Exploring the Spectral Shape of Gravitational Waves Induced by Primordial Scalar Perturbations and Connection with the Primordial Black Hole Scenarios

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There is a growing expectation that the gravitational wave detectors will start probing the stochastic gravitational wave backgrounds in the following years. We explore the spectral shapes of gravitational waves induced to second order by scalar perturbations and presumably have been produced in the early universe. We calculate the gravitational wave spectra generated during radiation and kination era together with the associated primordial black hole counterpart. We employ power spectra for the primordial curvature perturbation generated by $\alpha$-attractors and non-minimal derivative coupling inflation models as well as Gaussian and delta-type shapes. We demonstrate the ability of the tensor modes to constrain the spectrum of the primordial curvature perturbations and discriminate among inflationary models. Gravitational wave production during kination and radiation era can also be distinguished by their spectral shapes and amplitudes.

I. INTRODUCTION

A network of operating and designed gravitational wave detectors raise expectations that, sooner or later, the relic gravitational radiation background will be detected. Gravitational waves (GWs) are thought to be generated by several processes in the early universe \cite{11-13}. Among them there is a model independent source, the primordial density scalar perturbations, that unambiguously source tensor modes at second order perturbation theory \cite{4-8}, the so-called induced gravitational waves (IGW).

Tensor modes are induced to second order by scalar perturbations despite their decoupling at first order. The cosmological implications of the scalar induced GWs to second order were firstly discussed in the pioneering works \cite{9}, where the effects of IGWs on CMB polarization was computed, and \cite{10}, where the spectrum of the IGWs on small scales was studied. Further notable results followed that extended our understanding and formulation of IGW physics \cite{11-13} and gave us insights of how to probe the primordial power spectrum, $P_\mathcal{R}(k)$, at small scales.

Primordial scalar density perturbations are directly measured only at the very large scales of the CMB with wavenumber $k_{\text{cmb}} \sim 10^{-2}$ Mpc$^{-1}$ and with amplitude $\delta \rho / \rho \sim 10^{-5}$. At smaller scales $k \gg k_{\text{cmb}}$, there are weak constraints coming mainly from bounds on the abundance of primordial black holes (PBHs) \cite{13-16}. Additional constraints on the scalar density amplitude can come from gravitational wave experiments. Already, at the frequency range $f \sim 10^{-9}$ Hz and $f \sim 10^2$ Hz the PTA and LIGO/VIRGO experiments respectively have provided upper bounds.

The weak bounds on the primordial $P_\mathcal{R}(k)$ has prompted the building of several scenarios that exhibit an enhanced power at small scales. A subclass of these scenarios predict a significant PBH abundance, see Ref. \cite{17} for a review. The new observational window to the very early universe through the IGWs can test these scenarios \cite{18-33}. The interpretation of a GW signal is nevertheless a non-trivial task and one needs to take into account several parameters, such as the reheating temperature and the equation of state of the early universe, as well as whether the primordial fluctuations are Gaussian or a non-negligible non-Gaussianity exists \cite{21, 24, 27, 34}. From a different point of view, detection of the relic GWs stochastic background is a direct probe of the very early cosmic history, which is unknown for $t \lesssim 1$s \cite{35}. In this context, the recent works \cite{26, 38} have studied the impact of different early cosmological evolution on the IGWs.

In this work we explore the shape of the IGWs produced in the early universe assuming different scenarios and models. Prior to BBN there are no direct tests of the thermodynamical state of the universe and we choose to presume either the standard radiation equation of state or that of the stiff fluid, as benchmark cases. We focus on a $P_\mathcal{R}(k)$ shape with a peak. $P_\mathcal{R}(k)$ with a peak is a rather interesting scenario because it predicts IGWs and might also generate PBHs that can be part of the dark matter in the galaxies. We take into account the transition details and we target at cosmological parameter values that can be tested by the existing and designed GW experiments, such as LIGO and LISA. We also derive the scalar-induced GW spectral shapes for particular models of inflation: $\alpha$-attractors inflation \cite{39, 40} and a generalized Galileon-type model of inflation \cite{40}. In addition to the IGW spectra we examine the abundance and distribution of the PBHs associated with the $P_\mathcal{R}(k)$. We examine scenarios that predict a sizable amount of both PBHs and IGWs, and scenarios that predict significant IGWs with the PBH counterpart being either negligibly small or promptly evaporating. We present analytic and numerical results and manifest the connection between the PBH mass and the GW counterpart making use of figures and tables.

The paper is organized as follows. In Section \cite{11} we re-
view the method that is usually followed in the literature to calculate the spectra of scalar-induced GWs. In Section [11] we focus on particular inflationary models that predict $\mathcal{P}_R(k)$ with a peak as well as general Gaussian $\mathcal{P}_R(k)$ types, and we specialize into radiation and kinetic domination early universe scenarios. In Section [14] we turn into the PBH counterpart and calculate mass distributions and abundances. Our results are presented in Section [15] and finally, in Section [16] we draw our conclusions.

II. METHODOLOGY FOR CALCULATING INDUCED GRAVATIONAL WAVES

In this section we review the formulae required to calculate the energy density of the induced gravitational waves (IGWs) by first order scalar perturbations and adjust the notation in our context. We follow Ref. [11] and for a further study in a relevant context we refer the reader to Ref. [38, 41, 42].

We consider a flat Friedmann-Robertson-Walker model with first order scalar perturbations $\Phi$, $\Psi$ and second order (induced) tensor perturbations $h_{ij}$, and ignore vector perturbations and first order tensor perturbations. We describe our perturbations in the Newtonian gauge. The line element of the perturbed metric is written as,

$$\begin{align*}
    ds^2 &= a^2(\eta) \left[-(1+2\Phi) \, d\eta^2ight. \\
    & \left.+ \left((1-2\Psi) \, \delta_{ij} + \frac{1}{2} h_{ij}\right) \, dx^i \, dx^j\right],
\end{align*}$$

(1)

where $a(\eta)$ is the scale factor and $\delta_{ij}$ is the Kronecker tensor. For future reference we also denote the conformal Hubble function by $\mathcal{H}(\eta) = \frac{a(\eta)}{a(\eta)}$. Here, and later, prime (′) denotes conformal time differentiation. We will also ignore anisotropies and set $\Phi = \Psi$. In the Newtonian gauge the scalar perturbations play the role of the gravitational potential and the tensor modes describe gravitational waves.

We define $x \equiv k\eta$ and express the evolution of the scalar perturbations in terms of the scalar transfer function, $\Phi$, defined by $\Phi_{k}(\eta) \equiv \Phi(x) \phi_{k}$ in momentum space, where $\phi_{k}$ the Fourier mode of the primordial scalar perturbations. Considering the field equations of general relativity for a single perfect cosmological fluid with the standard density-pressure equation of state (EoS) relation $\rho = u \rho$, the evolution equations for the gravitational potential is obtained by

$$\Phi''(x) + 3(1+w) \mathcal{H}(\eta) \Phi'(x) + w k^2 \Phi(x) = 0,$$

(2)

We consider adiabatic perturbations (zero entropy perturbations) and initial conditions $\Phi(0) = 1$ and $\Phi'(0) = 0$ for the scalar transfer function. The general solution of Eq. (2) is given in terms of the Bessel function of the first kind, $J$, and the gamma function, $\Gamma$, [12],

$$\Phi(x) = \frac{2^\alpha \Gamma(\alpha + 1)}{(\sqrt{w}x)^\alpha} J_\alpha(\sqrt{w}x),$$

(3)

where $\alpha = \frac{5+3w}{2(1+3w)}$.

The evolution of the induced tensor modes is given by the equation,

$$h_{k}''(\eta) + 2\mathcal{H}(\eta) h_{k}'(\eta) + k^2 h_{k}(\eta) = S_{k}(\eta),$$

(4)

where the source function $S_{k}$ at the left hand side plays a critical role. It is a convolution of scalar perturbations at different wavenumbers given by,

$$S_{k}(\eta) = 4 \int \frac{d^3q}{(2\pi)^{3/2}} \, e^{ij}(\mathbf{k}) \, q_i \, q_j \, \phi_{k-q} \times f(|q|/k, |\mathbf{k} - \mathbf{q}|/k, \eta, k).$$

(5)

Here, $e^{ij}$ is the polarization tensor, and $f$ is an auxiliary function defined by,

$$f(u, v, \eta, k) = 2 \, \Phi(ux)\Phi(vx) + \frac{4}{3(1+w)} \left[ \Phi(ux) + \frac{uk}{\mathcal{H}(\eta)} \Phi'(ux) \right] \left[ \Phi(vx) + \frac{vk}{\mathcal{H}(\eta)} \Phi'(vx) \right].$$

(6)

Gaussian fluctuations are best described with the power spectral density. For scalar and tensor perturbations the power spectral densities, called $\mathcal{P}_\Phi$ and $\mathcal{P}_h$ respectively, are defined in terms of two point correlation functions as,

$$\langle \phi_{q} \phi_{k} \rangle \equiv \frac{2}{k^3} \mathcal{P}_\Phi(k) \delta^{(3)}(\mathbf{q} + \mathbf{k}),$$

(7)

and

$$\langle h_{q}^{\oplus} \cdot h_{k}^{\oplus} \rangle \equiv \frac{2}{k^3} \mathcal{P}_h(\eta, k) \delta^{(3)}(\mathbf{q} + \mathbf{k}),$$

(8)

where $\delta^{(3)}$ is the three-dimensional delta distribution and $\oplus, \odot$ denote the two polarization modes. The power spectral density of the comoving curvature perturbation, $\mathcal{P}_R$, would be

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1 For the effects of the anisotropic stress due to $O(10)^%$ difference between $\Phi$ and $\Psi$ see Ref. [12].

2 Here, we neglect the contribution from the Bessel function of the second kind, $Y_n(\sqrt{w}x)$, to avoid the singularity at $\eta = 0$. 

is related to $\mathcal{P}_R$ via,

$$\mathcal{P}_R(k) = \left( \frac{5 + 3w}{3 + 3w} \right)^2 \mathcal{P}_\Phi(k). \quad (9)$$

The density parameter of IGWs is defined to be the ratio of the energy density in IGWs over the total energy density, taking into account both $\oplus$ and $\otimes$ GW polarizations. A useful definition for the energy density of IGWs per unit logarithmic frequency interval is given in terms of the tensor power spectrum as,

$$\Omega_{\text{IGW}}(\eta, k) \equiv \frac{1}{24} \left( \frac{k}{\mathcal{H}(\eta)} \right)^2 \mathcal{P}_\hG (\eta, k), \quad (10)$$

where the over-line denotes the oscillation average. The power spectral density of the tensor perturbations is expressed as a double integral involving the power spectrum of the curvature perturbations,

$$\mathcal{P}_\hG (\eta, k) = \int_0^\infty dv \int_{[1-v]}^{1+v} du \mathcal{T}(u, v, \eta, k) \mathcal{P}_R(u k) \mathcal{P}_R(v k). \quad (11)$$

The $\mathcal{T}$ is the tensor transfer function given by

$$\mathcal{T}(u, v, \eta, k) = 4 \left( \frac{4\nu^2 - (1 + u^2 - v^2)^2}{4uv} \right)^2 \times \left( \frac{3 + 3w}{5 + 3w} \right)^2 \mathcal{P}_T(u, v, \eta, k), \quad (12)$$

where we have defined the kernel function by,

$$I(u, v, \eta, k) = \int_0^x dy \frac{a(y/k)}{a(\eta)} kG_k(x, y) f(u, v, y/k, k), \quad (13)$$

and have used the Green’s function $kG_k$ given in terms of Bessel functions, as $kG_k(x, y) = \frac{\mu}{2\sqrt{-\eta}} \cdot [Y_\nu(x) J_\nu(y) - Y_\nu(y) J_\nu(x)],$ with $\nu \equiv \sqrt{\frac{3(1-w)}{2(1+3w)}}.$ The oscillation average is given by,

$$\mathcal{P}_T(u, v, \eta, k) = \frac{1}{2\pi} \int_x^{x+2\pi} dy I^2(u, v, y/\eta). \quad (14)$$

The IGWs are sourced gravitational waves. At the time $t_c$ the production of IGWs ceases and afterwars the GWs propagate freely. In a radiation dominated background the energy density parameter of the GWs also remains constant. The energy density parameter of the IGW today, $t_0$, is therefore given by Eq. (10) at the moment $t_c^\text{th}$ times the current radiation density parameter, $\Omega_{\gamma,0} h^2 = 4.2 \times 10^{-5},$ modulo changes in the number of the relativistic degrees of freedom $g_*$ in the radiation fluid,

$$\Omega_{\text{IGW}}(t_0, f) h^2 = 0.39 \times \left( \frac{g_*}{106.75} \right)^{-\frac{1}{2}} \Omega_{\gamma,0} h^2 \times \Omega_{\text{IGW}}(t_c, f). \quad (15)$$

For the total IGW density parameter, we integrate equation (15) over the logarithmic interval of frequency,

$$\Omega_{\text{IGW}}(t_0) = \int_{f_{\min}}^{f_{\max}} d\ln f \Omega_{\text{IGW}}(t_0, f). \quad (16)$$

Since our spectra will be peaked at some frequency $f^o_{\text{IGW}}$ we may take $f_{\min} = f^o_{\text{IGW}}/100$ and $f_{\max} = 10f^o_{\text{IGW}}$. Here $\Omega_{\text{IGW}}(t_0)$ should not be confused with $\Omega_{\text{IGW}}(t_0, f)$ where the later function is frequency dependent and denotes the density parameter of IGWs per logarithmic frequency interval.

In the following we will examine particular $\mathcal{P}_R(k)$ types and we will focus on two benchmark early universe cosmological scenarios: the radiation and kination dominated eras.

### III. STUDY OF THE IGWs PRODUCED BY EXPLICIT MODELS AND IN DIFFERENT COSMOLOGICAL ERAS

It is well known that the equation of state of the Universe has not been directly probed for times prior to BBN $t \sim 1$ s. IGWs have amplitude and spectral shape that depend on the details of cosmological era at the time of their production. Indeed, the $\Phi$ that sources the GWs evolves in a different way for different equation of state $w$ and, additionally, the energy density of the produced GWs scales differently with respect to the background.

In order to study the IGW spectrum and demonstrate its properties and behaviour we assume that the curvature power spectrum $\mathcal{P}_R(k)$ features a peak at the wavenumber $k_p$. If the amplitude of the $\mathcal{P}_R(k)$ peak is significant it produces a strong IGW signal, that is possibly detectable, and additionally, enhances exponentially the production probability for PBHs. Therefore, explicit models can be tested and upper bounds on the IGWs can be derived.

We will study $\mathcal{P}_R(k)$ peaks generated by complete and explicit inflationary models, namely the $\alpha$-attactors and Horndeski general non-minimal derivative coupling (GNMDC) inflation introduced in Refs. [39] and [44] respectively. The qualitative element of the first model is that features a near-inflection point [45, 46] and the later function is frequency dependent and denotes the density parameter of IGWs per logarithmic frequency interval.

The qualitative element of the first model [39, 44] have a non-canonical kinetic term,

$$\mathcal{L}^\alpha-\text{attactor} = \frac{\mathcal{L}^\alpha}{\sqrt{-\eta}} = R - \frac{(\partial_\mu \Phi)^2}{2(1 - \frac{2w}{3})^2} + V(\Phi) \quad (17)$$

$$\mathcal{L}^\text{GNMDC} = \frac{\mathcal{L}^\text{GNMDC}}{\sqrt{-g}} = R - \tilde{f}(\varphi)G^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \quad (18)$$

We note that we examine the second order induced tensor perturbations, not the first order tensor modes produced by the inflationary stage itself; for the later see e.g. [48, 49] for relevant studies. What is of interest for our
Gaussian and Dirac assume general shapes for the \( \mathcal{P}_R(k) \) spectrum at some wavenumber \( k_p \). For a comprehensive description and details about the inflationary Lagrangians [17] and [15], as well as the generated \( \mathcal{P}_R(k) \) shapes we refer the reader to the original works [39] [44]. A brief description can be found in the Appendix B.

Moreover, irrespective of the inflationary theory, we assume general shapes for the \( \mathcal{P}_R(k) \) spectra. We utilize Gaussian and Dirac \( \delta \)-type distributions,

\[
\mathcal{P}_R^G(z) = A_R e^{-(z/\epsilon)^2} = \frac{A_0}{\epsilon \sqrt{\pi}} e^{-(z/\epsilon)^2} \\
\mathcal{P}_R^D(z) = A_0 \delta(z)
\]

where \( z \equiv \ln(k/k_p) \) and \( A_R = \mathcal{P}_R(k_p) \). In the limit \( \epsilon \to 0 \) the Gaussian distribution approaches the \( \delta \)-distribution. The advantage of these distributions is that analytic or semi-analytic results can be obtained and, in addition, realistic models can be parametrized. Gaussian spectra has been also studied recently in [12].

We are interested in three characteristics for the power spectra \( \mathcal{P}_R(k) \): i) the amplitude \( A_R \), ii) the wavenumber of the peak, \( k_p \), where \( \mathcal{P}_R(k_p) = A_R \) and iii) the width \( \epsilon \). We choose a large amplitude \( A_R \) so that the IGW signal is significant. We slightly modulate the \( A_R \) in order to maximize or minimize to a negligible amount the PBH abundance. The wavenumber \( k_p \) is chosen so that either the PBHs constitute a significant fraction of the dark matter in the galaxies, or the frequency of the IGWs lays in the sensitivity range of the gravitational detectors.

Let us now examine two benchmark cosmological scenarios separately: the radiation and the kination.

### A. The radiation domination scenario

Let us assume that the \( \mathcal{P}_R(k) \) peak enters the horizon during the radiation era. This is the standard early universe cosmological scenario. The IGWs produced during radiation era have been thoroughly studied, thus we will not repeat known results and technical details. We remind the reader that the potential \( \Phi(x) \) oscillates with a \( z^{-2} \) decaying amplitude in subhorizon scales. The formalism of the Section 11 is applied for \( w = 1/3 \). We quote the analytic expression that we use to obtain the IGWs results for the models (17), (18) and (19). Relying on analytical methods we obtain the tensor transfer function in late times as was done in [14]: the resulting expression is,

\[
\lim_{x \to \infty} x T_{RD}(u, v, x) = 2 \left( \frac{4v^2 - (1 + v^2 - u^2)^2}{4uv} \right)^2 \left( \frac{3(u^2 + v^2 - 3)}{4w^3} \right)^2 \left[ \frac{3 -(u + v)^2}{3 -(u - v)^2} \right]^2 \\
+ \pi^2(u^2 + v^2 - 3)^2 \Theta \left( u + v \sqrt{3} \right)
\]

where \( \Theta \) the unit step function.

We then perform a numerical integration to calculate the \( \mathcal{P}_R(\eta, k) \) as the Eq. (17) dictates. In order this to be done, an input for the power spectrum shape \( \mathcal{P}_R(k) \) is required. We assume two Gaussian power spectra with medium and narrow widths, \( \epsilon = 1 \) and \( \epsilon = 0.1 \) respectively; as well as power spectra produced by \( \alpha \)-attractor and Horndeski-type inflationary theories, see Fig. 4. The \( \mathcal{P}_R(k) \) produced by these inflationary theories are found after solving the Mukhanov-Sasaki equation numerically.

### B. The kination domination scenario

In this subsection, let us assume the cosmological scenario that the very early universe has been dominated by a phase whose equation of state is stiffer than radiation. Kination is a regime where the kinetic energy of a scalar field is dominant against its potential energy. In the limiting case \( w = 1 \), where the sound velocity is equal to the speed of light, the energy density of the stiff fluid scales as \( \rho \sim a^{-6} \). Such a scenario is natural in theories with runaway potentials, e.g. theories with moduli fields.

We examine the kination scenario because it has striking implications both for the PBH formation and the IGW produced. Our numerical results complement previous studies [38] and we show that an early universe kination era (eKD) shapes in a different way the spectrum of the tensor perturbations. This fact renders the eKD era testable.

We comment that the attribute "early" to kination might sound redundant since there is no kination era observed in the universe. Nevertheless we use it to emphasize that the kination era assumed precedes the standard radiation era.

The redshift of the stiff fluid energy density is the fastest and any ambient radiation will sooner or latter dominate the early universe. Hence, a kination era ends naturally if there is an extra fluid with softer equation of state. Besides this gradual transition, an eKD era can end suddenly. The later is simpler to examine because the scalar transfer function can be computed explicitly...
due to the discrete \( w \) values and single fluid analysis\(^3\)

Let us consider an early kination domination scenario that transits suddenly into the RD phase. The transition takes place at the reheating temperature \( T_{r\text{h}} \). The transfer function in the kination domination cannot be written in closed analytical expressions. One can only rely on approximate methods, since the Bessel functions of order one involved can only be written as infinite series and not further simplified. Hence, we treat the scenario eKD to RD numerically. Further, in order to simplify our calculations we utilize a monochromatic power spectrum of scalar perturbations modeled by a delta-distribution peaked at \( k_p \), given by Eq. (20). Such a choice models a narrow and sharply peaked \( P_R \). In the conclusions, we will discuss a correspondence between \( \delta \)-type and Gaus-

sian distributions, and anticipate features for the IGWs produced by a wider \( P_R(k) \).

In a sudden eKD→RD scenario the scale factor is a piecewise defined function, that is continuous at the point of transition \( \eta = \eta_{\text{f}} \),

\[
a(\eta) = \begin{cases} \frac{\eta}{\eta_{\text{f}}}, & \eta < \eta_{\text{f}} \\ \eta + \eta_{\text{f}}, & \eta \geq \eta_{\text{f}} \end{cases}
\]  \( (22)\)

A similar expression can be obtained for the conformal Hubble parameter, \( H \). The scalar transfer function, Eq. (2) is given by,

\[
\Phi(x) = \begin{cases} \frac{2}{x} J_1(x), & x < x_{\text{f}} \\ \frac{3}{x^2} \left[ C_1 \left( \frac{\sin(x/\sqrt{3})}{x/\sqrt{3}} - \cos(x/\sqrt{3}) \right) + C_2 \left( \frac{\cos(x/\sqrt{3})}{x/\sqrt{3}} + \sin(x/\sqrt{3}) \right) \right], & x \geq x_{\text{f}} \end{cases}
\]  \( (23)\)

\( \Phi(x) \) remains constant as long as \( x \ll 1 \). At the horizon crossing, \( \eta = \eta_{\text{entry}} \), the gravitational potential starts decaying as the Eq. (23) dictates: roughly as \( x^{-3/2} \) during eKD and after the transition to the RD phase, as \( x^{-2} \).

The reheating conformal time \( \eta_{\text{f}} \), is inversely proportional to the reheating temperature, \( \eta_{\text{f}} \propto T_{r\text{h}}^{-1} \). This means that higher reheating temperature scenarios predict a faster reheating transition into RD. During eKD the growth of IGWs goes proportional to conformal time for a given wave mode \( k \); that is \( \Omega_{\text{IGW}}^{\text{KD}} \propto (k_p \eta)^2 T_{r\text{h}}^2 \propto \eta \propto \eta_{\text{f}} \propto T_{r\text{h}}^{-1} \). This growth stalls once the transition is completed and radiation takes over. In the cases that \( \eta_{\text{entry}} \ll \eta_{\text{f}} \), we neglect the effect from RD era, because by that time the gravitational potential sourcing the tensor modes is negligibly small. Numerically, in those cases we calculate the IGWs at the time of transition\(^4\). Therefore, sudden eKD to RD scenarios may show up in the experiments with an enhanced IGW spectrum.

When there is a fast reheating, i.e right after the horizon entry of the perturbation the universe transits into the radiation regime, the growth, \( \Omega_{\text{IGW}} \propto a^2 \), of IGWs ceases early. However, the gravitational potential has not been suppressed significantly, and the contribution from RD era is important. In such a case of fast

\footnote{A mechanism that can implement a sudden transition can be easily conceived: an extra field direction that ends the kination stage via a waterfall transition to a global minimum where the field decays and reheats the universe.}

\( \footnote{After the transition, a sharp peak appears at \( k = \frac{2}{\sqrt{3}} k_p \) superimposed on the spectral shape obtained at \( \eta = \eta_{\text{f}} \). This corresponds to a logarithmic divergence manifest in the radiation domination as can be seen in Eq. (21). We neglect such a resonance peak as it has no physical meaning, but it is rather a mathematical singularity associated to the delta power spectrum we used, Eq. (20).} \)
reheating we calculate the IGWs at a conformal time \( \eta_c = 50 \eta_{rh} \gg \eta_{rh} \), thus taking into account the non-negligible contribution from the RD era. In Fig. 2 we present the evolution of the time dependent piece of the IGW spectral shape with respect to the conformal time for the transition eKD-RD.

![Graph showing the behaviour of the time dependent piece of the IGW spectrum under a sudden eKD-RD transition. The transition occurs when \( \eta/\eta_{rh} = 1 \). Here we plot the combination \( (\eta/\eta_{rh}) I(k) \) for \( k_{rh}/k = 1 \) and for four choices of the ratio \( k_{rh}/k_p \) corresponding to different colors in this figure. The growth is proportional to \( \sqrt{\eta} \) during eKD. After the transition, this function always approaches an oscillatory state with a constant amplitude in the late times.]

The final formula for the density parameter of IGWs in the eKD-RD is given by

\[
\Omega_{\text{IGW}}^{\text{eKD-RD}}(\eta_c, k) = \frac{1}{6} A_R^2 \left( \frac{k_p}{\mathcal{H}(\eta_c)} \right)^2 \left[ 1 - \left( \frac{k}{2 k_p} \right)^2 \right]^2 \\
\times \int_{k_{\text{eKD-RD}}}^k \left( \frac{k_p}{k} \right)^2 \Theta \left( 1 - \frac{k}{2 k_p} \right) \, \text{d}k.
\]

Here the unit step function \( \Theta \) is included by conservation of momentum, so that tensor modes with \( k > 2 k_p \) are cut-off.

Finally, let us mention that the existence of an early kination phase implies that the energy density of the produced IGWs gets enhanced and might backreact on the geometry. This backreaction must be limited in order that the energy density of the IGWs during the BBN does not increase the expansion rate to a disturbing level for the observed abundances of the relic nuclei. In order to comply with this constraint we find out a new bound on the reheating temperature that must be satisfied,

\[
T_{rh} \gtrsim 10^7 \text{ GeV} \, g_*^{1/16} A_R^{3/2} \left( \frac{M/\gamma}{10^{16} \text{g}} \right)^{-1/2}.
\]

This is a rough, conservative bound that connects horizon mass \( M/\gamma \) and the \( A_R \) with the \( T_{rh} \), and guarantees the success of the BBN predictions. Further discussion and derivation details can be found in the Appendix A.

IV. PRIMORDIAL BLACK HOLES AND THE ASSOCIATED GRAVITATIONAL WAVE SIGNAL

PBHs form from the collapse of large-amplitude primordial inhomogeneities [53, 54]. An inhomogeneity decouples from the background expansion if the power spectrum \( \mathcal{P}(k) \) is enhanced at a scale \( k^{-1} \), characteristic of the PBH mass. During RD, typical values for the \( \mathcal{P}(k) \sim O(10^{-2}) \) are required. If PBHs have mass \( M < 10^{15} \text{ g} \) evaporate at timescales less than the age of the universe, whereas PBHs with \( M > 10^{15} \text{ g} \) survive till today being dynamically cold component of the dark matter in galactic structures.

The PBH dark matter scenario is constrained in a wide range for the mass parameter \( M \) by several observational experiments, labeled with EG\( \gamma_\), Subaru HSC, EROS, X/R. CMB in the Fig. 3. In the present universe, black holes of mass above \( 10^{15} \text{ g} \) are subject to gravitational lensing constraints [54–56]. The abundance of light PBH is constrained from the extra galactic gamma-ray background (EG\( \gamma_\)) [57–61]. The CMB constrains the PBH with mass above \( 10^{17} \text{g} \) [62]. It has been claimed that the CMB bounds on massive PBHs might be relaxed [63, 64]. For a recent update on the PBH constraints see Ref. 65.

A. PBH abundance for general EoS

The present relic energy density parameter of primordial black holes with mass \( M \) produced at the cosmic time \( t \) is

\[
\Omega_{\text{PBH}}(M) = \Omega_{\text{m}} \gamma \beta(M) \left( \frac{M_{eq}}{M/\gamma} \right)^{2/3} \left( \frac{M_{rh}}{M_{eq}} \right)^{1/2} \tilde{g}(g_*),
\]

where \( \tilde{g}(g_*) = 2^{1/4} (g_*(t)/g_*(t_rh))^{1/4} (g_*(t_rh)/g_*(t_m))^{-1/4} \), and \( g_* \) the thermalized degrees of freedom. The parameter \( p \) is equal to 1 for \( t > t_{rh} \) and 0 for \( t < t_{rh} \). \( \Omega_{\text{m}} \) is the total matter density parameter today, \( M_{eq} \) is the horizon mass at the moment of matter radiation equality and \( M_{rh} \) the horizon mass at the moment of reheating.

The \( \beta(M) \) is the mass fraction of the universe with horizon mass \( M/\gamma \) that collapsed and formed PBHs; it can be interpreted as the black hole formation probability. Assuming Gaussian statistics, for a spherically symmetric region it is

\[
\beta(M) = \int_{\delta_c} d\delta \frac{1}{\sqrt{2\pi \sigma^2(M)}} e^{-\frac{\delta^2}{2\sigma^2(M)}},
\]

where \( \sigma(M) \) is the variance of the density perturbations and \( \delta \) the density contrast. The PBH abundance has an exponential sensitivity to the variance of the perturbations \( \sigma(M) \) and the threshold value \( \delta_c \), which is \( w \)-dependent. In this work we assign values to \( \delta_c \) following the findings of Ref. 67.
The horizon mass increases with a different rate for different expansion rates. Using the relation $f = k/(2\pi)$ we find the frequency-PBH mass correspondence for general equation of state $w$ and reheating temperature $T_{rh}$,

$$f_{hor}(M, T_{rh}, w) \simeq 2.7 \times 10^2 \text{Hz} \left(\frac{T_{rh}}{10^{10}\text{GeV}}\right)^{1-3w/(3+w)} \left(\frac{M}{10^{12}g}\right)^{-\frac{3w}{1-w}} \left(\frac{g_*}{106.75}\right)^{\frac{3}{3+3w}} \tilde{R}(g, w)$$

(28)

where $\tilde{R}(g, w) = (g_*/106.75)^{-1/12}$ for $w = 1/3$ and one for $w \neq 1/3$. In the above relation we have assumed a one-to-one correspondence between $k$ (or $f$) and the PBH mass $M$. This it true for the approximation of a monochromatic PBH mass spectrum, which is practically the case in many models. We note that the PBH mass distribution peaks at a value $M(k)$ that is in a slight offset with the value $M(p_h)$, as our following numerical analysis shows. In the $k$-space the peak of the $\sigma(k)$ is about at $0.7k_{p_h}$ [30]. The $f(M, T_{rh}, w)$ relation is depicted in Fig. 3. The $f_{hor}$ frequency and $f_{IGW}$ frequency, where the IGW spectrum maximizes, have similar size but do not coincide.

Therefore, searching for GWs with frequency $f_{IGW}$ one can probe PBH scenarios and the reheating temperature of the universe.

B. GW detectors and PBH scenarios

Currently, there is a network of operating ground-based GW detectors that focus on the high and low frequency regime, roughly at $10^{2}$ and $10^{-9}$ Hz, of the GW spectrum. Additionally, there are several designed and proposed experiments sensitive enough to detect or constrain the stochastic GW background at a large range of frequencies. In this subsection, we outline the GW experiments labeled in the Fig. 6 that can test the predictions of the models described in this work.

The operating ground-based GW detector system is the advanced Laser Interferometer Gravitational wave Observatory (aLIGO). In Fig. 6 we show for reference O1 and O5 which correspond to the first observing run and the design sensitivity respectively [68]. The future ground-based laser third generation interferometers is the Einstein Telescope (ET) [69], that probes the range of frequencies near 100 Hz. The space-based proposed experiments are DECIGO [70, 71] and BBO [72], that will probe in the deciherz frequency window. In the milli-herz band of the spectrum there is the scheduled Laser Interferometer Space Antenna (LISA) [73], and proposed space-based experiments such as the TianQin [74]. Pulsar Timing Arrays (PTA), with the International PTA (IPTA) [75], and the planned Square Kilometer Array (SKA) [76] are projects that probe and already pose constraints on the nanohertz regime.

Fitting functions regarding the sensitivity curves for LISA, aLIGO and ET are provided by [77] for DECIGO and BBO we consulted [78] and adjusted the sensitivities of these experiments to current values and expectations; for TianQin we used the fitting function provided in Ref. [79].

IGWs can constrain a large window of the PBHs masses. Currently, there are indirect constraints from the pulsar timing array experiments on IGWs associated with the formation of relatively massive PBHs at the epoch of horizon entry. Notably, a very severe constraint exists, $\beta(M) \lesssim 10^{-52}$, in the solar mass range, from pulsar timing data [85, 86]. We note that Ref. [81] pointed out that the tension with PTA constraints can be relieved if the perturbations are locally non-Gaussian.

![FIG. 3. The figure depicts the relation between the PBH mass $M_{PBH}$ and the corresponding frequency of horizon with size $k^{-1} = (2\pi f_{hor})^{-1}$ and mass $M_{PBH} / \gamma$ for two equation of states $w = 1/3$ (orange line) and $w = 1$ (green lines); see Eq. (28). The reheating temperature is $T_{rh} = 10^4$ GeV ($5 \times 10^3$ GeV) for the dashed (dot-dashed) lines. The gray strip highlights the frequency band of LISA optimal sensitivity.](image)

V. RESULTS

In this section we outline our findings. We assume $\mathcal{P}_R(k)$ amplitudes and wavenumbers having as a rular the PBH abundance, and then we calculate IGWs aiming at constraining early universe cosmological scenarios and inflationary models.

We consider $\mathcal{P}_R(k)$ peaks generated by the $\alpha$-attactors [69] and non-minimal derivative coupling Horndeski inflation [44], as well as, irrespectively of the inflationary theory, Gaussian and Dirac $\beta$-type distributions. The peak choice of the $\mathcal{P}_R$ is spanning a wide range of fre-

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5 For the case of kination domination the minor correction of replacing $T_{rh}$ by $2^{1/3}T_{rh}$ should be considered if there is equipartition of the energy density between the radiation and the scalar field at the time of reheating.
quencies in the GW spectrum, see Fig. 4. We compare the resulting IGW shape with the sensitivity of current, planned and proposed GW experiments. We choose amplitudes and shapes for the $P_R(k)$ that induce GWs and PBH abundances of cosmologically significant amount. We always make sure that our input parameters respect the current constraints for the PBH abundance and the GWs. We also take care the position and the width of the $P_R(k)$ peak to respect constraints coming from the Hawking radiation [14].

We assemble and present our results considering firstly $P_R(k)$ that generate a sizable amount of both PBHs and IGWs and secondly $P_R(k)$ that generate only IGWs with the PBH counterpart being either negligibly small, $\Omega_{PBH} \lesssim 10^{-10}$, or promptly evaporating.

A. IGWs in scenarios with dominant and subdominant PBH dark matter component

1. Significant PBH abundance

In the mass window $M = 10^{17} - 5 \times 10^{22}$ g the PBH abundance can reach its maximum value $\Omega_{PBH}/\Omega_{DM} = 1$, see Ref. [16]. If PBHs constitute a significant fraction of the total dark matter then an IGW counterpart must exist that can be tested by LISA and by proposed experiments such as BBO and DECIGO.

Another motivated PBH mass range is $M = \mathcal{O}(1 - 100) M_{\odot}$ where LIGO detected several coalescence events in the last years with the most recently published the intriguing event [87]. For that PBH mass range the IGWs are expected to have a nanohertz frequency detectable by PTA experiments [80]

The exact PBH mass parameters that we choose can be found in Tables VII and VIII and in the Fig. 5 There, for the PBH abundance the order of magnitude value is given, since this number is exponentially sensitive to the model dependent threshold value $\delta_c$.

2. Negligible PBH abundance

Contrary to the exponential sensitivity of the $\Omega_{PBH}$ on the $A_R$ the sensitivity of the IGWs amplitude on the $A_R$ is of a power-law type. Therefore, a minute decrease in the $P_R(k)$ amplitude can nearly disappear the PBHs abundance while decrease the IGW amplitude by a minor amount. This is rather fascinating because primordial density perturbations washed out completely by the RD era have left a relic GW behind that reveals their presence.

In addition, there are PBH masses that evaporate promptly in the early universe. These PBHs do not survive until today and, unless a stable remnant is left behind, the corresponding $\Omega_{PBH}$ is zero and can not account for the dark matter in the galaxies. Nevertheless, the associated IGWs might be strong enough to be detectable. Such a possibility is rather interesting because the PBH remnants can constitute a significant fraction of the total $\Omega_{DM}$ and, moreover, the inflationary scenarios that predict mini-PBHs can have a spectral index $n_s$ in full accordance with the Planck-2018 data [40], see Eq. (27), for the $\alpha$-attractors potential that we use.

Our results, for zero or negligible PBH abundance are listed in Tables VII and VIII. We note that, instead of the $\Omega_{PBH}$, we write the $\beta(M)$ values, see Eq. (27), since PBH with mass $M \lesssim 10^{15}$ grams have already evaporated.

6 In that range there are feeble constraints coming from white dwarfs and neutron stars [82,85] but these constraints, together with others in the same mass range such as femtolensing and piconelensing, are seen as insecure [16,80].
B. Cosmological Eras

1. Radiation

The spectral shapes of the IGWs produced during the radiation era are depicted in the upper panels of Fig. 6. For narrow Gaussian $\mathcal{P}_R(k)$ peaks, $\epsilon = 0.1$, the spectral shape obtained has a double peak structure with a sharp peak at $f \approx 2 f_p / \sqrt{3}$, resembling the IGW spectrum generated by a $\delta$-distribution for $\mathcal{P}_R(k)$. For a broader $\mathcal{P}_R(k)$ peak a single wide peak forms in the IGWs at approximately $2 f_p / \sqrt{3}$, see top panels of Fig. 6.

The $\alpha$-attractor inflationary model, described by the potential $V_2$, generates an enhanced $\mathcal{P}_R(k)$ as depicted in Fig. 4. The predicted IGW spectrum resembles more that of the broad Gaussian, $\epsilon = 1$, with a peak shifted towards $k_p$, i.e. $f_{\text{IGW}}^p \approx f_p$. The general non-minimal derivative coupling model, see Eq. (15), (14) and (15), features an IGW spectrum that resembles the broad Gaussian; hence it also resembles the $\alpha$-attractor model around the peak. However, the IGW spectra of the two inflationary models deviate at large frequencies following the different scaling at the high-$k$ tale of the $\mathcal{P}_R(k)$, as the inspection of the Fig. 4 and 5 shows.

For the case the $A_R$ amplitude is decreased to a level that the PBH abundance becomes negligible, $\Omega_{\text{PBH}} \lesssim 10^{-10}$, the amplitude of the IGWs decreases a little and the spectral shape characteristics remain unchanged. For the case of evaporating mini PBHs, we examine a different $\alpha$-attractors model, described by the potential $V_3$. This model generates a $\mathcal{P}_R(k)$ peak that is relatively narrow. The spectrum of the IGWs is found to feature a double peak structure, manifest in the Fig. 4 and 5.

Regarding the energy density, we find that the growth of the IGWs amplitude occurs rapidly, for a period $\eta_e \sim O(10) k_p^{-1}$, and then the growth ceases. At that time the $\Omega_{\text{IGW}}$ produced by a Gaussian $\mathcal{P}_R(k)$ peak with width $\epsilon$ and with frequency $f_p$ is

$$\Omega_{\text{IGW}}^{(RD)}(\eta_e) \approx (A_R \epsilon \sqrt{\pi})^2. \quad (29)$$

We further comment on the scaling of the IGW spectrum, at scales beyond the peak, in the next subsection.

2. Early Kination

The IGWs produced during the early kination era and sourced by $\delta$-distributions for the $\mathcal{P}_R(k)$ are depicted in the lower panels of Fig. 6. What is of interest is that the shape of the IGW spectrum depends on the reheating temperature, always assuming a sudden change from eKD to RD at the temperature $T_{\text{rh}}$.

If the $\delta$ peak enters well before the reheating moment, $\eta_{\text{entry}} \ll \eta_{\text{rh}}$, the induced tensor power spectrum has a distinctive shape, characteristic of the kination stage. It is distinctive because the scalar perturbations decay as $r^{3/2}$ experiencing a maximal pressure. No sharp peak in the IGWs appears even for the monochromatic $\mathcal{P}_R(k)$. This feature has been also noted in [38]. Additionally, the energy density of the IGWs gets enhanced due to the redshift of the stiffer background. We call this transition "slow eKD $\rightarrow$ RD".

If, on the other hand, the reheating happens fast after the horizon entry of the $\delta$-peak, the scalar perturbations experience only for a while the maximal pressure of the stiff fluid. It is actually the radiation phase that mostly forms the IGW spectrum. We call this transition "fast eKD $\rightarrow$ RD". This is readily seen for the IGW spectrum in the PTA frequency range, as depicted in the Fig. 6 where the BBN bound for the $T_{\text{rh}}$ limits the duration of the kination era. The parameters chosen for this case are $k_p \eta_{\text{rh}} \approx 1.5$ which means that transition essentially occurs at the moment of entry of the curvature perturbation. Apparently, the distinction between the pure RD
and the very fast eKD→RD transition is hard to be made.
By decreasing the $\eta_{\text{entry}}$ the differences in the amplitude and the $f$-scaling become manifest.

We note that, in our examples, the amplitude $A_0$ for the $\delta$-distribution together with the $T_{\text{rh}}$ values has been chosen so that either the $\Omega_{\text{PBH}}$ or the $\Omega_{\text{IGW}}$ is maximized, given the observational constraints. Thus, these eKD scenarios can be probed in the near future by gravitational wave observatories. Also, a change in the $T_{\text{rh}}$, apart from uplifting/downlifting the $\Omega_{\text{IGW}}$ shifts the position of the spectrum changing the frequency $f_{\text{IGW}}$.

Regarding the energy density, the IGWs at the time $\eta_c$, in the radiation era, have an energy density parameter of

$$\Omega_{\text{IGW}}(\eta_c) \approx \frac{\eta_{\text{rh}}}{\eta_{\text{entry}}} A_0^2,$$

This result is found for a monochromatic scalar power spectrum. Utilizing the correspondence between the $\delta$ and the Gaussian distribution with width $\epsilon$,

$$A_0 \leftrightarrow A_\delta \equiv \frac{A_0}{\epsilon \sqrt{\pi}},$$

one can find the energy density parameter of the IGWs produced during the kination era for wide $\mathcal{P}_R(k)$ distributions as well.

C. Fitting broken power-laws for IGW spectra

In this sub-section we make a few notes on the power laws that describe the IGW spectral curves in Fig. 6 and facilitate the detectability of our results. For a recent investigation regarding the infrared scaling of the IGWs spectra see also [SS] and [II] for a broken power law analysis for general stochastic GW backgrounds.

- The IGWs produced during RD for both of the Gaussian models scale as a power-law $f^2$ for $f < f_{\text{IGW}}$. Regarding the large-$f$ part, $f > f_{\text{IGW}}$, the $\epsilon = 0.1$ Gaussian is almost cut-off whereas the $\epsilon = 1$ falls roughly as $f^{-10}$.

- The Horndeski general non-minimal derivative coupling (GNMC) inflation model has a compound scaling in the small-$f$ band: it scales about $f^2$ at smaller frequencies. The large-$f$ drop-off is very well approximated by the $f^{-7}$ power law.

- The first $\alpha$-attractors inflationary model, given by Eq. (E2), follows a very similar power-law scaling to the GNMDC model at the small-$f$ band, and scales like a $f^{-2.7}$ in the large-$f$. The IGW spectrum is broader.

- The second $\alpha$-attractors inflationary model, given
by Eq. \[B3\], triggers evaporating mini-PBH formation and induced GWs near the LIGO frequency band. The IGW scale like \(f^{-9}\) in the large-\(f\) while in the small-\(f\) band it scales similarly to the Gaussian models. This is a consequence of the fact \(P_R(k)\) is narrow like our Gaussians.

Apparently, the distinction between \(\alpha\)-attractors and the Horndeski GNMDC inflationary models can be made in the large-\(f\) region.

- For the eKD case, the scaling of the IGWs follows a power-law behaviour, proportional to \(f\) in the small-\(f\) band. The large-\(f\) part drops abruptly, consistent with our choice of delta distribution for \(P_R\). In the case of a fast transition into RD, the scaling, as expected, goes like \(f^2\) for \(f < f^p_{\text{IGW}}\).

We summarize the scaling for the IGWs in Table I.

| era        | \(P_R\) scenario | small-\(f\) | large-\(f\) |
|------------|------------------|------------|-----------|
| RD         | Gaussian 0.1      | \(f^3\)    | cut-off   |
|            | Gaussian 1.0      | \(f^5\)    | \(f^{-10}\) |
| \(\alpha\)-attr. (DECIGO) | \(f^3\)    | \(f^{-2.7}\) |
| \(\alpha\)-attr. (LIGO)   | \(f^3\)    | \(f^{-9}\) |
| GNMD-Horndeski   | \(f^3\)    | \(f^{-7}\) |
| \(\delta\)   | \(f^5\)    | cut-off   |
| fast eKD→RD   | \(\delta\)    | \(f^*\)  | cut-off   |
| slow eKD→RD   | \(\delta\)    | \(f\)    | cut-off   |

TABLE I. We list the approximate power-law scalings that describes the IGW spectral shapes for the models we studied. The "cut-off" means that the IGW spectrum falls of very abruptly in the large-\(f\) band.

As a general rule, GWs induced during the radiation era the spectrum follows the \(f^3\) power-law under a broad \(P_R\) and the \(f^2\) power-law under a monochromatic \(P_R\) in the small-\(f\) band. In the slow eKD→RD scenario, where most of the GWs are induced during the kination era, the spectrum increases as \(f\) in the small-\(f\) band. In almost all cases, with the exception of the first \(\alpha\)-attractors inflation model \[B2\], the spectrum falls of very abruptly and is cut-off in the monochromatic cases.

Finally, we comment that our IGW spectra are found to be broad enough and are not expected to experience a deformation, mentioned recently in \[B9\], via the Sachs-Wolfe and integrated Sachs-Wolfe effect as the GWs travel towards the detectors.

VI. CONCLUSIONS

In this work we have explored the features of the scalar-induced GW spectrum produced by different types of \(P_R(k)\) peaks and for two different early universe cosmological scenarios: radiation and kination domination. In addition, we examined two explicit inflationary models that generate PBHs, \(\alpha\)-attractors and Horndeski general non-minimal derivative coupling models, and tested the predicted IGW signals against the observational constraints.

Assuming a Gaussian \(P_R(k)\) with amplitude \(A_R\) and width \(\epsilon\) we find that if the IGWs produced during radiation era have a spectral energy density parameter today, at the frequency \(f_p\) where the \(P_R\) maximizes, given by

\[
\Omega^{(RD)}_{IGW}(t_0, f_p) \approx 5.2 \times 10^{-9} \epsilon^2 \left( \frac{g_*}{106.75} \right)^{-1/3} \left( \frac{A_R}{10^{-2}} \right)^2.
\]

(32)

For IGWs produced during kination domination, with \(T_{rh}\) the reheating temperature, the corresponding expression for the energy density today is

\[
\Omega^{(KD)}_{IGW}(t_0, f_p) \approx \pi \Omega_{IGW}^{(RD)}(t_0, f_p) \left( \frac{10^7 \text{ GeV}}{T_{rh}} \right) \left( \frac{f_p}{\text{Hz}} \right)\
\]

(33)

where, \(\Omega_{IGW}^{(RD)}(t_0, f_p)\) is given by Eq. \[32\].

A radiation (RD) and an early kination domination (eKD) era may be distinguished by their power-law scaling in the small-\(f\) band, \(f < f^p_{\text{IGW}}\). An eKD scenario with a slow reheating and sudden transition to RD predicts spectral shapes with large amplitudes for IGWs. Over the next years aLIGO, reaching its design sensitivity, will put constraints on the eKD scenario in the high frequency band of the spectrum. Hints for the reheating temperature can be found if the IGW spectrum has been modified due to the transition into the radiation era.

A calculation involving a \(\delta\)-distribution is simpler to implement, compared to the Gaussian or any other realistic distribution such as those of \(\alpha\)-attractors, since the integrals \[11\] can be computed analytically. We used \(\delta\), i.e monochromatic, distributions only for the eKD case, because a broad distribution is computationally more costly in that case. Utilizing the correspondence, Eq. \[31\], between \(\delta\) and Gaussian distributions, \(A_0 \leftrightarrow A_R\), the maximum \(\Omega_{IGW}\) can be found following analytic steps and in a good approximation, either for the RD or the eKD case.

The spectral shape for the \(\Omega_{IGW}(t_0, f)\), depends on the features of the source, the scalar spectrum \(P_R(k)\). It maximizes at a frequency \(f^p_{\text{IGW}}\), in a little offset from \(f_p\), depending on the width of the \(P_R(k)\), as the results listed in the Tables \[VI\] and \[VII\] demonstrate. It is interesting to mention that, for the radiation domination case at least, the \(P_R(k)\) shape is projected in a much more informative manner onto the IGW spectrum than on the PBH mass distribution, which is predominantly monochromatic. By observing the IGW spectral shape and the power law scaling in the large-\(f\) band one can infer the width and the amplitude of the scalar spectrum, the generator of the IGWs.

Consequently, we can say that the detection of the IGW spectrum is a portal to the primordial power spectrum of curvature perturbations, \(P_R(k)\). It can be used to discriminate inflationary models, and our analysis aimed at contributing to this direction.
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Appendix A: GWs during kination era and constraints on the \( P_R(k) \) and the reheating temperature.

The gravitational wave energy density gets enhanced during the kination regime \([90–93]\). The energy density of the GWs does not alter BBN predictions if

\[
I \equiv h^2 \int_{k_{\text{BBN}}}^{k_{\text{max}}} \Omega_{GW}(k, \eta_0) d\ln k \leq 2 \times 10^{-6}.
\]  

(A1)

Equivalently, the above constraint can be written in terms of the \( \Omega_{GW}(k, \eta_c) \). The duration of the kination era is constrained due to energy density of the runaway field \( \varphi \) and the GWs, see \([81,85]\) for a recent discussion. Here, we derive a bound on the duration of the kination era coming from the amplitude of the IGWs, thus the \( P_R(k) \) at small scales, see also \([38,41]\). Let us assume that the energy of GWs is stored mainly in a narrow wave band \((k_1, k_2)\) with central wavenumber \( k_p \) that enters the horizon at \( \eta_{\text{entry}} \). During kination regime the GW energy density parameter scales as \( \Omega_{GW} \propto a^2 \). At the time of reheating it is

\[
\Omega_{GW}(\eta_{\text{rh}}) \simeq \frac{1}{2} \frac{\rho_{GW}(\eta_{\text{entry}})}{\rho_{\text{tot}}(\eta_{\text{entry}})} \left( \frac{a(\eta_{\text{entry}})}{a(\eta_{\text{rh}})} \right)^2
\]

where we assumed equipartition of energy densities at \( \eta_{\text{rh}} \); in the case of sudden transition the 1/2 factor should be dropped. Therefore, \( \Omega_{GW}(\eta_c) \simeq \Omega_{GW}(\eta_{\text{rh}}) \simeq \frac{1}{2} \Omega_{GW}(\eta_{\text{entry}})(k_p/k_{\text{rh}}) \). Let us make the top-hat approximation for the GW spectrum, \( \Omega_{GW} = A_{GW} \) in the interval \((k_1, k_2)\). Then, the integral \((A1)\) can be estimated. Plugging in numbers we find approximately the constraint

\[
\frac{k_p}{k_{\text{rh}}} A_{GW} \ln \left( \frac{k_2}{k_1} \right) \lesssim 0.4.
\]  

(A2)

For a ballpark analytic estimation, let us assume the GW spectrum width \( k_2/k_1 = 10 \) and consider the scaling \( \Omega_{GW}(\eta_c) \sim A_{GW}^2 \) during radiation domination era.

From the relation \( k_{\text{rh}} = 2 \times 10^7(T_{\text{rh}}/\text{GeV}) \text{Mpc}^{-1} \) and the kination era relation \( k_p = 5.4 \times 10^{28} g_*^{-1/12}(T_{\text{rh}}/\text{GeV})^{-1/3}(M/\text{grams})^{-2/3} \gamma^{-2/3}\text{Mpc}^{-1} \), see Eq. (23), a rough conservative lower bound on the reheating temperature for the kination regime is found,

\[
T_{\text{rh}} \gtrsim 10^7 \text{GeV} g_*^{1/6} A_{\text{hor}}^{3/2} \left( \frac{M}{10^{20} \text{g}} \right)^{-1/2},
\]  

(A3)

where \( A_R \equiv P_R(k_p) \). If this bound is violated the BBN predictions are endangered.

Appendix B: Explicit PBH generating inflation models

In inflationary cosmology the primordial perturbations are produced from quantum fluctuations with a vast range of wavelengths. Here, we briefly present two sort of inflationary models that generate an enhanced amplitude for the \( P_R(k) \) at small scales: the \( \alpha \)-attractors \([39,40]\) and the GNMDC inflation models \([41,46]\).

1. \( \alpha \)-attractors inflation

In the \( \alpha \)-attractors inflation scenario \([97]\) the effective Lagrangian for the inflaton field \( \varphi \) turns out to be

\[
\mathcal{L} = \frac{1}{2} R - \frac{1}{2} \left( \partial_{\mu} \varphi \right)^2 - f^2 \left( \tanh \frac{\varphi}{\sqrt{6} \alpha} \right)^2,
\]  

(B1)

where \( \text{Re} \Phi = \ddot{\varphi} = \sqrt{3} \tanh(\varphi/\sqrt{6} \alpha) \) is a chiral superfield, and \( R \) is the Ricci scalar. We also took \( M_{\text{Pl}} = 1 \). The canonically normalized inflaton potential that determines the inflationary trajectory is \( V(\varphi) = f^2(\tanh \varphi/\sqrt{6} \alpha)^2 \). Polynomial, trigonometric and exponential forms for the function \( f(\dot{\varphi}) \) can feature an inflection point plateau sufficient to generate a significant dark matter abundance in accordance with the observational constraints \([39]\). In this work we presented tensor power spectra induced by scalar power spectra \( P_R(k) \) predicted by the inflationary potentials

\[
V(\varphi) = f_0^2 \left[ \sum_{n=0}^{3} c_n \left( \tanh(\varphi/\sqrt{6} \alpha)^n \right)^2 \right],
\]  

(B2)

and

\[
V(\varphi) = f_0^2 \left[ c_0 + c_1 e^{\lambda_1 \tanh \varphi/\sqrt{6}} + c_2 e^{\lambda_2 (\tanh(\varphi/\sqrt{6}) - \tanh(\varphi/\sqrt{6}))} \right]^2.
\]  

(B3)

The form of the potentials \([B2]\) and \([B3]\) are depicted in Fig. \( [\ldots] \). The inflection point plateau is the field region where the curvature perturbations get enhanced. The potential \([B2]\) implements an RD cosmological post-inflationary phase with high reheating temperature together with large amplitudes for \( P_R(k) \) that induce GWs.
| PBH Mass | $\mathcal{P}_R(k)$ type | $A_R$ | $\frac{\Omega_{PBH}}{\Omega_{DM}}$ | $h^2 \Omega_{IGW}$ | $f_{IGW}^0$ | Experiment                  |
|----------|--------------------------|-------|-------------------------------|-----------------|-------------|-----------------------------|
| $10^{18}$ g | Gaussian 0.1             | $1.3 \times 10^{-1}$ | $\mathcal{O}(1)$ | $9.3 \times 10^{-10}$ | $1.4 \times 10^{-1}$ Hz | DECIGO + WD/Lensing          |
|          | Gaussian 1               | $4.1 \times 10^{-2}$ | $\mathcal{O}(1)$ | $3.3 \times 10^{-9}$ | $1.4 \times 10^{-1}$ Hz |                             |
|          | $\alpha$-attractors      | $2.1 \times 10^{-2}$ | $\mathcal{O}(1)$ | $1.1 \times 10^{-9}$ | $9.1 \times 10^{-2}$ Hz |                             |
|          | Galileon                 | $2.5 \times 10^{-2}$ | $\mathcal{O}(1)$ | $1.1 \times 10^{-9}$ | $1.4 \times 10^{-1}$ Hz |                             |
| $10^{22}$ g | Gaussian 0.1             | $1.5 \times 10^{-1}$ | $\mathcal{O}(1)$ | $1.4 \times 10^{-9}$ | $1.2 \times 10^{-3}$ Hz | LISA + Lensing               |
|          | Gaussian 1               | $4.8 \times 10^{-2}$ | $\mathcal{O}(1)$ | $4.4 \times 10^{-9}$ | $1.5 \times 10^{-3}$ Hz |                             |
| $10^{35}$ g | Gaussian 0.1             | $2.5 \times 10^{-1}$ | $\mathcal{O}(10^{-2})$ | $9.0 \times 10^{-9}$ | $3.7 \times 10^{-10}$ Hz | PTA + Lensing, X-rays        |
|          | Gaussian 1               | $8.0 \times 10^{-2}$ | $\mathcal{O}(10^{-2})$ | $2.7 \times 10^{-8}$ | $3.8 \times 10^{-10}$ Hz |                             |

TABLE II. The values for the PBHs and the associated IGWs produced during radiation domination are listed.

| PBH Mass | $\mathcal{P}_R(k)$ type | $A_0$ | $\frac{\Omega_{PBH}}{\Omega_{DM}}$ | $T_{rh}$ | $h^2 \Omega_{IGW}$ | $f_{IGW}^0$ | Experiment                  |
|----------|--------------------------|-------|-------------------------------|----------|-----------------|-------------|-----------------------------|
| $10^{18}$ g | $\delta$-distribution    | $1.6 \times 10^{-2}$ | $\mathcal{O}(1)$ | $10^6$ GeV | $6.2 \times 10^{-8}$ | $1.1 \times 10^{-1}$ Hz | DECIGO + WD/Lensing          |
|          |                          | $1.2 \times 10^{-2}$ | $\mathcal{O}(10^{-5})$ | $5 \times 10^4$ GeV | $1.3 \times 10^{-6}$ | $5.4 \times 10^{-1}$ Hz |                             |
| $10^{22}$ g | $\delta$-distribution    | $1.7 \times 10^{-2}$ | $\mathcal{O}(10^{-1})$ | $10^4$ GeV | $1.1 \times 10^{-6}$ | $1.0 \times 10^{-3}$ Hz | LISA + Lensing               |
|          |                          | $1.5 \times 10^{-2}$ | $\mathcal{O}(10^{-2})$ | $5 \times 10^2$ GeV | $2.2 \times 10^{-6}$ | $4.2 \times 10^{-3}$ Hz |                             |
| $10^{35}$ g | $\delta$-distribution    | $2.8 \times 10^{-2}$ | $\mathcal{O}(10^{-2})$ | $0.01$ GeV | $3.2 \times 10^{-8}$ | $5.3 \times 10^{-10}$ Hz | PTA + Lensing, X-rays        |

TABLE III. The values for the PBHs and the associated IGWs produced during kination domination and for different reheating temperatures are listed.

| $f_{hor}$ | $\mathcal{P}_R(k)$ type | $A_R$ | $M_{PBH}$ | $\beta_{PBH}$ | $h^2 \Omega_{IGW}$ | $f_{IGW}^0$ | Experiment                  |
|-----------|--------------------------|-------|-----------|----------------|-----------------|-------------|-----------------------------|
| $10^2$ Hz | Gaussian 0.1             | $7.8 \times 10^{-2}$ | $7 \times 10^{12}$ g | $\mathcal{O}(10^{-24})$ | $3.4 \times 10^{-10}$ | $1.1 \times 10^{2}$ Hz | LIGO           |
|          | Gaussian 1               | $2.5 \times 10^{-2}$ | $7 \times 10^{12}$ g | $\mathcal{O}(10^{-24})$ | $1.2 \times 10^{-9}$ | $1.0 \times 10^{2}$ Hz |               |
|          | $\alpha$-attractors      | $2.0 \times 10^{-2}$ | $7 \times 10^{12}$ g | $\mathcal{O}(10^{-24})$ | $1.9 \times 10^{-10}$ | $3.7 \times 10^{3}$ Hz |               |
| $10^{-1}$ Hz | Gaussian 0.1             | $9.0 \times 10^{-2}$ | $7 \times 10^{18}$ g | $\mathcal{O}(10^{-22})$ | $4.5 \times 10^{-10}$ | $1.1 \times 10^{-1}$ Hz | DECIGO         |
|          | Gaussian 1               | $2.7 \times 10^{-2}$ | $7 \times 10^{18}$ g | $\mathcal{O}(10^{-22})$ | $1.4 \times 10^{-9}$ | $1.0 \times 10^{-1}$ Hz |               |
|          | $\alpha$-attractors      | $1.4 \times 10^{-2}$ | $7 \times 10^{18}$ g | $\mathcal{O}(10^{-22})$ | $5.2 \times 10^{-10}$ | $3.4 \times 10^{-2}$ Hz |               |
| $10^{-3}$ Hz | Gaussian 0.1             | $8.7 \times 10^{-2}$ | $7 \times 10^{22}$ g | $\mathcal{O}(10^{-20})$ | $4.2 \times 10^{-10}$ | $1.1 \times 10^{-3}$ Hz | LISA           |
|          | Gaussian 1               | $3.0 \times 10^{-2}$ | $7 \times 10^{22}$ g | $\mathcal{O}(10^{-20})$ | $1.7 \times 10^{-9}$ | $1.0 \times 10^{-3}$ Hz |               |
| $10^{-9}$ Hz | Gaussian 0.1             | $1.2 \times 10^{-1}$ | $7 \times 10^{34}$ g | $\mathcal{O}(10^{-16})$ | $1.7 \times 10^{-9}$ | $1.1 \times 10^{-9}$ Hz | PTA            |
|          | Gaussian 1               | $3.8 \times 10^{-2}$ | $7 \times 10^{34}$ g | $\mathcal{O}(10^{-16})$ | $5.9 \times 10^{-9}$ | $1.0 \times 10^{-9}$ Hz |               |

TABLE IV. The values for IGWs produced during radiation domination with negligible or zero PBH abundance, $\Omega_{PBH}/\Omega_{DM} \lesssim 10^{-9}$, are listed.
The potential \[ \text{(B3)} \] implements an inflationary phase followed by a kination phase. The kination phase can end gradually, as in the original proposal \[ \text{(B0)} \], or via a sudden transition due to e.g. an extra field direction towards a global minimum. The power spectra generated are depicted in Fig. 4. Details about the range of the parameters’ values for the inflationary models \[ \text{(B2)} \] and \[ \text{(B3)} \] as well as details about the reheating temperature and the \( P_K(k) \) amplitude and the PBH abundances can be found in the works \[ \text{(39)} \] and \[ \text{(40)} \].

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