SEMI-ANALYTIC MODELS FOR ELECTRON ACCELERATION IN WEAK ICM SHOCKS

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Abstract: We propose semi-analytic models for the electron momentum distribution in weak shocks that accounts for both in situ acceleration and re-acceleration through diffusive shock acceleration (DSA). In the former case, a small fraction of incoming electrons are assumed to be reflected at the shock ramp and pre-accelerated to the so-called injection momentum, \( p_{\text{inj}} \), above which particles can diffuse across the shock transition and participate in the DSA process. This leads to the DSA power-law distribution extending from the smallest momentum of reflected electrons, \( p_{\text{ref}} \), all the way to the cutoff momentum, \( p_{\text{eq}} \), constrained by radiative cooling. In the later case, fossil electrons, specified by a power-law spectrum with a cutoff, are assumed to be re-accelerated also from \( p_{\text{ref}} \) up to \( p_{\text{eq}} \) via DSA. We then show that, in the in situ acceleration model, the amplitude of radio synchrotron emission depends strongly on the shock Mach number, whereas it varies rather weakly in the re-acceleration model. Considering rather turbulent nature of shocks in the intracluster medium, such extreme dependence for the in situ acceleration might not be compatible with relatively smooth surface brightness of observed radio relics.

Key words: acceleration of particles — cosmic rays — galaxies: clusters: general — shock waves

1. INTRODUCTION

Cosmological hydrodynamic simulations predicted that the intracluster medium (ICM) could encounter shocks on average several times during the formation of the large-scale structures in the Universe (e.g., Ryu et al. 2003; Vazza et al. 2009). As in the case of astrophysical shocks such as the Earth’s bow shock and supernova remnants, ICM shocks are expected to produce cosmic-ray protons (CRp) and electrons (CRe) via diffusive shock acceleration (DSA) (e.g., Bell 1978; Drury 1983; Brunetti & Jones 2014). Many of merger-driven shocks have been observed and identified as “radio relic shocks” in the outskirts of galaxy clusters through radio synchrotron radiation from shock-accelerated CRe with Lorentz factor \( \gamma_e \sim 10^3 - 10^4 \) (e.g. van Weeren et al. 2010, 2014; Kang et al. 2012). Shocks formed in the hot ICM are weak with the sonic Mach number, \( M_s \lesssim 4 \), which can be inferred from the observed radio spectral index, \( \alpha_v = (M^2 + 3)/2(M^2 - 1) \), using the test-particle prediction of DSA (e.g. Kang 2011, 2016).

In this discussion, the shock is specified by the sonic Mach number, \( M_s \), and preshock temperature, \( T_1 \), where the subscripts, 1 and 2, denote the preshock and postshock states, respectively. The momentum distribution, \( f(p) \), scales with the upstream gas density, \( n_1 \), and so it does not need to be specified. For quantities related with synchrotron emission and cooling, the preshock magnetic field strength, \( B_1 = 1 \mu \text{G} \), is adopted. The plasma beta refers the ratio of thermal to magnetic pressures, \( \beta = P_{\text{gas}}/P_B \), in the background ICM. Common symbols in physics are used: e.g., \( m_e \) for the electron mass, \( m_p \) for the proton mass, \( c \) for the speed of light, and \( k_B \) for the Boltzmann constant.

Suprathermal particles above the so-called injection momentum, \( p_{\text{inj}} \), have gyroradii long enough to diffuse across the shock transition and may participate in DSA, aka Fermi 1st-order acceleration, if scattering MHD/plasma waves of sufficient amplitudes are present (e.g., Drury 1983). Hence the pre-acceleration of thermal particles to \( p_{\text{inj}} \), i.e. the ‘injection problem’, has been a longstanding key problem in the DSA theory (e.g., Malkov & Drury 2001; Kang et al. 2002; Marucovich et al. 2014). According to plasma simulations of quasi-parallel shocks (Caprioli & Spitkovsky 2014; Caprioli et al. 2015; Ha et al. 2018), some of incoming protons are specularly reflected by the overshoot in the shock potential and undergo shock drift acceleration (SDA) at the shock front, resulting in the self-excitation of upstream waves via both resonant and non-resonant streaming instabilities. Then the protons are scattered around the shock by those waves, which leads to the formation of the DSA power-law spectrum above \( p_{\text{inj}} \sim (3.0 - 3.5)P_{\text{th}} \), where \( P_{\text{th}} = (2m_p k_B T_2) \) is the postshock thermal proton momentum.

On the other hand, electrons are known to be injected and accelerated preferentially at quasi-perpendicular shocks, which involves kinetic processes on electron kinetic scales much smaller than ion scales (e.g., Balogh & Truemann 2013). Guo et al. (2014) showed, through particle-in-cell (PIC) simulations of quasi-perpendicular shocks, that a small fraction of incoming electrons are reflected due to the magnetic mirror and energized via SDA, while the backstreaming
electrons excite oblique waves by the electron firehose instability (EFI). Due to scattering of electrons between the shock ramp and EFI-induced waves in the shock foot, the pre-accelerated electrons seem to form a DSA power-law spectrum through a Fermi i-like acceleration. However, Kang et al. (2019) showed that such pre-acceleration is effective only in supercritical shocks with $M_s \gtrsim 2.3$. Moreover, they suggested that suprathermal electrons may not be energized all the way to $p_{\text{ini}} \sim 150p_{\text{th},\text{e}}$ (where $p_{\text{th},\text{e}} = (2m_e k_B T_e)^{1/2}$), because the growth of longer waves via the EFI is saturated. On the other hand, Trotta & Burgess (2019) and Kobzar et al. (2019) have demonstrated through hybrid simulations with test-particle electrons ($\beta \approx 1$) and PIC simulations ($\beta \approx 5$), respectively, that at supercritical quasi-perpendicular shocks the rippling of shock surface excited by Alfvén Ion Cyclotron (AIC) instability could induce multi-scale fluctuations, leading to the pre-acceleration of electrons beyond $p_{\text{ini}}$. Whereas the critical Mach number above which the shock rippling becomes active was estimated to be $M_{\text{crit}} \approx 3.5$ for $\beta \approx 1$ shocks (Trotta & Burgess 2019), this problem needs to be investigated for higher $\beta$ shocks.

Although the DSA model seems to provide a simple and natural explanation for some observed properties of radio relics, such as thin elongated shapes, post-shock spectral steepening due to aging electron population, and polarization vectors indicating perpendicular shock spectral steepening due to aging electron populations, there remain some unresolved problems that need further investigation. First of all, the pre-acceleration of thermal electrons to suprathermal energies and the subsequent injection into the DSA process still remains rather uncertain, especially at sub-critical shocks with $M_s \lesssim 2.3$ (Kang et al. 2019). Secondly, the fraction of observed merging clusters with detected radio relics is only $\sim 10\%$ (Feretti et al. 2012), while numerous quasi-perpendicular shocks are expected to form in the ICM (Wittor et al. 2017; Roh et al. 2019). Thirdly, in a few cases, the sonic Mach number inferred from X-ray observations is smaller than that estimated from radio spectral index of radio relics, i.e., $M_s < M_{\text{radio}}$ (Akamatsu & Kawahara 2013; Kang 2016). Thus re-acceleration of fossil CRe, pre-existing in the ICM, has been suggested as a possible resolution for these puzzles that the DSA model with ‘in situ injection only’ leaves unanswered (e.g., Kang et al. 2012, 2017; Kang 2016).

Based on what we have learned from the previous studies, here we propose semi-analytic models for the momentum distribution function of CRe, $f(p)$, in the two scenarios of DSA at quasi-perpendicular weak shocks in the test-particle regime: (1) in situ acceleration model in which electrons are injected directly from the background thermal pool at the shock, and (2) re-acceleration model in which pre-existing fossil CRe are accelerated. Although it remains largely unknown if and how CRe are accelerated at subcritical shocks, in this paper we take a heuristic approach and assume that DSA operates at shocks of all Mach numbers.

In the next section we describe in details the semi-analytic models for $f(p)$ along with in-depth discussion on underlying physical justification. In Section 2 we demonstrate how our model can be applied to weak shocks in the ICM and discuss observation implications.

A brief summary will be given in Section 1.

2. Semi-Analytic DSA Model

The physics of collisionless shocks depends on various shock parameters including the sonic Mach number, $M_s$, the plasma beta, $\beta$, and the obliquity angle, $\theta_{\text{inj}}$, between the upstream background magnetic field direction and the shock normal (e.g., Balogh & Truemann 2013). For instance, collisionless shocks can be classified as quasi-parallel ($Q_{\parallel}$, hereafter) shocks with $\theta_{\text{inj}} \lesssim 45^\circ$ and quasi-perpendicular ($Q_{\perp}$, hereafter) shocks with $\theta_{\text{inj}} \gtrsim 45^\circ$. CRs are known to be accelerated efficiently at $Q_{\parallel}$ shocks, while CRe are accelerated preferentially at $Q_{\perp}$ shocks (Gosling et al. 1989; Burgess 2007; Caprioli & Spicketovskiy 2014; Guo et al. 2014).

In this study, we focus on the electron acceleration at $Q_{\perp}$ shocks with $M_s \lesssim 4$ that are expected to form in the ICM. Most of the kinetic problems involved in the electron acceleration, including the shock criticality, excitation of waves via microinstabilities, and wave-particle interactions, have been investigated previously for shocks in $\beta \sim 1$ plasma such as the solar wind and the interstellar medium (see Balogh & Truemann 2013; Marcowith et al. 2016). Although a few studies, using kinetic PIC simulations, have recently considered weak shocks in the high $\beta$ ICM environment (Guo et al. 2014; Matsukiyo & Matsumoto 2015; Kang et al. 2019; Kobzar et al. 2019), full understanding of the electron injection and acceleration into the genuinely diffusive scattering regime has yet to come.

The main difficulty in reaching such a goal is the severe computational requirements to perform PIC simulations for high $\beta$ shocks; the ratio of the proton Larmor radius to the electron skin depth increases with $\beta^{-1/2}$. Moreover, to properly study these problems, PIC simulations in at least two-dimensional domains extending up to several proton Larmor radii are required, because kinetic instabilities induced by both protons and electrons may excite waves on multi-scales that propagate in the direction oblique to the background magnetic fields.

2.1. Particle Injection to DSA

In this section, we review the current understandings of the injection problem that have been obtained previously through plasma hybrid and PIC simulations. Suprathermal particles, both protons and electrons, with $p \gtrsim 3p_{\text{th},p}$ could diffuse across the shock both upstream and downstream, and participate in the DSA process, because the shock thickness is of the order of the gyroradius of postshock thermal protons. Thus the injection momentum is often parameterized as

$$p_{\text{ini}} = Q_{1,p} \cdot p_{\text{th},p},$$

where the injection parameter is estimated to range $Q_{1,p} \sim 3.0 - 3.5$, according to the hybrid simulations
of $Q_\parallel$ shocks in $\beta \sim 1$ plasma (Caprioli & Spitkovsky 2014; Caprioli et al. 2014; Hu et al. 2018). On the other hand, Ryu et al. (2019) showed that the DSA power-law with $Q_{p,\parallel} \approx 3.8$ gives the postshock CRp energy density less than 10 % of the shock kinetic energy density for $M_s \lesssim 4$, i.e., $E_{\text{CRp}} \ll 0.1E_\text{sh}$ (where $E_\text{sh} = p_\perp u_\perp^2/2$).

The electron injection at $Q_\perp$-shocks involves somewhat different processes, which can be summarized as follows: (1) the reflection of some of incoming electrons at the shock ramp due to magnetic deflection, leading to a beam of backstreaming electrons, (2) the energy gain from the motional electric field in the upstream region through shock drift acceleration (SDA), (3) the trapping of electrons near the shock due to the scattering by the upstream waves, which are excited by backstreaming electrons via the EFI, and (4) the formation of a suprathermal tail for $p \gtrsim p_{\text{ref}}$ with a power-law spectrum, which seems consistent with the test-particle DSA prediction (Guo et al. 2014; Matsukiyo & Matsumoto 2013). Here, $p_{\text{ref}}$ represents the lowest momentum of the reflected electrons above which the suprathermal power-law tail develops. This is again parameterized as

$$p_{\text{ref}} = Q_{i,e} \cdot p_{\text{inj,e}},$$

with the injection parameter, which is assumed to range $Q_{i,e} \sim 3.5 - 3.8$ as in the case of $p_{\text{inj}}$ (e.g., Guo et al. 2014; Kang et al. 2019).

Recently, Kang et al. (2019) showed that the electron pre-acceleration through the combination of reflection, SDA, and EFI may operate only in supercritical $Q_\perp$-shocks with $M_s \gtrsim 2.3$ in $\beta \sim 100$ plasma. In addition, they argued that the EFI alone may not energize the electrons all the way to $p_{\text{inj}}$, unless there are pre-existing turbulent waves with wavelengths longer than those of the EFI-driven waves. As mentioned earlier, on the other hand, Trotta & Burgess (2019) and Kobzar et al. (2019) showed through 2D simulations that the suprathermal tail may extend to beyond $p_{\text{inj}}$ in the presence of multi-scale turbulence excited by the shock rippling instability. But Trotta & Burgess (2019) suggested that the critical Mach number, at which the shock surface rippling starts to develop, is $M_e \approx 3.5$ in $\beta \approx 1$ plasma. Hence, we still need to answer the following questions in future studies: (1) if and how the electron injection occurs at subcritical shocks with $M_s \lesssim 2.3$, and (2) how the critical Mach number for the shock surface rippling varies with shock parameters such as $\beta$ and $\theta_B$.

On the other hand, X-ray and radio observations of several radio relics indicate the efficient electron acceleration even at subcritical shocks with $1.5 \lesssim M_e \lesssim 2.3$ (van Weeren et al. 2013). Hence, in the discussion below, we heuristically assume that the DSA power-law spectrum of the accelerated electrons, $f_{e,\text{inj}} \propto p^{-q}$ (where $q = 4M_f^2/(M_f^2 - 1)$), develops from $\sim p_{\text{ref}}$ all the way to the cutoff momentum $p_{\text{cutoff}}$ (see below) at $Q_\perp$-shocks of all Mach numbers. This hypothesis needs to be examined in future studies for high $\beta$ shocks.

![Figure 1.](https://example.com/figure1.png)

Figure 1. Semi-analytic functions for the momentum distribution, $f(p)p^\gamma$, in a $M_s = 3.0$ shock, based on the test-particle DSA model. The black dashed line shows a power-law spectrum of pre-existing fossil electrons, $f_{e,\text{foss}}(p)$, with the slope, $\gamma = 4.7$, and the cutoff momentum, $p_c/m_e = 10^5$. The blue dot-dashed line shows the spectrum of re-accelerated electrons, $f_{e,\text{reacc}}(p)$ in Equation (12). The red line shows the injected population, $f_{e,\text{inj}}(p)$, in Equation (6). The green vertical line denotes $p_{\text{cutoff}} = Q_{\perp}p_{\text{inj}}$, with $Q_{\perp} = 3.8$, above which suprathermal electrons are reflected at the shock ramp and accelerated by Fermi-I acceleration. Note that the amplitude of $f_{e,\text{reacc}}(p)$ scales with the adopted normalization factor, $f_0$, and so the relative importance between $f_{e,\text{reacc}}$ and $f_{e,\text{inj}}$ depends on it. The proton spectrum, including both the postshock Maxwellian and DSA power-law components, is shown by the black solid line for comparison. The magenta vertical line demarcates the injection momentum, $p_{\text{inj}} = Q_{\perp}p_{\text{inj},p}$ with $Q_{\perp} = 3.8$, above which particles can undergo the full DSA process across the shock transition.

Nevertheless, we refer Figure 4 of Park et al. (2015), in which the acceleration of both protons and electrons at strong $Q_\parallel$-shocks ($M_s = 40$) were investigated through 1D PIC simulations. There, electrons form a DSA power-law for $p \gtrsim p_{\text{ref}}$, because local fields become quasi-perpendicular at some parts of the shock surface due to turbulent magnetic field amplification driven by the strong non-resonant Bell instability.

### 2.2. Test-Particle Solutions for Injection-only Case

Here we adopt the test-particle solutions of DSA, because dynamical feedbacks of CRp and CRe are expected to be insignificant at weak ICM shocks (e.g., Ryu et al. 2019). Then the isotropic part of the momentum distribution function at the shock position can be approximated by a power-law spectrum with a super-exponential cutoff. For CRp spectrum,

$$f_{p,\text{inj}}(p) \approx f_{\text{inj}} \cdot \left( \frac{p}{p_{\text{inj}}} \right)^{-q} \exp \left( -\frac{p^2}{p_{\text{max}}^2} \right).$$

(3)
The normalization factor at \( n \) is given as

\[
f_{\text{i}} = \frac{n_{p,2}}{\pi^{3/2}} P_{\text{th},p} \exp(-Q_{\text{th},p}^2),
\]

where \( n_{p,2} \) is the postshock proton number density. The maximum momentum of CRp achieved by the shock age of \( t \) can be estimated as

\[
\frac{p_{\text{max}}}{m_p c} \approx \frac{\sigma - 1}{\kappa^* t},
\]

where \( \sigma = n_2/n_1 \) is the shock compression ratio and \( \kappa^* \) is the diffusion coefficient at \( p = m_p c \) (Kang & Ryu 2011). For ICM shocks, \( p_{\text{max}}/m_p c \gg 1 \), so the exponential cutoff at \( p_{\text{max}} \) is not important for weak shocks.

Similarly, the test-particle spectrum of CRE can be expressed as

\[
f_{\text{e}, \text{i}}(p) \approx f_{\text{ref}} \left( \frac{p}{p_{\text{ref}}} \right)^{-q} \exp \left( -\frac{p^2}{p_{\text{eq}}^2} \right).
\]

The normalization factor at \( p_{\text{ref}} \) is given as

\[
f_{\text{ref}} = \frac{n_{e,2}}{\pi^{3/2}} P_{\text{th},e} \exp(-Q_{\text{th},e}^2),
\]

where \( n_{e,2} \) is the postshock electron number density. The cutoff momentum, \( p_{\text{eq}} \), can be derived from the equilibrium condition that the DSA momentum gains per cycle are equal to the synchrotron/iC losses per cycle (Kang 2011):

\[
p_{\text{eq}} = \left( \frac{m_e^2 c^2 u_s}{4 \pi e^2 q} \frac{B_1}{B_{\text{e},1}^2 + B_{\text{e},2}^2} \right)^{1/2},
\]

where the ‘effective’ magnetic field strength \( B_{\text{e},1} = B^2 + B_{\text{rad}}^2 \) takes account for radiative losses due to both synchrotron and iC processes, where \( B_{\text{rad}} = 3.24 \mu G (1+z)^2 \) corresponds to the cosmic background radiation at redshift \( z \). Here, we assume the Bohm diffusion for DSA, and set \( z = 0.2 \) as a reference epoch and so \( B_{\text{rad}} = 4.7 \mu G \). For typical ICM shock parameters it becomes

\[
\frac{p_{\text{eq}}}{m_p c} \approx 6.75 \times 10^9 \left( \frac{u_s}{1000 \text{ km s}^{-1}} \right) \left( \frac{B_1}{B_{\text{e},1}^2 + B_{\text{e},2}^2} \right)^{1/2},
\]

where the magnetic field strength is expressed in units of \( \mu G \). Again, \( p_{\text{eq}}/m_p c \gg 1 \), so the exponential cutoff is not important for weak shocks.

With the DSA model spectra given in Equations (3) and (6), if \( Q_{\text{th},p} = Q_{\text{th},e} \) as assumed here, then the ratio of \( f_{\text{p}, \text{i}}(p) \) to \( f_{\text{e}, \text{i}}(p) \) at \( p = p_{\text{eq}} \) can be estimated as

\[
K_{p/e} = \frac{f_{\text{p}, \text{i}}(p_{\text{eq}})}{f_{\text{e}, \text{i}}(p_{\text{eq}})} \left( \frac{p_{\text{th},p}}{p_{\text{th},e}} \right)^{q-3} \left( \frac{m_p}{m_e} \right)^{(q-3)/2},
\]

where \( K_{p/e} \) is equivalent to the CRp-to-CRe number ratio. For example, in the case of a \( M_s = 3.0 \) shock with \( q = 4.5 \), \( K_{p/e} = 280 \), but with a caveat that protons (electrons) are accelerated at \( Q_{\text{th},p} \) (\( Q_{\text{th},e} \)) shocks.

In Figure 2 we illustrate the thermal Maxwellian distribution and the test-particle power-law spectrum, \( f_{\text{p}, \text{i}}(p) \), for protons, which are decelerated by the line of \( p_{\text{eq}} \). Here the shock parameters adopted are \( Q_{\text{th},p} = 3.8 \), \( M_s = 3 \) and \( T_1 = 5.8 \times 10^7 \) K, and \( q = 4.5 \). Also, the thermal Maxwellian distribution and \( f_{\text{p}, \text{i}}(p) \) for electrons are decelerated by the line of \( p_{\text{ref}} \) with \( Q_{\text{th},e} = 3.8 \). This clearly demonstrates that, in order to get injected to DSA, the reflected electrons need to be energized by a factor of \( p_{\text{eq}}/p_{\text{ref}} \approx \sqrt{m_p/m_e} \) or so.
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2.3. Re-acceleration of Fossil CR Electrons

For the preshock population of fossil CRe, we adopt a power-law spectrum with the slope $s$ and a cutoff momentum $p_c$ for $p > p_{\text{ref}}$:

$$f_{e,\text{foss}}(p) = f_0 \left( \frac{p}{p_{\text{ref}}} \right)^{-s} \exp \left( -\frac{p^2}{p_c^2} \right), \quad (11)$$

where $p_c/m_c = 10^3 - 10^5$ is considered in this discussion, and the normalization factor, $f_0$, determines the amount of fossil CRe. Then, the re-accelerated population at the shock can be calculated semi-analytically by the following integration:

$$f_{e,\text{reacc}}(p) = q \cdot p^{-q} \int_{p_{\text{ref}}}^{p} p'^{q-1} f_{e,\text{foss}}(p') dp' \quad (12)$$

Druy 1983; Kang & Ryu 2011. In Figure 1 we show an example of $f_{e,\text{foss}}(p)$ with $s = 4.7$, and $p_c/m_c = 10^3$ in the black dashed line, while its re-accelerated spectrum, $f_{e,\text{reacc}}(p)$, at a $M_s = 3$ shock is shown in the blue dot-dashed line. Note that the normalization factor is set as $f_0 = f_{\text{ref}}$ in Equation (11) with $Q_{1,e} = 3.8$ just for illustration purpose only.

Note that for a power-law fossil population without a cutoff, $f_{e,\text{foss}} \propto p^{-s}$, the re-accelerated spectrum can be obtained by direct integration Kang & Ryu 2011: for $p \geq p_{\text{ref}}$

$$f_{e,\text{reacc}}(p) = \begin{cases} \frac{q}{(q-s)} \left[ 1 - (p/p_{\text{ref}})^{-q+s} \right] f_{e,\text{foss}}(p), & \text{if } q \neq s; \\ q \ln(p/p_{\text{ref}}) f_{e,\text{foss}}(p), & \text{if } q = s. \end{cases} \quad (13)$$

Although we do not explicitly show it here, the re-accelerated spectrum of pre-existing protons, $f_{p,\text{reacc}}(p)$, can be described by the same integration as Equation (12), except that the lower bound should be replaced with $p_{\text{inj}}$ and $f_{e,\text{foss}}(p)$ should be replaced with a pre-existing proton population, $f_{p,\text{pre}}(p)$, with appropriate parameters, $s$, $p_c$, and $f_0$.

3. Application to Radio Relics

3.1. DSA Model Spectrum

We apply the DSA models given in Equations (8) and (12) to calculate the energy spectrum of accelerated electrons at weak shocks propagating into the preshock gas with $T_1 = 5.8 \times 10^7$ K. Panel (a) of Figure 2 shows the injection spectrum, $f_{e,\text{inj}}(p)$, for shocks with $M_s = 1.6 - 4.0$ (with the increment $\Delta M_s = 0.2$). Considering that the synchrotron emission from mono-energetic electrons with the Lorentz factor, $\gamma_e$, peaks around the characteristic frequency, $v_{\text{peak}} \approx 130\text{MHz}(\gamma_e/10^3)^3/(B/1 \mu G)$, we compare the amplitude of $f_{e,\text{inj}}(p_{\text{rad}})$, where $p_{\text{rad}} = 10^3 m_c c$, by the green vertical dashed line. Then, panel (a) of Figure 3 illustrates how $f_{e,\text{inj}}(p_{\text{rad}})$ depends on the shock Mach number by the magenta dotted line. In the case of the in situ acceleration model, $f_{e,\text{inj}}(p_{\text{rad}})$ increases by a factor of $4.2 \times 10^3$ for the range of $M_s = 2.0 - 3.0$. This strong dependence is even stronger at lower Mach number, so that $f_{e,\text{inj}}(p_{\text{rad}})$ decreases almost by a factor of 90, when $M_s$ decreases from 2.0 to 1.8. This implies that the radio surface brightness could vary extremely sensitively with $M_s$, when a radio relic consists of multiple shocks with slightly different Mach numbers Roh et al. 2019.

Panels (b) and (c) of Figure 2 show the re-accelerated spectra, $f_{e,\text{reacc}}(p)$, of the fossil electron spectrum, $f_{e,\text{foss}}(p)$, with $s = 4.5$ and $p_c/m_c = 10^3$ and $10^4$, respectively. Again, we set $f_0 = f_{\text{ref}}$ with $Q_{1,e} = 3.8$ as in Figure 1. For stronger shocks with $M_s \geq 3.0$, the re-accelerated spectrum is flatter the fossil spectrum, i.e., $q \leq s$. Hence fossil CRs serve only as seed particles, and so $f_{e,\text{reacc}}(p)$ does not de-

Figure 3. (a) Amplitude of $f(p_{\text{rad}})$ at $p_{\text{rad}} = 10^3 m_c c$ for the re-acceleration case with the slope, $s = 4.5$, and the cutoff momentum, $p_c/m_c = 10^3$ (black circles), $10^4$ (red circles), and $10^5$ (blue circles). The magenta line with asterisks shows $f(p_{\text{rad}})$ for the injection-only case. The semi-analytic DSA spectra ($Q_{1,e} = 3.8$) shown in Figure 2 are used. (b) CRe pressure, $E_{\text{CRe}}$, in units of $E_{\text{sh}} = \rho_1 u_1^2/2$. The preshock $E_{\text{CRe},1}$ due to fossil CRe with $s = 4.5$ and $p_c/m_c = 10^3$ is shown by open circles, while the postshock $E_{\text{CRe},2}$ due to re-accelerated CRe is shown by closed circles. The magenta line with asterisks shows $E_{\text{CRe},2}$ for the injection-only case. Again, the semi-analytic DSA spectra in Figure 2 are used.
due to the following three CRe spectra: (1) the postshock region, (2) the re-acceleration model with the slope, \( s = 4.5 \), and the cutoff momentum, (a) \( p/m_c = 10^3 \), (b) \( p/m_c = 10^4 \), (c) \( p/m_c = 10^5 \). Note that the results for the in situ acceleration model are not shown, because the corresponding synchrotron spectrum is a simple power-laws with a cutoff. The semi-analytic DSA spectra (\( Q_s = 3.8 \)) shown in Figure 2 are used. The emissivity \( \nu_{jr} \) is plotted in arbitrary units. The blue dashed lines correspond to the models with \( M_s = 3.0 \) and \( q = s = 4.5 \). The green vertical denotes \( \nu = 153 \) MHz. Lower panels: Synchrotron spectral index, \( \alpha_{\nu} = -d\ln \nu_{jr}/d\ln \nu \), for the radiation spectra shown in the upper panels.

Figure 4. Upper panels: Synchrotron spectrum, \( \nu_{jr} \), due to \( f_{e,\text{reacc}}(p) \) re-accelerated at shock with \( M_s = 1.6 - 4.0 \) and \( T_1 = 5.8 \times 10^7 \) K in the presence of fossil CRe with the slope, \( s = 4.5 \), and the cutoff momentum, (a) \( p/m_c = 10^3 \), (b) \( p/m_c = 10^4 \), (c) \( p/m_c = 10^5 \). The semi-analytic DSA spectra (\( Q_s = 3.8 \)) shown in Figure 2 are used. The emissivity \( \nu_{jr} \) is plotted in arbitrary units. The blue dashed lines correspond to the models with \( M_s = 3.0 \) and \( q = s = 4.5 \). The green vertical denotes \( \nu = 153 \) MHz. Lower panels: Synchrotron spectral index, \( \alpha_{\nu} = -d\ln \nu_{jr}/d\ln \nu \), for the radiation spectra shown in the upper panels.

3.2. Radio Synchrotron Emission

We then calculate the synchrotron volume emissivity, \( j_{\nu}(\nu) \), due to \( f_{e,\text{reacc}}(p) \), shown in Figure 2 with the postshock magnetic field strength, \( B_2 = B_1 \sqrt{1/3 + 25/3} \) (where \( B_1 = \mu G \)), in order to illustrate how the radio spectrum changes with the shock Mach number. We do not show explicitly the emissivity spectrum for the in situ acceleration case, since both \( f_{e,\text{inj}}(p) \) and \( j_{\nu}(\nu) \) are simple power-laws with exponential a cutoff.

The top panels of Figure 4 shows \( \nu_{jr} \), while the bottom panel shows its spectral index, \( \alpha_{\nu} = -d\ln \nu_{jr}/d\ln \nu \). Note that \( \nu_{jr} \) scales with the adopted normalization factor, \( f_o \), and is plotted in arbitrary units here. For stronger shocks with \( q < s \), \( j_{\nu}(\nu) \) is a power-law with \( \alpha_{\nu} = (q - 3)/2 = (M_s^2 + 3)/2(M_s^2 - 1) \) with a cutoff. For weaker shocks with \( q > s \), on the other hand, the radio spectrum depends on the cutoff momentum, \( p_c \), as well as \( M_s \), as expected obviously from \( f_{e,\text{reacc}}(p) \) in Figure 2. At these weaker shocks, the slope, \( \alpha_{\nu} \), gradually increases from \( (s - 3)/2 \) to \( (q - 3)/2 \), as the
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Figure 5. Upper panels: Amplitudes of \( j_\nu \) at 153 MHz, 608 MHz, and 1.38 GHz for the synchrotron spectra shown in Figure 4. Three models with fossil CRe with the slope, \( s = 4.3 \), and the cutoff momentum, \( p/m_c = 10^8 \) (black circles), \( 10^9 \) (red circles), and \( 10^{10} \) (blue circles) are shown. The magenta lines with asterisks show the same quantities for the injection-only case. Note that \( j_\nu \) scales with the adopted normalization factor, \( j_0 \), and is plotted in arbitrary units here. Lower panels: Synchrotron spectral index, \( \alpha_\nu = -d\ln j_\nu/d\ln \nu \), for the same models shown in the upper panels. Note that the black line (re-acceleration case with \( p/m_c = 10^8 \)) coincides with the magenta line (injection-only case), which also corresponds to \( \alpha_\nu(M_\ast) = (s - 3)/2 \). The green horizontal lines denote \( \alpha_\nu = (s - 3)/2 = 0.75 \).

...frequency increases. Hence, the fossil CRe power-law should extend to well above \( p/m_c \gtrsim 10^9 \), in order for the spectral index to be determined by the slope of fossil CRe, i.e., \( \alpha_\nu = (s - 3)/2 \), for \( \nu \lesssim 10 \) GHz (see panel (f) of Figure 4). For example, if the power-law spectrum of fossil CRe extends only up to \( p/m_c \lesssim 10^8 \), the radio spectral index due to postshock CRe becomes \((q - 3)/2\) instead of \((s - 3)/2\) for \( \nu \gtrsim 153 \) MHz (see panel (d) of Figure 4)....

...models have the spectral index that follows the injection index, \( \alpha_\nu \) (magenta asterisks). So in the bottom panels of Figure 5 all symbols (black, red, blue, and magenta) overlap with each other for \( M_\ast \gtrsim 3.0 \). For weaker models with \( q > s \), again the spectral index depends on \( p_\nu \). For the models with \( p/m_c = 10^8 \) (blue filled circles), \( \alpha_\nu \approx (s - 3)/2 \) for \( M_\ast \lesssim 3 \). In the case of the models \( p/m_c = 10^9 \) (black filled circles), fossil CRe serve as only seed particles, so the spectral indices at the three observational frequencies becomes the same as the injection index, \( \alpha_\nu(M_\ast) \).

In conclusion, if a radio relic is composed of multiple shocks with slightly different mach numbers (Roh et al. 2019), the surface brightness fluctuations could be much larger in the in situ acceleration model, compared to the re-acceleration model. But the variations in the spectral index profile should be much smaller. Relatively smooth profiles of radio flux along the edge of some observed radio relics, such as the Sausage relic (Hoang et al. 2017) and the Toothbrush relic (van Weeren et al. 2010), probably indicate that the re-acceleration might play a significant role there.

4. SUMMARY

Based on the recent studies using plasma kinetic simulations (e.g., Guo et al. 2014; Matsukibo & Matsumoto 2017; Park et al. 2015; Kang et al. 2019; Trotta & Burgess 2019; Kobzar et al. 2019), here we suggest semi-analytic DSA models for the electron...
(re)-acceleration at weak \( Q_{\perp} \) shocks in the test-particle regime. They rely on the following working assumptions: (1) at \( Q_{\perp} \) shocks of all Mach numbers in the test-particle regime (i.e., \( M_\parallel \lesssim 4 \)), electrons can be pre-accelerated from thermal pool by both electron and ion kinetic instabilities and injected to the DSA process, and (2) the momentum distribution function of (re)-accelerated electrons follows the prediction of the DSA theory for \( p \geq p_{\text{ref}} = Q_{\perp} e \rho_{\text{b}} v_{\text{he}} \) with \( Q_{\perp} e \approx 3.5 \)–3.8. However, it remains uncertain that subcritical shocks with \( M_\parallel \lesssim 2.3 \) could inject electrons to the DSA process [Kang et al. 2015] or re-accelerate pre-existing fossil CR electrons through DSA. We include the electron (re)-acceleration in subcritical shocks here, because, in some of observed radio relics, the shock Mach number is estimated to be less than 2.3 (e.g., van Weeren et al. 2016).

Then, the momentum distribution of accelerated electrons can be represented by the simple power-law with a cutoff given in Equation (6) for the \textit{in situ} acceleration model. For the re-acceleration model with fossil CR electrons, which is specified by the three parameters, the slope, \( s \), the cutoff, \( p_c \), and the normalization factor, \( f_0 \), the re-accelerated spectrum can be integrated semi-analytically as in Equation (12).

We explore how our model spectrum of CR\( e \) varies with the parameters such as \( M_\parallel \), \( s \), and \( p_c \) in the case of weak shocks with \( M_\parallel = 1.6 \)–4.4 for the two types of DSA models: the \textit{in situ} acceleration model and the re-acceleration model. The main findings can be summarized as follows:

1. For stronger shocks with \( q = 4M_\parallel^2/(M_\parallel^2 - 1) \leq s \), the re-accelerated spectrum becomes a power-law, \( f_{e, \text{reacc}}(p) \propto p^{-q} \), and it does not depend on \( p_c \). The radio synchrotron spectrum becomes also a power-law with \( \alpha_e \approx \alpha_e = (q-3)/2 = (M_\parallel^2 + 3)/(2M_\parallel^2) - 1 \) and an appropriate cutoff.

2. For weaker shocks with \( q > s \), on the other hand, \( f_{e, \text{reacc}}(p) \) depends on the cutoff \( p_c \). Only for \( p_c/m_e c \gtrsim 10^3 \), the radio synchrotron spectrum has the spectral index, \( \alpha_e \approx \alpha_e = (s-3)/2 \) for the observation frequencies in the range of \( \nu_{\text{obs}} \approx 100 \text{ MHz} \)–10 GHz.

3. If \( p_c/m_e c \lesssim 10^3 \), the fossil CR\( e \)s provide only seed particles to DSA and so the spectral index is similar to the injection index, \( \alpha_v \approx \alpha_e \).

4. In the \textit{in situ} acceleration model, the radio synchrotron emissivity, \( j_\nu \), depends strongly on \( M_\parallel \), and it increases by a factor of \( 10^3 \)–\( 10^4 \), as \( M_\parallel \) increases from 2.0 to 3.0. But it varies by a factor of only 15 or so in the re-acceleration model with \( p_c/m_e c = 10^3 \) for the same range of \( M_\parallel \). In the case of a lower cutoff \( p_c/m_e c = 10^3 \), \( j_{153 \text{ MHz}} \) increases by a factor of 48, and \( j_{1.38 \text{ GHz}} \) by a factor of 120 for the same range of \( M_\parallel \).

Considering that the profiles of radio flux and spectral index vary rather smoothly along the edge of some observed radio relics (e.g., van Weeren et al. 2016), and that giant radio relics on Mpc scales are likely to consist of multiple shocks with different \( M_\parallel \) (e.g., Roh et al. 2019), our results imply that the re-acceleration of fossil CR\( e \)s is important in understanding the origin of radio relics.

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