New Developments in Cosmology

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Abstract

A brief review is given of the present observational data in cosmology. A review of a new bimetric gravity theory with multiple light cones is presented. The physical consequences of this gravity theory for the early universe are analyzed.

1 Status of Observational Cosmology

The ten most significant parameters to be determined in observational cosmology are:

1. Age of the universe: \( t_0 \)
2. The Hubble constant \( H_0 \) from the Hubble relation: \( v = H_0d \)
3. Density parameter : \( \Omega_m = \frac{\rho_m}{3H_0^2/8\pi G} \)
4. Deceleration parameter: \( q_0 = -\frac{\ddot{R}(t_0)R(t_0)}{R^2(t_0)} \)
5. The baryon density \( \Omega_B \) and the vacuum density \( \Omega_\Lambda \)
6. The parameters associated with microwave background fluctuations: \( n, \sigma_8, T/S, N_T \)

The strongest lower limit for \( t_0 \) is determined from studies of the stellar populations of globular clusters. The main error in the globular clusters age estimate comes from the uncertain distance to the globular clusters. A 0.25 magnitude error in the distance translates into a 22% error in the cluster age \([1]\). Independent age limits come from the cooling of white dwarfs. The best estimates give \( t_0 \sim 13 \) Gyr, with a lower limit of \( \sim 11 \) Gyr. For \( t_0 > 13 \) Gyr, we have \( h \leq 0.50 \) (where \( h \) is defined by the Hubble parameter: \( H_0 = 100 \, h \, \text{km s}^{-1} \text{Mpc}^{-1} \)) for matter density \( \Omega_m = 1 \), and for \( h \leq 0.73 \) we have \( \Omega_m \sim 0.3 \) in spatially flat cosmologies with \( \Omega_m + \Omega_\Lambda = 1 \).

The Hubble parameter is now better determined (it used to be known to within a factor of two). Most measurements are now consistent with a value: \( h = 0.65 \pm \)
It is remarkable that data obtained from several different methods for determining $H_0$ lead to similar results, which gives hope for an ultimate convergence of measurements. For $\Omega_m = 0.4$ and $\Omega_\Lambda = 0.6$ and $h = 0.65 \pm 0.08$, the age of the universe would be $t_0 = 13 \pm 2$ Gyr, in agreement with globular cluster age estimates. This result is one of the strong arguments for a low matter density $\Omega_m \sim 0.3$ and a non-zero cosmological constant $\Omega_\Lambda \sim 0.7$.

A most promising new way of measuring $\Omega_m$ and $\Omega_\Lambda$ on cosmological scales is to use small-angle anisotropies in the CMB radiation and high-redshift Type Ia supernovae (SNe Ia). The Supernovae Cosmology Project (CSP) and the High-Z Supernovae team have found a significant number of Type Ia supernovae [3, 4, 5]. The more recent larger SCP data set of 42 high redshift data gives for the flat case $\Omega_m = 0.28^{+0.09}_{-0.08}^{+0.05}$. The High-Z Supernovae group has also measured $\Omega_m$ giving in the flat case $\Omega_m = 0.4 \pm 0.3$. Two possible sources of problems are the dimming by dust and the assumption made that evolution for nearby and far supernovae is uniform.

In CMB anisotropy studies, the location of the first acoustic Doppler peak at angular wave number $l \sim 250$ is a strong indication of a flat universe $\Omega_m + \Omega_\Lambda = 1$. The MAXIMA and BOOMERANG balloon flights seem to confirm this result, and the existence of a second and possible third peak would appear to be consistent with the predictions of simple inflation models. New data from the NASDA Microwave Anisotropy Probe satellite will hopefully strengthen these results.

We can summarise the main observational results:

1. Age of universe $t_0 = 9 - 16$ Gyr (from globular clusters) = 9 - 17 Gyr*
2. Hubble parameter $H_0 = 100 h \, \text{s}^{-1} \, \text{Mpc}^{-1}$, $h = 0.65 \pm 0.08$.
3. Baryon density $\Omega_b h^2 = 0.019 \pm 0.001$ (from $D/H$)
   > 0.015 from Ly$\alpha$ forest opacity*
4. Matter density $\Omega_m = 0.4 \pm 0.2$ (from cluster baryons)
   $= 0.34 \pm 0.1$ from Ly$\alpha$ forest $P(k)^* (P(k) = Ak^n$ with $n = 1$ for the Harrison-Zel’dovich spectrum)
   $= 0.4 \pm 0.2$ from cluster evolution*
   > $\frac{3}{4} \Omega_\Lambda - \frac{1}{4} \pm \frac{1}{8}$ from SN Ia > 0.3 (2.4\sigma from flows)
5. Total density $\Omega_m + \Omega_\Lambda = 1 \pm 0.3$ (from CMB peak location)
6. Dark vacuum energy density $\Omega_\Lambda = 0.8 \pm 0.3$ (from last two lines)
7. Neutrino density $\Omega_\nu \geq 0.001$ (from Superkamiokande) $\leq 0.1^*$
Here, the cosmological parameters with * are obtained by assuming ΛCDM i.e.
cold dark matter models with non-zero cosmological constant.

The Type Ia reshift measurements have indicated the remarkable result that the
universe is presently undergoing an acceleration.

2 Bimetric Gravity Theory

A new kind of vector-tensor and scalar-tensor theory of gravity, which exhibits a
bimetric structure and contains two or more light cones [6, 7, 8], has been introduced,
recently. This type of model has attracted some attention [9, 10, 11], and similar
effects have been noted elsewhere [12, 13, 14]. The motivation for considering
these models is derived from earlier work [15], which provided a scenario in which
some of the outstanding issues in cosmology can be resolved. These models provide a
fundamental dynamical mechanism for varying speed of light theories and generate a
new mechanism for an inflationary epoch that could solve the initial value problems
of early universe cosmology. In the following, we shall review some of the main
features of this new kind of gravity theory and its application to cosmology.

In these models matter that satisfies the strong energy condition can nevertheless
contribute to the cosmic acceleration. Our cosmological model can be mapped to
a model with varying fundamental constants [16, 17, 18], albeit not uniquely and
requiring some care in the interpretation of the varying constants that appear.

It is hoped that the models can shed some light on the new observational data
that suggests the expansion of the universe at present is undergoing an acceleration [3, 4, 19]. Although there has been some success in understanding the latter
problem by the inclusion of a class of very particular scalar field potentials [20], it
is fair to say that not all issues have been resolved. Using the scalar field version of
the model, we expect that not only will we be able to generate sufficient inflation,
but that a quintessence-like solution should be achievable.

We shall be considering models with an action of the form

\[ S = \mathcal{S}_{gr}[\tilde{g}] + S[g, \psi] + \hat{S}[\hat{g}, \hat{\phi}]. \]  

The first term is the usual Einstein-Hilbert action for general relativity constructed
from a metric \( \tilde{g}_{\mu\nu} \), and the final term is the contribution from the non-gravitational
(matter) fields in spacetime \( \hat{\phi} \), and is built from a different but related metric \( \hat{g}_{\mu\nu} \).

The contribution \( S[g, \psi] \) is constructed from a metric \( g_{\mu\nu} \) and includes kinetic
terms for a field or fields (unspecified as yet) \( \psi \) that may be considered to be part of
the gravitational sector, modifying the reaction of spacetime to the presence of the
matter fields in \( \hat{S}[\hat{g}, \hat{\phi}] \). The manner in which \( \psi \) accomplishes this is by modifying
the metric that appears in each of the actions. For example, in [4] \( \psi \) was a vector
field, \( \tilde{g}_{\mu\nu} = g_{\mu\nu} \) and \( \tilde{g}_{\mu\nu} = g_{\mu\nu} + b \psi_\mu \psi_\nu \), whereas in [7] \( \psi \) was a scalar field, \( \tilde{g}_{\mu\nu} = g_{\mu\nu} \)
and \( \tilde{g}_{\mu\nu} = g_{\mu\nu} + b \partial_\mu \psi \partial_\nu \psi \). These relations imply that matter and gravitational fields
propagate at different velocities if \( \psi \) is non-vanishing.
Since the matter action $S$ is built using only $\hat{g}_{\mu\nu}$, it is the null surfaces of $\hat{g}_{\mu\nu}$ along which matter fields propagate. If we assume that other than the presence of a “composite” metric the matter action is otherwise a conventional form (perfect fluid, scalar field, Maxwell, etc.), then variation of the matter action yields the matter energy-momentum tensor $\hat{T}_{\mu\nu}$, which will be conserved

$$\nabla_\nu \hat{T}^{\mu\nu} = 0,$$

by virtue of the matter field equations $\hat{F}_I = 0$. Throughout we will write, for example, $\nabla_\nu$ for the covariant derivative constructed from the Levi-Civita connection of $\hat{g}_{\mu\nu}$. Since we also assume that the matter fields satisfy the dominant energy condition, we therefore know (assuming appropriate smoothness of $\hat{g}_{\mu\nu}$) that matter fields cannot travel faster than the speed of light as determined by $\hat{g}_{\mu\nu}$.

The gravitational action is written

$$\bar{S}_{\text{gr}}[\bar{g}] = -\frac{1}{\kappa} \int d \bar{\mu} \bar{R},$$

where we use a metric with $(+---)$ signature and have defined $\kappa = 16\pi G/c^4$. We will denote the metric densities by, e.g., $\bar{\mu} = \sqrt{-\det(\bar{g}_{\mu\nu})}$ and in addition write $d \bar{\mu} = \bar{\mu} dt d^3x$. We will not consider a cosmological constant, since it can easily be included later. We can identify the metric $\bar{g}_{\mu\nu}$ as providing the light cone for the gravitational system.

We consider a Proca model with arbitrary potential

$$S[g, \psi] = -\frac{1}{\kappa} \int d \mu \left( \frac{1}{4} B^2 - V(X) \right),$$

where we will use the definition

$$X = \frac{1}{2} \psi^2,$$

and $V'(X) = \partial V(X)/\partial X$. We will also use $B_{\mu\nu} = \partial_\mu \psi_\nu - \partial_\nu \psi_\mu$, $\psi^2 = g^{\mu\nu} \psi_\mu \psi_\nu$ and $B^2 = g^{\alpha\beta} g^{\theta\varphi} B_{\alpha\beta} B_{\theta\varphi}$. We will assume that as $\psi_\mu \rightarrow 0$ we have $V(X) \sim m^2 X$ and therefore the linearized (in $\psi_\mu$) limit of our model is identical to Einstein-Proca field equations coupled to matter. The standard energy-momentum tensor for the vector field is

$$T^{\mu\nu} = -B^{\alpha\nu} B^{\mu\alpha} + \frac{1}{4} g^{\mu\nu} B^2 + V' \psi^\mu \psi^\nu - V g^{\mu\nu}.$$

Although there exists a more general class of models, we will limit ourselves to the choice

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + b \psi_\mu \psi_\nu, \quad \bar{g}_{\mu\nu} = g_{\mu\nu} + g \psi_\mu \psi_\nu,$$

where $b$ and $g$ are constants, so that the variations of $\hat{g}_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ are related to those of $g_{\mu\nu}$ and $\psi_\mu$.

The field equations are given by

$$\nabla_\mu B^{\mu\nu} + V' \psi^\nu + g T^{\mu\nu} \psi_\mu + \kappa \bar{\mu} (g - b) \bar{T}^{\mu\nu} \psi_\mu = 0,$$
\[ \hat{\mu} \bar{G}^{\mu\nu} = \frac{1}{2} \mu T^{\mu\nu} + \frac{1}{2} \kappa \hat{T}^{\mu\nu}. \]  

(9)

It is clear that \( \hat{g}_{\mu\nu} \) and \( \bar{g}_{\mu\nu} \) provide the characteristic surfaces for matter and gravitational fields, respectively.

We can prove that any matter model that conserves energy-momentum with respect to \( \hat{g}_{\mu\nu} \) is consistent with the gravitational structure that we have introduced \([7]\).

The “most physical” metric is clearly \( \hat{g}_{\mu\nu} \), since it describes the geometry on which matter propagates and interacts. Because all matter fields are coupled to the same metric \( \hat{g}_{\mu\nu} \) in exactly the same way, the weak equivalence principle is satisfied. Furthermore, because one can work in a local Lorentz frame of \( \hat{g}_{\mu\nu} \), in which non-gravitational physics takes on its special relativistic form, the Einstein equivalence principle is also satisfied. However, because \( \hat{g}_{\mu\nu} \) does not couple to matter in the same way as in general relativity unless \( \psi_\mu = 0 \), the strong equivalence principle will be violated.

The main motivation for considering these theories is that they should have something to say about the horizon problem in the early universe. If \( \psi_\mu \neq 0 \), then if we choose \( b > g \), matter fields will propagate outside the light cone of the gravitational field. As \( \psi_\mu \to 0 \) the matter light cone will ‘contract’ and matter and gravitational disturbances will eventually propagate at the same velocity. If one considers a frame in which gravitational waves propagate at a constant speed, then as the light cone of matter contracts, the universe will appear to material observers to expand acausally.

3 Cosmology

Implicit in the idea of a varying light speed is that the speed of light is changing with respect to some fixed frame of reference. If one introduces a fundamental frame for this, then it is perhaps sensible to introduce a function \( c(t, x) \) to describe this variability \([16, 18]\). The models that we are considering are based on the idea that the speed of light can be changing with respect to the speed of gravitational disturbances, and therefore any indication of the speed of light as a function of spacetime is frame-dependent. In particular, we will see that a frame in which the speed of light is constant and the speed of gravitational disturbances is changing is connected via a diffeomorphism to a frame where the speed of gravitational disturbances is constant, and the speed of light is changing. Quantities of interest such as the local light cone, horizons, etc. are derived directly from the relevant metric, thereby avoiding any guesswork as to which ‘speed of light’ to use—the gravitational or electromagnetic \([8, 9]\). The constant \( c \) is fixed in the present universe by making measurements of the electromagnetic field.

In a homogeneous and isotropic (FRW) universe, the vector field \( \psi_\mu \) has components \( \psi_\mu = (c\psi_0(\tau), 0, 0, 0) \). We will begin with the metric \( g_{\mu\nu} \) in comoving form

\[ g_{\mu\nu} dx^\mu \otimes dx^\nu = c^2 d\tau \otimes d\tau - R^2(\tau) \gamma_{ij} dx^i \otimes dx^j, \]  

(10)
and therefore
\begin{align*}
\hat{g}_{\mu\nu} dx^\mu \otimes dx^\nu &= \hat{\Theta}^2(\tau) c^2 d\tau \otimes d\tau - R^2(\tau) \gamma_{ij} dx^i \otimes dx^j, \\
\bar{g}_{\mu\nu} dx^\mu \otimes dx^\nu &= \bar{\Theta}^2(\tau) c^2 d\tau \otimes d\tau - R^2(\tau) \gamma_{ij} dx^i \otimes dx^j.
\end{align*}
(11)
The spatial metric in spherical coordinates has the standard form
\[ \gamma_{ij} = \text{diag}(1/(1-kr^2), r^2, r^2 \sin^2 \theta), \]
(12)
and we have defined
\[ \hat{\Theta} = \sqrt{1 + 2bX}, \quad \bar{\Theta} = \sqrt{1 + 2\bar{g}X}, \]
(13)
where from (5) we have \( X = \frac{1}{2} \psi_0^2 \).

Although we begin with the choice (10), once we have derived the field equations, we will make a coordinate transformation in order to put \( \hat{g}_{\mu\nu} \) in comoving form and thereby make a comparison with the standard cosmological results a simpler matter. Note that we are reversing the definitions of \( t \) and \( \tau \) as used in our previous article [6].

The matter energy-momentum tensor will have a perfect fluid form
\[ \hat{T}^{\mu\nu} = \left( \rho + \frac{p}{c^2} \right) \hat{u}^\mu \hat{u}^\nu - p \hat{g}^{\mu\nu}, \]
(14)
where we have written the velocity field as \( \hat{u}^\mu \) to emphasize that it is normalized using the metric \( \hat{g}_{\mu\nu} \), so that
\[ \hat{g}_{\mu\nu} \hat{u}^\mu \hat{u}^\nu = c^2. \]
(15)

The matter conservation laws (2) lead to the usual relation
\[ \partial_\tau \rho + 3 \frac{\partial_\tau R}{R} \left( \rho + \frac{p}{c^2} \right) = 0. \]
(16)
The Friedmann equations take the form
\begin{align*}
\left( \frac{\partial_\tau R}{R} \right)^2 + \frac{kc^2 \hat{\Theta}^2}{R^2} &= \frac{k\epsilon}{6} \hat{\Theta}^3 \left[ \rho \hat{\Theta} + \frac{1}{k\epsilon^2}(2XV' - V) \right], \\
2\frac{\partial^2 R}{R} + \left( \frac{\partial_\tau R}{R} \right)^2 + \frac{kc^2 \hat{\Theta}^2}{R^2} - 2 \frac{\partial_\tau R}{R} \frac{\partial_\tau \hat{\Theta}}{\hat{\Theta}} &= -\frac{k\epsilon}{2} \hat{\Theta} \left[ \hat{\rho} + \frac{1}{\kappa} \hat{V} \right].
\end{align*}
(17)
(18)
The single remaining Proca field equation from (8) is
\[ \frac{1}{c^{2}} \psi_0 \left[ \hat{\Theta} (\hat{\Theta}^2 V' - gV) - \kappa (b - g)c^2 \rho \right] = 0. \]
(19)

We now perform the coordinate transformation
\[ dt = \hat{\Theta} d\tau, \]
(20)
and defining
\[
\eta = \frac{\Theta}{\bar{\Theta}} = \sqrt{\frac{1 + 2gX}{1 + 2bX}},
\] (21)
we see that the metric \( \hat{g}_{\mu\nu} \) is put into comoving form
\[
\hat{g}_{\mu\nu} dx^\mu \otimes dx^\nu = c^2 dt \otimes dt - R^2(t)\gamma_{ij} dx^i \otimes dx^j,
\] (22)
\[
\bar{g}_{\mu\nu} dx^\mu \otimes dx^\nu = \eta^2(t)c^2 dt \otimes dt - R^2(t)\gamma_{ij} dx^i \otimes dx^j.
\] (22)
We have
\[
\left( \frac{\dot{R}}{R} \right)^2 + \frac{k c^2 \eta^2}{R^2} = \eta^2 \frac{\kappa c^4}{6} \rho_{\text{eff}},
\] (23)
\[
2 \frac{\ddot{R}}{R} + \left( \frac{\dot{R}}{R} \right)^2 + \frac{k c^2 \eta^2}{R^2} - 2 \frac{\dot{R} \dot{\eta}}{R \eta} = -\eta^2 \frac{\kappa c^2}{2} p_{\text{eff}},
\] (24)
where \( \dot{\varphi} = \partial_t \rho \). In (23), we have defined the effective energy and pressure densities as
\[
\rho_{\text{eff}} = \eta \left( \rho + \frac{1}{\kappa c^2} \hat{\Theta}(2XV' - V) \right), \quad p_{\text{eff}} = \frac{1}{\eta} \left( p + \frac{1}{\kappa \hat{\Theta}} V \right).
\] (25)
The reason for making these definitions is that (23) has exactly the form of the Friedmann equations for the metric \( \bar{g}_{\mu\nu} \), and therefore these effective energy and momentum densities will also satisfy the conservation laws (16).

The function \( R(t) \) is written in comoving coordinates and, therefore, the speed of light is constant. This emphasizes that having a ‘varying speed of light’ is a frame-dependent statement. In a frame where the speed of matter propagation (including electromagnetic fields) is constant, the speed of gravitational waves will be changing. In a frame where the speed of gravitational waves is constant, the speed of matter propagation will be changing. This, of course, is as it should be, since we have not introduced any nondynamical preferred frame into our model.

In the following we will specialize to a model where the vector field potential is a simple mass term:
\[
V = m^2 X, \quad V' = m^2.
\] (26)
In this case (25) becomes
\[
\rho_{\text{eff}} = \eta \left( \rho + (b - g) \rho_{\text{pt}} \hat{\Theta} X \right),
\]
\[
p_{\text{eff}} = \frac{1}{\eta} \left( p + c^2 (b - g) \rho_{\text{pt}} \frac{X}{\Theta} \right).
\] (27)
The nontrivial solution \( (\psi_0 \neq 0) \) of the field equation (19) leads to
\[
\rho = \rho_{\text{pt}} \hat{\Theta}(1 + gX),
\] (28)
where
\[
\rho_{\text{pt}} = \frac{m^2}{\kappa c^2 (b - g)}, \quad H_{\text{pt}} = \sqrt{\frac{c^2 m^2}{6(b - g)}},
\] (29)
are the density at which $\psi_0^2 = 0$ is reached, and the inverse Hubble time at which this occurs (assuming that $k = 0$).

We can now write the acceleration parameter as observed by material observers from (23) as

$$\dot{q} = -\frac{\ddot{R}}{H^2 R} = \frac{\kappa c^4}{12} \frac{\eta^2}{H^2} (\rho_{\text{eff}} + \frac{3}{c^2} \rho_{\text{eff}}) - \frac{\dot{\eta}}{H \eta},$$

(30)

where we have defined the Hubble function $H = \dot{R}/R$. We have

$$\frac{\dot{\eta}}{H \eta} = 3 \frac{(b-g)}{\rho_{\text{pt}} \Theta \Theta (g + b + 3bgX)} \left( \rho + \frac{p}{c^2} \right).$$

(31)

4 The Very Early Universe

For very short times following the initial singularity, we expect that $\psi_0$ is large, and if we assume that $gX \gg 1$ and $bX \gg 1$, then from (28) we find that

$$\rho = \rho_{\text{pt}} \sqrt{2bgX^{3/2}}.$$  

(32)

This results in the Friedmann equation

$$\left( \frac{\dot{R}}{R} \right)^2 + \frac{kc^2}{R^2} = \frac{\bar{c} \bar{c}^4}{6} \rho,$$

(33)

where

$$\bar{c} = c \sqrt{\frac{g}{b}}, \quad \bar{G} = G \sqrt{\frac{g}{b}}, \quad \bar{\kappa} = \frac{16\pi \bar{G}}{c^4}.$$  

(34)

Although the behaviour of the solutions are well-known, it is worth pointing out that the ‘effective’ constants $\bar{c}$ and $\bar{G}$ are not interpretable as the effective speed of light and gravitational constant, rather they are effective constants that dictate how the gravitational field reacts to the presence of matter. Matter fields continue to propagate with speed $c$ consistent with (22). It is the gravitational field perturbations that propagate with speed $\bar{c}$, which is the justification for the notation.

During this phase there is clearly no inflation, but the horizon scales of the gravitational field and matter fields are related by

$$\tilde{d}_H(t) = \frac{\bar{c}}{c} \tilde{d}_H(t) = \sqrt{\frac{g}{b}} \tilde{d}_H(t), \quad \text{where} \quad \tilde{d}_H(t) = cR(t) \int_0^t \frac{ds}{R(s)},$$

(35)

with a similar definition for $\tilde{d}_H(t)$ using the metric $\bar{g}_{\mu\nu}$. Because we have $g < b$ we expect that not only is the speed of gravitational disturbances slower than that of matter, but also that the coupling between matter and the gravitational sector is also lessened.

What we have here is very close to what was originally envisaged by one of us in [15]. This is part of the motivation for including the $g \neq 0$ possibility, the other
is that the approach to the initial singularity in this phase follows the same path as in ordinary GR+matter models, with a re-interpretation of the parameters. In this case we have a model that interpolates between this initial period where $\bar{c} > c$ and the later universe where $\bar{c} = c$.

5 Inflation and Light Cone Contraction

As $\psi_0$ decreases towards the point where $gX \sim 1$ the solution will no longer be a good approximation. If we now consider the solution when $gX \ll 1$, from (28) we have

$$\hat{\Theta} = \frac{\rho}{\rho_{pt}}, \quad \text{or} \quad X = \frac{1}{2b} \left[ \left( \frac{\rho}{\rho_{pt}} \right)^2 - 1 \right],$$

and the Friedmann equation (23) becomes

$$\left( \frac{\dot{R}}{R} \right)^2 + \frac{k c^2 \eta^2}{R^2} \left( \frac{\rho_{pt}}{\rho} \right)^2 = \kappa c^4 \frac{12}{12} \rho_{pt} \left[ 1 + \left( \frac{\rho_{pt}}{\rho} \right)^2 \right].$$

In this limit

$$\rho_{eff} + \frac{3}{c^2} p_{eff} = \frac{1}{\rho_{pt}} \left[ \rho \left( \rho + 3 \frac{c^2}{c^2} p \right) + \rho^2 - \rho_{pt}^2 \right],$$

which is greater than zero if the strong energy condition is satisfied, since $\rho \geq \rho_{pt}$, and (30) reduces to

$$\hat{q} = \frac{\kappa c^4 \rho_{pt}}{12 H^2} \left[ \frac{1}{\rho} \left( \rho + 3 \frac{c^2}{c^2} p \right) + 1 - \left( \frac{\rho_{pt}}{\rho} \right)^2 \right] - \frac{3}{\rho} \left( \rho + \frac{p}{c^2} \right).$$

Since we expect that $H^2$ is large in the early universe (we can arrange that $\rho_{pt} \ll \rho_c$ where $\rho_c = 12 H^2 / (\kappa c^4)$), it is clear from (39) that even if matter satisfies the strong energy condition, the final term will dominate and $\hat{q} < 0$ (unless, perhaps, the weak energy condition is also violated). This is the expansion of the universe as seen by material observers. The acceleration of the gravitational geometry $\bar{q}$ would lack the final term and therefore $\bar{q} > 0$.

That we get inflation was demonstrated previously [6], where an exact solution for $k = 0$ and $g = 0$ was found. Although we discovered that we could not get enough expansion to solve the horizon problem with pure radiation, a slowly rolling scalar field could provide the necessary negative pressure. The role that the extra structure of our model plays is that the fine-tuning that is required in a simple scalar field, potential-driven model is alleviated.

The flatness problem requires a bit more explanation. Dividing (23) by $H^2$ and defining

$$\bar{\epsilon} = \frac{kc^2 \eta^2}{(R)^2},$$

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we find a differential equation that \( \bar{\epsilon} \) satisfies by taking a derivative and using (23) to give
\[
\dot{\bar{\epsilon}} = \frac{\kappa c^4 \eta^2}{6H} \bar{\epsilon} \left( \rho_{\text{eff}} + \frac{3}{c^2} p_{\text{eff}} \right).
\]
(41)

Therefore, since \( \bar{\epsilon} > 0 \) and \( H > 0 \) in the early universe, the only way for \( \bar{\epsilon} = 0 \) to be an attractor for (23) is for \( \rho_{\text{eff}} + \frac{3}{c^2} p_{\text{eff}} < 0 \) at least for part of the history of the universe. What is not so obvious is whether the quantity \( \bar{\epsilon} \) as defined in (40) is of physical relevance.

The quantity of geometrical importance is the 3-curvature of the spacelike slices, \( 6k/R^2 \), which suggests that the physically meaningful quantity to examine would be
\[
\dot{\hat{\epsilon}} = \frac{k c^2}{(R)^2},
\]
(42)
which has the equation of motion
\[
\dot{\hat{\epsilon}} = 2\hat{\epsilon} \hat{q}.
\]
(43)

Another way of stating this is that the curvature radius defines \( \Omega \) through
\[
R_{\text{curv}} = \frac{R}{|k|^{1/2}} = \frac{c}{H(|\Omega - 1|)^{1/2}},
\]
(44)

and so \( \dot{\hat{\epsilon}} = |\Omega - 1| \). Since we found from (39) that \( \hat{q} < 0 \) in the early universe, clearly \( \dot{\hat{\epsilon}} = 0 \) is an attractor for (23), and since it is most-likely the quantity of physical importance for matter physics, we can also claim to have solved the flatness problem once the horizon problem is solved.

### 6 Conclusions

In our bimetric model, the universe generically accelerates (\( \hat{q} < 0 \)) during some period in the early universe, and in the same period the physical importance of spatial curvature diminishes (\( |\Omega - 1| \) is decreasing). This can occur even when the matter fields satisfy the strong energy condition.

The model that we have considered generalizes that which appeared in [6, 7] in a way that more closely follows the scenario discussed in [15]. In the very early universe, matter and gravitational fields propagate with different and approximately constant velocities. During a period in which the matter light cone, originally much larger than the light cone of gravity, contracts, material observers will see an acausal expansion of the universe similar to inflation. Because the light cone of gravity does not undergo the same contraction, we expect there to be an observable difference in the scalar versus tensor contributions to the cosmic microwave background anisotropies.
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