Performance analysis of $M$-ary OQAM/FBMC with impact of nonlinear distortion over compound Rician $K$-factor unshadowed/$\kappa-\mu$ shadowed fading channels

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1 | INTRODUCTION

Nowadays, offset QAM with filter-bank multicarrier (OQAM/FBMC) is being a good candidate modulation scheme for fifth-generation (5G) communication systems, which is generally known as OQAM/OFDM system. It has been proposed in [1, 2] as an alternate waveform solution to reduce limitations of spectral efficiency and synchronisation requirements in the cyclic prefix (CP)-OFDM for 3GPP applications. That made the OQAM/FBMC viewed as an attractive solution for wireless broadband systems [3, 4].

On the other hands, like to the others multicarrier communication systems, the OQAM/FBMC system generates a high peak-to-average power ratio at the transmitter, which makes the OQAM susceptible to nonlinear distortion (NLD) induced by the high-power amplifier (HPA) and have the same sensitivity of the OQAM/OFDM even when a perfect channel is perfectly equalised [5]. One of the possible solution to prevent the NLD caused by the HPA can be made by forcing the RF transmitting amplifier to operate in its linear area by using low back-off values. Unfortunately, this would reduce the amplifier efficiency and introduce additional interference between the subcarriers, especially in the satellite communication system [6].

Different works have been investigated the effects of NLD-HPA on the OFDM system and FBMC system. According to [7], a complete theoretical analysis of the OFDM system in the presence of NLD induced by three HPA models has been presented via the AWGN channel. The study presented in [8] used the findings presented in [7] for the transmission of multiple-input multiple-output (MIMO) diversity method. The effect of HPA on out-of-band spectral has been studied in [9, 10] for OFDM and OQAM/FBMC systems. The closed-form BER analysis presented in [11] has been used a full rank polynomial order of all coefficients of the approximated memoryless NLD for OFDM system and OQAM/FBMC system over multipath fading channel using numerical integration method. The closed-form BER expressions of $M$-ary (QAM or OQAM) for NLD-HPA-OFDM system over AWGN and multipath fading channel...
channel amplitude with Rayleigh distribution have been shown in [5] without numerical integration method.

In fact, the propagation of a communication signal over realistic wireless channels is a complex phenomenon characterised by small-scale (SC) fading induced by unshadowing effects and large-scale (LS) fading induced by shadowing effects [12]. So, realistic communication channels should be modelled as time-shared superimposed of unshadowing and shadowing fading distributions. Different works have been investigated the communication over fading channels. According to [13], multipath fading channels have been modelled by using Rician, Rayleigh and Nakagami-\( m \) distributions. While the effect of shadowing fading has been captured by using log-normal distribution in [14]. A closed-form of an average SER for QAM over gamma-shadowed Nakagami and Nakagami-log-normal fading channel has been derived by using exponential approximation in [15]. The outage probability over a combination \( \eta - \mu \) fading model and log-normal shadowing fading model has been derived by using gamma distribution as a shadowing fading model instead of log-normal shadowing fading model in [16]. Many models of shadowing fading have been presented in [17].

In recent literature, various generalised distributions of fading amplitude have been suggested, such as the \( \kappa - \mu \) model, the Fisher–Snedecor \( F \) (FS \( F \)) model, and the fluctuating two-ray (FTR) model for modelling several communication channels scenarios. This is because they can be modelled all conventional distributions with results closer to practical data measurements [18–20]. As a result, these distributions have been widely used in analytical communication systems. Consequently, the \( \kappa - \mu \) shadowed fading model has been received much attention for modelling multipath fading channels, such as Rician, Rayleigh, Nakagami-\( m \) and one-sided Gaussian fading channels by changing the coefficients of \( \kappa \) and \( \mu \) to particular real numbers greater than or equal to zero for creating SC or LS propagation effects [18]. This model has been shown great fits for modelling all of the mentioned distributions in [21, 22]. In addition to fitting to underwater acoustic communication (UAC) channels [18, 23] and land-mobile satellite (LMS) channels [24] with good analytical properties [18]. For example, and for the Rician shadowed fading channel, the multipath fading in the \( \kappa - \mu \) model is followed the Rician distribution and the shadowing is followed the Nakagami-m distribution [18]. While in FS \( F \) model, it is assumed that the multipath fading and shadowing fading follows Nakagami-m distribution and inverse Nakagami-m distribution, respectively [19]. Whereas in the case of the FTR model, the shadowed fading amplitudes of the \( N \) specular components follow the Nakagami-\( m \) random variable (rv) [20, 24] and the diffuse waves follow the complex Gaussian rv [20]. Hence, the amplitudes of the shadowing fading components of the three described models can be modulated by using inverse Nakagami-\( m \) distribution in the case of FS \( F \) model or Nakagami-\( m \) distribution in both \( \kappa - \mu \) model and FTR model.

The best of our knowledge, the unshadowing line-of-sight (LOS) Rician \( K \)-factor (RK-\( \delta \)), shadowing non-line-of-sight (NLOS) \( \kappa - \mu \) and compound unshadowing/shadowing fading channel models have got little attention until now and no closed-form expressions for the BER, the ergodic capacity, the outage probability and the outage capacity probability have been derived in the presence of NLD caused by HPA. Therefore, the paper main contributions are listed below.

- Instantaneous SNR distributions due to NLD-HPA over unshadowing RK-\( \delta \) fading channel, shadowing \( \kappa - \mu \) model and compound model have been derived.
- The theoretical BER expressions for the \( M \)-ary OQAM/FBMC system in NLD-HPA were derived in a closed-form without any numerical integration method over different environments and fading channel scenarios such as RK-\( \delta \) unshadowing fading channel plus Gaussian noise, \( \kappa - \mu \) shadowing fading model with AWGN, compound time slots unshadowing RK-\( \delta \) /shadowing \( \kappa - \mu \) fading model plus Gaussian noise.
- Theoretical expressions of ergodic capacity, outage probability and outage probability for the \( M \)-ary OQAM/FBMC system in NLD-HPA were derived in closed form without any numerical integration method in different environments and fading channel scenarios as described above.

The remainder of the paper is structured as follows. The OQAM/FBMC system model is described in Section 2 in the presence of NLD-HPA. Section 3 presents the exact theoretical derivations of the BER for \( M \)-ary OQAM/FBMC system over different fading channel scenarios. Section 4 presents the theoretical derivations of the ergodic capacity for the \( M \)-ary OQAM/FBMC over different fading channel scenarios. Sections 5 and 6 present the exact outage probability and outage capacity probability derivations, respectively, for the \( M \)-ary OQAM/FBMC over different fading channel scenarios. Section 7 is then used to evaluate the derived equations described in Sections 3–6 by the Monte-Carlo simulation and finally the analyses and simulations are concluded in Section 8.

2 | THE MODEL OF THE OQAM/FBMC SYSTEM OVER MULTIPATH FADEING CHANNEL

The block diagram for the OQAM/FBMC communication system is shown in Figure 1.

The \( m \)-th symbol on subcarrier \( n \) modulated OQAM/FBMC signal can be generated by staggering the real and imaginary components in the time domain by half a symbol period \( (T/2) \), and then passing through a bank of \( N \) contiguous bandpass
filters. Consequently, the time domain baseband continuous signal, \(i(t)\), can be represented as [2, 5, 11]

\[
i(t) = \sum_{m=0}^{N-1} \sum_{n=-\infty}^{\infty} a_{n,m} e^{j(\pi(n+m)/2 + \varphi_{n,m})/\beta_{n,m}^c(t)},
\]

where \(N\) is the number of sub-carrier, \(a_{n,m}\) is a real-valued symbol transmitted corresponding to \(\sqrt{M}\) pulse-amplitude modulation, \(b(t - mT/2)\) is a filter impulse response, \(\varphi_{n,m} = \pi(n + m)/2 - \pi nm\) is a phase term and \(\beta_{n,m}^c(t)\) is a shifted prototype filter impulse response. In a distortion-less and noise-free communication channel, the demodulated symbol can be expressed as

\[
\gamma_{n,m} = a_{n,m} + \sum_{n \neq n, m \neq m} a_{n,m} \int_{-\infty}^{\infty} \beta_{n,m}^c(t) \beta_{n,m}^e(t) dt
\]

\[
= a_{n,m} + j\mu_{n,m}.
\]

The PHYDYAS [1, 11] prototype filter coefficients, \(\beta_{n,m}^c(t)\) in (2), has been designed to achieve the interference from non-adjacent subcarriers at the same symbol time to be orthogonal, that is, equal to zero. Unfortunately, even with distortionless, noiseless, perfect time and frequency synchronisation, the PHYDYAS processing includes a multiplication by the phase term \(e^{j\varphi_{n,m}},\) which introduces some intercarrier interference (and/or intercarrier coupling) at the output of the PHYDYAS, leading to \(z_{n,m}\) always seen as pure imaginary-valued. Therefore, an almost complete reconstruction of the transmitted real symbol \(a_{n,m}\) can be reconstructed simply by taking the real part of the demodulated symbol \(\gamma_{n,m}\).

The transmitted modulated OQAM/FBMC signal, \(i(t)\), in (1) is then amplified by the memoryless nonlinear (NL)-HPA operating in the nonlinear region. Consequently, the \(u(t)\) signal can be expressed at the memoryless NL-HPA output as

\[
u(t) = F_u(\varphi(t))(i(t)) e^{j(\varphi(t))} e^{j(\varphi(t))},
\]

where \(\varphi(t)\) and \(\phi(t)\) are the magnitude and phase of \(i(t)\), \(F_u(\cdot)\) and \(F_a(\cdot)\) are the AM/AM and AM/PM conversions, respectively. In our work, we adopt the Saleh model for modelling the NLD casing by HPA, whose AM/AM and AM/PM functions are presented in [25] as

\[
F_a(\varphi(t)) = A_{sat}^2 \frac{\varphi(t)}{\varphi(t)^2 + A_{sat}^2},
\]

\[
F_p(\varphi(t)) = \varphi_0 \frac{\varphi(t)^2}{\varphi(t)^2 + A_{sat}^2},
\]

where \(A_{sat}\) denotes the input saturation amplitude and \(\varphi_0\) is the phase distortion controller. Thus, the NLD output due to HPA according to the Bussgang theorem [26] in the time domain can be expressed as [5, 11]

\[
u(t) = K_0(t)i(t) + d(t),
\]

where \(K_0(t) = |K_0|e^{i\varphi_{n,m}(t)}\) is a complex-valued gain, \(d(t)\) is an additive noise with \(\mu_d = 0\) and \(\sigma_d^2 = E(|d(t)|^2),\) which is independent to the OQAM/FBMC signal, \(i(t)\), and \(E\) is the expectation value. However, for large number of sub-carriers, \(N\), the mean of Gaussian random process signal, \(i(t)\), will approach to zero.

The nonlinearly amplified (NLA) signal in (5) will propagate through the multipath fading channel. The received signal at the input of OQAM/FBMC demodulator in the time domain can be presented as [5, 11]

\[
z(t) = b_i(t) \otimes u(t) + w(t)
\]

\[
= i(t) \otimes [K_0 b_i(t)] + d(t) \otimes b_i(t) + w(t),
\]

where \(b_i(t)\) and \(\otimes\) denote the channel impulse response and the convolution operation, respectively. So the complex received signal obtained in the frequency domain can be expressed as

\[
Z(f) = I(f)[K_0 H_i(f)] + D(f)H_i(f) + W'(f),
\]

where \(I(f), H_i(f)\) and \(D(f)\) are the OQAM/FBMC signal in frequency domain, the frequency response of the fading channel and the NLD in the frequency domain, respectively. Assuming a complete understanding of the \(H(f)\) channel state information (CSI), and \(K_0\). Hence, the detection of real component samples in the frequency domain, \(\{Z(f)\}_{K_0 H_i(f)}\) at zero forcing (ZF) equaliser output can be defined as [5]

\[
\left\{ \left\{ \frac{Z(f)}{K_0 H_i(f)} \right\} \right\} = \left\{ I(f) \right\} R + \left\{ \frac{D(f)}{K_0} \right\} R + \left\{ \frac{W'(f)}{K_0 H_i(f)} \right\} R.
\]

Thus, the instantaneous signal to noise ratio (\(\gamma\)) after OQAM/FBMC demodulator equaliser, can be expressed as [11]

\[
\gamma = \frac{E_b|K_0|^2|H_i|^2}{\sigma_d^2 + |H_i|^2\sigma_w^2},
\]

where \(E_b\) is the energy of a bit and and \(\sigma_w^2\) is the variance of the AWGN. The NL parameter of \(K_0\) and \(\sigma_d^2\) due to the NL noise, \(d(t)\), have been derived in [11] as

\[
K_0 = \sqrt{\frac{\pi}{8}} \sum_{n=1, m \text{ odd}}^{N-1} (m + 1)a_m \sigma_n^{m+1} \prod_{i=0}^{m-1} (2i + 1) + \frac{1}{2} \sum_{n=2, m \text{ even}}^{N-1} (m + 1)a_m \sigma_n^{m+1} \left( \frac{m}{2} \right)!,
\]

\[
K_0 = \sqrt{\frac{\pi}{8}} \sum_{n=1, m \text{ odd}}^{N-1} (m + 1)a_m \sigma_n^{m+1} \prod_{i=0}^{m-1} (2i + 1) + \frac{1}{2} \sum_{n=2, m \text{ even}}^{N-1} (m + 1)a_m \sigma_n^{m+1} \left( \frac{m}{2} \right)!.
\]
TABLE 1 | Theoretical values of $K_0$ and $\sigma_d^2$ for $L = 10$, $N = 64$ and 10$^6$-4OQAM/FBMC signals [11]

| IBO (dB) | $\varphi_0$ | $K_0$ (Equation 10) | $\sigma_d^2$ (Equation 11) |
|----------|------------|------------------|------------------|
| SSPA     | 4  | 0.7690 | $4.4431 \times 10^{-3}$ |
|          | 6  | 0.8297 | $1.8722 \times 10^{-3}$ |
|          | 8  | 0.8785 | $7.0583 \times 10^{-4}$ |
| TWTA     | 4  | 0.6036 | $1.0317 \times 10^{-2}$ |
|          | 6  | 0.6909 | $4.9654 \times 10^{-3}$ |
|          | 8  | 0.7775 | $2.1030 \times 10^{-2}$ |
|          | $\pi/6$ | 0.5904 + 0.1068j | $1.1271 \times 10^{-2}$ |
|          | $\pi/6$ | 0.6870 + 0.0948j | $5.5812 \times 10^{-3}$ |
|          | $\pi/6$ | 0.7727 + 0.0850j | $2.4345 \times 10^{-3}$ |

where $L$, $\sigma$ and $a$ represents the polynomial order, the signal variance at the HPA input and the complex coefficients of the approximated HPA polynomial, respectively. In this paper, we have simply used the values of $K_0$ and $\sigma_d^2$ that are tabulated in [11] that were computed based on the complex valued polynomial coefficients $a_w$ and $a_j$ in (10) and (11) using the least square (LS) method presented in [27]. The theoretical equations of $K_0$ and $\sigma_d^2$ have been computed in [11] based on (10) and (11), respectively, as a function of input back-off ratio (IBO) representing the ratio of maximum to mean input power at the HPA operating near the saturation area where $\text{IBO} = 10 \log_{10} \frac{I_d^2}{\sigma^2}$. The IBO values for NLA-OQAM/FBMC system are tabulated in Table 1, for two HPA models named solid state power amplifier (SSPA) when $\varphi_0$ is fixed to 1 and traveling wave tube amplifier (TWTA) when $A_{out}^2 = 1$ as given in [11].

3 | EXACT BER DERIVATIONS FOR M-ARY OQAM/FBMC IN THE PRESENCE OF HPA

3.1 Unshadowing Rician K-factor fading channel

In the wireless communication systems, there is a possibility of receiving the signal from both LOS and NLOS. The Rician distribution is therefore used to model the envelope of the short-term fading channel in wireless systems when there is a LOS path in addition to the NLOS paths between the transmitter and the receiver. In this case, we are assumed that the fading channel amplitude follows the Rician K-factor (RK-f) distribution with a random variable (rv) $x$. This distribution can be expressed as [12]

$$p(x) = \frac{2(1 + K)x^{-K}}{\gamma_u} e^{\left(-\frac{x^2}{2}\right)} I_0\left(\sqrt{4K(1 + K)x^2}\right),$$

(12)

where $K$ denote the direct path to scattered paths power ratio, that is, $K = \frac{\sigma^2_d}{\sigma^2_{dw}}$, and $\gamma_u = \mathbb{E}(x^2)$ is the average fading power. In fact, as the $K$-factor becomes zero/infinit, the Rician distribution becomes Rayleigh/Gaussian PDF, respectively. The distribution of instantaneous SNR per symbol, $\gamma_s$, of the unshadowing channel is given by [12] as

$$p_{\gamma_s}(\gamma_s) = \frac{1}{\gamma_u} \left(1 + \frac{1}{K}\right) e^{-\frac{\gamma_s}{\gamma_u}} I_0\left(\sqrt{4K\gamma_s}\right), \quad \gamma_s > 0,$$

(13)

where $I_0(\cdot)$ is the modified Bessel function of the first kind of order 0. Due to the NLD-HPA, the signal to noise power ratio over the unshadowing fading channel, $\gamma_u$, can be computed based on (7), where the signal power can be determined from the first term as $\mathbb{E}(|\mathcal{A}|^2)|K_0|^2\mathbb{E}(|h|^2)$ and the noise power can be calculated from the second term as $\sigma_d^2\mathbb{E}(|h|^2) + \sigma_d^2$. So the SNR can be computed as

$$\gamma_s = \frac{\text{Signal power}}{\text{Noise power}} = \frac{\gamma_u \alpha_s}{\alpha_s \sigma_d^2 + \sigma_w^2},$$

(14)

where $\gamma_u = \mathbb{E}(|\mathcal{A}|^2)|K_0|^2\mathbb{E}(|h|^2)$, $\alpha_s = \mathbb{E}(|h|^2)$. Let $\gamma_s = f(\alpha_u)$ and $\alpha_u = g(\gamma_s) = \frac{\sigma_d^2}{\gamma_u}$. The cumulative distribution function of $\gamma_s$ can be computed as $F_{\gamma_s}(\gamma_s) = F_{\gamma_s}(f(\alpha_u) \leq \gamma_s) = F_{\gamma_s}(g(\gamma_s) \leq \alpha_u) = F_{\gamma_s}(\alpha_u)$ for $\gamma_s \geq 0$. Therefore, $p_{\gamma_s}(\gamma_s) = \frac{d}{d\gamma_s} F_{\gamma_s}(\alpha_u) = p_{\gamma_s}(\alpha_u) \frac{d}{d\alpha_u}(f(\alpha_u)) = \frac{d}{d\alpha_u}(f(\alpha_u))$, where $\frac{d}{d\alpha_u}(f(\alpha_u)) = \frac{\sigma_d^2}{(\gamma_u - \sigma_d^2)^2}$. Therefore, $p_{\gamma_u}(\gamma_u)$ can be expressed as

$$p_{\gamma_u}(\gamma_u) = p_{\gamma_s}\left(\frac{\sigma_d^2}{(\gamma_u - \sigma_d^2)^2}\right),$$

(15)

At $\sigma_d^2 = 0, \gamma_u$ in (14) will be $\gamma_u = \frac{\gamma_u}{\sigma_d^2}$, hence, $p_{\gamma_u}(\gamma_u)$ in (15) can be expressed as a distribution in the range $0 \leq \gamma_u < \frac{\gamma_u}{\sigma_d^2}$. Substituting (13) in (15), yield the instantaneous SNR distribution due to NLD-HPA over unshadowing RK-f fading.
channel as

\[ p_{\text{BER}}(y_u) = \frac{(1 + K)e^{-K}}{\bar{y}_u}, \frac{\sigma_y^2}{\bar{y}_u}, \left( \frac{1}{\gamma_y - \sigma_y^2} \right)^{\gamma_y} \times I_0 \left( \frac{4K(1 + K)}{\bar{y}_u}, \frac{\sigma_y^2}{\gamma_y - \sigma_y^2} \right), \]

where \( C_1 = \frac{2}{\log_2(M)}(1 - 1/\sqrt{M}) \), \( C_2 = \frac{3\log_2(M)}{M - 1} \) and \( \text{erfc}(\cdot) \) is a complementary error function expressed as \( \text{erfc}(1) = 1 - \text{erf}(1) \), where the error function is \( \text{erf}(\cdot) \). So Equation (17) can be expressed as

\[ p_{\text{BER}}^e = C_1 \left[ \int_0^{\frac{y_u}{\sigma_y^2}} \text{erfc}(\sqrt{C_2 y_u}) p_{\text{BER}}(y_u) dy_u - \int_0^{\frac{y_u}{\sigma_y^2}} \text{erf}(\sqrt{C_2 y_u}) p_{\text{BER}}(y_u) dy_u \right] = C_1[I_{n1} - I_{n2}], \]

where

\[ I_{n1} = \int_0^{\frac{y_u}{\sigma_y^2}} \left( \frac{1 + K)e^{-K}}{\bar{y}_u}, \frac{\sigma_y^2}{\bar{y}_u}, \left( \frac{1}{\gamma_y - \sigma_y^2} \right)^{\gamma_y} \times I_0 \left( \frac{4K(1 + K)}{\bar{y}_u}, \frac{\sigma_y^2}{\gamma_y - \sigma_y^2} \right) \right] dy_u, \]

and

\[ I_{n2} = \int_0^{\frac{y_u}{\sigma_y^2}} \text{erf}(\sqrt{C_2 y_u}) \left( \frac{1 + K)e^{-K}}{\bar{y}_u}, \frac{\sigma_y^2}{\bar{y}_u}, \left( \frac{1}{\gamma_y - \sigma_y^2} \right)^{\gamma_y} \times I_0 \left( \frac{4K(1 + K)}{\bar{y}_u}, \frac{\sigma_y^2}{\gamma_y - \sigma_y^2} \right) \right] dy_u. \]

Unfortunately, the literature does not provide a closed-form expression for the integral of \( I_{n1} \) in (19). Hence, the closed-form expression of \( I_{n1} \) can be expressed using a Taylor series. Assuming \( \delta(y_u) = e^{-\frac{y_u}{\sigma_y^2}} \) or \( \delta(y_u) = e^{-\frac{y_u}{\gamma_y - \sigma_y^2}} \), the Taylor series can be expressed using \( e^{-\gamma} = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!} e^r \) [28] as

\[ \delta(y_u) = \sum_{r=0}^{\infty} \frac{(-1)^r (1 + K)\sigma_y^2}{\bar{y}_u}, \left( \frac{1}{\gamma_y - \sigma_y^2} \right)^{\gamma_y} \times I_0 \left( \frac{4K(1 + K)}{\bar{y}_u}, \frac{\sigma_y^2}{\gamma_y - \sigma_y^2} \right), \]

even when substituting the Taylor series of \( \delta(y_u) \) in (19), no closed-form is available in the literature for \( I_{n1} \), therefore, we assume \( \eta(y_u) = I_0 \left( \frac{4K(1 + K)}{\bar{y}_u}, \frac{\sigma_y^2}{\gamma_y - \sigma_y^2} \right) \), the Taylor series can be expressed using \( I_0(\gamma) = \sum_{r=0}^{\infty} \frac{1}{r! \gamma^{r+1}} (\gamma^{r+1} / (2)) \) in [28, eq. 8.445] as

\[ \eta(y_u) = \sum_{r=0}^{\infty} \frac{(-1)^r K^r e^{-K}}{r! \gamma_{\sigma_y^2}^{r+1}}, \frac{1}{\gamma_y - \sigma_y^2} \right) \times \int_0^{\frac{y_u}{\sigma_y^2}} \text{erf}(\sqrt{C_2 y_u}) p_{\text{BER}}(y_u) dy_u, \]

Comparing (23) with the integral form [28, eq. 3.194] where both conditions of the integral are satisfied, and after some mathematical manipulations, we get

\[ I_{n1} = \sum_{r=0}^{\infty} \sum_{r=0}^{\infty} \frac{(-1)^r K^r e^{-K}}{r! \gamma_{\sigma_y^2}^{r+1}}, \left( \frac{1}{\gamma_y - \sigma_y^2} \right) \times \int_0^{\frac{y_u}{\sigma_y^2}} \text{erf}(\sqrt{C_2 y_u}) p_{\text{BER}}(y_u) dy_u, \]

Comparing (26) with the integral in [28, eq. 3.194] where both conditions of the integral are satisfied, and after some
mathematical manipulations, we get

\[
I_{22} = \frac{1}{\sqrt{\pi}} \sum_{\varepsilon=0}^{\infty} \sum_{\varepsilon=0}^{\infty} \sum_{\varepsilon=0}^{\infty} r! d! \varepsilon^\varepsilon e^{-\varepsilon} (-1)^{r+\varepsilon} K^r e^{-K} = \frac{1}{\sqrt{\pi}} \sum_{\varepsilon=0}^{\infty} \sum_{\varepsilon=0}^{\infty} \sum_{\varepsilon=0}^{\infty} r! d! \varepsilon^\varepsilon e^{-\varepsilon} (-1)^{r+\varepsilon} K^r e^{-K}.
\]

Thus, the BER over unshadowing RK-f fading channel can be computed by substituting (24) and (27) in (18), yield

\[
p_{sh} = \frac{1}{\sqrt{\pi}} \sum_{\varepsilon=0}^{\infty} \sum_{\varepsilon=0}^{\infty} \sum_{\varepsilon=0}^{\infty} r! d! \varepsilon^\varepsilon e^{-\varepsilon} (-1)^{r+\varepsilon} K^r e^{-K} = \frac{1}{\sqrt{\pi}} \sum_{\varepsilon=0}^{\infty} \sum_{\varepsilon=0}^{\infty} \sum_{\varepsilon=0}^{\infty} r! d! \varepsilon^\varepsilon e^{-\varepsilon} (-1)^{r+\varepsilon} K^r e^{-K}.
\]

Substituting (29) in (31), yield the instantaneous SNR distribution due to NLD-HPA over \(\kappa-\mu\) shadowed fading model as

\[
\begin{aligned}
\rho_{\kappa-\mu} (y_{sh}) &= \frac{1}{\Gamma (\mu)} \left( \frac{m}{\mu + m} \right)^{m} \left( \frac{\mu (1 + k) \sigma_w^2}{\gamma_{sh}} \right)^{\mu} \\
&\times \frac{\gamma_{sh}^{\mu-1}}{(\gamma_{sh} - \sigma_{y_{sh}}^2)^{\mu+1}} e^{-\frac{\gamma_{sh}^{\mu+1}}{(\gamma_{sh} - \sigma_{y_{sh}}^2)^{\mu+1}}} \\
&\times \int_{\sigma_{y_{sh}}^2}^{\gamma_{sh}} F_1 \left( m, \mu; \frac{\mu^2 k^2 (1 + k) \gamma_{sh}}{(\mu + m) \sigma_{y_{sh}}^2} \right) d\gamma_{sh}.
\end{aligned}
\]

So the BER of \(M\)-ary OQAM/FBMC over \(\kappa-\mu\) shadowed fading model with AWGN can be computed by using (32) as

\[
p_{\kappa-\mu} = C_1 \int_{0}^{\infty} \frac{\gamma_{sh}}{\sigma_{y_{sh}}^2} \text{erfc} (\sqrt{C_2 y_{sh}}) p_{\kappa-\mu} (y_{sh}) d\gamma_{sh}
\]

3.2 Shadowing \(\kappa-\mu\) fading channel model

The \(\kappa-\mu\) shadowed fading model is presented as the generalisation of the physical \(\kappa-\mu\) fading model presented in [18, 29, 30]. The distribution of instantaneous SNR, \(y_{sh}\), with a mean value \(\bar{y}_{sh}\), for the \(\kappa-\mu\) shadowed model with real positive shaping parameters \(\kappa, \mu, m\) can be expressed as [18, 29, 31]

\[
p_{y_{sh}} (y_{sh}) \propto \frac{\mu^m m^m (1 + k)^m}{\Gamma (\mu) \Gamma (\mu + m)} \left( y_{sh} \right)^{\mu-1} \frac{e^{-\mu (1 + k) y_{sh}}}{\gamma_{sh}} \\
\times \int_{0}^{\gamma_{sh}} F_1 \left( m, \mu; \frac{\mu^2 k^2 (1 + k) \gamma_{sh}}{(\mu + m) \gamma_{sh}} \right) d\gamma_{sh},
\]

where \(F_1(\cdot, \cdot, \cdot)\) is the confluent hypergeometric function defined in [28, eq. 9.210]. Due to the HPA, the SNR, \(y_{sh}\), over shadowing fading channel can be expressed as

\[
y_{sh} = y \frac{\alpha_{sh}}{\alpha_{sh} \sigma_y^2 + \sigma_w^2}.
\]
Unfortunately, no closed-form expression for the integral $I_{db}$ in (34) is available in the literature. Hence, the Taylor series of
\[
(-\frac{\mu(1+k)}{\nu}\frac{\sigma^2_y}{\nu^2})
\]
can be expressed as
\[
\delta(y_{db}) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!} \left( \frac{\mu(1+k)}{\nu} \frac{\sigma^2_y}{\nu^2} \right)^r \frac{y_{db}}{(y_r - \sigma^2_y)^r}.
\]
(36)

Even when substituting the Taylor series of (36) in (34), no closed-form is available in the literature for $I_{db}$. Therefore, the
Taylor series of $F_1(m, \mu; \frac{\mu^2(1+k)}{(\mu+k+m)^2} \frac{\sigma^2_y}{\nu^2})$ can be expressed using $F_1(\sigma, k; \infty) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!} \frac{\sigma^2_y}{\nu^2} \frac{y_{db}}{(y_r - \sigma^2_y)^r}$, as [28, eq. 9.21010, 1]
\[
\eta(y_{db}) = \sum_{r=0}^{\infty} \frac{(m)}{(\nu)} \frac{\mu^2(1+k)}{(\mu+k+m)^2} \frac{\sigma^2_y}{\nu^2} \frac{y_{db}}{(y_r - \sigma^2_y)^r},
\]
(37)
where $(-1)^r \frac{\nu^r}{r!}$ is the Pochhammer symbol. After substituting (36) and (37) in (34) and after some mathematical manipulations in the equation, we get
\[
I_{db} = \sum_{r=0}^{\infty} \sum_{m=0}^{\infty} \frac{(-1)^r}{r!} \frac{\mu^2(1+k)}{(\mu+k+m)^2} \frac{\sigma^2_y}{\nu^2} \frac{y_{db}}{(y_r - \sigma^2_y)^r}
\]
\[
\times \left( \frac{\mu(1+k)}{\nu} \frac{\sigma^2_y}{\nu^2} \left( \frac{y_{db}}{y_r - \sigma^2_y} \right)^{r+1} \int_{0}^{\frac{y_{db}}{y_r - \sigma^2_y}} \left( 1 - \frac{\sigma^2_y}{y_r} \right)^{r+1} dy_{db} \right)^{r+1}
\]
(38)

Thus,
\[
I_{db} = \frac{1}{\sqrt{\pi}} \sum_{r=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^r}{r!} \frac{\mu^2(1+k)}{(\mu+k+m)^2} \frac{\sigma^2_y}{\nu^2} \frac{y_{db}}{(y_r - \sigma^2_y)^r}
\]
\[
\times \left( \frac{\mu(1+k)}{\nu} \frac{\sigma^2_y}{\nu^2} \left( \frac{y_{db}}{y_r - \sigma^2_y} \right)^{r+1} \int_{0}^{\frac{y_{db}}{y_r - \sigma^2_y}} \left( 1 - \frac{\sigma^2_y}{y_r} \right)^{r+1} dy_{db} \right)^{r+1}
\]
\[
\times F_1(\mu + r + s + 1, \mu + r + s, \mu + r + s + 1, 1).
\]
(39)

Moreover, to compute the integration of $I_{db}$ in (35), no closed-form is available in the literature without expression $\Lambda(y_{db}) = \text{erf}(\sqrt{C}y_{db})$ as a Taylor series, the Taylor series of $\Lambda(y_{db})$ can be expressed when substituting $y_{db}$ instead of $y_{a}$ in (25).
Therefore, the Rician shadowed fading channel with shaping parameters $K$ and $m$ can be generated by substituting $\mu = 1, K = K, m = m$ in (42). Table 2 list all the fading channels that can be generated from the general BER derivation of $\kappa-\mu$ shadowed distribution over NLD-HPA as presented in [18, 29].

### 3.3 Compound unshadowing/shadowing fading channel

There is a possibility in the land-mobile satellite communication channel to receive the signal from both LOS with unshadowing multipath fading during a one-time period, and to receive the signal from both LOS plus shadowing multipath fading on the other time slot as presented in [12], Lutz et al. in [32] and Barts and Stutzman [33]. Therefore, the received superimposed LOS-unshadowed/LOS-shadowed fading environment should take the instantaneous composite multipath/shadowed signal instead of the average magnitude of the multipath signal. For example, if there is no shadowing we assume that the fading follows a RK-factor distribution. Otherwise, when shadowing is present, we assume that the fading follows a shadowed Rice distribution modelled using the $\kappa-\mu$ model. The compound distribution of the instantaneous SNR can therefore be defined by the shadowing time-sharing factor, $A$ as [12]

$$\gamma_i = (1 - A) \gamma_c + A \gamma_s, \quad \sigma_w^2 = \frac{\sigma_c^2 + \sigma_w^2}{2}$$.  

(45)

With the help of (13) and (32), the PDF of $\gamma_i$ can be defined as

$$p_{\text{comp}}(\gamma_i) = (1 - A) \frac{\sigma_c^2 \gamma_i}{\gamma_i - \sigma_c^2} + A \frac{\sigma_w^2 \gamma_c}{\gamma_c - \sigma_w^2}$$

(46)

The BER of $M$-ary OQAM/FBMC over compound unshadowing/shadowing fading channel with AWGN can be computed by using (28) and (42) as

$$p_r^{\text{comp}} = (1 - A) \times p_r^t + A \times p_r^h$$.  

(47)

Different scenarios of RK-f distribution and the time-sharing factor $A$ have been considered for the unshadowing/shadowing cases and presented in Table 3 [34].

### 4 ERGODIC CAPACITY FOR OQAM/FBMC WITH IMPACT OF HPA

The expectation of information rate overall fading channel states indicates the ergodic capacity [35, 36]. The ergodic
capacity in (bit/s/Hz) can be expressed as [35].

\[
C = E[\log_2(1 + \gamma)] = \int_0^\infty \log_2(1 + \gamma) p_\mathcal{R}(\gamma) d\gamma. \tag{48}
\]

### 4.1 Unshadowing Rician K-factor fading channel

The C over RK-f fading channel in the presence of NLD-HPA can be computed as

\[
C^\mathcal{R} = \int_0^\infty \log_2(1 + \gamma) p_\mathcal{R}(\gamma) d\gamma,
\]

where \( p_\mathcal{R}(\gamma) \) expressed in (16) for the interval \( 0 \leq \gamma < \frac{\gamma_r}{\sigma_d^2} \).

Therefore, the \( C^\mathcal{R} \) can be computed as

\[
C^\mathcal{R} = \int_0^{\gamma_r} \log_2(1 + \gamma) p_\mathcal{R}(\gamma) d\gamma = \sum_{r=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^r K^r e^{-K} \left(\frac{1 + K}{\gamma_r} \sigma_w^2\right)^{r+1}}{r! \sigma^2} + \gamma_r \sigma_w^2 d\gamma
\]

\[
\times \left[ \int_0^{\gamma_r} \frac{\gamma_r^{r+1}}{(1 - \frac{\gamma_r}{\sigma_d^2})^{r+2}} d\gamma \right] \tag{50}
\]

Unfortunately, there is no closed-form expression available in the literature for the integral of (52). A good approximation of \( \log_2(\gamma_w) \) can be suggested based on MATLAB program as

\[
\log_2(\gamma_w) \approx \frac{\gamma_w}{\ln(2)} \left( e^{\gamma_w} - 1 \right) \text{ which gives a mean square error} = \frac{1}{9990} \sum_{\gamma_w=10}^{\gamma_w=1000} (\log_2(\gamma_w) - \frac{\gamma_w}{\ln(2)} (e^{\gamma_w} - 1))^2 = 7.3031 \times 10^{-6} \text{ for all values of } \gamma_w \text{ from 10 to 1000 and for } a = 10^4. \]

Hence, \( C^\mathcal{R} \) can be computed after some mathematical manipulations as

\[
C^\mathcal{R} = \sum_{r=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^r K^r e^{-K} \left(\frac{1 + K}{\gamma_r} \sigma_w^2\right)^{r+1}}{r! \sigma^2} + \gamma_r \sigma_w^2 \left[ \int_0^{\gamma_r} \frac{\gamma_r^{r+1}}{(1 - \frac{\gamma_r}{\sigma_d^2})^{r+2}} d\gamma \right] \tag{53}
\]

### 4.2 Shadowed Rician fading channel using \( \kappa-\mu \) model

The \( C^{\kappa-\mu} \) can be derived by using (38) and after following the same previous steps mentioned in the previous sub-section for \( \gamma_{sh} < 10 \) dB, we get

\[
C^{\kappa-\mu} = \int_0^{\gamma_r} \sqrt{\gamma_{sh} \kappa-\mu} d\gamma_{sh}
\]

\[
\times \left( \frac{\gamma_r}{\sigma_d^2} \right)^{1/2} F_2(\gamma_r + s + 2, r + s + 2 + 3/2; r + s + 5/2; 1).
\]

Moreover, in [37], it has been suggested that \( \log_2(1 + \gamma) \approx \log_2(\gamma) \) for \( \gamma > 10 \) dB. Hence, \( C^\mathcal{R} \) can be computed as

\[
C^\mathcal{R} = \int_0^{\gamma_r} \log_2(1 + \gamma) p_\mathcal{R}(\gamma) d\gamma = \sum_{r=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^r K^r e^{-K} \left(\frac{1 + K}{\gamma_r} \sigma_w^2\right)^{r+1}}{r! \sigma^2} + \gamma_r \sigma_w^2 \left[ \int_0^{\gamma_r} \frac{\gamma_r^{r+1}}{(1 - \frac{\gamma_r}{\sigma_d^2})^{r+2}} d\gamma \right] \tag{52}
\]
Moreover, $C^{\kappa-\mu}$ can be computed for $\gamma_{th} \geq 10$ dB as

$$C^{\kappa-\mu} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^n a(m)}{(m+1)!} \frac{\Gamma(\mu+k+m)^{m+n}}{\Gamma(\mu)^{m+n}} \left( \frac{\mu(1+k)\sigma_v^2}{\tilde{\gamma}_s\sigma_d^2} \right)^{\mu+r+s} \left( \frac{\gamma_e}{\tilde{\gamma}_s\sigma_d^2} \right)^{\mu+r+s+1/\sigma_v} \times \left( \frac{\gamma_e}{\tilde{\gamma}_s\sigma_d^2} \right)^{\mu+r+s+1/\sigma_v} \frac{r+1}{r+1+1} \right), \tag{55}$$

4.3 Compound unshadowing/shadowing fading channel

It is easy to compute $C^{\text{comp}}$ over compound fading channel for $\gamma_{\text{comp}} < 10$ dB by using (51) and (54) and for $\gamma_{\text{comp}} \geq 10$ dB by using (53) and (55) as

$$C^{\text{comp}} = (1 - A) \times C^{Ric} + A \times C^{\kappa-\mu}. \tag{56}$$

5 | OUTAGE PROBABILITY FOR OQAM/FBMC WITH IMPACT OF HPA

The outage probability, $P_{\text{out}}$, is known as the probability of the output SNR, $\gamma$, which does not exceed a specified threshold defined, $\gamma_{th}$. It is very important for the fading channel to give the probability of a certain BER that cannot be achieved.

5.1 Unshadowing Rician $K$-factor fading channel

The $P_{\text{out}}$ over RK-f for M-ary OQAM/FBMC system with impact of NLD-HPA can be computed by using (16). It can be mathematically defined as

$$P_{\text{out}}^{Ric}(\gamma_{th}^R) = \int_{0}^{\gamma_{th}^R} p_{\text{out}}^{Ric}(\gamma_s) d\gamma_s, \quad 0 \leq \gamma_{th}^R \leq \frac{\gamma_c}{\sigma_d^2}. \tag{57}$$

After substituting the (16) in Taylor series in (57) and after some mathematical manipulations, we get

$$P_{\text{out}}^{Ric}(\gamma_{th}^R) = \int_{0}^{\gamma_{th}^R} \gamma_s^r \left( \frac{1}{\tilde{\gamma}_s\sigma_d} \right)^{r+1} d\gamma_s, \tag{58}$$

comparing (58) with [28, eq. 3.194] where both conditions of the integral are satisfied, the $P_{\text{out}}^{Ric}(\gamma_{th}^R)$ of unshadowing RK-f can be written in closed form after some mathematical manipulations as

$$P_{\text{out}}^{Ric}(\gamma_{th}^R) = \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{(-1)^r K' \gamma_{th}^R}{r! s! \tilde{\gamma}_s\sigma_d} \left( \frac{(1+K)\sigma_v^2}{\tilde{\gamma}_s\sigma_d} \right)^{r+1} \times \left. \left( \frac{\gamma_e}{\tilde{\gamma}_s\sigma_d} \right)^{r+1} \right), \tag{59}$$

5.2 Shadowed Rician fading channel using $\kappa-\mu$ model

The $P_{\text{out}}$ of M-ary OQAM/FBMC over $\kappa-\mu$ shadowing Rician fading channel in NLD-HPA can be derived with the help of (32) as

$$P_{\text{out}}^{Ric}(\gamma_{th}^R) = \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{(-1)^r K' \gamma_{th}^R}{r! s! \tilde{\gamma}_s\sigma_d} \left( \frac{(1+K)\sigma_v^2}{\tilde{\gamma}_s\sigma_d} \right)^{r+1} \times \left. \left( \frac{\gamma_e}{\tilde{\gamma}_s\sigma_d} \right)^{r+1} \right), \tag{59}$$

After substituting the (32) in Taylor series in (60) and after some mathematical manipulations, we get

$$P_{\text{out}}^{Ric}(\gamma_{th}^R) = \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{(-1)^r K' \gamma_{th}^R}{r! s! \tilde{\gamma}_s\sigma_d} \left( \frac{(1+K)\sigma_v^2}{\tilde{\gamma}_s\sigma_d} \right)^{r+1} \times \left. \left( \frac{\gamma_e}{\tilde{\gamma}_s\sigma_d} \right)^{r+1} \right), \tag{61}$$

comparing (61) with integral form in [28, eq. 3.194], and after some mathematical manipulations, the $P_{\text{out}}^{Ric}(\gamma_{th}^R)$ of $\kappa-\mu$ shadowing model can be written in closed as

$$P_{\text{out}}^{Ric}(\gamma_{th}^R) = \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{(-1)^r K' \gamma_{th}^R}{r! s! \tilde{\gamma}_s\sigma_d} \left( \frac{(1+K)\sigma_v^2}{\tilde{\gamma}_s\sigma_d} \right)^{r+1} \times \left. \left( \frac{\gamma_e}{\tilde{\gamma}_s\sigma_d} \right)^{r+1} \right), \tag{62}$$

5.3 Compound unshadowing/shadowing fading channel

It is easy to compute the outage probability over compound LOS-unshadowed/LOS-shadowed multipath fading channel by
using (43) as

\[ P_{\text{out}}^{\text{comp}}(\gamma_{\text{th}}) = (1 - A) \times P_{\text{out}}^{Ric}(\gamma_{\text{th}}) + A \times P_{\text{out}}^{K-\mu}(\gamma_{\text{th}}), \]

\[ 0 \leq \gamma_s < \frac{\gamma}{\sigma_{d}^2}, \quad 0 \leq \gamma_{th} < \frac{\gamma}{\sigma_{d}^2}, \quad (\gamma_s) \quad (63) \]

where \( P_{\text{out}}^{Ric}(\gamma_{\text{th}}) \) and \( P_{\text{out}}^{K-\mu}(\gamma_{\text{th}}) \) denotes the outage probability of unshadowing Rician channel in (59) and \( K-\mu \) shadowing fading channel in (62).

## 6.1 Unshadowing Rician K-factor fading channel

The \( P_{\text{out}}(C_{\text{out}}) \) over RK-f channel for \( M \)-ary OQAM/FBMC system with NLD-HPA can be computed by using (16) as

\[ P_{\text{out}}^{Ric}(C_{\text{out}}) = \int_0^{2^{C_{\text{out}}/R} - 1} p_{Ric}(\gamma_s) d\gamma_s. \quad (65) \]

As mentioned above, the \( p_{Ric}(\gamma_s) \) can be expressed in series form. Therefore, the integral of (65) can be expressed as

\[ P_{\text{out}}^{Ric}(C_{\text{out}}) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^n K^r e^{-K}}{r! \mu^n \gamma_s^{r+1}} (1 + K) \frac{\sigma_{w}^2}{\sigma_{d}^2} \frac{d}{d\gamma_s} \gamma_{th}^{r+1} d\gamma_s. \quad (66) \]

Comparing (66) with [28, eq. 3.194] where both conditions of the integral are satisfied, the \( P_{\text{out}}^{Ric}(C_{\text{out}}) \) can be written in closed form after some mathematical manipulations as

\[ P_{\text{out}}^{Ric}(C_{\text{out}}) = (1 - A) P_{\text{out}}^{Ric}(C_{\text{out}}) + A \times P_{\text{out}}^{K-\mu}(C_{\text{out}}), \quad (71) \]
where $p_{\text{out}}^{Ric}(C_{\text{out}})$ and $p_{\text{out}}^{\kappa-\mu}(C_{\text{out}})$ denotes the outage capacities computed in (67) and (70), respectively.

7 | SIMULATION RESULTS

In this section, the performance evaluation of the derived closed-form expressions for $M$-ary OQAM/FBMC in the presence of memoryless HPA nonlinearity distortion over three different fading channel scenarios, such as $\kappa-\mu$ shadowing model and compound unshadowing/shadowing fading channels with AWGN are examined by Monte-Carlo simulation. In our simulations, we consider using the FBMC system with $N = 64$ subcarriers for 16-OQAM constellations. And we considered TWTA-HPA with AM/PM distortion over three different IBO scenarios as presented in Table 1.
unshadowing RK-f fading channel scenario, we considered three values of $K$-factor as shown in Table 3 and a constant value of $\bar{\gamma} u = 10$ dB. In addition, we considered Rician shadowed fading channel based on $\kappa-\mu$ shadowing fading model with configuration parameters depending on the $K$-factor presented in Table 3, where $\kappa = K, \mu = 1$ and $m = 3$ and a constant value of $\bar{\gamma} sh = 10$ dB. In addition, we considered compound unshadowing/shadowing channel scenarios based on unshadowing RK-f distribution and Rician shadowed fading channel for three different scenarios of shadowing time-share factor $A$ depending on the value of $K$-factor presented in Table 3 in this work. It is worth noting that the derived equations are plotted by solid lines and their corresponding simulation results are plotted by markers.
7.1 Impact of changing the IBO on OQAM/FBMC system for urban environment

Figure 2 shows the impact of the HPA with AM/PM distortion for the urban environment on the BER performance of 16-OQAM/FBMC systems. Simulation comparisons were made for different nonlinear parameters of IBO = 4, 6, 8 dB as shown in Table 1, where values of $\phi_0$, $K_0$, and $\sigma_d^2$ are taken for each corresponding value of IBO.

Based on each value of IBO, we have recognised that the BER performance of the 16-OQAM/FBMC system over $\kappa$-$\mu$ fading channel gives better performance than the system over $\kappa$-$\mu$ shadowed fading model, but when considering the compound model, the performance will be worse than the
performance of the RK-f fading channel and better than the performance of the \( \kappa-\mu \) shadowed model. It is worth noting that the BER performance improves when increasing IBO values due to reducing the variance of the signal at the input of the HPA from \( \sigma = 0.6310 \) at IBO = 4 dB to \( \sigma = 0.5012 \) at IBO = 6 dB and then \( \sigma = 0.3981 \) at IBO = 8 dB. In addition, it can be seen that the BER performance is very close to each others at low \( \frac{E_b}{N_0} \) dB for each IBO scenario due to the high AWGN variance, \( \sigma^2_w \), compared to the \( \sigma^2_d \) caused by the nonlinear HPA. There is therefore no gap between BER results at low \( \frac{E_b}{N_0} \). The gap appears at high \( \frac{E_b}{N_0} \) due to high \( \sigma d^2 \) compared to low level \( \sigma w^2 \), leading to constant BER. Indeed, using a low value of \( K = 3 \) dB of RK-f distribution, leading to a high impact of the fading channel, resulting a high impact of the shadowed \( \kappa-\mu \) model, and for a high value of \( A = 0.6 \), leading the BER is dominated by \( \kappa-\mu \) shadowed mode, resulting in a closer result to the performance of the compound system.

### 7.2 Impact of changing the IBO on OQAM/FBMC system for suburban environment

Figure 3 presents the results of BER when the user moves from the urban to the suburban environment, the overall system performance will be better than the overall system performance in the urban environment, due to a high value of \( K = 9.5 \) dB leading to the Rician distribution approaching Gaussian distribution, resulting in a lesser impact of the RK-f fading channel. As a result, the performance of the \( \kappa-\mu \) shadowed model will be better than the results of the urban environment. Leading, the impact of the compound unshadowing/shading fading model is less but still dominated by the \( \kappa-\mu \) shadowing model due to the use of the high value of \( A = 0.59 \).

### 7.3 Impact of changing the IBO on OQAM/FBMC system for highway environment

Figure 4 shows the BER results when the user moves from a suburban to a highway environment. The overall performance of the BER is better than the overall performance of the system in previous environments. In the highway environment, the performance of BER improves as IBO increases and the performance of the compound model is dominated by the distribution of RK-f due to the use of a low value of \( A = 0.25 \).

On the other hand, by comparing the results of Figures 2, 3 and 4, a significant gap appears between the results when the environment changes due to an increase in the value of \( K \)-factor leading to the RK-f approaching the Gaussian distribution and

### Table 4 \( \gamma_{th} \) at BER = \( 10^{-3} \) for different channel scenarios over suburban environment

| \( \gamma_{th} \) | IBO = 4 dB | IBO = 6 dB | IBO = 8 dB |
|----------------|------------|------------|------------|
| \( \gamma_a^d \) | 10.5       | 7.5        | 5.5        |
| \( \gamma_b^d \) | 18         | 14         | 11.5       |
| \( \gamma_{comp}^d \) | 15.5       | 12.25      | 9.75       |

### Table 5 \( \gamma_{th} \) at BER = \( 10^{-3} \) for different channel scenarios over highway environment

| \( \gamma_{th} \) | IBO = 4 dB | IBO = 6 dB | IBO = 8 dB |
|----------------|------------|------------|------------|
| \( \gamma_a^d \) | 9.5        | 6.5        | 4.5        |
| \( \gamma_b^d \) | 16.5       | 13         | 10.5       |
| \( \gamma_{comp}^d \) | 13.25     | 9.75       | 7.75       |
a decrease in the value of $A$ leading to the error is dominated by $\text{R}_K\text{-f}$ distribution. As a result, the BER performance of the urban environment is worse than the suburban environment and the suburban environment is worse than the highway environment, resulting in the BER being dominated by a shading fading channel for high values of $A$ as presented in Figures 2 and 3 and dominated by unshadowing fading channel with a low value of $A$ as presented in Figure 4. Finally, it is worth noting that the derived BER equations plotted by solid lines have closely match with their corresponding simulation results plotted by markers in all figures.

7.4 Impact of changing the IBO on the ergodic capacity for OQAM/FBMC system

Figures 5 and 6 demonstrate an ergodic capacity $C$ on the 16-OQAM/FBMC system over different fading channel scenarios with impact of NLD-HPA in suburban and highway environments. We used derived equations (51) and (53) for $\text{R}_K\text{-f}$, as well as (54) and (55) for shadowing $\kappa-\mu$ model and (56) for the compound model. It can be seen from both figures as IBO increases the ergodic capacity increases and that, as a user moves from a suburban to a highway environment, the overall ergodic capacity increases.

7.5 Impact of changing the IBO on the outage probability for OQAM/FBMC system

In this section, we demonstrate the derived outage probability in (59), (62) and (63) over $\text{R}_K\text{-f}$ distribution, $\kappa-\mu$ shadowed model and compound unshadowing/shadowing fading model for 16-OQAM/FBMC with impact of HPA-NL distortion over suburban and highway environments. In our simulation, we consider the same parameters used for plotting the Figures 3 and 4.

The $P_{\text{out}}^{\text{Ric}}(\gamma_{th})$, $P_{\text{out}}^{\kappa-\mu}(\gamma_{th})$ and the $P_{\text{out}}^{\text{comb}}(\gamma_{th})$ can be computed after the target BER is assumed and then taken the corresponding value of $\gamma_{th}$ (dB). Figures 7 and 8 demonstrate the outage probability of different threshold levels in suburban environment and highway environment, respectively. At $\text{BER} = 10^{-3}$, the values of $\gamma_{th}$ (dB) can be calculated from Figures 3 and 4, respectively, as presented in Tables 4 and 5. Both figures show that $P_{\text{out}}(\gamma_{th})$ increases with an increase in the threshold value of $\gamma_{th}$. In fact, for received $\frac{E_b}{N_0}$ (dB) less than $\gamma_{th}$ (dB), the receiver cannot decode the received symbol with probability $p_e$, and the system declares an outage.

7.6 Impact of changing the IBO on the outage capacity probability for OQAM/FBMC system

Figures 9 and 10 demonstrate an outage capacity probability of $C_{\text{out}}/R = 6[\text{b/s/Hz}]$ on the 16-OQAM/FBMC system over different fading channel scenarios with impact of NLD-HPA in suburban and highway environments. We used derived equations (67) for $\text{R}_K\text{-f}$, (70) for shadowing $\kappa-\mu$ model and (71) for the compound model. It can be seen from both figures as IBO increases the outage capacity probability decreases and that, as a user moves from a suburban to a highway environment, the overall of an outage capacity probability decreases.
8 | CONCLUSION

In this paper, the closed-form expressions of BER, ergodic capacity, outage probability and outage capacity probability for M-ary OQAM/FBMC system over different channel scenarios, such as $R_K$-$f$, $\kappa-\mu$ shadowing model and compound (time-shared) $\kappa-\mu$ shadowing and unshadowing $R_K$-$f$ fading channels have been derived. In this study, closed-form BER, ergodic capacity, outage probability and outage capacity probability expressions were derived in the presence of in-band NLD caused by memoryless HPA. The derivations of the BER, the ergodic capacity, the outage probability and the outage capacity probability are valid for any memoryless HPA model. The results of the Monte-Carlo simulation show an excellent agreement between the derived expressions and the simulation on the different channel scenarios for IBO and AM/PM distortion. In addition, the derived expressions for the $\kappa-\mu$ model and the compound model have been provided as a general derivation and all distributions generated by the $\kappa-\mu$ shadowed fading model can be obtained by controlling the values of $\kappa$, $\mu$ and $m$ as presented in Table 2.

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