Viscoelastic Theory Representation Of Gravitational Strain Fields In The $^{+}\Lambda$-CDM Vacuum

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In recent years, investigations of gravitational interactions has led us to discover new facets of the fundamental force. With these discoveries the general theory of relativity is under greater scrutiny now than it was 100 years ago. Development of a more advanced theory is needed for experimental tests and extensive predictions of gravitational interactions in local and cosmological settings. In this work, we present axioms of a novel theory that describes spacetime as a relativistic viscoelastic continuum; retaining the properties of the natural $^{+}\Lambda$-CDM vacuum. With these axioms, we provide a foundation for a tensor field theory of Gravitational Strain; introduced as an advanced formulation of an elastic interpretation of gravitational interactions. This viscoelastic definition of gravity is a natural advancement of general relativity to an observable measure of a proper relativistic gravitational field.

Keywords: elastic, spacetime, gravitational strain, tensor field

INTRODUCTION

Since the genesis of experimental tests of general relativity, the field of astrophysics has slowly veered away from being solely an “observational” science. Experimental astrophysics is emerging as a propagator of novel and innovative constructs that helps to transform concepts in theoretical physics into tangible tests of physical processes. This builds a long sought after bridge between theory and experiment that has been considered elusive in regards to testing any relativistic theory of gravity. During the last few years the largest experimental astrophysics project to date, the LIGO/LSC/VIRGO collaboration, has been a champion of this effort in bridging theory with experiment. Experimentation spearheads and propels theoretical concepts and predictions into a broader consensus. The recent activities by this major collaboration has given motivation to redefine what we interpret the fabric of spacetime as. Einstein and his contemporaries insisted that a much more advanced theory is needed to describe the strangeness that is observed in the universe (e.g. black holes, wormholes, and “dark” objects). With this being said, the development of a more advanced theory based on observations of nature is needed for experimental tests of gravitational interactions in a local and cosmological setting.

In this work we introduce an innovative approach to bridge theory and experimentation by presenting a novel interpretation of spacetime as a relativistic viscoelastic fluid. Considering the results of the first LIGO observations [1, 2], it is logical to hypothesize an elastic nature to the interactions of space-time geometry and the measure associated with the gravitational field; strain. Considering this interpretation of spacetime geometry in terms of gravitational strain provides for a more physical interpretation of the properties of the gravitational field interactions and subsequent propagation through space-time. This can be seen by the observations of gravitational strain in the gravitational wave astronomy community. Thus, a diversion from Riemannian geometry and general relativity is not needed. The geometric constructs in Riemannian geometry supports this extension of general relativity. A Gravitational Strain Field Theory finds itself at the junction of the metric-theory general relativity and the results from contemporary methodologies (LISA, and LIGO[1–3]), and observations (Hulse-Taylor Binary Pulsar [4]).

Since Newton’s Philosophiæ Naturalis Principia Mathematica in 1687 we perceive the fundamental force of Gravity as having static properties; lacking a time-dependent nature to its field equations and resulting interactions. Even with the rise of Einstein’s remarkable metric theory [5] of gravity in 1916 and its numerous extensions and interpretations, a dynamic theory of gravitation has eluded physicists to date. Given that the general theory of relativity is a metric theory of gravity, and that the metric tensor is not a direct observable [6, 7], it is an obvious approach to consider spacetime strain as an observable that would have a strongly correlated relationship

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with the metric of spacetime. A closer examination of the formulations in continuum mechanics shows that Einstein’s relativity can be considered as one interpretation and application of a generalized theory concerning dynamics of a continuum. An elastic definition of gravitational interactions would be a natural extension of general relativity to an observable measure of a proper gravitational field.

By considering a covariant formulation with the axioms defined in section III we demonstrate that a generalized equation can be described to permit this proposed elastic nature of spacetime. This formulation which is imposed upon an ambient dynamic background which includes a new approach to using a combination of superimposed metrics to construct a background measure. The geometrical representation of a gravitational field, generated by a mass distribution, is thus described as a set of coupled elastic deformations of the distribution. This is stated with respect to variations in the volume, surface shape, and rotation. The respective tensor fields are the Dilation (D), Shear-Tidal (S) and Vorticity Induction (V) of the source distribution. The concentration of this work is on the formulation of the two tensor fields D and S, excluding the rotational contributions. Rotational attributes of the gravitational strain field will be addressed later, but is mentioned here for the completion of the strain fields. Terms corresponding to deformations like that of a covariant strain tensor (ε_{μν}) and cosmological Bulk Incompressibility (β) will be used in the formulation of the two respective strain fields in section III.

I. STRONG AND WEAK BACKGROUND METRIC FORMS

Traditionally, when one is solving the Einstein field equations a single metric solution is sought after that supports a description of the spacetime structure for a singular static source of mass-energy. This metric solution generally gives a linear measure of distance and causality within the local spacetime under the influence of said mass-energy source. If we consider that this (matter) source distribution is contained within an ambient environment, with respective energetic properties (nonzero energy density), then there must also exist a metric solution to account for this ambient geometry and properties. Thus, interpreting the local spacetime environment surrounding a matter source is described by a composition of metric solutions representing strong and weak matter sources. This is done by imposing a non-static background that includes using a superposition of cosmological and matter metric solutions to construct a composite background measure.

Strong and weak forms are constructed by an imposed combination of local and cosmological (Frédéman-Láhimé-âreston-Walker, FRLW metric) spacetime metrics with the Λ-Vacuum metric. The strong and weak form background metrics are expressed as external direct sums (⊕) of the characteristic parts pertaining to the spacetime background. As can be seen, the strong form metric contains the combination of the Λ-vacuum (+Λη_{μν}) with the local geometric measure of the spacetime associated with a matter distribution (χ_{μν}).

$$\Lambda_{μν}^{(background)} = +Λη_{μν}^{(Λ−Vacuum)} ⊕ χ_{μν}^{(source)}$$

(1)

with the weak form background metric tensor as

$$\lambda_{μν}^{(background)} = +Λη_{μν}^{(Λ−Vacuum)} ⊕ χ_{μν}^{(source)} ⊕ f_{μν}^{(FLRW)}$$

(2)

At spatial infinity from the source, the weak form includes the relative expansion of spacetime pertaining to cosmological length scales, and subsequent redshift.

Determining which background form is appropriate for use within the local spacetime structure is done by considering the proximity to the matter distribution. When considering the volume or extension of a closed surface surrounding the distribution, we can set a bounds on the extension of the local field. Thus, determining which background form is appropriate for the description of the spacetime structure. Equation (3) below represents a generalization of these requirements on the background form choice.

$$g_{μν} = \begin{cases} 
\Lambda_{μν}^{(strong)} & \text{(local to source)} \\
\Lambda_{μν}^{(weak)} & \text{(far from source)} 
\end{cases}$$

(3)

The source metric will take on a metric solution that is respective of describing the geometry of a local distribution, in this representation of a background metric tensor. The inclusion of the Λ−vacuum in both strong and weak background measures is a statement of the zero state gravitational vacuum for all space. Excluding the contribution of curvature by sources of matter, this is the fundamental metric of the vacuum with nonzero energy density under this representation. As for the source metric, χ_{μν}, this metric is left undefined. As it can take on representation of any metric solution corresponding to the Einstein Field Equations for confined masses and compact objects (e.g. Kerr, Schwarzschild, Kerr-Newman, etc.).
II. THE VISCOELASTICITY OF SPACETIME

A. Axioms of Viscoelastic Theory

The following axioms are defined for a background spacetime represented as a relativistic Viscoelastic fluid:

1. Physical spacetime is described as a smooth continuous Pseudo-Riemannian manifold (Lorentzian manifold), with appropriate rotational and translational symmetries. The spacetime vacuum can be assumed to be “fluid-like” because of its attributes as a smooth continuous manifold, satisfying the symmetries of relativity.

2. The Lorentzian manifold is described by a metric tensor solution ($\Lambda_{\mu\nu}$). Due to Lorentz transformations of the spacetime coordinates, the resulting length contraction and time dilation hold along with a deformable “local” spacetime volume.

3. A defined flow of energy-momentum through the vacuum is expressed in terms of a defined set of current densities $\{J_{\mu}\}$.

4. An intrinsic zero state compressibility/expansion arises in the form of a nonzero vacuum pressure, the Cosmological Constant, $+\Lambda$. Taken for positive values of $\Lambda$ (not to be confused with the strong-form background metric tensor, $\Lambda_{\mu\nu}$, as mentioned above.).

With these assumptions of the vacuum fluidity, the background spacetime can naturally give rise to dynamic properties of elasticity in n-dimensions. The following formulation of the fundamental properties of continuum elasticity is given as derived in the text [10]. Applications of this description of elasticity serves to provide a fundamental basis from which a viscoelastic theory of spacetime can be formulated. From this, an analogy of viscoelasticity can be applied towards interpreting gravitational field interactions.

B. Formulation of Spacetime Viscoelasticity

Firstly, a notion of the comparison of reference and deformed configurations is made. Providing a statement of the material deformation and unit volume expansion/compression. The Lagrangian (material) Deformation Gradient Tensor $\Delta_{\hat{\mu}\nu}$, as a two-point tensor, characterizes the local deformation at a continuum point, with local coordinates ($x_\mu$) described by

$$\Delta_{\hat{\mu}\nu} = \Delta (e_{\hat{\mu}} \otimes I_{\nu})$$

(4)

The deformation at neighboring points is given by transferring a continuum line element (ds) emanating from that point in the reference configuration (labeled by indices without a hat) to the current deformed (hat) configuration. With continuity condition, the mapping function $u(x_\mu)$, is a function of the local coordinates.

$$\partial_\beta u(x_\alpha) dx_\alpha = \Delta_{\alpha\beta} dx_\alpha$$

(5)

where,

$$\partial_\alpha u_\beta (x_\alpha) = \frac{du_\beta}{dx_\alpha} = \Delta_{\alpha\hat{\beta}}$$

(6)

describes the “motion” of the continuum. The metric in the reference configuration has components, $g_{\mu\nu} (\partial_\mu, \partial_\nu)$; and relative to its new configuration in the deformed coordinate system ($u_\hat{\nu} (x_\hat{\mu})$) the metric will determine a different tensor of coefficients: $g_{\hat{\mu}\hat{\nu}} (\partial_{\hat{\mu}}, \partial_{\hat{\nu}})$. The new coordinates are related to the metric in the reference configuration by the transformation.

$$g_{\hat{\mu}\hat{\nu}}[u] = \Delta_{\hat{\mu}\alpha} g_{\alpha\beta}[x] \Delta_{\beta\hat{\nu}}$$

(7)

Where the line elements for each of the configurations (reference and deformed) are:

$$ds^2 = g_{\hat{\mu}\hat{\nu}}[u] dU_{\hat{\mu}} dW_{\hat{\nu}}$$

(8)

$$ds^2 = g_{\alpha\beta}[x] dU^\alpha dW^\beta$$

(9)

For the deformation gradient tensor, a notion of spacetime (continuum) compressibility is made via the determinant and its relationship to the local volume:

$$\det [\Delta_{\hat{\mu}\nu}] = \begin{cases} > 1, & \text{expansion} \\ = 1, & \text{constant} \\ < 1, & \text{compression} \end{cases}$$

(10)

Fundamentally, the deformation gradient tensor serves as a notion of a background continuum transformation.
C. Cauchy Deformation and Metric Equality

In order to further define deformations in a background spacetime, a statement of the metric compatibility (describing the definition of deformed paths and metric equality) must be made for the succeeding Cauchy deformation tensor in 4-dimensions. On a Lorentzian manifold the Cauchy deformation tensor $E_{\mu\nu}$ has the form stating,

$$E_{\mu\nu} = \Delta^{\mu\alpha} \Delta_{\alpha\nu} = \frac{\partial u_{\alpha}}{\partial x_{\mu}} \frac{\partial u_{\alpha}}{\partial x_{\nu}}$$

(11)

where the deformation tensor gives the square of the “local” change in coordinate distances due to deformation:

$$du^2 = E_{\mu\nu} dx^\mu dx^{\nu}$$

(12)

Such that changes in parametrized length are characterized by this deformation tensor. Prior to deformation the path is found by the integral below defining the parameterized path with $\lambda$ as an affine parameter. Before deformation,

$$s[\lambda] = \int_{0}^{\lambda} \sqrt{dx_{\mu} \cdot \eta_{\mu\nu} \cdot dx_{\nu}} d\lambda$$

(13)

and after deformation

$$\hat{s}[\lambda] = \int_{0}^{\lambda} \sqrt{\hat{E}_{\mu\nu} dx^\mu dx^{\nu}} d\lambda$$

(14)

giving a relative coordinate displacement of $\Delta S = \hat{S} - S$. With these definitions of the Cauchy deformation tensor an equality can be made such that the deformation tensor behaves like that of a deformed metric tensor. In which it can be defined for the coordinate displacement field on a smooth manifold with the required relationship, $E_{\mu\nu} \equiv \hat{g}_{\mu\nu}$, where $\hat{g}_{\mu\nu}$ is the metric in the deformed configuration with respect to the reference configuration coordinates. From this relationship, an expression for defining an affine connection in the deformation configuration can be given for such a metric equality,

$$\hat{\Gamma}_{\gamma\delta}^{\alpha} = \frac{1}{2} \hat{E}^{\alpha\lambda} \left( \partial_{\sigma} E_{\gamma\lambda} + \partial_{\lambda} E_{\alpha\sigma} - \partial_{\lambda} E_{\gamma\sigma} \right)$$

(15)

Naturally, we can then use the constructs of Riemannian geometry to construct the resulting deformed Riemann curvature tensor:

$$\hat{R}_{\mu\nu\alpha\beta} = \partial_{\beta} \hat{\Gamma}_{\nu\alpha}^{\delta} + \hat{\Gamma}_{\gamma\beta}^{\delta} \hat{\Gamma}_{\nu\alpha}^{\gamma} - \partial_{\alpha} \hat{\Gamma}_{\nu\beta}^{\delta} - \hat{\Gamma}_{\gamma\alpha}^{\delta} \hat{\Gamma}_{\nu\beta}^{\gamma}$$

(16)

and Ricci curvature tensor

$$\hat{R}_{\mu\nu} = \partial_{\beta} \hat{\Gamma}_{\mu\nu}^{\beta} - \partial_{\sigma} \hat{\Gamma}_{\mu\nu}^{\sigma} + \hat{\Gamma}_{\gamma\beta}^{\sigma} \hat{\Gamma}_{\mu\nu}^{\beta} - \hat{\Gamma}_{\gamma\alpha}^{\sigma} \hat{\Gamma}_{\mu\nu}^{\gamma} - \hat{\Gamma}_{\beta\alpha}^{\gamma} \hat{\Gamma}_{\mu\nu}^{\gamma}$$

(17)

This deformed Ricci tensor represents the amount of spacetime curvature generated by the stress attributed to the deformation of the sourced local mass distribution.

When the background is represented by a zero-vacuum metric, for $\Lambda_{\mu\nu}$(background) $\equiv \Lambda_{\mu\nu}$(zero) or $\lambda_{\mu\nu}$(background) $\equiv \lambda_{\mu\nu}$(zero), we can see that even in the absence of a mass (matter) distribution, the scalar curvature is intuitively nonzero for this spacetime in the presence of a nonzero vacuum energy density. Now, when considering this zero-vacuum as somewhat of a zero state configuration, the scalar curvature is inherently nonzero for this spacetime. This deformed Ricci tensor is extended to pseudo-Riemannian geometries and non-euclidean geometries, this compatibility conditions for the Cauchy deformation tensor states that in Euclidean space the Riemann tensor is assigned to be zero, stating that local curvature is zero. But for the case of non-euclidean geometries, this compatibility condition is extended to pseudo-Riemannian geometries $\Gamma_{\mu\alpha\beta}(E_{\mu\nu}) \neq 0$. This contributes to the metric equivalence of the Cauchy deformation tensor.

Using the changes in the spacetime metrics (Cauchy and background), large local displacements described by an analogous Lagrange finite strain tensor in $n = 4$ dimensions can be constructed. Here the finite spacetime strain, $\varepsilon_{\alpha\beta}$, is used as a measure of how much a given deformation in the local spacetime differs from the original configuration described by the background metric. For the most fundamental case involving the minkowski metric for flat space the strain tensor is represented as

$$\varepsilon_{\alpha\beta} = \frac{1}{2} (E_{\alpha\beta} - \eta_{\alpha\beta})$$

(18)

For the case applicable to this work we use the strong-form background, where the resulting strain is

$$\varepsilon_{\alpha\beta} = \frac{1}{2} (E_{\alpha\beta} - \Lambda_{\alpha\beta})$$

(19)

with the deformation line element as

$$(ds_1)^2 - (ds_0)^2 = 2 \varepsilon_{\mu\nu}(\eta, \tau) d\eta^{\nu} d\eta^{\mu}$$

(20)

where the parameters $\eta$ are the local coordinates of the space and $\tau$ the proper time associated with the
local spacetime. For an arbitrarily rotating frame, the continuum line element of the space is

\[(ds_1)^2 - (ds_0)^2 = c^2 \varepsilon_{00} dt^2 - \varepsilon_{ii} (d\eta^i)^2 - 4\varepsilon_{ij} d\eta^i d\eta^j + 2\varepsilon_{i0}\cos\gamma dt d\eta^i \] (21)

This strain tensor completely defines deformations of the resulting “elastic” spacetime that it describes. In the following sections it will be shown how this fundamental strain is used to define the overall gravitational strain fields that describe deformations of mass-energy distributions contained in a curved local spacetime, with nonzero scalar curvature and an introduced constant of spactime elasticity; the Bulk Incompressibility \((\beta)\) defined in accordance to the \(\Lambda\)-CDM cosmological model.

### III. GRAVITATIONAL STRAIN TENSOR

#### A. Constant of Incompressibility and Resistance

A bulk modulus as a measure of the incompressibility of spacetime needs to be a resistive measure of local contractions and bulk shearing in spacetime. This modulus incorporates local and cosmological properties of the space, which would include the use of a cosmological scale factor \((\alpha(\tau))\) as a measure of the local expansion of spacetime and densities associated with the \(\Lambda\)-CDM background for the vacuum density, \(\rho_{\text{vac}}\), and cold dark matter, \(\rho_{\text{CDM}}\). With these densities, we constrict an approximation to consider these densities as homogeneous with respect to the background measure. By including cosmological homogeneities that arise, we can begin to construct a modulus constant that describes the resistive nature of the cold dark matter cosmological background. With respect to this background, this constant accompanies the nonlinear nature of the gravitational field vacuum under a viscoelastic description as described in this work.

Beginning with the Friedmann equations for a monotonically expanding or contracting universe\[14,12\] we use cosmological parameters to describe the resistive nature of the vacuum. This gives a much more physically intuitive approach; as opposed to relying solely on a geometric description in totality. Now, one could develop an expression for a bulk modulus in terms of deriving the relative length contraction for a moving frame using the Lorentz transformations; however, this method will ultimately cause infinities to arise when applying the hyperbolic transformations to the energy density as volume changes under rarefaction (compression) and dilation (contraction)\[13\]. This would give an indirect relationship of the relative change in volume due to a governing pressure in the comoving frame. Along with this, a purely geometric representation of the incompressibility will give an inappropriate description of cosmological implications across cosmological distances. With that being said, an expression of the incompressibility, in dimensions of pressure \((M \cdot L^{-1} \cdot T^2)\), can be made via the Friedmann equations and a volumetric velocity, \(V = V \left(\frac{3\alpha}{\sigma}\right)\). The square of the Hubble constant \(H\) is approximated, neglecting the contributions from non-gravitational radiation, extrinsic spatial curvature and baryonic matter densities.

\[H^2 = \frac{8\pi G}{3} (\rho_{\text{CDM}} + \rho_\Lambda) \] (22)

From this approximation, we can solve for the density of cold dark matter \((\rho_{\text{CDM}})\) in equation \(25a\) and make a substitution for the vacuum density with respect to the time-dependent volume, \(V(\tau)\), and an effective vacuum mass, \(m_{\text{vac}}\). Where, \(\tau\), is the cosmological time for the current epoch.

\[\rho_{\text{CDM}} = \frac{1}{8\pi G} \left(3H^2 - (\rho_\Lambda)c^2\right) \] (23a)

\[\rho_{\text{CDM}} \equiv \frac{dM_{\text{vac}}}{dV(\tau)} = \frac{dM_{\text{vac}}}{\alpha^3\sqrt{-g}dV} \] (23b)

Collecting all constants after rearrangement, including the dimensionless scale factor \((\alpha)\), we have

\[\frac{c^2}{\alpha^4}\rho_{\text{CDM}} = -\frac{\sqrt{-g}}{8\pi G} \left(3H^2 - (\rho_\Lambda)c^2\right) \] (24)

Lastly giving a Cosmological Bulk Incompressibility constant \((\beta)\) of:

\[\beta = -\frac{\sqrt{-g}}{\alpha^8\pi G} \left(3H^2 c^2 - (\rho_\Lambda) c^4\right) \] (25)

This constant is suited for the description of a proposed value of the inability of the dense \(\Lambda\)-CDM vacuum to be compressed. Note that this resistance should always be negative corresponding to the pressure felt due to the relative expansion of space. Combining this constant with that of an effective vacuum mass \((m_{\text{vac}})\) that is a measure of the inertial resistance gives a formulation of a characteristic permittance of a gravitational field; \(\beta/2m_{\text{vac}}\). This formulation is loosely analogous to that of the electromagnetic vacuum permittivity constant, \(\epsilon_0\), accompanying Maxwell’s equations for the coupled description.
of electrodynamics. With this, the modulus for bulk gravity is given by equation \ref{eq:25} using values of cosmological parameters in H and $^{+}\Lambda$\cite{14,16}, where an approximated value of ($\beta$) is:

$$
\beta = -\frac{\sqrt{-\text{det}(|\Lambda_{\mu\nu}|)}}{8\pi G} \left( 3H^2c^2 - (^{+}\Lambda)c^4 \right) \quad (26)
$$

$$
= 8.30977 \times 10^{29} N \cdot m^{-2}
$$

### B. Field Tensor Components

From this definition of \textit{spacetime strain} and a modulus for describing the resistive nature of the \Lambda-CDM vacuum we now introduce the resulting \textit{Gravitational Strain Fields}. Analyzing the fundamental components of the strain tensor explained in previous sections of this work, we can parse these terms and construct corresponding gravitational strain fields; fundamentally based upon the elastic nature of spacetime. Thus, creating two differential strain fields; fundamentally based upon the longitudinal and shear (tangential) deformations. These, respectfully, are the \textit{Dilation} ($D_{\mu\nu}$)

$$
D_{\mu\nu} = \frac{\beta R_{\text{vac}}}{2m_{\text{vac}}} A_{\mu\nu} (\Lambda_{\sigma\gamma} e^{\sigma\gamma}) \quad (27)
$$

and \textit{Shear-Tidal} ($S_{\mu\nu}$) tensor fields

$$
S_{\mu\nu} = \frac{\nu_{\text{irr}} R_{\text{vac}}}{2m_{\text{vac}}} \left[ e_{\mu\nu} - \frac{1}{4} A_{\mu\nu} (\Lambda_{\sigma\gamma} e^{\sigma\gamma}) \right] \quad (28)
$$

These separated fields can then be recoupled or repackaged together to form the gravitational strain field tensor, analogous to the Faraday tensor for the electromagnetic field. The table below gives a statement of the symmetries of the coupled gravitational strain fields, represented in terms of \Psi$^{(\mu\nu)}$, where ($c_g$) is the propagation speed of the fields in gravitational vacuum. Much like the properties of the electromagnetic field tensor (or Faraday tensor) $F_{\mu\nu}$, with the exception of the trace being nonzero for \Psi$^{(\mu\nu)}$. The inclusion of diagonal terms for this strain tensor.

For \Psi$^{(\mu\nu)}$ in terms of only the shear-tidal and dilation fields, the diagonal elements represent the longitudinal components of the general tensor field in terms of the dilation field, $D_{\mu\nu}$. The off-diagonal elements of \Psi$^{(\mu\nu)}$ are the components of the shear strain of the field, where (n) is the dimensionality of the space and ($\nu_{\text{irr}}$) is the irrotational \textit{viscosity} constant associated with viscous deformations. Without rotations about the spatial axes, the shear-tidal field

$$
\Psi^{(\mu\nu)} = \left( \begin{array}{cccc}
-\mathcal{D}_{00} & -\mathcal{S}_{01} & -\mathcal{S}_{02} & -\mathcal{S}_{03} \\
-\mathcal{S}_{01} & -\mathcal{S}_{11} & -\mathcal{S}_{12} & -\mathcal{S}_{13} \\
-\mathcal{S}_{02} & -\mathcal{S}_{12} & -\mathcal{S}_{22} & -\mathcal{S}_{23} \\
-\mathcal{S}_{03} & -\mathcal{S}_{13} & -\mathcal{S}_{23} & -\mathcal{S}_{33} \\
\end{array} \right) 
$$

$$
= \left( \begin{array}{cccc}
0 & -\mathcal{S}_{01} & -\mathcal{S}_{02} & -\mathcal{S}_{03} \\
\mathcal{S}_{01} & 0 & -\mathcal{S}_{12} & -\mathcal{S}_{13} \\
\mathcal{S}_{02} & \mathcal{S}_{12} & 0 & -\mathcal{S}_{23} \\
\mathcal{S}_{03} & \mathcal{S}_{13} & \mathcal{S}_{23} & 0 \\
\end{array} \right) 
$$

As can be seen, the total strain is comprised of two coupled fields. Much like the electric field, the gravitational field behaves in the same manner. Thus, it is then logical to associate a symmetric (non-rotating) tensor representation of the field\cite{17,18}. As for a complete solenoidal (rotating) field for gravitation, an elusive magnetic analog concerning a proposed gravitational \textit{vorticity} is not so intuitive. The

| Symmetry | \Psi$^{(\mu\nu)}$ |
|----------|------------------|
| Symmetry | $\Psi^{(\mu\nu)} = \frac{1}{2} (\Psi^{\mu\nu} + \Psi^{\nu\mu})$ |
| Antisymmetry | $\Psi^{[\mu\nu]} = \frac{1}{2} (\Psi^{\mu\nu} - \Psi^{\nu\mu})$ |
| Inner Product | $\Psi^{\mu\nu} \Psi^{\nu\mu} = (\mathcal{D}_{\mu\nu})^2 + 3(\mathcal{S}_{\mu\nu})^2 - \frac{2c_g^2}{3} S_{\mu\nu} S^{\mu\nu}$ |

### TABLE I. Symmetries of the skew-symmetric Gravitational Strain Field tensor, ($\Psi^{(\mu\nu)}$).
table below groups these coupled fields in terms of the respective transverse and longitudinal parts of the gravitational strain tensor.

| Irrotational Field | $S^{0i} \equiv c_0 \Psi^{0i}$ |
| Solenoidal Part | $S^{ij} \equiv \Psi^{[ij]}$, for $i \neq j$ |
| Longitudinal Field | $D^{\mu\nu} \equiv \Lambda^{\mu\nu} (\Lambda_{\alpha\gamma} \Psi^{\alpha\gamma})$ |

TABLE II. Components and properties of the skew-symmetric Gravitational Strain Field tensor, ($\Psi^{\mu\nu}$).

IV. INTERPRETING GRAVITATIONAL STRAIN

Realizing volume deformations of source mass distributions, the dilation field constitutes deformations when considering the principle directions or local volumes. In terms of gravitational physics, mass densities and scalar curvatures are considered the sources of this longitudinal field. Under static conditions this scalar-valued tensor field is described by the finite volumetric gravitational strain for a constant local mass distribution. Expansion of the dilation tensor is given in terms of the finite volumetric strain and the trace of the background vacuum Ricci curvature tensor is given in terms of the finite volumetric gravitational strain for a constant local mass distribution. Under these cosmological conditions the irrotational part of the tensor are the anti-planar strain much like the linearized Einstein field equations. The scalar curvature and effective mass of the background vacuum are represented as previously explained, with the exception of the field constant in ($\nu_{tr}$). This constant is a representation of the viscous-like flow that accompanies the shearing deformations of distributions. On a cosmological scale, the irrotational permittivity of a gravitational field is governed by the bulk field constant, ($\beta$). Over these cosmological distances, the bulk field constant is the only significant parameter that regulates this proposed permittivity of a gravitational field.

The Shear-Tidal field represents irrotational shape or surface deformations of a mass distribution. In relation to gravitational field theory, the shear-tidal part represents the tidal field generated by a source of gravitational stress acting upon a central mass. This is expressed as the following for reiteration:

$$S^{\mu\nu} = \frac{\beta R_{\text{vac}}}{2m_{\text{vac}}} \left[ \varepsilon^{\mu\nu} - \frac{1}{4} \Lambda^{\mu\nu} (\Lambda_{\sigma\gamma} \varepsilon^{\sigma\gamma}) \right]$$

Where, again for reiteration, the components ($S^{0i}$) of the tensor are the anti-planar strain much like the linearized Einstein field equations. The scalar curvature and effective mass of the background vacuum are represented as previously explained, with the exception of the field constant in ($\nu_{tr}$). This constant is a representation of the viscous-like flow that accompanies the shearing deformations of distributions. On a cosmological scale, the irrotational permittivity of a gravitational field is governed by the bulk field constant, ($\beta$). Over these cosmological distances, the bulk field constant is the only significant parameter that regulates this proposed permittivity of a gravitational field.

V. CONCLUSIONS

Having established a dynamic background metric tensor in the first section allows for a non-trivial use of the metric tensor as a reference measure. Using the background metric (equations 1 and 2) in this fashion aides in the design of the overall field theory.

Historically, as is explained above, using a purely geometrical (mathematical) theory of gravity excludes a notion or intuition of implementing empirical experiments that test for the dynamics outlined in a metric theory of gravity [19, 20]. Currently in the field of gravitational theory, there is no consistent theory of gravitation that produces time-dependent solutions that also admit a wave nature for gravitational interactions. This work presented seeks to extend our interpretations of gravity as a fundamental force, such that it can then be used to aid a more robust formulation of the wave nature of the field. With the axioms of the vacuum fluidity in section III the background spacetime naturally gives rise to properties of elasticity in 4-dimensions. The next steps are to formulate the full rotational contributions to this viscoelastic representation of gravitation. While exploring variations of these fields; when investigating a formal Lagrangian density and respective action integral.
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