Constraining extra dimensions on cosmological scales with LISA future gravitational wave siren data

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Abstract. The Universe is undergoing a late time acceleration. We investigate the idea that this acceleration could be the consequence of gravitational leakage into extra dimensions on cosmological scales rather than the result of a non-zero cosmological constant, and consider the ability of future gravitational-wave (GW) siren observations to probe this phenomenon and constrain the parameters of phenomenological models of this gravitational leakage. A gravitational space interferometer such as LISA will observe massive black hole binary (MBHB) merger events at cosmological distances, and will also provide sky localization information that may permit optical and other electromagnetic (EM) surveys to identify an EM counterpart of these events. In theories that include additional non-compact spacetime dimensions, the gravitational leakage into extra dimensions leads to a reduction in the amplitude of observed gravitational waves and thereby a systematic discrepancy between the distance inferred to such sources from GW and EM observations. We investigate the capability of LISA to probe this modified gravity on large scales, specifically in Dvali, Gabadadze and Porrati (DGP) model. Additionally, we include a SNeIa sample at lower redshift in order to explore the efficacy of this cosmological probe across a range of redshifts. We use previously published simulated catalogues of cosmologically distant MBHB events detectable by LISA, and which are likely to produce an observable EM counterpart. We find that the extent to which LISA will be able to place limits on the number of spacetime dimensions and other cosmological parameters characterising modified gravity will strongly depend on the actual number and redshift distribution of sources, together with the uncertainty on the GW measurements. A relatively small number of sources ($\sim 1$) and high measurement uncertainties would strongly restrict the ability of LISA to place meaningful constraints on the parameters in cosmological scenarios where gravity is only five-dimensional and modified at scales larger than about $\sim 4$ times the Hubble radius. Conversely, if the number of sources observed amounts to a four-year average of $\sim 27$, then in the most favourable cosmological scenarios LISA has the potential to place meaningful constraints on the cosmological parameters with a precision of $\sim 1\%$ on the number of dimensions and $\sim 7.5\%$ on the scale beyond which gravity is modified, thereby probing the late expansion of the universe up to a redshift of $\sim 8$, i.e. on scales not yet tested by present EM observations.
1 Introduction

The expansion of the universe is undergoing a late time acceleration, the reality of which was for the first time directly confirmed in the late 1990s by surveys of cosmologically distant Type 1a supernovae and then further supported with more recent SNIa observations [1, 2]. Low-redshift data of baryon acoustic oscillations (BAO) [3], CMB data from the Planck Collaboration et.al [4] and data from other cosmological probes provide further, independent support for an accelerating universe.

In the standard model of cosmology where General Relativity (GR) is the underlying framework, this acceleration may be explained by a non-zero but very small cosmological constant, $\Lambda$, that opposes the self-attraction of pressureless matter and causes the expansion of the universe to accelerate at the largest scales [5]. Although the $\Lambda$-Cold-Dark-Matter model ($\Lambda$CDM) is very successful in explaining almost all observations, it has some theoretical issues. These involve the mysterious physical origin of the two largest contributions to the energy content of the late-time universe: cold dark matter (CDM) and the cosmological constant ($\Lambda$), together with the unsatisfactory predictivity and testability of the inflation theory [6]. Therefore it is important to explore alternative explanations for the late-time acceleration of the universe. There are several proposals in the literature in this regard, e.g. [7] presented a study using data from the Hubble Space Telescope and the large-scale galaxy distribution data to demonstrate the disadvantages of $\Lambda$CDM in specific cosmic acceleration scenarios over the redshift range of these observations, thus providing the motivation to approach other modified models such as the Randall-Sundrum proposal, wherein GW data set new constraints on extra dimensional cosmological parameters [8]. Therefore, in this work we will focus on the
Dvali, Gabadadze and Porrati (DGP) braneworld model proposed in [9], in which our observed four-dimensional universe is considered to be a surface (called a brane) embedded in an infinite-volume, five-dimensional space (called the bulk). This proposed model explains the late time acceleration of the expansion of the universe through a large-scale modification of gravity arising from the slow evaporation of gravitational degrees of freedom into the infinite-volume extra dimension, and without requiring a non-zero cosmological constant. The reason for first studying the DGP model is that we will then borrow notions from it to consider cosmological models including additional non-compact spacetime dimensions.

Until now, by precisely mapping the distance-redshift relation up to a redshift of $z \sim 1$, Type Ia SNe measurements were the most sensitive probe of the late time acceleration of the universe. Another potentially very powerful probe of the universe are GWs emitted from merging binary black holes and neutron stars. In particular, since the propagation of gravitons and photons could differ at a fundamental level, we argue here that GWs emitted by coalescing pairs of massive black holes, at the centre of distant galaxies, observed with an EM counterpart may be used as an alternative way to test modified theories of gravity. To measure GWs emitted by these massive black hole binary (MBHB) mergers, with high signal to noise ratio (SNR) and in the previously unobserved redshift range $1 < z < 8$, ESA and NASA will build the Laser Interferometer Space Antenna (LISA [1]). Such GW sources can be thought as standard sirens (gravitational analogues of standard candles such as SNIe) in the sense that the GWs emitted by a compact binary directly encode information about the gravitational-wave luminosity of the binary system, and thus its luminosity distance [10]. Still, MBHBs have the big disadvantage that the redshift needs to be measured independently, e.g. by optically identifying the host galaxy. However, if we do successfully identify the host galaxy then one question we might be able to answer with such joint events is whether long-wavelength gravitational waves and short-wavelength EM radiation experience the same number of spacetime dimensions [11]. In higher dimensional theories such as the DGP model, as the GWs propagate through spacetime, they leak into the extra dimensions, leading to the effect that cosmologically distant sources appear dimmer than they truly are – hence resulting in a systematic error in the inferred distance to the gravitational wave source. Assuming, as is the case for the DGP model, that light and matter propagate in four spacetime dimensions only, EM waves remain unaffected. Hence, if we can make an independent measurement of the luminosity distance to the GW source by measuring the redshift of its EM counterpart then this allows us to place limits on the gravitational leakage.

In this paper we investigate the consequences of higher dimensional theories with non-compact extra dimensions, borrowing notions from the DGP model to forecast the capability of LISA to constrain theories with gravitational leakage from gravitational wave standard sirens with an observable EM counterpart. We find that LISA’s ability to set bounds on the number of additional non-compact spacetime dimensions and the other parameters characterising the theory, namely the screening scale and the transition steepness, will depend strongly on the actual redshift distribution of massive black hole binary merger events, the corresponding efficiency in identifying their host galaxy and the uncertainty on these measurements. Also, to compare our forecast at high redshift, we perform a DGP test with the standard Pantheon SNeIa sample at low redshifts, in the sense that this test will be our pivot model in order to carry out a model selection dependent on the number of dimensions and screening scale.

This paper is organised as follows: in Sec.2 we introduce the dynamics of the background metric of the universe in the DGP model. Borrowing notions from the DGP model, in Sec.3 we write expressions for the luminosity distance expressions for supernovae and GW standard sirens, and then describe the effects of gravitational leakage on the GW waveform in higher-dimensional theories with non-compact dimensions. In Sec.4 we investigate the predicted ability of LISA to constrain higher-dimensional gravity. First, in Sec.4.1 we summarise the properties of the MBHB population catalogues that we use to simulate realistic GW data. In Sec.5 we then briefly review our statistical methods and perform the model selection comparison on the Pantheon SNe sample, using the DGP model as a pivot model. Finally, in Sec.6 we present and analyse our results for the predicted LISA constraints on the parameters characterising higher-dimensional theories. In addition, we present the analyses

\hspace{1cm}1http://lisamission.org/
for the joint Pantheon and LISA samples. Our main conclusions are then presented and summarised in Sec.7.

2 Extra-dimensional theory: DGP cosmological solutions

In the DGP braneworld theory, matter and all standard model forces and particles are pinned to a $(3 + 1)$-dimensional brane universe, while gravity is free to explore the full five-dimensional empty bulk. For the purposes of our analysis we are mainly interested in the geometry of the 4-D braneworld universe, which (assuming a homogeneous, isotropic expanding universe) is at all times described by a standard Friedmann-Lemaitre-Robertson-Walker (FLRW) metric

$$ds^2 = -d\tau^2 + a^2(\tau)\gamma_{ij}d\lambda^i d\lambda^j,$$

where the chosen coordinates are the cosmological time $\tau$ and the spatial comoving coordinates of the observable universe $\lambda^i$. The parameter $a(\tau)$ corresponds to the familiar scale factor of the four-dimensional cosmology in our braneworld, $d\psi^2$ is an angular line element, $k = -1, 0, 1$ parametrizes the spatial curvature of the brane universe and $S_k(r)$ is given by

$$S_k(r) = \begin{cases} 
\sin r & (k = 1), \\
\sinh r & (k = -1), \\
r & (k = 0).
\end{cases}$$

Notice that we recover the standard four-dimensional Friedmann equation from (2.3) when whenever $\rho/M_P^2$ is large compared to $1/r_0^2$. However, at late times we may no longer assume $\rho/M_P^2 >> 1/r_0^2$ and the brane universe has a generically different behaviour from the ordinary four-dimensional cosmology. Depending on the sign of $\epsilon$ in the modified Friedmann equation (2.3) one can identify and distinguish two distinct late time behaviours. Firstly, in the case where $\epsilon = -1$, it was shown in [13] that $a(\tau)$ diverges at late times such that the energy density $\rho$ is driven to smaller values and reaches a regime where it is small in comparison with $M_P^2/r_0^2$. One then has a transition to a pure five-dimensional regime where the Hubble scale parameter is linear in the energy density $\rho$. This phase is referred to as the FLRW phase and will not be the subject of our present analysis.

On the other hand, if $\epsilon = +1$, we find that as the energy density crosses the threshold $M_P^2/r_0^2$ one has a transition from the usual four-dimensional FLRW cosmology to a brane, self-inflationary solution where the Hubble scale parameter is approximately constant. Thus, the cosmology of DGP gravity provides an alternative explanation for the contemporary cosmic acceleration. In terms of the Hubble scale, if we neglect the spatial curvature term, $H(\tau)$ is found to evolve according to the standard Friedmann equation at early times and when $H(\tau)$ is large, but is modified when $H(\tau)$ becomes comparable to $r_0^{-1}$. In the case of choosing a cosmological solution associated with $\epsilon = +1$
in (2.3) and assume that the crossover scale is of the order of the Hubble radius \( H_0^{-1} \), where \( H_0 \) is today’s Hubble constant, we will obtain a scenario where DGP gravity could explain the current cosmic acceleration, despite the fact there is no cosmological constant on the brane, through the existence of an infinite-volume extra dimension which modifies the laws of gravity at distances larger than \( r_0 \).

The set of Eqs. (2.3)-(2.4) may be used to characterise the cosmology of the DGP model and connect it to observational constraints of the real \((3+1)\)-dimensional world. Therefore, we will first briefly review dark energy models in order to carry out this possible connection.

In dark energy cosmological models, we assume that GR is still valid and that the late-time acceleration phase of the cosmic expansion is driven by a cosmological constant component with sufficient negative pressure, the so-called dark energy, interpreted as a dark fluid whose energy density is \( \rho_{DE} \) and pressure \( p_{DE} \). The content of the universe is then related to its expansion through the standard Friedmann equation. The total energy density is given by

\[
\rho = \rho_M + \rho_{DE},
\]

where in the \( \Omega_M = \frac{\rho_M}{\rho} \) is specified, one may define the cosmological parameters \( \Omega_M, \Omega_{DE} \) and \( \Omega_k \) in terms of the redshift \( 1 + z \equiv a_0/a \) as usual:

\[
\Omega_M = \Omega_M^0 (1 + z)^3, \quad \Omega_{DE}^0 = \frac{8\pi G \rho_M^0}{3H_0^2}, \quad \Omega_k = -\frac{k}{H_0^2 a_0^2}.
\]

The Friedmann equation can be re-written as

\[
H^2 = H_0^2 \left[ \Omega_k (1 + z)^2 + \Omega_M^0 (1 + z)^3 + \Omega_{DE}^0 (1 + z)^3(1 + w) \right],
\]

where in the \( \Lambda \)CDM model \( \Omega_{DE}^0 (1 + z)^3(1 + w) = \Omega_\Lambda \).

Let us now consider the evolution behaviour of the modified Friedmann equation of DGP gravity (2.3). In this work we are concerned in the case where the universe only consists of pressureless energy-momentum constituents, \( \rho_M \), while still accelerating in its late time expansion. We must then focus on the self-inflationary phase of the Friedmann equation such that \( \epsilon = 1 \) in (2.3). By analogy with the dark energy \( w \)-model one may introduce a new cosmological parameter \( \Omega_{\epsilon} \)

\[
\Omega_{\epsilon} \equiv \frac{1}{4\pi^2 H_0^2},
\]

such that the self-inflationary phase of the Friedmann equation (2.3) can be written as

\[
H^2 (z) = H_0^2 \left[ \Omega_k (1 + z)^2 + \sqrt{\Omega_{\epsilon} + \Omega_M^0 (1 + z)^3} \right].
\]

Comparing this expression with the conventional Friedmann equation (2.3) we notice that \( \Omega_{\epsilon} \) mimics a dark energy parameter \( \Omega_{DE}^0 \).

At the present time \( (z = 0) \) we can find the corresponding normalisation condition for the cosmological parameters

\[
\Omega_k + \sqrt{\Omega_{\epsilon} + \Omega_M^0} = 1,
\]
which is clearly different from the normalization condition of the standard four-dimensional Friedmann equation \( (2.8) \), \( \Omega_k + \Omega_M^0 + \Omega_{DE}^0 = 1 \). Note also that in the case where the universe is assumed to be flat (\( \Omega_k = 0 \)), \( (2.11) \) gives
\[
\Omega_\tau = \left( \frac{1 - \Omega_M^0}{2} \right)^2 \quad \text{and} \quad \Omega_\tau < 1. \quad (2.12)
\]

3 Luminosity distances in higher dimensional theories with non-compact extra dimensions

3.1 Supernova sample

The first dataset considered in this work is the ‘Pantheon’ SNIe sample [14], which consists of 1048 objects in 40 bins compressed. With these data we can estimate the observed SNIe distance moduli \( \mu \), which is related to the luminosity distance, \( d_L \), as follows:
\[
\mu(z) = 5 \log \left[ \frac{d_L(z)}{1 \text{Mpc}} \right] + 25, \quad (3.1)
\]

where \( d_L \) is given in units of Mpc. In the standard statistical analysis, wherein one uses observed SNIe distance moduli to constrain cosmological model parameters, one adds to the distance modulus the nuisance parameter \( M \) – an offset comprising the sum of the supernovae absolute magnitude (and other possible systematics). Note that \( M \) is degenerate with \( H_0 \), which can be taken to have a uniform prior. In the case where spatial flatness is assumed, then \( d_L \) is related to the comoving distance as follows:
\[
d_L(z) = \frac{c}{H_0} (1 + z) D(z), \quad D(z) = \frac{H_0}{c} (1 + z)^{-1} \left( \frac{\mu(z)}{5} - 5 \right), \quad (3.2)
\]

where \( c \) is the speed of light. Therefore, the normalised Hubble function \( H(z)/H_0 \) can be inferred by taking the inverse of the derivative of \( D(z) \) with respect to the redshift using the relation
\[
D(z) = \int_0^z H_0 d\tilde{z}/H(\tilde{z}), \quad (3.3)
\]

where \( H_0 \) is the Hubble constant, considered as a prior value to normalise \( D(z) \). From this point forward, it is possible to carry out a standard statistical analysis by minimizing the quantity
\[
\chi^2_\mu = \sum_{i=1}^N \frac{[\mu_{\text{DGP}}(z_i; \Omega_m, \Omega_r, \tau_r) - \mu_{\text{obs}}(z_i)]^2}{\sigma^2_{\mu,i}}, \quad (3.4)
\]

where the \( \sigma^2_{\mu,i} \) are the measurement variances and \( N \) is the number of SNIe in the total sample. Using the latter, in Table 1 we report the values inferred for the DGP model parameters using the full Pantheon 2019 dataset. We performed a MCMC analysis using binned Pantheon data points to fit the distance moduli (having first subtracted a value of \( M = -19.3 \), informed by the prior on \( H_0 \) from SH0ES) to a linear model at first, second and third order. The resulting fits are showed in Figure 1; it can be seen from this Figure that a third-order model gives an excellent fit to the data. The joint posterior on the DGP model parameters is then also illustrated in Figure 2.

**Table 1:** Median values and 95% confidence intervals for the parameters of the DGP model inferred from the Pantheon supernova sample with \( M = -19.3 \).

| Parameter | \( \Omega_m \) | \( \Omega_r \) | \( \Omega_k \) | \( rH_0 \) | \( H_0 \) |
|-----------|---------------|---------------|---------------|-----------|-----------|
|           | 0.22 \( ^{+0.10}_{-0.10} \) | 0.17 \( ^{+0.05}_{-0.05} \) | \(-0.11^{+0.38}_{-0.36} \) | 1.20 \( ^{+0.22}_{-0.18} \) | 71.54 \( ^{+0.90}_{-0.86} \) |
Figure 1: Distance modulus $\mu$ as a function of $z$. The dots are the observational Pantheon sample (after subtracting the fiducial value of $M = -19.3$). The solid lines are the reconstructed fits of $\mu(z)$ using the linear model formalism.

3.2 Gravitational wave damping

In the absence of a unique GW model for higher dimensional theories with non-compact extra dimensions, we introduce a phenomenological ansatz for the GW amplitude scaling based on the DGP model. In GR the GW amplitude goes as

$$h_{GR} \propto \frac{1}{d_{GW}^{GR}} \tag{3.5}$$

where $d_{GW}^{GR}$ is the luminosity distance of the GW source. Note that in GR, $d_{GW}^{GR}$ is identical to the distance $d_{EM}^{EM}$ that would be inferred from EM observations which, without loss of generality, we assume to be the ‘true’ distance to the source.

Based on DGP gravity one may expect higher dimensional theories to contain a new length scale: a so-called screening scale $R_c$, beyond which they deviate from GR and exhibit gravitational leakage, leading (by flux conservation) to a reduction in the amplitude of the observed gravitational waves which now takes the form

$$h_{NGR} \propto \frac{1}{d_{GW}^{NGR}} = \frac{1}{d_{EM}^{EM} \left[ 1 + \left( \frac{d_{EM}^{EM}}{R_c} \right)^n \right]^{(D-4)/2n}} \tag{3.6}$$

where $D$ denotes the number of dimensions and $n$ is the transition steepness allowing for a more general shape for the transition at the crossover scale $[15]$. Note that (3.6) reduces to the standard GR scaling at distances much shorter than $R_c$ and when $D = 4$ as required.

Neglecting all other effects of modified gravity on the GW amplitude, the gravitational leakage will then simply result in a measured GW distance to the source that is greater than its true luminosity distance. An event only measured in GWs would not allow us to distinguish the measured GW distance from its true distance. However, if we can independently measure $d_{EM}^{EM}$ by detecting an EM counterpart, then by comparing the two measured distances, we can place limits on the number of spacetime dimensions and the screening scale.

4 Forecasting LISA data: constraints on Gravitational Wave leakage

A gravitational observatory such as LISA will measure GWs emitted by coalescing pairs of massive black holes up to a high redshift, providing an accurate measurement of the luminosity distance to
Figure 2: Joint posterior PDF for parameters of the full DGP four-dimensional parameter space, inferred from the Pantheon supernova sample with $M = -19.3$.

those sources. The excellent sky localization capabilities of LISA should also greatly assist with the identification of a host galaxy or EM counterpart, thus permitting the redshift of the siren to be measured. In this section we investigate and quantify the ability of LISA to place constraints on large-scale extra-dimensional leakage of gravity from cosmologically distant standard sirens.

4.1 MBHB synthetic catalogues

The first step towards forecasting LISA’s ability to place limits on the number of spacetime dimensions is to identify a sufficient number of synthetic catalogues of MBHB merger events, the gravitational radiation emitted by which would be detectable by LISA and which are likely to produce an EM counterpart observed by future optical and/or radio surveys – with each merger event then being assigned a corresponding redshift, GW luminosity distance (assuming GR) and corresponding 1-$\sigma$ error on the luminosity distance. Such catalogues have already recently been published in the context
of another cosmological study [16] and we re-employ these catalogues in the present study too. We now briefly describe the main steps involved in creating and using these synthetic catalogues and we refer to [16] for more technical details about how they were generated.

The first step consists of predicting the rate and redshift distribution of the MBHB events. In [16] these were derived using semi-analytical simulations of the evolution of the massive BHs during the formation and evolution of their host galaxies, and they were computed for three distinct scenarios regarding the initial conditions for the massive BH population at high redshift, namely:

1. **Model popIII**: A ‘realistic’ light-seed scenario in which the first massive BHs are assumed to form from the remnants of population III stars (popIII) [17, 18], and including a delay between the coalescence of MBHB host galaxies and that of the BHs themselves [19].

2. **Model Q3d**: A ‘realistic’ heavy-seed scenario in which the first massive BHs are assumed to form from the collapse of protogalactic disks [20–22], also including delays.

3. **Model Q3nod**: Same model as Q3d but ignoring delays, an assumption which significantly increases the BH merger rate. This model is thus considered to be an ‘optimistic’ scenario for LISA’s observed merger rates.

The resulting catalogues of MBHB merger events for each variant MBHB model include all information about the MBHBs (masses, spins, redshift, etc.) and their host galaxies. We refer to [23, 24] for a more detailed description of these models and simulations.

The next step in the analysis consists of using the parameters of the MBHB systems of each catalogue as an input to a gravitational waveform model that, assuming GR, computes the SNR of each merger event and corresponding errors on the MBHB’s waveform parameters. Since ultimately we want to use the sources as standard sirens we are mostly interested in the error on the luminosity distance $\Delta d_L$ and sky location $\Delta \Omega$. The reason for the latter is that the following and ultimate step is to select those events that are likely to provide a detectable optical counterpart and hence provide a measurement of the redshift to the source. To do so, merger events with a SNR $\geq 8$ and sky location error of $\Delta \Omega < 10 \text{ deg}^2$ were identified in [16], where $10 \text{ deg}^2$ corresponds to the field of view of e.g. the optical Large Synoptic Survey Telescope (LSST) survey \(^2\).

In summary, the catalogues of sources presented in [16] consist of MBHBs whose gravitational radiation would be detectable by the LISA configuration [25] with a SNR $> 8$ and $\Delta \Omega < 10 \text{ deg}^2$ and whose EM counterpart could be visible either directly by an optical survey telescope such as the LSST or in the radio band by e.g. the Square Kilometer Array (SKA) \(^3\) (with follow-up observations of the host galaxy in the optical band). As discussed in [16], because there is a significant scatter in the characteristics of the MBHB population between different catalogues, 22 simulated four-year catalogues were considered for each MBHB formation scenario.

Figure 3 shows an example of simulated data points for a random catalogue for the model Q3d. Each MBHB merger event corresponds to a point in the $[z, d_L(z)]$ plane with a 1-$\sigma$ error on the luminosity distance which takes into account the following factors: the experimental error of LISA measurement, the errors due to weak gravitational lensing and peculiar velocities [26, 27] and finally the error in estimating the redshift of the host galaxy or EM counterpart which is only non-negligible if the measurement has to be done photometrically.

Following [16] the average numbers of GW standard sirens that would be observable over LISA’s 4 year mission, which can thus be used to populate the redshift-distance diagram, are reported in Table 2 together with the average relative error on the luminosity distance for each of the MBHB formation models. These numbers indicate that the number of ‘useful’ standard sirens is substantially larger for the Q3nod model, thereby potentially improving the precision with which the parameters of our cosmological model can be inferred. Regarding the other two models one expects them to produce very similar results, with the popIII being slightly worse.

\(^2\)http://www.lsst.org

\(^3\)http://www.skatelescope.org
Figure 3: Hubble diagram for an example synthetic catalogue of MBHB standard sirens, assumed to simultaneously detectable with LISA and an optical survey telescope, as described in the text and as originally presented in [16]. Here we illustrate a random catalogue within the model Q3d.

Table 2: Key numbers summarising the synthetic four-year catalogues for all three MBHB formation models. The middle column gives average values (4 years) for the number of LISA detections with an EM counterpart and the last column presents average values of the relative error on luminosity distance $\Delta d_L$.

| Model   | Total number | $\Delta d_L$ |
|---------|--------------|--------------|
| popIII  | 12.9         | 0.0897       |
| Q3d     | 14.0         | 0.0616       |
| Q3nod   | 27.2         | 0.0632       |

4.2 Generating GW data

Equipped with the synthetic catalogues of standard sirens generated for $\Lambda$CDM cosmology, and with the required methodology (3.6) to describe the effect of gravitational damping on the GW waveform in higher dimensional theories with non-compact extra dimensions, the next step is to generate realisations of MBHB siren catalogues for various possible higher-dimensional theories. Since we always assume that light and matter is restricted to the four-dimensional universe, we first infer the EM luminosity distance $(d_{EM}^L)_i$ for each MBHB event of each catalogue by plugging its corresponding redshift $z_i$ into the usual distance-redshift relation [28]. Note that $d_{EM}^L$ depends on the assumed value of the Hubble constant which we assume to be given by the SH0ES $H_0 = 73.24 \pm 1.74$ km s$^{-1}$ Mpc$^{-1}$ value [29] and on the assumed value of the cosmological parameters $\Omega_M^0$ and $\Omega_\Lambda$, which will be fitted with the Pantheon supernovae sample.

We note, of course, that to be fully self-consistent, then ideally we should compute the MBHB merger rates and redshift distributions in our particular chosen higher dimensional theory and not in $\Lambda$CDM. However, we do not expect the rates and distributions obtained in that manner to be significantly different from those obtained assuming $\Lambda$CDM, since the dominant effect is instead the details of the galaxy formation and evolution model adopted. Thus, for simplicity we will continue to adopt the merger rates and redshift distribution in each formation model previously calculated for the $\Lambda$CDM case.

The next step is then to compute the corresponding GW luminosity distance $(d_{GW})_i \equiv d_{GW}^L ((d_{EM}^L)_i, D, R_c, n)$ of each event which is simply obtained by inverting the scaling relation (3.6). The GW distance depends not only on the ‘true’ EM distance, but also on the ‘cosmological’ parameters of the higher-dimensional gravity. We consider $D = 5, 6$ or 7, $n = 1$ or 10 and $R_c = R_H \sim 4$ Gpc or $4R_H$ where $R_H = H_0^{-1}$ is the current Hubble radius. The reason for these particular choices of screening scale is that, based on the DGP model, one may expect the characteristic
length scale for higher-dimensional theories of gravity to be of the order of the Hubble radius. The values for the steepness parameter on the other hand were chosen arbitrarily but such that they have not yet been ruled out by the comparison of distance measurements from GW and EM observations of GW170817 [15].

Finally, assuming stationary Gaussian noise with zero mean and standard deviation given by

\[\sigma_i = (\Delta d_L)_i \times (d_L^{GW})_i,\]

where \((\Delta d_L)_i\) is the relative error on corresponding EM distance, we computed for each MBHB event a ‘measured’ GW luminosity distance \((x_{GW})_i\), given by

\[(x_{GW})_i = (d_L^{GW})_i + \sigma_i,\]

and an independently measured redshift \(z_i\) or equivalently an EM distance \((d_L^{EM})_i\).

5 Statistical analysis method

Given two sets of \(j\) statistically independent GW and EM measurements for MBHB merger events from a given catalogue \(C\): \(x_{GW}^C = \{(x_{GW})_1, (x_{GW})_2, \ldots, (x_{GW})_j\}\) and \(z^C = \{z_1, z_2, \ldots, z_j\}\) respectively, we carried out a Bayesian analysis to infer the posterior of the number of spacetime dimensions \(D\), the screening scale \(R_c\), the steepness parameter \(n\) and Hubble constant \(H_0\). We then carried out a Bayesian model selection analysis to compare the evidence for, or against, each higher-dimensional model. These results allow us to assess the capability of LISA to place constraints on higher-dimensional theories of gravity. In our case, we consider the full analyses in comparison to DGP scenarios as pivot models.

We first briefly describe the statistical framework of Bayesian parameter estimation, before outlining the Bayesian model selection technique used.

5.1 Parameter Estimation

The joint posterior of the parameters \(\theta^C = (D, R_c, n, H_0)\) for a given catalogue is obtained by applying Bayes’ theorem [30]

\[p(\theta^C|x_{GW}^C, M) = \frac{p(z^C, x_{GW}^C|\theta^C, M)p(\theta^C|M)}{Z},\]

where \(p(z^C, x_{GW}^C|\theta^C)\) is the joint likelihood for GW and EM data from the catalogue, \(p(\theta^C|M)\) the prior probability distribution, \(M\) represents all the background information that has gone into defining our model and \(Z = p(z^C, x_{GW}^C|M)\) is a normalization constant – the so-called evidence, obtained by multiplying the likelihood by the prior and integrating over all parameters that define the model. We will make use of \(Z\) when comparing the models in (5.2) but since \(Z\) does not depend on \(\theta^C\), it is a constant for a given model and the posterior PDF is essentially proportional to the product of the prior distribution on the parameters and the likelihood of the data given the parameters.

We can write the joint likelihood for the GW data \(x_{GW}^C\) and EM observable \(z^C\) given \(\theta^C = (D, R_c, n, H_0)\) as

\[p(z^C, x_{GW}^C|\theta^C, M) \propto \exp \sum_{i=1}^{j} \left[ -\frac{1}{2\sigma_i^2} \right] \frac{(x_{GW})_i - (d_L^{GW})_i}{},\]

where the sum runs over all MBHB events in a given catalogue and the theoretical GW luminosity distance \(d_L^{GW}(d_L^{EM}(z_i, H_0), D, R_c, n)\) is computed with respect to the cosmological parameters evaluated at their fiducial values.

In order to represent complete ignorance about the parameters defining the higher-dimensional theory we take uniform uninformative priors in the range \(D \in [3, 11], n \in [0, 100]\) and \(R_c \in [20, \infty]\) where the lower limit on the screening scale is set by distances ruled out by GW170817 [11] and the range on \(D\) and \(n\) was chosen to limit the computational cost of the parameter estimation method. For the prior on the Hubble constant we consider the SH0ES measurement which assumes a Gaussian prior centred on \(H_0 = 73.24\text{~km~s}^{-1}\text{Mpc}^{-1}\) with a standard deviation of \(\sigma_{H_0} = 1.74\text{~km~s}^{-1}\text{Mpc}^{-1}\). The prior distribution on the parameter vector \(\theta^C\) is then

\[p(\theta^C|M) = p(D)p(R_c)p(n)p(H_0) \propto \mathcal{N}(H_0, \sigma_{H_0}).\]
Figure 4: Joint posterior PDF for the full four-dimensional parameter space for the catalogue corresponding to FoM of the cosmological scenario $\theta^1 = (D = 5, R_c = R_H, n = 1, H_0)$ within the Q3d formation model. On the axes are superimposed the one-dimensional marginalised distributions with the dashed lines marking the 95% confidence intervals for the PDFs and the true values indicated by a vertical blue line. The two-dimensional plot shows the contours at 0.5, 1, 1.5, and 2-$\sigma$.

Results for parameters of interest are found by marginalising the joint posterior over any unwanted parameters,

$$p(\theta_1|x_{GW}^C, z^C, M) = \int d\theta_2...d\theta_N p(\theta^C|x_{GW}^C, z^C, M).$$

(5.4)

The marginalised posterior PDF of a given parameter can then be used to find its median and construct confidence regions.

As can be seen from (5.4) computing the marginalised PDFs requires the evaluation of multi-dimensional integrals which is computationally intensive for the size of the parameter space and the amount of data to consider. This is addressed by using a stochastic sampling engine based on the MCMC algorithm which samples random draws from the target posterior distributions allowing us to approximate the desired integrals.\footnote{In this analysis we used the python package emcee by Foreman-Mackey et al.(2013) \url{http://dan.iel.fm/emcee}, which implements the affine-invariant ensemble sampler of Goodman & Weare (2010) \url{https://github.com/grinsted/gwemcee} to perform the MCMC algorithm.} The results are presented in Sec. 6.
To further quantify the ability of future LISA observations to place constraints on modified gravity theories – or extra-dimensional theories – we want to compute the probability that, for data simulated for a particular non GR (NGR)\(^5\) model \(M_{\text{NGR}}\) characterised by the parameters \(\boldsymbol{\theta}_{\text{NGR}} = (D, R_c, n, H_0)\), the data will indeed favour the NGR model over the nested DGP model \(M_{\text{DGP}}\) i.e. the pivot model parametrised by \(\boldsymbol{\theta}_{\text{DGP}} = (D = 5, R, H_0 = 1.2, n, H_0)\). This step is achieved by computing the posterior odds defined as the ratio of the posterior probabilities of the two competing models:

\[
O_{\text{NGR}/\text{GR}} = \frac{p(M_{\text{NGR}}) p(z, x_{\text{GW}} | M_{\text{NGR}})}{p(M_{\text{DGP}}) p(z, x_{\text{GW}} | M_{\text{DGP}})} = \frac{M_{\text{NGR}}}{M_{\text{DGP}}} B_{\text{NGR}/\text{DGP}},
\]

which is simply the prior odds multiplied by the ratio of the evidence for each model, the so-called Bayes factor in favor of \(M_{\text{NGR}}\).

If we further assume that the two models are equally probable a priori, such that \(p(M_{\text{NGR}}) = p(M_{\text{DGP}})\), then the posterior odds is simply equal to the Bayes factor which from Bayes’ theorem considered above (5.1) may be now expressed as

\[
B_{\text{NGR}/\text{DGP}} = \frac{Z_{\text{NGR}}}{Z_{\text{DGP}}} = \frac{\int d\boldsymbol{\theta}_{\text{NGR}} p(\boldsymbol{\theta}_{\text{NGR}} | M_{\text{NGR}}) p(z, x_{\text{GW}} | \boldsymbol{\theta}_{\text{NGR}}, M_{\text{NGR}})}{\int d\boldsymbol{\theta}_{\text{DGP}} p(\boldsymbol{\theta}_{\text{DGP}} | M_{\text{DGP}}) p(z, x_{\text{GW}} | \boldsymbol{\theta}_{\text{DGP}}, M_{\text{DGP}})}
\]

According to Jeffrey’s scale: if \(B_{\text{NGR}/\text{DGP}}\) is larger (smaller) than unity then this tells us that \(M_{\text{NGR}}\) (\(M_{\text{DGP}}\)) is more strongly supported by the data under consideration than the alternative model.

6 Results and discussion

The MCMC and nested sampling implementations described above were run on the simulated GW and EM data for each of the 22 catalogues of the three MBHB formation scenarios and for each cosmological scenario presented in Sec.(4.2). We first discuss the results of the parameter estimation, then move on to the results of the Bayesian model selection.

6.1 Parameter estimation using MCMC

To quantify the capability of LISA to constrain a particular cosmological parameter for a given cosmological scenario and MBHB formation model, we first record the median values of the marginalised PDFs of each cosmological parameter for all the catalogues; we then use the median as a Figure-of-Merit (FoM) for the parameter estimate. For an estimate of the LISA error on the FoM we first take the 95% confidence interval (CI) around the median value for each catalogue and then pick the median of these intervals to represent the 95% CI of the FoM.

There is also a significant scatter in the characteristics of the MBHB population between different catalogues. To have an idea of the scatter of cosmological constraints that we can place between the different catalogues we refer to Figures 6-7 where we show box plots representing the distributions of these constraints for each cosmological scenario and MBHB formation model. When we compare the capability of LISA to place constrains on a given parameter for different cosmologies and MBHB models, we will always use the median FoM together with 95% CI, which essentially captures the scatter in the FoM and hence provides a realistic estimate of the statistical uncertainty we can expect.

Tables 3-4-5-6 list the FoMs of the cosmological parameters \(\boldsymbol{\theta} = (D, R_c, n, H_0)\) for all MBHB formation scenarios and for all cosmological models considered. The results for the steepness parameter and Hubble constant will not be considered in great detail as they are not of astrophysical interest to our analysis and do not add any information about LISA’s ability to constrain modified gravity that we cannot infer by considering the results for \(D\) and \(R_c\). Figure 4 shows an example of a joint posterior PDF through the four-dimensional space for a random catalogue, cosmological scenario and MBHB formation model.

\(^5\)In this paper the NGR are the scenarios with \(D \geq 6\).
In each entry of Tables 3-4-5-6, the top row shows the FoM and 95% CI for light seeds (popIII), the central row for heavy seeds with delays (Q3d) and the bottom row for heavy seeds without delays (Q3nod). Q3nod systematically gives better results than the other two scenarios, which are roughly comparable to each other, due to their lower number of detectable standard sirens, as shown in Table 2. Clearly, this shows that the extent to which LISA can be used to perform meaningful constraints on theories of modified gravity defined by the scaling (3.6) will strongly depend on the actual redshift distribution of MBHB merger events and the corresponding efficiency in identifying an EM counterpart. In the following, whenever we need to restrict to one MBHB formation model, we always choose Q3d, since it is the intermediate scenario (as far as the number of standard sirens is concerned) among those we consider. The constraints would be comparable or slightly worse for popIII and slightly better for the model Q3d. Also whenever we need to show likelihood contours we restrict to the contour of the catalogue representing the FoM.

Table 3: Figures of merit and 95% confidence intervals summarising the marginalised posterior PDFs of the number of dimensions for the various cosmological scenarios and MBHB formation models considered. In each row of the Table separated by dashed lines, the top sub-row shows the FoMs and CI for ligh-seeds (popIII), the central sub-row for heavy seeds with delays (Q3d) and the bottom sub-row for heavy seeds without delays (Q3nod).

| Parameter | $D = 5$ | $D = 6$ | $D = 7$ |
|-----------|---------|---------|---------|
| Model     | $R_c = R_H$ | $R_c = 4R_H$ | $R_c = R_H$ | $R_c = 4R_H$ | $R_c = R_H$ | $R_c = 4R_H$ |
|           | $n = 1$ | $n = 10$ | $n = 1$ | $n = 10$ | $n = 1$ | $n = 10$ |
| $D = 5$   | 5.02±0.65 | 5.01±0.55 | 5.02±0.43 | 6.00±0.63 | 6.00±0.56 | 6.97±0.40 |
|           | ±0.27   | ±0.35   | ±0.29   | ±0.35   | ±0.33   | ±0.35   |
| $D = 6$   | 4.99±0.13 | 5.01±0.10 | 5.02±0.07 | 6.02±0.11 | 6.01±0.08 | 6.99±0.068 |
|           | ±0.99   | ±0.98   | ±0.96   | ±0.99   | ±0.86   | ±0.68   |
| $D = 7$   | 5.32±1.08 | 5.15±1.36 | 5.16±3.56 | 6.28±3.40 | 6.12±1.79 | 6.08±1.30 |
|           | ±0.32   | ±0.56   | ±0.51   | ±0.40   | ±0.68   | ±0.62   |
|           | 5.17±1.81 | 5.06±0.24 | 5.03±0.44 | 6.08±0.63 | 5.98±0.34 | 6.04±0.29 |
|           | ±0.38   | ±0.24   | ±0.21   | ±0.36   | ±0.25   | ±0.22   |

6.2 Constraints on the number of spacetime dimensions

We first consider LISA’s ability to place limits on the number of spacetime dimensions. The FoMs and 95% CI for all MBHB formation models can be found in Table 3. We see that, except for the ($R_c = 4R_H, n = 1$) cosmological scenario, the estimates of the number of dimensions are very close to their true fiducial values for all MBHB models, indicating that LISA will probably be able to place limits on the number of dimensions in the future. Note that although the FoMs are very close to their true values for all MBHB models, the spread around the FoMs is narrower for the Q3nod model giving constraints on $D$, for the most favourable cosmological scenario ($D = 7, n = 10, R_c = R_H$), at the level of 0.86% compared to 1.43% and 1.57% for the Q3d and popIII formation models respectively.

Furthermore, we can see from Table 3 that the steeper the transition and the smaller the screening scale the more accurate the estimates on $D$ and the smaller the confidence interval, hence the reason why LISA will not be able to place meaningful constraints on $D$ in the case where $R_c = 4R_H$ and $n = 1$. In Figure 5 we plot the likelihood contours of $D$ and $R_c$ for $D = 5, 6, 7$ when ($n = 1, R_c = R_H$), ($n = 10, R_c = R_H$) and ($n = 10, R_c = 4R_H$) allowing us to understand the effect of the steepness.
Figure 5: Marginalised contours at 0.5, 1, 1.5, and 2-σ for $D$ and $R_c$ in the heavy-seeds (Q3d) model for the following cosmological scenarios: from left to right ($n = 1, R_c = H_d$), ($n = 10, R_c = H_d$) and ($n = 10, R_c = 4H_d$) and from top to bottom $D = 5, 6, 7$. The blue cross hairs indicates the true/best values of the parameters.

As a final note to the constraints we can place on $D$, we deduce from Table 3 that the precision with which we can estimate $D$ gets marginally better as the number of dimensions increases, going from 1.2% to 0.86% in the most favourable cosmological scenario, but is not very sensitive to it.

6.3 Constraints on the screening scale

Let us now consider LISA’s ability to place limits on the screening scale. The FoMs and 95% CI for all MBHB formation models can be found in Table 4. This latter and Figure 5 show that just like for the constraints on the number of dimensions, the constraints on the screening scale improve the steeper
The Hubble radius is of the screening scale for the various cosmological scenarios and MBHB formation models considered. The Hubble radius is $R_H = 4.093$ Gpc.

| Parameter | $R_e = R_H$ | $R_e = 4R_H$ |
|-----------|-------------|--------------|
| Model     | $n = 1$     | $n = 10$     | $n = 1$     | $n = 10$     |
| $D = 5$   | 1.08$^{+3.403}_{-0.579}$ | 1.007$^{+0.257}_{-0.181}$ | 7.259$^{+10.722}_{-5.648}$ | 4.450$^{+4.894}_{-0.081}$ |
|           | 1.065$^{+2.646}_{-0.697}$ | 0.997$^{+0.236}_{-0.156}$ | 5.450$^{+7.478}_{-4.048}$ | 4.021$^{+2.524}_{-0.783}$ |
|           | 1.050$^{+1.903}_{-0.479}$ | 1.004$^{+0.152}_{-0.135}$ | 5.359$^{+5.846}_{-3.763}$ | 4.133$^{+1.558}_{-0.640}$ |
| $D = 6$   | 1.000$^{+1.233}_{-0.437}$ | 1.014$^{+0.132}_{-0.103}$ | 5.044$^{+20.453}_{-2.918}$ | 4.143$^{+0.973}_{-0.556}$ |
|           | 1.000$^{+1.113}_{-0.421}$ | 1.001$^{+0.137}_{-0.097}$ | 4.346$^{+12.107}_{-2.389}$ | 4.000$^{+0.536}_{-0.428}$ |
|           | 0.999$^{+0.596}_{-0.326}$ | 1.004$^{+0.092}_{-0.086}$ | 4.052$^{+7.712}_{-2.133}$ | 4.036$^{+0.463}_{-0.330}$ |
| $D = 7$   | 1.000$^{+0.917}_{-0.372}$ | 1.000$^{+0.093}_{-0.084}$ | 4.122$^{+9.580}_{-2.203}$ | 4.051$^{+0.483}_{-0.380}$ |
|           | 0.974$^{+0.692}_{-0.363}$ | 1.000$^{+0.102}_{-0.0799}$ | 4.600$^{+10.939}_{-2.358}$ | 4.000$^{+0.387}_{-0.310}$ |
|           | 1.000$^{+0.524}_{-0.269}$ | 1.001$^{+0.076}_{-0.068}$ | 4.038$^{+4.377}_{-1.690}$ | 4.003$^{+0.272}_{-0.230}$ |

The transition and the lower the screening scale. However, unlike for the number of dimensions, it appears that LISA’s capability to constrain the screening scale strongly depends on the hypothetical value of $D$ and which formation model is considered. Considering the $D = 5$ cosmological scenario first, we see that LISA is is only capable of constraining the screening scale in the most favourable scenario ($n = 10$, $R_e = R_H$) of the Q3nod and Q3d model at the level of 14.3% and 19.6% respectively. Then as the number of dimensions increases the FoMs get closer to their true values and the error gets significantly smaller for all the MBHB formation models. For the $D = 6$ scenario one can place meaningful constraints on the screening scale in all cosmological scenarios and MBHB formation models, with a precision of 8.8% for the most favourable scenario and Q3nod model, except for the ($n = 1$, $R_e = 4R_H$) as you would expect as it is the hardest scenario to probe. Finally, for $D = 7$, the ($n = 1$, $R_e = 4R_H$) scenario is still not possible to constrain but the constrains we can place on the screening scale for the other scenarios are very good given the scales considered, reaching 7.2% for ($n = 10$, $R_e = R_H$) and the Q3nod model. Again these results clearly show the high improvement that can be obtained if a few more standard sirens are available. Indeed, we saw that the constraints on the screening scale are much tighter in the model Q3nod where the number of standard sirens is much higher, than in the other two scenarios where the number of sources is lower.

### 6.4 Constraints on the steepness parameter and Hubble parameter

Tables 5-6 list the FoMs of the cosmological parameters $n$ and $H_0$ for all MBHB formation scenarios and for all cosmological models considered. These results will not be discussed further since as they are fully consistent with the results obtained for $D$ and $R_e$. Note however that the results from Table 6 imply that LISA could give a fully independent constraint on the Hubble constant, complementing the other optical measurements. Even though the errors on $H_0$ are slightly higher compared to other cosmological probes, even in the most optimistic cosmological and MBHB formation scenario it is still extraordinary that LISA could potentially constrain this parameter which cannot be independently measured by observations of Pantheon sampler.
Table 5: Figures of merit and 95% confidence intervals summarising the marginalised posterior PDFs of the steepness parameter for the various cosmological scenarios considered.

| Parameter | $R_c = R_H$ | $R_c = 4R_H$ |
|-----------|-------------|-------------|
|           | $n = 1$     | $n = 10$    | $n = 1$ | $n = 10$ |
| $D = 5$   | 0.958$^{+1.319}_{-0.284}$ | 9.13$^{+5.53}_{-6.11}$ | 0.892$^{+0.592}_{-0.238}$ | 8.10$^{+6.50}_{-5.90}$ |
|           | 0.96$^{+0.99}_{-0.47}$ | 9.10$^{+5.57}_{-8.29}$ | 0.93$^{+0.81}_{-0.25}$ | 9.37$^{+5.32}_{-5.21}$ |
|           | 0.96$^{+0.60}_{-0.24}$ | 9.59$^{+5.10}_{-5.30}$ | 0.94$^{+0.51}_{-0.22}$ | 9.37$^{+5.18}_{-5.18}$ |
| $D = 6$   | 1.00$^{+0.57}_{-0.22}$ | 9.63$^{+5.06}_{-5.43}$ | 0.95$^{+0.36}_{-0.18}$ | 9.66$^{+5.00}_{-5.38}$ |
|           | 1.00$^{+0.64}_{-0.23}$ | 8.96$^{+5.68}_{-5.89}$ | 1.00$^{+0.41}_{-0.20}$ | 9.91$^{+4.80}_{-4.13}$ |
|           | 1.01$^{+0.26}_{-0.18}$ | 9.72$^{+4.98}_{-4.70}$ | 0.99$^{+0.24}_{-0.17}$ | 10.01$^{+4.77}_{-4.77}$ |
| $D = 7$   | 0.99$^{+0.26}_{-0.17}$ | 10.0$^{+4.71}_{-5.13}$ | 0.98$^{+0.27}_{-0.16}$ | 10.14$^{+4.60}_{-4.89}$ |
|           | 1.02$^{+0.39}_{-0.19}$ | 9.73$^{+5.60}_{-5.37}$ | 0.95$^{+0.21}_{-0.15}$ | 10.52$^{+4.24}_{-4.63}$ |
|           | 0.99$^{+0.17}_{-0.13}$ | 10.01$^{+4.44}_{-4.52}$ | 0.99$^{+0.19}_{-0.13}$ | 10.56$^{+4.37}_{-4.37}$ |

Table 6: Figures of merit and 95% confidence intervals summarising the marginalised posterior PDFs of the Hubble constant $H_0 = 73.24$ km s$^{-1}$ Mpc$^{-1}$ for the various cosmological scenarios considered.

| Parameter | $H_0$ (km s$^{-1}$ Mpc$^{-1}$) |
|-----------|--------------------------------|
|           | $R_c = R_H$ | $R_c = 4R_H$ |
|           | $n = 1$     | $n = 10$    | $n = 1$ | $n = 10$ |
| $D = 5$   | 73.15$^{+3.33}_{-3.54}$ | 73.19$^{+3.21}_{-3.08}$ | 73.05$^{+3.37}_{-3.48}$ | 73.40$^{+2.18}_{-1.85}$ |
|           | 73.14$^{+3.35}_{-3.53}$ | 73.23$^{+3.29}_{-3.33}$ | 73.10$^{+3.37}_{-3.48}$ | 73.31$^{+2.29}_{-2.04}$ |
|           | 73.17$^{+3.28}_{-3.31}$ | 73.23$^{+2.97}_{-2.64}$ | 73.28$^{+2.97}_{-2.63}$ | 73.25$^{+1.37}_{-1.37}$ |
| $D = 6$   | 73.23$^{+3.39}_{-3.50}$ | 73.24$^{+3.20}_{-3.17}$ | 73.18$^{+3.36}_{-3.44}$ | 73.29$^{+2.11}_{-1.85}$ |
|           | 73.25$^{+3.37}_{-3.48}$ | 73.22$^{+3.25}_{-3.32}$ | 73.24$^{+3.40}_{-3.44}$ | 73.29$^{+2.19}_{-1.85}$ |
|           | 73.22$^{+3.35}_{-3.48}$ | 73.21$^{+2.97}_{-2.91}$ | 73.23$^{+2.97}_{-2.91}$ | 73.15$^{+1.33}_{-1.33}$ |
| $D = 7$   | 73.23$^{+3.36}_{-3.47}$ | 73.25$^{+3.26}_{-3.13}$ | 73.23$^{+3.39}_{-3.42}$ | 73.23$^{+1.94}_{-1.82}$ |
|           | 73.22$^{+3.35}_{-3.46}$ | 73.23$^{+3.28}_{-3.31}$ | 73.16$^{+3.35}_{-3.46}$ | 73.29$^{+1.82}_{-1.74}$ |
|           | 73.24$^{+3.35}_{-3.42}$ | 73.27$^{+3.00}_{-2.89}$ | 73.21$^{+3.35}_{-3.00}$ | 73.29$^{+1.36}_{-1.36}$ |

6.5 Statistical spread of the catalogues

Although we employed 22 synthetic catalogues for each MBHB formation model in order to improve the statistics of our analysis, the scatter of the properties of the massive binary black hole population across the catalogues is still quite high. This can be understood by looking at the distribution of the median values of the marginalised PDFs of the cosmological parameters. In Figures 6-7 we plot the distributions of the median values of the 22 catalogues, for each MBHB formation scenario, each cosmological model and the cosmological parameters $D$ and $R_c$. As one would expect from our results
of (6.1) for both parameters the median values are closer to their true value and the spread of the values is smaller the steeper the transition and the lower the screening scale. Also, whereas the distribution of the median values of the screening scale is more peaked as the number of dimensions increases, the spread in the median values of $D$ is not very sensitive to the number of dimensions in agreement with our earlier results. Finally, for any cosmological model the Q3nod model gives systematically better results than the other two models, the reason being that more BH binaries are formed in this model than in the other two.

6.6 Bayesian model selection results, using nested sampling

To quantify LISA’s ability to constrain higher-dimensional theories with a screening scale and steepness parameter we calculate the Akaike Information Criteria (AIC) in favour of each non GR model for every catalogue and MBHB formation model. For an estimate of the error we state the 95% confidence interval around this median value. The reason for this is that the individual errors on the log evidence are much smaller than the scatter in log evidence between the catalogues within a MBHB formation model. Just like for the parameter estimation, when comparing different cosmologies and MBHB models, we will use the $\Delta$AIC. Table 7 shows the results of the model comparison of all the alternative non GR models considered against the DGP model for all MBHB formation scenarios.
Figure 7: Boxplot distributions for the median values of the cosmological parameter $R_c$ of the 22 catalogues for all the cosmological models and MBHB formation scenarios. Note that the outliers were left out for clarity but the number decreases the steeper the transition and the lower screening scale.

When the $\Delta AIC$ factor is greater than $\sim 5$ then this suggests strong evidence in favour of the NGR model in this case. Since the data was simulated for cosmological models where gravity is modified according to (3.6), we would expect, if LISA will be able to constrain alternative theories of gravity in the future, the data to favour the NGR model over the DGP model. Indeed one sees from Table 7 that $\Delta AIC_{NGR/DGP}$ is larger than 5 for all cosmological scenarios and MBHB models. Therefore, we will first discuss the trend in the $\Delta AIC$ factor between different cosmology models and MBHB formation models.

The first thing to note is that as expected the Q3nod model gives systematically higher evidence towards the true NGR model compared to the other two formation models – again confirming the importance of the number of sources LISA will observe. Then, just like for the constraints we can place on the cosmological parameters, the stronger the modification from GR i.e. the higher the number of dimensions, the lower the screening scale and the steeper the transition, the higher the evidence towards the true model.

Consider the reason why the $\Delta AIC$ factor is so high: the first plausible reason would be the
choice of priors which we chose to be uniform in the range $D \in [3,11]$, $n \in [0,100]$ and $R_c \in [20,\infty]$ for $D$, $R_c$ and $n$ respectively and Gaussian for the Hubble constant aforementioned. Increasing the extent of the uniform priors will not affect the prior much as these are already quite large, but let us increase the standard deviation of the SHoES prior to, say twice its value. Table 8 shows the resulting $\Delta \text{AIC}$ when the prior is modified for the catalogue representing FoM for three arbitrarily chosen cosmology models $\theta_1^8 = (D = 6, R_c = R_H, n = 1, H_0)$, $\theta_2^8 = (D = 6, R_c = R_H, n = 10, H_0)$ and $\theta_3^8 = (D = 6, R_c = 4R_H, n = 1, H_0)$ within the Q3d formation scenario. In all cases, increasing the prior variance slightly shifts the $\Delta \text{AIC}$ factor towards a lower value, but not enough to explain the high value of the evidence favouring the NGR model.

Table 8: AIC factors for catalogue 8 for three arbitrarily chosen cosmology models $\theta_1^8, \theta_2^8, \theta_3^8$ within the MBHB formation model Q3d, assuming a prior $p(H_0) \propto N(\mu_{H_0},\sigma_{H_0})$ and $p(H_0) \propto N(\mu_{H_0},2\sigma_{H_0})$ on the Hubble constant. A positive AIC factor is evidence in favour of the NGR model.

| $\Delta \text{AIC}_{\text{NGR/DGP}} = \text{AIC}_{\text{NGR}} - \text{AIC}_{\text{DGP}}$ |
|------------------|---|---|---|
| $\theta_1^8$ | $\theta_2^8$ | $\theta_3^8$ |
| Prior on $H_0$ | $N(\mu_{H_0},\sigma_{H_0})$ | 836 | 1249 | 146 |
| $N(\mu_{H_0},2\sigma_{H_0})$ | 596 | 1066 | 141 |

The other and more probable reason for the very high $\Delta \text{AIC}$ factor is the very small uncertainty in the simulated GW luminosity distances. Indeed, to approximate the error in the GW measurement we assumed that the relative error on the GW luminosity distance is equal to the relative error on the corresponding EM distance ($\Delta d_L$), such that the standard deviation of a particular GW measurement $x_{GW}$ was approximated to be $\sigma_i = (\Delta d_L) \times (d^{GW}_i)$. Table 2 shows that $\Delta d_L$, ranges from 0.0632 for the Q3nod model to 0.0897 for the popIII model i.e. is very small and hence it is not surprising that our implementation favours the true model over the DGP model. However if one propagates the error on the EM distance to an error on the GW luminosity distance using the scaling (3.6) such that the error on a GW measurement is now $\sigma_i = \frac{\Delta d^{GW}_i}{d^{EM}_i} \sigma_{EM}$ where $(\sigma_{EM})_i = (\Delta d_L)_i \times (d^{EM}_i)_i$, is the uncertainty in the corresponding EM distance, then we find that the errors are larger by a factor of ~ 1 to ~ 2.5 depending on how far the source is and which cosmological scenario is considered. More precisely, the further away the sources and the more pronounced the modification
from GR, i.e. the larger the number of dimensions, the lower the screening scale and the steeper the transition, the larger the discrepancy between the GW and EM luminosity distance and hence the larger the factor by which the error on $x^{GW}$ is increased. To understand this, Figure 8 shows simulated GW data scattered around their ‘true’ GW distance for both $\sigma$ stated above for the less ‘extreme’ $\theta^{14} = (D = 5, R_c = 4R_H, n = 1, H_0)$ and most extreme $\theta^{11} = (D = 7, R_c = R_H, n = 10, H_0)$ cosmological scenarios within the Q3d formation model.

We computed the $\Delta AIC$ factor for the originally assumed ‘optimistic’ uncertainties and the more realistic uncertainties for the data shown in 8 and we find that $\Delta AIC_{NGR/DGP}$ goes from $1092$ to $16$ in the most extreme cosmological scenario and from $97$ to $-63$ in the less extreme scenario. This requires further investigation but these preliminary results show the large effect the errors can have on the $\Delta AIC$ factor. In the case where the modification from DGP is the most pronounced the data still favours the NGR model, but in the least modified cosmological scenario, however, increasing the standard deviation up to a factor of $\sim 2.5$ leads to a negative $\Delta AIC_{NGR/DGP}$, thereby implying that the data favours the DGP model. This is in agreement with the observation in Sec. 6.1 that LISA will not be able to place meaningful constraints on the cosmological parameters for this particular cosmological scenario and MBHB formation model.

### 6.7 Constraints on the DGP model using the full LISA and Pantheon samples

We can perform the posterior statistical analysis using the full seven-dimensional parameter space for the catalogues corresponding to the cosmological scenarios with $\theta = (D = 5, R_c = 1.2R_H, n = 1, H_0)$ and $\theta = (D = 5, R_c = 1.2R_H, n = 10, H_0)$ for the Pantheon and LISA data combined. The main constraints are reported in Table 9 and the joint posterior for a random catalogue corresponding to the cosmological scenario $\theta^8 = (D = 5, R_c = 1.2R_H, n = 1, H_0)$ within the Q3 model is shown in Figure 9.

### 7 Conclusions

We have investigated the potential capability of the LISA satellite to place constraints on higher-dimensional cosmological theories with non-compact spacetime dimensions, by using GW standard
Figure 9: Joint posterior PDF for the full seven-dimensional parameter space for the catalogue corresponding to FoM of the cosmological scenario $\theta^a = (D=5, R_c = 1.2R_H, n = 1, H_0)$ within the Q3d formation model for Pantheon and LISA data combined.

sirens observed with an EM counterpart. In the absence of a complete, unique GW model for these theories, we used a phenomenological ansatz for the GW amplitude, which is based on the physics of the DGP model and includes a screening scale beyond which the GWs leak into the higher dimensions, leading to a reduction in the amplitude of the observed GWs and hence a systematic error in the inferred distance to the source. Considering various plausible cosmological scenarios ($D = 5, 6$ or $7$, $n = 1$ or $10$ and $R_c = R_H \sim 4\text{Gpc}$ or $4R_H$) and three models for the MBHB formation (heavy seeds without delays, heavy seeds with delays and popIII stars) we have produced catalogues of MBHB events for which an optical counterpart may be observed, each composed of a GW luminosity distance, redshift of corresponding EM counterpart and an estimate of the error on measured distance.

We have found that in general the heavy seeds with no delays model – where the number of detectable standard sirens amounts to a four-year average of $\sim 27$ – gives systematically better
Table 9: Figures of merit and 95% confidence intervals summarising the marginalised posterior PDFs of the Hubble parameter for the various MBHB formation models when considering Pantheon plus LISA data.

| Parameter | $H_0 (\text{km s}^{-1} \text{Mpc}^{-1})$ |
|-----------|-----------------------------------|
|           | $n = 1$                          | $n = 10$       |
| $D = 5$   | $71.56^{+0.89}_{-0.88}$          | $71.38^{+0.76}_{-0.73}$ |
|           | $71.61^{+0.89}_{-0.88}$          | $71.41^{+0.76}_{-0.73}$ |
|           | $71.59^{+0.86}_{-0.86}$          | $71.41^{+0.69}_{-0.68}$ |

results than the other two scenarios, where the four-year average only amounts to about 12 and 14 for the popIII and Q3nod respectively. Thus, we conclude that the ability of LISA to place meaningful constraints on the number of spacetime dimensions and screening scale will strongly depend on the actual number and redshift distribution of MBHB merger events and the corresponding efficiency in identifying a host galaxy redshift.

We also found that LISA’s ability to constrain higher-dimensional theories defined by the scaling (3.6) will depend on the cosmological parameters defining the theory, namely the number of dimensions, screening scale and transition steepness – the $(D = 7, R_c = R_H, n = 10)$ scenario, where modification from GR is the most pronounced, being much better constrained than the others.

In the cosmological scenario where the modification from gravity is the least pronounced, namely $(R_c = 4R_H, n = 1)$, we have found that LISA is not sensitive enough to provide meaningful constraints on the parameters. This is true for $D = 5, 6$ and 7 and all formation models, though popIII and Q3d give slightly worse results. On the other hand, if any of the other cosmological scenarios is considered, then we found that LISA is able to place meaningful constraints on the number of spacetime dimensions and that this power considerably improves, not only for greater numbers of standard sirens observed but also the steeper the transition and the smaller the screening scale – reaching a precision at the level of 0.86% for the most favorable cosmological scenario and formation model.

As for the constraints on the screening scale, we found that $R_c$ is not as well constrained as $D$, but that unlike the constraints on $D$, the constraints on $R_c$ are very sensitive to the number of spacetime dimensions. Indeed, LISA is unable to place meaningful constraints on $R_c$ when $D = 5$, unless $(n = 10, R_c = R_H)$ and the Q3nod or Q3 model is considered, but for $D \geq 6$, except for the least favourable cosmological scenario, LISA can place meaningful constraints on $R_c$ in all cosmological scenarios and MBHB formation models reaching a precision of 7.2% for $(D = 7, n = 10, R_c = R_H)$ within the Q3nod model.

Following our parameter estimation analysis, we then investigated whether the simulated data would favour the particular model of modified gravity from which it was simulated from when compared with the DGP model. To that end we calculated the Bayesian evidence in favour of each higher-dimensional model considered, and found that the log evidence favoured the true NGR model in every case, but the evidence was unexpectedly larger – ranging from 97 in the least favourable scenario and formation model to 4025 in the most favourable scenario and formation model. We believe this is due to the uncertainties on the luminosity measurements being so small and this requires further investigation.

In summary, standard sirens observed with LISA in the redshift range $1 < z < 8$ have the potential to test higher-dimensional theories with non compact extra-dimensions in a completely different way than current EM probes. However, our analysis is a phenomenological one, giving the GW damping considering a very general type of leakage for large extra dimensions. Our results do not hold for higher-dimensional theories with compact extra dimensions such as string theory. Additionally, even though the GW amplitude scaling used applies to the DGP model, the so-called ‘infrared transparency’ effect can be shown to result in a distance at which GW damping is manifested in the DGP model much beyond the distances to sources observable with frequencies relevant to LISA.
Our analysis still provides a good measure of the constraints we can place on extra-dimensional theories with large (≥ 100km) extra dimensions.

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