Numerical research of influence of residual stress on interface crack propagation

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Abstract. The crack propagation was simulated by a generalized potential-based mixed-mode (PPR) cohesive interface model. Based on the principle of equivalence of thermal and mechanics, the effects of residual stress on interface crack propagation was researched by finite element model. The results demonstrate that residual compressive stress is more likely to cause material interface crack propagation. The material interface is more likely to fail when there is residual compressive stress in the material. The residual compressive stress is more likely to cause the interface crack propagation when the crack propagation length caused by certain residual compressive stress value is seen an important influence parameter.

1. Introduction
Residual stresses are unavoidable in welding, coating and other components in practical engineering applications. The Residual stress even exceeds the yield strength of its bulk material. The interface position is very easy to delaminate, and finally causes the failure due to interface crack propagation. The research of influence of residual stress on interface crack propagation is of great importance.

In last several decades, the cohesive interface model has undergone the great improvements and developments in describing the materials fracture process benefiting by the development of the finite element methods [1]. With the continuous improvement of computer performance and the maturity of finite element method, the finite element method was combined with the cohesive model to simulate crack propagation of materials by more and more researchers [2, 3].

At present, there are few researches on the evolution of interfacial crack propagation process. In the present investigation, based on the principle of equivalence of thermal and mechanics, the effects of residual stress on interface crack propagation was researched by finite element model combined with the PPR mixed-mode cohesive model.

2. The PPR mixed-mode cohesive interface model
The cohesive traction–separation relationship was obtained from a potential-based cohesive zone model, so-called the PPR model [4]. The potential of cohesive fracture was given by

$$\Psi_{(A_n, A_m)} = \min \left( \phi_h, \phi_f \right) + \left[ \Gamma_n \left( 1 - \frac{\Delta_n}{\alpha_n} \right)^{\alpha_n} + \left( \frac{m}{\beta - \frac{\Delta_n}{\alpha_n}} \right)^{m} + \left( \frac{n}{\beta - \frac{\Delta_m}{\alpha_m}} \right)^{n} \right] \left[ \phi_h \left( 1 - \frac{\Delta_n}{\alpha_n} \right)^{\alpha_n} + \left( \frac{m}{\beta - \frac{\Delta_n}{\alpha_n}} \right)^{m} + \left( \frac{n}{\beta - \frac{\Delta_m}{\alpha_m}} \right)^{n} \right]$$

where $\langle \bullet \rangle$ is the Macauley bracket

$$\langle x \rangle = \begin{cases} 0 & x \leq 0 \\ x & x > 0 \end{cases}$$
The derivatives of the PPR potential with respect to the normal and tangential separations lead to the normal and tangential cohesive tractions

\[
T_n(\Delta_n, \Delta_t) = \frac{\Gamma_n}{\delta_n} \left[ m \left( 1 - \frac{\Delta_n}{\delta_n} \right)^\alpha \left( \frac{m + \Delta_n}{\alpha + \delta_n} \right)^{m-1} - \alpha \left( 1 - \frac{\Delta_n}{\delta_n} \right)^{\alpha-1} \left( \frac{m + \Delta_n}{\alpha + \delta_n} \right)^m \right] \times 
\left[ \Gamma_i \left( 1 - \frac{|\Delta_i|}{\delta_i} \right)^\beta \left( \frac{n + |\Delta_i|}{\beta + \delta_i} \right)^n + (\phi_i - \phi_i) \right]
\]

\[
T_i(\Delta_n, \Delta_t) = \frac{\Gamma_i}{\delta_i} \left[ n \left( 1 - \frac{|\Delta_i|}{\delta_i} \right)^\beta \left( \frac{n + |\Delta_i|}{\beta + \delta_i} \right)^{n-1} - \beta \left( 1 - \frac{|\Delta_i|}{\delta_i} \right)^{\beta-1} \left( \frac{n + |\Delta_i|}{\beta + \delta_i} \right)^n \right] \times 
\left[ \Gamma_n \left( 1 - \frac{\Delta_n}{\delta_n} \right)^\alpha \left( \frac{m + \Delta_n}{\alpha + \delta_n} \right)^m + (\phi_n - \phi_i) \right] \frac{\Delta_n}{|\Delta_i|}
\]

(3)

The PPR potential-based model satisfies the following boundary conditions associated with cohesive fracture.

The complete normal separation occurs \((T_n = 0)\) when either normal or tangential separation reaches a certain length scale,

\[
T_n(\delta_n, \Delta_t) = 0 \quad \text{or} \quad T_n(\Delta_n, \delta_t) = 0
\]

(4)

where \(\delta_n\) is a normal final crack opening width, and \(\delta_t\) is a tangential conjugate final crack opening width.

The complete tangential separation occurs \((T_i = 0)\) when either normal or tangential separation reaches a certain length scale,

\[
T_i(\delta_n, \Delta_t) = 0 \quad \text{or} \quad T_i(\Delta_n, \delta_t) = 0
\]

(5)

where \(\delta_n\) is a normal conjugate final crack opening width, and \(\delta_t\) is a tangential final crack opening width.

The area under the pure normal and tangential traction–separation curves provides the fracture energy in the normal \((\phi_n)\) and tangential \((\phi_t)\) directions, respectively,

\[
\phi_n = \int_0^{\delta_n} T_n(0, \Delta_n) d\Delta_n, \quad \phi_t = \int_0^{\delta_t} T_i(\Delta_t, 0) d\Delta_t
\]

(6)

The traction–separation curves reach a peak point at a critical crack opening width \((\delta_{nc}, \delta_{tc})\),

\[
\frac{\partial T_n}{\partial \Delta_n} \bigg|_{\Delta_n = \delta_{nc}} = 0, \quad \frac{\partial T_i}{\partial \Delta_t} \bigg|_{\Delta_t = \delta_{tc}} = 0
\]

(7)

The traction value at the critical separation corresponds to the cohesive strength \((\sigma_{max}, \tau_{max})\),

\[
T_n(\delta_{nc}, 0) = \sigma_{max}, \quad T_i(0, \delta_{tc}) = \tau_{max}
\]

(8)

The energy constants \(\Gamma_n\) and \(\Gamma_i\) are related to the fracture energies (e.g. modes I and II). When the modes I and II fracture energies are different, one obtains the energy constants

\[
\Gamma_n = (-\phi_n)^{\left(\frac{\delta_{nc} - \delta_{tc}}{\delta_{nc}}\right)} \left(\frac{\alpha}{m}\right)^m \Gamma_i = (-\phi_i)^{\left(\frac{\delta_{tn} - \delta_{tc}}{\delta_{tn}}\right)} \left(\frac{B}{n}\right)^n \quad (\phi_n \neq \phi_i)
\]

\[
\Gamma_n = (-\phi_n)^{\left(\frac{\delta_{nc} - \delta_{tc}}{\delta_{nc}}\right)} \left(\frac{\alpha}{m}\right)^m \Gamma_i = \left(\frac{B}{n}\right)^n \quad (\phi_n = \phi_i)
\]

(9)

The exponents \(m\) and \(n\) are associated with the initial slope
\[ m = \alpha (\alpha - 1) \lambda_n^2 - n = \beta (\beta - 1) \lambda_t^2 \]
\[ \lambda_n = \frac{\delta_n}{\delta_n}, \lambda_t = \frac{\delta_t}{\delta_t} \]
(10)

The normal final crack opening width (\( \delta_n \)) is given as
\[ \delta_n = \frac{\phi_n}{\sigma_{\text{max}}} \alpha \lambda_n (1 - \lambda_n)^{m-1} \left( \frac{\alpha}{m} + 1 \right)^{-1} \]
while the tangential final crack opening width (\( \delta_t \)) is expressed as
\[ \delta_t = \frac{\phi_t}{\tau_{\text{max}}} \beta \lambda_t (1 - \lambda_t)^{n-1} \left( \frac{\beta}{n} + 1 \right) \]
(11)

The tangential conjugate final crack opening width is the solution of the following nonlinear function,
\[ f_t(\Delta) = \Gamma_t \left( 1 - \frac{\Delta}{\delta_t} \right)^\beta \left( \frac{n}{\beta} + \frac{\Delta}{\delta_t} \right)^m + (\phi_t - \phi_n) = 0 \]
(13)

The normal conjugate final crack opening width is the solution of the nonlinear function,
\[ f_n(\Delta_n) = \Gamma_n \left( 1 - \frac{\Delta_n}{\delta_n} \right)^m \left( \frac{\alpha}{\delta_n} + \frac{\Delta_n}{\delta_n} \right)^n + (\phi_n - \phi_t) = 0 \]
(14)

Figure 1 PPR mixed-mode cohesive model and its gradients with \( \phi_n = 200 \text{N/m}, \phi_t = 100 \text{N/m}, \sigma_{\text{max}} = 40 \text{MPa}, \tau_{\text{max}} = 30 \text{MPa}, \alpha = 5, \beta = 2, \lambda_n = 0.1, \lambda_t = 0.2 \)

As shown in Figure 1, there are eight control parameters (\( \phi_n, \phi_t, \sigma_{\text{max}}, \tau_{\text{max}}, \alpha, \beta, \lambda_n, \lambda_t \)) in the PPR mixed-mode cohesive interface model. Compared with the widely used bilinear mixed-mode cohesive model\(^{[5]}\), the PPR mixed-mode cohesive interface model has two more shape control parameters (\( \alpha, \beta \)), which can better simulate the interfacial crack propagation.

3. Finite element model

Finite element method (FEM) was used in numerical simulations for interface crack propagation. The 2D finite element model and finite element mesh are shown in Figure 2. The PPR mixed-mode cohesive interface model was used to describe the interface crack propagation. The plain strain element (CPE4) was adopted in material 1 and 2 which constitutive relation is elastic. The Abaqus user element subroutine (UEL) was adopted in interface. The under surface of substrate was fixed.

![Figure 2 2D finite element model and finite element mesh](image_url)
In order to simulate the effects of residual tensile and compressive stress, the positive and negative temperature fields are applied to the whole model respectively.

4. Results and discussions

4.1. The effects of residual stress on interface crack propagation

![Crack propagation with the increase of residual compressive stress](image)

Figure 3 Crack propagation with the increase of residual compressive stress (a)-(f) 100, 170, 200, 300, 400, 500MPa.

The propagation of the interface crack is given in Figure 3 at different residual compressive stress state. With the increase of the residual compressive stress, the interface cracks gradually expand until the final failure. For residual tensile stress, the law of crack propagation is also similar.

4.2. Comparison of the effects of residual tensile stress and compressive stress on crack propagation

![Comparison of the effects of residual tensile stress and compressive stress on crack propagation](image)

Figure 4 Comparison of the effects of residual tensile stress ($\sigma_R^+\text{)}$) and compressive stress ($\sigma_R^-\text{)}$) on crack propagation.

According to Figure 4, it can be seen that crack propagation accelerates with the increase of residual stress, and interface crack propagation is faster under the action of equivalent residual compressive stress ($\sigma_R^-\text{)}$). When the residual stress required for the same length of crack
propagation is used as a comparative parameter to evaluate the influence of external factors on crack propagation, the residual compressive stress can cause crack propagation more.

According to the buckling theory model of Evans \cite{Evans2001}, the residual compressive stress of thin films is more likely to lead to the delamination of thin film/substrate interface. The simulation results in this paper validate the buckling theory model of Evans.

5. Conclusions
The crack propagation was simulated by a generalized potential-based (PPR) mixed-mode cohesive interface model which has two more shape control parameters and can better simulate the interfacial crack propagation. Based on the principle of equivalence of thermal and mechanics, the effects of residual compressive and tensile stress on interface crack propagation was studied. The results demonstrate that residual compressive stress is more likely to cause material interface crack propagation. The material interface is more likely to fail when there is residual compressive stress in the material. In a words, residual compressive stress is more likely to cause the interface crack propagation when the crack propagation length caused by certain residual compressive stress value is seen an important influence parameter.

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