Dark Energy and QCD Ghost

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Abstract

It has been suggested that the dark energy that explains the observed accelerating expansion of the universe may arise due to the contribution to the vacuum energy of the QCD ghost in a time-dependent background. The argument uses a four-dimensional simplified model. In this paper, we put the discussion in more realistic model keeping all components of the QCD vector ghost and show that indeed QCD ghost produces dark energy proportional to the Hubble parameter $H\Lambda_{\text{QCD}}^3$ ($\Lambda_{\text{QCD}}$ is the QCD mass scale) which has the right magnitude $\sim (3 \times 10^{-3} \text{ eV})^4$. 
1 Introduction

The recent cosmological observations have confirmed the existence of the early inflationary epoch and the accelerated expansion of the present universe [1]-[3]. Observational result is consistent with the picture that the universe has an unknown form of energy density, named the dark energy, about 75% of the total energy density. The simplest possibility is the existence of the vacuum energy or cosmological constant whose origin is yet to be identified.

Such vacuum energy is easily incorporated in the quantum field theory. In the standard model of particle physics, we have Higgs field which produces electroweak phase transition, which changes the value of the vacuum energy. It has also been known that the vacuum fluctuations in quantum field theory naturally induce such a vacuum energy, but the problem is how to control the size of it. The contribution of quantum fluctuations in known fields up to 300 GeV, which is about the highest energy at which the current theories have been verified, gives a vacuum energy density of order \((300 \text{ GeV})^4\). This is vastly larger than the observed dark energy density \((3 \times 10^{-3} \text{ eV})^4\) by a factor of order \(10^{56}\). Assuming the tree-level contribution is zero, it is a great challenge how to understand the origin of this tiny energy density.

Recently a very interesting suggestion on the origin of a cosmological constant is made, without introducing new degrees of freedom beyond what are already known, with the cosmological constant of just the right magnitude to give the observed expansion [4]. In this proposal, it is claimed that the cosmological constant arises from the contribution of the ghost fields which are supposed to be present in the low-energy effective theory of QCD [5, 6, 7, 8, 9]. The ghosts are required to exist for the resolution of the U(1) problem, but are completely decoupled from the physical sector [9]. The above claim is that the ghosts are decoupled from the physical states and make no contribution in the flat Minkowski space, but once they are in the curved space or time-dependent background, the cancellation of their contribution to the vacuum energy is off-set, leaving a small energy density \(\rho \sim H \Lambda_{\text{QCD}}^3\), where \(H\) is the Hubble parameter and \(\Lambda_{\text{QCD}}\) is the QCD mass scale of order a hundred MeV. With \(H \sim 10^{-33} \text{ eV}\), this gives the right magnitude. This coincidence is remarkable and suggests that we are on the right track.

However in this proposal, the authors discuss a four-dimensional model similar to the one based on the Schwinger model (proposed by Kogut and Susskind [10]), keeping only the longitudinal and scalar components of the QCD ghost. These scalar fields have positive and negative norms and cancel with each other, leaving no trace in the physical subspace, but it is argued that they have small contribution to the vacuum energy in the curved space or time-dependent background. (Similar system is used in a mechanism of supersymmetry breaking in Ref. [11].) However it is known that the QCD ghost must be intrinsically vector field in order for the U(1) problem to be consistently resolved within the framework of QCD [12]. It is thus an interesting and important question to examine if the mechanism works even if we formulate the proposal keeping all the modes of the vector ghost.

In the next section, we briefly recapitulate how the U(1) problem is resolved by the vector ghost following [9], and also show how the ghost decouples from the physical sector. In sect. 3, following the discussions in Ref. [13], we discuss the mechanism of generating a tiny contribution to the vacuum energy in the Rindler space as a typical example of the time-dependent background [14, 15]. We argue that due to the change of the definition of the vacuum, we indeed obtain small contribution to the vacuum energy proportional to the Hubble parameter. We find that our result is factor 2 larger than the previous estimate. Sect. 4 is devoted to the discussions and conclusions.
2 Decoupling of the vector ghost in the Minkowski space

We consider the low-energy effective Lagrangian [7, 8, 9]

\[
\mathcal{L} = \frac{1}{2} (\partial_\mu S)^2 - \frac{1}{2} m^2_{NS} S^2 + \frac{1}{2 F_S^2 (m_S^2 - m_{NS}^2)} (\partial_\mu K^\mu)^2 - \frac{1}{F_S} S \partial_\mu K^\mu,
\]

where \( S \) is a flavor-singlet pseudoscalar field with the decay constant \( F_S \), \( K^\mu \) is an axial vector “field”, which in QCD corresponds to the gluonic current

\[
K^\mu = 2 N_f \frac{g^2}{16 \pi^2} \epsilon^{\nu\lambda\sigma} A^a_\nu \left( \partial_\lambda A^a_\sigma + \frac{1}{3} g f_{abc} A^b_\lambda A^c_\sigma \right),
\]

where \( N_f \) is the number of flavors.

The Lagrangian (1) is invariant under the gauge transformation

\[
K^\mu \rightarrow K^\mu + \epsilon^{\nu\lambda\sigma} \partial_\nu \Lambda_{\lambda\sigma},
\]

where \( \Lambda_{\lambda\sigma} \) denotes an arbitrary antisymmetric tensor. In fact this transformation reflects the color gauge invariance of the underlying QCD. Under the QCD gauge transformation, the gluonic current transforms as [9, 12]

\[
[Q_B, K^\mu] = 2 i N_f \frac{g^2}{16 \pi^2} \epsilon^{\nu\lambda\sigma} \partial_\nu (C^a \partial_\lambda A^a_\sigma),
\]

where \( Q_B \) is the BRST charge and \( C^a \) is the Faddeev-Popov ghost.

To quantize this system, we have to break the gauge invariance under (3). This can be done by adding the term

\[
\frac{1}{4 F_S^2 (m_S^2 - m_{NS}^2) \alpha} (\partial_\mu K^\nu - \partial_\nu K^\mu)^2
\]

where \( \alpha \) is a gauge parameter. The simplest case is to choose \( \alpha = 1 \). One can then derive the Feynman rules as follows:

- \( K^\mu \)-propagator: \( \frac{i \eta^{\mu\nu} k^2}{k^2 - m^2_{NS} F_S^2 (m_S^2 - m^2_{NS})} \)
- \( S \)-propagator: \( \frac{i}{k^2 - m^2_S} \)
- \( S - K^\mu \)-mixing: \( \frac{1}{F_S} k^\mu \)

It appears that the system (1) describes a scalar field \( S \) with mass \( m_{NS} \) and a massless vector. However it is not difficult to show that the mass of the scalar \( S \) gets shifted to \( m_S \) due to the mixing of the scalar and vector modes. This is the approach that Veneziano took in [6]. Alternatively, in the general gauge, one can derive the two-point functions

\[
T(K_\mu(x)K_\nu(y))_M = \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot (x-y)} \left\{ \frac{k^2 - m_{NS}^2}{k^2 - m_S^2} \left( \frac{k\mu k_\nu}{k^2 - m^2_S} \right) \right. \\
+ \left. \frac{\alpha}{k^2} \left( \eta^{\mu\nu} - \frac{k\mu k_\nu}{k^2} \right) \right\} F_S^2 (m_S^2 - m_{NS}^2),
\]

\[
T(S(x)S(y))_M = \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{i}{k^2 - m^2_S},
\]

\[
T(S(x)K_\mu(y))_M = \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{F_S (m_S^2 - m^2_{NS})}{k^2 (k^2 - m^2_S)},
\]

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where $T$ denotes the time-order and the subscript $M$ stand for the expectation value by the Minkowski vacuum. We see that the two-point function of $S$ clearly shows that it has the shifted mass $m_S$ instead of the original $m_{NS}$, confirming the above claim. This is one of the consequences of the massless mode $K_\mu$ and gives a resolution of the $U(1)$ problem. What happens to the massless mode in the system? Because we observe no massless field in the low-energy world, it must decouple from the physical sector. The precise mechanism of this was not clear in the approach of Ref. [6] but it was simply assumed that it decouples because it is gauge-variant, as indicated above. We now show how this can be achieved in the Minkowski space.

One can derive the field equations from the Lagrangian (1):
\[
\Box K_\mu - \partial_\mu \{(1-\alpha)\partial^\nu K_\nu + F_\nu (m^2_S - m^2_{NS}) S\} = 0,
\]
\[
\Box S + m^2_{NS} S + \frac{1}{F_\nu} \partial^\mu K_\mu = 0.
\]
(8)

The first equation in (8) tells us that $\Box K_\mu$ is expressed as a gradient of a field, so we find
\[
\Box (\partial_\mu K_\nu - \partial_\nu K_\mu) = 0.
\]
(9)

Hence we can consistently impose the subsidiary condition on the physical states:
\[
(\partial_\mu K_\nu - \partial_\nu K_\mu)^{(+)}|_{\text{phys}} = 0.
\]
(10)

In the gauge $\alpha = 1$, we can write the mode expansion of the vector field. In terms of the canonically normalized field $K'_\mu(x) \equiv F_\nu \sqrt{m^2_S - m^2_{NS}} K_\mu(x)$, it takes the form
\[
K'_\mu(x) = \int \frac{d^3k}{(2\pi)^{3/2} \sqrt{2k_0}} e^{(\lambda)}_\mu [e^{-ik\cdot(x-y)} a(k, \lambda) + e^{ik\cdot(x-y)} a^\dagger(k, \lambda)],
\]
(11)

where $e^{(1)}_\mu = (0, e^{(1)})$ and $e^{(2)}_\mu = (0, e^{(2)})$ represent the transverse modes, $e^{(3)}_\mu = (0, e^{(3)})$ the longitudinal mode, and $e^{(0)}_\mu = (1, 0)$ the time component with
\[
e^{(3)} = \frac{k}{|k|}, \quad e^{(1)} \cdot e^{(2)} = e^{(1)} \cdot e^{(3)} = e^{(2)} \cdot e^{(3)} = 0.
\]
(12)

Canonical quantization of the system then gives
\[
[a(k, \lambda), a^\dagger(k', \lambda')] = \eta_{\lambda\lambda'} \delta^3(k - k'),
\]
(13)

We see that the transverse modes have the opposite sign to the usual gauge fields. It is then easy to see that the condition (10) means that the two transverse components $a(k, 1), a(k, 2)$ and the combination $\frac{1}{\sqrt{2}}[a(k, 3) - a(k, 0)]$ should annihilate the physical state. This forbids the states generated by the two transverse components and by $\frac{1}{\sqrt{2}}[a^\dagger(k, 3) + a^\dagger(k, 0)]$. The remaining component $\frac{1}{\sqrt{2}}[a^\dagger(k, 3) - a^\dagger(k, 0)]$ can only produce zero norm states, so that all the components of $K_\mu$ are completely decoupled from the physical state. Nevertheless it produces the physical effect of shifting the mass of the singlet pseudoscalar $S$ and resolves the problem associated with the $\eta'$ meson decay [9]. In the Kogut-Susskind model, similar subsidiary condition can be imposed [4, 11].

We are now going to see what effects this massless mode may produce if our space is not just the Minkowski but curved space or time-dependent.
3 Vector ghost in the Rindler space

In this section, we consider the QCD vector ghost in the Rindler space. Consider the Minkowski space

\[ ds^2 = dt^2 - dx^2 - dy^2 - dz^2 = d\bar{u}d\bar{v} - dx^2 - dy^2, \] (14)

where we have defined

\[ \bar{u} = t - z, \quad \bar{v} = t + z. \] (15)

Under the transformation

\[ t = \frac{1}{a} e^{a\xi} \sinh a\eta, \quad z = \frac{1}{a} e^{a\xi} \cos a\eta, \quad (\infty < \eta, \xi < \infty, a > 0), \] (16)

we obtain

\[ ds^2 = e^{2a\xi} (v - u)dudv - dx^2 - dy^2 = e^{2a\xi} (dy^2 - d\xi^2) - dx^2 - dy^2, \]

\[ \bar{u} = -\frac{1}{a} e^{a(\xi - \eta)} \equiv -\frac{1}{a} e^{-au}, \quad \bar{v} = \frac{1}{a} e^{a(\xi + \eta)} \equiv \frac{1}{a} e^{av}, \] (17)

The Rindler coordinates \( \eta \) and \( \xi \) in (16) describe only the quadrant part \( z > |t| \) called R. The opposite quadrant part L: \( z < -|t| \) is described by changing the signs in (16). The rest of our Minkowski space are described by analytic continuation of these coordinates [14, 15].

The wave function to be used in our massless vector field can be obtained from the solutions for the scalar wave equation

\[ \phi^{\alpha \beta} = 0. \] (18)

Explicitly this becomes in our coordinate system

\[ [e^{-2a\xi} (\partial^2_\eta - \partial^2_\xi) - \partial^2_x - \partial^2_y] \phi = 0. \] (19)

We denote by \( R_u(k) \) the wave function which asymptotes

\[ R_u(k) = \begin{cases} e^{-ik_0u} e^{-i(k_1x + k_2y)} & \text{in R} \\ 0 & \text{in L} \end{cases} \] (20)

along the surface \( v = -\infty, \bar{v} = 0 \), the past horizon of the Rindler coordinate. Similarly the wave function in the L region is defined as

\[ L_u(k) = \begin{cases} 0 & \text{in R} \\ e^{ik_0v} e^{-i(k_1x + k_2y)} & \text{in L} \end{cases} \] (21)

The positive-frequency Minkowski modes are characterized by the condition that they are analytic and bounded in the lower half complex \( \bar{u} \) plane on \( \bar{v} = 0 \). The combinations

\[ \frac{1}{2 \sinh(\pi k_0/(2a))^{1/2}} \left( e^{\pi k_0/(2a)} R_u(k) + e^{-\pi k_0/(2a)} L_u(-k)^* \right), \] (22)

\[ \frac{1}{2 \sinh(\pi k_0/(2a))^{1/2}} \left( e^{-\pi k_0/(2a)} R_u(-k)^* + e^{\pi k_0/(2a)} L_u(k) \right), \] (23)

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where \( L u(-k) \) and \( R u(-k) \) denote the wave function with minus momenta, have precisely this property [14, 15], so we can use these modes to express our Minkowski space field:

\[
K_\mu^\nu(x) = \int \frac{d^3k}{(2\pi)^{3/2}2k_0} \frac{\delta^{(3)}(k)}{2 \sinh(\pi k_0/(2a))^{1/2}} \left[ e^{\pi k_0/(2a)} R u(k) + e^{-\pi k_0/(2a)} L u(-k)^* \right] a^{(1)}(k, \lambda) + \text{h.c.},
\]

and the Minkowski vacuum is defined as

\[
a^{(i)}(k, 1)|0_M\rangle = a^{(i)}(k, 2)|0_M\rangle = a^{(i)}(k, 3) - a^{(i)}(k, 0)|0_M\rangle = 0, \quad (i = 1, 2).
\]

The field in the Rindler space is written as

\[
K_\mu^\nu(x) = \int \frac{d^3k}{(2\pi)^{3/2}2k_0} \frac{\delta^{(3)}(k)}{2 \sinh(\pi k_0/(2a))^{1/2}} \left[ e^{\pi k_0/(2a)} R u(k) b^{(1)}(k, \lambda) + L u(k)^* b^{(1)}(k, \lambda) \right.
\]

\[
+ \left. R u(k) b^{(2)}(k, \lambda) + L u(k)^* b^{(2)}(k, \lambda) \right],
\]

Comparing (24) and (26), we see that

\[
b^{(1)}(k, \lambda) = \frac{1}{\sqrt{2 \sinh(\pi k_0/a)}} \left[ e^{\pi k_0/(2a)} a^{(2)}(k, \lambda) + e^{-\pi k_0/(2a)} a^{(1)*}(k, \lambda) \right],
\]

\[
b^{(2)}(k, \lambda) = \frac{1}{\sqrt{2 \sinh(\pi k_0/a)}} \left[ e^{\pi k_0/(2a)} a^{(1)}(k, \lambda) + e^{-\pi k_0/(2a)} a^{(2)*}(k, \lambda) \right].
\]

The resulting energy for each mode is then given by

\[
\langle 0_M | \int d^3k' \sum_{\lambda, \lambda'} \frac{3}{k_0} b^{(1)}(k, \lambda) b^{(1)*}(k', \lambda') |0_M\rangle
\]

\[
= \langle 0_M | \int d^3k' \sum_{\lambda, \lambda'} \frac{3}{k_0} b^{(2)}(k, \lambda) b^{(2)*}(k', \lambda') |0_M\rangle = \frac{4k_0}{e^{2\pi k_0/a} - 1}.
\]
The contribution of high frequency modes is suppressed by the factor $e^{-2\pi k_0/a}$ and the main contribution comes from $k_0 \sim a$. In the cosmological context, $a \sim H$ and hence $k_0 \sim H$, giving the vacuum energy proportional to the Hubble parameter. In the context of strongly interacting confining QCD with topological nontrivial sector, this effect occurs only in the time direction and their wave function in other space directions is expected to have the size of QCD energy scale. As a result, this ghost gives the vacuum energy density $H\Lambda_{QCD}^3$ of the right magnitude $\sim (3 \times 10^{-3} \text{ eV})^4$. Thus this vacuum energy arises due to the mismatch between the vacuum energies computed in slowly expanding universe and Minkowski space.

For the modes with $k_0 \sim 1 \text{ K} \sim 10^{-4} \text{ eV}$ as in our present universe, the contribution is suppressed by $\exp(-\frac{k_0}{H}) \sim \exp(-10^{29})$. The deviation from Minkowski space starts only for modes with large wave length $\lambda \sim a^{-1}$. Thus the effect is infrared in nature. The local physics with $k_0 \gg a$ is not affected by the unphysical modes with high accuracy.

4 Discussions and conclusions

In this paper, correcting the argument of Ref. [4] in accordance with QCD, we have first clarified the decoupling mechanism of the QCD vector ghost in the flat Minkowski space, and then evaluated the contribution of the QCD vector ghost to the vacuum energy density in the Rindler space as a typical example of time-dependent spacetime, and found that it gives the vacuum energy proportional to the Hubble parameter.

This model has extremely interesting feature. First of all, this does not assume new degrees of freedom only to produce nonzero cosmological constant. Rather it is induced by the already existing field just because the universe is expanding. Secondly it gives the amazing result of the cosmological constant of right magnitude without artificial fine tuning. Note that the vacuum energy is not just a constant but depends on the Hubble parameter.

One may wonder that the unphysical modes or polarization of QED photon may also contribute to the dark energy of the similar amount if the QCD vector ghost makes such contribution. However, QED is weakly interacting theory unlike QCD and also does not have nontrivial topological structure, and hence there is no restriction on the wave function as in QCD. So the contribution to the energy density is very small of order $H^4$ by dimensional reason and does not need to be considered [4]. However, see [16] for alternative suggestion.

Other possible origin of such a “vacuum energy” was also suggested based on the QCD condensate [17], assuming that there is no contribution for the flat Minkowski spacetime. Our mechanism is different in that such an assumption is not necessary.

It has been suggested that the same ghost may also generate magnetic field in an expanding universe [18]. This is also discussed keeping only two components of the QCD vector ghost. It would be interesting to check if this mechanism makes sense with the vector ghost. Another interesting question is to try to find if there is any other effects to check the proposed mechanism. There are already some discussions on this type of dark energy [19, 20]. These problems will be discussed elsewhere.
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