Manipulation of optical memory bits in atomic vapors and Bose-Einstein condensates

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Abstract

We provide an exact analytic description of decelerating, stopping and re-accelerating optical solitons in atomic media. By virtue of this solution we describe in detail how spatially localized optical memory bits can be written down, read and moved along the atomic medium in a prescribed manner. Dynamical control over the solitons is realized via a background laser field whose intensity controls the velocity of the slow light in a similar way as in the linear theory of electromagnetically induced transparency (EIT). We solve the nonlinear model when the controlling field and the solitons interact in an inseparable nonlinear superposition process. This allows us to access results beyond the limits of the linear theory of EIT.
Today, significant theoretical and experimental efforts are made to better understand the physics of light propagation in atomic vapors and Bose-Einstein condensates whose interaction with light is well described by the nonlinear Λ-model. This process is accompanied by striking effects such as electromagnetically induced transparency [1], which permits the propagation of light through an otherwise opaque medium; slow-light phenomena when the pulse is slowed down to velocities of a few meters per second [2], and optical information storage when the pulse is completely stopped inside the dielectric [3, 4]. Even though the linear approach to describing these effects is developed in detail, it has reached its validity limits, because modern experiments require more adequate nonlinear descriptions. However, the theory of the nonlinear regime is still very incomplete.

We investigate the problem of control over optical memory bits created by the slow-light solitons. Our theoretical model corresponds to a gas of alkali atoms, whose interaction with light is approximated by the structure of levels of the Λ-type (see Ref. [1]). The two lower levels are the states \(|1\rangle\) and \(|2\rangle\), while \(|3\rangle\) is the excited state. The medium is described by the $3 \times 3$ density matrix $\rho$ in the interaction picture. In order to cancel the residual Doppler broadening, two optical beams are chosen to be co-propagating. The fields are described by the Rabi-frequencies $\Omega_{a,b}$. The field $\Omega_a$ corresponds to $\sigma^-$ polarization, while the second $\Omega_b$ corresponds to $\sigma^+$ polarization. Within the slowly varying amplitude and phase approximation (SVEPA), dynamics of the atom-field system is well described by the reduced Maxwell-Bloch equations [5]:

$$\partial_\tau \rho = i \left[ \frac{\Delta}{2} D - H_I, \rho \right].$$

Here $\zeta = (z - z_0)/c, \tau = t - (z - z_0)/c, \Delta$ is the detuning of the resonance, and $\nu_0$ is the coupling constant. The matrix $H_I = -\frac{1}{2} (\Omega_a |3\rangle \langle 1| + \Omega_b |3\rangle \langle 2|) + h.c.$ represents the interaction Hamiltonian.

The system of equations Eq.(1) is exactly solvable in the framework of the Inverse Scattering (IS) method [5, 6, 7, 8]. We define the initial and boundary conditions for the system of equations Eq.(1) as follows. We consider a semi-infinite $\zeta \geq 0$ active medium with a pulse of light incident at the point $\zeta = 0$ (initial condition). This means that the evolution
is considered with respect to the space variable $\zeta$, while the boundary conditions should be specified with respect to the variable $\tau$. The boundary conditions are defined by the asymptotic values of the density matrix $\rho$ and the matrix $H_I$ at $\tau \to \pm \infty$.

In our previous work [5] we proposed a method to realize quantum memory bits in BEC by slow-light optical solitons. There we reported a slow-light soliton solution on a constant background field $\Omega_0$, which was created by the auxiliary laser on entrance into the medium ($\zeta = 0$). The group velocity of the slow-light soliton depends explicitly on the field $\Omega_0$. For the time-dependent background field some approximate solutions based on the methods of scaled time are studied in [3, 9, 10, 11]. In a simplifying approximation the velocity is $v_g \approx c \frac{\Omega_0^2}{2 \nu_0}$. This expression immediately suggests that the soliton can be stopped through switching off the controlling laser field [5, 12]. In the present paper we provide a full analytical description of how one can perform controlled preparation, manipulation and readout of optical memory bits formed in atomic vapors and Bose-Einstein condensates by slow-light solitons.

Specifically, we consider a gas of rubidium atoms. To make parameters dimensionless, we measure the time in units of optical pulse length $t_p = 1 \mu s$ typical for the experiments on the slow-light phenomena in rubidium vapors [3, 13]. The retarded time $\tau$ is measured in microseconds and the Rabi frequencies are normalized to MHz. The spatial coordinate is normalized to the spatial length of the pulse slowed down in the medium, i.e. $l_p = v_g t_p \approx c \frac{\Omega_0^2}{2 \nu_0} t_p$. In experiments with rubidium the controlling background field is of order of a few megahertz. We choose $\Omega_0 = 3$. This corresponds to the group velocity of several meters per second, depending on the density of the atoms. We take the group velocity to be $10^{-7}c$, so the pulse spatial length is $30 \mu m$, and $\zeta$ is normalized to $10^{-13}s$. Then, in the dimensionless units, the coupling constant $\nu_0 = \frac{\Omega_0^2}{2} = 4.5$.

We consider the following scenario for the dynamics (see Fig.1). To begin with, in a remote past we create in the medium a slow-light soliton, and assume it is propagating on the constant background $\Omega_0$. In the next step, we slow down the soliton by switching off the laser source of the background field. We assume an exponential decay of the background field with a decay constant $\alpha$, i.e. $\Omega_0 e^{-\alpha \tau}$. At a certain, say $T_1 = 4/\alpha$, the field becomes negligible. Therefore, we cut off the exponential tail and approximate it by zero. At this step the soliton is completely stopped. The position where the soliton stops, depends on the decay constant $\alpha$ and on the moment when we switch off the laser [12]. The process of
FIG. 1: Time-dependence of the controlling background field. We choose $\alpha = 4$, and the delay interval is $T - T_1 = 3$. The dimensionless units are defined in the text.

stopping the soliton corresponds to writing a memory bit into the medium. The information borne by the soliton is stored in the form of spatially localized polarization. This formation can live a relatively long period of time in atomic vapours or BEC. At the moment of time $T$ we restore the slow-light soliton by abruptly switching on the laser. The whole dynamics is divided into four time intervals

$$
\bigcup_{i=0}^{3} \mathcal{D}_i = (-\infty, 0) \cup [0, T_1] \cup (T_1, T) \cup (T, \infty).
$$

The time-dependence of the intensity of the background field at entrance into the medium is given in Fig.1.

Before the soliton enters the medium, the atoms are assumed to be in the state $|1\rangle$. Notice that this state is a dark-state for the controlling field, which means that the atoms do not interact with the field created by the auxiliary laser. We specify the initial state of the system as follows:

$$
\Omega_a^{(0)} = 0, \quad \Omega_b^{(0)} = \Omega(\tau), \quad |\psi_{at}\rangle = |1\rangle.
$$

This configuration corresponds to a typical experimental setup (see e.g. [2, 3, 14]). The function $\Omega(\tau)$ is given in Fig.1. The state Eq.(2) satisfies the Maxwell-Bloch equations Eqs.(1). Using the methods of our previous works [5, 12], we construct the one-soliton solution corresponding to the background profile (see Fig.1), viz.

$$
\tilde{\Omega}_a = \frac{(\lambda^* - \lambda)w(\tau, \lambda)}{\sqrt{1 + |w(\tau, \lambda)|^2}} e^{i\tilde{\phi}_s} \text{sech}\tilde{\phi}_s,
$$

$$
\tilde{\Omega}_b = \frac{(\lambda - \lambda^*)w(\tau, \lambda)}{1 + |w(\tau, \lambda)|^2} e^{i\tilde{\phi}_s} \text{sech}\tilde{\phi}_s - \Omega(\tau),
$$

\[\frac{\lambda - \lambda^*}{1 + |w(\tau, \lambda)|^2} e^{i\tilde{\phi}_s} \text{sech}\tilde{\phi}_s - \Omega(\tau),\]
with the atomic state $\tilde{\rho} = |\tilde{\psi}_{at}\rangle\langle\tilde{\psi}_{at}|$, where

$$
|\tilde{\psi}_{at}\rangle = \frac{\text{Re}\lambda - \Delta - i\text{Im}\lambda \tanh \tilde{\phi}_s}{|\lambda - \Delta|} |1\rangle + \frac{\tilde{\Omega}_a}{2|\lambda - \Delta| w(\tau, \lambda)} |2\rangle - \frac{\tilde{\Omega}_a}{2|\lambda - \Delta|} |3\rangle.
$$

(4)

Here,

$$
\tilde{\phi}_s = \tilde{\phi}_0 + \frac{\nu_0 \zeta}{2} \text{Im} \frac{1}{\lambda - \Delta} + \text{Re}(z(\tau, \lambda)) + \ln \sqrt{1 + |w(\tau, \lambda)|^2},
$$

$$
\tilde{\theta}_s = \tilde{\theta}_0 - \frac{\nu_0 \zeta}{2} \text{Re} \frac{1}{\lambda - \Delta} + \text{Im}(z(\tau, \lambda)),
$$

where $\lambda$ is an arbitrary complex parameter. The functions $w(\tau, \lambda)$, $z(\tau, \lambda)$ are of piecewise form, specific to each time region $D_i$.

| $D$ | $\Omega(\tau)$ | $w(\tau, \lambda)$ | $z(\tau, \lambda)$ | $C$ |
|---|---|---|---|---|
| $D_0$ | $\Omega_0$ | $w_0$ | $i\Omega_0 w_0 \tau$ | 0 |
| $D_1$ | $\Omega_0 e^{-\alpha \tau}$ | $\frac{i}{e} J_{\lambda-\gamma} \left(\frac{\Omega(\tau)}{\alpha}\right) - J_{\gamma-\alpha} \left(\frac{\Omega(\tau)}{\alpha}\right)$ | $-\alpha \gamma \tau + \ln \frac{\frac{\Omega(\tau)}{\alpha}}{J_{\lambda-\gamma}\left(\frac{\Omega(\tau)}{\alpha}\right) + J_{\gamma-\alpha}\left(\frac{\Omega(\tau)}{\alpha}\right)}$ | $-\Omega_0 J_{\lambda-\gamma}(-\frac{\Omega_0}{\alpha}) + J_{\gamma-\alpha}(-\frac{\Omega_0}{\alpha})$ |
| $D_2$ | 0 | 0 | $\ln \frac{c}{e^{-\alpha \tau}} \frac{\Omega(\tau)}{\alpha}$ | $C_2 = C_1$ |
| $D_3$ | $\Omega_0$ | $\frac{\Omega_0 \tan \left(\frac{\Omega_0}{\alpha}\right) \gamma \tau - \Omega_0}{\lambda \tan \left(\frac{\Omega_0}{\alpha}\right) \gamma \tau - i \sqrt{\lambda^2 + \Omega_0^2}}$ | $\frac{\Omega_0}{\alpha} \frac{1}{e^{-\alpha \tau}} \frac{\Omega(\tau)}{\alpha} - \left(\frac{\Omega_0}{\alpha} \frac{1}{e^{-\alpha \tau}} \frac{\Omega(\tau)}{\alpha} \right)$ | $\frac{\Omega_0^2 + 2\lambda (\lambda - \sqrt{\lambda^2 + \Omega_0^2})}{\Omega_0^2}$ |

For clarity we organize elements of the solution corresponding to different time regions in Table I. We use an auxiliary function $w_0 = \Omega_0/(\lambda + \sqrt{\lambda^2 + \Omega_0^2})$, the index of Bessel functions is defined as $\gamma = (\alpha + i\lambda)/(2\alpha)$. The values $C_i$ of the constant $C$ for each time region $D_i$ are specified in the rightmost column of the table, the moment of time $T$ is chosen as in Fig I i.e. $T = 4/\alpha + 3$. Notice that in the table $w_2 = w_1(\infty, \lambda)$ and $z_2 = z_1(\infty, \lambda)$. Therefore the solution in the region $D_2$ is parameterized by the asymptotic values of the data for the region $D_1$ corresponding to the absence of cut-off of the exponentially vanishing tail. The region $D_2$ describes the phase when the slow-light soliton is stopped, the fields vanish, while the information borne by the soliton is stored in the medium in the form of spatially localized polarization. At the moment of time $T$ the laser is instantly turned on again. The stored localized polarization then generates a moving slow-light soliton. This process is described by the solution in the region $D_3$. Except for the point $T_1$, the functions
$w, z$ are continuous in $\tau$. This ensures that the physical variables such as the wave-function and field amplitudes evolve continuously.

![Contour plot of the intensity of $\tilde{\Omega}$](image)

**FIG. 2:** Contour plot of the intensity of $\tilde{\Omega}_a$. We choose $\lambda = -4.1i$ and zero detuning, $\Delta = 0$. The break-up area in between the two solitonic trails manifests the creation of a standing memory bit in the medium.

We demonstrate typical dynamics of the intensity of the field in the channel $a$ in Fig.2. This contour plot shows that in the process of rapid deceleration the solitonic trail forms end sharply. It is interesting to note that the restored pulse, however, has the same characteristics, i.e. the width and group velocity, as the input signal existed in the medium before the stopping. The dynamics of the corresponding localized polarization is presented in Fig.3. Notice that in the presence of the soliton the population flip from level $|1\rangle$ to $|2\rangle$ in the center of the peak is almost complete. A small fraction of the total population is located in the upper level $|3\rangle$ and provides for some atom-field interaction. In [5] we show that the population of level $|3\rangle$ is proportional to $|\Omega_0|^2$. Therefore when the field vanishes the population of the second level reaches unity, while the destructive influence of relaxation becomes negligible. This behavior of the system points to a sound possibility for realizing optical memory.

To conclude, with an exactly solvable example, we have demonstrated a possibility to manipulate memory bits by slow-light solitons. We have proposed a method to prepare, control and read out optical memory bits in atomic vapors and Bose-Einstein condensates.
FIG. 3: Population of level $|2\rangle$. Here, $\lambda = -4.1i$ and $\Delta = 0$. The time interval $1 \leq t \leq 4$ corresponds to a standing localized polarization flip.

in the regime of strong nonlinearity.

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