The elastic property of water and penstock’s effect on the governor stability

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Abstract. To gain stable operation of a hydro power plant, it is mostly a matter of having the right ratio between the time constant of the rotating masses, $T_a$, and the time constant for the water masses, $T_w$. If $T_a/T_w > 6$ (or at least $>4$), the stability is normally not a problem. However, for power plants with long penstocks, this criterion is not enough. The elastic property of the penstock becomes an issue. The solution of the wave equation includes a term, which mathematically is defined as $\tanh$ (tangents hyperbolicus). This function is notoriously unstable. It has a similarity to the tan-function, which goes from to $\pm \infty$ as it approaches $\pm 90^\circ$. The cross frequency defines the frequency up to which the governor will function. Above the cross frequency, any disturbance will go without any interference from the governor. Therefore, the issue is to make sure that the elastic frequency is well above the cross frequency. In this paper, the transfer functions are solved analytically and the pressure response calculation is partly verified by measurements. However, the quality of the measurements are not quite adequate and will be repeated this spring.

1. Introduction

Simulations with the purpose of controlling system stability is preferably done in the frequency domain, because then all the Eigen frequencies of the system are identified. If there is a stability problem, the cause can easily be detected, which is not the case if one do the simulations and analysis only in time domain.

The method of such analysis implies that the differential equations for the system are linearized around the point of operation, i.e. at a given flow $Q_0$, head $H_0$ and speed of rotation $n_0$, and then Laplace transformed to frequency domain. The system is hereby defined by its transfer functions presented in a block diagram. This is a well-documented method. There are numerous methods to analyse the result by graphical representation, like Bode, Nyquist, Nichols, Root-locus to mention a few. They all ends up finding the stability margins, i.e. the Phase margin and the Gain margin [1].

In Norway, there is a long tradition of analysing the stability of hydro power plants by means of frequency analysis presented in Bode plots. In Hermod Brekke’s Dr. Techn Thesis [2], he described the transference of both turbine and system equations to frequency domain. He developed the Matrix method for simulations. (His main focus was to define a frequency depended dampening factor in the equations).

The transfer functions of a hydro power plant assumed rigid penstock is shown in the block diagram Figure 1. The most important time constants are $T_w$ and $T_a$. $T_w$ is the time constant for the inertia of the
water masses, which is defined as the time it takes to accelerate the water masses from zero to nominal flow. The water masses participating in the governing process is from the nearest surface up-stream the turbine to the nearest water surface down-stream the turbine. $T_a$ is the time constant for the rotating masses, which is defined as the time it takes to accelerate the rotating masses, i.e. mainly the generator, from zero to nominal speed of rotation with full torque.

$$T_w = \frac{Q_0 \cdot L}{H_0 \cdot gA}$$  \hspace{1cm} (1)

$T_w$ is highly dependent on the hydraulic design of the power plant, while $T_a$ is more or less given by the generator manufacturer. For large aggregates, $T_a$ is 5 – 7 seconds.

To gain stable operation of a hydro power plant, it is mostly a matter of having the right ratio between the time constant of the rotating masses, $T_a$, and the time constant for the water masses, $T_w$. If $T_a/T_w > 6$ (or at least >4), the stability is normally not a problem. However, for power plants with long penstocks, this criterion is not enough. The elastic property of the penstock becomes an issue.

The cross frequency defines the frequency up to which the governor will function. Above the cross frequency, any disturbance will go without any interference from the governor. Therefore, the issue is to make sure that the elastic frequency is well above the cross frequency. But it can be shown, that the elasticity also influences the stability below the water hammer frequency.

2. Comparison of the rigid and elastic transfer function
Assuming rigid water and penstock, the block diagram, between the guide vane position, $y$, and the hydraulic power, $p$, is shown in more detail in Figure 2.

Including the elastic properties, the solution of the wave equation, or Allievi equation, includes a term which mathematically is defined as tanh (tangents hyperbolicus). This function is notoriously unstable. It has a similarity to the tan-function, which goes between $\pm \infty$ as it approaches $\pm 90^\circ$. 

Figure 1: Block diagram for hydro power plant, rigid equations.

Figure 2: Block diagram between guide vane position $y$ and power, $p$, rigid penstock
The block diagram with elastic properties of the penstock and water will be as shown in Fig. 3. The constant, $h_w$, is the Allievi constant, defined as:

$$h_w = \frac{Q_o a}{2AgH_o}$$  \hspace{1cm} (2)

![Figure 3 Block diagram between guide vane position, y, and hydraulic power, p. Elastic penstock.](image)

The two block diagrams shown in Figure 2 and Figure 3, give the two following transfer functions between guide vane position and power:

**Rigid:**

$$\frac{p}{y} = \frac{1-T_e s}{1+0.5T_p s}$$  \hspace{1cm} (3)

**Elastic:**

$$\frac{p}{y} = \frac{1-2h_w \tanh \left( \frac{L}{a} s \right)}{1+h_w \tanh \left( \frac{L}{a} s \right)}$$  \hspace{1cm} (4)

Where $s$ is the complex variable: $s = j\omega$.

3. **Analytical solution of the transfer function**

According to methods described in control theory [1] the amplitude and the phase angle can be solved analytically. In general, for a transfer function with time constants $T_1 - T_4$:

$$A(s) = A(j\omega) = \frac{(1+T_1 s)(1+T_3 s)}{(1+T_2 s)(1+T_4 s)}$$  \hspace{1cm} (5)

The amplitude is:

$$|A(j\omega)| = \sqrt{\frac{(1+(\omega T_1)^2)(1+(\omega T_3)^2)}{(1+(\omega T_2)^2)(1+(\omega T_4)^2)}}$$  \hspace{1cm} (6)

The phase angle is:

$$\angle A(j\omega) = \tan^{-1}(\omega T_1) + \tan^{-1}(\omega T_3) - \tan^{-1}(\omega T_2) - \tan^{-1}(\omega T_4)$$  \hspace{1cm} (7)

For the rigid transfer function, the calculation is rather straightforward. For the elastic transfer function, the complex variable $s$ in the expression $\tanh \left( \frac{L}{a} s \right)$ makes a problem. The complex “$j$” has to be outside the parenthesis. In [3] it is shown that:
tanh\left(\frac{L}{a} s\right) = j \tan\left(\frac{L}{a} \omega\right) \quad (8)

Hence, the tanh(Ls/a) term has transformed to j tan(Lω/a) and the amplitude and angle can be calculated.

For the rigid transfer function:

Amplitude: \[ |A(j\omega)| = \sqrt{\frac{1 + (\omega T_u^2)}{1 + (0.5\omega T_u^2)}} \] \quad (9)

Phase angle: \[ \angle A(j\omega) = -\arctan(\omega T_u) - \arctan(0.5\omega T_u) \] \quad (10)

For the elastic transfer function:

Amplitude: \[ |A(j\omega)| = \sqrt{\frac{1 + (2h_n \tan(\frac{L}{a} \omega))^2}{1 + (h_n \tan(\frac{L}{a} \omega))^2}} \] \quad (11)

Phase angle: \[ \angle A(j\omega) = -\arctan(2h_n \tan(\frac{L}{a} \omega)) - \arctan(h_n \tan(\frac{L}{a} \omega)) \] \quad (12)

Solving the equations for increasing \( \omega \), the result is shown in Figure 4, left Figure rigid and right Figure with elastic property.

![Figure 4](image)

**Figure 4** Rigid (left) and elastic transfer function plotted in a Re-Im plane.

The difference is that the rigid function goes from 1 and stops at -2 on the Re-axes, while the elastic function takes the whole turn all the way back to 1 again. The intuitive explanation of this is that the rigid just stops, while the elastic one bounces back due to the elasticity.

4. **Measurements**

In a pipe loop in Stavanger, an oscillating valve was installed and the pressure pulsations at various frequencies were measured. The valve was driven by a rotating disc connected to a frequency controlled motor. The test rig is shown in Figure 5.
In this paper, the pressure transmitter positions p5 and p8 are focused on. The length between these transmitters is 567 m. In this paper, the differential pressure is defined as $h_8 - h_5$ corresponding to the transmitter positions p8 and p5 on the figure. Unfortunately, the valve position was not registered. The flow meter reading, however, followed the valve frequency at low frequencies, see Figure 6. For higher frequencies, the flow meter, which is an electromagnetic type, was of course not reliable. In the analysis of higher frequencies, it is reasonable to assume that valve opening degree is the same.

Figure 5 Test rig for pressure pulsation measurements.
Figure 6 Results from the measurement

Comments to the Figures 6a – 6f:

6a At low frequency, 0.3 rad/s, the flow measurements follows the valve opening. Nearly no pressure difference, \(h_8-h_5\), and the amplitude: \(|A| = -20\) dB

6b At frequency 0.6 rad/s the amplitude: \(|A| = -15\) dB

6c At frequency 1.3 rad/s the pressure amplitude get less dampened. The amplitude \(|A| = -8\) dB

6d Frequency near the elastic wave frequency, 2.7 rad/s, the pressure difference curve shows a shape which looks like a water hammer wave. The amplitude: \(|A| = 0\) dB

6e Frequency 5.3 rad/s is above the water hammer. The amplitude: \(|A| = -10\) dB

6f Increasing the frequency to 7.8 rad/s, the amplitude is: \(|A| = -5.5\) dB

Figure 6d, which is frequency near the resonance frequency, shows that the pressure, \(h_5\), is approximately constant, while the pressure \(h_8\) follows the valve movement. It is obvious that there is a node at pressure position 5 and an anti-node at 8. The distance between these positions is 567 m, which gives a propagation speed of \(a = 1134\) m/s, which is quite reasonable.

5. Comparing simulations and measurements

To compare the measurements and the simulations, the pressure response, i.e. the transfer function between pressure, \(h\), and valve position, \(y\), is established by:

Rigid: \[
\frac{|h|}{|y|} = 20 \log \left( \frac{-T_w s}{1 + 0.5T_w s} \right) \text{ dB} \tag{13}
\]

Elastic: \[
\frac{|h|}{|y|} = 20 \log \left( \frac{-2h_w \tanh(\frac{s L}{a})}{1 + h_w \tanh(\frac{L a}{s})} \right) \text{ dB} \tag{14}
\]

The simulated Bode plot is shown in Figure 7. The corresponding measurement for the amplitude is:

\[
|A| = 20 \log \left( \frac{h_8 - h_5}{y} \right) \text{ dB} \tag{15}
\]

And the angle will be the phase angle between \((h_8-h_5)\) curve and the valve opening, \(y\), which cannot be obtained by these measurements because the valve position was not registered. At very low frequency, though, the flow measurement is reliable and it follows the valve opening. The phase difference between valve position and pressure is 90°, see Figure 6a.
6. Consequence for the stability simulation of a power plant

To establish if a power plant is stable or not, is a question of checking the stability margins, i.e. the amplifying margin and the Phase margin. The complete transfer function of the power plant must be established and the amplitude and phase angle calculated as a function of the frequency. There are many ways of plotting the result in order to find a conclusion on whether the stability margin is satisfactory. The author prefers the Bode plot, however to plot the result in a Re-Im plan, i.e. Nyquist diagram, might give additional information.

The complete transfer functions for rigid and elastic model is:

Rigid: \[ A(j\omega) = \frac{K_p}{T_1 T_a} \frac{(1 + T_1 s)(1 + T_2 s)}{s^2} \frac{(1 - T_2 s)}{1 + 0.5 T_2 s} \] (16)

Elastic: \[ A(j\omega) = \frac{K_p}{T_1 T_a} \frac{(1 + T_1 s)(1 + T_2 s)}{s^2} \frac{(1 - 2 h_a \tanh(\frac{L}{a} s))}{(1 + h_a \tanh(\frac{L}{a} s))} \] (17)
For the rigid function the Amplitude is:

\[ |A(j\omega)| = \frac{K_p}{T \omega_a} \sqrt{\frac{1 + (\omega T_p)^2}{\omega^2} \frac{1 + (\omega T_d)^2}{\omega^2} \frac{1 + (\omega T_u)^2}{\omega^2} \frac{1 + (0.5\omega T_u)^2}{\omega^2}} \]  

(18)

And the Phase angle is:

\[ \angle A(j\omega) = \text{atan}(\omega T_p) + \text{atan}(\omega T_d) - \text{atan}(\omega T_u) - \text{atan}(0.5\omega T_u) - \pi \]  

(19)

Because of the negative sign in the expression \((1-T_w)\) in the numerator of eq.18, the phase shift is negative. This is the key issue regarding stability and \(T_w\). It makes the phase go faster towards \(-180^\circ\).

The \(-\pi\) term in eq. 21 comes in because of the two poles at \(s=0\).

For the elastic function the Amplitude is:

\[ |A(j\omega)| = \frac{K_p}{T \omega_a} \sqrt{\frac{1 + (\omega T_p)^2}{\omega^2} \frac{(1 + (\omega T_d)^2)}{\omega^2} \frac{(1 + (2h \tan(\omega L/a))^2)}{\omega^2} \frac{(1 + (h \tan(\omega L/a))^2)}{\omega^2}} \]  

(20)

And the phase angle is:

\[ \angle A(j\omega) = \text{atan}(\omega T_p) + \text{atan}(\omega T_d) - \text{atan}(2h \tan(\omega L/a)) - \text{atan}(h \tan(\omega L/a)) - \pi \]  

(21)

Figure 8 shows Bode plots for both rigid an elastic model. Ignoring the elasticity of the penstock, the hydro power plant seems to be stable as both Phase margin and Gain margin is sufficient. Including the elasticity, the system becomes instable, the phase margin is negative, shown in Figure 8, right.

![Bode plots](image_url)

Figure 8 Bode plots. Left side shows the rigid simulation and right side the elastic simulation, \(T_w=0.4\sec, T_a=6\sec, h_w=0.61\).
The PID parameters for both simulations are the same and so is the system geometry. The PID parameters used for the simulations shown in Figure 8 are $K_p = 10$, $T_i = 4$ sec, $T_d = 0.0$. The rigid simulation shows that the system is stable with good stability margins, while the elastic simulation shows instability. The phase margin is negative. It is of course possible to stabilize the system by tuning the PID parameters, however, the cross frequency will easily be at too low frequency, which means that the governing will be too slow and standing oscillations will occur. Increasing the inertia of the generator, i.e. increasing $T_a$ will have the same effect.

7. Conclusion
High head power plants often have long penstocks, which makes the reflection time, $T_r$, too big compared to $T_w$. It is quite possible to obtain $T_w < 1$, which is often the design criterion used, and still get instability because of the elastic property of the penstock. This paper shows that the elasticity influences the stability also for frequencies below the frequency of the water hammer waves.

In order to design a stable system with sufficient stability margins, Allievi’s constant must be checked. If $h_w > 1$, preferable a good deal larger than 1, the elasticity can be disregarded. This means that the frequency of the elastic wave must be larger than the cross frequency, which is very near $1/T_w$.

If this criterion is not fulfilled, it might still be possible to make the system stable by optimizing the governor settings; however, the quality of the governing system will be lousy.

The Allievi constant is in fact the ratio $T_w/T_r$, where $T_r$ is the reflection time of the elastic wave:

$$T_r = \frac{2L}{a}$$

(22)

Where $L$ is the length of the penstock and $a$ is the pressure propagation speed. Allievi’s constant is in fact:

$$h_w = \frac{T_w}{T_r}$$

(23)

So the criterion $h_w > 1$ can then be interpreted as:

$$\frac{1}{T_r} > \frac{1}{T_w}$$

(24)

Unfortunately, the measurements were not entirely satisfactory. The valve position was not registered and in this long horizontal pipe, it was difficult to get rid of the air. The plan is to repeat the tests this spring, and, hopefully, better measurement will be obtained for verification of the theory in this paper.

References
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Nomenclature

| Symbol | Unit        | Description                           |
|--------|-------------|---------------------------------------|
| $T_w$  | (s)         | Penstock time constant                |
| $T_a$  | (s)         | Time constant for rot. masses         |
| $T_i$  | (s)         | Integral time constant                |
| $T_D$  | (s)         | Derivative time constant              |
| $K_p$  | (-)         | Proportional constant                 |
| $T_r$  | (s)         | Reflection time                       |
| $y$    | (-)         | Guide vane position                   |
| $\mu$  | (-)         | Dimensionless speed of rotation       |
| $a$    | (m.s$^{-1}$)| Pressure propagation speed            |
| $L$    | (m)         | Length of penstock                    |
| $h_w$  | (-)         | Allievie’s constant                   |
| $s$    | (-)         | Complex variable                      |
| $q$    | (-)         | Dimensionless flow                    |
| $h$    | (-)         | Dimensionless head                    |
| $p$    | (-)         | Dimensionless power                   |