Beating the Standard Model

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This report, adapted from my talk at the 1998 Ettore Majorana Subnuclear School at Erice, proffers speculative explanations of the strong CP problem and the existence of cosmic rays beyond the GZK bound. It is based on works done with Sidney Coleman and Howard Georgi.

1. Introduction

Although our beloved standard model of quarks and leptons offers a complete, consistent and correct description of most particle phenomena, lots of vexing questions remain unanswered, such as: Why are the gauge group and the fermion masses what they are? Why three families of quarks and leptons? What breaks electroweak symmetry? How about gravity? Leaving such profound meta-questions to supersymmetrists, string theorists and their successors, we shall beat on two unrelated and more modest puzzles: the origin of CP violation and the reported observation of cosmic-ray events with unexpectedly high energies.

2. The Strong CP Problem

The standard model admits two kinds of CP violation: (1) Complex Yukawa couplings of the Higgs doublet produce CP-violating mass matrices for up-like and down-like quarks and generate the complex phase $\delta$ appearing in the Kobayashi-Maskawa (KM) matrix. This mechanism offers a plausible explanation of all observed CP-violating effects. (2) The CP-odd self interaction of the QCD gauge fields characterized by the $\theta$ parameter (together with complex quark masses) can induce additional CP-violating effects such as a neutron electric dipole moment. Although the complex phase $\delta$ is large in the standard model, this parameter must be unnaturally small, to wit: $\bar{\theta} \equiv \theta + \text{Arg det } M < 3 \times 10^{-10}$, where $M$ is the quark mass matrix.

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The strong CP problem is often addressed, but many proposed solutions seem to me either contrived, inelegant, empirically unacceptable, or all three. Several recent papers approach the problem in a novel manner [1] [2]. I shall discuss one model of this kind developed with Howard Georgi. The key notion is for CP to be softly broken: conserved by all terms in the Lagrangian with mass dimension four, but not by terms with lower mass dimension. Of course, the only lower dimension term in the standard-model Lagrangian is the Higgs mass, which cannot violate CP. Our hypothesis excludes both sources of symmetry violation; $\theta = 0$ and the KM matrix is real. We’ve lost the baby with the bathwater; there is no strong CP problem because CP is unbroken.

To do better, additional architecture is needed: new particles with new interactions, but not too many of them! The new particles are heavy because they are not seen. They—unlike quarks and leptons—have gauge invariant mass terms with mass dimension less than four. Therein lurks the origin of CP violation!

Wait a second! Heavy unobserved particles are often posited for other reasons. The aesthetic appeal of left-right symmetry, or of $SO(10)$ over $SU(5)$, suggests that the 15-member families of chiral fields be extended to include massive singlet neutrinos with large Majorana masses. These heavy neutrinos generate see-saw Majorana masses for the three weak-doublet neutrinos, from the interplay between the Higgs mass terms linking doublet and singlet neutrinos and much larger gauge-invariant Majorana mass terms linking singlet neutrinos to one another.

The bare mass term of the left-handed singlet neutrinos $N_i$ has mass-dimension three and (according to our hypothesis) need not, indeed should not, conserve CP. Thus heavy neutrino masses are described by a complex $3 \times 3$ symmetric matrix $M_{ij}$. Ordinary neutrinos acquire the masses $m^\dagger M^{-1} m$, where $m$ is the unknown Higgs-generated real matrix with eigenvalues sometimes, without much reason, taken comparable to charged lepton masses. A unitary matrix analogous to the complex KM matrix of the quark sector in the standard model relates neutrino mass and flavor eigenstates. Thus the see-saw mechanism leads directly to CP violation in the realm of neutrino physics, where with luck it may be seen, but hardly at all in the relevant realm of hadron physics.

To save ourselves, the seemingly useless CP violation buried in the heavy neutrino mass matrix somehow must contaminate the quark sector. Remarkably, just one more unobserved particle does the trick: a heavy boson $\zeta$ which is a color triplet and a weak doublet with electric charges $\frac{2}{3}$ and $-\frac{1}{3}$. The $\zeta$ has Yukawa interactions with coupling constants $g_{ia}$ linking each flavor $i$ of quark doublet to each flavor $a$ of Majorana neutrino.
To make order-of-magnitude estimates, we tentatively take the masses of the $\zeta$ meson and the three singlet neutrinos comparable and $\sim M_\zeta$. We also take the $g_{ia}$ comparable and $\sim g$. What we know about quark and lepton masses suggests that this approximation is unsound: we use it only to get a feel for the model.

As in the standard model, the leading contribution to CP violation among kaons comes from a box diagram by which two $s$ quarks become two $d$ quarks. But it’s not the usual box diagram with internal quark and $W$-boson lines because the KM matrix is real in tree approximation, and as we shall see, remains nearly real when radiatively corrected. Rather, it’s the box with internal $N$’s and $\zeta$’s. For this diagram to generate the observed value of $\epsilon_K$, the common mass and coupling constant must satisfy the constraint:

$$\frac{\alpha_g}{M_\zeta} \approx 2 \times 10^{-8} \text{ GeV},$$

with $\alpha_g = g^2/4\pi$.

CP-violating radiative corrections to the renormalizable interactions of the standard model must be small if our unconventional box diagram is to yield all observable CP violation. In particular, they induce finite complex phases in the KM matrix. Some phases can be removed by field redefinitions, but the area $\mathcal{A}$ of the unitarity triangle, given by

$$2\mathcal{A} = \left| \text{Im} \left( V_{ub} V_{ud}^* V_{cb} V_{cd}^* \right) \right|,$$

is an invariant measure of CP violation in the KM matrix. For models such as this one, we have shown $\mathcal{A}$ to be tiny compared to its value in the standard model. The radiatively corrected KM matrix is nearly real and the unitarity triangle is an essentially straight line. Salvatore Mele finds this result to be compatible with all available experimental data. If a model of this kind is correct, experimenters at BELLE and BABAR are in for a big surprise.

What about strong CP? Although $\bar{\theta} = 0$ in the bare Lagrangian, radiative corrections $\Delta M$ to the quark mass matrix can be complex. Current constraints on the neutron electric dipole moment require: $\Delta \bar{\theta} \equiv \text{Arg det } M < 3 \times 10^{-10}$. Because corrections to quark masses are small, this condition becomes:

$$\text{Im} \left[ \text{tr} \left( \Delta M_U M_U^{-1} + \Delta M_D M_D^{-1} \right) \right] < 3 \times 10^{-10}.$$
For the model at hand, we have shown that the first non-vanishing contribution to $\Delta \bar{\theta}$ appears at three loops. Estimating this diagram, we found that the strong CP problem is solved provided that $\alpha_g < 0.0024$.

Making use of Eq. (2.1), we find the bound $M_\zeta < 1.2 \times 10^5$ GeV, which is a bit awkward from the point of view of see-saw neutrino masses. However, the present analysis is merely a proof of principle that our model can solve the problem. We can relax our assumption that all heavy-sector masses are comparable. Surely there may be a hierarchy of singlet neutrino masses, whereupon the leading contribution to $\Delta \bar{\theta}$ is suppressed by an additional ratio of singlet neutrino masses. Much larger values of $\alpha_g$, and much smaller see-saw neutrino masses, can be obtained without encountering a strong CP problem.

3. Ultra-High Energy Cosmic Rays

Primary nucleons with sufficient energy will collide inelastically with CBR photons, thereby losing energy. This results in the GZK cutoff, saying that nucleons with energies $> 5 \times 10^{19}$ eV cannot reach us from distances greater that $\sim 50$ Mpc. However, cosmic rays are seen well above this energy. Indeed, there are a handful of events with energies significantly above $10^{20}$ eV. In this connection, a remarkable correlation has been discovered by Farrar and Biermann: that the five highest energy cosmic ray events seem to be closely aligned in space with compact radio-loud quasars at cosmological distances.

We argue that these events may have been produced by ultra-high-energy (UHE) primary neutrons that are both stable and immune to the GZK cutoff. To accomplish these miracles, we invoke tiny departures from strict Lorentz invariance, too small to have been detected otherwise. The results in this section are abstracted from a recent paper with Sidney Coleman. Many observable consequences of Lorentz violation are described in terms of modified energy-momentum relations for freely moving particles. To each particle species $a$ there corresponds a mass $m_a$ and a maximum attainable velocity $c_a$. (Here we neglect the possibility that $c_a$ may be helicity-dependent and flavor non-diagonal. Furthermore, we do not consider violations of TCP symmetry.) The dispersion relations become $E^2 = c_a^2 \vec{p}^2 + m_a^2 c_a^2$. Lorentz invariance is recaptured iff all $c_a$ are the same.

Ordinarily, free neutrons can beta decay but protons cannot. Departures from Lorentz invariance can affect the kinematics of decay processes and even invert this pattern! To see how, let’s examine the case $c_p = c_e = c_\nu < c_n$, where conventional relativistic kinematics may be used with $c_p$ as “the speed of light,” provided the neutron is assigned an effective
mass \( m_{\text{eff}} \) given by: 
\[
m_{\text{eff}}^2 \equiv m_n^2 - (c_p^2 - c_n^2) \vec{p}^2,
\]
where \( \vec{p} \) is its momentum in the preferred frame. Neutron beta-decay is allowed iff 
\( m_{\text{eff}} > m_p + m_e \). Expressed in terms of the neutron energy \( E \) in the preferred frame, this condition becomes:
\[
E < E_1 = \sqrt{\frac{m_n^2 - (m_p + m_e)^2}{c_p^2 - c_n^2}} \approx 2.7 \times 10^{19} \left( \frac{10^{-24}}{c_p - c_n} \right)^{1/2} \text{ eV}.
\]

With our choice of Lorentz-violating parameters, \textit{neutrons with energies exceeding} \( E_1 \) \textit{are stable particles that can be present among UHE cosmic rays.} Conversely, we find that a proton with energy \( E \) can beta decay iff:
\[
E > E_2 \approx \sqrt{\frac{m_n^2 - (m_p - m_e)^2}{c_p^2 - c_n^2}} \approx 4.1 \times 10^{19} \left( \frac{10^{-24}}{c_p - c_n} \right)^{1/2} \text{ eV}.
\]

For our example, \textit{protons with energies exceeding} \( E_2 \) \textit{are unstable particles that cannot be present among UHE cosmic rays.} The above results are expressed in terms of a nominal choice, \( c_p - c_n = 10^{-24} \), lying beyond the sensitivity of current tests of Lorentz invariance. Perhaps highest energy cosmic-ray primaries are stable neutrons.

Next we point out that there may not be a GZK cutoff. Effects of departures from Lorentz invariance increase rapidly with energy and can kinematically prevent cosmic-ray nucleons from undergoing inelastic collisions with CBR photons. The cutoff thereby undone, a deeply cosmological origin of UHE cosmic rays becomes tenable. To see how this goes, consider the formation reaction yielding the first pion-nucleon resonance:
\[
p + \gamma \ (\text{CBR}) \rightarrow \Delta(1232),
\]
by a proton of energy \( E \) colliding with a CBR photon of energy \( \omega \). The target photons are thermal with \( T = 2.73 \) K or \( kT = \omega_0 = 2.35 \times 10^{-4} \) eV. For a head-on impact, \( \Delta \) formation is allowed iff:
\[
2\omega + \frac{M_p^2}{2E} \geq (c_\Delta - c_p) E + \frac{M_\Delta^2}{2E},
\]
where \( c_\Delta - c_p \) is the relevant Lorentz-violating parameter. If \( c_\Delta = c_p \), Eq. (3.2) yields the usual threshold, \( E_f = (M_\Delta^2 - M_p^2)/4\omega \). Otherwise, Eq. (3.2) yields a quadratic inequality in \( E \) which can be satisfied iff \( c_\Delta - c_p < \hat{\delta}(\omega) \equiv \omega/2E_f \). As \( c_\Delta - c_p \) increases toward \( \hat{\delta} \), the threshold for \( \Delta \) formation grows. If it exceeds its critical value,
\[
c_\Delta - c_p > \frac{2\omega^2}{M_\Delta^2 - M_p^2} \approx 1.7 \times 10^{-25} \left[ \omega/\omega_0 \right]^2,
\]
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reaction (3.1) is forbidden for all $E$. Recalling that the photons are thermal, we see that if $c_{\Delta} - c_p \sim \tilde{\delta}(\omega_0)$, the GZK cutoff due to resonant $\Delta(1232)$ formation would be relaxed. Should it much exceed this value, formation would be precluded off virtually all CBR photons.

Reaction (3.1) is the dominant process leading to the GZK cutoff. If it is forbidden, a weakened version of the cutoff may result from non-resonant photo-production:

$$p + \gamma \ (CBR) \rightarrow p + \pi .$$

If $c_\pi = c_p$, the threshold energy is $E_p = M_\pi(2M_p + M_\pi)/4\omega$. If $c_\pi - c_p > 0$ the threshold is larger. As $E \rightarrow \infty$, the pion energy $E_\pi$ must remain finite. Energy conservation yields the kinematic condition:

$$2\omega \geq (c_\pi - c_p) E_\pi + \frac{m_\pi^2}{2E_\pi},$$

which may be satisfied iff:

$$c_\pi - c_p < \tilde{\delta}(\omega) \equiv \frac{2\omega^2}{m_\pi^2} \simeq 5 \times 10^{-24} \left[\omega/\omega_0\right]^2 .$$

For $c_\pi - c_p > \tilde{\delta}(\omega)$, reaction (3.4), as well as multiple pion production, is kinematically forbidden off photons of energy $\omega$ at all proton energies. For the actual thermal photons, $c_\pi - c_p \sim \tilde{\delta}(\omega_0)$ would suppress photo-pion production, or even eliminate it entirely so that no vestige of the GZK cutoff survives. Much larger, and experimentally intolerable, violations of Lorentz invariance would be needed to affect the interactions of UHE cosmic rays with nuclei in the atmosphere.

A tiny value of the Lorentz-violating $c_n - c_p$ stabilizes UHE neutrons. Tiny values of the parameters $c_{\Delta} - c_p$ or $c_\pi - c_p$ forbid the processes underlying the GZK cutoff. Let’s go for broke, and suppose both Lorentz-violating effects are present. Then cosmic-ray events of the highest energies could be produced by primary UHE neutrons from sources at cosmological distances. They can have been stabilized and made GZK-resistant by departures from strict Lorentz invariance. They are electrically neutral, so that they are undeflected by magnetic fields and can reveal their distant origins.

Existing tests of special relativity are far too weak to exclude these dramatic effects on UHE cosmic rays. (See ref. [6] for a list of current bounds.) Fortunately, some bounds can be strengthened considerably. Laboratory tests of Lorentz invariance far more precise than any done before are now feasible [7]. Dedicated searches for velocity oscillations of solar neutrinos, or of accelerator-produced $\sim$ TeV neutrinos at baselines of $\sim$1000 km, can reveal neutrino velocity differences as small as $10^{-25}$. 

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References

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