How stable are transport model results to changes of resonance parameters?
A UrQMD model study

Jochen Gerhard,∗ Bjørn Bäuchle, Volker Lindenstruth, and Marcus Bleicher
Frankfurt Institute for Advanced Studies, Ruth-Moufang-Straße 1, 60438 Frankfurt am Main
Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität,
Max-von-Laue-Straße 1, 60438 Frankfurt am Main and
Institut für Informatik, Johann Wolfgang Goethe-Universität,
Robert-Mayer-Straße 11–15, 60054 Frankfurt am Main

The Ultrarelativistic Quantum Molecular Dynamics [UrQMD] model is widely used to simulate heavy ion collisions in broad energy ranges. It consists of various components to implement the different physical processes underlying the transport approach. A major building block are the shared tables of constants, implementing the baryon masses and widths. Unfortunately, many of these input parameters are not well known experimentally. In view of the upcoming physics program at FAIR, it is therefore of fundamental interest to explore the stability of the model results when these parameters are varied. We perform a systematic variation of particle masses and widths within the limits proposed by the particle data group (or up to 10%). We find that the model results do only weakly depend on the variation of these input parameters. Thus, we conclude that the present implementation is stable with respect to the modification of not yet well specified particle parameters.

I. INTRODUCTION

One of the major themes of today’s high energy physics is the exploration of Quantum Chromo-Dynamics (QCD). QCD predicts that at sufficiently high temperatures and densities, nuclear matter could exhibit a phase transition into a new state of matter the Quark Gluon-Plasma (QGP). In this state the usual color confinement is relaxed and the constituents of the matter, namely quarks and gluons, are allowed to move over distances larger than the scale of a single nucleon. Indeed, experiments at the CERN-SPS, the BNL-RHIC and the LHC at CERN have collected an impressively large body of data that is consistent with the interpretation that a QGP was formed for a short period of time. While the current LHC program is running at the highest available energies and therefore provides insights into the properties of the QGP at high temperatures and very low baryon densities, the RHIC program is now focused on the exploration of the QGP phase transition with a low energy scan program. Here one hopes to find the existence and location of the critical end point of the first order transition line that is expected at high baryon densities. From the year 2018 on, this scan for the onset of deconfinement and the search for the critical end point will also be a top priority of the FAIR facility currently under construction in Germany.

Apart from the experimental difficulties, a major obstacle to pin down the properties of strongly interacting matter is the unambiguous interpretation of the experimental results. Unfortunately, first principle lattice QCD calculations are currently only feasible in thermal equilibrium and for very moderate T/µB values (T being the temperature and µB being the baryo-chemical potential). Therefore, transport approaches like the Ultra-relativistic Quantum Molecular Dynamics model (UrQMD), the Parton-Hadron-String-Dynamics (PHSD) [1], the Multi-Phase-Transport model (AMPT) [2] and many other dynamical models are employed to link the final state observables to the physics properties of the hot and dense stage of the reaction. All these models have in common that they rely as input on measured quantities like the hadron masses, the hadron decay widths, individual branching ratios and cross sections. Unfortunately, these quantities are very often not exactly known, as one can see from an inspection of the Particle Data Group (PDG) tables.

In this paper we explore systematically how a variation of some of these parameters influences the results obtained in transport simulation. As an example, we employ the UrQMD model[3–5]. In the long run, these investigations will allow to obtain a systematic error of the simulations, which is needed to quantify the quality of the model results. While we restrict ourselves to the UrQMD model in this study, the results should (qualitatively) also be transferable to other transport simulations based on similar physics assumptions as the ones mentioned above.

II. COMPUTATIONAL SET-UP

The idea is to replace the hard-coded hadron masses and widths with automatically generated tables with varied parameters. Then a sufficient number of simulations is performed and evaluated. To this aim, we have designed Python modules to read the up-to-date data from PDG web page[6]. The usage of Python as glue code for high performance computations allows for very short development cycles by combining rather slow, but powerful
scans for variations of mass and width of each baryon family are performed. Typically up to 10’000 events are simulated to stay clear of statistical errors.

III. CROSS SECTIONS

Let us start by investigating the total pion-nucleon cross section as a function of energy. This cross section is of special importance for the dynamics of nuclear matter at intermediate energies. It also serves as a direct benchmark to adjust the parameter sets since it is well measured experimentally. The total cross section is given by

\[
\sigma_{N\pi}^{\text{tot}} = \sum_{R=\Delta,N^*} \langle j_N, m_N, j_{\pi}, m_{\pi} | J_R, M_R \rangle \times \frac{2 S_R + 1}{(2 S_N + 1)(2 S_{\pi} + 1)} \frac{\pi}{p_{CMS}^2} \frac{\Gamma_{R\rightarrow N\pi} \Gamma_{R\rightarrow N\pi}^*}{(M_R - \sqrt{s})^2 + \frac{\Gamma_{R\rightarrow N\pi}^2}{4}} ,
\]

with the total and partial decay widths \( \Gamma_{\text{tot}} \) and \( \Gamma_{R\rightarrow N\pi} \). Thus, the cross-section depends on the widths and masses of all nucleon- and delta-resonances \( N^* \) and \( \Delta^{(*)} \). Figure 1 (top) depicts the total pion-nucleon cross section \( \sigma_{N\pi}^{\text{tot}} \) as a function of the center of mass energy \( \sqrt{s} \) for a systematic variation of the nucleon resonance masses. The full line depicts the PDG averages, the dotted line shows a variation of the PDG parameters by +10%, the dashed line a variation by -10%.

The full line depicts the PDG averages, the dotted line shows a variation of the PDG parameters by +10%, the dashed line a variation by -10%.

---

**FIG. 1.** Top: Total pion-nucleon cross section as a function of \( \sqrt{s} \) for a systematic variation of the nucleon resonance masses. The full line depicts the PDG averages, the dotted line shows a variation of the PDG parameters by +10%, the dashed line a variation by -10%. Bottom: Total pion-nucleon cross section as a function of \( \sqrt{s} \) for a systematic variation of the nucleon resonance widths. The full line depicts the PDG averages, the dotted line shows a variation of the PDG parameters by +10%, the dashed line a variation by -10%.

---

We explore the dependence of the UrQMD results on the particle data, within the estimated errors provided by the PDG. In addition we vary the parameters within an overall range of \( \pm 10\% \) of mass and width. Separate scripting parts of the program with very fast compiled parts in another language. This possibility is often used in graphics card computations. We use Python to automatically rewrite the Fortran source code of UrQMD according to recalculated variations of the PDG-Data. This form of meta-programing is often preferable to using configuration files. With the parameters being hard-coded in the Fortran source they are known at compile time, which enables the compiler to do more optimizations. We employ the Frankfurt LOEWE-CSC to carry out a systematical parameter scan and to check of the stability of UrQMD. After the computation the data files from different UrQMD runs are parsed and compressed to statistics files, which can easily be interpreted on local systems.

We explore the dependence of the UrQMD results on the particle data, within the estimated errors provided by the PDG. In addition we vary the parameters within an overall range of \( \pm 10\% \) of mass and width. Separate
FIG. 2. Top: Total pion-nucleon cross section as a function of $\sqrt{s}$ for a systematic variation of the $\Delta$ masses. The full line depicts the PDG averages, the dotted line shows a variation of the PDG parameters by +10%, the dashed line a variation by -10%. Bottom: Total pion-nucleon cross section as a function of $\sqrt{s}$ for a systematic variation of the $\Delta$ widths. The full line depicts the PDG averages, the dotted line shows a variation of the PDG parameters by +10%, the dashed line a variation by -10%.

FIG. 3. Top: Relative pion yield in Pb+Pb collisions at 2 AGeV beam energy. For a systematic variation of the masses and widths of the nucleon resonances by ±10%. Bottom: Relative pion yield in Pb+Pb collisions at 30 AGeV beam energy. For a systematic variation of the masses and widths of the nucleon resonances by ±10%.

FIG. 4 investigates the pion production as a function of varying masses of the delta-resonances $m_{\Delta}$ and their widths $\Gamma_{\Delta}$. The masses of all delta resonances have been scaled with the same factor. Here we limit the variation to -8% – +10%, because the code becomes unstable for too low masses. Again a variation of the width leaves the results unchanged. The variation of the $\Delta^{(*)}$ masses, however results in a strong variation of the pion yield at 2 AGeV. This effect is mainly attributed to the $\Delta(1232)$ resonance that is pushed towards the kinematic limit ($m_{\Delta(1232)} \rightarrow m_p + m_\pi$). At 30 AGeV, the variance of the yield stays generally moderate. However, the pion yield shows a pronounced step if the masses are shifted by $\sim 1.8%$ upwards. While the magnitude of the effect is small it indicates that complex simulation models may exhibit discontinuous behaviors. Let us now have a closer look into the origin of the step.

IV. PION PRODUCTION

A. Pion yields

Let us next turn to the investigation of full Pb+Pb collisions and focus on the FAIR energy range of 2 AGeV and 30 AGeV. Here we investigate the total pion yield for a systematic variation of all nucleon resonance masses $m_{N^*}$ by up to 10%. We show the deviation of the pion yield compared to a UrQMD calculation with the mean values of the PDG-Data files. Figure 3 (top) shows the relative pion yield in Pb+Pb collisions at 2 AGeV beam energy for a systematic variation of the masses and widths of the nucleon resonances by ±10%. Figure 3 (bottom) shows the pion yield in Pb+Pb collisions at 30 AGeV beam energy for a systematic variation of the masses and widths of the nucleon resonances by ±10%.

Even at the lowest energy, which is strongly dominated by resonance dynamics, the model results do at worst vary linearly with the variation of the model parameters. At 30 AGeV, the model results are stable against a variation of the resonance parameters. The variation of the particle widths has no significant effect on the model results.

Figure 4 investigates the pion production as a function of varying masses of the delta-resonances $m_{\Delta}$ and their widths $\Gamma_{\Delta}$. The masses of all delta resonances have been scaled with the same factor. Here we limit the variation to -8% – +10%, because the code becomes unstable for too low masses. Again a variation of the width leaves the results unchanged. The variation of the $\Delta^{(*)}$ masses, however results in a strong variation of the pion yield at 2 AGeV. This effect is mainly attributed to the $\Delta(1232)$ resonance that is pushed towards the kinematic limit ($m_{\Delta(1232)} \rightarrow m_p + m_\pi$). At 30 AGeV, the variance of the yield stays generally moderate. However, the pion yield shows a pronounced step if the masses are shifted by $\sim 1.8%$ upwards. While the magnitude of the effect is small it indicates that complex simulation models may exhibit discontinuous behaviors. Let us now have a closer look into the origin of the step.
FIG. 4. Top: Relative pion yield in Pb+Pb collisions at 2 AGeV beam energy. For a systematic variation of the masses and widths of the Δ resonances by ±10%. Bottom: Relative pion yield in Pb+Pb collisions at 30 AGeV beam energy. For a systematic variation of the masses and widths of the Δ resonances by ±10%.

For further analyses, we group different production processes into 5 classes, discriminating the decay of delta-resonances (Δ), the decay of nucleon-resonances (N*), the decay of strange baryons (accounting for all unstable baryons not included in the former two classes) (B_s), the decay of meson resonances (m) and scatterings (XY → π + R). In Figure 5 (top) one observes that at E_{lab} = 2 AGeV, the number of pions from scatterings stays constant as a function of delta mass, while the production of pions from delta decays rises linearly. At very low delta masses (less than -6% of the standard value), the formation of deltas absorbs on average more pions than are being produced by the decays thereof, while at higher delta masses, the opposite is true. The increase of pion production from deltas is counteracted by a decrease of pion production from nucleon resonances N*, which start to be net-absorbing above +6% of the standard (delta-)masses. This can be explained in a picture of detailed balance: When the delta resonances produce more pions, the equilibrium value of pion- and N*-multiplicity is shifted towards the N*. Thus, the N*-phase space is populated more quickly than it is depleted. The same effect, though much weaker and not turning around completely, can be seen in the decrease of the number of pions from mesonic decays. In total, the rise of pions from deltas counteracts the fall of pions from N*, thus leading to a weak overall rise of the pion production.

At 30 AGeV we focus now on the step like behavior at a Δ-mass shift of ∼ +1.8%. Figure 5 (bottom) shows the contributions of different Δ-resonances to the final number of pions for varying delta masses between 0 and +10% at high impact energy E_{lab} = 30 AGeV. I an analysis similar to the one from Figure 5 (top), we trace the step to the delta contribution. The step we discovered earlier consists of an increased pion production (less absorption) from the Δ_{1950}-resonance, which rises from -20 to 0, and a corresponding decreased production (increased net absorption) from the Δ_{1920}-resonance, which drops from 0 to -16. The √s-distribution of the underlying πN collision remains essentially unchanged as a function of μΔ. Therefore, at the point of the discontinuities, the Δ_{1920} takes the role that Δ_{1950} had at lower masses. Since the branching ratios Δ_{1920} → πΔ_{1232} and Δ_{1950} → πΔ_{1232} differ by a factor of 2 (40% vs. 20%), the number of Δ_{1232} and thus the number of pions are changed over a small mass intervall. We find a similar behavior at ∼ +5%,...
where the pion production from $\Delta_{1950}$ decreases, while the production from $\Delta_{1900}$ increases. Superimposed is an approximately linear rise of pion production from the lowest delta resonance $\Delta_{1232}$.

**B. $p_T$-spectra of $\pi^+$**

Finally, we discuss the transverse momentum distributions. Here we investigate the pions transverse momentum distributions in Pb+Pb collisions at 2 and 30 AGeV. Figure 6 (top) shows the deviation of the pion transverse momentum spectra ($p_t$) in Pb+Pb collisions at 2 AGeV for variations of the nucleon resonance masses. At 2 AGeV, a decrease of the nucleon resonance masses shifts the pions to lower $p_t$, while an increase of the masses shifts it to higher transverse momenta. However, variations of the yields at higher $p_t$ maybe up to $\pm 20\%$ for a variation of $\pm 5\%$.

Figure 6 (bottom) displays the deviation of the pion transverse momentum spectra ($p_t$) in Pb+Pb collisions at 30 AGeV for variations of the nucleon resonance masses. Figure 7 (top) shows the deviation of the pion transverse momentum spectra ($p_t$) in Pb+Pb collisions at 2 AGeV for variations of the $\Delta$ resonance masses. Again, we do not observe and effect on the calculations at $E_{lab} = 30$ AGeV is within the statistical fluctuations. Figure 7 (bottom) displays the deviation of the pion transverse momentum spectra ($p_t$) in Pb+Pb collisions at 30 AGeV for variations of the $\Delta$ resonance masses.

The effect of variations in delta resonance masses is strongly non-linear. Both at high beam energies and at low beam energies, we can distinguish three transverse momentum regions. Pions from intermediate transverse momentum $0.2 < p_t < 0.6$ GeV are being shifted to low transverse momentum $p_t < 0.2$ GeV, if the masses are decreased and vice versa, if the masses are increased. This is expected, since the available kinetic energy in a delta decay decreases with decreasing delta mass. At higher transverse momenta, lower masses lead to higher pion yields, while higher masses lead to lower pion yields, which is a reversal from the behavior observed from varying the nucleon resonance masses. Furthermore, we observe the effects to be a lot stronger in low-energy collisions. Also the variation of the $\Delta$ masses by $\pm 5\%$ results in modifications of the pion yield by $\pm 20\%$ in given $p_t$. 

**FIG. 6.** Top: Deviation of pion transverse momentum spectra ($p_t$) in Pb+Pb collisions at 2 AGeV for variations of the nucleon resonance masses. Bottom: Deviation of pion transverse momentum spectra ($p_t$) in Pb+Pb collisions at 30 AGeV for variations of the nucleon resonance masses.

**FIG. 7.** Top: Deviation of pion transverse momentum spectra ($p_t$) in Pb+Pb collisions at 2 AGeV for variations of the $\Delta$ resonance masses. Bottom: Deviation of pion transverse momentum spectra ($p_t$) in Pb+Pb collisions at 30 AGeV for variations of the $\Delta$ resonance masses.
regions.

V. SUMMARY

In light of the upcoming high precision experiments at FAIR, it is highly desirable to obtain better estimates on the systematic errors of transport simulations. We addressed this question by using the UrQMD transport approach in nucleus-nucleus reactions in the FAIR energy regime from 2 to 30 AGeV. We have analysed elementary cross sections in pion-nucleon reactions, as well as lead-lead collisions for various sets of input parameter variations of the hadron masses and widths. Although the analyzed quantities show globally only a weak dependence, discontinuities like in Figure 4 (bottom) may occur and influence predictions made by the applied models. The dependence is strongest in low-energy collisions, where the collision dynamics is dominated by resonance production and decay. Here, one may encounter systematic errors on the order of ±20%. At higher energies, the systematic errors are much smaller. One should note, that the present study explored a worst case scenario where all parameters were shifted simultaneously in one direction. The error on the masses (and widths) are however uncorrelated and should therefore induce smaller systematic bias into the simulations as compared to this study. Therefore, we conclude that the predictive and analysis power of the present approach is better than a systematic error of 20%.

VI. ACKNOWLEDGEMENTS

The computational resources were provided by the LOEWE-CSC. This work was supported by HIC for FAIR within the Hessian LOEWE initiative.

[1] E. Bratkovskaya, W. Cassing, V. Konchakovski, and O. Linnyk, Nucl.Phys. A856, 162 (2011), arXiv:1101.5793 [nucl-th].
[2] Z.-W. Lin, C. M. Ko, B.-A. Li, B. Zhang, and S. Pal, Phys. Rev. C 72, 064901 (2005).
[3] M. Bleicher et al., J. Phys. G25, 1859 (1999), arXiv:hep-ph/9909407.
[4] S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255 (1998), arXiv:nucl-th/9803035.
[5] H. Petersen, J. Steinheimer, G. Burau, M. Bleicher, and H. Stöcker, Phys. Rev. C 78, 044901 (2008).
[6] K. Nakamura and P. D. Group, Journal of Physics G: Nuclear and Particle Physics 37, 075021 (2010).
[7] K. Aehlig, H. Dietert, T. Fischbacher, and J. Gerhard, (2011), arXiv:1110.5936 [hep-th].
[8] M. Bach, M. Kretz, V. Lindenstruth, and D. Rohr, Computer Science - Research and Development 26, 153 (2011), 10.1007/s00450-011-0161-5.
[9] We also analyzed the rapidity spectrum distributions, but found now significant deviation.