When radiative corrections are finite but undetermined

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Abstract

In quantum field theory radiative corrections can be finite but undetermined.

1 Introduction

The following situation is familiar in quantum field theory. One begins with a field-theoretic Lagrange density that contains, in addition to its kinetic term, various further contributions, which at “tree level” describe masses of excitations and coupling strengths of interactions. One might suppose that these take on definite “classical” values and attempt to compute corrections that arise when the fields are quantized. While the hope is that such corrections are small – $O(h)$ – mostly one finds that they are infinite. It is then said that these quantities – masses and coupling strengths – are not calculable, and the classical plus (infinite) quantal contributions are defined to take on definite, experimentally determined values. In this way, the program of renormalization, renormalization group, running coupling constants, and so on, becomes an essential part of the theory.

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But this is not the only scenario. There are also favorable situations where a tree-level value can be assigned; the quantum correction is calculable and finite, giving a definite (usually small) contribution. A physical example is the magnetic moment of leptons, where the tree-level value of 2 for the $g$-factor acquires precisely determined quantal corrections, which these days are calculated and measured to six significant figures. A theoretical example is provided by the Schwinger model – two-dimensional QED with massless fermions. The massless “photon” of the tree approximation acquires at one loop (which is exact in this simple model) the mass $e^2/\pi$, where $e$ is the coupling strength with dimensionality of (mass)$^{1/2}$. [A similar effect is believed to happen in three-dimensional non-Abelian gauge theories, with radiatively induced mass $O(g)$, where $g$ is the coupling strength with dimensionality of mass.]

In this essay, presented to Duggy Rajaraman on the occasion of a significant birthday, I call attention to a third possibility. It can happen that radiative corrections are finite, but not determined by the theory; so, just as for infinite radiative corrections, their values are only fixed by experiment. Rajaraman and I discussed an example of this 15 years ago [1], and more recently another instance has been encountered [2].

## 2 Schwinger model

Although the Schwinger model is simple, much studied, and well understood, a close examination is useful for my further discussion. The quantity of interest is the effective action, radiatively induced by fermions:

$$\Gamma_S(A) = -i \ln \det(i\slashed{D} - eA) .$$

(1)

Possible mass generation is seen in the $O(A^2)$ contribution, which for this model determines $\Gamma_S(A)$ completely. To this end one computes the vacuum polarization, given in momentum space by

$$\Pi^{\mu\nu}_S(p) = i \text{tr} \int \frac{d^2k}{(2\pi)^2} i^{\mu} \frac{\gamma^{\nu}}{k^2 + p^2} .$$

(2a)

The integral is logarithmically divergent, hence shifting integration variables does not alter its value. After the divergent part is identified and separated, the
convergent part is evaluated, with the result

$$\Pi^\mu_\nu_S(p) = \tr \frac{1}{2} \gamma^\mu \gamma^\nu \Pi^\mu_\nu_\infty + \frac{1}{\pi} \left( \frac{g^\mu_\nu}{2} - \frac{p^\mu p^\nu}{p^2} \right)$$  \hspace{1cm} (2b)$$

where

$$\Pi^\mu_\nu_\infty \equiv 2i \int \frac{d^2k}{(2\pi)^2} \frac{(-k^2 g^\mu_\nu + 2k^\mu k^\nu)}{(k^2 - \mu^2)^2}$$  \hspace{1cm} (2c)$$

and $\mu^2$ is an arbitrary infrared cutoff, whose value does not affect $\Pi^\mu_\nu_\infty$. Note that in general $\Pi^\mu_\nu_S(p)$ is not gauge invariant – it is not transverse to $p^\mu$. In fact the tensor is traceless in $(\mu, \nu)$, a feature already recognized in the integral representation (2a) when it is remembered that in two dimensions, $\gamma^\mu k^\gamma^\mu = 0$.

To make progress I must assign a value to $\Pi^\mu_\nu_\infty$. But no unique value can be given, because the integral is divergent, that is, undefined. By Lorentz invariance, $\Pi^\mu_\nu_\infty$ should be proportional to $g^\mu_\nu$. In two dimensions any Lorentz-invariant prescription for calculating the integral will give a vanishing value, $\Pi^\mu_\nu_\infty = 0$, consistent with its being proportional to $g^\mu_\nu$ and traceless. Gauge invariance is regained by using, for instance, Pauli-Villars regularization: the Pauli-Villars regulator fields give an additional contribution $\Pi^\mu_\nu_{PV} = \frac{1}{2\pi} g^\mu_\nu$, so that $\Pi^\mu_\nu_S(p) + \Pi^\mu_\nu_{PV}$ is transverse. Alternatively, an evaluation of $\Pi^\mu_\nu_\infty$ by dimensional regularization gives $\Pi^\mu_\nu_\infty = \frac{1}{2\pi} g^\mu_\nu$, which also leads to a gauge-invariant result for $\Pi^\mu_\nu_S(p)$. But I shall not adopt these regularization procedures. The viewpoint that I prefer will be used in the analysis of the models considered below, for which the above regularization methods are problematical.

Since different evaluations of $\Pi^\mu_\nu_\infty$ produce different results, I propose that $\Pi^\mu_\nu_\infty = ag^\mu_\nu$, where $a$ is dimensionless and as yet undetermined. This Ansatz is consistent with the fact that in $\Gamma_S(A)$, terms cubic and higher in $A$ are well-defined (actually they vanish), while the quadratic term can have an undetermined, local contribution.

Thus within my viewpoint, the Feynman graphs of the Schwinger model need not be regulated, but they give a vacuum polarization with an undetermined local part:

$$\Pi^\mu_\nu_S(p) = \frac{1}{\pi} \left( g^\mu_\nu \left( \frac{1 + a}{2} \right) - \frac{p^\mu p^\nu}{p^2} \right).$$  \hspace{1cm} (3)$$
Now I make use of the formal gauge invariance of the Schwinger model and enquire whether it is possible to fix the ambiguity in (3) by insisting that this symmetry is preserved: $\Pi^{\mu\nu}_S(p)$ should be transverse. Indeed this is possible; the choice $a = 1$ yields the conventional result for the vacuum polarization in this model

$$\Pi^{\mu\nu}_S(p) = \frac{1}{\pi} \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right). \quad (4)$$

and a photon mass

$$m^2 = \frac{\epsilon^2}{\pi}. \quad (5)$$

Note that by adopting the transverse expression for the vacuum polarization, in order to agree with the constraint of gauge invariance, I have abandoned another formal feature: tracelessness of $\Pi^{\mu\nu}_S(p)$.

I have passed slowly and laboriously over familiar ground so that new territories can be explored quickly. The key point is that regularization has been avoided. While usual regularization methods, for example, Pauli-Villars or dimensional, can be successfully used in the Schwinger model, I shall now examine models for which these regularizations are unavailable or inappropriate. Nevertheless, finite but undetermined radiative corrections can be calculated, and then further properties of the theory are brought to bear on the question of whether or not the arbitrariness can be removed.

3 First example: Chiral Schwinger model

My first example is the one that Rajaraman and I studied in 1985 \[1\]: the chiral Schwinger model, where the vector interaction of the Schwinger model is replaced by a chiral interaction $e(1 + \gamma_5)A$, $\gamma_5 = \gamma^0\gamma^1$. The relevant induced action now reads

$$\Gamma_{CS}(A) = -i \ln \det \left( i \partial - e(1 + \gamma_5)A \right). \quad (6)$$

Evaluation of the vacuum polarization proceeds as above in the Schwinger model to the end

$$\Pi^{\mu\nu}_{CS}(p) = \frac{1}{\pi} \left( g^{\mu\nu} a - (g^{\mu\alpha} + \varepsilon^{\mu\alpha}) \frac{p_\alpha p_\beta}{p^2} (g^{\beta\nu} - \varepsilon^{\beta\nu}) \right). \quad (7)$$
Here \( a \) is once again a dimensionless parameter, not determined uniquely by the different procedures for calculating the fermion determinant; it gives a local, \( O(A^2) \), contribution to \( \Gamma_{CS}(A) \). Note further that since the usual Schwinger model is the sum of two chiral models with opposite chirality, combining (7) with its chiral partner \((\varepsilon^{\mu\nu} \rightarrow -\varepsilon^{\mu\nu})\) reproduces formula (3) for \( \Pi_{S}^{\mu\nu}(p) \).

Can a regulator method be implemented here, as it can be for the Schwinger model, to remove the ambiguity? Pauli-Villars regulators with massive fermions are obviously inappropriate because the chiral interaction requires massless fermions. Dimensional regularization is problematic with a \( \gamma_5 \) matrix, which is dimension specific. So there only remains the possibility of enforcing gauge invariance – a formal property of the theory.

However, unlike for the Schwinger model, imposing transversality on \( \Pi_{CS}^{\mu\nu}(p) \) does not determine \( a \), because the longitudinal part does not vanish for any value of \( a \):

\[
p_{\mu} \Pi_{CS}^{\mu\nu}(p) = \frac{1}{\pi} \left( p^\nu (a - 1) + p_\mu \varepsilon^{\mu\nu} \right).
\]

This of course is another face of the two-dimensional chiral anomaly \( [3] \) – owing to the anomalous nonconservation of the chiral current, the quantized chiral Schwinger model is not gauge invariant. Nevertheless, it possesses a physical spectrum for \( a > 1 \), with radiatively induced photon mass \( m \):

\[
m^2 = \frac{e^2 a^2}{\pi (a - 1)}.
\]

Thus here radiative corrections are finite but undetermined, so that if a physical setting for this model can be found (perhaps in a description of edge states in the quantum Hall effect), the value of \( a \) and \( m \) is fixed only by experiment.

Finally we note that for \( a \neq 0 \), a formal property of the chiral Schwinger model is abandoned: owing to the two-dimensional identity \( \gamma^5 \gamma^\mu = \varepsilon^{\mu\nu} \gamma_\nu \), the gauge field \( A_\mu \) enters (3) only on the combination \( (g^{\mu\nu} + \varepsilon^{\mu\nu}) A_\nu \), which is consistent with the unique, absorptive part of (4), but not with the real part. [In the present context, this corresponds to abandoning the tracelessness of \( \Pi_{S}^{\mu\nu}(p) \) for the vector Schwinger model.]
4 Second example: Triangle graphs

My second example of finite but undetermined radiative corrections is even older – I recall the massless, fermionic triangle-loop graphs in four dimensions with vector, vector, and axial vector vertices: $\Gamma^{\mu \nu \alpha}(p, q)$. (The incoming vector momenta are $p^\mu$ and $q^\nu$, while the outgoing axial vector momentum is $p^\alpha + q^\alpha$.) Because three fermion propagators determine the triangle, the Feynman graphs are (superficially) linearly divergent (even though an eventual evaluation, relying on a Lorentz-invariant calculation, yields a finite answer). However, owing to the linear divergence, shifting the integration momentum in the closed loop changes the value of the integral, so that there is an essential ambiguity in $\Gamma^{\mu \nu \alpha}(p, q)$: an evaluation of the integral produces some preferred form, plus an undetermined contribution proportional to $\varepsilon^{\alpha \beta \mu \nu} (p - q)_\nu$:

$$\Gamma^{\mu \nu \alpha}(p, q) \sim \Gamma^{\mu \nu \alpha}(p, q) + i a \varepsilon^{\mu \alpha \beta \nu} (p - q)_\beta .$$  

Here $a$ is a dimensionless constant, controlling the magnitude of an arbitrary local part. Turning to symmetries/formal properties to fix $a$, I try to make use of the conservation of the vector current and (since fermions are massless) of the axial vector current, thereby requiring transversality of $\Gamma^{\mu \nu \alpha}(p, q)$ in each index. But as is well known, for no value of $a$ can this condition be satisfied, and this is another face of the four-dimensional chiral anomaly. The situation is completely analogous to the chiral Schwinger model. So we must abandon some of the formal properties, and settle for transversality in the vector indices or in the axial vector, but not in all three. Moreover, the calculation of the radiative correction cannot decide which option to choose – this must come from elsewhere in the theory. In other words, the “correct” answer for the triangle graph is not intrinsic to it, but depends on the context in which it arises. Thus, for example, when the vector indices couple to photons and refer to gauge currents, while the axial vector refers to a global chiral symmetry, the choice is made to preserve transversality of the vector indices and to abandon axial-vector transversality. This is the situation for $\pi^0 \rightarrow 2\gamma$ decay. On the other hand, in the standard model of particle physics, when axial vertices are part of the chiral coupling to gauge fields and a vector index refers to a global fermion-number current, transversality of the former rather than the latter is enforced. This is the situation with ’t Hooft’s celebrated calculation of proton decay in the standard model.
I must emphasize that both Pauli-Villars and a specific dimensional regularization \cite{footnote} preserve vector gauge invariance. Nevertheless, as explained above, this need not be the correct choice if chiral invariance should be enforced.

Note that once a decision is made about which symmetry (transversality) should be preserved, a unique value for $\Gamma(p,q)$ is established. An arbitrary value persists only when no symmetry is enforced.

5 Third example: Induced Lorentz-PTC symmetry breaking

It is known that if one adds to conventional four-dimensional Maxwell electrodynamics the Lorentz- and PTC-violating Chern-Simons term

$$\Delta L = \frac{1}{2} c_\mu \varepsilon^{\mu\alpha\beta\gamma} F_{\alpha\beta} A_\gamma$$

where $c_\mu$ selects a fixed direction in spacetime, light from distant galaxies undergoes a Faraday-like rotation \cite{footnote}. Observation of distant galaxies puts a stringent bound on this “vacuum birefringence”: $c_\mu$ should effectively vanish \cite{footnote, footnote}. An important feature of the Chern-Simons term is that its Lagrange density is not gauge invariant: $\Delta L$ depends on $A_\mu$. However, the action, the spacetime integral of the density, is gauge invariant because under a gauge transformation $\Delta L$ changes by a total derivative. Correspondingly, the Euler-Lagrange equations remain gauge invariant, even in the presence of the gauge-noninvariant Chern-Simons density \cite{footnote}.

A natural question is whether such a term could be induced through radiative corrections when Lorentz and PTC symmetries are violated in other parts of a larger theory.

For definiteness, consider the fermionic Lagrange density of conventional QED, extended to include a Lorentz- and PTC-violating axial vector term \cite{footnote, footnote, footnote}.

$$\mathcal{L}_{\text{extended}} = \overline{\psi} \left( i \gamma \cdot \partial - e A - m - \gamma_5 \right) \psi$$

where $\gamma_5$ is Hermitian with $\text{tr} \gamma_5 \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta = 4 i \varepsilon^{\alpha\beta\gamma\delta}$ and $b_\mu$ is a constant, prescribed 4-vector. One is then led to enquire whether the effective action

$$\Gamma(A) = -i \ln \det \left( i \gamma \cdot \partial - e A - m - \gamma_5 \right)$$

(13)
contains the Chern-Simons term $\Gamma^\mu_\alpha$ with $c_\mu$ determined by $b_\mu$. Since $\Gamma^\mu_\alpha$ is bilinear in $A_\mu$, I examine only the $O(A^2)$ portion of $\Gamma(A)$.

A plausible approach is to calculate the lowest order in $b_\mu$. But then it is clear that one again encounters triangle graphs, with two vector vertices coupling to the two $A_\mu$'s and the axial vertex contracted with $b_\mu$. Moreover, the axial vector carries zero momentum: in the notation of the previous subsection $p = -q$, and the relevant amplitude is $b_\alpha \Gamma^{\mu\alpha}(p, -p)$ where now $\Gamma^{\mu\alpha}(p, q)$ is calculated with massive fermions. The coefficient of the induced Chern-Simons terms is determined by $b_\alpha \Gamma^{\mu\alpha}(p, -p)|_{p^2 = 0}$.

It follows from the property of the triangle graphs, which I reviewed previously, that $\Gamma^{\mu\alpha}(p, -p)$ is undetermined as in (10):

$$\Gamma^{\mu\alpha}(p, -p) \sim \Gamma^{\mu\alpha}(p, -p) + 2i\alpha \varepsilon^{\mu\nu\alpha\beta} p_\beta .$$

(14)

Of course gauge invariance, that is, transversality of $\Gamma^{\mu\alpha}(p, -p)$ to $p_\mu$ and $p_\nu$, must hold. But unlike the case when the axial vector carries nonvanishing momentum, $\Gamma^{\mu\alpha}(p, -p)$ is transverse in its photon indices for all routings of the integration momentum, as is also seen from the form of the ambiguity: $\varepsilon^{\mu\nu\alpha\beta} p_\beta$ is transverse to $p_\mu$ and $p_\nu$. Therefore, the requirement of gauge invariance does not fix a value for the graphs in the present context.

The conclusion is that the radiatively induced Chern-Simons term, to lowest order in $b_\mu$, is as in (11), with $c_\mu$ proportional to $b_\mu$, but the numerical proportionality constant is undetermined when the theory is considered perturbatively in $b_\mu$ [2, 11].

What is the situation with regulators? Pauli-Villars regularization removes the induced Chern-Simons term. This is true because the induced undetermined coefficient is a mass-independent pure number, and an equal amount is subtracted by Pauli-Villars regulators. Dimensional regularization is problematic in the presence of a $\gamma_5$ matrix, and a variety of results can be obtained when a variety of prescriptions is made for the dimensional generalization of $\gamma_5$. One can arrange a dimensional extension [7] so that the induced Chern-Simons coefficient vanishes.

However, these regulators are inappropriate for the following subtle reason [2]. We are seeking an induced density, $\Gamma^\mu_\alpha$, which is not gauge invariant, while its spacetime integral is gauge invariant. An alternative, equivalent momentum-space statement is that we seek a term that is gauge invariant at zero momentum,
but not at arbitrary momenta. Pauli-Villars regularization and gauge-invariant dimensional regularization enforce gauge invariance at all momenta, and therefore exclude \textit{a priori} the generation of a Chern-Simons term, which does not have this property. These regulators are not sufficiently fine to enforce gauge invariance at zero momentum only. Thus an undetermined but finite answer remains.

Is there any other criterion that can be brought to bear on the problem? Coleman and Glashow \cite{12} have suggested that the axial vector current be regulated so it is gauge invariant at all momenta. This gives a unique result for the induced Chern-Simons term – it vanishes. However, as stated above, such a regularization requirement (which can be implemented by Pauli-Villars or dimensional procedures \cite{4}) is \textit{ad hoc} and unjustified since the axial vector enters into the theory only at zero momentum (it couples to an external constant, $b_\mu$). Demanding gauge invariance for all momenta is equivalent to demanding gauge invariance for the unintegrated densities, and this would exclude \textit{a priori} the Chern-Simons term, which is not a gauge-invariant density.

Curiously, a unique answer does emerge when the interaction $\overline{\psi} \gamma_5 \psi$ is treated nonperturbatively. Note that the $b_\mu$-exact, $O(A^2)$ contribution to the effective action is determined by the vacuum polarization tensor constructed with $b_\mu$-exact fermion propagators

\begin{equation}
\Pi^{\mu\nu}(p) = \text{tr} \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \frac{i}{k - m - \gamma_5} \gamma^\nu \frac{i}{k + \gamma - m - \gamma_5}.
\end{equation}

The $b_\mu$-exact propagator

\begin{equation}
G(p) = \frac{i}{k - m - \gamma_5}
\end{equation}

may be decomposed as

\begin{equation}
G(p) = S(p) - iG(p)\gamma_5 S(p) \equiv S(p) + G_b(p)
\end{equation}

where $S(p)$ is the free propagator

\begin{equation}
S(p) = \frac{i}{\gamma - m}.
\end{equation}
With decomposition $\Pi^{\mu\nu}(p)$ splits into three terms

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}_0 + \Pi^{\mu\nu}_b + \Pi^{\mu\nu}_{bb}.$$  \hspace{1cm} (19)

$\Pi^{\mu\nu}_0$ is the usual, lowest-order vacuum polarization tensor of QED, which I shall not discuss further. $\Pi^{\mu\nu}_{bb}$, containing $G_b$ twice, is at least quadratic in $b$; it is at most logarithmically divergent and suffers no ambiguities in routing the internal momenta. The $b_\mu$-linear contribution to the induced Chern-Simons term arises from $\Pi^{\mu\nu}_b$, which is explicitly given by

$$\Pi^{\mu\nu}_b(p) = i \text{tr} \int \frac{d^4k}{(2\pi)^4} \left\{ \gamma^\mu S(k)\gamma^\nu G_b(k + p) + \gamma^\mu G_b(k)\gamma^\nu S(k + p) \right\}.$$  \hspace{1cm} (20)

There are several important features of this expression. Each of the two integrals is (superficially) linearly divergent. However, the divergences cancel when the two terms are taken together and the traces are evaluated. As a consequence, there is no momentum-routing ambiguity in the summed integrand. When the integration momentum is shifted by the same amount on both integrands, the value of the integral does not change, even though shifting separately by different amounts in each of the two integrands changes the value of the integral by a surface term. It follows that different momentum routings in the entire integrand leave the value of $\Pi^{\mu\nu}_b$ unchanged, because they produce a simultaneous shift of integration variable by the same amount in each of the two integrands. Therefore, a unique value can be attached to $\Pi^{\mu\nu}_b(p)$.

Next we evaluate $\Pi^{\mu\nu}_b(p)$ to lowest order in $b_\mu$, by replacing $G_b(k)$ with $-i S(k) \gamma_5 S(k)$. This gives $\Pi^{\mu\nu}_b(p) \simeq b_\mu \Pi^{\mu\alpha\nu\alpha}(p)$, where

$$\Pi^{\mu\alpha\nu\alpha}(p) = \text{tr} \int \frac{d^4k}{(2\pi)^4} \left\{ \gamma^\mu S(k)\gamma^\nu S(k + p)\gamma^\alpha\gamma_5 S(k + p) \right\}.$$  \hspace{1cm} (21)

When Lorentz invariance is enforced, this integral may be evaluated unambiguously. It leads to the finite result

$$\Pi^{\mu\alpha\nu\alpha}(p) = -2i\varepsilon^{\mu\nu\alpha\beta} p_\beta \left( \frac{1}{8\pi^2} + \frac{2}{\pi^2} \int_2^{\infty} \frac{d\sigma}{\sqrt{\sigma^2 - 4m^2}} \frac{1}{\frac{\sigma^2 - \sigma^2 - 4m^2}{4}} \right) = i\varepsilon^{\mu\nu\alpha\beta} p_\beta \left( \frac{\theta}{\sin \theta} - \frac{1}{4} \right).$$  \hspace{1cm} (22)
where $\theta = 2 \sin^{-1} \sqrt{p^2/2m}$, so that

$$\Pi^{\mu\nu\alpha}(p)\big|_{p^2=0} = \frac{3i}{8\pi^2} \epsilon^{\mu\nu\alpha\beta} p^\beta$$

and the Chern-Simons coefficient is unambiguously \[2, 13\]

$$c^\mu = \frac{3}{16\pi^2} b^\mu . \quad (23b)$$

What is the difference between the present calculation and the perturbative ones that leave $c^\mu$ undetermined? Eq. (21) does indeed exhibit the vector, vector, axial vector triangle graphs, with zero 4-momentum in the axial vertex, as in the $b_\mu$-perturbative calculation. However, here the two triangle graphs have their integration momenta routings correlated, since they descend from the single, $b_\mu$-exact formula (15). In $b_\mu$-perturbation theory, no correlation is determined \textit{a priori} between the momentum routings of the two graphs. If in the perturbative calculation the relative routings are as in (21), the resulting expression coincides with (22). Otherwise a shift of integration variables in one integrand relative to the other produces the configuration (21), but generates a surface contribution. Therefore, a $b_\mu$-perturbative calculation gives (22) with an undetermined polynomial contribution

$$\Pi^{\mu\nu\alpha}(p) = -i\epsilon^{\mu\nu\alpha\beta} p^\beta \left( \frac{a}{2\pi^2} + \frac{2}{\pi^2} \int_2^{\infty} \frac{m^2}{\sigma^2 - 4m^2} \frac{1}{p^2 - \sigma^2 + i\varepsilon} \right)$$

$$= i\epsilon^{\mu\nu\alpha\beta} \frac{p^\beta}{2\pi^2} \left( \frac{\theta}{\sin \theta} - a \right)$$

leading to a Chern-Simons density with strength

$$c^\mu = \frac{1}{4\pi^2} (1 - a) b^\mu .$$

Note that gauge invariance is preserved for all $a$, in the sense that $p_\mu \Pi^{\mu\nu\alpha}(p) = 0$. (In momentum space one does not see the gauge noninvariance of the position-space Chern-Simons density.)

As in our other examples, the arbitrary term is a local contribution to the effective action \[13\] and in a dispersive representation, as in (24), it contributes a real subtraction, even though the unique absorptive part permits presenting an unsubtracted integral.
An open question remains whether the unique result obtained by the non-perturbative evaluation has any deeper significance.

[The demand that triangle graphs be evaluated so that they are gauge invariant with arbitrary momentum in the axial vector vertex [12], can be met by a particular routing of the integration momentum [2, 3] or by Pauli-Villars regularization, or by a particular dimensional regularization [7]; one finds \( a = 1 \) and \( c^\mu = 0 \). But there is no \textit{a priori} reason for placing this requirement on the theory, and one may view its consequence (\( c^\mu = 0 \)) as tautological, since a Chern-Simons term does not enjoy such a strong form of gauge invariance.]

Experiment effectively prohibits a Chern-Simons term in QED, and I conclude that the conjectural noninvariant addition to fermion dynamics, \( \bar{\psi} \gamma_5 \psi \), is absent altogether (if the nonperturbative approach is taken) or that the ambiguity is fixed so that \( c^\mu = 0 \) (if the undetermined perturbative calculation is taken). When several different fermion species participate in the Lorentz- and PTC-violating interaction, another possibility emerges: contributions from different fermions may sum to zero [11].

6 Conclusion

The various radiative corrections that I surveyed (\( g - 2 \), induced mass in the Schwinger and chiral Schwinger models, triangle graphs of the chiral anomaly, induced Lorentz and PTC-violating Chern-Simons term) behave variously in their calculability. Can one formulate a criterion that will settle \textit{a priori} whether the radiative correction produces a definite or indefinite result? The above examples suggest the following rule: If the form of the radiative correction is such that inserting it into the bare Lagrangian would interfere with symmetries of the model or would spoil renormalizability, then the radiative result will be finite and uniquely fixed. Alternatively, if modifying the bare Lagrangian by the radiative correction preserves renormalizability and retains the symmetries of the theory, then the radiative calculation will not produce a definite result – it is as if the term in question is already present in the bare Lagrangian with an undetermined strength, and the radiative correction adds a further undetermined contribution. With this criterion, the radiatively induced \( g - 2 \) Pauli term in QED and the photon mass in the Schwinger model are unique, because in the former case a Pauli
term in the bare Lagrangian would spoil renormalizability, and in the latter, a photon mass term would spoil gauge invariance. Correspondingly, in the quantified chiral Schwinger model, the chiral anomaly spoils gauge invariance, so there is no symmetry prohibiting inserting a photon mass in the bare Lagrangian, and indeed the radiatively induced mass is undetermined. Similarly with the symmetry-violating Chern-Simons term: ignoring experimental constraints and allowing symmetry violation in the fermion sector permits insertion of a Chern-Simons term in the Lagrange density, so the radiatively induced Chern-Simons term can have an arbitrary coefficient. (The subtlety here is that one must allow for calculational schemes that permit gauge noninvariance on the level of a position-space density.) Finally, the triangle graphs of the axial vector anomaly take on a unique value once it is decided which symmetries are preserved, vector or chiral. In either case they induce a process that, if inserted as a vertex in the bare Lagrangian, would destroy renormalizability: the induced $\pi^0$ decay vertex is $\pi^0 \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$ and the induced baryon decay amplitude involves high powers of Fermi fields. Thus the radiatively induced values are unique.

When a radiative correction is infinite, there must be a term in the bare Lagrangian of similar form to absorb the infinity through renormalization. The finite but undetermined radiative corrections, which I have discussed, are seen to be similar to the infinite ones, and their finite value does not enhance the predictability of the theory.

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