The problem of relativity of motion in quantum vacuum is addressed by considering a cavity moving in vacuum in a monodimensional space. The cavity is an open system which emits photons when it oscillates in vacuum. Qualitatively new effects like pulse shaping in the time domain and frequency conversion in the spectrum may help to discriminate motion-induced radiation from potential stray effects.

1 Introduction

Relativity of motion is one of the basic principles of physics since Galileo. In classical physics this principle applies to motion in vacuum which is just another name for empty space. But the face of the problem today is changed by quantum theory. Quantum vacuum is no longer empty. It contains field fluctuations which lead to mechanical effects for any scatterer in vacuum. In this paper we will discuss some observable effects associated with the motion of mirrors in vacuum.

Here we restrict our attention to the vacuum fluctuations of the electromagnetic field. These fluctuations are known to exert a mechanical action on scatterers. Their coupling to electrons in atoms lead to phenomena like spontaneous emission and the Lamb shift of energy levels for a single atom, or van der Waals forces between two atoms or molecules.

For macroscopic objects, the most famous effect induced by vacuum fluctuations is the Casimir force arising between two mirrors in vacuum. For a single mirror moving in vacuum there also exists a dissipative force opposing itself to the mirror’s motion. Even when the mirror is at rest in vacuum, it experiences a fluctuating force due to the radiation pressure of field fluctuations. The dependence of the dissipative force is directly connected to the spectral properties of the fluctuating force through the fluctuations-dissipation relations.

2 The dissipative force

Let us begin with the simple model of a perfect mirror in a two-dimensional spacetime. In a thermal field, the dissipative force \( F_{\text{diss}}(t) \) is proportional to the mirror’s...
velocity $q'(t)$

$$F_{\text{diss}}(t) = -\frac{\hbar \theta^2}{6\pi c^2} q'(t) \quad (1)$$

The force may equivalently be written in the frequency domain

$$F_{\text{diss}}[\omega] = \frac{\hbar \theta^2}{6\pi c^2} i\omega q[\omega] \quad (2)$$

where $F_{\text{diss}}[\omega]$ and $q[\omega]$ are the Fourier transform of the force and mirror’s displacement. In both formulas, $\theta$ is the field temperature expressed in frequency units

$$\theta = \frac{2\pi k_B T_{\text{field}}}{\hbar} \quad (3)$$

$\hbar$, $k_B$ and $c$ are the Planck constant, the Boltzmann constant and the speed of light respectively. This force is a classical expression which tends towards zero when temperature goes to zero. In fact, it neglects the effect of vacuum fluctuations.

When this effect is taken into account, the linear susceptibility is found to scale as the third power of frequency at the limit of zero temperature

$$F_{\text{diss}}[\omega] = \frac{\hbar}{6\pi c^2} i\omega^3 q[\omega] \quad (4)$$

This result, which could be expected from mere dimensional arguments, implies that the force is proportional to the third order time derivative of the mirror’s position

$$F_{\text{diss}}(t) = \frac{\hbar}{6\pi c^2} q'''(t) \quad (5)$$

The linear susceptibilities (2, 4) are directly connected to the spectral properties of the fluctuating force exerted upon a mirror at rest through the fluctuations-dissipation relations. At an arbitrary temperature the dissipative force is just the sum of the 2 contributions (2) and (4). These expressions can also be generalized to the case of a real mirror with frequency-dependent reflection and transmission amplitudes.

The dissipative force arising for a mirror moving in quantum vacuum has interesting consequences with respect to the problem of relativity of motion. This force vanishes for a uniform velocity, as expected from the Lorentz invariance of quantum vacuum. It also vanishes for a uniform acceleration. The appearance of vacuum in an accelerated frame is a much debated question. For the present problem of motion of a mirror in vacuum we have a clear answer at our disposal. No dissipative force arises for a motion with uniform acceleration and this fact may be explained as a consequence of the conformal invariance of electromagnetic vacuum.

Now, there exists a non-vanishing dissipative force for a mirror moving in vacuum with a non-uniform acceleration. This means that vacuum fluctuations have observable mechanical effects on a mirror moving without any further reference than vacuum fluctuations themselves. In other words, this implies that quantum vacuum may be considered as defining privileged reference frames for motion.
3 Observation of dissipative effects?

The dissipative effects related to motion in vacuum have never been observed for macroscopic objects like mirrors. As a matter of fact, the orders of magnitude are exceedingly small for the fluctuating force as well as for the dissipative force. This raises the question which we discuss in this paper: how can one observe dissipative effects of vacuum fluctuations on mirrors?

A first idea is to observe changes in the field rather than in the mechanical forces. Indeed, due to energy conservation photons are emitted into vacuum when the mirror’s motion is damped. In other words, the dissipated energy is transformed into radiation. Let us consider a mirror oscillating in vacuum at a frequency $\Omega$ and with an amplitude $a$ (cf. figure 1). The number of emitted photons $N$ during the measurement time $T$ can then be written in the following way

$$N = \frac{\Omega^3 a^2 T}{3\pi c^2} = \frac{\Omega T \nu^2}{3\pi c^2}$$

$\nu = a\Omega$ (6)

$\Omega^3$ characterizes the already discussed motional susceptibility and $\nu$ is the mirror’s maximal velocity. Since $N$ scales as the square of the ratio between the mirrors mechanical velocity and the speed of light, it remains very small for any possible macroscopic motion. If we consider a macroscopic velocity to be bound by the sound velocity in typical materials (e.g. quartz), one obtains at most one emitted photon per $10^{10}$ oscillation periods.

A second idea for improving the orders of magnitude of this motion-induced radiation is to study a cavity oscillating in vacuum instead of a single mirror. This configuration allows one to profit from the resonant amplification of the optomechanical coupling between the field and the moving mirrors. The resonant enhancement is determined by the cavity finesse $F$ which gives the number of roundtrips of the field before it leaves the cavity. Hence the cavity has to be treated as an open system with mirrors having reflection coefficients smaller than unity so that the field can escape it by transmission through the mirrors. This distinguishes

Figure 1. Single mirror oscillating in vacuum. The arrows represent the vacuum field which, in a monodimensional space, may be considered as two counterpropagating fields.
these calculations from the numerous works devoted to photon production between a pair of perfectly reflecting mirrors \( \text{in which case the amount of radiation emitted outside the cavity cannot be evaluated.} \)

We have shown that the motion-induced radiation is effectively enhanced under opto-mechanical resonance conditions. For a motion of the cavity as a whole (see figure 2), this occurs when the mechanical frequency \( \Omega \) is an odd multiple of the fundamental cavity resonance frequency \( \pi/\tau \)

\[
\Omega = \frac{3\pi}{\tau}, \frac{5\pi}{\tau}, \frac{7\pi}{\tau}, \ldots
\]

\[
\tau = \frac{L}{c}
\]

(7)

\( \tau \) is the time of flight of photons between the two mirrors separated by a distance \( L \). At perfectly tuned resonance, the number of photons emitted by the cavity is the product of the number (6) corresponding to a single mirror by the cavity finesse \( F \)

\[
N = F \frac{\Omega T v^2}{3\pi c^2}
\]

(8)

Since the cavity finesse can be a very large number, up to \( 10^9 \) for instance for microwave cavities, this makes the experimental observation of motion-induced radiation a much less difficult challenge.

In order to make this experimental observation feasible, we have also looked for signatures which might help to discriminate motion induced radiation from potential stray effects. To this aim, we have performed a detailed study of the temporal and spectral features of this radiation.

4 Pulse shaping

The multiple scattering of the field by the moving cavity is represented on the space-time diagram of figure 3. On this diagram light rays are presented by lines making a 45 degree angle with the space and the time axis. The moving mirrors
correspond to the sinusoidal lines. The scale of the mirrors oscillations is largely exaggerated.

The point is that multiple scattering gives rise to periodic orbits. An incoming light ray is attracted to the neighboring stable orbit while it is repelled from the neighboring unstable orbit. When considering a fixed number of scattering processes one obtains the input-output transformation shown in figure 3. This process leads to the formation of regularly spaced field pulses bouncing back and forth the cavity. At each scattering on one of the mirrors, there is a small probability for a photon for escaping the cavity and therefore being detected outside the cavity. This probability is given by the inverse of the cavity finesse. Figure 4 shows the energy density into vacuum emitted by the oscillating cavity as a function of time. We have plotted the energy density for three different values of \( \eta \), which is the ratio of the effective phase velocity \( Fv \) characterizing the efficiency of the multiple scattering to the velocity of light

\[
\eta = \frac{Fv}{c}
\]  

As already discussed, the single scattering parameter \( \frac{v}{c} \) is necessarily very small for macroscopic motions but this is not the case for the multiple scattering parameter
\( \eta \) thanks to the multiplication by the cavity finesse. Figure 4 shows that the pulses become sharp and high when the effective phase velocity approaches the velocity of light. The plot is based on an analytical solution of the multiple scattering problem in terms of homographic mappings of phase exponentials. This approach remains valid in the case of interest \( \eta \sim 1 \) whereas a linearized approach would be restricted to \( \eta \ll 1 \).

5 Frequency conversion

Another interesting feature is the frequency spectrum of the emitted radiation shown in figure 5. Radiation is emitted at the resonance frequencies of the cavity. The spectrum shown here is plotted for a cavity oscillating at a frequency

\[
\Omega = \frac{5\pi}{\tau} \tag{10}
\]

This means that the cavity performs five oscillations during one roundtrip of the field inside the cavity. Photons are emitted at multiple integers of the fundamental cavity frequency, that is at specific rational multiples of the mechanical excitation frequency \( \Omega \)

\[
\omega = \frac{\pi}{\tau}, \frac{2\pi}{\tau}, \frac{3\pi}{\tau}, \frac{4\pi}{\tau}, \frac{6\pi}{\tau}, \ldots \]

\[
= \frac{\Omega}{5}, \frac{2\Omega}{5}, \frac{3\Omega}{5}, \frac{4\Omega}{5}, \frac{6\Omega}{5}, \ldots \tag{11}
\]
A striking feature is that no radiation is emitted at multiple integers of $\Omega$. In addition, photons are emitted not only for frequencies lower but also for frequencies higher than the oscillation frequency $\Omega$.

6 Orders of magnitude

Clearly, the specific temporal and spectral signatures of the emission may help to discriminate motion-induced radiation from potential stray effects in an experimental observation.

To be more specific about the orders of magnitude, let us recall that we have assumed the input fields to be in the vacuum state. This assumption requires the number of thermal photons per mode to be smaller than 1 in the frequency range of interest

$$\hbar \omega \ll k_B T$$  \hspace{1cm} (12)

Low temperature requirements thus point to experiments using small mechanical structures with optical resonance frequencies as well as mechanical oscillation frequencies in the GHz range. This corresponds to an operation temperature

$$T \sim 10\text{mK}$$  \hspace{1cm} (13)

In these conditions, the finesse of a superconducting cavity can reach $10^9$. A peak velocity

$$v \sim 0.3\text{m/s}$$  \hspace{1cm} (14)
would thus be sufficient to obtain a multiple scattering parameter $\eta$ close to unity. The radiated flux of 10 photons per second outside the cavity could be detected by efficient photon-counting detection available in the GHz range. Alternatively, the photons produced inside the cavity could be probed with the help of Rydberg atoms.

It is important to emphasize that the peak velocity considered here is only a small fraction of the typical sound velocity in materials so that fundamental breaking limits do not oppose to these numbers. This velocity corresponds to a small amplitude

$$\frac{v}{\Omega} \sim 10^{-11} \text{m}$$

but to a very large acceleration

$$\Omega v \sim 10^{10} \text{m/s}^2$$

The observation of motional radiation in vacuum seems to be achievable by an experiment of this kind. The difficulty remains to find means for exerting a very large force to excite the motion of the cavity while keeping the optical part of the experiment at a very low temperature and unaffected by the stray fields induced by the excitation.

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