GIMPs from Extra Dimensions

Martin Holthausen\textsuperscript{1} and Ryo Takahashi\textsuperscript{2}

Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany

Abstract

We study a scalar field theory in a flat five-dimensional setup, where a scalar field lives in a bulk with a Dirichlet boundary condition, and give an implementation of this setup to the Froggatt-Nielsen (FN) mechanism. It is shown that all couplings of physical field of the scalar with the all brane localized standard model particles are vanishing while realizing the usual FN mechanism. This setup gives the scalar a role as an only Gravitationally Interacting Massive Particle (GIMP), which is a candidate for dark matter.

\textsuperscript{1}E-mail: martin.holthausen@mpi-hd.mpg.de
\textsuperscript{2}E-mail: ryo.takahashi@mpi-hd.mpg.de
The Large Hadron Collider (LHC) experiments is just being started. One of the important missions of this big project is the discovery of the Higgs particle. The coupling to this particle is the origin of fermion masses in the standard model (SM) and it plays a crucial role in the electroweak (EW) symmetry breaking. The SM will be completed as a renormalizable theory when the Higgs is discovered. However, a theoretical problem still exists within the SM which is so-called the gauge hierarchy problem about the big desert between the EW and Planck scales. Theories with additional space dimensions are interesting approaches towards solving this problem [1, 2]. Generally, extra dimensional theories lead to rich phenomenologies, for instance, new heavy particles with masses of the compactification scale, the so-called Kaluza-Klein (KK) particles. In the Universal Extra Dimensions (UED) model [3], the lightest KK particle with an odd parity is stable, and can be a candidate for the dark matter. The compactification scale is generally constrained to be larger than a few TeV by the EW precision measurements for the brane localized fermion scenario [4]-[8] but it can be weakened to a few hundred GeV in the UED case due to the five-dimensional Lorentz symmetry [3, 9, 10]. Another interesting phenomenological consequence is the top Yukawa deviation, which is a deviation of the Yukawa coupling between top and physical Higgs fields from naive SM expectation, induced from an existence of the brane localized Higgs potentials [11, 12] leading to Dirichlet type boundary conditions [13], and the $SO(5) \times U(1)$ warped gauge-Higgs unification model [14]. The LHC experiment may give some suggestions for the TeV or few hundred GeV scale KK resonances and resultant phenomena induced from bulk fields.

On the other hand, the flavour problem, that is the origin of the three generations of the SM fermions and their Yukawa couplings, which determine the masses and mixings, is also one of the most important problems in the SM. A fascinating approach to explain the flavour structure of the SM is to introduce some flavour symmetries broken at a high energy scale by an additional scalar (Higgs) field, the so-called flavon [15]. Such a field is introduced in a number of flavour models, e.g. for the purpose of realizing tri-bimaximal generation mixing [16] via non-abelian discrete flavour symmetries [17], etc..

In this letter, we study a scalar field theory in a five-dimensional setup, where a scalar field lives in a bulk with a Dirichlet boundary conditions. We then further discuss implementations of this setup to the Froggatt-Nielsen (FN) mechanism.

We consider a complex scalar field theory in a five-dimensional spacetime compactified on a flat line segment. The bulk-scalar kinetic action is given by

$$S = - \int d^4x \int_{-L/2}^{+L/2} dz |\partial_M \Phi|^2,$$

where we write five-dimensional coordinates as $x^M = (x^\mu, z)$ with $\mu = 0, 1, 2, 3$ and the extra
Our metric convention is $(-++++)$. In a case of a free complex scalar field, $\Phi = (\Phi_R + i\Phi_I)/\sqrt{2}$, the variation of the action is given by

$$\delta S = \int d^4x \int_{-L/2}^{+L/2} dz \left[ \delta\Phi_X (\mathcal{P}\Phi_X) + \delta(z - \frac{L}{2})\delta\Phi_X (-\partial_z\Phi_X) + \delta(z + \frac{L}{2})\delta\Phi_X (+\partial_z\Phi_X) \right],$$

(2)

where $\mathcal{P} \equiv \Box + \partial_z^2$. The vacuum expectation value (VEV) of the scalar field, $\Phi^c$, is determined by the action principle, $\delta S = 0$, that is, $\mathcal{P}\Phi^c_X = 0$. Assuming unbroken 4D Lorentz invariance, the general solution of this equation (EOM) is $\Phi^c(z) = A + Bz$. The undetermined coefficients $A$ and $B$ can be fixed by taking boundary conditions (BCs) at $z = \pm L/2$. We obviously have four choices of combination of Dirichlet and Neumann BCs at $z = (-L/2, +L/2)$, namely, $(D, D), (D, N), (N, D)$, and $(N, N)$, where $D$ and $N$ denotes Dirichlet and Neumann BCs, respectively. These BCs are written as $\delta\Phi(x, z)|_{z=\xi} = 0$ for the Dirichlet BC, and $\partial_z\Phi(x, z)|_{z=\xi} = 0$ for the Neumann BC, where $\xi$ is taken as $+L/2$ or $-L/2$ in each case. A different choice of BCs corresponds to a different choice of the theory.

The theory is fixed once one chooses one of the four conditions. In this letter, we suppose that all SM particles are localized on $z = +L/2$ brane and focus on a Dirichlet BC at the brane. Therefore, the discussed BCs are restricted to $(D, D)$ or $(N, D)$. The most general BCs for each case can be written as

$$(\Phi(x, z)|_{z=-L/2}, \Phi(x, z)|_{z=+L/2}) = (v_-, v_+),$$

(3)

$$(\partial_z\Phi(x, z)|_{z=-L/2}, \Phi(x, z)|_{z=+L/2}) = (0, v_+),$$

(4)

for $(D, D)$ and $(N, D)$, respectively, where $v_-$ and $v_+$ are constants of mass dimension $[3/2]$. The BCs (3) and (4) fix the VEV to be the value $v_+$ on $z = +L/2$ brane, while requiring the quantum fluctuation to be vanishing at the boundary. These BCs also determine the coefficients $A$ and $B$, that is, the VEV profile in the extra-dimensional direction as

$$\Phi^c(z) = \begin{cases} 
\frac{v_++v_-}{2} + \frac{v_+-v_-}{L}z & \text{for } (D, D), \\
\frac{v_+}{L}z & \text{for } (N, D).
\end{cases}$$

(5)

It is easily seen that the resultant VEV profile for $(N, D)$ case becomes flat in the extra dimension due to the Neumann BC at the $z = -L/2$ brane while the one for $(D, D)$ case linearly depends on the extradimensional coordinate. In a case of $v_- = v_+$, the profile for $(D, D)$ BCs also becomes flat, $\Phi^c(z) = v_+$.

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1One can consider in the usual extradimensional coordinate, $y$. In this case, the fundamental region becomes $0 \leq y \leq L$, and $z$ is defined as $z \equiv y - L/2$. 

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Next, let us consider the profile of quantum fluctuation of the scalar field. We utilize the background field method, separating the field into the classical field and quantum fluctuation:

\[ \Phi(x, z) = \Phi^c(z) + \frac{1}{\sqrt{2}} [\phi(x, z) + i\chi(x, z)]. \]  

We put separation (6) into (1) and expand up to the quadratic terms of the field \( \phi \) as

\[ S_\phi = \int d^4x \int_{-L/2}^{+L/2} dz \left[ \frac{1}{2} \phi (\Box + \partial_z^2) \phi + \frac{\delta(z - L/2)}{2} \phi(-\partial_z \phi) + \frac{\delta(z + L/2)}{2} \phi(\partial_z \phi) \right]. \]  

Hereafter we focus on only \( \phi(x, z) \) field for our purpose. The KK expansion for \( \phi(x, z) \) is given by

\[ \phi(x, z) = \sum_{n=0}^{\infty} f^\phi_n(z) \phi_n(x), \]

where \( f^\phi_n(z) \) are eigenfunction of differential operator in the free action (7):

\[ \partial_z^2 f^\phi_n(z) = -\mu_{\phi n}^2 f^\phi_n(z). \]

The general solution of this equation for each \( n \)th mode is written as \( f^\phi_n(z) = \alpha_n \cos(\mu_{\phi n} z) + \beta_n \sin(\mu_{\phi n} z) \). In total there are now three unknown constants for each \( n \)th mode, \( \alpha_n, \beta_n, \) and \( \mu_{\phi n} \). Two of the three are fixed by the two BCs at \( z = \pm L/2 \) while the last one is fixed by the normalization \( \int_{-L/2}^{+L/2} dz f^\phi_n(z) f^\phi_m(z) = \delta_{nm} \). In the following we are focusing on two specific choices of BCs, namely the \( (D, D) \) and \( (N, D) \) cases. Then, the EOM under such BCs determines the VEV profiles given in (5) and expansion (6) leads to the following BCs for quantum fluctuation (substituting both (3) and (4) to (3) and (4)),

\[ (f^\phi_n(z)|_{z=-L/2}, f^\phi_n(z)|_{z=+L/2}) = 0, \]

\[ (\partial_z f^\phi_n(z)|_{z=-L/2}, f^\phi_n(z)|_{z=+L/2}) = 0, \]

for \( (D, D) \) and \( (N, D) \), respectively. Therefore, we can obtain the wave function profile of the quantum fluctuations as

\[ f^\phi_n(z) = \begin{cases} 
\sqrt{2 \frac{L}{\tau}} \cos \left( \frac{(n+1)\pi}{\tau} \right) & \text{for even } \ n, \\
\sqrt{2 \frac{L}{\tau}} \sin \left( \frac{(n+1)\pi}{\tau} \right) & \text{for odd } \ n, 
\end{cases} \]  

\[ f^\phi_n(z) = \sqrt{\frac{1}{L}} \left[ \cos \left( \frac{(2n+1)\pi}{2\tau} \right) - (-1)^n \sin \left( \frac{(2n+1)\pi}{2\tau} \right) \right]. \]

for \( (D, D) \) and \( (N, D) \) BCs, respectively. The wave function profiles of \( n = 0, 1, \) and \( 2 \) modes in the extra-dimension are shown in FIG. 1. They mean that a flat zero-mode profile

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2The same expansion is taken for \( \chi(x, z) \). See e.g. Appendix B in [11] for the derivation of action.
in the Neumann BC case is deformed to the cosine function of 
\[ f_0^\phi(z) = \sqrt{2/L} \cos(\pi z/L) \]
through the Dirichlet BCs at \( z = \pm L/2 \) for the \((D, D)\) case. The profiles are described by a combination of the sine and cosine functions due to the Neumann BC at \( z = -L/2 \) and the Dirichlet at \( z = +L/2 \) for the \((N, D)\) case.

The \( n \)th scalar mass is calculated to be

\[
m_{\phi_n}^2 = \begin{cases} 
\left( \frac{(n+1)\pi}{L} \right)^2 & \text{for } (D, D), \\
\left( \frac{(2n+1)\pi}{2L} \right)^2 & \text{for } (N, D).
\end{cases}
\]  

which shows the lowest \((n = 0)\) mode has a KK mass, \( m_{\text{KK}} \equiv \pi/L \), for the \((D, D)\) case. On the other hand, the mass of the lowest mode for the \((N, D)\) case becomes a half of KK mass, \( m_{\text{KK}}/2 \). These features are just results from the Dirichlet BC, that is, the lowest mode mass is pushed up to the KK mass in the case of Dirichlet BCs at both boundaries, and the mass is pushed up to only a half of KK mass when the Dirichlet BC is taken at one boundary.

The above discussions can be similarly applied to the \( \chi \) field. The important point of this type of setup is that with the Dirichlet BC(s) the wave function profile of the physical field is vanishing at the boundary while the VEV of the scalar field can be obtained due to the Dirichlet BC without contradiction to the action principle. The mass of this physical field is of the order of the compactification scale as massless modes are forbidden by the BCs.

Next, we propose an implementation of this setup. We investigate an identification of the bulk scalar field with a flavon. The origin of the three generations of the SM fermions and their Yukawa couplings determining the masses and mixings is one of the most important problems in particle physics. An introduction of family symmetries is a common approach in order to explain the origin of fermion masses and mixings. Such symmetries must be broken at a high energy scale, and additional scalar fields, the so-called flavons, are required to break the symmetries. Then effective Yukawa couplings can be induced from non-renormalizable
operators, which generates hierarchical Yukawa structures through the Froggatt-Nielsen (FN) mechanism [15].

In the FN mechanism, scalar fields (flavons) are introduced that are charged under a family symmetry which acts on the different generations of SM fermions. Once the symmetry is broken at a high energy scale, the effective Yukawa coupling can be determined by the charges of fields and the VEV of flavons. Here, we investigate a case that the flavons are bulk scalar fields described by the above setup (11) with the Dirichlet BC (3) or (4) and the SM fermions are localized at the $z = +L/2$ brane. When we introduce a abelian family symmetry and one flavon for simplicity, the effective mass term for the SM fermions in four dimensions is given by

$$\int_{-L/2}^{+L/2} dz \delta(z - L/2) c_{ij} \left( \frac{\Phi}{\Lambda^{3/2}} \right)^{N_{ij}} \bar{L}_{i} F_{Rj} H + h.c.,$$

(15)

where $c_{ij}$ is a dimensionless coupling of order one and $N_{ij}$ are determined by the charges of flavon and SM fermions. After expanding $\Phi$ as in (6) and integrating the five dimensional direction, we obtain

$$c_{ij} \epsilon^{N_{ij}} \bar{L}_{i} F_{Rj} H + h.c.,$$

(16)

where $\epsilon \equiv v_+ / \Lambda^{3/2}$. This is the usual result of the FN mechanism except for the different mass dimension of the VEV in $\epsilon$. However, notice that the lowest modes of the physical fields $\phi_0(x)$ in the KK expansion (8) certainly exist with the KK (a half of KK) mass induced from the bulk kinetic term (11) with $(D(N), D)$ BC. The physical states do not have any couplings with the brane localized SM fields because the wave function profiles are vanishing at boundary while the VEV can be obtained due to the Dirichlet BC. Therefore, the lowest mode of the flavon remains as a stable particle. The physical state of flavon becomes only Gravitationally Interacting Massive Particle (GIMP) which can be a candidate for the dark matter.

One should note, however, that the stability of the dark matter candidate would be spoilt by the introduction of terms of the type

$$\int d^5 x \delta(z - L/2) (\partial_5 \phi) \times \text{SM fields}$$

(17)

which would couple the KK modes to the SM fields. Any bulk interactions will generally induce such disastrous brane interactions [18]. The interpretation of the lightest KK mode of the scalar as the dark matter particle therefore requires that there must not be any bulk interactions of the scalar, meaning a free theory in the bulk. It is clear that since the KK sector is completely decoupled from the SM, there will be no interactions between
the two sectors induced by quantum corrections and the lightest KK mode is therefore stable. The stability is due to a ’superselection rule’ between the two decoupled sectors whose decoupling is a consequence of the Dirichlet BCs even in the presence of Yukawa-type brane interactions. The relevant observation in this paper is thus the fact that the brane interactions might lead to a non-vanishing VEV of the field, which together with the Froggatt-Nielsen type interactions will lead to an explanation of the fermion mass hierarchy without the introduction of a FN field in the 4D spectrum of the theory. The stable lightest KK mode is an only gravitationally interacting dark matter candidate.

In the above implementation of Dirichlet BCs, the family symmetry is broken by the BC. The VEV related with the symmetry breaking scale and the cutoff scale could be taken as arbitrary high energy scales as long as \( \epsilon \ll 1 \) is satisfied. The variant of the FN mechanism discussed above is only one example of this setup with the bulk flavon with Dirichlet BC and it can be applied to practically any flavour models with non-SM Higgs fields [16, 17]. The implementation gives the fields a role as the GIMP. In some cases, constraints on flavour models from the electroweak precision measurements would be relaxed because the wave functions of non-SM bulk Higgs fields vanish at boundary.

Finally, we comment on other extra-dimensional backgrounds which make it possible to identify our stable particle as a dark matter candidate. Our stabilization was based on a flat five-dimensional spacetime. If we extend the setup with the Dirichlet BC(s) to a larger number of extra-dimensions such as a six-dimensional model [11] or to a warped extra-dimension model [2], the hierarchy problem can be solved. A realistic model, which does not suffer from the hierarchy problem, must be constructed on such backgrounds in studies of extra-dimensions. To get a viable DM candidate from our stabilization, we also have to extend the mechanism to a model with larger number of extra-dimension. Our KK-flavon, which has only gravitational interactions, has survived until the present epoch, and its energy density can dominate but must not exceed the present DM one. Thus, we need \( \Omega_\phi \leq \Omega_{\text{DM}} \), where \( \Omega_\phi \) and \( \Omega_{\text{DM}} \) are density parameters of KK-flavon and DM, respectively. The interaction rate of the KK-flavon can be roughly estimated as \( \Gamma_\phi \sim T^5/M_{\text{pl}}^4 \) with four-dimensional Planck mass, \( M_{\text{pl}} \). The decoupling and non-exceeding conditions for KK-flavon constrain the KK-flavon mass to be smaller than \( \mathcal{O}\text{(keV)} \), \( m_\phi \sim \Omega_\phi h^2 g (4.4\text{eV}) \lesssim \mathcal{O}\text{(keV)} \), where \( h \) and \( g \) are the current dimensionless Hubble constant and degrees of freedom of order a few hundred at the decoupling temperature. This means that we require a larger number of extra-dimensions in order to obtain a correct energy density of KK-flavons and to solve the gauge hierarchy problem without contradiction with the current cosmological observations and experimental

\[3\text{Notice that the KK scale in } 4+\delta \text{ dimensional corresponds to } m_\phi \sim \mathcal{O}(10^{-1})\text{meV}, \mathcal{O}(10)\text{keV}, \mathcal{O}(10)\text{MeV} \text{ for } \delta = 2, 4, 6 \text{ with } 4+\delta \text{ dimensional Planck mass of order } \mathcal{O}(\text{TeV}), \text{ respectively.} \]
limits for a deviation of the Newton’s law. More detailed phenomenological predictions of this scenario should be compared with a DM model of sterile (lightest right-handed) neutrino with keV mass scale, and such discussions will be given in a separate publication.

We have studied a scalar field theory in a flat five-dimensional setup with the Dirichlet type BC at \( z = +L/2 \) brane. The wave-function profiles of the physical field are deformed by the Dirichlet BC(s) in the setup. As the results, the physical field profiles vanish at the brane while a finite VEV can generally be obtained without contradiction to the action principle. The lowest mode masses of these fields are pushed up to a KK (a half of KK) scale by the \((D(N), D)\) BC.

We have also proposed an implementation of this setup to flavon physics. The bulk scalar and all SM fields have been assumed to be a flavon in the FN mechanism and brane localized fields, respectively. It has been shown that all couplings of the physical field of the flavon with the SM particles are vanishing due to the Dirichlet BC while realizing the usual FN mechanism in the free theory on the bulk. However, the physical field of the flavon with a KK mass induced from the bulk kinetic term certainly exist in the theory. This setup gives the flavon a role as an only Gravitationally Interacting Massive Particle (GIMP), which is a candidate for dark matter. This mechanism can be implemented in a number of flavor models with non-SM Higgs fields apart from the FN mechanism presented here.

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