A Temperature Propagation Calculation Method of Slender Energy Pile and Surrounding Soil Based on the Infinite Line Source Model

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Abstract. Based on the infinite line source model, the article proposes a temperature propagation calculating method of the energy pile and surrounding soil considering multiple heating-cooling cyclic temperature. The calculated soil temperatures under different distances at 1.3 D, 2.7 D and 4.7 D (D means diameter of the pile) from the pile center were compared with the centrifuge test results, and the relative errors were 8.57 %, 3.11 % and 1.04 % respectively. The results indicated that under the action of a single cycle of temperature, the difference between the highest and the lowest temperature is 10.62 °C at 1.3D from the center of the pile. The temperature fluctuations at 2.7D and 4.7D are relatively flat, and the temperature differences reach 3.75°C and 1.81°C. In addition, the heat transfer rate per linear meter is relatively stable under heating condition, but it decreases with the increase of pile depth under cooling condition. With the increase of time, the heat transfer rate also has the trend of downward. The calculated results of soil temperature under the action of multi-cycle temperature and the measured value of the centrifugal model test have little error, which can provide a reference for engineering applications.

1. Introduction

Based on the characteristics of thermostatic, low-temperature (compared to geothermal resources), and shallow, the combination of piping system, heat pump equipment room, and buried heat exchanger forms an effective use of shallow geothermal energy[1]. The pile foundation is combined with the vertical
buried pipe heat exchanger, and the heat exchange tube is buried in the pile foundation of the building. On the one hand, it avoids the high cost of drilling and backfilling materials required for the reburied pipe heat exchanger, and has higher economic benefits. On the other hand, since the heat transfer pipe is buried in the concrete, the construction quality is stable. Pile foundation which buried pipe to heat exchanger is also called the energy pile.

Regarding the model of heat transfer inside the pile foundation, a unified technical specification has not yet been formed. Some commercial software such as GeoStar[2], PILESIM2[3], Ground Loop Design(GLD)[4] are all designed based on vertical analytical model of ground source heat pumps. The slender energy pile is a branch of the vertical heat pumps. Both have similar geometric characteristics, and the length is much larger than the diameter. Therefore, the existing foreign standards on the heat transfer performance of energy piles mostly draw on the traditional heat transfer model of the vertical ground source heat pumps.

Analytical models based on traditional vertical ground source heat pump that meet the heat transfer characteristics of slender piles include: infinite line source model[5], infinite cylindrical source model[6-8] and finite line source model[9, 10]. On account of the research of Philippe[11], the three models are applicable over certain ranges. The slender energy pile can be used for times greater than 34h and up to 1.6years of operation, within the relative error is less than 2% between any of the two models. In view of the above, based on the infinite line source model, this paper proposes a method of temperature propagation considering multiple heating-cooling cyclic temperature conditions, and compares it with the results of the centrifuge model test. The variation law of heat transfer rate per meter was discussed at different pile depths and different cycle times in the temperature change process of soil around piles.

2. Pile-soil temperature propagation model in multiple cycle mode

2.1 Infinite line source model (ILS)

Ingersoll[12, 13] proposed that the Kelvin[5]Model is used to calculate the soil temperature field, and the heat conduction form is simplified to a one-dimensional heat transfer state. Assuming a constant heat transfer rate per unit length is $q_l$.

According to the Green function, the excess temperature field of the point heat source in space can be obtained as:

$$
\theta = \frac{q_l}{\rho c} \frac{1}{8\pi a (\tau - \tau')} \exp \left[ -\frac{\left( x^2 + y^2 + (z - z')^2 \right)}{4a(\tau - \tau')} \right] \int dz',
$$

expressed in cylindrical coordinates: $r = \sqrt{x^2 + y^2}$, with $u = \frac{-r^2}{4a(\tau - \tau')}$, after integration:

$$
\theta = \frac{q_l}{\rho c} \frac{1}{4\pi a(\tau - \tau')} \exp \left[ -\frac{r^2}{4a(\tau - \tau')} \right] \int_0^{\infty} \exp(u) du
$$

Equation (3) can be re-written as:
\[ \theta(r, \tau) = \frac{-q l}{4 \pi k} E_0 \left( -\frac{r^2}{4 a \tau} \right) \]  

(3)

2.2 Pile-soil temperature propagation under heating-cooling cyclic temperature

Assuming that heating and cooling have the same heat transfer rate per meter, in order to balance the heating and cooling load and reduce the adverse effect of long-term operation on the performance of the pile foundation heat exchanger.

It is assumed that the initial heat transfer rate per linear meter is \( q l \) under heat cycles, theoretically under cold cycles of heat transfer per meter is \( -q l \).

When the temperature rise is completed, the temperature at that moment is used as the initial value to calculate the temperature propagation during the cooling process. By analogy, a pile-soil temperature propagation model considering multiple heating-cooling cyclic temperature is formed.

3. Comparison with centrifugal model test

3.1 Layout of centrifugal model test

The acceleration of gravity used in a centrifuge experiment\(^{[14]}\) is 70g, and the internal dimensions of the model container are 1000mm \( \times \) 400mm \( \times \) 1000mm (length \( \times \) width \( \times \) height), which are equivalent to the length, width, and height of the full-scale model of 70m, 28m, and 70m. The layout of model instruments are shown in Figure 1.
The test carried out 30 heating-cooling cyclic temperature, each process lasted about 35.27 min (equivalent to the full-scale model for 4 months). This article selects some of the four cycles for discussion. Convert the results of the centrifugal model test into the full-scale model, and then analyze the test results.

As shown in figure 2, within a certain four cycles, the fluctuation trends of soil temperature at different pile depths at 2.7D and 4.7D from the pile center are consistent. The soil at pile depth of 38.15 m meets the maximum soil temperature difference, which is about 3.97°C and 2.14°C, and the slope is the largest, that is, the temperature change rate is the largest. Also, as the number of cycles increases, the temperature range of the soil continues to move to lower temperatures. The soil temperature range at 2.7D away from the center of the pile dropped from 11.02°C ~16.25°C to 10.47°C ~15.38°C. It may be due to the continuous decrease of the ambient temperature during the test, which caused the soil temperature difference to continuously shift to lower temperature.

**Figure 1. Layout of model instruments**

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Figure 2. Results of soil temperature response at different pile depths.

Figure 3 presents the results of the change trend of the pile wall temperature in the heating phase remains the same, and the maximum soil temperature relative errors in each cycle are less than 2%. During the cooling phase, the soil temperature response at different pile depths is quite different, and the relative errors of the lowest temperature in each cycle are 10.0%, 10.57%, 10.19%, and 10.81%, respectively. Therefore, the heat transfer rate per linear meter in the heating phase is maintained at a constant value, and the cooling phase decreases as the depth increases.

Figure 3. Temperature response of the soil near pile wall at different pile depths.
3.2 Comparison of calculation results of a single cycle

Assuming that the thermal conductivity of the soil around the pile is $1.5 W/(m\cdot K)$, the initial heat transfer rate per linear meter of the energy pile is $50 W/m$. The soil layer at the pile depth of $14.35 m$ is selected from a single cycle to multiple cycles, and the difference between the measured value and the calculated value under heating-cooling cyclic temperature was analyzed.

Figure 4 shows that at a distance of $1.3D$ from the center of the pile, the temperature changes rapidly and the curve fluctuates greatly. The difference between the maximum and the minimum temperature of the cycle is $10.62 \degree C$, and the curve at $2.7D$ and $4.7D$ positions are relatively flat, with a difference of $3.747 \degree C$ and $1.813 \degree C$. In the heating circulation phase, the measured heat transfer rate is originally slightly higher than the calculated value rate; in the end, it is significantly lower than the calculated value rate. Also due to the sudden change of the heat transfer rate per, the calculated value curve has pinnacle, which are obviously higher than the peak value and lower than the valley value.

However, the measured soil temperature also fluctuates in a small range within a short period of time, which is due to the certain fluctuations in the temperature of the water tank during the test. As the distance between the measuring point and the center of the pile increases, the inflection point of the measured soil temperature continues to lag behind, which is consistent with the hysteresis of the soil temperature propagation under the normal time scale.

![Comparison of calculated and measured values of soil temperature at different distances from the center of the pile within a single cycle (14.35m).](image)

**Figure 4.** Comparison of calculated and measured values of soil temperature at different distances from the center of the pile within a single cycle (14.35m).

3.3 Comparison of calculation results of multiple cycles

As depicted in figure 5, the slope of the double-cycle analytical solution of temperature change in the heating phase is higher than that in the cooling phase, indicating that the efficiency of the thermal phase is higher than that of the cold phase, which may be caused by the continuous decrease of the ambient temperature too.
Since the initial temperature of the soil in the laboratory is about 12°C, which affects the cooling conditions to a certain extent. However, without considering the change of heat transfer rate during the transition period of the heating-cooling cyclic temperature, resulting in the slope of the calculated value is always higher than that of the measured value at the end of each cooling and heating phase.

![Graph](image)

**Figure 5.** Comparison of calculated and measured values of soil temperature at different distances from the center of the pile within double-cycle (14.35m).

Select a certain continuous four heating-cooling cycle intervals, and compare the fluctuation curves of the calculated and the measured value at different distances from the center of the pile. Figure 6 illustrates that the heat transfer rate per linear meter changes over time, showing a downward trend. This rule is consistent with the results simulated by Wang Zhe et al.[15] with COMSOL.

In addition, the initial temperature of the soil around the pile continued to decrease, which is also one of the factors that led to the continuous decrease of cooling and heat transfer rate, also the difference in heat transfer power between heating-cooling stages.
4. Conclusion

Based on the infinite line source model, the heat transfer model of the slender energy pile under the action of multiple heating-cooling cyclic temperature is established. And compared with the results of the centrifuge model test, it is found that under certain conditions, the use of the infinite line source model can better simulate the measurement results. It is shown that at the end of the heating phase, the calculated value change rate is significantly higher than the measured value change rate. In addition, due to the sudden change of the heat transfer rate, the calculated value curves have pinnacles; the slope of the calculated value curve at the end of each stage is higher than the measured value.

When the temperature is raised, the heat transfer rate remains the same, and when the temperature is declined, the heat transfer rate decreases with the increase of the depth. At the same time, as time increases, the heat exchange rate also decreases.

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