Bilarge leptonic mixing from Abelian horizontal symmetries

Tommy Ohlsson\textsuperscript{a*}, Gerhart Seidl\textsuperscript{a†}

\textsuperscript{a}Institut für Theoretische Physik, Physik-Department, Technische Universität München, James-Franck-Straße, 85748 Garching bei München, Germany

Abstract

We construct and present a model for leptonic mixing based on higher-dimensional operators, using the Froggatt–Nielsen mechanism, and Abelian horizontal symmetries (flavor symmetries) of continuous and discrete type. Our model naturally yields bilarge leptonic mixing, coming from both the charged leptons and the neutrinos, and an inverted neutrino mass hierarchy spectrum. The obtained values of the parameters, i.e., the leptonic mixing parameters and the neutrino mass squared differences, are all consistent with the atmospheric neutrino data and the Mikheyev–Smirnov–Wolfenstein large mixing angle solution for the solar neutrino problem.

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1. Introduction

Predicting the pattern of fermion masses and mixings from a fundamental gauge theory is one of the major challenges in particle physics. In such an approach, the observed hierarchy of the fermion masses is usually understood in terms of some symmetry breaking interaction. In fact, due to their plausible Majorana nature, the extreme smallness of the neutrino masses could be associated with a violation of the $B-L$ symmetry. Thus, the neutrinos can shed light on the origin of the fermion masses and mixings, since most grand unified theories (GUTs) based on SO(10) or $E_6$ and also string theories indeed expect the $B-L$ symmetry to be broken. In building models, it is therefore particularly important to naturally reproduce the neutrino mass squared differences and the leptonic mixing parameters that have been determined by the atmospheric and solar neutrino data.

The neutrino mass squared differences are generally defined as

$$\Delta m^2_{ab} \equiv m^2_a - m^2_b,$$

where $m_a$ is the mass of the $a$th neutrino mass eigenstate. We will here assume that there are three neutrino flavors, and therefore, three neutrino flavor states $\nu_\alpha$ ($\alpha = e, \mu, \tau$) and also three neutrino mass eigenstates $\nu_\alpha$ ($\alpha = 1, 2, 3$). The unitary leptonic mixing matrix is then given by

$$U = (U^\ell) U^{\nu},$$

where the unitary mixing matrix $U^\ell$ ($U^{\nu}$) rotates the left-handed charged lepton fields (the neutrino fields) so that the charged lepton mass matrix (the neutrino mass matrix) becomes diagonal. Thus, the leptonic mixing matrix acquires contributions from both the charged leptons and the neutrinos. These contributions usually add in a non-trivial way (see App. A). For three neutrino flavors, in the so-called standard parameterization, the leptonic mixing matrix reads

$$U = \begin{pmatrix} e^{i\delta} & c_2 c_3 & s_2 s_3 \\ -s_2 c_3 & c_1 c_2 - s_1 s_3 e^{i\delta} & c_1 s_3 - s_1 c_3 e^{i\delta} \\ s_1 c_3 - s_2 c_1 c_3 e^{i\delta} & s_1 s_3 + s_2 c_1 c_3 e^{i\delta} & c_1 c_3 + s_2 s_3 c_3 e^{i\delta} \end{pmatrix},$$

where $S_\alpha = \sin \theta_\alpha$, $C_\alpha = \cos \theta_\alpha$ (for $\alpha = 1, 2, 3$), and $\delta$ is the physical $CP$ phase. Here $\theta_1 \equiv \theta_{23}$, $\theta_2 \equiv \theta_{13}$, and $\theta_3 \equiv \theta_{12}$ are the leptonic mixing angles. Recent results suggest that among the possible solutions to the solar neutrino problem, the Mikheyev–Smirnov–Wolfenstein (MSW) large mixing angle (LMA) solution is somewhat preferred to the MSW small mixing angle (SMA) solution, the MSW low mass (LOW) solution, and the vacuum oscillation (VAC) solution. Actually, a global solar two flavor neutrino oscillation analysis including the latest SNO

\textsuperscript{†}The leptonic mixing matrix is sometimes called the Maki-Nakagawa-Sakata (MNS) mixing matrix.

\textsuperscript{*}E-mail: tohansson@ph.tum.de
\textsuperscript{†}E-mail: gseidl@ph.tum.de
data strongly favors the MSW LMA solution \(^5\). However, the MSW LMA solution excludes maximal solar mixing at the 95\% confidence level \(^6\) (and now even at the 99.73\% confidence level \(^7\)), and therefore, it also disfavors the scenario of so-called bimaximal mixing \(^8\). Thus, we will instead, most probably, have a bilarge mixing scenario in which the solar mixing angle \(\theta_{12}\) is large, but not maximal, and the atmospheric mixing angle \(\theta_{23}\) is approximately maximal. It is interesting to observe that there are very strong indications that the leptonic mixing is large, whereas, on the other hand, it has turned out experimentally that the quark mixing is small \(^9\).

In this paper, we will investigate a model, which yields in a technically natural way bilarge leptonic mixing, reproduces the observed mass hierarchy of charged leptons, and leads to an inverted neutrino mass hierarchy spectrum. This will be achieved by generating lepton mass matrix textures, where the mixing of the charged leptons is comparable with the mixing of the quarks and the mixing of the neutrinos is essentially bimaximal. The striking difference between the bilarge leptonic mixing and the small quark mixing will then be accounted for by the neutrinos (mainly) and the charged leptons (partly).

This paper is organized as follows: In Sec. 2, we will introduce a model by adding to the standard model (SM) a set of extra fields and horizontal symmetries that will give rise to specific effective Yukawa interactions for the charged leptons. Next, minimizing the corresponding scalar potential, we will naturally obtain a mass matrix texture for the charged leptons, which is in agreement with experimental data. In Sec. 3 we will extend the representation content of the model in order to also obtain a realistic mass matrix texture for the neutrinos. (In App. A we will explicitly calculate the leptonic mixing angles coming from the diagonalizations of the charged lepton and neutrino mass matrices, respectively.) Finally, in Sec. 4, we will present a summary as well as our conclusions.

2. Charged leptons

2.1. Horizontal symmetries

We will here consider an extension of the SM in which the lepton masses arise from higher-dimensional operators \(^10\) via the Froggatt–Nielson mechanism \(^11\). (For recent studies, see, e.g., Ref. \(^12\).) We will write, in a self-explanatory notation, the lepton doublets as \(L_\alpha = (\nu_{\alpha L}, e_{\alpha L})\), where \(\alpha = e, \mu, \tau\), and the right-handed charged leptons as \(E_\alpha = e_{\alpha R}\), where \(\alpha = e, \mu, \tau\). Suppose that the part of the scalar sector, which transforms non-trivially under the SM gauge group, consists of two Higgs doublets \(H_1\) and \(H_2\), where \(H_1\) couples to the neutrinos and \(H_2\) to the charged leptons \(^12\). (For simplicity and without loss of generality, the quark sector will be left out in our entire discussion.) Let us first restrict our discussion to the generation of the charged lepton masses. In order to obtain the structure of the charged lepton mass matrix from an underlying symmetry principle, we will further extend the scalar sector by SM singlet scalar fields \(\phi_i\) \((i = 1, 2, \ldots, 8)\) and \(\theta\) and assign the fields gauged horizontal U(1) charges \(Q_1\), \(Q_2\), and \(Q_3\) as follows:

| \(L_{e, E_e}\) | \((1, 0, 0)\) |
| \(L_{\mu, L_\tau, E_\mu, E_\tau}\) | \((0, 1, 0)\) |
| \(\phi_1, \phi_2\) | \((-1, 1, 2)\) |
| \(\phi_3, \phi_4\) | \((1, -1, 2)\) |
| \(\phi_5, \phi_6\) | \((0, 0, 0)\) |
| \(\phi_7, \phi_8\) | \((0, 0, 1)\) |
| \(\theta\) | \((0, 0, -1)\) |

In the rest of the paper, it is always understood that the Higgs doublets \(H_1\) and \(H_2\) are total singlets under transformations of the additional symmetries. Note that our model is kept anomaly-free, since the fermions transform as vector-like pairs under the extra U(1) charges. Next, the charges \((Q_1, Q_2)\) of the charged lepton-

\(^{12}\)This can easily be achieved by imposing a discrete \(Z_2\) symmetry under which \(H_2\) and \(E_\alpha\) \((\alpha = e, \mu, \tau)\) are odd and \(H_1\) and the rest of the SM fields are even.
We observe that these charges forbid dimension-four Yukawa coupling terms in the first generation to the second and third generations. A realistic charged lepton mass matrix will arise if we, in addition to the $U(1)$ charges, introduce a set of discrete symmetries $D_i$ ($i = 1, 2, \ldots, 5$), which are, at this level, not plagued with chiral anomalies. The $\mathbb{Z}_4$ symmetry

\[
D_1 = \begin{cases} 
E_\mu \rightarrow i E_\mu, & E_\tau \rightarrow i E_\tau, \\
\phi_3 \rightarrow -i \phi_3, & \phi_4 \rightarrow -i \phi_4, \\
\phi_5 \rightarrow -i \phi_5, & \phi_6 \rightarrow -i \phi_6, \\
\phi_7 \rightarrow -i \phi_7, & \phi_8 \rightarrow -i \phi_8 
\end{cases} \tag{1}
\]

forbids dimension-four Yukawa coupling terms in the $\mu$-$\tau$-subsector of the charged lepton sector. The $\mathbb{Z}_2$ symmetry

\[
D_2 = \begin{cases} 
E_e \rightarrow -E_e, \\
\phi_1 \rightarrow -\phi_1, & \phi_2 \rightarrow -\phi_2
\end{cases} \tag{2}
\]

sets to leading order the $e$-$e$-element of the charged lepton mass matrix equal to zero. It has been pointed out that bimaximal leptonic mixing corresponds to a permutation symmetry of the second and third generation leptons. Thus, we will introduce the three permutation symmetries

\[
D_3 = \begin{cases} 
L_\mu \rightarrow -L_\mu, & E_\mu \rightarrow -E_\mu, \\
\phi_1 \leftrightarrow \phi_2, & \phi_3 \leftrightarrow \phi_4
\end{cases} \tag{3a}
\]

\[
D_4 = \begin{cases} 
L_\mu \rightarrow -L_\mu, \\
\phi_1 \leftrightarrow \phi_2, & \phi_5 \leftrightarrow \phi_6
\end{cases} \tag{3b}
\]

\[
D_5 = \begin{cases} 
L_\mu \leftrightarrow L_\tau, & E_\mu \leftrightarrow E_\tau, \\
\phi_2 \rightarrow -\phi_2, & \phi_4 \rightarrow -\phi_4, \\
\phi_6 \rightarrow -\phi_6, & \phi_7 \leftrightarrow \phi_8
\end{cases} \tag{3c}
\]

Then, the most general charged lepton mass terms, which are invariant under all symmetry transformations of our model, are given by the higher-dimensional operators

\[
\mathcal{L} = \overline{L}_e H_2 \left[ (Y^e_{\text{eff}})_{\alpha\beta} + (Y^e_{\text{hor}})_{\alpha\beta} \right] E_\beta + \text{h.c.}, \tag{4}
\]

where the relevant effective Yukawa interaction matrices $Y^1_{\text{eff}}$ and $Y^2_{\text{eff}}$ are on the forms

\[
Y^1_{\text{eff}} = \begin{pmatrix} 0 & B(\phi_3 - \phi_4) & B(\phi_3 + \phi_4) \\ A(\phi_1 - \phi_2) & C(\phi_5 - \phi_6) & 0 \\ A(\phi_1 + \phi_2) & 0 & C(\phi_5 + \phi_6) \end{pmatrix}, \tag{5a}
\]

\[
Y^2_{\text{eff}} = \text{diag}(0, D\phi_\tau, D\phi_8). \tag{5b}
\]

Here, the dimensionful coefficients $A$, $B$, $C$, and $D$ are given by

\[
A = Y_a \frac{\theta^2}{M_1}, \tag{6a}
\]

\[
B = Y_b \frac{\theta^2}{M_1^2}, \tag{6b}
\]

\[
C = Y_c \frac{1}{M_1}, \tag{6c}
\]

\[
D = Y_d \frac{\theta}{M_1^2}, \tag{6d}
\]

where the quantities $Y_a$, $Y_b$, $Y_c$, and $Y_d$ are arbitrary order unity coefficients and $M_1$ is the high mass scale of the intermediate Froggatt–Nielsen states. Actually, in Sec. 2.3, the mass scale $M_1$ will be related to the breakdown scale of the extra symmetries by a small expansion parameter.

### 2.2. The scalar potential

The most general renormalizable scalar potential, involving only the fields $\phi_i$ ($i = 1, 2, \ldots, 6$), which is invariant under transformations of the horizontal symmetries given in Sec. 2.1, reads

\[
V = \mu_1^2 |\phi_1|^2 + |\phi_2|^2 + \mu_2^2 (|\phi_3|^2 + |\phi_4|^2) + \mu_3^2 (|\phi_5|^2 + |\phi_6|^2) + \kappa_1^2 (|\phi_1^\dagger \phi_1|^2 + |\phi_2^\dagger \phi_2|^2) + \kappa_2^2 (|\phi_3^\dagger \phi_3|^2 + |\phi_4^\dagger \phi_4|^2) + \kappa_3^2 (|\phi_5^\dagger \phi_5|^2 + |\phi_6^\dagger \phi_6|^2) + a |\phi_1^\dagger \phi_2|^2 + b |\phi_3^\dagger \phi_4|^2 + c |\phi_5^\dagger \phi_6|^2 + d (|\phi_1^\dagger \phi_1|^2 + |\phi_2^\dagger \phi_2|^2 + |\phi_3^\dagger \phi_3|^2 + |\phi_4^\dagger \phi_4|^2) + e (|\phi_1^\dagger \phi_1|^2 + |\phi_5^\dagger \phi_5|^2 + |\phi_6^\dagger \phi_6|^2 + |\phi_2^\dagger \phi_2|^2) + f (|\phi_1^\dagger \phi_5|^2 + |\phi_3^\dagger \phi_6|^2 + |\phi_2^\dagger \phi_4|^2 + |\phi_4^\dagger \phi_2|^2)
\]
\[ + \lambda_1 \left( \phi_1^t \phi_2 \right) + \lambda_2 \left( \phi_3^t \phi_4 \right) + \lambda_3 \left( \phi_5^t \phi_6 \right) \]
\[ + m_1 \left( \phi_1^t \phi_2 + \phi_3^t \phi_4 \right) \left( \phi_1 \phi_4 + \phi_3 \phi_4 \right) \]
\[ + m_2 \left( \phi_1^t \phi_2 + \phi_3^t \phi_4 \right) \left( \phi_1 \phi_6 + \phi_3 \phi_6 \right) \]
\[ + m_3 \left( \phi_3^t \phi_4 + \phi_3^t \phi_3 \right) \left( \phi_3 \phi_6 + \phi_3 \phi_6 \right) . \tag{7} \]

where all coefficients are real. Due to the symmetries of our model, the remaining scalar fields enter relevant terms in the potential $V$ only via quartic couplings in form of absolute squares of these fields, which means that they can be combined into the coefficients $\mu_i$ ($i = 1, 2, 3$). Therefore, we can choose the coefficients in the potential to fulfill $\mu_i^2 < 0$ ($i = 1, 2, 3$), $\kappa_i > 0$ ($i = 1, 2, 3$), and $a, b, \ldots, f > 0$, which yield after spontaneous symmetry breaking (SSB) non-vanishing vacuum expectation values (VEVs) that satisfy

\[ |\langle \phi_1 \rangle| = |\langle \phi_2 \rangle|, \quad |\langle \phi_3 \rangle| = |\langle \phi_4 \rangle|, \quad |\langle \phi_5 \rangle| = |\langle \phi_6 \rangle|. \]

If $\lambda_i < 0$ ($i = 1, 2, 3$), then we obtain pairwise relatively real VEVs, i.e.,

\[ \frac{\langle \phi_1 \rangle \langle \phi_3 \rangle \langle \phi_5 \rangle}{\langle \phi_2 \rangle \langle \phi_4 \rangle \langle \phi_6 \rangle} \in \{-1, 1\} . \]

Next, choosing $m_1 < 0$ and $m_2, m_3 > 0$, we obtain

\[ \frac{\langle \phi_1 \rangle \langle \phi_3 \rangle}{\langle \phi_2 \rangle \langle \phi_4 \rangle} = 1 \quad \text{and} \quad \frac{\langle \phi_1 \rangle \langle \phi_5 \rangle}{\langle \phi_2 \rangle \langle \phi_6 \rangle} = -1 , \]

i.e., the relative sign between $\langle \phi_1 \rangle$ and $\langle \phi_2 \rangle$ is equal to the relative sign between $\langle \phi_3 \rangle$ and $\langle \phi_4 \rangle$ and opposite to the relative sign between $\langle \phi_5 \rangle$ and $\langle \phi_6 \rangle$. In Sec. 2.3 we will show that this alignment mechanism reconciles the permutation symmetry $D_5$ with an approximate diagonal form of the charged lepton mass matrix.

### 2.3. The charged lepton mass matrix

Suppose that the SM singlet scalar fields acquire their VEVs at a high mass scale and thereby give rise to a small expansion parameter

\[ \epsilon \sim \frac{\langle \phi_1 \rangle}{M_1} \sim \frac{\langle \theta \rangle}{M_1} \sim 10^{-1} , \tag{8} \]

where $i = 1, 2, \ldots, 8$. Such small hierarchies can arise from large hierarchies in supersymmetric theories when the scalar fields acquire their VEVs along a “D-flat” direction [17]. As a consequence of the permutation symmetries $D_3$, $D_4$, and $D_5$, the lowest energy state is two-fold degenerate. Applying the results of Sec. 2.2 and inserting Eq. (6) into Eqs. (3) and the result thereof into Eqs. (8), we obtain the two possible charged lepton mass matrices

\[ M_\ell \sim m_\tau \begin{pmatrix} 0 & \epsilon^2 & 0 \\ \epsilon^2 & \epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{9} \]

and

\[ M_\ell \sim m_\tau \begin{pmatrix} 0 & 0 & \epsilon^2 \\ 0 & 1 & 0 \\ \epsilon^2 & 0 & \epsilon \end{pmatrix} , \tag{10} \]

where $m_\tau$ is the tau mass and only the order of magnitude of the matrix elements have been indicated. Note that a permutation of the second and third generations, $L_\mu \leftrightarrow L_\tau$, $E_\mu \leftrightarrow E_\tau$, leads from one solution to the other. Let us take the first one. Diagonalization of $M_\ell$ [in Eq. (6)] then gives for the charged lepton masses the order-of-magnitude relations

\[ m_\tau / m_\mu \approx \epsilon^2 \approx 10^{-2} \quad \text{and} \quad m_\mu / m_\tau \approx \epsilon \approx 10^{-1} , \]

which approximately fit the experimentally observed values [i.e., $(m_\mu / m_\mu)_\text{exp} \approx 4.8 \cdot 10^{-3}$ and $(m_\mu / m_\tau)_\text{exp} \approx 5.9 \cdot 10^{-2}$ [2]]. Here $m_\tau$ and $m_\mu$ are the electron and muon masses, respectively. The charged lepton mass matrix $M_\ell$ is diagonalized by a rotation of the left-handed charged lepton fields in the 1-2-plane by an angle $\theta_{12} \approx 6^\circ$

\[^3\text{The charged lepton mass spectrum is: } m_e \equiv |\lambda_1|, \quad m_\mu \equiv |\lambda_2|, \quad m_\tau \equiv |\lambda_3|, \quad \text{where } \lambda_1 = \frac{\theta}{2} \left( 1 - \sqrt{1 + 4\epsilon^2} \right) m_\tau \approx -\epsilon^2 m_\tau, \quad \lambda_2 = \frac{\theta}{2} \left( 1 + \sqrt{1 + 4\epsilon^2} \right) m_\tau \approx \epsilon m_\tau, \quad \text{and } \lambda_3 = m_\tau \text{ are the eigenvalues of the matrix } M_\ell.\]
(θ_{12}^\nu = \epsilon - \frac{1}{3} \epsilon^3 + O(\epsilon^5), \text{ where } \epsilon \simeq 0.1, \theta_{13}^\nu = 0, \text{ and } \theta_{23}^\nu = 0), \text{ which will finally give a contribution to all leptonic mixing angles (see App. A).}

3. The neutrino mass matrix

Let us now turn our discussion to the neutrino mass matrix. As intermediate Froggatt–Nielsen states we will assume two heavy SM singlet Dirac fermions $F_1$ and $F_2$, which have masses of the order $M_1$. In order to account for the smallness of the neutrino masses, we will furthermore introduce three SM singlet Dirac fermions $N_e$, $N_\mu$, and $N_\tau$, which have masses of the order of some relevant high mass scale $M_2$. From the assignment of the charges $(Q_1, Q_2)$ to the lepton doublets, the structure of the $(Q_1, Q_2)$ charges associated with the effective neutrino mass matrix follows immediately:

| $(Q_1, Q_2)$ | $L_e$ | $L_\mu$ | $L_\tau$ |
|--------------|----------|----------|----------|
| $(1,0)$      | $(1,0)$  | $(0,1)$  | $(0,1)$  |
| $(0,1)$      | $(2,0)$  | $(1,1)$  | $(1,1)$  |
| $(0,1)$      | $(0,1)$  | $(1,1)$  | $(0,2)$  |

Note that the given representation content so far forbids any neutrino mass term. We therefore introduce the additional SM singlet scalar fields $\phi_9$, $\phi_{10}$, $\phi_{11}$, and $\phi_{12}$ and we assign the charges $Q_1$, $Q_2$, and $Q_3$ to the fields as follows:

| $\phi_{9,10}$ | $(Q_1, Q_2, Q_3)$ |
|--------------|------------------|
| $(1,0,0)$    | $(1,0,0)$        |
| $(0,1,0)$    | $(0,1,0)$        |
| $(1,0,0)$    | $(1,0,0)$        |
| $(-1,0,1)$   | $(-1,0,1)$       |

Note that our model is again kept free from chiral anomalies, since the heavy fermions are vector representations under transformations of all U(1) charges. The Dirac neutrino fields and the scalar fields transform under the discrete symmetries $D_3$, $D_4$, $D_5$, and the additional discrete symmetry $D_6$ as

$$D_3 : \ldots, N_\mu \rightarrow -N_\mu, \phi_9 \rightarrow -\phi_9, \quad (11a)$$
$$D_4 : \ldots, N_\mu \rightarrow -N_\mu, \phi_9 \rightarrow -\phi_9, \quad (11b)$$
$$D_5 : \ldots, N_\mu \leftrightarrow N_\tau, \phi_9 \leftrightarrow \phi_{10}, \quad (11c)$$
$$D_6 : \left\{ \begin{array}{l} N_e \rightarrow iN_e, \\ \phi_{11} \rightarrow -i\phi_{11}, \quad \phi_{12} \rightarrow i\phi_{12}. \end{array} \right. \quad (11d)$$

It is easily verified that the results for the charged lepton mass matrix remain unchanged by this new representation content. The leading order tree-level realizations of the higher-dimensional operators, which generate the neutrino masses, are shown in Figs. 1 and 2. After SSB, the effective neutrino mass matrix $M_\nu$ will be on the approximate bimaximal mixing form $\left[1\right]$

$$M_\nu = \begin{pmatrix} A' & B' & -B' \\ B' & 0 & 0 \\ -B' & 0 & 0 \end{pmatrix}, \quad (12)$$

![Figure 1](image1.png)  
**Figure 1.** The dimension six operator for $\alpha = \mu, \tau$ and $\phi_\mu \equiv \phi_9$, $\phi_\tau \equiv \phi_{10}$, generating the $e-\mu$- and $e-\tau$-elements in the effective neutrino mass matrix.

![Figure 2](image2.png)  
**Figure 2.** The dimension eight operators, generating the $e-e$-element in the effective neutrino mass matrix.
Thus, diagonalizing the neutrino mass matrix \( M_{\nu} \) therefore, we can parameterize the effective neutrino expression ratio which is on the inverted hierarchical form \((\Delta m_{32}^2)\), and hence, the magnitudes of the \( e-\mu \) and \( e-\tau \)-entries in the effective neutrino mass matrix \( M_{\nu} \) are exactly degenerate. The possible relative phase \( \varphi \) between these entries can be eliminated by, e.g., the field redefinition \( L_{\tau} \rightarrow e^{i\varphi} L_{\tau} \), since it does not affect the mixing angles in the charged lepton sector. If we assume that all SM singlet scalar fields affect the mixing angles in the charged lepton sector, which is about 6\(^\circ\) (see Sec. 2.3), resulting in a change of the mixing angles \( \theta_{12} \) and \( \theta_{13} \) by approximately 4\(^\circ\), while the atmospheric mixing angle \( \theta_{23} \) practically stays maximal (see App. A). Thus, the leptonic mixing angles are

\[
\theta_{12} \simeq 41^\circ, \quad \theta_{13} \simeq 4^\circ, \quad \text{and} \quad \theta_{23} \simeq 45^\circ.
\]

Hence, our model predicts the charged lepton mass hierarchy spectrum, an inverted neutrino mass hierarchy spectrum, bilarge leptonic mixing, as well as it reproduces the mass squared differences to lie within the ranges preferred by the MSW LMA solution and atmospheric neutrino data. In particular, it yields a significant deviation from maximal solar mixing. However, the solar mixing angle is bounded from below by approximately 41\(^\circ\) and it is therefore still too close to maximal to be in the 95\% (or 99.73\%) confidence level region of the MSW LMA solution.

### 4. Summary and conclusions

In summary, we have presented a model built upon extra fields, horizontal (flavor) symmetries, and higher-dimensional operators including the Froggatt–Nielsen mechanism. This model naturally yields the well-known mass matrix textures

\[
\begin{pmatrix} 0 & c^2 & 0 \\ c^2 & \epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \epsilon^2 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}
\]

for the charged leptons and the neutrinos, respectively, which involve the same small expansion parameter \( \epsilon \simeq 0.1 \). These textures reproduce the mass hierarchy among the charged leptons very

\[ \text{at 99.73\% C.L.: } 1.9 \cdot 10^{-5} \text{eV}^2 \lesssim \Delta m_{21}^2 \lesssim 2.7 \cdot 10^{-4} \text{eV}^2 \]

\[ \text{and at 90\% C.L.: } 1.6 \cdot 10^{-3} \text{eV}^2 \lesssim \Delta m_{21}^2 \lesssim 4.0 \cdot 10^{-3} \text{eV}^2 \]

\[ \text{with best-fit: } \Delta m_{21}^2 \simeq 2.5 \cdot 10^{-3} \text{eV}^2 \]

\[ \text{at } 99.73\% \text{ C.L.: } 0.22 \lesssim \tan^2 \theta_{13} \lesssim 0.71 \Rightarrow 25^\circ \lesssim |\theta_{13}| \lesssim 31^\circ \]

\[ \text{at 99.73\% C.L.: } \theta_{12} \simeq 40^\circ \Rightarrow \text{best-fit: } tan^2 \theta_{12} \simeq 0.37 \Rightarrow |\theta_{12}| \simeq 25^\circ \]

\[ \text{and at 99.73\% C.L.: } |\theta_{13}| \simeq 45^\circ \Rightarrow \text{best-fit: } |\theta_{13}| \simeq 45^\circ \]

\[ \text{and maximal solar mixing, which is of the order } 0.1^\circ \]

\[ \theta_{12} = 25^\circ + \frac{1}{4} \epsilon^2 - \frac{1}{96} \epsilon^4 + O(\epsilon^6) \]

where \( \epsilon \simeq \sqrt{A/B} \approx 0.1, \theta_{13} = 0, \) and \( \theta_{23} = 45^\circ \).
accurately as well as they give rise to the most probable values of the mass squared differences for the neutrinos coming from solar and atmospheric neutrino data. In addition, assuming no CP violation, i.e., \( \delta = 0 \), the model gives bilarge leptonic mixing, i.e., \( \theta_{12} \simeq 41^\circ \), \( \theta_{13} \simeq 4^\circ \), and \( \theta_{23} \simeq 45^\circ \), which is in very good agreement with the present experimental data. The mixing angle \( \theta_{12} \) is on the borderline of being compatible with the MSW LMA solution, which has, however, been further strengthened by recent SNO results \[19\], whereas \( \theta_{13} \) is well below the CHOOZ upper bound, and \( \theta_{23} \) fits perfectly the best-fit value from the Super-Kamiokande collaboration of their atmospheric neutrino data.

As a final conclusion, we have shown in App. \[A\] that the leptonic mixing angles are all dependent on the “solar” mixing angle \( \theta'_{12} \) in the charged lepton sector in a non-trivial way.

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### A. The leptonic mixing matrix - A special case

The leptonic mixing matrix can be written as

\[
U = O_{23}(\theta_{23})U_{13}(\theta_{13}, \delta)O_{12}(\theta_{12})
\]

\[
= \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix} \times \begin{pmatrix} C_{13} & 0 & S_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -S_{13}e^{i\delta} & 0 & C_{13} \end{pmatrix} \times \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

where \( C_{ab} \equiv \cos \theta_{ab}, \: S_{ab} \equiv \sin \theta_{ab}, \) and \( O_{ab}(\theta_{ab}) \) is a rotation by an angle \( \theta_{ab} \) in the ab-plane. If \( \delta = 0 \), then \( U_{13}(\theta_{13}, 0) = O_{13}(\theta_{13}) \). Assuming that all the CP phases are zero, i.e., \( \delta = \delta' = \delta'' = 0 \), then the charged lepton and neutrino mixing matrices, \( U^\ell \) and \( U^\nu \), can, of course, also be written in the same form as the above complete leptonic mixing matrix, \( U \), i.e., we have

\[
U^\ell = O_{23}(\theta_{23})U_{13}^\ell(\theta_{13}^\ell, \delta^\ell)O_{12}^\ell(\theta_{12}^\ell), \quad (15a)
\]

\[
U^\nu = O_{23}(\theta_{23}^\nu)U_{13}^\nu(\theta_{13}^\nu, \delta^\nu)O_{12}^\nu(\theta_{12}^\nu). \quad (15b)
\]

Thus, inserting Eqs. (15) into the definition of the leptonic mixing matrix, \( U = (U^\nu)^T U^\ell \), we find that

\[
U = O_{12}^\ell(\theta_{12}^\ell)^T O_{13}(\theta_{13}^\ell) O_{23}(\theta_{23}^\ell)^T 
\]

\[
\times \: O_{13}^\nu(\theta_{13}^\nu) O_{23}^\nu(\theta_{23}^\nu) O_{12}^\nu(\theta_{12}^\nu). \quad (16)
\]

Furthermore, assuming that we have only a small mixing coming from the mixing angle \( \theta_{12}^\ell \) in the charged lepton sector \( \left( \theta_{13}^\nu = 0, \: \theta_{23}^\nu = 0 \right) \) and bimaximal mixing in the neutrino sector \( \left( \theta_{12}^\nu = 45^\circ, \: \theta_{13}^\nu = 0, \: \theta_{23}^\nu = 45^\circ \right) \), we then obtain

\[
U = O_{12}^\ell(\theta_{12}^\ell)^T O_{23}^\nu(\theta_{23}^\nu = 45^\circ)O_{12}^\nu(\theta_{12}^\nu = 45^\circ)
\]

\[
= \begin{pmatrix} c_{12}^2/\sqrt{2} + s_{12}^2/2 & s_{12}^2/\sqrt{2} - c_{12}^2/2 \\ s_{12}^2/\sqrt{2} - c_{12}^2/2 & c_{12}^2/\sqrt{2} + s_{12}^2/2 \end{pmatrix}
\]

\[
= \begin{pmatrix} c_{12}^2/\sqrt{2} + s_{12}^2/2 & s_{12}^2/\sqrt{2} - c_{12}^2/2 \\ c_{12}^2/\sqrt{2} + s_{12}^2/2 & -s_{12}^2/\sqrt{2} \end{pmatrix}
\]

\[
× \begin{pmatrix} c_{12}^2/\sqrt{2} - s_{12}^2/2 \\ c_{12}^2/\sqrt{2} + s_{12}^2/2 \end{pmatrix}
\]

\[
(17)
\]
The mixing angles (in the standard parameterization) of a $3 \times 3$ orthogonal mixing matrix can be read off as follows \cite{20}:

\[\begin{align*}
\theta_{12} &= \arctan \frac{U_{e2}}{U_{e1}}, \\
\theta_{13} &= \arcsin U_{e3}, \\
\theta_{23} &= \arctan \frac{U_{\mu 3}}{U_{\tau 3}}.
\end{align*}\]

(18) (19) (20)

Thus, inserting the appropriate matrix elements of the matrix $U$ in Eq. (17) into Eqs. (18) - (20), we finally obtain

\[\begin{align*}
\theta_{12} &= \arctan \frac{\cos \theta_{12} - \frac{1}{\sqrt{2}} \sin \theta_{12}^c}{\cos \theta_{12} + \frac{1}{\sqrt{2}} \sin \theta_{12}^c}, \\
\theta_{13} &= -\arcsin \left(\frac{1}{\sqrt{2}} \sin \theta_{12}^c\right), \\
\theta_{23} &= \arctan \cos \theta_{12}^c.
\end{align*}\]

(21) (22) (23)

When $\theta_{12}^c$ is small ($\theta_{12}^c \ll 1$), we have\footnote{Introducing a small deviation $\eta$ from maximal solar mixing in the neutrino sector (i.e., $\theta_{12}^c = 45^\circ \rightarrow \theta_{12}^c - \eta = 45^\circ - \eta$, where $\eta \ll 1$), we find that $\theta_{12}^c = \pi/4 - \frac{1}{4 \sqrt{2}} \theta_{12}^c + \frac{1}{6 \sqrt{2}} \theta_{12}^c^3 + \mathcal{O}(\theta_{12}^c^5)$, (24) \[\begin{align*}
\theta_{13} &= -\frac{1}{\sqrt{2}} \theta_{12}^c + \frac{1}{12 \sqrt{2}} \theta_{12}^c^3 + \mathcal{O}(\theta_{12}^c^5), \\
\theta_{23} &= \frac{\pi}{4} - \frac{1}{4} \theta_{12}^c^2 - \frac{1}{24} \theta_{12}^c^4 + \mathcal{O}(\theta_{12}^c^6).
\end{align*}\] (25) (26)}

\[\begin{align*}
\theta_{12} &= \frac{\pi}{4} - \frac{1}{\sqrt{2}} \theta_{12}^c + \frac{1}{6 \sqrt{2}} \theta_{12}^c^3 + \mathcal{O}(\theta_{12}^c^5), \\
\theta_{13} &= -\frac{1}{\sqrt{2}} \theta_{12}^c + \frac{1}{12 \sqrt{2}} \theta_{12}^c^3 + \mathcal{O}(\theta_{12}^c^5), \\
\theta_{23} &= \frac{\pi}{4} - \frac{1}{4} \theta_{12}^c^2 - \frac{1}{24} \theta_{12}^c^4 + \mathcal{O}(\theta_{12}^c^6).
\end{align*}\]

(24) (25) (26)

Note that all leptonic mixing angles receive contribution from the small mixing angle $\theta_{12}^c$ in the charged lepton sector. Furthermore, we observe that $\theta_{12}$ has first order corrections in $\theta_{12}^c$, whereas $\theta_{23}$ has only second order corrections in $\theta_{12}^c$. The mixing angle $\theta_{13}$ is directly proportional to the mixing angle $\theta_{12}$ (when $\theta_{12}^c$ is small), which means that if $\theta_{12}^c$ is small, then $\theta_{13}$ will also be small. In fact, $|\theta_{12}^c| \lesssim 13.1^\circ$ has to be fulfilled in order for the mixing angle $\theta_{13}$ to be below the CHOOZ upper bound $\sin^2 2\theta_{13} \lesssim 0.10$ (i.e., $|\theta_{13}| \lesssim 9.2^\circ$) \cite{21}.

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