Asymmetric Cellular Automaton Modeling Earthquakes

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We propose an asymmetric modification of the sand-pile-like cellular automaton for earthquake modeling. The cumulative event distribution is shown to be dependent on the asymmetry parameter.

I.

The ideas of self-organized criticality (SOC) and the observation of SOC-like behavior of simple dynamical systems, like cellular automata (CA), have significantly warmed the interest in the application of a CA to earthquake simulation. A number of papers on the construction of CA counterpart of the well known Burridge-Knopoff (BK) model earthquake faults \cite{?} have been published \cite{?,?}. Both, the P.Bak & C.Tang model \cite{?} of the complete stress release and the H.Nakanishi model \cite{?} with more elaborated stress relaxation function show the power law behavior which closely resembles the Gutenberg-Richter power law. Both of them are evidently too simplistic in comparison to the BK system of differential equations for the spring-block model. What can be done to make the existing CA models closer to the BK model? In this paper we present a modification of the existing CA which partially answers this question.

II.

One of the basic principles of the construction of cellular automata for physical applications consists in conserving the symmetry of the original physical system as precisely as possible. Let us compare the CA models \cite{?,?} and the BK spring-block model in this aspect.

- The BK system is a system of $N$ blocks of mass $m_i, i = 1, N$ resting on a rough surface and connected to each other by harmonic springs of stiffness $k_c$; each block is attached by a leaf spring of stiffness $k_p$ to the moving upper line, see Fig. 1.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{The geometry of the BK model: the system is composed of $N$ identical blocks of mass $m_i$, $k_c$ is the stiffness of the “horizontal” springs, $k_p$ is the stiffness of the pulling springs, $v$ is the constant velocity of the pulling line.}
\end{figure}

Initially (at $t = 0$) the system is at rest, and the elastic energy accumulated in “horizontal” springs is only due to randomly generated small initial displacements of the blocks from their neutral positions. The moving upper line, which simulates the movement of an external driving plate, exerts a force $f_n = -k_p(x_n - vt)$ on each $n$-th block. The nonlinear friction is defined in such a way that it holds each block at rest until the sum of all the forces applied to this block exceeds a certain critical value $F_0$. Then the block makes a slip inhibited by nonlinear friction to a new position. A pause between two slips is believed to account for a pause between earthquakes.
The CA of P. Bak and C. Tang for artificial earthquake simulation \cite{Bak95} is a simple sandpile-like system which obeys the following rules:

1. The array of \( N \) values ("blocks") is initiated by certain randomly generated values \( f_i^0, i = 1, N \) (the upper indices are used for discrete time).

2. If the sum of the forces on \( i \)-th block exceeds a certain threshold value (usually \( f_{Th} = 1 \), without loss of generality), then "the accumulated stress" is shared with the nearest neighbors according to the rule

\[
 f_{i \pm 1} \rightarrow f_{i \pm 1} + \frac{k_c}{2k_c + k_p} \delta f_i, \quad (1)
\]

where \( \delta f_i = f_i - f_i' \) is the stress drop of the over-threshold \( i \)-th block, which is evaluated according to the law

\[
 f_i' = \phi(f_i - f_{Th}). \quad (2)
\]

The relaxation function \( \phi \) is model dependent: it may be taken to be zero as in the P. Bak & C. Tang model, or it may be some decreasing function as in the Nakanishi cellular automaton.

3. When the evolution is completed the forces applied to all the blocks are incremented by the tectonic force

\[
 f_i' \rightarrow f_i' + k_p v \Delta t, \quad i = 1, N \quad (3)
\]

Now let us turn to the symmetries of the BK and CA models. Due to the presence of the tectonic driving force \( k_p \vec{v} t \), which implies the existence of a preferable space direction, namely, the direction of the moving plate velocity \( \vec{v} \), the BK system is apparently not invariant under inversion \( \vec{x} \rightarrow -\vec{x} \). The stress redistribution law \( (1) \) of the CA, in contrast, is completely symmetric with respect to this inversion \( f_{i+1} \leftrightarrow f_{i-1} \). Therefore, the above mentioned CA has more symmetries than the parent BK model. What can be done about this? The answer is evident. An asymmetry should be introduced in the stress redistribution law \( (1) \) as follows:

\[
 f_{i+1} \rightarrow f_{i+1} + (1 - \gamma)\alpha \delta f_i, \quad f_{i-1} \rightarrow f_{i-1} + \gamma \alpha \delta f_i. \quad (4)
\]

The value of the asymmetry parameter \( \gamma = 1/2 \) corresponds to equal sharing \( (1) \), \( \gamma = 0 \) leads to completely asymmetric sharing. The factor \( \alpha = \frac{2k_c}{2k_c + k_p} \) is chosen to comply with \cite{Bak95, Nakanishi90}.

We have performed computer simulations with one-dimensional 35 blocks CA for different values of the asymmetry parameter \( \gamma = 0.25, 0.4, 0.45, 0.5 \). The cumulative event distribution, i.e. the number of events of magnitude not less than a given value, which is often expected to have the form of the Gutenberg-Richter law, is presented in Fig.2.
FIG. 2. Cumulative event distribution \( N(M > M') \) for one-dimensional 35 cell CA calculated for different values of the asymmetry parameter \( \gamma = 0.25, 0.4, 0.45, 0.5 \) (\( \gamma = 0.5 \) corresponds to the symmetric case) plotted in logarithmic coordinates vs. the magnitude \( m = \lg M \) with the event size understood to be the total stress relaxation \( M = \sum_i (f_i - f_i') \). The significance level was taken as 0.1 of the maximal magnitude for each run. The relaxation function \( \phi \) was taken as in the above cited Nakanishi’s paper.

As it can be seen from the picture, the asymmetry parameter \( \gamma \), significantly affects the slope of the curve (the logarithm of the cumulative event number vs. the magnitude). We also observed the amplitude of events to increase with asymmetry increasing. This means that \( \gamma \) is a control parameter of the system, the appropriate choice of which can tune the system close or far from the realistic value of \( b \approx 1 \) in the Gutenberg-Richter law [?]

\[
\lg N(M > M') = a - mb, \quad m = \lg M = \lg \sum_i (f_i - f_i').
\]

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