INCLUSIVE $B_c$ DECAYS AS A QCD LAB

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Abstract

Phenomenological models of heavy flavour decays differ significantly in their predictions of global features of $B_c$ decays, like the $B_c$ lifetime or the relative weight of $c \to s$ and $b \to c$ transitions. The $1/m_Q$ expansion which is directly based on QCD allows predictions on the pattern to be expected, namely $\tau(B_c)$ to lie well below 1 psec with $c \to s$ dominating over $b \to c$ and a reduced semileptonic branching ratio. Due to interference effects one also predicts a lower charm content in the final states of $B_c$ decays than naively anticipated. The numerical aspect of the predictions, however, has to be viewed with considerable caution since one cannot expect the $1/m_c$ expansion to converge readily for $\Delta C = 1$ transitions.

1 Introduction

$B_c$ mesons consisting of two heavy quarks – $B_c = (b\bar{c})$ – are not easily produced. On the other hand it is highly desirable to obtain large samples of them. For their study would deepen our quantitative understanding of the inner workings of QCD in a significant way: one expects a rich spectroscopy for the $(b\bar{c})$ boundstates probing the inter-quark potential at distances intermediate to those determining quarkonia spectroscopy in the charm and beauty systems [1]; the Isgur-Wise function for the striking channel $B_c \to l\nu\psi$ can be calculated; the weak $B_c$ decays reflect a multifaceted interplay of various dynamical mechanisms. It is this last aspect I will analyze in this note: in Sect.2 I will review previous phenomenological descriptions of $B_c$ decays; in Sect.3 I introduce a genuine QCD treatment based on the heavy quark expansion and apply it to inclusive $B_c$ transitions; in Sect.4 I discuss the final states in $B_c$ decays before presenting my conclusions in Sect.5.
A First Phenomenological Look at $B_c$ Decays

There are three classes of transitions contributing to $B_c$ decays with roughly comparable strength; they appear to be easily distinguishable on the diagrammatic level. The first two are the decay of the $b$ quark and that of the $\bar{c}$ antiquark. Since $b \to c$ and $\bar{c} \to \bar{s}$ transitions do not interfere with each other in any appreciable way, one can cleanly separate their widths. The only subtlety here is that $b \to \bar{c}\bar{c}s$ decays lead to two $\bar{c}$ antiquarks in $B_c$ decays; there arises then an interference between different decay amplitudes that is usually referred to as PI. The third class of transitions is produced by Weak Annihilation (WA) of $b$ with $\bar{c}$. To lowest order in the strong interactions the WA amplitude suffers helicity and wavefunction suppression (the latter reflecting the practically zero range of the weak interactions). Yet for $B_c \to s\bar{c}$ they are represented by $m_c/m_b$ and $f(B_c)/m_b$ and thus relatively mild ($f(B_c) \sim 450 - 700$ MeV [1]); furthermore these reductions are partially offset by the factor $16\pi^2$ reflecting the enhancement of two-body phase space – relevant for WA – over three-body phase space appropriate for the spectator decay. As explained later, interference between $b$ decay and WA can arise; yet this is usually ignored in phenomenological analyses.

One thus writes down for the nonleptonic width:

$$\Gamma_{NL}(B_c) = \Gamma_{\text{spect}}^{\bar{c} \to \bar{s}d\bar{u}}(B_c) + \Gamma_{\text{spect}}^{b \to c\bar{c}u}(B_c) + \Gamma_{\text{spect}}^{b \to c\bar{c}s}(B_c) - |\Delta \Gamma_{b \to c\bar{c}s}^{PI}(B_c)| + \Gamma_W(B_c) \quad (1)$$

The expression simplifies for the semileptonic width since no interference occurs there and WA does not contribute (at least to lowest order in the strong coupling)\footnote{Only $B_c \to \tau^+\nu$ leading to a purely leptonic final state possesses an appreciable rate.}:

$$\Gamma_{SL}(B_c) = \Gamma_{\text{spect}}^{\bar{c} \to \bar{s}d\bar{u}}(B_c) + \Gamma_{\text{spect}}^{b \to c\bar{d}u}(B_c) \quad (2)$$

Very naively one might equate $\Gamma_{\bar{c} \to \bar{s}}(B_c)$ with $\Gamma(D^0)$ and $\Gamma_{b \to c}(B_c)$ with $\Gamma(B_d)$ and thus

$$\Gamma(B_c) \simeq \Gamma(D^0) + \Gamma(B_d) \quad (3)$$

If eq.(3) were to hold, the $B_c$ lifetime would be rather short, namely

$$\tau(B_c) \sim 3 \cdot 10^{-13} \text{ sec} \quad (4)$$

and $B_c$ decays would be dominated by $\bar{c} \to \bar{s}$ over $b \to c$ in the ratio of roughly 4:1. It is quite natural, though, to suspect that eq.(3) represents a gross oversimplification. Two specific alternatives have been suggested:

(i) The phase space in $B_c = (b\bar{c}) \to bs\bar{d}\bar{u} \simeq B_s(d\bar{u})$ is more limited than in $\bar{D} = (q\bar{c}) \to q\bar{s}d\bar{u} \simeq K(d\bar{u})$. This could – due to the high sensitivity of the $c$ decay width to the available phase space – reduce $\Gamma_{\bar{c} \to \bar{s}}(B_c)$ significantly relative to $\Gamma(D)$. No such reduction is expected for $\Gamma_{b \to c}(B_c)$. Therefore the relative weight of $b \to c$ transitions...
in $B_c$ decays gets enhanced. Using a simple recipe for estimating the phase space dependance of the quark decay width the authors of ref.\[2\] estimate
\[\tau(B_c) \sim 5 \times 10^{-13} \text{ sec}\] (5)
with – and that is the major difference to the naive guestimate given above – $\bar{c} \to \bar{s}$ transitions now holding only a slight edge over $b \to c$ decays.

(ii) It has been advocated by Eichten and Quigg that in the expression for the decay width $-\Gamma_Q \propto G_F^2 m_Q^5$ one should use a quark mass reduced by the binding energy inside the hadron. For the ordinary mesons $B_d$, $B_u$ and $B_s$ this can be seen effectively as a redefinition of the quark mass. Yet for the more tightly bound system $B_c$ there arises an observable difference: the binding energy $\mu_{BE}$ being the same for the charm and for the beauty mass represents a larger fraction of the charm mass than of the beauty mass: $(m_c - \mu_{BE})/m_c = 1 - \mu_{BE}/m_c < (m_b - \mu_{BE})/m_b = 1 - \mu_{BE}/m_b$. Inside the $B_c$ meson the $\bar{c} \to \bar{s}$ rate will therefore be more reduced than the $b \to c$ rate. Furthermore the impact of this mass shift on the width is greatly enhanced: $\Delta \Gamma_Q/\Gamma_Q \simeq 5 \cdot \mu_{BE}/m_Q$ due to $\Gamma_Q \propto m_Q^5$! For $\mu_{BE} = 500$ MeV one finds $\Gamma_{\bar{c} \to \bar{s}}$ and $\Gamma_{b \to c}$ reduced by a factor of 6 and 1.7, respectively! This leads to the guestimate
\[\tau(B_c) \sim 1.3 \text{ psec},\] (6)
i.e., a considerably longer $B_c$ lifetime; the $b \to c$ transitions now occur somewhat more frequently than the $\bar{c} \to \bar{s}$ ones.

The two questions raised above – (i) whether the $B_c$ lifetimes is short, i.e. well below 1 psec, or ‘long’, i.e. roughly 1 psec or longer, and (ii) whether $B_c$ decays are driven mainly by $b \to c$ or by $\bar{c} \to \bar{s}$ transitions – are highly important and deserve study with the best available theoretical tool, the $1/m_Q$ expansion.

3 Treating $B_c$ Decays through a $1/m_Q$ Expansion

3.1 General Methodology

In analogy to the treatment of $e^+e^- \to hadrons$ one describes the transition rate into an inclusive final state $f$ through the imaginary part of a forward scattering operator evaluated to second order in the weak interactions \[3, 4\]:
\[\hat{T}(Q \to f \to Q) = i \text{Im} \int d^4x \{\mathcal{L}_W(x)\mathcal{L}_W^\dagger(0)\}_T\] (7)
where $\{.\}_T$ denotes the time ordered product and $\mathcal{L}_W$ the relevant effective weak Lagrangian expressed on the parton level. If the energy release in the decay is sufficiently large one can express the non-local operator product in eq.(7) as an infinite
sum of local operators \( O_i \) of increasing dimension with coefficients containing higher and higher powers of \( 1/m_Q \). The width for \( H_Q \to f \) is obtained by taking the expectation value of \( \hat{T} \) between the state \( H_Q \). For semileptonic and nonleptonic decays treated through order \( 1/m_Q^3 \) one arrives at the following generic expression\[4\]:

\[
\Gamma(H_Q \to f) = \frac{G_F m_Q^5}{192 \pi^3} |KM|^2 c_f^3 \langle H_Q | \bar{Q}Q | H_Q \rangle + c_f^5 \frac{\langle H_Q | \bar{Q}i \sigma \cdot GQ | H_Q \rangle}{m_Q^2} + \sum_i c_f^6 \frac{\langle H_Q | (\bar{Q}\Gamma_i q)(\bar{q}\Gamma_i Q) | H_Q \rangle}{m_Q^3} + O(1/m_Q^4)
\]

(8)

where the dimensionless coefficients \( c_f^i \) depend on the parton level characteristics of \( f \) (such as the ratios of the final-state quark masses to \( m_Q \)); \( KM \) denotes the appropriate combination of KM parameters, and \( \sigma \cdot G = \sigma_{\mu\nu} G_{\mu\nu} \) with \( G_{\mu\nu} \) being the gluonic field strength tensor. The last term in eq.(8) implies also the summation over the four-fermion operators with different light flavours \( q \). It is through the quantities \( \langle H_Q | O_i | H_Q \rangle \) that the dependence on the decaying hadron \( H_Q \), and on non-perturbative forces in general, enters; they reflect the fact that the weak decay of the heavy quark \( Q \) does not proceed in empty space, but within a cloud of light degrees of freedom – (anti)quarks and gluons – with which \( Q \) and its decay products can interact strongly. These are matrix elements for on-shell hadrons \( H_Q \); \( \Gamma(H_Q \to f) \) is thus expanded into a power series in \( \mu_{\text{had}}/m_Q < 1 \). For \( m_Q \to \infty \) the contribution from the lowest dimensional operator obviously dominates; here it is the dimension-three operator \( \bar{Q}Q \). Since \( \langle H_Q | \bar{Q}Q | H_Q \rangle = 1 + O(1/m_Q^2) \) holds, one reads off from eq.(8) that the leading contribution to the total decay width is universal for all hadrons of a given heavy-flavour quantum number; i.e., for \( m_Q \to \infty \) one has derived – from QCD proper – the spectator picture. Contributions from what is referred to as WA and PI in the original phenomenological descriptions are systematically and consistently included through the dimension-six four-fermion operators in eq.(8).

Yet the \( 1/m_Q \) expansion goes well beyond reproducing familiar results. It shows the leading nonperturbative corrections to integrated inclusive rates to arise in order \( 1/m_Q^2 \) controlled by the expectation values of dimension-five operators \[4\]. These terms had been overlooked before. What is crucial for our subsequent analysis is the absence of contributions of order \( 1/m_Q \). This is due to the fact that there is no relevant dimension-four operator that cannot be removed by applying the equation of motion \[4 \]. It can also be understood as due to a subtle intervention of the local colour gauge symmetry. A phenomenological ansatz on the other hand where the quark mass appearing in the decay width is reduced by a ‘binding energy’ leads to large corrections of order \( 1/m_Q \); see the discussion above eq.(6). This is in clear conflict with what holds in QCD!

Using the equation of motion one can obtain a \( 1/m_Q \) expansion also for the leading
operator in eq. (8), $\bar{Q}Q$. Its expectation value can then be expressed as follows:

$$\langle H_Q | \bar{Q}Q | H_Q \rangle = 1 - \frac{\langle (\vec{p}_Q)^2 \rangle_{H_Q}}{2m_Q^2} + \frac{\langle \mu_G^2 \rangle_{H_Q}}{2m_Q^2} + O(1/m_Q^3)$$

(9)

where $\langle (\vec{p}_Q)^2 \rangle_{H_Q} \equiv \langle H_Q | \bar{Q}(i\vec{D})^2Q | H_Q \rangle$ denotes the average kinetic energy of the quark $Q$ moving inside the hadron $H_Q$ and $\langle \mu_G^2 \rangle_{H_Q} \equiv \langle H_Q | \bar{Q}\frac{1}{2} \sigma \cdot GQ | H_Q \rangle$.

Eqs. (8,9) show that the nonperturbative contributions to the width through order $1/m_Q^3$ can be expressed through three expectation values: $\langle \mu_G^2 \rangle_{H_Q}$, $\langle (\vec{p}_Q)^2 \rangle_{H_Q}$ and $\langle H_Q(p) | \bar{Q}_L \gamma_\mu q_L (\bar{q}_L \gamma_\nu Q_L) | H_Q(p) \rangle$. The size of the mesonic matrix element of the chromomagnetic operator is obtained from the hyper-fine splitting:

$$\langle \mu_G^2 \rangle_{P_Q} \simeq \frac{3}{4} (M_{V_Q}^2 - M_{P_Q}^2)$$

(10)

where $P_Q$ and $V_Q$ denote the pseudoscalar and vector mesons, respectively. For the average kinetic energy we have the model-independent bound [3]

$$\langle (\vec{p}_Q)^2 \rangle_{H_Q} \geq \langle \mu_G^2 \rangle_{H_Q}$$

(11)

and we know it cannot be much larger than that. The $1/m_Q^2$ corrections thus largely cancel in the expectation value of the operator $\bar{Q}Q$ between meson states:

$$\langle P_Q | \bar{Q}Q | P_Q \rangle \simeq 1 + O(1/m_Q^3)$$

The expectation values for the four-quark operators taken between meson states can be expressed in terms of a single quantity, namely the decay constant:

$$\langle H_Q(p) | \bar{Q}_L \gamma_\mu q_L (\bar{q}_L \gamma_\nu Q_L) | H_Q(p) \rangle \simeq \frac{1}{4} f_{H_Q}^2 p_\mu p_\nu$$

(12)

where factorization has been assumed.

### 3.2 $B_c$ Decays through Order $1/m_Q^2$

Since $b \to c$ and $\bar{c} \to \bar{s}$ decays do no interfere with each other in any practical way, one can cleanly separate their widths:

$$\Gamma(B_c) = \Gamma_{b\to c}^{decay}(B_c) + \Gamma_{\bar{c}\to \bar{s}}^{decay}(B_c) + O(1/m_{b,c}^3)$$

(13)

These widths are denoted as $\Gamma^{decay}$ rather than $\Gamma^{spect}$ for a reason: they describe the quark decays as proceeding in an environment shaped by the other components of the

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2 I use the relativistic normalization for the states.
decaying hadron $H_Q$ as expressed by the expectation values $\langle (\vec{p}_Q)^2 \rangle_{H_Q}$ and $\langle \mu^2_{G} \rangle_{H_Q}$; thus they go beyond the simple spectator picture.

The decay widths include $1/m^2_{Q}$ corrections which consist of the semileptonic and nonleptonic components:

\[ \Gamma^{\text{decay}}_{b \rightarrow cQ}(B_c) = \Gamma_{0}^{(b)} \cdot \langle B_c | \bar{c}c | B_c \rangle \left[ I_0(x_c, 0, 0) + \frac{\langle \mu^2_{G} \rangle_{B_c}}{m_b^2} (x \frac{d}{dx} - 2) I_0(x_c, 0, 0) \right] , \quad (14a) \]

\[ \Gamma^{\text{decay}}_{b \rightarrow cQ}(B_c) = \Gamma_{0}^{(b)} \cdot N_C \cdot \langle B_c | \bar{c}c | B_c \rangle \left[ A_0 \sum I_0(x_c) + \frac{\langle \mu^2_{G} \rangle_{B_c}}{m_b^2} (x \frac{d}{dx} - 2) \sum I_0(x_c) \right] - 8A_2 \frac{\langle \mu^2_{G} \rangle_{B_c}}{m_b^2} \cdot \left[ I_2(x_c, 0, 0) + I_2(x_c, x_c, 0) \right] \right] \cdot (14b) \]

\[ \Gamma_{0}^{(b)} = \frac{G_F m_b^5}{192 \pi^3} |V(cb)|^2 \quad (14c) \]

where the following notations have been used: $I_0$ and $I_2$ are phase-space factors:

\[ I_0(x, 0, 0) = (1 - x^2)(1 - 8x + x^2) - 12x^2 \log x \quad (15a) \]

\[ I_2(x, 0, 0) = (1 - x)^3 , \quad x_c = (m_c/m_b)^2 \quad (15b) \]

\[ I_0(x, x, 0) = v(1 - 14x - 2x^2 + 12x^3) + 24x^2(1 - x^2) \log \frac{1 + v}{1 - v} , \quad v = \sqrt{1 - 4x} \quad (15c) \]

\[ I_2(x, x, 0) = v(1 + \frac{x}{2} + 3x^2) - 3x(1 - 2x^2) \log \frac{1 + v}{1 - v} , \quad (15d) \]

with $I_{0,2}(x, x, 0)$ describing the $b \rightarrow c\bar{c}s$ transition, and $\sum I_0(x) \equiv I_0(x, 0, 0) + I_0(x, x, 0)$; $A_0 = \eta J$, $A_2 = (c^2 - c^2)/6$, where $\eta = (c^2 + 2c^2)/3$, and $J$ represents the effect of the subleading logarithms. With $x_c \simeq 0.08$ one obtains

\[ I_0(x, 0, 0)|_{x=0.08} \simeq 0.56 , \quad I_2(x, 0, 0)|_{x=0.08} \simeq 0.78 \text{ for } b \rightarrow c\bar{u}d \]

\[ I_0(x, x, 0)|_{x=0.08} \simeq 0.24 , \quad I_2(x, x, 0)|_{x=0.08} \simeq 0.32 \text{ for } b \rightarrow c\bar{c}s . \]

Since these functions are normalized to unity for $x = 0$, one notes that the final-state quark masses reduce the available phase space quite considerably in this reaction. The expressions are simpler for $c \rightarrow s$:

\[ \Gamma^{\text{decay}}_{c \rightarrow s Q}(B_c) = \Gamma_{0}^{(c)} \cdot \langle B_c | \bar{c}c | B_c \rangle \left[ I_0(x_s, 0, 0) + \frac{\langle \mu^2_{G} \rangle_{B_c}}{m_c^2} (x \frac{d}{dx} - 2) I_0(x_s, 0, 0) \right] , \quad (16a) \]

\[ \Gamma^{\text{decay}}_{c \rightarrow s Q}(B_c) = \Gamma_{0}^{(c)} \cdot N_C \cdot \langle B_c | \bar{c}c | B_c \rangle \left[ A_0 \sum I_0(x_s) + \frac{\langle \mu^2_{G} \rangle_{B_c}}{m_c^2} (x \frac{d}{dx} - 2) \sum I_0(x_s) \right] - 8A_2 \frac{\langle \mu^2_{G} \rangle_{B_c}}{m_c^2} \cdot I_2(x_s, 0, 0, 0) \right] \cdot (16b) \]
\[ \Gamma_0^{(c)} \equiv \frac{G_F^2 m_c^5}{192\pi^3} |V(c)s)|^2, \quad x_s = \frac{m_s^2}{m_c^2} \] (16c)

and the radiative corrections lumped into \( A_0 \) and \( A_2 \) are given by the appropriate values for \( c_+ \) and \( c_- \). With \( x_s \approx 0.012 \) one finds:

\[ I_0(x, 0, 0)|_{x=0.012} \simeq 0.91, \quad I_2(x, 0, 0)|_{x=0.012} \simeq 0.96, \]

i.e. there is much less phase space suppression than for \( b \to c \) transitions.

The transition operators driving \( B_c \) decays are the same that generate \( B \) and \( D \) decays. However their expectation values are evaluated for the \( B_c \) state, rather than the \( B \) and \( D \) state reflecting that the \( b \to c \) and \( \bar{c} \to \bar{s} \) transitions proceed in a different environment. The expectation value of the chromomagnetic operator is again given by hyperfine splitting between the masses of \( B_c^* \) and \( B_c \). Those have not been measured yet; on the other hand the theoretical predictions should be quite reliable for those. With \( M(B_c^*) \approx 6.33 \text{ GeV} \) and \( M(B_c) \approx 6.25 \text{ GeV} \) one obtains

\[ \langle B_c|\bar{b}\frac{i}{2}\sigma \cdot G b|B_c\rangle \simeq \langle \bar{B}_c|\bar{c}\frac{i}{2}\sigma \cdot G c|\bar{B}_c\rangle \simeq 0.75 \,(\text{GeV})^2, \] (17)

which is twice the value as for mesons with light antiquarks:

\[ \langle B|\bar{b}\frac{i}{2}\sigma \cdot G b|B\rangle \simeq 0.37 \,(\text{GeV})^2, \quad \langle D|\bar{c}\frac{i}{2}\sigma \cdot G c|D\rangle \simeq 0.41 \,(\text{GeV})^2. \] (18)

Thus

\[ \frac{\langle B_c|\bar{b}\frac{i}{2}\sigma \cdot G b|B_c\rangle}{m_b^2} \simeq 0.033 \] (19a)

\[ \frac{\langle \bar{B}_c|\bar{c}\frac{i}{2}\sigma \cdot G c|\bar{B}_c\rangle}{m_c^2} \simeq 0.38; \] (19b)

i.e., this correction becomes quite large in the \( c \to s \) transition.

Putting everything together one finds for the \( B_c \) width through order \( 1/m_{b,c}^2 \):

\[ \Gamma(B_c) \simeq 0.95 \cdot \Gamma(B_d) + 0.75 \cdot \Gamma(D^0) + \mathcal{O}(1/m_{b,c}^3) \sim (4.1 \cdot 10^{-13} \text{ sec})^{-1} \] (20)

\[ \frac{\Gamma_{b \to c}(B_c)}{\Gamma(B_c)} \sim 0.26; \] (21)

i.e., a short lifetime with \( c \to s \) transitions dominating all \( B_c \) decays! One also obtains a rather low semileptonic branching ratio

\[ BR_{SL}(B_c) \sim 6\% \] (22)

with half of the semileptonic \( B_c \) decays being generated by \( b \to cl\nu \). However these numbers have to be taken with quite a grain of salt. For the nonperturbative corrections in the \( c \to s \) component of the \( B_c \) width are very large, as indicated by eq.(19b). Thus one can hope only for a semi-quantitative treatment of that component.
3.3 Order $1/m_Q^2$ Contributions

In order $1/m_Q^2$ the explicitly flavour dependant terms appear that had been anticipated in previous phenomenological studies. Due to the large value predicted for $f(B_c)$ they are quite sizable: as indicated in eq.(1) PI reduces the rate for $b \to c\bar{c}s$ to proceed inside $B_c$ mesons by $\sim 20 - 40\%$ and WA contributes in an only mildly suppressed manner. In addition a more subtle effect arises that had not been incorporated into eq.(1): $\Gamma_{WA}(B_c)$ and $\Gamma_{spect}^{\Delta B=1}(B_c)$ can no longer be separated in a strict manner. For those two classes of reactions -- for $b \to c\bar{u}d$ as well as for $b \to c\bar{c}s$ modes -- can interfere with each other once gluon emission generates a $c\bar{c}$ pair in WA: $B_c \to b\bar{c}g^*g^* \to s\bar{c}(c\bar{c})g^*g^*$. While this observation has no relevant impact on the predicted overall lifetime, it becomes very important in the analysis of the final states to be given in the next section.

As far as the total lifetime is concerned, the most relevant effect is produced by WA which to lowest order in the strong interactions leads to a moderate reduction in lifetime:

$$\tau(B_c) \sim 4 \cdot 10^{-13} \text{sec}.$$  \hfill (23)

4 On the Pattern in the Final States

As stated already in eq.(1) PI reduces $\Gamma_{b \to c\bar{c}s}(B_c)$ considerably. The interference between WA and $b$ decays sketched above also reduces the charm content in the final state of $\Delta B = 1$ $B_c$ decays in general. The argument goes as follows: To describe the impact of WA on the decay rate beyond the lowest order in the strong interactions one has to include the emission of ‘off-shell’ as well as ‘on-shell’ gluons with the former hadronizing into a $c\bar{c}$ pair:

$$b\bar{c} \to d\bar{u}/s\bar{c} + g^*g^* \to d\bar{u}/s\bar{c} + c\bar{c}$$

For this reaction can interfere with the lowest order decay process. As shown in ref.\footnote{8}, this interference is destructive and it actually will reduce the rate for $B_c \to s\bar{c}c\bar{c}$ and quite possibly also for $B_c \to d\bar{u}s\bar{c}$ by roughly 5-10\%. This decrease in the decay rate is largely compensated for by $B_c \to d\bar{u}/s\bar{c} + gg$ where the gluon hadronizes mainly into light-flavour hadrons. Details will be discussed in a future publication.

5 Conclusions

$B_c$ decays represent a particularly intriguing lab to study the interplay of strong and weak forces in a non-trivial environment. The $1/m_Q$ expansion derived from QCD makes clear predictions on the global pattern:
• a short $B_c$ lifetime well below 1 psec;
• a preponderance of charm over beauty decays among the non-leptonic modes; and
• a reduced semileptonic branching ratio with roughly equal contributions from $b \to c l \nu$ and $c \to s l \nu$.

Essential for the analysis is the observation that in a treatment genuinely based on QCD there can be no corrections of order $1/m_Q$ that have been introduced in purely phenomenological models and play a central role there.

As far as the numerical predictions are concerned, one has to keep an important caveat in mind: the weak link in the analysis is the fact that the charm quark mass does not provide a parameter that is very large compared to ordinary hadronic scales. Thus the $1/m_c$ expansion cannot be expected to be quickly convergent. In principle it is conceivable that it might actually fail in charm transitions, say through quark-hadron duality becoming inoperational there [9].

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