Anticipating the Sun’s heavy-element abundance

D. O. Gough*  
Institute of Astronomy and Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0HA, UK  
Physics Department, Stanford University, CA 94305, USA

ABSTRACT

Much of our understanding of the internal structure of the Sun derives from so-called standard theoretical solar models. Unfortunately, none of those models agrees completely with observation. The discrepancy is commonly associated with chemical abundance, and has led to what is now called the solar abundance problem, the resolution of which has previously been out of sight. But now the Borexino Collaboration, who recently announced measurements of the pp-chain solar neutrinos, are optimistic that they will be able to measure the flux \( \Phi_{\text{CNO}} \) of the neutrinos emitted by the relatively weak CNO cycle. Since C, N and O constitute the majority of the heavy elements, that measurement will permit a crucial determination of the heavy-element abundance \( Z_c \) in the Sun’s energy-generating core, thereby shedding important light on the problem. To accomplish that determination, a robust relation between \( Z_c \) and \( \Phi_{\text{CNO}} \) will be required. That relation is \( Z_c = 0.400 \Phi_{\text{CNO}} \), where \( \Phi_{\text{CNO}} \) is in units of \( 10^{10} \text{ cm}^{-2} \text{s}^{-1} \).

Key words: Sun: neutrinos – Sun: helioseismology – Sun: abundances – opacity

1 INTRODUCTION

The recent report by Agostini & the Borexino Collaboration (2018) of the impressive measurement of pp-chain solar neutrinos has provided the first observational confirmation of the agreement (to within about 10%) between the nuclear energy production rate in the Sun and the radiant luminosity, confirming that the Sun is more-or-less in thermal balance, as it is normally assumed to be. The result is a significant step towards deepening our understanding of the inner workings of a star.

The collaboration relate their results to two modern sets of standard solar models (SSMs) published by Vinyoles et al. (2017), one with the commonly adopted GS98 (Grevesse & Sauval 1998) abundances, the other with the more recent, and lower, AGSS09 (Asplund et al. 2009) values, which appear to be better representations of photospheric values (see also Caffau et al. 2011). By comparing \(^7\)Be and \(^8\)B fluxes, suitably adjusted for flavour transitions, with theoretical values from the models, the collaboration report a preference for the former composition, which is in conflict with the standard assumption that, aside from gravitational settling and radiative levitation, abundances in the radiative envelope are the same as those in the photosphere. However, the preference is at least comforting because it has long been known that solar models with low heavy-element abundance, \( Z \), are ruled out by helioseismology (Gough 1983; Duvall & Harvey 1983; Christensen-Dalsgaard & Gough 1998; Basu & Antia 2008). Vinyoles et al. (2017) present a careful demonstration of this issue in the specific case of the GS98 and AGSS09 abundances. The conflict has been named the solar abundance problem.

2 CIRCUMVENTING OPACITY

Opacity, which depends directly on \( Z \), is the principal cause of the model differences, via its control of the flow of heat in the Sun’s radiative envelope. A lower opacity requires a smaller temperature gradient, and hence a lower temperature, leading to a lower sound speed in the radiative envelope, although the latter is offset in the energy-generating core by the higher hydrogen abundance, \( X \), required to fuel the otherwise slower nuclear reactions. A reliable knowledge of opacity is therefore a crucial ingredient for understanding the ‘so-called’ standard structure and evolution of the Sun. Its calculation involves very complicated physics, and the outcome has commonly been questioned.

It should be appreciated, however, that nuclear reactions do not themselves depend directly on opacity. Moreover, the pertinent properties of the internal structure of the Sun can be determined, subject to relatively minor additional assumptions, by seismological analysis of acoustic modes of oscillation. Acoustic propagation depends on relatively simple physics, rendering correctly interpreted, yet ad-
mittedly more limited, inferences from helioseismology more reliable than those from SMMs.

Setting aside an acoustic glitch that is present immediately beneath the convection zone, whose origin at least in part is material redistribution in the tachocline (Elliott & Gough 1999; Christensen-Dalsgaard et al. 2018), ignored in SMMs but having a relatively minor impact on the overall stratification of the deep interior of the star (Vinyoles et al. 2017), the principal flaw in the construction of SMMs is likely to be either an error in the opacity calculation, a possibility that is now not wholly accepted (but there is, in particular, a degree of acceptance resulting from recent laboratory measurements by Bailey et al. (2015), or it is the standard assumption that photospheric abundances directly reflect the abundances in the radiative envelope (Guzik & Mussack 2010). A potential resolution could be, for example, that mechanical waves generated off-resonance at the base of the convection zone (e.g. Press 1981) have amplitudes enough to carry a significant, yet uncertain, fraction of the total luminosity. If that were so, the role of opacity would be substantially diminished. The associated wave momentum flux would have negligible influence on the hydrostatic balance, which alone (aside from the adiabatic exponent $\gamma_1$, the theory of which is relatively robust in the radiative interior) determines the Sun’s seismic structure. However, the thermal structure would then depend on the uncertain details of the wave spectrum. Without knowing the thermal structure, no SMM can be trusted.

3 DETERMINING $Z$ VIA SEISMOLOGY

As the Borexino Collaboration point out, their anticipated measurement of the neutrino flux $\Phi_{\text{CNO}}$ produced by the CNO cycle will provide a direct evaluation of the CNO abundances, the dominant contributors to $Z$, irrespective of opacity. To accomplish that, a relation between $Z$ and $\Phi_{\text{CNO}}$ will be needed. One might be tempted to interpolate between SMMs such as those provided by Vinyoles et al. (2017), but the outcome would be subject to their reliability, which, as I pointed out above, is in doubt. However, an adequate estimate of pertinent conditions in the core can be obtained more reliably from helioseismology, provided one adopts an assumption such as a core that was initially homogeneous and which suffered no material redistribution of the products of the nuclear reactions during the subsequent main-sequence evolution, an assumption that is adopted also in creating SMMs. For the purpose of estimating $Z$, it is adequate to linearize the difference between the seismic structure of the Sun and an appropriate accurately computed SSM which, in the case of the representation I use here (Gough 2004), was Model S of Christensen-Dalsgaard et al. (1996). The outcome is an estimate of the sound speed $c$ and the density $\rho$, which are subject to an uncertainty of a few parts in $10^3$ (cf. Takata & Gough 2003; Basu & Antia 2008), from which the pressure $p$ can be determined from the constraint of hydrostatic support. It is then necessary to estimate the helium abundance $Y$ and the temperature $T$, neither of which is seismically accessible. To this end a shell representing the tachocline in Model S was homogenized with the convection zone, and then a constant $\delta Y$ was added to $Y$, enabling $T$ to be determined implicitly from $c$, $p$ and $\rho$ with the help of the equation of state: $\delta Y$ and $T$ were determined simultaneously by requiring that the nuclear energy generation rate in the core is equal to the observed luminosity at the surface. The resulting structure, Model Ss, is presented by Gough (2004). To the precision required here, that structure is independent of the reference SSM adopted (cf. Basu et al. 2000), and of the imperfect (cf. Christensen-Dalsgaard et al. 2018) representation of the tachocline.

The relation between the central heavy-element abundance $Z_c$ and the CNO neutrino fluxes was obtained using the cross-sections adopted by Vinyoles et al. (2017). It was achieved by scaling model B16-GS98 of Vinyoles et al. (2017) to the seismic structure of the Sun, in the form of Model Ss, using the functional forms of the CNO reaction rates, presumed to be in equilibrium. The relative abundances of C, N and O are thereby determined, irrespective of their initial values (e.g. Clayton 1983). The outcome is that the Sun produces neutrino fluxes $\Phi_{\text{13}} = 1.41$, $\Phi_{\text{15}} = 1.06$ and $\Phi_{\text{17}} = 0.027$ per $Z_c$, all in units of $10^{10}$ cm$^{-2}$ s$^{-1}$, from the beta decays of $^{13}$N, $^{15}$O and $^{17}$F respectively. The central heavy-element abundance is related to the total CNO neutrino flux according to

$$Z_c = \alpha \Phi_{\text{CNO}}, \quad (1)$$

where $\alpha = 0.400$ and $\Phi_{\text{CNO}}$ is also in units of $10^{10}$ cm$^{-2}$ s$^{-1}$. The uncertainty in $\alpha$ is dominated by uncertainties in the nuclear reaction rates, which are detailed by Agostini & the Borexino Collaboration (2018) and Vinyoles et al. (2017).

Unlike hydrogen and helium abundances, the total CNO abundance is unaltered by the nuclear reactions, so the relation between $Z_c$ and the heavy-element abundance $Z$ throughout the radiative envelope is relatively secure, depending only on the weak variation resulting from gravitational settling. Hence one can be assured that the analysis leading to equation (1) is reliable. We now await the promised future measurement by the Borexino Collaboration to resolve the abundance issue.

I thank T. Sekii and the referee for useful suggestions.

REFERENCES

Agostini M., the Borexino Collaboration 2018, Nature, 562, 505
Asplund M., Grevesse N., Sauval A. J., Scott P., 2009, ARA&A, 47, 481
Bailey J. E., et al., 2015, Nature, 517, 56
Basu S., Antia H. M., 2008, Phys. Rep., 457, 217
Basu S., Pinsonneault M. H., Bahcall J. N., 2000, ApJ, 529, 1084
Caffau E., Ludwig H.-G., Steffen M., Freytag B., Bonifacio P., 2011, Sol. Phys., 268, 255
Christensen-Dalsgaard J., Gough D. O., 1998, The Observatory, 118, 25
Christensen-Dalsgaard J., et al., 1996, Science, 272, 1286
Christensen-Dalsgaard J., Gough D. O., Knudstrup E., 2018, MNRAS, 477, 3845
Clayton D. D., 1983, Principles of stellar evolution and nucleosynthesis, University of Chicago Press
Duval Jr T. L., Harvey J. W., 1983, Nature, 302, 24
Elliott J. R., Gough D. O., 1999, ApJ, 516, 475
Gough D. O., 1983, Nature, 302, 18
Gough D. O., 2004, in V. Čelikovski, D.O. Gough, & W. Däppen ed., American Institute of Physics Confer-
Anticipating the Sun’s heavy-element abundance

ence Series Vol. 731, Equation-of-State and Phase-Transition in Models of Ordinary Astrophysical Matter. pp 119–138, doi:10.1063/1.1828398

Grevesse N., Sauval A. J., 1998, Space Sci. Rev., 85, 161

Guzik J. A., Mussack K., 2010, ApJ, 713, 1108

Press W. H., 1981, ApJ, 245, 286

Takata M., Gough D. O., 2003, in Sawaya-Lacoste H., ed., ESA Special Publication Vol. 517, GONG+ 2002. Local and Global Helioseismology: the Present and Future. pp 397–400

Vinyoles N., et al., 2017, ApJ, 835, 202

This paper has been typeset from a TeX/LaTeX file prepared by the author.