Anti-Grand Unification and Critical Coupling Universality

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Abstract

The present work considers the phase transition between the confinement and "Coulomb" phases in $U(1)$, $SU(2)$ and $SU(3)$-sectors of Anti-grand unified theory described by regularized Wilson loop action. It was shown the independence of the critical coupling constants of the regularization method ("universality").

Standard model unifying QCD with Glashow–Salam–Weinberg electroweak theory well describes all experimental results known today. Most efforts to explain the Standard model are devoted to Grand unification theory (GUT). The precision of the LEP-data allows to extrapolate three coupling constants of the Standard model to high energies with small errors and we are able to perform consistency checks of the Grand unification theories.

In the Standard model based on the group

$$SMG = SU(3)_c \otimes SU(2)_L \otimes U(1)$$

the usual definitions of the coupling constants are used:

$$\alpha_1 = \frac{5}{3} \frac{\alpha}{\cos^2 \theta_{MS}}, \quad \alpha_2 = \frac{\alpha}{\sin^2 \theta_{MS}}, \quad \alpha_3 \equiv \alpha_S = \frac{g^2}{4\pi},$$

(2)

where $\alpha$ and $\alpha_S$ are the electromagnetic and strong fine structure constants, respectively. Using experimentally given parameters:

$$\sin^2 \theta_{MS}(M_Z) = 0.2316 \pm 0.0003, \quad \alpha_S(M_Z) = 0.118 \pm 0.003, \quad \alpha^{-1}(M_Z) = 127.9 \pm 0.1,$$

(3)

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it is possible to extrapolate the experimental values of three inverse running
constants $\alpha_i^{-1}(\mu)$ to the Planck scale: $\mu_{Pl} = 1.22 \times 10^{19}\text{GeV}$.

The comparison of the evolutions of the inverses of the running coupling
constants to high energies in the Minimal Standard model (MSM) (with one
Higgs doublet) and in the Minimal Supersymmetric Standard model (MSSM)
(with two Higgs doublets) gives rise to the existence of the grand unification point
at $\mu_{GUT} \sim 10^{16}\text{GeV}$ only in the case of MSSM (see Ref.[1]). This observation is
true for a whole class of GUT’s that break to the Standard model group in one
step, and which predict a “grand desert” between the weak (low) and the grand
unification (high) scales. If grand desert indeed exists, and the supersymmetry is
established at future colliders then we shall eventually be able to use the coupling
constant unification to probe the new physics near the unification and Planck
scales.

Scenarios based on the Anti-grand unification theory (AGUT) was developed
in Refs.[2]-[5] as an alternative to GUT’s.

Anti-grand unified theory suggests that at the Planck scale $\mu_{Pl}$, considered as
a fundamental scale, there exists the more fundamental gauge group $G$, containing
$N_{gen}$ copies of the Standard model group $SMG$:

$$ G = SMG_1 \otimes SMG_2 \otimes \ldots \otimes SMG_{N_{gen}} = (SMG)^{N_{gen}}, \quad (4) $$

where the integer $N_{gen}$ designates the number of quark and lepton generations.

The theory predicts and experiment confirms, that $N_{gen} = 3$. Subsequently,
the fundamental gauge group $G$ is:

$$ G = (SMG)^3 = SMG_1 \otimes SMG_2 \otimes SMG_3. \quad (5) $$

The AGUT assumes that Nature seeks a special point – the multiple critical
point (MCP) where the group $G$ undergoes spontaneous breakdown to the
diagonal subgroup:

$$ G \rightarrow G_{\text{diag.subgr.}} = \{g, g, g \mid g \in SMG\} \quad (6) $$

which is identified with the usual (lowenergy) group SMG.

This means that at the Planck scale the fine structure constants $\alpha_Y \equiv \frac{2}{3}\alpha_1$,
$\alpha_2$ and $\alpha_3$, as chosen by Nature, are just the ones corresponding to the multiple
critical point (MCP) which is a point where all action parameter (coupling) values
meet in the phase diagram of the regularized Yang-Mills $(SMG)^3$ - gauge theory.

The extrapolation of the experimental values of the inverses $\alpha_{1,2,3}^{-1}(\mu)$ to the
Planck scale $\mu_{Pl}$ by the renormalization group formulas (under the assumption of
a "desert" in doing the extrapolation with one Higgs doublet) is shown in Fig.1
and gives us the following result:

$$ \alpha_Y^{-1}(\mu_{Pl}) = 55.5; \quad \alpha_2^{-1}(\mu_{Pl}) = 49.5; \quad \alpha_3^{-1}(\mu_{Pl}) = 54. \quad (7) $$
Properties of Anti-grand unification can be studied by means of Monte Carlo simulations, which indicate the existence of the multiple critical point.

Using theoretical corrections to the Monte Carlo results on lattice, Anti-grand unified theory gives the following predictions:

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Group & AGUT predictions & "Experiment" – the extrapolation of the SM results to the Planck scale \\
\hline
\( U(1) \) & \( \alpha_Y^{-1}(\mu_{MCP}) = 55 \pm 6 \) & \( \alpha_Y^{-1}(\mu_{Pl}) \approx 55.5 \) \\
\hline
\( SU(2) \) & \( \alpha_2^{-1}(\mu_{MCP}) = 49.5 \pm 3 \) & \( \alpha_2^{-1}(\mu_{Pl}) \approx 49.5 \) \\
\hline
\( SU(3) \) & \( \alpha_3^{-1}(\mu_{MCP}) = 57 \pm 3 \) & \( \alpha_3^{-1}(\mu_{Pl}) \approx 54 \) \\
\hline
\end{tabular}
\end{table}

For \( U(1) \) - gauge lattice theory the authors of Ref.\[6\] have investigated the behaviour of the effective fine structure constant in the vicinity of the critical point and they have obtained:

\[ \alpha_{\text{crit}} \approx 0.2. \] (8)

We gave put forward the calculations of the fine structure constant in \( U(1) \) - gauge theory, suggesting that the modification of the action form might not change too much the critical value of the effective coupling constant.

Instead of the lattice hypercubic regularization we have considered rather new regularization using Wilson loop (nonlocal) action \[7\]:

\begin{equation}
S = \int d^4x \int_{0}^{\infty} d\log(\frac{R}{a}) \beta(R) R^{-4} \sum_{\text{average}} \text{Re} T \text{exp} \left[ i \oint_{C(R)} \hat{A}_\mu(x) dx^\mu \right] \tag{9}
\end{equation}

in approximation of circular Wilson loops \( C(R) \) of radius \( R \geq a \). In the last equation we have the average (\( \sum_{\text{average}} \)) over all positions and orientations of the Wilson loops \( C(R) \) in 4-dimensional (Euclidean) space.

With this action we have investigated the partition function, giving the free energy \( F \):

\[ Z = \int [DA_\mu] e^{-S} = e^{-F}. \] (10)

At the phase transition point the change of the free energy is equal to zero:

\[ \Delta F = 0. \] (11)
This condition allows us to obtain the Balance equation between energy and entropy:

$$\Delta F = \Delta <S> - \Delta E_{\text{entr}} = 0, \quad (12)$$

where $<S>$ is the vacuum mean value of the action which plays role of energy, and $E_{\text{entr}}$ is the entropy.

We have investigated the behaviour of $\Delta <S>$, which gave us the following function:

$$f_1(x) = \left(\frac{\sqrt{8}}{x}\right)^4, \quad (13)$$

where $x = R p_{\text{cutoff}}$ and $p_{\text{cutoff}}$ is the cutoff momentum value of the gauge field $A_\mu(p)$.

The second function was obtained from the change of the entropy ($\Delta E_{\text{entr}}$):

$$f_2(x) = 4\pi \frac{1 - J_1^2(x) - J_0^2(x)}{x^2} = 4\pi h(x), \quad (14)$$

and contains two Bessel functions $J_0(x)$ and $J_1(x)$.

The intersection of these two functions $f_1(x)$ and $f_2(x)$ at $x_{\text{crit}}$ corresponds to the Balance equation (12) and gives rise to: $x_{\text{crit}} \approx 2.57$.

The critical value of the fine structure constant for the regularized $U(1)$-gauge theory is related with the value of $h(x_{\text{crit}})$ in the following way:

$$\alpha_{\text{crit}}^{-1} - \alpha_{\text{max}}^{-1} = \frac{1}{2h(x_{\text{crit}})} \quad (15)$$
where $\alpha_{\text{max}} \approx 0.26$. According to Eq. (15) the critical value of the fine structure constant is:

$$\alpha_{\text{crit}} \approx 0.204.$$  \hspace{1cm} (16)

The result (16) confirms the Monte Carlo simulation result (8) on the lattice.

The further investigations of $SU(2) \otimes U(1)$ and $SU(3)$ gauge theories with the regularization using fermion string action confirm the "universality" of the critical coupling constants. Such an approximate regularization independence – "universality" – of the critical coupling constants is needed for the fine structure constant predictions claimed from Anti-grand unified theory \[2\]-\[5\].

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