Strangeness of the proton

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Abstract. The contribution of strange quarks to the radius and the magnetic moment of the proton is discussed in the framework of the unquenched quark model in which the effects of the $s\bar{s}$ pairs (strange sea) are taken into account through a $^3P_0$ quark-antiquark pair creation mechanism. The results for the the strange magnetic moment and the strangeness radius of the proton are found to be compatible with zero, in agreement with the latest experimental results from parity-violating electron scattering experiments and recent lattice calculations.

1. Introduction
Electromagnetic and weak form factors are key ingredients to the understanding of the internal structure of the nucleon, since they contain the information about the distributions of electric charge and magnetization. Deep-inelastic scattering experiments have demonstrated that the structure of the proton cannot be described by its $uud$ valence structure alone: the valence quarks carry about half of the proton momentum and about one third of the proton spin, and the observed violation of the Gottfried sum rules indicates that there are more $\bar{d}$ quarks in the proton than there are $\bar{u}$. The contribution of strange quarks to the nucleon structure is of special interest because it is exclusively part of the quark-antiquark sea.

Recently, parity-violating electron scattering (PVES) experiments were used to probe the contribution of strange quarks to the structure of the nucleon. The strange quark content of the form factors is determined by assuming charge symmetry and combining parity-violating asymmetries with measurements of the electric and magnetic form factors of the proton and neutron [1, 2]. The first experimental results from the SAMPLE collaboration indicated that the strangeness content of both the magnetic moment and the radius of the proton was rather large and positive [3, 4], an unexpected and surprising finding, since a majority of theoretical studies favored a negative value for both quantities [2]. The situation has changed with the
The latest set of PVES experiments by the HAPPEX [5, 6, 7], PVA4 [8] and G0 [9] collaborations which showed much smaller values for the strange form factors. The most recent measurements show that the contribution of the strange quarks to the electric and magnetic form factors, $G_S^E$ and $G_S^M$, is compatible with zero within the experimental errors.

Theoretically, it has been argued that the effects of strange quarks in the proton are not dominated by the lowest-lying virtual meson-baryon intermediate states, but rather are the result from cancellations between a large number of such intermediate states [10]. The aim of this contribution is to present a study of the contribution of strange quarks to the magnetic moment and the radius of the proton in an unquenched quark model in which the effects of the sea quarks are taken into account in an explicit way [11, 12].

2. Unquenched quark model

In the constituent quark model (CQM) hadrons are described as a system of constituent (or valence) quarks and antiquarks, $qqq$ for baryons and $q\bar{q}$ for mesons. Despite the success of the quark model there is strong evidence for the existence of exotic degrees of freedom (other than valence quarks) in hadrons from CQM studies of electromagnetic and strong couplings of baryons that are on average underpredicted by CQMs [13, 14, 15, 16]. More direct evidence for the importance of quark-antiquark components in the proton comes from measurements of the $d/u$ asymmetry in the nucleon sea [17, 18], the proton spin crisis [19, 20] and PVES experiments [7, 8, 9].

The role of higher Fock components in baryon wave functions has been studied by many authors in the context of meson cloud models, the cloudy bag model, meson convolution models and chiral models [17, 21]. There have also been several attempts to study the importance of higher Fock components in the context of the constituent quark model. In this respect we mention the work by Riska and coworkers who introduce a small number of selected higher Fock components which are then fitted to reproduce the experimental data [22]. However, these studies lack an explicit model or mechanism for the mixing between the valence and sea quarks.

The impact of $q\bar{q}$ pairs in hadron spectroscopy was originally studied by Törnqvist and Zencykowskim in a quark model extended by the $^3P_0$ model [23]. Even though their model only includes a sum over ground state baryons and ground state mesons, the basic idea of the importance to carry out a sum over a complete set of intermediate states was proposed in there. Subsequently, the effects of hadron loops in mesons was studied by Geiger and Isgur in a flux-tube breaking model in which the $q\bar{q}$ pairs are created in the $^3P_0$ state with the quantum numbers of the vacuum [24, 25, 26]. In this approach, the quark potential model arises from an adiabatic approximation to the gluonic degrees of freedom embodied in the flux-tube [27]. It was shown that cancellations between apparently uncorrelated sets of intermediate states occur in such a way that the modification in the linear potential can be reabsorbed, after renormalization, in the new strength of the linear potential [25]. In addition, the quark-antiquark pairs do not destroy the good CQM results for the mesons [25] and preserve the OZI hierarchy [26] provided that the sum be carried out over a large tower of intermediate states. A first application of this procedure to baryons was presented in [10] in which the importance of $ss$ loops in the proton were studied combining a $^3P_0$ pair-creation mechanism with harmonic oscillator wave functions for the baryons and mesons. This approach has the advantage that the effects of quark-antiquark pairs are introduced explicitly via a QCD-inspired pair-creation mechanism, which opens the possibility to study the importance of $q\bar{q}$ pairs in baryons and mesons in a systematic and unified way.

The present approach, called the unquenched constituent quark model (UCQM), is motivated by these earlier studies on extensions of the quark model to include the effects of $q\bar{q}$ pairs [10, 23]. Our approach is based on a CQM to which the quark-antiquark pairs with vacuum quantum numbers are added as a perturbation employing a $^3P_0$ model for the $q\bar{q}$ pair creation.
[10, 11, 12]. The pair-creation mechanism is inserted at the quark level and the one-loop diagrams are calculated by summing over a complete set of intermediate baryon-meson states. Under these assumptions, the baryon wave function consists of a zeroth order three-quark configuration \(|A\rangle\) plus a sum over all possible higher Fock components due to the creation of \(3P_0\) quark-antiquark pairs

\[
| \psi_A \rangle = N \left[ |A\rangle + \sum_{BCI} \int d\vec{k} \langle BC\vec{k}lJ| T^\dagger | A\rangle \right].
\]  

(1)

Here \(A\) denotes the initial baryon, \(B\) and \(C\) represent the intermediate baryon and meson, and \(M_A, E_B\) and \(E_C\) are their respective energies, \(k\) and \(l\) the relative radial momentum and orbital angular momentum of \(B\) and \(C\), and \(J\) is the total angular momentum \(J = J_B + J_C + l\). The operator \(T^\dagger\) creates a quark-antiquark pair in the \(3P_0\) state with the quantum numbers of the vacuum: \(L = S = 1\) and \(J = 0\) [11, 12, 28]

\[
T^\dagger = -3\gamma_0 \sum_{ij} \int d\vec{p}_i d\vec{p}_j \delta(\vec{p}_i + \vec{p}_j) C_{ij} F_{ij} e^{-r_i^2(\vec{p}_i - \vec{p}_j)^2/\mu} \frac{[\chi_{ij} \times \chi_{ij}]}{6} b_i^\dagger(\vec{p}_i) d_j^\dagger(\vec{p}_j).
\]  

(2)

Here, \(b_i^\dagger(\vec{p}_i)\) and \(d_j^\dagger(\vec{p}_j)\) are the creation operators for a quark and antiquark with momenta \(\vec{p}_i\) and \(\vec{p}_j\), respectively. The quark pair is characterized by a color singlet wave function \(C_{ij}\), a flavor singlet wave function \(F_{ij}\) and a spin triplet wave function \(\chi_{ij}\) with spin \(S = 1\). The solid harmonic \(\chi_{ij}(\vec{p}_i - \vec{p}_j)\) indicates that the quark and antiquark are in a relative \(P\) wave. The operator \(T^\dagger\) creates a pair of constituent quarks with an effective size, the pair creation point is smeared out by a Gaussian factor. The strong coupling vertex \(\langle BC\vec{k}lJ| T^\dagger | A\rangle\) can be derived in explicit form in the harmonic oscillator basis [28, 29]. In the present calculations, we use harmonic oscillator wave functions with a single oscillator parameter for the baryons and another one for the mesons [30].

In order to calculate the effects of quark-antiquark pairs on an observable, one has to evaluate the contribution of all possible intermediate states. By using a combination of group theoretical and computational techniques, the sum over intermediate states is carried out up to saturation and not only for the first few shells as in previous studies [23, 10]. Not only does this have a significant impact on the numerical result, but it is necessary for consistency with the OZI-rule and the success of CQMs in hadron spectroscopy. In addition, the contributions of quark-antiquark pairs can be evaluated for any initial baryon (ground state or resonance) and for any flavor of the \(q\bar{q}\) pair (not only \(s\bar{s}\) as in [10], but also \(u\bar{u}\) and \(d\bar{d}\), and for any model of baryons and mesons, as long as their wave functions are expressed in the basis of harmonic oscillator wave functions [11, 12].

In the calculations presented in this contribution, we use harmonic oscillator wave functions up to four oscillator shells for the intermediate baryons and mesons. All parameters were taken from the literature without attempting to optimize their values in order to improve the agreement with experimental data [11, 12, 30].

3. Strange magnetic moment and strangeness radius

Even though the nucleon carries no net strangeness, it may have a nonvanishing distribution of strangeness. Kaplan and Manohar [31] observed that neutral current experiments could provide information on the strange matrix elements of the nucleon. Around the same time, it was suggested by Beck and McKeown [1] to use parity-violating electron scattering experiments to determine the strangeness contribution to the nucleon by combining the weak form factors.
$G^{z,p}$ of the proton with the electromagnetic form factors of the nucleon, $G^{e,p}$ and $G^{m,n}$. The strangeness contribution may then be extracted by performing a flavor decomposition \[1, 2\]

\[
G_{E,M}^{z,p}(Q^2) = \frac{2}{3} G_{E,M}^{u} - \frac{1}{3} \left( G_{E,M}^{d} + G_{E,M}^{s} \right), \\
G_{E,M}^{m,n}(Q^2) = \frac{2}{3} G_{E,M}^{d} - \frac{1}{3} \left( G_{E,M}^{u} + G_{E,M}^{s} \right), \\
G_{E,M}^{z,p}(Q^2) = \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) G_{E,M}^{u} + (-1 + \frac{4}{3} \sin^2 \theta_W) \left( G_{E,M}^{d} + G_{E,M}^{s} \right).
\]

The first measurements showed large and positive values of $G_M^s$, e.g. the SAMPLE collaboration in 1999 found $G_M^s = 0.61 \pm 0.17 \pm 0.21 \pm 0.19$ at $Q^2 = 0.1$ (GeV/c)$^2$ \[3\]. However, the more recent values for $G_M^s$ and $G_E^s$ obtained by the HAPPEX \[6, 7\], PVA4 \[8\] and G0 \[9\] collaborations are much smaller. The most recent measurements show that the contribution of strange quarks to the electric and magnetic form factors is compatible with zero within the experimental errors.

Another method to determine the strangeness in the proton was suggested by Pate \[32\] which consists in combining experimental data on neutrino scattering with the electromagnetic form factors of the nucleon.

Theoretically, the strangeness of the proton has been addressed mostly in terms of two static observables, the strange magnetic moment $\mu_s$ and the strangeness radius $r_s^2$. The first theoretical calculation of the strange form factors of the nucleon performed by Jaffe in 1989 \[33\] reported quite large results for $r_s^2$ which triggered a lot of interest in the contribution of strange quarks in the proton. Subsequent theoretical calculations of $\mu_s$ and $r_s^2$, obtained in lattice QCD calculations, hadronic models and effective hadronic theory, vary widely both in absolute value and sign \[2, 34\]. Fig. 1 shows a compilation of theoretical values of these two quantities. Most studies agree on a small negative strangeness radius and a moderately negative strange magnetic moment \[2\]. Recent lattice calculations give small negative values for both $\mu_s$ and $r_s^2$ \[35, 36, 37\] (see Tables 1 and 2).

The strange magnetic moment and the strangeness radius can be determined from the behavior of the strange form factors of the proton near the origin $Q^2 = 0$ as \[2\]

\[
\mu_s = G_M^s(0), \\
r_s^2 = -6 \frac{dG_E^s}{dq^2}|_{Q^2=0}.
\]

The values of $\mu_s$ and $r_s^2$ can be extracted from the experimental data as follows. In PVES experiments one determines a linear combination of the strange electric and magnetic form factors, $G_E^s + \eta G_M^s$ which can be separated by changing the scattering angle and/or the target (hydrogen, deuterium or $^4$He).

Since the strangeness radius is proportional to the slope of $G_E^s$ in the origin, the evaluation of $r_s^2$ requires a measurement of the strange electric form factor at a small value of $Q^2$ in combination with $G_M^s(0) = 0$ (the nucleon carries no net strangeness). Two global analyses of the available experimental data at low values of $Q^2$ give $r_s^2 = 0.014 \pm 0.096$ fm$^2$ \[38\] and $r_s^2 = 0.019 \pm 0.038$ fm$^2$ \[39\]. In Table 1 we show a compilation for the strangeness radius of the proton.

Since the strange magnetic moment is defined as $\mu_s = G_M^s(0)$, it involves an extrapolation of $G_M^s(Q^2)$ to $Q^2 = 0$. For example, in Ref. \[40\] the measured value of $G_M^s$ at $Q^2 = 0.1$ (GeV/c)$^2$ is extrapolated by considering the momentum dependence of $G_M^s(Q^2)$ from \[40\]. The values obtained in a global analysis of the experimental data a low $Q^2$ are $\mu_s = 0.12 \pm 0.55 \pm 0.07 \mu_N$ \[38\] and $\mu_s = 0.29 \pm 0.21 \mu_N$ \[39\] (see Table 2).

Finally, we discuss the calculation of the strange magnetic moment and radius of the proton in the unquenched quark model. The results are obtained in a calculation involving a sum over
Figure 1. Theoretical values of the strange magnetic moment and the strangeness radius [2, 34]

Table 1. Strangeness radius of the proton

| Reference          | $r_s^2(p)$ [fm$^2$] |
|--------------------|----------------------|
| HAPPEX [6]        | 0.098 ± 0.108 ± 0.026 |
| HAPPEX [7]        | −0.006 ± 0.042 ± 0.021 |
| HAPPEX [7]        | 0.012 ± 0.044        |
| PVA4 [8]          | −0.053 ± 0.040 ± 0.020 |
| G0 [9]            | 0.015 ± 0.038 ± 0.019 ± 0.018 |
| Global Analysis [38] | 0.014 ± 0.096         |
| Global Analysis [39] | 0.019 ± 0.038         |
| LQCD [36]         | −0.007 ± 0.004 ± 0.002 ± 0.021 |
| LQCD [37]         | −0.0024 ± 0.0015 ± 0.0007 |
| Unquenched QM [30] | 0.012               |

Table 2. Strange magnetic moment of the proton

| Reference          | $\mu_s(p)$ [$\mu_N$] |
|--------------------|----------------------|
| SAMPLE [4]         | 0.01 ± 0.29 ± 0.31 ± 0.07 |
| Global Analysis [38] | 0.12 ± 0.55 ± 0.07   |
| Global Analysis [39] | 0.29 ± 0.21           |
| LQCD [35]          | −0.046 ± 0.019       |
| LQCD [37]          | −0.017 ± 0.025 ± 0.007 |
| Unquenched QM [30] | 0.0018               |
intermediate states up to four oscillator shells for both baryons and mesons. In the UCQM formalism, the strange magnetic moment of the proton is given by the expectation value of the operator

\[ \vec{\mu}_s = \frac{1}{e_s} \sum_i \mu_{i,s} \left[ 2\vec{s}(q_{i,s}) + \vec{\ell}(q_{i,s}) - 2\vec{s}(\bar{q}_{i,s}) - \vec{\ell}(\bar{q}_{i,s}) \right] . \] (5)

Here \( \mu_{i,s} \) is the magnetic moment of the strange quark \( i \) (not be confused with the strange magnetic moment of the proton \( \mu_s \)). In the UCQM the strange magnetic moment of the proton arises from the sea quarks. There is a contribution from the quark spins of the \( s\bar{s} \) pair 0.0012 \( \mu_N \), as well as from its orbital motion 0.0006 \( \mu_N \). Both contributions are small and give a total strange magnetic moment \( \mu_s = 0.0018 \mu_N \) [30].

Similarly, the strange radius of the proton is calculated as the expectation value of the operator

\[ r_s^2 = \sum_{i=1}^{5} \left( \vec{r}_{i,s} - \vec{R}_{CM} \right)^2 , \] (6)

where \( \vec{r}_{i,s} \) and \( \vec{R}_{CM} \) are the coordinates of the strange quark \( i \) and the center of mass, respectively. The strangeness radius of the proton is calculated to be \( r_s^2 = 0.012 \text{ fm}^2 \) [30].

In conclusion, the effects of the higher Fock components on the strange magnetic moment and the strange radius of the proton are found to be negligible. Tables 1 and 2 show that the results for the unquenched quark model are compatible with the latest experimental data and recent lattice calculations.

4. Summary and conclusions

In this contribution, we presented a study of the strangeness of the proton via two static observables, the strange magnetic moment and the strangeness radius, in the framework of an unquenched quark model for baryons in which the effects of sea quarks are taken into account in an explicit form via a \( ^3P_0 \) creation mechanism of the quark-antiquark pairs (\( u\bar{u}, d\bar{d} \) and \( s\bar{s} \)). The contribution of \( s\bar{s} \) pairs to the magnetic moment and the radius of the proton was found to be small, in agreement with the latest experimental results and recent lattice calculations.

The results of the unquenched quark model obtained for the magnetic moments, the spin and flavor content of octet baryons [11, 12] and the strangeness of the proton [30] are very promising and encouraging. The inclusion of the effects of quark-antiquark pairs in a general and consistent way, as in the UCQM, may provide a major improvement to the constituent quark model which increases considerably its range of applicability.

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References

[1] McKeown R D 1989 Phys. Lett. B 219 140
Beck D H 1989 Phys. Rev. D 39 3248
[2] Beck D H and McKeown R D 2001 Ann. Rev. Nucl. Part. Sci. 51 189
Beck D H and Holstein B R 2001 Int. J. Mod. Phys. E 10 1
[3] Spayde D T et al 2000 Phys. Rev. Lett. 84 1106
Spayde D T et al 2004 Phys. Lett. B 583 79
[4] Hasty R et al 2000 Science 290 2117
[5] Aniol K A et al 1999 Phys. Rev. Lett. 82 1096
Aniol K A et al 2001 Phys. Lett. B 509 211
Aniol K A et al 2004 Phys. Rev. C 69 065501
[6] K. A. Aniol et al [HAPPEX Coll.], Phys. Rev. Lett. 96 (2006) 022003.
[7] Acha A et al 2007 Phys. Rev. Lett. 98 032301
[8] Baunack S et al 2009 Phys. Rev. Lett. 102 151803
[9] Androic D et al 2010 Phys. Rev. Lett. 104 012001

See http://www.npl.uiuc.edu/exp/G0/Backward

[10] Geiger P and Isgur N 1997 Phys. Rev. D 55 299
[11] Bijker R and Santopinto E 2009 Phys. Rev. C 80 065210
[12] Santopinto E and Bijker R 2010 Phys. Rev. C 82 062202(R)
[13] Capstick S and Roberts W 1994 Phys. Rev. D 49 4570
[14] Bijker R, Iachello F and Leviatan A 1994 Ann. Phys. (N.Y.) 236 69

Bijker R, Iachello F and Leviatan A 1996 Phys. Rev. C 54 1935
[15] Bijker R, Iachello F and Leviatan A 1997 Phys. Rev. D 55 2862

Bijker R, Iachello F and Leviatan A 2000 Ann. Phys. (N.Y.) 284 89

[16] Aznauryan I G et al 2008 Phys. Rev. C 78 045209

Aznauryan I G et al 2009 Phys. Rev. C 80 055203

Aznauryan I G, Burkert V D, Lee T S H and Mokeev V 2011 J. Phys.: Conf. Ser. 299 012008

[17] Kumano S 1998 Phys. Rep. 303 183
[18] Garvey G T and Peng J C 2001 Prog. Part. Nucl. Phys. 47 203
[19] Ashman J et al 1988 Phys. Lett. B 206 366
[20] Bass S D 2008 The spin structure of the proton (Singapore: World Scientific)

Thomas A W 2009 Int. J. Mod. Phys. E 18 1116

Kuhn S E, Chen J P and Leader E 2009 Progr. Part. Nucl. Phys. 63 1

[21] Speth J and Thomas A W 1998 Adv. Nucl. Phys. 24 83
[22] Zou B S and Riska D O 2005 Phys. Rev. Lett. 95 072001

An C S, Riska D O and Zou B S 2006 Phys. Rev. C 73 035207

Riska D O and Zou B S 2006 Phys. Lett. B 636 265

Li Q B and Riska D O 2007 Nucl. Phys. A 791 406

[23] Törnqvist N A and Zenczykowski P 1984 Phys. Rev. D 29 2139

Törnqvist N A 1985 Acta Phys. Polon. B 16 503 and 683

Zenczykowski P 1986 Ann. Phys. (N.Y.) 169 453

[24] Kokoski R and Isgur N 1987 Phys. Rev. D 35 907
[25] Geiger P and Isgur N 1990 Phys. Rev. D 41 1595
[26] Geiger P and Isgur N 1991 Phys. Rev. Lett. 67 1066

Geiger P and Isgur N 1991 Phys. Rev. D 44 799

Geiger P and Isgur N 1993 Phys. Rev. D 47, 5050

[27] Isgur N and Paton J 1983 Phys. Lett. B 124 247

Isgur N and Paton J 1985 Phys. Rev. D 31 2910

[28] Roberts W and Silvestre-Brac B 1992 Few-Body Systems 11 171
[29] Ferreretti J 2011 Ph. D. Thesis University of Genova, unpublished
[30] Bijker R, Ferreretti J and Santopinto E 2012 Phys. Rev. C 85 035204
[31] Kaplan D B and Manohar A 1988 Nucl. Phys. B 310 527
[32] Pate S F 2004 Phys. Rev. Lett. 92 082002

Pate S F, McKee D W and Papavassilion V 2008 Phys. Rev. C 78 015207

[33] Jaffe R L 1989 Phys. Lett. B 229 275
[34] Bijker R 2006 J. Phys. G: Nucl. Part. Phys. 32 L49
[35] Leinweber D B et al 2005 Phys. Rev. Lett. 94 212001
[36] Leinweber D B et al 2006 Phys. Rev. Lett. 97 022001
[37] Doi T et al 2009 Phys. Rev. D 80 094503
[38] Young R D et al 2006 Phys. Rev. Lett. 97 102002
[39] Liu J et al 2007 Phys. Rev. C 76 025202
[40] Hemmert T R, Meissner U G and Steininger S 1998 Phys. Lett. B 437 184