Emergent scenario and different anisotropic models

Sudeshna Mukerji\textsuperscript{1}
Nairwita Mazumder\textsuperscript{2}
Ritabrata Biswas\textsuperscript{3}
Subenoy Chakraborty\textsuperscript{4}

Department of Mathematics, Jadavpur University, Kolkata-32, India.

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Abstract

In this work, Emergent Universe scenario has been developed in general homogeneous anisotropic model and for the inhomogeneous LTB model. In the first case, it is assumed that the matter in the universe has two components - one is perfect fluid with barotropic equation of state $p = \omega \rho$ ($\omega$, a constant) and the other component is a real or phantom (or tachyonic) scalar field. In the second case, the universe is only filled with a perfect fluid and possibilities for the existence of emergent scenario has been examined.

Keywords: Emergent Scenario, General anisotropic model, LTB model.

1 Introduction

The idea of emergent universe is the result of the search for singularity-free inflationary model in classical general relativity. An emergent universe model can be defined as a singularity free universe which is ever existing with an almost static nature in the infinite past ($t \to -\infty$) and then evolves into an inflationary stage. In fact, an extension of the original Lemaitre-Eddington model can be termed as emergent universe. There are several features for the emergent universe viz. (i) the universe is almost static at the infinite past, (ii) there is no timelike singularity, (iii) the universe is always large enough so that classical description of space time is adequate (iv) the universe has accelerated expansion, etc.

However, there was singularity-free solution in the literature since 1967 - Harrison described a closed model of the universe with radiation, which coincides with Einstein static model in infinite past. But actual search for Emergent model of the universe was started (about 40 years back) by Ellis and collaborators (2004, 2004). They formulated a closed model of the universe filled with two non-interacting fluids - one is a minimally coupled scalar field having self-interaction potential and the other is a perfect fluid with

\textsuperscript{1}mukerjisudeshna@gmail.com
\textsuperscript{2}nairwita15@gmail.com
\textsuperscript{3}biswas.ritabrata@gmail.com
\textsuperscript{4}schakraborty@math.jdvu.ac.in
equation of state \( p = \omega \rho \). In fact, they studied only the asymptotic behavior to characterize the emergent scenario without finding exact analytic solutions. Then Mukherjee et al. (2005) solved semi-classical equations in the Starobinsky model for flat FRW space-time and examined the features of emergent scenario. Subsequently, Mukherjee and others (2006) were able to obtain nonsingular (i.e., geodesically complete) inflationary solution where a part of the matter is in exotic form. Mulryne et al. (2005) have discussed the existence and stability of emergent models in the context of Loop Quantum Cosmology. Debnath (2008) has formulated an emergent model of the universe for exotic matter in the form of phantom or tachyonic field. Banerjee et al. (2007, 2008) have shown a model of emergent universe in braneworld scenario while Campo et al. (2007) have studied a model of emergent universe for self-interacting Brans-Dicke theory. Recently, Mukerji et al. (2010, 2010) have formulated an emergent universe model in the context of Einstein-Gauss-Bonnet theory and in Horava Gravity.

In section 2, we consider a homogeneous and anisotropic space-time models described by the line element

\[
\text{ds}^2 = -\text{dt}^2 + a^2 \text{dx}^2 + b^2 d\Omega^2_k
\]  

where the scale factors \( a \) and \( b \) are functions of time \( t \) alone. We note that

\[
d\Omega^2_k = dy^2 + dz^2, \quad \text{when} \quad k = 0 \quad (\text{Bianchi I model})
\]
\[
= d\theta^2 + \sin^2 \theta d\phi^2, \quad \text{when} \quad k = +1 \quad (\text{Kantowski - Sachs model})
\]
\[
= d\theta^2 + \sinh^2 \theta d\phi^2, \quad \text{when} \quad k = -1 \quad (\text{Bianchi III model})
\]

Here \( k \) is the scalar curvature and the above three types are characterized by Thorne (1967) as flat, closed and open respectively.

Now, the expression for the Hubble parameter \( H \), the deceleration parameter \( q \) and the anisotropy scalar \( \sigma \) in terms of the scale factors are:

\[
H = \frac{1}{3} \left[ \frac{\dot{a}}{a} + 2 \frac{\dot{b}}{b} \right]
\]
\[
q = -1 - \frac{\dot{H}}{H^2}
\]

and

\[
\sigma^2 = \frac{1}{3} \left( \frac{\dot{a}}{a} - \frac{\dot{b}}{b} \right)^2
\]

In section 3, we consider our universe as inhomogeneous Lemaitre - Tolman - Bondi (LTB) Model. The LTB model is one of the most well-known spherically symmetric models in general relativity. It was described by Lemaitre (1933), Tolman (1934) and Bondi (1947) during the period of time from 1933 to 1947. The exact solution have been obtained by Bonnor (1972, 1974). This simple inhomogeneous cosmological model agrees with current supernova and some other data (Moffat 2005; Moffat 2006; Alnes 2006; Alnes 2007; Romano 2010, 2010). Also very recently Clarkson and Marteens (2010) give a justification for inhomogeneous model from the point of view of perturbation analysis.

The inhomogeneous spherically symmetric LTB space-time model is described by the metric ansatz in a co-moving frame as

\[
\text{ds}^2 = -\text{dt}^2 + \frac{R^2}{1 + f(r)} dr^2 + R^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right)
\]
where $R = R(r,t)$ is the area radius of the spherical surfaces and $f(r)(> -1)$ is the curvature scalar that classifies the space-time as

(i) bounded, if, $-1 < f(r) < 0$,
(ii) marginally bounded, if, $f(r) = 0$,
(iii) unbounded if $f(r) > 0$.

The energy-momentum tensor for perfect fluid is given by

$$T_{\mu\nu} = (\rho + p) u_{\mu} u_{\nu} + p g_{\mu\nu}$$  \hspace{1cm} (3)

where the fluid 4-velocity $u^\mu$ has normalization $u^\mu u_\mu = -1$ and $\rho$, $p$ are respectively the usual matter density and pressure of the fluid. Finally, the paper ends with a brief discussion in section 4.

2 General anisotropic model : Basic Equations for Emergent Scenario :

The Einstein field equations for homogeneous and anisotropic model of the universe with non-interacting two fluid system can be written as :

$$\frac{\ddot{a}}{a} + 2 \frac{\ddot{b}}{b} = -\frac{1}{2} [\rho_m + \rho_\phi + 3p_m + 3p_\phi]$$  \hspace{1cm} (4)

and

$$\frac{\dot{b}^2}{a^2} + 2 \frac{\dot{a} \dot{b}}{ab} + \frac{k}{b^2} = [\rho_m + \rho_\phi]$$  \hspace{1cm} (5)

where $\rho_m$ and $p_m$ are the energy density and pressure of a perfect fluid having equation of state $p_m = \omega \rho_m$, ($\omega$ a constant), $\rho_\phi$ and $p_\phi$ are the energy-density and pressure of a scalar field $\phi$ having expressions

$$\rho_\phi = \frac{\varepsilon}{2} \dot{\phi}^2 + V(\phi)$$  \hspace{1cm} (6)

$$p_\phi = \frac{\varepsilon}{2} \dot{\phi}^2 - V(\phi)$$  \hspace{1cm} (7)

Here $V(\phi)$ is the potential for the scalar field $\phi$ and $\varepsilon = \pm 1$ corresponds to normal or phantom scalar field.

As the two components of the matter field are non-interacting, so the energy conservation equations are given by

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0$$  \hspace{1cm} (8)

and

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0$$  \hspace{1cm} (9)

For emergent scenario the appropriate form of the scale factors will be according to Debnath (2008)

$$a(t) = a_0 [\beta + e^{\alpha t}]^n$$  \hspace{1cm} (10)

and

$$b(t) = b_0 [\delta + e^{\gamma t}]^m$$  \hspace{1cm} (11)

where $a_0$, $\beta$, $\alpha$, $n$ and $b_0$, $\delta$, $\gamma$, $m$ are positive constants. The justification for such choices are the following :
1. $a_0 > 0$ and $b_0 > 0$ for scale factor to be positive definite.
2. $\beta > 0$ and $\delta > 0$, otherwise there will be big rip singularity.
3. $\alpha > 0$, $n > 0$, $\gamma > 0$ and $m > 0$ for expanding model of the universe.
4. $\alpha < 0$, $n < 0$ or $\gamma < 0$, $m < 0$ implies that there was a big bang singularity at infinite past.

For the above choice of the scale factors, the universe started with a finite volume at $t = -\infty$, grows gradually without encountering any singularity for any $t$ and finally, the universe will be of infinite volume at future infinity.

For this choice of the scale factors 'a' and 'b', the Hubble parameter, its derivatives and the deceleration parameter 'q' have the following expressions:

$$H = \frac{1}{3} \left[ \frac{[\alpha e^{\beta t}]}{[\beta + e^{\gamma t}]} + \frac{[2m\gamma e^{\gamma t}]}{[\delta + e^{\gamma t}]} \right],$$
$$\dot{H} = \frac{1}{3} \left[ \frac{[\beta e^{\gamma t}]}{[\beta + e^{\gamma t}]} + \frac{[2m\delta e^{\gamma t}]}{[\delta + e^{\gamma t}]} \right],$$
$$\ddot{H} = \frac{1}{3} \left[ \frac{[\beta e^{\gamma t}]}{[\beta + e^{\gamma t}]} + \frac{[2m\delta e^{\gamma t}]}{[\delta + e^{\gamma t}]} \right],$$
$$q = -1 - \frac{3n\beta e^{\gamma t}(\delta + e^{\gamma t})^2 + 6m\delta e^{\gamma t}(\beta + e^{\gamma t})^2}{[n\alpha e^{\gamma t} e^{\gamma t} + 2m\gamma e^{\gamma t}(\beta + e^{\gamma t})^2]}.$$

(12)

We note that $H$ and $\dot{H}$ are positive definite while $q$ is negative definite throughout the evolution but $\ddot{H} = 0$ when $t$ satisfies

$$\frac{(\delta + e^{\gamma t})^3 (\beta - e^{\gamma t}) e^{\gamma t}}{(\beta + e^{\gamma t})^3 (e^{\gamma t} - \delta) e^{\gamma t}} = \frac{2m\delta e^{\gamma t}}{n\beta e^{\gamma t}}.$$

Asymptotically, as $t \to -\infty$, $H$, $\dot{H}$, and $\ddot{H}$ all tend to zero but $q \to -\infty$ while the model becomes a de Sitter universe as $t \to \infty$.

In figure 1 we have plotted the deceleration parameter, $q$, against time $t$. As $\dot{H}$ is positive throughout the evolution so $q$ is always less than $-1$ while asymptotically as $t \to +\infty$, $\dot{H} \to 0$ and hence from (12), $q \to -1$ as shown in figure 1.

Now using the equation of state for the perfect fluid the energy conservation relation (5) can be integrated to obtain

$$\rho_m = \rho_0 (ab^2)^{-(1+\omega)},$$

(13)

where $\rho_0$ is an integration constant.

Also, writing the explicit form of $\rho_\phi$ and $p_\phi$, from the field equations (11) and (15)

$$\rho_\phi = \frac{\dot{b}^2}{b^2} + 2\frac{\dot{a}}{a} \frac{\dot{b}}{b} + \frac{k}{b^2} - \rho_m,$$

(14)

and

$$p_\phi = -\frac{1}{3} \left[ \frac{2\dot{a}}{a} + \frac{4\dot{b}}{b} + \dot{b}^2 + 2\frac{\dot{a}}{ab} + \frac{k}{b^2} \right] - \omega \rho_m,$$

(15)

i.e., we have

$$\ddot{\phi}^2 = \frac{1}{3} \left[ \frac{2}{3} \left( \frac{k}{b^2} - 3\dot{H} - 3\sigma^2 \right) - (1 + \omega)\rho_0 (ab^2)^{-(1+\omega)} \right].$$

(16)
Fig. 1 represents the graph of the deceleration parameter, $q$, against the cosmic time, $t$, in anisotropic model of universe for the constants $\alpha = 0.1$, $\beta = 0.2$, $\gamma = 0.3$, $\delta = 0.4$, $m = 4$, $n = 6$. 
\[ V(\phi) = \left[ \dot{H} + 3H^2 + \frac{2k}{3b^2} \right] - \frac{1}{2}(1 - \omega)\rho_0(ab^2)^{-(1+\omega)} \]  

(17)

We note that \( V(\phi) \) is independent of \( \varepsilon \), i.e. it has the same value for real or phantom scalar field.

2.1 Possibility of Emergent Scenario: The restrictions.

In this section, we discuss the possibility of emergent universe and present the restrictions for its validity. In other words, we examine whether the choices of \( a(t) \) and \( b(t) \) given by equations (10) and (11) are possible solutions for the field equations for (i) the real scalar field and (ii) the phantom scalar field.

2.1.1 Real Scalar field \((\varepsilon = +1)\):

In this case, the expression for \( \dot{\phi}^2 \) is

\[ \dot{\phi}^2 = \frac{2}{3} \left[ \frac{k}{b^2} - 3H - 3\sigma^2 \right] - (1 + \omega)\rho_0(ab^2)^{-(1+\omega)} \]

When the perfect fluid matter component is not of exotic nature (i.e. does not violate the weak energy condition) then for real scalar field, emergent scenario is not possible for \( k = 0, -1 \) (i.e. for flat and open model) while for Kantawski-Sachs model (with \( k = +1 \)), emergent scenario is possible provided

\[ \frac{1}{b^2} > 3H - 3\sigma^2 + \frac{3}{2}(1 + \omega)\rho_0(ab^2)^{-(1+\omega)} \]  

(18)

On the other hand, if the perfect fluid matter component is of phantom nature, then emergent scenario is possible for all three models provided

\[ |1 + \omega|\rho_0(ab^2)^{-(1+\omega)} > 3\dot{H} + 3\sigma^2 - \frac{k}{b^2} \]  

(19)

and \( \phi \) can be obtained as

\[ \phi = \int \sqrt{ \frac{2}{3} \left[ \frac{\dot{b}^2}{b^2} + \frac{2\dot{a}\dot{b}}{ab} - \frac{\ddot{a}}{a} - \frac{2\ddot{b}}{b} + \frac{k}{b^2} \right] - (1 + \omega)\rho_0(ab^2)^{-(1+\omega)} } dt \]

As \( a \) and \( b \) are given functions of \( t \), (equation (10) and (11)), so in principle the above integral can be evaluated to obtain \( \phi \) as a function of \( t \).

The graphical representation of \( V \) for various choices of the parameters and the curvature scalar \( k \) are shown in figure fig 2(a)-2(h). The first five figures are for \( k = +1 \) then next two are for \( k = -1 \) and the last one is for \( k = 0 \). The curves for the potential show some distinct features for different choices of the parameters.

2.1.2 Phantom Scalar field \((\varepsilon = -1)\):

In this case, \( \dot{\phi}^2 \) can be written as

\[ \dot{\phi}^2 = \frac{2}{3} \left[ 3\dot{H} + 3\sigma^2 - \frac{k}{b^2} \right] + (1 + \omega)\rho_0(ab^2)^{-(1+\omega)} \]
Fig. 2a represents the graph of the scalar field potential $V$ against cosmic time, $t$, for the variables $\alpha = 0.05$, $\beta = 0.05$, $\gamma = 0.1$, $\delta = 0.3$, $m = 1.05$, $n = 1.71$, $a_0 = 1$, $b_0 = 2$, $\rho_0 = 1$, $\omega = 1/3$ at $k=1$, i.e., for the closed universe. Fig. 2b represents the graph for the same value of the variables except for $m = 1.38$, $n = 1$. Fig. 2c represents the graph for the same value of the variables except for $m = 2.98$. Fig. 2d represents the graph for the same value of the variables except for $m = 4.72$, $n = 1.71$. Fig. 2e represents the graph for the same value of the variables except for $m = 1$, $n = 2.48$, $\alpha = 0.218$, $\beta = 0.228$, $\gamma = 0.158$, $\delta = 0.3419$, and a very low value of $\rho_0 = 0.001$. Fig. 2f represents the graph of the scalar field potential $V$ against cosmic time, $t$, for the variables $\alpha = 0.812$, $\beta = 0.456$, $\gamma = 0.496$, $\delta = 0.596$, $m = 2.85$, $n = 2.95$, at $k=0$, i.e., for the flat universe. Fig. 2g represents the graph of the scalar field potential $V$ against cosmic time, $t$, for the variables $\alpha = 0.608$, $\beta = 0.336$, $\gamma = 0.1$, $\delta = 1.39$, $m = 2.81$, $n = 3.55$, at $k=-1$, i.e., for the open universe. Fig. 2h represents the graph of the scalar field potential $V$ against cosmic time, $t$, for the same value of the variables except $\gamma = 0.442$ at $k=-1$, i.e., for the open universe.
when the perfect fluid matter component is not of exotic nature, the emergent scenario is possible for phantom scalar field when $k = 0$, $-1$ (i.e. Bianchi I and Bianchi III model) while for Kantowski Sachs model ($k=1$) emergent scenario is possible provided
\[
\frac{1}{b^2} < 3H + 3\sigma^2 + \frac{3}{2}(1 + \omega)\rho_0(ab^2)^{-(1+\omega)}\tag{20}
\]

But when the perfect fluid matter component is of phantom nature, then emergent scenario is always possible for all three models provided
\[
|1 + \omega|\rho_0(ab^2)^{-(1+\omega)} < 3H + 3\sigma^2 - \frac{k}{b^2}\tag{21}
\]

2.2 Emergent Scenario for Tachyonic Scalar field:

For a tachyonic scalar field $\psi$, with potential $B(\psi)$, the energy density $\rho_\psi$ and pressure $p_\psi$ are given by
\[
\rho_\psi = \frac{B(\psi)}{\sqrt{1 - \varepsilon |\dot{\psi}|^2}}\tag{22}
\]
and
\[
p_\psi = -B(\psi)\sqrt{1 - \varepsilon |\dot{\psi}|^2}\tag{23}
\]
where as before $\varepsilon = \pm 1$ corresponds to normal and phantom tachyonic field. So, $\dot{\psi}^2$ and $B(\psi)$ can be written as
\[
\dot{\psi}^2 = \frac{\rho_\psi + p_\psi}{\varepsilon(\rho_\psi)}\tag{24}
\]
and
\[
B(\psi) = \sqrt{-\rho_\psi p_\psi}\tag{25}
\]

Hence from the field equations $\dot{\psi}^2$ and $B(\psi)$ have the expressions
\[
\dot{\psi}^2 = \left(\frac{1}{\varepsilon}\right)^2 \frac{\frac{k}{b^2} - 3\dot{H} - 3\sigma^2 - (1 + \omega)\rho_0(ab^2)^{-(1+\omega)}}{\frac{\dot{a}^2}{a^2} + \frac{5\dot{b}^2}{2b^2} + \frac{\dot{a}\dot{b}}{ab} + 3\dot{H} + \frac{k}{b^2}}\tag{26}
\]
and
\[
B(\psi) = \sqrt{\frac{2}{3} \left[ \frac{\dot{a}^2}{a^2} + \frac{5\dot{b}^2}{2b^2} + \frac{\dot{a}\dot{b}}{ab} + 3\dot{H} + \frac{k}{b^2} \right] + \omega\rho_0(ab^2)^{-(1+\omega)} \times \frac{\dot{b}^2}{b^2} + \frac{2ab}{ab} + \frac{k}{b^2} - \rho_0(ab^2)^{-(1+\omega)}}\tag{27}
\]
We note that as before $B(\psi)$ is independent of $\varepsilon$ i.e. it has the same value for normal or phantom tachyonic field.

The variation of $B$ with cosmic time $t$ for different choices of the parameters and the curvature scalar $k$ are shown in fig. 3(a)-(h). The first four figures are for $k = 1$, then two for $k = -1$ and the last two for $k = 0$. In this case also the curves for the potential show some special feature for different choices of the parameters.
Fig. 3a represents the graph of the tachyonic field potential $B$ against cosmic time, $t$, for the constants $\alpha = 0.1$, $\beta = 0.2$, $\gamma = 0.1$, $\delta = 0.2$, $m = 1$, $n = 1$, $a_0 = 0.2$, $b_0 = 0.2$, $\rho_0 = 0.01$, $\omega = -1$ at $k=1$, i.e., for the closed universe. Fig. 3b represents the graph of the tachyonic field potential $B$ against cosmic time, $t$, for the constants $\alpha = 1$, $\beta = 16.2$, $\gamma = 0.59$, $\delta = 0.2$, $m = 1$, $n = 1$, $a_0 = 0.2$, $b_0 = 0.2$, $\rho_0 = 0.01$, $\omega = -1$ at $k=1$, i.e., for the closed universe. In 3c and 3d all the data are same with 3b except for $\gamma$ which has values respectively 0.92 and 3.51 in these two cases. 3e and 3f represents the graph of the tachyonic field potential $B$ against cosmic time $t$ at $k=-1$, i.e. open universe and 3g-3h represents the tachyonic field potential $B$ at $k=0$, i.e. flat universe.
2.2.1 Normal Tachyonic Scalar field ($\varepsilon = +1$): 

Here for real $\psi$ both numerator and denominator in equation (26) must have the same sign. The denominator cannot be negative because then the expression within the second square root in equation (27) becomes negative and hence $B(\psi)$ becomes imaginary. So, the numerator has to be positive. Hence, when the perfect fluid matter component is not of exotic nature (i.e., does not violate the weak energy condition) then emergent scenario is not possible in this case for flat Bianchi I ($k = 0$) model and open Bianchi III ($k = -1$) model whereas for closed Kantowski Sachs model ($k = +1$) emergent scenario is possible provided inequality (18) holds. When the perfect fluid matter component is of phantom nature then emergent scenario is possible for all three models provided restriction (19) holds.

Also $\psi$ can be obtained as

$$
\psi = \int \left[ \frac{2}{3} \left\{ \frac{k}{\rho} - 3\dot{H} - 3\sigma^2 \right\} - (1 + \omega) \rho_0 (ab^2)^{-(1+\omega)} \right]^{\frac{1}{2}} dt
$$

2.2.2 Phantom Tachyonic Scalar Field ($\varepsilon = -1$):

In this case, for real $\psi$ the numerator and the denominator of equation (26) should have opposite signs. But the denominator can not be negative as stated earlier. So, the only possibility is that the numerator must be negative definite. In this case, $\dot{\psi}^2$ can be written as

$$
\dot{\psi}^2 = \frac{2}{3} \left\{ 3H + 3\sigma^2 - \frac{\dot{H}}{\rho} \right\} + (1 + \omega) \rho_0 (ab^2)^{-(1+\omega)}
$$

So, when the perfect fluid matter component is not of exotic nature then emergent scenario is always possible when $k = 0$, $-1$. While for $k = +1$, emergent scenario is possible when restriction (20) holds. When the perfect fluid matter component is of phantom nature, then emergent scenario is possible for flat, closed and open models provided restriction (21) holds.

3 Lemaitre-Tolman-Bondi model

By introducing the mass function $F(r, t)$ (Joshi 1979; Banerjee 2003; Debnath 2003,2004) (related to the mass contained within the co-moving radius $r$) as

$$
F(r, t) = R(\dot{R}^2 - f(r))
$$

(28)

the Einstein’s equations in LTB model are

$$
8\pi G\rho = \frac{F'(r, t)}{R^2 R'}
$$

(29)

and

$$
8\pi Gp = -\frac{\dot{F}(r, t)}{R^2 R'}
$$

(30)

and the evolution equation for $R$ is

$$
2\dot{R} + \dot{R}^2 + 8\pi GpR^2 = f(r)
$$

(31)
The conservation equation is
\[ \dot{\rho} + 3H (\rho + p) = 0 \quad (32) \]
where
\[ H = \frac{1}{3} \left( \frac{\dot{R}}{R} + 2 \frac{\dot{R}'}{R'} \right) \quad (33) \]
is the Hubble parameter. The equation of state is given by
\[ p = \epsilon \rho, \text{ where } \epsilon \text{ is a constant.} \quad (34) \]
Using equations (33) and (34), equation (32) can be integrated to give
\[ \rho = \rho_0 \left( R^2 R' \right)^{-(1+\epsilon)} \quad (35) \]
with \( \rho_0 \), a constant of integration.

Using (34) and (35) in equation (31), the differential equation for \( R \) is obtained as
\[ 2\dot{R} + \ddot{R} + \frac{8\pi G \epsilon \rho_0}{(R^2 R')^{(1+\epsilon)}} = f(r) \quad (36) \]
Now, taking \( 8\pi G = 1 \) and writing
\[ R = h(r)g(t) \quad (37) \]
we can write (36) as
\[ 2g(t) (g(t))^{3(1+\epsilon)} \frac{d^2}{dt^2} g(t) + \left( \frac{d}{dt} g(t) \right)^2 (g(t))^{3(1+\epsilon)} + \frac{\epsilon \rho_0}{(h(r))^2 \left( (h(r))^2 \frac{d h(r)}{d r} \right)^{(1+\epsilon)}} = \frac{f(r)}{h^2} (g(t))^{3(1+\epsilon)} \quad (38) \]
Choosing \( \epsilon = -1 \) (i.e., matter as a cosmological constant) the above differential equation can be written in separable form as
\[ 2g(t) \frac{d^2}{dt^2} g(t) + \left( \frac{d}{dt} g(t) \right)^2 = \frac{1}{(h(r))^2} \{ \rho_0 + f(r) \} = \lambda \quad \text{(say)} \quad (39) \]
i.e.,
\[ 2g(t) \frac{d^2}{dt^2} g(t) + \left( \frac{d}{dt} g(t) \right)^2 = \lambda \quad (39) \]
and
\[ \frac{1}{(h(r))^2} \{ \rho_0 + f(r) \} = \lambda \quad (40) \]
Solving (39) we have
\[ \left( \sqrt{\lambda g(t) - 1} + \sqrt{\lambda g(t)} \right)^5 \exp \left\{ (\lambda g(t) - 5) \sqrt{\frac{\lambda g(t)}{\lambda g(t) - 1}} \right\} = \exp \left\{ \lambda \frac{2}{5} (t - t_0) \right\} \quad (41) \]
and
\[ h(r) = \sqrt{\frac{\rho_0 + f(r)}{\lambda}} \quad (42) \]
where \( t_0 \) appears as the constant of integration. From the solution (41), we note that in the asymptotic region (i.e. \( t \to -\infty \)), \( R(t, r) \) has a finite non-zero value. \( (g(t) \to \frac{1}{\lambda}) \). Also, the asymptotic behaviour of \( g(t) \) is shown in figure 4.

Thus, emergent scenario is possible in LTB model with matter only in the form of a cosmological constant.
Fig 4 shows the variation of the function $g(t)$ w.r.t. time $t$. 
4 Discussion:

In this work, first we have examined the possibility of emergent scenario for general homogeneous and anisotropic model of the universe filled with matter having non-interacting two components—one in the form of perfect fluid with linear equation of state \( p = \omega \rho \) and a scalar field (real, phantom or tachyonic) with potential as the other component. For real scalar field and tachyonic scalar field, when the perfect fluid matter component is not of exotic nature (i.e. does not violate the weak energy condition) then emergent scenario is not possible for flat Bianchi I model \( (k = 0) \) and open Bianchi III model \( (k = -1) \) but for closed Kantowaski-Sachs model \( (k = +1) \), emergent scenario is possible when inequality (18) holds. When the perfect fluid matter component is of phantom nature, then emergent scenario is possible for all models in real scalar field and tachyonic scalar field provided restriction (19) holds. For phantom scalar field and phantom tachyonic scalar field, when the perfect fluid matter component is not of exotic nature, then emergent scenario is always possible for flat and open models while for closed model it is possible when inequality (20) holds. But when the perfect fluid matter component is of phantom nature, then emergent scenario is possible for all three models provided restriction (21) holds. As these restrictions for emergent scenario depend on the anisotropy scalar \( \sigma \) so the results are distinct from the isotropic model.

In the second case we have studied the emergent scenario for LTB model. Here it is possible to have a solution assuming the scale factor to be inseparable (product form) and the matter is purely a cosmological constant. The asymptotic analysis shows that in the infinite past there is no singularity and the scale factor grows with time. In future it will be interesting to study the emergent scenario with general form of matter, particularly, with exotic matter.

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