Temporal analysis of dissipative constructions with disconnecting links

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Abstract. The oscillation process of the dissipative structure is modeled on the basis of the "elastic force-relative displacement" deformation diagram for a random constructive nonlinear element, which rigidity changes suddenly at the moment of disconnection (breaking) of the connection. It is assumed that before the connection is switched off, the system is in a state of motion. The equations of the system reaction are represented in the nontrivial matrix form of the Duhamel integral. At the moment of transition through the critical time point, the parameters of the calculated dynamic model are corrected and changes are made to the parameters of the reaction of the damaged model. For the critical time point, the kinematic and force parameters of the reaction in two states of the computational model are determined: before and after switching off the connection. From these expressions, the analytical dependencies, that determine the magnitude of the leap in the parameters of the dynamic reaction at the moment of coupling failure, follow. As an example, we consider the problem of oscillations of a two-span steel beam, modeled by nine degrees of freedom, with a sudden destruction of the intermediate support, which is caused by impulsive action.

1. Introduction
Modern structures are exposed to complex dynamic loads associated with man-made or natural impacts, which often leads to the destruction of one or more load-bearing elements. The problem of damage to the structures of buildings and constructions is extremely acute and attracts more attention of engineering and scientific workers. The increased interest is caused by the requirements of reliability and safety of structures, in connection with which the issues of the survivability of structures and estimates of their safe operation are widely discussed in the scientific and technical literature [1-4]. Sudden failures of structural elements can lead to a change in the design scheme of the structure and cause a progressive collapse [5,6], so a large amount of research is mainly aimed at developing measures to increase the system's resistance to destruction [7-10], as well as the creation of methods for assessing the reliability of structures using nonlinear fracture models [11-13].

To ensure the safety of buildings and structures, normative documents are being developed [14-17]. However, in the part of protecting buildings from progressive collapse, there are practically no methods for calculating structures with sudden undesigned impact. There is a gap "between the level of protection provided by regulatory requirements and the level of danger" [18]. Therefore, the development of analytical methods for constructing system responses that take into account the effect of bond breaking is of exceptional importance.
In the article, equations of the reaction of the calculated dynamic model (CDM) of the system, which is before the disconnection in the process of oscillations, are presented on the basis of the theory of time analysis of discrete dissipative systems (DDS). An analysis is made of the state of the system before and after the connection failure [19]. This work is a continuation of the studies begun in the article [20].

2. Deformation diagram and basic relationships

When the connection is broken, the rigidity of the \( j \)-th constructive nonlinear element of the system changes suddenly, changing from one level to the next. On the deformation diagram "Elastic force - relative displacement": \( R_j(t) \sim v_j(t) \) (Figure 1) the stiffness levels of the \( j \)-th element are shown at the moments when the connections are switched off at \( t_1, t_2, \ldots \) These levels are determined by the slopes of the straight lines \( ab, cd, ef, \ldots \) to the horizontal axis.

![Figure 1. Deformation diagram of the \( j \)-th constructive nonlinear element of the system.](image)

At the time \( t_1 \) there is a replacement of the "old" CDM parameters (damping matrix \( C \) and rigidity matrix \( K \)) formed at \( t_0 = 0 \), for "new" – matrices \( C_1, K_1 \), formed at \( t_1 \). We analyze the response of the system on the intervals \( t \in [t_0, t_1] \) and \( t \in [t_1, t_2] \) under external exposure \( f(t) = f_0 + p(t) \), where \( f_0, p(t) - \) vectors of static and dynamic loads. Let at \( t \in [t_0, t_1] \) all the \( j \)-th elements of the calculated model work linearly, then the equation of motion and the initial conditions of the DDS take the form

\[
M \ddot{v}(t) + Cv(t) + Kv(t) = f(t),
\]

\[
v_0(t_0) = v(t_0), \quad \dot{v}_0(t_0) = \dot{v}(t_0).
\]

where \( M = \text{diag}(m_1, \ldots, m_n) \), \( C = C^T \), \( K = K^T \); \( v(t), f(t) - \) displacement and external load vectors.

In the general case, for \( t_0 \) the system can be in a state of motion, then in the initial conditions (2):

\[
v_0(t_0) \neq 0, \quad \dot{v}_0(t_0) \neq 0.
\]

If at \( t_0 \) the system is in the static equilibrium position (\( p(t_0) = 0 \)), then the initial conditions take the form:

\[
v_0(t_0) = v_{st}, \quad \dot{v}_0(t_0) = 0,
\]

where \( v_{st} - \) vector of static displacements of a given system, associated with a vector \( f_0 \) by the dependency:

\[
v_{st} = K^{-1} f_0.
\]

The formation of fundamental solutions of a homogeneous ODE that follows from (1) is related to the function \( \Phi(t) = e^{St} \), where \( S \in M_n(C) \) satisfies the equation of motion of its own forms:

\[
MS^2 + CS + K = 0.
\]

3. Reaction of the dynamical system at the moment of connection failure

For the dynamical problem represented by equation (1) and the initial conditions (2), the DDS reaction equations have a nontrivial matrix form of the Duhamel integral. On the interval \( t \in [t_0, t_1] \) under the exposure \( f(t) = f_0 + p(t) \) of the reaction equation, according [19], there will be the following form:

\[
\dot{v}(t) = 2 \Re \{ x(t) \} + v_{st}, \quad \ddot{v}(t) = 2 \Re \{ Sx(t) \}, \quad \dddot{v}(t) = 2 \Re \{ S^2 x(t) \} + M^{-1} \dot{p}(t),
\]
where
\[ x(t) = x^0(t - t_0) + z^0(t - t_0), \quad x^0(t - t_0) = \Phi(t - t_0)U^{-1}M[-\vec{S}(v_0(t_0) - v_0) + \vec{v}_0(t_0)], \]
\[ z^0(t - t_0) = U^{-1}\int_{t_0}^{t} \Phi_o(t - \tau)^T P(\tau) d\tau. \]  

(6)

When the connection is turned off, the formation of equation (4) follows with the help of matrices \( C_1, K_1 \) and the calculation of new values \( S_1, 1 \). Since the process of oscillations of the damaged DDS is also linear, the reaction equations will be similar to equations (5), (6) \((t \geq t_1)\):
\[ v(t) = 2 \text{Re}\{x(t)\} + v_\alpha(t), \quad \dot{v}(t) = 2 \text{Re}\{S_i x(t)\}, \]
\[ \ddot{v}(t) = 2 \text{Re}\{S_i^2 x(t)\} + M^{-1}p(t), \quad x(t) = x^0(t - t_1) + z^0(t - t_1). \]

(7)

where
\[ x^0(t - t_1) = \Phi_i(t - t_1)U_i^{-1}M[-\vec{S}_i(v_0(t_1) - v_0) + \vec{v}_0(t_0)], \]
\[ v_\alpha(t_1) = K^{-1}_1 f_Q, \quad z^0(t - t_1) = U_i^{-1}\int_{t_1}^{t} \Phi_i(t - \tau)^T P(\tau) d\tau. \]

(8)

Elements of the vector \( v_\alpha(t_1) \) represent static displacements of DDS nodes in the new state. For the initial conditions \( v_0(t_1), \dot{v}_0(t_1) \) the values of vectors \( v(t), \dot{v}(t) \) in (5) at \( t = t_1 \) are taken.

The dynamic reaction in the form (7), (8) provides an exact solution of the equation of motion of the damaged system on the interval \( t \in [t_1, t_2] \). Indeed, when substituting (7), (8) into the left-hand side of equation (1) (with new matrices \( C_1, K_1 \)) and grouping the summands, we obtain \((t \geq t_1)\):
\[ 2M[\text{Re}\{S_i x(t)\} + M^{-1}p(t)] + 2C_i \text{Re}\{S_i x(t)\} + 2K_i[\text{Re}\{x(t)\} + v_\alpha(t)] = \]
\[ = 2[\text{Re}\{MS_i^2 + C_i S_i + K_i\} x(t) + p(t) + K_i v_\alpha(t)] = p(t) + f_Q = f(t). \]

4. DDS reaction at the moment of connection failure
Consider the calculation model in two states: before \((t^-_1)\) and after \((t^+_1)\) the connection is off. The reaction parameters of the model take the form similar to the formulas given in the article [20]:
\[ v(t^-_1) = v_0(t_1), \quad \dot{v}(t^-_1) = \dot{v}_0(t_1), \quad \ddot{v}(t^-_1) = -M^{-1}K[v_0(t_1) - v_\alpha] - M^{-1}C_1 v_0(t_1) + M^{-1}p(t_1), \]
\[ f_\delta(t^-_1) = K v_0(t_1), \quad f_c(t^-_1) = C v_0(t_1), \quad f_f(t^-_1) = K[v_0(t_1) - v_\alpha] - C v_0(t_1) + p(t_1). \]

(9)

\[ v(t^+_1) = v_0(t_1), \quad \dot{v}(t^+_1) = \dot{v}_0(t_1), \quad \ddot{v}(t^+_1) = -M^{-1}K[v_0(t_1) - v_\alpha] - M^{-1}C_1 v_0(t_1) + M^{-1}p(t_1), \]
\[ f_\delta(t^+_1) = K v_0(t_1), \quad f_c(t^+_1) = C v_0(t_1), \quad f_f(t^+_1) = K[v_0(t_1) - v_\alpha(t_1)] - C \dot{v}_0(t_1) + p(t_1). \]

(10)

5. Analysis of results
We will present the residuals for the reaction parameters at \( t_1 \) by means of the following formulas:
\[ \Delta v(t_1) = v(t^+_1) - v(t^-_1), \quad \Delta \dot{v}(t_1) = \dot{v}(t^+_1) - \dot{v}(t^-_1), \quad \Delta \ddot{v}(t_1) = \ddot{v}(t^+_1) - \ddot{v}(t^-_1). \]

Taking into consideration the formulas (9), (10) at the critical point, we will have:
\[ \Delta v(t_1) = 0, \quad \Delta \dot{v}(t_1) = 0, \quad \Delta \ddot{v}(t_1) = M^{-1}(\Delta K \dot{v}(t_1) + \Delta C \ddot{v}(t_1)), \]
\[ \Delta f_\delta(t_1) = 0, \quad \Delta f_c(t_1) = 0, \quad \Delta f_f(t_1) = M^{-1}(\Delta K \dot{v}(t_1) + \Delta C \ddot{v}(t_1)). \]

(11)

where the differences \( \Delta K = K - K_1, \Delta C = C - C_1 \) are obtained for the corresponding matrices before and after the damage to the model; \( v(t_1), \dot{v}(t_1), \ddot{v}(t_1) \) – vectors of displacements and velocities.
Similar expressions are given in [20] for the particular case of a system that is in the state of quiescence before a coupling fails. In contrast to these expressions, in (11) for $t_1$, only the displacements and velocities are continuous functions of time from all reaction parameters, which is ensured by setting the initial conditions. The remaining parameters have leaps that depend on the discrepancies of the stiffness matrix and / or the damping matrix $\Delta K(t_1)$, $\Delta C(t_1)$ and nonzero components of the vectors $v(t_1)$, $\dot{v}(t_1)$.

6. Example of steel beam vibrations caused by the destruction of the middle support

The design scheme of the beam (I-bar 50, steel 14G2) is given in figure 2. The rigidity of the beam $EJ = 79.45$ MN·m², rigidity of the middle support $r_0 = kEJ/l^3$ ($k = 10^3$). The span $l = 15$ m is divided into 10 sections ($a = 1.5$ m). Number of degrees of freedom $n = 9$. Mass $m_i = 0.423$ kN·s²/m are in the nodes of the design model.

![Figure 2](image-url)

**Figure 2.** Design scheme of the beam before (a) and after (b) the destruction of the support.

Destruction of the support comes from the combined effect of static and dynamic loads. Elements of the vector of static nodal load $f_Q$ are: $Q_1 = 4,1478$ kN. The impulse load acts according to the law of the sine and is applied in the 4th node ($p_{04} = 100$ kH). Vector of impulse forces $p(t) = \sin (\theta(t-t_0))p_0$, where $p_0 = [0 0 0 100 0 0 0 0 0]^{T}$ kN; $\theta = E\pi / 0,2$.

The oscillations at $t \in [t_0, t_m]$ ($t_0 = 0$, $t_m = 1.5$ s) are considered. First, the problem (1), (2) is solved on the interval $t \in [t_0, t_1]$, where $t_1 = 0.093$ s – time at which the middle support suddenly breaks down at maximum reaction $R_{05} = 100.6$ kN. Then on the interval $t \in [t_1, t_m]$ the problem for the damaged system with the changed parameters of the CDM is solved. In the first case, equations (5), (6) are used, in the second – (7), (8), where $z'_i(t-t_i)$ is as follows [19]:

$$z'_i(t-t_i) = \{\Phi(t-t_i) \cdot \Theta - S \cdot \sin \theta(t-t_i) - \theta \cdot \cos \theta(t-t_i))\} [U(S^2 + \Theta^2)]^{-1} p_0 (i = 0, 1)$$

At $i = 0$ the parameters of the CDM correspond to the given system, at $i = 1$ – to the damaged system.

Figure 3 a, b output displacements and elastic forces of the 4th, 5th, 8th beam nodes. Static displacements of nodes are shown with dash-dotted lines; at $t \leq t_1$: $v_{st}(4, 5, 8) = [7.8 7.39 5.24]/100$ (cm); at $t > t_1$: $v_{st}(4, 5, 8) = [0.42 0.46 0.19]^T$ (cm).

![Figure 3](image-url)

**Figure 3.** Oscillograms of the reaction parameters for the sudden destruction of the middle support: a - displacements of beam nodes; b - elastic forces.
Figure 4 shows the oscillogram of the absolute values of the maximum bending moment. After the free damping oscillations of the system are completed, this moment tends to a static value $M_{\text{max}} = ql^2/12 = 51,847$ kN-m (horizontal asymptote). The greatest normal stresses are equal to $\sigma_{\text{max}} = 257,8$ MPa.

The accuracy of the solution of problem (1), (2) is estimated using the vector function $\varphi(t)$, which represents the algebraic sum of all the forces of the left part of the ODE of motion (1), and the vector $f_Q$ for the convenience of analysis, it is carried over to the left side of equation: $\varphi(t) = f_s(t) + f_c(t) - f_1(t) - f_Q$. The character of the convergence of the solution $\varphi(t)$ to the given functions of the right-hand side of the equation is given on the oscillogram $\varphi_4(t)$ for the loaded node 4 (Figure 5). The graph indicates a high accuracy of the solution, the error of which does not exceed the value $\varepsilon \leq 2,8 \times 10^{-12}$ kN.

7. Conclusion

Within the theory of time analysis, a method is proposed for calculating dissipative structures with disconnected connections under the exposure of a dynamic load.

1. It is shown that the formed dynamic response equations of DDS with switched off connections have a closed form. Consequently, the integration of the ODE of motion with an random damping matrix makes it possible to obtain an exact solution of the DDS oscillation problem for the entire reaction interval both before and after the sudden disruption of connection.

2. For the parameters of the dynamic response of the dissipative structure, which is before the moment of switching off the connection ($t = t_1$) in the state of oscillation:
   - analytic expressions of leaps at the moment of switching off the connection are obtained (at $t_1$);
   - it is shown that the displacements and velocities are continuous functions of time, and the accelerations and the force parameters of the reaction have leaps that depend on the discrepancies of the stiffness matrix and / or the damping matrix and on the values of the vectors $\nu(t_1), \nu_1(t_1)$ at the moment of disconnection.

The results of the analysis of beam oscillations during the destruction of the support testify to the high efficiency of the method. There opens the possibility of a detailed study of the behavior of building structures in the conditions of failure of their bearing elements.

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