Numerical simulation of Vortex-induced Vibration of A Top-tensioned Riser

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Abstract. The objective purpose of the paper is explore the prediction the vortex-induced vibration (VIV) of a top-tensioned riser. The riser model is established based on vector form intrinsic finite element (VFIFE) method. The riser is discretized into a set of mass points whose motion follow the Newton’s second law and the central difference explicit integration method is used to solve the control equation of the mass point at each time. The fluid hydrodynamic forces are simulated by a Landl-modified Rayleigh wake oscillator. The calculation result is compared with the experiment result to validate the accuracy of the model. Time history of the axial displacement indicates that the model is a real three-dimensional model.

1. Introduction
With the development of offshore oil exploration, the safety of the riser is essential to the development of the oil industry. The vortex-induced vibration (VIV) is the main factor of riser fatigue damage, which seriously threatens the safety of the riser. Therefore, accuracy of prediction of VIV has always been an important issue in the field of marine engineering.

Over the past few decades, analysis of VIV prediction have extensively progressed. Hassan (1964) firstly proposed to use wake oscillator to simulate the force of VIV. Facchinetti et al. (2004) analyzed three ways of coupling of the structure and the fluid, which are displacement, velocity and acceleration. The results showed that acceleration coupling is the most suitable for the prediction of VIV. Bai and Qin (20014) predicted the in-line and cross-flow VIV with a new oscillator. Kurushina et al. (2018) investigated different types of nonlinear damping in the liquid equation and got a suite of wake oscillators which are suitable for mass ratio of 2-4.

In this paper, the model is established by vector form intrinsic finite element (VFIFE) method and the wake oscillator calibrated by Kurushina et al. (2018) will be used to predict the fully-coupled in-line, cross-flow and axial direction VIV.

2. Model of the riser

2.1. Governing equation
Assuming that the length of riser is L. The riser is divided into N segments by N+1 points to describe the riser model, and the points are connected by beam elements without mass. All the mass, internal force, and external forces are superimposed on the points.

The motion equations of mass point ̅ are:
2.2. Internal force and external force

Since the force of the three-dimensional structure is complicated. The internal forces include axial force, torque, two sets of bending moments and shear force. Compared with Euler-Bernoulli beam theory, in VFIFE method, there are two coordinate systems: global coordinate system and principal coordinate system.

The pure deformation and angles are calculated in the principal coordinate. The internal force are calculated according to the Hulk's law, shear Hooke's Law, deflection theory of asymmetric sections. The external force are expressed the following:

\[ F_D = \frac{\rho_f C_D D}{2} \left[ \hat{U}_R \right]^2 \left[ \hat{U}_R \right] \left( C_{D0} + \frac{\omega \rho C_D^2}{2} \right) \] (3)

\[ F_L = \frac{\rho_f D}{2} C_L \left| \hat{U}_R \right|^2 \left( \frac{\pi}{2} , \hat{K} \right) \left[ \hat{U}_R \right] = \frac{\rho_f D q C_{L0}}{2} \left[ \hat{U}_R \right]^2 \left( \frac{\pi}{2} , \hat{K} \right) \left[ \hat{U}_R \right] \] (4)

The equations of Landl-modified Rayleigh wake oscillator are expressed as following:

\[ \ddot{\omega} + 2 \Omega_R \dot{\omega} (\varepsilon_{x1} - \varepsilon_{y2} \omega^2 + \varepsilon_{x3} \omega^4) + 4 \Omega_R^2 \omega = \frac{\dot{X}}{D} \] (5)

\[ \ddot{q} - \varepsilon_{y2} \Omega_R \dot{q} + \frac{\varepsilon_{y2}^2}{\Omega_R} q^3 + \Omega_R^2 q = \frac{\dot{Y}}{D} \] (6)

The value of the coefficients set as \( \varepsilon_{x1} = 0.7055, \varepsilon_{x2} = 1.0783, \varepsilon_{x3} = 0.7287, \varepsilon_{y1} = 0.1692, \varepsilon_{y2} = 0.0519, A_x = 12.02, A_y = 5.92, C_{L0} = 0.82, C_{D0} = 1.47, C_{D0}^f = 0.01. \)

2.3. Numerical solution

The central difference method as an explicit time integration method is used in this paper. To synchronize with finite element calculation, the wake oscillator equation as a differential equation is transformed to the form of algebraic equation. The solution the equation can be obtained by Matlab symbolic computation.

The central difference method are following as:

\[ X_i^{n+1} = C_1 \frac{h^2}{m_j} F_i^n + 2 C_1 X_i^n - C_2 X_i^{n-1} \] (7)
3. Numerical simulation

3.1. validation of the model

The experiment carried out by Chaplin et al. (2005) is set as a benchmark with a current velocity 0.16m/s and top tension 405N. The length of the riser is 13.12m, the external and internal diameter are 0.028m and 0.023m.

\[
\theta^{n+1} = C_1 \frac{h^2}{I_n} M^n + 2C_2 \theta^n - C_2 \theta^{n-1}
\] (8)

Fig.1 shows the non-dimensional response of the riser with the VFIFE method. The maximum in-line displacement is 3.8 which coincides with the result of the experiment.

The in-line and cross-flow dominant mode is 7 and 4. Although the RMS in-line displacement is much lower than that of cross-flow, considering that the frequency of in-line is about twice that of the cross-flow, the vibration of the in-line can not be neglected in the calculating of fatigue failure.

Fig 1. Non-dimensional response of the riser with current velocity 0.16m/s and top tension 457N

Fig 2. Time-history of top tension and axial direction at Z=6.56m
3.2. Time-varying top tension
The VFIFE model is a real three-dimensional model with varying axial displacement and varying axial tension. Fig.2(a) shows the time-history of the top tension. It can be observed that the top tension is much larger than the initial value of the top tension which is 457N. The changing of the top tension can be explained by Fig.2(b) by Hooke’s Law. Fig.3 shows that the existence of standing wave and the dominant mode is 4.

4. Conclusion
In this paper, a fully-coupled in-line, cross-flow and axial direction VIV model is established based on VFIFE method. The hydrodynamic forces are simulated by a Landl-modified Rayleigh wake oscillator. The numerical simulation results have good consistency with the experimental results. The top tension is much larger than the initial value with considering of the axial displacement of the riser. In a word, the model is validated to be reliable in the prediction of the VIV response of the riser.

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