ELECTROWEAK BARYOGENESIS
WITH TOPOLOGICAL DEFECTS

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1. Introduction

Electroweak baryogenesis has become a topic of much recent activity [1]. Here we discuss a new scenario which has the advantage of being insensitive to the order of the electroweak phase transition. We briefly review a mechanism [2] using unstable electroweak strings [3] and discuss in detail a mechanism [4] using topological defects (in particular cosmic strings) left behind after a previous phase transition.

In the standard electroweak theory all three necessary conditions to generate a net baryon number are satisfied. Sphaleron transitions violate baryon number [5]. The electroweak theory explicitly violates C invariance and in extensions of the standard model with non-minimal Higgs structure there is explicit CP violation [6]. Finally, out of equilibrium field configurations may result as remnants of the phase transition.

The key issue is how the ‘out of equilibrium’ condition is realised. In most previous work [7,8] use was made of bubble walls which form if the electroweak phase transition is first order. However, at present it is unclear [9] whether the electroweak phase transition is sufficiently strongly first order for baryogenesis mechanisms involving bubble walls to be effective. In [2] it was pointed out that topological or non-topological defects may play a role similar to bubble walls in triggering electroweak baryogenesis. Such a mechanism is independent of the order of the phase transition.

In the next section we show how electroweak baryogenesis can be implemented in models with a second order phase transition and compare it with the first order case. A mechanism with metastable electroweak strings is then briefly reviewed. A more robust mechanism using topological defects produced in a previous phase transition is discussed and the resulting baryon asymmetry estimated. Finally, we discuss future extensions to our work and conclusions.
2. ELECTROWEAK BARYOGENESIS WITH A SECOND ORDER PHASE TRANSITION

Let us review how Sakharov’s conditions are realised in electroweak baryogenesis scenarios and compare the implementations in first and second order phase transitions.

Baryon number violation occurs via sphaleron transitions. The transition rate is exponentially suppressed in the broken phase. However, in the symmetric phase transitions are copious. Their rate per unit volume is \[ \Gamma_B \sim \alpha_W^4 T^4 \] (2.1)

where \( \alpha_W = g^2 / 4\pi \), \( g \) being the \( SU(2) \) gauge coupling constant. In extensions of the standard electroweak theory with non-minimal Higgs structure containing explicit CP violation, there exists a CP violating phase which changes by an amount \( \Delta \theta \) during the phase transition.

In Fig.1 we compare the ways in which the out of equilibrium condition is realized in models with first and second order phase transitions. The key role is played by expanding bubble walls and contracting topological defects respectively. In the scenarios of Refs.1,2, baryogenesis takes place in the outer edge of the bubble wall, i.e.

\[ |\phi| < g \eta_{EW} \] (2.2)

\( |\phi| \) being the order parameter of the transition. The amplitude \( |\phi| \) is increasing at any point in space which the bubble wall crosses. This may be related to the change of the CP-odd phase and hence CP violation has a preferred sign. Finally, as long as the bubble wall moves at relativistic speed, there will be no time to establish thermal equilibrium inside the walls.
With a second order phase transition, the role of the bubble wall is played by topological defects. Baryogenesis takes place inside the core of the defect and the surrounding region where (2.2) is satisfied. If the defects contract (and eventually evaporate), then there will be an overall increase in $|\phi|$ and hence net baryon number generation. The field configurations within contracting topological defects are out of thermal equilibrium.

In the next section we consider a specific implementation of this using electroweak strings.

3. BARYOGENESIS WITH ELECTROWEAK STRINGS

In [2] it was suggested that electroweak strings could provide the out of equilibrium condition necessary to generate a baryon asymmetry. The electroweak theory doesn’t admit topologically stable strings. However, it has been shown that it is possible to embed the Nielsen-Olesen vortex [11] in the electroweak theory [12]. Such strings could be metastable, in which case their formation would be similar to that of topological strings. The core of the string would play a similar role to the bubble wall in first order phase transitions. In the core of the string the electroweak symmetry is restored and baryon violating processes are unsuppressed with rate as in (2.1). Outside the string the electroweak symmetry is broken and baryon violating transitions are exponentially suppressed. Since the strings are at best metastable and of finite length, terminating on a monopole anti-monopole pair, they will collapse along their axis. This collapse of the string provides the out of thermal equilibrium condition. If this is the case in the 2-doublet model then in the tip of the string the $CP$ violating phase is rapidly changing. Hence, we have all the necessary conditions to generate a baryon asymmetry.

An optimistic estimate of the baryon asymmetry can be obtained by assuming that the mean length and average separation of electroweak strings at $t_G$, the time corresponding to the Ginsburg temperature of the phase transition, is the
correlation length, $\xi(t_G) \sim \lambda^{-1}\eta^{-1}$. The strings will collapse along their axes and decay in time interval

$$\Delta t_s \sim v^{-1}(\lambda\eta)^{-1}$$

where $v$ is the velocity of collapse, taken to be $\sim 1$. By considering the rate of change of the volume in which $CP$ violation is effective we can estimate the rate of baryon number generation per string to be

$$\frac{dN_B}{dt} \sim w^2 v^\Gamma_B \Delta \theta \Delta t_c$$

where $w \sim \lambda^{-\frac{1}{2}}\eta^{-1}$ is the string width and $\Delta t_c$ is the length of time a fixed point in space is in the transition region. Taking one string per correlation volume $\xi(t_G)^3$ and integrating from the Ginsburg time $t_G$ to $t_G + \Delta t_s$ gives

$$n_B \sim \frac{9}{2\pi^2 g^*} \frac{\lambda}{\gamma(v) v^\Gamma_B} \Delta \theta g^3 \alpha_W^4$$

where $g^*$ is the effective number of degrees of freedom. In our estimate we have included a suppression for the fact that baryon violating processes are only unsuppressed for $|\phi| < g\eta$.

For our mechanism to work we require the core radius to be large enough to support sphaleron processes. In addition, sphaleron transitions must be suppressed in the broken phase for $T = T_G$. Finally, we require the electroweak string to be metastable in the 2-doublet model. Unfortunately, it has been shown [13] that this is not the case for physical Weinberg angle [14]. Attempts to stabilise it with quark and lepton condensates have been unsuccessful [15]. Hence, a more robust method of electroweak baryogenesis could involve topological defects formed at a phase transition at a scale above the electroweak scale.
4. BARYOGENESIS WITH TOPOLOGICAL DEFECTS

In order to obtain topological defects – cosmic strings to be specific – we assume that at an energy scale $\eta$, larger than $\eta_{EW}$, there is another symmetry breaking which produces strings. One possibility is to embed $SU(2) \times U(1)$ in some larger simply connected group $G$ such that at a scale $\eta$ 

$$G \rightarrow SU(2) \times U(1)$$

and

$$\Pi_1 (G/(SU(2) \times U(1))) \neq 1$$

A second possibility is to assume that electroweak symmetry breaking is induced dynamically by having a technifermion condensate form at the scale $\eta_{EW}$:

$$\langle \bar{\psi}_{TC} \psi_{TC} \rangle \neq 0, \ T \leq \eta_{EW}.$$

Here, $\psi_{TC}$ denotes the technifermion. In this case we can assume that fermion masses are induced by a second phase transition in the technifermion sector at a scale $\eta$ which in general is only slightly higher than $\eta_{EW}$. It is possible that strings form in this transition. In the core of these strings the fermion condensates vanish, the electroweak symmetry is unbroken, and hence baryon number violating processes are unsuppressed.

The region of electroweak symmetry restoration is actually larger than the core of the string. In [16] it was shown that cosmic strings formed at a previous phase transition and coupled to the Weinberg-Salam model restore the electroweak symmetry out to a region of order $\eta_{EW}^{-1}$. This is a result of the coupling between
the string fields and the electroweak gauge fields. If the string is superconducting then the region of symmetry restoration is much larger, being given by \[17,16\].

\[ R_s \sim \frac{I}{\eta_{EW}^2} \]  

(4.1)

where \(I\) is the current carried by the string. The maximal current is of order \(\eta\). (For grand unified strings this region is macroscopic, being of order \(10^{-5}\) m!)

Let us now give a rough estimate of the baryon to entropy ratio which can be generated using the proposed mechanism and compare the result with that obtained by the mechanisms of Refs.7,8 which rely on bubble wall expansion. To simplify the calculations we assume that the phase transition is rapid and that the strings move relativistically (in order that the out-of-equilibrium condition is satisfied).

The important parameters in our calculation are the total volume \(V\), the volume \(V_{BG}\) in which net baryon number violating processes are taking place, the rate \(\Gamma_B\) of these processes (see (2.1)), and the net change

\[ \Delta \theta = \int dt \dot{\theta} \]

in the \(CP\) violating phase \(\theta\). We are making the plausible assumption that the electroweak symmetry is restored inside the core of the string. In this case, the mean value of the \(CP\) violating phase vanishes in the core. In the broken phase, the distinguished value of \(\theta\) will be nonvanishing. Hence, for points in space initially inside the string core, the net change in \(\theta\) will have a preferred direction. In this respect there is no difference between our mechanism and the ones of Refs.7,8.

The net baryon number density \(\Delta n_B\) is then given by
\[ \Delta n_B = \frac{1}{V} \frac{\Gamma_B}{T} V_{BG} \Delta \theta. \]  \hfill (4.2)

The volume \( V_{BG} \) is determined by the mean separation \( \xi \) of the strings and the radius \( R_s \) of the string core (strictly speaking the part of the core where (2.2) is satisfied and hence \( n_B \) violating processes are unsuppressed).

The key to the calculation is a good estimate of \( V_{BG} \). Note that the translational motion of a topological defect does not lead to any net baryogenesis since \( \Delta \theta = 0 \) integrated over time. At the leading edge of the moving defect, a baryon number with one sign will be produced, but at the trailing edge baryogenesis will have the opposite sign. We will return to this point later. Contraction, on the other hand, does produce a net \( \Delta n_B \). Integrated over time, there is a net \( \Delta \theta \neq 0 \) in the entire volume corresponding to the initial defect configuration. Hence, a lower bound on the volume \( V_{BG} \) is obtained by taking the volume occupied initially by the collapsing defects at the time when baryogenesis commences (see below). We focus on string loops. Their mean separation is \( \xi(t) \). Hence, in one horizon volume

\[ V = \frac{4\pi}{3} t^3 \]  \hfill (4.3)

the corresponding volume where net baryon number generation takes place is

\[ V_{BG} \sim R_s^2 \xi(t) \left( \frac{t}{\xi(t)} \right)^3. \]  \hfill (4.4)

The last factor on the right hand side is the number of string loops per horizon volume, the second factor is the length of a loop.

Most of the contribution to the baryon to entropy ratio is generated at a time \( t_U \) soon after \( t_{EW} \) when sphaleron processes cease to be thermally excited in the true vacuum. To simplify the equations, we will set the two times equal in the
following. Thus, to obtain an order of magnitude estimate of the strength of our baryogenesis mechanism we will evaluate all quantities at $t_{EW}$. Combining (4.2)-(4.4) and (2.1) yields

$$
\Delta n_B(t_{EW}) \sim \alpha_W^4 \Delta \theta \left( \frac{R_s}{\xi(t_{EW})} \right)^2 T_{EW}^3
$$

(4.5)

or

$$
\frac{\Delta n_B}{s} \sim g^*-1 \alpha_W^4 \Delta \theta \left( \frac{R_s}{\xi(t_{EW})} \right)^2 \equiv g^*-1 \alpha_W^4 \Delta \theta (SF),
$$

(4.6)

with $g^*$ being the number of spin degrees of freedom in radiation, and with

$$
(SF) = \left( \frac{R_s}{\xi(t_{EW})} \right)^2.
$$

(4.7)

Apart from the factor $(SF)$, this is the same order of magnitude as obtained in the mechanisms using a first order phase transition [7,8]. Hence, we call $(SF)$ the “suppression factor”.

Above, we implicitly assumed that all strings have the same radius. This is a good approximation for strings in the friction dominated epoch [18], but not for a string network in the scaling regime. In the latter case we need to integrate over all loop sizes to obtain $(SF)$.

The above analysis does not depend on the topology of the defect in a key way. For collapsing domain walls, our mechanism is stronger since the suppression factor $(SF)$ would be

$$
(SF) \sim \frac{R_c}{\xi(t_{EW})},
$$

$R_c$ being proportional to the domain wall core radius. For collapsing monopoles,
however, the mechanism is weaker since

\[(SF) \sim \left( \frac{R_c}{\xi(t_{EW})} \right)^3.\]

There are two ways to increase \((SF)\): either we decrease \(\xi(t_{EW})\) or we increase \(R_s\). The obvious way to decrease \(\xi(t_{EW})\) is to decrease the scale \(\eta\) of the string producing phase transition. The earlier we are in the friction dominated epoch, the closer the strings are relative to the horizon since [18]

\[\xi(t) \sim \xi(t_f) \left( \frac{t}{t_f} \right)^{5/4},\] (4.8)

\(t_f\) being the time of string formation (given by \(\eta\)). According to the Kibble mechanism

\[\xi(t_f) \simeq \lambda^{-1} \eta^{-1},\] (4.9)

where \(\lambda\) is the string scalar field self coupling constant. As mentioned previously, a cosmic string coupled to the Weinberg-Salam model restores the electroweak symmetry in a region

\[R_s \sim \eta_{EW}^{-1}\] (4.10)

and therefore

\[(SF) \sim \left( \frac{\eta_{EW}}{\eta} \right)^3.\] (4.11)

The second way to increase \((SF)\) is to make the strings superconducting. For maximal string current this region is
In general, however, the initial current on a superconducting string will be much less than \( \eta \) and thus \( R_s \) smaller than (4.12). To obtain a more realistic estimate consider the superconductivity to be bosonic and the winding of the boson condensate at formation to be \( \sim 1 \). The winding at later times is \( \sim N^{\frac{1}{2}} \), where \( N \) is the ratio of comoving volumes corresponding to loop sizes at \( t \) and \( t_f \) respectively. This winding in the field giving rise to superconductivity induces a current of order

\[
I \sim I_{\text{max}} (T/\eta)^{\frac{1}{2}}
\]

(4.13)

giving

\[
R_s \sim (T/\eta)^{\frac{1}{2}} \frac{\eta}{\eta_{\text{EW}}} \frac{\eta}{\eta_{\text{EW}}}
\]

(4.14)

and resulting suppression factor of

\[
SF \sim \lambda^2 \left( \frac{\eta_{\text{EW}}}{\eta} \right)^{\frac{3}{2}}
\]

(4.15)

For \( \eta \) just above the electroweak transition and \( \lambda \) close to unity the strength of this mechanism for baryogenesis is comparable to the first order one.
5. FUTURE DEVELOPMENTS

In the previous section we ignored the effect of the string’s transverse motion and just concentrated on the initial collapse of the loop. For superconducting strings this is a gross underestimate. In this case the region of symmetry restoration, $R_s$, is significantly larger than the mean free path of baryons within the string. As the string moves, the $CP$ violating phase, $\Delta \theta$, is equal and opposite on the leading and trailing edges of the string. At the leading face anti-baryons are produced. However, before the trailing face arrives the anti-baryons have already decayed to anti-leptons and thus do not annihilate with the baryons produced at the trailing edge. The result of this is that the volume of baryogenesis is

$$V_{BG} \sim \xi(t)^2 R_s \left(\frac{t}{\xi(t)}\right)^3$$

rather than (4.4). The resulting suppression factor becomes

$$SF \sim \frac{R_s}{\xi(t_{EW})} \sim \lambda \left(\frac{\eta_{EW}}{\eta}\right)$$

This makes our mechanism even more attractive.

Finally, we note that the baryogenesis mechanism discussed by Barriola [19] in the context of electroweak strings can also be applied to topological defects. Barriola noted that integrating the anomaly in the baryon current gives the change in baryon number in a given volume and time interval

$$\Delta B = \frac{N_f}{32\pi^2} \int d^4x [g^2 E_W.B_W + G^2 \cos(2\theta_W)E_Z.B_Z + \frac{G^2}{2} \sin(2\theta_W)(E_Z.B_A + E_A.B_Z)]$$

where $E$ and $B$ refer to the electric and magnetic fields of the physical fields after symmetry breaking, $N_f$ refers to the number of families and $G^2 = g^2 + g'^2$, where
$g$ and $g'$ are the $SU(2)$ and $U(1)$ coupling constants respectively. In (5.3) the first term is small in the broken phase, the second term is only non-zero in defects with a non-zero helicity [20], but the third term could be large. In the region $R_s$ around a cosmic string there is a non-trivial $Z$-flux [16]. As discussed at this meeting [20,21] the electroweak phase transition produces a background magnetic field of order $B_A \sim gT^2$. In the two-doublet model this gives a contribution to the action of

$$\Delta S = \frac{N_f G^2}{64\pi^2} \sin(2\theta_W) \int d^4x \theta(E_A B_Z + B_A E_Z)$$  \hspace{1cm} (5.4)$$

Integrating by parts this gives a contribution to the free energy density

$$F_B = -\dot{\theta} B$$  \hspace{1cm} (5.5)$$

In the core of collapsing topological defects $\dot{\theta} > 0$ and so $B$ is driven positive to minimise the free energy.

As a first estimate the rate per unit volume of baryon violation for electroweak strings is

$$\Gamma_B \sim \gamma_v v_t g^2 T^4$$  \hspace{1cm} (5.6)$$

where $v_t$ is the speed at which the loop is collapsing and $\gamma_v$ the associated Lorentz factor. This rate is considerably larger than that resulting from sphaleron processes in (2.1).

The change in baryon number is again given by (4.2), but with rate $\Gamma_B$ as in (5.6) and volume $V_{BG}$ of (5.1). Unlike the mechanism discussed in section 4, this increased rate of baryon violation allows the exciting possibility of electroweak baryogenesis with GUT strings. By the time of the electroweak phase transition
the GUT string network has reached a scaling solution. Considering the scaling solution for the number of loops per unit volume and integrating over loop size, we have estimated the suppression factor to be $\sim 10^{-1}$ for a scale of symmetry breaking of $10^{15}$ GeV. This allows the exciting possibility that the same cosmic strings that may participate in the formation of large scale structure could also mediate electroweak baryogenesis [22].

To conclude, we have discussed a new mechanism for electroweak baryogenesis which operates even if the phase transition is second order. Topological defects produced in a previous phase transition can play a role analogous to bubble walls. We have discussed two mechanisms for generating an asymmetry of order $n_B/s \sim 10^{-10}$. The first involves a transition shortly before the electroweak one and sphaleron processes. The second, and more speculative, involves a new, stronger mechanism of baryogenesis and allows the exciting possibility of GUT strings mediating electroweak baryogenesis.

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Figure Captions
**Figure 1** A comparison between the electroweak baryogenesis mechanisms using first order phase transitions (top) and our mechanism (bottom). The contracting topological defect (bottom) plays a role similar to that of the expanding bubble wall (top) in that it is the location of extra CP violation and of baryon number violating processes taking place out of equilibrium.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9406355v1