LATTICE SUPERSYMMETRY
WITH DOMAIN WALL FERMIONS

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Supersymmetry, like Poincaré symmetry, is softly broken at finite lattice spacing provided the gaugino mass term is strongly suppressed. Domain wall fermions provide the mechanism for suppressing this term by approximately imposing chiral symmetry. We present the first numerical simulations of \( N = 1 \) supersymmetric SU(2) Yang-Mills on the lattice in \( d = 4 \) dimensions using domain wall fermions.

Supersymmetric (SUSY) field theories may play an important role in describing the physics beyond the Standard Model. Non-perturbative numerical studies of these theories could provide confirmation of existing analytical calculations and new insights on aspects of the theories not currently accessible to analytic methods. One such SUSY field theory that can be formulated and studied numerically on the lattice is the four dimensional \( N = 1 \) super Yang-Mills (SYM), which is just QCD with one flavor of adjoint Majorana fermions, called gluinos. In the traditional approach, the Wilson fermions are used to simulate the gluinos. Since Wilson fermions break chiral symmetry the gluinos will be massive, breaking SUSY, unless fine-tuned mass counterterms are introduced. Pioneering work using these methods has already produced very interesting numerical results.

In this work, we summarize recent results of the first simulations using the lattice domain wall fermion formulation to represent the massless gluinos of SYM. The idea behind domain wall fermions is to start with a formulation of massless fermions in one higher dimension and introduce domain walls so that chiral surface modes become exponentially bound to the walls. Pulling apart domain walls of opposite chirality, by increasing the size of the extra dimension, exponentially suppresses chiral symmetry breaking at a cost proportional to the size of the extra dimension. For reviews on DWF please see the LATTICE ’00 review talk of Vranas and references therein. The possible use of DWF in SUSY theories has been discussed in earlier works and the methods used here are along these lines. For lists of references not included here for lack of space, please see the cited articles.

The Dirac operator in the adjoint representation of SU(\( N \)) has an index \( 2N\nu \), where \( \nu \) is the winding of the gauge field. Classical instantons have integer winding and cause condensations of operators with \( 2N \) gluinos which anomalously breaks the U(1) \( R \)-symmetry to \( Z_{2N} \). For \( \langle \overline{\chi} \chi \rangle \) to condense, the remaining \( Z_{2N} \) symmetry must further break either spontaneously or anomalously to \( Z_2 \). If the breaking is
anomalous, then the responsible gauge configurations must have fractional winding. The existence of such gauge configurations has already been established. It is our goal to distinguish between these two scenarios.

All numerical simulations were performed on $8^4$ and $4^4$ Euclidean spacetime volumes with periodic boundary conditions using the inexact hybrid molecular dynamics (HMD) $R$ algorithm. The algorithm numerically integrates the classical equations of motion as part of generating a statistical ensemble with weights proportional to the fourth root of a two adjoint flavor Dirac determinant. For DWF, this weight is proportional to the weight for a single adjoint Majorana flavor. The integration step sizes were chosen such that systematic uncertainties due to numerical integration errors are negligible compared to statistical uncertainties. The $8^4$ volume simulations were run with $\beta=4/g^2=2.3$, chosen as large as possible without entering the finite volume transition region. The $4^4$ volume simulations were run with $\beta=2.1$. Scaling arguments appropriate for weak coupling suggest that the lattice spacing at $\beta=2.1$ is twice as large as at $\beta=2.3$.

To extrapolate the measured values of $\langle \chi \chi \rangle$ to the chiral limit, $L_s \to \infty$ and $m_f \to 0$, simulations were performed in $8^4$ volumes at fixed $\beta=2.3$ while the size of the extra dimension $L_s$ was set to 12, 16, 20 or 24 and the bare mass $m_f$ was set to 0.02, 0.04, 0.06 or 0.08. If the formation of the gluino condensate is due to spontaneous symmetry breaking, the lattice volume limits how small the dynamical gluino mass can be set without losing the condensate: $12m_{\text{eff}} \langle \chi \chi \rangle V \gg 1$ (the 12 is just normalization). As $m_{\text{eff}} \gtrsim m_f$, this limit is satisfied for all $8^4$ simulations with $m_f \geq 0.02$.

To estimate the gluino condensate in the chiral limit, we first extrapolate at fixed $m_f$ to the $L_s \to \infty$ limit using the fit function $\langle \chi \chi \rangle = c_0 + c_1 \exp(-c_2 L_s)$. The values of the extrapolated gluino condensate (with propagated errors) appear as points in figure 1. These extrapolated values are further extrapolated to the $m_f \to 0$ limit using a linear function $\langle \chi \chi \rangle|_{L_s \to \infty} = b_0 + b_1 m_f$. The best fit function appears as the line in figure 1, with $b_0=0.00455(21)$. It is also reassuring to note that reversing the order...
of limits, i.e., first $m_f \to 0$ at fixed $L_s$ then $L_s \to \infty$, yields a statistically consistent answer.

Another approach to estimating the gluino condensate in the chiral limit is to actually perform dynamical simulations with $m_f = 0$. Since finite $L_s$ will induce an exponentially small breaking of chiral symmetry, the effective gluino mass will not be zero. However, the gluino mass should be too small to support spontaneous symmetry breaking. Additional simulations were run with $L_s$ set to 12, 16, 20 or 24. The data are shown in figure 2. The curve is the best fit to an exponential fitting function with the extrapolated value of the condensate as shown.

Surprisingly, both methods for estimating the gluino condensate produce consistent results within the statistical errors. Note that this is inconsistent with the notion of spontaneous symmetry breaking. Operationally, this result reinforces our claim that systematic uncertainties are still relatively small despite limited statistical precision. Further, it gives us some confidence that our fit functions are valid over the region of interest.

To further check for spontaneous symmetry breaking of the $Z_4$ symmetry, we measured $\langle \chi \chi \rangle$ on smaller $4^4$ lattices with $m_f = 0$ and even larger values for $L_s$. The data are shown in figure 2 with the best exponential fit and the extrapolated value for the condensate. This provides even stronger evidence that spontaneous symmetry breaking is not responsible for the formation of a gluino condensate, at least in finite volumes. On these lattices $12m_f V \langle \chi \chi \rangle < 1$, so analytical considerations suggest the support of $\langle \chi \chi \rangle$ must come primarily from topological sectors with fractional winding of $\nu = \pm 1/2$.

The spectrum of the theory is of great interest but it was not possible to measure it on the small lattices considered here. Also, the gluino condensate was measured at only two different lattice spacings so extrapolation to the continuum limit to compare with analytical results is not possible. Future work could explore these very interesting topics.

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