SOLVING QCD VIA MULTI-REGGE THEORY

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Abstract
A high-energy, transverse momentum cut-off, solution of QCD is outlined. Regge pole and "single gluon" properties of the pomeron are directly related to the confinement and chiral symmetry breaking properties of the hadron spectrum. This solution, which corresponds to a supercritical phase of Reggeon Field Theory, may only be applicable to QCD with a very special quark content.

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1 Introduction

The QCD pomeron is usually discussed without much attention paid to the scattering states. States containing only elementary constituents are normally considered. As a matter of principle, a full solution of QCD at high-energy requires that we find both the true hadronic states and the exchanged pomeron giving scattering amplitudes. Unitarity must be satisfied in both the s-channel and the t-channel.

Experimentally the pomeron appears, approximately, to be a Regge pole at small $Q^2$ and a single gluon at larger $Q^2$. Neither property is present in QCD perturbation theory. In the high-energy, transverse momentum cut-off, “solution” of QCD that I outline in this talk the experimental “non-perturbative properties” of the pomeron are directly related to the confinement and chiral symmetry breaking properties of hadrons. That is, experimental properties of the pomeron are directly related to special properties of the scattering states. It is particularly interesting that our solution may only be applicable to QCD with a very special quark content.

Our arguments involve the techniques of multi-regge QCD calculations, the dynamics of the massless quark U(1) anomaly, which will be the main focus of this talk, and reggeon field theory phase-transition analysis, which we will avoid almost completely. Our results show how confinement and chiral symmetry breaking, normally understood as consequences of the vacuum, can instead be produced by a “wee-parton” distribution. This is a very non-trivial property that provides, I hope, a deeper basis for the parton model (and even the constituent quark model) in QCD!

Multi-Regge Theory is an abstract formalism that we developed a major part of in the 70’s. The formalism is based on the existence of asymptotic analyticity domains for multiparticle amplitudes derived via “Axiomatic Field Theory” and “Axiomatic S-Matrix Theory”. All the assumptions made are expected to be valid in a completely massive spontaneously-broken gauge theory. Since we begin with massive reggeizing gluons, this is effectively the starting point for our analysis of QCD.

For our purposes, the most important component of multi-regge theory is the reggeon unitarity equations. Using these equations, well-known Regge limit QCD calculations can be extended to obtain amplitudes, of the form illustrated in Fig. 1, involving multiple exchanges of reggeized gluons and quarks in a variety of channels. The central idea of our work is that we can,

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‡In general our work leads us to doubt very strongly that “We now know that there are an infinite number of consistent S-Matrices that satisfy all the sacred principles. ... any gauge group, and many sets of fermions.”

by considering infra-red limits in which gluon mass(es) $M$ and quark mass(es) $m \to 0$, find hadrons (e.g. the pion) and the IP as (coupled) Regge pole bound states of reggeons. Presently the simultaneous study of bound states and their scattering amplitudes is impossible in any other formalism.

![Diagram](image)

**Fig. 1 The Anticipated Formation of Pion Scattering Amplitudes**

In general, limits wrt all mass, gauge symmetry, and cut-off parameters are crucial. The most important feature, however, is the dynamical role played by new “reggeon helicity-flip” vertices that appear in the amplitudes we discuss. The hadron amplitudes we obtain are initially isolated via a (“volume”) infra-red divergence that appears when SU(3) gauge symmetry is partially broken to SU(2) and the limit of zero quark mass is also taken. The divergence involves quark loop helicity-flip vertices containing chirality violation (c.f. instanton interactions). The chirality violation survives the massless quark limit because of an infra-red effect closely related to the triangle anomaly. The divergence produces a “wee parton condensate” that is directly responsible, while the gauge symmetry is partially broken, for confinement and chiral symmetry breaking. The pomeron is a reggeized gluon in the wee parton condensate and so is obviously a Regge pole. We will not give a description of the supercritical pomeron in this talk, however, all the essential features of this RFT phase are present in the solution of partially-broken QCD that we present.

We discuss the restoration of SU(3) gauge symmetry only briefly. The increase in the gauge symmetry is closely related to the critical behaviour of the pomeron and the associated disappearance of the supercritical condensate. We note that the large $Q^2$ of deep-inelastic scattering provides a finite volume constraint that can keep the theory (locally) in the supercritical RFT phase as the full gauge symmetry is restored. A single gluon (in the background wee parton condensate) should then be a good approximation for the pomeron. Finally we discuss the circumstances under which our solution can be realized in QCD.
2 Reggeon Diagrams in QCD

Leading-log Regge limit calculations of elastic and multi-regge production amplitudes in (spontaneously-broken) gauge theories show that both gluons and quarks “reggeize”, i.e. they lie on Regge trajectories. Non-leading log calculations are described by “reggeon diagrams” involving reggeized gluons and quarks. Reggeon unitarity implies that a complete set of reggeon diagrams arise from higher-order contributions.

Gluon reggeon diagrams involve a reggeon propagator for each reggeon state and also gluon particle poles e.g. the two-reggeon state

\[ \int \frac{d^2k_1}{(k_1^2 + M^2)} \frac{d^2k_2}{(k_2^2 + M^2)} \frac{\delta^2(k'_1 + k'_2 - k_1 - k_2)}{J - 1 + \Delta(k_1^2) + \Delta(k_2^2)} \]

The BFKL equation\(^8\) corresponds to 2-reggeon unitarity i.e. iteration of the 2-reggeon state with a 2-2 reggeon interaction \( R_{22} = \frac{[(k_1^2 + M^2)(k_2^2 + M^2) + (k_1' + M^2)(k_2' + M^2)]}{[(k_1 - k_1')^2 + M^2]} + \cdots \)

We will be interested in the limit \( M \to 0 \) and will assume that two leading-order properties of reggeon diagrams generalize to all orders. The first is that infra-red divergences exponentiate to zero all diagrams that do not carry zero color in the \( t \)-channel. The second property is that infra-red finiteness implies canonical scaling (\( \sim Q^{-2} \)) for color zero reggeon amplitudes when all transverse momenta are simultaneously scaled to zero (this requires \( \alpha_s(Q^2) \to \infty \) when \( Q^2 \to 0 \)).

3 Reggeon Diagrams for Helicity-Pole Limit Amplitudes

The generality of reggeon unitarity is most powerful when applied to the amplitudes appearing in “maximal helicity-pole limits”. Amplitudes of this kind contain the bound-state reggeon scattering amplitudes we are looking for and, since the appropriate Sommerfeld-Watson representation shows that only a single (analytically-continued) partial-wave amplitude is involved, reggeon unitarity implies that such limits can be described by reggeon diagrams. (The physical significance of such diagrams is subtle\(^3\) in that “physical” \( k_\perp \) planes in general contain lightlike momenta !)

As an example, we consider a maximal helicity-flip limit for an 8-pt amplitude. We introduce angular variables variables as illustrated in Fig. 2. We consider the “helicity-flip” limit \( z, u_1, u_2, u_3, u_4 \to \infty \). The behavior of invariants in this limit is

\[ \]
Reggeon unitarity determines that the helicity-flip limit is described by reggeon diagrams of the form shown in Fig. 3. The amplitudes \( A \) contain all elastic scattering reggeon diagrams. The \( T^F \) are new “reggeon helicity-flip” vertices that play a crucial role in our QCD analysis. (These vertices do not appear in elastic scattering reggeon diagrams).

4 Reggeon Helicity-Flip Vertices

The \( T^F \) vertices are most simply isolated kinematically by considering a “non-planar” triple-regge limit which, for simplicity, we will define by introducing three distinct light-cone momenta. (This limit actually gives a sum of three \( T^F \) vertices of the kind discussed above, but in this talk we will not elaborate on this subtlety.) We use the tree diagram of Fig. 4(a) to define momenta and study the special kinematics

\[
\begin{align*}
P_1 &\rightarrow (p_1, p_1, 0, 0), \quad p_1 \rightarrow \infty \\
P_2 &\rightarrow (p_2, 0, p_2, 0), \quad p_2 \rightarrow \infty \\
P_3 &\rightarrow (p_3, 0, p_3), \quad p_3 \rightarrow \infty \\
Q_1 &\rightarrow (0, 0, q_2, -q_3) \\
Q_2 &\rightarrow (0, -q_1, 0, q_3) \\
Q_3 &\rightarrow (0, q_1, -q_2, 0)
\end{align*}
\]

Consider, first, three quarks scattering via gluon exchange with a quark loop coupling for three quark scattering.

\[
\begin{align*}
P_1 &\rightarrow (p_1, p_1, 0, 0), \quad p_1 \rightarrow \infty \\
P_2 &\rightarrow (p_2, 0, p_2, 0), \quad p_2 \rightarrow \infty \\
P_3 &\rightarrow (p_3, 0, p_3), \quad p_3 \rightarrow \infty \\
Q_1 &\rightarrow (0, 0, q_2, -q_3) \\
Q_2 &\rightarrow (0, -q_1, 0, q_3) \\
Q_3 &\rightarrow (0, q_1, -q_2, 0)
\end{align*}
\]
loop coupling as in Fig. 4(b). The non-planar triple-regge limit gives

$$\rightarrow g^6 \frac{p_1 p_2 p_3}{t_1 t_2 t_3} \Gamma_{1+2+3}^\mu (q_1, q_2, q_3) \leftrightarrow g^3 \frac{p_1 p_2 p_3}{t_1 t_2 t_3} T^F(Q_1, Q_2, Q_3)$$

where $\gamma_i^+ = \gamma_0 + \gamma_i$ and $\Gamma_{\mu_1 \mu_2 \mu_3}$ is given by the quark triangle diagram i.e.

$$\Gamma_{\mu_1 \mu_2 \mu_3} = i \int \frac{d^4k}{[(q_1 + k)^2 - m^2][(q_2 + k)^2 - m^2][(q_3 + k)^2 - m^2]} \left( \gamma_{\mu_1} (q_3 + k + m) \gamma_{\mu_2} (q_1 + k + m) \gamma_{\mu_3} (q_2 + k + m) \right)$$

where $m$ is the quark mass. We denote the $O(m^2)$ chirality-violating part of $T^F \equiv \Gamma_{1+2+3}^\mu (q_1, q_2, q_3)$ by $T^F, m^2$ and note that the limits $q_1, q_2, q_3 \sim Q \to 0$ and $m \to 0$ do not commute, i.e.

$$T^F, m^2 \sim \frac{T^F, 0}{Q \to 0} = \frac{Q^2}{m^2} \int \frac{d^4k}{[k^2 - m^2]^3} = R Q$$

where $R$ is independent of $m$. This non-commutativity is an “infra-red anomaly” due to the triangle Landau singularity[11].

5 The Infra-Red Anomaly in Helicity-Flip Vertices

After color factors are included and all related diagrams summed, $T^F_0$ survives only in very special vertices coupling reggeon states with “anomalous color parity”. We define color parity ($C_c$) via the transformation $A^{a}_{i} \rightarrow -A^{b}_{i}$ for gluon color matrices and say that a reggeon state has anomalous color parity if the signature $\tau$ (i.e. whether the number of reggeons is even or odd) is not equal to the color parity.

In general, the $T^F$ vertices coupling multiple-reggeon states contain many reduced quark loops. The vertices containing $T^F_0$ also contain ultra-violet divergences associated with the anomaly. To maintain the reggeon Ward identities that ensure gauge invariance[10], we introduce Pauli-Villars fermions as a regularization. (Note that we take the regulator mass $m_{\Lambda} \to \infty$ after $m \to 0$. This implies that the initial theory with $m \neq 0$ is non-unitary for $k_{\perp} \gtrsim m_{\Lambda}$.)

The regulated vertex, $T^F, m^2$, satisfies

$$T^F, m^2 (Q) \sim T^F, m^2 - T^F, m^2 \xi \sim Q^2, \quad T^F, 0 (Q) \sim T^F_0 \sim Q$$

implying that imposing gauge invariance for $m \neq 0$ gives a slower vanishing as $Q \to 0$ when $m = 0$. 

5
We will be particularly interested in the “anomalous odderon” three-reggeon state with color factor $f_{ijk}A_i^j A_k^l A_k^l$ that has $\tau = -1$ but $C_c = +1$ (c.f. the winding-number current $K_\mu = \epsilon_{\mu
u\gamma\delta}f_{ijk}A_i^\nu A_j^\gamma A_k^\delta$).

$T^{F,0}(Q)$ appears in the triple coupling of three anomalous odderon states as in Fig. 5.

6 A Quark Mass Infra-Red Divergence

A vital consequence of the “anomalous” behavior of $T^{F,0}$ as $Q \to 0$ is that an additional infra-red divergence is produced (as $m \to 0$) in massless gluon reggeon diagrams. The divergence occurs in diagrams involving the $T^F$ where $Q_1 \sim Q_2 \sim Q_3 \sim 0$ is part of the integration region. This requires that $T^F$ be a disconnected component of a vertex coupling distinct reggeon channels, as in Fig. 6. In this diagram an anomalous odderon reggeon state (\equiv \ldots) is denoted by \textcolor{red}{\rule{5mm}{5mm}} while \textcolor{red}{\rule{5mm}{5mm}} denotes any normal reggeon state.

Fig. 6 is of the general form illustrated in Fig. 1.

The canonical scaling of the anomalous odderon states gives the infra-red behaviour

$$\int \cdots \frac{d^2 Q_1 d^2 Q_2 d^2 Q}{Q_1^2 Q_2^2 Q_3^2 (Q - Q_1)^2 (Q - Q_2)^2} V_1(Q_1)V_2(Q_2)V_3(Q - Q_2)V_4(Q - Q_1)\times T^F(Q_1, Q)T^F(Q, Q_2) \times \text{[regular vertices and reggeon propagators]}$$

for Fig. 6. Depending on the behaviour of the $V_i$, it is clear that a divergence may indeed occur when $Q \sim Q_1 \sim Q_2 \to 0$.

The divergence of Fig. 6 is preserved and a possible cancelation eliminated if we partially break the SU(3) gauge symmetry to SU(2). In this case, a divergence can occur in any diagram of the form of Fig. 8 in which \textcolor{red}{\rule{5mm}{5mm}} is any SU(2) singlet combination of massless gluons with $C_c = -\tau = +1$ (i.e. a generalized SU(2) anomalous odderon) and \textcolor{red}{\rule{5mm}{5mm}} is any normal reggeon state containing one or more SU(2) singlet massive reggeized gluons (or quarks).
A-priori reggeon Ward identities imply $V_i \sim Q_i$ when $Q_i \to 0$, $\forall \ i$, which would be sufficient to eliminate any divergence in Fig. 6. However, if we impose the "initial condition" that $V_1, V_2 \to 0$, the divergence is present in a general class of diagrams, including those having the general structure illustrated in Fig. 7. In this diagram there are $n + 3$ multi-reggeon states of the form $\ldots$. Imposing $V_1, V_2 \to 0$ and assuming that reggeon Ward identities are satisfied by the remaining vertices, i.e.

$$V_i(Q_i) \sim V(Q_i) = Q_i$$

$i \neq 1, 2$, gives that Fig. 7 has the infra-red behavior

$$\int \frac{d^2Q}{Q^2} \left[ \int \frac{d^2Q}{Q^4} \right]^n [V(Q) T^F(Q)]^n$$

giving (as $m \to 0$) an overall logarithmic divergence. In general, this divergence occurs in just those multi-reggeon diagrams which contain only SU(2) color zero states of the form $\ldots$ coupled by regular and $T^{F,0}$ vertices, as in the examples we have discussed.

7 Confinement and a Parton Picture

We define physical amplitudes by extracting the coefficient of the logarithmic divergence. There is "confinement" in that a particular set of color-zero reggeon states is selected that contains no massless multigluon states and has the necessary completeness property to consistently define an S-Matrix. That is, if two or more such states scatter via QCD interactions, the final states contain only arbitrary numbers of the same set of states. Since $k_{\perp} = 0$ for the anomalous odderon component of each reggeon state, an "anomalous odderon condensate" is generated. The form of physical amplitudes is illustrated in
Fig. 8. In addition to the $k_{\perp} = 0$ (“wee-parton”) component, each physical reggeon state has a finite momentum “normal” parton component carrying the kinematic properties of interactions. We emphasize that the “scattering” of the $k_{\perp} = 0$ condensate is directly due to the infra-red quark triangle anomaly.

The breaking of the gauge symmetry has produced physical states in which the “partons” are separated into a universal wee-parton component and a normal reggeon parton component which is distinct in each distinct physical state. However, the condensate has the important property that it switches the signature compared to that of the normal parton component. The following are a direct consequence.

i) The “pomeron” has a reggeized gluon normal parton component, but is a Regge pole with $\tau = -C_c = +1$ and intercept $\neq 0$.

ii) There is a bound-state reggeon formed from two massive SU(2) doublet gluons, giving an exchange-degenerate partner to the pomeron. The SU(2) singlet massive gluon lies on this trajectory.

iii) There is chiral symmetry breaking. Studies of $\tau = -1$ quark-antiquark exchange to demonstrate a “reggeization cancelation” generating a Regge pole with zero intercept and with $\tau = -1$, $C_c = +1$ and $P = -1$. In the condensate this gives a Regge pole with $\tau = +1$, $P = -1$, resulting in the massless pion associated with chiral symmetry breaking.

Although we have not discussed Reggeon Field Theory, we note that all the features of my supercritical RFT solution are present. (This solution was very controversial 20 years ago - although it was supported by Gribov!)

8 Restoration of SU(3) Gauge Symmetry

We make only a few brief comments on this, obviously important, subject. Because of complimentarity restoring SU(3) symmetry (decoupling a color triplet Higgs scalar field) should be straightforward if we impose $k_{\perp} < \Lambda_{\perp}$. Restoring the symmetry removes the mass scale that produces the reggeon condensate. If the (partially) broken theory can be mapped completely onto supercritical RFT then the condensate and the odd-signature partner for the pomeron will disappear simultaneously and the critical pomeron will be the result. The wee-parton condensate will be replaced by a universal, small $k_{\perp}$, wee parton, critical phenomenon that merges smoothly with the large $k_{\perp}$.
normal (or constituent) parton component of physical states, just as originally envisaged by Feynman [2]. (Note that, because of the odd SU(3) color charge parity of the pomeron, the two-gluon BFKL pomeron will not contribute.)

RFT implies that $\Lambda_\perp$ mixes with the symmetry breaking mass scale and becomes a “relevant parameter” for the critical behavior. After the symmetry breaking scale is removed, there will (for a general number of quark flavors) be a $\Lambda_\perp$ such that $\Lambda_\perp > \Lambda_\perp^c$ implies the pomeron is in the subcritical phase, while $\Lambda_\perp < \Lambda_\perp^c$ will give the supercritical phase. This implies that the supercritical phase can be realized with the full gauge symmetry restored, if $\Lambda_\perp$ is taken small enough. $\alpha_P(0)$ will be a function of $\Lambda_\perp$. In deep-inelastic diffraction $Q^2$ will act as an additional (local) lower $k_\perp$ cut-off and produce a “finite volume” effect that can keep the theory supercritical as the SU(3) symmetry is restored. This implies that the pomeron should be well approximated by a single (reggeized) gluon (in a soft gluon background) in DIS diffraction.

To remove $\Lambda_\perp$ requires $\Lambda_\perp^c = \infty$. This requires a specific quark flavor content. For any quark content, we can take $\Lambda_\perp << \Lambda_\perp^c$, and go deep into the supercritical phase. We obtain a picture in which constituent quark hadrons interact via a massive composite “gluon” (and an exchange degenerate pomeron). Confinement and chiral symmetry breaking are realized via a simple, universal, wee parton component of physical states. This is remarkably close to the realization of the constituent quark model via light-cone quantization that has been advocated by light-cone enthusiasts [2].

9 When is this Solution Realized in QCD ??

We have found a high-energy S-Matrix via a transverse momentum infra-red phenomenon involving massless gluons and quarks. At first sight, this should not occur in QCD since non-perturbative effects should eliminate massless gluons for $k_\perp < \Lambda_QCD$! Our solution requires that massless QCD remain weak-coupling at $k_\perp = 0$. This is generally anticipated to be the case only if there are a sufficient number of massless quarks in the theory to give an infra-red fixed point for $\alpha_s$.

There is such an infra-red fixed point when the number of flavors is the maximum allowed by asymptotic freedom. In this case SU(3) symmetry can be broken to SU(2) with an asymptotically-free scalar field. This implies that $\Lambda_\perp^c = \infty$, so that critical pomeron scaling occurs for all $k_\perp$, allowing a smooth match with perturbative QCD.

The above arguments suggest that if “single gluon” supercritical pomeron behavior is actually observed at HERA then new QCD physics, in the form of a new fermion sector, remains to be discovered above the (diffractive) $Q^2$
range presently covered. Everything is consistent if the electroweak scale is a QCD scale, i.e. the “Higgs sector” of the Standard Model, that is yet to be discovered, is composed of higher-color (sextet) quarks. A special definition of QCD is necessarily involved, but we will not discuss this here.

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