Creating, moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice

Leticia Tarruell1, Daniel Greif1, Thomas Uehlinger1, Gregor Jotzu1 & Tilman Esslinger1

Dirac points are central to many phenomena in condensed-matter physics, from massless electrons in graphene to the emergence of conducting edge states in topological insulators12–14. At a Dirac point, two energy bands intersect linearly and the electrons behave as relativistic Dirac fermions. In solids, the rigid structure of the material determines the mass and velocity of the electrons, as well as their interactions. A different, highly flexible means of studying condensed-matter phenomena is to create model systems using ultracold atoms trapped in the periodic potential of interfering laser beams15–17. Here we report the creation of Dirac points with adjustable properties in a tunable honeycomb optical lattice. Using momentum-resolved interband transitions, we observe a minimum bandgap inside the Brillouin zone at the positions of the two Dirac points. We exploit the unique tunability of our lattice potential to adjust the effective mass of the Dirac fermions by breaking inversion symmetry. Moreover, changing the lattice anisotropy allows us to change the positions of the Dirac points inside the Brillouin zone. When the anisotropy exceeds a critical limit, the two Dirac points merge and annihilate each other—a situation that has recently attracted considerable theoretical interest18–20 but that is extremely challenging to observe in solids21. We map out this topological transition in lattice parameter space and find excellent agreement with ab initio calculations. Our results not only pave the way to model materials in which the topology of the band structure is crucial, but also provide an avenue to exploring many-body phases resulting from the interplay of complex lattice geometries with interactions18–21.

Ultracold gases in optical lattices have become a versatile tool with which to simulate a wide range of condensed-matter phenomena14. For example, the control of interactions has led to the observation of Mott insulating phases14–16. In fermionic systems, this provides new access to the physics of strongly correlated materials. However, the topology of the band structure is equally important for the properties of a solid. A prime example is the honeycomb lattice of graphene, where the presence of topological defects in momentum space—the Dirac points—leads to remarkable transport properties, even in the absence of interactions1. In quantum gases, a honeycomb lattice has recently been realized and investigated using a Bose–Einstein condensate17,18, but no signatures of Dirac points were observed. Here we study an ultracold Fermi gas of 40K atoms in a two-dimensional, tunable optical lattice, which can be continuously adjusted to create square, triangular, dimer and honeycomb structures. In the honeycomb lattice, we identify the presence of Dirac points in the band structure by observing a minimum bandgap inside the Brillouin zone using interband transitions. Our method is closely related to a technique recently used with bosonic atoms to characterize the linear crossing of two high-energy bands in a one-dimensional, bichromatic lattice19, but also provides momentum resolution.

To create and manipulate Dirac points, we have developed a two-dimensional optical lattice of adjustable geometry. It is formed by three retro-reflected laser beams of wavelength $\lambda = 1.064$ nm create the two-dimensional lattice potential of equation (1). Beams X and Y interfere and produce a chequerboard pattern, and beam Z creates an independent standing wave. Their relative position is controlled by the detuning $\delta$. a, b, Top: different lattice potentials can be realized depending on the intensities of the lattice beams. White regions correspond to lower potential energies and blue regions to higher potential energies. Bottom: diagram showing the accessible lattice geometries as a function of the lattice depths $V_X$ and $V_Y$. The transition between triangular (T) and dimer (D) lattices is indicated by a dotted line. When crossing the dashed line into the honeycomb (Hc) regime, Dirac points appear. The limit $V_3 \gg V_X, V_Y$; $V_3 \gg V_X$ corresponds to weakly coupled, one-dimensional chains (1D c). c, The real-space potential of the honeycomb lattice has a two-site unit cell (sites A and B) and the primitive lattice vectors are perpendicular. d, Left: sketch of the first and second Brillouin zones (BZs) of the honeycomb lattice, indicating the positions of the Dirac points. Right: three-dimensional view of the energy spectrum showing the linear intersection of the bands at the two Dirac points. The colour scale illustrates lines of constant energy. We denote the full bandwidth, $W$, the minimum energy gap at the edges of the Brillouin zone, $E_{CZ}$, and the Bloch wavevector, $q_0 = 2\pi/\lambda$. 

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1Institute for Quantum Electronics, ETH Zurich, 8093 Zurich, Switzerland.

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where $V_X$, $V_Y$ and $V_V$ denote the single-beam lattice depths (proportional to the laser beam intensities), $x$ is the visibility of the interference pattern and $k = 2\pi/\lambda$. We can adjust the two phases continuously, and choose $\theta = \pi$ and $\phi = 0$ (Methods). Varying the relative intensities of the beams allows us to realize various lattice structures (Fig. 1b). In the following, we focus on the honeycomb lattice, whose real-space potential is shown in Fig. 1c.

The honeycomb lattice consists of two sublattices, A and B. Therefore, the wavefunctions are two-component spinors. Tunnelling between the sublattices leads to the formation of two energy bands, which are well separated from the higher bands and have a conical intersection at two quasi-momentum points in the Brillouin zone—the Dirac points. These points are topological defects in the band structure, with respective associated Berry phases of $\pi$ and $-\pi$. This guarantees their stability with respect to lattice perturbations, such that a large range of lattice anisotropies change only the positions of the Dirac points inside the Brillouin zone. In contrast, breaking the inversion symmetry of the potential by introducing an energy offset, $\Delta$, between the sublattices opens an energy gap at the Dirac points, proportional to $\Delta$. In our implementation, $\Delta$ depends only on the value of the phase $\theta$ and can be precisely adjusted (Methods). As shown in Fig. 1c, d, the primitive lattice vectors are perpendicular, leading to a square Brillouin zone with two Dirac points inside. Their positions are symmetric around the centre and are fixed to quasi-momentum $q_x, q_y = 0$, owing to the time-reversal and reflection symmetries of the system. The band structure for our lattice implementation is in the two lowest bands topologically equivalent to that of a hexagonal lattice with six-fold symmetry. For deep lattices, both configurations then also map to the same tight-binding Hamiltonian.

We characterize the Dirac points by probing the energy splitting between the two lowest-energy bands through interband transitions. The starting point of the experiment is a non-interacting, ultracold gas of $N = 50,000$ fermionic $^{40}$K atoms in the $|F, m_F\rangle = |9/2, -9/2\rangle$ state, where $F$ denotes the hyperfine manifold and $m_F$ the Zeeman state. The cloud is prepared in the lowest-energy band of a honeycomb lattice with $V_X/E_F = 4.0(2)$, $V_Y/E_F = 0.28(1)$ and $V_V/E_F = 1.8(1)$, which also causes a weak harmonic confinement with trapping frequencies $\omega_x/2\pi = 17.6(1)$ Hz, $\omega_y/2\pi = 31.8(5)$ Hz and $\omega_z/2\pi = 32.7(5)$ Hz. Here $E_F = \hbar^2k^2/2m$ is the recoil energy, $\hbar$ denotes Planck’s constant and $m$ is the mass of a $^{40}$K atom. Throughout the manuscript, errors in parenthesis denote the standard deviation. On application of a weak magnetic field gradient, the atomic cloud is subjected to a constant force, $F$, in the $x$ direction, with an effect equivalent to that produced by an electric field in solid-state systems. The atoms are hence accelerated such that their quasi-momentum $q_x$ increases linearly up to the edge of the Brillouin zone, where a Bragg reflection occurs. The cloud eventually returns to the centre of the band, performing one full Bloch oscillation. Then we measure the quasi-momentum distribution of the atoms in the different bands (Methods).

Owing to the finite momentum width of the cloud, trajectories with different quasi-momenta $q_y$ are simultaneously explored during the Bloch cycle (Fig. 2a). For a trajectory far from the Dirac points, the atoms remain in the lowest-energy band (trajectory 1). In contrast, when passing through a Dirac point (trajectory 2), the atoms are transferred from the first band to the second because of the vanishing energy splitting at the linear band crossing. When measuring the quasi-momentum distribution, these atoms are missing in the first Brillouin zone and appear in the second band (Fig. 2a). We identify the points of maximum transfer with the Dirac points. The energy resolution of the method is set by the characteristic energy of the applied force, $E_g/h = F\lambda/2h = 88.6(7)$ Hz, which is small compared with the full bandwidth, $W/h = 4.6$ kHz, and the minimum bandgap at the edges of the Brillouin zone, $E_c/h = 475$ Hz.

To investigate how breaking the inversion symmetry of the lattice affects the Dirac points, we vary the sublattice offset, $\Delta$, which is controlled by the frequency detuning, $\delta$, between the lattice beams, and measure the total fraction of atoms transferred to the second band,
The results obtained for a honeycomb lattice with $V_X/E_R = 3.6(2)$, $V_Y/E_R = 0.28(1)$ and $V_J/E_R = 1.8(1)$ are displayed in Fig. 2b and show a sharp maximum in the transferred fraction. We identify this situation as the point of inversion symmetry, where $\Delta = 0$ ($\theta = \pi$), in good agreement with an independent calibration (Methods). At this setting, the bandgap at the Dirac points vanishes. The population in the second band decreases symmetrically on both sides of the peak as the gap increases, indicating the transition from massless to massive Dirac fermions.

The relative strength of the tunnel couplings between the different sites of the lattice fixes the position of the Dirac points inside the Brillouin zone, as well as the slope of the associated linear dispersion relation $\sim \sqrt{q}$. However, the tunability of our optical lattice structure allows for independent adjustment of the tunnelling parameters in the $x$ and $y$ directions simply by controlling the intensity of the laser beams. For isotropic tunnelling, the slope of the dispersion relation around the Dirac points is the same in all directions, but is anisotropic otherwise. The distance from the Dirac points to the corners of the Brillouin zone along $q_y$ can be varied between 0 and $q_y/2$, whereas $q_x = 0$ is fixed by reflection symmetry. Here $q_R = 2\pi/\Lambda$ denotes the Bloch wave vector.

We exploit the momentum resolution of the interband transitions directly to observe the movement of the Dirac points. Starting from a honeycomb lattice with $V_X/E_R = 5.4(3)$, $V_Y/E_R = 0.28(1)$ and $V_J/E_R = 1.8(1)$, we gradually increase the tunnelling in the $x$ direction by decreasing the intensity of $X$. The position of the Dirac points continuously approaches the corners of the Brillouin zone (Fig. 3), as expected from an ab initio two-dimensional band structure calculation (Methods). The deviations close to the merging point are possibly caused by the flattening of the dispersion relation between the two Dirac points as they approach each other.

When they reach the corners of the Brillouin zone, the two Dirac points merge, annihilating each other. There the dispersion relation becomes quadratic along the $q_x$ axis, remaining linear along $q_y$. Beyond this critical point, a finite bandgap appears for all quasi-momenta of the Brillouin zone. This situation signals the transition between band structures of two different topologies, one containing two Dirac points and the other containing none. For two-dimensional honeycomb lattices at half-filling, it corresponds to a Lifshitz phase transition from a semimetallic phase to a band-insulating phase $\sim \Delta^2$.

We experimentally map out the topological transition line by recording the fraction of atoms transferred to the second band, $\xi$, as a function of the lattice depths $V_X$ and $V_Y$, while keeping $V_J/E_R = 1.8(1)$. The results are shown in Fig. 4a. There the onset of population transfer to the second band signals the appearance of Dirac points in the band structure of the lattice. For a given value of $V_X$, the transferred fraction, $\xi$, decreases again for large values of $V_Y$, as the Dirac points lie beyond the momentum width of the cloud.

To extend the range of our measurements and probe the Dirac points even in this region, we apply a force in the $y$ direction. We hence explore a new class of trajectories in quasi-momentum space. This allows for the investigation of very anisotropic Dirac cones, which become almost flat in the $q_y$ direction as we approach the crossover to a one-dimensional lattice structure ($V_X \gg V_Y$). Along the $q_y$ trajectories, the centre of the cloud successively passes the two Dirac points during the Bloch cycle, effectively realizing a Stueckelberg interferometer in a two-dimensional band structure. As shown in Fig. 4b, we again identify the topological transition by the onset of population transfer to the second band. The results for the transition lines obtained for the

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**Figure 3 | Movement of the Dirac points.** a. Distance from the Dirac points to the corners of the Brillouin zone, as measured through momentum-resolved interband transitions. The tunnelling in the $x$ direction increases when the lattice depth $V_X$ is decreased. The distance is extracted from the second-band quasi-momentum distribution after one Bloch cycle (insets). The merging of the two Dirac points at the corners of the Brillouin zone is signalled by a single line of missing atoms in the first band. Data show mean ± s.d. of three to nine measurements. The solid line is the prediction of a two-dimensional band structure calculation without any fitting parameters. b. Energy splitting between the two lowest bands. It shows the displacement of the Dirac cones inside the Brillouin zone, as well as their deformation depending on the lattice depth $V_X$.

**Figure 4 | Topological transition.** Fraction of atoms transferred to the second band, $\xi$, as a function of lattice depths $V_X$ and $V_Y$, with $V_J/E_R = 1.8(1)$. Different lattice geometries (square, chequerboard, triangular, dimer and honeycomb) are realized (Fig. 1b). We consider trajectories in quasi-momentum space in the $q_y$ (a) and $q_x$ (b) directions. Each data point is a single measurement, and as a result there are at least 1,200 points per diagram. To maximize the transfer for the $q_x$ trajectories, where the cloud successively passes the two Dirac points, we set $\theta = 1.013(1)\pi$. For both trajectories, the onset of population transfer to the second band signals the topological transition, where the Dirac points appear. The dashed line is the theoretical prediction for the transition line without any fitting parameters, and the dotted line indicates the transition from the triangular lattice to the dimer lattice. The bottom diagrams show cuts of the band structure along the $q_y$ axis ($q_x = 0$; a) and $q_x$ axis ($q_y = 0$; b) for the values of $V_X$ and $V_Y$ indicated.
two measurement series are in excellent agreement with an \textit{ab initio} band structure calculation. At the transition, the transfer is expected to be lower by a factor of two along the central $q_y$ trajectory, as compared to the central $q_x$ trajectory. This is caused by the dispersion relation at the corners of the Brillouin zone being quadratic in the $q_x$ direction and linear in $q_y$. For the data points on the (dashed) transition line in Fig. 4, we find an average ratio for the transfer along $q_x$ and $q_y$ of $\xi = 0.52(14)$, in good agreement with the simple model.

In this work, we have realized Dirac points with highly tunable properties using ultracold fermionic atoms in a honeycomb optical lattice. A new class of physical phenomena based on complex lattice topologies is now within the domain of quantum gas experiments. The versatility and novel observables of these systems will provide new insights. For example, the unique coherence of quantum gases offers possibilities of directly measuring, by interferometric methods, the Berry phase associated with the Dirac points. Topological order could be achieved by introducing artificial gauge fields, either through Raman transitions or time-dependent lattice modulation. Moreover, the highly tunable lattice potential we have developed opens many new avenues for optical lattice experiments. For spin mixtures with repulsive interactions, the dynamic transition between dimer and square lattices should facilitate the adiabatic preparation of an antiferromagnetic phase and allow the study of quantum criticality. Additionally, the triangular and honeycomb lattices provide the possibility of exploring magnetic frustration and spin liquid phases.

**METHODS SUMMARY**

To prepare a non-interacting, quantum-degenerate Fermi gas of $^{40}$K atoms, we perform evaporative cooling of a balanced spin mixture of the $|F, m_f\rangle = |9/2, -9/2\rangle$ and $|9/2, -7/2\rangle$ states in an optical dipole trap at a magnetic field of $197.6(1)$ G. After reaching typical temperatures of $0.2T_F$, where $T_F$ is the Fermi temperature, we apply a magnetic field gradient in the $y$ direction to remove the $|9/2, -7/2\rangle$ component while levitating the $|9/2, -9/2\rangle$ atoms. The polarized Fermi gas is loaded into the optical lattice, where the dipole trap is switched off. We then apply an additional magnetic field gradient in either the $x$ or the $y$ direction, to excite Bloch oscillations. The two-dimensional lattice lies in the $x-y$ plane, whereas in the $z$ direction the atoms are harmonically trapped. The time phase, $\phi$, between the interfering beams $X$ and $Y$ in equation (1) is stabilized interferometrically at a value of $\phi/\pi = 0.00(3)$. The measured visibility of the interference pattern is $\eta = 0.90(5)$. The phase $\phi$ depends on the optical path length between the atomic cloud and the retro-reflecting mirror, and can be precisely controlled through the detuning, $\delta$, between beams $X$ and $X$. We infer the precise value of $\phi/\pi = \pm 384.7(6)$ MHz$^{-1}$ from the peak position in Fig. 2b. This is in good agreement with an independent calibration obtained using Raman–Nath diffraction on a $^{23}$Rb Bose–Einstein condensate, which yields $\phi/\pi = 388.4(7)$ MHz$^{-1}$. The theory curves are extracted from an \textit{ab initio} single-particle, two-dimensional, numerical band structure calculation for the homogeneous system. It therefore also takes into account higher-order tunnelling terms, which are relevant for the regime studied here. In particular, they cause an asymmetry between the two lowest bands and lead to a tilt of the Dirac cones in certain parameter regimes.

**Full Methods** and any associated references are available in the online version of the paper at www.nature.com/nature.

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Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of this article at www.nature.com/nature. Correspondence and requests for materials should be addressed to T.E. (esslinger@phys.ethz.ch).
The phase between the two beams at the position of the atoms is stabilized in the vicinity of the Feshbach resonance at 202.1 G. We obtain typical temperatures of \( \lambda \approx 0.2 \) nm from each other and from each axis. The interference of X and Y produces a chequerboard potential of the form\(^{29}\)

\[
V(x, y) = -V_X \cos^2(kx) - V_Y \cos^2(ky) - 2\sqrt{V_X V_Y} \cos(kx) \cos(ky) \cos(\phi)
\]

The phase between the two beams at the position of the atoms is stabilized interferometrically at a value of \( \phi/\pi = 0.00(3) \) using a pair of additional beams detuned from each other and from X and Y. This results in a weak additional lattice along each axis of about 0.1 \( \text{E}_\text{R} \). We use a heterodyne locking technique\(^{30}\), where the two interfering arms are formed by the optical lattice beam paths and are each compared to a common reference beam.

Beam X creates a potential \( V_X(x, y) = -V_X \cos^2(kx + \phi/2) \), where the phase \( \phi \) determines the relative position of the chequerboard pattern and the one-dimensional standing wave. We control the value of \( \phi \) in the centre of the cloud by adjusting the frequency detuning, \( \Delta \), between X and Y. We infer the precise value \( \theta/\delta = 384.7(6) \text{ MHz}^{-1} \) from the peak position in Fig. 2b. This is in good agreement with an independent calibration obtained using Raman–Nath diffraction on a \(^{87}\text{Rb}\) Bose–Einstein condensate, which yields \( \theta/\delta = 388(4) \text{ MHz}^{-1} \). At the edges of the cloud, the phase differs by approximately \( \pm 10^{-4} \pi \).

The total lattice potential is given by \( V_\text{f}(x, y) + V_\text{f}(x) \) and, depending on the relative intensities of the beams, gives rise to square, chequerboard, triangular and honeycomb lattices, as well as to a staggered arrangement of dimers and an array of weakly coupled, one-dimensional chains.

The visibility, \( \alpha = 0.9(5) \), and the lattice depths, \( V_\text{f}, V_X \) and \( V_Y \), are calibrated using Raman–Nath diffraction. The method has a systematic uncertainty of 10% for the lattice depths; the statistical uncertainties are given in the main text. The two-dimensional lattice lies in the x–y plane, whereas in the z direction the atoms are harmonically trapped. Owing to the absence of interactions, the z direction decouples.

The underlying trap frequencies in our system scale with the lattice depths according to the approximate expressions \( \omega_x \propto \sqrt{V_X}, \omega_y \propto \sqrt{V_X + V_Y} / V_X \), and \( \omega_z \propto \sqrt{V_X + V_Y} / V_Y \). For the parameters used in Fig. 2a, we find that \( \omega_x/2\pi = 17.6(1) \text{ Hz}, \omega_y/2\pi = 31.8(5) \text{ Hz} \) and \( \omega_z/2\pi = 32.7(5) \text{ Hz} \), as calibrated from dipole oscillations of the cloud.

Detection. The quasi-momentum distribution of the gas is probed using a band-mapping technique. The optical lattice beams are linearly ramped down in 500 \( \mu \text{s} \), that is, slowly enough for the atoms to stay adiabatically in their band while quasi-momentum is approximately conserved\(^{34}\). We then allow for 15 ms of ballistic expansion before making an absorption image of the cloud.

Band structure calculations. The energy spectrum is obtained using an \textit{ab initio} single-particle, two-dimensional, numerical band structure calculation for the homogeneous system. It therefore also takes into account higher-order tunnelling terms, which are relevant for the regime studied here. In particular, they cause an asymmetry between the two lowest bands and lead to a tilt of the Dirac cones in certain parameter regimes.

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