Short review about the MSSM with three right-handed neutrinos (MSSM3RHN).

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Abstract

We give a review about the Minimal Supersymmetric Standard Model with three right-handed neutrinos (MSSM3RHN). We, first introduce the minimal set of fields to built this model in their superfields formalism. After it, we build the lagrangian of the model in the superspace formalism and also introduce the soft terms to break SUSY. We show how to get masses to the neutrinos and sneutrinos in this model.

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1 Introduction

Although the Standard Model (SM) describes the observed properties of charged leptons and quarks it is not the ultimate theory. However, the necessity to go beyond it, from the experimental point of view, comes at the moment only from neutrino data [1].

The experimental results that suggest that the neutrinos have non-zero masses and oscillations. The best-fit values at 1σ error level for these neutrino oscillation parameters in the three-flavor framework are summarised as follows [2]

\[
\begin{align*}
\sin^2 \theta_{12} &= \sin^2 \theta_{\text{solar}} = 0.304^{+0.022}_{-0.016}, \quad &\Delta m^2_{21} &= \Delta m^2_{\text{solar}} = 7.65^{+0.23}_{-0.26} \times 10^{-5} \text{ eV}^2, \\
\sin^2 \theta_{23} &= \sin^2 \theta_{\text{atm}} = 0.50^{+0.07}_{-0.06}, \quad &|\Delta m^2_{23}| &= |\Delta m^2_{\text{atm}}| = 2.40^{+0.12}_{-0.11} \times 10^{-3} \text{ eV}^2, \\
\sin^2 \theta_{13} &= \sin^2 \theta_{\text{CHOOZ}} = 0.01^{+0.016}_{-0.011} .
\end{align*}
\]
To explain the data presented above, we need three massive neutrinos. In order to see how to fit these data with three massive neutrinos see [2, 3].

However, if we take seriously, the data coming from Los Alamos Liquid Scintillation Detector experiment (LSND) [4] imply we need to introduce at least one right-handed neutrinos\(^1\). If we take into account cosmological constraint into account we get the following upper limit \(\sum m_\nu < 1.01\, \text{eV}\) if we have three neutrinos and it change to \(\sum m_\nu < 2.12\, \text{eV}\) in the case of five neutrinos as presented at [6, 7]

The aim of the present paper is to propose a modification Minimal Supersymmetric Standard Model to give masses in a simple way to neutrinos and get nice Dark Matter candidate (right-handed sneutrinos) and also get flat directions in scalar potential that can generate the cosmological inflation. Also, the matter anti-matter asymmetry could be obtained from the letogenesis mechanism.

I started this studies to make the articles presented at [8, 9], my first goal was to include it on my previous review about some generalization about the MSSM with singlets [10], but this model is so interesting due it I have done this review.

This paper is organized as follows. In Section 2 we present the minimal supersymmetric model with three right-handed neutrinos. We calculate the masses to neutrinos and sneutrinos (with one right-handed neutrinos). Next, we show some flat directions in this model.

2 The Minimal Supersymmetric Model with three right-handed neutrinos (MSSM3RHN)

On the Minimal Supersymmetric Model with three right-handed neutrinos (MSSM3RHN) [11, 12, 13, 14, 15], the particle content is very similar to the Minimal Supersymmetric Standard Model (MSSM) [11, 16], it contains three families of left-handed quarks \(Q_{iL}, i = 1, 2, 3\) is family indices, three families of leptons \(L_{iL}\) plus the Higgs fields \(H_1\). We have also to introduce three families of right-handed quarks, given by \((u_{iR}, d_{iR})^2\), three families of right-handed charged leptons \((l_{iR})\) and another Higgs fields \(H_2\) as shown at

\(^1\)They are known as sterile neutrinos, for more detail see [5]

\(^2\)We are using similar notation as presented at [10]
| Superfield          | Usual Particle | Spin | Superpartner | Spin |
|---------------------|----------------|------|--------------|------|
| $\hat{L}_{iL} \sim (1, 2, -1)$ | $L_{iL} = (\nu_{iL}, l_{iL})^T$ | $\frac{1}{2}$ | $\tilde{L}_{iL} = (\tilde{\nu}_{iL}, \tilde{l}_{iL})^T$ | 0    |
| $\hat{Q}_{iL} \sim (3, 2, 1/3)$ | $Q_{iL} = (u_{iL}, d_{iL})^T$ | $\frac{1}{2}$ | $\tilde{Q}_{iL} = (\tilde{u}_{iL}, \tilde{d}_{iL})^T$ | 0    |
| $\hat{\ell}_{iR} \sim (1, 1, 2)$ | $l_{iR} = \overline{\ell}_{iL}$ | $\frac{1}{2}$ | $\tilde{l}_{iR} = \tilde{\ell}_{iL}$ | 0    |
| $\hat{N}_{iR} \sim (1, 1, 0)$ | $\nu_{iR} = \overline{\nu}_{iL}$ | $\frac{1}{2}$ | $\tilde{\nu}_{iR} = \tilde{\nu}_{iL}$ | 0    |
| $\hat{d}_{iR} \sim (3^*, 1, 2/3)$ | $d_{iR} = \overline{d}_{iL}$ | $\frac{1}{2}$ | $\tilde{d}_{iR} = \tilde{d}_{iL}$ | 0    |
| $\hat{u}_{iR} \sim (3^*, 1, -4/3)$ | $u_{iR} = \overline{u}_{iL}$ | $\frac{1}{2}$ | $\tilde{u}_{iR} = \tilde{u}_{iL}$ | 0    |
| $\hat{H}_1 \sim (1, 2, -1)$ | $H_1 = (H^0_1, H^-_1)^T$ | 0    | $\tilde{H}_1 = (\tilde{H}^0_1, \tilde{H}^-_1)^T$ | $\frac{1}{2}$ |
| $\hat{H}_2 \sim (1, 2, 1)$ | $H_2 = (H^+_2, H^0_2)^T$ | 0    | $\tilde{H}_2 = (\tilde{H}^+_2, \tilde{H}^0_2)^T$ | $\frac{1}{2}$ |
| $\hat{V}_c^a (SU(3))$ | $G^a_m$ | 1    | $\tilde{g}^a$ | $\frac{1}{2}$ |
| $\hat{V}^i (SU(2))$ | $V^i_m$ | 1    | $\lambda^i_A$ | $\frac{1}{2}$ |
| $\hat{V}' (U(1))$ | $V_m$ | 1    | $\lambda_B$ | $\frac{1}{2}$ |

Table 1: Particle content of MSSM3RHN. The families index for leptons and quarks are $i, j = 1, 2, 3$. The parentheses are the transformation properties under the respective representation of $(SU(3)_C, SU(2)_L, U(1)_Y)$.

Tab. (1). The vacuum expectation values (vev) of our Higgses fields are given by:

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \quad (2)$$

The superpotential of the MSSM is define in the following way

$$W_{MSSM} = \left( \frac{W_H + W_{2RV}}{2} \right) + \left( \frac{W_Y + W_{3RV}}{3} \right) + hc, \quad (3)$$

where

$$W_H = \mu_H (\hat{H}_1 \hat{H}_2),$$
\[ W_Y = \sum_{i,j=1}^{3} \left[ f'_{ij} \left( \hat{H}_1 \hat{L}_{iL} \right) \hat{l}_{jR} + f''_{ij} \left( \hat{H}_1 \hat{Q}_{iL} \right) \hat{d}_{jR} + f'_{ij} \left( \hat{H}_2 \hat{Q}_{iL} \right) \hat{u}_{jR} \right], \quad (4) \]

we have suppressed the \( SU(2) \) indices and the supersymmetric parameter \( \mu_H \) is a complex number and has mass dimension. In general all the parameters \( f \) are, in principle, complex numbers and they are symmetric in \( ij \) exchange and they are dimensionless parameters \([11, 16]\). Moreover, \( f^d \) and \( f^u \) can give account for the mixing between the quark current eigenstates as described by the CKM matrix. In this model, we can also explain the mass hierarchy in the charged fermion masses as showed recently in \([17, 18]\).

We we consider only the gauge symmetry in the MSSM it will allow terms which allow to break the baryon number and the lepton number conservation laws and in the SM all the interactions conserve both laws and no physical process with this property has been discovered so far. This phenomenological fact suggest imposing a discrete symmetry in the model. This symmetry is the \( R \)-parity \([11, 16]\). The terms that break the \( R \)-parity are given by

\[
\begin{align*}
W_{2RV} &= \sum_{i=1}^{3} \mu_i \left( \hat{H}_2 \hat{L}_{iL} \right), \\
W_{3RV} &= \sum_{i,j,k=1}^{3} \left[ \lambda_{ijk} \left( \hat{L}_{iL} \hat{L}_{jL} \right) \hat{l}_{kR} + \lambda'_{ijk} \left( \hat{L}_{iL} \hat{Q}_{jL} \right) \hat{d}_{kR} + \lambda''_{ijk} \hat{u}_{iR} \hat{d}_{jR} \hat{d}_{kR} \right].
\end{align*}
\]

(5)

When we consider \( R \) Parity scenarios, this model has viable candidates to be the Dark Matter. One of most studied candidate is the lightest neutralino. However, some years ago, there were some interesting studies with the lightest sneutrino\(^3\). There are some authors study the gravitino as Dark Matter candidate, more details about SUSY Dark Matter Candidates is presented in nice way in \([20]\).

The mass matrix of neutrinos arise when we allow a mixing between the usual leptons with the higgsinos and its mixings is generated by

\[
\left( \hat{H}_2 \hat{L}_{iL} \right) \supset \left( \bar{\hat{H}}_2 L_{iL} \right) = l_{iL} \bar{H}_2^+ - \nu_{iL} \bar{H}_2^0,
\]

\(^3\)Unfortunately this particle have been ruled out by the combination of collider experiment as LEP and direct searches for cosmological relics as discussed at \([19]\).
the second term at right size mixing the neutrinos with the neutral higgsinos (neutralinos as shown at Fig.(1)) and it is the mechanism to generate masses to two neutrinos at tree level and one neutrino get mass at one loop level as discuss at [21, 22, 23, 24, 25].

Figure 1: Diagrams for the neutrino mass generation at tree level in the model with the R-parity violation figure taken from [25].

More realistic neutrino masses require radiative corrections as shown at Fig.(2) and presented at [23, 24]. In this model the neutrinos are Majorana particles, therefore we can have neutrinoless double beta decay, see [8]. As we broke $L$-number conservation we can have Leptogenesis [26], provides an attractive scenario to explain the baryon asymmetry [8].

Figure 2: One loop diagrams for the neutrino mass generation in the model with R-parity violation, figure taken from [25].

The main motivation on this model is we want to generate, masses to all neutrinos. Therefore, we add also three additional gauge-field left-chiral scalar superfields $\tilde{N}_{1,2,3}$, see Tab.(1)$^4$

$$\tilde{N}_{iR}(y, \theta) = \tilde{\nu}_{iL}^c(y) + \sqrt{2}(\theta\nu_{iL}^c(y)) + (\theta\theta)F_{\nu_{iL}^c}(y), \quad (7)$$

$^4$In chiral superfields all the fermion fields are left-handed [8]
the fields \( \nu^c_i \) are the right-handed neutrinos, known as sterile neutrinos [5]. As a consequence of supersymmetric algebra beyond the three right-handed neutrinos \( \nu^c_i \equiv \nu^c_i \), or sterile neutrinos, we should also introduce three right-handed sneutrinos \( \tilde{\nu}^c_i \), and we will call them as sterile sneutrinos and they are good dark matter candidate [12]. The new coordinate \( y \) is defined as [27]

\[
y^m = x^m + i \left( \theta \sigma^m \bar{\theta} \right).
\]  

(8)

The particle content of each this model is presented in the Tab.(1).

The Lagrangian of this model is written as

\[
\mathcal{L}_{\text{MSSM3RH}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}},
\]  

(9)

where \( \mathcal{L}_{\text{SUSY}} \) is the supersymmetric piece and can be divided as follows

\[
\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{lepton}} + \mathcal{L}_{\text{Quarks}} + \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Higgs}},
\]  

(10)

where each term is given by

\[
\mathcal{L}_{\text{lepton}} = \int d^4 \theta \left[ \hat{L}_{iL} e^{2 g v^c + g' (\frac{1}{2}) \bar{\nu}^c} \hat{L}_{iL} + \hat{\nu}^c_{iR} e^{g' (\frac{1}{2}) \bar{\nu}^c \hat{\nu}^c} \hat{\nu}^c_{iR} + \hat{\nu}^c_{iR} e^{g' (\frac{1}{2}) \bar{\nu}^c \hat{\nu}^c} \hat{\nu}^c_{iR} \right],
\]

\[
\mathcal{L}_{\text{Quarks}} = \int d^4 \theta \left[ \hat{Q}_{iL} e^{2 g \nu^c + g' (\frac{1}{2}) \bar{\nu}^c} \hat{Q}_{iL} + \hat{\nu}^c_{iR} e^{2 g \nu^c + g' (\frac{1}{2}) \bar{\nu}^c} \hat{\nu}^c_{iR} \right]
\]

\[
\mathcal{L}_{\text{Gauge}} = \frac{1}{4} \left\{ \int d^2 \theta \left[ \sum_{\alpha=1}^{8} W^a_{\alpha i} W^a_{\alpha i} + \sum_{i=1}^{3} W^i \nu^c + W^i \nu^c \right] + h.c. \right\},
\]

\[
\mathcal{L}_{\text{Higgs}} = \int d^4 \theta \left[ \hat{H}_1 e^{2 g v^c + g' (\frac{1}{2}) \bar{\nu}^c \hat{H}_1} + \hat{H}_2 e^{2 g v^c + g' (\frac{1}{2}) \bar{\nu}^c \hat{H}_2} \right]
\]

\[
+ \int d^2 \theta \ W + \int d^2 \bar{\theta} \ W.
\]  

(11)

Therefore, our right handed neutrinos have any interactions with the usual gauge bosons then we will refere them as “fully sterile” [5]. We use the notation \( \hat{V} = T^i \hat{V}^i \) where \( T^i \equiv \sigma^i / 2 \) (with \( i = 1, 2, 3 \)) are the generators of \( SU(2)_L \) and \( \hat{V}^c \equiv T^a \hat{V}^a \) and \( T^a = \lambda^a / 2 \) (with \( a = 1, \cdots, 8 \)) are the generators of \( SU(3)_C \). As usual, \( g_s, g \) and \( g' \) are the gauge couplings for the \( SU(3)_C \), \( SU(2)_L \) and \( U(1)_Y \) groups, and \( W \) is the superpotential of this model and we
will present it below. The field strength are given by [27]

\[ W^a_{\alpha} = -\frac{1}{8g_s} D D e^{-2g_s \bar{V}_i} D e^{2g_s \bar{V}_i} \alpha = 1, 2 , \]
\[ W^i_{\alpha} = -\frac{1}{8g} D D e^{-2g \bar{V}^i} D e^{2g \bar{V}^i} , \]
\[ W'^{\alpha} = -\frac{1}{4} D D D \alpha \bar{V}' . \]  

(12)

The masses of charged gauge boson are the same as given at MSSM and it is given by

\[ M^2_W = \frac{g^2}{4} (v_1^2 + v_2^2) = \frac{g^2 v_1^2}{4} (1 + \tan^2 \beta) = \frac{g^2 v_1^2}{4} \sec^2 \beta , \]  

(13)

the new free parameter \( \beta \) is defined in the following way

\[ \tan \beta \equiv \frac{v_2}{v_1} , \]  

(14)

where \( v_2 \) is the vev of \( H_2 \) while \( v_1 \) is the vev of the \( H_1 \). Due the fact that \( v_1 \) and \( v_2 \) are both positive, it imples that

\[ 0 \leq \beta \leq \left( \frac{\pi}{2} \right) \text{ rad.} \]  

(15)

Using the free parameter \( \beta \), we can get the following relation to the vev o the Higgses defined at Eq.(2)

\[ v_1 = \frac{2M_W \cos \beta}{g} , \]
\[ v_2 = \frac{2M_W \sin \beta}{g} . \]  

(16)

In this case, the most general superpotential \( W \) of this model is given by

\[ W_{MSSM3RHN} = \left[ W_{1N} + \left( \frac{W_H + W_{2RV} + W_{2N}}{2} \right) \right] + \left( \frac{W_Y + W_{3RV} + W_{3NRC} + W_{3NRV}}{3} \right) + hc , \]  

(17)
where $W_H, W_Y$ are defined at Eq.(4), while $W_{2RV}, W_{3RV}$ from Eq.(5) and we have defined

\[
W_{1N} = \sum_{i=1}^{3} \kappa_i' \hat{N}_{iR},
\]

\[
W_{2N} = \sum_{i,j=1}^{3} M_{ij} \hat{N}_{iR} \hat{N}_{jR},
\]

\[
W_{3NR} = \sum_{i,j=1}^{3} f_{ij}'' \left( \hat{H}_2 \hat{L}_{iL} \right) \hat{N}_{jR},
\]

\[
W_{3NRV} = \sum_{i,j,k=1}^{3} \left[ \kappa_i'' \left( \hat{H}_1 \hat{H}_2 \right) \hat{N}_{iR} + \kappa_{ijk} \hat{N}_{iR} \hat{N}_{jR} \hat{N}_{kR} \right].
\]

(18)

We can conclude the masses of charged fermions are the same as in the MSSM, and we can write [11, 16]

\[
f_{d_{ij}} = \frac{g m_{d_{ij}}}{\sqrt{2} M_W \cos \beta}, \quad f_{l_{ij}} = \frac{g m_{l_{ij}}}{\sqrt{2} M_W \cos \beta},
\]

\[
f_{u_{ij}} = \frac{g m_{u_{ij}}}{\sqrt{2} M_W \sin \beta}.
\]

(19)

The fact that $m_u, m_d, m_s$ and $m_e$ are many orders of magnitude smaller than the masses of the other fermions may well be indicative of a radiative mechanism at work for these masses as considered at [21, 28]. We can explain the mass hierarchy in the charged fermion masses as showed in [17, 18].

We defined at Tab.(2) the $R$-charges of the superfields in the MSSM3RHN. In this model the right-handed sneutrinos are viable candidate to Dark Matter was showed at [12, 29]

Using the $R$-charges defined at Tab.(2), will forbid the Majorana Mass term as well the $\Xi_i, \lambda_i, \kappa_{ijk}, \mu_i, \lambda_{ij}, \lambda'_{ijk}, \lambda''_{ijk}$ couplings in the superpotential defined at Eq.(17). In this case our superpotential become

\[
W = \left( \frac{W_H}{2} \right) + \left( \frac{W_Y + W_{3NR}}{3} \right) + h_c,
\]

(20)

see Eq.(17) and we can now write, in similar way a we have done at Eq.(19)

\[
f_{\nu_{ij}} = \frac{g m_{\nu_{ij}}}{\sqrt{2} M_W \sin \beta}.
\]

(21)
\[
\begin{array}{|c|c|c|}
\hline
\text{Superfield} & R\text{-charge} & (B - L)\text{-charge} \\
\hline
\hat{L}_i L = (\hat{\nu}_i L, \hat{l}_i L)^T & n_L = + \left(\frac{1}{2}\right) & - \left(\frac{1}{2}\right) \\
\hat{Q}_i L = (\hat{u}_i L, \hat{d}_i L)^T & n_Q = + \left(\frac{1}{2}\right) & + \left(\frac{1}{3}\right) \\
\hat{H}_1 = (H_1^0, H_1^-)^T & n_{H_1} = 0 & 0 \\
\hat{H}_2 = (H_2^+, H_2^0)^T & n_{H_2} = 0 & 0 \\
\hat{l}_i R & n_l = - \left(\frac{1}{2}\right) & + \left(\frac{1}{2}\right) \\
\hat{N}_i R & n_N = - \left(\frac{1}{2}\right) & + \left(1\right) \\
\hat{d}_i R & n_d = - \left(\frac{1}{2}\right) & - \left(\frac{1}{3}\right) \\
\hat{u}_i R & n_u = - \left(\frac{1}{2}\right) & - \left(\frac{1}{3}\right) \\
\hline
\end{array}
\]

Table 2: \( R\)-charge and \((B - L)\)-charge assignment to all superfields in the MSSM3RHN.

It is a nice result because now neutrinos are Dirac fermions [12] and it was shown recently that the right-handed sneutrinos can be non-thermal in the presence of a Majorana mass term, as showed at [30].

From Eqs.\((20,21)\), we can write the simple relation between Yukawa coupling to neutrinos and their physical masses

\[
m_{\nu} = f_{\nu} v,
\]

where \( v \sim \mathcal{O}(10^{11})\text{eV} \) is the vev of the electroweak breaking scale and if we impose

\[
m_{\nu} \sim \mathcal{O}(10^{-2} - 10^{-1})
\]

it imposes that the Yukawa coupling of neutrinos should be

\[
f_{\nu} \sim \mathcal{O}(10^{-12} - 10^{-13}),
\]

and in this case the non-thermal sterile sneutrino can be a good dark matter candidate see [12, 31, 32].

The experimental evidence suggests that the supersymmetry is not an exact symmetry. Therefore, supersymmetry breaking terms should be added to the Lagrangian defined by the Eq.(9). The most general soft supersymmetry breaking terms, which do not induce quadratic divergence, where described...
by Girardello and Grisaru [33]. They found that the allowed terms can be
categorized as follows: a scalar field $A$ with mass terms

$$L^\text{SMT} = -A^\dagger m_{ij}^2 A_j,$$  \hspace{1cm} (25)

a fermion field gaugino $\lambda$ with mass terms

$$L^\text{GMT} = -\frac{1}{2}(M_\lambda \lambda^a \lambda^a + h c),$$  \hspace{1cm} (26)

and finally trilinear scalar interaction terms

$$L^\text{INT} = B_{ij} A_i A_j + A_{ijk} f_{ijk} A_i A_j A_k + h c.$$ \hspace{1cm} (27)

The terms in this case are similar with the terms allowed in the superpoten-
tial of the model we are going to consider next. As we are not doing
any phenomenology with the strong sector, we are omitting the squark contribution because they are the same as in the MSSM.

The general soft SUSY breaking terms are given as

$$L^\text{SMT}_{\text{MSSM}3RHN} = -\sum_{i=1}^3 \left[ \bar{L}_{iL}^\dagger \left( M_L^2 \right)_{ij} \tilde{L}_{jL} + \bar{L}_{iR}^\dagger \left( M_L^2 \right)_{ij} \tilde{L}_{jR} + \bar{\nu}_{iR}^\dagger \left( M_\nu^2 \right)_{ij} \tilde{\nu}_{jR} \right]$$

$$+ M_{H_1}^2 |H_1|^2 + M_{H_2}^2 |H_2|^2, \hspace{1cm} (28)$$

we do not write the masses to squarks. We can suppose there is just one soft
SUSY breaking slepton mass for each generation of left-slepton, it means

$$\left( M_L^2 \right)_{11} = M_{\tilde{e}_L}^2 = M_{\tilde{\nu}_e L}, \hspace{1cm} \left( M_L^2 \right)_{11} = M_{\tilde{\nu}_e R}, \hspace{1cm} \left( M_N^2 \right)_{11} = M_{\tilde{\nu}_e R}^2.$$ \hspace{1cm} (29)

the terms similar to our superpotential, defined at Eq.(20), is written as

$$L^\text{INT}_{\text{MSSM}3RHN} = B_{ij} H_1 H_2 + A_{ij} f_{ij} \bar{L}_{iL} H_1 \tilde{L}_{jR} + A_{ij} f_{ij} \bar{L}_{iL} H_2 \tilde{\nu}_{jR} + h c,$$ \hspace{1cm} (30)

again we do not write the terms to squarks. The term $L^\text{GMT}$ is the same as
given at MSSM due it we do not presented it here and there expansion can
be found at [11, 16].

The only interaction to the right-handed sneutrinos are given by is given
by $\bar{L}_{iL} H_2 \tilde{\nu}_{jR}$ and the last term at Eq.(30) and due this fact we can call the
right-handed neutrinos as sterile sneutrinos, see comment after Eq.(11), as
done at [32].
3 Masses

We will here consider the masses to neutrinos and sneutrinos. The masses to charged leptons and charged sleptons are the same as in the MSSM, and can be found at [10, 11, 16], due it we do not discuss them in this review.

4 Masses to Neutrinos

In general case, we have three left-handed neutrinos and three right-handed neutrinos and taken into account our superpotential, see Eq.(20), the most general terms to neutrinos are given by:

\[- \left[ \frac{1}{3} f_{ij}^\nu (L_iLH_2) \nu_{jR} + \frac{1}{2} (M_N)_{ij} \nu_{iR} \nu_{jR} \right] + \text{hc}, \quad (31)\]

The parameters $f_{ij}^\nu$ are symmetric in $i, j$ exchange and we have chosen a basis for the right-handed neutrinos superfields so that the superpotential terms are diagonal, and $M \sim M_{\text{Planck}}$ or comparable to the SO(10) breaking scale as presented at [11]. After the electroweak symmetry is broken we get $6 \times 6$ mass matrix to diagonalize. This matrix can be write as

\[
\begin{pmatrix}
0 & M_{\nu}^{\text{Dirac}} \\
(M_{\nu}^{\text{Dirac}})^T & M
\end{pmatrix}, \quad (32)
\]

where

\[
(M_{\nu}^{\text{Dirac}})_{ij} = \frac{1}{3} f_{ij}^\nu v_2, \\
(M)_{ij} = \frac{1}{2} (M_N)_{ij}. \quad (33)
\]

We get the usual see saw mechanism (type I) [34] if $M_{\nu}^{\text{Dirac}} \ll M$, the mass spectrum is one heavy Majorana neutrino, with mass $M$. We get three light Dirac neutrinos with their masses given by

\[
m_{\nu} \simeq \frac{(M_{\nu}^{\text{Dirac}})^2}{M}. \quad (34)
\]

and the neutrinos are Majorana particles as happen in MSSM.
One interesting case, happen when we consider three left-handed neutrinos and only one right-handed neutrinos. Under this consideration we get the mass matrix from Eq.(31)

\[- \left[ \frac{v_2}{3\sqrt{2}} (f_1^\nu \nu_1 L + f_2^\nu \nu_2 L + f_3^\nu \nu_3 L) + \frac{1}{2} M_N \nu_R \nu_R \right] + h c, \]  

(35)

in the base \((\nu_1 L, \nu_2 L, \nu_3 L, \nu_R)^T\) we get the mass matrix presented at Eq.(32), and in this case we can write \[ [35, 36] \]

\[ M_{\nu}^{\text{Dirac}} = \frac{v_2}{3\sqrt{2}} \begin{pmatrix} (f_1^\nu)^2 & f_1^\nu f_2^\nu & f_1^\nu f_3^\nu \\ f_1^\nu f_2^\nu & (f_2^\nu)^2 & f_2^\nu f_3^\nu \\ f_1^\nu f_3^\nu & f_2^\nu f_3^\nu & (f_3^\nu)^2 \end{pmatrix}, \]  

(36)

we can draw at this case a diagramatic representation similar to ones shown at Fig.(1). We make the additional hipotesis, all the Yukawa coupling \(f_i^\nu\) are real. From Eq.(36) it is simple to show the following results

\[ \det \left( M_{\nu}^{\text{Dirac}} \right) = 0, \]

\[ \text{Tr} \left( M_{\nu}^{\text{Dirac}} \right) = (f_1^\nu)^2 + (f_2^\nu)^2 + (f_3^\nu)^2. \]  

(37)

The characteristic equation obtained from Eq.(36) is

\[ \det \left( M_{\nu}^{\text{Dirac}} - \lambda I_{3\times3} \right) = -\lambda^3 + \lambda^2 \left[ (f_1^\nu)^2 + (f_2^\nu)^2 + (f_3^\nu)^2 \right] = 0, \]  

(38)

it means we have one more zero eigenvalue. Therefore, we can conclude, Eq.(36) has two zero eigenvalues and one non-zero eigenvalue, as presented at \[ [35, 36] \].

The mass eigenstate is defined in the following way \[ [35, 36] \]

\[ \nu_H = \frac{f_1^\nu \nu_1 L + f_2^\nu \nu_2 L + f_3^\nu \nu_3 L}{\sqrt{(f_1^\nu)^2 + (f_2^\nu)^2 + (f_3^\nu)^2}}, \]  

(39)

and their masses is

\[ m_{\nu_H} = \left[ (f_1^\nu)^2 + (f_2^\nu)^2 + (f_3^\nu)^2 \right] \frac{v_2}{3\sqrt{2}}. \]  

(40)

It is the seesaw mechanism (type I) \[ [34] \] to give mass to neutrinos, alternatives to the seesaw mechanism was presented at \[ [25] \].
Figure 3: One-loop diagram generating a Majorana neutrino mass adapted from [36, 37].

The others two neutrinos in this model get their masses from 1-loop level. Having generated a sneutrino Majorana mass, see Sec.(4.1), it is straightforward to see that such a mass will lead to one-loop radiative corrections to neutrino masses via the self-energy diagram drawn in Fig.(3) [37].

We can with this mechanism explain neutrinos data as shown at [35, 36].

4.1 Masses of Sneutrinos

The masses and mixing of sparticles are of crucial importance both experimentally and theoretically [10, 11, 16]. All the terms that give contribution to sneutrino mass matrix came from Eq.(30), it means that the mixing in this sector is unrelated with the mixing we get at neutrino sector. In general on this sector we get one $6 \times 6$ mass matrix (we get Super-CKM and Super-PMNS matrix [16, 38]), but when we ingone the mixing into families this matrix can be devided into three blocks of $2 \times 2$ for each family, we will use this fact here, as discussed at [11, 16].

The mass term to sneutrinos is given by

$$\mathcal{L}_{\text{sneutrinos}} = - \left[ m_{\tilde{\nu}_e L}^2 \tilde{\nu}^\dagger_e L \tilde{\nu}_e L + m_{\tilde{\nu}_e R}^2 \tilde{\nu}^\dagger_e R \tilde{\nu}_e R + A^\nu f^\nu v_2 (\tilde{\nu}_e L \tilde{\nu}_e R + h.c) \right].$$

We have defined

$$m_{\tilde{\nu}_e L}^2 = M_L^2 + \frac{1}{2} \cos (2\beta) M_Z^2, \quad m_{\tilde{\nu}_e R}^2 = M_N^2,$$

(42)
where $M_Z$, as usual, is the $Z$ boson mass. The first line above is similar as done in the MSSM when we calculate the sneutrinos masses [10, 11, 16]. More general case, with Majorana mass term to neutrinos is discussed at [37]. We must to emphasize that the parameters $M_{\tilde{\nu}_L}^2, M_{\tilde{\nu}_R}^2$ and $\nu$ are defined at electroweak scale because they break softly the SUSY.

In order to get the sneutrinos mass eigenstates, we have, first, to diagonalize the following mass matrix

$$M_{\text{sneutrinos}}^2 = \begin{pmatrix} m_{\tilde{\nu}_L}^2 & A\nu f\nu v_2 \\ A\nu f\nu v_2 & m_{\tilde{\nu}_R}^2 \end{pmatrix}, \quad (43)$$

it is simple to show the following results

$$\det (M_{\text{sneutrinos}}^2) = m_{\tilde{\nu}_L}^2 m_{\tilde{\nu}_R}^2 - (A\nu f\nu v_2)^2,$$

$$\text{Tr} (M_{\text{sneutrinos}}^2) = m_{\tilde{\nu}_L}^2 + m_{\tilde{\nu}_R}^2. \quad (44)$$

The characteristic equation obtained from Eq.(36) is

$$\det (M_{\text{sneutrinos}}^2 - \lambda I_{2\times2}) = \lambda^2 - (m_{\tilde{\nu}_L}^2 + m_{\tilde{\nu}_R}^2) \lambda + m_{\tilde{\nu}_L}^2 m_{\tilde{\nu}_R}^2 - (A\nu f\nu v_2)^2 = 0. \quad (45)$$

it means we have two massives eigenstates.

The eigenvalues of $M_{\text{sneutrinos}}^2$ are given by

$$m_{\pm}^2 = M^2 \pm \sqrt{(m^2)^2 + (A\nu f\nu v_2)^2}, \quad (46)$$

where we have defined the following parameters

$$M^2 = \frac{(m_{\tilde{\nu}_L}^2 + m_{\tilde{\nu}_R}^2)}{2},$$

$$m^2 = \frac{(m_{\tilde{\nu}_L}^2 - m_{\tilde{\nu}_R}^2)}{2}. \quad (47)$$

The eigenvectors are given by [12]

$$\begin{pmatrix} \tilde{\nu}_{\epsilon+} \\ \tilde{\nu}_{\epsilon-} \end{pmatrix} = \begin{pmatrix} \cos \theta_\nu & \sin \theta_\nu \\ -\sin \theta_\nu & \cos \theta_\nu \end{pmatrix} \begin{pmatrix} \tilde{\nu}_{\epsilon L} \\ \tilde{\nu}_{\epsilon R} \end{pmatrix},$$
where we define the mixing angle as

$$
\cos \theta_\nu = \frac{m^2 + \sqrt{(m^2)^2 + (A^\nu f^\nu v_2)^2}}{|\tilde{\nu}|^2},
$$

$$
\sin \theta_\nu = \frac{(A^\nu f^\nu v_2)}{|\tilde{\nu}|^2},
$$

$$
|\tilde{\nu}|^2 = \left(m^2 + \sqrt{(m^2)^2 + (A^\nu f^\nu v_2)^2}\right)^2 + (A^\nu f^\nu v_2)^2, \tag{48}
$$

in this model the $\tilde{\nu}_{e-}$ is LSP [12].

5 Flat direction of MSSM3RHN Model

We can calculate all flat directions in the MSSM using the prescription given in [39]. The flat directions $\hat{N}$ and $\hat{L}\hat{H}_2$ will generate a left-right asymmetry in the sneutrino sector [12, 29].

The flat directions shown in Tab.(3) was getting using the most general superpotential of this model presented at Eq.(17).

| Flat direction | $(B - L)$ |
|----------------|-----------|
| $N$            | 1         |
| $NN$           | 2         |
| $H_1H_2$       | 0         |
| $LH_2$         | -1        |
| $NNN$          | 3         |
| $H_1H_2N$      | 1         |
| $LH_2N$        | 0         |

Table 3: Flat direction of the model MSSM3RHN.
6 Conclusions

In this article we have presented the MSSM3RHN lagrangian in terms of superfields. Then we presented the mass spectrum of neutrinos and sneutrinos with three right handed neutrinos and with only one right-handed neutrino and we can get neutrinos are Majorana particles or Dirac particles. At the end we presented some flat directions of these model.

We hope this review can be useful to all the people wants to learn about Supersymmetry.

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A Lagrangian

The Lagrangian of this model is defined at Eq.(9). The $\mathcal{L}_{\text{Quarks}}, \mathcal{L}_{\text{Gauge}}$ and $\mathcal{L}_{\text{Higgs}}$ are the same as in the MSSM and due it we do not write their expressions [11, 16, 10].

B Lepton Lagrangian

The lagrangian defined at Eq.(11), has the following components

$$\mathcal{L}_{\text{Lepton}} = \mathcal{L}_{\text{ lept}}^F + \mathcal{L}_{\text{ lept}}^D + \mathcal{L}_{\text{cin}}^\text{lep} + \mathcal{L}_{\text{iIV}}^\text{lep}.$$  

(49)

The first two terms in Eq.(49) are the usual $F$ terms given by:

$$\mathcal{L}_{\text{ lept}}^F = \sum_{i=1}^{3} \left( |F_{L_iL}|^2 + |F_{L_iR}|^2 + |F_{N_{iR}}|^2 \right),$$  

(50)

while for the $D$ terms

$$\mathcal{L}_{\text{ lept}}^D = g \left[ \tilde{L}_{iL} \left( \frac{g^a}{2} \right) \tilde{L}_{iL} \right] D^a + g' \left[ \tilde{L}_{iL} \left( \frac{-1}{2} \right) \tilde{L}_{iL} \right] D' + g' \left[ \tilde{l}_{iR} \left( \frac{2}{2} \right) \tilde{l}_{iR} \right] D',$$  

(51)
where $\sigma^a$ (with $a = 1, 2, 3$) are the generators of $SU(2)_L$.

The others terms in the lagrangian are given by

$$\mathcal{L}_{\text{cin}}^{\text{lep}} = \left\{ \begin{array}{l} (\mathcal{D}_m \bar{L}_i)^\dagger \left( \mathcal{D}_m \bar{L}_{iL} \right) + (\mathcal{D}_{mR} \bar{L}_{iR})^\dagger \left( \mathcal{D}_{mR} \bar{L}_{iR} \right) + \left( \partial_m \bar{N}_{iR} \right)^\dagger \left( \partial_m N_{iR} \right) + \bar{l}_i \sigma^m (\mathcal{D}_m L_i) \right. \\
\left. + \bar{L}_{iR} \sigma^m (\mathcal{D}_{mR} l_i) + \bar{N}_{iR} \sigma^m (\partial_m N_{iR}) \right\} \right., \tag{52}
$$

where the covariant derivatives are defined as

$$\mathcal{D}_m \bar{L}_i = \partial_m \bar{L}_i + ig \left( \frac{\sigma^a}{2} V^a_m \right) \bar{L}_i + ig' \left[ \left( \frac{-1}{2} \right) V_m \right] \bar{L}_i$$

and

$$\mathcal{D}_{mR} \bar{L}_{iR} = \partial_m \bar{L}_{iR} + ig' \left[ \left( \frac{2}{2} \right) V_m \right] \bar{L}_{iR}. \tag{53}\$$

The last term is

$$\mathcal{L}_{\text{lep}}^{\text{W}_2} = -ig\sqrt{2} \left( \bar{L}_{iL} \left( \frac{\sigma^a}{2} \lambda_A^a \right) \bar{L}_{iL} - \bar{L}_{iL} \left( \frac{\sigma^a}{2} \lambda_A^a \right) L_{iL} \right) - ig' \sqrt{2} \left( \bar{L}_{iL} \left( \frac{-1}{2} \lambda_B^i \right) \bar{L}_{iL} \right)$$

$$- \left( \frac{-1}{2} \lambda_B^i \right) L_{iL} - ig' \sqrt{2} \left( \bar{L}_{iR} \left( \frac{2}{2} \lambda_B^i \right) \bar{L}_{iR} - \bar{L}_{iR} \left( \frac{2}{2} \lambda_B^i \right) \bar{L}_{iR} \right). \tag{54}\$$

Our superpotential $W$ is defined at Eq.(20). We can write it in field components as

$$W_2 = \mathcal{L}_F^{W_2} + \mathcal{L}_{HMT}^{W_2},$$

$$W_3 = \mathcal{L}_F^{W_3} + \mathcal{L}_{HMT}^{W_3} + \mathcal{L}_{\text{lep}}^{W_3}, \tag{55}\$$

Where the $F$ terms are

$$\mathcal{L}_F^{W_2} = \mu_H (F_H H_2 + F_{H_2} H_1) +hc,$$

$$\mathcal{L}_F^{W_3} = f_{ij}^L (F_H L_{iL} \bar{L}_{jR} + F_{1L} H_{1L} \bar{L}_{iL}) + f_{ij}^L (F_{2L} H_{2L} \bar{L}_{iL} + F_N H_{2L}) + hc,$$  

while the others parts

$$\mathcal{L}_{HMT}^{W_2} = -\mu_H \bar{H}_1 \bar{H}_2 +hc,$$  

$$\mathcal{L}_{\text{lep}}^{W_3} = - \left\{ f_{ij}^L \left( H_{1L} \bar{L}_{iL} \right) l_{jR} + f_{ij}^L \left( H_{2L} \bar{L}_{iL} \right) N_{iR} \right\} +hc.$$

Here we have omit the expansion to quarks.
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