THE CONTRIBUTION OF THE INTERGALACTIC MEDIUM TO COSMIC MICROWAVE BACKGROUND ANISOTROPIES

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ABSTRACT

We compute the power spectrum of the cosmic microwave background (CMB) temperature anisotropies generated by the intergalactic medium (IGM). To estimate the electron pressure along the line of sight and its contribution to the Sunyaev-Zel’dovich component of the CMB anisotropies, we assume that the nonlinear baryonic density contrast is well described by a lognormal distribution. For model parameters in agreement with observations and for an experiment operating in the Rayleigh-Jeans regime, the largest IGM contribution corresponds to scales \( l \approx 2000 \). The amplitude is rather uncertain and could be as large as 100–200 \( \mu K^2 \), comparable to the contribution of galaxy clusters. The actual value is strongly dependent on the gas polytropic index \( \gamma \), the amplitude of the matter power spectrum \( \sigma_8 \), namely, \( C_{[\text{IGM}]} \sim (\gamma^2 \sigma_8)^{12} \). At all redshifts, the largest contribution comes from scales very close to the baryon Jeans length. This scale is not resolved in numerical simulations that follow the evolution of gas on cosmological scales. The anisotropy generated by the IGM could make compatible the excess of power measured by Cosmic Background Imager (CBI) on scales of \( l \geq 2000 \) with \( \sigma_8 = 0.9 \). Taking the CBI result as an upper limit, the polytropic index can be constrained to \( \gamma < 1.5 \) at 2 \( \sigma \) level at redshifts \( z \approx 0.1–0.4 \). With its large frequency coverage, the \( \textit{Planck} \) satellite will be able to measure the secondary anisotropies coming from hot gas. Cluster and IGM contributions could be separated by cross-correlating galaxy/cluster catalogs with CMB maps. This measurement will determine the state of the gas at low and intermediate redshifts.

Subject heading: cosmic microwave background — cosmology: observations — cosmology: theory

1. INTRODUCTION

In the last decade, numerical simulations (Cen & Ostriker 1999; Dave et al. 1999, 2001) and observational evidence (Rauch 1998; Stocke et al. 2004) have indicated that the highly ionized intergalactic gas has evolved from the initial density perturbations into a complex network of mildly nonlinear structures in the redshift interval \( 0 < z < 6 \) called the cosmic web. This structure could contain most of the baryons in the universe (Rauch et al. 1997; Schaye 2001; Fukugita & Peebles 2004). With cosmic evolution, the fraction of baryons in these structures decreases as more matter is concentrated in compact virialized objects. The \( \text{Ly}\alpha \) forest absorbers at low redshifts are filaments (with low \( \text{H}\ i \) column densities) containing about 30% of all baryons (Stocke et al. 2004). Hydrodynamic simulations predict that another large fraction of all baryons resides in mildly nonlinear structures that are partly shock-confined gas filaments heated up to temperatures of \( 10^5–10^7 \) K, called the warm-hot intergalactic medium (WHIM). The amount of baryons within the WHIM is estimated to reach 20%–40% (Cen & Ostriker 1999; Dave et al. 1999, 2001). Maloney & Bland-Hawthorn (1999) discussed whether the intragroup medium in groups of galaxies could contain a cosmological significant fraction of gas compatible with \( \text{H}\alpha \) observations.

In this paper we compute the electron pressure along the line of sight of both the ionized gas in the \( \text{Ly}\alpha \) forest and the hot gas in the WHIM. Even if the temperature of the gas in these absorbing filaments is relatively low (usually less than \( 10^7 \) K), the total amount of ionized gas is large. This electron pressure induces distortions and temperature anisotropies on the cosmic microwave background (CMB) spectrum by the thermal Sunyaev-Zel’dovich (TSZ) effect (Sunyaev & Zel’dovich 1972, 1980), as do clusters of galaxies. Early calculations of the TSZ power spectrum used analytical approaches (Atrio-Barandela & Mücket 1999; Komatsu & Kitayama 1999; Hernández-Monteagudo et al. 2000; Molnar & Birkinshaw 2000). Numerical simulations were soon used to make more accurate predictions (Refregier et al. 2000; Refregier & Teyssier 2002; Zhang et al. 2002). Da Silva et al. (2001) studied the relative contribution from high- and low-density gas and found that in all models considered, the high-density gas in halos dominated the TSZ signal. White et al. (2002) noticed that at \( l = 2000 \) about 25% of the signal came from gas in regions with density smaller than 100 times the cosmological mean. At \( l = 6000 \) the contribution from the diffuse gas was less than 2%.

Numerical simulations currently agree with the predictions based on the halo model, and it is reasonable to expect that the physics of baryons on dense environments has been well described in cosmological simulations. What has not been proven is whether numerical simulations have reached convergence, i.e., whether further increments in resolution will not increase the signal (Bond et al. 2005). To study numerically the evolution of the complex network of filaments that develops when the evolution of the gas in low-density regions becomes nonlinear requires computer resources that are at present unavailable. Soft particle hydrodynamics (SPH) codes do not have enough resolution, and adaptive refinement methods (ARM) are controlled by density and follow the evolution of high dense regions. To study the low dense regions with similar accuracy is computationally very expensive.
In this paper we study the contribution coming from baryons located in the low dense regions that constitute the IGM. The web of cosmic filaments corresponds to scales between large-scale fluctuations and strongly nonlinear scales inside halos. The halo model is likely to fail to account for its complicated geometry, and an analytical treatment would require a different approach. We use a lognormal probability distribution function (PDF) to describe the nonlinear evolution of baryons in low-density environments. Briefly, in § 2 we describe the model and in § 3 we derive the expressions that give the contribution of the IGM to the average γ-parameter and CMB temperature anisotropies. In § 4 we discuss our results and their dependence with cosmological and physical parameters. In § 5 we present our conclusions and the observational prospects to measure the anisotropies generated by the IGM.

2. THE LOGNORMAL BARYON DISTRIBUTION MODEL

Inverse Compton scattering of CMB photons by hot gas along the line of sight produces both temperature anisotropies and distortion of the CMB spectrum. We shall assume that baryons are distributed like a lognormal random field. The lognormal distribution was introduced by Coles & Jones (1991) as a model for the nonlinear distribution of matter in the universe. Bi & Davidsen (1997) have used it as a model to describe the Lyα forest and found that it reproduced the observations well. Choudhury et al. (2001) have introduced an analytical formalism that correctly describes the clustering properties of the neutral hydrogen in the mildly nonlinear regime. We shall follow their approach to describe the distribution of neutral gas but of the highly ionized phase of the IGM.

If the distribution of baryons is given by a lognormal random field then the probability $P(\xi)$ that at any spatial position $x$ at redshift $z$ the baryon (nonlinear) density contrast has the value $\xi = n_B(x,z)/n_0(z)$ is given by

$$P(\xi) = \frac{1}{\xi \sqrt{2\pi}} e^{-[\log(\xi) - \Delta_0] / 2} \Delta_0^2,$$  

(1)

Hereafter we shall represent the linear density contrast by $\delta$ and the nonlinear lognormal random field by $\xi$. The baryon number density $n_B(x,z)$ is given by

$$n_B(x,z) = n_0(z) e^{\delta_B(x,z) - \Delta_0^2 / 2},$$  

(2)

where $x$ denotes the spatial position at redshift $z$ and $|x(z)|$ is the proper distance; $\delta_B(x,z)$ is the baryon (linear) density contrast, $n_0(z) = \rho_B(1 + z)^{3/2} \rho_m$, and $\rho_B$, $\rho_m$ are the baryon density and proton mass, respectively. The linear baryon power spectrum is related to the DM power spectrum by (Fang et al. 1993)

$$P_B^{(3)}(k,z) = \frac{P_{DM}^{(3)}(k,z)}{[1 + x_B^2(z) k^2]^2},$$  

(3)

where

$$x_B(z) = \frac{1}{H_0} \left[ \frac{2 \gamma \rho_B T_m(z)}{3 \mu m_p \rho_m (1 + z)} \right]^{1/2},$$  

(4)

is the comoving Jeans length, $T_m$ is the averaged temperature of the IGM, $\gamma$ is the polytropic index, $\Omega_m$ is the cosmological fraction of matter density, $\mu = 4/(8 - 5Y)$ is the mean molecular weight of the IGM, and $Y = 0.24$ is the helium weight fraction. The Jeans length defines the scale below which baryon perturbations are suppressed with respect to those of the DM. Only scales larger are allowed to grow. We use the lognormal statistics to describe the nonlinear evolution of those perturbations at any given epoch $z$. Finally,

$$\Delta_0^2(z) = \langle \delta_B^2(x,z) \rangle = D^2(z) \int \frac{d^3k}{(2\pi)^3} \frac{P_{DM}(k)}{[1 + x_B^2(z) k^2]^2},$$  

(5)

where $D(z) = D(z, \Omega_\Lambda, \Omega_m)$ is the linear growth factor.

At each redshift, the fraction $\eta(\xi_{\text{max}})$ of matter in regions with overdensities smaller or equal than a fixed value $\xi_{\text{max}}$ is

$$\eta(\xi_{\text{max}}) = 1 - \int_{\xi_{\text{max}}}^{\infty} P(\xi) d\xi.$$  

(6)

At each redshift, the fraction of matter in regions with overdensity larger than a fixed value $\xi_{\text{max}} = 1 - \eta(\xi_{\text{max}})$ is. In Figure 1 we plot the fraction of matter that resides in regions with overdensity $\xi \geq 10, 50, 100$, and 500 at different redshifts. Since the lognormal PDF is rather skewed, the fraction of matter in high dense regions is not negligible. As the figure indicates, when the expansion of the universe starts to accelerate (around $z = 0.5$), this fraction becomes constant. Below this redshift, the fraction of matter with density contrasts $\xi > 100$ is less than 0.3%.

The number density of electrons $n_e$ in the IGM can be obtained by assuming ionization equilibrium between recombination and photoionization and collisional ionization. At the conditions valid for the IGM (temperature in the range $10^3$–$10^7$ K, and density contrast $\xi < 100$) the degree of ionization is very high. Commonly $n_e = cn_B$, with $0.9 < c \leq 1$ depending on the degree of ionization. To compute the temperature of the IGM at each position and redshift we use a polytropic equation of state,

$$T(x, z) = T_0(z) \left[ \frac{n_B(x, z)}{n_0(z)} \right]^{-1},$$  

(7)

where $T_0$ is the temperature of the IGM at mean density $n_0$ at given redshift $z$. 

![Figure 1](image-url)
3. COMPTONIZATION PARAMETER AND RADIATION POWER SPECTRUM

The contribution to the Comptonization parameter of a patch of hot gas of size \( L \) at a proper distance \( l = |x| \) is (Sunyaev & Zel’dovich 1972)

\[
\Delta \gamma_c(x, z) = y_0 \int_0^L T_e(x, z) n_e(x, z) d l,
\]

where \( y_0 = k_B \sigma_T / m_e c^2 g(\nu) \). Constants have their usual meaning, and \( g(\nu) \) is the frequency dependence of the SZ effect. The average line-of-sight contribution coming from structures located at \( z \) is

\[
\Delta \gamma_c(z) = y_0 \int_0^L \langle n_e(x, z) T_e(x, z) \rangle d l.
\]

The average is carried out over the whole spatial volume at redshift \( z \). The total contribution up to redshift \( z_f \) at which the universe is fully reionized is

\[
y_{c, av} = y_0 \int_0^{z_f} \langle n_e(x, z) T_e(x, z) \rangle \frac{d l}{d z} d z.
\]

Substituting the expressions of \( n_e \) and \( T_e \) obtained assuming a lognormal gas distribution gives

\[
y_{c, av} = y_0 \int_0^{z_f} n_0(z) T_0(z) e^{(\xi - 1) \Delta^2(\nu)/2} \frac{d l}{d z} d z.
\]

Numerically, we shall restrict the average on equation (9) to baryons residing in overdensities \( \xi \leq \xi_{\text{max}} \) (which for concreteness we shall take equal to 100) to exclude those baryons that could not be correctly described by the lognormal model (see below). Therefore, at each redshift we compute

\[
\langle f(\xi) \rangle_{\xi_{\text{max}}} = \int_0^{\xi_{\text{max}}} f(\xi) P(\xi) d \xi.
\]

The power spectrum contribution of the CMB temperature anisotropies induced by the IGM can be obtained from the two-point correlation function of the spatial variations of the electron density:

\[
\Delta \gamma_c(z) B(\theta, z, z') \Delta \gamma_c(z') \frac{d l}{d z} \frac{d l'}{d z'} d z d z'.
\]

In this expression \( \Delta \gamma_c(z) \) is given by equation (9), \( B(\theta, z, z') = e^{(\xi - 1) Q(z, z') \Delta^2(\nu)/2} - 1 \) is the normalized two-point correlation function with

\[
Q(z, z') = \frac{D(z, \Omega_{\Lambda}, \Omega_m) D(z', \Omega_{\Lambda}, \Omega_m)}{2 \pi^2} \int_0^\infty \frac{P_{\text{DM}}(k) k^2}{[1 + x_2^2(z') k^2][1 + x_2^2(z) k^2]} \frac{\sin(k|z - z'|)}{k|z - z'|}.\]

In here \( |z - z'| \) denotes the proper distance between two patches at positions \( x(z) \) and \( x(z') \) separated by the angle \( \theta \). In the flat-sky approximation

\[
|z - z'| \approx \sqrt{l_1(\theta, z)^2 + |r(z) - r(z')|^2},
\]

where \( l_1(\theta, z) \) is the transversal distance of two points located at the same redshift. Within this approximation, the correlation function is dominated by patches that are physically very close. For small \( \theta \), the correlation between patches at different redshifts is negligible and \( B(\theta, z, z') \sim B(\theta, z, z') \text{corr} |z - z'| \) is accurate at the 1% level. Equation (13) can be simplified to give

\[
C(\theta) = y_0^2 \int_0^{z_f} \left( \frac{d l}{d z} \right)^2 n_0^2(z) T_0^2(z) e^{\xi - 1} \Delta^2(\nu)|e^{\nu} \hat{Q}(\theta, z) - 1| d z.
\]

This approximation to equation (13) fails at large angular scales, but since at those scales the correlation is negligible, it does not affect the numerical results, while it greatly speeds up the computer code. As for the Comptonization parameter \( y_{c, av} \), we restrict the average to baryons within the mildly nonlinear regime, i.e., \( \xi \leq \xi_{\text{max}} = 100 \). Finally, the power spectrum can be obtained by Fourier transform:

\[
C_{l}^{\text{IGM}} = 2\pi \int_{-\infty}^{\infty} C(\theta) P_l(\cos \theta) d(\cos \theta),
\]

where \( P_l \) denotes the Legendre polynomial of multipole \( l \).

Since \( y_{c, av} \) and \( C_{l}^{\text{IGM}} \) depend on the electron pressure and not separately on the IGM temperature or density, equations (11) and (16) scale with IGM mean temperature \( T_0 \) and ionization fraction \( \epsilon \) as \( (T_0 \epsilon) \) to some power. The uncertainty in the degree of gas ionization is smaller than the one on the temperature \( T_0 \) at mean density, so we do not consider it any further. In our numerical results we shall take \( \epsilon = 1 \).

4. NUMERICAL RESULTS

To compute the contribution of the IGM to CMB temperature anisotropies, we take the concordance model as our fiducial cosmological model: \( \Omega_{\Lambda} = 0.73 \), \( \Omega_{\text{DM}} = 0.23 \), \( \Omega_B = 0.04 \), \( h = 0.71 \), and \( \sigma_8 = 0.9 \), in agreement with Wilkinson Microwave Anisotropy Probe (WMAP) results (Spergel et al. 2003). Except when specified otherwise, we present our results for \( g(\nu) = 1 \).

The physical parameters describing IGM thermal evolution history, i.e., the temperature \( T_0 \) at mean density, the Jeans length — fixed by \( T_0 \) —, and the polytropic index \( \gamma \), are the free parameters of our model. When the temperature of the IGM increases (e.g., during the reionization of He at \( z \approx 3 \)), so does the Jeans length. Then, perturbations that were previously evolving are frozen or partly damped. Suppression of power on smaller scales can also happen at an early epoch by energy injection (Springel et al. 2001). Since the contribution of those scales will be erased, one would require a detailed study of the evolution of \( T_m \) with redshift to estimate the CMB temperature anisotropies. To be conservative, we take it as equal to the largest admissible value instead of the average IGM temperature. The He II reionization at \( z \approx 3 \) requires temperatures larger than \( 5 \times 10^4 \) K (Schaye et al. 2000). Hui & Haiman (2003) argued that the IGM reached higher values during its thermal history. Analyzing the Sloan Digital Sky Survey data, Viel & Haehnelt (2006) found that the temperature range is weakly constrained and gave an upper bound of \( T_m \approx 2 \times 10^5 \) K. For our numerical estimates we adopt a maximum value of \( T_m = 1 \times 10^5 \) K. With respect to the temperature at mean density, we take an average of \( T_0 = 1.4 \times 10^4 \) K according to the lower values obtained by Hui & Haiman (2003). This temperature is mainly determined by the equilibrium of the photoionization due to the UV background radiation and recombination of hydrogen at mean density. This value is also in
the range obtained from the analysis of the QSO absorption lines (Stocke et al. 2004). Finally, our numerical estimates depend critically on the redshift evolution of \( \gamma \), which is very uncertain and strongly model dependent. Since our results show (see below) that most of the contribution to the CMB temperature anisotropy is generated in the redshift interval \( z \approx 0.1–0.4 \), we can fix \( \gamma \) to be the average value in that redshift interval.

Our current ideas of galaxy formation suggest that above \( z = 6 \) an increasing fraction of the gas is neutral and with low temperature; on those grounds, we do not expect a large contribution from earlier epochs. Therefore, all integrations were carried up to \( z_f = 6 \), the epoch when the reionization can be considered complete.

### 4.1. Mean Comptonization

Equation (11) gives the average \( y \)-parameter distortion produced by the IGM. In Figure 2 we show the dependence of the amplitude of the average Comptonization parameter \( y_{av} \) with respect to the parameter \( \gamma \). In the figure, \( \sigma_8 \) varies from 0.7 (bottom) to 1.1 (top) in units of one-tenth.

### 4.2. Radiation Power Spectrum

In Figure 3a we show several spectra for different values of \( \sigma_8 \gamma^2 \). The amplitude of the radiation power spectrum at all scales is strongly dependent on this product (see eqs. [13]–[16]). The small wiggle at \( l = 10–50 \) is caused by using the approximated correlation function given by equation (16) at large angular scales. In Figure 3b diamonds show the variation of the power spectrum maximum amplitude as a function of \( \sigma_8 \gamma^2 \). It corresponds to a scaling \( C_0^{\text{IGM}} \sim (\sigma_8 \gamma)^2 \). Due to this strong dependence, if \( \sigma_8 \) is known, even an order-of-magnitude estimate of the IGM contribution to temperature anisotropies will give a rather accurate measurement of the polytropic index \( \gamma \). In the same figure, asterisks show the location of the radiation power spectrum maxima and the dashed line corresponds to the best fit. Here the dependence is much weaker and, in our range of cosmological parameters, the maximum anisotropy corresponds to angular scales ranging from \( 5' \) to \( 10' \).

### 4.3. Contribution of Different Redshift Intervals

Due to its definition, \( T_0 \) is expected to vary little with redshift, and any time dependence can be easily incorporated into the analytical expressions. On the other hand, the polytropic index \( \gamma \)

is strongly dependent on the thermal history of the IGM. In our calculations we have assumed that \( T_0 \) and \( \gamma \) are strictly constant during the cosmic evolution of the IGM from reionization until today. Even in this simplified model not all redshifts contribute equally to CMB distortions and temperature anisotropies. In Figure 4a we show the differential growth of the Comptonization parameter with respect to redshift \( dy_{av}/dz \) for different \( \gamma \). In the figure, the redshift intervals are \( \Delta z = 0.001 \). Let us note that when \( \gamma \) is small, the contribution of the IGM to \( y_{av} \) at high redshifts \( (z > 0.3) \) is much higher than at smaller redshifts. For large values of \( \gamma \), the gas located at redshifts \( z > 1 \) gives approximately equal contributions and even the low-redshift gas contributes significantly. In all cases, the contribution increases close to \( z = 6 \) and the choice of the upper limit of integration can affect the average distortion. As far as the IGM is well described by our model, out to \( z_f = 6 \), our results must be taken as lower limits. For comparison, we also show the contribution of the different redshift intervals to the correlation function: \( dC(0,z)/dz \)

![Fig. 2.—Mean Comptonization parameter \( y_{av} \) as a function of the polytropic index \( \gamma \). Curves correspond to different values of \( \sigma_y \). From top to bottom, curves decrease in units of one-tenth.](image)

![Fig. 3.—(a) CMB radiation power spectrum. The middle curve corresponds to \( \gamma = 1.4, \sigma_8 = 0.9 \). (b) Best fit to the amplitude (dotted line) and location (dashed line) of the radiation power spectrum maxima as a function of combined gas and cosmological parameters \((\sigma_8 \gamma^2)\). Asterisks and diamonds correspond to the models actually computed. The \( y \)-axis gives the maximum value in \( k^2 \) (diamonds) and the multipole \( l_{max} \) corresponding to the maxima (asterisks).](image)

![Fig. 4.—(a) Contribution of the IGM at different redshift intervals to the Comptonization parameter in redshift bins of \( \Delta z = 0.001 \) for different parameters. The dash-dotted line shows the time dependence of \( dC(0,z)/dz \) (normalized to unity at \( z = 0 \)). (b) Contribution to the radiation power spectrum of redshift bins of width \( \Delta z = 0.001 \) as a function of redshift \( d[\Delta(l+1)/2]/\Delta l \) for fixed multipoles \( l \) \((\gamma = 1.4, \sigma_8 = 0.9)\).](image)
for γ = 1.4, normalized to unity at z = 0. In Figure 4b we show the differential redshift contribution to the radiation power spectrum: \( \frac{dP}{dz}/[(l + 1)C_l^{\mathrm{IGM}}/2\pi] \), for different multipoles. There is a clear difference with the behavior shown in Figure 4a: at each multipole the signal comes preferentially from a narrow redshift range. Even if the maximum value decreases for increasing \( l \), the effective width of redshift intervals dominating the contribution increases and the overall maximum in the full spectrum occurs at \( l \approx 1000–3000 \), as indicated by Figure 3a.

In Figures 2 and 3 we are implicitly assuming that all baryons are in the IGM (see eq. [2]). At high redshifts (\( z = 2–4 \)) the entire baryon content of the universe can be accommodated within the warm (~\( 10^4 \) K) photoionized IGM. At low redshifts, the combined fraction of baryons in warm photoionized IGM together with those in the WHIM could be as large as 70%–80%. As indicated in Figure 4b, the radiation power spectrum originates on a very narrow redshift range, and if during that period the fraction \( f \) of baryons in the IGM is kept constant at, say, 70%, the amplitude of the power spectra, which scales as \( f^2 \), would be reduced by a factor of 2. The effect would be very small for the \( \gamma \)-parameter since first \( \gamma_{\mathrm{CL}1} \) scales linearly with \( f \) and second with the gas at higher redshifts, where \( f \approx 1 \), also contributes. Considering the strong dependence of \( C_l^{\mathrm{IGM}} \) with \( \gamma \) and \( \sigma_8 \), the effect of this uncertainty in the upper limit derived above is not significant.

### 4.4. Contribution of Different Scales

If the main drawback of analytical treatments is to be based on simplifying assumptions and to require scaling relations derived from observations, current numerical simulations lack spatial resolution in large enough volumes to resolve the complex gas dynamics (Bond et al. 2005). Our results, detailed above, seemingly contradict those of numerical simulations carried out up until now. For example, White et al. (2002) and da Silva et al. (2001) analyzed whether the majority of the contribution to the TSZ angular power spectrum came from diffuse gas or gas within halos. Figure 5 is useful for understanding the different outcome between our analytical estimates and the results obtained using cosmological hydro simulations. In the figure we plot the contribution of the different scales to the radiation power spectrum. We particularize for \( l \approx 2000 \), which in all models is close to the largest amplitude of the radiation power spectrum, \( C_l \). In the top two curves we integrate equation (14) from \( k = 0 \) to \( k_{\text{max}} \), expressed in units of the inverse Jeans length \( 1/x_0 \). The solid line corresponds to \( \gamma = 1.4 \), \( T_m = 1 \times 10^5 \) K, and the dashed line corresponds to \( \gamma = 1.3 \) at the same \( T_m \). Let us note that even for a fixed \( T_m \) the Jeans length varies with the polytropic index \( \gamma \). The bottom two curves show the differential contribution to \( C_{2000} \) of different scale intervals. We integrate equation (14) in bins of width \( \Delta k = 0.2/x_0 \). Lines correspond to the same parameters as before. The amplitudes are different depending on model parameters, but both curves have very similar shapes. The plot clearly shows that the main contribution to \( C_l \) comes from scales in the range \( k x_0 \simeq (0.5–3) \). About 80%–90% of the total power comes from scales in that range, the exact figure depending on model parameters.

In a model with \( \gamma = 1.4 \), \( T_m = 1 \times 10^5 \) K, the Jeans length is \( x_0 = 470 \) h\(^{-1} \) kpc. High-resolution hydrodynamic simulations with ARM techniques are extremely good in dealing with the gas behavior in high-density regions since the refinement is governed by the local density. For example, Refregier & Teyssier (2002) studied the evolution of DM and gas with resolutions of 96 h\(^{-1} \) kpc at \( z \approx 5 \) to 12 h\(^{-1} \) kpc at \( z = 0 \) in the densest regions using an adaptive mesh refinement algorithm. But their resolution was much smaller in low-density regions, where most of the baryons reside. SPH codes do not yet account for contributions that comes from scales \( k \geq 2/x_0 \) or are just on the edge of the necessary resolution. One would think that an SPH code evolving \( 2 \times (216)^3 \) particles in a box of 100 h\(^{-1} \) Mpc and cell size of 370 h\(^{-1} \) kpc (i.e., White et al. 2002) could reach enough spatial resolution to follow the gas dynamics on scales close to the Jeans length, but this is not the case. Spatial resolution is not cell size but the minimum scale below which the code is not able to solve the evolution equations. Particle mesh codes include a force cutoff at small distances, resulting in a damping of the power spectrum up to 8 times the cell size (Refregier & Teyssier 2002). Even though White et al. (2002) had an effective resolution much larger than their cell size and could not account for the effect of gas on scales close to the Jeans length, they did find a 25% contribution coming from regions with density contrast \( \leq 100 \) at \( l = 2000 \). This contribution was less than 2% at \( l = 6000 \) much as would be expected if the signal were due to diffuse gas. As Figure 3a shows, between those multipoles the power decreases by a factor of 3–10, depending on the IGM temperature and polytropic index.

### 4.5. Contribution of Different Density Contrasts and Jeans Length

Analytic calculations based on the halo model correctly account for the SZ contribution of gas in clusters of galaxies and collapsed objects. On the other hand, a lognormal description of baryons in the IGM is only valid for a limited range of overdensities. In the redshift interval [0.1, 0.4], where most of the temperature anisotropy is generated, the fraction of matter with \( \xi > 100 \) is less than 0.3% (see Fig. 1). Since the lognormal PDF weights heavily the high-density regions, even this fraction could have a large contribution. Observationally, it has been established that the model describes rather well Ly\( \alpha \) clouds with overdensities \( \xi \leq 50 \) by \( z = 3 \). Recent analyses (see, e.g., Tatekawa 2005; Kayo et al. 2001), have shown that as a result of cosmic evolution, the density distribution becomes increasingly better described by a lognormal PDF at \( z \to 0 \). In particular, it was shown by Kayo et al. (2001) that it accurately describes the density distribution even in the nonlinear regime up to \( \xi < 100 \). Baryons at much higher densities (\( \xi > 500 \)) will cluster in halos and experience different astrophysical processes, such as shock heating, radiative effects, star formation, energy injection through
supernovae explosions, etc., occurring on galaxy and cluster scales. Those baryons cannot be described as a gas with uniform temperature and a polytropic equation of state. As remarked in § 3, averages in equations (9) and (13) have been carried out excluding regions with overdensities $\xi > 100$. In Figure 6a we examine how the maximum amplitude of the radiation power spectrum $C_{l,\text{max}}$ scales with $\xi_{\text{max}}$ for two different values of $T_m$. The scaling is rather weak compared to the dependence on the polytropic index $\gamma$.

The Jeans length measures the minimum scale that is gravitationally unstable. It varies with redshift as different physical processes heat and cool the gas. Then, the evolution of scales close to the Jeans length becomes rather complex. We simplify their treatment assuming $T_m$ to be fixed and equal to its maximum value throughout its thermal evolution. Since the baryon power spectrum in equation (3) is damped on scales smaller than the Jeans length, this assumption reduces the contribution of those intermediate scales. In Figure 6b we plot the dependence of the maximum amplitude of $C_{l,\text{max}}$ with $T_m$. As expected, larger $T_m$ lead to a smaller contribution since a smaller number of scales is included. To summarize, the scaling behavior represented in Figures 3b and 6 is

$$C_{l,\text{max}} \sim \sigma_8^{12} \gamma^{24} T_m^{-6} \xi_{\text{max}}^{12}. \quad (18)$$

For comparison, $C_{l,\text{clusters}} \sim \sigma_8^6$ obtained by Komatsu & Kitayama (1999) from analytical estimates. Small variations on $\gamma$ can produce a strong change in the amplitude of the radiation power spectrum. Even if $T_m$ is only known up to an order of magnitude and our model can only be applied up to overdensities $\xi \lesssim 20$, we could still obtain some constraints on $\gamma$ from observations of CMB temperature anisotropies.

4.6. IGM Contribution to CBI Scales

In Figure 7 we compare the CMB temperature anisotropy power spectrum of cosmological origin (solid line) with TSZ contribution of clusters of galaxies (dotted line) and the IGM (dashed line) with $\gamma = 1.4$, $T_m = 10^6$ K and $\gamma = 1.27$. $T_m = 5 \times 10^4$. We adopted $\sigma(32$ GHz) $= -1.96$ to rescale the TSZ power spectra to the operating frequency of the Cosmic Background Imager (CBI) experiment (Readhead et al. 2004). CBI showed an excess over the cosmological radiation power spectrum at $l > 2000$. The result and its 1 $\sigma$ error box is included in the figure. Depending on model parameters, TSZ cluster and IGM components could have similar or very different amplitudes and shapes. Sadeh & Rephaeli (2004) have demonstrated that the radiation power spectrum generated by clusters depends on the assumed mass-temperature relation and gas evolution. The cluster power spectrum represented in Figure 7 has been generated using an analytical model that does not peak at $l = 2000$. Bond et al. (2005) found that to explain the CBI excess with the anisotropy generated by clusters requires $\sigma_8 = 1.0$ or larger. Making an account the contribution of the IGM, it can be seen in Figure 7 that these TSZ and IGM components are consistent with the CBI observations. Turning the argument around, if the CBI data are assumed to be an upper limit to the total anisotropy on scales $l > 2000$, then $\gamma > 1.5$ is ruled out at the 2 $\sigma$ level. Since the contribution to those scales comes mostly from $z = 0.1-0.4$ (see Fig. 6b), this upper limit applies to the gas at that redshift.

5. DISCUSSION

In this article we have shown that ionized gas, in the deep potential wells of clusters of galaxies and in the IGM, have a significant effect on the CMB, generating both temperature anisotropies and spectral distortions. While the latter is proportional to the electron pressure along the line of sight, the former depends on the clustering properties of the hot gas (Hernández-Monteagudo et al. 2000). The high amplitude of the radiation power spectrum, similar in magnitude to the contribution of clusters, is due to the nonlinear evolution and high degree of clustering of the baryonic matter at low redshift. It is the lognormal distribution of the ionized gas that drives the strong dependence of the amplitude of the radiation power spectrum with $\gamma$ and $\sigma_8$. This extra component can make compatible the CBI measured power excess at $l \sim 2000$ with $\sigma_8 = 0.9$. At all redshifts, the largest contribution comes from scales around the baryon Jeans length. In cosmological simulations set to compute the TSZ contribution to the CMB temperature anisotropies, this scale is not well resolved, which explains why this contribution has not yet been identified in numerical simulations.
The lognormal model is remarkable for predicting a strong
dependence of the radiation power spectrum with two parame-
ters: $C_{l, \text{max}} \sim (\gamma \sigma_\gamma)^{-1}$. Even if the range of scales, densities, and
redshifts to which the model can be applied is rather uncertain,
those parameters produce minimal variations in shape of the
radiation power spectrum and the variations in the amplitude are
much smaller than that of $\gamma$ and $\sigma_\gamma$. Remarkably, the anisotropy
due to the IGM is generated in a narrow redshift interval. Assum-
ing $\sigma_\gamma = 0.9$, one can obtain a firm upper limit of $\gamma \leq 1.5$
at the 2 $\sigma$ confidence level in the redshift range [0.1–0.4]. As
different redshift intervals dominate the anisotropy at different
angular scales, measurements of the power spectrum at different
$l$ will allow us to determine the polytropic index $\gamma$ and the state
of the ionized gas at different redshifts.

By hypothesis, the gas obeys a polytropic equation of state.
This cannot account for the effect of the shock-heated gas of the
WHIM leading to high temperatures ($T \approx 10^5–10^7$ K). Thus,
this baryon component might provide an extra TSZ contribution
at redshifts close to zero. To distinguish the IGM TSZ effect from
the one coming from clusters of galaxies, it is necessary to use
their different statistical properties. Since the TSZ effect is in-
dependent of redshift, the cluster signal will correlate with
cluster positions on the sky. Cross-correlation of cluster catalogs
with CMB maps opens the possibility of determining the cluster
contribution and separating the IGM component. Hernández-
Monteagudo et al. (2004) and Afshordi et al. (2005) have re-
cently carried out such analyses using WMAP data. They found
strong evidence (at the 5 and 8 $\sigma$ levels, respectively) of a TSZ
contribution to the radiation power spectrum due to clusters, but
the data were not sensitive enough to yield the radiation power
spectrum. The Planck satellite, with its large frequency cover-
age, will be well suited for measuring the TSZ power spectrum.
It is worth exploring correlation techniques that will permit to
separate the cluster from the IGM component. Measurements of
the IGM power spectrum at different multipoles will provide a
measurement of the state of the gas (temperature and polytropic
index) at different redshifts.

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