Negative Effective Density in An Acoustic Metamaterial

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Abstract

Abstract. We report theoretical and experimental results for a new type of homogenized acoustic metamaterials with negative effective mass density. We constructed one-dimensional metamaterial, which is a tube with an array of very thin elastic membranes placed inside. This structure exhibited negative effective density in the frequency range from 0 to 735 Hz. The experimental result is in excellent agreement with our theoretical model that predicts negative effective density below a cut-off frequency. The frequency characteristics of this effective density is analogous to that of the permittivity of the electromagnetic plasma oscillation.

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The realization of double negative electromagnetic metamaterials has opened a new area of science in the field of electromagnetic waves \[1, 2, 3, 4, 5, 6\]. In the electromagnetism, the two constitutive parameters, the electric permittivity, \(\epsilon\), and magnetic permeability, \(\mu\), determine the phase velocity, \(v_{ph} = \sqrt{1/\epsilon\mu}\), in a medium. If \(\epsilon\) and \(\mu\) are simultaneously negative (double negativity, DNG), the waves propagate through the media with a phase velocity which is antiparallel to the Poynting vector. Progress in electromagnetic metamaterials also stimulated researches in acoustic metamaterials \[7, 8, 9, 10, 11, 12, 13\]. If the density and the bulk modulus are simultaneously negative, the phase velocity of sound is expected to also become negative. Because no naturally occurring material exhibits such negative parameters, acoustic DNG has to be achieved using engineered subwavelength structures. Negative bulk modulus has been realized using an array of Helmholtz resonators \[14\]. As for the negative density, a single resonator consisting of a rubber membrane with a central disk has been demonstrated to exhibit negative dynamic mass \[9\]. However, a negative mass density in a medium has not been realized yet.

In this paper we present a new type of homogenized acoustic metamaterial that exhibits negative effective density for a wide range of frequencies. This metamaterial is based on a new type of elasticity not observed in naturally occurring materials. This elasticity is generated in a fluid by using an array of very thin tensioned membranes. We present a calculation showing that this elasticity creates negative effective density for all frequencies below a cut-off frequency instead of a stop band. So far, all known negative constituent parameters, either for electromagnetic waves or for acoustic waves, have stop bands except for the effective permittivity of the electromagnetic plasma oscillation. Our model predicts the frequency characteristics of negative density exactly of the same form as that of the effective permittivity in a plasma. A one-dimensional version of the material was constructed as shown in figure 1(a). It is a tube with an array of very thin elastic membranes placed inside. The negative effective density has been observed in the frequency range from 0 to 735 Hz, confirming the theoretical prediction.

The metamaterial consists of the unit cells shown in figure 1(a). A low density polyethylene membrane with thickness of 0.01 mm is placed inside a tube of 32.3 mm inner diameter and 70 mm long. The membrane is stretched and the periphery is attached leak-tight to the inner surface of the tube. The resulting tension of the membrane is 65 N/m. When the fluid (air for the present case) moves along the tube, the membrane is pushed to form a
FIG. 1: (color online) (a) Structures of metamaterials; One-dimensional structure consisting of thin tensioned elastic membranes in a tube. Negative effective density is observed in this system. (b) Experimental setup for the transmission and phase velocity measurements.

When the unit cells are connected, the one dimensional metamaterial is formed as shown in figure 1(a). The acoustic wave in the tube is described by the longitudinal displacement of the fluid, $\vec{\xi}(x,t)$. If the length of the unit cell is much smaller than the wavelength (the long-wavelength limit), the restoring forces of the membranes generate static pressure gradient proportional to $\vec{\xi}$,

$$\nabla p = -\kappa \vec{\xi}$$

where $p$ and $\kappa$ are the pressure and the new elastic modulus, respectively. In three dimensions, several types of membrane structures are possible. The simple cubic type, for example, consists of cubic cells with windows of thin elastic membranes on all six walls. Generally speaking, the new elasticity can be regarded as an intrinsic property that characterizes the
metamaterial according to equation (1), independent of the particular structure that generates it. Because the fluid is elastically anchored in space by the membranes, we will refer to this type of elasticity as ‘spatially anchored elasticity’ (SAE).

In the dynamic case, due to the force from the membranes, Newton’s equation becomes
\[-\nabla p = \rho' \partial \vec{u} / \partial t + \kappa \vec{\xi},\]
where \(\vec{u} = \partial \vec{\xi} / \partial t\) is the velocity of the fluid particle. The volume-averaged density \(\rho'\) of the fluid and the membranes, giving the inertia term in the equation, is significantly different from the dynamic effective mass density \(\rho_{eff}\) defined below. Using the harmonic expressions \(\vec{u}(x, t) = \vec{U} e^{-i\omega t}\), the equation can be written in a convenient form,
\[-\nabla p = \left(\rho' - \frac{\kappa}{\omega^2}\right) \frac{\partial \vec{u}}{\partial t}.\]

Now the proportionality constant of the acceleration to the pressure gradient force is defined as the effective density,
\[\rho_{eff} = \rho' - \frac{\kappa}{\omega^2} = \rho' \left(1 - \frac{\omega^2_{SAE}}{\omega^2}\right),\]
which becomes negative below the critical frequency \(\omega_{SAE} = \sqrt{\kappa / \rho'}\) \((f = \omega / 2\pi)\). Interestingly equation (3) has essentially the same form as the expression for the permittivity \(\epsilon\) in a plasma, with \(\omega_{SAE}\) corresponding to the plasma frequency \(\omega_p\). The acoustic wave equation obtained from equation (2) and the continuity equation \(\nabla \cdot \vec{u} = -(1/B) \partial p / \partial t\) give the frequency-dependent phase velocity,
\[v_{ph} = \sqrt{B / \rho_{eff}} = \sqrt{\frac{B}{\rho' \left(1 - \omega^2_{SAE} / \omega^2\right)}}.\]

For frequencies below \(\omega_{SAE}\), the acoustic waves do not propagate because the phase velocity assumes an imaginary value. Above this frequency, the phase velocity assumes a large value and decreases down to \(\sqrt{B / \rho'}\) as the frequency becomes high.

To experimentally verify this result, the pressure is measured in each cell using the setup shown in figure 1 (b). A small sound source is placed at the beginning end of the metamaterial, and the other end is terminated by an absorber to prevent any reflection so that the acoustic wave propagating in the metamaterial behaves as if the metamaterial extends to infinity. The absorber is a long metamaterial of the same kind with dissipation elements additionally placed in each cell. The dissipation element is a sponge-like plate placed across the tube to generate a drag force against the longitudinal motion of the fluid. Pressure measurement confirmed that the sound decayed exponentially away as it propagated in the
absorber, and there was almost no reflection at the metamaterial/absorber boundary (data not shown). This eliminates concerns about the effect of the finite number of cells used in the experiment, as well as the interference effect from the reflected waves. For the phase velocity data, pressure was measured as a function of time and position in the pass band. The detector was inserted into the cell through the tube wall using feed-through plugs and moved from cell to cell. The detector did not alter the characteristics of the media.

The average density of the air loaded with the membrane in the tube was $\rho' \sim 1.34 \text{ kg/m}^3$, about 10% higher than the density of air which is $\rho_0 \sim 1.21 \text{ kg/m}^3$ [15]. The modulus in equation (1) was calculated from the tension of the membranes to be $\kappa = 2.85 \times 10^7 \text{ N/m}^4$. From these values a critical frequency of $\omega_{SAE} = \sqrt{\kappa/\rho'}$ was calculated to be about 735 Hz ($f = \omega/2\pi$).

Figure 2 (a) shows propagation data of acoustic waves in the metamaterial for several frequencies. For the frequencies below $\omega_{SAE}$ (100 Hz, 300 Hz, 500 Hz, 700 Hz) the sound intensity decayed exponentially with the distance, $x$, from the sound source. Lines connecting data points are drawn for eye guides. For the frequencies above $\omega_{SAE}$ (only 900 Hz is shown for clarity) the sound waves propagated well without decay. The sound intensities were normalized to the intensity level at $x = 0$. The transmissions of the acoustic waves in the metamaterial were measured as the ratios of pressure intensities at two positions; at $x = 0$ and $x = 1.3$ m. The cut-off frequency of the transmission data agrees with the calculated value of $\omega_{SAE}$ (indicated with a broken line). Experimental and theoretical values of the phase velocities are shown in figure 2 (c). The theoretical phase velocity predicted by equation (4) agrees excellently with the experimental data.

Note that even though the SAE is a kind of elasticity, it has no effect on the value of the effective modulus, but it strongly alters the value of $\rho_{eff}$. The mechanism for the formation of $\rho$-negativity from the SAE is different from that of local-resonant type acoustic metamaterials [7, 9]. In addition, the SAE offers a substantially wider frequency range for negative effective density.

In conclusion, we described the fabrication of a new class of acoustic metamaterials in this paper. We introduced the novel concept of 'spatially anchored elasticity' which uses a homogenized structure of membranes to produce negative effective density. The constructed structure exhibited negative effective density characteristics in the spectral range from 0 to 735 Hz. Above this frequency, the phase velocity is highly dispersive. Experimental trans-
FIG. 2: (color online) (a) Sound intensities as functions of the distance from the source, $x$. (b) Transmittance data in the metamaterial. The cut-off frequency of the transmission data agrees with the calculated value of $f_{SAE}$ (indicated with a broken line). (c) Phase velocities in the metamaterial.
mission and phase velocity measurement agree excellently with the theoretical predictions. We expect the acoustic density negativity presented in this paper to provide a useful basis for applications in acoustic superlensing and cloaking [16, 17, 18, 19, 20, 21].

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