Interacting viscous ghost tachyon, K-essence and dilaton scalar field models of dark energy

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Abstract

We study the correspondence between the interacting viscous ghost dark energy model with the tachyon, K-essence and dilaton scalar field models in the framework of Einstein gravity. We consider a spatially non-flat FRW universe filled with interacting viscous ghost dark energy and dark matter. We reconstruct both the dynamics and potential of these scalar field models according to the evolutionary behavior of the interacting viscous ghost dark energy model, which can describe the accelerated expansion of the universe. Our numerical results show that the interaction and viscosity have opposite effects on the evolutionary properties of the ghost scalar field models.

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1. Introduction

Observations and experiments are consistent with the hypothesis that the majority of the energy of the universe is in the form of a heretofore undiscovered substance, referred to simply as ‘dark energy’ (DE) that causes the cosmic expansion to accelerate [1]. Although the physical origin of DE is still unknown, various models of DE have been proposed in the literature (for review see [2]). The cosmological constant is the most obvious theoretical candidate of DE. Although the cosmological constant is consistent with the observational data, at the fundamental level it suffers from the two well-known difficulties containing the ‘fine tuning’ and the ‘cosmic coincidence’ problems [3].

Recently, a new model of DE called Veneziano ghost DE (GDE) was introduced to describe the accelerated expansion of the universe [4]. This model has been motivated from the Veneziano ghost of chromodynamics (QCD). Veneziano ghost is supposed to exist for solving the \(U(1)\) problem in low-energy effective theory of QCD. The key ingredient of this new model is that the Veneziano ghost, being unphysical in the quantum field theory formulation in the Minkowski spacetime, exhibits important non-trivial physical effects in an expanding universe such as our Friedmann–Robertson–Walker (FRW) universe, or in a
spacetime with a non-trivial topological structure [5]. The QCD ghost has a small contribution to the vacuum energy density proportional to $\Lambda_{\text{QCD}}^3 H$, where $\Lambda_{\text{QCD}} \sim 100$ MeV is the QCD mass scale and $H$ is the Hubble parameter. This small contribution can play an important role in the evolutionary behavior of the universe. For instance, taking $H \sim 10^{-33}$ eV at the present, $\Lambda_{\text{QCD}}^3 H$ gives the right order of magnitude of the DE density. This remarkable coincidence implies that the GDE model gets rid of the fine tuning problem [4]. In addition, the appearance of the QCD scale could be relevant for a solution to the cosmic coincidence problem, as it may be the scale at which dark matter (DM) forms [6].

The other advantage of the GDE with respect to other DE models includes the fact that it can be completely explained within the standard model and general relativity (GR), without recourse to any new field, new symmetries or modifications of GR. Several aspects of this new paradigm have been investigated in the literature [7–12]. For instance, the GDE model has been fitted with current observational data including type Ia supernovae (SNeIa), baryon acoustic oscillation (BAO), cosmic microwave background (CMB), big bang nucleosynthesis (BBN), Hubble parameter and growth rate of matter perturbation and it was found that the GDE model cannot be discriminated by the data, and this model like the $\Lambda$CDM model fit the data very well [7].

One clarification is in order since there are some DE models where the ghost plays the role of DE (see, e.g., [13]) and becomes a real propagating physical degree of freedom subject to some severe constraints. However, the Veneziano ghost is not a new physical propagating degree of freedom and therefore the GDE model does not violate unitarity, causality, gauge invariance and other important features of renormalizable quantum field theory, as advocated in [14]. At the same time, it has been emphasized in [4], and especially later in [14], that in fact the description of GDE in terms of the Veneziano ghost is just a matter of convenience to describe very complicated infrared dynamics of strongly coupled QCD. One can describe the same dynamics using some other approaches (e.g. direct lattice simulations) without using the ghost.

Another interesting proposal for describing DE is scalar field models such as tachyon, K-essence and dilaton (for review see [15] and references therein). The scalar field models can alleviate the fine tuning and coincidence problems [16]. Scalar fields are popular not only because of their mathematical simplicity and phenomenological richness, but also because they naturally arise in particle physics including supersymmetric field theories and string/M theory. Therefore, the scalar field is expected to reveal the dynamical mechanism and the nature of DE. Although fundamental theories such as string/M theory do provide a number of possible candidates for scalar fields, they do not uniquely predict their potential $V(\phi)$ or kinetic term [17]. This is one of the main problems regarding the scalar field models. To determine the potential or kinetic term of a scalar field, there are two approaches in the literature. First, for a given potential motivated purely on phenomenological considerations, its viability is examined by finding the stable attractor solutions of the evolution equations for the system of scalar field plus a background fluid. For instance, Halliwell [18] studied a homogeneous isotropic cosmological model driven by a scalar field with an exponential potential. It was shown that there is a solution with power-law inflation, and this solution is an attractor. van den Hoogen et al [19] investigated the stability of cosmological scaling solutions describing a barotropic fluid with a non-interacting scalar field with an exponential potential. They studied homogeneous and isotropic spacetimes with non-zero spatial curvature and found three possible asymptotic future attractors in an ever-expanding universe. Copeland et al [20] analyzed the dynamics of a single scalar field in an FRW universe with spatial curvature. For a wide class of potentials, they obtained the fixed point solutions which are shown to be late time attractors.
In the second approach, which is used in this work, the scalar field model is regarded as an effective description of an underlying theory of DE. Then, one should be capable of using the scalar field model to mimic the evolving behavior of the dynamical DE and reconstructing this scalar field model according to the evolutionary behavior of DE. With this strategy one can reconstruct $V(\phi)$ or the kinetic term of the scalar fields from some DE models such as holographic [21], agegraphic [22] and ghost [10–12]. Note that although from a given equation of state (EoS) describing a perfect fluid, a minimally coupled scalar field in GR can be derived [23]; this correspondence is not in general one to one. From the conceptual point of view, it is clear that a scalar field and a perfect fluid are very different physical systems. A fluid is obtained by averaging microscopic quantities associated with its constituent particles, and the fluid laws can be obtained via kinetic theory using microscopic models of the fluid particles and of their interactions. A scalar field, instead, is more fundamental and does not derive from an average. It is not made of point-like particles and does not need an average to be defined. It is already a continuum. Hence, one should note that the fluid description of a scalar field is not an identification, but only a convenient correspondence for formal purposes.

Viewing the scalar field models as an effective description of the underlying theory of DE motivates us to establish different scalar field models according to the evolutionary behavior of the GDE scenario. To do so, in section 2 we investigate GDE in a spatially non-flat FRW universe. In section 3, we reconstruct both the dynamics and potential of the tachyon, K-essence and dilaton scalar field models of DE according to the evolution of GDE density. Section 4 is devoted to conclusions.

2. GDE scenario

Following [4], the GDE density is proportional to the Hubble parameter

$$\rho_D = \alpha H,$$

where $\alpha$ is a constant with dimension [energy]$^3$, and roughly of order of $\Lambda_{QCD}^3$. In general, it is very difficult to accept such a linear behavior in the energy density because QCD is a theory with a mass gap determined by the scale $\Lambda_{QCD} \sim 100$ MeV. Therefore, it is generally expected that there should be exponentially small corrections rather than linear corrections $\sim H$. This question has been elaborated in detail in [14] where it has been argued that the linear scaling $\sim H$ is due to the complicated topological structure of strongly coupled QCD, not related to the physical massive propagating degrees of freedom. However, the Veneziano ghost is not a new physical propagating degree of freedom.

Here, we consider a spatially non-flat FRW universe filled with GDE and DM. Within the framework of FRW cosmology, the first Friedmann equation takes the form

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}(\rho_D + \rho_m),$$

where $M_p = (8\pi G)^{-1/2}$ is the reduced Planck mass. Here $k = 0, 1, -1$ represent a flat, closed and open FRW universe, respectively. Also, $\rho_D$ and $\rho_m$ are the energy densities of GDE and DM, respectively.

Using the dimensionless energy densities defined as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3M_p^2H^2}, \quad \Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{\rho_D}{3M_p^2H^2}, \quad \Omega_k = \frac{k}{a^2H^2},$$

the Friedmann equation (2) can be rewritten as

$$1 + \Omega_k = \Omega_D + \Omega_m.$$
Substituting equation (1) into $\rho_D = 3M_p^2H^2\Omega_D$ yields

$$\Omega_D = \frac{\alpha}{3M_p^2H^2}. \quad (5)$$

Using the above relation, the curvature energy density parameter can be obtained as

$$\Omega_k = \left(\frac{9M_p^4k}{\alpha^2}\right) \left(\frac{\Omega_D}{a}\right)^2 = \left(\frac{\Omega_D}{\Omega_D^0}\right) \left(\frac{\Omega_D}{a}\right)^2, \quad (6)$$

where we take $a_0 = 1$ for the present value of the scale factor.

Note that at late time when the DE is dominated, equation (2) yields

$$\rho_D = 3M_p^2H^2.$$

Using this and equation (1), we obtain $H = \alpha/(3M_p^2) =$ constant. Substituting this into equation (5) gives $\Omega_D = 1$ corresponding to the DE dominated epoch at late time. Also at late time when $H =$ constant, the deceleration parameter reads $q = -1 - H/H^2 = -1$ which behaves like the de Sitter universe. This is in agreement with that obtained by Cai et al [7]. They showed that for a flat FRW universe containing the GDE and DM, the universe transits from a matter dominated epoch at early time to the de Sitter phase in the future, as expected.

Here, we would like to generalize our study to the case where the GDE model has viscosity property. According to the WMAP observations it was shown that in an isotropic and homogeneous FRW universe, the shear viscosity has no contribution to the energy–momentum tensor and the bulk viscosity behaves like an effective pressure [24]. It was also pointed out that the bulk viscosity can play a significant role in the formation of the large scale structures (LSS) of the universe [25]. DE with bulk viscosity has a peculiar property to cause accelerated expansion in the late evolution of the universe [26]. It can also alleviate several cosmological puzzles such as age problem and coincidence problem. It was pointed out that the viscous fluid may give a unified description of DE and DM, under the name ‘dark fluid’ [27]. The energy–momentum tensor of the viscous fluid is

$$T_{\mu\nu} = u_\mu u_\nu \rho_D + (g_{\mu\nu} + u_\mu u_\nu) \tilde{p}_D, \quad (7)$$

where $u_\mu$ is the 4-velocity vector, $g_{\mu\nu}$ is the background metric and

$$\tilde{p}_D = p_D - 3H \xi, \quad (8)$$

is the effective pressure of DE, and $\xi = \epsilon H^{-1}\rho_D$ is the bulk viscosity coefficient in which $\epsilon$ is a constant parameter [28]. A viscosity $\epsilon > 0$ will be able to drive acceleration [28].

We further assume that there is an interaction between viscous GDE and DM. The interaction between DE and DM can be detected in the formation of LSS. It was suggested that the dynamical equilibrium of collapsed structures such as galaxy clusters would be modified due to the coupling between DE and DM. The recent observational evidence provided by the galaxy cluster Abell A586 supports the interaction between DE and DM [29]. The other observational signatures on the dark sectors’ mutual interaction can be found in the probes of the cosmic expansion history by using the SNeIa, BAO, CMB shift and BBN data [30]. In the presence of interaction, $\rho_D$ and $\rho_m$ do not conserve separately and the energy conservation equations for interacting viscous GDE and DM are

$$\dot{\rho}_D + 3H(1 + \omega_D)\rho_D = 9\epsilon H \rho_D - Q, \quad (9)$$

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (10)$$

where $\omega_D = p_D/\rho_D$ is the EoS parameter of the interacting viscous GDE and $Q$ stands for the interaction term. Following [31], we shall assume $Q = 3b^2H(\rho_m + \rho_D)$ with the coupling constant $b^2$. This expression for the interaction term was first introduced in the study of the suitable coupling between a quintessence scalar field and a pressureless cold DM field [28, 32].
Taking the time derivative of equation (2) and using equations (3) and (10) gives
\[
\frac{\dot{H}}{H^2} = \frac{\dot{\rho}_D}{6M_p^2 H^3} + \frac{3}{2} b^2 (\Omega_m + \Omega_D) - \frac{3}{2} \Omega_m + \Omega_k. \tag{11}
\]

Taking the time derivative of equation (1) yields
\[
\frac{\dot{H}}{H} = \frac{\dot{\rho}_D}{\rho_D}. \tag{12}
\]

Substituting equation (12) into (11) and using equation (4), we obtain
\[
\frac{1}{H} \left( 1 - \frac{a}{6M_p^2 H} \right) \frac{\dot{\rho}_D}{\rho_D} = \frac{3}{2} b^2 (1 + \Omega_k) - \frac{3}{2} (1 + \Omega_k - \Omega_D) + \Omega_k. \tag{13}
\]

Inserting equation (5) into the above relation, one can obtain
\[
\frac{\dot{\rho}_D}{\rho_D} = 3H \left[ \frac{\Omega_D - 1 - \frac{\Omega_k}{3} + b^2 (1 + \Omega_k)}{2 - \Omega_D} \right]. \tag{14}
\]

Taking the time derivative of equation (5) and using (1) and (14) one can obtain the evolution of the interacting viscous GDE density parameter as
\[
\frac{d\Omega_D}{d\ln a} = \left( \frac{3\Omega_D}{\Omega_D - 2} \right) \left[ \Omega_D - 1 - \frac{\Omega_k}{3} + b^2 (1 + \Omega_k) \right], \tag{15}
\]
which is the same as that obtained for the interacting GDE in a non-flat universe in the absence of viscosity [9]. It is interesting to note that the viscosity constant \( \epsilon \) does not affect the evolution of the GDE density parameter (15). Substituting equation (6) into (15) yields a differential equation for \( \Omega_D(a) \) which can be solved numerically with a suitable initial condition such as \( \Omega_{D_i} = 0.72 \). The numerical results obtained for \( \Omega_D(a) \) are displayed in figure 1 for different coupling constant \( b^2 \). The figure shows that: (i) for a given \( b^2 \), \( \Omega_D \) increases when the scale factor increases and (ii) at early and late times, \( \Omega_D \) increases and decreases with increasing \( b^2 \), respectively.

![Figure 1. The evolution of the GDE density parameter, equation (15), for different coupling constants \( b^2 \). Auxiliary parameters are \( \Omega_{D_i} = 0.72 \) and \( \Omega_{k_i} = 0.01 \).](image-url)
Substituting equation (14) into (9) gives the EoS parameter of the interacting viscous GDE model as
\[ \omega_D = \frac{1 - \Omega_D}{\Omega_D - 2} \left( b^2 \frac{1 + \Omega_D}{\Omega_D} \right) + 3\epsilon, \]  
which shows that in the absence of interaction and viscous terms, i.e. \( b^2 = \epsilon = 0 \), at early \( (\Omega_D \to 0) \) and late \( (\Omega_D \to 1) \) times \( \omega_D \) goes \(-1/2\) and \(-1\), respectively, and cannot cross the phantom divide line [7]. For the present time \( (a_0 = 1) \), taking \( \Omega_D = 0.72 \) and \( \Omega_k = 0.01 \) [33] equation (16) gives
\[ \omega_{D_0} = -0.78 - 2.19b^2 + 3\epsilon, \]  
which clears that for \( \epsilon = 0 \) the phantom EoS parameter \( (\omega_{D_0} < -1) \) can be obtained provided \( b^2 > 0.1 \). This value for the coupling constant \( b^2 \) is consistent with the observations in which we have that \( b^2 \) could be as large as 0.2 [34]. Also, the phantom divide crossing is compatible with the recent observations [35].

The evolution of the EoS parameter (16) for different \( b^2 \) and \( \epsilon \) is plotted in figures 2 and 3, respectively. Figure 2 shows that: (i) for \( b^2 = 0 \), \( \omega_D \) decreases from \(-0.5\) at early time and approaches to \(-1\) at late time and (ii) for \( b^2 \neq 0 \), \( \omega_D \) increases at early time and decreases at late time. The results of \( \omega_D \) in the absence of viscosity \( (\epsilon = 0) \) are in agreement with those obtained by [9]. Figure 3 shows that: (i) for a given \( \epsilon \), \( \omega_D \) decreases with increasing scale factor and (ii) for a given scale factor, \( \omega_D \) increases when \( \epsilon \) increases.

### 3. Interacting viscous ghost scalar field models of DE

Here, our main aim is to investigate whether a minimally coupled scalar field with a specific action/Lagrangian can mimic the dynamics of the QCD ghost DE model so that this model can be related to some fundamental theory (such as string/M theory), as it is for a scalar field. For this task, it is then meaningful to reconstruct the dynamics \( \phi \) and potential \( V(\phi) \) of a scalar field model possessing some significant features of the underlying theory of DE, such as the GDE model. To do so, we establish a correspondence between the interacting viscous GDE and various scalar field models by identifying their respective energy densities and EoS and then reconstructing both the dynamics and potential of the field.
3.1. Ghost tachyon model

The tachyon field is another approach for explaining DE. The tachyonic condensate in a class of string theories can be described by an effective scalar field with a Lagrangian of the form [36]

$$\mathcal{L} = -V(\phi)\sqrt{1 + \partial_\mu \phi \partial^\mu \phi},$$  

(18)

where \(\phi\) is a tachyon scalar field and \(V(\phi)\) is a potential of \(\phi\). When \(\phi = \phi(t)\), the energy density and pressure of a tachyonic source are given by [36]

$$\rho_T = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}},$$  

(19)

$$p_T = -V(\phi)\sqrt{1 - \dot{\phi}^2}.$$  

(20)

The EoS parameter of the tachyon scalar field is obtained as

$$\omega_T = \frac{p_T}{\rho_T} = \dot{\phi}^2 - 1.$$  

(21)

To establish the correspondence between the interacting viscous GDE and tachyon field, equating (16) with (21), i.e. \(\omega_D = \omega_T\), yields

$$1 - \frac{\Omega_D}{3} + 2b_2^2 \left(\frac{\Omega_D}{3\Omega_0}\right) + 3\epsilon = \dot{\phi}^2 - 1.$$  

(22)

Also equating equation (1) with (19), i.e. \(\rho_D = \rho_T\), gives

$$\alpha H = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}.$$  

(23)

From equations (22) and (23), one can obtain the kinetic energy term and the tachyon potential energy as follows:

$$\dot{\phi}^2 = \frac{\Omega_D - 1 - \frac{\Omega_0}{3} + 2b_2^2 \left(\frac{\Omega_D}{3\Omega_0}\right)}{\Omega_D - 2} + 3\epsilon.$$  

(24)
Figure 4. The evolution of the ghost tachyon scalar field, equation (26), for different coupling constants $b^2$ with $\epsilon = 0$. Auxiliary parameters are $\Omega_{D_0} = 0.72$, $\Omega_{k_0} = 0.01$ and $\phi(1) = 0$.

Figure 5. The same as figure 4 for different viscosity constants $\epsilon$ with $b^2 = 0$. Auxiliary parameters as in figure 4.

\begin{equation}
V(\phi) = \frac{\alpha^2}{3M_p^2\Omega_D} \left[ \frac{1 - \frac{\Omega_i}{3} + 2b^2 \left( \frac{1+\Omega_k}{\Omega_D} \right)}{2 - \Omega_D} - 3\epsilon \right]^{1/2}.
\end{equation}

Note that equations (24) and (25) for the flat case, i.e. $\Omega_k = 0$, and in the absence of viscosity ($\epsilon = 0$) reduce to the results obtained by [11].

From equation (24) and using (5), one can obtain the evolutionary form of the ghost tachyon scalar field as

\begin{equation}
\phi(a) - \phi(1) = \frac{3M_p^2}{\alpha} \int_1^a \Omega_D \left[ \frac{\Omega_D - 1 - \frac{\Omega_i}{3} + 2b^2 \left( \frac{1+\Omega_k}{\Omega_D} \right)}{2 - \Omega_D} + 3\epsilon \right]^{1/2} \frac{da}{a}.
\end{equation}

where we take $a_0 = 1$ for the present time. The evolution of the ghost tachyon scalar field, equation (26), for different values of $b^2$ and $\epsilon$ is plotted in figures 4 and 5, respectively. The
The evolution of the ghost tachyon kinetic energy $\chi = \dot{\phi}^2/2$, equation (24), for different coupling constants $b^2$ with $\epsilon = 0$. Auxiliary parameters as in figure 4.

Figure 7. The same as figure 6 for different viscosity constants $\epsilon$ with $b^2 = 0$. Auxiliary parameters as in figure 4.

Figures show that: (i) for a given $b^2$ or $\epsilon$, $\phi(a)$ increases with increasing scale factor and (ii) for a given scale factor, $\phi(a)$ decreases and increases with increasing $b^2$ and $\epsilon$, respectively. Note that figure 4 shows only the real scalar field, i.e. $\dot{\phi}^2 > 0$. Indeed, for $b^2 = 0$, 0.02 and 0.04 the scalar field $\phi$ becomes pure imaginary ($\dot{\phi}^2 < 0$) at $a > 43.5$, 2.8 and 2.1, respectively, and it does not show itself in figure 4. To investigate this problem in ample detail, the evolution of the ghost tachyon kinetic energy $\chi = \dot{\phi}^2/2$, equation (24), for different values of $b^2$ and $\epsilon$ is plotted in figures 6 and 7, respectively. Figure 6 confirms that for $b^2 = 0$, 0.02 and 0.04 the kinetic energy becomes negative ($\chi < 0$) at $a > 43.5$, 2.8 and 2.1, respectively. Figures 6 and 7 show that: (i) for a given $b^2$ or $\epsilon$, the kinetic energy $\chi$ decreases when the scale factor increases and (ii) for a given scale factor, the kinetic energy decreases and increases with increasing $b^2$ and $\epsilon$, respectively.

It is worth noting that from equation (19) due to having a real tachyon energy density we need to have $\dot{\phi}^2 < 1$ which is in accordance with figures 6 and 7. Moreover, from equation (21)
Figure 8. The ghost tachyon potential, equation (25), versus the scalar field $\phi$ for different coupling constants $b^2$ with $\epsilon = 0$. Auxiliary parameters as in figure 4.

Figure 9. The same as figure 8 for different viscosity constants $\epsilon$ with $b^2 = 0$. Auxiliary parameters as in figure 4.

for $\dot{\phi}^2 < 0$ and $0 < \dot{\phi}^2 < 1$ we have $\omega_T < -1$ and $-1 < \omega_T < 0$, respectively, corresponding to the phantom [37] and quintessence [38] DE, respectively. In the absence of interaction ($b^2 = 0$), the kinetic energy of the ghost tachyon scalar field is always positive (see figure 7) and behaves like quintessence DE with $\omega_T = \omega_D > -1$ (see figure 3).

The ghost tachyon potential, equation (25), versus the scalar field (26) for different $b^2$ and $\epsilon$ is plotted in figures 8 and 9, respectively. The figures illustrate that: (i) for a given $b^2$ or $\epsilon$, $V(\phi)$ decreases with increasing $\phi$. This behavior is in agreement with the scaling solution $V(\phi) \propto \phi^{-2}$ obtained for the tachyon field corresponding to the power law expansion [39]. (ii) For a given scalar field, $V(\phi)$ increases and decreases with increasing $b^2$ and $\epsilon$, respectively.
3.2. Ghost K-essence model

The K-essence scalar field model of DE is characterized by a scalar field with a non-canonical kinetic energy. The most general scalar field action which is a function of $\phi$ and $\chi = \dot{\phi}^2/2$ is given by [40, 41]

$$S = \int d^4x \sqrt{-g} p(\phi, \chi),$$

(27)

where the Lagrangian density $p(\phi, \chi)$ corresponds to a pressure with non-canonical kinetic terms as

$$p(\phi, \chi) = f(\phi)(-\chi + \chi^2),$$

(28)

and the energy density of the K-essence field $\phi$ is

$$\rho(\phi, \chi) = f(\phi)(-\chi + 3\chi^2).$$

(29)

One of the motivations to consider this type of Lagrangian originates from considering low-energy effective string theory in the presence of a high order derivative terms.

It is important to note that the K-essence models constructed to solve the coincidence problem inevitably give rise to the superluminal propagation of the field ($c^2 = \frac{\partial s_{\phi,\chi}}{\partial s_{\phi,\chi}} > 1$) at some stage of the cosmological evolution. In case when K-essence DE is reconstructed to reproduce the behavior of the GDE, it should not be treated as a signal for instability of the theory as the Veneziano ghost is not a propagating physical degree of freedom. Appropriate interpretation is that a notion of the speed of sound does not exist for a non-physical, non-propagating degree of freedom [12].

The EoS parameter of the K-essence scalar field is obtained as

$$\omega_K = \frac{\rho(\phi, \chi)}{\rho(\phi, \chi)} = \frac{\chi - 1}{3\chi - 1}.$$  

(30)

Equating (30) with (16), $\omega_K = \omega_{DE}$, we find $\chi$ as follows:

$$\chi = \frac{3 - \frac{\Omega_D}{\Omega_K} + 2b^2 \left(\frac{1+\Omega_D}{\Omega_K}\right) - \Omega_D + 3\epsilon(\Omega_D - 2)}{5 - \Omega_K + 6b^2 \left(\frac{1+\Omega_D}{\Omega_K}\right) - \Omega_D + 9\epsilon(\Omega_D - 2)}.$$  

(31)

Using equation (31) and $\phi^2 = 2\chi$, we obtain the ghost K-essence scalar field as

$$\phi(a) - \phi(1) = \frac{3M_p^2}{\alpha} \int_1^a \frac{d\Omega_D}{\Omega_D^2} \left[ \frac{6 - 2\Omega_D + 4b^2 \left(\frac{1+\Omega_D}{\Omega_K}\right)}{5 - \Omega_K + 6b^2 \left(\frac{1+\Omega_D}{\Omega_K}\right) - \Omega_D + 9\epsilon(\Omega_D - 2)} \right]^{1/2} \frac{da}{a},$$

(32)

where its evolution for different $b^2$ and $\epsilon$ is displayed in figures 10 and 11, respectively. The figures present that: (i) for a given $b^2$ or $\epsilon$, $\phi(a)$ increases with increasing scale factor and (ii) for a given scale factor, $\phi(a)$ decreases and increases with increasing $b^2$ and $\epsilon$, respectively.

The evolution of the ghost K-essence kinetic energy, equation (31), for different values of $b^2$ and $\epsilon$ is plotted in figures 12 and 13, respectively. The figures clarify that: (i) for a given $b^2$ or $\epsilon$, the ghost K-essence kinetic energy like the tachyon field decreases when the scale factor increases and (ii) for a given scale factor, the kinetic energy of the ghost K-essence field like the tachyon model decreases and increases with increasing $b^2$ and $\epsilon$, respectively. If we compare figure 12 with figure 6 we see that the kinetic energy of the ghost K-essence model in contrast with the ghost tachyon field is always positive. Note that the result of figure 12 is in contrast with that obtained by authors of [12] who showed that for a given $b^2$, the kinetic energy of the ghost K-essence field increases with increasing scale factor. This difference may come back to this fact that the K-essence field selected by [12] is a purely kinetic model in which the action (27) is independent of $\phi$. This yields the energy density and pressure of a purely kinetic K-essence which are different from those considered in equations (28) and (29).
3.3. Ghost dilaton model

The dilaton scalar field model is also an interesting attempt to explain the origin of DE using string theory. The pressure and energy density of the dilaton scalar field model are given by \[ p_D = -\chi + ce^{\lambda \phi} \chi^2, \] \[ \rho_D = -\chi + 3ce^{\lambda \phi} \chi^2, \] where \( c \) and \( \lambda \) are positive constants and \( \chi = \phi^2 / 2 \). This is motivated by dilatonic higher order corrections to the tree-level action in low-energy effective string theory. The EoS parameter of the dilaton scalar field takes the form

\[ \omega_D = \frac{p_D}{\rho_D} = \frac{c e^{\lambda \phi} \chi - 1}{3c e^{\lambda \phi} \chi - 1}. \]
Figure 12. The evolution of the ghost K-essence kinetic energy \( \chi = \dot{\phi}^2/2 \), equation (31), for different coupling constants \( b^2 \) with \( \epsilon = 0 \). Auxiliary parameters as in figure 4.

Figure 13. The same as figure 12 for different viscosity constants \( \epsilon \) with \( b^2 = 0 \). Auxiliary parameters as in figure 4.

Equating (35) with (16) we find the solution

\[
 ce^{i\phi} \chi = \frac{3 - \frac{\Omega_\Lambda}{3} + 2b^2 \left( \frac{1+\Omega_\Lambda}{\Omega_D} \right) - \Omega_D + 3\epsilon (\Omega_D - 2)}{5 - \Omega_\Lambda + 6b^2 \left( \frac{1+\Omega_\Lambda}{\Omega_D} \right) - \Omega_D + 9\epsilon (\Omega_D - 2)}; \tag{36}
\]

then using \( \chi = \dot{\phi}^2/2 \), we obtain

\[
 e^{\dot{i}\phi} \dot{\phi} = \sqrt{\frac{2}{c}} \left( \frac{3 - \frac{\Omega_\Lambda}{3} + 2b^2 \left( \frac{1+\Omega_\Lambda}{\Omega_D} \right) - \Omega_D + 3\epsilon (\Omega_D - 2)}{5 - \Omega_\Lambda + 6b^2 \left( \frac{1+\Omega_\Lambda}{\Omega_D} \right) - \Omega_D + 9\epsilon (\Omega_D - 2)} \right)^{1/2}. \tag{37}
\]
Integrating with respect to $a$, we obtain

$$
\phi(a) = \frac{2}{\lambda} \ln \left\{ \frac{\lambda}{e^{\lambda(1)}} + \frac{3M_2^2 \lambda}{2\alpha \sqrt{c}} \right\}
\times \int_1^a \Omega_D \left[ \frac{6 - 2(\Omega_D - 2) 2b^2}{5 - \Omega_D + 6b^2 \left( \Omega_D - 2 \right)}\right]^{1/2} \frac{da}{a} \right\}. \quad (38)
$$

The evolution of the ghost dilaton scalar field (38) for different $b^2$ and $\epsilon$ is displayed in figures 14 and 15, respectively. The figures present that: (i) for a given $b^2$ or $\epsilon$, $\phi(a)$ increases with increasing scale factor and (ii) for a given scale factor, $\phi(a)$ decreases and increases with increasing $b^2$ and $\epsilon$, respectively.

Figure 14. The evolution of the ghost dilaton scalar field, equation (38), for different coupling constants $b^2$ with $\epsilon = 0$. Auxiliary parameters are $\Omega_{D_0} = 0.72$, $\Omega_{k_0} = 0.01$, $\phi(1) = 0$ and $\frac{3M_2^2}{2\alpha \sqrt{c}} = 1$.

Figure 15. The same as figure 14 for different viscosity constants $\epsilon$ with $b^2 = 0$. Auxiliary parameters as in figure 14.
With the help of equation (36) we plot the evolution of the ghost dilaton kinetic energy for different $b^2$ and $\epsilon$ in figures 16 and 17, respectively. The figures show that for a given $b^2$ or $\epsilon$, the kinetic energy of the ghost dilaton field like the tachyon and K-essence models decreases with increasing scale factor.

4. Conclusions

Here, we investigated the interacting viscous GDE model in the framework of standard FRW cosmology. We considered a spatially non-flat FRW universe filled with GDE and DM. We obtained the evolution of the fractional energy density and EoS parameters of the interacting viscous GDE model throughout the history of the universe. Furthermore, we reconstructed both the dynamics and potential of the tachyon, K-essence and dilaton scalar field models
of DE according to the evolutionary behavior of the interacting viscous GDE model. Our numerical results show the following.

(i) The evolution of the interacting viscous GDE density parameter $\Omega_D$ is independent of viscosity constant $\epsilon$. But for a given coupling constant $b^2$, $\Omega_D$ increases with increasing scale factor. Also at early and late times, $\Omega_D$ increases and decreases, respectively, with increasing $b^2$.

(ii) The EoS parameter $\omega_D$ of the GDE model in the absence of viscosity can cross the phantom divide line ($\omega_D < -1$) at the present provided $b^2 > 0$.1 which is compatible with the observations. Also, in the absence of viscosity for a given coupling constant $b^2$, $\omega_D$ increases and decreases at early and late times, respectively. Moreover, in the absence of interaction for a given viscosity constant $\epsilon$, $\omega_D$ decreases when the scale factor increases. For a given scale factor, $\omega_D$ increases with increasing $\epsilon$.

(iii) The ghost tachyon scalar field for a given $b^2$ or $\epsilon$ increases with increasing scale factor. Also for a given scale factor, it decreases and increases with increasing $b^2$ and $\epsilon$, respectively. For a given $b^2$ or $\epsilon$, the ghost tachyon kinetic energy $\chi(\phi)$ and potential $V(\phi)$ decrease with increasing scale factor and scalar field, respectively. For a given scale factor, $\chi(\phi)$ decreases and increases and decreases with increasing $b^2$ and $\epsilon$, respectively. For a given scalar field, $\chi(\phi)$ decreases and increases and decreases with increasing $b^2$ and $\epsilon$, respectively.

(iv) The ghost K-essence scalar field for a given $b^2$ or $\epsilon$ increases with increasing scale factor. But its kinetic energy decreases. For a given scale factor, the ghost K-essence scalar field increases and decreases with increasing $b^2$ and $\epsilon$, respectively. This behavior also holds for the kinetic energy of the ghost K-essence model.

(v) The ghost dilaton scalar field and its corresponding kinetic energy for a given $b^2$ or $\epsilon$ behave like the ghost K-essence model.

The points mentioned above illustrate that the interaction and viscosity have opposite effects on the dynamics of ghost tachyon, K-essence and dilaton scalar field models of DE.

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