Making objective decisions in mechanical engineering problems

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Abstract. Decision making process has a great influence in the development of a given project, the goal being to select an optimal choice in a given context. Because of its great importance, the decision making was studied using various science methods, finally being conceived the game theory that is considered the background for the science of logical decision making in various fields. The paper presents some basic ideas regarding the game theory in order to offer the necessary information to understand the multiple-criteria decision making (MCDM) problems in engineering. The solution is to transform the multiple-criteria problem in a one-criterion decision problem, using the notion of utility, together with the weighting sum model or the weighting product model. The weighted importance of the criteria is computed using the so-called Step method applied to a relation of preferences between the criteria. Two relevant examples from engineering are also presented. The future directions of research consist of the use of other types of criteria, the development of computer based instruments for decision making general problems and to conceive a software module based on expert system principles to be included in the Wiki software applications for polymeric materials that are already operational.

1. Introduction
Knowledge acquiring process in engineering is based on the extensive use of the models. In this way there are used analytical models, numerical models or experimental studies. Most of the models in engineering are deterministic, this means that we have the same results for a certain set of input data. However, in the development of a process there are certain stages when an analyst or an expert must make decisions. If the best decision is not taken at the proper time, the final result will be suboptimal. Usually humans use both objective and intuitive grounds in the decision making process. If several criteria must be taken into account, the fuzzy decisional environment may influence the decision making process. Moreover, if in the automatic data processing there must be made decisions at certain moments inside the loops, the human that analysis the problem may produce important delays. For this reasons, acquiring knowledge in the decision making process in order to create an appropriate instrument to be used in the subsequent projects becomes an important goal.

2. Theoretical background
The current conditions of increasing complexity and of increasing speed of the phenomena require accurate decisional instruments to provide the hierarchy of options from which an optimal solution may be chosen in a given context.
Game theory offers basic concepts used to assess the influences on the alternatives from the set of possible options.

2.1. Ideas regarding the game theory

The results of the decision making processes are influenced by a series of conditions depending either on the wilfulness and actions of the people, or on the context of natural conditions of the ongoing events. The mathematical modelling of the influences onto the decision making processes are done by the use of the ‘strategic game’ concept.

The game is a competition between several participants designated players; among them at least one is analysing and rationally deciding what optimal option is the preferred variant of action. The two players’ games are considered the most important games.

The play, i.e. an instance of the game, is the succession of the players’ decisions, also designated moves or actions. The moves may be done according to a set of rules. A given play has a start and an end; for each of these moments the system has a given state. By the end of the game each participant has a result which may be either ‘win’ or ‘loss’.

The strategy is the plan used by a player to select a set of actions, in order to win the game, taking into account the moves of the other players.

A game with two players is expressed in a matrix form in the following table.

|         | \( N_1 \) | \( N_2 \) | ... | \( N_j \) | ... | \( N_n \) |
|---------|-----------|-----------|-----|-----------|-----|-----------|
| \( P_1 \) | \( c_{11} \) | \( c_{12} \) | ... | \( c_{1j} \) | ... | \( c_{1n} \) |
| \( P_2 \) | \( c_{21} \) | \( c_{22} \) | ... | \( c_{2j} \) | ... | \( c_{2n} \) |
| ...     | ...       | ...       | ... | ...       | ... | ...       |
| \( P_i \) | \( c_{i1} \) | \( c_{i2} \) | ... | \( c_{ij} \) | ... | \( c_{in} \) |
| ...     | ...       | ...       | ... | ...       | ... | ...       |
| \( P_m \) | \( c_{ml} \) | \( c_{m2} \) | ... | \( c_{mj} \) | ... | \( c_{mn} \) |

where:
- \( P \) is the first player;
- \( N \) is the opponent that sometimes may be the nature;
- \( P_i = \{P_1, P_2, \ldots, P_m\} \) is the set of strategies of the \( P \) player;
- \( N_j = \{N_1, N_2, \ldots, N_n\} \) is the set of strategies of the \( N \) player;
- \( c_{ij}, (i = 1..m; j = 1..n) \) represents the consequence of the \( P_i \) strategy selection belonging to the \( P \) player and of the \( N_j \) strategy selection done by the \( N \) player.

In order to evaluate the consequences of a strategy’s selection, there must be considered the utility of each consequence, being employed the hypothesis that it represents either a financial value, or a pure utility.

Games may be classified according to the existence of the so called “saddle point”, [1], a.k.a. equilibrium point. Specific to the saddle point games is the fact that a rational reasoning determines the two players to have an optimal strategy. The pair made of the both strategies represents a solution of the game and defines a so called saddle point.

The generalization of such problem is based on the maxi.min principle. In this way, let us consider an \( m \times n \) order game, having the following matrix:
By applying the maxi.min principle, the first user selects the strategy that offers the minimum profit:

\[ v_1 = \max_i \left( \min_j a_{ij} \right), 1 \leq i \leq m, 1 \leq j \leq n \]

To determine the \( v_1 \) value and the strategy that yields this value, we use the following method: we determine all the minimum values along the lines, \( \left( \min_j a_{ij} \right) \), and from these values we choose the \( \left( \max, a_{ij} \right) \) maximum value. The second player applies the same procedure and it results:

\[ v_2 = \max_j \left( \min_i a_{ij} \right), 1 \leq i \leq m, 1 \leq j \leq n \]

The \( v_2 \) value and the strategy used to yield this value consist in selecting the \( \left( \max, a_{ij} \right) \) maximum values along the columns and then choosing the \( \left( \min_i a_{ij} \right) \) minimum value.

One can notice that \( v = v_1 = v_2 = \max_j \left( \min_i a_{ij} \right) \left( \min_j a_{ij} \right), 1 \leq i \leq m, 1 \leq j \leq n \).

In comparison with the saddle point games, in all the other games a rational reasoning way to analyse and to make decisions doesn’t identifies a pair of strategies, i.e. to an equilibrium point. In these cases there must be determined optimal composite strategies of the both players using algebraic, geometric and iterative methods.

The saddle point games are useful to model the conflictual situations between several parties, these situations being simplified to the two-players games by creating ‘alliances’. Each such upper-level player uses optimal strategies to maximize his profit at the expense to the opposite alliance.

Similarly, there may be conceived decision support models where a single player must choose an optimal strategy, by considering factors that may not be influenced by other players, these factors being states of the nature. Thus, the “nature” is not considered an intelligent player that wishes to get a maximum profit at the expense of the opponent, therefore there cannot be identified rules regarding its behaviour. Instead, there may be used statistical information regarding the nature and there may be done probabilistic forecasts. Based on the statistical information, the player is able to conceive his own strategy.

For the games where the opponent is the “nature”, decisions may be classified in the following classes:

- decisions in certitude conditions, the consequences being known;
- decisions in uncertainty conditions, when there are no information regarding the probabilities of occurrence of the consequent state of nature;
- decision in risk conditions, when the probabilities of occurrence of the consequent state of nature are known.

2.2. Decisions in uncertainty conditions – definition of the ‘utility’

For this class of conditions several criteria to make the decisions may be employed.

**Prudent criterion** or **Wald’s pessimism criterion**

According to Wald, the most play-safe making decision policy is to choose the variant which returns the maximum of the minimal profits or payoffs. In this way, there are determined the minimal utilities for each line, and among these, it will be selected the largest value.

This criterion is also known as Wald’s maxi.min principle, rule or paradigm.
Hurwicz’s optimism criterion
According to Hurwicz, there is assigned the \( p_1 \) probability of occurrence of the highest payoff situation and the \( p_2 \) probability of occurrence of the lowest profit case, where \( p_1 + p_2 = 1 \). Using these two probabilities there are computed the mathematical expectations and, further on the strategy that yields the largest expectation will be chosen.

Laplace’s criterion
According to this criterion there is considered the assumption of equiprobability of all nature’s states and the mathematical expectations are compared.

Savage’s minimum regret criterion
According to Savage, the “regret” is the difference between the maximum profit attainable for a given strategy (or state of nature) and the current payoff. In this way the payoff matrix is transformed into a “regret” matrix. This criterion selects the “regret” having the smallest value.

Definition of ‘utility’
Hurwicz considers the following criterion to define the utility, notion that replaces the notion of profit or payoff. Let us define the “optimism” of a player as a \( 0 \leq \alpha \leq 1 \), random number. On each line of the payoff matrix is found the \( (a_{\min i}) \) smallest payoff value and the \( (a_{\max i}) \) largest profit value. The \( h_i \) utility for each line is defined as

\[
h_i = \alpha \cdot a_{\max i} + (1 - \alpha) \cdot a_{\min i}.
\]

Finally is selected the \( “i” \) line for which we have the \( \max \{h_i\} \), i.e. the maximum value of the utility.

For a person in charge to make decisions is important to know for what value of \( \alpha \) a line, i.e. a strategy, becomes preferable in despite of another. This means that the deciding person must know how the strategies are sorted with respect to \( \alpha \). This analysis is paramount in the creation of the top level decisions’ hierarchy within a project.

2.3. Decisions in risk conditions
For extensive decision making processes, there must be conceived scenarios that include various situations where the risk may occur, cases that may be further on grouped according to various criteria. Risk may be attached to activities that will be performed at different time frames. There are several types of risks (economical, commercial, political etc.) and the risk can be attached to various functionalities of a project or of a company.

In the running conditions of the complex actions, the multitude of aspects concerning the risk will intersect each other and, from the decision making person standpoint, there may be analysed the so called “risk package”. In these complex cases the evaluation of the individual risk will be subordinated to the global risk analysis.

Because of the high complexity of this approach, the solution is to employ simulation techniques.

2.4. Ideas regarding the utility theory in decisional problems
The elements of a decision making process that are identified by modelling its general structure are:
- the decision making entity, i.e. a person or a board;
- formulation of the decision making problem;
- the set of variants of action specific to a decision making situation;
- the set of consequences, evaluated for each variant of action;
- the set of criteria employed to make the decision;
- the goal of the decision, i.e. to maximise or to minimise some indicators;
the states of the nature that represent circumstantial factors, out of the control of the decision making entity.

From the set of variants of actions there must be chosen a unique variant, which is the optimal option, therefore it is necessary to compare the variants of action that yield to various consequences. In order to have a unique unit employed to quantify the consequences of the variants of action, there was defined the concept of ‘utility’.

The von Neumann-Morgenstern axioms offer the grounds of the expected utility in decisional problems.

A1: Completeness - if we compare two variants of action, \( V_1 \) and \( V_2 \), there may be one of the following options:
- \( V_1 \) variant is preferred with respect to the \( V_2 \) variant (\( V_1 \).Pr. \( V_2 \));
- \( V_2 \) variant is preferred with respect to the \( V_1 \) variant (\( V_2 \).Pr. \( V_1 \));
- None of the variants is preferred, between the variants being a relation of indifference, i.e. (\( V_2 \).In. \( V_1 \));

A2: Transitivity - the relation of preference is transitive and the relation of indifference is transitive and symmetrical; if \( (V_i \text{.Pr.} V_j) \) and \( (V_j \text{.Pr.} V_k) \) → \( (V_i \text{.Pr.} V_k) \);

Starting from two simple variants of options, V1 and V2, there may be created a new variant if two probabilities, \( p \) and \( (1-p) \) are assigned to the initial variants. As mentioned before, the sum of these two probabilities is 1. The resulting variant of action is a mixture between the initial two variants, i.e. \( V_{12} = \{p \cdot V_1; (1-p) \cdot V_2\} \);

Starting from three variants of action, \( V_1, V_2, V_3 \) and the \( (V_1 \text{.Pr.} V_2 \text{.Pr.} V_3) \) relation of preference, there may be created two combinations or a mixtures, \( V' = \{p'V_1;(1-p')V_2\} \), where \( V' \text{ P} V_2 \), and \( V'' = \{p''V_1;(1-p'')V_3\} \), where \( V_2 \text{ P} V'' \). Using this method, starting from two variants of action, for instance \( V_1 \) and \( V_3 \), there may be created an infinite number of mixtures, also known as probabilistic combinations, that continuously vary along the scale of preferences between \( V_1 \) and \( V_3 \).

Starting from three distinct variants of action \( V_1, V_2, V_3 \), and the relation of preference \( V_1 \text{ P} V_2 \), then, it is also implicitly expressed the relation \( \{pV_1;(1-p)V_2\} \text{ P} \{pV_2;(1-p)V_3\} \). Therefore, if a \( V_1 \) variant is preferred to a \( V_2 \) variant of action, then a mixture of \( V_1 \) with a third variant, \( V_3 \), will be preferred to a mixture of \( V_2 \) with \( V_3 \).

A3: Continuity - if \( (V_i \text{.Pr.} V_j \text{.Pr.} V_k) \), there is ‘p’ in \([0,1]\) so that \( \{V_j \text{.In.} \{p \cdot V_i ;(1-p) \cdot V_k\}\} \).

A4: Independence - for any ‘p’ in \([0,1]\) and \( V_i, V_2 \) and \( V_3 \), for which \( (V_i \text{.Pr.} V_2) \), we have the relation \( \{p \cdot V_i ;(1-p) \cdot V_2\} \text{.Pr.} \{p \cdot V_2 ;(1-p) \cdot V_3\} \).

Using these propositions it is defined the function of utility that assigns to each variant of action a certain value, i.e. a real number.

The utilities are subjectively expressed by the decision making person, therefore the problem is difficult and controversial. The notion of utility is indispensable in the decision making processes.

If there are two or several states of the nature, the decision making depends on the information regarding the probabilities assigned to the states of the nature. In this way, being known the \( p_1 \) probability to achieve the \( N_1 \) state of the nature and the \( p_2 \) probability to achieve the \( N_2 \) state of the nature, the decision will be done under risk conditions. In this case there must be used the mathematical expectations of the utilities for each state of the nature and there will be selected the state that has the maxim mathematical expectation of the utility.

To determine the optimal variant under risk conditions, there is computed the weighted average utility, as a sum of the factors resulted by multiplying the probability of achievement and the according utility for each state of the nature.
3. Discussion
In many situations the decision making process must take into account several states of the nature and several criteria. The criteria may optimise the results by choosing either the maximum values, or the minimum ones. In such cases the problem to be solved is a multiple-criteria decision-making (MCDM), that is a component of the operations research science.

3.1. Basic ideas regarding the multiple-criteria decision analysis and the calculus method
The multiple-criteria decision problem was studied from several points of view, being conceived solutions that start from mathematical complex approaches [2], to practical calculus methods. In what follows we consider the theoretical background useful to solve practical applications.

Let us consider a decision making problem having “m” variants of action, \( (V_1, V_2, \ldots, V_m) \), “n” criteria, \( (C_1, C_2, \ldots, C_n) \) and “r” states of the nature, \( (S_1, S_2, \ldots, S_r) \).

Table 2. Matrix of the consequences for a multiple-criteria decision analysis.

| Criteria Variants | \( C_1 \) | \( C_2 \) | \( \ldots \) | \( C_n \) | \( C_1 \) | \( C_2 \) | \( \ldots \) | \( C_n \) | \( C_1 \) | \( C_2 \) | \( \ldots \) | \( C_n \) |
|-------------------|--------|--------|-------------|--------|--------|--------|-------------|--------|--------|-------------|--------|
| \( V_1 \)         | \( x_{1,1} \) | \( x_{1,2} \) | \( \ldots \) | \( x_{1,n} \) | \( x_{1,1} \) | \( x_{1,2} \) | \( \ldots \) | \( x_{1,n} \) | \( x_{1,1} \) | \( x_{1,2} \) | \( \ldots \) | \( x_{1,n} \) |
| \( V_2 \)         | \( x_{2,1} \) | \( x_{2,2} \) | \( \ldots \) | \( x_{2,n} \) | \( x_{2,1} \) | \( x_{2,2} \) | \( \ldots \) | \( x_{2,n} \) | \( x_{2,1} \) | \( x_{2,2} \) | \( \ldots \) | \( x_{2,n} \) |
| \( \ldots \)     | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( V_m \)        | \( x_{m,1} \) | \( x_{m,2} \) | \( \ldots \) | \( x_{m,n} \) | \( x_{m,1} \) | \( x_{m,2} \) | \( \ldots \) | \( x_{m,n} \) | \( x_{m,1} \) | \( x_{m,2} \) | \( \ldots \) | \( x_{m,n} \) |

For this problem, the matrix of the consequences is \( X = (x_{i,j,k}) \), \( i = 1, m \), \( j = 1, n \), \( k = 1, r \), where \( x_{i,j,k} \) is the consequence of the “i” variant of action, according to the “j” criterion, in the “k” state of the nature.

For a unique state of the nature, \( S_k \), the decision making problem is under certainty conditions and the according matrix of the consequences is presented in the table 3, where \( (u_{i,j}) \) are the utilities that replace the \( (x_{i,j}) \) consequences.

For the “i” variant of action, by applying the “j” criterion, transition from consequences to utilities is done using the following method:

- if the “j” criterion is an optimisation by maximisation criterion, then \( u(V_j) = \frac{x_j - x_{\min}}{x_{\max} - x_{\min}} \);
- if the “j” criterion is an optimisation by minimisation criterion, then \( u(V_j) = \frac{x_{\max} - x_j}{x_{\max} - x_{\min}} \),

where \( x_{\min} \) and \( x_{\max} \) are the extreme values of the current criterion, for all the variants of action to be considered.
Table 3. Matrix of the consequences for a multiple-criteria decision analysis under certainty conditions.

| Criteria Variants | $S_k$ | $C_1$ | $C_2$ | ... | $C_n$ |
|-------------------|-------|-------|-------|------|-------|
| $V_1$             | $(u_{1,1})_{S_1}$ | $(u_{1,2})_{S_1}$ | ... | $(u_{1,n})_{S_1}$ |
| $V_2$             | $(u_{2,1})_{S_1}$ | $(u_{2,2})_{S_1}$ | ... | $(u_{2,n})_{S_1}$ |
| ...               | ... | ... | ... | ... | ... |
| $V_m$             | $(u_{m,1})_{S_1}$ | $(u_{m,2})_{S_1}$ | ... | $(u_{m,n})_{S_1}$ |

The general utility may be determined using either the weighted sum model (WSM), or the weighted product model (WPM).

By applying the weighted sum model, the general utility for the “$i$” variant of action may be computed using the relation

$$u_{gl,i} = \sum_{j=1}^{m} (\gamma_j \cdot u_{ij}),$$

where $\gamma_j$ is the relative weight of importance of the “$j$” criterion.

In this way, the multiple-criteria decision problem is changed into a single criterion decision problem for which the maximum general utility must be selected. Therefore, the optimal variant of action is

$$V_{optimal} = V(\max(u_{gl,i})).$$

According to the weighted product model (WPM), the first stage is to assign a mark, for instance $H_{T_{abs}} = 1000$, which may be interpreted as an absolute level of performance assigned to a variant of action, whose $x_{ref,j}$ consequences are considered the references for all the other variants of action.

The level of performance of any variant of action is

$$H_{abs,i} = 1000 \cdot \left[ \prod_{j=S_{min}}^{x_{ref,j}} \left( \frac{x_{i,j}}{x_{ref,j}} \right)^{\gamma_j} \right] \left[ \prod_{j=S_{max}}^{x_{ref,j}} \left( \frac{x_{i,j}}{x_{ref,j}} \right)^{\gamma_j} \right],$$

where the meanings of the according parameters are:

- $i$ - index of the variant of action
- $j$ - index of the criterion
- $x_{ref,j}$ - the “$j$” consequence of the variant of action selected as reference, for which $H_{T_{abs}} = 1000$;
- $S_{min}$ - the optimisation by minimisation set of criteria;
- $S_{max}$ - the optimisation by maximisation set of criteria;
- $\gamma_j$ - the relative weight of importance of the “$j$” criterion.

The according relative level of performance is

$$H_{rel,i} = \frac{H_{abs,i}}{\max(H_{abs})}.$$
Again, the multiple-criteria decision problem is changed into a single criterion decision problem with respect to the maximum relative level of performance. Therefore, the optimal variant of action is

\[ V_{optimal} = V \left( \max \{ H_{rel i} \} \right). \]

One can notice that for both models, i.e. the weighted sum model and the weighted product model there are used relative weights of importance of the criteria.

For each criterion we have the following propositions:

a) Each criterion is either an optimisation by maximisation condition, like profit, or an optimisation by minimisation such as fuel consumption, production costs or purchase price;

b) Each criterion must be independent with respect to the others; if this condition is not respected the same feature is considered several times and the analysis uses redundant aspects;

c) The number of criteria to be considered may not be very large, because the resulting hierarchy of variants of action of the decision making problem becomes irrelevant.

The analyst defines a relationship of preference between all the criteria of the decision making problem using one of the following operators, i.e. types of relations between two criteria:

- Strong preference relationship - .PP.;
- Preference relationship - .PR.;
- Indifference relationship - .IN..

The relative weight of importance of the criteria are computed using the so called ‘Step method’ applied for the relationship of preference between all the criteria. For instance, let us consider five criteria and the following relationship ‘C03.PP.C01.IN.C02.PR.C04.IN.C05’. We create a “\( n \times n \)” matrix of the preferences, \( a_{j_1,j_2} \), \( j_1 = 1 + n \), \( j_2 = 1 + n \), where \( n \) is the number of criteria; in this case \( n = 5 \). The elements of the matrix of preferences are defined using the following rule:

\[
a_{j_1,j_2} = \begin{cases} 4, & C_{j_1,PP}.C_{j_2} \\ 2, & C_{j_1,PR}.C_{j_2} \\ 1, & C_{j_1,IN}.C_{j_2} \\ 0, & C_{j_2,PP}.C_{j_1} \text{ or } C_{j_2,PR}.C_{j_1} \end{cases}
\]

For the ‘C03.PP.C01.IN.C02.PR.C04.IN.C05’ relationship, the matrix of the preferences is

\[
\begin{align*}
\begin{bmatrix}
 j_1 = 1 & j_2 = 2 & j_2 = 3 & j_2 = 4 & j_2 = 5 \\
 j_1 = 2 & & & & \\
 j_1 = 3 & 1 & 4 & 1 & 4 & 4 \\
 j_1 = 4 & 0 & 0 & 0 & 1 & 1 \\
 j_1 = 5 & 0 & 0 & 0 & 1 & 1 \\
\end{bmatrix}
\end{align*}
\]

\[
\sum_{j_1} \left[ \sum_{j_2} (a_{j_1,j_2}) \right] = 33
\]

The relative weight of importance of the criteria are computed using the relation

\[
\gamma_{j_1} = \frac{\sum_{j_2} (a_{j_1,j_2})}{\sum_{j_1} \sum_{j_2} (a_{j_1,j_2})}.
\]
For the matrix of preference previously presented the relative weight of importance of the criteria are \( \gamma_1 = \gamma_2 = \frac{6}{33} = 0.1818, \gamma_3 = \frac{17}{33} = 0.5151, \gamma_4 = \gamma_5 = \frac{2}{33} = 0.0606 \).

These theoretical aspects are used to solve multi criteria decision making problems.

3.2. Application 1: hierarchy of some polymeric materials

Polymeric materials are a long run concern of the authors. One of the important accomplishments was the development of web enabled advisor programs based on script engine/database pair PHP/MySQL and a WikiAdvisor concept using computer virtualization, [3, 4, 5] the original application being entitled Polymeric Material Adviser (the acronym PMA). This was the starting point as primary database and describes the moulding and the processing conditions for the thermoplastic and for the thermoset materials. Polymeric Material Adviser describes also the typical values for temperatures, of specific pressures on material, and the material change over and cleaning of plasticizing cylinder.

![Polymeric Material Adviser interface](image)

**Figure 1.** Polymeric Material Adviser interface.

Polymeric Material Adviser improves the process solution including the selection of the injection moulding material, the mechanical and the technological properties of the polymeric material. Polymeric Material Adviser includes many and different polymeric materials from several various suppliers, so we can choose easily which material is best suited for a given technological application.

Let us consider the polymeric materials presented in the following table. In this case study the criteria consist of the properties of the materials: density, softening temperature (VICAT - 5kg), water absorption (24h-23°C), tensile strength (at break), Rockwell hardness, elongation (at break), the elastic modulus - E, the impact strength (Izod-notch) and the dielectric strength. All these criteria may be optimised by maximization, except the water absorption, the elongation and the elastic modulus.

In this case study we must create the hierarchy of variants of actions, i.e. of the polymeric materials for a given relation of preference between the criteria.
Table 4. Properties of a given set of polymeric materials.

| Polymeric material | Density [g/cm³] | Softening temperature (VICAT - 5kg) [°C] | Water absorption (24h-23°C) [%] | Tensile strength (at break) [MPa] | Rockwell hardness (at break) [R scale] | Elongation (at break) [%] | Elastic modulus E [MPa] | Impact strength (Izod-notch) [mJ/mm²] | Dielectric strength [kV/mm] |
|--------------------|-----------------|-----------------------------------------|---------------------------------|----------------------------------|----------------------------------------|--------------------------|-------------------------|----------------------------------------|--------------------------|
| PA 6               | 1.14            | 165.00                                  | 2.80                            | 65.00                            | 110.00                                 | 180.00                   | 1500.00                 | 25.00                                  | 18.00                    |
| PA 6 glass-fibre filled 25% | 1.30          | 185.00                                  | 1.40                            | 105.00                           | 110.00                                 | 8.00                     | 6000.00                 | 12.00                                  | 19.00                    |
| PA 610             | 1.08            | 170.00                                  | 0.50                            | 55.00                            | 110.00                                 | 110.00                   | 1450.00                 | 30.00                                  | 20.00                    |
| PA 66              | 1.14            | 200.00                                  | 1.70                            | 73.00                            | 109.00                                 | 140.00                   | 2500.00                 | 13.00                                  | 15.00                    |
| PA 66 glass-fibre filled 25% | 1.32          | 220.00                                  | 1.00                            | 110.00                           | 125.00                                 | 6.00                     | 7000.00                 | 8.00                                  | 19.00                    |
| Max value          | 1.32            | 220.00                                  | 2.80                            | 110.00                           | 125.00                                 | 180.00                   | 7000.00                 | 30.00                                  | 20.00                    |
| Min value          | 1.08            | 165.00                                  | 0.50                            | 55.00                            | 109.00                                 | 6.00                     | 1450.00                 | 8.00                                  | 15.00                    |

The according utilities of the properties in the table above are presented in the following table.

Table 5. Utilities of a given set of polymeric materials.

| Polymeric material | Criterion's symbol | Criterion's type | Density | Softening temperature (VICAT - 5kg) | Water absorption (24h-23°C) | Tensile strength (at break) | Rockwell hardness (at break) | Elongation (at break) | Elastic modulus E | Impact strength (Izod-notch) | Dielectric strength |
|--------------------|-------------------|------------------|---------|--------------------------------------|-----------------------------|-----------------------------|-------------------------------|----------------------|-------------------|-------------------------------|----------------------|
| PA 6               | C01               | MAX              | 0.25    | 0                                    | 0.1818182                  | 0.0625                      | 0                            | 0.990991            | 0.772727          | 0.6                            |                      |
| PA 6 glass-fibre filled 25% | 0.916667      | MAX              | 0.3636363 | 0.6086956                             | 0.9090909                  | 0.0625                      | 0.9885077                  | 0.18018             | 0.181818          | 0.8                            |                      |
| PA 610             | 0                 | MIN              | 0.0909091 | 1                                    | 0                           | 0.0625                      | 0.4022988                  | 1                    | 1                | 1                             |                      |
| PA 66              | 0.25              | MAX              | 0.6363636 | 0.4782609                             | 0.3272727                  | 0                           | 0.2298851                  | 0.810811            | 0.227273          | 0                             |                      |
| PA 66 glass-fibre filled 25% | 1                | MAX              | 0.7826087 | 1                                    | 1                           | 1                           | 0                            | 0                   | 0                | 0.8                            |                      |

We consider the following relation of preference between the criteria:  
C04 . PP. C07 . PP. C03 . PP. C01 . PP. C02 . PP. C06 . PP. C05 . PP. C08 . PP. C09
and we compute the according relative weights of importance which are presented in the following table.
Table 6. Calculus of the relative weight of importance of the criteria.

| C01 | C02 | C03 | C04 | C05 | C06 | C07 | C08 | C09 | Sum   | Relative weight of importance |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-------|--------------------------------|
| Density | C01 | 1   | 4   | 0   | 0   | 4   | 4   | 0   | 4     | 21     | 0.137254902                  |
| Softening temperature (VICAT - 5kg) | C02 | 0   | 1   | 0   | 0   | 4   | 4   | 0   | 4     | 17     | 0.111111111                  |
| Water absorption (24h-23°C) | C03 | 4   | 4   | 1   | 0   | 4   | 4   | 0   | 4     | 25     | 0.163398693                  |
| Tensile strength (at break) | C04 | 4   | 4   | 4   | 1   | 4   | 4   | 4   | 4     | 33     | 0.215686275                  |
| Rockwell hardness | C05 | 0   | 0   | 0   | 0   | 1   | 0   | 0   | 4     | 4     | 9      | 0.058823529                  |
| Elongation (at break) | C06 | 0   | 0   | 0   | 0   | 4   | 1   | 0   | 4     | 4     | 13     | 0.08496732                   |
| Elastic modulus E | C07 | 4   | 4   | 4   | 0   | 4   | 4   | 1   | 4     | 4     | 29     | 0.189542484                  |
| Impact strength (Izod - notch) | C08 | 0   | 0   | 0   | 0   | 0   | 0   | 1   | 4     | 5      | 0.032679739                  |
| Dielectric strength | C09 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 1     | 1      | 0.006535948                  |
| Sum | 153 |     |     |     |     |     |     |     |       | 1       | 1                      |

Once we have the matrix of utilities and the values of the relative weights of importance, we compute the general utility for each material and it results the following hierarchy.

Table 7. Hierarchy of the variants of action.

| Variant of action (materials) | Weighted sum model |
|------------------------------|--------------------|
| 1 PA 66 glass-fibre filled 25% | 0.740949133        |
| 2 PA 6 glass-fibre filled 25% | 0.594749022        |
| 3 PA 610                      | 0.440116599        |
| 4 PA 66                       | 0.434399258        |
| 5 PA 6                       | 0.29421487         |

As it can be noticed, for the relation of preference that considers the materials’ strength, elastic modulus and water absorption as the most important criteria, the best variant of action is “PA 66 glass-fibre filled 25%” followed by “PA 6 glass-fibre filled 25%” at a rate of 19.73%, which is a significant large value.

3.3. Application 2: hierarchy of the running conditions of an internal combustion engine
An important problem in engineering is to select the optimal running conditions of a given equipment. Let us consider an internal combustion engine (abbreviation ICE) that is actually an energy converter. Beside the minimum fuel consumption, an important actual concern is the emission of harmful substances, i.e. the pollution. This research topic, [6, 7, 8], is influencing the decisions regarding the selection of a given ICE or of the most appropriate set of parameters in running conditions.

In this case study, let us consider the running conditions presented in the following table, together with the specific parameters for each angular speed, i.e. variant of action.
### Table 8. Parameters of an ICE for various angular speeds.

| Angular Speed [rpm] | Power [kW] | Torque [N·m] | Consumption [kg/h] | NOx emission [ppm] | CO Emission [ppm] | Hydrocarbon emission [ppm] | Smoke [%] |
|---------------------|------------|--------------|--------------------|---------------------|------------------|-----------------------------|----------|
| 2400                | 52         | 206.9        | 12.38              | 1107                | 735              | 127                         | 3.45     |
| 2300                | 50.97      | 211.6        | 11.977             | 1150                | 589              | 126                         | 2.63     |
| 2200                | 49.54      | 215          | 11.567             | 1174                | 536              | 121                         | 2.69     |
| 2100                | 48.17      | 219.1        | 11.174             | 1205                | 528              | 124                         | 2.21     |
| 2000                | 46.68      | 222.9        | 10.735             | 1291                | 515              | 123                         | 2.22     |
| 1900                | 44.97      | 226.1        | 10.264             | 1390                | 532              | 114                         | 2.38     |
| 1800                | 43.1       | 228.7        | 9.749              | 1469                | 621              | 110                         | 2.5      |
| 1700                | 41.13      | 231          | 9.22               | 1555                | 686              | 109                         | 2.58     |
| 1600                | 39.08      | 233.2        | 8.701              | 1620                | 690              | 102                         | 2.31     |
| 1500                | 36.85      | 234.6        | 8.24               | 1663                | 805              | 102                         | 3.06     |
| 1440                | 35.35      | 234.4        | 7.924              | 1679                | 875              | 101                         | 2.97     |
| 1300                | 31.29      | 229.8        | 7.006              | 1753                | 825              | 108                         | 2.99     |
| 1199                | 28.57      | 227.6        | 6.466              | 1778                | 1030             | 108                         | 2.87     |
| 1100                | 25.77      | 223.8        | 5.843              | 1946                | 720              | 111                         | 2.2      |
| 1000                | 22.79      | 217.7        | 5.184              | 2004                | 1027             | 126                         | 2.85     |

Max value | 52 | 234.6 | 12.38 | 2004 | 1030 | 127 | 3.45 |
Min value | 22.79 | 206.9 | 5.184 | 1107 | 515 | 101 | 2.2 |

### Table 9. Matrix of utilities.

| Angular speed [rpm] | Power | Torque | Consumption | NOx emission | CO Emission | Hydrocarbon emission | Smoke |
|---------------------|-------|--------|-------------|--------------|-------------|----------------------|-------|
| Criterion’s symbol  | 01    | 02     | 03          | 04           | 05          | 06                   | 07    |
| Criterion type      | MAX   | MAX    | MIN         | MIN          | MIN         | MIN                  | MIN   |
|---------------------|-------|--------|-------------|--------------|-------------|----------------------|-------|
| 2400                | 0.964738 | 0.16968 | 0.056003335 | 0.95206243 | 0.85631068 | 0.038461538          | 0.0   |
| 2300                | 0.915782 | 0.29242 | 0.112979433 | 0.925306577 | 0.959223301 | 0.230769231          | 0.608 |
| 2200                | 0.868881 | 0.44043 | 0.167593107 | 0.890746934 | 0.974757282 | 0.115384615          | 0.992 |
| 2100                | 0.817871 | 0.57762 | 0.228599222 | 0.794871795 | 1            | 0.153846154          | 0.984 |
| 2000                | 0.759329 | 0.69314 | 0.294052251 | 0.684503902 | 0.966990291 | 0.5                   | 0.856 |
| 1900                | 0.69531  | 0.787   | 0.365619789 | 0.596432553 | 0.794174757 | 0.653846154          | 0.76  |
| 1800                | 0.627867 | 0.87004 | 0.439132852 | 0.500557414 | 0.667961165 | 0.692307692          | 0.696 |
| 1700                | 0.557686 | 0.94946 | 0.511256253 | 0.428093645 | 0.660194175 | 0.96138462          | 0.912 |
| 1600                | 0.481342 | 1       | 0.575319622 | 0.380156076 | 0.436893204 | 0.96138462          | 0.312 |
| 1500                | 0.429999 | 0.99278 | 0.619232907 | 0.362318841 | 0.30970874    | 1                    | 0.384 |
| 1400                | 0.290996 | 0.82671 | 0.746803787 | 0.279821628 | 0.398058252 | 0.730769231          | 0.368 |
| 1300                | 0.197877 | 0.74729 | 0.82184547   | 0.251950948 | 0            | 0.730769231          | 0.464 |
| 1199                | 0.10202  | 0.61011 | 0.908421345 | 0.064659978 | 0.601941748    | 0.615384615          | 1     |
| 1100                | 0       | 0.38989 | 1             | 0             | 0.005825243    | 0.38461538          | 0.48  |

Using the method previously presented we compute the utilities, table 9. We consider the following relation of preference between the criteria: 

\[ C04 \cdot PP \cdot C07 \cdot PP \cdot C03 \cdot PP \cdot C01 \cdot PP \cdot C02 \cdot PP \cdot C06 \cdot PP \cdot C05 \]
and the according relative weights of importance are presented in the following table.
Table 10. Calculus of the relative weight of importance of the criteria.

| Criteria         | C01 | C02 | C03 | C04 | C05 | C06 | C07 | Sum | Relative weight of importance |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|--------------------------------|
| Power            | C01 | 1   | 4   | 0   | 0   | 4   | 4   | 0   | 13 | 0.143 |
| Torque           | C02 | 0   | 1   | 0   | 0   | 4   | 4   | 0   | 9  | 0.099 |
| Consumption      | C03 | 4   | 4   | 1   | 0   | 4   | 4   | 0   | 17 | 0.187 |
| NOx emission     | C04 | 4   | 4   | 4   | 1   | 4   | 4   | 25  | 0.275 |
| CO emission      | C05 | 0   | 0   | 0   | 0   | 1   | 0   | 0   | 1  | 0.011 |
| Hydrocarbon emission | C06 | 0   | 0   | 0   | 0   | 4   | 1   | 0   | 5  | 0.055 |
| Smoke            | C07 | 4   | 4   | 4   | 0   | 4   | 4   | 1   | 21 | 0.231 |
| Sum              |     |     |     |     |     |     |     |     | 91 | 1 |

We compute the general utilities for each variant of action, table 11.

Table 11. Hierarchy of the variants of actions.

| Variant of action | Weighted sum model |
|-------------------|--------------------|
| n = 2100 rpm      | 0.689678925        |
| n = 2000 rpm      | 0.681561385        |
| n = 1900 rpm      | 0.657237634        |
| n = 1800 rpm      | 0.65648574         |
| n = 1700 rpm      | 0.629360615        |
| n = 2200 rpm      | 0.601288918        |
| n = 2300 rpm      | 0.59858963         |
| n = 1100 rpm      | 0.58952645         |
| n = 1440 rpm      | 0.53357978         |
| n = 1500 rpm      | 0.521700943        |
| n = 1199 rpm      | 0.509212825        |
| n = 1300 rpm      | 0.47215419         |
| n = 2400 rpm      | 0.469170259        |
| n = 1000 rpm      | 0.33832042         |

As it can be noticed from the previous table, the optimal choice for the NOx low emission is an angular speed \( n = 2100 \text{ rpm} \). However, the next choice is \( n = 2000 \text{ rpm} \) and it is at a rate of 1.78% of the first option, which is a small value. This means that in certain conditions the first two options may exchange places. The third option, \( n = 1600 \text{ rpm} \), is at a relative distance of 3.24% with respect to the first variant of action.

4. Conclusions
Decision making process is not anymore a white spot on the map of science. However, an overall goal in our existence is to reach a level of wisdom and to make intelligent decisions. The theoretical background, the method and the applications presented in the paper are relevant for our attempt to acquire knowledge in this field, the decision making processes being a long run research topic of the authors [9, 10, 11, 12]. In this context, using the computer to make optimal decisions was a general objective of the worldwide researchers, an example regarding the selection of the appropriate metallic material using an information technology related method being presented in [13].

The future directions of research identified by us are:
- acquiring knowledge regarding new types of optimisation criteria;
- creation of a standalone application which solves the multiple-criteria decision problem using the nowadays information technologies;
- creation of a material selection module based on expert system principles, to be used within the already operational software dedicated to the polymeric materials.

**Figure 4.** Role of a database and of an expert system in the context of the intelligent CAE.

These directions are included in the general trend regarding the development of inter-disciplinary and inter-domain computer based models, [14, 15]. Moreover, the ideas, methods and case studies previously presented may be used in more complex decision making processes and in complex optimisation studies, [16, 17, 18, 19, 20]

Finally, we remark that the theoretical and practical instruments offer support and suggestions regarding the optimal choice in a given scenario and they cannot eliminate or replace the analyst or the expert in the decision-making process.

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