Kinetic helicity decay in linearly forced turbulence

A. Brandenburg1,2,* and A. Petrosyan3,4

1 NORDITA, AlbaNova University Center, Roslagstullsbacken 23, SE-10691 Stockholm, Sweden
2 Department of Astronomy, AlbaNova University Center, Stockholm University, SE-10691 Stockholm, Sweden
3 Space Research Institute of the Russian Academy of Sciences Profsoyuznaya 84/32, Moscow 117997, Russia
4 Moscow Institute of Physics and Technology, State University, Institutsky lane 9, Dolgoprudny 141700, Russia

Received 2012 Feb 24, accepted 2012 Mar 9
Published online 2012 Apr 5

Key words stars: interiors – hydrodynamics – turbulence

The decay of kinetic helicity is studied in numerical models of forced turbulence using either an externally imposed forcing function as an inhomogeneous term in the equations or, alternatively, a term linear in the velocity giving rise to a linear instability. The externally imposed forcing function injects energy at the largest scales, giving rise to a turbulent inertial range with nearly constant energy flux while for linearly forced turbulence the spectral energy is maximum near the dissipation wavenumber. Kinetic helicity is injected once a statistically steady state is reached, but it is found to decay on a turbulent time scale regardless of the nature of the forcing and the value of the Reynolds number.

1 Introduction

The physical properties of astrophysical turbulence are often studied by solving the hydrodynamic equations in a periodic domain with an assumed forcing function. In particular, isotropic homogeneous turbulence is often studied by solving the hydrodynamic equations in a periodic domain with an assumed forcing function. In particular, isotropic homogeneous turbulence is often studied by solving the hydrodynamic equations in a periodic domain with an assumed forcing function.

As mentioned above, in a polytropic flow, the quadratic interactions conserve kinetic helicity, \( \langle \omega \cdot u \rangle \), where \( \omega = \nabla \times u \) is the vorticity, \( u \) the velocity, and angular brackets denote volume averaging over a closed or periodic domain. This conservation property becomes evident when writing the Navier-Stokes equation in the form

\[
\frac{\partial u}{\partial t} = u \times \nabla - \nabla p + \nu [\frac{1}{3} \nabla (\nabla \cdot u) - \nabla \times \nabla \times u + G],
\]

where \( \nabla \times u \) is the curl of the vorticity. In the ideal case, \( \nu = 0 \), kinetic helicity is just subject to viscous decay, because the nonlinear term \( u \times \omega \) is perpendicular to \( \omega \), i.e., we have

\[
\frac{d}{dt} \langle \omega \cdot u \rangle = -2\nu \langle q \cdot \omega \rangle,
\]

where \( q = \nabla \times \omega \) is the curl of the vorticity. In the ideal case, \( \nu = 0 \), we have \( \langle \omega \cdot u \rangle = \text{const} \). However, in fluid dynamics the ideal case is hardly representative of the limit of large Reynolds numbers, where \( \nu \to 0 \). Indeed, for a self-similar decay of kinetic energy, the wavenumber of the energy-carrying eddies, \( k_t \), decreases with time such that \( \nu k_t^2 t \approx \text{const} \). This implies that the rate of energy decay, \( \nu \langle \omega^2 \rangle \), is essentially independent of \( \nu \) and hence independent of the Reynolds number. Given that \( \langle u^2 \rangle \) is related to the kinetic energy, which is also independent of the Reynolds number, we expect that the ratio

\[
\langle \omega^2 \rangle / \langle u^2 \rangle \equiv k_T^{2/Tay},
\]

which is related to the Taylor micro-scale wavenumber \( k_{Tay} \), should be proportional to \( Re \), and therefore

\[
k_{Tay} \sim Re^{1/2}.
\]
However, for helical flows the rate of kinetic helicity dissipation is proportional to $\nu \langle \mathbf{q} \cdot \omega \rangle$. Thus, if we define

$$\langle \mathbf{q} \cdot \omega \rangle / (\omega \cdot \mathbf{u}) \equiv k^2_{\text{eff}},$$

we see that kinetic helicity dissipation is related to kinetic energy by altogether 3 wavenumber factors. If all these factors scale like in Eq. (4), we may expect the rate of kinetic helicity dissipation to diverge with decreasing $\nu$ like $\nu^{-3/2}$. This is in stark contrast to the related case of magneto-hydrodynamic turbulence where, following similar reasoning, the kinetic helicity dissipation converges to zero like $\eta^{1/2}$ as the magnetic dissipation $\eta$ goes to zero (Brandenburg & Subramanian 2005b; see also the appendix of Brandenburg & Käpylä 2007) for a clear exposition of these differences.

A problem with the simple argument above is that in cases of practical relevance the forcing function $f$ usually breaks kinetic helicity conservation. This is particularly evident for the so-called linear forcing model of Lundgren (2003) where

$$f = A u$$

is a positive multiple of the velocity vector. In that case we have

$$\frac{d}{dt} \langle \omega \cdot \mathbf{u} \rangle = 2A \langle \omega \cdot \mathbf{u} \rangle - 2\nu \langle \mathbf{q} \cdot \omega \rangle,$$

so that $\langle \omega \cdot \mathbf{u} \rangle$ could even exhibit exponential growth. An aim of this paper is thus to investigate to what degree kinetic helicity is conserved in forced turbulence using both the linear forcing model and compare it with the more traditional stochastic forcing in a narrow wavenumber band. Another motivation is the fact that, by analogy, magnetic helicity is conserved in forced turbulence using both the linear forcing model using Eq. (6) with a positive constant $A$, or, alternatively, it is given by a random forcing function consisting of plane transversal waves with random wavevectors $k$ such that $|k|$ lies in a band around a given forcing wavenumber $k_f$. The vector $k$ changes randomly from one timestep to the next, so $f$ is $\delta t$ correlated in time. The forcing amplitude is chosen such that the Mach number, $Ma = u_{\text{rms}}/c_s$, is about 0.1. Here, $u_{\text{rms}} = (\langle u^2 \rangle)^{1/2}$ is the root-mean-square (rms) velocity. We use triply-periodic boundary conditions in a Cartesian domain of size $L^3$. The smallest wavenumber that fits into the computational domain is $k_f = 2\pi/L$.

For the linear forcing model we choose for the amplitude $A/c_0 k_1 = 0.02$, while for the random forcing function, $f$ is of the form

$$f(x,t) = \text{Re} \{ N f_k(t) \exp[i k(t) \cdot x + i \phi(t)] \},$$

$x$ is the position vector and

$$f_k = (k \times e) / \sqrt{\mathbf{k}^2 - (k \cdot e)^2},$$

where $e$ is an arbitrary unit vector not aligned with $k$; note that $|f_k|^2 = 1$. The wave vector $k(t)$ and the random phase $-\pi < \phi(t) \leq \pi$ change at every time step. For the time-integrated forcing function to be independent of the length of the time step $\delta t$, the normalization factor $N$ has to be proportional to $\delta t^{-1/2}$. On dimensional grounds it is chosen to be $N = f_0 c_s (k_f c_s / \delta t)^{1/2}$, where $f_0$ is a nondimensional forcing amplitude, which is chosen to be $f_0 = 0.02$. For the monochromatically forced simulations we choose $k_f = 1.5 k_1$. For the linear forcing module, on the other hand, we compute $k_1$ is the integral scale from the resulting kinetic energy spectrum (see below).

Our simulations are characterized by the value of the Reynolds number,

$$Re = u_{\text{rms}}/\nu k_1,$$

where the wavenumber $k_1$ is either the forcing wavenumber the case of monochromatic random forcing, or it is evaluated as the wavenumber of the energy-carrying eddies,

$$k_1^{-1} = \int k^{-1} E(k) \, dk / \int E(k) \, dk,$$

with $E(k)$ being the kinetic energy spectrum, which is here normalized such that

$$\int_0^\infty E(k) \, dk = \frac{1}{2} \langle u^2 \rangle.$$
tra collapse onto each other in the dissipation range if the
Reynolds number the spectra begin to sketch out a continuation to-
ward smaller wavenumber \( k \). Note that in the linear forc-
ing model, energy is being injected at all length scales and not just at the largest scale of the domain. This is a property that may also be responsible for the fact that the effective driving scale tends to be smaller than in other-
wise equivalent monochromatically forced simulations
(Rosales & Meneveau 2005).

In the statistically steady state, the kinetic helicity fluc-
tuates around zero. However, after adding a finite perturba-
tion in the form of Eq. (17), the helicity begins to grow for
about 5 turnover times, \( \delta t u_{\text{rms}}k_{\parallel} \approx 5 \). After that time there is a systematic decline of the kinetic helicity. By inspecting
the results for different values of Re, we see that the time it takes for the kinetic helicity to relax to previous levels
becomes longer as the Reynolds number is increased from
44 to 125; see Fig. 2. However, for Re = 300, which is
our largest Reynolds number for which we have performed
experiments with added Beltrami fields, the decline of he-
licity has happened on a similar time scale as for Re = 70.
This suggests that there may not be a systematic Reynolds
number dependence of kinetic helicity decay.

It should be noted that during the first 2–3 eddy turnover
times after adding the Beltrami field perturbation, the ki-
etic helicity shows an exponential increase; see also Fig. 3.
This is connected with the fact that in the absence of any
other effective damping, Eq. (7) would imply an exponen-
tial growth of \( \langle \omega \cdot u \rangle \propto \exp \lambda t \) with growth rate
\[ \lambda = 2 (A - \nu k_{\parallel}^2), \]
where \( k_{\parallel} \) quantifies the involvement of high wavenum-
bers in the expression for the kinetic helicity dissipation; see Eq. 5.

One would expect strong perturbations to survive this
exponential growth for longer, but this is not the case, as
is demonstrated in Fig. 3, where we show the evolution of
the kinetic helicity for different values of \( \epsilon \) after adding the
Beltrami field perturbation. The reason for the subsequent
decay of magnetic helicity lies in the fact that \( k_{\parallel} \) scales
like \( k_{\parallel} \propto \text{Re}^{1/2} \), so the rate of kinetic helicity dissipation
remains significant even in the limit \( \text{Re} \to \infty \), i.e. \( \nu \to 0 \).
This is quantified in terms of \( k_{\parallel} \), whose scaling will with
Reynolds number will be considered below.

3.2 Monochromatic random forcing

In order to see whether the results presented above are a
special consequence of the linear forcing model, we now
perform simulations using the more traditional monochro-
matic forcing in a narrow wavenumber interval. In Fig. 4 we
show energy spectra for different Reynolds numbers.
The results suggest that the decline in spectral power toward
the smallest wavenumbers seen in Fig. 1 for the linear forc-
ing model is now absent. In other words, while in Fig. 4 we
clearly see that the compensated energy spectrum is flat, this

\[ k_{\parallel} = (\epsilon/\nu^3)^{1/4}, \]
\[ \frac{\rm d}{\rm dx} (\sin k z, \cos k z, 0), \]
\[ \int_0^\infty F(k) \, \text{d}k = \langle \omega \cdot u \rangle. \]

We note that the kinetic helicity spectrum can have either
sign and its modulus obeys the well-known realizability
condition, \[ |F(k)| \leq 2kE(k) \] (Moffatt 1969).

In the statistically steady state, the kinetic helicity fluc-
tuates around zero. We consider the evolution of kinetic heli-
city after adding an instantaneous finite perturbation in the
form of a Beltrami field,

\[ u \to u + \epsilon c_0 (\sin kz, \cos kz, 0), \]

\[ k_{\parallel} = (\epsilon/\nu^3)^{1/4}. \]

Here, \( \epsilon = \nu (\omega^2) \) is the rate of energy dissipation. We also
analyze spectra of kinetic helicity, \( F(k) \), which are normal-
ized such that

\[ \int_0^\infty F(k) \, \text{d}k = \langle \omega \cdot u \rangle. \]

We note that the kinetic helicity spectrum can have either
sign and its modulus obeys the well-known realizability
condition, \[ |F(k)| \leq 2kE(k) \] (Moffatt 1969).

In the statistically steady state, the kinetic helicity fluc-
tuates around zero. We consider the evolution of kinetic heli-
city after adding an instantaneous finite perturbation in the
form of a Beltrami field,

\[ u \to u + \epsilon c_0 (\sin kz, \cos kz, 0), \]

\[ \frac{\rm d}{\rm dx} (\sin k z, \cos k z, 0), \]

\[ \int_0^\infty F(k) \, \text{d}k = \langle \omega \cdot u \rangle. \]

We note that the kinetic helicity spectrum can have either
sign and its modulus obeys the well-known realizability
condition, \[ |F(k)| \leq 2kE(k) \] (Moffatt 1969).

In the statistically steady state, the kinetic helicity fluc-
tuates around zero. We consider the evolution of kinetic heli-
city after adding an instantaneous finite perturbation in the
form of a Beltrami field,

\[ u \to u + \epsilon c_0 (\sin kz, \cos kz, 0), \]

\[ \frac{\rm d}{\rm dx} (\sin k z, \cos k z, 0), \]

\[ \int_0^\infty F(k) \, \text{d}k = \langle \omega \cdot u \rangle. \]

We note that the kinetic helicity spectrum can have either
sign and its modulus obeys the well-known realizability
condition, \[ |F(k)| \leq 2kE(k) \] (Moffatt 1969).

In the statistically steady state, the kinetic helicity fluc-
tuates around zero. We consider the evolution of kinetic heli-
city after adding an instantaneous finite perturbation in the
form of a Beltrami field,

\[ u \to u + \epsilon c_0 (\sin kz, \cos kz, 0), \]

\[ \frac{\rm d}{\rm dx} (\sin k z, \cos k z, 0), \]

\[ \int_0^\infty F(k) \, \text{d}k = \langle \omega \cdot u \rangle. \]

We note that the kinetic helicity spectrum can have either
sign and its modulus obeys the well-known realizability
condition, \[ |F(k)| \leq 2kE(k) \] (Moffatt 1969).

In the statistically steady state, the kinetic helicity fluc-
tuates around zero. We consider the evolution of kinetic heli-
city after adding an instantaneous finite perturbation in the
form of a Beltrami field,

\[ u \to u + \epsilon c_0 (\sin kz, \cos kz, 0), \]

\[ \frac{\rm d}{\rm dx} (\sin k z, \cos k z, 0), \]

\[ \int_0^\infty F(k) \, \text{d}k = \langle \omega \cdot u \rangle. \]

We note that the kinetic helicity spectrum can have either
sign and its modulus obeys the well-known realizability
condition, \[ |F(k)| \leq 2kE(k) \] (Moffatt 1969).

In the statistically steady state, the kinetic helicity fluc-
tuates around zero. We consider the evolution of kinetic heli-
city after adding an instantaneous finite perturbation in the
form of a Beltrami field,

\[ u \to u + \epsilon c_0 (\sin kz, \cos kz, 0), \]

\[ \frac{\rm d}{\rm dx} (\sin k z, \cos k z, 0), \]

\[ \int_0^\infty F(k) \, \text{d}k = \langle \omega \cdot u \rangle. \]

We note that the kinetic helicity spectrum can have either
sign and its modulus obeys the well-known realizability
condition, \[ |F(k)| \leq 2kE(k) \] (Moffatt 1969).

In the statistically steady state, the kinetic helicity fluc-
tuates around zero. We consider the evolution of kinetic heli-
city after adding an instantaneous finite perturbation in the
form of a Beltrami field,

\[ u \to u + \epsilon c_0 (\sin kz, \cos kz, 0), \]

\[ \frac{\rm d}{\rm dx} (\sin k z, \cos k z, 0), \]

\[ \int_0^\infty F(k) \, \text{d}k = \langle \omega \cdot u \rangle. \]

We note that the kinetic helicity spectrum can have either
sign and its modulus obeys the well-known realizability
condition, \[ |F(k)| \leq 2kE(k) \] (Moffatt 1969).

In the statistically steady state, the kinetic helicity fluc-
tuates around zero. We consider the evolution of kinetic heli-
city after adding an instantaneous finite perturbation in the
form of a Beltrami field,

\[ u \to u + \epsilon c_0 (\sin kz, \cos kz, 0), \]

\[ \frac{\rm d}{\rm dx} (\sin k z, \cos k z, 0), \]

\[ \int_0^\infty F(k) \, \text{d}k = \langle \omega \cdot u \rangle. \]
Fig. 2 Evolution of the normalized kinetic helicity for different values of Re after adding a Beltrami field perturbation at $t = t_\ast$.

Fig. 3 Evolution of the kinetic helicity for different values of $\epsilon$ after adding the Beltrami field perturbation at $t = t_\ast$.

Fig. 4 Kinetic energy spectra compensated with $k^{5/3} \epsilon^{-2/3}$ for a range of different Reynolds numbers and numerical resolutions using monochromatic forcing.

is not the case in Fig. 1 for the linear forcing model. However, there is still an uprise near $0.1 k_d$ that one may generally associate with the bottleneck effect (Falkovich 1994; Dobler et al. 2003).

Next, we study the effect of adding a helicity perturbation also in this case. Figure 5 gives time series for three values of Re. There is no evidence for a prolonged relaxation to zero. The reason for this could be that a helical wave cannot interact with itself; see a corresponding discussion following Eq. (11) of Kraichnan (1973). This also suggests that kinetic helicity conservation in the inviscid case, $\nu = 0$, is of no relevance to the inviscid limit, $\nu \to 0$, in which case the kinetic helicity dissipation diverges. This behavior was less clear in the previous case with the linear forcing model. This might either be a matter of coincidence, but it could also be a consequence of the linear forcing model which has exponentially growing solutions.

The time series in Fig. 5 reveals another interesting aspect in comparison to Fig. 2 for the linear forcing model in that the level of fluctuations of $\langle \omega \cdot u \rangle$ is generally larger for the monochromatic forcing function than for the linear forcing model. Furthermore, the time series show much stronger short-term fluctuations while for the linear forcing model the time traces of $\langle \omega \cdot u \rangle$ are smoother.

In Table 1 we summarize integral and dissipation wavenumbers as well as the normalized energy fluxes for both linear and monochromatic random forcings. In wind tunnel turbulence one usually expresses the energy flux in units of a quantity $C_\epsilon = u'^3/L$, where $u' = u_{rms}/\sqrt{3}$ is the one-dimensional rms velocity and $L = 3\pi/4k_f$ is the customary definition of the integral scale. The standard result of $C_\epsilon \approx 0.5$ (Pearson et al. 2004) corresponds then to $\epsilon \approx 0.04 k_f u_{rms}^3$. Comparing the normalized energy fluxes for linear and monochromatic random forcings we see hardly any differences. This suggests that the nature of the forcing in hydrodynamic turbulence might not be of great qualitative importance, although it is still possible that the bottleneck effect (Falkovich 1994; Dobler et al. 2003)
Table 1  Summary of the normalized characteristic wavenumbers $\tilde{k}_{f} = k_{f}/k_{1}$, $\tilde{k}_{d} = k_{d}/k_{1}$, $k_{T_{ay}} = k_{T_{ay}}/k_{1}$, and $\tilde{k}_{\text{eff}} = k_{\text{eff}}/k_{1}$, for linear and monochromatic forcings. The numerical resolution is given in the second column.

| Re   | Res. | $\tilde{k}_{f}$ | $\tilde{k}_{d}$ | $k_{T_{ay}}$ | $\tilde{k}_{\text{eff}} | $\epsilon/k_{1}u_{rms}^{3}$ | linear forcing | $\tilde{k}_{f}$ | $\tilde{k}_{d}$ | $k_{T_{ay}}$ | $\tilde{k}_{\text{eff}} | $\epsilon/k_{1}u_{rms}^{3}$ | monochromatic forcing |
|------|------|-----------------|-----------------|-------------|-----------------|--------------------------|----------------|----------|----------|-------------|-----------------|----------------|-----------------|-
| 40   | 64   | 1.6             | 15              | 3           | 3               | 0.080                    | linear forcing | 1.6      | 12       | 2           | 3               | 0.072           | monochromatic forcing |
| 70   | 128  | 1.9             | 23              | 4           | 5               | 0.069                    | linear forcing | 1.9      | 20       | 3           | 4               | 0.052           | monochromatic forcing |
| 130  | 256  | 2.0             | 40              | 6           | 6               | 0.073                    | linear forcing | 1.9      | 34       | 4           | 7               | 0.045           | monochromatic forcing |
| 300  | 512  | 2.2             | 79              | 9           | 7               | 0.067                    | linear forcing | 2.8      | 68       | 7           | 8               | 0.028           | monochromatic forcing |
| 600  | 1024 | 2.2             | 133             | 13          | 10              | 0.069                    | linear forcing | 1.2      | 130      | 8           | 3               | 0.064           | monochromatic forcing |

Fig. 5  Evolution of the normalized kinetic helicity using the monochromatic forcing scheme for different values of Re after adding a Beltrami field perturbation at $t = t_{*}$.

near the dissipative scale might be connected with the nature of the forcing at large scales (Davidson 2004).

In Table 1 we also list the values of $k_{T_{ay}}$ and $k_{\text{eff}}$, as defined in Eqs (3) and (5). Both wavenumbers are clearly proportional to Re$^{1/2}$, as can be seen from Fig. 7, where we plot the Reynolds number dependence of the ratios $k_{T_{ay}}/k_{1}$ and $k_{\text{eff}}/k_{1}$ for linear and monochromatic forcings.

To compare the linearly and monochromatically forced turbulence simulations in real space, we present in Fig. 6 visualizations of the logarithmic density on the periphery of the computational domain. The logarithmic density is a convenient scalar quantity characterizing the pressure fluctuations resulting from the Reynolds stress. These visualiza-

Fig. 6  (online colour at: www.an-journal.org) Visualization of the logarithmic density (proportional to logarithmic pressure fluctuations) on the periphery of the domain for linear and random forcings. In both cases we have Re $\approx$ 600 at a resolution of 512$^{3}$ mesh points.
Fig. 7  (online colour at: www.an-journal.org) Scalings of $k_{\text{Tay}}/k_1$ (solid lines) and $k_{\text{eff}}/k_1$ (dashed lines) with Re for linear (filled symbols) and monochromatic (open symbols and red lines) forcings. The dotted line indicates $\text{Re}^{1/2}$ scaling.

The linear forcing model is based on the driving of turbulence by a linear instability instead of a forcing function that is independent of the flow. This type of forcing might be more physical, because it does not change abruptly and depends on the local flow properties. Nevertheless, both types of forcing are Galilean invariant. This would change if the flow-independent monochromatic forcing were no longer $\delta$ correlated in time. A disadvantage of linearly forced turbulence is that energy injection occurs at all wavenumbers. Our simulations show that the energy spectrum is perhaps slightly shallower than with the flow-independent monochromatic forcing function. Part of this is explained by the bottleneck effect (Falkovich 1994; Dobler et al. 2003), and that the energy spectra compensated by $k^{5/3}$ show a stronger uprise toward the dissipation wavenumber. It is unclear whether this result would persist at larger resolution.

In those patches where the kinetic helicity is negative, the modulus of the value is plotted as a dotted line (red for the linear forcing model and blue for the monochromatic random forcing function).

4 Conclusions

With regards to geophysical and astrophysical applications we can say that in a turbulent system, kinetic helicity is no longer a conserved quantity, even though it would be if $\nu = 0$ were strictly true. The latter requirement is of course not really possible in a turbulent system, because kinetic energy would then accumulate at the smallest possible scale resolved within the hydrodynamics framework and kinetic energy would not be able to decay, which is unphysical. While this should not be surprising, it is important to remember that this is quite different in the case of magnetohydrodynamics, where magnetic helicity dissipation really does go to zero in a turbulent system – even for finite (but small) values of the magnetic diffusivity. At the same time, magnetic energy dissipation does stay finite and is able to accomplish magnetic reconnection on the smallest resolved scales of the turbulent cascade (Galsgaard & Nordlund 1996, Lazarian & Vishniac 1999).

This paper has also shown that, regardless of the nature of the forcing, there are fairly strong helicity fluctuations. They appear to be coherent over many turbulent eddy timescales. One may wonder how generic
such fluctuations are and if such fluctuations could be relevant for say the incoherent dynamo effects that have been investigated by several authors in recent years (Vishniac & Brandenburg 1997; Sur & Subramanian 2009; Heinemann et al. 2011; Mitra & Brandenburg 2012; Richardson & Proctor 2010). It will therefore be interesting the associate the kinetic helicity fluctuations with those of $\alpha$, which have already been determined in simulations of turbulent shear flows (Brandenburg et al. 2008).

Acknowledgements. We thank two anonymous referees for providing useful comments that have helped improving the presentation of the paper. We acknowledge the allocation of computing resources provided by the Swedish National Allocations Committee at the Center for Parallel Computers at the Royal Institute of Technology in Stockholm and the National Supercomputer Centers in Linköping as well as the Norwegian National Allocations Committee at the Bergen Center for Computational Science. This work was supported in part by the European Research Council under the AstroDyn Research Project No. 227952 and the Swedish Research Council Grant No. 621-2007-4064.

References

André, J.-C., Lesieur, M.: 1977, JFM 81, 187
Borue, V., Orszag, S.A.: 1997, PhRvE 55, 7005
Brandenburg, A.: 2001, ApJ 550, 824
Brandenburg, A., Subramanian, K.: 2005a, A&A 439, 835
Brandenburg, A., Subramanian, K.: 2005b, Phys. Rep. 417, 1
Brandenburg, A., Käpylä, P.J.: 2007, New J. Phys. 9, 305
Brandenburg, A., Rädler, K.-H., Rheinhardt, M., Käpylä, P.J.: 2008, ApJ 676, 740
Chkhetiani, O. G.: 1996, JETP Lett. 63, 808
Davidson, P.A.: 2004, Turbulence: an introduction for scientists and engineers (Oxford: Oxford University Press)
Ditlevsen, P.D., Giuliani, P.: 2001a, PhRvE 63, 036304
Ditlevsen, P.D., Giuliani, P.: 2001b, PhFl 353, 550
Dobler, W., Haugen, N.E.L., Yousef, T.A., Brandenburg, A.: 2003, PhRvE 69, 026304
Falkovich, G.: 1994, PhFl 6, 1411
Galsgaard, K., Nordlund, Á.: 1996, JGR 101, 13445
Heinemann, T., McWilliams, J.C., Schekochihin, A.A.: 2011, PrRvL 107, 255004
Kraichnan, R.H.: 1973, JFM 59, 745
Lazarian, A., Vishniac, E.T.: 1999, ApJ 517, 700
Lundgren, T.S., “Linearly forced isotropic turbulence,” in Annual Research Briefs Center for Turbulence Research, Stanford, 2003, pp. 461-473
Mitra, D., Brandenburg, A.: 2012, MNRAS 420, 2170
Moffatt, H.K.: 1969, JFM 35, 117
Pearson, B.R., Yousef, T.A., Haugen, N.E.L., Brandenburg, A., Krogstad, P.A.: 2004, PhRvE 70, 056301
Richardson, K.J., Proctor, M.R.E.: 2010, GApFD 104, 601
Rosales, C., Meneveau, C.: 2005, PhFl 17, 095106
Stepanov, R.A., Frick, P.G., Shestakov, A.V.: 2009, Fluid Dyn. 44, 658
Sur, S., Subramanian, K.: 2009, MNRAS 392, L6
Vishniac, E.T., Brandenburg, A.: 1997, ApJ 475, 263