Numerical study of stress-strain state of reinforced concrete slab in punching zone

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Abstract. The article presents the analysis of the stress state of a flat reinforced concrete slab under punching through a numerical study on finite element models. Three models with different cross-section shape of the column-square, rectangular and L-shaped—are considered. The calculation of the punching according to the standard method is carried out under the assumption of a uniform distribution of shear stresses on the control perimeter of the punching. The results of the study allow drawing a conclusion about a significant uneven distribution of shear stresses on the control perimeter, depending on the shape of the cross-section of the column. The conclusion corresponds to the results of previously performed experimental studies on test samples. According to the results of the study, the effective length of the control perimeter of the plate punching for the considered forms of the column cross-section is determined.

1. Introduction
The calculation for punching both according to the normative method [1] and in the proposals for its improvement [2-4] is performed under the assumption of a uniform distribution of shear stresses on the control perimeter of punching. This is true for the square or round shape of the column cross-section [5-7]. The results of experimental studies on test samples when the plate punching by a rectangular column [8-11] showed that there is a significant drop in shear stresses on the long side of the control perimeter.

The numerical study of the stress-strain state of the reinforced concrete slab was performed on finite element models in the environment of the PC "LIRA-CAD 2013". The study was carried out in order to determine the influence of the column cross-section shape on the stress state of the plate in the punching zone.

2. Materials and methods
The present study was performed on three finite element models in an elastic formulation. The models are constructed from volumetric finite elements FE41-44 and represent a fragment of the interface node of a flat slab and a column. The dimensions of the plate in the plan are 2.2 × 2.2 m, the height of the cross section of the plate is 140 mm. The plate was loaded with a uniformly distributed load. The value of the load on the plate was selected based on the unit value of tensile stresses on the punching control perimeter. The value of the control perimeter was determined in accordance with the requirements [1]. The models differed in the shape of the column cross section. Model No. 1 had a
square column with a cross section of 200×200 mm, model No. 2 had a rectangular column with a
cross section of 200×800 mm, model No. 3 had an L-shaped column with a short side size of 200 mm,
a long side of 500 mm. The center of gravity of the section of the L-shaped column is located in the
center of gravity of the plate. The developed finite element models are presented in figure 1.

![Fig 1](image1)

**Figure 1.** Finite element models: a – model No. 1; b – model No. 2; c – model No. 3.

The analysis of the plate stress state was performed on the control perimeter, located at a distance
of 0,5h0 from the face of the column, in accordance with the requirements [1]. The distribution of
shear stresses in the middle part of the slab section height, where their value reaches the maximum
value, was analyzed.

3. **Analysis of the stress state at the punching control perimeter**
The distribution of shear stresses on the calculated contour of punching for the column of square
section (model No. 1) is shown in figure 2. The distribution is symmetrical along the length of the
calculated contour, so it is drawn from the angle of the column to the symmetry axis of the section.

![Fig 2](image2)

**Figure 2.** Distribution of shear stresses on the control perimeter for model No. 1.

The distribution of shear stresses on the calculated contour of punching for a column of rectangular
cross-section (the ratio of the sides of the column 1 to 4) is shown in figure 3.

![Fig 3](image3)

**Figure 3.** Distribution of shear stresses on the control perimeter for model No. 2: a – short side of
control perimeter; b – long side of control perimeter.
The distribution is symmetrical along the short and long sides of the calculated contour, so it is drawn from the angle of the column to the symmetry axis of the section.

The distribution of shear stresses on the calculated contour of punching for the column of L-shaped section (the ratio of the sides of the column 1 to 2.5) is shown in figures 4, 5. The distribution is asymmetric along the short and long sides of the calculated contour, so it is built on the entire length of the corresponding side of the column section.

The analysis of the stress state on the control perimeter was performed using the coefficient of completeness of the shear stress diagram $\omega$, which is calculated for each side of the estimated contour punching. The obtained values of the effective length of the calculated contour of punching $u_{\text{calc.}}$ were compared with the value of the calculated contour $u_{\text{code}}$ obtained by the standard method [1]. The value $u_{\text{calc.}}$ was defined as the product of the length of the control perimeter by the coefficient $\omega$ for each face of the column. The results of the analysis are presented in table 1.

**Table 1. Results of stress state analysis on the control perimeter**

| Model No. | $\omega$ | $u_{\text{calc.}}$, mm | $u_{\text{code}}$, mm | $u_{\text{code}} / u_{\text{calc.}}$ |
|-----------|----------|------------------------|-----------------------|----------------------------------|
| Model No. 1 | $\omega = 0.94$ | 1152 | 1200 | 1.04- |
| Model No. 2 | $\omega_1 = 0.97; \omega_2 = 0.5$ | 1688 | 2400 | 1.42 |
| Model No. 3 | $\omega_3 = 0.86; \omega_2 = 0.47; \omega_3 = 0.58$ | 2114 | 2224 | 1.05 |

The results of the numerical study were compared with the experimental data obtained during the testing of samples [10]. The samples with a column of square and rectangular cross section were tested.
Failure of the samples was brittle due to punching plate by column. The values of failure loads reached 272 kN and 328.4 kN for CMR-1 and CMR-4 samples, respectively.

![Figure 6. Testing of sample CMR-4.](image)

For a sample with a square column (CMR-1), the experimental value of failure load has a good correspondence with the calculated value obtained by the standard method [1]. For a sample with a rectangular cross-section column (CMR-4), the calculated strength value obtained by the standard method [1] exceeds the experimental value by 47%. This value has a good match with the value $u_{\text{code}} / u_{\text{calc}} = 1.42$ for model No. 2, which is geometrically similar to sample CMR-4.

4. Conclusion

The results of numerical studies of the stress-strain state of reinforced concrete slabs show that the effective length of control perimeter $u_{\text{calc}}$ for models No. 1 and No. 3 satisfactorily coincides with the magnitude of the calculated contour $u_{\text{code}}$ obtained by the standard method [1].

For model No. 2, the value of the calculated contour $u_{\text{code}}$ exceeds the effective length of control perimeter $u_{\text{calc}}$. This indicates a significant unevenness of the shear stresses on the calculated contour of the plate when punching by a rectangular column, which is the reason for the decrease in the strength of the plate under punching [10] compared to the calculation by the standard method [1].

The results of the present and previous studies on samples with rectangular columns allow concluding that the uneven distribution of plate deformations along the perimeter of the column section, that is, the presence of a “zone of constrained deformations” of the plate in the immediate vicinity of the column. Taking into account this feature in the calculation of plates for punching by the method of code [1] allows improving the convergence of experimental and calculated values of strength.

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