Online Motion Planning with Soft Metric Interval Temporal Logic in Unknown Dynamic Environment

Zhiliang Li, Mingyu Cai, Shaoping Xiao, Zhen Kan

Abstract—Motion planning of an autonomous system with high-level specifications has wide applications. However, research of formal languages involving timed temporal logic is still under investigation. Furthermore, many existing results rely on a key assumption that user-specified tasks are feasible in the given environment. Challenges arise when the operating environment is dynamic and unknown since the environment can be found prohibitive, leading to potentially conflicting tasks where pre-specified timed missions cannot be fully satisfied. Such issues become even more challenging when considering time-bound requirements. To address these challenges, this work proposes a control framework that considers hard constraints to enforce safety requirements and soft constraints to enable task relaxation. The metric interval temporal logic (MITL) specifications are employed to deal with time-bound constraints. By constructing a relaxed timed product automaton, an online motion planning strategy is synthesized with a receding horizon controller to generate policies, achieving multiple objectives in decreasing order of priority 1) formally guarantee the satisfaction of hard safety constraints; 2) mostly fulfill soft timed tasks; and 3) collect time-varying rewards as much as possible. Another novelty of the relaxed structure is to consider violations of both time and tasks for infeasible cases. Simulation results are provided to validate the proposed approach.

Index Terms—Formal Method, Model Predictive Control, Multi-Objective Optimization, Timed Automaton

I. INTRODUCTION

Complex rules in modern tasks often specify desired system behaviors and timed temporal constraints that require mission completion within a given period. Performing such tasks can be challenging, especially when the operating environment is dynamic and unknown. For instance, user-specified missions or temporal constraints can be found infeasible during motion planning. Therefore, this work is motivated for online motion planning subject to timed high-level specifications.

Linear temporal logic (LTL) has been widely used for task and motion planning due to its rich expressivity and resemblance to natural language [1]. When considering timed formal language, as an extension of traditional LTL, timed temporal languages such as metric interval temporal logic (MITL) [2], signal temporal logic (STL) [3], time-window temporal logic (TWTL) [4], are often employed. However, most existing results are built on the assumption that user-specified tasks are feasible. New challenges arise when the operating environment is dynamic and unknown since the environment can become prohibitive (e.g., an area to be visited is found later to be surrounded by obstacles), leading to mission failure.

To address these challenges, tasks with temporal logic specifications are often relaxed to be fulfilled as much as possible. A least-violating control strategy is developed in [5]–[10] to enforce the revised motion planning close to the original LTL specifications. In [11]–[13], hard and soft constraints are considered so that the satisfaction of hard constraints is guaranteed while soft constraints are minimally violated. Time relaxation of TWTL has been investigated in [14]–[16]. Receding horizon control (RHC) is also integrated with temporal logic specifications to deal with motion planning in dynamic environments [17]–[22]. Other representative results include learning-based methods [23]–[27] and sampling-based reactive methods [28], [29]. Most of the results mentioned above do not consider time constraints in motion planning.

MITL is an automaton-based temporal logic that has flexibility to express general time constraints. Recent works [30]–[34] propose different strategies to satisfy MITL formulas. The works of [30], [31] consider cooperative planning of a multi-agent system with MITL specifications and the work of [34] further investigates MITL planning of a MAS subject to intermittent communication. When considering dynamic environments, MITL with probabilistic distributions is developed in [33] to express time-sensitive missions, and a Reconfigurable algorithm is developed in [32]. However, the aforementioned works assume that the desired MITL specifications are always feasible for the robotic system. Sofie et al. [12], [13] first take into account the soft MITL constraints and studies the interactions of human-robot, but only static environments are considered. It is not yet understood how timed temporal tasks can be successfully managed in a dynamic and unknown environment, where predefined tasks may be infeasible.

Motivated by these challenges, this work considers online motion planning of an autonomous system with timed temporal specifications. Unlike STL defined over predicates, MITL provides more general time constraints and can express tasks over infinite horizons. Furthermore, MITL can be translated into timed automata that allow us to exploit graph-theoretical approaches for analysis and design. Therefore, MITL is used in this work.

The contributions of this work are multi-fold. First, the operating environment is not fully known a priori and dynamic in the sense of containing mobile obstacles and time-varying areas of interest that can only be observed locally. The dynamic and unknown environment can lead to potentially conflicting tasks (i.e., the pre-specified MITL missions or
time constraints cannot be fully satisfied). Inspired by our previous work [21], we consider both hard and soft constraints. The motivation behind this design is that safety is crucial in real-world applications; therefore, we formulate safety requirements (e.g., avoid obstacles) as hard constraints that cannot be violated in all cases. In contrast, soft constraints can be relaxed if the environment does not permit such specifications so that the agent can accomplish the tasks as much as possible. Second, to deal with time constraints, we apply MITL specifications to model timed temporal tasks and further classify soft constraints by how they can be violated. For instance, the mission can fail because the agent cannot reach the destination on time, or the agent visits some risky regions. Therefore, the innovation considers violations of both time constraints and task specifications caused by dynamic obstacles, which can be formulated as continuous and discrete types, respectively.

Our framework is to generate controllers achieving multiple objectives in decreasing order of priority: 1) formally guarantee the satisfaction of hard constraints; 2) mostly satisfy soft constraints (i.e., minimizing the violation cost); and 3) collect time-varying rewards as much as possible (e.g., visiting areas). Finally, we demonstrate the effectiveness of our approach by a complex infinite time simulation.

II. PRELIMINARIES

A dynamical system with finite states evolving in an environment can be modeled by a weighted transition system.

**Definition 1.** [35] A weighted transition system (WTS) is a tuple \( T = (Q, q_0, \delta, AP, L, \omega) \), where \( Q \) is a finite set of states; \( q_0 \in Q \) is the initial state; \( \delta \in Q \times Q \) is the state transitions; \( AP \) is the finite set of atomic propositions; \( L : Q \rightarrow 2^{AP} \) is a labeling function, and \( \omega : \delta \rightarrow \mathbb{R}^+ \) assigns a positive weight to each transition.

A timed run of a WTS \( T \) is an infinite sequence \( r = (q_0, \tau_0)(q_1, \tau_1) \ldots \), where \( q = q_0q_1 \ldots \) is a trajectory with \( q_i \in Q \), and \( \tau = \tau_0\tau_1 \ldots \) is a time sequence with \( \tau_0 = 0 \) and \( \tau_{i+1} = \tau_i + \omega(q_i, q_{i+1}), \forall i \geq 0 \). The timed run \( r \) generates a timed word \( w = (\sigma_0, \tau_0)(\sigma_1, \tau_1) \ldots \), where \( \sigma = \sigma_0\sigma_1 \ldots \) is an infinite word with \( \sigma_i = L(q_i) \) for \( i \geq 0 \). Let \( R_h(q) \) denote the time-varying reward associated with a state \( q \) at time \( k \). The reward reflects the time-varying objective in the environment. Given a predicted trajectory \( q_k = q_0q_1 \ldots q_N \) at time \( k \) with a finite horizon \( N \), the accumulated reward along the trajectory \( q_k \) can be computed as \( R_k(q_k) = \sum_{i=1}^{N} R_h(q_i) \).

Note that this paper mainly studies high-level planning and decision-making problems. Similar to [9], we assume low-level controllers can achieve go-to-goal navigation, which can be abstracted by WTS. We further assume that the workspace boundaries are known, which is a common assumption in many existing works [11–13, 18, 30, 31].

### A. Metric Interval Temporal Logic

Metric interval temporal logic (MITL) is a specific temporal logic that includes timed temporal specification [31]. The syntax of MITL formulas is defined as \( \phi := p \mid \neg \phi \mid \phi_1 \wedge \phi_2 \mid \Diamond \phi \mid \Box \phi \mid \phi_1 U_i \phi_2 \), where \( p \in AP \), \( \wedge \) (conjunction), \( \neg \) (negation) are Boolean operators and \( \Diamond \phi \) (eventually), \( \Box \phi \) (always), \( U_i \phi \) (until) are temporal operators bounded by the non-empty time interval \( I = [a, b] \) with \( a, b \in \mathbb{R}_{\geq 0}, b > a \). They are called temporally bounded operators if \( b \neq \infty \), and non-temporally bounded operators otherwise. A formula \( \phi \) containing a temporally bounded operator will be called a temporally bounded formula. The same holds for non-temporally bounded formulas.

Given a timed run \( r \) of \( T \) and an MITL formula \( \phi \), let \( (r, i) \) denote the indexed element \( (q_i, \tau_i) \). Then the satisfaction relationship \( \models \) of MITL can be defined as:

\[
(r, i) \models p \iff p \in L(q_i)
\]

\[
(r, i) \models \neg \phi \iff (r, i) \not\models \phi
\]

\[
(r, i) \models \phi_1 \land \phi_2 \iff (r, i) \models \phi_1 \text{ and } (r, i) \models \phi_2
\]

\[
(r, i) \models \Diamond \phi \iff \exists j, i \leq j, s.t. (r, j) \models \phi, \tau_j - \tau_i \in I
\]

\[
(r, i) \models \Box \phi \iff \forall j, i \leq j, s.t. (r, j) \models \phi, \tau_j - \tau_i \in I \text{ and } (r, k) \models \phi \text{ for every } i \leq k \leq j
\]

### B. Timed Büchi Automaton

Let \( X = \{x_1, x_2, \ldots, x_M\} \) be a finite set of clocks. The set of clock constraints \( \Phi(X) \) is defined by the grammar \( \varphi := X | \neg \varphi | \varphi_1 \land \varphi_2 | \varphi \mid c \mid c \geq e \mid c \leq e \mid c \geq e \mid c \leq e \mid \ldots \) is a clock constant and \( b, e \in \{<, >, \geq, \leq, =\} \). A clock valuation \( \nu : X \rightarrow \mathbb{R}^+ \) assigns a real value to each clock. We denote by \( \nu \models \varphi \) if the valuation \( \nu \) satisfies the clock constraint \( \varphi \), where \( \nu = (\nu_1, \ldots, \nu_M) \) with \( \nu_i \) being the valuation of \( x_i \), \( \forall i \in 1, \ldots, M \). An MITL formula can be converted into a Timed Büchi Automaton (TBA) [36].

**Definition 2.** A TBA is a tuple \( A = (S, S_0, AP, L, X, I_X, E, F) \) where \( S \) is a finite set of states; \( S_0 \subseteq S \) is the set of initial states; \( 2^{AP} \) is the alphabet where \( AP \) is a finite set of atomic propositions; \( L : S \rightarrow 2^{AP} \) is a labeling function; \( X \) is a finite set of clocks; \( I_X : S \rightarrow \Phi(X) \) is a map from states to clock constraints; \( E \subseteq S \times \Phi(X) \times 2^{AP} \times S \) represents the set of edges of form \( e = (s, g, a, s') \) where \( s, s' \) are the source and target states, \( g \) is the guard of edge via an assigned clock constraint, and \( a \in 2^{AP} \) is an input symbol; \( F \subseteq S \) is a set of accepting states.

**Definition 3.** An automata timed run \( r_A = (q_0, \tau_0) \ldots (q_n, \tau_n) \) of a TBA \( A \), corresponding to the timed run \( r = (q_0, \tau_0) \ldots (q_n, \tau_n) \) of a WTS \( T \), is a sequence...
and only if \(q_j \models g_a, s_g, a_s \in E\) such that i) \(\tau_j \models g_j, j \geq 0\), and ii) \(L(q_j) \subseteq L(s_j), \forall j\).

**Definition 4.** Given a WTS \(\mathcal{T} = (Q, q_0, \delta, AP, L, \omega)\) and a TBA \(\mathcal{A} = (S, S_0, AP, L, X, I_X, E, F)\), the product automaton \(\mathcal{P} = \mathcal{T} \times \mathcal{A}\) is defined as a tuple \(\mathcal{P} = (P, P_0, AP, L_P, \delta_P, I_{\mathcal{P}}^X, \mathcal{F}_{\mathcal{P}}, \omega_{\mathcal{P}})\), where \(P \subseteq \{(q, s) \in Q \times S : L(q) \subseteq L(s)\}\) is the set of states; \(P_0 = \{(q_0) \times S_0\}\) is the set of initial states; \(L_P = P \rightarrow 2^{AP}\) is a labeling function, i.e., \(L_P(p) = L(q)\); \(\delta_P \subseteq P \times P\) is the set of transitions defined such that \(((q, s), (q', s')) \in \delta_P\) if and only if \((q, q') \in \delta\) and \(\exists g, a, s, s' \text{ such that } (s, g, a, s') \in E; I_{\mathcal{P}}^X(p) = I_X(s)\) is a map of clock constraints; \(\mathcal{F}_{\mathcal{P}} = Q \times F\) is the set of accepting states; \(\omega_{\mathcal{P}}: \delta_P \rightarrow \mathbb{R}^+\) is the positive weight function, i.e., \(\omega_{\mathcal{P}}(p, p') = \omega(q, q')\).

**III. Problem Formulation**

To better explain our motion planning strategy, we use the following running example throughout this work.

**Example 1.** Consider a motion planning problem for a simplified Pac-Man game in Fig. 1(a) The maze is abstracted to a named grid-like graph, and the set of atomic propositions \(AP = \{\text{obstacle}, \text{grass}, \text{pear}, \text{cherry}\}\) indicates the labeled properties of regions. In particular, obstacle represents areas that should be totally avoided, grass represents risky areas that should be avoided if possible, and pear and cherry represent points of interest. The environment is dynamic in the sense of containing mobile obstacles and time-varying rewards \(R_k(q) \in \mathbb{R}^+\) that are randomly generated. Cyan dots represent the rewards with size proportional to their value.

We make the following assumptions: 1) the environment is only partially known to Pac-Man, i.e., the locations of pear, cherry, and grass are known, but not the obstacles it may encounter; 2) the Pac-Man has limited sensing capability, i.e., it can only detect obstacles, sense region labels, and collect rewards within a local area around itself. The motion of the Pac-Man is modeled by a weighted transition system \(\mathcal{T}\) as in Def. 1 with four possible actions, “up,” “down,” “right,” and “left.” The timed temporal task of Pac-Man is specified by an MITL formula \(\phi = \phi_h \land \phi_s\), where the hard constraints \(\phi_h\) enforce safety requirement (e.g., \(\phi_h = \neg \text{obstacle}\)) that has to be fully satisfied while the soft constraints \(\phi_s\) represent tasks that can be relaxed if the environment does not permit (e.g., \(\phi_s = \neg \text{grass} \land \exists t \leq 10 \text{pear}\)).

In Example 1, the motion planning problem is challenging since \(\phi_s\) can be violated in multiple ways. For instance, suppose that grass is in between pear and Pac-Man, and it takes more than 10 seconds to reach pear if Pac-Man circumvents grass. In this case, Pac-Man can either violate the mission \(\neg \text{grass}\) by traversing grass or violate the time constraints \(\exists t \leq 10 \text{pear}\) by taking a longer but safer path.

To consider potentially infeasible specifications, we define the total violation cost of an MITL formula as follows.

**Definition 5.** Given a time run \(r = (q_0, \tau_0) \ldots (q_n, \tau_n)\) of a WTS \(\mathcal{T}\), the total violation cost of an MITL formula \(\phi\) is defined as

\[
W(r, \phi) = \sum_{k=0}^{n-1} \omega(q_k, q_{k+1}) \omega_i(q_k, q_{k+1}, \phi),
\]

where \(\omega(q_k, q_{k+1}) = \tau_{k+1} - \tau_k\) is the time required for the transition \((q_k, q_{k+1})\) and \(\omega_i(q_k, q_{k+1}, \phi)\) is defined as the violation cost of the transition with respect to \(\phi\). Then, the formal statement of the problem is expressed as follows.

**Problem 1.** Given a weighted transition system \(\mathcal{T}\), and an MITL formula \(\phi = \phi_h \land \phi_s\), the control objective is to design a multi-goal online planning strategy, in decreasing order of priority, with which 1) \(\phi_h\) is fully satisfied; 2) \(\phi_s\) is fulfilled as much as possible if \(\phi_s\) is not feasible i.e. minimize the total violation cost \(W(r, \phi_s)\); and 3) the agent collects rewards as much as possible over an infinite horizon task operation.

**IV. Relaxed Automaton**

Sec. IV-A presents the procedure of constructing the relaxed TBA to allow motion revision. Sec. IV-B presents the design of energy function that guides the satisfaction of MITL specifications. Sec. IV-C gives the online update of environment knowledge for motion planning.

**A. Relaxed Timed Büchi Automaton**

To address the violation of MITL tasks, the relaxed TBA is defined to contain two extra components (i.e., a continuous violation cost and a discrete violation cost) compared with the original TBA. This section presents the procedure of constructing a relaxed TBA for an MITL formula \(\phi = \phi_h \land \phi_s\).

First, we explain how to build the set of states in a relaxed TBA (see Alg. 1). Given the hard constraints \(\phi_h\), which have to be fully satisfied and cannot be violated at any time, we add a sink state \(s_{\text{sink}}\) in the relaxed TBA to indicate the violation of hard constraints.

Before developing soft constraints \(\phi_s\), a more detailed classification of temporal operators for MITL formulas is introduced. An MITL specification \(\phi\) can be written as \(\phi = \bigwedge_{i \in \{1, 2, \ldots, n\}} \phi_i\ s.t. \phi_i \neq \phi_j, \forall i \neq j\). For each sub-formula \(\phi_i\), if it is temporally bounded, \(\phi_i\) can be either satisfied, violated, or uncertain (12). If \(\phi_i\) is non-temporally bounded, it can...
be either satisfied/uncertain or violated/uncertain. Specifically, a non-temporally bounded formula \( \phi_i \) is of Type I (i.e., satisfied/uncertain) if \( \phi_i \) cannot be concluded to be violated at any time during a run since there remains a possibility for it to be satisfied in the future. In contrast, it is of Type II (i.e., violated/uncertain) if \( \phi_i \) cannot be concluded to be satisfied during a run, since it remains possible to be violated in the future. For instance, when \( b = \infty \), \( \Diamond_{[a,b]} \) is of Type I and \( \Box_{[a,b]} \) is of Type II. The operator \( \mathcal{U}_{[a,b]} \) is special since it results in two parts of semantics, which can be classified as Type I and II, respectively. Hence we treat formulas like \( \mathcal{A}\mathcal{U}_{[a,b]}\mathcal{B} \) as a combination of two non-temporally bounded sub-formulas.

Based on above statement, for the soft constraints \( \phi_s = \bigwedge_{i=1,\ldots,n} \phi_i \), an evaluation set \( \varphi_i \) of a sub-formula \( \phi_i \) which represent possible satisfaction for a sub-formula is defined as:

\[
\varphi_i = \begin{cases} 
\{ \phi_i^{\text{vio}}, \phi_i^{\text{sat}}, \phi_i^{\text{unc}} \}, & \text{if } \phi_i \text{ is temporally bounded,} \\
\{ \phi_i^{\text{sat}}, \phi_i^{\text{unc}} \}, & \text{if } \phi_i \text{ is non-temporally bounded of Type I,} \\
\{ \phi_i^{\text{vio}}, \phi_i^{\text{unc}} \}, & \text{if } \phi_i \text{ is non-temporally bounded of Type II.}
\end{cases}
\]

Based on (2), a subformula evaluation \( \psi_s \) of \( \phi_s \) is defined as:

\[
\psi_s = \bigwedge_{i=1,\ldots,n} \varphi_i^{\text{state}}, \varphi_i^{\text{state}} \in \varphi_i.
\]

In (3), \( \psi_s \) represents one possible outcome of the formula, which can be obtained by taking an element from the evaluation set \( \varphi_i \) for each sub-formula \( \phi_i \), and then operating the conjunction of all these elements. Each different combination corresponds to a sub-formula evaluation \( \psi_i \). Let \( \Psi_s \) denote the set of all sub-formula evaluations \( \psi_i \) of \( \phi_s \), the number of \( \psi_i \in \Psi_s \) is equal to the product of the number of elements in the evaluation set \( \varphi_i \), which can be defined as \( \Psi_s = \{ \psi_i, j = 0, 1, \ldots, n - 1 \} \) with \( n = \prod |\varphi_i| \) where \( |\varphi_i| \) represents the number of elements in set \( \varphi_i \). The set \( \Psi_s \) represents all possible outcomes of \( \phi_s \) at any time. Every possible \( \psi_i \in \Psi_s \) is associated with a state \( s \). The initial state \( s_0 \) is the state whose corresponding sub-formulas are uncertain, which indicates no progress has been made. The accepting state \( s_F \) is the state whose corresponding temporally bounded sub-formulas and non-temporally bounded sub-formulas of Type I are satisfied, while all non-temporally bounded sub-formulas of Type II are uncertain.

The construction of the set of atomic propositions \( \mathcal{AP} \), labeling function \( \mathcal{L} \), clocks \( X \) and the map from states to clock constraints \( I_X \) in relaxed TBA is the same as in TBA. Here consider two different types of violation cost, i.e., a state \( s \neq s_{sink} \) can violate soft constraints \( \phi_s \) by either continuous violation (e.g., visiting time constraints) or discrete violation (e.g., visiting risky regions). To measure their violation degrees, the outputs of continuous violation cost \( v_c(s) \) and discrete violation cost \( v_d(s) \) for each state \( s \neq s_{sink} \) are defined, respectively, as:

\[
v_c(s) = \begin{cases} 
k, & \text{if } \exists \phi_1^{\text{vio}}, \phi_2^{\text{vio}}, \ldots, \phi_k^{\text{vio}} \in \psi_s \text{ that is temporally bounded,} \\
0, & \text{otherwise,}
\end{cases}
\]

\[
v_d(s) = \begin{cases} 
1, & \text{if } \exists \phi_1^{\text{vio}} \in \psi_s \text{ that is non-temporally bounded,} \\
0, & \text{otherwise.}
\end{cases}
\]

At the sink state \( s_{sink} \), the continuous and discrete violation costs are defined as \( v_c(s_{sink}) = v_d(s_{sink}) = \infty \).

The next step is to define violation-based edges connecting states, and the following definitions and notations are introduced.

**Definition 6.** Given soft constraints \( \phi_s \), the distance set between \( \psi_s \) and \( \psi_s' \) is defined as \( (\psi_s - \psi_s') = \{ \phi_i : \phi_i^{\text{state}} \neq \phi_i^{\text{state}} \} \). That is, it consists of all sub-formulas \( \phi_i \) that are under different evaluations.

We use \( (\psi_s, g, a) \to \psi' \) to denote that all sub-formulas \( \phi_i \in |\psi_s - \psi_s'| \) are (i) evaluated as uncertain in \( \psi_s \) (i.e., \( \phi_i^{\text{unc}} \in \psi_s \)) and (ii) re-evaluated to be either satisfied or violated in \( \psi_s' \) (i.e., \( \phi_i^{\text{state}} \in \psi_s' \), where \( \text{state}' \in \{ \text{vio, sat} \} \) if symbol \( a \) is read at time \( t \), satisfies guard \( g \).

The edge construction can be summarized into four steps:

1. Construct all edges corresponding to progress regarding the specifications (i.e., the edges that a TBA would have).
2. Construct edges \( E \) of non-temporally bounded soft constraints that are no longer violated, such that \( (s, g, a, \dot{s}) \in E \) satisfying all of the following conditions: (i) \( \forall \phi_i \in |\psi_s - \psi_s'| \), \( \phi_i^{\text{sat}} \in \psi_s \) where \( \dot{s} \) corresponds to \( \psi_s \), \( s' \) corresponds to \( \psi_s' \) and \( \phi_i \) is non-temporally bounded, and (ii) \( (s'', g, a, \dot{s}') \in E \) for some \( s'' \) where \( |\psi_s - \psi_s'| = |\psi_s - \psi_s''| \) or \( (s', g, a, \dot{s}) \in E \) where \( \dot{a}' = 2^{|\mathcal{AP}|} \cdot a \).
3. Construct edges \( E \) of temporally bounded soft constraints that are no longer violated, such that \( (\dot{s}, g, a, \dot{s}) \in E \).
satisfying all the following conditions: (i) \( \exists \phi_i \in \{ \psi_s - \psi' \} \), \( \phi_i^{\text{vio}} \in \psi_s, \phi_i^{\text{sat}} \in \psi'_s, \phi_i^{\text{unc}} \in \psi''_s \) where \( s \) corresponds to \( s' \), \( s'' \) corresponds to \( s'_i \) and \( \phi_i \) is temporally bounded, (ii) \( (s', g', a, s') \in \hat{E}, (s'', g, a, s) \in \hat{E} \) and \( g = g' \setminus \Phi(X_1) \), where \( X_1 \) is the set of clocks associated with \( \phi_i \), s.t. \( \phi_i^{\text{unc}} \in \psi''_s \) and \( \phi_i^{\text{vio}} \in \psi_s \).

(4) Construct self-loops such that \( (\hat{s}, g, a, s) \in \hat{E} \) if \( \exists (g, a) \) s.t. \( g \subseteq g' \), \( a \subseteq a' \) where \( (s', g', a', s') \in \hat{E} \) for some \( s' \) and \( (\hat{s}, g', a', s'') \notin \hat{E} \) for any \( s'' \).

In the first step, the edges of the original TBA are constructed except self-loops, i.e., transitions from and to the same state. Then, we construct edges from states where \( v_d = 1 \), i.e., states corresponding to discrete violation (step 2). These edges can be considered as alternative routes to the ones in step 1, where some non-temporally bounded sub-formula/formulas are violated at some points. Similarly, we construct edges from states with \( v_c > 0 \), i.e., states corresponding to continuous violations (step 3). This ensures that the accepting states can be reached when the time-bound action finally occurs, even after the deadline is exceeded. Finally, we consider self-loops to ensure no deadlocks in the automaton except the sink state \( \hat{s}_{\text{sink}} \). Compared with TBA, the relaxed TBA allows more transitions and enables task relaxation when \( \phi_a \) is not fully feasible.

**Definition 7.** An automata timed run \( r_{\hat{A}} = (\hat{s}_0, \tau_0) \ldots (\hat{s}_n, \tau_n) \) of a relaxed TBA \( \hat{A} \), corresponding to the timed run \( r = (q_0, \tau_0) \ldots (q_n, \tau_n) \) is a sequence where \( \hat{s}_0 \in \hat{S}_0, \hat{s}_1 \in \hat{S}, \) and \( (\hat{s}_j, g_j, a_j, \hat{s}_{j+1}) \in \hat{E} \) \( \forall j \geq 0 \) such that (i) \( \tau_j = g_j, j \geq 0 \), and (ii) \( L(q_j) \subseteq L(\hat{s}_j), \forall j \).

The continuous violation cost for the automata timed run is

\[
\sum_{k=0}^{n-1} v_c(\hat{s}_{k+1}) (\tau_{k+1} - \tau_k)
\]

and similarly the discrete violation cost is

\[
\sum_{k=0}^{n-1} v_d(\hat{s}_{k+1}) (\tau_{k+1} - \tau_k).
\]

**Example 2.** As a running example in Fig. 2. Consider an MITL specification \( \phi = \phi_h \land \phi_s \) with \( \phi_h = \Box \neg \text{obs} \) and \( \phi_s = \Box \neg g \land P_{t<10} \), where \( \text{obs} \) represents obstacles, and \( g \) and \( p \) represent the grass and peer, respectively. The TBA and the corresponding relaxed TBA are shown in Fig. 2. The soft constraint \( \phi_s \) is composed of two subformulas: \( \phi_1 = \Box \neg g \) and \( \phi_2 = \Box_{t<10} p \), where \( \phi_1 \) is non-temporally bounded of Type II and \( \phi_2 \) is temporally bounded. Hence \( \phi_1 \) can be evaluated as violated or uncertain while \( \phi_2 \) can be evaluated as violated, uncertain or satisfied, i.e., the corresponding evaluation sets are \( \varphi_1 = \{ \phi_1^{\text{unc}}, \phi_1^{\text{vio}} \} \) and \( \varphi_2 = \{ \phi_2^{\text{unc}}, \phi_2^{\text{vio}}, \phi_2^{\text{sat}} \} \), respectively. By operating the conjunction of the first element in set \( \varphi_1 \) and set \( \varphi_2 \), a sub-formula evaluation \( \psi^0 = \phi_2^{\text{vio}} \land \phi_1^{\text{vio}} \) is obtained. Similarly, we can enumerate all sub-formula evaluations. Therefore, the set of all sub-formula evaluations of the formula \( \phi_s \) is \( \Psi_s = \{ \phi_s^{\text{unc}} \land \phi_s^{\text{vio}} \} \). Then, \( \varphi_2 \) is considered as uncertain. For the rest of the states, we denote by \( \hat{s}_1 \sim \phi_2^{\text{vio}} \land \phi_2^{\text{sat}}, \hat{s}_2 \sim \phi_2^{\text{vio}} \land \phi_2^{\text{sat}}, \hat{s}_3 \sim \phi_2^{\text{vio}} \land \phi_2^{\text{sat}} \). There are two clock constraints in this example: \( t < 10 \) associated with states corresponding to \( \phi_2^{\text{sat}} \), and \( t \geq 10 \) associated with \( \phi_2^{\text{vio}} \). The first clock constraint is then mapped to \( \hat{s}_1 \) and \( \hat{s}_2 \), and the second to \( \hat{s}_2 \) and \( \hat{s}_3 \). The continuous and discrete violation costs are mapped such that \( v_c(\hat{s}) = [0 1 1 0 0 0 \infty] \) and \( v_d(\hat{s}) = [0 1 1 0 0 1 \infty] \).

Compared with TBA, the relaxed TBA allows more transitions, enables task relaxation and measure its violation when \( \phi_a \) is not fully feasible. Since a traditional product automaton \( P = T \times \hat{A} \) cannot handle the infeasible case, a relaxed product automaton is introduced as follow.

**Definition 8.** Given a WTS \( T = (Q, q_0, \delta, AP, L, \omega) \) and a relaxed TBA \( \hat{A} = (\hat{S}, \hat{S}_0, AP, L, X, I_X, v_c, v_d, E, F) \), the relaxed product automaton (RPA) \( \hat{P} = T \times \hat{A} \) is a labeling function, i.e., \( \hat{L}_P(p) = L(q) \setminus \delta_P \subseteq \hat{P} \times \hat{P} \) is the set of transitions defined such that \( ((q, s), (q', s')) \in \delta_P \) if and only if \( (q, q') \in \delta \) and \( \exists q, a, s \text{. s.t.} (s, q, a, s') \in E; \hat{L}_P^X(p) = I_X(\hat{s}) \) is a map of clock constraints; \( v_c^X(p) = v_c(\hat{s}) \) is the continuous
violation cost; \(v^p_\omega(p) = v_d(s)\) is the discrete violation cost; \(F_p = Q \times F\) are accepting states; \(\omega_p: \delta_p \rightarrow \mathbb{R}^+\) is the positive weight function, i.e., \(\omega_p(p, q') = \omega(q, q')\).

By accounting continuous and discrete violation simultaneously, the violation cost with respect to \(\phi_s\) is defined as
\[
\omega^p_\nu(p_k, p_{k+1}, \phi_s) = (1 - \alpha) v^p_\nu(p_{k+1}) + \alpha v^p_\nu(p_k),
\]
where \(\alpha \in [0, 1]\) measures the relative importance between continuous and discrete violations. Then based on \(W(r, \phi)\) defined in [1], the total weight of a path \(\hat{\rho} = (q_0, \hat{s}_0) \ldots (q_n, \hat{s}_n)\) for \(\hat{\rho}\) is
\[
W(\hat{\rho}) = \sum_{k=0}^{n-1} \omega^p_\nu(p_k, p_{k+1}, \phi_s),
\]
where \(W(\hat{\rho})\) measures the total violations with respect to \(\phi_s\) in the WTS. Hence, by minimizing the violation of \(\phi_s\) a run \(\hat{\rho}\) of \(\hat{\rho}\) can fulfill \(\phi_s\) as much as possible.

### B. Energy Function

Inspired by previous work [21], we design a hybrid Lyapunov-like energy function consisting of different violation costs. Such a design can measure the minimum distance to the accepting sets from the current state and enforce the accepting condition by decreasing the energy as the system evolves.

Based on [7], \(d(\hat{p}_i, \hat{p}_j) = \min_{\varphi \in \mathcal{D}(\hat{p}_i, \hat{p}_j)} W(\hat{\rho})\) is the shortest path from \(\hat{p}_i\) to \(\hat{p}_j\), where \(\mathcal{D}(\hat{p}_i, \hat{p}_j)\) is the set of all possible paths.

For \(\hat{p} \in P\), we design the energy function as
\[
J(\hat{p}) = \begin{cases} 
\min_{\hat{p} \in F^*} d(\hat{p}, \hat{p}'), & \text{if } \hat{p} \notin F^*, \\
0, & \text{if } \hat{p} \in F^*,
\end{cases}
\]
where \(F^*\) is the largest self-reachable subset of the accepting set \(F_p\). Since \(\omega_p\) is positive by definition, \(d(\hat{p}, \hat{p}') > 0\) for all \(\hat{p}, \hat{p}' \in P\), which implies that \(J(\hat{p}) \geq 0\). Particularly, \(J(\hat{p}) = 0\) if \(\hat{p} \in F^*\). If a state in \(F^*\) is reachable from \(\hat{p}\), then \(J(\hat{p}) \neq \infty\), otherwise \(J(\hat{p}) = \infty\). Therefore, \(J(\hat{p})\) indicates the minimum distance from \(\hat{p}\) to \(F^*\).

**Theorem 1.** For the energy function designed in [8], if a trajectory \(\hat{p} = \hat{p}_1 \hat{p}_2 \ldots \hat{p}_n\) is accepting, there is no state \(\hat{p}_i, \forall i = 1, 2, \ldots n\) with \(J(\hat{p}_i) = \infty\), and all accepting states in \(\hat{p}\) are in the set \(F^*\) with energy 0. In addition, for any state \(\hat{p} \in P\) with \(\hat{p} \notin F^*\) and \(J(\hat{p}) \neq \infty\), there exists at least one state \(\hat{p}'\) with \(\hat{p}, \hat{p}' \in \delta_p\), such that \(J(\hat{p}') < J(\hat{p})\).

**Proof:** Considering an accepting state \(\hat{p}_i \in F_p\). Suppose \(\hat{p}_i \notin F^*\). By definition [8], \(\hat{p}\) intersects \(F_p\) infinitely many times which indicates there exists another accepting state \(\hat{p}_j \in F_p\) reachable from \(\hat{p}_i\). If \(\hat{p}_j \in F^*\), then by definition of \(F^*\), \(\hat{p}_j\) must be in \(F^*\) which contradicts the assumption that \(\hat{p}_i \notin F^*\). For the case \(\hat{p}_j \notin F^*\), there must exist a non-trivial strongly connected component (SCC) composed of accepting states reachable from \(\hat{p}_j\). All states in SCC belong to \(F^*\). Since the SCC is reachable from \(\hat{p}_j\), it implies \(\hat{p}_j \in F^*\), which contradicts the assumption. Thus all accepting states in \(\hat{p}\) must be in \(F^*\) with energy zero based on [8]. Since \(F^*\) is reachable by any state in \(\hat{p}\), \(J(\hat{p}_i) \neq \infty, \forall i = 1, 2, \ldots n\).

### C. Automaton Update

The system model needs to be updated according to the sensed information during the runtime to facilitate motion planning. The update procedure is outlined in Alg. 2. Let \(\text{Info}(\hat{p}) = \{L_p(p') \mid p' \in \text{Sense}(\hat{p})\}\) denote the newly observed labels of \(\hat{p}'\) that are different from the current knowledge, where \(\text{Sense}(\hat{p})\) represents neighbor states that the agent at current state \(\hat{p}\) can detect and observe. Denote the sensing range is \(N_s\). If the sensed labels \(L_p(p')\) are consistent with the current knowledge of \(\hat{p}'\), \(\text{Info}(\hat{p}) = \emptyset\); otherwise, the properties of \(\hat{p}'\) have to be updated. Let \(J \in \mathbb{R}_+^{|P|}\) denote the stacked \(J\) for all \(\hat{p} \in P\). The terms \(J\) are initialized from the initial knowledge of the environment. At each step, if \(\text{Info}(\hat{p}) \neq \emptyset\), the weight \(\omega_p(p', p'')\) and \(\omega_p(p', p')\) for states that satisfy \(p' \in \text{Sense}(\hat{p})\) and \(p', p'' \in \delta_p\) are updated. Then the energy function \(J\) is updated.

**Lemma 1.** The largest self-reachable set \(F^*\) remains the same during the automaton update in Alg. 2.

**Proof:** Given \(\hat{P}(p, \delta_p)\), the graph induced from \(\hat{P}(p, \delta_p)\) by neglecting the weight of each transition is denoted by \(\hat{G}(p, \delta_p)\). Similar to [21], Alg. 2 only updates the cost of each transition so that the topological structure of \(\hat{G}(p, \delta_p)\) and its corresponding \(F^*\) remain the same.

**Lemma 2** indicates that \(F^*\) doesn’t need to be updated whenever newly sensed information caused by unknown obstacles is obtained. Therefore, it reduces the complexity. As a result, the \(F^*\) is computed off-line, and the construction of \(F^*\) involves the computation of \(d(\hat{p}, \hat{p}')\) for all \(\hat{p}' \in F_p\) and the check of terminal conditions [18].
V. CONTROL SYNTHESIS OF MITL MOTION PLANNING

The control synthesis of the MITL motion planning strategy is based on receding horizon control (RHC). The idea of RHC is to solve an online optimization problem by maximizing the utility function over a finite horizon $N$ and produces a predicted optimal path at each time step. With only the first predicted step applied, the optimization problem is repeatedly solved to predict optimal paths. Specifically, based on the constraint $\beta$ fulfilling soft constraints $\phi$, a predicted path of horizon $N$ at time $k$ starting from $\hat{p}_k$, where $\hat{p}_{i|k} \in \hat{P}$ satisfies $(\hat{p}_{i|k}, \hat{p}_{i+1|k}) \in \delta_p$ for all $i = 1, \ldots, N-1$, and $(\hat{p}_k, \hat{p}_{1|k}) \in \delta_p$. Let Path($\hat{p}_k, N$) be the set of paths of horizon $N$ generated from $\hat{p}_k$. Note that a predicted path $\hat{p}_k \in \text{Path}(\hat{p}_k, N)$ can uniquely project to a trajectory $\gamma(\hat{p}_k) = q = q_1 \cdots q_N$ on $\mathcal{T}$, where $\gamma(\hat{p}_{i|k}) = q_i$, $\forall i = 1, \ldots, N$. The choice of the finite horizon $N$ depends on the local sensing range $N_k$ of the agent. The total reward along the predicted path $\hat{p}_k$ is $R(\gamma(\hat{p}_k)) = \sum_{i=1}^{N} R_k(\gamma(\hat{p}_{i|k}))$.

Based on (9), for every predicted path $\hat{p}_k$ the total violation cost is $W(\hat{p}_k)$. Then the utility function of RHC is designed as

$$U(\hat{p}_k) = R(\gamma(\hat{p}_k)) - \beta W(\hat{p}_k),$$

where $\beta$ is the relative penalty.

By applying large $\beta$, maximizing the utility $U(\hat{p}_k)$ tends to bias the selection of paths towards the objectives, in the decreasing order, of 1) hard constraints $\phi_h$ satisfaction, 2) fulfilling soft constraints $\phi_s$ as much as possible, and 3) collecting time-violation rewards as much as possible. Note that continuous and discrete violations are optimized simultaneously based on the preference weight $\alpha$ in $W(\hat{p}_k)$. To satisfy the acceptance condition of $\hat{P}$, we consider the energy function-based constraints simultaneously.

The initial predicted path from $\hat{p}_0$ can be identified by solving

$$\hat{p}_{0, \text{opt}} = \arg \max_{\hat{p}_0 \in \text{Path}(\hat{p}_0, N)} U(\hat{p}_0),$$

subject to: $J(\hat{p}_0) < \infty$. (10)

The constraint $J(\hat{p}_0) < \infty$ is critical because otherwise, the path starting from $\hat{p}_0$ cannot be accepting.

After determining the initial state $\hat{p}_{0, \text{opt}} = \hat{p}_{1|0, \text{opt}}$, where $\hat{p}_{1|0, \text{opt}}$ is the first element of $\hat{p}_{0, \text{opt}}$, RHC will be employed repeatedly to determine the optimal states $\hat{p}_k^*$ for $k = 1, 2, \ldots$. At each time instant $k$, a predicted optimal path $\hat{p}_{k, \text{opt}} = \hat{p}_{1|k, \text{opt}}\hat{p}_{2|k, \text{opt}} \cdots \hat{p}_{N|k, \text{opt}}$ is constructed based on $\hat{p}_k^*$ and $\hat{p}_{k-1, \text{opt}}$ obtained at time $k - 1$. Note that only $\hat{p}_{1|k, \text{opt}}$ will be applied at time $k$, i.e., $\hat{p}_k^* = \hat{p}_{1|k, \text{opt}}$, which will then be used with $\hat{p}_{k, \text{opt}}$ to generate $\hat{p}_{k+1, \text{opt}}$.

**Theorem 2.** For each time $k = 1, 2, \ldots$ provided $\hat{p}_{k-1}^*$ and $\hat{p}_{k-1, \text{opt}}$ from previous time step, consider a RHC

$$\hat{p}_{k, \text{opt}} = \arg \max_{\hat{p}_k \in \text{Path}(\hat{p}_{k-1}^*, N)} U(\hat{p}_k),$$

subject to the following constraints:

1) $\hat{J}(\hat{p}_N|k) < J(\hat{p}_{N|k-1, \text{opt}})$ if $J(\hat{p}_{k-1}^*) > 0$ and $\hat{J}(\hat{p}_{i|k-1, \text{opt}}) \neq 0$ for all $i = 1, \ldots, N$;

2) $\hat{J}(\hat{p}_i(\hat{p}_{k-1, \text{opt}}) - 1|k) = 0$ if $J(\hat{p}_{k-1}^*) > 0$ and $\hat{J}(\hat{p}_{i|k-1, \text{opt}}) = 0$ for some $i = 1, \ldots, N$, where $i_{\text{opt}}(\hat{p}_{k-1, \text{opt}})$ is the index of the first occurrence that satisfies $\hat{J}(\hat{p}_{i_{\text{opt}}(\hat{p}_{k-1, \text{opt}})} - 1|k) = 0$ in $p_{k-1, \text{opt}}$;

3) $\hat{J}(\hat{p}_N|k) < \infty$ if $J(\hat{p}_{k-1}^*) = 0$.

Applying $\hat{p}_k^* = \hat{p}_{1|k, \text{opt}}$ at each time $k$, the optimal path $\hat{p}_k^* = \hat{p}_0^*\hat{p}_1^* \cdots$ is guaranteed to satisfy the acceptance condition.

**Proof:** Consider a state $\hat{p}_{k-1}^* \in P, \forall k = 1, 2, \ldots$ and Path($\hat{p}_{k-1}^*, N$) represents the set of all possible paths starting from $\hat{p}_{k-1}^*$ with horizon $N$. Since not all predicted trajectories maximizing the utility function $U(\hat{p}_k^*)$, $\hat{p}_k \in \text{Path}(\hat{p}_{k-1}^*, N)$ in (11) are guaranteed to satisfy the acceptance condition of $\hat{P}$, additional constraints need to be imposed. The key idea about the design of the constraint for (11) is to ensure the energy of the states along the trajectory eventually decrease to zero. Therefore, we consider the following three cases.

1) Case 1: if $J(\hat{p}_{k-1}^*) > 0$ and $\hat{J}(\hat{p}_{i|k-1, \text{opt}}) \neq 0$ for all $i = 1, \ldots, N$, the constraint $\hat{J}(\hat{p}_N|k) < J(\hat{p}_{N|k-1, \text{opt}})$ is enforced. The energy $J(\hat{p}_{k-1}^*) > 0$ indicates there exists a trajectory from $\hat{p}_{k-1}^* \rightarrow F^*$, and $\hat{J}(\hat{p}_{i|k-1, \text{opt}}) \neq 0$ for all $i = 1, \ldots, N$ indicates $\hat{p}_{k-1, \text{opt}}$ does not intersect

![Figure 3. Snapshots of the motion planning. The red dotted arrow line represents the predicted trajectory at the current time.](attachment://image.png)
The constraint $J(\hat{p}_{N|k}) < J(\hat{p}_{N|k-1,opt})$ enforces that the optimal predicted trajectory $\hat{p}_{N|k}$ must end at a state with lower energy than that of the previous predicted trajectory $\hat{p}_{N|k-1,opt}$, which indicates the energy along $\hat{p}_{k,opt}$ decreases at each iteration $k$.

2) Case 2: if $J(\hat{p}_{i|k-1,opt}) = 0$ for some $i = 1, \ldots, N$, $\hat{p}_{k-1,opt}$ intersects $\mathcal{F}^*$. Let $i_0(\hat{p}_{k-1,opt})$ be the index of the first occurrence in $\hat{p}_{k-1,opt}$ where $J(\hat{p}_{i_0|k-1}) = 0$. The constraint $J(\hat{p}_{i_0|k-1,opt}) < 0$ enforces the predicted trajectory at the current time $k$ to have energy 0 if the previous predicted trajectory contains such a state.

3) Case 3: if $J(\hat{p}_{k-1}^*) = 0$, it indicates $\hat{p}_{k-1}^* \in \mathcal{F}^*$. The constraint $J(\hat{p}_{k|k}) < \infty$ only requires the predicted trajectory $\hat{p}_{k}$ to be at a state with bounded energy, where Cases 1 and 2 can then be applied to enforce the following sequence $\hat{p}_{k+1}^* \hat{p}_{k+2}^* \ldots$ converging to $\mathcal{F}^*$.

Since the environment is dynamic and unknown, the agent will update the environment according to the detected information at each time step. In addition, by selecting the predictive horizon $N$ to be less than or equal to the sensor range $N_s$, we can ensure the existence of the solutions, since the local environment can be regarded as static. As a result, lemmas in [16] can be applied directly and the proof of the existence is omitted here.

Similar as [21], the energy function based constraints in Theorem 2 ensure an optimal trajectory $\hat{p}^* = \hat{p}_{0|opt} \hat{p}_{1} \ldots$ is obtained which satisfies the acceptance condition. Since the hard constraint is not relaxed, we can restrict the agent to avoid collisions at each time-step based on the sensor information. We assume the local information of WTS can be accurately updated such that the hard constraint is guaranteed. The system will return no solution in cases where no feasible trajectories satisfy the hard constraint, e.g., obstacles surrounding the agent. Note that the optimality mentioned in this paper refers to local optimum since RHC controllers only optimize the objective within finite predictive steps.

The control synthesis of the MITL online motion planning strategy is presented in the form of Algorithm 3. Lines 2-3 are responsible for the offline initialization to obtain an initial $J$. The rest of Algorithm 3 (lines 4-16) is the online receding horizon control part executed at each time step. In Lines 4-6 the receding horizon control is applied to determine $\hat{p}_0$ at time $k = 0$. Since the environment is dynamic and unknown, Algorithm 3 is applied at each time $k > 0$ to update $J$ based on local sensing in Lines 7-9. The RHC is then employed based on the previously determined $\hat{p}_{k-1}^*$ to generate $\hat{p}_{k,opt}$, where the next state is determined as $\hat{p}_{k} = \hat{p}_{1|k,opt}$ in Lines 10-12. The transition from $\hat{p}_{k-1}^*$ to $\hat{p}_{k}^*$ applied on $\hat{P}$ corresponds to the movement of the agent at time $k$ from $\gamma_T(\hat{p}_{k-1}^*)$ to $\gamma_T(\hat{p}_{k}^*)$ on $\mathcal{T}$ in Line 11. By repeating the process in Lines 7-13, an optimal path $\hat{p}^* = \hat{p}_{0|opt} \hat{p}_{1} \ldots$ can be obtained that satisfies the acceptance condition of $\hat{P}$.

Complexity Analysis: Since the off-line execution involves the computation of $\hat{P}, \mathcal{F}^*$ and the initial $J$, its complexity is $O(|\mathcal{F}_P|^2 + |\mathcal{F}_P|^2 + |\hat{P}|^2 \times |\mathcal{F}_P|)$. For online execution, since $\mathcal{F}^*$ remains the same from Lemma 1, Algorithm 2 requires $|\hat{P}|$ runs of Dijkstra’s algorithm. Suppose the number of Sense($\hat{p}$) is bounded by $|N_1|$, therefore, the complexity of Algorithm 2 is at most $O(|N_1| \times |\hat{P}| + |\hat{P}|)$. Suppose the number of total transitions between states is $|\Delta|$. In Algorithm 3, the complexity of recursive computation at each time step is highly dependent on the horizon $N$ and is bounded by $|\Delta|^N$. Overall, the maximum complexity of the online portion of RHC is $O(|N_1| \times |\hat{P}| + |\hat{P}| + |\Delta| N)$.

VI. Case Studies

The simulation was implemented in MATLAB on a PC with 3.1 GHz Quad-core CPU and 16 GB RAM. We demonstrate our framework using the Pac-Man setup shown in Section III. Consider an MITL specification $\phi = \phi_h \land \phi_s$, where $\phi_h = \square\neg\text{obstacle} \land \phi_s = \square(\neg\text{grass}) \land \square(\text{cherry} \lor \diamond\text{cherry})$. In this case, $\phi_h$ means the agent has always to avoid obstacles, and $\phi_s$ indicates the agent needs to repeatedly and sequentially eat pears and cherries within the specified time intervals while avoiding the grass. The tool [57] allows converting MITL into TBA. Fig. 3 shows the snapshots during mission operation. The simulation video is provided [37].

Simulation Results: As for the priorities of violations, we set up that avoiding grass is more critical than eating fruits within the specified time, i.e., we prefer to avoid discrete violation rather than the continuous violation when $\phi_s$ is infeasible. Therefore, we set the parameters $\alpha = 0.8$ and $\beta = 10$. The Pac-Man starts at the bottom left corner and can move up, down, left, and right. In the maze, the time-varying reward $R_k(q)$ is randomly generated at region $q$ from a uniform distribution at time $k$.

[37] https://youtu.be/S_jf4vMfIMO
Since the WTS $\mathcal{T}$ has $|Q| = 100$ states and the relaxed TBA $\hat{\mathcal{A}}$ has $|\hat{S}| = 15$ states, the relaxed product automaton $\hat{\mathcal{P}}$ has $|\hat{\mathcal{P}}| = 1500$ states. The computation of $\hat{\mathcal{P}}$, the largest self-reachable set $\mathcal{F}^*$, and the energy function took 0.62s. The control algorithm outlined in Algorithm [3] is implemented for 50 time steps with horizon $N = 4$.

Fig. 5 shows the snapshots during mission operation. Fig. 3 (a) shows that Pac-Man plans to reach cherry within the specified time interval. Fig. 3 (b) shows that $\phi_s$ is relaxed, and Pac-Man has two choices: go straight to the left, pass the grass, and eat the pear within the specified time or go up first and then to the left to avoid the grass and eat pear beyond the specified time. The former choice means discrete violation while the latter means continuous violation. Since the avoidance of discrete violations has higher priority in our algorithm, the agent chooses the second plan as the predicted optimal path illustrated. Note that, due to the consideration of dynamic obstacles, the deployment of black blocks can vary with time. Fig. 3 (c) and (d) show that on the second completion of the MITL task Pac-Man detects that the cherry at the right top corner is blocked by obstacles and chooses to eat the bottom one.

Fig. 4 (a) shows the evolution of the energy function during mission operation. Each time the energy $J(\hat{\mathcal{P}}) = 0$ in Fig. 4 (a) indicates that an accepting state has been reached, i.e., the desired task is accomplished for one time. The jumps of energy from $t = 30s$ to $35s$ (e.g., $t = 30s$) in Fig. 4 (a) are due to the violation of the desired task whenever the soft task is relaxed. Nevertheless, the developed control strategy still guarantees the decrease of energy function to satisfy the acceptance condition of $\hat{\mathcal{P}}$. Fig. 4 (b) shows the collected local time-varying rewards.

**Computation Analysis:** To demonstrate our algorithm’s scalability and computational complexity, we repeat the control synthesis introduced above for workspace with different sizes. The sizes of the resulted graph, WTS $\mathcal{T}$, the relaxed product automaton $\hat{\mathcal{P}}$, and the meantime taken to solve the predicted trajectories at each time-step are shown in Table I. We also analyze the effect of horizon $N$ on the computation. From Table I, we can see that in the cases with the same horizon $N$, the computational time increases gradually along with the increased workspace size. It is because trajectory updating involves recomputing the energy function based on the updated environment knowledge. In this paper, the proposed RHC-based algorithm only needs to consider the local optimization problem, and the energy constraints will ensure global task satisfaction. Therefore, the mean computation time at each time step does not increase significantly. It shall be noted that in general RHC optimizations, the computations are influenced by the pre-defined horizon $N$.

**VII. CONCLUSION**

In this paper, we propose a control synthesis under hard and soft constraints given as MITL specifications. A relaxed timed product automaton is constructed for task relaxation consisting of task and time violations. An online motion planning strategy is synthesized with a receding horizon controller to deal with the dynamic and unknown environment and achieve multi-objective tasks. Simulation results validate the proposed approach. Future research will consider building the deterministic system online based on the real-time sensing information and develop online robust planning methods for stochastic systems.

**REFERENCES**

[1] C. Belta, A. Bicchi, M. Egerstedt, E. Frazzoli, E. Klavins, and G. J. Pappas, “Symbolic planning and control of robot motion,” IEEE Robot. Autom. Mag., vol. 14, no. 1, pp. 61–70, 2007.

[2] R. Alur, T. Feder, and T. A. Henzinger, “The benefits of relaxing punctuality,” Journal of the ACM (JACM), vol. 43, no. 1, pp. 116–146, 1996.

[3] O. Maler and D. Nickovic, “Monitoring temporal properties of continuous signals,” in Proc. Formal Techn. Model. Anal. Timed Fault Tolerant Syst. Springer, 2004, pp. 152–166.

[4] C.-I. Vasile, D. Aksaray, and C. Belta, “Time window temporal logic,” Theoretical Computer Science, vol. 691, pp. 27–54, 2017.

[5] L. I. R. Castro, P. Chaudhari, J. Tunova, S. Karaman, E. Frazzoli, and D. Rus, “Incremental sampling-based algorithm for minimum-violation motion planning,” in IEEE Conf. Decis. Control, 2013, pp. 3217–3224.

[6] J. Tunova, S. Karaman, C. Belta, and D. Rus, “Least-violating planning in road networks from temporal logic specifications,” in Proc. Int. Conf. Cyber-Phys. Syst. IEEE, 2016, pp. 1–9.

[7] M. Lahijanian, M. R. Maly, D. Fried, L. E. Kavraki, H. Kress-Gazit, and M. Y. Vardi, “Iterative temporal planning in uncertain environments with partial satisfaction guarantees,” IEEE Transactions on Robotics, vol. 32, no. 3, pp. 583–599, 2016.

[8] C.-I. Vasile, J. Tunova, S. Karaman, C. Belta, and D. Rus, “Minimum-violation sctm motion planning for mobility-on-demand,” in Proc. Int. Conf. Robot. Autom. IEEE, 2017, pp. 1481–1488.

[9] M. Cai, Z. Li, H. Gao, S. Xiao, and Z. Kan, “Optimal probabilistic motion planning with partially infeasible LTL constraints,” arXiv preprint arXiv:2007.14325, 2020.
[10] M. Cai, S. Xiao, and Z. Kan, “Reinforcement learning based temporal logic control with soft constraints using limit-deterministic generalized buechi automata,” arXiv preprint arXiv:2104.10284, 2021.

[11] M. Guo and D. V. Dimarogonas, “Multi-agent plan reconfiguration under local LTL specifications,” Int. J. Robotics Res., vol. 34, no. 2, pp. 218–235, 2015.

[12] S. Andersson and D. V. Dimarogonas, “Human in the loop least violating robot control synthesis under metric interval temporal logic specifications,” in Proc. Europ. Control Conf. IEEE, 2018, pp. 453–458.

[13] A. Nikou, D. Boskos, J. Tumova, and D. V. Dimarogonas, “Cooperative task planning of multi-agent systems under timed temporal specifications,” in Proc. Int. Conf. Autom. Sci. Eng. IEEE, 2019, pp. 788–793.

[14] P. Peterson, A. T. Buyukkocak, D. Aksaray, and Y. Yazıcıoğlu, “Distributed safe planning for satisfying minimal temporal relaxations of twtl specifications,” Robotics and Autonomous Systems, vol. 142, p. 103801, 2021.

[15] D. Kamale, E. Karyofylli, and C.-I. Vasile, “Automata-based optimal planning with relaxed specifications,” arXiv preprint arXiv:2107.13650, 2021.

[16] M. Hasanbeig, Y. Kantaros, A. Abate, D. Kroening, G. J. Pappas, and I. Lee, “Reinforcement learning for temporal logic control synthesis with probabilistic satisfaction guarantees,” in 2019 IEEE 58th Conference on Decision and Control (CDC). IEEE, 2019, pp. 5338–5343.

[17] M. Cai, H. Peng, Z. Li, H. Gao, and Z. Kan, “Receding horizon control based motion planning with partially infeasible LTL constraints,” IEEE Control Syst. Lett., vol. 5, no. 4, pp. 1279–1284, 2020.

[18] E. Aasi, C. I. Vasile, and C. Belta, “A control architecture for provably-correct autonomous driving,” arXiv preprint arXiv:2105.02759, 2021.

[19] M. Hasanbeig, Y. Kantaros, A. Abate, D. Kroening, G. J. Pappas, and I. Lee, “Reinforcement learning for temporal logic control synthesis with probabilistic satisfaction guarantees,” in 2019 IEEE 58th Conference on Decision and Control (CDC). IEEE, 2019, pp. 5338–5343.

[20] Q. Lu and Q.-L. Han, “Mobile robot networks for environmental monitoring: A cooperative receding horizon temporal logic control approach,” IEEE Trans. Cybern., vol. 49, no. 2, pp. 698–711, 2018.

[21] M. Cai, H. Peng, Z. Li, H. Gao, and Z. Kan, “Receding horizon control based motion planning with partially infeasible LTL controls,” IEEE Control Syst. Lett., vol. 5, no. 4, pp. 1279–1284, 2020.

[22] E. Aasi, C. I. Vasile, and C. Belta, “A control architecture for provably-correct autonomous driving,” arXiv preprint arXiv:2105.02759, 2021.

[23] M. Hasanbeig, Y. Kantaros, A. Abate, D. Kroening, G. J. Pappas, and I. Lee, “Reinforcement learning for temporal logic control synthesis with probabilistic satisfaction guarantees,” in 2019 IEEE 58th Conference on Decision and Control (CDC). IEEE, 2019, pp. 5338–5343.

[24] M. Cai, H. Peng, Z. Li, and Z. Kan, “Learning-based probabilistic ltl motion planning with environment and motion uncertainties,” IEEE Transactions on Automatic Control, vol. 66, no. 5, pp. 2386–2392, 2020.

[25] M. Cai, S. Xiao, B. Li, Z. Li, and Z. Kan, “Reinforcement learning based temporal logic control with maximum probabilistic satisfaction,” in Int. Conf. Robo. Auton. IEEE, 2021, pp. 806–812.

[26] M. Cai, M. Hasanbeig, S. Xiao, A. Abate, and Z. Kan, “Modular deep reinforcement learning for continuous motion planning with temporal logic,” IEEE Robotics and Automation Letters, vol. 6, no. 4, pp. 7973–7980, 2021.

[27] M. Cai and C.-I. Vasile, “Safe-critical modular deep reinforcement learning with temporal logic through gaussian processes and control barrier functions,” arXiv preprint arXiv:2109.02791, 2021.

[28] C. I. Vasile, X. Li, and C. Belta, “Reactive sampling-based path planning with temporal logic specifications,” Int. J. Robot. Res., pp. 1002–1028, 2020.

[29] Y. Kantaros, M. Malencia, V. Kumar, and G. J. Pappas, “Reactive temporal logic planning for multiple robots in unknown environments,” in 2020 IEEE International Conference on Robotics and Automation (ICRA). IEEE, 2020, pp. 11479–11485.

[30] A. Nikou, J. Tumova, and D. V. Dimarogonas, “Cooperative task planning of multi-agent systems under timed temporal specifications,” in Proc. Am. Control Conf. IEEE, 2016, pp. 7104–7109.

[31] A. Nikou, D. Boskos, J. Tumova, and D. V. Dimarogonas, “On the timed temporal logic planning of coupled multi-agent systems,” Automatica, vol. 97, pp. 339–345, 2018.

[32] C. K. Verginis, C. Vrohidis, C. P. Bechlioulis, K. J. Kyriakopoulos, and D. V. Dimarogonas, “Reconfigurable motion planning and control in obstacle cluttered environments under timed temporal tasks,” in 2019 International Conference on Robotics and Automation (ICRA). IEEE, 2019, pp. 951–957.