Theoretical research of helium pulsating heat pipe under steady state conditions

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Abstract. As a new-type pipe, pulsating heat pipe (PHP) has several outstanding features, such as great heat transport ability, strong adjustability, small size and simple construction. PHP is a complex two-phase flow system associated with many physical subjects and parameters, which utilizes the pressure and temperature changes in volume expansion and contraction during phase changes to excite the pulsation motion of liquid plugs and vapor bubbles in the capillary tube between the evaporator and the condenser. At present time, some experimental investigation of helium PHP have been done. However, theoretical research of helium PHP is rare. In this paper, the physical and mathematical models of operating mechanism for helium PHP under steady state are established based on the conservation of mass, momentum, and energy. Several important parameters are correlated and solved, including the liquid filling ratio, flow velocity, heat power, temperature, etc. Based on the results, the operational driving force and flow resistances of helium PHP are analysed, and the flow and heat transfer is further studied.

1. Introduction
As a highly effective two-phase cooling technique, cryogenic heat pipes can transport several orders of magnitude larger heat loads than heat conduction in solids such as copper, and it could be used widely for cooling of superconducting magnet and electronic devices and for harvesting energy. Among the different types of heat pipes, pulsating heat pipes (PHP) invented by Akachi [1-4] in 1990 belong to a new-type of heat pipes which have several outstanding features, such as great heat transport ability, strong adjustability, small size and simple construction. For PHP at liquid helium temperature, some experimental investigation has been done. Gully and Bonnet [5-6] had developed a helium based cryogenic PHP at 4.2 K, and the maximum power of 145 mW was transferred by the PHP made of Cu/Ni pipe with 5 turns at 40°. The measured effective thermal conductivities have reached 11700 W/m·K at tilt angle 10° with 75 mW, and with the increasing of tilt angle, the maximum capability increased. Luis and John [7] had studied helium based cryogenic PHP made of stainless steel with 32 turns and found that the measured effective thermal conductivities reached 2200-2500 W/m·K. We had developed a mechanical-thermal switch working as a novel pre-cooling system for the helium PHP, and the measured effective thermal conductivity of helium PHP was 16760 W/m·K with a heating power of 49.2 mW[8].

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It is thus clear that the experimental investigation for helium PHP has been widely conducted. For PHP at room temperature, a number of theoretical models for PHP have been developed to study the oscillating motion and the heat transfer performance. However, the theoretical research for helium PHP is rare. The objective of this study is to conduct a theoretical analysis of a helium PHP under steady state, and study the flow and heat transfer performance.

2. Physical model

As shown in figure 1, the physical model which consists of a tube with a condenser, an evaporator and an adiabatic section filled with both the liquid and vapor phases is established. The model is developed on the following assumptions and simplifications:

(1) Without regard for the transient state in the condenser and evaporator, the liquid helium and helium gas in the tube could be considered under steady state condition, which means the speed of helium plugs is constant.

(2) The liquid helium and helium gas in the tube is under saturation state condition, and the gas could be taken as an ideal gas.

![Figure 1. Physical model of helium pulsating heat pipe.](image)

3. Mathematical model

Mathematical modelling of helium pulsating heat pipe involves many processes, including phase changes, capillary force, wall shear stress, gravity and so on, which is coupled with fundamental laws of the conservation of mass, momentum and energy.

3.1. Conservation of mass

As a result of steady state, the increment of liquid helium at the condenser is equal to the decrement of helium gas, and the decrement of liquid helium at the evaporator is equal to the increment of helium gas. There is no input power and phase change in the adiabatic section, which means the mass of liquid helium and helium gas is constant.

For the liquid helium at the evaporator and the condenser:

\[
\phi_l \rho_l AV + \Delta \dot{m}_{l,e} = \phi_l \rho_l AV
\]

(1)

\[
\phi_l \rho_l AV + \Delta \dot{m}_{l,c} = \phi_l \rho_l AV
\]

(2)

For the helium gas at the evaporator and the condenser:

\[
(1 - \phi_l) \rho_{v,e} AV + \Delta \dot{m}_{v,e} = (1 - \phi_l) \rho_{v,e} AV
\]

(3)
\[(1 - \phi_1)\rho_{v,e} AV + \Delta \dot{m}_{v,e} = (1 - \phi_0)\rho_{v,c} AV \quad (4)\]

Where: \(\phi_0\) and \(\phi_1\) are the liquid filling ratio of inlet and outlet of evaporator, \(\rho_1\) is the density of liquid helium, \(\rho_{v,e}\) and \(\rho_{v,c}\) are the density of helium gas of inlet and outlet of evaporator, \(\rho_{v,e}'\) and \(\rho_{v,c}'\) are the density of helium gas of inlet and outlet of condenser, \(\Delta \dot{m}_{v,e}\) and \(\Delta \dot{m}_{v,c}\) are the mass change of liquid helium of evaporator and condenser, \(\Delta \dot{m}_{v,e}'\) and \(\Delta \dot{m}_{v,c}'\) are the mass change of helium gas of evaporator and condenser, \(A\) is the area of the tube, \(V\) is the velocity of vapour bubbles and liquid plugs.

### 3.2. Conservation of Momentum

As a result of steady state, the speed of helium plugs is constant. Thus, the resultant of forces is zero. The pressure exerted on the plugs is given by:

\[
\Delta P_q = \Delta P_f + \Delta P_c + \Delta P_g
\]

\[
\Delta P_q = \rho_{v,e}' RT_h - \rho_{v,e} RT_c
\]

\[
\Delta P_f = \frac{32LV}{d^2} \left[ \mu_l \frac{\phi_0 + \phi_1}{2} + \mu_v (1 - \frac{\phi_0 + \phi_1}{2}) \right]
\]

\[
\Delta P_c = \pi d \sigma (\cos \theta_{rec} - \cos \theta_{adv})
\]

\[
\Delta P_g = \pm \rho g L \sin \alpha
\]

Where: \(\Delta P_q\), \(\Delta P_f\), \(\Delta P_c\) and \(\Delta P_g\) are the differential pressure between the two sides of the plug, the viscous force, the capillary pressures, and the gravitational force, \(R\) is the gas constant, \(T_h\) and \(T_C\) are the temperature of the condenser and the evaporator, \(L\) and \(d\) are the equivalent length and inside diameter of the PHP, \(\mu_l\) and \(\mu_v\) are the viscosity of liquid and vapour, \(\sigma\) is the surface tension, \(\theta_{rec}\) and \(\theta_{adv}\) are the receding and advancing contact angles, and \(\alpha\) is the tilt angle.

### 3.3. Conservation of Energy

The energy results in the changes of mass of both the vapour bubble and the liquid film. Thus, the change of mass flow is given by:

\[
\Delta \dot{m}_{v,e} = -\Delta \dot{m}_{l,e} = \frac{\beta Q}{r}
\]

\[
\Delta \dot{m}_{l,e} = -\Delta \dot{m}_{v,e} = \frac{Q_e}{r}
\]

\[
\phi_i = \phi_0 - \frac{\beta Q}{\rho AV r}
\]
Where: $\beta$ is the proportion of the latent heat transfer to the total heat transfer in the evaporator, $Q$ is the total heat transfer, $Q_c$ is the condensation heat transfer, and $r$ is the latent heat.

The condensation heat transfer in the tube could be given by:

$$\delta = 1.16\left[ \frac{d \mu \lambda_l (T_h - T_c)}{g \sin \alpha r \rho}$ \right]^{0.25}$$

$$Q_c = \pi (d - 2\delta) \lambda_l \left( \frac{T_h + T_c}{2} - T_c \right) \frac{L_c [1 - (\phi_h + \phi_a) / 2]}{\delta}$$

Where: $\delta$ is the liquid film thickness, $\lambda_l$ is the conductivity of the liquid helium, and $L_c$ is the length of the condenser.

Heat transferred from the bubble at temperature $T_h$ to the liquid at temperature $T_c$ is in the form of sensible and latent heat transfer. Thus, the sensible heat could be given by:

$$(1 - \beta)Q = \frac{\phi_h + \phi_a}{2} \rho \gamma V c_p (T_h - T_c)$$

Where: $c_p$ is the specific heat of liquid helium.

We have the initial values: $d=5\times10^{-4}$ m, $T_c=4.2$ K, $L=1$ m (10 turns), and $L_c=0.5$ m (10 turns). By trying to incorporate the constitutive relationships, the equations above from (1) to (15) have merged into three equations with three variable ($Q_c$, $\beta$, and $V$) and could be solved by using the method of least square.

4. Results and discussion

For helium pulsating heat pipe, the proportion of the latent heat transfer to the total heat transfer in the evaporator $\beta$ represents the heat transfer characteristics and the velocity of plugs represents the flow characteristics. Figure 2 shows the proportion $\beta$ as a function of temperature with a parameter of liquid filling ratio. As shown in figure 2, the proportion of the latent heat transfer $\beta$ is low (1%~20%), which means most of the heat transfers through the helium PHP is in the form of sensible heat.

Moreover, with the increase of the temperature of the evaporator, the proportion decreases, and the sensible heat is playing an ever-growing role in the heat transfer process. However, the sensible heat transfer depends on the difference in temperature. That is the reason why the temperature of the evaporator rapidly rises along with the increase of the heat power in our previous experiment. With the increase of liquid filling ratio, the area of phase transformation decreases. Therefore, the phase change heat transfer decreases, and the proportion of the latent heat transfer to the total heat transfer in the evaporator $\beta$ decreases.
Figure 2. $\beta$ as a function of temperature with a parameter of liquid filling ratio.

Figure 3 shows the velocity as a function of heat power with a parameter of liquid filling ratio. As shown in figure 3, the velocity increases when increasing the heat power. That is because the increased heat power results in the increase of temperature difference and then elevates the driving force. As is known, the smaller the liquid filling ratio is, the less the flow resistance is. Therefore, the velocity will increase with the decrease of the liquid filling ratio.

Figure 3. Velocity as a function of heat power with a parameter of liquid filling ratio.

5. Conclusion

The physical and mathematical model was established which consists of a tube with a condenser, an evaporator and an adiabatic section filled with both the liquid helium and vapour. Then, equations are listed based on fundamental laws of the conservation of mass, momentum and energy. Three equations with three variables are obtained and solved by using the method of least square. The results show that:
(1) For helium, the proportion of the latent heat transfer to the total heat transfer in the evaporator $\beta$ is low (1%~20%), which means most of the heat transfer through the helium pulsating heat pipe is in the form of sensible heat transfer. The proportion decreases with the increase of the temperature and liquid filling ratio.

(2) The velocity increases with the increase of the heat power, and the decrease of the liquid filling ratio. That is because the increased heat power results in the increase of temperature difference and then elevates the driving force.

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