Analysis of Key Problems in the Optimal Design of Satellite-based Precise Point Positioning Messages

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Abstract. Precise point positioning (PPP) technology for satellite navigation has received considerable attention because of its wide area coverage, uniform accuracy, and small number of ground reference sites. The use of satellite-based broadcasting technique enables the realization of PPP at the globe, even for users on the oceans. However, due to limited and valuable satellite downlink resources, the optimal design of satellite-based broadcasting message is of great significance. In this study, the influences of some key factors on the accuracy loss of products are analyzed, including the PPP message broadcasting interval, the message data bit width, and the scale factor. On this basis, the effect on the final positioning accuracy of the user is evaluated. Broadcasting design ideas and suggestions for major augmentation products, including Global Navigation Satellite System satellite orbits and clock corrections, through a simulation calculation based on a precise ephemeris are given, thereby providing references for the optimal design of satellite-based augmentation PPP service messages.

1. Introduction
Precise point positioning (PPP) technology for satellite navigation is an important technique of achieving enhanced high-precision positioning for satellite navigation because of its wide area signal coverage, uniform accuracy distribution, and small number of ground reference stations[11, 22]. Early PPP systems are mainly developed by commercial companies, which provide paid services, such as StarFire of NavCom, OmniSTAR and SeaStar of Trimble, StarFix of Furgo, VERIPOS of Subcea 7, and TerraStar of Hexagon. These commercial systems usually use the L-band downlink of Inmarsat Maritime Satellite for GNSS (Global Navigation Satellite Systems) satellite correction product broadcasting, which can provide high-precision positioning service in the service area from decimeter to centimeter level. With the improvement in constellation capability of the basic satellite navigation systems in recent years, satellite-based augmentation PPP services have gradually shifted from the commercial payment mode to the basic navigation constellation function, thereby providing a mode of public or free service. Among them, Galileo in Europe has announced that it will provide free PPP services with global coverage and accuracy of 20 cm on its E6B signal[3]; Japan’s QZSS system recently announced an enhanced version of the L6 signal system, and Centimeter-level Augmentation Service (CLAS) covering Japan and surrounding areas will be provided[4]. China’s BeiDou-2 regional satellite navigation system designed D2 navigation message in Geostationary satellite (GEO) to broadcast information, including the integrity of the BeiDou system and the differential correction and grid ionosphere products, to ensure and improve system performance. However, this system is still different from the standard PPP technology system[5]. The BeiDou-3 global system will utilize the B2b frequency signals of three GEO satellites to broadcast correction information, such as satellite orbit,
satellite clock error, and satellite pseudorange code biases, and provide PPP services at static centimeter and dynamic decimeter levels for users in China and surrounding areas[6].

PPP can achieve higher positioning accuracy than satellite-based augmentation systems (SBASs) because it can eliminate ionospheric errors and perform positioning solutions using dual-frequency carrier phase observations. However, unlike SBAS, satellite-based PPP systems have not formed a unified signal system and a standard message format agreement under a framework like the International Civil Aviation Organization because they were initially commercial activities and limited to different satellite link resources[7–9]. On the basis of the State Space Representation (SSR), formulated by the Radio Technical Commission for Maritime Services (RTCM), each PPP system customizes the simplification and design according to its own actual downlink resources and service performance requirements. A comparison of published broadcast schemes for relevant precision positioning messages shows that in addition to the broadcast content, the schemes mainly differ in message broadcast interval and data bit width, which are important factors that affect the design of the message protocol and performance of PPP services[4, 10]. On the one hand, the message broadcast interval determines the accuracy loss of the corrected product, which affects the user’s time-to-first-fix and the final positioning accuracy; on the other hand, different message broadcast intervals and data bit widths have different requirements on the required byte length of the message protocol, thereby affecting the broadcast rate requirement of satellite downlinks.

This study uses the GNSS precise ephemeris and broadcast ephemeris to simulate the main PPP corrections, such as satellite orbits and clock errors. The loss of product accuracy under different broadcast intervals and data bit widths and its influence on the final PPP service positioning accuracy of the GNSS are calculated and analyzed. Broadcasting design ideas and suggestions about the major corrections, including GNSS satellite orbits and clock errors, are given, thereby providing references for the optimal design of satellite-based PPP service messages.

2. Broadcast Interval and Data Width of Correction

2.1. Influences of the Broadcast Interval of Correction

Similar to the basic broadcast message of a satellite navigation system, the satellite-based PPP service message is generally composed of multiple message types, and the satellite precise orbit, clock error, and other corrections are cyclically broadcasted in the corresponding message types. Therefore, according to differences in the downlink broadcast rate of the satellite and the specific byte length of each message type, the user will receive the update correction of a certain navigation satellite in a certain period, and the length of the period is the broadcast interval of the correction.

The satellite orbits obtained from the PPP message and the rate-of-change correction vectors are \([\delta O_r, \delta O_a, \delta O_c]^T\) and \([\delta \dot{O}_r, \delta \dot{O}_a, \delta \dot{O}_c]^T\), respectively, where subscripts R, A, and C represent the radial, along, and cross directions, respectively; and the clock error correction message polynomial coefficient obtained from the PPP message is \(C_i (i=0,1,2)\). At time \(t\), the recovered precise orbit correction vector \(\delta O\) used by the user is

\[
\delta O = \begin{bmatrix} \delta O_{\text{radial}} \\ \delta O_{\text{along}} \\ \delta O_{\text{cross}} \end{bmatrix} + \begin{bmatrix} \delta \dot{O}_{\text{radial}} \\ \delta \dot{O}_{\text{along}} \\ \delta \dot{O}_{\text{cross}} \end{bmatrix} (t-t_0)
\]

where \(t_0\) is the reference time. The recovered clock error parameter \(\delta C\) is

\[
\delta C = C_0 + C_1 (t-t_0) + C_2 (t-t_0)^2
\]

Equations (1) and (2) show that the actual accuracy of the corrections used is inversely proportional to the delay time \((t - t_0)\) because the user can only use the historical correction extrapolation of the previous reference time before rereceiving the new correction parameter. Short correction update and broadcast intervals result in short delays and high accuracy; on the contrary, long update and broadcast
intervals cause large delays, which lead to larger loss of precision. The accuracy loss of the orbit correction with the delay mainly relate to the satellite orbital motion characteristics, and the accuracy loss of the clock error correction relate to the satellite atomic clock frequency stability (mainly short stability).

2.2. Influences of the Bit Width of Correction
The correction digit width (or bit) refers to the length of the bytes allocated in the PPP message for expressing and transmitting the correction, and it is jointly determined by the value range and scale factor. The value range determines the upper and lower limits of the maximum and minimum of the expressed correction; that is, an excessively large value will waste the byte, and a small value will cause truncation error. The scale factor determines the resolution of the described correction in that an excessively large scale factor can lead to coarseness and reduce the precision of the correction; if it is too small, substantial byte resources will be occupied in the same numerical range.

The design of the correction digit width needs to be integrated and balanced from performance and byte length. If the correction digit width is \( n \) bits and the scale factor is \( m \) (unit: m), then the valid data expression range of the correction \( R \) is

\[
-2^{m-1}m \leq R \leq +2^{m-1}m
\]  

(3)

The bits and data range of the key parameters advertised in the currently published standard SSR format and QZSS CLAS service format are given in Table 1.

Table 1. Comparison of correction bits, scale factor, and data range.

| Type of correction       | Standard SSR | QZSS CLAS |
|-------------------------|--------------|-----------|
|                         | Bits | Scale factor | Data range | Bits | Scale factor | Data range |
| Orbital correction (R)  | 22   | 0.1 mm  | ±209.7151 m | 15   | 1.6 mm   | ±26.2128 m |
| Orbital correction (A)  | 20   | 0.4 mm  | ±209.7148 m | 13   | 6.4 mm   | ±26.208 m  |
| Orbital correction (C)  | 20   | 0.4 mm  | ±209.7148 m | 13   | 6.4 mm   | ±26.208 m  |
| Clock error correction C0 | 22   | 0.1 mm  | ±209.7151 m | 15   | 1.6 mm   | ±26.2128 m |
| Clock error correction C1 | 21   | 0.001 mm/s | ±1.048575 m/s | --   | --      | --         |
| Clock error correction C2 | 27   | 0.00002 mm/s² | ±1.34217726 m/s² | --   | --      | --         |
| Code biases             | 14   | 0.01 m  | ±81.91 m    | 11   | 0.02 m   | ±20.46 m   |
| User range accuracy     | 6    | --      | --          | 6    | --      | --         |

The RTCM standard SSR format uses a small scale factor and a large data bit width. Thus, the range of data that can be represented is wide. By contrast, the QZSS CLAS service uses a small data bit width and defines a large scale factor (mm level) to save the byte resources of messages. Designers of the QZSS CLAS service believe that a millimeter-level correction scale factor is sufficient for enhanced positioning accuracy at the centimeter level.

3. User Differential Range Error (UDRE) Accuracy Loss Analysis
The accuracy loss of satellite orbit and clock error correction determines the satellite orbit and clock residual error corrected by the user and is reflected in the UDRE, which ultimately affects PPP accuracy.

Similar to the effect of standard point positioning (SPP), that of satellite orbit correction residual on UDRE is largest in the radial direction and smaller in the along and cross directions, and the residual influence of satellite clock error correction is also mainly reflected in the radial direction[7]. Therefore, an empirical model of the satellite ranging error (URE) in the SPP can be used to evaluate the effect of the satellite orbit and residual error of clock error correction on the UDRE for satellite-based PPP.

Different empirical models may have subtle discrepancies in specific parameters but will not considerably affect the evaluation of UDRE accuracy loss. With reference to a spatial signal error model that considers the characteristics of the BeiDou hybrid constellation[11], [12], the effect of satellite orbit and the residual error of clock error correction on UDRE can be expressed as
\[
\text{UDRE}_{\text{GPS}} = \sqrt{(0.98\Delta R - \Delta C \times \Delta T)^2 + 0.141^2(\Delta A^2 + \Delta C^2)}
\]
\[
\text{UDRE}_{\text{GLO}} = \sqrt{(0.98\Delta R - \Delta C \times \Delta T)^2 + 0.149^2(\Delta A^2 + \Delta C^2)}
\]
\[
\text{UDRE}_{\text{BDS,Gi}} = \sqrt{(0.98\Delta R - \Delta C \times \Delta T)^2 + \frac{1}{127}(\Delta A^2 + \Delta C^2)}
\]
\[
\text{UDRE}_{\text{BDS,MEO}} = \sqrt{(0.98\Delta R - \Delta C \times \Delta T)^2 + \frac{1}{54}(\Delta A^2 + \Delta C^2)}
\]
\[
\text{UDRE}_{\text{GAL}} = \sqrt{(0.98\Delta R - \Delta C \times \Delta T)^2 + 0.1208^2(\Delta A^2 + \Delta C^2)}
\]

(4)

where the subscript GPS, GLO, BDS_Gi, BDS_MEO, GAL denotes GPS satellite, GLONASS satellite, BDS GEO/IGSO satellite, BDS MEO satellite and Galileo satellite, respectively. \(\Delta R\), \(\Delta A\), and \(\Delta C\) are the components of the satellite orbit error correction residual at the radial, along, and cross directions, respectively; and \(\Delta T\) is the correction residual of satellite clock error.

4. Assessment of Positioning Accuracy Loss

User range error (UERE) is composed of two parts, namely, signal-in-space range error (URE) and user equipment error (UEE). The relationship is

\[
\text{UERE} = \sqrt{\text{URE}^2 + \text{UEE}^2}
\]

(5)

In the PPP mode, URE is transformed into UDRE; thus, Equation (5) can be written as

\[
\text{UERE} = \sqrt{\text{UDRE}^2 + \text{UEE}^2}
\]

(6)

Therefore, in PPP mode, the relationship between the user three dimensional positioning mean square error \(\sigma_u\) and the UERE can be expressed as

\[
\sigma_u = \text{UERE} \cdot \text{PDOP}
\]

(7)

The focus is on the loss effect of the UDRE on the positioning accuracy. Hence, UEE=0 can be set; that is, the influences of path propagation and terminal error can be ignored.

5. Simulation Calculation and Analysis

The accuracy of satellite ephemeris parameters and signal-in-space range error can be directly evaluated by comparing the clock error parameters in the precise ephemeris and broadcast ephemeris [13][15]. This study uses multi-GNSS broadcast ephemeris data for 31 days in January 2017, and the precise orbits and clock error products of multi-GNSSs (GBM) released by the Helmholtz-Centre Potsdam-German Research Centre for Geosciences (GFZ) are used as the reference in obtaining PPP satellite orbit and clock error corrections (1 s for sampling). The multi-GNSS constellation PDOP value of the corresponding time period is given by STK software simulation to calculate the user positioning accuracy loss.

5.1. Influence Analysis of Broadcast Interval on the Accuracy Loss

The accuracy loss of products when the user is using the PPP correction increases as the message broadcast interval increases. For each GNSS satellite, the accuracy loss at the broadcast interval of 1–179 s is calculated in each 180 s period. The average value of each period is calculated to obtain the average orbital and clock error loss of each satellite with the interval of 1–179 s. Each GNSS satellite value is then averaged to obtain the correction accuracy loss of each GNSS. Figures 1–3 show the accuracy loss caused by the message broadcast interval to the components of the satellite orbit correction in three directions. Figure 4 presents the accuracy loss caused by the message broadcast interval to the satellite clock correction.
**Figure 1.** Delay accuracy loss of satellite orbit in radial direction.

**Figure 2.** Delay accuracy loss along satellite orbit.

**Figure 3.** Delay accuracy loss across satellite orbit.
Figure 4. Delay accuracy loss of satellite clock error.

The simulation results show that the broadcast interval of the correction has the least impact on the accuracy loss of the Galileo system and affects the GLONASS system loss mostly. The accuracy loss of the BeiDou MEO satellite is basically the same as that of the GPS satellite, and the accuracy loss of the BeiDou GEO and IGSO satellites is slightly smaller than that of the GPS satellite.

The specific numerical statistics are shown in Tables 2 and 3.

Table 2. Accuracy loss statistics of GNSS orbits at intervals of 30, 60, and 120 s (95%).

| Direction | Broadcast interval | GPS/m | GLO/m | GAL/m | BDS/m |
|-----------|--------------------|-------|-------|-------|-------|
| Radial (R)| 30 s               | 0.002 | 0.014 | 0.001 | 0.002 |
|           | 60 s               | 0.004 | 0.029 | 0.001 | 0.004 |
|           | 120 s              | 0.007 | 0.062 | 0.002 | 0.007 |
| Cross (C) | 30 s               | 0.006 | 0.015 | 0.001 | 0.011 |
|           | 60 s               | 0.012 | 0.030 | 0.003 | 0.023 |
|           | 120 s              | 0.025 | 0.063 | 0.005 | 0.046 |
| Along (A) | 30 s               | 0.004 | 0.014 | 0.001 | 0.012 |
|           | 60 s               | 0.009 | 0.029 | 0.002 | 0.025 |
|           | 120 s              | 0.018 | 0.062 | 0.004 | 0.050 |

Table 3. Accuracy loss statistics of GNSS clock error at intervals of 5, 10, and 20 s (95%).

| Broadcast interval | GPS/m | GLO/m | GAL/m | BDS/m |
|--------------------|-------|-------|-------|-------|
| 5 s                | 0.004 | 0.005 | 0.000 | 0.002 |
| 10 s               | 0.008 | 0.009 | 0.001 | 0.004 |
| 20 s               | 0.017 | 0.018 | 0.001 | 0.007 |

5.2. Bit width Design of Correction

The same 31-day precise ephemeris and broadcast ephemeris in January 2017 are used to calculate the orbital clock errors and clock error corrections for the different GNSS satellites. The standard SSR format and QZSS CLAS service format are used as the reference. The probabilities of satellite orbit and clock error correction falling in the limit range of two different types of messages are shown in Table 4.

According to Table 4, even in the QZSS CLAS service format, which has a considerably small data expression range, the probability that each GNSS satellite orbit and clock error correction falls in the interval is still greater than 99.9%, and minimal effect will be exerted on PPP service performance. The byte resources of messages are saved, and the scale factor at the millimeter level can meet the accuracy requirements of the centimeter- to decimeter-level PPP service. Therefore, when the satellite downlink resources are limited, the QZSS CLAS service format can be considered a good reference design and approach.
Table 4. Probability of GNSS satellite correction falling in limit range of different types of messages.

| GNSS | Reference format of message | Orbital correction | Clock error correction (%) |
|------|-----------------------------|--------------------|---------------------------|
|      |                             | Radial (%)         | Cross (%)                 | Along (%)    |                     |
| GPS  | Standard SSR                | 100.0000           | 99.9882                   | 99.9878      | 100.0000             |
|      | QZSS CLAS                   | 100.0000           | 99.9874                   | 99.9874      | 100.0000             |
| BeiDou| Standard SSR                | 99.9382            | 99.9784                   | 99.9388      | 99.9340              |
|      | QZSS CLAS                   | 99.9382            | 99.9784                   | 99.9388      | 99.9340              |
| GLO  | Standard SSR                | 99.9866            | 99.9866                   | 99.9866      | 100.0000             |
|      | QZSS CLAS                   | 99.9866            | 99.9866                   | 99.9866      | 100.0000             |
| GAL  | Standard SSR                | 100.0000           | 100.0000                  | 100.0000     | 100.0000             |
|      | QZSS CLAS                   | 100.0000           | 99.9391                   | 99.9903      | 99.9909              |

5.3. Positioning Accuracy Loss

The probability that the correction is in the data expression range can be more than 99.9% or even higher when an appropriate data bit width is selected. This condition makes the occurrence of data truncation a minimal-probability event; hence, the effect of data truncation error on PPP accuracy can be basically ignored. This study focuses on the analysis of the positioning accuracy loss caused by the broadcast update delay of correction.

The loss of user positioning accuracy can be calculated and evaluated by Equation (7) on the basis of the analysis of satellite orbit and clock error correction loss. Figures 5 and 6 show the loss of final positioning accuracy caused by satellite orbit and clock error correction of the broadcast interval, respectively.

The specific numerical statistics are shown in Tables 5 and 6.

Figure 5. Influences of broadcast interval of satellite orbit correction on positioning accuracy loss.

Figure 6. Influences of broadcast interval of satellite clock correction on positioning accuracy loss.
The broadcast intervals of the orbital and clock error correction of different GNSS or satellites can be clarified according to the specific design requirements of PPP service performance through the simulation results. For example, for the China BeiDou Navigation Satellite System, if the PPP error caused by the delay accuracy loss of the correction needs to be within 1 cm, then the maximum intervals between the satellite orbit and clock error correction should not exceed 60 and 10 s, respectively.

Table 5. Positioning accuracy (3D) loss of GNSS orbits with interval of 30, 60, and 120 s (95%).

| Interval | GPS/m | BDS/m | GLO/m | GAL/m |
|----------|-------|-------|-------|-------|
| 30 s     | 0.001 | 0.005 | 0.046 | 0.003 |
| 60 s     | 0.017 | 0.010 | 0.100 | 0.007 |
| 120 s    | 0.036 | 0.022 | 0.199 | 0.014 |

Table 6. Positioning accuracy (3D) loss of GNSS clock errors with interval of 5, 10, and 20 s (95%).

| Interval | GPS/m | BDS/m | GLO/m | GAL/m |
|----------|-------|-------|-------|-------|
| 5 s      | 0.013 | 0.004 | 0.013 | 0.003 |
| 10 s     | 0.024 | 0.009 | 0.026 | 0.003 |
| 20 s     | 0.050 | 0.020 | 0.053 | 0.007 |

6. Conclusions
PPP is an important technique of achieving enhanced high-precision positioning for satellite navigation because of its wide area signal coverage, uniform accuracy distribution, and small number of ground reference stations. However, for the satellite-based PPP, satellite downlink resources are valuable and limited. Therefore, adaptive optimal design of PPP message protocol that is in accordance with the actual service performance requirements is necessary. In this study, major factors that affect the message broadcast interval, data bit width, and scale factor of PPP message broadcast rate and service performance are analyzed on the basis of a comparative analysis of public PPP service message formats (RTCM standard SSR and QZSS CLAS). The influences of the different factors on the user range error and final positioning accuracy of GNSS satellite are studied. On the basis of the simulation calculation of precise ephemeris, the related reference relationship and analysis conclusion are obtained, which can provide references for the optimal design of satellite-based PPP messages.

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