Application of Quantum Density Matrix in Classical Question Answering and Classical Image Classification

Xing-Qiang Zhao, Hai Wan

Abstract: Quantum density matrix represents all the information of the entire quantum system, and novel models of meaning employing density matrices naturally model linguistic phenomena such as hyponymy and linguistic ambiguity, among others in quantum question answering tasks. Naturally, we argue that applying the quantum density matrix into classical Question Answering (QA) tasks can show more effective performance. Specifically, we (i) design a new mechanism based on Long Short-Term Memory (LSTM) to accommodate the case when the inputs are matrixes; (ii) apply the new mechanism to QA problems with Convolutional Neural Network (CNN) and gain the LSTM-based QA model with the quantum density matrix. Experiments of our new model on TREC-QA and WIKI-QA data sets show encouraging results. Similarly, we argue that the quantum density matrix can also enhance the image feature information and the relationship between the features for the classical image classification. Thus, we (i) combine density matrices and CNN to design a new mechanism; (ii) apply the new mechanism to some representative classical image classification tasks. A series of experiments show that the application of quantum density matrix in image classification has the generalization and high efficiency on different datasets. The application of quantum density matrix both in classical question answering tasks and classical image classification tasks show more effective performance.

Keywords: Application, Quantum density matrix, Question answering, Image classification, CNN, LSTM.

1. Introduction

In a quantum system, the quantum density matrix represents all the information of the entire quantum system. Earlier work has already demonstrated potential quantum advantage for natural language processing (NLP) [1] in a number of manners, and novel models of meaning employing density matrices naturally model linguistic phenomena.

X.-Q. Zhao, H. Wan

1 School of Computer Science and Engineering, Sun Yat-Sen University, Guangzhou 510006, China

E-mail: zhaoxq23@mail2.sysu.edu.cn, wanhai@mail.sysu.edu.cn.
such as hyponymy and linguistic ambiguity, among others [2]. In addition, the methods of solving classical problems in accordance with the ideas from quantum theory [3] to improve the performance of classical algorithms have already emerged [4-8]. Naturally, we argue that applying the quantum density matrix into classical question answering tasks can show more effective performance. To test our conjecture, we (i) design a new mechanism based on Long Short-Term Memory (LSTM) [9] to accommodate the case when the inputs are matrixes; (ii) apply the new mechanism to Question Answering (QA) [10] problems with Convolutional Neural Network (CNN) [11] and gain the LSTM-based QA model with the quantum density matrix. Experiments of our new model on both TREC-QA [12] and WIKI-QA [13] data sets show encouraging results.

Similarly, we argue that the quantum density matrix can also enhance the image feature information and the relationship between the features for the classical image classification [14]. Thus, we (i) combine density matrices and CNN to design a new mechanism that is called QIM for convenience; (ii) apply the new mechanism to some representative classical image classification tasks. Our experiments on Intel dataset [15] have verified our conjecture that it can also enhance the image feature information and the relationship between the features by applying the quantum density matrix method to the classical image classification tasks. Besides, a series of experiments on Mnist [16], Fashion-Mnist [17], Cifar 10/100 [18], ...., have shown that the quantum density matrix fits these different datasets in image classification tasks.

In a nutshell, the application of quantum density matrix both in classical question answering and classical image classification tasks show more effective performance.

The structure of this paper is described as follows. In Section 2, the application of quantum density matrix in classical question answering is introduced, and the corresponding experimental results are provided. In Section 3, the application of quantum density matrix in classical image classification is introduced, and the corresponding experimental results are provided. In Section 4, a brief conclusion and the introduction of our future work are given.

2. Application of Quantum Density Matrix in Classical Question Answering

2.1 Scheme Design

A quantum language model (QLM) [19] represents a word or a compound dependency between words by a quantum elementary event [20]. For each single word
\( w_i \), the corresponding projector \( \Pi_i = |e_i\rangle\langle e_i| \), where \( |e_i\rangle \) is an one-hot vector and it is the standard basis vector associated to a word. In 2013, Sordoni et al. utilized the Maximum Likelihood Estimation (MLE) to estimate the density matrices \( \rho_q \) and \( \rho_d \), which represented the query \( q \) and document \( d \), respectively [19].

Inspired by the quantum language model and quantum density matrix, we argue that applying the quantum density matrix into classical question answering tasks can show more effective performance. At first, we represent the quantum density matrix with linear algebra to generate the density matrices. The mathematical formalism of quantum theory is based on linear algebra. In quantum probability [21], the probabilistic space is naturally represented in a Hilbert space, denoted as \( H^n \). For practical reasons, the previous models with the method of quantum limited the problem in the real space, denoted as \( R^n \) [19]. The Dirac’s notation is often utilized, which denotes a unit vector \( \bar{u} \in R \) as a ket \( |u\rangle \) and its transpose \( \bar{u}^T \) as a bra \( \langle u| \). The inner product between two state vectors is denoted as \( \langle u|v\rangle \). The projector onto the direction \( |u\rangle \) is \( |u\rangle\langle u| \), which is an outer product (dyad) of \( |u\rangle \) itself. Each rank-one projector \( |u\rangle\langle u| \) can represent a quantum elementary event. Density matrix is a generalization of the conventional finite probability distributions. A quantum density matrix \( \rho \) can be defined as a mixture of dyads \( |\varphi_i\rangle\langle \varphi_i| \)

\[
\rho = \sum_i p_i |\varphi_i\rangle\langle \varphi_i|
\]  

(1)

where \( |\varphi_i\rangle \) is a pure state vector with probability \( p_i \). \( \rho \) is symmetric, positive semidefinite, and of trace 1 \( (\text{tr}(\rho) = 1) \). Similarly, we provide the expression of the density matrix \( \rho' \) in our work:

\[
\rho' = \sum_i k_i \bar{u} \cdot \bar{u}^T
\]

(2)

where \( k_i \) denotes the weight of vector \( \bar{u} \) and \( \sum_i k_i = 1 \).

We design a new mechanism based on LSTM to accommodate the case when the inputs are matrixes, and the new mechanism is shown in Figure 1.
Then we apply the new mechanism to question answering problems with CNN and gain the LSTM-based QA model with quantum density matrix shown in Figure 2.

2.2 Experimental Evaluation

A series of experiments are carried out for a systematic evaluation, and
experiments of our new model on the TREC-QA [12] and WIKI-QA [13] data sets show encouraging results. The experiment results are summarized in Table 1, where LSTM(V) represents the model without the new mechanism we designed and LSTM(M) represents LSTM-based question answering model with the quantum density matrix. It is clear that our new model has advantages in both two datasets, which shows the effectiveness of our new model with the density matrix together. Therefore, applying the quantum density matrix into classical question answering tasks can show more effective performance.

| Method  | TREC-QA           | WIKIQA          |
|---------|-------------------|-----------------|
|         | MAP               | MRR             | MAP             | MRR             |
| Ref [22]| 0.5693            | 0.6613          | 0.6190          | 0.6281          |
| QLM [19]| 0.6784            | 0.7265          | 0.5109          | 0.5148          |
| QLM_T [19]| 0.6683          | 0.7280          | 0.5108          | 0.5145          |
| LSTM(V) | 0.7031            | 0.7375          | 0.6405          | 0.6374          |
| LSTM(M) | 0.7124            | 0.7388          | 0.6567          | 0.6477          |

3. Application of Quantum Density Matrix in Classical Image Classification

3.1 Image Classification

Image classification, one of the main applications of computer vision (CV) [23], is the task of assigning labels to images through image analysis. The process of traditional image classification algorithms (refer to Figure 3 (a)) consists of two steps: (i) extracting features manually from the image; (ii) making use of the training support vector machine (SVM) [24] or other classifiers to execute image classification according to the extracted features. In general, the accuracy of the whole system of traditional image classification algorithms is limited to the method of extracting features. To solve the problem of feature extraction, AlexNet [25] is designed by applying deep learning [26] to large-scale image classification tasks and achieves better results. After that, many methods based on AlexNet emerged, such as LeNet [27], ZFNet [28], VGG-Net [29], ResNet [30], and so on.
3.2 Scheme Design

Similar to the application of quantum density matrix in classical question answering tasks, we consider making the density matrix to enhance image features and the correlation information between features in classical image classification tasks. In a quantum system, every particle $i$ is in its own state $|s_i\rangle$, and the tensor product $|s_i\rangle\langle s_i|$ is its own quantum density matrix. The quantum density matrix of the entire system is generated by adding the quantum density matrix of each particle in the quantum system according to a certain weight $p_i$, the generation of quantum density matrix $\sum_i p_i |s_i\rangle\langle s_i|$ about the entire system is shown in Figure 4.

Considering that the inputs of image information are pixel matrices in CV, we directly flatten these extracted feature matrices to obtain their own flattened feature map vectors $\{\tilde{m}_i\}$. Then we perform the operation shown in Equation (2) on each $\tilde{m}_i$ to obtain the corresponding density matrix $p^{m}_i \tilde{m}_i \tilde{m}_i^T$ with its own weight $p^{m}_i$. The generation process of density matrices is shown in Figure 5.
Finally, we add density matrices to CNN with 2D convolution kernels to generate our scheme of employing quantum density matrix in classical image classification tasks. We name our scheme as Quantum Inspired Mechanism (QIM) for convenience, and the QIM is shown in Figure 6.

Suppose the number of filters is $c$. The $j^{th}$ convolution operation is

$$ C_j = \delta \left[ (\vec{m}_j \cdot \vec{m}_j^\top) \ast p_m^j + b_j \right] $$

(3)

where $1 \leq j \leq c$, $\delta$ is the non-linear activation function, $*$ is the 2D convolution, $p_m^j$ and $b_j$ are the weight and bias respectively for the $j^{th}$ convolution kernel, and $C_j$ is the feature map. After the convolution layer obtains the feature maps, we then
use row-wise and column-wise max-pooling to generate vectors $\tilde{r}_t \in R^{d-k+1}$ and $\tilde{c}_t \in R^{d-k+1}$, respectively.

$$\tilde{r}_t = \max \left( C_{j,g} \right), 1 \leq g \leq k - d + 1$$

$$\tilde{c}_t = \max \left( C_{g,j} \right), 1 \leq g \leq k - d + 1$$

We concatenate these vectors as follow:

$$\tilde{f} = [\tilde{c}_1; \tilde{r}_1; \tilde{c}_2; \tilde{r}_2; \ldots; \tilde{c}_t; \tilde{r}_t]$$

where $1 \leq t \leq c$. The above convolution operation aims to extract useful similarity patterns and each convolution kernel corresponds to a feature. We can adjust the parameters of the kernels when the model is being trained.

---

**Figure 6** The quantum-inspired mechanism (QIM)

### 3.3 Experimental Evaluation

To verify our scheme, we apply QIM to the classical image classification tasks. The specific process of the classical image classification task based on CNN with quantum density matrix is shown in Figure 3 (b), where QIM is marked blue. We design a series of preliminary ablation experiments on Intel dataset to test whether the density
matrices enhance the image feature information and the relationship between the features. Furthermore, we design a series of ablation experiments according to the current better results achieved by typical models on different datasets to verify whether QIM can improve the accuracy of various image classification tasks.

We choose the classical image classification datasets by considering the following reasons:

- they have covered both gray-scale and colored images;
- both small-sized (28x28, 32x32) images and large-sized (150x150) images are considered;
- categories of image data range from 6 to 100.

Taking all above reasons, we apply QIM to the following numerous public image classification datasets.

- **MNIST** [16]: Mnist is a dataset of handwritten digits, which has a training set of 60k examples, and a test set of 10k examples. The digits have been size normalized to 28x28 and centered in a fixed-size image, all of which are black-and-white. Since information inside those images is relatively simple, we apply StandardCNN, LeNet-5, AlexNet, and ZFNet on convolution, not involving deeper CNN such as VGG-19, ResNet-18.

- **FASHION-MNIST** [17]: Fashion-Mnist is a dataset consisting of images related to clothe-ware, shoes, and bag. It also has a training set of 60k examples and a test set of 10k examples. Like Mnist dataset, each example in Fashion-Mnist is a gray-scale image of size 28x28, associated with a label from 10 classes. Though image data are more complex, we still only use StandardCNN, LeNet, AlexNet, and ZFNet on convolution work, excluding deeper CNN.

- **CIFAR-10** [18]: Cifar-10 dataset consists of 60k 32x32 color images divided in 10 classes of objects in reality (e.g. bird, cat), with 6k images per class. There are 50k training images and 10k test images in total. Compared with Mnist-series, Cifar-10 has similar image shape, but more channels (3 instead of 1), which means more abundant information. So we apply StandardCNN, AlexNet, ZFNet to it, but yet not considering deeper CNN such as VGG-19.

- **CIFAR-100** [18]: Cifar-100 is a largescale dataset for image classification, and Cifar-100 task accesses much more categories. Just like Cifar-10, it also consists of 60k 32x32 color images, but divided into 100 classes, with 600 images per class.

- **INTEL** [15]: Intel classification task was published on https://datahack.
Analyticsvidhya.com by Intel initially to host a image classification challenge. Relative resources is an image dataset of natural scenes around the world, which contains around 25k images of size 150x150 distributed under 6 categories. There are around 14k images in train, 3k in test and 7k in prediction. Intel image dataset fits small label size but plentiful image information, thus we consider all kinds of CNN structures.

To verify whether QIM helps classical image classification tasks, we do some preliminary ablation experiments with StandardCNN [31] on Intel dataset at first, and the results are shown in Table 2.

| Approach          | Density Feature Maps Number | Density Feature Map Size | Accuracy  |
|-------------------|-----------------------------|--------------------------|-----------|
| StandardCNN       | None                        | None                     | 77.01     |
|                   | 8                           | 75.69                    |
|                   | 10                          | 76.61                    |
|                   | 12                          | 77.62                    |
|                   | 16                          | 77.02                    |
|                   | 8                           | 77.28                    |
|                   | 10                          | 78.19                    |
|                   | 12                          | 77.04                    |
|                   | 16                          | 76.69                    |
| StandardCNN+QIM   | 32                          | 8                        | 77.54     |
|                   | 10                          | 78.32                    |
|                   | 12                          | 77.62                    |
|                   | 16                          | 76.54                    |
|                   | 8                           | 76.84                    |
|                   | 10                          | 76.91                    |
|                   | 12                          | 76.01                    |
|                   | 16                          | 75.89                    |

We noticed that under many groups of parameters, CNN with QIM could outperform StandardCNN. Such phenomena prove that QIM helps classical image classification tasks, which verifies our guess that density matrices can enhance the image feature information and the relationship between the features. Results of those experiments also show a regular pattern that 128 feature maps with shape of 10x10 may
be the optimum parameters. So we apply this group of parameters to the follow-up ablation experiments. To further verify whether QIM is helpful for kinds of classical image classification tasks with the following models on different datasets, we only add QIM to the current model with the better results, and the comparison results between these better results and the results obtained by adding QIM to these typical models are shown in Table 3. As for Cifar series, Cifar10 dataset contains small-sized images, as well as smaller classification space compared to Cifar 100, so we do not apply large models (e.g. VGG, ResNet) on it.

-StandardCNN [31]: A CNN model carried out for our baseline experiments. Its structure is designed in particular, including 3 convolutional layers, 3 pooling layers and 2 fully connected layers.

-LeNet-5 [27] LeNet is a CNN model. In general, LeNet-5 refers to lenet-5 and it is a simple CNN, built up with 2 convolutional layers, 2 pooling layers and 2 fully connected layers. Though simpler than StandardCNN, it still performs better on some datasets such as Mnist-series.

-AlexNet [25]: AlexNet is a CNN model, and it contains 11 layers: 5 layers are convolutional layers, 3 layers are max-pooling layers, and the last 3 layers are fully connected layers. It used the non-saturating ReLU activation function, which showed improved training performance over tanh and sigmoid. In our experiments, it has the ability to perform well in many tasks.

-ZFNet [28]: ZFNet is a CNN model, which is very similar to AlexNet in structure. Its design was motivated by visualizing intermediate feature layers and the operation of the classifier. ZFNet also contains 11 layers: 5 layers are convolutional layers, 3 layers are max-pooling layers, and the last 3 are fully connected layers. Compared to AlexNet, the filter sizes in ZFNet are reduced and the stride of the convolutions are reduced. Theoretically, ZFNet can be seen as an enhanced version of AlexNet.

-VGG [29]: VGG is a CNN model, and it may be one of the deepest traditional CNNs up to now. It has many types such as VGG-16 and VGG-19. Take VGG-19 as example, it is built up with 24 layers, including 16 convolutional layers, 5 pooling layers and 3 fully connected layers. Its design suits well for multi-sort image classification tasks with large-sized image information, but failed to match small-sized image classification tasks in our experiments. Such result has also proved that a wider and deeper neural network isn’t always equivalent to a better model.
Table 3 Comparison results between the better results of typical models with the results obtained by adding QIM to these typical models.

| Datasets        | Image Size | Category Number | Approach         | Batch Size | Learning Rate | Accuracy  | Comparison |
|-----------------|------------|-----------------|------------------|------------|---------------|-----------|------------|
| Mnist           | 28x28      | 10              | StandardCNN      | 64         | 0.0005        | 97.2622   | +1.8496    |
|                 |            |                 | StandardCNN+QIM  | 64         | 0.0005        | 99.1118   |            |
|                 |            |                 | LeNet            | 64         | 0.0005        | 99.1209   | +0.0373    |
|                 |            |                 | LeNet+QIM        | 64         | 0.0005        | 99.1582   |            |
|                 |            |                 | AlexNet          | 64         | 0.0005        | 99.3395   |            |
|                 |            |                 | AlexNet+QIM      | 64         | 0.0005        | 99.4943   | +0.1813    |
|                 |            |                 | ZFNet            | 64         | 0.0005        | 99.3531   |            |
|                 |            |                 | ZFNet+QIM        | 64         | 0.0005        | 99.5536   | +0.2005    |
| Fashion-Mnist   | 28x28      | 10              | LeNet            | 64         | 0.0005        | 91.3962   | +0.1425    |
|                 |            |                 | LeNet+QIM        | 64         | 0.0005        | 91.5117   |            |
|                 |            |                 | AlexNet          | 64         | 0.0005        | 92.0373   | +0.4006    |
|                 |            |                 | AlexNet+QIM      | 64         | 0.0005        | 92.4379   |            |
|                 |            |                 | ZFNet            | 64         | 0.0005        | 91.9371   | +0.4107    |
|                 |            |                 | ZFNet+QIM        | 64         | 0.0005        | 92.3478   |            |
| Cifar 10        | 32x32      | 10              | StandardCNN      | 64         | 0.0005        | 67.5381   | +8.3834    |
|                 |            |                 | StandardCNN+QIM  | 64         | 0.0005        | 75.9215   |            |
|                 |            |                 | AlexNet          | 64         | 0.0005        | 78.5757   | +1.1919    |
|                 |            |                 | AlexNet+QIM      | 64         | 0.0005        | 79.7676   |            |
|                 |            |                 | ZFNet            | 64         | 0.0005        | 80.1983   | +0.3452    |
|                 |            |                 | ZFNet+QIM        | 64         | 0.0005        | 80.5435   |            |
| Cifar 100       | 32x32      | 100             | AlexNet          | 64         | 0.0005        | 64.5532   | +0.4708    |
|                 |            |                 | AlexNet+QIM      | 64         | 0.0005        | 65.0240   |            |
|                 |            |                 | StandardCNN      | 64         | 0.0005        | 79.4387   | +3.5097    |
|                 |            |                 | StandardCNN+QIM  | 64         | 0.0005        | 82.9484   |            |
|                 |            |                 | AlexNet          | 64         | 0.0005        | 77.6835   | +7.0651    |
|                 |            |                 | AlexNet+QIM      | 64         | 0.0005        | 84.7486   |            |
|                 |            |                 | ZFNet            | 64         | 0.0005        | 83.6956   | +1.8000    |
|                 |            |                 | ZFNet+QIM        | 64         | 0.0005        | 85.4959   |            |
|                 |            |                 | VGG              | 16         | 0.0001        | 76.2433   | +6.6196    |
|                 |            |                 | VGG+QIM          | 16         | 0.0001        | 82.8629   | -0.6304    |
| Intel           | 150x150    | 6               | ResNet           | 32         | 0.0025        | 80.8763   |            |
|                 |            |                 | ResNet+QIM       | 32         | 0.0025        | 80.2459   |            |

-ResNet [30]: Residual neural network (ResNet) is an artificial neural network (ANN) of a kind that builds on structures known from pyramidal cells in the cerebral cortex. Different from how traditional CNNs extract features, ResNet utilizes block connections, or shortcuts to jump over some layers. Typical ResNet models (e.g.
ResNet-18, ResNet-50) are implemented with double or triple layer blocks that contain nonlinearities (ReLU) and batch normalization in between. In our experiments, we have applied ResNet-18 on different tasks. Like VGG-19, ResNet-18 performed well in large-sized image classification tasks, and our QIM still succeed in making it go up a storey still higher.

From Table 3, it is obviously that most models with QIM have higher accuracy on different datasets than before. Fortunately, we have obtained two discoveries about QIM as follows through these experimental results:

- QIM achieves higher performance improvement on accuracy when dimension of image data is obviously broader. In most instances, some attributes of the image data can result in high dimensionality (e.g. size of image, number of categories). Traditional CNNs are not always competent in extracting features of image data, even if the structure of those networks is very colossal, and thus perform not better than satisfactory. As our experiments have shown, QIM has provided an effective way to juice more detailed features of images, which helps to activate residual potential of convolution layers. For example, the experimental results on dataset Cifar 10 are obviously better than the experimental results on datasets Mnist and Fashion-Mnist.

- QIM achieves higher performance improvement on accuracy when the original CNNs have insufficient features for image extraction. On each former image datasets, there is always a novel model that has achieved greatest performance. However, it is impossible to build a network model that has the ability to triumph over every image classification task. When facing with a new-coming image dataset, existing leading models may become unable to do the best. In this situation, QIM could act as a benchmark on how convolution layers fit those images. For example, the accuracy of StandardCNN with QIM is significantly better than other models with QIM on datasets Mnist and Cifar 10 since StandardCNN is the simplest model compared to other models.

As we can see from Table 3, QIM can significantly improve the accuracy of original models (StandardCNN, LeNet, AlexNet, ZFNet, VGG, and ResNet) with better results on different datasets (Mnist, Fashion-Mnist, Cifar 10, Cifar 100, and Intel). All the experiments show that QIM is really helpful for classical image classification tasks and it is applicable with above models on different datasets. The performance of model without/with QIM on different datasets is shown in Figure 7, where the abscissa
represents different image datasets, and the ordinate represents image classification accuracy with different models. It is clear that QIM does help to improve the performance in majority of cases.

![Figure 7 Accuracy of models without/with QIM on different datasets of classifiers trained.](image)

The abscissa represents different image datasets, and the ordinate represents image classification accuracy with different models.

In most cases, image classifiers gained higher accuracy when attached to QIM. Besides, we have noticed that QIM benefits more when original CNN has difficulty in extracting image features effectively. However, there is still another exception. For example, applying ResNet-18 to Intel dataset, QIM has made it more difficult in achieving higher accuracy. A possible reason is that QIM emphasizes mainly on carrying CNN models to extract more detailed information, without making original CNN models much deeper or wider. So QIM may not work well on those tasks in which the original model itself fits well in extracting image information. We stress that QIM is a proof-of-concept, and there are many possible optimizations but they are beyond the scope of this paper.

### 3.4 Case Study

The comparative experiments with StandardCNN on Cifar10 are shown in Figure 8, where the upper part stands for the initial experiment, and the lower part represents the experiment with QIM. In image classification tasks, traditional CNN models usually make consideration of just applying fully connect layers on feature maps extracted by convolution layers, but QIM tends to juice more valid information from those feature maps. Therefore, after convolution layers have produced feature maps, QIM firstly flattens those feature maps and does an outer product calculation within it, in which step the density matrices are built. Secondly, QIM applies another group of large-sized
convolution layers on the density matrices mentioned above. This operation usually makes those matrices resize to 10x10 called the product density feature maps. Thirdly, once density feature maps are produced, row-pooling and column-pooling operations are attached. In this step, all the density feature maps are extracted to rows of features and columns of features. Finally, those row and column features are concatenated together, forming linear features just as the StandardCNN does. QIM has just extended the original model a 3-layer depth, but it is able to boost the final result obviously. In the majority of cases in Table 3, QIM achieves higher performance improvement on accuracy when dimension of image data is obviously broader or original CNN could hardly suit a designated image classification task.

4. Conclusion and Future Work

To investigate whether the quantum density matrix helps the classical question answering problems and the classical image classification tasks, we have designed two schemes. On the one hand, we have applied the quantum density matrix into classical question answering tasks. We have designed a new mechanism based on LSTM with quantum density matrix and further proposed a new LSTM-based question answering model with the quantum density matrix. Experiments of our new model on different data sets have shown encouraging results. On the other hand, we have also applied the quantum density matrix into classical image classification tasks. We have proposed the quantum inspired mechanism, i.e., QIM, which re-represented the quantum density
matrix with linear algebra and combined CNN with density matrices. To the best of our knowledge, this is the first time for the quantum method applied to image classification tasks. In QIM, density matrices enhance image features and the correlation between features in classical CV algorithms, which has been verified by a series of preliminary ablation experiments. We gained the optimum parameters for QIM from preliminary ablation experiments and applied these parameters to the further ablation experiments. Results have shown that QIM has the generalization in classical image classification tasks on different datasets.

To sum up, the application of quantum density matrix both in classical question answering problems and classical image classification tasks have shown more effective performance, which confirms our argues about the quantum density matrix.

For future work, we plan to develop optimization techniques to improve the performance and try to apply the quantum density matrix to other classical tasks.

Acknowledgements This work is supported by the National Natural Science Foundation of China (No. 61976232, and No. 51978675).

References
[1] Martin J H.: Speech and language processing: An introduction to natural language processing, computational linguistics, and speech recognition [M]. Pearson/Prentice Hall, 2009.
[2] Meichanetzidis K, Toumi A, de Felice G, et al.: Grammar-aware question-answering on quantum computers [J]. arXiv preprint arXiv:2012.03756, 2020.
[3] Nielsen M A, Chuang I.: Quantum computation and quantum information [J]. AAPT, College Park, Maryland, 2002.
[4] Beloborodov D, Ulanov A E, Foerster J N, et al.: Reinforcement learning enhanced quantum-inspired algorithm for combinatorial optimization [J]. Machine Learning: Science and Technology, 2020, 2(2): 025009.
[5] Narayanan A, Moore M.: Quantum-inspired genetic algorithms [C]. In Proceedings of IEEE international conference on evolutionary computation. IEEE, 1996: 61-66.
[6] Zhang Y, Li Q, Song D, et al.: Quantum-inspired interactive networks for conversational sentiment analysis [C]. In 28th International Joint Conference on Artificial Intelligence (IJCAI2019), 10-16 Aug 2019, Macau, China, pp. 5436–5442.
[7] Gkoumas D, Li Q, Dehdashti S, et al.: Quantum cognitively motivated decision fusion for video sentiment analysis [C]. In Proceedings of the AAAI Conference on Artificial Intelligence. 2021, 35(1): 827-835.
[8] Zhang Y, Song D, Li X, et al.: A quantum-like multimodal network framework for modeling interaction dynamics in multiparty conversational sentiment analysis [J]. Information Fusion, 2020, 62: 14-31.
[9] Hochreiter S, Schmidhuber J.: Long short-term memory [J]. Neural computation, 1997, 9(8): 1735-1780.
[10] Hirschman L, Gaizauskas R.: Natural language question answering: the view from here [J]. Natural Language Engineering, 2001, 7(4): 275-300.
[11] Fukushima K, Miyake S. Neocognitron: A self-organizing neural network model for a mechanism of visual pattern recognition [M]. Competition and cooperation in neural nets. Springer, Berlin, Heidelberg, 1982: 267-285.
[12] Wang M, Smith N A, Mitamura T. What is the Jeopardy model? A quasi-synchronous grammar for QA [C]. In Proceedings of the 2007 joint conference on empirical methods in natural language processing and computational natural language learning (EMNLP-CoNLL). 2007: 22-32.
[13] Yang Y, Yih W, Meek C.: Wikiqa: A challenge dataset for open-domain question answering [C]. In Proceedings of the 2015 conference on empirical methods in natural language processing. 2015: 2013-2018.
[14] David A, Jean P.: Computer vision: a modern approach [J]. Prentice Hall, 2002: 654-659.
[15] Laakom F, Raitoharju J, Nikkanen J, et al.: Intel-tau: A color constancy dataset [J]. IEEE Access, 2021, 9: 39560-39567.
[16] Byerly A, Kalganova T, Dear I. No routing needed between capsules [J].
Neurocomputing, 2021, 463: 545-553.

[17] Tanveer M S, Khan M U K, Kyung C M. Fine-tuning darts for image classification [C]. In 2020 25th International Conference on Pattern Recognition (ICPR). IEEE, 2021: 4789-4796.

[18] Foret P, Kleiner A, Mobahi H, et al.: Sharpness-aware minimization for efficiently improving generalization [J]. arXiv preprint arXiv:2010.01412, 2020.

[19] Sordoni A, Nie J Y, Bengio Y.: Modeling term dependencies with quantum language models for ir [C]. In Proceedings of the 36th international ACM SIGIR conference on Research and development in information retrieval. 2013: 653-662.

[20] Zhang P, Niu J, Su Z, et al.: End-to-end quantum-like language models with application to question answering [C]. In Proceedings of the AAAI Conference on Artificial Intelligence. 2018, 32(1).

[21] Neumann J, Wigner E P, Hofstadter R.: Mathematical foundations of quantum mechanics [M]. Princeton university press, 1955.

[22] Yu L, Hermann K M, Blunsom P, et al.: Deep learning for answer sentence selection [J]. arXiv preprint arXiv:1412.1632, 2014.

[23] Forsyth D A. J.: Ponce Computer Vision–A Modern Approach [J]. PHPTR, 2002.

[24] Ebrahimi M A, Khoshtaghaza M H, Minaei S, et al.: Vision-based pest detection based on SVM classification method [J]. Computers and Electronics in Agriculture, 2017, 137: 52-58.

[25] Krizhevsky A, Sutskever I, Hinton G E. Imagenet classification with deep convolutional neural networks [C]. In ACM, 60(6):84–90, May 2017.

[26] Hinton G E, Salakhutdinov R R.: Reducing the dimensionality of data with neural networks [J]. science, 2006, 313(5786): 504-507.

[27] LeCun Y, Boser B, Denker J S, et al.: Backpropagation applied to handwritten zip code recognition [J]. Neural computation, 1989, 1(4): 541-551.

[28] Zeiler M D, Fergus R.: Visualizing and understanding convolutional networks [C]. In European conference on computer vision. Springer, Cham, 2014: 818-833.

[29] Ioffe S, Szegedy C. Batch normalization: Accelerating deep network training by reducing internal covariate shift [C]. In International conference on machine learning.
[30] He K, Zhang X, Ren S, et al. Deep residual learning for image recognition [C]. In Proceedings of the IEEE conference on computer vision and pattern recognition. 2016: 770-778.

[31] Chua L O, Yang L.: Cellular neural networks: Theory [J]. IEEE Transactions on circuits and systems, 1988, 35(10): 1257-1272.