A de Sitter tachyonic braneworld revisited

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Abstract. Within the framework of braneworlds, several interesting physical effects can be described in a wide range of energy scales, starting from high-energy physics to cosmology and low-energy physics. An usual way to generate a thick braneworld model relies in coupling a bulk scalar field to higher dimensional warped gravity. Quite recently, a novel braneworld was generated with the aid of a tachyonic bulk scalar field, having several remarkable properties. It comprises a regular and stable solution that contains a relevant 3-brane with de Sitter induced metric, arising as an exact solution to the 5D field equations, describing the inflationary eras of our Universe. Besides, it is asymptotically flat, despite of the presence of a negative 5D cosmological constant, which is an interesting feature that contrasts with most of the known, asymptotically either dS or AdS models. Moreover, it encompasses a graviton spectrum with a single massless bound state, accounting for 4D gravity localized on the brane, separated from the continuum of Kaluza-Klein massive graviton modes by a mass gap that makes the 5D corrections to Newton’s law to decay exponentially. Finally, gauge, scalar and fermion fields are also shown to be localized on this braneworld. In this work, we show that this tachyonic braneworld allows for a nontrivial solution with a vanishing 5D cosmological constant that preserves all the above mentioned remarkable properties with a less amount of parameters, constituting an important contribution to the construction of a realistic cosmological braneworld model.

Keywords: extra dimensions, string theory and cosmology, cosmological applications of theories with extra dimensions, cosmology with extra dimensions

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1 Introduction

In the last two decades, the paradigm underlying the observable Universe as a 3-brane, embedded in a higher-dimensional spacetime, has become a fruitful scenario for addressing several questions in physics, that cover a comprehensive plethora of phenomena, ranging from low energy physics [1–4], gravity [5–9], astrophysics [10–13], and high energy physics [14–18] to cosmology [19–23]. Regarding some prominent cases, the braneworld paradigm leads to plausible reformulations or even complete solutions of these problems (see refs. [24–27] for complete reviews). A striking property of these models consists of pointing out that a higher dimensional world could encode our universe, without any conflict with 4D current experiments and recent observational data.

In the context of the braneworld physics, there is a branch of models in which the fifth dimension is modeled by bulk scalar fields, extending the idea of thin branes to thick brane configurations. Several different thick brane configurations, whose dynamics is generated by a 5D gravitational action with a bulk scalar field and a self-interacting potential have been proposed. The nature of the scalar field setup leads to different specific scenarios (see the reviews [24–26]).

Despite the usefulness of the braneworld paradigm for approaching the above mentioned problems, the quest for a more realistic approach imposes a series of important tests and physical constrains. Namely, the braneworld models, whose signatures must be reflected as small corrections to the 4D physical laws according to experimental and observational data, must be able to recover the 4D physics of our Universe for precise physical limits. Hence, it is essential to define a robust physical notion of 4D gravity and matter fields localization on the brane within these models with the less possible amount of parameters.

In order to get a consistent physical model the braneworld configuration must be stable under small fluctuations of all the gauge, matter, and gravity background fields involved in the setup [28]. Checking for this kind of stability is a highly non-trivial task, strongly dependent on the field content of the model. Another important aspect is the localization of the Standard Model matter (gauge, scalar and fermion fields) onto the brane [29–36]. So far, different scenarios of thick branes have been studied in the literature, revealing that the localization mechanism of matter fields directly depends on the warp factor into the 5D metric [37].

Within this braneworld framework, a tachyonic scalar field has been used to generate such models with recent applications to cosmology [38–40], localization of 4D gravity on expanding 3-branes [41–43] and localization of the Standard Model matter fields [44–47].
The original form of the tachyonic effective action was proposed along a series of articles [48–51] as a supersymmetric generalization of the Dirac-Born-Infeld action that describes the dynamics of light modes (massless and tachyonic) on the world-volume of a non-BPS D-brane within the context of type II string theory in Minkowski spacetime [52, 53]. This tachyonic effective action has found several applications within string cosmology [54–59] and supergravity [60]. Moreover, solar system constraints were imposed on its parameters by considering this action as a scalar-tensor model [61].

From the cosmological point of view, the early Universe and the observed accelerating expansion can be qualitatively described within this de Sitter braneworld model. Thus, these attempts to construct a consistent inflationary braneworld take into account the fact that the cosmic inflationary theory is in good agreement with temperature fluctuation properties of the Cosmic Microwave Background Radiation, and that the inflationary epoch likely took place at very high temperatures [39].

Thus, the construction of a tachyonic braneworld model, that can fulfill the full set of aforementioned tests and physical constraints, is not an easy task. In fact, the corresponding Einstein and field equations are extremely nonlinear, due to the Lagrangian associated with the tachyon field. Despite these difficulties, a great effort has been made in [41] to provide an interesting tachyonic thick braneworld supported by a tachyon scalar field coupled to gravity with a bulk cosmological constant that yields certain phenomenological aspects of our 4D Universe. This model possesses several appealing properties: a) it contains an expanding metric induced on the 3-brane which arises as an exact solution to the 5D field equations; b) the field configuration is completely regular and stable under small perturbations [62]; c) The de Sitter 3-brane describes the inflationary epochs of our universe; d) the braneworld is asymptotically flat, despite the presence of a negative 5D cosmological constant (usually braneworlds are asymptotically dS or AdS); e) it contains a graviton spectrum with a single massless bound state that accounts for 4D gravity localized on the brane; f) it has a mass gap that makes the 5D corrections to Newton’s and Coulomb’s laws decay exponentially; g) finally, gauge, scalar and fermion fields were shown to be localized on this braneworld.

Thus, in this work we show that this tachyonic braneworld model allows for a more general nontrivial solution with no bulk cosmological constant, that preserves all the above mentioned remarkable properties with a less amount of parameters, constituting an important contribution to the construction of a realistic cosmological braneworld model.

The paper is organized as follows: in section 2 the tachyonic thick braneworld model with the 5D cosmological constant is briefly reviewed. In section 3, we obtain a second exact solution without the bulk cosmological constant, also analyzing its underlying parameters and some relevant derived quantities (the tachyonic self-interaction potential, the effective 4D Planck mass, and the curvature scalar), hence comparing them with their expressions when the bulk cosmological constant was present. Finally, a brief discussion of the new solution for our tachyonic braneworld model is presented and elucidated as a set of conclusions in section 4.

2 Review of the tachyonic de Sitter thick braneworld

The 5D action for a thick braneworld model that is produced by gravity and a bulk tachyonic scalar field in the presence of a 5D cosmological constant reads [41]

\[
S = \int d^5x \sqrt{-g} \left( \frac{1}{2\kappa_5^2} R - \Lambda_5 - V(T) \sqrt{1 + g^{AB} \partial_A T \partial_B T} \right),
\]  

(2.1)
where $R$ is the 5D scalar curvature, $\Lambda_5$ is the bulk cosmological constant, the tachyon field $T$ represents the matter in the 5D bulk, $V(T)$ denotes its self-interaction potential, $\kappa_5^2 = 8\pi G_5$ with $G_5$ being the 5D Newton constant, and $A, B = 0, 1, 2, 3, 5$. Besides, a 5D metric ansatz with an induced 3-brane of FLRW background type is be taken to be
\[
ds^2 = e^{2f(w)} [-dt^2 + a^2(t) (dx^2 + dy^2 + dz^2) + dw^2],
\]
where $e^{2f(w)}$ and $a(t)$ are the warp factor and the scale factor of the brane, respectively, and $w$ represents the extra-dimensional coordinate. The corresponding Einstein’s equations for this model read
\[
G_{AB} = -\kappa_5^2 \Lambda_5 g_{AB} + \kappa_5^2 T_{AB},
\]
where $T_{AB}$ is the energy-momentum tensor for the bulk tachyonic scalar field, described by
\[
T_{AB} = -g_{AB} V(T) \sqrt{1 + (\nabla T)^2} + \frac{V(T)}{\sqrt{1 + (\nabla T)^2}} \partial_A T \partial_B T.
\]
The matter field equation corresponding to the action (2.1) is expressed in the following form:
\[
\Box T - \frac{\nabla C \nabla_D T \nabla^C T \nabla^D T}{1 + (\nabla T)^2} = \frac{1}{V} \frac{\partial V(T)}{\partial T}.
\]
By using the metric ansatz (2.2) and the fact that the tachyon field depends only on the extra coordinate in (2.3) and (2.5) (i.e. $T = T(w)$), it is straightforward to write a set of nonlinear coupled field equations, which have the following complete solution:
\[
a(t) = e^{H t}, \quad f(w) = \frac{1}{2} \ln [s \text{sech} (H (2w + c))],
\]
\[
T(w) = \pm \sqrt{-\frac{3}{2 \kappa_5^2 A_5}} \text{arctanh} \left( \frac{\text{sinh} \left[ \frac{H (2w+c)}{2} \right]}{\sqrt{\cosh \left[ H (2w+c) \right]}} \right),
\]
\[
V(T) = -\Lambda_5 \text{sech} \left( \sqrt{-\frac{2}{3 \kappa_5^2 A_5}} T \right) \left( 6 \text{sech}^2 \left( \sqrt{-\frac{2}{3 \kappa_5^2 A_5}} T \right) - 1 \right)
= -\Lambda_5 \sqrt{(1 + \text{sech} [H (2w + c)]) \left( 1 + \frac{3}{2} \text{sech} [H (2w + c)] \right)},
\]
where $H, c$ and $s > 0$ are arbitrary constants. By consistency of the above presented field equations, the constant $s$ must be
\[
s = -\frac{6H^2}{\kappa_5^2 A_5},
\]
where the arbitrary 5D cosmological constant is negative, $\Lambda_5 < 0$, to ensure the real nature of the tachyonic field and its potential. It is important to note that we have an explicit expression for the self-interaction potential in terms of the tachyon field. This potential has a maximum at the position of the brane. Since the tachyonic field has a bounded domain in order to be real, it then leads to a real and bounded potential as well.

By looking at the structure of eq. (2.9) one can see that the field configuration has a limiting case when the bulk cosmological constant vanishes $\Lambda_5 \to 0$ with the same rapidity as.
$H^2 \to 0$ in such a way that $s$ remains finite. In this two-fold limit, the tachyon field becomes linear $T = \pm \frac{s}{4} (2w + c)$, the self-interaction potential vanishes $V(T) = 0$, whereas the scale and warp factor become a constant that can be further ignored through a rescaling of the metric coordinates, leading to a flat 5D spacetime \cite{41}.

The corresponding 5D curvature scalar reads

$$R = -\frac{14}{3} \kappa_5^2 \Lambda_5 \operatorname{sech} [H (2w + c)],$$

which is positive definite and asymptotically flat along the fifth dimension. This assertion is easy to see by observing the action (2.1) along with the asymptotic behavior of the potential (2.8) which cancels the value of the 5D cosmological constant, while $T$ remains constant at infinity.

An important point that should be stressed is that this 5D spacetime is completely free of naked singularities, that usually arise when the mass spectrum of Kaluza-Klein tensorial fluctuations display a mass gap, giving rise to an important feature of the graviton mass spectrum from the phenomenological point of view (see refs. \cite{5–7} for further details).

A detailed analysis of the stability of the tensorial metric fluctuations of the tachyonic braneworld was performed in \cite{41}, showing that 4D gravity is consistently localized on it. Moreover, the corrections to the Newton’s law, due to the 5D massive fluctuations, were computed in the thin brane limit and shown to be exponentially suppressed. On the other hand, the stability of the model under small fluctuations of the tachyon field and the scalar metric modes was successfully achieved with the aid of an auxiliary Sturm-Liouville eigenvalue problem for any value of the bulk cosmological constant in \cite{62}.

### 3 The thick brane with $\Lambda_5 = 0$ model and its solution

The action (2.1) that describes our tachyonic braneworld model presents a second exact solution that can be obtained from the Einstein’s equations when the bulk cosmological constant vanishes

$$G_{AB} = \kappa_5^2 T_{AB}. \quad (3.1)$$

By making use of the metric ansatz (2.2), the Einstein tensor components read

$$G_{00} = 3 \frac{\ddot{a}}{a^2} - 3 \left( f'' + f' \right),$$

$$G_{\alpha \alpha} = -2 \ddot{a} a - \dot{a}^2 + 3 a^2 \left( f'' + f' \right),$$

$$G_{55} = -3 \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) + 6 f'^2, \quad (3.2)$$

where "$\ddot{\cdot}$" and "$\dot{\cdot}$" are derivatives with respect to the extra dimension and time, respectively, whereas the index $\alpha$ labels the spatial dimensions $x$, $y$ and $z$.

The main goal of this work is to construct a new relevant solution in the context of the above mentioned tachyonic thick braneworld with a vanishing 5D cosmological constant and analyze whether the appealing properties of the brane field configuration (2.6)–(2.8) are still present in the new solution.

By taking into account the fact that the tachyon field depends only on the fifth dimension, $T(w)$, eq. (2.5) adopts the form

$$T'' - f' T' + 4 f T' (1 + e^{-2f T'^2}) = (e^{2f} + T'^2) \frac{\partial f}{V(T)} V(T). \quad (3.3)$$
At the same time the Einstein’s equations (3.1) can be rewritten as

\[
f'' - f' + \frac{\ddot{a}}{a} = -\kappa_5^2 \frac{V(T)T^2}{3\sqrt{1 + e^{-2f}T^2}},
\]

\[
f'' - \frac{1}{2} \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) = -\kappa_5^2 \frac{e^{2f} V(T)}{6\sqrt{1 + e^{-2f}T^2}}.
\]

By demanding consistency of this system of equations, the 3-brane solution corresponds to a 4D de Sitter cosmological background defined by

\[
a(t) = e^{H(t-t_0)},
\]

however, the constant factor \(a_0 = 1/e^{Ht_0}\) can be absorbed by a rescaling of the spatial coordinates. Thus, this result in a braneworld model in which the induced metric on the 3-brane is described by a \(dS_4\) geometry.

By further taking into account the form of the scale factor \(a(t) = e^{Ht}\), we can recast (3.4) and (3.5) as follows

\[
f'' - f' + H^2 = -\kappa_5^2 \frac{V(T)T^2}{3\sqrt{1 + e^{-2f}T^2}},
\]

\[
f'' - H^2 = -\kappa_5^2 e^{2f} V(T)
\]

Now it is straightforward to obtain separate equations for the derivative of the scalar field \(T\) and the self-interaction potential \(V(T)\) from eqs. (3.7) and (3.8), leading to:

\[
T' = \pm e^f \sqrt{\frac{f'' - f'^2 + H^2}{2(f'^2 - H^2)}},
\]

\[
V(T) = \pm \frac{3}{\kappa_5^2} e^{-2f} \sqrt{2 (f'' + f'^2 - H^2) (f'^2 - H^2)}.
\]

Therefore, by determining the nature of the scaling and warp factors, the self-interaction potential \(V(T)\) and the derivative of the tachyon field \(T\) are completely determined. However, care must be taken, since the tachyon scalar field must be real and have a form that ensures the localization of 4D gravity, whereas the self-interaction potential must be real and well defined in the sense that ensures the stability of the whole braneworld field configuration. These restrictions are very demanding, several warp factors with “convenient” localizing behavior lead to a complex tachyon field \(T\). On the other hand, ensuring a real and stable tachyon potential often yields either a complex tachyon field \(T\) or a warp factor that does not enable the localization of 4D gravity. Hence, we propose the most general form for the warp factor as follows

\[
f(w) = -n \ln \left[ \frac{\cosh \left( \frac{H(w + c)}{s} \right)}{s} \right],
\]

where \(H, c, s,\) and \(n\) are constants. Here we should demand \(s > 0\) and \(n > 0\) in order to have a warp factor that ensures the localization of 4D gravity and other matter fields. It is straightforward to realize that this warp factor is a solution of the Einstein and field
equations if the tachyon scalar field adopts the general form\footnote{The case \(n = 1\) yields a constant tachyon scalar field, and therefore a constant tachyon potential \(V(T)\), that at the level of the Lagrangian \((2.1)\) corresponds to recovering the Einstein-Hilbert action with a positive cosmological constant that was already shown to localize gravity with the warp factor \((3.11)\) without the aid of matter fields [63].}

\[
T(w) = \pm \sqrt{\frac{n}{2(1-n)}} s^n 2F_1 \left( \frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cosh^2 \left[ H \left( \frac{w}{n} + c \right) \right] \right) \frac{H \cosh^{n-1} \left[ H \left( \frac{w}{n} + c \right) \right]}{k}, \quad n \neq 1, \tag{3.12}
\]

where the hypergeometric function is defined as

\[
2F_1 (a, b; c; z) = \frac{\Gamma(b-a)\Gamma(c)(-z)^{-a}}{\Gamma(b)\Gamma(c-a)} \sum_{k=0}^{\infty} \frac{(a)_k(a-c+1)_k z^{-k}}{k!(a-b+1)_k} + \frac{\Gamma(a-b)\Gamma(c)(-z)^{-b}}{\Gamma(a)\Gamma(c-b)} \sum_{k=0}^{\infty} \frac{(b)_k(b-c+1)_k z^{-k}}{k!(b-a+1)_k} \tag{3.13}
\]

outside the unit circle, i.e. \(|z| \geq 1\), which is the case corresponding to \(z = \cosh^2 \left[ H \left( \frac{w}{n} + c \right) \right]\); here \(a - b\) cannot be an integer, the Gamma function \(\Gamma(x)\) is undefined for non-positive integers (these conditions restrict \(n\) to adopt non-integer values); the Pochhammer symbol reads \((y)_m = y(y+1) \cdots (y+m-1)\) for \(m > 0\) and \((y)_0 = 1\); and \(k\) denotes a complex constant in general.

In eq. \((3.12)\) the product \(\cosh^{1-n} \left[ H \left( \frac{w}{n} + c \right) \right] 2F_1 \left( \frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cosh^2 \left[ H \left( \frac{w}{n} + c \right) \right] \right)\) is a purely imaginary function plus a real constant. Therefore, within the domain \(0 < n < 1\) the tachyon scalar field is a purely real field configuration added with an imaginary constant contribution that can be eliminated with the aid of the constant \(k\) for a given value of \(n\); note that in this case the radicand is positive. On the other hand, for \(n > 1\) this radicand becomes negative and eq. \((3.12)\) renders an imaginary tachyon scalar field configuration plus a real constant that can be made away with the aid of the constant \(k\).

Thus, for non-integer positive values of \(n\), the tachyon scalar field can be purely real only within the domain \(0 < n < 1\), since eq. \((3.12)\) yields a purely imaginary tachyon field for non-integer values of \(n > 1\).

From the relation \((3.10)\) the tachyon potential is given by the following expression

\[
V(w) = \frac{3H^2 \sqrt{2(n+1)}}{k^2 s^{2n}} \sech^{2(1-n)} \left[ H \left( \frac{w}{n} + c \right) \right]. \tag{3.14}
\]

It is convenient to have at hand a real tachyon scalar field that allow us to explicitly write the self-interaction potential \(V(w)\) in terms of the scalar field \(T\). In order to satisfy both conditions we consider the particular case when \(n = \frac{1}{2}\). For this specific value, the warp factor, the tachyon scalar field and the self-interaction potential read

\[
f(w) = -\frac{1}{2} \ln \left[ \frac{\cosh \left[ H \left( 2w + c \right) \right]}{s} \right], \tag{3.15}
\]

\[
T(w) = \pm \frac{1}{\sqrt{2b}} i \text{EllipticF} \left( iH \left( w + \frac{c}{2} \right), 2 \right), \tag{3.16}
\]

\[
V(T) = \frac{3\sqrt{6} b^2}{k^2 s} \sech \left[ 2\text{JacobiAmplitude} \left( i\sqrt{2b} T, 2 \right) \right] = \frac{3\sqrt{6} b^2}{k^2 s} \sech \left( H \left( 2w + c \right) \right), \tag{3.17}
\]
Figure 1. The profile of the tachyonic scalar field $T$. The thin line represents the positive branch of the field and the thick line is associated to the negative branch of the tachyon. Here we have set $n = 1/2$, $c = 0$, $H = 1$, $2 \kappa^2_5 = 1$ and $s = 1$ for simplicity.

Figure 2. The shape of the self-interaction potential of the tachyonic scalar field $V(T)$. We set $n = 1/2$, $c = 0$, $H = 1$, $2 \kappa^2_5 = 1$ and $s = 1$ for simplicity.

where the JacobiAmplitude$(u, m)$ gives the amplitude $am(u|m)$ for Jacobi elliptic functions, i. e., it is the inverse of the elliptic integral of the first kind. In the last two equations we set

$$s = \frac{H^2}{b^2}. \tag{3.18}$$

where $b$ is an arbitrary constant and the scale factor remains the same.

Therefore, the set of equations (3.6), (3.15), (3.16), and (3.17) provide a new relevant tachyonic thick braneworld configuration. By comparing with the previous solution (2.6), (2.7) and (2.8), the new warp factor (3.11) also has a decaying and vanishing asymptotic behavior for $0 < n < 1$ and coincides with (2.6) when $n = 1/2$. Similarly, the new tachyon scalar field (3.16) is real and possesses a kink– or antikink-like profile as shown in figure 1, but with the positive and negative branches opposite to those presented by the field (2.7). On the other hand, the potential (3.17) also has a maximum at the position of the brane, it is positive definite and bounded (from below and above), but unlike (2.8), it vanishes as $w$ tends to infinity or, equivalently, as $T$ approaches its asymptotic value (see figure 2), emphasizing the fact that the braneworld is asymptotically flat.

It is important to recall that the new solution presented in this work cannot be recovered from the solution (2.6)–(2.8) with the aid of a two-fold limit. On the other hand, it can be noticed as well that when $\Lambda_5 = 0$, the tachyon scalar field (2.7) diverges, while the potential (2.8) vanishes, indicating that this is not a physical limit.
By substituting eqs. (3.12) and (3.14) into eqs. (3.7) and (3.8), we can see that there is no restriction on the $s$ parameter coming from the field equations of the system. This contrasts with the first solution generated by the warp factor (2.6) with $\Lambda_5 \neq 0$, where the field equations demand the relation $s = -\frac{6H^2}{\kappa_5^2 \Lambda_5}$.

However, when we compute the effective 4D Planck mass $M_{Pl}$ for the special value $n = \frac{1}{2}$ with no bulk cosmological constant $\Lambda_5 = 0$, then we obtain $s = \frac{H^2}{b^2}$, where $b$ is an arbitrary constant, yielding

$$M_{Pl}^2 \sim M_s^3 \frac{\sqrt{\frac{2}{\pi}} \Gamma \left(\frac{3}{4}\right)^2 H^2}{b^3}.$$  (3.19)

This expression shows the relationship between the Planck mass in 5D and 4D, adjusted with the $b$ parameter. Comparing to the braneworld model presented in [41], the effective Planck mass in 4D is related to the 5D one by

$$M_{Pl}^2 \sim -M_s^3 \frac{\sqrt{\frac{2}{\pi}} \Gamma \left(\frac{3}{4}\right)^2 H^2}{\Lambda_5^3},$$  (3.20)

which diverges when taking the single limit $\Lambda_5 \to 0$. However, one can still perform a two-fold limit, leading to a finite value of the 4D Planck mass $M_{Pl}$.

Finally, computing the 5D curvature scalar for our solution (3.11) yields

$$R = \frac{4H^2(3n + 2)}{n s^{2n}} \text{sech}^{2(1-n)} \left[ H \left( \frac{w}{n} + c \right) \right],$$  (3.21)

in agreement with the asymptotic form dictated by the self-interaction potential (3.17). For the special value $n = 1/2$, it reads

$$R = 28 b^2 \text{sech} \left[ H \left( 2w + c \right) \right].$$  (3.22)

This 5D invariant is positive definite and asymptotically vanishes, yielding an asymptotically 5D Minkowski spacetime as the solution (2.6)–(2.8) does. However, unlike the curvature scalar corresponding to the braneworld [41], this 5D invariant is obtained within a simpler theory, since the original action (2.1) has one less input parameter ($\Lambda_5 = 0$).

4 Discussion and conclusions

We presented a new complete solution for the braneworld model (2.1), generated by a tachyonic scalar field minimally coupled to gravity, with no bulk cosmological constant, and a 5D warped metric ansatz (2.2), with respect to which the 3-brane metric defines a FLRW geometry. The resulting spacetime is regular and stable along the whole fifth dimension for certain values of the $n$ parameter. Analytic expressions were derived for the physically meaningful warp factor (3.11), the tachyon scalar field (3.12), the self-interaction potential (3.14) and the curvature scalar (3.21), for the significant values of the $n$ parameter (for $0 < n < 1$).

On the one hand, the profile of the warp factor (3.11) and the 5D Ricci scalar (3.21) show a regular structure within the domain $n \in (0, 1)$. These geometrical quantities show that 4D gravity, as well as other Standard Model matter fields, can be localized in the tachyonic de Sitter braneworld model, whenever $0 < n < 1$. On the other hand, the tachyon scalar field (3.12) and its self-interaction potential (3.14) are real only for the aforementioned
domain $0 < n < 1$. Moreover, the potential (3.14) leads to a completely stable braneworld configuration, in full compliance to the treatment presented in [62]. In fact, it has a maximum for precisely this range of the $n$ parameter.

It is worth noticing as well a physically consistent picture at the level of the Einstein’s equations: the geometrical left hand side localizes the known 4D interactions of our world and renders a regular, asymptotically flat braneworld for $0 < n < 1$, whereas the matter right hand side is real and makes the whole field configuration stable, for the same domain $0 < n < 1$ within our setup.

It is important to note that the 5D spacetime, for the special value $n = 1/2$, supports the same geometrical configuration as the solution presented in [41]. However, it has one parameter less involved in the initial braneworld action, since $\Lambda_5 = 0$. This fact yields the localization properties of the model, with $n = 1/2$ remaining the same, since the form of the warp factor determines the character of the 5D graviton spectrum of KK massive modes (which can be gapless or possess a mass gap, for instance) and its localizing properties.

Thus, the braneworld solution (3.15) presented in section 3 preserves all the aforementioned appealing properties that we look for in such a model for being realistic. In fact, it is completely regular and stable under small metric and tachyon field perturbations, it localizes 4D gravity and from the low-energy point of view recovers the Newton’s law [41] and the Coulomb’s law [44] in the thin brane limit with small, exponentially suppressed corrections that come from the extra dimension. Moreover, the structure of the warp factor given in eq. (3.15) for this model makes possible the localization of massive (massless) scalar fields with spin-0 [46], gauge boson fields with spin–1 [45], and spin–1/2 fermions [44].

It seems that the localization of these matter fields can be achieved for any value of the $n$ parameter within the interval $0 < n < 1$. However, it is not straightforward to invert the relation we obtain upon integration of (3.9) in order to get an explicit expression for the tachyonic scalar field in terms of the fifth dimension and an analytic expression for the self-interaction potential in terms of the tachyon field for a value $n \neq 1/2$, representing a major technical difficulty that we are currently trying to address.

Finally, one might use the information entropy on thick branes [64, 65], to determine a more precise bound on the $n$ parameter and to derive refinements of the model here presented.

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