First principles should predetermine physical geometry and dynamics both together. In the “algebrodynamics” they follow solely from the properties of biquaternion algebra $\mathbb{B}$ and the analysis over $\mathbb{B}$. We briefly present the algebrodynamics over the Minkowski background based on a nonlinear generalization to $\mathbb{B}$ of the Cauchi-Riemann analyticity conditions. Further, we consider the effective real geometry uniquely resulting from the structure of multiplication in $\mathbb{B}$ and found it to be of the Minkowski type, with an additional phase invariant. Then we pass to study the primordial dynamics that takes place in the complex $\mathbb{B}$ space and brings into consideration a number of remarkable structures: an ensemble of identical correlated matter pre-elements (“duplicons”), caustic-like signals (interaction carriers), a concept of random complex time resulting in irreversibility of physical time at a macrolevel, etc. In particular, the concept of “dimerous electron” naturally arises in the framework of complex algebrodynamics and, together with the above-mentioned phase invariant, allows for a novel approach to explanation of quantum interference phenomena alternative to the recently accepted paradigm of wave-particle duality.

### 1. Status of Minkowski geometry and the algebrodynamical paradigm

A whole century after German Minkowski introduced his famous conception of the 4D space-time continuum, we come to realize the restricted nature of this conception and the necessity of its revision, supplement and derivation from some general and fundamental principle.

Indeed, formalism of the 4D space-time geometry was indispensable to ultimately formulate the Special Theory of Relativity (STR), to ascertain basic symmetries of fundamental physical equations and related conservation laws. It was also the Minkowski geometry that served as a base for formulation of the concept of curved space-time in the framework of the Einstein’s General Theory of Relativity (GTR).

Subsequently, Minkowski geometry and its pseudo-Riemannian analog have been generalized via introduction of effective geometries related to correspondent field dynamics (in the formalism of fiber bundles), or via exchange of Riemannian manifold for spaces with torsion, non-metricity or additional “hidden” dimensions (in the Kaluza-Klein formalism). There have been considered also the models of discrete space-time, and a challenging scheme of the causal sets [1] among them.

However, none of modified space-time geometries has become generally accepted and able to replace the Minkowski geometry. Indeed, especial significance and reliability of the latter is stipulated by its origination from trustworthy physical principles of STR and, particularly, from the structure of experimentally verified Maxwell equations. None of its subsequent modifications can boast of such a firm and uniquely interpreted experimental ground.

From the epoch of Minkowski we did not get better comprehension of the true geometry of our World, its hidden structure and origin. In fact, we are not even aware whether physical geometry is Riemannian or flat, has four dimensions or more, etc. Essentially, we can say nothing definite about the topology of space (both global and at a microscale). And, of course, we still have no satisfactory answer to the sacramental question: “Why is the space three dimensional (at least, at a macrolevel)?” Finally, an “eternal” question about the sense and origin of physical time stands as before on the agenda.

Meanwhile, the Minkowski geometry suffers itself from grave shortcomings both from phenomenological and generic viewpoints. To be concrete, complex structure of field equations accepted in quantum theory results, generally, in a string-like structure of field singularities (perhaps, this was first noticed by Dirac [2]) and, moreover, these strings are unstable and, as a rule, radiate themselves to infinity (see, e.g., [3] and the example in Section 2).

Another drawback (exactly, insufficiency) of the Minkowski geometry is the absence of fundamental distinction of temporal and spatial coordinates within its framework. Time enters the Minkowski metrical form on an equal footing with the ordinary coordinates though with opposite sign. In other words, in the framework of the STR geometry, time does not reveal itself as an evolution parameter which it had been, in fact, even in the antecedent Newton’s picture of the World. At a pragmatic level this results, in particular, in the difficulty to coordinate “times” of various interacting (entangled) particles in an ensemble, in impossibility to introduce universal global time and to adjust the latter to proper times of different observers, or in the absence of clear comprehension of the passage of local time and dependence of its rate on matter. All these problems are widely discussed in physical literature (see, e.g., [4]) but

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are still far from resolution.

However, the main discontent with generally accepted Minkowski geometry is related to the fact that this geometry does not follow from some deep logical premises or exceptional numerical structures. This is still more valid with respect to generalizations of spacetime structure arising, in particular, in the superstring theories (11D-spaces) and in other approaches for purely phenomenological, “technical” reasons which in no way can replace the transparent and general physical principles of STR: the principle of relativity and of the universal velocity of interaction propagation.

At present, physics and mathematics are mature enough for construction of multidimensional geometries with different number of spatial and temporal dimensions. Moreover, they aim to create a general unified conception from which it would follow definite conclusions on the true geometry of physical space and on the properties and meaning of physical time, on the dynamics of Time itself!

In most of approaches of such kind the Minkowski space does not reproduce itself in its canonical form but is either deformed through some parameter (say, fundamental length and mass in the paradigm of Kadychevsky [5]), under keeping correspondence with canonical scheme, or changes its structure in a more radical way. The latter takes place, in particular, in the theory of Euclidean time developed by Pestov [6] (in this connection, see also [7]), in the concept of Clifford space-time of Hestenes-Pavsic (see, e.g., [8, 9]), in the framework of a special 6D geometry proposed by Urusovskii [10], etc.

At a still more fundamental level of consideration, one assumes to derive the geometry of physical spacetime from some primordial principle encoding it (perhaps, together with physical dynamics). One can try to relate such an elementary Code of Nature with some exceptional symmetry (theory of physical structures of Kulakov [11] and binary geometrophysics of Vladimirov [12]), special group or algebra (quaternionic theory of relativity of Yefremov [13] and algebrodynamics of Kassandrov [14, 15]), with an algebraically distinguished geometry (Finslerian anisotropic geometry of Bogoslovsky [16] and geometry of polynumbers of Pavlov [17]) as well as with some special “World function” (metrical geometry of Rylov [18]).

Generally, all the above-mentioned and similar approaches affecting the very foundations of physics differ essentially one from another in the character of the first principle (being either purely physical or abstract in nature), in the degree of confidence of their authors to recently predominant paradigms (Lorentz invariance, Standard model, etc.) and in their attitude towards the necessity to reproduce, in the framework of the original approach, the principal notions and mathematical instrumentation of the canonical schemes (Lagrangian formalism, quantization procedure, Minkowski space itself, etc.). In this respect the neo-Pythagorean philosophical paradigm professing by the author [19, 20, 21] seems most consistent and promising though difficult in realization.

Accordingly, under construction of an algebraic (logical, numerical) “Theory of Everything” one should forget all of the known physical theories and even experimental facts and to unprejudicedly read out the laws of physical World in the internal properties of some exceptional abstract primordial structure, adding and changing nothing in the course of this for “better correspondence with experiment”. In this connection, one should be ready that physical picture of the World arising at the output could have little in common with recently accepted one and that the real language of Nature might be quite different from that we have thought out for a more effective description of the observable phenomena. In this situation none principle of correspondence with former theories could be applied.

We have no opportunity to go into details of the neo-Pythagorean philosophy, quite novel and radical for modern science, sending the reader to Refs. [19, 20, 21]. Instead, in Section 2 we briefly present its realization in the framework of the “old” version of algebrodynamics developed during the period 1980-2005 [13, 15]. Therein an attempt had been undertaken to obtain the principal equations of physical fields and the properties of particle-like formations as the only consequence of the properties of the exceptional quaternions-like algebras, exactly, of the algebra of biquaternions B.

We have forcible arguments to regard this attempt successful. From the sole conditions of B analitivity (generalization of the Cauchy-Riemann equations, see Section 2) we were able to obtain an unexpectedly rich and rather realistic field theory. In particular, as a principal element of the arising picture of the World there turned to be a flow of light-like rays densely filling the space and giving rise to a sort of particle-like formations at caustics and focal points. Such a primordial, matter generating structure has been called the Flow of Prelight. From mathematical point of view, this flow is defined by the twistor structure of the first equations for the biquaternionic field, whereas geometrically it represents itself a congruence of null rays of a special type (namely, shear-free), below – the generating congruence.

Meanwhile, the “geometrical scene” on which the algebraic dynamics displays itself has been, in fact “by hands”, restricted to a subspace with canonical Minkowski metric for essence of the Lorentz invariance of the scheme. Such a procedure was in an evident contradiction with the whole philosophy of algebrodynamics, since corresponding subspace does not even form a subalgebra of B and is thus in no way distinguished in the structure of the B algebra. From a more general viewpoint, neither in our old works nor in those of other authors there has been found any space-time algebra, that is, ascertained an algebraic (“numerical”)
structure which could naturally induce the Minkowski geometry (or include the latter as its part) 2

However, in 2005 in [22] we have demonstrated that, under a thorough consideration, the primordial complex geometry of the B algebra unavoidably induces a real geometry just of the Minkowski type. In this scheme, the additional coordinates of (8D in reals) vector space of \( \mathbb{B} \) are naturally compactified and behave like a geometrical phase suggesting thus a geometrical explanation of the wave properties of matter in general. In the following, this geometry has been called the phase extension of the Minkowski space. Its derivation and simplest properties are presented in Section 3.

Discovery of a novel geometry induced by the primordial algebraic structure of complex quaternions 3 opens a wide perspective for the construction of a completed version of the algebrodynamics [23][24]. In particular, it turns out that just (and only!) in the primordial complex space there can be realized one of the most interesting and original ideas of Wheeler-Feynman about “all identical electrons as one and the same electron”, in its distinct positions on a unique world line. In [23] a set of (dynamically correlated) copies of a sole “generating charge” has been called the ensemble of “duplicons”. We consider the geometrodynamical properties of duplicons and the related particle-like formations in Section 4.

In Section 5 a naturally arising concept of complex time is presented. Indeed, already in the previous version of the algebrodynamics (in the real Minkowski background) the temporal coordinate is distinguished in a natural way as an evolution parameter of the primordial biquaternionic (and of associated twistor) field: the generating Prelight Flow is then identified with the Flow of Time [19][21]. Now, in the complex pre-space such a parameter unavoidably turns to be two-dimensional, and the related order of the sequence of events – indefinite. Thus in the framework of initially deterministic “classical” theory there arises inevitable uncertainty of evolution of states related to effectively stochastic alteration of the evolution parameter itself on the complex plane; we are led, therefore, to accept the concept of complex random time. On the other hand, existence of the geometrical phase makes it possible to suggest a novel treatment of the phenomena of quantum interference, alternative to the generally accepted concept of the wave-particle dualism. In particular, such a treatment relates the notion of the phase of wave function to the classical action of a particle, quite in the spirit of Feynman’s version of quantum theory. Considerations of these issues conclude the paper.

2. Algebrodynamics over the Minkowski space-time

Biquaternionic (\( \mathbb{B} \)) algebrodynamics is completely based on the (proposed by the author in 1980) version of noncommutative (including biquaternionic) analysis, that is, on the generalization of the theory of functions of complex variable to the case of noncommutative algebras of quaternionic type. This version is exposed in detail in the monograph [14] (where one can find references to the preceding works) and in the recent review [14].

Essentially, the whole structure of the theory of functions of \( \mathbb{B} \) variable \( Z \in \mathbb{B} \) follows from the invariant definition of a differential \( dF \) of such, a differentiable in \( \mathbb{B} \), function \( F : Z \mapsto Z \) (a direct analog of an analytical function in the complex analysis). Specifically, in account of the associativity yet noncommutativity of the algebra, one has

\[
dF = \Phi \ast dZ \ast \Psi, \tag{1}
\]

where \( \Phi(Z), \Psi(Z) \) are some two auxiliary functions formerly called (left and right) semi-derivatives of \( F(Z) \).

Relation (1) explicitly generalizes the well-known Cauchy-Riemann conditions. Indeed, in the case of the commutative algebra of complex numbers it acquires a familiar form

\[
dF = F' \ast dZ, \tag{2}
\]

with \( F' := \Phi \ast \Psi \) being the ordinary derivative of an analytical function \( F(Z) \) of complex variable \( Z \). Writing (2) down in components, one comes to the standard Cauchy-Riemann set of equations. Thus, requirement of invariance of the differential (2) is a basic relation for constructing complex analysis, one of a number of equivalent well-known ones but suitable, moreover, for its generalization to the noncommutative case in the form (1). Note that such a version was, perhaps, first proposed by Sheffers [25] for construction of the analysis over an arbitrary commutative-associative algebra, and nearly after a century, was used by Vladimirov and Volovich [26] for generalization to superalgebras.

Remarkably, in the case of real Hamilton quaternions \( \mathbb{Q} \) the proposed conditions (1) reproduce another exceptional property of complex analysis, namely, the conformity of the correspondent mapping implemented by any analytical function [27][28]. However, since the conformal group of the Euclidean space \( \mathbb{E}^k \) under \( k \geq 3 \) is finite (exactly, 15-parametrical for \( k = 4 \)), quaternionic analysis built on the base of relation (1), turns to be unattractive with respect to the physical applications.

When, however, one passes to the case of \( \mathbb{B} \) algebra (i.e., through the complexification of \( \mathbb{Q} \), the class of differentiable (in the sense of (1)) functions essentially expands due to the existence of special elements of \( \mathbb{B} \) – null divisors (see details in [14][15]; corresponding mappings have been called degenerate conformal ones). In
this way we naturally arrive at the formulation of the first “interpretational” principle of the algebrodynamics:

*In the paradigm of \( \mathbb{B} \) algebrodynamics, there exists a unique fundamental physical field. This is a (essentially complicated and even multivalued) function of the \( \mathbb{B} \) variable obeying the conditions of \( \mathbb{B} \) differentiability \( (1) \) – the only primordial “field equations”. All of the other “fields” arising in the scheme are secondary and can be defined through (semi)derivatives, contractions, etc. of the input \( \mathbb{B} \) field. Their equations also follow from the “master equations” \( (1) \).*

Realization of this programme requires, meanwhile, the resolution of the problem of relationship between the 4D complex coordinate space \( Z \), vector space of \( \mathbb{B} \)-algebra, and the Minkowski physical space-time. As it was already mentioned, the correct correspondence between these spaces has been ascertained only not long ago in our works and leads to a principally novel view on the geometry of space-time (see below). As to this section, we shall expose here only the former version of the algebrodynamics in which the coordinate space \( Z \) is forcibly restricted onto the subspace with Minkowski metric, in order to guarantee the Lorentz invariance of the scheme and to avoid the problem of prescription of a particular meaning to the additional “imaginary” coordinates of the vector space of \( \mathbb{B} \).

Specifically, it is well known that the biquaternion algebra \( \mathbb{B} \) is isomorphic to the full \( 2 \times 2 \) matrix algebra over \( \mathbb{C} \). Further on we shall use the following two equivalent matrix representations of an element \( Z \in \mathbb{B} \):

\[
Z = \begin{pmatrix} w & v \\ p & v \end{pmatrix} = \begin{pmatrix} z_0 + z_3 & z_1 - iz_2 \\ z_1 + iz_2 & z_0 - z_3 \end{pmatrix}
\]  

(3)

through the four complex “null” \( \{w, w, p, v\} \) or the “Cartesian” \( \{z_\mu\}, \mu = 0, 1, 2, 3 \) coordinates of a biquaternion \( Z \) respectively. Therefore, the 4D complex vector space of \( \mathbb{B} \) possesses a natural complex (quasi)metrical form correspondent to Hermitean matrices

\[
D = z_0^2 - z_1^2 - z_2^2 - z_3^2.
\]  

(4)

This form turns into the Minkowski pseudo-Euclidean metric if only one considers the coordinates \( z_\mu \mapsto x_\mu \in \mathbb{R} \) as reals. This corresponds to the restriction of a generic matrix \( (3) \) to a Hermitean one \( Z \mapsto X = X^+ \). However, we do not intend to restrict, in a similar way, the “field” matrices associated with functions \( F(X) \) of the space-time coordinates \( X = \{x_\mu\} \), since the principal physical fields (especially in the framework of quantum theory) are generally considered as complex-valued.

Thus we arrive at the second interpretational principle of the considered version of the algebrodynamics:

*In \( \mathbb{B} \) algebrodynamics, the coordinate physical space-time is represented by a subspace of the \( 4\mathbb{C} \) vector space of \( \mathbb{B} \) correspondent to Hermitean \( 2 \times 2 \) matrices \( X = X^+ \) with their determinant representing just the Minkowski metric. After such a restriction, the whole algebrodynamical scheme becomes manifestly Lorentz invariant.*

On the above described coordinate “cut” correspondent to the Minkowski space, the initial conditions of \( \mathbb{B} \) differentiability \( (1) \) take the form

\[
dF = \Phi \ast dX \ast \Psi
\]  

(5)

and represent themselves a sort of “master equations” for some algebraical field theory uniquely determining all derivable properties of the latter. It is also noteworthy that none Lagrangians, commutation relations or other additional structures are used in the theory under consideration. Moreover: system of equations \( (5) \) turns to be overdetermined and does not allow for any generating Lagrangian structure \( (4) \).

Consequently, the overdetermined character of the primordial algebrodynamical relations \( (5) \), together with the non-linearity of the arising field equations (see below), makes it possible to consider the \( \mathbb{B} \) algebrodynamics as a theory of interacting fields (and “particles”, with rigidly fixed, “self-quantized” characteristics, see below).

As for particles, in the classical theory (like the algebrodynamics in its original form) in the capacity of those one can obviously take either regular (soliton-like) or singular field formations localized in the 3-space. Now we are ready to formulate the last (third) interpretational principle completing the set of first statements of the algebrodynamical scheme:

*In the framework of \( \mathbb{B} \) algebrodynamics, “particles” (particle-like formations) correspond to the (point- or extended but bounded in 3-space) singularities of the biquaternionic field, or to its derivatives. The latter may be put into correspondence with singularities of the secondary (Maxwell, Yang-Mills and other) fields which can be associated with any distribution of the primary \( \mathbb{B} \) field. the shape, spatial arrangement, the characteristics and temporal dynamics of these particle-like formations are again completely determined by the properties of the master algebrodynamical system of equations for the \( \mathbb{B} \) field \( (2) \).*

It should be noted that symmetries (relativistic and conformal among them) of the system \( (5) \) are considered in detail in the review \( [15] \). The gauge and twistor structures specific for the system \( (5) \) are also described therein (below we shall return to consideration of these). Let us now briefly review the principal properties and consequences of the \( \mathbb{B} \) algebrodynamics (based solely on the conditions of \( \mathbb{B} \) differentiability \( (5) \)).

\[4\text{Lagrangian structure can be defined only for dynamical equations of secondary gauge fields associated with the “master equations” \( (1) \).} \]
1. Each matrix component $S(x, y, z, t)$ of a differentiable function $F(X)$ satisfies the complex eikonal equation

$$
\frac{\partial S}{c \partial t} - \frac{\partial S}{\partial x}^2 - (\frac{\partial S}{\partial y}^2 - \frac{\partial S}{\partial z}^2) = 0.
$$

(6)

This nonlinear, Lorentz and conformal invariant equation substitutes the Laplace equation in the complex analysis and form the basis of the algebraic field theory.

2. Primary conditions (3) can be reduced to a simpler set of equations of the form

$$
d_\xi = \Phi dX \xi
$$

(7)

for an effectively “interacting” 2-spinor field $\xi(X) = \{\xi_A\}, A = 1, 2$ and potentials $\Phi(X) = \{\Phi_{AA}\}$ of a complex gauge-like field (see for details [15]).

3. Integrability conditions for the reduced overdetermined system (7) are just the self-duality conditions

$$
F = i F^*
$$

(8)

for the field strengths correspondent to the gauge potentials $\Phi_{AA}$. Consequently, the complexified Maxwell and $SL(2, \mathbb{C})$ Yang-Mills free equations are both satisfied on the solutions of the “master system” (7).

4. A field of a null 4-vector $k_\mu : k^\mu k_\mu = 0$ can be constructed from the fundamental 2-spinor $\xi(X)$ as follows:

$$
k_\mu = \xi^+ \sigma_\mu \xi,
$$

(9)

where $\sigma_\mu = \{I, \sigma_a\}, a = 1, 2, 3$ is the canonical basis of the $2 \times 2$ matrix algebra. As a consequence of “master system” (7), the null congruence of rays tangent to $k_\mu$ is rectilinear (geodetic) and shear-free. This congruence of rays plays an extremely important role in the algebrodynamics; below we shall call it the generating congruence. [4]

In the context of algebrodynamics it is important that an effective Riemannian metric $g_{\mu \nu}$ of a special form

$$
g_{\mu \nu} = \eta_{\mu \nu} + h(X) k_\mu k_\nu
$$

(10)

(the so-called Kerr-Schild metric [32]) may be put in correspondence with any $\mathbb{B}$ field or with associated generating congruence. This is a deformation of the flat Minkowski metric $\eta_{\mu \nu}$ preserving all the defining properties of generating congruence. Note that a self-consistent algebrodynamical scheme over a curved (algebraically special) space-time background has been developed in [17].

5. In contrast to the ordinary nonlinear field models, in the algebrodynamics it turns out to be possible to obtain the general solution of the “master system” of equations (7) or (3) in an implicit algebraic form.

The procedure is based upon the (well-known in the framework of GTR) Kerr theorem [32, 33] that gives a full discription of the null shear-free congruences in the Minkowski or Kerr-Schild spaces, and makes also use of a natural generalization of this theorem [34, 35], namely, of the general solution to the complex eikonal equation obtained therein. Briefly, the procedure of searching the solutions of eikonal equation and the associated congruence can be described as follows (for details see [15, 35]).

Using gauge (projective) symmetry, one reduces the fundamental spinor $\xi(X)$ to the ratio of its two components choosing, say,

$$
\xi^T = (1, g(X)); \quad (11)
$$

then any solution of the algebrodynamical field theory is defined via the only complex function $g(x, y, z, t)$ – the component of the projective 2-spinor $\xi$.

In turn, any solution for $g(X)$ is obtained in the following way. Consider an arbitrary (almost everywhere smooth) surface in the 3D complex projective space $\mathbb{CP}^3$; it may be set by an algebraic constraint of the form

$$
\Pi(g, r^1, r^2) = 0,
$$

(12)

where $\Pi(\ldots)$ is an arbitrary (holomorphic) function of three complex arguments. Let now these latter be linearly linked with the points of the Minkowski space through the so-called incidence relation [33]

$$
\tau = X \xi
$$

(13)

or, in components,

$$
\tau_1 = wg + u, \quad \tau_2 = vg + p,
$$

(14)

where in the considered case of real Minkowski space the coordinates \{u, v\} or \{x, y\} are real and $p, w = x \pm iy$ complex conjugated. It is known that two spinors $\xi, \tau$ related with the points $X$ via the incidence relation (13) or (14) form the so called projective twistor of the Minkowski space [33].

After substitution of (14) into equation of generating surface (12) the latter acquires the form of an algebraic equation

$$
\Pi(g, wg + u, vg + p) = 0
$$

(15)

with respect to the only unknown $g$, whereas the coordinates \{u, v, p, w\} play the role of parameters. Resolving the equation above at each point of the Minkowski space $X$, one obtains some (generally multivalued) field distribution $g(X)$.

In a rather puzzling way (the proof may be found, say, in [36, 37]), for any generating function $\Pi$ and any continuous branch of the solution under consideration, the field $g(X)$ identically satisfies both fundamental relativistic equations, the linear wave equation...
\( q = 0 \) and the nonlinear equation of complex eikonal \( (2) \). The correspondent spinor \( \xi \) (in the gauge \( (11) \)) satisfies meanwhile the equations of shear-free null congruences and, according to the above-mentioned Kerr theorem, all such congruences can be obtained with the help of the exposed algebraic procedure.

6. It has been demonstrated in [33, 39] that the complexified electromagnetic field associated with fundamental spinor \( \xi \) (which identically satisfies the self-duality conditions \( 8 \)) and thus the homogeneous Maxwell equations can be directly expressed through the function \( g \) (obtained as a solution of the algebraic constraint \( (15) \)) and its derivatives \( \{ \Pi_A, \Pi_{AB} \} \) with respect to the twistor arguments \( \{ \tau_A \} \). Specifically, for the spin-tensor \( g \) of electromagnetic field strength \( \varphi_{AB} \) one gets

\[
\varphi_{AB} = \frac{1}{P} (\Pi_{AB} - \frac{d}{dg} (\Pi_A \Pi_B / P)),
\]

with \( P := d\Pi/dg \). The strengths of the associated Yang-Mills field can also be represented algebraically via \( (10) \) and the spinor \( g \) itself.

7. It can be seen from representation \( (16) \) that the electromagnetic field strength turns to infinity at the points defined by the condition

\[
P = \frac{d\Pi}{dg} \equiv \frac{\partial \Pi}{\partial g} + w\Pi_1 + v\Pi_2 = 0.
\]

Similar situation takes place for singularities of the associated Yang-Mills field and the curvature field of the effective Kerr-Schild metric \( (19) \) (see [32, 33, 21]). Therefore, in the context of B algebrodynamics one is brought to identify particles with locus of common singularities of the curvature and gauge fields. It is also reasonable to assume under this identification that, instantaneously, particle-like singularities are bounded in the 3-space \( \mathbb{R} \).

8. With respect to the primary \( \mathbb{C} \)-field \( g(X) \) obtained from the constraint \( (15) \) condition \( (17) \) defines its branching points. Geometrically, this corresponds to caustics of the light-like rays of the generating congruence. Generally speaking, instead of the primary \( \mathbb{B} \) field and correspondent multivalued field \( g(X) \), one can equivalently consider the fundamental congruence consisting, generically, of a (great) number of individual branches (“subcongruences” [39]) and forming caustics-particles at the points of merging of rays from some two of them, i.e., at the envelope. This all-matter-generating primordial structure in \( [19, 35, 21] \) has been called the pre-light flow, or the “Pre-light”.

9. At the same time, existence of the Prelight flow immediately distinguishes the temporal structure \( \mathbb{R} \). Indeed, the incidence relation \( (13) \) preserves its form under a one-parametrical coordinates transformation of the form

\[
\begin{align*}
x_a &\mapsto x_a + n_a s, \\
t &\mapsto t + s, \\
n_a n_a &= 1, \\
s &\in \mathbb{R},
\end{align*}
\]

\( x \mapsto \mathbb{R} \) corresponding to a translation in the 3-space along each of rectilinear rays of the congruence, i.e., along the spatial directions specified by the unit vector \( n = \{n_a\} \),

\[
\begin{align*}
\mathbf{n} &= \frac{\xi^+ \sigma \xi}{\xi^+ \xi} = \\
&= \frac{1}{1 + gg^*} \{g + g^*, \ i(g - g^*), \ 1 - gg^*\}.
\end{align*}
\]

Under such transformations physically correspondent to the process of propagation of the principal field with the universal velocity \( V = c = 1 \), all of the three components of the projective twistor are preserved in value, as well as the direction vector \( n \) itself. Then, in accord with representation \( (18) \), one can regard these transformation as a prototype of the course of time and the Prelight Flow itself as the Time Flow. In more details these issues were considered in [35, 21], and in Section 5 we shall see in what an interesting way they are refracted under the introduction of a complex pre-space.

10. Particles identified above with common singularities of the gauge and curvature fields exhibit a number of remarkable properties specific for the real matter constituents. The most interesting is, perhaps, that of the self-quantization of electric charge. This property follows from the over-determinance of “master system” \( (7) \) and the self-duality of associated field strength \( (8) \). It is, at least partially, of topological origin. According to the quantization theorem proved in \( (10) \), for any isolated and bounded (i.e., particle-like) singularity of the electromagnetic field \( (10) \) electric charge is either null or necessarily integer multiple to some minimal, elementary value, namely, to the charge of fundamental static solution to the \( \mathbb{B} \) equations \( (7) \). The latter is a direct analog of the well-known Kerr-Newman solution in the GTR. The solution follows from the twistor constraint \( (12) \) with generating function \( \Pi \) of the form

\[
\Pi = g\tau_1 - \tau_2 + 2ia g = \\
= wg^2 + 2(z + ia)g - p = 0, \quad z := \frac{u - v}{2},
\]

resolving which, one obtains the two-valued solution

\[
g = \frac{p}{z \pm \bar{r}} = \frac{x + iy}{z + ia \pm \sqrt{x^2 + y^2 + (z + ia)^2}},
\]

\( a = const \in \mathbb{R} \). With the above solution one can associate the famous Kerr congruence with the caustic locus of the form of a singular ring of radius a correspondent to the locus of branching points of function \( (21) \). Particularly, in the degenerate case \( a = 0 \) of a point-like singularity the associated via \( (10) \) electric field is the Coulomb one but the electric charge \( q \) of singularity is strictly fixed in absolute value (in the accepted

\[\text{For string-like singularities expanding to infinity and found, e.g., in [39]}\] another interpretation is needed (cosmic strings, etc.).

\[\text{Actually, this is true for any twistor structure in general.}\]
normalization \( q = \pm 1/4 \) \[14\] \[27\]. Correspondent effective metric \( g = \frac{1}{4} \) is just the Reissner-Nördstrem solution to the Einstein-Maxwell equations.

In the general case \( a \neq 0 \) solution \( 21 \) leads to the field and metric exactly correspondent (under additional requirement on the electric charge to be unit!) to the above-mentioned Kerr-Newman solution (in the regime of a naked singularity free of horizon). B. Carter \[11\] was the first to notice that correspondent gyromagnetic ratio for this field distribution is exactly equal to its anomalous value for the Dirac fermion. This observation stimulated subsequent studies (of Lopez, Israel, Burinskii, Newman et al.) in which the Kerr singular ring, with associated set of fields, has been regarded as a model of electron. Note that in the algebrodynamical scheme this consideration is still more justified since the electric charge therein is necessarily fixed in modulus and may be identified as the elementary one. Thus,

in the framework of \( \mathbb{B} \) algebrodynamics over Minkowski space the electron can be represented by the Kerr singular ring (of Compton size) related to a unique static axisymmetrical solution \( 21 \) of equations \( 7 \), or of the constraint \( 14 \).

11. A number of other exact solutions to the initial algebrodynamical equations and the related eikonal, Maxwell and Yang-Mills equations have been obtained in \[33\] \[34\], among them a bisingular solution and its toroidal modification \[37\]. They correspond to the case when generating function \( \Pi \) in \[15\] is quadratic in \( g \). More complicated solutions demand the computer assistance for solving the algebraic relation \[15\]. However, the (most interesting) structure of singular loci of these distributions can be determined through elimination of the unknown \( g \) from the set of two algebraic equations \[15\] and \[17\] \[8\]. The complex equation arising under the procedure,

\[
\Pi(x, y, z, t) = 0
\]

represents itself the equation of motion of particles-singularities and, moreover, at a fixed instant determines their spatial distribution and shape. In this way, we have examined the structure of singularities for a number of complicated solutions to “master equations” and equations of associated biquaternionic and electromagnetic fields. As for the latter, there has been obtained a peculiar solution to free Maxwell equations (?) describing the process of annihilation of two unlike (and necessarily unit) charges, with accompanying radiation of a singular wave front \[39\], a class of the wave-like singular solutions \[39\] etc.

11. If one restricts itself by generic solutions to “master equations” \( 7 \) or to associated Maxwell equations \( 8 \), then their singular locus will (instantaneously) represent itself as a number of one-dimensional curves – “strings”. Generally, these strings (though neutral or carrying unit charges) are unstable in shape and size with respect to a small variation of parameters of the generating function \( \Pi \). As an example, consider a special deformation of the Kerr solution and congruence \[8\] defined by the following modification of the Kerr generating function:

\[
\Pi = gr^1(1 - ih) - r^2(1 + ih) + 2iag,
\]

in which an additional parameter \( h \in \mathbb{R} \) enters in addition to the standard Kerr parameter \( a \in \mathbb{R} \). As a result, from the constraint \( \Pi = 0 \) one obtains a novel solution for the function \( g \) that defines still axisymmetrical but now time-dependent generating congruence of rays. Its caustic defined by the branching points of \( g \) is represented by a uniformly collapsing into a point and, afterwards, expanding to infinity singular ring:

\[
z = 0, \quad \rho := \sqrt{x^2 + y^2} = v(t - t_0),
\]

where \( t_0 = a/\sqrt{1 + h^2} \) and velocity of collapse/expansion \( v = h/\sqrt{1 + h^2} \) is always less than the light one \( c = 1 \). Thus,

the Kerr congruence is unstable with respect to a small perturbation of controlling parameters of the generating function. This let one expect also the instability of the Kerr (Kerr-Newman) solution of the (electro)vacuum Einstein equations, since the latter is defined, to a considerable degree, by the structure of a null congruence of the above-presented type.

Note in addition that the deformed ring still carries a fixed, elementary charge but, nonetheless, is finally radiated to infinity.

At this point we complete our brief review of the “old” algebrodynamics on the Minkowski background by the following remarks. In fact, from a single initial condition of \( \mathbb{B} \) differentiability we were able to develop a self-consistent theory of fields and particle-like formations possessing a whole set of unique and physically realistic properties. The only ad hoc assumption made during the construction of the algebrodynamical theory, in order to ensure its Lorentz invariance, was a rather artificial restriction of the coordinate 4C vector space of the \( \mathbb{B} \) algebra onto the subspace with the Minkowski metrical form. On the other hand, the structure of string-like singularities-particles arising on \( \mathbb{M} \) under this procedure turns out to be unstable and, perhaps, diffuses with time. Together, these considerations suggest the necessity of a more successive analysis of the geometry “hidden” in the algebraic structure of biquaternion algebra, and of the probable links of its 4C

\[8\]In the case of a polinomial form of the generating function \( \Pi \) the procedure reduces to determination of the resultant of two polinoms and can be easily algorithmized

\[9\]That is, by solutions free of any symmetry, in particular, being nonstatic and nonaxisymmetric.
vector space with the true physical geometry. On this way we immediately discover a completely novel geometry of the (extended) space-time presented in the next section.

3. Biquaternion geometry and phase extension of the Minkowski space

Let us return to matrix representation (3) of the elements $Z \in \mathbb{B}$ of the biquaternion algebra. Restriction to unitary matrices $Z \mapsto U : U^+ = U^{-1} \ast \det U$ reduces the algebra $\mathbb{B}$ to that of real Hamilton quaternions $\mathbb{Q}$. Recall that $\mathbb{Q}$ is one of the two exceptional associative division algebras, together with the complex numbers algebra. The transformations preserving, together with division algebras, together with the complex numbers $\mathbb{C}$, the unitarity property, the structure of multiplication in $\mathbb{Q}$ (the inner automorphisms) are of the form

$$U \mapsto S \ast U \ast S^{-1}, \quad S^{-1} = S^+, \quad S \in SU(2). \quad (25)$$

Under these, the diagonal (real) component of a matrix $U$ is invariant whereas the other three $\{x_1, x_2, x_3\}$ behave as the components of a rotating 3-vector (note that both $\pm S$ correspond to the same rotation: spinor structure). So the automorphism group of quaternion algebra $Aut(\mathbb{Q}) = SU(2) \cong SO(3)$ is 2 : 1 isomorphic to the group of 3D rotations, with the main invariant

$$l = x_1^2 + x_2^2 + x_3^2, \quad \sigma = z_1^2 + z_2^2 + z_3^2 \equiv |z|^2, \quad (26)$$

defining the Euclidean structure of geometry induced by the algebra $\mathbb{Q}$. In this sense, from the times of Hamilton, exceptional algebra of real quaternions is considered as the algebra of physical background space and, in the algebrodynamics, predetermines its dimensionality and observable Euclidean structure.

We can then apply the same “Hamilton’s logic” to the algebra of biquaternions $\mathbb{B}$. Now the elements $Z \in \mathbb{B}$ are represented by complex matrices of generic type $[3]$, and multiplication in $\mathbb{B}$ is preserved under the transformations

$$Z \mapsto M \ast Z \ast M^{-1}, \quad \det M = 1, \quad M \in SL(2, \mathbb{C}). \quad (27)$$

In full analogy with real case, the diagonal component $z_0$ in $[3]$ remains invariant, and the three others $z = \{z_1, z_2, z_3\}$ manifest themselves as a 3D complex vector under complex rotations. Thus one has: $Aut(\mathbb{B}) = SL(2, \mathbb{C}) \cong SO(3, \mathbb{C})$.

Some explanations must be presented at this point. It is well known that the 6D (in reals) group $SL(2, \mathbb{C})$ is a covering of the Lorentz group realizing its spinorial representation; the same is true for the 2 : 1 isomorphic group of 3D complex rotations $SO(3, \mathbb{C})$. Specifically, Lorentz transformations can be represented in the form analogous to (27),

$$X \mapsto M \ast X \ast M^+, \quad (28)$$

but act on the subspace $Z \mapsto X$ of Hermitean matrices $X = X^+$ with determinant representing the Minkowski metric. It is just this restriction that we considered in the previous section. Now, however, we are interested in a natural geometry induced by the full structure of the 8D (in reals) vector space $Z$ of the $\mathbb{B}$-algebra, in its hypothetical relationship to the Minkowski space and in the possible physical meaning of the four additional coordinates. It should be noted that, surprisingly, this geometry has not been discovered until now. As we shall see, corresponding construction is rather transparent and successive.

We have seen that the structure of $\mathbb{B}$ multiplication is preserved under the 3D complex rotations forming the $SO(3, \mathbb{C})$ group. The main complex invariant of these transformations, the analog of Euclidean invariant (20) of the real algebra $\mathbb{Q}$, is represented by a holomorphic (quasi)metrical bilinear form

$$\sigma = z_1^2 + z_2^2 + z_3^2 \equiv |z|^2, \quad (29)$$

the (squared) “complex length” of a vector $z$. It should be emphasized that all other metrical forms, the Hermitean metric among them, that could be canonically defined on the vector space $\mathbb{C}^4$ itself (or on its subspace $\mathbb{C}^3$) are, in fact, meaningless in the framework of the algebrodynamics since they do not preserve their structure under the $\mathbb{B}$ automorphisms.

On the other hand, from the complex invariant (29) one can naturally extract a positive definite (exactly, non-negative) Finslerian metrical form of the 4-th degree by taking the square of complex modulus of the considered invariant

$$S^2 := \sigma \sigma^* = |z|^2 |z^*|^2 \quad (30)$$

As the next step, one can make use of the following remarkable identity (see, e.g., [22]):

$$|z|^2 |z^*|^2 \equiv (z \cdot z^*)^2 - |i \; z \times z^*|^2 \quad (31)$$

that can be explicitly verified. Taking it into account, one can represent the positive-definite invariant (30) in the form of a Minkowski-like interval [22]:

$$S^2 = T^2 - |R|^2 \geq 0, \quad (32)$$

in which the quantities $T$ and $R$ defined through the scalar ($\cdot$) and vector ($\times$) multiplications of complex 3-vectors as

$$T := z \cdot z^*, \quad R := i \; z \times z^*, \quad (33)$$

aquire respectively the meaning of temporal and spatial coordinates of some effective 4D space with a Minkowski-type metric. Note also that such an identification is quite informal since under the $\mathbb{B}$ automorphisms acting as the 3D complex rotations the quantities $T, R$ transform one through the others just as the temporal
and spatial coordinates do under the Lorentz transformations.\(^\text{10}\)

Thus, the main real invariant of the biquaternion algebra, being positive definite, induces nonetheless the structure of causal domain of the Minkowski space corresponding to the interior of the light cone (together with its light-like boundary). In this scheme, the events that are not causally connected as if do not exist at all (just as this should be from a successive viewpoint of the STR). We arrive, therefore, at a paradoxical but much interesting, both from physical and mathematical viewpoints, concept of the physical space-time with a positive definite metric.

Consider now the phase part of complex invariant (29). The latter can be represented in the form

\[ \sigma = S \exp^{i\alpha}, \tag{34} \]

with absolute value \( S \) correspondent to the Minkowski interval and the phase \( \alpha \) also invariant under the 3D complex rotations (that is, in fact, under Lorentz transformations). In this connection, the non-compact (corresponding to modulus) part of the initial invariant is responsible for the macro-geometry explicitly fixed by an observer: remarkably, it turns to be exactly of the Minkowski type. At the same time, its phase, compact part determines geometry of the “fiber” and, perhaps, reveals itself at a micro-level being, in particular, related to the universal wave properties of matter (see Section 5). In the other respect, invariant \( \alpha \) has the meaning of the phase of the proper complex time as this can be seen from (34) and will be discussed below.

We accept thus a novel concept of the background space-time geometry as of the phase extension of a (causal part) of the Minkowski space predetermined by the initial complex-quaternionic structure, with coordinates bilinear in those of the primordial and “actually existing” \( \mathbb{C}^3 \) space.

Note that in literature dealing with various versions of complex extensions of the space-time geometry (see, e.g., \([52, 53]\)) one usually encounters the procedure of separation of complex coordinates into real “physical” and imaginary “unphysical” parts (alternative to their separation into “modulus” and “phase” parts in our approach). This procedure is, actually, inconsistent, since both parts are completely equivalent in their inner properties and should thus equally contribute to the induced real geometry one is going to construct.

Nonetheless, the above-mentioned linear separation of the complex coordinates is rather demonstrative. Specifically, consider a couple of the 3D real vectors \( \{ p, q \} \) associated with a complex vector \( z \):

\[ z = p + iq. \tag{35} \]

In this representation the principal invariant (29) takes the form

\[ \sigma = (|p|^2 - |q|^2) + i(2p \cdot q), \tag{36} \]

and corresponds to a pair of invariants in which one easily recognizes the two well-known invariants of electromagnetic field (with vectors \( p, q \) identified as the field strengths of electric and magnetic field, respectively). The noticed analogy of complex coordinates and electromagnetic field strengths seems much suggesting and requires thorough analysis.

Express now, through the vectors \( p, q \), the effective temporal and spatial coordinates (33):

\[ T = |p|^2 + |q|^2, \quad R = 2p \times q \tag{37} \]

and note that the temporal coordinate is positive definite (in Section 5 we shall relate this property with that of the time irreversibility). As to the three spatial coordinates, they form an axial vector so that the choice of sign corresponds to a reference frame of definite chirality.

Finally, let us write down a remarkable relation (22) that links the module \( V \) of the velocity \( V = \delta R/\delta T \) of motion of a material point in the induced Minkowski space with characteristics of the initial complex space \( \mathbb{C}^3 \), namely, with the invariant phase \( \alpha \) and the angle \( \theta \) between the vectors \( p \) and \( q \):

\[ \cos^2 \theta = \frac{1 - V^2}{1 + V^2 \coth^2 \alpha}. \tag{38} \]

In a limited case of motion with fundamental velocity \( V = c = 1 \) one gets \( \theta = \pi/2 \), so that vectors \( p, q \) are orthogonal to each other and to the direction of motion \( V \) (in analogy with an electromagnetic wave). From one obtains also that in this “light-like” case the two vectors are equal in modulus, invariant \( \sigma \) turns to zero, and the phase \( \alpha \) becomes indefinite.

In the opposite case of a “particle” at rest \( V = 0 \) one gets \( \theta = 0, \pi \), so that two different (“para” and “ortho”) relative orientations of vectors \( p, q \) are possible. This remarkable property might be related to two admissible projections of the spin vector onto an arbitrary direction in the 3-space.

4. Complex algebrodynamics and the ensemble of “duplicons”

According to the first principles of the algebrodynamical approach, the true dynamics takes place just in the biquaternionic “pre-space” \( \mathbb{C}^4 \). In fact, we are able to explicitly observe only a “shadow” of this primordial dynamics on the induced (via mapping (33)) real Minkowski-like space with the additional invariant phase and the causal structures.

As another ground for construction of complex algebrodynamics there serves a distinguished role of the
“complex null cone”, a direct analog of real Minkowski light cone. Specifically, consider two points $P, P^{(0)} \in \mathbb{C}^4$ with their coordinates $Z$ connected through the algebraic relation

$$[z_1 - z_1^{(0)}]^2 + [z_2 - z_2^{(0)}]^2 + [z_3 - z_3^{(0)}]^2 = [z_0 - z_0^{(0)}]^2.$$  
(39)

Then it is easy to demonstrate [23], using the incidence relation $\tau = Z\xi$ (comp. with real case [13]), that the twistor field (as well as the principal spinor field $g$ and the initial biquaternionic field) takes equal values along the complex null line connecting the two points under consideration, that is, along an element of the null cone. In this respect the positions and displacements of such points are correlated. It is noteworthy that in both sides of the null cone equation (39) there stands one of the two fundamental invariants of the $\mathbb{B}$ algebra so that the primordial complex geometry dynamically reduces to the geometry of smaller space $\mathbb{C}^3$ with holomorphic (quasi)metrical form [29]. As we are already aware of, this gives rise to a real effective geometry of the Minkowski type. Note also that for a fixed value of the (two equal) invariants equation (39) defines a complex 2-sphere. The latter manifold is $SO(3, \mathbb{C})$-invariant and closely related to the unitary representations of the Lorentz group [18].

Finally, in full analogy with the “old” version of algebrodynamics on $\mathbf{M}$, let us identify particles with singularities of the biquaternionic and the associated fields, geometrically – with caustics of the generating congruence. In this connection, recall that the generic type singularities on $\mathbf{M}$ have the structure of one-dimensional curves – “strings”. However, on the complexified $\mathbb{C}^3$ background singularities manifest a much richer structure.

Let us also emphasize from the beginning that the dynamical principles of complex algebrodynamics are completely the same as of its “old” version on $\mathbf{M}$. Specifically, we only make use of the biquaternionic fields obeying the $\mathbb{B}$-analyticity conditions or, equivalently, of twistor fields defining a generating congruence of complex null rays with zero shear. In particular, all the rules of definition of the set of relativistic fields (Section 2) do not require any modification in the complex case.

Consider now generating congruences (and, correspondingly, biquaternionic, twistor, and associated gauge fields) of a mathematically special and physically interesting type. These are congruences with a focal line – a world line of some virtual point charge “moving” in the complex extension $\mathbb{C}^4$ of the Minkowski space [14]. Structures like this have been first considered in the framework of GTR by Newman [29, 43]; further on, congruences with a focal line will be called Newman’s congruences [22].

In this connection, consider a point-like singularity–particle “moving” in the complex space $\mathbb{C}^4$ along a “trajectory” $z_\mu = z_\mu(\tau), \tau \in \mathbb{C}, \mu = 0, 1, 2, 3$. Points at which the primordial twistor (spinor) field takes the same values as in the vicinity of the “particle” are defined by the null cone equation (39). However, let these points belong themselves to the considered world line and represent thus other “particles”. Then the null cone equation (39) acquires the form

$$L := [z_1(\lambda) - z_1(\tau)]^2 + [z_2(\lambda) - z_2(\tau)]^2 + [z_3(\lambda) - z_3(\tau)]^2 - [z_0(\lambda) - z_0(\tau)]^2 = 0$$  
(40)

and for any $\tau$ has, in general, a great (or even infinite) number of roots $\lambda = \lambda^{(n)}(\tau)$ defining an ensemble of correspondent “particles” $z_\mu^{(n)}$. These are arranged in various points of the same complex world line and dynamically correlated (“interact”) with the initial (generating) particle – the so-called elementary observer, see below.

Such a set of “copies” of a sole pointlike particle “observing itself” (both in its past and future, see Section 5) was first considered in our works [23, 24] and was called therein the ensemble of duplicons. It is noteworthy that on the real Minkowski background equation (40) (which on $\mathbf{M}$ turns out to be an ordinary retardation equation), in the case when the point of observation belongs itself to the world line of a particle, has a unique solution independently on the form of the trajectory, namely, the trivial solution $\lambda = \tau$. Thus the concept of duplicons cannot be realized on the background of ordinary Minkowski space $\mathbf{M}$.

In [23, 24] we were guided by the old idea of R. Feynman and J.A. Wheeler [13] and considered each duplicon in the capacity of an electron model. Indeed, in full correspondence with the Feynman-Wheeler conjecture, in the arising picture “all of the electrons” are, essentially, “one and the same electron” in various locations on a unique world line. In fact, however, the arising structure of singularities suggests a more natural, though exotic, interpretation.

Indeed, let us consider a primordial “generating” duplicon in the capacity of an “elementary observer” $\mathbf{O}$ [14]. All other duplicons on the null cone of the latter (40), though dynamically correlated with $\mathbf{O}$, are in fact “invisible” and not perceived by the elementary observer: any “signal” is absent! It is thus natural to conjecture

11At present, this approach is intensively elaborating by Newman himself with collaborators [13] as well as by Burinskii [91]. Their construction, by virtue of the above-mentioned reason, cannot be realized on real $\mathbf{M}$.

12To model a real macroscopic observer, instead of a trajectory of an individual duplicon $z_\mu(\tau)$, one should introduce some averaged trajectory simplest of which is represented by a null complex line and, under its mapping into the real Minkowski space, corresponds to a uniform rectilinear motion of an inertial observer.

13Exact definition and specification of such congruences is presented, say, in [23].
that the act of “perception” (actually – of interaction) takes place when only a null complex line (an element of the complex null cone) connecting \( O \) with some duplicon becomes “material”, that is, a caustic of the generating congruence.

It is easy to determine the caustic locus of the congruence from the null cone equation \( M \). Similar to the case of general solution \( L \) of the \( B \) analitycity equations, in the case of a Newman’s congruence caustics coincide again with the branching points of the principal spinor field \( g \) or, equivalently, of the field of local time of a duplicon \( \tau(Z) \). At these points one observes an amplification of the principal twistor-biquaternionic field (preserved along the elements of the complex null cone), and this can be regarded as the process of propagation of a “signal” to (from) the observer \( O \).

In turn, branching points correspond to multiple roots of the null cone equation defined by the condition

\[
L^\prime := -\frac{1}{2} \frac{dL}{d\lambda} = z_0(\lambda) - z_0(\tau) = 0 \tag{41}
\]

(summation over \( a = 1, 2, 3 \) is assumed and prime denotes differentiation by \( \lambda \)). Together with the initial defining relation for duplicons \( H \), the above condition specifies a discrete set of positions of the observer (via its local times \( \tau = \tau(k) \)) and of a pair of duplicons joining at a correspondent instant (defined as one of the multiple roots \( \lambda = \lambda(k) \)). Thus, an elementary interaction act can be regarded as a merging of some two duplicons \((a, b)\) (with \( \lambda(a) = \lambda(b) \)) considered with respect to an observer \( O \) at some of his positions (with \( \tau = \tau(k) \)). At such instants a process of the field amplification occurs along a null complex line, a caustic, connecting the observer and the two instantaneously coinciding duplicons. As it was already mentioned, under its mapping into \( M \) this line corresponds to some rectilinear path of a field perturbation moving in uniform with a velocity \( V \leq c \).

We are now in a position to naturally distinguish particles-singularities arising in the scheme under consideration as the matter constituents and the interaction carriers, in full analogy with the generally accepted theoretical classification. First of them form the ensemble of identical duplicons and can move along a very complicated and mutually concordant trajectories, geometrically – along the focal curve of the generating congruence. As to the second, they always move along rectilinear line elements of the complex null cone connecting a pair of “interacting” duplicons. In this process, the two merging duplicons represent an entire particle (see below) and stand for an emitter whereas the observer – for a detector of the propagating “signal”; the problem of temporal ordering arising in this connection will be discussed in Section 5.

Thus, we are led to the conclusion that any elementary object (electron?) may be fixed by an observer only at some particular instants and represents itself a pair of pre-elements, duplicons, emitting a signal towards the observer when and only when their positions coincide in (complex) space. At all the rest time these pre-elements – duplicons – are separated in space, do not radiate and, consequently, can be detected by none observer.

Conjecture about duplicons as halves of the electron revealing themselves solely at the instants of pairwise fusion strongly correlates with the modern concept and observations of fractional charges (see, e.g., the review \( H \)) and, on the other hand, makes it possible to offer an alternative explanation for the wave properties of microobjects, particularly, for the quantum interference phenomena.

Indeed, let a pair of duplicons be identified as an electron at an instant of the first fusion, via the caustic-signal emitted towards an observer. In the following, these “twins” diverge in space and, in particular, can pass through different “slots” in an idealized interference experiment. As a result, they can again reveal themselves only at an instant of the next fusion accompanied by a new act of emission of a signal-caustic in the direction of the observer. Between the two fusions, each of the “twins” acquires a particular phase lag, namely, of the geometrical phase \( \alpha \) of the principal complex invariant \( C \). Since, however, the complex coordinates of both “twins” at the instant of fusion should be equal, for the acquired phase lag one has

\[
\Delta \alpha = 2\pi N, \quad N = 0, \pm 1, \pm 2, \ldots . \tag{42}
\]

Thus there exists only a certain set of points at which a microobject might be once more observed after some its primary “registration”. This strongly resembles the well-known procedure of preparation of a quantum-mechanical state and of the following QM measurement, respectively. However, in the above presented picture we do not encounter any sort of the wave-particle dualism, of the de Broglie wave concept, etc. Each matter pre-element, duplicon, manifests itself as a typical pointlike corpuscular, whereas phase relations are of a completely geometrical nature and relate to some internal space of a “fiber” over \( M \). Below we shall once more return to discuss the interference phenomenon.

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15. This field satisfies the complex eikonal equation \( M \). On the real Minkowski background this is analogous to the field of the “retarded” time.

16. In the complex algebrodynamics there exists also another class of singularities representing themselves as a sort of “three-element” formations. One can speculate about their probable relationship to the the quark content of the matter.

17. In our scheme, the fiber itself defines the structure of the effective Minkowski base. Such situation is indeed unique and, in particular, can find application in the theory of the Calabi-Yau manifolds with a 3C fiber structure.
5. Random complex time and quantum uncertainty

Essentially, it is meaningless to discuss the problem of dynamics in complex space before one specifies the notion of complex time. In fact, we have already seen that the evolution parameter \( \tau \in \mathbb{C} \) of an “elementary observer” \( \mathbf{O} \), that is, the parameter of the “world line” \( z_\mu(\tau) \) of a generating point singularity is now complex-valued. This means that the subsequent position of an observer on its “trajectory” (under \( \tau \mapsto \tau + d\tau \)) is in fact indefinite by virtue of arbitrariness of alteration of the phase of the parameter \( \tau \).

On the one hand, any value of \( \tau \) one-to-one corresponds to a certain position of the observer \( \mathbf{O} \) (and of the associated set of duplicons correlated with \( \mathbf{O} \) through the null cone constraint) and, therefore, to a definite “state of the Universe” with respect to a given observer.

On the other hand, a particular realization of those or other continuations of the trajectory is ambiguous being ruled by an unknown law of the “walk” of the evolution parameter \( \tau \) across the complex plane, that is, by the form of a curve \( \tau = \tau(t) \) with monotonically increasing real-valued parameter \( t \in \mathbb{R} \). In [23, 24] this was called the evolution curve.

Note that only after specification of the form of the evolution curve one can ascertain the order of succession of events and even distinguish past from future. It is just this form that defines the time arrow and predetermines, in particular, which of the (completely identical in dynamics) duplicons is “younger” and which “older” than a certain “elementary observer” \( \mathbf{O} \).

In the framework of the neo-Pythagorean ideology of algebrodynamics, the form of the universal evolution curve should follow from some general mathematical considerations and be exceptional with respect to its internal properties; unfortunately, at present the form is unknown. Up to now it only seems natural to expect that this “Time Curve” is extremely complicated and entangled (being, probably, of a fractal-like nature). Whether this is the case, for us the character of alteration of the evolution parameter on its complex plane would effectively represent itself a random walk. Moreover, one may conjecture that this walk is discrete, whereas the generating worldline \( z_\mu(\tau) \) itself remains complex analytical: these two are completely independent. In the latter case, in the scheme there arises the time quanta, the “chronon”. We shall see below that it has to be of the order of the Compton size, not of the Plank one. From different viewpoints the latter concept has been advocated in a number of works (see, e.g., [10] and references therein).

It is noteworthy that, despite its probable random character, the Time Curve gives rise to mutually correlated alterations of the locations of different particles or, more generally, to global synchronization of random processes of various nature. At a microlevel, this may be related to the quantum nonlocality and entanglement, at a macrolevel – to universal correlations already observed in the experiments of Shnoll (see, e.g., the review [50]).

Conjecture about random nature of the Time dynamics and resulting randomness of the motion of microobjects makes it possible to solve also the problem of concordance between the increments \( \delta T, \delta R \) of the effective space-time coordinates [53] and the differences of their values for final and initial states \( \Delta T, \Delta R \). Indeed, say, for the time coordinate one gets

\[
\Delta T := T' - T = \\
= \left( |p + dp|^2 - |p|^2 \right) + \left( |q + dq|^2 - |q|^2 \right) \\
= 2(p \cdot dp + q \cdot dq) + (dp \cdot dp + dq \cdot dq).
\]

Now under the averaging procedure the mixed term \( dt = 2(p \cdot dp + q \cdot dq) \) vanishes, and the time interval

\[
\delta T = dp \cdot dp + dq \cdot dq \equiv \Delta T
\]

at a “physically infinitesimal” scale behaves as a full differential, an actually holonomic entity. The same is true for the increments of averaged spatial coordinates \( \delta R \equiv \Delta R \).

Moreover, property of the increment of the time coordinate \( \delta T \geq 0 \) is positive definite “in average” leads immediately to a natural kinematical explanation of the irreversibility of physical time. Actually, any “macroscopic” alteration of the particles’ positions (of the state of a system of particles) in the primary complex space necessarily results in an increase of the value of time coordinate of the effective Minkowski space. Thus, in the algebrodynamical approach irreversibility of time seems to be of kinematical and statistical nature, and in the latter respect time resembles the entropy-like quantity in the orthodox scheme (if the latter is understood as a probability measure).

To conclude, in the context of initially deterministic “classical” dynamics there arises an unremovable uncertainty and, effectively, randomness of evolution of an observable ensemble of micro-objects. This uncertainty is of a global and universal character and is related to conjectural stochastic type of alteration of the complex time parameter, to complex and effectively random nature of the physical time itself. It is noteworthy that numerous problems and perspectives arising under introduction of the notion of two-dimensional time have been considered by Sakharov [51]; Keckhun and Asadov [55] studied the quantum mechanics with complex time parameter and introduced, in this connection, the notion of different alteration regimes of this parameter similar to the above-introduced notion of the “evolution curve”.

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18One can evidently represent this parameter by the length of the curve.

19Generally, these are not necessarily equal due to the bilinearity of the induced space-time coordinates with respect to the primary complex “holonomic” coordinates \( z_\mu \).
Remarkably, in the capacity of a rather unspecified parameter $\tau$ of the generating world line $z_\mu(\tau)$ one can (should!) use the principal invariant $\sigma$ of complex proper time \[33\], with its modulus $S$ corresponding to ordinary Minkowski proper time, and the phase $\alpha$ responsible for the uncertainty of evolution. This is the only parameter ensuring preservation of both the primary twistor field and the caustic structure (along straight null rays of the generating congruence) (see the proof in \[33\]). In this sense (despite its accepted name) complex “proper” time acquires the meaning of universal global time governing the concordant dynamics of the Universe.

We are now ready to return back to the analysis of quantum interference experiment started in the previous section. Recall that we have undertaken an attempt to relate the wave properties of matter to the conjecture of dimerous electron (formed by two pre-elements, duplicons, at the instants of their fusion) and to the geometrical phase (phase of the complex time $\alpha$ “attached” at each point of the generating world line and alternating along the latter. In the simplest case, assuming the linear proportionality of the (physically) infinitesimal increments of the modulus $dS$ and phase $d\alpha$ of complex time,

\[
de\alpha = \text{Const} \cdot dS,
\]

and choosing as the scale factor the inverse of a one-half of the Compton length $\lambda_0$ of the electron, \[34\], (this corresponds to the above-mentioned assumption about the quanta of complex time, the “chronon”), one obtains from the merging condition \[35\]

\[
\Delta \alpha = \frac{2Me}{\hbar} \Delta \int dS = \frac{\Delta A}{\hbar} = 2\pi N.
\]

Essentially, the above formula represents the condition for maxima of classical interference in the relativistic case. According to it, the phase lag for two “halves”-duplicons under interference are proportional to their path difference, with the Minkowski interval as the invariant measure. On the other hand, in this is a remarkable correspondence with famous Feynmann representation of the wave function $\Psi = R \exp(iA/\hbar)$ whose phase is proportional to the classical action $A$ (for free particle – to the proper time interval). On the other hand, in the nonrelativistic approximation, decomposing the interval $dS$ over the powers of velocity $V/c$ and taking into account the integrability of zero power term, one obtains as the condition of quantum interference the de Broglie relation

\[
\Delta \int \frac{dL}{\lambda} = N, \quad \lambda := \frac{h}{MV},
\]

with integer path difference of the two duplicons in fractions of the de Broglie wave length $\lambda$.

Thus, the phase invariant $\alpha$ seems to be of fundamental physical importance being at the same time a measure of uncertainty of the evolution of micro-objects and the measure of their wave properties. The latters have their origin in the peculiarities of the primordial complex geometry and do not appeal to the paradigm of wave-particle dualism.

To conclude, we have endeavored to demonstrate the following. Exceptional complex geometry based on the properties of a remarkable algebraic structures (biquaternions), when introduced into foundations of physics as the primordial “hidden” geometry of the space-time (instead of the habitual Minkowski geometry), results in a quite novel and unexpected picture of the World. As its principal elements one can distinguish the identical pre-elements of matter – duplicons – constituents of observable particles (electrons?), as well as the uniformly propagating interaction carriers (caustics) and the effectively random complex time that predetermines the kinematical irreversibility of physical time at a macrolevel. However, after 25 years of development of the algebrodynamical field theory on ordinary Minkowski background, the “new” complex algebrodynamics is just at the very beginning of its march. We expect that the properties of biquaternion algebra and of the associated mathematical structures are rich enough to encode in themselves the most fundamental laws of dynamics and geometry of the physical World.

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