Qudit-Based Measurement-Device-Independent Quantum Key Distribution Using Linear Optics

H. F. Chau,1, 2, Cardythy Wong,1 Qinan Wang,1 and Tieqiao Huang1
1 Department of Physics, University of Hong Kong, Pokfulam Road, Hong Kong
2 Center of Theoretical and Computational Physics, University of Hong Kong, Pokfulam Road, Hong Kong
(Dated: August 31, 2016)

Measurement-device-independent (MDI) method is a way to solve all detector side-channel attacks in quantum key distribution (QKD). However, very little work has been done on experimentally feasible qudit-based MDI-QKD scheme although the famous (qudit-based) round-robin differential-phase-shift (RRDPS) scheme is vulnerable to attacks on uncharacterized detectors. Here we report a mother-of-all QKD protocol on which all provably secure qubit-based QKD schemes known to date including the RRDPS and the so-called Chau15 schemes are based. We also report an experimentally feasible MDI system via optical implementation of entanglement swapping based on a recent qudit teleportation proposal by Goyal et al. In this way, we show that all provably secure qudit-based QKD schemes discovered to date can be made MDI.

PACS numbers: 03.67.Dd, 89.70.-a

In quantum key distribution (QKD), two cooperative agents (commonly called Alice and Bob) try to share a secret key by sending and measuring signals through a quantum channel. Realistic experimental apparatus, which is never ideal, posts a serious and non-trivial threat to the security of QKD as eavesdropper (commonly called Eve) may exploit loopholes due to apparatus imperfections. Even worse, such loopholes, some may yet to be found, could be experimental setup specific. One way to solve the imperfect detector problem is the so-called measurement-device-independent (MDI) method, which uses teleportation or entanglement swapping technique to close all detector side channel loopholes once and for all. The beauty of MDI method is that the teleportation or entanglement swapping measurement can be performed by a third untrustworthy party (commonly called Charlie). MDI method is applicable to all prepare-and-measure QKD schemes involving the transfer of qubits, qudits and continuous-variable quantum modes as long as these schemes can be reduced from certain entanglement-based ones. Several experiments have demonstrated the feasibility of qubit-based MDI-QKD. However, it is not clear how to reliably implement the qudit-based MDI proposal in Ref. [1], which relies on the entanglement swapping scheme by Bouda and Bužek [8], using photonics techniques. With the discovery of several promising qudit-based QKD schemes, study of their MDI version is no longer a pure academic issue.

One such scheme is the round-robin differential-phase-shift (RRDPS) scheme [9], which has attracted a few pioneer experiments [10–13]. While this scheme is robust against encoding errors [14] and does not need to monitor signal disturbance [9], it is insecure against several detector attacks [13, 15]. Moreover, it is not clear if the MDI version of the RRDPS scheme exists.

Another scheme is the so-called Chau15 scheme [17]. Information is transmitted in this scheme and its extension [18] via preparation and measurement of qudit-like qudits in the form \((|j\rangle \pm |k\rangle)/\sqrt{2}\) with \(|j\rangle\) and \(|k\rangle\) being distinct orthonormal states in a \(2^n\)-dimensional Hilbert space. Following Ref. [3], an naive MDI implementation of this scheme is for Charlie to perform entanglement swapping by measuring the states Alice and Bob send to him along \(\{j, j'\pm|k, k'\rangle, j, j'\pm|k, k'\rangle\}\) for some randomly chosen distinct pairs of \((j, k)\) and \((j', k')\). However, this naive implementation is insecure as the cheating Charlie may project the states Alice and Bob send to him along \(|j'\rangle\pm|k'\rangle\) and \(|j\rangle\pm|k\rangle\) respectively before performing the entanglement swapping to obtain the phase information of the two states without being caught. Furthermore, after entanglement swapping, Bob may have to change the value of his raw key based on Charlie’s measurement result. This step can only be performed without affecting the key rate if the teleportation procedure is compatible with the state preparation procedure of Alice and Bob in the sense that the quantum operations needed to transform between different teleportation measurement states used (in the qubit case, these are the four Pauli operations) can be deterministically mapped to the corresponding classical operations acting on Bob’s raw key (in the qudit case, this is the logical-NOT operation). Unfortunately, the teleportation procedure used in Ref. [4] is not compatible with the state preparation procedure used in the Chau15 scheme. Hence, even if one may optically implement the MDI protocol in Ref. [4] in future, it cannot be used make the Chau15 scheme MDI.

Here we first report a mother-of-all entanglement-distillation-based MDI-QKD scheme which can be reduced to all known provably secure qudit-based QKD schemes to date. Then we show the feasibility of this mother-of-all scheme by reporting a linear optics imple-
mentation of the required entanglement swapping operation.

The mother-of-all scheme.

1. Let \( n > 1, N = 2^n \) and \( GF(N) \) denotes the finite field of \( N \) elements. Alice and Bob each prepare an entangled state \( |\Phi_{00}\rangle = \sum_{i \in GF(N)} |i, i\rangle / \sqrt{N} \) and send the second half of the state to Charlie through an insecure quantum channel.

2. Charlie performs entanglement swapping by measuring the states he received from Alice and Bob along the basis \( \mathcal{B} = \{|\Phi_{ab}\rangle = \sum_{i \in GF(N)} (-1)^{\text{Tr}(bs)}|i, i + a\rangle / \sqrt{N} : a, b \in GF(N)\} \), where \( \text{Tr}(i) = i + i^2 + i^4 + \cdots + i^{N/2} \) is the absolute trace of \( i \). Note that all arithmetic in the state ket are done in the finite field \( GF(N) \). (See Ref. \[19\] for an introduction to finite field arithmetic.) Charlie publicly announces his measurement result, namely, the values of \( a, b \) he obtained. Bob applies the linear transformation \( |i\rangle \mapsto (-1)^{\text{Tr}(a-1)\bar{i}}|i-a\rangle \) for all \( i \in GF(N) \) to the first half of his state. (Or equivalently, Alice applies the linear transformation \( |i\rangle \mapsto (-1)^{-\text{Tr}(b)\bar{i}}|i+a\rangle \) to the first half of her state.) In the absence of Eve and noise, Alice and Bob should now share the entangled state \( |\Phi_{00}\rangle \).

3. Alice, Bob and Charlie repeat the above procedure some times to accumulate enough entangled states. Then Alice and Bob perform channel error estimation, if necessary, plus entanglement distillation to get the final almost perfect copies of \( |\Phi_{00}\rangle \)’s. They measure their shares of these distilled pairs to get their final key.

Note that if the state measurement procedure used by Alice and Bob to obtain their final key in step 3 above is compatible with the teleportation procedure in step 2, we obtain a provably secure qudit-based QKD scheme by the standard Shor-Preskill argument \[19\]. More importantly, all provably secure qudit-based QKD schemes to date can be deduced from this mother-of-all scheme in this way. For instance, if Alice and Bob both project each of their shared distilled pairs to states in the form \( (|j\rangle \pm |k\rangle) / \sqrt{2} \), we get the Chau15 scheme \[17\] and its extension \[18\]. If Alice and Bob project their states in the form \( \sum_{i \in GF(N)} (-1)^{s_i}|i\rangle / \sqrt{N} \) for \( s_i \in GF(2) \) and \( |j\rangle \pm |k\rangle / \sqrt{2} \) respectively, we obtain the RRDPS scheme using \( N \)-dimensional qudits \[2\]. And if Alice and Bob prepare their states using the method stated in Ref. \[20\], we arrive at the so-called Chau05 scheme. We state the MDI version of the RRDPS scheme obtained in this way below as illustration.

**The MDI version of the RRDPS scheme.**

1. Alice prepares \( \sum_{i \in GF(N)} (-1)^{s_i}|i\rangle / \sqrt{N} \) and sends it to Charlie. She jots down the values of \( s_i \)’s.

2. Bob prepares \( |j\rangle + (-1)^{s_k}|k\rangle / \sqrt{2} \) and sends it to Charlie. He jots down the values of \( t \in GF(2) \) and \( j \neq k \in GF(N) \). And he uses \( t \) as his raw bit.

3. Charlie jointly measures the states of Alice and Bob along the basis \( \mathcal{B} \) and announces the state \( |\Phi_{ab}\rangle \) he obtains.

4. Bob announces \( j \) and \( k \).

5. Alice uses \( s_j - s_k - \text{Tr}[b(k-j)] \) mod 2 as her raw bit.

6. Alice and Bob repeat the above steps to get sufficient raw key bits and then distill out their final key through error correction and privacy amplification.

Although it is not possible to perform the complete Bell-like measurements in step 2 of the mother-of-all scheme using linear optics \[22, 23\], we report a partial implementation below based on Goyal et al.’s qudit teleportation proposal. In Fig. 3 of Ref. \[24\], Goyal et al. reported way to project a \( N \) qudit state to the antisymmetric state \( \ket{\Psi} = \sum_{P \in S(N)} \varepsilon(P) \prod_{i \in GF(N)} a_i^{P(i)} \ket{0} / \sqrt{N!} \) by means of linear optics and photon number resolving detectors. Here \( S(N) \) is the group of permutations of a set of \( N \) elements, \( \varepsilon(P) \) is the sign of the permutation \( P \), \( a_i^{P(j)} \) is the creation operator for a photon propagating along path \( i \) and orbital angular momentum \( j \), and \( \ket{0} \) is the vacuum state. Since \( \sum_{P \in S(N)} \varepsilon(P) \ket{m}_i^{U^{-1}} \ket{P(j)} \ket{m}_j^{U^{-1}} \ket{P(k)} = 0 \) for all \( N \)-dimensional unitary operator \( U \) and \( j, k, m \in GF(N) \) with \( j \neq k \), measuring every qudit of the state \( \ket{\Psi} \) along the basis \( \{U|i\rangle : i \in GF(N)\} \) always yields \( N \) distinct outcomes. In this sense, \( \ket{\Psi} \) is the generalization of the singlet state for qubits. By choosing \( U \) to be a direct sum of \( N/2 \) Hadamard transformations, we have the following modified MDI-RRDPS scheme using linear optics. (Only those modified steps are shown.)

**The MDI version of the RRDPS scheme using linear optics.**

2’ Bob randomly group the \( N \) elements in \( GF(N) \) into \( N/2 \) pairs in the form \( \{(j_i, k_i)\} \). He prepares \( (N - 1) \) distinct qudit states each selected from the basis \( \overline{\mathcal{B}} = \{(|j_i\rangle \pm |k_i\rangle) / \sqrt{2}\} \) and sends them to Charlie. He jots down the state \( |j\rangle + (-1)^{s_k}|k\rangle / \sqrt{2} \) in \( \overline{\mathcal{B}} \) that he has not prepared and uses \( t \) as his raw bit.

3’ Charlie jointly projects the single qudit from Alice and \( (N - 1) \) qudits from Bob to \( \ket{\Psi} \) and informs Alice and Bob if the projection is successful.

5’ Alice uses \( s_j - s_k \) mod 2 as her raw bit.
Note that the connections between the above two MDI-RRDPS schemes is that the $|\Psi\rangle$ used in the latter can be identified with the $|\Phi_j\rangle$ of the former for some $j \neq 0$ through the logical encoding of each state in the basis set $\mathcal{B}$ by the tensor product of the other $N-1$ states in $\mathcal{B}$. Since the probability for Charlie to successfully obtain $|\Psi\rangle$ in step 3’ equals $1/N^2$, the above scheme is practical only when $N$ is small. It is instructive to find more efficient way to project a state to $|\Phi_{ij}\rangle$. Finally, we write down the MDI version of the Chau15 scheme for completeness.

The MDI version of the Chau15 scheme using linear optics.

Alice, Bob and Charlie follow all the steps in the MDI version of the RRDPS scheme using linear optics with the following modifications.

1’ Alice sends the state $|j'\rangle + (-1)^s|k'\rangle/\sqrt{2}$ to Charlie. She jots down $j' \neq k' \in GF(N)$ and $s \in GF(2)$.

5’ If $\{j, k\} = \{j', k'\}$, Alice and Bob uses $s$ and $t$ as their raw key bits, respectively.

This work is supported by the RGC grant 17304716 of the Hong Kong SAR Government.

* Corresponding author, email: hfchau@hku.hk

[1] V. Scarani, H. Bechmann-Pasquinucci, N. J. Cerf, M. Dušek, N. Lütkenhaus, and M. Peev, Rev. Mod. Phys. 81, 1301 (2009).
[2] E. Diamanti, H.-K. Lo, B. Qi, and Z. Yuan, “Practical challenges in quantum key distribution,” (2016), arXiv:1606.05853, to appear in NPJ Quant. Inform.
[3] H.-K. Lo, M. Curty, and B. Qi, Phys. Rev. Lett. 108, 130503 (2012).
[4] S. L. Braunstein and S. Pirandola, Phys. Rev. Lett. 108, 130502 (2012).
[5] S. Pirandola, C. Ottaviani, G. Spedalieri, C. Weedbrook, S. L. Braunstein, S. Lloyd, T. Gehring, C. S. Jacobsen, and U. L. Andersen, Nature Photonics 9, 397 (2015).
[6] Z. Tang, Z. Liao, F. Xu, B. Qi, L. Qian, and H.-K. Lo, Phys. Rev. Lett. 112, 190503 (2014).
[7] Y.-L. Tang, H.-L. Yin, S.-J. Chen, Y. Liu, W.-J. Zhang, X. Jiang, L. Zhang, J. Wang, L.-X. You, J.-Y. Guan, D.-X. Yang, Z. Wang, H. Liang, Z. Zhang, N. Zhou, X. Ma, T.-Y. Chen, Q. Zhang, and J.-W. Pan, Phys. Rev. Lett. 114, 069901 (2015).
[8] J. Bouda and V. Bužek, J. Phys. A 34, 4301 (2001).
[9] T. Sasaki, Y. Yamamoto, and M. Koashi, Nature 509, 475 (2014).
[10] J.-Y. Guan, Z. Cao, Y. Liu, G.-L. Shen-Tu, J. S. Pelle, M. M. Fejer, C.-Z. Peng, X. Ma, Q. Zhang, and J.-W. Pan, Phys. Rev. Lett. 114, 180502 (2015).
[11] H. Takesue, T. Sasaki, K. Tamaki, and M. Koashi, Nature Photonics 11, 827 (2015).
[12] S. Wang, Z.-Q. Yan, W. Chen, D.-Y. He, X.-T. Song, H.-W. Li, L.-J. Zhang, Z. Zhou, G.-C. Guo, and Z.-F. Han, Nature Photonics 11, 832 (2015).
[13] Y.-H. Li, Y. Cao, H. Dai, J. Lin, Z. Zhang, W. Chen, Y. Xu, J.-Y. Guan, S.-K. Liao, J. Yin, Q. Zhang, X. Ma, C.-Z. Peng, and J.-W. Pan, Phys. Rev. A 93, 030302(R) (2016).
[14] A. Mizutani, N. Imoto, and K. Tamaki, Phys. Rev. A 92, 060303(R) (2015).
[15] T. Iwakoshi, in Proc. of SPIE 9505 on Quantum Optics And Quantum Information Transfer And Processing (Prague, Czech Republic, 2015) p. 950504.
[16] Z. Cao, Z.-Q. Yin, and Z.-F. Han, Phys. Rev. A 93, 022310 (2016).
[17] H. F. Chau, Phys. Rev. A 92, 062324 (2015).
[18] H. F. Chau, Q. Wang, and C. Wong, (2016), arXiv:1603.02370.
[19] S. Lin and D. J. Costello, Jr., Error Control Coding, 2nd ed. (Prentice Hall, Upper Saddle River, NJ, 2004) Chap. 2.2–2.6, chap. 2.2–2.6.
[20] P. W. Shor and J. Preskill, Phys. Rev. Lett. 85, 441 (2000).
[21] H. F. Chau, IEEE Trans. Inf. Theo. 51, 1451 (2005).
[22] L. Vaidman and N. Yoran, Phys. Rev. A 59, 116 (1999).
[23] N. Lütkenhaus, J. Calsamiglia, and K.-S. Suominen, Phys. Rev. A 59, 3295 (1999).
[24] S. K. Goyal, P. E. Boukama-Dzoussi, S. Ghosh, F. S. Roux, and T. Konrad, Sci. Rep. 4, 4543 (2014).