Rapidly varying accretion and AGN feedback

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ABSTRACT
Accretion rates onto AGN are likely to be extremely variable on short timescales; much shorter than the typical cooling time of X-ray emitting gas in elliptical galaxies and galaxy clusters. Using the Langevin approach it is shown that, for a simple feedback system, this can induce variability in the AGN power output that is of much larger amplitude, and persists for longer timescales, than the initial fluctuations. An implication of this is that rich galaxy clusters are expected to show the largest and longest-lived fluctuations. Stochastic variations in the accretion rate also mean that the AGN injects energy across a wide range of timescales. This allows the AGN to maintain a much closer balance with its surroundings than if it was periodically activated. The possible non-linear correlation between Bondi accretion rate and jet power, found by Allen et al. (2006), can be explained if the instantaneous accretion rate, scaled by jet power, varies log-normally. This explanation also implies that the duty cycle of AGN activity increases with the radiative losses of the surroundings, in qualitative agreement with Best et al. (2005).

Key words: galaxies: cooling flows, active, jets

1 INTRODUCTION
It is well known that elliptical galaxies are commonly the hosts of powerful radio AGNs (e.g. McLure et al. 2004). These sources give rise to lobes of radio emission embedded in the X-ray emitting gaseous haloes surrounding the host galaxies. There is also growing theoretical evidence that outflows from AGNs play a significant role in the evolution of their surroundings (e.g. Brüggen & Kaiser 2002; Croton et al. 2006; Sijacki & Springel 2006; Bower et al. 2006).

In theoretical work, outflows from AGNs are commonly used to prevent catastrophic radiative cooling in both elliptical galaxies (e.g. Tabor & Binney 1993; Binney & Tabor 1993) and galaxy clusters (e.g. Churazov et al. 2001; Brüggen & Kaiser 2002; Brüggen 2003; Basson & Alexander 2003; Omma et al. 2004; Dalla Vecchia et al. 2004; Cattaneo & Teyssier 2006). However, for there to be a long term balance between the heating and cooling processes there must be some sort of feedback mechanism through which the cooling gas triggers AGN outbursts. The level of jet activity appears to be a function of the environment and black hole mass (e.g. Burns 1994; Best et al. 2002; Best 2007). In addition, observations also suggest that the heating must be relatively gentle and well matched to its environment (e.g. Fabian & Sanders 2000).

In numerical simulations, the jet power of an AGN is often related directly to the effective inflow rate of material through an inner boundary of the computational grid (e.g. Vernaleo & Reynolds 2006; Cattaneo & Teyssier 2006). In reality the picture is not so simple since the overall accretion rate onto a supermassive black hole, located in an elliptical galaxy, is probably a combination of its local, and large-scale, environments (e.g. Sokolov 2000; Hardcastle et al. 2007; Cen 2007). The local accretion rate is probably governed by Bondi accretion (e.g. Allen et al. 2006). However, numerical simulations suggest that Bondi accretion is not steady, but can be highly variable (e.g. Edgar 2004, and references therein). We also observe extremely powerful, cluster-scale outbursts, e.g. Hydra-A (Nulsen et al. 2005). These outbursts are probably triggered by accretion of material from the wider cluster environment, rather than Bondi accretion. In this scenario, one could envisage cold ‘blobs’ of material which drop out of the flow around a galaxy, ‘diffusing’ towards the central black hole. Clearly this process would be expected to be intrinsically variable.

Most of the infalling material will have appreciable angular momentum; somehow this gas must be funnelled down onto the black hole, presumably through an accretion disc. This process will mediate the accretion rate from the external system that surrounds the black hole. Depending on the disc processes, the instantaneous black hole accretion rate

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need not be strongly correlated with the external (systemic) accretion rate.

Another source of accretion rate variability would be tidal interactions and mergers with nearby galaxies (e.g. Hopkins et al. 2006). Furthermore, the dynamic state of the accreted medium will also affect the black hole accretion rate. In this respect, some advances in numerical models have been made using initial conditions based on the simulation of the Springel et al. (2001) by Heinz et al. (2003).

In short, it is easy to understand how the accretion rate onto an AGN could be highly variable, on short timescales, compared to the approach assumed in many hydrodynamic simulations. However, adequately resolving the required range of scales is clearly impractical with current computing constraints: most numerical simulations of AGN feedback do not even resolve scales as small as the Bondi radius, let alone the accretion disc.

Although there is not extensive literature, the importance of random, or stochastic, accretion is slowly becoming more apparent. Hopkins & Hernquist (2006) employ the phenomenon to investigate the fueling of low-level AGN, and its effect on the black hole - bulge relations. In their model, they assume that cold gas stochastically accretes onto a central supermassive black hole at a rate set by the dynamics of that gas. In particular, clouds of molecular gas collide with the supermassive black hole. This provides a distinction between local, low-luminosity quiescent AGN activity, and violent, merger-driven bright quasars.

King & Pringle (2006) and King & Pringle (2007) also study the effect of small-scale, randomly oriented accretion event, as a mechanism for feeding nearby AGN. King & Pringle (2006) conclude that supermassive black holes can grow rapidly if most of the mass comes from a sequence of randomly oriented episodes whose angular momenta are no larger than the angular momentum of the black hole. This means that the black hole has a low spin, and therefore a low radiative efficiency, so rapid growth is possible without exceeding the Eddington limit. Their model requires the accretion hydrodynamics to be chaotic at the level of the size of the disc: < 0.1pc. This model is extended by Navakshin & King (2007) who find that, due to the random orientation of the accretion disc in these events, energy can be feedback into the surroundings in an almost isotropic manner. This is important since it is much more efficient than anisotropic heating.

Nipoti & Binney (2005) also argue that the mechanical luminosity of an AGN is expected to fluctuate across a wide range of timescales, while the X-ray luminosity of the cluster gas will only vary slowly and weakly with time. Since energy injection by AGN is expected to balance the radiative losses of the cluster atmosphere, and \( K_1 \) is a constant. If we also assume that \( \dot{H} \) and \( \dot{m} \) vary on much shorter timescales than \( L_X \), we can treat \( L_X \) as approximately constant. In the absence of stochastic accretion we can write \( H = K_2 \dot{m} \), where \( K_2 = \eta c^2 \approx 10^{50} \)cm/s. Solving equation (1), assuming that the accretion rate is zero at \( t = 0 \), gives

\[
\frac{\dot{m}(t)}{\dot{m}} = K_1 \left( L_X - H \right),
\]

where \( H \) is the energy injection rate by the AGN, \( L_X \) are the radiative losses of the cluster atmosphere, and \( K_1 \) is a constant. If we also assume that \( \dot{H} \) and \( \dot{m} \) vary on much shorter timescales than \( L_X \), we can treat \( L_X \) as approximately constant. In the absence of stochastic accretion we can write \( H = K_2 \dot{m} \), where \( K_2 = \eta c^2 \approx 10^{50} \)cm/s. Solving equation (1), assuming that the accretion rate is zero at \( t = 0 \), gives

\[
\dot{m}(t) = L_X \frac{K_2}{K_1} \left[ 1 - \exp \left( -\frac{t}{\tau} \right) \right],
\]

where \( \tau \equiv 1/(K_1 K_2) \). In other words, the system tends towards a state with a steady accretion rate of \( L_X / K_2 \), as time passes.

If there is a time-delay in the AGN response we would expect the accretion rate to overshoot the steady-state value, and subsequently perform damped sinusoidal oscillations around this point. Of course, if the time-delay is so large that the overshoot is equal to the steady-state accretion rate, then the system will be unstable.

The value of \( K_1 \) is slightly harder to evaluate; we esti-
mate it in the following way,
\[
\frac{dm}{dt} \sim \frac{\dot{m}}{t_{\text{cool}}} = K_1 L_X. \tag{3}
\]
We can write \( L_X = 5k_b T m_c/(2\mu m_p) \), where \( m_c \) is the mass dropout rate from the ICM, which is distinct from \( \dot{m} \) - the accretion rate onto the black hole. Thus we have
\[
\tau \approx 4 \times 10^3 \left( \frac{T}{10^7 \text{K}} \right) \left( \frac{t_{\text{cool}}}{10^8 \text{yrs}} \right) \left( \frac{\dot{m}_c}{m_c} \right) \text{yrs}. \tag{4}
\]
We might reasonably expect \( \dot{m}_c/\dot{m} \sim 100 \) (e.g. Pope et al. 2004) so that \( \tau \sim 10^3 \text{yrs} \). This is much shorter than typical radiative cooling times, thus justifying our initial approximation that the accretion rate varied on comparatively short timescales. This also indicates that the heating response quickly establishes an equilibrium, but that \( \tau \) increases with the temperature of the ICM, suggesting that more massive clusters have a longer response time.

Now, if we also consider the effect of stochastic accretion, we can write \( H = K_2 \dot{m} + \Delta H \), where \( \Delta H \) is a random variable that can be positive or negative. Since the system also has a steady-state accretion rate, \( \langle \dot{m} \rangle = L_X/K_2 \), we can write \( \dot{m} = \langle \dot{m} \rangle + \Delta \dot{m} \),
\[
\frac{d\Delta \dot{m}}{dt} = -\frac{\Delta \dot{m}}{\tau} - K_1 \Delta H(t). \tag{5}
\]
Equation \( \text{(5)} \) has the form of the Langevin equation where \( -K_1 \Delta H = \sigma_N f(t) \). \( f(t) \) is a Gaussian ‘white-noise’ process, with zero mean and unit standard deviation; \( \sigma_N \), is the standard deviation of the noisy process. The variance of \( \Delta \dot{m} \) is given by,
\[
\sigma^2 = \frac{\sigma_N^2 \tau}{2}, \tag{6}
\]
which demonstrates that seed fluctuations of \( \langle \dot{m} \rangle/\sqrt{\tau} \ll \langle \dot{m} \rangle \) can produce AGN power fluctuations that are comparable with the mean accretion rate.

The transformation of a random variable through a differential equation, to obtain a probability distribution, is notoriously complex (see Chatterjee et al. 2002, for an example). However, in the case described the probability distribution is Gaussian,
\[
g(\langle \dot{m} \rangle) d\langle \dot{m} \rangle = \frac{1}{\sqrt{\pi \sigma_N^2 \tau}} \exp \left[ -\frac{(\langle \dot{m} \rangle - \langle \dot{m} \rangle)^2}{\sigma_N^2 \tau} \right], \tag{7}
\]
where the width of the distribution grows with \( \tau \). Equation \( \text{(4)} \) shows that \( \tau \) increases with cluster temperature, which in turn, increases with cluster mass. Therefore, from equation \( \text{(6)} \) we should expect to observe greater variability in more massive systems. However, the ratio \( \langle \dot{m} \rangle/\sigma \) may decrease with increasing cluster mass, although it is not clear how \( t_{\text{cool}} \) and \( \dot{m}_c/\dot{m}_c \) scale with cluster mass.

Discretising equation \( \text{(5)} \) gives the standard form of a first-order auto-regressive process (e.g. Karlin & Taylor 1972; King et al. 2004).
\[
X_t = \left( 1 - \frac{\Delta t}{\tau} \right) X_{t-1} + \sigma_N \Delta t f(t), \tag{8}
\]
where \( \Delta t \) is the time-step of integration. This is a first-order auto-regressive process because the variable at time \( t \) depends on only the value at \( t - \Delta t \), and is only physically meaningful if \( 0 \leq 1 - \frac{\Delta t}{\tau} \leq 1 \). Using equation \( \text{(8)} \) we can generate \( X_t \) as a function of time, variance given by (e.g. Karlin & Taylor 1972),
\[
\sigma_\xi^2 = \frac{\sigma_N^2}{1 - \phi^2}, \tag{9}
\]
where \( \phi = 1 - \Delta t/\tau \). Note that, in this case \( \Delta t = 1 \), and in the limit that \( \tau \gg 1 \), \( \sigma_\xi^2 = \sigma_N^2/2 \), in agreement with equation \( \text{(6)} \). Two light curves generated using equation \( \text{(10)} \) are shown in Figure 1.

Even if the AGN power output is predominantly described by a Gaussian probability distribution, the distribution is likely to be truncated somewhat. This is because the minimum AGN power is zero, while the maximum steady power output is governed by the Eddington limit. The truncation of the distribution will be negligible if the width of the probability distribution is small compared to the mean. However, this is probably not the case, otherwise we would expect to observe a high proportion of objects injecting energy at this rate. Thus, the distribution could only be symmetric if the Eddington limit is equal to twice the time-averaged AGN power output, which is approximately equal to \( L_X \). A simple inspection of the available data (e.g. Fujita & Reiprich 2004) demonstrates that this is probably not the case.

2.1 Non-linear, asymmetric feedback

In equation \( \text{(1)} \) we assumed that \( L_X \) varied slowly, so that it could be treated as approximately constant. However, if the radiative losses are not matched by AGN we heating we would reasonably expect that the radiative losses to increase. The growth of radiative losses is a positive feedback effect, so it is easy to see that the system overall could be non-linear and asymmetric for perturbations about the current, and temporary, steady-state. For example (e.g. von Storch & Zwiers 2004),
\[
\frac{dX}{dt} = c_0 - c_1 X - c_2 X^2 + f, \tag{10}
\]
where \( c_1 > 0 \), and \( f \) is the white-noise random variable. Regardless of these considerations, given that the accretion rate is positive, definite, it seems likely that a skewed probability distribution will result, whatever its origin.

In equation \( \text{(10)} \), the damping is stronger to the lower side of the equilibrium state, than for deviations to the larger side. As a result the probability density function (pdf) is no longer Gaussian, but will be positively skewed and deviations to larger values are more likely than to smaller. To better understand this, we can think of the feedback equation as a randomly forced, classical particle moving in a potential, \( U \). The deterministic forcing component is given by the negative derivative of the potential, \( \frac{dU}{dt} = -\frac{dU}{dt} + f \), thus for equation \( \text{(5)} \): \( U = X^2/(2\tau) \) is symmetric, so we would expect the probability distribution of \( X_t \) also to be symmetric. In contrast, for equation \( \text{(10)} \): \( U = -c_0 X + (1/2)c_1 X^2 + (1/3)c_2 X^3 \). This is clearly asymmetric and the pdf of the \( X_t \) values will be skewed towards the less steep slope.

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**Notes:**
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**Cross-references:**
- Karlin & Taylor 1972
- Pope et al. 2004
- Fujita & Reiprich 2004
- von Storch & Zwiers 2004
3 THE CORRELATION BETWEEN INSTANTANEOUS ACCRETION RATE AND JET POWER

AGN jets are commonly observed to inflate cavities in the X-ray emitting gas that surrounds galaxies and permeates clusters. The jet powers can be estimated by dividing the cavity enthalpy by a characteristic timescale, (e.g. Birzan et al. 2004; Dunn et al. 2005; Allen et al. 2006),

\[ P_{\text{jet}} = \frac{\gamma}{\gamma - 1} \frac{P V}{\tau} , \]  

(11)

where \( P \) is the ambient pressure, \( V \) is the volume of the cavity, \( \gamma \) is the adiabatic index of the material within the cavity, and \( \tau \) is the timescale. The pressure, and volume can be estimated from X-ray observations of galaxies and clusters. Note that this assumes that the cavity was inflated slowly, and in approximate pressure equilibrium with its surroundings.

The appropriate timescale is not clear, and there are several options to choose from: the time taken to buoyantly rise to the observed location, the time taken to refill the displaced volume as the cavity rises, and the sound crossing time for ICM material to traverse a distance comparable with the bubble radius.

The jet power, as given by equation (11), represents a time-averaged quantity over the duration of the cavity inflow. As a result it should correlate well with the mean accretion rate during this period. Allen et al. (2006) calculated the jet powers for a sample of 9 nearby X-ray luminous elliptical galaxies with good optical velocity dispersion measurements. The optical velocity dispersion measurements permit black hole mass estimates via the \( M_{\text{bh}} - \sigma \) relation (e.g. Tremaine et al. 2002). From this, and the X-ray data, Allen et al. (2006) also estimate the Bondi accretion rate in the vicinity of each supermassive black hole. It should be stated here that the value of the Bondi accretion rate is an instantaneous value which could vary significantly from the mean. They find the following correlation between the Bondi accretion power and jet power,

\[ \log(\frac{P_{\text{Bondi}}}{10^{43} \text{erg s}^{-1}}) = 0.65 + 0.77 \log(\frac{P_{\text{jet}}}{10^{43} \text{erg s}^{-1}}) \]  

(12)

Interestingly, the correlation is not linear which may simply be a consequence of the relatively small sample, or a systematic error in calculating the jet power, for example. Alternatively, the non-linear correlation may indicate that other important physical effects are operating; perhaps this is an indication of feedback between the jet and its fuel supply, or that the jet production efficiency is a function of jet power (e.g. Nemmen et al. 2007).

The possible explanation presented here is that the non-linear correlation arises directly from the distribution of the fluctuations in the instantaneous accretion rate. If the probability distribution of the accretion rate is symmetric about the mean then we would expect, on average, to observe a linear correlation between jet-power and instantaneous accretion rate. As a result this symmetric model of feedback given in equation (11) cannot provide an explanation for the possible non-linear correlation between jet power and Bondi accretion rate reported by Allen et al. (2006), even if the variance of the distribution is a function of the mean accretion rate. However, a skewed probability distribution of accretion rates may be a possible explanation, since the mode and mean will not be identical. Yet, even this fact is not enough to explain the observed non-linear correlation: if \( P(\bar{n}/\mu \geq 1) = \text{constant} \neq 0.5 \), and not a function of \( \mu \) (or the variance) the correlation between jet power (\( \mu \)) and instantaneous accretion rate \( \bar{n} \) will still be linear. If \( P(\bar{n}/\mu \geq 1) > 0.5 \), the best-fit line determined from observations may over-estimate the normalisation of the relationship since we expect to observe larger accretion rates more frequently. The reverse is also true. However, if the skewness of the probability distribution is not constant, but is a function of \( \mu \), this could lead to the observed non-linear correlation.

Nipoti & Binney (2005) assumed that the X-ray luminosity of the cluster gas was slowly varying, while the AGN power varied much more rapidly. In a similar manner we will assume that the jet power, estimated using equation (11), is also slowly varying compared to the instantaneous Bondi accretion rate. Following the same arguments as Nipoti & Binney (2005) further, it makes sense to define the variable \( y \) as the ratio of the accretion power obtained from the instantaneous Bondi accretion rate, \( P_{\text{Bondi}} \) and the jet power \( P_{\text{jet}} \),

\[ y = \frac{P_{\text{Bondi}}}{P_{\text{jet}}} \]  

(13)

Again, we can argue that \( y \) is log-normally distributed,

\[ g(y) = \frac{1}{\sqrt{2\pi} S_y} \exp\left[-\frac{(\ln y - M)^2}{2S^2}\right] , \]  

(14)

for \( y > 0 \), where \( M \) and \( S \) are the mean and standard deviation of the probability density function, and \( y \) is defined as the ratio of the accretion power obtained from the instantaneous Bondi accretion rate, \( P_{\text{Bondi}} \) and the jet power \( P_{\text{jet}} \),.
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ation of ln y. The expectation of y is

\[\langle y \rangle = \exp \left( M + \frac{S^2}{2} \right).\] (15)

This quantity can be directly estimated from the observed best-fit relation between \(P_{\text{Bondi}}\) and \(P_{\text{jet}}\). Taking the Bondi accretion rate-Jet power relation from equation (12) and assuming that \(M = 0\), for simplicity, we find,

\[\langle y \rangle \approx \text{best} - \text{fit} \left( \frac{P_{\text{Bondi}}}{P_{\text{jet}}} \right) = 3.4 \times 10^{10} P_{\text{jet}}^{-0.23} = \exp \left( \frac{S^2}{2} \right).\] (16)

This tells us that the variance of the distribution is a function of jet power,

\[S^2 \approx 3 - 0.46 \ln \left( \frac{P_{\text{jet}}}{10^{44} \text{erg s}^{-1}} \right).\] (17)

so that \(S^2 \sim 3\). Such values are comparable with those found by Nipoti & Binney (2003) in their model.

The duty cycle for a source with light curve \(L(t)\) is defined by Ciotti & Ostriker (2001) as,

\[\delta \equiv \frac{\langle L \rangle^2}{\langle L^2 \rangle} = \exp(S^2).\] (18)

Substituting equation (17) into equation (18) tells us that the duty cycle is also a function of jet power (probably indirectly),

\[\delta \approx 0.05 \left( \frac{P_{\text{jet}}}{10^{44}} \right)^{0.46}.\] (19)

Since there is probably a close balance between the jet power and the radiative losses of the X-ray halo around the galaxy we may assume that \(\langle P_{\text{jet}} \rangle \approx L_X\). Therefore, we can argue that this result strongly suggests that AGN in larger galaxies, hence larger \(L_X\), are active for a greater fraction of the time. In fact, in elliptical galaxies \(L_X\) is roughly proportional to the square of the mass of the black hole, \(m_{\text{bh}}\), (e.g. Best et al. 2005) at the centre of the galaxy. Therefore we might expect \(\delta \propto m_{\text{bh}}^{-1/2}\) in elliptical galaxies.

In reality the observed non-linear correlation given in Allen et al. (2006) may not be real, but only a statistical anomaly. Nevertheless, it is interesting to note that the implications for the duty cycle qualitatively agree with the observational result of Best et al. (2005). Their result shows that the fraction of AGN in elliptical galaxies, which are detected in the radio, increases with the mass of the black hole at the centre of the galaxy. This can be interpreted probabilistically as the fraction of time the AGN is active, i.e. the duty cycle. At low black hole masses \((10^6 - 10^7 M_\odot)\), the observed relationship given by Best et al. (2005) scales as \(m_{\text{bh}}^{-0.6}\), although this flattens significantly at higher masses. Interestingly, the estimate of the duty cycle presented here applies for black hole masses of \(~8-9 \times 10^7 M_\odot\), which agrees quantitatively with the flattening in the Best et al. (2005) relation at large black hole masses.

3.1 Stochastic variability and gentle AGN heating

Under certain circumstances, it is possible to imagine that AGN activity is triggered periodically, or exhibits periodic activity of some sort. \(^1\) This could lead to large variations in the temperature, and thus large deviations from equilibrium. Yet, observations of metal distributions and cooling time profiles in clusters seem to suggest that there is an extremely close balance between heating and cooling in galaxy clusters. This is what one would expect from continuous feedback-controlled heating. However, in the simplest case the ensuing steady-state could result in the AGN being permanently active with a constant power output (e.g. Hoelt & Brüggen 2004). (This example is of interest considering the temporal form of the accretion rate, which appears very similar to equation (11), in some cases). However, this is not what we expect from the stochastic feedback model discussed above. Instead, the rapid stochastic variations in the accretion rate mean that the AGN injects energy across a wide range of timescales. Thus, the AGN can maintain a much closer balance with its surroundings than if it was periodically activated.

For a skewed probability distribution of AGN power output, much of this heating must occur at relatively low powers and so will provide the gentle heating that is consistent with observations. The power spectral density of the equation (15) described above is typical of a ‘red-noise’ process (e.g. Vaughan et al. 2003), with \(\Gamma(\omega) \approx \frac{a \omega}{\omega^2 + b^2}\), where \(a\) and \(b\) are constants, and \(\omega\) is the angular frequency of the variations.

3.2 Stability of the system

Time-delay in control systems tends to reduce the stability of the system. Indeed, for sufficiently long delays the system will be completely unstable. In the galaxy cluster example, if the heating response from the AGN is longer than the local cooling time, the system is unlikely to be stable.

The main components that contribute to the time-delay are: 1) the time taken for inflowing gas to reach the black hole, and 2) the time taken for the energy feedback into the ICM to be dissipated. Both components are difficult to estimate, but the former more so since it is probably composed of at least two stages: the inflow from large radii, and inflow through an accretion disc. For example, the gas presumably has some angular momentum, and should stagnate at a radius (e.g. Sarazin 1986, and references therein),

\[r_{st} = \frac{l^2}{GM(< r_{st})},\] (20)

where \(l\) is the angular momentum per unit mass of the gas, \(G\) is Newton’s gravitational constant, and \(M(< r_{st})\) is the mass within the stagnation radius. This radius is expected to be of the order of a few kiloparsecs (Sarazin 1986). Dynamical interactions could then scatter the gas clouds and allow a fraction to move closer to the black hole.

The majority of the material will pass through an accretion disc that encircles the black hole; the inflow timescale

\[^1\] In an alternative model we could replace \((L_X - H)\) with \(\int (L_X - H)dt\) in equation (11), thus the AGN power output would vary sinusoidally between 0 and \(K_2 L_X / K_1\). The period of the AGN variability would be \(\tau = 2\pi / \sqrt{(K_1 K_2)}\). In this case \(K_1\) would have a different value to the model described in the main text, but would still probably increase with galaxy, or cluster, mass.
being given by the viscous drift timescale \( t_{\text{vis}} \sim r^2/\nu \), where \( \nu \) is the viscosity and \( r \) is the radial distance from the black hole. Czerny (2004) gives this time as \( \sim 1 \text{Myr} \) at roughly 0.1 pc for a disc around a \( 10^8 M_\odot \) black hole.

The time taken to dissipate the injected energy is also unknown, but it is conceivable that the heating response of an AGN to a particular event can have a significant time lag which could destabilise the system. This could have dramatic consequences were it not for the presence of small stochastic fluctuations which can give rise to extremely powerful, long lived AGN outbursts. Such outbursts can occur at any time and may be sufficient to stabilise the system. There is a slight irony in this possibility since in other systems, stochastic variability is usually thought of as a destabilising force.

4 SUMMARY

There are many processes in the accretion of material on to an AGN that occur on relatively short timescales compared to the cooling time of the X-ray emitting gas in and around elliptical galaxies and galaxy clusters. In a feedback system these stochastic variations have been shown to lead to much longer and larger fluctuations in the AGN power output. Larger fluctuations occur in systems with longer response times, e.g. rich galaxy clusters. The rapid variability also means that power is injected across a range of timescales allowing an intimate balance between heating and cooling.

The possible non-linear correlation between Bondi accretion rate and jet power in the Allen et al. (2006) sample can be explained by a skewed probability distribution for the ratio of the instantaneous accretion rate to the jet power. This explanation also predicts that the duty-cycle of AGN activity varies with its environment, or black hole mass.

Stochastic variability of the accretion rate may help to stabilise the system against the large time-delays that are probably inherent with AGN feedback.

5 ACKNOWLEDGEMENTS

The author would like to thank Christian Kaiser, David Pope, Georgi Pavlovski, Jim Hinton and Willem-Jan De Wit for useful discussions, and the anonymous referee for constructive comments.

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