Gravity, Nonlinear Gauge Fields and Charge Confinement/Deconfinement *

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Abstract

We discuss in some detail the properties of gravity (including $f(R)$-gravity) coupled to non-standard nonlinear gauge field system containing a square root of the usual Maxwell Lagrangian $-\frac{f_0}{2} \sqrt{-F^2}$. The latter is known to produce in flat spacetime a QCD-like confinement. Inclusion of gravity triggers various physically interesting effects: new mechanism for dynamical generation of cosmological constant; non-standard black hole solutions with constant vacuum electric field and with “hedge-hog”-type spacetime asymptotics, which are shown to obey the first law of black hole thermodynamics; new “tubelike” solutions of Levi-Civita-Bertotti-Robinson type; charge-“hiding” and charge-confining “thin-shell” wormhole solutions; dynamical effective gauge couplings and confinement-deconfinement transition effect when coupled to quadratic $R^2$-gravity.

1. Introduction

We consider gravity, including $f(R)$-gravity [1], coupled to non-standard nonlinear gauge field system containing a square root of the ordinary Maxwell Lagrangian $-\frac{f_0}{2} \sqrt{-F^2}$. In flat spacetime the latter model has been shown [2] to produce a QCD-like confinement.

We exhibit several interesting features of the above system (see also Refs.[3, 4]) :

- New mechanism for dynamical generation of cosmological constant due to nonlinear gauge field dynamics: $\Lambda_{\text{eff}} = \Lambda_0 + 2\pi f_0^2$ ($\Lambda_0$ bare cosmological constant, may be absent at all).

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Non-standard black hole solutions of Reissner-Nordström-(anti-)de-Sitter type containing a constant radial vacuum electric field (in addition to the Coulomb one), in particular, in electrically neutral black holes of Schwarzschild-(anti-)de-Sitter type. It is shown that these non-standard black holes obey the first law of black hole thermodynamics.

In case of vanishing effective cosmological constant $\Lambda_{\text{eff}}$ (i.e., $\Lambda_0 < 0$, $|\Lambda_0| = 2\pi f_0^2$) the resulting Reissner-Nordström-type black hole, apart from carrying an additional constant vacuum electric field, turns out to be non-asymptotically flat—a feature resembling the gravitational effect of a hedgehog [6].

Appearance of confining-type effective potential in charged test particle dynamics in the above black hole backgrounds.

New “tubelike” solutions of Levi-Civita-Bertotti-Robinson type, i.e., with spacetime geometry of the form $\mathcal{M}_2 \times S^2$, where $\mathcal{M}_2$ is a two-dimensional anti-de Sitter, Rindler or de Sitter space depending on the relative strength of the electric field w.r.t. the coupling $f_0$ of the square-root gauge field term.

When in addition one or more lightlike branes are self-consistently coupled to the above gravity/nonlinear-gauge-field system (as matter and charge sources) they produce (“thin-shell”) wormhole solutions displaying two novel physically interesting effects [4]:

- “Charge-hiding” effect - a genuinely charged matter source of gravity and electromagnetism may appear electrically neutral to an external observer—a phenomenon opposite to the famous Misner-Wheeler “charge without charge” effect [5];
- Charge-confining “tubelike” wormhole with two “throats” occupied by two oppositely charged lightlike branes—the whole electric flux is confined within the finite-extent “middle universe” of generalized Levi-Civita-Bertotti-Robinson type—no flux is escaping into the outer non-compact “universes”.

Additional interesting features appear when we couple the “square-root” confining nonlinear gauge field system to $f(R)$-gravity with $f(R) = R + \alpha R^2$ and a dilaton. Reformulating the model in the physical “Einstein” frame we find (cf. second Ref.[3]):

- “Confinement-deconfinement” transition due to appearance of “flat” region in the effective dilaton potential;
- The effective gauge couplings as well as the induced cosmological constant become dynamical depending on the dilaton v.e.v. In particular, a conventional Maxwell kinetic term for the gauge field is dynamically generated even if absent in the original theory;
- Regular black hole solution (no singularity at $r = 0$) with confining vacuum electric field: the bulk spacetime consist of two regions—an interior de Sitter and an exterior Reissner-Nordström-type (with “hedgehog asymptotics”) glued together along their common horizon.
occupied by a charged lightlike brane. The latter also dynamically determines the non-zero cosmological constant in the interior de-Sitter region. This result is analogous to the regular black hole solution in the case of ordinary Einstein gravity presented in Ref.[8] and will be discussed in more detail in a subsequent paper.

Concluding the introductory remarks, let us briefly mention the principal motivation for studying non-standard gauge field models with $\sqrt{-F^2}$. G. ’t Hooft has shown [9] that in any effective quantum gauge theory, which is able to describe linear confinement phenomena, the energy density of electrostatic field configurations should be a linear function of the electric displacement field in the infrared region (the latter appearing as an “infrared counterterm”).

The simplest way to realize these ideas in flat spacetime was proposed in Refs.[2]:

$$ S = \int d^4 x L(F^2) \quad , \quad L(F^2) = - \frac{1}{4} F^2 - \frac{f_0}{2} \sqrt{-F^2} , $$

$$ F^2 \equiv F_{\mu\nu} F^{\mu\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , $$

The square root of the Maxwell term naturally arises as a result of spontaneous breakdown of scale symmetry of the original scale-invariant Maxwell action with $f_0$ appearing as an integration constant responsible for the latter spontaneous breakdown. For static field configurations the model (1) yields an electric displacement field $\vec{D} = \vec{E} - \frac{f_0}{\sqrt{2}|E|}$ and the corresponding energy density turns out to be

$$ \frac{1}{2} \vec{E}^2 = \frac{1}{2} |\vec{D}|^2 + \frac{f_0^2}{4 |\vec{E}|} + \frac{1}{4} \frac{f_0^2}{|\vec{E}|} , $$

so that it indeed contains a term linear w.r.t. $|\vec{D}|$. The model (1) produces, when coupled to quantized fermions, a confining effective potential $V(r) = -\frac{\alpha}{r} + \gamma r$ (Coulomb plus linear one with $\gamma \sim f_0$) which is of the form of the well-known “Cornell” potential in the phenomenological description of quarkonium systems in QCD [10].

2. Einstein Gravity Coupled to Confining Nonlinear Gauge Field

The pertinent action is given by ($R$-scalar curvature; $\Lambda_0$ - bare cosmological constant, might be absent):

$$ S = \int d^4 x \sqrt{-G} \left[ \frac{R - 2 \Lambda_0}{16 \pi} + L(F^2) \right] \quad , \quad L(F^2) = - \frac{1}{4} F^2 - \frac{f_0}{2} \sqrt{-F^2} , $$

$$ F^2 \equiv F_{\kappa\lambda} F_{\mu\nu} G^{\kappa\mu} G^{\lambda\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . $$

Remark. One could start with the non-Abelian version of the gauge field action in (2). Since we will be interested in static spherically symmetric solutions, the non-Abelian gauge theory effectively reduces to an Abelian one.
The corresponding equations of motion read accordingly – Einstein equations:

\[ R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R + \Lambda_0 G_{\mu\nu} = 8\pi T^{(F)}_{\mu\nu}, \]  

\[ T^{(F)}_{\mu\nu} = \left( 1 - \frac{f_0}{\sqrt{-F^2}} \right) F_{\mu\kappa} F_{\nu\lambda} G^{\kappa\lambda} - \frac{1}{4} (F^2 + 2 f_0 \sqrt{-F^2}) G_{\mu\nu}, \]  

and nonlinear gauge field equations:

\[ \partial_\nu \left( \sqrt{-G} \left( 1 - \frac{f_0}{\sqrt{-F^2}} \right) F_{\kappa\lambda} G^{\mu\kappa} G^{\nu\lambda} \right) = 0. \]  

**Important remark.** Note the non-zero value of the trace of energy-momentum tensor unlike ordinary Maxwell theory:

\[ T(F)_{\mu\nu} = \left( 1 - f_0 \sqrt{-F^2} \right) F_{\mu\kappa} F_{\nu\lambda} G^{\kappa\lambda} - \frac{1}{4} (F^2 + 2 f_0 \sqrt{-F^2}) G_{\mu\nu}. \]  

Solving Eqs. (3)–(5) we find new non-standard Reissner-Nordström-(anti-)de-Sitter-type black holes depending on the sign of a dynamically generated cosmological constant \( \Lambda_{\text{eff}} \):

\[ ds^2 = -A(\eta) dt^2 + \frac{d\eta^2}{A(\eta)} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \]  

\[ A(\eta) = 1 - \sqrt{8\pi} |Q| f_0 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\text{eff}}}{3} r^2, \quad \Lambda_{\text{eff}} = 2\pi f_0^2 + \Lambda_0, \]  

with static spherically symmetric electric field containing apart from the Coulomb term an additional constant “vacuum” piece:

\[ F_{0\eta} = \frac{\varepsilon_F f_0}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi} r^2}, \quad \varepsilon_F \equiv \text{sign}(F_{0\eta}) = \text{sign}(Q). \]  

The latter corresponds to a confining “Cornell”-type [10] potential \( A_0 = -\frac{\varepsilon_F f_0}{\sqrt{2}} r + \frac{Q}{\sqrt{4\pi} r} \). When \( \Lambda_{\text{eff}} = 0 \), \( A(\eta) \to 1 - \sqrt{8\pi} |Q| f_0 \) for \( r \to \infty \), i.e., the black hole exhibits “hedgehog” [6] non-flat-spacetime asymptotics.

Furthermore, we find three distinct types of static solutions of “tube-like” Levi-Civita-Bertotti-Robinson [7] type with spacetime geometry of the form \( M_2 \times S^2 \), where \( M_2 \) is some 2-dimensional manifold ((anti-)de Sitter (A)dS2, Rindler Rind2):

\[ ds^2 = -A(\eta) dt^2 + \frac{d\eta^2}{A(\eta)} + r_0^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad -\infty < \eta < \infty, \]  

\[ F_{0\eta} = c_F = \text{const}, \quad \frac{1}{r_0^2} = 4\pi c_F^2 + \Lambda_0 (= \text{const}). \]
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(i) $AdS_2 \times S^2$ with constant vacuum electric field $|F_{0\eta}| \equiv |E| = |c_F|:

$$A(\eta) = 4\pi \left( c_F^2 - \sqrt{2} f_0 |c_F| - \frac{\Lambda_0}{4\pi} \right) \eta^2 \quad (\eta - \text{Poincare patch coordinate}),$$

provided either $|c_F| > \frac{f_0}{\sqrt{2}} \left( 1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0}} \right)$ for $\Lambda_0 \geq -2\pi f_0^2$ or $|c_F| > \sqrt{\frac{1}{4\pi}|\Lambda_0|}$ for $\Lambda_0 < 0$, $|\Lambda_0| > 2\pi f_0^2$.

(ii) $Rind_2 \times S^2$ with constant vacuum electric field $|F_{0\eta}| = |c_F|$, where $Rind_2$ is the flat 2-dimensional Rindler spacetime with:

$$A(\eta) = \eta \quad \text{for} \quad 0 < \eta < \infty \quad \text{or} \quad A(\eta) = -\eta \quad \text{for} \quad -\infty < \eta < 0 \quad (12)$$

provided $|c_F| = \frac{f_0}{\sqrt{2}} \left( 1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0}} \right)$ for $\Lambda_0 > -2\pi f_0^2$.

(iii) $dS_2 \times S^2$ with weak const vacuum electric field $|F_{0\eta}| = |c_F|$, where $dS_2$ is the 2-dimensional de Sitter space with:

$$A(\eta) = 1 - 4\pi \left( \sqrt{2} f_0 |c_F| - c_F^2 + \frac{\Lambda_0}{4\pi} \right) \eta^2 , \quad (13)$$

when $|c_F| < \frac{f_0}{\sqrt{2}} \left( 1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0}} \right)$ for $\Lambda_0 > -2\pi f_0^2$. Note that $dS_2$ has two horizons at $\eta = \pm \eta_0 \equiv \pm \left[ 4\pi \left( \sqrt{2} f_0 |c_F| - c_F^2 \right) + \Lambda_0 \right]^{-\frac{1}{4}}$.

3. Bulk Gravity/Nonlinear Gauge Field Coupled to Light-like Brane Sources

In the following two Sections we will consider bulk Einstein/non-linear gauge field system (2) self-consistently coupled to $N \geq 1$ (distantly separated) charged codimension-one lightlike $p$-brane ($LL$-brane) sources (here $p = 2$).

World-volume $LL$-brane actions in a reparametrization-invariant Nambu-Goto-type or an equivalent Polyakov-type formulation were proposed in Refs.[11]:

$$S_{LL}[q] = -\frac{1}{2} \int d^{p+1}\sigma \ T_{b_0}^{\frac{p-1}{2}} \sqrt{-\gamma} \left[ \gamma^{ab} \bar{g}_{ab} - b_0(p-1) \right] \quad (14)$$

$$\bar{g}_{ab} \equiv \partial_a X^\mu G_{\mu\nu} \partial_b X^\nu - \frac{1}{F^2} (\partial_a u + qA_a)(\partial_b u + qA_b) \quad , \quad A_a \equiv \partial_a X^\mu A_\mu \quad (15)$$

Here and below the following notations are used:

- $\gamma_{ab}$ is the intrinsic world-volume Riemannian metric;
- $\bar{g}_{ab} = \partial_a X^\mu G_{\mu\nu} \partial_b X^\nu$ is the induced metric on the world-volume, which becomes singular on-shell (manifestation of the lightlike nature);
- $b_0$ is world-volume “cosmological constant”.


• $X^\mu(\sigma)$ are the $p$-brane embedding coordinates in the bulk $D$-dimensional spacetime with Riemannian metric $G_{\mu\nu}(x)$ ($\mu, \nu = 0, 1, \ldots, D - 1$);
  
  $\sigma \equiv (\sigma^0 \equiv \tau, \sigma^i)$ with $i = 1, \ldots, p$; $\partial_a \equiv \partial_{\sigma^a}$.

• $u$ is auxiliary world-volume scalar field defining the lightlike direction of the induced metric;

• $T$ is dynamical (variable) brane tension;

• $q$ – the coupling to bulk spacetime gauge field $A_\mu$ is LL-brane surface charge density.

The on-shell singularity of the induced metric $g_{ab}$, i.e., the lightlike property, directly follows from the LL-brane equations of motion:

$$g_{ab} \left( g^{bc}(\partial_c u + qA_c) \right) = 0 . \quad (16)$$

Now, let us consider the full action of self-consistently coupled bulk Einstein/non-linear gauge field/LL-brane system ($L(F^2) = -\frac{1}{4}F^2 - \frac{F^2}{2}\sqrt{F^2}$):

$$S = \int d^4x\sqrt{-G} \left[ \frac{R(G) - 2\Lambda_0}{16\pi} + L(F^2) \right] + \sum_{k=1}^{N} S_{LL}[q^{(k)}] , \quad (17)$$

where the superscript $(k)$ indicates the $k$-th LL-brane.

The corresponding equations of motion are as follows:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R + \Lambda_0 G_{\mu\nu} = 8\pi \left[ T^{(F)}_{\mu\nu} + \sum_{k=1}^{N} T^{(k)}_{\mu\nu} \right] , \quad (18)$$

$$\partial_\nu \left[ \sqrt{-G} \left( 1 - \frac{f_0}{\sqrt{-F^2}} \right) F_{\kappa\lambda} G^{\mu\nu} G^{\kappa\lambda} \right] + \sum_{k=1}^{N} J^\mu_{(k)} = 0 . \quad (19)$$

The energy-momentum tensor and the charge current density of $k$-th LL-brane are straightforwardly derived from the pertinent LL-brane world-volume action (14):

$$T^{\mu\nu}_{(k)} = - \int d^3\sigma \frac{\delta^{(4)}(x - X_k(\sigma))}{\sqrt{-G}} T^{(k)}(\bar{g}_{(k)} \bar{g}^{ab} X_{(k)}^\mu \partial_b X_{(k)}^\nu) , \quad (20)$$

$$J^\mu_{(k)} = -q^{(k)} \int d^3\sigma \delta^{(4)}(x - X_k(\sigma)) \sqrt{[\bar{g}_{(k)} \bar{g}^{ab} X_{(k)}^\mu \partial_b X_{(k)}^\nu]} \frac{\partial_\nu u^{(k)} + q^{(k)} A_{(k)}^b}{T^{(k)}} . \quad (21)$$

Solving Eqs.(18)–(19) with (20)–(21) we find “thin-shell” wormhole solutions of static “spherically-symmetric” type (in Eddington-Finkelstein coordinates $dt = dv - \frac{dv}{A(\eta)}$, $F_{0\eta} = F_{\eta v}$):

$$ds^2 = -A(\eta)dv^2 + 2dv d\eta + C(\eta) h_{ij}(\theta)d\theta^i d\theta^j , \quad F_{\eta v} = F_{v\eta}(\eta) , \quad (22)$$

$$-\infty < \eta < \infty , \quad A(\eta_{(1)}^0) = 0 \text{ for } \eta_{(2)}^0 < \ldots < \eta_{(N)}^0 . \quad (23)$$
The derivation of these “thin-shell” wormhole solutions proceeds along the following main steps:

(i) Take “vacuum” solutions of (18)–(19) (without delta-function LL-brane terms) in each spacetime region (separate “universe”) given by $(-\infty < \eta < \eta_0^{(1)}), \ldots , \eta_0^{(N)} < \eta < \infty$ with common horizon(s) at $\eta = \eta_0^{(k)}$ ($k = 1, \ldots , N$).

(ii) Each $k$-th LL-brane automatically locates itself on the horizon at $\eta = \eta_0^{(k)}$ – intrinsic property of LL-brane dynamics [11].

(iii) Match discontinuities of the derivatives of the metric and the gauge field strength across each horizon at $\eta = \eta_0^{(k)}$ using the explicit expressions for the LL-brane stress-energy tensor and charge current density (20)–(21).

4. Charge “Hiding” and Charge Confining Wormholes

First we will construct “one-throat” wormhole solutions to (17) with the charged LL-brane occupying the wormhole “throat”, which connects (i) a non-compact “universe” with Reissner-Nordström-(anti)-de-Sitter-type geometry (where the cosmological constant is partially or entirely dynamically generated) to (ii) a compactified (“tubelike”) “universe” of (generalized) Levi-Civita-Bertotti-Robinson type with geometry $AdS_2 \times S^2$ or $Rind_2 \times S^2$.

These wormholes possess the novel property of hiding electric charge from external observer in the non-compact “universe”. Namely, the whole electric flux produced by the charged LL-brane at the wormhole “throat” is pushed into the “tubelike” “universe”. As a result, the non-compact “universe” becomes electrically neutral with Schwarzschild-(anti-)de-Sitter or purely Schwarzschild geometry. Therefore, an external observer in the non-compact “universe” detects a genuinely charged matter source (the charged LL-brane) as electrically neutral.

The explicit form $ds^2 = -A(\eta)dv^2 + 2dvd\eta + C(\eta)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$ for the metric and the nonlinear gauge theory’s electric field $F_{\nu\eta}(\eta)$ read:

- “Left universe” of Levi-Civita-Bertotti-Robinson (“tubelike”) type with geometry $AdS_2 \times S^2$ for $\eta < 0$:
  
  $A(\eta) = 4\pi \left(c_F^2 - \sqrt{2}f_0|c_F| - \frac{\Lambda_0}{4\pi}\right)\eta^2$, $C(\eta) \equiv r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda_0}$ (24)
  
  $|F_{\nu\eta}| \equiv |\vec{E}| = |c_F| > \frac{f}{\sqrt{2}}\left(1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0^2}}\right)$ for $\Lambda_0 > -2\pi f_0^2$,

  or  $|F_{\nu\eta}| \equiv |\vec{E}| = |c_F| > \sqrt{\frac{1}{4\pi}(|\Lambda_0|}$ for $\Lambda_0 < 0$, $|\Lambda_0| > 2\pi f_0^2$.

- Non-compact “right universe” for $\eta > 0$ comprising the exterior region of Reissner-Nordström-de-Sitter-type black hole beyond the middle (Schwarzschild-type) horizon $r_0$ when $\Lambda_0 > -2\pi f_0^2$ (in particu-
lar, when \( \Lambda_0 = 0 \), or the exterior region of Reissner-Nordström-anti-de-Sitter-type black hole beyond the outer (Schwarzschild-type) horizon \( r_0 \) in the case \( \Lambda_0 < 0 \) and \( |\Lambda_0| > 2\pi f_0^2 \), or the exterior region of Reissner-Nordström-“hedgehog” black hole for \( |\Lambda_0| = 2\pi f_0^2 \) (note: \( A(\eta) \equiv A_{\text{RN-}\text{-}(\text{AdS})}(r_0 + \eta) \)):

\[
A(\eta) = 1 - \sqrt{8\pi Q f_0} \frac{2m}{r_0 + \eta} + \frac{Q^2}{(r_0 + \eta)^2} - \frac{\Lambda_0 + 2\pi f_0^2}{3}(r_0 + \eta)^2,
\]

\[
C(\eta) = (r_0 + \eta)^2, \quad |F_{\nu\eta}| \equiv |\vec{E}| = \frac{f_0}{\sqrt{2}} + \frac{|Q|}{\sqrt{4\pi (r_0 + \eta)^2}}.
\]

The matching relations for the discontinuities of the metric and gauge field components across the LL-brane world-volume occupying the wormhole “throat” (which are here derived self-consistently from a well-defined world-volume Lagrangian action principle for the LL-brane) (14) determine all parameters of the wormhole solutions as functions of \( q \) (the LL-brane charge) and \( f_0 \) (coupling constant of \( \sqrt{-F^2} \)):

\[
Q = 0 \quad , \quad |c_F| = |q| + \frac{f_0}{\sqrt{2}}, \quad (26)
\]
as well as the allowed range for the “bare” cosmological constant:

\[
-4\pi \left( |q| + \frac{f_0}{\sqrt{2}} \right)^2 < \Lambda_0 < 4\pi \left( q^2 - \frac{f_0^2}{2} \right), \quad (27)
\]
The relations (26) (recall \( |F_{\nu\eta}| \equiv |\vec{E}| = |c_F| \) in the “tubelike” “left universe”) have profound consequences:

(A) The non-compact “right universe” (25) becomes exterior region of electrically neutral Schwarzschild-(anti-)de-Sitter or purely Schwarzschild black hole beyond the Schwarzschild horizon carrying a vacuum constant radial electric field \( |F_{\nu\eta}| \equiv |\vec{E}| = \frac{f_0}{\sqrt{2}} \).

(B) Recalling that the dielectric displacement field is \( \vec{D} = \left( 1 - \frac{f_0}{\sqrt{2}|\vec{E}|} \right) \vec{E} \), we find from the second relation (26) that the whole flux produced by the charged LL-brane flows only into the “tubelike” “left universe” (24) (since \( \vec{D} = 0 \) in the non-compact “right universe”). This is a novel property of hiding electric charge through a wormhole connecting non-compact to a “tubelike” universe from external observer in the non-compact “universe”.

The charge-“hiding” wormhole geometry is visualized on Fig.1 below.

Further, we find more interesting “two-throat” wormhole solution exhibiting QCD-like charge confinement effect – obtained from a self-consistent...
coupling of the gravity/nonlinear-gauge-field system with two identical oppositely charged LL-branes (Eq.(17) with \( N = 2 \)). The total “two-throat” wormhole spacetime manifold is made of:

(i) “Left-most” non-compact “universe” comprising the exterior region of Reissner-Nordström-de-Sitter-type black hole beyond the middle Schwarzschild-type horizon \( r_0 \) for the “radial-like” \( \eta \)-coordinate interval:

\[
-\infty < \eta < -\eta_0 \equiv -\left[ 4\pi \left( \sqrt{2}f_0 |c_F| - c_F^2 \right) + \Lambda_0 \right]^{-\frac{1}{2}},
\]

where:

\[
A(\eta) = A_{RN\text{dS}}(r_0 - \eta_0 - \eta) = 1 - \sqrt{8\pi |Q| f_0} - \frac{2m}{r_0 - \eta_0 - \eta} + \frac{Q^2}{(r_0 - \eta_0 - \eta)^2} - \frac{\Lambda_0 + 2\pi f_0^2}{3} (r_0 - \eta_0 - \eta)^2,
\]

\[
C(\eta) = (r_0 - \eta_0 - \eta)^2 , \quad |F_{\eta \eta}(\eta)| \equiv |\vec{E}| = \frac{f_0}{\sqrt{2}} + \frac{|Q|}{\sqrt{4\pi} (r_0 - \eta_0 - \eta)^2}.
\]

(ii) “Middle” “tube-like” “universe” of Levi-Civita-Bertotti-Robinson type with geometry \( dS_2 \times S^2 \) comprising the finite extent (w.r.t. \( \eta \)-coordinate)
region between the two horizons of $dS_2$ at $\eta = \pm \eta_0$:

$$-\eta_0 < \eta < \eta_0 \equiv \left[4\pi \left(\sqrt{2} f_0 |c_F| - c_F^2\right) + \Lambda_0\right]^{-\frac{1}{2}}, \quad (30)$$

where the metric coefficients and electric field are:

$$A(\eta) = 1 - \left[4\pi \left(\sqrt{2} f_0 |c_F| - c_F^2\right) + \Lambda_0\right] \eta^2, \quad A(\pm \eta_0) = 0, \quad (31)$$

$$C(\eta) = r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda_0}, \quad |F_{\nu\eta}| \equiv |\vec{E}| = \left|c_F\right| < \frac{f}{\sqrt{2}} \left(1 + \sqrt{1 + \frac{\Lambda}{2\pi f_0^2}}\right),$$

with $\Lambda_0 > -2\pi f_0^2$;

(iii) “Right-most” non-compact “universe” comprising the exterior region of Reissner-Nordström-de-Sitter-type black hole beyond the middle Schwarzschild-type horizon $r_0$ for the “radial-like” $\eta$-coordinate interval $\eta_0 < \eta < \infty$ ($\eta_0$ as in (30)), where:

$$A(\eta) = A_{\text{RN}}(r_0 + \eta - \eta_0) = 1 - \sqrt{8\pi Q} f_0 - \frac{2m}{r_0 + \eta - \eta_0} + \frac{Q^2}{(r_0 + \eta - \eta_0)^2} - \frac{\Lambda_0 + 2\pi f_0^2}{3} (r_0 + \eta - \eta_0)^2, \quad (32)$$

$$C(\eta) = (r_0 + \eta - \eta_0)^2, \quad |F_{\nu\eta}(\eta)| \equiv |\vec{E}| = \frac{f_0}{\sqrt{2}} + \frac{|Q|}{\sqrt{4\pi (r_0 + \eta - \eta_0)^2}}.$$ 

As dictated by the $LL$-brane dynamics [11] each of the two $LL$-branes locates itself on one of the two common horizons at $\eta = \pm \eta_0$ between “left” and “middle”, and between “middle” and “right” “universes”, respectively.

The matching relations for the discontinuities of the metric and gauge field components across the each of the two $LL$-brane world-volumes determine all parameters of the wormhole solutions as functions of $\pm q$ (the opposite $LL$-brane charges) and $f_0$ (coupling constant of $\sqrt{-F^2}$). Most importantly we obtain:

$$Q = 0, \quad |c_F| = |q| + \frac{f_0}{\sqrt{2}}, \quad (33)$$

and the bare cosmological constant must be in the interval:

$$\Lambda_0 \leq 0, \quad |\Lambda_0| < 2\pi (f_0^2 - 2q^2) \quad \rightarrow \quad |q| < \frac{f_0}{\sqrt{2}}, \quad (34)$$

in particular, $\Lambda_0$ could be zero.

Similarly to the charge-“hiding” case, relations (33) meaning:

$$|\vec{E}|_{\text{middle universe}} = |q| + |\vec{E}|_{\text{left/right universe}},$$

have profound consequences:
The “left-most” (29) and “right-most” (32) non-compact “universes” become two identical copies of the electrically neutral exterior region of Schwarzschild-de-Sitter black hole beyond the Schwarzschild horizon. They both carry a constant vacuum radial electric field with magnitude $|\vec{E}| = f_0/\sqrt{2}$ pointing inbound towards the horizon in one of these “universes” and pointing outbound w.r.t. the horizon in the second “universe”. The corresponding electric displacement field $\vec{D} = 0$, so there is no electric flux there (recall $\vec{D} = \left(1 - \frac{f_0}{\sqrt{2}|\vec{E}|}\right)\vec{E}$).

The whole electric flux produced by the two charged $LL$-branes with opposite charges $\pm q$ at the boundaries of the above non-compact “universes” is confined within the “tube-like” middle “universe” (31) of Levi-Civita-Robinson-Bertotti type with geometry $dS_2 \times S^2$, where the constant electric field is $|\vec{E}| = f_0/\sqrt{2} + |q|$ with associated non-zero electric displacement field $|\vec{D}| = |q|$. This is QCD-like confinement.

A simple visualization of the charge-confining wormhole geometry is given in Fig.2.
5. \(R^2\)-Gravity Coupled to Confining Nonlinear Gauge Field and Dilaton

Consider now coupling of \(f(R) = R + \alpha R^2\) gravity (possibly with a bare cosmological constant \(\Lambda_0\)) to a “dilaton” \(\phi\) and the nonlinear gauge field system containing \(\sqrt{-F^2}\):

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi} \left( f(R(g, \Gamma)) - 2\Lambda_0 \right) + L(F^2(g)) + L_D(\phi, g) \right],
\]

(35)

\[
f(R(g, \Gamma)) = R(g, \Gamma) + \alpha R^2(g, \Gamma), \quad R(g, \Gamma) = R_{\mu\nu}(\Gamma)g^{\mu\nu},
\]

(36)

\[
L(F^2(g)) = -\frac{1}{4e^2} F^2(g) - \frac{f_0}{2}\sqrt{-F^2(g)},
\]

(37)

\[
F^2(g) \equiv F_{\kappa\lambda}F^{\mu\nu}g^{\kappa\mu}g^{\lambda\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,
\]

(38)

\[
L_D(\phi, g) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi).
\]

(39)

\(R_{\mu\nu}(\Gamma)\) is the Ricci curvature in the first order (Palatini) formalism, i.e., the spacetime metric \(g_{\mu\nu}\) and the affine connection \(\Gamma^{\nu}_{\mu\lambda}\) are a priori independent variables.

The equations of motion resulting from the action (35) read:

\[
R_{\mu\nu}(\Gamma) = \frac{1}{f_R'} \left[ 8\pi T_{\mu\nu} + \frac{1}{2} f(R(g, \Gamma))g_{\mu\nu} \right], \quad f_R' \equiv \frac{df(R)}{dR} = 1 + 2\alpha R(g, \Gamma),
\]

(40)

\[
\nabla_\lambda \left( \sqrt{-g} f_R' g^{\mu\nu} \right) = 0,
\]

(41)

\[
\partial_\nu \left( \sqrt{-g} \left[ \frac{1}{e^2} - \frac{f_0}{\sqrt{-F^2(g)}} \right] F_{\kappa\lambda}g^{\kappa\mu}g^{\nu\lambda} \right) = 0.
\]

(42)

The total energy-momentum tensor is given by:

\[
T_{\mu\nu} = \left[ L(F^2(g)) + L_D(\phi, g) - \frac{1}{8\pi} \Lambda_0 \right] g_{\mu\nu}
\]

\[
+ \left( \frac{1}{e^2} - \frac{f_0}{\sqrt{-F^2(g)}} \right) F_{\mu\lambda}F^{\nu\lambda} + \partial_\mu \phi \partial_\nu \phi.
\]

(43)

Eq.(41) leads to the relation \(\nabla_\lambda \left( f_R' g_{\mu\nu} \right) = 0\) and thus it implies transition to the “physical” Einstein-frame metrics \(h_{\mu\nu}\) via conformal rescaling of the original metric \(g_{\mu\nu}\) [12]:

\[
g_{\mu\nu} = \frac{1}{f_R'} h_{\mu\nu}, \quad \Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} h^{\mu\kappa} (\partial_\nu h_{\lambda\kappa} + \partial_\lambda h_{\nu\kappa} - \partial_\kappa h_{\nu\lambda}).
\]

(44)

Using (44) the \(R^2\)-gravity equations of motion (40) can be rewritten in the form of standard Einstein equations:

\[
R^\mu_\nu(h) = 8\pi \left( T_{\text{eff}}^\mu_\nu(h) - \frac{1}{2} \delta^\mu_\nu T_{\text{eff}}^\lambda(h) \right)
\]

(45)
with effective energy-momentum tensor of the following form:

\[ T_{\text{eff} \mu \nu}(h) = h_{\mu \nu} L_{\text{eff}}(h) - 2 \frac{\partial L_{\text{eff}}}{\partial h_{\mu \nu}}. \]  

(46)

The effective Einstein-frame matter Lagrangian reads (the dilaton kinetic term \( X(\phi, h) \equiv -\frac{1}{2} h^{\mu \nu} \partial_\mu \phi \partial_\nu \phi \) will be ignored in the sequel):

\[
L_{\text{eff}}(h) = -\frac{1}{4e_{\text{eff}}^2(\phi)} F^2(h) - \frac{1}{2} e_{\text{eff}}(\phi) \sqrt{-F^2(h)}
+ \frac{X(\phi, h)(1 + 16\pi \alpha X(\phi, h)) - V(\phi) - \Lambda_0/8\pi}{1 + 8\alpha (8\pi V(\phi) + \Lambda_0)} \]  

(47)

with the following dynamical \( \phi \)-dependent couplings:

\[
e_{\text{eff}}^2(\phi) = \frac{1}{e^2} + \frac{16\pi \alpha f_0^2}{1 + 8\alpha (8\pi V(\phi) + \Lambda_0)} , \]  

(48)

\[
f_{\text{eff}}(\phi) = f_0 \frac{1 + 32\pi \alpha X(\phi, h)}{1 + 8\alpha (8\pi V(\phi) + \Lambda_0)} . \]  

(49)

Thus, all equations of motion of the original \( R^2 \)-gravity system (35)–(39) can be equivalently derived from the following Einstein/nonlinear-gauge-field/dilaton action:

\[
S_{\text{eff}} = \int d^4x \sqrt{-h} \left[ \frac{R(h)}{16\pi} + L_{\text{eff}}(h) \right] , \]  

(50)

where \( R(h) \) is the standard Ricci scalar of the metric \( h_{\mu \nu} \) and \( L_{\text{eff}}(h) \) is as in (47).

**Important observation.** Even if ordinary kinetic Maxwell term \( -\frac{1}{4} F^2 \) is absent in the original system (\( e^2 \to \infty \) in (37)), such term is nevertheless dynamically generated in the Einstein-frame action (47)–(50), which is a combined effect of \( \alpha R^2 \) and \( -\frac{f_0}{2} \sqrt{-F^2} \):

\[
S_{\text{maxwell}} = -4\pi \alpha f_0^2 \int d^4x \sqrt{-h} \frac{F_{\kappa \lambda} F_{\mu \nu} \eta^{\kappa \mu} \eta^{\lambda \nu}}{1 + 8\alpha (8\pi V(\phi) + \Lambda_0)} . \]  

(51)

In what follows we consider constant “dilaton” \( \phi \) extremizing the effective Lagrangian (47):

\[
L_{\text{eff}} = -\frac{1}{4e_{\text{eff}}^2(\phi)} F^2(h) - \frac{1}{2} e_{\text{eff}}(\phi) \sqrt{-F^2(h)} - V_{\text{eff}}(\phi) \]  

\[
V_{\text{eff}}(\phi) = \frac{V(\phi) + \frac{\Lambda_0}{8\pi}}{1 + 8\alpha (8\pi V(\phi) + \Lambda_0)} , \quad e_{\text{eff}}(\phi) = \frac{f_0}{1 + 8\alpha (8\pi V(\phi) + \Lambda_0)} \]  

(53)

\[
e_{\text{eff}}^2(\phi) = \frac{1}{e^2} + \frac{16\pi \alpha f_0^2}{1 + 8\alpha (8\pi V(\phi) + \Lambda_0)} . \]  

(54)
Important observation. The dynamical couplings and effective potential are extremized \textit{simultaneously}—this is an explicit realization of “least coupling principle” of Damour-Polyakov [13]:

\[
\frac{\partial f_{\text{eff}}}{\partial \phi} = -64\pi\alpha f_0 \frac{\partial V_{\text{eff}}}{\partial \phi}, \quad \frac{\partial}{\partial \phi} \frac{1}{e_{\text{eff}}^2} = -(32\pi\alpha f_0)^2 \frac{\partial V_{\text{eff}}}{\partial \phi} \rightarrow \frac{\partial L_{\text{eff}}}{\partial \phi} \sim \frac{\partial V_{\text{eff}}}{\partial \phi} .
\]

Therefore at the extremum of \( L_{\text{eff}} \) (52) \( \phi \) must satisfy:

\[
\frac{\partial V_{\text{eff}}}{\partial \phi} = \frac{V'(\phi)}{[1 + 8\alpha (\kappa^2 V(\phi) + \Lambda_0)]^2} = 0 .
\]

(56)

There are two generic cases:

(a) \textit{Confining phase}: Eq.(56) is satisfied for some finite-value \( \phi_0 \) extremizing the original potential \( V(\phi) \): \( V'(\phi_0) = 0 \).

(b) \textit{Deconfinement phase}: For polynomial or exponentially growing original \( V(\phi) \), so that \( V(\phi) \rightarrow \infty \) when \( \phi \rightarrow \infty \), we have:

\[
\frac{\partial V_{\text{eff}}}{\partial \phi} \rightarrow 0 , \quad V_{\text{eff}}(\phi) \rightarrow \frac{1}{64\pi\alpha} = \text{const} \quad \text{when} \quad \phi \rightarrow \infty ,
\]

(57)

i.e., for sufficiently large values of \( \phi \) we find a “flat region” in \( V_{\text{eff}} \). This “flat region” triggers a \textit{transition from confining to deconfinement dynamics}.

Namely, in the “flat-region” case \( V(\phi) \rightarrow \infty \) we have from (53)–(54):

\[
f_{\text{eff}} \rightarrow 0 , \quad e_{\text{eff}}^2 \rightarrow e^2
\]

and the effective gauge field Lagrangian (52) reduces to the ordinary \textit{non-confining} one (the “square-root” term \( \sqrt{-F^2} \) vanishes):

\[
L_{\text{eff}}^{(0)} = -\frac{1}{4e^2}F^2(h) - \frac{1}{64\pi\alpha}
\]

(59)

with an \textit{induced} cosmological constant \( \Lambda_{\text{eff}} = 1/8\alpha \), which is \textit{completely independent} of the bare cosmological constant \( \Lambda_0 \).

Within the physical “Einstein”-frame in the confining phase \( V'(\phi_0) = 0 \), \( \phi_0 = \text{finite} \) we find:

(A) Reissner-Nordström-(anti-)de-Sitter type black holes, in particular, non-standard Reissner-Nordström type with non-flat “hedgehog” asymptotics, generalizing solutions (6)–(8) in the ordinary Einstein-gravity case, where now the effective cosmological constant and the vacuum constant radial electric field read:

\[
\Lambda_{\text{eff}}(\phi_0) = \frac{\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2}{1 + 8\alpha (\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2)^2},
\]

(60)

\[
|\vec{E}_{\text{vac}}| = \left( \frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha (8\pi V(\phi_0) + \Lambda_0)} \right)^{-1} \frac{f_0/\sqrt{2}}{1 + 8\alpha (8\pi V(\phi_0) + \Lambda_0)}.
\]

(61)
(B) Levi-Civita-Bertotti-Robinson type “tubelike” spacetimes with geometries $AdS_2 \times S^2$, $Rind_2 \times S^2$ and $dS_2 \times S^2$ generalizing (9)–(13), where now (using short-hand notation $\Lambda(\phi_0) \equiv 8\pi V(\phi_0) + \Lambda_0$):

$$\frac{1}{r_0^2} = \frac{4\pi}{1 + 8\alpha \Lambda(\phi_0)} \left[ \left( 1 + 8\alpha \left( \Lambda(\phi_0) + 2\pi f_0^2 \right) \right) E^2 + \frac{1}{4\pi} \Lambda(\phi_0) \right]. \quad (62)$$

6. Discussion

Inclusion of the non-standard nonlinear “square-root” gauge field term provides explicit realization of the old “classic” idea of ’t Hooft [9] about the nature of low-energy confinement dynamics. Coupling of nonlinear gauge theory containing $\sqrt{-F^2}$ to gravity (Einstein or $f(R) = R + \alpha R^2$ plus scalar “dilaton”) leads to a variety of remarkable effects:

- Dynamical effective gauge couplings and dynamical induced cosmological constant;
- New non-standard black hole solutions of Reissner-Nordström-(anti-)de-Sitter type carrying an additional constant vacuum electric field, in particular, non-standard Reissner-Nordström type black holes with asymptotically non-flat “hedgehog” [6] behavior;
- “Cornell”-type [10] confining potential in charged test particle dynamics;
- Coupling to a charged lightlike brane produces a charge-“hiding” wormhole, where a genuinely charged matter source is detected as electrically neutral by an external observer;
- Coupling to two oppositely charged lightlike brane sources produces a two-“throat” wormhole displaying a genuine QCD-like charge confinement.
- When coupled to $f(R) = R + \alpha R^2$ gravity plus scalar “dilaton”, the $\sqrt{-F^2}$ term triggers a transition from confining to deconfinement phase. Standard Maxwell kinetic term for the gauge field is dynamically generated even when absent in the original “bare” theory. The above are cumulative effects produced by the simultaneous presence of $\alpha R^2$ and $\sqrt{-F^2}$ terms.

Let us conclude with a brief remark concerning the thermodynamic properties of the non-standard black hole solutions described above. To this end, let us recall that for any static spherically symmetric metric of the form (6) with Schwarzschild-type horizon $r_0$, i.e., $A(r_0) = 0$, $\partial_r A|_{r_0} > 0$, the so called surface gravity $\kappa$ proportional to Hawking temperature $T_h$ (e.g. [14], Ch. 12.5) is given by $\kappa = 2\pi T_h = \frac{1}{2} \partial_r A|_{r_0}$. With $A(r)$ of the general form $A(r) = 1 - c(Q_i) - 2m/r + A_1(r; Q_i)$, where $Q_i$ are the rest of the black hole parameters apart from the mass $m$, and $c(Q_i)$ is generically a non-zero constant as in (7) (responsible for the “hedgehog” non-flat spacetime asymptotics), one can straightforwardly derive the first
law of black hole thermodynamics for the above class of solutions:

$$\delta m = \frac{1}{8\pi}\kappa A_H + \Phi_i \delta Q_i, \quad A_H = 4\pi r_0^2, \quad \Phi_i = \frac{r_0}{2} \frac{\partial}{\partial Q_i} \left( A_1(r_0; Q_i) - c(Q_i) \right).$$

(63)

In the special case of non-standard Reissner-Nordström-(anti-)de-Sitter type black holes (6)–(7) with parameters \((m, Q)\) the conjugate potential in (63):

$$\Phi = \sqrt{4\pi} \left( \frac{Q}{\sqrt{4\pi r_0}} - \frac{f_0}{2} \right) = \sqrt{4\pi} A_0 \mid_{r=r_0}$$

(64)

is (up to a constant factor) the electric field potential of the nonlinear gauge system on the horizon.

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