Magnetic moments of exotic pentaquarks in the chiral quark-soliton model

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Abstract

We investigate the magnetic moments of the baryon antidecuplet within the framework of the chiral quark-soliton model in the chiral limit in a "model-independent" approach. Sum rules for the magnetic moments are derived. The magnetic moment of $\Theta^+$ is found to be about $0.2 \sim 0.3 \mu_N$.

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Recently, the pentaquark (uudd¯ s) baryon Θ+ was found by LEPS collaboration [1], motivated by a theoretical prediction of the chiral quark-soliton model [2]. In fact, exotic SU(3) representations containing exotic baryonic states are naturally accommodated in the chiral models [3, 4], where the quantization condition emerging from the Wess-Zumino-Witten term selects SU(3) representations of triality zero [5]. The early estimate of the Θ+ mass in the Skyrme model [6] is in very good agreement with the present experimental results.

A series of experiments confirmed the existence of Θ+ [7, 8, 9, 10]. The measured mass of Θ+ is about 1.54 GeV with a very narrow width below 25 MeV which is consistent with the prediction of Ref. [2]. In addition to Θ+, the NA49 collaboration at CERN has recently announced the finding of the new exotic baryons Ξ3/2 [11] with strangeness S = −2 and isospin T = 3/2, which are also the members of the antidecuplet.

The discovery of Θ+ has motivated theorists to look more closely into the possible exotic states in quark models [12], chiral models [13, 14], lattice QCD [15], nucleon-meson bound states models [16], constituent quark models [17], QCD sum rules [18, 19] and in group theoretical approach [20]. Some authors questioned the applicability of the rigid rotator approach to the exotic states within the chiral models [14, 21]. For a comprehensive review of different models, see Ref. [22].

The cross sections for the Θ+ production from nucleons induced by hadrons [23, 24] and photons [23, 25] have been already described theoretically. However, while it is crucial to know the magnetic moment of Θ+ in order to study its production via photo-reactions, it is yet to be determined. The magnetic moments of the baryon octet and decuplet have been successfully described within the chiral quark-soliton model (χQSM) [26, 27] in a “model-independent” approach. The “model-independent” analysis has an advantage over dynamical model calculations, since it only makes use of the underlying symmetries, with the experimental data of the octet magnetic moments used as an input. In Refs. [26, 27] we calculated the magnetic moments of Δ++ and Ω−, finding satisfactory agreement with the existing data. We have also predicted the magnetic moment of Δ+ which was measured quite recently [28].

In this letter, we extend the former investigation [26] to the magnetic moments of the baryon antidecuplet in the chiral limit. In particular, we find that the magnetic moments of the baryon antidecuplet are unexpectedly small, similarly to the recent finding of Ref.[19]. In addition, we obtain the sum rules for the antidecuplet magnetic moments similar to the Coleman-Glashow sum rules [29].

In the chiral limit the collective magnetic moment operator can be parametrized within the framework of the χQSM as follows:

\[ \hat{\mu} = v_1 D^{(8)}_{Q3} + v_2 d_{pq3} D^{(8)}_{Qp} \cdot \hat{J}_q + \frac{v_3}{\sqrt{3}} D^{(8)}_{Q8} \hat{J}_3, \]  

where the dynamical variables \( v_i \) encode dynamics of the chiral soliton and are independent of the baryon considered. They are generically expressed in terms of the inertia parameters of the soliton in the χQSM:

\[ \sum_{m,n} \langle n|\Gamma_1|m\rangle \langle m|\Gamma_2|n\rangle \mathcal{R}(E_n, E_m, \Lambda), \]  

where \( \Gamma_i \) are spin-isospin operators acting on the quark eigenstates \( |n\rangle \) of the one-body Dirac Hamiltonian in the soliton-background field. The double sum over all the eigenstates can be
evaluated numerically [30, 31, 32]. Since its sea part diverges, one requires the regularization expressed by $\mathcal{R}$ with the cut-off parameter $\Lambda$. However, instead of calculating the dynamical variables $v_i$ numerically, two linear combinations of them can be fitted to the experimental data of the octet magnetic moments [26, 27]. $D_{ab}^{(\mathcal{R})}(R)$ denotes the SU(3) Wigner function, $R(t)$ is the time-dependent SU(3) matrix responsible for the rotation of the soliton in the collective coordinate space [5, 30, 33]. $Q$ is the quark electric charge operator and $\hat{J}_a$ stands for an operator of the generalized spin acting on the baryonic wave functions $\psi_{BR}(R)$.

In order to evaluate the magnetic moments of the baryon antidecuplet, we need to calculate the following matrix elements:

$$\mu_{B_{10}} = \int dR \psi_{B_{10}}^*(R) \mu(R) \psi_{B_{10}}(R),$$  \hspace{1cm} (3)

where the wave functions $\psi_{B_{10}}(R)$ take the following form:

$$\psi_{B_{10}}(R) = \psi(\mathcal{R};Y,T,T_3)(\mathcal{R}^*;Y',J,J_3) = \sqrt{\dim(\mathcal{R})} (-1)^{J_3-Y'/2} D_{Y,T,T_3;Y',J,J_3}^{(R)*}(R).$$  \hspace{1cm} (4)

Here $\mathcal{R}$ stands for the allowed irreducible representations of the SU(3) flavor group, i.e. $\mathcal{R} = 8, 10, 10^* \cdots$ and $Y, T, T_3$ are the corresponding hypercharge, isospin, and its third component, respectively. Right hypercharge $Y'$ is constrained to be unity for the physical spin states for which $J$ and $J_3$ are spin and its third component. Note that under the action of left (flavor) generators $\hat{T}_a = -D_{\alpha\beta}^{(8)} \hat{J}_\beta \psi_{BR}$ transforms like a tensor in representation $\mathcal{R}$, while under the right generators $\hat{J}_a$ like a tensor in $\mathcal{R}^*$ rather than $\mathcal{R}$. This is the reason why operators like the one multiplied by $v_2$ in Eq.(1) have different matrix elements for the decuplet (which is spin 3/2) and antidecuplet (which is spin 1/2). The other two operators multiplied by $v_{1,3}$ have the same matrix elements between decuplet and antidecuplet states.

Using Eq.(4) and the known formulae [36] for the action of $V_\pm = \hat{J}_4 \pm i \hat{J}_5$, $U_\pm = \hat{J}_6 \pm i \hat{J}_7$

$$U_+ \leftrightarrow V_+$$

$$V_- \leftrightarrow U_-$$
on the antidecuplet ($\mathcal{R} = (0, q + 2)$ with $q = 1$) states, we can easily calculate the collective matrix elements of the magnetic moments of the baryon antidecuplet in Eq.(3). The matrix elements of Eq.(3) are expressed just in terms of SU(3) Clebsch-Gordan coefficients [36]. For example, the first term of the magnetic moment in Eq.(1) in the case of $\Theta^+$ can be calculated as follows:

$$\int dR \psi_{\Theta^+}(R) D_{Q3}^{(8)}(R) \psi_{\Theta^+}(R) = 10 \int dR D_{Q3}^{(10)*}(R) D_{Q3}^{(8)}(R) D_{Q3}^{(10)}(R) = 10 \int dR D_{Q3}^{(10)*}(R) D_{Q3}^{(8)}(R) D_{Q3}^{(10)}(R) = \frac{1}{2} \left[ \left( \begin{array}{c|c} 8 & 10 \\ \hline 0 & 200 \\ \frac{1}{\sqrt{3}} & 200 \end{array} \right) \right] \left( \begin{array}{c|c} 8 & 10 \\ \hline 0 & 200 \\ \frac{1}{\sqrt{3}} & 200 \end{array} \right) = \frac{1}{24}.$$  \hspace{1cm} (5)

We can compute all relevant matrix elements in a similar manner. Having scrutinized the results, we find the following simple expression:

$$\mu_{B_{10}} = -\frac{1}{12} \left( v_1 + \frac{5}{2} v_2 - \frac{1}{2} v_3 \right) Q_{B_{10}} J_3,$$  \hspace{1cm} (6)
where $Q_{B_{10}}$ is the charge of the antidecuplet expressed by the Gell-Mann–Nishijima relation:

$$Q_{B_{10}} = T_3 + \frac{Y}{2}. \quad (7)$$

$J_3$ is the corresponding third component of the spin.

In order to fit the parameters $v_i$, it is convenient to introduce two parameters consisting of $v_1$, $v_2$ and $v_3$:

$$v = \frac{1}{60} (v_1 - \frac{1}{2} v_2), \quad w = \frac{1}{120} v_3. \quad (8)$$

In Ref. [26] the octet and decuplet magnetic moments were expressed as follows:

$$\begin{align*}
\mu_p &= \mu_{\Sigma^+} = -8v + 4w, \\
\mu_n &= \mu_{\Xi^0} = 6v + 2w, \\
\mu_\Lambda &= -\mu_{\Xi^0} = 3v + w, \\
\mu_{\Sigma^-} &= \mu_{\Xi^-} = 2v - 6w, \\
\mu_{B_{10}} &= \frac{15}{2} (-v + w) Q_{B_{10}}.
\end{align*} \quad (9)$$

which are in fact the well-known SU(3) formulae for the magnetic moments.

The magnetic moments of the baryon antidecuplet (6) can be rewritten as:

$$\mu_{B_{10}} = \left[ \frac{5}{2} (-v + w) - \frac{1}{8} v_2 \right] Q_{B_{10}}. \quad (10)$$

Interestingly, the magnetic moments of the antidecuplet are different from those of the decuplet by the second term in Eq.(10). The factor three difference in the first term between Eq.(9) and Eq.(10) is due to the fact that the baryon antidecuplet has spin $1/2$, while the decuplet has $3/2$. Additional term including $v_2$ appears due to the different action of the second term in Eq.(1) on spin $1/2$ and $3/2$ states.
Using Eq. (10), we can derive the sum rules which are similar to the generalized Coleman and Glashow sum rules [29] in the chiral limit:

\[
\mu_{\Sigma^0_{10}} = \frac{1}{2} \left( \mu_{\Sigma^+_{10}} + \mu_{\Sigma^-_{10}} \right),
\]

\[
\mu_{\Xi^+_{3/2}} + \mu_{\Xi^-_{3/2}} = \mu_{\Xi^0_{3/2}} + \mu_{\Xi^-_{3/2}},
\]

\[
\sum \mu_{\Xi^0_{10}} = 0.
\]

(11)

As discussed in Ref. [26], there are different ways to fix the parameters \(v\) and \(w\) by using the experimental data of the octet magnetic moments. Here, we simply fit the proton and neutron magnetic moments (fit I):

\[
v = \frac{(2\mu_n - \mu_p)}{20} = -0.331,
\]

\[
w = \frac{(4\mu_n + 3\mu_p)}{20} = 0.037,
\]

(12)

and use the following "average" values (fit II):

\[
v = \frac{(2\mu_n - \mu_p + 3\mu_{\Xi^0} + \mu_{\Xi^-} - 2\mu_{\Sigma^-} - 3\mu_{\Sigma^+})}{60} = -0.268,
\]

\[
w = \frac{(3\mu_p + 4\mu_n + \mu_{\Xi^0} - 3\mu_{\Xi^-} - 4\mu_{\Sigma^-} - \mu_{\Sigma^+})}{60} = 0.060.
\]

(13)

to fix parameters \(v\) and \(w\). It was shown in Ref. [26] that combinations of Eq. (13) are independent of the linear corrections due to the nonzero strange quark mass \(m_s\). Thus, fit II is also valid when the SU(3)-symmetry breaking is taken into account, while fit I will be changed by the corrections of order \(O(m_s)\). The results of these fits are listed in Table I.

|       | exp. | fit I | fit II | \(\chi_{\text{QSM}}\) |
|-------|------|-------|--------|----------------------|
| \(p\) | 2.79 | 2.39  | 2.27   | 0.05                 |
| \(n\) | -1.91| -1.49 | -1.55  | 0.17                 |
| \(\Lambda\) | -0.61 | -0.96 | -0.74 | -0.78 |
| \(\Sigma^+\) | 2.46 | 2.79 | 2.38 | 2.27 |
| \(\Sigma^0\) | (0.65) | 0.96 | 0.74 | 0.78 |
| \(\Sigma^-\) | -1.16 | -0.89 | -0.90 | -0.71 |
| \(\Xi^0\) | -1.25 | -1.91 | -1.49 | -1.55 |
| \(\Xi^-\) | -0.65 | -0.89 | -0.90 | -0.71 |
| \(\Delta^{++}\) | 4.52 | 5.52 | 4.92 | 4.47 |
| \(\Omega^-\) | -2.02 | -2.76 | -2.46 | -2.23 |
| \(\Theta^+\) | ? | 0.30 | 0.20 | 0.12 |

We see that the quality of these fits is rather poor reaching in its worst case about 25% accuracy, which indicates the importance of the SU(3)-symmetry breaking corrections.

For the baryon antidecuplet we cannot predict the magnetic moments unambiguously, because they depend on the new parameter \(v_2\), unknown from nonexotic baryons. Therefore, we have to resort to model calculations (2) of parameters \(v_{1,2,3}\) which can be found in the literature [30, 31, 32]:

\[
v = -0.264, \ w = 0.029, \ v_2 \sim 5.
\]

(14)

We see that these parameters are in a rough agreement with the model-independent analysis. It is therefore reasonable to assume that \(v_2\) is approximately equal to 5. With this assumption
we get the predictions for the $\Theta^+$ magnetic moment which are listed in Table I. We see that in all three cases our prediction for $\Theta^+$ magnetic moment is very small, almost order of magnitude smaller than the magnetic moments for charge $Q = 1$ particles both in the octet and decuplet. Even if we take into account a 25% error, a typical error of the SU(3) symmetry fits, we still face yet another facet of exoticness of the baryon antidecuplet: excessively small magnetic moments.

The smallness of the $\Theta^+$ magnetic moment is due to the cancellation of two terms in Eq.(10) $(-v + w)$ (which is positive) and $v_2$. This is very similar to the cancellation which occurs in the case of the $\Theta^+$ width [2]. For these three fits discussed above we have:

$$
\begin{align*}
\mu_{\Theta^+} &= 0.92 - v_2/8 \quad \text{fit I,} \\
\mu_{\Theta^+} &= 0.82 - v_2/8 \quad \text{fit II,} \\
\mu_{\Theta^+} &= 0.75 - v_2/8 \quad \chi\text{QSM.}
\end{align*}
$$

(15)

Therefore, even if the model prediction for $v_2$ has 50% error, the largest value for $\mu_{\Theta^+}$ which we can get is the one from fit I for $v_2 = 2.5$, i.e. $\mu_{\Theta^+} = 0.61$, still a pretty small number.

The expression of Eq.(13) leads to the additional sum rule for the magnetic moments of the baryon antidecuplet:

$$
\mu_{\Theta^+} - \mu_{\Xi^{-}_{1/2}} = \frac{1}{4}(2\mu_p + \mu_n + 2\mu_{\Sigma^+} - \mu_{\Sigma^-} - \mu_{\Xi^0} - 2\mu_{\Xi^-}),
$$

(16)

which is very similar to Eq.(1) in Ref. [26].

3. If in the $\chi$QSM one artificially sets the soliton size $r_0 \to 0$, then the model reduces to the free valence quarks which, however, "remember" the soliton structure. In this limit, many quantities, for example the axial-vector couplings, are given as ratios of the group-theoretical factors [34]. In the case of magnetic moments the pertinent expressions are given as a product of the group-theoretical factor and the model-dependent integral which we shall in what follows denote by $K$ [35].

Constants $v_{1,2,3}$ entering Eq.(1) are expressed in terms of the inertia parameters in the following way

$$
\begin{align*}
v_1 &= M_0 - \frac{M_1^{(-)}}{I_1^{(+)}}, \quad v_2 = -2\frac{M_2^{(+)}}{I_2^{(+)}}, \quad v_3 = -2\frac{M_1^{(+)}}{I_1^{(+)}},
\end{align*}
$$

(17)

For the soliton size $r_0 \to 0$ we have [35]:

$$
\begin{align*}
M_0 \to -2K, \quad \frac{M_1^{(-)}}{I_1^{(+)}} \to \frac{4}{3}K, \quad \frac{M_2^{(+)}}{I_2^{(+)}} \to -\frac{4}{3}K, \quad \frac{M_1^{(+)}}{I_1^{(+)}} \to -\frac{2}{3}K,
\end{align*}
$$

(18)

which give

$$
\begin{align*}
v &= -\frac{7}{90}K, \quad w = \frac{1}{90}K, \quad v_3 = \frac{4}{3}K,
\end{align*}
$$

(19)

yielding the magnetic moments of the proton and neutron as follows:

$$
\begin{align*}
\mu_p = \frac{2}{3}K, \quad \mu_n = -\frac{4}{9}K.
\end{align*}
$$

(20)
Hence, the ratio of the proton magnetic moment to the neutron one takes the value from the nonrelativistic quark model:

$$\frac{\mu_p}{\mu_n} = -\frac{2}{3}. \quad (21)$$

For antidecuplet magnetic moments we get

$$\mu_{B_{\pi \pi}} = -\frac{1}{3} K Q_{B_{\pi \pi}} \quad (22)$$

which differs in sign from the phenomenological value of Table I (note that $K$ is positive in view of Eq.(20)). This is caused by the large value of $v_3$ in the quark-model limit. It would be interesting to see what the quark models discussed in the literature give for the magnetic moment of $\Theta^+$.  

4. In the present work, we determined the magnetic moments of the baryon antidecuplet in a “model independent” analysis, based on the chiral quark-soliton model in the chiral limit. Starting from the collective operators with dynamical parameters fixed by experimental data, we were able to obtain the magnetic moments of the baryon antidecuplet up to one unknown constant which we have estimated from the model calculations of Refs.[30, 31, 32]. The expression for the magnetic moments of the antidecuplet is different from those of the baryon decuplet. We found that the magnetic moment of $\mu_{\Theta^+}$ is about $0.2 \sim 0.3 \mu_N$ which is surprisingly small and is in line with the recent result of Ref. [19].  

In the present letter, we have worked in the chiral limit. The SU(3)-symmetry breaking effects will definitely make the magnetic moments of the baryon antidecuplet deviate from those of the present paper. There are two different sources of the SU(3)-symmetry breaking effects: one comes from the collective operator, the other arises from the fact that the collective wave functions of the baryon antidecuplet are mixed with the octet and eikosiheptaplet representations. Moreover, nonanalytical symmetry breaking effects are of importance [37]. The effect of the SU(3)-symmetry breaking on the magnetic moments of the antidecuplet baryons is under investigation, however, our previous experience shows that these effects do not exceed 25%. Therefore, we expect that the overall conclusion that the antidecuplet magnetic moments are small will remain unchanged.

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