Explosive synchronization in dynamic networks: A comparative study

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We present a comparative study on Explosive Synchronization (ES) in dynamic networks consisting of phase oscillators. Dynamic nature of the networks has been modeled with two configurations: (1) oscillators are allowed to move in a closed two dimensional box such that they couple with their neighbors, (2) oscillators are static and they randomly switch their coupling partners. (1) has been further studied under two possible scenarios: in the first case oscillators couple to fixed numbers of neighbors while in other they couple to all oscillators lying in their circle of vision. Under these circumstances, we monitor the degrees of dynamic networks, velocities and radius of circle of vision of the oscillators, and probability of forming connections in order to study and compare the critical values of the coupling required to induce ES in the population of phase oscillators.

I. INTRODUCTION

Synchronization is a phenomenon wherein simultaneous evolution in the dynamics of oscillators is observed by the virtue of coupling between them. After its discovery by Christian Huygens by performing an experiment with a pair of pendulum clocks hung on a common supporting beam, studies of synchronization remained dormant for a long period of time. The pioneering work of phase synchronization in chaotic oscillators in 1996 \cite{1} motivated scientists to explore synchronization in different systems. Since then, synchronization and its manifestations have been studied in diverse systems found in nature \cite{2-5} as well as in model experimental \cite{6-9} and numerical systems \cite{10-12}.

In their pioneering work Kuramoto et al. studied synchronization in a network of globally coupled phase oscillators and a second order transition from unsynchronized to synchronized state was observed. This was followed by a plethora of studies for synchronization in networks involving phase oscillators and other dynamical systems. Chimera state is a manifestation of synchronization in networks where the coupling is employed between neighbors of the oscillators \cite{13, 14}; this coupling scheme results in partial synchronization of the population of oscillators. Synchronization has also been reported in random networks wherein the oscillators randomly couple to each other \cite{15}. Finally, synchronization has also been reported in dynamic networks. The topology of a dynamic network changes over time \textit{i.e.} the oscillators keep forming new connections with other oscillators while maintaining and losing some of the old connections. Two different variations of dynamic networks have been studied and reported: in one case, the oscillators remain static while switching couplings with other oscillators and in the other case, the oscillators move in a given space and they couple to their nearby oscillators \cite{16, 17}. In nature, dynamic networks can be observed in a population of fireflies where a firefly observes the blinking of its neighbors and adjust its own frequency of blinking. The adjustment of rhythms by the entire population of fireflies results in the emergence of synchronized behavior. Dynamical networks also have its importance in studying synchronized dynamics of a population of robots in order execute a collective task.

ES is a manifestation of synchronization in a population of interacting oscillators which is characterized by a first order transition from unsynchronized to synchronized state when the strength of interaction ($\epsilon$) between the oscillators is gradually increased. Presence of hysteresis is another property which is associated with the first order transition in ES. This signifies that the value of coupling strength ($\epsilon = \epsilon_l$) at which oscillators unsynchronize while decreasing coupling between oscillators is lower than the critical value of coupling strength ($\epsilon = \epsilon_u$) to achieve synchronization while increasing coupling. The principle mechanism behind this phenomenon is to suppress the formation of synchronization clusters which eventually results in the global synchronization of the population. In the initial works on ES, it has been shown that this suppression can be achieved by implementing positive correlation between the natural frequencies of the oscillators and their degrees \cite{18, 19}. Following this article, several other mechanisms to induce first order transition have been reported \cite{20, 21}. In another report by Zhang et al. \cite{22}, the role of local order parameter of the oscillators in achieving ES has been studied. In this study, the dynamics of every oscillator is influenced by its local order parameter; this reduces the effective coupling between oscillators as the local order parameter started to augment, which further causes the suppression of synchronization clusters. Eventually, when the coupling strength between oscillators is sufficiently high (higher than critical $\epsilon$ to observe second order Kuramoto transition), ES can be observed in the population of oscillators. In another work \cite{23} this concept has been extended to modify the width of hysteresis loop formed during ES. The mechanism of ES has also been intended to explain a neurological disease called Fibromyalgia \cite{24}. A functional network from the EEG signals of the patients
was constructed and analyzed to identify the imprints of ES. Finally, ES has also been reported in a model experimental system consisting of Mercury Beating Heart oscillators [25].

In this work, we present a comparative study of ES in dynamical networks. Three different configurations have been considered to employ dynamic coupling in a population of phase oscillators. In one of the configurations, the oscillators remain static and randomly couple to other oscillators such that the average degree of the networks forming at any time moment fluctuates around a mean value. In other two configurations, the oscillators execute random walk in a two dimensional closed space and couple to their neighbors; degrees of the oscillators for these cases are further decided on the basis of two different schemes (discussed later). Depending on the configuration, parameters like coupling strength, velocity and vision size of oscillators and probability of forming a connection between two oscillators have been varied to study ES.

II. COUPLING MECHANISMS

We consider a population of \( N \) coupled phase oscillators to study ES in dynamical networks. A population of coupled phase oscillators is generally represented with the following equation:

\[
\frac{d\phi_p}{dt} = \omega_p + \epsilon \sum_{q=1}^{N} A_{pq} \sin(\phi_q - \phi_p).
\]

Here, \( \phi_p \) and \( \omega_p \) are the phase and the frequency of the \( p^{th} \) oscillator, \( \epsilon \) is the coupling strength between the oscillators, and \( A_{pq} \) is an element of adjacency matrix \( A \) giving details about the coupling links between the oscillators; \( A_{pq} = 1 \) if \( p^{th} \) and \( q^{th} \) are coupled and \( A_{pq} = 0 \) otherwise. Degree of an oscillator can be calculated from \( k_p = \sum_{q=1}^{N} A_{pq} \). The extent of synchronization of the population can be calculated using the following expression:

\[
r_{e^{i\Phi}} = \frac{1}{N} \sum_{q=1}^{N} e^{i\phi_q}.
\]

Here, \( 0 \leq r \leq 1 \) represents the extent of synchronization and \( \phi \) represents the average phase of the population.

Our scheme of implementing coupling in phase oscillators is similar to that proposed by [23] where the dynamics of the oscillators are influenced with their local order parameter. Under such circumstances, the model equation of the coupled population that we use is:

\[
\frac{d\phi_p}{dt} = \omega_p + \epsilon r \sum_{q=1}^{N} A_{pq} \sin(\phi_q - \phi_p).
\]

Similar to Eq. (2), local order parameter of an oscillator is defined as:

\[
r_{e^{i\Phi_p}} = \frac{1}{k_p} \sum_{q=1}^{N} A_{pq} e^{i\phi_q}.
\]

\( \Phi_p \) is the average phase of the oscillators coupled to \( p^{th} \) oscillator. Frequencies of the oscillators (\( \omega \)) have been chosen from a uniform distribution of numbers lying between 0-2. Finally, to simulate dynamic networks the instantaneous adjacency matrices \( A_{pq} \) have been obtained using following three configurations:

- **Configuration 1**: Moving oscillators with nearest neighbor coupling such that every oscillator of the population couple to same number of nearest oscillators i.e. \( k \) of every oscillators remains equal and constant.

- **Configuration 2**: Moving oscillators with nearest neighbor coupling such that every oscillator has a circular vision size and the oscillator couple to all oscillators lying in its vision. In this case \( k \) depends on the radius (\( R \)) of the circle of vision.

- **Configuration 3**: Static oscillators where every oscillator randomly couples to other oscillators such that the average degrees of the dynamic networks remains same throughout time. In this case, \( k \) depends on the probability (\( p \)) with which oscillators couple with each other.

![FIG. 1: Demonstration of instantaneous locations of \( N = 500 \) phase oscillators in the \( x-y \) plane along with the trajectories of oscillators with \( v = 0.002 \) (magenta), \( v = 0.05 \) (yellow), and \( v = 0.1 \) (green) curves and also the circle of vision (black) of one of the oscillators (red dot) with \( R = 0.1 \).](image-url)
In the case of configurations involving moving oscillators, their motion has been confined into a unit size two dimensional box with rigid boundaries. Oscillators are permitted to move a $\delta$ step in either $\pm x$ or $\pm y$ direction at every iteration. Value of the step moved by oscillators also defines their velocity ($v = \delta$) in the closed box. In Fig. 1 the initial positions (blue dots) of $N = 500$ oscillators have been shown for demonstration. The motion of three random oscillators with different velocities has been represented with magenta line ($v = 0.002$), yellow line ($v = 0.05$), and green line ($v = 0.1$). A circle of vision of an oscillator (marked with red dot) belonging to configuration 2 with radius of vision $R = 0.1$ has also been shown with black circle. Further details on the dynamic aspects of networks for each configuration are discussed later when their respective results are presented. Finally, Eq. (3) has been numerically simulated using RK4 algorithm with a step size of 0.1; in [26] authors report that this step size is sufficiently small to numerically simulate a population of coupled phase oscillators.

FIG. 2: (a-c) Results of ES in dynamic networks emulated with configuration 1 for three different velocities of the oscillators: (a) $v = 0.002$, (b) $v = 0.05$, (c) $v = 0.1$. (d) Critical values of the coupling strength required to synchronize oscillators, (e) critical values of coupling strength required to unsynchronize oscillators, and (f) width of the hysteresis loops formed during ES. • ($v = 0.002$), ○ ($v = 0.05$), and ■ ($v = 0.1$).

III. RESULTS

A. Configuration 1

In this configuration the dynamical nature of the networks of phase oscillators has been modeled such that the oscillators move in a closed unit size box with uniform velocities and all of the oscillators interact with fixed number of nearest neighbors. This implies that the instantaneous degree ($k$) of all oscillators remains same and fixed. Moreover, velocities ($v$) of oscillators are equal to the step $\delta$ that the oscillators move in every iteration.

Fig. 2 shows the results of ES in phase oscillators modeled with this configuration. In Fig. 2(a-c), we plot the variation of order parameter of the entire population as a function $\epsilon$ for three different velocities of the oscillators (a): $v = 0.002$, (b) $v = 0.05$, and (c) $v = 0.1$). Moreover, for each of these cases, order parameter has been calculated and plotted for various values of $k$. It can be noted that for sufficiently small degree of the oscillators ($k = 15$ and $v = 0.002$ in Fig. 2(a)), the population of the oscillators undergoes classical second order Kuramoto transition in the order parameter. However, as $k$ increases gradually, the population experiences explosive (first order) transitions between unsynchronized and synchronized states; the transition is also accompanied...
by its characteristic hysteresis loop. For the purpose of demonstration, the critical values of coupling strength at which oscillators synchronize (unsynchronize) have been marked with $\epsilon_u$ ($\epsilon_l$) on Fig. 2(a). Furthermore, similar dependences of ES on $k$ have also been observed when the oscillators move with higher velocities: $v = 0.05$ and $v = 0.1$ (Fig. 2(b and c)).

In Fig. 2(d), variation of $\epsilon_u$ (critical value of coupling required to observe ES) has been illustrated as a function of $k$ for three different velocities; for any $v$, lowest value of $k$ is the degree of the oscillators at which ES starts to appear. It can be noted that $\epsilon_u$ increases uniformly with $k$ for every velocity. In [27, 28] Kuramoto model has been studied by the perspective of complex networks. According to this study, if the time dependent local parameter of an oscillator in the present situation is defined with the following relation:

$$r_p^t e^{i\phi_p} = \sum_{q=1}^{N} A_{pq} \langle e^{i\phi_q} \rangle_t,$$  \hspace{1cm} (5)

where, $\langle \cdots \rangle_t$ is the time average, then, the state of an oscillator can be represented as:

$$\frac{d\phi_p}{dt} = \omega_p + \frac{e r_p^t \cdot \sin(\Phi_p - \phi_p)}{h_p} - c \epsilon_h.$$ \hspace{1cm} (6)

Here, $\Phi_p$ is the average phase of the oscillators coupled to $p^{th}$ oscillator and $h_p$ accounts for the time fluctuations in the dynamics of this oscillator by the virtue of dynamic adjacency matrices. The mathematical form of $h_p$ is given by $h_p = \text{Im} \{ e^{-i\phi_p} \sum_q A_{pq} \langle e^{i\phi_q} \rangle_t - e^{i\phi_p} \}$, where “Im” stands for imaginary and $\langle \cdots \rangle_t$ is the time average.

During the onset of synchronization, $r_p = r_p^{k \to k_p}$ and $h_p$ is expected to be of the order of $\sqrt{k_p}$ ($k_p$ is the degree of $p^{th}$ oscillator). Therefore, as the degree of the oscillators of the moving population increases, these fluctuations also increase causing the suppression of synchronization clusters (as explained in the introduction section) and requiring even larger $\epsilon$ for the population to synchronize. Another interesting behavior that can be observed in Fig. 2(d) is that the minimum value of $k$ at which ES starts to appear reduces as the velocities of the oscillators increase. The instantaneous time spent by an oscillator in a local cluster decreases as its velocity increases, causing less interaction among the members of the clusters. The results of Fig. 2(d) show that when the degree of oscillators is large then due to higher heterogeneity in the cluster, it would require more time or higher coupling strength to synchronize at higher velocities. Conversely, it can be said that the as the velocity of the oscillators increase, the population of the oscillators can exhibit ES at lower degrees. In Fig. 2(e) variation of $\epsilon_l$ (critical value of coupling at which oscillators unsynchronize) as a function of $k$ for three different velocities has been shown. While $\epsilon_l$ decreases with $k$ for $v = 0.002$, it does not show any particular trend for higher velocities. Furthermore, the critical values of $\epsilon_u$ and $\epsilon_l$ constitutes the hysteresis loops of ES and in agreement to results of Fig. 2(d and e), width of the hysteresis loops increases with $k$ (Fig. 2(f)).

![Fig. 3: (a) Results of ES in dynamic networks emulated with configuration 1 for $k = 15$. (b) Critical values of the coupling strength required to synchronize ($\epsilon_u$: ●), and critical values of coupling strength required to unsynchronize oscillators ($\epsilon_l$: ○). (c) width of the hysteresis loops formed during ES.](image)

From Fig. 2(a) it can be observed that when $k = 15$, ES does not exhibit in the population of oscillators for $v = 0.002$ but it appears at higher velocities for the same value of $k$ (Fig. 2(b and c)). To study this transition, we vary velocities of the oscillators keeping $k = 15$ and the results of explosive transitions are shown in Fig. 3(a). It can be observed that the oscillators exhibit ES as their velocities increase from $v = 0.02$ to $v = 0.03$. Furthermore, in Fig. 3(b), variation of $\epsilon_u$ has been shown as a function of $v$. In [17] it has been reported that similar to coupling strength ($\epsilon$), velocity of the oscillators also acts as a parameter to observe second order Kuramoto transition of synchronization. In the present case, when velocities of the oscillators augment, it results in the formation of synchronization clusters. From the analysis of dependence of $\epsilon_u$ on $k$ (Eq. (6)), it can be said that the local order parameter of the oscillators also depends on the velocities of the oscillators and it explains the increasing nature of $\epsilon_u$ with $v$ (Fig. 3(b)). Finally, variations of $\epsilon_l$ and width of the hysteresis loops have been shown on Fig. 3(b and c) and they are identical to their respective results in Fig. 2(e and f).
B. Configuration 2

In this section, the results of ES in dynamical networks obtained using configuration 1 have been compared with another configuration in which coupling has been implemented such that every moving oscillator interacts with other oscillators lying in its vision circle (radius: \( R \)). As a consequence, degree of oscillators \( k \) in this case depends on \( R \), does not remain fixed in time, and could be different for every oscillator. Similar to previous subsection, results have been obtained by varying \( R \), keeping \( v \) constant and vice versa.

![FIG. 4: (a) Results of ES in dynamic networks emulated with configuration 2 for \( v = 0.002 \). (b) Critical values of the coupling strength required to synchronize \( (\epsilon_u): \text{circles} \), and critical values of coupling strength required to unsynchronize oscillators \( (\epsilon_l): \text{stars} \), (c) width of the hysteresis loops formed during ES. \( (v = 0.002) \), \( (v = 0.05) \), and \( (v = 0.1) \).](image)

In Fig. 4(a), hysteresis loops for different values of \( R \) have been shown for \( v = 0.002 \). It can be observed that for smaller \( R \) ES is not observed and that it starts to appear as \( R \) increases. The variation of \( \epsilon_u \) and \( \epsilon_l \) as a function of \( R \) have been shown in Fig. 4(b) for \( v = 0.002 \), \( v = 0.05 \), and \( v = 0.1 \). It must be noted that, while \( \epsilon_u \) for \( v = 0.002 \) and \( \epsilon_l \) for all velocities show similar trend as their counterparts in Fig. 2(d and e), the response of \( \epsilon_u \) for \( v = 0.05 \) and for \( v = 0.1 \), however, is different. It shows that the critical value of \( \epsilon \) to achieve synchronization of oscillators moving with larger velocities show an initial fall before starting to increase with \( R \). Given that Eq. (6) explains the relationship between \( \epsilon_u \) and \( k \), this result deviates for smaller \( R \) \( (k \propto R^2) \) from its analogous results of Fig. 2(d) where the critical \( \epsilon_u \) for ES uniformly increased with \( k \). The possible reason for this deviation lies in the fact that in the present case \( k \) of oscillators is not constant. However, detailed theoretical and/or numerical analysis needs to be carried out to understand this behavior. Finally, in Fig. 4(c), width of the hysteresis loops as a function of \( R \) have been shown for different velocities and it can be observed that the width increases with \( R \).

For the purpose of completion, in Fig. 5, results of ES of moving oscillators have been shown by varying \( \epsilon \) and \( v \): keeping \( R \) of the oscillators fixed at 0.1. Results obtained in this case are identical to those presented in Fig. 3, where it was shown that the oscillators do not exhibit ES when they move slowly and at higher velocities width of the hysteresis loops of the ES increases as the velocities of the oscillators increase.

![FIG. 5: (a) Results of ES in dynamic networks emulated with configuration 2 for \( R = 0.1 \). (b) Critical values of the coupling strength required to synchronize \( (\epsilon_u): \text{circles} \), and critical values of coupling strength required to unsynchronize oscillators \( (\epsilon_l): \text{stars} \), (c) width of the hysteresis loops formed during ES.](image)

C. Configuration 3

In the final scenario, coupling has been implemented such that at every iteration the oscillators randomly switch coupling between each other. This signifies that coupling between two oscillators does not depend on the physical distance between them and that the degree of the oscillators changes in every iteration. If the probability with which an oscillator couples to another while switching coupling is given by \( p \) then the average degree of the network will be \( k = pN \). Moreover, when an oscillator switches its coupling partners, the algorithm does not preclude that this oscillator cannot couple again to same oscillators it was coupled in the previous iteration.

Fig. 6 shows the results of ES as a function of \( \epsilon \) for different values of \( p \). Similar to previous results, for a
sufficiently smaller $p$ (or $k$), ES is not observed in the population of oscillators and as $p$ increases, ES appears. In Fig. 6(b), variations of $\epsilon_u$ and $\epsilon_l$ have been shown. Surprisingly, similar to the result for $v = 0.05$ and $v = 0.1$ in Fig. 4(b), $\epsilon_u$ in the present case, initially decreases with $p$ before starting to increase. On one hand, a possible explanation of this behavior can be ascribed to the fact that the oscillators in this case switches coupling at every iteration and the coupling also does not depend on the physical distance between the oscillators. This is tantamount to the situation that the oscillators in this case are moving with very high velocities and frequently changing their coupling partners. On the other hand, the reason for the similarity of these results only with the corresponding results of configuration 2 suggests that the reason behind this behavior lies in the fact that unlike configuration 1, in configuration 2 and 3, degrees of the oscillators does not remain constant. However, we modified configuration 1 by introducing fluctuations in the degrees of the oscillators, but the behavior of $\epsilon_u$ similar to that in Fig. 4(b) and Fig. 6(b) was not obtained. Therefore, initial fall of $\epsilon_u$ with $k$ remains unclear. Finally, in Fig. 6(c), variation of width of the hysteresis loop has been shown as a function of $p$ and it increases monotonically with $p$.

IV. CONCLUSIONS

We presented our results on ES in dynamical networks of phase oscillators. The results were obtained and compared by implementing three different configurations in order to achieve dynamical nature of the oscillators. In two of these configurations, the oscillators were changing their coupling partners depending on the distances between them while in the third coupling partners were switched randomly (irrespective of physical distances). Different control parameters were monitored in order to observe and analyze ES in the oscillators. Using Eq. (6) the analytical understanding of the dependence of ES on $k$ and $v$ was established.

The most striking differences in three configurations were observed when degrees of the oscillators ($k \propto R^2$ in configuration 2 and $k \approx pN$ in configuration 3) were varied. In configuration 1 the critical value of coupling to observe ES ($\epsilon_u$) increased uniformly with $k$. However, in configuration 2 and 3, before starting to rise for larger values of $k$, $\epsilon_u$ decreased initially as $k$ started to increase from its lower values. Moreover, this behavior was observed only in the situations when the oscillators were moving with large velocities. It was also observed that $\epsilon_l$ decreased and the width of the hysteresis loops increased with $k$ in all the configurations.

In our future work we will explore ES by implementing dynamical nature of coupling in experimental systems. Configuration 1 and 2 although are more realistic to observe in nature, but they are difficult to realize in model experimental nonlinear oscillators. Configuration 3, however, can be established easily and ES can be explored. The fact that ES has already been reported in an experimental system consisting of static Mercury Beating Heart oscillators [25] makes this system a potential candidate to study ES in the present circumstances.

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