Masses of decuplet baryons treated within anyonic realization of the $q$-algebras $U_q(su_N)$

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Abstract

In the approach to hadronic flavour symmetries based on the $q$-algebras $U_q(su_N)$ and proved to be realistic, the known construction of $U_q(su_N)$ in terms of anyonic oscillators residing on 2d lattice is utilized. Anyonic Fock-like realization of basis state vectors is given for baryons $(3/2)^+$ from the 10-plet of $U_q(su_3)$ embedded, via 20-plet of $U_q(su_4)$, into the "dynamical" representation $[4000]$ of $U_q(su_5)$. Within the anyonic picture, we reobtain the universal $q$-deformed decuplet mass relation

$$M_{\Omega} - M_{\Xi^*} + M_{\Sigma^*} - M_\Delta = [2]_q(M_{\Xi^*} - M_{\Sigma^*}),$$

where $[2]_q = q + q^{-1} = 2 \cos \theta$. Consistency with data for baryon masses requires $\theta \simeq \frac{\pi}{14}$. As a result, anyons with anyonic statistics parameter $\nu = \frac{1}{14}$ can be put into correspondence, at least formally, with the constituent quarks of decuplet baryons.

1. Introduction

Quantum groups and quantum (or $q$-deformed) algebras $su_q(N) \equiv U_q(su_N)$ introduced more than a decade ago \cite{1, 2} remain to be a subject of intensive study. Interesting and very important for further applications aspect of the $q$-algebras consists in a variety of their possible realizations. Among these, there exist realizations in terms of $q$-deformed bosonic oscillators \cite{3, 4}, $q$-fermionic oscillators as well as mixed ($q$-bose and $q$-fermi) ones \cite{1, 3}. Few years ago, it was shown by Lerda and Sciuto \cite{6} that the $q$-algebra $U_q(su_2)$ admits a realization in terms of two modes of so-called anyonic oscillators - certain non-local two-dimensional objects defined on a 2d square lattice. Shortly after that, their results were extended \cite{7}-\cite{8} to the higher rank $U_q(sl_N)$ algebras and, moreover, to the $q$-anlogs of all semisimple Lie algebras from the classical series $A_r$, $B_r$, $C_r$ and $D_r$.

On the other hand, $q$-algebras find their phenomenological applications in (sub-)nuclear physics, e.g., in modelling rotational spectra of deformed nuclei \cite{9}, in describing some properties of hadrons, see \cite{10, 11} for relevant references. Concerning the use of higher rank $q$-algebras for deriving generalized hadron mass relations, let us mention the approach
based on replacing usual unitary groups of hadronic \((N \equiv N_f)\)-flavor symmetries, or their Lie algebras, by the corresponding quantum algebras \(U_q(su_N)\). Such replacement leads to a number of interesting implications. In conjunction with this, the attempt to utilize the possibility suggested by anyonic construction of the \(q\)-algebras, within just mentioned approach to hadron mass relations, seems quite natural.

Our goal in this note is to demonstrate that the \(U_q(su_N)\) based results concerning baryon masses and their mass sum rules, at least for the decuplet case, can be rederived in the framework of anyonic picture if the masses are defined in such way that one gets rid of anyons’ nonlocality in final results. As a consequence, it appears that constituent quarks within the present approach can be formally treated as anyons of definite statistics \(\nu\). A comparison of the obtained \(U_q(su_N)\) mass relation for decuplet baryons with data for empirical masses shows: one has to fix the anyonic statistics parameter (directly linked to the deformation strength) as \(\nu = \frac{1}{14}\).

2. Anyonic oscillators on 2d square lattice

In this section we recapitulate necessary minimum of details concerning \(d = 2\) lattice angle function, disorder operators, and anyonic oscillators (see \([6,7]\)). Let \(\Omega\) be a 2d square lattice with the spacing \(a = 1\). On this lattice we consider a set of \(N\) species (sorts) of fermions \(c_i(x)\), \(i = 1, \ldots, N\), \(x \in \Omega\), which satisfy the following standard anticommutation relations:

\[
\{c_i(x), c_j(y)\} = \{c_i(x)^\dagger, c_j(y)^\dagger\} = 0, \tag{1}
\]

\[
\{c_i(x), c_j^\dagger(y)\} = \delta_{ij} \delta(x, y). \tag{2}
\]

Here \(\delta(x, y)\) is the conventional lattice \(\delta\)-function: \(\delta(x, y) = 1\) if \(x = y\) and vanishes if \(x \neq y\).

We use the same as in \([3,4]\) definition of the lattice angle functions \(\Theta_{\gamma x}(x, y)\) and \(\Theta_{\delta x}(x, y)\) that correspond to the two opposite types of cuts (\(\gamma\)-type and \(\delta\)-type), and the same definition of ordering of lattice sites (\(x > y\) or \(x < y\)). The corresponding two types of disorder operators \(K_i(x_\gamma)\) and \(K_i(x_\delta)\), \(i = 1, \ldots, N\), are introduced in the form

\[
K_i(x_\gamma) = \exp \left( i\nu \sum_{y \neq x} \Theta_{\gamma x}(x, y) c_j^\dagger(y)c_j(y) \right),
\]

\[
K_i(x_\delta) = \exp \left( i\nu \sum_{y \neq x} \Theta_{\delta x}(x, y) c_j^\dagger(y)c_j(y) \right). \tag{3}
\]

Anyonic oscillators (AOs) \(a_i(x_\gamma)\) and \(a_i(x_\delta)\), \(i = 1, \ldots, N\), are defined \([4]\) as

\[
a_i(x_\gamma) = K_i(x_\gamma) c_i(x), \quad a_i(x_\delta) = K_i(x_\delta) c_i(x) \tag{4}
\]
(no summation over $i$), and the number $\nu$ appearing in (3)–(4) is usually called the statistics parameter. One can show that these AOs satisfy the following relations of permutation. For $i \neq j$ and arbitrary $x, y \in \Omega$,

$$\{a_i(x_\gamma), a_j(y_\gamma)\} = \{a_i(x_\gamma), a_j^\dagger(y_\gamma)\} = 0. \quad (5)$$

With $q \equiv \exp(i\pi \nu)$, for $i = j$ and for two distinct sites (i.e. $x \neq y$) on the lattice $\Omega$ one has

$$a_i(x_\gamma)a_i(y_\gamma) + q^{-\text{sgn}(x-y)}a_i(y_\gamma)a_i(x_\gamma) = 0, \quad (6)$$

$$a_i(x_\gamma)a_i^\dagger(y_\gamma) + q^{\text{sgn}(x-y)}a_i^\dagger(y_\gamma)a_i(x_\gamma) = 0, \quad (7)$$

whereas on the same site

$$(a_i(x_\gamma))^2 = 0, \quad \{a_i(x_\gamma), a_i^\dagger(x_\gamma)\} = 1. \quad (8)$$

The analogs of relations (5)–(8) for anyonic oscillators of the opposite type $\delta$ are obtained by replacing $\gamma \to \delta$ and $q \to q^{-1}$ in (5)–(8). Clearly, at $\nu = 0$ (i.e., $q = 1$) anyonic operators reduce to the above fermionic ones.

Note that it is the pair of relations (6), (7) (their analogs for the $\delta$-type of cut, and Hermitian conjugates of all them) which the statistics parameter $\nu$ does enter. In comparison with ordinary fermions (1)–(2), the basic feature of anyons is their nonlocality (the attributed cut) and their peculiar braiding property encoded in (6), (7). These relations imply that anyons of the same sort, even allocated at different sites of the lattice, nevertheless ‘feel’ each other due to the factor involving the parameter $q$ (or $\nu$).

Finally, the commutation relations for anyons of opposite types of non-locality, i.e. of $\gamma$-type and $\delta$-type, are to be exhibited ($x, y$ arbitrary):

$$\{a_i(x_\gamma), a_j(y_\delta)\} = 0, \quad \{a_i(x_\gamma), a_j^\dagger(y_\delta)\} = 0 \quad (x \neq y), \quad (9)$$

$$\{a_i(x_\gamma), a_j^\dagger(x_\delta)\} = \delta_{ij} q^{\sum_{y<x} - \sum_{y>x}} c_i^\dagger(y)c_i(y), \quad (10)$$

as well as the relations which result from (5)–(10) by applying Hermitian conjugation.

3. The algebra $U_q(sl_N)$ and its anyonic realization

We adopt the standard notation $[A]_q \equiv (q^A - q^{-A})/(q - q^{-1})$ for $A$ being either an operator or a number, with a complex number $q$. Let $a_{ij}, i, j = 1, \ldots, N-1$, be the Cartan matrix of the $sl_N$ algebras (i.e. $a_{ii} = 2, \quad a_{i,i+1} = a_{i+1,i} = -1$, all the other $a_{ij}$ equal
Quantum algebra $U_q(sl_N)$ is generated by the Chevalley-basis elements $E^\pm_i$, $H_i$, $i = 1, \ldots, N - 1$, which obey the commutation relations

$$[H_i, H_j] = 0,$$
$$[H_i, E^\pm_j] = \pm a_{ij} E^\pm_j,$$
$$[E^+_i, E^-_j] = \delta_{ij} [H_i]_q,$$

$$[E^\pm_i, E^\pm_j] = 0 \quad \text{if} \quad |i - j| > 1,$$

$$(E^\pm_i)^2 E^\pm_{i+1} - [2]_q E^\pm_i E^\pm_{i+1} E^\pm_i + E^\pm_{i+1}(E^\pm_i)^2 = 0,$$

$$(E^\pm_{i+1})^2 E^\pm_i - [2]_q E^\pm_{i+1} E^\pm_i E^\pm_{i+1} + E^\pm_i(E^\pm_{i+1})^2 = 0.$$ (11)

The $q$-deformed algebra $U_q(sl_N)$ becomes a Hopf algebra if one imposes a comultiplication $\Delta$, counit $\epsilon$, and antipode $S$ (we omit them since they will not be used below).

In what follows, we use the fact of principal importance: the fundamental representations of the algebras $U_q(sl_N)$ and $sl_N$ are the same, as can be seen from explicit formulas for representation operators [2]. By repeated use of comultiplication applied to the fundamental representation, all other representations can be obtained. Due to different comultiplication rules in these two algebras, the difference between $U_q(sl_N)$ and $sl_N$ emerges in all representations other than fundamental and trivial ones.

So, using the lattice of ordered points $\Omega$, we assign to each its point $x \in \Omega$ a fundamental $N$-dimensional representation of the algebra $U_q(sl_N)$.

The following statement is true [3]-[8].

**Proposition.** The set of $N$ anyons defined according to (4), through the formulae

$$E^+_j \equiv A_{j,j+1} = \sum_{x \in \Omega} A_{j,j+1}(x), \quad E^-_j \equiv A_{j+1,j} = \sum_{x \in \Omega} A_{j+1,j}(x),$$

$$H_j \equiv A_{jj} - A_{j+1,j+1} = \sum_{x \in \Omega} \{A_{jj}(x) - A_{j+1,j+1}(x)\}$$ (12)

where

$$A_{j,j+1}(x) = a_j^\dagger(x) a_{j+1}(x), \quad A_{j+1,j}(x) = a_{j+1}^\dagger(x) a_j(x),$$

$$A_{jj}(x) = a_j^\dagger(x) a_j(x) = a_j^\dagger(x) a_j(x) = N_j(x), \quad N_j(x) \equiv c_j^\dagger(x) c_j(x),$$ (13)

provides a (bilinear) realization of the $U_q(sl_N)$ algebra.

These same $N$ anyons, through the formulae

$$
\tilde{E}^+_j \equiv \tilde{A}_{j,j+1} = \sum_{x \in \Omega} \tilde{A}_{j,j+1}(x), \\
\tilde{E}^-_j \equiv \tilde{A}_{j+1,j} = \sum_{x \in \Omega} \tilde{A}_{j+1,j}(x), \\
\tilde{H}_j \equiv \tilde{A}_{jj} - \tilde{A}_{j+1,j+1} = \sum_{x \in \Omega} \{\tilde{A}_{jj}(x) - \tilde{A}_{j+1,j+1}(x)\}
$$

where

$$
\tilde{A}_{j,j+1}(x) = a_j^\dagger(x) a_{j+1}(x), \quad \tilde{A}_{j+1,j}(x) = a_{j+1}^\dagger(x) a_j(x), \\
\tilde{A}_{jj}(x) = a_j^\dagger(x) a_j(x) = N_j(x)
$$
(α denotes either γ or δ), provide the so-called dual bilinear realization of $U_q(sl_N)$. It is easy to see that the generators $\tilde{A}_{jj}$ and $\tilde{H}_j$ in the dual realization coincide with their counterparts $A_{jj}$ and $H_j$ from the realization (12), (13).

4. State vectors of decuplet baryons within anyonic realization

In order to construct state vectors for baryons $\frac{3}{2}^+$ that form the decuplet of $U_q(su_3)$, we exploit the chain of embeddings

$$U_q(su_3) \subset U_q(su_4) \subset U_q(su_5)$$

and the respective chain of embeddings of representation spaces for $[30] \subset [300] \subset [4000]$. Whereas in the case of adjoint representations the construction (within anyonic realization) of basis vectors in their carrier spaces involves only two sites of lattice, the baryonic case at hand requires four sites to be involved in the construction of states from the representation space of [4000]. For definiteness, let us fix the order $x_1 < x_2 < x_3 < x_4$ of the coordinates of the sites.

With the notation (note that each $n_i$ labels the species of anyons while the subscript $'i'$ labels the site of the lattice)

$$|n_1n_2n_3n_4⟩ ≡ a^\dagger_{n_1}(x_{1\delta})a^\dagger_{n_2}(x_{2\delta})a^\dagger_{n_3}(x_{3\delta})a^\dagger_{n_4}(x_{4\delta})|0⟩,$$  

the highest weight vector (h.w.v.) of [4000] can be realized as $|1111⟩$. One can verify this fact merely by checking that the representation operators $E_{1}^+, E_{2}^+, E_{3}^+, E_{4}^+$ annihilate this state, and the operators $H_i$, $i = 1, ..., 4$, acquire on it the appropriate eigenvalues.

Acting on $|1111⟩$ sequentially by the operators $E_{-1}, E_{-2}, E_{-3}, E_{-4}$ we obtain the h.w.v. of representation $[30] ≡ 10$ (decuplet) space of the subalgebra $U_q(su_3)$ as certain superposition of states differing by the position of 5-th sort anyon, and it is natural to put this into correspondence with the particle $\Delta^{++}$:

$$|\Delta^{++}⟩ \sim (q^{-3}|5111⟩ + q^{-2}|1511⟩ + q^{-1}|1151⟩ + |1115⟩).$$  

The other basis vectors of 10 can be obtained by acting with the lowering operators $E_{-1}, E_{-2}$.

In order to construct dual basis (obtainable by acting with lowering operators in dual anyonic realization) we start with the same h.w.v. $|1111⟩$ of the representation [4000]. This fact is reflected in the equality $|1111⟩ = |1111⟩$, i.e., the h.w.v. is common for both bases.

Since a state of real baryon must be insensitive to the choice of cut type, the state

$$|\tilde{\Delta}^{++}⟩ \sim (q^{-3}|5111⟩ + q^{-2}|1511⟩ + q^{-1}|1151⟩ + |1115⟩)$$  

(16)
from dual basis corresponds to the \( \Delta^{++} \) hyperon as well (let us emphasize that this tilded state serves as a h.w.v. for irrep 10 in dual realization).

We will denote the basis vectors of representation space of 10 by \( |B_i\rangle \) and \( \tilde{|B_i}\rangle \), \( i = 1, \ldots, 10 \), where tilded vectors correspond to dual realization.

The normalization for the vectors \( |B_i\rangle \) and \( \tilde{|B_i}\rangle \) is chosen in such a way that the condition

\[
\| |B_i\rangle \|^2 \equiv \langle \tilde{B}_i |B_i\rangle = 1
\]

holds. This also implies \( \| \tilde{|B_i}\rangle \|^2 \equiv \langle B_i |\tilde{B_i}\rangle = 1 \). Thus, nonlocality is absent in the norm.

For general state vector, we introduce the following \( q \)-weighted superposition (\( q \)-symmetrized state):

\[
|(n_1 n_2 \ldots n_l)\rangle = \sum_{\sigma \in P_l/(P_{k_1} \times P_{k_2} \times \ldots P_{k_r})} [n_{\sigma(1)}n_{\sigma(2)}\ldots n_{\sigma(l)}]q^{\sum_{j=1}^{l} \sum_{j+i+1}^{l} \theta(n_{\sigma(j)}-n_{\sigma(i)})} .
\]  

(17)

In this formula, \( r \) is the maximal number from the set \( (n_1, n_2, \ldots, n_l) \), \( k_s (s = 1, \ldots, r) \) is the multiplicity of numbers \( 's' \) in this set, \( P_l \) is the group of permutations of \( l \) elements (all the \( P_0 \) must be omitted), \( \theta(x) \) is the usual step function ( \( \theta(x) = 1 \) if \( x > 0 \), and \( \theta(x) = 0 \) otherwise). All possible permutations of the numbers \( (n_1, n_2, \ldots, n_l) \), which lead to non-coinciding basis vectors, are to be taken into account in the sum in (17). As a result, the vector \(|(n_1 n_2 \ldots n_l)\rangle \) depends only on the set of numbers \( (n_1, n_2, \ldots, n_l) \) but not on their order.

Using anyonic realization (12)–(13) of the generators \( E_i^\pm \), we have the following formulas of their action upon vectors (17):

\[
E_i^+ |(n_1 n_2 \ldots (i+1) \ldots n_l)\rangle = [k_i + 1] q |(n_1 n_2 \ldots (i) \ldots n_l)\rangle ,
\]

(18)

\[
E_i^- |(n_1 n_2 \ldots (i) \ldots n_l)\rangle = [k_{i+1} + 1] q |(n_1 n_2 \ldots (i+1) \ldots n_l)\rangle ,
\]

(19)

where \( k_i \) (resp. \( k_{i+1} \)) is the multiplicity of \( 'i' \) (resp. \( 'i+1' \)) in the initial vector.

It is possible to introduce the dual analogue of (17):

\[
|(n_1 \tilde{n_2} \ldots n_l)\rangle = \sum_{\sigma \in P_l/(P_{k_1} \times P_{k_2} \times \ldots P_{k_r})} [n_{\sigma(1)}n_{\sigma(2)}\ldots n_{\sigma(l)}]q^{\sum_{j=1}^{l} \sum_{j+i+1}^{l} \theta(n_{\sigma(j)}-n_{\sigma(i)})} .
\]

The dual analogs of the formulas (18) and (19) (obtained by changing the generators and vectors into dual ones) are also valid.

The norm of the vector (17) as well as of its dual counterpart is

\[
\| |(n_1 n_2 \ldots n_l)\rangle \|^2 = \| |(n_1 \tilde{n_2} \ldots n_l)\rangle \|^2 = \langle (n_1 \tilde{n_2} \ldots n_l)| (n_1 n_2 \ldots n_l) \rangle = [l]_q! \prod_{s=1}^{r} \frac{1}{[k_s]_q} .
\]

(20)

where the notation \( [m]_q! \equiv [m]_q[m-1]_q[m-2]_q \ldots [2]_q[1]_q \) for \( q \)-factorial \([m]_q\) is used.
5. Masses of decuplet baryons from anyonic realization

Mass $M_{B_i}$ of a particle $B_i$ is defined as diagonal matrix element

$$M_{B_i} = \langle B_i | \hat{M} | B_i \rangle$$

of the mass operator (see [14])

$$\hat{M} = M_0 \mathbf{1} + \alpha (A_{35} \tilde{A}_{35} + \tilde{A}_{35} A_{35}) + \beta (A_{53} \tilde{A}_{53} + \tilde{A}_{53} A_{53}) \ ,$$

where $A_{35} \equiv A_{35}(q) \equiv q^{1/2} A_{34} A_{45} - q^{-1/2} A_{45} A_{34}$, $A_{53} \equiv A_{53}(q) \equiv q^{1/2} A_{43} A_{54} - q^{-1/2} A_{54} A_{43}$, $\tilde{A}_{35} \equiv - A_{35}(q^{-1})$, $\tilde{A}_{53} \equiv - A_{53}(q^{-1})$. Clearly, the constants $M_0$, $\alpha$ and $\beta$ have the dimension of mass.

Due to particular choice of representations (mentioned above), the part of mass operator which gives nonvanishing contribution becomes

$$\hat{M} = M_0 \mathbf{1} + \alpha \hat{M}_\alpha + \beta \hat{M}_\beta$$

where $\hat{M}_\alpha \equiv E_3^+ E_4^+ E_3^- E_4^-$, $\hat{M}_\beta \equiv E_4^- E_3^- E_4^+ E_3^+$. With the notation

$$M_\alpha(B_i) \equiv \langle B_i | \hat{M}_\alpha | B_i \rangle, \quad M_\beta(B_i) \equiv \langle B_i | \hat{M}_\beta | B_i \rangle ,$$

the expressions for masses take the form

$$M_{B_i} = M_0 + \alpha M_\alpha(B_i) + \beta M_\beta(B_i) .$$

(23)

Then, using the relations $(E_i^\pm)^* = \tilde{E}_i^\mp$ where "*" denotes Hermitian conjugation, one rewrites the expressions $M_\alpha(B_i)$ and $M_\beta(B_i)$ as follows:

$$M_\alpha(B_i) \equiv ||E_3^- E_4^- | B_i ||^2, \quad M_\beta(B_i) \equiv ||E_3^+ E_4^+ | B_i ||^2 .$$

(24)

In other words, $M_\alpha(B_i)$ is given as scalar product of the vectors $E_3^- E_4^- | B_i \rangle$ and their duals $\tilde{E}_4^- E_3^- | B_i \rangle$; likewise, $M_\beta(B_i)$ is given by the scalar product of vectors $E_3^+ E_4^+ | B_i \rangle$ and their duals $\tilde{E}_4^+ E_3^+ | B_i \rangle$.

It is not hard to see that, with this specific form of mass operator, masses of baryons within each isomultiplet (multiplet of $U_q(su_2)$) in the decuplet are equal.
In order to obtain the expressions for masses of baryons with state vectors (21), we substitute these vectors in formulas (23),(24) and take into account (18)–(20). For example, for the $\Omega^-$ hyperon we obtain

$$M_{\Omega^-} = M_0 + \alpha M_\alpha(\Omega^-) + \beta M_\beta(\Omega^-)$$

with

$$M_\alpha(\Omega^-) = \frac{1}{|q|} |E_3^- E_3^-| |\Omega^-\rangle|^2 = \frac{[2]^2_q}{[4]^2_q} |(3355)\rangle^2 = [2]^q[3]_q$$

(here the formulae $E_3^- |(3335)\rangle = |(3345)\rangle, E_3^- |(3345)\rangle = [2]_q|(3355)\rangle$, and (20) were used) and

$$M_\beta(\Omega^-) = \frac{1}{|q|} |E_4^+ E_4^+| |\Omega^-\rangle|^2 = \frac{[4]^2_q}{[4]^2_q} |(3333)\rangle^2 = [4]^q$$

(here the formulae $E_4^+ |(3335)\rangle = |(3334)\rangle, E_3^+ |(3334)\rangle = [4]_q|(3333)\rangle$, and (20) were used). Hence,

$$M_{\Omega^-} = M_0 + [2]^q[3]_q\alpha + [4]^q\beta.$$  

(25)

In similar way, calculations for the other three isoplets yield the masses

$$M_\Delta = M_0 + \beta, \quad M_\Sigma^* = M_0 + [2]^q\alpha + [2]^q\beta, \quad M_\Xi^* = M_0 + [2]^q\alpha + [3]^q\beta.$$  

(26)

Eliminating the constants $M_0, \alpha, \beta$ from the expressions (25),(26) for isoplet masses of the decuplet, we arrive at the mass relation

$$M_{\Omega^-} - M_{\Xi^*} + M_{\Sigma^*} - M_\Delta = [2]^q (M_{\Xi^*} - M_{\Sigma^*})$$  

(27)

in the form of $q$-average. It is the same mass relation as that obtained previously in [14] using $q$-analogue of Gel’fand-Tsetlin formalism. In [14] it was also shown that the decuplet mass formula (27) results not only within this particular dynamical $U_q(su_5)$ representation [4000], but also within any admissible (such that contains the $U_q(su_4)$ 20-plet wherein the decuplet of $U_q(su_3)$ is placed) dynamical representation of $U_q(su_5)$ or $U_q(su_4,1)$. This is a kind of universality - independence of the result (27) on the choice of dynamical representation.

Comparison of (27) with empirical situation [15] shows that with $q = \exp(i\pi/14)$ (in which case we have $[2]^q \simeq 1.96$), the mass relation remarkably agrees with data. This is in contrast with the average-type formula $\frac{1}{2}(M_{\Omega^-} - M_{\Xi^*} + M_{\Sigma^*} - M_\Delta) = M_{\Xi^*} - M_{\Sigma^*}$ known long ago [16]. To see that, it is enough to “predict” $M_\Delta$, both from the latter formula and from the $q$-deformed one (27) with $[2]^q \simeq 1.96$, in terms of known masses $M_{\Omega^-}, M_{\Xi^*}, M_{\Sigma^*}$, and then to compare the results with the empirical value ($1232 \pm 2$) MeV.
of $M_\Delta$. Unlike the case of average-type formula, the value of $M_\Delta$ “predicted” on the base of (27) lies exactly in the range $(1232 \pm 2)$ MeV. Concerning physical meaning of the value $q = \exp(i\pi/14)$ of deformation parameter it was argued in [11] that $\frac{\pi}{14}$ may be identified with the Cabibbo angle.

6. Concluding remarks

We have shown in the decuplet case that the $U_q(su_N)$ based results [14] concerning decuplet baryon masses and their mass sum rule can as well be derived in the framework of anyonic realization of the (flavour symmetry) quantum algebras. To this end, we have first constructed, using anyonic operators, the state vectors corresponding to isoplet members of the decuplet. Then we have calculated baryon masses adopting the appropriate definition (22) which allowed us to get rid of nonlocality in final results for the masses. As a consequence it appears that constituent quarks (as modified within the present approach) may be, at least formally, considered as anyons of definite statistics $\nu$. From comparison of the $q$-relation (27) with empirical data for decuplet baryons we draw the conclusion that: the anyonic statistics parameter (connected with the deformation parameter $q$) should be $\nu = \frac{1}{14}$ in the decuplet case. Recall that the value of statistics parameter characterizes braiding of anyons (on the lattice $\Omega$), and these are used as building blocks for the state vectors of decuplet baryons, as seen in Sec. 5. Of course, there still remains an open question whether the quarks bound in baryons $\frac{3}{2}^+$ could be really considered as anyon-like, quasi-two-dimensional entities. This problem however has to be answered on the base of a field-theoretic setting which goes beyond the scope of present note. Let us only mention, as partial justification, that one can use the argumentation of Ref. [17] according to which the appropriately interacting (charge-flux) fermions, with nonzero probability to be found in the plane (in which they interact), exhibit to certain extent the properties of anyonic statistics.

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