Higgs mechanism as a collective effect due to extra dimension.

A.A.Slavnov.
Steklov Mathematical Institute
Gubkina st. 8, 119991 Moscow,
Moscow State University
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Abstract
A systematic analysis of the unitary electroweak model described by the higher derivative Lagrangian depending on extra dimension [1] is presented.

1 Introduction
The essential part of renormalizable models of weak and electromagnetic interactions, like Weinberg-Salam model [2], [3], or more advanced models describing neutrino oscillations is the spontaneous symmetry breaking via interaction with the spin zero Higgs meson [4], [5].

The Higgs meson interaction is described by the Lagrangian of the form

\[ L = |\partial_\mu \varphi + ig \frac{\tau^a}{2} A^a_\mu \varphi - \frac{ig_1}{2} B_\mu \varphi|^2 + G[(\bar{L}\varphi) R + \bar{R}(\varphi^+ L)] + \ldots + \frac{m^2}{2} (\varphi^+ \varphi) - \lambda^2 (\varphi^+ \varphi)^2 \]

Here \( A_\mu, B_\mu \) are the \( SU(2) \) and \( U(1) \) gauge fields, \( \varphi \) is the complex doublet, \( L, R \) denote the chiral lepton \( SU(2) \) multiplets and \( \ldots \) stand for the similar terms corresponding to quarks.

The minimum of the Higgs potential is achieved at some nonzero nonsymmetric value of \( \varphi \) and the perturbation theory near the stable minimum may be constructed in terms of the shifted scalar fields having the following real components

\[ \varphi_1 = \frac{iB_1 + B_2}{\sqrt{2}}; \quad \varphi_2 = \frac{\sqrt{2}m_1}{g} + \frac{1}{\sqrt{2}} (\sigma - iB_3) \]

where \( m_1 \) is the mass which the Yang-Mills field acquires via Higgs mechanism.
The gauge invariance of the model allows to choose the gauge condition $B_\alpha = 0$, corresponding to the manifestly unitary gauge, whereas another choice, for example $\partial_\mu A_\mu = 0$, provides a power-counting renormalizable perturbation theory \cite{6}.

Gauge invariance allows to eliminate from the spectrum all components $B_\alpha$, but the field $\sigma$ survives and must be observed experimentally. The mass of this field is $m_2^2 = 2\sqrt{2}\lambda m_1 g^{-1}$ and at first sight it seems that choosing the constant $\lambda$ big enough one may shift this mass to an unobservable region. However this mass enters also the self interaction of Higgs mesons, which in the unitary gauge acquires a form:

$$L_H = -\frac{gm_2^2}{4m_1}\sigma^3 - \frac{g^2m_2^2}{32m_1^2}\sigma^4$$

One sees that in the limit $m_2^2 \to \infty$ this interaction blows up violating the renormalizability of the theory.

Existing experimental data impose stringent limits on the mass of the Standard Model Higgs meson. These limits may be changed by different modifications of the Weinberg-Salam model. In particular the models with additional scalar gauge singlet mesons were discussed (see e.g. \cite{7}, \cite{8}, \cite{9}, \cite{10}). These modifications indeed change predictions concerning masses and other characteristics of scalar mesons, but at the same time they introduce considerable ambiguity in the choice of the Lagrangian, describing the interaction of the singlet spin zero particles.

An interesting idea was put forward by N.Krasnikov \cite{11}, who considered the limiting case when the number of scalar singlets is infinite. In this case one can avoid the appearance of one particle poles in the channels corresponding to the exchange by a spin zero particle and in this sense to get rid off fundamental scalar mesons.

A common deficiency of all above mentioned models is a lack of a guiding principle of choosing a particular form of the singlet field interactions.

The form of the interaction of additional gauge singlet fields may be fixed by some symmetry of a corresponding Lagrangian. This approach was adopted in our paper \cite{1} and is studied in more details in the present paper. Our model is similar to the model proposed in the paper \cite{11}, and in the same sense does not require the existence of fundamental spin zero bosons. We interpret the spontaneous symmetry breaking responsible for the mass generation of intermediate vector mesons as a collective effect related to the dependence of the singlet scalar fields on the additional coordinate. The Yang-Mills fields and the Higgs doublet fields are living on the four dimensional brane in the five dimensional space, whereas the gauge singlet scalar fields and their masses depend also on the fifth coordinate. The form of the singlet field interaction is fixed by requirement of invariance of the Lagrangian with respect to some supersymmetry transformations. This symmetry arises in a natural way in the higher derivative model \cite{1} and will be discussed below.
2 Spontaneous symmetry breaking via extra dimension.

To avoid trivial complications in this section we consider the gauge group $SU(2)$ and only vector meson - scalar sector.

Following the paper [1] we assume that the gauge fields $A_\mu$, as well as the fermions and the spin zero complex doublet $\varphi$ are living on the four dimensional "brane" and transform under the gauge transformations as in the Weinberg-Salam model. In addition we introduce the gauge singlet neutral scalar field, which depends on the extra coordinate $\lambda : X(x, \lambda)$.

The dynamics of the model is described by the higher derivative Lagrangian, which has a form:

$$L = L_{YM} + (D_\mu \varphi)^+(D_\mu \varphi) + \frac{g}{2m_1} \partial_\mu (\varphi^+ \varphi) \int_{-\pi/\kappa}^{\pi/\kappa} \partial_\mu X(x, \lambda) d\lambda + \frac{1}{2} \int_{-\pi/\kappa}^{\pi/\kappa} \partial_\mu X(x, \lambda) d\lambda \int_{-\pi/\kappa}^{\pi/\kappa} \partial_\mu X(x, \lambda) d\lambda + \frac{\pi}{\kappa} \int_{-\pi/\kappa}^{\pi/\kappa} \Box X(x, \lambda) \Box X(x, \lambda) a^{-2}(\lambda) d\lambda - \frac{\pi}{\kappa} \int_{-\pi/\kappa}^{\pi/\kappa} \partial_\mu X(x, \lambda) \partial_\mu X(x, \lambda) d\lambda + \int_{-\pi/\kappa}^{\pi/\kappa} \partial_\mu \bar{c}(x, \lambda) \partial_\mu c(x, \lambda) d\lambda$$

(4)

where $\varphi$ is again parameterized as in eq.(2). The fields $\bar{c}, c$ are anticommuting scalar ghosts, singlet with respect to gauge transformations. We assume that the extra dimension is compact $-\pi/\kappa \leq \lambda \leq \pi/\kappa$. The function $a^{-2}(\lambda)$, which determines the masses of the $X$-fields also may depend on $\lambda$.

To quantize the model we start with the discretized version, introducing the lattice in the fifth dimension. We take $\frac{2\pi}{\kappa} = Nb$, where $b$ is the lattice spacing, $\lambda_i = bi, \ X_i = X(\lambda_i), \ c_i = c(\lambda_i), \ \bar{c}_i = \bar{c}(\lambda_i)$. After rescaling the fields $\sqrt{b}c_i \to c_i, \sqrt{b}\bar{c}_i \to \bar{c}_i, bX_i \to X_i$ the discretized Lagrangian may be written in the form:

$$L = L_{YM} + (D_\mu \varphi)^+(D_\mu \varphi) + \frac{g}{2m_1} \partial_\mu \left( \sum_{i=1-N/2}^{N/2} X_i \right) \partial_\mu (\varphi^+ \varphi) + \frac{1}{2} \left( \sum_{i=1-N/2}^{N/2} \partial_\mu X_i \right)^2 - \frac{N}{2} \sum_{i=1-N/2}^{N/2} \partial_\mu X_i \partial_\mu X_i + \frac{N}{2} \sum_{i=1-N/2}^{N/2} a_i^{-2} \Box X_i \Box X_i + \sum_{i=1-N/2}^{N/2} \partial_\mu \bar{c}_i \partial_\mu c_i$$

(5)

The Lagrangian (5) includes higher derivatives and one may expect the appearance of negative norm states. They indeed appear, but due to invariance of this Lagrangian under supersymmetry transformations which will be described below the negative norm states decouple and the theory is unitary in the nonnegative norm subspace [12], [13].
To quantize the model we shall use the Ostrogradsky canonical formalism [14, 13]. First of all we diagonalize the quadratic form in the Lagrangian (5) by the shift

$$\sigma(x) \rightarrow \sigma(x) - \sum_{i=1-N/2}^{N/2} X_i$$

(6)

After such a shift the Lagrangian takes a form:

$$L = \tilde{L}(A_\mu, B, \tilde{\sigma}) + \frac{1}{2} \partial_\mu \sigma \partial_\mu \sigma - \frac{N}{2} \sum_{i=1-N/2}^{N/2} \partial_\mu X_i \partial_\mu X_i + \frac{N}{2} \sum_{i=1-N/2}^{N/2} a_i^2 \Box X_i + \sum_{i=1-N/2}^{N/2} \partial_\mu \bar{c}_i \partial_\mu c_i + \frac{g}{4m_1} \partial_\mu (B^2 + \tilde{\sigma}^2) \sum_{i=1-N/2}^{N/2} \partial_\mu X_i$$

(7)

Here $\tilde{L} + \frac{1}{2} \partial_\mu \sigma \partial_\mu \sigma$ is the usual Lagrangian for the massive Yang-Mills field interacting with the scalar fields $B_a, \sigma$, in which $\sigma$ in the interaction is replaced by

$$\tilde{\sigma} = \sigma - \sum_{i=1-N/2}^{N/2} X_i$$

(8)

This Lagrangian is obviously invariant with respect to the gauge transformations

$$\delta A^a_\mu = (D_\mu \varepsilon)^a$$

$$\delta \sigma = -\frac{g}{2} (B_a \varepsilon^a)$$

$$\delta B_a = -m_1 \varepsilon_a - \frac{g}{2} \varepsilon_{abc} B^b \varepsilon^c - \frac{g}{2} \sigma \varepsilon_a + \frac{g}{2} \sum_{i=1-N/2}^{N/2} X_i \varepsilon_a$$

(9)

Using this invariance we impose the gauge condition $B_a = 0$. Quantization of the fields $A^a_\mu, \sigma, \bar{c}_i, c_i$ is performed in a standard way.

The Lagrangian for $X_i$ field includes higher time derivatives and to quantize it we introduce the variables $Q^1_i = X_i, \quad Q^2_i = \dot{X}_i$ and conjugated momenta

$$P^1_i = -N \partial_0 X^i - a_i^2 N \partial_0 \Box X_i + \tilde{f}(A_\mu, \sigma, X^i); \quad P^2_i = a_i^2 N \Box X^i$$

(10)

Here $\tilde{f}$ denotes the terms of order $O(g)$.

In terms of these variables the Fourier transform of the Hamiltonian looks as follows

$$\tilde{H}(k) = \sum_i P^1_i Q^2_i + \sum_i P^2_i \dot{Q}^1_i - L = \sum_i \{P^1_i Q^2_i - k^2 P^2_i Q^1_i + \frac{(P^2_2 a_i)^2}{2N} + \frac{N}{2} (Q^2_2)^2 + \frac{N}{2} \sigma^2 (Q^1_2)^2 + \frac{1}{2} (P^1_\sigma)^2 + \frac{1}{2} k^2 \sigma^2 + P^2_\bar{c} \dot{P}^1_\bar{c} + k^2 \bar{c}_i c_i + \tilde{H}_1(A_\mu, \tilde{\sigma}, X)$$

(11)

In this equation $\tilde{H}_1$ denotes the Hamiltonian of the free massive vector field and the interaction terms.
The spectrum of the free Hamiltonian includes a spin zero massless one-particle state generated by the field \( \sigma \), as well as \( N \) spin zero massless and \( N \) spin zero massive one-particle states with the masses \( a_i \), generated by the fields \( X_i \). The corresponding creation and annihilation operators are

\[
\sigma^\pm(k) = \frac{1}{\sqrt{2\omega}}(\omega \sigma \mp ip); \quad [\sigma^-(k), \sigma^+(k')] = \delta(k - k'); \quad \omega = \sqrt{k^2}
\]

\[
a_i^\pm(k) = \frac{1}{\sqrt{2\omega N}}(P^i_1 \mp iN\omega Q^i_1 \mp i\omega P^i_2); \quad [a_i^-(k), a_i^+(k')] = -\delta(k - k')
\]

\[
b_i^\pm(k) = \frac{1}{\sqrt{2\omega N}}(P^i_1 + NQ^i_2 \mp i\omega P^i_2); \quad [b_i^-(k), b_j^+(k')] = \delta_{ij}\delta(k - k'); \quad \omega_i = \sqrt{k^2 + a_i^2}
\]

The operators \( a^+ \) create massless states with nonpositive norm. However due to invariance of the Lagrangian (7) with respect to the supersymmetry transformations

\[
\delta X_i(x) = c_i(x)\varrho; \quad \delta\sigma(x) = \sum_{i=1-N/2}^{N/2} c_i(x)\varrho;
\]

\[
\delta\bar{c}_i(x) = [NX_i(x) - \sigma(x) - Na^-_i\Box X_i(x) - \frac{g}{4m_1}(B(x)^2 + \tilde{\sigma}(x)^2)]\varrho,
\]

where \( \varrho \) is an anticommuting parameter, there exists a conserved operator \( Q \), which allows to select the nonnegative norm states by imposing the condition

\[
Q|\Phi \rangle = 0
\]

The asymptotic operator \( Q_0 \) has a form

\[
Q_0 = \int d^3k \sum_i (NQ^i_1\partial_0 c^i + P^i_1\partial_0 c^i + P^i_2\partial_0 c^i - P_\sigma c_i + \sigma\partial_0 c_i) = \int d^3k \sum_i [c_i^+(\sqrt{Na^-_i} - \sigma^-) + (\sqrt{Na^-_i} + \sigma^+)c_i^-]
\]

The solution of the eq.(16) for asymptotic states has a form

\[
|\Phi \rangle = [1 + \sum_n b_n \prod_{1\leq j\leq n} (\sigma^+(k_j) - \frac{1}{\sqrt{N}} \sum_{i=1-N/2}^{N/2} a_i^+(k_j))]|\Phi >_{A,b_i}
\]

where the vectors \( |\Phi >_{A,b_i} \) describe the states of the massive vector field and the scalar particles with the masses \( a_i \). The vectors \( |\Phi >_{A,b_i} \) have nonnegative norm and this subspace being factorized with respect to the null vector space coincides with the physical one.

It is worth to mention that for \( N \neq 1 \) the conserved operator \( Q \) is not nilpotent. However as in our case the ghost fields \( c_i, \bar{c}_i \) are noninteracting, the condition (16) guarantees the positivity of the physical states norm.
It completes the quantization of the model in the "unitary" gauge $B_a = 0$. In this gauge the theory is not power counting renormalizable as the vector field propagator

$$\tilde{D}^{ab}_{\mu \nu}(k) = \frac{\delta^{ab} \eta^{\mu \nu} - k^\mu k^\nu m_1^{-2}}{k^2 - m_1^2}$$

(19) does not decrease at infinity. However using the gauge invariance of the Lagrangian (7) one can pass in a standard way to some renormalizable gauge, i.e. the Lorentz gauge $\partial_\mu A_\mu = 0$. In this gauge the $S$-matrix is given by the path integral

$$S = \int \exp\{i \int L dx\} \delta(\partial_\mu A_\mu) \text{det}[M] dA_\mu dB^\alpha d\sigma dX_i$$

(20)

where $L$ is the gauge invariant Lagrangian (7), $\text{det}[M]$ is the Faddeev-Popov determinant and proper boundary conditions in the path integral are assumed. We omitted the ghost fields $\bar{c}_i, c_i$ as being free they do not influence the result.

The propagators, corresponding to the integral (20) are

$$\tilde{D}^{ab}_{\mu \nu}(k) = \frac{\delta^{ab} \eta^{\mu \nu} - k^\mu k^\nu m_1^{-2}}{k^2 - m_1^2}$$

$$\tilde{D}^{ab}_{FP}(k) = \frac{\delta^{ab}}{k^2}$$

$$\tilde{D}_\sigma(k) = \frac{1}{k^2}$$

$$\tilde{D}^{ab}_{B}(k) = \frac{\delta^{ab}}{k^2}$$

$$\tilde{D}^{ij}_{X}(k) = \frac{a_i^2}{k^2(k^2 - a_i^2)N} = \frac{\delta^{ij}}{N} \left[ \frac{1}{k^2 - a_i^2} - \frac{1}{k^2} \right]$$

(21)

All the propagators except for $\tilde{D}_X(k)$ decrease at $k \to \infty$ as $k^{-2}$, and the ultraviolet asymptotic of the propagator $\tilde{D}_X(k)$ is $k^{-4}$. Obviously the model is renormalizable.

The fields $X$ which may produce nonpositive norm states enter the interaction only in the combinations

$$\tilde{\sigma} = \sigma - \sum_i X_i, \quad \tilde{X}_i = \Box X_i$$

(22)

In the $\tilde{\sigma}$ propagators the contribution of the field $\sigma$ compensates exactly the contribution of zero mass components of $X_i$ and only positive norm components with the masses $a_i$ are propagating. The propagators of $\tilde{X}_i$ also do not have zero mass poles. Neither of these combinations generates negative norm asymptotic states in accordance with the previous proof of the unitarity in the physical subspace.

The probability of the creation of a spin zero state with the mass $a_i$ is suppressed by the factor $N^{-1}$ and in the limit $N \to \infty$ vanishes. However as the number of $X_i$ fields in the intermediate states in this limit goes to infinity, their total contribution is finite and describes some collective effect. This contribution depends on the particular form of the function $a(i)$. If the dependence of $a$ on $i$ is such that for $N \to \infty$ the series

$$\sum_{i=1-N/2}^{N/2} \frac{1}{k^2 - a_i^2}$$

(23)
is convergent, then in this limit

\[ \sum_{i,j=1-N/2}^{N/2} \tilde{D}_{\tilde{X}}^{ij}(k) \sim \frac{1}{k^2}. \]  

(24)

As the interaction of \( X \) fields includes the second derivative, one obtains a nonrenormalizable theory.

The other extreme case corresponds to \( a_i \equiv a \). In this case

\[ \sum_{i,j} \tilde{D}_{\tilde{X}}^{ij}(k) = \frac{a^2}{k^2(k^2 - a^2)} \]  

(25)

and the propagator of the \( \tilde{\sigma} \)-field is

\[ \tilde{D}_{\tilde{\sigma}}(k) = \frac{1}{k^2 - a^2} \]  

(26)

It is nothing but the usual Higgs model. All massive spin zero states have the same quantum numbers and are indistinguishable.

In a general case for large momenta \( k^2 \gg a_i^2 \), the propagator \( \tilde{D}_{\tilde{\sigma}}(k) \) coincides with the usual Higgs meson propagator \( \tilde{D}_{\tilde{\sigma}} \sim k^{-2} \), but for \( k^2 \approx a_i^2 \) the predictions of our model may be very different of the Higgs-Kibble model. For any finite \( N \) the Lagrangian describes a model with \( N \) neutral spin zero particles with the masses \( a_i \), which is unitary in the positive norm space and renormalizable. For \( N \) big enough the individual scalar states become unobservable, but their presence in the intermediate states produces some collective effects which may be checked experimentally.

The model described above may be also described by a Lagrangian without higher derivative terms introducing a gauge invariant interaction of the singlet fields with the Higgs meson as it was done in the papers cited in the introduction. In distinction of these papers in our approach the spin zero Higgs field, described by the complex doublet field \( \varphi \) is eliminated completely from the spectrum of observables and all physical spin zero excitations are associated with the gauge singlets. Even more important is the invariance of the Lagrangian with respect to the supersymmetry transformation, which together with the requirement of renormalizability fixes the form of the interaction of the singlet fields. In particular it forbids the vertices of the type \( X^3 \) or \( \varphi^+ \varphi X^2 \).

To come back to the theory with compact continuous extra dimension, described by the Lagrangian, one has to consider the limit \( N \to \infty, b \to 0, Nb = 2\pi\kappa^{-1} \). This theory will be discussed in the next section.

### 3 Continuous extra dimension.

We shall consider the model with the continuous extra dimension as a limiting case of the discrete model, whose quantization was discussed in the previous section.
Let us introduce the lattice spacing $b$ by rescaling the fields $X_i \rightarrow bX_i$. In the limit $N \rightarrow \infty$, $Nb = 2\pi \kappa^{-1}$ the $S$-matrix will take the form \( \text{eq}(20) \), where the Lagrangian is equal to

\[
L = \tilde{L}(A_\mu, B_a, \bar{\sigma}) + \frac{1}{2} \partial_\mu \sigma \partial_\mu \sigma + \frac{g}{4m^4} \partial_\mu (B^2 + \bar{\sigma}^2) \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda \partial_\mu X(x, \lambda) - \frac{\pi}{\kappa} \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda \partial_\mu X(x, \lambda) \partial_\mu X(x, \lambda) + \frac{\pi}{\kappa} \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda a^{-2}(\lambda) \square X(x, \lambda) \square X(x, \lambda)
\]

with

\[
\bar{\sigma} = \sigma - \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda X(x, \lambda)
\]

This Lagrangian is obtained from \( \text{eq}(11) \) by substituting the explicit expression for $\varphi$ in terms of $B, \sigma$, making the shift \( \text{eq}(28) \) and omitting the free ghost fields $\bar{c}(x, \lambda), c(x, \lambda)$.

The fields $A_\mu, B_a, \sigma$ and the Faddeev-Popov ghosts do not depend on the extra dimension, so their propagators have the standard form. The $X$-field propagator is easily calculated. It is equal to

\[
\tilde{D}_X(k, \lambda, \mu) = \delta(\lambda - \mu) \kappa \frac{a^2(\lambda)}{2\pi k^2(k^2 - a^2(\lambda))}
\]

As before the interaction terms depend only on the combinations

\[
\tilde{\sigma}(x) = \sigma(x) - \int_{-\pi/\kappa}^{\pi/\kappa} X(x, \lambda) d\lambda; \quad \tilde{X}(x) = \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda \square X(x, \lambda)
\]

The corresponding propagators

\[
\tilde{D}_{\sigma}(k) = \frac{1}{k^2} + \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda d\mu \frac{\kappa}{2\pi} [-\frac{1}{k^2} + \frac{1}{k^2 - a^2(\lambda)}] \delta(\lambda - \mu) = \frac{\kappa}{2\pi} \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda \frac{1}{k^2 - a^2(\lambda)}
\]

\[
\tilde{D}_{\tilde{X}, \tilde{X}} = \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda \frac{\kappa}{2\pi} \frac{k^2 a^2(\lambda)}{k^2 - a^2(\lambda)}
\]

\[
\tilde{D}_{\tilde{X}, \tilde{\sigma}}(k) = \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda \frac{\kappa}{2\pi} \frac{a^2(\lambda)}{k^2 - a^2(\lambda)}
\]

have no one-particle poles, and in this sense in our model there are no fundamental spin zero particles. The spin zero excitations in intermediate states produce some collective effects, described by the propagators \( \text{eq}(31, 32, 33) \). For a general $a(\lambda)$ in the limit $|k| \gg \max[a]$ the $\tilde{\sigma}$-field propagator coincides with the Higgs meson propagator. If $a(\lambda)$ is a constant $a(\lambda) = a$, the $\tilde{\sigma}$ propagator coincides with the Higgs meson propagator $\tilde{D}_H = (k^2 - a^2)^{-1}$ for any $k$, and as in the discrete case the model reduces to the usual Higgs-Kibble model.
An explicit form of the $\tilde{\sigma}$ propagator depends on the functions $a(\lambda)$, which should be chosen to fit the experiment. For example, if
\[ a^2(\lambda) = M^2 - \lambda^2, \quad M^2 > \frac{\pi^2}{\kappa^2} \]
then
\[ \tilde{D}_\sigma(k) = -\frac{\kappa}{2\pi\sqrt{M^2 - k^2}} \ln\left(\frac{\sqrt{M^2 - k^2} - \pi/\kappa}{\sqrt{M^2 - k^2} + \pi/\kappa}\right) \]
Instead of one particle poles we have branch points describing a collective excitation. Another example was considered in [1]. If
\[ a^2(\lambda) = \lambda + \frac{\pi}{\kappa} + M^2, \quad \lambda < 0; \quad a^2(\lambda) = -\lambda + \frac{\pi}{\kappa} + M^2, \quad \lambda > 0 \]
then
\[ \tilde{D}_\sigma(k) = -\frac{\kappa}{\pi} \ln\left(1 - \frac{\pi}{\kappa(k^2 - M^2)}\right) \]
As before the one particle poles are replaced by the branch points, and for $k^2 \gg \frac{\pi^2}{\kappa^2}$
\[ \tilde{D}_\sigma(k) \simeq k^{-2} \]
Choosing properly the function $a(\lambda)$ one can model a different behaviour of the propagators $\tilde{D}_\sigma, \tilde{D}_{\tilde{\chi}}, \tilde{D}_{\tilde{\chi}, \tilde{\sigma}}$.

4 Application to electroweak interactions.

The $SU(2) \times U(1)$ models of electroweak interactions are readily generalized to include five dimensional scalar fields. The gauge meson - fermion sector may be chosen the same as in the models with the usual Higgs mesons. In the Weinberg-Salam model it is described by the Lagrangian
\[ L_{GF} = \frac{1}{8} \text{tr} \left[ F_{\mu\nu}F_{\mu\nu} \right] - \frac{1}{4} G_{\mu\nu}G_{\mu\nu} + \bar{L}\gamma_\mu(\partial_\mu + ig\frac{\tau^a}{2}A^a_\mu + ig_1\frac{1}{2}B_\mu) L + \bar{R}\gamma_\mu(\partial_\mu + ig_1B_\mu) R + \ldots \]
where $B_\mu$ is the Abelian gauge field, $G_{\mu\nu}$ is the corresponding stress tensor and \ldots stands for the similar terms including quark fields.

The spin zero field Lagrangian is
\[ L = |\partial_\mu \varphi + ig \frac{\tau^a}{2} A^a_\mu \varphi - \frac{ig_1}{2} B_\mu \varphi|^2 - G(\bar{L}\varphi R + \bar{R}\varphi L) + \ldots + \frac{g}{2m_1} \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda \partial_\mu X(x, \lambda) \partial_\mu (\varphi^+ \varphi) + \frac{1}{2} \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda \partial_\mu X(x, \lambda)^2 - \frac{\pi}{\kappa} \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda \partial_\mu X(x, \lambda) \partial_\mu X(x, \lambda) + \frac{\pi}{\kappa} \int_{-\pi/\kappa}^{\pi/\kappa} d\lambda a^{-2}(\lambda) \Box X(x, \lambda) \Box X(x, \lambda) \]

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where again \ldots stand for the terms with quark fields, and the field $\varphi$ is parameterized as before

$$\varphi_1 = \frac{(iB_1 + B_2)}{\sqrt{2}}; \quad \varphi_2 = \frac{\sqrt{2}m_1}{g} + \frac{\sigma - iB_3}{\sqrt{2}}$$ (41)

All the discussion given above is applied to this model. It is gauge invariant, renormalizable and has no one particle excitations with spin zero. The interaction of the gauge singlet fields is fixed by the invariance with respect to the supersymmetry transformations (15).

Notice that the structure of fermion anomalies in our model remains the same as in the Weinberg-Salam model, so the compensation of lepton and quark anomalies works in the same way.

5 Conclusion.

In this paper we discussed the higher derivative formulation of the gauge invariant model of the massive Yang-Mills field, which is unitary in the positive norm space and at the same time does not produce one particle poles in the channels corresponding to the exchange by the spin zero particles, and in this sense does not require the existence of fundamental scalar mesons. A discretized version describes a gauge invariant model of a massive vector field with spontaneously broken symmetry and several neutral scalar mesons. In our model the form of the spin zero field interaction is fixed by the symmetries imposed on the theory.

This mechanism may be applied to the electroweak model of the Salam-Weinberg type to modify the predictions concerning the spin zero particles. The same mechanism may be used in electroweak models including several fermion generations and in the models with neutrino oscillations.

For discussion of some experimental consequences of the models with additional gauge singlet fields see [15], citeEsGu, [17] and other papers cited in the introduction.

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