Appraisal of excess Kurtosis through outlier-modified GARCH-type models

Emmanuel Alphonsus Akpan, Kazeem Etitayo Lasisi, Imoh Udo Moffat, and Ubon Akpan Abasiekwere

Department of Mathematical Science, Abubakar Tafawa Balewa University, Bauchi, Nigeria; Department of Basic Sciences, Federal School of Medical Laboratory Technology (Science), Jos, Plateau State, Nigeria; Department of Statistics, University of Uyo, Uyo, Akwa Ibom State, Nigeria; Department of Mathematics, University of Uyo, Uyo, Akwa Ibom State, Nigeria

ABSTRACT
The aim of this paper is to appraise if there is any improvement subtracting the effects of outliers from existing heteroscedastic models and whether this improvement makes difference with the existing models in achieving efficiency in capturing excess kurtosis in the returns series. The study employed both existing and outlier modified autoregressive conditional heteroscedastic (ARCH), generalized autoregressive conditional heteroscedastic (GARCH), exponential GARCH (EGARCH), Glosten, Jagannathan and Runkle GARCH (GJR-GARCH) models with respect to normal and student-t distributions to assess the portion of excess kurtosis of the returns series expressed compare to the theoretical value of kurtosis. The data applied were the share prices of Union bank of Nigeria and Unity bank from January 3, 2006 to November 24, 2016, comprising 2690 observations and were obtained from Nigerian Stock Exchange. The results obtained revealed that the Outlier Modified GARCH-type models chosen were adequate and sufficiently reducing the value of excess kurtosis in close proximity to the theoretical value. Therefore, the modification of existing GARCH-type models by subtracting the effects of outliers seems to show a substantive improvement in the portion of excess kurtosis captured and thus proves that the Outlier Modified GARCH-type models make difference with the existing ones.

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1. Introduction
It is a pertinent remark that one of the major setbacks to linear stationary models when applying to financial data (returns series) is their failure to account for changing variance. In other words, whenever the assumption of constant variance is violated, heteroscedasticity (changing variance conditional on the past information) has occurred, implying that the conditional distribution of the dependent variable has different degree of variability at different level. The relationship between the occurrence of heteroscedasticity in financial data and the violation of assumption of constant variance in linear time series has given birth to an extensive research area for professionals in Statistics, Economics and Finance.
The generalized autoregressive conditional heteroscedastic (GARCH-type) models were introduced to account for heteroscedasticity, a phenomenon which occurs as a result of violation of assumption of constant variance in time series. The GARCH-type models are further divided into symmetric and asymmetric. The symmetric GARCH models (for example ARCH and GARCH) rely on modeling the conditional variance as a linear function of squared past residuals. The strength of this specification is in allowing the conditional variance to depend only on the modulus of the past variables (past positive and negative innovations have the same effect on the current conditional variance). One important behavior not handled by GARCH model is the leverage effect which occurs when an unexpected drop in price (bad news) increases predictable volatility more than an unexpected increase in price (good news) of similar magnitude (Engle and Ng 1993; Francq and Zakoian 2010). The asymmetric specifications (for example EGARCH and GJR-GARCH) allow for the signs of the innovations (returns) to have impact on the volatility apart from magnitude (see also Akpan et al. 2019).

The GARCH-type models are commonly specified under the assumption that the error follows a normal distribution. But the assumption of normal distribution always appears to be insufficient in accommodating some characterizations of financial data especially fat-tailedness, which is due to excess kurtosis (Moffat and Akpan 2018). Kurtosis is purportedly, the measure of the degree of peakedness of distribution of real random variables while kurtosis coefficient is defined as the ratio of the fourth-order moment, which is assumed to exist, to the squared second-order moment. This coefficient is equal to 3 for a normal distribution and also serves as a gauge for other distributions (Francq and Zakoian 2010). Thus, distributions with heavy-tail probabilities compared to that of the normal are said to be heavy-tailed. If a distribution of returns has more returns clustered around the mean, it is referred to as leptokurtic or highly peaked, which leads to heteroscedasticity (changing variance). It is this feature of stock returns that provides a more practical and expedient reason for entertaining GARCH models (Franses and van Dijk 2003). GARCH-type models have received widest attention of application in modeling heteroscedasticity in financial time series and the studies of Caporin and Costola (2019), Opschoor et al. (2018), Siddiqui and Narula (2016), Huskaj and Larsson (2016), Feng and Shi (2017), Adu, Alagidede, and Karimu (2015), Tiopath and Gil-Alana (2015), Islam (2014), Zhang and Lin (2012), Escanciano(2010), Gil-Alana (2010), Jianhong and Lixing (2009), Maasoumi and McAleer (2008), So et al. (2008), He, Teräsvirta, and Malmsten (2002), Karanasos (2001), and Bollerslev and Mikkelsen (1996) applied GARCH-type models in modeling heteroscedasticity. Again, the student-t distribution was traditionally specified to remedy the weakness of the normal distribution in accommodating the heavy-tailed property, yet it also failed in many applications to account for excess kurtosis yet, so inadequate for capturing the fat-tailedness (Feng and Shi 2017; Franses and Ghijssels 1999). Meanwhile, the studies of Jiang, Song, and Xiong (2016), Muler and Yohai (2008), Duchesne (2004), and Jiang, Zhao, and Hui (2001) applied different robust methods without necessarily specifying any distribution targeting at overcoming the problem associated with the choice of distribution. On the other hand, the studies of Hotta and Trucios (2018), Kamranfar, Chinipardaz, and Mansouri (2017) and Park (2002) considered different outlier robust methods.

Moreover, it is necessary and worthy of emphasis that the presence of outliers is responsible for the existence of excess kurtosis in financial data. According to Alih and Ong (2015), the presence of outliers (an outlier is an observation that diverges from an overall pattern on a sample). If the data is normally distributed, a single observation may be classified as an outlier if it falls outside of 3 of standard deviation (± 3σ, about 99.8 percent of normally distributed data fall within this range) is a very common attribute in time series data, noting that outliers in homoscedastic model make the model heteroscedastic and in addition, outliers distort the diagnostic tools for heteroscedasticity such that it may not be correctly identified. Similarly, Carnero, Pena, and Ruiz (2007) further affirmed and maintained that outliers affect the identification of conditional heteroscedasticity and the estimation of GARCH models. Also, it is evident in Rana (2010) that
outliers have great impact on the existing heteroscedasticity tests and the estimators of heteroscedastic model and such impact of outliers on the diagnostic tools for heteroscedasticity is well defined in Van Dijk, Franses, and Lucas (1999). They showed that both the asymptotic size and power properties of Lagrange (LM) test for ARCH/GARCH are adversely affected by outliers, particularly, additive outliers. Grossi and Laurini (2004) found that order of identification, t-statistics and corresponding p-values of the estimates of GARCH parameters are affected by outliers in an unexpected manner. Moreover, prior studies involving GARCH-type modeling in the presence of outliers include Urooj and Asghar (2017), Grane and Veiga (2014), Grane and Veiga (2010), Hotta and Tsay (2012), Van Dijk, Franses, and Lucas (1999), Bilen and Huzurbazar (2002), Carnero, Pena, and Ruiz (2001), Franses and Ghijsels (1999), and Akpan et al. (2019). So far, we identified that previous studies have tried in one way or the other to improve the forecasting ability of the GARCH-type models but failed to appraise the ability of the models to completely capture the heavy-tailedness (Excess kurtosis) of the data comparable to the theoretical value of kurtosis. Hence, this study seeks to improve (by subtracting the effects of outliers from existing models to achieve efficiency) the work of Akpan et al. (2019) that modeled the efficiency of heteroscedastic models by adjusted for the effects of outliers in the returns series of Nigerian Banks stocks without the corresponding and necessary modifications on the existing GARCH-type models to depict such adjustment.

2. Materials and methods

2.1. Returns

The return series $R_t$ can be obtained given that $P_t$ is the price of a unit share at time, $t$, and $P_{t-1}$ is the share price at time $t-1$ as follows:

$$R_t = \nabla \ln P_t = (1 - B) \ln P_t = \ln P_t - \ln P_{t-1}$$

(1)

The $R_t$ in equation (1) is regarded as a transformed series of the share price, $P_t$ meant to attain stationarity, that is, both mean and variance of the series are stable (Akpan and Moffat 2019; Moffat and Akpan 2018; Akpan and Moffat 2017). The letter $B$ is the backshift operator.

2.2. Autoregressive integrated moving average (ARIMA) model

According to Box, Jenkins, and Reinsel (2008) the autoregressive integrated moving average process is the general form of model used to describe time series

$$\varphi(B) = \phi(B) \nabla^d X_t = \theta(B) \epsilon_t$$

(2)

where $\varphi(B)$ is the nonstationary autoregressive operator with $d$ of the roots of $\varphi(B) = 0$ equal to unity, that is, $d$ unit roots. $\phi(B)$ is a stationary autoregressive operator and $\theta(B)$ is a moving average operator.

2.3. Heteroscedastic models

Heteroscedastic models are hybridized of both mean and variance equations. The mean equation is represented the ARIMA Model as shown in equation (3),

$$R_t = \mu_t + a_t,$$

(3)

where $\mu_t = \phi_0 + \sum_{j=1}^{p} \phi_j R_{t-j} + \sum_{i=1}^{q} \theta_i a_{t-i}$

$$a'_t = \sigma_t \epsilon_t,$$

(4)
where \( e_t \) is a sequence of independent and identically distributed (i.i.d.) random variables with mean zero, that is \( E(e_t) = 0 \) and variance, 1, while \( a'_{t-i} \) is the standardized residual term and follows ARCH(q), GARCH (q, p), EGARCH(q,p) and GJR-GARCH(q,p) models in (5–7) and (8), respectively.

### 2.3.1. Autoregressive conditional heteroscedastic (ARCH) model

ARCH(q) model provides a methodical structure for modeling hetroscedasticity and specified as

\[
\sigma^2_t = \omega + \sum_{i=1}^{q} \alpha_i \sigma^2_{t-i} + \sum_{j=1}^{p} \beta_j \sigma^2_{t-j}.
\]

(For more details, see Engle 1982; Tsay 2010).

### 2.3.2. Generalized autoregressive conditional heteroscedastic (GARCH) model

GARCH(q,p) model provides an alternative to ARCH model for the purpose of achieving parsimony and overcoming the weakness of ARCH model that requires many parameters to completely capture the heteroscedasticity (Bollerslev 1986). The model is defined as follows:

\[
\sigma^2_t = \omega + \sum_{i=1}^{q} \alpha_i a^2_{t-i} + \sum_{j=1}^{p} \beta_j \sigma^2_{t-j}.
\]

### 2.3.3. Exponential generalized autoregressive conditional heteroscedastic (EGARCH) model

EGARCH(q,p) model applies the natural logarithm to ensure that the conditional variance is positive and thus overcome the requirement of parameter restrictions(Nelson 1991). The EGARCH (q, p) is defined as,

\[
\ln\sigma^2_t = \omega + \sum_{i=1}^{q} \alpha_i \left| \frac{a'_{t-i}}{\sigma^2_{t-i}} \right| + \sum_{k=1}^{r} \gamma_k \left( \frac{a'_{t-k}}{\sqrt{\sigma^2_{t-k}}} \right) + \sum_{j=1}^{p} \beta_j \ln\sigma^2_{t-j}.
\]

\( \gamma_k \) is the asymmetric coefficient.

### 2.3.4. Glosten, Jagannathan and Runkle (GJR-GARCH) model

The GJR GARCH (q, p) model (Glosten, Jagannathan, and Runkle 1993) is a variant, represented by

\[
\sigma^2_t = \omega + \sum_{i=1}^{q} \alpha_i a^2_{t-i} + \sum_{i=1}^{p} \eta_i I_{t-i} a^2_{t-i} + \sum_{j=1}^{p} \beta_j \sigma^2_{t-j}
\]

where \( I_{t-1} \) is an indicator for negative \( a_{t-i} \), that is,

\[
I_{t-1} = \begin{cases} 
0 & \text{if } a'_{t-i} < 0, \\
1 & \text{if } a'_{t-i} \geq 0,
\end{cases}
\]

and \( \alpha_i, \gamma_i, \) and \( \beta_j \) are nonnegative parameters satisfying conditions similar to those of GARCH models. Also the introduction of indicator parameter of leverage effect, \( I_{t-1} \) in the model accommodates the leverage effect, since it is supposed that the effect of \( a^2_{t-i} \) on the conditional variance \( \sigma^2_t \) is different accordingly to the sign of \( a'_{t-i} \) (For more details, see Francq and Zakoian 2010; Tsay 2010).
2.4. Proposed modification

Different findings have indicated that integration of different models can be an effective way of improving upon their performances especially when the models in combination are quite different (Zhang 2003). However, efficiency of such hybridized model could still be threatened especially in the presence of outliers. Take for instance, the generalized autoregressive conditional heteroscedastic (GARCH-type) models that were introduced to account for heteroscedasticity and were specified based on the normal distribution for the innovations yet could not capture the heavy-tailed characterizations. Similarly, the student-t distribution which was traditionally stated to remedy the weakness of the normal distribution in accommodating the heavy-tailed property also failed in many applications to account for excess kurtosis and thus, the resulting estimates of GARCH models are not efficient. In addition, this heavy-tailed property indicates the presence of excess kurtosis which in turn is a measure of outliers. It is against this background that the following proposal is done to improve and adjust for outliers to obtain a more efficient model than the existing one.

\[ R_t - \mu_t - \sum_{k=1}^{m} \tau_k V_k(B) t_{t}^{(T)} = a_t^*, \]  

\[ a_t^{**} = \sigma_t e_t, \]  

where \( e_t \) is a sequence of independent and identically distributed (i.i.d.) random variables with mean zero, that is \( \text{E}(e_t) = 0 \) and variance 1, \( a_t^* \) is outlier adjusted residual term, \( \sum_{k=1}^{m} \tau_k V_k(B) t_{t}^{(T)} \) in equation (9) is an expression for multiple outliers introduced into the ARIMA model (mean equation), where \( V_k(B) = 1 \) for an additive outlier, \( V_k(B) = \frac{\delta(B)}{\sigma(B)} \) for an innovation outlier at \( t = T_k \), \( V_k(B) = (1 - B)^{-1} \) for a level shift, \( V_k(B) = (1 - \delta B)^{-1} \) for a temporary change, and \( \tau \) is the size of outlier (For more details on the types of outliers and estimation of the outliers effects (see Akpan et al. 2019; Moffat and Akpan 2018; Moffat and Akpan 2017; Sanchez and Pena 2003; Box, Jenkins, and Reinsel 2008; Wei 2006; Chen and Liu 1993; Chang, Tiao, and Chen 1988).

Moreover, in financial time series, the residual series is assumed to be uncorrelated with its own past, so additive, innovative, temporary change and level shift outliers coincide, and where both the mean and variance equations evolves together. \( a_t^{**} \) is the outlier free standardized residual which now enters into the variance equation. Therefore, the idea behind this modification is to show whether there is an improvement after subtracting the effects of the outliers and if it thus makes difference with the existing model. Hence, \( a_t^{**} \) follows outlier modified; ARCH\(q\), GARCH\(q,p\), EGARCH\(q,p\) and GJR-GARCH\(q,p\) models in (11–13) and (14), respectively.

2.4.1. Outlier modified ARCH model

\[ \sigma_t^{*^2} = \omega + x_1 a_{t-1}^{*^2} + \ldots + x_q a_{t-q}^{*^2}, \]  

(11)

2.4.2. Outlier modified GARCH model

\[ \sigma_t^{*^2} = \omega + \sum_{i=1}^{q} x_i a_{t-i}^{*^2} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{*^2}. \]  

(12)

2.4.3. Outlier modified EGARCH model

\[ \ln(\sigma_t^{*^2}) = \omega + \sum_{i=1}^{q} x_i \left| \frac{a_{t-i}^{*}}{\sqrt{\sigma_{t-i}^{*^2}}} \right| + \sum_{k=1}^{r} \gamma_k \left( \frac{a_{t-k}^{*}}{\sqrt{\sigma_{t-k}^{*^2}}} \right) + \sum_{j=1}^{p} \beta_j \ln(\sigma_{t-j}^{*^2}). \]  

(13)

Alternatively, EGARCH \(q, p\) model with respect to student-t distribution can be represented by
\[
\ln \sigma_t^2 = \omega + \sum_{k=1}^{r} \gamma_k a_{t-k}^v + \sum_{i=1}^{q} \alpha_i \left( |a_{t-i}^v| - \frac{2 \sqrt{v-2} \Gamma(\nu+1)/2}{(\nu-1)\Gamma(\nu/2)/\pi} \right) + \sum_{j=1}^{p} \beta_j \ln \sigma_t^2, \tag{14}
\]

where \( \gamma_k \) is the asymmetric coefficient.

### 2.4.4. Outlier modified GJR-GARCH model

\[
\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i a_{t-i}^v + \sum_{i=1}^{p} \gamma_i I_{t-i} a_{t-i}^v + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{15}
\]

where \( I_{t-1} \) is an indicator for negative \( a_{t-i}^v \), that is,

\[
I_{t-1} = \begin{cases} 
0 & \text{if } a_{t-i}^v < 0, \\
1 & \text{if } a_{t-i}^v \geq 0.
\end{cases}
\]

Going forward, this particular modification is based on the assumption that returns series are uncorrelated (no serial correlation). Serial correlations (a relationship between a variable and its lagged-value over a period of time) tend to exist in most financial series and these serial correlations are believed to be introduced by those in the time-varying heteroscedasticity process (Zhang, Wong, and Li 2016; Conrad and Karanasos 2015; Zhang et al. 2013; Tsay 2010; Dias 2017; Hong 1991).

### 2.5. Procedure for detecting outliers

The procedure for detecting outliers is based on the following steps:

Step 1. Derive initial estimates of the model parameters.

Step 2. Given the parameter values, for any \( t \) and for each type of outliers, assume that an outlier has occurred at time \( T \), and estimate its amplitude. If the largest absolute estimated amplitude is significant, that is, larger than an a priori fixed sensitivity level, usually 3.5 if \( T \leq 50 \) or \( \geq 4 \) if \( T \geq 450 \) times its estimated standard error which is called critical value, \( CV \), identify an outlier of that type at that time; otherwise stop (López-de-Lacalle 2019; Kaya 2010). Moreover, where the critical value of 4 is not sufficient, critical value of 5 is considered in this study.

Step 3. Remove the effects of the identified outlier by subtracting its estimated amplitude from \( R_t \) (and also correcting all subsequent observations according to the estimated model in case of innovational outlier).

Step 4. Estimate again the model parameters on the corrected series, and iterate step 2.

### 3. Results and discussion

Data collection was based on secondary source as documented in the records of Nigerian Stock Exchange. The documented data on the daily closing share prices of the sampled banks (Union bank and Unity bank) from January 3, 2006 to November 24, 2016 were purchased from the Nigerian Stock Exchange and delivered through contactcentre@nigerianstockexchange.com. The dataset used can be assessed as supplementary data. The time plots of share price series, ACF and PACF of share price series and time plots of the return series for all the banks were generated by Gretl version 1.10.1. The main analyses were carried out using R-project version 3.4.0.
3.1. Time plots

Figures 1 and 2 represent the share price series for the two banks. It could be observed that the share prices do not fluctuate around a common mean, which clearly indicate the presence of a stochastic trend in the share prices. This is also an evidence of non-stationarity of the series.

Since the share price series is found to be non-stationary, the first difference of the natural logarithm of the series is taken to obtain a stationary series. The inclusion of the log transformation is to stabilize the variance. Figures 3 and 4 show that the return series appear to be stationary and they suggest that heteroscedasticity is quite evident in the series.

3.2. Modeling ARIMA processes of return series

From Table 1, ARIMA(1,1,0) model was each selected for the return series of the banks considered based on the grounds of significance of the parameters and minimum AIC.

Evidence from Ljung-Box Q-statistics in Table 2 showed that each of ARIMA(1,1,0) model was adequate at 5% level of significance. That is, the hypothesis of no autocorrelation was not rejected.

3.3. Identification of ARCH effects in the residuals of ARIMA models fitted to the return series of the banks

Evidence from Portmanteau-Q (PQ) statistics in Table 3 showed that heteroscedasticity does not exist given that the null hypothesis of no autocorrelation is not rejected at 5% significance level which is due to the presence of outliers. Meanwhile, Lagrange-Multiplier (LM) test statistics in Table 3 showed that heteroscedasticity exists given that the null hypotheses of no autocorrelation and no ARCH effect are rejected at 5% significance level.
3.4. Modeling ARIMA-GARCH-type processes of the return series of the banks

The following hybridized ARIMA-GARCH-type models with respect to both normal (norm) and student-t (std) distributions were considered and selected on the grounds of smallest information criteria (Table 4).

The models were found to be adequate at 5% level of significance according to evidence provided by weighted Ljung-Box Q-statistics on standardized residuals, weighted Ljung-Box Q-statistics on standardized squared residuals, and weighted Lagrange-Multiplier statistics [see Table 5]. This implies that the hypotheses of no autocorrelation and no remaining ARCH effects were not rejected.

3.5. Identification of outliers in the residual series of ARIMA models fitted to return series of the banks

Several different outliers were identified to have contaminated the residual series of ARIMA models using the critical value (a priori fixed sensitive level), $CV = 5$ on the condition that the sample size is very large. An observation is regarded as outlier if its corresponding t-statistic is greater than 5 times the estimated standard deviation in absolute value [see Tables 6 and 7].

Moreover, in financial time series, it is assumed that the error is uncorrelated with its past values, and then all the outliers are classified as innovation outliers with a unified effect.

Outlier adjusted series was obtained by removing the effects of outliers from the return series.
Figure 3. Return series of Union Bank share prices.

Figure 4. Return series of Unity Bank share prices.
Table 1. ARIMA models for return series.

| Bank   | Model       | Parameter | Estimate | s.e  | z-ratio | p-value    | AIC     |
|--------|-------------|-----------|----------|------|---------|------------|---------|
| Union  | ARIMA(1,1,0)| \( \phi_1 \) | 0.1014   | 0.0192| 5.2866  | 1.246e-07 | -9132.2 |
| Unity  | ARIMA(1,1,0)| \( \phi_1 \) | 0.0786   | 0.0192| 4.0895  | 4.3230e-05| -7588.08|

Table 2. Ljung-Box Test on ARIMA models for return series.

| Bank   | Lag | Ljung-Box Q-statistics | p-value |
|--------|-----|------------------------|---------|
| Union  | 1   | 0.0133                 | 0.9082  |
|        | 4   | 2.3753                 | 0.6671  |
|        | 8   | 4.318                  | 0.8274  |
|        | 24  | 7.9309                 | 0.9991  |
| Unity  | 1   | 0.0119                 | 0.9133  |
|        | 4   | 2.0207                 | 0.7320  |
|        | 8   | 6.6927                 | 0.5701  |
|        | 24  | 17.6390                | 0.8202  |

Table 3. ARCH heteroscedasticity test for residuals of ARIMA models.

| Bank   | Lag | Portmanteau-Q Test | Lagrange -Multiplier Test |
|--------|-----|--------------------|---------------------------|
|        |     | PQ Value            | p-value                   | LM Value            | p-value   |
| Union  | 4   | 0.736               | 0.947                     | 420097              | 0.0000    |
|        | 8   | 0.743               | 0.999                     | 208864              | 0.0000    |
|        | 12  | 0.753               | 1.000                     | 138360              | 0.0000    |
|        | 16  | 0.760               | 1.000                     | 103158              | 0.0000    |
|        | 20  | 0.766               | 1.000                     | 82035               | 0.0000    |
|        | 24  | 0.777               | 1.000                     | 67942               | 0.0000    |
| Unity  | 4   | 0.0820              | 0.9990                    | 5897                | 0.0000    |
|        | 8   | 0.0881              | 1.0000                    | 2928                | 0.0000    |
|        | 12  | 0.0933              | 1.0000                    | 1943                | 0.0000    |
|        | 16  | 0.0995              | 1.0000                    | 1448                | 0.0000    |
|        | 20  | 0.1082              | 1.0000                    | 1150                | 0.0000    |
|        | 24  | 0.1163              | 1.0000                    | 9514                | 0.0000    |

Table 4. Output of ARIMA-GARCH-type models of returns series.

| Bank   | Model       | Parameter | Estimate | s.e  | t-ratio | p-value    | AIC     | BIC     | HQIC    |
|--------|-------------|-----------|----------|------|---------|------------|---------|---------|---------|
| Union  | ARIMA(0,1,1)- GARCH(2,0)-std | \( \mu \) | \(-7.22e^{-4}\) | 3.34e^{-4} | \(-2.1632\) | 0.0305 | -4.3974 | -4.3843 | -4.3927 |
|        |             | \( \phi_1 \) | 0.0622   | 0.0238| 2.6181  | 8.84e^{-2} |         |         |         |
|        |             | \( \omega \) | 2.17e^{-4} | 2.8e^{-5} | 7.6906 | 0.0000 |         |         |         |
|        |             | \( \chi_1 \) | 0.5907   | 0.0658| 8.9797  | 0.0000  |         |         |         |
|        |             | \( \chi_2 \) | 0.4083   | 0.0529| 7.7051  | 0.0000  |         |         |         |
| Unity  | ARIMA(0,1,1)- GARCH(1)-norm | \( \mu \) | \(1.38e^{-4}\) | 1.9e^{-5} | 7.1606 | 0.0000 | -4.8711 | -4.8601 | -4.8671 |
|        |             | \( \theta_1 \) | 0.1009   | 0.0236| 4.2767  | 1.9e^{-5} |         |         |         |
|        |             | \( \omega \) | 4.0e^{-6} | 0.0000 | 115.7800 | 0.0000 |         |         |         |
|        |             | \( \chi_1 \) | 0.2368   | 0.0117| 20.2352 | 0.0000  |         |         |         |
|        |             | \( \beta_1 \) | 0.7622   | 7.927e^{-1} | 96.1509 | 0.0000 |         |         |         |

Table 5. Diagnostic Checking for ARIMA-GARCH-type models of returns series.

| Bank   | Model       | Standardized Residuals | Standardized Squared Residuals |
|--------|-------------|------------------------|------------------------------|
|        |             | Weighted LB p-value | Weighted LB p-value | Lagged ARCH-LM p-value | Lagged ARCH-LM p-value |
| Union  | ARIMA(0,1,1)- GARCH(2, 0)-std | 1 | 0.0615 | 0.8042 | 1 | 0.0008 | 0.9768 | 3 | 0.0008 | 0.9770 |
|        |             | 2 | 0.0726 | 1.0000 | 5 | 0.0025 | 1.0000 | 5 | 0.0020 | 1.0000 |
|        |             | 5 | 0.1900 | 0.9998 | 9 | 0.0040 | 1.0000 | 7 | 0.0027 | 1.0000 |
| Unity  | ARIMA(0,1,1)- GARCH (1)-norm | 1 | 0.0031 | 0.9557 | 1 | 0.0008 | 0.9776 | 3 | 0.0008 | 0.9776 |
|        |             | 2 | 0.0031 | 1.0000 | 5 | 0.0024 | 1.0000 | 5 | 0.0019 | 1.0000 |
|        |             | 5 | 0.0035 | 1.0000 | 9 | 0.0039 | 1.0000 | 7 | 0.0028 | 1.0000 |

*LB = Ljung-Box; LM = Lagrange-Multiplier.*
Table 6. Detected outliers in the residual series of ARIMA (1, 1, 0) model of union bank.

| Type | Observation index | Location | Estimate of outlier | T-statistic |
|------|-------------------|----------|---------------------|-------------|
| IO   | 458               | 16/11/2007| -0.20259320         | -9.867965   |
| IO   | 1472              | 23/12/2011| -0.22031597         | -10.731210  |
| IO   | 1831              | 07/06/2013| 0.10533439          | 5.130683    |
| IO   | 1843              | 25/06/2013| 0.10950627          | 5.158511    |
| AO   | 150               | 15/08/2006| 0.13856541          | -6.783874   |
| AO   | 705               | 14/11/2008| 0.20066454          | -9.833910   |
| AO   | 1471              | 22/12/2011| 1.67935140          | 82.217553   |
| AO   | 1830              | 06/06/2013| 0.11483241          | -5.621956   |
| AO   | 1842              | 24/06/2013| -0.10581300         | -5.123098   |
| AO   | 1984              | 21/01/2014| 0.10581300          | 5.180384    |
| AO   | 1994              | 04/02/2014| 0.16239480          | 7.950512    |
| TC   | 691               | 27/10/2008| 0.08071046          | -5.129738   |
| TC   | 901               | 31/08/2009| 0.08274861          | -5.259278   |
| TC   | 1470              | 22/12/2011| 0.03378545          | 33.925958   |
| TC   | 1523              | 09/03/2012| 0.08218825          | -5.232663   |
| TC   | 1541              | 04/04/2012| 0.07869209          | 5.001456    |
| TC   | 1824              | 28/05/2013| 0.11353246          | 7.215815    |
| TC   | 2534              | 08/04/2016| 0.08059290          | -5.122266   |
| AO   | 1748              | 06/02/2013| 0.11923771          | 5.160464    |
| IO   | ¼                 | Innovation Outlier; AO = Additive Outlier; TC = Temporary Change |

Table 7. Detected outliers in the residual series of ARIMA (1, 1, 0) of unity bank.

| Type | Observation index | Location | Estimate of outlier | T-statistic |
|------|-------------------|----------|---------------------|-------------|
| IO   | 2293              | 20/04/2015| -0.180979695        | -7.444781   |
| AO   | 248               | 10/01/2007| 1.996612289         | 45.331893   |
| AO   | 1906              | 24/09/2013| -0.200532990        | -8.274566   |
| AO   | 2292              | 17/04/2015| 0.302585093         | 95.011264   |
| TC   | 247               | 09/01/2007| 0.365004553         | 19.903211   |
| TC   | 1736              | 18/01/2013| 0.107790532         | 5.876764    |
| TC   | 1745              | 01/02/2013| 0.112419182         | 6.130068    |
| TC   | 1753              | 13/02/2013| -0.118923561        | -6.484743   |
| TC   | 1762              | 26/02/2013| 0.091893800         | 5.010932    |
| TC   | 2291              | 16/04/2015| 0.758297010         | 41.348923   |
| TC   | 2298              | 27/04/2015| -0.142415961        | -7.765752   |
| TC   | 2304              | 06/04/2015| -0.098876918        | -5.391626   |
| TC   | 2446              | 30/11/2015| -0.093629262        | -5.105479   |
| TC   | 2458              | 16/12/2015| 0.112419118         | 6.130064    |
| TC   | 2460              | 18/12/2015| 0.104980801         | 5.724463    |
| TC   | 2467              | 04/01/2016| -0.106045605        | -5.782525   |
| TC   | 2469              | 06/01/2016| -0.119002493        | -6.489047   |
| IO   | 1905              | 23/09/2013| 0.127354627         | 5.132279    |
| AO   | 1904              | 20/09/2013| -0.163097022        | -6.592937   |
| LS   | 243               | 29/12/2006| -0.003141767        | -5.772181   |
| LS   | 251               | 15/01/2007| -0.002771753        | -5.084049   |
| LS   | 347               | 11/06/2007| -0.002837928        | -5.102009   |
| LS   | 520               | 19/06/2008| -0.003114010        | -5.387808   |
| LS   | 598               | 13/06/2008| -0.003027395        | -5.143002   |
| LS   | 613               | 04/07/2008| -0.003035068        | -5.137530   |
| LS   | 631               | 30/07/2008| -0.002988789        | -5.037234   |
| LS   | 635               | 05/08/2008| -0.003001055        | -5.052994   |
| LS   | 2286              | 09/04/2015| -0.008202488        | -6.130741   |
| TC   | 1901              | 17/09/2013| 0.097493034         | 5.208012    |
| TC   | 2477              | 18/01/2016| 0.096324642         | 5.145597    |
| LS   | 607               | 26/06/2008| 0.022239276         | 28.623733   |
| AO   | 1336              | 09/06/2011| -0.128187865        | -5.110079   |
| AO   | 1872              | 05/08/2013| -0.144642972        | -5.766045   |

IO = Innovation Outlier; AO = Additive Outlier; TC = Temporary Change; LS = Level Shift.
3.6. Modeling the outlier-modified ARIMA-GARCH-type processes of the return series

The following hybridized outlier-modified ARIMA-GARCH-type models with respect to both normal (norm) and student-t (std) distributions were considered and selected on the grounds of smallest information criteria (Table 8).

The Outlier Modified models were found to be adequate at 5% level of significance according to evidence provided by weighted Ljung-Box Q-statistics on standardized residuals, weighted Ljung-Box Q-statistics on standardized squared residuals, and weighted Lagrange-Multiplier statistics (see Table 9). That is to say, the hypotheses of no autocorrelation and no remaining ARCH effect are not rejected.

### Table 8. Output of Outlier Modified ARIMA-GARCH-type models of adjusted returns series.

| Bank       | Model            | Parameter | Estimate | s.e  | t-ratio | p-value | AIC          | BIC          | HQIC         |
|------------|------------------|-----------|----------|------|---------|---------|--------------|--------------|--------------|
| Union      | Outlier Modified | $\mu$     | $-4.10e^{-4}$ | $9.4e^{-5}$ | $-4.3524$ | $1.3e^{-5}$ | $-4.5619$ | $-4.5509$ | $-4.5579$   |
|            | ARIMA(1,1,0)-GARCH(1,1)-norm | $\phi_1$ | $-0.0627$ | $0.0219$ | $-2.8559$ | 0.0043 |
|            |                  | $\omega$  | $5.0e^{-6}$ | $0.0000$ | $22.4824$ | 0.0000 |
|            |                  | $\alpha_1$| $0.1521$ | $0.0102$ | $14.9393$ | 0.0000 |
|            |                  | $\beta_1$ | $0.8469$ | $7.864e^{-3}$ | $106.6974$ | 0.0000 |
| Unity      | Outlier Modified | $\mu$     | $-2.284e^{-3}$ | $1.0e^{-5}$ | $-220.0970$ | 0.0000 | $-4.2296$ | $-4.2406$ | $-4.2336$   |
|            | ARIMA(1,1,0)-GJR-GARCH(1,0)-norm | $\phi_1$ | $0.0559$ | $2.66e^{-4}$ | $209.9780$ | 0.0000 |
|            |                  | $\omega$  | $1.0e^{-6}$ | $0.0000$ | $3.4940$ | 0.0005 |
|            |                  | $\alpha_1$| $0.8566$ | $3.888e^{-3}$ | $220.3020$ | 0.0000 |
|            |                  | $\gamma_1$| $0.2175$ | $0.0114$ | $19.0470$ | 0.0000 |

### Table 9. Diagnostic Checking for ARIMA-GARCH-type models of outlier adjusted returns series.

| Bank       | Model            | Standardized Residuals | Standardized Squared Residuals |
|------------|------------------|------------------------|--------------------------------|
|            |                  | Lag | Weighted LB | p-value | Lag | Weighted LB | p-value | Lag | Weighted ARCH–LM | p-value |
| Union      | Outlier Modified | 1   | 1.147       | 0.2841 | 1   | 3.333       | 0.0679 | 3   | 0.3721         | 0.5419 |
|            |                  | 2   | 1.764       | 0.3053 | 5   | 3.948       | 0.2606 | 5   | 0.4175         | 0.9077 |
|            |                  | 5   | 5.729       | 0.0586 | 9   | 5.867       | 0.3138 | 7   | 2.5696         | 0.5982 |
| Unity      | Outlier Modified | 1   | 0.0081      | 0.9283 | 1   | 0.3091      | 0.5782 | 2   | 0.0990         | 0.7530 |
|            |                  | 2   | 0.0158      | 1.0000 | 2   | 0.3587      | 0.7639 | 4   | 2.5450         | 0.3332 |
|            |                  | 5   | 4.3656      | 0.1730 | 5   | 2.3489      | 0.5386 | 6   | 2.8354         | 0.5386 |

LB = Ljung-Box; LM = Lagrange-Multiplier.

### Table 10. Portion of theoretical value of Kurtosis not captured by outiler modified GARCH-type models.

| Banks | Theoretical Value of Kurtosis | Existing GARCH-type Models | Number of Outliers Identified | Outlier Modified GARCH-type Models | Portion of Theoretical Value of Kurtosis not Captured by Outlier Modified GARCH-type Models (%) |
|-------|-------------------------------|-----------------------------|--------------------------------|-----------------------------------|--------------------------------------------------------------------------------------------------|
| Union | 3 ARIMA(1,1,0)-GARCH(2,0)-std | 755.4995 (19) Outliers ARIMA(1,1,0)-GARCH(1,1)-norm | 3.0891                          | -8.91                             |
| Unity | 3 ARIMA(1,1,0)-GARCH(1,1)-norm | 888.5032 (32) Outliers ARIMA(1,1,0)-GJR-GARCH(1,1)-norm | 3.2678                          | -26.78                            |

3.6. Modeling the outlier-modified ARIMA-GARCH-type processes of the return series

The following hybridized outlier-modified ARIMA-GARCH-type models with respect to both normal (norm) and student-t (std) distributions were considered and selected on the grounds of smallest information criteria (Table 8).

The Outlier Modified models were found to be adequate at 5% level of significance according to evidence provided by weighted Ljung-Box Q-statistics on standardized residuals, weighted Ljung-Box Q-statistics on standardized squared residuals, and weighted Lagrange-Multiplier statistics (see Table 9). That is to say, the hypotheses of no autocorrelation and no remaining ARCH effect are not rejected.
3.7. Comparison of Kurtosis value captured by the outlier modified GARCH-type models to the theoretical value of Kurtosis

The proximity of kurtosis value captured by the outlier modified GARCH-type model for each bank to the theoretical value of kurtosis in percentage is presented in Table 10. It is revealed that for Union bank and Unity bank, the respective values are 8.91% and 26.78%, which are above the theoretical value of kurtosis.

4. Conclusion

Outstandingly, our study revealed that there is an improvement after subtracting the effects of the outliers and it makes difference with the existing models. Hence, all the outliers modified models selected successfully provided the needed improvement on the work of Akpan et al. (2019) by efficiently capturing the excess kurtosis in proximity to the theoretical value of kurtosis in the returns series irrespective of the choice of the distribution of the innovations. Given that the modification in this work was based on the assumption that returns series are uncorrelated and the selected outlier modified models could not completely capture the excess kurtosis that is, arriving at exact 3, a value required for a normal distribution, showed that there is still room for improvement. One possible way of improving upon this study is to account for the existence of serial correlations which is believed to be introduced or caused by the presence of heteroscedasticity and thus forms the basis for further studies.

ORCID

Emmanuel Alphonsus Akpan  http://orcid.org/0000-0003-3809-0702
Kazeem Etitayo Lasisi  http://orcid.org/0000-0001-9306-5994
Imoh Udo Moffat  http://orcid.org/0000-0001-7478-9890
Ubon Akpan Abasiekwere  http://orcid.org/0000-0002-2339-3543

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