A Study on Teaching the Kinematic Analysis of Structures (Based on the Approach Applied in Learning Process at Industrial University of Tyumen)

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Abstract. 2020, the year marked by global pandemic, revealed a number of systemic weak points, including those in higher education industry in general and in teaching approaches to the discipline of Structural Engineering in particular, which would have remained uncovered under usual conditions. Earlier articles highlighted the positive impact of applying the workbook-based teaching method, developed by the authors of this article, on student learning. However, mass transitioning to online education has helped identify a series of issues that students had to face in the course of their studies. One of these problems included conceptual understanding of how the calculating model, especially a model of a statically indeterminate structure (building), was formed. This article is dedicated to the new methods of teaching kinematic analysis, which have already been put into practice in the virtual classroom of Tyumen Industrial University (TIU).

1. Introduction
Proceeding with kinematic analysis of structures through its initial stages, students often experience difficulties in estimating the necessary amount of links, joints, etc. The main reason for this is choosing an incorrect way to lay out the connections between links forming the structure, and (or) to split the structure into separate elements. Further on, in solving statically indeterminate structures and analyzing them with the help of the displacement method, mistakes are mostly made in choosing the fixing method or the primary structure.

Classical textbooks and guides [10-19] (this article is mainly aimed at Russian-speaking students) contain sufficient amount of material on the topic, which is, however, mostly laid out mostly with regard to various rod systems. It became necessary to make deliberate efforts to develop a method to cover this issue in the process of teaching the discipline of Structural Engineering. The new method is based on enabling the reflection process of choosing the calculation model and its constructive elements.
2. Relevance
The key part of the problem turned out to be as follows. The official textbooks (and the first-class hours of the subject within the University’s educational process) described kinematic analysis for various types of buildings, but did not provide any method for splitting the whole structure into members and analyzing if the connections between the elements themselves as well as their attachment to the ground had been performed in the correct fashion. The newly-introduced method, especially when further applied to the analysis of statically indeterminate structures, allowed the students to choose not only the optimal and most beneficial force method calculation scheme, but also the appropriate solution procedure. What looked comprehensible at the start of the course, failed to provide insight into kinematic analysis of the structures, and limited the opportunities of future graduates to solve complex and challenging tasks in a creative way in real life.

This article is dedicated to the new methods of teaching kinematic analysis, which have already been put into practice in the virtual classroom of Tyumen Industrial University (TIU). The material on kinematic analysis was reviewed when solving statically indeterminate structures via force method. The layout received positive response from both students and teachers.

3. Main provisions of the methodology
Let us take a closer look at the concept of developing key skills for performing kinematic analysis of the structures.

The course in Structural Mechanics provides the following terms and their definitions:
1. link – any rigid body which preserves its geometric stability;
2. support rod – an element which connects the structure to the ground;
3. primary (first-order) joint and (or) higher-order joint (Figure 1).

Taking into consideration the classical formula for determining the joint’s order, i.e. its equivalence when substituted for primary joints, we get:

\[ J = L - 1, \]

where \( J \) - number of primary joints, \( L \) - number of links contained in the linkage.

![Figure 1](image)

As set out in theory, it is clear and obvious that the joint in Fig. 1a) is a higher-order joint as it contains three pins (links): \( J = 3 - 1 = 2 \), and is, accordingly, a second-order joint. Figures 1b-d contain first-order attached joints, connecting two links each.
Let us analyze one frame construction (Figure 2) by breaking it down into separate links, connecting them to each other in various ways and then attaching them to the ground. To calculate the number of degrees of freedom for the frames [20], we use Chebyshev formula:

\[ W = 3 \cdot L - 2 \cdot J - G_0, \quad (1) \]

where \( L \) - number of links in the system, \( J \) - number of primary joints, connecting the inner parts of the structure, \( G_0 \) – number of support rods, connecting the structure to the ground. To allow for more complex structures, this formula should be modified with the inclusion of closed loops:

\[ W = 3 \cdot L - 2 \cdot J - G_0 - 3 \cdot \text{Number of c.l.} \quad (2) \]

where \( \text{Number of c. l.} \) - number of closed loops with no added joints. By design, they are statically indeterminate to the third degree.

Let us analyze the scheme in Fig.2. This structure does not contain joints and is therefore considered a complex link with three closed loops. (Fig.3):

- bdc – 1;
- abc – 2;
- cefg – 3.

Each of these loops has three redundant internal forces and therefore they are internally statically indeterminate. They possess no external static indeterminacy, featuring only three ground supports necessary.
Let us apply the modified Chebyshev formula (2) to calculate the number of degrees of freedom:

$$W = 3 \cdot L - 2 \cdot J - G_0 - 3 \cdot Number \ of \ c.l. = 3 \cdot 1 - 2 \cdot 0 - 3 - 3 \cdot 3 = -9$$

As seen from above, this scheme is statically indeterminate to the 9-th degree. Let us illustrate this on Fig. 4. By splitting each closed loop, we obtain three internal forces in each case: bending moment, shear force and axial force.

Adding a joint reduces the degree of indeterminacy by 1. To identify patterns in formation of different systems with regard to the type of input joints, we shall alter the scheme in Fig.2, adding joints in a number of various ways (Fig. 5a-6a), and analyze the resulting schemes (Fig.5b-6b).
Let us perform the calculations for the scheme in Fig. 5a. Structure members are outlined in Fig. 5b.

\[ W = 3 \cdot L - 2 \cdot J - G_0 = 3 \cdot 4 - 2 \cdot 6 - 3 = -3, \]

which implies that the frame is statically indeterminate to the third degree.
Below the calculations for the scheme in Fig. 6а, structure members are outlined in Fig. 6b.

\[ W = 3 \cdot L - 2 \cdot J - G_0 = 3 \cdot 3 - 2 \cdot 5 - 3 = -4, \]

which implies that the frame is statically indeterminate to the fourth degree. The frames in Figures 5 и 6 contain a different amount of added joints, which are then placed in different parts of the structure. Therefore, the number of degrees of freedom in both frames should also be different.

Let us add three joints to the frame in Figure 2, in a fashion similar to Figure 6a, but fix them in a different way; add supports to the inner structure (Fig.7).
Let us determine the number of degrees of freedom for the scheme in Figure 7a. Structure members are outlined in Figure 7b:

\[ W = 3 \cdot L - 2 \cdot J - G_0 = 3 \cdot 5 - 2 \cdot 7 - 3 = -2, \]
which implies that the frame is statically indeterminate to the second degree.

As we can see from the picture, the joints fell into the same line, but the degree of indeterminacy was reduced by half. This phenomenon indicates that setting the supports in and attaching them provides different results.

It should be noted that changing the input for a joint, located in the middle of a vertical post, also changes the way the structure is being split into links, which is explicitly shown in Figures 6 and 7. Let us take the next step (Fig. 8a) and perform a thorough input of the middle and lower joints to the frame displayed in Fig. 7a.
Determining the number of degrees of freedom for the frame in Fig. 8a, structural members are outlined in Fig. 8b:

\[ W = 3 \cdot L - 2 \cdot J - G_0 = 3 \cdot 6 - 2 \cdot 8 - 3 = -1, \]

which implies that the frame has become statically indeterminate to the first degree. Thus, increasing the joint order diminishes the degree of static indeterminacy of the frame and increases the number of links.

Afterwards, the teacher can suggest the students play around and outline a correct, but now statically determinate structure based on a joint triangle or one link, supplemented with additional links attached to the structure via joints.

Let us illustrate the process with two examples in Fig. 9 and 10. The basic structure remains the same (Fig. 2).

A joint triangle \( a-b-c \) (Fig.9a) is joined with two pins \( d-e \) and \( e-f \) under dyad method [20], as well as with link \( g-h \) and pin \( h-i \), altogether forming a statically determinate structure (Fig.9b).
This scheme does not contain closed loops and, in this regard, a common Chebyshev formula should be applied (1):

\[ W = 3 \cdot L - 2 \cdot J - G_0 = 3 \cdot 7 - 2 \cdot 9 - 3 = 0. \]

A pin structure in Fig. 9b is statically determinate.

Let us turn our attention to another connection method (Fig. 10a). The triangle \( a-b-c \) is joined with two links \( d-e \) and \( f-g \) under dyad method. Points \( d-e-f \) form a triangle.
Similar to the frame in Fig. 9b, the resulting structure (Fig. 10b) is statically determinate:
\[ W = 3 \cdot L - 2 \cdot J - G = 3 \cdot 5 - 2 \cdot 6 - 3 = 0. \]

After performing this procedure, students should be encouraged to find one more connecting method, without teacher’s assistance.

4. Addition
The final issue to consider while performing kinematic analysis of the structure is checking, if the structure is fixed to the ground in a correct way. Any rod system outlined in Fig. 9b or 10b can serve as a basic configuration. The appropriate connection forms were demonstrated in Fig. 2-8. Let us analyze the incorrect ones (Figures 11-12). Fig. 11 demonstrates an obviously inappropriate connection, because all three rods are in a vertical position and provide no constraint to the horizontal displacement of the structure.
The most common mistake the students make in choosing the primary structure when applying the force method (Figure 12), is placing support rods in the way that three support hinges fall into one line. Figure 12 demonstrated that a slight vertical displacement of the frame is possible in the roller support attachment point (point A). When rotated, two parts of the frame will share a common slope. This type of structure is called instantaneously variable.

Figure 13 features another kind of an instantaneously variable structure. In this scheme, the structure can slightly rotate around the common center appearing at the continuation of the lines (point A), set by support rods.
5. Conclusion
When the principles of kinematic and structural analysis are demonstrated on the same rod structure, students develop an understanding of the correct ways to form systems and split structures into links, the functioning of various joint types and their appropriate fixing methods. They learn how to distinguish between set-in and attached supports and choose the corresponding modes of installation.

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