Nature-inspired Hybrid Optimization Algorithms for Load Flow Analysis of Islanded Microgrids

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Abstract—Load flow analysis is a significant tool for proper planning, operation, and dynamic analysis of a power system that provides the steady-state values of voltage magnitudes and angles at the fundamental frequency. However, due to the absence of a slack bus in an islanded microgrid, modified load flow algorithms should be adopted considering the system frequency as one of the solution variables. This paper proposes the application of nature-inspired hybrid optimization algorithms for solving the load flow problem of islanded microgrids. Several nature-inspired algorithms such as genetic algorithm (GA), differential evolution (DE), flower pollination algorithm (FPA), and grasshopper optimization algorithm (GOA) are separately merged with imperialistic competitive algorithm (ICA) to form four hybrid algorithms named as ICGA, ICDE, ICFPA, and IC-GOA. Performances of these algorithms are tested on the 6-bus test system and the modified IEEE 37-bus test system. A comparison among the proposed algorithms is carried out in terms of statistical analysis conducted using SPSS statistics software. From the statistical analysis, it is identified that on an average, ICDE takes less number of iterations and consequently needs less execution time compared with other algorithms in solving the load flow problem of islanded microgrids.

Index Terms—Nature-inspired hybrid algorithm, global optimization, islanded microgrid, load flow analysis.

I. INTRODUCTION

In modern time, microgrid systems have evolved as an organized and flexible architecture comprising of distributed energy resources (DERs), which can be a potential replacement of the aging electrical infrastructure with enhanced operability and reliability, and reduced CO2 emission to alleviate the environmental change. Due to its functionality as an aggregated distributed generation (DG) unit in both islanded and grid-connected modes, microgrid has gained much acceptance over the years [1]-[3]. Load flow analysis is a potential tool for proper planning and operation of any power system. It is also a prerequisite for transient stability analysis, optimal power flow, and contingency studies [4]. In grid-connected mode, the conventional methods of load flow analysis are easily applicable as the concept of the slack bus can be easily adopted. Whereas, in islanded mode, droop controllers are used to regulate the voltage and frequency at each generator bus, and due to the variable nature of the voltage and frequency, the concept of the slack bus becomes invalid [5]-[7]. Thus, the solution of the steady-state operating points of islanded microgrid through load flow analysis remains a challenge. Hence, both conventional and non-conventional approaches are adopted by the researchers to date.

In [8], a conventional two-step load flow approach was used considering the highest-rated local bus as the slack bus. In [9], a conventional three-phase power flow method with a sequence frame power solver was used for distributing the responsibility of the slack bus among different DERs. However, it is essential to have regulation and stabilization of voltage and frequency levels along with real and reactive power sharing for islanded microgrid [10], [11]. Considering steady-state system frequency as one of the power flow variables, a three-phase power flow method was proposed in [6] using the Newton-trust region method considering decentralized droop control schemes for the DERs. A modified version of the conventional Gauss-Seidel method was used in [12], where load demands were shared by the DGs, keeping the system frequency and voltage within specified limits. Modified Newton-Raphson method was used in [13], where generator bus was formulated as droop bus considering the absence of the slack bus for an islanded microgrid.

In most of these studies, the system model was developed in the stationary reference frame considering the voltages and currents as phasors, which only allowed steady-state analysis of the system. However, these studies lacked necessary information for linearizing a dynamic model of the system. There are angular differences between synchronous reference frames of the generation units, which are not determined by these methods. For consistent system-level operation points, these angle differences are critical to obtain. In [14], the system model was developed in the synchronous reference frame, and a quasi-Newton method was introduced to solve the load flow analysis considering the system frequency, reference frame angles, and voltage magnitudes as the load flow variables. Most of these load flow techniques use gradient-based algorithms that require evaluation of derivatives for a series of complex equations. Gradient-based algorithms have a tendency of getting stuck in a local optimum if the initial starting point is not selected close to the global optimum. Furthermore, these derivative-based approaches often fail to converge in the case of nonlinear and...
discontinuous functions [15].

Nature-inspired optimization algorithms can be good alternatives to the gradient-based techniques in obtaining a global solution. Multi-solution-based evolutionary algorithms and swarm-intelligence-based algorithms have a better possibility of avoiding a local optimum by exploring a larger portion of the search space [16]. These algorithms lie in a special class of optimization algorithms called metaheuristics because these algorithms operate in a stochastic manner involving absolute randomness in the optimization process. For droop-controlled islanded microgrid, a load flow algorithm was introduced in [7], where particle swarm optimization (PSO) technique was used to determine the droop parameters. Hybridization among different evolutionary algorithms can result in a better exploration of the search space. In [17], hybridized ICDE was used for load flow analysis by incorporating the imperialistic competitive algorithm (ICA) with the multi-solution-based genetic algorithm (GA). Reasonably good performance was obtained in the work mentioned above. However, the system modeling was developed in the stationary reference frame, and only one hybrid algorithm was used. Comparison among different hybrid algorithms was not considered, and hence there is still room for further research.

In this paper, the application of hybrid nature-inspired optimization algorithms was proposed to solve the load flow problem of droop-controlled islanded microgrids in the synchronous reference frame. To come up with a fairly optimal result, several multi-solution-based algorithms such as GA, differential evolution (DE) algorithm, flower pollination algorithm (FPA) and grasshopper optimization algorithm (GOA) were adopted, and each of them was separately combined with ICA to obtain four hybrid algorithms, namely ICGA, ICDE, ICFPA, and ICGOA. Statistical analysis was conducted to compare the performance of each of the hybrid algorithms in obtaining the optimal solution.

The rest of the paper is organized as follows. Section II contains a brief discussion on the mathematical model of the droop-controlled microgrids based on different literatures. In Section III, discussions regarding the load flow approach and the proposed hybrid algorithms are presented. Comparison among the proposed hybrid algorithms, along with the load flow results obtained for two case study systems, is carried out in Section IV. Finally, the conclusions are drawn in Section V.

II. MATHEMATICAL MODEL OF MICROGRID

Multiple DERs are aggregated in a microgrid system. Due to the nature of energy produced, in most cases, it is not suitable to directly connect these DERs to the distribution network. Thus, to convert the energy produced by a DER to the desired form, power electronic inverters are associated with these DERs before connecting to a distribution bus. As a result, developing the mathematical model of the inverter, along with its associated controllers, is important for the analysis of microgrid systems. The control strategy of an inverter coupled with an individual DER, is shown in Fig. 1. The output from the inverter is coupled to the distribution bus after passing through an LC filter.

![Fig. 1. Block diagram of control strategy of droop-controlled inverter for individual DER.](image_url)

The three-phase output voltage (v<sub>P</sub>) across the filter capacitor (C<sub>f</sub>), the three-phase output current (i<sub>P</sub>) through coupling inductor (L<sub>f</sub>), and the three-phase current (i<sub>P</sub>) through the filter inductor (L<sub>i</sub>) are transformed to synchronous reference frame through abc to dq transformation. As shown in Fig. 1, C<sub>f</sub> and L<sub>f</sub> are considered to be associated with parasitic resistances r<sub>f</sub> and r<sub>i</sub>, respectively. The d- and q-axis components of v<sub>P</sub> and i<sub>P</sub> are then passed to the power controller where the instantaneous active power (P) and reactive power (Q) are calculated. After low-pass filtering (LPF), the average active power (P) and reactive power (Q) are obtained, which are then passed to the droop controller. The droop controller provides the set points of voltage (v<sub>P</sub>*) and frequency (ω*) for the voltage controller. The voltage controller then provides the reference values of the d- and q-axis components of i<sub>q</sub> for the current controller. The current controller generates d- and q-axis components of voltage reference points of the inverter output voltage (v<sub>i</sub>), which are then transferred to abc-frame through dq to abc transformation. These reference voltages are then used to provide switching signals for the inverter. For this system model, a dq-based phase-locked loop (PLL) is used to measure the transformation angle (θ) and frequency (ω<sub>PLL</sub>). The modeling technique followed in this work is based on the studies carried out in [14], [18], [19]. Detailed discussion on the mathematical model of the microgrid is presented in the supplementary material.
### III. Load Flow Analysis

In the conventional load flow analysis, the voltage and frequency of the slack bus are constant. However, in the case of islanded microgrid, the concept of the slack bus is not applicable as the system frequency is variable. As a result, for an islanded microgrid with the droop-controlled inverter, the system frequency has to be considered as one of the load flow variables along with the voltage magnitudes and reference angles contributed by each inverter in the system. The state variable \(x\) can be described in terms of the load flow variables as:

\[
x = \begin{bmatrix}
\omega & \delta_1 & \delta_2 & \cdots & \delta_K & v_{oq1} & v_{oq2} & \cdots & v_{oqK}
\end{bmatrix}
\tag{1}
\]

where \(\omega\) is the system frequency; \(\delta_i\) (\(i=2, 3, \ldots, K\)) is the angular difference between the local reference frame and the global reference frame for the \(i^{th}\) inverter, \(K\) is the total number of inverters in the system; and \(v_{oq_i}\) (\(i=1, 2, \ldots, K\)) is the \(q\)-axis component of the voltage magnitude across the filter capacitor branch of the \(i^{th}\) inverter in its local reference frame. For this study, as the first inverter is considered as the global reference frame, \(\delta_1\) is considered to be 0 throughout the whole study. Furthermore, the PLL is used to tune the \(d\)-axis component of \(v_c\) to become 0, which results in the steady-state voltage magnitude to be equal to its \(q\)-axis component. The constraints of the objective function can be defined as:

\[
\omega_{\text{min}} \leq \omega \leq \omega_{\text{max}}
\tag{2}
\]

\[
\delta_{\text{min}} \leq \delta_i \leq \delta_{\text{max}}
\tag{3}
\]

\[
v_{oq\text{min}} \leq v_{oqi} \leq v_{oq\text{max}}
\tag{4}
\]

#### A. Problem Formulation

The objective of the load flow analysis is to minimize the square of the absolute summation of errors in the active and reactive power mismatches of the inverters. The objective function can be written as:

\[
\min f(x) = \sum_{i=1}^{K} \Delta P_i^2 + \sum_{i=1}^{K} \Delta Q_i^2
\tag{5}
\]

where \(\Delta P_i\) and \(\Delta Q_i\) are the active and reactive power mismatches, respectively.

For a droop-controlled inverter, the power mismatch equations indicate the difference between the output power of the inverter calculated at the global reference frame and the reference values set by the droop controllers. For the \(i^{th}\) inverter, the active and reactive power mismatch equations are:

\[
\Delta P_i = \frac{3}{2} (V_{ad} I_{ad} + V_{aq} I_{aq}) - \frac{\omega_a - \omega}{m_i}
\tag{6}
\]

\[
\Delta Q_i = \frac{3}{2} (V_{aq} I_{ad} - V_{ad} I_{aq}) - \frac{V_a - V_{oa}}{n_i}
\tag{7}
\]

where \(V_{ad}\) and \(V_{aq}\) are the \(d\)- and \(q\)-axis components of voltage magnitude across the filter capacitor branch of the \(i^{th}\) inverter transferred in the global reference frame, respectively; \(I_{ad}\) and \(I_{aq}\) are the \(d\)- and \(q\)-axis components of current magnitude through the coupling inductor of the \(i^{th}\) inverter bus transferred in the global reference frame, respectively; \(\omega_a\) and \(V_a\) are the nominal values of system frequency and bus voltage, respectively; and \(m_i\) and \(n_i\) are the coefficients of the droop controller associated with the \(i^{th}\) inverter.

To determine the power mismatch values, a set of equations has to be solved, which includes the calculation of bus voltages and output currents of inverters. A simplified equivalent circuit is given for this purpose, considering the output voltage of inverter across the filter capacitor as a voltage source behind its coupling impedance, as shown in Fig. 2. The process of determining the power mismatch values is described in the following steps.

**Step 1:** following (S27) and (S28) in the supplementary material, the \(d\) and \(q\) component of the output voltage of the \(i^{th}\) inverter are transformed in the global reference frame using the reference angle \(\delta_i\). Then, the output voltage of the \(i^{th}\) inverter in terms of a complex quantity can be calculated as:

\[
V_{ad} = V_{oq} \sin \delta_i
\tag{8}
\]

\[
V_{aq} = V_{oq} \cos \delta_i
\tag{9}
\]

\[
V_{oi} = V_{ad} + j V_{aq}
\tag{10}
\]

where \(V_{ad}\) and \(V_{aq}\) are the \(d\)- and \(q\)-axis components of the voltage of \(i^{th}\) inverter in its local reference frame, respectively.

**Step 2:** the current injected by the inverters can be easily calculated by transforming the circuit shown in Fig. 2 to its Norton equivalent shown in Fig. 3. For this study, only constant impedance loads are considered. Hence, the inverters are the only sources to inject current to their respective buses. The current injected by the \(i^{th}\) inverter can be calculated as:

\[
I_{oi} = I_{SC} = \frac{V_{oi}}{Z_c(\omega)}
\tag{11}
\]

where \(I_{SC}\) is the Norton equivalent current source; and \(Z_c(\omega)\) is the coupling impedance as shown in Fig. 2 and Fig. 3.
Step 3: the bus voltages can be calculated from the injected currents as shown in (12).

\[ V_b = Z_{bus}(\omega)I_{inj} \quad (12) \]

where \( Z_{bus}(\omega) \) is the bus impedance matrix; and \( V_b \) and \( I_{inj} \) are the vectors of voltages and injected currents at different buses of an islanded microgrid, respectively. For an \( N \)-bus system, \( V_b \), \( Z_{bus}(\omega) \) and \( I_{inj} \) will have the dimensions of \( N \times 1 \), \( N \times N \) and \( N \times 1 \), respectively. For islanded microgrids, the bus impedance matrix is a function of frequency, and it has to be updated at each iteration. For an \( N \)-bus system, the injected current at each bus \( p \) (\( p = 1, 2, \ldots, N \)) is given by:

\[ I_{inj,p} = \begin{cases} I_{inj,p} & \text{if } \text{bus } p \text{ is the location of the } i^{th} \text{ inverter} \\ 0 & \text{otherwise} \end{cases} \quad (13) \]

Step 4: after determining the bus voltages, the output current of the \( i^{th} \) inverter can be determined by (14).

\[ I_{oi} = \frac{V_{oi} - V_b}{Z_{oi}(\omega)} \quad (14) \]

The \( d \)- and \( q \)-axis components of the output current of the \( i^{th} \) inverter in the global reference frame is given by:

\[ I_{oi} = \text{Re}\{I_{oi}\} \quad (15) \]

\[ I_{qi} = \text{Im}\{I_{oi}\} \quad (16) \]

Step 5: the equations from (8) to (16) are sufficient to calculate the mismatch values of active and reactive power for each inverter by solving (6) and (7).

Thus, these values of the power mismatch equations can be used to evaluate the objective function, as indicated in (5).

B. Algorithm for Load Flow Analysis

The flowchart of the hybrid algorithms employed in this study is given in Fig. 4. Several hybrid algorithms based on nature-inspired optimization algorithms have been employed to perform the load flow analysis of islanded microgrids. These hybrid algorithms have been designed keeping the ICA as the main frame. Inspired by the concept of imperialistic competition, ICA was first introduced by Atashpaz-Gargari and Lucas. Imperialism is the policy where the more developed countries try to establish their dominance over underdeveloped countries to possess control over their resources. Thus, several empires are formed where the developed countries act as the imperialists, and the countries under the control of an imperialist are termed as the colonies to that imperialist. There always exists a competition among different empires to take control over the colonies of other empires to enhance their own power. Apart from that, the developing colonies also try to liberate themselves from the authority of the imperialist to regain the control over their own resources. This phenomenon results in an imperialistic competition that is emulated in case of ICA in the form of an optimization algorithm.

Four other metaheuristic algorithms, GA, DE, FPA, and GOA, were separately combined with ICA to obtain four hybrid algorithms, namely ICGA, ICDE, ICFPA, and ICGOA. The detailed explanations for GA, DE, FPA, and GOA can be found in [15], [16], [20]-[29]. The process of applying these hybrid algorithms for the load flow analysis is described in the following steps.

Step 1: initialize the system data of islanded microgrids.

Step 2: generate the initial population for the state variables in (1). In the case of ICA, the population individuals are called countries. For this study, the total number of countries in the population \( n_{pop} \) is set to 100.

Step 3: for each country, (8) to (16) are solved, and the mismatch values of active and reactive power for each inverter are calculated using (6) and (7). Then, for each country in the population, the value of the objective function is determined using (5).

Step 4: the countries are sorted according to their objective function values. Then, depending on the fitness values of the countries, the empires are generated by setting a specific number of countries \( n_{imp} \) as the imperialists and assigning the rest of the countries \( n_{col} \) as colonies to them. For this study, among the 100 countries, 5 are chosen as imperialists, and the rest are assigned as colonies to the imperialists.

Step 5: the positions of the colonies are then moved towards the position of the imperialist by a process called assimilation. How much a particular colony will move towards
the imperialist depends upon the assimilation co-efficient $\beta$ and the distance between the colony and the imperialist.

Step 6: the positions of some of the colonies are perturbed randomly by performing an operation called revolution. First, whether a particular colony will undergo revolution or not is determined through the probability of revolution, and then the perturbation is performed randomly based on the revolution rate $\mu$.

Step 7: if there is a colony that has a lower fitness value than the imperialist, its position is interchanged. This process is referred to as intra-empire competition.

Step 8: the hybridization process of ICA with GA, DE, FPA, and GOA is carried out. Four distinct hybrid algorithms are obtained by following the four cases, as indicated in Fig. 4.

Case 1: ICGA. GA is one of the most popular evolutionary optimization algorithms which was inspired by Charles Darwin’s theory of natural evolution. The real-coded version of GA is employed in this study. In GA, the initial solution sets are termed as parent chromosomes. For this study, the updated empires from the previous step are set as the parent chromosomes of GA. In order to generate offspring from the parent chromosomes, the extended-line crossover is performed for each child, two parents are randomly selected, and the crossover operation is carried out in terms of the extension rate for crossover $\gamma$. The generated offspring undergoes the mutation process to add diversity to the population individuals. A gaussian mutation operation is performed with respect to the mutation rate $\mu_m$ in order to generate the mutants. The number of offspring and the number of mutants to be generated in each iteration is determined in terms of the percentage of crossover $p_c$ and the percentage of mutation $p_m$, respectively. Then, through a selection process, the fittest solutions are chosen for the next steps [15], [21]-[24].

Case 2: ICDE. DE is a real-parameter optimization algorithm that also falls into the category of evolutionary algorithms. Here, the empires are assigned as parents of the DE algorithm. For a particular parent vector in the population, a differential mutation process is performed by randomly selecting three other distinct solution vectors. Then, a scaled difference is taken between any two of these three vectors in terms of a scaling factor $F$, and the scaled difference is added with the third vector to obtain the mutant vector. Finally, the offspring is generated from the mutant and parent vector by exchanging components of the parent and mutant vector based on crossover probability $p_{cx}$. If the fitness of the offspring is better than the parent vector, their positions are interchanged. In this way, the positions of the countries (imperialists and colonies) are updated through the mutation, crossover, and selection process of DE [15], [25], [26].

Case 3: ICFPA. FPA is another evolutionary algorithm inspired by the pollination process of flowers. In this case, the empires are assigned as the population of flowers of FPA, and the positions of the population individuals are then modified by mimicking either global or local pollination process depending upon a probability switch $p$. By imitating the concept of global pollination, a global searching process is introduced where Levy distribution is typically used to indicate the jump or fly distance of pollinators. On the other hand, the concept of local pollination is utilized in developing a local search operator which plays a vital role in exploiting the search area in the vicinity of the current solution [27], [28].

Case 4: ICGOA. GOA is a swarm intelligence based algorithm which was proposed by mathematically modeling the swarming behavior of grasshoppers in nature. This mathematical model includes the model for the social interaction between grasshoppers. The attractive and repulsive forces between two grasshoppers are simulated through the social interaction function. Based on the intensity of attraction $f$ and the attractive length scale $l_{at}$, the social interaction function is calculated, which is utilized in updating the positions of grasshoppers. Apart from that, the best solution up to the current iteration is considered as the target solution, which simulates the tendency of grasshoppers to move towards the source of food. While updating the positions of grasshoppers in each iteration, a deceleration coefficient $c$ is introduced to gradually obtain a balance between the exploration and the exploitation while chasing the target solution [16], [29].

Selecting any one of the four cases in Step 8 is the principal difference among the four hybrid algorithms. The rest of the steps are similar for each algorithm.

Step 9: intra-empire competition is performed again, as mentioned in Step 7.

Step 10: first of all, the total cost of each empire is calculated in terms of the cost of the imperialist, the mean cost of the colonies, and their corresponding mean cost co-efficient $\xi$. Then, in imperialistic competition, the weakest colonies are identified and given to the empires which have the most likelihood to possess them. If an empire ends up with no colonies, it will be eliminated.

Step 11: the solution set, which provides the best fitness value, is identified.

Step 12: if the stopping criteria are satisfied, the whole process will be terminated. Otherwise, the calculations will be repeated from Step 5. For this study, the optimization process will be terminated if one of the following criteria is satisfied:

1) The value of the best fitness is less than a pre-specified threshold value $c$, which is set to be $10^{-4}$.

2) The total number of iteration is equal to a pre-specified value of the maximum number of iterations. For this study, the maximum number of iteration is set to be 50.

The values of the different parameters of these hybrid algorithms used for the simulations conducted in this study are summarized in Appendix A Table A1.

IV. SIMULATION RESULTS AND DISCUSSIONS

In Section II, the dynamic model of the microgrid system is discussed, and the proposed hybrid algorithms are outlined in Section III. In order to validate the applicability of these algorithms and to make a comparative analysis of these algorithms, the 6-bus test system and the modified IEEE 37-bus system are considered as the case study systems for load flow analysis. For both the systems, the simulations are performed using a personal computer with a processor of Intel Core i7-8550 of 1.8 GHz and with an in-
stalled RAM of 8 GB.

A. Case Study 1

The single-line diagram of the 6-bus test system is shown in Fig. 5. This system contains three identical droop controller based inverters interfaced with DG units, which are coupled to buses 4, 5 and 6. The load and line parameters and the droop coefficients are considered the same as [6]. A nominal voltage of \( V_n = 127 \) V is considered for all the inverters, and the nominal frequency is set to be \( \omega_n = (2\pi \times 60) \) rad/s as in [6]. The bus associated with inverter 1 is considered as the reference bus. The \( q \)-axis components of the output voltage of the inverter, the reference frame angles of inverters 2 and 3, and the system frequency are considered as the load flow variables according to (1).

![Fig. 5. Single-line diagram of 6-bus test system.](image-url)

Then, the load flow analysis is performed by applying ICGA, ICDE, ICFPA, and ICGOA to this case study system separately. These algorithms follow a stochastic process, and due to the inherent randomness of these algorithms, it is most likely that the number of iterations and the execution time needed to complete the optimization process may vary for each independent run. Thus, each algorithm is executed for 30 independent runs to make an overall comparison among the algorithms. For each run, the number of iterations to reach the stopping criterion and the overall execution time are recorded. These data are summarized in Table I in terms of the best, the average, and the worst results. As shown in Table I, the ICDE takes a lower number of iterations and less run time on an average to complete the load flow analysis. Compared with the ICGOA, the ICDE exhibits significantly better performance. However, with respect to the ICGA and the ICFPA, there is not much difference with the performance of ICDE.

B. Case Study 2

The standard IEEE 37-bus system is modified by connecting seven inverters at different bus locations, as indicated in [30]. The single-line diagram of the modified IEEE 37-bus system is shown in Fig. 6. The seven inverters are connected at buses 15, 18, 22, 24, 29, 33 and 34, as indicated by the black dots in Fig. 6.

![Fig. 6. Single-line diagram of modified IEEE 37-bus system.](image-url)

Only constant impedance loads are considered in this case study. The branch and load parameters are considered to be the same as [30]. For all the inverters, a nominal voltage of \( V_n = 170 \) V is chosen, and the nominal frequency is set to be \( \omega_n = (2\pi \times 60) \) rad/s. The maximum power ratings and the droop coefficients for each inverter are given in Table II [14].

![Table II](image-url)

| \( i \) | Bus | \( P_{\text{max}} \) (kW) | \( Q_{\text{max}} \) (kvar) | \( m_i \) | \( n_i \) |
|---|---|---|---|---|---|
| 1 | 15 | 15 | 15 | 2387.3 | 1250.0 |
| 2 | 18 | 8 | 8 | 1273.2 | 666.7 |
| 3 | 22 | 10 | 10 | 1591.5 | 833.3 |
| 4 | 24 | 15 | 15 | 2387.3 | 1250.0 |
| 5 | 29 | 8 | 8 | 1273.2 | 666.7 |
| 6 | 33 | 10 | 10 | 1591.5 | 833.3 |
| 7 | 34 | 15 | 15 | 2387.3 | 1250.0 |

As done for the 6-bus test system, each of the four hybrid algorithms are applied for 30 independent runs. For each independent run, the number of iterations and the overall execution time to complete the load flow analysis are recorded. These data are summarized in Table III in terms of the best, the average, and the worst results. As shown in Table III, for the best results, the ICGA and the ICDE achieve convergence with the minimum number of iterations of 16, and the
ICDE completes the optimization process 1.597 s faster than the ICGA. On the other hand, for the worst results, the ICDE completes the load flow analysis in 21 iterations with an execution time of 78.109 s. Whereas, the other algorithms require more iterations and higher execution time to complete the load flow analysis. To summarize, considering the average results, the ICDE can be regarded as the algorithm with better performance.

TABLE III

| Result       | Algorithm | No. of iteration | Running time (s) |
|--------------|-----------|------------------|------------------|
| **Best result** | ICGA      | 16               | 58.553           |
|              | ICDE      | 16               | 56.956           |
|              | ICFPA     | 17               | 63.373           |
|              | ICGOA     | 19               | 69.371           |
| **Average result** | ICGA    | 20               | 74.282           |
|              | ICDE      | 18               | 67.699           |
|              | ICFPA     | 22               | 79.005           |
|              | ICGOA     | 28               | 101.466          |
| **Worst result** | ICGA     | 26               | 94.262           |
|              | ICDE      | 21               | 78.109           |
|              | ICFPA     | 28               | 99.310           |
|              | ICGOA     | 49               | 180.412          |

To further support the above discussion, the convergence graphs of each algorithm considering their best and worst results are shown in Fig. 7 and Fig. 8, respectively.

![Convergence graphs for the best results of each algorithm. (a) Original version. (b) Enlarged version.](image1)

![Convergence graphs for the worst results of each algorithm. (a) Original version. (b) Enlarged version.](image2)

From Fig. 7(a) and Fig. 8(a), it can be seen that all the algorithms attain very high fitness values in the initial iterations, and after completing several iterations, the fitness values obtain convergence gradually. In order to show the exact point of convergence for each algorithm, the enlarged versions of the graphs shown in Fig. 7(a) and Fig. 8(a) are depicted in Fig. 7(b) and Fig. 8(b), respectively. From these two figures, it is also evident that for both the cases, the ICDE provides faster convergence than the other algorithms.

For further analysis of the acquired results, statistics software SPSS is used to perform statistical analysis of the obtained data from 30 independent runs. To demonstrate the uniqueness of each algorithm, the independent samples $t$-test is performed to compare the means of the data obtained from each algorithm. In this study, the data from two algorithms are defined as grouping variables at a time. Whenever independent $t$-test samples are performed in SPSS, the software generates corresponding $F$-test results which determine whether the data samples of two groups have equal variances or not. Table IV and Table V show the results of $F$-test and $t$-test considering the required number of iterations and the execution time as the comparison variables, respectively. In both the tables, the $F$ value and the significant factor ($p$ value) of the $F$-test are highlighted for different groups. Whereas for the $t$-test, the mean difference, $t$ value, degree of freedom $df$ and $p$ value for different grouping variables are presented. Here, $p$ value indicates the difference between the means of the $t$-test variables for a pair of grouping variable and $df$ indicates the number of quantities associated with the statistical distribution that can vary independently.

**TABLE IV**

| Algorithm      | $F$  | $p$     | Mean difference | $t$   | $df$   | $p$ (2-tailed) |
|----------------|------|---------|-----------------|-------|--------|----------------|
| ICGA-ICDE      | 7.309| 0.009   | -3.067          | -5.763| 45.730 | 0.000          |
| ICDE-ICGOA     | 20.134| 0.000  | -1.467          | -2.472| 42.181 | 0.018          |
| ICGA-ICGOA     | 21.531| 0.000  | -9.200          | -6.959| 31.352 | 0.000          |
| ICFPA-ICDE     | 1.713| 0.196   | 1.600           | 2.266 | 58.000 | 0.027          |
| ICFPA-ICGOA    | 12.608| 0.001  | -6.133          | -4.456| 36.301 | 0.000          |
| ICGA-ICGOA     | 9.106| 0.004  | -7.733          | -5.519| 38.530 | 0.000          |

**TABLE V**

| Algorithm      | $F$  | $p$     | Mean difference | $t$   | $df$   | $p$ (2-tailed) |
|----------------|------|---------|-----------------|-------|--------|----------------|
| ICGA-ICFPA     | 7.064| 0.010  | -11.305         | -5.924| 46.490 | 0.000          |
| ICDE-ICGOA     | 20.501| 0.000  | -6.582          | -3.141| 43.271 | 0.003          |
| ICFPA-ICDE     | 20.005| 0.000  | -33.766         | -7.108| 31.446 | 0.000          |
| ICGA-ICGOA     | 1.686| 0.199   | 4.723           | 1.896 | 58.000 | 0.063          |
| ICFPA-ICGOA    | 11.782| 0.001  | -22.461         | -4.549| 36.191 | 0.000          |
| ICGA-ICGOA     | 8.658| 0.005  | -27.184         | -5.423| 38.081 | 0.000          |

For the $F$-test, if the $p$ value is higher than the significance level of 0.05, the group variances are considered to be equal. Otherwise, equal variances cannot be assumed. For
the t-test, the null hypothesis $H_0$ assumes that the mean values of the data sets are equal, and the alternative hypothesis $H_1$ assumes that the mean values of the data sets are not equal. Whether the null hypothesis can be accepted or not depends on the $p$ value of the $t$-test. From Table IV, it can be seen that the $p$ value of the $t$-test with respect to the required number of iterations is smaller than 0.05 for all the pairs of the data sample, which indicates that the null hypothesis can be rejected. In this context, there is significant difference among all the algorithms. On the other hand, considering the $p$ value of the $t$-test in Table V, it can be observed that all pairs except ICFPA-ICGA contain significant differences with respect to execution time. Thus, considering both execution time and the number of iterations required to complete the optimization process, all the algorithms can be regarded to have unique characteristics. Furthermore, the ICDE performs relatively better than the other algorithms considering the average number of iterations and the average execution time.

The results of the load flow analysis of the two case study systems using ICDE are presented in Table VI and Table VII, respectively. In these two tables, the optimized values of the load flow variables, such as the system frequency, the reference frame angles, and the $q$-axis component of the output voltages of the inverters, are recorded.

**TABLE VI**

**LOAD FLOW RESULTS OBTAINED BY ICDE FOR 6-BUS SYSTEM**

| Load flow variable | Calculated value | Load flow variable | Calculated value |
|--------------------|------------------|--------------------|------------------|
| $\omega$ (rad/s)   | 376.8256         | $v_{q_i}$ (V)     | 126.844          |
| $\delta_2$ (°)     | 2.5751           | $v_{q_i}$ (V)     | 123.359          |
| $\delta_3$ (°)     | -2.9404          | $v_{q_i}$ (V)     | 125.687          |

**TABLE VII**

**LOAD FLOW RESULTS OBTAINED BY ICDE FOR MODIFIED IEEE 37-BUS SYSTEM**

| Load flow variable | Calculated value | Load flow variable | Calculated value |
|--------------------|------------------|--------------------|------------------|
| $\omega$ (rad/s)   | 375.5875         | $v_{q_i}$ (V)     | 168.685          |
| $\delta_2$ (°)     | -0.2692          | $v_{q_i}$ (V)     | 168.764          |
| $\delta_3$ (°)     | -1.3196          | $v_{q_i}$ (V)     | 168.934          |
| $\delta_4$ (°)     | -0.0119          | $v_{q_i}$ (V)     | 169.037          |
| $\delta_5$ (°)     | 0.5006           | $v_{q_i}$ (V)     | 169.836          |
| $\delta_6$ (°)     | 1.1234           | $v_{q_i}$ (V)     | 169.926          |
| $\delta_7$ (°)     | -0.3126          | $v_{q_i}$ (V)     | 165.162          |

Among the 30 independent runs, the best results are tabulated here. In [14], the load flow solution of the modified IEEE 37-bus system was obtained through a quasi-Newton method. In order to make a comparison among ICDE and the quasi-Newton method, the per-unit values of the output voltages of the inverter at each inverter bus are tabulated in Table VIII.

| Bus | Output voltage (p.u.) |
|-----|-----------------------|
| ICDE | Quasi-Newton [14]  |
| 1    | 0.9923                | 0.9789          |
| 2    | 0.9927                | 0.9601          |
| 3    | 0.9937                | 0.9655          |
| 4    | 0.9943                | 0.9844          |
| 5    | 0.9991                | 0.9745          |
| 6    | 0.9996                | 0.9673          |
| 7    | 0.9715                | 0.9700          |

From Table VIII, it can be observed that the per-unit values of the output voltages of inverter obtained through both algorithms are very close to each other. Meanwhile, all the bus voltages lie within 5% of the rated bus voltage, which satisfies the IEEE standard of voltage regulation, as stated in [31]. Furthermore, it can be observed that the per-unit values of the output voltages of the inverters are close to unity in the case of ICDE, which indicates that the voltages in this case are close to the nominal value. The results obtained so far are sufficient to calculate the voltages and phase angles at other buses of the network. These information is important for proper monitoring and operation of the whole system. The obtained load flow results can also be used to calculate steady-state operation points that can be used to linearize the nonlinear equations of the system model, which is necessary for the control and the small-signal stability analysis of the system.

V. CONCLUSION

In this paper, the application of nature-inspired hybrid optimization algorithms is demonstrated for the efficient solution of the load flow problem of islanded microgrids. For solving the load flow problem, an objective function is formulated based on the square of the absolute summation of errors in the real and reactive power generations from the inverter-based microgrid sources. Using hybrid optimization techniques, namely ICGA, ICDE, ICFPA, and ICGOA, the objective function is solved for minimization. The hybridization is performed with a view to improving the global searching capability by an enhanced exploration of the search space. The 6-bus test system and the modified IEEE 37-bus system are considered to conduct the load flow analysis. The performances of the aforementioned hybrid algorithms are compared through a series of statistical tests. Based on the statistical tests, the ICDE is found to exhibit better performance than the other algorithms in terms of the required number of iterations and the execution time. Therefore, ICDE can be regarded as a prospective alternative to the conventional load flow techniques in the case of islanded microgrids.

Possible future research scope could be the consideration of the uncertainties of renewable energy resources and loads where the probabilistic models for the source and load have
to be incorporated. Considering the load flow problem as a multi-objective optimization problem could be another possible future research direction where the power mismatch equations at each bus can be considered as separate objective functions.

**APPENDIX A**

**TABLE A1**

| Algorithm | Parameters of ICGA, ICDE, ICFPA, ICGOA |
|-----------|-------------------------------------|
| ICA       | $n_{pop} = 100$, $n_{max} = 5$, $n_{col} = 95$, $\beta = 1.5$, $p_c = 0.05$, $\mu = 0.1$, $\xi = 0.2$ |
| GA        | $p_c = 0.7$, $p_m = 0.3$, $\mu_s = 0.1$, $\gamma = 0.4$ |
| DE        | $P_{CR} = 0.5$, $F \in [0.4, 1]$ |
| FPA       | $r = 0.8$ |
| GOA       | $c_{max} = 1$, $c_{min} = 0.00004$, $f = 0.5$, $I_{opt} = 1.5$, $105$, pp. 30-47, Mar. 2017. |

**REFERENCES**

[1] A. Hirsch, Y. Parag, and J. Guerrero, “Microgrids: a review of technologies, key drivers, and outstanding issues,” *Renewable and Sustainable Energy Reviews*, vol. 90, pp. 402-411, Jul. 2018.

[2] T. L. Vandoorn, J. C. Vasquez, J. de Kooning et al., “Microgrids: hierarchical control and an overview of the control and management strategies,” *IEEE Industrial Electronics Magazine*, vol. 7, no. 4, pp. 42-55, Dec. 2013.

[3] F. Khatami, M. R. Iravani, and P. W. Lehn, “Micro-grid autonomous operation during and subsequent to islanding process,” *IEEE Transactions on Power Delivery*, vol. 20, no. 1, pp. 248-257, Jan. 2005.

[4] H. Saadat, *Power System Analysis*, New York: McGraw-Hill, 2009.

[5] Z. Shuai, Y. Sun, Z. J. Shen et al., “Microgrid stability: classification and a review,” *Renewable and Sustainable Energy Reviews*, vol. 58, pp. 167-179, May 2016.

[6] M. M. A. Abdelaziz, H. E. Farag, E. F. El-Saadany et al., “A novel and generalized three-phase power flow algorithm for islanded microgrids using a Newton trust region method,” *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 190-201, Feb. 2013.

[7] A. Elraryah, Y. Sozer, and M. E. Elbuluk, “A novel load-flow analysis for stable and optimized microgrid operation,” *IEEE Transactions on Power Delivery*, vol. 29, no. 4, pp. 1709-1717, Aug. 2014.

[8] H. Nikkhajoei and R. Iravani, “Steady-state model and power flow analysis of electronically-coupled distributed resource units,” *IEEE Transactions on Power Delivery*, vol. 22, no. 1, pp. 721-728, Jan. 2007.

[9] M. Z. Kamh and R. Iravani, “A sequence frame-based distributed slack bus model for energy management of active distribution networks,” *IEEE Transactions on Smart Grid*, vol. 3, no. 2, pp. 828-836, Jun. 2012.

[10] T. Senjyu, M. Miyazato, A. Yona et al., “Optimal distribution voltage control and coordination with distributed generation,” *IEEE Transactions on Power Delivery*, vol. 23, no. 2, pp. 1236-1242, Apr. 2008.

[11] J. He and Y. W. Li, “An enhanced microgrid load demand sharing strategy,” *IEEE Transactions on Power Electronics*, vol. 27, no. 9, pp. 3984-3995, Sept. 2012.

[12] F. Mumtaz, M. H. Syed, M. A. Hosani et al., “A simple and accurate approach to solve the power flow for balanced islanded microgrids,” in *Proceedings of 2013 IEEE 15th International Conference on Environment and Electrical Engineering (EEICE)*, Rome, Italy, Jun. 2015, pp. 1852-1856.

[13] F. Mumtaz, M. H. Syed, M. A. Hosani et al., “A novel approach to solve power flow for islanded microgrids using modified Newton Raphson with droop control of DG,” *IEEE Transactions on Sustainable Energy*, vol. 7, no. 2, pp. 493-503, Apr. 2016.

[14] J. A. Mueller and J. W. Kimball, “An efficient method of determining operating points of droop-controlled microgrids,” *IEEE Transactions on Energy Conversion*, vol. 32, no. 4, pp. 1432-1446, Dec. 2017.

[15] X. S. Yang, *Nature-inspired Optimization Algorithms*. Amsterdam: Elsevier, 2014.

[16] S. Saremi, S. Mirjalili, and A. Lewis, “Grasshopper optimisation algorithm: theory and application,” *Advances in Engineering Software*, vol. 105, pp. 30-47, Mar. 2017.

[17] M. Abedini, “A novel algorithm for load flow analysis in island microgrids using an improved evolutionary algorithm,” *International Transactions on Electrical Energy Systems*, vol. 26, no. 12, pp. 2727-2743, Jun. 2016.

[18] N. Pogaku, M. Prodanovic, and T. C. Green, “Modeling, analysis and testing of autonomous operation of an inverter-based microgrid,” *IEEE Transactions on Power Electronics*, vol. 22, no. 2, pp. 613-625, Mar. 2007.

[19] M. Rashidianzaman, J. A. Mueller, and J. W. Kimball, “An accurate small-signal model of inverter-dominated islanded microgrids using dq reference frame,” *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 2, no. 4, pp. 1070-1080, Dec. 2014.

[20] E. Atashpaz-Gargari and C. Lucas, “Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition,” in *Proceedings of 2007 IEEE Congress on Evolutionary Computation*, Singapore, Sept. 2007, pp. 4661-4667.

[21] J. H. Holland, *Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence*. Michigan: University of Michigan Press, 1992.

[22] A. H. Wright, “Genetic algorithms for real parameter optimization,” in *Foundations of Genetic Algorithms*, vol. 1. Amsterdam: Elsevier, 1991, pp. 205-218.

[23] K. Deep and M. Thakur, “A new crossover operator for real coded genetic algorithms,” *Applied Mathematics and Computation*, vol. 188, no. 1, pp. 895-911, May 2007.

[24] Y. Yoon and Y. H. Kim, “The roles of crossover and mutation in real-coded genetic algorithms,” in *Bio-inspired Computational Algorithms and Their Applications*. Makati: IntechOpen, 2012.

[25] R. Storn and K. Price, “Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces,” *Journal of Global Optimization*, vol. 11, no. 4, pp. 341-359, Jan. 1997.

[26] S. Das and P. N. Suganthan, “Differential evolution: a survey of the state-of-the-art,” *IEEE Transactions on Evolutionary Computation*, vol. 15, no. 1, pp. 4-31, Feb. 2011.

[27] X.-S. Yang, “Flower pollination algorithm for global optimization,” in *Proceedings of International Conference on Unconventional Computing and Natural Computation*, Berlin, Germany, Sept. 2012, pp. 240-249.

[28] E. Nabil, “A modified flower pollination algorithm for global optimization,” *Expert Systems with Applications*, vol. 57, pp. 192-203, Sept. 2016.

[29] S. M. Ismael, S. H. A. Aleem, A. Y. Abdelaziz et al., “Optimal conductor selection of radial distribution feeders: an overview and new application using grasshopper optimization algorithm,” in *Classical and Recent Aspects of Power System Optimization*. Amsterdam: Elsevier, pp. 185-217, 2018.

[30] L. Luo and S. V. Dhople, “Spatiotemporal model reduction of inverter-based islanded microgrids,” *IEEE Transactions on Energy Conversion*, vol. 29, no. 4, pp. 823-832, Dec. 2014.

[31] IEEE Guide for Identifying and Improving Voltage Quality in Power Systems, IEEE Standard 1250-2011, 2011.