rf magnetic noise driving unwanted Zeeman transitions, and if that noise has a steep frequency dependence (say, due to low-pass filters on the trap-magnet coils), then atoms at the lower magnetic fields at trap center will undergo Zeeman transitions and leave the trap at a faster rate.

In many experimental situations, heating is observed that is far too large to be accounted for by antievaporation alone. In anti-evaporation’s limiting case, only atoms with zero mean energy would undergo decay. A differential change of number $\delta N$ would then give rise to a differential increase in temperature $\delta T$ such that $\delta T/T = -\delta N/N$. This is an upper limit on the amount of heating that can arise from anti-evaporation. When the observed fractional rate of heating is larger than the observed rate of trap loss, one must look elsewhere to identify the dominant source of heating.

7.2.2. Field noise/Shaking trap. Atoms in a magnetic trap are by intention highly isolated from the external environment. The one mechanical force that must be present is from the trapping itself, and thus in investigating heating we naturally suspect the trapping fields. In our experience, however, mechanical noise on the magnetic coils, or electric noise in the coils, is seldom the dominant source of unwanted heating. There are two reasons for this — (i) moving the coils, or modulating the current in them, affects all the atoms nearly the same, and thus does not couple well to a heating effect, except to the extent that it drives macroscopic pulsing or sloshing modes of the cloud. (ii) In magnetic traps at least, and at low temperatures, the atoms see a very harmonic potential. This has the effect of sharply constricting the bandwidth of the noise that can harm the atoms. Unless the noise is near a trap oscillation frequency, or a harmonic thereof, the energy can’t couple to the atoms.

If the noise is at the second harmonic of trap frequency, the motion of the atoms can be driven parametrically, and the resulting “pulsing” of the cloud can convert via collisions to heat. If the noise is directly at a trap frequency, “sloshing” can be driven, which due to anharmonicity also converts to heat. Directly driven sloshing is usually only a problem in the vertical direction, for which gravity breaks symmetry. In an effort to characterize the heating from noise-driven pulsing, we have intentionally applied electrical and mechanical noise to coils. If the noise is broad-band, it takes a surprisingly high power density to accomplish any heating.

One week at JILA in 1997, when we were having a particularly bad problem with electrical ground loops, our evaporation stopped working well. It turned out that during our evaporation procedure (which includes an adiabatic ramp of the trapping frequencies) we were unintentionally pausing for a second at the value of the trap parameters which caused one of the trap frequencies to be resonant with a multiple of 60 Hz (the North
American power-line frequency). Eventually we fixed the power supply, but we were able to implement a successful “quick fix” simply by ramping very quickly over the offending region of trap parameters. In nine years of JILA experience with magnetic traps, this was the only occasion in which magnetic coil noise was confirmed to be the dominant source of unwanted heating. We have used parametric driving to deliberately heat the atoms (see e.g. ref. [74]), but to accomplish this we had to apply a modulation tone of considerable amplitude compared to background noise.

For a thorough, quantitative discussion of heating from noise on the trapping field, see Savard et al. [219]. While they analyze the same mechanisms we mention above, they come to the opposite qualitative conclusion — they assert that noise on a trap is likely to be an important source of heating. The source of the disagreement lies in the technical details — their model system was an optical rather than a magnetic trap, and optical traps tend to have higher confining frequencies. As Savard et al. show, the higher the confining frequency, the more susceptible a system is to parametric heating [219]. Moreover, electrical and mechanical stability of electromagnets is likely to be better than intensity and pointing stability in laser beams.

Noise-induced heating is likely to be a very important effect in “composite traps,” traps made of magnetic and optical forces [111] or of more than one laser beam. In those cases noise can lead to relative motion of the various components of the confining potential, and thus heat the atoms more directly.

7.2.3. Stray rf and optical fields. If care is not taken to keep radio and optical frequency photons out from where they do not belong, they can cause trap loss and presumably, under the right conditions, heating as well. In practice it is often necessary to place an opaque box around either the lasers (with their associated absorption cells) or the magnetic trap.

7.2.4. Black-body radiation. In conventional cryogenic experiments, black-body radiation is a major source of heat transfer, but it is difficult to imagine that it will ever be important for the case of trapped atoms. The cross-section for scattering a photon, at least for atoms, is simply too small. The question may need to be revisited if one were to attempt to evaporatively cool objects more complicated than individual atoms, objects which may couple more strongly to long-wavelength radiation.

7.3. Collisions with background atoms. – Another common source of unwanted heating in conventional cryogenic experiments is imperfect vacuum: residual gas atoms move back and forth between warmer and colder surfaces, transferring heat with each bounce. In trapped-atoms experiments as well the room-temperature residual gas atoms can provide heat to the confined atoms, but the effect can not be understood as a simple shuttling of heat between a warm surface and a cold sample. When a 300 Kelvin background atom collides with a trapped atom, the most probable result will be a clean ejection – both the incoming and the impacted atoms leave the trap without further collisions. These are the events that give rise to the background loss rate $\gamma_{bg}$ discussed in section 8.1 below.
But there exists also the possibility of glancing collisions which transfer a relatively small amount of energy to the trapped atom. These events lead to heating rather than to loss.

### 7.3.1 Direct creation of “hot” atoms

The distinction between heating and loss can be somewhat arbitrary. Imagine a magnetic trap which is 5 mK deep, confining at its center a sample of atoms with temperature 20 µK. Now suppose a trapped atom undergoes a collision that leaves it with 4 mK of kinetic energy. Is this heating, or loss? If one were to measure the total number and total energy of the atoms confined in the potential, this particular collision looks like heating. A more common experimental procedure, however, would be to image the cloud, and fit its profile to a Gaussian lineshape. If one identifies the energy of the cloud as the mean-square width of the fit lineshape, and the number of atoms in the cloud as the area under the lineshape, then our hypothetical collision looks like loss. Typically, then, “heating” results from background collisions which transfer energy less than three or four times the mean energy of the trapped cloud. At the opposite extreme, collisions which leave the impacted atom with energy larger than the trap depth clearly result in loss. Collisions which transfer an intermediate range of energy give rise to a large-area, diffuse cloud of atoms that are still trapped but whose optical density is below detection threshold. These intermediate collisions appear to give rise to loss, rather than heating, at least at short times. The longer-term effects of the diffuse cloud are discussed in subsection 7.5 below.

To get a rough estimate of the size of these heating effects, we need a simple model for small-angle collisions with background gas. The following simplified explanation draws on thorough treatments by Helbing and Pauly [220] and by Anderson [221]; our results are not meant to be accurate to within a factor of two. Small-angle collisions are the result of trajectories with large impact parameters. Therefore only the long-range tail of the inter-atomic potential is relevant. We write the inter-atomic potential as $U_{\text{int}}(r) = \frac{C_6}{r^6}$.

The value of $C_6$ depends on the scattering species. For helium on rubidium, $C_6$ is 36 atomic units (au) [222]. For rubidium on rubidium, the value is 4700 au. Treating the scattering event classically, in the impact approximation, and working in the lab frame (in which the trapped atom is initially at rest, and the incoming background atom has energy $E_{\text{col}}$), we find the partial cross section for a glancing collision to transfer energy $E_t$

$$\sigma_{\text{g}}^{\text{class}}(E_t) = \frac{\pi}{6} \left( \frac{9C_6^2}{E_{\text{col}}} \right)^{1/6} E_t^{-7/6}.$$ (6)

Note that the integral of $\sigma_{\text{g}}^{\text{class}}(E_t) dE_t$ diverges at low energy; quantum mechanics intervenes to keep the true total cross-section finite. We define a cross-over energy $E_c$: for $E_t < E_c$, the deBroglie wavelength of the transferred momentum becomes comparable to or longer than the classically determined impact parameter, and diffraction effects dominate. $E_c$ is given by
where $m$ is the mass of the trapped atom, $m_b$ is the mass of the background atom, and it is assumed that $m_b \leq m$. For helium on rubidium, $E_c$ is about 40 mK. For rubidium on rubidium it is about 3 mK.

As discussed in ref. [221], for $E_t < E_c$, $\sigma_g(E_t)$ shows oscillatory behavior due to diffraction. In our simplified model, we use the following approximation:

\[
\sigma_g(E_t) = \begin{cases} 
\alpha & \text{if } E_t \leq E_c; \\
\alpha \left( \frac{E_c}{E_t} \right)^{7/6} & \text{if } E_t > E_c;
\end{cases}
\]

where $\alpha = \sigma_{\text{class}}^g(E_c)$. To convert the cross-section into a rate, one needs to know the density of the residual 300 K atoms. In the lab, it is easier to measure the lifetime of a trapped sample than it is to get an accurate estimate of the local residual vapor pressure at the trap. If we define $\gamma_{bg}$ to be the loss rate measured in the limiting case of a very low-density sample and a very shallow trap, we know that $\gamma_{bg}$ will be proportional to the integrated collision rate and to the background pressure. Assuming there is one dominant species of background gas, we can normalize eqs. 8 to get the following partial rate results, for the rate per atom, per differential energy, of energy transfer collisions:

\[
\gamma_g(E_t) = \begin{cases} 
\gamma_{bg} & \text{if } E_t \leq E_c; \\
\gamma_{bg} \left( \frac{E_c}{E_t} \right)^{7/6} & \text{if } E_t > E_c.
\end{cases}
\]

What then is the resulting heating rate from such collisions? As discussed above, depending on exactly how one measures “heating,” one can estimate a threshold value, call it $E_h$, for the transfer energy. For $E_t > E_h$, the collisions result in “loss.” For $E_t < E_h$ collisions result in “heating.” The fractional rate of heating is then

\[
\frac{1}{E} \frac{d\overline{E}}{dt} = \frac{1}{\overline{E}} \int_0^{E_h} E_t \gamma_g(E_t) \, dE_t
\]

where $\overline{E}$ is the mean energy per atom in the trapped sample. If we take a plausible estimate for the $E_h$, say $E_h \approx 3\overline{E}$, then $(d\overline{E}/dt)/\overline{E} \approx 2\gamma_{bg} kT/E_c$, for a sample with temperature $T$.

This simple model of direct heating from glancing collisions is qualitatively successful in accounting for heating observed in early JILA experiments on trapped Cs [74, 218]. By heating the vacuum chamber, we caused the dominant background species to be cesium.
In this limit, we observed that the fraction heating rate of a 200 microKelvin cloud scaled linearly with the background loss rate, with a constant of proportionality of about 0.5. For cesium on cesium, $E_c$ is about 1 mK, and the predicted constant of proportionality is between heating and loss is about 0.2. Given the qualitative nature of the model, the agreement is pretty good. When we instead cooled the vacuum chamber the dominant background species became helium, and (consistent with the large value of $E_c$ for helium on cesium) the heating rate essentially vanished, even though we continued to observe trap loss.

In most modern BEC experiments, however, the lab conditions are quite dissimilar to the early JILA Cs experiments, and the simple model is empirically found to be completely inadequate. In a clean vacuum chamber, the dominant background species is usually a low-mass, low-$C_6$ species such as helium or hydrogen, such that the appropriate value for $E_c$ may be 40 mK or higher. The observed value of $\gamma_{bg}$ is typically less than 0.01 sec$^{-1}$. In addition, the sample temperature is typically 1 $\mu$K or lower. Under these conditions, eq. 10 typically predicts heating rates four orders of magnitude smaller than what is observed.

### 3.2. Heating from secondary scattering

As a first attempt to improve the model, we revisit its implicit assumption that collision products can pass unimpeded through the sample, i.e. that the sample size $l$ is much less than the mean-free-path $\lambda_{mf}$.

If the opposite limit applied, if $l \gg \lambda_{mf}$, then an atom which has acquired an energy $E_t$ from a background event would collide with other atoms in the sample many times along its trajectory out of the sample. If the sample is thick enough, the entire energy $E_t$ could be distributed among a shower of atoms, all with energies less than $3E_t$. We can get a rough estimate of the resulting heating rate in this limit from eq. 10, with $E_h$ now defined as the maximum value of energy an atom in the middle of the sample can have and still be trapped by multiple collisions in the dense sample. $E_h$ is now a function of the column density of the sample, $nl$, and to estimate its value we look at the energy dependence of $\lambda_{mf} = (n\sigma_{hc}(E))^{-1}$, where $\sigma_{hc}(E)$ is the energy-dependent cross-section for large-angle scattering. In the limit of very large $nl$, $E_h$ will be sufficiently large that $\sigma_{hc}(E_h)$ will be in the classical limit, for which $\sigma_{hc}(E) \propto C_6^{1/3}E^{-1/3}$. With each collision, the particle loses perhaps 1/2 its energy, so that $\sigma_{hc}(E)$ becomes larger and the corresponding value of $\lambda_{mf}$ becomes shorter. The sum over the sequence of successively shorter values of $\lambda_{mf}$ converges, such that most of the shower of atoms resulting from the original energy-transfer event should be brought to rest within a distance proportional to $E_t^{1/3}/n$. Therefore the maximum energy atom that can be trapped, $E_h$ scales as $(nl)^3$.

Substituting this value for $E_h$ into eq. 10, we see that in the limit of a trapped cloud with very high column density, $nl$, the heating rate can scale as $(nl)^6$.

A heating rate that scales as the sixth power of the column density is an alarming prospect, but when reasonable numerical values are plugged in, we find that our multiple-scattering model predicts dramatic effects only for very large trapped samples – with column-densities of perhaps $10^{13}$ atoms/cm$^2$ or greater. We include the above discussion only because it illustrates how, even with perfect rf-shielding (see subsection 7·5 below),
heating may eventually prove to be a formidable upper limit to the sizes of condensates one can create. We wish to reemphasize that the models above are very crude in nature. We made a number of simplifying assumptions which we did not explicitly state. For instance, an implicit assumption in the previous paragraph is that, after a particular background scattering event, either all the energy will be captured in the sample by multiple rescattering, or all the energy will escape. In actual fact, most scattering events will leave behind as heat some fraction of their initially transferred energy, a fraction that is neither zero nor unity. Such events are not properly accounted for in our model.

To conclude this subsection, we should emphasize that for the column-densities of the trapped samples in currently ongoing BEC experiments, the amount of heating that can be attributed to the explicit scenario described above (a 300 K atom scatters from a trapped atom, which in turn rescatters on its way out of the sample) is relatively small. On the other hand, while it is true that most of the atoms which are perturbed by background gas will exit the sample without additional scattering, a significant fraction of them may remain within the much larger volume of the magnetic trap. These atoms may cause trouble later on, as we discuss in subsection 7.5 below.

7.4. Collisions with the products of inelastic decay. – When an atom is lost from the sample due to a collision with a 300 K background atom, the chances are 6/7 that it will have an energy larger than $E_c \approx 40$ mK (see eq. 9). This means, in turn, that the large majority of atoms lost due to background collisions will have energies greater than the depth of the confining magnetic trap. With inelastic decay, however, the story is reversed. If we assume most three-body decay populates the least-bound vibrational state, then the decay products will have energies mostly under 1 mK. Dipolar decay products may also have energy under 1 mK. Inelastic loss then, leads to production of atoms with energies typically much greater than the trapped sample, but much less than the depth of the confining magnetic potential. Given that lifetimes due to background decay often exceed 100 seconds, and given that the three-body decay constants for alkalis are in the vicinity of $10^{-29}$, it is often the case that, as the density of a trapped cloud approaches the critical value for BEC, the dominant loss process is inelastic decay, rather than background collisions. The combined result then is that as a sample approaches the critical density, the dominant mechanism for the production of atoms with “dangerous” energies (i.e. $1\mu K < E < 2\mu K$) will be inelastic decay, rather than glancing collisions with background atoms.

The decay products will help populate a diffuse cloud of hot atoms, which can cause heating, as discussed below. Depending on the column-density of the trapped sample, they may also contribute to direct heating, exactly as background-scattered atoms can. There is a widely-held assumption that the decay products of three-body recombination are so energetic that they have a relatively small cross-section for colliding with a trapped atom. This is completely incorrect. Given that the dominant decay channel is into the highest vibrational state, a simple calculation for Rb-87 shows that the outgoing “witness” atom will have a cross-section for scattering which is about the same as the zero-energy s-wave cross-section [223]. Beyond the specifics of the rubidium potential,
general considerations on the relationship between s-wave scattering and the binding energy of the highest vibrational level suggest that the witness atom will usually have an energy low enough that its cross-section will be comparable to the zero-energy value. As for the outgoing dimer, the kinetics of three-body recombination is such that the dimer will have per-atom translational energy four times smaller than the witness atom’s energy. There is no reason to believe that the dimer’s cross-section on its atomic counterpart will be small. After all, the loosely bound dimer is presumably no less polarizable than a single atom.

7.5. The Oort cloud paradigm. – The usable volume of a Ioffe-Pritchard magnetic trap can be quite significant, perhaps 2 cm³ or more, and the depth of the confining potential is typically on the order of several millikelvin. A typical sample of ultra-cold atoms, with temperature on the order of a microkelvin, occupies only a tiny fraction of the available volume: the ratio of sample volume to the trap volume may be $10^{-5}$ or less. There exists considerable evidence that, towards the end of an evaporative cooling cycle when the central sample is nearing degeneracy, the outer region of the trap is filled with a very dilute, very high-energy halo of trapped atoms which we refer to as the “Oort cloud” [224]. The millikelvin Oort cloud is of course not in thermal equilibrium with the microkelvin ultra-cold sample, and the coupling between them is weak, due to the Oort cloud’s very low density. Nonetheless occasional collisions may transfer energy (and atoms) between the two trapped components, and we believe this coupling provides the dominant mechanism for sample heating in many experimental situations [225].

The Oort cloud is not readily imaged. Its radius may be ten to 100 times larger than that of the ultracold sample, which means that its cross-sectional area may be 100 to $10^4$ times larger. Thus even if the Oort cloud comprises more atoms than the ultracold sample, its optical depth may be so small as to make it easy to overlook. In our experience, a thermal cloud that is allowed to remain in a magnetic trap for several seconds in the absence of evaporative cooling will sometimes develop “wings,” which cannot be fit with a single Gaussian density profile. While these wings are usually thought of as a high-energy tail to the ultracold distribution, they may as well be described as a low-energy tail to the Oort cloud.

Direct evidence for the existence of the Oort cloud can be found in discrepancies between two distinct methods of measuring the number of trapped atoms in a magnetic trap. The first method is to pass a probe beam through the trap and image the two-dimensional absorption profile. The number of atoms in the trap is then assumed to be proportional to the integrated signal under the central feature in the shadow cast by the ultra-cold sample. This method is not sensitive to the presence of an additional, spatially dispersed population of atoms. The second method is suddenly to turn off the magnetic trap and to turn on the MOT, and then to collect the total fluorescence emitted from the atoms recaptured in the optical trap. The MOT laser beams are several cm in diameter and sweep up any atoms that had been trapped anywhere within the magnetic trapping volume. The two methods can be performed on similar clouds of trapped atoms, and the results compared. What is often found is that when a cloud
of trapped atoms has just been freshly loaded into the magnetic trap, the two methods give similar estimates of the total number of trapped atoms. Later, after extensive evaporation has been performed and the number of atoms in the sample reduced 100-fold, the MOT-recapture method may show perhaps five times more atoms than the shadow-image method. In this example, then, only 20% of the trapped atoms are in the ultracold sample. The remainder populate the Oort cloud.

The strongest evidence that the Oort cloud is responsible for much of the typically observed heating comes from the effect of an “rf shield.” Both at JILA [217, 148, 204] and at MIT [113, 118] it has been noted that the application of an rf field can affect the lifetime and heating rate of trapped samples. During evaporation, an rf magnetic field is applied to the trapping region, and its frequency is gradually ramped downward as evaporation proceeds. There is a 2-dimensional surface surrounding the center of the trap, a surface over which the magnetic field has the correct magnitude to bring the atoms on resonance with the rf-field. Atoms whose trajectories cross this surface have some probability of being transferred into an untrapped spin state. As the frequency of the rf is decreased, the radius of the rf-surface decreases, cutting into the edge of the ultracold sample, and driving the evaporative process. In a typical experiment, the rf surface is smoothly brought inward until the ultracold sample reaches the desired temperature, then moved back out again and left on while experiments are performed on the sample. This post-evaporation configuration is referred to as an “rf shield.” The radius of the rf surface is typically sufficiently large that the rate of evaporation essentially vanishes; no ultracold atoms have trajectories with sufficiently high energy to reach the rf shield. Yet the shield is observed to have a profound effect – turn the rf field off, and the rates of heating and loss from the ultracold sample can increase by an order of magnitude. Presumably the rf shield depletes the Oort cloud or prevents it from being populated in the first place.

How is the Oort cloud populated? We speculate that there may be several mechanisms. First, there is the mechanism of glancing collisions. This is discussed in section 7.3. above. Second, there is the generation of inelastic decay products. This is described in Section 7.4, above. Third, “incomplete evaporation.” In experiments with atoms in the $F = 2, m_F = 2$ state, rf evaporation may remove an atom from the $m_F = 2$ state but fail to transfer it into an untrapped state, leaving it instead in the $m_F = 1$ state. Some of these atoms can later cause trouble. Fourth, “primordial remnants.” At any given instant, a small fraction of atoms in an optical molasses have energies which are extremely superthermal. These atoms may end up loaded into the magnetic trap with trajectories that are outside the initial radius of the evaporative rf knife. While these atoms may initially represent less than 1% of the total sample, the process of evaporation reduces the size of the trapped sample by a factor of 100, which increases proportionally the relative significance of the high-energy remnants. Fifth, the Oort cloud can populate itself through erosion of the ultracold sample. For example, an Oort atom with 500 $\mu$K of energy can plunge into the ultracold sample and undergo a collision, with the result that two, 250 $\mu$K atoms emerge. The net population of the Oort cloud is increased, and it is left colder and denser. The colder is the Oort cloud, the more tightly is it coupled
to ultracold sample, and the more heating it will cause.

How is the Oort cloud depopulated? The presence of the rf shield (or equivalently, ongoing rf evaporation, which amounts to the limiting case of a close-in rf shield) will tend to deplete the Oort cloud. As the atoms pass through the surface in space on which the rf is resonant, they are transferred into untrapped states, and may leave the trap. In a TOP trap, atoms in the Oort cloud may encounter as well the orbiting zero-field point, which can induce Majorana transitions [109] and detrap atoms.

The rf shield is an imperfect device for removing the Oort cloud. If the amplitude of the rf is too low, then the Oort atoms (whose average velocity is high compared to the atoms undergoing evaporation) may pass through the rf’s resonant surface too rapidly to undergo a transition. The trajectories of such Oort atoms may carry them through the ultra-cold cloud many times before they are finally ejected by the weak rf field. On the other hand, if the amplitude of the rf is too high, such that there is unity probability for an Oort atom to undergo a transition as it passes through the surface, the situation is equally unsatisfactory. As the Oort atom passes through the resonant surface, towards the center of the trap, it is flipped into an untrapped state. But its energy will be such that it can still reach the ultracold atoms at the center of the trap. To leave the trap, it must pass once again through the resonant surface, where with high probability it will be flipped back into a trapped spin-state! See fig. 5b of ref. [226] and accompanying text for a nice description of the relevant physics.

A quantitative understanding of the interplay between the rf shield and the Oort cloud, and of the consequences for trap loss and heating, may require a fairly elaborate model. Certainly any attempt to create a condensate with say $10^9$ Rubidium atoms will need to consider the problem of heating very carefully. Perhaps optical dipole traps, which are so shallow that they can not support an Oort cloud [118], may play an important role in the creation of extremely large condensates.

8. – Collisions and Evaporation: an Optimistic note

The crucial issue in planning a successful dilute-gas BEC experiment is collisions. Until a new technology comes along, the last step in a BEC experiment will be evaporation, and evaporation works only if the collisional properties of one’s system are favorable. In practice, this means that planning a BEC experiment requires learning to cope with ignorance. So much is currently known about the elastic and inelastic collisional properties of the atoms in the first column of the periodic table [227] that it is easy to forget that essentially nothing is known about the low-temperature collisional properties of any other atomic or molecular species.

One can not expect theorists to relieve one’s ignorance – inter-atomic potentials derived from room-temperature spectroscopy are not adequate to allow theoretical calculations of cold elastic and inelastic collision rates, even at the order-of-magnitude level. Cold-atom spectroscopy can do the job (as we have seen with lithium, sodium, and rubidium), but starting an experimental program to investigate the cold collisional properties of a new atom is a major and uncertain endeavor. In most cases, the easiest way to
discover if evaporation will work for a particular atom or molecule is simply to try it. Launching a major new project without any assurances of success is a daunting prospect, but we believe that, if one works hard enough, the probability that any given species can be evaporatively cooled to the point of BEC is actually quite high. In this section we will review the scaling laws behind this optimistic assertion.

8.1. Collisional scaling predicts success! – The extensive literature on evaporation will not be reviewed here. See refs. [31, 65] for important early work. The results of the earliest Monte Carlo trajectory simulations are presented in refs. [74, 218]. See ref. [226] for a useful review paper. The literature can be summarized in the following requirement: In order for evaporative cooling to succeed, there must be a great many elastic collisions per atom in trap per lifetime of the atoms in the trap. Exactly what is meant by a “great many” depends on how evaporation is implemented, but a reasonable guess is “200,” give or take a factor of four. Since the lifetime of the atoms in the trap is usually limited by collisions [228], the requirement can be restated: the rate of elastic collisions must be about two orders of magnitude higher than the rate of bad collisions.

At sufficiently low collision energy, the cross-section for elastic collisions becomes energy-independent, and the rate per atom can be written

\[ \gamma_e = n\sigma v \]  

(11)

where \( n \) is the mean density, \( \sigma \) is the zero-energy s-wave cross-section, and \( v \) is the mean relative velocity. Assuming atoms are trapped in a state for which there is no spin-exchange, there are three bad collisional processes: background collisions, three-body recombination, and two-body dipolar relaxation, with the per-atom loss rate respectively denoted by \( \gamma_{bg} \), \( \gamma_{3b} \) and \( \gamma_{2b} \). Loss due to background collisions is to first approximation independent of the cloud density and temperature. (We examine the break-down of this approximation in section 7 above.) We write the rate as

\[ \gamma_{bg} = \alpha \]  

(12a)

where \( \alpha \) is proportional to an effective total residual pressure in the vacuum chamber. The constant of proportionality depends on the composition of the residual gas. The other two rates are inelastic processes among the trapped atoms. At low energies these rates become independent of collision energy, and we can write

\[ \gamma_{3b} = \lambda n^2 \]  

(12b)

\[ \gamma_{2b} = \beta n \]  

(12c)
where $\beta$ is the dipolar rate constant and $\lambda$ is the three-body rate constant.

The total rate of bad collisional processes is $\gamma_{\text{bad}} = \gamma_{\text{bg}} + \gamma_{3b} + \gamma_{2b}$. For good evaporation, the three inequalities,

\begin{align}
(13a) & \quad n\sigma v \gg \alpha \\
(13b) & \quad n\sigma v \gg \lambda n^2 \\
(13c) & \quad n\sigma v \gg \beta n
\end{align}

must be separately satisfied. Moreover, the inequalities must be satisfied not only initially, (i.e. immediately after the atoms are loaded into the trap) but also as evaporation proceeds towards ever greater phase-space density, towards larger $n$ and smaller $v$.

With respect to evaporating rubidium 87 and the lower hyperfine level of sodium 23, nature has been kind. The respective values of $\sigma$ are sufficiently high, and $\lambda$ and $\beta$ sufficiently low, that the collisional rate inequalities (13b) and (13c) are routinely met along a reasonable evaporation path through $n-T$ space to BEC. One need “only” arrange for the initial trapped cloud to have sufficiently large $n$, and design a vacuum chamber with sufficiently small $\alpha$, and evaporation works. The main point of this section, however, is that evaporation is likely to be possible even with less favorable values of $\sigma$, $\beta$, and $\lambda$. We deal with the constraints (13) in order:

8'1.1. Background loss. The technical difficulties of ensuring that the elastic rate is much greater than the background loss rate can be formidable. All the same, eq. (13a) does not represent a fundamental collisional limit, for the simple reason that with sufficient laboratory effort $\alpha$ can be made almost arbitrarily small. Lifetimes of $10^4$ s are attainable in cryogenic magnetic traps. Cryogenic experiments are notoriously difficult, but if one is willing to go to the effort and the expense, one should ultimately succeed. The challenge is only to satisfy (13a) at the beginning of evaporation. Once efficient evaporation is established, $nv$ increases, while $\alpha$ remains the same (except see section 7 above.)

8'1.2. Three-body recombination. It is unlikely that $\lambda$ could ever be so large that three-body recombination is a problem when the atoms are first loaded into the trap. As evaporation proceeds, however, density increases and velocity decreases, such that inequality (13b) may well fail, if one is working with a species with small $\sigma$ or large $\lambda$. The solution lies in manipulating the trapping potential. Adiabatically ramping the mean harmonic confinement frequency $\omega_t$ has no effect on the phase-space density — the ramp takes atoms neither closer to nor further from the BEC transition. The mean density and mean velocity are both affected by the ramp, however, such that $\gamma_{3b}/\gamma_e = \lambda n^2/(n\sigma v) \propto \omega_t$. Therefore, as long as one can continue to turn down the
confining strength of one’s trap, one can ensure that (13b) remains satisfied all the way to the BEC transition.

8.1.3. Dipolar relaxation. As with three-body recombination, if dipolar relaxation is to be a problem, it will likely not be until late in the evaporative process when the decrease in velocity may threaten the validity of (13c). Adiabatically turning down the confining potential is not helpful, because the scaling with \( \omega_t \) has the opposite sign: 
\[
\frac{\gamma_{2b}}{\gamma_e} \propto \omega_t^{-1/2}.
\]
Evidently one can win by turning up the trap. While this may be a useful tactic in a particular laboratory situation, it is unsatisfactory in terms of our “Existence Proof” for evaporation. One may always make a trap weaker, but there is likely to be an engineering-based upper limit to trap strength, and if \( \beta \) is particularly large, a problem can remain. More fundamentally, if one is cursed with a species for which both \( \beta \) and \( \lambda \) are large, there may be no value of \( \omega_t \) for which (13b) and (13c) can be simultaneously satisfied in a cloud near degeneracy.

Fortunately, one is not required to accept the value of \( \beta \) nature provides. In fact, all one really has to do is to operate the magnetic trap with a very low bias field. Below a threshold field of perhaps five gauss, the value of \( \beta \) for the lowest hyperfine level drops rapidly to zero. This behavior is simple to understand. At low temperature, the incoming collisional channel must be purely s-wave. Dipolar relaxation changes the projection of spin angular momentum, so to conserve angular momentum the outgoing collisional channel must be at d-wave or higher. The nonzero outgoing angular momentum means that there must be a angular momentum barrier in the effective molecular potential, a barrier which rises a few hundred microkelvin above the the \( r = \infty \) limit. This barrier is irrelevant for atoms trapped in the stretch-state, for instance (\( F = 2, m_F = 2 \) in rubidium 87). Dipolar relaxation releases the (relatively large) hyperfine energy, and the outgoing atoms barely feel the the angular momentum barrier. But if the atoms are trapped in the lower state (\( F = 1, m_F = -1 \), in rubidium 87) the outgoing energy is only the Zeeman energy in the trapping fields. For fields below about 5 Gauss, this energy is insufficient to get the atoms back out over the angular momentum barrier. If relaxation is to occur, it can happen only at inter-atomic radii larger than outer turning-point of the angular momentum barrier. For smaller and smaller fields, the barrier gets pushed further out, with correspondingly lower transition rates [229]. This effect turns out to be irrelevant in atomic hydrogen, since the lower hyperfine state is \( F = 0 \), and is not magnetically trappable. In most magnetically trappable atoms and molecules, however, the lowest state has \( F \neq 0 \), and one of the Zeeman sublevels will have no spin-exchange, and suppressed dipolar relaxation.

To summarize this subsection, given (i) a modestly flexible magnetic trap, (ii) an arbitrarily good vacuum, (iii) a true ground-state with \( F \neq 0 \) and \( S \neq 0 \) and (iv) non-pathological collisional properties, almost any species can be successfully evaporated to BEC.

8.2. Except of course . . . - The assumption of “nonpathological properties” may actually be fairly constraining. For example: i) Rb-87 has a d-wave resonance [230],

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ii) Cs-133 appears to have a zero-field Feshbach resonance [231]. iii) The elastic cross-section of Rb-85 has a pronounced energy dependence and drops to near-zero at a very inconvenient temperature [197, 79]. One is forced to confront the fact that at least in the alkalis, pathology seems almost to be the rule. Also, for technical reasons having to do with low-frequency electronics noise, it can be inconvenient to trap atoms at fields below about 0.5 G, which may not always be low enough to completely suppress dipolar relaxation. Also the scaling arguments discussed above assume T-independent elastic cross-section; optical molasses temperatures are not always low enough to ensure that one is in this regime. Further, we have completely neglected the issue of heating. Therefore we invite you to adopt an appropriately skeptical attitude towards the universality of the arguments outlined in this section.

8.3. But on the other hand . . . . – Requiring that a particular atomic or molecular species be able, on its own, to cool to BEC is unnecessarily restrictive. Sympathetic cooling has been shown to work well. As far as we know, thorough modeling of the collisional requirements for sympathetic cooling has not been performed, but simple estimates suggest that in the presence of a robustly evaporating working fluid, the collisional requirements on the species to be cooled, and on interspecies collisions, are down by a factor of ten or more from those outlined in the first paragraph of section 8.1.

Once you have cooled to the BEC transition, your ability to accumulate a significant number of atoms in the condensate depends on the scattering length’s being positive. Here again, even in total ignorance, your odds are pretty good: see the argument in the footnote [97].

* * *

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