HARD INELASTIC INTERACTIONS AT PARTON AND HADRON LEVEL

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In the study of multiple scattering of partons in hadron-hadron collisions the possibility of a hard inelastic process at the parton level is included in its simplest possible way, i.e. including the $2 \rightarrow 3$ transition. The specific physical process to which the treatment is applied is the inelastic collision of a nucleon with a heavy nucleus.

1 Motivations

As a part of a wider investigation on the contribution of multiple hard scattering of partons in hadron-hadron collisions the production of parton already at hard, perturbative level is considered. A definite part of the problem is investigated: the elementary parton collision can give rise only to one more parton, the production and reabsorption are taken into account, the process has presumably its largest relevance in nucleus-nucleus collisions, but for simplicity only the hadron-nucleus collision is investigated. For the moment, what is shown is the possibility of studying the problem with the re-interactions taken into account and, as intermediate conclusion, of building up the final spectrum of the partons produced in the hard processes.

A former version of this investigation was presented at a Workshop in Turin,[1] in order to save space we shall make systematic reference to this paper, that will be called T, where also a larger bibliography is listed.

2 Transport equation for the produced partons

The guiding idea can be stated in this way: the incoming hadron has a distribution of partons that interact with the partons of the heavy nucleus, the produced partons may be also partially reabsorbed, so we write a transport equation for the population of the secondary partons that go forward i.e. in the direction of the hadron. We introduce a parameter $\tau$: the depth at which the nucleon has drilled the nucleus, or the mean number of hit partons of the nucleus. The parameters $u_i$ are the relevant coordinated of the primary partons, usually impact parameter and rapidity, $u = (b, y)$ the parameters
\[ v, w \] are the relevant coordinates of the secondary partons they may be either again impact parameter and rapidity or transverse momentum and rapidity \( v = (k, y) \). As it was done in a simpler form in Ta transport equation is written:

\[
P^n_r ([u]; v_1, \ldots, v_r; \tau + \Delta \tau) = P^n_r ([u]; v_1, \ldots, v_r; \tau) + \sum_{i=1}^{n} \sum_{s=1}^{r} P^n_{r-1} ([u]; v_1, \ldots, v_{s-1}, v_{s+1}, \ldots, v_r; \tau) E(u_i; v_s; \tau) \Delta \tau \\
+ \sum_{i=1}^{n} \int dw P^n_{r+1} ([u]; v_1, \ldots, v_r; w; \tau) A(u_i; w; \tau) \Delta \tau \\
- \sum_{i=1}^{n} \sum_{s=1}^{r} P^n_r ([u]; v_1, \ldots, v_r; \tau) A(u_i; v_s; \tau) \Delta \tau \\
- \sum_{i=1}^{n} \int dw P^n_r ([u]; v_1, \ldots, v_r; \tau) E(u_i; w; \tau) \Delta \tau\]

(1)

\( E, A \) are emission and absorption probabilities, which may depend on \( \tau \).

Only one emission or absorption is involved, four basic steps are foreseen

\[
\begin{align*}
\tau - 1 & \to \tau \\
r + 1 & \to r \\
r & \to \tau - 1 \\
r & \to \tau + 1
\end{align*}
\]

(The overall impact parameter \( \beta \), is fixed and it will not be written). The equations solved by defining a generating functional for every given set of primary coordinates \([u]\):

\[
F^n([u]; I; \tau) = \sum_r \frac{1}{r!} \int dv_1, \ldots, dv_r I(v_1) \ldots I(v_r) P^n_r ([u]; v_1, \ldots, v_r; \tau) 
\]

(2)

and an equation in the continuum limit \( \Delta \tau \to 0 \).

\[
\frac{\partial}{\partial \tau} F^n([u]; I; \tau) = \sum_{i=1}^{n} F^n([u]; I; \tau) \int dw I(w) E(u_i, w; \tau) \\
+ \sum_{i=1}^{n} \int dw A(u_i, w; \tau) \frac{\delta}{\delta I(w)} F^n([u]; I; \tau) \\
- \sum_{i=1}^{n} F^n([u]; I; \tau) \int dw E(u_i, w; \tau) \\
- \sum_{i=1}^{n} \int dw A(u_i, w; \tau) I(w) \frac{\delta}{\delta I(w)} F^n([u]; I; \tau) 
\]

(3)
The solution for $\mathcal{F}$ with the initial condition that there are no secondaries at all for $\tau = 0$ yields the expression for $P$ which corresponds to a Poisson distribution, in condensed form

$$P_r = p_1 \ldots p_r e^{-\int p dv}$$

where

$$p^{(n)}(\{u\}; v; \tau_f) = \sum_{i=1}^{n} \int_{0}^{\tau_f} dt E(u_i; v; t) \exp \left[ - \sum_{i=1}^{n} \int_{t}^{\tau_f} dt' A(u_i; v; t') \right]. \quad (4)$$

The expression of $p^{(n)}$ is the one-body distribution of secondaries at fixed distribution of the primaries, is not yet an observable result, one must in fact sum over the different partonic structure of the colliding hadron. If one assumes, as it is sometimes done, a Poissonian distribution for the primary partons:

$$G_{1\ldots n} = C(u_1) \ldots C(u_n) \exp \left[ - \int C(u) du \right] = \sum_n \frac{1}{n!} \int G_{1\ldots n} du,$$

one gets an explicit although complicated expression per the distribution of the secondaries; in particular the one-body distribution is

$$D_1(v; \tau_f) = \sum_{n=0}^{\infty} \frac{1}{n!} \exp \left[ - \int du C(u) \right] \int du_1 \ldots du_n C(u_1) \ldots C(u_n) p^{(n)}(\{u\}; v; \tau_f)$$

$$= \int_{0}^{\tau_f} dt \int du C(u) E(u; v; t) \exp \left[ - \int_{t}^{\tau_f} dt' A(u; v; t') \right]$$

$$\times \exp \left\{ - \int du C(u) \left( 1 - \exp \left[ - \int_{t}^{\tau_f} dt' A(u; v; t') \right] \right) \right\}. \quad (6)$$

By calculating the 2-body function $D_2(v_1, v_2)$, it is found that the distribution is no longer Poissonian even at fixed $\beta$; in fact

$$C_2(v_1, v_2) = D_2(v_1, v_2) - D_1(v_1)D_1(v_2) \neq 0$$

The choice of the Poissonian distribution for the primary partons is only a simplified example, one could deal with much more general distributions by using the formulation in term of generating functionals [2].
3 Emission and absorption coefficients

When only the elastic scattering among partons was taken as the elementary dynamical process the basic ingredient was the elastic cross section, now it is necessary to feed into the formalism the elementary $2 \leftrightarrow 3$ processes.

If we want this process to be described in perturbative terms there are kinematical limitations (in particular the sub-energy of every pair of parton must be large) so that the process may be described in term of amplitude $M_{gg \rightarrow ggg}$ of the the nonlocal Lipatov vertex[3], its absolute square may be brought into the form:

$$\left| M_{gg \rightarrow ggg} \right|^2 = 54 g^6 \frac{s^2}{k_0^2 k_1^2 k_2^2} \sum_i k_{i\perp} = \mathbf{p}_a + \mathbf{p}_b = 0 \quad (7)$$

Now the rest of the calculation may be only indicated: since we work, see T, in the impact parameter space the Fourier transform of $M_{gg \rightarrow ggg}$ with respect to the momentum transfer between the primary partons must be calculated, with this procedure the coefficient $E$ is obtained, then pure kinematics must yield the absorption coefficient $A$.

The spectrum of the produced particles is then obtained, this result can be also read as a modification of the primaries' population induced by the hard scattering among partons.

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