Bianchi -VI inflationary model with flat potential in general relativity

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Abstract. We have investigated Bianchi Type VI inflationary model in presence of flat potential to study the accelerating behavior of the physical universe in detail. To obtain the deterministic solution we consider the supplementary relation the component $\sigma_2^2$ of shear tensor $\sigma_i^j$ is proportional to expansion parameter $\theta$ which yield to appropriate condition $c = (ab)^N$ between metric coefficients, where $N$ is non-negative constant. Special case in Bianchi Type VI model provide Bianchi I, III and V space-time. Geometrical and dynamical aspects of models are also discussed in different cases.

Keywords: Bianchi Type VI, Inflationary Cosmological Model, Flat Potential, General Relativity

1. Introduction

In modern research, Einstein theory becomes a subject of interest due to its attainment in explaining accelerated cosmic expansion of the physical universe. Many cosmological problems like isotropy, homogeneity, monopole and flatness can be successfully explained by inflationary theory. Bianchi space VI investigated familiar standard models like Robertson-walker (RW) universe [1], the de-sitter universe [2], and taub-nut solution [3]. Bianchi space VI also leads to closed FRW space time. This model allows not only universe expansion but also shearing, rotation and anisotropic more generally. The initial idea of inflation in early universe is proposed by Guth [4] in context of GUT. Modern cosmological models provide a framework for investigation of early evolution of universe. Standard models indicates universe is purely isotropic and homogeneous which agreed with astronomical facts about early stage of universe, but this model are unstable near cosmological singularities so the choice of anistrophic model for Einstein’s field equations provides a systemic explanation of physical universe rather than standard model.

Inflation is consider as highly rapid expansion of early cosmos by a factor of $10^{78}$ in volume drives under effect of false vacuum energy density. Inflationary scenario in different aspects is developed by many cosmologist [5-12]. Vaidya and Patel [13] have developed spatially homogenous bianchi IX space that provides exact solution of system of field equation for perfect fluid and the pure radiation. Bali and Goyal [14] have resulted various model to explain inflation under consideration of distribution in perfect fluid by assuming the appropriate condition that shear scalar is proportional to expansion scalar. The effect of Higgs field together with potential play vital role in this discussion. Various aspect of inflationary scenario and importance of the scalar field in the study of universe evolution are observed by many researchers [15-22] Motivated by the situations discussed above, in
this work we have derived Synchronized Bianchi space VI cosmological model in existence of flat potential under scalar fields which is purely mass less and in which \( V(\varphi) \) is constant. Bianchi Type VI model provide Bianchi I, III and V space-time as special cases. We have assumed the appropriate relation between metric coefficients to find deterministic solutions. Physical and geometrical features of models are also discussed in different cases.

2. The metric and field equations

Spatially homogeneous Bianchi Type VI in Synchronous coordinate is given by line element

\[
 ds^2 = dt^2 - a^2(t) e^{-2pz} dx^2 - b^2(t) e^{2qz} dy^2 - c^2(t) dz^2
\]  

(2.1)

Where \( a, b \) and \( c \) are metric coefficients

Proper volume of model (2.1) is given by

\[
 V = \sqrt{-g} = abc \ e^{(q-p)z}
\]  

(2.2)

The Shear scalar for model is given by

\[
 \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij}
\]  

(2.3)

where

\[
 \sigma_{ij} = \frac{1}{2} \left( v_{i,k} D^k_j + v_{j,k} D^k_i - \frac{1}{3} \theta (g_{ij} + v_i v_j) \right)
\]  

(2.4)

and

\[
 D^i_j = \delta^i_j - v^i v_j
\]  

The Non vanishing component of shear tensor is given by

\[
 \sigma^1_1 = \frac{1}{3} \left( 2 \frac{a_4}{a} - \frac{b_4}{b} - \frac{c_4}{c} \right)
\]  

(2.5)

\[
 \sigma^2_2 = \frac{1}{3} \left( 2 \frac{b_4}{b} - \frac{a_4}{a} - \frac{c_4}{c} \right)
\]  

(2.6)

\[
 \sigma^3_3 = \frac{1}{3} \left( 2 \frac{c_4}{c} - \frac{a_4}{a} - \frac{b_4}{b} \right)
\]  

(2.7)

\[
 \sigma^4_4 = 0
\]  

(2.8)

The shear scalar is obtained as

\[
 \sigma^2 = \frac{1}{3} \left[ \left( \frac{a_4}{a} \right)^2 + \left( \frac{b_4}{b} \right)^2 + \left( \frac{c_4}{c} \right)^2 - \frac{a_4 b_4}{ab} - \frac{b_4 c_4}{bc} - \frac{a_4 c_4}{ac} \right]
\]  

(2.9)

The Expansion scalar of model is given by

\[
 \theta = v^i_i = \frac{a_4}{a} + \frac{b_4}{b} + \frac{c_4}{c}
\]  

(2.10)

we obtained deceleration parameter by given relation
The Hubble Parameter is given by

\[ H = \frac{1}{3} \left( \frac{a^4}{a} + \frac{b^4}{b} + \frac{c^4}{c} \right) \]  

(2.12)

The co-moving coordinates is given by \( v^i = (0,0,0,1) \)

The action of field of gravitation coupled minimally to scalar field region with potential \( V(\varphi) \) is given by

\[ S = \int \left( R - \frac{1}{2} \varphi_{,i} \varphi_{,j} g^{ij} - V(\varphi) \right) \sqrt{-g} \ dx^4 \]  

(2.13)

which on variation on \( S \) with respect to dynamical field provided Einstein field Equation is given by

\[ R^I_l - \frac{1}{2} R g^I_l = -8\pi T^I_l \]  

(2.14)

(In geometrical unit \( G = c = 1 \))

Here energy momentum Tensor \( T^I_l \) is given by

\[ T_{ij} = \varphi_{,i} \varphi_{,j} - \left( \frac{1}{2} \varphi_{,i} \varphi^i + V(\varphi) \right) g_{ij} \]  

(2.15)

Einstein Fields equations (3) for metric (1) is given by

\[ -b_{44} \frac{c_{44}}{c} + \frac{q^2}{c^2} b_{44} c_{44} = 8\pi \left[ \frac{1}{2} \varphi_4^2 + V(\varphi) \right] \]  

(2.16)

\[ -a_{44} \frac{c_{44}}{c} + \frac{p^2}{c^2} a_{44} c_{44} = 8\pi \left[ \frac{1}{2} \varphi_4^2 + V(\varphi) \right] \]  

(2.17)

\[ -b_{44} \frac{a_{44}}{a} - \frac{pq}{c^2} a_{44} b_{44} = 8\pi \left[ \frac{1}{2} \varphi_4^2 + V(\varphi) \right] \]  

(2.18)

\[ \frac{a_{44} c_{44}}{ab} + \frac{b_{44} c_{44}}{bc} + \frac{a_{44} c_{44}}{ac} + \frac{pq-(p^2+q^2)}{c^2} = 8\pi \left[ -\frac{1}{2} \varphi_4^2 + V(\varphi) \right] \]  

(2.19)

\[ p \left( \frac{a_{44}}{a} \right) + q \left( \frac{b_{44}}{b} - \frac{c_{44}}{c} \right) = 0 \]  

(2.20)

other symbols have their usual meaning, the equation of conservation energy is given by

\[ \varphi_{,i} = -\frac{dV}{d\varphi} \]  

(2.21)

which provide

\[ \varphi_{44} + \left( \frac{a_{44}}{a} + \frac{b_{44}}{b} + \frac{c_{44}}{c} \right) \varphi_4 = -\frac{dV}{d\varphi} \]  

(2.22)
3. Solution of field equations

Steinshabes observed that scalar field $\phi$ with potential $V(\phi)$ has the flat region and field evolves at slow rate but universe is expanding because of vacuum field energy. It is considered that scalar field will cover sufficient time in order to cross the flat region leads to expansion in universe to become more homogeneous and isotropic. We considered the flat region where potential $V$ is constant.

\[ V(\phi) = \beta \text{ (constant)} \]  

(3.1)

System of field equation can be written as

\[
-b_4 \frac{a_{44}}{b} - c_4 \frac{a_{44}}{c} + \frac{q^2}{c^2} \frac{b_4 c_4}{bc} = 8\pi \left[ \frac{1}{2} \phi_4^2 + \beta \right] 
\]  

(3.2)

\[
-a_4 \frac{a_{44}}{a} - c_4 \frac{a_{44}}{c} + \frac{p^2}{c^2} \frac{a_4 c_4}{ac} = 8\pi \left[ \frac{1}{2} \phi_4^2 + \beta \right] 
\]  

(3.3)

\[
-b_4 \frac{a_{44}}{b} - a_4 \frac{a_{44}}{a} - \frac{pq}{c^2} = 8\pi \left[ \frac{1}{2} \phi_4^2 + \beta \right] 
\]  

(3.4)

\[
\frac{a_4 b_4}{ab} + \frac{b_4 c_4}{bc} + \frac{a_4 c_4}{ac} + \frac{pq (p^2 + q^2)}{c^2} = 8\pi \left[ -\frac{1}{2} \phi_4^2 + \beta \right] 
\]  

(3.5)

\[
p \left( \frac{c_4}{c} \frac{a_{44}}{a} + q \left( \frac{b_4}{b} - \frac{c_4}{c} \right) = 0 
\]  

(3.6)

In order to obtained inflationary solution of equations (3.2-3.6), we need one extra physical condition: since component of shear $\sigma_3^2$ is proportional to scalar expansion ($\theta$) of universe which leads to relation between matric coefficients

\[
\frac{1}{3} \left( 2 \frac{c_4}{c} \frac{a_{44}}{a} - \frac{b_4}{b} \right) = \gamma \left( \frac{a_4}{a} + \frac{b_4}{b} + \frac{c_4}{c} \right) 
\]  

(3.7)

where

\[
N = \frac{3\gamma + 1}{2 - 3\gamma} 
\]  

(3.8)

From equation (3.6) and (3.8) we get $b = c_4 a^k$

(3.9)

where $k = \frac{p(N-1)-qN}{q(N-1)-pN}$

Equation (3.2) and (3.3) leads to

\[
\frac{a_4}{a} \frac{a_{44}}{b} + \frac{b_4}{b} c_4 + \frac{q^2 - p^2}{c^2} \frac{b_4 c_4}{bc} = 0 
\]  

(3.10)

using equation (3.8) and (3.9), equation (3.10) become

\[
a_4 + \left( k + N(1+k) \right) \frac{a_4^2}{a} = \delta a^{-2N(1+k)+1} 
\]  

(3.11)
where \( \delta = \frac{p^2 - q^2}{c_1^2N(1+k)} \)

Assume \( a_4 = f(a) \) \( (3.12) \)

Equation (3.11) and (3.12) leads to

\[
\begin{align*}
\left[ \frac{\delta}{1+k} a^{2-2N(1+k)} + D a^{-2(N(1+k)+k)} \right]^{\frac{1}{2}}
\end{align*}
\]

(3.13)

where D is the integration constant.

Equation (19) gives

\[
\int \left[ \frac{\delta}{1+k} a^{2-2N(1+k)} + D a^{-2(N(1+k)+k)} \right]^{\frac{1}{2}} da = \pm(t - t_0)
\]

(3.14)

The line element (2.1) reduced in to form

\[
\begin{align*}
\left[ \frac{\delta}{1+k} T^{2-2N(1+k)} + D T^{-2(N(1+k)+k)} \right]^{-\frac{1}{2}} dT^2 - T^2 e^{-2pZ} dX^2 - c_1^2 T^{2k} e^{2qZ} dY^2
\end{align*}
\]

\[
- c_1^{2N} T^{2N(1+k)} dZ^2
\]

(3.15)

in which \( a = T, \ x = X, \ y = Y \) and \( z = Z \).

Equation (2.22) and (3.1) leads to

\[
\varphi = \int \frac{\mu}{c_1^{(1+N)T(1+N)(1+k)}} dT + C_2
\]

(3.16)

3.1 Physical and dynamical aspects of model

The Proper volume of model is given by

\[
V = c_1^{1+N} T^{(1+N)(1+k)} e^{(q-p)Z}
\]

(3.17)

The Scalar expansion for the developed model is obtained as

\[
\theta = (1 + N)(1 + k) \left[ \frac{\delta}{1+k} T^{-2N(1+k)} + D T^{-2(l+N)(1+k)} \right]^\frac{1}{2}
\]

(3.18)

The Shear Scalar \( (\sigma^2) \) is given by

\[
\sigma^2 = \frac{1}{3} [(\sigma_1^2)^2 + (\sigma_2^2)^2 + (\sigma_3^2)^2 + (\sigma_4^2)^2]
\]

\[
\sigma = \frac{1}{\sqrt{3}} (1 + k)[(N^2 - N + 1) - 3k] \left[ \frac{\delta}{1+k} T^{-2N(1+k)} + D T^{-2(l+N)(1+k)} \right]^\frac{1}{2}
\]

(3.19)

The Hubble parameter for model is given by
The Deceleration parameter is given by

\[ q = -1 + \frac{3}{(1+N)(1+k)} \]  

(3.21)
since \( \frac{a}{b} = \text{constant} \), the developed model maintained anisotropic condition for large T. since the model expanding at \( T > 0 \) and the expansion stops for infinite large T. The Proper volume is increases function of T and become infinite for large T represent inflationary phenomenon of physical universe from early stage of evolution. since constant \( N > 1 \) and \( k > 0 \) so the deceleration parameter turns to negative has astronomical significance. The Higgs field become infinite for \( T= 0 \) and constant for large T. The Hubble parameter decrease with time. The model start with infinite shear and tends to zero as T approaches infinity.

4. Case I: \( q = 0 \)

The metric (2.1) reduced in Bianchi Type III type is defined as

\[ ds^2 = dt^2 - a^2(t)e^{-2pz} dx^2 - b^2(t)dy^2 - c^2(t)dz^2 \]  

(4.1)

where \( a, b \) and \( c \) are metric coefficients

Equation (3.1) gives

\[ b = c_3 a^r \]  

(4.2)

where \( r = \frac{1-N}{N} \) and \( c_3 \) is integration constant

Equation (3.10) and (3.6) provide

\[ \frac{a_{44}}{a} - \frac{b_{44}}{b} + \frac{a_4 c_4}{ac} - \frac{p^2}{c^2} - \frac{b_4 c_4}{bc} = 0 \]  

(4.3)

\[ c = k_4 a \]  

(4.4)

where \( k_4 \) is constant

from equations (3.8),(4.2), (4.3) and (4.4)

\[ 2 \frac{a_{44}}{a} + 2(N(1+r) + r) \left( \frac{a_4}{a} \right)^2 = a a^{-2N(1+r)} \]  

(4.5)

where \( \alpha = \frac{2p^2}{k_4^2 N(1-r)} \)

Equation (4.5) leads to

\[ dt^2 = \left[ \frac{a}{1+r} a^{-2(N(1+r)-1)} + M a^{-2(N(1+r)+r)} \right]^{-1} da^2 \]  

(4.6)

where \( M \) is constant of integration
The line element (4.1) can be reduced to the form
\[
ds^2 = \left[\frac{\alpha}{1+r} T_1^{-2(N(1+r)-1)} + MT_1^{-2(N(1+r)+r)} \right]^{-1} dt_1^2 - T_1^2 e^{-2p_2} dX^2 - c_3^2 T_1^{2r} e^{2q_2} dY^2 - c_3^2 N T_1^{2N(1+r)} dZ^2
\]
(4.7)
where
\[
a = T_1, \quad x = X, \quad y = Y \quad \text{and} \quad z = Z
\]

4.1 Physical and geometrical properties

The Proper volume of the model is given by
\[
V = c_3^{1+N} T_1^{(1+N)(1+r)} e^{(q-p)Z}
\]
(4.8)
The scalar expansion for the developed model is obtained as
\[
\theta = (1+N)(1+r) \left[\frac{\alpha}{1+r} T_1^{-2(N(1+r)-1)} + MT_1^{-2(N(1+r)+r)} \right]^{\frac{1}{2}}
\]
(4.9)
The shear scalar \((\sigma)\) is given by
\[
\sigma = \frac{1}{\sqrt{3}} (1+r) \left[(N^2 - N + 1) - 3r \right]^{\frac{1}{2}} \left[\frac{\alpha}{1+r} T_1^{-2(N(1+r)-1)} + MT_1^{-2(N(1+r)+r)} \right]^{\frac{1}{2}}
\]
(4.10)
The Hubble parameter for the model is given by
\[
H = \frac{1}{3} (1+N)(1+r) \left[\frac{\alpha}{1+r} T_1^{-2(N(1+r)-1)} + MT_1^{-2(N(1+r)+r)} \right]^{\frac{1}{2}}
\]
(4.11)
The deceleration parameter is given by
\[
q = -1 + \frac{3}{(1+N)(1+r)}
\]
(4.12)
The Higgs field is given by
\[
\varphi = \int \frac{\mu}{c_3^{1+N} T_1^{(1+N)(1+r)}} dt_1 + C_4
\]
Here \(\frac{\alpha}{\sigma} = \text{constant}\), the model does not approach isotropy at late time. Model expanded after initial epoch and stops for infinite large \(T\). The volume \(V\) increases with time represent inflationary universe. The Higgs field evolves at a slow rate while expanding universe is observed. The Hubble parameter decrease with time. The model have point type singularities for initial time. This model mostly resembles with bianchi VI model and leads to de-sitter space.

5. Case II: \(p = q = 0\)

In this case model (2.1) reduces to bianchi Type I line element is given by
\[
ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)dy^2 - c^2(t)dz^2
\]
(5.1)
Where $a$, $b$ and $c$ are metric coefficients.

The System of field’s equation is obtained as

\begin{equation}
\frac{b_{44}}{b} - \frac{c_{44}}{c} - \frac{b_{4} c_{4}}{bc} = 8 \pi \left[ \frac{1}{2} \phi_{4}^{2} + \beta \right] \tag{5.2}
\end{equation}

\begin{equation}
\frac{a_{44}}{a} - \frac{c_{44}}{c} - \frac{a_{4} c_{4}}{ac} = 8 \pi \left[ \frac{1}{2} \phi_{4}^{2} + \beta \right] \tag{5.3}
\end{equation}

\begin{equation}
\frac{b_{44}}{b} - \frac{a_{44}}{a} - \frac{b_{4} a_{4}}{ab} = 8 \pi \left[ \frac{1}{2} \phi_{4}^{2} + \beta \right] \tag{5.4}
\end{equation}

\begin{equation}
\frac{a_{4} b_{4}}{ab} + \frac{b_{4} c_{4}}{bc} + \frac{a_{4} c_{4}}{ac} = 8 \pi \left[ \frac{1}{2} \phi_{4}^{2} + \beta \right] \tag{5.5}
\end{equation}

To obtain relativistic solution, we consider the suitable relation between metric coefficients

\begin{equation}
a = b^{\frac{1}{2}} \tag{5.6}
\end{equation}

from equation (3.8) \quad c = (ab)^{N} \tag{5.7}

Equation (5.2) and (5.3) leads to

\begin{equation}
\frac{a_{44}}{a} - \frac{b_{44}}{b} + \frac{a_{4} c_{4}}{ac} - \frac{b_{4} c_{4}}{bc} = 0 \tag{5.8}
\end{equation}

using equation (5.6),(5.7) in eq. (5.8) we obtain

\begin{equation}
a a_{44} + (3N + 2) a_{4}^{2} = 0 \tag{5.9}
\end{equation}

on solving (5.9), line element (5.1) become

\begin{equation}
ds^2 = \frac{1}{c_{6}^{2}} T_{2}^{2(N+2)} dT_{2}^{2} - T_{2}^{2} dX^{2} - T_{2}^{4} dY^{2} - T_{2}^{6N} dZ^{2} \tag{5.10}
\end{equation}

where \quad a = T_{2}, \quad x = X, \quad y = Y, \quad z = Z

and \quad c_{6} = \sqrt{c_{5}} \quad is \quad constant

\section*{5.1 Physical and geometrical aspects}

The Proper volume of model (5.10) is given by

\begin{equation}
V = T_{2}^{3(N+1)} \tag{5.11}
\end{equation}

The scalar of expansion for model is given by

\begin{equation}
\theta = 3( N + 1 ) \frac{c_{6}}{T_{2}^{2(N+1)}} \tag{5.12}
\end{equation}

The Shear scalar for model is given by
The Deceleration parameter for model is given by

\[ q = - \frac{N}{N+1} = \text{constant} \quad (5.14) \]

The Hubble Parameter for model is given by

\[ H = (N + 1) \frac{c_6}{T_2^{3(N+1)}} \quad (5.15) \]

The Higgs field for model is given by

\[ \varphi = \int \frac{e}{T_2^{3(N+1)}} dT_2 + C_3 \quad (5.16) \]

Since volume is increased with time and become infinite for large \( T_2 \) indicates the inflationary universe in present scenario. Also ratio of shear and expansion is constant show the model is failure to approach isotropy for infinite \( T_2 \) i.e. anisotropic phase of physical universe. The Hubble parameter in sufficiently large initially and approaches to zero for large \( T_2 \). The Higgs field tends to constant value at late \( T_2 \). The universe initiate with infinite shear and expansion and decreases as \( T_2 \) increases. The model obeys point type singularity initially. Since \( N > 1 \) so deceleration parameter turned to negative favors the accelerating phase of space time.

6. Case III: \( p = -q \)

In this case model (2.1) reduces to Bianchi Type V line element is given by

\[ ds^2 = dt^2 - (a^2(t)dx^2 + b^2(t)dy^2)e^{2qz} - c^2(t)dz^2 \quad (6.1) \]

In this case equation (3.6) and (3.8) provide

\[ b = k_3a^{-1} \quad (6.2) \]

and

\[ c = k_3^N \quad (6.3) \]

where \( k_3 = k_2^{-\frac{1}{2N-1}} \) is constant

In this case equation (3.10) leads to

\[ \frac{a_{44}}{a} - \frac{b_{44}}{b} + \left( \frac{a_{44}}{a} - \frac{b_{44}}{b} \right) \frac{c_4}{c} = 0 \quad (6.4) \]

using equation (5.2)-(5.4), line element (5.1) leads to

\[ ds^2 = \frac{1}{\omega^2}T_3^2dT_3^2 - \left[ T_3^2dX^2 + k_3^2T_3^{-2}dY^2 \right]e^{2qZ} - k_3^{2N}dZ^2 \quad (6.5) \]

using transformation \( a = T_3, \quad x = X, \quad y = Y, \quad z = Z \)
6.1 Physical and geometrical aspects

The Proper volume of model (6.5) is given by

$$V = \text{constant}$$

(6.6)

The scalar of expansion for model is given by

$$\theta = 0$$

(6.7)

The Shear scalar for model is given by

$$\sigma = \text{constant}$$

(6.8)

since model start with constant volume and independent of parameter $T_3$. Here scalar of expansion is zero i.e. non-expanding universe is observed. Also constant shear scalar has constant value. so inflation is not possible in this case and stationary model of universe is investigated.

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