Meta-Cognition. An Inverse-Inverse Reinforcement Learning Approach for Cognitive Radars

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Main Idea. Detecting utility maximization ≡ Checking linear feasibility

\textit{How to make checking linear feasibility difficult?}

Cognitive radar → Choose optimal waveform for target tracking
Adversarial Target → Malicious maneuvers to ‘estimate’ radar’s utility

\textit{How to spoof adversarial attacks on radar’s utility function?}

Ans. Cognition Masking
Intelligently perturbed radar actions successfully hide radar’s utility
Background. Cognitive Radar and Revealed Preference

Cognitive Radar (Utility $u$)

Cognitive Radar [1–3]: Optimal waveform adaptation. For target maneuvers (probe) $\{\alpha_k\}_{k=1}^K$, radar chooses waveforms (response) $\{\beta_k\}_{k=1}^K$ that maximize utility $u$:

$$\beta_k = \arg\max_{\beta \in \mathbb{R}^m} u(\beta), \quad \alpha'_k \beta \leq 1 \quad (1)$$

Radar Bayesian tracker: Linear Gaussian dynamics

(i) $\alpha_k$: state noise covariance
(ii) $\beta_k$: observation noise covariance
(iii) $\alpha'_k \beta_k \leq 1$ (1): Bound on radar SNR $\equiv$ Bound on radar’s asymptotic predicted Kalman precision [3]

‘Choose best waveform subject to resource constraints’

Utility Estimation via Revealed Preference (RP):

RP Test [4, 5]: For dataset $\mathcal{D} = \{\alpha_k, \beta_k\}_{k=1}^K$, linear feasibility test is equivalent to checking for utility maximization (1):

$$\text{RP}(u, \mathcal{D}) \leq 0, \quad u = \{u_k, \lambda_k\} \in \mathbb{R}^{2m}_+, \quad (2)$$

$$u_{\text{est}}(\beta) = \min_k \{u_k + \lambda_k \alpha'_k (\beta - \beta_k)\} \quad (3)$$

What if $\mathcal{D}$ is noisy?

RP Test (2) generalizes to statistical hypothesis test to detect feasibility [6] (discussed in slide 4).

Cognition Masking

How to mitigate adversarial RP test and ensure poor reconstruction of radar’s utility function
Result 1. Deterministic Inverse RP for Masking Cognition

**Assumption:** “Radar and adversary have accurate probe-response measurements.”

Adversarial target $^{IRL} \rightarrow$ RP Feasibility test (2) (Set-valued estimate of radar’s utility)

How to rank utility functions in the feasible set?

Rank via Margin of RP test - **max. perturbation to fail RP test** (based on [7])

\[
\text{Margin}_D(u) = \max_{\epsilon \geq 0} \epsilon, \quad \text{RP}(u, D) + \epsilon \geq 0, \quad u \in \text{Feasible set}
\]

- **Margin:** Closeness to edge of feasible set (infeasibility of RP test)
- **Center of feasible set:** **max. margin**, edge of feasible set: **zero margin**
- $\uparrow$ Margin $\iff \uparrow$ Goodness-of-fit to RP test
- **Deterministic Cognition masking:** Deliberately perturb radar’s response to push radar’s utility **towards** edge of feasible set from RP test
Suppose radar faces adversarial constraints \( \{\alpha'_k \beta_k \leq 1\}_{k=1}^K \). The radar’s deterministic I-IRL algorithm to hide its utility \( u \) is:

**Step 1.** Choose margin \( \epsilon_{\text{thresh}} \in \mathbb{R}_+ \)

**Step 2.** Compute naive response \( \beta^*_k \) (1)

**Step 3.** Compute optimal perturbation \( \{\delta^*_k\} \) for I-IRL:

\[
\{\delta^*_k\} = \arg\min_{\{\delta_k\} \in \mathbb{R}^m} \sum_{k=1}^K \|\delta_k\|_2^2, \quad \text{Margin} \{\alpha_k, \beta^*_k + \delta_k\}(u) \leq \epsilon_{\text{thresh}}
\]

(Mitigating adversarial RP Test) (4)

**Step 4.** Transmit engineered sub-optimal responses \( \{\beta^*_k + \delta^*_k\} \).

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**Summary**

**Deterministic I-IRL:** Small margin \( \epsilon_{\text{thresh}} \)

\( \iff \) Closer to failing RP test (2)

\( \iff \) Larger deviation from radar’s optimal strategy

- Margin Constraint in (4) is non-convex (bilinear).

**Current research:** Formulate convex relaxations of bilinear constraints in (4).
Numerical Results: Deterministic Inverse IRL

- Simulation-based datasets to illustrate I-IRL for 2 utility functions
- Parameters: Time horizon $K = 50$, Probe/Response dimension $m = 2$

**Key Insights:**

- **Small deviation** from *optimal strategy* masks utility by a large extent.
- Radar’s performance degradation ↑ with $\epsilon$. 

![Graphs](image)

(a) $u(\beta) = \sqrt{\beta(1)} + \sqrt{\beta(2)}$

(b) $u(\beta) = \beta^2(1) + \beta^2(2)$
Result 2. Stochastic Inverse RP for Masking Cognition

Assumption: “Adversary has noisy measurements of the radar’s response.”
(Adversary side): \( \hat{\beta}_k = \beta_k + w_k, \ w_k \sim f_w \) (\( f_w \) known to radar) \( \tag{5} \)

Adversarial target \( \overset{\text{IRL}}{\rightarrow} \) Feasibility Detector (see also [3] for details)

\( H_0 : \) RP Test (2) has a feasible solution for \{\( \alpha_k, \beta_k \)\}

\( H_1 : \) RP Test (2) has NO feasible solution for \{\( \alpha_k, \beta_k \)\}

IRL Feasibility Detector: \( \phi^\ast(\hat{\mathcal{D}}) \overset{H_1}{\leq} \overset{H_0}{F_L}^{-1}(1 - \eta) \) \( (\hat{\mathcal{D}} = \{\alpha_k, \hat{\beta}_k\}) \), \( \tag{6} \)

\( \phi^\ast(\hat{\mathcal{D}}) : \max_{\{\bar{u} > 0, \bar{u}(\beta_1)\}} \text{Margin}_{\bar{u}}(\hat{\mathcal{D}}), \) r.v. \( L := \max_{j,k} \alpha_j'(w_j - w_k), \)

\( \eta : \) Adversary chosen bound for \( \mathbb{P}(H_1|H_0) \)

“Radar is non-cognitive if margin is under a threshold”

- Radar **can no more** manipulate margin of RP test.
- Can **at best** manipulate \( \mathbb{P}(H_1|\{\alpha_k, \beta_k\}, u) \) (Cond. Type-I error prob.)
- **Stochastic Cognition masking**: Deliberately perturb radar’s response to mitigate IRL detector (**increase** conditional Type-I error probability).
Stochastic Inverse IRL for Masking Cognition (Theorem 5)

Adversary’s sensor is noisy; everything else the same as deterministic case. Radar’s stochastic I-IRL algorithm is:

**Step 1.** Choose sensitivity parameter $\lambda > 0$

**Step 2.** Compute naive response $\beta^*_k$ (1)

**Step 3.** Compute optimal perturbation $\{\delta^*_k\}$ for I-IRL:

$$\{\delta^*_k\} = \arg\min_{\{\delta_k\} \in \mathbb{R}^m} \sum_{k=1}^{K} (u(\beta^*_k) - u(\beta^*_k + \delta_k)) - \lambda \mathbb{P}(H_1|\{\alpha_k, \beta^*_k + \delta_k\}, u)$$

(Mitigating adversarial IRL detector)

**Step 4.** Transmit engineered sub-optimal responses $\{\beta^*_k + \delta^*_k\}$

(7): Ensuring low margin of RP Test with high probability

**Summary**

- **Stochastic I-IRL**: Trade-off between $\uparrow$ QoS and $\uparrow$ adversarial obfuscation.
- Radar can only estimate $\mathbb{P}(H_1|H_0, u)$ (7) via Monte-Carlo methods.
- Stochastic approximation based algorithms like SPSA [8] can be used for implementing optimization problem (7).
- SPSA $\rightarrow$ Fewer (only 2) computations/update wrt finite diff. methods.
Numerical Results: Stochastic Inverse IRL

- Simulations for a single utility function $u(\beta) = \sqrt{\beta_1} + \sqrt{\beta_2}$
- Parameters: Time horizon $K = 50$, Probe/Response dimension $m = 2$

Key Insights:
- Small performance loss sufficiently confuses IRL detector (large cond. Type-I error).
- Both adversarial confusion and radar’s performance degradation ↑ with $\lambda$.
- Interestingly, performance degradation ↓ with $\eta$ (error bound).
Conclusion and Extensions

Summary:

- Radar counter-countermeasure to mitigate an adversarial countermeasure
- Cognition Masking: *Deliberately perturb optimal radar waveforms to sufficiently reduce margin of RP test and ‘hide’ radar’s utility.*
- Sub-optimality in response trades-off between Privacy and Performance
- Methodology inspired from adversarial obfuscation [9] in deep learning and differential privacy [10]

Applications of Inverse IRL:

*Online Ad Design.* Deliberately tweak meta-data to incentivize user clicks

*Survey Design.* Deliberate abnormality in questions to incentivize truthfulness

Extensions (Current research):

1. Finite sample results for spoofing the adversary’s IRL detector
2. Convex relaxations of the I-IRL objective function
3. **Counter**-(counter-)”measure: What if adversary knows radar’s spoofing strategy? *Game theoretic approach?*
Thank You!
Miscellaneous
• How justified is the constrained utility maximization abstraction for radar operation?

Quite prevalent in literature:

(i) Multi-UAV network [11]: Utility $\rightarrow$ Fairness and downlink data rate, Constraint $\rightarrow$ Transmission power, Cramer-Rao bound on localization accuracy

(ii) Q-RAM (Resource Allocation) [12]: Utility $\rightarrow$ QoS for tracking and search, Constraint $\rightarrow$ Bandwidth, Short-term and Long-term constraints

(iii) Radar Tracking with ECM [13]: Utility $\rightarrow$ Neg. of weighted mean of radar energy and dwell time, Constraint $\rightarrow$ 4% Cap on lost tracks due to ECM
Is conditional Type-I probability the only I-IRL metric for adversarial obfuscation in stochastic I-IRL?

No fixed formula, does need more work. Some intuitive alternatives: (a) Use deterministic I-IRL as is. Formulate concentration inequalities for margin of the noisy dataset. (b) Manipulate the average margin instead of margin. BUT, might be underplaying robustness of IRL detector. (c) [Speculative] Use a neural network to learn IRL method on the fly and disrupt.

Remark: I-IRL hinges delicately on IRL methodology.

Other heuristic ideas to hide utility?
• What’s next after IRL, and inverse IRL? I2-IRL?

Game-theoretic formulation.

Key challenge: Formulate a utility function in terms of both adversary probes and radar response.

*Anticipated outcome:* Inverse game theory - Detecting play from the Nash equilibrium of a game between adversary and radar.
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