Measurements of $D^0$-$\bar{D}^0$ Mixing and Searches for $CP$ Violation: HFAG Combination of all Data

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Abstract We present world average values for $D^0$-$\bar{D}^0$ mixing parameters $x$ and $y$, $CP$ violation parameters $|q/p|$ and $\text{Arg}(q/p)$, and strong phase differences $\delta$ and $\delta_{K\pi\pi}$. These values are calculated by the Heavy Flavor Averaging Group (HFAG) by performing a global fit to relevant experimental measurements. The results for $x$ and $y$ differ significantly from zero and are inconsistent with no mixing at the level of 6.7σ. The results for $|q/p|$ and $\text{Arg}(q/p)$ are consistent with no $CP$ violation. The strong phase difference $\delta$ is less than 45° at 95% C.L.

Key words mixing, $CP$ violation

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1 Introduction

Mixing in the $D^0$-$\bar{D}^0$ system has been searched for for more than two decades without success — until last year. Three experiments – Belle,$^{[1]}$ Babar,$^{[2]}$ and CDF$^{[3]}$ – have now observed evidence for this phenomenon. These measurements can be combined with others to yield World Average (WA) values for the mixing parameters $x \equiv (m_1 - m_2)/\Gamma$ and $y \equiv \Delta(\Gamma_1 - \Gamma_2)/(2\Gamma)$, where $m_1, m_2$ and $\Gamma_1, \Gamma_2$ are the masses and decay widths for the mass eigenstates $D_1 \equiv p(D^0) - q(\bar{D}^0)$ and $D_2 \equiv p(D^0) + q(\bar{D}^0)$, and $\Gamma = (\Gamma_1 + \Gamma_2)/2$. Here we use the phase convention $CP(D^0) = -|D^0|$ and $CP(\bar{D}^0) = -|\bar{D}^0|$. In the absence of $CP$ violation ($CPV$), $p = q = 1/\sqrt{2}$ and $D_1$ is $CP$-even, $D_2$ is $CP$-odd.

Such WA values have been calculated by the Heavy Flavor Averaging Group (HFAG)$^{[4]}$ in two ways: (a) adding together three-dimensional log-likelihood functions obtained from various measurements for parameters $x$, $y$, and $\delta$, where $\delta$ is the strong phase difference between amplitudes $A(D^0 \rightarrow K^+\pi^-)$ and $A(D^0 \rightarrow K^-\pi^+)$; and (b) doing a global fit to measured observables for $x$, $y$, $\delta$, an additional strong phase $\delta_{K\pi\pi}$, and $R_D \equiv |A(D^0 \rightarrow K^+\pi^-)/A(D^0 \rightarrow K^-\pi^+)|^2$. For this fit, correlations among observables are accounted for by using covariance matrices provided by the experimental collaborations. The first method has the advantage that non-Gaussian errors are accounted for, whereas the second method has the advantage that it is easily expanded to allow for $CPV$. In this case three additional parameters are included in the fit: $|q/p|$, $\phi \equiv \text{Arg}(q/p)$, and $A_D \equiv (R_D - R_{\bar{D}})/(R_D + R_{\bar{D}})$, where the $+(-)$ superscript corresponds to $D^0(\bar{D}^0)$ decays. When both methods are applied to the same set of observables, almost identical results are obtained. The observables used are from measurements of $D^0 \rightarrow K^+\ell^-\nu$, $D^0 \rightarrow K^+K^-/\pi^+\pi^-$, $D^0 \rightarrow K^+\pi^-$, $D^0 \rightarrow K^+\pi^-\pi^0$, $D^0 \rightarrow K^+\pi^-\pi^0$, and $D^0 \rightarrow K_S^0\pi^+\pi^-$ decays, and from double-tagged branching fractions measured at the $\psi(3770)$ resonance.

Mixing in heavy flavor systems such as that of $B^0$ and $B_s^0$ is governed by the short-distance box diagram. In the $D^0$ system, however, this diagram is doubly-Cabibbo-suppressed relative to amplitudes dominating the decay width, and it is also GIM-suppressed. Thus the short-distance mixing rate is tiny, and $D^0-\bar{D}^0$ mixing is expected to be dominated by long-distance processes. These are difficult to calculate reliably, and theoretical estimates for $x$ and $y$ range over two-three orders of magnitude.$^{[5, 6]}$

With the exception of $\psi(3770) \rightarrow DD$ measurements, all methods identify the flavor of the $D^0$ or $\bar{D}^0$ when produced by reconstructing the decay $D^+ \rightarrow D_0^0\pi^+$ or $D^+ \rightarrow \bar{D}_0^0\pi^-$; the charge of the accompanying pion identifies the $D$ flavor. For signal decays, $M_{D^+} - M_{D^0} - M_{\pi^+} \equiv Q \approx 6$ MeV, which is rela-
tively close to the threshold. Thus analyses typically require that the reconstructed $Q$ be small to suppress backgrounds. For time-dependent measurements, the $D^0$ decay time is calculated via $(\ell/p) \times M_{D^0}$, where $\ell$ is the distance between the $D^*$ and $D^0$ decay vertices and $p$ is the $D^0$ momentum. The $D^*$ vertex position is taken to be at the primary vertex$^3$ $(pp)$ or is calculated from the intersection of the $D^0$ momentum vector with the beamspot profile $(e^+e^-)$. 

2 Input Observables

The global fit determines central values and errors for eight underlying parameters using a $\chi^2$ statistic constructed from 26 observables. The underlying parameters are $x, y, \delta, R_D, A_D, [q/p], \phi$, and $\delta_{K^\pi\pi}$. The parameters $x$ and $y$ govern mixing, and the parameters $A_D, [q/p]$, and $\phi$ govern CPV. The parameter $\delta_{K^\pi\pi}$ is the strong phase difference between the amplitude $A(D^0 \to K^+\pi^-\pi^0)$ evaluated at $M_{K^+\pi^-} = M_{K^*}(890)$, and the amplitude $A(D^0 \to K^-\pi^+\pi^0)$ evaluated at $M_{K^-\pi^+} = M_{K^*}(890)$. 

All input values are listed in Table 1. The observables to the $\chi^2$ is calculated as $\vec{V}(M^{-1}) \cdot \vec{V}^T$, where $M^{-1}$ is the inverse of the covariance matrix for the measurement. All covariance matrices used are listed in Table 1.

![Fig. 1](image1.png)

Fig. 1. WA value of $R_M$ from Ref. [4], as calculated from $D^0 \to K^+\ell^-\nu$ measurements$^7$.

![Fig. 2](image2.png)

Fig. 2. WA values of $y_{CP}$ (top) and $A_T$ (bottom) from Ref. [4], as calculated from $D^0 \to K^+K^-/\pi^+\pi^-$ measurements$^4,8$.

Published $D^0 \to K^+\pi^-\pi^0$ results. The $D^0 \to K^+\pi^-\pi^0$ and $D^0 \to K^+\pi^-\pi^-\pi^-$ results are from Babar$^{[12]}$ and the $\psi(3770) \to DD$ results are from CLEO$^{c, [12]}$.

The relationships between the observables and the fitted parameters are listed in Table 2. For each set of correlated observables, we construct the difference vector $\vec{\nu}$, e.g., for $D^0 \to K^0_s\pi^+\pi^-$ decays $\vec{\nu} = (\Delta x, \Delta y, \Delta [q/p], \Delta \phi)$, where $\Delta$ represents the difference between the measured value and the fitted parameter value. The contribution of a set of measured observables to the $\chi^2$ is calculated as $\vec{V}(M^{-1}) \cdot \vec{V}^T$, where $M^{-1}$ is the inverse of the covariance matrix for the measurement. All covariance matrices used are listed in Table 1.
3 Fit results

The global fit uses MINUIT with the MIGRAD minimizer, and all errors are obtained from MINOS. Three separate fits are performed: (a) assuming CP conservation ($A_D$ and $\phi$ are fixed to zero, $|q/p|$ is fixed to one); (b) assuming no direct CPV ($A_D$ is fixed to zero); and (c) allowing full CPV (all parameters floated). The results are listed in Table 3. For the CPV-allowed fit, individual contributions to the $\chi^2$ are listed in Table 4. The total $\chi^2$ is 23.5 for $26 - 8 = 18$ degrees of freedom; this corresponds to a confidence level of 0.17.

Confidence contours in the two dimensions ($x,y$) or in $(|q/p|,\phi)$ are obtained by letting, for any point in the two-dimensional plane, all other fitted parameters take their preferred values. The resulting 1σ-5σ contours are shown in Fig. 3 for the CP-conserving case, and in Fig. 4 for the CPV-allowed case. The contours are determined from the increase of the $\chi^2$ above the minimum value. One observes that the ($x,y$) contours for no-CPV and for CPV-allowed are almost identical. In both cases the $\chi^2$ at the no-mixing point ($x,y$) = (0,0) is 49 units above the minimum value; this has a confidence level corresponding to 6.7σ. Thus, no mixing is excluded at this high level. In the $(|q/p|,\phi)$ plot, the point (1,0) is on the boundary of the 1σ contour; thus the data is consistent with no CPV.

One-dimensional confidence curves for individual parameters are obtained by letting, for any value of the parameter, all other fitted parameters take their preferred values. The resulting functions $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ (where $\chi^2_{\text{min}}$ is the minimum value) are shown in Fig. 5. The points where $\Delta \chi^2 = 2.70$ determine 90% C.L. intervals for the parameters as shown in the figure. The points where $\Delta \chi^2 = 3.84$ determine 95% C.L. intervals; these are listed in Table 3.
| Observable               | Value               | Comment                                                                 |
|--------------------------|---------------------|-------------------------------------------------------------------------|
| \( y_{CP} \)            | \((1.132 \pm 0.266)\)% | WA \( D^{0} \rightarrow K^{+}K^{-}/\pi^{+}\pi^{-} \) results [4]       |
| \( A_{\phi} \)          | \((0.123 \pm 0.248)\)% |                                                                          |
| \( x \) (no CPV)        | \((0.811 \pm 0.334)\)% |                                                                          |
| \( y \) (no CPV)        | \((0.309 \pm 0.281)\)% | No CPV:                                                                 |
| \(|q/p|\) (no direct CPV)| \(0.95 \pm 0.22^{+0.10}_{-0.09} \) | WA \( D^{0} \rightarrow K_{S}^{0} \pi^{+}\pi^{-} \) results [4]        |
| \( \phi \) (no direct CPV)| \((-0.035 \pm 0.19 \pm 0.09) \) rad |                                                                          |

| Table 1. Input values used for the global fit, from Refs. [1, 2, 7–12]. |
|---------------------------------------------------------|

| \( R_{M} \) | \((0.0173 \pm 0.0387) \)% | WA \( D^{0} \rightarrow K^{+}\ell^{-}\nu \) results [4] |
|--------------------------|---------------------|-------------------------------------------------------------------------|
| \( x'\prime \)          | \((2.39 \pm 0.61 \pm 0.32) \)% | Babar \( D^{0} \rightarrow K^{+}\pi^{-}\pi^{0} \) result. Correlation coefficient = –0.34. |
| \( y'\prime \)          | \((-0.14 \pm 0.60 \pm 0.40) \)% |                                                                          |

| \( R_{M} \) | \((0.019 \pm 0.0161) \)% | Babar \( D^{0} \rightarrow K^{+}\pi^{-}\pi^{+}\pi^{-} \) result. |

| \( R_{D} \) | \((8.878 \pm 3.369 \pm 1.579) \)% | CLEOc results from “double-tagged” branching fractions measured in \( \psi(3770) \rightarrow DD \) decays. Correlation coefficients: |
|--------------------------|---------------------|-------------------------------------------------------------------------|
| \( x'^{+} \)            | \((-0.241 \pm 0.052) \)% | Babar \( D^{0} \rightarrow K^{+}\pi^{-}\pi^{+} \) results. Correlation coefficients: |
| \( y'^{+} \)            | \((0.98 \pm 0.78) \)% |                                                                          |

| \( A_{\phi} \)          | \((-2.1 \pm 5.4) \)% | Babar \( D^{0} \rightarrow K^{+}\pi^{-} \) results; correlation coefficients same as above. |
|--------------------------|---------------------|-------------------------------------------------------------------------|
| \( x'^{-} \)            | \((-0.20 \pm 0.050) \)% |                                                                          |
| \( y'^{-} \)            | \((0.96 \pm 0.75) \)% |                                                                          |

| \( R_{D} \) | \((0.364 \pm 0.018) \)% | Babe \( D^{0} \rightarrow K^{+}\pi^{-} \) results. Correlation coefficients: |
|--------------------------|---------------------|-------------------------------------------------------------------------|
| \( x'^{+} \)            | \((-0.12 \pm 0.58) \)% | Babar \( D^{0} \rightarrow K^{+}\pi^{-} \) results; correlation coefficients same as above. |
| \( y'^{+} \)            | \((0.06 \pm 0.034) \)% |                                                                          |
| \( A_{\phi} \)          | \((-2.3 \pm 4.7) \)% |                                                                          |

Note: \( \alpha = (|q/p|+1)^2/2 \) is a variable transformation factor.
Table 2. Left: decay modes used to determine fitted parameters $x$, $y$, $\delta$, $\delta_{K\pi\pi}$, $R_D$, $A_D$, $|q/p|$, and $\phi$. Middle: the observables measured for each decay mode. Right: the relationships between the observables measured and the fitted parameters.

| Decay Mode | Observables | Relationship |
|------------|-------------|--------------|
| $D^0 \rightarrow K^+ K^- / \pi^+ \pi^-$ | $y_{CP}$, $A_\Gamma$ | $2y_{CP} = (|q/p| + |p/q|) y \cos \phi - (|q/p| - |p/q|) x \sin \phi$  
$2A_\Gamma = (|q/p| - |p/q|) y \cos \phi - (|q/p| + |p/q|) x \sin \phi$ |
| $D^0 \rightarrow K^0_S \pi^+ \pi^-$ | $x$, $y$, $|q/p|$, $\phi$ | $R_M = (x^2 + y^2)/2$ |
| $D^0 \rightarrow K^+ \ell^- \nu$  
(Dalitz plot analysis) | $R_M$, $x''$, $y''$ | $x'' = x \cos \delta_{K\pi\pi} + y \sin \delta_{K\pi\pi}$  
$y'' = y \cos \delta_{K\pi\pi} - x \sin \delta_{K\pi\pi}$ |
| $D^0 \rightarrow K^+ \pi^- \pi^0$  
(“Double-tagged” branching fractions measured in $\psi(3770) \rightarrow DD$ decays) | $R_M$, $y$, $R_D$, $\sqrt{R_D \cos \delta}$ | $R_M = (x^2 + y^2)/2$  
$R_D = (R_D^+ + R_D^-)/2$  
$A_D = (R_D^+ - R_D^-)/(R_D^+ + R_D^-)$  
$x' = x \cos \delta + y \sin \delta$  
$y' = y \cos \delta - x \sin \delta$  
$A_M \equiv (|q/p|^4 - 1)/(|q/p|^4 + 1)$  
$x'' \pm = [(1 \mp A_M)/(1 \mp A_M)]^{1/4}(x' \cos \phi \mp y' \sin \phi)$  
$y'' \pm = [(1 \mp A_M)/(1 \mp A_M)]^{1/4}(y' \cos \phi \mp x' \sin \phi)$ |
Fig. 5. The function $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ for fitted parameters $x, y, \delta, \delta_K, |q/p|$, and $\phi$. The points where $\Delta \chi^2 = 2.70$ (denoted by the dashed horizontal line) determine a 90% C.L. interval.
Table 3. Results of the global fit for different assumptions concerning CPV.

| Parameter | No CPV | No direct CPV | CPV-allowed | CPV-allowed 95% C.L. |
|-----------|--------|---------------|-------------|---------------------|
| $x$ (%)   | 0.98±0.26 | 0.97±0.27 | 0.97±0.27 | 0.39−1.48           |
| $y$ (%)   | 0.75±0.18 | 0.78±0.18 | 0.78±0.18 | 0.41−1.13           |
| $\delta$ (°) | 21.6±11.6 | 23.4±11.6 | 21.9±11.5 | −6.3−44.6          |
| $R_D$ (%) | 0.335±0.009 | 0.334±0.009 | 0.335±0.009 | 0.316−0.353        |
| $A_D$ (%) | − | − | −2.2±2.5 | −7.10−2.67        |
| $|q/p|$ | − | 0.95±0.15 | 0.86±0.18 | 0.59−1.23          |
| $\phi$ (°) | − | −2.7±5.4 | −9.6±9.5 | −30.3−6.5         |
| $\delta_{K^+\pi}$ (°) | 30.8±25.0 | 32.5±25.0 | 32.4±25.1 | −20.3−82.7       |

Table 4. Individual contributions to the $\chi^2$ for the CPV-allowed fit.

| Observable | $\chi^2$ | $\sum\chi^2$ |
|-----------|----------|---------------|
| $y_{CP}$ | 2.06 | 2.06          |
| $A_{\Gamma}$ | 0.10 | 2.16          |
| $x_{K^0\pi^+\pi^-}$ | 0.20 | 2.36          |
| $y_{K^0\pi^+\pi^-}$ | 1.94 | 4.30          |
| $|q/p|_{K^0\pi^+\pi^-}$ | 0.00 | 4.30          |
| $\phi_{K^0\pi^+\pi^-}$ | 0.46 | 4.76          |
| $R_{M}(K^+\ell^-\nu)$ | 0.06 | 4.83          |
| $x_{K^+\pi^-\pi^0}$ | 1.24 | 6.06          |
| $y_{K^+\pi^-\pi^0}$ | 1.62 | 7.69          |
| $R_M/y/R_D/\sqrt{R_D} \cos \delta$ (CLEOc) | 5.59 | 13.28        |
| $R_D^{+}/x^{2+}/y'^{+}$ (Babar) | 2.54 | 15.82        |
| $R_D^{±}/x^{2±}/y'^{±}$ (Babar) | 1.75 | 17.57        |
| $R_D^{+}/x^{2+}/y'^{+}$ (Belle) | 3.96 | 21.53        |
| $R_D^{±}/x^{2±}/y'^{±}$ (Belle) | 1.43 | 22.95        |
| $R_M(K^+\pi^-\pi^0\pi^-)$ | 0.49 | 23.45        |
4 Conclusions

From the global fit results listed in Table 3 and shown in Figs. 4 and 5, we conclude the following:

- the experimental data consistently indicate that $D^0$ mesons undergo mixing. The no-mixing point $x = y = 0$ is excluded at $6.7\sigma$. The parameter $x$ differs from zero by $3.0\sigma$; the parameter $y$ differs from zero by $4.1\sigma$. The effect is presumably dominated by long-distance processes, which are difficult to calculate. Thus unless $|x| \gg |y|$ (see Ref. [5]), it may be difficult to identify new physics from mixing alone.

- Since $y_{CP}$ is positive, the $CP$-even state is shorter-lived, as in the $K^0$-$\bar{K}^0$ system. However, since $x$ also appears to be positive, the $CP$-even state is heavier, unlike in the $K^0$-$\bar{K}^0$ system.

- It appears difficult to accommodate a strong phase difference $\delta$ larger than $45^\circ$.

- There is no evidence yet for $CPV$ in the $D^0$-$\bar{D}^0$ system. Observing $CPV$ at the level of sensitivity of the current experiments would indicate new physics.

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