Could we rotate proton decay away?

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In this work we investigate the possibility to completely rotate away proton decay. We show that by choosing specific mass matrices for fermions it is possible to accomplish this in flipped $SU(5)$.

Proton decay \(^1\) is the most important prediction of grand unified theories \(^2,3,4,5,6\). For phenomenological studies of its signatures see references \(^7,8,9,10,11,12,13\). It is usually dominated by the gauge $d = 6$ operators in any non-supersymmetric grand unifying scenario. On the other hand, in supersymmetric scenarios, proton decay is dominated by the $d = 4$ and $d = 5$ \(^14,15,16,17\) operators. However, the $d = 4$ contributions can be forbidden by imposing the so-called matter parity \(^18\) and there is always a way to suppress the $d = 5$ contributions \(^19,20,21,22\). Therefore, it looks as if the $d = 6$ operators are the most promising sources to test both supersymmetric and non-supersymmetric scenarios. It is thus crucial if we can establish how sensitive these operators to the parameters entering in a grand unified theory are since they represent the source for such an important signature of grand unification.

The idea of using the gauge $d = 6$ dominated branching ratios for the two-body nucleon decays to distinguish between different fermion mass models has been around since the pioneering work of De Rujula, Georgi and Glashow \(^4\). More recently, the possibility to make a clear test of any grand unified theory based on $SU(5)$ and $SO(10)$ with symmetric Yukawa couplings through the decay channels into antineutrinos via $d = 6$ gauge contributions has been put forth \(^23\). Similar program has also been carried out in the context of flipped $SU(5)$ \(^4,5,6,24\). Namely, the minimal flipped $SU(5)$ \(^25,26,27,28\) scenario can be tested by looking simultaneously at the decay $p \to \pi^+\tau$ and the ratio $\tau(p \to K^0 e^+)/\tau(p \to \pi^0 e^+)$ \(^29\). It is thus interesting to investigate how these conclusions change if one departs from the flavor structure of the minimal renormalizable theory.

It is well known that the gauge $d = 6$ proton decay cannot be rotated away, i.e., set to zero via particular choice of parameters entering in a grand unified theory, in the framework of conventional $SU(5)$ \(^2,3\) theory with the Standard Model particle content \(^30,31,32\). So, one might think that
the gauge $d = 6$ operators and proton decay they govern are genuine features of matter unification.
In this work we show that this might not be true. Namely, we demonstrate that it is possible to completely rotate away the gauge $d = 6$ contributions for proton decay imposing simple conditions on fermion mixing. We accomplish this in the framework of flipped $SU(5)$.

In order to appreciate all the difficulties involved in trying to rotate proton decay away, let us first revisit the case of a theory based on conventional $SU(5)$ \cite{2,3}. In the ordinary $SU(5)$ the integration of off-diagonal gauge bosons, $V = (X, Y) = (3, 2, 5/3)$, yields the following operators \cite{15,16,17} in the physical basis \cite{23}:

\[
O(e^C_a, d^C_\beta)_{SU(5)} = c(e^C_a, d^C_\beta)_{SU(5)} \epsilon_{ijk} u^C_i \gamma^\mu u_j \bar{d}^C_\beta \gamma^\mu d_k \beta, \quad (1a)
\]
\[
O(e_\alpha, d^C_\beta)_{SU(5)} = c(e_\alpha, d^C_\beta)_{SU(5)} \epsilon_{ijk} \bar{u}^C_i \gamma^\mu u_j \bar{d}^C_\beta \gamma^\mu e_\alpha, \quad (1b)
\]
\[
O(\nu_i, d_\alpha, d^C_\beta)_{SU(5)} = c(\nu_i, d_\alpha, d^C_\beta)_{SU(5)} \epsilon_{ijk} \bar{u}^C_i \gamma^\mu d_j \alpha \bar{d}^C_\beta \gamma^\mu \nu_k, \quad (1c)
\]
\[
O(\nu^C_i, d_\alpha, d^C_\beta)_{SU(5)} = c(\nu^C_i, d_\alpha, d^C_\beta)_{SU(5)} \epsilon_{ijk} \bar{d}^C_\beta \gamma^\mu u_j \nu^C_i \gamma^\mu d_k \alpha, \quad (1d)
\]

where the coefficients that enter in the decay rate formulas take the form (for the relevant decay formulas see \cite{28,29}):

\[
c(e^C_a, d^C_\beta)_{SU(5)} = k_1^2 \left[V_1^{11} V_2^{a \beta} + (V_1 V_{UD})^{1 \beta} (V_2 V_{UD})^{a 1}\right], \quad (2a)
\]
\[
c(e_\alpha, d^C_\beta)_{SU(5)} = k_2^2 V_1^{11} V_3^{\beta \alpha}, \quad (2b)
\]
\[
c(\nu_i, d_\alpha, d^C_\beta)_{SU(5)} = k_3^2 (V_1 V_{UD})^{1 \alpha} (V_3 V_{EN})^{\beta i}, \quad \alpha = 1 \text{ or } \beta = 1, \quad (2c)
\]
\[
c(\nu^C_i, d_\alpha, d^C_\beta)_{SU(5)} = 0. \quad (2d)
\]

In the above expressions $k_1 = g_5 M_V^{-1}$, where $M_V \sim M_{GUT} \approx 10^{16}$ GeV and $g_5$ are the masses of the superheavy gauge bosons and the coupling at the GUT scale. $i$, $j$ and $k$ are the color indices, $a$ and $b$ are the family indices, and $\alpha, \beta = 1, 2$. The mixing matrices are: $V_1 = U_C^U$, $V_2 = E_C^D$, $V_3 = D_C^E$, $V_{UD} = U^D$, and $V_{EN} = E^N$. Our convention for the diagonalization of the up, down and charged lepton Yukawa matrices is specified by $U_C^T Y_U U = Y_U^{\text{diag}}$, $D_C^T Y_D D = Y_D^{\text{diag}}$, and $E_C^T Y_E E = Y_E^{\text{diag}}$. The quark mixing is given by $V_{UD} = U^D = K_1 V_{CKM} K_2$, where $K_1$ and $K_2$ are diagonal matrices containing three and two phases, respectively. $V_{CKM}$ is the Cabibbo-Kobayashi-Maskawa quark-mixing matrix \cite{33,34}.

The leptonic mixing $V_{EN} = K_3 V_{1}^{D} K_4$ in case of Dirac neutrino, or $V_{EN} = K_3 V_{1}^{M}$ in the Majorana case. $V_{1}^{D}$ and $V_{1}^{M}$ are the leptonic mixing matrices at low energy in the Dirac and Majorana case, respectively.

We now show that the demand to rotate away proton decay leads the conflict with the experimental data \cite{32}. In order to set Eq. (2b) to zero, the only possible choice is $V_1^{11} = 0$. [Setting
Notice that we use the subscripts $SU_0$ the coefficient entering in the decay channel into antineutrinos. Namely, we have to choose $(V_1 V_U)_{1\alpha} = 0$. This, however, is not possible since it would imply that, at least, $V_{CKM}^{13}$ is zero in conflict with the data.

Let us now investigate the same issue in flipped $SU(5)$ [23, 24]. In this case the gauge $d = 6$ proton decay is mediated by $V' = (X', Y') = (3, 2, -1/3)$. This time $d = 6$ operators in the physical basis are [23]:

\begin{align}
O(e^C, d_\beta)_{SU(5)'} &= c(e^C, d_\beta)_{SU(5)'} \epsilon_{ijk} \overline{u}^C_i \gamma^\mu u_j e^C_\alpha \gamma_\mu d_k \beta, \quad (3a) \\
O(e_\alpha, d^C_\beta)_{SU(5)'} &= c(e_\alpha, d^C_\beta)_{SU(5)'} \epsilon_{ijk} \overline{u}_i \gamma^\mu u_j \overline{d}^C_k \gamma_\mu e_\alpha, \quad (3b) \\
O(\nu_l, d_\alpha, d^C_\beta)_{SU(5)'} &= c(\nu_l, d_\alpha, d^C_\beta)_{SU(5)'} \epsilon_{ijk} \overline{u}^{C}_i \gamma^\mu d_j \alpha \overline{d}^C_\beta \gamma_\mu \nu_l, \quad (3c) \\
O(\nu^C_l, d_\alpha, d^C_\beta)_{SU(5)'} &= c(\nu^C_l, d_\alpha, d^C_\beta)_{SU(5)'} \epsilon_{ijk} \overline{d}^C_\beta \gamma^\mu u_j \overline{\nu}^{C}_l \gamma_\mu d_k \alpha, \quad (3d)
\end{align}

where

\begin{align}
c(e^C, d_\beta)_{SU(5)'} &= 0, \quad (4a) \\
c(e_\alpha, d^C_\beta)_{SU(5)'} &= k_2^2 (V_4 V_U^T)_{1\beta} (V_1 V_U V_4^d V_3)^{1\alpha}, \quad (4b) \\
c(\nu_l, d_\alpha, d^C_\beta)_{SU(5)'} &= k_2^2 V_4^{\beta\alpha} (V_1 V_U V_4^d V_3 V_E)^{\gamma l}, \quad \alpha = 1 \text{ or } \beta = 1, \quad (4c) \\
c(\nu^C_l, d_\alpha, d^C_\beta)_{SU(5)'} &= k_2^2 \left[(V_4 V_U^T)_{1\beta} (U_{EN}^T V_2)^{\gamma l} + V_4^{\beta\alpha} (U_{EN}^T V_2 V_U^T)^{\gamma l}\right], \quad \alpha = 1 \text{ or } \beta = 1. \quad (4d)
\end{align}

Notice that we use the subscripts $SU(5)'$ for flipped $SU(5)$. In the above equations, the mixing matrices $V_4 = D_C^T D$, and $U_{EN} = E_C^T N_C$. The factor $k_2 = g_5'^{-1} / M_{Y'}$, where $g_5'$ is given by the unification of $\alpha_2$ and $\alpha_3$.

Let us see if it is possible to rotate away the proton decay in flipped $SU(5)$. To set Eq. (4e) to zero, we can only choose $V_4^{\beta\alpha} = (D_C^T D)^{\beta\alpha} = 0$, where $\alpha = 1$ or $\beta = 1$. We could think about possibility of making both Eqs. (4f) and (4g) zero, choosing $(V_4 V_U^T)^{\beta 1} = 0$, however, this is in contradiction with the measurements of the CKM angles. Since in flipped $SU(5)$ the neutrino is Majorana, we only have to suppress Eq. (4f). This can be accomplished by setting $(V_1 V_U V_4^d V_3)^{1\alpha} = (U_C^T E)^{1\alpha} = 0$. Notice that this is completely unrelated to our condition on $V_4$. Thus, there is no contradiction with unitarity constrains nor conflict with any experimental measurements of mixing angles. As you can appreciate, in the context of flipped $SU(5)$, it is possible to completely rotate away the gauge $d = 6$ contributions in a consistent way, if we impose these two conditions at 1 GeV.

We stress that in minimal renormalizable flipped $SU(5)$ [25, 26, 27, 28] it is not possible to satisfied the first condition, since $Y_D = Y_D^T$ implies $V_4 = K_d^*$, where $K_d$ is a diagonal matrix.
containing three CP violating phases. However, as we know, in general we have to take into account the nonrenormalizable operators, which are very important for fermion masses and which invariably lead to modification of naive predictions. Therefore in general, in the context of flipped SU(5), we are allowed to impose our conditions and remove the gauge operators for proton decay.

Note that the main difference between the SU(5) analysis and flipped SU(5) one is that the unitary constraint that prevents us to rotate away proton decay in conventional SU(5) does not operate in the latter case. In other words, the coefficients which depend on \( \alpha \) and \( \beta \) with \( \alpha = 1 \) or \( \beta = 1 \) have different impact in those two scenarios (see Eqs. (2b) and (4c)).

What these two conditions that remove \( d = 6 \) operators imply for the structure of the fermion sector? We give one example. Let us choose the basis where the up quark mass matrix is diagonal. In this case we have:

\[
Y_D = K_1^* V_{CKM}^{*} K_2^* A Y_D^{\text{diag}} K_2^* V_{CKM}^{*} K_1^*, \tag{5a}
\]

\[
Y_E = E_C^* Y_E^{\text{diag}} E^\dagger, \tag{5b}
\]

\[
Y_N = K_3^* V_1^* Y_N^{\text{diag}} E^\dagger V_1^* K_3^*, \tag{5c}
\]

where \( |E^{13}| = 1 \) and \( A \) is a unitary matrix, with \( |A^{13}| = |A^{22}| = |A^{31}| = 1 \).

In order to understand if it is possible to suppress all contributions to proton decay we assume the matter parity to be an exact symmetry and proceed with the analysis of the Higgs \( d = 6 \) and \( d = 5 \) contributions. In SUSY flipped SU(5) the interactions for triplets are given by:

\[
W_T = \int d^2 \theta \left[ \hat{Q} A \hat{Q} \hat{T} + \hat{D}^C B \hat{N}^C \hat{T} + \hat{Q} C \hat{L} \hat{T} + \hat{D}^C D \hat{U} \hat{C} \hat{T} \right] + \text{h.c.}, \tag{6}
\]

where the matrices \( A, B, C, \) and \( D \) are:

\[
A = Y_D^{\text{ren}} + \frac{M_{\text{GUT}}}{M_{\text{Planck}}} Y_1 + \frac{M_{\text{GUT}}^2}{M_{\text{Planck}}^2} Y_2, \tag{7a}
\]

\[
B = Y_D^{\text{ren}} + \frac{M_{\text{GUT}}}{M_{\text{Planck}}} Y_3 + \frac{M_{\text{GUT}}^2}{M_{\text{Planck}}^2} Y_4, \tag{7b}
\]

\[
C = Y_U^{\text{ren}} + \frac{M_{\text{GUT}}}{M_{\text{Planck}}} Y_5 + \frac{M_{\text{GUT}}^2}{M_{\text{Planck}}^2} Y_6, \tag{7c}
\]

\[
D = Y_U^{\text{ren}} + \frac{M_{\text{GUT}}}{M_{\text{Planck}}} Y_7 + \frac{M_{\text{GUT}}^2}{M_{\text{Planck}}^2} Y_8, \tag{7d}
\]

up to the second order in \( M_{\text{GUT}}/M_{\text{Planck}} \) expansion. \( Y_D^{\text{ren}} \) and \( Y_U^{\text{ren}} \) are the Yukawa matrices at the renormalizable level for down and up quarks, respectively. \( Y_i, i = 1..8 \), are the contributions coming from the non-renormalizable terms.
Now, notice that we could forbid the $d = 5$ and Higgs $d = 6$ contributions, imposing the conditions $A^T = -A$ and $(D^T C D U C)^{ij} = 0$, except for $i = j = 3$. For similar approach see [35, 36]. This confirms that it is possible to completely rotate proton decay away in flipped $SU(5)$ context.

In this work we review the possibilities to suppress all operators for proton decay. We revisit conventional $SU(5)$ to show that in this scenario it is not possible to rotate away proton decay. We further investigate the case of flipped $SU(5)$ finding that there it is possible to completely eliminate the gauge $d = 6$ operators. In the same context we show the way to remove the Higgs $d = 6$ and $d = 5$ contributions. Our main result—the possibility to rotate away proton decay in flipped $SU(5)$—shows the lack of robustness of the gauge $d = 6$ contributions under departure from the “naive” assumptions for the parameters entering matter unifying theories.

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