TRANSIENT ANALYSIS OF N-POLICY QUEUE WITH SYSTEM
DISASTER REPAIR PREVENTIVE MAINTENANCE
RE-SERVICE BALKING CLOSEDOWN AND SETUP TIMES

A. AZHAGAPPAN
Department of Mathematics
St. Anne’s College of Engineering and Technology
Anna University, Panruti, Tamilnadu - 607 110, India

T. DEEPA
Department of Mathematics
Idhaya College of Arts and Science for Women
Pondicherry University, Pudukudiyarpet, Puducherry - 605 008, India

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Abstract. This paper investigates the transient behavior of a M/M/1 queueing model with N-policy, system disaster, repair, preventive maintenance, balking, re-service, closedown and setup times. The server stays dormant (off state) until N customers accumulate in the queue and then starts an exhaustive service (on state). After the service, each customer may either leave the system or get immediate re-service. When the system becomes empty, the server resumes closedown work and then undergoes preventive maintenance. After that, it comes to the idle state and waits N accumulate for service. When the \( N^{th} \) one enters the queue, the server commences the setup work and then starts the service. Meanwhile, the system suffers disastrous breakdown during busy period. It forced the system to the failure state and all the customers get eliminated. After that, the server gets repaired and moves to the idle state. The customers may either join the queue or balk when the size of the system is less than N. The probabilities of the proposed model are derived by the method of generating function for the transient case. Some system performance indices and numerical simulations are also presented.

1. Introduction. Queueing models with disaster and repair have been studied by many researchers in the past few decades as they possess wide applications in modeling many practical situations related to computer networks, communication systems, etc (refer [3], [4], [7], [8], [13], [14], [15], [19], [20], [25]). Closedown the system when it becomes empty and setup the system before starting the service, play a key role in various real life situations as they support economically to minimize the expenses of an organization. The preventive maintenance of the server is essential as it extends the life of the server. Only few works are investigated in literature related to closedown, preventive maintenance and setup times (refer [2], [10], [16]).
In many practical situations, the service will not be started whenever an arrival occurs. The server fixes a threshold value (say, 'N') to start the service and waits idle till 'N' to reach. This is called the queueing model with \textit{N-policy} (refer [7], [18], [20]). During this time, new arrivals are getting discouraged and may decide not to join (\textit{balk}) the queue (refer [6], [8], [11], [24]). When a customer is dissatisfied with the quality of service what he received, the necessity of immediate re-service/feedback is unavoidable to the customer's point of view. If he is not permitted for immediate re-service/feedback, the trading company may lose a potential customer which leads to the reduction of profit (refer [5], [23]). There are many practical situations where the steady state solutions are not applicable. As a result, the development of transient solutions for various queueing models is mainly focused by many researchers for the past three decades (refer [1], [9], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22]).

Kumar and Madheswari [13] considered an M/M/1 queueing model where the system faces catastrophe disaster and the server becomes failure. They computed the probabilities of system size for both stationary and time-dependent cases. Yechiali [25] discussed the impatient behavior of customers when the system undergoes disaster and derived the probabilities for the steady-state case. Kumar et al. [14] obtained the transient solutions for a two heterogeneous server queue where the servers undergo catastrophic failure. Kumar et al. [15] derived the transient and stationary probabilities for a single server model with catastrophes, failures and repairs of server using continued fraction method.

A single server queue where the system undergoes disastrous breakdown and customers behave impatience is analyzed by Sudhesh [19]. He obtained the transient probabilities for that model. Chakravarthy [3] studied the steady state analysis of a catastrophic queueing model with delayed action using matrix-analytic method and also he deduced the mean system size. Chang et al. [4] analyzed an unreliable server retrial queueing model with impatience and feedback nature of customers. They developed a recursive solver algorithm to find the steady state probabilities using quasi-birth and death process.

The solution of a single server queueing model is obtained by Parthasarathy [17] for the time-dependent case. Sudhesh et al. [21] investigated a Markovian single server queue in which the server starts working vacation when the system becomes empty and the impatience of customers due to the vacation of server. They obtained the time-dependent system size distributions, average and variance of size of the system explicitly. Sudhesh and Azhagappan [22] obtained the transient system size probabilities for the M/M/1 queueing model with working vacation and variant impatient behavior of customers using continued fraction. They also derived the expected system size and variance of system size for the time-dependent case.

Jain et al. [7] studied a multi component machine system with N-policy where the repairman starts the repair when there were N number of the repairable items accumulated in the system. They solved the system state equations by Range-Kutta method. Parthasarathy and Sudhesh [18] computed the transient and steady-state solutions for a multi-server queueing model with N-policy. Sudhesh et al. [20] investigated a single server N policy queue with system breakdown. They obtained both stationary and transient solutions.

Ke [10] investigated a batch arrival vacation queueing model with server breakdown, closedown and setup times. He derived the probability generating function of the queue size for that model under the steady state case. Kumar et al. [16]
considered a Markovian queueing model where the server undergoes closedown and then preventive maintenance whenever the system becomes empty. They derived the transient solutions and some system measures such as asymptotic behavior of various system state probabilities, average system size, average workload, etc. Arumuganathan and Jeyakumar [2] considered a batch arrival bulk service N-policy vacation queueing model where the server closedown the system when it becomes empty and setup the system after the completion of vacation to resume the service.

Haight [6] introduced the balking behavior of customers in queueing models. Kumar et al. [11] derived the time-dependent system size probabilities of the M/M/1 queueing model. Jain [8] investigated a batch arrival queueing model with unreliable server and balking. She obtained the probability generating function of the queue size at an arbitrary epoch. Vijayalaxmi and Jyothsna [24] studied a renewal input finite buffer multiple working vacation queueing model with balking.

Takacs [23] introduced the queue with feedback customers where each customer either immediately joins the queue for another service or leaves the system, after the service completion. He obtained the steady state queue size distribution and distribution function of sojourn time of a customer in the system. Choi and Kim [5] analyzed a two phases vacation queueing model with feedback customers where the first phase is batch service followed by a single service in the second phase. They obtained the steady state system size probabilities of that model.

The significance of the proposed research work is in the following points: Close-down and setup periods are introduced in a Markovian queueing model and the transient system size probabilities are obtained in terms of modified Bessel function of first kind using the method of generating function. The main contribution of this research work is to derive the most significant transient system size probabilities of an M/M/1 queueing model with the presence of closedown, setup periods, preventive maintenance, balking, re-service, N-policy, system disaster and repair. In the literature, so far no work is carried out to derive the time-dependent system size probabilities of the proposed model. This gives us the motivation to carry out this research work.

The remaining sections are as follows. The M/M/1 queueing model with N-policy, system disaster, repair, preventive maintenance, balking, re-service, closedown and setup times is described and the transient probabilities are derived in section 2. The system performance measures such as expectation, variance, probabilities of closedown, maintenance, failure, setup and empty state are also obtained in section 3. Numerical simulations of the proposed model are presented in section 4. The conclusion and future scope are provided in section 5.

2. Model description. We consider a single server Markovian queueing model with N-policy, system disaster, repair, preventive maintenance, balking, re-service, closedown and setup times. Figure 1 gives the state transition diagram of the present model. The assumptions to derive the system size probabilities under transient state for the model corresponding to the Figure 1 are described as follows:

- When the server is busy in on state (idle in off state), customers join the queue at the rate of $\lambda_1$ ($\lambda_0$) which is exponentially distributed. It is assumed that the arrival rate during the off state is less than that of the on state of the server. That is, $\lambda_0 < \lambda_1$. 
The server stays dormant until \( N \) customers accumulate in the queue and then starts an exhaustive service which is exponentially distributed with the rate \( \mu \).

- After the service, each customer may either move out from the system with probability \( \eta \) or get immediate re-service with probability \( 1 - \eta \).
- When the system becomes empty, the server resumes closedown work exponentially at the rate \( \delta \) and then undergoes preventive maintenance exponentially with parameter \( \xi \).
- After that, it comes to the idle state and waits \( N \) accumulate for service. When the \( N \)th one enters the queue, the server commences the setup work exponentially at the rate \( \gamma \) and then starts the service.
- Meanwhile, the system suffers disastrous breakdown during busy period which is distributed exponentially with parameter \( \theta \).
- It forced the system to the failure state and all the customers get eliminated. After that, the server gets repaired exponentially at the rate \( \omega \) and moves to the idle state.
- The customers may either join the queue with probability \( \sigma \) or balk the queue with probability \( 1 - \sigma \) when the size of the system is less than \( N \) and the server is idle in off state.
- The service of customers is based on first-come first-served.
- Assume that inter-arrival, service, failure, repair, closedown, setup and maintenance times are all independent.

2.1. **Practical applications of the proposed model.** In the process of *Plasma Arc Welding*, the welding of metal pieces (job) takes place one by one. It will be performed if there are sufficient quantity of work pieces (jobs) available for welding. In order to meet the operating cost, it will be put in waiting for the arrival of \( N \) (threshold value) work pieces (jobs). During this waiting period, as there is no service available to the arriving jobs, some of the arriving jobs decide to leave the work place without waiting in the queue (balking). After the arrival of \( N^{th} \) job, the tungsten electrodes are connected and the position of nozzle is corrected in order to start the welding process. Then the welding process begins and completes service for all the work pieces exhaustively. After the completion of service, immediate re-service can be given to the work piece if it is required. If there is no work piece waiting for welding after welding the last work piece, all the connections of welding torch are removed and then the electrodes as well as nozzle are cleaned. During the welding process, the torch utilized for welding may be breakdown. Then immediately all the work pieces (jobs) are removed from the work place and the torch will be sent for repair. After the repair, the welding torch becomes ready to start the welding process and waits for the arrival of \( N \) work pieces. This situation is modeled mathematically in this research work.

Let \( \{ \Omega(\tau), \tau \geq 0 \} \) and \( \zeta(\tau) \) are size of the system and the server state respectively at time \( \tau \). Let

\[
\zeta(\tau) = \begin{cases} 
0, & \text{off state of the server}, \\
1, & \text{on state of the server at time } \tau.
\end{cases}
\]
Then \(\{\zeta(\tau), \Omega(\tau), \tau \geq 0\}\) is a Markov process with state space \(S = \{V\} \cup \{C\} \cup \{F\} \cup \{S\} \cup \{0, j : j = 0, 1, 2, \ldots, N - 1\} \cup \{1, j : j = 1, 2, 3, \ldots\}\). Let

\[
\begin{align*}
\pi_{1,j}(\tau) &= P\{\zeta(\tau) = 1, \Omega(\tau) = j\}, \ j \geq 1, \\
\pi_{0,j}(\tau) &= P\{\zeta(\tau) = 0, \Omega(\tau) = j\}, \ 0 \leq j \leq N - 1, \\
\pi_F(\tau) &= P\{\text{the server is in failure state at time } \tau\}, \\
\pi_S(\tau) &= P\{\text{setup state of the server at time } \tau\}, \\
\pi_C(\tau) &= P\{\text{closedown state of the server at time } \tau\}, \\
\pi_V(\tau) &= P\{\text{maintenance state of the server at time } \tau\}.
\end{align*}
\]

From Figure 1, the steady state equations are obtained with the assumption of \(\lambda_1 < \mu\) as

\[
\begin{align*}
\xi \pi_V &= \delta \pi_C, \\
\delta \pi_C &= \eta \mu \pi_{1,1}, \\
(\lambda_1 + \eta \mu + \theta) \pi_{1,1} &= \eta \mu \pi_{1,2}, \\
(\lambda_1 + \eta \mu + \theta) \pi_{1,j} &= \lambda_1 \pi_{1,j-1} + \eta \mu \pi_{1,j+1}, \ j \geq 2, \ j \neq N, \\
(\lambda_1 + \eta \mu + \theta) \pi_{1, N} &= \lambda_1 \pi_{1, N-1} + \eta \mu \pi_{1, N+1} + \gamma \pi_S, \\
\omega \pi_F &= \theta \sum_{j \geq 1} \pi_{1,j}, \\
\sigma \lambda_0 \pi_{0,0} &= \omega \pi_F + \xi \pi_V, \\
\sigma \lambda_0 \pi_{0,j} &= \sigma \lambda_0 \pi_{0,j-1}, \ 1 \leq j \leq N - 1, \\
\gamma \pi_S &= \sigma \lambda_0 \pi_{0, N-1}.
\end{align*}
\]
Then the transient system state equations are established as follows:

$$\pi'_1(\tau) = -\xi \pi_V(\tau) + \delta \pi_C(\tau),$$  \hspace{1cm} (1)
$$\pi'_C(\tau) = -\delta \pi_C(\tau) + \eta \mu \pi_{1,1}(\tau),$$  \hspace{1cm} (2)
$$\pi'_{1,1}(\tau) = -(\lambda_1 + \eta \mu + \theta) \pi_{1,1}(\tau) + \eta \mu \pi_{1,2}(\tau),$$  \hspace{1cm} (3)
$$\pi'_{1,j}(\tau) = -(\lambda_1 + \eta \mu + \theta) \pi_{1,j}(\tau) + \lambda_1 \pi_{1,j-1}(\tau) + \eta \mu \pi_{1,j+1}(\tau),$$  \hspace{1cm} \( j \geq 2, \ j \neq N, \) \hspace{1cm} (4)
$$\pi'_{1,N}(\tau) = -(\lambda_1 + \eta \mu + \theta) \pi_{1,N}(\tau) + \lambda_1 \pi_{1,N-1}(\tau) + \eta \mu \pi_{1,N+1}(\tau) + \gamma \pi_S(\tau),$$  \hspace{1cm} (5)
$$\pi'_F(\tau) = -\omega \pi_F(\tau) + \theta \sum_{j=1}^N \pi_{1,j}(\tau),$$  \hspace{1cm} (6)
$$\pi'_{0,0}(\tau) = -\sigma \lambda_0 \pi_{0,0}(\tau) + \omega \pi_F(\tau) + \xi \pi_V(\tau),$$  \hspace{1cm} (7)
$$\pi'_{0,j}(\tau) = -\sigma \lambda_0 \pi_{0,j}(\tau) + \sigma \lambda_0 \pi_{0,j-1}(\tau), \ 1 \leq j \leq N - 1, \hspace{1cm} (8)
$$\pi'_S(\tau) = -\gamma \pi_S(\tau) + \sigma \lambda_0 \pi_{0,N-1}(\tau),$$  \hspace{1cm} (9)

with \( \pi_{0,0}(0) = 1. \)

2.2. Transient probabilities. The transient system size probabilities are derived for the proposed model in this section.

**Theorem 2.1.** The expressions for the probabilities \( \pi_{1,j}(t), \) for \( j \geq 1 \) are derived from (3), (4) and (5) as

$$\pi_{1,j}(\tau) = \gamma \int_0^\tau \pi_S(x)e^{-(\lambda_1 + \eta \mu + \theta)(\tau-x)}h^{j-N}[I_{j-N}(r(\tau-x)) - I_{j+N}(r(\tau-x))]dx,$$

\hspace{1cm} (10)

where \( I_j(\tau) \) is the modified Bessel function of the first kind of order \( j. \)

**Proof.** Define the generating function as

$$\psi(z, \tau) = \sum_{j=1}^\infty \pi_{1,j}(\tau)z^j.$$  \hspace{1cm} (11)

From the equations (3), (4) and (5), we obtain

$$\frac{\partial \psi(z, \tau)}{\partial \tau} = \left[-(\lambda_1 + \eta \mu + \theta) + \frac{\eta \mu}{z} + \lambda_1 z \right] \psi(z, \tau) - \eta \mu \pi_{1,1}(\tau) + \gamma \pi_S(\tau).$$

Solving the above partial differential equation, we obtain

$$\psi(z, \tau) = \gamma \int_0^\tau \pi_S(x)z^Ne^{-(\lambda_1 + \eta \mu + \theta)(\tau-x)}e^{(\lambda_1 z + \frac{\eta \mu}{z})\tau}dx$$

$$-\eta \mu \int_0^\tau \pi_{1,1}(x)e^{-(\lambda_1 + \eta \mu + \theta)(\tau-x)}e^{(\lambda_1 z + \frac{\eta \mu}{z})(\tau-x)}dx.$$  \hspace{1cm} (12)

Let us assume that \( r = 2\sqrt{\lambda_1 \eta \mu}, \ h = \sqrt{\frac{2\sqrt{\lambda_1 \eta \mu}}{\eta \mu}}. \) Then

$$e^{(\lambda_1 z + \frac{\eta \mu}{z})\tau} = \sum_{-\infty \leq j \leq \infty} (hz)^j I_j(r\tau).$$  \hspace{1cm} (13)
Using (13) in (12) and simplifying, for \( j \geq 1 \), we get

\[
\pi_{1,j}(\tau) = \gamma \int_0^\tau \pi_S(x)e^{-(\lambda_1+\eta\mu+\theta)(\tau-x)}h_j^{-N}I_{j-N}(r(\tau-x))dx
- \eta\mu \int_0^\tau \pi_{1,1}(x)e^{-(\lambda_1+\eta\mu+\theta)(\tau-x)}h_l^j(r(\tau-x))dx.
\]  

(14)

The above expression holds for \( j = -1, -2, -3, \ldots \) Using \( I_{-j}(y) = I_j(y) \), for \( j \geq 1 \), we have

\[
0 = \gamma \int_0^\tau \pi_S(x)e^{-(\lambda_1+\eta\mu+\theta)(\tau-x)}h_j^{-j-N}I_{j+N}(r(\tau-x))dx
- \eta\mu \int_0^\tau \pi_{1,1}(x)e^{-(\lambda_1+\eta\mu+\theta)(\tau-x)}h_j^{-j}(r(\tau-x))dx.
\]  

(15)

From (14) and (15), for \( j \geq 1 \), we obtain (10).

Theorem 2.2. The expressions for the probabilities \( \pi_{0,j}(t) \), for \( 1 \leq j \leq N-1 \), \( \pi_S(\tau) \), \( \pi_F(\tau) \), \( \pi_C(\tau) \) and \( \pi_V(\tau) \) are derived from (8), (9), (6), (2) and (1) respectively as

\[
\pi_{0,j}(\tau) = (\sigma\lambda_0)^j\pi_0(\tau) e^{-\sigma\lambda_0\tau} \frac{\tau^j}{(j-1)!} \pi_0(\tau), \quad 1 \leq j \leq N-1,
\]  

(16)

\[
\pi_S(\tau) = (\sigma\lambda_0)^N e^{-\gamma\tau} \pi_0(\tau) e^{-\sigma\lambda_0\tau} \frac{\tau^N-2}{(N-2)!} \pi_0(\tau),
\]  

(17)

\[
\pi_F(\tau) = \xi e^{-\omega\tau} \sum_{j=1}^\infty A_j(\tau) \pi_0,0(\tau),
\]  

(18)

\[
\pi_C(\tau) = \eta\mu e^{-\delta\tau} \pi_{1,1}(\tau),
\]

(19)

\[
\pi_V(\tau) = \delta\eta\mu e^{-\xi\tau} e^{-\delta\tau} \pi_{1,1}(\tau),
\]  

(20)

where

\[
\pi_{0,0}(\tau) = Q(\tau) + \xi\delta\eta\mu e^{-\sigma\lambda_0\tau} e^{-\xi\tau} e^{-\delta\tau} \pi_{1,1}(\tau) \pi(\tau),
\]  

(21)

\[
\pi_{1,1}(\tau) = \sum_{j=0}^\infty \xi^j e^{-\sigma\lambda_0\tau} \frac{\tau^j}{(j-1)!} e^{-\eta\tau} \frac{\tau^j}{(j-1)!} \pi_{1,1}(\tau) \pi(\tau)
+ \xi^j \delta\eta\mu e^{-\xi\tau} e^{-\delta\tau} \pi_{1,1}(\tau) \pi(\tau)
+ \sum_{j=0}^\infty A_1(\tau) Q(\tau)^{j+1},
\]

(22)

\[
Q(\tau) = \sum_{k=0}^\infty (\theta\omega)^k e^{-\sigma\lambda_0\tau} \frac{\tau^{k-1}}{(k-1)!} e^{-\omega\tau} \frac{\tau^{k-1}}{(k-1)!} \left( \sum_{j=1}^\infty A_j(\tau) \right)^k.
\]

‘*’ denotes the convolution and ‘*m’ represents the m-fold convolution.
Proof. Taking Laplace transform of (8) and rearranging the terms, for $1 \leq j \leq N-1$, we get

$$\hat{\pi}_{0,j}(s) = \left(\frac{\sigma \lambda_0}{s + \sigma \lambda_0}\right)^j \hat{\pi}_{0,0}(s).$$

(23)

On Laplace inversion of (23), for $1 \leq j \leq N-1$, we obtain (16). Laplace transform of (9) and using (23), we get

$$\hat{\pi}_S(s) = \left(\frac{\sigma \lambda_0}{s + \gamma}(s + \sigma \lambda_0)^{N-1}\hat{\pi}_{0,0}(s).$$

(24)

Laplace inversion of (24) gives (17). Taking Laplace transform of (10), for $j \geq 1$ and using (24), we get

$$\hat{\pi}_{1,j}(s) = \hat{A}_j(s)\hat{\pi}_{0,0}(s),$$

(25)

where

$$\hat{A}_j(s) = \frac{\gamma \sigma \lambda_0}{s + \gamma} \left(\frac{\sigma \lambda_0}{s + \sigma \lambda_0}\right)^{N-1} \frac{h^j-N}{\sqrt{u^2 - r^2}}$$

$$\times \left[\left(u - \sqrt{u^2 - r^2}\right)^{j-N} - \left(u - \sqrt{u^2 - r^2}\right)^{j+N}\right]$$

and $u = s + \lambda_1 + \eta \mu + \theta$. Laplace inversion of (25), for $j \geq 1$, we obtain

$$\pi_{1,j}(\tau) = A_j(\tau) * \pi_{0,0}(\tau),$$

(26)

where

$$A_j(\tau) = \gamma (\sigma \lambda_0)^N e^{-\gamma \tau} * e^{-\sigma \lambda_0 \tau} - \frac{\tau^{N-2}}{(N-2)!}$$

$$* \ h^j-N e^{-(\lambda_1+\eta \mu+\theta) \tau} [I_{j-N}(h \tau) - I_{j+N}(h \tau)].$$

Taking Laplace transform of (6) and using (25), we get

$$\hat{\pi}_F(s) = \frac{\theta}{s + \omega} \sum_{j=1}^{\infty} \hat{A}_j(s)\hat{\pi}_{0,0}(s).$$

(27)

Taking inverse Laplace transform of (27), we obtain (18). Taking Laplace transform of (2), we get

$$\hat{\pi}_C(s) = \frac{\eta \mu}{s + \delta} \hat{\pi}_{1,1}(s).$$

(28)

Laplace inversion of (28) yields (19). Laplace transform of (1) and using (28), we get

$$\hat{\pi}_V(s) = \frac{\delta \eta \mu}{(s + \xi)(s + \delta)} \hat{\pi}_{1,1}(s).$$

(29)

Laplace inversion of (29), we obtain (20). Laplace transform on (7), using (27) as well as (29) and after some simplification, we get

$$\hat{\pi}_{0,0}(s) = \hat{Q}(s) + \frac{\xi \delta \eta \mu}{(s + \sigma \lambda_0)(s + \xi)(s + \delta)} \hat{\pi}_{1,1}(s)\hat{Q}(s),$$

(30)
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where

\[
\hat{Q}(s) = \sum_{k=0}^{\infty} \frac{(\omega \xi)^k}{(s + \sigma \lambda_0)^k (s + \omega)^k} \left( \sum_{j=0}^{\infty} \hat{A}_j(s) \right)^k.
\]

Laplace inversion of (30), we obtain (21). Using (25), for \(j = 1\), (30) and on simplification, we get

\[
\hat{\pi}_{1,1}(s) = \sum_{j=0}^{\infty} \frac{(\xi \delta \eta \mu)^j}{(s + \sigma \lambda_0)^j (s + \xi)^j (s + \delta)^j} (\hat{A}_1(s) \hat{Q}(s))^{j+1}.
\]

Taking Laplace inversion of (31), we get (22). The expressions given in (10), (16), (17), (18), (19), (20), (21) and (22) together represent the transient system size probabilities of the proposed model.

2.3. **Special case.** When \(\theta = 0, \gamma = 0, \omega = 0, \sigma = 1, \lambda_0 = 0, \eta = 1, \lambda_1 = \lambda\), then

\[
\pi_{1,j}(\tau) = \frac{\tau^{N-2}}{(N-2)!} \cdot \sum_{i=1}^{N-1} \pi_{0,i-1}(\tau) \cdot Q(\tau) + \sum_{i=1}^{N-1} \lambda_1 - \eta \mu - i \theta \cdot \pi_{1,i}(\tau).
\]

Integrating the above equation, we get

\[
m(\tau) = \gamma N_{\pi_S(\tau)} - \sigma \lambda_0 (N-1) \pi_{0,N-1}(\tau) + \sigma \lambda_0 \sum_{1 \leq i \leq N-1} \pi_{0,i-1}(\tau) + \sum_{i \geq 1} (\lambda_1 - \eta \mu - i \theta) \pi_{1,i}(\tau).
\]

(II) The variance, \(v(\tau)\), of \(\{\Omega(\tau)\}\) at time \(\tau\) is

\[
v(\tau) = n(\tau) - (m(\tau))^2,
\]
where
\[ n(\tau) = \sum_{k=1}^{\infty} k^2 \pi_{1,k}(\tau) + \sum_{k=1}^{N-1} k^2 \pi_{0,k}(\tau). \]

From (3), (4), (5) and (8), we get
\[ n(\tau) = \gamma N^2 \pi_S(\tau) - \sigma \lambda_0 (N-1)^2 \pi_{0,N-1}(\tau) + \sigma \lambda_0 \sum_{1 \leq i \leq N-1} (2i - 1) \pi_{0,i-1}(\tau) \]
\[ + \sum_{i \geq 1} \left[ -\theta i^2 + 2(\lambda_1 - \eta \mu) i + (\lambda_1 + \eta \mu) \right] \pi_{1,i}(\tau). \]

On integration of the above equation, we obtain
\[ n(\tau) = \gamma N^2 \int_0^\tau \pi_S(x) \, dx - \sigma \lambda_0 (N-1)^2 \int_0^\tau \pi_{0,N-1}(x) \, dx \]
\[ + \sigma \lambda_0 \sum_{1 \leq i \leq N-1} (2i - 1) \int_0^\tau \pi_{0,i-1}(x) \, dx \]
\[ + \sum_{i \geq 1} \left[ -\theta i^2 + 2(\lambda_1 - \eta \mu) i + (\lambda_1 + \eta \mu) \right] \int_0^\tau \pi_{1,i}(x) \, dx. \]

(III) The probability of closedown, \( \pi_C(\tau) \) is
\[ \pi_C(\tau) = \eta \mu e^{-\delta \tau} \sum_{j=0}^{\infty} (\xi \delta \eta \mu)^j e^{-\sigma \lambda_0 \tau} \frac{\tau^{j-1}}{(j-1)!} \ast e^{-\eta \tau} \frac{\tau^{j-1}}{(j-1)!} \ast e^{-\delta \tau} \frac{\tau^{j-1}}{(j-1)!} \]
\[ \ast (A_1(\tau) \ast Q(\tau))^{\ast(j+1)}. \]

(IV) The probability of maintenance, \( \pi_V(\tau) \) is
\[ \pi_V(\tau) = \delta \eta \mu e^{-\xi \tau} \ast e^{-\delta \tau} \sum_{j=0}^{\infty} (\xi \delta \eta \mu)^j e^{-\sigma \lambda_0 \tau} \frac{\tau^{j-1}}{(j-1)!} \ast e^{-\eta \tau} \frac{\tau^{j-1}}{(j-1)!} \ast e^{-\delta \tau} \frac{\tau^{j-1}}{(j-1)!} \]
\[ \ast (A_1(\tau) \ast Q(\tau))^{\ast(j+1)}. \]

(V) The probability of failure, \( \pi_F(\tau) \) is
\[ \pi_F(\tau) = \xi e^{-\omega \tau} \ast \sum_{j \geq 1} A_j(\tau) \ast \left[ Q(\tau) + \xi \delta \eta \mu e^{-\sigma \lambda_0 \tau} \ast e^{-\xi \tau} \ast e^{-\delta \tau} \right. \]
\[ \ast \sum_{j \geq 0} (\xi \delta \eta \mu)^j e^{-\sigma \lambda_0 \tau} \frac{\tau^{j-1}}{(j-1)!} \ast e^{-\eta \tau} \frac{\tau^{j-1}}{(j-1)!} \ast e^{-\delta \tau} \frac{\tau^{j-1}}{(j-1)!} \]
\[ \ast (A_1(\tau) \ast Q(\tau))^{\ast(j+1)} \ast Q(\tau) \right]. \]
(VI) The probability of setup, \( \pi_S(\tau) \) is
\[
\pi_S(\tau) = (\sigma \lambda_0)^N e^{-\gamma \tau} * e^{-\sigma \lambda_0 \tau} \frac{\tau^{N-2}}{(N-2)!} \left[ Q(\tau) + \xi \eta e^{-\sigma \lambda_0 \tau} * e^{-\xi \tau} * e^{-\delta \tau} \right. \\
\left. \sum_{j \geq 0} (\xi \eta)^j e^{-\sigma \lambda_0 \tau} \frac{\tau^{j-1}}{(j-1)!} \right] \\
* e^{-\delta \tau} \frac{\tau^{j-1}}{(j-1)!} \left( A_1(\tau) * Q(\tau) \right) * (j+1) * Q(\tau). \]

(VII) The empty state probability, \( \pi_0,0 (\tau) \) is
\[
\pi_0,0 (\tau) = \left[ Q(\tau) + \xi \eta e^{-\sigma \lambda_0 \tau} * e^{-\xi \tau} * e^{-\delta \tau} \right. \\
\left. \sum_{j \geq 0} (\xi \eta)^j e^{-\sigma \lambda_0 \tau} \frac{\tau^{j-1}}{(j-1)!} \right] \\
* e^{-\delta \tau} \frac{\tau^{j-1}}{(j-1)!} \left( A_1(\tau) * Q(\tau) \right) * (j+1) * Q(\tau). \]

(VIII) The expression for the steady state mean system size, \( E(\Omega) \) is given as
\[
E(\Omega) = \sum_{k \geq 1} \lambda_1 (1 + 2k + \eta \mu (1 - 2k) - \theta) \pi_1,k + \gamma \pi_S - \sigma \lambda_0 \pi_{0,N-1} + \sigma \lambda_0 \sum_{0 \leq k \leq N-1} \pi_0,k. \]

(IX) The expression for the steady state variance of system size, \( V(\Omega) \) is
\[
V(\Omega) = E(\Omega^2) - (E(\Omega))^2, \]
where
\[
E(\Omega^2) = \sum_{k \geq 1} \lambda_1 (1 + 2k + \eta \mu (1 - 2k) - \theta) \pi_1,k + \gamma \pi_S + \sigma \lambda_0 \pi_{0,N-1} \\
- 2\sigma \lambda_0 \sum_{1 \leq k \leq N-1} k \pi_0,k + \sigma \lambda_0 \sum_{0 \leq k \leq N-2} \pi_0,k. \]

Here, the probabilities \( \pi_1,k, k \geq 1, \pi_S \) and \( \pi_0,k, 0 \leq k \leq N-1 \) can be obtained from the steady state equations given in section 2.1.

4. Numerical Illustration. The numerical simulation is carried over for the model under consideration using MATLAB software in this section.

Figure 2 and 3 give the probability curves corresponding to the off and on state of the server respectively for \( \lambda_0 = 0.5, \lambda_1 = 0.75, \mu = 1, \eta = 0.5, \delta = 0.5, \theta = 0.3, \)
ξ = 0.2, γ = 0.6, ω = 0.4, σ = 0.5 and N = 5. All the probability curves, except π_{0,0}(τ) increase initially and become steady state in the long run. Figure 4 and 5 depict that the average and variance of number of jobs in the system increase when the rate σ of joining the queue increases. It is natural in every practical situation that the customers get accumulated when their joining rate increase. Figure 6 and
7 show that the expected value and variance of customers in the system go down with the increment of leaving rate of the system. This happens as more customers leave the system while \( \eta \) increases.

5. Conclusion and future scope. The analysis of a single server Markovian queueing model with N-policy, system disaster, repair, preventive maintenance, balking, re-service, closedown and setup times is carried over. Using the method of generating function, the probabilities of number present in the system are computed for the transient state. Numerical illustrations motivate us to visualize the influence of various system parameters. The future extension of this model may be a multi-server queueing model with closedown, setup times and maintenance.

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E-mail address: azhagappanmaths@gmail.com
E-mail address: deepatmaths@gmail.com