Orientation control of rodlike objects by flow

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Suspensions of rodlike objects in a liquid are encountered in many areas of science and technology and the need to orientate them is extremely important to enhance or inhibit certain chemical reactions between them, other chemicals or with the walls of vessels holding the flowing suspension. Orientation control is feasible by altering velocity and nature of the liquid, its flow and the geometry of the channel containing the flow. In this work we consider the simplest possibility of orientation control with flow in two dimensions on the basis of the orientation distribution of rodlike objects. The simple differential equation satisfied by the orientation probability density function is derived and its solution discussed. In addition, we show how birefringence and dichroism ("Maxwell effect") of the suspension provide a direct experimental test of orientation control.

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I. INTRODUCTION

In many physical, chemical, biological processes, the behavior and orientation of rodlike objects (such as fibers, nanotubes, nanowires, DNA, macromolecules) suspended in a flowing liquid affect the transport, rheology, chemical and hydrodynamic characteristics of the suspension. Orientation control for the sake of aligning or separating the rodlike objects (RLO), enhancing reaction between them, with other chemicals or vessel walls is important in many areas of science and technology such as sedimentation, blood flow, pulp and paper, polymer processing, liquid crystal flow, microfluidic devices, ferrofluids...

This work is relevant to students who have completed an undergraduate Statistical Physics course of the Reif 1 level and are interested in applications of stochastic processes or graduate students whose level corresponds to Landau and Lifshitz course 3 and are interested in the general applications of Fluid Mechanics. The history of dilute solutions of RLO (from the polymeric point of view) is detailed in Chapter 8 of Doi and Edwards book "The Theory of Polymer Dynamics" 4 as well as in a review paper by Forest et al. 5. Students interested in various other materials and applications are encouraged to look at the "Soft Matter" four volume series edited by G. Gompper and M. Schick 6.

In this work, we are interested in dilute suspensions of rigid and long RLO with a negligible cross-sectional area and concentrate on the study of orientation control by flow in two dimensions. Our goal is to investigate in the simplest, yet physical case, and with a self-contained pedagogical point of view, the effect of liquid shear flow on the rotational diffusion of a dilute suspension of non-interacting RLO in a planar contraction. We derive in a straightforward manner the main equation describing the statistical distribution of orientation in two dimensions (2D), solve it analytically and show how orientation control might be detected optically with a measurement such as birefringence 7 (double refraction) and dichroism of the suspension.

Objects suspended in flow undergo two types of motion: smooth motion due to the average fluid velocity field and erratic random motion produced by the fluctuating fluid velocity, temperature and inertia driven motion. The resulting change in the suspension microstructure can have a significant effect on the mechanical, thermal, optical, electrical, magnetic and chemical properties. The equation that accurately models the orientation state of non-interacting RLO in hydrodynamic nonhomogenous flow depends on the rotational Peclet number $\alpha$ that represents the interplay between the randomizing effect of temperature induced rotational diffusion and the orientating effect of streamwise mean rate of strain due to the liquid flow.

Two types of flow are of particular importance: simple shear flow and elongational (extensional) flow. Simple shear flow corresponds to a velocity profile where the gradient in the fluid flow velocity is constant. In the case of elongational flow the sample is compressed in one direction and elongated in the other. In this work we consider a dilute concentration of non-interacting RLO suspended in a liquid performing shear flow exclusively in a planar contraction of width $L$ (see fig. 1).

The rotational Peclet number $\alpha$ measuring the relative strength of hydrodynamic interactions and Brownian forces is defined mathematically as the ratio of hydrodynamic shear flow and the rotational diffusion constant: 

\[ \alpha = \frac{\text{hydrodynamic shear flow}}{\text{rotational diffusion constant}} \]
FIG. 1: 2D geometry of the shear flow and the RLO of length $\ell$ making an angle $\theta$ with the flow direction along $x$. The shear rate of the flow is $\dot{\gamma} = \frac{\partial v_x}{\partial y}$. The fluid velocity profile $v_x$ is shown explicitly as a function of the coordinate $y$. In the frame moving with the rod, the velocity field is antisymmetric (resp. symmetric) above (and resp. below) the rod.

$$\alpha = \frac{\dot{\gamma}}{D_r}$$  \hspace{1cm} (1)

$\dot{\gamma}$ is the (hydrodynamic) shear rate and $D_r$ the (thermal) rotational diffusion coefficient (see the Appendix for an explanation about the difference between ordinary diffusion [1] and rotational diffusion).

The flow shear rate defined by $\dot{\gamma} = \frac{\partial v_x}{\partial y}$ with $v_x$ the velocity field in the $x$ direction of the flow (see fig. 1) is related to translational degrees of freedom whereas $D_r$, the Brownian diffusion coefficient, governs the rotational motion of the RLO around its center of mass and hence relates to rotational degrees of freedom (see Appendix).

The RLO suspended in the liquid are under the action of two orientating forces: one that comes from the hydrodynamic flow and the other due to thermal effects. Despite the apparent simplicity of the problem, its physics is extremely rich and one might stumble upon some unusually complex behaviour; for instance the following set of states may be observed:

1. An **aligned** state where the RLO is aligned with the flow at a fixed angle;
2. A **tumbling** state, in which the RLO lies in the shear $(x - y)$ plane and rotates about the $z$ axis.
3. A **wagging** state, in which the RLO lies in the shear plane, but oscillates between two values.
4. A **Kayak-tumbling** state, equivalent to the tumbling state, but in which the RLO is out of the shear plane.
5. A **Kayak-wagging** state where the RLO is out of plane, but the projection of the RLO on the shear plane oscillates between two values.
6. A **mixed** state, typically found close to the boundaries between wagging and tumbling states and where the RLO undergoes both oscillation and complete rotation.
7. A **Chaotic** state, in which the RLO angle with the flow behaves with time in an apparently random way.

While the above describe the time dependence of the RLO orientation, it would be rather interesting to evaluate the statistical distribution of the angles the RLO makes with the flow. Specializing to 2D, Boeder [8] is the first to have studied the RLO stationary angular distribution problem in a flowing liquid from a theoretical and experimental point of view. In the shear flow case, he derived an ordinary differential equation (ODE) satisfied by $P(\theta)$, the stationary PDF (see Appendix) describing the average orientations of the RLO assuming that the motion of RLO, of negligible cross-sectional area occurs in the plane of the flow, without any boundary conditions.

The resulting ODE reads:

$$\frac{d^2}{d\theta^2}P(\theta) + \frac{d}{d\theta}[\alpha \sin^2(\theta) P(\theta)] = 0$$  \hspace{1cm} (2)

In order to derive the above ODE, the RLO angular time evolution induced by shear flow is needed. We assume that the effects of gravity and inertia on the RLO are negligible as well as any disturbance motion resulting from the RLO presence. Taking a unit vector $\hat{u}$ along the RLO axis, $\hat{u} = \hat{x}\cos \theta + \hat{y}\sin \theta$, its time derivative is given by $\frac{d\hat{u}}{dt} = \frac{d\hat{u}}{d\theta}(-\hat{x}\sin \theta + \hat{y}\cos \theta)$. This yields the angular time evolution of the RLO as $\hat{z} \frac{d\theta}{dt} = \hat{u} \times \frac{d\hat{u}}{d\theta}$ where $\hat{z}$ is the unit vector perpendicular to the $x - y$ plane. In addition, the fluid velocity field $v = v_x \hat{x}$ is along the $x$ direction and depends solely on the coordinate $y$. The above simplifying assumptions imply that the RLO follows the surrounding
fluid velocity gradient and that the time derivative of $\mathbf{u}$ along the $x$ direction (see fig. 1) is proportional to the shear rate ($v_x$, change along $y$) and the $u_y$ component (see additional note [9]). This means $\frac{d u_y}{d t} = \dot{\gamma} u_y$, i.e. $\frac{d u_y}{d t} = \dot{\gamma} \sin \theta \hat{x}$. Therefore, the RLO angular time evolution induced by shear flow becomes:

$$\frac{d \theta}{d t} = -\dot{\gamma} \sin^2 \theta$$  \hfill (3)

Thermal disturbances to the RLO motion, are accounted for by adding to the above a random term proportional to a time-dependent White noise amplitude $\xi(t)$ possessing the following statistical properties:

$$< \xi(t) > = 0 \quad \text{and} \quad < \xi(t) \xi(t') > = \delta(t - t')$$  \hfill (4)

where $< \xi >$ is the statistical average of $\xi$, meaning that $\xi(t)$ has zero mean and zero correlation with another amplitude occurring at a different time. The angular time evolution becomes:

$$d\theta = Adt + \sqrt{B} \xi(t) dt$$  \hfill (5)

with $A = -\dot{\gamma} \sin^2(\theta)$ and $B$ a constant independent of time and the angle $\theta$. This implies that the deterministic hydrodynamic forces tend to act on the RLO rotating it in the shear flow with an average angular speed $\frac{d\theta}{dt}$ while the fluctuating term $\xi(t)$ originating from thermal effects produce random disorientations.

The differential equation $Adt + \sqrt{B} \xi(t) dt$ in the variable $\theta$ containing the random term $\xi(t) dt$ is called a Langevin equation [1] that can easily be transformed into a 1D Fokker-Planck equation (see the derivation in Chapter 15 of Reif’s book [1] or the paper titled "Simplified Derivation of the Fokker-Planck equation" by Siegman [2] in this Journal):

$$- \frac{d}{d\theta} [AP(\theta)] + \frac{1}{2} \frac{d^2}{d\theta^2} [BP(\theta)] = 0$$  \hfill (6)

Straightforward comparison with the ODE given by eq. [2] leads to $B = 2D_r$ and in the limit of no shear ($\alpha = 0$) we recover the stationary planar rotational diffusion equation $D_r \frac{d^2}{d\theta^2} P(\theta) = 0$ (see Appendix).

Having derived the Boeder ODE from simple assumptions, this work is meant to provide a solution in closed form for the ODE (eq. 2) using standard analytical methods (pertaining to Freshman calculus level) to obtain $P(\theta)$ for any Peclet number $\alpha$ and assess later on the impact with regard to orientation control from the measurement of optical properties.

This paper is organised as follows: In Section II, we present an exact analysis of the ODE (see eq. 2) to obtain the PDF, for a wide range of $\alpha \in [10^{-4} - 10^8]$. In section III optical properties are evaluated to test liquid flow effect on orientation control. Conclusions are given in Section IV. The Appendix explains in detail the difference between ordinary and rotational diffusion.

II. ANALYTICAL SOLUTION

In this section, an analytic procedure for deriving the solution of the ODE (see eq. 2) as the probability density function (PDF), $P(\theta)$ is presented. The PDF solution of the ODE is $\pi$-periodic since the RLO’s are indistinguishable when orientated at $\theta$ or $\theta + \pi$. Since the PDF is $\pi$-periodic one might write, in principle, a Fourier series solution valid for small and large values of $\alpha$.

Since we are interested in finding an exact solution we differ from this approximate approach by viewing the problem as a boundary condition one, more precisely:

$$P(0) = P(\pi) \quad \text{and} \quad P'(0) = P'(\pi)$$  \hfill (7)

where $P'(\theta) = \frac{dP(\theta)}{d\theta}$.

Since the PDF ought to be normalised over the interval $[0, \pi]$:

$$\int_0^\pi P(\theta) d\theta = 1$$  \hfill (8)
Consequently, the determination of the PDF is a constrained periodic boundary value problem eq. 2, the constraint originating from the normalization condition.

In order to get closed form solutions, we write eq. 2 as a first-order ODE:

\[ P'(\theta) + \alpha \sin^2(\theta)P(\theta) = C \]  

with initial condition: \( P(\theta = 0) = P(0) \). This is mathematically sound provided the initial value \( P(0) \) and the constant \( C \) are known for any value of \( \alpha \).

The constant \( C \) appears in eq. 9 and is equal to \( P'(0) \) by substituting \( \theta = 0 \) and assuming finiteness of \( P(\theta \to 0) \).

The formal solution of eq. 9 may be given generally as:

\[ P(\theta) = C \exp\left[\frac{\alpha}{2} \left( \sin\left(\frac{2\theta}{\alpha}\right) - \frac{\alpha}{2} \sin\left(\frac{2x}{\alpha}\right)\right)\right] \int_{-\infty}^{\theta} \exp\left[ -\frac{\alpha}{2} \left( \sin\left(\frac{2x}{\alpha}\right) - x\right) \right] dx \]  

The lower limit \( -\infty \) is not obvious since the problem is defined over the angular interval \([0, \pi]\). Analytically, the lower limit \( -\infty \) is the only possibility compatible with the boundary conditions given by eq. 7 since the factor \( \exp(\alpha x/2) \) appearing in the integrand as given by eq. 10 enlarges the angular interval from \([0, \pi]\) to the interval \([-\infty, \theta]\). The reason behind this transformation comes from imposing a zero integration constant that can be obtained only when \( x \to -\infty \) given the factor \( \exp(\alpha x/2) \) and the positive sign of \( \alpha \).

Performing a change of variables, the solution may then be written as:

\[ P(\theta) = \left( \frac{2C}{\alpha} \right) \exp\left[\frac{\alpha}{4} \left( \sin(2\theta) \right) \right] \int_{0}^{\infty} \exp(-x) \exp\left[ \frac{\alpha}{4} \sin\left(\frac{4x}{\alpha}\right) - 2\theta\right] dx \]  

The form in eq. 11 is superior to the previous form given by eq. 10 since the exponential factors \( \exp(\theta) \) and \( \exp(-\theta) \) terms (that can vary rapidly by several orders of magnitude) are not used 10. The constant \( C \) is determined from the normalisation condition of the PDF (eq. 8) yielding:

\[ \frac{1}{C} = \frac{2}{\alpha} \int_{\theta}^{\frac{\pi}{2}} d\theta \int_{0}^{\infty} dx \exp(-x) \exp\left[ \frac{\alpha}{2} \sin\left(\frac{2x}{\alpha}\right) \cos(2\theta - \frac{2x}{\alpha}) \right] \]  

From the general expression eq. 11, the initial value \( P(0) \) is given by:

\[ P(0) = \left( \frac{2C}{\alpha} \right) \int_{0}^{\infty} \exp(-x) \exp\left[ \frac{\alpha}{4} \sin\left(\frac{4x}{\alpha}\right) \right] dx \]  

The PDF depends on \( \alpha \) which we may want to vary over several orders of magnitude.

Surprisingly, the difficulty in solving the ODE stems from the fact that its nature may be modified when \( \alpha \) increases, with the constraints of \( \pi \)-periodicity and PDF normalisation maintained. For small values of \( \alpha \) the PDF is flat as observed in fig. 2 and this can be shown with a simple argument.

When \( \alpha \) approaches zero, we should satisfy the pure rotational diffusion equation in the plane given by \( D_r \frac{d^2}{d\theta^2} P(\theta) = 0 \) whose solution is \( P(\theta) = a\theta + b \) with \( a, b \) constant and determined from \( \pi \)-periodicity eq. 7 and normalisation condition eq. 8 yielding \( a = 0, b = \frac{1}{\pi} \).

In the opposite case of large \( \alpha \) (as in fig. 3) the PDF displays a strong peak around an angle \( \theta_{max} \) indicating that orientation of the RLO is setting in.

III. OPTICAL PROPERTIES

In general, even if a solution of RLO’s is optically isotropic at equilibrium, under flow conditions, the distribution of orientation will be affected in a way such that the optical properties become anisotropic because of the existence of some privileged direction due to the flow. This phenomenon is called flow birefringence or Maxwell effect 3.

Shear-induced birefringence and dichroism measurements of suspensions of RLO consists of detecting phase and intensity variation (due to the flow) of light travelling across the sample.
The flow produces a modification in the real and imaginary part of the optical index $\Delta n', \Delta n''$ of the suspension. $\Delta n'$ is extracted from the phase difference between the transmitted and reference signal whereas $\Delta n''$ is extracted from the attenuation of light intensity. $\Delta n', \Delta n''$ are both proportional \[4\] to a single quantity, $S$ the nematic order parameter \[11\] encountered in the physics of liquid crystals.

$S$ is 0 for the unorientated (disordered) state and 1 for the fully orientated (ordered) state. Writing $S = a < \cos^2 \theta > + b$ (with $a, b$ constants to be determined) and noting that in the ordered state $< \cos^2 \theta > = 1$ since $\theta \sim 0$ whereas in the disordered state $< \cos^2 \theta > = 1/2$. Hence we find $S = 2 < \cos^2 \theta > -1 = < \cos 2\theta >$ \[12\].

When $S = < \cos 2\theta >$ saturates to one, perfect alignment is reached i.e. complete orientation control is achieved as observed in Fig. 4 for very large values of the Peclet number $\alpha$.

When $S$ saturates to 1, the orientational PDF becomes sharply peaked around $\theta_{\text{max}}$, the value of the angle that corresponds to the PDF maximum as the Peclet number increases. At the value $\theta = \theta_{\text{max}}$, the first-order ODE (eq. 9) writes:

$$\alpha \sin^2(\theta_{\text{max}}) P(\theta_{\text{max}}) = C \tag{14}$$

Since, it is required that the PDF should be normalised according to eq. 8 when $\alpha$ is large, we get after using eq. 14:

$$\theta_{\text{max}} P(\theta_{\text{max}}) \sim \text{constant} \tag{15}$$

The above is the area under the PDF curve since the distribution function becoming sharply peaked around $\theta_{\text{max}}$ when $\alpha$ is large, leading to a PDF approximately triangular in shape with a height $P(\theta_{\text{max}})$ and a base equal to $2\theta_{\text{max}}$. This implies that $\theta_{\text{max}}$ is close to the standard deviation of the PDF $P(\theta)$ when $\alpha$ is large.
Using eq. 14 and eq. 15 we find the following leading behaviour:

$$\theta_{\text{max}} \sim \alpha^{-1/3}, \quad P(\theta_{\text{max}}) \sim \alpha^{1/3}$$

(16)

Hence $S$ behaves for large $\alpha$ as:

$$S = \int_{-\theta_{\text{max}}}^{\theta_{\text{max}}} P(\theta) \cos 2\theta d\theta \sim \int_{-\theta_{\text{max}}}^{\theta_{\text{max}}} P(\theta) \cos 2\theta d\theta$$

(17)

which yields:

$$S \sim \theta_{\text{max}} P(\theta_{\text{max}}) \cos 2\theta_{\text{max}}$$

(18)

implying that $S$ behaves as $\cos(2\theta_{\text{max}}) \sim 2 \cos^2(\theta_{\text{max}}) - 1 \sim 1 - 2\alpha^{-\frac{2}{3}}$ which is close to the asymptotic behaviour found in fig. 4 and obtained from full numerical integration.

FIG. 4: Nematic order parameter $S = \langle \cos(2\theta) \rangle$ versus Peclet number $\alpha$ spanning the interval $[10^{-4} - 10^8]$. For small values of $\alpha$ we have a "disordered" state characterized by $S \sim 0$ whereas $S$ approaches 1 ("orientated" state) for large values of $\alpha$. The asymptotic regime $1 - 2\alpha^{-\frac{2}{3}}$ is shown for comparison.

Since $\theta_{\text{max}}$ is a measure of the standard deviation (for large values of $\alpha$) in the fluctuations of the angle $\theta$ about the main flow direction, the RLO orientation controllability with flow can be analysed with the variation of the angle $\theta_{\text{max}}$ with the Peclet number $\alpha$.

In fig. 5 the variation of $\theta_{\text{max}}$ with the Peclet number $\alpha$ is depicted. From the figure, one infers that orientation controllability is achieved when $\theta_{\text{max}}$ drops below a small specified value. For instance, one has to have $\alpha \sim 400$ in order to achieve $\theta_{\text{max}} \leq 0.1$ radian (see fig. 5).

IV. CONCLUSIONS

This work is the simplest illustration of the interplay between shear (due to liquid flow) and rotational diffusion (due to thermal agitation) on the planar orientation of a dilute concentration of rigid non-interacting thin and long RLO suspended in a liquid shear flow in a simple geometry.

The PDF describing the RLO orientations is analytically determined in 2D and accurately evaluated as a function of $\alpha$, the rotational Peclet number given by the ratio of shear rate and rotational diffusion constant.

The solutions found for the PDF in the bulk of the flowing liquid for arbitrary values of $\alpha$ confirm the validity of the approach used for small values of $\alpha$. In this regime flat angular distributions (PDF) are expected: this means we have isotropic PDF due to rotational diffusion dominated motion.

On the other, when $\alpha$ increases, we expect sharply peaked angular distributions (PDF) the peak occurring around an orientation angle $\theta_{\text{max}}$: this means we are in the shear dominated motion case and the PDF is anisotropic with a single mode.
FIG. 5: Variation of the angle $\theta_{\text{max}}$ (in radians) with the Peclet number $\alpha$. For small values of $\alpha$, $\theta_{\text{max}}$ saturates at a value of about 0.8 radian that is close to $\pi/4$ as seen in fig. 2. When $\alpha$ is increased beyond 400, $\theta_{\text{max}}$ decreases as $\alpha^{-1/3}$ dropping to small values $\leq 0.1$ radian indicating that orientation control has started to set in.

The impact of $\alpha$ on orientation control and its signature through optical properties is examined and the variation of the nematic order parameter $S$ is obtained for a wide range of values of $\alpha$ spanning the interval $[10^{-4} - 10^5]$. Since Boeder’s pioneering work, many improvements have been made to remove restrictions on the cross-sectional area and size of the RLO, introduce internal vibrational and rotational degrees of freedom within the RLO [4], consider rotational diffusion in 3D, turbulent flow [13], more complex flow geometries [4] and interactions between the RLO’s. The Boeder equation provides orientation control requiring a wide variation of the Peclet number whereas practically a much smaller range is demanded. From fig. 5 the transition from $\theta_{\text{max}} \sim \pi/4$ (almost flat and isotropic behaviour leading to no alignment) to the case $\theta_{\text{max}} \sim 0$ (peaked behaviour, anisotropic meaning controlled orientation) spans an interval $\alpha$ of $[0.1 - 400.]$ in order to switch from no alignment to orientation control in the 0.1 radian range. If, on the other hand, we require $\theta_{\text{max}} \leq 0.01$ radian, then $\alpha \sim 6 \times 10^5$ which is a very large value.

Practically speaking, a small interval of variation of $\alpha$ is required in order to achieve a fast and precise control of alignment (with a small standard deviation), hence the question: Would it be possible, within a model similar to this one and under certain conditions to vary slightly $\alpha$ (by a few units only) and achieve a standard deviation smaller than 0.01 radian?

Another question is: Would it be possible, within a model similar to this one and under certain conditions to vary $\alpha$ and obtain several peaks (multi-modal) behaviour with several orientations, in other words make a transition with $\alpha$ from single-mode to multi-mode behaviour?

V. APPENDIX: ROTATIONAL DIFFUSION

Ordinary diffusion describing the random walk of a particle in space is given by:

$$\frac{\partial f(\vec{r}, t)}{\partial t} = D \Delta f(\vec{r}, t) \quad (19)$$

with $f(\vec{r}, t)$ the probability function for the particle to be at position $\vec{r}$ at time $t$ and $\Delta f$ the full Laplacian of $f$. $D$ is the ordinary (translational) diffusion coefficient. In rotational diffusion, a particle wanders on the surface of the unit sphere such that the diffusion equation becomes:

$$\frac{\partial f(\theta, t)}{\partial t} = D_r \Delta_\theta f(\theta, t) \quad (20)$$

The angular Laplacian $\Delta_\theta$ and the probability function $f = f(\theta, t)$ depends now solely on the angular coordinates represented collectively by $\theta$. $\Delta_\theta$ contains (second-order) derivatives with respect to $\theta$ with the radial variable fixed ($r = 1$) and with the rotational diffusion coefficient $D_r$ replacing the ordinary diffusion coefficient $D$.

In the 2D case, the rotational diffusion equation on the unit circle reads:

$$\frac{\partial f}{\partial t} = D_r \frac{\partial^2 f}{\partial \theta^2} \quad (21)$$
The stationary PDF $P(\theta)$ studied in this work is obtained from the long-time limit: $P(\theta) = \lim_{t \to \infty} f(\theta, t)$.

In contrast to translational diffusion, $D_r$ units are not $\text{Length}^2/\text{Time}$ but simply $1/\text{Time}$ since the random walks on the unit sphere are purely angular with no dependence on any characteristic length. Similarly to translational diffusion where the average displacement squared $<|\vec{r}(t) - \vec{r}(0)|^2> \sim D_t t$ at time $t$, in rotational diffusion, we have $<|\vec{u}(t) - \vec{u}(0)|^2> \sim D_r t$ with $\vec{u}(t)$ representing the angular position $\theta(t)$ at time $t$ (while radial position $r = 1$ is kept fixed for all times).

[1] F. Reif, *Statistical and Thermal Physics*, McGraw-Hill, New-York (1985).
[2] A. E. Siegman, Am. J. Phys. 47, 545 (1979).
[3] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, Pergamon, Oxford (1975).
[4] M. Doi and S.F. Edwards, *The Theory of Polymer Dynamics*, Clarendon Press, Oxford (1986).
[5] M. G. Forest, R. Zhou, Q. Wang, J. Non-Newtonian Fluid Mech. 116 183 (2004).
[6] *Soft Matter*, Vols.1-4 edited by G. Gompper and M. Schick Wiley-VCH Verlag GmbH and Co. KGaA, Weinheim (2006).
In particular, Vol. 2: *Complex Colloidal Suspensions* treats subjects of interest to the present work.
[7] E. Hecht, *Optics*, Fourth edition, Addison-Wesley, San Francisco (2002).
[8] P. Boeder, Z. Physik 75, 258 (1932).
[9] The RLO being rigid, its rotation motion is described by: $\frac{d\hat{u}}{dt} = \Omega \times \hat{u}$ where $\Omega$ is the rotation vector about the center. Assuming [3] the RLO is thin, long and follows the macroscopic velocity gradient, we have in the shear flow case: $\Omega = \hat{u} \times (\kappa \cdot \hat{u})$ where $\kappa$ is the fluid velocity gradient tensor with components given by: $\kappa_{ij} = \frac{\partial v_i}{\partial x_j}$; $i, j \in [x, y]$. The double cross product yields $\frac{d\hat{u}}{dt} = \kappa \cdot \hat{u} - (\hat{u} \cdot \kappa \cdot \hat{u}) \hat{u}$ and the angular time evolution becomes: $\hat{z} \frac{d\hat{u}}{dt} = \hat{u} \times \frac{d\hat{u}}{dt} = \hat{u} \times (\kappa \cdot \hat{u})$. Note that the tensor $\kappa$ has a single component $\kappa_{xy} = \frac{\partial v_x}{\partial y} = \dot{\gamma}$, the hydrodynamic shear rate, given the fluid velocity field $v = v_x \hat{x}$ is along the $x$ direction and depends on the coordinate $y$, only (see fig. 1).
[10] W. H. Press, W. T. Vetterling, S. A. Teukolsky and B. P. Flannery, *Numerical Recipes in C: The Art of Scientific Computing* Second Edition, Cambridge University Press, New-York (1992).
[11] M. J. Stephen and J. P. Straley, Rev. Mod. Phys. 46, 617 (1974).
[12] In $n$ dimensions, we have in the disordered state $<\cos^2 \theta> = 1/n$ since we have $n$ equivalent vector components. Writing as previously $S = a <\cos^2 \theta> + b$ and since in the ordered state $<\cos^2 \theta> = 1$ we find $S = \frac{a}{n} - \frac{b}{n} (n <\cos^2 \theta> - 1)$.
[13] The Reynolds number defined as [3]: $Re = \frac{\nu \dot{\gamma} L^2}{\eta_s}$ must be smaller than $\sim 2000$ in order to have a non-turbulent flow. $\rho_s$ and $\eta_s$ are solvent density and viscosity whereas $L$ is the width of the planar contraction (see fig. 1).